Some issues in a gauge model of unparticles

Yi Liao
Department of Physics, Nankai University, Tianjin 300071, China

Abstract

We address in a recent gauge model of unparticles the issues that are important for consistency of a gauge theory, i.e., unitarity and Ward identity of physical amplitudes. We find that non-integrable singularities arise in physical quantities like cross section and decay rate from gauge interactions of unparticles. We also show that Ward identity is violated due to the lack of a dispersion relation for charged unparticles although the Ward-Takahashi identity for general Green functions is incorporated in the model. A previous observation that the unparticle’s (with scaling dimension $d$) contribution to the gauge boson self-energy is a factor $(2 - d)$ of the particle’s has been extended to the Green function of triple gauge bosons. This $(2 - d)$ rule may be generally true for any point Green functions of gauge bosons. This implies that the model would be trivial even as one that mimics certain dynamical effects on gauge bosons in which unparticles serve as an interpolating field.

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1liaoy@nankai.edu.cn
1 Introduction

As the era of the Large Hadron Collider is approaching, many new theoretical ideas have been contemplated that could potentially be tested there. One of radical suggestions is perhaps that of unparticle by Georgi \cite{1}. Such an object is by definition not a particle, but some stuff that follows scale invariance, though it may well arise from certain high energy scale theory of particles. The scale invariance makes a dispersion relation generally not possible for an unparticle; instead, it determines its kinematics in terms of a parameter, called scaling dimension, of its corresponding field. The very nature of the invariance also implies that the field is generically non-local. The latter results in novel features not seen in the particle world, for instance, non-trivial interference in the time-like regime \cite{2} (see also Ref \cite{3}), one particle to one unparticle transitions \cite{4}, and non-integral power laws of long distance forces between particles mediated by unparticles \cite{5} (see also Ref \cite{6}), etc.

Unparticles must interact with particles to be physically relevant since we manipulate particles in experiments, and the interactions can be systematically organized in effective field theory. Although most studies, both phenomenological and theoretical, cope with bosonic unparticles that couple as a standard model singlet to particles \cite{7}, it is hard to imagine that unparticles must not carry the standard model charges. As a matter of fact, the first gauge model of unparticles has been constructed in Ref \cite{8}. In this circumstance, fermionic unparticles \cite{9, 10, 11, 12, 13} can equally well couple to particles, and their phenomenology could be even more interesting than their bosonic counterparts \cite{12}.

In this work we continue our theoretical investigation on the gauge model of \cite{8} and address some issues that have only been lightly touched upon in \cite{14}. A gauge model of unparticles must pass the standard consistency criteria like unitarity and Ward identities for scattering amplitudes. We make these checks and find the answer is negative for both. This means that the model is not yet amenable to computing physical amplitudes involving unparticles in the initial or final state. In \cite{14}, we also observed that the scalar unparticle contribution to the complete (not just the imaginary part of as shown in \cite{8}) gauge boson self-energy is exactly \((2 - d)\) times that of a scalar particle in the same representation, where \(d\) is the scaling dimension of the unparticle. We extend this to the case of triple gauge bosons. This seems to indicate that this \((2 - d)\) rule is generally true. In that case, the model of \cite{8} would be naive even as one that mimics certain dynamical effects on gauge bosons in which unparticles serve as an interpolating field.

The paper is organized as follows. We describe briefly the gauge model in the next section and catalog the Feynman rules to be employed in later sections. The derivation of the rules is outlined in the appendix. In section \ref{sec:triple} we show explicitly that the \((2 - d)\) rule holds true for the Green function of triple gauge bosons. This is then followed in section \ref{sec:unitarity} by comparative unitarity checks for ungauged and gauged unparticles using the simplest two-point Green functions of particle fields. Although the Green functions in the gauge model fulfil Ward-Takahashi identities, we demonstrate in section \ref{sec:ward} that physical amplitudes involving unparticles and physical gauge bosons do not satisfy Ward identities. We conclude with some remarks in the final section.
2 Gauge model and Feynman rules

The scale symmetry of a scalar unparticle field of scaling dimension \( d \) demands its inverse propagator to be proportional to \((-p^2 - i\epsilon)^{2-d}\), with \( p \) being its momentum [2, 3]. This is a non-integral power for a general real number \( d \geq 1 \), and thus corresponds to a non-local field. The non-locality makes the conventional minimal gauging not work. Fortunately, a similar non-local problem was successfully dealt with some years ago in the context of reproducing low energy Goldstone dynamics from dynamical quarks in QCD [15, 16]. The lesson has been recently applied to gauging unparticle fields in [8].

A scalar unparticle multiplet \( \mathcal{U} \) may be coupled to gauge fields \( A_{\mu}^a \) via the Wilson line. The action is [8]

\[
S = \int d^4x \, d^4y \, \mathcal{U}^\dagger(x) E(x-y) F(x,y),
\]

\[
F(x,y) = P \exp \left[ -i g T^a \int_x^y A_{\mu}^a \, dw^\mu \right] \mathcal{U}(y),
\]

where \( P \) denotes path-ordering that affects on the group generators \( T^a \) in the unparticle representation, and \( g \) is the gauge coupling. \( i^{-1} E(z) \) is the Fourier transform of the inverse propagator:

\[
E(z) = \int \frac{d^4 p}{(2\pi)^4} e^{-ip \cdot z} \tilde{E}(p),
\]

\[
\tilde{E}^{-1}(p) \equiv D(p) = \frac{A(d)}{2 \sin(\pi d)} \frac{1}{(-p^2 - i\epsilon)^{2-d}},
\]

with \( A(d) \) a \( d \)-dependent constant not essential for our purpose. Note that putting an infrared cut-off in the propagator does not modify our subsequent conclusions.

The action (1) contains gauge interactions that are quadratic in unparticle fields but involve gauge fields to an arbitrary order. There is no obstacle to derive their Feynman rules though the procedure rapidly becomes more and more involved as the number of gauge fields increases. Some details of it are given in the appendix. We list below the vertices up to three gauge fields that will be required in later sections.

