Schooling Choice, Labour Market Matching, and Wages

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ABSTRACT. This paper develops an empirical two-sided matching model with endogenous pre-investment. The model can be used to measure the impact of frictions in labour markets using a single cross-section of matched employer-employee data. The observed matching of workers to firms is the outcome of a discrete, two-sided matching process where firms with heterogeneous preferences over education sequentially choose workers according to an index correlated with worker preferences over firms. The distribution of education arises in equilibrium from a Bayesian game: workers, knowing the distribution of worker and firm types, invest in education prior to the matching process. I propose an inference procedure combining discrete choice methods with simulation. Counterfactual analysis using Canadian data shows that changes in matching frictions can lead to economically significant equilibrium changes in both inequality and the probability of investing in higher education. These effects are more pronounced when worker and firm attributes are complements in the match surplus function.

KEY WORDS. Wage inequality; Two-sided matching; Pre-match investments; Matching technology; Structural estimation; Bayesian game estimation; Order statistics

JEL CLASSIFICATION: C51, C57, J31
1. Introduction

Since the 1980s, economists have attributed rising wage inequality to a number of sources. One possible source of such inequality is positive assortative matching between workers and firms - the tendency for the quality of workers and firms who match with one another to be positively correlated.\(^1\) Unfortunately, studying matching in labour markets presents a serious challenge when the decisions of individual job seekers affect each other’s hiring outcomes. This paper develops a methodology to address this challenge. In particular, I show how cross-sections of matched employer-employee data can be used to study the role that a labour market matching technology plays in shaping the equilibrium distributions of education and wages. A key result is that - even in the absence of complementarities between worker and firm types in the match production function - the model can capture assortative matching between workers and firms.\(^2\)

A general overview of the labour market in my model is as follows. Agents from one side of the market sequentially choose agents from the other side according to their preferences. Preference rankings of the choosers depend on a preference parameter, along with the capital of both types of agents. The order in which the choosers pick depends on the chooser’s capital and a matching technology parameter. Before matching, the agents who will be chosen are allowed to simultaneously decide their capital given the distribution of the chooser’s capital and the underlying parameters of the economy (including the frictions).

The structural approach I develop allows me to examine counterfactual distributions of education and wages under different matching technologies and preferences. My model also offers a novel explanation for a longstanding empirical puzzle - namely, how college wage premia can increase without an associated increase in the supply of highly educated workers. I show that this pattern is captured in a special case of my model in which matching frictions become less severe over time.

This paper contributes to the econometric literature concerned with inference in two-sided matching models. I propose a two-stage approach for inference on the agents’ preferences and the matching technology. In the first stage, I fix the matching technology and construct confidence regions for the preference parameter by estimating the Bayesian game associated with the workers’ pre-match investment in education decision. I show

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\(^1\)Recent empirical papers examining the role of sorting on wage inequality include Card, Heining, and Kline (2013), Barth et al. (2016), and Kantenga and Law (2016).\(^2\)The value of a match between any type of worker, \(h\), and any type of firm, \(k\), can be represented using a positive, increasing function, \(f(h, k)\). We say that the types are complements in \(f\) when the marginal product of an \(h\) type is higher when matched with a higher \(k\) type (and vice versa).
that this problem can be cast in a discrete choice framework that is tractably estimable using maximum likelihood when the workers’ educational decision takes one of two values (college, or no college). In the second stage, I construct confidence intervals for the matching technology using a simulation-based inference approach. In the first stage, the presence of the matching function in workers’ expected utility function makes estimating workers’ equilibrium expectations highly non-trivial. Nevertheless, under reasonable assumptions, I show that workers’ equilibrium expectations can be written in a closed form suitable for estimation. The second-stage inference on the matching technology uses the following insight: once the matching process is specified, the finite sample distribution of the observed matching is known up to a parameter. I construct a test statistic that measures the distance between the observed joint distribution of worker education and matched firm capital to simulated counterparts. A confidence interval for the matching technology can then be constructed by inverting the test.

A unique feature of such a model is that worker and firm types need not be complements in the match production function for sorting between workers and firms to occur. This paper builds on the fundamental insights of Becker (1973) and Gale and Shapley (1962) to highlight the fact that, under certain circumstances, a meaningful notion of sorting can be captured in an empirical model with additive worker and firm effects. In his seminal 1973 paper on the marriage market, Becker argued the following: when the match production function is supermodular and utility is transferable between matched agents, high types can outbid low types for the best partners, leading to an equilibrium with positive assortative matching. The reason why positive assortative matching can occur in my model even when such complementarities are absent is answered in Becker’s same 1973 paper. Becker notes that sorting can arise in an non-transferable utility (NTU) framework when the payoffs of the agents on both sides of the market are monotonic in the other agent’s type. To explain why this is so, Becker invokes the logic of pairwise stability (Gale and Shapley, 1962). For example, consider an economy with four agents where a low-type firm is paired with a high-type worker and vice-versa. Such a matching is unstable when high types are preferred, because both high-types will agree to abandon their low-type partners for one another. In a special case of my model where

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3This idea of using a structural model to characterize the joint distribution of a discrete matching model that can then be used for inference on the model parameters builds from work-in-progress I am pursuing with Taehoon Kim, Kyungchul Song, and Yoon-Jae Whang. Although computationally intractable when the dimension of the parameter is large, this approach is attractive for inference on the matching technology parameter in the second stage of my approach.

4When \( f \) is differentiable, (strict) supermodularity is equivalent to \( \partial^2 f(h, k)/\partial h \partial k > 0 \).

5This insight comes to me by way of Chade, Eeckhout, and Smith (2017)’s excellent review of the search and matching literature.
preferences are indeed monotonic, sorting - and some inequality - may emerge. In this case, complementarities are not necessary for sorting but merely amplify the effects of sorting, since interactions between worker and firm types in the wage function lead to more wage dispersion than when such interactions are absent.

I estimate my model using matched employer-employee data on the finance and manufacturing industries using Canada’s Workplace-Employee Survey (WES). I find that frictions in the matching technology rose in the middle of the sample period, a time corresponding with relatively stable wage inequality. My model counterfactuals imply that the matching technology frictions matter: for example, in the 2001 finance industry, the effect of eliminating frictions causes a roughly 8% increase in the equilibrium probability of high education in the complementarities case and only a 3% increase in the specification without.

The empirical findings highlight the importance of production complementarities to wage inequality. The model-predicted level of wage inequality is much higher (and more reasonable for Canada) in the case that complementarities are present - for example, the predicted Gini coefficient for the 2002 finance industry is 0.2542 for the case of complementarities, while it is only 0.1655 in the additive case. The effect of information frictions on wage inequality are complicated by the presence of two competing effects: when frictions are lowered, the number of workers investing in education rises (a supply effect), but assortative matching between worker and firms also increases (a sorting effect). The sorting effect tends to increase inequality while the supply effect tends to decrease it. I argue that the role of the latter effect is relevant in the subgroups I study, where the equilibrium probability of investment in education is typically quite high. For example, among high skilled workers in the manufacturing industry in 2005, the Gini is 0.2220 and the investment in education is 77%. This rises to 0.2507 (education investment 76%) when information frictions are highest and 0.2452 (education investment 85%) when frictions are lowest. Overall, however, the result results suggest that changes in wage inequality over time are mostly driven by changes in exogenous worker and firm characteristics and preferences (including shifts in the underlying production technology) rather than changes in the matching technology.

The key features of the model are explored in Section 2.3. In addition to illustrating the model’s main implications concerning the relationship between complementarities, frictions, and sorting, the section also suggests that my model may be able to shed some light on other empirical puzzles. In particular, we see how the model predicts that a fall in information frictions can lead to a dramatic rise in the education wage premium through sorting while at the same time, the same fall in information frictions leads to a much more modest increase in the supply of highly educated workers. Thus, changes
in informational frictions may be a useful way to explain a puzzling empirical findings concerning the relationship between wage premia and educational attainment.\(^6\)

1.1. Background and Related Literature

Since Abowd and Margolis (1999) (AKM), the availability of matched employer-employee data has allowed researchers to study the role that unobserved worker and firm attributes play in driving wage variation over time. In AKM, the correlation of worker and firm fixed effects from wage regressions is taken to capture a notion of sorting. Although popular for investigating the wage structure, a burgeoning literature has criticised the viability of AKM for detecting sorting on unobservables. In particular, the additive structure of AKM implies that wages are monotone in firm type - an implication that is difficult to reconcile with equilibrium models of sorting with and without frictions (Eeckhout and Kircher (2011), Lopes de Melo (2013)).\(^7\) For example, in Eeckhout and Kircher (2011), a low-type worker can receive a lower wage at a high-type firm since the worker must implicitly compensate the high-type firm in equilibrium for forgoing the opportunity to fill a vacant job with a higher-type worker.\(^8\)

Search and matching models have emerged as the leading alternative to the AKM framework for studying sorting in labour markets.\(^9\) In this literature, the standard matching technology is one that converts aggregates of vacancies and unemployed workers into matches. Although treating matching at the aggregate level simplifies the analysis considerably, any strategic interdependence that may be present in the matching process is assumed away (Chade, Eeckhout, and Smith (2017)). In many labour markets, it is unrealistic to suppose that the decisions of individual workers do not affect the outcomes of other workers.\(^10\) One contribution of this paper is to develop and estimate a model that takes such strategic interdependence in the matching process seriously. In the equilibrium of my model, the probability that a worker matches to a given firm typically depends on the decisions of all the other agents in the economy.

Another key facet of search models is that workers direct their job search based on the wages that employers set for them. However, many recent studies of online job markets

\(^{6}\)See Card and Lemieux (2001).

\(^{7}\)Gautier and Teulings (2006) was an early empirical study that detected a concave relationship between wages and firm type.

\(^{8}\)There are other reasons wages may be non-monotonic in firm type. In Postel-Vinay and Robin (2002), workers may be willing to accept lower wages at higher type firms when they expect to receive higher wages in the future.

\(^{9}\)Hagedorn et al. (2017), Bagger and Lentz (2014), Lise et al. (2016) and Lopes de Melo (2013) all find evidence of positive sorting when an AKM approach finds negligible sorting.

\(^{10}\)For example, a worker’s decision to get a master’s degree in finance will not only affect the likelihood that he gets a job at an investment bank, but also the likelihood that his competitors get the job.
have found that it is relatively uncommon for positions to explicitly post wages. This paper differs from the traditional search literature by supposing that workers do not observe posted wages directly. Instead, workers know the underlying distributions of job characteristics and the matching process prior to simultaneously investing in education. In this sense, the worker’s decision to invest in education is the channel by which workers are able to direct their search.

This paper is related to a literature studying the role that match function complementarities play in driving sorting patterns. Since Becker’s seminal study of the marriage market, it has been known that in two-sided matching markets with non-transferable utility (NTU), a sufficient condition for a stable matching to exhibit positive assortativity is that the agents’ payoff be monotonic in their partners’ type. NTU arises naturally in my model from the assumption that wages for any matched pair are determined exogenously (in fact, by imposing Nash bargaining). In my approach, sorting can arise in the absence of direct interactions between worker and firm types in the (post-match) surplus function in the special case that workers and firms have monotone preferences over one another.

This paper contributes to the literature concerned with inference in two-sided matching models. A key feature of my setup is that the characteristics of agents on one side of the market are endogenous - in particular, arising in equilibrium from a pre-matching investment game. I show how, rather than making the empirical analysis intractable, accommodating such pre-matching investments provides the researcher useful information for inference.

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11For example, Marinescu and Wolthoff (2015) study the role of job titles in directing the search of workers and report that only 20% of job the advertisements CareerBuilder.com report a wage.

12More generally, Cardoso, Loviglio, and Piemontese (2015) is another recent paper that studies the impact of information frictions in search and matching models.

13Becker remarks this without explicitly invoke a notion of pairwise stability, such as that in Gale and Shapley (1962) (Chade, Eeckhout, and Smith (2017)).

14A desire to accommodate complementarities does not necessarily require us to abandon the AKM framework entirely. Bonhomme et al. (2015) incorporate worker-firm complementarities into a framework resembling AKM, while relaxing the exogenous mobility assumption. While providing empirical support for the existence of complementarities, they also note that the additive specification does not appear to be a bad approximation in practice.

15See Chiappori and Salanié (2016) for a review of this literature. A seminal paper in this literature is Choo and Siow (2006), which considers inference in a transferable utility setup with a continuum of agents.

16A popular approach for estimating two-sided matching models builds on the notion that the observed matching is pairwise stable. For example, see Fox and Bajari (2013), Echenique, Lee, and Shum (2013), and Menzel (2015). Requiring that the observed matching be pairwise stable may be unrealistic in the context of a frictional labour matching market of the sort that is the focus of this paper.
My framework supposes that each equilibrium gives rise to a single large matching between workers and firms.\textsuperscript{17} Under familiar assumptions (e.g., iid and separable private information), I follow similar arguments to Aguirregabiria and Mira (2016) to prove that an equilibrium exists. My setup, however, also allows me to provide sufficient conditions for equilibrium uniqueness.

This paper is also part of the literature concerned with estimating cross-sectionally dependent observations. In my setup, the observed matching of workers to firms exhibits cross-sectional dependence of an unknown form due to the matching process. This means that asymptotic inference approaches that appeal to the the law of large numbers and central limit theorems will not work. The approach I pursue builds on an idea from work-in-progress that demonstrates how inference in structural matching models are possible when knowledge of the matching process can be used to characterize the joint distribution of the observed matching. This paper shows how such a simulation-based inference approach, cumbersome when the dimension of the parameter space is high dimensional or complex, is useful for estimating a subset of the parameters in structural models with cross-sectional dependence.\textsuperscript{18}

Section 2 introduces the model of two-sided labour market matching with frictions. In the baseline model of Section 2.1, workers and firms with exogenous characteristics match with one another and split the match surplus according to a Nash bargaining rule. The rest of the paper is organized as follows. Section 2.2 extends the baseline model to allow for endogenous worker characteristics - after observing their type, workers simultaneously invest in education prior to entering the labour market. Section 3 outlines an approach for inference on the parameters of the structural model of Section 2.2. Section 4 applies the structural inference methodology to matched employer-employee data from Statistics Canada’s Workplace Employee Survey to study labour markets in Canada. Section 5 concludes. Mathematical proofs, additional details of the empirical application, and a simulation study illustrating reasonable performance of the estimators, are confined to the Appendix of this paper.

\textsuperscript{17}This contrasts with cases in which the researcher sees many independent copies of games involving few players, such as those studied by Bresnahan and Reiss (1991), Ciliberto and Tamer (2009), Berry (1992) and many others. See Xu (2015), Song (2014), Menzel (2016) for more papers discussing the estimation of large Bayesian games.

\textsuperscript{18}The simulation-based approach used in the second-stage of the inference procedure is known as a Monte Carlo test. Monte Carlo tests have a history in econometrics dating back at least to the 1950s, as discussed by Dufour and Khalaf (2001) in their overview of the technique.
2. The Labour Market As a Two-Sided Matching Market

Our goal is to study the distribution of education and wages using separate cross-sections of matched employer-employee data. The first subsection introduces the core elements of the model that will serve as the basis for the structural model in the second subsection.

2.1. Baseline model

Let \( N_h = \{1, \ldots, n_h\} \) be the set of workers and \( N_f = \{1, \ldots, n_f\} \) be the set of firms, where \( n_h \) and \( n_f \) are used to denote the total number of workers and firms, respectively. Each worker seeks one job and each firm seeks to hire one worker.

The matching of workers to firms will be determined by the preference rankings of workers and firms. Workers value the capital of firms, \( K = (K_j)_{j \in N_f} \), and firms value the human capital of workers \( H = (H_i)_{i \in N_h} \), where \( K_j \) and \( H_i \) are scalars. Any worker \( i \) who is matched with firm \( j \) receives wage \( w_{ij} \geq 0 \) while firm \( j \) receives profit \( \rho_{ji} \geq 0 \), where both wages and profits may also depend on a parameter, \( \theta \in \mathbb{R}^d \). Since our framework supposes that wages and profits are always non-negative for any worker and firm that could match, I will assume throughout the paper that no agent will ever unilaterally dissolve a match to become unmatched. This requirement that any matching satisfy an individual rationality constraint is embodied in the following condition:

**Condition IR** (Individual rationality of matches): For each \( i \in N_h \), and \( j \in N_f \) \( w_{ij} \geq 0 \) and \( \rho_{ji} \geq 0 \).

Based on the values of \( \rho_j = (\rho_{ji})_{i \in N_h} \) each firm \( j \) can construct preference rankings over the workers. We suppose that if the firm is ever indifferent between one or more workers, then the firm picks preference rankings over these workers at random. Next, we introduce a condition on the worker’s wage function that will be useful for interpreting the matching process (along with our notion of information frictions).

