Brane fluctuation and the electroweak chiral Lagrangian*

Qi-Shu Yan† and Dong-Sheng Du‡

Institute high energy physics, Chinese Academy of Science,
Beijing 100039, P.R.China

Abstract

We use the external field method to study the electroweak chiral Lagrangian of the extra dimension model with brane fluctuation. Under the assumption that the contact terms between the matter of the standard model and KK excitations of the bulk gauge fields are heavily suppressed, we use the standard procedure to integrate out the quantum fields of these KK excitations and the equation of motion to eliminate the classic fields of these KK excitations. At one-loop level, we find that up to the order $O(p^4)$, due to the momentum conservation of the fifth dimension and the gauge symmetry of the zero modes, there is no constraint on the size of extra dimension. This result is consistent with the decoupling theorem. However, meaningful constraints can come from those operators in $O(p^6)$, which can contribute considerably to some anomalous vector couplings and can be accessible in the LC and LHC.

12.39.Fe, 12.60.cn

*Supported in part by National Natural Science Foundation of China

†E-mail : qsy@mail.ihep.ac.cn

‡E-mail : dsdu@mail.ihep.ac.cn
I. INTRODUCTION

Extra dimension scenario is one kind of interesting candidates for the possible new physics beyond the standard model (SM). As we known, for a higher dimensional quantum field theory, there exist several theoretical problems, the unitarity violation [1], the ultraviolet cutoff dependence, the non-renormalizability [2], and so on. The contribution of the infinite Kaluza-Klein (KK) towers of the bulk fields always violates the unitarity condition of S matrix and makes it even harder to evaluate loop effects. The reference [3] provided one way to suppress the contribution of KK excitations by considering the power running of gauge coupling constants of non-Abelian gauge groups. The reference [4, 5] provided another ingenious mechanism to suppress the contribution of massive KK excitations by assuming that the 3-brane is flexible. In this mechanism, due to the momentum conservation of fifth dimension, the contact interaction of matter fields localized on the 3-brane and KK excitations of the bulk fields would be exponentially suppressed. Then at least at the tree level, the contribution of the infinite KK towers can be well regularized. There are papers to discuss phenomenologies of this mechanism [6]. And it seems that due to this suppression mechanism, the constraint on the size of the extra dimensions imposed by the present experimental researches can be considerably relaxed. However, in this brane fluctuation suppression mechanism those couplings which respect the momentum conservation of the fifth dimension will not be suppressed, say couplings among KK modes in the gauge bosonic part. This part might suffer those aforementioned theoretical problems of higher dimensional quantum field theory which could not be solved by the brane fluctuation. Then it seems that only the string theory could provide the radical solutions [7].

Recently, the reference [8] used the technicolor way (the Moose diagram) to deconstruct the extra dimensions and the reference [9] used the latticed extra dimensions to construct the renormalizable effective theoretical description of the extra dimension models. One of the important features is that the extra components of the bulk vector gauge bosons can act as the Goldstone and Higgs bosons. Based on these two works, there are papers [10] to construct realistic models. We would like to mention that the effective Lagrangian obtained by the Refs. [8, 9] doesn’t have the contact structure as assumed in [11]

\[ g^2 |\phi_2|^2 (W_\mu + \sqrt{2} \sum_{n=1}^{\infty} W^n_{\mu})^2, \]

where \( g \) is the gauge coupling constant, and \( W_\mu \) is the zero mode and \( W^n_{\mu} \) is the \( n \)-th KK excitations of gauge bosons. Furthermore, it seems that the interaction terms among zero and KK modes have been ignored by these authors.

After taking into account the brane fluctuation given in [4, 5], it seems that the contact structure is more likely modified to be

\[ |\phi_2|^2 \left[ (gW_\mu)^2 + \sqrt{2}gg_nW_\mu \sum_{n=1}^{\infty} W^n_{\mu} + \cdots \right], \]

where...
where \( g_n \) is the effective coupling constant of the \( n \)-th KK excitations of gauge bosons to matter of the SM, and it’s actual form will be given in the second section. If assuming that the the effective coupling constant \( g_n \) is heavily suppressed, we see the contact term between Higgs field and KK excitations would be very very small.

Then, a common feature in the deconstructing and brane fluctuation extra dimension models is that there could be no large tree level mixing among zero mode and KK excitations, and the constraint on extra dimensions put by LEP and SLAC could be considerably relaxed. It is natural to ask that if the world is indeed as described by [4, 5] and [8, 9], then whether there still exist a way to find the traces of KK excitations at low energy region near the threshold of the first KK modes. Fortunately, thanks to the couplings of the gauge bosonic part between the 0 mode and KK excitations! Since these couplings are not exponentially suppressed and can be large, they can help us to probe KK excitations. So in the deconstructing and brane fluctuation extra dimension models, the bosonic part of the bulk gauge fields will act as the main probe to discover the signal of extra dimensions.

The electroweak chiral Lagrangian (EChL) is the model-independent way to describe the spontaneous symmetry breaking of the \( SU(2) \times U(1) \) symmetry of the standard model \([12]\). It can be regarded as the effective theory of the underlying theory in its low energy region after integrating out those heavy degree of freedoms (DOF), where the dynamic degree of freedoms are the particle contents of the SM. The operators in the Lagrangian are consisted of the low energy DOF and can be arranged by the momentum expansion, where the external momentum is assumed to be small compared with the mass of the integrated-out particles. These operators can be classified as the relevant, the marginal, and the irrelevant. And the relevant and the marginal operators are the most interesting and heavily studied, which are normally collected in and referred as the \( O(p^2) \) and \( O(p^4) \) part. The irrelevant operators are normally suppressed by the mass of heavy degree of freedoms according to the decoupling theorem \([13]\). The complete set of operators in \( O(p^6) \) has been given by \([14]\). The coefficients of these operators form the generic theoretical parameter space of all possible new physics at low energy scale. The dimension of this parameter space at \( O(p^6) \) is quite large. After integrating-out those heavy DOFs, a specified underlying theory will occupy a corner of this large parameter space.

Two prices must be paid for this generality owned by the effective theory: the first one is that the renormalizability of a underlying theory is sacrificed and the couplings of these operators must be determined from experiments. Another one is that the theory is invalid for the momentum larger than the scale \( \Lambda_{UV} \), and above this scale the unitarity of the S matrix might be explicitly broken down.

The reference \([15]\) used the EChL to study the effects of KK excitations of the graviton and of the dilaton in large extra dimension scenarios. In this paper, we will use the EChL to analyze the effects of KK excitation of gauge bosons of the SM in the small extra dimension scenarios \([16]\). We will conduct our computation in the background field gauge method. This method has several advantages compared with the standard Feynman diagram method. The computation is manifestly gauge invariant at every step, the relevant diagrams are much less, etc. And we find that, under the brane fluctuation
suppression assumption and due to the momentum conservation of the fifth dimension and the gauge symmetry of the zero mode, except for contributing to the renormalization of gauge coupling and wave-function, KK excitations have no effect up to $O(p^4)$ and this result is consistent with the decoupling theorem \[13\]. However we know that the meaningful contributions of KK excitations can still come from operators higher than $O(p^4)$, say $O(p^6)$.

The paper is organized as follows. We will briefly describe the brane fluctuation in the second section and give the gauge boson sector using the external field method in the third section. We will emphasize some of its features that has been ignored. We will compute the electroweak chiral Lagrangian of KK excitations in the fourth section by using the path integral method. Finally, the a brief discussion and conclusion is made.

