Evaluation of distribution of void ratio and degree of saturation in partially saturated triaxial sand specimen using micro x-ray tomography

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ABSTRACT

Partially saturated soils are composed of three phases: soil particle, pore water and pore air. The pore water forms menisci among soil particles at which suction works. Suction is higher at the upper part of partially saturated soils due to the higher negative pressure head. It is therefore likely that degree of saturation in partially saturated soils distributes in a vertical direction with different suction levels. Additionally, partially saturated soils show more brittle mode of failure compared with fully saturated soils for which investigation of distribution changes in degree of saturation and void ratio is important. In this present study, triaxial compression tests for partially saturated Toyoura sand specimens are conducted and x-ray computed tomography scans are performed at initial state and after tests. Trinarisation technique by taking into account partial volume effect is applied to evaluate the volume of each three phase, through which the distribution of void ratio and degree of saturation in a vertical direction of partially saturated triaxial sand specimens are discussed. It is found that degree of saturation is lower in larger voids while degree of saturation was higher in smaller voids. At the middle of specimen where large deformation is observed, void ratio and degree of saturation significantly changes while those at the other position where less deformation is observed slightly changes.

Keywords: partially saturated soil, distribution, x-ray CT, trinarisation, deformation

1 INTRODUCTION

Partially saturated soils are three-phase mixture of soil particles, pore air and pore water. The pore water between soil particles of partially saturated soils exists as menisci at which negative water pressure works. The negative water pressure is called ‘suction’, which depends on density and degree of saturation in partially saturated soils. Suction is higher at the upper part of partially saturated soils due to higher negative pressure head, e.g.\(^1\). It denotes that degree of saturation of partially saturated soils probably distributes in a vertical direction. The relation between suction and degree of saturation, i.e., the water-retention curve, has been understood macroscopically through water-retention test. Up until now, however, distribution of degree of saturation in a vertical direction has been not sufficiently clear.

Partially saturated soils exhibit more brittle mode of failure with clear shear bands than fully saturated soils, especially in the case of lower confining pressure, e.g.,\(^2\). This is probably caused by loss of contribution of suction to the strength and stiffness of partially saturated soils due to the shear deformation. In order to understand the mechanism of brittle failure of partially saturated soils, it is important to investigate the distribution changes in void ratio and degree of saturation.

In the present study, three triaxial specimens of partially saturated sand have been observed using micro x-ray computed tomography (CT) which allows us to observe three phases in three dimensional conditions. X-ray CT images have been segmented into three phases by a trinarisation technique. Trinarisation technique used in this study takes into account partial volume effect, which affects the accuracy of separating each phase\(^3, 4\). Using the trinarised images, the distribution of void ratio and degree of saturation of partially saturated triaxial sand specimens at the initial state and deformed state have been quantitatively evaluated.

2 MATERIAL AND METHODS

2.1 Sample and testing program

The sample used in this study was Toyoura sand. The physical properties included a soil density of 2.64 g/cm\(^3\), a maximum void ratio of 0.975, a minimum void ratio of 0.614, a mean diameter of 0.187 mm. In this present study, three specimens with individually different initial

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degree of saturation were prepared by water pluviation technique. The diameter of the specimens was 35.00 mm, height was 70.00 mm and the relative density was 90.00 %. Suction was applied by negative water column\(^3\) in order to desaturate the specimens. Initial degree of saturation for the three specimens were 70.2 % (Case-1), 60.2 % (Case-2) and 43.9 % (Case-3), respectively.

Each of three specimens at the initial state and an axial strain of 21 % after triaxial test was observed by micro focus x-ray computed tomography equipment (KYOTOGEO-\(\mu\)XCT). Flat Panel Detector (FPD) was installed on this equipment, which provides the high resolution CT images with low noise. Specification of x-ray equipment is listed in Table 1. In this study, two kinds of scans were performed. One of the two was global tomography to observe the entire specimen with spatial resolution of 72.30 \(\mu\)m, while the other was local tomography to observe microstructure of the specimen in a vertical direction with relatively high spatial resolution of 13.22 \(\mu\)m as shown in Fig. 1. For Case-3, triaxial test was conducted under drained condition for air and undrained condition for water. Axial strain rate was 0.500 \%/min and confining pressure was 50 kPa. Subsequently, two kinds of scans were performed to observe deformation behaviour.

