New two-sided bound on the isotropic Lorentz-violating parameter of modified Maxwell theory

F.R. Klinkhamer and M. Schreck

Institute for Theoretical Physics, University of Karlsruhe (TH), 76128 Karlsruhe, Germany

Abstract

There is a unique Lorentz-violating modification of the Maxwell theory of photons, which maintains gauge invariance, CPT, and renormalizability. Restricting the modified-Maxwell theory to the isotropic sector and adding a standard spin-$\frac{1}{2}$ Dirac particle $p^\pm$ with minimal coupling to the nonstandard photon $\tilde{\gamma}$, the resulting modified-quantum-electrodynamics model involves a single dimensionless "deformation parameter," $\tilde{\kappa}_{tr}$. The exact tree-level decay rates for two processes have been calculated: vacuum Cherenkov radiation $p^\pm \to p^\pm \tilde{\gamma}$ for the case of positive $\tilde{\kappa}_{tr}$ and photon decay $\tilde{\gamma} \to p^+ p^-$ for the case of negative $\tilde{\kappa}_{tr}$. From the inferred absence of these decays for a particular high-quality ultrahigh-energy-cosmic-ray event detected at the Pierre Auger Observatory and a well-established excess of TeV gamma-ray events observed by the High Energy Stereoscopic System telescopes, a two-sided bound on $\tilde{\kappa}_{tr}$ is obtained, which improves by eight orders of magnitude upon the best direct laboratory bound. The implications of this result are briefly discussed.

PACS numbers: 11.30.Cp, 12.20.-m, 98.70.Rz, 98.70.Sa

Keywords: Lorentz violation, quantum electrodynamics, gamma-ray sources, cosmic rays

∗Electronic address: frans.klinkhamer@physik.uni-karlsruhe.de
†Electronic address: schreck@particle.uni-karlsruhe.de
I. INTRODUCTION

Spacetime is a dynamic entity and quantum mechanics inescapably leads to randomness. Combined, this suggests [1] that space may not be perfectly smooth at very small length scales and that Lorentz invariance may be fundamentally violated.

Precision tests of Lorentz invariance are, therefore, of paramount importance and the ideal testing ground is the photonic sector. If space, on the whole, is homogeneous and isotropic, there is essentially one dimensionless parameter that describes Lorentz violation in the photonic sector (in a first approximation, this nonstandard photon is taken to be coupled to standard Lorentz-invariant charged particles such as the electron).

The aim of the present article is to obtain a new bound on this isotropic Lorentz-violating parameter. Needless to say, the parameter may also have an entirely different origin than the nontrivial spacetime structure mentioned above. For the purpose of obtaining the bound, we remain “agnostic” as to the potential origin of this particular Lorentz-violating parameter.

II. ISOTROPIC MODIFIED-MAXWELL THEORY AND CURRENT BOUNDS

In this article, we consider an isotropic Lorentz-violating (LV) deformation of quantum electrodynamics (QED) [2] given by the following action:

\[ S_{\text{modQED}}[\tilde{\kappa}_{\text{tr}}, e, M] = S_{\text{modMaxwell}}[\tilde{\kappa}_{\text{tr}}] + S_{\text{Dirac}}[e, M], \] (1)

with a modified-Maxwell term [3, 4, 5, 6, 7] for the gauge field \( A_\mu(x) \) and the standard Dirac term [2] for the spinor field \( \psi(x) \) relevant to a spin \( \frac{1}{2} \) particle (electric charge \( e \) and mass \( M \)) and its antiparticle (opposite charge and equal mass):

\[ S_{\text{modMaxwell}}[\tilde{\kappa}_{\text{tr}}] = \int_{\mathbb{R}^4} d^4 x \left( -\frac{1}{4} \left[ \eta^{\mu\rho} \eta^{\nu\sigma} + \kappa^{\mu\nu\rho\sigma} \right] F_{\mu\nu}(x) F_{\rho\sigma}(x) \right), \] (2a)

\[ S_{\text{Dirac}}[e, M] = \int_{\mathbb{R}^4} d^4 x \bar{\psi}(x) \left( \gamma^\mu \left( i \partial_\mu - e A_\mu(x) \right) - M \right) \psi(x), \] (2b)

\[ F_{\mu\nu}(x) \equiv \partial_\mu A_\nu(x) - \partial_\nu A_\mu(x), \] (2c)

\[ \kappa^{\mu\nu\rho\sigma} \equiv \frac{1}{2} \left( \eta^{\mu\rho} \tilde{\kappa}^{\nu\sigma} - \eta^{\mu\sigma} \tilde{\kappa}^{\nu\rho} + \eta^{\nu\sigma} \tilde{\kappa}^{\rho\mu} - \eta^{\nu\rho} \tilde{\kappa}^{\mu\sigma} \right), \] (2d)

\[ \left( \tilde{\kappa}^{\mu\nu} \right) \equiv \frac{3}{2} \tilde{\kappa}_{\text{tr}} \text{ diag}(1, 1/3, 1/3, 1/3), \] (2e)

where condition (2d) on the dimensionless deformation parameters \( \kappa^{\mu\nu\rho\sigma} \) restricts the theory to the nonbirefringent sector and the further condition (2e) to the isotropic sector. Here, and
in the following, natural units are used with $c = \hbar = 1$. The fundamental constant $c$ now corresponds to the maximum attainable velocity of the Dirac particle or, more importantly, to the causal velocity from the underlying Minkowski spacetime with Cartesian coordinates $(x^\mu) = (x^0, \mathbf{x}) = (ct, x^1, x^2, x^3)$ and metric $g_{\mu\nu}(x) = \eta_{\mu\nu} \equiv \text{diag} (+1, -1, -1, -1)$.

The particular Ansätze (2d)–(2e) reduce the number of independent parameters in the real background tensor $\kappa^{\mu\nu\rho\sigma}$ from 19 to 1. Physically [6], the single remaining LV parameter $\tilde{\kappa}_{tr}$ changes the phase velocity of light to $\sqrt{(1 - \tilde{\kappa}_{tr})/(1 + \tilde{\kappa}_{tr})} c$, as will become clear from (6)–(7) below.

