Influence of high precision telescopic instrument characterization on multilateration points accuracy

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Abstract: Currently accuracy of manufacturing machines is a must. Verification is the main way to obtain it; highlighting the volumetric verification as the best technique to improve machine tool position accuracy along all its workspace in the shortest time possible. In this way, different measurements based on multilateration techniques like laser tracker and laser tracer are used. This paper studies how characteristics of a new high precision telescopic system consisting in three lines, with measuring principle based on simultaneous laser affect multilateration accuracy to obtain 3D coordinates. The paper analyses instrument characterising both the design, the components and their operation. Moreover, tests carried out study how instruments behavior affect to the accuracy of data capture using analytical and optimization techniques, proving an error estimation depending on the technique used.

Keywords: Verification, Multilateration, Accuracy, Model, Laser interferometer.

1. Introduction

The development of machining and measuring technology requires process productivity optimization considering also the accuracy and repeatability requirements of new applications. For this purpose, calibration and verification of machine tools (MTs) is a clear example of increase MT working capability assuring their consistence and repeatability.

Verification techniques are divided into two groups depending on how MT geometric errors are measured. The first group, known as geometric verification techniques is composed of those who are able to measure individually each geometrical error for a particular position of the MT workspace [1].UNE-ISO 230-1:2014 [2] is an international standard that specifies the methods for testing the accuracy of MTs by using direct measurements, operating under either no-load or quasi-static conditions. Systems used are classified in three subgroups depending on their working principle: specific methods with standard systems [3], specific methods using the linear propagation of the laser and its wavelength as a reference, as well as multidimensional artefacts such as telescopic bars [1, 4] or specific methods based on gravity such as levels. The second verification group, known as volumetric verification techniques, is able to provide a global compensation of whole MT workspace through an indirect measurement of all MT geometric errors and the kinematic model of the MTs [5]; reducing verification time and improving MT capability [1, 6]. Volumetric verification is a mathematical, not physical, compensation that improves MT accuracy through approximation functions obtained using
optimization techniques that characterize MTs geometric errors. So, adequacy of approximation functions used depends on several factors such as accuracy of measurement systems used, optimization techniques designed or environment conditions. [6]. More common measurement systems used for MTs volumetric verification are ball bars [7], laser trackers (LT) [8] and laser tracers (LC) [9]. LT accuracy is affected by encoders accuracy and LC accuracy by changes on measurement conditions as environmental one. To avoid these uncertainty sources a new measurement system known as High Precision Telescopic Instrument (HPTI) has been developed [10] based on multilateration techniques.

In the 90s, multilateration technique began to be used as a technique to improve data accuracy of robots positioning and measurement coordinate machine verification. In the recent years, multilateration has proven to be the most appropriate way to provide accuracy data from MTs verification using LT and LC. Several studies have been done in this field from equipment modelling, to spatial distribution equipment location, or adequacy of multilateration techniques to use [11, 12]. This paper analyses the relationship between HPTI characteristics, multilateration and 3D coordinates accuracy. To do that, we developed a new simulation software able to model the HPTI measurement system principles to determine length measurement, equipment location and measurement uncertainty from real tests. Moreover, the software allows applying different multilateration techniques taking into account the characteristics of the equipment. Thus, influence of system characteristics and multilateration technique on data accuracy can be obtained.

2. High Precision Telescopic System – working principle
The measurement system consists of a multi-arm telescopic system for volumetric verification of small and medium-sized MT based on high-precision simultaneous multilateration. It integrates three telescopic arms that measure at the same time enabling the volumetric verification of MTs with linear and/or rotary axes in an accurate, fast and economical way [10].

The main advantage of the system is that it allows data capture in a single cycle thanks to the simultaneous operation with at least three telescopic arms that are simultaneously registered at a single point. Each telescopic arm integrates an interferometer in one side of the arm, and a retroreflector located at the other arm’s end, which is close to the sphere fixed to the MT spindle nose (Figure 1). From each position, the displacement between a fixed sphere located in the MT table and the sphere attached to the MT head is measured by interferometry with the telescopic system. Therefore, a significant time reduction is generated in the data capture process since only one cycle is required.

