The HTL resumed propagators in the light cone gauge

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The expression of the HTL resumed gluon propagator in the light cone gauge is derived. In the real time mechanism, using the Mandelstam-Leibbrandt prescription of \((n \cdot K)^{-1}\), we calculate the transverse and longitudinal parts of the gluon HTL self-energy and prove the transverse and longitudinal parts do not have divergence. We also calculate the quark self energy in the HTL approximation, and find it gauge independent. We analytically calculate the damping rates of the hard quark and gluon with this HTL resumed gluon propagator.

I. INTRODUCTION

The bare QCD perturbative theory breaks down at high temperature. There are some serious problems of gauge theories at finite temperature, such as IR singularity and gauge dependent results, when the bare propagators (vertices) are used. The HTL (Hard Thermal Loop) resumed propagators have been developed by Braaten and Pisarski \cite{1}. Some gauge independent physical quantities are given with the HTL resumed propagators in the calculation, other than the bare propagators at finite temperature. If the momentum of the propagators are soft at finite temperature, we should use the HTL resumed propagators. High order loop HTL diagrams can give a low order contribution in the coupling constant at finite temperature, which should be resummed. Because of the HTL resummation, the medium effects are taken into account, such as the Debye screening caused by the color charges of the QGP. The HTL resummation technique represents a great progress compared to the bare perturbation theory at finite temperature.

The light cone gauge is one of non-covariant and physical gauges \cite{2},\cite{3}, and is also ghost-free. When the multiple gluon emission is calculated in the light cone gauge, because the interference terms among different tree diagrams do not contribute to the leading order in the process of calculating the diagram amplitude in the light cone gauge, the differential cross section with \(n\)-gluon emission in the leading pole approximation has a simple ladder structure at zero temperature \cite{4}. These nice properties simplify the calculation. However, the light cone gauge has its disadvantage, such as the spurious singularity of \((n \cdot K)^{-1}\), the renormalization and so on.

In the experiment of the Heavy Ion Collisions, the longitudinal momentum of the generated parton is very large, however, the transverse momentum is very small, and it is more suitable to calculate some physical quantities in the light cone gauge with light cone variables \(K^\mu = (k^+, k^-, k_\perp)\), than to do in the Coulomb gauge in the Minkowski space \(K^\mu = (k^0, \vec{k})\). Many good theoretical works have been done in light cone gauge in vacuum which are well consistent with the experiment data.

Recently we can only calculate the evolution equation of parton distribution functions (PDFs) \cite{5},\cite{6} and parton fragmentation functions (FFs)\cite{7},\cite{8},\cite{9} by pertubative QCD, which are both non-perturbative physical quantities. The evolution equations govern the running of PDFs and FFs with the scale \(Q\). Correspondingly, we can extract PDFs and FFS from the experimental data, which are taken some fixed value for the relevant hard scale \(Q\).

For some physical quantities such as the FFs \cite{10} and so on, we can extend the case of high energy in vacuum to the case at high temperature. For hard processes, we can use the bare propagators in the light cone gauge at finite temperature. With the HTL resumed gluon propagator in the light cone gauge, we can consider multiple soft gluon scattering among hard partons and hot medium in the Heavy Ion Collisions, which contains soft process on the basis of the hard process. The soft process can give a significant correction compared with the hard process. This is the motivation of this paper. We work out the transverse and longitudinal parts of the gluon HTL self energy in the light cone gauge, derive the HTL resumed gluon propagator and demonstrate it is gauge independent for further research in the future.

The remainder of the paper is organized as follows. In Sec.II we review the HTL resumed gluon propagator in the Coulomb gauge. In Sec.III the HTL resumed gluon propagator in the light cone gauge is worked out, and then we calculate and analyze the transverse and longitudinal parts of the gluon HTL self energy in the light cone gauge. Via the HTL resumed gluon propagator we show the transverse and longitudinal spectral functions and the equations of

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the dispersion relation. In Sec.IV we calculate the quark self energy in the HTL approximation, and show the HTL
resumed quark propagator gauge independent. In Sec.V we analytically calculate the damping rates of the hard quark
and gluon with this HTL resumed gluon propagator in the light cone gauge in a particular limit, and demonstrate in
general case we can have the same result in the light cone gauge and the Coulomb gauge. Our conclusion is given in
Sec.VI. We define the notation $P^\mu = (p^0, \vec{p})$, etc.

II. HTL RESUMED GLUON PROPAGATOR IN THE COULOMB GAUGE

At zero temperature, covariant gauge has a definite advantage over non-covariant gauges such as the Coulomb
gauge or axial gauges. Calculations are simplified considerably due to Lorentz invariance, and the renormalization
program can be implemented in practice only in covariant gauge. At finite temperature, Lorentz invariance is broken
because the heat bath defines a privileged frame, and renormalization program is of secondary importance, so that
non-covariant gauges may present useful alternatives to covariant gauge [11].

The HTL resumed gluon propagator has been derived in the Coulomb gauge [12,13]. The HTL gluon self energy
$\Pi^{\mu\nu}(P)$ is expressed as the transverse part and the longitudinal part. The gluon self energy is given by

$$\Pi^{\mu\nu}(P) = -\Pi_T(P)T_P^{\mu\nu} - \frac{1}{n_P^2} \Pi_L(P)L_P^{\mu\nu},$$

(1)

where the transverse projection tensor $T_P^{\mu\nu}$, the longitudinal projection tensor $L_P^{\mu\nu}$, and the four vector $n_P^\mu$ are defined
as

$$T_P^{\mu\nu} = g^{\mu\nu} - \frac{P^\mu P^\nu}{P^2} - \frac{n_P^\mu n_P^\nu}{n_P^2},$$

$$L_P^{\mu\nu} = \frac{n_P^\mu n_P^\nu}{n_P^2},$$

$$n_P^\mu = n^\mu - \frac{n \cdot P}{P^2} P^\mu.$$  

(2)

The axial vector is

$$n_\xi^\mu = (n^0, n^1, n^2, n^3) = (1, 0, 0, 0),$$

(3)

which specifies the thermal rest frame.

The inverse propagator for general $\xi$ in the Coulomb gauge is

$$\Delta_\xi^{-1}(P)^{\mu\nu} = \Delta^{-1}(P)^{\mu\nu} - \frac{1}{\xi} (P^\mu - P \cdot n n^\mu)(P^\nu - P \cdot n n^\nu),$$

(4)

where $\xi$ is an arbitrary gauge parameter.

The inverse propagator reduces in the limit $\xi \to \infty$ to

$$\Delta_\infty^{-1}(P)^{\mu\nu} = -P^2 g^{\mu\nu} + P^\mu P^\nu - \Pi^{\mu\nu}(P).$$

(5)

$\Delta_\infty^{-1}(P)^{\mu\nu}$ can also be written as

$$\Delta_\infty^{-1}(P)^{\mu\nu} = \frac{1}{\Delta_T(P)} T_P^{\mu\nu} + \frac{1}{n_P^2 \Delta_L(P)} L_P^{\mu\nu},$$

(6)

where $\Delta_T(P)$ and $\Delta_L(P)$ are the transverse and longitudinal propagators:

$$\Delta_T(P) = \frac{1}{P^2 - \Pi_T(P)},$$

$$\Delta_L(P) = \frac{1}{-n_P^2 P^2 + \Pi_L(P)}.$$  

(7)

The HTL resumed gluon propagator in the Coulomb gauge [13] is

$$\Delta_\xi^{\mu\nu}(P) = -\Delta_T(P) T_P^{\mu\nu} + \Delta_L(P) n^\mu n^\nu - \frac{P^\mu P^\nu}{n_P^2 P^2}.$$

(8)
By calculating, we can find

\[ T_p^{00} = 0, T_p^{0i} = T_p^{0i} = 0 . \]  

(9)

So the HTL resumed gluon propagator in the Coulomb gauge can be simplified into

\[ G_{00}^{\text{Ret}}(P) = \frac{1}{p^2 + \Pi_L(P)}, \]
\[ G_{ij}^{\text{Ret}}(P) = \frac{\delta_{ij} - \hat{p}_i \hat{p}_j}{P^2 - \Pi_T(P)}, \]

(10)

where \( \hat{p} \) is a unit vector on the direction of \( \hat{p}^i \), \( \hat{p}^i = \frac{p^i}{p} \) and \( \hat{p}^i = (\hat{p}^1, \hat{p}^2, \hat{p}^3) \).

The longitudinal and transverse gluon HTL self energy \[14\] are

\[ \Pi_L(P) = m_D^2 \left[ 1 - \frac{p_0}{2p} \ln \frac{p_0 + p}{p_0 - p} + i\pi \frac{p_0}{2p} \theta(p^2 - p_0^2) \right], \]
\[ \Pi_T(P) = \frac{m_D^2 p_0^2}{2 p^2} \left[ 1 - (1 - \frac{p_0^2}{p_0^2}) \frac{p_0}{2p} \ln \frac{p_0 + p}{p_0 - p} - i\pi \theta(p^2 - p_0^2) \right] , \]

(11)

where the gluon screening mass \( m_D^2 = \frac{1}{3}(C_A + \frac{1}{2}N_f)g^2 T^2 \) and \( \theta(p^2 - p_0^2) \) is the step function.

