SUPERMASSIVE BLACK HOLES IN GALACTIC NUCLEI WITH TIDAL DISRUPTION OF STARS. II. AXISYMMETRIC NUCLEI

SHIYAN ZHONG1, PETER BERCZIK1,2,3, and RAINER SPURZEM1,2,4,5

1 National Astronomical Observatories of China and Key Lab for Computational Astrophysics, Chinese Academy of Sciences, 20A Datun Rd., Chaoyang District, 100012, Beijing, China
2 Astronomisches Rechen-Institut, Zentrum für Astronomie, University of Heidelberg, Mönchhofstrasse 12-14, D-69120, Heidelberg, Germany
3 Main Astronomical Observatory, National Academy of Sciences of Ukraine, 27 Akademika Zabolotnogo St., 03680, Kyiv, Ukraine
4 Kavli Institute for Astronomy and Astrophysics, Peking University, Beijing, China
5 Key Lab of Frontiers in Theoretical Physics, Institute of Theoretical Physics, Chinese Academy of Sciences, Beijing, 100190, P.R. China

Received 2015 March 30; accepted 2015 July 31; published 2015 September 16

ABSTRACT

The tidal disruption (TD) of stars by supermassive central black holes from dense rotating star clusters is modeled by high-accuracy direct N-body simulations. As in a previous paper on spherical star clusters, we study the time evolution of the stellar tidal disruption rate and the origin of tidally disrupted stars, which are now accorded to several classes of orbits that only occur in axisymmetric systems (short-axis tube and saucer orbits). Compared with that in spherical systems, we found a higher TD rate in axisymmetric systems. The enhancement can be explained by an enlarged loss cone in phase space that stems from the fact that the total angular momentum \( J \) is not conserved. As in the case of spherical systems, the distribution of the last apocenter distance of tidally accreted stars peaks at the classical critical radius. However, the angular distribution of the origin of the accreted stars reveals interesting features. Inside the influence radius of the supermassive black hole the angular distribution of disrupted stars has a conspicuous bimodal structure with a local minimum near the equatorial plane. Outside of the influence radius this dependence is weak. We show that the bimodal structure of orbital parameters can be explained by the presence of two families of regular orbits, namely short-axis tube and saucer orbits. Also, we present the consequences of our results for the loss cone in axisymmetric galactic nuclei.

Key words: galaxies: kinematics and dynamics -- galaxies: nuclei -- methods: numerical -- quasars: supermassive black holes -- stars: kinematics and dynamics

1. INTRODUCTION

A large fraction of galaxies show evidence of supermassive black holes (henceforth SMBH) residing in their center. They are typically embedded in nuclear star clusters (NSC); if resolution allows the observation of the NSCs, they are among the densest clusters known. Their size is similar to galactic globular clusters, but they are much heavier and brighter (Böker et al. 2002, 2004). In massive galaxies NSCs may not be significant or may not even exist; however, the SMBHs are still surrounded by an enormous number of stars. SMBHs residing in these NSCs will tidally disrupt stars that come close to their tidal radii and eventually accrete the gaseous debris, which can light up the central SMBH for a period of time (Rees 1988; Evans & Kochanek 1989). This kind of event is a useful tool for examining the relativistic physics near SMBHs because the disruption occurs at a place that is very close to the BH’s Schwarzschild radius. Also, it can help us to investigate SMBHs in non-active galactic centers. Although the tidal disruption (TD) of stars has been proposed for almost half a century, only in the last decade have people realized the importance of such events, after the discovery of a dozens of TD candidates (Komossa 2002; Komossa & Merritt 2008). Liu et al. (2014) discovered a candidate binary SMBH system by analyzing the break in the light curve of a TD event, demonstrating that the TD of stars can be a promising tool for searching hidden SMBH binaries in quiescent galactic centers. In order to compute the tidal disruption event rate, many theoretical works have been performed in the past few decades (Frank & Rees 1976; Lightman & Shapiro 1977; Magorrian & Tremaine 1999; Wang & Merritt 2004). The core of the story is loss cone theory, which was first established in the case of spherical symmetric systems.

Stars with an orbital pericenter smaller than the tidal radius \( r_t \) are defined as being inside the loss cone, with \( r_t \) expressed by

\[
r_t = \alpha r_s \left( \frac{M_{bh}}{m_*} \right)^{1/3},
\]

where \( r_s \) and \( m_* \) are the radius and mass of a star, \( n \) is its polytropic index (assuming the stellar structure can be approximated by a polytropic sphere), and \( \alpha \) is a free parameter that is used by us for scaling. Stars with an angular momentum \( J < J_{lc} \approx \sqrt{2GM_r r_t} \) are inside the loss cone. Typically, loss cone stars are consumed in dynamical timescales. If no new star is supplied to the loss cone, there will be no more tidal disruption events. Based on the status of the loss cone, it can be divided into two regimes, namely the empty and the full loss cone. Due to the short “lifetime” of the loss cone stars, the loss cone will quickly become empty. The refilling of the loss cone happens on relaxation timescales and is often referred to as diffusion process in angular momentum space. Thus in empty loss cone regimes it is the refilling rate that controls the disruption rate. Note that throughout this paper, and as in most, if not all, of the cited papers on the tidal accretion of stars into SMBHs, we assume that a star is disrupted completely at \( r_t \) and that all of its mass, energy, and angular momentum are momentarily absorbed by the SMBH. We know that this is not realistic, and more detailed numerical models of the process of disruption, possible disk formation, and accretion show that...
only fractions of the material are absorbed into the SMBH after a number of orbits (Guillochon & Ramirez-Ruiz 2013; Hayasaki et al. 2013). However, the assumption that the process is fast is reasonable compared to the orbital timescales of stars further out in the cluster.

In a previous paper (Zhong et al. 2014, henceforth Paper I) we showed that for a spherical symmetric system in the diffusive empty loss cone regime, the classical loss cone approximation can be well-reproduced using large direct N-body models of the tidal accretion of stars into SMBHs. Now we are focusing on the generalization to axisymmetric galactic nuclei and compare our new results in an otherwise very similar study to that in Paper I.

The TD of stars is one possible way of growth for SMBHs, especially in quiescent galactic nuclei. Since most models assumed spherical stellar clusters, SMBH growth rates via TD are very low, limited by the very long relaxation time needed to refill the loss cone, and the contribution of the TD process to the overall growth of SMBHs is considered to be relatively insignificant. However, the stellar distribution in real galactic nuclei might not be spherically symmetric. Many galactic nuclei show evidence of rotation in their centers, even very close to the SMBH (Greenhill et al. 1995; Miyoshi et al. 1995; Neufeld & Maloney 1995).

According to the current standard model of structure formation, massive galaxies have undergone quite significant mergers (in number and mass ratio). Numerical models of the merging process of galaxies show that the merger remnant shows rotation, axial symmetry, or even triaxiality in the central regions (Khan et al. 2011; Preto et al. 2011; Gualandris & Merritt 2012; Bois et al. 2013).

