Testing isotropy of Cosmos with WMAP and PLANCK data

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Abstract
In recent years, there have been a large number of studies which support violation of statistical isotropy. Meanwhile, there are some studies which also found inconsistency. We use the power tensor method defined earlier in the literature to study the new CMBR data. The orientation of these three orthogonal vectors, as well as the power associated with each vector, contains information about possible violation of statistical isotropy. This information is encoded in two entropy measures, the power-entropy and alignment-entropy. We apply this method to WMAP 9-year and PLANCK data. Here, we also revisit the statistics to test the high-\(l\) anomaly reported in our earlier paper and find that the high-degree of isotropy seen in earlier WMAP 5-year data is absent in the revised WMAP-9 year data.

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1 Introduction

The existence of a preferred axis from various data, particularly the polarization of radio waves, optical polarization from quasars, cluster peculiar velocity and also low multipoles in CMB \(l = 1, 2, 3\) which are pointing towards Virgo is supported by many observations \cite{1,3,5,10}. The low-\(l\) anomalies are tested using various statistics \cite{9,11}. Here, we used the power tensor technique \cite{4,6,7} to study the alignment between CMB quadrupole and octopole in PLANCK data in comparison to WMAP 9-year data. Here, we revisit the statistics defined earlier in \cite{6,7} to study the low-\(l\) alignment as well as the high-\(l\) anomaly seen in \cite{7} for WMAP-9 year foreground cleaned data.

The measured temperature anisotropy of CMB can be expanded in spherical harmonics as
\[
\Delta T(\hat{n}) = \sum_{lm} a_{lm} Y_{lm}(\hat{n}).
\]  

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Statistical isotropy implies,
\[ (a_{lm}a_{l'm'}) = C_l \delta_{ll'} \delta_{m,m'} . \]  
(2)

The second rank power tensor that can be formed from the products of the spherical harmonics coefficients \( a_{lm} \) is defined as
\[ A_{ij}(l) = \frac{1}{l(l+1)} \sum_{m,m'} a_{lm}^* \langle J_i J_j \rangle_{mm'} a_{l'm'} , \]  
(3)

where \( J_i (i = 1, 2, 3) \) are the angular momentum operators in representation \( l \). The ensemble average of the power tensor is
\[ \langle A_{ij}(l) \rangle = \frac{C_l}{3} \delta_{ij} \]  
(4)

For each multipole \( l \geq 2 \), there exist three rotationally invariant eigenvalues of \( A_{ij}(l) \). The sum of these three eigenvalues gives the usual power \( C_l \). The isotropic CMB data predicts all three equal eigenvalues should be degenerate and equal to \( C_l/3 \). Taking into account the eigenvalues of the power tensor, the power entropy \([7]\) is defined as,
\[ S = - \sum_i (\hat{\lambda}_i) \log(\hat{\lambda}_i) \]  
(5)

where \( i = 1, 2, 3 \) corresponds to three eigenvalues of the power tensor matrix. The normalized eigenvalues of the power tensor are given as
\[ \hat{\lambda}_i = \frac{\lambda_i}{\sum_i \lambda_i} \]  
(6)

The range of power entropy \( S \) is \( 0 \leq S \leq \log(3) \). The power entropy has maximum value \( \log(3) \) for isotropic CMB prediction, where all three eigenvalues are equal. Any excess of power along one of the eigenvector of power tensor results in low power entropy compared to isotropic prediction. Since the largest eigenvalue makes the largest contribution to power spectrum, we single out the eigenvector corresponding to the largest eigenvalue and call it as the principal eigenvector (PEV). If the data is anisotropic, a large number of vectors may lie along one direction or in preferred plane. Here, the sign of the eigenvectors is meaningless. To study the alignment of vectors over a range of multipoles, the alignment matrix is defined as
\[ X_{ij}(l_{max}) = \sum_{l=2}^{l_{max}} \hat{e}_i^l \hat{e}_j^l , \]  
(7)

