Intuitionistic Fuzzy Hub Location Problems: Model and Solution Approach

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ABSTRACT

One of the most important problems in network design applications is the hub location problem, which is an extension of the facility location problem. The purpose of the problem is to select the least hub nodes from the available nodes so by establishing faster connections between hub nodes, the cost of transferring the entire network traffic is minimised. To deal with uncertainty and hesitation, the traffic amount between origin and destination nodes, the transfer cost, and the cost of establishing hub nodes are considered to be trapezoidal intuitionistic fuzzy numbers. The problem is formulated, and a new approach and a linearisation technique are shown to transform the Intuitionistic Fuzzy Hub Location Problem into a classical one. The transformed problem is solved using integer linear programming algorithms. The feasibility and efficiency of the obtained solutions applied to some airline passenger distribution problem applications are illustrated.

1. Introduction

In a hub location problem, a set of hub nodes is located to allocate other non-hub nodes to the located hub nodes to optimally collect and distribute traffic by hub nodes and its fast transfer between the special origin and destination nodes. The problem has many applications in the airline, maritime and telecommunication industry, logistical systems, freight transportation, postal companies, etc. See [1,2] for a complete review of all variants of hub location problems and their applications.

Real-life network design cases deal with many uncertainties and imprecision in the information and data follows from lack of information, high cost of information extraction, unpredictable and predictable changes, such as traffic changes and changes in social and economic conditions or climate change, judgment uncertainty, etc. [3,4]. To handle this type of uncertainty an ambiguous fuzzy set theory is introduced. Many researchers applied the fuzzy set theory in many real application problems to consider vague linguistic parameters and variables [5–13]. Dynamic virtual hub location, in which unexpected situations, including demand flow, factors of original hubs, and capacities of virtual hubs were modelled by fuzzy numbers, is considered in [14,15]. They proposed a fuzzy integer linear...
programming approach to minimise transportation costs. The effectiveness of the model and their solution approach are examined using the CAB dataset. A Hub-covering location problem was studied in which the location of demands was considered unknown and formulated by fuzzy variables [16]. To solve the problem a Variable Neighbourhood Search (VNS) method was proposed. Hub location problem in a fuzzy-stochastic environment, in which risk factors, such as delay time, air pollution, availability, and security, are considered using triangular fuzzy-stochastic numbers, was studied [17]. They transformed the problem into a crisp equivalent model and solved it by the Benders decomposition method. To show the efficiency of the proposed Benders decomposition method, the results are compared with those of commercial optimisation software. A multi-objective fuzzy goal programming approach for the P-hub location problem, concerning the survivability of the network and budget restrictions, was proposed [18]. Congestion in the hub location was considered, and was proposed an M/M/c/K queue system for a multi-objective multi-modal hub network design under uncertainty [19]. To show the efficiency of the proposed Benders decomposition method, the results are compared with those of GAMS commercial software for small-sized instances and simulated annealing (SA) algorithm for large instances. The authors have studied a real case of passenger transportation problem in Iran. An interactive multi-objective fuzzy linear programming model was proposed for a multi-capacity p-hub median problem under uncertainty [12]. Demand, establishment cost, and transportation costs were considered to be fuzzy. Time uncertainty in a hierarchical multi-modal hub location problem was studied in [20]. The authors proposed a credibility-based fuzzy programming approach to consider uncertainty in travel time and handling time and a heuristic procedure to solve the transformed crisp problem.

Furthermore, to consider uncertainty and hesitation in real application problems, the concept of the intuitionistic fuzzy set (IFS) theory was introduced [21] to measure the degree of belongingness and the degree of non-belongingness. Researchers highly regarded this theory due to its wide application in solving real problems referred to as an Intuitionistic Fuzzy problem [22,23]. In this way, Intuitionistic Fuzzy Hub Location Problem considered in this paper is a new research problem in which the traffic amount between origin and destination nodes, the transfer cost, and the cost of establishing hub nodes are considered to be trapezoidal intuitionistic fuzzy numbers. In this paper a new approach based on accuracy function is applied to transform the IFHLP into a crisp HLP problem to be able to use classical integer linear programming algorithms to solve the final problem.

