Dark radiation from a unified dark fluid model

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We present a unified dark fluid model to describe the possible evolutionary behavior of \(\Delta N_{\text{eff}}\) in dark radiation. This model can be viewed as an interacting model for the dark sectors, in which dark matter interacts with dark radiation. We show that the evolution of \(\Delta N_{\text{eff}}\) can be nicely explained without some drawbacks, such as the blowup of \(\Delta N_{\text{eff}}\) and the non-vanishing interaction at the late time.

Subject Index E45, E70, E75

1. Introduction

The \(\Lambda\)CDM model has successfully explained many important cosmological observations such as the acceleration of the universe and the radial velocity distribution of galaxies, as well as the cosmic microwave background (CMB) fluctuations [1,2]. Besides the motivation of theoretical completeness, from the viewpoint of the observational data there is still some room for the existence of physics beyond \(\Lambda\)CDM. Recently, the analysis of the pure CMB data has shown that the effective number of relativistic degrees of freedom is \(N_{\text{eff}} = 3.36^{+0.68}_{-0.64}\) (95\% confidence level (CL)) [2], which accommodates the standard model (SM) prediction of \(N_{\text{eff}}^{\text{SM}} = 3.046\) [3,4] within the 1\(\sigma\) range, while the combined analysis with the measurement of \(H_0\) gives \(N_{\text{eff}} = 3.62^{+0.50}_{-0.48}\) (95\% CL) [2], which is larger than the SM value at around the 2\(\sigma\) level. The extra degree of freedom is usually referred to as dark radiation (DR). It is worth noticing that the extra radiating component can be extracted by a probe of the primordial deuterium and helium abundances at the big bang nucleosynthesis (BBN) epoch [5]. For instance, it has recently been shown that \(N_{\text{eff}} = 3.71^{+0.47}_{-0.45}\) and \(3.50 \pm 0.20\) in Refs. [6] and [7], respectively.

Many models have been used to describe \(\Delta N_{\text{eff}} = N_{\text{eff}} - N_{\text{eff}}^{\text{SM}}\). Among them, imposing a new relativistic degree of freedom beyond the SM is straightforward [8–17], but such a scenario can only explain the case in which DR is equal at the BBN and CMB epochs, namely, \(N_{\text{eff}}^{\text{BBN}} = N_{\text{eff}}^{\text{CMB}}\). Note that there may be tension between BBN and CMB for \(N_{\text{eff}}\), as the current data seem to indicate that \(N_{\text{eff}}^{\text{CMB}} < N_{\text{eff}}^{\text{BBN}}\). In order to understand such a decrease (or increase) of \(N_{\text{eff}}\) at CMB, various subtle models have been proposed in which some interactions between DR and dark matter (DM)
are assumed. For example, if heavy DM particles\textsuperscript{1} can decay into relativistic states, the increase in DR could be interpreted; see, e.g., Refs. [20–33] for model-dependent and -independent analyses.

Since there is no evidence that the dark sectors are independent of each other, an interaction between DM and DR is quite possible. Models related to this possibility have been widely discussed in the literature [34–51]. However, it should be pointed out that there are still some drawbacks in these models: some of them blow up $\Delta N_{\text{eff}}$ at the late time, which is also equivalent to the existence of a non-vanishing interaction between DM and DR in the present. In this paper, we propose a unified dark fluid model describing both DM and DR, which can nicely yield the decrease (or increase) in $\Delta N_{\text{eff}}$ without the above drawbacks.

This paper is organized as follows. In Sect. 2, we introduce the unified dark fluid model. In Sect. 3, we discuss the extra effective relativistic degree of freedom $\Delta N_{\text{eff}}$. Conclusions are given in Sect. 4.

2. A unified dark fluid

We start with a dark fluid, in which the energy density is expressed as

$$\rho_{\text{dark}} = (Aa^{-4(1+\alpha)} + Ba^{-3(1+\alpha)})^{1/(1+\alpha)},$$  \hspace{1cm} (1)

where $a$ is the scale factor of the universe, $\alpha$ is a small real number, and $A$ and $B$ are positive, which can be determined by the initial condition at some specific time. Note that this dark fluid can be viewed as a mixture of DM and DR. In fact, it is a special case of the new generalized Chaplygin gas (NGCG) model with the equation of state (EOS) $w = 1/3$ proposed in Ref. [52]. We remark that this model is also inspired by the generalized Chaplygin gas (GCG) scenario, which unifies DM and dark energy (in the case of the cosmological constant) in a single fluid [53–55]. For $\alpha = 0$ in Eq. (1), the energy density reduces to the sum of the matter and radiation forms. For $\alpha$ being a small real number, the fluid can exhibit the behavior of both matter and radiation.

