A Homogenization Procedure for the Numerical Simulation of Mechanical Behavior of CFCC by Considering the Overall and Local Anisotropic Damage

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Abstract: An anisotropic damage constitutive model was presented to characterize mechanical behavior of continuous fiber-reinforced ceramic matrix composites (CFCC) with two-scale damage. An overall fourth-rank damage effect tensor was introduced to account for the local effects of damage experienced by both the matrix and fibers. The overall and local damage tensors were correlated together using homogenization procedure. In terms of the homogenization methods, the effective elastic properties were obtained, and the stress and strain concentration factors were derived for damaged composites. The model was applied in detail to the unidirectional laminate that was subjected to uni-axial tension. The results were compared well with experimental data. The effects of important parameters such as the fiber volume fraction and the damage material parameters on the nonlinear behavior of the composites were investigated. The model provided a useful tool for understanding the overall dependence of stress-strain behavior on all the underlying constituent material properties.

Key words: Homogenization method, Anisotropic damage, Continuous fiber-reinforced ceramic matrix composites (CFCC), Mechanical behavior, Constitutive model

1. INTRODUCTION

Continuous fiber-reinforced ceramic matrix composite materials (CFCC) play an important role in the industry today through the design and manufacture of advanced materials capable of attaining higher stiffness-density and strength-density ratios. Since the constituent materials are both brittle, and the non-linear response associated with ceramic-matrix composites is a direct result of damage interactions between its two linear-elastic constituents, fiber and matrix. Micro-structural interactions associated with the constituents of the composite prevent catastrophic failure and promote toughness through load transfer and energy dissipation, and it is important to analyze and capture these phenomena in CFCC plates. Although the literatures were rich in the developments of the CFCC technology [1-5], most of them were focus on the experimental work, and the theoretical analyses were limited on classical analysis, such as shear-lag model or modified shear-lag model. These models attempt to estimate the material response by assuming simplified damage configurations (such as uniformly-spaced, infinitely-long matrix cracks, regular array of fibers, etc.) within the composites. The stochastic nature of brittle failure, however, has forced these models to rely on empirical data, thereby reducing the utility of the numerical models.

The overall deformation of the composite depends on the microscopic flaws in the matrix and fibers. Micromechanics is widely employed to relate the overall properties of strongly heterogeneous media with the properties of the constituents and the microstructure. In this context, the homogenization method usually qualifies the passage from the micro-to macro-scale. The advantage of homogenization procedure is that the physical details contained on the smaller scale are not lost, whereas they are not present at all when the phenomenological is utilized [6-8]. Furthermore, the macro-scale model may in fact be easier to be constructed by this method since the macro-scale properties are usually dependent on simpler micro-scale properties. This method has been applied in a variety of applications, for example, in studying mechanical behavior of a heterogeneous lamina [9-10] and laminates structures for armored vehicles [11], in predicting the effective elastic properties of composites [12], and in analyzing the mechanical behavior of elastic-perfectly plastic and strain hardening fiber-reinforced composites [13-14].

The number of literatures covering micro-mechanics applied to CFCC is rather limited. Hild et al. [15] employed continuum damage mechanics to derive the macroscopic stress-strain law for the damaged composites: matrix cracking and interface debonding are described through micro-scopic damage variables. Zhang et al. [16] focused their attention upon the evaluation of stress intensity factors associated with cracks in the matrix and at the fiber-matrix interface, assuming the un-cracked matrix to be linear elastic. It is noticeable that most of the studies have assumed that the fibers do not fail during matrix cracking, and there is little work on the application of the method to CFCC with anisotropic damage in the matrix and fiber simultaneously.

The importance of damage in the mechanical behavior depends upon a balance between the respective elastic properties of the matrix and fibers. The factors to determine the mechanical responses are critical for the
development of optimized composite systems and the design of structural components using composites. By means of suitable selection and combination of single components, the properties of the composite can be varied over a wide range.

