Core formation in high-z massive haloes: heating by post-compaction satellites and response to AGN outflows

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Abstract

Observed rotation curves in star-forming galaxies indicate a puzzling dearth of dark matter in extended flat cores within haloes of mass $\geq 10^{12} M_\odot$ at $z \sim 2$. This is not reproduced by current cosmological simulations, and supernova-driven outflows are not effective in such massive haloes. We address a hybrid scenario where post-compaction merging satellites heat up the dark-matter cusps by dynamical friction, allowing active galactic nucleus (AGN)-driven outflows to generate cores. Using analytic and semi-analytic models (SatGen), we estimate the dynamical friction heating as a function of satellite compactness for a cosmological sequence of mergers. Cosmological simulations (VELA) demonstrate that satellites of initial virial masses $> 10^{11.3} M_\odot$, which undergo wet compactions, become sufficiently compact for significant heating. Constituting a major fraction of the accretion to haloes $\geq 10^{12} M_\odot$, these satellites heat up the cusps in half a virial time at $z \sim 2$. Using a model for outflow-driven core formation (CuspCore), we demonstrate that the heated dark-matter cusps develop extended cores in response to removal of half the gas mass, while the more compact stellar systems remain intact. The mergers keep the dark matter hot, while the gas supply, fresh and recycled, is sufficient for the AGN outflows. AGNs indeed become effective in haloes $\geq 10^{12} M_\odot$, where the black hole growth is no longer suppressed by supernovae and its compaction-driven rapid growth is maintained by a hot circumgalactic medium. For simulations to reproduce the dynamical friction effects, they should resolve the compaction of the massive satellites and avoid artificial tidal disruption. AGN feedback could be boosted by clumpy black hole accretion and clumpy response to AGN.

Key words: black holes – galaxies: discs – galaxies: formation – galaxies: haloes – galaxies: mergers – dark matter.

1 Introduction

As seen in cosmological gravitating N-body simulations with no baryons, dark-matter (DM) haloes robustly produce cuspy inner density radial profiles with a central negative log slope $\alpha \sim 1$ (Navarro, Frenk & White 1997, NFW). In contrast, dwarf galaxies are observed kinematically to have inner flat DM cores, $\alpha \sim 0$ (e.g. Flores & Primack 1994; Burkert 1995; de Blok et al. 2001, 2008; Oh et al. 2011a, b, 2015). This is commonly understood both analytically and in simulations in terms of bursty supernova (SN) feedback (e.g. Pontzen & Governato 2012; Dutton et al. 2016; Freundlich et al. 2020a), which is capable of effectively ejecting the gas from the central regions in haloes below a critical halo virial velocity of $V_\text{c} \sim 100 \text{ km s}^{-1}$, where the supernovae energy deposited in the interstellar medium (ISM) is comparable to the binding energy of the gas in the central halo potential well (Dekel & Silk 1986). The inner DM halo responds to the shallowing of the gravitational potential well due to the central mass loss by expanding and producing a flat core.

Very surprisingly, kinematic observations and careful analysis by Genzel et al. (2020), following earlier indications (e.g. van Dokkum et al. 2015; Wuyts et al. 2016; Genzel et al. 2017; Lang et al. 2017), point to low central DM fractions within the stellar effective radii ($r_{\text{e}}$), associated with DM cores that may extend to $\sim 10 \text{ kpc}$, in a non-negligible fraction of the galaxies they studied. The Genzel et al. (2020) sample consists of 41 massive star-forming disc galaxies at $z = 0.65-2.5$. The data include near-infrared observations with SINFONI and KMOS at the ESO–VLT, as well as LBT–LUCI, and sub-millimetre observations from IRAM–NOEMA. The kinematics is determined from ionized gas traced by Hα and molecular gas traced by CO. The baryons in the selected galaxies are rotationally
supported \((V_{\text{rot}}/\sigma > 2.3)\), lying in the main sequence of star-forming galaxies (SFGs), with stellar masses \(M_* = 10^{8.8} - 11.4 M_\odot\). There is a correlation between low \(f_{\text{dm}}\) and a high bulge-to-disc ratio. By properly inverting the relations based on abundance matching of observed galaxies and simulated LCDM DM haloes (Behroozi, Wechsler & Conroy 2013; Moster, Naab & White 2018), the corresponding halo virial mass range is \(M_h = 10^{11.6} - 12.9 M_\odot\). In the high part of the redshift range, \(z \sim 1.2 - 2.5\), more than two-thirds of the galaxies show low DM fractions within the effective radius \((r_{\text{e}} < 0.3)\), of which about one-half are indicated to have extended cores. The cores tend to appear in this sample for \(M_h \geq 10^{12} M_\odot\), with the median at \(M_h = 10^{12.3} M_\odot\), while less massive galaxies tend to have higher DM fractions. At the lower redshift band, \(z \sim 0.6 - 1.2\), the portion of low DM fraction galaxies is reduced to about a third, but their vast majority are with extended cores. At this lower redshift range, the DM fractions are typically not as low as at higher redshifts, and the mass dependence of this phenomenon is clearer, with several galaxies of high DM fractions below \(10^{12} M_\odot\). A similar analysis is being extended to a sample of 100 galaxies (Price et al. 2021), which seems to reinforce the existence of DM-deficient extended cores in massive SFGs at \(z \sim 2\).

The appearance of low DM fractions in extended cores is very surprising from a theory point of view because in such massive haloes, and at such redshifts, the gravitational potential is likely to be too deep for SN feedback to effectively remove central gas for gravitationally pushing the DM out and generating cores (Dekel & Silk 1986). The fact that current cosmological simulations do not produce such extended cores makes this a non-trivial challenge.

Beyond the unavoidable observational uncertainties, the non-trivial dynamical analysis of the observed rotation curves (RCs), which reveals the apparent low DM fractions and extended cores, is based on various assumptions and is thus associated with further uncertainties. These include, for example, the assumption of an exponential disc and an uncertain correction term to the centripetal force by a pressure gradient (‘asymmetric drift’ Burkert et al. 2010). Furthermore, a cuspy NFW profile is assumed in the fit of a DM halo to the data, and when the corresponding central DM fraction conflicts with the observations, the conclusion is that the assumed NFW cusp should be replaced by a core. These uncertainties are expected to be reduced in a subsequent analysis, e.g. by using a more flexible model profile that allows a core. However, the uncertainties that have been carefully considered so far do not seem to systematically weaken the general need for a dearth of central DM and extended cores. Thus, despite the uncertainties, the intriguing implications of these results call for a special theoretical effort to explore potential processes that may push the DM out and generate cores in massive galaxies, while the stellar systems remain intact.

There are seemingly conflicting reports of larger \(f_{\text{dm}}\) values based on galaxies from the KROSS survey (Tiley et al. 2019; Sharma, Salucci & van de Ven 2021). It can be argued (as reviewed by Förster Schreiber & Wuyts 2020, Section 4.4.3) that these results are actually largely consistent with Genzel et al. (2020), who find a range of RC shapes and \(f_{\text{dm}}\) depending on galaxy properties. The apparently different results may be due to differences between the samples and the analysis methods, such as the following: (a) The larger \(f_{\text{dm}}\) values in KROSS are obtained within radii of \((1.9 - 3.6) R_e\), compared to \(1 R_e\) in Genzel et al. (2020), in agreement with the expectation that the DM fraction should increase with radius. (b) The KROSS sample is mostly at \(z \sim 1\), while the low \(f_{\text{dm}}\) values are observed mostly at \(z \sim 2\), with a typical reduction of 0.2–0.3 between these redshifts. (c) The KROSS galaxies are typically of lower masses than the galaxies of Genzel et al. (2020), while it is shown (and emphasized in Price et al. in preparation) that \(f_{\text{dm}}\) is strongly decreasing with mass. (d) The selection of the KROSS galaxies is not as restricted to disc galaxies with high \(V/\sigma\) values, thus including galaxies where the RC is less reliable. (e) When stacking RCs for the typical shape, in KROSS it is done at \(R_e\), thus smearing the RC peak, which appears in different galaxies at different radii with respect to \(R_e\). The stacking in Genzel et al. (2020) attempts to take into account the different shapes of the RCs such that the peak may be resolved better. While this debate may continue, the evidence for low DM fractions and extended cores within the effective radii of massive, \(z \sim 2\) discs seems rather convincing, and the puzzle it introduces calls for a theoretical understanding.

Several different mechanisms could in principle work towards generating cores in massive galaxies. These include, for example, mergers of SN-driven cored building blocks (Dekel, Devor & Hetzroni 2003), dynamical friction (DF) by the DM on merging satellites (El-Zant, Shlosman & Hoffman 2001), and active galactic nucleus (AGN)-driven outflows (e.g. Martizzi et al. 2012; Peirani et al. 2017), which may work in massive galaxies in analogy to SN-driven outflows that generate cores in low-mass galaxies. The puzzling issue is that none of the currently available cosmological simulations seem to give rise to such low central DM fractions and extended cores in massive haloes, despite the fact that all the above-mentioned physical elements, including mergers, DF, supernovae, and AGNs, are supposed to be incorporated in the simulations. Examples showing cusps in massive galaxies are in the TNG simulation (Wang et al. 2020) and in the FIRE-2 simulations (Lazar et al. 2020).

The effect of AGN feedback on the DM profile in the central regions of clusters of galaxies has been studied using cosmological simulations (e.g. Martizzi et al. 2012) and isolated-halo simulations (Martizzi, Teyssier & Moore 2013). It has been found that the repeating episodes of gas ejection and recycling can indeed generate a core of radius \(\sim 10\) kpc, but with the simulated halo masses of \(10^{14} M_\odot\) and \(1.4 \times 10^{13} M_\odot\) in these studies, the virial radii are \(\sim 1.2\) Mpc and \(\sim 0.6\) Mpc, respectively, so the core radii extend only to 1–2 per cent of the virial radii as opposed to the desired \(\sim 10\) per cent. No cores, or cores of limited extent, have also been found in massive haloes in Dubois et al. (2013a, fig. 4), in the Horizon–AGN simulation (Peirani et al. 2017, 2019), and in the NIHAO–AGN simulations (Macciò et al. 2020), all including AGN feedback. A preliminary inspection of the haloes of massive galaxies in the TNG50 cosmological simulation, where a relatively strong AGN feedback is implemented (Weinberger et al. 2018), indicates cores that extend to only 1–2 per cent of the virial radius (Sandro Tacchella, private communication). It seems that AGN feedback in cosmological simulations, as implemented so far, fails to reproduce cores of \(\sim 0.1 R_e\), extent as deduced from observations by Genzel et al. (2020). Potential ways to remedy this could be to have the DM halo pre-heated and thus be closer to escape before the mass ejection and/or to boost the strength of the AGN feedback itself and tighten its coupling with the gas in the inner halo. The issue of whether AGN feedback can remove large amounts of cold gas is controversial, as it depends on the uncertain subgrid model assumed for this feedback in the simulations. Certain simulations find that the AGN feedback quenches the gas accretion but does not remove cold gas efficiently (e.g. Dubois et al. 2013b; Gabor & Bournaud 2014), while other simulations find efficient gas removal by AGN (e.g. Biernacki & Teyssier 2018).

The effect of DF on the cusps of massive haloes has been discussed by El-Zant et al. (2001). They used semi-analytic simulations with hundreds of satellites that sum up to 10 per cent of the host mass of \(10^{12} M_\odot\) during \(2 - 3\) Gyr, which is roughly a halo virial time at \(z = \ldots\)
0. This led to almost-flat cores that extend to 4–6 kpc, which at $z = 0$, with $R_c \sim 300$ kpc, corresponds to only $(0.01–0.02)R_c$. In El-Zant et al. (2004), they used $N$-body simulations with multiple satellites that amount to 3–20 per cent of the host-halo mass. Semicores of log slope $-0.35$ that extend out to $R_c \sim 0.06R_c$ were generated in $\sim 6$ Gyr. However, in both cases, as well as in El-Zant (2008), where a universal profile is maintained under satellite sinking, the models assumed point-mass satellites and ignored their tidal stripping, thus overestimating the effect of DF on the host cusp. While this has been a stimulating proof of concept, the situation studied is not a representative cosmological population of satellites that could reproduce the realistic effect of DF on the host cusps. Further simulations of the effect of DF moved beyond the earlier studies, e.g. by introducing sub-haloes or stellar systems (Ma & Boylan-Kolchin 2004; Laporte et al. 2012; El-Zant 2019). The impression from all these studies is that DF by itself is not likely to generate flat cores as extended as observed.

Clearly, efficient satellite penetration that may result in DM heating requires that the incoming satellites would be massive, compact, and on relatively radial orbits, such that tidal stripping would be minimal prior to reaching the host central regions where the DF is to increase the energy and angular momentum of the DM cusp. The questions to be verified are whether the realistic satellites that merge with the host haloes of the relevant masses at the relevant redshifts are sufficiently massive, compact, and on proper orbits for effective DF heating or flattening of the host DM cusps. It is also interesting to find out how the deposited energy is divided between heating the cusp and partially flattening the profile into a core.

We study here a promising two-staged scenario where DF heating or flattening and AGN feedback are both boosted, and they are put together in a hybrid scenario where the DF that acts on compact merging satellites heats up or flattens the inner DM halo, making it more susceptible to expansion under clumpy AGN-driven outflows.

We note that standard NFW cusps are kinematically cold, as Jeans equilibrium implies that the (isotropic) velocity dispersion profile is decreasing towards the centre like $\sigma^2 \propto r$, while the gravitational potential gets deeper. This means that the dark matter is more tightly bound in the inner cusp, which implies that it would be hard to unbind it by central mass ejection. This is why kinematically heating the DM particles by DF prior to the mass ejection, bringing them closer to the escape velocity from the cusp, could be a key to DM expansion and core formation.