II. THE BRANE FLUCTUATIONS

The total action given by \[5\] has by two parts: 1) the bulk part $S_{\text{bulk}}$, where gravity and vector gauge bosons are assumed to propagate in the bulk, 2) the brane part $S_{\text{brane}}$, where fermion and scalar matter are assumed to be localized on the brane. The SM is consisted of the zero mode of gravity and vector gauge bosons, matter fields confined on the brane, and their interactions. New physics include KK excitations of gravity and vector gauge bosons, the Nambu-Goldstone bosons, and their interactions with each other and with the particles of the SM. In the convention of the reference \[5\], the bulk part action defined in D dimension takes the form

$$S_{\text{bulk}} = \int d^D X \det E \left[ -\Lambda + \frac{M^{D-2}}{2} R - \frac{1}{4} G^{MN} G^{RS} \text{tr}(F_{MN} F_{RS}) + \cdots \right],$$  \hspace{1cm} (3)

where $\Lambda$, $M$ and $R$ are the cosmological constant, the D-dimensional fundamental scale, and the D-dimensional scalar curvature, respectively. $F_{MN}$ are the Yang-Mills field strength defined in D dimensions.

The matter fields on the brane couple to the bulk fields through the induced vielbein and Yang-Mills fields. The d-dimension brane is assumed to embedded in the D dimension space-time, and its action can be formulated in the following form

$$S_{\text{brane}} = \int d^d x \det e \left[ -\tau + e^\mu_\alpha(x) \bar{\psi}(x) i\gamma^\mu \left( \frac{\nabla_\mu}{2} - ig a_\mu(x) \right) \psi(x) - m \bar{\psi}(x) \psi(x) + \cdots \right],$$  \hspace{1cm} (4)

where $\psi(x)$ be a fermion field on the brane which is charged under the Yang-Mills gauge group. The original paper \[5\] does not consider the scalar case. If we assume there are scalar fields in the theory, we should add their corresponding terms to $S_{\text{brane}}$.

In the flat space-time metric, the $S_{\text{brane}}$ can be reduced to

$$\int d^d x \det e (-\tau) = \int d^d x \left[ -\tau + \frac{1}{2} \partial^\mu \phi^m(x) \partial_\mu \phi^m(x) + \frac{1}{8\tau} (\partial^\mu \phi^m(x) \partial_\mu \phi^m(x))^2 \right.$$

$$- \frac{1}{4\tau}(\partial^\mu \phi^m(x) \partial_\nu \phi^m(x))(\partial^\nu \phi^m(x) \partial_\mu \phi^m(x)) + \cdots],$$  \hspace{1cm} (5)
where $\phi$ is the Nambu-Goldstone boson corresponding to the spontaneous breaking of the translation symmetry.

Assuming that the $D - d$ dimensions are compactified, the bulk gauge field can be Fourierly expanded in their KK modes:

$$A_M(X^\mu = x^\mu, X^m = Y^m) = \frac{1}{\sqrt{V}} \sum_n A_M^{(n)}(x)e^{inY/R}. \quad (6)$$

Then the gauge interaction term on the brane reads

$$\int d^d x \sum_n g \overline{\psi}(x)\gamma^\mu \psi(x)A_\mu^{(n)}(x) \exp\left(\frac{in \cdot \phi(x)}{R\sqrt{\tau}}\right). \quad (7)$$

Considering that the Nambu-Goldstone bosons have their fluctuations, the gauge interaction term should be rewritten as

$$\int d^d x \sum_n g e^{-\frac{1}{2} \frac{n^2}{\tau} \Delta(M^{-1})} \overline{\psi}(x)\gamma^\mu \psi(x)A_\mu^{(n)}(x) : \exp\left(\frac{in \cdot \phi(x)}{R\sqrt{\tau}}\right) :, \quad (8)$$

where $\Delta$ is the free propagator of $\phi$

$$\Delta(x - y) \equiv \langle \phi(x)\phi(y) \rangle = -\frac{1}{4\pi^2} \frac{1}{(x - y)^2}. \quad (9)$$

The most interesting phenomena owned by the brane fluctuation is that the effective coupling $g_n$ of the level $n$ KK mode to the four-dimensional field is suppressed exponentially:

$$g_n \equiv g \cdot e^{-\frac{1}{2} \frac{n^2}{\tau} \frac{M^2}{\tau^2}}. \quad (10)$$

The origin of this suppression is a recoil effect of the brane. It is this suppression mechanism that makes the constraints on the extra dimensions being substantially loosen.

According to the analysis of [5, 17], although there exists a constraint on the tension of brane when taking into account the effects of the Nambu-Goldstone boson, it seems that KK excitations might escape our detections.

However, thanks to the couplings in the bosonic part between the 0 mode and KK excitations! Since these couplings will not be exponentially suppressed, they can help us to probe KK excitations. Below we will assume this suppression mechanism for the $S^1/Z_2$ case\footnote{The suppression mechanism given by the reference [3] is valid for compactified $S^1$, and it is not very clear whether this assumption can be proper for the $S^1/Z_2$ case. However, for the sake of simplicity, we show in this paper how to conduct calculation under this assumption in the orbifold compactification case. The computational procedure can be extended to the $S^1$ compactification straightforward.}, and investigate the effects of KK excitations to the bosonic sector of the SM in the
brane-fluctuation extra dimension model. For the sake of simplicity, we omit the gravity part, which should be expected to be small when compared with the Yang-Mills part in the small extra dimension scenarios. In order to compare and contrast with the SM, we assume that there is a Higgs doublet field. To get the electroweak symmetry breaking, the linear Higgs mechanism is assumed. However, considering exponential suppression of the coupling of the matter on the brane and KK excitations, the tree level mixing angle among zero mode and KK excitations will be neglected.

III. THE GAUGE BOSONIC SECTOR IN THE EXTERNAL FIELD METHOD

To simplify the consideration, we study the 5D compactified on $M_4 \times S^1/Z_2$. And we will use the dimension reduction procedure to get the effective theory in 4D. The total action is formulated as

$$S_5 = \int d^5x (L_{YM} + L_\text{contact} \delta(x_5)),$$

where $L_{YM}$ contains the contact terms of the SM to the KK excitations except for the vector boson field part. The Lagrangian is formally invariant under the gauge transformation in 5D.

In order to get the effective Lagrangian which is manifestly gauge covariant to the symmetry of the SM, we use the background field method and split the vector gauge field $\tilde{V}_M$ into two parts as (here $V = W$ and $B$, respectively):

$$\tilde{V}_M = \bar{V}_M + \hat{V}_M,$$

where $\bar{V}_M$ is the classic part, and $\hat{V}_M$ is the quantum fluctuation. In the background field gauge, we have the freedom to choose different gauges for the classic and quantum vector boson field, respectively. For the classic field, we will use the unitary gauge, which means $\bar{V}_5 = 0$ (this will be more manifest in the deconstructing model), where the Goldstone boson field is realized in the non-linear way, $U = \exp \int \bar{V}_5 dx^5$. $U = 1$ is the unitary gauge, and this corresponds to $\bar{V}_5 = 0$ and $V_5$ will not appear in the Lagrangian. While for the quantum field, we will use the $R_\xi$ gauge and the $\bar{V}_5$ does not vanish.

The gauge fixing terms are chosen to be

$$F(W) = D^\mu \bar{W}_\mu - \xi_W \partial^5 \bar{V}_5,$$

$$F(B) = \partial^\mu \hat{B}_\mu - \xi_B \partial^5 \hat{B}_5,$$

where $D^\mu = \partial^\mu + g \bar{A}^\mu$. To write these two gauge fixing term, we haven’t taken into account the spontaneous symmetry breaking of the SM. The variation of the gauge fixing terms under the gauge transformation is given as
\[
\frac{\delta F(W)}{\delta \alpha_W} = \bar{D}_\mu (D_\mu + g f \hat{W}_\mu) + \xi W \partial_5 (\partial_5 + g f \hat{W}_5),
\]
(16)
\[
\frac{\delta F(B)}{\delta \alpha_B} = \partial_\mu \partial_\mu + \xi_B \partial_5 \partial_5.
\]
(17)

By requiring that the field is unchanged under the orbifold transformation, we can decompose vector bosons \(V\) as
\[
\hat{V}(\hat{V})_\mu(x, x_5) = \sum_{i=0}^\infty \hat{V}(\hat{V})_\mu^i(x) \cos i \theta_5,
\]
(18)
\[
\hat{V}_5(x, x_5) = \sum_{i=1}^\infty \hat{V}_5^i(x) \sin i \theta_5,
\]
(19)
where \(\theta_5 = M_c x_5, M_c = 2\pi / R_c, R_c\) is the radius of the compactified fifth dimension. To compare and contrast with the SM, below we will omit the index 0 of zero modes and represent \(\hat{W}^0 = W\) and \(\hat{B}^0 = B\), respectively. \(W\) and \(B\) are the vector gauge bosons of the SM, respectively. Below we will suppress the bar of the classic background fields.

