The Improvement on Noise Attenuation Performance of a Duct-resonator System

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Abstract
The Helmholtz resonator (hereafter resonator) is qualified as a silencer with a narrow noise attenuation band at its designed resonance frequency. Combining several resonators in line is a possible way to produce a broader noise attenuation band. This paper focuses on improving the noise attenuation performance of a duct-resonator system at low frequency. Two types of periodic duct-resonator system are analyzed theoretically and numerically: a duct-resonator system with identical resonators and a modified duct-resonator system with periodic two-resonator arrays. The planar wave theory and the transfer matrix method are used to investigate wave propagation in the duct-resonator system. The theoretical prediction results yield satisfactory agreement with the Finite Element Method simulation results. The results indicate that both the periodic duct-resonator system and the modified duct-resonator system can broaden the noise attenuation band. Furthermore, the proposed modified duct-resonator system in this paper contributes to a relatively broader noise attenuation band than the periodic duct-resonator system. The modified duct-resonator system provides a useful method for the design of such a system, in order to obtain a relatively broadband noise attenuation.

Keywords: noise control; resonator; transfer matrix; periodic

1. Introduction
A ventilation ductwork system is an essential component of buildings that includes both the exchange of fresh air indoors as well as circulation of air within a building. It plays a significant role in maintaining good indoor air quality. However, the accompanied unpleasant duct-borne noise produced by in-duct elements of a ventilation system results in psychological and physiological ailments to humans. Engineers are very concerned about whether they can reduce the noise, especially the low frequency and broadband noise in ventilation ductwork systems.

The traditional passive noise control methods for the duct-borne noise problem can be divided into two categories: dissipative silencers that particularly try to absorb sound energy as it propagates through the duct and reactive silencers that specifically aim to prevent sound propagation through the duct. However, a dissipative silencer can hardly absorb any noise at low frequency. On the contrary, a reactive silencer such as a Helmholtz resonator (hereafter resonator) is known as an efficient silencer to reduce low frequency duct-borne noise and has been widely used in noise control applications for the ventilation ductwork system.

A resonator is qualified as a silencer with a narrow noise attenuation band at its designed resonance frequency. The resonance frequency of a resonator is only determined by the geometries of the neck and the cavity. It is therefore easy to tune a desired resonance frequency. To obtain a broader noise attenuation band, combining several resonators in line is a possible way. Numerous studies have been conducted following this idea using two methods. One uses a serial arrangement of resonators with different resonance frequencies to obtain a wide band of noise control in ducts. Terao et al. proposed an onsite tuning system for resonator arrays for HVAC ducts and conducted an experimental resonator array tuning system. The effectiveness of the tuning system was confirmed by their experiment. Seo and Kim investigated the acoustic characteristics of multiple array resonators and optimized the arrangement of resonators to broaden the noise attenuation band. Coulon et al. examined the effects of distance between resonators on the attenuation performance of the whole duct-resonator system and proposed an optimization procedure to handle noise attenuation on a required frequency band. The other utilizes the coupling effects of the resonator and the structure to achieve broadband noise attenuation. The wave propagation...
in a periodic structure exhibits peculiar characteristics results from the Bragg reflections\(^{15-16}\). Owing to the coupling of the periodic structure and the resonator, the resonators distributed periodically on the duct provide a much broader noise attenuation compared to a single resonator\(^{17}\). Sugimoto and Horioka\(^{18}\) revealed the mechanisms of stopbands and passbands in a spatially periodic structure. Wang and Mak\(^{19}\) presented three types of stopbands in a duct loaded with identical resonators and predicted their bandwidth theoretically. Li et al.\(^{20}\) investigated the acoustic wave propagation and sound transmission in a metamaterial-based piping system with resonators attached periodically and examined the effects of resonator parameters on the passband, including bandwidth, location and attenuation performance.

