Black hole wind speeds and the $M-\sigma$ relation

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ABSTRACT

We derive an $M_{\rm BH}-\sigma$ relation between supermassive black hole mass and stellar velocity dispersion in galaxy bulges, that results from self-regulated, energy-conserving feedback. The relation is of the form $M_{\rm BH}v_w \propto \sigma^3$, where $v_w$ is the velocity of the wind driven by the black hole. We take a sample of quiescent early-type galaxies and bulges with measured black hole masses and velocity dispersions and use our model to infer the wind speeds they would have had during an active phase. This approach, in effect, translates the scatter in the observed $M_{\rm BH}-\sigma$ relation into a distribution of $v_w$. There are some remarkable similarities between the distribution of black hole wind speeds that we obtain and the distributions of outflow speeds observed in local AGN, including a comparable median of $v_w = 0.035c$.

Key words: galaxies: nuclei — galaxies: formation — galaxies: evolution

1 INTRODUCTION

Self-regulated feedback from accreting supermassive black holes (SMBHs) in gaseous protogalaxies is thought to play a key role in establishing the $M_{\rm BH}-\sigma$ relation observed in local quiescent galaxies, between SMBH mass and bulge-star velocity dispersion: $M_{\rm BH} \propto \sigma^x$ with $x = 4-5$ (Ferrarese & Merritt 2000; Gebhardt et al 2000; Ferrarese & Ford 2005; Gültekin et al 2009; McConnell & Ma 2013). The accreting SMBH drives a wind, which sweeps the surrounding ambient medium into a shell. There is then a critical SMBH mass above which the wind thrust pushing the shell outwards (proportional to $M_{\rm BH}$) can overcome the inward gravitational pull of the dark matter (related to $\sigma$) and the SMBH itself. At this critical mass, the shell may be blown out of the galaxy, cutting off fuel to the SMBH and locking in an $M_{\rm BH}-\sigma$ relation (Silk & Rees 1998; Fabian 1999; King 2003). Supporting this scenario are observations of strong outflows in local active galactic nuclei (AGN), both on large scales (e.g., Sturm et al 2011) and closer to the SMBHs (e.g., Pounds et al 2003; Tombesi et al 2011; Gofford et al 2013). The latter in particular have speeds and mechanical luminosities similar to those needed for SMBH winds to have cleared the gas from now-normal spheroids at high redshift, when the systems were active.

The dynamics of a swept-up shell of gas depend on whether or not the region of shocked wind material immediately behind the shell is able to cool. If the shocked gas cools efficiently then the region is geometrically thin and the swept-up shell is pushed outwards by the ram pressure of the wind. This momentum-driven regime is expected to be the case initially in the case of SMBH feedback (King 2003), and thus many authors have considered the $M_{\rm BH}-\sigma$ relation that results if the feedback is entirely momentum-driven (e.g., Fabian 1999; King 2003; 2005; McQuillin & McLaughlin 2012). In McQuillin & McLaughlin (2012) we considered shells moving outwards in non-isothermal, spherical dark matter haloes that have peaked circular speed curves. We showed that the critical SMBH mass above which any shell can escape tends to the limiting value (for haloes much more massive than the SMBH, independent of any further details of the dark matter density profile)

$$M_{\rm crit} = \frac{f_0 \kappa}{\pi G^2} \frac{V_{c,pk}^4}{4} \simeq 1.14 \times 10^9 M_\odot \left( \frac{f_0}{0.2} \right) \left( \frac{V_{c,pk}}{200 \, \text{km} \, \text{s}^{-1}} \right)^4.$$  

(1)

Here, $V_{c,pk}$ is the peak value of the circular speed in the dark matter halo; $\kappa$ is the electron scattering opacity; and $f_0$ is a spatially constant gas-to-dark matter mass fraction. The peak circular speed defines a natural “characteristic” velocity dispersion for a non-isothermal galaxy: $\sigma_0 \equiv V_{c,pk}/\sqrt{2}$. Equation (1) then implies an $M_{\rm BH}-\sigma$ relation, which has a slope and an intercept that are near the observed values (see Figure 2 below).

If the shocked gas cannot cool then the region behind the shell is geometrically thick and hot. The outflow is energy-driven and the shell is pushed outwards by the thermal pressure of the shocked material. In the context of SMBH feedback, an initially momentum-driven shell is expected to transition to energy-driven, probably quite early on when the shell is still at relatively small galactocen-
Thus, in this paper we investigate the implications of energy-driven feedback for the $M_{\text{BH}}-\sigma$ relation.

