A Review of Dynamic Modeling and Fault Identifications Methods for Rolling Element Bearing

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Abstract

The rolling elements bearings are widely used in industrial and domestic machines. The existence of even tiny defects on the mating surfaces of the bearing components can lead to failure through passage of time. Their failure leads to economical and personal losses. The vibration monitoring technique is mostly used in the industries for health monitoring of bearings. Significant studies are available in open literature for vibration analysis of healthy and defective rolling elements bearings. Various researchers have studied the vibrations generated by bearings through theoretical model and experimentations. The researchers have developed the dynamic model of shaft bearing systems for the theoretical studies. This paper reviews different dynamic models for rolling bearing in presence and absence of local and distributed defects. Moreover, the techniques used for the improvement of fault detection have also been summarized. The signal processing techniques like wavelet transform, high frequency resonance technique (HFRT), envelope analysis and cyclic autocorrelation have improved the fault detection.

Keywords: Rolling element bearing; Vibration; Dynamic model; Fault Identification; Signal Processing method.

1. Introduction

The rolling element bearings are used in any rotating machinery to support the load and reduce the friction. The failure of these bearings resulted in sudden break down or catastrophic failure of machine. Major causes of premature bearing failure in the machinery are dirt, misassemble, misalignment, insufficient lubrication, overloading, corrosion and manufacturing error. The bearing defects are mainly categorized in ‘Localized defect’ and ‘Distributed defect’.

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The faults due to surface roughness, waviness, misaligned races, manufacturing error, and off-size rolling elements are categorized under distributed defect while bearing faults like spalls, pits, dirt, dent, and crack on the rolling surfaces, brinelling and contaminations in lubricant are considered as localized defect in a rolling element bearing. Fatigue is the predominant mode of failure in rolling element bearings; the life of bearings is governed by its rolling contact fatigue life [1].

To prevent the financial and personal losses condition-monitoring (CM) techniques like vibration and acoustic measurements, temperature measurement, and wear debris analysis are used in the industries. In the industries vibration monitoring has proved a reliable and effective technique for defect detection in bearings. The most commonly used vibration analysis methods for mechanical fault diagnosis are time domain analysis, frequency domain analysis; time frequency analysis, high frequency resonance technique (HFRT) and wavelet transform methods which have been presented by Tandon and Nakara [2]. To understand the dynamic behavior of healthy and the defective bearings researchers have developed dynamic model of rolling element bearing with various considerations. The objective of the present study is to summarize and updates the reviews of dynamic modeling for healthy and defective rolling elements bearings.

2. Dynamic Modeling of rolling element bearing.

The key components of any vibratory system are mass, stiffness, damping and external forces. Therefore, in the dynamic model of bearing researchers have studied and derived the expressions for bearing stiffness coefficients, dynamic forces, damping coefficients and effect of lubrications. The localized contact stresses in ball and rolling element bearings are extremely high as compared with stresses acting on rotating structural components. In absence of lubricant these contact stress in bearing are governed by the Hertzian theory.

Gupta [3-7] has proposed a six degrees-of-freedom model for the movement of rolling element around the inner ring. The author has considered the masses of rolling elements and Hertzian load displacement effect. In these studies of ball and ball-cage interaction vibrations, author has considered only the masses of rolling elements. The mass of the shaft, races and housing have not been accounted. However, Gupta [6] has included rolling elements and cage interactions as simulated by hydrodynamically lubricated metallic contacts. Gupta [7] has also suggested an advanced model which is capable of handling geometrical imperfections such as variations in rolling element size, race curvature, bearing element imbalance and cage geometry, allowing various bearing defects to be simulated. A comprehensive general analysis of the motions of balls and cage with lubrication has been presented by Walters [8].

In the dynamic analysis, the equations of motion have been obtained by considering four degrees-of-freedom for balls and six degrees-of-freedom for the cage. The pertinent relationships for the evaluation of rolling element bearing dynamic performance in any application have been presented by Harris and Mindel [9]. Boesiger et al. [10] where carried out tests with precision angular-contact ball bearings verified important features such as friction threshold for stability, cage motion and instability frequency. Tomovic et al. [11] have proposed a vibration model of a rotor-rolling bearings system to find the effect for internal radial clearance value and number of rolling elements influence on rigid rotor vibrations in unloaded rolling element bearing.

