Cosmological Black Holes

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Abstract

In this paper we propose a model for the formation of the cosmological voids. We show that cosmological voids can form directly after the collapse of extremely large wavelength perturbations into low-density black holes or cosmological black holes (CBH). Consequently the voids are formed by the comoving expansion of the matter that surrounds the collapsed perturbation. It follows that the universe evolves, in first approximation, according to the Einstein-Straus cosmological model. We discuss finally the possibility to detect the presence of these black holes through their weak and strong lensing effects and their influence on the cosmic background radiation.

key words cosmology: theory, dark matter, black hole physics

1 Introduction

One of the most intriguing features of the universe is that galaxies tend to lie on sheet-like structures surrounding voids with typical sizes of about 40 – 50$h^{-1}$ Mpc [1].

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The existence of voids has been evident after the discovery by Kirshner et al. of a 60 Mpc large void in the Böotes constellation. Systematic surveys have shown the existence of many regions with similar characteristics. Today it is believed that voids occupy about 50 per cent of the volume of the universe.

From the observational point of view, one of the most important issues is whether the voids are or are not really empty regions. IRAS surveys indicate the absence of infra-red galaxies, other considerations lead to exclude the presence of dark matter inside them.

But as observed by Peebles in the low dispersion of the velocities of the galaxies indicates, when $\Omega_m = 1$, that most of the matter must be inside the voids, but the author suggests that this could also be true even in the case of small $\Omega_m$. Moreover, very recent observations indicate that a possible value can be $\Omega = 1$ and that likely the correct value of $\Omega_m$ is about $1/3$ and therefore, that about 90% of this mass is not luminous.

It needs to be stressed that the visual inspection of galaxy’s distribution suggests nothing else that the absence of large amount of luminous matter in vast regions. Furthermore it is not very clear whether the voids are physically empty approximately spherical regions or larger underdense regions with arbitrary shapes. Many definition of voids have been proposed, but a definitive conclusion has not been reached yet.

Two solutions of the Einstein equations are generally employed to study the theoretical properties of the cosmological voids. The Lemaître-Tolman (L-T) metric and the Einstein-Straus Swiss Cheese model. The first is a spherically symmetric metric for irrotational dust. A complete account of the developments of this model is discussed in.

The Swiss Cheese model is obtained by cutting out spherical regions from a Friedmann-Lemaître-Robertson-Walker (FLRW) model, with null pressure, and substituting them with regions with a spherically symmetric metric such as the Schwarzschild or the L-T solutions. Appropriate junction conditions have to be imposed in order to join the solutions of the Einstein equations. A problem which was solved by Einstein and Straus in order to study the effect of the universal expansion on the planetary orbits (McVittie and Järnefeldt studied the same problem some time before, see references).

The L-T and the Swiss Cheese models do not predict the formation of voids, which must be contained in the initial data.

In a series of papers, Piran and his coworkers attribute the void formation to the evolution of negative perturbations. In particular in
it is shown that these negative fluctuations behave as if they have a negative effective mass; in Friedmann and Piran show that the underdense regions can result as the combined effect of the gravitational expansion of negative density perturbations and biasing, because galaxies are less likely to form in an underdense region.

In this paper, we show that void formation can be the result of the collapse of positive perturbations. To this purpose, we consider an Einstein-Straus model with a distribution of spherical voids of fixed comoving radius $R_v$. In the centre of each void we assume that a black hole, whose mass $M$ compensates the mass that the void would have if it were completely filled by matter with the average cosmological mass-energy density $\rho$. In what follows we calculate the mass, the Schwarzschild radius and the densities of these black holes. Due to their cosmological properties, we shall call them cosmological black holes (CBH). As they have apparently very low densities, these CBHs may have likely formed directly from the collapse of very large wavelength perturbations. Therefore voids are the result of the comoving expansion of the matter surrounding the CBHs.

This scenario explains the existence of a large amount of dark matter, which is hidden in these black holes without perturbing the cosmic background radiation. In this simple model the CBHs do not affect the galaxy motion due to the Birkhoff theorem, they just take part in the collective universal expansion.

In the last part of the paper we discuss some observational implications of the proposed scenario, in particular the gravitational lensing effects induced by these CBHs behave and their influence on the cosmic background radiation.

