Measurement of the Partial Branching Fraction for Inclusive Charmless Semileptonic $B$ Decays and Extraction of $|V_{ub}|$

The $BaBar$ Collaboration

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Abstract

Charmless semileptonic decays, $\bar{B} \rightarrow X_u \ell \bar{\nu}$, are studied in a sample of 232 million $B\bar{B}$ decays recorded with the $BaBar$ detector, in events where the decay of the second $B$ meson is fully reconstructed. Inclusive charmless decays are selected in kinematic regions where the dominant background from semileptonic $B$ decays to charm is reduced by requirements on the hadronic mass $M_X$ and the momentum transfer $q^2$. The partial branching fraction for $\bar{B} \rightarrow X_u \ell \bar{\nu}$ decays for $M_X < 1.7$ GeV/$c^2$ and $q^2 > 8$ GeV$^2$/c$^4$ is measured to be $\Delta B(\bar{B} \rightarrow X_u \ell \bar{\nu}) = (0.87 \pm 0.09_{\text{stat}} \pm 0.09_{\text{sys}} \pm 0.01_{\text{th}}) \times 10^{-3}$. The CKM matrix element $|V_{ub}|$ is determined by using theoretical calculations of phase space acceptances. Theoretical uncertainties in this extrapolation are reduced by using the inclusive $b \rightarrow s\gamma$ photon spectrum and moments of the $b \rightarrow c\ell\bar{\nu}$ lepton energy and hadronic invariant mass.

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1 Introduction

The principal physics goal of the \textit{BaBar} experiment is to establish \textit{CP} violation in \textit{B} mesons and to test whether the observed effects are consistent with the predictions of the Standard Model (SM). \textit{CP} violating effects result in the SM from an irreducible phase in the Cabibbo-Kobayashi-Maskawa (CKM) matrix which describes the couplings of the charged weak current to quarks. An improved determination of the magnitude of the matrix element $|V_{ub}|$, the coupling strength of the \textit{b} quark to the \textit{u} quark, will contribute critically to tests of the consistency of the measured angles of the unitarity triangle of the CKM matrix.

The precise determination of $|V_{ub}|$ is very difficult as, due to the large charmed backgrounds, we are forced to measure partial branching fractions and extrapolate them to the full phase space by relying on QCD based theoretical calculations. These calculations make use of specific assumptions and are affected by different uncertainties. It is therefore important to make redundant measurements by using several experimental techniques, and different theoretical frameworks. Measurements of the inclusive and exclusive charmless semileptonic decays of \textit{B} mesons are sensitive to different theoretical calculations, and therefore to different sources of systematic uncertainties. In addition, exploiting the available kinematic variables which discriminate between rare charmless semileptonic decays and the much more abundant decays involving charmed mesons, gives different sensitivities to the underlying theoretical calculations and assumptions.

In inclusive measurements, three kinematic variables are discussed in the literature, each having its own advantages: the lepton energy ($E_\ell$), the hadronic invariant mass ($M_X$), and the leptonic invariant mass squared ($q^2$). The first measurements were restricted to the high end of the lepton spectrum where theoretical uncertainties are very large and therefore the extrapolation to the full spectrum becomes uncertain. Event selection based on $M_X$ and $q^2$ allows us to select larger portions of phase space, but the underlying theoretical assumptions need to be carefully evaluated.

Theoretical studies indicate that it is possible to reduce the theoretical error on the extrapolation by taking advantage of other kinematic variables or applying simultaneous cuts on $M_X$ and $q^2$ in inclusive $\overline{B} \rightarrow X_u \ell \bar{\nu}$ \footnote{Charge-conjugate modes are implied throughout this paper.} decays [1]. In fact, while the $M_X$ distribution has a large usable fraction of events, of the order of 70\%, but depends on the shape function (SF) describing the Fermi motion of the \textit{b} quark inside the \textit{B} meson, the $q^2$ distribution is less sensitive to non-perturbative effects and less dependent on the calculation. Unfortunately, only a small fraction of events (about 20\%) is usable with a pure $q^2$ selection. The study presented in [1] shows that a combined cut on $M_X$ and $q^2$ may mitigate the drawbacks of the two methods while retaining good statistical and systematic sensitivities.

\textit{BaBar} has already published a determination of $|V_{ub}|$ from a measurement of the inclusive charmless semileptonic branching fraction $\mathcal{B}(\overline{B} \rightarrow X_u \ell \bar{\nu})$ [2] based on the study of the recoil of fully reconstructed \textit{B} mesons and applying a kinematic cut on $M_X < 1.55$ GeV$/c^2$, which resulted in the most precise determination of this quantity. Nevertheless the measurement was dominated (17\% on the branching ratio, \textit{i.e.} 8.5\% on $|V_{ub}|$) by the theoretical uncertainty on the underlying kinematic variable distributions, and therefore on the extrapolation to the full phase space. This result was obtained on a limited dataset, corresponding to about 80 fb$^{-1}$ of integrated luminosity.

