Long Distance Contribution to $s \to d\gamma$ and Implications for $\Omega^- \to \Xi^-\gamma$, $B_s \to B^*_d\gamma$ and $b \to s\gamma$.

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Abstract

We estimate the long distance (LD) contribution to the magnetic part of the $s \to d\gamma$ transition using the Vector Meson Dominance approximation ($V = \rho, \omega, \psi_i$). We find that this contribution may be significantly larger than the short distance (SD) contribution to $s \to d\gamma$ and could possibly saturate the present experimental upper bound on the $\Omega^- \to \Xi^-\gamma$ decay rate, $\Gamma_{\Omega^-\to\Xi^-\gamma}^{\text{MAX}} \simeq 3.7 \times 10^{-9}\text{eV}$. For the decay $B_s \to B^*_d\gamma$, which is driven by $s \to d\gamma$ as well, we obtain an upper bound on the branching ratio $\text{BR}(B_s \to B^*_d\gamma) < 3 \times 10^{-8}$ from $\Gamma_{\Omega^-\to\Xi^-\gamma}^{\text{MAX}}$.

Barring the possibility that the Quantum Chromodynamics coefficient $a_2(m_s)$ be much smaller than 1, $\Gamma_{\Omega^-\to\Xi^-\gamma}^{\text{MAX}}$ also implies the approximate relation

$$\sum_i \frac{g_{\psi_i}^2(0)}{m_{\psi_i}^2} \simeq \frac{1}{2} \frac{g_{\rho}^2(0)}{m_{\rho}^2} + \frac{1}{6} \frac{g_{\omega}^2(0)}{m_{\omega}^2}.$$ 

This relation agrees quantitatively with a recent independent estimate of the l.h.s. by Deshpande et al., confirming
that the LD contributions to $b \to s\gamma$ are small. We find that these amount to an increase of $(4 \pm 2)\%$ in the magnitude of the $b \to s\gamma$ transition amplitude, relative to the SD contribution alone.

1. Introduction and Overview

The investigation of the quark radiative transition $b \to s\gamma$ has been an important focus of attention in recent years [1] both because of experimental measurements [2] and because long distance (LD) corrections to the Standard Model (SM) predictions for the short-distance (SD) contributions are estimated to be small [3]. (For exclusive $B \to K^*\gamma$ decays see Ref. [4]). Thus, this transition constitutes an excellent laboratory to test the SM or possible high energy deviations thereof [5]. It has been pointed out recently [6] that for the $c \to u\gamma$ transition the situation is reversed, with the LD contributions dominating over the SD ones by many orders of magnitude.

In this paper we investigate the analogous quark transition $s \to d\gamma$ and two exclusive hadronic processes, $\Omega^- \to \Xi^-\gamma$ and $B_s \to B_d^*\gamma$, where it plays an important role. Throughout this paper we are concerned with the magnetic transition only, since the charge-radius one vanishes for real photons.

The SD contribution to $s \to d\gamma$ has been investigated before [see e.g.: 7,8,9] and we simply repeat the calculations, using updated values for the relevant QCD coefficients. Applying the quark model formalism of Ref. [10] we find that the SD $s \to d\gamma$ contribution (by itself) to the $\Omega^- \to \Xi^-\gamma$ decay
rate is far below (by a factor of order 600) the present experimental upper limit [11]:

$$\Gamma(\Omega^- \to \Xi^- \gamma) < 3.7 \times 10^{-9} \text{ eV (90\%CL)}.$$ (1)

Hadronic LD effects that involve light mesons in loops are estimated to be small[8, 12], comparable to the SD contributions.

On the other hand, by using a Vector Meson Dominance (VMD) approximation for the LD contribution to the $s \to d \gamma$ transition (along the lines discussed by Deshpande et al. [3] for $b \to s \gamma$), we find LD contribution that are likely to be significantly larger than the SD ones. In fact, the rate for $\Omega^- \to \Xi^- \gamma$ may not be far from the experimental bound (1), due to this VMD contribution. The resulting VMD amplitude is approximately proportional to

$$a_2(m_s) \left[ \frac{2}{3} \sum_i \frac{g_{\psi_i}(0)}{m_{\psi_i}^2} - \frac{1}{2} \frac{g_{\rho}(0)}{m_{\rho}^2} - \frac{1}{6} \frac{g_{\omega}(0)}{m_{\omega}^2} \right]$$ (2)

where $a_2(m_s)$ is a Quantum Chromodynamics (QCD) coefficient [13] and the $g_V(0)$’s are the usual vector meson-photon couplings, evaluated at $q^2 = 0$.

