Dynamic Reliability and Reliability-Based Sensitivity of Transmission System in Shearer

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Abstract. Transmission system in shearer, such as traction unit and cutting arm, is the essential mechanism in the mining process, which supports the whole shearer and shock vibration. Considering various uncertainties, the evolving process is established to describe the dynamic system response with uncertain factors. Considering the parametric randomness and material strength, the failure domain is regarded as the dynamic extreme value of transmission system response exceeding the threshold in operation. According to the saddle-point approximation method, the dynamic extreme distribution is investigated to measure the safe probability and parametric sensitivity of transmission system in shearer. The dynamic reliability and reliability-based sensitivity are analysed to evaluate the evolving process of transmission system in shearer. According to the transmission model of traction unit, the influence degree of each parameter’s fluctuation can be quantified based on the indices of dynamic mean-value reliability-based sensitivity and standard deviation reliability-based sensitivity. The crude Monte Carlo simulation is taken as the benchmark to validate the reasonability.

1. Introduction
Shearer equipment plays an increasingly important role in the industrial development of the mining industry. According to the hostile environment and shock vibration, system reliability guarantees the safe power transmission process and reliable structure. The maintenance information was considered to analyze the failure logic of a long-wall shearer for pinpointing the weak links [1]. Also, a case study was presented to describe the reliability and availability analysis on the subsystems of crushing plant at Jajarm Bauxite Mine [2]. The statistical approach was investigated to locate critical subsystems in order to enhance the optimum operation of the shearer machine [3]. Various uncertainties were considered to establish the dynamic transmission system and analyze the fluctuation of the dynamic responses [4]. Fracture failure analysis was investigated to describe the useful life and dynamic performance of internal ring gear in the planetary transmission system [5]. The worst case of the limit state in the failure-safety boundary was analyzed to evaluate the planetary transmission system in shearer [6]. Also, the response surface method was investigated to evaluate the vibration signal to describe the system responses [7]. The dynamic operation process needs to be considered to evaluate the performance of the transmission system for the safety and robustness of shearer loader.
Dynamic reliability is critical to guarantee the safe system performance in the moving process, which includes the external excitation and various system uncertainties. The saddle-point approximation method (SPA) has extensive applications in many fields, and provides the probability approximations with high accuracy [8]. The line sampling method and the Nataf transformation were combined to analyze the structural reliability in case of non-normal random variables [9]. The mean-value first-order SPA was implemented to investigate the epistemic uncertainty and aleatory uncertainty in engineering structures [10]. The sequence of cycles of deterministic optimization was presented to investigate the reliability-based design via SPA [11]. The stochastic response surface method and SPA were investigated for the structural reliability [12]. The third-moment SPA approach was proposed to quantify the probability of failure [13]. Also, fourth-moment SPA was investigated to calculate the structural reliability [14]. The SPA method was developed for the limit state function with a quadratic form with standard normal variables [15]. The SPA method and the second-order Taylor expansion were investigated for the system reliability-based design optimization [16]. Also, the second-order SPA was developed for the system reliability in cases with correlated non-Gaussian variables [17]. The approximate SPA method is investigated to evaluate the system reliability and the important index [18]. The SPA approach can evaluate the probability distributions of multi-variate system responses with various uncertainties. Based on SPA method, the dynamic characteristics can be investigated for the safety of traction unit in shearer.

The objective of this paper is to analyze the dynamic reliability and dynamic reliability-based sensitivity of the transmission system in shearer. The paper is organized as follows. Section 2 presents the dynamic evolving process by the SPA method for the dynamic system. Section 3 proposes the dynamic reliability and dynamic reliability-based sensitivity methods for the evolving process. Then, the dynamic reliability and sensitivity indices of the transmission system of traction unit in shearer are both analyzed in section 5. Finally, conclusions are presented in section 6.

2. Dynamic evolving process of transmission system in shearer

Dynamic transmission system in the transmission system is one of the most important sub-systems in the transmission chains of the shearer. The complex mechanical structure and nonlinear features have important effects on the dynamic system responses and statistical characteristics. Considering vibration mechanical model and rigid motion model, the dynamic transmission model of traction unit can be established under the time-varying speed. In this section, a dynamic evolving process of system response is investigated by the saddle-point approximation (SPA) method for reliability analysis in the transmission system.