In order to integrate out the fifth dimension, we decompose the field strength by using Eqs. (18) and (19). The \(\hat{W}_{\mu\nu}\) can be decomposed by cos modes and we have

\[
\text{0 mode} : W_{\mu\nu} + (D_\mu W_\nu - D_\nu W_\mu)
\]
\[
+ \frac{1}{2} g f \sum_{i=1}^{\infty} \left[ W_{\mu}^i W_\nu^i + \hat{W}_\mu^i \hat{W}_\nu^i + W_{\mu}^i \hat{W}_\nu^i + \hat{W}_\mu^i \hat{W}_\nu^i \right],
\]
(20)
\[
\text{n mode} : (D_\mu W_\nu^n - D_\nu W_\mu^n + D_\mu \hat{W}_\nu^n - D_\nu \hat{W}_\mu^n)
\]
\[
+ g f \left( \hat{W}_\mu W_\nu^n - \hat{W}_\nu W_\mu^n \right) + g f \left( \hat{W}_\mu \hat{W}_\nu^n - \hat{W}_\nu \hat{W}_\mu^n \right)
\]
\[
+ \frac{1}{2} g f \sum_{i=1}^{n-1} \left[ W_{\mu}^i W_\nu^{n-i} + \hat{W}_\mu^i \hat{W}_\nu^{n-i} + W_{\mu}^i \hat{W}_\nu^{n-i} + \hat{W}_\mu^i \hat{W}_\nu^{n-i} \right]
\]
\[
+ \frac{1}{2} g f \sum_{i=1}^{\infty} \left[ W_{\mu}^i W_\nu^{n+i} + \hat{W}_\mu^i \hat{W}_\nu^{n+i} + W_{\mu}^i \hat{W}_\nu^{n+i} + \hat{W}_\mu^i \hat{W}_\nu^{n+i} \right]
\]
\[
+ \hat{W}_\mu^{n+i} \hat{W}_\nu^i + W_{\mu}^{n+i} \hat{W}_\nu^i + \hat{W}_\mu^{n+i} \hat{W}_\nu^i \right],
\]
(21)
where \(W_{\mu\nu} = \partial_\mu W_\nu - \partial_\nu W_\mu + g f W_\mu W_\nu, \) and \(D_\mu = \partial_\mu + g f W_\mu.\) While for \(\hat{B}_{\mu\nu},\) we have

\[
\text{0 mode} : B_{\mu\nu} + \hat{B}_{\mu\nu},
\]
(24)
\[
\text{n mode} : B_{\mu\nu}^n + \hat{B}_{\mu\nu}^n.
\]
(25)

where \(V_{\mu\nu} = \partial_\mu V_\nu - \partial_\nu V_\mu,\) with \(V = B, \hat{B}, B^n, \hat{B}^n.\)

The \(\hat{W}_{5\mu}\) can be decomposed by sin modes and we have
\[ n \text{ mode} : D_\mu \hat{W}_5^n + g f \hat{W}_\mu \hat{W}_5^n + n M_c W_\mu^n + n M_c \hat{W}_\mu^n \]
\[ + \frac{1}{2} g f \sum_{i=1}^{n-1} (W_i^\mu + \hat{W}_i^\mu) \hat{W}_5^{n-i} \]
\[ + \frac{1}{2} g f \sum_{i=1}^{\infty} \left[ (W_i^\mu + \hat{W}_i^\mu) \hat{W}_5^{n+i} + (W_{n+i}^\mu + \hat{W}_{n+i}^\mu) \hat{W}_5^i \right], \quad (26) \]

while for \( \hat{B}_5 \), we have
\[ n \text{ mode} : \partial_\mu \hat{B}_5^n + n M_c B_\mu^n + n M_c \hat{B}_\mu^n. \quad (27) \]

It is remarkable that there is no zero mode for the sin KK modes for \( V_5^{\mu} \) and it is related with the assumption of the compactified space-time.

The gauge fixing term of SU(2) is decomposed by cos modes and we have
\[ 0 \text{ mode} : D^\mu \hat{W}_\mu, \quad (28) \]
\[ n \text{ mode} : D^\mu \hat{W}_\mu^n - n \xi_{W} M_c \hat{W}_5^n \]
\[ + \frac{1}{2} g f \sum_{i=1}^{n-1} (W_i^{\mu i} + \hat{W}_i^{\mu i}) \hat{W}_\mu^{n-i} \]
\[ + \frac{1}{2} g f \sum_{i=1}^{\infty} \left[ (W^{\mu i} + \hat{W}^{\mu i}) \hat{W}_\mu^{n+i} + (W_{n+i}^{\mu i} + \hat{W}_{n+i}^{\mu i}) \hat{W}_\mu^i \right], \quad (29) \]

and that of U(1) is decomposed as
\[ 0 \text{ mode} : \partial^\mu \hat{B}_\mu, \quad (30) \]
\[ n \text{ mode} : \partial^\mu \hat{B}_\mu^n - n \xi_{B} M_c \hat{B}_5^n. \quad (31) \]

Since we are only interested in low energy physics where zero modes play the main part, so we will only keep those terms containing zero modes and neglect those pure interactions of KK excitations. Then after integrating out the fifth dimension, we get the reduced YM Lagrangian which reads
\[
L_{YM}^{\text{eff}} = -\frac{1}{4}(2\pi R_c) \left[ (W_{\mu \nu} + D_\mu \hat{W}_\nu - D_\nu \hat{W}_\mu + g f \hat{W}_\mu \hat{W}_\nu)^2 \right.
\[ + g f \left( W_{\mu \nu} + D_\mu W_\nu - D_\nu W_\mu + g f \hat{W}_\mu \hat{W}_\nu \right)
\[ \times \sum_{n=1}^{\infty} \left( W_\mu^n W_\nu^n + \hat{W}_\mu^n \hat{W}_\nu^n + W_\mu^n \hat{W}_\nu^n + \hat{W}_\mu^n \hat{W}_\nu^n \right) \left. \right]
\[ - \frac{1}{4}(\pi R_c) \sum_{n=1}^{\infty} \left[ D_\mu W_\nu^n - D_\nu W_\mu^n + D_\mu \hat{W}_\nu^n - D_\nu \hat{W}_\mu^n \right.
\[ + g f \left( \hat{W}_\mu W_\nu^n - \hat{W}_\nu W_\mu^n + \hat{W}_\mu W_\nu^n - \hat{W}_\nu W_\mu^n \right) \right]^2 \left. \right]
\[ + \frac{1}{2}(\pi R_c) \sum_{n=1}^{\infty} \left[ D_\mu \hat{W}_5^n + g f \hat{W}_\mu \hat{W}_5^n + n M_c W_\mu^n + n M_c \hat{W}_\mu^n \right]^2 \right. \]
the final effective Lagrangian of 4D reads