In the center of our own Milky Way the NSC can be observed in unparalleled high resolution (Feldmeier et al. 2014; Schödel et al. 2014). It consists of $1.4 \times 10^7 M_\odot$ within its effective radius (4.2 pc); kinematic data indicate that it possesses bulk rotation (Feldmeier et al. 2014). The formation mechanism of NSCs is still under debate. There are two scenarios: in situ formation (Milosavljević 2004) and a sinking scenario (globular clusters sink to the center and merge; Tremaine et al. 1975; Lotz et al. 2001). NSCs in a sample of nearby galaxies observed by Seth et al. (2006, 2008) show that these objects are non-spherical and even contain multi-component (younger disk plus an older spherical component), which favors the in situ scenario. However, Antonini et al. (2012) have performed a series of N-body simulations to study the formation of NSCs, which support the sinking scenario. The model NSCs formed in their simulations via the merging of infalling globular clusters initially have a mildly triaxial shape. After the final infall, the shape of the NSCs gradually becomes axisymmetric, as a consequence of dynamical evolution. Finally, we note that dissipative (in situ) and dissipationless (sinking) formation mechanisms must not be mutually exclusive, and that it is indeed quite possible that both have played an important role in the growth of the nuclei (Antonini et al. 2015).

Despite the debate between different formation scenarios, we think that it is quite likely that NSCs are non-spherical. This provides a good motivation for studying the tidal disruption rate (TDR) in axisymmetric (and triaxial) clusters. Some studies have already been performed but the mission is not over. Fiestas & Spurzem (2010) used a two-dimensional (2D) Fokker–Plank model (Einsel & Spurzem 1999; Kim et al. 2002) to study rotating dense stellar clusters with black holes (BHs) and cross-checked with N-body models (Fiestas et al. 2012). Both works find that BHs embedded in rotating models have higher TDRs compared to spherical models. The BH mass at the end of the simulation is roughly 20% higher in the rotating case. Fiestas et al. (2012) find an excess of accreted prograde rotating stars that mainly originate outside of the influence radius $r_b$ and call for a further investigation of the roles of stars with non-conserved $J_z$, $J$ angular momentum. As shown by the works of Merritt & Poon (2004), in non-spherical systems chaotic orbits (existing in regions outside $r_b$) can keep the loss cone full for a sufficiently long time, thus tidal disruption can contribute a lot of mass within Hubble time and could play an important role in a BH’s growth across cosmic time.

On the other hand, the loss cone itself might be enlarged, as pointed out by Magorrian & Tremaine (1999), due to the fact that the angular momentum $J$ is not conserved in axisymmetric potential. Vasiliev & Merritt (2013) confirmed this picture in a detailed analysis of the loss cone problem in axisymmetric galactic nuclei. They analyzed the depletion and refilling of loss cone orbits and found that TDRs could be increased by a moderate factor due to axisymmetry, compared to spherical symmetry. In their work, chaotic orbits with low-angular momenta can reach just outside the influence radius at apocenter, but also get close to the central SMBH at pericenter, causing some difficulty when comparing with Fokker–Planck models, as was already discovered by Malkov et al. (1993, pp. 139–152; note that the last author of this paper is the same person as the last author of Malkov et al. 1993, pp. 139–152 because there was a mistake in retranslating the name from Russian).

In this work we follow an experimental numerical approach to the problem, following Paper I for the case of spherically symmetric systems. We treat particle number and tidal radius as free parameters and analyze the tidal accretion rate of the system as a function of the strength of deviation from spherical symmetry. We measure the shape of the loss cone in axisymmetric potential and and characterize the characteristic orbits of stars in the loss cone. We find that it is indeed enlarged and can account for the higher TDR as compared to spherically symmetric galactic nuclei.

This paper is organized as follows. We describe the model setup of the simulation in Section 2 and present the results of TDR measurement in Section 3. Section 4 is devoted to the measurement of loss cone shape in axisymmetric potential and we demonstrate the enlargement of loss cone. In Section 5, we present the results for the origin and orbital classifications of disrupted stars. In Section 6, we discuss the potential application of our results.

2. N-BODY MODEL

We adopt the standard N-body unit definitions from Heggie & Mathieu (1986), namely $G = M = 1$ and $E = -1/4$, where $G$ is the gravitational constant, $M$ is the total mass of the model cluster, and $E$ is the total energy. In our N-body models we assume that all of the particles have the same mass, so $m = 1/N$, where $m$ is the particle mass and $N$ is the total particle number. To preserve the scale invariance of our N-body simulations we fix the initial black hole mass relative to the total mass of the star cluster (0.01) and use the particle number and the tidal radius $r_i$ in N-body units (which is a
dimensionless number) as free parameters. We have shown in Paper I that the method of scaling to realistic parameters for $N$ and $r_1$ can be used to obtain astrophysically meaningful results from the collection of our models. In order to support our scaling procedure we do not even change the tidal radius during the simulation, relative changes in tidal radius are small (notice that $r_1 \propto (M/R)^{1/3}$).

The initial distribution of particles follows a generalized King model with rotation. The distribution function is (Einsel 
& Spurzem 1999; Ernst et al. 2007)

$$f(E, J_z) = C \cdot \left[ \exp \left( -\frac{E}{\sigma_K^2} \right) - 1 \right] \cdot \exp \left( -\frac{\Omega_0 J_z}{\sigma_K^2} \right),$$ (2)

where $\sigma_K$ is the King velocity dispersion and $\Omega_0$ is a characteristic angular velocity. Since we are considering an isolated system, the $\Phi_i$ is set to 0. This rotating King model has two dimensionless parameters: $W_0$ and $\omega_0$. The King parameter $W_0 = -\Phi_0 / \sigma_K^2$, where $\Phi_0$ is the central potential, controls the degree of central concentration. And the rotation parameter $\omega_0 = \sqrt{9(4\pi G \rho_0)} \cdot \Omega_0$, where $\rho_0$ is the central density, controls the degree of rotation. $\omega_0 = 0$ will reduce the model to a usual non-rotating spherically symmetric King model.

We limit our current study to only one concentration parameter $W_0 = 6$ and two rotation parameters $\omega_0 = 0.3, 0.6$; the density profile of the King model with this concentration is similar to that of the Plummer model used in Paper I, so it is possible to compare with the previous results and focus on the effects of rotation and axial symmetry only. The rotation is moderate (see, e.g., Einsel 
& Spurzem 1999) and resembles that of Milky Way globular clusters.

For completeness we also employ a non-rotating King model with $W_0 = 6$ and $\omega_0 = 0.0$, which is used as a fiducial model and also as a bridge to the results of Paper I, confirming our claim that it indeed closely resembles the results of the Plummer model used in Paper I (e.g., in the evolution of the TDR). In another test run we used a larger rotation with $\omega_0 = 0.9$—it experienced an unstable stage during which a bar formed but quickly disappeared. This bar formation could probably be identified with the radial orbit instability of Aguilar 
& Merritt (1990). We note that our standard models with $\omega_0 = 0.3, 0.6$ remain fully axisymmetric during the entire simulation; to study TD in triaxial systems with bars is beyond the scope of our current paper.

