The Overshoot Problem in Inflation after Tunneling

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A brief introduction to inflation

What is ‘overshoot problem’?

Overshoot in the scenario of bubble nucleation

Estimation of overshoot – universal behavior

Some generalizations

Conclusions and implications
Einstein's General Relativity is the standard model of gravitation

Curvature

Matter-Energy

Cosmological Principle: Homogenous and isotropic Universe

\[ ds^2 = -dt^2 + a^2(t) \left( \frac{dr^2}{1 - kr^2} + r^2 d\theta^2 + r^2 \sin^2\theta d\phi^2 \right) \]

Friedmann, Lemaitre, Robertson, Walker

\[ H^2 = \left( \frac{\dot{a}}{a} \right)^2 = \frac{\rho}{3M_P^2} - \frac{k}{a^2} \]

\[ \frac{\ddot{a}}{a} = -\frac{\rho + 3p}{6M_P^2} \]
Hot Big Bang started in a particular way!
What we know..

\[-0.0178 < \Omega_k < 0.006\]

Flatness problem.
What we know..

Homogeneous and isotropic

\[ \Delta T/T \sim 10^{-5} \]
Who ordered these?

Who ordered in this particular way!
Nearly exponential expansion: \( a(t) \sim e^{\alpha t} \)

Standard Big-Bang expansion: \( a(t) \sim t^n \)

H nearly constant

\[
H \sim \frac{1}{t}
\]
How does it help?

\[ \Omega - 1 \]

Flatness problem solved.
How does it help?

Horizon problem solved.
Conditions for Inflation

\[ \frac{\ddot{a}}{a} = -\frac{\rho + 3p}{6M_P^2} \]

\[ p < -\frac{\rho}{3} \]

Scalar Fields

\[ p = \frac{1}{2} \dot{\phi}^2 - V(\phi) \]

\[ \rho = \frac{1}{2} \dot{\phi}^2 + V(\phi) \]

\[ \frac{1}{2} \dot{\phi}^2 \ll V(\phi) \]

\[ p \simeq -\rho \]
Classical Dynamics

\[ h \rightarrow 0 \]

\[ \ddot{\phi} + 3H \dot{\phi} = -V''(\phi) \]

\[ H^2 = \frac{1}{3} \left( V(\phi) + \frac{1}{2} \dot{\phi}^2 \right) \]

Slow-roll parameters:

\[ \epsilon = \frac{1}{2} \left( \frac{V'}{V} \right)^2 \]

\[ \eta = \frac{V''}{V} \]
Goodbye and Hello!

Curvature perturbations:

$$\mathcal{P}_{R}^{1/2} = \frac{1}{2\sqrt{3\pi}} \frac{V^{3/2}}{|V'|}$$

$$n_s = 1 - 6\epsilon + 2\eta$$

WMAP data: $n_s \sim 0.96$

Nearly scale invariant perturbations during inflation
Not so trivial
Models of Inflation

old, new, pre-owned,
chaotic, quixotic, ergodic,
ekpyrotic, autoerotic,
faith-based, free-based,
D-term, F-term, summer-term,
brane, braneless, brainless,
supersymmetric, supercilious,
natural, supernatural, au natural,
hybrid, low-bred, white-bread,
one-field, two-field, left-field,
eternal, internal, infernal,
self-reproducing, self-promoting,
dilaton, dilettante, ........
Models of Inflation

\[ V(\phi) = V_0 \left[ 1 - \left( \frac{\phi}{\mu} \right)^p \right] + \ldots \]

\[ V(\phi) = \lambda_p \phi^p \]

\[ \Delta \phi < M_P \]

\[ \Delta \phi > M_P \]
Attractor Solutions and Overshoot

\[ V(\phi) = V_0 (1 + \lambda_1 (\phi - \phi_0) + \lambda_2 (\phi - \phi_0)^3) \]

On the attractor slow-roll parameters are small

Meeting point to the attractor depends on the initial kinetic energy

OVERSHOOT PROBLEM
Setting up the problem

Steep slope

Large kinetic energy

inflationary plateau

reheating

\[ \phi = 0 \]
Setting up the problem

What determines the initial K.E., in general, initial conditions and subsequent dynamics?
UV Physics