**Condition H** (Homogeneous worker preferences): For each \( i \in N_h \), the wage of worker \( i \) is increasing in the capital of their matched firm.

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19 In this setup, \( \theta \) represents the preferences of both workers and firms. As we will see, \( w_{ij} \) and \( \rho_{ji} \) depend on the output of worker \( i \) at firm \( j \), and the production function that gives rise to this output will depend on a part of \( \theta \).

20 The current setup is tailored to settings where the researcher has at least one cross-section of matched employer-employee data and the agents who are unmatched are not of primary interest in the analysis. An interesting (and challenging) extension of the current framework would accommodate the possibility of unmatched agents, and hence unemployment.
The assumption is tantamount to a notion of worker preference homogeneity, implying that all workers prefer higher capital firms. Supposing that workers accurately observe the capital of firms, this assumption implies that a matching algorithm in which the highest capital firm, $j_1$, choose his preferred worker, $i_1 \in N_h$, the second highest capital firm, $j_2$, choose his preferred worker $i_2 \in N_h \setminus \{i_1\}$ and so on is an example of the serial dictatorship mechanism (Abdulkadiroğlu and Sönmez (1998)) and would produce a stable matching.

In order to build a model that accounts for the possibility of mismatches between workers and firms, we suppose there are information frictions in the market. Specifically, workers do not directly observe realizations of the firm’s capital. Instead, each worker sees $v = (v_j)_{j \in N_f}$, where $v_j$ is a ‘noisy’ measure of firm $j$’s capital. In particular, suppose that workers see

$$v_j = \beta K_j + \eta_j,$$

for each $j$, where $\beta \in B$, $B \subset \mathbb{R}$ is the parameter space of $\beta$, and $\eta_j$ is a random variable that is independent across $j$. The size of the variance of $\eta_j$ relative to the magnitude $\beta$ represents the magnitude of information frictions in the matching process. It is clear that when $\beta$ is zero and the variance of $\eta_j$ is positive, then this setup yields random matching from firm to worker the characteristics, since variation across firm capital plays no role in determining the realizations of $v$. Furthermore, when $\beta \neq 0$ and $\text{Var}(\eta_j) = 0$, it will be as if firm capital is observed by the worker, since $v_j$ is determined entirely by the firm’s capital. In the latter case, when $\beta > 0$ workers would favour firms with the largest realizations of $v$, while in the case that $\beta < 0$, workers would favour firms with the smallest realizations of $v$. However, even in the case that $\text{Var}(\eta_j) > 0$, $v_j$ still conveys some useful information to the worker under certain circumstances. When $\beta > 0$, $\eta_j$’s are iid, and workers see $v_{j_1} > v_{j_2}$, then workers would still prefer matching with Firm $j_1$ over Firm $j_2$ since the distribution of $K_{j_1}$ stochastically dominates distribution of $K_{j_2}$.

The following condition specifies the matching process we will use throughout the paper.

**Condition SD** (Matching process): The matching of workers to firms in the economy arises as follows. The highest $v$ firm, $j_1$, chooses his preferred worker, $i_1 \in N_h$, the second highest $v_2$ firm, $j_2$, chooses his preferred worker $i_2 \in N_h \setminus \{i_1\}$, and so on, until the lowest $v$ firm, $j_{nf}$, chooses his preferred worker among those not chosen by any higher ranked firms.

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21See Section 2.2. of Roth and Sotomayor (1992).

22We will impose such an assumption in a later section.
One way of understanding this matching algorithm in economic terms is to consider the following thought experiment. Imagine a situation in which a group of job-seekers have assembled in a large room on the day of a job fair. Workers do not observe the true quality of any of the firms, (represented by $K$), but they do see each firm’s value of $v$. When $\beta > 0$ and $\eta_j$’s are iid, each worker is happiest to match with the highest $v$ firm, since the distribution of capital associated with the highest $v$ firm stochastically dominates the distribution of capital associated with any of the lower $v$ firms. A procedure in which the highest $v$ firm, $j_1$, chooses his preferred worker, $i_1 \in N_h$, the second highest capital firm, $j_2$, chooses his preferred worker $i_2 \in N_h \setminus \{i_1\}$ and so on, will have no complaints from any of the participants at the job fair – that is, until uncertainty associated with $K$ is revealed. In this world, agents will typically have more regret (and hence a greater desire to rematch) when the frictions in $v$ are large. However, rematching is outside the scope of the model.

Next, we add some further structure to wages and profits. In particular, we will assume that the payoffs for any two matched agents follow a Nash bargaining structure. Let $\tau \in (0, 1)$ be the bargaining weight. A worker $i$ who matches with a firm $j$ receives

$$w_{ij} = \tau f(H_i, K_j) + (1 - \tau) g(H_i)$$

and

$$\rho_{ji} = (1 - \tau) (f(H_j, K_j) - g(H_i)),$$

where $f$ is the worker-firm output function and $g(H_i)$ is an outside option function, both of which may depend on elements of $\theta$. In a subsequent section, we will allow worker covariates, $X_i$, to effect wages through the outside option function, $g$.\footnote{$X_i$’s have support $X \subset \mathbb{R}^d$, where $d$ is an integer greater than or equal to one.} The following condition requires $f$ to satisfy some intuitive properties with respect to the worker and firm capital variables.

**Condition F (Production function):** $f$ is increasing in human capital and firm capital.

Condition F merely requires that more capital leads to more output - it does not impose that the worker and firm attributes be complements in $f$. Section 2.2 goes into further detail about the role of $f$ in this model.

### 2.2. Frictional Matching Model with Worker Investments

We now introduce a structural model where workers simultaneously invest in education prior to the serial dictatorship matching process as outlined in the previous section. A general overview of the matching process is as follows: i) workers, observing only their
type, simultaneously choose a level of education, ii) $v$ is realized, iii) firms, seeing only the education of workers, match according to Condition SD.

Although firms select their preferred workers in the serial dictatorship phase after constructing preference rankings over the workers, firms are not considered strategic agents within the context of the investment game itself. In addition, I will implicitly assume throughout this section that the worker’s expected utility with respect to the distributions of $K$ and $\eta$ conditional on their information is always finite. In other words, this section supposes that the worker can resolve the uncertainty with the serial dictatorship matching process. In Section 3.2 we will demonstrate the precise form that these expectations take under particular lower-level assumptions.

There are $n_h$ players indexed by $i \in N_h$. Each player chooses an education level, $h_i$, from the discrete set $\mathcal{H} \equiv \{1, \ldots, J\}$ to maximize their expected payoff. Let $\lambda = (\theta', \beta)$, where $\beta$ is the matching frictions parameter and $\theta \in \mathbb{R}^d$ is a preference parameter. The payoff function of player $i$ comprises the wage less a cost of education,

$$u(h_i, h_{-i}, x_i, k, \eta, \varepsilon_i; \lambda) = \omega(h_i, h_{-i}, x_i, k, \eta; \lambda) - c(h_i, x_i, \varepsilon_i; \lambda),$$

where $h_{-i} \in \mathcal{H}_{-i}$ are the choices of the other agents, $x_i \in \mathcal{X}$ and $\varepsilon_i \in \mathbb{R}^J$ are the private information of worker $i$, and $k \in \mathbb{R}^{n_f}$ and $\eta \in \mathbb{R}^{n_f}$ are vectors of exogenous firm variables that are unobserved by the workers. Although $\varepsilon_i$ and $x_i$ are private information of the worker, we will assume $x_i$ is observed by the econometrician in a subsequent section. The variable $\varepsilon_i$ represents the worker's private cost associated with each of the $J$ education levels. In Section 3.2 we will supply explicit assumptions on worker and firm information that illustrates why, given the matching process, the components of the payoff function depend on model’s underlying variables in the way stipulated by equation (3).

We now provide additional conditions that establish the existence of a Bayesian Nash equilibrium for our game (which we prove in Section 6.1).

**Assumption 2.1.** (a) $K_j$’s, $\eta_j$’s are independent across $j$. $X_i$’s, $\varepsilon_i$’s are independent across $i$. $X$, $K$, $\varepsilon$, and $\eta$ are independent. (b) $\varepsilon_i$’s are continuously distributed.

**Assumption 2.2.** The cost function is separable in private information:

$$c(h_i, x_i, \varepsilon_i; \lambda) = c_0(h_i, x_i; \lambda) + \varepsilon_i^j d(h_i),$$

where $d(h_i)$ is a $J$ dimensional vector with one in the $h_i$-th row and zero otherwise.

In Section 6.1, we show that Assumptions 2.1 and 2.2 are sufficient for establishing the existence of the Bayesian Nash equilibrium for the game of this section. For now, we will

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24Since the set of pure strategies for each agent is $\mathcal{H}$, it follows that $\mathcal{H}_{-i} = \mathcal{H}^{n_h-1}$ for each $i$, where $\mathcal{H}^{n_h-1}$ denotes the $(n_h - 1)$-ary Cartesian power of $\mathcal{H}$. 
provide some intuition into the worker’s education decision problem. First, we define the set of pure strategies as \( \sigma = \{ \sigma_i(x_i, \varepsilon_i) : i \in \mathcal{N}_h \} \) where \( \sigma_i \) is a function that maps from \( \mathcal{X} \times \mathbb{R}^{J-1} \) into \( \mathcal{H} \). Assumption 2.2 says that we can write the expected utility of agent \( i \) with covariates \( x_i \), who chooses \( h_i \) under beliefs \( \sigma \) as

(4) \[ U_i(h_i, x_i, \sigma, \varepsilon_i) = \tilde{U}_i(h_i, x_i, \sigma) + \varepsilon_i d(h_i), \]

where the first term in the expected utility is

(5) \[ \tilde{U}_i(h_i, x_i, \sigma) = \sum_{h_{-i} \in \mathcal{H}_{-i}} \tilde{u}_i(h_i, h_{-i}, x_i) P_{-i}(h_{-i} | \sigma), \]

and

(6) \[ \tilde{u}_i(h_i, h_{-i}, x_i) \equiv \tilde{\omega}_i(h_i, h_{-i}, x_i) - c_0(h_i, x_i), \]

where \( \tilde{\omega}_i(h_i, h_{-i}, x_i) \) is given by

\[ \tilde{\omega}_i(h_i, h_{-i}, x_i) = \mathbb{E}[\omega(H_i, H_{-i}, X_i, K, \eta; \lambda) | H_i = h_i, H_{-i} = h_{-i}, X_i = x_i], \]

and expectation is taken with respect to the distributions of \( K \) and \( \eta \). By Lemma (6.1) (on page 33) we can rewrite equation 5 as

\[ \tilde{U}_i(h_i, x_i, \sigma) = \sum_{h_{-i} \in \mathcal{H}_{-i}} \tilde{u}_i(h_i, h_{-i}, x_i) \prod_{j \in \mathcal{N}_h \setminus \{i\}} P_j(h_j | \sigma_j). \]

Throughout this paper, we will consider the case in which the wages of workers are determined by Nash bargaining. As in equation 2, we will suppose that firm capital only enters the worker’s payoff through the production function. Denote \( \mathcal{M}(i) \) as the identity of the firm that worker \( i \) matches to once all the uncertainty associated with the matching process has been resolved, and \( K_{\mathcal{M}(i)} \) as the level of capital associated with firm \( \mathcal{M}(i) \). Under these assumptions, we may write \( \tilde{\omega}_i(h_i, h_{-i}, x_i) \) as

(7) \[ \tilde{\omega}_i(h_i, h_{-i}, x_i) = \tau \tilde{f}_i(h_i, h_{-i}) + (1 - \tau) g(h_i, x_i), \]

where

\[ \tilde{f}_i(h_i, h_{-i}) = \mathbb{E}[f(H_i, K_{\mathcal{M}(i)}) | H_i = h_i, H_{-i} = h_{-i}, X_i = x_i], \]

the expectation is taken with respect to the distributions of \( K \) and \( \eta \), and we have allowed the worker’s characteristics to enter the payoff function through the outside option function, \( g \).

\[ ^{25} \text{Here, } \tau_i = \tau \text{ for each } i. \text{ My framework can be extended to incorporate heterogeneity in worker bargaining positions. In the empirical results, however, I ignore this channel. Bagger and Lentz (2014) emphasize the role that disentangling variation such as endogenous search intensity from matching variation (e.g., Postel-Vinay and Robin (2002)) plays in understanding the causes of wage inequality.} \]
Education affects the worker's expected utility in a number of ways. The first two are obvious: since $f$ is increasing in $h_i$ by Condition F, the worker who invests in a higher level of education obtains a higher wage at any firm he matches to. The worker's choice of education also affects his payoff through the outside option function, $g$. The novel channel in this setup is that $h_i$ also determines the expected quality of the firm that $i$ matches to. Even though (as mentioned before) firms in this model are non-strategic agents, the functional form of the production function, $f$, plays a key role in determining whether or not firms with different levels of capital exhibit different preferences for workers of differing levels of education. To see how $f$ determines whether or not firms' preferences are heterogeneous, consider the Nash bargaining preferences of a firm for any worker who chooses education level $h$:

\[
\rho(k, h; \theta) = (1 - \tau)(f(h, k; \theta) - \bar{g}(h; \theta)),
\]

where $\bar{g} = \mathbb{E}g(h, X_i)$ and the expectation is taken with respect to the distribution of $X_i$.\(^{26}\)

Suppose that $X_i$ is iid, $K$ takes two values $k_1, k_2$ and there are two levels of education, $h_1, h_2$ with $h_2 > h_1$. Let us denote the set of firms that prefer high education ($h_2$) as

\[M_2^+(\theta) = \{m \in \{1, 2\} : \rho(k_m, h_2; \theta) \geq \rho(k_m, h_1; \theta)\}.
\]

If $f(h, k)$ is of the form $a(h) + b(k)$, where $a$ and $b$ are two functions that map the capital variables to the real numbers, then $M_2^+(\theta)$ will be either $\{1, 2\}$ or $\emptyset$. In this case we say that firms have homogeneous preferences, since both types of firms in the economy prefer the higher educated workers. Alternatively, if $f(h, k)$ is of the form $a(h)b(k)$ then $M_2^+(\theta)$ will be either $\{1, 2\}$, $\emptyset$, or $\{2\}$. This is the case of heterogeneous firm preferences. In this latter case where $f$ exhibits complementarities in worker and firm types, the set of firms types that prefer high to low education is more finely partitioned. Moreover, the presence or absence of complementarities will play a key role in determining the severity of wage inequality, as we will see in Section 4. More general than all these points, however, is the following fact about the model: as long as $k$ appears somewhere in $f$, $k$ does not have to interact directly with $h$ in $f$ for the information frictions represented by $\beta$ to matter in worker's investment decision.

### 2.3. Some Implications of Frictional Matching Model

In this section, we explore some key features of the model. We will suppose that the functional forms, underlying distributions, and firm preferences are such that firms

\(^{26}\)Here, we implicitly assume that firms do not observe workers' covariates and rank workers only in terms of their education. We make these assumptions concerning firm information explicit in a subsequent section.
always strictly prefer higher educated workers. In the following subsection, we will illustrate sorting without any direct interactions between worker and firm types in the production function.

2.3.1. Sorting Without Complementarities

In Figure 1 and Figure 2, we compare the equilibrium probability of investing in education and the equilibrium Gini coefficient for a range of the friction parameters under two specifications of the production function: Specification 1 allows direct interaction between worker and firm types, \( f = \theta_1 h k \), while such interactions are absent in Specification 2, \( f = \theta_1 (h + k) \).\(^{27}\) Each point on the plot is the average of 500 simulations of endogenous variable from the equilibrium of the model. The outside option parameter is set to \( \theta_2 = (−.75, .25, .5) \). There are 100 workers and firms. In Specification 1, the high value of \( \theta_1 \) is 3.5, and the low value of \( \theta_1 \) is 1.5. In Specification 2, the high value of \( \theta_1 \) is 1.8, and the low value of \( \theta_1 \) is 0.8. There are two levels of of firm capital: \( K = 1/2 \) and \( K = 1 \). The fraction of each type of firm is .5 in the economy.

A number of implications are straightforward: the equilibrium probability of investing in high education is higher when \( \theta_1 \) is higher and frictions are lower. When \( \theta_1 \) is higher, workers will be compensated more for higher levels of education. When \( \beta \) is higher, the probability of matching to a higher type firm when they choose high education is higher.

The effect of increasing \( \beta \) (lowering matching frictions) on both the education and wage inequality is typically much more dramatic in Specification 1. A rise in \( \beta \) (a lessening in matching frictions) increases sorting in both specifications, though the effect is more dramatic in the complementarities case: in Figure 1, the correlation between worker and firm types rises from 60% to 68% when \( \theta_1 \) is high, but from 64% to 81% when \( \theta_1 \) is lower; in Figure 2, the correlation between worker and firm types rises from 84% to 90% in the high theta case whereas it rises from 77% to 87% in the low theta case. The overall level of inequality in Specification 1 is also higher since whatever sorting is present is amplified to a greater extent when the types interact in the wage equation than when they do not.