In the final remarks we discuss the limits of this cosmological model. For instance we expect to gain some improvement by weakening the condition of exact sphericity of the CBHs and of the voids.

## 2 Properties and Origin of the Cosmological Black Holes

We consider here an Einstein-Straus universe characterised by the following properties,

a) For sake of simplicity we shall limit ourselves to consider a flat Friedmann-Lemaître-Robertson-Walker universe;

b) in all the voids there is a central spherical black hole with mass $M$

$$M = \frac{4}{3} \pi \Omega_{\text{cbh}} \rho_{\text{crit}} R_v^3,$$  \hspace{1cm} (1)
\[
\Omega_{cbh} = \frac{\rho_{cbh}}{\rho_{\text{crit.}}} \tag{2}
\]
represents the fraction of density due to all these black holes to the total density of the universe; where \(\rho_{\text{crit.}} = 1.888h^2 \times 10^{-29} \text{ g cm}^{-3}\) is the critical density of the universe; and
c) all the voids are spherical.

We also note that the Schwarzschild radius can be related to the mass-energy density \(c^2 \rho = \epsilon\), see \[18\].

It is known that a black hole forms when a body with mass \(M\) collapses entirely within a sphere of radius
\[
R_s = \frac{2GM}{c^2}. \tag{3}
\]
This statement is equivalent to say that its density satisfies the relation,
\[
R_s(\rho) = \sqrt{\frac{3c^2}{8\pi G \rho}}. \tag{4}
\]
Conversely, Eq. (3) relates to any density a corresponding Schwarzschild radius. In other words any space-like sphere of matter with uniform density \(\rho\) and radius at least equal to \(R_s(\rho)\) is a black hole.

Whereas astrophysical and primordial black holes form at very high densities \[38\], relation (3) implies that it is possible to consider also the formation of low-density black holes. The transition of collapsing matter to a black hole at low densities is described in \[13\].

From Eqs. (3) and (4) we determine the mass \(M\) and the corresponding Schwarzschild radius and consequently from Eq. (4) the mass-energy density of the central black hole. In Table 1, we list, in solar units, the results for three typical diameters of voids.

| Void diameter | Mass \(\times h/\Omega_{cbh}\) | \(R_s \times h/\Omega_{cbh}\) | density |
|---------------|-------------------------------|----------------|---------|
| 30h\(^{-1}\) Mpc | \(3.9 \times 10^{15} M_\odot\) | 0.37 kpc | \(5.02 \times 10^{-15} \text{g/cm}^3\) |
| 50h\(^{-1}\) Mpc | \(1.8 \times 10^{16} M_\odot\) | 1.7 kpc | \(2.34 \times 10^{-16} \text{g/cm}^3\) |
| 100h\(^{-1}\) Mpc | \(1.4 \times 10^{17} M_\odot\) | 13.9 kpc | \(3.66 \times 10^{-18} \text{g/cm}^3\) |

Table 1: CBHs corresponding to different size voids.
The low densities given in the Table 1 are those reached by the collapsing matter when it crossed the Schwarzschild radius. Loeb in [20], observes that massive black holes can form from the collapse of primordial gas clouds after the recombination epoch \(200 \leq (1 + z) \leq 1400\). We expect that the process of formation of a CBH started with large wavelength perturbations at cosmological densities smaller than \(10^{-19} \text{g/cm}^3\) i.e for \(1 + z \approx 10^3\). According to the inflationary scenario, we only need to suppose that the inflation occurred during a time long enough to provide such perturbations.

The Oppenheimer-Snyder model [21] describes the spherical symmetric collapse at zero pressure. A growing perturbation, in a homogeneous and isotropic universe, with initial wavelength \(\lambda_i\) and amplitude \(\delta_i \ll 1\), it first expands and then collapses according to the equations,

\[
\left(\frac{\dot{a}_p}{a_i}\right)^2 = H_i^2 \left(\Omega_p(t_i) \frac{a_i}{a_p} + 1 - \Omega_p(t_i)\right),
\]

\[
\Omega_p(t_i) = \frac{\rho(t_i)(1 + \delta_i)}{\rho_c(t_i)} = \Omega(t_i)(1 + \delta_i).
\]