We demonstrate below that in such a scenario the observed preferred host-halo masses and preferred redshifts for DM cores are natural outcomes. We note that for host haloes of $M_\odot \geq 10^{12}M_\odot$ at $z \sim 1–3$, the likely massive merging satellites are with haloes of $m_\odot \geq 10^{11.3}M_\odot$, which are above the mass threshold for drastic compaction events into blue nuggets (Zolotov et al. 2015; Tomassetti et al. 2016; Tacchella et al. 2016a, b). This golden mass (Dekel, Lapiner & Dubois 2019) is determined by the combined effect of SN feedback at lower masses (Dekel & Silk 1986) and virial shock heating of the circumgalactic medium (CGM) aided by AGN feedback at higher masses (Birnboim & Dekel 2003; Dekel & Birnboim 2006). The satellites above the golden mass are therefore likely to be more compact than lower-mass satellites that typically feed lower-mass hosts. These compact satellites could be optimal for deep penetration into the host cusp with less tidal stripping, allowing effective DF heating or flattening in the host cusp. The highly compact baryonic cusp of the satellite should allow the buildup of a central stellar system that would remain intact under the AGN-driven gas outflows that push the less-concentrated DM outward. AGN feedback is indeed expected to be most effective above the golden mass of $M_\odot \sim 10^{12}M_\odot$, where the black hole (BH) growth is no longer suppressed by SN feedback (Dubois et al. 2015), the BH growth is made possible by the hot CGM (Bower et al. 2017; Anglés-Alcázar et al. 2017b), and the rapid BH growth is triggered by a wet compaction event (Dekel et al. 2019; Lapiner, Dekel & Dubois 2020). This is confirmed by observations of luminous AGN fractions above the golden mass (Förster Schreiber et al. 2019). The preferred appearance of massive bulges in low $\lambda$ galaxies is indeed consistent with the association of cores with the bulge-forming compaction events that tend to occur above the golden mass.

The preferred high redshift for DM cores could naturally emerge from the higher merger rate, higher gas fraction, and therefore more effective compaction events at higher redshifts. Furthermore, at higher redshifts, the DM cusps are maintained ‘hot’ under the DF by a cosmological sequence of satellites and are thus more susceptible to core formation by AGN outflows, while at lower redshifts the cusps cool and expand and thus make the AGN outflows less effective.

A companion paper (Burkert et al. in preparation) addresses the same problem of DM core formation emphasizing somewhat different angles and a similar hybrid scenario of DF heating, followed by response to outflows. This version of the scenario appeals instead to DF heating by inward migrating clumps in violently unstable discs, and it bases the feasibility tests of the two parts of the scenario on simple $N$-body simulations tailored for the purpose.

The paper is organized as follows. In Section 2, we derive from the VELA simulations the fiducial mass profiles of pre-compaction and post-compaction satellites below and above the golden mass. In Section 3, we use a toy model to make analytic estimates of the DF heating as a function of satellite compactness. In Section 4, we utilize SatGen semi-analytic simulations to explore the DF heating as a function of satellite compactness and orbit, as well as for a cosmological sequence of satellites. In Section 5, we address the post-heating relaxation of the cusp, cooling, and expansion, and use simplified $N$-body simulations to study the interplay between DF heating and density flattening and evaluate the cooling timescale. In Section 6, we use the CuspCore analytic model to study the response of the pre-heated haloes to gas ejection by AGN feedback. In Section 7, we discuss our results and refer to the prospects of reproducing them in cosmological simulations. In Section 8, we summarize our conclusions. In Appendices A–J (available as supplementary material online), we bring supportive descriptions of the analytic models and simulations used and add figures relevant to cases that are complementary to our fiducial cases.

## 2 COMPACT SATELLITES ABOVE A THRESHOLD MASS IN COSMOLOGICAL SIMULATIONS

A key for the strength of the cusp heating by DF is the initial compactness of the merging satellite, which determines the mass with which it penetrates into the host-halo cusp. A mass threshold for compact satellites would be translated to a host-halo mass threshold for the formation of extended cores in the hosts. Indeed, cosmological simulations reveal that most galaxies undergo a major event of wet compaction to a ‘blue nugget’ when they are near a golden halo mass of $m_\odot \sim 10^{11.3}M_\odot$ (Zolotov et al. 2015). Following Tacchella et al. (2016b), we use galaxies from the VELA zoom-in cosmological simulations (briefly described in Appendix B, supplementary material online), in order to derive the mass profiles of haloes pre-compaction versus post-compaction, namely below and above the golden mass, respectively (Zolotov et al. 2015; Tomassetti et al. 2016; Huertas-Company et al. 2018). These will serve us in...
the merging satellites as a function of their mass.

Fig. 1 shows the evolution of mass profiles for four representative simulated VELA galaxies, at a sequence of output times before and after the major compaction events, which occur in different galaxies at different redshifts but typically when the halo mass is near $M_\star \approx 10^{11.3} M_\odot$. The figure shows the profiles of the total mass, consisting of dark matter, stars, and gas, in comparison with the mass profiles of the DM alone. In both cases, we see rather universal profiles both before and after the compaction events, showing that the overall compactness is growing significantly during the wet compaction process, both for the mass profiles.

In order to quantify the universal mass profiles, relevant here in particular to the merging satellites, we use the Dekel–Zhao profile (Dekel et al. 2017; Freundlich et al. 2020b, DZ)\(^1\) (summarized in Appendix A, supplementary material online). This is a two-parameter functional form, with a flexible inner slope, and with analytic expressions for the profiles of density, mass, and velocity as well as potential and kinetic energies (and lensing properties). It has been shown to fit DM haloes in the NHI AO cosmological simulations with baryons better than the other commonly used two-parameter profiles (such as the generalized NFW and the Einasto profiles). This is being confirmed in the Auriga, Apostle, and EAGLE simulations (Marius Cautin, private communication). In the DZ profile, the mean density contrast of the haloes with respect to the cosmological background in the Einstein–deSitter (EdS) cosmological regime (approximately valid at $z > 1$) is $\bar{\rho}_v/\rho_0 \sim 200$, and by definition $\bar{\rho}_v$ is the same for the host and for the satellites when they were still isolated. To be used below, the associated mass encompassed within a sphere of radius $r$ is

\[
M(r) = \frac{1}{\bar{\rho}_v} \int_0^r \rho(x) \, dV = \frac{1}{\bar{\rho}_v} \int_0^r \frac{\rho(x)}{x^2} \, x^3 \, dx = \frac{\mu}{\bar{\rho}_v} \, \frac{M_{\star}}{(4\pi/3) R_{\star}^3},
\]

where $\mu$ is the mass-to-light ratio.

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\]

and the log slope of the mass profile is

\[
\nu(r) = \frac{d}{d \log r} \left( \frac{\rho(r)}{\bar{\rho}_v} \right) = \frac{3 - \alpha}{1 + x^{1/2}}.
\]

We fit a Dekel–Zhao profile with the free parameters $(c, \alpha)$ to each of the VELA-simulated mass profiles, separately for the DM and the total mass. The best fit is obtained by minimizing residuals in equally spaced log bins in the range $(0.01 - 1)R_\star$ and $(0.03 - 1)R_\star$ for the DM and total mass, respectively, assuming that the most relevant mass profile is near $(0.03 - 0.1)R_\star$. Fig. 2 shows the median and standard deviation for the two profile parameters over all the 34

\[1\text{Available for implementation in https://github.com/JonathanFreundlich/Dekel_profile.} \]
simulated galaxies as a function of time, via the expansion factor $a = (1 + z)^{-1}$, with respect to the time of the blue-nugget peak at the end of the wet compaction process. Based on this figure, we adopt for the fiducial pre-compaction DM profile the DZ parameters $(c, \alpha) = (3, 0.5)$, and for the fiducial post-compaction profile $(c, \alpha) = (7, 1)$. Other choices, relevant at different times before or after the compaction, make only little differences to the effective diffuseness and compactness of the mass distribution in these two distinct phases of evolution. For the total mass, we adopt $(c, \alpha) = (5, 0.5)$ and $(3, 1.8)$ for the diffuse and compact satellites, respectively.

While the DZ best-fitting parameters are quite different for the DM and for the total mass profiles, the actual mass profiles turn out not to be that different. The diffuse haloes roughly match an NFW profile with $c_{\text{NFW}} \approx 5$, with the total mass about 50 per cent higher than the DM mass interior to $0.067 R_{\text{v}}$. The compact haloes, interior to the same radius, have more mass than the diffuse haloes by a factor of a few, with the total mass twice as large as the DM mass. We find below that the effect of using the DM mass in the merging satellites is not very different from the effect of using the total mass. In order for our analysis to be on the conservative side in terms of the DF heating, we adopt as our fiducial satellite profile the typical profile of the dark matter alone.

As described in equations (A12)–(A15) of Appendix A (supplementary material online), based on equations (11)–(14) of Freundlich et al. (2020b), an alternative, sometimes more accessible pair of shape parameters is the concentration $c_2$, referring to the radius where the log local density slope is $-2$ (as in the NFW profile), and the negative log local density slope $s_1$ at some small radius $r_1$ (e.g. at 1 per cent of the virial radius). While there is a valid DZ profile for any values of $c (>0)$ and $\alpha (<3)$, a valid profile is not guaranteed for all arbitrary values of $c_2$ and $s_1$. For example, $c_2$ is not defined for an isothermal sphere or a steeper density profile.

The alternative DZ parameters corresponding to the universal profiles are $(c_2, s_1) = (3, 0.94)$ and $(15.6, 1.52)$ for the fiducial diffuse and compact satellite haloes, respectively, using the DM mass alone. The inner slopes $s_1$ are, respectively, somewhat flatter and steeper than an NFW cusp (which has $s_1 = 1.06$ and $1.27$ for $c_{\text{NFW}} = 3$ and $15.6$, respectively), with a smaller and larger concentration compared to the $c_{\text{NFW}} \approx 5$ that fits typical simulated haloes in the relevant mass range at $z \sim 2$. For the total mass, the alternative DZ parameters are $(c_2, s_1) = (4.5, 1.31)$ and $(100, 2.2)$, the latter indicating that the slope is steeper than two in the whole relevant radius range.

The above implies that deeper satellite penetration, which is associated with more significant host-cusp heating by DF, is expected once the host halo is more massive than $M_\ast \sim 10^{12} M_\odot$, such that a significant fraction of the accreting mass is in post-compaction nuggets above $m_\ast \sim 10^{11.3} M_\odot$. On the other hand, much less penetration and heating is expected for hosts of $M_\ast < 10^{12} M_\odot$, where most of the merging satellites tend to be pre-compaction. This will be studied next.

### 3 Dynamical Friction Heating by Satellites: A Toy Model

We start with a simple analytic toy model in order to obtain a crude estimate for the energy deposited in the host-halo cusp as a result of the DF exerted on penetrating satellites as a function of their compactness. These estimates will be improved in Section 4 using semi-analytic simulations.

#### 3.1 Satellite penetration – tidal stripping

In order to evaluate the energy deposited in the host cusp by DF, we should first estimate the satellite mass as it penetrates the cusp after it had suffered tidal stripping along its orbit. We first evaluate the mass within the satellite stripping radius when the satellite is at the cusp half-mass radius, crudely assuming an onion-shell outside-in stripping outside the stripping radius. When doing so, we neglect changes in the density and kinematics inside the tidal radius (crudely referred to as ‘tidal heating’) that may affect the stripping. These are incorporated in the treatment by SatGen in Section 4. We then apply a simple correction to the mass within the tidal radius, to be calibrated by simulations, in order to take into account effects such as the deviations of the tidal force from spherical symmetry about the satellite and for the fact that the stripping is not instantaneous but rather takes a dynamical time.

##### 3.1.1 The tidal radius

The tidal radius (King 1962), where self-gravity balances the tidal and centrifugal forces along the line connecting the centres of host and satellite, obeys the equation

$$\tilde{\rho}_{\text{ad}}(\tilde{\ell}_s) = \tilde{\rho}(r) \left[ 2 - v(r) + e^2(r) \right],$$

Figure 2. Dekel–Zhao profile fits to the DM mass profiles in all 34 VELA simulated galaxies. Shown are the median and standard deviation of the parameters $c$ and $\alpha$ as a function of time (expansion factor $a = (1 + z)^{-1}$) with respect to the time of the blue-nugget peak at the end of the major compaction. This quantity is a proxy for mass, below and above a halo mass of $m_\ast = 10^{11.3} M_\odot$. We see different typical profiles well before and after the compaction event, namely below and above the critical mass. Left: for the DM mass profiles, where we crudely adopt $(c, \alpha) = (3, 0.5)$ and $(7, 1)$ as our fiducial profiles pre-compaction and post-compaction, respectively. Right: for the total mass profiles, where we crudely adopt $(c, \alpha) = (5, 0.5)$ and $(3, 1.8)$, respectively. The corresponding profiles are shown in Fig. 1.
where $\tilde{\rho}_{\text{sat}}(r)$ and $\tilde{\rho}(r)$ are the mean density profiles of the satellite (before it entered the host virial radius $R_v$) and the host, respectively. Here, $v(r)$ is the local log slope of the host mass profile $M(r)$, namely $v(r) = \frac{\text{dln} M}{\text{dln} r}$. It is $v(r) = 3 - \alpha(r)$, where $\alpha(r)$ is minus the local log slope of $\tilde{\rho}(r)$. For example, $v = 3$, 2, and 1 for a flat core, an NFW cusp, and an isothermal sphere for which $\alpha = 0$, 1, and 2, respectively.