2.1. Saddlepoint approximation method

The SPA method has the advantage of high accuracy in the tails of multivariate distribution functions [8,15]. The approach can be utilized to describe the output distribution of a complex system with various distributions, such as the traction unit in shearer. For clarity, symbol $X$ denotes a random variable, symbol $x$ represents an observation of $X$, and vector $X$ represents $[X_1,\ldots,X_n]^T$. Considering the saddle point $s$, the moment-generating function (MGF) of $X_j (j=1,\ldots,n)$ can be presented as

$$M_{X_j}(s) = \int_{-\infty}^{+\infty} \exp(sx_j)f_{X_j}(x_j)dx_j$$

in which $f_{X_j}(x_j)$ can be regarded as the probability density function. The cumulant-generating function (CGF) of variable $X_j$ can be depicted as

$$K_{X_j}(s) = \ln\left[M_{X_j}(s)\right]$$

The CGFs for different kinds of probability distributions are summarized in reference [6]. According to the CGF $K_{X_j}(s)$ of random variable $X_j$, the useful property of the CGF can be depicted expressed as


\[ K_Y(s) = \sum_{j=1}^{n} K_{X_j}(\xi_j, s) + \gamma s, \quad \text{if } Y = \sum_{j=1}^{n} \xi_j X_j + \gamma \]  

(3)

According to the constant parameters \( \xi \) and \( \gamma \), the property of the CGF is listed in Equation (3). Considering the first-order Taylor expansion series of function \( Y = g(X) \), the CGF of function \( Y \) can be expressed as

\[ K_Y(s) = K_X\left(g(x^*) + \sum_{j=1}^{n} \frac{\partial g}{\partial x_j} \mid_{x=x^*} (X_j - x_j^*) \right) = g\left(x^*\right)s - \sum_{j=1}^{n} \frac{\partial g}{\partial x_j} \mid_{x=x^*} x_j^*s + \sum_{j=1}^{n} K_{X_j} \left(\frac{\partial g}{\partial x_j} \mid_{x=x^*} \right) s \]  

(4)

in which the expansion point \( x^* \) takes the mean values in the defined domain of \( X \). Based on the dynamic transmission model of traction unit, the state function \( Y \) can be established by HOSRM (High Order Stochastic Respond Surface Method) [7]. For any multivariate function \( Y = g(X) \), a closed expression [8] of probability density function (PDF) can be defined as

\[ f_y(y) = \left[ 2\pi K_y^\prime(s) \right]^{-1/2} \exp\left[K_y^\prime(s) - sy\right] \]  

(5)

in which the saddle point \( s \) can be computed by the first order derivative \( K_y^\prime(\cdot) \), as below

\[ K_y^\prime(s) = y \]  

(6)

Moreover, the cumulative distribution function (CDF) can be depicted as

\[ F_y(y) = P\{Y \leq y\} = \Phi(w) + \varphi(w)\left(w^{-1} - v^{-1}\right) \]  

(7)

in which parameters \( \Phi(\cdot) \) and \( \varphi(\cdot) \) are the CDF and PDF of the standard normal distribution, respectively. Parameters \( w \) and \( v \) in Equation (8) and Equation (9) can be expressed by

\[ w = \text{sgn}\left(s\right)\left[2\left[ sy - K_y^\prime(s)\right]\right]^{1/2} \]  

(8)

\[ v = s\left[K_y^\prime(s)\right]^{1/2} \]  

(9)

Hence, according to SPA method, the PDF and CDF can be utilized to indicate the output distribution information of a multi-variate system. Parameter uncertainties can be considered to analyse the statistical characteristic of the transmission system in shearer by SPA method.