In [2] we derive the large-radius coasting speed, $v_\infty$, of an energy-conserving shell in a dark-matter halo modelled as a singular isothermal sphere with velocity dispersion $\sigma_0$. We find that for the shell to coast at the escape speed of a truncated isothermal halo (i.e., $v_\infty = 2\sigma_0$) requires

$$
\left(\frac{M_{\text{BH}}}{10^8 M_\odot}\right)^\frac{1}{2} \left(\frac{v_\infty}{c}\right) \approx 6.68 \times 10^{-2} \left(\frac{\sigma_0}{0.2}\right) \left(\frac{200 \text{ km s}^{-1}}{2}\right)^2,
$$

where $M_{\text{BH}}$ is the (fixed) SMBH mass driving a wind of speed $v_\infty$. This $M_{\text{BH}}-\sigma$ relation differs from that resulting from momentum-driven outflows (equation [1]), both in the power on $\sigma_0$ and in the explicit dependence on $v_\infty$.

In [3] we apply our escape condition for energy-conserving feedback to the $M_{\text{BH}}-\sigma$ relation defined observationally by a standard sample of low-redshift, quiescent early-type galaxies and bulges ([Gültekin et al. 2008]. We use equation (2) to infer the black hole wind speeds that would have had to occur during the main epoch of galaxy and SMBH formation, if this simple model is to account for the individual $M_{\text{BH}}$ and $\sigma$ values for each galaxy or bulge in the Gültekin et al. sample. This gives a distribution of $v_\infty/c$ for these galaxies in the past. In [3] we compare this distribution directly to the distributions of $v_\infty/c$ observed for fast outflows in different samples of local AGN ([Tombesi et al. 2011; Gofford et al. 2013]). Our main result is a remarkable similarity between these distributions. In particular, the median SMBH wind speed we infer for the normal galaxies of Gültekin et al. is $v_\infty = 0.035c$, while the median of the outflow speeds in low-redshift AGN is $v_\infty = 0.1c$ according to Tombesi et al. or $v_\infty = 0.056c$ according to Gofford et al.

2 ENERGY-DRIVEN OUTFLOWS

In the self-regulated feedback scenario the black hole wind sweeps up a shell of ambient gas as it moves outwards. This gives rise to two shock fronts, one propagating forwards into the ambient medium and one propagating back into the wind material. The resulting shock pattern has a four-zone structure: 1) the freely flowing wind; 2) the shocked wind region lying between the wind shock and the contact surface that separates material originally in the wind from material originating in the ambient medium; 3) the shocked ambient medium, lying between the contact surface and the ambient shock, also containing the original swept-up shell that gave rise to the shock fronts; and 4) the undisturbed ambient medium.

In detail, the dynamics of the swept-up shell depend on three timescales: the flow time of the shell, $t_{\text{flow}} = r_s/v_s$, where $r_s$ is the radius of the shell and $v_s$ is the shell velocity; the dynamical time of the wind, $t_{\text{dyn}} = r_{sw}/v_{sw}$, where $r_{sw}$ is the radius of the wind shock and $v_{sw}$ is the wind velocity; and the cooling time of the shocked wind, $t_{\text{cool}}$ ([Koo & McKee 1992; Faucher-Giguère & Quataert 2012]).

If $t_{\text{cool}} \ll t_{\text{dyn}}$, then the shocked wind region cools before more energy is injected into the region from the freely flowing wind. The material in the region is then confined to a thin shell (so $r_{sw} \sim r_s$) and the shell is effectively driven outwards by a transfer of momentum from the wind impacting on its inner side, corresponding to a momentum-driven outflow.

If, instead, $t_{\text{cool}} \gg t_{\text{flow}}$, then the most recently shocked material cannot cool in the time it takes to travel across the shocked wind region. The region is thick and hot and drives the shell outwards with its thermal pressure, corresponding to an energy-driven outflow.

In the intermediate case, $t_{\text{dyn}} \lesssim t_{\text{cool}} \lesssim t_{\text{flow}}$, the shell is in a partially radiative phase where most of the material cools and condenses into a thin shell but the most recently shocked material has not cooled and occupies most of the volume of the region. In this regime the outflow conserves neither energy nor momentum.

