Chiral effective Lagrangian for excited heavy-light mesons from QCD

Qing-Sen Chen, Hui-Feng Fu, Yong-Liang Ma and Qing Wang

1 Center for Theoretical Physics, College of Physics, Jilin University, Changchun 130012, China
2 School of Fundamental Physics and Mathematical Sciences, Hangzhou Institute for Advanced Study, UCAS, Hangzhou, 310024, China
3 International Center for Theoretical Physics Asia-Pacific (ICTP-AP) (Beijing/Hangzhou), UCAS, Beijing 100190, China
4 Department of Physics, Tsinghua University, Beijing 100084, China
5 Center for High Energy Physics, Tsinghua University, Beijing 100084, China

E-mail: huifengfu@jlu.edu.cn, ylma@ucas.ac.cn and wangq@mail.tsinghua.edu.cn

Received 19 February 2021, revised 8 March 2021
Accepted for publication 15 March 2021
Published 20 April 2021

Abstract

We derive the chiral effective Lagrangian for excited heavy-light mesons from QCD under proper approximations. We focus on the chiral partners with $j_0^h = \frac{3}{2}^+$ and $j_0^l = \frac{3}{2}$ which amounts to $(1^+, 2^+)$ and $(1^-, 2^-)$ states respectively. The low energy constants including the masses of the chiral partners are calculated. The calculated spectrum for the excited mesons are found roughly consistent with experimental data. In addition, our results indicate that quantum numbers of $B_J(5970)$ can be identified with $1^-$ or $2^-$. Keywords: chiral effective Lagrangian, heavy-light meson, QCD, chiral partner

(Some figures may appear in colour only in the online journal)

1. Introduction

The heavy quark spin-flavor symmetry that exact under the $m_Q \to \infty$ limit with $m_Q$ being the heavy quark mass plays an important role in the hadronic systems containing one heavy quark/antiquark [1–3]. Due to this symmetry, the total angular momentum of a heavy-light meson has eigenvalues $j = j_0 \pm 1/2$ with $j_0$ being the angular momentum of the light degrees of freedom, and the corresponding eigenstates form a degenerate doublet $(j_-, j_0)$ for each $j_0$. The angular momentum $j$ can be decomposed as $j = s + \ell$, where $s$ is the spin of light quark/antiquark and $\ell$ is the orbital angular momentum. For S-wave heavy-light mesons, $l = 0$ and $j_0^h = \frac{1}{2}^-$ represents the degenerate doublet $(j_0^h, j_0^l) = (0^-, 1^-)$. For P-wave, $l = 1$ and $j_0^h = \frac{3}{2}^+$ or $j_0^l = \frac{3}{2}^-$ represent two distinct doublets $(j_0^p, j_0^l) = (0^+, 1^+)$ or $(j_0^p, j_0^l) = (1^+, 2^+)$, respectively. For D-wave, $l = 2$ and $j_0^h = \frac{3}{2}^-$ or $j_0^l = \frac{5}{2}^-$, we have two distinct doublets $(j_0^p, j_0^l) = (1^-, 2^-)$ and $(j_0^p, j_0^l) = (2^-, 3^-)$. The similar argument applies to other excited states [4].

For the light quarks, the chiral symmetry, which is dynamically (and explicitly) broken, is essential. If the chiral symmetry were preserved, the heavy-light mesons with the same $j_0$ but opposite parities would be degenerated. We call them chiral partners. The mass splitting between the chiral partners reflects the chiral symmetry breaking [5, 6].

Since heavy-light mesons capture the features of both heavy quark symmetry and chiral symmetry, it is ideal to study heavy-light meson phenomena using the chiral Lagrangian incorporating heavy quark symmetry. Since 2000, a series of researches on deriving the chiral effective Lagrangian from QCD have been carried out in [7–15]. The advantage of this method is that it can establish the analytic relationships between the low energy constants (LECs) of the chiral effective theory and QCD. Recently, we used the same methodology to derive the chiral Lagrangian incorporating heavy quark symmetry from QCD to study heavy-light mesons [16, 17]. In these works, we focused on the chiral partners $j_0^h = \frac{1}{2}^-$ and $j_0^l = \frac{1}{2}^+$. The low energy constants of the effective Lagrangian

---

* Authors to whom any correspondence should be addressed.
are expressed in terms of the light quark self-energy which can be calculated by using Dyson–Schwinger equations or lattice QCD. Numerical results of the low energy constants are globally in agreements with the experiment data.

In recent years, more and more excited heavy-light mesons have been observed. In the charm sector, many new excited states such as \( D_{s0}(2550), D_{s1}^*(2680), D_1(2740), D_s^*(3000), D_J(3000), D_s^*(3000), \) etc. have been announced by LHCb [18–20] and BABAR [21]. In the bottom sector, new bottom states \( B_0(5721), B_0^*(5747), B_1(5840) \) and \( B_J(5970) \) were observed by CDF [22] and LHCb [23]. The properties of these hadrons have drawn extensive attractions in recent years [24–31].

Here, we extend our approach to the effective field theory of excited heavy-light mesons with chiral partner structure [32]. Special interests are given to the states with quantum numbers \( j^p = \frac{3}{2}^+ \) and \( j^p = \frac{3}{2}^- \) to lay the foundation for researches on arbitrary spin heavy-light mesons.

