Sea level and deep-sea temperature reconstructions suggest quasi-stable states and critical transitions over the past 40 million years

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Sea level and deep-sea temperature variations are key indicators of global climate changes. For continuous records over millions of years, deep-sea carbonate microfossil–based δ¹⁸O (δw) records are indispensable because they reflect changes in both deep-sea temperature and seawater δ¹⁸O (δw); the latter are related to ice volume and, thus, to sea level changes. Deep-sea temperature is usually resolved using elemental ratios in the same benthic microfossil shells used for δw, with linear scaling of residual δw to sea level changes. Uncertainties are large and the linear-scaling assumption remains untested. Here, we present a new process-based approach to assess relationships between changes in sea level, mean ice sheet δ¹⁸O, and both deep-sea δw and temperature and find distinct nonlinearity between sea level and δw, changes. Application to δw records over the past 40 million years suggests that Earth’s climate system has complex dynamical behavior, with threshold-like adjustments (critical transitions) that separate quasi-stable deep-sea temperature and ice-volume states.

INTRODUCTION

Ice sheets are a pivotal component of the climate system during icehouse periods. The current icehouse period started at the Eocene–Oligocene Transition (EOT), 34 million years (Ma) ago, when the Antarctic Ice Sheet (AIS) developed, followed by fluctuations in its volume with northern hemisphere glacial variations (1–4). Modern warming drives climate in the opposite direction, and improved estimates are needed for the associated long-term equilibrium sea level rise. This requires continuous sea level records that extend at least through the Middle Pliocene (about 3 to 3.3 Ma ago), the last time atmospheric CO₂ levels were similar to those of today (4–7). Records should preferably continue into more ancient times to ensure coherent results across the entire range of full glacial to completely ice-free climate states, which is essential for evaluating fundamental aspects such as climate-state dependencies of feedbacks, and climate tipping points.

Ice-volume changes have dominated sea level amplitudes since the EOT. Sea level per se, however, is only reasonably constrained in continuous records for the past 500 to 800 thousand years (ka), on the basis of records of marginal-sea isolation from the open ocean (8–11) and on statistical assessment across various methods (12). For more ancient times, deep-sea oxygen isotope records provide the dominant information source. We want to know seawater δ¹⁸O (δw), but the data come in the form of carbonate microfossil δ¹⁸O (δc) (13, 14), which represents combined influences of δw and Tw. Analytically, Tw influences may be corrected using Mg/Ca data (15, 16), but there are complications from oceanic Mg-concentration changes over time scales longer than a million years (17–19). Moreover, Mg/Ca thermometry has 1° to 1.5°C uncertainties (1σ) in cold deep-sea environments (20), while Tw signals commonly amount to only a few degrees, causing unfavorable signal-to-noise ratios (16). Even if Tw could be resolved without uncertainties, the resolved δw record would still need to be translated into ice-volume/sea level changes. This is commonly done by assuming a linear Δδw–ΔSL relationship, which is often “calibrated” to the Last Glacial Maximum (LGM)–to-modern δw gradient measured in marine sediment pore waters (20–22). However, these assumptions remain untested.

The ubiquitous reliance on linear Δδw–ΔSL approximations is unexpected given that, from first principles, the primary expectation should be that it is nonlinear. This is because ice sheets accumulate precipitation of increasingly negative δ¹⁸O composition as they grow because of increasingly intense Rayleigh distillation and because different ice sheets begin to form and develop their δ¹⁸O signature at different times in moment in time (and, hence, at different moments in global ΔSL history). Thus, the global mean ice δ¹⁸O will change with sea level, which imposes nonlinearity on the Δδw–ΔSL relationship. Here, we evaluate this issue quantitatively using a new approach based on straightforward representations of underlying process relationships and interdependencies, and we apply our mutually consistent reconstruction method for key parameters in paleoceanographic reconstructions. We start with a simplified analytical assessment of the Δδw–ΔSL relationship to illustrate its fundamental nature and sensitivity to key assumptions and uncertainties. Thereafter, we present applied assessments that use published sea level records to determine mutually consistent variations in ΔSL, ice sheet volumes (Vic, δIce), Tw, and δw, culminating in a detailed assessment over the past 40 Ma. Sensitivity tests and a range of independent validation criteria are used to assess the robustness of results, and key avenues for further improvements are highlighted.

Our mutually consistent, process-based framework for ΔSL, Vic, δIce, Tw, and δw permits complete system validation using multiple parameters rather than only one or two as in traditional approaches. We obtain new records for these parameters, which are of fundamental interest in (paleo)climate science, and that are independently cross-validated over at least the past 22 Ma. This approach provides a testable way to understand relationships among measurable parameters and underlying processes of interest in understanding climate change.

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RESULTS

Relationship between precipitation $\delta^{18}O$ and ice sheet volume

The volume ($V$) of ice sheets is a key determinant in the $\delta^{18}O$ of their newly accumulating precipitation ($\delta_p$). Past work has related low $\delta_p$, reflected in the $\delta^{18}O$ of accumulated ice ($\delta_{ic}$), to intense Rayleigh distillation of atmospheric vapor through successive vapor saturation/condensation cycles with decreasing temperature along vapor pathways from evaporative sources to condensation/precipitation sites [e.g., (23, 24), review in (25)]. Rayleigh distillation intensifies with cooling during atmospheric vapor transport from oceanic evaporation sites to high latitudes, from coastal regions to cold (winter) continental interiors, and from lower to higher altitudes. This is because vapor saturation drops nonlinearly with decreasing temperature (Clausius-Clapeyron relationship), while Rayleigh distillation follows a natural logarithmic relationship to more negative values with reduction in the fraction of remaining atmospheric vapor (25). As a result, relatively low-altitude seasonal snow over ground or small snow patches has much less negative $\delta_p$ than snow falling at the center of a large, high-altitude continental ice sheet. While this is an idealized representation and there are additional controlling factors (26), we address this both here with a sensitivity test (below) and in our final outlook for further method development.

Seasonal snow $\delta_p$ typically reaches −10 to −20‰ [e.g., (27–29)]. Values become more negative with distance from the coast and with increasing altitude; for example, a trans-Alaskan continental gradient exists of −8.3‰/1000 km from the coast, with a superimposed orographic $\delta^{18}O$ decrease of −6.8‰/km of altitude (28). As regional ice-albedo effects combine with high altitude, intense cooling in cold mountain regions with permanent ice (glaciers and ice caps) cause $\delta_p$ to typically reach −25‰ or even −30‰, with lowest values recorded in winter [e.g., (30–33)]. Hence, a sharp initial $\delta_p$ drop is apparent between initial snow and sustained precipitation over nascent high-altitude, permanent ice bodies. Eventually, $\delta_p$ can reach −60‰ or lower in the coldest, most isolated locations of the intensely cold hearts of major, kilometers-high continental ice sheets, such as the Antarctic interior (34). Rayleigh distillation between evaporative sources and precipitation sites was less extreme for the North American Laurentide Ice Sheet (LIS) and the Eurasian Ice Sheet (EIS) during ice ages because of their positions in lower (“warmer”) latitudes closer to relatively warm oceanic moisture sources. For example, five isotope-enabled global climate circulation models indicate that the LIS reached a minimum LGM $\delta_p$ of about −38‰ (35).

We represent $\delta_p$ changes over ice sheets as a generic function of $V$, starting with an initial snow $\delta_p = -15/0$‰ that drops rapidly with initial ice sheet volume buildup, followed by an exponential change with $V$ toward very negative values over large ice sheets (Methods, Eq. 1, and fig. S1). We use two sets of equation constants to account for more intense Rayleigh distillation over high-latitude “cold” ice sheets than over lower-latitude warmer ice sheets. In our analytical assessment, sensitivity test 1 uses a different relationship that is simply linear (Methods and Eq. 2)—albeit physically implausible—to evaluate the importance of the selected function shape for the conclusions, given that our idealized representation may overlook additional controls on $\delta_p$ (26).

