EFFECTIVE FERMION MODELS IN SYMMETRY-BREAKING PHASE AND QUANTUM CHROMODYNAMICS

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Abstract

In our lecture we discuss the fermion models with quasilocal interaction implemented by derivatives and a momentum cutoff as substitutes of QCD at low energies. They are investigated in the strong coupling regime when several coupling constants are matched to their critical values. It is found that around polycritical points there appear a number of resonances with the same quantum numbers. Respectively the particular change of environment caused by gluon condensate results in the mass splitting independent of the cutoff. Such models are supposed to be an essential ingredient in the description of quark matter at high baryon densities.

1. Introduction

The effective quark models with four-fermion interaction are widely used as substitutes of the low-energy QCD in the hadronization regime\(^1\)\(^2\)\(^3\). They are applied also to describe the minimal extensions of the Standard Model (Top-mode SM\(^4\)\(^5\)\(^6\)). At strong coupling they reveal the dynamic breaking of chiral symmetry (DCSB) which is essential in description of light hadrons. As well as in QCD in the quark models DCSB arises due to fermion condensation \(\langle \bar{\psi}\psi \rangle \neq 0\) that corresponds to the creation of dynamic quark mass \(m_{\text{dyn}} \neq 0\).

In the common approach the local four-fermion interaction only is employed\(^7\)\(^9\). Meanwhile in these models very complicated gluon forces between quarks are replaced by a color-singlet self-interaction of quarks. Such an effective action should generally contain quasilocal vertices with derivatives\(^6\)\(^10\). Moreover as we will see the simplest quark model does not take the main properties of QCD which are important to reproduce the variety of hadron states. Namely an effective quark model that reflects reasonably QCD-born mass spectrum at low and intermediate energies should obey the following requirements:

(i) at strong couplings the DCSB should arise and the expected set of hadron states should be generated by the universal DCSB (including radial excitations with

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the same quantum numbers, $\pi$, $\pi'$, $\pi''$; $\sigma$, $\sigma'$, $\sigma''$ etc., see Review of Particle Properties);

(ii) both DCSB and hadron state formation should be derived in the large-$N_c$ approach, i.e. in the mean-field approximation for fermion fields;

(iii) the mass splitting of hadron states should be induced by a single force in order to provide the universal mass scale $m_{\text{hadron}} \sim \Lambda_{\text{QCD}} \sim m_{\text{dyn}}$.

We are going to show that above requirements can be realized in fermion models with quasilocal vertices with derivatives. In order to generate "radial excitations" one has to admit strong (critical) coupling constants for them. Thereby we come to the generalized Nambu-Jona-Lasinio (NJL) model in the vicinity of polycritical point for several coupling constants.

There are two important reasons to make this generalization:

(i) it extends the energy range of applicability of such a quark model and improves its accuracy at low energies;

(ii) such models are thought of as more realistic for investigations of hadron matter at high temperatures and nuclear densities near the restoration of CSB and deconfinement.

2. Dynamic Symmetry Breaking in Models with Four-fermion Interaction and the Critical Point (Fine-tuning)

Let us remind how the DCSB arises in a model with local 4-fermion interaction due to strong attraction in the scalar channel. The simplest, Gross-Neveu model retaining the scalar channel only can be presented by the lagrangian density, in two forms (the euclidean-space formulation is taken here),

$$
\mathcal{L} = \bar{\psi} \mathcal{D}\psi + \frac{g^2}{4N_c} (\bar{\psi}\psi)^2 = \bar{\psi}(\mathcal{D} + im(x))\psi + \frac{N_c}{g^2} m^2(x),
$$

where $\mathcal{D} = i\gamma_\mu \partial_\mu$ and $\psi \equiv \psi_i$ stands for color fermion fields with $N_c$ components.

We simplify our analysis and set up the number of flavours $N_F = 1$ and the current quark mass $m_q = 0$. In Eq.1 the scalar auxiliary field $m(x)$ is introduced in order to describe the CSB phenomenon in the large-$N_c$ limit.

