Low-Rank Representation with Graph Constraints for Robust Visual Tracking

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SUMMARY  Subspace representation model is an important subset of visual tracking algorithms. Compared with models performed on the original data space, subspace representation model can effectively reduce the computational complexity, and filter out high dimensional noises. However, for some complicated situations, e.g., dramatic illumination changing, large area of occlusion and abrupt object drifting, traditional subspace representation models may fail to handle the visual tracking task. In this paper, we propose a novel subspace representation algorithm for robust visual tracking by using low-rank representation with graph constraints (LRGC). Low-rank representation has been well-known for its superiority of handling corrupted samples, and graph constraint is flexible to characterize sample relationship. In this paper, we aim to exploit benefits from both low-rank representation and graph constraint, and deploy it to handle challenging visual tracking problems. Specifically, we first propose a novel graph structure to characterize the relationship of target object in different observation states. Then we learn a subspace by jointly optimizing low-rank representation and graph embedding in a unified framework. Finally, the learned subspace is embedded into a Bayesian inference framework by using the dynamical model and the observation model. Experiments on several video benchmarks demonstrate that our algorithm performs better than traditional ones, especially in dynamically changing and drifting situations.
key words: visual tracking, low-rank representation, graph embedding

1. Introduction

In recent years, model based tracking algorithms have received significant attention due to their good performance of handling appearance variability of the target object, such as 3D models [1], integration of shape and color [2], foreground/background models [3], kernel-based filters [4] and subspace learning models [5], [6]. The common part of these algorithms is that all of them build or learn a model of the target object at first and then use it for tracking. To some extent, therefore, how to learn the initial model decides the tracking performance of each algorithm.

Subspace representation model is an important subset of visual tracking algorithms. Compared with models performing on the original data space, subspace representation model can effectively reduce the computational complexity due to its fast feature extraction. The subspace representation model provides a compact notion of the target rather than treating the target as an unclear sparse feature representation. It can also filter out high dimensional noises. The classical subspace tracking approach of Black and Jepson [7] was enhanced by incremental subspace updating in [6], [8]. Ho et al. [5] considered the general adaption problem as a subspace adaption problem, where the visual appearance variations at a short time period are represented as a linear subspace. Ross et al. [6] proposed an incremental visual tracker (IVT) with adaptive appearance model that aims to account for rigid appearance variations and deformable motions.

In various subspace learning methods, graph embedding [9] has been widely exploited for its superiority of preserving samples manifold structures. Recently, different kinds of approaches have been proposed to apply graph embedding for visual tracking [10], [11]. In order to preserve discriminative characteristics of target object after feature extraction, Zhang et al. [10] used some labeled foreground and background samples to construct the topological graph. Qiao et al. [11] focused on preserving the short distance among some labeled rotating head samples for face tracking. The essence of graph embedding is that two graphs, a within-cluster/class graph and a between-cluster/class graph, are constructed to preserve the relationship among samples [9], [12]. Before tracking starts, there may be some labeled samples in different states (e.g., different extents of rotation, different levels of occlusion and illumination conditions) available. If we can utilize the label information to design an optimal graph structure to extract the relationship among object samples, we can learn a subspace which can preserve the properties of the object distributions. In this paper, we classify samples into different clusters according to their states. In the tracking process, object samples obtained consecutively in a short time usually show high similarities when the object moves smoothly and regularly. In this situation, these object samples should be close to each other in the geometric structure. This neighboring relationship can be captured well by the within-cluster graph. The object may also change abruptly from one state to another state. In this case, there should also be a close connection between the object samples although they are in different states. This connection can be handled well by the between-cluster graph to degrade the drift problem.

The very assumption behind graph embedding used in visual tracking is that it can preserve the geometric manifold structure within the same state and among different states. Actually, the relationship among different samples not only can be formulated as geometric structure, it can also be ana-

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lyzed from the view of feature selection. Specifically, in the tracking process, the object may have different modalities due to the change of pose, rotation, illumination or occlusion. However, the target objects obtained consecutively in a short time usually share a lot of features. If we can dig out the common factors shared among objects, we can obtain the latent low-dimensional space and employ it for the object tracking. Low-rank representation (LRR) [13] has been proven to be effective for revealing common latent factors from different appearances of one object. LRR also stands out for its superiority of robustness. Therefore, it is reasonable to introduce low-rank representation into our model to handle the challenging visual tracking task.

The “statistical properties” and “geometric structure” are two observations of the data from different viewpoints. The two viewpoints are not mutually exclusive and combining them normally could transcend the specific limitations of each perspective. Some influential work [14], [15] has demonstrated that the two properties are complementary and according experiments have shown the benefits of jointly optimizing them. Low-rank constraint is useful for feature selection and can technically uncover the shared features among different objects; while the graph allows the samples to follow the data manifold. Since the neighborhoods would be changed if we use a different feature space learned by feature selection method, we employ the graph embedding as a regularization to preserve the properties of the samples distributions. Finally, the main contributions of this work are listed as follows:

1) A novel graph structure is proposed under the framework of graph embedding to capture the relationship among target objects in the same observation state or different observation states.

