Electroweak Precision Observables, New Physics and the Nature of a 126 GeV Higgs Boson

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ABSTRACT: We perform the fit of electroweak precision observables within the Standard Model with a 126 GeV Higgs boson, compare the results with the theoretical predictions and discuss the impact of recent experimental and theoretical improvements. We introduce New Physics contributions in a model-independent way and fit for the $S$, $T$ and $U$ parameters, for the $\epsilon_{1,2,3,b}$ ones, for modified $Zb\bar{b}$ couplings and for a modified Higgs coupling to vector bosons. We point out that composite Higgs models are very strongly constrained. Finally, we compute the bounds on dimension-six operators relevant for the electroweak fit.
1 Introduction

Electroweak Precision Observables (EWPO) have played a key role in constraining New Physics (NP) for the past twenty years [1–11]. The most striking examples of the power of these indirect constraints are the prediction of the top and Higgs masses. Concerning physics beyond the Standard Model (SM), the $\epsilon_{1,2,3,b}$ parameterization [6–8] allowed to extract interesting information without knowing the Higgs mass, although the constraining power of EWPO was somewhat diluted by the missing information on the Higgs boson and by the approximations necessary to write all LEP observables in terms of the $\epsilon_{1,2,3,b}$ parameters.
The experimental situation improved dramatically in the past year, with the precise measurement of the Higgs mass at the LHC [12–15]. In addition, the information on other key SM parameters such as the top and W boson masses has increased considerably, leading altogether to a sizable progress in the electroweak (EW) fit. It is therefore phenomenologically relevant to reassess the constraining power of the EW fit in the light of these recent experimental improvements. To this aim, we perform the EW fit in the SM and update the constraints on oblique NP and on modified $Zb\bar{b}$ couplings. Although the direct measurement of the Higgs boson mass completes the SM parameters relevant for the EW fit and thus makes the use of the $\epsilon_{1,2,3,b}$ parameters unnecessary, for the sake of comparison with previous analyses we will present also results for NP in this parameterization.

On the theory side, the full two-loop fermionic EW contributions to the $R_0^b$ observable have been recently numerically calculated in ref. [16]. The implementation of this result in the global fit has a large impact but represents a nontrivial problem, as we illustrate in detail below.

A very interesting question that can be tackled with present data is whether the Higgs boson is elementary or composite. Using a general effective Lagrangian for Higgs boson interactions [17–20], we analyze the constraints on the Higgs coupling to vector bosons, and find out that this coupling can be determined from the fit with an uncertainty of 5% at 95% probability, while much larger departures from the SM value are expected in generic composite Higgs models. Thus, the EW fit points to an elementary Higgs or to composite Higgs models in which additional contributions are present to restore the agreement with EWPO.

Finally, we consider the most general effective Lagrangian relevant for EWPO and compute the constraints on the coefficients of dimension six operators, which can be translated into lower bounds on the NP scale assuming a given value for the couplings.

To obtain our results, we perform a Bayesian analysis using the BAT library [21] and our own implementation of the EWPO formulæ. We have tested the agreement of our code with the ZFITTER (v6.43) one [22–25] and with outputs from the formulæ in refs. [26, 27].

The paper is organized as follows. In Section 2 we present the ingredients of the SM fit, the fitting procedure and the SM results. In Section 3.1 we present the results for the oblique parameters $S$, $T$ and $U$. In Section 3.2 we discuss the results for $\epsilon_{1,2,3,b}$ parameters. In Section 3.3 we report the constraints on modified $Zb\bar{b}$ couplings. In Section 3.4 we present constraints on the Higgs coupling to vector bosons. In Section 3.5 we discuss the constraints on the effective Lagrangian relevant for EWPO and the bounds on the NP scale. Finally, in Section 4 we summarize our findings. Some technical details are presented in Appendices A and B, while more information on the fit results is reported in Appendices C, D and E.

## 2 Standard Model fit

The part of the SM Lagrangian relevant for the computation of EWPO can be defined in terms of the following free parameters: the fine structure constant $\alpha$, the muon decay constant $G_\mu$, the Z boson mass $M_Z$, the strong coupling $\alpha_s(M^2_Z)$, the top quark mass $m_t$
and the Higgs mass $m_h$. In addition, we introduce the effective parameter $\Delta \alpha_{\text{had}}^{(5)}(M_Z^2)$ to take into account the hadronic contribution to the running of $\alpha$. In terms of the seven parameters above, the SM prediction for all other EWPO can be computed.\footnote{\textsuperscript{1}While they are negligible in most cases, we have kept all fermion masses whenever relevant. Furthermore, we have neglected fermion mixing.}

In the Bayesian approach we are following (see ref. \cite{28} for details on the statistical treatment), prior distributions for the parameters $\alpha$, $G_\mu$, $M_Z$, $\alpha_s(M_Z^2)$, $m_t$, $m_h$ and $\Delta \alpha_{\text{had}}^{(5)}(M_Z^2)$ have to be specified. However, given the very accurate experimental measurements of these parameters (see below), the results are insensitive to the choice of any reasonable prior.\footnote{\textsuperscript{2}In practice, any reasonable prior, convoluted with the experimental measurement, will coincide with the experimental likelihood. Thus, we can use directly as prior for the above parameters their experimental gaussian likelihood.}

The numerical results presented in the following are derived computing the region containing 68\% of a marginalized probability distribution function (p.d.f.) starting from the mode and then symmetrizing the error, \textit{i.e.}, the central value corresponds to the center of the 68\% probability region and not to the mode. Since all p.d.f.’s obtained from the fit are almost gaussian, there is very little dependence on the prescription adopted.

### 2.1 Experimental values of SM parameters

The recent measurements of $m_h$ by the ATLAS \cite{12} and CMS \cite{14} experiments are given by

$$m_h = \begin{cases} & 125.5 \pm 0.2 \text{ (stat) } ^{+0.5}_{-0.6} \text{ (syst) GeV ATLAS,} \\ & 125.7 \pm 0.3 \text{ (stat) } \pm 0.3 \text{ (syst) GeV CMS.} \end{cases}$$

We adopt the average $m_h = 125.6 \pm 0.3$ GeV in the current study.\footnote{This na"ive average might underestimate the error neglecting possible correlations in the systematics, however even doubling the error would not affect any of the results in this paper.}

According to ref. \cite{29, 30}, the world average of $\alpha_s(M_Z^2)$ from the fit to various data, excluding the EW precision measurements, is given by $\alpha_s(M_Z^2) = 0.1184 \pm 0.0006$.

For the hadronic contribution to the running of the electromagnetic coupling, we adopt the recent evaluation $\Delta \alpha_{\text{had}}^{(5)}(M_Z^2) = 0.02750 \pm 0.00033$ in ref. \cite{31}. Note that other recent studies have reported much smaller uncertainties, e.g., $\Delta \alpha_{\text{had}}^{(5)}(M_Z^2) = 0.02757 \pm 0.00010$ \cite{32}, $0.027626 \pm 0.000138$ \cite{33} and $0.027498 \pm 0.000135$ \cite{34}, where the first result relies on pQCD, and the last one has been derived with the Adler function approach. The result of ref. \cite{33} differs from ref. \cite{31} mainly in the use of exclusive (instead of inclusive) data in the range 1.2–2 GeV. Since exclusive determinations suffer from an unknown systematic uncertainty, we use the conservative result of ref. \cite{31}. We prefer not to rely on the model-dependent results of refs. \cite{32} and \cite{34}, although they are consistent with the values we are using.

In the absence of a world average for the top pole mass, we adopt the Tevatron average $m_t = 173.18 \pm 0.56 \text{ (stat) } \pm 0.75 \text{ (syst) GeV} = 173.2 \pm 0.9 \text{ GeV} \cite{35}$, fully compatible with the LHC result $m_t = 173.3 \pm 0.5 \text{ (stat) } \pm 1.3 \text{ (syst) GeV} \cite{36}$. Since there might be subtleties related to the precise definition of the pole mass measured at Tevatron and LHC, we also use for comparison the determination of the $\overline{\text{MS}}$ mass $m_t(\overline{m}_t) = 163.3 \pm 2.7$ GeV obtained

$$\overline{m}_t(\overline{m}_t) = 163.3 \pm 2.7 \text{ GeV obtained}$$
from the measurement of the $t\bar{t}$ production cross-section \[37\]. This value corresponds to $m_t = 173.3 \pm 2.8$ GeV.

For completeness, the other quark masses are taken to be $m_u(2\text{ GeV}) = 0.0023$ GeV, $m_d(2\text{ GeV}) = 0.0048$ GeV, $m_s(2\text{ GeV}) = 0.095$ GeV, $m_c(m_c) = 1.275$ GeV and $m_b(m_b) = 4.18$ GeV \[29\].

The renormalization group runnings of the strong coupling constant and the fermion masses are taken into account up to three-loop level \[38–40\].

The measurement of the $Z$ boson mass is taken from LEP: $M_Z = 91.1875 \pm 0.0021$ GeV \[41\]. Finally, the parameters $G_{\mu}$ and $\alpha$ are fixed to be constants: $G_{\mu} = 1.1663787 \times 10^{-5}$ GeV$^{-2}$ and $\alpha = 1/137.035999074$, respectively \[29\].

### 2.2 Theoretical expressions for EWPO

The SM contributions to the EWPO have been calculated very precisely including higher-order radiative corrections. We adopt the on-mass-shell renormalization scheme \[42–45\], where the weak mixing angle is defined in terms of the physical masses of the gauge bosons:

$$ s_W^2 \equiv \sin^2 \theta_W = 1 - \frac{M_W^2}{M_Z^2}, $$ (2.2)

and $c_W^2 = 1 - s_W^2$.

The Fermi constant $G_{\mu}$ in $\mu$ decay is taken as an input quantity instead of the $W$-boson mass, since the latter has not been measured very precisely compared to the former. The relation between $G_{\mu}$ and $M_W$ is written as

$$ G_{\mu} = \frac{\pi \alpha}{\sqrt{2} s_W^2 M_W^2} (1 + \Delta r), $$ (2.3)

where $\Delta r$ represents radiative corrections. From eq. (2.3), the $W$-boson mass is calculated as

$$ M_{W}^2 = \frac{M_Z^2}{2} \left( 1 + \sqrt{1 - \frac{4 \pi \alpha}{\sqrt{2} G_{\mu} M_Z^2} (1 + \Delta r)} \right). $$ (2.4)

The radiative corrections to $\Delta r$ are known very precisely. In the current study, we employ the approximate formula for $M_W$, equivalently for $\Delta r$, in ref. \[46\], which includes the full one-loop EW corrections of $O(\alpha)$ \[42, 43\], the full two-loop QCD corrections of $O(\alpha \alpha_s)$ \[47–53\], three-loop QCD corrections of $O(G_{\mu} \alpha_s^2 m_t^2 (1 + M_Z^2/m_t^2 + (M_Z^2/m_t^2)^2))$ \[54–56\], the full two-loop EW corrections of $O(\alpha^2)$ \[46, 57–70\], and leading three-loop corrections of $O(G_{\mu}^2 \alpha_s m_t^4)$ and $O(G_{\mu}^3 m_t^6)$ \[71, 72\]. Further higher-order corrections are known to be negligibly small \[73–78\]. The remaining theoretical uncertainty in $M_W$ coming from missing higher-order corrections is estimated to be 4 MeV \[46\]. Since this residual uncertainty is much smaller than the present experimental one, we do not take it into account.\(^4\) A comprehensive summary of the radiative corrections can be found in ref. \[79\].

\(^4\)This uncertainty should however be added to the SM prediction quoted in table 3.
The interaction between the $Z$ boson and the neutral current can be written in terms of the effective $Zf\bar{f}$ couplings $g_V^f$ and $g_A^f$, of $g_R^f$ and $g_L^f$, or of $\rho_Z^f$ and $\kappa_Z^f$:

$$\mathcal{L} = \frac{e}{2s_W c_W} Z_\mu \sum_f f \left( g_V^f \gamma_\mu - g_A^f \gamma_\mu \gamma_5 \right) f,$$

$$= \frac{e}{2s_W c_W} Z_\mu \sum_f f \left[ g_R^f \gamma_\mu (1 + \gamma_5) + g_L^f \gamma_\mu (1 - \gamma_5) \right] f,$$

$$= \frac{e}{2s_W c_W} \sqrt{\rho_Z^f} Z_\mu \sum_f f \left[ (I_3^f - 2Q_f \kappa_Z^f s_W^2) \gamma^\mu - I_3^f \gamma^\mu \gamma_5 \right] f,$$

where $e^2 = 4\pi\alpha$, $Q_f$ is the electric charge of the fermion $f$ and $I_3^f$ is the third component of weak isospin. The effective mixing angle for a given fermion $f$ is defined through the relation

$$\sin^2 \theta_{\text{eff}}^f = \text{Re}(\kappa_Z^f) s_W^2 = \frac{1}{4|Q_f|} \left[ 1 - \text{Re} \left( \frac{g_V^f}{g_A^f} \right) \right].$$

The radiative corrections to the effective couplings and the weak mixing angle depend on the flavour of final-state fermions in general. The corrections to $\sin^2 \theta_{\text{eff}}^f$ are given in the forms of approximate formulae [80–82], including the full two-loop EW corrections of $O(\alpha^2)$ as well as leading $O(G^{2}_\mu \alpha_s m_t^2)$ and $O(G^{3}_\mu m_t^4)$ corrections, where the bosonic two-loop EW contribution is still missing only in the $Z \rightarrow b\bar{b}$ channel. The theoretical uncertainty from missing higher-order corrections is estimated to be $4.7 \times 10^{-5}$ for the leptonic channels [80, 81], and we neglect it in the following. We use those formulae to calculate the coupling $\text{Re}(\kappa_Z^f)$ through eq. (2.8), while the imaginary part of $O(\alpha)$ is also included.