Analytical assessment

We use simple growth descriptions for the AIS as a single entity that represents both the East and West AIS, Greenland Ice Sheet (GrIS), EIS, and LIS, where the latter refers collectively to the LIS and Canadian Ice Sheet. $V_{AIS}$ today is 57.8 m sea level equivalent (msea) and $V_{GrIS}$ is 7.3 mseq (36) so that $\Delta SL = +65.1$ m in an ice-free state. In this analytical assessment, we assume that AIS and GrIS grow proportionally between $\Delta SL = +65.1$ and 0 m. Growth of continental ice sheets (i.e., glaciation) relative to the present caused negative $\Delta SL$, mainly due to growth of LIS and EIS (which do not exist today) along with slight AIS and GrIS expansions. Common approximations for the LGM are that $\Delta SL$ was roughly −125 m because LIS had reached of the order of 70 mseq and EIS about half that, while AIS grew by roughly 15 mseq [see overview in (37)]. Here, we simply assume that GrIS grew another ~5 mseq, which may be an overestimate, but this does not appreciably affect results, especially because less GrIS growth would need to be compensated by more growth in other ice sheets to keep the sea level budget closed. For different glacial maxima, size dominance may have alternated between LIS and EIS [see overview in (37)], but this does not affect results either because both are similar types of “warm” ice sheet in our calculations. We assume initially that all expansions were linearly proportional to $\Delta SL$. This gives individual ice sheet $V$ variations relative to $\Delta SL$ (Fig. 1A) and their sum $V_{tot}$ (Fig. 1C).

The schematic ice sheet $V$ variations can now be combined with the chosen relationship between $\delta_p$ and ice sheet volume (Methods and Eq. 1 or 2) to determine $\delta_p$ developments for each ice sheet (Fig. 1B). We assume that all ice sheets are equilibrated with $\delta_p$ that is, the mass-weighted mean $\delta_p$ of each ice sheet (indicated by $\delta_{ic}$) is assumed to equal $\delta_p$ (this simple instantaneous equilibration assumption is addressed in detail below in our applied assessments). Next, we calculate the mass-weighted mean global $\delta_{ice}$ value for all accumulated ice (i.e., $\delta_{ic}$; Methods and Eq. 3). Last, relative $\delta_w$ changes (i.e., $\Delta \delta_w$) are determined using an ice-to-water density ratio of 0.9, a value for global ocean volume ($V_w$) of 3700 mseq and $\Delta SL = -[(V_{AIS} - 57.8) + (V_{GrIS} - 7.3) + V_{LIS} + V_{EIS}]$ (Methods and Eq. 4). In sensitivity test 2, the AIS is set to grow nonlinearly during glaciation ($\Delta SL < 0$ m), giving it a somewhat increased expansion rate during final glaciation stages. This test investigates the possible influences of a hypothetically enhanced AIS contribution to a marked final sea level drop (38) that coincided roughly with AIS expansion to its maximum LGM extent (39); the overall sea level budget is kept unchanged by setting LIS to grow nonlinearly in the opposite sense (Fig. 1A). We find (below) that this nonlinearity is too small to significantly affect the outcome of our analysis and the same holds if we invert the nonlinearity between AIS and LIS [the condition suggested in (38)].

The main case and both sensitivity tests reveal pronounced nonlinearity between $\Delta SL$ and $\delta_{ic}$* and, thus, between $\Delta SL$ and $\Delta \delta_w$ (Fig. 1, C and D). Similar $\Delta \delta_w$:$\Delta SL$ relationships for all three cases indicate that the pattern is robust with respect to the choice of $V$: $\delta_p$ relationship, details of ice sheet growth histories, or both (Fig. 1D). This is because the nonlinearity arises principally from addition of two much less negative ice sheets at $\Delta SL < 0$ (which increases $\delta_{ic}$*) to the virtually fully formed and isotopically very negative AIS. Thus, the nonlinearity is a robust, unavoidable consequence of different glaciation thresholds for different ice sheets at different latitudes. Even if the same $\delta_p$ function were used for LIS and EIS as for AIS and GrIS, considerable nonlinearity would remain because LIS and EIS in their initial stages (up to 50 mseq) have more positive $\delta_p$ than AIS.

Applied assessments

Here, we use published sea level records to determine mutually consistent variations in ice sheet volumes, $\delta_{ic}$, $T_w$, and $\delta_w$. A flow diagram
for the entire method is presented in fig. S2. From this workflow, the initial step of regression-based conversion of $\delta_c$ changes into $\Delta S_L$ is omitted when results are produced for published sea level records for comparison with $\delta_c$-based results. Our method uses a straightforward model for growth of generic ice sheets with circular plano-convex lens shapes, axially symmetric parabolic profiles (40), and a constant height:radius aspect ratio $\varepsilon = 2 \times 10^{-3}$ (roughly the mean aspect ratio of the modern AIS). Unless specified otherwise, global ice-volume variations are determined in terms of sea level–equivalent changes, accounting for the density difference between ice and water. Model ice sheets retain their shape characteristics throughout, while sea level change between individual time steps is used to determine global net ice-volume growth or loss, which is partitioned over the different ice sheets as described in Methods (Eqs. 5 to 11). Thermo-steric sea level changes are not considered.

The model contains one large and one smaller ice sheet that both disappear at the present-day sea level; these approximate LIS and EIS. It is irrelevant which of the two grows larger and which stays smaller, as all model ice sheets are generic, without geographic attribution, and both LIS and EIS approximations are taken to be warmer ice sheets for Rayleigh distillation (Methods and Eq. 1). The model approximates GrIS with $V_{GrIS} = 7.3 \text{ m}_{\text{eq}}$ at $\Delta S_L = 0$ m and AIS with $V_{AIS} = 57.8 \text{ m}_{\text{eq}}$ at $\Delta S_L = 0$ m (36). Ice-volume changes are driven in accordance with the applied sea level records and are attributed to the various ice sheets as discussed in Methods (Eqs. 5 to 11). $\delta_c$ changes over the ice sheets are calculated as in the main case of the analytical assessment above, using Methods and Eq. 1.

At this stage, we abandon the simplistic assumption that $\bar{\delta}_{ice}$ for each ice sheet is instantaneously similar to $\delta_p$ and instead calculate its time evolution (Methods and Eqs. 12 to 14). The initial assumption needed to be abandoned because $\delta_p$ instantaneously tracks ice sheet dimension changes, whereas it takes time for $\bar{\delta}_{ice}$ to equilibrate with changing conditions. Internal ice sheet properties reflect surface accumulation, downward and outward ice flow (ice dynamics), and ice loss through melting and calving. Thus, ice sheets contain contiguous ancient ice sequences within their interiors; in GrIS, this dates back to at least 125 ka ago (41) and in AIS to at least 800 ka ago (42, 43). AIS has been at (almost) its modern size long enough that even ancient ice formed from precipitation with $\delta_p$ close to present values, but in short-lived ice sheets, especially LIS and EIS, a noticeable lag is expected between ice-volume buildup and full $\bar{\delta}_{ice}$ equilibration to the volume change. This lag is roughly 17 to 20 ka/m seq, based on modern maximum ice ages and the GrIS and AIS sizes, but may be ice sheet size and, thus, time dependent. Given this uncertainty, we refrain from applying a standard lag function and instead resolve $\bar{\delta}_{ice}$ evolution directly per time step (Methods and Eq. 14).
Here, $\bar{\delta}_{icr}^j$ is calculated at time step $j$ using ice sheet volume and $\bar{\delta}_{icr}^{j-1}$ of preceding time step $j - 1$, and both gross mass accumulation with more negative $\delta_w$ (determined from the $\delta_r$ relationship above) and gross mass loss with $\delta_w$ that is (before achieving isotopic equilibrium) less negative between time steps $j - 1$ and $j$. We use $\delta_{icr}^{j-1}$ for the latter term, along with a gross mass loss term that is determined by gross accumulation minus net accumulation/loss (where loss is negative). Net accumulation/loss is obtained from sea level variations, and gross accumulation ($a_{grs}$) is calculated in an ice sheet surface-area-dependent manner directly proportional to modern global annual $a_{grs}$ of $-0.008$ m seq $^{-1}$ (following rates in (44, 45)) for a global ice volume of 65.1 m seq.