2) Low-rank representation and graph embedding are jointly optimized in a unified framework to learn a robust subspace, and this subspace learning is incorporated into the Bayesian framework for robust visual tracking. Thus, the benefits from both feature selection and geometric structure preservation are exploited.

The rest of this paper is organized as follows. Section 2 presents the background information and related work. The propose method, low-rank representation with graph constraints for robust visual tracking, is detailed in Sect. 3. Experiments are reported in Sect. 4, and Sect. 5 is the conclusion.

### 2. Background Information and Related Work

In this section, we first introduce two related techniques, graph based subspace learning and low-rank representation. Then, we review some related work, especially tracking algorithms based on subspace model.

#### 2.1 Graph Based Subspace Learning

Graph based subspace learning aims to learn a compact subspace where the data manifold structure can be captured by constructing appropriate graph. The graph is used to define the relationship among samples. From the view of spectral graph theory [16], the learned subspace should mostly preserve the topological structure of vertices in the graph. Consider the problem of mapping the weighted graph $G$ to a line. Let $y = (y_1, y_2, \ldots, y_n)^T$ be such a map. The optimal $y$ is given by minimizing [17]:

$$
\sum_{i,j=1}^{m} (y_i - y_j)^2 W_{ij}
$$

subject to $y^T Dy = 1$

where $W$ is the weight matrix and $D$ is a diagonal matrix whose entries are column (or row since $W$ is symmetric) sums of $W$: $D_{ii} = \sum_{j \neq i} W_{ij}$, and $L = D - W$ is the Laplacian matrix of graph $G$ [16]. In Eq. (1), the constraint $y^T Dy = 1$ removes an arbitrary scaling factor in the embedding. Then the minimization problem reduces to find

$$
y^* = \arg \min_{y^T Dy = 1} y^T Ly.
$$

Suppose $y^T = p^T X$ is the linear projection from $X$ to $y$, where $p$ is a projection vector and $X = [x_1, \ldots, x_n]$ is a $d \times n$ matrix. Equation (2) can be rewritten as

$$\min_p p^T XLX^T p
$$

subject to $p^T XDX^T p = 1$.

When $P$ is the projection that maps the data points from the original space to a low-dimensional space, we can rewrite Eq. (3) to the following equation

$$\min_P tr(P^T XLX^T P)
$$

subject to $P^T XDX^T P = I$.

Finally, the optimal $P$ can be obtained by solving eigen-decomposition

$$XLX^T P = \Lambda XDX^T P,$

where $\Lambda$ is a diagonal matrix whose diagonal elements are eigenvalues.

#### 2.2 Low-Rank Representation

Low-Rank Representation (LRR) [13] has been proven to be effective for many machine learning problems, such as image classification [18], [19], subspace segmentation [13] and transfer learning [20]. A representative practical of LRR is the Robust PCA [21]. To discover the global subspace structures of data, LRR optimizes the following objective function:

$$\min_{Z,E} \text{rank}(Z) + \lambda \|E\|_F
$$

subject to $X = AZ + E$,

where $X$ is the data matrix and $A$ is a dictionary that linearly
spans the data space. \( \text{rank}(Z) \) is the rank of coefficients matrix \( Z \). \( A > 0 \) is a balanced parameter and \( \| \cdot \|_F \) indicates certain regularization strategy, such as the \( \ell_1, \ell_2 \)-norms, used for modeling the noise. By choosing an appropriate dictionary \( A \), LRR can recover the underlying row space so as to reveal the true segmentation of data.

Recently, low-rank representation has been incorporated with subspace learning, e.g., Low-rank Transfer Subspace Learning (LTSL) [22], Supervised Regularization based Robust Subspace (SRRS) [18], Low-rank Common Subspace (LRCS) [23] and Low-Rank Discriminate Embedding (LRDE) [19], aiming to find a more robust subspace with low-rank constraint.

2.3 Subspace Based Visual Tracking Algorithms

Subspace learning has been widely exploited in visual tracking tasks due to its good performance. Here we review some previous work reported in recent literatures. The earlier work [7] deployed classical subspace learning method Principal Component Analysis (PCA) [24] to tackle the visual tracking problem. However, PCA is not optimal to preserve the manifold structure of data samples. Qiao et al. [11] proposed a method to better preserve the manifold for dynamic visual tracking. To address the robustness of tracking algorithm, sparse representation [25], [26] and low-rank representation [27] were introduced into the visual tracking models. Graph models [25], [28] were also incorporated to preserve the relationship among samples. In the tracking process, most subspace models were combined with Bayesian framework to predict the location of the target object [6].