String Theory   Landscape of vacuua
UV Physics

String Theory     Landscape of vacuua

Tunneling from a close-by metastable vacuum via Coleman de Luccia instanton
String Theory  Landscape of vacuua

Tunneling from a close-by metastable vacuum via Coleman de Luccia instanton

bubble nucleation with negative curvature \((k = -1)\)

very special initial condition  \(a(t) = t\)  \(\phi_0 = \phi(t = 0) = 0\)
Problem at Hand

\[ V_L(\phi) = (-1)^n \frac{\lambda_n}{n} \phi^n \]

\[ V_R(\phi) = V_-(1 - \sqrt{2} \epsilon \phi) \]

\[ H^2 = \frac{1}{3 M_P^2} \left( \frac{\dot{\phi}^2}{2} + V(\phi) \right) + \frac{1}{a^2} \]

\[ \ddot{\phi} + 3H \dot{\phi} + \partial_\phi V = 0 \quad H = 1/t \]

Large initial damping due to the nature of CdL solution
$n = 1$, linear case

\[ V(\phi) = V_-(1 - \lambda \phi) \]

\[ \phi(t) = \phi_0 + \frac{\lambda V_-}{8} t^2 \]

\[ t_f = 2 \sqrt{\frac{-2\phi_0}{\lambda V_-}} \]

\[ \dot{\phi}(t_f) = \sqrt{-\left(\lambda V_0\phi_0\right)/2} \]
\( n = 1, \) linear case

\[
V(\phi) = V_-(1 - \lambda \phi) \quad \phi(t) = \phi_0 + \frac{\lambda V_-}{8} t^2
\]

\[
t_f = 2\sqrt{\frac{-2\phi_0}{\lambda V_-}}
\]

\[
\dot{\phi}(t_f) = \sqrt{-\frac{(\lambda V_\phi_0)/2}{2}}
\]

Curvature term = Potential energy

\[
\frac{1}{t_c^2} = \frac{1}{3V_-}
\]

\[
\phi(t_c) = \frac{3}{4\sqrt{2}} \sqrt{\epsilon - \phi_0}
\]

the amount of overshoot

\[
\phi_{overshoot} = \frac{3}{4\sqrt{2}} \sqrt{\epsilon + 2|\phi_0|}
\]
\( n = 2, \text{ quadratic case} \)

\[
V(\phi) = \frac{1}{2} m^2 \phi^2
\]

\[
\phi(t) = 2\phi_0 \frac{J_1(mt)}{mt}
\]

\( t_f \approx \frac{3.83}{m} \)

\[
\dot{\phi}(t_f) = \phi_0 \frac{J_0(mt_f) - J_1(mt_f)}{t_f} \approx -0.21m\phi_0
\]

\[
\phi(t_c) = \frac{3}{4\sqrt{2}} \sqrt{\epsilon} - 0.403\phi_0
\]

the amount of overshoot

\[
\phi_{\text{overshoot}} = \frac{3}{4\sqrt{2}} \sqrt{\epsilon} + 1.4\phi_0
\]

For linear:

\[
\phi_{\text{overshoot}} = \frac{3}{4\sqrt{2}} \sqrt{\epsilon} + 2|\phi_0|
\]
\[ n = 3, \text{ cubic case} \]

\[ \ddot{\phi} + \frac{10}{3t} \dot{\phi} - \lambda \phi^2 = 0 \]

First integral

\[ C_0 = \frac{1}{2} \phi^2 t^4 + \frac{4}{3} \phi \phi t^3 - \frac{1}{3} \phi^3 t^4 + \frac{2}{9} \phi^2 t^2 - \frac{4}{9} \phi t - \frac{28}{27} \]

Non-zero velocity at the bottom

Solutions in terms elliptic Weierstrass function

Upper and lower limit of velocity at the bottom can be found!
A brief summary

For linear, quadratic and cubic

\[ \phi(t_c) = \frac{3}{4\sqrt{2}} \sqrt{\epsilon} - \mathcal{O}(1) \phi_0 \]