The main source of the vibration in the shaft bearing system is the presence of the defect on the interacting bearing components. Many research papers have been published in last few decades on the detection of the defects in rolling element bearings. The dynamic models of rolling element bearing with local and distributed defects have been reviewed in sections 2.1 and 2.2 respectively. Moreover, in section 2.3 different signal processing techniques used for improvements of fault detection have been reviewed.

2.1 Vibration response due to local defect on bearing element.

The vibration signal generated by the faulty bearing can be analysed in time domain or frequency domain. The time domain vibration analysis is depend on the estimation of statistical parameters like crest factor, skewness, kurtosis, probability density curve, etc. Among this kurtosis is the most effective parameter in a time domain which is calculated using following expression (1). For the healthy bearing the kurtosis is closer to 3 and for defective bearing it is more than 3.
Kurtosis = \frac{(N-1)\sum_{i=1}^{N}(x_i - \bar{x})^4}{\sum_{i=1}^{N}(x_i - \bar{x})^2} \quad (1)

Where \( x_i \) = instantaneous amplitude, \( \bar{x} \) = mean, \( N \) = speed in rpm.

In the frequency domain analysis the fault signal can be identified based on bearing fundamental frequencies which depend on the bearing geometry and rotor speed which have been calculated using following eq. (2-5).

Inner race ball pass frequency (BPFI) = \frac{n}{2} f_r (1 + \frac{d_b}{d_p \cos \varphi}) \quad (2)

Outer race ball pass frequency (BPFO) = \frac{n}{2} f_r (1 - \frac{d_b}{d_p \cos \varphi}) \quad (3)

Ball spin frequency (BSF) = \frac{n}{2} f_r (1 - \frac{d_b^2}{d_p^2 \cos^2 \varphi}) \quad (4)

Fundamental train frequency (FTF) = \frac{f_r}{2} (1 - \frac{d_b}{d_p \cos \varphi}) \quad (5)

Where \( n \) = number of balls in rolling element bearing, \( d_b \) = ball diameter, \( d_p \) = bearing pitch diameter, \( \varphi \) = bearing contact angle, \( f_r \) = rotor frequency.

Fig. 1(a) and 1(b) shows the frequency response of defective outer race and inner race. The peaks at shaft rotational frequency \( f_s \), BPFO with its harmonics (Fig. 1(a)) and BPFI with side bands (Fig. 1(b)) are clearly visible.

In the review paper presented by Kim [12] and Tandon [13] have noticed that mainly two approaches have been adopted by researchers to monitor the vibration response due to localized defect, one is to run the bearing for its entire life and wait to observe the changes in vibration response until bearing failure occurs and the other approach is to prepared artificially defective bearing by using acid etching, spark erosion, scratching or mechanical indentation and measure their vibration response and compare it with that of good bearings. For the simulation of defects researchers have introduced external force through series of impulses using Delta function or Fourier series. Fig. 2 shows the Gaussian impulse train that can be used for simple dynamic model of defective bearing. The impulses generated due to interaction of defect are shown in Fig. 2(b). The rate of repetition of impulses is observed at 200 Hz in the frequency domain as shown Fig. 2(b).
Another way of defect consideration adopted by the researchers is addition of extra displacement in total deflection. Initially a simple dynamic model was proposed by McFadden and Smith [14-15]. In their theoretical and experimental studies they have considered single point defect and multiple point defects on the inner race. The impulses generated by interaction of defect with inner race has been generated by delta function while, variations in the load around the bearing has been computed by Stribeck equation under radial load. Su and Lin [16] have extended the original work of McFadden to characterize the vibrations measured from bearings subject to various loading conditions and with defects located on any bearing components. They have determined the periodic characteristics of various loading, transmission path and their influence on the vibration amplitude. Su et al. [17] have obtained a reliable model to predict the possible bearing frequencies, harmonics and sidebands for the various types of localized fatigue damage, the pattern of expected frequencies can be searched for as part of routine bearing condition monitoring.