3. Low-Rank Representation with Graph Constraints for Robust Visual Tracking

In this section, we introduce the details about how we learn the robust subspace for visual tracking. Firstly, we give the details of how to construct an appropriate graph under the framework of graph embedding. Then, we formulate our objective as a low-rank optimization problem with graph constraints. Thirdly, we use the Alternating Direction Method of Multiplier (ADMM) [29] to solve the objective. In the last, we incorporate our model into the Bayesian framework to tackle visual tracking tasks.

3.1 Construct Graphs

Given a set \( x_1, x_2, \cdots, x_h \) in \( \mathbb{R}^d \), where \( x_i \) is the sample, \( h \) is the sample number and \( d \) is the feature dimension. The samples are classified into \( K \) different clusters, e.g., \( C_1, C_2, \cdots, C_K \), according to the states. Samples with similar modalities are grouped into the same cluster. Figure 1 shows some clusters of 2 data sequences. Clearly, \( h = \sum_{q=1}^K n_q \), where \( n_q \) is the number of samples for cluster \( C_q \).

In the tracking process, the target object is consecutively changing, that is, the features of the target object in current frame would usually show high similarity with that in the previous frame and that in the next frame. From the intuition and experience of human beings, we know that the most relevant factors of tracking in current frame are the target object state in the latest frames. If we say that global information (long-term memory) is predominant in recognition and classification tasks, we would say that active information (short-term memory) is important in tracking tasks. Therefore, it’s significant to preserve the short-term memory for visual tracking. To address this problem, we use a cluster, denoted by \( C_{est} \), to memorize the target objects obtained from the latest several frames in the tracking process. The size of \( C_{est} \) is limited to \( n_{est} \), e.g., three or five, to ensure that it contains up to \( n_{est} \) target objects which are only obtained recently. \( C_{est} \) is updated every \( n_{est} \) observations obtained in the tracking process. Given data matrix \( X = [X_1, X_2, \ldots, X_K, X_{est}] \), where data matrix \( X_q (q = 1, 2, \ldots, K) \) corresponds to cluster \( C_q \) and \( X_{est} \) corresponds to cluster \( C_{est} \), we aim to construct a graph which can reflect relationship among samples in the same state or different states. Suppose \( X \) contains \( n \) data points, we have \( n = \sum_{q=1}^K n_q + n_{est} \). Let \( G = (\mathcal{V}, W) \) be the graph with vertex set \( \mathcal{V} \) and similarity matrix \( W \), where \( \mathcal{V} = C_1 \cup C_2 \cup \cdots \cup C_K \cup C_{est} \) and \( W \) is an \( n \times n \) matrix used to define the relationship between each pair of vertices. The graph structure is modeled as the combination of the within-cluster graph and the between-cluster graph, which are detailed in the next two subsections, respectively.

3.1.1 Construct Within-Cluster Graph

The within-cluster graph is constructed according to relationship among samples in each cluster. Let \( G_w = (\mathcal{V}, W_w) \) be the within-cluster graph, where \( \mathcal{V} \) is the vertex set and \( W_w \) is the \( n \times n \) within-cluster matrix. Suppose \( x_i \) and \( x_j \) are any two vertices in \( \mathcal{V} \), an edge is added between \( x_i \) and \( x_j \) if \( x_i \) and \( x_j \) are in the same cluster and \( x_j \) is one of \( x_i \)'s \( k \)-nearest neighbors. Examples are shown in Fig. 2. The connection between vertices can reflect the transition of the modality in the same cluster. Thus this method can preserve the local manifold structure by keeping objects with similar modalities closer. We apply the heat kernel method to define
We set $k = 2$ in nearest neighbor searching to preserve the local structure. Here we only take one vertex in each cluster as the example.

$W_w$. The element $w_{ij}^w$ representing the similarity between $x_i$ and $x_j$ is defined as $w_{ij}^w = \exp\left(-\frac{|x_i - x_j|^2}{2\sigma^2}\right)$ if there is an edge between $x_i$ and $x_j$, otherwise it is 0.

The within-cluster graph can be used to preserve the distribution of data points when the target object changes smoothly and regularly. However, in some cases, the target object may drift abruptly. To address this problem, we further construct the between-cluster graph as follows.

### 3.1.2 Construct Between-Cluster Graph

The between-cluster graph is used to preserve the distribution of samples in different states. Since samples in $C_{est}$ are obtained recently and can reflect current state of the object, the between-cluster graph is constructed according to the relationship between $C_{est}$ and other clusters. This method aims to set up connections between current state and other states, thus it can capture the abrupt changes of the object in the tracking process.