From the continuity equation, $\dot{\rho} + 3H(\rho + P) = 0$, the pressure of the dark fluid $P_{\text{dark}}$ can be derived, and then the EOS parameter of the dark fluid can be obtained as

$$w_{\text{dark}} = \frac{P_{\text{dark}}}{\rho_{\text{dark}}} = \frac{Aa^{-4(1+\alpha)}}{3(Aa^{-4(1+\alpha)} + Ba^{-3(1+\alpha)})}.$$  \hspace{1cm} (2)

For small $a$, we have $w_{\text{dark}} \simeq 1/3$, which is the same as the radiation fluid, while, for large $a$, the fluid behaves like matter with $w_{\text{dark}} \simeq 0$. Similar to GCG, this fluid can be naturally decomposed into two interacting components with constant EOS parameters, $w = 1/3$ and $w = 0$, respectively. As a result, this unified dark fluid model can be also regarded as an interacting dark-sector model in which DM interacts with DR.

Subsequently, we can write $\rho_{\text{dark}} = \rho_{\text{dm}} + \rho_{\text{dr}}$ and $P_{\text{dark}} = P_{\text{dm}} + P_{\text{dr}}$. By using $P_{\text{dm}} = 0$ and $P_{\text{dr}} = (1/3)\rho_{\text{dr}}$, we derive

$$\rho_{\text{dm}} = K \frac{1}{1+\alpha} \left( 1 - \frac{Aa^{-4(1+\alpha)}}{K} \right), \quad \rho_{\text{dr}} = K \frac{1}{1+\alpha} \frac{Aa^{-4(1+\alpha)}}{K},$$  \hspace{1cm} (3)

where $K = Aa^{-4(1+\alpha)} + Ba^{-3(1+\alpha)}$. Evidently, $A$ and $B$ can be naturally determined by the initial condition of the two components, e.g., the DR and DM densities at the present time.

\textsuperscript{1}For ultra-light DM candidates, some interesting properties were discussed in Refs. [18,19].
The energy transfer from DM to DR in unit volume and in unit time can be derived as
\[
Q = -3\alpha H \frac{P_{\text{dark}}}{\rho_{\text{dark}}}(\rho_{\text{dark}} - 3P_{\text{dark}}) = -\alpha H \frac{\rho_{\text{dm}} \rho_{\text{dr}}}{\rho_{\text{dm}} + \rho_{\text{dr}}},
\]
where the sign of \(\alpha\) fixes the direction of the energy flow. A positive \(\alpha\) makes the energy flow from DR to DM, whereas a negative one reverses the direction. By this definition, the energy continuity equations for DM and DR are given by \(\dot{\rho}_{\text{dm}} + 3H\rho_{\text{dm}} = -Q\) and \(\dot{\rho}_{\text{dr}} + 4H\rho_{\text{dr}} = +Q\), respectively. Note that, if \(\rho_{\text{dr}} \gg (\ll) \rho_{\text{dm}}\), the energy transfer \(Q\) can be reduced to \(Q = -\alpha H \rho_{\text{dm}}(\text{dr})\). This kind of interaction simultaneously involves the two important forms \(Q\) and \(-\alpha H \rho_{\text{dr}}\), studied extensively in the interacting dark energy models \([56-60]\). These forms, similar to those obtained from the GCG fluid, are crucial features of the GCG-like model \([53-55]\). We remark that, once \(Q\) is proportional to the Hubble expansion rate \(H\), there is a factor of \(T^2\) in the radiation-dominated epoch. We will discuss the effect of the interactions on the time evolution of \(\Delta N_{\text{eff}}\) in the next section.