**Definition 1.1:** [24]: A fuzzy set $\tilde{A}$ in the universe set $X$ is defined by \( \{(x, \mu_{\tilde{A}}(x)) | x \in X, 0 \leq \mu_{\tilde{A}}(x) \leq 1\} \) in which $\mu_{\tilde{A}}$ is the membership function of $\tilde{A}$. Also, an intuitionistic fuzzy set (IFS) $\tilde{A}^I$ in the universe set $X$ is defined by \( \{(x, \mu_{\tilde{A}^I}(x), \vartheta_{\tilde{A}^I}(x)) | x \in X, 0 \leq \mu_{\tilde{A}^I}(x) \leq 1, 0 \leq \vartheta_{\tilde{A}^I}(x) \leq 1\} \) in which $\mu_{\tilde{A}^I}(x)$ and $\vartheta_{\tilde{A}^I}(x)$ represent the membership degree and non-membership degree of $x$ in $\tilde{A}^I$, respectively and $\mu_{\tilde{A}^I}(x) + \vartheta_{\tilde{A}^I}(x) \leq 1$.

**Definition 1.2:** [24]: An intuitionistic fuzzy number in $R$ is an intuitionistic fuzzy set satisfied the following four conditions:

(i) The set $\text{supp}(\tilde{A}^I) = \{ x \in R: \vartheta_{\tilde{A}^I}(x) < 1 \}$ is bounded.

(ii) $\tilde{A}^I$ is the intuitionistic fuzzy normal.
Figure 1. An intuitionistic trapezoidal fuzzy number $\tilde{A}^l = (a_1, a_2, a_3, a_4; a'_1, a'_2, a'_3, a'_4)$ [26].

(iii) $\tilde{A}^l$ is the intuitionistic fuzzy convex.

(iv) $\vartheta_{\tilde{A}^l}$ is the lower semi-continuous and $\mu_{\tilde{A}^l}$ is the upper semi-continuous.

$\tilde{A}^l$ is the intuitionistic fuzzy normal if for at least one $x \in X$, $\mu_{\tilde{A}^l}(x) = 1$. Also, $\tilde{A}^l$ is the intuitionistic fuzzy convex if the condition of being convex for the membership function and the condition of being concave for the non-membership function are met.

Definition 1.3: [25]: A trapezoidal intuitionistic fuzzy number $\tilde{A}^l = (a_1, a_2, a_3, a_4; a'_1, a'_2, a'_3, a'_4)$ is an intuitionistic fuzzy number in which

$$\mu_{\tilde{A}^l}(x) = \begin{cases} \frac{x - a_1}{a_2 - a_1} & a_1 < x \leq a_2 \\ \frac{1}{a_4 - a_3} & a_2 < x \leq a_3 \\ \frac{a_4 - x}{a_4 - a_3} & a_3 < x \leq a_4 \\ 0 & O.W. \end{cases}$$

and $a'_1 \leq a_1 \leq a'_2 \leq a_2 \leq a'_3 \leq a_3 \leq a'_4 = a_4$.

Figure 1 shows an intuitionistic trapezoidal fuzzy number. If $a'_2 = a_2 = a_3 = a'_3$, then $\tilde{A}^l$ represents a triangular fuzzy number. Furthermore, if $a'_1 = a_1 = a'_2 = a_2 = a'_3 = a_3 = a'_4 = a_4$ a real number is obtained.

Definition 1.4: [25]: The arithmetic operations between trapezoidal intuitionistic fuzzy numbers $\tilde{A}^l = (a_1, a_2, a_3, a_4; a'_1, a'_2, a'_3, a'_4)$ and $\tilde{B}^l = (b_1, b_2, b_3, b_4; b'_1, b'_2, b'_3, b'_4)$ are defined
as follows:

\[ \tilde{A}^l \oplus \tilde{B}^l = (a_1 + b_1, a_2 + b_2, a_3 + b_3, a_4 + b_4; a'_1 + b'_1, a'_2 + b'_2, a'_3 + b'_3, a'_4 + b'_4) \]

\[ \tilde{A}^l - \tilde{B}^l = (a_1 - b_4, a_2 - b_3, a_3 - b_2, a_4 - b_1; a'_1 - b'_4, a'_2 - b'_3, a'_3 - b'_2, a'_4 - b'_1) \]