Let $G_b = \{V, W_b\}$ be the between-cluster graph, where $V$ is the vertex set and $W_b$ is the $n \times n$ between-cluster matrix. To construct the between cluster graph $G_b$, we connect $C_{est}$ and each of other clusters. Specifically, suppose $x_i$ and $x_j$ are any two vertices in $V$, an edge is added between $x_i$ and $x_j$ if $x_i \in C_{est}$ and $x_j \in C_q$ ($q = 1, 2, \ldots, K$) and $\{x_i, x_j\}$ is one of the $k$-nearest pairs of neighbors between $C_{est}$ and $C_q$. Examples are shown in Fig. 3. In Fig. 3 (a), when $k=2$, the two-nearest pairs of neighbors between $C_{est}$ and $C_1$ are selected and each pair of neighbors are connected. $C_{est}$ and $C_2$ are connected in the same way. In Fig. 3 (b), when $k = 1$, the nearest pair of neighbors between $C_{est}$ and $C_1$ are selected and connected. $C_{est}$ and $C_2$ are also connected in the same way. Similarly, we apply the heat kernel method to define $W_b$. The element $w_{ij}^b$ representing the similarity between $x_i$ and $x_j$ is defined as $w_{ij}^b = \exp\left(-\frac{|x_i - x_j|^2}{2\sigma^2}\right)$ if there is an edge between $x_i$ and $x_j$, otherwise it is 0.

### 3.1.3 Graph Combination

The within-cluster graph $G_w$ and between-cluster graph $G_b$ are two complementary parts of graph $G$. Finally, we can obtain the graph structure by combining the two graphs. The similarity matrix $W$ is the combination of $W_w$ and $W_b$.

$$W = W_w + W_b. \quad (7)$$

### 3.2 Learning Subspace with Low-Rank Representation

Suppose the data matrix $X \in \mathbb{R}^{d \times n}$ consists of $n$ data points in graph $G$, where $d$ is the original data dimension. Given a complete basis matrix $A = [a_1, a_2, \ldots, a_m] \in \mathbb{R}^{d \times m}$, it can linearly spans the data space as

$$X = AZ, \quad (8)$$

where $Z \in \mathbb{R}^{m \times n}$ is the coefficient matrix. To achieve our goal of seeking a robust subspace $P \in \mathbb{R}^{d \times l}$ ($l \ll d$), which maps the $n$ samples to the low-dimensional space, we first denote the projected low-dimensional data matrix as

$$\bar{X} = P^T X = P^T AZ. \quad (9)$$

Due to the fact that the samples belong to different clusters of the same class, the coefficient vectors corresponding to samples within the same cluster should be highly correlated, and therefore the coefficient matrix $Z$ is expected to be low-rank. By incorporating the low-rank constraint and the subspace learning into a single problem, we formulate our problem as the following objective function:

$$\min \text{rank}(Z) + \lambda_1 \text{tr}(P^T XLX^T P) + \lambda_2 \|P^T AZ\|_F^2,$$

subject to $P^T X = P^T AZ, P^T DX^T P = I$. \quad (10)

where $\lambda_1 > 0$ and $\lambda_2 > 0$ are trade-off parameters used to balance different parts of the objective and $L = D - W$ is the Laplacian matrix of graph $G$ and $D_{ii} = \sum_{j=1}^{n} W_{ij}$. The regularization term $\|P^T AZ\|_F^2$ is introduced to learn a more compact subspace. It should be noticed that the rank($\cdot$) minimization problem in objective Eq. (10) is difficult to solve. Fortunately, nuclear norm can be used as a good surrogate for the rank minimization problem [13]. Then, the objective becomes

$$\min \|Z\|_n + \lambda_1 \text{tr}(P^T XLX^T P) + \lambda_2 \|P^T AZ\|_F^2,$$

subject to $P^T X = P^T AZ, P^T DX^T P = I$. \quad (11)
We convert Eq. (11) to the equivalent Eq. (12) to make the optimization easier.

\[
\min_{Z,P} \left[ ||Z||_2 + \lambda_1 ||X^T P - H||_F^2 + \lambda_2 ||P^T A Z||_F^2 \right] \\
\text{s.t.} P^T X = P^T A Z,
\]

where \( H \) is a matrix whose rows are eigenvectors of the eigen-problem \( WH = \Lambda DH \), and \( \Lambda \) is a diagonal matrix of which diagonal elements are eigenvalues. The proof of the equivalence of Eq. (11) and Eq. (12) can be found in [30].

In real world applications, the samples often mix with large amount of noise. To learn a robust subspace, we need to measure the noise and learn discriminative subspaces from the noiseless samples. Recently, [13] has shown that \( l_{2,1}\)-norm can be used to successfully model the samplespecific corruptions and other types of noise. Therefore, we introduce an additional term into our objective to model errors. Based on the above observations, we come up with the final objective function:

\[
\min_{Z,P,E,J} \left[ ||Z||_2 + \lambda_0 ||E||_{2,1} + \lambda_1 ||X^T P - H||_F^2 \\
+ \lambda_2 ||P^T A Z||_F^2 \right] \\
\text{s.t.} P^T X = P^T A Z + E, \ P^T P = I,
\]

where \( \lambda_0 > 0 \) is a trade-off parameters. The constraint \( P^T P = I \) is introduced to learn a compact subspace.