Although a direct estimate of $a_2(m_s)$ is not reliable because we are well into the low energy region where perturbation theory cannot be trusted, we can use “smoothness” arguments to extrapolate from the phenomenologically determined values $a_2(m_b^2) = 0.24 \pm 0.04$ [3] and $a_2(m_c^2) = 0.55 \pm 0.1$ [13] to obtain $a_2(m_s) \geq O(0.5)$. We then apply the formalism of Ref. [10] to get an expression for the $\Omega^- \to \Xi^- \gamma$ decay rate from our SD+VMD $s \to d \gamma$ amplitude. (Notice that there are no pole contributions to this decay). It turns out that if the above rough estimate $a_2(m_s) \geq O(0.5)$ is correct
then the experimental limit (1) can be satisfied only if the contribution of the $\psi_i$ resonances in the parenthesis of eq. (2) cancels, at a level of 30% or better accuracy, the $\rho$ and $\omega$ meson contributions, which can be reliably obtained from the $\rho$ and $\omega$ leptonic widths [14]. The limit (1) then forces the approximate relation at $q^2 = 0$:

$$\frac{2}{3} \sum_i \frac{g_{\psi_i}^2(0)}{m_{\psi_i}^2} \simeq \frac{1}{2} \frac{g_{\rho}^2(0)}{m_{\rho}^2} + \frac{1}{6} \frac{g_{\omega}^2(0)}{m_{\omega}^2}$$

which is highly nontrivial, and may be interpreted as a remnant of the badly broken SU(4)$_F$ symmetry.

The relation (3) turns out to be very useful for the $b \to s\gamma$ decays. As noted in Ref. [3], in the VMD approximation the main LD contributions to this decay can be expressed in terms of the l.h.s. of eq. 3. In Ref. [3], the sum in the l.h.s. of eq. (3) is estimated by using measured leptonic widths of the $\psi_i$ states and $\psi$ photoproduction data as well as an assumption about the higher $\psi$ excitations. We estimate this sum with better accuracy by replacing experimental values for $g_{\rho}(0)$ and $g_{\omega}(0)$ in the r.h.s. of eq. (3) and find very good quantitative agreement with the central value obtained in Ref. [3]. We thus confirm the main result of Ref. [3] that LD contributions to $b \to s\gamma$ are of order of a few percent. According to our explicit estimate, these corrections amount to an increase of $(4 \pm 2\%)$ in the magnitude of the $b \to s\gamma$ transition amplitude, relative to the SD contribution alone.

Finally, we also apply the SD + VMD approximation for $s \to d\gamma$ to the unusual decay mode $B_s \to B^*_d\gamma$, where the $b$ quark plays the “spectator” role. We point out that this decay (followed by $B^*_d \to B_d\gamma$) has a clear experimental signature of 2 monochromatic photons of energies $\simeq 50\text{MeV}$ each. We
find, using the limit of eq. (1), a small but hopefully measurable branching ratio $BR(B_s \to B_d^* \gamma) < 3 \times 10^{-8}$.

2. SD Contribution to the $s \to d \gamma$ Amplitude

The SD amplitude relevant to the $s \to d \gamma$ transition can be expressed as

$$A_{SD} = -\frac{e G_F}{8\pi^2} \sqrt{2} F_2(\mu^2) \bar{d} \sigma^{\mu\nu} [m_s R + m_d L] s F_{\mu\nu},$$

(4)

where $m_s, m_d$ are current quark masses and $F_2(\mu^2)$ is a form factor evaluated at a low scale $\mu \geq 0(m_s)$ which includes (dominant) QCD corrections. Early estimates of $F_2(\mu^2)$ [7,8,12] were in the approximate range $0.15 - 0.36$ [15] while we obtain by explicit calculation, using $\alpha_s(m_c^2) \simeq 0.3$, $\alpha_s(\mu^2) = 0.9$ in the formulas given in Ref. [16], a somewhat smaller value $F_2(\mu^2) \simeq 0.1$, which will be used below (see also Ref. [9]).