\[
- \frac{1}{2\xi_W} (2\pi R_c) \left( D^\mu \hat{W}_\mu \right)^2 - \frac{1}{2\xi_W} (\pi R_c) \sum_{n=1}^\infty \left( D^\mu \hat{W}_\mu^n - \xi_W n M_c \hat{A}_5^n \right)^2
+ (2\pi R_c) \bar{c} \left[ -D^\mu \left( D_\mu + g f \hat{W}_\mu \right) \right] c
+ (\pi R_c) \sum_{n=1}^\infty \bar{c} c \left[ -D^\mu \left( D_\mu + g f \hat{W}_\mu \right) - n^2 \xi_W M_c^2 \right] c^n
+ (\pi R_c) g f \sum_{n=1}^\infty \left( D^\mu \hat{c} n \hat{W}_\mu^n c + D^\mu \hat{c} W_\mu^n c^n \right)
+ \ldots
- \frac{1}{4} \left( 2\pi R_c \left[ B_{\mu\nu} + \hat{B}_{\mu\nu} \right] \right)^2 - \frac{1}{4} \left( \pi R_c \right) \sum_{n=1}^\infty \left[ B_{\mu\nu}^n + \hat{B}_{\mu\nu}^n \right]^2
+ \frac{1}{2} \left( \pi R_c \right) \sum_{n=1}^\infty \left[ n M_c B_\mu + n M_c \hat{B}_\mu + \partial_\mu \hat{B}_5 \right]^2
- \frac{1}{2\xi_B} (2\pi R_c) (\partial^\mu \hat{B}_\mu)^2 - \frac{1}{2\xi_B} (\pi R_c) \sum_{n=1}^\infty (\partial^\mu \hat{B}_\mu^n)^2
+ (2\pi R_c) \bar{c}_B \left( -\partial^\mu \partial_\mu - n^2 \xi_B M_c^2 \right) c^n_B ,
\]

where the omitted terms are only related to the non-Abelian SU(2) gauge symmetry and the U(1) part is exact.

By utilizing the rescaling relations

\[
W(B, c_W, c_B) \rightarrow \sqrt{2\pi R_c} W(B, c_W, c_B) , \quad g(g') \rightarrow \frac{1}{\sqrt{2\pi R_c}} g(g') ,
\]

\[
W^n(B^n, c^n_W, c^n_B, W_5^n, B_5^n) \rightarrow \sqrt{\pi R_c} W^n(B^n, c^n_W, c^n_B, W_5^n, B_5^n) ,
\]

the final effective Lagrangian of 4D reads

\[
L_{\text{YM,4D}}^{\text{eff}} = -\frac{1}{4} \left[ \left( W_{\mu\nu} + D_{\mu} \hat{W}_\nu - D_{\nu} \hat{W}_\mu + g f \hat{W}_\mu \hat{W}_\nu \right) \right]^2
+ R g f \left( W_{\mu\nu} + D_{\mu} \hat{W}_\nu - D_{\nu} \hat{W}_\mu + g f \hat{W}_\mu \hat{W}_\nu \right)
\times \sum_{n=1}^\infty \left( W_{\mu}^n W_\nu^n + \hat{W}_\mu^n \hat{W}_\nu^n + W_\mu^n \hat{W}_\nu^n + \hat{W}_\mu^n \hat{W}_\nu^n \right)
- \frac{1}{4} \sum_{n=1}^\infty \left[ D_{\mu} W_\nu^n - D_{\nu} W_\mu^n + D_{\mu} \hat{W}_\nu^n - D_{\nu} \hat{W}_\mu^n
+ g f \left( \hat{W}_\mu W_\nu^n - \hat{W}_\nu W_\mu^n + \hat{W}_\mu \hat{W}_\nu^n - \hat{W}_\nu \hat{W}_\mu^n \right) \right]^2
+ \frac{1}{2} \sum_{n=1}^\infty \left[ D_{\mu} \hat{W}_5^n + g f \hat{W}_\mu \hat{W}_5^n + n M_c W_5^n + n M_c \hat{W}_5^n \right)^2
- \frac{1}{2\xi_W} (D^\mu \hat{W}_\mu)^2 - \frac{1}{2\xi_W} \sum_{n=1}^\infty \left( D^\mu \hat{W}_\mu^n - \xi_W n M_c \hat{A}_5^n \right)^2
+ \bar{c} c \left[ -D^\mu \left( D_\mu + g f \hat{W}_\mu \right) \right] c_W
\]

\[ + \sum_{n=1}^{\infty} \bar{c}_W^n \left[ -D^\mu \left( D_\mu + gf \hat{W}_\mu \right) - n^2 \xi \bar{c}_W^n \right] c_W^n \]

\[ + gf \sum_{n=1}^{\infty} \left( D^\mu \bar{c}_W^n c + D^\mu \bar{c}_W^n c^n \right) \]

\[ + \cdots \]

\[ - \frac{1}{4} \left[ B_{\mu\nu} + \hat{B}_{\mu\nu} \right]^2 - \frac{1}{4} \sum_{n=1}^{\infty} \left[ B_{\mu\nu}^n + \hat{B}_{\mu\nu}^n \right]^2 \]

\[ + \frac{1}{2} \sum_{n=1}^{\infty} \left[ nM_c B_{\mu} + nM_c \hat{B}_{\mu} + \partial_{\mu} \hat{B}_5 \right]^2 \]

\[ - \frac{1}{2\xi_B} \left( \partial^\mu \hat{B}_5 \right)^2 - \frac{1}{2\xi_B} \sum_{n=1}^{\infty} \left( \partial^\mu \hat{B}_5^n \right)^2 \]

\[ + \bar{c}_B \left( -\partial^\mu \partial_\mu \right) c_B + \bar{c}_B^n \left( -\partial^\mu \partial_\mu - n^2 \xi_B \bar{c}_B^n \right) c_B^n , \]

where \( R = 2 \), which arises from the different normalization factor of zero mode and KK excitations.

Several features are quite remarkable of the reduced effective Lagrangian in 4D given in the Eq. (35): 1) In the dimension reduction procedure, the zero modes are still massless, and the corresponding gauge symmetry is unbroken and is explicit in the background field gauge. And KK excitations are the adjoint representations of the SU(2) symmetry in 4D, as pointed out in [19]. To break the symmetries of the zero mode, other assumptions should be introduced.

2) There are infinite KK excitations. For each massive KK mode, the spectrum is consisted of a massive quantum field, its corresponding Goldstone field, its corresponding ghost field, and a massive background field;

3) There are infinite interaction terms among KK excitations which is controlled by only two gauge coupling constants, \( g \) and \( g' \). This structure can not sustain the quantum corrections even we truncate the infinite KK tower to finite. The intrinsic reason is that the underlying theory defined in 5D is non-renormalizable, as already pointed out in [2].

4) For the vector boson field of U(1) symmetry, there is no interaction among KK modes. While for the vector boson field of SU(2) symmetry, there do exist gauge interactions between different KK modes. This fact will bring into some interesting phenomenologies, as we will show below.

5) Due to the momentum conservation of the extra dimensions, every of the interaction terms between the 0-mode \( A_0 \) and a KK excitation \( A^n \) contains at least two \( A^n \)s, as shown in the Eq. (35).

IV. INTEGRATING-OUT THE KK EXCITATIONS AT 1-LOOP LEVEL

In this section we will extract the effective Lagrangian up to 1-loop level by integrating out KK excitations. The method we will use is the functional integral. The functional method to integrate out a heavy DOF is quite standard, and the references [20, 21] provide
the detailed procedure. Normally, the background field method and Stuckeberg transformation are used to integrate out the quantum DOF. After that, the equation of motion of the heavy fields are used to eliminate the classic heavy DOF from the Lagrangian. In [20, 21], the authors use this method to investigate the effect of heavy Higgs bosons, and in [22] the authors use this method to study that of the heavy Fermion. To integrate out KK excitations, we assume that KK excitations are massive and heavier than all particles of the SM.

A. Tree level relations

First we provide the classic equation of motion (EOM) of those background field (BF). The EOM of the BF of the zero mode is given as

\[ D^\mu W_\mu - M^2_{W_0} W_\mu = g f \sum_{n=1}^{\infty} W^\mu (D_\nu W^n_\mu - D_\mu W^n_\nu) + J_\mu (light), \]  

Here \( J_\mu (light) \) means the currents of light DOFs of the SM which are light compared with massive KK excitations. While the EOM of the BF of the nth KK excitation is given as

\[ D^\mu (D_\mu W^n_\nu - D_\nu W^n_\mu) - n^2 M^2_c W^n_\mu = W_\mu W^n_\mu + \frac{g_n}{g} J_\mu (light) + \cdots, \]  

The omitted terms are terms of KK excitations which can be safely neglected. For vector gauge boson field of U(1), the EOM is simple. Considering that there is no interaction among KK excitations of U(1) symmetry and the brane fluctuation greatly suppresses the interactions between KK excitations and light DOFs, below we will omitted the KK excitations of U(1) part.

