Explanation, Evolution and Subjective Probability in Everett Quantum Mechanics with Positive Preclusion *

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Abstract

The usual interpretational rule of quantum mechanics which states that outcomes do not occur when their weights are zero is changed so as to preclude outcomes with weights less than a small but positive value. With this “positive preclusion” rule, and in the absence of any notion of objective probability, Everett quantum mechanics has the explanatory power to account for the evolution of organisms with subjective expectations of probability that are in accord with the Born rule. Positive preclusion also allows for the derivation of a connection between weight and relative frequency in situations involving a finite number of measurements.

Key words: Everett interpretation, probability, preclusion, evolution, Heisenberg picture

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1 Introduction

An Everett interpretation of quantum mechanics in which subjective probability is present but objective probability (or some surrogate for it) is absent is incapable of explaining probabilistic outcomes, most importantly the biological evolution leading to organisms capable of making decisions or having subjective expectations such as “likely” or “unlikely” in approximate agreement with the Born rule—i.e., in agreement with experience, something which we believe to be governed by quantum mechanics including the Born rule. In some branches of the wavefunction organisms with Born-rule-consistent subjective-probabilistic expectations will evolve and perceive that their expectations are often met; in other branches organisms will evolve with expectations different from the Born rule and perceive that their expectation are often met; in others non-Born-rule-expecting organisms will be in constant states of surprise as their expectations are not met; and in still others organisms expecting to find that the Born rule holds will be surprised on a regular basis. From our Born-rule-branch point of view the goings-on in these other non-Born branches are highly improbable, and ours highly probable. But those in other branches would have different opinions on the matter, and without some objective notion of probability we cannot claim that one opinion is correct and another is not. There is no sense in which one can explain why, in our branch, we expect Born-rule-consistent outcomes, and are unsurprised when our expectations are met. The most one can say is “This is how we are.”

One might be able to say more if one could show, in a way not relying on any concept of probability, that, e.g., evolution of non-Born-rule-expecting organisms is in some sense impossible, inherently contradictory. The decision-theoretic program initiated by Deutsch aims to show that, in a world (multiverse) obeying the nonprobabilistic parts of quantum mechanics, an organism which obeys certain rules of rationality will act in accordance with the Born rule (presumably from Born-rule-consistent expectations of outcome). Assume for the sake of argument that this connection is correct. Then essentially the same problem reemerges: How can we explain why we are in a branch in which organisms (at least from time to time) act rationally? Admittedly, the proposed rules of rationality are extremely natural, and it may indeed be possible to show that no organism capable of making decisions could evolve that did not in some sense incorporate these rules, and is therefore compelled to act in accordance with the Born rule (when acting rationally). If that were shown it would constitute an “anthropic” argument: We are organisms capable of making decisions, and any such organism must incorporate the rules of rationality, hence make decisions (when they are made rationally) based on the Born rule. But I do not believe that anything of this sort has yet been shown.

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1 See, e.g., [6] for a discussion of the relation between subjective and objective probability.
2 Zurek’s approach to probability in quantum mechanics defines and quantifies probability based on the impossibility of exhibiting a preference between states related by the envariance symmetry. It thus falls into the decision-theoretic category of approaches to probability and is also subject to this
The concept of subjective probability, i.e., probability as an expectation, a cognitive state experienced by a living organism, has two great merits. It is a noncircular definition of probability, even when applied to the probability of the outcome of a single experiment. And it is in accord with observation, both introspective and experimental (see following paragraph). In any version of Everett quantum mechanics that provides a notion of explanation, the existence of evolution leading to organisms with a sense of subjective probability consistent with the Born rule should follow from the theory as unambiguously as it does in orthodox quantum mechanics. There, objective probability is a primitive concept not defined in terms of anything else and interacting with the Hilbert-space formalism of the nonprobabilistic parts of quantum mechanics as a coequal partner. The Hilbert-space formalism determines the numerical value of the probability in any given situation. And the probability concept, in conjunction with wavefunction collapse, gives the theory explanatory power. Specifically, we do not observe organisms with subjective probabilistic expectations inconsistent with the Born rule because the evolution of such organisms would be extreme improbable—and the term “improbable” cannot be analyzed further.

I am of course not arguing that somewhere in the brains of living organisms are neural circuits equipped by evolution to compute Hilbert-space inner products. What I am arguing is that subjective probability has as its basis tendencies acquired through biological evolution which cause the organism to act (and, if sufficiently high on the evolutionary ladder, to anticipate and plan) in accordance with at least some features of the physical universe which, according to the Born rule, are highly probable. For example, a plant turning its leaves to face the sun has “learned,” through evolution, that it is likely (i.e., has greater Born-rule weight) that in doing so it will increase the number of photons of light that it absorbs. And recent experiments indicate that humans have an inborn tendency to learn by induction; that is, to anticipate that the statistics found in past observations will persist in future observations (see, e.g., [20]-[22]).