Annual $a_{grs}$ is $\sim 0.01\%$ relative to ice volume and is set to change proportionally with ice sheet size, so the model ice sheets have isotopic equilibrium time scales of the order of 10$^4$ years. This time scale is especially relevant for LIS and EIS, which existed only during glacial stages with life spans of order 10$^4$ to at most 10$^5$ years, and that typically reached maximum sizes only millennia before rapid glacial terminations. As a result, LIS and EIS were continually playing catch-up in terms of isotopic equilibrium to their marked size changes. In the Late Pleistocene, the more permanent AIS (and, to a lesser extent, also GrIS) largely maintained conditions close to isotopic steady state. This did not hold during the Pliocene and older times, when considerable sea level changes well above 0 m imply major

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in our analytical assessment (Fig. 1D). This substantially affects the way Δδ\(_w\) is reconstructed through time relative to traditional approaches that assume a linear Δδ\(_w\)-ΔSL relationship (Fig. S7). By resolving the mass-weighted mean global δ\(_{ice}\) value for all accumulated ice (δ\(_{ice}^*\)) at every time step in accordance with evolving characteristics of the ice sheets, our reconstruction allows Δδ\(_w\) to shift seamlessly among lines obtained from Δδ\(_w\)-ΔSL relationships based on different δ\(_{ice}\) values. This removes the need to assume which δ\(_{ice}^*\) value applies at specific times and how and when shifts between these values occurred. Our method thus provides a major step toward the necessity—specified in the most recent long-term sea level assessment—that “changes in the […] Δδ\(_w\)-ΔSL […] calibration with evolution of ice-sheet size should be modeled” (3). We note that the shape of the Δδ\(_w\)-ΔSL nonlinearity identified from our modeling remains robust even when combining data points from model runs for all ΔSL records, with all of their different underpinning assumptions and uncertainties (Fig. 6D). This indicates that the shape is largely insensitive to the distinct V\(_\delta\)\(_{ice}\) hysteresis for individual ice sheets, which was omitted from our analytical assessment. There are impacts only on the overall slope of the lines and data distributions around the regression. However, such robustness does not imply that the model approximates reality well; it merely reflects internal model consistency. Validation against
Fig. 3. Comparison of our $\Delta_{SL}$, $T_w$, and $\delta_w$ with other observations. (A) Sea level change from the sea level stack [(12), blue]; from the Red Sea record, $\times$ 1.1 to approximate global mean sea level variations [(11), orange]; from model-based $\delta_c$ deconvolution [(4), green]; and from lag-optimized, quadratic regression between (13) and (12) (fig. S3) forced to peak at $\Delta_{SL} = 65.1$ m (black), and from the regression’s upper 95% confidence limit (pink). Cyan boxes indicate maximum amplitude envelopes of New Zealand relative sea level fluctuations (S2), vertically positioned for comparison. Green dots are western Mediterranean benchmarks (49, 50), with highlighted Middle Pliocene range (yellow box). (B) $T_w$ changes derived using $\delta_c$ and $\delta_w$, where $\delta_w$ is calculated from $\Delta_{SL}$ [this study; black and pink using $\Delta_{SL}$ in (A); orange using $\Delta_{SL}$ from the unconstrained regression; fig. S3]. (C) to (E) all on the age scale of (E). (C) Last 1.6 Ma of (B) versus Antarctic (EDC) temperature on its independent chronology (54). Red dot represents a noble gas–based estimate of LGM global ocean cooling, plotted against the $T_w$ axis (57). (D) As (C) but versus Mg/Ca-based $T_w$ [(16), blue, individual data and 4-ka Gaussian smoothing]. (E) As (D), but for $\delta_w$. We show ODP Site 1123 (16) alone at 0 to 0.35 and 1.45 to 1.6 Ma ago and a three-record $\delta_w$ stack including ODP 1123 for 0.35 to 1.45 Ma ago, with 1× and 2× bootstrap errors (58). Red dot indicates LGM sediment pore-water–based $\delta_w$ (20–22).
The other assumption concerns the chosen regression-based derivation of $\Delta_{SL}$ from $\delta_c$ (fig. S3); and previous model-based $\delta_c$ deconvolution ([14], green). (B) Our results for AIS (black, pink, and orange as above) and northern ice volume (LIS + EIS + GrIS; red, only for the main regression for clarity). (C) Model-derived $\delta_w$ based on our $\Delta_{SL}$ in (A) (black, pink, and orange) versus $\delta_c$ ([14], lilac). (D) $T_w$ changes based on $\delta_c$ residuals in (C) (black, pink, and orange). Regular italic numbers indicate positive validations: 1, western Mediterranean benchmarks (49, 50); 2, best- and maximum-amplitude sea level ranges (53); 3a and 3b, first major iceberg calving in Nordic Seas (59) and LIS calving onset (60), respectively; 4, AIS-volume variability range (61); 5, onset partial/ephemeral northern ice (1); 6, end of last intermittently ice-free period (3); 7, AIS glaciation onset (1, 3, 4, 64, 66–70); 8, MMCT $\delta_w$ change of 0.35 ± 0.12‰ (62); 9a and 9b, loess fit through Mg/Ca $T_w$ with 95% confidence bounds; at 34.5 to 37.5 Ma ago, data were questioned (67). Roman numerals indicate discrepancies: i, amplitudes of sea level change in North West Australia (63); ii, EOT sea level drop of 70 to 80 m (64) versus ~30 m in this study (Discussion); iii, 2.5°C EOT cooling (66) versus 3.5°C cooling here (Discussion); iv, MMCT cooling of 1.5° ± 0.5°C (62) versus 2.5° ± 0.5°C here.

**DISCUSSION**

**Validation of results**

Clear linear progression in our method illustrates an absence of circularities; fundamental assumptions exist only at two stages (fig. S2). One assumption concerns regression-based derivation of $\Delta_{SL}$ from $\delta_c$ (fig. S3). Yet, sensitivity tests with realistically different regressions (fig. S3) indicate that our $\delta_c$-based $\Delta_{SL}$ record is robust within about 10 m (total range; Figs. 3A and 4A). Diverse validations of the mutually consistent $\Delta_{SL}$, $\delta_{ice}$, $\delta_{ev}$, and $T_w$ records support this robustness (below). The other assumption concerns the chosen $\delta_w$ versus ice-volume relationships. Sensitivity tests with alternative linear relationships indicate that our conclusions are robust regardless of the relationship shape (Fig. 1, C and D).

Below, we compare our reconstructions with independent observations. A first validation against observations is that even at the largest LIS volumes, LIS $\delta_{ice}$ barely reached −35‰ (Fig. 5), while values integrated over 100-ka glacial cycles within the past 500 ka are consistently between about −25 and −35‰ (fig. S5). Groundwaters in North America that reflect subglacial or preglacial recharge typically reach −26‰, with a possible −28‰ end member, while ground ice in Yukon and the Barnes Ice Cap, Nunavut (both in Canada) has values of −29 to −35‰ (35). These values agree well with our LIS $\delta_{ice}$ simulations.