The Feynman rules for the vertices up to two gauge fields are known in [8]. The \( A_{\mu}^a \mathcal{U} \mathcal{U}^\dagger \) vertex is (with \( ig \) to be attached on both sides)

\[
\Gamma_{\mu}^a(-p-q,p;q) = T^a(2p+q)_{\mu} E_1(p;q),
\]

where momenta before the semicolon are for unparticles and those after it for gauge bosons, with all momenta being incoming. The \( A^a_{\mu} A^b_{\nu} \mathcal{U} \mathcal{U}^\dagger \) vertex is (with \( ig^2 \) to be attached on both sides and differing by a factor of \( i \) from [8])

\[
\Gamma_{\mu\nu}^{ab}(-p-q_{12},p;q_1,q_2) = g_{\mu\nu} \{T^a, T^b\} E_1(p;q_{12}) + T^a T^b (2p+q_2)_\nu (2p+q_2+q_{12})_{\mu} E_2(p;q_{12},q_2) + T^b T^a (2p+q_1)_\mu (2p+q_1+q_{12})_\nu E_2(p;q_{12},q_1),
\]
where \( q_{12} = q_1 + q_2 \), etc, and the following notations are used,

\[
E_1(a;b) = \frac{\tilde{E}(a + b) - \tilde{E}(a)}{(a + b)^2 - a^2},
\]

\[
E_2(a;b_1,b_2) = \frac{E_1(a;b_1) - E_1(a;b_2)}{(a + b_1)^2 - (a + b_2)^2},
\]

\[
E_3(a;b_1,b_2;c) = \frac{E_2(a;b_1,c) - E_2(a;b_2,c)}{(a + b_1)^2 - (a + b_2)^2}.
\]

The notations are slightly improved over those in \([14]\) to better display symmetry.

Finally, the \( A^a A^b A^c \gamma \mathcal{U} \mathcal{U}^\dagger \) vertex is (with \( ig^3 \) to be attached on both sides)

\[
\Gamma^{abc}_{\alpha\beta\gamma}(-p - q_{123}, p; q_1, q_2, q_3)
= T^c \{ T^a, T^b \} g_{\alpha\beta} (q_{123} + q_{12} + 2p) \gamma E_2(p; q_{123}, q_{12})
+ \{ T^a, T^b \} T^c g_{\alpha\beta} (q_3 + 2p) \gamma E_2(p; q_{123}, q_3)
+ T^a T^b T^c (q_{123} + q_{23} + 2p) \alpha (q_{23} + q_3 + 2p) \beta (q_3 + 2p) \gamma E_3(p; q_{123}, q_{23}; q_3)
+ T^b T^a T^c (q_{123} + q_{31} + 2p) \beta (q_{31} + q_3 + 2p) \alpha (q_3 + 2p) \gamma E_3(p; q_{123}, q_{31}; q_3)
+ 2 \text{ perms}. \tag{8}
\]

3 \((2 - d)\) rule for triple gauge bosons

When the action in \((1)\) is exponentiated and integrated in the path integral over the unparticle fields, we obtain an effective action of the gauge fields. It is in this sense that the gauge model discussed here is parallel to the non-local chiral quark model in low energy hadronic physics \([15, 16]\). In the latter case, integration over chiral quarks with a momentum-dependent dynamical mass results in an effective action for the Goldstone bosons and external sources or gauge fields. The chiral quark fields serve as an interpolating field that mimics the strong dynamics of QCD as manifested in the low energy constants in chiral Lagrangian. It therefore sounds reasonable to expect that the unparticle fields in the gauge model should at least play a similar role in the context of certain new strong dynamics at a high energy scale.

Figure 1. Diagrams contributing to triple gauge boson function.

In the previous work \([14]\), we observed that the unparticle contribution to the gauge boson self-energy follows a simple rule; namely, it is \((2 - d)\) times the contribution from scalar particles in the same representation. Its possible impact on the running and unification of gauge
couplings was also studied. In this section we examine whether the rule applies to the Green function of triple gauge bosons which is kinematically more varied than a self-energy. If it does apply, it would unlikely be accidental but might be generally true.

The contributing diagrams to the function are shown in Fig. 1, in which the double dashed line stands for the scalar unparticle and the three gauge bosons carry the group \((a, b, c)\) and Lorentz indices \((\alpha, \beta, \gamma)\) with the incoming momenta \(q_i\). The vertices in section 2 yield

\[
\mathcal{G}^{abc:1}_{\alpha \beta \gamma} = ig^3 \text{tr} \int \frac{d^4 p}{(2\pi)^4} \Gamma_\alpha (-p + q_2, p + q_3; q_1) \Gamma_\alpha^\gamma (-p - q_3, p; q_3) \Gamma_\beta (-p, p - q_2; q_2) \\
\times D(p)D(p + q_3)D(p - q_2) + (q_2, \beta, b) \leftrightarrow (q_3, \gamma, c),
\]

\[
\mathcal{G}^{abc:2}_{\alpha \beta \gamma} = -ig^3 \text{tr} \int \frac{d^4 p}{(2\pi)^4} \Gamma_\alpha (-p - q_1, p; q_1) \Gamma_\beta^c (-p, p + q_1; q_2, q_3) D(p)D(p + q_1) \\
+ 2 \text{ perms},
\]

\[
\mathcal{G}^{abc:3}_{\alpha \beta \gamma} = ig^3 \text{tr} \int \frac{d^4 p}{(2\pi)^4} \Gamma_\alpha^{abc} (-p, p; q_1, q_2, q_3) D(p).
\]

(9)

The integrals can be defined in \(n\) dimensions for regularization, but our subsequent algebraic manipulation does not depend on it. We stress again that introducing an infrared cut-off to the propagator does not modify our discussion either. The particle case can be recovered in the limit \(d \to 1\) whence \(E_1 \to 1\): The graph (3) vanishes identically and the graph (2) vanishes due to symmetry, while the two terms in graph (1) combine to

\[
\left[ \mathcal{G}^{abc}_{\alpha \beta \gamma} \right]_{\text{particle}} = g^3 f^{abc} C(r) \int \frac{d^4 p}{(2\pi)^4} \frac{2p + q_3 - q_2}{p^2 + i\epsilon} \frac{\alpha(2p - q_2)\beta(2p + q_3)\gamma}{(p + q_3)^2 + i\epsilon},
\]

(10)

where \(\text{tr}T^aT^b = C(r)\delta^{ab}\) for particles in representation \(r\).