The solution of Eq. (5) and the relation

\[
\lambda(t) = \lambda_i \frac{a(t)}{a_i}
\]

give, in a parametric form, the evolution of \(\lambda(t)\),

\[
\lambda(\theta) = \frac{\lambda_i}{2} \frac{\Omega_p}{\Omega_p - 1}(1 - \cos \theta)
\]

and

\[
t(\theta) = \frac{1}{2H_i(\Omega_p - 1)^{\frac{3}{2}}}(\theta - \sin \theta),
\]

where \(a_i\) is the expansion factor at the beginning of the perturbation formation, \(H_i\) is the corresponding Hubble constant and \(\Omega_p\) is the ratio of the perturbation density and the background critical density. It is important to note that, borrowing a sentence in [13], due to spherical symmetry, it follows that inside the collapsing matter and in a certain neighborhood behaves as if the collapsing matter were embedded in a space “with no cosmic expansion or curvature”.

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A serious objection to this picture could be that very unlikely perturbations develop with an exact spherical symmetry. As a matter of fact, one of the principal processes that can prevent the collapse of a perturbation into a black hole is the acquisition of angular momentum through the tidal torques produced by interaction with other structures. The presence of an angular momentum produces a centrifugal barrier at typical scales of six order of magnitude larger than the Schwarzschild radius. But it has been shown by Loeb [20] that, for large perturbations, the friction between the collapsing matter and the cosmic background radiation is capable to extract angular momentum and energy and reduce the centrifugal barrier below the Schwarzschild radius. We assume that the total mass of the perturbation

$$M = \frac{\pi}{6} \Omega_p \rho_i \lambda_i^3$$  \hspace{1cm} (10)

remains constant during the whole process.

The Schwarzschild radius of the spherical perturbation is equal to

$$R_s = H_i^2 \left( \frac{\lambda_i}{2} \right)^3 \Omega_p.$$  \hspace{1cm} (11)

We can therefore distinguish two cases. First, the case in which the relation

$$\frac{2R_s}{\lambda_i} \geq 1,$$  \hspace{1cm} (12)

is satisfied, the perturbation is in the linear regime and, according to the evolution equations in a universe with constant equation of state, it is frozen when \(\lambda_i\) is larger than the Hubble radius [22]. After crossing the Hubble horizon, it collapses and becomes a black hole when

$$\left( \frac{\lambda_i}{2} \right)^2 \geq \frac{c^2}{H_i^2 \Omega_p}.$$  \hspace{1cm} (13)

In the second case

$$\frac{2R_s}{\lambda_i} < 1,$$  \hspace{1cm} (14)

the perturbation evolves according to Eqs. (8) and (9). During the contraction, it becomes unavoidably a black hole, since the final density is very low and the internal pressure and temperature can not raise to values large
enough to prevent the collapse, even when the perturbation enters in a non-linear regime. In addition, any possible centrifugal barrier is reduced to values smaller than the Schwarzschild radius.

Let us establish a limit for the black hole formation. To this aim we fix a threshold value $\hat{\rho}$ of the mass-energy density, above which one can expect that the equation of state sensibly changes and then equilibrium conditions can be established, thus preventing any further collapse, see a similar discussion in [18].

To $\hat{\rho}$ we can associate a final wavelength $\hat{\lambda}$ defined by the equation

$$M = \frac{\pi}{6} \hat{\rho} \hat{\lambda}^3.$$  \hspace{1cm} (15)

Comparison with Eq. (10) yields

$$\hat{\lambda}^3 = \frac{\rho_i \Omega_p}{\hat{\rho}} \lambda_i^3.$$  \hspace{1cm} (16)

A perturbation does not collapse to a black hole when $\hat{\lambda}/2 > R_s$, i.e. if its initial wavelength satisfies the equation

$$\left(\frac{\lambda_i}{2}\right)^2 < \frac{3c^2}{8\pi G (\rho_i \Omega_p)} \left(\frac{\rho_i \Omega_p}{\hat{\rho}}\right)^{\frac{1}{2}}.$$  \hspace{1cm} (17)

According to the Einstein-Straus model, after the formation of the black hole, the matter around it expands in a comoving way leading to the formation of an empty region between it and the rest of the universe. As the central black hole cannot be seen, the whole region appears to an external observer as a void.

In conclusion, among all wavelengths of the cosmological perturbations spectrum, only those structures which satisfy (17) appear in the observed universe as luminous matter or exotic dark matter. The rest of the matter is confined in very massive cosmological black holes.

