Light custodians in natural composite Higgs models

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Abstract

We present a class of composite Higgs models arising from a warped extra dimension that can satisfy all the electroweak precision tests in a significant portion of their parameter space. A custodial symmetry plays a crucial role in keeping the largest corrections to the electroweak observables below their experimental limits. In these models the heaviness of the top quark is not only essential to trigger the electroweak symmetry breaking, but it also implies that the lowest top resonance and its custodial partners, the custodians, are significantly lighter than the other resonances. These custodians are the trademark of these scenarios. They are exotic colored fermions of electromagnetic charges $5/3$, $2/3$ and $-1/3$, with masses predicted roughly in the range $500 - 1500$ GeV. We discuss their production and detection at the LHC.
1 Introduction

Theories of warped extra dimensions, with their holographic interpretation in terms of 4D strongly coupled field theories [1, 2], have recently given a new twist to the idea of Higgs compositeness [3, 4]. Calculability is one of the main virtues of this new class of models. Differently from the old approach [5], physical quantities of central interest can be computed in a perturbative expansion. This opened up the possibility of building predictive and realistic theories of electroweak symmetry breaking (EWSB).

The minimal composite Higgs model (MCHM) of Ref. [4] is extremely simple to define based only on symmetry considerations. It consists in a 5D theory on AdS space-time compactified by two boundaries, respectively called infrared (IR) and ultraviolet (UV) boundary [6]. An SO(5)×U(1)×SU(3)c gauge symmetry in the bulk is broken down to SO(4)×U(1)×SU(3)c on the IR boundary, (with SO(4)∼SU(2)L×SU(2)R), delivering four pseudo-Goldstone bosons that transform as a 4 of SO(4) and are identified with the Higgs doublet. On the UV boundary the bulk symmetry is reduced to the standard Model (SM) gauge group $G_{SM}=SU(2)_L×U(1)_Y×SU(3)_c$, where hypercharge is defined as $Y=X+T^R_3$.

Once the SO(5) bulk representations in which the SM fermions are embedded and their boundary conditions are chosen, the model is completely determined. One can write down the most general Lagrangian compatible with the above symmetries, compute the one-loop Higgs potential and determine the region of the parameter space with the correct EWSB and SM fermion masses.

In Ref. [4] the SM fermions were embedded in spinorial representations of SO(5). This choice leads generically to large corrections to the $Zb_L\bar{b}_L$ coupling, with the result that a sizable portion of the parameter space of the model ($\sim 95\%$) is ruled out [7]. However, it was recently realized [8] that the $Zb_L\bar{b}_L$ constraint is strongly relaxed if the fermions are embedded in fundamental (5) or antisymmetric (10) representations of SO(5), with $b_L$ belonging to a $(2,2)$ of $SU(2)_L×SU(2)_R$, and the boundary symmetry SO(4) is enlarged to O(4). In this case a subgroup of the custodial symmetry $O(3)⊂O(4)$ protects the $Zb_L\bar{b}_L$ coupling from receiving corrections.

In this paper we investigate the predictions of the MCHM with fermions in 5’s or 10’s of SO(5) and the IR symmetry enlarged to O(4). We will determine the region of the parameter space with successful EWSB, and study the constraints imposed by the electroweak precision tests (EWPT). The most relevant constraint comes from the Peskin-Takeuchi $S$ parameter,

\footnote{Although these boundaries act as sharp cutoffs of the extra dimension, they can be considered as an effective description of some smoother configuration that can arise in a more fundamental (string) theory.}
which excludes $\sim 50 - 75\%$ of the parameter space of the model.\footnote{This must be compared with the most popular supersymmetric models where experimental constraints have already excluded large portions of the parameter space ($\sim 99\%$ in the case of universal soft masses\cite{9}).} A sizable portion of the latter is still allowed, and awaits to be explored at the LHC. An important prediction of the model is that the heaviness of the top quark implies that the lowest top Kaluza-Klein (KK) resonance and its O(4)-custodial partners, the “custodians”, are significantly lighter than the other KK resonances. The custodians are color triplets and transform as $2_{7/6}$ of $\text{SU}(2)_L \times \text{U}(1)_Y$ when the SM fermions are embedded in 5’s of SO(5), and as $2_{7/6} \oplus 3_{2/3} \oplus 1_{5/3} \oplus 1_{-1/3}$ in the case of 10’s of SO(5). They have electromagnetic charges $Q_{\text{em}} = 5/3, 2/3, -1/3$ and their masses are predicted roughly in the range $500 - 1500$ GeV. The excitations of the SM gauge bosons are always heavier, with masses around $2 - 3$ TeV. The Higgs mass is predicted in the range $m_{\text{Higgs}} \simeq 115 - 190$ GeV. Its value is correlated with the mass of the custodians, since top loops give the largest contribution to the Higgs potential, and are thus responsible for triggering the EWSB. Being at the reach of the LHC, the custodians offer the best signature for distinguishing 5D composite Higgs from other scenarios of EWSB. We will discuss their most important production mechanisms and decay channels.

Other models of EWSB in which $b_L$ is embedded in a $(2, 2)$ of $\text{SU}(2)_L \times \text{SU}(2)_R$ to protect $Zb\bar{b}$ have been proposed in Refs.\cite{10, 11}.

\section{Higgs potential and EWSB}

At the tree level, the massless spectrum of the bosonic sector of the MCHM consists of the SM gauge bosons, plus four real scalar fields that correspond to the SO(5)/SO(4) degrees of freedom of the fifth component of the 5D gauge field. The presence of these scalars is dictated by the symmetry breaking pattern of the model: they are pseudo-Goldstone bosons and have the right quantum numbers to be identified with the SM Higgs, $h^a (a = 1, 2, 3, 4)$. In addition to the massless sector, the theory also contains an infinite tower of massive resonances: the KK states. We can integrate out all the massive states and obtain an effective low-energy Lagrangian for the massless modes. We do this by following the holographic approach of Ref.\cite{4}. The form of the effective Lagrangian for the gauge bosons is completely determined by the symmetries of the model. It can be found in Ref.\cite{4}, and it will not be repeated.
We only report the following relations:

\[ v \equiv \epsilon f_\pi = f_\pi \sin \frac{\langle h \rangle}{f_\pi} = 246 \text{ GeV}, \quad f_\pi = \frac{1}{L_1} \frac{2}{\sqrt{g_5^2 k}}, \quad m_\rho \simeq \frac{3\pi}{4L_1}, \quad (1) \]

where \( h \equiv \sqrt{(h_a)^2} \). Here \( L_1 \) denotes the position of the IR boundary in conformal coordinates and sets the mass gap \( (1/L_1 \sim \text{TeV}) \); \( g_5 \) is the SO(5) gauge coupling in the bulk; \( 1/k \) is the AdS\(_5\) curvature radius; and \( m_\rho \) is the mass of the lightest gauge boson KK. Following Ref. [4], we define

\[ \frac{1}{N} \equiv \frac{g_5^2 k}{16\pi^2} \quad (2) \]

as our expansion parameter. The fermionic sector of the model depends on our choice for the 5D bulk multiplets. We want to study the case in which the SM fermions are embedded in fundamental (5) or antisymmetric (10) representations of SO(5). To this aim, we consider two different choices of multiplets and boundary conditions. In the first choice, which we will refer to as the MCHM\(_5\), the bulk fermions transform as 5’s of SO(5) and are defined by Eq. (22) of the Appendix. In the second choice, which we will denote as the MCHM\(_{10}\), the bulk fermions are defined by Eq. (35) and transform as 10’s of SO(5). For all the technical details, we refer the reader to the Appendix. Here it will suffice to say that in both cases the low-energy effective Lagrangian for the quarks can be written, in momentum space and at the quadratic order, as:

\[
\mathcal{L}_{\text{eff}} = \bar{q}_L \hat{\rho} \left[ \Pi_0^i(p^2) + \frac{s_h^2}{2} \left( \Pi_1^i(p^2) \hat{H} \hat{H}^\dagger + \Pi_1^0(p^2) \hat{H} \hat{H}^\dagger \right) \right] q_L \\
+ \bar{u}_R \hat{\rho} \left( \Pi_0^u(p^2) + \frac{s_h^2}{2} \Pi_1^u(p^2) \right) u_R + \bar{d}_R \hat{\rho} \left( \Pi_0^d(p^2) + \frac{s_h^2}{2} \Pi_1^d(p^2) \right) d_R \\
+ \frac{s_h c_h}{\sqrt{2}} M_1^u(p^2) \bar{q}_L \hat{H}^c u_R + \frac{s_h c_h}{\sqrt{2}} M_1^d(p^2) \bar{q}_L \hat{H} d_R + h.c. . \quad (3)
\]