2.2 Trinarisation technique

X-ray CT images are assembly of discrete gray values, and thus they have a specific artifact such as partial volume effect; the voxel including two phases, which is called ‘mixel’, has a gray value based on the weighted average value of the two materials in a voxel. The CT images of partially saturated soils contain three types of mixels: the voxels sharing the soil phase and the water phase; the voxels sharing the water phase and the air phase; and the voxels sharing the soil phase and the air phase. In particular, a mixel between the soil phase and the air phase often gives a gray value similar to that of the water phase. It often leads to misidentify as the water phase resulting in overestimation of degree of saturation. Hence, partial volume effect should be taken into account to perform trinarisation with less misidentification and less overestimation of degree of saturation.

In the present study, superposition of the weighted distributions for the three phases and the three types of mixels is determined by means of the maximum likelihood estimation method (MLE). Fig. 2 shows a conceptual illustration of MLE method. The gray value distributions for the soil phase, the water phase and the air phase are assumed to be normal distributions, while those for three types of mixels are assumed to be uniform distributions.

In order to distinguish the soil phase, the water phase and the air phase, we applied region growing method\(^3,6\) in which a tolerance value for extracting a continuous one phase is needed (Fig. 3). Median filter

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**Table 1. Specification of x-ray equipment installing FPD**

| X-ray source | Max. voltage [kV] | Max. current [mA] | Max. consumption power [W] | Min. focus size [\(\mu\)m] | Size of matrices | Resolution performance [\(\mu\)m] | Integration time [ms] | Projection view | Integration Images |
|--------------|-------------------|-------------------|-----------------------------|-----------------------------|-----------------|-------------------------------|---------------------|-----------------|-----------------|
| FPD          | 225               | 0.888             | 200                         | 4                           | 1024\(^2\)      | 5                             | 66-999              | 600-4800        | 10-50           |

**Fig. 1. Schematic illustration of local tomography**

(a) Axial strain 0 % (b) Axial strain 21 % for only case-3

**Fig. 2. Conceptual illustration of MLE method**

(a): distribution of six phases including mixels
(b): superposition of six distribution by MLE method
with 5^3 voxels was applied to CT images of the present study to reduce noises before trinarisation. Once the superposition of weighted distribution is determined, a tolerance to extract the air phase is given by an interval between mean value of normal distribution for the air phase and the intersection of the air phase and the water phase. Similarly, a tolerance to extract the soil phase is given by an interval between mean value of normal distribution for the soil phase and the intersection of the water phase and the soil phase. It should be noted that the intersections are not used for thresholding phases but used to determine tolerance values for the region growing method. The water phase is given as the remaining voxels after region growing for the other phases.

Through the trinarisation method, we calculated local void ratio and degree of saturation by counting the number of voxel for the soil phase, the water phase and the air phase. In this study, the distribution of void ratio and degree of saturation of the specimens in a vertical direction are investigated with calculated values applying the same tolerance to all of local scan areas. The example of original CT image and trinarised image obtained in this study is shown in Fig. 4.

2.3 Validation for trinarisation technique

We prepared a validation sample whose height was 12 mm, diameter was 6 mm, void ratio was 0.846 and degree of saturation was 0.662 as shown in Fig. 5 to confirm the validation for trinarisation technique. The diameter was small enough to observe whole sample with even high spatial resolution of up to 6 μm. Two kinds of x-ray CT scans with 6.72 μm (Case-A) and 12.25 μm (Case-B) in resolution were performed and trinarisation was applied to the CT images. Then, calculated void ratio and degree of saturation were

| Case | Measured value | Calculated value |
|------|----------------|------------------|
|      |                | A                | B                |
| Void ratio | 0.846 | 0.831 | 0.820 |
| Degree of saturation | 0.662 | 0.656 | 0.725 |
| Water content | 0.212 | 0.206 | 0.225 |

![Fig. 3. Determination of tolerance for region growing method](image)

![Fig. 4. Example of the original CT image and the trinarised image](image)

![Fig. 5. Validation sample for trinarisation technique and application to the different resolution CT images](image)
were compared with those of the validation sample.