The high \( \theta_1 \) case in the right hand panel of Figure 1 also illustrates the role that two competing effects of changes in \( \beta \) play on the level of wage inequality. When \( \beta \) rises from 0 to 1, the level of inequality increases through the sorting channel. However, as \( \beta \) continues rises, the equilibrium probability of investing in education also continues to rise. As the fraction of highly educated surpasses 80%, the level of inequality begins to level off (at \( \beta = 2 \)) and then begins to fall. This phenomenon is also illustrated to a lesser degree in the high \( \theta_1 \) case of the right hand side panel of Figure 2.

\(^{27}\) The precise functional forms are the same as the ones used in Section 6.3.
Figures 1 and 2 plot the equilibrium probability of high education investment and the Gini coefficient for a range of values of the matching frictions parameter, $\beta$. We consider two specifications for the production function: Specification 1 includes interactions between worker and firm types while Specification 2 does not. Lowering matching frictions (increasing $\beta$) increases the equilibrium level of education across specifications. A rise in $\beta$ impacts inequality through two competing effects: a sorting effect that increases inequality and an a supply effect that lowers inequality. This can be seen most dramatically in Figure 1: as $\beta$ rises past a value of three, the fraction of highly educated rises more and more and inequality falls, dominating the effects of sorting on inequality.

2.3.2. Supply of Highly Educated Workers and Education Premia

In this section, we show how our model can address a puzzling finding discussed in Card and Lemieux (2001): how can dramatic increases in the education wage premium lead to only modest increases in the supply of highly educated workers? The authors note that, over a roughly 30 year period beginning in the early 1970s, the college-high school
Figures 3 offers an explanation to an empirical puzzle discussed in Card and Lemieux (2001): why are increases in wage premia not associated with large increases in the supply of highly educated workers? We plot the equilibrium probability of high education investment and the returns to education for a range of values of the matching frictions parameter, $\beta$. We consider Specification 1. In the case that $\theta_1$ is very low, the effect of increasing $\beta$ is to dramatically increase sorting while keeping the returns to education for any particular worker reasonably low.

wage gap rose considerably in the United States, Canada, and the United Kingdom, and that this rise occurred mostly for younger workers. They argue that an important source of this trend is a stagnation in the rate of educational attainment among workers born in the 1950s and thereafter.

In Figure 3, we show how this pattern can be driven entirely by changes in the matching technology over time. The wage premium is measured as the difference between the average wages of the workers with high education and the average wages of workers with low education. Each point on the plot represents the average of 500 simulations of the model. We use Specification 1, $f = \theta h k$, under the same setup as before with only one difference; we choose the low value of $\theta_1$ to be 0.6 and the high $\theta_1$ to be 3.5. In the case that $\theta_1$ is very low, the effect of raising $\beta$ is to dramatically increase sorting without a large benefit to any particular worker.

### 3. Econometric Inference

In this section, we outline the general empirical strategy for performing inference on the underlying model parameters. In Section 3, we describe how the main model can be used to characterize the observed distribution of the matching of workers to firm and
hence the wages of all the workers in the economy. The goal is to then use these representations to construct confidence regions for the preference and matching technology parameters.

However, if the model is high dimensional, the Monte Carlo inference approach may be cumbersome to apply in practice. For this reason, we propose a two-stage inference approach that relies on the construction of a first-stage confidence interval for a subset of the model parameters. We demonstrate this approach in practice in Section 3.2 by estimating the Bayesian game from 2.2 for fixed values of $\beta$.

3.1. Two-Stage Inference Accommodating Cross-Sectional Dependence of Observed Matching

The econometrician observes a matching of workers to firms, $M = (M(i))_{i \in N_h}$, where for each $i \in N_h$, $M(i)$ takes values in the set of firms. The main challenge associated with inference is the fact that the distribution of $M$ exhibits cross-sectional dependence of a complicated form. The matching of workers to firms can be thought of as discrete choice problem on the part of the firm where the choice sets of firms are endogenously constrained by the choices of firms with higher $v$-indices, which depends on $\beta$, $\eta$ and $k$. Hence, the event that worker $i$ matches to firm $j$ cannot be considered independent from the event that a worker $i' \neq i$ matches to firm $j$. Also, the fact that firm preferences are heterogeneous means we cannot condition on the $v$-index and firm preferences in a way to remove the cross-sectional dependence as was done by Agarwal and Diamond (2014).

The econometrician observes the vector $M \in \mathbb{R}^{n_h}$, which represents a matching of workers to firms. Given the serial dictatorship matching process, the joint distribution of $M$ is known up to a parameter. Let $K = (K(i))_{i \in N_h}$, where $K(i) = K_{M(i)}$; i.e., the capital of the firm matched to by worker $i$.

Our model also implies that the finite sample distribution of wages, $(W(i))_{i \in N_h}$, is known up to a parameter. Under Nash bargaining (and a specification of the post-match wage function based off an equation such as 2), we have for each $i \in N_h$

$$W(i) = w(H_i, K(i)).$$

We denote all the match-related observables as $Y = (K, M)$. $M$ is observed whenever the researcher has matched employer-employee data. $K$ is observed when the researcher can use the matching data, $M$, and the firm capital data, $K$, to find the capital of the firm.

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28Throughout this paper, we will suppose that the matching is one-to-one between workers and firms. In practice, “firms” in this context can be viewed as positions at particular firms.
each worker in the sample is employed at. Using $Y$ and worker observables $H$ and $X$, the econometrician wishes to infer $\lambda_0$.

3.1.1. Finite Sample Inference on Parameters

Next, we consider a test statistic that matches the moments of the distribution of the matched-related observables with their simulated counterparts. To simplify the exposition, we discuss the construction of a confidence interval for $\beta_0$ alone, i.e., supposing that we knew the true values of $\theta_0$. Denote $R + 1$ as the total number of simulations in the Monte Carlo inference procedure. Drawing $\eta_r$ from some continuous parametric distribution $F_{\theta}$, we simulate a version of the matching for each $\beta \in B$ and each $r = 1, \ldots, R + 1$, which we write as $M_r(\beta) = \{M_r(i; \beta) : i \in N_h\}$. The simulated wages are then $W_r(i; \beta) = w(H_i, K_{M_r(i; \beta)})$.

It is convenient to define

\[ Y_r(\beta) = \{Y_r(i; \beta) : i \in N_h\}, \]
\[ Y_{R+1}(\beta) = \{Y_r(i; \beta) : i \in N_h, r = 1, \ldots, R + 1\}, \text{ and} \]
\[ Y_{-r}(\beta) = Y_{R+1}(\beta) \setminus Y_r(\beta). \]

Next, we will propose a test statistic that depends on both the observed matching data, $Y$, and the simulated matching data, (along with simulated versions of this test statistic). That is,

\[ T(\beta) = \phi_n(Y, Y_{R}(\beta)), \quad \text{and} \]
\[ T_r(\beta) = \phi_n(Y_r(\beta), Y_{-r}(\beta)). \]

An example of such a test statistic is one that compares the observed joint distribution of worker human capital and matched firm capital with simulated counterparts. For example, we may consider the test statistic

\[ T(\beta) = \frac{1}{R} \sum_{r=1}^{R} \left\| \hat{P} - \hat{P}_r(\beta) \right\|, \]

where $\hat{P}$ is an $M \times J$ matrix whose $(m, j)$ element is the estimated probability that a worker of education level $h_j$ matches to a firm of capital level $m$, $\hat{P}_r(\beta)$ is defined similarly.

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\(^{29}\) We will specify a particular parametric family that $F_\theta$ belongs to, along with additional assumptions, in Section 3.2.

\(^{30}\) In this example, we are implicitly assuming that the distribution of $K$ is discrete and has $M$ support points. We will make this assumption explicit in a subsequent section.
to $\hat{P}$, except we replace the observed matching with the $r$th simulated matching, $\mathbf{M}_r(\beta)$, and $\|\cdot\|$ is the matrix norm.\(^{31}\)

Using our test statistic, we may compute a confidence region for $\beta$ as

$$C_{\alpha, R}^\beta = \{\beta \in B : T(\beta) \leq c_{\alpha, R}(\beta)\},$$

where the critical value is computed as the $(1 - \alpha)$-quantile of the empirical distribution of $\{T_r(\beta) : r = 1, ..., R\}$:

$$c_{\alpha, R}(\beta) = \inf \left\{ c \in \mathbb{R} : \frac{1}{R} \sum_{r=1}^{R} 1\{T_r(\beta) \leq c\} \geq 1 - \alpha \right\}.$$

Under Assumption 3.2, it can easily be shown that finite sample inference on $\beta_0$ satisfies $P\{\beta_0 \in C_{\alpha, R}^\beta\} \geq 1 - \alpha$ when the procedure outlined above involves the true parameter, $\theta_0$.

In practice, we do not know the true value of $\theta_0$. In situations in which the full parameter vector $\lambda_0$ is not very large, it may be feasible to construct a $(1 - \alpha)100\%$ confidence region for this parameter that exhibits finite sample validity. That is, we construct

$$C_{\alpha, R}^\lambda = \{\lambda \in \Lambda : T(\lambda) \leq c_{\alpha, R}(\lambda)\},$$

where $T(\lambda)$ and $c_{\alpha, R}(\lambda)$ are defined analogously to $T(\beta)$ and $c_{\alpha, R}(\beta)$. In the case that $\Lambda$ is high-dimensional, the finite sample procedure outlined above may not be practical due to the unreasonable computational cost. In the following subsection, we explore a two-stage inference approach that admits inference on $\beta_0$ when the researcher is able to construct a first-stage confidence region for a subset of the parameters, $\theta_0$.

Plugging in a consistent estimator of $\theta_0$, $\hat{\theta}_n$, for the true value in inference procedure outlined above will (in general) not lead to valid inference on $\beta_0$. This is because there is no reason to suspect that plugging in $\hat{\theta}_n$ for $\theta_0$ will make the distribution of the simulated matching, $\mathbf{M}_r$, equal to the distribution of the observed matching, $\mathbf{M}$. The fact that $\mathbf{M}_r$ is not equal in distribution to $\mathbf{M}$, in turn implies that $\mathbf{K}_r$ does not follow the same distribution as $\mathbf{K}$. The severe consequences of estimation error in $\hat{\theta}_n$ occur because the firm preferences are typically misspecified at all values of $\theta$ other than the true value, $\theta_0$. Moreover, this problem is not alleviated by conditioning on $H, K$, or exogenous variables.

In the following section, we discuss a general two-stage inference approach when the econometrician can construct an (asymptotically) valid confidence first-stage confidence

\(^{31}\)See equation 28 in the empirical section for more on constructing $\hat{P}$ and $\hat{P}_r(\beta)$. In this section, we also choose the matrix norm to be the Frobenius norm. That is, for an $m \times n$ matrix $A$, $\|A\| = \left(\sum_{i=1}^{m} \sum_{j=1}^{n} |a_{ij}|^2\right)^{1/2}$, where $a_{ij}$ denotes the $(i, j)$-element of $A$. 
interval for $\theta_0$. In Section 2.2, we extend our baseline economic model of Section 2 in a manner that admits the application of this two-stage inference approach to our setup.

3.1.2. Two-Stage Inference on $\beta$ using Test-Inversion Confidence Interval

Suppose that we wish a $(1 - \alpha)$-level asymptotic confidence interval for $\beta_0$, and can construct a confidence interval for $\theta_0$. Let us denote the test statistic and its simulated counterpart from the previous section, where the $\theta$ arguments make explicit the test statistic’s dependence upon a given value of $\theta \in \Theta$:

$$T(\beta; \theta_0, \theta_1) = \phi_n(Y(\beta_0, \theta_0), Y_R(\beta, \theta_1)),$$

and

$$T_r(\beta; \tilde{\theta}, \theta_1) = \phi_n(Y_r(\beta, \tilde{\theta}), Y_{-r}(\beta, \theta_1)).$$

Note that according to the notation we used in the last section we have $T(\beta; \theta_0, \theta_0) = T(\beta)$. Our inference on $\beta$ proceeds in two steps:

**Step 1.** Using the first stage estimates of $\hat{\theta}(\beta)$, we construct a confidence region for $\theta_0$, $\hat{C}_{\alpha/2}(\beta)$, with $(1 - (\alpha/2))$ asymptotic coverage.

**Step 2.** Next, we construct a test statistic that doesn’t involve $\theta$. Define

$$S(\beta) = \inf_{\theta_1 \in \hat{C}_{\alpha/2}(\beta)} T(\beta; \theta_0, \theta_1),$$

and

$$S^*_r(\beta) = \sup_{\tilde{\theta} \in \hat{C}_{\alpha/2}(\beta)} \inf_{\theta_1 \in \hat{C}_{\alpha/2}(\beta)} T_r(\beta; \tilde{\theta}, \theta_1).$$

We now construct a confidence set for $\beta$ as

$$\hat{C}_{\alpha,R} = \{\beta \in B : S(\beta) \leq c^*_{1-(\alpha/2),R}(\beta)\},$$

where the critical value $c^*_{1-(\alpha/2),R}(\beta)$ is computed as the $(1 - (\alpha/2))$-quantile of the empirical distribution of $\{S^*_r(\beta) : r = 1, \ldots, R\}$; that is,

$$c^*_{1-(\alpha/2),R}(\beta) = \inf \left\{ c \in \mathbb{R} : \frac{1}{R} \sum_{r=1}^{R} 1\{S^*_r(\beta) \leq c\} \geq 1 - (\alpha/2) \right\}.$$

The following lemma establishes the asymptotic validity of the two-stage inference procedure.

**Lemma 3.1.** Suppose that the econometrician can construct $\hat{C}_{\alpha/2}(\beta_0)$ such that

$$\lim_{n \to \infty} P \left( \theta_0 \in \hat{C}_{\alpha/2}(\beta_0) \right) \geq 1 - (\alpha/2).$$

Then

$$\lim_{n \to \infty} P \left( \beta_0 \in \hat{C}_{\alpha,R} \right) \geq 1 - \alpha.$$
Proof. By the definition of $\hat{C}_{\alpha,R}$, $P\left(\beta_0 \in \hat{C}_{\alpha,R}\right)$ is equal to

$$P(S(\beta_0) \leq c^*_1 - (\alpha/2),R(\beta_0))$$

$$= P\left(\inf_{\theta_1 \in \hat{C}_{\alpha/2}(\beta)} T(\beta_0; \theta_0, \theta_1) \leq c^*_1 - (\alpha/2),R(\beta_0)\right)$$

(12)

$$\geq P\left(\left\{\inf_{\theta_1 \in \hat{C}_{\alpha/2}(\beta)} T(\beta_0; \theta_0, \theta_1) \leq c^*_1 - (\alpha/2),R(\beta_0)\right\} \cap A_1\right).$$

where $A_1 \equiv \left\{\theta_0 \in \hat{C}_{\alpha/2}(\beta_0)\right\}$. Then, the right hand side of the right hand side of (12) is greater than or equal to

$$P\left(\left\{\sup_{\theta_0 \in \hat{C}_{\alpha/2}(\beta)} \inf_{\theta_1 \in \hat{C}_{\alpha/2}(\beta)} T_r(\beta; \hat{\theta}, \theta_1) \leq c^*_1 - (\alpha/2),R(\beta_0)\right\} \cap A_1\right),$$

$$\geq P\left(\sup_{\theta_0 \in \hat{C}_{\alpha/2}(\beta)} \inf_{\theta_1 \in \hat{C}_{\alpha/2}(\beta)} T_r(\beta; \hat{\theta}, \theta_1) \leq c^*_1 - (\alpha/2),R(\beta_0)\right) - P(A^\complement_1).$$

Now since

$$\lim_{n \to \infty} P\left(\theta_0 \notin \hat{C}_{\alpha/2}(\beta_0)\right) \leq \alpha/2,$$

we have

$$\lim_{n \to \infty} P\left(\beta_0 \in \hat{C}_{\alpha,R}\right) \geq 1 - \alpha.$$

In the following section, we describe how to construct $\hat{C}_{\alpha/2}(\beta)$ for each $\beta \in B$.\(\textsuperscript{32}\)

3.2. First-Stage Estimation of $\theta$

In this section, we show how $\theta$ can be estimated for a particular fixed value of $\beta$. We will write an estimator of such an object as $\hat{\theta}(\beta)$. The main challenge associated with this problem is that of estimating the worker’s expected utility from equation 6. The problem is difficult because the workers must somehow resolve uncertainty associated with the serial dictatorship matching process in order to compute the expected output under the equilibrium education choices. In spite of these complications, it turns out that, under reasonable assumptions, the parameters are tractably estimable using discrete choice

\(\textsuperscript{32}\)In the empirical application of this paper, we estimate $\theta_0$ by maximum likelihood and construct confidence regions using numerical derivatives of the likelihood function. A Monte Carlo simulation (reported in the Appendix) - illustrates acceptable finite sample performance of this inference approach.
methods with a fixed point constraint when there are only two education choices. We now provide and discuss these assumptions.