The complete two-loop formulæ for the coupling $\rho_Z^f$ are currently missing. Recently, the complete fermionic two-loop EW corrections have been calculated for $R_0^b = \Gamma_b/\Gamma_h$ in ref. [16], where an approximate formula has been presented. However, from this approximate formula alone we cannot extract the values of $\rho_Z^f$ including fermionic two-loop corrections, that are necessary to compute other $\rho_Z^f$-dependent observables such as $R_0^b$, $R_0^c$, $\Gamma_Z$ and the hadronic cross section (see below for their definitions). The authors of ref. [16] have kindly provided us with the approximate formulæ for $\Gamma_u/\Gamma_b$ and $\Gamma_d/\Gamma_b$ [83], which allow us to use the experimental information on one more observable in addition to $R_0^b$. To illustrate the impact of these two-loop corrections, we present our results for the SM fit in two scenarios. First, we use only the previously known leading and (where available) next-to-leading two-loop EW contributions of $O(G^{2}_\mu m_t^4)$ and $O(G^{3}_\mu m_t^2 M_Z^2)$ in the large-$m_t$ expansion, together with the leading three-loop corrections of $O(G^{2}_\mu \alpha_s m_t^4)$ and $O(G^{3}_\mu m_t^4)$. Second, we use the approximate formulæ for $\Gamma_u/\Gamma_b$ and $\Gamma_d/\Gamma_b$ adding three free parameters to the fit, which represent the unknown corrections to $\rho_Z^u$, $\rho_Z^d$ and $\rho_Z^\nu$. The corrections to $\rho_Z^{u,d,\nu}$ can then be determined using the formulæ for $\Gamma_u/\Gamma_b$ and $\Gamma_d/\Gamma_b$. This is the optimal use we can make of the presently available theoretical information. It will be interesting to compare the fitted values of $\delta \rho_Z^{u,d,\nu}$, $\delta \rho_Z^u$, $\delta \rho_Z^d$ and $\delta \rho_Z^\nu$ with the theoretical expressions, once
these will be available. As we shall see below, the corrections computed in ref. [16, 83] are surprisingly large, so that an independent check of the computation would be very useful.

In the following, we consider so-called pseudo observables at the $Z$ pole [84, 85], which are not directly measurable in experiments but can be extracted from real observables by subtracting initial-state QED corrections and a part of final-state QED/QCD corrections.

The asymmetry parameter $A_f$ for a channel $Z \to f \bar{f}$ is defined in terms of the effective couplings:

$$A_f = \frac{2 \Re \left( \frac{g_f^v}{g_A^f} \right)}{1 + \left[ \Re \left( \frac{g_f^v}{g_A^f} \right) \right]^2}.$$  \hfill (2.9)

The left-right asymmetry, the forward–backward asymmetry and the longitudinal polarization of the $\tau \bar{\tau}$ channel are written in terms of the asymmetry parameters:

$$A_{LR}^0 = A_e,$$  \hfill (2.10)

$$A_{FB}^0 = A_e A_f,$$  \hfill (2.11)

$$P_{\text{pol}}^\tau = A_{\tau}.$$  \hfill (2.12)

The partial width of $Z$ decaying into a charged-lepton pair $\ell \bar{\ell}$, including contribution from final-state QED interactions, is given in terms of the effective couplings by [22, 79]:

$$\Gamma_\ell = \Gamma_0 \left| \rho_\ell \right| \sqrt{1 - \frac{4m_\ell^2}{M_Z^2}} \left[ \left( 1 + \frac{2m_\ell^2}{M_Z^2} \right) \left( \frac{|g_\ell|^2}{g_A^f} \right)^2 + 1 \right] - \frac{6m_\ell^2}{M_Z^2} \left( 1 + \frac{3\, \alpha(M_Z^2)}{4\, \pi} Q_\ell^2 \right),$$  \hfill (2.13)

where $\Gamma_0 = G_\mu M_Z^3/(24\sqrt{2}\pi)$ and $m_\ell$ is the mass of the final-state lepton. In the case of the $Z \to q \bar{q}$ channels, final-state QCD interactions have to be taken into account in addition to the QED ones:

$$\Gamma_q = N_c \left| \rho_\ell \right|^2 \left[ \frac{|g_\ell|^2}{g_A^f} \left( R_\ell^q(M_Z^2) + R_A^q(M_Z^2) \right) \right] + \Delta_{\text{EW/QCD}},$$  \hfill (2.14)

where $N_c$ is the color factor, and $R_\ell^q(s)$ and $R_A^q(s)$ are the so-called radiator factors for which we refer to refs. [22, 79, 86]. We add recent results for $O(\alpha_4)$ corrections [87] to the radiator functions. The last term $\Delta_{\text{EW/QCD}}$ denotes non-factorizable EW-QCD corrections [22, 88, 89]: $\Delta_{\text{EW/QCD}} = -0.113$ MeV for $q = u, c$, $-0.160$ MeV for $q = d, s$ and $-0.040$ MeV for $q = b$.\footnote{The non-factorizable EW-QCD corrections have been neglected in the results of ref. [16, 83].}

The total decay width of the $Z$ boson, denoted by $\Gamma_Z$, is then given by the sum of all possible channels:

$$\Gamma_Z = 3 \Gamma_\nu + \Gamma_e + \Gamma_\mu + \Gamma_\tau + \Gamma_h,$$  \hfill (2.15)

where we have defined the hadronic width $\Gamma_h = \sum_q \Gamma_q$. Moreover the ratios of the widths

$$R_{\ell}^0 = \frac{\Gamma_h}{\Gamma_\ell}, \quad R_q^0 = \frac{\Gamma_q}{\Gamma_h},$$  \hfill (2.16)
and the cross section for $e^+e^- \to Z \to \text{hadrons}$ at the $Z$ pole

$$\sigma^0_h = \frac{12\pi \Gamma_e \Gamma_h}{M_Z^2 \Gamma_Z^2}$$  \hspace{1cm} (2.17)

are part of the EWPO.

For the $W$-boson decay width $\Gamma_W$, we use the one-loop formula in refs. [45, 79, 90].

### 2.3 Experimental data for EWPO and fit results

The latest Tevatron average of the $W$-boson mass is $M_W = 80.385 \pm 0.015$ GeV [91]. We use the results for $\Gamma_Z$, $\sigma^0_h$, $P^\text{pol}_f$, $A_f$, $A^{0,f}_\text{FB}$ and $R^0_f$ from SLD/LEP-I [41, 84] and $\Gamma_W$ from LEP-II/Tevatron [85]. All experimental inputs are summarized in the second column of table 1, where we take into account the correlations among the inputs that can be found in ref. [84].

In the third column of table 1 we present the results of the SM fit obtained using the top pole mass and the expressions for $\Gamma_{u/b}$ and $\Gamma_{d/b}$ from refs. [16, 83]. As discussed above, in this case we do not have enough information to compute $\Gamma_Z$, $\sigma^0_h$ and $R^0_\ell$, at the same level of accuracy of $R^0_b$ and $R^0_c$. We therefore add three free parameters to the fit, representing the fermionic two-loop corrections $\delta \rho^Z_u$, $\delta \rho^Z_d$ and $\delta \rho^Z_b$. These parameters affect only the observables $\Gamma_Z$, $\sigma^0_h$ and $R^0_\ell$, since we have

$$R^0_b = \frac{\Gamma_b}{\Gamma_h} = \frac{1}{1 + 2 \left( \frac{\Gamma_u}{\Gamma_b} + \frac{\Gamma_d}{\Gamma_b} \right)}, \quad R^0_c = \frac{\Gamma_c}{\Gamma_h} = \frac{\Gamma_u}{\Gamma_b} \left( \frac{\Gamma_u}{\Gamma_b} + \frac{\Gamma_d}{\Gamma_b} \right),$$  \hspace{1cm} (2.18)

where we have used the approximation $\Gamma_u = \Gamma_c$ and $\Gamma_d = \Gamma_s$. In this way, while we cannot predict $\Gamma_Z$, $\sigma^0_h$ and $R^0_\ell$, we obtain a posterior for the parameters $\delta \rho^Z_u$, $\delta \rho^Z_d$ and $\delta \rho^Z_b$, which can be compared to the theoretical expressions once they become available. The other parameters $\delta \rho^Z_u$ and $\delta \rho^Z_d$ are determined from $\delta \rho^Z_b$ through $\Gamma_{u/b}$ and $\Gamma_{d/b}$, respectively. Notice that fits performed using the formula for $R^0_b$ from ref. [16] and the formulæ for $\rho^f_Z$ from ref. [63] are inconsistent, since the change in $R^0_b$ implies a change in $R^0_c, \ell$, $\Gamma_Z$ and $\sigma^0_h$. Furthermore, the results of ref. [16] imply much larger two-loop fermionic corrections than expected from the expansion in ref. [63]. In fact, we can estimate the size of the unknown two-loop corrections as follows:

$$\delta \rho^Z_u - \delta \rho^Z_b \approx \frac{\Gamma_{u}/\Gamma_b - \Gamma'_{u}/\Gamma'_b}{\Gamma'_{u}/\Gamma'_b} = \begin{cases} 4.8 \times 10^{-3} & \text{for } q = u, \\ 4.4 \times 10^{-3} & \text{for } q = d, \end{cases}$$  \hspace{1cm} (2.19)

where $\Gamma_f$ ($\Gamma'_f$) denotes a partial width including (omitting) the contribution from $\delta \rho^f_Z$, and the approximation $\rho^Z_{u/d} \approx 1$ has been used. Since these corrections are comparable in size to one-loop contributions, it would be desirable to have an independent confirmation of the calculation of ref. [16].

From the fit we also obtain posteriors for the SM parameters $\alpha_s(M_Z^2)$, $\Delta\alpha^{(5)}_{\text{had}}(M_Z^2)$, $M_Z$, $m_l$ and $m_h$ (see table 1). As can be seen in figure 1, while the posteriors are dominated...
indirect determination with a compatible result and a remarkable accuracy (with the well-
known exception of the Higgs mass which is poorly indirectly determined). The correlation
matrix for the posteriors is given in table 11.

|                | Data            | Fit             | Indirect        | Pull |
|----------------|-----------------|-----------------|-----------------|------|
| $\alpha_s(M_Z^2)$ | 0.1184 ± 0.0006 | 0.1184 ± 0.0006 | 0.078 ± 0.024   | −1.9 |
| $\Delta \alpha^{(5)}_{\text{had}}(M_Z^2)$ | 0.02750 ± 0.00033 | 0.02742 ± 0.00026 | 0.02728 ± 0.00043 | −0.4 |
| $M_Z$ [GeV]    | 91.1875 ± 0.0021 | 91.1878 ± 0.0021 | 91.204 ± 0.013  | +1.2 |
| $m_t$ [GeV]    | 173.2 ± 0.9     | 173.5 ± 0.8     | 175.7 ± 2.6     | +0.9 |
| $m_b$ [GeV]    | 125.6 ± 0.3     | 125.6 ± 0.3     | 98.5 ± 27.7     | −0.8 |
| $\delta p_{Z}^2$ | —              | −0.0052 ± 0.0031 | —               | —    |
| $\delta p_{Z}^3$ | —              | −0.0002 ± 0.0010 | —               | —    |
| $\delta p_{Z}$  | —              | −0.0021 ± 0.0011 | —               | —    |
| $\delta p_{Z}^2$ | 0.0026 ± 0.0012 | —               | —               | —    |
| $\delta p_{Z}^3$ | 0.0023 ± 0.0012 | —               | —               | —    |
| $M_W$ [GeV]    | 80.385 ± 0.015  | 80.366 ± 0.007  | 80.361 ± 0.007  | −1.4 |
| $\Gamma_W$ [GeV] | 2.085 ± 0.042  | 2.0890 ± 0.0006 | 2.0890 ± 0.0006 | +0.1 |
| $\Gamma_Z$ [GeV] | 2.4952 ± 0.0023 | 2.4952 ± 0.0023 | —               | —    |
| $\delta h$ [nb] | 41.540 ± 0.037  | 41.539 ± 0.037  | —               | —    |
| $\sin^2 \theta_{\text{eff}}^{\text{had}}(Q_{\text{FB}})$ | 0.2324 ± 0.0012 | 0.23145 ± 0.00009 | 0.23145 ± 0.00009 | −0.8 |
| $P_{\text{FB}}^{\text{pol}}$ | 0.1465 ± 0.0033 | 0.1476 ± 0.0007 | 0.1476 ± 0.0007 | +0.3 |
| $A_t^0$ (SLD)  | 0.1513 ± 0.0021 | 0.1476 ± 0.0007 | 0.1470 ± 0.0008 | −1.9 |
| $A_c$           | 0.670 ± 0.027   | 0.6681 ± 0.0003 | 0.6681 ± 0.0003 | −0.1 |
| $A_b$           | 0.923 ± 0.020   | 0.93466 ± 0.00006 | 0.93466 ± 0.00006 | +0.6 |
| $A_{FB}^{0,t}$  | 0.0171 ± 0.0010 | 0.0163 ± 0.0002 | 0.0163 ± 0.0002 | −0.8 |
| $A_{FB}^{0,c}$  | 0.0707 ± 0.0035 | 0.0739 ± 0.0004 | 0.0740 ± 0.0004 | +0.9 |
| $A_{FB}^{0,b}$  | 0.0992 ± 0.0016 | 0.1034 ± 0.0005 | 0.1038 ± 0.0005 | +2.7 |
| $R_0^{0,t}$     | 20.767 ± 0.025  | 20.768 ± 0.025  | —               | —    |
| $R_0^{0,c}$     | 0.1721 ± 0.0030 | 0.17247 ± 0.00002 | 0.17247 ± 0.00002 | +0.1 |
| $R_0^{0,b}$     | 0.21629 ± 0.00066 | 0.21492 ± 0.00003 | 0.21492 ± 0.00003 | −2.1 |

Table 1. Summary of experimental data and fit results in the SM, including the subleading two-loop fermionic EW corrections to $\rho_Z$, with the results of ref. [16, 83] and introducing the parameters $\delta \rho_{Z}^{c,b}$. The values in the column “Indirect” are determined without using the corresponding experimental information. The last column shows the pulls in units of standard deviations evaluated from the p.d.f.’s of “Data” and “Indirect” as explained in ref. [92]. For completeness we also report the fit result for $\delta \rho_{Z}^{u,d}$ computed from $\delta \rho_{Z}$ using $\Gamma_{u,d}/\Gamma_b$.

by the experimental input (as desirable for fit input parameters), the fit would provide an indirect determination with a compatible result and a remarkable accuracy (with the well-known exception of the Higgs mass which is poorly indirectly determined). The correlation matrix for the posteriors is given in table 11.