The quantity $\epsilon(r)$, representing the centrifugal force that helps stripping, is the local circularity of the satellite orbit at $r$, namely the ratio of tangential to circular velocity

$$\epsilon(r) = \frac{V_{\text{tan}}(r)}{V_{\text{circ}}(r)},$$

with $\epsilon = 0$ and 1 for local radial and circular orbits, respectively. This local circularity is typically not the same as its value at $R_v$, upon entry to the halo, which commonly serves as one of the two parameters characterizing the orbit. The value of the local $\epsilon$ may vary as the satellite is orbiting within the host halo, and we do not know a priori its effective range of values. We therefore use $\epsilon = 0.5$ as a fiducial value, recalling that it could range from zero to above unity.

3.1.2 The penetrating satellite mass

We use for the satellites the Dekel–Zhao profile, equation (1). As shown in equation (A11) of Appendix A (supplementary material online), the fraction $f = m_M/m_{\text{sat}}$ of satellite mass within a sphere of radius $\ell$ about the satellite centre with respect to the initial total satellite mass obeys

$$(f/\mu)[(f/\mu)^{-12(3-\alpha)} - 1]^6 = \tilde{\rho}_{\text{sat}}(\ell)/\tilde{\rho}_{\text{sat},c}.$$ (7)

Using the tidal condition, equation (6), we obtain an equation for the mass fraction $f(\ell)$ within the tidal radius,

$$\left(\frac{\ell}{\mu}\right)^{12(3-\alpha)-1} = \frac{[2v(r) + \epsilon^2(r)]\tilde{\rho}(r)}{\mu c^3 \tilde{\rho}_c}. $$ (8)

Recall that $\mu$ is a function of the satellite parameters $c$ and $\alpha$. For a given host mass profile $M(r)$, and its log slope $v(r)$, and for a given satellite orbit circularity $\epsilon(r)$, this equation can be solved numerically for $f(\ell, \epsilon)$. We note that the solution depends only on the profiles, and it does not depend explicitly on the total masses or the ratio $m_{\text{sat}}/M_h$.

One can see that for a given $\alpha$ (<3), in the limit $c \to \infty$, where $\mu \to 1$, the equation yields $f_{\text{sat}} \to 1$. Similarly, for a given $c$, in the limit $\alpha \to 3$ (from below), again $\mu \to 1$ and $f_{\text{sat}} \to 1$. Thus, the mass fraction within the tidal radius is increasing with either $c$ or $\alpha$, approaching no stripping as $c \to \infty$ or $\alpha \to 3$.

Analytic solutions of equation (9) can be obtained for certain values of $\alpha$, e.g. there is a simple explicit solution for $\alpha = 0$, and solutions for $\alpha = 1$ and $\alpha = 2$ via solutions of cubic polynomial equations.

The actual bound satellite mass at $r$, for the purpose of DF, may deviate from the mass within the tidal radius. Among other effects, this is because the tidal stripping is not spherical symmetric, because it is not instantaneous, occurring over a dynamical time-scale at $r$, and because of the effects on the inner satellite. For example, Green, van den Bosch & Jiang (2021) assumed that the actual mass stripping rate is crudely modelled as $\dot{m} = A \text{m} / t_{\text{dyn}}$, where $t_{\text{dyn}}$ is approximated by the dynamical time of the halo at $r$, and found a best fit to N-body simulations with $A \approx 0.55$. We can therefore consider the mass within the tidal radius to be an underestimate of the actual satellite mass, possibly by a factor of ~2 or more at the relevant radii within the cusp if the modelling above is valid. We crudely model this correction in the cusp as

$$f = B f_1.$$ (10)

The value of $B$ can be calibrated by matching the results of the toy model to the SatGen simulations described in Section 4, where any evolution of the inner satellite is included. We adopt $B = 2$ as our fiducial value, but this choice has no qualitative effect on the results.

3.1.3 The satellite mass in a given host cusp

The host halo is also described by a Dekel–Zhao profile, equation (1) and equation (A1) (supplementary material online), with parameters $(C_h, \alpha_h)$. The log slope of the mass profile, $v(r)$, to be used in equation (9), is given for the DF profile by equation (5). We evaluate $\tilde{\rho}(r)$ and $v(r)$ at the half-mass radius of the cusp of radius $R_c = 10$ kpc, which is roughly $R_v = 7$ kpc, and insert them in equation (9) in order to solve for the satellite mass fraction there.

Our fiducial case is an initial host halo of $M_h = 10^{12.5} M_\odot$ at $z = 2$ (with virial radius and velocity $R_v = 150$ kpc and $V_v = 300$ km s$^{-1}$). Such a halo, if unperturbed by baryons, is well fit by an NFW profile, equation (A21) (supplementary material online), Navarro et al. (1997), with a moderately steep cusp, and an NFW concentration $c_{\text{NFW}} = 5$ (Bullock et al. 2001). This NFW profile is matched best across $r = (0.01 - 1) R_v$ by a DF profile of $(c_{\text{DF}}$, $s_{\text{DF}}) = (5.04, 0.908)$, or $(C_h$, $\alpha_h) = (7.13, 0.216)$. In this case, we neglect earlier baryonic effects on the DF halo, either steepening by adiabatic contraction or partial flattening by outflows.

We note that the assumed `cusp radius' of $R_c = 10$ kpc is rather crude, motivated by the desired `core radius' based on the observational estimates. Just for reference, in the fiducial NFW host, the local negative log slope of the density profile at this radius is $\simeq 1.5$. As an alternative case with an initially steeper cusp, we use $(C_h$, $\alpha_h) = (1.13, 1.29)$ or $(c_{\text{DF}}$, $s_{\text{DF}}) = (5.04, 1.50)$. This steep-cusp profile is typical to haloes of $M_h \geq 10^{12} M_\odot$ from the NIHAO cosmological simulations with baryons, as analyzed in Freundlich et al. (2020b). We will mention the results for the steep-cusp halo below, but the figures referring to this case are presented in Appendix G (supplementary material online).

The left-hand panel of Fig. 3 shows the bound mass fraction $f$ as a function of the DF parameters $(c, \alpha)$ for the satellite profile, as obtained by solving equation (9) [with the correction of equation (10) assuming $B = 2$]. This is shown here for the fiducial NFW host and in Fig. G1 (supplementary material online) for the steep-cusp host.

As described in Section 2, we learn from the VELA simulations that the DM profiles of typical diffuse and compact haloes, below and above $m_{\text{c}} \sim 10^{11.3} M_\odot$, are well fit by a DF profile with $(c, \alpha) = (3, 0.5)$ and $(c, \alpha) = (7, 1)$, respectively. These parameters for the satellites are marked in the figure. Also marked are the parameters for the total mass profiles, $(c, \alpha) = (5, 0.5)$ and $(3.18)$, for the diffuse and compact satellites, respectively.

In these simulations, the satellite-bound mass is defined using an iterative unbinding algorithm (van den Bosch & Ogiya 2018), which considers the energy in the satellite frame, with the gravitational potential energy involving satellite particles only.

2The very inner slope at 0.01$R_v$, $s_{\text{DF}}$, is apparently slightly smaller than unity, but the fit is overall good across the cusp and in the most relevant region of $r \leq 0.1 R_v$. Different DF fits, that enforce a perfect fit at 0.01$R_v$, and have an almost similar overall quality, are possible, but they do not significantly affect the results.
For the NFW host halo, with the fiducial DM satellite profiles, we obtain $f = \frac{m_{\text{sat}}}{m_{\text{host}}} \approx 0.38$ and $0.05$ for the compact and diffuse satellites, respectively. Thus, the fiducial massive, post-compaction satellite penetrates into the cusp with a significant fraction of its original mass, which promises significant cusp heating. On the other hand, the fiducial low-mass, pre-compaction satellite is more significantly stripped and it penetrates to the halo cusp with a small fraction of its mass, such that it is not expected to provide much cusp heating.

For the NFW halo but with the somewhat more compact total satellite profiles, we obtain $f = \frac{m_{\text{sat}}}{m_{\text{host}}} \approx 0.74$ and $0.12$. This is a deeper penetration, as expected from the larger satellite compactness due to the baryons, but the qualitative difference between the compact and diffuse satellites remains the same.

For the steep-cusp halo, we obtain $f \approx 0.23$ and $0.01$ for the fiducial compact and diffuse satellites, respectively. Here, as expected from the steeper cusp, the satellites penetrate to the cusp with a somewhat smaller fraction of their mass. Nevertheless, the penetration of the compact satellite to the steep cusp is still with a noticeable fraction of its original mass, so it still has the potential to provide significant heating, to be estimated next.

3.2 Energy deposited in the host cusp

Given the penetrating mass fraction $f$, we wish to estimate the energy deposited in the host cusp by DF, with respect to the kinetic energy within this cusp. This will tell whether DF is capable of significantly heating the cusp, and possibly partly flatten the density profile, in preparation for core formation by AGN feedback.

### 3.2.1 A single satellite

We crudely approximate the work done by DF on a satellite within the cusp by

$$ W_c = s R_c F, $$

where $F$ is the typical DF force exerted by the dark matter on the satellite in the cusp, $R_c$ is the characteristic cusp radius, encompassing mass $M_c$, and $s \approx 1$ is a factor that characterizes the effective length of the path of the satellite inside the cusp. The DF force is approximated by the Chandrasekhar formula (Chandrasekhar 1943) (as long as the cusp has not flattened to a core),

$$ F = 4\pi \ln \Lambda G^2 \frac{\rho m^2}{V^2} H(u), $$

where $\rho$ is the typical DM density within the cusp, $m$ is the bound satellite mass when in the cusp, $V$ is the satellite orbital velocity, and $\ln \Lambda$ is the Coulomb logarithm. The latter is commonly assumed to be approximately $\ln (M_c/m) \sim 3$. The correction factor $H(u)$ that limits the effect to particles that move slower than the satellite, assuming a Maxwellian distribution of velocities, is

$$ H(u) = \text{erf}(u) - \frac{2u}{\sqrt{\pi}} e^{-u^2}, \quad u = \frac{V}{\sqrt{2} \sigma}, $$
where \( \sigma \) is the velocity dispersion in the cusp. For example, \( H(\mu = 0.55) = 0.1 \) and \( H(\mu = 0.7) = 0.2 \).

We note that the use of the Chandrasekhar formula involves certain inaccuracies, beyond the uncertainties in the values adopted for the parameters, which could either strengthen or weaken the force. One is an uncertainty in the satellite mass that is relevant for the purpose.

For example, when the mass is measured in a given way, Green et al. (2021) indicate that a multiplicative factor \( \beta \approx 0.75 \) in equation (12) yields best fit to results deduced from the Bolshoi cosmological N-body simulation. This should be better calibrated using carefully designed simulations where the DF is studied in more detail. On the other hand, self-friction within the satellite is ignored, and so is the binding effect of the baryons within the satellite. Dynamical friction is known to be significantly reduced when the satellite is orbiting in a very flat host core, but here we limit ourselves to affecting a steep cusp in its development towards a somewhat flatter cusp. In summary, our toy-model treatment of DF should be considered as a crude approximation only, good for an order-of-magnitude estimate.

Adopting the mean density within \( R_c \) and the circular velocity at \( R_c, \rho = (4\pi/3)M_c/R_c^3 \) and \( V_c = (GM/R_c)^{1/2} \), we get

\[
W_c = 1.8 s_2 (\ln \Lambda_1) H(u,0.1) \frac{Gm^2}{R_c},
\]

where the subscripts denote the fiducial values assumed for the different quantities in this expression (\( s_2 = s/2 \) etc.).

The DF work estimated in equation (14) is to be compared to the kinetic energy in the cusp, \( K_c = (1/2) M_c \sigma^2 \). We assume for the host \( M_c = 10^{12} M_\odot \), whose virial radius is \( R_c = 150 \text{kpc} M_{12.5}^{1/3} (1+z)_0^{-1} \), in which \( M_{12.5} = M/10^{12} M_\odot \) and \( (1+z)_0 = (1+z)/3 \). For an NFW profile with \( c_{\text{NFW}} = 5 \) at \( z = 2 \), a cusp of \( R_c = 10 \text{kpc} \) corresponds to \( r = 0.067 R_c \) and \( x_c = 0.33 \), so from equation (A21) (supplementary material online), the cusp mass is \( M_c = 0.04 M_* \). This implies that \( V_c \sim 0.77 V_\odot \). We read from Fig. 1 of Freundlich et al. (2020b) that at \( r = 0.067 R_c \) we have \( (1+z)_0 \simeq V_c \) (and that indeed \( V_c \geq 0.78 V_\odot \)). These imply for the NFW host that

\[
K_c \simeq 0.5 M_* V_c^2 \simeq 0.83 M_* V_c^2 \simeq 6 \times 10^{15} M_\odot \text{kpc}^2 \text{Gyr}^{-2}. \tag{15}
\]

We also learn that \( u \simeq 0.54 \), such that \( H(u) \simeq 0.1 \) is a reasonable estimate. For the steep-core host, we find that \( K_c \) is 1.25 times larger.

Using equation (14) and equation (15), we obtain

\[
W_c/K_c = 2.2 s_2 (\ln \Lambda_1) H(u,0.1) \frac{m^2}{M_*^2} \tag{16}
\]

Thus, for a single satellite of initial mass \( m_c \) in the NFW host of mass \( M_* \), we obtain

\[
W_c/K_c = 0.55 s_2 f_{0.2} \left( \frac{m_c}{M_*} \right)^2 \tag{17}
\]

The value of \( s \) here absorbs the uncertainties in the three parameters \( s, \ln \Lambda_1 \), and \( H(u) \). Here, \( f = 0.2 f_{0.2} \) to be obtained, e.g. by solving equation (9), given the profile shapes of the halo and the satellite, with the possible correction of equation (10). Note that \( f \) in this model is independent of \( m_c/M_* \). We learn that a single satellite of \( m_c \geq 0.13 M_* \) may be capable of heating the cusp, with \( W_c/K_c \geq K_c \), once more than 0.2 of its mass survives tidal stripping before entering the cusp. For the steep-core host, the numerical factor in equation (17) is 1.25 times lower because of the higher \( K_c \).