3. LD Contribution to $s \to d \gamma$

To estimate the LD contribution to $s \to d \gamma$ we use the VMD approximation in analogy to the formalism used in Ref. [3] for $b \to s \gamma$. As an intermediate step one defines a transverse amplitude $A(s \to dV(q))_T (V = \psi, \rho, \omega$ in this case) and then introduces the $V$ to $\gamma$ conversion vertices, setting $q^2 = 0$. Using Gordon decomposition we find that the LD amplitude for the $s \to d \gamma$ transition is

$$A_{LD} = -\frac{e G_F}{\sqrt{2} V_{cs} V_{cd} a_2(\mu^2)} \left( \frac{2}{3} \sum_i \frac{g_{\psi_i}^2(0)}{m_{\psi_i}^2} - \frac{1}{2} \frac{g_{\rho}^2(0)}{m_{\rho}^2} - \frac{1}{6} \frac{g_{\omega}^2(0)}{m_{\omega}^2} \right)$$

$$\cdot \frac{1}{M_s^2 - M_d^2} \bar{d} \sigma^{\mu\nu} [M_s R - M_d L] s F_{\mu\nu},$$

(5)
where we have used $V_{cs}V_{cd}^* \simeq -V_{us}V_{ud}^*$. $a_2(\mu^2)$ is a QCD coefficient the value of which is taken from phenomenology in the context of the factorization approximation [13], and the $g_V(q^2)$ factors are defined in the usual way, e.g. $\langle \psi(q)|\bar{c}\gamma_\mu c|0 \rangle = ig_\psi(q^2)\epsilon^{+}_\mu(q)$. We have not included possible contributions from the $\rho$ and $\omega$ radial excitations ($\rho', \rho'', \ldots, \omega', \omega'', \ldots$) because we think that their contribution is much smaller and is already taken into account to a significant degree in the SD amplitude (4). The $\psi$ excitations should be included however, because they are narrow resonances that are clearly distinguished from the $c\bar{c}$ continuum. Note that due to the hadronic nature of the VMD approximation, $M_s$ and $M_d$ should correspond to “constituent” mass parameters. (The use of “constituent quark” spinors in deriving (5) should take into account to some extent non-perturbative effects such as chiral symmetry breaking and confinement). In any case, it turns out that only the combination $\sqrt{M_s^2 + M_d^2}$ which has a similar magnitude for “constituent” or “current” $s, d$ quark masses, appears in our applications (see Sects. 4 and 5) when the interference between the (presumably) dominant LD contribution and the SD contribution is neglected.

It is difficult to estimate the coefficient $a_2(\mu^2)$ for $\mu \geq O(m_s)$ appearing in eq. (5). However, a smooth extrapolation from the phenomenologically obtained values $a_2(m_s^2) \simeq 0.24 \pm 0.04$ and $a_2(m_s^2) = 0.55 \pm 0.1$ [3,13] leads to $a_2(m_s^2) \geq 0.5$.

The couplings $g_{\psi_i}(m_{\psi_i}^2), g_{\rho}(m_R^2), g_{\omega}(m_\omega^2)$ are readily obtained from leptonic decays of these mesons, but their extrapolated values at $q^2 = 0$ are
less trivial, especially for the $\psi_i$ states. Photoproduction data seems to indicate that $g_\rho^2(0) \simeq g^2(m_\rho^2)$, $g_\omega^2(0) \simeq g^2(m_\omega^2)$ [17,18]. On the other hand, estimates in Ref. [3] using $\psi$ photoproduction data [18-20] give $g_\psi^2(0) = (0.12 \pm 0.04) g_\psi^2(m_\psi^2)$. In Ref. [3] it is also assumed that the same ratio holds for the excitations $\psi'$, $\psi''$, etc.