To show the impact on the fit of the new calculation of $R_0^{0,b}$ [16], we present in table 2 the results obtained using instead refs. [57–63] for the leading and next-to-leading terms in

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6 Actually the indirect determination of $\alpha_s(M_Z^2)$ is not very precise when we use the results of ref. [16], due to the uncertainty related to $\delta \rho_{Z}^{c,b}$, $\delta \rho_{Z}^{c}$ and $\delta \rho_{Z}^{b}$. This can be seen by comparing the first and the next-to-last plots in figure 1.
Figure 1. Comparisons between the direct measurement and the posterior probability distributions for the input parameters in the SM fit, together with their indirect determinations from the EWPO measurements, obtained by assuming a flat prior for the single parameter under consideration. Using the results of ref. [16, 83] and introducing the parameters $\delta \rho_{f, e, b}$, the subleading two-loop fermionic EW corrections to $\rho_f^{L}$ have been taken into account in the plots, except for the bottom-centre and bottom-right plots, in which the corrections have been omitted. Here and in the following, the dark (light) regions correspond to 68% (95%) probability. In the bottom-right plot, we report the indirect determinations of the Higgs mass excluding the observables $M_W$, $\Gamma_Z$, $P_{pol}^0$, $A_l$ and $A_{0,b}^F$, except for the one specified in each row. The vertical blue (red) band represents the one obtained from the fit with all the observables (from the direct measurement). We assume a flat prior for the Higgs mass ranging from 10 MeV to 1 TeV.

Using as SM input $m_t = 173.3 \pm 2.8$ GeV obtained from the $\overline{\text{MS}}$ mass instead of the Tevatron pole mass average, one obtains the posterior $m_t = 174.6 \pm 1.9$ (174.9 $\pm$ 1.9) GeV using the results of ref. [16, 83] (using the large-$m_t$ expansion), on the upper end of the Tevatron result. Concerning the EWPO fit, the main observables affected by the change in $m_t$ are $M_W$, $\Gamma_W$ and $R_0^0$, for which we obtain $M_W = 80.371 \pm 0.011$ (80.373 $\pm$ 0.010) GeV,
The compatibility of $M_W$, $\mathcal{A}_\ell$ and $A_{FB}^{0,b}$ is shown in figure 2. While these results are stable against the inclusion of the recently calculated two-loop fermionic corrections to $R_b^0$, the compatibility of $R_b^0$ is worsened by the inclusion of the results in ref. [16], as can be seen by comparing the plots in figure 3.

In the bottom-right plot in figure 1 we report the indirect determinations of the Higgs mass obtained considering the constraints from $M_W$, $\Gamma_Z$, $P_{\tau}^{pol}$, $A_{FB}^{0}$ and $A_{FB}^{0,b}$ one at a time, as well as the full fit result and the direct measurement, omitting the results of ref. [16].

Our numerical results agree with those obtained using the ZFITTER package [22–25]. Our fit results are compatible with the ones obtained by the LEP Electroweak Working Group [85] and also with the ones in refs. [93, 94]. A comparison with the recent Gfitter group fits [95, 96] is not straightforward since the result for $R_b^0$ of ref. [16] has been used without correspondingly modifying other $\Gamma_q$-related observables and without accounting for other possibly large fermionic two-loop corrections.

Table 2. Same as table 1, but using the large-$m_t$ expansion for the two-loop fermionic EW corrections to $\rho_Z$.

|                   | Data    | Fit     | Indirect | Pull   |
|-------------------|---------|---------|----------|--------|
| $\alpha_s(M_Z^2)$| 0.1184 ± 0.0006 | 0.1184 ± 0.0006 | 0.1193 ± 0.0027 | +0.3   |
| $\Delta\alpha_s^{(5)}(M_Z^2)$ | 0.0275 ± 0.00033 | 0.02740 ± 0.00026 | 0.02725 ± 0.00042 | −0.5   |
| $M_Z$ [GeV]      | 91.1875 ± 0.0021 | 91.1878 ± 0.0021 | 91.197 ± 0.012  | +0.8   |
| $m_t$ [GeV]      | 173.2 ± 0.9   | 173.5 ± 0.8  | 176.3 ± 2.5   | +1.1   |
| $m_h$ [GeV]      | 125.6 ± 0.3  | 125.6 ± 0.3  | 97.3 ± 26.9   | −0.9   |
| $M_W$ [GeV]      | 80.385 ± 0.015 | 80.367 ± 0.007 | 80.362 ± 0.007 | −1.4   |
| $\Gamma_W$ [GeV] | 2.085 ± 0.042 | 2.0891 ± 0.0006 | 2.0891 ± 0.0006 | +0.1   |
| $\Gamma_Z$ [GeV] | 2.4952 ± 0.0023 | 2.4953 ± 0.0004 | 2.4953 ± 0.0004 | +0.0   |
| $\sigma_h^0$ [nb] | 41.540 ± 0.037 | 41.484 ± 0.004 | 41.484 ± 0.004 | −1.5   |
| $\sin^2 \theta_{lep}^{t\bar{t}}(Q_{FB}^{pol})$ | 0.2324 ± 0.0012 | 0.23145 ± 0.00009 | 0.23144 ± 0.00009 | −0.8   |
| $P_{FB}^{pol}$    | 0.1465 ± 0.0033 | 0.1476 ± 0.0007 | 0.1477 ± 0.0007 | +0.3   |
| $\mathcal{A}_\ell$ (SLD) | 0.1513 ± 0.0021 | 0.1476 ± 0.0007 | 0.1471 ± 0.0008 | −1.9   |
| $A_\ell$         | 0.670 ± 0.027 | 0.6682 ± 0.0003 | 0.6682 ± 0.0003 | −0.1   |
| $A_0$            | 0.923 ± 0.020 | 0.93466 ± 0.00006 | 0.93466 ± 0.00006 | +0.6   |
| $A_{FB}^{0,\ell}$ | 0.0171 ± 0.0010 | 0.0163 ± 0.0002 | 0.0163 ± 0.0002 | −0.8   |
| $A_{FB}^{0,c}$    | 0.0707 ± 0.0035 | 0.0740 ± 0.0004 | 0.0740 ± 0.0004 | +0.9   |
| $A_{FB}^{0,b}$    | 0.0992 ± 0.0016 | 0.1035 ± 0.0005 | 0.1039 ± 0.0005 | +2.8   |
| $R_\ell^0$       | 20.767 ± 0.025 | 20.735 ± 0.004 | 20.734 ± 0.004 | −1.3   |
| $R_{FB}^0$       | 0.1721 ± 0.0030 | 0.17223 ± 0.00002 | 0.17223 ± 0.00002 | +0.0   |
| $R_b^0$          | 0.21629 ± 0.00066 | 0.21575 ± 0.00003 | 0.21575 ± 0.00003 | −0.8   |

$\Gamma_W = 2.0894±0.0009\ (2.0896±0.0008)$ GeV and $R_b^0 = 0.21488±0.00007\ (0.21570±0.00007)$. Let us now discuss the compatibility of the SM prediction with experimental data. To this aim, we use the compatibility plots introduced in ref. [92], where the difference in standard deviations between the fit prediction and the experimental result is given by the color coding.

\[
\Gamma_W = 2.0894\pm0.0009 \ (2.0896\pm0.0008) \text{ GeV and } R_b^0 = 0.21488\pm0.00007 \ (0.21570\pm0.00007).
\]
The predictions are computed with the large-

\[ m_h \] expansion for the two-loop fermionic

EW corrections to \[ m_h \] is always

negligible. The predictions are computed with the large-

\[ m_t \] expansion for the two-loop fermionic

EW corrections to \[ m_t \], except for \( R_{c}^{5} \) and \( R_{b}^{5} \) in the last two rows, which are computed with the results of ref. [16, 83].

![Compatibility plots of \( M_W \), \( A_t \) and \( A_{FB}^{0,b} \). Any direct measurement corresponds to a point in the (central value, experimental error) plane, and its compatibility with the indirect determination is given in numbers of standard deviations by the color coding. The present experimental result is indicated by a star.](image)

**Table 3.** SM predictions computed using the theoretical expressions for EWPO without the experimental constraints on the observables, and individual uncertainties associated with each input parameter: \( \alpha_s (M_Z^2) = 0.1184 \pm 0.0006 \), \( \Delta \alpha_{\text{had}}^{(5)} (M_Z^2) = 0.02750 \pm 0.00003 \), \( M_Z = 91.1875 \pm 0.0021 \) GeV and \( m_t = 173.2 \pm 0.9 \) GeV, where the uncertainty associated to \( m_t \) = 125.6 \pm 0.3 GeV is always negligible. The predictions are computed with the large-

\[ m_t \] expansion for the two-loop fermionic

EW corrections to \( m_t \), except for \( R_{c}^{5} \) and \( R_{b}^{5} \) in the last two rows, which are computed with the results of ref. [16, 83].

| Prediction | \( \alpha_s \) | \( \Delta \alpha_{\text{had}}^{(5)} \) | \( M_Z \) | \( m_t \) |
|------------|----------------|----------------|------------|----------|
| \( M_W \) [GeV] | 80.362 \pm 0.008 | \pm 0.000 | \pm 0.006 | \pm 0.003 | \pm 0.005 |
| \( \Gamma_W \) [GeV] | 2.0888 \pm 0.0007 | \pm 0.0002 | \pm 0.0005 | \pm 0.0002 | \pm 0.0004 |
| \( \Gamma_Z \) [GeV] | 2.4951 \pm 0.0005 | \pm 0.0003 | \pm 0.0003 | \pm 0.0002 | \pm 0.0002 |
| \( \sigma_b^0 \) [nb] | 41.484 \pm 0.004 | \pm 0.003 | \pm 0.000 | \pm 0.002 | \pm 0.001 |
| \( \sin^2 \theta_{\text{eff}}^{0,R_{\text{FB}}} \) | 0.23149 \pm 0.00012 | \pm 0.00000 | \pm 0.00012 | \pm 0.00001 | \pm 0.00003 |
| \( R_{c}^{5} \) | 0.1472 \pm 0.0009 | \pm 0.0000 | \pm 0.0009 | \pm 0.0001 | \pm 0.0002 |
| \( A_c \) | 0.6680 \pm 0.0004 | \pm 0.0000 | \pm 0.0004 | \pm 0.0001 | \pm 0.0001 |
| \( A_b \) | 0.93464 \pm 0.00008 | \pm 0.00000 | \pm 0.00007 | \pm 0.00001 | \pm 0.00001 |
| \( A_{FB}^{0,b} \) | 0.0163 \pm 0.0002 | \pm 0.00000 | \pm 0.0002 | \pm 0.00000 | \pm 0.00000 |
| \( A_{FB}^{0,b} \) | 0.0738 \pm 0.0005 | \pm 0.00000 | \pm 0.0005 | \pm 0.0001 | \pm 0.0001 |
| \( R_{c}^{5} \) | 0.13032 \pm 0.00007 | \pm 0.00000 | \pm 0.00006 | \pm 0.00001 | \pm 0.00002 |
| \( R_{b}^{5} \) | 0.20734 \pm 0.0004 | \pm 0.00000 | \pm 0.0002 | \pm 0.00000 | \pm 0.00000 |
| \( R_{c}^{5} \) | 0.17222 \pm 0.00002 | \pm 0.00000 | \pm 0.00001 | \pm 0.00000 | \pm 0.00000 |
| \( R_{b}^{5} \) | 0.21576 \pm 0.00003 | \pm 0.00000 | \pm 0.00000 | \pm 0.00000 | \pm 0.00000 |
| \( R_{c}^{5} \) | 0.17247 \pm 0.00002 | \pm 0.00000 | \pm 0.00001 | \pm 0.00000 | \pm 0.00000 |
| \( R_{b}^{5} \) | 0.21493 \pm 0.00004 | \pm 0.00000 | \pm 0.00000 | \pm 0.00000 | \pm 0.00000 |
Figure 3. Compatibility plot of $R_0^b$ computed using the results of ref. [16] (left) or the large $m_t$ expansion for the two-loop fermionic EW corrections to $\rho_f^Z$ (right).