In this paper, the asymptotic homogenization method is first used to determine the effective elastic properties of the undamaged CFCC with different fiber volume fractions. The results are verified by the results of the rule of mixture and the average method. Following the obtained effective elastic properties, the continuum damage mechanics is used with a micro-mechanical composite model to analyze two-scale damage in CFCC. Both overall and local damage variables are introduced to model the overall and local damage effects. The local damage relations are linked to the overall response by a homogenization procedure, and the stress and strain concentration factors are derived for the damaged composites. The subject of this paper is to predict the stress-strain and failure behavior of unidirectional CFCC as a function of the matrix and fiber damages, and to investigate the influence of important parameters on the mechanical behavior of composites. The effects of important parameters such as the fiber volume fraction and damage material parameters on the nonlinear mechanical behavior of the CFCC are examined by modeling the macro-structure.

2. THEORETICAL ANALYSIS OF THE ANISOTROPIC DAMAGE

2.1. Definitions and Assumptions

Considering a body of CFCC in the initial undeformed and undamaged configuration $C_0$. Let $C$ be the configuration of the body that is both damaged and deformed after a set of external loading act on it, and $\bar{C}$ is the state of the body after it had deformed without damage. Assume that the representative volume element (RVE) in $C_0$ is statistically homogeneous, and is free of voids and cracks initially. Assume also that the composite material is loaded by an overall stress or strain field which is followed by increments of loading. The overall stress and strain fields are assumed to be uniform. The effective overall stress is defined in the configuration $C$ as the stress in a perfectly-bonded two-phase composite free of cracks or voids.

The composite material is assumed to consist of elastic fibers and an elastic matrix. The fibers are continuous, aligned and equally spaced. In the following, quantities are defined in the configuration $C$ of the overall composite system. Barred quantities are defined in the configuration $\bar{C}$ of the overall composite system. Quantities with a superscript $M$ or $F$ refer to matrix or fiber related quantities, respectively. $\sigma$ is the composite (overall) Cauchy stress in $C$, $\bar{\sigma}$ is the effective composite Cauchy stress in $\bar{C}$, $\sigma^M$ and $\sigma^F$ are the matrix and fiber stresses in $C$, respectively, and $\bar{\sigma}^M$ and $\bar{\sigma}^F$ are the effective matrix and fiber stresses in $\bar{C}$, respectively.

2.2. The Mechanical Behavior of Composites

In this section, the relations between the local (matrix and fiber) and overall (composites) relations are presented in the configuration $C$ [17].

In the configuration $C$, the effective stress tensor $\bar{\sigma}^\eta (\eta = M, F)$ is related to the effective composite stress tensor $\bar{\sigma}$ by

$$\bar{\sigma}^\eta = B^\eta_{\mu \nu} \bar{\sigma}_\nu,$$

(1)

where $B^\eta_{\mu \nu}$ is a fourth-rank tensor indicating the elastic phase stress concentration factor.

As a result of volume integration and averaging of the local stress fields, the following relation is obtained between the local (matrix and fiber) stresses and the overall stress in $C$:

$$\bar{\sigma}_\nu = V^M \bar{\sigma}^M_\nu + V^F \bar{\sigma}^F_\nu.$$

(2)

where $V^M$ and $V^F$ are the matrix and fiber volume fractions, respectively, given by:

$$V = V^M + V^F.$$

(3)

where $V$ is the total volume of the RVE.

As the same method, the local-overall relationships for the effective strain tensor $\bar{\varepsilon}$ in the configuration $C$ are presented. Upon volume integrating and averaging the local stress fields [18], the following local-overall relation is obtained for the effective spatial strain tensor:

$$\bar{\varepsilon}_\nu = V^M \bar{\varepsilon}^M_\nu + V^F \bar{\varepsilon}^F_\nu.$$

(4)

where the appropriate relations for the effective matrix and fiber strain tensors are used as follows:

$$\bar{\varepsilon}^\eta = A^\eta_{\mu \nu} \bar{\varepsilon}_\nu.$$

(5)

where $A^\eta_{\mu \nu}$ is a fourth-rank tensor denoting the elastic phase strain concentration factor.