Figure 1 shows the axial ratio ($c/a$) of the model clusters as a function of radius up to $r = 2.0$ (within which most of stars are located). We estimate the axial ratio for both rotating models, using the moment of inertia tensor measured in concentric shells. One can see that $c/a$ is close to 1 at the innermost part and decreases outward: the $\omega_0 = 0.3$ model decreases slowly to its minimum value of 0.9; the $\omega_0 = 0.6$ model decreases faster and has a minimum value of 0.71. If we measure the $c/a$ for the whole cluster, the results of the two models are $0.9$ ($\omega_0 = 0.3$) and $0.75$ ($\omega_0 = 0.6$). Figure 1 also shows that $c/a$ is almost unchanged during its long time evolution, except for the inner part of the $\omega_0 = 0.6$ model, which exhibits a slight decrease.

In rotating systems, there is a phenomenon called gravo-gyro instability, which is caused by the negative specific moment of inertia (Inagaki 
& Hachisu 1978; Hachisu 1979, 1982). This kind of instability happens in long term evolution of rotating clusters, which is much longer than our integration time (Ernst et al. 2007).

The model set is summarized in Table 1.

### Table 1

| Model | $N/K$ | $\omega_0$ | $r_1$ | $T$ |
|-------|-------|------------|-------|-----|
| R20w00 | 64    | 0.0        | $10^{-3}$ | 1500 |
| R30w00 | 128   | 0.0        | $10^{-3}$ | 1600 |
| R21w00 | 64    | 0.0        | $10^{-4}$ | 1500 |
| R31w00 | 128   | 0.0        | $10^{-4}$ | 1300 |
| R20w03 | 64    | 0.3        | $10^{-3}$ | 1500 |
| R30w03 | 128   | 0.3        | $10^{-3}$ | 1500 |
| R21w03 | 64    | 0.3        | $10^{-4}$ | 2600 |
| R31w03 | 128   | 0.3        | $10^{-4}$ | 2000 |
| R20w06 | 64    | 0.6        | $10^{-3}$ | 1500 |
| R30w06 | 128   | 0.6        | $10^{-3}$ | 1500 |
| R21w06 | 64    | 0.6        | $10^{-4}$ | 1600 |
| R31w06 | 128   | 0.6        | $10^{-4}$ | 2000 |

Note. Column 1: model code name; Column 2: particle number in the unit of K ($=1024$); Column 3: dimensionless rotation parameter; Column 4: black hole’s tidal radius; Column 5: total integration time. $r_1$ and $T$ are in the model unit.

### Table 2

| $t/t_{th}$ | $N_0$ | $N_1$ | $N_0/N_0$ | $N_k$ | $N_0/N_0$ |
|------------|-------|-------|------------|-------|------------|
| 0.25       | 14.96 | 15.41 | 1.03       | 16.30 | 1.09       |
| 0.50       | 12.89 | 13.67 | 1.06       | 17.30 | 1.34       |
| 0.75       | 10.44 | 12.07 | 1.16       | 14.52 | 1.39       |
| 1.00       | 8.73  | 10.32 | 1.18       | 12.65 | 1.45       |
| 1.25       | 7.97  | 9.73  | 1.22       | 11.09 | 1.39       |
| 1.50       | 6.99  | 7.93  | 1.13       | 10.04 | 1.44       |

Note. Measured TDRs for models with the same $N$ (128 K) and $r_1 (10^{-3})$ but different rotation parameters, at different evolution times. $N_0$ is TDR in the classic King model ($\omega_0 = 0$); $N_1$ and $N_0$ are the results for the $\omega_0 = 0.3$ and $\omega_0 = 0.6$ models. We also give the boost factors $N_k/N_0$ and $N_0/N_0$. 

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**Figure 1.** Axial ratio for rotating models as a function of radius. For each model we show the axial ratio measured at different evolution stages: $T = 0$ (red); $T = 500$ (green); and $T = 1000$ (blue). The lines with symbols are the results for the $\omega_0 = 0.3$ model; the lines without symbols are the results for the $\omega_0 = 0.6$ models.
We run the simulation for more than one initial half-mass relaxation time \( t_{\text{rh}} \), which is estimated using the same formula in Paper I, where the values can also be found (in Table 2 of Paper I).

All simulations are running with the \( \varphi \)GRAPE code (Berczik et al. 2011), which runs with high performance (up to 350 Gflop/s per GPU) on our GPU clusters in Beijing (NAOC/ CAS). This code is a direct \( N \)-body simulation package, with a high-order Hermite integration scheme and individual block time steps. In principle, a direct \( N \)-body code evaluates all pairwise forces between the gravitating particles, and its computational complexity per crossing time scales asymptotically with \( N^2 \); however, it is not to be confused with a simple brute force shared time step code, due to the block time steps. We refer more interested readers to a general discussion about \( N \)-body codes and their implementation in Spurzem et al. (2011a, pp. 35–58, 2011b). The present code is well-tested and has already been used to obtain important results in our earlier large-scale few-million-body simulation (Khan et al. 2012).

3. TIDAL DISRUPTION RATE (TDR)

3.1. Results of Our Work

In this section, we present the TDR measured in simulations with our rotating King models and compare it with the TDR of the non-rotating model of Paper I. In Figure 2, we show the TDR (both in terms of mass and particle number) as it evolves with time for two different tidal radii; in each panel two different rotation parameters are plotted together with the data of the non-rotating system. The time is given in units of initial half-mass relaxation time \( t_{\text{rh}} \), which is convenient for comparing simulations with different particle numbers. To smooth out fluctuations due to particle noise we have plotted in the figure the TDR averaged over a time interval (here \( 1/4t_{\text{rh}} \)).

The TDR with a large tidal radius (i.e., \( r_{\text{t}} = 10^{-4} \)) initially quickly rises in the \( N = 64 \) K model to its peak value and then decreases; for the \( N = 128 \) K model the TDR almost decreases from the beginning. The initial phase is connected with the formation of a central density cusp in the surrounding stellar system and with the process of a transition from an initially full to an empty loss cone. The BH gains mass from the accreted stars, thus the mass ratio between stars and the BH \( (\gamma = m_{\text{BH}}/M) \) decreases with time, and as a result the BH’s random motion damps. In Paper I we noted that the status of the loss cone is connected with the BH’s Brownian motion in the sense that once the amplitude of Brownian motion is smaller than \( 10^{-3}r_{\text{t}} \), the system enters the empty loss cone regime, during which the cusp and central density are still growing but the TDR begins to fall. In the \( N = 128 \) K model, the mass ratio \( \gamma \) is smaller, so the initial loss cone depletion is very short, practically invisible in the plots, and the subsequent evolution is determined by cusp formation and the damping of BH motion.