The purpose of this study is to improve the noise attenuation performance of a duct-resonator system at low frequency. Two types of periodic duct-resonator system are analyzed theoretically and numerically: a duct-resonator system with identical resonators and a modified duct-resonator system with periodic two-resonator arrays. The modified periodic duct-resonator system proposed in this paper takes into account both the special dispersion characteristics of periodic structure and the effects of tuned resonators with different resonance frequencies to achieve a broader noise attenuation band. The planar wave theory and the transfer matrix method can offer initial prototype solutions quickly for a duct-resonator system. Besides, the frequency range considered in this paper is well below the cut-off frequency of the duct. Therefore the planar wave and the transfer matrix method are used to investigate wave propagation in the duct in this study. Then, the theoretical prediction results are compared with the Finite Element Method (FEM) simulation results.

2. Theoretical Analysis

2.1 Acoustic Performance of a Side Branch Resonator

The traditional approach in modelling a resonator leads to an equivalent mass-spring model. The mass of air in the neck is driven by an external force and the cavity is regarded as the spring\(^{8}\). To improve the accuracy of the mass-spring model, an end correction factor is introduced to account for the multidimensional wave effects at the discontinuity interface\(^{21}\). Considering the wave propagation in both the neck and the cavity could provide a more accurate resonator model. However, this paper focuses on proposing a novel design technique for a duct-resonator system. For this reason, the traditional modelling approach is adopted, using a mass-spring model for the resonator. The acoustic impedance of a HR \(Z_r\) is given as:

\[
Z_r = j \frac{\rho_0 c_0}{S_c \omega} (\omega^2 - \omega_r^2)
\]  
(1)

where \(\omega_r = c_0 \sqrt{S_c / I_0}\) is the circular resonance frequency, \(c_0\) is the speed of the sound in the air, \(\rho_0\) is the air density, \(l_0\) is "corrected" neck length by adding a term of end correction factor.

As indicated in Fig.1, a side branch HR, with neck cross-section \(S_n\), neck length \(l_n\) and cavity volume \(V_c\), mounted on the duct with cross section area \(S_c\). The frequency range considered in this paper is well below the cut-off frequency of the duct. This means that the non-planar wave excited at the duct-neck interface will be delayed exponentially. Only a planar wave is assumed to exist in the duct\(^{22}\). Assuming a time-harmonic disturbance in the form of \(e^{\omega t}\) is neglected, the sound pressure and particle velocity can be expressed as:

\[
P_1 = A_1 e^{-j k x} + B_1 e^{j k x}, \quad P_2 = A_2 e^{-j k x}
\]  
(2-3)

\[
u_1 = \frac{A_1 e^{j k x}}{\rho_0 c_0} - B_1 e^{j k x} / \rho_0 c_0, \quad \nu_2 = \frac{A_2 e^{j k x}}{\rho_0 c_0}
\]  
(4)

where \(A_1, B_1, A_2\) represent respective complex wave amplitudes, \(k\) is the number of waves.

Combining the continuity of the sound pressure and volume velocity at the duct-neck interface, the relation of point 1 to point 2 could be obtained through the transfer matrix method\(^{22}\) as:

\[
\begin{bmatrix} P_1 \\ u_1 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix}^{-1} \begin{bmatrix} P_2 \\ u_2 \end{bmatrix} \quad \begin{bmatrix} P_1 \\ u_1 \end{bmatrix} = T_1 \begin{bmatrix} P_2 \\ u_2 \end{bmatrix}
\]  
(6)

Once the transfer matrix \(T_1\) of the side branch resonator has been obtained, the transmission loss can be calculated from the expression as:

\[
TL = 20 \log_{10} \left| \frac{1}{I_n + t_1 + \rho_0 c_0 I_{12} + t_2} \right| = 20 \log_{10} \left| 1 + \frac{\rho_0 c_0}{S_c Z_r} \right|
\]  
(7)

![Fig.1. A Side Branch Helmholtz Resonator](image1)

![Fig.2. A Two Helmholtz Resonators Array](image2)
2.2 Acoustic Performance of a Two-resonator Array