Next, we observe reasonable agreement between our new $\delta_c$ regression-based $\Delta_{SL}$ and independent multiproxy sea level reconstructions for interglacial Marine Isotope Stages (MIS) 5e and 11 (48), the Red Sea sea level record (11), and Middle Pliocene sea level benchmarks (Figs. 2, A and C, and 3A) (49, 50). The $\delta_c$ records that underpin our regression-based $\Delta_{SL}$ do not fully resolve millennial-scale features, so our new ($\delta_c$ regression-based) $\Delta_{SL}$ records are likely to miss short-term extremes of several-meters amplitude that can be important in interglacials like MIS 5e and 11 [e.g., (51)]. Compared with the Red Sea record (11), our new $\Delta_{SL}$ records agree much better with the multiproxy assessment for MIS 11 (48), a further indication independent observations is needed to assess whether the calculated records are realistic (Discussion and Figs. 3, A, C, and D, and 4, A to D).
that the Red Sea record may underestimate sea level at that time. All Middle Pliocene sea level benchmarks shown (49, 50) agree with our \( \Delta S \), although the youngest does so only at its extreme age uncertainty. Our \( \Delta S \) may offer a means to refine age estimates for benchmarks with relatively large dating uncertainties at times of large \( \Delta S \) variations.

Agreement with the 800-ka multiproxy sea level stack (Figs. 2B and 3A) (12) cannot be used for validation because it underpins the regression that provides our \( \Delta S \) (Methods and fig. S3). In addition, we note that a previous \( \delta_n \)-based \( \Delta S \) deconvolution (4) also agrees well with our \( \Delta S \) between 0 and 3 Ma ago, albeit at somewhat muted amplitudes with glacials only reaching −100 m (Fig. 3A). Between 3 and 5.3 Ma ago (Fig. 3A), and extending to 13 Ma ago (Fig. 4A), however, that record (4) has much less \( \Delta S \) variability than our results, which requires comparison with additional independent evidence.

In the 3.3– to 3.0– and 2.9– to 2.65–Ma ago intervals, amplitude variations in both our \( \Delta S \) record and the previous \( \delta_n \)-based record (4) agree with maximum relative sea level amplitude ranges of up to 27 and 35 m in New Zealand, respectively (Fig. 3A) (52). However, Middle Pliocene sea level benchmarks (Figs. 3A and 4A) (49, 50) and Miocene best and maximum sea level amplitude estimates from New Jersey and the Delaware Coastal Plain (Fig. 4A) (53) indicate that the previous \( \delta_n \)-based \( \Delta S \) deconvolution (4) is anomalously flat and low between 3 and 13 Ma ago and that its amplitude variations between 13 and 22 Ma ago exceed independently established amplitude constraints (53). In contrast, our \( \Delta S \) record agrees well with both these independent lines of evidence (Figs. 3A and 4A). The difference may arise from the fact that our method operates in the same way across all time frames, based on generic warm and cold ice sheets. In contrast, that of de Boer et al. (4) was primarily set up to account for LIS and EIS variations, driven by mid-latitude to subpolar northern hemisphere temperatures inferred from \( \delta_n \) and used a tuning factor for AIS so that a large \( V_{AIS} \) response was obtained across the EOT. Possibly, the tuning factor used by de Boer et al. (4) is too strong, resulting in overly sensitive \( V_{AIS} \) responses through the Oligocene-Miocene, achieving a “full” AIS state that allowed no further expansions, which results in a flat \( \Delta S \) curve until northern hemisphere glaciation began. Alternatively, the assumption that mid-latitude to subpolar northern hemisphere temperatures could be inferred from \( \delta_n \) (4) may be incorrect.

We favor the latter explanation, on the basis of comparison of our calculated \( T_w \) variations with independently determined Antarctic temperature variations at European Project for Ice Coring in Antarctica (EPICA) Dome C (EDC) (Fig. 3C) (54). This reveals excellent amplitude structure agreement between our \( T_w \) and \( T_{EDC} \); scaled as \( \Delta T_w \approx 0.25 \Delta T_{EDC} \). Offsets in this comparison only concern the chronologies between the two records, between 0.4 and 0.7 Ma ago. We infer that global mean deep-ocean temperature has consistently been set by temperature fluctuations in high southern latitudes rather than in the mid-latitudes to subpolar northern hemisphere as was assumed previously (4). This is an important observation for understanding ocean-atmosphere feedbacks on these time scales. In addition, this observation may help in developing chronologies for old (partially deformed) ice-core sections and blue-ice sections, which are difficult to date because they lack stratigraphic continuity [e.g., (55, 56)], by adding independent age controls from \( T_w \) matching.

We also compare our \( T_w \) record with a 1.6-Ma Mg/Ca reconstruction for Ocean Drilling Program (ODP) Site 1123 from Chatham Rise, east of New Zealand (3290 m in water depth) (16). Reasonable agreement exists over the interval 0 to 1.2 Ma ago (Fig. 3C). Offsets occur at 1.3 to 1.45 Ma ago, where the Mg/Ca record suggests \( T_w \) drops to −4°C relative to present. Given a modern in situ bottom-water temperature of just above 1°C at Site 1123 (15) and adjustment to potential temperature of less than −0.2°C, a −4°C \( T_w \) drop at Site 1123 would suggest deep-water formation at surface temperatures just under −3°C. This is considerably below the seawater freezing temperature of about −1.8°C, which suggests that there is an issue with the
Mg/Ca record in this interval. Additional temperature validation is provided by ice-core noble gas analyses, which indicate an LGM global mean ocean cooling of $-2.57^\circ \pm 0.3^\circ$C (57), which agrees with our LGM global deep–sea cooling of $-2.5^\circ \pm 0.3^\circ$C (Fig. 3, C and D). However, Mg/Ca temperatures at Site 1123 suggest LGM cooling of only $-1^\circ$ to $-1.5^\circ$C (Fig. 3D).

The ODP Site 1123 Mg/Ca-based $T_w$ has been used with coregistered $\delta_c$ to calculate $\delta_{w}$ (16). These data were later used as part of a three-record stack for the interval 0.35 to 1.45 Ma ago (58). These $\delta_{w}$ records compare reasonably with our $\delta_c$ results, especially in glacial intervals (Fig. 3E). This is a valid comparison to add to the $T_w$ comparison because observational studies primarily resolve $T_w$ and secondarily infer $\delta_c$, while the opposite applies to the present study (fig. S2). Moreover, the studies use independent $\delta_c$ records. Additional $\delta_{w}$ validation comes from marine-sediment pore-water $\delta_{w}$ measurements. These reveal an LGM–modern gradient of about $1 \pm 0.1\%$ (20–22), while we find 1.1 to 1.4% across the different runs; namely, 1.4% in the Red Sea–based example (8–11), 1.4% in the example for the statistical multiproxy sea level stack (12), 1.2% in the Mediterranean-based example (46), 1.1% based on our $\delta_c$-to-$\Delta SL$ conversion using the benthic stack (13), and 1.2% based on our $\delta_c$-$\Delta SL$ conversion using the benthic megasplice (14). The difference likely falls within assumptions and uncertainties in both methods, especially given that reevaluation of the pore-water method has indicated larger uncertainties (22) than considered originally (20, 21).