2.3. Anisotropic Damage Analysis

There are two steps that can be followed in order to develop a continuum damage model for a composite system consisting of fibers and a matrix. First, one considers that damage in the overall composite system is whole continuous. At this step, the model will reflect various types of damage mechanisms such as void growth and coalescence in the matrix, fiber fracture, debonding and delamination, etc. It should be noted that at this step, no distinction is made between these types of damage as they are all reflected through the fourth-rank overall damage effect tensor $M_{\mu \nu}$. In the second step, one considers the damage that the matrix and fibers undergo separately such as nucleation and growth of voids and void coalescence for the matrix and fracture for the fibers. In this case, two fourth-rank matrix and fiber damage effect tensors $M^M_{\mu \nu}$ and $M^F_{\mu \nu}$ are introduced that reflect all types of damage that the matrix and fibers undergo. Subsequently, the local-overall relations are used to transform these local damage effects to the whole composite system.

Following the first step outlined above and utilized an overall damage effect tensor $M$ for the whole composite system, the overall effective Cauchy stress tensor $\bar{\sigma}$ is given by
Substituting Eq. (6) into Eq. (1), we can obtain:

\[
\bar{\sigma}_0 = M_{ijkl} \sigma_{ij}.
\] (6)

Following the second step discussed at the beginning of this section, we can establish the following local transformation equation:

\[
\bar{\sigma}_0 = B_{ijkl} M_{ijkl} \sigma_{ij}.
\] (7)

Comparing Eqs. (7) and (8), the following relation between the phase stress tensor and the overall stress tensor can be obtained:

\[
\sigma_0 = B_{ijkl} \sigma_{ij} = M_{ijkl} \sigma_{ij}.
\] (8)

where

\[
B_{ijkl} = (M_{ijkl})^{-1} B_{ijkl} M_{ijkl}.
\] (10)

The fourth-rank tensor \( B_{ijkl} \) is the damaged phase stress concentration factor that includes geometrical and damage related effects as can be seen from Eq. (10).

Similar relations are exist for strain concentration factor:

\[
\varepsilon_0 = A_{ijkl} \varepsilon_{ij},
\] (9)

where

\[
A_{ijkl} = (M_{ijkl})^{-1} A_{ijkl} M_{ijkl}.
\] (12)

Substituting Eq. (8) into Eq. (2) and simplifying, the required relationship between the local damage effect tensors \( M^M, M^F \) and the overall damage effect tensor \( M \) becomes:

\[
M_{ijkl} = V^M M_{ijkl}^M B_{ijkl}^M + V^F M_{ijkl}^F B_{ijkl}^F.
\] (13)

The above equation is an explicit relation between the effective local concentration factors and the overall damage effect tensor. It is clear that once the local (matrix and fiber) damage mechanisms have been described through the tensors \( M^M, M^F \), then the overall damage in the composite system can be described which includes the matrix and fiber related damage as well as the damage resulting from the interaction of the two phases such as debonding.

The local (matrix and fiber) damage tensors are given by Voyiadjis and Kattan as follows [19]:

\[
[M^*] = \begin{bmatrix}
1 - D^M_{11} & 0 & D^M_{12} \\
\Delta & 1 - D^M_{22} & D^M_{23} \\
0 & \Delta & 1 - D^M_{22} \\
D^M_{12} & D^M_{13} & D^M_{23} \\
2\Delta & 2\Delta & 2\Delta
\end{bmatrix}
\] (14)

where \( \Delta = (1 - D^M_{11})(1 - D^M_{22}) - (D^M_{12})^2 \), and \( D^M_{11}, D^M_{22}, D^M_{12} \) represent the damage variables parallel to the stress directions of \( \sigma_{11}, \sigma_{22}, \sigma_{12} \) in the matrix and fiber, respectively.

2.4. Constitutive Equations

Assuming the material obeys generalized Hooke's law in the undamaged configuration \( \vec{C} \), and \( E_{ijkl} \) is the component of elastic stiffness without damage:

\[
\sigma_{ij} = E_{ijkl} \varepsilon_{kl}.
\] (15)

In the damaged composite configuration \( C \), the elastic constitutive relation takes the form

\[
\sigma_{ij} = \tilde{E}_{ijkl} \varepsilon_{kl},
\] (16)

where the fourth-rank tensor \( \tilde{E}_{ijkl} \) is no longer constant but depends on the damage effect tensor \( M_{ijkl} \). Using the hypothesis of elastic energy equivalence, \( \tilde{E}_{ijkl} \) can be obtained:

\[
\tilde{E}_{ijkl} = M_{ijkl}^{M} E_{ijkl}^{M} + M_{ijkl}^{F} E_{ijkl}^{F}.
\] (17)