In the unparticle case, the graph (3) contains two classes of terms. The integrand in the first (class I) is proportional to

\[
E_1(p; 0) = \lim_{q \to 0} E_1(p; q) = \frac{2 - d}{p^2 + i\epsilon} \frac{1}{D(p)},
\]

(11)

while the remaining terms constitute the class II whose integrand is not proportional to \(E_1(p; 0)\). We demonstrate that the class II terms in graph (3) cancel completely the graphs (1) and (2). First, the terms in class II that involve a single signature tensor cancel in pair upon taking the traces and doing integration, so do the similar terms in graph (2). Second, by inspection, the remaining terms in II and graph (2) can be combined in pair. Choosing judiciously the routing momenta and making use of identities of \(E_1\), we found that those terms condense to a sum of two ‘form factors’ with the common Lorentz structure, \((2p - q_2 + q_3)\alpha(2p + q_3)\gamma(2p - q_2)\beta\). This is the same Lorentz structure exactly for the first term in \(\mathcal{G}^{abc:1}_{\alpha \beta \gamma}\) and up to a minus sign for the second upon flipping the sign of \(p\). The first form factor is (with the prefactors \(ig^3\) understood),

\[
\text{tr}T^aT^bT^c \sum_{(ijk)} \frac{d_j}{n_j - n_i} \left[ d_k e_i (e_i - e_j) + \frac{e_i}{n_j - n_k} + \frac{e_k - e_i}{n_i - n_j} \right],
\]

(12)
where the sum is over the set (123), (231), (312). We have introduced some abbreviations:

\[
\begin{align*}
n_1 &= p^2, \quad n_2 = (p - q_2)^2, \quad n_3 = (p + q_3)^2; \\
d_1 &= D(p), \quad d_2 = D(p - q_2), \quad d_3 = D(p + q_3); \\
e_i &= \frac{d_{i \rightarrow 1} - d_{k \rightarrow 1}}{n_j - n_k},
\end{align*}
\]  

(13)

where \( ijk \) is again cyclic in 123. The second form factor is obtained from the first by the interchanges of indices, \( b \leftrightarrow c \) and \( 2 \leftrightarrow 3 \), but with a global minus sign. Third, using identities of fractions, it is straightforward to show that the sum in eq (12) is equal to \( d_1 d_2 d_3 e_1 e_2 e_3 \). This is again the integrand in the second term of \( \mathcal{A}_{\alpha\beta\gamma}^{abc1} \) upon extracting the Lorentz structure displayed above so that the sum in (12) (i.e., the first form factor) completely cancels the second term in \( \mathcal{A}_{\alpha\beta\gamma}^{abc1} \). Similar cancellation occurs between the second form factor and the first term in eq (12). To summarize, the complete graphs (1) and (2) are cancelled by class II terms in graph (3).

Now we are left with class I terms in graph (3). First of all, eqs (11,9) imply that the integrand is of a particle type. The terms with a signature tensor are again cancelled in pair. Using the fraction identity, \( \sum_{(ijk)} [n_i (n_i - n_j) (n_i - n_k)]^{-1} = [n_1 n_2 n_3]^{-1} \), some algebra verifies our claim:

\[
\left[ \mathcal{A}_{\alpha\beta\gamma}^{abc} \right]_{\text{unparticle}} = (2 - d) \left[ \mathcal{A}_{\alpha\beta\gamma}^{abc} \right]_{\text{particle}}.
\]  

(14)

We end this section with a remark. It seems unlikely that the above relation is specific to two- and three-point functions of gauge fields. Our explicit demonstration of it might suggest a way to reach the general result for any point functions: The cancellation mechanism witnessed in two- and three-point functions might indicate that the only contribution for any point function comes exclusively from the tad-pole like graph involving the highest point vertex in each case.

## 4 Unitarity

Unitarity of the scattering matrix is one of the fundamental criteria that any quantum theory must meet. This is especially true of a gauge theory in which additional delicacies may occur. The purpose of this section is to show using the simplest possible process that the gauge model of unparticles proposed in [8] breaks unitarity. For comparison, we also examine unitarity in non-gauge interactions between unparticles and particles, and we find that these interactions generally preserve unitarity in the conventional sense.

Consider the one-loop self-energy of a scalar particle field arising from interactions with scalar unparticles. To the graphs (1) and (2) shown in Fig. 2 there correspond the two effective interactions:

\[
\mathcal{L}_1 = \lambda_1 \Phi \Phi \mathcal{U}, \quad \mathcal{L}_2 = \lambda_2 \Phi \mathcal{U}_1 \mathcal{U}_2,
\]  

(15)  

(16)
where $\Phi$, $\phi$ are the scalar particle fields of mass $M$, $m$, and $\mathcal{U}$, $\mathcal{U}_1$, $\mathcal{U}_2$ the scalar unparticle fields of scaling dimension $d$, $d_1$, $d_2$, respectively.

\[
\begin{split}
\text{Figure 2. Self-energy of scalar particle field arising from eqs (15,16).}
\end{split}
\]

The imaginary part of the self-energy in graph (2) is found to be, for $M > m$,

\[
\begin{align*}
\text{Im } \mathcal{A}_1(M^2) &= \frac{\lambda^2 M^{2(d-1)}}{(4\pi)^2} \frac{A(d)}{2(d-1)} \int_{r^2}^1 dx \frac{x^{1-d}(1-x)^{d-1}(x-r^2)^{d-1}}{r^2} \\
&= \frac{\lambda^2 M^{2(d-1)}}{(4\pi)^2} \frac{A(d)}{2(d-1)} (1-r^2)^{2d-1} _2F_1 (d-1, d; 2d; 1-r^2) B(d, d), \quad (17)
\end{align*}
\]

where $r = m/M$ and $_2F_1$ and $B$ are the standard special functions. This should be compared to the decay width for $\Phi \rightarrow \phi + \mathcal{U}$ for unitarity check. Finishing all phase space integrals but that of the unparticle energy yields

\[
\begin{align*}
\Gamma_1 &= \frac{\lambda^2 M^{2(d-1)}}{(2\pi)^2} \frac{A(d)}{2M} (1-r)^d \int_{\frac{1}{2}(1+r)}^1 dt \left(1-t\right) \left(\frac{1+r}{1-r} - t\right) \frac{1}{2} \left[t - \frac{1}{2}(1+r)\right]^{d-2} \quad (18)
\end{align*}
\]