The validation results for different resolution CT images are listed in Table 2. The gray value histograms for Case-A and Case-B are shown in Fig. 6 and Fig. 7. It can be seen that the shape of superposition of weighted distributions estimated by MLE method is similar to that of original distribution in both cases. As shown in Fig. 7, for Case-B, it is clearly seen that the proposed technique reasonably describes the influence of mixels which leads to be relatively a larger amount of frequency between the water phase and the soil phase.

3 RESULTS AND DISCUSSION

3.1 Distribution of local values at initial state

Table 3 and Table 4 list measured values and calculated values of mean value \( \mu \) and standard deviation \( \sigma \) with respect to void ratio and degree of saturation. The calculated values are obtained by using trinarised images of the area of local tomography as shown in Fig. 1, while the measured values are global values obtained for the whole specimen.

Calculated mean values of void ratio \( \mu_0 \) are similar to measured values in each case. Calculated mean values of degree of saturation \( \mu_d \) for Case-1 show good agreement with the measured values whereas those for Case-2 and Case-3 are larger than measured values. This is attributed to the partial volume effect (mixel) on trinarisation results, which is consistent with the previous findings: trinarisation for higher pore saturation region provides better estimation of degree of saturation than that for lower pore saturation region.

It is also found that \( \mu_d \) for Case-2 is larger than that for Case-1, while measured value of degree of saturation for Case-2 is lower than that for Case-1. This is possibly because of heterogeneity of the specimen, i.e., the local scan area contains larger amount of water that the other area.

Fig. 8 to Fig. 10 demonstrate the distribution of local void ratio and degree of saturation of partially saturated triaxial sand specimens in a vertical direction for each case. These values were calculated at totally 10 portions of local tomography. Note that trinarisation technique was not applied to the CT images obtained at the number of 8, 9 in Fig.6 and the number from 0 to 3 in Fig.8, since the resolution was not good enough to perform trinarisation due to inherent instability of x-ray beam.

As can be seen in Fig. 8 and Fig. 9, for Case-1 and Case-2 in which degree of saturation is relatively high, degree of saturation tends to be lower in larger void ratio and degree of saturation tends to be higher in smaller void ratio. These suggest that the pore water drains faster from larger voids than smaller voids.

On the other hand, it is also seen that degree of saturation is lower even in smaller void ratio. This trend is more remarkable for Case-3 as shown in Fig. 10. This is probably because the pore water is initially easy to drain from mainly larger voids as long as the specimen keeps relatively high degree of saturation, while the pore water starts desaturating from not only large but also small voids when the specimen reaches relatively lower degree of saturation.

Degree of saturation at the lower part of the specimen is relatively higher as shown in Fig. 8 and Fig. 9. This is because the pore water tends to drain faster from the upper part of the specimen compared with lower part due to the higher negative pressure head.

| Case | Measured values | Calculated values |
|------|----------------|------------------|
|      | Void ratio \( \mu \) | \( \sigma \) | \( \mu' \) | \( \sigma' \) |
| 1    | 0.650          | 0.641            | 0.012    | 0.632    | 0.016 |
| 2    | 0.650          | 0.647            | 0.025    | 0.642    | 0.041 |
| 3    | 0.650          | 0.668            | 0.037    | 0.695    | 0.014 |

Table 3. Measured and calculated values of void ratio

| Case | Measured values | Calculated values |
|------|----------------|------------------|
|      | Degree of saturation \( \mu \) | \( \sigma \) | \( \mu' \) | \( \sigma' \) |
| 1    | 0.702          | 0.698            | 0.052    | 0.690    | 0.061 |
| 2    | 0.602          | 0.734            | 0.070    | 0.700    | 0.073 |
| 3    | 0.439          | 0.665            | 0.036    | 0.685    | 0.036 |

Table 4. Measured and calculated values of degree of saturation
Fig. 8. The distribution of void ratio and degree of saturation in vertical direction of the specimen for Case-1: global degree of saturation is equals to 0.702.

