Resonant Monopole Oscillation in the Bose-Fermi Mixed System

Tomoyuki Maruyama†,1,2 Hiroyuki Yabu,3 and Toru Suzuki3

1 Institute for Nuclear Theory, University of Washington, Seattle, Washington 98195, USA
2 College of Bioresource Sciences, Nihon University, Fujisawa 252-8510, Japan
3 Department of Physics, Tokyo Metropolitan University, 1-1 Minami-Ohsawa, Hachioji, Tokyo 192-0397, Japan

We study the monopole oscillation in the bose-fermi mixed condensed system by performing the time-dependent Gross-Pitaevskii and Vlasov equations. We study the resonant oscillation where the intrinsic frequencies of boson and fermion oscillations are same.

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Over the last few years there has been significant progress in the production of ultracold gases, which realize the Bose-Einstein condensates (BEC) [1, 2, 3], degenerate atomic fermi gases [4] and bose-fermi mixtures [5]. These systems offer great promise for studies of new, interesting-driven phenomena. Collective oscillations are some of the most prominent phenomena common to a variety of many-body system. Theoretical studies on the collective motion in bose-fermi mixtures were performed in the normal phase with the sum-rule approach [6] and the random-phase approximation (RPA) [7]. In Ref. [8] we constructed a transport mode for the bose-fermi mixing system by combining the time-dependent Gross-Pitaevskii(GP) equation and the Vlasov equation, and studied the monopole oscillation by solving the time-evolution of the system directly. Then we found a rapid damping in the fermion oscillation and a blurring of the beat in the boson oscillation. We considered the system where the boson number \( N_b \) is much larger than the fermion number \( N_f \) (\( N_b \gg N_f \)), which optimizes the overlap of bosons and fermions. In such systems the boson intrinsic frequency is \( \sqrt{5} \) which has been shown in the system including the condensed bosons only [2]. On the other hand the fermion intrinsic frequency becomes larger as the attractive boson-fermion coupling increases. This is suggestive that the fermion intrinsic frequency becomes equivalent to the boson one for a certain value of the boson-fermion coupling.

In this work we choose the value of the boson-fermion coupling where the boson and fermion intrinsic frequencies are same. Then we investigate resonant behavior of the monopole oscillation by performing the numerical simulation.

Here we briefly explain our formalism. First we define the Hamiltonian for boson-fermion coexistent

† e-mail: tomo@brs.nihon-u.ac.jp
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system as follows.

\[ H = H_B + H_F + H_{BF} \]  
(1)

with

\[ H_B = \int d^3x \left\{-\frac{1}{2} \phi^\dagger(x) \nabla^2 \phi(x) + \frac{1}{2} x^2 \phi^\dagger(x) \phi(x) + \frac{g_B}{2} (\phi^\dagger \phi)^2 \right\}, \]  
(2)

\[ H_F = \int d^3x \left\{-\frac{\hbar^2}{2m_f} \psi^\dagger \nabla^2 \psi + \frac{1}{2} m_f \omega_f^2 x^2 \psi^\dagger \psi \right\}, \]  
(3)

\[ H_{BF} = h_{BF} \int d^3x \{\phi^\dagger \phi \psi^\dagger \psi\}, \]  
(4)

where \( \phi \) and \( \psi \) are boson and fermion fields, respectively. Fermion mass \( m_f \) and the trapped frequency \( \omega_f \) are normalized with the boson mass \( M_B \) and the boson trapped frequency \( \Omega_B \), respectively. The coordinates are normalized by \( \xi_B = (\hbar/M_B \Omega_B)^{1/2} \). The coupling constants \( g_B \) and \( h_{BF} \) are given as

\[ g_B = 4\pi a_{BB} \xi_B^{-1}, \]  
(5)

\[ h_{BF} = 2\pi a_{BF} \xi_B^{-1}(1 + m_f^{-1}), \]  
(6)

where \( a_{BB} \) and \( a_{BF} \) are the scattering lengths between two bosons and between the boson and the fermion, respectively.