For a wind from an SMBH, with cooling primarily by inverse Compton scattering ([King 2003]), the cooling rate is (e.g., [Longair 2011])

$$
\frac{dE}{dt} = \frac{4}{3} \kappa m_p c u_{\text{rad}} \left(\frac{v_e}{c}\right)^2 \left(\frac{E}{m_e c^2}\right)^2,
$$

where $v_e$ is the velocity of a post-shock electron; $E$ is the post-shock electron energy; $u_{\text{rad}}$ is the radiation energy density; and $\kappa$ is the electron-scattering opacity. We take $u_{\text{rad}} = L_{\text{Edd}}/(4\pi r_s^2 c)$, where $L_{\text{Edd}} = 4\pi G M_{\text{BH}} c/\kappa$ is the Eddington luminosity of a black hole of mass $M_{\text{BH}}$, and $E \approx (9/16)m_p v_e^2$ for the electron energy. Then, the cooling time, $t_{\text{cool}} \equiv E/(dE/dt)$, is less than the dynamical time of the wind, $t_{\text{dyn}} \equiv r_{sw}/v_{sw}$, at radii

$$
r_s \gtrsim \frac{3}{4} \frac{GM_{\text{BH}}}{c v_s} \left(\frac{m_p}{m_e}\right)^2 \left(\frac{v_{sw}}{c}\right)^2 \left(\frac{v_s}{c}\right)^2 \approx 0.26 \text{ pc} \left(\frac{M_{\text{BH}}}{10^8 M_\odot}\right)^2 \left(\frac{v_{sw}}{0.03 c}\right)^2 \left(\frac{v_s}{0.85 c}\right)^2.
$$

When the wind shock is inside this radius, the shocked wind region is thin, so $r_{sw} \sim r_s$ and the shell is momentum-driven.

The cooling time exceeds the flow time of the shell, $t_{\text{flow}} \equiv r_s/v_s$, at radii

$$
r_s \gtrsim \frac{3}{4} \frac{GM_{\text{BH}}}{c v_s} \left(\frac{m_p}{m_e}\right)^2 \left(\frac{v_{sw}}{c}\right)^2 \left(\frac{v_s}{c}\right)^2 \approx 11 \text{ pc} \left(\frac{v_{sw}}{200 \text{ km s}^{-1}}\right)^{-1} \left(\frac{M_{\text{BH}}}{10^8 M_\odot}\right) \left(\frac{0.03 c}{v_{sw}}\right)^2 \left(\frac{0.85 c}{v_s}\right)^2,
$$

for typical shell velocities $v_{sw} \sim \sigma_0 \sim 200 \text{ km s}^{-1}$. This is in rough agreement with [Zubovas & King 2012] see their equation [6]), although they replace $v_s$ with an estimate for the terminal velocity of a momentum-driven shell and normalize to a higher fiducial $v_s$ than we do (see Sections 3 and 4 below for more about typical wind speeds). In any case, the radius in equation (5) is comparable to the sphere of influence of a $10^8 M_\odot$ black hole in a stellar distribution with velocity dispersion 200 km s$^{-1}$. As such, SMBH outflows can be energy-conserving over much of their evolution, and accordingly we focus on purely energy-driven feedback in what follows.

McLaughlin et al. (2006) noted that a self-regulated feedback scenario can also be applied to nuclear star clusters in galaxy centres to explain the $M_{\text{sc}}-\sigma$ relation observed by Ferrarese et al. (2000). In that case, cooling by atomic processes gives a shorter cooling timescale with a strong dependence on the wind speed (equation [9]) of McLaughlin et al.
and a slower wind speed results in a longer dynamical time. Thus, outflows from nuclear clusters can cool efficiently and be momentum-driven to much larger radii than in the black hole case.

Whether momentum- or energy-driven, the equation of motion for a shell of swept-up gas moving out into the dark-matter halo of a protogalaxy against the inwards gravitational pull of both the SMBH and the dark matter behind the shell can be written as (see also King 2005)

\[
\frac{d}{dt} [M_k(r) v(r)] + \frac{G M_k(r)}{r^2} [M_{BH} + M_{DM}(r)] = 4 \pi r^2 P .
\]

(6)

Here \( r \) is the instantaneous radius of the shell; \( M_{DM}(r) \) is the dark matter mass inside radius \( r \); \( M_k(r) \) is the mass of ambient gas initially inside radius \( r \) (i.e., the mass that has been swept up into the shell when it has radius \( r \)); \( v(r) = dr/dt \) is the velocity of the shell; and \( P \) is the outward pressure on the shell.