Making use of the above estimates as well as of the leptonic widths of the relevant vector mesons [14] we obtain the numerical values $g_\rho^2(0)/m_\rho^2 \simeq 0.047\text{GeV}^2$, $g_\omega^2(0)/m_\omega^2 \simeq 0.038\text{GeV}^2$ and $\sum_i g_{\psi_i}^2(0)/m_{\psi_i}^2 \simeq 0.041\text{GeV}^2$. The first two estimates should be accurate to about 10% while the latter must be considered only as a rough estimate, with an uncertainty of at least 40%. Once we derive the approximate relation (3) we will be able to give a far more reliable estimate of $\sum_i g_{\psi_i}^2(0)/m_{\psi_i}^2$, which is consistent with the above central value.

4. Application to the Decay $\Omega^- \to \Xi^-\gamma$ and Consequences

We use the quark model of Ref. [10] to estimate the rate for the decay $\Omega^- \to \Xi^-\gamma$, from the SD and LD contributions to the $s \to d\gamma$ quark decay amplitude obtained in previous sections.

For notational convenience, we define the constants $v \equiv |V_{cs}V_{cd}^*| \simeq 0.22$ and $C_{\text{VMD}} \equiv \left(\frac{2}{3} \sum_i \frac{g_{\psi_i}^2(0)}{m_{\psi_i}^2} - \frac{1}{2} \frac{g_\rho^2(0)}{m_\rho^2} - \frac{1}{6} \frac{g_\omega^2(0)}{m_\omega^2}\right)$. The relative sign of the SD and LD contributions is determined by the theory [3] so that the full amplitude for the $s \to d\gamma$ transition can be written as
\[ A_{\text{TOT}}(s \rightarrow d\gamma) = A_{\text{SD}} + A_{\text{LD}} \]
\[ = -\frac{eG_F}{\sqrt{2}} d\sigma^{\mu\nu} \left[ \left( \frac{m_s F_2}{8\pi^2} + \frac{v a_2 C_{\text{VMD}} M_s}{M_s^2 - M_d^2} \right) R \right. \]
\[ \left. + \left( \frac{m_d F_2}{8\pi^2} - \frac{v a_2 C_{\text{VMD}} M_d}{M_s^2 - M_d^2} \right) L \right] s F_{\mu\nu}. \]  

(6)

Following Ref. [10] we then obtain
\[ \Gamma(\Omega^{-} \rightarrow \Xi^{-}\gamma) = \frac{\alpha G_F^2}{12\pi^4} \left( \frac{m_{\Xi^{-}}}{m_{\Omega^{-}}} \right) |\vec{q}|^3 \]
\[ \cdot \left\{ \left( m_s F_2 + \frac{8\pi^2 v a_2 C_{\text{VMD}} M_s}{M_s^2 - M_d^2} \right)^2 + \left( m_d F_2 - \frac{8\pi^2 v a_2 C_{\text{VMD}} M_d}{M_s^2 - M_d^2} \right)^2 \right\}, \]  

(7)

where \(\vec{q}\) is the photon momentum in the \(\Omega^{-}\) rest frame and the separate SD and LD contributions are exhibited explicitly.

In the absence of LD (VMD) contributions, we would obtain (for \(m_s \simeq 175\text{MeV}, m_d \simeq 10\text{MeV}, F_2 \simeq 0.1\), see Sect. 2)
\[ \Gamma_{\text{SD}}(\Omega^{-} \rightarrow \Xi^{-}\gamma) \simeq 6.4 \times 10^{-12} \text{eV} \]  

(8)

which is far below the present experimental bound of \(\Gamma_{\text{exp}}(\Omega^{-} \rightarrow \Xi^{-}\gamma) < 3.7 \times 10^{-9}\text{eV}\). On the other hand, the large theoretical uncertainty of over 40% in the value of the sum \(\sum_i \frac{g_{\psi_i}^2(0)}{m_{\psi_i}^2}\) (see Sect. 3) which appears in \(C_{\text{VMD}}\), would allow the LD contribution to saturate this experimental bound. In fact, the experimental limit can be used to constrain \(C_{\text{VMD}}\) and hence \(\sum_i \frac{g_{\psi_i}^2(0)}{m_{\psi_i}^2}\).
Using typical values \( M_s \simeq 0.5 \text{GeV}, M_d \simeq 0.35 \text{GeV} \) for the constituent quark masses and \( a_2 > 0.5 \) (see Sect. 3), we find
\[
\left| C_{\text{VMD}} \right| = \left| \frac{2}{3} \sum_i \frac{g_{\psi_i}^2(0)}{m_{\psi_i}^2} - \frac{1}{2} \frac{g_{\rho}^2(0)}{m_{\rho}^2} - \frac{1}{6} \frac{g_{\omega}^2(0)}{m_{\omega}^2} \right| < 0.01 \text{GeV}^2. \tag{9}
\]
This constraint would be only slightly different, had we used current quark mass parameters instead of \( M_s \) and \( M_d \).