**Assumption 3.1.** (a) Firms observe (i) workers’ education decisions, $H$, and (ii) the distribution of characteristics, $X$. (b) Workers observe (i) the distribution of firm capital, (ii) the distribution of $\eta$, (iii) the distribution of $X$, and (iv) the distribution of the number of firms preferring each education level $h_j \in \mathcal{H}$.

Under part (a) of Assumption 3.1, firms do not take covariates into account when forming their preference rankings over workers. Thus, workers with the same education level are equally desirable to any given firm. When worker $i$ considers the desirability of choosing education $h_j$, he need only consider the capital a generic agent who chooses level $h_j$ expects to receive in the matching process. In many contexts, (a) will be reasonable for a host of variables that affects the worker’s education decision (e.g., marital status, number of dependent children). Part (b) says that workers know only the distribution of firm capital without knowing the precise realizations of capital. Assumption 3.1 (b) also stresses that the worker’s knowledge of the distribution of capital is not sufficient for knowledge of the distribution of the number of firms that prefers each education class, which will turn out to be crucial for our results of this section.

**Assumption 3.2.** (a) $K$ is discrete with probability mass $q = (q_m)_{m=1}^M$ where $q_m = P(K = k_m)$ for $m = 1, ..., M$. (b) $\eta_j$’s are iid $N(0, \sigma^2)$ (c) $\varepsilon_i$’s follow the Type I extreme value distribution.

Part (a) says the distribution of firm capital has discrete support. In practice, we can let $M$ be as large as our application requires. In concert with (b) and the parametric structure for $v$ stipulated by equation 1, (a) allows us to express the unconditional distribution of $v_j$ as a mixture of normals, $G \equiv \sum_{m=1}^M q_m F_m$, where $F_m$ is $N(\beta k_m, \sigma^2)$. Part (c) is an assumption on the worker’s unobserved costs that allows us to estimate the model parameters using conventional discrete choice methods.

We wish to obtain a convenient representation of each worker’s conditional expectation of the production function, for each education level that the worker can choose. Under the model of Section 2.2 the identity of the firm that worker $i$ matches with, $\mathcal{M}(i)$, depends on $K$, $H$, $\beta$, and, $\theta$. Therefore, for each $i \in N_h$ and $h_j \in \mathcal{H}$, we wish to estimate

$$\tilde{f}_{ij} \equiv \mathbb{E}[f(H_i, K_{\mathcal{M}(i)}) | H_i = h_j, X_i = x_i].$$

---

$^{33}$In some cases in which employers do see these worker characteristics, they are prohibited from discriminating based on them due to state or federal anti-discrimination laws.

$^{34}$In the simulations and empirical sections of the paper we normalize $\sigma^2 = 1$ when we perform inference on the model parameters.
where the expectation is taken with respect to the distribution of $K$, $H_{-i}$ and $\eta$. Under Assumption 3.2 (a), we can express the expectation on the preceding line as

$$\tilde{f}_{ij} = f'_{j} \pi^{(i)}_{j},$$

where $f_{j} = (f_{j1}, ..., f_{jm})'$ is an $M \times 1$ vector with the $m$-th element of $f_{j}$ given as $f_{jm} = f(h_{j}, k_{m})$ and $\pi^{(i)}_{j} = (\pi^{(i)}_{1j}, ..., \pi^{(i)}_{Mj})'$ is an $M \times 1$ vector with the $m$-th element of $\pi^{(i)}_{j}$ given as

$$\pi^{(i)}_{mj} = \sum_{h_{-i} \in H_{-i}} P(M(i) = m | H_{i} = h_{j}, H_{-i} = h_{-i}, X_{i} = x_{i}) P(h_{-i} | x_{i}).$$

This is the probability that worker $i$ matches to a firm of capital level $k_{m}$ when he has chosen education level $h_{j}$.\(^{35}\) Given that there are $M$ education levels, $J$ choices, and $n_{h}$ workers, the dimensionality of the problem appears daunting. However, under our assumptions the problem is simplified considerably, and we can show that for each $j$ and $m$, $\pi^{(i)}_{mj} = \pi_{mj}$, and hence, $\tilde{f}_{ij} = \tilde{\tilde{f}}$.\(^{36}\)

Although it is unclear how to represent $\pi_{mj}$’s analytically when the worker faces a choice between a large number of education levels, the problem becomes tractable when there are only two (i.e., $J = 2$). Proposition 6.1 shows that under our informational assumptions, firms (and workers) cannot distinguish between workers with the same education level during the matching process. As a consequence, we find that a worker is only concerned with the number of other workers who picked one of the two education levels (and not which particular workers chose what). Independence and identical distributions assumptions imply that the probability that $n_{j}$ workers picked education level $h_{j}$ can be represented using the binomial probability mass function. However, the number of workers choosing education level $h_{j}$ is unknown to workers, so they must take expectations. Thus, instead of having to sum over $n_{h} - 1$ indices associated with actions of each of the other workers to compute the worker’s expectation, we need only sum over one: the number of workers choosing a particular education level.

We will also allow $\theta$ to enter $\pi_{mj}$’s through the distribution of the number of firms that prefer high (or low) education. The following assumption is a natural way to specify this distribution. We use the notation $M^{+}_{j}(\theta)$ to denote the set of firm types that prefer education level $h_{j}$.\(^{37}\)

\(^{35}\)Note that although these terms depend on $\theta$ and $\beta$, we will occasionally omit these from our notation for convenience.

\(^{36}\)The argument for why this is the case is given in the proof of Proposition 6.1.

\(^{37}\)That is, $M^{+}_{j}(\theta) = \{m \in \{1, ..., M\} : \rho(k_{m}, h_{j}; \theta) \geq \rho(k_{m}, h_{j'}; \theta), j \neq j'\}$. See also the discussion before Proposition 6.2.
Assumption 3.3. In the model with $J = 2$, the probability that exactly $n^{(j)}$ firms prefer workers with education level $h_j$ follows the binomial distribution with probability $\sum_{m \in \hat{M}_j^+} \hat{q}_m$.

The explicit representation of the matching probabilities are given in Propositions A.1.2, A.1.3, and Lemma 6.3. These results can be used to construct estimates of the $\pi_{mj}$’s - and hence the $\hat{f}_j$’s - for fixed values of $\theta$ and $\beta$. Using a given functional form for the production function, we denote an estimate of the expected production function when the worker chooses education level $h_j$ as

$$\hat{f}_j(\theta, \beta) = \tilde{f}_j(\theta, \beta),$$

where our notation emphasizes the dependence of the objects upon the parameter values. To construct $\hat{\pi}_{mj}$’s we must estimate the terms of equation 25. $\hat{P}(n_j)$ is constructed as $B(n_j; n_h - 1, \hat{p}_j)$ where the latter denotes the binomial probability mass function with $\hat{p}_j = P(H_i = h_j)$.

Similarly, $\hat{P}(n^{(j)}; \theta)$ is constructed as $B(n_j; n_h - 1, \hat{q}_j(\theta))$, where $\hat{q}_j(\theta) = \sum_{m \in \hat{M}_j^+(\theta)} \hat{q}_m$, with $\hat{q}_m = \hat{P}(K_j = m)$,

$$\hat{M}_j^+(\theta) = \{m \in \{1, \ldots, M\} : \hat{\rho}(k_m, h_j; \theta) \geq \hat{\rho}(k_m, h_{j'}; \theta), j \neq j'\},$$

and $\hat{\rho}(k_m, h_j; \theta)$ is as in equation (8), except we use $\hat{g}_j = \frac{1}{n} \sum_{i=1}^n g(h_j, X_i)$ in place of $\tilde{g}$.

Lastly, the $P_{h_j, n_j, n^{(j)}}(m)$’s, from equation 25 - that is, the probability that a worker matches to a firm of type $m$ when they choose education level $h_j$, $n_j$ other workers choose $h_j$, and $n^{(j)}$ firms prefer $h_j$ - can be simulated for fixed values of $\theta$ and $\beta$. Proposition 6.2 and Proposition 6.3 show how these can be represented using probabilities involving order statistics. Under Assumption 3.2 (b), we can construct $\hat{P}_{h_j, n_j, n^{(j)}}(m)$’s by averaging functions of simulated draws of beta-distributed random variables (in particular, see Corollaries A.1.1 and A.1.2, which follow the order statistic result in Lemma 6.3).

Once we have estimated $\hat{f}_j(\theta, \beta)$ for each education level, we may use the specification of the wage from equation 7 to write the expected wage as

$$\hat{\omega}_{ji}(\theta, \beta) = \tau \hat{f}_j(\theta, \beta) + (1 - \tau) g(H_i, X_i; \theta).$$

When there are two choices ($J = 2$), the worker chooses high education ($h_j = 1$) if and only if

$$U_{1i}^* - U_{10}^* > 0.$$

---

38In so doing, we pursue a two-step approach for estimating the choice probabilities, such as Bajari and Nekipelov (2013). See for example Kasahara and Shimotsu (2012) for an alternative approach.
Under the assumption that ε_i’s follow the extreme value distribution (Assumption 3.2), the probability that worker i chooses high education can be written as

\[ \hat{p}_i(\theta, \beta) = \frac{\exp(\hat{\omega}_{1i}(\theta, \beta) - \hat{\omega}_{0i}(\theta, \beta))}{1 + \exp(\hat{\omega}_{1i}(\theta, \beta) - \hat{\omega}_{0i}(\theta, \beta))}. \]

Since the covariates \( \{X_i\}_{i=1}^n \) are iid we can write the joint likelihood as the product of the marginal likelihoods. We can then define the estimator of \( \theta \) (for a fixed value of \( \beta \)) as the minimizer of the standard logit likelihood function:

\[ \ln L_n(\theta, \beta) = -\sum_{i=1}^n (h_i \ln \hat{p}_i(\theta, \beta) + (1 - h_i) \ln(1 - \hat{p}_i(\theta, \beta))). \]

### 3.3. Matching Probabilities

In this section, we consider the role of frictions, or the magnitude of \( \beta \) relative to the variance of \( \eta \), in shaping matching patterns between workers and firms. Note that these frictions play no role in determining firm preferences, or which firm types prefer high education.\(^{40}\) Nevertheless, because the frictions do affect sorting patterns, they are of considerable importance to workers when they decide how much to invest in education.

In the following example, we will suppose that that the set of firms that prefer education level \( h_j \), \( M_j^+ \), contains at least two types of firms, \( m \) and \( \bar{m} \) with \( k_m \neq k_{\bar{m}} \). Suppose we fix \( N_j \), the number of workers who chose education level \( h_j \), at some \( n_j \) and we fix \( N^{(j)} \), the number of firms who prefer highly-educated workers at some \( n^{(j)} \) such that \( n_j + 1 < n^{(j)} \). In this situation, there are strictly more firms who prefer type \( h_j \) workers than there are workers of this type. Let \( \kappa = n^{(j)} - n_j + 1 \), and denote \( p_{mk} \equiv P(v_m > v(\kappa)) \) for each \( m \) in \( M_j^+ \). Proposition 6.2 says that the difference in the probability of matching to a type \( \bar{m} \) versus a type \( m \) firm at these values of \( n_j \) and \( n^{(j)} \) in such a situation is given by

\[ (p_{\bar{m}\kappa} - p_{mk}) q_{m \kappa}^+ / c_{\kappa} + p_{mk} (q_{m \kappa}^+ - q_{\bar{m}\kappa}^+ ) / c_{\kappa}, \]

\(^{39}\)When \( \beta \) is fixed, maximizing the likelihood by computing the \( f_j(\theta, \beta) \)'s for each candidate value of \( \theta \) can be slow. The following strategy can be used to estimate \( \theta \) for fixed \( \beta \) more quickly provided that the support of \( K \) is not too large. First, note that \( \theta \) enters \( f_j(\theta, \beta) \) only through the set of firm types that prefer education level \( h_j \), \( M_j^+(\theta) \). Given our assumptions on the production function and firm preferences, \( M_j^+(\theta) \) must take one of \( M+1 \) possible values. Therefore, for fixed \( \beta \), we can avoid simulating \( f_j(\theta, \beta) \) for each candidate value of \( \theta \) by pre-allocating the \( \hat{f}_j(\theta)'s \) and \( P_{h_j, n_j, n^{(j)}}(m)'s \) for each of the \( M+1 \) cases for \( M_j^+(\theta) \). It then suffices to evaluate \( M_j^+(\theta) \), select the appropriate dimension of the array of terms, then assemble the terms according to equation 25.

\(^{40}\)We discuss the role of firm preferences on matching patterns at the end of Section 2.2.
with
\[ c_κ \equiv \sum_{m \in M_j^+} p_{mk} q_m^+, \]
where \( q_m^+ = q_m / \sum_{m \in M_j^+} q_m \). Under Assumption 3.2, the case of \( β = 0 \) gives us that \( p_{mk} = p_{mk} \), implying that the first term in the parentheses of equation 16 is zero. This means that when matching frictions are highest (i.e., when \( β = 0 \)), the difference in the probability of matching to one type of firm that prefers \( h_j \) over another is captured by the relative prevalence of those types of firms in the economy.

In the case that \( β > 0 \), under Assumption 3.2, \( p_{mk} - p_{mk} \) becomes larger as \( k_m - k_m \) becomes larger. This means that higher capital firms have a better chance of matching with the high education workers when \( β > 0 \). On the other hand, in the case that \( n_j + 1 > n_j \) (i.e., \( h_j \) is demanded by fewer firms than there are in the economy), then the above probabilities are independent of firm capital and \( β \) once again depend solely on the relative prevalence of the each type of firm.

4. Analysis of a Labour Matching Market in Canada

4.1. Background

In this section, we investigate the role of a labour matching technology in shaping education and wage patterns for the Canadian economy. Since the 1980’s, many scholars studying the Canadian economy have focused on the rise of wage inequality, contrasting it to similar trends in the United States and Western Europe Fortin et al. (2012); Saez and Veall (2005); Lemieux (2008). To explain this recent growth in wage inequality, some have emphasized forces on the demand side of the labour market, such as increasing wage premia for highly-skilled workers Boudarbat and Riddell (2010), and a declining demand for jobs in the middle of the skill distribution Green and Sand (2015). Another set of explanations emphasizes the role of institutional changes and government policy; namely, the effect of minimum wages and falling unionization rates Fortin et al. (2012); Lemieux (2008). One less-explored cause of this inequality may lie in the underlying process by which firms find workers to hire. This paper emphasizes such a channel, focusing on the role of the labour market matching mechanism in recent wage patterns in Canada. As in the study of the German labour market in Card, Heining, and Kline (2013), this channel considers the role that sorting patterns play in wage patterns, with a particular focus on the process by which firms find workers to hire.
4.2. Data

The matched employer-employee data we consider come from the Workplace Employee Survey (WES) of Statistics Canada (Statscan). WES is a longitudinal survey of Canadian firms and the workers they employ. WES allows researchers to study how the characteristics and outcomes of workers and firms are related. Thus, WES goes beyond other surveys that track only one side of the market, such Labour Force Survey (LFS) in the case of workers, or the Logitudinal Employment Analysis Program (LEAP) in the case of firms. Furthermore, the WES allows, in more detail than in previous surveys, to understand how firms adopted new technology and what the impacts of this was (Statscan). The WES is especially rich in terms of information concerning worker-firm bargaining, outside options, and technology use. As the 2006 release only contains employer data, I only consider WES panels for the years 1999-2005.

WES only collects data on firms and workers in Canadian provinces whose information was obtainable from Statcan’s Business Register. The target population of the study was all non-governmental firms aside from agricultural and religious organizations. Furthermore, the focus was only on firms that hired more than one worker (who was not the owner or the employer). A firm employee is defined as a person associated with that firm who is working or on paid leave in March of the survey year who receives a T-4 slip from Canada Revenue Agency.

The workplace component of WES was conducted from 1999-2006. The firms were followed throughout the course of the study. Every two years, a sample of firms which are new to the Business Register are added to the base sample. The employee component of WES was conducted from 1999-2005. In each workplace survey firm that employs more than four workers, up to 24 workers are randomly sampled. All firms with fewer than four workers are included in the sample. Workers are only followed for two years in the workplace survey. For this reason, every second year, workers are resampled from the firms.