3 Constraints on New Physics

Let us now discuss the EW fit beyond the SM, using several widely adopted model-independent parameterizations of NP contributions. Before dwelling into the details of the different analyses, a discussion on the inclusion of the results of ref. [16, 83] is mandatory. In our SM fit (see Section 2), we parameterized the unknown two-loop fermionic EW corrections to $\rho_f^Z$ with three free parameters. The fit result selects values of these corrections that are as large as the ones computed by Freitas and Huang, and much larger than naively expected from the large-$m_t$ expansion. Waiting for a complete calculation of these corrections, we cannot use consistently the results of ref. [16, 83] in NP fits where the use of $R_0^\ell$, $\Gamma_Z$ and $\sigma^0_h$ is necessary to constrain NP contributions. Thus, in these cases we only present results obtained using the large-$m_t$ expansion for the two-loop fermionic EW corrections to $\rho_f^Z$, while in other cases we present results using both the large-$m_t$ expansion and the expressions in ref. [16, 83], leaving the choice of the preferred option to the reader. In the latter case, we do not use the observables $\Gamma_Z$, $R_0^\ell$ and $R_0^c$ in the fit. In all the NP fits reported below, the fit result for SM parameters practically coincides with the input reported in table 1.

3.1 Constraints on the oblique parameters

In several NP scenarios, the dominant NP effects appear in the gauge-boson vacuum-polarization corrections, called oblique corrections [97, 98]. If the NP scale is sufficiently higher than the weak scale, the oblique corrections are effectively described by the three independent parameters $S$, $T$ and $U$ [4, 99]:

\begin{align}
S &= -16\pi \Pi_{30}^{NP}(0) = 16\pi \left[ \Pi_{33}^{NP}(0) - \Pi_{3Q}^{NP}(0) \right], \\
T &= \frac{4\pi}{s_W^2 c_W M_Z^2} \left[ \Pi_{11}^{NP}(0) - \Pi_{33}^{NP}(0) \right],
\end{align}

(3.1, 3.2)
Using ref. [16, 83]

| Parameter | Large-\(m_t\) expansion | Using ref. [16, 83] |
|-----------|--------------------------|---------------------|
| \(STU\) fit | \(ST\) fit with \(U = 0\) | \(ST\) fit with \(U = 0\) |
| \(S\) | \(0.04 \pm 0.10\) | \(0.08 \pm 0.10\) |
| \(T\) | \(0.05 \pm 0.12\) | \(0.10 \pm 0.08\) |
| \(U\) | \(0.03 \pm 0.09\) | — |

Table 4. Fit results for the oblique parameters with floating \(U\) or fixing \(U = 0\), using the large-\(m_t\) expansion or with the results of ref. [16, 83] for the two-loop fermionic EW corrections to \(\rho^Z\). In the latter case, we do not consider constraints from \(\Gamma_Z, \sigma_h^0\) and \(R_\ell^0\).

\[
U = 16\pi \left[ \Pi_{11}^{NP}(0) - \Pi_{33}^{NP}(0) \right],
\]

(3.3)

where \(\Pi_{XY}^{NP}\) with \(X, Y = 0, 1, 3, Q\) denotes NP contribution to the vacuum polarization amplitude of the gauge bosons defined, e.g., in ref. [4], \(\Pi'_{XY}(q^2) = \frac{d\Pi_{XY}(q^2)}{dq^2}\), and \(s_W^2\) and \(c_W^2\) represent their SM values. NP contributions to an observable, parameterized by the above oblique parameters, add up to the SM contribution:

\[
O = O_{SM} + O_{NP}(S, T, U),
\]

(3.4)

where \(S = T = U = 0\) in the SM, and we linearize the NP contribution in terms of the oblique parameters [4, 99–102]. Explicit formulæ for the observables are summarized in Appendix A. Actually, all EWPO can be expressed in terms of the following combinations of oblique parameters:

\[
A = S - 2c_W^2 T - \frac{(c_W^2 - s_W^2)U}{2s_W^2},
\]

(3.5)

\[
B = S - 4c_W^2 s_W^2 T,
\]

\[
C = -10(3 - 8s_W^2)S + (63 - 126s_W^2 - 40s_W^4)T.
\]

Note that the parameter \(C\) describes the NP contribution to \(\Gamma_Z\), the parameter \(A\) (the only one containing \(U\)) describes the NP contribution to \(M_W\) and \(\Gamma_W\), and NP contributions to all other EWPO are proportional to \(B\). Clearly, for \(S, T\) and \(U\) all different from zero, \(\Gamma_Z\) is necessary to obtain bounds on the NP parameters, so in this case we only use the large-\(m_t\) expansion. We fit the three oblique parameters together with the SM parameters to the EW precision data in table 1. The fit results are summarized in the second column of table 4, and the correlation matrix is given in table 13. The two-dimensional probability distribution for \(S\) and \(T\) is shown in the left plot of figure 4.

If one fixes \(U = 0\), which is the case in many NP models where \(U \ll S, T\), the fit yields the results in the third (fourth) column of table 4, with correlation matrices given in table 14 (15) omitting (using) the formulæ of ref. [16, 83]. The corresponding two-dimensional distribution is given in the center and right plots in figure 4. As expected, the results in the case \(U = 0\) do not depend sizably on the choice made for the two-loop fermionic EW corrections.
Figure 4. Left: Two-dimensional probability distribution for the oblique parameters $S$ and $T$ obtained from the fit with $S$, $T$, $U$ and the SM parameters, with the large-$m_t$ expansion for the two-loop fermionic EW corrections to $\rho_Z^I$. Center: Two-dimensional probability distribution for the oblique parameters $S$ and $T$ obtained from the fit with $S$, $T$ and the SM parameters with $U = 0$, with the large-$m_t$ expansion for the two-loop fermionic EW corrections to $\rho_Z^I$. The individual constraints from $M_W$, the asymmetry parameters $\sin^2 \theta_{\text{eff}}$, $P_{\text{pol}}$, $A_f$ and $A_0^{f,f}$, with $f = t, c, b$, and $\Gamma_Z$ are also presented, corresponding to the combinations of parameters $A_A$, $A_B$ and $C_A$ in eq. (3.5). Right: Same as center, but using the results of ref. [16, 83]. In this case, the constraint from $\Gamma_Z$ cannot be used.

### 3.2 Constraints on the $\epsilon$ parameters

Aiming at a fully model-independent analysis of EWPO in the absence of experimental information on the Higgs sector, Altarelli and Barbieri introduced the parameters $\epsilon_1$, $\epsilon_2$ and $\epsilon_3$ [6, 7]:

$$\begin{align*}
\epsilon_1 &= \Delta \rho' , \\
\epsilon_2 &= c_0^2 \Delta \rho' + \frac{s_0^2}{c_0^2} \Delta r_W - 2 s_0^2 \Delta \kappa', \\
\epsilon_3 &= c_0^2 \Delta \rho' + (c_0^2 - s_0^2) \Delta \kappa',
\end{align*}$$

(3.6)\hspace{1cm}(3.7)\hspace{1cm}(3.8)

where $\Delta r_W$, $\Delta \rho'$ and $\Delta \kappa'$ are defined through the relations

$$\begin{align*}
s^2_W c^2_W &= \frac{\pi \alpha(M_Z^2)}{\sqrt{2} G_\mu M_Z^2 (1 - \Delta r_W)}, \\
\sqrt{\text{Re} \rho_Z^I} &= 1 + \frac{\Delta \rho'}{2}, \\
\sin^2 \theta_{\text{eff}}^Z &= (1 + \Delta \kappa') s_0^2
\end{align*}$$

(3.9)\hspace{1cm}(3.10)\hspace{1cm}(3.11)

with

$$s_0^2 c_0^2 = \frac{\pi \alpha(M_Z^2)}{\sqrt{2} G_\mu M_Z^2},$$

(3.12)

and $c_0^2 = 1 - s_0^2$. Unlike the oblique parameters $S$, $T$ and $U$ discussed in Section 3.1, the $\epsilon$ parameters include the SM contribution in addition to possible NP contributions. Moreover, they involve not only oblique corrections, but also vertex corrections. The
\[ \begin{array}{c|c|c}
\text{Parameter} & \text{Large-}m_t \text{ expansion} & \text{\epsilon}_{1,3} \text{ fit} \\
\hline
\epsilon_1 [10^{-3}] & 5.6 \pm 1.0 & 6.0 \pm 0.6 \\
\epsilon_2 [10^{-3}] & -7.8 \pm 0.9 & -5.8 \pm 1.3 \\
\epsilon_3 [10^{-3}] & 5.6 \pm 0.9 & 5.9 \pm 0.8 \\
\epsilon_b [10^{-3}] & -5.8 \pm 1.3 & -5.8 \pm 1.3 \\
\end{array} \]

Table 5. Fit results for the \( \epsilon \) parameters, with floating \( \epsilon_{1,2,3,b} \), or with assuming \( \epsilon_2 = \epsilon_2^{SM} \) and \( \epsilon_b = \epsilon_b^{SM} \). The non-universal vertex corrections and the SM values for the \( \epsilon \) parameters are computed with the large-\( m_t \) expansion for the two-loop fermionic EW corrections to \( \rho_Z \).

\( \epsilon \) parameters are defined in such a way that the logarithmic corrections are separated from the large quadratic corrections proportional to the top-quark mass. The quadratic corrections are then parameterized by \( \epsilon_1 \), while the other corrections are included in \( \epsilon_2 \) and \( \epsilon_3 \).

In the SM, the \( Z \to b\bar{b} \) vertex receives large corrections from the top-quark loop, which can be parametrized by an additional parameter \( \epsilon_b \) [8]. However, given the present experimental accuracy on EWPO, the flavour non-universal vertex corrections in the SM have to be taken into account in all channels. We define

\[ \begin{align*}
\rho_f^Z &= \rho_Z^c + \Delta \rho_f^Z, \\
\kappa_f^Z &= \kappa_Z^c + \Delta \kappa_f^Z
\end{align*} \]

for \( f \neq b \), and

\[ \begin{align*}
\rho_b^Z &= \left( \rho_Z^c + \Delta \rho_b^Z \right) (1 + \epsilon_b)^2, \\
\kappa_b^Z &= \frac{\kappa_Z^c + \Delta \kappa_b^Z}{1 + \epsilon_b}
\end{align*} \]

where the non-universal corrections \( \Delta \rho_f^Z \) and \( \Delta \kappa_f^Z \) are defined in Appendix B. In refs. [103, 104], the relations between the observables and the \( \epsilon \) parameters are linearized. However, in the case of the \( W \)-boson mass, the difference between the values derived with and without the linearization is comparable in size to the current experimental uncertainty. Therefore, we do not employ any linearization in our analysis.

We fit the four \( \epsilon \) parameters together with the SM parameters to the precision observables listed in table 1, except for \( \Gamma_W \), which is not directly related to \( \epsilon \)’s. The fit results are given in the second column of table 5, and the corresponding correlation matrix is summarized in table 16. Fixing \( \epsilon_2 = \epsilon_2^{SM} \) and \( \epsilon_b = \epsilon_b^{SM} \) in the fit, we obtain the results in the third column of table 5 with the correlation matrix in table 17. The two-dimensional probability distributions for \( \epsilon_1 \) and \( \epsilon_3 \) in both fits are shown in figure 5, where in the case of \( \epsilon_2 = \epsilon_2^{SM} \) and \( \epsilon_b = \epsilon_b^{SM} \) we also plot the individual constraints. To show the impact of including non-universal vertex corrections, we also report in figure 5 the probability regions obtained omitting these terms.
The corresponding SM predictions for the \( \epsilon \) parameters with the large-\( m_t \) expansion for the two-loop fermionic EW corrections to \( \rho^f_b \) are given by:

\[
\begin{align*}
\epsilon_{1}^{SM} &= (5.21 \pm 0.08) \times 10^{-3} \quad ([5.04, 5.37] \times 10^{-3} \text{ @95\% prob.}), \\
\epsilon_{2}^{SM} &= -(7.37 \pm 0.03) \times 10^{-3} \quad ([{-7.43, -7.32}] \times 10^{-3} \text{ @95\% prob.}), \\
\epsilon_{3}^{SM} &= (5.279 \pm 0.004) \times 10^{-3} \quad ([5.271, 5.288] \times 10^{-3} \text{ @95\% prob.}), \\
\epsilon_{b}^{SM} &= -(6.94 \pm 0.15) \times 10^{-3} \quad ([{-7.24, -6.64}] \times 10^{-3} \text{ @95\% prob.}),
\end{align*}
\]  

(3.15)

where the uncertainties are dominated by the top-quark mass, and the quadratic dependence in \( \epsilon_{1}^{SM} \) and \( \epsilon_{b}^{SM} \) results in the larger uncertainties. The 95\% ranges of \( \epsilon_{1}^{SM} \) and \( \epsilon_{b}^{SM} \) become \([4.71, 5.72] \times 10^{-3}\) and \([-7.49, -6.41] \times 10^{-3}\), respectively, if adopting \( m_t = 173.3 \pm 2.8 \) GeV instead of \( m_t = 173.2 \pm 0.9 \) GeV. Notice that one can define \( \epsilon_{b}^{SM} \) either from the first or from the second of eq. (3.14). We choose to define it from \( \kappa_{Z}^{b} \), so that the prediction is insensitive to the inclusion of two-loop fermionic contributions to \( \rho^f_{Z} \) (this is possible within the approximations inherent in the \( \epsilon \) parameterization). In figure 5 we report the one-dimensional 95\% probability range of the SM predictions for \( \epsilon_{1} \) and \( \epsilon_{3} \), where the latter is invisible due to the tiny error band.