2.2. Evolving process of dynamic system

According to the statistical information, the failure domain can be considered as the extreme value of system response exceeding the threshold in operation. The number of extreme responses is gradually increasing in the working process. Considering the output system data \( Y_i \), the safety domain \( \Theta \) can be described as the maximum values below the corresponding threshold \( H \), which can be expressed as

\[ \Theta = \{Y_1 < H, \ldots, Y_i < H, Y_{i+1} < H, \ldots\} = \{\max\{Y_i\} < H\} = \{Y_{(m)} < H\} \]  

(10)

in which parameter \( Y_{(m)} \) is the maximum value of output response \( Y_i \). The probability of output variable \( y \) in the safety domain can be expressed as

\[ F_M(y) = P\left\{\max\{Y_1, \ldots, Y_i, Y_{i+1}, \ldots\} \leq y\right\} = \prod_{k=i}^{m} F(y) = F^m(y) \]  

(11)

in which \( M = Y_{(m)} \) and parameter \( m \) can be regarded as a dynamic factor or frequency number of actions. Considering the SPA method from Equation (5) to Equation (7), the dynamic evolving process of output system variable \( y \) can be expressed as
\[ F_M(y) = P^n[Y \leq y] = \left[ \Phi(w_0) + \varphi(w_0) \left( w_0^{-1} - v_0^{-1} \right) \right]^n \]  

in which the parameters \( w_0 \) and \( v_0 \) can be calculated by Equation (8) and Equation (9). The statistical information of dynamic system response can be utilized to depict the evolving process based on the SPA method. The dynamic process indicates the evolving process of dynamic response in the safety domain of multivariate system. Also, the evolving process can be utilized to evaluate the dynamic system reliability in operation.

3. Dynamic reliability-based sensitivity

According to the dynamic evolving process, a dynamic reliability model (DRM) based on SPA method can be established to analyse the system performance. Also, dynamic reliability-based sensitivity is investigated by SPA method.

3.1. Dynamic reliability model

Considering the system performance function, the CDF and PDF can be established to depict the transmission system in operation. Also, the load-strength interference is investigated to analyse the system reliability. For a structural strength \( h(\bullet) \) and probability density function \( f(\bullet) \), the reliability index \( R \) of the static load-strength interference model [18] can be defined as

\[
R = \int_{-\infty}^{\infty} f(X) \int_{-\infty}^{\infty} h(\varepsilon) d\varepsilon dX = \int_{-\infty}^{\infty} h(\varepsilon)\int_{-\infty}^{\infty} f(X) dX d\varepsilon
\]

in which parameter \( \varepsilon \) can be considered as a vector including system random variables; and parameter \( \varepsilon \) can be regarded as the material strength of any system structure. Also, the dynamic factor \( m \) is investigated to evaluate the system performance in operation. Considering the dynamic evolving process \( F_M(\bullet) \) and the strength distribution \( h(\bullet) \), the dynamic reliability model (DRM) can be depicted by

\[
R(m) = \int_{-\infty}^{\infty} h(\varepsilon)\int_{-\infty}^{\infty} f(X) dX d\varepsilon = \int_{-\infty}^{\infty} h(\varepsilon)F_M(X, \varepsilon, m) d\varepsilon
\]

The dynamic load-strength interference model is established to describe the evolving process of transmission system in operation. According to Equation (12), the dynamic reliability model via SPA method can be expressed as

\[
R(m) = \int_{-\infty}^{\infty} h(\varepsilon)\left[ \Phi(w_0) + \varphi(w_0) \left( w_0^{-1} - v_0^{-1} \right) \right]^n d\varepsilon
\]

in which the parameters \( w_0 \) and \( v_0 \) are defined by Equation (8) and Equation (9). The relationship is established to evaluate the system parameters and system reliability. The dynamic model in Equation (15) can be utilized to evaluate the system reliability with frequency \( m \) of actions, and it can be considered as the dynamic reliability model with action frequency \( m \). Also, the dynamic evolving processes of system parameter and material parameter can be analysed to describe the system performance in operation. Hence, the dynamic performance of the transmission system in shearer can be evaluated by dynamic reliability model Equation (15) with the mesh frequency \( m \).

4. Dynamic Reliability-Based Sensitivity

In engineering, the probability distribution of load has no connection with the structural strength in the system. For the structural and load parameters, the mean-value dynamic reliability-based sensitivity can be computed by

\[
S_{\mu_j} = \frac{\partial R}{\partial \mu_j} = \int_{-\infty}^{\infty} h(\varepsilon)\frac{\partial F_M(X, \varepsilon, m)}{\partial \mu_j} d\varepsilon = \int_{-\infty}^{\infty} h(\varepsilon)\left[ \frac{\partial}{\partial \mu_j} \left[ \Phi(w_0) + \varphi(w_0) \left( w_0^{-1} - v_0^{-1} \right) \right]^n \right] d\varepsilon
\]

