NONET SYMMETRY AND TWO-BODY DECAYS OF CHARMED MESONS

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ABSTRACT

The decay of charmed mesons into pseudoscalar (P) and vector (V) mesons is studied in the context of nonet symmetry. We have found that it is badly broken in the PP channels and in the P sector of the PV channels as expected from the non-ideal mixing of the $\eta$ and the $\eta'$. In the VV channels, it is also found that nonet symmetry does not describe the data well. We have found that this discrepancy cannot be attributed entirely to SU(3) breaking at the usual level of 20–30%. At least one, or both, of nonet and SU(3) symmetry must be very badly broken. The possibility of resolving the problem in the future is also discussed.
1. Introduction

The original motivation for nonet symmetry [1] is the near ‘ideal’ mixing of the \( \omega \) and \( \phi \) mesons and the extension of nonet symmetry to the pseudoscalar sector goes back to the original study of charmed meson decays of ref. 2. In this paper, we investigate the validity of and nonet symmetry in Cabibbo-allowed two-body decays of charmed mesons (D) into pseudoscalar (P) and vector (V) mesons.

In an earlier paper [3], one of us has already shown that nonet symmetry does not work in the decays of charmed mesons to two pseudoscalars. We repeat the analysis here in light of more recent data and we also show that it does not work for \( D \to PV \), possibly through a breakdown in the P sector. As a further test, we will look at \( D \to VV \) where it is expected to work. However, we find that it does not! An alternative explanation using the breakdown of SU(3) symmetry is also explored. The present status of experimental data does not yet allow us to draw any definite conclusions.

The discussion of charm decay in the context of flavor SU(3) [4] and nonet symmetry [5] has been treated in great detail in the literature. Here we adopt the notations of ref. [3]. The tensor structure of the Hamiltonian governing Cabibbo-allowed decays gives rise to two representations, a \( 15 \) and a \( 6^* \). As before, we construct these from the two final state nonets and the charm meson triplet. The incorporation of the singlet into the octet to form the nonet does not alter the Clebsch-Gordon series of \( 8 \otimes 8 = 27 + 10 + 10^* + 8 + 8 + 1 \). We will label the reduced matrix elements obtained from the \( 27 \) as \( T \), those from the \( 10 \) and \( 10^* \) as \( D \) and those from the symmetric and antisymmetric combinations of the two \( 8 \)'s as \( S \) and \( A \) respectively. Finally, the overall representation will be denoted by a subscript so that \( T_{15} \) would be the reduced matrix element of the \( 15 \) obtained from \( 27 \otimes 3 \).

The \( D \)'s and the \( A \)'s are antisymmetric under the exchange of the final state mesons so that they do not contribute to PP and VV channels. It is convenient to introduce the combinations \( S_\pm = S_{15} \pm S_6 \) and \( A_\pm = A_{15} \pm A_6 \) because each of these occurs only in the decays of either the \( D^0 \) meson or the \( D_s \) meson but not both. The decay amplitudes are summarized in tables 1, 2, and 3 along with the relevant phase space factors and experimentally measured branching ratios.

When nonet symmetry is broken, amplitudes involving the singlets are no longer related to those involving only members of the octets. To parametrize the extent of the breaking, we keep the original amplitudes under nonet symmetry and introduce a
new amplitude $B$ for each channel involving a singlet. Two such amplitudes are used in each of tables 1 and 3 for the discussion of the PP and VV channels.

For computational purposes, we will define all amplitudes in terms of branching ratios expressed in percent by the following relation

$$B(D_i \to XY) = \frac{|A(D_i \to XY)|^2 \times \text{phase space factor}}{\Gamma(D_i)/\Gamma(D^0)}.$$  \hspace{1cm} (1)

To allow for final state interactions, we have taken all amplitudes to be complex. To solve for both the real and imaginary parts of one, we will need two branching ratios. Also, because of the quadratic nature of eq. (1), there will in general be a two-fold ambiguity in each solution.