The bound in eq. (9) represents a remarkable cancellation at the 30\% level, considering that \( \frac{1}{2} \frac{g_{\rho}^2(0)}{m_{\rho}^2} + \frac{1}{6} \frac{g_{\omega}^2(0)}{m_{\omega}^2} \simeq 0.030 \text{GeV}^2 \) (see Sect. 3). We presume that this effect may stem from the combination of the GIM [21] mechanism and the underlying SU(4)_F symmetry, which if exact would give a full cancellation (after inclusion of \( \rho', \rho'', \ldots, \omega', \omega'' \ldots \) states). The SU(4)_F symmetry is known to be badly broken by the large mass of the \( c \) quark. However, here we are comparing the form factors \( g_{\psi_i}^2(q^2), g_{\rho}^2(q^2), g_{\omega}^2(q^2) \) at a common scale \( q^2 = 0 \), which seems to “restore” this symmetry to some extent. We have noticed that if \( |g_{\phi}(0)| \simeq |g_{\phi}(m_{\phi}^2)| \) [17,18], leading through \( \phi \) leptonic width data [14] to \( |g_{\phi}(0)| \simeq 0.24 \text{GeV}^2 \), a completely analogous near cancellation occurs for the quantity \( C'_{\text{VMD}} \equiv -\frac{1}{3} \frac{g_{\phi}^2(0)}{m_{\phi}^2} + \frac{1}{2} \frac{g_{\rho}^2(0)}{m_{\rho}^2} - \frac{1}{6} \frac{g_{\omega}^2(0)}{m_{\omega}^2} \), which is relevant to LD effects in \( c \to u \gamma \) decay [6]. We obtain \( C'_{\text{VMD}} \simeq -1.8 \times 10^{-3} \text{GeV}^2 \), which represents a cancellation at a level better than 10\% for which presumably the SU(3)_F symmetry is responsible.

We note that the upper bound (9) on \( |C_{\text{VMD}}| \) tells us that although the LD effects are likely to dominate \( s \to d \gamma \), they can be at most a factor of about 25 larger than the SD contribution in the amplitude. This represents
an intermediate situation between the $b \to s \gamma$ decays where the SD contribution clearly dominates [3,4,22] and the $c \to u \gamma$ decays where the SD effects are completely negligible relative to the LD ones.

5. Implications for the LD Contribution to $b \to s \gamma$

Because $\left( \frac{1}{2} g_\rho^2(0) + \frac{1}{6} g_\omega^2(0) \right) \approx 0.030 GeV^2$, eq. (9) implies that the approximate relation given in eq. (3) must hold to an accuracy of order 30%. This then independently determines

$$\sum_i \frac{g_{\psi_i}^2(0)}{m_{\psi_i}^2} = 0.045 \pm 0.016 GeV^2,$$

where our uncertainty in the values of $g_\rho(0)$ and $g_\omega(0)$ has been folded in. Notice that this result is in very good agreement with the central value ($\approx 0.041$) estimated from $\psi$ photoproduction data in Ref. (3), but the uncertainties there were larger (above 40%). Our results thus confirm previous assertions that the LD corrections are at the few percent level only [3,4] and further show that these contributions are well under control. The amplitude for $b \to s \gamma$ including SD and LD contributions can be expressed as [3]

$$A_{TOT}(b \to s \gamma) = -\frac{eG_F}{\sqrt{2}} V_{tb} V_{ts}^* \left[ \frac{1}{4\pi^2} m_b C_7^{\text{eff}}(m_b) - a_2(m_b) \frac{2}{3m_b} \sum_i \frac{g_{\psi_i}^2(0)}{m_{\psi_i}^2} \right] \cdot \bar{s}\sigma^{\mu\nu} RbF_{\mu\nu},$$

where $m_s$, $M_s$ have been neglected compared to $m_b$. Using $a_2(m_b) \approx 0.24 \pm 0.04[3]$, $C_7^{\text{eff}}(m_b) = -0.30 \pm 0.03$ [16] and $m_b = 4.8 \pm 0.2 GeV$, we find that the LD contribution increases the magnitude of the amplitude by $(4 \pm 2)\%$.  