The imaginary parts in the above equations correspond to the Landau damping, which means that one particle is emitted from the thermal medium and absorbed by the medium.

In the static limit, \( p_0 \to 0 \), the longitudinal HTL self energy

\[ \Pi_L^R(p_0 \to 0, p) = m_D^2 , \]

(12)

which means the Debye screening of the gluon in the plasma.

However, in the static limit, the transverse HTL self energy

\[ \Pi_T^R(p_0 \to 0, p) = 0 , \]

(13)

which shows no static magnetic screening.

The self-energy tensor \( \Pi^{\mu\nu} \) is symmetric in \( \mu \) and \( \nu \) and satisfies

\[ P_\mu \Pi^{\mu\nu}(P) = 0 , \]
\[ g_{\mu\nu} \Pi^{\mu\nu}(P) = -2\Pi_T(P) - \frac{1}{n_T^2} \Pi_L(P) = -m_D^2 . \]

(14)

III. THE HTL RESUMED GLUON PROPAGATOR IN THE LIGHT CONE GAUGE

In this section, we derive the HTL resumed gluon propagator in the light cone gauge, and compute the transverse and longitudinal parts of gluon HTL self energy in the real time formalism. In the static limit, we discuss \( \Pi^{00}(P) \) in the light cone gauge and the Coulomb gauge. We obtain the pole terms and the cut terms of the transverse and longitudinal spectral functions.

The light cone gauge is one of axial type gauges and non-covariant gauges \[2, 3\],

\[ n_l^2 = 0, n_l \cdot A = 0 . \]

(15)

The axial vector in the light cone gauge is

\[ n_l^\mu = (n_0^1, n_1^2, n_2^3) = \left( \frac{\sqrt{2}}{2}, 0, 0, -\frac{\sqrt{2}}{2} \right) . \]

(16)

The bare gluon propagator in the light cone gauge is

\[ i(-g^{\mu\nu} + n_\mu K^\nu + n_\nu K^\mu) \]
\[ K^2 + i\epsilon . \]

(17)
Here we use the Mandelstam-Leibbrandt (ML) prescription of \((n \cdot K)^{-1}\) instead of the usual principal-value prescription,
\[
\frac{1}{n \cdot K} = \frac{n^* \cdot K}{n \cdot K n^* \cdot K + i\epsilon} = \frac{1}{n \cdot K + isgn(n^* \cdot K)\epsilon} = \frac{n_0 k_0 + \overrightarrow{n} \cdot \overrightarrow{k}}{(n_0 k_0)^2 - \overrightarrow{n}^2 \overrightarrow{k}^2 + i\epsilon}.
\]
where \(n_0^* = (n^0, n^1, n^2, n^3) = (\sqrt{n^2}, 0, 0, \sqrt{n^2})\).

The usual principal-value prescription of \((n \cdot K)^{-1}\) leads to some serious problems, such as violating power counting and other basic criteria, when we calculate the integral of the loop diagram [2].

In the real time mechanism, the time of the field goes from \(t = 0\) to \(t = -i\beta\). The contour can be deformed in order to include the real time axis by going first from \(t = 0\) to \(t = \infty\) above the real time axis and then back to \(t = -i\beta\) below real time axis. So we have double degrees of freedom, one exists above the real time axis and the other one exists below the real time axis. We get the propagator in the real time formalism [11], [15], which is a 2 × 2 matrix,
\[
\Delta(K) = \begin{pmatrix}
\Delta_{11} & \Delta_{12} \\
\Delta_{21} & \Delta_{22}
\end{pmatrix} = \left( \begin{array}{cc}
\frac{1}{K^2 - n^2 + i\epsilon} & 0 \\
0 & \frac{1}{K^2 - m^2 - i\epsilon}
\end{array} \right) - 2\pi i\delta(K^2 - m^2) \left( \begin{array}{cc}
n_B(k_0) & \theta(-k_0) + n_B(k_0) \\
\theta(-k_0) - n_B(k_0) & -n_B(k_0)
\end{array} \right).
\]

The Bose-Einstein distribution function \(n_B(k_0)\) is
\[
n_B(k_0) = \frac{1}{e^{\frac{|k_0|}{\beta}} - 1}.
\]

For fermions, we have
\[
F(K) = (\mathcal{K} + m)\Delta(K) = (\mathcal{K} + m) \left( \begin{array}{cc}
\Delta_{11} & \Delta_{12} \\
\Delta_{21} & \Delta_{22}
\end{array} \right) = (\mathcal{K} + m) \left[ \begin{array}{cc}
\frac{1}{K^2 - n^2 + i\epsilon} & 0 \\
0 & \frac{1}{K^2 - m^2 - i\epsilon}
\end{array} \right] - 2\pi i\delta(K^2 - m^2) \left( \begin{array}{cc}
f(k_0) & \theta(-k_0) - f(k_0) \\
\theta(-k_0) - f(k_0) & f(k_0)
\end{array} \right),
\]

The Fermi-Dirac distribution function \(f(k_0)\) is
\[
f(k_0) = \frac{1}{e^{\frac{k_0}{\beta}} + 1}.
\]

We use the Keldysh representation in real time formalism [15], [16]. The retarded propagator, advanced propagator and symmetric propagator for bosons are
\[
\Delta_R(K) = \Delta_{11} - \Delta_{12} = \frac{1}{K^2 - m^2 + isgn(k_0)\epsilon},
\]
\[
\Delta_A(K) = \Delta_{11} - \Delta_{21} = \frac{1}{K^2 - m^2 - isgn(k_0)\epsilon},
\]
\[
\Delta_S(K) = \Delta_{11} + \Delta_{22} = -2\pi i\delta(K^2 - m^2)[1 + 2n_B(k_0)].
\]

The inverse relation for bosons is
\[
\Delta_{11} = \frac{1}{2} [\Delta_S(K) + \Delta_A(K) + \Delta_R(K)],
\]
\[
\Delta_{12} = \frac{1}{2} [\Delta_S(K) + \Delta_A(K) - \Delta_R(K)],
\]
\[
\Delta_{21} = \frac{1}{2} [\Delta_S(K) - \Delta_A(K) + \Delta_R(K)],
\]
\[
\Delta_{22} = \frac{1}{2} [\Delta_S(K) - \Delta_A(K) - \Delta_R(K)].
\]

For fermions, in the Keldysh representation, we only replace the Bose-Einstein distribution function \(n_B(k_0)\) by the Fermion-Dirac distribution function \(-f(k_0)\) in the symmetric propagator, and the retarded propagator and advanced propagator are the same as that of bosons.
\[ \hat{\Delta}_S(K) = \hat{\Delta}_{11} + \hat{\Delta}_{22} = -2\pi i\delta(K^2 - m^2)[1 - 2f(k_0)]. \]  

The inverse relation in Eq. (23) is also applicable for fermions.

A. The gluon HTL self energy in the light cone gauge

It has been checked by explicit computation in different gauges (covariant, Coulomb, temporal) whose axial vectors are all the same, \( n^\mu_a = (1, 0, 0, 0) \), that the gluon HTL self energy does not depend on the choice of gauge. However, the axial vector in light cone gauge is different, that brings about some changes.

A massive boson gives rise to the longitudinal polarization state, so that the boson self energy is separated into the longitudinal and transverse parts [4]. The massive gluon self energy in the light cone gauge is made up of the transverse and longitudinal parts,

\[ \Pi^{\mu\nu}(P) = - \left[ \hat{T}^{\mu\nu}_P \Pi_T(P) + \frac{\hat{L}^{\mu\nu}_P}{n^\mu_P} \Pi_L(P) \right]. \]  

The transverse projection tensor is

\[ \hat{T}^{\mu\nu}_P = g^{\mu\nu} - \frac{n^\mu P^\nu + n^\nu P^\mu}{n \cdot P} + \frac{n^\mu n^\nu P^2}{(n \cdot P)^2}. \]  

The longitudinal projection tensor is

\[ \hat{L}^{\mu\nu}_P = - \left[ \frac{n^\mu n^\nu P^2}{(n \cdot P)^2} - \frac{n^\mu P^\nu + n^\nu P^\mu}{n \cdot P} + \frac{P^\mu P^\nu}{P^2} \right]. \]  

The four-vector \( n^\mu_P \) is

\[ n^\mu_P = \left( g^{\mu\nu} - \frac{P^\mu P^\nu}{P^2} \right) n_\nu = n^\mu - \frac{n \cdot P}{P^2} P^\mu. \]