Here \( c_h \equiv \cos(h/f_\pi), \) \( s_h \equiv \sin(h/f_\pi), \) and

\[
\hat{H} = \frac{1}{h} \left[ \begin{array}{c} h^1 - ih^2 \\ h^3 - ih^4 \end{array} \right], \quad \hat{H}^c = \frac{1}{h} \left[ \begin{array}{c} -(h^3 + ih^4) \\ h^1 + ih^2 \end{array} \right]. \quad (4)
\]

The form factors \( \Pi_{0,1}^i \) and \( M_1^i \) in Eq. (3) can be computed in terms of 5D propagators using the holographic approach of Ref. [4]. Their explicit form is given in the Appendix. From Eq. (3) one can derive the SM up and down fermion masses, \( m_{u,d} \). A reasonably good

\footnote{The only difference between the gauge sector of the model presented here and that of Ref. [4] is the symmetry on the IR boundary. Enlarging SO(4) to O(4) forbids the otherwise allowed IR-boundary kinetic term \( \epsilon^{ijkl} F_{ijkl} F_{(kl)\mu\nu} \), where \( i,j,k,l \) are SO(4) indices. Since this term was not included in Ref. [4], we can use the results for the gauge sector presented there.}
approximation to the exact expressions can be obtained by setting \( p^2 = 0 \) in the form factors, the error being of order \((m_{u,d} L_1)\). We obtain

\[
m_u \simeq \frac{s_h c_h}{\sqrt{2}} \frac{M_u^q(0)}{\sqrt{Z_{uL}Z_{uR}}}, \quad m_d \simeq \frac{s_h c_h}{\sqrt{2}} \frac{M_d(0)}{\sqrt{Z_{dL}Z_{dR}}},
\]

where \( Z_{uL,dL} = \Pi_0^q(0) + (s_h^2/2) \Pi_1^{q_1,q_2}(0) \) and \( Z_{uR,dR} = \Pi_0^{u,d}(0) + (s_h^2/2) \Pi_1^{u,d}(0) \). An explicit version of Eq. (5) in terms of the 5D input parameters is given in Eqs. (34) and (42) of the Appendix, respectively for the MCHM

At the tree level the Higgs field is an exact Goldstone boson, and as such it has no potential. At the one-loop level, the virtual exchange of the SM fields transmits the explicit breaking of SO(5) and generates a potential for \( h \). The largest contribution comes from \( t_L, b_L \) and \( t_R \), and from the SU(2)\(_L\) gauge bosons, since these are the fields that are most strongly coupled to the Higgs. They give

\[
V(h) = \frac{9}{2} \int \frac{d^4 p}{(2\pi)^4} \log \left( \Pi_0 + \frac{s_h^2}{4} \Pi_1 \right) - 2N_c \int \frac{d^4 p}{(2\pi)^4} \left\{ \log \left( \Pi_0^q + \frac{s_h^2}{2} \Pi_1^q \right) \right. \\
+ \log \left[ p^2 \left( \Pi_0^u + \frac{s_h^2}{2} \Pi_1^u \right) \left( \Pi_0^d + \frac{s_h^2}{2} \Pi_1^d \right) - \frac{s_h^2 c_h^2}{2} (M_1^u)^2 \right] \right\}.
\]

Here, and from now on, the fermionic form factors \( \Pi_0^q, \Pi_1^{q_1,q_2}, \Pi_{0,1}^u, M_1^u \) stand for those of the 3rd quark family, \( q_L = (t_L, b_L) \) and \( t_R \). The gauge form factors \( \Pi_{0,1} \) can be found in Ref. [4]. Since the \( \Pi_1 \)'s and \( M_1^u \) drop exponentially for \( pL_1 \gg 1 \), the logarithms in Eq. (6) can be expanded and the potential is well approximated by

\[
V(h) \simeq \alpha s_h^2 - \beta s_h^2 c_h^2,
\]

where \( \alpha \) and \( \beta \) are integral functions of the form factors. In particular, \( \alpha \) receives contributions from loops of the gauge fields and of \( q_L \) or \( t_R \) alone. We will denote these contributions respectively by \( \alpha_{\text{gauge}}, \alpha_L \) and \( \alpha_R \). On the other hand, \( \beta \) receives contributions from loops where both \( t_L \) and \( t_R \) propagate. For \( \alpha < \beta \) and \( \beta \geq 0 \) we have that the electroweak symmetry is broken: \( \epsilon \neq 0 \). If \( \beta > |\alpha| \), the minimum of the potential is at

\[
s_h = \epsilon = \sqrt{\frac{\beta - \alpha}{2\beta}},
\]

while for \( \beta < |\alpha| \) the minimum corresponds to \( c_h = 0 \), and the EWSB is maximal: \( \epsilon = 1 \). In this latter case the fermion masses vanish – see Eq. (5) – due to an accidental chiral symmetry
that is restored in the limit $\epsilon \to 1$. The model is thus realistic only for $0 < \epsilon < 1$, i.e. $\beta > |\alpha|$. The coefficients $\alpha$ and $\beta$ can be computed in terms of the relevant 5D parameters

$$N, \ c_q, \ c_u, \ \tilde{m}_u, \ \tilde{M}_u, \quad (9)$$

where $c_q, c_u$ are the bulk masses (in units of $k$) of the 5D multiplets $\xi_q, \xi_u$ that contain the SM $q_L$ and $t_R$, and $\tilde{m}_u, \tilde{M}_u$ are mass terms localized on the IR boundary that mix $\xi_q$ with $\xi_u$ (see Appendix). The scale $L_1$ has been traded for $v$. The five parameters of Eq. (9) are not completely determined by the present experimental data. The are only two constraints coming from fixing the top quark mass to its experimental value, $m_t^{pole} = 173$ GeV, and by requiring $0 < \epsilon < 1$.

Let us outline the viable region of this five-dimensional space of parameters. The large Yukawa coupling of the top quark is reproduced by having the wave functions of the $t_L$ and $t_R$ zero modes peaked towards the IR boundary, where the Higgs lives. This constrains the 5D bulk masses to lie in the interval $|c_{q,u}| < 1/2$, see Eqs. (33) and (42). In the 4D dual interpretation, this corresponds to say that the elementary fields $t_L$ and $t_R$ couple to relevant operators $\mathcal{O}$ of the strongly coupled conformal field theory (CFT), with conformal dimension $3/2 < \text{Dim}[\mathcal{O}] < 5/2$ (see [12]). Since the operators $\mathcal{O}$ have the quantum numbers to excite the fermionic composite resonances, the elementary top will have a sizable mixing with these massive states. The stronger the mixing, the larger will be the degree of compositeness of the physical top quark. The requirement $|c_{q,u}| < 1/2$ is also necessary in order to have EWSB. In this region the top quark contribution to the Higgs potential dominates over the gauge one, which would otherwise align the vacuum in an $(\text{SU}(2)_L \times \text{U}(1)_Y)$-preserving direction (since $\alpha_{\text{gauge}}$ is always positive). The conditions $\alpha < 0$ and $\beta > 0$ can then be easily satisfied thanks to the top contribution. In other words, the EWSB, in our model, is a direct consequence of the heaviness of the top.