This deformation parameter $\tilde{\kappa}_{tr}$ is, however, difficult to measure. The first bound by the Ives–Stilwell experiment [8] was at the 1 percent level (see, e.g., Ref. [9] for a general discussion of this type of experiment). At present, the best direct laboratory bound [10] at the two-$\sigma$ level is

$$|\tilde{\kappa}_{tr}| < 2 \times 10^{-7},$$

as follows from the penultimate unnumbered equation in Ref. [10]. The difficulty of bounding $\tilde{\kappa}_{tr}$ in laboratory experiments is that its effects always appear in combination with a quadratic Doppler factor (“$v/c$”)$^2$, where the quotation marks point to the subtle issue of identifying the relevant velocities [9, (b)].

A precision measurement [11] of the electron anomalous magnetic moment $a_e \equiv (g_e - 2)/2$, combined with a four-loop calculation of standard QED [12], gives an indirect bound [13] on $\tilde{\kappa}_{tr}$ which is approximately a factor 6 better than the direct bound (3).

Recently, an entirely different type of indirect bound [14, 15] has been obtained from an ultrahigh-energy-cosmic-ray (UHECR) event, which, at the two-$\sigma$ level, is given by

$$\tilde{\kappa}_{tr} < 1.4 \times 10^{-19}.$$  

(4)

Clearly, this bound improves upon the laboratory bound (3) by many orders of magnitude, but bound (4), as it stands, is only one-sided. The main goal of the present article is to obtain a two-sided bound.

III. LORENTZ-NONINvariant DECAY PROCESSES

The modified–QED model defined by (1)–(2) is gauge invariant, CPT invariant, and power-counting renormalizable (renormalizability has been verified at the one-loop level [5]). The violation of Lorentz invariance leads to modified propagation properties of the photon $\tilde{\gamma}$,
FIG. 1: Feynman diagrams with time running from left to right for (a) the vacuum-Cherenkov-radiation process and (b) the photon-decay process, both evaluated for the isotropic Lorentz-violating deformation of quantum electrodynamics given by [1]–[2].

where a nonstandard symbol is used to emphasize its unusual properties. The modified photon propagation, in turn, allows for new types of particle decays.

In this article, we consider two such decay processes (Figs. 1ab), whose occurrence depends on the sign of the LV parameter in (1)–(2):

\[
\tilde{\kappa}_{\text{tr}} > 0 : \quad p^\pm \rightarrow p^\mp \tilde{\gamma},
\]

\[
\tilde{\kappa}_{\text{tr}} < 0 : \quad \tilde{\gamma} \rightarrow p^- p^+,
\]

where \( p^- / p^+ \) stands for the electron/positron particle (\( e^- / e^+ \) in standard notation) of the vectorlike \( U(1) \) gauge theory considered, that is, pure QED [2]. It is also possible to take the charged particles \( p^+ / p^- \) in (5) to correspond to a simplified version of the proton/antiproton (namely, a Dirac particle with partonic effects neglected in first approximation). Process (5a) has been called “vacuum Cherenkov radiation” in the literature and process (5b) “photon decay.” See Ref. [16] for a general review of LV decay processes and Ref. [17] for a detailed discussion starting from Lorentz-noninvariant scalar models.

For both processes (5a) and (5b) in the modified–QED model (1)–(2), we have calculated the exact tree-level decay rates and, in particular, the threshold energies \( E_{\text{thresh}}^{(a,b)} \). This work builds on earlier calculations of Ref. [18], which also contains an extensive list of references on standard and nonstandard Cherenkov radiation. Certain technical details of the calculation can be found in Ref. [19] and some of the results presented here have already been mentioned in a recent review article [20].

In order to write the results compactly, define

\[
\xi \equiv 2 \tilde{\kappa}_{\text{tr}}, \quad A \equiv 1/B \equiv \sqrt{(2 + \xi)/(2 - \xi)}.
\]

Precisely this combination \( B \) enters the photon dispersion relation (leaving out the tilde on
the suffix \( \gamma \), for clarity):

\[
E_\gamma(\vec{k}) = B |\vec{k}|, \quad E_p(\vec{k}) = \sqrt{|\vec{k}|^2 + M^2},
\]

(7)

where, for completeness, also the standard dispersion relation of the charged Dirac particle \( p \) has been given.

For the vacuum-Cherenkov-radiation process \([\text{la}]\) at \( \xi > 0 \) with \( E \) standing for the energy \( E_p \) of the initial charged Dirac particle (Fig. \([\text{la}]\)), the exact tree-level result for the radiated-energy rate is

\[
\left. \frac{dW^{(a)}}{dt} \right|_{E \geq E^{(a)}_{\text{thresh}}} = \frac{e^2}{4\pi} \frac{1}{3\xi^3 E \sqrt{E^2 - M^2}} \left( B E - \sqrt{E^2 - M^2} \right)^2 \times \left\{ 2 \left( \xi^2 + 4 \xi + 6 \right) E^2 - (2 + \xi) \left( 3 \left(1 + \xi\right) M^2 + 2 \left(3 + 2 \xi\right) B E \sqrt{E^2 - M^2} \right) \right\},
\]

(8)

where, from now on, the suffix ‘exact’ on a rate abbreviates ‘exact tree-level.’ The threshold energy corresponds to the highest-energy zero of the above expression and is given by

\[
E^{(a)}_{\text{thresh}} = M \sqrt{(1 + \xi/2)/\xi},
\]

(9a)

\[
= M/\sqrt{\xi} + \mathcal{O}(M/\sqrt{\xi}),
\]

(9b)

where the expansion holds for sufficiently small but positive \( \xi \). The threshold kinematics has the final fermion carrying the full three-momentum of the initial fermion.