A single HR has a high transmission loss amplitude with narrow band at its resonance frequency. Combining several tuned resonators in line is a possible way to obtain a broader noise attenuation band. Fig.2. illustrates a two-resonator array configuration. The acoustic impedance of these two HRs can be calculated by Eq. (1), expressed as \( Z_1 \) and \( Z_2 \) respectively. The property of the sound pressure and the particle velocity between these two duct-neck interfaces could be described by Eq. (6) in a matrix form as:

\[
\begin{bmatrix}
P_1 \\
u_1
\end{bmatrix} = \begin{bmatrix} 1 & 0 \\ \frac{1}{S_d Z_1} & 1 \end{bmatrix} \begin{bmatrix} P_2 \\
u_2
\end{bmatrix} = T_{12} \begin{bmatrix} P_2 \\
u_2
\end{bmatrix}
\]

(8)

\[
\begin{bmatrix}
P_3 \\
u_3
\end{bmatrix} = \begin{bmatrix} 1 & 0 \\ \frac{1}{S_d Z_2} & 1 \end{bmatrix} \begin{bmatrix} P_4 \\
u_4
\end{bmatrix} = T_{32} \begin{bmatrix} P_4 \\
u_4
\end{bmatrix}
\]

(9)

Only a planar wave is assumed to propagate in the duct. This means that there is only a phase delay when the wave propagation in the straight duct is from point 2 to point 3. Thus the relation of point 2 to point 3 could be expressed as:

\[
\begin{bmatrix}
P_3 \\
u_3
\end{bmatrix} = \begin{bmatrix} \cos(kL_3) & j \rho_3 c_3 \sin(kL_3) \\ j(-1) \sin(kL_3) & \cos(kL_3) \end{bmatrix} \begin{bmatrix} P_2 \\
u_2
\end{bmatrix} = T_{32} \begin{bmatrix} P_2 \\
u_2
\end{bmatrix}
\]

(10)

where \( L_3 \) represents the distance between two resonators, \( T_{duct} \) is the transfer matrix of the straight duct. Then, the transfer matrix of sound pressure and particle velocity between point 1 and point 4 can be given as:

\[
\begin{bmatrix}
P_1 \\
u_1
\end{bmatrix} = T_1 T_{duct} T_2 \begin{bmatrix} P_1 \\
u_1
\end{bmatrix}
\]

(11)

Combining Eq. (11) with Eq. (7), the transmission loss of the two-resonator array could be calculated.

2.3 Acoustic Performance of a Periodic Duct-resonator System with Identical Resonators

Wave propagation in a duct-resonator system with identical resonators mounted periodically is investigated in this section, as shown in Fig.3. The diameter of the resonator’s neck is assumed to be negligible compared to the distance between two nearby resonators. This means that the length of the duct segment \( d \) is regarded as the periodic distance. Owing to the coupling of the resonators and the duct segment, it is found that the resonators distributed periodically can provide a much broader noise attenuation band due to Bragg reflection. In practice, the half-wavelength is often chosen as the periodic distance when the first Bragg reflection band coincides with the desired resonance frequency. In such case, a wider noise attenuation band could be achieved.

The sound pressure and particle velocity in the duct segment of the \( nth \) periodic cell can be described as \( p_n(x) \) and \( u_n(x) \) with a suffix of \( n \). By introducing Eq. (6) and Eq. (10), the relation of sound at the beginning of the \( nth \) and \( (n+1)th \) periodic cell could be expressed as:

\[
\begin{bmatrix}
P_x \\
u_x
\end{bmatrix} = \begin{bmatrix} \cos(kd) & j \rho_3 c_3 \sin(kd) \\ j(-1) \sin(kd) & \cos(kd) \end{bmatrix} \begin{bmatrix} 1 \\
0
\end{bmatrix} \begin{bmatrix} P_{x+1} \\
u_{x+1}
\end{bmatrix}
\]

(12)

Eq. (12) could be simplified as:

\[
\begin{bmatrix}
P_x \\
u_x
\end{bmatrix} = T_{duct} \begin{bmatrix} P_{x+1} \\
u_{x+1}
\end{bmatrix} = T_{net} \begin{bmatrix} P_{x+1} \\
u_{x+1}
\end{bmatrix}
\]

(13)

where \( T_{net} \) is the transfer matrix.