We adopt the simple description by King & Pounds (2003) of a wind driven by radiation (continuum scattering) from an accreting SMBH, such that the wind thrust is

\[
\dot{M}_{out} v_w = \tau \frac{L_{Edd}}{c} .
\]

(7)

Here \( \dot{M}_{out} \) is the mass outflow rate in the wind and \( v_w \) is the wind velocity when it escapes the black hole; these are distinct from the mass growth rate \( dM_k/dt \) and the expansion speed \( v \) of the shell of swept-up ambient gas that the wind drives. The parameter \( \tau \) is the electron-scattering optical depth in the wind, measured down to its escape radius from the black hole (thus, \( \tau \approx 1 \) in the single-scattering limit), multiplied by a geometrical factor (which is also \( \approx 1 \)) allowing for some non-sphericity in the wind; see King & Pounds (2003) for more detail. In what follows, we retain \( \tau \approx 1 \) in our calculations, although ultimately we assume that \( \tau \approx 1 \).

The pressure on the right-hand side of equation (6) is just the wind ram pressure, \( 4 \pi r^2 P = \dot{M}_{out} v_w \approx L_{Edd}/c \), for a momentum-driven shell. This is the case we solved in McQuillin & McLaughlin (2012) for isothermal and non-isothermal dark-matter halo models. For an energy-conserving shell, the driving pressure is instead the thermal pressure of the shocked-wind region behind the shell. In this case, \( P \) in equation (6) satisfies the energy equation,

\[
\frac{d}{dt} \left[ \frac{4}{3} \pi r^3 \frac{P}{\gamma - 1} \right] = \dot{E} - P \frac{d}{dt} \left[ \frac{4}{3} \pi r^3 \right] - \frac{G M_k(r) v(r)}{r^2} [M_{BH} + M_{DM}(r)] .
\]

(8)

In this equation, \( \gamma \) on the left-hand side is the ratio of specific heats. The last three terms on the right-hand side give the rates of work done by the expanding shell (both \( P \frac{dV}{dt} \) work and the work against the gravity of the SMBH and the dark matter behind the shell; cf. King 2005). The first term on the right-hand side is the rate of energy input to the shocked wind region, which is given by the kinetic energy flux of the wind:

\[
\dot{E} = \frac{1}{2} \dot{M}_{out} v_w^2 = \tau \frac{v_w}{c} \frac{L_{Edd}}{2} .
\]

(9)

Note that this differs slightly from, e.g., King (2005, 2010) and King et al. (2011), where it is either stated or implied that \( \dot{E} = \eta L_{Edd}/2 \) with \( \eta \) the radiative efficiency of accretion onto the black hole. These other papers make the additional assumption that \( \dot{M}_{out} = M_{Edd} = L_{Edd}/(\eta c^2) \). In combination with equation (7) above, this requires \( v_w/c = \eta \tau \); and putting this plus \( \tau \approx 1 \) into equation (7) is what gives \( \dot{E} = \eta L_{Edd}/2 \). However, in this paper we do not assume that \( \dot{M}_{out} = M_{Edd} \), nor that \( v_w/c = \eta \tau \) necessarily; thus, \( v_w/c \) remains as an explicit parameter in our analysis.

Now we specialise to the case of a shell expanding into a dark-matter halo modelled as a singular isothermal sphere (SIS), with the ambient protogalactic gas tracing the dark matter exactly. The density of an SIS is given by \( \rho_{DM}(r) = \sigma_0^2/(2\pi G r^2) \), so that the mass inside radius \( r \) is

\[
\dot{M}_{DM}(r) = 2\pi r^2 \sigma_0^2 ,
\]

(10)

and \( \dot{M}_{bh}(r) = f_0 M_{DM}(r) \) with a fiducial (cosmic) \( f_0 \approx 0.2 \). As in McQuillin & McLaughlin (2012), we then define characteristic mass and radius units in terms of the characteristic velocity dispersion of the halo, \( \sigma_0 \):

\[
M_* \equiv \frac{f_0 \pi \sigma_0^2}{\pi G} \approx 4.56 \times 10^8 M_\odot \left( \frac{f_0}{0.2} \right) \left( \frac{\sigma_0}{200 \, \text{km s}^{-1}} \right)^4
\]

and \( r_* \equiv \frac{G M_*}{\sigma_0^2} \approx 49.25 \, \text{pc} \left( \frac{f_0}{0.2} \right) \left( \frac{\sigma_0}{200 \, \text{km s}^{-1}} \right)^2 \).

With the identification \( \sigma_0 \equiv V_{esc}/\sqrt{2} \), the mass unit \( M_* \) is just the critical SMBH mass from equation (1) for the breakout of momentum-driven shells from non-isothermal dark matter haloes with peaked circular speed curves. In singular isothermal spheres, the critical mass required for momentum-driven shells to coast at large radii with the escape speed \( 2\sigma_0 \approx 3 M_\odot \) McQuillin & McLaughlin (2012; see also Silk & Nusser 2010).

