COORDINATING A MULTI-ECHELON SUPPLY CHAIN UNDER PRODUCTION DISRUPTION AND PRICE-SENSITIVE STOCHASTIC DEMAND

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Abstract. This paper considers a three-echelon supply chain system with one raw-material supplier, one manufacturer and one retailer in which both the manufacturer and the raw-material supplier are exposed to the risk of production disruptions. The market demand is assumed to be uncertain but sensitive to the retail price. The objective is to determine the optimal lot sizes of the supplier and the manufacturer, and the selling price of the retailer when the wholesale prices of the upstream entities are prescribed and the retailer’s order quantity is chosen before the actual demand is realized. As the benchmark case, the expected total profit of the centralized channel is maximized. The decentralized supply chain is coordinated under pairwise and spanning revenue sharing mechanisms. Numerical study shows that disruptions have remarkable impact on supply chain decisions.

1. Introduction. Uncertainties in demand and supply are two major challenges that most of the supply chain managers face today. Both the uncertain scenarios are highly undesirable in the prevailing competitive business environment. If the demand uncertainty is not appropriately taken into consideration, companies may lose market share as a result of unsatisfied customer demand or may incur higher holding cost due to excessive stock of goods (Petkova and Maranas [31]). Similarly, improper or inaccurate supply of goods due to natural hazards, such as flood, hurricane, tornado, snowstorm, earthquake or man-made hazards, such as transportation delay, custom delay and labor strike may affect the retailer’s profitability as well as the company’s goodwill. Thus, it often becomes very difficult for the managers to accomplish demand-supply balancing in the supply chain (Jones et al. [20]). Several researchers (Li et al. [25], Iyer and Bergen [19], Mantrala and Raman [26], Chen and Xu [6], Lau and Lau [22] and Huang and Lin [18]) discuss the negative impacts of demand uncertainty on the inventory/supply chain decisions. Wang and Gerchak [41] use the concept of random yield to model supply uncertainty and derive the optimal policy for the inventory model under variable production capacity. Güllü et al. [12] use Bernoulli process to address the supply uncertainty in which

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the ordered quantity may arrive or not. They consider a periodic review inventory model and obtain a non-stationary order-up-to policy. Kazaz [21] studies a single period two-stage decision-making problem under random yield and demand uncertainty, and determines the optimal production quantity as well as the optimal resource order when the retail price is yield dependent. Li and Zheng [24] derive the optimal price and production quantity which maximize the total discounted profit, in the presence of random yield and stochastic demand. Dada et al. [7] discuss the multi-supplier newsvendor problem when suppliers are unreliable. Wilson [43] investigates the impact of transportation disruption on the performance of a supply chain. Azad et al. [1] study the design problem of a stochastic supply chain network in the presence of random disruptions in the location of distribution centers (DCs) and the transportation modes between DCs and customers. They assume that the impact of disruption in DCs depends on the amount of investment for opening and operating. Singh et al. [37] model a multi-stage global supply chain network incorporating a set of risk factors (such as late shipment, exchange rates, quality problems, logistics and transportation breakdown, and production risks) and their effects on the supply chain operation.

Serel [36] develops a single period model to identify the best stocking policy for a retailer with uncertain demand and supply. Hsieh and Wu [15] study coordinated decisions in a decentralized supply chain with uncertainties in both demand and supply. Liu et al. [25] consider the impact of supply uncertainty on the performance of the firm under joint marketing and inventory decisions. They develop a single period model and show that reducing variance of supply uncertainty improves the profit of the firm. Schmitt and Snyder [35] consider an inventory system that faces yield uncertainty and the risk of complete supply disruption. They demonstrate the importance of analyzing the model over a sufficiently long time horizon. He and Zhao [13] study a three-echelon supply chain with a wholesale price contract under supply and demand uncertainties. Yeo and Yuan [44] develop a periodic review model in which the firm manages its inventory under supply uncertainty and demand cancellation. Chen and Fan [5] discuss a bioethanol supply chain planning problem with both supply and demand uncertainties. Hu et al. [16] and Güler and Keskin [11] study coordination in a two-echelon supply chain under yield and demand uncertainties. Meena et al. [27] determine the optimal number of suppliers assuming equal allocation among those selected under supplier disruption due to catastrophic events. They focus on minimising the total costs subject to the satisfaction of a service level and maximising the service level subject to constraint on the total costs. Merzifonluoglu [28] consider two-stage supply selection portfolio under the risks of supply and demand volatility. In the first stage, the size of the forward contract with the primary supplier and reserve quantity with an option contract with the backup supplier are made. After monitoring the risk of the primary supplier, in the second stage, the decision of using the reserved quantity via option contract or directly purchasing from the spot market is made. Zeng and Xia [46] develop a dual sourcing supply chain where the backup supplier is used as a contingent supply when the supply from the primary supplier is disrupted. They derive the backup contract parameters through a Nash game. Sawik [34] suggests two portfolio approaches to supplier selection under the risks of disruption - an integrated approach with perfect information about the future disruption of supply chain and hierarchical approach with no such information available.
One of the common assumptions in supply chain researches is that the market demand, while uncertain, is not influenced by price or non-price factor(s) (Wan et al. [39], Zhang et al. [45]). Since the retail price of many consumer goods plays a major role to generate greater market demand, in this paper, we assume that the demand is stochastic but sensitive to retail price. In practice, endogenous factors like machine failure or breakdown, process shift and labor strike are found to be responsible for disruptions in production process while exogenous factors like natural calamities and transportation/custom delay are responsible for complete cut-off in supply to retailers. In case of supply disruption, the main source is totally unavailable while in the event of production disruption, the production yield may be less than the expected volume. Paul et al. [30] develop a disruption recovery model for a single-stage production-inventory system under the risk of disruptions during the production up-time. They propose a dynamic solution approach to deal with multiple disruptions on a real time basis.

The issue of disruption has been addressed in the literature mainly in the context of single-echelon supply uncertainty. In a more realistic situation dealing with multi-echelon supply chain system, a disruption at the upper echelon (upstream) can trigger a series of disruptions at the lower levels (downstream). For example, consider the following disruption event that took place in Nokia cell phone manufacturing industry. The New Mexico based company Royal Philips Electronics was a major supplier of semiconductors for Nokia cell phones. On 17 March, 2000, the semiconductor plant caught fire due to thunderstorms and lightning, and millions of microchips were damaged due to direct fire and/or smoke in temporary storages and nearby warehouses. As a result, more than five percent of the Nokia’s annual production was disrupted (Hopp et al. [14] and Azimi et al. [2]). This example shows that production disruption has tremendous impact on the manufacturer who relies on supplies from his supplier and loses its significant market advantage to its competitor(s). However, the manufacturer may have independent causes of production disruption due to labor strike, fire, explosion and machine breakdown, even though flow of raw materials from the supplier is perfect. Irregular supplies of garments to Wal-Mart and departmental stores (such as Dillards, J.C. Penny, Sears, etc.) from overseas and their production disruptions due to political problem and/labor unrest are some causes of price changes of these products in those stores. Besides many multi-echelon manufacturing systems, similar disruptions in supply and production are not uncommon in many food production industries (such as drinks, sauces, etc.) which are affected by the damage of the seasonal fruits (apples, oranges, grapes, tomatoes, etc.) by excessive cold weather. So disruption risk management has become an extremely important part of the supply chain management strategy. Recently, Go and Gao [10] develop a system dynamics model for remanufacturing/manufacturing integrated supply chain and use the methods of setting multi-echelon inventory levels before disruption and back-up plans after disruption to deal with production disruption in supply chain.