Assuming the generalized Hooke's law to hold for each of the phases in the configuration \( C \), then

\[
\sigma_0 = \tilde{E}_{ijkl} \varepsilon_0 = \tilde{E}_{ijkl} \varepsilon_{kl},
\] (18)

where the tensor \( \tilde{E}_{ijkl} \) ( \( \eta = M, F \) ) are the damaged elasticity tensors for phase materials (matrix and fiber). Substituting Eqs. (16) and (18) into an equation similar to Eq. (2), written in the configuration \( C \), we can obtain:

\[
\tilde{E}_{ijkl} = V^M A_{ijkl}^{M} E_{ijkl}^{M} + V^F A_{ijkl}^{F} E_{ijkl}^{F}.
\] (19)

It should be noted that there is no change in the phase volume fractions is assumed in Eq. (19).

2.5. Evolution of Damage

The criterion for damage evolution used here is that proposed by Lee et al [20] and is given by the function \( g(y,B) \) defined by:

\[
g(y,B) = \frac{1}{2} J_{ijkl} y_{ij} y_{kl} - B^y (\beta) = 0,
\] (20)

where \( B^y (\beta) \) is a function of the local damage parameters and \( B^y (\beta) = 1/2 K_1^2 \beta^2 \). \( J \) is a constant fourth-rank tensor that can be represented by a constant \( 6 \times 6 \) matrix, and \( y^y \) is the generalized thermodynamic force.

The generalized thermodynamic force conjugate to the kinematic damage variable [21] is:

\[
-\cdot = \frac{1}{2} \sigma_{ij} \delta \cdot (\cdot) \sigma_{ij} = \sigma : (\tilde{E}_{ijkl}^{(y)})^{-1} : (M^*)^{-1} \cdot \delta M^* : \sigma, (21)
\]

\(-\cdot\) is called as the damage energy release rate, and it
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follows that $-Y^* \geq 0$, so the dissipation inequality becomes:

$$(-Y^*) \cdot \dot{D}^* \geq 0.$$  \hspace{1cm} (22)

The evolution laws of anisotropic damage are characterized by S. Murakami \cite{22}:

$$D_{11}^\alpha = \frac{\alpha}{2K_d} \left( \frac{\gamma_{11}^2}{\gamma_{11}^2 + \gamma_{22}^2 + \gamma_{12}^2} \right),$$ \hspace{1cm} (23a)

$$D_{22}^\alpha = \frac{\alpha}{2K_d} \left( \frac{\gamma_{22}^2}{\gamma_{11}^2 + \gamma_{22}^2 + \gamma_{12}^2} \right),$$ \hspace{1cm} (23b)

$$D_{12}^\alpha = \frac{\alpha}{2K_d} \left( \frac{\gamma_{12}^2}{\gamma_{11}^2 + \gamma_{22}^2 + \gamma_{12}^2} \right).$$ \hspace{1cm} (23c)

where, $\alpha$ is a damage governing parameter, and $\alpha=1$ for damage state, $\alpha=0$ for undamaged state, $K$ are the damage material parameters in matrix and fiber, respectively.

2.6. The Criterion of Damage

In this paper, the damage equivalent strain, $\theta^{(e)}$, is defined as the square root of the matrix damage energy release rate \cite{9}:

$$\theta^{(e)} = \sqrt{(-Y^*)}.$$ \hspace{1cm} (24)