### 3.3 Constraints on the \( Zb\bar{b} \) couplings

Motivated phenomenologically by the long-standing pull in \( A_{FB}^{0,b} \) and by the more recent pull in \( R_{0}^{b} \), and theoretically by the larger coupling to NP in the third generation realized in many explicit models, the possibility of modified \( Zb\bar{b} \) couplings has been extensively studied (see for example refs. [105–118]).
Using ref. [16, 83] to figure 6) where tables 18 and 19. There is also a second region in the fit (not shown in table 6 nor in are summarized in table 6, where the correlation matrices for the posteriors are given in |δg|

| Parameter       | Large-m_t expansion | Using ref. [16, 83] |
|-----------------|---------------------|-------------------|
| δg^R_L          | 0.018 ± 0.007       | 0.019 ± 0.007     |
| δg^L_L          | 0.0028 ± 0.0014     | 0.0016 ± 0.0015   |
| δg^R_A          | 0.021 ± 0.008       | 0.020 ± 0.008     |
| δg^L_A          | -0.015 ± 0.006      | -0.017 ± 0.006    |

Table 6. Fit results for the shifts in the Zb¯b couplings, using the large-m_t expansion or the results in ref. [16, 83] for the two-loop fermionic two-loop EW corrections. In the latter case, we do not consider constraints from Γ_Z, σ_h^0 and R^0_t.

We parameterize NP contributions to the Zb¯b vertex by modifying the couplings in eq. (2.5) in the following way:

\[ g^b_V = (g^b_V)^{SM} + δg^b_V, \quad g^b_A = (g^b_A)^{SM} + δg^b_A, \]

or equivalently by introducing δg^b_R = (δg^b_V - δg^b_A)/2 and δg^b_L = (δg^b_V + δg^b_A)/2. We may assume flat priors either for δg^b_V and δg^b_A or for δg^b_R and δg^b_L, but both choices yield almost identical results. Here we perform a fit with flat priors for δg^b_R and δg^b_L. The results are summarized in table 6, where the correlation matrices for the posteriors are given in tables 18 and 19. There is also a second region in the fit (not shown in table 6 nor in figure 6) where g_R flips its sign.\(^7\)

As shown in the left plots in figure 6, the asymmetries A_h and A^0_{FB} are mainly sensitive to δg^b_R, since their shifts are given in terms of the combination (g^b_L)^{SM}δg^b_R - (g^b_R)^{SM}δg^b_L with \(|(g^b_R)^{SM}| ≪ |(g^b_L)^{SM}|\). On the other hand, R^0_h is associated with (g^b_R)^{SM}δg^b_R + (g^b_L)^{SM}δg^b_L, and mainly constrains δg^b_L.

3.4 Constraints on a non-standard Higgs coupling

A key question to understand the mechanism of EWSB is whether the underlying dynamics is weak or strong. As we shall see below, EWPO strongly constrain the Higgs coupling to vector bosons, and this hints either at a weakly interacting Higgs or at a non-trivial strongly interacting sector in which additional contributions to EWPO are present and restore the agreement with experimental data.

To investigate the question above, it is useful to consider a general Lagrangian for a light Higgs-like scalar field h [17–20]. Under the assumption of an approximate custodial symmetry, the longitudinal W and Z polarizations can be described by the two-by-two matrix Σ(x) = \(\exp(\mathbf{i} \tau^a \chi^a(x)/v)\), with \(\tau^a\) the Pauli matrices and \(v^2 = 1/(\sqrt{2}G_μ)\). Then, assuming that there are no other light states and no new sources of flavour violation, the most general Lagrangian for h can be written as [18, 19]:

\[
\mathcal{L} = \frac{1}{2}(\partial_μ h)^2 - V(h) + \frac{v^2}{4} \text{Tr}(D_μΣ^†D^μΣ) \left(1 + 2a \frac{h}{v} + b \frac{h^2}{v^2} + \cdots \right) \\
- m_{u,i}(\bar{u}_{L,i}, \bar{d}_{L,i}) Σ \begin{pmatrix} u_{R,i} \end{pmatrix} \left(1 + c_u \frac{h}{v} + c_{2u} \frac{h^2}{v^2} + \cdots \right) + \text{h.c.}
\]

\(^7\)The other two allowed regions from the EWPO fit are disfavored by the off Z-pole data [107].
Figure 6. Two-dimensional probability distributions for the parameters $\delta g_R^b$ and $\delta g_L^b$ (left), or $\delta g_V^b$ and $\delta g_A^b$ (right), using the large-$m_t$ expansion (top) or the results of ref. [16, 83] (bottom) for the two-loop fermionic EW corrections to $\rho_Z$. The individual constraints in the left plots are computed by omitting $A_0$, $A_{FB}^0$, $\Gamma_Z$, $\sigma^0_h$, $R^0$, $R^0_c$ and $R^0_b$ except for the one specified in the legend.

$$-
\begin{align*}
-m_d,i (\bar{u}_{L,i}, \bar{d}_{L,i}) & \Sigma \begin{pmatrix} 0 \\ d_{R,i} \end{pmatrix} \left( 1 + c_d \frac{h}{v} + c_{2d} \frac{h^2}{v^2} + \cdots \right) + \text{h.c.} \\
-m_\ell,i (\bar{\nu}_{L,i}, \bar{\ell}_{L,i}) & \Sigma \begin{pmatrix} 0 \\ \ell_{R,i} \end{pmatrix} \left( 1 + c_\ell \frac{h}{v} + c_{2\ell} \frac{h^2}{v^2} + \cdots \right) + \text{h.c.},
\end{align*}
$$

(3.17)

where $V(h)$ is the potential of the scalar field

$$V(h) = \frac{m^2_h}{2} h^2 + \frac{d_3}{6} \left( \frac{3m^2_h}{v} \right) h^3 + \frac{d_4}{24} \left( \frac{3m^2_h}{v^2} \right) h^4 + \cdots.
$$

(3.18)

The SM corresponds to the choice $a = b = c_u = c_d = c_\ell = d_3 = d_4 = 1$ and $c_{2u} = c_{2d} = c_{2\ell} = 0$. The dominant deviations from the SM in EWPO are induced by
the non-standard coupling $a \neq 1$. This generates extra contributions to the $S$ and $T$ parameters [119]:

\begin{align}
S &= \frac{1}{12\pi} (1 - a^2) \ln \left( \frac{\Lambda^2}{m_h^2} \right), \\
T &= -\frac{3}{16\pi c_W^2} (1 - a^2) \ln \left( \frac{\Lambda^2}{m_h^2} \right),
\end{align}

where $\Lambda = 4\pi v/\sqrt{|1 - a^2|}$ is the cutoff of the light Higgs effective Lagrangian. A sum rule for $1 - a^2$ can be written in terms of the total cross sections in different isospin channels of longitudinal EW gauge boson scattering [120], implying $a^2 \leq 1$ unless the $I = 2$ channel dominates the cross section. Thus, we expect in general a positive $S$ and a negative $T$.

We fit the coupling $a$ together with the five SM parameters to the precision observables using the large-$m_t$ expansion for the two-loop fermionic EW corrections to $\rho_Z^f$, and obtain the results shown in the left plot in figure 7 and reported in table 7. The correlation matrices for the posteriors are given in tables 20 and 21 for the case of $m_t$ from Tevatron pole mass average. In table 7 we also present the result obtained using $m_t$ from the $\overline{\text{MS}}$ mass and including the subleading two-loop fermionic EW corrections to $\delta\rho_Z^f$ with the results of ref. [16, 83]. As is evident from the table, the results are stable against the treatment of $\delta\rho_Z^f$, but the error is sensitive to the uncertainty in $m_t$. This can be understood by looking at the impact of the individual constraints on $a$ shown in the center plot in figure 7, from which it is evident that $M_W$ is giving the strongest bound on the nonstandard Higgs coupling. Our result is compatible with the analysis of ref. [121].

Since the fit prefers values of $a > 1$, while the sum rule of ref. [120] gives in general $a < 1$, additional contributions to the EWPO, for example from additional light fermions [119, 122], are required in order to restore the agreement with experimental data in composite Higgs models. If one takes literally the model with no new particles below the cutoff and assuming $a \leq 1$, from the 95% probability range $a \in [0.984, 1.070]$ ([0.981, 1.071]) one can derive a lower bound on $\Lambda$:

\[ \Lambda > 17 (16) \text{ TeV} @95\% \text{ probability}, \]

using the large-$m_t$ expansion (using the results of ref. [16, 83]). One can generalise the analysis allowing for $\Lambda < 4\pi v/\sqrt{|1 - a^2|}$ and assuming that the dynamics at the cutoff does not contribute sizably to $S$ and $T$. In this case one can determine regions in the $a$–$\Lambda$ plane as shown in right plot of figure 7. Clearly the value of $a$ is tightly constrained for values of $\Lambda$ compatible with direct searches.

\begin{table}[h]
\centering
\begin{tabular}{|c|c|c|}
\hline
$m_t$ [GeV] & large-$m_t$ expansion & Using ref. [16, 83] \\
\hline
173.2 ± 0.9 & 1.024 ± 0.021 & 1.024 ± 0.022 \\
173.3 ± 2.8 & 1.025 ± 0.030 & 1.027 ± 0.031 \\
\hline
\end{tabular}
\caption{Fit result for the $HHV$ coupling $a$, obtained with different choices for $m_t$ and for the two-loop fermionic EW corrections to $\rho_Z^f$. When using ref. [16, 83], we do not impose constraints from $\Gamma_Z$, $\sigma_0^h$ and $R_0^\ell$.}
\end{table}
3.5 General bounds on the New Physics scale

Before concluding, let us take a more general approach and consider the contributions to the EW fit of arbitrary dimension-six NP-induced operators [11, 20, 123]:

\[ \mathcal{L}_{\text{eff}} = \mathcal{L}_{\text{SM}} + \sum_i \frac{C_i}{\Lambda^2} \mathcal{O}_i, \]  

(3.22)

For concreteness, let us use the same operator basis of ref. [11]:

\[
\begin{align*}
\mathcal{O}_{WB} &= (H^\dagger \tau^a H) W^a_{\mu\nu} B^{\mu\nu}, & \mathcal{O}_H &= |H^\dagger D_\mu H|^2, \\
\mathcal{O}_{LL} &= \frac{1}{2} (\bar{\tau} \gamma^\mu \tau^a L)^2, & \mathcal{O}'_{HL} &= i(H^\dagger D_\mu \tau^a H)(\bar{\tau} \gamma^\mu \tau^a L), \\
\mathcal{O}'_{HQ} &= i(H^\dagger D_\mu H)(\bar{Q} \gamma^\mu Q), & \mathcal{O}_{HL} &= i(H^\dagger D_\mu H)(\bar{\tau} \gamma^\mu L), \\
\mathcal{O}_{HU} &= i(H^\dagger D_\mu H)(\bar{U} \gamma^\mu U), & \mathcal{O}_{HE} &= i(H^\dagger D_\mu H)(\bar{E} \gamma^\mu E), \\
\mathcal{O}_{HD} &= i(H^\dagger D_\mu H)(\bar{D} \gamma^\mu D),
\end{align*}
\]

(3.23)

where we add the contribution of the Hermitian conjugate for operators \( \mathcal{O}'_{HL} \) to \( \mathcal{O}_{HD} \).

The Higgs field gets a vev \( \langle H \rangle = (0, v/\sqrt{2})^T \). For fermions, we do not consider generation mixing, and assume lepton-flavour universality: \( C'_{HL} = C'_{HLi}, C_{HL} = C_{HLi} \) and \( C_{HE} = C_{HEi} \) for \( i = 1, 2, 3 \).

The first two operators contribute to the oblique parameters \( S \) and \( T \):

\[
\begin{align*}
S &= \frac{4 s_W c_W C_{WB}}{\alpha(M_Z^2)} \left( \frac{v}{\Lambda} \right)^2, \\
T &= -\frac{C_H}{2 \alpha(M_Z^2)} \left( \frac{v}{\Lambda} \right)^2,
\end{align*}
\]

(3.24, 3.25)

where \( \mathcal{O}_H \) violates the custodial symmetry, since it gives a correction to the mass of the \( Z \) boson, but not to that of the \( W \) boson. The next two operators yield non-oblique
corrections to the Fermi constant:

\[ G_\mu = G_{\mu,\text{SM}} \left[ 1 - C_{LL} \left( \frac{v}{\Lambda} \right)^2 + 2 C'_{HL} \left( \frac{v}{\Lambda} \right)^2 \right], \]  

(3.26)

where \( G_{\mu,\text{SM}} \) denotes the Fermi constant in the SM. The corrections to the Fermi constant affect the mass and width of the \( W \) boson and the \( Z f \bar{f} \) couplings as shown in Appendix A.

The width of the \( W \) boson also receives the corrections from the operators \( \mathcal{O}'_{HL} \) and \( \mathcal{O}'_{HQ} \):

\[ \Gamma_W = \Gamma_{W,\text{SM}} \left[ 1 + (3 C'_{HL} + C'_{HQ_1} + C'_{HQ_2}) \left( \frac{v}{\Lambda} \right)^2 \right]. \]  

(3.27)

Finally, the operators from \( \mathcal{O}'_{HL} \) to \( \mathcal{O}_{HD} \) contribute to the \( Z f \bar{f} \) couplings:

\[
\begin{align*}
\delta g^v_L & = \frac{C'_{HL_1} - C_{HL_1}}{2} \left( \frac{v}{\Lambda} \right)^2, & \delta g^d_L & = -\frac{C'_{HL_1} + C_{HL_1}}{2} \left( \frac{v}{\Lambda} \right)^2, \\
\delta g^u_L & = \frac{C'_{HQ_1} - C_{HQ_1}}{2} \left( \frac{v}{\Lambda} \right)^2, & \delta g^d_L & = -\frac{C'_{HQ_1} + C_{HQ_1}}{2} \left( \frac{v}{\Lambda} \right)^2, \\
\delta g^e_L & = -\frac{C_{HE}}{2} \left( \frac{v}{\Lambda} \right)^2, & \delta g^u_R & = -\frac{C_{HU}}{2} \left( \frac{v}{\Lambda} \right)^2, & \delta g^d_R & = -\frac{C_{HD}}{2} \left( \frac{v}{\Lambda} \right)^2, \\
\end{align*}
\]

(3.28)

where the shifts in the vector and axial-vector couplings are given by \( \delta g^v_L = \delta g_L^v + \delta g_R^v \) and \( \delta g_A = \delta g_L^A - \delta g_R^A \), respectively.