It should be pointed out that, although our model is inspired by the GCG and NGCG models, there are some significant differences between the DM–DR interacting model and the DM–dark energy interacting model, in particular when the cosmological perturbations are considered. For example, for the GCG model, when it is considered as a unified model, the perturbation calculations force it to be extremely close to the \(\Lambda\)CDM model (\(\alpha < 10^{-6}\)) \([61,62]\), whereas a much wider range of \(\alpha\) is allowed, i.e., \(\alpha\) may be of the order \(\mathcal{O}(10^{-1})\) \([63]\), when it is treated as a model of vacuum energy interacting with DM. The case of the NGCG model is discussed in Ref. \([64]\). The primary cause is that dark energy is a non-adiabatic fluid so that the proper treatment of its pressure perturbation is obscure to some extent\(^2\). Nevertheless, for the model considered in this paper, since both DM and DR are adiabatic fluids, our model can be treated as a model of unified dark fluid as well as a model of DM interacting with DR. As a result, we expect that the constraints on our model from “geometry measurements” and “structural growth measurements” will be consistent owing to the fact that both DM and DR are adiabatic fluids with well defined sound speeds and well treated pressure perturbations.

3. \(N_{\text{eff}}\) in interacting models

From the definition of \(N_{\text{eff}}\), the extra relativistic energy density exceeding the \(\Lambda\)CDM model is given by
\[
\Delta \rho_v \equiv \Delta N_{\text{eff}} \frac{7}{8} \left( \frac{T_v}{T_\gamma} \right)^4 \rho_0^0 a^{-4},
\]
where \((T_v/T_\gamma) = (4/11)^{1/3}\) after the photon was heated at the \(e^+e^-\) annihilation epoch.

In terms of the description of our model, DR interacts with DM, which results in the deviation \(\Delta \rho_v\) from the standard evolution behavior in the \(\Lambda\)CDM model. Consequently, if we identify \(\rho_{\text{dr}}\) with \(\Delta \rho_v\) in Eq. \((5)\), we obtain a time-evolutionary \(\Delta N_{\text{eff}}\). On the other hand, since DM also differs from the standard scaling \(a^{-3}\), we can write \(\rho_{\text{dm}} = \rho_{\text{dm}}^0 f(a)a^{-3}\), where \(f(a)\) represents the departure from

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\(^2\) In the case of dark energy, since \(w\) is negative, its adiabatic sound speed \(c_a\) would be imaginary due to \(c_a^2 = \frac{\partial p}{\partial \rho}/\frac{\partial \rho}{\partial \rho} = \frac{w}{3}\) (for the example of constant \(w\)), leading to instability in the theory. In order to fix this problem, it is necessary to assume that dark energy is a non-adiabatic fluid and to impose a physical sound speed \(c_a^2 > 0\) by hand. Usually, \(c_a\) is set to be the light speed as if the dark energy fluid is realized by a scalar field, which is what is done in the CAMB and CMBFAST codes. But such a treatment would also lead to some instabilities, in particular for the \(w = -1\)-crossing models and some specific interacting dark energy models. For more detailed discussions, see Ref. \([65]\).
the standard result. The explicit form of $f(a)$ can be extracted from Eq. (3). Hence, according to the decomposition of the model, we have

$$\rho_{\text{dark}} = \rho_{\text{dr}} + \rho_{\text{dm}} = \rho_{\text{dark}}^0 \left( r a^{-4(1+\alpha)} + (1 - r) a^{-3(1+\alpha)} \right)^{1/(1+\alpha)}$$

$$= \Delta N_{\text{eff}}(a) \frac{7}{8} \left( \frac{T_r}{T_y} \right)^4 \rho_r^0 a^{-4} + \rho_{\text{dm}} f(a) a^{-3}, \quad (6)$$

where $\rho_{\text{dark}}$ has been re-parameterized by the value $\rho_{\text{dark}}^0$ at the present time and a dimensionless parameter $r$, taken around $10^{-5}$, which is of the same order as the radiation fractional density now.