The energy behavior of the radiated-energy rate \([\text{8}]\) is not quite obvious and the following two expansions at fixed Dirac-particle mass \( M \) and fixed LV parameter \( 0 < \xi \ll 1 \) may be helpful:

\[
\left. \frac{dW^{(a)}}{dt} \right|_{E \geq E^{(a)}_{\text{thresh}}} \approx \frac{e^2}{4\pi} \left( E - E^{(a)}_{\text{thresh}} \right)^2 \left\{ \frac{4}{3} \xi \left[ \left( E/E^{(a)}_{\text{thresh}} - 1 \right) - \left( E/E^{(a)}_{\text{thresh}} - 1 \right)^2 \right. \right.
\]

\[
\left. + \mathcal{O} \left( E/E^{(a)}_{\text{thresh}} - 1 \right)^3 \right) + \mathcal{O} \left( \xi^2 \left( E/E^{(a)}_{\text{thresh}} - 1 \right) \right) \right\}
\]

(10a)

\[
\approx \frac{e^2}{4\pi} E^2 \left\{ \left( \frac{7}{24} \xi - \frac{1}{16} \xi^2 + \mathcal{O}(\xi^3) \right) + \left( -1 + \frac{1}{48} \xi - \frac{3}{32} \xi^2 + \mathcal{O}(\xi^3) \right) \frac{M^2}{E^2} + \mathcal{O} \left( \frac{M^4}{\xi E^4} \right) \right\},
\]

(10b)

where the first expression holds for energies \( E \) sufficiently close to but above the threshold energy [using the exact result \([\text{9a}]\) throughout] and the second expression holds for sufficiently large energies \( E \).\(^1\) The corresponding expressions for the decay rate \( \Gamma^{(a)} \) are relegated to Appendix \([\text{A}]\).

\(^1\) The approximation sign ‘\( \approx \)’ has been used in \([\text{10}]\) because the remaining \( \xi \) dependence is indicated only symbolically, not exactly.
For the photon-decay process (5b) at $\xi < 0$ with $E$ standing for the energy $E_\gamma$ of the initial photon (Fig. 1b), the exact tree-level result for the decay rate is

$$
\Gamma^{(b)} \bigg|_{E \geq E_{\text{thresh}}^{(b)}} = \frac{e^2}{4\pi} \frac{2 - \xi}{6 (2 + \xi)^{7/2}} \sqrt{(1 - \xi/2) E^2}
\times \left\{ C_4 (C_1 - C_2) + 2 A E C_3 (C_1 + C_2) + C_5 (C_1^2 - C_2^2) \right\},
$$

(11)

with

$$
C_1 \equiv \sqrt{\frac{(2 + \xi)^2 M^2 + A E (2 A \xi E - C_3)}{\xi (2 + \xi)}},
$$

(12a)

$$
C_2 \equiv \sqrt{\frac{(2 + \xi)^2 M^2 + A E (2 A \xi E + C_3)}{\xi (2 + \xi)}},
$$

(12b)

$$
C_3 \equiv \sqrt{\xi (4 - \xi^2) \left[ 2 (2 + \xi) M^2 + A^2 \xi E^2 \right]},
$$

(12c)

$$
C_4 \equiv 16 \left[ (1 + \xi) M^2 - A^2 \xi E^2 \right] + 4 \xi^2 M^2,
$$

(12d)

$$
C_5 \equiv 3\sqrt{2} \xi (2 + \xi) E.
$$

(12e)

The corresponding threshold energy is

$$
E_{\text{thresh}}^{(b)} = 2 M \sqrt{(1 - \xi/2)/( - \xi)}
$$

(13a)

$$
= 2 M/\sqrt{( - \xi)} + O(M \sqrt{-\xi}),
$$

(13b)

where the exact result (13a) has the same basic structure as (9a) and the expansion in (13b) holds for sufficiently small $|\xi|$. The threshold kinematics has the three-momentum of the initial photon split equally over the two final fermions.

The energy behavior of (11) is clarified by the following two expansions at fixed $M$ and fixed parameter $0 < -\xi \ll 1$:

$$
\Gamma^{(b)} \bigg|_{E \geq E_{\text{thresh}}^{(b)}} \approx \frac{e^2}{4\pi} \sqrt{E_{\text{thresh}}^{(b)} \left( E - E_{\text{thresh}}^{(b)} \right)} \left\{ (-\xi/\sqrt{2}) \left[ 1 - \frac{5}{12} \left( E/E_{\text{thresh}}^{(b)} - 1 \right) \right] \right.
\left. + O \left( \left( E/E_{\text{thresh}}^{(b)} - 1 \right)^2 \right) \right\} + O(\xi^3)
$$

(14a)

$$
\approx \frac{e^2}{4\pi} E \left\{ \left( -\frac{1}{3} \xi + O(\xi^3) \right) + O \left( M^4/\xi E^4 \right) \right\},
$$

(14b)

with the first expression holding for energies $E$ just above threshold and the second for sufficiently large energies $E$. 

6
Recalling the definitions (6) of $B$ and $A$, the explicit presence of these square roots in the rates (8) and (11) makes clear that, purely theoretically, the LV parameter $\tilde{\kappa}_{tr}$ cannot have an absolute value larger than 1. (See, e.g., Refs. [21, 22, 23] for a general discussion of causality, unitarity, and stability in Lorentz-violating theories.) But how small $|\tilde{\kappa}_{tr}|$ really is, is up to experiment to decide.

As a final technical remark, we repeat that the rates calculated in this article apply to Dirac point particles. This may be reasonable for photon decay into an electron-positron pair but the results are only indicative for processes involving hadrons (see also related remarks in, e.g., Ref. [20] and Appendix C of the present article).

IV. NEW INDIRECT BOUNDS ON THE ISOTROPIC LV PARAMETER

In this section, we turn to the experimental data in order to obtain bounds on $\tilde{\kappa}_{tr}$. For positive $\tilde{\kappa}_{tr}$, the argument [24, 25] is well-known (see also the review [20] for further discussion): from a single observed cosmic-ray event with primary energy $E_{prim}$, the inferred absence of vacuum Cherenkov radiation (5a) implies $E_{prim} < E_{(a)\text{thresh}}$, which, with expression (9), gives an upper bound on $\tilde{\kappa}_{tr}$. For negative $\tilde{\kappa}_{tr}$, the argument is similar: from a single observed gamma-ray event with initial photon energy $E_{\gamma}$, the inferred absence of photon decay (5b) implies $E_{\gamma} < E_{(b)\text{thresh}}$ which, with expression (13), gives a lower bound on $\tilde{\kappa}_{tr}$.