This model is implemented by a cutoff $\Lambda$ for fermion energy spectrum. Namely we define low-energy fermion fields by means of the projection $\psi_i = \Theta(\Lambda^2 - \mathcal{D}^2)\psi$ where $\Theta(x)$ is a step-function\(^{12}\) (or any other regulator). For a quark model the cutoff $\Lambda$ can be thought of as a separation scale which appears when constructing the QCD low-energy effective action. From this viewpoint the observables should not depend on the latter one. The scale invariance is achieved by appropriate prescription of cutoff dependence for effective coupling constants $g$. 
The regularized effective action $S_{\text{eff}}$ for auxiliary field,

$$Z_F^\Lambda(m) = \exp(-S_{\text{eff}}) = \left\langle \exp\left(-\int d^4x \mathcal{L}_F(m(x))\right) \right\rangle_{\bar{\psi}\psi},$$

possesses the mean-field extremum on constant configurations $m = \text{const}$.

The relevant effective potential $V_{\text{eff}}$ can be obtained by intergration over fermions,

$$V_{\text{eff}} = \frac{S_{\text{eff}}}{(\text{vol.})} = \frac{N_c}{8\pi^2} \left\{ \frac{\Lambda^4}{2} \left( \frac{1}{2} - \ln \frac{\Lambda^2 + m^2}{\mu^2} \right) - \frac{m^2\Lambda^2}{2} + \frac{m^4}{2} \ln \frac{\Lambda^2 + m^2}{m^2} + \frac{8\pi^2 m^2}{g^2} \right\}. \quad (3)$$

Its extrema are arrived from the mass-gap equation,

$$R(m) \equiv \frac{4\pi^2}{N_c} \cdot \partial V_{\text{eff}} / \partial m = m \left( \frac{8\pi^2}{g^2} + m^2 \ln \frac{\Lambda^2 + m^2}{m^2} - \Lambda^2 \right) = 0. \quad (4)$$

The main contribution into Eq.4 is given by a tadpole term in the one-fermion loop which is related to v.e.v of scalar fermion operator,

$$R(m) = m \frac{8\pi^2}{g^2} + i \frac{4\pi^2}{N_c} \langle \bar{\psi}\psi \rangle. \quad (5)$$

The cutoff independence is realized with aid of fine-tuning $8\pi^2 / g^2 = \Lambda^2$. In the language of the theory of critical phenomena it is equivalent to developing of our model around critical or scaling point. By definition the critical coupling constant is $g_{\text{crit}}^2 = 8\pi^2 / \Lambda^2$. When $g^2 < g_{\text{crit}}^2$ the only solution of mass-gap Eq.4 is $m = 0$, while for $g^2 > g_{\text{crit}}^2$ there exists a nontrivial dynamic mass solution $m_{\text{dyn}} \neq 0$ which brings the true minimum for $V_{\text{eff}}$.

The fine-tuning states that the strong $\Lambda^2$-dependence should be compensated by the corresponding term in the coupling constant,

$$\frac{8\pi^2}{g^2} = \Lambda^2 - m_0^2 \quad (6)$$

The deviation scale $m_0^2$ determines the physical mass of scalar meson. Namely its kinetic term can be obtained from the second variation of $S_{\text{eff}},$

$$S_{\text{eff}} \simeq S_{\text{eff}}(m = m_{\text{dyn}}) + \frac{1}{2} \int \frac{d^4p}{(2\pi)^4} \sigma(-p) \int \sigma(p) f_\sigma^2 [p^2 + m_\sigma^2] \sigma(p); \quad m = m_{\text{dyn}} + \sigma. \quad (7)$$

The scalar meson mass is given by the remarkable Nambu relation $m_\sigma \simeq 2m_{\text{dyn}}$. Respectively the scale $m_0^2$ should be weakly dependent $m_0^2 \sim m_{\text{dyn}}^2 \ln(\Lambda^2 / m_{\text{dyn}}^2)$ on the cutoff $\Lambda$ in order that the physical mass parameters were decoupled on $\Lambda,$ $\partial_\Lambda m_{\text{dyn}} = 0.$

What have we learned from this model?
(i) As a result of DSB in this model only one scalar meson is created in the large-$N_c$ approach.

(ii) In such a simple model the radial excitations are not present in the large-$N_c$ approach.

(iii) The mass scale of scalar state is assumed to be different from $\Lambda$ and related to the basic QCD scale $\Lambda_{QCD}$.

(iv) Still the hadron coupling constants, e. g. $f_\pi$, $f_\sigma$, remain weakly depending on $\Lambda$ in accordance to the log-divergence of self-energy diagram, $f^2_{\pi,\sigma} \sim \ln \Lambda^2/m^2_{\text{dyn}}$.