The middle column of Fig. 3 shows the toy-model estimates of \( W_c/K_c \), based on equation (17), for a single satellite of \( m_c = 0.1 M_* \) as a function of the satellite’s DZ profile parameters \((c, \alpha)\), and for the NFW host, assuming \( s = 2 \) (and \( \epsilon = 0.5 \) and \( B = 2 \) as before). The same for the steep-cusp host is shown in Fig. G1 (supplementary material online). The contours \( W_c/K_c \simeq 1 \) mark the boundaries for significant cusp heating. The fiducial diffuse and compact satellites, as evaluated from the DM in the VELA simulations, Section 2, are marked by open and filled circles at \((c, \alpha) = (3, 0.5) \) and \((7, 1) \), respectively. We read for the NFW host \( W_c/K_c \simeq 2.05 \) and 0.03 for the fiducial compact and diffuse satellite, respectively. For the fits to the more compact total satellite mass, we obtain \( W_c/K_c \simeq 7.5 \) and 0.2, respectively. This is more heating, but with a similar difference between the compact and diffuse satellites. For the steep-cusp host, and the DM satellite profiles, the respective results are \( W_c/K_c \simeq 0.56 \) and 0.001, namely less heating, as expected. Overall, we learn quite robustly that a single compact satellite of \( m_c \sim 0.1 M_* \) is clearly capable of significantly heating up the cusp in an NFW host and is expected to do so also in a steep-cusp host. In contrast, a fiducial diffuse satellite is expected to have only a negligible effect on the host cusp in the two types of hosts, even if the satellite compactness is evaluated via its total mass profile. According to equation (17), the energy deposited by a massive satellite, \( m_c = 0.1 \mu M_* \), is expected to provide more heating by a factor \( \mu^2 \).

A note of caution is that the deposited kinetic energy may not all be in the form of ‘heat’, e.g. it could partly be in the form of bulk motion and in particular rotational energy. In this case, the analysis should involve angular momentum, both in the estimation of the effect of DF on the DM cusp and in the evaluation of the DM response to outflows (Section 6). For satellite entry to the cusp on a rather tangential orbit, it can be shown that the angular momentum deposit by DF with respect to the maximum angular momentum of the cusp, had it been a disc with circular orbits, is comparable to the relative energy deposit of equation (17). This implies a likely significant effect of the compact satellite on the angular momentum. We defer an analysis involving angular momentum to future work.

### 3.2.2 A cosmological sequence of satellites

For a cosmological sequence of satellites, we assume that all satellites of mass \( m_s > 10^{11} M_\odot \) are post-compaction (Zolotov et al. 2015; Tomassetti et al. 2016), with a high concentration and a cusp (Section 2), which make them penetrate to the host cusp with a non-negligible mass fraction \( f \). For a host halo of \( M_c = 10^{12} M_\odot \), these are mergers of mass ratio \( m_s/M_* \geq 1:10 \). We read from fig. 7 of Neistein & Dekel (2008), based on EPS theory, that \( \sim 60 \) per cent of the total accretion rate is in such satellites (at all redshifts). For the average total accretion rate, we adopt the analytic estimate of Dekel et al. (2013) in the EdS cosmological regime (a good approximation at \( z > 1 \)),

\[
M = 0.47 \text{Gyr}^{-1} M_* (1+z)^{5/2}. \tag{18}
\]

The rate of total energy deposited in the cusp by DF involving all these satellites can be computed by integration over their Extended Press-Schechter (EPS) conditional satellite mass function normalized to a total that equals \( 0.6M_* \) convolved with the energy per satellite as given by equation (17). The mass function of first-order subhaloes at infall could be approximated by \( \nu dm/dm \sim m^{-1.5} \) at \( m \simeq 0.02 M_* \) with a truncation at \( m_{\text{max}} \simeq 0.5 M_* \) (Jiang & van den Bosch 2014). For a crude conservative estimate, we assume here that each of the relevant satellites above \( 0.1 M_* \) is of a given mass \( m_* \), somewhat above \( 0.1 M_* \) and write the accreted mass deposit in these satellites as \( N m_* = 0.6 M_* \), where \( N \) is the number of relevant satellites that
merge during 1 Gigayear. This gives
\[ N = 1.4 \text{ Gyr}^{-1} \left( \frac{m_i}{M_h} \right) \left( 1 + z \right)^{3/2}. \]  
(19)

Multiplying the result for a single satellite from equation (17) by \( N \), we obtain an approximation for the rate of relative work done on the host cusp by all the compact satellites
\[ \frac{W}{K_c} = 3 \text{ Gyr}^{-1} f_{1.2}^2 s_2 \left( \frac{m_i}{M_h} \right) \left( 1 + z \right)^{3/2}. \]  
(20)

In t halo virial crossing times, each given by \( t_c \approx 0.5 \text{ Gyr} \left( 1 + z \right)^{3/2} \) (Dekel et al. 2013), the relative DF work becomes
\[ \frac{W_c(\Delta t = t_c)}{K_c} = 1.5 \tau f_{1.2}^2 \left( \frac{m_i}{M_h} \right) \left( 1 + z \right)^{3}. \]  
(21)

Based on fig. 7 of Neistein & Dekel (2008), we adopt \( m_i/M_h = 0.2 \) as our fiducial value.

An upper limit for the total energy deposited could be estimated by integrating over a whole Hubble time, \( \tau \approx 6.6 t_c \approx 3.3 \text{ Gyr} \left( 1 + z \right)^{3/2} \), namely about a factor of 6.6 times the energy in equation (21). During a time window of 0.5, the time-scale for the ‘heat’ deposited by DF to ‘dissipate’ as estimated below in Section 5.2 from an N-body simulation, we estimate \( W/K_c \approx 0.75 \left( 1 + z \right). \)

The right column of Fig. 3 shows the toy-model estimates for \( W_c/K_c \) for a cosmological sequence of satellites as a function of the satellite’s profile parameters \( (c, \alpha) \), for the NFW host, based on equation (21). This is assuming \( m_i = 0.2 M_h \), at \( z = 2 \), and a duration of one virial time \( \sim 0.5 \text{ Gyr} \), namely \( \tau = 1 \) [with \( s = 2, B = 2, \) and \( \epsilon = 0.5 \) when solving for \( f_i \) in equation (9), as before]. The same for the steep-cusp host is shown in Fig. G1 (supplementary material online). We read for the NFW host, using the satellite DM mass profiles, \( W_c/K_c \approx 5.6 \tau \) and 0.08 for the fiducial compact and diffuse satellite, respectively. When using the satellite total mass, the results are \( W_c/K_c \approx 20 \tau \) and 0.5, \( \tau \). For the steep-cusp host, with the satellite DM profiles, the respective results are \( W_c/K_c \approx 1.5 \tau \) and 0.003 \( \tau \). We learn that the median energy deposited by a sequence of satellites in one virial time is crudely estimated to be comparable to and slightly larger than the energy deposited by a single satellite of \( m_i \sim 0.1 M_h \). During a period of \( \sim 0.5 \text{ Gyr} \), the satellites are expected to significantly heat up the cusp in an NFW host, and they are estimated to do so also in a steep-cusp host. In contrast, a sequence of diffuse satellites is expected to have only a negligible effect on the host cusp for the two types of hosts during a period of order a virial time.

Based on the proportionality to \((1 + z)\) in equation (21), a somewhat higher satellite compactness is needed for heating the cusp to the same level over a similar period at lower redshifts, while a somewhat lower compactness would be sufficient at higher redshifts.

Fig. 11 (supplementary material online) complements Fig. 3 by showing the same toy-model estimates for satellite penetration and the energy deposited in the host cusp by DF but in the plane of the alternative DZ parameters \((c_2, s_1)\) instead of the natural DZ parameters \((c, \alpha)\).

We note that, with a satellite initial mass function that declines flatter than \( m_i^{−2} \) up to a drop at \( m_{\text{max}} \), the dependence on \( m_i^2 \) of the energy deposited in the cusp, equation (17), indicates that the total heating of the cusp is dominated by the more massive satellites. One could comment that this dominance of massive satellites may be strengthened because, beyond the crude tidal-stripping approximation of equation (6), the actual satellite mass loss also depends on the time the satellite spends in the strong-tide regime before entering the cusp. In particular, satellites that make it to the cusp in the first or second pericentre of their orbit are expected to bring in more mass than satellites that hang around and are subject to stripping for a longer time (see Fig. 4 below). One can estimate that, for a given satellite density profile, the rate of decay of the satellite orbit is increasing with \( m_i/M_h \). This can be obtained by relating the decay rate to either the ratio of AM loss by the DF torque to the orbital AM of the satellite or the ratio of work done on the satellite to its energy. These ratios are both proportional to \( m/M \), because the DF acceleration is proportional to \( m \), while the specific angular momentum or energy is proportional to \( M \), and the tidal stripping is not strongly dependent on either.

### 4 Dynamical Friction Heating: Semi-Analytic SatGen Simulations

In order to explore more accurately the satellite properties required for efficient penetration and strong DM heating by DF, we utilize the dedicated Semi-Analytic Satellite Generator, SatGen (Jiang et al. 2020),\(^5\) (described in Appendix C, supplementary material online). It simulates tidal stripping while the satellite of an assumed initial density profile orbits in the potential well of the host halo and the central galaxy and penetrates to the inner halo subject to DF by the host dark matter. The code utilizes EPS merger trees and physical prescriptions for the relevant mechanisms. The tidal mass loss and the structural evolution of the satellites depend on the initial structure of the satellites, following Errani, Peñarrubia & Walker (2018), such that a dense, cuspy satellite at infall is more resistant to tidal disruption than a cored satellite. In the version used here, we model the satellite with non-dissipative particles, without explicitly treating the gas component.\(^6\)

The energy deposited in the radius shell \((r \pm dr)\) of the host is computed either by the work done by DF on the satellite along its orbit or by the orbital energy loss of the satellite, with similar results. Dynamical friction is modelled using equation (12) with \( \ln \Lambda = \ln \left( M_c/m_i \right) \). In both cases, rather crudely, the energy is assumed to be deposited locally.

#### 4.1 A single satellite

Fig. 4 summarizes the results of running SatGen with a single satellite, our fiducial compact or diffuse, penetrating into an NFW host of \( c_{\text{NFW}} = 5 \). The DZ mass profile parameters of the initial satellites at the host virial radius are the fiducial \((c, \alpha) = (7, 1)\) and \((3,0.5)\), respectively, consistent with the DM VELA profiles after and before the compaction in Figs 1 and 2. The initial satellite mass at \( R_c \) is \( m_i = 10^{11.5} M_\odot \) in a host of \( M_c = 10^{12.5} M_\odot \). The orbit has a typical circularity at \( R_c \) of \( \epsilon = 0.4 \).

The orbit (top) shows that the compact satellite enters the 10 kpc cusp near its second pericentre, while the diffuse satellite enters only near its third pericentre. The satellite bound mass fraction at the half-mass radius of the core \((7 \text{ kpc})\) is \( m/m_i \approx 0.32 \) and 0.09, respectively. These are to be compared to the toy model predictions of \( m/m_i \approx 0.39 \) and 0.046. The energy deposited in the cusp is \( W_c/K_c \approx 2.0 \) and 0.32 for the compact and diffuse satellite, respectively. In comparison, substituting the mass fractions from SatGen in equation (17) of the toy model yields \( W_c/K_c \approx 2.05 \) and 0.029, emphasizing the qualitative

\(^5\)Available for implementation in https://github.com/shergreen/SatGen.

\(^6\)We expect this to underestimate the heating of DM by dynamical friction, as the baryonic component of each satellite is usually more compact than the DM component. Furthermore, the inflowing baryonic clumps, that may form, e.g. in the inflowing cold streams and in tidal tails, are expected to add to the dynamical friction heating and they are not included in our current analysis.
difference between the effects of the compact and diffuse satellites. The semi-analytic simulation thus confirms the qualitative estimates from the toy model that the fiducial compact satellite is expected to significantly heat up the host cusp, while the fiducial diffuse satellite is not expected to generate major heating.

In order to evaluate the effect of including the central baryons in the satellites, we re-ran SatGen with our fiducial NFW host halo but with the satellites following the more compact DZ-profile fits to the VELA simulated galaxies using the total mass rather than the dark matter alone. The DZ profiles are here \((c, \alpha) = (5, 0.5)\) and \((3, 1.8)\) for the diffuse and compact satellites, respectively, following the VELA pre- and post-compaction galaxies from Fig. 1. We learn from Fig. F1 (supplementary material online), which summarizes this SatGen run compared to the fiducial case of Fig. 4, that the more compact satellites indeed penetrate to the host cusp with a higher mass and deposit more energy there accordingly, as expected. However, the difference is rather small, with \(m/m_v \simeq 0.5\) compared to 0.4, and \(W_c/K_c \simeq 2.5\) compared to 2, for the compact satellites. This is a smaller difference than expected based on the toy model of Section 3. These results indicate that our main analysis using the fiducial satellite profiles as derived from the DM mass in the VELA simulated galaxies should provide good, conservative estimates for the cusp heating by DF.