In Fig. 1(a), we show $\Delta N_{\text{eff}}$ as a function of the scale factor $a$ for $\alpha = 0.1$ (blue), $-0.1$ (red), $-0.3$ (green), and $0$ (gray), and $r = 0.5 \times 10^{-5}$ (solid) and $3 \times 10^{-5}$ (dashed), respectively. All curves with $r$ fixed approach the same $\Delta N_{\text{eff}}$ at $a = 1$, and are sensitive to the initial condition $\rho_{\text{dark}}^0$ for $a \gtrsim 10^{-2}$. For $\alpha = 0.1$, $\Delta N_{\text{eff}}$ is a decreasing function, while, for $\alpha = -0.1$ or $\alpha = -0.3$, it behaves as an increasing one. Notice that $\alpha = 0$ gives a constant value of $\Delta N_{\text{eff}}$ due to the vanishing of the interacting term $Q$. Different choices of $r$ will lead to different results. From the figure, it is clear that $\alpha > 0$ with the energy flowing from DM to DR is favored. In Fig. 1(b), we illustrate the correlations between the two parameters $r$ and $\alpha$ with different choices of $\Delta N_{\text{eff}}$ in the CMB and BBN epochs, respectively. In the figure, the contours with the cyan, black, brown, and purple curves stand for $\Delta N_{\text{eff}} = 0.1, 0.3, 0.5, \text{ and } 0.8$, while the solid and dashed ones correspond to $z = 1100$ and $10^{-9}$, respectively. From Fig. 1(b), we can roughly determine the model parameters. For example, if we assume that $\Delta N_{\text{eff}}^{\text{CMB}} = 0.3$ and $\Delta N_{\text{eff}}^{\text{BBN}} = 0.5$, which are consistent with the current observations, we obtain $\alpha \simeq 0.15$ and $r \simeq 0.5 \times 10^{-5}$, which are reasonable parameters for the model. Note that a positive value of $\alpha$ is required if the result of the smaller $\Delta N_{\text{eff}}$ in CMB persists in the future observations.

In Fig. 2(a), we plot the ratio of $H/H_{\text{SC}}$ as a function of the scale factor $a$, where $H$ and $H_{\text{SC}}$ are the Hubble parameter in our model and $\Lambda$CDM, respectively. The parameters are taken as $r = 0.5 \times 10^{-5}$, and $\alpha = 0.1$ (blue), $-0.1$ (red), $0$ (gray), and $-0.3$ (green) for the plots. It is shown that the cosmic expansion becomes faster in the early time for any value of $\alpha$. This can be easily understood since $\Delta N_{\text{eff}}$ is always positive, as shown in Fig. 1(a), which implies the existence of extra energy density apart from that given by $\Lambda$CDM. In addition, the EOS parameter $w_{\text{eff}}$ versus $a$ is given.

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**Fig. 1.** (a) $\Delta N_{\text{eff}}$ versus the scale factor $a$, where the blue, red, green, and gray curves represent $\alpha = 0.1$, $-0.1$, $-0.3$, and $0$, and the solid and dashed ones denote $r = 0.5 \times 10^{-5}$ and $3 \times 10^{-5}$, respectively, while the black dashed line indicates the scale corresponding to the CMB epoch. (b) Correlations between $r$ and $\alpha$, where the cyan, black, brown, and purple curves stand for $\Delta N_{\text{eff}} = 0.1, 0.3, 0.5, \text{ and } 0.8$, while the solid and dashed ones correspond to the CMB and BBN epochs, respectively.
Fig. 2. (a) $H/H_{\mathrm{SC}}$ and (b) $w_{\text{eff}}$ versus the scale factor $a$ with $r = 0.5 \times 10^{-5}$, where the blue, red, green, and gray curves represent $\alpha = 0.1, -0.1, -0.3$, and 0, respectively, while the black dashed curve in (b) represents $w_{\text{eff}}$ in the $\Lambda$CDM model.

Fig. 3. The CMB temperature power spectrum in models, where the upper to lower (blue solid, blue dashed, red, and green) curves correspond to $\Lambda$CDM, $\Lambda$CDM with one massless sterile neutrino, $(\alpha, r) = (0.15, 5 \times 10^{-6})$, and $(0.01, 5.1 \times 10^{-5})$, respectively.

in Fig. 2(b). In the figure, we also show the result (dashed curve) for $\Lambda$CDM. It is worth noting that the evolution of the Hubble parameter $H$ could also provide some effect on the anisotropic CMB power spectrum. The increasing of $H$ at the CMB epoch ($\log_{10} a \simeq -3$) for any value of $\alpha$ would not only suppress the damping tail to equivalently solve the anomaly of DR, but also shift the acoustic peak slightly toward a smaller angular scale (larger $\ell$), while the value of the first acoustic peak could be increased. Typically, by taking $r \simeq 5 \times 10^{-6}$ and $\alpha = 0.15$, the first peak could rise about the same amount as that in the scenario of adding an additional massless sterile neutrino into $\Lambda$CDM. For a larger $r$, the amplitude of the power spectrum increases rapidly. These effects are illustrated in Fig. 3, where several models including $\Lambda$CDM, $\Lambda$CDM with an extra massless sterile neutrino, and our unified fluid model with $(\alpha, r) = (0.15, 5 \times 10^{-6})$ and $(0.01, 5.1 \times 10^{-5})$ are presented. A similar discussion on the anisotropic spectrum for other interacting models was also given in Ref. [34].