WES data has been used by other researchers. Dionne and Dostie (2007) use WES data from 1999-2002 to study the impact of work arrangements on employee absenteeism. Dostie and Jayaraman (2008) investigate the role of computer use on firm productivity gains. Pendakur and Woodcock (2010) study the extent to which immigrant and minority access to high-paying jobs is determined by barriers to becoming hired at high-paying firms.
4.3. Model Estimates and Counterfactuals

In this section, we explore the evolution of the matching technology and preferences in two industries from the WES sample: Secondary Products Manufacturing (WES industry 4), and the Finance and Insurance (WES industry). Table 3 reports results for the matching technology for a subgroup of higher skilled workers: namely, managers (WES occupation category 1) and professionals (WES occupation category 2), while tables 4 and 5 report preference estimates for the same subgroup of workers for the manufacturing and finance industries respectively. The estimates of preferences are reported at the minimum distance estimate of $\beta$. We consider the two specifications for the expected wage equation 7 that are found to behave reasonably in the simulation studies of Section 6.3. Specification 1 uses the production function where worker and firm types are multiplicative while the production function in Specification 2 is additive. The results in this section provide similar insights on parameter inference. However, the results from 6.7 illustrate the importance consequences of production function interactions in wage inequality.

The results for the matching technology suggest a period of low frictions in 1999-2000, followed by an increase in frictions. Whereas the frictions remain high towards the end of the sample in the finance industry, the frictions fall in the manufacturing industry towards the end of the sample (2004-2005). Note that the standard errors are so small in general that we report confidence intervals for $\beta$ using the plug-in values of $\hat{\theta}$ (that is, we are not strictly taking into account estimation error of $\hat{\theta}$ into account).

In the preference estimates for both industries, we typically estimate $\theta_2$ - the coefficient on female$_i$ in the worker’s outside option function - to be negative. The estimated coefficients on $\theta_3$ (marital status), and $\theta_4$ (number of dependent children) are less conclusive. In both industries, $\theta_1$ is found to be largest towards the end of the sample - 2004 in the case of the manufacturing industry, 2005 in the finance industry. The production technology appears more stable in the finance industry than it does in the manufacturing industry over time.

In section 6.7, we use the structural model developed in this paper along with the structural estimates of section to 6.6 to generate statistics from two key counterfactual distributions of interest: wages and education. We consider the counterfactual implications of different (in-sample) estimated levels of matching frictions. For example, we can see what level of inequality would have prevailed in 1999 if the matching frictions had been as low as they were in 2005. Counterfactuals education levels in the two specifications are generated in a similar way as the outcome variables generated in the Monte Carlo study from in Section 6.3, and the wage is generated using these values along with
the simulated matchings (that involve iid draws of \( \eta \)). Here, of course, we use the relevant covariates, firm capital data, and parameter estimates for each of the cell in the tables. Overall, the results highlight the important role that matching technology and the production complementarities play in educational decisions and wage patterns.

Tables 6 and 7 shows results for counterfactual education levels. Production complementarities lead to higher investment in education in both cases, but appear to matter more in the finance industry. For instance, in 1999, the effect of switching to a multiplicative production function from an additive one increases the equilibrium investment in education by about 2% in the manufacturing industry but by about 5% in finance industry. We also see the substantial role that both preferences and the matching technology play in the decision to obtain higher education. In the manufacturing sector in 1999 (a high \( \beta \) year), the effect of switching to the matching technology from 2001 causes a fall in the equilibrium probability of attending college by roughly 8%. Overall, however, there is evidence that - taken together - changes to preferences (including the parameter in the production function) - matter much more to the worker’s college decision than changes in the matching frictions. For example, in the year 2001, the effect of switching to 1999’s preference parameter is a fall in the probability of investing in education of almost 20%.

Tables 10 and 11 report counterfactual (weighted) Gini coefficients for the WES sample years along with two counterfactual levels: maximal frictions (\( \beta = 0 \)) and very low frictions (\( \beta = 5 \)). The Gini coefficients in the row \( \hat{\beta}_{year} \) were simulated from the equilibrium of the model taking the exogenous variables and preference estimates from that year.

In Tables 10 and 11 we see that in both industries, inequality is typically much higher in the case with production complementarities (Specification 1). In the finance industry in Specification 2, the effect of lowering matching frictions raises wage inequality in every year. In this case, the sorting effect raises inequality and dominates the inequality-lowering effects of a greater supply of highly educated workers. In other cases, however, the effect is ambiguous. In Specification 1 in the manufacturing industry, the level of inequality at the estimated value of the frictions is lower than at the counterfactual levels for most years (except 2002). For example, in 2005 the simulated Gini is 0.222 and the investment in education is 77%. This rises to 0.2507 (education investment 76%) when information frictions are highest and 0.2452 (education investment 85%) when frictions are lowest. The opposite is the case in Specification 1 in the finance industry, where the level of inequality at the estimated value of the frictions is higher than at the counterfactual levels for each year (except 1999).
5. Conclusion

This paper presents an empirical strategy for studying wages and education in a labour market where the decisions of workers matter in the matching process. In particular, I perform inference on a labour market matching technology using matched employer-employee data. I demonstrate the feasibility of my approach in the case that the worker faces a choice between two education levels.

The methodology I develop in this paper can be extended in a number of useful directions. Although the data used in this paper did not include information on firms’ profit, Bartolucci and Devicienti (2012) have shown that such data is useful for investigating sorting. Another natural extension of the current setup is to consider the role that heterogeneity in the worker’s bargaining strength plays in driving wage variation.

A number of intriguing extensions to the model would prove much more challenging. One limitation of the current approach is its reliance on cross-sectional variation alone for inference. In effect, useful information concerning unemployment and job-to-job transitions by workers is unused in my framework.

This paper has also demonstrated how the decision to invest in education - and wage inequality - is sensitive to the presence of a particular source of matching frictions in the economy. Although firm capital is exogenous in this paper, the role of information frictions on capital accumulation in an extended framework could be a fruitful way to study not only wage inequality, but also economic growth.

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6. Appendix

6.1. Equilibrium of Investment and Matching Game

In this section, we characterize the equilibrium of the incomplete information game of Section 2.2. First, we introduce a representation of the worker’s expected utility function that proves useful for establishing the existence of the Bayesian Nash equilibrium of the game as a fixed point of a best probability response operator. We begin by defining relevant terms. A profile of strategy functions (or decision rules) is

\[
\sigma = \{\sigma_i(x_i, \varepsilon_i) : i \in N_h\},
\]

where the functions \(\sigma_i : \mathcal{X} \times \mathbf{R}^{j-1} \to \mathcal{H}\). The conditional probability that a worker with covariates \(x_i\) chooses action \(h_i\) can be written

\[
P_i(h_i|x_i, \sigma_i) \equiv \int 1\{\sigma_i(x_i, \varepsilon_i) = h_i\} dF(\varepsilon_i).
\]

Since \(X_i\)’s are private information in this model, each agent \(i\) must take expectations with respect to the distribution of \(X_{-i}\). The following result shows that under the independence assumptions embodied by Assumption 2.1, the agent’s expected utility has a very convenient form - it is only affected by the behaviour of the other agents through the choice probabilities.
Lemma 6.1. In the model of Section (2.2) and Assumptions 2.1 and 2.2, we can represent the first term in the expected utility of agent $i$ from equation 4 as

$$
\tilde{U}_i(h_i, x_i, \sigma) = \sum_{h_{-i} \in \mathcal{H}_{-i}} \tilde{u}_i(h_i, h_{-i}, x_i) \prod_{j \neq i} P_j(h_j|\sigma_j).
$$

Proof. First, we write equation 4 as

$$
\tilde{U}_i(h_i, x_i, \sigma) = \sum_{x_{-i} \in \mathcal{X}_{-i}} \sum_{h_{-i} \in \mathcal{H}_{-i}} \tilde{u}_i(h_i, h_{-i}, x_i) P_{-i}(h_{-i}|x_{-i}, \sigma) P(x_{-i}),
$$

where $x_{-i} = (x_j)_{j \in \mathcal{N}_h \setminus \{i\}}$ and we use the shorthand $P(x_{-i}) \equiv P(X_{-i} = x_{-i})$. Without loss of generality, let $i = 1$. Then we write $\tilde{U}_1(h_1, x_1, \sigma)$ as

$$
\sum_{h_{-1} \in \mathcal{H}_{-1}} \sum_{x_2 \in \mathcal{X}} \ldots \sum_{x_n \in \mathcal{X}} \tilde{u}_1(h_1, h_{-1}, x_1) P_{-1}(h_{-1}|x_2, \ldots, x_n, \sigma) \prod_{j=2}^{n_h} P_j(x_j),
$$

where we used the independence of $X_i$’s from Assumption 2.1. Next, since Assumption 2.1 says that $X_i$’s and $\varepsilon_i$’s are independent, we know that the actions of each of the agents are independent and depend only on their personal value of $X_i$ and $\varepsilon_i$. Therefore,

$$
P(h_{-1}|x_2, \ldots, x_n, \sigma) = \prod_{j=2}^{n_h} P_j(h_j|x_2, \ldots, x_n, \sigma_j) = \prod_{j=2}^{n_h} P_j(h_j|x_j, \sigma_j).
$$

Plugging (19) back into (18) yields that $\tilde{U}_1(h_1, x_1, \sigma)$ is equal to

$$
\sum_{h_{-1} \in \mathcal{H}_{-1}} \tilde{u}_1(h_1, h_{-1}, x_1) \sum_{x_2 \in \mathcal{X}} \ldots \sum_{x_n \in \mathcal{X}} \prod_{j=2}^{n_h} P_j(h_j|x_j, \sigma_j) \prod_{j=2}^{n_h} P_j(x_j).
$$

Grouping the sums in (20) and restoring the generic $i$ index gives

$$
\sum_{h_{-i} \in \mathcal{H}_{-i}} \tilde{u}_i(h_i, h_{-i}, x_i) \prod_{j \neq i} \sum_{x_j \in \mathcal{X}} P_j(h_j|X_j = x_j, \sigma_j) P_j(x_j)
$$

and hence we have the desired result. \qed

We will show the existence of the equilibrium for our model. The solution concept for the game described in Section 2.2 is Bayesian Nash Equilibrium (BNE), which we now define.

Definition 6.1. A Bayesian Nash Equilibrium (BNE) of the game described in Section (2.2) is a profile of decision rules $\sigma^*$ such that for any player $i$ and for any $(x_i, \varepsilon_i)$:

$$
\sigma^*_i(x_i, \varepsilon_i) = \arg\max_{\hat{h}_i \in \mathcal{H}} \{U_i(h_i, x_i, \varepsilon_i, \sigma^*)\}.
$$

The notation and arguments in this section follow Aguirregabiria and Mira (2016), but we include them here for completeness. Under Assumption 2.2, we write the expected
utility of $i$ as

$$U_i(h_i, x_i, \sigma, \varepsilon_i) = \tilde{U}_i(h_i, x_i, \sigma) + \varepsilon'_i d(h_i).$$

By Lemma 6.1 we can express the first term on the right hand side of the preceding equation as

$$\tilde{U}_i(h_i, x_i, \sigma) = \sum_{h_{-i} \in \mathcal{H}_{-i}} \tilde{u}_i(h_i, h_{-i}, x_i) \prod_{j \in \mathcal{N} \setminus \{i\}} P_j(h_j | \sigma_j).$$

Note that $\tilde{U}_i(h_i, x_i, \sigma)$ only depends on the choices of other agents through the choice probabilities of the other players that are induced by $\sigma$. We write the choice probabilities of the people other than $i$ as $P_{-i} \equiv \{P_j(h_j) : (j, h_j) \in N \setminus \{i\} \times \mathcal{H} \setminus \{1\}\}$. For any $P_{-i}$, we can define a best response probability function as:

$$\tilde{\Psi}_i(h_i | x_i, P_{-i}) \equiv \int \{\argmax_{h_i \in \mathcal{H}} \tilde{U}_i(h_i, x_i, \sigma) + \varepsilon'_i d(h_i) = h_i\} dF(\varepsilon_i).$$

$\tilde{\Psi}_i$ tells us the probability that a particular action is optimal for $i$ with covariates $x_i$ when others choose according to probabilities $P_{-i}$. Let

$$\Psi_i(h_i | P_{-i}) = \sum_{x_i \in \mathcal{X}} \tilde{\Psi}_i(h_i | x_i, P_{-i}) P(x_i).$$

An equivalent to Definition 1.21 is that the equilibrium probabilities, $P^\ast \equiv P(\sigma^\ast)$, satisfy the fixed point constraint, $P^\ast = \Psi(P^\ast)$, where $\Psi$ is the best response probability mapping:

$$(22) \quad \Psi(P) = \{\Psi_i(h_i | P_{-i}) : (i, h_i) \in N \times \mathcal{H} \setminus \{1\}\}.$$  

**Lemma 6.2.** Under Assumption 2.1 and Assumption 2.2 the game described in Section (2.2) has a Bayesian Nash Equilibrium.

**Proof.** Since $\Psi(\cdot)$ maps from a compact convex set, $[0, 1]^{n \times (J-1)}$, to itself and is continuously differentiable (by the continuity of $\varepsilon_i$’s (Assumption 2.1)), $\Psi(\cdot)$ has a fixed point by Brouwer’s fixed point theorem. \hfill \Box

We can appeal to a result of Kellogg (1976) to show that the Bayesian equilibrium is unique under a mild condition on the derivatives of the best response probability mapping. Let $I_n$ be an identity matrix of size $n$. Kellogg’s result - as stated in Konovalov and Sándor

\footnote{Note that when $\varepsilon_i$’s have the extreme value distribution (as in Assumption 3.2) then we have $\tilde{\Psi}_i(h_i | x_i, P_{-i}) = \frac{\exp(\tilde{U}_i(h_i, x_i))}{\sum_{j=1}^J \exp(\tilde{U}_i(h_j, x_i))}$.}
(2010) stipulates that the equilibrium is unique if the determinant of \( \tilde{J}_n \equiv \partial \Psi(P)/\partial P' - I_n \) is nonzero and the mapping \( \Psi \) has no fixed points on the boundary of \([0,1]^{n\times(J-1)}\). In our setup, the former condition holds provided that the best response of any agent is not excessively sensitive to a change in the probability of any other agent.\(^{42}\)

### 6.2. Additional Mathematical Results

The remaining results of this section allow us to represent the matching probabilities from equation 14, hence workers’ expectations, in a convenient way. These representations can then be used to estimate \( \theta(\beta) \) using maximum likelihood.

**Proposition 6.1.** Suppose that \( J = 2 \), that Assumptions 2.1, 2.2, 3.1, and 3.2 hold. Then for each \( m = 1, \ldots, M \) and each \( h_j \in \mathcal{H} \) the conditional probability of an arbitrary worker \( i \) matching to firm with capital class \( m \) after choosing education level \( h_j \) is

\[
\pi_{m_j} = \sum_{n_{h_j} = 0}^{n_{h_j} - 1} P(\mathcal{M}(i) = m | H_i = h_j, N_j = n_j) B(n_j; n_h - 1, p_j),
\]

where \( N_j \) is the number of workers other than \( i \) who picked education level \( h_j \), \( B(n_j; n_h - 1, p_j) \) is the binomial p.m.f. and \( p_j = P(H_i = h_j) \).

**Proof.** Part (a) of Assumption 3.1 that says firms do not consider the workers’ covariates when ranking them in the matching process. This means that for each \( m = 1, \ldots, M \) we have that

\[
P(\mathcal{M}(i) = m | H_i = h_j, H_{-i} = h_{-i}, X_i = x_i) = P(\mathcal{M}(i) = m | H_i = h_j, H_{-i} = h_{-i}).
\]

Combining this with equation 14, we can write

\[
\pi_{m_j}^{(i)} = \sum_{h_{-i} \in \mathcal{H}_{-i}} P(\mathcal{M}(i) = m | H_i = h_j, H_{-i} = h_{-i}) P(H_{-i} = h_{-i} | X_i = x_i).
\]

\(^{42}\)Under our assumptions, \( \tilde{J}_n \) is a matrix with \(-1\)'s on the diagonal and \( \tilde{p} \equiv \partial \Psi_i/\partial p_j \) for all \( i \neq j \) on the off diagonals. Using the fact that \( \tilde{J}_n \) is a circulant matrix, we can express its determinant explicitly as \( (\tilde{p}(n - 1) - 1) (-1 + \tilde{p})^{n-1} \). A sufficient condition for \( \det(\tilde{J}_n) \neq 0 \) is thus \( \tilde{p} < 1/(n - 1) \).
Next, it is straightforward to see that

\[(24) \quad P(H_{-i} = h_{-i}|X_i = x_i) = \prod_{j \neq i}^n P_j(h_j).\]

Since \(\varepsilon_i\) are identically distributed by Assumption 2.1, for each \(j\) and \(m\), have \(\pi^{(i)}_{m,j} = \pi_{m,j}\).