In Fig. 4B, additional validation criteria labeled 3a, 3b, and 5 indicate broad validation of key northern hemisphere (LIS + EIS + GrIS) glaciation stages (red) with ice-raftered debris (IRD) records from the Nordic Seas and wider North Atlantic (59, 60) and previous inferences of an onset of ephemeral or partial northern hemisphere ice development, respectively (1). Criterion 4 corroborates $V_{AIS}$ fluctuations (61), while 6 marks agreement about the end of the most recent intermitently ice-free period (3); we note that 4 and 6 are not fully independent because they partly rely on $\delta_c$ information (3, 61). AIS glaciation onset at the EOT (criterion 7) marks a well-studied greenhouse–icehouse transition that provides binary validation (ice versus no ice) although some suggest somewhat earlier major glaciation from ~36 Ma ago (47). In addition, a $\delta_{w}$ change of 0.35 ± 0.12% has been inferred across the Middle Miocene Climate Transition (MMCT) (62), which agrees well with our results (criterion 8). Last, a Mg/Ca-based $T_w$ compilation over a broad Eocene–Oligocene interval indicates excellent agreement with our result between 37.5 and 40 Ma ago (criterion 9a) and reasonable agreement for the lowest $T_w$ values around the EOT (criterion 9b; allowing for potential chronological offsets). In the Early and Middle Oligocene, values in the Mg/Ca compilation generally fall $\sim 1^\circ$C above our estimates, albeit just within uncertainties. Between 31 and 28 Ma ago and after 26 Ma ago, this offset is reduced noticeably so that values agree well within uncertainties.

Use of multiparameter approaches to validate our reconstructions goes a long way to excluding potential diagenetic impacts on
benthic $\delta_{\text{w}}$. Following our method, diagenetic alteration of $\delta_{\text{w}}$ to more positive (negative) values would cause spuriously low (high) $\Delta \delta_{\text{w}}$ anomalies that would imply larger (smaller) ice volume. This, in turn, would drive positive (negative) $\Delta \delta_{\text{w}}$ anomalies, whose subtraction from more positive (negative) $\delta_{\text{w}}$ would have a canceling effect, resulting in reconstruction of minor to negligible $T_{\text{w}}$ changes. The only interval with limited (though still substantial) $T_{\text{w}}$ variability is between 13 and 5 Ma ago (Fig. 4D), but $\Delta \delta_{\text{SL}}$, AIS volume, and onset of northern hemisphere ice presence from our method remain well validated (criteria 1, 2, 4, and 5; Fig. 4, A and B), which indicates that low $T_{\text{w}}$ variability in this interval does not arise from diagenetic alteration of $\delta_{\text{w}}$.

There are also some discrepancies. Deep-sea cooling at the MMCT was estimated at $1.5^\circ \pm 0.5^\circ$C (62), which is $1^\circ$C less than inferred here (discrepancy iv, Fig. 4D). In addition, at around that time, large-amplitude $\Delta \delta_{\text{SL}}$ fluctuations have been inferred from stratigraphic analyses of northeastern Australian sequences (Fig. 4A, discrepancy i) (63). However, these disagree not only with our results but also with the $\Delta \delta_{\text{SL}}$ record of de Boer (4) and with criterion 2 (53). Discrepancies ii and iii are now evaluated in more detail.

Previous cumulative EOT $\Delta \delta_{\text{w}}$ estimates reach $-70$ to $-80$ m (3, 64), which greatly exceed our estimate of about $-30$ m (discrepancy ii). The $\Delta \delta_{\text{SL}}$ record of de Boer (4) indicates about $-45$ m of sea level change across the EOT (accounting for different pre-EOT values in Fig. 4A), similar to $-30$ to $-50$-m estimates of relative sea level fall from New Jersey Coastal Plain backstripping analysis (65); both are closer to our estimate than to the $-70$ to $-80$-m estimates. This EOT $\Delta \delta_{\text{SL}}$ discrepancy must be considered together with the difference between Mg/Ca-based $-2.5^\circ$C EOT cooling (66) and our inferred $-3.5^\circ$C cooling (Fig. 4D, discrepancy iii). The $1^\circ$C difference scales to $0.25\%$ in $\delta^{18}$O. Given that the AIS at the time was only beginning to grow, its $\delta_{\text{w}}$ ice would not have been very negative. This is evident in Fig. 4 where $\Delta \delta_{\text{w}} = 0.25\%$ for $\Delta \delta_{\text{SL}} = -30$ m at the EOT. If the $-2.5^\circ$C EOT cooling (66) is correct—which remains to be settled in view of agreement between our results and both high Eocene and lowest EOT $T_{\text{w}}$ (criteria 9a and 9b (67))—then the implied $0.25\%$ of “missing” $\delta^{18}$O would correspond to another $-30$ m of $\Delta \delta_{\text{SL}}$, giving a total of $-60$ m that is similar to the modern $V_{\text{AIS}}$. Moreover, AIS may, at the time, have behaved more like a warm ice sheet (similar to the Pleistocene LIS and EIS), with early-growth $\delta_{\text{w}}$ only around $-25\%$ and full-size $\delta_{\text{w}}$ only around $-40\%$ (68). This would make the $\Delta \delta_{\text{SL}}$ of $-60$ m an underestimate and bring potential values closer to previous $-70$ to $-80$-m estimates (64). If this were the case, then a fundamental shift is implied in the global $\Delta \delta_{\text{w}}$/$\Delta \delta_{\text{SL}}$ relationship at some stage between 34 and 22 Ma ago. This might be related to a Rayleigh distillation change over the AIS from warm ice sheet behavior to cold ice sheet behavior so that the $\Delta \delta_{\text{w}}$/$\Delta \delta_{\text{SL}}$ relationship in Fig. 6 would apply only to later times ($\geq 22$ Ma ago). It is also possible that Mg/Ca-based $T_{\text{w}}$ estimates across the EOT are problematic; for example, because of ocean Mg concentration changes and carbonate saturation changes (69). Western Ross Sea sedimentary cycles suggest that sea level oscillations only reached $\sim 20$-m amplitudes across the EOT, similar to our $\Delta \delta_{\text{SL}}$ (70). Given literature disagreement about sea level and $T_{\text{w}}$ changes across the EOT, it is not yet possible to either accept or reject our $\Delta \delta_{\text{SL}}$ results for that event.

Important evidence that is used to argue for large EOT $V_{\text{AIS}}$ buildup (or starting 36 Ma ago) concerns an onset of IRD deposition in marine sediments around Antarctica (47, 71), which indicates that ice large enough to support calving had (in places) made it to the coast. On the basis of the size of the continent and the general parabolic cross-sectional profile of ice sheets, this suggests, at first glance, that an ice sheet of roughly modern proportions had built up. However, similar arguments have been rejected for the Early Pleistocene LIS, on the basis of the fact that ice sheets on slippery regolith would remain much lower (because of reduced bed friction) and less voluminous than ice sheets positioned on bedrock after earlier glaciations removed most regolith (72–74). We propose that consideration must be given to the possibility that the EOT AIS may have been less like the present-day AIS and more like an Early Pleistocene low-slung, slippery LIS (72–75), which might explain IRD deposition even with $V_{\text{AIS}}$ only about half its present-day value. Our results suggest that a consistently large-volume/high AIS existed only since about 13 Ma ago (Figs. 4D and 7A).

**Wider implications**

In our reconstructions, before deep northern hemisphere glaciations started 3 to 2.5 Ma ago, mean global deep-sea temperatures remained a few degrees above freezing, which indicates substantial deep-water supply from nonfreezing regions (Figs. 3, C and D, 4D, and 7D). Relative to the Late Pleistocene mean, $T_{\text{w}}$ was $2^\circ$ to $3^\circ$C higher at 3 to 13 Ma ago, $3^\circ$ to $5^\circ$C higher at 15 to 33.5 Ma ago, and $7^\circ$ to $9^\circ$C higher at 34.5 to 40 Ma ago, marking distinct quasi-stable states that remain evident when considering variability around those means (pale red, orange, and blue rectangles in Fig. 7D). These ranges and an absence of outlier patterns in our mutually consistent $\Delta \delta_{\text{w}}$, $\Delta \delta_{\text{SL}}$, and $T_{\text{w}}$ reconstructions (Supplementary Materials) indicate that no major, warm, saline deep-water contributions existed throughout the time scales investigated and that high-latitude deep-water formation processes were dominant.