Measurements of $|V_{ub}|$ through inclusive charmless semileptonic decays $\overline{B} \rightarrow X_u \ell \bar{\nu}$ using a combination of $M_X$ and $q^2$ ("$M_X$-$q^2$" analysis) have also been presented by \textit{BaBar} at ICHEP 2004 [3]. In this case, the extrapolation of the branching fraction measurements from a limited region of phase space to the full spectrum was done following Bauer \textit{et al} [1] (BLL in the following).
The calculation for charmless semileptonic decays [4] implemented in the BaBar Monte Carlo simulation was also used to evaluate theoretical uncertainties. The preliminary result presented in this paper extends the analysis published in [3]. This new study is based on the same analysis strategy, but on a much larger dataset.

This paper is organized as follows: Section 2 describes the detector, the data sample and the Monte Carlo simulation, including a description of the theoretical model on which our efficiency calculations are based. Section 3 describes the event reconstruction and selection, while in Section 4.1 the results of the $M_X-q^2$ analysis are presented. These results are interpreted by using the calculations from BLL and Neubert et al (BLNP) [5, 6, 7].

The SF parameters $m_b$ and $\mu_\pi^2$ described in Section 5 are an essential input for the determination of kinematic acceptances. In our previous result [3] we used two set of values for $m_b$ and $\mu_\pi^2$, as determined from the analysis of the photon spectrum in $B \to X_u \gamma$ decays from CLEO and Belle, respectively. For the present studies, we use the Belle result. The SF parameters can also be determined from the analysis of the electron energy and hadronic mass moments in $b \to c\ell\bar{\nu}$ decays. Therefore, in Section 5.1 we quote results based on the BaBar measurements of SF parameters with $b \to c\ell\bar{\nu}$ moments.

2 Data Sample and Simulation

The data used in this paper were recorded with the BaBar detector [8] at the PEP-II collider. The total integrated luminosity of the data set is 210.7 fb$^{-1}$ collected on the $\Upsilon(4S)$ resonance. The corresponding number of produced $B\bar{B}$ pairs is 232 million. We use Monte Carlo (MC) simulations of the BaBar detector based on GEANT [9] to optimize selection criteria and to determine signal efficiencies and background distributions.

2.1 Simulation of $B \to X_u\ell\bar{\nu}$ decays

Charmless semileptonic $B \to X_u\ell\bar{\nu}$ decays are simulated as a combination of both exclusive three-body decays to narrow resonances, $X_u = \pi, \eta, \rho, \omega, \eta'$, and inclusive decays to non-resonant hadronic final states $X_u$. The simulation of the inclusive charmless semileptonic $B$ decays into hadronic states with masses larger than $2m_\pi$ is based on a prescription by De Fazio and Neubert [4] (DFN), which calculates the triple differential decay rate, $d^3\Gamma / dq^2 dE_\ell ds_H \ (s_H = M_H^2)$, up to $O(\alpha_s)$ corrections. The motion of the $b$ quark inside the $B$ meson is incorporated in the DFN formalism by convolving the parton-level triple differential decay rate with a non-perturbative SF. The SF describes the distribution of the momentum $k_+$ of the $b$ quark inside the $B$ meson. The two free parameters of the SF are $\Lambda_{SF}^2$ and $\lambda_1^{SF}$. The first relates the $B$ meson mass, $m_B$, to the $b$ quark mass, $m_b^{SF} = m_B - \Lambda_{SF}^2$, and $\mu_\pi^2 = -\lambda_1^{SF}$ is the average momentum squared of the $b$ quark in the $B$ meson. The SF parameterization used in the generator is of the form

$$F(k_+) = N(1 - x)^a e^{(1+a)x},$$

where $x = \frac{k_+}{\Lambda_{SF}^2} \leq 1$ and $a = -3(\Lambda_{SF}^2)^2/\lambda_1^{SF} - 1$. The first three moments of the SF must satisfy: $A_0 = 1, A_1 = 0$ and $A_2 = -\lambda_1^{SF}/3$.

In the simulation the hadron system $X_u$ is produced with a non-resonant and continuous invariant mass spectrum according to the DFN model. The fragmentation of the $X_u$ system into final state hadrons is performed by JETSET [10]. The exclusive charmless semileptonic decays are simulated using the ISGW2 model [11]. The resonant and non-resonant components are combined
such that the total branching fraction is consistent with the measured value [2] and that the integrated spectra agree with the prediction of Ref. [4]. In the evaluation of the associated uncertainty all branching fractions and theory parameters are varied within their errors.