The researchers have developed the lumped mass model with various considerations about mass, linearity of bearing stiffness and damping coefficients, lubrication, clearance, defect size and slip of balls. Choudhury and Tandon [18] have proposed simplified lumped-mass model and linear bearing stiffness to obtain the vibration response due to localized defects considering the effects of shaft and housing mass. While Arslan and Akturk [19] have neglected the effect of housing and considered the mass of ball during modeling. Moreover the bearing stiffness coefficients are assumed to be non linear. The deflections of ball clearly indicate its deformation in load zone and defect region. Patil et al. [20] have varied the defect size and studied its effect on vibration response. Authors have noticed that the vibration amplitude increases with defect size. In two different studies Patel et al. [21-22] have developed 6 DOF dynamic model of deep groove ball bearing in presence of defects on either of races under steady and dynamic loading condition. They have concluded that the amplitude of vibration velocity in case of multiple defects is more as compared to single defect on either race. Although, the defect detection for multiple defects is difficult due to same vibration spectra as single defect. To improve defect detection researchers [23-33] have used techniques like high frequency resonance technique (HFRT), bond graph theory, Minimum Shannon Entropy Criterion (MEC) and wavelet transform. A new method based on the HFRT and the envelope has been proposed by Mohammadi and Safizadeh [23] to improve the defect detection for multiple defects. Cong et al. [24] have observed that the dynamic bearing fault model can also be improved by the combination of the decaying oscillation fault signal model and rotor dynamic response.

Weinzapfel et al. [25] have presents a 3D topologically equivalent model of granular material microstructures, which can be used to investigate subsurface Hertzian stresses fields that arise in RCF. Recently Weinzapfel and Sadeghi [26] have developed a 3D FEM model for sub-surface initiated spalling in cylindrical roller bearings for investigating the influence of 3D microstructure topology on the stochastic phenomenon of rolling contact fatigue. Brie [27] has proposed a model for vibration study of spalled rolling element bearing. The author pointed out that the excitation due to spalled bearing is quasi periodic due to load distribution and approximation in the evaluation of contact angle. Raje et al. [28] have developed statistical damage mechanics based fatigue model for subsurface initiated spalling in rolling contacts that incorporates micro crack initiation, coalescence, and
propagation into a unified framework. A detailed 33 degree of freedom model for rolling element bearings using vector bond graphs incorporating multi body dynamics of elements, centrifugal effects, dynamics of contacts, and surface defects has been developed by Nakhaeinejad and Bryant [29].

Kankar et al. [30-31] have developed the dynamic model of rotor-bearing system based on the method of Minimum Shannon Entropy Criterion (MEC) for selection of most appropriate wavelet and characteristic defect frequency. Authors have further extended the developed model to obtain the vibration response for high speed rotor bearing system in presence of defect using Response Surface Methodology (RSM). The modified Hertzian force-deflection relationship for dented surfaces to study the effects of dent size, location, inner race speed, and dent cluster on bearing performance has been investigated by Ashtekar et al. [32]. Tadina and Boltezar [33] have developed an improved multi degree of freedom model for radial ball bearing due to dent and lubricant contaminations during run up of shaft. In improved model include detailed fault modelling considering the non stationary speed of the shaft and the deformable outer race.

The effects of lubricant on bearing stiffness and damping have been studied by Sarangi et al. [34-37]. Generally in industries the actual bearing signal merges in strong background noise. To understand the effect of background noise Randall et al. [38] have proposed a simple statistic model. This developed model is further extended by Antoni et al. [39] for highly variable speed.

2.2 Vibration response due to distributed defect on bearing element.

The distributed bearing defects such as waviness of races, surface roughness, off size rolling elements, cage imbalance, and misalignment of cages generate the additional vibration which can excite the resonance in rotating machines. The roundness profile of a bearing ring which is also known as race waviness can be presented by the following eq. (6).

\[ R(\theta) = \sum_{m=1}^{n} A_m \cos(m\theta + \varphi_m) \]  

Where \( \theta \) = angular coordinate of the bearing ring, \( \varphi_m \) = Phase angle of \( m \)th order waviness, \( A_m \) = Amplitude.