3. Modelling of equipment influence on multilateration points
To model the measurement system, it is necessary to take into consideration same relevant aspects. When a point is measured, each laser provides the distance between its laser source and its retroreflector. Therefore, to obtain the coordinates of a point in 3D, it is necessary to define the relationship of each laser with respect to a common coordinate reference system (CRS). This system can be created taking as a reference a system external to the instrument, such as the reference system of the MT to be verified or a new-created reference system.

If an external system is used, for example the machine tool coordinate system (MTCRS), the relationship between lasers and MTCRS depends on coordinates \((x_{i,MTCRS}, y_{i,MTCRS}, z_{i,MTCRS})\) with \(i\) number of laser as shown in figure 1. However, if the reference system is created in the own instrument, the number of coordinates required can be reduced from 9 to 3. To do that, it is necessary to create the coordinate reference system of the instrument (ICRS) following the next generation conditions, as shown in figure 2:

1. The origin of ICRS is at the origin of the master laser interferometer, laser 1.
2. The direction of the X axis is defined by joining the origin of the new system with the origin of the laser 2.
3. The origin of laser interferometer 3 is in the plane created by the origin of lasers 1 and 2.
In this case, to relate ICRS and MTCRS coordinates it will be necessary to perform a least square adjustment between both systems.

Next aspect to consider for modelling the equipment lies in its manufacture and assembly. As shown in figure 3, distance between moorages that materializes the position to be measured $d_{mm}$ is different to the distance measured by laser interferometer $d_{l}$. Each laser has their own offsets related with its manufacture and assembly. As a preliminary hypothesis it was considered that HPTI is rigid enough to consider that the offsets have the same direction as the laser beam. Therefore, the real distance between moorages can be modelled as equation (1) presents.

$$d_{mm} = d_{m} + d_{l} + d_{r}$$  \hspace{1cm} (1)

Another factor to model in the HPTI is its measurement uncertainty. The interferometer used has a resolution provided by the manufacturer of 5 pm. However, assembly uncertainty and conditions in which tests are carried out should be taken into consideration. In addition, the developed model must consider the influence of the thermal variation in the MT and their influence on the measurement, as well as vibrations generated by the instrument and the machine.
Because these factors differ among the tests, the algorithms developed in this work allow to model jointly the measurement uncertainty and the MT’s vibrations. Moreover, these factors should be independent from changes in the testing ambient temperature. Therefore, it is necessary to measure the same point continuously for a specific period of time. The sequence of work, is the following, illustrating same points also in figure 4:

1. Filter the information leaving only the capture time and the measured distance.
2. Eliminate those points that are considered spurious data as result of not capturing a single position, undocking the equipment, etc.
3. Calculate the slope resulting from the temperature variation during data accuracy.
4. Elimination of temperature influence on captured data.
5. Characterization of the influence of the measurement noise together with vibrations of the machine (figure 4) using sine series.

The regression function that best suits the behavior of the points distribution of figure 4 is a sinusoidal function of degree 8 expressed in the equation (2). This equation is used to model the influence of this errors in the tests presented afterwards in section 5.

\[
f(t) = 6.058e^{-6} \sin(6.283 t - 1.362) + 1.833e^{-6} \sin(1263 t + 1.377) + 8.939e^{-6} \sin(1257 t + 0.3743) + 2.593e^{-6} \sin(62.83t - 2.088) + 1.833e^{-6} \sin(1250 t + 2.565) + 2.133e^{-6} \sin(75.4t + 2.734) + 1.870e^{-6} \sin(18.85t + 2.042) + 1.77e^{-6} \sin(31.42t + 3.104)
\]

(2)

