Charmed hadron physics in quenched anisotropic lattice QCD

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We investigate the anisotropic lattice with $O(a)$ improved quark action as a candidate of framework in which we can treat both the heavy and light quark region in the same manner and systematically reduce the systematic uncertainties. To examine applicability of anisotropic lattice, we calculate the charmed meson spectrum and decay constants in quenched approximation. We find consistent result with most advanced results on isotropic lattices.

1. Introduction

Recent experimental progress in flavor physics to look for the effect of new physics strongly requires precise theoretical predictions from the standard model. However, model independent calculation of hadronic matrix elements is difficult because of nonperturbative nature of QCD. The lattice QCD simulation is one of most promising approaches in which the systematic uncertainties can be reduced systematically. The ultimate goal of this work is to construct a framework for lattice calculations of hadronic matrix elements in a few percent systematic accuracy, as required by the experiments in progress.

In a lattice calculation of heavy quarks such as the charm and bottom, one need to either avoid or control the large discretization error of $O((m_Q a)^n)$. Although the effective theoretical approaches successfully describe the matrix elements with heavy quarks within 10\% accuracy, further improvement of accuracy is difficult due to perturbative error. The nonperturbative renormalization method are not directly applicable to a determination of the coefficient of $O(a)$ improving term in the region of $m_Q \sim a^{-1}$ because of a strong mass dependence. Therefore a new framework is desired for which a systematic nonperturbative improvement can be performed.

The anisotropic lattice, on which the temporal lattice spacing $a_\tau$ is finer than the spatial one $a_\sigma$, is a candidate of such a framework. A high temporal lattice cutoff may enable a relativistic treatment of heavy quark while keeping computational requirement tractable. The $O(a)$ improvement coefficients determined in the light quark region in good precision may directly be available for a heavy quark with $m_Q \ll a_\tau^{-1}$. Whether these observations practically hold should be examined numerically, as well as in the perturbation theory.

In this study, we examine the applicability of anisotropic lattice approach numerically by computing the charmed meson masses and decay constants. Although presently achieved accuracy is far from our goal, the result is encouraging for further development in this direction. This paper shows our preliminary result without technical details and evaluation of the systematic uncertainties, which are fully discussed in other publications.

2. Anisotropic lattice quark action

The quark action we adopt has the following structure in the hopping parameter form

\begin{equation}
S_F = \sum_{x,y} \tilde{\psi}(x) \left\{ \delta_{x,y} - \kappa_\tau \left[ (1 - \gamma_4) U_4(x) \delta_{x+i,y} + (1 + \gamma_4) U_4(x) \delta_{x-i,y} \right] \\
- \kappa_\sigma \sum_i \left[ (r-\gamma_i) U_i(x) \delta_{x+i,y} + (r+\gamma_i) U_i(x) \delta_{x-i,y} \right] \\
- \kappa_\sigma \sum_{i>j} \sigma_{ij} F_{ij}(x) \delta_{x,y} \right\} \psi(y),
\end{equation}

where $\kappa_\sigma$ and $\kappa_\tau$ are the spatial and temporal hopping parameters, and are related to the bare

\begin{align}
\kappa_\sigma &= \frac{\kappa_0}{1 - \gamma^2} \\
\kappa_\tau &= \frac{\kappa_0}{1 - \lambda^2},
\end{align}
quark mass and bare anisotropy as given below. 
\( r = \) the spatial Wilson parameter, and \( c_E \) and \( c_B \) are the coefficients of clover terms which remove \( O(a^2) \) errors. Although the explicit Lorentz symmetry is removed due to the anisotropy, in principle it can be restored for physical observables up to errors of \( O(a^2) \) by proper tuning of \( \kappa_\sigma/\kappa_\tau \), \( r \), \( c_E \) and \( c_B \) for a given \( \kappa_\sigma \). The action is constructed in accord with the Fermilab approach and hence applicable to an arbitrary quark mass. However, a mass dependent tuning of parameters is difficult beyond perturbation theory. This may be circumvented by taking \( a_\tau^{-1} \gg m_Q \), with which the mass dependence of parameters are expected to be small, and it suffices with those values in the chiral limit.

In present study, we vary only two parameters \( \kappa_\sigma \) and \( \kappa_\tau \) with fixed other parameters. We set the Wilson parameter as \( r = 1/\xi \) and the clover coefficients as the tadpole-improved tree-level values, \( c_E = 1/\kappa_\sigma a^2 \), and \( c_B = 1/\kappa_\tau a^2 \). The tadpole improvement is achieved by rescaling the link variable as \( U_i(x) \rightarrow U_i(x)/u_\sigma \) and \( U_4(x) \rightarrow U_4(x)/u_\tau \), with the mean-field values of the spatial and temporal link variables, \( u_\sigma \) and \( u_\tau \), respectively. Instead of \( \kappa_\sigma \) and \( \kappa_\tau \), we introduce \( \kappa \) and \( \gamma_F \) as
\[
\frac{1}{\kappa} = \frac{1}{\kappa_\sigma u_\sigma} - 2(\gamma_F + 3r - 4) = 2(m_0 \gamma_F + 4), \quad \gamma_F = \frac{\kappa_\tau u_\tau}{\kappa_\sigma u_\sigma}. \tag{2}
\]
The former plays the same role as on the isotropic lattice, and the latter corresponds to the bare anisotropy.