Supposing the damage evolution at time $t$ can be expressed as

$$D_{ij}(x,t) = f(k^{(e)}_{ij}(x,t), ((i,j=1,2),$$ \hspace{1cm} (25)

where, $k^{(e)}_{ij}(x,t)$ is the deformation history parameter, and it is determined by the evolution of damage equivalent strain $\theta^{(e)}$ as follows:

$$k^{(e)}_{ij}(x,t) = \max(\theta^{(e)}_{ij}(x,t), k_0),$$ \hspace{1cm} (26)

where the threshold value for damage initiation, $k_0$, represents the extreme value of the equivalent strain prior to the initiation of damage which is also a damage material parameter. Here, we assume that the initiated damages are the same in three damage directions in matrix and fiber. The evolution of damage equivalent strain can be also expressed by the Kuhn-Tucker relations:

$$\dot{k}_{ij}^{(e)} \geq 0, \theta^{(e)}_{ij} - k^{(e)}_{ij} \leq 0, \theta^{(e)}_{ij} - k^{(e)}_{ij} = 0.$$ \hspace{1cm} (27)

Combining the energy dissipation inequality (22), and the definition of damage evolution (25) with Kuhn-Tucker relations, the damage evolution conditions become:

If $\theta^{(e)}_{ij} - k^{(e)}_{ij} = 0$, $k^{(e)}_{ij} > 0 \Rightarrow$ damage process:

$$D_{ij}^\alpha > 0$$ \hspace{1cm} (28)

If $\theta^{(e)}_{ij} - k^{(e)}_{ij} \leq 0$, $\dot{k}_{ij}^{(e)} = 0 \Rightarrow$ elastic process:

$$D_{ij}^\alpha = 0$$ \hspace{1cm} (29)

3. HOMOGENIZATION METHOD AND ITS APPLICATION

3.1. The Theory of Homogenization Method

The homogenization procedure has already been fully derived in Pellegrino et al. \cite{6}, therefore we only briefly recall it here. We take into consideration a periodic composite material, which is a material made of a large number of regularly distributed and equal unit cells with linear boundary conditions on them, and only a unit cell need be considered for microscopic analysis.

An asymptotic expansion of the displacement field is:

$$u^x(x) = u_0^x(x,y) + \epsilon u_1^x(x,y) + \epsilon^2 u_2^x(x,y) + ... ,$$ \hspace{1cm} (30)

where, $x$ is a macroscopic coordinate, and $y \equiv x/\zeta$ is a microscopic position vector, and $\zeta$ is the characteristic length, which is the scaling factor between the two length scales (see Fig.1(a) and (b)).

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With consideration of the indirect differentiation rule, the strain field can be expanded as:

$$\varepsilon(x,y) = \frac{1}{\zeta} \epsilon_{x}(x,y) + \epsilon_{y}(x,y) + \epsilon_{xy}(x,y) + ... ,$$ \hspace{1cm} (31)

The stress field:

$$\sigma^x(x,y) = \frac{1}{\zeta} \sigma_{x}(x,y) + \sigma_{y}(x,y) + \sigma_{xy}(x,y) + ... .$$ \hspace{1cm} (32)

The stresses and displacements fields satisfy the following equations on macroscopic domain:

$$\sigma_{x}^{\Omega}(x,y) = -f^x,$$ \hspace{1cm} (33a)

$$\sigma_{y}^{\Omega} = \frac{k_{ij}(\gamma_{ij})}{E_{ij}(\omega_{ij})},$$ \hspace{1cm} (34a)

$$\sigma_{ij}^{\Omega}(x,y) = F^x, \text{ on } \Gamma_F,$$ \hspace{1cm} (34b)

$$u_0^x(x,y) = 0, \text{ on } \Gamma_0,$$ \hspace{1cm} (34c)

where, $F^x$ is a prescribed stress on the portion $\Gamma_F$ of the boundary of $\Omega$, and $F^0$ is the prescribed displacement
boundary portion (as seen in Fig. 1(a)). The subscript pairs with parentheses denote the symmetric gradients defined as:

\[ u_{\xi(x,y)} = \frac{1}{2} (u_{\xi(x,y)} + u_{\xi(\xi,y)}) \]  

Substituting Eqs. (31), (32) into Eqs. (33) and (34), we have:

\[ \sigma_{\xi} = 0, \sigma_{\eta} = \sigma_{\xi} = (E_{x}^{u} + \varepsilon_{\eta}(u^{*})) \]  

\[ \sigma_{\eta(\xi)} + \sigma_{\eta(\eta)} + f_{i} = 0. \]  

Due to the linearity of Eq. (36), the following characteristic function is introduced:

\[ u_{\xi(x,y)} = H_{\eta(y)}^{(A)}(\varepsilon_{\eta}(u^{*})), \]  

where \( H_{\eta(y)}^{(A)} \) is a \( y \)-periodic function, and \( \varepsilon_{\eta}(u^{*}) \) is the macroscopic strain.