Based on this knowledge, it is possible to simulate the measurement of the system with respect to the MTCRS according to equations (3) and (4). Where \(i\) is the number of the laser with \(i=1..3\), \((x_i, y_i, z_i)\) are the coordinates of the origin of laser \(i\) respect MTCRS, \((x_{p,MCSR}, y_{p,MCSR}, z_{p,MCSR})\) are the coordinates of the point to measure in MTCRS and \(p\) is the number of point measure with \(p=1..n\).

\[
d_{i,p,ir} = \sqrt{(x_{p,MCSR} - x_i)^2 + (y_{p,MCSR} - y_i)^2 + (z_{p,MCSR} - z_i)^2 + f(t)}
\]

(3)

\[
d_{i,p,mm} = d_{i,mi} + d_{i,p,ir} + d_{i,rn}
\]

(4)

If measured points are calculated related to ICRS, \((x_{p,MCSR}, y_{p,MCSR}, z_{p,MCSR})\), the coordinates should be transformed from MT to instrument coordinate reference system (MTCRS to ICRS). Developed algorithms allow the users to define the roto-translation matrix that links both systems. So, measured distances can be simulated by equations (5) to (8), where \((x_{p,ICRS}, y_{p,ICRS}, z_{p,ICRS})\) are the points coordinates in the ICRS, \(R_{MTCRS}^{ICRS}\) is the rotational matrix that links MTCRS to ICRS, \(T_{MTCRS}^{ICRS}\) is the translational vector between both origins and \(x_{1,ICRS}, x_{2,ICRS}, y_{2,ICRS}\) are the coordinates of figure 5.
\[
\begin{bmatrix}
X_{p,ICRS} \\
Y_{p,ICRS} \\
Z_{p,ICRS}
\end{bmatrix} = \frac{R_{ICRS}^{MTCRS}}{1} \times \begin{bmatrix}
X_{p,MTCRS} \\
Y_{p,MTCRS} \\
Z_{p,MTCRS}
\end{bmatrix} + T_{ICRS}^{MTCRS}
\]

(5)

\[d_{1,p,ir} = \sqrt{x_{p,ICRS}^2 + y_{p,ICRS}^2 + z_{p,ICRS}^2 + f(t)}\]

(6)

\[d_{2,p,ir} = \sqrt{(x_{p,ICRS} - x_1)^2 + y_{p,ICRS}^2 + z_{p,ICRS}^2 + f(t)}\]

(7)

\[d_{3,p,ir} = \sqrt{(x_{p,ICRS} - x_2)^2 + (y_{p,ICRS} - y_2)^2 + z_{p,ICRS}^2 + f(t)}\]

(8)

\[d_{i,p,mm} = d_{i,m1} + d_{i,pl1} + d_{i,rm}\]

(9)

This way, the measurement of a mesh of points with the HTPI can be simulated, allowing us to generate synthetic tests to analyze the adequacy of different multilateration techniques and the relevance of the influence factors.

4. Multilateration techniques to telescopic system

Multilateration is a widely extended technique based on determination of the position of a point in space measuring it from three or more different positions. There are two different approaches. First approach is based on analytical resolution of spheres intersection defined by radial distance of measurement equipment, its origin and the point to measure. Second approach is based on an optimization process, it identifies the location of the measurement system location and points coordinates minimizing the difference between interferometer measurement and distances of obtained on optimization process modifying identification parameters values [12].

Focusing on the analytical resolution through intersection of spheres, intersection problem has been analysed in depth on [12]. It is the same as in the LT case. However, previous to calculate the intersection problem, the location of the measurement system needs be accurately obtained, for example by means of an external instrument However, if it is obtained using only lasers information, multilateration problem cannot be obtained related to the MTCRS directly. Moreover, to solve the HPTI location problem related to the ICRS, additional information is required.