\[ k\tilde{A}^l = (ka_1, ka_2, ka_3, ka_4; ka'_1, ka'_2, ka'_3, ka'_4), k > 0 \]

\[ k\tilde{A}^l = (ka_4, ka_3, ka_2, ka_1; ka'_4, ka'_3, ka'_2, ka'_1), k < 0 \]

\[ \tilde{A}^l \otimes \tilde{B}^l = (c_1, c_2, c_3, c_4; c'_1, c'_2, c'_3, c'_4), \text{ in which} \]

\[ c_1 = \min\{a_1b_1, a_1b_4, a_4b_1, a_4b_4\} \quad c'_1 = \min\{a'_1b'_1, a'_1b'_4, a'_4b'_1, a'_4b'_4\} \]

\[ c_2 = \min\{a_2b_2, a_2b_3, a_3b_2, a_3b_3\} \quad c'_2 = \min\{a'_2b'_2, a'_2b'_3, a'_3b'_2, a'_3b'_3\} \]

\[ c_3 = \max\{a_2b_2, a_2b_3, a_3b_2, a_3b_3\} \quad c'_3 = \min\{a'_2b'_2, a'_2b'_3, a'_3b'_2, a'_3b'_3\} \]

\[ c_4 = \min\{a_1b_1, a_1b_4, a_4b_1, a_4b_4\} \quad c'_4 = \min\{a'_1b'_1, a'_1b'_4, a'_4b'_1, a'_4b'_4\} \]

Also, the approximate multiplication of \( \tilde{A}^l \) and \( \tilde{B}^l \) is reduced to

\[ \tilde{A}^l \otimes \tilde{B}^l = (a_1b_1, a_2b_2, a_3b_3, a_4b_4; a'_1b'_1, a'_2b'_2, a'_3b'_3, a'_4b'_4) \]

**Definition 1.5:** [25]: The degree of the hesitancy of \( x \) in \( \tilde{A}^l \) is defined by \( h_{\tilde{A}^l}(x) = 1 - \mu_{\tilde{A}^l}(x) - \nu_{\tilde{A}^l}(x) \). In the case of trapezoidal intuitionistic fuzzy numbers \( \tilde{A}^l = (a_1, a_2, a_3, a_4; a'_1, a'_2, a'_3, a'_4) \), its accuracy function is defined as follows:

\[ H_{\tilde{A}^l} = \frac{(a_1 + a_2 + a_3 + a_4) + (a'_1 + a'_2 + a'_3 + a'_4)}{8} \]

which is a linear function on the set of trapezoidal intuitionistic fuzzy numbers. The accuracy function is applied to compare trapezoidal intuitionistic fuzzy numbers.

### 2. Intuitionistic Fuzzy Hub Location Problems

In this section the integer linear programming formulation of hub location problem in a crisp and fuzzy environment is presented.

#### 2.1. Hub Location Problems in Crisp Environment

The main concept in the hub location problem is to select the least hub nodes and connect other nodes to the selected hub nodes. By establishing faster connections between hub nodes, the cost of transferring the entire network traffic between origin and destination nodes is minimised. Each node is connected to at least one hub node. The cost includes the collection and transfer cost from the origin node to the hub node connected to it, the transfer cost between hub nodes, and the transfer and distribution cost from the hub node to the destination node. Of course, if the origin or destination node is the hub itself, or origin and destination nodes are connected to a common hub node, part of this cost will be eliminated. Transfer between hub nodes is done by direct links. One way for economic savings in network design problems is to create high-quality links between hub centres and normal links between other nodes. For example, in transportation network design problems, highways of multiple traffic lines are constructed in congested zones, and normal lines are connected to manage traffic and reduce costs and time. The integer linear programming
Table 1. Description of parameters and variable of location hub problem.

| Notation | Description |
|----------|-------------|
| $N = \{1, 2, \ldots, n\}$ | The set of nodes |
| $w_{ij}$ | The amount of traffic should be transferred between $i$ and $j$. |
| $c_{ij}$ | Transfer cost of the link $(i, j)$. |
| $\alpha$ | Collection cost coefficient |
| $\gamma$ | Transfer cost coefficient between hubs |
| $\delta$ | Distribution cost coefficient |
| $z_{ij}$ | The decision variable $z_{ij} = 1$ if the node $i$ is connected to hub $j$, $z_{ij} = 0$. Obviously $z_{kk} = 1$ if and only if node $k$ is selected as a hub node. |