\( \hat{L}^{\mu\nu}_P \) and \( \hat{T}^{\mu\nu}_P \) satisfy the following relations,

\[ \hat{L}^{\mu\nu}_P \hat{L}^{\rho\sigma}_P = \hat{L}^{\mu\rho}_P \hat{L}^{\nu\sigma}_P, \]
\[ \hat{T}^{\mu\nu}_P \hat{T}^{\rho\sigma}_P = \hat{T}^{\mu\rho}_P \hat{T}^{\nu\sigma}_P, \]
\[ \hat{T}^{\mu\rho}_P \hat{L}^{\nu\sigma}_P = 0. \]  

These equations are also suitable for \( L^{\mu\nu}_P \) and \( T^{\mu\nu}_P \) in Eq. (21) in the Coulomb gauge. The axial vector \( n^\mu \) in the light cone gauge is defined in Eq. (10),

\[ n^\mu \hat{T}^{\mu\nu}_P = 0, \]
\[ n^\mu \hat{L}^{\mu\nu}_P \neq 0. \]

\( \hat{L}^{\mu\nu}_P \) and \( \hat{T}^{\mu\nu}_P \) are the longitudinal and transverse projection tensors with respect to the axial vector \( n^\mu \) in the light cone gauge in Eq. (10). Because the axial vector in the light cone gauge in Eq. (10) is different from the axial vector in the Coulomb gauge in Eq. (3), so the longitudinal and transverse projection tensors \( \hat{L}^{\mu\nu}_P \) and \( \hat{T}^{\mu\nu}_P \) are different from those in the Coulomb gauge. Finally, these differences make the expression of the HTL resumed gluon propagator in the light cone gauge changed.

The inverse propagator in the light cone gauge in the limit \( \xi \to \infty \) is

\[ \Delta^{-1}_L(P)^{\mu\nu} = -P^2 g^{\mu\nu} + P^\mu P^\nu - \Pi^{\mu\nu}(P), \]
\[ = \left[ -P^2 + \Pi_T(P) \right] \hat{T}^{\mu\nu}_P + \left[ -P^2 + \frac{1}{n^\mu_P} \Pi_L(P) \right] \hat{L}^{\mu\nu}_P. \]  

Applying Eq. (30) (31), we can get the HTL resumed gluon propagator in the light cone gauge

\[ \Delta^{\mu\nu}(P) = \frac{\hat{T}^{\mu\nu}_P}{-P^2 + \Pi_T(P)} + \frac{-n^\mu n^\nu P^2}{n^\mu_P - P^2 + \frac{1}{n^\mu_P} \Pi_L(P)}, \]
where we do not consider the terms containing the gauge parameter $\xi$. 

When $\Pi_T(P) = 0$ and $\Pi_L(P) = 0$, the HTL resumed gluon propagator returns back to the bare gluon propagator in Eq. (17).

Due to the relation in Eq. (26), (27), (28), (30), the transverse and longitudinal gluon HTL self-energies are given by

$$\Pi_T(P) = -\frac{1}{2} T_{\mu\nu} \Pi^{\mu\nu}(P) = -\frac{1}{2} \left[ g_{\mu\nu} - \frac{n_\mu P_\nu + n_\nu P_\mu}{n \cdot P} + \frac{n_\mu n_\nu P^2}{(n \cdot P)^2} \right] \Pi^{\mu\nu}(P),$$  \hspace{1cm} (35)

$$\frac{1}{n_P^2} \Pi_L(P) = -\tilde{L}_{\mu\nu} \Pi^{\mu\nu}(P) = \left[ \frac{P_\mu P_\nu}{P^2} - \frac{n_\mu P_\nu + n_\nu P_\mu}{n \cdot P} + \frac{n_\mu n_\nu P^2}{(n \cdot P)^2} \right] \Pi^{\mu\nu}(P).$$  \hspace{1cm} (36)

$\Pi^{\mu\nu}(P)$ is the sum of the quark loop, the gluon loop and the gluon tadpole in the light cone gauge. Multiply $\Pi^{\mu\nu}(P)$ by the projection tensors $-\frac{1}{2} T_{\mu\nu}$ and $-\tilde{L}_{\mu\nu}$, and we can calculate $\Pi_T(P)$ and $\frac{1}{n_P^2} \Pi_L(P)$ in the HTL approximation.

### B. The quark loop of the gluon HTL self energy in the light cone gauge

The quark loop in Fig. 1 can be expressed as

$$\Pi^{\mu\alpha}_{ab}(P) = -\frac{1}{2} N_f g^2 \delta_{ab} \int \frac{d^4 K}{(2\pi)^4} \text{Tr}[\gamma^{\mu} F(K) \gamma^\alpha F(K - P)],$$  \hspace{1cm} (37)

where $N_f$ is the active quark flavors and $F(K)$ is the bare quark propagator.

The retarded self energy in the real time formalism [6], [17] is expressed as

$$\Pi^{\mu\alpha}_R(P) = \Pi^{\mu\alpha}_{11}(P) + \Pi^{\mu\alpha}_{12}(P) = \frac{i}{2} g^2 N_f \int \frac{d^4 K}{(2\pi)^4} \text{Tr}[\gamma^{\mu} K \gamma^\alpha(K - P)] \left[ \tilde{\Delta}_{11}(K) \tilde{\Delta}_{11}(K - P) - \tilde{\Delta}_{12}(K) \tilde{\Delta}_{21}(K - P) \right],$$  \hspace{1cm} (38)

where the RTF Green function $\tilde{\Delta}_{ij}(K)$ refers to the component of the propagator in the real time formalism in Eq. (21).

Multiply $\Pi^{\mu\alpha}(P)$ by the transverse projection tensor $-\frac{1}{2} T_{\mu\alpha}$, we can get the transverse self energy $\Pi_T(P)$,

$$\Pi_T(P) = \frac{i}{4} N_f g^2 \int \frac{d^4 K}{(2\pi)^4} 8 \left[ K \cdot P - 2 \frac{n \cdot K}{n \cdot P} K \cdot P + \frac{(n \cdot K)^2}{(n \cdot P)^2} P^2 \right] \left[ \tilde{\Delta}_{11}(K) \tilde{\Delta}_{11}(K - P) - \tilde{\Delta}_{12}(K) \tilde{\Delta}_{21}(K - P) \right].$$  \hspace{1cm} (39)

Using the relation in Eq. (24), we can obtain

$$\tilde{\Delta}_{11}(K) \tilde{\Delta}_{11}(K - P) - \tilde{\Delta}_{12}(K) \tilde{\Delta}_{21}(K - P) = \frac{1}{2} \left[ \tilde{\Delta}_S(K - P) \tilde{\Delta}_R(K) + \tilde{\Delta}_A(K - P) \tilde{\Delta}_S(K) + \tilde{\Delta}_A(K - P) \tilde{\Delta}_A(K) + \tilde{\Delta}_R(K - P) \tilde{\Delta}_R(K) \right]$$

$$= \frac{1}{2} \left[ \tilde{\Delta}_S(K - P) \tilde{\Delta}_R(K) + \tilde{\Delta}_A(K - P) \tilde{\Delta}_S(K) \right],$$  \hspace{1cm} (40)

where the minus sign in front of the term $\tilde{\Delta}_{12}(K) \tilde{\Delta}_{21}(K - P)$ comes from the vertex of the type 2 fields [18]. The $k_0$ integral of $\tilde{\Delta}_A(K - P) \tilde{\Delta}_A(K)$ and $\tilde{\Delta}_R(K - P) \tilde{\Delta}_R(K)$ reduce to zero.
Replace $K$ by $P - K$ in the first term and using $\tilde{\Delta}_R(P - K) = \tilde{\Delta}_A(K - P)$, and this expression can be simplified further on,

$$
\Pi_T(P) = \frac{i}{4} N_f g^2 \int \frac{d^4 K}{(2\pi)^4} \left[ K \cdot P - \frac{2 n}{n \cdot p} K \cdot P + \frac{(n \cdot K)^2}{(n \cdot P)^2} P^2 \right] \tilde{\Delta}_S(K) \tilde{\Delta}_A(K - P)
$$

In the calculation, we use the HTL approximation, i.e. high temperature limit. The internal momentum $K$ is hard, and the external momentum momentum $P$ is soft. The transverse part of the quark loop in the light cone gauge in the HTL approximation from Eq. (35) is obtained by

$$
\Pi_T(P) = \frac{i}{4} N_f g^2 \int \frac{d^4 K}{(2\pi)^4} \left[ K \cdot P - 2 \frac{n \cdot K}{n \cdot P} K \cdot P + \frac{(n \cdot K)^2}{(n \cdot P)^2} P^2 \right] \tilde{\Delta}_S(K) \tilde{\Delta}_A(K - P)
$$