In the models with a small tidal radius (\( r_{\text{t}} = 10^{-4} \)), there is always an initial growth phase for the TDR, followed by a convergent approach to a stationary state. Due to the small \( r_{\text{t}} \), their BH growth is slow, thus they need more time to achieve the mass required to limit their Brownian motion.
Figure 2 also shows the TDR dependence on the rotation parameter $\omega_0$ as a new result, compared to Paper I. For a large tidal radius ($r_t = 10^{-3}$), faster rotation will result in a higher TDR; note that these models are in an empty loss cone regime. Table 2 list the numbers for TDR measurement. One can see that, on average, the $\omega_0 = 0.3$ model has a TDR that is 13% higher than the $\omega_0 = 0.0$ model. And the TDR in the $\omega_0 = 0.6$ model is, on average, 35% higher than that in the $\omega_0 = 0.0$ model. The BH masses of these three models, measured at $T = 1500$, are 0.131, 0.143, and 0.167. The fractional increase of the final BH mass with the increasing degree of rotation is consistent with the result of Fiestas et al. (2012). The reason for this dependence of $\omega_0$ is that in these systems the effective loss cone is larger than the classical one in spherical systems. We will investigate such an enlarged loss cone in more detail in the next section. For a small tidal radius ($r_t = 10^{-4}$), however, we observe a different behavior for the TDR. From beginning to about $\sim 1.5r_{\text{in}}$, faster rotation results in a smaller TDR! The argument presented by Magorrian & Tremaine (1999) may provide some hints: if a BH’s wandering timescale is shorter than its dynamical timescale, a decrease in the TDR will occur. We note that at this early stage in the simulation the BH is quickly wandering due to its small mass and slow growth. Furthermore, in axisymmetric systems, a star’s pericenter distance changes with time (even ignoring irregular perturbations from other stars). So when the BH comes back to the place where it was, it may still miss the star that is supposed to be disrupted shortly before.

Afterward the system begins to enter the empty loss cone regime, and all of the TDR curves converge with each other; for small tidal radii more tidally disrupted stars originate from inside the BH influence radius, where the system is approximately spherically symmetric. Any deviation from spherical symmetry in our rotating models prevails near and outside the influence radius. The convergence of the TDR reflects the original results obtained in Paper I for spherical systems.

Figure 3 compares the TDR of the classic King model ($W = 6$, $\omega_0 = 0.0$) with that in the Plummer model. In $r_t = 10^{-3}$ models, except for the initial higher accretion rate in the King model, the two models have similar TDRs following evolution. In the case of $r_t = 10^{-4}$, most of the time the King model has a higher accretion rate; but later on the TDRs of the King model gradually come to the same level as the Plummer model. The higher rate in the King model could be explained by the slightly higher density in the core region at the beginning. In the following evolution of $r_t = 10^{-3}$ models, the two models form cusps that are similar to each other, so they have roughly same accretion rate. In the case of $r_t = 10^{-4}$, the initial accretion rate ratio $N_{\text{King}}/N_{\text{Plum}}$ is higher than that in $r_t = 10^{-3}$. BHs inside the King cluster grow faster, and the growth of the cusp following the evolution in the King model always results in a higher density in the cusp, which in turn gives a higher accretion rate. Only after the BH gains enough mass and becomes a “static” object does the accretion rate slowly reach a maximum and begin to drop afterward.

Up to this point, all of the results have been presented with model units, though observers have voiced concerns about the TDR in a real galactic environment expressed with physical units, which is one of the topics in Paper I (see Section 5 and the appendix of that paper). To predict the TDR in real galactic nuclei, we need to scale up the TDR obtained in low $N$ simulations ($N \sim 10^5$) to a high-$N$ ($\sim 10^8$) real cluster, which can be done using the scaling relations of $N$; also, TDR results obtained with artificially enhanced $r_t (10^{-3}, 10^{-4})$ have to be scaled down to the small $r_t (10^{-7})$ regime, which can be done by applying the $r_t$ scaling relation. As we have shown above, in the empty loss cone regime, the TDRs in the classical King model and the Plummer model are similar, thus we expect the scaling formula (A10) in Paper I to also be valid for the classical King model. Then we can multiply the boost factor by this formula and find the scaled result for axisymmetric nuclei. For example, in Paper I we estimated that the TDR of the Milky Way SMBH is $1.09 \times 10^{-5}$ yr$^{-1}$. By fitting a surface brightness profile to a mid-infrared image of the nuclear cluster in the Milky Way, Schödel et al. (2014) reported the mean ratio between minor and major axes is 0.71, which is closer to the $\omega_0 = 0.6$ model. Thus we increase the spherical result by 35% and the TDR in this axisymmetric nuclei rises to $1.47 \times 10^{-5}$ yr$^{-1}$.

3.2. Comparing with Other Works

While we have shown that our results are in agreement with Fiestas et al. (2012), in the meantime we also note that our results regarding the enhancement of the TDR in axisymmetric systems seem to contradict some existing works in the literature. For example, numerical simulations performed by Vasiliev & Merritt (2013) and Vasiliev (2014) showed that the TDR in axisymmetric nuclei can be larger than that in the spherical case by a factor of a few. Li et al. (2014) analyzed the orbital component of an axisymmetric galaxy model and found that the total number of stars that can interact with the central SMBH binary is six times larger in their flattened system than in the spherical system. All of them give a much larger enhancement factor than ours. We will discuss possible reasons for this discrepancy in this subsection.

The main difference between our work and theirs is the model cluster. In all of the above works, they used n flattened Dehnen model (or Hernquist model; given the parameters they chose, the models are identical in the sense of density profile). Their models possess a fixed axial ratio ($c/a = 0.75$) throughout the whole cluster and an initial central cusp, while our rotating model shows a gradual changing of the axial ratio from inside to out (Figure 1) and a central density core. However, in the radius range where most of the disrupted stars originate (see Section 5 and Figure 9), the system is well-deviated from spherical symmetry, thus we can say that the enhancement is connected with the non-spherical geometry. The maximum deviation in our $\omega_0 = 0.6$ models is less than their models, so the effect of flattening might be weaker in our case compared to theirs.

We also note that even their works with the same initial density profile show some discrepancies. For example, the enhancement of the number of accreted stars in Vasiliev & Merritt (2013) was smaller than 100% (see Table 2 in their paper), while Li et al. (2014) found a factor of six. Meanwhile, we also see that some of the models in Vasiliev & Merritt (2013) only show mild enhancement, which is at the same level as ours. Another example comes from the debate regarding the “final parsec problem” in SMBH binary evolution. Based on their simulation results, Khan et al. (2013) claimed that the “final parsec problem” is not a problem in axisymmetric host galaxy. While Vasiliev et al. (2014) reached an opposite conclusion according to their simulation. We note that both of
these works employ similar flattened galaxy models; however, they used different methods to generate the initial model. Vasiliev & Merritt (2013), Vasiliev (2014), and Vasiliev et al. (2014) utilized an orbital superposition method (Schwarzschild 1979) to construct their model. On the other hand, Khan et al. (2013) and Li et al. (2014) used another method called the “adiabatic squeeze technique” developed by Holley-Bockelmann et al. (2001). We note that in the process of adiabatic squeeze, there is a step that applies a slow and smooth velocity drag on the stars in the \( z \) direction. This step may artificially reduce the energy and angular momentum of the stars in the model cluster. Although the radius and velocity vectors of the stars are rescaled after the squeeze, it is not clear how the rescaling affects the phase space distribution. Thus it might be possible that the process still overproduces low-energy and low-angular-momentum stars. Evidence of overproducing comes from Vasiliev (2014), who, in that work, still uses the orbital superposition method but changes the scheme for generating initial model so that it creates more low-energy- and low-angular-momentum stars. In their test run (Figure 2 in their paper) we see a much larger enhancement of the number of accreted stars compared to Vasiliev & Merritt (2013). So adding more low-energy- and low-angular-momentum stars seems to be promising for abridging the different enhancement factors between Vasiliev & Merritt (2013) and Li et al. (2014). We suggest that a detailed comparison (e.g., phase space distribution) between models constructed with these two methods should be performed in order to explain the discrepancy.