2.2 Simulation of Background Processes

To estimate the shape of the background distributions we make use of simulations of $e^+e^- \rightarrow \Upsilon(4S) \rightarrow B\bar{B}$ with the $B$ mesons decaying inclusively. The most relevant backgrounds are due to $B \rightarrow X_e\ell\bar{\nu}$ events. The simulation of these processes uses a Heavy Quark Effective Theory (HQET) parametrization of form factors for $B \rightarrow D^*\ell\bar{\nu}$ [12], and models for $B \rightarrow D(\ast)\pi\ell\bar{\nu}$ [13], and $B \rightarrow D\ell\bar{\nu}, D^{\ast\ast}\ell\bar{\nu}$ [11]. We also make use of a JETSET simulation of “continuum” $e^+e^- \rightarrow q\bar{q}$ ($q = u, d, s, c$) events.

3 Event Selection and Reconstruction

The event selection and reconstruction and the measurements of branching fractions follow closely the strategy described in Ref. [2] and represent the basis for the $M_X$-$q^2$ analysis presented here.

In this paper we study the recoiled $B$ candidate opposite of a fully reconstructed $B$ in hadronic decay ($B_{\text{reco}}$), which is a moderately pure sample of $B$ mesons with known flavor and four-momentum. We select $B_{\text{reco}}$ decays of the type $B \rightarrow D^0Y$, where $D$ refers to a charm meson, and $Y$ represents a collection of hadrons with a total charge of $\pm 1$, composed of $n_1\pi^\pm + n_2K^\pm + n_3K^0 + n_4\pi^0$, where $n_1 + n_2 < 6$, $n_3 < 3$, and $n_4 < 3$. Using $D^-$ and $D^{*-}$ ($\bar{D}^0$ and $\bar{D}^{*0}$) as seeds for $B^0$ ($B^+$) decays, we reconstruct about 1000 different decay chains. Overall, we correctly reconstruct one $B$ candidate in 0.3% (0.5%) of the $B^0\bar{B}^0$ ($B^+B^-$) events. The kinematic consistency of a $B_{\text{reco}}$ candidate with a $B$ meson decay is checked using two variables, the beam-energy-substituted mass $m_{\text{ES}} = \sqrt{s/4 - \vec{p}_B^2}$ and the energy difference, $\Delta E = E_B - \sqrt{s}/2$. Here $\sqrt{s}$ refers to the total energy in the $\Upsilon(4S)$ center of mass frame, and $\vec{p}_B$ and $E_B$ denote the momentum and energy of the $B_{\text{reco}}$ candidate in the same frame. For signal events the $m_{\text{ES}}$ distribution peaks at the $B$ meson mass, while $\Delta E$ is consistent with zero. We require $\Delta E = 0$ within approximately three standard deviations, as determined from each decay channel.

A semileptonic decay of the other $B$ meson ($B_{\text{recoil}}$) is identified by the presence of a charged lepton with momentum in the $B_{\text{recoil}}$ rest frame ($p^*$) above 1 GeV/c. In addition, the detection of missing energy and momentum in the event is taken as evidence for the presence of a neutrino. The hadronic system $X$ is reconstructed from charged tracks and energy depositions in the calorimeter that are not associated with the $B_{\text{reco}}$ candidate or the identified lepton. Care is taken to eliminate fake charged tracks, as well as low-energy beam-generated photons and energy depositions in the calorimeter from charged and neutral hadrons due to beam backgrounds. The neutrino four-momentum $p_\nu$ is estimated from the missing momentum four-vector $p_{\text{miss}} = p_{\Upsilon(4S)} - p_{B_{\text{reco}}} - p_X - p_\ell$, where all momenta are measured in the laboratory frame, and $p_{\Upsilon(4S)}$ refers to the $\Upsilon(4S)$ meson.

Undetected particles and measurement uncertainties affect the determination of the four-momenta of the $X$ system and neutrino, and lead to a leakage of $B \rightarrow X_e\ell\bar{\nu}$ background from the high $M_X$ into the low $M_X$ region ($M_X < 1.86$ GeV/c$^2$).

In the sample of reconstructed $B$ decays ($B_{\text{reco}}$), two backgrounds need to be considered: the combinatorial background from $B\bar{B}$ and continuum events, due to random association of tracks and neutral clusters, which does not peak in $m_{\text{ES}}$, and the $B\bar{B}$ background whose $m_{\text{ES}}$ distribution has the same shape as the signal. After applying all selection criteria, the remaining combinatorial
background is subtracted by performing an unweighted likelihood fit to the $m_{ES}$ distribution. In this fit, the combinatorial background originating from $e^+e^- \rightarrow q\bar{q}$ $(q = u, d, s, c)$ continuum and $B\bar{B}$ events is described by an empirical threshold function [14], and the signal is described by a modified Gaussian [15] peaked at the $B$ meson mass. In addition, to further reduce the effects of the combinatorial background, only events with $m_{ES} > 5.27$ GeV/c$^2$ are considered.