We eliminate \( P \) from equation (6) using equation (9), then combine with the dark-matter and gas mass profiles of an SIS from equation (10) and the energy input from equation (9), together with \( L_{Edd} = 4\pi G M_{out} c^2/\kappa \). Also, we write \( d/dt = v \, dr/dv \) in order to solve for the velocity fields of shells, \( v \), rather than for \( r(t) \) explicitly. Then, defining dimensionless variables

\[
\tilde{M} \equiv M/M_* , \quad \tilde{r} \equiv r/r_* \quad \text{and} \quad \tilde{v} \equiv v/\sigma_0 ,
\]

the equation of motion for energy-driven shells in an SIS is

\[
\frac{d^2}{d\tilde{r}^2} \left[ \frac{3}{2} \tilde{v}^2 \left( \frac{\tilde{r}}{\gamma - 1} \right) \right] + 3(\gamma - 1) \frac{d}{d\tilde{r}} \left[ \tilde{v} \tilde{v}^2 \left( \frac{\tilde{r}}{\gamma - 1} \right) \right]
\]

\[
+ 12(\gamma - 1) \frac{\tilde{v}}{\gamma - 1} - 6(\gamma - 1) \frac{\tilde{M}_{BH} \tilde{v}}{\tilde{r}}
\]

\[
= -4(\gamma - 5) .
\]

(11)

In the limit of large radius, the term \( \tilde{M}_{BH}/\tilde{r} \to 0 \) in equation (11), and the remaining terms imply that the velocity of the shell tends to a constant:

\[
\tilde{v} \to \tilde{v}_\infty , \quad (\tilde{r} \gg 1) \quad (12)
\]

where \( \tilde{v}_\infty \) satisfies

\[
(3\gamma - 2) \tilde{v}_\infty^3 + 2(\gamma - 5) \tilde{v}_\infty = 3(\gamma - 1) \tilde{M}_{BH} \tilde{v}_\infty .
\]

(13)

Thus, any energy-conserving shell at sufficiently large radius
tends to a coasting speed that depends on the black hole mass, the velocity dispersion of the halo and the velocity of the black hole wind.

A natural criterion for the escape of the feedback is that it reach a coasting speed equal to the escape speed from a truncated isothermal sphere, \( v_\infty = 2\sigma_0 \). Thus, we set \( v_\infty = 2 \) in equation (13) and obtain a critical value for the product of black hole mass and wind speed:

\[
\left[ M_{\text{BH}} v_w \right]_{\text{crit}} = \frac{1}{\tau} \frac{4(4\gamma - 3)}{(\gamma - 1)}
\]

or, with all units restored,

\[
\left[ M_{\text{BH}} v_w \right]_{\text{crit}} = \frac{1}{\tau} \frac{4(4\gamma - 3)}{(\gamma - 1)} \frac{\sigma_0}{\pi G^2} \propto \sigma_0^5.
\]

Setting \( \gamma = 5/3 \) then gives

\[
\left( \frac{M_{\text{BH}}}{10^8 M_\odot} \right) \left( \frac{v_w}{c} \right) =

6.68 \times 10^{-2} \frac{1}{\tau} \left( \frac{f_0}{0.2} \right) \left( \frac{\sigma_0}{200 \text{ km s}^{-1}} \right)^5.
\]

This is what we will compare to the observed \( M_{\text{BH}} - \sigma \) relation in Figure 1 below.

Equation (16) shows explicitly how the escape of energy-conserving shells from an isothermal galaxy requires \( M_{\text{BH}} v_w \propto \sigma_0^5 \) in general. If \( v_w \) were effectively the same in all galaxies (or at least uncorrelated with SMBH mass or halo velocity dispersion), then the implication is an observable relation \( M_{\text{BH}} \propto \sigma^5 \), as has been argued many times (e.g., Silk & Rees 1998, King 2003). In more detail, however, if \( v_w \) did in fact depend on black hole mass as, say, \( v_w \propto M_{\text{BH}}^{\gamma} \), then equation (16) would actually imply

\[
M_{\text{BH}} \propto \sigma_0^{5/(1+\gamma)}.
\]

That is, if \( v_w \) and \( M_{\text{BH}} \) were correlated by even a weak

power, the logarithmic slope of the \( M_{\text{BH}} - \sigma \) relation from energy-driven outflows could differ measurably from 5.