3.3 Problem Optimization

To address the optimization problem of Eq. (13), we write it as the following equivalent problem by introducing a relax variable \( J \):

\[
\min_{Z,P,E,J} \left[ ||J||_2 + \lambda_0 ||E||_{2,1} + \lambda_1 ||X^T P - H||_F^2 \\
+ \lambda_2 ||P^T A Z||_F^2 \right] \\
\text{s.t.} P^T X = P^T A Z + E, \ Z = J,
\]

Eq. (14) can be solved by Augmented Lagrangian Multiplier (ALM) [31]. Firstly, we transform Eq. (14) into the augmented Lagrangian function as follows:

\[
\min_{Z,P,E,J} \left[ \frac{1}{2} ||J||_2 + \lambda_0 ||E||_{2,1} + \lambda_1 ||X^T P - H||_F^2 \\
+ \lambda_2 ||P^T A Z||_F^2 + \frac{\mu}{2} \left( ||P^T X - P^T A Z - E||_F^2 + ||Z - J||_F^2 \right) \right] \\
\text{s.t.} P^T X = P^T A Z + E, \ Z = J,
\]

where \( \mu > 0 \) is a penalty parameter, \( Y \) and \( U \) are two Lagrange multipliers. To optimize the variables in Eq. (15), we use the Alternating Direction Method of Multiplier (ADMM) [29] since previous work [18] and [23] have demonstrated that ADMM works well in solving similar problems. By using ADMM, we can alternately update variables one by one with an iterative method. Specifically, once the target variable is determined, the others can be regarded as constants. For example, when we optimize \( P \), the irrelevant terms in Eq. (15), e.g., \( ||J||_2, \lambda_0 ||E||_{2,1} \), \( \text{tr}(U^T(Z - J)) \) and \( ||Z - J||_F^2 \) can be regarded as constants.

For clarity, we use subscript \( t \) to represent the \( t \)-th iteration and we optimize the variables in the \((t+1)\)-th iteration as shown in Algorithm 1. For simplicity, we use the data matrix \( X \) itself as the dictionary \( A \) in Algorithm 1.

3.4 Complexity Analysis

The main time consuming of Algorithm 1 depends on the following parts:

1) Nuclear norm calculation in step 4;

2) Matrix multiplication and inverse in step 6 and 7.

As aforementioned, \( X \) is a \( d \times n \) matrix, where \( d \) is the original data dimension and \( n \) is the size of the dataset. The SVD computation in Step 4 takes \( O(n^3) \). The cost of the inverse of an \( n \times n \) matrix is \( O(n^3) \). The general matrix multiplication of an \( n \times n \) matrix takes \( O(n^3) \). If there are \( \gamma \) multiplications, the total cost is \( \gamma O(n^3) \). For simplicity, suppose the matrixes in step 6 and 7 are \( n \times n \) matrixes, then the computation in step 6 and 7 each takes \( (\gamma + 1)O(n^3) \). If the number of samples is large, the computation cost cannot be ignored. Fortunately, step 4 can be solved more efficiently by using the way proposed by [13] and the time complexity of matrix multiplication can be reduced to \( O(n^2\sqrt{n}) \) by using the method provided by [32]. Moreover, as the vertex set of graph \( G \) is composed by samples achieved currently and other samples with possible appearances, the number of
data points in $X$ is fixed every time when the subspace is updated. Therefore, the time cost would not increase with the accumulation of observations in the tracking process. We clarify this in detail in Sect. 3.6.

3.5 Tracking Model

Bayesian framework has provided a robust and effective framework in many tracking algorithms [6], [10]. We introduce our low-rank graph embedding into Bayesian framework to learn a flexible and effective tracking algorithm. The goal is to find the best configuration of the target with a Gaussian distribution around its counterpart in point (i.e. proportional to the distance from the sample to the reference of a sample being generated from this subspace is inversely computed by [6]):

$$p(X|I_{1:t}) \propto p(I|X_t)$$

where $p(I|X_t)$ denotes the observation model that measures how much the target and observation at the proposed state coincide, and $p(X|I_{t-1},X_t)$ denotes the dynamical model between two states $X_t$ and $X_{t-1}$.

3.5.1 Dynamical Model

Each parameter in $X_t$ is modeled independently by a Gaussian distribution around its counterpart in $X_{t-1}$, and thus the motion between frames is itself an affine transformation. Specifically,

$$p(X_t|X_{t-1}) = N(X_t; X_{t-1}, \Sigma),$$

where $\Sigma$ is a diagonal covariance matrix whose elements are the variances of the $x$, $y$ position, height, width and rotation angle of the object.

