Antigravity and the Big Crunch/Big Bang Transition

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- a proposal for continuing time through cosmological singularities

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arXiv/0365509 (gr-qc): today!
Success!

fluctuation level: temperature

Z'eldovich, Peebles+Yu 70’s
Bond+Efstathiou 80’s

polarization

Coulson, Crittenden, NT 94

\[ (l = 2\pi/\theta) \]

Angle on Sky (Degrees)
good evidence for ...

nearly flat FRW universe:
\[ \Omega_\Lambda : \Omega_{CDM} : \Omega_B : \Omega_\gamma : \Omega_\nu \sim 0.7 : 0.25 : 0.05 : 0.003 : 0.0003 \]

primordial perturbations
  * linear
  * growing mode
  * nearly scale-invariant
  * nearly “adiabatic”
  * nearly Gaussian

universe is: geometrically astonishingly simple compositionally complex
The inflationary paradigm has several basic conceptual difficulties
inflation

* initial conditions
* fine-tuned potentials
* $\Lambda \sim 10^{-120}; \quad \Lambda_I \sim 10^{-15}$

$V(\phi)$

$\phi$
* eternal inflation
  “anything that can happen will happen: and it will happen an infinite number of times” A. Guth, 2002
* string landscape: measure problem

-> reliance on anthropic arguments

- see P. Steinhardt, Sci Am 304, 36, 2011
  NT, http://pirsa.org/11070044
Inflation is based on the idea that the big bang singularity was the beginning.

But this may contradict unitarity.

What if the singularity was instead a bounce from a pre-bang universe?

An attractive cyclic universe scenario then becomes feasible.
The “big” puzzles:
- flatness, homogeneity and isotropy
- origin of perturbations
are solved via a pre-big bang period of ultra-slow contraction with an equation of state $w = P/\rho \gg 1$.

Since $\rho \sim a^{-(1+w)}$ rises rapidly as $a \rightarrow 0$ this nearly homogeneous and isotropic component* - rapidly dominates as the universe contracts to a “big crunch.”

Quantum fluctuations can generate scale-invariant, Gaussian, adiabatic perturbations.

*e.g. a scalar field with a steep negative potential.
For this scenario to be viable, we have to understand whether the universe can bounce from a “crunch” into a “bang.”

We shall try to do this largely using classical GR-scalar theory: we do not yet know how to properly include quantum corrections.

Our method is to introduce a new gauge symmetry - Weyl symmetry - allowing us to move the problem of $\det(g_{\mu\nu})$ vanishing to a sector where it appears milder. The field space in the “lifted” theory is larger and Newton’s constant is not necessarily positive.
A certain Weyl-invariant quantity passes analytically through the singularity, causing $M_{\text{PL}}$ to vanish momentarily*, and $G_N$ to briefly become negative.

We shall take this seriously, and study the resulting dynamics. We find the antigravity phase is brief, and the universe quickly recovers normal gravity.

Through a combination of analytic continuation and symmetry arguments we shall argue the outcome is unique: a completely predictable bounce.

*and hence Weyl symmetry to be restored.
Starting point: Einstein-scalar gravity

\[ S = \int d^4x \sqrt{-g} \left[ \frac{1}{2\kappa^2} R(g) - \frac{1}{2} (\partial \sigma)^2 - V(\sigma) \right] \]

Initial conditions: nearly homogenous, isotropic, flat universe with small perturbations.

As long as \( V(\sigma) \) is bounded, it becomes negligible as singularity nears.

Kinetic energy of scalar \( \sigma \) dominates, removes mixmaster chaos, ensures smooth ultralocal (locally Kasner) dynamics

Belinski+Khalatnikov+Lifshitz,Anderson+Rendall
In the final approach to the singularity, scalar kinetic energy density, scaling as $\sim a^{-6}$, dominates over anisotropies (also $\sim a^{-6}$), radiation ($\sim a^{-4}$), matter ($\sim a^{-3}$), pot energy($\sim a^0$).