Changing the variable to $u = \left[t - \frac{1}{2}(1+r)\right]/\left[\frac{1}{2}(1-r)\right]$ works out the integral to

\[
\begin{align*}
\Gamma_1 &= \frac{\lambda^2 M^{2(d-1)}}{(4\pi)^2} \frac{A(d)}{2M} (1-r)^{2d-1} (1+r) \\
&\times _2F_1 \left(\frac{1}{2}, d-1; d+1; \frac{1-r}{1+r}\right) B\left(d-1, \frac{3}{2}\right). \quad (19)
\end{align*}
\]

The unitarity relation $\text{Im } \mathcal{A}_1(M^2) = M\Gamma_1$ is verified using the relation

\[
F\left(d-1, d; 2d; \frac{4z}{(1+z)^2}\right) = 2^{(d-1)(1+r)^{-2(d-1)}} F\left(-\frac{1}{2}, d-1; d+1; \frac{1-r}{1+r}\right), \quad (20)
\]

and a relation for $B$ function.

The interaction $\mathcal{L}_2$ involves two unparticles and has been less discussed in the literature. Its contribution to the imaginary part of graph (2) is easily worked out to be

\[
\begin{align*}
\text{Im } \mathcal{A}_2(M^2) &= -\frac{\lambda^2 M^{2(d_1+d_2-2)}}{4(4\pi)^2} \frac{A(d_1)A(d_2)}{M^{2(d_1+d_2-2)}} \\
&\times \frac{\Gamma(2-d_1-d_2)}{\Gamma(2-d_1)\Gamma(2-d_2)} B(d_1, d_2) \frac{\sin(d_1+d_2)\pi}{\sin(d_1\pi)\sin(d_2\pi)}. \quad (21)
\end{align*}
\]
The phase space for the decay $\Phi \rightarrow \mathcal{U}_1 \mathcal{U}_2$ is more involved. Finishing integrals of one unparticle momentum and the angles of the other, we obtain

$$\Gamma_2 = \frac{\lambda^2}{(2\pi)^3M} A(d_1)A(d_2)M^{2(d_1+d_2-2)} I_{d_1-2,d_2-2},$$

where

$$I_{\alpha,\beta} = \int\int_R dv_0 dv (v_0^2 v^2)^\alpha [(1-v_0)^2-v^2]^{\beta}. \tag{23}$$

Here the integration region $R$ in the $v_0v$ plane is bounded by the lines, $v = 0$, $v = v_0$, and $v + v_0 = 1$. As the integrand is even in $v$, we make the region symmetric under $v \rightarrow -v$. The integrals are then factorized by the new variables, $v_0 - v = x$, $v_0 + v = y$ with $x \in [0,1]$ and $y \in [0,1]$ so that, for $\alpha > -1$, $\beta > -1$ (i.e., $d_{1,2} > 1$ in our case),

$$I_{\alpha,\beta} = 2^{-d} \int_0^1 dx \int_0^1 dy ((y-x)^2 (xy)^\alpha [(1-y)(1-x)]^{\beta}$$

$$= 2^{-d-3} \left[ B(\alpha + 3, \beta + 1) B(\alpha + 1, \beta + 1) - (B(\alpha + 2, \beta + 1))^2 \right]$$

$$= \frac{\Gamma(\alpha + 1) \Gamma(\alpha + 2) \Gamma(\beta + 1) \Gamma(\beta + 2)}{8 \Gamma(\alpha + \beta + 3) \Gamma(\alpha + \beta + 4)}.$$ \tag{24}

The unitarity relation $M\Gamma_2 = \text{Im } \mathcal{M}_2(M^2)$ is confirmed using $\Gamma(z)\Gamma(1-z) \sin(z\pi) = \pi$.

It is not surprising that unitarity is preserved by non-gauge interactions of unparticles because the unparticle propagator has the correct cut structure by construction \[2\] and because those interactions are Hermitian. As we pointed out in \[14\], the gauge interactions of unparticles are Hermitian. As we pointed out in \[14\], the gauge interactions of unparticles in \[8\] are actually non-Hermitian in the time-like regime. This may be a source of unitarity violation in the model. In Ref \[14\], we reached this conclusion by symmetry analysis for the process, $q\bar{q} \rightarrow \mathcal{U} \mathcal{U}$ via gluon exchange assuming $\mathcal{U}$ is charged under QCD. In what follows, we demonstrate the violation analytically by the simplest possible process of the gauge boson decay, $A^a_\mu(p) \rightarrow \mathcal{U}(k_1)\mathcal{U}(k_2)$, with the gauge boson momentum $p$ in the time-like regime.

The imaginary part of the gauge boson self-energy from the unparticle loop can easily be obtained from that of the scalar particle’s using the $(2-d)$ rule:

$$\Pi^{ab\mu\nu}_\mu(p) = \delta^{ab} \left( \frac{p_\mu p_\nu}{p^2} - g_{\mu\nu} \right) \Pi(p^2),$$

$$\text{Im } \Pi(p^2) = (2-d) \frac{g^2}{48\pi} C(r) p^2.$$ \tag{25}

The amplitude for the decay is basically the vertex shown in eq (5). The properly summed and averaged decay rate is

$$\Gamma = \frac{2}{3\pi^3} g^2 C(r) \sqrt{p^2} \cos^2(d\pi)J(d),$$

where

$$J(d) = \int\int_R dv_0 dv (1-2v_0)^2 \left[ \left( \frac{v_0^2 - v^2}{(1-v_0)^2-v^2} \right)^{2-d} + \left( \frac{(1-v_0)^2 - v^2}{v_0^2 - v^2} \right)^{2-d} - 2 \right]. \tag{27}$$
with the same region $R$ as in $\Gamma_2$ above. The terms in the square brackets arise from combination of phase space factors and the numerator of $E_1$ in the vertex. The potential singularity at $v_0 = \frac{1}{2}$ is due to the denominator in $E_1$, and makes it impossible to factorize the integral into two separate ones as we did in $I_{\alpha, \beta}$.