For positive $\tilde{\kappa}_{tr}$, this is indeed how the previously mentioned bound (4) was obtained from the detection at the Pierre Auger Observatory [26] of an $E_{prim} = 148$ EeV UHECR event [27]. Here, we use another Auger event [28, 29] with a slightly higher energy, $E_{prim} = 212$ EeV = $2.12 \times 10^{20}$ eV. This particular Auger event is of the so-called hybrid type [26], which means that it has been observed by both surface detectors and fluorescence telescopes; see Table I for further details. Demanding $E_{prim} < E_{(a)\text{thresh}}$ and using (9) with $M$ set to the value 52 GeV of an iron nucleus (an absolutely conservative choice as discussed in Refs. [14, 15]), the following two–σ (98 % CL) bound is obtained:

$$\tilde{\kappa}_{tr} < 6 \times 10^{-20}, \quad (15a)$$

which is lower than (4), mainly because of the larger primary energy used. For completeness, the three–σ (99.9 % CL) bound is $\tilde{\kappa}_{tr} < 8 \times 10^{-20}$.

For negative $\tilde{\kappa}_{tr}$, we use the detection by the High Energy Stereoscopic System (HESS) imaging atmospheric Cherenkov telescopes [30] of $E_{\gamma} \geq 30$ TeV photons from the shell-type supernova remnant RX J1713.7–3946 [31, 32]. These gamma-ray photons with energies
TABLE I: Experimental results from the Pierre Auger Observatory [26] and the HESS Cherenkov-telescope array [30] for the energy values (with relative errors for the two individual events) used for bounds (15a) and (15b). The value of the Auger-event energy [28] has been increased by 5% to account for the missing energy [29] of a hadronic primary (the energy value given in Ref. [28] corresponds to a hypothetical photon primary). For this hybrid Auger event, also the shower-maximum atmospheric depth $X_{\text{max}}$ has been measured (the measured $X_{\text{max}}$ value rules out a photonic primary at the three–σ level [28]). The target position of the HESS events has values for the right ascension $\alpha$ and declination $\delta$ that correspond to those of the supernova remnant RX J1713.7–3946 [31, 32].

| experiment | observation | energy $E$ | $\Delta E/E$ | additional characteristics |
|------------|-------------|------------|---------------|-----------------------------|
| Auger      | 737165 [28] | $E_{\text{prim}} = 212 \text{ EeV}$ | 25 %          | $X_{\text{max}} = 821 \text{ g cm}^{-2}$ |
| HESS       | 2003–2005 [32] | $E_{\gamma} = 30 \text{ TeV}$ | 15 %          | $(\alpha, \delta) = (17^h 13^m 33^s, -39^\circ 45' 44'')$ |

above 30 TeV have been detected at the five–σ level [32]. For definiteness, we consider a fiducial photon event with $E_{\gamma} = 30 \text{ TeV} = 3.0 \times 10^{13} \text{ eV}$ and a relative error in the individual reconstructed energy of the order of 15% as quoted in Ref. [31]; see Table I for further details. Demanding $E_{\gamma} < E_{\text{thresh}}^{(b)}$ and using (13) with $M$ set to the value 511 keV of an electron, the following two–σ (98 % CL) bound is obtained:

$$-\tilde{\kappa}_{\text{tr}} < 9 \times 10^{-16}.$$  \hspace{1cm} (15b)

For completeness, the three–σ (99.9 % CL) bound is $-\tilde{\kappa}_{\text{tr}} < 1.1 \times 10^{-15}$.

Related bounds in a modified-QED model with additional isotropic Lorentz violation in the fermionic sector are given in Appendix B.

At this moment, it may be appropriate to mention that the orders of magnitude of bounds (15a) and (15b) [the first one scaled to $M = M_{\text{proton}}$; see below] agree with the qualitative limits of Coleman and Glashow [25]. The improvement, here, is that the exact tree-level decay rates have been calculated (and found to be nonvanishing) and that a proper error analysis has been performed. Still, the fact remains that the qualitative analysis of Ref. [25] has been confirmed remarkably well by the quantitative results of the present article.

Three last remarks are as follows. First, the above threshold bounds only make sense if the travel length of the decaying particle is less than the source distance. In general terms, this was already noticed in Ref. [25]. With the specific results of Sec. III and the
methodology of Ref. [17], it indeed follows that the average travel lengths $L^{(a)}$ and $L^{(b)}$ from the processes (5a) and (5b), for the $\tilde{\kappa}_{tr}$ and $M$ values considered in this article, are of the order of meters (or even less) rather than the parsec-scale distances of the astronomical sources involved ($D \sim 1 \text{ Mpc}$ for the case of UHECRs [27] and $D \sim 1 \text{ kpc}$ for the case of supernova remnant RX J1713.7–3946 [31]).

Second, the events from Table I used for the above bounds are far from unique. Similar events have also been seen by other experiments, for example, the UHECR event with $E_{\text{prim}} = 320 \text{ EeV}$ and $X_{\text{max}} = 815 \text{ g cm}^{-2}$ observed by the Fly’s Eye detector [33] and the $E_\gamma = 8 \text{ TeV}$ gamma-rays from the galaxy M87 detected by VERITAS [34].

Third, the right-hand side of bound (15a) or (15b) scales as $(M/E)^2$, with $E$ the energy of the incoming particle and $M$ the mass of the charged Dirac particle in the respective process (5a) or (5b). This functional dependence shows the potential for improvement if higher energy events become available in the future\(^2\) and, for the UHECR case, if light hadronic primaries can be selected.\(^3\)

V. CONCLUSION

From the results of the previous section, the final two-sided two-$\sigma$ bound on the single isotropic Lorentz-violating parameter of modified–QED theory (1)–(2) is

$$-9 \times 10^{-16} < \tilde{\kappa}_{tr} < 6 \times 10^{-20}.$$ (16)

The simple model (1)–(2) can be considered to be embedded in a Standard Model Extension [4] with all physical (renormalized) Lorentz-violating parameters set to zero, except for one, namely, the isotropic, CPT–even, Lorentz-violating parameter for the dimension-four term with two Maxwell field strength tensors of the photon field.\(^4\)

\(^2\) Bound (15b), for example, can be reduced by a factor 7 if the 1.5–$\sigma$ detection at HESS [32] of 80 TeV gamma-ray photons from RX J1713.7–3946 is confirmed by further observations.