In the literature, production disruption is typically considered at the manufacturer. In this paper, production disruption is assumed to occur at the manufacturer and the raw materials supplier of the manufacturer. To get rid of the negative impact of production disruption on the immediate downstream member, we assume that both the supplier and the manufacturer utilize the available spot market or reliable sources. Our primary objective of this study is to focus on the following questions: How to coordinate the supply chain under uncertain demand influence
by the retail price and unwanted production disruptions at the upstream levels, and how the performance of the whole supply chain is affected due to production disruptions at upper echelons of the supply chain? To find answers of these questions, we formulate a three-echelon supply chain model under both centralized and decentralized settings. We then implement pairwise and spanning revenue sharing mechanisms to coordinate the decentralized system and compare the performance of the supply chain with respect to the centralized system.

The rest of the paper is organised as follows. Section 2 presents notations and key assumptions of the proposed model. The centralized model is developed in Section 3 as the benchmark case. The decentralized model is studied in the next section. Section 5 deals with the decentralized model under pairwise and spanning revenue sharing contracts. In Section 6, optimal results are obtained and managerial insights are discussed for numerical examples. Finally, Section 7 draws conclusions and future research directions.

2. Notations and model assumptions. 2.1. Notations. The following notations are used throughout the paper:

- $p$ : unit retail price of the finished product ($/unit)
- $X_p$ : stochastic market demand (units)
- $Y$ : a random variable with range $[0, 1]$, pdf $g(\cdot)$, cdf $G(\cdot)$ and mean $\bar{y}$
- $Z$ : a random variable with range $[0, 1]$, pdf $h(\cdot)$, cdf $H(\cdot)$ and mean $\bar{z}$
- $\bar{x}_p$ : expected market demand at the retail price $p$ (units)
- $Q(\geq \bar{x}_p)$ : manufacturer’s production quantity of finished goods (units)
- $R(\geq Q)$ : supplier’s production quantity of the raw materials (units)
- $w_s$ : wholesale price of the raw material supplier ($/unit$)
- $w'_s$ : unit price of the raw material in the spot (secondary) market ($/unit$)
- $c_s$ : unit production cost of the raw material ($/unit$)
- $v_s$ : salvage value of each item at the supplier ($/unit$)
- $w_m$ : wholesale price of the manufacturer ($/unit$)
- $w'_m$ : unit price of the finished product at the reliable manufacturer ($/unit$)
- $c_m$ : unit production cost of the finished product ($/unit$)
- $v_m$ : salvage value of each item at the manufacturer ($/unit$)
- $v_r$ : salvage value of each item at the retailer ($/unit$)
- $c_{ur}$ : under stocking (opportunity) cost for each item at the retailer ($/unit$)

2.2. Model assumptions. We consider a three-level supply chain consisting of a single raw material supplier, single manufacturer and single retailer. This industrial setup is common in practice and is often opted by the companies who would like to reduce the number of vendors they use (Munson and Rosenblatt [29]). The supply chain under consideration is designed to trade for a single product over a single period of time.

We assume that processing for raw materials at the supplier and the production for finished goods at the manufacturer are subject to disruptions. Without any loss of generality, we assume that one unit of raw material is required to produce one unit of finished good. The market demand is random (randomness is price independent) but influenced by the retail price. It can be modeled either in an additive or a multiplicative form (Huang et al. [17]). The additive model is appropriate when the variance of demand is unaffected by the expected demand and multiplicative model is appropriate when the variance of demand increases with the expected demand in such a way that the coefficient of variation remains unaffected. In this paper, we choose the multiplicative model as it is more tractable than the additive
model (see Wang et al. [42]). We take the market demand \( X_p = \phi(p)X \) where \( \phi(p) \) is a deterministic, positive and strictly decreasing function in the retail price \( p \) and \( X \) is a positive random variable with pdf \( f(\cdot) \), cdf \( F(\cdot) \) and mean \( \mu \) (Petruzzi and Dada [32], Wang [40]). Therefore, we have \( F(\cdot) = 1 - F(\cdot) \). We suppose that the retailer estimates the market demand for the product as \( \bar{x}_p \) and places the order to the manufacturer well before the season starts. To fulfill the retailer’s order quantity, the manufacturer plans to produce \( Q(\geq \bar{x}_p) \) quantity of finished goods. If a production disruption occurs, the manufacturer gets only a fraction \( y \) of his planned quantity \( Q \); \( y \) \((0 \leq y \leq 1)\) being the value which the non-negative random variable \( Y \) takes. The random variable \( Y \) has the support \([0, 1]\) and it is characterized by the probability density function \( g(\cdot) \) and cumulative distribution function \( G(\cdot) \). Similarly, in order to fulfill the manufacturer’s demand, the raw material supplier plans to process raw materials of size \( R(\geq Q) \). We assume that if disruption occurs then his actual process quantity would be \( zR \) instead of \( R \), where the non-negative random variable \( Z \), having support \([0, 1]\), probability density function \( h(\cdot) \) and cumulative distribution function \( H(\cdot) \), takes the value \( z \). In case of yield deficit, both the raw material supplier and the manufacturer buy from the spot market or reliable sources at prices higher than those of their individual wholesale prices in order to maintain company’s goodwill with the immediate downstream customer. Let \( \alpha \) \((0 \leq \alpha \leq 1)\) and \( \beta \) \((0 \leq \beta \leq 1)\) denote the probabilities of production disruption at the manufacturer and the supplier, respectively. In order to validate the proposed models, we further assume that the following cost-price relations hold good: \( c_m + w_s < w_m < w_m' \), \( c_s < w_s < w_s' \) and \( v_r < c_{ur} \).

3. **Centralized system - The benchmark case.** In this section, we characterize the optimal behavior of the supply chain system under the assumption that there is a single decision maker who has to decide the retail price, production quantities of the supplier and the manufacturer simultaneously before observing the market demand. At the retail price \( p \), the estimated market demand is \( \bar{x}_p = \phi(p)\mu \) while the realized demand is \( x_p = \phi(p)x \). The retailer’s net profit is then given by

\[
\Pi_r(p) = \phi(p)\left[\int_0^\mu ([p - v_r]x - \mu(w_m - v_r)\right)dF(x) + (p - w_m + c_{ur})\mu F(\mu) - c_{ur} \int_\mu^\infty xdF(x)\right]
\]

From (1), the expected profit of the retailer can be obtained as

\[
\Pi_r(p) = \phi(p)\left[(p - w_m)\mu - (p + c_{ur} - v_r)\int_0^\mu F(x)dx\right]
\]

Since the manufacturer is exposed to the risk of production disruption with probability \( \alpha \) \((0 \leq \alpha \leq 1)\) and no production disruption with probability \((1-\alpha)\), therefore, the total weighted expected profit of the manufacturer is given by

\[
\Pi_m(p, Q) = (1 - \alpha)[w_m\bar{x}_p - (w_s + c_m)Q + v_m(Q - \bar{x}_p)] + \alpha\left[w_m\bar{x}_p - (w_s + c_m)Q - w_m'\int_0^{\bar{x}_p/Q}(\bar{x}_p - yQ)dG(y)\right]
\]
From (5), we have by partial differentiation

\[ \frac{\partial \Pi}{\partial p} = 0 = \frac{\partial \Pi}{\partial Q} = \frac{\partial \Pi}{\partial R} \] (6)