In the limit of small radius, equation (11) admits solutions of the form

\[
v^2 \tilde{r}^2 \longrightarrow C - 4M_{\text{BH}} \tilde{r} - \frac{2(6\gamma - 5)}{(3\gamma - 2)} \tilde{r}^2 + O(\tilde{r}^3),
\]

where the constant \( C \) represents the square of the shell momentum, \( [M_g(r)v(r)]^2 \propto \tilde{v}^2 \tilde{r}^2 \), at \( \tilde{r} = 0 \). In order for equation (14) to apply, a shell moving out from \( \tilde{r} = 0 \) must have an initial momentum large enough to keep \( \tilde{v}^2 \tilde{r}^2 \gg 0 \) and avoid stalling before it reaches the large radii where the coasting speed in equation (13) applies.

Figure 1 shows the velocity fields, \( \tilde{v}(\tilde{r}) \), that solve equation (11) with \( \gamma = 5/3, \tau = 1 \) and dimensionless \( M_{\text{BH}} \tilde{v}_w = 43/16 \) (2.7), 6.5 and 22. The different curves in each panel represent different initial shell momenta, i.e., different values of \( C \) in equation (15). We have specified a fixed wind speed in all cases: \( \tilde{v}_w = 45 \), which corresponds to \( v_w = 0.03c \) for \( \sigma_0 = 200 \text{ km s}^{-1} \). The dimensionless black hole masses are then (again, assuming \( \tau = 1 \)) \( M_{\text{BH}} \approx 0.06, 0.14 \) and 0.49. These are all below the critical SMBH masses for the escape of momentum-conserving shells from either non-isothermal haloes (\( M_{\text{crit}} = 1 \)) or an SIS (\( M_{\text{crit}} = 3 \)). Given any of the black hole masses represented in Figure 1 all purely momentum-driven shells would stall at relatively small radii and go into collapse until the SMBH grew substantially (see Figure 1 of McQuillin & McLaughlin 2012).

With \( \gamma = 5/3 \) and \( \tau = 1 \), equation (13) gives the final coasting speeds of the energy-driven shells illustrated in Figure 1 as \( v_\infty/\sigma_0 = 0.5, 1 \) and 2 (independent of initial conditions) in the three panels from left to right. These are confirmed by our numerical solutions for the full \( \tilde{v}(\tilde{r}) \).

![Figure 1](image-url)
In particular, all of the energy-driven solutions in the case \( M_{\text{BH}} = 22 \) eventually attain the speed for escape from a truncated SIS, \( v_{\infty} = 2\sigma_0 = v_{\text{esc}} \). Energy-conserving feedback can blow out of an isothermal halo if driven by a wind at speed \( v_w \sim 0.3\sigma_0 \) (of the order of the nuclear outflows observed in local AGN; see below) from an SMBH significantly less massive than that required to expel momentum-conserving shells from isothermal or non-isothermal haloes.

3 THE OBSERVED \( M_{\text{BH}} - \sigma \) RELATION

The left-hand panel of Figure 2 shows \( M_{\text{BH}} \) versus bulge-star velocity dispersion \( \sigma \) for 51 normal (quiescent) early-type galaxies and bulges in Table 1 of Gültekin et al. (2009). The dashed line on the plot traces the relation

\[
\frac{M_{\text{BH}}}{10^8 M_\odot} = 4.56 \left( \frac{f_0}{0.2} \right) \left( \frac{\sigma}{200 \text{ km s}^{-1}} \right)^4.
\]

This represents the SMBH mass \( M_{\text{crit}} \) of equation (1) above, which is sufficient for the escape of any purely momentum-driven shell from any non-isothermal dark-matter halo, if the peak circular speed in the halo of an observed galaxy can be estimated as \( v_{\text{pk}} = \sqrt{2\sigma} \) (McQuillin & McLaughlin 2012). In a singular isothermal sphere, for a momentum-driven shell to reach the escape speed of \( 2\sigma \) at large radii requires \( M_{\text{BH}} \geq 3M_{\text{crit}} \)—that is, SMBH masses a further 0.5 dex above the dashed line in Figure 2, which already represents an upper limit to the data. Relaxing the isothermal assumption alleviates some of this difficulty, and additional momentum input from bulge-star formation triggered by the outflow could further reduce the requirement on \( M_{\text{BH}} \) from that in equation (19) (see, e.g., Silk & Nusser 2010; and further discussion in McQuillin & McLaughlin 2012).