The existence of quasi-stable $T_{\text{w}}$ states is consistent with similar reconstructed ice volume behavior, which suggest virtually no ice before 34 Ma ago, small-to-mid size Antarctic ice at 15 to 33.5 Ma ago, and small-to-full size Antarctic ice with small northern hemisphere ice at 3 to 13 Ma ago (pale red, orange, and blue rectangles in Fig. 7A). These quasi-stable periods are separated by major transitions: the EOT, the end of the Middle Miocene Climatic Optimum (MMCO) and the Plio-Pleistocene transition to extensive bipolar glaciation (yellow bars in Fig. 7D). This resembles critical transition behavior in complex dynamical systems, where variance and autoregression increases occur before transitions (76). Variance analysis (Methods and Fig. 7D) reveals that $\Delta T_{\text{w}}$ variance increased well before each transition—at $-38.2$ Ma ago (previous work suggests $\sim 37$ Ma ago (76) and longer time series need to be considered to more precisely identify this onset in our method), 17.2 or even $24.8$ Ma ago, and 6 Ma ago (dashed purple lines and magenta arrows in Fig. 7D)—and culminated during the transitions. Given that this is consistent with expectations for critical transitions in complex dynamical systems (76), we explore the potential drivers behind this behavior.

Earth’s climate system has diverse feedbacks that operate over different time scales, including short-term variability/”noise” (77, 78). Insolation, a key external climate forcing, is governed by astronomical cycles that lack multimillion-year trends (79) but that can precondition the system for major state changes at certain times (80, 81). Ice-albedo feedbacks operate over time scales that are too short to cause multimillion-year secular changes, but they will amplify any initial change as part of a feedback cascade, aiding transitions in regimes with multiple stable states. Plate tectonics is a suitable candidate for causing multimillion-year secular atmospheric $CO_2$ changes and
hence climate, through subtle but long-lasting modifications of the balance between volcanic CO₂ outgassing and weathering-related CO₂ sequestration (82). Note that this concerns long-term exogenic CO₂ trends related to plate tectonics rather than shorter endogenic CO₂ variations related to carbon-cycle feedbacks. CO₂ levels changed from ~1000 parts per million (ppm) at around 40 Ma ago to ~180 ppm during Late Pleistocene glacial stages and ~280 ppm during interglacials (83–85), which spans roughly two 2× CO₂ changes. Given that radiative forcing change for each 2× CO₂ change amounts to almost 4 Wm⁻² (86), the CO₂ reduction since 40 Ma ago amounts to roughly 8 Wm⁻² reduction in radiative climate forcing. This represents 5° to 9°C cooling based on a 5 to 95% range for equilibrium climate sensitivity (86), in agreement with the 6°C cooling in our reconstructed T_w (Fig. 7D).

While major climate transitions separating relatively stable states are known from benthic δ₁₈O compilations (1–3, 14), we have deconvolved these records into the contributing ice-volume and deep-sea temperature components. Adjusting more rapidly than ice volume,
Δw represents a more direct reflection of global climate variability, while ice volume provides a longer time-integrated view of the changing climate state. We propose that long periods of increasing Δw variance (“flickering”) before the detected state transitions (Fig. 7) represent periods when gradually declining CO₂ reached levels where multiple stable climate states exist (78, 87), the existing warmer state that supports less ice and a colder state that supports more ice. Stochastic shocks play an important role in triggering transitions before bifurcation points (87). We propose that cold- or warm-based organization of climate system noise during cold or warm orbital extremes provided these shocks [akin to stochastic or nonlinear resonances; (78, 88)], similar to model-based findings for the EOT (81).

We emphasize that the entire past 40 Ma investigated here is dominated by cooling; apart from the MMCO onset, the interval contains no significant long-term warming. Our evidence, therefore, is largely indicative of transitions in the cooling direction, although inferences can also be made about potential warming transitions. Existence of long (2 Ma or more) flickering intervals preceding the detected transitions (Fig. 7) suggests that bistable regimes existed over long time periods. The eventual transition to a colder stable state with greater ice volume, therefore, likely occurred at a considerably lower CO₂ threshold value than the return to a warmer stable state with smaller ice volume because of stabilizing feedbacks associated with large ice masses (89). Similar behavior has been inferred from Antarctic ice sheet modeling experiments, where the hysteresis relative to CO₂ levels spans hundreds of parts per million (90). Using the bounds of flickering intervals in our reconstructions (Fig. 7), we infer that bistable regimes existed between at least 0 and 6 Ma ago, between 12.6 and 17.2 Ma ago, and—less well defined in our results—between 34 and 38.2 Ma ago. Precise CO₂ reconstructions for those intervals may help to quantify climate state hysteresis relative to CO₂ with implications for understanding potential future climate transitions.

Application and outlook
Our mutually consistent, process-based framework for reconstructing ΔSL, VICE, δICE, T, and δw permits complete system validation using multiple parameters rather than only one or two as in traditional approaches. It also helps to identify unrealistic single-parameter fluctuations and/or to fill in information across data gaps so that more complete records can be obtained. Last, our results offer continuous, quantitative context to data from analytical methods such as sea level benchmarking or novel T, assessment (e.g., clumped isotopes) with low temporal resolution and/or age uncertainties that preclude precise attribution to specific climate cycles.

Our mutually consistent ΔSL, δICE, T, and δw records are well validated over at least the past 22 Ma, which comprise the full −130- to +65.1-m range of sea level variation. Validation is not (yet) as convincing for earlier times, so we infer that our ΔSL record (black in Fig. 4A) offers a well-validated sea level record only for the past 22 Ma; this record is obtained using ΔSL = −8.4 δc² + 6.8 δc + 65.1, where δc is normalized to the most recent value of 3.23‰ in the Lisiecki and Raymo benthic δc stack (13). The two regression extremes (pink and orange in Fig. 4A) provide realistic total range bounds (for equations, see fig. S3). Before 22 Ma ago, our ΔSL results require further validation. Note that our assessment concerns orbital and longer time scale sea level variability and that no conclusions should be drawn with respect to millennial sea level variability without further extensive testing.

We find close amplitude structure agreement between our calculated Δw variations and independently determined Antarctic temperature variations at EDC (54), scaled as ΔT = 0.25 ΔTEDC (Fig. 3C). This stunning independent agreement provides strong validation of our approach. We infer that global mean deep-ocean temperature has consistently been set by high southern-latitude temperature fluctuations.

Diverse positive validations also indicate that our inferred Δδw-ΔSL relationships offer reasonable approximations of reality. Hence, we propose that ice-volume/sea level reconstruction based on δw records from T-corrected δc data, or ice-volume corrections of δw records to identify regional hydrological influences, should no longer use linear transformations. Instead, we propose that improved results will be obtained using the nonlinear relationship found here through all instances (N = 54,644). The optimum regression fit to capture the shape of the data cloud is Δδw = 9.6 × 10⁻¹¹ ΔSL⁵ + 1.9 × 10⁻⁷ ΔSL⁴ + 2.5 × 10⁻⁵ ΔSL³ − 1 × 10⁻⁴ ΔSL² − 0.015 ΔSL − 0.133. While polynomial orders higher than cubic do not statistically significantly improve the relationship, the fifth-order fit visually best captures the non-linearity that is theoretically evident, with the least edge effects, and offers the best conversion equation.