The version of Everett quantum mechanics I present here explains biological evolution leading to Born-rule-consistent subjective expectation of probability without invoking objective probability. It accomplishes this by modifying an interpretational rule that is used, explicitly or implicitly, in most interpretations of quantum mechanics: namely, that

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3“We report six experiments investigating whether 8-month-old infants are ‘intuitive statisticians.’ …Our findings provide evidence that infants possess a powerful mechanism for inductive learning, either using heuristics or basic principles of probability This ability to make inferences based on samples of information about the population develops early and in the absence of schooling or explicit teaching. Human infants may be rational learners from very early in development [21].”

4Inductive learning cannot by itself constitute all that is involved in the sense of subjective probability, as was implied in an earlier version of this paper. I am grateful to Rainer Plaga for pointing this out to me [23].
outcomes with zero weight are certain not to occur. This rule is replaced with the idea, a version of which was first promoted in the context of the Everett interpretation by Geroch and termed by him preclusion, that outcomes with sufficiently small weights are certain not to occur. The preclusion rule plays the role, in Everett quantum mechanics, that objective probability plays in orthodox quantum mechanics, in that it provides a notion of explanation for (at least) the results of biological evolution, in particular the evolution of Born-rule-consistent subjective expectations of probability.

2 Weight and zero preclusion

The weight associated with an event $E$ idealized as occurring at a moment of time $t$ is

$$W_E(t) = \langle \psi(t)|\hat{P}_E|\psi(t)\rangle,$$  \hspace{1cm} (1)

where $|\psi(t)\rangle$ is the Schrödinger-picture state at time $t$ and $\hat{P}_E$ is the Schrödinger-picture projection operator corresponding to the event $E$.

In orthodox quantum mechanics (1) is given meaning via the combined concepts of probability and wavefunction reduction. Immediately after a measurement with possible outcomes $E_i$, $i = 1, 2, \ldots, n$, one of these outcomes, say $E_j$, randomly occurs. The others, $E_i$ for $i \neq j$, do not occur. Immediately after the measurement is made, the wavefunction is an eigenfunction of $\hat{P}_{E_j}$ with unit eigenvalue:

$$\hat{P}_{E_j}|E_j\rangle = |E_j\rangle.$$  \hspace{1cm} (2)

The probability $P_{E_j}(t)$ that outcome $E_j$ occurs is equal to the weight $W_{E_j}(t)$ calculated via (1):

$$P_{E_j}(t) = W_{E_j}(t).$$  \hspace{1cm} (3)

We may not be able to define this last statement regarding probability—or, indeed, any statement regarding probability—in terms of anything more primitive, but we feel that we understand what it means.

The idea of randomness or unpredictability, which is one part of the concept of probability, emerges naturally from the Everett interpretation. In general, there is no possibility of an observer Alice who is about to make a measurement at time $t$ predicting what “she” will see at time $t'$ immediately following $t$, because there will in general not be

5Preclusion rules identical or similar to the one used in the present paper have been employed as key components of the single-outcome interpretations of quantum mechanics of Sorkin and of Galvan. Hanson has employed the related idea that low-weight worlds are “mangled” in his argument for objective probability from “world counting” in the Everett interpretation. In classical probability theory the idea that sufficiently small probability is in some sense equivalent to certain nonoccurrence is referred to as Cournot’s principle.
a unique “she” in existence at $t'$ but, rather, several “shes,” “Alice-who-saw-outcome-$E_1$,” “Alice-who-saw-outcome-$E_2$,” etc. But what meaning can we give to the quantitative idea of probability in the Everett interpretation? What meaning can we attach to the value of $W_E(t)$?

Actually, there is a special case in which the meaning of $W_E(t)$ is completely unproblematic, in the Everett or any other interpretation of quantum mechanics; namely, the case in which

$$W_E(t) = 0.$$  \hspace{1cm} (4)

In this case there is no component of the state vector $|\psi(t)\rangle$ corresponding to outcome $E$. The meaning of the weight, in this case is: After the measurement is made, Alice will not see outcome $E$ (i.e., no Everett copy of Alice will see $E$).

Furthermore, in the Everett interpretation, there is a meaning that can be attached to the complementary case

$$W_E(t) > 0.$$  \hspace{1cm} (5)

In this case, after the measurement an Everett copy of an observer who has observed the event $E$ will exist.