Fig. 9. The distribution of void ratio and degree of saturation in vertical direction of the specimen for Case-2: global degree of saturation is equals to 0.602.

Fig. 10. The distribution of void ratio and degree of saturation in vertical direction of the specimen for Case-3: global degree of saturation is equals to 0.439.

Fig. 11. The distribution change of void ratio and degree of saturation in vertical direction of the specimen for Case-3.
Through the proposed method of the present study, it is found that void ratio and degree of saturation of partially saturated sand triaxial specimens in a vertical direction distribute with a certain value of standard deviation as shown in Table 3 and Table 4. Mean value and standard deviation calculated based on the trinarised images obtained at the same region in each case, from 4 to 7 as shown in Fig. 8 to Fig. 10 are also listed as $\mu$ and $\sigma$ in Table 3 and Table 4.

3.2 Distribution changes due to deformation

Triaxial test under drained condition for air and undrained condition for water was conducted until an axial strain of 21% and x-ray scan was performed, through which changes in distribution of void ratio and degree of saturation due to the deformation are discussed.

Void ratio and degree of saturation of triaxial specimen in a vertical direction as well as CT image at an axial strain of 21% are shown in Fig. 11. In the CT images, black portion indicates the lower density region than the other white portion, i.e., dilation due to shear deformation occurs in the black portion. As can be seen in the image of Fig. 11, triaxial specimen shows bulging mode and shear bands are clearly observed.

Comparing Fig. 10 with Fig. 11, it is clear that void ratio becomes significantly larger than initial condition at the middle of the specimen where the larger deformation was observed. On the other hand, distribution of void ratio remains as dense as initial one at the upper part of the specimen where less shear deformation was observed.

Degree of saturation at the middle of specimen becomes lower than the other part and initial condition, whereas that at the upper part of specimen becomes slightly larger.

These results are consistent with the fact that the larger voids have lower water retention capability, while smaller voids have higher water retention capability.

4 CONCLUSIONS

In the present study, we have proposed the trinarisation technique taking into account partial volume effect (pixel) by the MLE approach, and validated the technique. The void ratio and degree of saturation calculated for the trinarized images provides good agreement with the measured data. Micro x-ray tomography with the proposed trinarisation technique gives the initial distribution of void ratio and degree of saturation of the three specimens with different initial degree of saturation in a vertical direction. In addition, the distribution of void ratio and degree of saturation after triaxial test are also calculated so that the changes in the local values have been discussed. The main conclusions obtained in this study are as follows:

1) Degree of saturation tends to be lower in larger voids when the initial degree of saturation of partially saturated sand specimen is relatively high. On the other hand, degree of saturation tends to be lower in smaller voids as well as larger ones, in the case that the initial degree of saturation is relatively low. These suggest that the pore water drains faster from larger voids as long as the specimen remains relatively high degree of saturation, while the pore water drains not only large but also small voids when the specimen reaches relatively lower degree of saturation.

2) Degree of saturation at the lower part of the specimen tends to be relatively higher than that at the upper part. This is because the pore water tends to easily drain from the upper part due to higher negative pressure head.

3) Both the void ratio and degree of saturation distribute with a certain standard deviation in the triaxial specimens by using the proposed method of this study.

4) Void ratio significantly increases and degree of saturation decreases at the middle of the specimen where the larger shear deformation was observed. On the other hand, the variations in those values are relatively small where the less shear deformation was observed. These denote that the distribution of void ratio and degree of saturation locally change because of the shear deformation.

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