Counter-diabatic driving for fast spin control in a two-electron double quantum dot

Yue Ban & Xi Chen

1Department of Electronic Information Materials, Shanghai University, 200444 Shanghai, People’s Republic of China, 2Department of Physics, Shanghai University, 200444 Shanghai, People’s Republic of China.

The techniques of shortcuts to adiabaticity have been proposed to accelerate the “slow” adiabatic processes in various quantum systems with the applications in quantum information processing. In this paper, we study the counter-diabatic driving for fast adiabatic spin manipulation in a two-electron double quantum dot by designing time-dependent electric fields in the presence of spin-orbit coupling. To simplify implementation and find an alternative shortcut, we further transform the Hamiltonian in terms of Lie algebra, which allows one to use a single Cartesian component of electric fields. In addition, the relation between energy and time is quantified to show the lower bound for the operation time when the maximum amplitude of electric fields is given. Finally, the fidelity is discussed with respect to noise and systematic errors, which demonstrates that the decoherence effect induced by stochastic environment can be avoided in speeded-up adiabatic control.

Electron spins in quantum dots (QDs) have been extensively investigated for potential applications in quantum information processing, as spins in QDs are expected as a possible realization of qubit in quantum information science and technology. Especially, a two-electron double QD can be further regarded as the smallest network to implement quantum computation, in which the highly entangled spin state, i.e., the singlet, can be generated. Requirements of precisely controlled qubits have intensively stimulated the detailed studies of the interactions in double-dot systems and the observations of phenomena thereby, such as Pauli spin blockade and Coulomb blockade. Furthermore, the demands for achieving efficient quantum computations and avoiding decoherence motivate us to manipulate spin states in double QDs in a fast and robust way. There are several methods to manipulate spin in QDs, such as electron spin resonance induced by magnetic field oscillating at the Zeeman transition frequency and electric control with spin-orbit (SO) coupling. Recently, conventional “rapid” adiabatic passages in quantum optics, for example, Landau-Zener scheme, have been extensively used to spin control in single QD, coupled double QD, tripled QD, which can be applied to prepare entanglement states and quantum logical gates, such as NOT and CNOT gates.

Shortcuts to adiabaticity have been proposed to speed up the adiabatic process without final excitation with many applications in atomic, molecular, optical physics, many-body physics, and even spintronics, see recent review. In a single QD, we applied the inverse engineering method to design a fast and robust protocol of spin flip in the nanosecond timescale, based on the Lewis-Riesenfeld invariant theory. Furthermore, in a two-electron QD, more freedom in the applied electric fields provides the flexibility to control spin states by the invariant dynamics and controllable Lewis-Riesenfeld phases. An alternative shortcut is provided by counter-diabatic control proposed by Demirplak and Rice, equivalent to transitionless quantum driving. This technique was originally utilized to fast adiabatic control in two-level quantum systems theoretically and experimentally. Short afterwards, it has been extended to multi-level systems, and even many-body systems.

In this Report, we propose a fast and reliable protocol to generate the entangled spin states by using counter-diabatic driving. The external electric fields are designed for rapid spin control in a two-electron double QD in the presence of a static magnetic field and SO coupling. We apply the electric fields, instead of magnetic fields, and take advantage of SO coupling, since the time-dependent electric fields are easy to be generated on the nanoscale by adding local electrodes. In addition, as comparing to a single QD, counter-diabatic driving is applicable in a two-electron double QD, as there exists more freedom with four controllable parameters, x and y components of the external electric fields for each dot. To simplify the experimental setup and reduce the device-dependent noise, we further apply the concept of multiple Schrödinger pictures to find an alternative shortcut with only x...
and the triplet, we focus on the state transition between these lowest one of the triplet states is much less than the gap between the singlet growth axis. If the energy difference between the singlet and the InAs QDs.