We can summarize this state of affairs by defining the *existence indicator function* for event $E$ at time $t$, $X_E(t)$:

$$X_E(t) = 0, \quad W_E(t) = 0,$$

$$X_E(t) = 1, \quad W_E(t) > 0.$$  \hspace{1cm} (6), (7)

The outcome $E$ does not occur at time $t$, $X_E(t) = 0$, if the weight is zero, and it occurs, $X_E(t) > 0$, if the weight is nonzero. We will refer to (6), (7) as the *zero preclusion rule*.

3 The two-level ontology of Everett quantum mechanics in the Heisenberg picture

In the Heisenberg picture of quantum mechanics it is the operators rather than the state vector that evolve in time. The elements of the formalism are a time-independent state vector $|\psi_0\rangle$ related to the time-dependent Schrödinger-picture state vector by

$$|\psi_0\rangle = \hat{U}^\dagger(t)|\psi(t_0)\rangle,$$  \hspace{1cm} (8)

where $t_0$ is an initial time that can be chosen arbitrarily, and time-dependent operators $\hat{A}(t)$, $\hat{B}(t)$, etc. The operators evolve in time according to the rule

$$\hat{A}(t) = \hat{U}^\dagger(t) \hat{A} \hat{U}(t).$$  \hspace{1cm} (9)

\hspace{1cm} 6^Of course there is another special case, the case of certainty, $W_E(t) = 1$, in which the only Everett copy of the observer will observe $E$. This case will not play a preferred role in what follows.
where $\hat{U}(t)$ is the unitary time-evolution operator, a function of other operators, satisfying

$$\hat{U}(t_0) = 1,$$  \hspace{2cm} (10)

and operators without explicit time arguments are Schrödinger-picture operators equal to their Heisenberg-picture counterparts at $t = t_0$:

$$\hat{A} = \hat{A}(t_0), \quad \hat{B} = \hat{B}(t_0), \quad \text{etc..}$$ \hspace{2cm} (11)

The operators are defined at all times (or at all places and times, if they are operators in quantum field theory), and have values at all times (e.g., they can be explicitly represented as infinite-dimensional matrices) regardless of the zero or nonzero value obtained when computing a weight or any other matrix element. Particularly in the Deutsch-Hayden version of the Heisenberg picture it is natural to think of the operators as being “real,” as “existing” in the world. For example, in a recent model of measurement in Deutsch-Hayden quantum field theory, the locality of the theory depends on taking this viewpoint.

If we accept the operators as real, then, in light of the zero preclusion rule, Everett quantum mechanics in the Heisenberg picture has a two-level ontology. The operators are real at all times, but events, in particular observers’ awareness of the outcomes of measurements, are only real when the corresponding existence indicator functions, which obtain their values from the operators and state vector, are nonzero. The connection between the operators and the reality, or lack thereof, of objects is given by the existence indicator functions in a one-way manner, since operators affect the existence of objects but operators evolve according to equations of motion which are unaffected by the existence of objects.

4 Positive preclusion, evolution, and subjective probability

So, we can modify Everett quantum mechanics without in any way affecting the underlying equations of motion—equations which in a myriad of experiments have been born out to a high degree of accuracy—if we modify only the rule governing the value of the existence indicator function. Rather than the zero preclusion rule (6), (7), we will take as a postulate the positive preclusion rule:

$$X_{\epsilon_p,E}(t) = 0, \quad W_E(t) \leq \epsilon_p,$$ \hspace{2cm} (12)

$$= 1, \quad W_E(t) > \epsilon_p,$$ \hspace{2cm} (13)

The quantity which plays the role of the weight in [46] is not an expectation value of a projection operator but, rather, the expectation value of the integral of the field-theoretic number-density operator.

In the Deutsch-Hayden picture we can simply say “obtain their values from the operators,” since the state vector is a purely conventional constant containing no information.
where the preclusion parameter $\epsilon_P$ is a small positive number,

$$0 < \epsilon_P \ll 1. \quad (14)$$

The outcome $E$ does not occur at time $t$, $X_{\epsilon_P,E}(t) = 0$, if the weight is less than or equal to $\epsilon_P$, and it occurs, $X_{\epsilon_P,E}(t) = 1$, if the weight is greater than $\epsilon_P$.

There is no shortage of events of low probability which are known to have occurred. Therefore the preclusion parameter $\epsilon_P$ must be extremely small, so as not to be in conflict with experience. It will therefore play almost no role in most events, in and out of the physics laboratory. Improbable events—winning the lottery, contracting a rare disease, etc.—will all exist in the sense of (13), and to just the same degree as highly probable or even inevitable occurrences of the death-and-taxes variety.