By contrast, the solid line running through the data in Figure 2 is the SMBH mass required for energy-driven shells to escape singular isothermal spheres, from equation (16) with a fixed SMBH wind speed of \( v_w/c = 0.035 \) (and assuming a wind optical depth \( \tau = 1 \) and a gas-to-dark matter mass fraction \( f_0 = 0.2 \)). With \( v_w/c \) set to a constant to draw this line, it has a slope \( M_{\text{BH}} \propto \sigma^2 \), the usual expectation for energy-conserving feedback. The numerical value of \( v_w/c \) then sets the intercept, and the value that we have applied is in fact the median of a distribution of wind speeds that we have estimated individually for every galaxy in Gültekin et al. (2009).

These are all quiet, non-active galaxies and bulges. But if their black hole masses were frozen in as part of the feedback process clearing ambient gas from the proto-spheroids, and if this feedback was energy-driven, then equation (16) can be used to infer the SMBH wind speeds in the past, when the galaxies were young and active. For each point in the left-hand panel of Figure 2 we have taken the measured values of \( M_{\text{BH}} \) and \( \sigma \) (and set \( \tau = 1, f_0 = 0.2 \)) to solve equation (16) for \( v_w/c \). The results are shown as the normalised histogram in the right-hand panel of Figure 2. The arrow there points to the median speed, \( v_w/c = 0.035 \). The minimum is \( v_w/c = 0.005 \), and the maximum (with Circinus...
Figure 3. Inferred $v_w/c$ vs. observed $M_{BH}$ for the normal early-type galaxies and bulges in Gültekin et al. (2009), with $v_w/c$ obtained from equation (16) for each $(M_{BH}, \sigma)$ measurement. The solid line shows the correlation $v_w \propto M_{BH}^{0.2}$, which could explain the slope of the best-fit power-law $M_{BH}-\sigma$ relation according to Gültekin et al. (2009). The dashed line shows the weaker correlation $v_w \propto M_{BH}^{0.12}$ suggested by the steeper power-law fit to $M_{BH}$ versus $\sigma$ by Ferrarese & Ford (2005).

Uncertainties in the $v_w/c$ values follow from the uncertainties in $M_{BH}$ and $\sigma$ tabulated by Gültekin et al. and the median errorbar, $\Delta(v_w/c) \approx \pm 0.02$, is also shown in Figure 2.

It is often reported that power-law fits to $M_{BH}-\sigma$ data return exponents that are closer to 4 than to 5; and, as we noted in 13 even a weak correlation between black hole mass and wind speed could result in an $M_{BH}-\sigma$ relation from energy-conserving feedback having a slope $<5$. In any case, the main and most robust result here is our value for the median black hole wind speed, $v_w/c \approx 0.035$. This not only gives a very credible fit of a simple energy-driven feedback model to the $M_{BH}-\sigma$ relation; it also similar to the typical speeds of nuclear outflows in samples of nearby, currently active galaxies having no overlap with the Gültekin et al. sample of quiescent early types and bulges.

4 OBSERVED AGN OUTFLOW VELOCITIES

Highly ionised, "ultra-fast" outflows have been observed from the centres of many local active galactic nuclei since the prototypes of the phenomenon were found by Pounds et al. (2003) and Reeves et al. (2003). These outflows are very massive and have high kinetic powers of the order needed, in simple scenarios of the type discussed in this paper, for the clearing of gaseous protogalaxies by SMBH-powered winds. As pointed out originally by King (2003), they appear to be an observable, present-day analogue of the processes that may have worked to establish the $M_{BH}-\sigma$ relation among now-inactive galaxies.

Two recent studies, by Tombesi et al. (2011) and Gofford et al. (2013), give the velocities for samples of 20 and 21 AGN outflows respectively, with 6 sources in common. We can now compare the distributions of these observed outflow speeds to the distribution that we inferred in 13 for SMBH wind speeds in the past, in the normal spheroids that define the $M_{BH}-\sigma$ relation.

Figure 4 shows this comparison, with the AGN outflow velocity distribution from Tombesi et al. (2011) in the left-hand panel, and with that from Gofford et al. (2013) in the right-hand panel. In each panel, the solid-line histogram is that from Figure 2 above, obtained from equation (16) assuming that the black holes in the Gültekin et al. (2009) galaxies were just able to drive energy-conserving supershells to the escape speeds of their dark-matter haloes. The dashed histograms represent the AGN data.