Further research is needed to hone the approach presented here. A particularly promising route is to develop VICE-δw transforms using geographically specific, less idealized, δ¹⁸O-enabled ice sheet models that account for additional controls on δw (26), which can then be applied to the various ice sheet growth histories to determine more realistic δw histories for each ice sheet. We advocate use of these transforms because integrating fully coupled δ¹⁸O-enabled ice sheet models over many millions of years will be computationally challenging. Making the suggested transform functions will provide a sound middle way between that and our use of idealized scenarios. In addition, there is a need for further paleoclimate and paleoceanographic benchmarks; for example, to resolve the origins of changes around the EOT and to provide further Oligocene-Miocene data for improved validation. Last, more detailed understanding of Mg/Ca changes over the past 40 Ma would improve deep-sea temperature reconstructions and facilitate more advanced validation of the approach presented here.

Our data-driven reconstructions suggest that climate-system changes during the descent from Eocene greenhouse to Late Pleistocene ice-house conditions were not gradual but comprised sharp threshold-like adjustments (critical transitions) between quasi-stable states. By analogy, it seems important for future climate change projections to allow for historically unprecedented adjustments, such as deep-water formation shifts, abrupt (partial) ice sheet collapse, or other cascading feedbacks.

METHODS

ΔSL from δw records
Calculation of sea level change (ΔSL) from benthic δc records (13, 14) was undertaken using second-order polynomial regressions based on the lag-optimized comparison between δc and a statistical sea level assessment based on a variety of input data (12). The regressions and their equations are given in fig. S3. Uncertainty bounds account for both the regression uncertainty and additional data uncertainties in both X and Y directions. For the latter, we used normal distributions with 1σ = 0.1‰ for δc and 1σ = 2 m for ΔSL.
Isotope fractionation over the ice sheets

\( \delta_p \) is determined in the main case of the analytical assessment using Eq. 1 and for sensitivity test 1 using Eq. 2. Constants were selected to achieve a reasonable visual fit through modern ice sheet values (fig. S1). For the main case of our analytical assessment and the subsequent applied assessments, we use a heuristically fitted function representative of the described trends that start with an initial snow \( \delta_p = -15\%o \) that drops rapidly with initial ice sheet volume buildup, followed by an exponential change with \( V \) toward very negative values over large-scale ice sheets (fig. S1). To this end, we use an equation of the form

\[
\delta_p = \alpha e^{\frac{(0.9V-10)}{43}} (100 - V) -(C + 15)
\]

Here, \( C \) is a constant that is adjusted along with amplitude parameter \( \alpha \) so that initial precipitation (for ice sheet volume \( V = 0 \ m_{eq} \)) is precisely \(-15\%o\) in all cases. Extreme Rayleigh distillation at the isolated, high-latitude AIS causes very negative \( \delta_p \) (42), which, in Eq. 1, requires \( \alpha = 1 \) (with \( C = 102.55 \)). GrIS is set to the same parameters. In contrast, a lower \( \alpha \) is needed for LIS and EIS to account for less negative \( \delta_p \) in response to more limited Rayleigh distillation between evaporative sources and precipitation sites, related to their positions in lower latitudes closer to relatively warm oceanic moisture sources. An \( \alpha = 0.55 \) value (with \( C = 56.4 \)) is used for LIS to reach a minimum LGM \( \delta_p \) of roughly \(-35\%o\), as suggested by five isotope-enabled global climate circulation models (35). EIS is set to the same parameters.

Sensitivity test 1 instead uses linear \( \delta_p \) changes (Eq. 2), although this is physically implausible. For AIS and GrIS, the slope-defining constants \( J \) and \( K \) are set to \(-60 \) and \( 66.3 \), respectively, and for LIS and EIS, these are set to \(-25 \) and \( 26 \), respectively, to get reasonable agreement with \( \delta_p \) observations (fig. S1)

\[
\delta_p = \sqrt{\frac{V}{K}} - 15
\]

In sensitivity test 2, AIS glaciation is set to an arbitrary quadratic function so that AIS grows less rapidly during initial glaciation stages and more rapidly during later stages. The sea level budget is closed by setting an opposite LIS-volume anomaly. This is explored to mimic the ice-to-water density ratio, \( V_{oc} \) is global ocean volume (3700 m$_{eq}$), and \( \Delta_{SL} = -(V_{AIS} - 57.8) + V_{GrIS} - 7.3 + V_{LIS} + V_{EIS} \). Note that, throughout, we consider ice-volume change to be inversely proportional to sea level change, accounting for density differences between water and ice. This is a simplification because some ice sheet portions may displace water so that they do not contribute to sea level change, whereas they do count toward ice-volume (and thus \( \delta_p \)) change [e.g., (37)].

Individual ice sheet volume histories

In our applied assessments, \( \Delta_{SL} \) (in meters relative to present) is used to determine global net ice loss or gain per time step \( j \) (forward in time). Ice sheets are grown in relation to \( \Delta_{SL} \) using prescribed conditional functions. Note that we do not fully reduce the functions below, so it will be clearer what is calculated. AIS volume per time step is given by

\[
\begin{align*}
V_{AIS_j} &= \left\{ \begin{array}{ll}
57.8 + \frac{-\Delta_{SL_j}}{2} & \text{if } 0 < 7.3 + \frac{-\Delta_{SL_j}}{2} \leq 7.3 \\
57.8 - \Delta_{SL_j} & \text{if } 7.3 < \Delta_{SL_j} \leq 57.8 \\
0 & \text{if } 57.8 < \Delta_{SL_j}
\end{array} \right.
\end{align*}
\]

Here, 57.8 stands for modern \( V_{AIS} \) (in m$_{eq}$), 7.3 for modern \( V_{GrIS} \) (in m$_{eq}$), 15 for the assumed LGM \( V_{AIS} \) addition (in m$_{eq}$), \( \Delta_{SL} \) for sea level position per time step (in meters relative to present), \( z_{min} \) for the maximum sea level drop in the \( \Delta_{SL} \) record used, and \( V_{AIS_j-1} \) at run initialization is simply set at modern \( V_{AIS} \) (hence, all runs have an initialization period during which the solution stabilizes). The squared term in the last row provides a nonlinear AIS-excess buildup during further glaciation, relative to present, to approximate observations of less excess-ice growth at earlier stages and greater growth at later glacial stages (38).

GrIS volume is calculated similarly (with assumed LGM \( V_{GrIS} \) addition of 5 m$_{eq}$), as

\[
\begin{align*}
V_{GrIS_j} &= \left\{ \begin{array}{ll}
7.3 + \frac{-\Delta_{SL_j}}{2} & \text{if } 0 < 7.3 + \frac{-\Delta_{SL_j}}{2} \leq 7.3 \\
0 & \text{if } 7.3 + \frac{-\Delta_{SL_j}}{2} \leq 0
\end{array} \right.
\end{align*}
\]

For LIS and EIS combined, which exist only when \( \Delta_{SL} < 0 \) m, linear growth is determined in proportion to the \( \Delta_{SL} \) record. However, between them, we assume that one dominates growth in initial glaciation stages, with the other catching up at later stages, to roughly mimic moisture scavenging effects between ice sheets due to large-scale atmospheric circulation changes (91). This is done in a similar fashion to sensitivity test 2 of the analytical assessment with an arbitrary nonlinear gradient function, calculated for each time step

\[
G_j = 1 - (0.5e^{\frac{2\Delta_{SL_j}}{z_{min}}})
\]

EIS volume is simply calculated taking into account this gradient function, and \( V_{LIS} \) in addition compensates for the nonlinearity stipulated in \( V_{AIS} \) at \( \Delta_{SL} < 0 \) m, to ensure that total ice-volume changes remain balanced with \( \Delta_{SL} \) (within rounding errors).