The formulation of the location hub problem is stated as follows:

$$
\min \sum_{i \in N} \sum_{k \in N} \sum_{l \in N} \sum_{j \in N} w_{ij}(\alpha c_{ik}z_{ik} + \gamma c_{kl}z_{ik}z_{jl} + \delta c_{jl}z_{jl}) + \sum_{k \in N} f_kz_{kk}
$$

$$
\sum_{k \in N} z_{ik} = 1, \quad \forall i \in N
$$

$$
z_{ik} \leq z_{kk}, \quad \forall i, k \in N
$$

$$
z_{ik} \in \{0, 1\}, \quad \forall i, k \in N
$$

The parameters and variables of the hub location problem are illustrated in Table 1. The objective function minimises the total cost, including the transfer and the hub establishing costs. Usually, the cost between hubs is less than the collection cost and the distribution cost. So, the value of $\gamma$ is much less than $\alpha$ and $\delta$. The first constraint states that each node is only connected to one hub node. The second constraint demonstrates that a node $i$ can be connected to the node $k$ if the node $k$ is a hub node.

2.2. Hub Location Problem in Intuitionistic Fuzzy Environment

In real applications of network design problems, one has to deal with uncertainty and hesitation. In these situations, intuitionistic fuzzy programming can be applied to consider impreciseness in the parameters of the problems. Location hub problem considering uncertainty and hesitation in the amount of traffic, transfer cost, and hub establishing cost can be formulated as follows:

$$
\min \tilde{Z}^I = \sum_{i \in N} \sum_{k \in N} \sum_{l \in N} \sum_{j \in N} \tilde{w}_{ij}^I \otimes (\alpha \tilde{c}_{ik}^I z_{ik} + \gamma \tilde{c}_{kl}^I z_{ik}z_{jl} + \delta \tilde{c}_{jl}^I z_{jl}) + \sum_{k \in N} \tilde{f}_k^I z_{kk}
$$

$$
\sum_{k \in N} z_{ik} = 1, \quad \forall i \in N
$$

$$
z_{ik} \leq z_{kk}, \quad \forall i, k \in N
$$

$$
z_{ik} \in \{0, 1\}, \quad \forall i, k \in N
$$

where $\tilde{w}_{ij}^I = (w_{ij1}, w_{ij2}, w_{ij3}, w_{ij4}, w'_{ij1}, w'_{ij2}, w'_{ij3}, w'_{ij4})$ is the intuitionistic fuzzy traffic amount should be transferred between $i$ and $j$. $\tilde{c}_{ik}^I = (c_{ik1}, c_{ik2}, c_{ik3}, c_{ik4}, c'_{ik1}, c'_{ik2}, c'_{ik3}, c'_{ik4})$ is the intuitionistic fuzzy collection and transfer cost from node $i$ to the connected hub node $k$. $\tilde{c}_{kl}^I = (c_{kl1}, c_{kl2}, c_{kl3}, c_{kl4}, c'_{kl1}, c'_{kl2}, c'_{kl3}, c'_{kl4})$ is the intuitionistic fuzzy transfer cost between the hub node $k$ and the hub node $l$. $\tilde{c}_{jl}^I = (c_{jl1}, c_{jl2}, c_{jl3}, c_{jl4}, c'_{jl1}, c'_{jl2}, c'_{jl3}, c'_{jl4})$ is the intuitionistic fuzzy transfer and distribution cost from the hub node $l$ to the node $j$. 
Based on Definition 1.4, the objective function of the intuitionistic fuzzy hub location problem $Z^I$ can be rewritten as follows:

$$Z^I = \left( \sum_{i \in N} \sum_{k \in N} \sum_{j \in N} w_{ij} (\alpha c_{ik} z_{ik} + \gamma c_{kj} z_{kj} + \delta c_{jl} z_{jl}) + \sum_{k \in N} f_{ik} z_{kk}, \right. \right. \left. \right. \right. \left. \sum_{i \in N} \sum_{k \in N} \sum_{j \in N} w'_{ij} (\alpha' c_{ik} z_{ik} + \gamma' c_{kj} z_{kj} + \delta' c_{jl} z_{jl}) + \sum_{k \in N} f'_{ik} z_{kk}, \right)$$

