Comment on “$B \to M_1 M_2$: Factorization, charming penguins, strong phases, and polarization”

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We show that the factorization formula for non-leptonic $B$ decays to two light flavor non-singlet mesons derived by Bauer et al. in the context of soft-collinear effective theory is equivalent to the corresponding formula in the QCD factorization approach. The apparent numerical differences in the analysis of $B \to \pi\pi$ data performed by these authors, as compared to previous QCD factorization analyses, can largely be attributed to the neglect of known perturbative and power corrections.

The extent to which hadronic decays of $B$ mesons to two light hadrons can be computed from first principles in QCD has been the subject of many investigations, following the statement [1, 2] that the decay amplitudes factorize in the limit of very large $B$-meson mass. Taking the final state to consist of two pions, the amplitude can be represented schematically in the form

$$A(B \to \pi\pi) = F^{B\to\pi} T^{I} \Phi_\pi + F^{B\to\pi} T^{II} \Phi_B \Phi_\pi \Phi_\pi.$$  \hspace{1cm} (1)

In this equation, $F^{B\to\pi}$ denotes a physical form factor, and $\Phi_B$ and $\Phi_\pi$ are the leading-twist light-cone distribution amplitudes of the mesons. The quantities $T^{I,II}$ are perturbative hard-scattering kernels involving the two scales $m_b$ (hard) and $\sqrt{m_b} \Lambda$ (hard-collinear), which are linked to the other elements of the formula by convolution integrals (indicated by an asterisk).

The authors of [3] suggest a factorization formula (their Eq. (24)) similar to (1) in the framework of soft-collinear effective theory (SCET) and imply that it is conceptually different. They also argue that, even at leading order in the $1/m_b$ expansion, there may be an additional term on the right-hand side of (1), corresponding to long-distance contributions from $c\bar{c}$ penguins. They do not disprove factorization of charm-penguin loops by providing a counter-example to factorization; rather, they state that they were not able to demonstrate factorization.

In this Comment, we explain why the formula given in [3] is identical in content to the QCD factorization formula (1), and why non-factorizable charm-penguin contributions are of higher order in the $1/m_b$ expansion. We also point out that the phenomenological analysis of [3] neglects important perturbative and power corrections which are already known. Once these are included, there is little room for significant additional contributions to the QCD penguin amplitude.

I. EQUVALENCE OF THE SCET AND QCD FACTORIZATION FORMULAE

We first note that the coefficient function of the spectator-scattering term can be represented as a convolution $T^{II} = C^{II} \ast J$ of hard and hard-collinear coefficient functions. The formula quoted in [3] follows from (1) by rewriting

$$T^{II} \ast \Phi_B \ast \Phi_\pi \ast \Phi_\pi = C^{II} \ast \zeta^{B\pi} \ast \Phi_\pi,$$  \hspace{1cm} (2)

with $\zeta^{B\pi}$ defined as $J \ast \Phi_B \ast \Phi_\pi$, and $C^{II}$ defined by the decomposition of $T^{II}$ shown above. In addition, in [3] the SCET form factor $\zeta^{B\pi}$ rather than the physical QCD form factor $F^{B\to\pi}$ is used. As discussed in [2], this implies another rearrangement of this type.

In general SCET provides a powerful tool to simplify factorization proofs (a task that has not yet been completed for the case of $B \to \pi\pi$ considered here), but the resulting QCD factorization formulae can also be obtained using traditional factorization methods, as is frequently done in practice.

The authors of [3] entertain the possibility that the hard-collinear scale may be non-perturbative, and hence they choose not to factorize $\zeta^{B\pi}$ into $J \ast \Phi_B \ast \Phi_\pi$ as indicated above. This is a logical possibility in the QCD factorization approach. However, we show below that it is not supported by theoretical calculations.

We disagree with [3] on the statement that $\zeta^{B\pi} \ll F^{B\to\pi}$ is a prediction of QCD factorization, whereas the SCET treatment suggests $\zeta^{B\pi} \sim F^{B\to\pi}$. The factorization formula states that both terms in (1) are of the same order in $1/m_b$ power counting and that $T^I$ and $T^{II}$ start at $O(\alpha_s^3)$ and $O(\alpha_s^2 \sqrt{m_b} \Lambda)$, respectively. However, it does not predict the relative size of the two terms, and as we explain below, the numerical value of $\zeta^{B\pi}$ depends on several hadronic input parameters, which are rather uncertain at present.