To illustrate this point, we show in Figure 1 the contour plots of $\epsilon$ in the planes $(c_q, c_u)$ and $(\tilde{m}_u, \tilde{M}_u)$ respectively for the MCHM$_{10}$ and the MCHM$_5$. The region with no EWSB ($\epsilon = 0$) is depicted in black, and the dashed black curve corresponds to $m_t^{pole} = 173$ GeV. In each plot, we have set $N = 8$ and kept the remaining 5D parameters fixed. The condition $\beta > |\alpha|$, i.e. $0 < \epsilon < 1$, further selects a smaller region of the planes $(c_q, c_u)$ and $(\tilde{m}_u, \tilde{M}_u)$. A naive estimate shows that $|\alpha_{L,R}|$ is parametrically larger than $\beta$ by a factor $(1/4 - c_{u,q}^2)$. A reduction in $\alpha$, however, can be obtained in the region where $\alpha_L \simeq -\alpha_R$. As already pointed out in Ref. [4], this is possible since $\alpha_L$ and $\alpha_R$ have generally opposite sign. In the case of the MCHM$_{10}$, the region with smaller $\alpha$ are the two gray areas with the “boomerang” shape shown in the left plot of Fig. 1 plus a specular region under $c_q \to -c_q$ which is not showed.
Figure 1: Contour plots of $\epsilon$ in the plane $(c_q, c_u)$ with $\tilde{m}_u = 1$, $\tilde{M}_u = -2$, $N = 8$ for the MCHM$_{10}$ (left plot), and in the plane $(\tilde{m}_u, \tilde{M}_u)$ with $c_q = 0.35$, $c_u = 0.45$, $N = 8$ for the MCHM$_5$ (right plot). The two gray areas correspond to the region with EWSB and non-zero fermion masses, $0 < \epsilon < 1$. The lighter gray area is excluded when the bound $S \lesssim 0.3$ is imposed. The dashed black line represents the curve with $m_t^{\overline{MS}}(2\,\text{TeV}) = 150\,\text{GeV}$, equivalent to $m_t^{\text{pole}} = 173\,\text{GeV}$.

These two solutions correspond to $q_L$ almost elementary ($c_q \simeq +1/2$) or almost composite ($c_q \simeq -1/2$). We found that the case of the MCHM$_5$ is analogous, but with the role of $c_q$ and $c_u$ interchanged: the two possible solutions are for $t_R$ almost elementary ($c_u \simeq -1/2$) or almost composite ($c_u \simeq +1/2$).

A second circumstance in which $\beta > |\alpha|$ is when $\tilde{m}_u \simeq -1/\tilde{M}_u$. As one can directly check, by using their expressions in terms of 5D propagators, the fermionic form factors $\Pi^q_1$, $\Pi^u_1$, and consequently $\alpha_{L,R,1}$, identically vanish for $\tilde{m}_u = -1/\tilde{M}_u$ (both in the MCHM$_5$ and in the MCHM$_{10}$). Therefore, for $\tilde{m}_u \simeq -1/\tilde{M}_u$ one can have $\beta > |\alpha|$. This is shown for the MCHM$_5$ in Fig. 1 right plot, and a similar result holds for the MCHM$_{10}$. Even though one can reach the $0 < \epsilon < 1$ region by moving along the plane $(\tilde{m}_u, \tilde{M}_u)$ for almost any choice of $c_q$, $c_u$, the additional constraint of having the top quark mass equal to its experimental value (the black dashed line in the plots of Fig. 1) disfavors in most of the cases solutions with both $q_L$ and $t_R$ elementary (that is, with $c_q \simeq 1/2$ and $c_u \simeq -1/2$). This is especially true for the MCHM$_{10}$, since the formula for the top mass has an extra suppression factor

\[ \text{Notice that } \Pi^q_2 = 2\Pi^u_2 \text{ in the MCHM}_{10}, \text{ while in the MCHM}_5 \Pi^q_2 \text{ is always suppressed and its contribution to } \alpha_L \text{ can be neglected, see Appendix.} \]
$1/\sqrt{2}$ compared to the MCHM$_5$, see Eqs. (34) and (42) of the Appendix.

Our investigation of the structure of the Higgs potential has then revealed that there are specific regions of the parameter space in which the electroweak symmetry is broken and the SM quarks get a mass. Part of these regions, however, is excluded by the precision tests. How large is this portion gives us a measure of the “degree of tuning” required in our model. This is the subject of the next section.

3 Electroweak precision tests

There are two types of corrections to the electroweak observables that any composite Higgs model must address, since they are usually sizable: Non-universal corrections to the $Z\bar{b}b$ coupling, and universal corrections to the gauge boson self-energies. The results of Ref. [8] show that for both our choices of fermionic 5D representations, Eqs. (22) and (35), non-universal corrections to $Z\bar{b}b$ are small, due to the custodial O(3) symmetry of the bulk and IR boundary. Therefore, we need to consider only universal effects, which can be parametrized in terms of four quantities: $S$, $T$, $W$ and $Y$ [13]. The last two parameters are suppressed by a factor $\sim (g^2 N/16\pi^2)$ compared to $S$ and $T$, and can be neglected [4]. The parameter $T$ is zero at tree level due to the custodial symmetry. Loop effects can be estimated to be small ($T \ll 0.3$), and explicit calculations in similar 5D models confirm this expectations [7, 10].

We defer a full calculation of the $T$ parameter in the MCHM$_5$ and MCHM$_{10}$ to a future study.

The Peskin-Takeuchi $S$ parameter gives the most robust and model-independent constraint. Neglecting a small correction from boundary kinetics terms, one has [4]:

$$S = \frac{3}{8} \frac{N}{\pi} \epsilon^2. \quad (10)$$

The 99% CL experimental bound $S \lesssim 0.3$ [13] then translates into

$$\epsilon^2 \lesssim \frac{1}{4} \left( \frac{10}{N} \right). \quad (11)$$

For $N = 5$ ($N = 10$) this rules out the values $1/2 \lesssim \epsilon^2 < 1$ ($1/4 \lesssim \epsilon^2 < 1$), which we naively expect to correspond to $\sim 1/2$ ($\sim 3/4$) of the region with EWSB and non-zero fermion

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5 In Ref. [10] the SM fermions were also embedded in the 5 and 10 representations of SO(5), but with different boundary conditions from ours. This implies that, for example, while in the MCHM$_5$ there is one SU(2)$_L$-doublet KK state with hypercharge $Y = 7/6$ that becomes light in the limit of $t_R$ composite, in the model of Ref. [10] this happens in the limit of $t_R$ mostly elementary.

6 In order to fully saturate the bound $S \lesssim 0.3$, a positive $T$ of the same size is required. The results of Refs. [7, 10] suggest that this could be possible in certain regions of the parameter space. Otherwise, the 99% CL experimental bound on $S$ becomes stronger: $S \lesssim 0.2$ (0.1) for $T \lesssim 0.1$ (0).
masses (the region $0 < \epsilon^2 < 1$). The exact numerical results – see Fig. 1 for a case with $N = 8$ – reasonably agree with this rough estimate. This means that there is still a large portion of the parameter space which is not ruled out by the constraint from $S$, and no large fine tuning is hence required. Notice that smaller values of $N$ imply larger fractions of allowed parameter space, although $N$ cannot be too small if we want to remain in the perturbative regime.

4 Spectrum of fermionic resonances and the Higgs mass

An important prediction of our model is that the requirement of a large top mass always forces some of the fermionic KK states to be lighter than their gauge counterpart (a similar property is found in the model of Ref. [14]). The reason is the following. The embedding of $t_L$ and $t_R$ into SO(5) bulk multiplets implies that some of their SO(5) partners have $(\pm, \mp)$ boundary conditions, an assignment that is necessary to avoid extra massless states (see Eqs. (22) and (33)). Consider for example the case in which the left-handed chirality of the 5D field is $(+, -)$, (hence the right-handed one is $(-, +)$): for values of the 5D mass $c_i = u, q > 1/2$, the lightest KK mode, denoted by $q^*$, has its left-handed chirality exponentially peaked on the UV boundary, while the right-handed one is localized on the IR boundary. This implies that the mass of $q^*$ is exponentially suppressed. On the other hand, for $c_i < -1/2$ both chiralities are localized on the IR boundary and the mass of $q^*$ is of the same order of that of the other KKs: $m_{q^*} \approx m_{\rho}$. In the intermediate region $-1/2 < c_i < 1/2$, the one in which the large mass of the top can be reproduced, one finds that $m_{q^*}$ is well interpolated by

$$m_{q^*} \simeq \frac{\kappa}{L_1} \sqrt{\frac{1}{2} - c_i},$$

where $\kappa \sim 2$ is a numerical coefficient with a mild dependence on the values of the boundary masses. This means that $m_{q^*}$ is still parametrically smaller than $m_{\rho}$ by a factor $\sqrt{1/2 - c_i}$. Analogous results hold for left-handed 5D fermions with $(+, +)$ boundary condition if they mix with additional 4D fermions localized on the IR boundary. In the case of right-handed 5D fields with $(+, -)$ boundary condition the same argument above also applies if $c_i \to -c_i$. Eq. (12) has a clear interpretation in the 4D dual description of the theory where the left- and right-handed chiralities of the lightest massive state correspond respectively to an elementary and a composite state [12]. For $-1/2 < c_i < 1/2$, it can be shown (see also the Appendix) that the coupling of the elementary state to the CFT flows to a fixed point value proportional to $\sqrt{-\gamma} = \sqrt{1/2 - c_i}$, where $\gamma$ is the anomalous dimension of the CFT operator to which
the elementary field couples. Naive dimensional analysis then immediately gives Eq. (12).