The frequency of the waviness depends on the order as well as the location of waviness (outer race/inner race). Researchers [40-46] have studied about the race waviness for different waviness order and off size rolling element. The waviness orders close to the number of balls \( (z \pm 1) \) and \( z \) generate vibrations at the bearing defect frequencies [40]. Sopanen and Mikkola [40-41] have proposed a general purpose dynamic model considering non idealities for race waviness, localized defect in races and misalignment. In another study [41] they have reported theoretical results generated through dynamic model for high and low order waviness at inner and outer race location. Authors have observed that level of vibration is very high in case of combined waviness order of both races than a single waviness order. Meyer et al. [42] have proposed an analytical model for flexural vibration of the stationary race under axial load due to waviness on the moving race or unequal ball diameter. Fig. 3 (a) and (b) shows the vibration generated due to unequal ball diameter and misaligned outer races respectively. The series of tones in Fig. 3(a) indicates variations of unequal ball diameter while, the series of tones in Fig. 3 (b) indicates the misaligned rotating outer race and a stationary inner race.

![Fig. 3 Frequency Spectrum (a) for unequal ball diameter (b) for misaligned outer race [42].](image-url)
Authors of references [43-44] have developed 3 DOF model and studied the effect of waviness on the vibration amplitude and also observed that the amplitude of vibration increased due to waviness of outer race as compared to the presence of waviness on inner race. Cao and Xiao [45] have extended existing 3 DOF model of single raw deep groove ball bearing (DGB) into a 5 DOF comprehensive dynamic model of double-row spherical roller bearing (SRB) with the consideration of races and ball surface waviness, radial clearance, surface defects, point defects and loading conditions. However, Babu et al. [46] have noticed that inner race radial waviness produce high amplitude of vibration as compared to outer race waviness. In their 6-DOF dynamic model of rigid rotor-angular contact ball bearings system the effect of frictional moments have been incorporated. Ghaisas et al. [47] have developed 6 DOF dynamic model for rigid cylindrical roller bearing cage to investigate the effect of bearing clearances, inner-race rotational speed, inner-race misalignment, single-roller size variation, cage asymmetry, and roller profile on cage dynamic motion in a lightly loaded and high-speed application. While, Ashtekar and Sadeghi [48] have developed a 3D explicit finite element model (EFEM) of the cage and combined with an existing discrete element dynamic bearing model (DBM) with six degrees of freedom to study the ball bearing dynamics in the presence of a flexible cage.

Any high speed rotating machinery produces high temperature during running for long period. It is essential to measure and check the tolerable temperature range on the rotating machine parts to avoid the failure due to high heat generation. To estimate the heat generation Tarawneh et al. [49] have developed thermal model of a railroad tapered-roller bearing using finite element analysis. The stochastic model with the capability of predicting vibrations for known defective conditions has been proposed by Behzad et al. [50]. This stochastic model includes the effect of back ground noise.

2.3 Improvements of fault detection.

The mode of bearing failure and type of machine are the most important criteria for the vibration monitoring of the bearing. The vibrations generated due to fatigue failure are less complex as compared to vibrations generated due to wear, lubrication starvation, corrosion or faulty installation. Simple rotating machinery require simple diagnostic techniques (time domain or frequency domain) while the complex rotating machinery require sophisticated signal processing techniques [51]. The researchers have identified the type, size and location of faults in a rolling bearing using various signal processing methods like time frequency analysis, high frequency resonance technique (HFRT), wavelet transform, Haar transform, S transform, cepstrum analysis, bispectrum analysis, higher order spectral analysis, adaptive noise cancellation (ANC), artificial neural network (ANN), and cyclic autocorrelation.