Based on the decomposition given in Eq. (38), Eq. (36) takes the following form:

\[ \{E_{ijkl}(I_{\eta(\xi,y)} + G_{ijkl}(\varepsilon_{\eta}(u^{*})))\}_{\eta(\xi,y)} = 0, \text{in } \Omega. \]  

The weak form boundary value problem defined by Eq. (39) can be solved by finite element method. This process has been implemented into the ANSYS program through a user-supplied material model subroutine [23].

The elastic homogenized stiffness is

\[ E_{ijkl}^{(H)} = \frac{1}{\Theta} \int_{\Omega} E_{ijkl}^{n} \tilde{A}_{ijkl}^{n} d\Omega, \]  

where \( \Theta \) is the area (volume for three-dimension) of a homogeneous unit cell, and \( \tilde{A}_{ijkl}^{n} \) are the elastic strain concentration functions which are defined as

\[ \tilde{A}_{ijkl}^{n} = I_{ijkl} + G_{ijkl}. \]  

The asymptotic expansion of strain can be expressed in terms of the macroscopic strain \( \varepsilon_{\eta}(u^{*}) \) as follows:

\[ \varepsilon_{\eta} = \tilde{A}_{ijkl}^{n} \varepsilon_{\eta}(u^{*}) + O(\zeta). \]  

We integrate equilibrium equation (37) over \( \Theta \), and the \( \int_{\Omega} \sigma_{\eta(\xi)} d\Omega \) term vanishes due to periodicity, so

\[ (\frac{1}{\Theta} \int_{\Omega} \sigma_{\eta(\xi,y)} d\Omega)_{\xi(y)} + f_{i} = 0 \text{ in } \Omega. \]  

Substituting the asymptotic expansion of the strain field Eq. (42) and Eq. (36) into Eq. (43) yields the macroscopic equilibrium equation:

\[ \left(\frac{1}{\Theta} \int_{\Omega} E_{ijkl}^{n} \varepsilon_{\eta}(u^{*}) d\Theta\right)_{\xi(y)} + f_{i} = 0. \]  

We define the macroscopic stress

\[ \sigma_{\eta} = \frac{1}{\Theta} \int_{\Omega} \sigma_{\eta(\xi,y)} d\Theta = E_{ijkl}^{n} \varepsilon_{\eta}(u^{*}). \]  

The average strains in each sub-domain in RVE are obtained by integrating Eq. (42) over \( \Theta \):

\[ \varepsilon_{\eta(v)}^{(v)} = \frac{1}{\Theta} \int_{\Omega} \varepsilon_{\eta(\xi,y)} d\Theta = \bar{A}_{ijkl}^{(v)} \varepsilon_{\eta}(u^{*}) + O(\zeta). \]  

\[ \bar{A}_{ijkl}^{(v)} = \frac{1}{\Theta} \int_{\Omega} A_{ijkl}^{n} d\Theta. \]  

The local average stress in is defined as

\[ \sigma_{\eta(v)}^{(v)} = \frac{1}{\Theta} \int_{\Omega} \sigma_{\eta(\xi,y)}^{(v)} d\Theta. \]  

Substituting Eqs. (34a), (38), (42) and (46) into Eq. (48), we have following phase stresses:

\[ \sigma_{\eta}^{(v)} = E_{ijkl}^{(v)} \varepsilon_{\eta(v)}^{(v)} + O(\zeta), (\eta = M, F). \]  

Substituting Eqs. (46) and (49) into Eq. (2), the homogenized elastic stiffness defined in Eq. (45) becomes:

\[ E_{ijkl} = E_{ijkl}^{M} E_{ijkl}^{M} A_{ijkl}^{M} + V_{ijkl}^{F} E_{ijkl}^{F} A_{ijkl}^{F}. \]  

Equation (50) is the same with Eq. (19). \( E_{ijkl}^{M} \) and \( A_{ijkl}^{M} \) are the homogenized damaged elastic concentration factor for matrix and fiber, respectively, and they are determined by the homogenization procedure. \( E_{ijkl}^{M} \) and \( A_{ijkl}^{M} \) are the damaged elastic stiffness of fiber and matrix phase, respectively.