To observe the non-integrability that the singularity may cause, we finish the $v$ integral first. Using the symmetry with respect to $v_0 = \frac{1}{2}$, we restrict ourselves to the left half of $R$ and obtain

$$J(d) = \frac{1}{2} \int_0^1 dt \frac{t^2}{(1 + \sqrt{t})^3(1 - t)^2} f(d - 2; t),$$

where

$$f(a; t) = t^a B \left( \frac{5}{2}, 1 + a \right) \, _2F_1 \left( a, \frac{5}{2}, \frac{7}{2}; a + t \right) + t^{-a} B \left( \frac{5}{2}, 1 - a \right) \, _2F_1 \left( -a, \frac{5}{2}, \frac{7}{2} - a; t \right) - \frac{4}{5}.$$  

Note that $J(d)$ is even in $(2 - d)$ (so is $\Gamma$) while $\text{Im} \, \Pi(p^2)$ is odd. This is the argument employed in [14] to signify unitarity breakdown. But what really occurs is even worse: The singularity introduced by the vertex in eq (5) is logarithmically non-integrable. To see this, one needs to expand $f(a; 1 - z)$ at $z = 0$:

$$f(a; 1 - z) = a \left[ \psi(1 + a) - \psi(1 - a) \right] z + \left( \frac{3}{2} a^2 \ln z + a^2 [C - \psi(1 - a) - \psi(1 + a)] \right. $$

$$\left. - \frac{7}{4} a(1 - a) \psi(2 - a) + \frac{7}{4} a(a + 1) \psi(2 + a) \right) z^2 + O(z^3),$$

where $\psi(\xi) = \Gamma'(\xi)/\Gamma(\xi)$ and $C$ is a constant. The unitarity is thus badly violated in this case by non-integrable singularities introduced in interaction vertices of the gauge model. Note in passing that there is no problem with the particle limit of $d \to 1$ although it is better to take the limit at the very start to avoid the ambiguity between the $\sin^2(d\pi)$ factor in $\Gamma$ and the singularity. We stress that this breakdown of unitarity occurs at all energy scales in the gauge model of unparticles, in contrast to the conventional effective field theory in which unitarity starts to be violated at energy scales close to its ultraviolet cut-off. This also implies that unitarity cannot be simply recovered by including new degrees of freedom in the gauge model as we do in a conventional effective theory.

### 5 Ward identity

Now we address the issue of Ward identity necessary for a consistent gauge theory. We do not expect any problem with Ward-Takahashi identity for generally off-shell Green functions as it is built in by construction of the gauge model. As examples, we list below the first few identities for Green functions involving up to three gauge bosons.
The simplest one is
\[
q^{\mu} \Gamma^{a}_{\mu}(-p-q,p;q) = T^{a}[D^{-1}(p+q) - D^{-1}(p)],
\]
while the next one requires a little rearrangement of terms,
\[
q^{\mu} \Gamma^{ab}_{\mu}(p-q_{12},p;q_{1},q_{2}) \\
= \Gamma^{b}_{\nu}(-p-q_{12},p+q_{1};q_{2}) T^{a} - T^{a} \Gamma^{b}_{\nu}(-p-q_{2},p;q_{2}) \\
+ i f^{abc} \Gamma^{c}_{\nu}(-p-q_{12},p;q_{12}).
\]

The derivation of identity for the triple gauge boson vertex is much more involved. The main trick is to use partial fraction. But when the cloud clears up, the answer is simple:
\[
q^{\alpha\Gamma^{abc}}_{\alpha\beta\gamma}(-p-q_{123},p;q_{1},q_{2},q_{3}) \\
= \Gamma^{bc}_{\beta\gamma}(-p-q_{123},p+q_{1};q_{2},q_{3}) T^{a} - T^{a} \Gamma^{bc}_{\beta\gamma}(-p-q_{23},p;q_{2},q_{3}) \\
+ i f^{abc} \Gamma^{d}_{\beta\gamma}(-p-q_{123},p;q_{12},q_{3}) - i f^{cab} \Gamma^{d}_{\beta\gamma}(-p-q_{123},p;q_{2},q_{31}).
\]

In the conventional gauge theory of particles, the Ward identity for physical amplitudes is obtained from the Ward-Takahashi identity by going to the physical limit of charged particles. In an Abelian theory like QED it is sufficient to require electrons to be on-shell. But in a non-Abelian theory like QCD, it is necessary that gluons be physical as well since they are also charged. In a gauge theory of unparticles however, an on-shell condition (dispersion relation) is missing for unparticles; this may endanger the Ward identity for physical amplitudes. If this happens, unphysical states of gauge bosons can be produced by unparticles, which is of course not acceptable. We show below by a simple process that this happens indeed in the considered model.