The remaining part of this paper is organized as follows. In section 2, for convenience, we give the general form of the chiral Lagrangian of the excited heavy-light mesons. In section 3 we derive the excited heavy-light meson Lagrangian from QCD and determine the expression of low energy constants. The numerical results calculated by using the quark self-energy obtained from a typical Dyson–Schwinger equation and from lattice QCD are given in section 4. Section 5 is devoted to discussions. The expressions of the LECs with the contribution from the renormalization factor of quark wave function are given in appendix.

2. Chiral effective Lagrangian for excited heavy-light mesons

For the convenience of the following description, we present the chiral effective Lagrangian for excited heavy-light meson doublets \( T^\mu = (1^+, 2^-) \) and \( R^\mu = (1^-, 2^+) \) here. They correspond to the \( j^p = \frac{3}{2}^+ \) in P-wave and \( j^p = \frac{3}{2}^- \) in D-wave, respectively. The excited heavy-light meson doublets \( T^\mu \) and \( R^\mu \) can be expressed as [32]

\[
T^\mu(x) = \frac{1 + \gamma_\mu}{2} \left\{ P_{2^+}^{\mu\nu} \gamma_\nu - \frac{3}{2} P_{1^+}^{\mu\nu} \gamma_5 \gamma_\nu \right\},
\]

\[
R^\mu(x) = \frac{1 + \gamma_\mu}{2} \left\{ P_{2^-}^{\mu\nu} \gamma_\nu - \frac{3}{2} P_{1^-}^{\mu\nu} \gamma_5 \gamma_\nu \right\},
\]

where \( (P_{2^+}^{\mu\nu}, P_{1^+}^{\mu\nu}) \) refer to \( J^P = (1^+, 2^-) \) states, and \( (P_{2^-}^{\mu\nu}, P_{1^-}^{\mu\nu}) \) refer to \( J^P = (1^-, 2^+) \) states, respectively. \( v^\mu \) is the velocity of an on-shell heavy quark, i.e. \( p^\mu = m_Q v^\mu \) with \( v^2 = 1 \). Then the chiral effective Lagrangian for excited heavy-light mesons can be written as [33, 32]

\[
\mathcal{L} = \mathcal{L}_T + \mathcal{L}_R + \mathcal{L}_{TR},
\]

where

\[
\mathcal{L}_T = -i \text{Tr} (T^\mu \cdot \nabla T^\mu) - m_T \text{Tr} (T^\mu T^\mu),
\]

\[
\mathcal{L}_R = -i \text{Tr} (R^\mu \cdot \nabla R^\mu) + m_R \text{Tr} (R^\mu R^\mu),
\]

\[
\mathcal{L}_{TR} = J^I_{TR} \text{Tr} (R^\mu T^\mu \gamma_5 \gamma_\mu A_{\mu}),
\]

(3)

The covariant derivative \( \nabla_\mu = \partial_\mu - i V_\mu \) with \( V_\mu = \frac{i}{2} (\Omega \partial_\mu \Omega + \Omega \partial_\mu \Omega^T - \Omega^T \partial_\mu \Omega) \). The Ω field is related to the chiral field \( U(x) = \exp(\epsilon(x) f_\epsilon) \) through \( U = \Omega^2 \). The parameters \( g_T, g_R, g_{TR}, m_T, m_R \) are the LECs of the Lagrangian. They are free ones at the level of the effective theory.

As mentioned previously, the states associated with \( T^\mu \) and \( R^\mu \) are called chiral partners, and their mass splitting arises from the chiral symmetry breaking. In the \( D \) meson family, \( T^\mu \) may be associated to \( T^\mu = (1^+, 2^-) = (D_{1}(2430), D_{s}^{*}(2460)) \) and a possible identification of the \( R^\mu \) doublet may be \( R^\mu = (1^-, 2^+) = (D_{s}^{*}(2600), D(2740)) \) with respect that the fact that the states in the \( R^\mu \) doublet can decay to their chiral partners in the \( T^\mu \) doublet so that should have broad widths, so the spin-averaged masses of the chiral partners are [34, 20]

\[ m_T \approx 2448 \text{ MeV}, \quad m_R \approx 2703 \text{ MeV}, \]

(4)

which yields the mass splitting

\[ \Delta m = m_R - m_T \approx 255 \text{ MeV}. \]

(5)

In the \( B \) meson family, \( T^\mu \) may be associated to \( (B_0(5721), B_0^*(5747)) \), so the spin-averaged mass is [34]

\[ m_T \approx 5735 \text{ MeV}. \]

(6)

For the \((1^-, 2^+)\) doublet, the situation is subtle since the quantum numbers of the possible candidates in PDG are not well determined. With respect to the mass splitting between chiral partners in the charmed meson sector, the masses of the states in the \( R^\mu \) doublet should have masses \( \sim 6000 \text{ MeV} \). This means that it is reasonable to identify the quantum numbers of the state \( B_0(5970) \) as \( 1^- \) or \( 2^- \) or maybe the \( 1^- \) and \( 2^- \) states have nearly degenerate masses.

3. Chiral effective Lagrangian of excited heavy-light mesons from QCD

In this section, we follow our previous work [16] to derive the chiral effective Lagrangian for excited heavy-light mesons with \( j^p = \frac{3}{2} \) and its low energy constants.