The authors of [3] point out that the hard-collinear kernel $J$ is universal, so that only a single function $\zeta^{B\pi}$ appears in [2], which is the same that appears in the factorization of the $B \to \pi$ form factors. This fact is important for phenomenology when one opts to treat the hard-collinear scale as non-perturbative, but does not by itself represent a conceptual difference between the formulae given in [1, 2] and [3]. Furthermore, the usefulness of the universality of $J$ is limited to the approxima-
tion where one neglects radiative corrections to the hard-scattering kernels $C^{\Pi}$, since only then does the function $\zeta_j^{B\pi}$ reduce to a single number $\mathbb{A}$, $\mathbb{B}$. In other words, a phenomenological treatment of the hard-collinear scale as non-perturbative relies on the approximation that the kernels are restricted to their tree-level approximations, whereas one of the key features of QCD factorization (as opposed to naive factorization) is that one can consistently include radiative corrections.

II. CHARM-PENGUIN LOOPS

In $\mathbb{B}$ it is claimed that there is a possible exception to factorization from diagrams with charm-quark loops (see Figure I). The argument is based on the observation that when the gluon virtuality is near the $c\bar{c}$ threshold, $q^2 \approx 4m_c^2$, the non-relativistic scales $m_v$ and $m_c v^2$ become important. Since $m_c v^2 \approx \Lambda$ numerically, this appears to introduce a sensitivity to non-perturbative scales without power-suppression in $1/m_b$. The authors of $\mathbb{B}$ suggest that these diagrams do not factorize, in the sense that the long-distance physics at leading-power in the $1/m_b$ expansion cannot be factored into form factors or light-cone distribution amplitudes. We emphasize that the question of non-factorizable $c\bar{c}$ effects at leading order in the heavy-quark expansion is a different issue than the one raised in $\mathbb{B}$, where it is speculated that power corrections to the QCD charm-penguin amplitudes may be numerically large.

Factorization statements, be they derived diagrammatically or with soft-collinear effective theory, always concern properties of the amplitude in certain asymptotic limits, here an expansion in $1/m_b$, independent of the actual size of the expansion parameter. It is therefore important to clearly distinguish the issue of factorization at leading order in the $1/m_b$ expansion from the question of whether there are non-factorizable contributions which are formally power-suppressed, but which nevertheless may be numerically significant. In the following discussion, we focus on the question of factorization in the formal heavy-quark limit.

The most intuitive way of understanding why the threshold region does not require special treatment is based on quark-hadron duality $\mathbb{B}$. The integration over the gluon virtuality in the range $0 \leq q^2 \leq m_b^2$ is weighted by the pion distribution amplitude, which is smooth over the entire integration region. This provides the necessary smearing of the loop amplitude, which ensures that the result is given by a simple partonic calculation up to power corrections, in complete analogy with the standard justification of the partonic interpretation of inclusive heavy-meson decays, or cross sections in general. The smearing also ensures that one can apply the same power-counting arguments that demonstrate factorization of other diagrams to charm-penguin diagrams with no need to single out the threshold region. In particular, in the non-relativistic situation implied in $\mathbb{B}$ there is no need to sum Coulomb ladder diagrams, since these do not result in large perturbative corrections after the integration over $q^2$.

Even without invoking the duality argument, the fact that the charm threshold region comprises only a parametrically small portion of the entire integration implies a phase-space suppression. This fact has been neglected in the argument of $\mathbb{B}$. More precisely, writing $q^2 = \tilde{x} m_b^2$, where $\tilde{x}$ denotes the longitudinal momentum fraction of the anti-quark in one of the pions, this region is $\Delta \tilde{x} \sim v^2 (m_c/m_b)^2$ or $\Delta \tilde{x} \sim \Lambda m_c/m_b^2$, whichever is larger. In order to study the question of whether long-distance $c\bar{c}$ loop effects are of leading order or not, it is necessary to decide how the limit $m_b \rightarrow \infty$ is to be taken. If we define the heavy-quark limit by $m_b \rightarrow \infty$ with $m_c$ fixed, one may distinguish several possibilities such as $m_c \sim \Lambda$, $m_c v^2 \sim \Lambda$, or $m_c v^2 \gg \Lambda$. While the physics of the threshold region is very different for all these cases, they share the common feature that, for the purposes of power counting, the charm quark can be considered to be a light quark, and the suppression of long-distance $c\bar{c}$ effects has the same origin as that for the corresponding diagrams with light-quark loops, which implies $\Delta \tilde{x} \sim 1/m_c^2$. In addition, since in this region $\tilde{x} \sim m_c^2/m_b^2$, there is a further suppression due to the end-point behavior of the pion distribution amplitude (which vanishes linearly as $\tilde{x} \rightarrow 0$).