3 Some phenomenological aspects of the CBHs

A CBH can be detected through its lensing properties, it must behave as a Schwarzschild gravitational lens. Since, according to our hypothesis, a CBH sits in the centre of a void, the Einstein angle is $\frac{\pi}{2}$. 
\[ \alpha_0 = 4.727 \times 10^{-4} \Omega_{CBH}^{1/2} \sqrt{\frac{R_V^3 D_{ds}}{D_d D_s}} \]  

(18)

where \( R_V \) is the radius of the void, \( D_s \) is the distance of the source from the observer, \( D_{ds} \) is the distance of the source from the CBH, and \( D_d \) is the distance of the CBH from the observer, all this quantities are expressed in Mpc. The characteristic length

\[ \xi_0 = \alpha_0 D_d. \]  

(19)

For a 50 Mpc void of diameter, with the centre placed at a distance of 80 Mpc from the Sun and with the source at the opposite edge of the void, we expect

\[ \alpha_0 \simeq 3.2 \times 10^{-3} \Omega_{CBH}^{1/2} \]

and

\[ \xi_0 \simeq 2.6 \times 10^{-1} \Omega_{CBH}^{1/2} \text{Mpc}. \]

This value is the expected extension of an Einstein ring, but to observe it is required that the source must lie exactly on top of the resulting degenerate point-like caustic [24]; so, in order to falsify our model, a first point is to analyze the probability that a galaxy in the background can satisfy this demand. We expect a general magnification effect in the neighborhood of the CBH [25]. Considering a galaxy with an effective luminous part long about 6 kpc, we expect in case of a perfect alignment a magnification factor of the order of \( 1.4 \times 10^2 \). Numerical calculations indicate that the magnification factor decays to 6.8 in the range of \( 6 \times 10^{-2} \) Mpc and reaches the value of 1.37 at 0.6 Mpc.

The high masses involved produce also strong field lensing effects [26] [27] [28], in the range of distances of about 3\( R_s \). In the case of a 50 Mpc void we expect, theoretically, the production of an infinite sequence of relativistic images, on both sides of the optical axis, at scales of the order of \( \sim 6 \) kpc.

A third observational effect is the influence of a CBH on the cosmic background radiation. In [29] Zeldovich and Sazhin point out that generally static structures can raise the temperature of the cosmic background radiation by an amount proportional to the Hubble parameter and the gravitational time delay. By considering the Swiss Cheese model case, they find a fluctuation of temperature \( \delta T / T \sim 10^{-10} \) for a giant galaxy \((M = 4 \times 10^{12} M_\odot)\). As this
result is proportional to the mass, it follows that it corresponds to a fluctuation of temperature $\delta T/T \sim 10^{-5}$ for a large CBH with mass $(M \sim 10^{17} M_\odot)$, which does not contradict the recent COBE measurements.

Very recent observations of type Ia supernovae showed that the expansion of the universe is accelerating [8]. This acceleration can be justified by admitting that the universe is dominated by a source of “dark energy” with negative pressure. Constraints on this source require that $\Omega_{\text{dark ener.}} \simeq 2/3$ and dark and baryonic matter contribute to the total energy density of the universe with $\Omega_M \sim 1/3$ [31].

Measurements, performed using different methods, led to a wide range of values results of $\Omega_M$. According to the mass-to-light ratio Bahcall and coworkers obtained $\Omega_M (M/L) = 0.16 \pm 0.05$ [31] and recently, after some corrections, $\Omega_M (M/L) = 0.17 \pm 0.05$ [32]. It is important to note that according to [32] it was assumed that the voids do not contribute with additional dark matter. On the other side estimates of the baryon fraction in clusters yield $\Omega_M \leq 0.3 \pm 0.05$ and the evolution of cluster abundance gives $\Omega_M \simeq 0.25$ (see [14]).

Finally Turner [35] infers that $\Omega_M = 0.33 \pm 0.035$ together with $\Omega_B = 0.039 \pm 0.0075$ according to recent measurements of the physical properties of clusters, CMB anisotropy and the power spectrum of mass inhomogeneity.

We think that our model could reconcile the discrepancy between the mass-to-light ratio method and the others which find larger values of $\Omega_M$. As a matter of fact the mass-to-light ratio cannot trace the presence of the CBHs.