In order to evaluate the effect of a steep-cusp host halo, we re-ran SatGen with the fiducial DM satellite profiles but with the steep-cusp host instead of the fiducial NFW. We read from Fig. G2...
(supplementary material online) that for the steep-cusp host the same satellites penetrate with \(m/v \approx 0.30\) and 0.075 for the compact and diffuse satellite, respectively. These values are only slightly smaller than for the NFW host, probably because the two host haloes are not very different outside the cusp. The satellite mass becomes significantly smaller only in the inner cusp, inside \(\sim 5\ \text{kpc}\), where the tidal stripping is more efficient due to the higher density of the steep-cusp halo. The corresponding toy-model estimates are \(m/v \approx 0.23\) and 0.01. The energy deposited in the steep cusp according to SatGen is \(W/K_{c} \approx 2.0\) and 0.32. These values are rather similar to those in the NFW host. Apparently, the lower satellite mass that has penetrated into the cusp is balanced by the higher density in the cusp, making the work by DF almost the same in the two cases. The toy-model estimates are \(W/K_{c} \approx 0.56\) and 0.001, indicating a stronger dependence on the host-cusp properties than is actually produced by SatGen, but emphasizing again that the effects of the compact and diffuse satellites are very different, and that the compact satellite is expected to provide non-negligible heating also in the steep-cusp halo.

4.2 A cosmological sequence of merging satellites

We next run SatGen with a cosmological distribution of satellites over time \(\tau_{c}\), where \(\tau \approx 1\), from \(z = 2.3-2\). We actually generate random merger trees for a target halo of a given mass at \(z = 2\) starting at \(z = 20\), evolve these cosmological sequences of satellites in time, and keep track of the DF heating induced by them over the last virial time at each given halo-centric radius. The satellite masses are drawn from the EPS conditional mass function (e.g. Lacey & Cole 1993; Neistein & Dekel 2008; Parkinson, Cole & Helly 2008; Benson 2017). We assume that the orbital energy at infall through \(R_{c}\) is the same as that of a circular orbit of radius \(R_{c}(t)\) and draw the circularity \(\epsilon\) at random from the distribution \(d\epsilon/\epsilon = \pi \sin(\pi \epsilon)/2\), which approximates the distribution measured in cosmological simulations (e.g. Wetzel 2013; van den Bosch 2017), where \(\epsilon \approx 0.45\) at \(z = 2\). We consider the infall locations to be isotropically distributed on the virial sphere. In the fiducial run, the host halo is again of mass \(M_{h} = 10^{12.5} M_{\odot}\) with an NFW profile and \(c_{\text{NFW}} = 5\). Every satellite with \(m_{v} > 10^{11} M_{\odot}\) is assumed to be post-compaction, with the fiducial compact DZ profile of \((c, a) = (7, 1)\), while all the less massive satellites are assumed to be diffuse, pre-compaction, with \((c, a) = (3, 0.5)\). In a reference run, with the same distribution of masses and initial orbits, all the satellites are assumed to be diffuse. This reference case is supposed to approximate the real situation for less massive hosts, say \(M_{h} < 10^{11.5} M_{\odot}\), where no compact satellites (of \(m_{v} > 10^{11} M_{\odot}\)) are expected.

Fig. 5 shows the energy deposited within the sphere of radius \(r\) about the halo centre, during a period of one virial time at \(z = 2\), \(\tau_{c} \approx 0.5\ \text{Gyr}\), corresponding to a redshift interval \(\Delta z \approx 0.3\).\(^7\) We read for the NFW host halo \(W/K_{c} \approx 1.8\) and 0.35 for the run with and without post-compaction satellites, respectively. This is compared to the toy-model predictions of \(W/K_{c} \approx 5.6\) and 0.08. They both point to a similar qualitative difference where the post-compaction satellites generate significant cusp heating while the diffuse satellites provide only minor heating during a time interval comparable to the virial time at \(z \approx 2\).

For the steep-cusp halo run of SatGen with a sequence of satellites, shown in Fig. G3 supplementary material online), we obtain \(W/K_{c} \approx 1.8\) and 0.35, respectively. This is very similar to the SatGen results for the NFW host, consistent with the similarity of the results obtained for a single satellite. The corresponding toy-model predictions are \(W/K_{c} \approx 1.5\) and 0.003, consistent with the SatGen result for the compact satellite, though estimating less energy deposit in the steep-cusp halo compared to the NFW halo. We learn that the cosmological sequence of compact satellites during half a virial time is expected to provide significant cusp heating in the steep-cusp host halo as well.

5 POST-HEATING RELAXATION

Before we proceed to the response of the heated DM to AGN outflows in Section 6, we address the behaviour of the cusp after the DF heating. In the spirit of our simple toy modelling, we first assume that the work done by DF on the satellite is instantaneously deposited in the dark matter as kinetic energy (`heating'), in terms of increased velocity dispersion, while the potential energy and the associated density profile remain unchanged at that instant.\(^8\) In the case of a single satellite, after it has disrupted or settled in the centre, we expect the host halo to relax over a certain time window \(\tau_{c}\) into a new Jeans equilibrium. In this process, the dark matter ‘cools’ while it expands, namely the potential energy grows (becoming less negative) at the expense of the kinetic energy, and the cuspy density profile partly flattens. We first address this process in Section 5.1 via simplified toy models and then in Section 5.2 using an \(N\)-body simulation.

5.1 Toy models

For a first crude qualitative impression of this process, we consider the cusp to be an isolated shell of mass \(M\) in virial equilibrium, where the kinetic and potential energies are related via \(K = -U/2 = GM^{2}/(2R)\), both at the initial radius \(R_{i}\) and at the final radius \(R_{f}\), after a deposit of energy \(W\) and a relaxation process. Conservation of energy implies

\[
\frac{GM^{2}}{2R_{i}} + W = -\frac{GM^{2}}{2R_{f}},
\]

so

\[
\frac{K_{i}}{K_{f}} = \frac{R_{i}}{R_{f}} = 1 - \frac{W}{K_{i}}.
\]

The system expands, while it cools, from a peak kinetic energy of \(K_{\text{peak}} = K_{i} + W\), immediately after the action of DF, to below the initial kinetic energy, \(K_{i} = K_{f} = W\). In the relaxation process, the kinetic and potential energy changes are \(\Delta K = -\Delta U = -2W\). Thus, after an energy deposit due to DF by a single satellite, the system may be found in one of two configurations. During the time window for expansion and cooling, \(\tau_{c}\) (to be estimated below), the system is ‘hotter’ than before with the cuspy profile practically unchanged. After this period, the system has a flatter density profile but it is ‘cooler’ than in its initial state.

For a more quantitative estimate, we assume for the host halo a Dekel–Zhao density profile both at the initial and at the final configurations, both in Jeans equilibrium, as described in section 3.3 of Freudlich et al. (2020a). The transition between the two states is assumed to start with an instantaneous addition of kinetic energy to

\(^{7}\)In the EdS regime, approximately valid at \(z > 1\), the relation is \(\Delta \tau \approx -1.68\ \text{Gyr}\ 1 + z_{c}^{5/2} \Delta z\).

\(^{8}\)This approximation is valid for the massive satellites that provide most of the DM heating, like the ones simulated above, which sink through the cusp on a dynamical time-scale.
the dark matter at every radius according to an assumed input profile $W(<r)$, mimicking the work by DF, while the mass distribution and therefore the potential energy are fixed. This is followed by a relaxation to a new equilibrium where shells enclosing given DM masses are assumed to conserve energy. This is analogous to the CuspCore model used in Section 6 and described in Appendix E (supplementary material online) for the response to outflows, except that the first instantaneous change is in the kinetic energy rather than in the potential energy. We assume here an initial host halo of $M_h = 10^{12} M_\odot$, with an NFW profile of $c_{\text{NFW}} = 5$. The fiducial run (blue) assumes compact satellites of $m_s > 10^{11} M_\odot$, while the reference run (red) assumes that all satellites are diffuse. Shown are the medians and the 68 per cent and 95 per cent percentiles over 600 random merger trees. The DZ profiles for the compact and diffuse satellites have the fiducial parameters $(c, \alpha) = (7, 1)$ and $(3, 0.5)$, respectively. We see significant mass and energy deposit in the cusp for the compact satellites (valid for hosts of $M_h > 10^{12} M_\odot$ at $z > 1$) and low mass and energy deposit for the case of diffuse satellites (approximating the situation in host haloes of $M_h < 10^{11.5} M_\odot$ or at low redshifts).

**Figure 5.** SatGen simulations of a cosmological sequence of satellites during one halo virial time at $z \sim 2$, $\tau_v \approx 0.5$ Gyr, corresponding to a redshift interval $\Delta z 
\sim 0.3$. Left: the energy deposited within radius $r$ with respect to the kinetic energy in the 10 kpc cusp, $W(<r)/K_c$, with the dashed lines representing the fit used in Section 6 and listed in Table 1. Right: the satellite mass deposited inside $r$ with respect to the host-cusp mass within 10 kpc, $\Delta m(<r)/M_c$. The host halo mass is $M_h = 10^{12} M_\odot$, with an NFW profile of $c_{\text{NFW}} = 5$. The same for the steep-cusp profile is shown in Fig. G3 (supplementary material online). The fiducial run (blue) assumes compact satellites of $m_s > 10^{11} M_\odot$, while the reference run (red) assumes that all satellites are diffuse. Shown are the medians and the 68 per cent and 95 per cent percentiles over 600 random merger trees. The DZ profiles for the compact and diffuse satellites have the fiducial parameters $(c, \alpha) = (7, 1)$ and $(3, 0.5)$, respectively. We see significant mass and energy deposit in the cusp for the compact satellites (valid for hosts of $M_h > 10^{12} M_\odot$ at $z > 1$) and low mass and energy deposit for the case of diffuse satellites (approximating the situation in host haloes of $M_h < 10^{11.5} M_\odot$ or at low redshifts).

5.2 Heating and flattening in $N$-body simulations

We realized in Section 5.1 that a halo cusp, after being heated by a single satellite, would relax into a new Jeans equilibrium in an energy-conserving process that involves expansion and cooling, yielding a cold cusp with a somewhat flatter density profile. Here, we use an $N$-body simulation in order to verify these heating and cooling processes. We wish in particular to estimate the time it takes for the heated cusp to cool down. This will be used as the effective duration for evaluating the heating effect of a cosmological sequence of satellites, expressed as $\tau$ in equation (21) of the toy model and as the proper duration of the SatGen simulation in Section 4.2. According to these estimates, if the cooling process takes a significant fraction of a virial time or more, namely $\tau \approx 0.5$ much smaller than unity, then we can expect the cusp to be maintained hot by the sequence of mergers with compact satellites. If, on the other hand, $\tau \ll 1$, the cusp will drop to a cool phase between the individual merger episodes. The different responses of a hot cusp and a cooled-flattened cusp to AGN-driven outflows will be discussed in Section 6.

The $N$-body simulation uses the public GADGET-2 code (Springel 2005), with a particle mass of $m_p = 3 \times 10^6 M_\odot$ and a softening length of $r_{\text{soft}} = 0.0003 (N_h/10^8)^{1/3} L_c$, where $N_h = m_c / m_p$ is the number of particles within $L_c$ (following van Kampen ; Ogiya et al. 2019), namely $r_{\text{soft}} = 20$ pc. We consider a satellite of $m_s = 10^{11} M_\odot$ orbiting a host of $M_h = 10^{12.5} M_\odot$ near $z \approx 2$ along a rather eccentric orbit of circularity $\epsilon = 0.1$ at the halo virial radius and a velocity virial velocity at that radius. We compare the cases of a compact and a diffuse satellite, where the compact satellite is initialized with an NFW profile of $c_{\text{NFW}} = 50$, and the diffuse satellite is initialized with a Burkert profile (Burkert 1995) of a scale radius $r_s = 12$ kpc, by AGN outflows (Section 6). We will study in Section 6 the DM response to AGN-driven outflows when starting from either the ‘hot’ or the ‘cold’ configuration.
the same as the inner scale radius of an NFW profile with $c_{\text{NFW}} = 6$. These profiles are not the same as the fiducial profiles used in the rest of the paper, but they are rather close and serve us for our qualitative purpose here. More details on this N-body simulation are provided in Appendix D (supplementary material online).

Fig. 7 shows the time evolution of the relative excess of kinetic energy of the dark matter within the cusp with respect to its initial kinetic energy. It also shows the distance of the satellite from the host centre, indicating the successive pericentres and apocentres with the orbit decay into the cusp, and the mass of the satellite at that time. We see hot-phase periods near the first three orbit apocentres, relaxing to cold-phase periods in between as well as after the coalescence. The overall time in the hot phase is $t_{\text{hot}} \sim 0.5t_{\text{cusp}}$.

Fig. 8 shows the density profile and kinetic-energy profile of the host halo at different times through the merger process compared to the initial cold NFW profile. In the kinetic energy, we see hot phases in the cusp during periods about the first, second, and third pericentres, cooling down in between. The maximum heating is near and interior to the cusp radius of $\sim 10$ kpc. The density profiles show that the initial profile hardly changes until after the first pericentre. Very minor flattening starts appearing after the first pericentre, and a more noticeable but still partial flattening develops after the second pericentre, becoming more significant after the third pericentre. The latter may be subject to uncertainties due to difficulties in identifying a centre for the halo at these stages of satellite coalescence with the host halo centre.

Substituting $\tau = 0.5$ in the toy-model estimate equation (21), we expect a total heating of $W_c/K_c \simeq 0.75(1 + z)$ in the hot phase. This being of order unity at $z \sim 2$ indicates that, at these high halo masses and redshifts, the cusp can be maintained for most of the time in the ‘hot’ phase, and thus be more susceptible to core formation by AGN outflows, to be demonstrated in Section 6.

6 DM RESPONSE TO AGN OUTFLOW

6.1 Modelling the response to outflows

In order to model the DM response in the inner halo to an episode of AGN-driven gas outflow, we use the CuspCore analytic model presented in Freundlich et al. (2020a), which has been originally implemented for SN-driven outflows in low-mass galaxies. The model, as summarized in Appendix E (supplementary material online), consists of a two-stage response, starting from an instantaneous change of potential due to an assumed central mass removal while the velocities are frozen, and following by an energy-conserving relaxation to a new Jeans equilibrium. The model assumes a spherical halo, isotropic velocities, Jeans equilibrium in the initial and the final stages, and energy conservation for shells enclosing a given DM mass. This model proved successful when compared to cosmological simulations (NIHAO) except in situations of major mergers. It uses the DZ density profile with flexible concentration and inner slope, utilizing its analytic expressions for the potential and kinetic energies, as described in Appendix A (supplementary material online) and Freundlich et al. (2020b).