We use Bianchi IX as an illustration:

$$ds^2 = a_E^2(\tau) \left( -d\tau^2 + ds_3^2 \right)$$

$$ds_3^2 = e^{-2\sqrt{2/3}\kappa_1} d\sigma_z^2 + e^{\sqrt{2/3}\kappa_1} \left( e^{\sqrt{2}\kappa_2} d\sigma_x^2 + e^{-\sqrt{2}\kappa_2} d\sigma_y^2 \right)$$

($d\sigma_{x,y,z}$ are $SU(2)$ left-invariant one-forms)

$\alpha_{1,2}$ parameterise the anisotropy
Generic solutions with anisotropy ($\alpha_{1,2}$)

\[
\frac{\dot{a}_E^2}{a_E^4} = \frac{\kappa^2}{3} \left[ \frac{\dot{\sigma}^2 + \dot{\alpha}_1^2 + \dot{\alpha}_2^2}{2a_E^2} + V(\sigma) + \frac{\rho_r}{a_E^4} \right] - \frac{Kv(\alpha_1, \alpha_2)}{a_E^2}
\]

\[
\frac{\ddot{a}_E}{a_E^3} - \frac{\dot{a}_E^2}{a_E^4} = -\frac{\kappa^2}{3} \left[ \frac{\dot{\sigma}^2 + \dot{\alpha}_1^2 + \dot{\alpha}_2^2}{a_E^2} - V(\sigma) + \frac{\rho_r}{a_E^4} \right],
\]

\[
\frac{\ddot{\sigma}}{a_E^2} + 2\frac{\dot{a}_E\dot{\sigma}}{a_E^3} + V'(\sigma) = 0,
\]

\[
\frac{\ddot{\alpha}_1}{a_E^2} + 2\frac{\dot{a}_E\dot{\alpha}_1}{a_E^3} - \frac{6K}{\kappa^2 a_E^2} \partial_{\alpha_1} v(\alpha_1, \alpha_2) = 0,
\]

\[
\frac{\ddot{\alpha}_2}{a_E^2} + 2\frac{\dot{a}_E\dot{\alpha}_2}{a_E^3} - \frac{6K}{\kappa^2 a_E^2} \partial_{\alpha_2} v(\alpha_1, \alpha_2) = 0.
\]
Near singularity, reduce to:

\[
\begin{align*}
\frac{\ddot{a}_E^2}{a_E^4} &= \frac{\kappa^2}{3} \left[ \frac{\dot{\sigma}^2 + \dot{\alpha}_1^2 + \dot{\alpha}_2^2}{2a_E^2} + \frac{\rho_r}{a_E^4} \right], \\
\ddot{\sigma} + 2\frac{\dot{a}_E}{a_E} \dot{\sigma} &= 0; \quad \ddot{\alpha}_i + 2\frac{\dot{a}_E}{a_E} \dot{\alpha}_i = 0
\end{align*}
\]

following from the effective action:

\[
\int d\tau \left\{ \frac{1}{2e} \left[ -\frac{6}{\kappa^2} \dot{a}_E^2 + a_E^2 (\dot{\sigma}^2 + \dot{\alpha}_1^2 + \dot{\alpha}_2^2) \right] - e\rho_r \right\}
\]
Our approach: “lift” Einstein-scalar to a Weyl-invariant theory

\[ \int d^4 x \sqrt{-g} \left[ \frac{1}{2} \left( (\partial \phi)^2 - (\partial s)^2 \right) + \frac{1}{12} (\phi^2 - s^2) R \right] \]

- add scalar ghost plus new gauge symmetry:
  \[ g_{\mu \nu} \rightarrow \Omega^2 g_{\mu \nu}, \quad \phi \rightarrow \Omega^{-1} \phi, \quad s \rightarrow \Omega^{-1} s \]
  (original motivation: brane picture/2T physics)

- gravitational trace anomaly cancels

- global $O(1,1)$ symmetry*: $\phi'^2 - s'^2 = \phi^2 - s^2$

* a closely related classical, approximate, shift symmetry appears in string theory - at tree level in $g_s$, but to all orders in $\alpha'$
Special quantity: Weyl and $O(1,1)$-invariant:

\[ \chi \equiv \frac{\kappa^2}{6} (-g)^{\frac{1}{4}} (\phi^2 - s^2) \quad (a^2_E = |\chi|) \]

- obeys Friedmann-like equation:

\[ \dot{\chi}^2 = \frac{2\kappa^2}{3} \left( p^2 + 2\rho_r \chi \right) \quad p \equiv \sqrt{p_{\sigma}^2 + p_1^2 + p_2^2} \]

- analytic at kinetic-dominated cosmological singularities
Gauges:

1. Einstein gauge $\phi^2 - s^2 = 6\kappa^{-2}$:
   \[
   \phi_E = \pm \left( \frac{\sqrt{6}}{\kappa} \right) \cosh(\kappa \sigma / \sqrt{6}) \\
   s_E = \left( \frac{\sqrt{6}}{\kappa} \right) \sinh(\kappa \sigma / \sqrt{6})
   \]