Consider the process of unparticle pair production by the fusion of a gauge boson pair, \(A_{\alpha}^{a}(k_{1})A_{\beta}^{b}(k_{2}) \rightarrow \not{q}_{U}(p_{1})\overline{\not{q}}_{U}(p_{2})\), whose Feynman diagrams are depicted in Fig. 3. Putting the gauge bosons on-shell, \(k_{1}^{2} = k_{2}^{2} = 0\), the once contracted component amplitudes are
\[
\begin{align*}
 k_{1}^{\alpha\beta} & = - [T^{a}, T^{b}] \left[k_{1\beta}(p_{1}^{2} - p_{2}^{2} ) + k_{2\beta}k_{1} \cdot (p_{1} - p_{2}) + (p_{2} - p_{1})\beta 2k_{1} \cdot k_{2} \right] \\
 & \times \frac{E_{1}(-p_{1};p_{1}+p_{2})}{2k_{1} \cdot k_{2}}, \\
 k_{1}^{\alpha\beta} & = \{T^{a}, T^{b}\} k_{1\beta} E_{1}(-p_{1};p_{1}+p_{2}) \\
 & + T^{a} T^{b}(-2p_{1} + k_{2})_{\beta} \left[E_{1}(-p_{1};p_{1}+p_{2}) - E_{1}(-p_{1};k_{2}) \right] \\
 & + T^{b} T^{a}(-2p_{1} + 2k_{1} + k_{2})_{\beta} \left[E_{1}(-p_{1};p_{1}+p_{2}) - E_{1}(-p_{1};k_{1}) \right], \\
 k_{1}^{\alpha\beta} & = - T^{b} T^{a}(2p_{2} - k_{2}) k_{1\beta} \left[1 - \frac{D(p_{2} - k_{2})}{D(p_{2})} \right] E_{1}(-p_{1};k_{1}), \\
 k_{1}^{\alpha\beta} & = T^{a} T^{b} \left[1 - \frac{D(p_{2} - k_{1})}{D(p_{2})} \right] (2p_{1} + k_{2})\beta E_{1}(-p_{1};k_{2}),
\end{align*}
\]
where a $g^2$ factor is implied on the right hand side. The particle case is recovered by sending $D^{-1}(q) \rightarrow q^2$ and $E_1 \rightarrow 1$. As the above result seems hopeless, we examine the simpler, less restrictive, doubly contracted amplitude. The sum is

$$k_1^\alpha k_2^\beta \mathcal{A}_{\alpha \beta} = \frac{T^a T^b}{D(p_1)D(p_2)} \left( D(k_1 - p_2) - \frac{1}{2} [D(p_1) + D(p_2)] \right)$$

$$+ \frac{T^b T^a}{D(p_1)D(p_2)} \left( D(k_2 - p_2) - \frac{1}{2} [D(p_1) + D(p_2)] \right),$$

(35)

which does not vanish as the Ward identity requires. The situation does not improve in the Abelian case.

A simple way out seems to require $D^{-1}(p) = 0$ for unparticles appearing in the initial and final states. While the meaning of this is obscure by itself, it implies a dispersion relation for unparticles with $d < 2$. But this is obviously not a consistent concept as there would be no difference between a physical particle and unparticle. Furthermore, it would break unitarity established for unparticles that have no gauge interactions.

6 Conclusion

It looks natural that unparticles are charged under the standard model gauge group. But it is rather difficult to couple them to gauge fields since they are generically nonlocal in nature. Nevertheless, the first gauge model has been attempted in Ref [8] (for recent discussions about the work and development, see [17, 18, 19]). We have investigated in this work whether the model fulfils the standard requirements that a consistent gauge theory must do; namely, the unitarity and the Ward identity for physical amplitudes involving unparticles. We find that the answers to both are negative.

In non-gauge interactions no surprise is expected for unitarity as the unparticle propagator incorporates correct analyticity properties and the interactions are usually trivial in analyticity structure. The latter is no longer the case in the gauge model considered here. We find that its gauge interactions introduce non-integrable singularities in phase space that make physical quantities like cross section meaningless. This failure in unitarity occurs at any energy scale and thus cannot be cured by incorporating new degrees of freedom as one does in a conventional effective field theory. The result on Ward identity may be surprising at first sight since the
Ward-Takahashi identity for general Green functions has been built in the model. The point here is that to obtain the Ward identity for physical amplitudes some physical conditions have to be imposed to delete the contact terms in the Ward-Takahashi identity. These are the on-shell condition for charged particles and the transversality condition for gauge bosons if charged. It is the lack of a dispersion relation for charged unparticles that the passage is blocked. These two defects might easily be blamed upon the wisdom of conformal field theory that no consistent scattering matrix is known. But this does not explain why we seem to obtain reasonable results with non-gauge interactions of unparticles by following the standard procedure in field theory.

Even if a gauge model of unparticles is afflicted with these diseases, it could still serve as a useful tool to mimic certain strong dynamical effects on gauge bosons at low energies. In that case, unparticles appear as an interpolating field confined to the virtual loops of gauge bosons. This situation is parallel to the relationship between the nonlocal chiral quark model \[15,16\] and the low energy dynamics of Goldstone bosons. Because of this, we have considered the Green function of triple gauge bosons due to unparticle loops and found that an earlier observation on the self-energy of gauge bosons also applies here. Namely, the unparticle contribution to the Green function is a factor $(2 - d)$ of the scalar particle's in the same representation of the gauge group. We conjecture that this $(2 - d)$ rule may be generally true. If this were the case, the model would be too naive even if the unparticles are considered as an interpolating field. It looks fair to say that the challenge of gauging unparticles still remains.

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**Appendix. Derivation of Feynman rules**

We outline the derivation of the vertices in eqs (5, 6, 8) for completeness. The basic technique was developed in Ref \[15\]. The vertices with up to two gauge bosons were known previously \[15,8\] for which our manipulation of series is slightly simpler, while the vertex with three gauge bosons is not yet available in the literature. Introducing Fourier transforms of the functions

\[
\mathcal{U}(x) = \int (dk)e^{-ik\cdot x} \tilde{U}(k),
\]

\[
F(x,y) = \int (dq_1)(dq_2)e^{-i(q_1\cdot x + q_2\cdot y)} \tilde{F}(q_1, q_2),
\]

and assuming $\tilde{E}(p)$ is a function of $p^2$ (which is the case here), expansion at $p^2 = 0$ yields for the action in eq (1):

\[
S = \sum_{n=0} \frac{\tilde{E}^{(n)}(0)}{n!} \int (dk)(dp)\tilde{U}^\dagger(k) (p^2)^n \tilde{F}(k - p, p)
\]

\[
= \sum_{n=0} \frac{\tilde{E}^{(n)}(0)}{n!} \int (dk)(dp) \int dxdy e^{i[-k\cdot x + (k-p)\cdot z + p\cdot y]} \mathcal{U}^\dagger(x) (p^2)^n F(z, y). 
\]
Replacing $e^{ip\gamma}(p^2)^n = \left(-\partial^2\right)^n e^{ip\gamma}$ and doing integration by parts, the action becomes

$$S = \sum_{n=0}^{\infty} \frac{\tilde{E}^{(n)}(0)}{n!} \int dx dy \, \delta(x-y) \mathcal{H}^{\dagger}(x) \left(-\partial^2\right)^n \left[ P \exp \left(-igT^a \int_x^y A^a_{\mu} \, dw^\mu \right) \mathcal{H}(y) \right], \quad (38)$$

which is the starting point to all vertices. We have used the following abbreviations:

$$(dk) = \frac{d^4k}{(2\pi)^4}, \, dx = d^4x, \, \delta(x) = \delta^4(x),$$

and the derivatives always refer to $y$ unless otherwise stated.