The equation of the motion of a classic KK excitation can be formulated in momentum presentation as

\[ [(p^2 - n^2 M^2_c) + f(W^0)] W^n_\mu = g_n J_\mu, \]  

where \( p^2 \) is the momentum of the \( A^n \), and \( nM_c \) is its mass, \( f(W^0) \) includes the terms of interactions between the 0-mode \( W^0 \) and the KK mode \( W^n \). The \( J_\mu \) is the current of the matters of the SM and \( g_n \) is the brane fluctuation suppression factor. In the low energy region, the terms with momentum \( p \) will be set to zero, and the \( W^n_\mu \) can be represented by the light degree of freedom as given below

\[ W^n_\mu \approx -\frac{g_n}{(n^2 M^2_c)} J_\mu [1 + f(W^0)/(n^2 M^2_c) + \cdots]. \]  

Therefore, at tree level, after integrating out the massive KK excitations, we will get terms like

\[ \frac{(g_n)^2}{(n^2 M^2_c)} J_\mu J^\mu [1 + f(W^0)/(n^2 M^2_c) + \cdots]. \]  

11
By invoking the heavy exponential suppression argument, we regard these terms belong to order $O(1/M^4_c)$ and neglect them in our consideration.

At tree level up to $O(1)$, to integrate out KK excitations means to set the field of KK excitations (both classic and quantum field) to zero, we get the tree level effective Lagrangian

$$L_{Y_M}^{\text{eff,tree}} = -\frac{1}{4} \left[ (W_{\mu\nu} + D_\mu \tilde{W}_\nu - D_\nu \tilde{W}_\mu + g f \tilde{W}_\mu \tilde{W}_\nu)^2 \right]$$

$$- \frac{1}{2\xi_W} \left( \tilde{D}_\mu \tilde{W}_\mu \right)^2 + \tilde{c}_W \left[ - D_\mu \left( D_\mu + g f \tilde{W}_\mu \right) \right] c_W .$$

(41)

This Yang-Mills Lagrangian is the standard one in the background gauge.

Up to the order $O(1/M^2_c)$, after integrating out massive KK excitations, we will get terms like

$$\sum_{n=1}^{\infty} \frac{g^2_n}{(nM_c)^2} J_\mu J_\mu + \cdots .$$

(42)

Under the assumption of brane fluctuation suppression, we regard these terms as terms higher than $O(1/M^4_c)$ and will omit them in the below analysis.

### B. Integrating out KK excitations

To extract the effective Lagrangian at 1-loop level, we reformulate the effective Lagrangian given in (35) and only keep those bilinear terms.

$$\mathcal{L} = \tilde{W}_\mu \Delta^{\mu\nu}_{WW} \tilde{W}_\nu + \tilde{c}_W \Delta_{cwCW} c_W$$

$$+ \sum_{n=1}^{\infty} \tilde{W}_n W W_n \tilde{W}_n + \sum_{n=1}^{\infty} \tilde{W}_n W W_n \tilde{W}_n + \sum_{n=1}^{\infty} \tilde{W}_n W W_n \tilde{W}_n$$

$$+ \sum_{n=1}^{\infty} \tilde{W}_n W W_n \tilde{W}_n$$

$$+ \sum_{n=1}^{\infty} \tilde{c}_W \Delta^{\mu} W W_n c_W + \sum_{n=1}^{\infty} \epsilon W \Delta_{cwCW} c_W c_W + \sum_{n=1}^{\infty} \tilde{c}_W \Delta_{cwCW} c_W c_W + \cdots$$

(43)

$$\Delta^{\mu\nu}_{WW} = \frac{1}{2} \left[ D^2 g^{\mu\nu} - \left( 1 - \frac{1}{\xi_w} \right) D_\mu D_\nu - g W^{\rho\sigma} J^{\mu\nu}_{\rho\sigma} \right] ,$$

(44)

$$\Delta^{\mu\nu}_{W n W n} = \frac{1}{2} \left[ (D^2 + n^2 M_c^2) g^{\mu\nu} - \left( 1 - \frac{1}{\xi_w} \right) D_\mu D_\nu - g W^{\rho\sigma} J^{\mu\nu}_{\rho\sigma} \right] ,$$

(45)

$$\Delta^{\mu\nu}_{W n W n} = \frac{1}{2} g f \left[ (W^{\mu}\nu D^\nu - g W^{\mu\nu} D_\alpha + (D_\mu W^{\nu}) - (D_\nu W^{\mu}) \right] ,$$

(46)

$$\Delta^{\mu\nu}_{W n W n} = \Delta^{\mu\nu}_{W W n} ,$$

(47)

$$\Delta^{\mu\nu}_{W 5 W 5} = \frac{1}{2} \left( -D^2 - \xi w n^2 M_c^2 \right) ,$$

(48)

$$\Delta_{cwCW} = -D^2 ,$$

(49)
\[
\Delta c_W^\mu e_W^\nu = -D^2 - \xi \omega n^2 M_c^2 ,
\]
\[
\Delta c_W^\mu e_W = -gfD^\mu W_n^\nu ,
\]
\[
\Delta c_W^\mu e_W = -gfD^\mu W_n^\nu ,
\]
where \( W_{\mu\nu} = W_{\mu\nu}^a t_a G^a \), \((t_a^G)_{bc} = i f^{bac}\) is structure constants and the generator adjoint representations of the non-Abelian group, and \( J_{\rho\sigma}^\mu\nu \) is the generator of Lorentz transformations on 4-vectors and is defined as
\[
J_{\rho\sigma}^\mu\nu = i (\delta^\mu_\rho \delta^\nu_\sigma - \delta^\mu_\sigma \delta^\nu_\rho) .
\]

Linear terms can be eliminated by using the classic EOMs. From the result listed above, it is apparent that the quadratic operators of KK excitations are very similar to that of the zero mode.

We have omitted those terms which contribute at two loop level. One feature is worthy of mention: KK excitations always appear at least in pair due to the momentum conservation of the fifth dimension. This fact is very important for us to understand the decoupling behavior of KK excitations. It is also remarkable that there exist mixings among the quantum fields of KK modes, and in order to integrate out quantum part of KK excitations we must diagonalize the bilinear terms. (There are also mixings among different quantum fields of KK excitations which have been omitted, and it is reasonable according to the auxiliary power counting rule which will be introduced below). Then we get the one-loop effective Lagrangian by integrating out the massive KK excitations
\[
L_{Y M, KK}^{\text{eff},1\text{-loop}} = \frac{1}{2} \sum_{n=1}^{\infty} \ln \text{Det} \left( \Delta W_n W_n \delta^{(4)}(x-y) \right) + \frac{1}{2} \sum_{n=1}^{\infty} \ln \text{Det} \left( \Delta c_W^\mu e_W^\nu \delta^{(4)}(x-y) \right)
\]
\[
- \sum_{n=1}^{\infty} \ln \text{Det} \left( \Delta c_W^\mu e_W^\nu \delta^{(4)}(x-y) \right)
\]
\[
= \frac{1}{2} \sum_{n=1}^{\infty} \text{Tr} \ln \left( \Delta W_n W_n \delta^{(4)}(x-y) \right) + \frac{1}{2} \sum_{n=1}^{\infty} \text{Tr} \ln \left( \Delta c_W^\mu e_W^\nu \delta^{(4)}(x-y) \right)
\]
\[
- \sum_{n=1}^{\infty} \text{Tr} \ln \left( \Delta c_W^\mu e_W^\nu \delta^{(4)}(x-y) \right) ,
\]
where \( \Delta W_n W_n = \Delta W_n W_n - \Delta W_n W_n \Delta W_n \Delta W_n \), \( \Delta c_W^\mu e_W^\nu = \Delta c_W^\mu e_W^\nu - \Delta c_W^\mu e_W^\nu \Delta c_W^\mu e_W^\nu \Delta c_W^\mu e_W^\nu \).

