Abstract

The electoral college of voting system for the US presidential election is analogous to a coarse graining procedure commonly used to study phase transitions in physical systems. In a recent paper, opinion dynamics models manifesting a phase transition, were shown to be able to explain the cases when a candidate winning more number of popular votes could still lose the general election on the basis of the electoral college system. We explore the dependence of such possibilities on various factors like the number of states and total population (i.e., system sizes) and get an interesting scaling behavior. In comparison with the real data, it is shown that the probability of the minority win, calculated within the model assumptions, is indeed near the highest possible value. In addition, we also implement a two step coarse graining procedure, relevant for both opinion dynamics and information theory.

1. Introduction

The concept that human society with a large number of mutually interacting agents can be regarded as a complex system consisting of an ensemble of basic units following simple rules, has led to the investigation of several social phenomena using models commonly studied in statistical physics. One of the most studied social phenomena has been the formation of opinion, where interaction between agents are considered at a microscopic level in the models, with or without external fields. The aim is often to find out the mechanism by which it is possible to achieve an agreement or consensus. The first step in such modeling approaches is to quantify the opinions of the agents involved. This task is easier \[1, 2, 3, 4\] when the choice of the opinion is binary. Examples of such cases are yes/no referenda, a two party voting etc., where the said quantification is straight-forward with the use of a binary variable to represent the support for either side. It is often done with $\pm 1$ values, i.e., like spins with Ising symmetry, for example in the voter model \[5, 6, 7\]. Depending on the situation, it may also be crucial to have neutral opinions - which can be assigned a zero value - reflecting the population that does not support either of the two groups or sides \[8\]. Then by including suitable interactions among the agents the dynamics of opinion formation, particularly the formation of a majority on either side through a symmetry breaking transition, are studied.

The formation of the majority opinion is very important as it often determines the final outcome of the election process (for example, the question of the UK leaving the EU \[9\]). However, in situations where the direct majority is neither the necessary nor the sufficient condition for winning the election, the spatial fluctuations in the voting patterns play a crucial role. Particularly, in this paper, we are concerned about the voting process of the US president-ship. This is not a direct election process for the post of the president; an intermediate body, namely the electoral college
delegates, ultimately decide who will be the president. In the electoral college system, in most cases, all the delegates from a particular state are assigned to the winner from that state and the overall winning candidate is then decided on the basis of the support of the majority of those delegates. This process has led to the rather rare cases when the popular candidate (PC henceforth), i.e., the one with the maximum number of individual votes could turn out to be the loser after the electoral college system is applied. Clearly, this is a result of strong local fluctuations in the voting pattern of the different states and more likely to happen in closely fought elections. If there are two candidates only, say A and B, the voters may be assigned the state values $\pm 1$ to express their support, e.g., state 1 means supporting A, and in a closely fought election the opinions values 1 and -1 are almost equal in number. A closely fought election with two candidates and the strong fluctuations together resemble the behavior of (Ising symmetric) systems near their critical point. Indeed, some of the present authors had shown, using a suitable model dynamics, that close to the critical point of the model, a finite probability exists for the minority win situation, which becomes vanishingly small in the parameter range of the actual voting data when a (spatially) completely uncorrelated random distribution of the opinions are considered [8].

It was further argued in Ref. [8] that the election process with the electoral college system is analogous to a coarse graining procedure commonly used to study phase transitions in, for example, magnetic materials. In reality, most of the US presidential elections involve two major candidates only. Thus, the scenario is similar to the Ising model, as mentioned earlier, and in that language, the sign of the magnetization (which is defined as sum over all the spins/opinions), based on all the individual votes decides the PC. Magnetization is the order parameter which for a fully disordered system will vanish. In the case of a two party voting, its sign determines the winning candidate. The process of electoral college voting then corresponds to a one step coarse graining procedure, implemented by a block transformation [10], where a block represents a state. If the magnetization before and after the coarse graining have the same sign, the PC emerges as the global winner (GW henceforth) and becomes the President. In the opposite case, PC loses and such rare cases have been witnessed four times in the history of the US presidential elections, the last time it happened was in 2016. Such cases can happen if the election is closely fought, or in other words, the magnetization (the order parameter in general) is small before the block transformation. Hence, in order to capture this behavior, one has to consider a model manifesting a phase transition and apply the coarse graining close to criticality.