The generating functional of QCD with an external source \( J(x) \) is

\[
Z[J] = \int Dq D\bar{q} DQ D\bar{Q} D\bar{G}_\mu D\bar{G}_\mu \Delta_\mu (G_\mu) \times \exp \left\{ i \int d^4x \left[ \mathcal{L}_{\text{QCD}}(q, \bar{q}, Q, \bar{Q}, G_\mu) + \bar{q} J q \right] \right\},
\]

(7)

where \( q(x), Q(x) \) and \( G_\mu(x) \) are the light-quark, heavy-quark and gluon fields, respectively.
By integrating in the chiral field $U(x)$ and the heavy-light meson fields, and integrating out gluon fields and quark fields, we obtain the effective action for the chiral effective theory as [16]

$$S[U, \Pi_2, \bar{\Pi}_2] = -iN_c \text{Tr} \ln \left\{ (i\not\partial - \Sigma)I_4 + J_{12} + (i\not\partial + m_Q\gamma - m_Q)I_4 \right\} - \Pi_2 - \bar{\Pi}_2,$$

(8)

where $\Pi_2$ and $\bar{\Pi}_2$ are the heavy-light meson field and its conjugate, respectively and, $m_Q$ is the mass of heavy quark $Q$. $\Sigma$ is the self-energy of the light quark propagator. $I_4 = \text{diag}(1, 1, 0)$ and $I_4 = \text{diag}(0, 0, 1)$ are matrices in the flavor space. $J_{12}$ is the chiral rotated external source.

Before going further, we would like to make a few comments on the approximations made during the derivation of equation (8). The details are given in the appendix A of [16].

• When integrating in the heavy-light meson fields, we have taken the heavy quark limit which brings in an uncertainty of order $1/m_Q$. And, to integrate out the gluon and quark fields, we left infinitely many gluon Green functions in the action.

• Further approximations, i.e. the chiral limit, the large $N_c$ limit and the leading order in dynamical perturbation, are made in order for the derived action could serve practical purposes. The chiral limit causes an uncertainty of order $m_{u,d}$ and large $N_c$ limit suffers from $1/N_c$ corrections. Taking the leading order in dynamical perturbation is a similar technique as the rainbow-ladder approximation in the Dyson–Schwinger–Bethe–Salpeter (DSBS) formulism. The errors brought in by this last approximation is hard to be estimated quantitatively, but its justification is supported by the relatively success of the rainbow-ladder approximation in the DSBS formulism.

Since the introduced heavy-light meson field $\Pi_2$ is a bilocal field, we need a suitable localization on $\Pi_2$ fields to get a local effective Lagrangian. Here we follow the approach in [32], and take the following localization conditions

$$\Pi_2(x_1, x_2) = \Pi_2(x_1)\delta(x_1 - x_2) + \Pi_2'(x_1) \frac{\partial}{\partial x_2} \delta(x_1 - x_2),$$

$$\bar{\Pi}_2(x_1, x_2) = \bar{\Pi}_2(x_1)\delta(x_1 - x_2) + \delta(x_1 - x_2) \frac{\partial}{\partial x_2} \bar{\Pi}_2'(x_1),$$

(9)

where

$$\Pi_2(x) = H(x) + G(x),$$

$$\Pi_2'(x) = T^\mu(x) + R^\mu(x).$$

(10)

Here, the fields $H$ and $G$ refer to $J^\rho(0^-, 1^-)$ states and $J^\rho(0^+, 1^-)$ states and $T^\mu$ and $R^\mu$ refer to $J^\rho(1^+, 2^+)$ states and $J^\rho(1^-, 2^-)$ states respectively. Since we are interested in the heavy-light meson doublets with $j^\rho = \frac{3}{2}^\pm$ in this work, we focus on the chiral effective Lagrangian for $\Pi_2'$ alone. So the chiral effective action is reduced to

$$S[U, \Pi_2', \bar{\Pi}_2'] = -iN_c \text{Tr} \ln \{ (i\not\partial - \Sigma)I_4 + J_{12} + (i\not\partial + m_Q\gamma - m_Q)I_4 \}
\quad - \Pi_2' - \bar{\Pi}_2'.$$

(11)

In order to obtain the chiral effective Lagrangian, we need expand the action $S[U, \Pi_2', \bar{\Pi}_2']$ with respect to the fields $U, \Pi_2'^{\mu}$ and $\bar{\Pi}_2'^{\mu}$. The kinetic terms of the excited heavy-light meson fields $\bar{T}^\mu$ (a similar equation holds for $R^\mu$) arise from

$$S_2 = \frac{\delta^2 S}{\delta T^\mu \delta T^\nu} \bar{T}^\nu T^\mu
\quad = iN_c \int d^4x d^4y \times \text{Tr} \{ (i\not\partial - \Sigma)^{-1} \delta(x_2 - x_1) \not\partial \bar{T}^\mu(x_1) \}
\quad \times (i\not\partial + m^2 + \Sigma) \not\partial \bar{T}^\nu(x_2) \delta(x_1 - x_2).$$

(12)

Taking the derivative expansion up to the first order, we obtain

$$S_2 = iN_c \int d^4x \int \frac{d^4p}{(2\pi)^4} \times \left[ \frac{p^2 - (v \cdot p)^2}{p^2 - \Sigma^2} + \frac{p^2 \Sigma}{(p^2 - \Sigma^2) v \cdot p} \right] \text{Tr} \{ T^\mu(x) T^\nu(x) \}
\quad + iN_c \int d^4x \int \frac{d^4p}{(2\pi)^4} \times \left[ \frac{p^2 \Sigma}{(p^2 - \Sigma^2) v \cdot p} \right] \text{Tr} \{ T^\nu(x) v \cdot \partial T^\mu(x) \}
\quad + iN_c \int d^4x \int \frac{d^4p}{(2\pi)^4} \times \left[ \frac{2\Sigma'(p^2 + \Sigma^2)p^2 - (v \cdot p)^2}{(p^2 - \Sigma^2) v \cdot p} \right] \text{Tr} \{ T_\mu(x) v \cdot V_\nu(x) \}. $$

(13)

In the calculation, we have used the relation $T^\mu v^\nu = -T^\nu v^\mu$. It is easy to identify the mass term and the kinetic term in equation (13). In addition, an interaction term between the $T^\mu$ fields and $V_\nu$ fields also appear in equation (13) because we have taken $\Sigma(x - y) = \Sigma(\nabla \cdot \delta(x - y) = \Sigma$ the calculation to retain the correct chiral transformation properties in the theory [8].