We begin with the derivation of eq (5). The vertex in coordinate space is

$$\delta^3 S \left|_{\delta A^a_\mu(x_1)\delta \mathcal{H}(y)\delta \mathcal{H}(z)} \right. = \sum_{n=0}^{\infty} \frac{\tilde{E}^{(n)}(0)}{n!} \int dy \, \delta(y-z) \left(-\partial^2\right)^n [(-ig)L^a_1 \delta(y-v)], \quad (40)$$

where from now on the following short-cuts will be used,

$$L^a_{i\alpha} = T^a \int_z^y \delta(x_i-u)du^\alpha, \, \delta_i = \delta(x_i-y).$$

Its Fourier transform yields

$$\int dx_1dydz \, e^{ip'z-py-qx_1} = \sum_{n=0}^{\infty} \frac{\tilde{E}^{(n)}(0)}{n!} \int dx_1dydz \, e^{ip'z-py-qx_1} \left[ \left(-\partial^2\right)^n \delta(z-y) \right] [-igL^a_{1\mu}]. \quad (42)$$

The basic trick here is integration by parts. The $n = 0$ term vanishes, while the $n = 1$ term gives, using $\partial^y L^a_{1\mu} = T^a \delta_1 g^{\mu\nu}$,

$$(n = 1 \text{ term}) = E^{(1)}(0)gT^a (p+p')^\mu (2\pi)^4 \delta(p'-p-q). \quad (43)$$

The general term is obtained by induction. Assuming

$$(n-\text{th term}) = \frac{\tilde{E}^{(n)}(0)}{n!}gT^a (p+p')^\mu f_n (2\pi)^4 \delta(p'-p-q), \quad (44)$$

with $f_0 = 0$, $f_1 = 1$, the coefficient for the next term is found to be

$$f_{n+1} = p^2 f_n + (p+q)^2 n, \quad (45)$$

with $k^{2n} \equiv (k^2)^n$; namely,

$$\frac{f_{n+1}}{(p+q)^{2n}} = 1 + r \frac{f_n}{(p+q)^{2(n-1)}} = 1 + r + \cdots + r^n = \frac{1 - r^{n+1}}{1 - r}, \quad r \equiv \frac{p^2}{(p+q)^2}. \quad (46)$$
The general coefficient is thus

$$f_n = \frac{(p+q)^{2n} - p^{2n}}{(p+q)^2 - p^2}.$$  \hfill (47)

Using

$$\sum_n \tilde{E}^{(n)}(0) \frac{(p+q)^{2n} - p^{2n}}{(p+q)^2 - p^2} = \tilde{E}(p + q) - \tilde{E}(p) \frac{(p+q)^2 - p^2}{(p+q)^2 - p^2},$$  \hfill (48)

eq \text{(5)} \text{ is obtained.}

Now we derive the vertex in eq \text{(5)}:

$$\int dx_1 dx_2 dv dz \, e^{i[p'z-\nu-y-q_1 x_1-q_2 x_2]} \frac{\delta^4 S}{\delta A^a_\mu(x_1) \delta A^b_\nu(x_2) \delta \mathcal{V}(v) \delta \mathcal{V}^+(z)} \bigg|_0$$

$$= -g^2 \sum_{n=0} \tilde{E}^{(n)}(0) \frac{n!}{n!} \int dx_1 dx_2 dv dz \, e^{i[p'z-\nu-\Sigma q_i x_i]} \left[ (-\partial^2)^n \delta(y-z) \right] P \left[ L_1^{a\mu} L_2^{b\nu} \right].$$ \hfill (49)

The \( n = 0 \) term again vanishes, while the \( n = 1 \) term gives

\( (n = 1) \text{ term} = g_1 \tilde{E}^{(1)}(0) g^{\mu\nu} \{ T^a, T^b \} (2\pi)^4 \delta(p' - p - q_{12}), \)

with \( q_{12} = q_1 + q_2 \). Since not all possible Lorentz structures have appeared, we have to go one step further. The \( n = 2 \) term yields after some algebra

\( (n = 2) \text{ term} = \frac{g^2 \tilde{E}^{(2)}(0)}{2!} (2\pi)^4 \delta(p' - p - q_{12}) \left\{ g^{\mu\nu} \{ T^a, T^b \} (p'^2 + p^2) \right. \)

$$+ T^b T^d (2p + q_1)^\mu (2p + q_1 + q_{12})^\nu$$

$$+ T^d T^b (2p + q_2)^\nu (q_2 + q_{12})^\mu \right\}.$$ \hfill (50)

Assuming the coefficients for the \( \{ T^a, T^b \} \), \( T^b T^a \) and \( T^a T^b \) terms to be \( g_n(p, q_1, q_2) \), \( h_n(p, q_1, q_2) \) and \( h_n(p, q_2, q_1) \) respectively, with the initial conditions:

\( g_0 = 0, \ g_1 = 1, \ g_2 = p'^2 + p^2; \ h_0 = h_1 = 0, \ h_2 = 1, \)

we find out their \( (n+1) \)-th expressions after some work:

\( g_{n+1}(p, q_1, q_2) = p^2 g_n(p, q_1, q_2) + (p + q_{12})^{2n}, \)

\( h_{n+1}(p, q_2, q_1) = p^2 h_n(p, q_2, q_1) + \frac{(p + q_{12})^{2n} - (p + q_{12})^{2n}}{(p + q_{12})^2 - (p + q_{12})^2}. \) \hfill (52)