We assume here as before a halo of mass $M_\ast = 10^{12.5}M_\odot$ with a given initial density profile. We add a central point mass representing baryons of $M_b = M_\ast = 4 \times 10^{10}M_\odot$, following the average stellar-to-halo mass ratio for such a halo mass at $z = 2$ (Behroozi et al. 2019) and assuming a gas fraction of 0.5 for SFGs at that redshift (Tacconi et al. 2018). The corresponding DM velocity dispersion profile is determined by requiring Jeans equilibrium (Freundlich et al. 2020b).

The outflowing mass is assumed to be a fraction $\eta$ of the gas mass, ranging from zero to unity, namely a fraction $0.5\eta_t$ of the central baryonic mass. We estimate in Section 7.2 that the typical gas mass in the galaxy of $M_{\text{gas}} \sim 4 \times 10^{11}M_\odot$ is comparable to the typical mass of the fresh cosmologically accreted gas during $\sim 0.5t_\text{cusp} \sim 0.25$ Gyr. Taking into account accreting recycled gas that is three times more massive than the accreting fresh gas, the gas available for ejection by AGN feedback during that period is $\sim 1.2 \times 10^{11}M_\odot$. Recall that we learned from the N-body simulation of Section 5.2 that $\sim 0.5t_\text{cusp}$ is the ‘cooling’ time for the cusp heated by DF, and we will show below that a pre-heated cusp is most responsive to outflows, so this is the time interval within which AGN feedback is most effective in producing a core. We model here only a single episode of $\eta = 0.5$ or $\eta = 1$ and consider it to provide a lower limit for the effect of core formation.

9The simplest version of CuspCore is available online for application in https://github.com/Jonathanfreundlich/CuspCore.
Figure 7. Time evolution in an N-body simulation of a compact satellite (blue) and a diffuse satellite (red) of $m_v = 10^{11.5} M_\odot$ on a rather radial orbit in a host halo of $M_v = 10^{12.5} M_\odot$ at $z = 2$. Left: the satellite distance from the host centre. Middle: the bound mass of the satellite. Right: the kinetic energy within the host cusp with respect to its initial value. We read that the heated episodes near the pericentres last a total of $\sim 0.25$ Gyr, which is $\sim 0.5 t_v$.

Figure 8. Evolution of the profiles of kinetic energy (left) and density (right) in an N-body simulation of a compact satellite in a massive halo, the same as Fig. 7. The initial NFW profile is shown (dashed). The times shown are: (a) $t = 0.86$ Gyr (black), virial crossing, cold, cuspy. (b) $t = 1.17$ Gyr (blue), first pericentre, hot, cuspy. (c) $t = 1.52$ Gyr (green), first apocentre, cold, cuspy. (d) $t = 1.91$ Gyr (yellow), second pericentre, hot, cuspy. (e) $t = 2.10$ Gyr (red), second apocentre, slightly hot, slightly flattened. (f) $t = 2.32$ Gyr (grey), third pericentre, hot, more flattened.

6.2 A single satellite

Fig. 9 shows the change in the DM density profile due to an instantaneous outflow episode based on the CuspCore model. The initial DM halo is our standard, with $M_c = 10^{12.5} M_\odot$ and an NFW profile of $c_{\text{NFW}} = 5$ at $z = 2$. The initial kinetic energy obeys Jeans equilibrium (following Freundlich et al. 2020a), with an additional presence of a $10^{10.9} M_\odot$ central point mass representing a baryonic component, half gas and half stars. The pre-outflow configuration is the result of DF heating by a single satellite of mass $m_v$, either the compact or the diffuse fiducial cases deduced from the cosmological simulations described in Section 2. The energy deposited by DF is read from the SatGen simulation for a satellite of $m_v = 0.1 M_v$, as shown in Fig. 4. We actually use the functional fit that is shown in the figure and presented in Table 1. A satellite mass of $0.1 \mu m_v$ is mimicked by multiplying the deposited energy from Fig. 4 by $\mu^2$, following equation (17). The mass removed by the outflow is assumed to be a fraction $\eta$ of the $10^{10.6} M_\odot$ gas (a fraction $0.5 \eta$ of the baryonic mass), and the halo expands differentially while conserving energy into its final Jeans equilibrium according to the CuspCore model. The two cases shown, both ending up with an extended core, are for $\mu = 1$ with $\eta = 1$ and for $\mu = \sqrt{2}$ with $\eta = 0.5$.

Three cases are shown in each panel of Fig. 9, as follows. The first (black) is a reference case of an outflow ignoring any pre-heating by DF, starting from the initial cold NFW halo (short-dashed black). One can see that the post-outflow cusp (long-dashed grey) is somewhat flatter than the original cusp, but even with $\eta = 1$ it is not as extended and as flat as deduced from observations.

In the second case in each panel (red), the pre-outflow configuration is set to mimic the hot phase soon after the energy deposit by DF, namely the density profile is the same initial NFW profile while all the input energy is deposited as kinetic energy. We see that in this case the cusp heated by the compact satellite generates an extended flat core, as desired (solid red). On the other hand, the cusp heated by the diffuse satellite shows only a minor flattening. The unrealistic positive slope at small radii is an artefact of the CuspCore model and should be interpreted as a flat inner core.

In the third case in each panel (blue), the pre-outflow configuration is set to mimic the cooled, slightly flattened phase after relaxation, which may in principle be reached a while ($\sim 0.5 t_v$) after the heating
6.3 A cosmological sequence of satellites

Fig. 10 shows the same as Fig. 9 but now starting with the DF energy deposit from the SatGen simulation of a cosmological sequence of satellites, the fiducial compact and diffuse, based on Fig. 5, which refers to a duration of $\tau \Omega_c$ with $\tau = 1$. The functional fits used here and shown in the figure are listed in Table 1. Fig. 10 assumes $\tau = 0.5$, namely half the energy deposited by DF at each radius in Fig. 5, to match the duration of $\sim 0.5t_v$ for a heated cusp, as discussed in Section 5, based on the estimates from the N-body simulation described in Section 5.2, Fig. 7. The positive slope at small radii is an artefact of the CuspCore model and should be interpreted as a flat inner core.

We learn that the overall effect of an outflow after heating by a sequence of satellites over half a virial time is comparable to, and slightly stronger than, the effect of an outflow after heating by a single satellite of $m_c/M_v \sim 0.14$ based on Fig. 9. This implies that the case of a pre-heated cusp (red) is realistic for the median galaxy, generating a flat core extending to $\sim 10$ kpc once the halo mass is above the golden mass, such that the satellites are above the mass threshold for compaction, and where the halo allows effective AGN feedback.

6.4 Other cases

Figs G4 and G5 (supplementary material online) are analogous of Figs 9 and 10 but for an initial host halo with a cusp steeper than NFW, a DZ profile with $(c_{NFW}, s_{NFW}) = (5, 1.5)$, the same as studied in Figs G1, G2, and G3 (supplementary material online). We learn that core formation in response to AGN-driven outflow is more challenging in the steep-cusp halo than in the NFW halo, both in terms of less heating by DF and a weaker response to an outflow. In order to demonstrate the formation of an extended core by compact satellites in this case, one needs a larger energy deposit by DF and/or a stronger outflow. For a single satellite, Fig. G4 (supplementary material online), we model a satellite of $m_s = 0.1m_c$ with $s = \sqrt{3}$ and an outflow that involves all the gas mass of $10^{10.5} M_\odot$, $\eta = 1$. In the case of DF heating by a cosmological sequence of compact satellites, Fig. G5

Table 1. We use as input to the outflow model CuspCore the energy deposited in the host cusp by the dynamical friction exerted on satellites as obtained from the corresponding SatGen simulations. We actually use a functional fit (shown in the SatGen figures) of the form $log(\rho) W(c/r) = A - B (log(r/R_{max}))^2$, with the parameters listed in this table, where $A$ and $B$ are log energy in units of $M_\odot kpc^2 Gyr^{-2}$, and $R_{max}$ is in kpc, with $R_t = 150$ kpc.

| Host   | Satellite | Compactness | A    | B    | $R_{max}$ |
|--------|-----------|-------------|------|------|-----------|
| NFW    | Single    | Compact     | 16.81| 0.62 | $R_v$     |
| NFW    | Single    | Diffuse     | 16.61| 0.71 | $R_v$     |
| NFW    | Cosmo     | Compact     | 16.80| 0.24 | 692       |
| NFW    | Cosmo     | Diffuse     | 16.70| 0.45 | 610       |
| Steep  | Single    | Compact     | 16.81| 0.58 | $R_v$     |
| Steep  | Single    | Diffuse     | 16.57| 0.65 | $R_v$     |
| Steep  | Cosmo     | Compact     | 16.70| 0.70 | $R_v$     |
| Steep  | Cosmo     | Diffuse     | 16.50| 1.00 | $R_v$     |

by DF (short dashed blue). Now, in the case of a compact satellite, the outflow generates a flattened profile (solid blue) but not as flat and extended as in the case of a hot pre-outflow cusp. We learn that the response of a pre-heated inner DM halo to central mass ejection is indeed much stronger than that of a standard cold NFW halo and stronger than that of a cooled flattened cusp that may in principle be reached late after the heating by DF.

Figure 9. Dark-matter density-profile response to an outflow according to the CuspCore model. The initial halo is NFW with $M_v = 10^{12.5} M_\odot$ and $c_{NFW} = 5$ with $R_t = 150$ kpc at $z = 2$. The DF heating is by a single fiducial satellite of initial mass $m_s$, compact or diffuse, based on the energy-deposit profile as revealed by SatGen, using the fit shown in the bottom left-hand panel of Fig. 4 and Table 1. The instantaneous outflow involves a fraction $\eta$ of the central gas mass of $10^{10.5} M_\odot$. Left: a compact satellite $m_s = 0.1 M_v$ and $\eta = 1$. Middle: a compact satellite and $m_s = 0.1 \sqrt{2} M_v$ (obtained by doubling the energy deposited by DF according to Fig. 4) and $\eta = 0.5$. Right: the same but for a diffuse satellite. Three cases are shown in each panel as follows. Grey: the dotted line is the pre-DF initial cuspy DM profile, cold NFW in Jeans equilibrium. It is assumed to be the pre-outflow profile, with the dashed grey line referring to the corresponding post-outflow profile. Red: the pre-outflow density profile is the same grey dotted line, but in this case the halo has been heated by dynamical friction, and the solid red line is the corresponding post-outflow profile, showing a flat core extending to the core radius of $\sim 10$ kpc (vertical line). Blue: the dotted line is the somewhat flatter density profile late after the heating by dynamical friction, after relaxation involving expansion and cooling. With this cold pre-outflow density profile, the solid blue line refers to the post-outflow profile, showing a core but not as flat as in the pre-heated case. Results for a steep-cusp halo are shown in Fig. G2 (supplementary material online). We learn that the diffuse satellite causes a much weaker effect than the compact satellite, as predicted by the toy model. We also learn that a heated cusp is more responsive to an outflow than a flattened, cold cusp, though both can end up as extended cores under pre-heating by the compact satellite.
(supplementary material online), we learn that an outflow of $\eta = 2$, namely involving at least two successive episodes of outflow, is sufficient for producing a moderate core without DF heating. A flatter core is produced with $\tau = 1$ and $\eta = 2$ if the slope of $K(< r)$ within the cusp is artificially flattened in order to make a better fit to the DZ profile in CuspCore and thus allow convergence with $\tau = 1$. The high $\tau$ can be interpreted as representing the top $1/3$ of the random realizations of the satellites in the sequence (in terms of mass and circularity) during $\sim 0.5 t_c$. Alternatively, it can be interpreted as the median during a longer period of $\sim t_c$. Thus, we learn that an extended core can be reproduced even in an initially steep cusp, at least in a fraction of the galaxies.

We explored the general dependence of the formation of a core on various parameters of the model. This was done using the fiducial case of an NFW host of $M_h = 10^{12.3} M_\odot$ and a single satellite of $m_s = 0.1 M_h$, with the fiducial compact or diffuse DM profile on a typical orbit, and with an outflow of $\eta = 0.5$ of the gas. The general conclusions are as follows. (1) While a compact satellite with an outflow of $\eta = 0.5$ is needed for an extended core, even a diffuse satellite can lead to a core with a stronger outflow of $\eta \sim 1$, but it is at best a small core of a few kiloparsecs. (2) While a gas mass of $\sim 10^{10} M_\odot$ leads to a core by an $\eta = 0.5$ outflow even if it occurs after post-heating relaxation, Section 5, a gas mass smaller than $\sim 10^{10} M_\odot$ may be enough for generating a core provided that the outflow occurs during the hot phase of the DM. It should be noted that the direction of the dependence on the baryonic mass is not obvious a priori, because a lower baryonic mass is associated with both a shallower potential well and a weaker outflow, which work to strengthen and weaken the formation of a core, respectively. (3) Spreading the baryons from a central point mass to an extended configuration, out to $0.2 R_c$, makes only a small difference to the core formation. This is due to the competing effects mentioned above, where a more extended baryon distribution is associated with both, a shallower potential and a weaker effect of the outflow.