Similarly, the longitudinal part of the quark loop in the light cone gauge in the HTL approximation is

$$
\frac{1}{n_p} \Pi_L(P) = \frac{i}{2} g^2 N_f \int \frac{d^4 K}{(2\pi)^4} \left[ 8 \frac{(n \cdot K)^2}{(n \cdot P)^2} P^2 - 16 \frac{n \cdot K}{n \cdot P} K \cdot P + 8 \frac{(K \cdot P)^2}{P^2} - 4 K \cdot P + 4 K^2 \right] \tilde{\Delta}_S(K) \tilde{\Delta}_A(K - P)
$$

C. The gluon loop and the gluon tadpole of the gluon HTL self energy in the light cone gauge

The gluon loop and the gluon tadpole in Fig. 2 are expressed as

$$
\Pi_\mu^{\alpha} \left( P \right) = \frac{i}{2} \int \frac{d^4 K}{(2\pi)^4} V^{\mu \nu}(P, -K, K - P) i d_{\nu\beta}(K) V^{\beta \gamma}(K, P - K, -P) i d_{\rho\gamma}(K - P) G(K) G(K - P)
$$

$$
+ \frac{i}{2} \int \frac{d^4 K}{(2\pi)^4} i d_{\rho\sigma}(K) G(K) \delta^{\rho \sigma} V^{\mu \alpha \kappa}_{ab} \left( K \right) .
$$

where $d_{\nu\beta}(K) G(K)$, $d^{\nu\alpha}(K - P) G(K - P)$ and $d_{\rho\sigma}(K) G(K)$ are all the bare gluon propagators.

Below are the tensor of the bare gluon propagator in the light cone gauge

$$
d_{\nu\beta}(K) = -g_{\nu\beta} + \frac{n_\nu K_\beta + n_\beta K_\nu}{n \cdot K} .
$$

The three gluon vertexes are

$$
V^{\mu \nu}(P, -K, K - P) = g f^{abcd} \left[ g^{\mu \nu} (P + K)^\rho + g^{\nu \rho} (2K - P) \rho \right] \left[ g^{\nu \rho} (2K - P) \mu + g^{\mu \rho} (K - 2P) \rho \right] ,
$$

$$
V^{\beta \gamma \alpha}(K, P - K, -P) = g f^{cdef} \left[ g^{\beta \gamma} (2K - P) \alpha + g^{\gamma \alpha} (2P - K) \beta + g^{\alpha \beta} (P - K) \gamma \right] .
$$

The four gluon vertex is

$$
V^{\mu \alpha \rho \sigma}_{ab} = -i g^2 \left[ f^{abcdef} \left( g^{\mu \alpha} g^{\rho \sigma} - g^{\rho \sigma} g^{\mu \alpha} \right) + f^{ade} f^{bce} \left( g^{\mu \alpha} g^{\sigma \rho} - g^{\rho \sigma} g^{\mu \alpha} \right) + f^{ade} f^{bec} \left( g^{\mu \alpha} g^{\sigma \rho} - g^{\rho \sigma} g^{\mu \alpha} \right) \right] .
$$

The transverse part of the gluon self energy in the HTL approximation from Eq. (35) is
$$\Pi_T(P) = \frac{i}{4} C_A g^2 \int \frac{d^3 K}{(2\pi)^3} \left[ -8 K \cdot P - 12 P^2 + 16 \frac{n-K}{nKW} - 8 \frac{(n-K)^2}{P^2} P^2 \right. \\
+ 8 \frac{n-P}{n(K-P)} (K^2 - P^2) - 8 \frac{n-P}{nK} (K^2 - 2K \cdot P) \right] G(K) G(K-P) \\
= \frac{1}{6} C_A g^2 T^2 \left[ \frac{(p_0)^2}{p^2} - \frac{p_0 P^2}{2p^3} \ln \frac{p_0 + p + ie}{p_0 - p - ie} \right] \\
+ \frac{i}{4} C_A g^2 \int \frac{d^3 K}{(2\pi)^3} \left[ 8 \frac{n-P}{n(K-P)} (K^2 - P^2) - 8 \frac{n-P}{nK} (K^2 - 2K \cdot P) \right] G(K) G(K-P).$$

The last two light cone terms in the fourth line come from the two tensors of the gluon propagators \((n^\nu K^\beta + n^\beta K^\nu)/(n \cdot K)\) and \([n^\rho (K - P)\nu + n^\gamma (K - P)\nu]/((n \cdot (K - P))\). Replace \(K\) by \(P - K\) in the first light cone term, and the sum of the two light cone terms is

$$- \frac{i}{4} C_A g^2 \int \frac{d^3 K}{(2\pi)^3} 16 \frac{n-P}{nKW} (K^2 - 2K \cdot P) / (K^2 - (K - P)^2).$$

At zero temperature, the divergence of the integral calculation of one loop diagram in Fig. 11 has been renormalized successfully. Here we only consider the contribution at finite temperature. Use the ML prescription of 1/(\(n \cdot K\)) in Eq. 13. By calculating with the contour integral, we find there is no divergence in the HTL approximation, and the power of the part is \(g^2 T^2\) order, which can be ignored in the result. The proof is in the Appendix.

Similarly, the longitudinal part of the gluon self energy in the HTL approximation is obtained by

$$\frac{1}{n_p} \Pi_L(P) = \frac{i}{2} C_A g^2 \int \frac{d^3 K}{(2\pi)^3} \left[ 4K^2 - 4K \cdot P + 2P^2 + 8 \frac{(K^2 - P^2)}{P^2} - 16 \frac{n-K}{nKW} - 8 \frac{(n-K)^2}{P^2} \right] K \cdot P + 8 \frac{(n-K)^2}{P^2}$$

$$+ \frac{n-P}{n(K-P)} 2K^2 (K-P)^2 / (n-K)^2 \right] G(K) G(K-P)$$

$$= - \frac{1}{3} C_A g^2 T^2 \left[ \frac{P^2}{p^2} - \frac{p_0 P^2}{2p^3} \ln \frac{p_0 + p + ie}{p_0 - p - ie} \right] + \frac{i}{2} C_A g^2 \int \frac{d^3 K}{(2\pi)^3} \left[ \frac{1}{2} \frac{n-P}{n(K-P)} - \frac{2n-P}{n-K} \right].$$

There are two light cone terms in the third line. By calculating, the integral with the light cone terms is zero.

$$\frac{i}{2} C_A g^2 \int \frac{d^3 K}{(2\pi)^3} \frac{1}{2} \frac{n-P}{n(K-P)} - \frac{2n-P}{n-K} = 0.$$

During the calculation, we set the momentum \(\vec{p}\) is on the positive direction of \(z\) axis. \(\vec{t}\) in Eq. 10 is on the negative direction of \(z\) axis. The angle between \(\vec{k}\) and \(\vec{p}\) is \(\theta\). Here we use the Kelydsh representation in the real time formalism and consider the \(T > 0\) contribution. In the HTL approximation, the internal momentum \(K\) is soft, and the external momentum \(P\) is hard.

D. The transverse and longitudinal parts of the gluon HTL self energy

Adding up the results from the longitudinal and transverse parts of the quark loop in Fig. 1, the gluon loop and the gluon tadpole in Fig. 2 in the HTL approximation, we obtain the following expression,

$$\Pi_T(P) = \frac{1}{6} (C_A + \frac{1}{2} N_f) g^2 T^2 \left[ \frac{(p_0)^2}{p^2} - \frac{p_0 P^2}{2p^3} \ln \frac{p_0 + p + ie}{p_0 - p - ie} \right],$$

$$\frac{1}{n_p} \Pi_L(P) = - \frac{1}{3} (C_A + \frac{1}{2} N_f) g^2 T^2 \left[ \frac{p^2}{p^2} - \frac{p_0 P^2}{2p^3} \ln \frac{p_0 + p + ie}{p_0 - p - ie} \right].$$

The factor \(\frac{1}{2}\) in the coefficient \((C_A + \frac{1}{2} N_f)\) stems from that these integrals \(\int_0^\infty kn(k)dk\) and \(\int_0^\infty k f(k)dk\) which have different distribution functions. We find we get the same result of the transverse and longitudinal HTL gluon self energy in the light cone gauge and the Coulomb gauge, although these two gauges have different projection tensors.
In the static limit $p_0 \to 0$, the longitudinal and transverse parts of gluon HTL self energy in the light cone gauge reduce to

$$
\lim_{p_0 \to 0} \frac{1}{n_T} \Pi_L(P) = \frac{1}{3} g^2 T^2 (C_A + \frac{1}{2} N_f) = m_D^2 , \\
\lim_{p_0 \to 0} \Pi_L(P) = \frac{1}{6} g^2 T^2 (C_A + \frac{1}{2} N_f) = \frac{1}{2} m_D^2 , \\
\lim_{p_0 \to 0} \Pi_T(P) = 0.
$$

(53)

In the static limit $p_0 \to 0$, the longitudinal and transverse parts of the gluon HTL self energy in the Coulomb gauge reduces to

$$
\lim_{p_0 \to 0} \Pi_L(P) = \frac{1}{3} (C_A + \frac{1}{2} N_f) g^2 T^2 = m_D^2 , \\
\lim_{p_0 \to 0} \Pi_T(P) = 0.
$$

(54)

We think the axial vector $n_t^\mu = (\frac{\sqrt{2}}{2}, 0, 0, -\frac{\sqrt{2}}{2})$ in the light cone gauge is rotated with respect to the axial vector $n_c^\mu = (1, 0, 0, 0)$ in the Coulomb gauge, so that it gives rise to some changes.