Switching on one operator at a time (thus barring accidental cancellations), one can constrain the coefficient of each of the above operators using the EW fit. Clearly, as is the case for all indirect constraints, one can either interpret this as a bound on the NP scale.

---

**Table 8.** Fit results for the coefficients of the dimension six operators at 95% probability in units of \( 1/\Lambda^2 \, \text{TeV}^{-2} \), with quark-flavour universality in NP contribution. The fit is performed switching on one operator at a time. The corresponding lower bounds on the NP scale in TeV obtained by setting \( C_i = \pm 1 \) are also presented. When using the results from ref. [16, 83], we do not consider constraints from \( \Gamma_Z, \sigma_R^0 \) and \( R_E^0 \).

| Coefficient | \( C_i/\Lambda^2 \, \text{[TeV]}^{-2} \) at 95% | \( \Lambda \, \text{TeV} \) | \( C_i/\Lambda^2 \, \text{[TeV]}^{-2} \) at 95% | \( \Lambda \, \text{TeV} \) |
|-------------|--------------------------------|----------------|--------------------------------|----------------|
| \( C_{WB} \) | \([-0.0096, 0.0042]\) | 10.2 | \([-0.0095, 0.0045]\) | 10.3 |
| \( C_H \) | \([-0.030, 0.007]\) | 5.8 | \([-0.031, 0.008]\) | 5.7 |
| \( C_{LL} \) | \([-0.011, 0.019]\) | 9.5 | \([-0.016, 0.023]\) | 8.0 |
| \( C'_{HL} \) | \([-0.012, 0.005]\) | 9.2 | \([-0.017, 0.009]\) | 7.6 |
| \( C'_{HQ} \) | \([-0.010, 0.015]\) | 10.2 | \([-0.40, 0.20]\) | 1.6 |
| \( C_{HL} \) | \([-0.007, 0.010]\) | 12.2 | \([-0.034, 0.022]\) | 5.5 |
| \( C_{HQ} \) | \([-0.023, 0.046]\) | 6.6 | \([-0.01, 0.11]\) | 11.7 |
| \( C_{HE} \) | \([-0.014, 0.008]\) | 8.4 | \([-0.029, 0.019]\) | 5.9 |
| \( C_{HU} \) | \([-0.061, 0.087]\) | 4.0 | \([-0.37, 0.08]\) | 1.6 |
| \( C_{HD} \) | \([-0.15, 0.05]\) | 2.6 | \([-1.1, -0.2]\) | 1.0 |

---
fixing the coupling or as a bound on the coupling for fixed NP scale. In tables 8 and 9, we list for all the operators the 95% probability regions of the coefficients and the lower bound on the NP scale in TeV obtained by setting $C_i = \pm 1$, with and without quark-flavour universality for the operators. Comparing these results with the ones of ref. [11], we see that the recent experimental improvements strengthen the bounds on NP contributions, pushing the lower bound on $\Lambda$ to scales as large as 15 TeV.

Moreover, we also fit multiple coefficients simultaneously by dividing the operators into three categories: the oblique operators $O_{WB}$ and $O_H$, the four-fermion operator $O_{LL}$, and the operators with scalars and fermions. Since one cannot determine all the operators simultaneously from the EWPO alone, we fit a part of them turning on the operators in each category. The fit results are summarized in table 10, with and without assuming quark-flavour universality (the results for $O_{LL}$ can be found in table 8). When we use the results of ref. [16, 83] dropping $\Gamma_Z$, $\sigma^0_h$ and $R^0_\ell$ from the fit, we cannot determine individually the coefficients $C_{HL}$ and $C_{HE}$, but only the combination

$$C[\mathcal{A}_C] = \left(g_L^{\ell}g_R^{\ell} - g_R^{\ell}g_L^{\ell}\right) \left(\frac{\Lambda}{v}\right)^2,$$

which is associated with $\mathcal{A}_C$, can be constrained. For the fit without universality, we float the coefficients $C'_{HL}$, $C'_{HQ_1}$, $C_{HL}$, $C_{HQ_2}$, $C_{HE}$, $C_{HU_1}$, $C_{HD_1}$, $C_{HD_2}$, $C_{HD_3}$ for $i = 1, 2$ and 3, except for $C_{HU_3}$, together with the SM parameters, and obtain the posteriors listed in table 10. The combinations $C'_{HQ_1} + C'_{HQ_2}$, $C'_{HQ_2} - C_{HQ_2}$, $C_{HU_3} + C_{HQ_3}$ and $C[\Gamma_{ads}]$, are associated with $W^\ell$, $g_L^{b\ell}$, $g_L^{b\ell}$ and the light-quark contribution to $\Gamma_Z$ respectively, where the last combination is defined as

$$C[\Gamma_{ads}] = \sum_{f=u,d,s} \left(g_L^{\ell}g_R^{\ell} + g_R^{\ell}g_L^{\ell}\right) \left(\frac{\Lambda}{v}\right)^2.$$

The correlations of the fit results are summarized in tables 22-27.
Using ref. [16, 83]

| Coefficient | Large-\(m_t\) expansion \(C_i/\Lambda^2\) [TeV\(^{-2}\)] at 95\% | Using ref. [16, 83] \(C_i/\Lambda^2\) [TeV\(^{-2}\)] at 95\% |
|-------------|-------------------------------------------------|-------------------------------------------------|
| \(C_{WB}\) | \([-0.009, 0.018]\) | \([-0.009, 0.021]\) |
| \(C_H\) | \([-0.058, 0.015]\) | \([-0.068, 0.016]\) |
| \(C'_{HL}\) | \([-0.026, 0.008]\) | \([-0.029, 0.006]\) |
| \(C'_{HQ}\) | \([-0.18, 0.00]\) | \([-0.34, 0.31]\) |
| \(C_{HL}\) | \([-0.013, 0.020]\) | — |
| \(C_{HQ}\) | \([-0.11, 0.07]\) | \([-0.07, 0.12]\) |
| \(C_{HE}\) | \([-0.022, 0.018]\) | — |
| \(C_{HU}\) | \([-0.22, 0.41]\) | \([-0.26, 0.49]\) |
| \(C_{HD}\) | \([-1.2, -0.2]\) | \([-1.2, -0.2]\) |
| \(C[A_4]\) | — | \([-0.0021, 0.0050]\) |
| \(C'_{HL}\) | \([-0.026, 0.008]\) | \([-0.029, 0.006]\) |
| \(C_{HL}\) | \([-0.013, 0.020]\) | — |
| \(C_{HE}\) | \([-0.022, 0.018]\) | — |
| \(C_{HU_2}\) | \([-0.22, 0.45]\) | \([-0.32, 0.55]\) |
| \(C_{HD_3}\) | \([-1.2, -0.2]\) | \([-1.2, -0.2]\) |
| \(C'_{HQ_1} + C'_{HQ_2}\) | \([-0.59, 0.51]\) | \([-0.68, 0.61]\) |
| \(C'_{HQ_2} - C_{HQ_2}\) | \([-0.30, 0.17]\) | \([-0.67, 0.55]\) |
| \(C'_{HQ_3} + C_{HQ_3}\) | \([-0.22, -0.01]\) | \([-0.77, 0.63]\) |
| \(C[A_4]\) | — | \([-0.0021, 0.0050]\) |
| \(C[\Gamma_{uds}]\) | \([-0.039, 0.044]\) | \([-0.42, 0.43]\) |

**Table 10.** Fit results for the coefficients of the dimension six operators at 95\% probability in units of \(1/\Lambda^2\) TeV\(^{-2}\). We perform three separate fits, for the oblique operators \(O_{WB}\) and \(O_H\), for the non-oblique operators, except for \(O_{LL}\), with quark-flavour universality, and for the non-oblique operators, except for \(O_{LL}\), without quark-flavour universality. When we use the results of ref. [16, 83], we cannot determine individually the coefficients \(C_{HL}\) and \(C_{HE}\) but only the combination \(C[A_4]\), since the observables \(\Gamma_Z, \sigma_h^0\) and \(R^0_T\) have been neglected.

A similar analysis was recently performed in ref. [20]. The constraints on \(C_{WB}/\Lambda^2\) and \(C_H/\Lambda^2\) correspond to those on \(\tan \theta_W(\bar{c}_W + \bar{c}_B)/v^2\) and \(-2\bar{c}_T/v^2\) in ref. [20], respectively, while the other coefficients satisfy the relations \(C_i/\Lambda^2 = \bar{c}_i/v^2\). Our results in table 10 are generally similar to theirs, although one cannot directly compare the results since we have floated a larger set of operators simultaneously. Our fit results are also compatible with the ones of ref. [124], considering that in the latter work \(m_h\) was not yet available and that in the fit the other SM parameters were not floated.

All the results presented here refer to coefficients computed at the weak scale. While other choices of operator basis could be more convenient to study running effects (see refs. [125–127]) or additional observables such as in ref. [20], for our purpose the basis of ref. [11] is perfectly adequate.
4 Summary

With the recent discovery of the Higgs boson and the persistent absence of any direct signal of NP, indirect searches represent even more than before the best strategy to probe physics beyond the SM. In particular, EWPO offer a very powerful handle on the mechanism of EWSB and allow us to strongly constrain any NP relevant to solve the hierarchy problem. In this context, we have presented an updated fit of EWPO in the SM and beyond, obtained using a new code tested against the ZFITTER one. We have discussed in detail the impact of the recently computed two-loop fermionic EW corrections to the $Z f \bar{f}$ vertices, stressing the need for an independent evaluation of these corrections for individual fermions. Our results in the SM are summarized in tables 1, 2 and 3. We have obtained bounds on oblique NP contributions (see table 4) and on $\epsilon$ parameters (see table 5), as well as SM predictions for $\epsilon_i$ in eq. (3.15). We have derived constraints on modified $Z b \bar{b}$ couplings, see table 6. We have studied the bounds from EWPO on the Higgs coupling to vector bosons, obtaining the results in table 7, hinting at an elementary Higgs boson or at a nontrivial composite Higgs model. Finally, we have updated the constraints on the NP-induced dimension-six operators relevant for the EWPO, reported in tables 8, 9 and 10.

A graphical summary of the result for each observable is presented in Appendices D and E.

While the results we obtained are consistent with the non-observation of NP at the 7 and 8-TeV runs, the possibility of weakly-interacting NP hiding behind the corner remains unscathed.

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A NP contributions to the EW precision observables

We express each observable as a linear function of the NP parameters as in refs. [4, 99–102]. Here we use $s_W^2$, $c_W^2$, $g_V^f$ and $g_A^f$ for the corresponding SM values, and write the shift to
the Fermi constant as $G_\mu = G_{\mu,\text{SM}}(1 + \Delta G)$. The corrections to the mass and width\(^8\) of the $W$ boson are then given by

$$M_W = M_{W,\text{SM}} \left[ 1 - \frac{\alpha(M_Z^2)}{4(c_W^2 - s_W^2)} \left( S - 2c_W^2 T - \frac{c_W^2 - s_W^2}{2s_W^2} U \right) - \frac{s_W^2}{2(c_W^2 - s_W^2)} \Delta G \right],$$ (A.1)

$$\Gamma_W = \Gamma_{W,\text{SM}} \left[ 1 - \frac{3\alpha(M_Z^2)}{4(c_W^2 - s_W^2)} \left( S - 2c_W^2 T - \frac{c_W^2 - s_W^2}{2s_W^2} U \right) - \frac{1 + c_W^2}{2(c_W^2 - s_W^2)} \Delta G \right],$$ (A.2)

where $\Gamma_{W,\text{SM}}$ is given in terms of $G_\mu$, $M_{W,\text{SM}}$ and so forth. Moreover, the shifts in the $Z f \bar{f}$ couplings read

$$\delta g_f^l = \frac{g_f^l}{2} \left[ \alpha(M_Z^2) T - \Delta G \right] + \frac{\left(g_f^l - g_A^l\right)}{4} \left[ \alpha(M_Z^2) \left(S - 4c_W^2 s_W^2 T\right) + 4c_W^2 s_W^2 \Delta G \right],$$ (A.3)

$$\delta g_f^A = \frac{g_A^l}{2} \left[ \alpha(M_Z^2) T - \Delta G \right],$$ (A.4)

where we neglect the imaginary parts of the SM couplings in NP contributions below. Using these couplings and defining the following quantities

$$G_f \equiv (g_f^l)^2 + (g_A^l)^2, \quad \delta G_f \equiv 2(g_f^l \delta g_f^l + g_A^l \delta g_A^l),$$ (A.5)

the $Z$-pole observables are written as

$$\sin^2 \theta_{\text{eff}}^{\text{lept}} = \sin^2 \theta_{\text{eff,SM}}^{\text{lept}} - \frac{g_f^l \delta g_f^l - g_A^l \delta g_A^l}{4(g_A^l)^2},$$ (A.6)

$$A_f = A_{f,\text{SM}} - \frac{2[(g_f^l)^2 - (g_A^l)^2]}{G_f^2} \left(g_A^l \delta g_f^l - g_f^l \delta g_A^l\right),$$ (A.7)

$$A_{0f}^{0,f} = A_{0f,\text{SM}} - \left[ \frac{3g_f^l g_A^l \left( (g_f^l)^2 - (g_A^l)^2 \right)}{G_f G_A^l} \left(g_A^l \delta g_f^l - g_f^l \delta g_A^l\right) + (e \leftrightarrow f) \right],$$ (A.8)

$$\Gamma_Z = \Gamma_{Z,\text{SM}} + \frac{\alpha(M_Z^2)M_Z}{12s_W^2 c_W^2} \sum_f N_f^l \delta G_f,$$ (A.9)

$$\sigma^0_h = \sigma_{h,\text{SM}}^0 + \frac{12\pi G_e}{M_Z^2} \left( \sum_f N_{f,\text{SM}}^l G_f \right)^2 \left( \delta G_e \sum_f N_{f,\text{SM}}^l G_f + \frac{2}{\sum_f N_{f,\text{SM}}^l G_f} \right),$$ (A.10)

$$R_{\ell}^0 = R_{\ell,\text{SM}}^0 + \frac{\sum_q N_{f,\text{SM}}^l \delta G_q}{G_\ell} - \frac{\left( \sum_q N_{f,\text{SM}}^l G_q \right)^2}{G_\ell^2},$$ (A.11)

$$R_{c}^0 = R_{c,\text{SM}}^0 + \frac{\delta G_c}{\sum_q G_q} - \frac{G_c \sum_q \delta G_q}{\left( \sum_q G_q \right)^2},$$ (A.12)

$$R_{b}^0 = R_{b,\text{SM}}^0 + \frac{\delta G_b}{\sum_q G_q} - \frac{G_b \sum_q \delta G_q}{\left( \sum_q G_q \right)^2},$$ (A.13)

where $N_f^l = 3$ for quarks and $N_{e}^l = 1$ for leptons.