We also tested multiple episodes of outflow, each with $\eta = 0.2$, following one event of DF heating and no further supply of central baryons during the sequence of outflows. The total fractional outflow mass in $N = 3$ such episodes is comparable to the outflow mass in a single episode of $\eta = 0.5$. We learn that under these conditions, the successive three weaker episodes and the stronger single episode lead to a similar core. This is in line with the toy model of shell response to an outflow (Dutton et al. 2016, Section 4), which predicts that the expansion of the shell radius is approximately by a factor of $1 + N f$, where $f \ll 1$ is the fraction of the total mass ejected in each episode, such that $N f$ is the same for all $N$. However, if each outflow is preceded by an inflow of the same gas mass, the net expansion factor (after certain contraction) becomes $1 + N f^2$. If the outflow events are preceded by a cosmological inflow of the same total gas mass $f_{\text{in}}$, then $f = f_{\text{in}} N$, such that the single outflow is expected to be more effective than a sequence. If, on the other hand, each outflow is preceded by an inflow of recycled gas, then $f$ is the same for all episodes, and a sequence of weak outflows is expected to be more effective than a single outflow.

7 DISCUSSION

7.1 Cores at large masses and high redshifts

We showed that the observed preferred appearance of cores in haloes of $M_h > 10^{12} M_\odot$ is a natural outcome of the characteristic threshold for compaction events at $m_c > 10^{11.3} M_\odot$, as seen in simulations (Zolotov et al. 2015; Tomassetti et al. 2016; Tacchella et al. 2016b) and observations (Huertas-Company et al. 2018). Once the host halo is above $10^{13} M_\odot$, satellites above $10^{11.3} M_\odot$ constitute most of the accretion into the host halo, and these are the post-compaction compact satellites that allow 20 per cent or more of their mass to penetrate intact into the host cusp and significantly heat up the cusp by DF. The transition in the satellite compactness as a function of mass translates to a threshold for haloes that can generate extended DM cores.

The preference of cores at redshifts above $z \sim 1$ arises from several different sources. First, as estimated by our toy model, the relative energy deposited by DF in the cusp in a Gyr is $\propto (1 + z)^{52}$, equation (20), and in a halo crossing time it is $\propto (1 + z)$, equation (21).
With a typical bound fraction $f \sim 0.2$ surviving stripping for a post-compaction satellite, the deposited energy is indeed comparable to the cusp energy near $z \sim 2$, where the majority of the observed haloes show cores. Secondly, the compaction events, which tend to occur near the characteristic mass, tend to be deeper at higher redshifts due to the higher gas fraction, which allows more dissipation and angular momentum loss and therefore more compact satellites. Thirdly, the satellite orbits tend to be more radial at higher redshifts, where a given halo mass represents a higher-sigma peak. According to Wetzel (2011), the mean orbit circularity is $\epsilon \sim 0.6, 0.5$, and $0.4$ at $z \sim 0, 2$, and 4, respectively. The more radial orbits at higher redshift give rise to less tidal stripping, deeper satellite penetration, and larger energy deposit by DF in the host cusp. Further discussion of the redshift and mass dependence of the orbit circularity is shown in Appendix J (supplementary material online).

The second-stage AGN-driven core formation gives rise to a similar mass threshold for cores. This is because the central BH growth is suppressed by SN feedback when the halo is less massive than the golden mass $\sim 10^{12} M_\odot$ (Dubois et al. 2015; Bower et al. 2017; Anglés-Alcázar et al. 2017b), and it turns into a rapid growth triggered by a compaction event (Dekel et al. 2019; Lapiner et al. 2020) in the hot-CGM phase once above the golden mass, consistent with observations (Kocevski et al. 2017; Förster Schreiber et al. 2019). In turn, the AGN-driven winds also contribute to the preference of cores at high redshifts, because the winds tend to be more effective at high redshifts due to the higher gas fraction, providing more gas for activating the AGN and as a substance to be ejected.

### 7.2 Gas availability for AGN-driven outflows

For a halo of mass $M_h = 10^{12.3} M_\odot$, we assume a galaxy gas mass of $M_g \approx 4 \times 10^{10} M_\odot$, following the average stellar-to-halo mass ratio for such a halo mass at $z = 2$ (Behroozi et al. 2019) and assuming a gas fraction of 0.5 (Tacconi et al. 2018). The outflowing mass in each ejection episode is assumed to be a fraction $\eta$ of the gas mass in the galaxy, between zero and unity, namely a fraction $\sim 0.5 \eta$ of the central baryonic mass at $z \sim 2$. An upper limit on $\eta$ is provided by the gas mass available for being ejected, which we estimate from cosmological accretion plus recycling during a period of, e.g., $0.5 t_c$, the typical time for the hot phase as estimated in Section 5.2. The typical cosmological specific accretion rate in the EdS regime ($z > 1$) can be approximated by equation (18), $\dot{M}/M \approx 0.47 \text{Gyr}^{-1} (1 + z)^{5/2}$, where the mass can refer successfully either to the total mass or to the baryonic mass (Dekel et al. 2013).

Assuming a cosmological baryon fraction $f_b = 0.17$, with an accreting gas fraction with respect to the total accreting baryons of $f_g = 0.5$ at $z = 2$, and assuming that the accretion rate of fresh and recycled gas is a factor $f_{\text{rec}}$ times the fresh gas accretion rate, the total gas accretion rate into the galaxy inside a halo of $M_h = 10^{12.3} M_\odot$ is

$$M_g \approx f_{\text{rec}} 1.3 \times 10^{11} M_\odot \text{Gyr}^{-1}. \quad (24)$$

This provides $M_g \approx f_{\text{rec}} 3.2 \times 10^{10} M_\odot$ of gas that is available for ejection during 0.5$t_c \approx 0.25 \text{Gyr}$. Given the comparable gas mass of $\sim 4 \times 10^{10} M_\odot$ in the galaxy, one can expect about $f_{\text{rec}}$ outflow episodes of as much as $\sim 1$ each during that period. Based on FIRE simulations, the recycling factor at $z \sim 2$ is estimated to be as high as $f_{\text{rec}} \sim 3$ (Anglés-Alcázar et al. 2017a, fig. 3). We therefore assume that from the point of view of gas availability for ejection, by modelling the response to one outflow episode of $\eta = 1$ we obtain a conservative lower limit to the effect of outflows during 0.5$t_c$. This is provided that the duty cycle of AGN outbursts is appropriate for the purpose, and that their energy/momentum and coupling to the available gas are sufficient for a removal of a significant fraction of the available gas. The observed high velocities of AGN-driven outflows, which can be larger by an order of magnitude than the galaxy escape velocity (e.g. Förster Schreiber et al. 2019), justify the approximation of an instantaneous gas removal in our model.

### 7.3 Central stars, gas, and dark matter

For the proposed scenario to produce DM cores with a low fraction of dark matter and with a stellar system that remains intact, several conditions have to be fulfilled. First, the accretion rate of dark matter to the cusp by satellites should not overcome the expansion rate induced by DF heating and/or AGN-driven outflows. Secondly, the stellar system should be more compact than the gas that is to be removed from the galaxy, and the dark matter should be less compact. Jeans equilibrium for each component implies in this case that the dark matter is kinematically hotter than the stars. This spatial and kinematic segregation would make the dark matter more susceptible for expansion (or even removal) due to gas removal.

#### 7.3.1 Dark-matter accretion

The galaxies with DM cores are observed to have a low fraction of dark matter with respect to the total core mass, $f_{\text{DM}} \sim 0.00$–0.3 within the effective radius $R_e$ (Genzel et al. 2020, fig. 10). The compact merging satellites that provide the DF heating bring into the cusp a mass that is comparable to the original cusp mass. For example, according to Fig. 4, for a typical single compact satellite of $m_\ast \sim 0.1 M_\odot$, the mass that enters the 10kpc host cusp is $m_\ast \sim 0.4 m_\odot = 0.04 M_\odot$, indeed comparable to the mass inside the cusp. For our scenario of core formation to work, the dark matter that dominates this incoming mass has to be pushed away from the cusp, together with the dark matter that has already been in the cusp, either by the post-heating expansion discussed in Section 5 or by the response to AGN-driven outflows discussed in Section 6. In order to compute the time-scale for doubling the cusp mass by incoming satellites, we recall that the specific accretion rate into the halo is $\dot{M}_c/M_c \approx 0.5 \text{Gyr}^{-1} (1 + z)^{5/2}$ (Dekel et al. 2013). If about 60 per cent of this input mass is in compact satellites of $m_\ast \sim 0.1 M_\odot$, as (Neistein & Dekel 2008), and if the satellite mass that penetrates to the cusp is $m_\ast \sim 0.4 m_\odot$, while the cusp mass in an NFW host is $M_\ast \sim 0.04 M_\odot$, we can obtain a specific DM accretion rate into the cusp of $m_\ast/M_\ast \sim 3 \text{Gyr}^{-1} (1 + z)^{5/2}$, namely a DM cusp doubling time of

$$t_c \sim 0.3 \text{Gyr} (1 + z)^{5/2} \sim 0.6(1 + z)^{1/2} t_c. \quad (25)$$

According to Fig. 4, this DM mass is mostly deposited in the outer cusp at $z \sim 10$ kpc. This time-scale turns out to be comparable to the time-scale for post-heating relaxation (Section 5) and to the timescale for fresh gas supply for an AGN-driven outflow that generates a core (Section 7.2). This implies that the DM accretion rate into the cusp is slow enough for the dark matter to be maintained hot by DF, and that the gas accretion rate (especially when adding recycling) is more than needed for producing AGN-driven outflows that push away the additional dark matter, once the AGN duty cycle, energy/momentum, and coupling to the gas are appropriate.

Fig. H1 (supplementary material online) shows the CuspCore results with the DM satellite mass that has been added to the host according to SatGen, Figs 4 and 5, prior to the outflow event. Shown are cases with the same parameters as in Figs 9 and 10, confirming core formation in the presence of the added mass. The core is slightly less flat than without the additional mass for a single satellite, but it
is as flat for a sequence of satellites, where the total added mass is only 0.2 of the cusp mass. This justifies ignoring the added mass in our main analysis.

7.3.2 The stellar system

What may lead to the initial compactness and the associated coldness of the stellar systems? The cored galaxies tend to have massive compact stellar bulges, with masses $10^{10.5} - 10^{11} M_{\odot}$ (Genzel et al. 2020, fig. 11). These bulges have to be compact enough such that they are not significantly heated by the merging satellites, and especially such that their compactness compared to the gas and the (possibly heated) dark matter, and their self-gravity, allow them to survive intact while the more extended dark matter expands in response to the AGN-driven outflows. These bulges are likely to have formed by earlier dissipative compaction events that occurred when the host halo crossed the threshold mass, generating a stellar nugget, blue that turned red, inside $\sim 1$ kpc (Zolotov et al. 2015; Tacchella et al. 2016b).

Furthermore, the stars that come in with the merging post-compaction satellites are expected to be more compact than the gas and DM at the centres of these satellites and thus to penetrate by DF deep into the host halo cusp with little stripping. The stellar-to-halo mass ratio at $z \sim 2$, for our dominant post-compaction satellite mass of $m_s \sim 10^{11.2} M_{\odot}$, is expected to be $m_s/m_h \sim 0.004$ (Behroozi et al. 2019). According to Fig. 4, the bound mass of a typical satellite of $m_s = 0.1 M_*$ that enters the inner 1 kpc of the host is $m_1 \sim 0.09 m_s = 0.009 M_*$. We learn that if the satellite stars are indeed confined to and dominate the central regions of the satellites, they are all expected to penetrate intact into the inner 1 kpc of the host halo, while the mass that is stripped within the outer cusp is primarily dark matter.

With a steep stellar density profile in Jeans equilibrium, in a given potential well that is determined by the total mass, the stellar system is kinematically colder than the dark matter. In a spheri-symmetric system, with isotropic velocity dispersion, the Jeans equation for each component is

$$\frac{d}{dr} \left( \rho \sigma_r^2 \right) = -\rho \frac{d \Phi}{dr}$$

(26)

The left-hand side represents the outward force due to the pressure gradient for the given component. It balances the gravitational force inward, the gradient of the gravitational potential $\Phi$, given by the total mass profile. Typically, the dominant term on the left-hand side is $\sigma_r^2 (d \rho/dr)$. Therefore, for a given total mass profile, the component with a steeper density profile needs a lower velocity dispersion in order to balance the same gravitational force.

The compact, cold stellar nugget is not expected to heat up due to DF. The gravitational potential of the stars in the host central $\sim 1$ kpc nugget is deeper by a factor of a few than that of the dark matter at several kiloparsecs. According to Fig. 4, the energy deposited by DF inside the inner 1 kpc of the host is two order of magnitude less than at a radius of a few kiloparsecs. These indicate that the heating effect of DF on the inner nugget is negligible.

7.3.3 The gas distribution

In order for the stellar nugget to remain intact while the dark matter is expanding in response to AGN-driven outflows, the gas to be expelled has to lie mostly outside the compact stellar system. We learn from the VELA hydro cosmological simulations that in post-compaction galaxies, even without AGN feedback, the gas half-mass radius is typically $\sim 3$ times the stellar half-mass radius (Tacchella et al. 2016b; Kretschmer et al. 2021)\(^\text{10}\). With $R_{e, \text{stars}} \sim 1$ kpc, we estimate $R_{e, \text{gas}}/R_{e, \text{stars}} \sim 3$ kpc, which is inside most of the stellar system but inside most of the DM cusp of $R_{e, \text{dm}} \sim 7$ kpc that is to be pushed away, as required. The gas is more diffuse than the stars in these simulations because of the depletion by central star formation and partial effects of SN feedback when near the golden mass and because the gas might have been segregated from the stars already in the merging satellites due to similar effects as well as ram-pressure stripping. Considering the effect of earlier AGN-driven outflows may make the gas even more diffuse.

Clearly, the above discussion is based on crude estimates. A quantitative analysis of the stellar system, the gas distribution, and the DM distribution in the presence of baryons is deferred to a future study that will properly address the baryonic components of the satellites and the host halo. In particular, the CuspCore model will be generalized to deal with a three-component system.