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6. Application to $B_s \rightarrow B_d^* \gamma$

Another process where the $s \rightarrow d \gamma$ quark transition will dominate is $B_s \rightarrow B_d^* \gamma$. There are no pole contributions and we explicitly estimated the LD contribution from light meson loops to be smaller but comparable with the SD $s \rightarrow d \gamma$ contributions. This is an unusual $B_s$ meson decay in the sense that it represents the decay of the light quark in a $\bar{Q}q$ system. Also, it has a clear signature: two photons with energies of about 50MeV and 46MeV (the second one coming from the decay $B_d^* \rightarrow B_d \gamma$), followed by a usual $B_d$ decay.

We roughly estimate the $B_s \rightarrow B_d^* \gamma$ decay rate from our $s \rightarrow d \gamma$ amplitude (6) by assuming that the spatial wavefunctions of the $s$ quark in the $B_s$ meson and the $d$ quark in the $B_d^*$ meson are similar, and noting that the photon energy (= 50MeV) is small compared to the average momentum ($O(700\text{MeV})$) of the light quark in the bound state. A “free quark” approximation should then give a reasonable estimate of the transition amplitude. In terms of the effective $s \rightarrow d \gamma$ Hamiltonian (6) we obtain for the decay rate:

$$\Gamma(B_s \rightarrow B_d^* \gamma) = \frac{\alpha}{16\pi^4} G_F^2 |\vec{q}|^2 \left\{ \left( m_s F_2 + \frac{8\pi^2 v a_2 C_{\text{VMD}} M_s}{M_s^2 - M_d^2} \right)^2 + \left( m_d F_2 - \frac{8\pi^2 v a_2 C_{\text{VMD}} M_d}{M_s^2 - M_d^2} \right)^2 \right\}$$

(12)

where $\vec{q}$ is the photon momentum in the $B_s$ rest frame. Comparing to eq. (7)
and using the upper bound (1) we obtain

$$\Gamma(B_s \rightarrow B_s^* \gamma) < 1.4 \times 10^{-20} \text{ GeV}.$$  \hspace{1cm} (13)

Then, the present central value for the $B_s$ lifetime $\tau_{B_s} \simeq 1.34 \times 10^{-12} \text{s}$ [14] gives a bound on the branching ratio, $BR(B_s \rightarrow B_s^* \gamma) < 3 \times 10^{-8}$. Although this is a very rare decay mode, its unique signature and the large number of $B_s$ mesons expected at $B$ meson factories and at LHC-B $O(2 \times 10^{-11})$ [23] make it interesting.

7. Conclusions

Using a VMD approximation, we found that the LD contribution to the $s \rightarrow d \gamma$ transition may be significantly larger than the SD one, and could even lead to a saturation of the present experimental upper limit on the decay rate for $\Omega^- \rightarrow \Xi^- \gamma$ (eq. (1)). This result throws new light on this decay mode. A further tightening of this upper limit or a measurement of the $\Omega^- \rightarrow \Xi^- \gamma$ rate would provide us with very useful information about the relative importance of the LD and SD contributions to $s \rightarrow d \gamma$. The present upper bound already implies a non-trivial cancellation at a level of 30% or better in the LD contribution. The resulting approximate relation (eq. (3)) allowed us to estimate the relative importance of the LD contribution to the $b \rightarrow s \gamma$ transition amplitude. Our estimate of $(4 \pm 2)\%$ for this relative LD contribution agrees with earlier ones, which had larger uncertainties. Because the unusual process $B_s \rightarrow B_s^* \gamma$ is also dominated by an $s \rightarrow d \gamma$ transition, its decay rate is related to that of $\Omega^- \rightarrow \Xi^- \gamma$. We find
that a present limit on the latter (eq. (1)) implies an upper bound for the branching ratio $BR(B_s \rightarrow B_s^* \gamma) < 3 \times 10^{-8}$, which is small but hopefully accessible in future experiments.

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