where \( \hat{e}_i, i = 1, 2, 3 \) are the PEVs of power tensor. The eigenvalues of the alignment matrix \( X \) are a probe of the shape of the bundle collected from \( 2 \leq l \leq l_{max} \). The anisotropy in the data can be probed by computing the alignment entropy \( S_X \) using the alignment matrix \( X \). The alignment entropy \( S_X \) is defined as
\[ S_X = -tr(\hat{\rho}_X \log(\hat{\rho}_X)) , \]  
(8)
where $\rho_X = \frac{X}{\text{tr}(X)}$. For isotropic CMB, $S_X \sim \log(3)$. The alignment entropy is independent of the power entropy. A very low value of $S_X$ compared to $\log(3)$ indicates a violation of isotropy. Here we mainly focus on the low-$l$ anomaly such as alignment between CMB quadrupole and octopole in PLANCK data in comparison to WMAP 9-year data and also the high-$l$ anomaly seen in [7] using the foreground cleaned maps provided by WMAP team.

### 2 Statistics

The significance of anisotropy is determined by comparing the result for real data with that corresponding to 10,000 isotropic randomly generated CMBR data. The significance is quoted in terms of the $P$-value, which is defined as the probability that a random isotropic CMB map may yield a statistic larger than that seen in data. We set preliminary level of statistical significance using $P-values 0.05$. The significance of $P-values$ is calculated using the binomial distribution. So the probability to encounter $k$ instances of passing defined by probability $p$ in $n$ trials is

$$P_{\text{bin}}(k, p, n) = p^k (1-p)^{(n-k)} n!/(n-k)! k!.$$  \hfill (9)

In assessing many $P$-values, we find the cumulative probability as calculated in [7]. We calculate the cumulative binomial probabilities as

$$P_{\text{bin}}(k \geq k^*, p, n) = \sum_{k=k^*}^{n} P_{\text{bin}}(k, p, n).$$  \hfill (10)

Both the probability and cumulative probability tell us how the observed data support the isotropic random realizations.

### 3 CMB data

We use WMAP 9-year ILC map [14] henceforth known as WILC9 and PLANCK’s NILC, SMICA and SEVEM CMBR maps [16]. Here we restrict ourselves to the multipole range $2 \leq l \leq 50$, since the statistical and the systematic errors lie within the cosmic variance in this range. For WILC9, we use the $KQ85$ mask and in the case of PLANCK maps, we use the CMB-union mask ($U73$) to eliminate the contribution from the galactic foregrounds. We filled the masked region by simulated isotropic CMB data with appropriate noise for WILC9. While in the case of PLANCK data, we do not consider the contribution of noise as it is too bulky. Since the filling of the masked region with random realization produces different result for different realizations for the full sky “data” map, we obtain the final results by taking their average over 100 such filled data maps for both WILC9 and SMICA maps.

At large-$l$, the WILC9 map is not reliable. Hence, we use the individual foreground cleaned Differencing Assembly (DA) maps, $Q1, Q2, V1, V2, W1, W2, W3, W4$, provided by the WMAP team to study the anisotropies in the large multipoles. We divide the high multipole region into three regions as $2 \leq l \leq 300, 150 \leq l \leq 300$ and $250 \leq l \leq 300$. We compute the $P$-values from random realizations including the appropriate detector noise for each band.
4 Result

4.1 Alignment of Quadrupole and octopole

We test the alignment for WILC9 and PLANCK maps (NILC, SMICA, SEVEM) using the power tensor technique [4, 6, 7]. We compute the PEVs for both quadrupole \( l = 2 \) and octopole \( l = 3 \) and extract the angle between these two vectors. The PEVs of quadrupole and octopole for different maps and the corresponding angle between them are given in the Table 1. The probability of the distribution for two different axes \( \hat{n} \) and \( \hat{n}' \) to align within an angle \( \theta \) which is given as

\[
P(\cos \theta) = (1 - \cos \theta)
\]

(11)

where \( \cos \theta = |\hat{n} \cdot \hat{n}'| \). The probability of alignment also quoted in Table 1.