To illustrate our results, we now compare our model with two other interacting models, Models A and B, in which the energy transfers are $Q_A = \alpha_1 H \rho_{\text{dm}}$ and $Q_B = \rho_{\text{dm}} / \tau_{\text{dm}}$, respectively, with $\alpha_1$ and $\tau_{\text{dm}}$ being the free parameters. Model A is a simple interacting scenario between DM and DR, which is studied in Ref. [34], while Model B is examined in Refs. [20–23], in which the interacting term $\rho_{\text{dm}} / \tau_{\text{dm}}$ can be directly interpreted as the energy transferring into the DR component from the decaying of heavy particles with the lifetime $\tau_{\text{dm}}$, around the BBN epoch. Unlike our model and
Model A, the energy density $\rho_{\text{dm}}$ for a heavy particle in Model B is unlikely to be linked with DM due to the short lifetime $\tau_{\text{dm}}$ of the order of only a few seconds [23].

In Fig. 2(a), we present $\Delta N_{\text{eff}}$ as a function of the scale factor $a$ in different models. For Model A (B), we will use $\alpha_1 = 0.03$ and $0.01$ ($\tau_{\text{dm}} = 2000 \text{ s}$ and $500 \text{ s}$) as input parameters. $\rho_{\text{dm}}^0$ in Model A is identical to the DM density in the present, while for Model B we will fix the comoving energy density ($\rho_{\text{dm}}/s = (2 \times 10^{-5}) \text{ MeV}$ at BBN, with $s$ being the entropy density at that time. In both models, $\rho_{\text{dr}} \times a^4$ at very early time is taken to be zero as the other initial condition. We see that, in Model A, $\Delta N_{\text{eff}}$ coincides with the observation at the CMB era, but it blows up in the late time. In Model B, $\Delta N_{\text{eff}}$ only increases at a very early time and behaves as a constant after $a \gtrsim 10^{-8}$. The average rate of change in $\Delta N_{\text{eff}}$ from BBN to CMB for our model is faster than in Model B, but slower than in Model A. Moreover, both increasing and decreasing behaviors of $N_{\text{eff}}$ can be described in our model, which could be a potential target for probing this model in future observations. In addition, the dimensionless relative energy transfer $q \equiv |Q|/\rho_H$, with $\rho_H$ being the sum of the energy densities of DM and DR, is plotted in Fig. 4(b) for each case. With the same parameter values in Fig. 4(a), $q$ in Model A always behaves as a constant due to the crucial feature $(\rho_{\text{dr}}/\rho_{\text{dm}}) \sim \alpha$ [34], whereas in the late time the nonzero value of $q$ indicates that the interaction between DM and DR is still rather strong, even at present. In Model B, the region of the nonzero $q$ centralizes at the beginning of BBN with the order of magnitude around the peak as large as order unity. In our model, $|Q|/H$ is proportional to $\rho_{\text{dm}}$ and $\rho_{\text{dr}}$ in very early and late times, respectively, so that a nonzero value of $q$ can only be confined in some range of time. Obviously, the behaviors of $\Delta N_{\text{eff}}$ and $q$ in our model are more reasonable than in Model A.

4. Conclusions

We have proposed a unified dark fluid model to understand the possible evolutionary behavior of $\Delta N_{\text{eff}}$ in DR. Inspired by the GCG model, the dark fluid can be viewed as a scheme for the unification of DM and DR. Such a fluid behaves like radiation and matter in the radiation- and matter-dominated epochs, respectively. Interestingly, this model can also be regarded as an interacting model in the dark sectors as DM interacts with DR with the form explicitly obtained. Moreover, we have evaluated the evolution of $\Delta N_{\text{eff}}$ in DR, which is favored by the current observational data for $\alpha > 0$. Comparisons with the other two interacting models, $Q = \alpha_1 \rho_{\text{dm}} H$ and $\rho_{\text{dm}}/\tau_{\text{dm}}$, have been also given. We have shown that our predicted values of $\Delta N_{\text{eff}}$ and $q$ in the unified dark fluid model are more reasonable than Model A. In particular, in our model there are no drawbacks, such as the blowup of $\Delta N_{\text{eff}}$ and the...
non-vanishing interaction at the late time. Clearly, more accurate analyses on $N_{\text{eff}}$ and its evolution in the future could help to identify if our model is a viable scenario.

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