To reject the background in the sample of semileptonic decays we require exactly one charged lepton with $p^* > 1$ GeV/c, a total event charge of zero, and a missing mass consistent with zero ($m_{miss}^2 < 0.5$ GeV$^2$/c$^4$). These criteria partly suppress the dominant $B \rightarrow X_c \ell \bar{\nu}$ decays, many of which contain an additional neutrino or an undetected $K_L^0$ meson.

In order to reject the peaking background coming from $D^*$ we use a dedicated method, the partial $D^*$ reconstruction. We explicitly veto the $B^0 \rightarrow D^{*+} \ell \nu$ background by searching candidates for such a decay with a partial reconstruction technique, that is only identifying the $\pi^+_s$ from the $D^{*+} \rightarrow D^0 \pi^+_s$ decay and the lepton: since the momentum of the $\pi^+_s$ is almost collinear with the $D^{*+}$ momentum in the laboratory frame, we can approximate the energy of the $D^{*+}$ as $E_{D^{*+}} \simeq m_{D^{*+}} \cdot E_{\pi^+_s}/145$ MeV/c$^2$ and estimate the neutrino mass as $m_{\nu}^2 = (p_B - p_{D^{*+}} - p_\ell)^2$. Events with $m_{\nu}^2 > -3$ GeV$^2$/c$^4$ are likely to be background events and are rejected. Finally, we veto events with charged or neutral kaons in the recoiling $B$ to reduce the peaking background from $B \rightarrow X_c \ell \bar{\nu}$ decays. Charged kaons are identified [8] with an efficiency varying between 60% at the highest (4 GeV/c) and almost 100% at the lowest momenta. The pion misidentification rate is about 2%. The $K_S^0 \rightarrow \pi^+\pi^-$ decays are reconstructed with an efficiency of 80% from pairs of oppositely charged tracks with an invariant mass between 486 and 510 MeV/c$^2$.

4 Measurement of Charmless Semileptonic Branching Ratios

To reduce the systematic uncertainties in the derivation of branching fractions, the observed number of signal events, corrected for peaking background and efficiency, is normalized to the total number of semileptonic decays $B \rightarrow X\ell\nu$ in the recoil of the $B_{reco}$ candidates. The number of observed $B_{reco}$ events which contain a charged lepton with $p^* > 1$ GeV/c is denoted as $N_{sl}^\text{meas}$. It can be related to the true number of semileptonic decays, $N_{sl}^\text{true}$ and the remaining peaking background $BG_{sl}$, estimated with Monte Carlo simulation, by $N_{sl}^\text{true} = (N_{sl}^\text{meas} - BG_{sl})/\epsilon_{sel}^sl \epsilon_t^{sl} = N_{sl}/\epsilon_t^{sl} \epsilon_{sel}^sl$. Here $\epsilon_{sel}^sl$ refers to the efficiency for selecting a lepton from a semileptonic B decay with a momentum above $p_{cut}$ in an event with a reconstructed $B$ with efficiency $\epsilon_t^{sl}$. Figure 1 shows the result of the $m_{ES}$ fit used to determine $N_{sl}^\text{meas}$.

If we denote as $N_u^\text{meas}$ the number of events fitted in the sample after all requirements, and with $BG_u$ the peaking background coming from semileptonic decays other than the signal, the true number of signal events $N_u^\text{true}$ is related to them by

$$N_u^\text{meas} - BG_u = \epsilon_u^{sel} \epsilon_t^{u} \epsilon_{sel}^{u} N_u^\text{true},$$

where the signal efficiency $\epsilon_{sel}^{u}$ accounts for all selection criteria applied on the sample after the requirement of a high momentum lepton.

To measure $BG_u$ in the inclusive studies, the peaking background ($BG_u$) is estimated by performing a $\chi^2$ fit on the $M_X$-$q^2$ distributions, resulting from $m_{ES}$ fits in individual $M_X$-$q^2$ bins, with the shape of the background estimated from Monte Carlo simulation, and its normalization free to vary.
The ratio between the partial branching fractions for the signal and $\bar{B} \to X\ell\bar{\nu}$ decays is

$$R_{u/sl} = \frac{\Delta B(\text{signal})}{B(\bar{B} \to X\ell\bar{\nu})} = \frac{N_{u}^{\text{true}}}{N_{sl}^{\text{true}}} = \frac{(N_{u}^{\text{meas}} - BG_{u})/\epsilon_{u}}{(N_{sl}^{\text{meas}} - BG_{sl})} \times \frac{\epsilon_{sl}^{\nu} \epsilon_{sl}}{\epsilon_{u}^{\nu} \epsilon_{u}}.$$  

(3)