We will concentrate on the region \(-\frac{1}{2} < c_u < \frac{1}{2}\) with \(c_u\) slightly smaller than \(\frac{1}{2}\) (\(t_L\) mostly elementary). From the argument above and by inspecting Eqs. (22) and (35), one can deduce that the light KK modes are all the SO(5) partners of \(t_R\) inside \(\xi_u\). In the case of the MCHM5, \(\xi_u\) transforms as a 5 of SO(5) and the partners of \(t_R\) form a \((2, 2)\) of \(SU(2)_L \times SU(2)_R\), equivalent to two \(SU(2)_L\) doublets of hypercharge \(Y = 1/6\) and \(Y = 7/6\). The first is the lightest resonance with the quantum numbers of \(t_L\), while the second is its O(4)-custodial companion. In the case of the MCHM10, \(\xi_u\) transforms as a 10 of SO(5) and the light partners of \(t_R\) also include its own O(4) custodians, a \(3_{2/3} \oplus 1_{5/3} \oplus 1_{-1/3}\) of \(SU(2)_L \times U(1)_Y\).

Figure 2 shows the spectrum of the lowest fermionic KK states in the MCHM5 (upper plot), and in the MCHM10 (lower plot). The light states are those predicted. Their mass is around 500 – 1500 GeV for \(\epsilon = 0.5\) and \(N = 8\), much smaller than that of the lightest gauge KK, \(m_\rho = 2.6\) TeV, and of other fermionic excitations. The custodian \(2_{7/6}\) turns out to be the lightest among all the light fermionic resonances and therefore the most accessible.

There is an alternative way to understand why in this type of models one expects light colored fermionic resonances. From Eq. (17), we have that the Higgs mass is given by

\[
m^2_h \simeq \frac{8\beta}{f^2} s_h^2 c_h^2,
\]

where

\[
\beta \simeq N_c \int \frac{d^4 p}{(2\pi)^4} \frac{F(p)}{p^2}, \quad F(p) \equiv \frac{(M^2)}{\left(\Pi^0 + (s^2/2) \Pi^1\right) \left(\Pi^0 + (s^2/2) \Pi^1\right)}.
\]

Using Eq. (5), we have \(m_t^2 = F(0)s_h^2 c_h^2/2\) and hence

\[
m^2_h \simeq \frac{N_c m_t^2}{\pi^2 \epsilon^2} \epsilon^2 \Lambda^2,
\]

where we have defined

\[
\Lambda^2 \equiv 2 \int_0^\infty dp \frac{p F(p)}{F(0)}.
\]

Eq. (15) shows the relation between the Higgs mass and \(\Lambda\), which is, roughly speaking, the scale at which the momentum integral is cut off. On general grounds, one would expect this.
Figure 2: Masses of the lightest colored KK fermions in the MCHM$_5$ (upper plot), and in the MCHM$_{10}$ (lower plot). Different symbols denote KKs with different quantum numbers under $SU(2)_L \times U(1)_Y$, as specified in the plots. Both plots are for $\epsilon = 0.5$, $N = 8$. In the upper one we have varied $0.28 < c_q < 0.38$, $0 < c_u < 0.41$, $0.32 < \tilde{m}_u < 0.42$, $-3.5 < \tilde{M}_u < -2.2$ (filled points), or $0.2 < c_q < 0.35$, $-0.25 < c_u < -0.42$, $-1.3 < \tilde{m}_u < 0.2$, $0.1 < \tilde{M}_u < 2.3$ (empty points). In the lower plot we have varied $0.36 < c_q < 0.45$, $0 < c_u < 0.38$, $0.8 < \tilde{m}_u < 3$, $-3 < \tilde{M}_u < -0.3$. The black continuous line is the fit to the mass of the lightest resonance according to Eqs. (15) and (18).
cutoff scale to be of the order of the mass of the lowest fermionic resonance:

\[ m_{q^*} \simeq \Lambda \simeq 900 \text{ GeV} \left( \frac{m_h}{150 \text{ GeV}} \right) \left( \frac{0.5}{\epsilon} \right), \tag{17} \]

where in the last equality Eq. (15) has been used. Eq. (17) shows that in composite Higgs models with a light Higgs and no tuning (\( \epsilon \sim 1 \)) colored resonances are expected to be not heavier than \( \sim 1 \) TeV. In our model, the relation between the Higgs mass and the mass of the lowest fermionic KK turns out to be more complicated than that of Eq. (17). We find that the points of Fig. 2 are better reproduced by a relation of the form

\[ \Lambda^2 = a_1 m_{q^*}^2 + a_2 m_{q^*} M + a_3 M^2, \tag{18} \]

where \( a_i \) are numerical coefficients, \( M \equiv m_\rho \) parametrizes the mass of the heavier resonances and by \( m_{q^*} \) we denote the mass of the KK weak doublet with hypercharge \( Y = 7/6 \) (the lightest among the fermionic KKs in Fig. 2). This means that in our model the integral \( \int_0^\infty dp p [F(p)/F(0)] \) is not completely cut off at \( p \sim m_{q^*} \), and that other (heavier) resonances also play a role. A fit to the points of Fig. 2 gives: \( a_{i=1,2,3} = (-0.10, 0.35, 0.007) \) for the MCHM5 (upper plot) and \( a_i = (-0.14, 0.24, 0.06) \) for the MCHM10 (lower plot). The dispersion of the points around the fitted curve (shown in each plot) can be explained as follows. In Fig. 2 we have fixed \( N = 8 \), \( \epsilon = 0.5 \) and \( m_t^{pole} = 173 \) GeV, which leaves two of the five parameters of Eq. (9) free to vary. If \( c_u \) is traded for \( m_{q^*} \), we are left with one free parameter, for example, \( c_q \). The coefficients \( a_i \) of Eq. (18) will thus depend on \( c_q \). To generate the points in Fig. 2 we have scanned over the values \( 0.2 < c_q < 0.38 \) (upper plot) and \( 0.36 < c_q < 0.45 \) (lower plot) and therefore the fitted \( a_i \) given above should be considered as average values over these intervals of \( c_q \).

5 Production and detection of the lightest fermionic resonances at the LHC

The most promising way to discover these models is by detecting their lowest fermionic KKs at the LHC. In particular, detecting the custodian with electric charge \( 5/3 \), \( q_{5/3}^* \), that arises

\[ m_h^2 \simeq \frac{N_c}{\pi^2} \frac{m_h^2}{2 v^2} \Lambda^2 - \frac{4 e_h^2 \alpha}{f_5^2}, \]

The first term is the formula for the Higgs mass one obtains in the SM by defining \( \Lambda^2/2 \equiv \int dp p \) in the top loop. The degree of cancellation between the first and second term gives a measure of the degree of “tuning” needed in our model. This exactly corresponds to \( \epsilon^2 \).
from the $2_{7/6}$ multiplet of SU(2)$_L \times$ U(1)$_Y$, would be the smoking-gun signature of the model. This exotic state is a direct consequence of the custodial symmetry required to forbid large corrections to $Zb\bar{b}$. For not-too-large values of its mass $m_{q_{5/3}^*}$, roughly below 1 TeV, this new particle will be mostly produced in pairs, via QCD interactions,

$$q\bar{q}, gg \rightarrow q_{5/3}^* \bar{q}_{5/3}^*, \quad (19)$$

with a cross section completely determined in terms of $m_{q_{5/3}^*}$ (see for example [15, 16]). Once produced, $q_{5/3}^*$ will decay to a (longitudinally polarized) $W^+$ plus a $t_R$, with a coupling of order $4\pi/\sqrt{N}(c_u + 1/2)$. Decays to SM light quarks will be strongly suppressed, with a coupling of order of the square root of their Yukawa couplings. In general, colored resonances will mostly decay to tops and bottoms, since these are the SM quarks more strongly coupled to them, as the result of the large top mass. The process of Eq. (19) then leads to a final state of four $W$’s and two $b$-jets:

$$q_{5/3}^* q_{5/3}^* \rightarrow W^+t W^-\bar{t} \rightarrow W^+W^+b W^-\bar{b}. \quad (20)$$

The same final state also comes from pair production of KKs with charge $-1/3$. A way to discriminate between the two cases consists in reconstructing the electric charge of the resonance. For example, one could look for events with two highly-energetic leptons of the same sign, coming from the leptonic decay of two of the four $W$’s, plus at least six jets, two of which tagged as $b$-jets. Demanding that the invariant mass of the system of the two hadronically-decaying $W$’s plus one $b$-jet equals $m_{q_{5/3}^*}$ then identifies the process and gives evidence for the charge $5/3$ of the resonance. Furthermore, indirect evidence in favor of $q_{5/3}^*$ would come from the non-observation of the decays to $Zb, Hb$ that are allowed for resonances of charge $-1/3$.