We design the electric fields in achieve fast adiabatic transition between the triplet and the singlet. Finally, the stability of designed shortcuts are discussed the lower bound of operation time for a given maximal amplitude of potential in Fig. 1, where they are isolated by Coulomb blockade n. In

Results

Two electrons are confined in a double QD, described as a quartic potential in Fig. 1, where they are isolated by Coulomb blockade n. In the presence of the applied magnetic fields, the lowest four eigenstates of the system can be expressed by singlet and triplet for S = 0 and S = 1 in the basis of |S, Sz⟩. This report presents a method to achieve fast adiabatic transition between the triplet and the singlet. We design the electric fields in x − y plane to manipulate spin states with static magnetic fields along z direction in each dot, considering structure-related Rashba (z) and bulk-originated Dresselhaus (β) for [110] growth axis. If the energy difference between the singlet and the lowest one of the triplet is much less than the gap between the singlet and the triplet, we focus on the state transition between these lowest two, as shown in Fig. 1, where Landé factor g < 0 like in GaAs and InAs QDs.

By choosing |1⟩ = (1, 0) and |−1⟩ = (0, 1) t, referring to the states |0, 0⟩ and |1, 1⟩, respectively, we may first take the reference Hamiltonian as

\[ H_0 = \frac{\hbar}{2} \begin{pmatrix} Z & iY \\ -iY & -Z \end{pmatrix}, \]

where \( Y = -\sqrt{2}e\sigma_z(A_{ij}^x - A_{ij}^y) / \hbar c, Z = (J - \Delta) / h + e\beta(A_{ij}^x + A_{ij}^y) / \hbar c, \) and \( A_{ij}^x \) is determined by the electric fields, \( E_j(t) = -(1/c)\partial \mathcal{E}_j / \partial t. \)

The subscriptions \( j = L, R \) represent the left and the right dots, respectively. Here we assume the ansatz of the vector potentials is \( A_j^t = A_0 \{ \tanh \left[ (t - a_j t_f) / (w_j t_f) \right] + 1 \}, \) where \( a_L = 0.54, a_R = 0.48, \) \( w_L = w_R = 0.1. \) The ansatz of vector potentials satisfies the condition \( A_j^t(0) = 0 \) and guarantees that the electric fields \( \mathcal{E}_j^t \) start to be driven from \( t = 0, \) that is, \( \mathcal{E}_j^t \equiv 0, \) when \( t \leq 0. \) When the adiabatic condition

\[ \left| \frac{ZY - YZ}{(Y^2 + Z^2)^{3/2}} \right| \ll 1 \]

is fulfilled, the spin state will evolves from \(|-1⟩ \) to \(|1⟩ \) adiabatically along one of instantaneous eigenstates. When the final time is \( t_f = 11 \) ns, the spin state is completely inverted, and the final population of \(|1⟩ \) is larger than 0.9999.

Shortening the manipulation time to \( t_f = 2 \) ns, shrinking \( A_j^t \) into this time duration and keeping the same amplitude, we can find the state evolution is no longer adiabatic and the final state cannot reach \(|1⟩ \) at the final time. The same profiles of time-dependent Y and Z terms in \( H_0 \) are shown in Fig. 2 (a) for different operation times, \( t_f. \)

Counter-diabatic driving, equivalent to transitionless quantum driving17,22,24 provides supplementary time-dependent interactions \( H_1 \) to cancel the diabatic couplings of \( H_0 \), and make the process fast and adiabatic, where \( H_1 \) is

\[ H_1 = \frac{\hbar}{2} \begin{pmatrix} X & 0 \\ 0 & -X \end{pmatrix}, \]

with \( X = \sqrt{2}e\sigma_z(A_{ij}^x - A_{ij}^y) / \hbar c, \) driven by \( \mathcal{E}_j^t, \) the difference between y component of two electric fields. As a result, the exact dynamical evolution of total Hamiltonian \( H = H_0 + H_1 \) coincides with adiabatic approximation of the reference Hamiltonian \( H_0 \). However, to implement accelerated adiabatic transitions more energy price has to pay, that is, the maximal amplitude of \( A_j^t \) in the \( X \) term increases when the finally time \( t_f \) is shortened. This can be intuitively understood from time-energy uncertainty principle, that is, \( A_j^t \) is proportional to \( 1/t_f \). Since \( \mathcal{E}_j^t(t) = -(1/c)\partial \mathcal{E}_j / \partial t, \) the larger value of \( \mathcal{E}_j^t \) and \( \mathcal{E}_j^0 \) are finally required for the shorter time, \( t_f, \) as shown in Fig. 2 (c).