(v) Due to the fine-tuning the fermion condensate has a strong $\Lambda$ dependence,

$$\langle \bar{\psi} \psi \rangle \sim N_c m \Lambda^2.$$ (8)

Hence the conventional NJL (or GN) quark models do not contain a consistent part of QCD effective action and at best can be used as a truncation$^3$ of the latter one at low energies or should be extended with inclusion of higher dimensional vertices which are needed to cure enumerated defects$^{10}$.

In order to simulate the mass-splitting by QCD forces one can start from a quark model precisely at the critical point $g^2 = g^2_{\text{crit}}$, $m_{\text{dyn}} = 0$, $m_{\text{hadron}} = 0$ and drive the mass spectrum by coupling to gluon medium (gluon condensates)$^3$. This scenario can be realized in a Gauged NJL (or GN) model around critical point.

Let us consider its particular GN version in the gluon condensate expansion. We replace,

$$\varphi \Rightarrow D = \varphi + G, \quad G_\mu = ig G^a_\mu t^a,$$ (9)

with low-energy gluon fields saturating condensates.

In the large mass limit $V_{\text{eff}}$ is supplemented with

$$\Delta V_{\text{eff}}(G) = \frac{1}{48\pi^2} \ln \frac{m^2}{\Lambda^2} (tr(C^a_{\mu\nu})^2)_0 = -\frac{C_g}{24} \ln \frac{m^2}{\Lambda^2},$$ (10)

where $C_g = (\alpha_s/\pi) (tr(C^a_{\mu\nu})^2)_0 \approx (350 \div 400 \text{MeV})^4$ is a gluon condensate.

The modified mass-gap Eq. is,

$$\frac{8\pi^2}{g^2} - \Lambda^2 + m^2 \ln \frac{\Lambda^2 + m^2}{m^2} = \frac{C_g \pi^2}{3N_c m^2} \equiv \frac{\gamma m^2}{2}.$$ (11)

When one drives to the overcritical region the positive gluon condensate is assumed to induce the mass $m_{\text{dyn}} \approx 300 \text{MeV}$ obeying the Eq. $\ln(\Lambda^2/m^2) \simeq \gamma/2$. Then one has $\gamma = 4 \div 6$ and consequently $\Lambda \simeq (0.8 \div 1.0) \text{GeV}$. 

Thus numerical estimations seem to support the concept of GNJL model with critical four-quark constant. However the calculation of scalar meson decay constant $f_\sigma$ shows too strong sensitivity to values of gluon condensate,

$$f_\sigma^2 \simeq \frac{N_cm^2}{8\pi^2} \left( -\frac{17}{12} + \frac{3\gamma}{20} \right) \ln \frac{\Lambda^2}{m^2} = \frac{\gamma}{2},$$

i.e. $f_\sigma$ is negative for $\gamma = 4 \div 6$. It makes such a model unrealistic.

We will see later on that the two-channel generalization of GN model fits noticeably better the decay constants and is less sensitive to the values of $\gamma$.

3. Dominant Effective Vertices in the DSB regime and Generalization of Gross-Neveu Model

We consider the DSB pattern in the mean-field approach (large-$N_c$ limit) and estimate the vertices with any number of fermion legs and derivatives. The main rule to select out relevant vertices is derived from the requirement of indifference in choice of separation scale $\Lambda$ following the conception of low-energy effective action.

We assume that:

(i) $\Lambda^2$-order vertices are dominant in creating the DSB-critical surface that is achieved when all contributions of $\Lambda^2$-order are cancelled;

(ii) $\Lambda^0$-orders in vertices assemble in the mean-field action to supply fermions with dynamic mass independent on $\Lambda$ (up to log,s);

(iii) $\Lambda^{-2}$ (etc.)-orders are irrelevant (though they are subject to compensation as well);

(iv) the real dimension in $\Lambda$ is estimated according to the large-$N_c$ analysis of DSB.

The large-$N_c$ approach leads to the following approximation for v.e.v. of fermion operators,

$$\langle (\bar{\psi}\psi)^n \rangle = \left( \langle \bar{\psi}\psi \rangle \right)^n \left( 1 + O(1/N_c) \right),$$

where any number of derivatives is accepted between fermion operators.