Because the axial vector $n_t^\mu$ in the light cone gauge is different from $n_c^\mu$ in the Coulomb gauge, we compare $\Pi^{00}(P)$ in the light cone gauge with $\Pi^{00}(P)$ in the Coulomb gauge.

In the static limit, $n_\mu n_\nu \Pi^{\mu\nu}(P)$ of the gluon HTL self energy in the Coulomb gauge in Eq. (1) is

$$
\lim_{p_0 \to 0} n_\mu n_\nu \Pi^{\mu\nu}(P) = \lim_{p_0 \to 0} \Pi^{00}(P) = -\lim_{p_0 \to 0} \frac{g_0^0}{n_T} \Pi_L(P) = -m_D^2 ,
$$

(55)

where the axial vector $n_t^\mu = (1, 0, 0, 0)$ in Eq. (5).

The transverse and longitudinal spectral functions are expressed as

$$
\rho_T(P) = 2 \pi Z_T \text{sgn}(p_0) \left[ \delta(p_0 - w_T) + \delta(p_0 + w_T) \right] + \beta_T(P), \\
\rho_L(P) = 2 \pi Z_L \text{sgn}(p_0) \left[ \delta(p_0 - w_L) + \delta(p_0 + w_L) \right] + \beta_L(P).
$$

(58)

The spectral function $\rho_{T/L}(P)$ is made up of the pole term and the cut term $\beta_{T/L}(P)$. For $P^2$ space-like, $i.e.$ $p_0^2 < p^2$, the function $\ln \frac{p_0 + p \pm i \epsilon}{p_0 - p \pm i \epsilon}$ generates the imaginary part,
So the cut terms of the transverse and longitudinal part of the HTL resumed gluon propagator in the light cone gauge are obtained by

\[
\beta_T(P) = \frac{1}{4\pi m_D^2} \frac{m_D^2}{p^2} \theta(p^2 - m_D^2),
\]

\[
\beta_L(P) = \frac{-\pi m_D^2}{p^2} \frac{m_D^2}{p^2} \theta(p^2 - m_D^2),
\]

(60)

where \( \theta(p^2 - m_D^2) \) is the step function, and the Debye screening mass \( m_D^2 = \frac{1}{3} (C_A + \frac{2}{3} N_f) g^2 T^2 \).

Via the expression of the HTL resumed gluon propagator in the light cone gauge, the transverse dispersion relation is given by

\[
\omega_T^2 - p^2 = -m_D^2 \left( \frac{\omega_T^2 - p^2}{2p^2} - \frac{\omega_T^2 - p^2}{2p^2} \ln \frac{\omega_T + p}{\omega_T - p} \right) = 0,
\]

(61)

where \( \omega_T \) is the solution of the above transverse dispersion relation.

The longitudinal dispersion relation is given by

\[
p^2 + m_D^2 \left[ 1 - \frac{\omega_L^2}{2p^2} \ln \frac{\omega_L + p}{\omega_L - p} \right] = 0,
\]

(62)

where \( \omega_L \) is the solution of the above longitudinal dispersion relation.

Obviously the transverse and longitudinal parts have the same dispersion relation as that of the HTL resumed gluon propagator in the Coulomb gauge. However, this two kinds of gauge have different expression of the transverse and longitudinal projection tensor, and then have different expression of the HTL resumed gluon propagator. Due to the same dispersion relation, for more analyses you can refer to [11].

The residue for the transverse part is

\[
Z_T = -\left( \left[ \frac{\partial (p^2 - \Pi_T)}{\partial p^0} \right]_{p^0 = \omega_T(p)} \right)^{-1}
\]

\[
= \frac{\omega_T (\omega_T^2 - p^2)}{m_D^2 \omega_T - (\omega_T^2 - p^2)^2}.
\]

(63)

The residue for the longitudinal part is

\[
Z_L = -\left( \left[ \frac{\partial^2 \Pi_L}{\partial p^0 \partial p^1} \right]_{p^0 = \omega_L(p)} \right)^{-1}
\]

\[
= \frac{\omega_L (\omega_L^2 - p^2)}{p^2 (p^2 - m_D^2 - \omega_L^2)}.
\]

(64)

About the transverse and longitudinal residues you can find more discussion in [11] too.

Below is the proof to tell the reason why we can get the same dispersion relation. From the famous Ward identity,

\[
P^\mu \Pi_{\mu \nu}(P) = p_0 \Pi_{0 \nu} - p_3 \Pi_{3 \nu} = 0,
\]

(65)

we can get the below relation,

\[
\Pi_{3 \nu} = \frac{p_0}{p_3} \Pi_{0 \nu},
\]

(66)

where the external momentum \( P^\mu = (p_0, 0, 0, p_3), p_3 > 0 \), and \( \vec{p} \) is on the positive direction of \( z \) axis.
Using the above relation, the longitudinal part of the gluon HTL self energy can become

\[
\hat{L}_P^{\mu\nu}(P) = -\frac{n^\mu n^\nu P^2}{(n \cdot P)^2} \Pi_{\mu\nu}(P)
\]

\[
= -\frac{n^\mu P^2}{(n \cdot P)^2} \frac{\sqrt{2}}{2} (\Pi_{0\nu} + n_3 \Pi_{3\nu})
\]

\[
= -\frac{P^2}{(n \cdot P)^2} \frac{\sqrt{2}(p_0 + p_3)}{2p_3} n^\nu \Pi_{0\nu}
\]

\[
= -\frac{P^2}{(n \cdot P)^2} \frac{(p_0 + p_3)^2}{2(3^2)} \Pi_{00}(P)
\]

\[
= -\frac{P^2}{(p_3)^2} \Pi_{00}(P),
\]

where \( L_P^{\mu\nu} \) is the longitudinal projection tensor in the light cone gauge in Eq.(28), \( \Pi_{00}(P) \) is the longitudinal part of the gluon HTL self energy in the Coulomb gauge in Eq.(11), and the axial vector in the light cone gauge \( n^\mu = (\sqrt{2}, 0, 0, -\sqrt{2}) \). The result tells us the longitudinal part of the gluon HTL self energy in the light cone gauge is the same as that in the Coulomb gauge.

Similarly, the transverse part of the gluon HTL self energy can become

\[
\frac{1}{2} \hat{T}_P^{\mu\nu}(P) = \frac{1}{2} \left[ g^{\mu\nu} + \frac{n^\mu n^\nu P^2}{(n \cdot P)^2} \right] \Pi_{\mu\nu}(P)
\]

\[
= -\frac{1}{2} m_D^2 + \frac{P^2}{2(p_3)^2} \Pi_{00}(P).
\]

where \( T_P^{\mu\nu} \) is the transverse projection tensor in the light cone gauge in Eq.(27). So the transverse part of the gluon HTL self energy in the light cone gauge is the same as that in the Coulomb gauge.

In the proof, we use the famous Ward identity. \( \vec{n} \) is on the positive direction of the z axis, and \( \vec{n} \) is on the negative direction of the z axis. We can get the same transverse and longitudinal parts of the gluon HTL self energy in the two gauges, and finally get the same dispersion relation of the transverse and longitudinal parts.

**IV. THE HTL RESUMED QUARK PROPAGATOR IN THE LIGHT CONE GAUGE**

The quark self energy in Fig. 3 can be expressed as

\[
\Sigma(P) = iC_F g^2 \int \frac{d^4 K}{(2\pi)^4} \gamma^\mu F(P - K) \gamma^\nu d_{\mu\nu}(K) G(K),
\]

where \( C_F = \frac{4}{3} \) is the color factor, \( F(P - K) \) is the bare quark propagator, and \( d_{\mu\nu}(K) G(K) \) is the bare gluon propagator.

The retarded quark self energy in the real time formalism is expressed as

\[
\Sigma_R(P) = \Sigma_{11}(P) + \Sigma_{12}(P)
\]

\[
= iC_F g^2 \int \frac{d^4 K}{(2\pi)^4} \gamma^\mu (J^\mu - \mathcal{K}) \gamma^\nu d_{\mu\nu}(K) [\Delta_{11}(P - K) \Delta_{11}(K) + \Delta_{12}(P - K) \Delta_{12}(K)].
\]
Using the relation in Eq. (24), we get

\[
\Delta_{11}(P-K)\Delta_{11}(K) - \Delta_{12}(P-K)\Delta_{12}(K) \\
= \frac{1}{2} [\tilde{\Delta}_R\Delta_S + \tilde{\Delta}_R\Delta_A + \tilde{\Delta}_S\Delta_R + \tilde{\Delta}_A\Delta_R] \\
= \frac{1}{2} [\tilde{\Delta}_R\Delta_S + \tilde{\Delta}_A\Delta_R],
\]

where \( \tilde{\Delta} \) and \( \Delta \) respectively represent the Green functions of quark and gluon. The terms \( \tilde{\Delta}_R\Delta_A \) and \( \tilde{\Delta}_A\Delta_R \) are both zero temperature parts, and we neglect them here.