**Proposition 1.** The function \( \Pi(p, Q, R) \) is concave provided that \( p\phi''(p) + 2\phi'(p) \leq 0 \).

**Proof.** From (5), we have by partial differentiation

\[ \Pi_{RR} = -\beta(w'_s - v_s)Q^2 \frac{R}{R^3} h(Q/R) < 0, \text{ as } v_s < w'_s \]

\[ \Pi_{QQ} = -\beta(w'_s - v_s) \frac{R}{Q^3} h(Q/R) - \frac{\alpha(w'_m - v_m)x_p}{Q^3} g(x_p/Q) \]

\[ \Pi_{pp} = -\phi''(p) \left[ (1 - \alpha)v_m\mu + \alpha\mu w'_m G(x_p/Q) \right] + \alpha v_m \mu \int_{x_p/Q}^{\mu} g(y)dy + (c_{ur} - \nu_r) \int_0^\mu F(x)dx \]

\[ -\alpha \mu^2 (w'_m - v_m)g(x_p/Q)[\phi'(p)]^2/Q + [p\phi''(p) + 2\phi'(p)] \int_0^\mu F(x)dx \]

< 0, provided that \( p\phi''(p) + 2\phi'(p) \leq 0 \)

\[ \Pi_{RQ} = \frac{\beta(w'_s - v_s)Q}{R^2} h(Q/R) > 0 \]

\[ \Pi_{Qp} = \frac{\alpha(w'_m - v_m)\mu^2}{Q^2} \phi(p)\phi'(p)g(x_p) < 0, \text{ as } \phi'(p) < 0 \]

\[ \Pi_{pR} = 0. \]

The Hessian matrix is

\[ +v_m \int_{x_p/Q}^{\mu} (yQ - x_p) dG(y) \] (3)
Clearly $D_2 = \Pi_{RR}\Pi_{QQ} - (\Pi_{QR})^2 > 0$.

Now $D_3 = |\mathcal{H}|$

$= \Pi_{pp}D_2 - \Pi_{pQ}(\Pi_{RR}\Pi_{pQ})$

$= \phi(p)\Pi_{pp} + Q\phi'(p)\Pi_{pQ}$

$= -\phi(p)\phi''(p)\left[(1 - \alpha)v_m\mu + \alpha\mu w'_m G(\bar{x}_p/Q) + \alpha v_m\mu \int_{\bar{x}_p/Q}^{1} g(y)dy \right.

\left. + (c_{ur} - v_r) \int_{0}^{\mu} F(x)dx \right] + \phi(p)[p\phi''(p) + 2\phi'(p)] \int_{0}^{\mu} F(x)dx. \tag{7}$

Therefore, $D_3 < 0$ when $p\phi''(p) + 2\phi'(p) \leq 0$. Then the Hessian matrix $\mathcal{H}$ is negative definite and hence the function $\Pi(p,Q,R)$ is concave.

Equations (6) yield

$\beta(w'_s - v_s) \int_{0}^{Q/R} z h(z)dz = c_s - (1 - \beta + \beta\bar{z})v_s, \tag{8}$

$\alpha(w'_m - v_m) \int_{0}^{\bar{x}_p/Q} y g(y)dy = c_s + c_m - v_m + \beta(1 - \bar{z})v_s + \alpha(1 - \bar{y})v_m, \tag{9}$

$\phi'(p) \left[ \mu\{p - c_s - c_m - \beta(1 - \bar{z})v_s - \alpha(1 - \bar{y})v_m\} - (p + c_{ur} - v_r) \int_{0}^{\mu} F(x)dx \right]$

$+ \phi(p) \int_{0}^{\mu} \bar{F}(x)dx = 0. \tag{10}$

which can be solved by ‘reverse inductive method’ to find the optimal values $p^c, Q^c$ and $R^c$ of $p, Q$ and $R$, respectively. These values, when substituted in (5), give the optimal expected profit ($\Pi^c$) of the integrated supply chain system.

**Note 1.** If $\phi(p)$ is linear in $p$ i.e. $\phi(p) = a - bp$, where $a, b > 0$, and $p < a/b$ then the condition in Proposition 1 is clearly satisfied.

**Proposition 2.** *In case of no production-disruption at the manufacturer and the supplier, we have $Q \rightarrow \bar{x}_p$ and $R \rightarrow Q$.***

**Proof.** Putting $\beta = 0$ in (8), we get $c_s = v_s$ which means that the unit production cost of the supplier is merely recovered by its salvage value. So, there is no financial loss on the part of the supplier for any excess production whatsoever. However, management will not be interested for any such over-production as there is no profit margin. This implies that $R$ should tend to $Q$ in such a case (make-to-order policy). With similar arguments, it can be said from Eq. (9) that $Q$ should tend to $\bar{x}_p$ in case of no disruption at the manufacturer. Hence the proposition is proved. \qed

4. **Decentralized system.** Though it is proven that the centralized channel yields the highest system profit, it is ideally impossible to implement in practice in general. So, the decentralized policy under a contract which binds the participating members in cooperation is a more realistic option to enhance supply chain performance. The optimal decisions of the centralized system are usually taken as the benchmark solution for the decentralized system. First, we model the decentralized scenario without a contract where there is no coordination among the supply chain entities.
The objective of each entity is to maximize its own profit regardless of the entire supply chain's profit. The expected profits of the retailer, manufacturer and supplier are given by

\[
\Pi_r(p) = \phi(p) \left[ (p - w_m)\mu - (p + c_{ur} - v_r) \right] \int_0^\mu F(x)dx, \quad (11)
\]

\[
\Pi_m(p, Q) = w_m\bar{x}_p - (w_s + c_m)Q + (1 - \alpha)v_m(Q - \bar{x}_p) - \alpha w_m' \int_{\bar{x}_p/Q}^{\bar{x}_p} (y'Q - \bar{x}_p) dG(y) + \alpha v_m \int_{\bar{x}_p/Q}^{1} (yQ - \bar{x}_p) dG(y), \quad (12)
\]

\[
\Pi_s(Q, R) = w_sQ - c_sR + (1 - \beta)v_s(R - Q) - \beta w_s' \int_{Q/R}^{1} (Q - zR) dH(z) + \beta v_s \int_{Q/R}^{1} (zR - Q) dH(z), \quad (13)
\]

respectively. We now study some characteristics of the model in the following properties:

**Proposition 3.** If the function \(\psi_2(p) = [p + \phi(p)/\phi'(p)]\) is strictly increasing then there exists a unique retail price \(p^d\) which maximizes \(\Pi_r(p)\).