3.5.2 Observation Model

The observation model is an important part of the Bayesian framework for visual tracking. Consider an image patch $I_t$ predicted by $X_t$, and $I_t$ was generated from a subspace of the target spanned by $P$ and centered at $\alpha$. The probability of a sample being generated from this subspace is inversely proportional to the distance from the sample to the reference point (i.e. $\alpha$) of the subspace. The probability can be computed by [6]:

$$p(I_t|X_t) = N(I_t; \alpha, PP^T + \epsilon I).$$

where $\epsilon I$ corresponds to the additive Gaussian noise in the observation process, and $I$ is an identity matrix with proper size.

3.6 Tracking Procedures

In this section, we introduce our low-rank graph embedding into the Bayesian framework to complete the tracking task. The corresponding diagram of the tracking algorithm is shown in Fig. 4. We use $f$ to indicate the number of observations accumulated in $C_{est}$ in the tracking process. To learn the subspace for tracking, the graph is initialized when $f = 1$ and $C_{est}$ contains only one object (the first object). We now specify the steps of the proposed tracking algorithm as follows.

S1: Locate the target object in the first frame. Initialize $C_{est}, X_1$ and $f$. Initialize the graph structure to learn subspace $P$.

S2: Advance to the next frame. Draw particles according to the dynamical model.

S3: Calculate the likelihood of each particle under the observation model.

S4: Store the new object into $C_{est}$ Update $\alpha$ and $f \leftarrow f+1$

S5: Update $X$ and update the graph structure

S6: Run step 2 through step 11 of Algorithm 1 to update subspace $P$. Empty $C_{est}$ and set $f=0$

Fig. 4 Diagram of the tracking algorithm
likely particle as the new object into $C_{est}$, and update $\alpha$ to be the appearance of the object newly obtained. Increase the effective number of the objects in $C_{est}$, i.e., $f = f + 1$. If $f < n_{est}$, go to S2. Otherwise go to S5.

S5: Update $X$ since $n_{est}$ new objects have been accumulated in $C_{est}$. That is, the old $X_{est}$ in $X$ is replaced by the new $X_{est}$. Run the first step of Algorithm 1 to update the graph structure. Specifically, $W_v$ is updated by updating the elements corresponding to the similarities between each pair of samples in $C_{est}$, and $W_b$ is updated according to the relationship between $C_{est}$ and each cluster of $C_1, C_2, \ldots, C_K$. $W$ is obtained by combining $W_v$ and $W_b$.

S6: Run step 2 through step 11 of Algorithm 1 to update subspace $P$. Empty $C_{est}$ and set $f = 0$. Go to S2.

Just as we have discussed in Sect. 3.1 that the active information of the object states is predominant in tracking tasks. It can be seen from the above tracking procedure that after the graph is initialized in S1, each time when the graph is updated, $C_{est}$ only contains the $n_{est}$ newly obtained objects in the tracking process. In other word, the number of vertices in the graph is fixed each time when the graph is updated. Thus the computational cost would not increase and the subspace learning would not become slow with the accumulation of observations in the tracking process.

4. Experiments

In this section, we conduct several experiments to demonstrate the effectiveness of our algorithm. All experiments are carried out on a standard 2.50GHz computer with 8GB memory. Four representative video sequences, i.e., Dudek, David, CarDark and Fish, are used in our experiments. Table 1 shows the characteristics of these sequences. As there is no extra data of these sequences provided for training, for each video sequence, we divide it into several sub-sequences and each sub-sequence contains 30-50 consecutive frames. The sub-sequences are then separated into two sets, e.g., set $A$ and set $B$, according to the objects (samples) states (the definition of the samples states will be clarified in the next paragraph). Each set contains a few sub-sequences and both sets have similar distributions of the samples states. We first use the sub-sequences in $A$ as the training sequences and use the sub-sequences in $B$ as the testing sequences. Likewise, we then use the latter as the training sequences and use the former as the testing sequences. For each testing sub-sequence, we locate the object in the first frame and implement the tracking algorithm presented in Sect. 3.6. The final results on a video sequence are obtained by combining the test results of all sub-sequences in both sets.

We extract a frame every a few frames (e.g., five frames) from the training sequences to collect the training frames. The smallest rectangle containing the object is cropped from the selected frame and resized to $32 \times 32$ pixels to fit the default size of the tracking window. After being selected, the training samples are grouped into different clusters according to the samples states. The definition of the samples states of different video sequences may vary. Generally, it depends on the specific modalities, such as different poses, expressions, extents of rotation, levels of occlusion, lighting conditions, etc. For instances, the front face and the profile can be regarded as different states and an object in different lighting conditions can be regarded as in different states. To make clusters, $K$ typical samples in $K$ different states are selected from the training samples as the centers of $K$ clusters. Then, the $K$-means algorithm is used to make clusters. Finally, we adjust the clustering results manually if it exists obviously incorrect clustering. If the number of samples in a cluster is restricted to $n_q$ ($n_q$ is set to 3 by default), we select only $n_q$ samples randomly from each cluster as the cluster members. The number of clusters ($K$) varies for different sequences. We evaluate the results under different number of clusters and different number of samples in a cluster in Sect. 4.1. We also attempt to use the hierarchical clustering method to determine the number of clusters and make clusters in our future work.