The efficiency ratio $\epsilon_{sl}/\epsilon_{u}$ is expected to be close to, but not equal to unity. Due to the difference in multiplicity and the different lepton momentum spectra, we expect the tag efficiency $\epsilon_{t}$ and lepton efficiency $\epsilon_{l}$ to be slightly different for the two classes of events, the largest effect coming from $\epsilon_{l}$. The ratio was measured to be $1.204 \pm 0.033$. The signal branching fraction is then obtained from $R_{u/sl}$ using the total semileptonic branching fraction of $B(\bar{B} \to X\ell\bar{\nu}) = (10.83 \pm 0.19)\%$, which is the sum of the charm semileptonic branching ratio $B_{c}(\bar{B} \to X_{c}\ell\bar{\nu}) = (10.61 \pm 0.16(\text{exp}) \pm 0.06(\text{th}))\%$ [16] and the charmless semileptonic branching ratio $B(\bar{B} \to X_{u}\ell\bar{\nu}) = (0.22 \pm 0.04(\text{exp}) \pm 0.04(\text{th}))\%$ [2], both measured in $\text{BaBar}$.

4.1 Measurement of the Partial Branching Fraction

Measurements done using only a $M_{X}$ kinematic cut to reject the $b \to c\ell\bar{\nu}$ background are limited by the dependence on the SF. This can be overcome by selecting a phase space region where the SF effects are small, namely the region at large $q^{2}$ values [17]. In this way we find a trade-off between the statistical and theoretical uncertainties by losing the $M_{X}$ cut and applying a cut on $q^{2}$. Moreover, since most of the theoretical uncertainties are due to the extrapolation from a selected kinematic region to the full phase space, measurements of partial branching fractions in different regions of phase space and their extrapolation to the full phase space can serve as tests of the theoretical calculations and models.

In order to extract the partial charmless semileptonic branching ratio, $\Delta B(\bar{B} \to X_{u}\ell\bar{\nu})$, in a given region of the $M_{X}$-$q^{2}$ plane, we define as signal the events with true values of the kinematic variables in the chosen region, treating as background those that migrate from outside this region.
because of the resolution. This means that in applying Eq. 3 we include the $b \rightarrow ul\nu$ events outside the signal region in $BG_u$ and the quoted efficiencies refer only to events generated in the chosen ($M_X-q^2$) region. These efficiencies are computed on simulation based on the DFN model. However, the associated theoretical uncertainty on the final result is small compared to the extrapolation error to the full phase space. We divide the events into 32 non-equidistant two-dimensional bins of $M_X$ and $q^2$ (4 bins in $M_X$ and 8 in $q^2$), we fit the $m_{ES}$ distribution to extract the yield in each bin, and we perform a two-dimensional binned fit of the entire $M_X-q^2$ distribution in order to extract the signal and background components. The result of the fit is shown in Fig. 2. We measure, out of $103590 \pm 474$ background-subtracted semileptonic events ($N_{sl}^{meas} - BG_{sl}$), $317 \pm 34$ signal events ($N_{u}^{meas} - BG_{u}$), above a background of $270 \pm 5$ events ($BG_u$). This, with $e_{sel}^{u} = 0.319 \pm 0.006$, corresponds to a partial branching fraction in the signal region $q^2 > 8\text{GeV}^2/c^4$, $M_X < 1.7\text{GeV}/c^2$ of:

$$R_{u/sl}(B \rightarrow X_u \ell \bar{\nu}, M_X < 1.7\text{GeV}/c^2, q^2 > 8\text{GeV}^2/c^4) = (0.80 \pm 0.09_{\text{stat}} \pm 0.09_{\text{syst}} \pm 0.01_{\text{th}}) \times 10^{-2},$$

(4)

where the errors are due to statistics, experimental systematics and theoretical systematics, respectively. This gives the following value for the partial branching fraction:

$$\Delta B(B \rightarrow X_u \ell \bar{\nu}, M_X < 1.7\text{GeV}/c^2, q^2 > 8\text{GeV}^2/c^4) = (0.87 \pm 0.09_{\text{stat}} \pm 0.09_{\text{syst}} \pm 0.01_{\text{th}}) \times 10^{-3}.$$  

(5)

### 4.2 Systematic Uncertainties

A breakdown of the systematic uncertainties is presented in Table 1.

Uncertainties related to the reconstruction of charged tracks are determined by removing randomly a fraction of tracks corresponding to the uncertainty in the track finding efficiency (1.4% per track).
For photons, we correct for differences between data and MC in energy resolution, energy scale, and EMC crystal edge effects and assign the systematic uncertainty by repeating the analysis without applying the corrections. For single photon reconstruction no efficiency correction is applied, but a systematic uncertainty of 1.8% per photon is assigned.