The interaction between the heavy-light meson field $T^\mu$ and the light Goldstone boson field $S_1$ can also be obtained by expanding the action $S[U, \Pi_2', \bar{\Pi}_2']$ with respect to the external $J_{12}$, $\Pi_2'$ and $\bar{\Pi}_2'$ as

$$S_3 = \frac{\delta^3 S}{\delta J_{12} \delta T^\mu \delta V^\mu} \text{Tr} I_3 T^\mu
\quad = -iN_c \int d^4x d^4x_1 d^4x_2 \times \text{Tr} \{ (i\not\partial - \Sigma)^{-1} \delta(x_2 - x_1)J_{12}(x_1) \}
\quad \times (i\not\partial - \Sigma)^{-1} \delta(x_1 - x_2) \not\partial \bar{T}^\mu(x_1) (i\not\partial) \not\partial \bar{T}^\mu(x_2) \delta(x_1 - x_2).$$

(14)
Upto the first order of the derivative expansion, one obtains

\[ S_1 = -i \mathcal{N} \int d^{4}x \int \frac{d^{4}p}{(2\pi)^{4}} \left[ \frac{p^2 (\Sigma^2 + \frac{1}{2}p^2)}{(p^2 - \Sigma^2)^2} \right] \]

\[ -2\Sigma \frac{p^2 - (v \cdot p)^2}{(p^2 - \Sigma^2)^2} \text{Tr} [\bar{T}(x)\gamma^\mu \gamma_5 A_\mu(x)] \]

\[ + i \mathcal{N} \int d^{4}x \int \frac{d^{4}p}{(2\pi)^{4}} \left[ \frac{p^2}{(p^2 - \Sigma^2)^2} \right] \text{Tr} [\bar{T}_\mu(x)v \cdot V_{\mu}(x)]. \]

(15)

Summing up equations (13) and (15), we obtain the expressions of the constants \( m_T \) and \( g_T \) as

\[ m_T = \frac{i \mathcal{N}}{Z_T} \int d^{4}p \left[ \frac{p^2 - (v \cdot p)^2}{(p^2 - \Sigma^2)^2} - \frac{p^2 \Sigma}{(p^2 - \Sigma^2)^2} \right], \]

\[ g_T = -\frac{i \mathcal{N}}{Z_T} \int d^{4}p \left[ \frac{p^2 (\Sigma^2 + \frac{1}{2}p^2)}{(p^2 - \Sigma^2)^2} \right] \]

\[ - 2\Sigma \frac{p^2 - (v \cdot p)^2}{(p^2 - \Sigma^2)^2}. \]

(16)

Following the same procedure, one can get the LECs associated to \( R^\mu \). The only difference in the calculation is the \( \frac{p^2}{(p^2 - \Sigma^2)^2} \) and \( \frac{1}{(p^2 - \Sigma^2)^2} \) terms.

\[ Z_T = \frac{i \mathcal{N}}{Z_T} \int d^{4}p \left[ \frac{p^2 - (v \cdot p)^2}{(p^2 - \Sigma^2)^2} + \frac{2\Sigma (\Sigma^2 + \frac{1}{2}p^2)}{(p^2 - \Sigma^2)^2} \right], \]

\[ Z_R = \frac{i \mathcal{N}}{Z_R} \int d^{4}p \left[ \frac{-p^2}{(p^2 - \Sigma^2)^2} \right] \]

\[ + \frac{2\Sigma (\Sigma^2 + \frac{1}{2}p^2)}{(p^2 - \Sigma^2)^2} \right]. \]

(17)

We also obtain the coupling constant between the chiral partner fields \( T^\mu \) and \( R^\mu \) as

\[ g_{TR} = -\frac{i \mathcal{N}}{\sqrt{Z_T Z_R}} \int d^{4}p \left[ \frac{p^2 (\Sigma^2 + \frac{1}{2}p^2)}{(p^2 - \Sigma^2)^2} \right]. \]

(18)

In the above expressions, \( \Sigma(-p^2) \) stands for self-energy of light quarks to be calculated from QCD. This is an apparent indicator that we have established a connection between the low energy constants of the chiral effective theory and the Green functions of QCD.

### 4. Numerical results and discussions

In order to obtained the explicit numerical values of the LECs, we first need to obtain the light quark self-energy \( \Sigma(-p^2) \). As in our previous works [16, 17], we use two method to determine \( \Sigma(-p^2) \) for comparison, namely, the gap equation, i.e. the Dyson–Schwinger equation for quarks, and the fittings from lattice QCD.