\( g_n \) has the same structure as \( f_n \) while \( h_n \), when multiplied by \( [(p + q_{12})^2 - (p + q_{2})^2] \), becomes a difference of two series each having the same structure as \( f_n \) again. We thus find

\( g_n(p, q_1, q_2) = \frac{(p + q_{12})^{2n} - p^{2n}}{(p + q_{12})^2 - p^2}, \)

\( h_n(p, q_2, q_1) = \frac{1}{(p + q_{12})^2 - (p + q_2)^2} \left[ \frac{(p + q_{12})^{2n} - p^{2n}}{(p + q_{12})^2 - p^2} - \frac{(p + q_{2})^{2n} - p^{2n}}{(p + q_{2})^2 - p^2} \right]. \) \hfill (53)
Then eq (6) obtains readily.

Finally, we describe briefly the derivation of the vertex with three gauge bosons. The algebra blows up rapidly as the number of gauge bosons increases. The vertex to compute is

\[ \int \Pi dx_i dy dz \ e^{i [p' z - p v - \Sigma q_i x_i]} \frac{\delta^3 S}{\delta A^a_\alpha(x_1) \delta A^b_\beta(x_2) \delta A^c_\gamma(x_3) \delta \ Σ \delta \ Σ \delta \ Σ (z)} \bigg|_0 \]

\[ = i g^3 \sum_{n=0}^{\infty} \frac{\dot{E}^{(n)}(0)}{n!} \int \Pi dx_i dy dz \ e^{i [p' z - \Sigma q_i x_i]} \delta (y - z) \left\{ e^{-i p y} P[L^a_1 L^b_2 L^c_3] \right\}. \tag{54} \]

Both \( n = 0 \) and \( n = 1 \) terms vanish. For \( n = 2 \) term, we use

\[ (-\partial^2)^2 \left\{ e^{-i p y} P[L^a_1 L^b_2 L^c_3] \right\} = e^{-i p y} \left[ (p^2)^2 - 4 p \sigma p \partial \sigma \partial \rho + 4 i p^2 \rho \partial \rho - 2 p^2 \partial^2 \right. \]

\[ -2 i p \mu \partial^2 \partial^2 - 2 i p \rho (\partial^2)^2 + (\partial^2)^2 \right] P[L^a_1 L^b_2 L^c_3], \tag{55} \]

and note that only terms with three or more derivatives can contribute because of the \( \delta (y - z) \) and that for the same reason only those without \( L_i \) after differentiation survive. Extracting out \( (-i) i g^3 \frac{1}{2} \dot{E}^{(2)}(0)(2\pi)^4 \delta (p' - p - \Sigma q_i) \), the result is

\[ \left[ T^c \{ T^a, T^b \} (q_{123} + q_{12} + 2p)^\gamma + \{ T^a, T^b \} T^c (q_3 + 2p)^\gamma \right] g^\alpha \beta + 2 \text{ perms}. \tag{56} \]

Since the non-\( g \) terms have not appeared, we have to compute explicitly the \( n = 3 \) term. This is the most tedious part for the vertex as it involves derivatives up to the sixth order. One should be very careful that derivatives do not commute because of the path ordering operation. We skip the detail of the calculation but recording the result. Leaving aside the common factors \( (-i) i g^3 \frac{1}{3!} \dot{E}^{(3)}(0)(2\pi)^4 \delta (p' - p - \Sigma q_i) \), the \( n = 3 \) term is

\[ \left[ \left( T^c \{ T^a, T^b \} [p^2 + (p + q_{12})^2 + (p + q_{123})^2] (q_{123} + q_{12} + 2p)^\gamma \right. \right. \]

\[ + \{ T^a, T^b \} T^c [p^2 + (p + q_3)^2 + (p + q_{123})^2] (q_3 + 2p)^\gamma \right] g^\alpha \beta + 2 \text{ perms} \]

\[ + \left. T^a T^b T^c (q_{123} + q_{23} + 2p) \alpha (q_{23} + q_3 + 2p) \beta (q_3 + 2p)^\gamma + 5 \text{ perms} \right] \tag{57} \]

Now we assume the above structure is valid for the \( n \)-th term and denote the coefficients of the displayed three terms as \( b_n(p, q_1, q_2, q_3) \), \( c_n(p, q_1, q_2, q_3) \), \( d_n(p, q_1, q_2, q_3) \). The initial conditions are therefore

\[ b_2 = c_2 = d_3 = 1, \]

\[ b_3(p, q_1, q_2, q_3) = p^2 + (p + q_{12})^2 + (p + q_{123})^2, \]

\[ c_3(p, q_1, q_2, q_3) = p^2 + (p + q_3)^2 + (p + q_{123})^2, \tag{58} \]

while those not listed vanish. The coefficients in the next term are found to be (with arguments
\[ p, q_1, q_2, q_3 \text{ suppressed}), \]

\[
b_{n+1} = p^2 b_n + \frac{(p + q_{123})^{2n} - (p + q_{12})^{2n}}{(p + q_{123})^2 - (p + q_{12})^2},
\]

\[
c_{n+1} = p^2 c_n + \frac{(p + q_{123})^{2n} - (p + q_3)^{2n}}{(p + q_{123})^2 - (p + q_3)^2},
\]

\[
d_{n+1} = p^2 d_n + \frac{1}{(p + q_{123})^2 - (p + q_{23})^2} \times \left[ \frac{(p + q_{123})^{2n} - (p + q_3)^{2n}}{(p + q_{123})^2 - (p + q_3)^2} - \frac{(p + q_{23})^{2n} - (p + q_3)^{2n}}{(p + q_{23})^2 - (p + q_3)^2} \right]. \tag{59}
\]

\[ b_n \text{ and } c_n \text{ have the same structure as } h_n, \text{ while } d_n, \text{ when multiplied by } [(p + q_{123})^2 - (p + q_{23})^2], \]

is a difference of two series each having the structure of \( h_n \). They are worked out to yield the final answer shown in eq (58).

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