As voids occupy about 50% of the volume of the universe, we claim that the CBHs would simply double the mass observed by the mass-to-light ratio method, i.e. $\Omega_{CBH} \sim \Omega_M (M/L)$ and

$$\Omega_M = \Omega_{CBH} + \Omega_M (M/L)$$ (20)

This hypothesis can be confirmed by measuring the Einstein angles produced by the CBHs, according to equation (18).

4 Remarks and conclusions

Before concluding this work we remind that the idea that black holes can be generated by cosmological perturbations has been already used to predict the existence of primordial black holes [36] [37] [38] [39] [40]. But as dark matter candidates their role is limited by very severe constraints [42].
The formation of supermassive black holes from the collapse of a primordial cloud was proposed by Loeb in [20]. Recent observations provide some evidence of the existence of supermassive black holes [41]. These objects have been revealed in the centers of the galaxies with masses up to $10^9 M_\odot$.

In this paper we considered the existence of the CBHs, with larger masses and residing at the center of the voids, isolated from any other form of matter. This model explains part of the dark matter problem and provides new observational predictions, it is possible to detect the presence of the cosmological black holes by observing their lensing properties on a background of galaxies.

Moreover we remark that the existence of these black holes is compatible with the Zeldovich perturbation spectrum and with any other spectrum. A more precise information on the spectrum can be obtained by analyzing the void distribution, which reflects the black hole distribution.

But we think that our model can be improved. First of all it is based on exact spherical symmetry, which is not a realistic condition when dealing with the collapse of a perturbation. Second since the CBHs are generated by perturbations, their mass does not necessarily correspond to the value rated by (1), even if the severe restrictions on the initial amplitudes of the perturbations, imposed by the COBE observations, allow us to take the values of mass, energy-mass density, and the Schwarzschild radius given in Table 1 as good approximations of the real ones. Third the Einstein-Straus model is not stable with respect to the variations of mass and to radial perturbations (see [11] and the references within).

Moreover as stated by the Birkhoff theorem, a spherical CBH, that compensates a spherical void, does not influence the motions of the galaxies outside the void. But, according to Peebles in [37], some peculiarities of the galaxy motion can be explained by admitting the presence of large amounts of mass in the voids. This requires that the effective mass of a cosmological black hole may be larger than the mass given by (10). In this case that the void expansion must not be comoving as in the Einstein-Straus model (for a review on this problem see [11]).

The previous considerations and the observation that presently voids present an underdense distribution of galaxies (see [3]), suggest that the CBHs can affect the motion of the surrounding galaxies. This can be explained only relaxing the conditions of the Birkhoff theorem and considering the non uniform distribution of matter on lower scales. This problem will be analyzed by computer simulations.
Finally we think to develop our model extending the results obtained from the Einstein-Straus Swiss-Cheese model with the recent results obtained by Bonnor [43][44], Senovilla and Vera [45], Mars [46][47].

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References

[1] J. A. Peacock, *Cosmological Physics* (Cambridge Univ. Press, Cambridge, 1999).

[2] R.P. Kirshner, A. Oemler, P.L. Schechter and S.A. Shectman, Astrophys. J. 248, L57 (1981).

[3] V. de Lapparent, M. J. Geller and J. P. Huchra, Astrophys. J. 302, L1 (1986).

[4] M. J. Geller and J. P. Huchra, Science 246, 897 (1989).

[5] H. El-Ad and T. Piran, Astrophys. J. 491, 421 (1997) [astro-ph/9702135].

[6] P.J.E. Peebles, Astrophys. J. 557, 495 (2001).

[7] P. de Bernardis et al., Nature 404, 955 (2000).

[8] S Perlmutter et al. Astrophys. J. 517, 565(1998).

[9] S. Khalil and C. Munoz, [arXiv:hep-ph/0110122](http://arxiv.org/abs/hep-ph/0110122).

[10] J. Schmidt, B. S. Ryden and A. Mellott Astrophys. J. 546, 609 (2001).

[11] A. Krasiński, *Inhomogeneous Cosmological Models* (Cambridge Univ. Press, Cambridge, 1997).
A. Einstein and E.G. Straus, Rev. Mod. Phys. 17, 120 (1945) and Rev. Mod. Phys. 18, 148 (1946).