The resulting problem is a multi-objective integer programming problem with eight quadratic objective functions. The accuracy function of the intuitionistic fuzzy objective function can be used to convert the multi-objective programming problem into a single objective integer programming problem. In this way the following single objective integer optimisation problem is obtained:

$$\min H \left( \sum_{i \in N} \sum_{k \in N} \sum_{j \in N} \tilde{w}_{ij} \otimes (\alpha \tilde{c}_{ik} z_{ik} + \gamma \tilde{c}_{kj} z_{kj} + \delta \tilde{c}_{jl} z_{jl}) + \sum_{k \in N} \tilde{f}_{ik} z_{kk}, \right)$$

$$\sum_{k \in N} z_{ik} = 1, \forall i \in N$$

$$z_{ik} \leq z_{kk}, \forall i, k \in N$$

$$z_{ik} \in \{0, 1\}, \forall i, k \in N$$

So, the optimal solution of the converted problem $z^* = (z_{ik})_{i,k \in N}$ is defined as a feasible

### Table 2. The cost of establishing hub nodes.

| Id | Node       | The cost of establishing the hub          |
|----|------------|------------------------------------------|
| 1  | Atlanta    | (26290, 26302, 26309, 26319, 26284, 26299, 26314, 26321) |
| 2  | Baltimore  | (68258, 68263, 68271, 68280, 68248, 68260, 68273, 68287) |
| 3  | Boston     | (26018, 26025, 26033, 26040, 26011, 26022, 26038, 26044) |
| 4  | Chicago    | (150764, 150777, 150781, 150793, 150763, 150774, 150789, 150796) |
| 5  | Cincinnati | (27283, 27289, 27293, 27306, 27278, 27287, 27299, 27310) |
| 6  | Cleveland  | (30191, 30197, 30205, 30214, 30187, 30192, 30209, 30219) |
| 7  | Dallas-Ft Worth | (21999, 22006, 22011, 22023, 221993, 22002, 22016, 22024) |
| 8  | Denver     | (24287, 24299, 24301, 24314, 24284, 24295, 24311, 24321) |
| 9  | Detroit    | (47316, 47324, 47330, 47341, 47307, 47318, 47333, 47345) |
| 10 | Houston    | (42029, 42040, 42044, 42050, 42024, 42030, 42045, 42058) |
solution such that for any other feasible solution $z = (z_{ik})_{i,k \in N}$:

$$H \left( \sum_{i \in N} \sum_{k \in N} \sum_{j \in N} \sum_{j' \in N} \tilde{w}_{ij} \otimes \left( \alpha_i c_{ik} z_{ik}^+ + \gamma_i c_{kj} z_{jk}^+ \right) + \sum_{k \in N} \tilde{f}_{kk} z_{kk}^+ \right) \leq H \left( \sum_{i \in N} \sum_{k \in N} \sum_{j \in N} \sum_{j' \in N} \tilde{w}_{ij} \otimes \left( \alpha_i c_{ik} z_{ik}^+ + \gamma_i c_{kj} z_{jk}^+ \right) + \sum_{k \in N} \tilde{f}_{kk} z_{kk}^+ \right)$$
Table 4. Solution of the AP problem.

| Node             | Allocated hub node |
|------------------|--------------------|
| Atlanta          | Chicago            |
| Baltimore        | Cleveland          |
| Boston           | Cleveland          |
| Chicago          | Chicago            |
| Cincinnati       | Chicago            |
| Cleveland        | Cleveland          |
| Dallas-Ft Worth  | Dallas-Ft Worth    |
| Denver           | Dallas-Ft Worth    |
| Detroit          | Cleveland          |
| Houston          | Dallas-Ft Worth    |

After this transformation the final model can be converted to a linear integer programming model using the Linearisation Technique [27]:

\[
\lambda_{ijkl} = z_{ik}z_{jl} \quad \forall i, j, k, l \in \{1, \ldots, N\}
\]

Then it is sufficient to add these constraints to the problem model:

\[
\begin{align*}
2\lambda_{ijkl} & \leq z_{ik} + z_{jl} \leq 1 + \lambda_{ijkl} & \forall i, j, k, l \in \{1, \ldots, N\} \\
\lambda_{ijkl} & \in \{0,1\} & \forall i, j, k, l \in \{1, \ldots, N\}
\end{align*}
\]