\(^8\)Our formula for the $W$-boson width in eq. (A.2) differs from that in ref. [100, 101], since we have expressed the $W$-boson mass appearing in the phase-space factor in terms of the NP parameters.
B Non-universal vertex corrections

As shown in eqs. (3.6)-(3.11), the parameters $\epsilon_1$, $\epsilon_2$ and $\epsilon_3$ are defined from the $Zee$ effective couplings. To apply the same parameters to other decay channels, flavour non-universal vertex corrections have to be taken into account. Below we summarize the formulæ of the non-universal corrections at one-loop level, which can be found in ref. [79] and references therein.

The non-universal corrections to the effective couplings $\rho_f^Z$ and $\kappa_f^Z$ are given by

$$
\Delta \rho_f^Z = \rho_f^Z - \rho_e^Z = \frac{\alpha}{2\pi s_W^2} (u_f - u_e),
$$

$$
\Delta \kappa_f^Z = \kappa_f^Z - \kappa_e^Z = \frac{\alpha}{4\pi s_W^2} \left( \frac{\delta_f^2 - \delta_e^2}{4\epsilon_W^2} F_Z(M_Z^2) - u_f + u_e \right),
$$

(B.1)

respectively, where $u_f$ and $\delta_f$ are defined as

$$
u_f = \frac{3v_f^2 + a_f^2}{4\epsilon_W^2} F_Z(M_Z^2) + F_W(M_Z^2),
$$

$$
\delta_f = v_f - a_f
$$

(B.2)

with the tree-level vector and axial-vector couplings $v_f = I_f^L - 2Q_f s_W^2$ and $a_f = I_f^L$. The so-called unified form factors $F_Z$ and $F_W$, associated with the radiative corrections to the $Zff$ vertices with a virtual $Z$ boson and with a virtual $W$ boson(s), respectively, are given as follows:

$$
F_Z(s) = F_{Za}(s),
$$

$$
F_W(s) = c_W^2 F_{Wn}(s) - \frac{1}{2}|\sigma_f| F_{Wa}(s) - \frac{1}{2} F_{Wa}(s),
$$

(B.3)

where $|\sigma_f| = |v_f + a_f|$ with $f'$ being the partner of $f$ in the $SU(2)_L$ doublet, and the subscripts “$a$” and “$n$” stand for contributions from abelian and non-abelian diagrams, respectively. In the limit of massless fermions, the form factors are written with the loop functions $B_0$ and $C_0$:

$$
F_{Va}(s) = 2(R_V + 1)^2 s C_0(s; 0, (M_V^2)^{1/2}, 0) - (2R_V + 3) \ln \left( -\frac{M_V^2}{s} \right) - 2R_V - \frac{7}{2},
$$

$$
F_{Wn}(s) = -2(R_V + 2) M_V^2 C_0(s; M_V, 0, M_V) + 2R_V + \frac{9}{2} - \frac{11}{18R_V} + \frac{1}{18R_V^2},
$$

$$
- \left( 2R_V + \frac{7}{3} - \frac{3}{2R_V} - \frac{1}{12R_V^2} \right) B_0(\mu; s; M_V, M_V),
$$

$$
F_{Wa}(s) = 0,
$$

(B.4)

where $\tilde{M}_V^2 \equiv M_V^2 - i M_V \Gamma_V \approx M_V^2 - i \epsilon$, and $R_V = M_V^2/s$ for $V = Z, W$. The scalar two-point function $B_0$ and the scalar three-point function $C_0$ are defined by

$$
B_0(\mu; p^2; m_0, m_1) = \frac{(2\pi \mu)^{4-d}}{i \pi^2} \int \frac{d^4k}{(k^2 - m_0^2 + i \epsilon) \left[ (k + p)^2 - m_1^2 + i \epsilon \right]},
$$

(B.5)
\[ C_0(p^2; m_0, m_1, m_0) = -\frac{1}{i\pi^2} \int d^4k \frac{1}{(k^2 - m_1^2 + i\epsilon)[(k + p_2)^2 - m_0^2 + i\epsilon][(k - p_1)^2 - m_0^2 + i\epsilon]}, \] (B.6)

where \( \mu \) is the renormalization scale in the former, and \( p_1^2 = p_2^2 = 0 \) and \( p^2 = (p_1 + p_2)^2 \) in the latter. Note that the contributions from \( F_{Wn}(s) \) cancel out in eq. (B.1).

In the case of \( f = b \), the additional non-universal corrections associated with the heavy top-quark loop are parameterized by \( \epsilon_b \) as shown in eq. (3.14).

C Correlation matrices for fit results

| \( \alpha_s \) | \( \Delta\alpha^{(5)}_{\text{had}} \) | \( M_Z \) | \( m_t \) | \( m_h \) | \( \delta\rho_Z^\nu \) | \( \delta\rho_Z^\ell \) | \( \delta\rho_Z^b \) |
|-----------|----------------|-------|-------|-------|----------------|----------------|----------------|
| \( \alpha_s \) | 1.00 | | | | | | |
| \( \Delta\alpha_{\text{had}}^{(5)} \) | -0.01 | 1.00 | | | | | |
| \( M_Z \) | 0.00 | 0.08 | 1.00 | | | | |
| \( m_t \) | 0.01 | 0.18 | -0.05 | 1.00 | | | |
| \( m_h \) | 0.00 | -0.01 | 0.00 | 0.00 | 1.00 | | |
| \( \delta\rho_Z^\nu \) | 0.00 | -0.01 | -0.05 | -0.02 | 0.00 | 1.00 | |
| \( \delta\rho_Z^\ell \) | 0.00 | 0.02 | -0.10 | -0.08 | 0.00 | 0.49 | 1.00 |
| \( \delta\rho_Z^b \) | -0.18 | 0.10 | -0.02 | -0.06 | 0.00 | -0.28 | 0.38 | 1.00 |

Table 11. Correlation matrix for the fit in table 1.

| \( \alpha_s \) | \( \Delta\alpha_{\text{had}}^{(5)} \) | \( M_Z \) | \( m_t \) | \( m_h \) |
|-----------|----------------|-------|-------|-------|
| \( \alpha_s \) | 1.00 | | | | |
| \( \Delta\alpha_{\text{had}}^{(5)} \) | 0.01 | 1.00 | | | |
| \( M_Z \) | 0.00 | 0.09 | 1.00 | | | |
| \( m_t \) | 0.01 | 0.19 | -0.06 | 1.00 | | | |
| \( m_h \) | 0.00 | 0.00 | 0.00 | 0.00 | 1.00 | | |

Table 12. Correlation matrix for the fit in table 2.

| \( S \) | \( T \) | \( U \) | \( \alpha_s \) | \( \Delta\alpha_{\text{had}}^{(5)} \) | \( M_Z \) | \( m_t \) | \( m_h \) |
|-------|-------|-------|-----------|----------------|-------|-------|-------|
| \( S \) | 1.00 | | | | | | |
| \( T \) | 0.85 | 1.00 | | | | | |
| \( U \) | -0.48 | -0.79 | 1.00 | | | | |
| \( \alpha_s \) | -0.07 | -0.10 | 0.10 | 1.00 | | | |
| \( \Delta\alpha_{\text{had}}^{(5)} \) | -0.30 | 0.00 | -0.09 | 0.00 | 1.00 | | | |
| \( M_Z \) | -0.05 | -0.10 | 0.03 | 0.01 | 0.00 | 1.00 | | |
| \( m_t \) | 0.02 | -0.07 | -0.04 | 0.01 | 0.00 | 0.00 | 1.00 | |
| \( m_h \) | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 1.00 |

Table 13. Correlation matrix for the fit results in the second column of table 4.
### Table 14. Correlation matrix for the fit results in the third column of table 4.

|   | $S$ | $T$ | $\alpha_s$ | $\Delta\alpha_{\text{had}}^{(5)}$ | $M_Z$ | $m_t$ | $m_h$ |
|---|-----|-----|-------------|---------------------------------|------|------|------|
| $S$ | 1.00 |     |             |                                 |      |      |      |
| $T$ | 0.86 | 1.00|             |                                 |      |      |      |
| $\alpha_s$ | -0.03 | -0.03 | 1.00 |                                 |      |      |      |
| $\Delta\alpha_{\text{had}}^{(5)}$ | -0.40 | -0.12 | 0.01 | 1.00 |      |      |      |
| $M_Z$ | -0.04 | -0.12 | 0.00 | 0.00 | 1.00 |      |      |      |
| $m_t$ | 0.00 | -0.17 | 0.01 | 0.00 | 0.00 | 1.00 |      |      |
| $m_h$ | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 1.00 |

### Table 15. Correlation matrix for the fit results in the fourth column of table 4.

|   | $\epsilon_1$ | $\epsilon_2$ | $\epsilon_3$ | $\epsilon_b$ | $\alpha_s$ | $\Delta\alpha_{\text{had}}^{(5)}$ | $M_Z$ | $m_t$ | $m_h$ |
|---|-------------|--------------|--------------|-------------|-----------|---------------------------------|------|------|------|
| $\epsilon_1$ | 1.00 |     |             |             |           |                                 |      |      |      |
| $\epsilon_2$ | 0.79 | 1.00|             |             |           |                                 |      |      |      |
| $\epsilon_3$ | 0.86 | 0.50 | 1.00 |             |           |                                 |      |      |      |
| $\epsilon_b$ | -0.32 | -0.31 | -0.21 | 1.00 |           |                                 |      |      |      |
| $\alpha_s$ | -0.06 | -0.06 | -0.04 | -0.12 | 1.00 |                                 |      |      |      |
| $\Delta\alpha_{\text{had}}^{(5)}$ | 0.00 | 0.07 | -0.30 | 0.00 | 0.00 | 1.00 |                                 |      |      |      |
| $M_Z$ | -0.10 | -0.03 | -0.05 | 0.02 | 0.00 | 0.00 | 1.00 |      |      |      |
| $m_t$ | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 1.00 |      |      |      |
| $m_h$ | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 1.00 |      |      |      |

### Table 16. Correlation matrix for the fit results in the second column of table 5.

|   | $\epsilon_1$ | $\epsilon_3$ | $\alpha_s$ | $\Delta\alpha_{\text{had}}^{(5)}$ | $M_Z$ | $m_t$ | $m_h$ |
|---|-------------|--------------|-----------|---------------------------------|------|------|------|
| $\epsilon_1$ | 1.00 |     |             |                                 |      |      |      |
| $\epsilon_3$ | 0.87 | 1.00|             |                                 |      |      |      |
| $\alpha_s$ | -0.04 | -0.03 | 1.00 |                                 |      |      |      |
| $\Delta\alpha_{\text{had}}^{(5)}$ | -0.10 | -0.39 | 0.01 | 1.00 |                                 |      |      |      |
| $M_Z$ | -0.12 | -0.04 | 0.02 | 0.00 | 1.00 |                                 |      |      |      |
| $m_t$ | -0.03 | -0.01 | 0.01 | -0.01 | -0.01 | 1.00 |                                 |      |      |      |
| $m_h$ | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 1.00 |      |      |      |

### Table 17. Correlation matrix for the fit results in the third column of table 5.
\begin{tabular}{cccccccc}
\hline
$\delta g^b_R$ & $\delta g^b_L$ & $\alpha_s$ & $\Delta \alpha^{(5)}_{\text{had}}$ & $M_Z$ & $m_t$ & $m_h$ \\
\hline
$\delta g^b_R$ & 1.00 & & & & & \\
$\delta g^b_L$ & 0.90 & 1.00 & & & & \\
$\alpha_s$ & 0.04 & -0.02 & 1.00 & & & \\
$\Delta \alpha^{(5)}_{\text{had}}$ & -0.22 & -0.21 & 0.00 & 1.00 & & \\
$M_Z$ & 0.01 & 0.01 & 0.00 & 0.08 & 1.00 & \\
$m_t$ & 0.01 & 0.02 & 0.00 & 0.18 & -0.06 & 1.00 \\
$m_h$ & 0.00 & 0.00 & 0.00 & -0.01 & 0.00 & 0.00 & 1.00 \\
\hline
\end{tabular}

Table 18. Correlation matrix for the fit results in the second column of table 6.