A Universal factor

Coefficient decreases for higher order
n = 4, quartic case

\[ \phi(t) = \frac{8\phi_0}{8 + t^2 \lambda \phi_0^2} \quad \phi \to 0, \quad t \to \infty \]

\[ \dot{\phi}(t) = -\frac{16t\lambda \phi_0^3}{(8 + t^2 \lambda \phi_0^2)^2} \quad t \to t_f \to 0 \]

No Overshooting at all for n = 4

\[ V > 0 \text{ causes vacuum energy domination for some large } t > 0 \]

already on the left for \( \phi < 0 \)!
\( \gamma = \frac{m + 3}{m - 1} \)

First integral

\[
C_0 = \frac{t^{\gamma - 1}}{2} \left( \dot{\phi}^2 t^2 + \ddot{\phi} \phi (\gamma - 1) \right) + \left( \phi t^\gamma \right)^{m+1} \frac{1}{m+1}
\]

\( t = 0 \quad C_0 = 0 \)

\( \phi = 0 \quad \dot{\phi} \neq 0 \)
No overshooting for quartic and higher monomials

An universal behavior of the amount of overshooting for $n < 4$
On the right hand side SR begins when the curvature becomes subdominant at

\[ t_c = \sqrt{\frac{3}{V_-}} \]

\[ \ddot{\phi} + \frac{3}{t} \dot{\phi} - V_- \sqrt{2\epsilon} = 0 \]

at \( t_f \) the field crosses \( \phi = 0 \) with speed \( \dot{\phi}_0 = \dot{\phi}(t_f) \)

Universal feature: Why?

\[ \phi(t_c) = \frac{3}{4\sqrt{2}} \sqrt{\epsilon} + \frac{1}{2} \dot{\phi}_0 t_f - \frac{1}{2\sqrt{2}} \sqrt{\epsilon V_- t_f^2} - \frac{1}{6} \dot{\phi}_0 V_- t_f^3 + \frac{1}{12\sqrt{2}} \sqrt{\epsilon V_-^2 t_f^4} \]

\[ \sim \frac{3}{4\sqrt{2}} \sqrt{\epsilon} + \frac{1}{2} \dot{\phi}_0 t_f + \mathcal{O}(V_- / V_0) \]
\[ \phi(t_c) \sim \frac{3}{4\sqrt{2}} \sqrt{\epsilon} + \frac{1}{2} \phi_0 t_f + \mathcal{O}(V_-/V_0) \]

\[ t_f \sim \begin{cases} \frac{|\phi_0|}{\sqrt{V_0}}, & 1 \leq n \leq 3 \\ \infty, & n \geq 4 \end{cases} \]

\[ \dot{\phi}_0 \sim \sqrt{V_0} \]

\[ \phi(t_c) = \frac{3}{4\sqrt{2}} \sqrt{\epsilon} - \mathcal{O}(1) \phi_0 \]

does not depend on \( \dot{\phi}_0 \)
Binomials

\[ V(\phi) = (-1)^m \frac{\lambda_m}{m} \phi^m + (-1)^n \frac{\lambda_n}{n} \phi^n, \quad n > m \]

match solutions where force becomes equal

\[ (-1)^m \lambda_m \phi^{m-1} \bigg|_{\phi=\phi_*} = (-1)^n \lambda_n \phi^{n-1} \bigg|_{\phi=\phi_*} \]

An example: linear + quadratic

\[ \phi(t_c) = \frac{3}{4\sqrt{2}} \sqrt{\epsilon} - \left( \frac{\phi_*}{\phi_0} \right)^{3/2} \phi_0 \]
A Plausibility argument

Higher monomial steeper at the beginning, but shallower at the end!

For steeper part ‘time’ is smaller, therefore friction is larger!
Final point

\[ \Delta \phi_{\text{Tot}} = \Delta \phi_{\text{overshoot}} + \Delta \phi_{\text{inflation}}(N = 60) \]

\[ \phi_{\text{overshoot}} = \frac{3}{4\sqrt{2}} \sqrt{\epsilon} + O(1) |\phi_0| \]
Conclusions

Inflationary regions followed by tunneling has negligible overshoot:

a) slow down of speed due to negative curvature of the bubble

b) Universal behavior of the overshoot

For small-field inflation models ‘overshoot problem’ is not severe if inflation is fed by the CdL tunneling
!!OBRIGADO!!