*Proof.* The necessary condition for optimality of \(\Pi_r(p)\) gives

\[
\psi_2(p) \int_0^\mu \tilde{F}(x)dx = w_m\mu + (c_{ur} - v_r) \int_0^\mu F(x)dx \quad (14)
\]

Since the right hand side of (14) is independent of \(p\), the above equation indicates that if \(\psi_2(p)\) is strictly increasing in \(p\) then there exists a unique value of \(p\), say \(p^d\), which satisfies (14). Using (14), the expression for \(\frac{d^2\Pi_r(p)}{dp^2}\) can be simplified as

\[
\frac{d^2\Pi_r(p)}{dp^2} = \left[ 2 - \frac{\phi(p)\phi''(p)}{[\phi'(p)]^2} \right] \phi'(p) \int_0^\mu \tilde{F}(x)dx
\]

\[
< 0, \text{ if } \psi_2(p)\text{ is strictly increasing in } p.
\]

Hence, the proposition is proved. \(\square\)

**Note 2.** If \(\phi(p)\) is linear in \(p\) i.e. \(\phi(p) = a - bp\), where \(a, b > 0\), and \(p < a/b\) then the condition in Proposition 3 is clearly satisfied. However, if \(\phi(p)\) is in exponential form i.e. \(\phi(p) = ap^{-\epsilon}\) then the condition in Proposition 3 is satisfied for \(\epsilon > 1\).

**Note 3.** Eq.(8) and RHS of (13) imply that the ratios of \(Q\) and \(R\) in the centralized and decentralized systems are equal. This indicates that the manufacturer's production quantity and consequently, the supplier’s raw material process quantity both increase uniformly in the centralized system.

**Proposition 4.** When the retail price \(p = p^d\) is known, the optimal production quantity \((Q^d)\) of the manufacturer and subsequently the optimal production quantity \((R^d)\) of the supplier are uniquely determined.

*Proof.* When \(p = p^d\), we have from (12)

\[
\frac{\partial^2 \Pi_m}{\partial Q^2} = -\frac{\alpha (w_m' - v_m)(\bar{x}_{p^d})^2}{Q^3} g(\bar{x}_{p^d}/Q) < 0,
\]
implying that \(\Pi_m\) is concave with respect to \(Q\). Therefore, there exists a unique production lot size \(Q^d\) which maximizes \(\Pi_m(p^d, Q)\). Similarly when \(Q = Q^d\), we have from (13)

\[
\frac{\partial^2 \Pi_m}{\partial R^2} = -\frac{\beta(w_s' - v_s)(Q^d)^2}{R^3} h(Q^d/R) < 0,
\]

implying that \(\Pi_s\) is concave with respect to \(R\). Hence the proposition follows. \(\square\)

**Proposition 5.** The manufacturer’s expected profit decreases in case of disruption and its production lot size increases with the disruption probability \(\alpha\).

**Proof.** The optimal production quantity of the manufacturer can be obtained from the equation

\[
\frac{d\Pi_m(p^d, Q)}{dQ} = 0
\]

which gives

\[
\alpha(w_m' - v_m) \int_{\bar{x}^d/Q} \gamma(y) dy = w_s + c_m - (1 - \alpha)v_m - \alpha v_m \bar{y}.
\]

Using (16) in (12), we get

\[
\Pi_m = [(w_m' - v_m) - \alpha(w_m' - v_m)G(\bar{x}^d/Q)]/G(\bar{x}^d)
\]

where \(G(\cdot)\) is an increasing function. Since \(w_s + c_m \geq v_m\), \(\bar{y} < 1\) and \(w_m' > v_m\), therefore, when \(\alpha\) increases, \(G(\bar{x}^d/Q)\) decreases and consequently \(Q\) increases. This completes the proof of the proposition.

Proceeding similarly as above, it can be shown that the raw material supplier’s profit decreases in case of a production disruption and that its production lot size increases with the disruption probability \(\beta\).

Propositions 3 and 4 suggest that the decentralized model can be solved as follows: The retailer’s optimal price \(p^d\) can be obtained from (11). Having known \(p^d\), the market demand can be estimated, depending on which the manufacturer determines his optimal production quantity \(Q^d\) from (12). After knowing \(Q^d\), the raw material supplier then decides his optimal production quantity \(R^d\) from (13). It is to be noted that unlike the centralized system, here the wholesale prices of the supplier and the manufacturer have significant impacts on the retail price as well as the total profit of the supply chain.

5. **Coordinating by revenue sharing.** In this section, we introduce two types of revenue sharing (RS) contract - pairwise and spanning revenue sharing contracts in the decentralized system. Our objective is to examine whether these contract mechanisms can achieve channel coordination and win-win outcome.

5.1. **Pairwise RS.** The traditional revenue sharing contract mechanism (Giannoccaro and Pontrandolfo [9], Chauhan and Proth [4], and Cachon and Lariviere [3]) is usually employed in a two-echelon supply chain involving a vendor and a buyer (retailer). However, in case of a multi-echelon supply chain, this idea can be generated between pairs of adjacent echelons. Such a contract is termed as pairwise
and supplier can be obtained as wholesale prices offered by the manufacturer and the supplier be \( w \) of the revenue to the raw material supplier on an agreement that the manufacturer will charge a lower wholesale price to the retailer. Similarly, the manufacturer keeps a quota \( \eta \) of the revenue to the retailer and the manufacturer to their immediate upstream entities are

\[
\tilde{\Pi}_r(p) = \phi(p) \left[ \xi \left( \mu - (p - v_r) \int_0^\mu F(x)dx \right) - \bar{w}_m \mu - c_{ur} \int_0^\mu F(x)dx \right], \tag{19}
\]

\[
\tilde{\Pi}_m(p, Q) = \eta \left[ (1 - \xi) \left( \mu - (p - v_r) \int_0^\mu F(x)dx \right) \phi(p) + \bar{w}_m \bar{x}_p \right.
+ \left. (1 - \alpha) v_m (Q - \bar{x}_p) + \alpha v_m \int_{\bar{x}_p/Q}^1 (yQ - \bar{x}_p) dG(y) \right]
\]

\[
-(\bar{w}_s + c_m)Q - \alpha w'_m \int_{\bar{x}_p/Q}^{\bar{x}_p/Q} (\bar{x}_p - yQ) dG(y) \tag{20}
\]

and

\[
\tilde{\Pi}_s(Q, R) = (1 - \eta) \left[ (1 - \xi) \left( \mu - (p - v_r) \int_0^\mu F(x)dx \right) \phi(p) \right.
+ \bar{w}_m \bar{x}_p + (1 - \alpha) v_m (Q - \bar{x}_p) + \alpha v_m \int_{\bar{x}_p/Q}^1 (yQ - \bar{x}_p) dG(y) \right]
\]

\[
+ \bar{w}_s Q - c_m R + (1 - \beta) v_s (R - Q) - \beta w'_s \int_0^{Q/R} (Q - zR) dH(z)
\]

\[
+ \beta v_s \int_{Q/R}^1 (zR - Q) dH(z), \tag{21}
\]

respectively. It is easy to verify that the profit functions \( \tilde{\Pi}_s(Q, R) \) and \( \tilde{\Pi}_m(p, Q) \) are strictly concave with respect to \( R \) and \( Q \), respectively, whereas the profit function \( \tilde{\Pi}_r(p) \) is concave in \( p \) if \( \psi_2(p) \) is strictly increasing in \( p \).