From Eqs. (45), (46), (49) and (9), we can derive the damaged stress concentration factors for matrix and fiber are:

\[ B_{ijkl}^{n} = E_{ijkl}^{n} A_{ijkl}^{M} E_{ijkl}^{(H)} A_{ijkl}^{M}^{-1}. \]  

The un-damaged stress concentration factors are

\[ B_{ijkl}^{n} = (M_{ijkl}^{n}) B_{ijkl}^{n} M_{ijkl}^{n^{-1}}. \]  

4. APPLICATION AND DISCUSSION

4.1. Elastic Material Properties for Undamaged Composites

The reliability of the adopted numerical model is first checked by the calculation of elastic properties for different fiber volume fractions in the linear elastic field. The considering composite materials are Nicalon/CAS composites.

In Fig. 2 the macroscopic elastic properties are plotted vs. the fiber volume fraction, \( V_{F} \). The data plotted in
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Fig. 2 include the numerical results given by the present model, the theoretical prediction by the rule of mixture and average method in [24]. The “rule of mixture” model has a low-estimation, and the average method has an over-estimation to the elastic properties of composites. The homogenization method gives a moderate result. With the increase of fiber volume fraction, the agreement between “rule of mixture” predictions and the homogenization method is definitely encouraging. However, the “rule of mixture” predictions are only limited on linear-elastic composites, and the results obtained from average method are actually the average relation between the stress and strain, and they are not the real material properties. Homogenization method is a rigorous approach to determine the effective material properties of composites with linear and non-linear mechanical behavior.

4.2. The Influence of Fiber Volume Fraction

The influence of fiber volume fraction on mechanical behavior of damaged composites is studied here. Figure 3 shows the dependence of the tensile strength on the fiber volume fractions. Four stages can be divided for Fig. 3. The laminate behaves linearly up to $V^f=30\%$, and it is controlled by the matrix strength, and the macroscopic brittle behaviors are obvious shown in Fig. 4, in which the macroscopic stress-strain curves show a strain-softening behavior. In this stage matrix cracking is the main damage. With the increase of fiber volume fraction, the stress increases gradually, and saturation of matrix damage tendency by shown by a “tough” stable stage of the curves. For $V^f=30-40\%$, the stress experiences a stable “tough” stage from $\varepsilon=0.3-0.55\%$, where the stress nearly keeps constant or even has a little decrease, and the stiffness (secant modulus) has a sudden drop. Then the stress continues to increase till fiber failure, which represents the stress of fiber experienced. For $V^f=40-60\%$, the stresses are controlled mainly by the fiber, and the stress-strain curve experiences another linear stage, which represent the fiber linear elastic deformation. For the case of $V^f \geq 70\%$, the tensile strength increases very slowly and the failure strength almost keeps constant. It means that it has no any meaning to improve the strength of composites by only increasing fiber volume fraction after the fiber volume fraction is big enough. Figure 3 shows that the stress-strain curves are very sensitive to fiber volume fractions, and the maximum stress of composites increases with the increase of fiber volume fraction. The loading sharing between the fibers and the matrix is modified with the change of fiber volume fraction, and the fiber volume fractions are optimized as $V^f=30-40\%$ by considering the strength changes of CFCC in parameter studied. This is consistent with the choice of typical specimen in the experiment.

4.3. The Influence of Damage Material Parameters

Damage material parameter is another important factor to influence the mechanical behavior of composites. The different stresses of matrix crack initiation can be observed in Fig. 5. The maximum stress of composites decreases with decreasing fiber damage parameter.

Fig. 2. The relationships between the $V^f$ and elastic properties.

Fig. 3. The relationship between tensile strength and fiber volume fraction.

Fig. 4. The stress-strain curves for different fiber volume fraction.