The impulses generated due to the interaction of defect and bearing elements excites the resonances periodically at the characteristic defect frequency. These excited resonances are amplitude modulated at the characteristic defect frequency. The demodulation of resonances eliminates the unwanted low frequency signals generated by other sources. The process of extraction of demodulated spectra is known as high frequency resolution or envelope analysis. The fault detection at the initial stage is difficult through envelope analysis. In recent years, the wavelet transform method has been suggested to extract very weak signals for which Fourier transform becomes ineffective [52]. The effectiveness of the envelope analysis depends on the selection of the centre frequency and band width. The short coming of the envelope analysis has been overcome by spectral kurtosis [53, 54], combination of squared envelope spectrum and computed over tracking analysis [55], the fusion of the wavelet transform and envelope spectrum [56]. Moreover, Yang [57] et al. have proposed the method for fault feature extraction based on intrinsic mode function (IMF) envelope spectrum which overcome the limitations of conventional envelope analysis method.

Hilbert-Huang transform is an adaptive type time-frequency analysis method. Due to its high time-frequency resolution capability for nonlinear and non-stationary signal, researchers [58-61] have used Hilbert-Hung transformation (HHT) to analyze the fault signal of the rolling bearing. Li and Wang [59] have summarizes the development in the present situation of application of Hilbert-Huang Transform for solving the problem of rolling bearing fault diagnosis from several aspects, analyzes the current rolling bearing fault diagnosis methods combined
with HHT, and sums up the practical applicability of HHT method through the comparison of rolling bearing fault diagnosis with other methods. In order to detect rolling element bearing faults from strong background noise, a method based on translation-invariant de-noising and HHT is proposed by Xu [61].

The information regarding frequency at any particular time is difficult to achieve either from frequency domain or time domain. The wavelets provide time-scale information of a signal, enabling the extraction of features that vary in time. The discrete wavelet transform (DWT) is derived from the discretization of continuous wavelet transform (CWT). The wavelet transform has proved its significance in bearing fault detection [62-68]. However, the effectiveness of the analysis depends on the selected mother wavelet. The defective frequencies of locally defective bearing were analyzed more accurately through DWT by Prabhakar et al. [62]. Rubini and Meneghetti [63] have proposed the method of envelope analysis and wavelet transform for the diagnosis of incipient bearing fault under very low radial load. The researchers of references [64-65] have used the Laplace wavelet transform for the defect detection. Hong and Liang [66] have proposed a new version of the Lempel–Ziv complexity as a bearing fault (single point) severity measure based on the continuous wavelet transform (CWT). The researchers [67-68] have proposed the method of singularity analysis using continuous wavelet transform for bearing fault diagnosis.

The non-stationary processes whose statistical characteristics vary periodically with time are called cyclostationary processes. The statistical properties of faulty bearing signal vary periodically with time due to the presence of slip or due to variation of forces. The cyclostationary analysis of these signals provides more information than the stationary analysis. The cyclostationary analysis provides the information regarding the content of a signal and its periodical variation. The degree of cyclostationarity (DCS) provides overall indication of appearance of the modulating frequencies. The researchers in references [69-72] have performed the cyclostationary analysis of faulty bearing signal. They have found the cyclic autocorrelation more effective than other methods in bearing diagnosis.

3. Conclusion

From a review of dynamic models of healthy and faulty rolling element bearing it has been observed that the vibration amplitude of the defective bearing are more compare to the healthy bearing. Moreover, the presence of bearing fault (local or distributed) and its location can be identified through the time and frequency domain analysis of the vibration signal. The accuracy of the dynamic model depends on the considerations like mass of shaft, bearing elements, housing, linear or non linear bearing stiffness, lubrication, speed, damping, defect, friction and presence of noise. The defect can be simulated by the addition of extra disturbing force or displacement. The fault detection can be improved by the signal processing techniques like envelope analysis, HHT, wavelet transform, cyclostationary analysis and noise cancellation. The effectiveness of envelope analysis and wavelet transform depends on the selection of the center frequency and mother wavelet, respectively.

Acknowledgements

Author acknowledges the sincere thanks to the SVIT, Vasad management for giving the permission for my research study.

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