We estimate the systematic error due to particle identification (PID) by varying the electron and kaon identification efficiencies by ±2% and the muon identification efficiency by ±3%. The misidentification probabilities are varied by 15% for all particles. Effects due to $K_L$ interactions have been estimated by removing all EMC deposits due to $K_L$ when reconstructing $M_X$.

The uncertainty of the $B_{\text{reco}}$ combinatorial background subtraction is estimated by changing the signal shape function to a Gaussian function instead of the empirical function of Ref. [15]. Furthermore, the parameters of the empirical function, which are kept fixed in the $m_{\text{ES}}$ fits, are varied within their uncertainties.

The size of the Monte Carlo sample limits the accuracy on the determination of the ratio $\frac{q^p e^p}{q^l e^l}$ to 3%.

The impact of the charm semileptonic branching fractions has been estimated by varying each of the exclusive branching fractions within one standard deviation of the current world average [18]. Similarly, the branching fractions of charm mesons for inclusive kaon production have been varied to estimate uncertainties in the kaon veto.

To study the mixture of resonant decays among the charmless modes we also varied the number of charmless exclusive semileptonic decays by 30% for $B \to \pi \ell \bar{\nu}$ and $B \to \rho \ell \bar{\nu}$, by 40% for $B \to \omega \ell \bar{\nu}$ and by 100% for the remaining exclusive charmless semileptonic $B$ decays. Using only the non-resonant model for the signal gives an estimate of the effects due to uncertainties in the hadronization model. Signal events where a gluon splits in a $s\bar{s}$ pair are varied by 30% in non-resonant events in order to obtain the associated systematic uncertainty.

The uncertainties related to the knowledge of the SF are calculated by changing the functional form and varying the SF parameters as described in Section 5.

5 Extraction of $|V_{ub}|$

Using the $M_X-q^2$ analysis we measure the partial branching fraction for charmless semileptonic decays in a selected phase space region. To translate this into a measurement of the total branching fraction, and therefore $|V_{ub}|$, we need the fraction of events inside the measurement region (referred to as “acceptance” in the rest of the paper) as an external input.

In the following we use two different theoretical calculations of Bauer, Ligeti and Luke [17] (BLL) and Bosch, Lange, Neubert and Paz [5, 6, 7] (BLNP) for calculating acceptance corrections. Both BLL and BLNP use operator product expansions (OPE) to calculate QCD effects.

5.1 Results using acceptances from BLL

Bauer, Ligeti and Luke perform an OPE-based calculation to second order in the strong coupling constant $\alpha_s$ and $b$-quark mass $m_b$. They focus on the region chosen for the measurement of the partial branching fraction where non-perturbative effects are small. In particular, they have shown that the theoretical uncertainties of the extrapolation to the full phase space are much reduced by restricting the selection to regions of higher values of $q^2$, rather than just restricting $M_X$ to a region below the charm meson mass.
Table 1: Systematic uncertainties in percent for the partial fraction $\Delta B(\bar{B} \to X_u \ell \bar{\nu})$.

| Source                                      | $\sigma(\Delta B(\bar{B} \to X_u \ell \bar{\nu})) / \Delta B(\bar{B} \to X_u \ell \bar{\nu})$ |
|---------------------------------------------|-------------------------------------------------------------------------------------------------|
| Statistical error                           | 10.7                                                                                           |
| Monte-Carlo statistics                      | 4.0                                                                                           |
| **Detector-related:**                       |                                                                                                |
| Tracking efficiency                         | 2.2                                                                                           |
| Neutral efficiency                          | 0.1                                                                                           |
| Neutral corrections                         | 0.6                                                                                           |
| $K_L$                                       | 2.0                                                                                           |
| PID efficiency & misidentification          | 2.5                                                                                           |
| Detector uncertainties                      | 3.9                                                                                           |
| **$B_{\text{reco}}$ & fit:**               |                                                                                                |
| $m_{\text{ES}}$ fit                         | 4.1                                                                                           |
| $\epsilon^u_i \epsilon^u_i / \epsilon^s_i \epsilon^s_i$ | 3.0                                                                                           |
| $B$ SL branching ratios                     | 4.9                                                                                           |
| $D$ branching ratios                         | 0.1                                                                                           |
| $B_{\text{reco}}$ & fit errors              | 7.1                                                                                           |
| **Signal:**                                 |                                                                                                |
| Composition of $\bar{B} \to X_u \ell \bar{\nu}$ decays | 3.0                                                                                           |
| Hadronization                               | 3.0                                                                                           |
| Gluon splitting to $s \bar{s}$              | 2.2                                                                                           |
| Shape function parameters                   | 1.6 (1.9)                                                                                     |
| Shape function form                         | 0.3                                                                                           |
| Signal efficiency                           | 5.1                                                                                           |
| Total systematic error                      | 10.4                                                                                          |
| Total error                                 | 14.9                                                                                          |
Based on these calculations we can convert the measured $\Delta B(\overline{B} \to X_u \ell \bar{\nu})$ into $|V_{ub}|$ by