In reality, the electron spin is subject to the device-dependent noise, which could be the amplitude noise of the electric fields20. It can be quite important, especially when the electric fields are relatively weak. From the above analysis, we find that four controllable parameters, \( \mathcal{E}_j^t \) and \( \mathcal{E}_j^0, \) x and y components of the electric fields for each electron in a double QD should be applied. If y component of the electric fields can be reduced, we can remove the amplitude noise from y component of the electric field. In addition to decreasing the total decoherent effects resulting from the device-dependent noise, the cancellation of y component of the electric field might be also useful to simplify the setup. To this end, we apply the concept of multiple Schrödinger pictures to find an alternative way to implement the shortcuts. Making unitary transformation of
Hamiltonian $H^{32,33}$ by a rotation around $z$ axis with the angle $\pi/2 - \phi$, we obtain

$$\tilde{H} = \frac{\hbar}{2} \left( Z + \tilde{\phi} iQ - iQ - Z - \tilde{\phi} \right),$$

(4)

without $\sigma_z$ term, where $\tan \phi = Y/X$ and $Q = \sqrt{X^2 + Y^2}$. Again, the maximal amplitude of $Q$ will increase when decreasing time $t_f$ due to the fact that $X$ becomes dominant (the maximal amplitude of $Y$ is unchanged). The Hamiltonian $\tilde{H}$ is equal to the original one $H$ at $t = 0$ and $t_f$, which guarantees that the initial (final) states of $H$ and $\tilde{H}$ coincide. However, the dynamics is not same during the intermediate process, although the populations are always equal. Accordingly, we may acquire two new controllable $x$ component of the electric fields, $E_L^{\text{max}}$ and $E_R^{\text{max}}$, calculated from Eq. (4), see Fig. 3.

**Discussion**

Comparisons of $E_L^{\text{max}}$ and $E_R^{\text{max}}$ provided by different times suggest that stronger electric fields have to be used for shorter times, though the amplitude of electric fields might be optimized by using superadiabatic iterations$^{31}$. However, the amplitude of electric fields cannot be arbitrarily large simply because strong fields may destroy the systems. In order to quantify the energy price mentioned above, we demonstrate the relation between the maximal values of electric fields and the operation time $t_f$, see Fig. 4. The maximal amplitude of electric fields, $E_{\text{max}} = \max(|E_L^{\text{max}}, E_R^{\text{max}}|)$, fulfills the scaling law at very short times,

$$E_{\text{max}} \propto \frac{1}{t_f^\gamma}.$$

(5)

since $E_L^{\text{max}} \propto A_L^\gamma / t_f$ and $A_R^\gamma \propto 1 / t_f$ go to infinity in the limit of $t_f \to 0$. The asymptotic exponent of $t_f$ implies that the minimal time should be $\propto E_{\text{max}}^{-1/2}$, which provides the lower bound of operation time when the maximal amplitude of electric fields is given. If the spin system in quantum dot, rather than the atom in harmonic trap, is considered as working medium in the cooling cycles of quantum refrigerator, the minimal time for the (accelerated) adiabatic process, bounded by the energy, could be relevant to the third law of thermodynamics and the unattainability principle$^{34,35}$.

For a realistic setup, the coupling to the stochastic environment is a general scenario to be considered, where the hyperfine interactions with the nuclear spin could play important role at low temperature. To study the decoherence effect, we present the master equation for the density matrix $\rho$, the operations of two electrons confined in each dot are $s$ and $x$ with the equal magnetic fields $B$ applied to the left dot and the right one in $z$ direction, and $A_{jz}$ are the vector potentials of the electric fields. The spin operators of two electrons confined in each dot are $s_j = \sigma_j / 2$ with $z$ component $s_j^z$. The Heisenberg term $J s_L^z s_R$ describes the exchange coupling $J$ between two spins. The example of a double QD of GaAs-based structure ($g = -0.44$) is taken with $B = 3.7$ T. The energy gap between the singlet and the triplet is $J = 0.1$ meV, so that

$$\dot{\rho} = -\frac{i}{\hbar} [\tilde{H}, \rho] + \frac{\gamma}{2} \sum_j [\sigma_j, [\sigma_j, \rho]].$$

(6)

Figure 3 | Electric fields of $E_L^{\text{max}}$ (solid blue line) and $E_R^{\text{max}}$ (dashed red line), designed from the Hamiltonian $\tilde{H}$, see Eq. (4).