For the Dyson–Schwinger equation method, we use the differential form of the gap equation [8]:

\[
\left( \frac{\alpha_s(\mu)}{\pi} \right)^\gamma \Sigma(\mu)^n - \left( \frac{\alpha_s(\mu)}{\pi} \right)^\gamma \Sigma(\mu)^n = -\frac{3C_2(R)}{4\pi} \left( \frac{\alpha_s(\mu)}{\pi} \right)^2 \int_0^{\Lambda^'} dx \frac{x \Sigma(\mu)}{x + \Sigma(\mu)} \left( \frac{\alpha_s(\mu)}{\pi} \right)^2 = 0,
\]

where the boundary conditions are

\[
\Sigma(0) = -\frac{3C_2(R)}{8\pi} \alpha_s(0),
\]

\[
\Sigma(\Lambda') = \frac{3C_2(R)\alpha_s(\Lambda')}{4\pi} \int_0^{\Lambda'} dx \frac{x \Sigma(\mu)}{x + \Sigma(\mu)} \left( \frac{\alpha_s(\mu)}{\pi} \right)^2.
\]

(19)

with \( \alpha_s \) being the running coupling constant of QCD. \( \Lambda' \) is an ultraviolet cutoff regularizing the integral, which is taken \( \Lambda' \to \infty \) eventually. To solve the equation (19), we take a model description for \( \alpha_s \) given in [35]

\[
\alpha_s(p^2) = \alpha_0 p^2 \frac{p^2 + M^2}{p^4 + (M^2 + m^2)p^2 + \lambda^2}.
\]

(20)

For convenience, we call it Gribov-Zwanziger (G-Z) Model. The parameters are \( M^2 = 4.303 \text{ GeV}^2, \langle M^2 + m^2 \rangle = 0.526 \text{ GeV}^2 \) and \( \lambda^2 = 0.49295 \text{ GeV}^4 \) [35]. \( \alpha_0 \) is a model parameter to be determined. Although the UV behavior of G-Z formalism is inconsistent with QCD, it should not have a sizable impact on our results because the LECs are mostly controlled by its low energy behavior.

Solving the gap equation, we obtain \( \Sigma(-p^2) \). Then the LECs are calculated according to equations (16)–(18). It is clear that the integrals of the LECs have a physical ultraviolet cutoff \( \Lambda_c \), which should be of the order of the chiral symmetry breaking scale and serves as another parameter in our calculations. Since we are studying excited states, the cutoff \( \Lambda_c \) should take a bit larger value than 1 GeV. For the excited states we are considering now, the energy of the light quark in the chromoelectrical fields generated by the heavy quark could be up to \( \sim 1.45 \text{ GeV} \), so \( \Lambda_c \) taking a value \( \gtrsim 1.5 \text{ GeV} \) would be more appropriate for the present situation. For the G-Z model, we take \( \Lambda_c = 1.6 \text{ GeV} \) and determine the model parameter \( \alpha_0 \) by requiring the calculated \( f_L \) be consistent with the corresponding experimental value. We find that using
are given for comparison. The masses and quantum numbers have not been confirmed yet, we focus on the mass of the chiral partners in our method in this work and hopefully could help identifying some excited mesons. The mass splitting $\Delta m$ is a direct indicator of the chiral symmetry breaking. In our calculations, it is completely originated from the non-vanishing of the quark condensate or the value of $\Sigma(0)$. The physical picture of this procedure is the ‘decoupling’ of the heavy quark from other degrees of freedom. So, in a heavy-light meson, (in the heavy quark limit) the heavy quark serves as a static color source, and the light degree of freedom interacts with this source. The ‘residue mass’ is just the energy of the light degree of freedom bounded by the color source. Thus the physical masses (denoted as $\tilde{m}_R$ and $\tilde{m}_L$) can be easily restored by adding the heavy quark mass to $m_T$ and $m_R$. For the $D$ meson sector, using $m_c \approx 1.27$ GeV [34], we obtain the physical masses to be $\tilde{m}_T \approx 2.35$ GeV and

| $a_0$ | $\Sigma(0)$ (GeV) | $f_\pi$(GeV) | $-\langle \bar{\psi}\psi \rangle$ (GeV$^3$) | $g_R$ | $g_T$ | $g_{TR}$ | $m_R$(GeV) | $m_T$(GeV) | $\Delta m$(GeV) |
|-------|------------------|-------------|-----------------------------------|------|------|--------|------------|-------------|-------------|
| 0.50  | 0.158            | 0.073       | (0.328)$^3$                       | 0.882| -0.479| -0.990 | 1.365      | 1.104       | 0.261       |
| 0.51  | 0.193            | 0.084       | (0.350)$^3$                       | 0.936| -0.442| -0.987 | 1.411      | 1.091       | 0.321       |
| 0.52  | 0.226            | 0.093       | (0.368)$^3$                       | 0.991| -0.409| -0.984 | 1.459      | 1.080       | 0.379       |
| 0.53  | 0.259            | 0.102       | (0.385)$^3$                       | 1.047| -0.377| -0.981 | 1.510      | 1.071       | 0.439       |
| 0.54  | 0.291            | 0.109       | (0.399)$^3$                       | 1.107| -0.347| -0.979 | 1.564      | 1.064       | 0.500       |

Exp. — 0.093 [34] — — — — 1.433$^a$ 1.178 0.255

$^a$ The experimental data of $m_R$, $m_T$ and $\Delta m$ are obtained by using the spin-averaged masses in the $D$ meson sector with $m_c$ subtracted.

$\alpha_0 = 0.52$ we can obtain the well-established quantity $f_\pi$. The results are listed in table 1. For comparison, we also list the numerical results obtained with different $\alpha_0$’s in the table. To give an intuitive impression, we draw the running coupling constant $\alpha_s$ calculated with the G-Z formalism at $\alpha_0 = 0.52$ in figure 1. The light quark self-energy $\Sigma(-p^2)$ solved by the gap equation (19) is shown in figure 2.