The signs of the contributions of ghost scalars and normal scalars are different, which is due to the fact that ghost fields satisfy anti-commutation relations.

To this step, the quantum fields of KK excitations have been integrated out and the functional trace and logarithm have to be evaluated. There are several methods to deal with this evaluation \[24\]–\[27\]. Below we will first use the method \[26\] to analyze those relevant terms. After doing this, we will use the heat kernel \[27\] to evaluate the trace and logarithm.

13
C. The auxiliary counting rule

We study the $Tr \ln \tilde{\Delta}_{W^mW^n}$ first. We have

$$
\tilde{\Delta}_{W^mW^n}(x, \partial_x)\delta^{(4)}(x-y) = \int \frac{d^4p}{(2\pi)^4} \tilde{\Delta}_{W^mW^n}(x, \partial_x) \exp[ip(x-y)] \\
= \int \frac{d^4p}{(2\pi)^4} \exp[ip(x-y)]\tilde{\Delta}_{W^mW^n}(x, \partial_x + ip).
$$

Then, the trace can be determined:

$$
Tr \ln \left[\tilde{\Delta}_{W^mW^n}(x, \partial_x)\delta^{(4)}(x-y)\right] = \int d^4x \int \frac{d^4p}{(2\pi)^4} tr \ln(\tilde{\Delta}_{W^mW^n}(x, \partial_x + ip)) ,
$$

here the "tr" means the sum over group and spin indices. $\tilde{\Delta}_{W^mW^n}(x, \partial_x + ip)$ can be expanded in terms of derivatives,

$$
\tilde{\Delta}_{W^mW^n}(x, \partial_x + ip) = \sum_{m=0}^{\infty} \frac{(-i)^m}{m!} \frac{\partial^m}{\partial p_{\mu_1} \cdots \partial p_{\mu_m}} \tilde{\Delta}_{W^mW^n}(x, ip) \partial_{\mu_1} \cdots \partial_{\mu_m}. \tag{60}
$$
in the 't hooft-Feynman gauge, it yields an expression like

$$
\tilde{\Delta}_{W^mW^n}(x, \partial_x + ip) = (p^2 - n^2M_c^2)\delta^{ab}(x, p, \partial_x). \tag{61}
$$

Dropping an irrelevant constant, we get

$$
tr\tilde{\Delta}_{W^mW^n}(x, \partial_x + ip) = \sum_{m=1}^{\infty} \frac{(-1)^m+1}{m} \frac{n}{n^2M_c^2} tr\left(\frac{\Pi}{p^2 - n^2M_c^2}\right)^m \tag{62}
$$

We are interested in those terms caused by the mixing among KK modes. According to the standard procedure given in [21], when expanding $\ln \tilde{\Delta}_{W^mW^n}(x, \partial_x + ip)$ we determine the leading powers of $p$, $W^m$ and $M_c$ for each term generated and introduce an auxiliary parameter $\zeta$, which counts these powers

$$
p_{\mu} \rightarrow \zeta, \quad M_c \rightarrow \zeta, \quad W^m \rightarrow \zeta^{-2} \frac{g_n}{n^2g}. \tag{63}
$$

We would like to mention that the $W^m$ is not only suppressed by its mass, but also by the brane fluctuation factor $g_n/g$. This counting rule tells us that the contribution of $\Delta_{W^mW^n}\Delta_{W^mW^n}^{-1}\Delta_{W^mW^n}$ is suppressed at least by $1/M_c^4(g_n/g)^2$. So we can neglect this term and extract terms reliably up to $1/M_c^2$. Then the procedure to evaluate the trace and logarithm is greatly simplified. For the operator $\Delta_{W^mW^n}(x, \partial_x + ip)$, we have the same conclusion. So we have

$$
S_{YM,KK}^{eff,1-loop} = \frac{i}{2} \int_x \left[tr\ln\Delta_{W^mW^n} - tr\ln\Delta_{W^mW^n}^{-1}\right] + O(\frac{1}{M_c^4}). \tag{64}
$$

To get the above equation we have used the relation $\Delta_{W^mW^n}^{-1} = \Delta_{W^nW^m}$.  

\textsuperscript{2}Even though the term $trX \equiv tr\Delta_{W^mW^n}^{-1}\Delta_{W^mW^n}\Delta_{W^mW^n}^{-1}\Delta_{W^mW^n}$ can provide contributions of order $M_c^2$ and $\ln M_c$, these contributions are proportional to $1/M_c^2(g_n/g)^2$ and $\ln M_c^2/M_c^4(g_n/g)^2$. Under the assumption of brane fluctuation suppression, we will omit them in the below analysis.
D. Evaluating the trace and logarithm by using the method of Heat kernel

Now, it becomes easy to evaluate the trace and logarithm by utilizing the method of heat kernel \([27]\), up to \(O(p^6)\), which reads

\[
S_{\text{loop}} = -\frac{1}{2(4\pi)^{d/2}} \int \left\{ \frac{m^d}{2} \left( \frac{d}{2} \right) \left( \text{tr}_0^W - \text{tr}_0^{cW} \right) + m^{d-2} \Gamma \left( 1 - \frac{d}{2} \right) \left( \text{tr}_{1}^W - \text{tr}_{1}^{cW} \right) 
\right.
\]
\[
+ m^{d-4} \Gamma \left( 2 - \frac{d}{2} \right) \left( \text{tr}_2^W - \text{tr}_2^{cW} \right) + m^{d-6} \Gamma \left( 3 - \frac{d}{2} \right) \left( \text{tr}_3^W - \text{tr}_3^{cW} \right) 
\left. + \cdots \right\}, \quad (65)
\]

where \(a_i^a\) are the Seeley-DeWitt coefficients of the corresponding quadratic operators. For the generic operator of the form \(\Delta = D^2 + M^2 + \sigma\), the Seeley-DeWitt coefficients in the coincidence limit read

\[
a_0 = 1, \quad (66)
\]
\[
a_1 = -\sigma, \quad (67)
\]
\[
a_2 = \frac{1}{2} \sigma^2 - \frac{g^2}{12} F^{\mu\nu} F_{\mu\nu} + \frac{1}{6} [D_\mu, [D^\mu, \sigma]], \quad (68)
\]
\[
a_3 = -\frac{1}{6} \sigma^3 + \frac{1}{12} \left\{ \{\sigma, D^2 \sigma\} + D^\mu \sigma D_\mu \sigma \right\} - \frac{1}{60} D^2 D^2 \sigma + i \frac{g}{60} [D_\alpha F^{\alpha\mu}, D_\mu \sigma] 
\]
\[
+ \frac{g^2}{60} (2 F_{\mu\nu} F^{\mu\nu}, \sigma) + F_{\mu\nu} \sigma F^{\mu\nu} - \frac{g^2}{45} D_\alpha F^{\alpha\mu} D^\beta F_{\beta\mu} 
\]
\[
- \frac{g^2}{180} D_\alpha F_{\beta\gamma} D^\alpha F^{\beta\gamma} - \frac{g^2}{60} \left\{ F_{\mu\nu}, D^2 F^{\mu\nu} \right\} - i \frac{g^3}{30} F_{\mu\nu} F^{\mu\alpha} F^\alpha_{\nu}. \quad (69)
\]