\(^3\) Bound (15a) can be reduced by a factor 12 if a mass value $M = 15 \text{ GeV}$ is used, which corresponds to the mass of an oxygen nucleus. For the single UHECR event of Table I this particular (conservative) mass value is suggested by the measured $X_{\text{max}}$ value (cf. Sec. III of Ref. [14]). Still, a proper analysis will require more events and perhaps other diagnostics in addition to $X_{\text{max}}$.

\(^4\) With nonzero weak mixing angle [35], the $W^\pm$ and $Z^0$ vector fields also display modified propagation (and interaction) properties, because of $SU(2) \times U(1)$ gauge invariance [the action is given by (C1)–(C2d) in Appendix C with $\pi_{tr}^{(1)} = \pi_{tr}^{(2)} \equiv \tilde{\kappa}_{tr}$ and $\pi_{tr}^{(3)} = 0$]. However, these weak-vector-boson modifications do not directly affect the tree-level calculations of processes (5a) and (5b), with the fermions considered as Dirac point particles.
If the same isotropic Lorentz-violating parameter applies equally to all gauge bosons of the Standard Model, bound (16) can be replaced by a stronger one, which has a lower bound at $-2 \times 10^{-19}$ (see Appendix C). But the derivation of this particular lower bound involves certain assumptions on the parton distributions (the error is, however, still dominated by the relatively large error from the primary-energy measurement). For the moment, bound (16) for the purely photonic parameter is to be preferred, as it does not involve a partonic calculation.

This new indirect bound (16) improves by 8 orders of magnitude upon the direct laboratory bound (3). Observe that, strictly speaking, (16) is also a “terrestrial bound,” as it relies on having detected a primary traveling over a few hundred meters in the Earth’s atmosphere, the precise astronomical origin of the primary being of secondary importance (see the first of the last three remarks in Sec. IV). For the type of experiments involved (Auger [26] and HESS [30]), the Earth’s atmosphere is an integral part of the instrument.

Combined with previous astrophysics bounds [6] on the 10 birefringent modified-Maxwell-theory parameters at the $10^{-32}$ level and “terrestrial” UHECR bounds [15] on the 8 non-isotropic nonbirefringent parameters at the $10^{-18}$ level, bound (16) gives the following two–σ bound on all entries of the background tensor $\kappa_{\mu\nu\rho\sigma}$ in the general modified-Maxwell action (2a):

$$\max_{\{\mu,\nu,\rho,\sigma\}} |\kappa_{\mu\nu\rho\sigma}| < 5 \times 10^{-16},$$

where the fact has been used that the largest entry of $|\kappa_{\mu\nu\rho\sigma}|$ has a value $(1/2) |\tilde{\kappa}_{11}|$ if the other 18 parameters are negligibly small. For completeness, the three–σ bound is $\max |\kappa_{\mu\nu\rho\sigma}| < 6 \times 10^{-16}$. Remark that bounds (16) and (17) hold in a Sun-centered, nonrotating frame of reference.

In order to put (17) in perspective, recall that the equality of gravitational and inertial mass was experimentally verified by Newton (1687), Bessel (1832), and Eötvös (1889) at the $10^{-3}$, $10^{-5}$, and $10^{-7}$ levels, respectively [36]. These experiments then led Einstein to the Principle of Equivalence, the cornerstone of the General Theory of Relativity [37]. In our case, the conclusion from (17) would be (elaborating on remarks in Refs. [38, 39, 40]) that the local Lorentz invariance left-over from a fundamental theory of quantum spacetime is exact and that the underlying principle for this apparent fact remains to be determined.
ACKNOWLEDGMENTS

It is a pleasure to thank W. Hofmann, V.A. Kostelecký, and M. Risse for helpful discussions.

NOTE ADDED

Recently, an indirect bound on $|\tilde{\kappa}_{tr}|$ at the $10^{-11}$ level was obtained from particle-collider data [41], which improves by 4 orders of magnitude upon the direct laboratory bound (3).

APPENDIX A: VACUUM–CHERENKOV DECAY RATE

In this appendix, the decay rate for the vacuum-Cherenkov process $p^+ \rightarrow p^+ \tilde{\gamma}$ is given, where $p^\pm$ stands for a Dirac point particle with charge $\pm e$ and mass $M$. The theory is defined by (1)–(2) and this particular decay process occurs for the case of a positive Lorentz-violating parameter $\xi \equiv 2 \tilde{\kappa}_{tr}$ (the relevant Feynman diagram appears in Fig. 1a).

With $E$ denoting the energy $E_p$ of the initial charged Dirac particle, the exact tree-level result for the decay rate is

$$\Gamma^{(a)}_{\text{exact}} = \frac{e^2}{4\pi} \frac{2}{3 \xi^2 \sqrt{4 - \xi^2} E \sqrt{E^2 - M^2}} \left( \mathcal{B} E - \sqrt{E^2 - M^2} \right)$$

$$\times \left\{ - \left( 3 \xi^2 + 6 \xi + 8 \right) E^2 + \left( 3 \xi^2 + 8 \xi + 4 \right) M^2 \right.\right.$$ 

$$\left. + \mathcal{B} \left( 3 \xi^2 + 10 \xi + 8 \right) E \sqrt{E^2 - M^2} \right\}, \quad (A1)$$

where $\mathcal{B} = B(\xi)$ has been defined in (6).

From (A1), the threshold energy is found to be the same as (9a) obtained from the radiated-energy rate (8). In fact, this threshold energy traces back to the common factor $(\mathcal{B} E - \sqrt{E^2 - M^2})$ in (A1) and (8).