Once the initial sound pressure is given, the sound pressure and particle velocity in an arbitrary cell can be determined successfully by Eq. (13). Moreover, the duct-resonator system can be expressed in a matrix form as:

\[
\begin{bmatrix}
P_1 \\
u_1
\end{bmatrix} = T_{net} \begin{bmatrix} P_1 \\
u_1
\end{bmatrix} = T_{net} \begin{bmatrix} P_2 \\
u_2
\end{bmatrix} = \ldots = T_{net} \begin{bmatrix} P_{x+1} \\
u_{x+1}
\end{bmatrix}
\]

(14)

Similarly, the transmission loss of the periodic duct-resonator system could also be calculated by Eq. (7) when the transfer matrix of the whole system has been derived from Eq. (14).

2.4 Acoustic Performance of a Modified Periodic Duct-Resonator System

Both the special dispersion characteristics of a periodic structure and the effects of tuned resonators with different resonance frequencies are taken into account in the proposed modified periodic duct-resonator system. A modified periodic duct-resonator system with periodic two-resonator arrays installed periodically is demonstrated in Fig.4.

Fig.4. A Modified Periodic Duct-Resonator System
A periodic system is composed of a number of identical components that are joined together one by one to form a whole complex [17,23]. As shown in Fig.4., sets of identical two-resonator arrays are mounted on the duct. A two-resonator array includes two tuned resonators of different resonance frequency. According to the definition of a periodic system, the modified duct-resonator system can also be considered as a periodic structure. A two-resonator array and a duct segment constitute a periodic cell. Similarly, the diameter of the resonator's neck is also assumed to be negligible compared to the distance between two periodic cells. The length of the duct segment d is regarded as the periodic distance. By introducing Eq. (11) into Eq. (13), the transfer matrix of the modified periodic duct-resonator system could be give as:

$$T_{\text{mod}} = T_1 T_{\text{duct1}} T_2 T_{\text{duct2}}$$

(15)

where $T_{\text{duct1}}$ represents the transfer matrix of a duct segment between two resonators in a periodic cell, $T_{\text{duct2}}$ represents the transfer matrix of a duct segment between two periodic cells.

Similarly, the transmission loss of the modified periodic duct-resonator system could be calculated through Eq. (7) after the transfer matrix of the whole system is acquired by introducing Eq. (15) into Eq. (14).

3. Results and Discussion

The three-dimensional Finite Element Method (FEM) is used in this paper to validate the theoretical predictions of the duct-resonator system. The FEM for time-harmonic acoustic governed by Helmholtz equation has been proved to be an effective and accurate verification tool [20]. The sound source is located at the beginning of the duct at a magnitude of $P_0 = 1$. The anechoic termination of the main duct is modeled with a non-reflective boundary condition. The temperature and the pressure in air are assumed as 20 degrees Celsius and 1 atmosphere respectively in both theoretical analysis and FEM simulation. This means that the speed of the sound in the air is $c_0 = 344 m/s$ and the air density is $\rho_0 = 1.21 kg/m^3$.

3.1 Validation of a Two-resonator Array

A two-resonator array is shown in Fig.2. The geometries of the neck are: neck cross-section area $S_n = 2.25\pi cm^2$, neck length $l_n = 2.3cm$. The cavity volume of HR1 and HR2 are: $V_1 = 198.42 cm^3$ and $V_2 = 572.87 cm^3$ respectively. The cross-section area of the main duct is $S_d = 25 cm^2$. The resonance frequency of a resonator is only determined by its geometries. It is therefore that the resonance frequencies of HR1 and HR2 are: 530Hz and 312Hz respectively. The comparison of transmission loss between the side branch resonator and the two-resonator array is illustrated in Fig.5. The average peak amplitudes of the two-resonator array decreased compared to a side branch resonator, as are the bandwidths around each resonance frequency. However, the two-resonator array provides a much broader noise attenuation band between the resonance frequencies of these two resonators. Fig.6. compares the predicted results to the FEM simulation results. It is shown that the theoretical predictions fit well with the FEM simulation results.