As figure 5 shows this problem is solved identifying three parameters \((x_1, x_2, y_2)\). For that, the distance between each laser \(d_{i,j}\) is measured with \(i\), being the laser used to measure and \(j\) the laser whose origin is measured as shown in figure 5. This way, location parameters can be obtained as shown in equations (10) to (12).

\[x_1 = d_{1,2}\]

(10)
\[ x_2 = \frac{d_{1,2}^2 + d_{1,3}^2 - d_{2,3}^2}{2d_{1,2}} \]  \hspace{1cm} (11) \\
\[ y_2 = \sqrt{\frac{d_{1,3}^2 + d_{1,2}^2 - d_{2,3}^2}{2d_{1,2}}} \]  \hspace{1cm} (12)

If multilateration points are obtained through optimization process, the multilateration options are strongly increased. The HPTI parameters to consider in the optimization may be location of HPTI related to ISC or MTCRS, the coordinates of multilateration points and the offsets between retroreflector – moorage and moorage-interferometer. Besides, these parameters can be identified all together or independently, being however equal the optimization scheme in all cases:

1. Select parameters to identify and create the identification vector [location parameter, equipment characteristics, multilateration point]. This vector will depend on the strategy defined by the users.
2. Define the initial values of the identification vector.
3. Define optimization characterised based on Levenberg Marquardt algorithm.
4. Define the objective function to minimize \( \Phi \) (equations (13) and (14)) with \( i \) the number of laser and \( P \) the number of point measured and the coordinates of multilateralized point \( P (x_{multi,p}, y_{multi,p}, z_{multi,p}) \).

\[ \Phi = \left[ d_{i,p \, measured} - d_{i,p \, calculated} \right] \]  \hspace{1cm} (13)

\[ d_{i,p \, calculated} = \sqrt{(x_{multi,p} - x_i)^2 + (y_{multi,p} - y_i)^2 + (z_{multi,p} - z_i)^2 + d_{i,mi} + d_{i,rm}} \]  \hspace{1cm} (14)

5. Extract final parameters from optimization and launch a new optimization if a phased optimization has been designed.

5. Test and results

This section studies the influence of the modelled parameters and the technique to be used using synthetic tests. To generate these tests we created several algorithms using commercial software MATLAB. Algorithm inputs are: location of measurement systems (MTCRS/ICRS), measurement offset (eq.4), distribution of nominal points to measure, number of tests to generate, and the measurement noise (eq.2). In this way \( d_{i,p} \) of each laser can be calculated based on eq.5-9 and eq.10-12, and method, analytical or optimization [12]. As outputs, algorithms provide the location of HPTI and the multilateralized coordinates of measured points.

We created a measurement mesh of 60 points uniformly distributed from 0 mm \( \leq x \leq 1000 \) with intervals of 250 mm, 0 mm \( \leq y \leq 600 \) mm with intervals of 200 mm and 300 mm \( \leq z \leq 600 \) mm with intervals of 200 mm. The tests carried out analyse the influence of spheres intersection and optimization method on ICRS with \( d_{1,2} = 1000 \) mm, \( d_{1,3} = 1166,1904 \) mm and \( d_{2,3} = 600 \) mm. As these values should be obtained with the own instrument, measured values have to be affected by equation (2). Therefore, instrument’s accuracy affects the accuracy of multilateralised point. To avoid it, 10,000 test for each multilateration configuration have been carried out. Figure 6 shows the mean error in distance between nominal and the multilateralised points for tests using sphere intersection (equations (3) to (12)). It can be observed how mean error is bounded around 6 micrometers.

Besides the accuracy on \( d_{i,j} \), the optimization sequence and its initial values are relevant parameters for the multilateration based on optimization process. Same configurations to analyse are the following:

- Joint optimization, where location coordinates \( (x_1, x_2, y_2) \) are obtained together with the multilateration points \( (x_{multi,p}, y_{multi,p}, z_{multi,p}) \).
Two-phase optimization. First of all, it is optimized the position of the HPTI with regard to the ICRS \((x_1, x_2, y_2)\). Then points \((x_{\text{multi},p}, y_{\text{multi},p}, z_{\text{multi},p})\) are calculated on a second optimization.

Only points optimization. Values obtained from \(d_{ij}\) are used as fixed values to calculate analytically \((x_1, x_2, y_2)\) as shown in equations (10) to (12). Multilateration points are calculated by optimization.