To simulate the closely fought election, the initial state can be considered to be random and the correlation is allowed to grow in time under the dynamical scheme. The value of the parameter driving the phase transition in the model is chosen to be close to criticality. Due to the large fluctuations near criticality, the order parameter oscillates about zero, the amplitude is finite in a finite sized system. The (small) difference in the votes therefore is a variable and the calculation is made for all values that occur over time.

Although two variables $\pm 1$ are sufficient to describe the scenario such that one can use the Ising model as a proper opinion dynamics model here, the results are closer to reality when one considers a third population with neutral opinion [8]. This was possible by taking a model having three opinion values, namely, the kinetic exchange model proposed in [11]. Note that the order parameter (magnetization) remains same even after including the neutral opinions.

Specifically, the parameter values were chosen to be as far as possible compatible to the actual case, e.g., the coarse graining was done by dividing the system into 49 blocks (the numerical method required a square number) close to the present number of states in the US which is 50. However, the number of states in the US has varied considerably over time as shown in Fig. [11]. Since the first ratification of admissions during 1787-88, the number of states in the US continued to increase, until the period of the civil war (1861-65), during which the confederate states were in existence,
Figure 1: In the US, the number of states \( M \) has changed over the years. As discussed in the text, while calculating the minority win probability, the relevant scaling variable turns out to be \( b/L \), which is \( 1/\sqrt{M} \) assuming uniform population density. The figure shows the variation of \( 1/\sqrt{M} \) with time in the US. The variation is non-monotonic in time during the period of the civil war, due to the confederate states. Other than that it varies from \( 1/\sqrt{11} \) in 1788 to \( 1/\sqrt{50} \) in 1959 and remained constant thereafter.

reducing the total number of states in the US. Since the end of the civil war, the total number of states kept increasing, until the present tally of 50 was reached in 1959 with the admission of Hawaii.

In this work, we therefore generalize the problem and ask the question: how does the probability that the PC turns out to be the loser after the so called coarse graining, depend on the number of states. We may thus predict what would be the probability of a PC losing if some states are joined/split in future. We have used the Kinetic exchange model (KEM) and the Ising model (IM) (both show an order disorder phase transition) on two dimensional square lattices for different system sizes \( N = L^2 \) and applied coarse graining with different scale factors \( b \). We note \( b/L \) emerges as the relevant scaling variable which is identical to \( \sqrt{1/M} \) where \( M \) is the number of states.

The entire procedure can also be regarded as a problem in information theory. Particularly, the time series of the sign of the opinion favoring the PC can be thought of as the input string and that of the GW the output string for the electoral college or the coarse graining process. The relative mutual information (see Eq. (4) in Ref. [8]) is then a measure of the probability of faithful translation of the popular opinion by the electoral college, which is close to unity in the ordered state and decreases sharply near the critical point.

Hence the estimate in which we are interested, the probability that the coarse grained result of the order parameter is opposite in sign is generally termed the error \( \epsilon(b, L) \), studied as a function of \( b \) and \( L \). \( b^2 \), the area of a block is an integer, \( b \) itself can be irrational. For a given \( L \), the maximum value of \( b \) is \( L/\sqrt{2} \). Two limiting cases are immediately identified. If \( b \) is trivially 1, then each individual represents a state and essentially no coarse graining is required and \( \epsilon(1, L) = 0 \). On the other hand, when \( b = L \), we are taking the whole population as a block and the coarse graining will merely yield the same value of the order parameter making \( \epsilon(L, L) = 0 \). Intermediate values of \( b \) will show a point of maximum error i.e., the highest probability for the minority win. Given that the number of states in the US has varied over the years, it is interesting to identify the proximity of the \( b/L \) value to the point where the maximum error is predicted from the model.
2. Models and quantities calculated

As mentioned earlier, we study the Ising model (IM) and a kinetic exchange model (KEM) in capturing the dynamics of essentially two-party voting systems. The Ising model represents a binary opinion case with $s_i = \pm 1$, the opinion held by the $i$th agent at the site $i$. The two values are used to define the support for the two candidates. The Ising model with the Hamiltonian $H = -J \sum_{<ij>} s_is_j$ is studied close to the exactly known critical point using Metropolis algorithm. Here $<ij>$ denotes nearest neighbors.