The \(a_0\) terms will contribute divergently but can be removed by redefining the vacuum. The \(a_1\) term simply vanishes for \(\Delta^{W\ast W}\) and \(\Delta^{cW\ast cW}\). The \(a_2\) term is non-zero and contributes to the hidden operators \([24]\) in \(O(p^4)\), which reads

\[
L_{\text{eff}}^{1-\text{loop}}(p^4) = -\frac{1}{2(4\pi)^{d/2}} \sum_{n=1}^{\infty} \left( n^2 M_c^2 \right)^{d-4} \Gamma \left( 2 - \frac{d}{2} \right) (EC_W - EC_{cw}) \frac{g^2}{4} W^{\mu\nu} W_{\mu\nu}, \quad (70)
\]

where

\[
EC_i^4 = \left[ \frac{1}{3} d_i(j) - 4 c_i(j) \right] C_i(G), \quad i = W, \ c_W, \quad (71)
\]

where the \(C_i(G)\) is the quadratic Casimir operator of the adjoint representation of the group, the \(d(j)\) is the number of spin components \([23]\) and

\[
d(j) = 1, \quad \text{for scalar(ghost)},
\]
\[
= 4, \quad \text{for vector boson}. \quad (72)
\]

While \(c(j)\) is the trace over spin indices and is defined as
\[ tr[J^{\alpha \beta} J^{\alpha \beta}] = (g^{\rho \alpha} g^{\sigma \beta} - g^{\rho \beta} g^{\sigma \alpha}) c(j), \]  
and \( c(j) \) has values as given below
\[ \begin{align*}
  c(j) &= 0, \text{ for scalar (ghost)}, \\
  &= 2, \text{ for vector boson}.
\end{align*} \]

The hidden operators can be eliminated by redefining the wave-function and gauge coupling of the zero mode. So we see that up to \( O(p^4) \), the KK excitations completely decouple from the low energy observables. The underlying reasons for this decoupling behavior of KK excitations can be traced back to the momentum conservation of the fifth dimension and the gauge structure of the Lagrangian given in Eq. (35).

However, up to \( O(p^6) \), the contribution of KK excitations is non-zero, and we have
\[ \begin{align*}
  L_{\text{eff}}^{1-\text{loop}}(p^6) &= -\frac{1}{2(4\pi)^{d/2}} \sum_{n=1}^{\infty} (nM_c)^{d-6} \Gamma \left( 3 - \frac{d}{2} \right) \left[ (EC_i^6 - EC_{W,c}^6) O_i^6 \right. \\
  &\quad \left. + (FC_i^6 - FC_{W,c}^6) O_i^6 \right] \\
  &= c_i^6 O_i^6 + c_i^2 O_i^6,
\end{align*} \]
where
\[ \begin{align*}
  O_1^6 &= g^2(D_\mu W^{\mu \nu})^a (D^a W_{\nu \omega})^a, \\
  O_2^6 &= g^3 W^{\alpha \mu \nu} W_\mu^b \alpha W_\nu^c f^{abc}, \\
  EC_i^6 &= \frac{1}{30} \left[ -d_i(j) + 10c_i(j) \right] C_i(G), \quad i = W, c_W, \\
  FC_i^6 &= \frac{1}{180} \left[ 2d_i(j) - 15 (c_i'(j) + 2c_i(j)) \right] C_i(G), \quad i = W, c_W,
\end{align*} \]
with
\[ \begin{align*}
  c'(j) &= 0, \text{ for scalar (ghost)}, \\
  &= 8, \text{ for vector boson}.
\end{align*} \]

which is defined from
\[ \begin{align*}
  tr(J^{\mu \nu} J^{\rho \sigma} J^{\alpha \beta}) W^a_{\mu \nu} W^b_{\rho \sigma} W^c_{\alpha \beta} f^{abc} &= -ic_i'(j) W^{a \mu \nu} W^b_{\mu \nu} W^c_{\nu \sigma} f^{abc},
\end{align*} \]
To get Eqs. (70) and (75), we have used the partial integration, the Bianchi identity which reads
\[ \begin{align*}
  D_\mu W_{\nu \rho} + D_\nu W_{\rho \mu} + D_\rho W_{\mu \nu} = 0,
\end{align*} \]
and the relations of adjoint representations
\[ \begin{align*}
  tr[t_{a_1 G_1}^{a_2 b_1}] &= C_2(G) \delta^{ab}, \quad tr[t_{G_1 G_2}^{a_1 b_1}] = i \frac{C_2(G)}{2} f^{abc}, \\
  [D_\mu, D_\nu] &= D_\mu^{ae} D_\nu^{eb} - D_\mu^{ae} D_\nu^{eb} = -ig W_{\mu \nu}^{c} (t_G^{eb}f_{ab}).
\end{align*} \]
It is remarkable that the contribution of a vector boson is much larger than that of a scalar (ghost), since a vector boson has four components and has a spin coupling with the background field, the contribution of which is represented by \( c(j) \) and \( c'(j) \).
V. DISCUSSIONS AND CONCLUSIONS

We know that the low energy oblique parameters, U, S, and T \[^{28}\] always put a very stringent constraint on the possible new physics \[^{29}\]. According to the standard electroweak chiral Lagrangian up to \(O(p^4)\) \[^{30}\], U, S, and T are related with the coefficients of operators up to \(O(p^4)\). While the result given in the Eq. (70) tells us that at the \(O(p^4)\) order, these low energy precision tests will not put any constraint on the brane-fluctuation and deconstructing-extra-dimension models.

The operators \(O_1^6\) and \(O_2^6\) belong to the contact operators in the complete set of operators of order \(O(p^6)\) \[^{14}\]. The EOM of the zero mode given in Eq. (36) can change the operator \(O_1^6\) to the following form

\[
O_1^6 = g^2 \left[ m_W^2 W_\mu + J_\mu (\text{light}) \right] \left[ m_W^2 W_\mu + J^\mu (\text{light}) \right]
\]  

Then form the Eq. (85) we know that KK excitations can contribute to the low energy fermion scattering processes.

The operator \(O_2^6\) will contribute to the anomalous trilinear vector couplings (say \(WWZ\) and \(WW\gamma\)), the anomalous quartic photonic vector couplings (\(WW\gamma\gamma\) and \(WWZ\gamma\)) and higher order gauge couplings.

In 5D, the operators \(O_i^6\) will contribute convergently even when the KK excitations are infinite, since the sum

\[
\sum_{n=1}^{\infty} \frac{1}{n^2} = \frac{\pi^2}{6},
\]

is finite. But in higher dimension, i.e. \((4+\delta)D\) and \(\delta \geq 2\), the sum is given by

\[
\text{sumKK} \equiv \frac{1}{2} \sum_{n=1}^{\infty} \frac{1}{n^2} \approx \frac{\pi^2}{2\Gamma(1+\frac{\delta}{2})} \int_{n=1}^{N_{UV}} n^{\delta-3} \, dn,
\]

\[
\approx \frac{\pi}{2} \ln N_{UV}, \quad \text{for } \delta = 2;
\]

\[
\approx \frac{\pi^{\frac{\delta}{2}}}{2\Gamma(1+\frac{\delta}{2})} \left( \frac{1}{\delta-2} \right) \left( N_{UV}^{\delta-2} - 1 \right), \quad \text{for } \delta \geq 3,
\]

then the operator \(O_i^6\) will contribute divergently and the meaningful theoretical prediction can only be made when the explicit ultraviolet cutoff \(M_{UV}\) is chosen (the relation between \(N_{UV}\) and \(M_{UV}\) is given as \(N_{UV} = M_{UV}/M_c\)). This fact reflects that the brane fluctuation suppression mechanism can work well at tree level. But at loop level and in the bosonic part, a more radical mechanism is needed in order to regularize the divergences brought into by the infinite KK towers.

For the general \((4+\delta)D\) extra dimensions model with brane fluctuation, the coefficients of the operators \(O_1^6\) and \(O_2^6\) will depend upon the number of extra dimensions \(\delta\), the size of the compactification scale \(M_c\), and the explicit ultraviolet cutoff \(M_{UV}\) of the effective theory.
The magnitude of $c_i^6$ is determined by the loop factor $1/(16\pi^2)$, the $M_c^2$, the symmetric factor 6, the $EC_6^0$, and the sum over KK excitations. The loop factor is about $10^{-2}$, the $M_c^2$ is assumed to be in the range of $0.5 \sim 1$ TeV and can provides a factor about $10^{-5} \sim 10^{-6}$ GeV$^{-2}$, and the $EC_6^0$ are about 3. If we take $M_c = 500$ GeV, $M_{UV} = 10$ TeV, and $\delta = 2$, the $c_i^6 \approx 10^{-6} \sim 10^{-7}$ GeV$^2$; if we take $M_c = 500$ GeV, $M_{UV} = 10$ TeV, and $\delta = 4$, the $c_i^6$ can reach $10^{-3} \sim 10^{-4}$ GeV$^{-2}$.