| Map    | \( l = 2 \)          | \( l = 3 \)          | \( \theta_{23} \) | \( 1 - \cos \theta_{23} \) |
|--------|----------------------|----------------------|-------------------|---------------------------|
| WILC9  | (0.248, 0.435, −0.865) | (0.240, 0.387, −0.890) | 3.87°             | 0.00185                   |
| NILC   | (0.087, 0.238, −0.967) | (0.240, 0.399, −0.884) | 13.91°            | 0.028                     |
| SMICA  | (0.144, 0.347, −0.926) | (0.284, 0.405, −0.868) | 9.87°             | 0.013                     |
| SEVEM  | (0.248, 0.435, −0.865) | (0.240, 0.386, −0.890) | 4.22°             | 0.00185                   |

Table 1: Alignments of PEVs for \( l = 2 \) and \( l = 3 \) for various maps and the corresponding angle between them.

The alignment of the eigenvectors in the WILC9 and PLANCK (SMICA, NILC and SEVEM) at low-\( l \) may be a signal of a fundamental anisotropy or could also be caused by foreground contamination. However, the possibility of the foregrounds for the observed alignment in CMBR data has been ruled out in Ref. [8] and hence it is most likely cosmological in nature. The alignment is better for WILC9 with \( \theta_{23} = 3.87° \) and the PLANCK SEVEM map with \( \theta_{23} = 4.22° \). But the alignment angle is quite large for SMICA and NILC map. Hence low-\( l \) alignment purely depends upon the cleaning pipeline used to obtain the foreground cleaned map.

4.2 Axial alignments

4.2.1 Region \( 2 \leq l \leq 11 \)

The signals obtained from the region \( 2 \leq l \leq 11 \) are highly significant as observed in Ref. [9]. So it is interesting to analyze this multipole range using both WILC9 and PLANCK data. We measure the alignment between multipoles by comparing the PEVs of the power tensor matrix. There are 3 PEVs corresponding to multipole \( l = 2, 3, 9 \) for WMAP 5-year ILC map as seen by Ref. [6]. We find that there are 2 PEVs with multipole \( l = 2, 3 \) for WILC9, 3 PEVs with \( l = 2, 3, 9 \) for NILC, 2 PEVs with \( l = 2, 3 \) for SMICA, 2 PEVs with \( l = 2, 3 \) for SEVEM maps which shows alignment with quadrupole. So WILC9, SMICA and SEVEM maps no longer support the earlier observation [6,9]. The only map which support the earlier observation is NILC. Hence, we find that WILC9 and the PLANCK data does not support the strong signal of alignment over the multipole range \( 2 \leq l \leq 11 \).
4.2.2 Region $2 \leq l \leq 50$

As reported earlier in Ref. [6], the WMAP 5-year ILC map also shows alignment with the quadrupole axis in the low multipole range $2 \leq l \leq 50$ with high significance. For 5-year map, there are 6 multipoles with $l = 3, 9, 16, 21, 40, 43$ aligned with quadrupole. Hence, the alignment with the quadrupole is seen over a relatively large range of $l$ values. We also test for the alignment of multipoles in this multipole range for WILC9 and PLANCK maps. The list of the multipoles whose P-value of alignment is less than 5% with quadrupole is given in Table 2. There are 4, 5, 5 and 5 multipoles which shows the alignment with the quadrupole for WILC9, NILC, SMICA, and SEVEM maps respectively. In the WILC9, $l = 9, 40$ multipoles are no longer aligned with the quadrupole as seen in the WMAP 5 year data. So there is a very mild signal of anisotropy in this range $2 \leq l \leq 50$ of multipole. The net significance of alignment with quadrupole is computed using binomial probability and cumulative binomial probability using Eq. (9) and Eq. (10) is given in Table 3.