Error analysis in this kind of calculation is rather complicated, but we try to take into account correlations as much as possible. Since we are always comparing amplitudes or branching ratios, the total widths that enter eq. (1) will appear in all quantities of interest. The uncertainties in the total widths have been set to zero in order to avoid artificially inflating the errors involved in such comparisons. We treat uncertainties in all measured branching ratios as independent, but will try to keep track of their propagation into various derived quantities correctly. In cases where a common normalization is used, such as in $D_s$ decays, the ratios instead of the absolute branching ratios will be treated as independent. For comparisons involving only $D_s$ modes, the uncertainty in the common normalization, $B_{\phi\pi^+}$, will again be dropped.

We will discuss the decay of charmed mesons into PP, PV and VV channels separately in sections 2, 3 and 4. Section 5 will be devoted to the study of possible SU(3) breaking in VV and PP channels. We conclude with a brief summary in section 6.

2. D $\to$ PP

Table 1 summarizes all relevant information about the PP channels. Almost all Cabibbo-allowed modes are very well measured, especially in the $D^0$ sector. Even in the presence of the nonet breaking amplitude $B_s$, we have enough data to solve for all other amplitudes. The $D^+$ branching ratio gives us directly the magnitude of $T_{15}$. Taking $T_{15}$ to be real, we can then solve for $S_+$ from mode 2 and 3. To solve for $B_+$, we need to express the $\eta_8, \eta_1$ amplitudes in terms of those of $\eta, \eta'$. We do this by
using a pseudoscalar mixing angle [11] of $-20^\circ$ and the solutions are

$$5T_{15} = 1.10 \pm 0.08,$$

$$S_+ = (0.10 \pm 0.23) + i(1.93 \pm 0.07),$$

$$B_+ = \begin{cases} 
+ (1.86 \pm 0.23) - i(1.61 \pm 0.49) \\
- (1.94 \pm 0.36) - i(2.70 \pm 0.41)
\end{cases}.$$

Another set of equally good solutions can be obtained by a reflection about the real axis. The large size of $B_+$ clearly indicates that nonet symmetry in this sector of charm decay is badly broken.

In the $D_s$ sector there are two new amplitudes, $S_-$ and $B_-$, but only three extra branching ratios. We do not have enough information to solve for everything, but there exist triangular sum rules which impose constraints on the amplitudes.

To simplify the discussion, we set $B_- = 0$ and see if this would leads to any contradictions. Table 1 gives us the following relations

$$\sqrt{3}A(\pi \eta_1) - \sqrt{6}A(\pi \eta_8) = 6T_{15}, \quad (2)$$

$$\frac{5}{2}\sqrt{3}A(\pi \eta_1) - \sqrt{6}A(\pi \eta_8) = 3A(K^+ \bar{K}^0). \quad (3)$$

Putting in the pseudoscalar mixing angle $\theta$ to convert $\eta_{1,8}$ into $\eta, \eta'$, we obtain from eq. (2):

$$(\cos \theta - \sqrt{2} \sin \theta)A(\pi \eta') - (\sqrt{2} \cos \theta + \sin \theta)A(\pi \eta) = 2\sqrt{3}T_{15}$$

$$\theta = -20^\circ: \quad 2.87 \pm 0.44 \quad 1.21 \pm 0.20 \quad 0.76 \pm 0.05$$

$$\theta = -10^\circ: \quad 2.48 \pm 0.38 \quad 1.50 \pm 0.25$$

The numbers under each term are the values of that term evaluated with the mixing angles indicated. For $\theta = -20^\circ$, we have $2.87 - 1.21 = 1.66 > 0.76$. Even if we take into account the errors in each term, there is no choice of phase in which the three amplitudes can form a closed triangle. This is strong evidence that $B_-$ has to be nonzero. For $\theta = -10^\circ$, the three amplitudes are barely consistent with $B_- = 0$, if the errors are stretched to their limits.