The quark self energy is decomposed into two parts,

\[
\Sigma_R(P) = -a(p_0, p)P' - b(p_0, p)\gamma^0, \\
a(p_0, p) = \frac{1}{4p^2} [\text{Tr}(P\Sigma_R) - p_0\text{Tr}[\gamma_0\Sigma_R]], \\
b(p_0, p) = \frac{1}{4p^2} [\text{Tr}[\gamma_0\Sigma_R] - \gamma_0\text{Tr}(P\Sigma_R)].
\]

With the above relation, we can do the following calculation,

\[
\text{Tr}[P\Sigma_R(P)] = iC_F g^2 \int \frac{d^4K}{(2\pi)^4} \left[ -8K \cdot P + \frac{n \cdot P}{n \cdot K}(-8K^2 + 16K \cdot P) \right] \frac{1}{2} \tilde{\Delta}_R(P-K)\Delta_S(K) + \tilde{\Delta}_S(P-K)\Delta_R(K). \tag{73}
\]

Replace \( K \) by \( P - K \) in this term \( \tilde{\Delta}_S(P-K)\Delta_R(K) \), this expression becomes

\[
\text{Tr}[P\Sigma_R(P)] = iC_F g^2 \int \frac{d^4K}{(2\pi)^4} \left[ \left[ 8(K \cdot P - P^2) + \frac{8n \cdot P}{n \cdot (K - P)}(K^2 - P^2) \right] \frac{1}{2} \tilde{\Delta}_S(P-K)\Delta_R(P-K) \\
+ \left[ -8K \cdot P + \frac{n \cdot P}{n \cdot K}(-8K^2 + 16K \cdot P) \right] \frac{1}{2} \tilde{\Delta}_R(P-K)\Delta_S(K) \right] \\
= iC_F g^2 \int \frac{d^4K}{(2\pi)^4} \left[ 4(K \cdot P - P^2)\tilde{\Delta}_S(P-K)\Delta_R(P-K) - 4K \cdot P\tilde{\Delta}_R(P-K)\Delta_S(K) \right] \\
+ \left[ \frac{4n \cdot P}{n \cdot (K - P)}(K^2 - P^2)\tilde{\Delta}_S(K)\Delta_R(P-K) + \frac{n \cdot P}{n \cdot K}(-4K^2 + 8K \cdot P)\tilde{\Delta}_R(P-K)\Delta_S(K) \right] \\
= 4m_F^2, \tag{74}
\]

In the HTL approximation, we can prove there is no spurious divergence from the light cone terms in the forth line, and these finite terms are power suppressed than the covariant terms, so we ignore these light cone terms.

In the same way, we can get

\[
\text{Tr}[\gamma_0\Sigma_R(P)] = C_F g^2 \int \frac{d^4K}{(2\pi)^4} [ -k_0 + \frac{n \cdot P}{n \cdot K} - \frac{n_0}{n \cdot K}(K^2 - K \cdot P) ] \frac{1}{2} [\tilde{\Delta}_R(P-K)\Delta_S(K) + \tilde{\Delta}_S(P-K)\Delta_R(K)] \\
= C_F g^2 \int \frac{d^4K}{(2\pi)^4} [ 4(k_0 - p_0)\tilde{\Delta}_S(K)\Delta_R(P-K) - 4k_0\tilde{\Delta}_R(P-K)\Delta_S(K) + 4\left( k_0 - p_0 \right) \frac{n \cdot P}{n \cdot (K - P)}(K^2 - K \cdot P)\tilde{\Delta}_S(K)\Delta_R(P-K) + 4\left( k_0 \frac{n \cdot P}{n \cdot K} - \frac{n_0}{n \cdot K}(K^2 - K \cdot P) \right)\tilde{\Delta}_R(P-K)\Delta_S(K) \right] \\
= 2m_F^2 \frac{1}{p} \ln \frac{p_0 + p + i\epsilon}{p_0 - p + i\epsilon}. \tag{75}
\]

In the HTL approximation, it can be proved that there is no spurious divergence from the light cone terms in the third and forth lines, and these finite terms are power suppressed than the covariant terms, so we ignore these light cone terms too.

So the result shows the quark HTL self energy is the same as that of covariant gauge \[19\]. With the quark HTL self energy, we can derive the same quark resumed propagator.
The HTL resumed quark propagator is
\[
S^*(P) = \frac{1}{D_+(P)} \frac{\gamma_0 - \not p \cdot \not \gamma}{2} + \frac{1}{D_-(P)} \frac{\gamma_0 + \not p \cdot \not \gamma}{2},
\]
\[
D_\pm(P) = -p_0 \pm p + \frac{m_F^2}{p} \left[ \frac{1}{2} \ln \frac{p_0 + p + i\epsilon}{p_0 - p + i\epsilon} + \frac{p_0}{2p} \ln \frac{p_0 + p + i\epsilon}{p_0 - p - i\epsilon} - 1 \right],
\]
where the effective quark mass \( m_F = g^2 T^2/6 \) in QCD, and the notation \( \not p = \frac{\not p}{|p|} \).

V. THE DAMPING RATES OF HARD QUARK AND GLUON IN THE LIGHT CONE GAUGE

The damping rates of the heavy fermion have been done \[14, 20\]. In a similar way, we use the HTL resumed gluon propagator in the light cone gauge to calculate the damping rates of the hard quark and gluon. By above analyses, we know the HTL resumed gluon propagator in the Coulomb gauge and the light cone gauge have the same denominator and different projection tensors in the nominator. But we can prove in general case, using the HTL resumed gluon propagator in the two gauges get the same result of the damping rates of the hard quark and gluon.

The quark self energy in Fig. 4 is expressed as
\[
\Sigma(Q) = ig^2 C_F \int \frac{d^4P}{(2\pi)^4} \left[ \gamma_\mu F(Q - P) \gamma_\nu G_{\mu\nu}(Q) \right],
\]
where \( G_{\mu\nu}(Q) \) is the HTL resumed gluon propagator in the light cone gauge, and \( F(Q - P) \) is the bare quark propagator.

In the real time formalism, the retarded quark self energy is expressed as
\[
\Sigma_R(Q) = \Sigma_{11}(Q) + \Sigma_{12}(Q)
\]
\[
= ig^2 C_F \int \frac{d^4P}{(2\pi)^4} \left[ \gamma_\mu (\not Q - \not P) \gamma_\nu \left[ \tilde{\Delta}_{11}(Q - P) \Delta_{11}^{\mu\nu}(P) - \tilde{\Delta}_{12}(Q - P) \Delta_{12}^{\mu\nu}(P) \right] \right].
\]

Using the relation of the Keldysh representation in Eq. (24), we have
\[
\tilde{\Delta}_{11}(Q - P) \Delta_{11}^{\mu\nu}(P) - \tilde{\Delta}_{12}(Q - P) \Delta_{12}^{\mu\nu}(P) = \frac{1}{2} \left[ \tilde{\Delta}_R \Delta_S^{\mu\nu} + \tilde{\Delta}_R \Delta_A^{\mu\nu} + \tilde{\Delta}_S \Delta_R^{\mu\nu} + \tilde{\Delta}_A \Delta_R^{\mu\nu} \right].
\]

By power counting the leading contribution at finite temperature comes from the first term in the bracket \( \tilde{\Delta}_R \Delta_S^{\mu\nu} \), which is \( O(1/g^3) \). \( \tilde{\Delta}_R \Delta_A^{\mu\nu} \) and \( \tilde{\Delta}_A \Delta_R^{\mu\nu} \) are both \( O(1/g^2) \), so we ignore them here. From the below equations, \( \tilde{\Delta}_S \Delta_R^{\mu\nu} \) with the Fermi-Dirac distribution function is \( O(g) \) power suppressed than \( \tilde{\Delta}_R \Delta_S^{\mu\nu} \) with the Bose-Einstein distribution function.