The number of the nearest neighbors for the within-cluster graph is set to 2 and the number of the nearest pairs of neighbors for the between-cluster graph is set to 1. $n_{est}$ is set to 5. The dimensionalities of the original space and the subspace are 1024 and 16, respectively.

| Sequence | Target type | # Frames | Description |
|----------|-------------|----------|-------------|
| Dudek | Human face | 573 | A person undergoing large pose, expression, appearance, and lighting changes, as well as partial occlusions. |
| David | Human face | 462 | A person moves from a dark to a bright area, undergoing large lighting and pose changes. |
| CarDark | Object | 393 | A vehicle moving in the night time with large illumination changes. |
| Fish | Object | 476 | A fish doll with lighting change and there exists large and unpredictable camera motion in the video. |

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1http://cvlab.hanyang.ac.kr/tracker_benchmark/datasets.html
Fig. 5 Average center distance (in pixels) with different balanced parameters on Dudek sequence.

Fig. 6 Average center distance (in pixels) with different number of clusters \((K)\) on (a) Dudek sequence and (b) Fish sequence. When the number of clusters changes, the number of samples for each cluster of \(C_1, C_2, \cdots, C_K\) is set to 3 by default.

Fig. 7 Average center distance (in pixels) with different number of samples for each cluster of \(C_1, C_2, \cdots, C_K\) on (a) Dudek sequence and (b) Fish sequence. When the number of samples in a cluster changes, the numbers of clusters are set to their default values (13 for Dudek and 6 for Fish).

clusters \((K)\) on two video sequences. We can see from Fig. 6 that the optimal parameters are different among situations. Basically, the average center distance decreases with the increase of number of clusters, as more observation states of the target object can be captured and extracted by the graph. Figure 7 show the results with different number of samples in a cluster on two video sequences, respectively. In this experiment, the number of samples for each cluster of \(C_1, C_2, \cdots, C_K\) is changed from 3 to 7 and \(n_{\text{ext}}\) is set to 5 by default. The results show that the optimal number of samples for each cluster is different for different sequences.

For fair comparison, we use the compromise values as the default settings of these parameters in our approach to compare it with other related work in Sect. 4.2. For instances, \(K\) is set to 13 for Dudek sequence and 6 for Fish sequence respectively, and the number of samples for each cluster of \(C_1, C_2, \cdots, C_K\) is set to 3 for all sequences.

4.2 Performance Comparison

To fully evaluate our approach, we compare it with 6 recently proposed algorithms. Incremental Visual Tracking (IVT) [6], Consistent Low-Rank Sparse Tracker (CLRST) [27], Multi-task Sparse Tracking (MTT) [26], Superpixel tracking (ST) [33], Kernelized Correlation Filters (KCF) [34] and Compressive Tracking (CT) [35]. IVT applies PCA to achieve and update the subspace online. CLRST represents particles as sparse linear combinations of dictionary templates to capture the low-rank structure of data. MTT formulates object tracking in a particle filter framework and deploys sparse learning to learn a robust subspace. ST is based on mean shift method. KCF formulate tracking as a ridge regression problem for correlation filter learning and apply HOG [36] in the visual model. CT uses an appearance model based on features extracted from the multi-scale image feature space with data-independent basis. For fair comparison, we use the source codes of all approaches with the default settings of the parameters provided by the authors. The initial parameters of the tracking window in the first frame of the testing sequence are same for all the approaches.

The running time of all approaches is shown in Table 2. It is worth noticing that the running time of all approaches in our experiment includes the time of saving the tracking result of each frame into the disk, thus the speeds of KCF and CT are not so fast as that are presented in [34]. We can see from Table 2 that the running time of IVT, KCF and CT is short and similar. The running time of our approach is close to that of these three approaches and it is much shorter than that of CLRST, MTT and ST. Our approach learns the subspace based on the samples achieved recently and other training samples. Thus the computation cost would not increase dramatically with the accumulation of observations.