Also, considering SPA method, the standard deviation dynamic reliability-based sensitivity can be
expressed as
\[
S_{\sigma_j} = \frac{\partial R}{\partial \mu_j} = \int_{-\infty}^{\infty} h(\varepsilon) \left[ \frac{\partial F_m(X, \varepsilon, m)}{\partial \mu_j} \right] d\varepsilon = \int_{-\infty}^{\infty} h(\varepsilon) \left[ \partial \left[ \Phi(w_0) + \varphi(w_0) \left( w_0^{-1} - v_0^{-1} \right) \right] / \partial \mu_j \right] d\varepsilon \quad (17)
\]

Additionally, the mean-value dynamic reliability-based sensitivity of the structural strength in the proposed model can be calculated by
\[
S_{\mu_j} = \frac{\partial R}{\partial \mu_j} = \int_{-\infty}^{\infty} \left[ \frac{\partial h(h)}{\partial F_m(X, \varepsilon, m)} \right] F_m(X, \varepsilon, m) d\varepsilon = \int_{-\infty}^{\infty} \left[ \frac{\partial h(h)}{\partial \mu_j} \right] \left[ \Phi(w_0) + \varphi(w_0) \left( w_0^{-1} - v_0^{-1} \right) \right] d\varepsilon \quad (18)
\]

The standard deviation dynamic reliability-based sensitivity of the structural strength can be depicted as
\[
S_{\sigma_j} = \frac{\partial R}{\partial \sigma_j} = \int_{-\infty}^{\infty} \left[ \frac{\partial h(h)}{\partial \sigma_j} \right] F_m(X, \varepsilon, m) d\varepsilon = \int_{-\infty}^{\infty} \left[ \frac{\partial h(h)}{\partial \sigma_j} \right] \left[ \Phi(w_0) + \varphi(w_0) \left( w_0^{-1} - v_0^{-1} \right) \right] d\varepsilon \quad (19)
\]

Hence, the dimensionless indices of the mean-value and standard deviation dynamic reliability-based sensitivity can be indicated by
\[
\tilde{S}_\mu = \frac{\mu_j}{R} \cdot S_{\mu_j}, \text{ if } S_{\mu_j} \neq 0 \text{ or } S_{\sigma_j} \quad (20)
\]
\[
\tilde{S}_\sigma = \frac{\sigma_j}{R} \cdot S_{\sigma_j}, \text{ if } S_{\sigma_j} \neq 0 \text{ or } S_{\sigma_j} \quad (21)
\]

Both the mean-value and standard deviation reliability-based sensitivity are investigated to present each parameter’s dynamic influence on the system performance with dynamic factor \( m \). The system parameters and strength parameters are taken into consideration. Also, based on SPA method, the dimensionless indices are presented to analyse the influences of various parameters’ numeric values in the dynamic reliability-based sensitivity model.

5. Dynamic reliability analysis on transmission system in shearer

The traction unit of MG300/700-WD shearer is shown in Figure 1. The transmission system is the imperative mechanism in the mining process, which supports the whole shearer and withstands the shock vibration. Gear drive with parallel axes and planetary gear train are widely utilized to constitute the whole transmission system in shearer mechanism, which is shown in Figure 2.

![Figure 1. Traction unit of the shearer mechanism.](image1)

![Figure 2. Transmission model of traction unit in shearer.](image2)

The transmission system has a significant impact on the system performance, even the coal mining process. Parameters of MG300/700-WD shearer [6] are considered to establish the dynamic transmission model of traction unit in shearer. Considering the variable speed and uncertain factors,
the dynamic model of transmission model [7] can be established to investigate the system performance and characteristic in shearer. Based on the established transmission model, dynamic reliability can be analysed to ensure the safety of coal mining. Dynamic responses, such as displacements and meshing forces, can be calculated to analyse the transmission system in shearer. Also, the experiment responses of force in the axle are compared with the result calculated by dynamic transmission system. The test position of transmission system is shown as Figure 3 to evaluate the system performance of traction unit in shearer. According to the FFT (Fast Fourier Transform) of corresponding results, the frequency and peak values have good coherence, as shown in Figure 4.