### Table 2: List of multipoles whose P-values of coincidence with quadrupole is less than 5% for $l \leq 50$.

| WILC9 | NILC | SMICA | SEVEM |
|-------|------|-------|-------|
| 3     | 3    | 3     | 3     |
| 16    | 9    | 16    | 16    |
| 21    | 16   | 28    | 28    |
| 44    | 28   | 40    | 40    |
| 40    | 46   | 44    |       |

### Table 3: Net significance of observing $P \leq 0.05$ for the multipoles that are well aligned with quadrupole $l = 2$ in the multipole range $2 \leq l \leq 50$.

|          | WILC9 | NILC | SMICA | SEVEM |
|----------|-------|------|-------|-------|
| Pro      | 0.12  | 0.06 | 0.05  | 0.03  |
| CumPro   | 0.20  | 0.09 | 0.07  | 0.04  |

5 Anomaly in high $l$

We next test for alignment of PEVs over a large range of multipoles. Our motivation is to verify whether the anisotropy found in low-$l$ multipole region continues to hold for a larger range of multipoles or are there any additional anomalies present in the data. In Ref. [7] it was found that the WMAP foreground cleaned 3-year and 5-year data in $W$ band for high multipoles shows unusual isotropy. The alignment entropy was so large that the probability to obtain this from a random samples exceeds 99.99% in the multipole range $150 \leq l \leq 300$ and almost the same probabilities are found for the multipole range $2 \leq l \leq 300$ and $250 \leq l \leq 300$. In contrast, the $Q$ band shows the signal of anisotropy with probability less than 0.01%. Since the $Q$ band is highly contaminated, one might assume that the anisotropy found in the $Q$ band would be due to the foreground contamination. We find that the WMAP foreground cleaned 9-year $Q$ band data also shows signal of anisotropy with same probability.
The high level of isotropy of $W$ band goes away when the authors lowered the level of noise in the simulated random realizations. Noise maps are generated by multiplying $\frac{\sigma_0}{\sqrt{N_p}}$, where $\sigma_0$ is the noise per observation and $N_p$ is the effective number of observation at each pixel, with a Gaussian distribution having zero mean. In generating the noise map they have used $\sigma_0$ value as 5.883($W_1$), 6.532($W_2$), 6.885($W_3$) and 6.744($W_4$). Reducing the value of $\sigma_0$ by two units they found that the P-value decreases to 92%. However such a large change in the value of $\sigma_0$ is not acceptable. To address this issue, we analyze these range of multipoles for WMAP 9-year foreground cleaned data. We find that the WMAP 9-year foreground cleaned data does not show such a high level of isotropy. The alignment entropy $S_X$ and the corresponding P-values for different range of multipoles are given in the Table 4. The P-value is less than 50% except the $W_1$ band in the multipole range $2 \leq l \leq 300$ and in $150 \leq l \leq 300$. For the multipole range $250 \leq l \leq 300$, the P-value for $W_1$ band is also less than 50%. Hence the WMAP 9-year data does not suffer from the anomaly seen in the $W_1$ band in the 3-year and 5-year data.

Table 4: Alignment entropy $S_X$ and corresponding P-values (in %) for WMAP 9 year foreground cleaned temperature DA maps over the three multipole ranges.

|            | $W_1$ | $W_2$ | $W_3$ | $W_4$ |
|------------|-------|-------|-------|-------|
| $S_X(2,300)$| 1.0934| 1.0925| 1.0945| 1.0948|
| $P(\%)$    | 55    | 44    | 18    | 24    |
| $S_X(150,300)$ | 1.0817| 1.0838| 1.0889| 1.090 |
| $P(\%)$    | 74    | 37    | 12    | 17    |
| $S_X(250,300)$ | 1.0657| 1.0570| 1.0736| 1.0827|
| $P(\%)$    | 45    | 34    | 9     | 6     |

6 Conclusion

We tested for the statistical low-$l$ alignments of the WILC9 and PLANCK CMBR data by extracting three orthogonal eigenvectors and the corresponding eigenvalues for each multipole $l$. We also measure the dispersion in the eigenvalues with the help of power entropy which provides the measure of the statistical isotropy. We study the dispersion in the PEV by constructing alignment matrix for a range of multipoles. In the multipole range $2 \leq l \leq 11$, we do not find any strong signal of anisotropy as seen in Refs. [9] and [6]. In the multipole range $2 \leq l \leq 50$, we find 4, 5, 5 and 5 multipoles which are aligned with the quadrupole for WILC9, NILC, SMICA, and SEVEM maps respectively. So there is very mild signal of anisotropy. The issue of highly isotropic nature of the CMBR data in the $W_1$ band seen in the 3-year and 5-year data is absent in the current 9-year data.

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