The most striking aspect of Figure 4 is the basic agreement, to within factors of a few at worst, in the typical $v_w/c$ of these different samples of galaxies: our median $v_w/c = 0.035$ for the normal early-type galaxies, versus a median $v_w/c = 0.1$ for the AGN outflows of Tombesi et al. (2011) and a median $v_w/c = 0.056$ for the AGN of Gofford et al. (2013). The overall ranges (i.e., the maxima) of the wind
speeds are also very similar. These facts are remarkable as much for the simplicity of the model we have used to estimate \( v_w/c \) in the normal galaxies, as for the complete disconnect between the Gültekin et al. galaxy sample and the Tombesi et al. or Gofford et al. AGN samples.

To be sure, the distributions as they stand in Figure 4 are not identical. Kolmogorov-Smirnov (KS) tests return a formal probability of only \( P_{KS} \approx 0.3\% \) that our distribution of \( v_w/c \) for the Gültekin et al. galaxies is drawn from the same parent distribution as the Tombesi et al. sample, and \( P_{KS} \approx 25\% \) for equality between our \( v_w/c \) values and the Gofford et al. sample. The main reason for this appears to be the relatively small numbers, in the present sample, of normal galaxies with inferred \( v_w/c \gtrsim 0.1 \)—or, conversely, a dearth of AGN (in the Tombesi et al. sample especially) with slower \( v_w/c \lesssim 0.1 \).

Whatever shortcomings our very simple analysis might have, it requires that normal galaxies with “underweight” black holes falling significantly below the mean \( M_{BH}–\sigma \) relation have higher-than-average \( v_w/c \). If several such galaxies were to be added to the Gültekin et al. sample, they could fill out the high-velocity tail of our model \( v_w/c \) distribution. As for the AGN, it is not clear how selection effects, observational biases or limitations due to instrumentation may have either affected the measurement of relatively slow outflows, or perhaps even prevented their inclusion in studies designed to focus on “ultra-fast” systems. It is also worth noting that the probability that the \( v_w/c \) measurements of Tombesi et al. and Gofford et al. are drawn from the same parent distribution is a formally inconclusive \( P_{KS} \approx 28\% \)—the same as in the comparison between the Gültekin et al. distribution of \( v_w \) for their AGN and ours for the normal galaxies. As such, it is not clear that any of the data suffice yet to allow a robust comparison at a very detailed level between distributions of observed SMBH wind speeds and those inferred from any model. This makes it even more noteworthy that the median of the \( v_w \) distribution we have obtained in this paper lies within a factor \( \approx 1.5–3 \) of the median \( v_w \) of two different observed distributions.

Ultimately, our results are encouraging for the general idea that there is a parallel between the strong nuclear outflows found in local AGN and the kind of black hole feedback that is routinely assumed to have been a key part of galaxy formation and the establishment of the \( M_{BH}–\sigma \) relation. They also lend support to the relevance of energy-driven feedback specifically, and to the simple sort of modelling that we have applied to assess its role quantitatively.

5 SUMMARY

We have looked at the behaviour of energy-conserving super-shells of swept-up ambient gas driven into isothermal protogalaxies by black hole winds. At large radii, such shells tend to a constant coasting speed, \( v_\infty \), that depends on the black hole mass, \( M_{BH} \), the black hole wind speed, \( v_w \), and the velocity dispersion of the halo, \( \sigma_0 \). For a shell to coast at the escape speed of a truncated isothermal halo (i.e., \( v_\infty = 2\sigma_0 \)) requires \( M_{BH} v_w \propto \sigma_0^5 \) as in equations (15) and (16).

We applied this escape condition for energy-conserving feedback to the observed \( M_{BH}–\sigma \) relation for the sample of quiescent early-type galaxies and bulges of Gültekin et al. (2009). We used equation (16) to infer the black hole wind speed that each galaxy would have had during an active phase if our simple model is to account for the measured value of \( M_{BH} \) in the galaxy, given its observed \( \sigma \). In this approach, scatter in the observed \( M_{BH}–\sigma \) relation directly reflects a distribution of wind speeds from the SMBHs in the protogalaxies. We compared the distribution of wind velocities we obtained for the normal galaxies in Gültekin et al. to the observed distributions of outflow velocities in two different samples of local AGN (Tombesi et al. 2011; Gofford et al. 2013). The distributions are strikingly similar. Most notably, the median of our inferred wind velocities, \( v_w = 0.035c \), is within a factor \( \approx 1.5–3 \) of the me-
dian of the observed distribution of wind speeds of both
Tombesi et al. ($v_w = 0.1c$) and Gofford et al. ($v_w = 0.056c$).

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