The energy behavior of rate (A1) is clarified by the following two expansions at fixed
mass $M$ and fixed LV parameter $0 < \xi \ll 1$:
\[
\Gamma^{(a)} \bigg|_{E \geq E_{\text{thresh}}^{(a)}} \approx \frac{e^2}{4\pi} \left( E - E_{\text{thresh}}^{(a)} \right) \left\{ 2\xi \left[ \left( E/E_{\text{thresh}}^{(a)} - 1 \right) - \frac{4}{3} \left( E/E_{\text{thresh}}^{(a)} - 1 \right)^2 \right. \\
+ O \left( \left( E/E_{\text{thresh}}^{(a)} - 1 \right)^3 \right) \left. \right] + O \left( \xi^2 \left( E/E_{\text{thresh}}^{(a)} - 1 \right) \right) \right\} \\
\approx \frac{e^2}{4\pi} E \left\{ \left( \frac{2}{3} \xi - \frac{1}{24} \xi^2 + O(\xi^3) \right) \\
+ \left( \frac{3}{2} - \frac{1}{6} \xi - \frac{23}{96} \xi^2 + O(\xi^3) \right) \frac{M^2}{E^2} + O \left( \frac{M^4}{\xi E^4} \right) \right\},
\]
where the first expression holds for energies $E$ just above threshold and the second expression for sufficiently large energies $E$.

**APPENDIX B: BOUNDS ON ISOlTRIC LOrentZ VIOLATION IN ANOTHER MODIFIED–QED MODEL**

In this appendix, bounds are given on isotropic Lorentz violation in a modified-QED model with additional isotropic $c$–type Lorentz violation in the fermionic sector [4].

For two fermion species, this modified–QED model has the following action:
\[
S_{\text{modQED2}} = S_{\text{modMaxwell}} + S_{\text{modDirac2}},
\]
with the modified–Maxwell term given by [2] and the modified–Dirac term by
\[
S_{\text{modDirac2}} = \int_{\mathbb{R}^4} d^4x \sum_{j=1}^{2} \bar{\psi}^{(j)}(x) \left[ \eta_{\mu\nu} + c_{(j)(\mu\nu)}^{(j)} \right] \gamma^\mu \left( i \partial^\nu - e^{(j)} A^\nu(x) \right) - M^{(j)}) \psi^{(j)}(x),
\]
\[
(c_{(j)(\mu\nu)}^{(j)}) = c_{(j)00} \text{ diag}(1, 1/3, 1/3, 1/3),
\]
where ‘$j$’ in [B2a] is the fermion species label. The action [B1] will be applied to an idealized physical situation with only photons, electrons, and protons.

From the identification of appropriate coordinate transformations [7], it then follows that, to leading order, bounds (15a) and (15b) are replaced by
\[
\left[ \tilde{\kappa}_{\text{tr}} - \frac{4}{3} c_{00}^{(p)} \right] < 6 \times 10^{-20},
\]
\[
- \left[ \tilde{\kappa}_{\text{tr}} - \frac{4}{3} c_{00}^{(e)} \right] < 9 \times 10^{-16},
\]
where the suffixes ‘$(p)$’ and ‘$(e)$’ refer to the possible isotropic $c$–type Lorentz violation of the proton and electron, respectively. Bound (B3a) is, most likely, rather conservative, because
it is based on the mass of an iron nucleus (which contains both protons and neutrons), as
discussed in the second paragraph of Sec. IV.

APPENDIX C: BOUNDS ON A UNIVERSAL ISOTROPIC LV PARAMETER

Consider a particular deformation \[4\] of the Standard Model of elementary particles,
where the same isotropic Lorentz-violating parameter applies universally to all gauge bosons
and other Lorentz-violating parameters are absent (or, at least, negligible). Specifically, the
action is given by

\[ S_{\text{modSM}} = S_{\text{modYM}} + S_{\text{restSM}}, \] (C1)

with the only modification occurring in the kinetic terms of the Lie-algebra-valued Yang–
Mills gauge fields \( A^{(i)}_{\mu}(x) \),

\[ S_{\text{modYM}} = \int_{\mathbb{R}^4} d^4x \left( \frac{1}{2} \sum_{i=1}^{3} \text{tr} \left( \left[ \eta^{\mu\rho} \eta^{\nu\sigma} + \bar{\kappa}^{(i)\mu\nu\rho\sigma} \right] F^{(i)}_{\mu\nu}(x) F^{(i)}_{\rho\sigma}(x) \right) \right), \] (C2a)

\[ F^{(i)}_{\mu\nu}(x) \equiv \partial_{\mu} A^{(i)}_{\nu}(x) - \partial_{\nu} A^{(i)}_{\mu}(x) + g^{(i)} \left[ A^{(i)}_{\mu}(x), A^{(i)}_{\nu}(x) \right] \], (C2b)

\[ \bar{\kappa}^{(i)\mu\nu\rho\sigma} \equiv \frac{1}{2} \left( \eta^{\mu\rho} \bar{\kappa}^{(i)\nu\sigma} - \eta^{\nu\sigma} \bar{\kappa}^{(i)\mu\rho} + \eta^{\mu\sigma} \bar{\kappa}^{(i)\nu\rho} - \eta^{\nu\rho} \bar{\kappa}^{(i)\mu\sigma} \right), \] (C2c)

\[ \left( \bar{\kappa}^{(i)\mu\nu} \right) \equiv \frac{3}{2} \bar{\kappa}^{(i)\mu\nu} \text{diag}(1, 1/3, 1/3, 1/3), \] (C2d)

\[ \bar{\kappa}^{\text{univ}}_{\text{tr}} \equiv \bar{\kappa}^{(1)}_{\text{tr}} = \bar{\kappa}^{(2)}_{\text{tr}} = \bar{\kappa}^{(3)}_{\text{tr}}, \] (C2e)

where ‘tr’ in (C2a) stands for the trace and \( F^{(i)}_{\mu\nu} \) in (C2b) is the standard (Lie-algebra-valued)
Yang–Mills field strength \[35\], with label \( i = 1, 2, \) and 3 referring to the gauge group \( U(1), SU(2), \) and \( SU(3) \), respectively. Since \( S_{\text{restSM}} \) in (C1) is not written down explicitly, there is
no need to say more about the normalization of the anti-Hermitian Lie-algebra generators,
except that setting \( \bar{\kappa}^{\text{univ}}_{\text{tr}} = 0 \) in (C1)–(C2) reproduces the Standard Model action; see
Refs. \[4, 35\] for details. Note that such a universal form of Lorentz violation could come
from a nontrivial small-scale structure of classical spacetime over which the gauge bosons
propagate freely \[40\].