Li et al. (2014) also report that the central 2 pc of their model galaxy exhibit slight triaxiality, which could introduce some additional centrophilic orbits, thus increasing the number of stars that can interact with the central SMBH binary.

Before finishing this section, we make a final remark about the results of Li et al. (2014). Their work actually is done in a static way, integrating individual orbits in a fixed model potential with one SMBH in the center. So the number of stars they claim that can interact with the central SMBH binary should be considered an upper limit. Once the two-body relaxation is turned on, some of the stars that were supposed to be inside the loss cone might be scattered out. And the presence of an SMBH binary in an evolving system may also affect how many stars can interact with them.

### 4. LOSS CONE IN AXISYMMETRIC POTENTIAL

First, we summarize the loss cone theory for stellar orbits in spherically symmetric gravitational potential in order to later discuss different behaviors in axisymmetric potential. If a stellar orbit has a pericenter distance less than the tidal radius, it is considered to be in the loss cone. In spherical symmetry, the boundary of the loss cone can be expressed in terms of a critical loss cone angular momentum \( J_G \), see, e.g., Amaro-Seoane et al. (2004). The loss cone is then defined as the region in phase space where the angular momentum \( J \) of a star fulfills \( J < J_G \). All stars inside the loss cone will reach the tidal radius within a dynamical (orbit) timescale. As a consequence the loss cone can become empty in that relatively short period of time. Once a star is inside the loss cone and
reaches the tidal radius, we assume that it will be destroyed by the BH’s tidal force instantaneously and add its total mass to the black hole at the same moment. Most authors studying stellar dynamics and TDRs of star clusters around a BH used similar approximations. Rees (1988) has already argued that the stellar debris after TD will make several orbits until it is finally accreted by the BH; nevertheless, the orbital time near the BH is very short compared to the original orbital time of the star before its disruption. Recent detailed simulations of tidal disruptions (Guillochon & Ramirez-Ruiz 2013; Hayasaki et al. 2013, 2015) show that in some cases not all material of the star may be accreted and that general assumptions about the tidal fallback rate are not correct; for example, a longer-lived accretion disk may form, which could delay the black hole growth. In a spherical system, without interactions between the stars, angular momentum \( J \) would be strictly conserved. So without any repopulation of the loss cone, the accretion process would stop after a few dynamical times. But stars do interact with each other while moving inside the star cluster via two-body relaxation through mutual encounters; in this process they can exchange angular momentum and energy, so the loss cone will be repopulated in the two-body relaxation timescale, which is generally long compared to the dynamical time (Cohn & Kulsrud 1978; Amaro-Seoane et al. 2004). The repopulation of the loss cone is modeled in these papers as a diffusive process using the Fokker–Planck approximation.

In an axisymmetric potential, the situation is more complex because \( J \) is not a conserved quantity. It changes continuously due to the non-central force resulting from the geometry of the potential. In this case, stars with \( J > J_k \) may have a chance to drift into the loss cone and get disrupted. In other words, the loss cone is enlarged in the \( J \) dimension in axisymmetric potential. However, the \( z \) component of angular momentum \( J_z \) is still conserved, so a solid boundary of the loss cone is \( J_z \leq J_k \). Magorrian & Tremaine (1999) investigated this topic using a symplectic map introduced by Touma & Tremaine (1997). In this work, we analyze the enlarged loss cone in phase space in terms of energy \( E \), modulus of angular momentum \( J \), and the \( z \) component of angular momentum \( J_z \) for stellar orbits near the BH. We use a different approach from Magorrian & Tremaine (1999) that is based on a numerical particle scattering experiment. In what follows, we describe the method we used in this experiment, then present our results.

For the first step, we need to know the smooth gravitational potential as a function of position without the fluctuations due to the discrete particle structure. We use a so-called self-consistent field code (SCF; Hernquist & Ostriker 1992) to generate the analytical function for the gravitational potential. The expansion coefficients \( C_{lm}, D_{lm}, E_{lm}, F_{lm} \) used in force calculations (Equations (3.21)–(3.23) in Hernquist & Ostriker 1992) are computed based on snapshot data generated during the direct N-body simulation. By default, the code uses radial basis functions labeled from \( n = 0 \) to \( n_{\text{max}} = 14 \), and spherical harmonic functions truncated at \( l_{\text{max}} = 10 \).

Our particle distribution is self consistently achieved as a consequence of the co-evolution of stars and the BH. Using the SCF code means that two-body interactions are smoothed out in the experiment because we assume that most of the perturbations (from other stars) occur during the apocenter passage. This assumption is also used by Touma & Tremaine (1997). After finding the coefficients, we can calculate the acceleration and the jerk, and perform orbit integration using a
Flat 4th integrator with variable time steps, which we developed. This code works very well and the energy and angular momentum errors of the test particle stay near the level of $10^{-9}$ over long time integration. In an axisymmetric system all coefficients with $m \neq 0$ should be 0. But in practice one will find some small numbers that are very close to 0 due to particle noise. We just ignore these terms, otherwise $J_z$ would no longer be conserved. We also ignore coefficients with odd $l$ because the rotating system should be symmetric about the equatorial plane and does not have a pear-like shape.

The next step is to generate initial positions and velocities for test particles. The basic idea of this experiment is to perform parameter space scanning. We uniformly sample $E$, $J$ and $J_z$ and all test particles are initially put at their apocenter. First, we choose a particular energy and calculate $J_{lc}$ through equation $J_{lc} = r_i \sqrt{2(\Phi(r_i) - E)}$. Then we choose a pair of $(J, J_z)$, $J$ can be a few times larger than $J_{lc}$ but $J_z$ remains smaller than $J_{lc}$. Given the combination of $(E, J, J_z)$ and the potential distribution, we can find the apocenter position given by $(r, \theta)$. Here $r$ is the distance to center and $\theta$ is the angle between position vector and $z$-axis. We note that there are actually four parameters $(E, J, J_z, \theta)$ to define the initial conditions for a particular orbit. So we further sampled 100 data points in the $\theta$ dimension. In order to plot the result in a 2D plane, we introduce a filling factor $P$ for every $(E, J, J_z)$ combination to describe this $\theta$ dependence, which is the fraction of stars in the loss cone for a given combination of $(E, J, J_z)$ (number of data points in the loss cone divided by the total sample size, i.e., 100), meaning that among all stars with the same $(E, J, J_z)$ only a fraction of $P$ are inside the loss cone.