The internal momentum \( P \) is soft, \( p_0 \sim gT \). The Bose-Einstein distribution function \( n_B(p_0) \) in the term \( \tilde{\Delta}_R \Delta_S^{\mu\nu} \) is \( O(1/g) \),
\[
\frac{1}{e^{\frac{p_0}{T}} - 1} \sim \frac{T}{|p_0|} \propto \frac{1}{g}.
\]

However, the Fermi-Dirac distribution function \( f(p_0 - q_0) \) in the term \( \tilde{\Delta}_S \Delta_R^{\mu\nu} \) is \( O(1) \). The external momentum \( Q \) is hard, \( q_0 \sim T \),
\[
\frac{1}{e^{\frac{|p_0 - q_0|}{T}} + 1} \sim O(1).
\]
The symmetric propagator of the soft gluon in the light cone gauge is

\[
\Delta_S^{\mu
u}(P) = -2\pi i \left[ -\delta^{\mu
u} \rho_T(P) + \frac{n^\mu n^\nu P^2}{(n \cdot P)^2} \rho_L(P) \right] \left[ 1 + 2n_B(p_0) \right],
\]

where we only consider the contribution at finite temperature.

Using the below equation, we can calculate the imaginary part of the quark self energy,

\[
\text{Im}\tilde{\Delta}(Q - P) = \text{Im}\left[ \frac{1}{(Q - P)^2 + i\text{sgn}(q_0 - p_0)\epsilon} \right] = -\pi\text{sgn}(q_0 - p_0)\delta[(Q - P)^2].
\]

In the integral, we use the \( \delta \) function to integrate out \( \cos\theta \),

\[
\delta[(Q - P)^2] = \frac{1}{2pq} \delta[\cos\theta - \frac{p_0}{p} + \frac{P^2}{2pq}] \approx \frac{1}{2pq} \delta[\cos\theta - \frac{p_0}{p}],
\]

where the term \( \frac{P^2}{2pq} \sim g \), which we can ignore.

The transverse projection tensor in the light cone gauge is

\[
\tilde{T}_P^{\mu\nu} = g^{\mu\nu} - \frac{n^\mu P^\nu + n^\nu P^\mu}{n \cdot P} + \frac{n^\mu n^\nu P^2}{(n \cdot P)^2},
\]

where the internal soft momentum \( P^\mu = (p_0, 0, 0, p_3) \).

By calculating we can find the relation \( T_P^{0\nu} = 0, \tilde{T}_P^{3\nu} = 0 \), so we have

\[
\tilde{T}_P^{ij} = -\delta^{ij}, \quad (i, j = 1, 2).
\]

The longitudinal and transverse spectral functions in the limit \( p_0 \to 0 \) are

\[
\rho_L(P) \approx \frac{p_0 m_2^2}{2p} \left( \frac{1}{(p^2 + m_2^2)^2} \right),
\]

\[
\rho_T(P) \approx \frac{p_0 m_2^2}{4} \left( \frac{1}{p^6 + \frac{1}{16}\pi^2 m_2^4 p_0^2} \right),
\]

where we find no static magnetic screening.

With above equations, we can calculate the imaginary part of \( \text{Tr}[\mathcal{Q}\Sigma_R(Q)] \),

\[
\text{Im}\left[ \text{Tr}[\mathcal{Q}\Sigma_R(Q)] \right] = -4\pi^2 C_F g^2 \int \frac{d^4 P}{(2\pi)^4} \left[ 4q_2^2 \rho_T(P) + 4 \frac{(n \cdot Q)^2 P^2}{(n \cdot P)^2} \rho_L(P) \right] n_B(p_0)\text{sgn}(q_0 - p_0)\delta[(Q - P)^2] = -\frac{1}{2\pi} C_F g^2 T q [1 + 2\ln\frac{1}{g}].
\]

The damping rate for the hard quark is

\[
\Gamma_q(Q) = -\frac{1}{2g}\text{Im}\left[ \text{Tr}[\mathcal{Q}\Sigma_R(Q)] \right] = \frac{1}{4\pi} C_F g^2 T [1 + 2\ln\frac{1}{g}].
\]

The first term in the bracket comes from the longitudinal contribution of the HTL resumed gluon propagator, and the second term comes from the transverse contribution. In the transverse part there is an IR-cutoff, which results from the magnetic mass of the order \( m_{magn} \sim g^2 T \).

We can express the gluon loop in Fig[3] as

\[
\Pi_{\mu\alpha}^{ab}(Q) = \frac{i}{2} \int \frac{d^4 P}{(2\pi)^4} V_{\mu\nu}(Q, -P, P - Q) iG_{\nu\beta}(P)V_{\beta\gamma\alpha}(P, Q - P, -Q) i\rho_\gamma(P - Q) G(P - Q),
\]

where \( \rho_\gamma(P - Q) \) is the magnetization of the HTL gluon.
where $G^\nu_\beta(P)$ is the HTL resumed gluon propagator in the light cone gauge in Eq.(57), $d^\rho_\gamma(P−Q)G(P−Q)$ is the bare gluon propagator in the light cone gauge, and the three gluon vertexes $V_\mu_\nu_\rho(Q,−P, P−Q)$ and $V_\beta_\gamma_\alpha(P, Q−P, −Q)$ are given in Eq.(16).

Using the relation of the Keldysh representation in (24), we have

$$\Delta_1^{\nu_\beta}(P)\Delta_1^{11}(Q−P) − \Delta_1^{\nu_\beta}(P)\Delta_2^{11}(Q−P) = \frac{1}{2}[\Delta_1^{\nu_\beta}S + \Delta_1^{\nu_\beta}A + \Delta_1^{\nu_\beta}A + \Delta_1^{\nu_\beta}R]$$

(91)

Similarly, by power counting the second term $\Delta_1^{\nu_\beta}A$ gives the leading order contribution $O(1/g^3)$ at finite temperature. $\Delta_1^{\nu_\beta}A$ and $\Delta_1^{\nu_\beta}R$ are both $O(1/g^2)$. The internal momentum $P$ is soft, $p_0 \sim gT$. The Bose-Einstein distribution function $n_B(p_0)$ in the term $\Delta_1^{\nu_\beta}A$ is $O(1/g)$,

$$\frac{1}{e^{\frac{ip_0}{T}} − 1} \sim \frac{T}{|p_0|} \propto \frac{1}{g}$$

(92)

However, the Bose-Einstein distribution function $n_B(p_0 − q_0)$ in the term $\Delta_1^{\nu_\beta}S$ is $O(1)$. The external momentum $Q$ is hard, $q_0 \sim T$,

$$\frac{1}{e^{\frac{ip_0}{T}} − 1} \sim O(1)$$

(93)

The three gluon vertexes are simplified into

$$V_\mu_\nu_\rho(Q,−P, P−Q) \approx gf^{acd}[Q_\rho g_{\mu\nu} + Q_\nu g_{\mu\rho} − 2Q_\mu g_{\rho\nu}]$$

$$V_\beta_\gamma_\alpha(P, Q−P, −Q) \approx gf^{cde}[−Q_\alpha g_{\beta\gamma} + 2Q_\beta g_{\gamma\alpha} − Q_\gamma g_{\alpha\beta}]$$

(94)

where the external momentum $Q$ is hard, and the internal momentum $P$ is soft, which is ignored in the above expression.

The nominator of the bare propagator in the light cone gauge $d^\rho_\gamma(P−Q)$ is

$$d^\rho_\gamma(P−Q) = −g^\rho_\gamma + \frac{n^\rho(P−Q)^\gamma + n^\gamma(P−Q)^\rho}{n \cdot (P−Q)} \approx −g^\rho_\gamma + \frac{n^\rho Q^\gamma + n^\gamma Q^\rho}{n \cdot Q}$$

(95)

In the calculation, we have

$$Q_\rho d^\rho_\gamma(P−Q) = \frac{n^\gamma Q^2}{n \cdot Q} = 0$$

(96)

where the external hard momentum $Q$ is on shell, $Q^2 = 0$. Due to the equation, we can simplify the calculation.
Now we calculate the imaginary part of the transverse part $\Pi_T(Q)$,

\[
\text{Im}\Pi_T(Q) = \frac{1}{2} \left( \delta^{ij} - \frac{q^i q^j}{q^2} \right) \text{Im}\Pi_T^j(Q) \\
= -C_A g^2 \pi^2 \int \frac{d^4P}{(2\pi)^4} \left[ 4q^2 \rho_T(P) + 4 \left( \frac{n \cdot Q}{(n \cdot P)^2} \rho_L(P) \right) n_B(p_0) \text{sgn}(q_0 - p_0) \delta((Q - P)^2) \right] \\
= -\frac{1}{8\pi} C_A g^2 T q [1 + 2 \ln \frac{1}{g}] .
\]

The damping rate for the hard gluon is

\[
\Gamma_g(Q) = -\frac{1}{2q} \text{Im}[\Pi_T(Q)] = \frac{1}{16\pi} C_A g^2 T |1 + 2 \ln \frac{1}{g}| .
\]

The first term in the bracket stems from the longitudinal contribution of the HTL resumed gluon propagator, and the second term stems from the transverse contribution. For the transverse part we take an IR-cutoff too.

We work out the result about the damping rates of the hard quark and gluon in the limit $p_0 \to 0$. Below we can prove the general case that the expression about the transverse and longitudinal projection in the above calculation of the damping rates is the same in the two gauges.