For increasing values of $m_{q_{5/3}^*}$ the cross section for pair production quickly drops, and single production might become more important. The relevant process is $tW$ fusion [17], where a longitudinal $W$ radiated from one proton scatters off a top coming from the other proton. The analogous process initiated by a bottom quark, $bW$ fusion, has been studied in detail in the literature and shown to be an efficient way to singly produce heavy excitations of the top quark [15, 18, 16]. To prove that the same conclusion also applies to the case of $tW$ fusion a dedicated analysis is required. The main uncertainty and challenge comes from the small top quark content of the proton, which however can be compensated by the large coupling involved, especially in the case of $t_R$ largely composite [4].

Besides $q_{5/3}^*$, the other components of the $2_{1/6}$ and $2_{7/6}$ multiplets of SU(2)$_L \times$ U(1)$_Y$ are also predicted to be light in both our models, see Fig. [2]. In the specific case of the
MCHM$_{10}$ there are also other states transforming as $3_{2/3}$, $1_{5/3}$, $1_{-1/3}$. These multiplets contain states with $Q_{em} = 5/3$ whose phenomenology will be similar to that of the $q^*_{5/3}$ described above. The states of electric charge $2/3$ or $-1/3$ will also be produced in pairs via QCD interactions or singly via $bW$ or $tW$ fusion. They will decay to a SM top or bottom quark plus a longitudinally polarized $W$ or $Z$, or a Higgs. When kinematically allowed, a heavier resonance will also decay to a lighter KK accompanied with a $W_{long}$, $Z_{long}$ or $h$. Decay chains could lead to extremely characteristic final states. For example, in the MCHM$_{10}$ the KK with charge $2/3$ from $3_{2/3}$ is predicted to be generally heavier than $q^*_{5/3}$; see Fig. 2. If pair produced, it can decay to $q^*_{5/3}$ leading to a spectacular six $W$'s plus two $b$-jets final state:

$$q^*_{2/3} \bar{q}^*_{2/3} \rightarrow W^- q^*_{5/3} W^+ \bar{q}^*_{5/3} \rightarrow W^- W^+ W^+ b W^- W^- \bar{b}.$$  

To fully explore the phenomenology of the fermionic resonances in our models a detailed analysis is necessary. One could for example adopt the simplifying strategy proposed in Ref. [19], where a 4D effective theory has been introduced to consistently describe the SM fields and the first KK excitations of a large class of warped models. Existing studies in the literature have focussed on the production and detection of SU(2)$_L$ singlets of hypercharge $Y = 2/3$ [16, 20], since this is a typical signature of Little Higgs theories. In our models, however, the singlet is not predicted to be light, except for specific regions of the parameter space. In conclusion, our brief discussion shows that there are characteristic signatures predicted by our models that will distinguish them from other extensions of the SM. While certainly challenging, these signals will be extremely spectacular, and will provide an indication of a new strong dynamics responsible for electroweak symmetry breaking.

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Appendix

Here we present in detail the fermion sector of our composite Higgs models. We have assumed that: (i) each SM fermion is embedded in a different 5D field; (ii) All the three SM families have the same embedding. Following these rules, we construct what we think are the minimal models with fermions embedded in 5 or 10 representations of SO(5).

A Defining the MCHM$_5$

The quark sector of the MCHM$_5$ is defined in terms of 5D bulk multiplets transforming as fundamentals of SO(5). Each SM generation is identified with the zero modes of the 5D fields

$$
\xi_{q_1} = \begin{pmatrix} (2, 2)_L^{q_1} \\ (1, 1)_L^{q_1} \end{pmatrix} = \begin{pmatrix} q_{1L}(++) \\ q_{1L}(++) \end{pmatrix}, \quad \xi_{q_2} = \begin{pmatrix} (2, 2)_L^{q_2} \\ (1, 1)_L^{q_2} \end{pmatrix} = \begin{pmatrix} q_{2L}(++) \\ q_{2L}(++) \end{pmatrix},
$$

where $\xi_{q_1}$, $\xi_u$ ($\xi_{q_2}$, $\xi_d$) transform as 5/23 (5/13) of SO(5)$\times$U(1)$_X$. A similar 5D embedding also works for the SM leptons, although with different U(1)$_X$ charges. Chiralities under the 4D Lorentz group have been denoted by $L$, $R$, and ($\pm$, $\pm$) is a shorthand notation to denote Neumann ($+$) or Dirichlet ($-$) boundary conditions on the two boundaries. We have grouped the fields of each multiplet $\xi_i$ in representations of SO(4)$\sim$SU(2)$_L\times$SU(2)$_R$, and used the fact that a fundamental of SO(5) decomposes as $5 = 4 \oplus 1 = (2, 2) \oplus (1, 1)$. Although each of $\xi_{q_1}$ and $\xi_{q_2}$ alone could account for the $q_L$ zero mode, we need them both to give mass to both the up and down SM quarks. We thus identify the SM $q_L$ field with the zero mode of the linear combination ($q_{1L} + q_{2L}$), and get rid of the extra zero mode by introducing a localized right-handed field on the UV boundary that has a mass mixing with the orthogonal combination. We denote by $c_i$, $i = q_1, q_2, u, d$, the bulk masses of each 5D field $\xi_i$ in units of $k$. Localized on the IR boundary, we consider the most general set of mass terms invariant under O(4)$\times$U(1)$_X$:

$$
\tilde{m}_u (2, 2)_L^{q_1} (2, 2)_L^{q_2} + \tilde{M}_u (1, 1)_L^{q_1} (1, 1)_L^{q_2} + \tilde{m}_d (2, 2)_L^{q_2} (2, 2)_L^{d} + \tilde{M}_d (1, 1)_L^{q_2} (1, 1)_L^{d} + h.c. \quad (23)
$$
A.1 The holographic description

The 5D field content of Eq. (22) has a very simple holographic interpretation in terms of three elementary chiral fields, $q_L = (u_L, d_L)$, $u_R$ and $d_R$, coupled to a CFT sector through composite operators $O_i$. An important difference from the model of Ref. [4], and also from the MCHM discussed in the next section, is that here the elementary $q_L$ couples to two different CFT operators $O_1$ and $O_2$ with couplings $\lambda_1, \lambda_2$. This is the consequence of having two different bulk fields in Eq. (22), $q_{1L}$ and $q_{2L}$, mixed on the UV boundary. The elementary fields $u_R$ and $d_R$, instead, couple to one operator each, $O_u$ and $O_d$, with couplings $\lambda_u$ and $\lambda_d$. The bulk gauge symmetry of the 5D model maps into an $SO(5) \times U(1)_X$ global symmetry of the CFT, and Eq. (22) implies that $O_1, O_u$ transform as $5_{2/3}$, while $O_2, O_d$ transform as $5_{-1/3}$. Schematically:

$$
\begin{align*}
\lambda_1 & \quad O_1 \quad \frac{H}{O_u} \quad \lambda_u \quad u_R \\
\lambda_2 & \quad O_2 \quad \frac{H}{O_d} \quad \lambda_d \quad d_R
\end{align*}
$$

The double line indicates that $(\bar{O}_1 O_u)$ and $(\bar{O}_2 O_d)$ have the correct quantum numbers to excite the Higgs and generate in this way the up and down Yukawa couplings of the 4D low-energy theory. Large hierarchies among the Yukawas can be explained naturally as the result of the RG evolution of the couplings $\lambda_i$ [4]. Notice that the CFT dynamics alone do not mix $O_1$ and $O_u$ with $O_2$ and $O_d$, since they have different $U(1)_X$ quantum numbers. Nevertheless, both $O_1$ and $O_2$ can couple to the external source $q_L$, since this latter coupling will only preserve the $SU(2)_L \times U(1)_Y$ elementary symmetry. This suggests that a hierarchy in the up and down Yukawa couplings, like in the case of the top and bottom quarks, can follow from the RG evolution if $\lambda_u \gg \lambda_d$ at low-energy, as already pointed out for the model of Ref. [4], or alternatively if $\lambda_1 \gg \lambda_2$.