Figure 4 | Dependence of $E_{\text{max}}$ on short time $t_f$ (solid blue line), where the dashed straight line shows the asymptotic exponent of $t_f$, i.e. $E_{\text{max}} \propto 1 / t_f^\gamma$.

where $\gamma$ is the dephasing rate. Solving the Bloch equation, we can obtain the final fidelity ($F = \rho_{11}$) for different times, see Fig. 5, and demonstrate that the faster manipulation increases the fidelity with fewer influences attributed to decoherence.

To demonstrate the feasibility of our protocol, we also check the stability with respect to systematic errors in $E_{\text{max}}$. The real electric fields can be $E_{\text{real}} = E_{\text{max}} (1 + \lambda)$, where $\lambda$ is the relative deviation. The dependence of fidelity $F$ on $\lambda$ is exhibited in Fig. 6 for different times. Different from decoherence affected by the stochastic environment, fidelity is more stable with larger $t_f$, since the systematic error considered here depends on the amplitude of electric fields. In general, the speeded-up adiabatic protocol has different stability with respect to different types of noise and systematic errors. Alternatively, one can combine the inverse engineering and optimal control theory to pick up the most robust protocol in quantum two-level systems in presence of different noise and errors$^{37-39}$.

**Methods**

**Effective Hamiltonian.** The total spin-dependent Hamiltonian consists of the Heisenberg term, Zeeman term, and interactions between the electric fields and the electrons, expressed as

$$H_{\text{total}} = J s_L^z s_R + \sum_j \Delta_j s_j^z - \sum_j \sum_i A_j v_j.$$  

(7)

The subscripts $j = L, R$ represent the left dot and the right one, respectively. Zeeman term is $\Delta = g_eB$, with the equal magnetic fields $B$ applied to the left dot and the right one in $z$ direction, and $A_j$ are the vector potentials of the electric fields. The spin operators of two electrons confined in each dot are $s_j = \sigma_j / 2$ with $z$ component $s_j^z$. The Heisenberg term $J s_L^z s_R$ describes the exchange coupling $J$ between two spins. The example of a double QD of GaAs-based structure ($g = -0.44$) is taken with $B = 3.7$ T. The energy gap between the singlet and the triplet is $J = 0.1$ meV, so that

Figure 5 | Fidelity $F$ versus dephasing rate $\gamma$ with respect to $t_f = 2$ ns (solid blue line), $t_f = 3$ ns (dashed red line), $t_f = 4$ ns (dot-dashed black line).
2. Rashba, E. I. "Theory of electric dipole spin resonance in quantum dots". *Nature* 442, 766–771 (2006).

3. Nowack, K. C., Koppens, F. H. L., Nazarov, Y. V. & Vandersypen, L. M. K. "Control of a single electron spin with external fields". *Science* 318, 1430–1433 (2007).

4. Cao, G. et al. "Ultrafast universal quantum control of a quantum-dot charge qubit using Landau-Zener-Stückelberg interference". *Nat. Commun.* 4, 1401 (2013).

5. Burkard, G., Loss, D. & Di Vincenzo, D. P. "Coupled quantum dots as quantum gates". *Phys. Rev. B* 59, 2070–2078 (1999).

6. Hanser, R., Kousenbrop, L. P., Petta, J. R., Tarucha, S. & Vandersypen, L. M. K. "Spins in few-electron quantum dots". *Rev. Mod. Phys.* 79, 1217–1265 (2007).

7. Taylor, J. M., Petta, J. R., Johnson, A. C., Yacoby, A., Marcus, C. M. & Lukin, M. D. "Relaxation, dephasing, and quantum control of electron spins in double quantum dots". *Phys. Rev. B* 76, 035315 (2007).