\begin{tabular}{cccccccc}
\hline
$\delta g^b_R$ & $\delta g^b_L$ & $\alpha_s$ & $\Delta \alpha^{(5)}_{\text{had}}$ & $M_Z$ & $m_t$ & $m_h$ \\
\hline
$\delta g^b_R$ & 1.00 & & & & & \\
$\delta g^b_L$ & 0.82 & 1.00 & & & & \\
$\alpha_s$ & -0.01 & 0.00 & 1.00 & & & \\
$\Delta \alpha^{(5)}_{\text{had}}$ & -0.19 & -0.22 & -0.01 & 1.00 & & \\
$M_Z$ & 0.01 & 0.01 & 0.00 & 0.08 & 1.00 & \\
$m_t$ & -0.01 & 0.02 & 0.01 & 0.17 & -0.05 & 1.00 \\
$m_h$ & 0.00 & 0.00 & 0.00 & 0.00 & 0.00 & 1.00 & 1.00 \\
\hline
\end{tabular}

Table 19. Correlation matrix for the fit results in the third column of table 6.

\begin{tabular}{cccccccc}
\hline
$a$ & $\alpha_s$ & $\Delta \alpha^{(5)}_{\text{had}}$ & $M_Z$ & $m_t$ & $m_h$ \\
\hline
$a$ & 1.00 & & & & & \\
$\alpha_s$ & -0.02 & 1.00 & & & & \\
$\Delta \alpha^{(5)}_{\text{had}}$ & 0.53 & 0.00 & 1.00 & & & \\
$M_Z$ & -0.17 & 0.00 & -0.02 & 1.00 & & & \\
$m_t$ & -0.32 & 0.01 & -0.02 & 0.00 & 1.00 & & \\
$m_h$ & 0.01 & 0.00 & 0.00 & 0.00 & 0.00 & 1.00 & 1.00 \\
\hline
\end{tabular}

Table 20. Correlation matrix for the fit result in top left entry of table 7.

\begin{tabular}{cccccccc}
\hline
$a$ & $\alpha_s$ & $\Delta \alpha^{(5)}_{\text{had}}$ & $M_Z$ & $m_t$ & $m_h$ \\
\hline
$a$ & 1.00 & & & & & \\
$\alpha_s$ & 0.02 & 1.00 & & & & \\
$\Delta \alpha^{(5)}_{\text{had}}$ & 0.53 & 0.00 & 1.00 & & & \\
$M_Z$ & -0.15 & 0.00 & -0.01 & 1.00 & & & \\
$m_t$ & -0.32 & 0.00 & -0.02 & 0.00 & 1.00 & & \\
$m_h$ & 0.01 & 0.00 & 0.00 & 0.00 & 0.00 & 1.00 & 1.00 \\
\hline
\end{tabular}

Table 21. Same as table 20 but for the top-right entry.
| $C_{WB}$ | $C_H$ | $\alpha_s$ | $\Delta\alpha^{(5)}_{\text{had}}$ | $M_Z$ | $m_t$ | $m_h$ |
|---|---|---|---|---|---|---|
| $C_{WB}$ | 1.00 | | | | | |
| $C_H$ | −0.86 | 1.00 | | | | |
| $\alpha_s$ | −0.03 | 0.03 | 1.00 | | | |
| $\Delta\alpha^{(5)}_{\text{had}}$ | −0.40 | 0.12 | 0.01 | 1.00 | | |
| $M_Z$ | −0.04 | 0.12 | 0.00 | 0.00 | 1.00 | |
| $m_t$ | 0.00 | 0.17 | 0.01 | 0.00 | 0.00 | 1.00 |
| $m_h$ | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 1.00 |

Table 22. Correlation matrix for the fit results in table 10 (first set of operators, using the large-$m_t$ expansion).

| $C'_{HL}$ | $C'_{HQ}$ | $C_H$ | $C_Q$ | $C_H$ | $C_H$ | $C_s$ | $\alpha_s$ | $\Delta\alpha^{(5)}_{\text{had}}$ | $M_Z$ | $m_t$ | $m_h$ |
|---|---|---|---|---|---|---|---|---|---|---|---|
| $C'_{HL}$ | 1.00 | | | | | | | | | | |
| $C'_{HQ}$ | 0.24 | 1.00 | | | | | | | | | |
| $C_H$ | 0.56 | 0.04 | 1.00 | | | | | | | | |
| $C_Q$ | 0.00 | −0.38 | −0.09 | 1.00 | | | | | | | |
| $C_H$ | 0.58 | −0.05 | 0.72 | −0.11 | 1.00 | | | | | | |
| $C_H$ | −0.01 | −0.78 | 0.05 | 0.62 | 0.04 | 1.00 | | | | | |
| $C_Q$ | 0.01 | 0.61 | −0.24 | 0.36 | −0.26 | −0.11 | 1.00 | | | | |
| $\alpha_s$ | −0.02 | −0.03 | −0.01 | 0.00 | −0.01 | 0.00 | 0.00 | 1.00 | | | |
| $\Delta\alpha^{(5)}_{\text{had}}$ | −0.36 | −0.04 | 0.12 | −0.01 | 0.12 | −0.01 | 0.01 | 0.00 | 1.00 | | |
| $M_Z$ | 0.16 | 0.02 | 0.06 | 0.00 | 0.12 | 0.00 | −0.01 | 0.00 | 0.00 | 1.00 | |
| $m_t$ | 0.32 | 0.06 | 0.12 | 0.02 | 0.17 | 0.00 | 0.00 | 0.00 | 0.00 | 1.00 | |
| $m_h$ | −0.01 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 1.00 | |

Table 23. Same as table 22, but for the second set of operators.

| $C'_{HL}$ | $C_H$ | $C_E$ | $C_{H_2}$ | $C_{H_3}$ | $C_{12}$ | $C_2$ | $C_3$ | $C[T_{uds}]$ | $\alpha_s$ | $\Delta\alpha^{(5)}_{\text{had}}$ | $M_Z$ | $m_t$ | $m_h$ |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| $C'_{HL}$ | 1.00 | | | | | | | | | | | | |
| $C_H$ | 0.56 | 1.00 | | | | | | | | | | | |
| $C_E$ | 0.58 | 0.72 | 1.00 | | | | | | | | | | |
| $C_{H_2}$ | −0.03 | 0.04 | 0.04 | 1.00 | | | | | | | | | |
| $C_{H_3}$ | 0.03 | −0.23 | −0.26 | −0.13 | 1.00 | | | | | | | | |
| $C_{12}$ | 0.02 | 0.02 | 0.01 | −0.02 | 0.00 | 1.00 | | | | | | | |
| $C_2$ | 0.11 | 0.05 | 0.02 | −0.30 | 0.14 | 0.02 | 1.00 | | | | | | |
| $C_3$ | 0.23 | −0.04 | −0.14 | −0.17 | 0.87 | 0.01 | 0.11 | 1.00 | | | | |
| $C[T_{uds}]$ | −0.01 | 0.08 | −0.03 | −0.30 | −0.03 | 0.00 | −0.75 | −0.01 | 1.00 | | | |
| $\alpha_s$ | −0.02 | −0.01 | −0.01 | 0.00 | 0.00 | −0.01 | −0.01 | −0.03 | −0.03 | 1.00 | | |
| $\Delta\alpha^{(5)}_{\text{had}}$ | −0.36 | 0.12 | 0.12 | −0.01 | 0.00 | 0.00 | −0.02 | −0.05 | 0.04 | 0.00 | 1.00 | |
| $M_Z$ | 0.16 | 0.06 | 0.12 | 0.00 | −0.01 | 0.00 | 0.01 | 0.02 | −0.02 | 0.00 | 0.00 | 1.00 |
| $m_t$ | 0.31 | 0.11 | 0.16 | −0.01 | 0.01 | 0.00 | 0.03 | 0.08 | −0.02 | 0.00 | 0.00 | 0.00 | 1.00 |
| $m_h$ | −0.01 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 1.00 | |

Table 24. Same as table 22, but for the third set of operators, where $C_{12}$, $C_2$ and $C_3$ denote $C_{HQ_1} + C_{HQ_2}$, $C_{HQ_2} - C_{HQ_3}$ and $C_{HQ_3} + C_{HQ_4}$, respectively.
Table 25. Same as table 22, using the results from ref. [16, 83].

| $C_{WB}$ | $C_{H}$ | $\alpha_s$ | $\Delta \alpha_{\text{had}}^{(5)}$ | $M_Z$ | $m_t$ | $m_h$ |
|---------|--------|------------|-----------------|------|------|------|
| $C_{WB}$ | 1.00  |            |                 |      |      |      |
| $C_{H}$  | -0.89 | 1.00       |                 |      |      |      |
| $\alpha_s$ | 0.00  | -0.01     | 1.00            |      |      |      |
| $\Delta \alpha_{\text{had}}^{(5)}$ | -0.40  | 0.15       | 0.00            | 1.00 |      |      |
| $M_Z$    | -0.02 | 0.09       | 0.00            | 0.00 | 1.00 |      |
| $m_t$    | -0.02 | 0.16       | 0.00            | 0.00 | 0.00 | 1.00 |
| $m_h$    | 0.00  | 0.00       | 0.00            | 0.00 | 0.00 | 1.00 |

Table 26. Same as table 25, but for the second set of operators.

| $C'_{HL}$ | $C'_{HQ}$ | $C_{HQ}$ | $C_{HU}$ | $C_{HD}$ | $C[A_4]$ | $\alpha_s$ | $\Delta \alpha_{\text{had}}^{(5)}$ | $M_Z$ | $m_t$ | $m_h$ |
|-----------|-----------|----------|----------|----------|-----------|------------|-----------------|------|------|------|
| $C'_{HL}$ | 1.00      |          |          |          |           |            |                  |      |      |      |
| $C'_{HQ}$ | 0.00      | 1.00     |          |          |           |            |                  |      |      |      |
| $C_{HQ}$  | -0.03     | 0.31     | 1.00     |          |           |            |                  |      |      |      |
| $C_{HU}$  | -0.04     | 0.35     | 0.70     | 1.00     |           |            |                  |      |      |      |
| $C_{HD}$  | 0.02      | -0.12    | 0.18     | -0.25    | 1.00      |            |                  |      |      |      |
| $C[A_4]$  | -0.23     | 0.00     | -0.09    | 0.10     | -0.36     | 1.00       |                  |      |      |      |
| $\alpha_s$ | -0.02   | -0.01    | 0.01     | 0.00     | 0.00      | 0.01       | 1.00            |      |      |      |
| $\Delta \alpha_{\text{had}}^{(5)}$ | -0.35   | 0.01     | 0.00     | 0.00     | 0.00      | 0.52       | 0.00            | 1.00 |      |      |
| $M_Z$     | 0.15      | 0.00     | -0.01    | 0.00     | 0.00      | -0.05      | 0.00            | 0.00 | 1.00 |      |
| $m_t$     | 0.31      | -0.01    | 0.02     | -0.02    | 0.00      | -0.13      | 0.00            | 0.00 | 0.00 | 1.00 |
| $m_h$     | -0.01     | 0.00     | 0.00     | 0.00     | 0.01      | 0.00       | 0.00            | 0.00 | 0.00 | 1.00 |

Table 27. Same as table 25, but for the third set of operators, where $C_{12}$, $C_2$ and $C_3$ denote $C'_{HQ_1} + C'_{HQ_2}$, $C'_{HQ_2} - C_{HQ_3}$ and $C'_{HQ_3} + C_{HQ_3}$, respectively.
D  Fit results for the observables with the large-\(m_t\) expansion for two-loop fermionic EW corrections to \(\rho_f^Z\)

In this Appendix we present a graphical summary of the fit results for all observables obtained within the various scenarios considered in this work, obtained using the large-\(m_t\) expansion for the two-loop fermionic EW corrections to \(\rho_f^Z\). The labels in the figures refer to the various fits performed with a self-explanatory notation. The blue band corresponds to the direct measurement, also reported with the “Data” label.

Figure 8. Fit results, with the large-\(m_t\) expansion for the two-loop fermionic EW corrections to the coupling \(\rho_f^Z\).
Figure 9. Same as figure 8.
Figure 10. Same as figure 8.
E  Fit results for the observables using the full two-loop fermionic EW corrections to $\rho_f^Z$

In this Appendix we present a graphical summary of the fit results for all observables obtained within the various scenarios considered in this work, obtained using the results from ref. [16, 83] for the two-loop fermionic EW corrections to $\rho_f^Z$. In the NP fits, we neglect the observables $\Gamma_Z$, $\sigma_h^0$ and $R_0^\ell$. The labels in the figures refer to the various fits performed with a self-explanatory notation. The orange band corresponds to the direct measurement, also reported with the “Data” label.

![Figure 11](image)

**Figure 11.** Fit results, including the full two-loop fermionic EW corrections to the coupling $\rho_f^Z$ with the results of ref. [16, 83] and the parameters $\delta_{\rho}^{\nu, e, b}$. 

| Observable | SM | S, T | $\delta g_R^b$, $\delta g_R^t$ | a |
|------------|----|------|----------------------|----|
| $M_W$ [GeV] | 80.36 | 80.38 | 80.4 | $80.385 \pm 0.015$ |
| $\Gamma_W$ [GeV] | $2.05$ | $2.1$ | $2.15$ | $2.085 \pm 0.042$ |
| $\sin^2 \theta_{\text{eff}}$ | $0.231$ | $0.232$ | $0.233$ | $0.234$ | $0.2324 \pm 0.0012$ |
| $A_L$ | $0.145$ | $0.15$ | $0.155$ | $0.1513 \pm 0.0021$ |
| $A_C$ | $0.64$ | $0.66$ | $0.68$ | $0.7$ | $0.64 \pm 0.027$ |
Figure 12. Same as figure 11.

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