If we define the heavy-quark limit as $m_b,c \rightarrow \infty$ with the ratio $m_c/m_b$ fixed, then there is no power suppression due to the phase-space or end-point behavior of the distribution amplitudes, but the threshold region is perturbative, up to a small non-perturbative contribution of order $v^2 \cdot v^3 \cdot (\Lambda/m_c)^4 \sim (\Lambda/m_b)^4$ $\mathbb{F}$.

We conclude that charm-penguin diagrams factorize at leading power in $1/m_b$. The argument for factorization remains valid also for more complicated higher-order penguin graphs whenever the threshold region is phase-space (and end-point) suppressed or the charm quark is heavy so that perturbation theory is applicable.

III. VALIDITY OF PERTURBATION THEORY AT THE HARD-COLLINER SCALE

In applying the QCD factorization formula to phenomenology the authors of $\mathbb{B}$ treat the hard-collinear scale $\sqrt{m_b \Lambda}$ as non-perturbative, and hence the quantity $\zeta_j^{B\pi}$ as an unknown phenomenological function. This is
justified \textit{a posteriori} following the result of a phenomenological fit (on which we comment below). This line of argument ignores the fact that perturbation theory at scales of order \( \sqrt{m_b \Lambda} \sim m_c \) has been used successfully in many important applications in B-physics, including all determinations of \( |V_{ub}| \) from inclusive B-decays, and studies of the hadronic decay rate of the \( \tau \) lepton. As already mentioned, a serious drawback of treating the hard-collinear scale as non-perturbative is that it renders the factorization approach unpredic\-tive beyond the tree approximation, because only the integral over \( \zeta_{B\pi} \) and not its functional form can be extracted from measurements.

A systematic way to address the question of the perturb\-ativity of the hard-collinear scale is to calculate higher-order corrections in \( \alpha_s \) to the jet function \( J \) in the product \( T^{H} = C^{H} \ast J \), defined as the Wilson coefficient function arising in the matching of certain (type-B) SCET\(_1\) current operators onto four-quark operators of SCET\(_{1p}\). The next-to-leading order terms have been computed recently and were found to be small \cite{5}. Specifically, for the case of light pseudoscalar mesons and an asymptotic light\-cone distribution amplitude \( \Phi_+(x) = 6x(1 - x) \), one finds that the convolution integrals over the jet function give rise to the series

\[
\frac{\alpha_s(\mu_i)}{\lambda_B} \left[ 1 + \frac{\alpha_s(\mu_i)}{\pi} \left( \frac{L^2}{3} - 1.31(L) + 1.00 \right) + \ldots \right],
\]

where \( \mu_i \sim \sqrt{m_b \Lambda} \) is the hard-collinear scale, \( \lambda_B \) is the first inverse moment of the \( B \)-meson distribution amplitude \( \Phi_B(\omega, \mu_i) \), \( L = \ln(m_B \omega / \mu_i^2) \), and \( \langle \ldots \rangle \) denotes an average over \( \Phi_B(\omega, \mu_i) \) with measure \( d\omega/\omega \). While the precise form of the \( B \)-meson distribution amplitude is unknown, the fact that \( \omega \sim \Lambda \) ensures that \( L \) cannot be large, giving a small coefficient to the next-to-leading term. For example, using the results of \cite{8} for the moment \( \langle L^2 \rangle \), \( \langle L \rangle \) the coefficient of \( \alpha_s / \pi \) in \( \langle L \rangle \) is \( 2.2 \pm 0.6 \) for \( \mu_i^2 = 0.5 \text{ GeV} m_b \). There is thus no evidence that perturbation theory cannot be applied at the hard-collinear scale. We also note in this context that the power corrections from the hard-collinear scale are \( 1/m_b \) suppressed (and not \( 1/\sqrt{m_b} \)) just as those from the hard scale.

Since the perturbative corrections to the jet function are well behaved, the quantity \( \zeta_{B\pi} \) can be factorized and expressed in terms of convolution integrals over light-cone distribution amplitudes. The question of the numerical value of \( \zeta_{B\pi} \), and whether it is a small contribution to the physical form factor \( F_{B \rightarrow \pi} \), rests on the properties of these amplitudes, as well as on other parameters such as the strange-quark mass. At leading order in perturbation theory, we obtain for \( \zeta_{B\pi} \) the result

\[
\zeta_{B\pi} = \frac{3\pi \alpha_s C_F}{N_c^2} \frac{f_B f_\pi}{M_B \lambda_B} \left( \frac{1}{r_N} + r_N^2 X_H \right)
\]