When there are only two education levels, any \(h_{-i} \in H_{-i}\) can be represented as a total number of workers other than \(i\) who picked education level \(h_j, n_j\). From worker \(i\)'s point of view, \(n_j\) is a particular realization of the random variable \(N_j\) that takes values in the set \(\{0, ..., n_h - 1\}\). Since there are \(n_h - 1\) agents other than \(i\) in the economy, the sum over \(h_{-i} \in H_{-i}\) amounts to a sum over the support of \(N_j\). Now consider any \(n_j\) in the support of \(N_j\). The assumption that \(\varepsilon_i\)'s are iid implies that the probability that exactly \(n_j\) out of \(n_h - 1\) workers pick \(h_j\) can be represented as

\[
\frac{(n_h - 1)!}{n_j!(n_h - 1 - n_j)!} p_j^{n_j}(1 - p_j)^{n_h - 1 - n_j},
\]

which is the binomial probability mass function, \(B(n_j; n_h - 1, p_j)\).

\[\square\]

**Characterization of Matching Probabilities when \(J\) equals 2**

When \(J = 2\) we can partition the types of firms, \(m = 1, ..., M\) into two sets: those who prefer \(h_j \in H\) and those who prefer \(h_{j'}\) with \(j' \neq j\). It is convenient to introduce the

\[\text{This can be shown using the same arguments as those in Lemma 6.1. The private information and independence of } X_i\text{'s (Assumption 2.1) implies that the left hand size of 24 equals}

\[(23) \quad \sum_{x_{-i} \in X_{-i}} P(h_{-i}|x_{-i}, x_i)P(x_{-i}|x_i) = \sum_{x_{-i} \in X_{-i}} P(h_{-i}|x_{-i})P(x_{-i}).\]

Suppose without loss of generality that \(i = 1\). It is convenient to rewrite the above as follows (using independence):

\[
\sum_{x_2 \in X} ... \sum_{x_n \in X} P(h_{-1}|x_2, ..., x_n) \prod_{j=2}^{n_h} P_j(x_j).
\]

Next, since \(X_i\)'s and \(\varepsilon_i\)'s are independent across \(i\) and each \(i\)'s strategy function is only a function of \(X_i\) and \(\varepsilon_i\) we have

\[
P(h_{-1}|x_{-1}) = \prod_{j \neq 1}^n P_j(h_j|x_{-1}) = \prod_{j \neq 1}^n P_j(h_j|x_j).
\]

Combining these two results we write 23 as

\[
\sum_{x_2 \in X} P_j(h_2|x_2)P_j(x_2) ... \sum_{x_n \in X} P_n(h_n|x_n)P_n(x_n) = \prod_{j \neq 1}^n P_j(h_j).
\]
following notation:

\[ M_j^+(\theta) = \{ m \in \{1, \ldots, M \} : \rho(k_m, h_j; \theta) \geq \rho(k_m, h_{j'}; \theta), j \neq j' \}, \text{ and} \]

\[ M_j^-(\theta) = \{1, \ldots, M \} \setminus M_j^+, \]

recalling that firm preferences are given in 8. The firm classes that prefer \( h_j \) are pinned down by the functional form for firm preferences, \( \rho \), along with the preference parameter, \( \theta \), and the distribution of \( X_i \). Furthermore let us denote

\[ \pi_{mj} \equiv \sum_{n(j)=0}^{n_j-1} \sum_{n_j=0}^{n_h-1} P_{h_j, n_j, n(j)}(m) P(n_j) P(n^{(j)}; \theta), \]

where

\[ P_{h_j, n_j, n(j)}(m) = P(M(i) = m | h_i = h_j, N_j = n_j, N^{(j)} = n^{(j)}). \]

Note that this object depends on both \( \beta \) and \( \theta \) through the matching function. For each firm type \( m = 1, \ldots, M \) let \( F_m \equiv N(\beta k_m, \sigma^2) \), and define the following for each education choice \( h_j \):

\[ G_{j+} \equiv \sum_{m \in M_j^+} q_m F_m \text{ and } G_{j-} \equiv \sum_{m \in M_j^+} q_m F_m. \]

Furthermore, define the posterior firm types as follows:

\[ q_{m+} \equiv q_m / \sum_{m \in M_j^+} q_m \text{ and } q_{m-} \equiv q_m / \sum_{m \in M_j^+} q_m. \]

We also define \( v_{(b_1, b_2; F)} \) as the \( b_1 \)-order statistic of \( b_2 \) random variables independently distributed according to cdf \( F \). Propositions 6.2 and 6.3 are characterizations of \( P_{h_j, n_j, n(j)}(m) \)'s of the model in the case that \( J = 2 \) and \( n_h = n_f = n \).

When considering these results, it is important to recall one core feature of the matching model as we outline it in Section 2: that there is no unemployment. Therefore, when reading the arguments, the reader should take for granted the fact that the probability that each worker matches to some firm occurs with probability one.

**Proposition 6.2.** (Heterogeneous firm preferences). Denote \( \bar{n}_j \equiv n_j + 1 \) and suppose that \( n_h = n_f = n \). Then under the assumptions of Proposition 6.1 we have the following for any \( n_j \) such that \( 1 \leq n_j \leq n \) and \( n^{(j)} \) such that \( 0 < n^{(j)} < n \):

1) For each \( m \in M_j^+ \),

\[ P_{h_j, n_j, n(j)}(m) = \begin{cases} 
q_{m+} n^{(j)}/\bar{n}_j & \text{if } \bar{n}_j \geq n^{(j)} \\
\frac{P(v_m > \theta) q_{m-}^+}{\sum_{m \in M_j^+} P(v_m > \theta) q_{m-}} & \text{if } \bar{n}_j < n^{(j)}.
\end{cases} \]
where \( \hat{v} \equiv v(a,b,F) \) with \( a = n^{(j)} - \bar{n}_j, b = n^{(j)}, \) and \( F = G_{j+}. \)

ii) For each \( m \in M^{-}_j, \)

\[
P_{h_j,n_j,n^{(j)}}(m) = \begin{cases} 
\frac{P(v_m < \hat{\vartheta}) q_m^-(n_j - n^{(j)})}{\Sigma_{m \in M^{-}_j} P(v_m < \hat{\vartheta}) q_m} & \text{if } n_j > n^{(j)}, \\
0 & \text{if } n_j \leq n^{(j)},
\end{cases}
\]

where \( \hat{v} \equiv v(a,b,F) \) with \( a = \bar{n}_j - n^{(j)} + 1, b = n - n^{(j)}, \) and \( F = G_{j-}. \)

**Proof.** We begin by introducing some notation. We denote the event that a worker who chose education level \( h_j \) matches to any firm of type \( m \in M^+_j \) or \( m \in M^-_j \) as \( M^+_{ij} \) and \( M^-_{ij} \) respectively.\(^{44}\)

First, we consider the probability that a worker who chose \( h_j \) matches to any firm in the class \( m \in M^+_j. \) Consider the case that \( n_j \geq n^{(j)}. \) In this case, there are at least as many workers who chose \( h_j \) as firms that prefer \( h_j. \) Given that Condition IR implies that no worker or firm will never unilaterally dissolve a match to become unmatched, the case of \( n_j \geq n^{(j)} \) implies that every firm in class \( m \) who wants a worker with \( h_j \) will hire one in the matching process. For each class of firm \( m \in M^+_j, \) the probability that a worker who chose \( h_j \) matches to a firm in the set of firms that prefers \( h_j \) and to the particular class \( m \in M^+_j \) is given as follows when \( n_j \geq n^{(j)}: \)

\[
P_j(M_i = m, M^+_{ij}) = P_j(M_i = m | M^+_{ij}) P_j(M^+_{ij}),
\]

where the \( j \)-subscript on the probabilities denote a probability conditional on the event \( H_i = h_j. \) \( P_j(M^+_{ij}) \) is equal to \( n^{(j)}/n_j \) because workers with the same \( h_j \) are indistinguishable to the firms that prefer them, so firms choose among these workers at random. The probability of matching to a firm of type \( m \in M^+_j \) given that the worker has already matched to some firm in \( M^+_j \) is equal to the relative proportion of type \( m \) firms in this category, \( q_m^{(j)}. \)

Next, we consider the case that \( n_j < n^{(j)}. \) Since there are strictly more firms that prefer \( h_j \) than workers who chose \( h_j, \) the probability that a worker who chose \( h_j \) matches to a firm that prefers workers with \( h_j \) occurs with probability one; that is \( P_j(M^+_{ij}) = 1. \)

Although \( P_j(M^+_{ij}) = 1, \) only the firms with the \( n_j \) largest \( v \)-indices will be able to match with a worker who chose \( h_j. \) Thus, a firm in \( M^+_{ij} \) matches to a worker with \( h_j \) if and only

\(^{44}\)That is, \( M^+_{ij} \equiv \{ M_i \in M^+_j \} \) and similarly for \( M^-_{ij} \equiv \{ M_i \in M^-_j \}. \)

\(^{45}\)This follows from Condition IR and the following two facts: i) \( h_j \) workers are scarce relative to the firms that prefer them ii) firms that prefer \( h_j \) will never choose a \( h_j \) worker in the matching process since the condition \( n_h = n_f = n \) and \( J = 2 \) implies that \( h_j \) workers are always available (i.e., when \( n_h = n_f = n, n^{(j)} > n_j \) implies that \( n_{j'} > n^{(j')} \), since \( n_{j'} = n - n_j \) and \( n^{(j')} = n - n^{(j)}). \)
if its $v$ statistic exceeded the $\kappa = n^{(j)} - \hat{n}_j$ order statistic among all $n^{(j)}$ firms in $M^+_j$. Thus, by Assumptions 3.1 and 3.2, the probability that a worker who chose $h_j$ matches with a firm from class $m \in M^+_j$ conditional on matching to some firm in $M^+_j$ is

\[
P(v(K) = v(k_m)|v(K) > \hat{v}, m \in M^+_j),
\]

which by Bayes’ rule equals

\[
\sum_{m \in M^+_j} P(v(K) > \hat{v}|v(K) = v(k_m), m \in M^+_j) P(v(K) = v(k_m)|m \in M^+_j),
\]

(27)

where $\hat{v} \equiv v_{(\kappa,n^{(j)};G_{+j})}$. Equation 27 represents the relative proportion of type $m$ firms represented among threshold crossers among all firms that prefer $h_j$. We next consider the probability of matching to each firm with $m \in M^-_j$. We consider first the case that $\bar{n}_j > n^{(j)}$. The relevant probability is

\[
P_j(\mathcal{M}_i = m, M^-_{ij}) = P_j(\mathcal{M}_i = m|M^-_{ij}) P_j(M^-_{ij}) = P_j(\mathcal{M}_i = m|M^-_{ij})(1 - n^{(j)}/\bar{n}_j).
\]

As stated above, the of case $\bar{n}_j > n^{(j)}$ combined with our assumption that $n_h = n_f = n$ implies that $n^{(j')} > n_{j'}$, since $n_{j'} = n - \bar{n}_j$ and $n^{(j')} = n - n^{(j)}$. Therefore by similar logic to before, firms who prefer $h_{j'}$ match to workers with $h_j$ if their $v$-index is lower than the $n^{(j')} - n_{j'} + 1 = \bar{n}_j - n^{(j)} + 1$ order statistic among those firms in $M^-_j$. Letting $\kappa \equiv \bar{n}_j - n^{(j)} + 1$, the probability of a worker who chose $h_j$ matching to a type $m \in M^-_j$ firm conditional on matching to some firm in $M^-_{ij}$ is given as the proportion of type $m$ firms whose $v$ index falls below this threshold:

\[
P_j(\mathcal{M}_i = m|M^-_{ij}) = \frac{P(v_m < \hat{v})q^-_m}{\sum_{m \in M^-_j} P(v_m < \hat{v})q^-_m},
\]

where $\hat{v} \equiv v_{(\kappa,n^{(j')};G_{-j})}$. Lastly, in the case that $\bar{n}_j \leq n^{(j)}$, $P(M^-_{ij}) = 0$. This completes the proof. 

Next we define $G \equiv \sum_{m=1}^M F_m q_m$. Proposition 6.3 characterizes the matching probabilities in the case that all firms types prefer one level of education; that is, in the case that firm preferences are homogeneous over worker education types. The arguments are abridged, since they are very similar to those used in the proof of Proposition 6.2.

**Proposition 6.3.** (Homogeneous firm preferences). Suppose that $n_h = n_f = n$. Then under the assumptions of Proposition 6.1 we have the following for the cases that $n^{(j)} = n$ and $n^{(j)} = 0$.

(1) if $n^{(j)} = n$, then $M^-_j = \emptyset$ and for each $m \in M^+_j = M$ we have
\[ P_{h_j,n_j,n(j)}(m) = \begin{cases} q_m & \text{if } \bar{n}_j = n \\ \frac{P(v_m > \hat{v}) q_m}{\sum_{m \in M} P(v_m > \hat{v}) q_m} & \text{if } \bar{n}_j < n, \end{cases} \]

where \( \hat{v} \equiv v(a_1, a_2; G) \), with \( a = n - \bar{n}_j \) and \( b = n \).

(2) If \( n^{(j)} = 0 \), then \( M^+_j = \emptyset \) and for each \( m \in M^-_j = M \) we have

\[ P_{h_j,n_j,n(j)}(m) = \begin{cases} q_m & \text{if } \bar{n}_j = n \\ \frac{P(v_m < \hat{v}) q_m}{\sum_{m \in M} P(v_m < \hat{v}) q_m} & \text{if } \bar{n}_j < n, \end{cases} \]

where \( \hat{v} \equiv v(a_1, a_2; G) \), with \( a = \bar{n}_j + 1 \) and \( b = n \).

Proof. When \( n^{(j)} = n \) and \( \bar{n}_j = n \) the probability of matching to firm \( m \) is simply equal to the marginal probability of that firm type in the economy, \( q_m \). When \( n^{(j)} = n \) and \( \bar{n}_j < n \), using logic identical to that employed in the proof of Proposition 6.2, we conclude that the probability of matching to a firm from class \( m \) is equal to the proportion of type \( m \) firms above the \( n - \bar{n}_j \) order statistic of the \( v \)'s.

When \( n^{(j)} = 0 \), we must have \( \bar{n}_j > n^{(j)} = 0 \) (since at least one person is assumed to choose \( h_j \)). Since the top \( n_j' = n - \bar{n}_j \) ranked firms in terms of \( v \) receive a worker with their preferred education, \( h_{j'} \), the probability of matching to a firm in class \( m \) is equal to the proportion of type \( m \) below the \( \bar{n}_j + 1 \) order statistic of the \( v \)'s. \( \square \)

The following result takes for granted a well-known fact that uniform order statistics follow the Beta distribution.

Lemma 6.3. Let: i) \( \{X_i\}_{i=1}^n \) be iid random variables from continuous distribution function \( G \); ii) \( Z \) be normally distributed with mean \( \mu \) and variance \( \sigma^2 \); iii) \( X^{(i)}_i \) be the \( i \)-th order statistic of \( \{X_i\}_{i=1}^n \); iv) \( U^{(i)}_i \) be the \( i \)-th order statistic of iid uniform random variables \( \{U_i\}_{i=1}^n \). Then,

\[ P(Z \geq X^{(i)}_i) = 1 - \mathbb{E} \Phi((G^{-1}(U^{(i)}_i) - \mu)/\sigma), \]

where \( \Phi(\cdot) \) is the standard normal cdf, and \( \mathbb{E}(\cdot) \) is taken over the distribution of \( U^{(i)}_i \), which follows the Beta distribution with parameters \( i \) and \( n + 1 - i \).

Proof. Note that since \( X_i \)'s are continuously distributed according to \( G \) it follows from the probability integral transformation result that for each \( i \)

\[ X_i = d G^{-1}(U_i). \]

\[ \text{For example, see Chapter 2 Ahsanullah and Shakil (2013).} \]
Also, since $G$ is monotone we have that for each $i$

$$X_{(i)} = G^{-1}(U_{(i)}).$$

The previous line implies that

$$P(Z \geq X_{(i)}) = P(Z \geq G^{-1}(U_{(i)})) = 1 - P(Z \leq G^{-1}(U_{(i)})) = 1 - \mathbb{E}\Phi((G^{-1}(U_{(i)}) - \mu)/\sigma),$$

where $\mathbb{E}(\cdot)$ is taken over the distribution of $U_{(i)}$. The last equality used the fact that $Z$ is normal with mean $\mu$ and variance $\sigma^2$.

The following results are direct application of the previous results. They are useful for constructing the $\pi_{mj}$'s that are used in the structural estimation of this paper. Recall the definitions of $G$, $G_{j+}$, $G_{j-}$, and $v(b_1, b_2, F)$ from before. We introduce the following notation:

$$a(\kappa, n, m; G) \equiv \mathbb{E}\Phi\left((G_{(i)} - 1(U_{(i)}) - \beta_k)/\sigma\right),$$

where $U_{(k:n)}$ is the $k$-order statistic of $n$ uniform random variables and $\mathbb{E}(\cdot)$ is taken over the distribution of $U_{(k:n)}$.