For the KEM, the opinion values can be $o_i = \pm 1$ and 0. For the $i$th agent, interacting with the $j$th agent (chosen randomly from one of the four nearest neighbors of the $i$th agent), the opinion value $o_i$ changes according to

$$o_i(t+1) = o_i(t) + \mu_{ij}o_j.$$  \hfill (1)

No sum over the index $j$ is implied, as the model is binary-exchange. A non-linearity enters the model from the imposed bounds in the opinion values for the extreme ends at $\pm 1$, i.e., $|o_i| \leq 1$, signifying the limit to an extreme opinion. $\mu_{ij}$ is an annealed variable assuming the values $\pm 1$, and is negative with probability $p$. An order disorder transition occurs close to $p_c \approx 0.11$ here [12, 13].

The normalized order parameter $m$ in the two models are given by $\sum_i s_i/N$ and $\sum_i o_i/N$ where $N$ is the total number of agents; $N = L^2$ in a $L \times L$ lattice. The initial state is chosen to be totally random for both models corresponding to a high noise value. The system gradually relaxes towards equilibrium state, which is still disordered but having large spatial correlation. The order parameter oscillates about zero close to the critical point as a function of time. For each time step, if the order
Figure 3: Variation of the minority win probability $\epsilon(b, L)$ for the Ising model. (a) The variation of the minority win probability $\epsilon(b, L)$ is shown for the Ising model in two dimensions for $L = 819$ at different temperatures above $T_c$. As the temperature is raised significantly higher than $T_c$, the curves approach the random configuration limit, as expected. The arrows indicate the points where the events of minority win did occur in the past. (b) The minority win probabilities are plotted for different system sizes near the critical point. A data collapse is seen for the scaling variable $b/L$. As before, the arrows indicate the events of minority win. It is clear that for further rise in the number of states would significantly reduce the minority win probability.

parameter $m$ is not exactly zero, the coarse graining is applied with different scale factors $b$. The magnetization $m_1$, after coarse graining, is calculated. We estimate the probability $\epsilon(b, L)$ that $mm_1 < 0$. The variation of this probability with $b$ for different values of $L$ are then studied.

Another quantity that is estimated is how the error $\epsilon$ propagates in successive coarse graining. For this we have considered a two step procedure. If the one step coarse graining is done with scale factor $b$, we now consider successive coarse graining procedure by factors $b_1$ and $b_2$ respectively with $b_1 b_2 = b$. It is of course well known in critical phenomena that the the two procedures lead to the same behavior as far as normalization of the parameters are concerned [10]. However, the question we ask here is somewhat different. In an election procedure, the winner is usually decided on a majority vote basis. Subsequently, the issues to be resolved are decided on the basis of the votes by the elected representatives. The outcome, however, could be different if all the individuals were allowed to vote on these issues. So the two step procedure measures how far the people’s decision could be propagated to a higher level. In the context of information theory, it could be regarded as a two step coding.

In the simulations on the square lattices, periodic boundary condition has been used and several configurations are run to get the average values.

3. Results

3.1. One step coarse graining

As mentioned before, the electoral college system in the US presidential election resembles a single step coarse graining process, with each coarse graining block representing a state. Here we report the results of a single step coarse graining when the number of coarse graining blocks covering the system is varied i.e., the block sizes are varied. In Fig. 2 the magnetization of the original lattice is plotted against that of the coarse grained lattice for the Ising model. For different values of $b$, the points where the signs of these two quantities are different, varies. In the extreme limits $b = 1$ and $b = L$ such points do not exist. We now go on to further quantify the variations in probabilities of such cases.
Figure 4: The variation of the minority win probability $\epsilon(b, L)$ of the KEM. (a) The variation of the minority win probability $\epsilon(b, L)$ is shown for the KEM in two dimensions for $L = 819$ at different values for the parameter $p$ above $p_c$. As the parameter $p$ is raised significantly higher than $p_c$, the curves approach the random configuration limit, as expected. The arrows indicate the points where the events of minority win did occur in the past. (b) The minority win probabilities are plotted for different system sizes near the critical point. A data collapse is seen for the scaling variable $b/L$. As before, the arrows indicate the events of minority win. It is clear that for further rise in the number of states would significantly reduce the minority win probability.