3. Numerical Results

Numerical examples are proposed in this section to illustrate the main concepts and the efficiency and applicability of the proposed approach. For this purpose, the AP problem set, which is a real-world hub location problem whose data belong to a postal company in Australia, is considered. This dataset is described in a crisp state in [28] and can be downloaded from the OR-Library set. The crisp dataset is converted into an intuitionistic fuzzy dataset so that the topology of the data set is preserved [29]. Also, the parameters are considered as some random trapezoidal intuitionistic fuzzy numbers \(\tilde{\lambda}^l = (a_1, a_2, a_3, a_4; a_1', a_2', a_3', a_4')\) in which

\[
\begin{align*}
a_1' & \in [a - 20, a - 15] & a_1 & \in [a - 15, a - 10] \\
a_2' & \in [a - 10, a - 5] & a_2 & \in [a - 5, a] \\
a_3 & \in [a, a + 5] & a_3' & \in [a + 5, a + 10] \\
a_4 & \in [a + 10, a + 15] & a_4' & \in [a + 15, a + 20]
\end{align*}
\]

and \(a\) is equal to the relevant parameter in the crisp hub location problem. The values of parameters \(a_1, a_2, a_3, a_4, a_1', a_2', a_3', a_4'\) are chosen randomly in the corresponding interval. The values of the trapezoidal intuitionistic fuzzy parameters are reported in Tables 2 and 3. Also the values of parameters \(\alpha, \gamma,\) and \(\delta\) are 3, 0.7, and 2, respectively.

AMPL (A Mathematical Programming Language) solver solves the converted linear integer programming problem. The solution to the problem is given in Table 4.
4. Conclusion and Future Directions

This paper deals with intuitionistic fuzzy hub location problems in which the parameters, including transfer cost, hub establishing cost, and traffic amount, are considered to be intuitionistic fuzzy trapezoidal numbers. A new approach is applied to convert the problem into a classical hub location problem. Also, the linearisation technique is used to convert the resulting problem into an integer linear programming problem which can be solved by AMPL software. Some instances from airline passenger (AP) distribution data are solved to illustrate the corresponding approach. It is interesting to note that, since the intuitionistic fuzzy hub location problem is a generalisation of fuzzy hub location problem, the approach presented in this paper can be applied to solve the fuzzy hub location problem. Also, other ranking methods for comparing intuitionistic fuzzy numbers can be used similarly.

Disclosure statement

No potential conflict of interest was reported by the author(s).

Notes on contributor

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References

[1] J.-Sharahi S, Khalili-Damghani K, Abtahi AR, et al. Type-II fuzzy multi-product, multi-level, multi-period location–allocation, production–distribution problem in supply chains: modelling and optimisation approach. Fuzzy Inf Eng. 2018;10(2):260–283.
[2] Farahani RZ, Hekmatfar M, Arabani AB, et al. Hub location problems: a review of models, classification, solution techniques, and applications. Comput Ind Eng. 2013;64(4):1096–1109.
[3] Karaşan A, Kahraman C. A novel intuitionistic fuzzy DEMATEL–ANP–TOPSIS integrated methodology for freight village location selection. J Intell Fuzzy Syst. 2019;36(2):1335–1352.
[4] Aggarwal A, Mehra A, Chandra S, et al. Solving Atanassov’s l-fuzzy linear programming problems using Hurwicz’s criterion. Fuzzy Inf Eng. 2018;10(3):339–361.
[5] Nafei A, Wenjun YUAN, Nasseri H. A new method for solving interval neutrosophic linear programming problems. Gazi Univ J Sci. 2020;33(4):796–808.
[6] Nafei AH, Yuan W, Nasseri H. Group multi-attribute decision making based on interval neutrosophic sets. Infinite Study. 2019;28(3):309–316.
[7] Nasseri SH, Ebrahimnejad A, Cao BY. Fuzzy linear programming. In: Fuzzy linear programming: solution techniques and applications (Studies in Fuzziness and Soft Computing). Cham: Springer; 2019. p. 39–61.
[8] Nasseri SH, Taghi-Nezhad N, Ebrahimnejad A. A note on ranking fuzzy numbers with an area method using circumcenter of centroids. Fuzzy Inf Eng. 2017;9(2):259–268.
[9] Nasseri SH, Bavandi S. Multi-choice linear programming in fuzzy random hybrid uncertainty environment and their application in multi-commodity transportation problem. Fuzzy Inf Eng. 2020;12(1):109–122.
[10] Nasseri SH, Darvishi D. Planning livestock diet with fuzzy requirements. J Inf Optim Sci. 2018;39(7):1527–1545.