![Figure 6. Mean error in distance between nominal and the multilateralised points using spheres intersection.](image)

Multilateration based on optimization process needs initial data to identify the point’s coordinates and the equipment location. First column of Table 1 shows the difference in percentage from initial optimization values and the actual point’s position (defined by their nominal values). Moreover, table 1 shows the highest values of average, maximum, minimum and standard deviation error of the 10,000 tests of each configuration. As it can be observed, when location parameters \((x_1, x_2, y_2)\) are introduced in the optimization process, there is a strong influence of the initial data accuracy. If the data accuracy is good enough, multilateration accuracy is improved. A two-phase optimization, process provides better results than the joint optimization method.

| Method                    | Initial data accuracy vs nominal point (%) | Average Error (µm) | Maximum Error (µm) | Minimum Error (µm) | Standard Deviation (µm) |
|---------------------------|------------------------------------------|--------------------|--------------------|--------------------|------------------------|
| Spheres Intersection      | 100                                      | 6.52               | 24.79              | 3.52e-5            | 5.43                   |
| Joint optimization        | 100                                      | 0.18               | 0.55               | 3.02e-5            | 0.08                   |
| Two phases optimization   | 100                                      | 0.07               | 0.35               | 3.64e-10           | 0.05                   |
| Only points optimization  | 100                                      | 6.04               | 23.09              | 3.21e-5            | 5.00                   |
| Joint optimization        | 95                                       | 139.57             | 216.50             | 0.05               | 63.76                  |
| Two phases optimization   | 95                                       | 1.49e4             | 5.034e4            | 126.99             | 1.14e4                 |
| Only points optimization  | 95                                       | 6.57               | 25.18              | 3.50e-5            | 5.41                   |
| Joint optimization        | 90                                       | 989.46             | 1.54e3             | 1.94               | 446.50                 |
| Two phases optimization   | 90                                       | 2.92e4             | 9.24e4             | 468.20             | 2.22e4                 |
| Only points optimization  | 90                                       | 6.50               | 25.18              | 3.46e-5            | 5.41                   |

However, changes on initial values have substantially worsened the points accuracy. When location parameters are included in the optimization, a small part of the initial error provided by equation (13) is modelled as location parameter \((x_1, x_2, y_2)\), providing a loss of accuracy on multilateration points, especially on two phases optimization. Nevertheless, if location parameters \((x_1, x_2, y_2)\) are considered as fixed values, accuracy of multilateration points is much more robust to changes on the initial values of optimization. Tests carried out related to MTCRS have a stronger behavior.
6. Conclusions
The use of a new measurement system such as HPTI for MT volumetric verification requires of an adequate system modelling to determine its best working conditions. It should be taken into consideration both the system components and its assembling. Tests carried out in this work show that the most adequate system to calculate 3D points coordinates is ICRS. Moreover, a real test carried out on MT stopped, shows that temperature affects to points measurement and therefore to the measured distance, being the laser's error limited to ±2.5 µm.

This error source is the main contribution to the final error when intersection spheres is used as multilateration techniques, obtaining an average error smaller than 6.52 µm and a maximum error smaller than 24.79 µm. Initial data accuracy has an huge influence both on optimization techniques and multilateration accuracy. If the initial data accuracy is good enough, intersection of sphere results including instrument location parameters on optimization, may improve. However, these results are worse if the initial parameters change. Finally, we could conclude that if location parameters are not included on the optimization, the optimization increases its robustness obtaining results similar to the analytical resolution.

Acknowledgements
This research was funded by the Ministerio de Economía, Industria y Competitividad with project number Reto 2017-DPI2017-90106-R, by Aragon Government (Department of Industry and Innovation) through the Research Activity Grant for research groups recognized by the Aragon Government (T56_17R Manufacturing Engineering and Advanced Metrology Group). This is co-funded with European Union ERDF funds (European Regional Development Fund 2014–2020, “Construyendo Europa desde Aragón”).

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