Although we are interested in the case when the system is close to criticality, one can calculate the errors for higher noise values as well. We calculate $\epsilon(b, L)$ for a fixed value of $L$ for different noise values for the Ising model and the KEM, shown in Fig. 3a and 4a. As expected, $\epsilon$ increases with the noise factor. We also note that $\epsilon$, which is zero at the extreme limits, increases with $b$ and shows a shallow peak at $b \sim b_p$ only very close to the maximum value of $b = L/\sqrt{2}$. For larger values of the noise, the results are almost independent of $b$ as the system approaches a random case, also shown for comparison.

Next, the results for different system sizes very close to criticality are presented in Figs 3b and 4b where $\epsilon$ is plotted for different system sizes. We note that the data collapse when they are plotted against $b/L$. It may be added that the same happens for the random case, and therefore presumably for all values of the noise factor above criticality.

Hence $b/L$ is identified as the scaling variable. We note that the scaling collapse becomes better as the system sizes are increased, in general there are quite strong finite size effects. Specifically, the error $\epsilon(b, L)$ decreases for smaller system sizes. Intuitively it is clear, as the effective ‘critical point’ of smaller systems (say, the peak of the heat capacity in the Ising model) is higher than that of the thermodynamic limit, the order parameter at a given distance from the thermodynamic critical point will increase with the decrease in the system size and hence will give lower values of $\epsilon$.

Now, clearly the variable $b/L = 1/\sqrt{M}$, where $M$ is the number of states. In the case of the US, as shown in Fig. 11 the number of states in the US has varied considerably over the years. Particularly, the quantity $1/\sqrt{M}$ has changed between $1/\sqrt{11} \approx 0.301$ in 1788 to $1/\sqrt{50} \approx 0.141$ from 1959 onward. Comparing these values with the plots in Figs 3b and 4b, we see that the probability of PC losing has changed with time and currently it is very close to the point beyond which the probability will decrease drastically if the number of states in increased, i.e., $b$ decreased in the coarse graining. In the Figs 3b and 4b the arrows indicate the years of PC losing.

3.2. Two step coarse graining

We now discuss the two step coarse graining done with scale factors (block size) $b_1$ and $b_2$ and compare the results with the one step process with $b = b_1b_2$. We also compare the results when the
order of the two step process is reversed, i.e., first with a scale factor $b_2$ and then $b_1$. We use the convention $b_2 > b_1$.

We first consider the random case which corresponds to infinite noise. Here it is found that the two step process significantly increases the error but the order hardly matters. Table 1 shows the errors in the two step process.

Let $\epsilon'(b_1, b_2, L)$ denote the error for the two step process, i.e., at the end of the two coarse grainings. After the first step, the fraction which retains the original sign of the order parameter is $(1 - \epsilon(b_1, L))$. The probability the sign is changed in the second step for this fraction is $\epsilon(b_2, L/b_1)$. For the fraction $\epsilon(b_1, L)$ for which the sign did change after the first step, the contribution to $\epsilon'$ will come if these configurations retain the signature in the second step. This happens with probability $(1 - \epsilon(b_2, L/b_1))$. Hence

$$\epsilon'(b_1, b_2, L) = \epsilon(b_2, L/b_1)(1 - \epsilon(b_1, L)) + (1 - \epsilon(b_2, L/b_1))\epsilon(b_1, L).$$

In the scaling regime, $\epsilon(b, L)$ is a function of $b/L$ only. So

$$\epsilon'(b_1, b_2, L) = \epsilon(b/L)(1 - \epsilon(b_1/L)) + (1 - \epsilon(b/L))\epsilon(b_1/L).$$

Let us call $\epsilon(b/L) = y$ and $\epsilon(b_1/L) = x$. Then the RHS of above eq becomes:

$$y(1 - 2x) + x$$

This quantity will increase with $x$ unless $y$ is greater than 0.5 which is usually not the case. One can proceed further for the random case and obtain an upper bound for the error. Here, we have noted that $\epsilon$ has a monotonic slow increase for small values of $b$ and attains a constant value for larger $b$ values. So an upper bound on $x$ is $y$ (since $b_1 < b$). $y$ has more or less a constant value $\approx 0.2$ (numerically obtained) and therefore $\epsilon' \leq 2y(1 - y) \approx 0.32$. We indeed get that the two step procedure gives values less than this upper bound (see Table 1).

According to Eq. (3), the results should depend on the order in which the 2 step coarse graining is done. For the reverse order we will get Eq. (4) where $x$ is replaced by $x' = \epsilon(b_2/L)$. However, for the random case, as $\epsilon$ remains almost constant for a considerable range of values of $b/L$ (see Figs 3, 4), we get negligible difference when the order is changed.