**Proposition 6.** If \( \bar{w}_m \leq c_s + c_m + \alpha (1 - \bar{y}) v_m + \beta (1 - \bar{z}) v_s < v_m + (1 - v_m/w'_m)(\bar{w}_s + c_m) \) where either \( \bar{w}_m \geq c_s + \beta (1 - \bar{z}) v_s > w'_m (1 - \alpha + \alpha \bar{y}) - c_m \) or, \( \bar{w}_s \leq c_s + \beta (1 - \bar{z}) v_s < w'_s (1 - \alpha + \alpha \bar{y}) - c_m \) then the pairwise revenue sharing contract can coordinate the supply chain provided that the revenue shares offered by the retailer and the manufacturer to their immediate upstream entities are

\[
\Delta_1 \equiv 1 - \xi = \frac{c_s + c_m - \bar{w}_m - \alpha (1 - \bar{y}) v_m + \beta (1 - \bar{z}) v_s}{c_s + c_m + \alpha (1 - \bar{y}) v_m + \beta (1 - \bar{z}) v_s + (c_{ur}/\mu) \int_0^\mu F(x)dx}, \tag{22}
\]

\[
\Delta_2 \equiv 1 - \eta = \left( \frac{w'_m - v_m}{v_m} \right) \left[ \frac{c_s + \beta (1 - \bar{z}) v_s - \bar{w}_s}{(1 - \alpha + \alpha \bar{y}) w'_m - (c_s + c_m) - \beta (1 - \bar{z}) v_s} \right], \tag{23}
\]
Proof. We have from equations (19)-(21), by partial differentiation

\[
\frac{\partial \hat{\Pi}_s}{\partial R} = -c_s + (1 - \beta + \beta \hat{z}) v_s + \beta (w'_s - v_s) \int_0^{Q/R} z h(z) dz \tag{24}
\]

\[
\frac{\partial \hat{\Pi}_m}{\partial Q} = -(\hat{w}_s + c_m) + \eta (1 - \alpha + \alpha \hat{y}) v_m + \alpha (w'_m - \eta v_m) \int_0^{Q/R} y g(y) dy \tag{25}
\]

\[
\frac{\partial \hat{\Pi}_r}{\partial p} = \phi'(p) \left[ \xi (p\mu - (p - v_r) \int_0^\mu F(x)dx) - \hat{w}_m \mu - c_{ur} \int_0^\mu F(x)dx \right] + \xi \phi(p) \int_0^\mu F(x)dx \tag{26}
\]

If the manufacturer chooses the optimal production quantity \( \hat{Q}^d = Q^c \) then it is clear from (8) and (24) that the raw material supplier’s optimal production quantity \( \hat{R}^d \) will be equal to \( R^c \). Again, from (9) and (25) we see that when the retail price \( \hat{p}^d = p^c \), the manufacturer’s optimal production quantity \( \hat{Q}^d \) will be equal to \( Q^c \) provided that he offers a revenue share \((1-\eta)\) given in (22). From (23), it could be verified that \( \eta > 0 \) if the condition \( c_s + c_m + \alpha (1 - \hat{y}) v_m + \beta (1 - \hat{z}) v_s < v_m + (1 - v_m/w'_m)(\hat{w}_s + c_m) \) is satisfied, and \( \eta \leq 1 \) provided that either \( \hat{w}_s \geq c_s + \beta (1 - \hat{z}) v_s > w'_m (1 - \alpha + \alpha \hat{y}) - c_m \) or, \( \hat{w}_s \leq c_s + \beta (1 - \hat{z}) v_s < w'_m (1 - \alpha + \alpha \hat{y}) - c_m \) hold. Finally, from (10) and (26), we have \( \hat{p}^d = p^c \) provided that the retailer offers the manufacturer a revenue share \((1-\xi)\) given in (22). The condition \( c_s + c_m + \alpha (1 - \hat{y}) v_m + \beta (1 - \hat{z}) v_s \geq w_m \) must hold in order that the revenue share of the retailer should not exceed unity. Hence the proposition follows.

**Proposition 7.** For a win-win outcome \((\hat{p}^d, \hat{Q}^d, \hat{R}^d)\), the revenue shares \( \xi \) and \( \eta \) where \( \xi < 1 - \Delta_1 \) and \( \eta < 1 - \Delta_2 \) would be such that

\[
\begin{align*}
(i) \quad \phi(\hat{p}^d) &\geq \phi(p^d) \cdot \max \left\{ \frac{1}{\sqrt{\xi} \hat{w}_m} \left( \frac{w_m - v_m}{\eta (\xi \hat{p}^d + \hat{w}_m)} - \frac{\alpha (w'_m - v_m)}{\eta (\xi \hat{p}^d + \hat{w}_m)} \right) \right\} \tag{27} \\
(ii) \quad \hat{Q}^d &\geq \frac{[(w_s - v_s) - \beta (w'_s - v_s)] Q^d}{\hat{w}_s + \eta (\xi \hat{p}^d + \hat{w}_m + \alpha v_m \hat{y})}. \tag{28}
\end{align*}
\]

**Proof.** From equations (11) and (19), \( \hat{\Pi}_r^* > \Pi_r^* \) (\( * \) indicates optimal value) implies that

\[
\frac{\xi \phi^2(\hat{p}^d)}{-\phi'(\hat{p}^d)} > \frac{\phi^2(p^d)}{-\phi'(p^d)}. \tag{29}
\]

In the event of pairwise revenue sharing, the retailer buys from the manufacturer at the reduced wholesale price \( \hat{w}_m \). It is, therefore, reasonable to assume that he can generate higher demand and more profit for the chain by setting his retail price not greater than that of the decentralized policy without revenue sharing, i.e. \( \hat{p}^d \leq p^d \). This gives \( \phi'(\hat{p}^d) \leq \phi'(p^d) \) and then equation (27) gives

\[
\phi(\hat{p}^d) \geq \phi(p^d) / \sqrt{\xi}. \tag{30}
\]

After simple calculations, it can be shown that \( \hat{\Pi}_m^* > \Pi_m^* \) implies

\[
[(w_m - v_m) - \alpha (w'_m - v_m)] \phi(p^d) \leq \eta [(1 - \xi) \hat{p}^d + \hat{w}_m - v_m] \phi(\hat{p}^d) \tag{31}
\]
Simplifying further, we get
\[ \phi(\tilde{p}^d) \geq \frac{(w_m - v_m) - \alpha(w'_m - v_m)}{\eta(\bar{\xi} + \bar{\omega}_m)} \phi(p^d) \] (32)

Eqs. (30) and (32) yield condition (i) given in (27). Again $\tilde{\Pi}^*_s > \Pi^*_s$ implies
\[ [(w_s - v_s) - \alpha(w'_s - v_s)]Q^d \leq (1 - \eta)(1 - \bar{\xi})\bar{p}^d + \bar{\omega}_m + \alpha v_m \bar{y}Q^d + \bar{w}_s - v_s \bar{Q}^d \] (33)
which, when simplified, yields condition (ii) given in Eqn. (28). This completes the proof of the proposition.