The total wave function \( |\Phi(\tau)\rangle \) has \( N_c \) condensed bosons, whose wave function \( \phi_c \) is defined as

\[ \phi_c(x, \tau) = \langle \Phi | \phi_B(x, \tau) | \Phi \rangle, \]  
(7)

where \( \tau \) is the time coordinate normalized with \( \Omega_B^{-1} \). In this work the wave function \( \phi_c \) is expanded with the harmonic oscillator wave function \( u_n(x) \) as

\[ \phi_c(x, \tau) = \sum_{n=0}^{N_{base}} A_n e^{i \theta_n} u_n(bx^2) e^{-\frac{1}{2}b^2 x^2}, \]  
(8)

where \( N_{base} + 1 \) is the number of the harmonic oscillator bases. We define the Lagrangian with the collective coordinates as

\[ L(A_n, \theta_n, b, \nu) = \langle \Phi(\tau) | \{ i \frac{\partial}{\partial \tau} - H \} | \Phi(\tau) \rangle. \]  
(9)

Instead of solving the time-dependent GP equation directly, we take \( A_n, \theta_n, b, \nu \) as time-dependent variables and solve the Euler-Lagrange equations with respect to these variables.

The many fermion system can be described with the Thomas-Fermi approximation, which is given in the limit \( \hbar \to 0 \). Here we define the phase-space distribution function as

\[ f(x, p, \tau) = \int d^3z < \Phi | \psi(x + \frac{1}{2}z, \tau) \psi^\dagger(x - \frac{1}{2}z, \tau) | \Phi > e^{-i p z}. \]  
(10)

In this classical limit this phase-space distribution function satisfies the following Vlasov equation [10]:

\[ \frac{d}{d\tau} f(x, p; \tau) = \left\{ \frac{\partial}{\partial \tau} + \frac{p}{m_f} \nabla_x - [\nabla_x U_F(x)] [\nabla_p] \right\} f(x, p; \tau) = 0. \]  
(11)
with
\[ U_F(x) = \frac{1}{2} m_f \omega_f^2 x^2 + h_{BF} \rho_B(x), \tag{12} \]

In the actual calculation we solve the above Vlasov equation with the test particle method \([11]\). Thus we can get the time-evolution of the condensed bosons and the fermions by solving the above time-dependent GP equation and the Vlasov equation.

We calculate the monopole oscillation of the system $^{39}$K $- ^{40}$K; the number of bosons ($^{39}$K) and fermions ($^{40}$K) are taken to be 100,000 and 1,000, respectively. The trapped frequencies are taken to be $\Omega_B = 100$ (Hz) and $\omega_f = 1$. The boson-boson interaction parameters is set to $g_B = 1.34 \times 10^{-2}$ which corresponds to $a_{BB} = 4.22$(nm) \([6]\); in addition we vary the boson-fermion interaction parameter $h_{BF}$ which is measured in the unit of $\hbar_0 = 7.82 \times 10^{-3}$ corresponding to $a_{BF} = 2.51$(nm) \([6]\).

Using the root-mean-square radius (RMSR) $R$, we here define
\[ \Delta x(\tau) = R(\tau)/R_0 - 1, \tag{13} \]

where $R_0$ is the RMSR of the ground state, and $\tau$ is the time variable normalized by $\Omega_B^{-1}$. In Fig. 1 we show the time-dependence of this observable for bosons ($\Delta x_B$) and fermions ($\Delta x_F$) with $h_{BF} = -\hbar_0$. In this calculation the initial condition is taken to be $\Delta x_B = 0$ and $\Delta x_F = 0.1$ at $\tau = 0$. We see a fast damping in the fermion oscillation as shown in Ref. [8]. However the the boson oscillation grows longer and, its amplitude becomes larger with $h_{BF} = -\hbar_0$ than when the boson-fermion coupling is repulsive, $h_{BF} > 0$.

When varying the boson-fermion coupling, the situation becomes different. In Fig. 2 we plot the results with $h_{BF} = -2\hbar_0$. The oscillation behaviors are quite different from those in Fig. 1. We can see the extension and shrink of the fermion density.

In order to study this phenomenon, further, we examine the spectrum which is obtained by the Fourier transformation
\[ F(\omega) = A_C \left| \int d\tau R^2(\tau)e^{i\omega \tau} \right|^2, \tag{14} \]

where $A_C$ is a normalization factor. In Fig. 3 we give the spectra for boson (a) and fermion (b) oscillations obtained by integrating over the regions $0 < \tau < 200$. We can see that these two spectra have peaks at the same frequencies.