\[5\] The particular calculation of Ref. \[40\] found spacetime “defects” to affect primarily gauge bosons, not
fermions. It remains to be seen if this conclusion holds generally.
A lower bound on $\kappa_{\text{univ}}^{\text{tr}}$ can be obtained from the proton-break-up process involving an electron-positron pair [the process will be labeled (c) in this appendix]:

$$\bar{\xi} \equiv 2 \kappa_{\text{univ}}^{\text{tr}} < 0 : \quad p^+ \to p^+ e^- e^+. \quad (C3)$$

This process can be calculated in the parton-model approximation (Fig. 2) and has already been considered in Ref. [39], to which the reader is referred for further details (for a general introduction to the parton model, see, e.g., Ref. [35]). The corresponding energy threshold for $|\bar{\xi}| \ll 1$ is given by

$$E^{(c)}_{\text{thresh}} = M_p / \sqrt{\left( -\bar{\xi} \right) \left( \chi_p - \chi_e \right)}, \quad (C4)$$

where $M_p$ is the proton mass and the numbers $\chi_{p,e} \in [0, 1]$ characterize the total $U(1)$, $SU(2)$, and $SU(3)$ gauge boson content of the proton and the electron, respectively.

Two remarks on (C4) may be helpful. First, the precise meaning of $\chi_p$ (or $\chi_e$) is that it is the sum of the first integral moments of the relevant (gauge-boson) parton distribution functions in the proton (or electron) [39]. Second, the heuristics behind (C4) is that, with only gauge bosons affected by Lorentz violation, the maximum speeds of proton and electron differ, because of their different gauge-boson content. These different speeds then allow for this particular proton-break-up process. It must be emphasized that the process occurs only due to the nontrivial parton content of proton and electron (remark that the analogous process $p^+ \to p^+ p^- p^+$ still does not occur).

Now turn to (C4) to get a bound on the parameter $\kappa_{\text{univ}}^{\text{tr}}$ by the same type of argument as employed in Sec. IV, namely, that the mere observation of a cosmic ray implies $E_{\text{prim}} < E^{(c)}_{\text{thresh}}$, which then gives the desired bound. For a proton $p$, neutron $n$, or electron $e$ of energy $E = 10^2$ EeV, one has $\chi_p - \chi_e \approx \chi_n - \chi_e \approx 0.35$, according to Table 1 of Ref. [39]. (The bulk of $\chi_{p,n} - \chi_e$ comes from the different gluon content of nucleon and electron, namely, $\chi_{p,n}^{(3)} - \chi_e^{(3)} \approx 0.45$.) Next, scale the proton mass $M_p$ in (C4) up to the value $M_{\text{prim}} = 52$ GeV.
and use the $E_{\text{prim}} = 212$ EeV event of Table 1, in order to get the following two-$\sigma$ bound:

$$-\kappa_{\text{univ}}^{\text{inv}} < 2 \times 10^{-19}. \quad (C5)$$

Combined with the previous bound \[15a\] which also holds for the theory considered in this appendix, the final two-sided two-$\sigma$ bound reads:

$$-2 \times 10^{-19} < \kappa_{\text{tr}}^{\text{inv}} < 6 \times 10^{-20}, \quad (C6)$$

where, for the lower bound, the relatively large error from the primary-energy measurement dominates the theoretical uncertainties of the partonic calculations.

[1] (a) J.A. Wheeler, “On the nature of quantum geometrodynamics,” Ann. Phys. (N.Y.) 2, 604 (1957); (b) J.A. Wheeler, “Superspace and the nature of quantum geometrodynamics,” in Battelle Rencontres 1967, edited by C.M. DeWitt and J.A. Wheeler (Benjamin, New York, 1968), Chap. 9.

[2] J.M. Jauch and F. Rohrlich, The Theory of Photons and Electrons, second edition (Springer, New York, 1976).

[3] S. Chadha and H.B. Nielsen, “Lorentz invariance as a low-energy phenomenon,” Nucl. Phys. B 217, 125 (1983).

[4] D. Colladay and V.A. Kostelecký, “Lorentz-violating extension of the standard model,” Phys. Rev. D 58, 116002 (1998), arXiv:hep-ph/9809521.

[5] V.A. Kostelecký, C.D. Lane, and A.G.M. Pickering, “One-loop renormalization of Lorentz-violating electrodynamics,” Phys. Rev. D 65, 056006 (2002), arXiv:hep-th/0111123.

[6] V.A. Kostelecký and M. Mewes, “Signals for Lorentz violation in electrodynamics,” Phys. Rev. D 66, 056005 (2002), arXiv:hep-ph/0205211.

[7] Q.G. Bailey and V.A. Kostelecký, “Lorentz-violating electrostatics and magnetostatics,” Phys. Rev. D 70, 076006 (2004), arXiv:hep-ph/0407252.

[8] H.E. Ives and G.R. Stilwell, “An experimental study of the rate of a moving atomic clock,” J. Opt. Soc. Am. 28, 215 (1938).

[9] (a) M.E. Tobar, P. Wolf, A. Fowler, and J.G. Hartnett, “New methods of testing Lorentz violation in electrodynamics,” Phys. Rev. D 71, 025004 (2005), arXiv:hep-ph/0408006; (b) M. Hohensee et al., “Erratum: New methods of testing Lorentz violation in electrodynamics,” Phys. Rev. D 75, 049902 (2007), arXiv:hep-ph/0701252.
[10] S. Reinhardt et al., “Test of relativistic time dilation with fast optical atomic clocks at different velocities,” Nature Phys. 3, 861 (2007).

[11] B. Odom, D. Hanneke, B. D’Urso, and G. Gabrielse, “New measurement of the electron magnetic moment using a one-electron quantum cyclotron,” Phys. Rev. Lett. 97, 030801 (2006).

[12] G. Gabrielse et al., “New determination of the fine structure constant from the electron g value and QED,” Phys. Rev. Lett. 97, 030802 (2006); 99, 039902(E) (2007).

[13] C.D. Carone, M. Sher, and M. Vanderhaeghen, “New bounds on isotropic Lorentz violation,” Phys. Rev. D 74, 077901 (2006), arXiv:hep-ph/0609150.

[14] F.R. Klinkhamer and M. Risse, “Ultrahigh-energy cosmic-ray bounds on nonbirefringent modified-Maxwell theory,” Phys. Rev. D 77, 016002 (2008), arXiv:0709.2502.