On an anisotropic lattice, one must tune the parameters so that the anisotropy of quark field, \( \xi_F \), equals that of the gauge field, \( \xi_G \):
\[
\xi_F(\beta; \gamma_F; \kappa, \gamma_F) = \xi_G(\beta; \gamma_G; \kappa, \gamma_F) = \xi. \tag{3}
\]
Although \( \xi_G \) and \( \xi_F \) are in general functions of both of gauge parameters \( (\beta, \gamma_G) \) and quark parameters \( (\kappa, \gamma_F) \), on a quenched lattice one can determine \( \xi_G = \xi \) independently of \( \kappa \) and \( \gamma_F \), and then tune \( \gamma_F \) so that a certain observable satisfies the condition (3). In this work, we define \( \xi_F \) through the relativistic dispersion relation of meson,
\[
E^2(p) = m^2 + p^2/\xi_F^2 + O[(p^2)^2], \tag{4}
\]
for calibration. In the above expression, the energy and mass \( E \) and \( m \) are in temporal lattice units while the momentum \( p \) is in spatial lattice units. \( \xi_F \) converts the momentum into that in temporal lattice units.

3. Numerical simulation

Numerical simulations are performed on two quenched lattices of sizes \( 16^3 \times 128 \) and \( 20^3 \times 160 \) with the Wilson plaquette action at \( \beta = 5.95 \) and 6.10, respectively, with the renormalized anisotropy \( \xi = 4 \). The values of bare anisotropy are chosen according to a numerical result performed in one percent accuracy in [10]. The lattice scales are set by \( K^+ \) meson mass, and result in \( a_{\tau}^{-1} = 1.525(27) \) GeV and 1.817(22) GeV for \( \beta = 5.95 \) and 6.10, respectively. These values deviate to about 10% from scales determined with other physical quantity, say hadronic radius, \( r_0 \), and represent the effect in neglecting dynamical quarks. The mean field values \( u_\tau \) and \( u_\sigma \) are obtained in the Landau gauge.

The calibration was done in the quark mass region below around charm quark mass in [3]. It was found that the quark mass dependence of bare anisotropy \( \gamma_F^* \), with which \( \xi_F = \xi \) holds, is actually small, and \( \gamma_F^* \) is well fitted to a linear form in \( m_q^2 \),
\[
\frac{1}{\gamma_F^*}(m_q) = \zeta_0 + \zeta_2 m_q^2, \quad m_q = \frac{1}{2\zeta} \left( \frac{1}{\kappa} - \frac{1}{\kappa_c} \right). \tag{5}
\]
The fit results in the values \( (\zeta_0, \zeta_2, \kappa_c) = (0.2490(8), 0.189(15), 0.12592(8)) \) at \( \beta = 5.95 \), and \( (0.2479(9), 0.143(14), 0.12558(9)) \) at \( \beta = 6.10 \). These values are determined in 1% statistical accuracy, while in the chiral limit additional 1% error exists due to the form of extrapolation. It was also shown that the systematic discretization errors decrease toward the continuum limit. The light hadron spectra with the obtained parameters are consistent with previous works on isotropic lattices.

For heavy quark, we use four values of \( \kappa \) covering the physical charm quark mass with \( \gamma_F \) according to Eq. (3). As the light quark, we use three values of \( \kappa \) with \( \gamma_F \) at the massless limit, which correspond to the lightest three values used.
Table 1
The heavy-light meson masses and decay constants for physical quark masses in physical units.

|       | $\beta = 5.95$ | $\beta = 6.10$ |
|-------|----------------|----------------|
| $m_{D_s}$ | 1.9740(28) | 1.9760(27) |
| $m_{D_s^*} - m_{D_s}$ | 0.1018(85) | 0.1022(74) |
| $m_{D_s^*} - m_D$ | 0.0980(42) | 0.0921(39) |
| $f_\pi$ | 0.1655(35) | 0.1462(35) |
| $f_K$ | 0.1868(29) | 0.1665(30) |
| $f_D$ | 0.2509(64) | 0.2245(58) |
| $f_{D_s}$ | 0.2863(40) | 0.2570(37) |
| $f_{D_s}/f_\pi$ | 1.516(44) | 1.536(49) |
| $f_{D_s}/f_D$ | 1.141(15) | 1.145(16) |

Figure 1. The heavy-light decay constant multiplied by $\sqrt{m}$ in physical units.

in [6] for light hadron spectroscopy. The meson correlator is calculated with the local meson operator. For the decay constant, we focus on the pseudoscalar mesons. While one needs the matching constants to convert the lattice result into the continuum theory, we only perform the tadpole-improved tree-level matching in this paper.

We first extrapolate the result to the chiral limit, linearly in light pseudoscalar meson mass squared. The physical ($u$, $d$) and $s$ quark masses are defined with the massless limit and the $K$ meson mass. Then they are fitted to the quadratic form in $1/m_H$, $m_H$ the PS heavy-light meson mass, and interpolated to the physical $m_D$ meson mass. The decay constant of heavy-light meson is extrapolated and interpolated in the form of $f_H \sqrt{m_H}$. The result after chiral extrapolation is shown in Figure 1. The gross feature of $m_H$ dependence is consistent with previous works [1]. The masses and decay constants for physical quark masses are listed in Table 1.

The hyperfine splittings are consistent with previous works, while 35% less than the experimental values. On the other hand, the splitting $m_{D_s} - m_D$ is close to the experimental value. The $\beta$ dependence of decay constants are rather large, and considered as of the discretization error and renormalization effect, as well as the uncertainty in setting the scale on quenched lattice. Taking ratios of decay constants, $\beta$ dependence is largely cancelled. The result is consistent with the previous lattice works [1]. For more quantitative discussion, we need to discuss more carefully on the systematic uncertainties due to the anisotropy, and to include the matching of current with continuum theory. As our conclusion, the result of numerical simulation is encouraging to pursue more quantitative analysis in this direction.

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