On the other hand, for both KEM and IM, close to criticality, we find that there is an appreciable dependence of $\epsilon$ on $b/L$ and the results for the two step process are found to be sensitive to the order (see Tables 2 and 3). In particular, we note that when the coarse graining is done with $b_1 < b_2$ first, the resultant errors are more or less same as that for the two step process. This might not be expected from eq. 4 however, it must be remembered that the above analysis is made for the thermodynamic limit, $L \to \infty$; the finite size effects mentioned before will be enhanced in a two step process.

When $b_2$ is used first, the error increases. This is not difficult to explain: a larger value of $b$ gives larger errors after the first step (unless it is beyond $b_p$ where a peak value occurs which has not been considered). Naturally, in the second step, when the system size has got reduced, even with $b_1$ smaller, the error is increased.

4. Discussion and conclusions

The indirect nature of the election of the US president highlights the importance of the coarse graining process while modeling the voting process using discrete Ising-symmetric models. So far as the dissimilarity of the final outcome of the election result and the popular vote is concerned, it
Table 1: Two step coarse graining in the random case

| $L$ | $b_1, b_2$ | $b_1 b_2 / L$ | $\epsilon'(b_1, b_2, L)$ | $\epsilon'(b_2, b_1, L)$ | $\epsilon(b_1 b_2, L)$ |
|-----|------------|----------------|--------------------------|--------------------------|--------------------------|
| 495 | 3.5        | 0.030          | 0.269                    | 0.270                    | 0.205                    |
| 495 | 3.15       | 0.091          | 0.272                    | 0.272                    | 0.204                    |
| 495 | 5.9        | 0.091          | 0.277                    | 0.276                    | 0.204                    |
| 495 | 3.55       | 0.333          | 0.262                    | 0.264                    | 0.191                    |
| 495 | 5.33       | 0.333          | 0.268                    | 0.270                    | 0.191                    |
| 495 | 11.15      | 0.333          | 0.271                    | 0.270                    | 0.191                    |
| 585 | 3.5        | 0.026          | 0.268                    | 0.268                    | 0.203                    |
| 585 | 3.39       | 0.200          | 0.268                    | 0.270                    | 0.201                    |
| 585 | 9.13       | 0.200          | 0.277                    | 0.277                    | 0.201                    |
| 585 | 3.65       | 0.333          | 0.263                    | 0.265                    | 0.193                    |
| 585 | 5.39       | 0.333          | 0.269                    | 0.269                    | 0.193                    |
| 585 | 13.15      | 0.333          | 0.274                    | 0.272                    | 0.193                    |
| 693 | 3.7        | 0.030          | 0.273                    | 0.269                    | 0.204                    |
| 693 | 3.33       | 0.143          | 0.273                    | 0.274                    | 0.204                    |
| 693 | 9.11       | 0.143          | 0.276                    | 0.276                    | 0.204                    |
| 693 | 3.77       | 0.333          | 0.266                    | 0.264                    | 0.194                    |
| 693 | 7.33       | 0.333          | 0.270                    | 0.272                    | 0.194                    |
| 693 | 11.21      | 0.333          | 0.270                    | 0.272                    | 0.194                    |