The corresponding results for eq. (3) are

$$(\frac{5}{2} \cos \theta - \sqrt{2} \sin \theta)A(\pi \eta') - (\sqrt{2} \cos \theta + \frac{5}{2} \sin \theta)A(\pi \eta) = \sqrt{3}A(K^+ \bar{K}^0)$$

$$\theta = -20^\circ: \quad 3.57 \pm 0.28 \quad 0.36 \pm 0.04 \quad 1.84 \pm 0.16$$

$$\theta = -10^\circ: \quad 3.41 \pm 0.27 \quad 0.74 \pm 0.07$$

(4)
Here, the common factor $B_{\phi\pi}, \Gamma(D_s)/\Gamma(D^0)$ in the three amplitudes has been taken out so that the listed errors can be treated as more or less independent. If we take the difference between the two terms on the left, we obtain $3.21 \pm 0.28$ for $\theta = -20^\circ$ and $2.67 \pm 0.28$ for $\theta = -10^\circ$. To form a triangle, these have to be less than $1.84 \pm 0.16$, which is certainly not true. Thus we have shown once again that prediction from nonet symmetry does not agree with data.

3. $D \to PV$

The sheer number of independent amplitudes in the PV channels make the analysis much more complicated. Because of the abundance of data, all quantities in the $D^0$ modes can be solved in terms of the relative phase between the two $D^+$ amplitudes. However, as mentioned earlier, there are discrete ambiguities in the solutions, and this makes precise predictions very difficult. So far, we have found no inconsistency between the data and amplitudes shown in table 2. We will see below that this is not the case with $D_s$ decays.

The experimental situation in the $D_s$ modes is somewhat less developed, but there are three sum rules that we can use. We will try to determine the effect they have on the amplitude $S_-$. First of all, from the $KK^*$ channels we have

$$A(K^+K^{*0}) + A(K^{0}K^{*+}) = 2S_- + \frac{2}{5}A(K^0\rho^+) + \frac{2}{5}A(\pi^+K^{*0}) .$$

Putting in numerical values for the amplitudes gives

$$|S_-| \leq 3.64 \pm 0.33 .$$

From the $\pi^+\omega, \phi$ sector, with only an upper limit for the $\omega$ mode, we have

$$2S_- = \sqrt{2}A(\pi^+\omega) + A(\pi^+\phi) ,$$

$$\Rightarrow \quad 0.27 \pm 0.17 \leq |S_-| \leq 2.33 \pm 0.17 .$$

Finally, expressing $\eta_1$ in terms of $\eta, \eta'$ gives

$$\frac{2}{\sqrt{3}}S_- = A(\eta^0\rho^+) \cos \theta - A(\eta\rho^+) \sin \theta$$

$$\theta = -20^\circ : \quad 6.02 \pm 1.13 \leq |S_-| \leq 8.54 \pm 1.38$$

$$\theta = -10^\circ : \quad 6.99 \pm 1.25 \leq |S_-| \leq 8.27 \pm 1.37$$

The relations (5) and (6) are compatible with each other but not with (7). The natural conclusion, of course, would be that there is large nonet symmetry breaking in the $\eta-\eta'$ sector but none in the $\phi-\omega$ sector.
To test the idea of nonet symmetry in the \( \phi \)-\( \omega \) sector we look at pure VV decays in the next section.

4. \( D \to VV \)

In VV channels partial waves with \( L = 0, 1, 2 \) can all contribute. Since the available phase space is generally small and the phase space factor depends on the center of mass momentum as \( p^{2L+1} \), s-waves tend to dominate. This is well supported by data of the \( \rho K^* \) modes [9]. Keeping only s-waves makes the SU(3) amplitudes of the VV modes look very similar to those of the PP modes with the exception that \( D^0 \to \phi \bar{K}^{*0} \) is kinematically forbidden. This means that we will not have enough information to determine both the phase and modulus of the \( B_+ \) amplitude.

Table 3 summarizes the situations. It is obvious that we can still determine \( T_{15} \) and \( S_+ \) from the three \( \rho K^* \) modes. By assuming \( B_+ \) to be zero, we can make a prediction for the \( \omega K^* \) mode:

\[
B(D^0 \to \omega \bar{K}^{*0}) = 2.6 \pm 2.0.
\] (8)

This prediction will, of course, change as the quality of the data improves. If, in the future, this branching ratio turns out to be significantly different from the predicted value, we will have to conclude that \( B_+ \) has to be big. We now turn to the \( D_s \) modes.