Quite interestingly, it is simple to show that $\lambda_1$ can grow much bigger (or smaller) than $\lambda_2$ in the infrared, even if both operators $O_1$ and $O_2$ are relevant. The argument goes as

---

We adopt a left-source holographic description for the fields $\xi_{q_1}$ and $\xi_{q_2}$, and a right-source description for $\xi_u$ and $\xi_d$. See Ref. [12].
follows. The RG equations of the two couplings $\lambda_1, \lambda_2$ form a coupled system:

\[ p \frac{d}{dp} \lambda_1(p) = \gamma_1 \lambda_1 + \frac{N}{16\pi^2} \left( a_1 \lambda_1^3 + a_{12} \lambda_1 \lambda_2^2 \right) + \ldots \quad (24) \]

\[ p \frac{d}{dp} \lambda_2(p) = \gamma_2 \lambda_2 + \frac{N}{16\pi^2} \left( a_2 \lambda_2^3 + a_{21} \lambda_2 \lambda_1^2 \right) + \ldots \quad (25) \]

The dots stand for subleading terms in a $1/N$ expansion, where the number of CFT colors $N$ is defined by Eq. (2). The duality with the 5D theory implies that the coefficients $a_1$ and $a_2$ are both positive (see [12]), and that the anomalous dimensions $\gamma_1, \gamma_2$ are linear functions of the 5D bulk masses: $\gamma_{1,2} = |c_{q_1, q_2} + 1/2| - 1$. Furthermore, it is easy to show that the leading contribution to the coefficients $a_{12}$ and $a_{21}$ comes from the wave function renormalization, which in turn implies $a_{21} = a_1, a_{12} = a_2$ at leading order in $1/N$. Let us then consider the case in which the operator with the smallest anomalous dimension, say $O_1$, is relevant, that is $c_{q_1} < 1/2, c_u < c_{q_2}$ (i.e. $\gamma_1 < 0, \gamma_1 < \gamma_2$). Then $\lambda_1$ will grow faster than $\lambda_2$ in the infrared, and at some energy $E_*$ it will reach a fixed-point value $\lambda_{1*} \simeq 4\pi/\sqrt{N} \sqrt{-\gamma_1/a_1}$. Below that energy, one can set $\lambda_1 = \lambda_{1*}$ in the RG equation for $\lambda_2$, Eq. (25), which becomes

\[ p \frac{d}{dp} \lambda_2(p) \simeq (\gamma_2 - \gamma_1) \lambda_2 + \ldots \quad (26) \]

Since $(\gamma_2 - \gamma_1) > 0$ by assumption, this implies that $\lambda_2$ will be suppressed at low energy,

\[ \lambda_2(E) \sim \left( \frac{E}{E_*} \right)^{\gamma_2 - \gamma_1}, \quad (27) \]

even if $\gamma_2 < 0$, that is, even if the operator $O_2$ is relevant.

The above argument shows that the small ratio $m_b/m_t$ follows naturally in the MCHM$_5$ by having $c_{q_2} > c_{q_1}$, even when $b_R$ is strongly coupled to the CFT sector. The large top mass, on the other hand, requires $|c_{q_1}|, |c_u| < 1/2$ for the third generation quarks, i.e. the operators $O_1$ and $O_u$ need to be both relevant. In our numerical analysis of the MCHM$_5$ (hence in all the results presented in the text), we have set $c_{q_2} = 0.4, c_u = -0.55$ for the third generation, and varied $-1/2 < c_u < 1/2, -1/2 < c_{q_1} < c_{q_2}$. Choosing $c_{q_1} < c_{q_2}$ also implies that the contribution of $\xi_{q_2}$ to the Higgs potential will be suppressed and hence negligible compared to that of $\xi_{q_1}$ (see also below). This justifies the following identification:

\[ \xi_q \equiv \xi_{q_1}, \quad c_q \equiv c_{q_1}, \quad (28) \]

where by $\xi_q$ we mean the field responsible for the contribution of $q_L$ to the Higgs potential to which we referred in the text.
Following the method of Ref. [4], one can derive the most general form of the holographic Lagrangian by introducing spurion fields and embedding the elementary sources in complete SO(5)\times U(1)\_X (chiral) multiplets. The fact that the elementary \( q_L \) couples to two different CFT operators implies that there are two different ways to embed it in a fundamental representation of SO(5), namely as the \( T^{3R} = +1/2 \) or the \( T^{3R} = -1/2 \) component of the internal (2, 2). More explicitly, grouping the entries of each 5 in SU(2)_L\times SU(2)_R representations:

\[
\Psi_{1L} = \begin{pmatrix} q_{1L}^L \\ q_{1L}^R \\ u'_{1L} \end{pmatrix}, \quad \Psi_{2L} = \begin{pmatrix} q_{2L}^L \\ q_{2L}^R \\ d'_{L} \end{pmatrix}, \quad \Psi_{uR} = \begin{pmatrix} q_{uR}^u \\ q_{uR}^d \\ u_{R} \end{pmatrix}, \quad \Psi_{dR} = \begin{pmatrix} q_{dR}^u \\ q_{dR}^d \\ d_{R} \end{pmatrix}. \tag{29}
\]

The multiplets \( \Psi_{1L}, \Psi_{uR}, \Psi_{dR} \) have \( U(1)\_X \) charge +2/3 (-1/3), and all components other than \( q_L, u_R \) and \( d_R \) are non-dynamical spurion fields. Therefore, the most general \( (\text{SO}(5) \times U(1)\_X) \)-invariant holographic Lagrangian, at the quadratic order and in momentum space, is

\[
\mathcal{L}^{(2)}_{\text{hologo}} = \sum_{r=1,2} \Psi_{iL}^r \slashed{p} \left( \delta^{ij} \hat{\Pi}_{0}^{L} (p) + \Sigma^{i} \Sigma^{j} \hat{\Pi}_{1}^{L} (p) \right) \Psi_{jL}^r + \sum_{r=u,d} \bar{\Psi}_{iR}^r \slashed{p} \left( \delta^{ij} \hat{\Pi}_{0}^{R} (p) + \Sigma^{i} \Sigma^{j} \hat{\Pi}_{1}^{R} (p) \right) \Psi_{jR}^r \\
+ \bar{\Psi}_{iL}^{u} \left( \delta^{ij} \hat{M}_{0}^{1L} (p) + \Sigma^{i} \Sigma^{j} \hat{M}_{1}^{1L} (p) \right) \Psi_{jL}^{u} + \bar{\Psi}_{iL}^{d} \left( \delta^{ij} \hat{M}_{0}^{2L} (p) + \Sigma^{i} \Sigma^{j} \hat{M}_{1}^{2L} (p) \right) \Psi_{jL}^{d} + h.c. \tag{30}
\]

Here \( i, j = 1, \ldots, 5 \) are SO(5) indices, and \( \Sigma \) is the non-linear realization of the Higgs field [4]:

\[
\Sigma = \frac{s_h}{\hbar} \left( h^1, h^2, h^3, h^4, h \frac{c_h}{s_h} \right). \tag{31}
\]