Table 2: Two step coarse graining in the Ising model

| $L$ | $b_1, b_2$ | $b_1 b_2 / L$ | $\epsilon'(b_1, b_2, L)$ | $\epsilon'(b_2, b_1, L)$ | $\epsilon(b_1 b_2, L)$ |
|-----|------------|----------------|--------------------------|--------------------------|--------------------------|
| 495 | 3.5        | 0.030          | 0.086                    | 0.091                    | 0.084                    |
| 495 | 3.15       | 0.091          | 0.145                    | 0.165                    | 0.142                    |
| 495 | 5.9        | 0.091          | 0.149                    | 0.154                    | 0.142                    |
| 495 | 3.55       | 0.333          | 0.177                    | 0.222                    | 0.175                    |
| 495 | 5.33       | 0.333          | 0.179                    | 0.214                    | 0.175                    |
| 495 | 11.15      | 0.333          | 0.188                    | 0.195                    | 0.175                    |
| 585 | 3.5        | 0.026          | 0.096                    | 0.099                    | 0.093                    |
| 585 | 3.39       | 0.200          | 0.186                    | 0.236                    | 0.185                    |
| 585 | 9.13       | 0.200          | 0.195                    | 0.201                    | 0.185                    |
| 585 | 3.65       | 0.333          | 0.187                    | 0.238                    | 0.186                    |
| 585 | 5.39       | 0.333          | 0.191                    | 0.233                    | 0.186                    |
| 585 | 13.15      | 0.333          | 0.203                    | 0.209                    | 0.186                    |
| 693 | 3.7        | 0.030          | 0.112                    | 0.118                    | 0.109                    |
| 693 | 3.33       | 0.143          | 0.180                    | 0.218                    | 0.178                    |
| 693 | 9.11       | 0.143          | 0.188                    | 0.192                    | 0.178                    |
| 693 | 3.77       | 0.333          | 0.181                    | 0.241                    | 0.179                    |
| 693 | 7.33       | 0.333          | 0.186                    | 0.222                    | 0.179                    |
| 693 | 11.21      | 0.333          | 0.193                    | 0.213                    | 0.179                    |
is expected that the effect will be most relevant when the elections are closely contested and there is spatial fluctuation in the voting pattern. In that case, if the votes of one candidate is heavily concentrated in a few states, while the other candidate wins in more number of states even though marginally, the latter would win the election due to the effective single step coarse graining coming from the electoral college system.

This then motivates the quantification of the probability of the minority win $\epsilon(b, L)$ as a function of the system size $L$ and the coarse graining block size $b$, in the models such as the Ising and the KEM. Interestingly, $b/L$ emerges as the relevant scaling variable, at least in the limit of the large system sizes (see Fig. 3, 4). However, it is obvious that in the two extreme limits $b = 1$ and $b = L$, the result of the coarse graining has no significance and $\epsilon$ is exactly zero (see Fig. 2). In the intermediate range, therefore, $\epsilon(b/L)$ will show a point of maximum. It is not obvious at which point the maximum would occur, but given that $b/L$ is the scaling variable, it is determined by the critical fluctuation of the model and not the system or block sizes.

Now, given that the number of states in the US has varied over the years and that the minority win probability $\epsilon(b/L)$ has a non-monotonic variation, it is interesting to check according to the prediction of this model, how has $\epsilon(b/L)$ changed over the years for the US presidential election. Indeed, as is indicated in Fig. 1, the effective $b/L$ values, estimated as $1/\sqrt{M}$ ($M$ is the number of states), is such that $\epsilon(b/L)$ has been close to its maximum. In fact, it can also be seen from Figs 3 and 4 that for larger values of $M$ i.e., by splitting up larger states, the minority win probability can be sharply reduced. However, it should also be mentioned here that it is an idealized situation and in practice there are some mechanisms in place to counter the effect introduced by the electoral college viz., the variation in the number of delegates according to the sizes of the states, which we did not consider here to keep things simple.

Finally, if the coarse graining process is repeated a second time, the scale factors being $b_1$ and $b_2$ in the first and second steps respectively, the errors $\epsilon'(b_1, b_2, L)$ are higher for the random cases.
Here, one can estimate an upper bound based on the numerical results. For the two models used here, the error depends on the order in which the two step process is implemented. In general, as can be seen from the tables, systematically $e'(b_2, b_1, L) > e'(b_1, b_2, L)$ if $b_2 > b_1$ near the critical points of the models in two dimensions. The results when the smaller of $b_1, b_2$ is taken first in fact yields an error not much different from the one step case with $b = b_1b_2$. This, however, is not true for random cases (or when the noise is far above the critical value), where the coarse graining processes commute. Therefore, this observation can also be attributed to the critical fluctuations of the model. The two step coarse graining is relevant in the real world situations where the elected members of a legislative body can further form coalitions among themselves. $e'$ measures the chances of their decisions not being aligned with that of their electorates. It is also relevant for coding in information theory, the error is expected to increase if the coding is done in two steps.

In conclusion, we have reported the effect of coarse graining in the elections where an intermediate body is present between the population and the winner, for example in the US presidential election. We showed, using finite size scaling of the Ising model and kinetic exchange opinion models near criticality that the probability of a candidate winning the election without winning the popular vote depends non-monotonically with the coarse graining block size. Furthermore, using the data for the number of states in the US, we show that according to the model studied here, the minority win probability is near to the maximum value and could sharply decrease, provided the number of states are increased further.

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