$$|V_{ub}| = \sqrt{\frac{192\pi^3}{\tau_B G_F^2 m_b^0} \frac{\Delta B(\overline{B} \to X_u \ell \bar{\nu})}{G}}$$  \hspace{1cm} (6)$$

where $\tau_B = 1.604 \pm 0.012$ ps [18] and $G$ is a theoretical parameter calculated in the BLL approach [17]. The first factor under the square root is $192\pi^3/(\tau_B G_F^2 m_b^0) = 0.00779$. To extract $|V_{ub}|$, we take $G = 0.27$ as computed by BLL for $m_b(1S) = 4.7$ GeV/$c^2$. We then infer the $b$-quark mass in the 1S scheme from the $\overline{B} \to X_s \gamma$ measurement of $m_b^{\text{kin}}$ [16] by using the prescription in [19], obtaining $m_b(1S) = 4.74$ GeV/$c^2$. $G$ is then recomputed by rescaling the original BLL value by the ratio $(4.74/4.7)^9$ [20], obtaining $G = 0.291 \pm 0.055$. The 19% error on $G$, which turns into a 9.5% error on $|V_{ub}|$, is the sum in quadrature of uncertainties due to: residual SF effects, higher order terms in the $\alpha_s$ perturbative expansion, a 80 MeV/$c^2$ uncertainty on the $b$ quark mass, and $\mathcal{O}(A_3^{QCD}/m^3)$ terms in the OPE expansion. The uncertainty on the $b$ quark mass is the dominant source, contributing about 15% to the uncertainty on $G$. Eq. 6 yields

$$|V_{ub}| = (4.82 \pm 0.26_{\text{stat}} \pm 0.25_{\text{syst}} \pm 0.46_{\text{th+SF}}) \times 10^{-3}.$$  \hspace{1cm} (7)$$

### 5.2 Results using the theoretical calculations by BLNP

Bosch, Lange, Neubert, and Paz have performed calculations of the differential decay rates for $\overline{B} \to X_u \ell \bar{\nu}$ and $B \to X_s \gamma$. The authors presented a systematic treatment of the SF effects, incorporating all known corrections to the rates, and provided an interpolation between regions of phase space that can be treated reliably by OPE calculations and others that depend on SF. They have introduced a parameterization of the SF. The parameters describing the SF cannot be calculated, rather they have to be taken from experiment.

On the basis of these SF parameters, the partial rate for $\overline{B} \to X_u \ell \bar{\nu}$ can be predicted for the measured phase space, and related to $|V_{ub}|$:

$$\Delta \zeta |V_{ub}|^2 = \int_0^{M_{\text{cut}}^A} d\zeta \int dq^2 dM_X \frac{d\sigma}{dq^2 dM_X} dq^2 dM_X,$$  \hspace{1cm} (8)$$

such that

$$|V_{ub}| = \sqrt{\frac{\Delta B(\overline{B} \to X_u \ell \bar{\nu})}{\Delta \zeta \tau_B}}.$$  \hspace{1cm} (9)$$

BLNP give results and uncertainties in terms of the reduced decay rate $\Delta \zeta$, defined in units of $|V_{ub}|^2$ ps$^{-1}$.

We rely on two measurements of these SF parameters, one based on the photon spectrum in $B \to X_s \gamma$ decays, the other on moments of the hadron mass and lepton energy spectrum in $\overline{B} \to X_u \ell \bar{\nu}$ decays. The analysis of $B \to X_s \gamma$ decays can be used to determine the SF parameters in a given renormalization scheme [21]. Likewise the moments of the lepton energy and hadronic invariant mass in $\overline{B} \to X_u \ell \bar{\nu}$ decays are sensitive to the heavy quark parameters, as shown in an OPE calculation [22] in the kinetic scheme. In both cases, the heavy quark parameters entering the calculations can be related to $\lambda_1^{SF}$ and $\lambda_1^{SF}$, see e.g. [23, 24].

The Belle Collaboration has measured the photon spectrum in $B \to X_s \gamma$ decays [25] and, based on a fit to the spectrum, has determined $\lambda_1^{SF} = 0.66$ GeV/$c^2$ and $\lambda_1^{SF} = -0.40$ GeV/$c^2$ [26]. They also provide a $\Delta \chi^2 = 1$ contour, which we use to estimate theoretical uncertainties.
These SF parameters translate to \( m_0^{SF} = 4.52 \pm 0.07 \) and \( \mu_\pi^{2, SF} = 0.27 \pm 0.23 \) \cite{27}. This results in \( \Delta \zeta = (21.6 \pm 4.0 \pm 2.4) \, |V_{ub}|^2 \) ps\(^{-1}\), where the first error is due to the limited experimental knowledge of the SF parameters and the second to theory uncertainties, and consequently

\[
|V_{ub}| = (5.00 \pm 0.27_{\text{stat}} \pm 0.26_{\text{syst}} \pm 0.46_{\text{SF}} \pm 0.28_{\text{th}}) \times 10^{-3},
\]

where the errors are due to statistics, experimental systematics, shape function parameters and theoretical systematics, respectively.

