Nonlinear Bound States in the Continuum in One-Dimensional Photonic Crystal Slab

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Abstract. We explore the nonlinear optical response of the silicon one-dimensional photonic crystal slab supporting bound states in the continuum (BIC). We show the emergence of such nonlinear effects as multistable behaviour, self-tuning of BIC and breaking of symmetry protected BIC. We define a class of the nonlinear solutions generated by the linear BIC state and analyze the modulation instability of the obtained solutions and the effect of the finite system size on the stability.

Bound states in the continuum \cite{1} (BICs) are localized states whose energy lies within the radiation continuum. Such states can be considered as ultra high quality resonance modes and therefore can be used to enhance optical nonlinear effects \cite{2, 3}. In particular, onset of multistable optical response is determined by the pump power density depending on the nonlinear susceptibility of the material and the quality factor of resonator. Hence, the threshold powers needed for the emergence of multistability in BIC-supporting structures are low and such structures may be used for the realization of all-optical switches.

In our work \cite{4}, we show that BIC supporting systems indeed allow to achieve strong nonlinear response without cavity at low pump power. For that we consider 1D photonic crystal slab with Kerr-type nonlinearity and analyze the role of BIC in formation of multistable states. Also we analyze the stability of found states with respect to longitudinal perturbations.

Considered system, schematically shown in Fig. 1(a), can be treated as a slab waveguide with grating on the top side, so it can be described by the effective refractive index $n(z)$:

$$n(z) = n_0 + \delta n \frac{2}{\pi} \sum_{j=1}^{\infty} \frac{1}{j} \sin \left( \frac{j \pi w}{d} \right) \cos \left( j G z \right) + n_2 I. \quad (1)$$

where $n_0$ is the average refractive index of core layer in the absence of grating, $\delta n$ is the modulation amplitude, $G = 2\pi/d$ is the reciprocal vector of the structure, $d$ is the period of lattice, $w$ is the width of the protrusions and $n_2 I$ is the term corresponding to the Kerr-type focusing nonlinearity. We consider the resonant excitation of the system, so it can be described by the coupled-mode theory within two-mode approximation \cite{5, 6, 7}. Hence, the electric field can be written in terms of forward and backward propagating waveguide modes with amplitudes $E_\pm$ and wavevector $\beta$:

$$E(z, t) = \left[ E_+(z, t) e^{i\beta z} + E_-(z, t) e^{-i\beta z} \right] e^{-i\omega_0 t} + c.c. \quad (2)$$
Figure 1. (a) Schematic picture of the considered system. (b) The total field as the function of pump intensity. Here $\delta \omega = -11.16$ meV. The green dots correspond to symmetric solutions and the blue dots correspond to asymmetric. The stable (unstable) solutions are shown by thick (tiny) dots. The red line corresponds to antisymmetric solutions $2|a_0|$.

Labeled points are used to distinguish between different branches of solutions. (c) Phase diagram of the solutions. Numerical indices $abcd$ have the following meaning: $a$ shows the total number of solutions, $b$ denotes how many of them are stable, $c$ is the number of symmetric solutions and $d$ is the number of stable symmetric solutions.

Assuming $\omega_0 = c k_0 = \beta c/n_0$, $\beta = G = 2\pi/d$ and using slow-varying approximation, we can get the resulting pair of the dimensionless coupled-mode equations with the pump and decay terms:

$$
\frac{\partial \tilde{E}_\pm}{\partial \tilde{t}} = i\delta \tilde{n} \tilde{E}_\pm - \gamma (\tilde{E}_+ + \tilde{E}_-) + i\tilde{E}_\pm (|\tilde{E}_\pm|^2 + 2|\tilde{E}_\pm|^2) + \sqrt{\gamma I_p} e^{i\phi_0 - i\delta \tilde{\omega} \tilde{t}}
$$

(3)

where $\tilde{I}_p = n_2 \tilde{E}_p^2/2$ is pump intensity, $\delta \tilde{\omega} = \delta \omega/\omega$ is the detuning between the pump beam, coupling constant is $\gamma \approx k_0^2 \delta \tilde{n}^2$ and the phase is $\phi_0 = -1/2i \log(-r_{slab} - t_{slab})$. The following normalized parameters were used:

$$
\tilde{z} = \beta z; \quad \tilde{E}_\pm = E_\pm (n_2/n_0)^{1/2}; \quad \tilde{t} = \omega t; \quad \tilde{\delta} \tilde{n} = \delta n \sin (2\pi w/d) / (2\pi n_0).
$$

(4)

For the linear case in the absence of pumping, system of equations (3) has symmetric solutions, corresponding to leaky modes, and antisymmetric solutions corresponding to the symmetry protected BIC:

$$
\tilde{\omega} = \delta \tilde{n}; \quad \tilde{E}_+ = -\tilde{E}_- \quad (5)
$$

$$
\tilde{\omega} = -\delta \tilde{n} - 2i\gamma; \quad \tilde{E}_+ = \tilde{E}_- \quad (6)
$$

However, in the nonlinear case, amplitude of BIC depends on the frequency:

$$
|\tilde{E}_\pm| = \sqrt{\frac{1}{3} (\delta \tilde{n} - \tilde{\omega})}.
$$

(7)
Hence, there always exists a purely antisymmetric (and thus non-radiating) state at all the frequencies smaller than the frequency of the linear BIC ($\omega < \delta n$). This is the manifestation of the self-tuning of BIC. If we pump the sufficient energy density in the system, then the resonant frequency is tuned such that the BIC coincides with pump frequency, which can be regarded as a way to excite BIC [8, 9].