where the notations of \cite{8} have been taken. Using the default values and uncertainties of the input parameters from this reference, and adding errors in quadrature, yields \( \zeta_{B\pi} = 0.016 - 0.064 \), which is small compared with typical values \( F_{B \rightarrow \pi} = 0.24 - 0.30 \). \( (F_{B \rightarrow \pi} = \zeta_{B\pi} + \zeta_{B\pi} \text{ when hard matching corrections are neglected}) \) Taking some correlated parameter variations so as to reproduce the data on \( B \rightarrow \pi\pi \) decays, scenario S2 of \cite{8} yields the somewhat increased value \( \zeta_{B\pi} = 0.080 \). To obtain significantly larger results would require a very small value of the hadronic parameter \( \lambda_B \). While this is a logical possibility, a recent QCD sum-rule calculation of \( \lambda_B \) gives a value around 0.45 GeV \cite{8}, which is in fact somewhat larger than the estimate adopted in \cite{1,2}.

These estimates are to be compared to the fit result \( \zeta_{B\pi} = 0.11 \pm 0.03 \) obtained by the authors of \cite{8}, who also find that the bulk of the \( B \rightarrow \pi \) form factor comes from \( \zeta_{B\pi} \), giving the very small result \( F_{B \rightarrow \pi} = 0.17 \pm 0.02 \). This picture contradicts the QCD sum rules for heavy-to-light form factors, in which \( \zeta_{B\pi} \) must be associated with a radiative correction \cite{11}. The preference of the data for a smaller \( B \rightarrow \pi \) form factor together with an increase of the hard-spectator scattering contribution to the color-suppressed tree amplitude \( a_2 \) has already been discussed in \cite{8}, which however did not arrive at a similarly extreme conclusion. This discrepancy can be traced to a few omissions in the calculation of \cite{8}, each of which has a minor effect: the absence of radiative corrections, the absence of phases in tree amplitudes, the absence of the scalar up-penguin amplitude in \( \tau \hbar \) decays, scenario S2 of \cite{9} yields the larger value of \( |V_{ub}| \). When these effects are taken into account and combined with the most recent experimental data, one finds a significantly smaller value of \( \zeta_{B\pi} \) and a larger value of \( F_{B \rightarrow \pi} \), in qualitative agreement with theoretical expectations.

IV. THE QCD PENGUIN AMPLITUDE

We now turn to the discussion of the phenomenological analysis of the \( B \rightarrow \pi\pi \) data performed in \cite{8}. Our principal criticism in addition to what has already been described concerns the evaluation of the QCD penguin amplitude. It may be written as

\[
P = a_4 + r_N a_6 + \beta_3 \approx -0.09,
\]

where \( a_4 \approx -0.023 \langle \alpha_s^0 \rangle - 0.002 \langle \alpha_s^1 \rangle \) represents the vector penguin contribution, \( r_N a_6 \approx -0.038 \langle \alpha_s^0 \rangle - 0.014 \langle \alpha_s^1 \rangle \) the \( 1/m_b \) suppressed scalar penguin contribution, and \( \beta_3 \approx -0.011 \) a power suppressed and rather uncertain penguin annihilation term. (The numbers are based on the analysis in \cite{8}. Without errors they should be taken only for illustration purposes. In particular, we neglected all phases, since they are unimportant for the following discussion.)

In the calculation of \cite{8} the lowest order \( \langle \alpha_s^0 \rangle \) and some of the \( \alpha_s \) contributions to \( a_4 \) (those included in the phenomenological parameters for hard scattering and charm penguins) are taken into account. The term \( r_N a_6 + \beta_3 \) is dropped, because it is power-suppressed. Now while it is true that the factorization properties of power-suppressed
contributions in general, and scalar penguin contributions in particular, have not yet been investigated to all orders in perturbation theory, the large tree-level contribution to $r_\chi a_6$ suggests that if one neglects power corrections entirely (as done in [3]) one is certain to obtain a poor approximation. We emphasize that this is unrelated to the charm-quark loops discussed above, which appear only in the small $\alpha_s$ corrections. By dropping the scalar penguin amplitude, the authors of [3] are forced to erroneously assign the QCD penguin amplitude almost entirely to the charm-quark loops.

There is considerable phenomenological evidence that the scalar penguin amplitude is in approximate agreement with our theoretical expectations. The suppression of the pseudoscalar-vector and vector-pseudoscalar penguin amplitudes relative to the pseudoscalar-pseudoscalar penguin amplitude [6], as well as the pattern of drastically different branching fractions for the decay modes $B \to \eta^{(')} K^{(*)}$ [11], can be attributed directly to the different size and sign of the $r_\chi a_6$ term relative to $a_4$ in the QCD penguin amplitude. We are unaware of any other theoretical framework that can explain these facts. From such studies of penguin dominated $B$-decays we are therefore led to the conclusion that there is little room for extra contributions to the QCD penguin amplitude.

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