To calculate \( V_{LIS} \), there is a first step

\[
V_{LIS} = \left\{ \begin{array}{ll}
\frac{-z_{min} - \Delta_{SL_j}}{25} & \text{if } -\frac{\Delta_{SL_j}}{25} < 0 \\
0 & \text{otherwise}
\end{array} \right.
\]
Here, 125 m\text{seq} stands for the total ice-volume increase corresponding to the assumed LGM $\Delta S_L$, 70 m\text{seq} for the LIS component (35 m\text{seq} for EIS), and 15 m\text{seq} for the LGM expansion of $V_{\text{AIS}}$, as before. Then, $V_{\text{LIS}}$ is given by

$$
V_{\text{LISj}} = \begin{cases} \frac{-\bar{\delta}_{\text{ice}} - 0.008}{125} \times 70 & \text{if } V_{\text{sj}} < 0 \\ \frac{-\bar{\delta}_{\text{ice}} - 0.008}{125} \times 70 & \text{otherwise} \end{cases}
$$

(9)

To calculate $V_{\text{EIS}}$, the first step is

$$
V_{\text{sj}} = \begin{cases} \frac{-\bar{\delta}_{\text{grs}}}{125} \times (70 + 35) & \text{if } -\Delta S_L > 0 \\ \frac{-\bar{\delta}_{\text{grs}}}{125} \times (70 + 35) & \text{otherwise} \end{cases}
$$

(10)

In addition, $V_{\text{EIS}}$ is then given by

$$
V_{\text{EISj}} = \begin{cases} \frac{-\bar{\delta}_{\text{grs}}}{125} \times (70 + 35) & \text{if } V_{\text{sj}} < 0 \\ \frac{-\bar{\delta}_{\text{grs}}}{125} \times (70 + 35) & \text{otherwise} \end{cases}
$$

(11)

### Ice sheet weighted mean $\delta^{18}O$ and seawater $\delta^{18}O$

For each ice sheet, the isotopic value of new precipitation ($\bar{\delta}_w$) is determined per time step using Eq. 1, for AIS and GrIS using $\alpha = 1$ ($C = 102.55$), and for LIS and EIS using $\alpha = 55$ (with $C = 56.4$). To calculate the volume-weighted mean $\bar{\delta}_{\text{ice}}$ (i.e., $\bar{\delta}_{\text{w}}$) for each ice sheet, we use a residence-time calculation. This starts with ice sheet gross accumulation rate ($a_{\text{grs}}$) determination based on the area defined by its volume (with constant aspect ratio $e$), simply scaled in proportion to a modern global annual $a_{\text{grs}}$ of $\sim 0.008$ m\text{seq} (44, 45) for a global ice volume of 65.1 m\text{seq}. Volume is obtained in relation to $\Delta S_L$ according to Eqs. 5 to 11; volume is true volume (in m$^3$) rather than scale volume (in m\text{seq}); this is indicated using $V$. The term $\Delta S_L$/LIS is used to indicate the sea level change component attributed to the different ice sheets (here LIS). For a world ocean area $A_{\text{oc}}$ of about $362 \times 10^2$ m$^2$, this gives

$$
V_{\text{LISj}} = \frac{V_{\text{LISj}}}{A_{\text{oc}}} = \frac{0.9}{125} \times 70
$$

(12)

For the vertical area under a parabolic profile (with maximum height $h_{\text{max}}$ which is $e$ times radius $r$), the mean height is $(\bar{\gamma})_j h_{\text{max}}$ or $(\bar{\gamma})_j e r$ so that the volume of our standard planoconvex lens-shaped ice sheet with parabolic cross section is $V = (\bar{\gamma})_j e r^2$. Solving for $r$ gives the information required to approximate the ice sheet area over which accumulation occurs (here, the example is given for LIS, but it is the same for all others)

$$
A_{\text{LISj}} = \pi \left( \frac{3}{2} \frac{V_{\text{LISj}}}{A_{\text{oc}}} \right)^{2}
$$

(13)

Note that this concerns simply the surface area of the circle occupied by the ice sheet, ignoring the relevant (half) arc length of the parabola because the aspect ratio is so small that both numbers are virtually identical; hence, the circle area was chosen for computational efficiency. The rate $a_{\text{grs}}$ over that ice sheet at time step $j$ then is $a_{\text{grs}} = 0.008 A_{\text{LISj}}/\Delta S_L$ (mod.), for a modern global annual $a_{\text{grs}} = \sim 0.008$ m\text{seq} when global ice volume is 65.1 m\text{seq}. Here, $A_{\text{LISj}}$ is the surface area given by Eq. 13 for the modern total global ice volume of 65.1 m\text{seq} ($10^7 \times 10^2$ m$^2$). The model calculates the isotopic contribution from that gross accumulation by its product with $\delta_c$ based on Eq. 1. At every time step, gross accumulation is balanced by a gross loss term, which is simply set to $a_{\text{grs}}$ minus the sea level change contribution from one time step to the next of the ice sheet considered (here, $a_{\text{net}} = \Delta S_L$/LIS). The model determines the isotopic impact of $a_{\text{net}}$ using $\bar{\delta}_{\text{ice}}$ (12). The calculation is conditional in that periods with negative $a_{\text{net}}$ do not lead to changes in $\bar{\delta}_{\text{ice}}$

$$
\bar{\delta}_{\text{icej}} = \begin{cases} a_{\text{grs}} \bar{\delta}_{\text{pr}}/(a_{\text{grs}} - a_{\text{net}}) & a_{\text{net}} < 0 \\ a_{\text{net}} + a_{\text{grs}} & a_{\text{net}} > 0 \\ a_{\text{net}} & a_{\text{net}} = 0 \end{cases}
$$

(14)

Last, a constraint is used that when $\bar{\delta}_{\text{ice}}$ from Eq. 14 drops below the model start value $\delta_c = -15^\circ$ for ice growth, and the $\bar{\delta}_{\text{ice}}$ is kept at $-15^\circ$. This is done to avoid spurious effects of rounding errors at extremely small ice volumes. We now have, per time step, ice volumes for each ice sheet and their weighted mean ice sheet $\bar{\delta}_{\text{ice}}$. Thus, we solve Eq. 4 per ice sheet, per time step, to calculate each ice sheet contribution to a change in mean ocean $\bar{\delta}_w$ using the world ocean volume $V_{\text{oc}} = 3700$ m\text{seq} giving (here for LIS)

$$
\Delta \bar{\delta}_w(LIS) = \frac{0.9 V_{\text{LISj}}}{V_{\text{oc}} + \Delta S_L(LIS)}
$$

(15)

Executed for each ice sheet, their cumulative effects can be determined per time step. The cumulative $\Delta \bar{\delta}_w$, normalized to the present-day value, can be subtracted from $\delta_c$ changes, giving a $\delta_c$ residual that indicates bottom-water temperature ($T_w$) changes based on a change of $-0.25^\circ$ per degree Celsius warming.

Thus, we have complete mutual consistency between all key parameters: $\Delta S_L$, ice sheet volumes, ice sheet weighted mean $\bar{\delta}_{\text{ice}}$ global $\Delta \bar{\delta}_w$, and $T_w$. Mutual consistency means that if one changes, so must others. Hence, validation of model results can be undertaken using multiple parameters.

### Variance analysis

For the variance analysis, we first detrended the records obtained using the benthic $\delta_c$ megasplice (14) with a 500-ka moving Gaussian smoothing window. We then calculated SDs of detrended records of residuals over 500-ka windows that were shifted in 100-ka steps across the record. For robustness, we consider only major signals revealed by this analysis. Autoregression analysis (76) is precluded because of the (partially interpolated and/or smoothed nature of the time series considered.

### SUPPLEMENTARY MATERIALS

Supplemental material for this article is available at http://advances.sciencemag.org/cgi/content/full/7/26/eabf5326/DC1

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