Considering inhomogeneous set of equations with solutions in the form $\tilde{E}_\pm = -ia_\pm e^{i\phi_0 - i\delta \omega t}$, we can obtain three classes of solutions. First of all, there are symmetric solutions $a_+ = a_- = a_s$, which generates a standard S-shaped bistability curve. The second class is asymmetric solutions, which have the form $a_+ = a_0 e^{i\phi_+}$ such that

$$a_0 = \sqrt{\frac{1}{3} (\delta n - \delta \omega)}, \quad \tan \left( \frac{\phi_+ + \phi_-}{2} \right) = -\gamma / \delta n, \quad \cos \left( \frac{\phi_+ - \phi_-}{2} \right) = \frac{\sqrt{\gamma I_p}}{2a_0 \sqrt{\gamma^2 + \delta n^2}},$$

(8)

It is important to note, that $a_0$ does not depend on pump intensity and as $\tilde{I}_p$ approaches zero, the solution approaches the fully antisymmetric solution $\phi_+ - \phi_- = \pi$. Moreover, the solution becomes fully symmetric at $I_p = \frac{4}{3\gamma 3\delta n} (\delta n - \delta \omega)(\gamma^2 + \delta n^2)$. This type of solutions was termed Josephson-like current [10]. Despite the fact that for infinite system these solutions are unstable, they may be stable in the finite-size system as we will show later [see Fig. 2]. Finally, there is a class of asymmetric solutions such that $|a_+| \neq |a_-|$, and $\phi_+ = \pi + \phi_-$. To analyze these solutions we have performed numerical calculations for a silicon slab waveguide in vacuum ($n_0 = 3.48, n_2 = 3 \cdot 10^{-18}$ m$^2$/W [11, 12]). The parameters are the following: depth of the grating is 10 nm, $h = 100$ nm, $\delta n \approx 0.0316$ and $w = d/4$. The lattice period $d$ was chosen in such a way that $\beta = 2\pi/d$ at wavelength $\lambda = 1$ $\mu$m, material losses are accounted. Results of the calculations are shown in Fig. 1(b),(c). If $\delta \omega > \delta n$ than only single symmetric solutions can exist. Then, two additional asymmetric solutions can emerge in the region $-\delta n < \delta \omega < \delta n$. One of this solutions is stable and therefore the nonlinearity could result in breaking of the symmetry protected BIC and its transformation into symmetric solution. But when the detuning is less than $-\delta n$, a multistable behavior is observed with three stable solutions.

Figure 2. Stability analysis of solutions. For each plot $I_p \approx 0.015$ W/m$^2$ and $\delta \omega = -11.16$ meV. All other parameters are given in the main text. Solutions in Fig. 1 corresponds to the case of $q = 0$.

To analyze the stability of the obtained solutions substitute the electric field in the form

$$\tilde{E}_\pm(\tilde{z}, \tilde{t}) = \left( a_\pm + \epsilon f_\pm e^{iqz} e^{i\tilde{t}} + \epsilon g_\pm e^{-iqz} e^{-i\tilde{t}} \right) e^{-i\delta \omega \tilde{t}},$$

(9)
to the initial differential equations and linearize them with respect to $\epsilon$ which is the amplitude of the small perturbation. Here $\tilde{q}$ is the wavevector of the perturbation, which is a parameter, and $\tilde{l}$ is the complex eigenfrequency. If at least one of calculated $\tilde{l}$ has positive real part, then the solution is unstable. We see that asymmetric and antisymmetric solutions are stable with respect to perturbations with wavevectors, larger than some finite critical value $\tilde{q}_{\text{max}}$, as it is shown in Fig. 2. Since only perturbations which have wavelength smaller than the system size may exist, the perturbations with $\tilde{q} < 1/N$ (where $N$ is the number of periods) will decay. Thus, if the system size is less than $\tilde{q}_{\text{max}}$, the solutions could be stabilized.

To conclude, we have demonstrated that the nonlinear BIC can exist in 1D photonic crystal slab and we have shown how the existence of the BIC leads to the emergence of the multistable behavior in the structure. Also, we have shown that the solutions unstable in the infinite system may be stable in the finite-size system. The simplicity of the structure and the moderate level of required pump intensities in BIC-supporting structures opens new avenues for the realization of all-optical switchers exploiting bound states in the continuum.

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