**Corollary 6.1.** Suppose the conditions of Proposition 6.2 hold and let $v_m$ be distributed according to $F_m$. Then, in the heterogeneous preferences case with $\bar{n}_j < n^{(j)}$,

1. For each $m \in M_{j+}$, $P(v_m > v_{(k,n^{(j)};G_{j+})}) = 1 - a(\kappa, n^{(j)}, m; G_{j+})$, where $\kappa = n^{(j)} - \bar{n}_j$.
2. For each $m \in M_{j-}$, $P(v_m < v_{(k,n^{(j)};G_{j-})}) = a(\kappa, n^{(j')}, m; G_{j-})$, where $\kappa = \bar{n}_j - n^{(j)} + 1$.

**Corollary 6.2.** Suppose the conditions of Proposition 6.3 hold and let $v_m$ be distributed according to $F_m$. Then, in the homogeneous preferences case with $\bar{n}_j < n$,

1. If $n^{(j)} = 0$, $P(v_m < v_{(k,n,G)}) = a(\kappa, n, m; G)$ for each $m \in M$, where $\kappa = \bar{n}_j + 1$.
2. If $n^{(j)} = n$, $P(v_m > v_{(k,n,G)}) = 1 - a(\kappa, n, m; G)$ for each $m \in M$, where $\kappa = n - \bar{n}_j$.

**Proof.** The proofs of Corollaries 1 and 2 follows directly from Lemma 6.3. □

### 6.3. A Monte Carlo Simulation Study

In this section, we investigate the finite sample size and power properties of the estimator of preferences, $\hat{\theta}_n(\beta)$, under a variety of parameters and functional form assumptions. The results in this section are for the case the matching technology, $\beta$, is known to the econometrician.
In this study, we choose the following general structure for the worker’s expected utility function:

\[
\tilde{U}_i = \left( f_i + g_i \right) / 2 + d(H_i)\varepsilon_i,
\]

where \( \theta = (\theta_1, \theta_2)' \in \mathbb{R}^4 \), \( X_i \in \mathbb{R}^3 \). \( d(H_i) \) be \( 2 \times 1 \) vector with one in the \( H_i \)-th row where \( H_i \in \{1, 2\} \). We suppose that \( \varepsilon_i \in \mathbb{R}^2 \) follows the extreme value distribution so that the best response probability function of each worker has the logit structure. We consider two functional forms for the production function \( f_i \), which we call Specification 1 \( (f_{i1}) \) and Specification 2 \( (f_{i2}) \):

\[
\begin{align*}
    f_{i1} &= \theta_1 H_i \cdot \pi_i(\theta, \beta)'k, \quad \text{and} \\
    f_{i2} &= \theta_1 (H_i + \pi_i(\theta, \beta)'k).
\end{align*}
\]

\( f_{i1} \) implies direct production complementarities between the worker and firm variables whereas any complementarities in \( f_{i2} \) are forced through the worker’s expectation of firm capital \( \pi_i'k \). We also choose the following \( g_i \) that ensures that the worker’s outside option is positive:

\[
g_i = \exp(H_i \cdot X_i'\theta_2).
\]

\( X_i = (X_{1i}, X_{2i}, X_{3i})' \) are drawn independently across \( i \) and one another from \( U[0, 1] \). For each simulation sample, the \( H_i \)'s are generated as follows. First, we solve for fixed point in the best response operator to obtain \( P^* \).\(^{48}\) Then we compute the best response at the simulated covariates

\[
\Psi_i(H_i|X_i, P^*_{\pi_i}) = \frac{\exp(\tilde{U}_i^*(H_i, X_i))}{\sum_{j=1}^2 \exp(\tilde{U}_j^*(H_j, X_i))}.
\]

Letting \( \Psi_i^*(X_i) \equiv \Psi_i(H_i|X_i, P^*_{\pi_i}) \) we generate the simulated actions as,

\[
H_i = 1\{\Psi_{i1}^* > \omega_i\}
\]

where \( \omega_i \)'s are drawn iid from the uniform distribution on \([0, 1]\).

\(^{47}\)Note that when \( g_i = H_i \exp(X_i'\theta_2) \) - that is, a functional form guaranteeing that \( g_i \) is increasing in \( H_i \) - also yielded comparable performance in the the simulation studies.

\(^{48}\)In experiments with different starting values, iterating the best response operator yielded the same fixed point each time.
Table 1. The Empirical Coverage Probability of Asymptotic Confidence Intervals for $a'\theta_0$ at 95% Nominal Level When $\beta_0$ is Known.

| $\beta_0$ | Specification 1 | Specification 2 |
|-----------|-----------------|-----------------|
|           | $M = 2$ | $M = 3$ | $M = 5$ | $M = 2$ | $M = 3$ | $M = 5$ |
| $-1$ | $n = 500$ | 0.9480 | 0.9430 | 0.9470 | 0.9510 | 0.9410 | 0.9430 |
|          | $n = 1000$ | 0.9480 | 0.9560 | 0.9490 | 0.9450 | 0.9450 | 0.9270 |
|          | $n = 2000$ | 0.9380 | 0.9300 | 0.9250 | 0.9590 | 0.9300 | 0.9090 |
| $-0.5$ | $n = 500$ | 0.9470 | 0.9490 | 0.9490 | 0.9530 | 0.9500 | 0.9310 |
|          | $n = 1000$ | 0.9480 | 0.9370 | 0.9550 | 0.9480 | 0.9310 | 0.9300 |
|          | $n = 2000$ | 0.9430 | 0.9430 | 0.9450 | 0.9170 | 0.9180 | 0.8860 |
| $0$   | $n = 500$ | 0.9460 | 0.9470 | 0.9530 | 0.9380 | 0.9350 | 0.9380 |
|          | $n = 1000$ | 0.9490 | 0.9490 | 0.9530 | 0.9370 | 0.9480 | 0.9440 |
|          | $n = 2000$ | 0.9360 | 0.9510 | 0.9400 | 0.9580 | 0.9530 | 0.9490 |
| $0.5$  | $n = 500$ | 0.9520 | 0.9530 | 0.9600 | 0.9420 | 0.9310 | 0.9330 |
|          | $n = 1000$ | 0.9410 | 0.9500 | 0.9360 | 0.9460 | 0.9450 | 0.9440 |
|          | $n = 2000$ | 0.9430 | 0.9260 | 0.9230 | 0.9560 | 0.9500 | 0.9280 |
| $1$    | $n = 500$ | 0.9440 | 0.9430 | 0.9380 | 0.9330 | 0.9360 | 0.9270 |
|          | $n = 1000$ | 0.9260 | 0.9180 | 0.8930 | 0.9470 | 0.9340 | 0.9220 |
|          | $n = 2000$ | 0.9170 | 0.9050 | 0.8850 | 0.9200 | 0.9240 | 0.9190 |

Notes: The table reports the empirical coverage probability of the asymptotic confidence interval for $\theta_0$. The simulated rejection probability at the true parameter is close to the nominal size of $\alpha = 0.05$. The simulation number is $R = 1000$.

6.4. Empirical Section

6.4.1. Variables

We use variables EDC_1 through EDC_12 to construct the indicator variable for high educated. A number of definitions are possible. The results record the value of $H_i = 1$ if the individual has completed both high school and a college degree. We rely on firm size as our measure of firm productivity. There is specific evidence that firm size is a useful proxy for firm productivity in both manufacturing and non-manufacturing sectors in the Canadian context. Leung et al. (2008), using Canadian administrative data for the period 1984-1997, argue that firm size is positively correlated with measures of labour and total factor productivity, particularly within the manufacturing sector.

For the outside option function, we use the worker’s marital status, the number of dependent children the worker has, and the worker’s gender. We drop the few individuals who reported having six dependent children (the maximum allowable in the sample). The form of the outside option is as reported in the simulations study.
**Table 2. Average Length of Confidence Intervals for $a\theta_0$ at 95% Nominal Level When $\beta_0$ is Known.**

| $\beta_0$ | M = 2 | M = 3 | M = 5 | M = 2 | M = 3 | M = 5 |
|-----------|-------|-------|-------|-------|-------|-------|
| -1 n = 500 | 15.5881 | 1.8333 | 1.8353 | 1.6223 | 1.4490 | 1.3557 |
| n = 1000  | 1.3573  | 1.2290 | 1.2920 | 1.1102 | 0.9449 | 0.9240 |
| n = 2000  | 0.9093  | 0.8762 | 0.8643 | 0.6812 | 0.6858 | 0.6385 |
| -0.5 n = 500 | 2.3023 | 2.0498 | 2.0104 | 2.4145 | 1.9593 | 1.6319 |
| n = 1000  | 3.4657  | 1.508  | 1.3757 | 1.6552 | 1.2743 | 1.0797 |
| n = 2000  | 1.0376  | 1.0841 | 1.0300 | 1.0803 | 0.8720 | 0.7451 |
| 0 n = 500 | 2.2851  | 2.2847 | 2.2773 | 1.1456 | 1.1542 | 1.2197 |
| n = 1000  | 1.5994  | 1.5993 | 1.6397 | 0.7371 | 0.8457 | 0.7375 |
| n = 2000  | 3.1524  | 1.1593 | 1.1208 | 0.5205 | 0.5196 | 0.5206 |
| 0.5 n = 500 | 3.0442 | 2.9003 | 2.9723 | 2.4889 | 3.0612 | 3.1814 |
| n = 1000  | 3.0954  | 1.9949 | 2.1289 | 2.0466 | 2.0044 | 2.3200 |
| n = 2000  | 1.5633  | 1.4310 | 1.5487 | 0.9286 | 1.3152 | 1.2469 |
| 1 n = 500 | 6.0386  | 11.0447 | 4.7156 | 4.8434 | 4.7347 | 4.3571 |
| n = 1000  | 2.7670  | 3.1329 | 4.6450 | 2.8590 | 3.1888 | 3.2592 |
| n = 2000  | 1.6956  | 2.0135 | 2.2905 | 2.1166 | 2.5055 | 2.2695 |

Notes: This table reports the average length of the asymptotic confidence interval for $\theta_0$. The lengths of the confidence intervals decrease with $n$. The simulation number is $R = 1000$.

6.5. Estimation

The estimation proceeds in two steps, broadly as outlined in 3. We begin by discussing the estimation of $\theta_0$, then we discuss the Monte-Carlo inference approach used to construct confidence intervals for $\beta$ in our samples of interest.

We set the parameter space for $\beta_0$ to be $B = [-1 : 0.075 : 3]'$. We normalize $\sigma = 1$ throughout. For each value of $\beta \in B$ we obtain $\hat{\theta}(\beta)$ using the employee-final weights provided for WES by Statistics Canada. Standard errors are constructed as the square roots of the diagonal elements of inverse of the sample Hessian, constructed using numerical differentiation of the log-likelihood function via a five-point stencil approach.

Much of computational difficulty associated with the estimation of $\theta$ involves the construction of the worker’s expectations, i.e., the $\hat{\pi}_j$’s. For the number of support points for the capital variable, we set a value of $M = 5$, but the results are not very sensitive to similar values of $M$ (i.e., $M = 3, M = 7$). The empirical distribution of firm size, $\hat{q}$, is constructed using the WES workplace weights after grouping the firms into categories based on log-firm size. We also use a value of fifty draws of random variables from the beta distribution for the construction of the threshold crossing-probabilities. We use the empirical distribution of high education as the equilibrium choice probabilities. For the distribution of, $N^{(j)}$, the number of firms that prefer workers who chose education level
we use the binomial probability mass function with probability equal to the expected fraction of firms who prefer education level $j$. In practice, we construct the matching probabilities using an interpolation of the support of $N^{(j)}$. A choice of forty support points is found to work well.

The test statistic of interest is based on matched observed characteristics in the data. In particular, we consider a test statistic that compares the observed joint distribution of worker human capital and the matched firm capital to the simulated counterpart. That is,

$$ T(\beta) = \frac{1}{R} \sum_{r=1}^{R} \left\| \hat{P} - \hat{P}_r(\beta, \hat{\theta}_n(\beta)) \right\|, $$

where $\hat{P}$ is an $M \times J$ matrix whose $(m, j)$ element is the probability that a worker of education level $j$ matches to a firm of capital level $m$ (i.e., $\hat{P}(M(i) = m, h_i = j)$), $\hat{P}_r$ is similarly defined except we use the the simulated matching, $M_r(i; \beta, \hat{\theta}(\beta))$, in place of the observed matching, $M(i)$, and $\| \cdot \|$ is the Frobenius norm. Note that when we construct $\hat{P}_r$ we use the $\hat{\theta}$ that were estimated using the employee-weights. However, no set of weights provided by Statistics Canada are appropriate at the level of the match itself. We also define an estimator of $\beta$ as the minimizer of the $T(\beta)$ as a heuristic measure of $\beta$.

6.6. Estimation Results
Table 3. Matching Technology In Canadian Manufacturing and Finance Industries, 1995-2005

| Year | Manufacturing Specification 1 | Manufacturing Specification 2 | Finance Specification 1 | Finance Specification 2 |
|------|-------------------------------|-------------------------------|-------------------------|-------------------------|
| 1999 | 1.475                         | 1.775                         | 0.950                   | 0.875                   |
|      | [0.5000, 2.450]               | [0.575, 2.450]               | [0.200, 1.625]          | [0.200, 1.625]          |
| 2000 | 1.475                         | 1.475                         | 1.625                   | 1.550                   |
|      | [0.2000, 2.525]               | [0.275, 2.600]               | [0.950, 1.850]          | [1.100, 1.925]          |
| 2001 | -1.000                        | -0.175                        | 0.575                   | 0.575                   |
|      | [-1.000, 1.325]               | [-1.000, 1.400]               | [-0.775, 1.700]         | [-0.700, 1.700]         |
| 2002 | -0.175                        | -0.100                        | 0.725                   | 0.650                   |
|      | [-1.000, 1.775]               | [-1.000, 1.625]               | [-0.550, 2.150]         | [-0.550, 2.225]         |
| 2003 | 0.725                         | 0.875                         | 0.875                   | 0.800                   |
|      | [-1.000, 2.375]               | [-1.000, 2.600]               | [0.0250, 1.750]         | [-0.025, 1.700]         |
| 2004 | 1.325                         | 1.100                         | 0.650                   | 0.575                   |
|      | [-0.325, 2.450]               | [-0.325, 2.525]               | [-0.550, 1.775]         | [-0.550, 1.850]         |
| 2005 | 2.300                         | 2.375                         | 0.425                   | 0.425                   |
|      | [0.950, 2.975]                | [0.950, 2.975]                | [-0.550, 1.550]         | [-0.550, 1.550]         |

This table reports minimum distances estimates of $\beta$ using the test statistics considered 28 along with 95% confidence intervals for $\beta$ for the years 1999-2005 using WES data for managers and professionals in the Secondary Products Manufacturing sector and the Finance and Insurance Industry. Specification 1 and Specification 2 refer to the cases in which worker and firm capital are multiplicative and additive, respectively. The results for $\beta$ are similar across specifications. The matching technology in the manufacturing industry exhibited the least frictions at the start of the sample, rising in the middle, then falling again towards the end of the sample. In the finance industry, we find an increase in matching frictions from 1999 onwards. Weighted sample sizes for relevant years and industries are reported in table 4 and 5.
Table 4. Estimation of Worker and Firm Preferences in Canadian Manufacturing Industry, 1999-2005

|                | Specification 1 |                      | Specification 2 |                      |
|----------------|-----------------|----------------------|-----------------|----------------------|
|                | 1999            | 2000                 | 2001            | 2002                 | 2003             | 2004             | 2005             |
| $\theta_1$    | 3.6061          | 1.9910               | 3.2229          | 0.0596               | 4.4090           | 5.4205           | 3.2859           |
|               | (0.0023)        | (0.0012)             | (0.0006)        | (0.0016)             | (0.0010)         | (0.0015)         |                  |
| $\theta_2$    | -0.9760         | 0.1460               | 0.0240          | -0.3201              | -0.4391          | 0.7434           | -29.7608         |
|               | (0.0269)        | (0.0011)             | (0.0002)        | (0.0005)             | (0.0012)         | (0.0003)         |                  |
| $\theta_3$    | 0.2742          | -30.7611             | 0.2259          | 0.6203               | 0.5536           | -1.3002          | 0.0877           |
|               | (0.0009)        | (3.132·10^9)†       | (0.0005)        | (0.0003)             | (0.0006)         | (0.0005)         | (0.0007)         |
| $\theta_4$    | -0.0889         | 0.5008               | 0.2017          | 0.1410               | -0.0299          | -0.3937          | 0.2015           |
|               | (0.0006)        | (0.0004)             | (0.0002)        | (0.0001)             | (0.0003)         | (0.0014)         | (0.0002)         |
| $b_i$         | 71361           | 72464                | 80738           | 76214                | 87026            | 153520           | 98879            |