By our definition a star in the loss cone will be disrupted by the BH within one dynamical time, so for every test particle we only integrate their orbits for one orbital cycle. If a particle comes back to its apocenter, we consider it to be outside of the loss cone and move to the next integration with new initial orbital data.

Figure 4 shows the results from the experiment in a slowly rotating model ($\omega_0 = 0.3$); it represents the loss cone shape in phase space. Since $J$ is not conserved we use its initial value at apocenter for the figure; at the time of disruption $J$ must be less than $J_{lc}$. From panels (a) to (d), the energies of the test particles are in a descending sequence, so their apocenter positions are moving closer and closer to the central BH. One can see that the whole plane comprises three regions: (1) an inner region where $P$ equals 1, meaning particles with these $(E, J, J_z)$ can hit the BH within one dynamical timescale; (2) a transition region where $P$ is non-zero but less than 1; particles with these $(E, J, J_z)$ have a chance to hit the BH depending on their apocenter position ($\theta$ value); and (3) an outer region where $P = 0$; none of the particles in this region can hit the BH. In panel (a) one can see that only a few points are red and many points are located in the transition region. From (a) to (d), the fraction of $P = 1$ points in the $(J, J_z)$-plane increases and the transition region is compressed by the inner and outer region in the horizontal direction ($J$ dimension). This is because test particles with high energy (loosely bound or unbound with respect to the BH) can go beyond the BH’s influence radius to the intermediate and outer regions of the cluster, where the axisymmetric stellar potential dominates. The angular momentum of these test particles will have large variations. So a wide transition region exists in high-energy cases. But in the low-energy case (stars strongly bound to the BH), e.g., panel (d), test particles are moving inside the BH’s influence sphere where the potential is dominated by the BH and thus approaches spherical symmetry. All loss cone stars, following the classical loss cone approximation, should have both $J$ and $J_z$ that are smaller than $J_{lc}$. In all panels of Figure 4, on the contrary, we see how stars with $J > J_{lc}$ can remain in the new, extended loss cone of an axisymmetric system with a certain non-zero probability.

For faster rotating models ($\omega_0 = 0.6$) the results are similar. Three regions are presented on the $(J, J_z)$ plane; however, the extent of each region is different from the counterpart of the same energy in a slow-rotating model. Figure 5 gives an example. The test particles in both the slow-rotating model (left panel) and the fast-rotating (right panel) have the same energy. However, the resulting appearances are quite different. In the left panel we see the the outer border extended to $J = 0.024$, while in the right panel the outer border goes to $J = 0.04$ and is not as clear as that in the left panel. Also, in the right panel the red region is almost disappeared. These results show how rotation modifies the loss cone shape in phase space. In both of these plots, the maximum radius that stars can achieve are roughly the same. However, faster rotation means that cluster shape is more flattened, which enhances the torque acting on stars, thus the variation in $J$ becomes larger. So the higher the degree of rotation in the stellar system, the larger the extension of the loss cone in the $J$ direction.

Upon first glance, Figure 4 (also Figure 5) might seem to indicate that the loss cone is generally enlarged by a significant factor. However, as we pointed out above, there is a filling factor $P$ for every point on the $(J, J_z)$ plane. To find the net enlargement of the loss cone in axisymmetric potentials we introduce an effective area $S$ of the loss cone in these plots by integrating the filling factor $P$ over the $(J, J_z)$ plane. For example, the effective area of the classical loss cone is just given by the size of the triangle $(J, J_z < J_{lc})$ in our plots, since in the classical case $P$ is of unity everywhere in this triangle region.

Now we compare the loss cone size by comparing the integrals $S$ with each other. We define the quotient $\alpha_{lc} = S_{eff}/S_{lc}$, where $S_{lc} = J_{lc}^2/2$ is the classical loss cone integral. $\alpha_{lc}$ is plotted in Figure 6 as a function of binding energy $|E|$. In the plot we show both slow and fast rotating models at two different evolution times. For the slow-rotating model the ratio $\alpha_{lc}$ is even smaller than unity for binding energies larger than 1.4–1.5, meaning that at large $|E|$ the loss cone is smaller compared to the classical one. This is caused by the reduction of the probability $P$ at the boundary and inside the classical loss cone region $J < J_{lc}$. $P$ is decreasing from inside toward outside. While for the intermediate $|E|$ case, although $P$ is still a decreasing function of $J$, the large number of valid points overwhelms, so the net effect is an increase of the effective area. However, if one goes further toward smaller $|E|$ the ratio will drop again, such as in the case of fast rotating model. This is just because $P$ is sufficiently small in this case. For a fast rotating model, another interesting feature is that the ratio drops below 1 at $|E| = 1.8$. From this figure we see the enlargement of the loss cone, quantified by the ratio $\alpha_{lc}$, as a function of binding energy. Interestingly, the change of the effective loss cone size in every energy slice is less than 5%–10%. These mild changes seem to be unable to raise the TDR to the amount observed in the simulation; to address this it will be useful if we can estimate the TDR based on the effective...
The component of angular momentum. The left panel corresponds to \( \omega = 0.3 \); the right panel corresponds to \( \omega = 0.6 \). Colors indicate the filling factor \( F \) as a percentage.

Figure 5. Comparison between two models with the same test particle energy \(-1.5\). The \( x \)-axis is a module of angular momentum in \( N \)-body unit. The \( y \)-axis is the \( z \) component of angular momentum. The left panel corresponds to \( \omega = 0.3 \); the right panel corresponds to \( \omega = 0.6 \). Colors indicate the filling factor \( F \) as a percentage.

Figure 6. Ratio between the effective area \( S_{\text{eff}} \) of the loss cone in an axisymmetric system and the \( S_{\text{th}} \) in a spherical system.

loss cone measurement and compare it with simulations. However, knowledge of how stars are distributed in energy and angular momentum is required. With the limited particle numbers of the model cluster, it is difficult to obtain an accurate and reliable distribution function. Also, in the current work, we sample the energy space with large intervals and obtain the energy and angular momentum. Thus we did not make the estimation. However, there is still plenty of work that could be done regarding this topic.

5. ORBITAL PROPERTIES OF DISRUPTED STARS

In this section, we investigate the origin of disrupted stars. Under the assumption that stars in a loss cone can survive for only one orbital period, the origin of these stars can be examined by looking at their energy and angular momentum, as well as their origin (apocenter) in spatial coordinates (radius and angle \( \theta \)). In spherical systems one can use effective potential to compute the apocenter of orbit, but in the axisymmetric case we do not have such convenient solutions except for running the simulation twice. In the first run we find out the ID for those stars that will be disrupted by the BH. In the second run we make records for these stars more frequently than other stars, in order to catch their last apocenter position.

In the beginning we found that the total TDR, especially for small \( r_i \) models, only marginally depends on the rotation of the system; consistent with this we found in the previous section that in given energy slices the loss cone structure does change significantly, but the total integral over the loss cone space for axisymmetric systems yields only relatively small changes.