5.2. Spanning RS. The major problem in implementing the pairwise revenue sharing contract is that contacts between the adjacent pairs are necessarily to be signed simultaneously because parameter setting in one contract depends on another contract. One member may enjoy the benefit of gain by even not participating in the contract when other members already signed the contract. This leads to an unrealistic situation. On the contrary, a spanning revenue sharing contract among the members is a more realistic option in which one member plays the leading role in establishing a single contract with the other members (Rhee et al. [32]). In our problem, we suppose that the retailer at the bottom level acts as the leader and he contributes certain fractions of his revenue to all upstream members provided that each upstream member offers a lower wholesale price to its immediate downstream member. Such a contract is valid as long as the coordination of the supply chain and win-win outcome are achieved. Under this contract, let the retailer (leader) contribute a revenue share $\xi_1$ ($0 < \xi_1 < 1$) to the manufacturer and a share $\xi_2$ ($0 < \xi_2 < 1$) to the supplier. In turn, the manufacturer offers a lower wholesale price $\bar{w}_m(\bar{w}_m)$ to the retailer and the supplier offers a lower wholesale price $\bar{w}_s(\bar{w}_s)$ to the manufacturer. Then the expected profits of the retailer, manufacturer and supplier can be obtained as
\[
\bar{\Pi}_r(p) = \phi(p) \left[ (1 - \xi_1 - \xi_2) \left( p\mu - (p - v_r) \int_0^\mu F(x)dx \right) \right. \\
-\bar{w}_m \mu - c_{ur} \int_0^\mu F(x)dx \right], \\
\bar{\Pi}_m(p, Q) = \xi_1 \phi(p) \left( p\mu - (p - v_r) \int_0^\mu F(x)dx \right) \\
+\bar{w}_m \bar{x}_p + (1 - \alpha)v_m(Q - \bar{x}_p) + \alpha v_m \int_{\bar{x}_p/Q}^1 (yQ - \bar{x}_p)dG(y) \\
-(\bar{w}_s + c_m)Q - \alpha w'_m \int_{\bar{x}_p/Q}^{\bar{x}_p/Q} (\bar{x}_p - yQ)dG(y), \tag{34}
\]
and
\[
\bar{\Pi}_s(R, Q) \\
= \xi_2 \phi(p) \left( p\mu - (p - v_r) \int_0^\mu F(x)dx \right) + \bar{w}_s Q - c_s R + (1 - \beta)v_s(R - Q) \\
-\beta w'_s \int_0^{Q/R} (Q - zR)dH(z) + \beta v_s \int_{Q/R}^1 (zR - Q)dH(z), \tag{35}
\]
respectively. Establishment of such a contract depends on how the retailer can negotiate with the upstream entities for the wholesale prices $\bar{w}_m$ and $\bar{w}_s$ with regard to his offer of revenue shares $\xi_1$ and $\xi_2$ so that each acting entity involved in the contract gains higher profit compared to that in the decentralized policy without RS contract. The following proposition suggests feasible ranges of the wholesale prices $\bar{w}_m$ and $\bar{w}_s$ for which a spanning revenue sharing contract can coordinate the supply chain.

**Proposition 8.** If $w_s > \bar{w}_s = c_s + \beta(1 - \bar{z})v_s$ and $w_m < \min\{w_m, \bar{w}_s + c_m + \alpha(1 - \bar{y})v_m\}$ then the spanning revenue sharing contract can coordinate the supply chain provided $\xi_1$ and $\xi_2$ are such that

$$\xi_1 + \xi_2 = \frac{\bar{w}_s + c_m - \bar{w}_m + \alpha(1 - \bar{y})v_m}{\bar{w}_s + c_m + \alpha(1 - \bar{y})v_m + \frac{\bar{c}v_m}{\bar{F}}} \int F(x)dx < 1 \quad (37)$$

**Proof.** The proof is similar to that of Proposition 6. We omit it for the sake of brevity.

6. A case scenario (garments supply chain). Suppose that a garment manufacturer plans to manufacture specially designed textile product for a mega-sporting event like Olympic, World Cup soccer or Super Bowl Championship and sell its products through a dedicated retailer. The major raw material for the product i.e., the cotton is produced and supplied by an external company (supplier) who is specialist in producing the kind of cotton required for the product. The supplier’s cotton production quantity and quality both depend on climate conditions. Also the firm is affected occasionally by natural disasters leading to cotton production disruption. However, the garment manufacturer faces different problem; his labors declare agitation or strikes time to time as a tactic to fulfill their demands. The garment production is then disrupted resulting in deviation from the targeted volume to fulfill retailer’s demand. To retain company goodwill, each upstream entity buys from spot market to make up the deficit while delivering to the immediate downstream echelon. The retailer estimates the quantity and price of the product well in advance of the event. The problem is thus to determine the desired raw material process quantity of the supplier, amount of finished goods of the manufacturer and the retail price of the retailer so that each player’s expected profit in the decentralized system is maximized. In the following examples, we assume in this context the parameter-values to illustrate the supply chain model developed in the previous section.

6.1. Example 1. Exponential demand. Let $X \sim \exp(\lambda)$, $\lambda > 0$; $Y \sim U[A, B], B > A > 0$ and $Z \sim U[C, D], D > C > 0$. Further, we suppose that $\phi(p) = ap^{-c}$ and the parameter-values are as follows: $a = 5000$, $\varepsilon = 3$, $\lambda = 0.02$, $v_r = \$4$/unit, $c_{ur} = \$5$/unit, $A = 0.65$, $B = 0.85$, $w_m = \$10$/unit, $w_m' = \$14$/unit, $c_m = \$2$/unit, $v_m = \$6$/unit, $\varepsilon = \$3$/unit, $w_s = \$5$/unit, $w_s' = \$9$/unit, $v_s = \$2$/unit, $C = 0.65$, $D = 0.85$, $\alpha = 0.2$, $\beta = 0.3$.

For the above data set, the optimal solution in the centralized scenario is obtained using equations (8)-(10) as $p^c = \$14.68$/unit, $Q^c = 125.82$ units, $R^c = 157.09$ units and from (5), the corresponding expected total profit of the integrated supply chain is obtained as $\Pi^c = \$244.30$.

In the decentralized case, the condition given in Proposition 3 for the existence of unique $p^d$ is clearly satisfied for the chosen data set. The optimal retail price is
obtained as \( p^d = \$24.60/\text{unit} \) and then by Proposition 4, the optimal lot sizes of the manufacturer and the supplier are obtained as \( Q^d = 19.42 \) units and \( R^d = 24.25 \) units, respectively. The expected profits of the supplier, manufacturer and retailer are \( \$27.48, \$38.33 \) and \( \$87.03 \), respectively. The sum of these expected profits gives the entire supply chain’s profit as \( \$152.84 \) which is about 37% lower compared to the expected profit in the centralized system.

When \( \alpha = \beta = 0 \), we get \( Q^d = R^d = 31.99 \) which verifies the result given in Proposition 2.

### Table 1. Effects of \( \alpha \) and \( \beta \) on the manufacturer’s and the supplier’s decentralized decisions.

| \( \alpha \) | \( \beta \) | \( Q^d \) | \( \Pi_m \) | \( R^d \) | \( \Pi_s \) |
|---|---|---|---|---|---|
| 0.0 | 0.0 | 16.79 | 50.36 | 0.0 | 19.42 | 38.84 |
| 0.2 | 0.2 | 19.42 | 38.33 | 0.2 | 22.62 | 29.94 |
| 0.4 | 0.4 | 21.28 | 29.82 | 0.4 | 25.20 | 25.45 |
| 0.6 | 0.6 | 22.03 | 21.98 | 0.6 | 26.27 | 21.91 |
| 0.8 | 0.8 | 22.43 | 14.33 | 0.8 | 26.87 | 18.65 |
| 1.0 | 1.0 | 22.69 | 6.76 | 1.0 | 27.24 | 15.51 |

The effects of disruption probabilities \( \alpha \) and \( \beta \) on the lot sizing decisions of the manufacturer and the supplier are shown in Table 1. Note that, as \( \alpha \) and \( \beta \) increase, both the manufacturer and the supplier suffer from increase in production uncertainty, in spite of their efforts to mitigate the effect of disruption with higher lot sizes.