The oscillation process is explained as follows. At first the fermion oscillation starts and trigger the boson oscillation. The boson oscillation grows while the fermion oscillation very rapidly damps. Once the fermion oscillation is stopped and grows up, again. The boson oscillation has a large amplitude at that time and scatter many fermions to the outside region. The boson density is distributed in a smaller region than the fermion density. Outside of the boson density the fermions feel only the harmonic oscillator potential, which is weaker than attractive force inside the boson density region. In addition the
scattered fermions lose the kinetic energy and remain there rather long time. So fermions move from the inside region to the outside region in the expansion time $30 \gtrsim \tau \gtrsim 150$.

Later fermions far from the boson region return to the central region, but some fermions moving out from the boson region continue to populate outside the boson region because a test particle in the harmonic oscillator potential moves on closed orbit, which does not cross in the boson region. The system approaches to being equilibrium around $\tau \approx 200$. Hence the averaged radius of fermion density remains to be 10% larger than that at the ground state.

In Fig. 4 we show the intrinsic frequencies of the boson, $\omega_B^M$, (crosses) and fermion, $\omega_f^M$, (full squares) oscillations as functions of the boson-fermion coupling $h_{BF}$ normalized with $h_0$. For reference we also plot the fermion intrinsic frequencies with the open circles which are connected by the solid line, and $\omega_f^M = \sqrt{5}$, which has been shown in the system only including the condensed bosons [2], with the dashed line. The boson intrinsic mode is almost independent of fermions because $N_b \gg N_f$.

When the boson-fermion coupling is $h_{BF} \gtrsim -h_0$ the intrinsic frequencies of the fermionic oscillation are almost equal to the frequency under the boson motion frozen, and their values are estimated to be

$$\omega_f^M \approx 2(\omega + h_{BF} \frac{\partial^2 \rho_B}{\partial x^2}).$$ (15)

The frequencies are determined by the boson density distribution, and these fermion oscillations are volume vibrations.

When the boson-fermion coupling is $-3h_0 \lesssim h_{BF} \lesssim -1.5h_0$, however, $\omega_f^M \approx \omega_M^b$, and the resonant oscillation occur. This oscillation behavior is similar in the interaction region, though $\Delta x_F$ reaches its maximum earlier as the interaction becomes more strongly attractive.

When the boson-fermion coupling becomes larger, $h_{BF} \lesssim -4h_0$, the fermion intrinsic frequency becomes $\omega_f^M \approx 2.1$, which is close to that only in the harmonic oscillator potential, $\omega_f^M = 2$. In strong attractive interactions the fermion motion loses collectivity, and RMSR of fermion density is affected only by the motion of fermions outside the boson density region. Namely the fermion oscillation is surmised to become a surface vibration.

In summary we study the collective monopole oscillation in the bose-fermi mixed condensed system when $h_{BF} = -2h_0$. In this coupling the intrinsic frequency of the fermion oscillation, $\omega_f^M$, is almost equal to with the intrinsic frequency of the boson oscillation, $\omega_M^b$, and we observed resonant behavior. In this resonant oscillation the amplitude of the boson oscillation becomes quite large even if the initial amplitude is taken to be zero. Furthermore the fermion density is largely extended because some fermions must move out from the boson density region to achieve the mixed system equilibrium.

Furthermore we find that the fermion oscillation becomes a surface vibration when the boson-fermion coupling is more strongly attractive $h_{BF} \lesssim -4h_0$, than that in the resonant region, $-3h_0 \lesssim h_{BF} \lesssim -1.5h_0$. While fermion oscillation exhibits the volume vibration when the coupling is attractive weaker...
or repulsive, $h_{BF} \gtrsim -1 \hbar_0$. In future we would like to examine this consideration more detailed.

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Figure 1: Time evolution of $\Delta x$ for boson (a) and fermion (b) at the initial condition that $s_b = 1.0$ and $s_f = 1.1$ with the boson-fermion coupling $h_{BF} = -h_0$. 
Figure 2: Time evolution of $\Delta x$ for boson (a) and fermion (b) with the boson-fermion coupling $h_{BF} = -2h_0$. The other conditions are same as those in Fig. 1.
Figure 3: Spectra of the boson (a) and fermion (b) oscillations shown in Fig.
Figure 4: Intrinsic frequencies of boson and fermion oscillations versus the boson-fermion coupling. Crosses and full squares represent the intrinsic frequencies of boson and fermion oscillations, respectively. The open circles, which are connected with the solid line, indicate the fermion intrinsic frequency with the boson motion frozen. The dashed line denotes $\omega_M = \sqrt{5}$.  

$\omega_M = \sqrt{5}$