Still, it is interesting to study how the orbital properties of stars, which are tidally disrupted, change with the rotation of the system. In order to address this, we now turn back to our full \( N \)-body simulations and study the distribution of \( |E| \) (Figure 7) and \( J \) (Figure 8) of the disrupted stars at their apocenter passage in three time intervals. From Figure 7 one can see that most of the tidally disrupted stars have a binding energy between 1 and 2, coincident with the small bumps in Figure 6 where \( \alpha_R > 1 \).

Further evidence comes from the distribution of \( J \) as shown in Figure 8, where one can see that the peaks are lying outside of the \( J_c \), which is roughly 0.015. The peaks are moving toward a larger \( J \), which is caused by the increase of BH mass (recall the expression for \( J_{\text{th}} \)). A significant fraction of stars comes from places outside of the classic loss cone in the \( (J, J_c) \) plane.

In spherical systems it is usually sufficient to describe the apocenter of an orbit by its radial distance from the center (the BH); the orientation of the orbit does not play any role for the orbital time and the nature of the encounter with the central BH. However, in axisymmetric systems, orbits with different angles \( \theta \) (the angle between position vector of the star at apocenter and the \( z \)-axis) will differ from each other significantly. Therefore we have to describe the distribution of apocenters of tidally disrupted stars in terms of both the \( r \) (Figure 9) and the \( \theta \) (Figure 10) dimension. From Figure 9 one can see that the peaks are quite far from the BH, in a region comparable to the BH influence radius, which is similar to the apocenter distribution in spherical systems (Paper I). The difference turns out to be in the \( \theta \) dimension, as shown in Figure 10. We compare the \( \theta \) distribution between spherical and axisymmetric systems. Imagine that we project all of the apocenter points onto a sphere with a radius equal to 1. The measured number counts in each \( \theta \) bin \( \Delta N(\theta) \) are computed by \( 2\pi \cdot \Sigma(\theta) \cdot \sin(\theta) \Delta \theta \), where \( \Sigma(\theta) \) is the surface density of projected points on the unit sphere. If apocenters are uniformly distributed with \( \theta \), \( \Sigma(\theta) \) is constant, then \( \Delta N(\theta) \propto \sin(\theta) \Delta \theta \). Here we choose an equal bin size, so the measured number count should follow a \( \sin(\theta) \) curve. The right panel of Figure 10 plots the \( \theta \) distribution for a spherical model, which is taken from our last work (Paper I). In the left panel we see that, compared to the curve, the last apocenter distribution has a deficit at the polar region, and excess at places beyond and below the equatorial plane, showing a double peak feature. The deficit at the polar region may have something to do with the flattening of the cluster; however, this is not the only reason.
Figure 7. Panels (a) and (b) show the normalized distribution of binding energy $|E|$ of tidally disrupted stars, for different rotating models and for three different time intervals (indicated by color) in the full $N$-body simulation with $n = 10^{-3}$. The distribution is normalized to the total number of disrupted stars in each time interval. Panels (c) and (d) show the cumulative fraction profiles corresponding to (a) and (b), respectively.

Figure 8. Same as Figure 7, but for the distribution of total angular momentum of the disrupted stars.
The double peak feature around the equatorial plane obviously does not relate to a geometrical origin, otherwise the peak should be placed at the equatorial plane. In Figure 11 we compare the $\theta$ distribution between slow and fast rotating models. One can see that in a fast rotating model, the double peaks are more significant, accompanied by a further deficit in the angle range $0.2-0.4\pi$ and $0.6-0.8\pi$.

In order to understand the double peak feature, we turn to the orbit structure of these disrupted stars. In a non-spherical symmetric stellar system with an SMBH in its center, the space populated by stars can be divided into three parts depending on the distance to the BH, namely the regular, chaotic, and mixing regions (Poon & Merritt 2001). Inside the BH’s influence radius $r_h$, the potential felt by the star is dominated by the BH, plus a small perturbation from the non-spherical stellar potential. In this region, the motion of stars is essentially regular, as in a spherical potential. Outside of $r_h$, stars passing the center will suffer a large angle deflection by the BH, which in conjunction with the non-spherical potential near and outside $r_h$, could make their orbits stochastic.

We are interested in stellar orbits in an axisymmetric stellar potential, which can get close to the central BH. These are typically two classes of orbits, short-axis tube (SAT) and saucer (see Vasiliev 2014 for example); they can be distinguished by their third integral of motion $I_3$. Although $I_3$ may help us quickly distinguish orbit families, finding the functional form of $I_3$ is difficult (see Lupton & Gunn 1987 and a discussion in Sridhar & Touma 1999) and is beyond the scope of this paper. We choose alternative ways to perform orbit classification, such as using a Surface of Section (SoS) plot and Fourier analysis of $J_x$ (see Appendix A).

Figure 12 gives examples of SAT and saucer orbits in configuration space. The plot is made in cylindrical coordinates so that one can catch the main point easily. For an SAT orbit, its apocenter can go both above and below the equatorial plane. In contrast, the apocenter of a saucer orbit can only exist on one side of the mid-plane, due to restrictions by the 3rd integral. We also check the value of $J$ at each apocenter passage. We find that the SAT orbit achieves its minimum $J$ at the equatorial plane; a saucer orbit cannot reach the equatorial plane, but its minimum $J$ is achieved at the place that is next to the equatorial plane as marked in the plot by the $A$ plane.

Recall in the last section that we said that no matter what $J$ one star has at the apocenter, at the time of disruption it must be smaller than $J_c$. So the last apocenter place should be around the $A$ plane. This seems to be promising for explaining the double peak in the $\theta$ distribution; however, this need to be confirmed. In order to see this we tried to perform orbit classification for the disrupted stars, which is computationally expensive. Thus we just randomly selected a sub-sample of...
disrupted stars and divided them into three orbit families: SAT, saucer, and others (here “others” means they do not belong to the former two families, and may be chaotic orbits). Among the 2943 sample stars, 1719 are classified as “others,” 757 as saucer, and 467 as SAT. Then we re-plot the $r$ and $\theta$ distributions for different orbit families in Figure 13.

The results show that the apocenter distribution of different orbit families not only differs in $\theta$, but also in $r$. One can see that the innermost region is dominated by SAT orbits, and is concentrated to the equatorial plane. The intermediate radius is mostly occupied by saucer orbits, and the distribution in $\theta$ shows double peaks as expected. Further out is the region dominated by orbits marked as others. These orbits can go outside of the influence radius and are basically chaotic orbits. From Figure 13 one can also see the fractions of each orbit family contributing to the budget of disrupted stars: the largest fraction comes from chaotic orbits; SAT orbits contribute the least to the budget because they are deeply buried in the cluster center where the total star number is small; the intermediate contribution is from saucer orbits, which create the two peaks in the $\theta$ distribution.