### 6.1.1. Decentralized decisions under pairwise RS contract.

For the model under pairwise revenue sharing contract, we take \( \tilde{w}_m = \$9/\text{unit} \), \( \tilde{w}_s = \$4/\text{unit} \), \( \xi = 0.95 \) and \( \eta = 0.90 \). The optimal policies for the retailer, manufacturer and supplier are obtained from (19), (20) and (21) as \( \tilde{p}^d = \$23.58/\text{unit} \), \( \tilde{\Pi}_r = \$89.98 \), \( \tilde{Q}^d = 24.12 \) units, \( \tilde{\Pi}_m = \$42.50 \) and \( \tilde{R}^d = 30.11 \) units, \( \tilde{\Pi}_s = \$31.18 \), respectively. The expected total profit of the chain is \( \$163.66 \). Clearly, each member enhances his profit and the total profit of the supply chain is also increased. Hence, it is a win-win situation but it does not necessarily mean that any pair of the values of \( \xi \) and \( \eta \) which increase the profit of the decentralized supply chain would provide a win-win outcome. As evident from Table 2, only the \( (\xi, \eta) \)-pairs \( (0.95, 0.90), (0.97, 0.90), (0.97, 0.92), \) and \( (0.99, 0.92) \) provide win-win situations, although the supply chain profit is increased for all \( \xi = 0.95, 0.97, 0.99 \) and \( \eta = 0.90, 0.92, 0.94 \).

### Table 2. Optimal results for different values of \( \xi \) and \( \eta \) in the decentralized system.

| \( \xi \) | \( \eta \) | \( \tilde{p}^d \) | \( \tilde{\Pi}_r \) | \( \tilde{Q}^d \) | \( \tilde{\Pi}_m \) | \( \tilde{R}^d \) | \( \tilde{\Pi}_s \) | Total profit |
|---|---|---|---|---|---|---|---|---|
| 0.95 | 0.90 | 23.58 | 89.98 | 24.12 | 42.50 | 30.11 | 31.18 | 163.66 |
| 0.92 | 23.58 | 89.98 | 24.60 | 46.76 | 30.71 | 27.24 | 163.98 |
| 0.94 | 23.58 | 89.98 | 25.12 | 51.07 | 30.71 | 23.06 | 164.11 |
| 0.97 | 0.90 | 23.03 | 96.38 | 25.89 | 39.40 | 32.32 | 32.78 | 168.56 |
| 0.92 | 23.03 | 96.38 | 26.41 | 43.83 | 32.97 | 28.80 | 169.01 |
| 0.94 | 23.03 | 96.38 | 26.97 | 48.32 | 33.67 | 24.74 | 169.44 |
| 0.99 | 0.90 | 22.49 | 103.11 | 27.80 | 35.89 | 34.71 | 34.48 | 173.48 |
| 0.92 | 22.49 | 103.11 | 28.35 | 40.51 | 35.39 | 30.35 | 173.97 |
| 0.94 | 22.49 | 103.11 | 28.96 | 45.19 | 36.16 | 26.14 | 174.44 |
Table 3 shows that when the manufacturer and the retailer do not reach to a RS agreement i.e. \( \xi = 1 \), the manufacturer’s profit is decreased but the retailer’s profit is increased, although the total profit of the supply chain is increased compared to that of the decentralized supply chain system. Similarly, when the supplier and the manufacturer do not sign the RS contract (i.e., when \( \eta = 1 \)), the profit of the supply chain is increased compared to that of the profit in the decentralized system but the raw material supplier becomes looser. This would enforce the supplier not to remain as a participating entity of the supply chain, resulting in a complete cut-off of the main raw material supply source. Alternately, the supplier can opt for further bargaining with the manufacturer for settling the wholesale price (\( \tilde{w}_m \)) in response to the manufacturer’s offer of revenue share (\( 1 - \eta \)) in order to remain operative in the supply channel.

### Table 3. A comparison of results in different scenarios of pairwise RS contract

| Decentralized model | Retailer’s profit | Manufacturer’s profit | Supplier’s profit | Total profit |
|---------------------|------------------|-----------------------|------------------|--------------|
| without RS contract | 87.03            | 38.33                 | 27.48            | 152.84       |
| \( \xi = 0.95, \eta = 0.90 \) | 89.98            | 42.50                 | 31.18            | 163.66       |
| \( \xi = 0.95, \eta = 1.0 \) | 89.98            | 64.38                 | 11.21            | 165.57       |
| \( \xi = 1.0, \eta = 0.90 \) | 106.60           | 33.96                 | 35.35            | 175.91       |

#### 6.1.2. Decentralized decisions under spanning RS contract.

To demonstrate the model under spanning revenue sharing contract, we take \( \tilde{w}_m = $8.5/unit, \tilde{w}_s = $4.6/unit, \xi_1 = 0.05 \) and \( \xi_2 = 0.02 \). The other parameter values remain the same. Then the optimal policies for the retailer, manufacturer and supplier are obtained from (34), (35) and (36) as (\( \bar{p}_d = $22.89/unit, \bar{\Pi}_r = $93.50 \)), (\( \bar{Q}_d = 25.91 \) units, \( \bar{\Pi}_m = $42.93 \)) and (\( \bar{R}_d = 32.58 \) units, \( \bar{\Pi}_s = $25.25 \)) which determine the expected total profit of the chain as $161.68. This shows that spanning revenue sharing contract enables to increase each participating individual’s profit in the decentralized system. As the expected total profit of the supply chain is also increased, it presents a win-win situation. When \( \alpha = 0 \) and \( \beta = 0 \), the expected total profit of the supply chain is $172.71 implying that the system suffers almost 6.38% loss of profit due to disruption.

### 6.2. Example 2. Normal demand.

We assume \( X \sim N(m, \sigma^2) \) where \( m = 200 \) and \( \sigma = 51 \), and keep all other parameters unchanged. The optimal results are summarized in Table 4.

| Model scenario | Retailer’s profit | Manufacturer’s profit | Supplier’s profit | Total profit |
|----------------|------------------|-----------------------|------------------|--------------|
| Centralized    | -                | -                     | -                | 463.60       |
| Decentralized without RS contract | 167.97            | 73.05                 | 52.35            | 293.37       |
| Decentralized with pairwise RS \( \xi = 0.95, \eta = 0.90 \) | 172.28            | 81.26                 | 58.84            | 312.38       |
| Decentralized with spanning RS \( \xi_1 = 0.05, \xi_2 = 0.02 \) | 178.30            | 81.92                 | 62.27            | 322.49       |
Figures 1-3 are drawn to investigate the effects of disruption probabilities $\alpha$ and $\beta$ on the optimal decisions in the decentralized system. Figure 1 shows that, as $\alpha$ increases, the manufacturer’s production quantity gradually increases over the expected market demand $\bar{x}_p$ in order to lessen the effect of disruption in the system, while his expected profit continues to decline. Figure 2 depicts that, for lower disruption probability $\beta$, the supplier’s expected profit is higher for higher values of $\alpha$. This is can be explained as follows. A higher value of $\alpha$ enforces the
manufacturer to order more from the supplier, which consequently increases sales volume and profit of the supplier. For any \( \alpha \geq 0 \), the supplier’s expected profit decreases as \( \beta \) increases. Figure 3 reflects that the supply chain’s expected total profit decreases with \( \beta \) as well as \( \alpha \).