Since there are more and more excited states observed experimentally whose masses and quantum numbers have not been confirmed yet, we focus on the mass of the chiral partners in our method in this work and hopefully could help identifying some excited mesons. The mass splitting $\Delta m$ is a direct indicator of the chiral symmetry breaking. In our calculations, it is completely originated from the non-vanishing of the quark condensate or the value of $\Sigma(0)$. At $\alpha_0 = 0.52$, we find $\Delta m = 379$ MeV which is a bit larger than the expected value $\sim 255$ MeV as given in (4). This is not surprising because on one hand the measured masses of the relevant excited mesons are not accurate, and on the other hand our results are suffering from uncertainties due to the ignorance of the $1/m_Q$ contributions, $1/N_c$ contributions, etc, as indicated in the previous section. Since $m_{Q,d}$ both are a few MeVs which are much smaller than the chiral scale, as we do not expect large errors from taking the chiral limit. For the $1/N_c$ contributions, one may expect relative errors of roughly $\sim 30\%$ (which is of course a too naive estimation and more precise values can only be obtained by calculating the $1/N_c$ corrections for LECs). For the $1/m_Q$ contributions, the results for the ground states given in [17] may give a quantitative intuition. It is shown in [17] that $1/m_Q$ corrections to $\Delta m$ of $p^2 = \frac{1.27}{2}$ heavy-light mesons are about $\sim 100$ MeV for $m_c$ and about tens MeVs for $m_b$.

The masses $m_T$ and $m_R$ listed in table 1 are the ‘residue masses’ with the heavy quark mass rotated away. Because, in the standard treatment for heavy quarks, a factor of the form $e^{-i m_Q x}$ is factored out from the heavy quark field, i.e. $Q(x) = e^{-i m_Q x} Q(x)$ in the heavy quark limit, and $m_Q$ is canceled out in the Lagrangian is expressed in terms of $Q(x)$, i.e. $Q(i \not{\partial} - m_Q)Q \rightarrow Q$. So $m_Q$ is rotated away and hadron masses in this formalism are free from $m_Q$. The physical picture of this procedure is the ‘decoupling’ of the heavy quark from other degrees of freedom. So, in a heavy-light meson, (in the heavy quark limit) the heavy quark

\[ F_0 = \frac{1}{3} F_F. \]
m_R ≈ 2.73 GeV with α_0 = 0.52. Keeping in mind the large uncertainties from both the theoretical side and the experimental side, these values are roughly comparable to the corresponding experimental data (4). For the B meson family, using m_R ≈ 4.66 GeV [34], we obtain the physical masses to be m_T ≈ 5.74 GeV and m_m ≈ 6.12 GeV. Comparing with the experimental data for the bottom mesons in (6), the numerical result of m_T is close to the corresponding experimental data. And we expect (1^+, 2^+) B meson states appear at around 6.12GeV. Actually, the excited state B_J(5970) reported in [22, 23] with the mass ~5970 MeV could be the candidate for the 1^- or 2^- states. Up to now, experiments have not yet determined the J^P quantum numbers for B_J(5970). We expect this could be found in the near future.

Now, let us turn to the results using the quark self-energy fitted from lattice QCD. We use the lattice fittings for the quark wave function renormalization Z(–p^2) and the running quark mass M(–p^2) given in [36]. The formula of LECs shown in equations (16)–(18) can be directly extended to incorporate the contributions from Z(–p^2). The relevant expressions are given in appendix. The functions Z(–p^2) and M(–p^2) are also plotted in figure 2. The numerical results of LECs calculated by using these lattice fittings are given in table 2. There is no free parameter in the functions of M(–p^2) and Z(–p^2). Typically, the running quark mass and the wave function renormalization are calculated in the negative p^2 region. Extrapolation of these functions into the complex p^2-plane is known to be technically difficult. Fortunately, we are calculating now only involve M(–p^2), Z(–p^2) and Σ(–p^2) in the negative p^2 region. Because, the existence of the cutoff Λ_c guarantees that the integrations in equations (16)–(18) and those in the appendix should only be performed within the range 0 ≤ |–p^2| ≤ Λ_c. This would be easier understood by translating the momentum p into the Euclidean momenta via Wick rotation.

From table 2, one can see that the numerical results of LECs are comparable to the results from the G-Z model when Λ_c = 1.6 GeV, which implies that Z(–p^2) does not have significant impacts on the excited states. The physical masses are m_T ≈ 2.37 GeV and m_m ≈ 2.73 GeV for excited D mesons, and m_T ≈ 5.76 GeV and m_m ≈ 6.12 GeV for excited B mesons. The conclusions could be drawn from these results are the same as we have in the G-Z model.

At last, given the coupling constants obtained in this work, we can calculate some of the decay widths involving excited D or B mesons. The mass difference between T^+ and T^- (R^- and R^+) is too small to allow T^- decay to T^+ + π (R^- decay to R^+ + π), where T^+ and T^- (R^- and R^+) represent the 2^+ and 1^+ (2^- and 1^-) states of the meson doublet T (R) respectively. In addition, at the O(p^2) order of chiral perturbation, R^- → T^-π and R^+ → T^+π decays vanish. So the only possible relevant decays are R^+ → T^-π and R^- → T^+π, which are induced by g_TR. Using equations (1)–(3), one can deduce the decay widths as

\[ \Gamma(R^- \rightarrow T^-\pi) = \frac{g^2_{TR} p m T E}{12 \pi m f} \left( 2 + \frac{E^2}{m^2} \right) \]

\[ \Gamma(R^+ \rightarrow T^+\pi) = \frac{g^2_{TR} p m T E}{180 \pi m f} \left( 19 + \frac{2 E^2}{m^2} + \frac{4 E^2}{m^2} \right) \]

where m is the mass of the decaying meson; m_T and E_T are the mass and the energy of the heavy-light meson in the final states respectively; E_π and p_π are the energy and the magnitude of the momentum of the pion, respectively.