The weights (11) for all events will continue to be determined by the initial state vector and the equations of motion for the operators, and will have the same values that they would have if we decided to interpret them as probabilities as in the orthodox interpretation. With only a preclusion rule at our disposal we cannot say which events among those which exist are “probable” or “improbable.” But, in any situation in which, in orthodox quantum mechanics, we would say that something is more or less probable, we can say that the same thing has a greater or lesser weight.

In particular, we can say that the biological evolution of an organism that learned a sense of subjective probabilistic expectation different from the Born rule would be of extremely low weight. To see the correctness of this statement, replace “weight” by “probability” and consider that an organism which could not learn an approximation to the Born rule would be at a disadvantage vis à vis one which could. We therefore make the

**Born evolution assumption:** The weight for the evolution of a sentient organism which has subjective expectations of probability significantly different from the Born rule is below the preclusion constant $\epsilon_P$. (The meaning of “significantly different” is, of course, “different enough so as to drop the weight for the evolution of the organism below $\epsilon_P$.”)

The explanation of subjective probability is then simply: Everett copies of organisms with subjective probability significantly different from the Born rule do not exist, since the weight for the event of their biological evolution, after a sufficiently long time, will be less than $\epsilon_P$.

This is all that a theory of probability need account for. All the phenomena that are usually described as related to, or examples of, probability ultimately reduce to the degree

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9 “Learning” refers both to hard-wiring of the nervous system produced over many generations by evolution as well as to the fruits of the ability, imbued in the organism by evolution, to learn from its own experience. E.g., visual systems have been shown to tune themselves, during the development of a given individual, to respond to features found in the environment to which the organism is exposed. 


of surprise, or lack thereof, experienced by a living organism at a turn of events. What does it mean, e.g., that photons directed at a 50:50 beamsplitter have an equal chance of being transmitted or reflected? Does it mean that in any group of photons exactly half will be transmitted? Does it mean that it will never be the case that all of the photons is a group of 100 will be reflected? The only objective fact about probability that can be described without using the term “probability” or a synonym is, in fact, the subjective experience of surprise at certain events and lack of surprise at others.

There is of course a long chain of association from basic cognitive processes to the cognition, e.g., of a physicist who learns quantum mechanics, calculates the cross-section for certain scattering process, gathers data in the laboratory and then is unsurprised (surprised) that the data is close to (far from) the predicted outcome. Still, it is reasonable to ask if we can construct a model, even if extremely simplified, of the preclusion of non-Born-expecting organisms. In fact, such a model already exists; namely the quantum-mechanical model of the measurement of relative frequency (see Appendix). This device observes the outcome of \( N \) identical quantum-mechanical experiments and records the relative frequency of the a given outcome. In this sense it “learns,” say, the relative frequency to expect for “spin up” in a Stern-Gerlach experiment. For \( N \) sufficiently large, the only Everett copy of the device that will not be precluded is the one corresponding to the outcome with the largest Born weight. Any organism or automaton that bases its expectations on a relative frequency device which has been trained in this manner will subsequently be more or less surprised at the outcome of a relative-frequency experiment on \( N' \ll N \) systems to the degree that the outcome is farther from or closer to the Born value.

5 Summary and discussion

Subjective expectation of probability is an observed fact of nature, but by itself does not allow us to explain probabilistic events. Objective probability does, and in particular enables us to explain the evolution of subjective expectation of probability consistent with objective probability. However, incorporating objective probability into the Everett interpretation is problematic; even in orthodox quantum mechanics it is an additional structure that exists alongside the Hilbert-space formalism. Positive preclusion posits, within the deterministic logical structure of the Everett interpretation, the nonexistence of extremely low-weight events. In conjunction with the Born evolution assumption, it can therefore explain why our subjective expectations of probability match the Born rule. It does so while adding to the theory only a minimal extension of the notion of zero preclusion. (Zero preclusion, in turn, arguably flows directly from the formalism, since, in all interpretations of quantum mechanics, absent components of the wavefunction correspond to nonoccurring events.)

\(^{10}\)See, e.g., [48]. However, this opinion is not universal; see [49].
The observation that a theory of probability need only explain the evolution of subjective probability is a critical one in the positive preclusion scheme. Competition acting over the long time scales of biological evolution causes species with evolutionary experience (and thus evolved behavior/expectation) significantly at variance to the Born rule to be precluded, while the smallness of the preclusion constant ensures that surviving, Born-rule-expecting, organisms will subsequently experience surprising low-weight as well as unsurprising high-weight events.