![Figure 3. Test position of transmission system in shearer.](image1.png)

![Figure 4. FFT results of dynamic transmission model and experiment test.](image2.png)

Hence, the dynamic transmission model can be utilized to investigate the system characteristic of the transmission system in shearer. The dynamic evolving process of the transmission system can be established on basis of section 2.2 to investigate the shearer in operation. The dynamic reliability and reliability-based sensitivity can be analysed for the worst part, which has the maximum displacement response. According to the dynamic transmission model, the dynamic factor $m$ is adopted to describe the frequency of tooth contact in the planetary 2 of the gear transmission system. The dynamic reliability of the traction unit is analysed by the dynamic reliability model based on SPA and Monte Carlo simulation (MCS), as shown in Figure 5. The standard deviation reliability-based sensitivity for structural strength is compared via the dynamic reliability-based sensitivity based on SPA and MCS, as shown in Figure 6.

![Figure 5. Dynamic reliability of the traction unit in shearer.](image3.png)

![Figure 6. Dynamic reliability-based of structural strength in traction unit.](image4.png)
The results calculated by SPA method are consistent with the MCS. The dynamic reliability is decreasing as time goes by. The relationship between dynamic reliability and coal production is established in Figure 5. The maintenance cycle can be set as three months to guarantee the safety of the transmission system in shearer. An increase in dispersion of structural strength enables the index of the parameter’s influence to increase and the dynamic reliability to decrease in operation.

The tooth width of planetary system 2 in the transmission system is taken into consideration. The mean value reliability-based sensitivity for tooth width $B_s$ with different strength values is described in Figure 7. Dynamic indices of reliability-based sensitivity are varying with different material strength and frequency. Different material strength can affect the system performance and system reliability. Also, the reliability-based sensitivity is varying with the motion of the transmission system in shearer. The standard deviation reliability-based sensitivity for tooth width $B_s$ with different strength values is described in Figure 8.

![Figure 7. Mean value dynamic reliability-based sensitivity for tooth width $B_s$.](image1)

![Figure 8. Standard deviation dynamic reliability-based sensitivity for tooth width $B_s$.](image2)

The standard deviation reliability-based sensitivity for parameter $B_s$ at fixed contact fatigue strength is negative and decreases as dynamic factor $m$ increases in Figure 8. The negative values imply that an increasing mean value of parameter $B_s$ decreases the dynamic reliability of the transmission system. The influence of the standard deviation on system reliability increases as dynamic factor $m$ increases. However, from another perspective, the index of sensitivity initially increases and then reduces on the condition of the structural strength at a specified factor $m$. The mean value and standard deviation reliability-based sensitivity for system torque $T$ are described in Figure 9 and Figure 10.

![Figure 9. Mean value dynamic reliability-based sensitivity for system torque $T$.](image3)

![Figure 10. Standard deviation dynamic reliability-based sensitivity for torque $T$.](image4)
In Figure 9 and Figure 10, considering the fixed material strength, the indices of reliability-based sensitivity for system torque $T$ are both negative and decrease as dynamic factor $m$. Considering the specified factor $m$, the indices of sensitivity initially increase and then decrease along with the structural strength. Hence, system parameter can be investigated to depict the dynamic performance of the transmission system in traction unit of shearer. Hence, the standard deviation’s influence on the system reliability can be analysed to represent the law of system structural parameter. A suitable material strength can be calculated and an appropriate material can be selected to ensure system dynamic reliability. The parameters can be monitored and controlled to keep the transmission system safety and reliability in operation.

Conclusions
In this work, the evolving process of the transmission system in shearer is established to represent the dynamic system performance in operation. The dynamic reliability model is proposed to evaluate the system characteristic with respect to different uncertain factors. Also, dynamic reliability-based sensitivity index is implemented to analyse each parameter’s influence on the dynamic transmission system in traction unit of shearer.

In conclusion, the results calculated by dynamic reliability and reliability-based sensitivity method coincide with those by the crude Monte Carlo simulation. Also, according to the dynamic reliability-based sensitivity, the plus-minus sign of index indicates the positive or negative influence on dynamic reliability of the transmission system. According to the different strength parameters, the reliability-based sensitivity index presents the degree of dynamic effect on the system dynamic reliability. The dynamic performance and characteristic can be evaluated by the proposed method. The suitable material and parameter can be selected and applied into the design of the transmission system in shearer.

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