2. "Supergravity-like" gauge $\phi = \phi_0 = \text{const}$:
   \[
   \int d^4 x \sqrt{-g} \left[ -\frac{1}{2} (\partial s)^2 + \frac{1}{12} (\phi_0^2 - s^2) R \right]
   \]
   - cf N=1 SUGRA models (e.g. S. Weinberg QFT III)

3. "$\gamma$-gauge": $\text{Det} g = -1$:
   \[
   \int d\tau \left[ -\frac{1}{2} \dot{\phi}_\gamma^2 + \frac{1}{2} \dot{s}_\gamma^2 + \frac{\kappa^2}{12} (\phi_\gamma^2 - s_\gamma^2) (\dot{\alpha}_1^2 + \dot{\alpha}_2^2) \right]
   \]
Weyl-extended superspace
**Isotropic case:**
\[ \alpha_1 = \alpha_2 = 0 \]

**Generic case w/anisotropy:**

Weyl restored at gravity/antigravity transition
Solution with radiation only

\[ \chi(\tau) = 2\overline{\tau}(p + \rho_r\overline{\tau}) \]

\[ \overline{\tau} = \kappa \tau / \sqrt{6} \]

\[ \frac{\kappa}{\sqrt{6}} \sigma(\tau) = \frac{p\sigma}{2p} \ln \left| \frac{\overline{\tau}}{T(p + \rho_r\overline{\tau})} \right| \]

, sim\( \gamma \alpha_{1,2} \)
Uniqueness of solution

1. Analytic continuation
2. Asymptotic symmetries
3. Stationary points of action
1: unique extension of $\sigma$, $\alpha_{1,2}$ around singularities in complex $\tau$-plane
2. Asymptotic symmetries

Recall: \[
\int d^4 x \sqrt{-g} \left[ \frac{1}{2} \left( (\partial \phi)^2 - (\partial s)^2 \right) + \frac{1}{12} (\phi^2 - s^2) R \right]
\]

Define: \( \alpha_0 \equiv \kappa^{-1} \sqrt{3/2} \ln |\chi|, \quad \alpha_3 \equiv \sigma \)

Effective action becomes:

\[
\int d\tau \left\{ \frac{\chi}{2e} \left[ -\dot{\alpha}_0^2 + \dot{\alpha}_1^2 + \dot{\alpha}_2^2 + \dot{\alpha}_3^2 \right] \right\} - e\rho_r
\]

Effect of last term negligible as \( \chi \) vanishes.

\( \rightarrow \) massless particle on a conformally flat background. Invariant under ...
Special Conformal Group

\[ p_\mu = \chi \eta_{\mu\nu} \dot{\alpha}^\mu / e \]

\[ M_{\mu\nu} = \alpha_\mu p_\nu - \alpha_\nu p_\mu \]

\[ D = \alpha_\mu p_\mu \]

\[ K_\mu = \alpha^2 p_\mu - 2\alpha_\mu \alpha \cdot p \]

-asymptotically conserved, and thus finite at singularity
-analytically continuing \( \chi \), and matching SCG generators uniquely fixes the solution
3. Stationary point of Action

action finite: calculation varying all parameters governing passage across singularity shows action is stationary only on this solution
Is vacuum unstable in antigen. region?

No: grav. vac in = grav. vac out

Negative energy graviton

Positive energy photons
No particle production!
(neglecting other effects)
In fact, there is a Euclidean instanton defining the global vacuum state.
1. Stable in UV due to analyticity

2. Any particle production only shortens antigravity phase: proper time spent in the antigravity loop is

\[ \int_{\tau_c}^{0} a_E(\tau) d\tau = \sqrt{3}\pi p^2 / (4\kappa \rho_{r}^{\frac{3}{2}}) \]
We have studied the same problem in the Wheeler-de Witt equation for (ultralocal) quantum gravity in the $M_{PL} \rightarrow 0$ limit.

The conclusion is the same: there was a brief antigravity phase between the crunch and the bang.
Conclusions

* There seems to be a more-or-less unique way to continue 4d GR-scalar theory through cosmological singularities.

* Most surprisingly, it involves a brief antigravity phase.

* Does it agree w/ fully quantum approaches? (eg using holography: Craps/Hertog/NT)
Thank you!