Fig.5. Comparison of the Side Branch Resonator and the Two-resonator Array

Fig.6. Comparison of the Theoretical Predictions and the FEM Simulation for a Two-resonator Array
(The Solid Line Represents the Theoretical Prediction, and Dotted Crosses Represent the FEM Simulation Results)

3.2 Validation of the Periodic Duct-resonator System with Identical Resonators

The periodic duct-resonator system with identical resonators mounted periodically is demonstrated in Fig.3. The number of identical resonators considered in this section is eight. The geometries of the resonator are: neck cross-section area $S_n = 2.25\pi cm^2$, neck length $l_n = 2.3cm$ and cavity volume $V = 424.12 cm^3$. The cross-section area of the main duct is $S_d = 25 cm^2$. Thus, the resonance frequency of the resonator is 400 Hz. As described above, the periodic distance $d$ is chosen as half-wavelength ($0.5 \lambda = 0.43m$). The comparison of transmission loss of a single resonator and the periodic duct-resonator system is illustrated in Fig.7. Owing to the characteristics of the periodic structure, it can be seen that the periodic system provides a much broader sound attenuation band around its resonance frequency than a single resonator.
The comparison of transmission loss between the theoretical predicted result and the FEM simulation result is shown in Fig. 8. The theoretical prediction fits well with the FEM simulation.

### 3.3 Validation of the Modified Periodic Duct-resonator System

The modified periodic duct-resonator system is illustrated in Fig. 4. A periodic cell is composed of a two-resonator array and a duct segment. As described above, the geometries of the two-resonator array are the same as in Section 3.1. The distance \( L \) between two resonators in a periodic cell is equal to the periodic distance \( d \). The optimal distance corresponds to the quarter wave of resonator mean frequency defined as:

\[
 f_m = 2f_{HR1} \times f_{HR2} / (f_{HR1} + f_{HR2})^{1/2}.
\]

As a consequence the distance used in this paper is \( L = d = 0.219 \) m. This means that the distance between any two identical resonators in the modified duct-resonator system is close to the half-wavelength of its resonance frequency. Due to the Bragg reflection effects, a broader noise attenuation band can be achieved at each resonance frequency. The periodic duct-resonator system with identical resonators is mainly designed to reduce the noise at a certain frequency. The coupling of the structure and the identical resonators provide a broadband noise attenuation due to Bragg reflection effects. The modified periodic duct-resonator system is compared to the periodic system with identical resonators of designed resonance frequency 400Hz, as illustrated in Fig. 9.

It can be seen that the modified periodic system can provide a broader noise attenuation band. Besides, the modified periodic system offers a relatively high transmission loss at each resonator’s resonance frequency. This is very useful in practice, especially as the noise source contains more than one frequency to be controlled. Fig. 10. exhibits the comparison of theoretical prediction result and the FEM simulation result. The predicted result agrees well with the FEM simulation.

### 4. Conclusion

This paper focuses on improving the noise attenuation performance of a duct-resonator system, and presents theoretical and numerical studies of the two types of periodic duct-resonator system. The periodic duct-resonator system with identical resonators is mainly designed to reduce the noise at a certain frequency. The coupling of the structure and the identical resonators provide a broadband noise attenuation due to Bragg reflection effects. By installing two-resonator arrays on the duct periodically, the modified periodic duct-resonator system can
achieve a broader noise attenuation at the desired frequency by fully utilizing both the effects of periodic structure and tuned resonators array. Furthermore, the modified periodic system can offer high transmission loss amplitudes corresponding to each resonator’s resonance frequency. This is very useful when the noise source contains more than one frequency component to control.

The periodic duct-resonator system and the modified periodic system both have potential applications in noise control at low frequency, the modified periodic system proposed in this paper especially, provides a useful method for the design of such a system to obtain a broader noise attenuation band. The periodic cell of the modified system in this paper only contains two tuned resonators, more tuned resonators could be taken into consideration to improve the noise attenuation performance of a duct-resonator system.

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