V.e.v. of bilinear operator is estimated in the assumption that quarks obtain a dynamic mass. Namely,

$$\langle \bar{\psi} \left( \frac{\partial^2}{\Lambda^2} \right)^n \psi \rangle \sim \frac{1}{\Lambda^{2n}} \int_{|p|<\Lambda} \frac{d^4p}{(2\pi)^4} \text{tr} \frac{p^{2n}}{p^2 + im} \sim N_cm\Lambda^2.$$
We omit the full classification of effective vertices relevant in the mass-gap Eq. (see\textsuperscript{10}) and report only the minimal structure of extended GN model which admits the polycritical regime,

\[ \mathcal{L} = \bar{\psi} \not D \psi + \frac{1}{4 N_c \Lambda^2} \sum_{m,n=0}^{\infty} a_{mn} \bar{\psi} \left( \frac{\partial^2}{\Lambda^2} \right)^n \psi \cdot \bar{\psi} \left( \frac{\partial^2}{\Lambda^2} \right)^m \psi. \]  

where \( a_{mn} \) is a real symmetric matrix. Let us diagonalize this matrix \( \sum a_{mn} f_n^{(i)} = \lambda_i f_m^{(i)} \), \( \sum f_m^{(i)} f_m^{(j)} = \delta^{ij} \) and consider the subspace of eigenvectors \( f^{(i)} \) with non-zero eigenvalues. We define the vertex functions \( \varphi^i(\tau) = \sum_{n=0}^{\infty} f_n^{(i)} \tau^n \).

Let us now introduce the appropriate set of auxiliary fields \( \chi_n(x) \sim \text{const} \) and develop the mean-field approach,

\[ \mathcal{L}(\chi) = \bar{\psi} \left( \not D + i M(\chi, \partial^2) \right) \psi + N_c \Lambda^2 \sum_i \chi_i \lambda_i^{-1} \chi_i. \]  

Herein the summation is extended over all non-zero eigenvalues \( \lambda_i \) (effective coupling constants). The dynamic mass functional is a linear combination of vertex functions

\[ M(\chi, \partial^2) \equiv \sum_i \chi_i \varphi^i \left( \frac{\partial^2}{\Lambda^2} \right). \]

The corresponding effective potential in the adiabatic approach can be derived with the regularization\textsuperscript{12} as in the simple GN model. Then the generalized mass-gap equation delivers an extremum to effective potential and in the \( \Lambda^2 \)-order (fine-tuning) reads,

\[ \frac{\partial V_{eff}}{\partial \chi_i} = 0 \simeq \frac{N_c \Lambda^2}{4 \pi^2} \sum_j \left( \frac{8 \pi^2}{\lambda_i} \delta_{ij} - \Phi_{ij} \right) \chi_j, \]

\[ \Phi_{ij} = \int_0^1 d\tau \varphi^{(i)}(\tau) \varphi^{(j)}(\tau) = \sum_{n,m=0}^{\infty} \frac{f_n^{(i)} f_m^{(j)}}{m+n+1}. \]  

It represents the most general condition for subspace of critical coupling constants. Namely the DSB occurs in the vicinity of zero-mode subspace for the matrix,\( G_{ij} = (8 \pi^2 / \lambda_i) \delta_{ij} - \Phi_{ij} \). By definition the \( (N+1) \)-critical surface (polycritical point) corresponds to \( N \) independent solutions. We remark that the larger the zero-mode subspace for \( G_{ij} \), the larger may be a symmetry in the DSB phase.

For the clarity we omit the further investigation of mass-gap Eq. in the general case and proceed to studying the two-channel model.

4. Two-channel Model around Tricritical Point and Formation of Two Resonances

This models has three coupling constants \( m, n = 1, 2 \). The selection of a model with DSB in all scalar channels together with requirement of scale independence
(tricritical condition) leads to the rigid set of coupling constants. After appropriate normalization the two-channel model around tricritical point can be represented by the interaction vertices,

\[
L_{\text{tricr}} = \frac{1}{4N_c\Lambda^2} \left[ \lambda_1^c (\bar{\psi}\psi)^2 + 3\lambda_2^c (\bar{\psi}(1 + 2\frac{\partial^2}{\Lambda^2})\psi)^2 + \frac{8\pi^2}{\Lambda^2} \sum_{i,k} \Delta_{ik} \cdot \bar{\psi} \phi_i (-\frac{\partial^2}{\Lambda^2})\psi \cdot \bar{\psi} \phi_k (-\frac{\partial^2}{\Lambda^2})\psi \right]
\]

where \( \lambda_1^c = 8\pi^2 \) and \( \phi_1 = 1, \phi_2 = \sqrt{3}(1 - 2\tau) \). The dynamic mass function is

\[
M(\tau) = \left[ \chi_1 + \sqrt{3}\chi_2 (1 - 2\tau) \right] \Theta(1 - \tau).
\]