6. CONCLUSIONS AND DISCUSSIONS

TD of stars by supermassive central black holes (SMBH) from dense rotating star clusters is modeled by high-accuracy direct $N$-body simulations. As in a previous paper on spherical star clusters, we study the time evolution of the stellar TDR and the origin of tidally disrupted stars, which are now accorded to several classes of orbits that only occur in rotating axisymmetric systems (short-axis tube and saucer). In an empty loss cone regime, when comparing spherically symmetric and axisymmetric systems we find a higher TD rate in large $r_t$ models in the axisymmetric case, but for the small $r_t$ case—somewhat surprisingly—there is virtually no difference in the TD rate, only a small increase that is perhaps due to axisymmetry.

We define an extended loss cone by the condition that stars in the axisymmetric potential reach the BH within one orbit. A detailed analysis shows that the structure of the loss cone significantly differs from the spherical case; if $J_{lc}$ is the critical angular momentum in the loss cone in a spherical system, and $J_c$, $J$ are the total and $z$ components of the angular momentum of a stellar orbit, there are many stars with $J > J_{lc}$ in the loss cone; however, there are also some stars with $J > J_{lc}$, which are not now in the loss cone. In the total balance the number of loss cone stars is only very moderately increased.
In the experiment of measuring the shape of the loss cone, we assume that the test star can survive only one dynamical time in collisional systems; after one orbit it will be “kicked” to another place in phase space due to interactions with other stars. However, in the collisionless limit, if we allow the test star to survive more orbit cycles, the test star with much higher \( J \) will also have a chance to get rid of its angular momentum and be disrupted by the BH. Then it is possible that an even larger loss cone region in phase space than what we presented here may exist, and result in a higher TDR. In order to check this, simulations with much more particles are needed and we would like to leave this task for future work.

The orbit type of disrupted stars strongly depends on energy, as we discuss in detail in the previous sections. The TDs of stars most strongly bound to the BH are dominated by SAT orbits. In intermediate regions saucer orbits dominate, which create a characteristic double peak structure in the last apocenter position of their orbit relative to the equatorial plane. Further out chaotic orbits dominate, and they do not leave any special signature in the distribution of the last apocenter position.

It has been known for almost half a century that the TD of stars should occur near SMBHs, but only recently has the X-ray emission of TD events been detected (Komossa 2002; Komossa & Merritt 2008). A simple argument on the fallback time for tidal debris by Rees (1988) has led to the prediction of a characteristic power law of the light curve with time, which can be used to distinguish TD events from other transients. It is interesting that an SMBH binary can cause characteristic disruptions in such an otherwise standard TD light curve (Liu et al. 2014). Hayasaki and collaborators claim that eccentric TD events lead to somewhat longer-lived central accretion disks (Hayasaki et al. 2013, 2015). It will be very interesting to see whether and how the evolution of tidal debris and the fallback rate are affected by different orbits of the disrupted stars as discussed here.

It has been claimed that rotation may help to quickly refill loss cones around binary SMBHs, which helps significantly to accelerate the shrinking and final coalescence of SMBH binaries in cosmologically short timescales (Berczik et al. 2006; Preto et al. 2011; Khan et al. 2013; Khan 2014). In our paper, using direct N-body simulations, we study the tidal accretion of stars and their orbital parameters in rotating axisymmetric systems. We confirm the results of Vasiliev & Merritt (2013) that there is an increase in the rate of refilling of the loss cone, but it is moderate. However, the situation deserves more detailed study because a SMBH binary creates a much stronger deviation from spherical symmetry than the one used in our models with single SMBHs. And second, the detailed structure of the rotation in a central NSC could affect the enhancement of the loss cone.

We acknowledge support by the Chinese Academy of Sciences through the Silk Road Project at NAOC, through the Chinese Academy of Sciences Visiting Professorship for Senior International Scientists, Grant Number 2009S1-5 (RS), and through the “Qianren” special foreign experts program of China.

The special GPU accelerated supercomputer laohu at the Center of Information and Computing at National Astronomical Observatories, Chinese Academy of Sciences, funded by Ministry of Finance of People’s Republic of China under the grant ZDY ZZ08-2, has been used for the simulations.

P.B. acknowledges the special support by the NAS Ukraine under the Main Astronomical Observatory GPU/GRID computing cluster project.

S.Z. thanks Yohai Meiron for providing the SCF source code, which is used in this work.

**APPENDIX A**

**ORBIT CLASSIFICATION**

**A.1. Surface of Section**

From Figure 14 we can see that the entire accessible region on the \((R, v_R)\) plane is divided into two parts (note that points with opposite \(v_R\) actually belong to the same orbit, so this plot is symmetric with the horizontal axis). Each part represents a family of orbit. Curves that intersect with the \(R\)-axis are footprints of SAT orbits; others are of saucer orbits.

**A.2. Fourier Analysis of \(J_x\) Evolution**

In axisymmetric potential, force is not centripetal, hence the force exerted a torque on the star which will change the \(x\) and \(y\) components of its angular momentum. Figure 15 show the time evolution of \(J_x\) for both SAT and saucer orbits. The patterns of \(J_x\) and \(J_y\) are the same but are shifted, with a phase of \(\pi/2\), so in the following discussion we only focus on \(J_x\). Furthermore, the evolution of \(J_x\) shows some quasi-periodicity. From eye inspection, one can guess the mathematical expressions for the curves.

As shown in Figure 15, the curve for SAT orbits seems to be represented by

\[
\sin(f_a(t(1+\cos(f_b*t))) = f_a < f_b)
\]

which can be further converted to

\[
\sin(f_a t) + \sin(f_b t) + \sin(f_t t)
\]

(ignoring coefficients before the trigonometric functions), with

\[
f_a = f_1, f_b = f_2 - f_1 \text{ and } f_t = f_2 + f_1
\]

If we perform a Fourier analysis on this curve, we expect to find three principal frequencies: \(f_a, f_b, f_t\) in ascending sequence. And these three frequencies satisfy the equation

\[
f_a = f_2 - f_1 \text{ and } f_b = f_2 + f_1
\]

For saucer orbits, the curve seems to be represented by

\[
\sin(f_a t)\cos(f_b t) = f_a < f_b
\]

Following the same procedure, we expected to find two principal frequencies: \(f_{a', b'}\), with

\[
f_{a'} = f_2 - f_{1'} \text{ and } f_{b'} = f_2 + f_{1'}
\]
A demonstration is given in Figure 16, and one can clearly see the three principal frequencies for SAT orbits and the two for saucer orbits. One can also see many small peaks located at higher frequencies, some of them are the high-order harmonics and the rest resulted from other sources.

We use both methods to cross-check the validity of the orbit classifications for the tidally disrupted stars.

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Figure 15. Time evolution of $J_x$ for the SAT orbit (left) and the saucer orbit (right).

Figure 16. Fourier frequency distribution of $J_x$. The horizontal axis is frequency in units of $T^{-1}$. The vertical axis is the amplitude of the corresponding component. The left panel represents the SAT orbit and the right panel represents the saucer orbit.

A demonstration is given in Figure 16, and one can clearly see the three principal frequencies for SAT orbits and the two for saucer orbits. One can also see many small peaks located at higher frequencies, some of them are the high-order harmonics and the rest resulted from other sources.

We use both methods to cross-check the validity of the orbit classifications for the tidally disrupted stars.
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