The form factors \( \hat{\Pi}, \hat{M} \) can be computed using the holographic technique of Ref. [4]; the result is:

\[
\hat{\Pi}_{0}^{L} = \Pi_{qL} (c_{q1}, c_{u}, \bar{m}_{u} ), \quad \hat{\Pi}_{1}^{L} = \Pi_{QL} (c_{q1}, c_{u}, \bar{M}_{u} ) - \Pi_{qL} (c_{q1}, c_{u}, \bar{m}_{u} ), \\
\hat{\Pi}_{0}^{2L} = \Pi_{qL} (c_{q2}, c_{d}, \bar{m}_{d} ), \quad \hat{\Pi}_{1}^{2L} = \Pi_{QL} (c_{q2}, c_{d}, \bar{M}_{d} ) - \Pi_{qL} (c_{q2}, c_{d}, \bar{m}_{d} ), \\
\hat{\Pi}_{0,1}^{uR} = \hat{\Pi}_{0,1}^{L} (c_{q1} \leftrightarrow c_{u}; L \leftrightarrow R), \quad \hat{\Pi}_{0,1}^{dR} = \hat{\Pi}_{0,1}^{2L} (c_{q2} \leftrightarrow c_{d}; L \leftrightarrow R), \tag{32}
\]

\[
\hat{M}_{0}^{1L} = M_{q} (c_{q1}, c_{u}, \bar{m}_{u} ), \quad \hat{M}_{1}^{1L} = M_{Q} (c_{q1}, c_{u}, \bar{M}_{u} ) - M_{q} (c_{q1}, c_{u}, \bar{m}_{u} ), \\
\hat{M}_{0}^{2L} = M_{q} (c_{q2}, c_{d}, \bar{m}_{d} ), \quad \hat{M}_{1}^{2L} = M_{Q} (c_{q2}, c_{d}, \bar{M}_{d} ) - M_{q} (c_{q2}, c_{d}, \bar{m}_{d} ),
\]

where \( \Pi_{qL,QL} \) and \( M_{Q,Q} \) are the form factors defined in the Appendix of Ref. [4]. After setting all non-dynamical fields to zero, the Lagrangian [30] reduces to that of Eq. [33] with

\[
\Pi_{0} = \hat{\Pi}_{0}^{1L} + \hat{\Pi}_{0}^{2L}, \quad \Pi_{1} = \hat{\Pi}_{1}^{1L}, \quad \Pi_{1}^{u} = -\frac{1}{2} \hat{\Pi}_{1}^{uR}, \quad M_{1}^{u} = \hat{M}_{1}^{1L}, \\
\Pi_{0}^{u} = \hat{\Pi}_{0}^{uR} + \hat{\Pi}_{1}^{1R}, \quad \Pi_{1}^{u} = \hat{\Pi}_{1}^{2L}, \quad \Pi_{1}^{d} = -\frac{1}{2} \hat{\Pi}_{1}^{dR}, \quad M_{1}^{d} = \hat{M}_{1}^{2L}. \tag{33}
\]
Since we set $c_{q_2} > c_{q_1}$, $c_d < -1/2$, the form factors $\hat{\Pi}^{2L}_1$, $\hat{\Pi}^d_1$ are suppressed compared to $\hat{\Pi}^{1L}_1$, $\hat{\Pi}^u_1$, and their effect in the Higgs potential can be neglected.

The fermionic spectrum of the SM fields and heavy resonances of the MCHM$_5$ can be expressed in terms of poles and zeros of the form factors (see Ref. [7] for the gauge spectrum).

Before EWSB, there are five towers of states:

- a tower of $q_L$’s ($2_{1/6}$ of SU(2)$_L$×U(1)$_Y$) with masses given by: zeros \( \{ \not p \Pi_0^q \} \).

- a tower of $u_R$’s ($1_{2/3}$ of SU(2)$_L$×U(1)$_Y$) with masses given by: zeros \( \{ \not p \Pi_0^u \} \).

- a tower of $d_R$’s ($1_{-1/3}$ of SU(2)$_L$×U(1)$_Y$) with masses given by: zeros \( \{ \not p \Pi_0^d \} \).

- a tower of $2_{7/6}$ of SU(2)$_L$×U(1)$_Y$ with masses given by: poles \( \{ \not p \left( \Pi_0^u + \frac{1}{2} \Pi_1^u \right) \} \).

- a tower of $2_{-5/6}$ of SU(2)$_L$×U(1)$_Y$ with masses given by: poles \( \{ \not p \left( \Pi_0^d + \frac{1}{2} \Pi_1^d \right) \} \).

After EWSB the different towers are mixed and the final spectrum consists of:

- a tower of charge $+2/3$ fermions with masses given by:

\[
\text{zeros} \left\{ p^2 \left( \Pi_0^u + \frac{2}{3} \Pi_1^u \right) \left( \Pi_0^d + \frac{2}{3} \Pi_1^d \right) - \frac{2}{3} (1 - \epsilon^2) (M_1^u)^2 \right\}.
\]

- a tower of charge $-1/3$ fermions with masses given by:

\[
\text{zeros} \left\{ p^2 \left( \Pi_0^d + \frac{2}{3} \Pi_1^d \right) \left( \Pi_0^u + \frac{2}{3} \Pi_1^u \right) - \frac{2}{3} (1 - \epsilon^2) (M_1^d)^2 \right\}.
\]

- a tower of charge $+5/3$ fermions with masses given by: poles \( \{ \not p \left( \Pi_0^u + \frac{1}{2} \Pi_1^u \right) \} \).

- a tower of charge $-4/3$ fermions with masses given by: poles \( \{ \not p \left( \Pi_0^d + \frac{1}{2} \Pi_1^d \right) \} \).

From the formulas above for the fermions of charge $2/3$ and $-1/3$ one recovers Eq. (5) by approximating the form factors with their values at $p^2 = 0$. Further use of Eqs. (32) and (33) gives

\[
m_u \simeq \frac{2}{L_1} \epsilon \sqrt{1 - \epsilon^2} \sqrt{\frac{(1/4 - c_q^2)(1/4 - c_u^2)}{(1/2 + c_u)(1 - \epsilon^2)} \tilde{M}_u (1 - \tilde{m}_u \tilde{M}_u)} \left[ \frac{\tilde{M}_u^2 (1/2 + c_q) + \epsilon^2 \tilde{m}_u^2 (1/2 + c_u)}{\tilde{M}_u^2 (1/2 + c_q) + \epsilon^2 \tilde{m}_u^2 (1/2 + c_u)} \right]^{1/2} \times \left[ \epsilon^2 (1/2 - c_q) + \tilde{M}_u^2 (2 (1/2 - c_u) + \tilde{m}_u^2 (1/2 - c_q)) \right]^{-1/2},
\]

and a similar formula for the down quark mass.
B Defining the MCHM_{10}

The quark sector of the MCHM_{10} is defined in terms of 5D bulk multiplets transforming as antisymmetric representations of SO(5). Each SM generation is identified with the zero modes of three 10_{2/3} of SO(5)\times U(1)_X,

\[ \xi_q = \begin{bmatrix} (2, 2)_L^{q} = \begin{bmatrix} q'_L(++) \\ q_L(++) \end{bmatrix} \\ (3, 1)_L^{q}(-) \\ (1, 3)_L^{q}(-) \end{bmatrix}, \quad \xi_r^{q} = \begin{bmatrix} (2, 2)_R^{q} = \begin{bmatrix} q'^+_R(-) \\ q_R(-) \end{bmatrix} \\ (3, 1)_R^{q}(+) \end{bmatrix}, \]

and an additional \( [\tilde{(3, 1)}_R \oplus \tilde{(1, 3)}_R] \) (an irreducible representation of O(4)) localized on the IR boundary. Bulk and boundary fields mix through the most general set of O(4)-symmetric IR-boundary mass mixing terms:

\[ \begin{bmatrix} (3, 1)^{u,d}_L \tilde{(3, 1)}_R + (1, 3)^{u,d}_L \tilde{(1, 3)}_R \end{bmatrix} + h.c. \]

and

\[ \tilde{M}_{u,d} \begin{bmatrix} (3, 1)^{u,d}_L \tilde{(3, 1)}_R + (1, 3)^{u,d}_L \tilde{(1, 3)}_R \end{bmatrix} + \tilde{m}_{u,d} \begin{bmatrix} (2, 2)^{q}_L \tilde{(2, 2)}_R + (1, 3)^q_R \end{bmatrix} + h.c. \]

We will denote by \( c_i, i = q, u, d \), the bulk masses of each 5D field \( \xi_i \) in units of \( k \). We have grouped the fields of each multiplet \( \xi_i \) in representations of SO(4)\sim SU(2)_L\times SU(2)_R, \) and used the fact that an antisymmetric of SO(5) decomposes as \( 10 = 4 \oplus 6 = (2, 2) \oplus (1, 3) \oplus (3, 1) \). A similar 5D embedding also works for the SM leptons, although with different U(1)_X charges.