The present experimental accuracy on the anomalous triple vector coupling $\lambda_V$ is of order $-6.2 \times 10^{-2} \sim 1.47 \times 10^{-1}$ [32]. The relation between $\lambda_V$ and $c_i^6$ is given as

$$g^2 c_i^6 = \frac{\lambda_V}{M_W^2},$$

(88)

And the $\lambda_V$ can be expressed as

$$\lambda_V = 6 \times \alpha_W \left( \frac{M_W}{M_c} \right)^2 \left( FC_6^6 - FC_6^6 \right) \text{sumKK},$$

$$\approx 1. \times 10^{-2} \left( \frac{\Lambda}{M_c} \right)^2 \text{sumKK},$$

(89)

where $\Lambda = 1$ TeV. If we take $M_c = 0.5$ TeV and $\delta = 1$, the value of $\lambda_V$ is $1. \times 10^{-3}$. The typical value of $\lambda_V$ is of order $10^{-3} \sim 10^{-4}$ which is within the reach of 500 GeV LC [33] and LHC.

About the anomalous quartic coupling, according to the analysis of [13, 34], operator $O_2^6$ can be in principle be detected via the process $e^+e^- \rightarrow W^+W^-\gamma$. Present experimental accuracy from LEP2 is of $10^{-2}$ GeV$^{-2}$ and will increase to $10^{-5}$ GeV$^{-2}$ at the LC and LHC.

We would like to mention that if the extra dimension(s) are compactified on $T^8$ torus, the contributions of KK excitations will double. Because there are not only the contribution of cosine modes but also sine modes for each field in the bulk.

In conclusion, we study the bosonic part in the brane fluctuation model where the couplings of the fermionic and bosonic currents on the brane and KK excitations are exponentially suppressed. Since the couplings among vector bosons do not suffer this substantially suppression, they could help us to probe extra dimension in the future LC and LHC. But due to the momentum conservation and the gauge structure of zero mode and KK excitations, up to $O(p^4)$, KK excitations decouple from the low energy physics. However, up to $O(p^6)$, it is still possible to detect the effects of KK excitations through precision measurement of the bosonic sector of the SM in LHC and LC.

ACKNOWLEDGMENTS

One of the authors, Qi-Shu Yan would like to thank Dr. X. J. Bi, Dr. M. Huang, Prof. Qing Wang, and Prof. Yu-Ping Kuang in physics department of the Tsinghua University, Prof. C.S. Huang in the ITP of CAS, and Prof. Xinning Zhang in the IHEP of CAS for helpful discussions. The work is supported in part by National Natural Science Foundation of China.
REFERENCES

[1] C. S. Kim, J.D. Kim, and J. Song, Phys. Lett. B 511 (2001) 251 [hep-ph/0103127]; R. S. Chivukula, D. A. Dicus, and H. J. He, [hep-ph/0111016].
[2] Qi-Shu Yan and Dong-Sheng Du, [hep-th/0112295].
[3] M. Masip, Phys. Rev. D. (2000), hep-ph/007048.
[4] M. Bando, T. Kugo, T. Noguchi, and K. Yoshioka, Phys. Rev. Lett. 83 (2000) 3601 [hep-ph/9906539]; J. Kubo, H. Terao, and G. Zoupanos, [hep-ph/9910277].
[5] T. Kugo, and K. Yoshioka, Nucl. Phys. B 594 (2001) 301.
[6] P. Creminelli, and A. Strumia, [hep-ph/0007267]; A. Dobado, and A. L. Maroto, [hep-ph/0007100]; S.C. Park and H.S. Song, [hep-ph/0109258, hep-ph/0109121].
[7] D.M. Ghilencea, H.P. Nilles, and S. Stieberger, [hep-th/0108183].
[8] N. Arkani-Hamed, A. G. Cohen, and H. Georgi, [hep-th/0104005].
[9] H.C. Cheng, C.T. Hill, S. Pokorski, and J. Wang, [hep-th/0104179].
[10] L. Hall, Y. Nomura, and D. Smith, [hep-ph/0107331]; I. Antoniadis, K. Benakli, and M. Quiros, [hep-th/0108003].
[11] M. Masip and A. Pomarol, Phys. Rev. D 60 (1999) 096005; A. Delgado, A. Pomarol, M. Quiros, JHEP 0001:030(2000); T.G. Rizzo, and J.D. Wells, Phys. Rev. D 61(2000)016007; C.D. Carone, Phys. Rev. D 61(2000)015008.
[12] H. Georgi, preprint HUTP-90/A077; F. Feruglio, Int. J. Mod. Phys. A 8 (1993) 4937.
[13] T. Appelquist and J. Carazzone, Phys. Rev. D 11 (1975) 2856.
[14] W. BuchMüller, and D. Wyler, Nucl. Phys. B 268(1986) 621; H. W. Fearing and S. Scherer, Phys. Rev. D 53 (1996) 315; J. Bijnens, G. Colangelo, and G. Ecker, [hep-ph/9902437].
[15] D. Dominici, [hep-ph/0012028].
[16] I. Antoniadis, Phys. Lett. B 246 (1990) 377; I. Antoniadis and K. Benakli, Phys. Lett. B 326 (1994) 69; I. Antoniadis, et. al., Nucl. Phys. B 544 (1999) 503.
[17] P. Creminelli and A. Strumia, [hep-ph/0007267].
[18] B.S. DeWitt, Phys. Rev. 162 (1967) 1195; Phys. Rev. 162 (1967) 1239.
[19] Qi-Shu Yan, [hep-th/0103189].
[20] A. Nyffeler and A. Schenk, [hep-ph/9409436] and [hep-ph/9907294].
[21] S. Dittmaier and C. Grosse-Knetter, [hep-ph/9505260] and [hep-ph/9501285].
[22] E. D’Hoker and E. Fahri, Nucl. Phys. B 248 (1984) 59; Nucl. Phys. B 248 (1984) 77.
[23] M.E. Peskin and D.V. Schroeder, An Introduction to Quantum Field Theory, Addison-Wesley Publishing Company, Chapter 16.6.
[24] J. Gasser and H. Leutwyler, Ann. Phys. 158 (1984) 142.
[25] M. Bilenky and A. Santamaria, Nucl. Phys. B 420 (1994) 47; L. H. Chan, Phys. Rev. D 36 (1987) 3755.
[26] L. H. Chan, Phys. Rev. Lett. 54 (1985) 1222 [Err. 56 (1986) 404]; 57 (1986) 1199.
[27] R. D. Ball, Phys. Rep. 182 (1989) 1.
[28] M.E. Peskin and T. Takeuchi, Phys. Rev. Lett. 65 (1990) 964; Phys. Rev. D 46 (1992) 381.
[29] M.E. Peskin and J.D. Wells, Phys. Rev. D 64 (2001) 093003 [hep-ph/0101342].
[30] T. Appelquist and G. H. Wu, Phys. Rev. D 48 (1993) 3235.
[31] K. Hagiwara, R. D. Peccei, D. Zeppenfeld, and K. Hikasa, Nucl. Phys. 283 (1987) 253; J. Ellison and J. Wudka, Ann. Rev. Nucl. Part. Sci. 48, (1998) 33.

[32] The ALEPH Collaboration, hep-ex/0104034.

[33] American Linear Collider Working Group, hep-ph/0007022.

[34] G. Bélanger, et. al., Eur. Phys. J. C. 13 (2000) 283.