[11] Nasseri H, Darvishi Salokolaei D, Yazdani A. A new approach for solving grey assignment problems. Contr Optim Appl Math. 2017;2(1):15–28.

[12] Rahmanniyyay F, Razmi J, Yu AJ. An interactive multi-objective fuzzy linear programming model for hub location problems to minimise cost and delay time in a distribution network. Int J Logist Syst Manag. 2020;37(1):79–105.

[13] Zadeh LA. Fuzzy sets. In: Fuzzy sets, fuzzy logic, and fuzzy systems: selected papers by Lotfi A Zadeh. United States: WSPC. 1996. p. 394–432.

[14] Taghipourian F, Mahdavi I, Mahdavi-Amiri N, et al. A fuzzy programming approach for dynamic virtual hub location problem. Appl Math Model. 2012;36(7):3257–3270.

[15] Pourghader Chobar A, Adibi MA, & Kazemi A. A novel multi-objective model for hub location problem considering dynamic demand and environmental issues. J Ind Eng Manage Stud. 2021;8(1):1–31.

[16] Davari S, Zarandi MF, Turksen IB. The incomplete hub-covering location problem considering imprecise location of demands. Sci Iran. 2013;20(3):983–991.

[17] Abbasi-Parizi S, Aminnayeri M, Bashiri M. Robust solution for a minimax regret hub location problem in a fuzzy-stochastic environment. J Ind Manage Optim. 2018;14(3):1271–1295.

[18] Masoumzadeh S, Solimanpur M, Kamran M. A multi-objective fuzzy goal programming P-hub location and protection model with back-up hubs considering hubs establishment fixed costs. Sci Iran. 2016;23(4):1941–1951.

[19] Rahimi Y, Tavakkoli-Moghaddam R, Mohammadi M, et al. Multi-objective hub network design under uncertainty considering congestion: An M/M/c/K queue system. Appl Math Model. 2016;40(5-6):4179–4198.

[20] Shang X, Jia B, Yang K, et al. A credibility-based fuzzy programming model for the hierarchical multimodal hub location problem with time uncertainty in cargo delivery systems. Int J Mach Learn Cybern. 2021;12(5):1413–1426.

[21] Atanassov KT. On intuitionistic fuzzy sets theory. Berlin: Springer; 2012.

[22] Rivaz S, Nasseri SH, Ziaseraji M. A fuzzy goal programming approach to multiobjective transportation problems. Fuzzy Inf Eng. 2020;12(2):139–149.

[23] SalooKolayi DD. Application of fuzzy optimization in diet formulation. J Math Comput Sci. 2011;2(3):459–468.

[24] Kumar PS. Algorithms for solving the optimization problems using fuzzy and intuitionistic fuzzy set. Int J Syst Assur Eng Manage. 2020;11(1):189–222.

[25] Niroomand S, Garg H, Mahmoodi A. An intuitionistic fuzzy two stage supply chain network design problem with multi-mode demand and multi-mode transportation. ISA Trans. 2020;107:117–133.

[26] Uluçay V, Deli I, Sahin M. Intuitionistic trapezoidal fuzzy multi-numbers and its application to multi-criteria decision-making problems. Compl Intell Syst. 2019;5(1):65–78.

[27] Nasseri SH, Zavieh H. A multi-objective method for solving fuzzy linear programming based on semi-infinite model. Fuzzy Inf Eng. 2018;10(1):91–98.

[28] Alumur S, Kara BY. Network hub location problems: the state of the art. Eur J Oper Res. 2008;190(1):1–21.

[29] Mohammadi M, Torabi SA, Tavakkoli-Moghaddam R. Sustainable hub location under mixed uncertainty. Transp Res Part E Logist Transp Rev. 2014;62:89–115.