Via this function $\delta((Q - P)^2)$, we have the relation about $\cos \theta$,

\[
\cos \theta \approx \frac{p_0}{p} = \frac{p_0}{|p_3|} ,
\]

where the internal soft momentum $P^\mu = (p_0, 0, 0, p_3)$, the angle $\theta$ is arbitrary.

When $p_3 > 0$, we have

\[
\cos \theta = \frac{q_3}{q} = \frac{q_3}{q_0} ,
\]

where $\theta$ is the angle between $\vec{p}$ and $\vec{q}$.

The expression of the longitudinal part in the calculation of the damping rate is

\[
\frac{(n \cdot Q)^2}{(n \cdot P)^2} P^2 = \frac{(q_0 + q_3)^2}{(p_0 + p_3)^2} P^2 = \frac{(q_0)^2 (1 + \cos \theta)^2}{(p_3)^2 (1 + \cos \theta)^2} P^2 = \frac{(q_0)^2}{(p_3)^2} P^2 = \frac{q_0^2}{p^2} P^2 .
\]

When $p_3 < 0$, we have

\[
\cos \theta = -\frac{q_3}{q} = -\frac{q_3}{q_0} .
\]

The expression of the longitudinal part in the calculation is

\[
\frac{(n \cdot Q)^2}{(n \cdot P)^2} P^2 = \frac{(q_0 + q_3)^2}{(p_0 + p_3)^2} P^2 = \frac{(q_0)^2 (1 - \cos \theta)^2}{(p_3)^2 (1 - \cos \theta)^2} P^2 = \frac{(q_0)^2}{(p_3)^2} P^2 = \frac{q_0^2}{p^2} P^2 .
\]

We can find, in the calculation we have the same expression about the longitudinal projection in the two gauges. The transverse projection tensor of the HTL resumed gluon propagator in the Coulomb gauge is

\[
\delta^{ij} - \frac{p^i p^j}{p^2} .
\]

By calculating, we have

\[
\delta^{3i} - \frac{p^3 p^i}{p^2} = 0 ,
\]

where the internal soft momentum $P^\mu = (p_0, 0, 0, p_3)$. So we can get the same transverse projection tensor in the two gauges.

We have the simple expression about the transverse projection tensor,

\[
\delta^{ij}, \quad (i, j = 1, 2) .
\]

Here we show the transverse and longitudinal projection tensors in the above calculation of the damping rates are the same in the two gauges when the transverse momentum of the internal momentum $P$ is zero. So we can get the same result about the damping rates of the hard quark and gluon in the two gauges.
VI. CONCLUSION

In this paper, we have derived the HTL resumed gluon propagator in the light cone gauge, and presented the results for the transverse and longitudinal gluon HTL self energy in the light cone gauge. We show the quark HTL energy is independent of the light cone gauge, and get the same HTL resumed quark propagator.

Although the longitudinal and transverse expression of the gluon HTL self energy both have the light cone terms with $1/(n \cdot K)$, there is no divergence in the transverse and longitudinal parts, which are different from the case at zero temperature. By calculating, we can find we obtain the same transverse and longitudinal gluon HTL self energies in the light cone gauge and the Coulomb gauge, although in the two gauges we have different transverse and longitudinal projection tensors. We think the axial vector $n^\mu_l = (\sqrt{2}/2, 0, 0, -\sqrt{2}/2)$ have the longitudinal part and is rotated with respect to the axial vector $n^\mu_c = (1, 0, 0, 0)$ in the Coulomb gauge, which brings about changes. Correspondingly, in the static limit, we compare the component $\Pi^{00}(P)$ of the gluon HTL self energy in the light cone gauge with $\Pi^{00}(P)$ in the Coulomb gauge, and find the result in the light cone gauge is the same as that in the Coulomb gauge.

We show the transverse and longitudinal spectral functions of the HTL resumed gluon propagator in the light cone gauge and the transverse and longitudinal dispersion relation. However, in the light cone gauge, the transverse and longitudinal projection tensors are both based on the the axial vector $n^\mu_l$, and they have different expression from the transverse and longitudinal projection tensors in the Coulomb gauge, so the expression of the HTL resumed gluon propagator in the light cone gauge is different from that in the Coulomb gauge.

With the HTL resumed gluon propagator in the light cone gauge, we calculate the damping rates of the hard on shell quark and gluon in a particular limit. We demonstrate in general case, we can get the same result about the damping rates in the two gauges. Although the expression of the the HTL resumed gluon propagator in the light cone gauge is different from that in the Coulomb gauge, we can find it is gauge independent. Using the propagator, we can further consider the correction from the soft process for some physical quantities at high temperature in the Heavy Ion Collisions.

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VIII. APPENDIX

At zero temperature, this kind of the integral in Eq. (49) has divergence, which has been renormalized successfully. However, we find this integral does not have divergence in the HTL approximation.

We use the contour integral to substitute the frequency sum of boson at finite temperature [21],

$$ T \sum_{n=-\infty}^{\infty} f(p_0 = 2n\pi T i) = \frac{1}{2\pi i} \int_{-i\infty}^{i\infty} dp_0 \left[ \frac{1}{2} f(p_0) + f(-p_0) \right] + \frac{1}{2\pi i} \int_{-i\infty + \epsilon}^{i\infty + \epsilon} dp_0 \left[ f(p_0) + f(-p_0) \right] \frac{1}{e^{\beta p_0} - 1}. \quad (107) $$
This equation has the zero temperature part and the finite temperature part, and we only consider the the finite temperature part here.

Below we prove this integral at finite temperature does not have the divergence in Eq. (49),

\[ \frac{1}{2\pi i} \int_{-\infty}^{\infty} dk_0 \int \frac{d^3 k}{(2\pi)^3} \frac{K^2 - 2K \cdot P}{n \cdot K K^2 (K - P)^2} \]  

where \( n \cdot K = n_0 k_0 - \vec{n} \cdot \vec{k} = n(\cos \theta) \), the angle \( \theta \) is the angle between \( \vec{k} \) and \( \vec{p} \), and the axial vector \( n^\mu = (\frac{\vec{n}}{\sqrt{n^2}}, 0, 0, -\frac{\vec{n}}{\sqrt{n^2}}) \).

When \( \cos \theta > 0 \), use the equation of the the integral contour in Eq. (107), we have

\[ \frac{1}{2\pi i} \int_{-\infty}^{\infty} dk_0 \int \frac{d^3 k}{(2\pi)^3} \left\{ \frac{K^2 - 2K \cdot P}{n(\cos \theta)K^2 (K - P)^2} \right\} e^{\beta k_0} \]  

The first term in the bracket has poles at \( k_0 = k \) and \( k_0 = p_0 + |\vec{k} - \vec{p}| \), and the residues of the two poles do not have the term of \( \frac{1}{\cos \theta - 1} \), so there is no divergence at \( \cos \theta = 1 \). The second term has poles at \( k_0 = k, k_0 = k \cos \theta \) and \( k_0 = -p_0 + |\vec{k} - \vec{p}| \). The residues with the two poles of \( k_0 = k \) and \( k_0 = k \cos \theta \) contain the terms of \( \frac{1}{\cos \theta - 1} \), but the sum of the two residues is finite when \( \cos \theta \to 1 \),

\[ \lim_{\cos \theta \to 1} \frac{1}{k^2(\cos^2 \theta - 1)} = \frac{1}{2k^2(\cos \theta + 1)} \]  

When \( \cos \theta < 0 \), we have

\[ \frac{1}{2\pi i} \int_{-\infty}^{\infty} dk_0 \int \frac{d^3 k}{(2\pi)^3} \left\{ \frac{K^2 - 2K \cdot P}{n(\cos \theta)K^2 (K - P)^2} \right\} e^{\beta k_0} \]  

The second term in the bracket has poles at \( k_0 = k \) and \( k_0 = -p_0 + |\vec{k} - \vec{p}| \), and the residues of the two poles do not have the term of \( \frac{1}{\cos \theta - 1} \), so there is no divergence at \( \cos \theta = 1 \). The first term has poles at \( k_0 = k, k_0 = -k \cos \theta \) and \( k_0 = p_0 + |\vec{k} - \vec{p}| \). The residues with the two poles of \( k_0 = k \) and \( k_0 = -k \cos \theta \) contain the terms of \( \frac{1}{\cos \theta + 1} \), but the sum of the two residues is finite when \( \cos \theta \to -1 \),

\[ \lim_{\cos \theta \to -1} \frac{1}{k^2(\cos^2 \theta - 1)} = \frac{1}{2k^2(\cos \theta + 1)} \]  

Combining the situation \( \cos \theta > 0 \) and \( \cos \theta < 0 \), the integral at finite temperature in Eq. (49) does not have divergence. The integral is \( O(\theta^3 T^2) \), which can be ignored.

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