McVittie and Järnefeldt studied the same problem some time before, see references in A. Einstein and E.G. Straus, Rev. Mod. Phys. 18, 148 (1946).

G. R. Blumenthal, L. Nicolaci da Costa, D. S. Goldwirth, M. Lecar and T. Piran, Astrophys. J. 388, 234 (1992).

J. Dubinski, L. Nicolaci da Costa, D. S. Goldwirth, M. Lecar and T. Piran, Astrophys. J. 410, 458 (1993)

T. Piran, Gen. Rel. Grav. 29 (1997) 1363.

Y. Friedmann and T. Piran, Astrophys. J. 548, 1 (2001)

S. Chandrasekhar, Phys. Rev. Lett. 12, 114 (1964).

S.W. Hawking and G.F.R. Ellis, The large scale structure of space-time (Cambridge Univ. Press, Cambridge, 1973), see pag.308. The reader can note that the densities of the black holes in this paper are far below the one considered by the authors.

A. Loeb, Astrophys. J. 403, 542 (1993)

J.R. Oppenheimer and H. Snyder, Phys. Rev. 57, 455 (1939).

V. F. Mukhanov, H. A. Feldman and R. H. Brandenberger, Phys. Rept. 215 (1992) 203.

P. Schneider, J. Ehlers, E.E. Falco, Gravitational lenses (Springer-Verlag, Berlin; New York, 1992).

Wambsganss, J., "Gravitational Lensing in Astronomy", Living Rev. Relativity 1, (1998), 12. [Online article]: cited on 22 Jan 2002. http://www.livingreviews.org/Articles/Volume1/1998-12wamb/

R. J. Bontz, Astrophys. J. 233, 402 (1979).

H.C. Ohanian, Am. J. Phys. 55, 428 (1987)
[27] V. Bozza, S. Capozziello, G. Iovane, G. Scarpetta, Gen. Rel. Grav. 33, 1535 (2001).

[28] K. S. Virbhadra and G. F. R. Ellis, Phys. Rev. D 62, 084003 (2000).

[29] Ya. B. Zeldovich and M. V. Sazhin, Sov. Astron. Lett. 13, 145 (1987).

[30] S. Perlmutter, M. Turner and M. White, Phys. Rev. Lett. 83, 670 (1999).

[31] N. A. Bahcall, R. Cen, R. Davé, J. P. Ostriker and Q. Yu, Astrophys. J. 541, 1 (2000).

[32] N. A. Bahcall and J. M. Comerford, Astrophys. J. 565, L5 (2002).

[33] N. A. Bahcall L. M. Lubin and V. Dorman, Astrophys. J. 447, L81 (1995).

[34] N. A. Bahcall “Constructing the Universe with Clusters of Galaxies” IAP meeting, Paris (France) July 2000, Florence Durret & Daniel GErbal eds.

[35] M. S. Turner, astro-ph/0106035, 2001.

[36] For a review on astrophysical and primordial black holes see e.g. R. D. Blanford and K.S. Thorne, General Relativity, edited by S. W. Hawking and W. Israel (Cambridge University Press, Cambridge, England 1979).

[37] P.J.E. Peebles, Principles of physical cosmology, (Princeton University Press, Princeton, New Jersey, 1993).

[38] S.W. Hawking, Mon. Noy. R. Astron. Soc. 52, 75 (1971).

[39] B.J Carr, Astrophys. J. 201, 1 (1975);

[40] B.J Carr, Astrophys. J. 206, 8 (1976).

[41] D. Richstone et al., Nature 395, A14 (1998).

[42] J. E. Gunn, Cosmology And Galaxy Formation: A Review, in “Jerusalem 1983/84, Proceedings, intersection between elementary particle physics and cosmology”, 1 (World Scientific, Singapore, 1986).
[43] W. B. Bonnor, Astrophys J. 316, 49 (1987).

[44] W. B. Bonnor, Class. Quantum Grav. 17, 2739, (2001).

[45] J. M. M. Senovilla and Raül Vera, Phys. Rev. Lett. 78, 2284 (1997).

[46] M. Mars, Phys. Rev. D 57, 3389 (1998).

[47] M. Mars, Class. Quantum Grav. 18, 3645 (2000).