Alternatively, the \( \text{BaBar} \) collaboration has determined \( m_0^{\text{kin}} \) and \( \mu_\pi^{2, \text{kin}} \) in the kinetic mass scheme from fits to moments measured for \( B \to X_c \ell \nu \) \cite{16}. The values have been translated into the SF scheme by following the prescription in \cite{23} resulting in \( m_0^{SF} = 4.61 \pm 0.08 \) GeV\(^2\) and \( \mu_\pi^{2, SF} = 0.15 \pm 0.07 \), with a correlation of -40\%. The systematic error due to the uncertainty of the SF parameters is reduced, due to the significantly better precision obtained in the \( \text{BaBar} \) moments analysis.

By using the results of the \( \text{BaBar} \) moments analysis we get \( \Delta \zeta = (25.04 \pm 4.9_{0.06\text{SF}} \pm 2.45_{\text{th}}) \) \( |V_{ub}|^2 \) ps\(^{-1}\). Again, the error is due to the limited experimental knowledge of the shape function parameters. This translates into

\[
|V_{ub}| = (4.65 \pm 0.24_{\text{stat}} \pm 0.24_{\text{syst}}^{+0.46}_{-0.38\text{SF}} \pm 0.23_{\text{th}}) \times 10^{-3}.
\]

6 Conclusions

We have presented a study of charmless semileptonic decays and a measurement of the \(|V_{ub}|\) CKM matrix element, by using the combined information of the \( M_X - q^2 \) distribution to discriminate signal and background and to minimize the theoretical uncertainties. We give a measurement of the partial branching fraction of charmless semileptonic decays \( \Delta B(B \to X_u \ell \nu) \) for \( M_X < 1.7 \text{GeV}/c^2 \) and \( q^2 > 8 \text{GeV}^2/c^4 \) and, by taking kinematic acceptances from two theoretical calculations by BLL and BLNP, extract \(|V_{ub}|\).

The measured partial branching fraction \( B(B \to X_u \ell \nu) \) in the region limited by \( M_X < 1.7 \text{GeV}/c^2 \), \( q^2 > 8 \text{GeV}^2/c^4 \) is

\[
\Delta B(B \to X_u \ell \nu, M_X < 1.7 \text{GeV}/c^2, q^2 > 8 \text{GeV}^2/c^4) = (0.87 \pm 0.09_{\text{stat}} \pm 0.09_{\text{syst}} \pm 0.01_{\text{th}}) \times 10^{-3}.
\]

We extract the CKM matrix element \(|V_{ub}|\) using different approaches. With acceptances calculated using the BLL calculations, we obtain:

\[
|V_{ub}|^{\text{BLL}} = (4.82 \pm 0.26_{\text{stat}} \pm 0.25_{\text{syst}} \pm 0.46_{\text{th+SF}}) \times 10^{-3}.
\]

Using the partial decay models calculated in the BLNP approach and by taking the shape function parameters from the Belle photon spectrum in \( B \to X_s \gamma \) and the \( \text{BaBar} \) analysis of \( B \to X_c \ell \nu \) moments, we find:

\[
|V_{ub}|^{\text{Belle} B \to X_s \gamma} = (5.00 \pm 0.27_{\text{stat}} \pm 0.26_{\text{syst}} \pm 0.46_{\text{SF}} \pm 0.28_{\text{th}}) \times 10^{-3},
\]

\[
|V_{ub}|^{\text{Belle} B \to X_c \ell \nu} = (4.65 \pm 0.24_{\text{stat}} \pm 0.24_{\text{syst}}^{+0.46}_{-0.38\text{SF}} \pm 0.23_{\text{th}}) \times 10^{-3},
\]

where the errors are due to statistics, experimental systematics, SF and theoretical systematics, respectively.
In conclusion, the total error on $|V_{ub}|$ is dominated by the experimental and theoretical uncertainties of the shape function. Our results of $|V_{ub}|$ using the two different calculations of BLL and BLNP are consistent with each other. For the BLNP calculations the two sets of shape function parameters coming from a fit to the photon energy spectrum and to the $\bar{B} \rightarrow X_c \ell \bar{\nu}$ moments are in good agreement and thus give consistent results on $|V_{ub}|$. Results based on partial branching fraction from the lepton spectrum and $q^2$ using BLNP appear to be consistent with our measurement but somewhat lower [28].

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