8. Chen, X., Torrontegui, E. & Muga, J. G. "Lewis-Riesenfeld invariants and counterdiabatic spin control in the presence of charge noise using tailored pulses". *Phys. Rev. Lett.* 108, 068804 (2013).

9. Koppens, F. H. L. "Drive coherent oscillations of a single electron spin in a quantum dot". *Nature* 442, 766–771 (2006).

10. Cao, G. "The Coulomb blockade in coupled quantum dots". *Science* 274, 1332–1335 (1996).

11. Chalk, M., Nowack, K. C., Reich, C., Wegscheider, W. & Vandersypen, L. M. K. "Resolving spin-orbit and hyperfine-mediated electric dipole spin resonance in a quantum dot". *Phys. Rev. Lett.* 110, 107601 (2013).

12. Ribeiro, H., Burkard, G., Petta, J. R., Loss, D. & Gossard, A. C. "Coherent adiabatic spin control in the presence of charge noise using tailored pulses". *Phys. Rev. Lett.* 110, 088804 (2013).

13. Ribeiro, H., Burkard, G., Petta, J. R., Loss, D. & Gossard, A. C. "Coherent adiabatic spin control in the presence of charge noise using tailored pulses". *Phys. Rev. Lett.* 110, 088804 (2013).

14. Koppens, F. H. L. "Driving coherent oscillations of a single electron spin in a quantum dot". *Nature* 442, 766–771 (2006).

15. Rashba, E. I. "Theory of electric dipole spin resonance in quantum dots: Mean field theory with Gaussian fluctuations and beyond". *Phys. Rev. B* 78, 195302 (2008).

16. Cao, G. "The Coulomb blockade in coupled quantum dots". *Science* 274, 1332–1335 (1996).
36. Virk, K. S. & Sipe, J. E. “Conduction electrons and the decoherence of impurity-bound electrons in a semiconductor”. *Phys. Rev. B* **72**, 155312 (2005).
37. Ruschhaupt, A., Chen, X., Alonso, D. & Muga, J. G. “Optimally robust shortcuts to population inversion in two-level quantum systems”. *New J. Phys.* **14**, 093040 (2012).
38. Lu, X. J., Chen, X., Ruschhaupt, A., Alonso, D., Guérin, S. & Muga, J. G. “Fast and robust population transfer in two-level quantum systems with dephasing noise and/or systematic frequency errors”. *Phys. Rev. A* **88**, 033406 (2013).
39. Daems, D., Ruschhaupt, A., Sugny, D. & Guérin, S. “Robust quantum control by a single-shot shaped pulse”. *Phys. Rev. Lett.* **111**, 050404 (2013).
40. Torrontegui, E., Martinez-Garaot, S. & Muga, J. G. “Hamiltonian engineering via invariants and dynamical algebra”. *Phys. Rev. A* **89**, 043408 (2014).

**Acknowledgments**

We appreciate E. Ya Sherman for his fruitful discussions. This work is partially supported by the NSFC (61176118, 61404079, 11474193), the Shanghai Rising-Star, Pujiang and Yangfan Program (12QH1400800, 13PJ1403000, 14YF1408400), the Specialized Research Fund for the Doctoral Program of Higher Education (2013310811003), and the Program for Professor of Special Appointment (Eastern Scholar) at Shanghai Institutions of Higher Learning.

**Author contributions**

Y.B. carried out the theoretical and numerical calculation; X.C. analyzed the theoretical results. Both authors wrote and reviewed the manuscript.

**Additional information**

Competing financial interests: The authors declare no competing financial interests.

How to cite this article: Ban, Y. & Chen, X. Counter-diabatic driving for fast spin control in a two-electron double quantum dot. *Sci. Rep.* 4, 6258; DOI:10.1038/srep06258 (2014).

This work is licensed under a Creative Commons Attribution-NonCommercial-ShareAlike 4.0 International License. The images or other third party material in this article are included in the article’s Creative Commons license, unless indicated otherwise in the credit line; if the material is not included under the Creative Commons license, users will need to obtain permission from the license holder in order to reproduce the material. To view a copy of this license, visit http://creativecommons.org/licenses/by-nc-sa/4.0/