[15] F.R. Klinkhamer and M. Risse, “Addendum: Ultrahigh-energy cosmic-ray bounds on nonbirefringent modified-Maxwell theory,” Phys. Rev. D 77, 117901 (2008), arXiv:0806.4351.

[16] T. Jacobson, S. Liberati, and D. Mattingly, “Lorentz violation at high energy: Concepts, phenomena and astrophysical constraints,” Ann. Phys. (N.Y.) 321, 150 (2006), arXiv:astro-ph/0505267.

[17] C. Kaufhold and F.R. Klinkhamer, “Vacuum Cherenkov radiation and photon triple-splitting in a Lorentz-noninvariant extension of quantum electrodynamics,” Nucl. Phys. B 734, 1 (2006), arXiv:hep-th/0508074.

[18] C. Kaufhold and F.R. Klinkhamer, “Vacuum Cherenkov radiation in spacelike Maxwell–Chern–Simons theory,” Phys. Rev. D 76, 025024 (2007), arXiv:0704.3255.

[19] C. Kaufhold, F.R. Klinkhamer, and M. Schreck, report KA–TP–32–2007 [available from http://www-itp.particle.uni-karlsruhe.de/prep2007.de.shtml].

[20] F.R. Klinkhamer, “UHECR bounds on Lorentz violation in the photon sector,” arXiv:0807.2147.

[21] C. Adam and F.R. Klinkhamer, “Causality and CPT violation from an Abelian Chern–Simons-like term,” Nucl. Phys. B 607, 247 (2001), arXiv:hep-ph/0101087.

[22] V.A. Kostelecký and R. Lehnert, “Stability, causality, and Lorentz and CPT violation,” Phys. Rev. D 63, 065008 (2001), arXiv:hep-th/0012060.

[23] J. Bros and H. Epstein, “Microcausality and energy positivity in all frames imply Lorentz invariance of dispersion laws,” Phys. Rev. D 65, 085023 (2002), arXiv:hep-th/0204118.

[24] E.F. Beall, “Measuring the gravitational interaction of elementary particles,” Phys. Rev. D 1, 961 (1970), Sec. III A.
[25] S.R. Coleman and S.L. Glashow, “Cosmic ray and neutrino tests of special relativity,” Phys. Lett. B 405, 249 (1997), arXiv:hep-ph/9703240.

[26] J. Abraham et al. [Pierre Auger Collaboration], “Properties and performance of the prototype instrument for the Pierre Auger Observatory,” Nucl. Instrum. Meth. A 523, 50 (2004).

[27] J. Abraham et al. [Pierre Auger Collaboration], “Correlation of the highest-energy cosmic rays with the positions of nearby active galactic nuclei,” Astropart. Phys. 29, 188 (2008), arXiv:0712.2843v1.

[28] J. Abraham et al. [Pierre Auger Collaboration], “An upper limit to the photon fraction in cosmic rays above $10^{19}$ eV from the Pierre Auger Observatory,” Astropart. Phys. 27, 155 (2007), arXiv:astro-ph/0606619.

[29] H.M.J. Barbosa, F. Catalani, J.A. Chinellato, and C. Dobrigkeit, “Determination of the calorimetric energy in extensive air showers,” Astropart. Phys. 22, 159 (2004), arXiv:astro-ph/0310234.

[30] (a) K. Bernloehr et al., “The optical system of the HESS imaging atmospheric Cherenkov telescopes. I: Layout and components of the system,” Astropart. Phys. 20, 111 (2003), arXiv:astro-ph/0308246; (b) R. Cornils et al., “The optical system of the HESS imaging atmospheric Cherenkov telescopes. II: Mirror alignment and point spread function,” Astropart. Phys. 20, 129 (2003), arXiv:astro-ph/0308247.

[31] F. Aharonian et al. [HESS Collaboration], “A detailed spectral and morphological study of the gamma-ray supernova remnant RX J1713.7–3946 with H.E.S.S.,” Astron. Astrophys. 449, 223 (2006), arXiv:astro-ph/0511678.

[32] F. Aharonian [HESS Collaboration], “Primary particle acceleration above 100 TeV in the shell-type supernova remnant RX J1713.7–3946 with deep H.E.S.S. observations,” Astron. Astrophys. 464, 235 (2007), arXiv:astro-ph/0611813.

[33] D.J. Bird et al., “Detection of a cosmic ray with measured energy well beyond the expected spectral cutoff due to cosmic microwave radiation,” Astrophys. J. 441, 144 (1995), arXiv:astro-ph/9410067.

[34] V.A. Acciari et al., “Observation of gamma-ray emission from the galaxy M87 above 250 GeV with VERITAS,” arXiv:0802.1951.

[35] M.E. Peskin and D.V. Schroeder, An Introduction to Quantum Field Theory (Addison–Wesley, Reading, USA, 1995).

[36] E. Fischbach and C.L. Talmadge, The Search for Non-Newtonian Gravity (Springer, New York, USA, 1999), Sec. 4.2.2.
[37] A. Einstein, “Die Grundlage der allgemeinen Relativitätstheorie,” Ann. Phys. (Leipzig) 49, 769 (1916); reprinted in Ann. Phys. (Leipzig) 14, S1, 517 (2005); English translation as “The foundation of the general theory of relativity” in: The Principle of Relativity, edited by H.A. Lorentz et al. (Dover Publ., New York, 1952), Chap. VII.

[38] J. Collins, A. Perez, D. Sudarsky, L. Urrutia, and H. Vucetich, “Lorentz invariance: An additional fine-tuning problem,” Phys. Rev. Lett. 93, 191301 (2004), gr-qc/0403053.

[39] O. Gagnon and G.D. Moore, “Limits on Lorentz violation from the highest energy cosmic rays,” Phys. Rev. D 70, 065002 (2004), hep-ph/0404196.

[40] S. Bernadotte and F.R. Klinkhamer, “Bounds on length scales of classical spacetime foam models,” Phys. Rev. D 75, 024028 (2007), arXiv:hep-ph/0610216.

[41] M.A. Hohensee, R. Lehnert, D.F. Phillips, and R.L. Walsworth, “Limits on isotropic Lorentz violation in QED from collider physics,” arXiv:0809.3442v1.