(20)

The matrix \( \Delta_{ik} \) describes the deviation from tricritical point induced by external forces and is responsible for the mass splitting of resonances. Let us display the set of mass-gap Eqs. with accuracy of \( O(1/\Lambda^2) \):

\[
\begin{align*}
\chi_1 \Delta_{11} + \chi_2 \Delta_{12} & = M_0^2 \ln \left( \frac{\Lambda^2}{M_0^2} \right) - 6\sqrt{3}\chi_1^2\chi_2 - 18\chi_2^2\chi_1 - 8\sqrt{3}\chi_2^3; \\
\chi_1 \Delta_{12} + \chi_2 \Delta_{22} & = \sqrt{3}M_0^3 \ln \left( \frac{\Lambda^2}{M_0^2} \right) - 2\sqrt{3}\chi_1^3 - 18\chi_1^2\chi_2 - 24\sqrt{3}\chi_2^2\chi_1 - 24\chi_2^3.
\end{align*}
\]

(21)

where \( M_0 = \chi_1 + \sqrt{3}\chi_2 \equiv m_{\text{dyn}} \). This is a set of highly nonlinear Eqs. which cannot be solved analytically. However we search for solutions around tricritical point which are smoothly governed by a small deviation \(|\Delta| << \Lambda^2\). Respectively these solutions are found in the large-log approximation.

Let us describe the mass spectrum of scalar states near a minimum of \( V_{\text{eff}} \). As in the one-channel model the effective kinetic term is defined from the second variation of effective action,

\[
\delta^2 S_{\text{eff}} \over \delta \chi_i(p)\delta \chi_k(p') \equiv \delta(p + p')(\hat{A}p^2 + \hat{B}).
\]

(22)

The scalar state masses are delivered by zeroes of Eq.

\[
\|\hat{A}p^2 + \hat{B}\| = 0.
\]

(23)

Among solutions one should select out those ones which ensure the positiveness of the second variation (the minimum) of \( S_{\text{eff}} \). It can be proven that such solutions exist and give rise to the physical resonances only with masses at \( p^2 = -m_\sigma^2 \) (no tachyons). Moreover in the vicinity of tricritical point two scalar resonances are always created when at least one of coupling constants exceeds its critical value. For an arbitrary deviation towards overcritical region one discovers two kind of mass spectra: the normal one with lighter mass \( m_1^2 \approx 4M_{\text{dyn}}^2 \) (NJL particle) and with heavier mass
\[ m_2^2 \sim m_1^2 \log \Lambda^2/M^2; \]
the abnormal one with \[ m_1^2 \simeq 6M_{dyn}^2; \]
\[ m_2^2 \sim m_1^2 \log^{2/3} \Lambda^2/M^2. \]
Therefore the mass splitting in general is not completely scale invariant.

Still there is a distinguished direction to drive the mass splitting independently of \( \log \Lambda \),
\[ \Delta_{ik}^{\text{scale}} = \mu^2 \left( \frac{1}{\sqrt{3}} \sqrt{\frac{3}{3}} \right), \quad \chi_i \Delta_{ik} \chi_k = \mu^2 M_0^2. \]  

The lighter scalar state is again a Nambu-GN particle, \[ m_1^2 \simeq 4M_{dyn}^2 \] but in the limit \( \log \Lambda^2 \to \infty \) one obtains the constant ratio, \[ m_2^2 = \frac{20}{3} \cdot m_1^2. \]

Thus we have started with three arbitrary coupling constants and after exploiting the cutoff independence of meson mass spectrum we end up with the single parameter \( \mu \) which governs universally the meson mass splitting. Its possible origin we clarify in the gauged effective quark model with low-energy gluons.