Fig. 5. The stress-strain curves for different fiber damage parameters.
When the fiber damage parameter is equal to matrix damage parameter, the fiber damage becomes so big that its influence on mechanical behavior of composites cannot be ignored, and this induces the brittleness of composites. When the fiber damage parameter is big sufficiently, the stress-strain curves tend to stable, and fiber damage is ignorable. From Fig.6, we can find that the damage is most sensitive to the fiber damage parameter between $k_{Fd}^{p}=0.15-10.0$.

Figure 7 shows that matrix damage parameter has no influence on the trends of stress-strain response, and it influences only the stress values in stable stage.

4.4. Stress-strain Curves for Unidirectional CFCC

For unidirectional composites with $V_f=35\%$, we suppose that the damage parameters are selected as $k_{Fd}^{p}=50.0$, and $k_{Md}^{p}=0.15$, respectively. The elastic constants of the constituents are listed in Table 1. The typical stress-strain curves are shown in Fig.8. Both the longitudinal and transverse strain responses are plotted. The numerical results are in good agreement with that in S. W. Wang's experiment [4]. They summarize important trends in the mechanical behavior of brittle-matrix composites. The first deviation from linearity in the curve of stress versus strain is at a longitudinal strain of 0.2%. Towards the higher stress end, the stress-strain curves show upward convexity. The non-linear stress-strain relations show that the CFCC is damage sensitive, and the curved domain of deformation results essentially from transverse cracking in the matrix. The cracks are arrested by the fibers and they are deflected at the fiber/matrix interfaces causing fiber debonding [25].

Table 1. Experimental values of material properties [4].

| Properties | Values |
|------------|--------|
| $E_f$     | 195.14Gpa |
| $E_m$     | 98.00Gpa  |
| $\nu_{mn}$| 0.30    |
| $t$       | 7.50\mu m |
| $V_f$     | 0.35    |
| $E_1$     | 132.00Gpa |
| $E_2$     | 101.90Gpa |
| $G$       | 48.27Gpa  |
| $\nu_{12}$| 0.258   |
| $\nu_{11}$| 0.223   |

4.5. The Damage Analysis

The basic damage phenomena in unidirectional CFCC involve multiple micro-cracks or crack is formed in the
Numerical Simulation of CFCCs with Multi-scale Damage

matrix, perpendicularly to the loading direction. These cracks are arrested by the fibers and deflected at the interface between the fiber and the matrix [25].

The main idea in this paper is to describe the local damage evolution in matrix and fiber, then the relationship between the overall damage and the local damage is established by using the stress concentration factors. Comparing the evolution of damage in the matrix and fibers in Fig. 9, we can find that the damages of composites are mainly controlled by the matrix damage. This is consistent with the experimental results in [4]. The experimental results also show that no fiber fracture and no clear indication of fiber/matrix debonding in unidirectional composites. Figure 10 is the relationship between overall (local) damages and strain. The overall damage redistribution is clearly indicated in Fig. 10, in which the fiber/matrix debonding can be described by $D_{12}$. The variations of the damaged elastic properties are shown in Fig. 11. There are obvious reductions in the elastic properties of the unidirectional composites as a consequence of damage. With the increase of the fiber volume fractions, the failure strains decrease, and the brittleness of composites is strengthened.

Fig. 10. The relationship between the overall (local) damage and strain.

5. CONCLUSIONS

The micro-mechanical constitutive model was established to predict the mechanical behavior of CFCC with anisotropic damage in the matrix and fiber. An overall damage variable was introduced to model damage in the composite system while two local damage variables were used to model damage in the matrix and fibers. The overall and local damage variables were then related to the matrix and fiber volume fractions and stress concentration factors. The homogenization method was used to derive the elastic material properties, and the stress and strain concentration factors of the undamaged and damaged composites. The main conclusions were summarized as follows:

(1) The homogenization method is a rigorous approach to determine the effective elastic properties and the stress and strain concentration factors of undamaged and damaged composites.

(2) The general influence of important model parameter such as the fiber volume fraction and damage material parameters on the mechanical behavior of the CFCC was studied. The optimization of parameters was analyzed by considering the strength variation of CFCC.

Fig. 11. The relations among the homogenized damaged elastic constants for different fiber volume ratios.

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