This table reports preference parameter estimates and standard errors (in parantheses) for professionals and managers in the Canadian manufacturing industry for two specifications for the years 1995-2005 (WES sample frame). $b_i$ denotes the weighted sample size in year $t$. For each year, we report the value of $\hat{\theta}_n(\hat{\beta})$, where $\hat{\beta}$ is the value of the minimum distance estimate for that year. Specification 1 and Specification 2 refer to the cases in which worker and firm capital are multiplicative and additive respectively. $\theta_1$ is the coefficient on worker and firm attributes in the production function. The remainder are coefficients on the outside option function $\theta_2$: $female_i$, $\theta_3$: marital status, $\theta_4$: number of dependent children. All coefficients other than ones with † are statistically significant at $\alpha = 0.01$. Both specifications suggest modest increases in the production technology parameter over time, $\theta_1$. Both specifications, particularly the additive, suggest a negative coefficient on $female_i$ in the worker’s wage equation.
Table 5. Estimation of Worker and Firm Preferences in Canadian Finance Industry, 1999-2005

| Specification 1 | 1999    | 2000    | 2001    | 2002    | 2003    | 2004    | 2005    |
|-----------------|---------|---------|---------|---------|---------|---------|---------|
| $\theta_1$      | 2.8284  | 2.5676  | 3.2546  | 2.9339  | 2.4986  | 2.5479  | 4.5527  |
|                 | (0.0014)| (0.0012)| (0.0009)| (0.0014)| (0.0013)| (0.0007)| (0.0016)|
| $\theta_2$      | -1.3669 | -1.2849 | 0.4988  | -1.2993 | -1.7819 | -0.9356 | -1.3559 |
|                 | (0.0055)| (0.011)| (0.0003)| (0.0072)| (0.0046)| (0.0015)| (0.0071)|
| $\theta_3$      | 0.0842  | 0.4970  | -1.1876 | 0.4788  | -0.3668 | -0.3690 | 0.6391  |
|                 | (0.0007)| (0.0004)| (0.0053)| (0.0003)| (0.0007)| (0.0007)| (0.0002)|
| $\theta_4$      | 0.1541  | 0.0397  | -0.3510 | -0.2085 | 0.4309  | 0.4245  | -0.0591 |
|                 | (0.0003)| (0.0002)| (0.0014)| (0.0004)| (0.0003)| (0.0003)| (0.0001)|
| $b_i$           | 193420  | 159320  | 191340  | 180890  | 188320  | 190750  | 185670  |

| Specification 2 | 1999    | 2000    | 2001    | 2002    | 2003    | 2004    | 2005    |
|-----------------|---------|---------|---------|---------|---------|---------|---------|
| $\theta_1$      | 1.0408  | 1.0157  | 1.1637  | 1.1011  | 0.9652  | 0.9886  | 1.7575  |
|                 | (0.0004)| (0.0005)| (0.0003)| (0.0005)| (0.0005)| (0.0003)| (0.0006)|
| $\theta_2$      | 0.5115  | -1.2849 | 0.4988  | -1.2993 | -1.7819 | -0.9356 | -1.3559 |
|                 | (0.0002)| (0.0110)| (0.0003)| (0.0072)| (0.0046)| (0.0015)| (0.0071)|
| $\theta_3$      | -0.4881 | 0.4970  | -1.1876 | 0.4788  | -0.3668 | -0.3690 | 0.6391  |
|                 | (0.0009)| (0.0004)| (0.0053)| (0.0003)| (0.0007)| (0.0007)| (0.0002)|
| $\theta_4$      | -1.0452 | 0.0397  | -0.3510 | -0.2085 | 0.4309  | 0.4245  | -0.0591 |
|                 | (0.0039)| (0.0002)| (0.0014)| (0.0004)| (0.0003)| (0.0003)| (0.0001)|
| $b_i$           | 193420  | 159320  | 191340  | 180890  | 188320  | 190750  | 185670  |

This table reports preference parameter estimates and standard errors (in parentheses) for professionals and managers in the Canadian finance and insurance industry for two specifications for the years 1995-2005 (WES sample frame). $b_i$ denotes the weighted sample size in year $t$. For each year, we report the value of $\hat{\theta}_n(\hat{\beta})$, where $\hat{\beta}$ is the value of the minimum distance estimate for that year. Specification 1 and Specification 2 refer to the cases in which worker and firm capital are multiplicative and additive respectively. $\theta_1$ is the coefficient on worker and firm attributes in the production function. The remainder are coefficients on the outside option function $\theta_2$: female$_i$, $\theta_3$: marital status, $\theta_4$: number of dependent children. All results are statistically significant at $\alpha = 0.01$. Both specifications suggest a relatively stable production technology, $\theta_1$, over time with increases in 2005. Both specifications suggest a negative coefficient on female$_i$ in the worker’s wage equation.
6.7. Model Counterfactuals

| Year | Specification 1 | Specification 2 |
|------|----------------|----------------|
|      | 1999 | 2000 | 2001 | 2002 | 2003 | 2004 | 2005 | 1999 | 2000 | 2001 | 2002 | 2003 | 2004 | 2005 |
| \( \hat{\beta}_{1999} \) | 0.7443 | 0.7310 | 0.8605 | 0.6723 | 0.8205 | 0.8276 | 0.7622 | 0.7253 | 0.6761 | 0.7036 | 0.6724 | 0.8066 | 0.8283 | 0.7762 |
| \( \hat{\beta}_{2000} \) | 0.7442 | 0.7310 | 0.8605 | 0.6723 | 0.8205 | 0.8275 | 0.7622 | 0.7224 | 0.6744 | 0.7042 | 0.6725 | 0.8040 | 0.8254 | 0.7738 |
| \( \hat{\beta}_{2001} \) | 0.6656 | 0.7263 | 0.8596 | 0.6736 | 0.7494 | 0.7436 | 0.7049 | 0.7036 | 0.6635 | 0.7074 | 0.6728 | 0.7860 | 0.8055 | 0.7582 |
| \( \hat{\beta}_{2002} \) | 0.6947 | 0.7281 | 0.8600 | 0.6732 | 0.7763 | 0.7758 | 0.7259 | 0.7045 | 0.6641 | 0.7072 | 0.6728 | 0.7868 | 0.8065 | 0.7590 |
| \( \hat{\beta}_{2003} \) | 0.7237 | 0.7298 | 0.8603 | 0.6727 | 0.8026 | 0.8066 | 0.7470 | 0.7160 | 0.6707 | 0.7053 | 0.6726 | 0.7980 | 0.8187 | 0.7685 |
| \( \hat{\beta}_{2004} \) | 0.7404 | 0.7308 | 0.8604 | 0.6724 | 0.8172 | 0.8235 | 0.7594 | 0.7185 | 0.6721 | 0.7049 | 0.6725 | 0.8003 | 0.8213 | 0.7705 |
| \( \hat{\beta}_{2005} \) | 0.7624 | 0.7321 | 0.8607 | 0.6719 | 0.8358 | 0.8400 | 0.7762 | 0.7306 | 0.6793 | 0.7027 | 0.6723 | 0.8116 | 0.8336 | 0.7806 |

This table reports the model-simulated probability of investing in high education at the estimated parameter values for the secondary products manufacturing sector (WES industry 4) for two specifications. Specification 1 and 2 are defined in the Section 6.6. The results demonstrate the importance of the matching technology and complementarities on education decisions. The equilibrium probability of education is typically higher in the complementarities case (Specification 1). In the manufacturing sector in 1999 (a high \( \hat{\beta} \) year), the effect of switching to the matching technology from 2001 causes a fall in the equilibrium probability of attending college by roughly 8% in Specification 1, and 2.5% in Specification 2. The variation of preferences and worker distribution of characteristics over time is also significant. In 2001, if preferences and characteristics and the were as they were in 2002, the probability of investing in higher education would plummet from 86% to 67%. The effect in greatly attenuated in Specification 2, which assumes no production complementarities between worker and firm types are present in production.
Table 7. Counterfactual Estimated Probabilities of Investing in High Education, Finance Industry

| Year | Specification 1 | Specification 2 |
|------|-----------------|-----------------|
|      | 1999 | 2000 | 2001 | 2002 | 2003 | 2004 | 2005 | 1999 | 2000 | 2001 | 2002 | 2003 | 2004 | 2005 |
| \( \hat{\beta}_{CF} \) |       |       |      |      |      |      |      | \( \hat{\beta}_{CF} \) |       |       |      |      |      |      |      |
| 1999 | 0.6903 | 0.6961 | 0.7163 | 0.6979 | 0.6833 | 0.6799 | 0.7889 | 0.6399 | 0.6619 | 0.6507 | 0.6415 | 0.6472 | 0.6431 | 0.7380 |
| 2000 | 0.7059 | **0.7097** | 0.7341 | 0.7146 | 0.6965 | 0.6932 | 0.8050 | 0.6449 | **0.6665** | 0.6565 | 0.6470 | 0.6514 | 0.6474 | 0.7442 |
| 2001 | 0.6807 | 0.6877 | **0.7052** | 0.6877 | 0.6751 | 0.6719 | 0.7787 | 0.6376 | 0.6598 | **0.6480** | 0.6390 | 0.6452 | 0.6411 | 0.7351 |
| 2002 | 0.6847 | 0.6911 | 0.7097 | **0.6918** | 0.6784 | 0.6752 | 0.7829 | 0.6382 | 0.6603 | 0.6487 | **0.6396** | 0.6457 | 0.6416 | 0.7358 |
| 2003 | 0.6885 | 0.6944 | 0.7142 | 0.6960 | **0.6817** | 0.6784 | 0.7870 | 0.6394 | 0.6614 | 0.6500 | 0.6409 | **0.6467** | 0.6426 | 0.7373 |
| 2004 | 0.6827 | 0.6894 | 0.7075 | 0.6898 | 0.6768 | **0.6735** | 0.7808 | 0.6376 | 0.6598 | 0.6480 | 0.6390 | 0.6452 | **0.6411** | 0.7351 |
| 2005 | 0.6767 | 0.6842 | 0.7005 | 0.6833 | 0.6717 | 0.6686 | **0.7744** | 0.6364 | 0.6587 | 0.6466 | 0.6376 | 0.6441 | 0.6401 | **0.7336** |

Specification 1 and 2 are as defined in Section 6.6. As in the manufacturing sector, the equilibrium probability of investing in education is higher in the case of complements (Specification 1). Changes in preferences and technology and characteristics matter to education patterns: for example, in 1999, the effect of switching to 2005’s preferences and exogenous characteristics leads to an increase of almost 10% in both specifications.
In this table we consider the effects of very low frictions ($\beta = 5$) and maximal frictions ($\beta = 0$) in the case that the production function exhibits interactions between worker and firm characteristics (Specification 1) and when they do not (Specification 2). The table illustrates the importance of both production complementarities and matching frictions to educational attainment. In 2004, the effect of lowering $\beta$ to 0 causes a fall in the probability of high education by 14% in Specification 1, but only by 2% in Specification 2. The overall level of investment in education is typically much lower in the case with high matching frictions (low $\beta$). $\beta$ has a greater effect on the outcome in the complementarities case (Specification 1).
### TABLE 9. Counterfactual Estimated Probabilities of Investing in High Education, Finance Industry

| Year | Specification 1 | Specification 2 |
|------|-----------------|-----------------|
|      | 1999 | 2000 | 2001 | 2002 | 2003 | 2004 | 2005 | 1999 | 2000 | 2001 | 2002 | 2003 | 2004 | 2005 |
| $\beta_{year}$ | 0.6903 | 0.7097 | 0.7052 | 0.6918 | 0.6817 | 0.6735 | 0.7744 | 0.6399 | 0.6665 | 0.6480 | 0.6396 | 0.6467 | 0.6411 | 0.7336 |
| $\beta = 0$ | 0.6648 | 0.6739 | 0.6867 | 0.6707 | 0.6618 | 0.6588 | 0.7615 | 0.6330 | 0.6555 | 0.6425 | 0.6339 | 0.6412 | 0.6372 | 0.7292 |
| $\beta = 5$ | 0.7509 | 0.7500 | 0.7830 | 0.7618 | 0.7366 | 0.7335 | 0.8492 | 0.6614 | 0.6817 | 0.6755 | 0.6650 | 0.6658 | 0.6618 | 0.7645 |

In this table we consider the effects of very low frictions ($\beta = 5$) and maximal frictions ($\beta = 0$) for Specifications 1 and 2 in the finance industry. The implications are similar to those for the manufacturing industry in the previous table. A rise in $\beta$ causes a much greater increase in the probability of high education in Specification 1: In 2001, the effect of a rise in the estimated $\beta$ to $\beta = 5$ leads to a 8% increase in the equilibrium probability of high education under Specification 1, but only a 3% increase in Specification 2. The level of investment in education is lower in the case without complementarities (Specification 2).
|           | Specification 1 |          |          |          |          |          |          |
|-----------|-----------------|----------|----------|----------|----------|----------|----------|
| Year      | 1999            | 2000     | 2001     | 2002     | 2003     | 2004     | 2005     |
| $\beta_{\text{year}}$ | 0.1916          | 0.4207   | 0.2112   | 0.4216   | 0.1646   | 0.2121   | 0.2220   |
| $\beta = 0$ | 0.2138          | 0.4299   | 0.2149   | 0.4218   | 0.2013   | 0.2534   | 0.2507   |
| $\beta = 5$ | 0.2112          | 0.4330   | 0.2280   | 0.4208   | 0.1951   | 0.2360   | 0.2452   |

|           | Specification 2 |          |          |          |          |          |          |
|-----------|-----------------|----------|----------|----------|----------|----------|----------|
| Year      | 1999            | 2000     | 2001     | 2002     | 2003     | 2004     | 2005     |
| $\beta_{\text{year}}$ | 0.1129          | 0.2203   | 0.3481   | 0.4206   | 0.1183   | 0.1268   | 0.1653   |
| $\beta = 0$ | 0.1181          | 0.2238   | 0.3499   | 0.4207   | 0.1301   | 0.1285   | 0.1710   |
| $\beta = 5$ | 0.1255          | 0.2311   | 0.3478   | 0.4201   | 0.1294   | 0.1251   | 0.1728   |

This table reports the model-simulated Gini coefficients under two specifications. Specification 1 and 2 are as defined in Section 6.6. The predicted level of wage inequality is typically much lower in Specification 2, where there are no production complementarities. In Specification 1, the level of inequality at the estimated value of the frictions is lower than at the counterfactual levels for most years (except 2002). For example, in 2005 the simulated Gini is 0.222 and the investment in education is 77%. This rises to 0.2507 (education investment 76%) when information frictions are highest and 0.2452 (education investment 85%) when frictions are lowest. Similar patterns can be seen in Specification 2.
This table reports the model-simulated Gini coefficients under two specifications. Specification 1 and 2 are as defined in Section 6.6. As in the manufacturing industry, the levels of inequality are typically much lower when there are no production complementarities (Specification 2). In Specification 1, the level of inequality at the estimated value of the frictions is higher than at the counterfactual levels for each year (except 1999). In Specification 2, the effect of lowering matching frictions raises wage inequality in every year. In this case, when frictions are lowered, the effect of increased sorting is stronger than the inequality-lowering effect of the greater supply of highly educated workers.

### Table 11. Matching Technology Counterfactuals, Gini Coefficient, Finance Industry

| Year | 1999   | 2000   | 2001   | 2002   | 2003   | 2004   | 2005   |
|------|--------|--------|--------|--------|--------|--------|--------|
| \( \hat{\beta}_{year} \) | 0.2602 | 0.2929 | 0.2769 | 0.2542 | 0.3594 | 0.2976 | 0.2607 |
| \( \beta = 0 \)          | 0.2516 | 0.2814 | 0.2502 | 0.2332 | 0.3408 | 0.2813 | 0.2495 |
| \( \beta = 5 \)          | 0.2647 | 0.2899 | 0.2523 | 0.2380 | 0.3421 | 0.2856 | 0.2432 |

### Specification 2

| Year | 1999   | 2000   | 2001   | 2002   | 2003   | 2004   | 2005   |
|------|--------|--------|--------|--------|--------|--------|--------|
| \( \hat{\beta}_{year} \) | 0.1998 | 0.2245 | 0.1756 | 0.1655 | 0.2801 | 0.2200 | 0.1815 |
| \( \beta = 0 \)          | 0.1984 | 0.2237 | 0.1714 | 0.1623 | 0.2759 | 0.2185 | 0.1793 |
| \( \beta = 5 \)          | 0.2075 | 0.2315 | 0.1803 | 0.1697 | 0.2827 | 0.2266 | 0.1837 |