7.4 Why do cosmological simulations fail?

We can see several reasons for the failure of DF to heat up the host cusps (and/or produce cores) in current cosmological simulations (e.g. Wang et al. 2020; Lazar et al. 2020, TNG and FIRE-2, respectively). First, the simulation resolution does not allow a full treatment of the compaction of satellites of $\sim 10^{11.3} M_{\odot}$. While the $\sim 10$ kpc environment in the satellite is resolved, the inner $\sim 1$ kpc is not properly resolved in most simulations (e.g. EAGLE, Illustris–TNG).

The compaction on these scales is important as it suppresses tidal heating in the inner satellites and reduces the overall tidal stripping of the satellites, which allows better penetration and stronger DF heating of the host cusp.

Secondly, the compaction process in the satellites may be suppressed due to feedback that is too strong. This may be the case in some of the popular cosmological simulations (e.g. FIRE, Illustris–TNG).

Thirdly, if the satellites are not properly resolved by more than $10^6$ particles and an optimal scale-dependent force-softening length, there is an artificial tidal disruption of the satellites (van den Bosch & Ogiya 2018; Errani & Navarro 2020). This mostly affects satellites in the inner host halo, where the satellite abundance could be artificially suppressed by a factor of 2 or more (Green et al. 2021), thus suppressing the DF heating of the host cusp.

The fact that the AGN environment is unresolved may suppress its efficiency in driving winds that could participate in generating DM cores. First, the simulations do not resolve any clumpiness in the accretion on to the BH. Such clumpiness should boost the BH growth rate and AGN activity, and it can be incorporated as a subgrid model (DeGraf et al. 2017). Secondly, the simulations may fail to resolve the coupling of the AGN with the ISM around the BH. For example, if the ISM is clumpy, the push of the dense clumps may boost the generated wind.

The key test for the proposed scenario for core formation in massive galaxies should eventually be cosmological simulations that reproduce such cores. A maximum resolution of $\sim 20$ pc will enable the formation of compact blue nuggets as merging satellites, and $\sim 10^6 M_{\odot}$ particles in their haloes will avoid artificial tidal disruption. Improvements of the BH and AGN subgrid recipes could help

\(^\text{10}\)For example, we read from table A1 of Kretschmer et al. (2021) typical values of $R_{e, \text{gas}}/R_{e, \text{stars}} \sim 5$ and obtain the desired ratio of effective radii using $R_{e, \text{gas}}/R_{e, \text{stars}} \sim 0.5$. 

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generating the desired bursty duty cycle. First, by introducing subgrid clumpiness in the accretion on to the BHs (e.g., following DeGraf et al. 2017), and secondly, by limiting the AGN-feedback push to cold clumps in the ISM/CGM. In addition, positive AGN feedback (Silk 2013) may trigger SN feedback to assist the driving of the wind.

7.5 Caveats

In the current absence of proper simulations, the analysis performed above is approximate, meant to provide a feasibility test for the proposed hybrid scenario of massive core formation at high redshift, so its quantitative predictions should be taken with a grain of salt. It is yet to be verified with proper cosmological simulations that resolve the satellite compactness and the processes of tidal stripping and DF and model realistic BH growth and AGN feedback. We mention below certain caveats of our analysis.

We derived fiducial satellite profiles from cosmological simulations (VELA), which indicate a compaction-driven transition from diffuse to compact when the galaxy is near a golden halo mass of \( M \sim 10^{11.3} M_\odot \). While this transition is rather robust for most galaxies, in simulations and observations, we adopted in our simplified analysis representative fiducial profiles below and above the golden mass and ignored the large scatter in these profiles and in the threshold mass. This scatter may weaken the systematic mass dependence of core formation.

The analytic toy model for DF heating is clearly very approximate, meant to motivate our qualitative expectations. The SatGen semi-analytic simulations are a step forward in approximating the evolution of satellites, single and in a cosmological sequence, under tidal stripping and DF. It has been successfully calibrated against cosmological simulations. However, the way we translate the satellite evolution to local heating of the dark matter in the host halo is very crude, as DF is not a local effect (Tremaine & Weinberg 1984).

The CuspCore modelling of the DM response to an outflow episode is only an approximation. In particular, the assumptions of a pre-outflow hot halo as the outcome of earlier DF heating, and of an instantaneous mass loss from a central point mass, may be oversimplified. Furthermore, the implementation of energy conservation in the subsequent relaxation process is not formally justified in the case of shell crossing. Here, the proof is in the pudding, in the sense that the model has been demonstrated to approximate the response of dark matter to outflow episodes in hydro cosmological simulations (NIHAO), except under merger situations. Finally, the CuspCore model as implemented here treats a single instantaneous mass loss, while realistic AGN feedback may involve a duty cycle of mass ejection episodes with mutual cross-talk. As pointed out in Section 1, it is yet to be verified that AGN feedback can indeed remove large amounts of gas from the halo centres.

In most of the above analysis, the focus was on the dark matter, ignoring certain effects that may involve the baryons. In particular, for the scenario to work, the baryons in the satellites should penetrate into the halo centre such that the gas is available for ejection by AGN feedback while the stars, incoming or forming in situ, remain intact at the centre as a visible galaxy. Clearly, a more sophisticated analysis that better addresses the baryons is required.

8 CONCLUSION

This work has been inspired by the puzzling observational indication for a dearth of dark matter and flat-density cores extending to \( \sim 10 \text{kpc} \) in about one-third of the DM haloes of mass \( M \geq 10^{12} M_\odot \) at \( z \sim 2 \) (Genzel et al. 2020). This is puzzling because such cores, which are not detected at low redshifts, are not reproduced by any current cosmological simulation, and because SN feedback, the common process assumed responsible for core formation in low-mass galaxies, is not expected to be energetic enough for producing cores in massive galaxies (Dekel & Silk 1986). Despite potential uncertainties in the non-trivial interpretation of the observations, we take it as a non-trivial theoretical challenge.

We study a hybrid scenario where two processes combine to generate such massive cores at high redshifts. These are ‘heating’ of the inner host-halo cusp by DF acting on compact merging satellites, followed by AGN-driven outflows that flatten the hot cusp into a core. Each of these processes by itself seems to be incapable of forming extended-enough cores (El-Zant et al. 2001, 2004; El-Zant 2008; Johansson, Naab & Ostriker 2009; Martizzi et al. 2012; Martizzi et al. 2013). For the scenario to work, they should act in concert, and for reproducing the desired combined effects in simulations, each should be treated properly.

The key for efficient dynamical friction heating is the compactness of the incoming satellites. This turns out to be a natural outcome of the wet compaction process that galaxies typically undergo when their haloes are near or above a golden mass of \( m_c \sim 10^{11.3} M_\odot \) (Zolotov et al. 2015; Barro et al. 2017; Huertas-Company et al. 2018). This is indeed the mass range for the dominant merging satellites in host haloes of \( M_\text{h} \geq 10^{12} M_\odot \), in which the cores are observed.

We use analytic toy modelling and dedicated semi-analytic simulations (SatGen, Jiang et al. 2020) to explore the energy deposit by DF as a function of satellite compactness for a single satellite on a typical orbit and for a cosmological sequence of satellites during a period comparable to the halo virial crossing time at \( z \sim 2 \).

Zoom-in hydro cosmological simulations (VELA) allow us to determine fiducial density profiles for the post-compaction high-mass satellites in contrast to the pre-compaction low-mass satellites. The Dekel–Zhaofunctional form (Dekel et al. 2017; Freundlich et al. 2020b), with a flexible concentration and inner slope and analytic expressions for the potential and kinetic energies, best fits these with the DZ parameter values \((c, \alpha) = (7, 1)\) and \( (3, 0.5) \) for the compact and diffuse satellites. These correspond to the equivalent pairs of parameters \((c_2, s_1) = (15, 1.5)\) and \((3, 1)\), respectively, where \( c_2 \) is the concentration where the density-profile slope is \(-2\) and \( s_1 \) is the density slope at \( 0.01 r_\text{vir} \).

For a massive host halo of an initial NFW cusp, we find using the toy model and SatGen simulations that the fiducial compact satellites, relevant for massive haloes, significantly heat up the host cusp. They typically penetrate into the cusp with one to a few tens of per cents of their original mass, and by DF they deposit there an energy comparable to the initial kinetic energy in the cusp during one-half of the virial time at \( z \sim 2 \), about 0.25 Gyr. On the other hand, the diffuse satellites, relevant for less massive haloes, penetrate with only a small fraction of their original mass and deposit energy that is a similarly small fraction of the cusp energy, causing a negligible effect.

We find, using toy models and N-body simulations, that the ‘hot’ phase of the cusp, driven by DF acting on a single compact satellite, is temporary, lasting for about half a virial crossing time. Later, the cusp would have relaxed to a new Jeans equilibrium by expanding and cooling into a cold and somewhat flatter cusp. However, at \( z \sim 2 \) and for \( M_c \geq 10^{12} M_\odot \), given the energy deposited by DF in half a virial time, the hot phase is expected to be maintained by the cumulative heating from a cosmological sequence of merging satellites.

In the second stage, using an analytic model for outflow-driven core formation (CuspCore, Freundlich et al. 2020a), we demonstrate
that the heated or flattened cusps respond strongly to removal of about half the central gas mass. This is a small fraction of the gas available, given the accretion and recycling in the available time of half a virial time at $z = 2$. A pre-heated cusp develops into a flat core extending to $\sim 10 \text{ kpc}$, while a cooled and somewhat flattened cusp generates a partly flattened core. AGN feedback is indeed expected to become effective in haloes of $M_r > 10^{13} M_{\odot}$, where the BH growth is no longer suppressed by SN feedback and its rapid growth, triggered by a wet compaction event, is maintained by a hot CGM (Bower et al. 2017; Dekel et al. 2019; Lapiner et al. 2020).

Repeating the analysis for a host halo with a steeper initial cusp of slope $s_1 = 1.5$, we find that core formation is more demanding but plausible for about one-third of the galaxies. For DF to sufficiently heat up the cusp in half a virial time, the actual sequence of accreting satellites should include larger masses and/or more radial orbits than typical in the top third of the distribution. For the AGN feedback to then generate a core, the gas mass removed should be comparable to the instantaneous gas mass in the galaxy, which is still only a fraction of the gas supplied by accretion and recycling during half a virial time.

The preference for cores in haloes of mass above the golden mass of $M_r \sim 10^{12} M_{\odot}$ is primarily due to the threshold mass for compaction events that (a) generate sufficiently compact satellites for significant DF cusp heating, and (b) trigger strong AGN-driven outflows in the host galaxies (Dekel et al. 2019; Lapiner et al. 2020). The preference for cores at $z > 1$ arises from the more frequent mergers that drive the DF heating and the stronger compaction events due to the higher gas fraction. The preferred mass and redshift are related to each other through the fact that the typical haloes at the Press–Schechter non-linear mass scale are $\sim 10^{12} M_{\odot}$ at $z \sim 2$.

We estimate the accretion rate of dark matter into the cusp via the merging satellites is slow compared to the hot-cusp relaxation rate and the rate of gas supply for AGN-driven outflows. This allows the formation of a core with a low central DM fraction, as observed. We anticipate that the central stellar nugget is likely to be more compact than the gas distribution such that the stellar system will not be significantly heated by DF and will not be strongly affected by the AGN-driven gas removal that generates the DM core. These should be verified via simulations including baryons.

For cosmological simulations to reproduce the required DF heating, they should resolve the wet compaction of the satellites of $m_1 > 10^{11} M_{\odot}$ on scales below ~100 pc and have at least ~10 particles in each subhalo in order to avoid artificial tidal disruption. AGN feedback could be boosted by resolving clumpy BH accretion and clumpy response to outflows. Simulations of this kind are not beyond reach in the near future, and they should be performed for a more realistic test of the proposed scenario.

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DATA AVAILABILITY

The codes used in this article are available online, as referenced in the article and in the online supplementary material. Data and results underlying this article will be shared on reasonable request to the corresponding author.

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**SUPPORTING INFORMATION**

Supplementary data are available at MNRAS online.

**Figure S1.** Semi-analytic SatGen simulations of single satellites, diffuse and compact, similar to Fig. 4, but where the satellite profiles are the best DZ fits to the total mass in the VELA pre- and post-compaction galaxies from Fig. 1, including the baryons, with DZ parameters \((c, \alpha) = (5, 0.5)\) and \((3, 1.8)\), respectively.

**Figure S2.** Same as Fig. 3 but for the fiducial steep-cusp host, with \(s_1 = 1.5\) and \(c_2 = 5\) in the DZ profile.

**Figure S3.** Same as Fig. 4, for an \(m_*/M_v = 0.1\) single satellite, but for a steep-cusp host halo with DZ parameters \((c_2, s_1) = (5, 1.5)\).

**Figure S4.** Same as Fig. 5 but for the steep-cusp host. Shown are the results of a SatGen simulation of a cosmological sequence of satellites during one halo virial time at \(z \sim 2\), where \(t_v \simeq 0.5\) Gyr.

**Figure S5.** Density profiles as in Fig. 9, for DF heating by single satellite, but for a steep-cusp initial host halo with \((c_{2h}, s_{1h}) = (5, 1.5)\) instead of NFW.

**Figure S6.** Same as Fig. 10, for DF heating by a cosmological sequence of satellites, but for a steep-cusp host halo with \((c_{2h}, s_{1h}) = (5, 1.5)\) instead of NFW.

**Figure S7.** The effect of added satellite mass to the cusp.

**Figure S8.** Toy-model estimates for satellite penetration and energy deposited in the host cusp by DF, similar to Fig. 3, but for the more accessible parameters \((c_2, s_1)\) instead of the natural Dekel–Zhao parameters \((c, \alpha)\).

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