Table 5. Optimal results for different values of \( e \) in the decentralized system.

| \( e \) | Retailer \((p^{d}, \Pi_{r}^{d})\) | Manufacturer \((Q^{d}, \Pi_{m}^{d})\) | Supplier \((R^{d}, \Pi_{s}^{d})\) | Total profit |
|---|---|---|---|---|
| 3.0 | (31.5, 167.97) | (37.01, 73.05) | (46.21, 52.35) | 293.37 |
| 3.1 | (31.0, 119.06) | (27.54, 54.37) | (34.38, 38.96) | 212.39 |
| 3.2 | (30.55, 84.52) | (20.47, 40.41) | (25.31, 28.68) | 153.61 |
| 3.3 | (30.13, 60.08) | (15.22, 30.05) | (19.0, 21.53) | 111.66 |
| 3.4 | (29.75, 42.77) | (11.31, 22.32) | (14.12, 16.0) | 81.09 |
| 3.5 | (29.40, 30.48) | (8.39, 16.58) | (10.47, 11.87) | 58.92 |

Table 6. Optimal results for different values of \( a \) in the decentralized system.

| \( a \) | Retailer \((p^{d}, \Pi_{r}^{d})\) | Manufacturer \((Q^{d}, \Pi_{m}^{d})\) | Supplier \((R^{d}, \Pi_{s}^{d})\) | Total profit |
|---|---|---|---|---|
| 5000 | (31.5, 167.97) | (37.01, 73.05) | (46.21, 52.35) | 293.37 |
| 6000 | (31.5, 201.56) | (44.41, 87.66) | (55.45, 62.83) | 352.05 |
| 7000 | (31.5, 235.16) | (51.81, 102.28) | (64.68, 73.30) | 410.74 |
| 8000 | (31.5, 268.75) | (59.21, 116.89) | (73.92, 83.77) | 469.41 |
| 9000 | (31.5, 302.34) | (66.61, 131.50) | (83.16, 94.24) | 528.08 |
| 10000 | (31.5, 335.94) | (74.01, 146.11) | (92.40, 104.71) | 586.76 |
Table 7. Optimal results for different values of \( \sigma \) in the decentralized system.

| \( \sigma \) | Retailer’s profit | Manufacturer’s profit | Supplier’s profit | Total profit |
|---------|-----------------|----------------------|------------------|-------------|
| 51      | 167.97          | 73.05                | 52.35            | 293.37      |
| 53      | 160.95          | 69.96                | 50.14            | 281.05      |
| 55      | 153.97          | 66.81                | 47.88            | 268.66      |
| 57      | 147.04          | 63.75                | 45.69            | 256.48      |
| 59      | 140.19          | 60.74                | 43.53            | 244.46      |

Tables 5, 6 and 7 show respectively the effects of the price sensitive parameters \( e \) and \( a \), and uncertainty of demand (standard deviation \( \sigma \)) on the optimal results. It is evident from Table 5 that the parameter \( e \) is quite sensitive to the optimal decisions. As \( e \) increases, the price of the retailer decreases gradually, and production quantities of the manufacturer and supplier decrease very sharply. Consequently, the individual profits of the retailer, manufacturer and the supplier decrease severely. On the other hand, Table 6 shows that the parameter \( a \) has no effect on the retail price. However, it has significant positive effect on the production quantities of the manufacturer and the supplier. As higher value of \( a \) generates greater demand (retail price remains unchanged), each member of the supply chain gains more. Table 7 shows that uncertainty in demand plays an important role in each entity’s profit. Larger the value of \( \sigma \) is, faster the optimal profit of each chain member decreases.

7. Managerial implications and conclusion. It is always a matter of great concern for the management how to coordinate a multi-echelon supply chain under uncertain demand and unwanted supply disruptions, especially when the disruptions may occur at multiple echelons of a supply chain. In managing inventory, uncertain demand can be taken care of primarily by safety stocks measure. However, uncertain supply due to disruptions needs proactive and reactive measures to mitigate the risk (Dani and Deep [8]). The consequence of disruption risk in the supply chain may be high as it may persist for a long duration of time affecting operational activities of the chain or may completely cut off the supply source from upstream to downstream and paralyze all activities of the company. In the past, there have been several high profile disruptions due to 9/11, Indian Ocean tsunami and Hurricane Katrina.

In this paper, we studied the impact of production disruptions at the upper echelons of a supply chain system that consists of a raw material supplier, a manufacturer and a retailer under price-sensitive stochastic demand. Taking into consideration uncertain demand influenced by retail price, and risks of multi-echelon disruption, we developed the centralized model as the benchmark case and characterized the optimal solution for the integrated system. We then considered the decentralized system and observed that the retail price can be significantly higher than that of the centralized system. As the retail price in this model plays a major role in estimating the market demand and consequently the manufacturer’s and the supplier’s production lot sizes, we are motivated to establish a revenue sharing contract to coordinate the supply chain. In case of pairwise revenue sharing contract, the upstream entity of a pair is invited to reduce his wholesale price on the condition that the lower stream entity will offer a share of his own revenue. However, this contract mechanism is not always acceptable in multi-echelon framework as there are certain concern over its feasibility. So, we employed spanning revenue sharing contract and
observed that the buyer can reduce his retail price substantially by setting the parameters $\xi_1$ and $\xi_2$ vis-a-vis negotiating the wholesale prices $\bar{w}_m$ and $\bar{w}_s$ and that a lower retail price aligns the incentives in the vertical chain. Of course, the choice of revenue shares $\xi_1$ and $\xi_2$ has a great impact on the performance of the alliance.

In the numerical study we observe that, along with contract parameters, the risks of production disruption at different echelons should be taken into consideration while optimizing the supply chain as those have significant negative impact on the supply chain performance. It is noted in Example 2 that a 20% chance of disruption at the manufacturer only reduces the supply chain’s expected profit by 4%, a 20% chance of disruption at the supplier only cuts down the supply chain’s expected profit by 4.5%, whereas 20% chance of disruption at both the upstream entities diminishes the supply chain’s expected profit as much as 12%. From management perspective, proper care should be given to estimate the probability of disruption. It is very difficult to estimate the probability as disruptive events do not occur regularly or frequently. Moreover, supply chain managers tend to underestimate or even neglect the probability of disruption because of its low value or non-occurrence of the event quite for a long time. However, such unjust/wrong estimation may have significant impact on supply chain performance (Tomlin [38]). So experience of the planner plays in this case an important role in ascertaining the values.

There are ample scope for further extending the paper. We have assumed that the order quantity for the retailer is equal to the average demand for each associated retailer price, i.e., $\bar{x}_p = \phi(p)\mu = \phi(p)E[X]$. In practice, the order quantity can be a decision variable for the retailer. One can consider disruption at the retailer level and more than one disruptive event at any echelon in the system. Consideration of multiple retailers with price and/or service level competition among themselves, determinations of optimal decisions for risk-averse entities for a multi-echelon supply chain, and extending the model to multi-period setting seem to be meaningful contributions from practical point of view.

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