5. Two-channel Model with Gluons

We develop the gauged two-channel model in a full analogy with the one-channel case (see Sect.3). Let us follow the large-\( N_c \) approach and neglect the \( 1/\Lambda^2 \) orders in the effective action. As a result modifications concern only the kinetic term of quark fields and gluon fields turn out not to contribute to higher dimensional vertices at the orders of \( \Lambda^2 \) and \( \Lambda^0 \). Thereby \( V_{\text{eff}} \) is extended with the same one-fermion loop functional of gluon condensates as in the one-channel model, Eq.10 with replacement \( m_0 \to M_0 = \chi_1 + \sqrt{3} \chi_2 \),

\[ \Delta V_{\text{eff}}(G) = \Phi(M_0, \langle G^n \rangle) \simeq \frac{1}{48\pi^2} \ln \frac{M_0^2}{\Lambda^2} \langle tr(C^a_{\mu\nu})^2 \rangle_0 + O \left( \frac{\langle G^3 \rangle}{M_0^2} \right) \]  

The latter property holds for any number of scalar channels. Hence the mass splitting driven by gluon condensates happens to be along the remarkable direction \( \Delta_{\text{scale}} \) and when \( \log \Lambda^2 \to \infty \) it takes the same fixed ratio \( m_2^2 = \frac{20}{3} \cdot m_1^2 \).

The mass gap Eqs. are modified respectively,

\[ \frac{\partial V_{\text{eff}}}{\partial \chi_i} = \frac{\partial V_{\text{eff}}}{\partial \chi_i} (G = 0) + \frac{\partial \Phi}{\partial M_0} \cdot (1, \sqrt{3}) + O(1/\Lambda^2). \]  

In the large mass expansion the solutions obey the relations,

\[ \chi_1 = O(1/\Lambda^2); \quad M_0 = \sqrt{3} \chi_2; \quad \ln \frac{\Lambda^2}{M_0^2} = \frac{8}{3} + \frac{\gamma}{2}. \]  

Evidently the solution \( M_0 \) exists for positive gluon condensates and if \( \gamma \simeq 4 \div 6 \) one obtains \( \Lambda/M_0 \simeq 10 \div 17 \) or when adopting \( M_0 \simeq 300 MeV \) one deals with \( \Lambda \simeq (3 \div 5) GeV \). We conclude that the range of applicability of two-channel model is consistently broader than of the one-channel model.
The scalar mass spectrum is characterized by,

\[ m_1^2 \simeq 12 \chi_2^2 = 4 M_{\text{dyn}}^2; \quad m_2^2 \simeq m_1^2 \cdot \frac{20(\gamma - 1)}{3\gamma + 5}. \]  

(28)

Numerically for \( M_0 \simeq (300 \div 350) \text{MeV} \) the scalar state masses are estimated as \( m_1 \simeq (600 \div 700) \text{MeV}, \quad m_2 \simeq (1.1 \div 1.6) \text{MeV} \) that is consistent with both the particle phenomenology and with low-energy expansion in powers of \( m^2_{1,2}/\Lambda^2 << 1 \).

We notice also that the decay constant (for the lighter meson) proves to be

\[ f_\sigma^2 \simeq M_0^2 \frac{N_c}{8\pi^2} \left( \frac{5}{4} + \frac{3\gamma}{20} \right)_{\gamma=4\div6} = \frac{N_c}{4\pi^2} (1 \pm 0.08). \]  

(29)

Hence they are positive and less sensitive to the choice of gluon condensate as compared to the one-channel model.

6. Conclusion

Let us summarize the lessons of studying effective fermion models with quasilocal vertices.

(i) The effective fermion models with quasilocal interaction may serve for interpolation of more complicated gauge theories of QCD-type in the hadronization regime. They contain the sufficient set of phenomenological coupling constants to describe the infinite spectrum of resonances in the large-\( N_c \) approach which is expected to appear due to confinement.

(ii) In the vicinity of tricritical (\( n \)-critical) point of a quasilocal fermion model two (respectively \( n - 1 \)) massive scalar states occur due to DSB mechanism that may simulate the variety of radial excitations of scalar mesons in accordance with the QCD concept.

(iii) When effective coupling constants are prescribed to provide the scale invariance of physical parameters (the condition of maximal polycriticality in open channels) then the position of polycritical point is uniquely determined in the space of coupling constants.

(iv) The minimal sensitivity of mass splitting of scalar states selects out the particular direction for DSB that leads to considerable reduction of the number of arbitrary parameters in the two-channel model \( a_{11}, a_{12}, a_{22} \rightarrow \mu \). Just in this direction the gluon medium drives the mass splitting.

(v) The inclusion of excited states into the model improves essentially the predictivity in description of light meson states especially for models with gluon condensates.

We realize that when constructing the realistic quark model in the hadronization regime one should include vertices with any spin and isospin structures and derivatives. In this case the selection rule based on the scale independence may be also applied to related coupling constants.
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