The holographic interpretation of the 5D theory defined above is that of three elementary
fields \( q_L, u_R, d_R \) coupled to a CFT sector via the composite operators \( O_q, O_u, O_d \). This is the same 4D description of the model of Ref. \[4\], though in this case the operators \( O_i \) transform as 10_{2/3} representations of the \( SO(5) \times U(1)_X \) global symmetry of the CFT. Following the usual procedure, we can embed the elementary fields into complete \( SO(5) \times U(1)_X \) multiplets (10_{2/3} of \( SO(5) \times U(1)_X \) in this case),

\[
\Psi_{qL} = \begin{pmatrix} q^L_L \\ q^L_R \\ (3, 1)^R_L \\ (1, 3)^R_L \end{pmatrix}, \quad \Psi_{uR} = \begin{pmatrix} (2, 2)^u_R \\ (3, 1)^u_R \\ \lambda^u_R \\ u^R_R \end{pmatrix}, \quad \Psi_{dR} = \begin{pmatrix} (2, 2)^d_R \\ (3, 1)^d_R \\ \lambda^d_R \\ d^R_R \end{pmatrix}
\]

(38)

and derive the most general holographic Lagrangian at the quadratic order and in momentum space: 10

\[
\mathcal{L}^{(2)}_{\text{holo}} = \sum_{r=qL,uR,dR} \left[ \text{Tr} \left( \bar{\Psi}_r \, \not\! \partial \hat{\Pi}_r(p) \Psi_r \right) + \Sigma \bar{\Psi}_r \, \not\! \partial \hat{\Pi}_r(p) \Psi_r \Sigma^T \right] + \sum_{r=uR,dR} \left[ \text{Tr} \left( \bar{\Psi}_{qL} \hat{M}_0^r(p) \Psi_r \right) + \Sigma \bar{\Psi}_{qL} \hat{M}_1^r(p) \Psi_r \Sigma^T \right] + h.c.
\]

(39)

The form factors \( \hat{\Pi}, \hat{M} \) can be computed using the holographic technique of Ref. \[4\]:

\[
\hat{\Pi}_0^L = \Pi_Q(c_u, c_u, \tilde{M}_u), \quad \hat{\Pi}_0^L = 2 \left[ \Pi_Q(c_q, c_u, \tilde{m}_u) - \Pi_Q(c_q, c_u, \tilde{M}_u) \right],
\]

\[
\hat{\Pi}_0^R = \hat{\Pi}_0^L(c_q \leftrightarrow c_u, L \leftrightarrow R), \quad \hat{\Pi}_1^R = \hat{\Pi}_1^L(c_q \leftrightarrow c_u, L \leftrightarrow R),
\]

\[
\hat{\Pi}_0^{dR} = \hat{\Pi}_0^L(c_q \leftrightarrow c_d, L \leftrightarrow R), \quad \hat{\Pi}_1^{dR} = \hat{\Pi}_1^L(c_q \leftrightarrow c_d, L \leftrightarrow R),
\]

\[
\hat{M}_1^{uR} = M_Q(c_q, c_u, \tilde{M}_u), \quad \hat{M}_1^{uR} = 2 \left[ M_Q(c_q, c_u, \tilde{m}_u) - M_Q(c_q, c_u, \tilde{M}_u) \right],
\]

\[
\hat{M}_1^{dR} = M_Q(c_q, c_d, \tilde{M}_u), \quad \hat{M}_1^{dR} = 2 \sqrt{2} \left[ M_Q(c_q, c_d, \tilde{m}_d) - M_Q(c_q, c_d, \tilde{M}_d) \right].
\]

Here \( \Pi_{qL,qL} \) and \( M_{qQ} \) are the form factors defined in the Appendix of Ref. \[4\]. After setting all non-dynamical fields to zero, the Lagrangian (39) reduces to that of Eq. (3) with

\[
\begin{aligned}
\Pi_0^2 &= \Pi_0^2 + \frac{1}{2} \hat{\Pi}_1^L, \\
\Pi_1^u &= -\frac{1}{2} \hat{\Pi}_1^L, \\
\Pi_1^d &= \frac{1}{2} \hat{\Pi}_1^d, \quad M_1^u = \frac{1}{2 \sqrt{2}} \hat{M}_1^{uR}, \\
\Pi_0^u &= \hat{\Pi}_0^u, \quad \Pi_1^u = \frac{1}{2} \hat{\Pi}_1^u, \quad M_1^u = \frac{1}{2 \sqrt{2}} \hat{M}_1^{uR}, \\
\Pi_0^d &= \hat{\Pi}_0^d, \quad \Pi_1^d = \frac{1}{2} \hat{\Pi}_1^d, \quad M_1^d = \frac{1}{2} \hat{M}_1^{dR}.
\end{aligned}
\]

\(^9\)We adopt a left-source holographic description for \( \xi_q \) and a right-source description for \( \xi_u \) and \( \xi_d \). See Ref. \[12\].

\(^{10}\)The operator \( \Psi_{ij} \Psi^{kl} \chi_{ijkl} \) is also \( SO(5) \) invariant, but its form factor identically vanishes due to the \( O(4) \) symmetry of the CFT. Also, we have omitted for simplicity a possible mixing term between \( \Psi_u \) and \( \Psi_d \), since it can be safely neglected in our analysis due to the small coupling of \( b_R \) to the CFT needed to explain the bottom quark mass.
The fermionic spectrum of the MCHM$_{10}$ can be expressed in terms of poles and zeros of the form factors. Before EWSB, there are six towers of states:

- a tower of $q_L$’s ($2_{1/6}$ of SU(2)$_L \times$ U(1)$_Y$) with masses given by: zeros $\{ p \Pi^q_0 \}$.
- a tower of $u_R$’s ($1_{2/3}$ of SU(2)$_L \times$ U(1)$_Y$) with masses given by: zeros $\{ p \Pi^u_0 \}$.
- a tower of $d_R$’s ($1_{-1/3}$ of SU(2)$_L \times$ U(1)$_Y$) with masses given by: zeros $\{ p \Pi^d_0 \}$.
- a tower of $2_{7/6}$ of SU(2)$_L \times$ U(1)$_Y$ with masses given by: poles $\{ p \Pi^q_0 \}$.
- a tower of $1_{5/3}$ plus a tower of $3_{2/3}$ of SU(2)$_L \times$ U(1)$_Y$ with masses given by: poles $\{ p \Pi^u_0 \}$.

The final spectrum after EWSB consists of:

- a tower of charge $+2/3$ fermions with masses given by:
  
  $$ \text{zeros} \left\{ p^2 \left( \Pi^q_0 + \frac{\epsilon^2}{2} \Pi^q_1 \right) \left( \Pi^u_0 + \frac{\epsilon^2}{2} \Pi^u_1 \right) - \frac{\epsilon^2 (1 - \epsilon^2)}{2} (M^u_1)^2 \right\}.$$  

- a tower of charge $-1/3$ fermions with masses given by:
  
  $$ \text{zeros} \left\{ p^2 \left( \Pi^q_0 + \frac{\epsilon^2}{2} \Pi^q_2 \right) \left( \Pi^d_0 + \frac{\epsilon^2}{2} \Pi^d_1 \right) - \frac{\epsilon^2 (1 - \epsilon^2)}{2} (M^d_1)^2 \right\}.$$  

- a tower of charge $+5/3$ fermions with masses given by: poles $\{ p \Pi^q_0 \}$ and poles $\{ p \Pi^u_0 \}$.

From the formulas above for the fermions of charge $2/3$ and $-1/3$, one recovers Eq. (5) by approximating the form factors with their values at $p^2 = 0$. Further use of Eqs. (40) and (41) gives the same explicit formula valid for the MCHM$_5$, Eq. (34), but a factor $\sqrt{2}$ smaller:

$$ m_u|_{\text{MCHM}_{10}} \simeq \frac{1}{\sqrt{2}} m_u|_{\text{MCHM}_{5}}. \quad (42) $$

A similar result is also valid for the down quark mass.

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