The internal consistency of the positive preclusion approach is assured by fact that the equations of motion are unaffected by the change from zero preclusion. However, it is certainly possible that Everett quantum mechanics with positive preclusion could be found to be inconsistent with experience; it must be regarded as a new theory, not simply a new interpretation. Observations that would falsify the theory are ones that would be inconsistent with the Born evolution assumption. E.g., it is principle possible to compute the weight for the evolution of various features of organisms [50]-[56]; in particular, one could in principle compute the weight for the evolution of a non-Born-expecting species. This would set a lower bound for $\epsilon_P$. If another event were then observed to occur for which the weight was below this lower bound, the theory would be falsified.

The explanatory power which positive preclusion gives to the Everett interpretation is limited compared to the explanatory power apparently afforded the orthodox interpretation by objective probability, in that positive preclusion seems to have no place for what might be called “normative probability.” If someone were to ask why it is that she is surprised at low-weight events, the positive preclusionist could answer: “You evolved that way, because Everett copies of you which are not surprised at such events are of too low weight to have survived.” But if that same person were to ask why she should be surprised at a low-weight event, the positive preclusionist would have no answer. With objective probability, on the other hand, we would of course simply say “because it’s improbable.” But it is not clear that the inability of positive preclusion to provide this sort of explanation is a deficiency. Rather, it may simply be the elimination of an illusion which, like absolute simultaneity, has no objective correlate in the physical world.

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11This assumes that $\epsilon_P$ is a fixed constant of nature. An alternative possibility is that $\epsilon_P$ emerges dynamically from the lower level, e.g. by virtue of nonlinearities in the basic equations arising from quantum-gravitational effects [57], [58]. In that case $\epsilon_P$ might vary from situation to situation. In such a scenario it might be possible to relate the smallness of $\epsilon_P$ to the weakness of the gravitational force relative to other forces.
Appendix: Relative frequency and positive preclusion

Starting from the zero preclusion rule, many derivations have been given of probability in the sense of relative frequency in the limit of an infinite number of identically-prepared measurement situations. The mathematics involved in considering an infinite number of measurements has been questioned and defended. If we start from the positive preclusion rule we can apply essentially the same calculational steps as in these previous derivations to prove, in a straightforward and rigorous fashion, a relative-frequency theorem which involves only a finite number of measurements.

Rather than taking the most general case of measurements involving an arbitrary discrete or continuous number of outcomes, we consider only measurements of two-state systems (qubits). Then the result for the infinite-number-of-copies limit in enables us to prove immediately the Positive preclusion relative frequency theorem: Let observers measure two-state systems, such that each of the systems is measured by one of the observers and such that all of the systems are prepared in the same quantum state

\[
|\psi(c_1, c_2)\rangle = c_1|1\rangle + c_2|2\rangle,
\]

where \(|c_1|^2 + |c_2|^2 = 1\). Subsequent to these measurements of systems by observers, let a single additional “relative-frequency observer” measure all \(N\) of the initial observers to determine the relative frequency of the result “1,” i.e. the fraction of the \(N\) observers who determined that the two-level system they measured was in state \(|1\rangle\). If the relative-frequency observer is of finite size and energy, and therefore can only record the results of measurements to a finite precision (so that there are only a finite number of possible outcomes to the relative frequency measurement), then there is a number \(N_B\) such that if \(N > N_B\) there will only exist one Everett copy of the relative-frequency observer subsequent to the relative-frequency measurement. This copy will have observed the Born-rule relative frequency \(|c_1|^2\) to within the precision of which it is capable.

**Proof:** I have previously shown, in , that in the limit \(N \to \infty\), the weight for the Everett copy of the relative-frequency observer which perceives relative frequency closest to the Born-rule value will remain nonzero (in fact, approach unity), while the weights for those which perceive relative frequencies farther from the Born-rule value will approach zero. But of course to say that \(\lim_{N \to \infty} W = 0\) is simply to say that for any positive real number \(\epsilon_P\), no matter how small, there is a number \(N_B\) such that, for \(N > N_B\), \(|W| < \epsilon_P\). From the positive preclusion rule we conclude that for \(N > N_B\) there will only be one surviving Everett copy of the relative frequency observer, the one which has obtained a relative-frequency measurement as close as possible to the Born-rule value.

\[\text{Q.E.D.}\]

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\(^{12}\)Or in some cases from the rule \(W_E(t) = 1 \Rightarrow \text{certain occurrence.}\)

\(^{13}\)If two possible measured relative frequencies are equally close to the Born-rule value then the Everett copies corresponding to each of these will survive. See .
For details of the calculation, the relative-frequency measurement model, and the ontology of the Everett interpretation in the Heisenberg picture, see [65].

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