We show the center distance (in pixels) between the tracking result and the ground truth over time for all approaches on 3 video sequences in Fig. 8. Some selected tracking results of all approaches are also shown in Fig. 9, Fig. 10, Fig. 11 and Fig. 12, respectively. Since the selected frames are the most representative or challenging ones in the video sequence, the results can reflect the global per-

Table 2 Running time (min) of all approaches on different sequences.

| Sequence | IVT | KCF | CT | CLRST | MTT | ST | LRGC |
|----------|-----|-----|----|-------|-----|----|-----|
| Dudek    | 1.95| 1.28| 2.51| 28.25 | 14.38| 30.25| 4.83 |
| David    | 0.95| 0.83| 0.91| 25.41 | 17.7 | 10.5 | 1.53 |
| CarDark  | 0.83| 0.65| 0.76| 9.5   | 4.36 | 13.65| 1.28 |
| Fish     | 1.01| 0.86| 0.99| 25.76 | 15.51| 19.97| 1.51 |
Fig. 8 Center distance (in pixels) between the center of the tracked target and the center of the ground truth over time for all approaches applied to 3 video sequences.

Fig. 9 Selected tracking results of (a) IVT, (b) CLRST, (c) MTT, (d) ST, (e) KCF, (f) CT and (g) our approach on Dudek sequence at frame #107, #184, #380, #470 and #570. The ‘×’ in (d) indicates tracking failure, i.e., the tracking window is out of the image.

Performance of different approaches. From the results, some discussions can be drawn as follows:

1) From the tracking results of faces in Fig. 9 and Fig. 10, we can see that our approach achieves good performances on both sequences. KCF perform good on...
Dudek sequence, but the locating rectangle is much bigger than the object on David sequence when the object moves abruptly, which indicates its ability of discriminating the object from the background becomes weaker when the object changes suddenly. CT works well in early stage when the object changes smoothly, but there are some biases on Dudek sequence and the locating rectangle is also much bigger than the object on both sequences when the object changes abruptly. IVT can successfully capture the smooth changes of object, and it can also handle small changes. However, there are still some biases when IVT tracks the profile of Dudek. The eigenbasis of IVT can capture the appearance details of the target in different poses, expressions, and with or without glasses, but the eigenbasis learned in IVT by PCA cannot preserve data manifold structure. Thus, when the target changes dramatically, e.g., from frontal face to profile, IVT may fail to effectively adapt to the changes. Since both CLRST and MTT deploy sparse learning to learn a robust subspace, their performances are also similar. ST works perfect in earlier stage. It uses an appearance model to distinguish the foreground target and the background, but it does not consider much about the change of the foreground target. Therefore, it fails when the foreground target changes abruptly.

2) Fig. 11 shows that both IVT and our approach perform well in dark environment. MTT also works well by dynamically updating particles. There are some biases in the tracking results (e.g., frame #123, #223, #280 and #355) of KCF. It seems that the tracking accuracy of KCF degrades when the object moves fast and the illumination changes frequently. Compared with IVT, MTT and our approach, the locating rectangle drawn by CLRST is much bigger, which means its discrimination ability is weak when features of the foreground and the

Fig. 10 Selected tracking results of (a) IVT, (b) CLRST, (c) MTT, (d) ST, (e) KCF, (f) CT and (g) our approach on David sequence at frame #31, #85, #120, #161 and #200.
background are similar. In addition, it seems that ST is sensitive to target which has obvious difference with background. CT cannot capture the object on CarDark sequence and it seems that CT is not suitable for the environment with large illumination changes.

3) It can be seen from Fig. 12 that IVT, CLRST, KCF and our approach can steadily handle the camera motion, but MTT, ST and CT perform comparatively bad when the differences between the foreground and the background are small.

4) IVT, KCF and CT runs fast, but the accuracy of tracking may degrade in some circumstances when the state of the object changes abruptly. The running time of our approach is close to that of these approaches. Moreover, our approach is robust to local changes of the target object by constructing a within-cluster graph to preserve the relationship of objects in similar modalities. It is also robust to abrupt changes of the target object by constructing a between-cluster graph to forcibly keep connections between current appearance ($C_{est}$) and other possible appearances. Besides, low-rank representation in our model helps to reveal the common latent features shared by different appearances.

5) ST needs to generate the super-pixel regions in each tracking, thus it needs a higher cost of computation and storage and a relatively long time to complete the tracking task, while our approach is computational friendly by constructing a compact graph and projecting samples to a low-dimensional space.

Although our approach is robust to both local changes...
and abrupt changes of the object in many circumstances, it needs to collect different observation states before tracking. This precondition may not be easy to satisfy in some circumstances. How to overcome this disadvantage is the work we need to do in the future.

5. Conclusion

This paper proposes a robust visual tracking algorithm by low-rank representation with graph constraints. It exploits the benefits from both feature selection and geometric structure preservation. Experimental evaluations on different faces and objects demonstrate that our approach is robust to both local changes and dramatically changes of the target object. Since our model needs some training samples to construct the graphs, in our future work, we will explore how to automatically generate training samples by only one sample, e.g., variation of pose and scale, shape deformation and occlusion.

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