Given the assignments of the D mesons, we find

\[ \Gamma(D^+(2600)^0 \rightarrow D^+(2430)^+\pi^-) = \begin{cases} 40 \text{ MeV (G-Z)}, \\ 43 \text{ MeV (Lat.)} \end{cases} \]

\[ \Gamma(D^+(2740)^0 \rightarrow D^+(2460)^+\pi^-) = \begin{cases} 123 \text{ MeV (G-Z)}, \\ 131 \text{ MeV (Lat.)} \end{cases} \]

where (G-Z) denotes results from the G-Z model with α_0 = 0.52; (Lat.) denotes results from the lattice-fittings with Λ_c = 1.6 GeV. For the B mesons, the assignments of R doublet is yet unclear. If B_J(5970) is the 1^- state in the R doublet, then we have

\[ \Gamma(B_J(5970)^0(R^-) \rightarrow B_J(5721)^+\pi^-) = \begin{cases} 98 \text{ MeV (G-Z)}, \\ 105 \text{ MeV (Lat.)} \end{cases} \]

(25)

If B_J(5970) is the 2^- state in the R doublet, then we have

\[ \Gamma(B_J(5970)^0(R^-) \rightarrow B_J^+(5747)^+\pi^-) = \begin{cases} 84 \text{ MeV (G-Z)}, \\ 90 \text{ MeV (Lat.)} \end{cases} \]

(26)
In the future, when the corresponding experimental results are available, our theoretical predictions could help testing the assignments of these excited mesons.

5. Conclusions

In this paper, we derive the chiral effective Lagrangian for excited heavy-light mesons from QCD. We focus on the chiral partners with $j_0^B = \frac{3}{2}^-$ and $j_0^B = \frac{3}{2}^+$ which amounts to ($1^+, 2^-$) and ($1^-, 2^+$) states respectively. The low energy constants are expressed as momentum integrals of the light quark self-energy, which in turn are obtained by using the gap equation or lattice QCD. The relevant LECs in the chiral Lagrangian are calculated, and the resulted masses for the excited $D$ mesons are found roughly consistent with experimental data. For the excited $B$ mesons, we obtain $\hat{m}_B \approx 5.74$ GeV and $\hat{m}_B \approx 6.12$ GeV. We find that $\hat{m}_B$ are consistent to the masses of $(B_1(5721), B_2^*(5747))$, and the result of $\hat{m}_B$ suggests that $B_J(5970)$ could be a good candidate for the states in the doublet ($1^-, 2^+$).

As more and more excited states of charmed mesons and bottom mesons are discovered in experiments, it will be an interesting and important research to extend our method to obtain the chiral effective Lagrangian of heavy-light mesons with arbitrary spins. Since this extension is straightforward, we will not go to details here.

We finally want to say that, so far, we did not discuss the chiral partner structure of the heavy-light mesons including a strange quark. One of the reasons is that the quark content of some of these mesons is still under debate. For example, the mesons $D_{so}(2317)$ and $D_{st}(2460)$ which can be arranged to the $G$ doublet with quantum numbers $G = (0^+, 1^-)$ are regarded as the chiral partners of $D$ and $D^*$, respectively, in the $H$ doublet with quantum numbers $H = (0^-, 1^+)$ [37]. The molecular state interpretation of these mesons and their bottom cousins cannot be ruled out [38–40]. We leave the discussion of these strange mesons as future works.

Acknowledgments

The work of YLM was supported in part by National Science Foundation of China (NSFC) under Grant No. 11 875 147 and No.11475071. H-F Fu was supported by NSFC under Grant No.12047569. QW was supported by the National Science Foundation of China (NSFC) under Grant No. 11 475 092.

Appendix. Formula for lecs with $Z(-p^2)$

The quark propagator can be generally written as

$$S(p) = \frac{i}{A(-p^2)} p + B(-p^2) = i Z(-p^2) p + M(-p^2) \frac{p^2 - M^2(-p^2)}{p^2 - M^2(-p^2)},$$

where $Z(-p^2) = 1/A(-p^2)$ stands for the quark wave function renormalization and $M(-p^2) = B(-p^2)/A(-p^2)$ is the renormalization group invariant running quark mass. After a series of calculations, we can get the LECs with $Z(-p^2)$ as follows:

$$m_T = \frac{i N_c}{Z_T} \int \frac{d^4p}{(2\pi)^4} \left[ \frac{p^2 - (v \cdot p)^2}{p^2 - M^2} - \frac{p^2 M}{(p^2 - M^2)v \cdot p} \right] Z,$$

$$g_T = -\frac{i N_c}{Z_T} \int \frac{d^4p}{(2\pi)^4} \left[ \frac{p^2}{p^2 - M^2} + \frac{2M(p^2 + v^2)}{(p^2 - M^2)^2v \cdot p} \right] Z,$$

$$m_R = \frac{i N_c}{Z_R} \int \frac{d^4p}{(2\pi)^4} \left[ \frac{p^2 - (v \cdot p)^2}{p^2 - M^2} + \frac{p^2 M}{(p^2 - M^2)v \cdot p} \right] Z,$$

$$g_R = -\frac{i N_c}{Z_R} \int \frac{d^4p}{(2\pi)^4} \left[ \frac{p^2}{p^2 - M^2} + \frac{2M(p^2 + v^2)}{(p^2 - M^2)^2v \cdot p} \right] Z,$$

$$Z_T = i N_c \int \frac{d^4p}{(2\pi)^4} \left[ \frac{-p^2}{(p^2 - M^2)v \cdot p} \right] \left[ \frac{2M + 2Z'M(p^2 + M^2)}{(p^2 - M^2)^2} \left( \frac{p^2}{p^2 - M^2} - \frac{2Z'M}{p^2 - M^2} \right) \right] Z,$$

$$Z_R = i N_c \int \frac{d^4p}{(2\pi)^4} \left[ \frac{-p^2}{(p^2 - M^2)v \cdot p} \right] \left[ \frac{2M + 2Z'M(p^2 + M^2)}{(p^2 - M^2)^2} \left( \frac{p^2}{p^2 - M^2} - \frac{2Z'M}{p^2 - M^2} \right) \right] Z,$$

$$g_{TR} = -\frac{i N_c}{\sqrt{Z_T Z_R}} \int \frac{d^4p}{(2\pi)^4} \left[ \frac{p^2(M^2 + p^2)}{(p^2 - M^2)^2v \cdot p} \right] Z^2. \quad (A1)$$

ORCID iDs

Yong-Liang Ma @ https://orcid.org/0000-0001-9154-529X

References

[1] Isgur N and Wise M B 1990 Phys. Lett. B 237 527
[2] Isgur N and Wise M B 1989 Phys. Lett. B 232 113
[3] Eichten E and Hill B R 1990 Phys. Lett. B 234 511
[4] Yuan T C 1995 Phys. Rev. D 51 4830
[5] Nowak M A, Rho M and Zahed I 1993 Phys. Rev. D 48 4370
[6] Bardeen W A and Hill C T 1994 Phys. Rev. D 49 409
[7] Wang Q, Kuang Y-P, Xiao M and Wang X-L 2000 Phys. Rev. D 61 054011
[8] Yang H, Wang Q, Kuang Y-P and Lu Q 2002 Phys. Rev. D 66 014019
[9] Jiang S-Z, Zhang Y, Li C and Wang Q 2010 Phys. Rev. D 81 014001
[10] Jiang S-Z and Wang Q 2010 Phys. Rev. D 81 094037
[11] Jiang S-Z, Wang Q and Zhang Y 2013 Phys. Rev. D 87 094014
[12] Jiang S-Z, Wei Z-L, Chen Q-S and Wang Q 2015 Phys. Rev. D 92 025014
[13] Wang X-L and Wang Q 2000 Commun. Theor. Phys. 34 519
[14] Wang X-L, Wang Z-M and Wang Q 2000 Commun. Theor. Phys. 34 683
[15] Ren K, Fu H-F and Wang Q 2017 Phys. Rev. D 95 074012
[16] Chen Q-S, Fu H-F, Ma Y-L and Wang Q 2020a Phys. Rev. D 102 034034
[17] Chen Q-S, Fu H-F, Ma Y-L and Wang Q 2020b arXiv:2012.03733
[18] Aaij R et al (LHCb) 2013 JHEP 09 145
[19] Aaij R et al (LHCb) 2016 Phys. Rev. D 94 072001
[20] Aaij R et al (LHCb) 2020 Phys. Rev. D 101 032005
[21] del Amo Sanchez P et al (BaBar) 2010 Phys. Rev. D 82 111101
[22] Aaltonen T A et al (CDF) 2014 Phys. Rev. D 90 012013
[23] Aaij R et al (LHCb) 2015 JHEP 04 024
[24] Godfrey S and Moats K 2016 Phys. Rev. D 93 034035
[25] Song Q-T, Chen D-Y, Liu X and Matsuki T 2015 Phys. Rev. D 92 074011
[26] Chen H-X, Chen W, Liu X, Liu Y-R and Zhu S-L 2017 Rep. Prog. Phys. 80 076201
[27] Gupta P and Upadhyay A 2018 Phys. Rev. D 97 014015
[28] Kher V, Devlani N and Rai A K 2017 Chin. Phys. C 41 073101
[29] Gandhi K and Rai A K 2019 arXiv:1911.06063
[30] Gupta P and Upadhyay A 2019 Phys. Rev. D 99 094043
[31] Godfrey S and Moats K 2019 Eur. Phys. J. A 55 84
[32] Nowak M A and Zahed I 1993 Phys. Rev. D 48 356
[33] Kilian U, Korner J G and Pirjol D 1992 Phys. Lett. B 288 360
[34] Zyla P et al (Particle Data Group) 2020 Prog. Theor. Exp. Phys. 2020 083C01
[35] Dudal D, Oliveira O and Rodriguez-Quintero J 2012 Phys. Rev. D 86 105005
[36] Oliveira O, de Paula W, Frederico T and de Melo J 2019 Eur. Phys. J. C 79 116
[37] Nowak M A, Rho M and Zahed I 2004 Acta Phys. Polon. B 35 2377
[38] Faessler A, Gutsche T, Lyubovitskij V E and Ma Y-L 2007a Phys. Rev. D 76 014005
[39] Faessler A, Gutsche T, Lyubovitskij V E and Ma Y-L 2007b Phys. Rev. D 76 114008
[40] Faessler A, Gutsche T, Lyubovitskij V E and Ma Y-L 2008 Phys. Rev. D 77 114013