In the absence of nonet symmetry breaking, table 3 tells us that \( A_1 \), the \( D^+ \) amplitude, would be 2.5 times as big as \( A_8 \), the \( \phi \rho^+ \) amplitude. Phase space for the two cases are comparable; with the lifetime of the \( D^+ \) being 2.5 times as big as that of the \( D_s \), this translates directly into a factor of more than fifteen for the branching ratios. This is in serious contradiction with data and we are left with the disturbing fact that \( B_- \) is indeed big—something that has not been born out by the PV analysis. Our reluctance to give up nonet symmetry for the vector mesons drives us to look for other explanations. In the next section we will investigate the possibility that a small SU(3) breaking may be responsible for the discrepancy.

5. SU(3) breaking

If we assume that the comparatively large \( s \) quark mass to be solely responsible for the breaking of flavor SU(3) in strong interactions, the effective Hamiltonian will transform as an octet [12]. Coupled to the original SU(3) conserving piece, it gives
us in the PP and VV cases two extra representations \([13]\): \(42^*\) and \(24\), both coming from the \(27\) of \(8 \otimes 8\). They appear in table 4 as \(T_{42}\) and \(T_{24}\).

In the wake of SU(3) breaking, the situation has become much more complicated, but two things remain unchanged. The first is the isospin relation of \(A_1\), \(A_2\), and \(A_3\); isospin is obviously still a good symmetry by design. The second is that the two amplitudes involving the singlet \(\omega_1\) are unaffected. Unfortunately, one of these involves the kinematically forbidden \(\phi K^*\) mode. The other one also gives us the following relation

\[
5B_- = \sqrt{2}A_7 + 3A_8 - 2A_6 .
\]

This relation holds regardless of whether there is SU(3) breaking or not. If we can show that the three amplitudes on the right hand side do not form a closed triangle, then we definitely have nonet symmetry breaking; the contrary, unfortunately, is not true. Presently, there is no measurement of the \(\rho^+ \omega\) branching ratio. We can always let \(B_- = 0\) and look forward to its value allowable under nonet symmetry:

\[
3|A_8| - 2|A_6| \leq \sqrt{2}|A_7| \leq 3|A_8| + 2|A_6|
\]

(9)
gives

\[
1.7 \pm 3.9 \leq \frac{B(D_s \to \rho^+ \omega)}{B(D_s \to \pi^+ \phi)} \leq 86 \pm 27 .
\]

The large upper limit is mainly due to the much larger phase space of the \(\rho \omega\) mode compared to those of the \(K^* K^*\) and the \(\rho \phi\) modes. With the improvement of data, the lower limit may prove to be useful if the central values of the present branching ratios remain unchanged. Returning to \(A_1\) and \(A_8\) and judging from the expressions in table 4, it is inevitable that at least one of \(B_-\), \(T_{24}\), and \(T_{42}\) must be big.

Relation (9) can also be used for the PP channels. The corresponding relation for the pseudoscalar case is exactly relation (4) we obtained earlier. Therefore in the case of \(D \to PP\) we have effectively shown that nonet symmetry is not obeyed regardless of whether there is SU(3) breaking or not.

6. Summary

We have shown in section 2 that nonet symmetry is badly broken in the PP channels. Even with the help of SU(3) breaking, one cannot evade this inevitable consequence. The situation with the PV channels is less clear. It seems that nonet symmetry may still be good in the vector sector but not in the pseudoscalar sector.
Also, we have not found any evidence for SU(3) breaking there. Compared with these two cases, the situation in the VV channels is much more confusing. First of all, we have shown by comparing $D^+ \to \bar{K}^{*0} \rho^+$ and $D_s \to \phi \rho^+$ that at least one of the three symmetry breaking amplitudes must be large. Secondly, we have not been able to rule out the possibility of having large SU(3) breaking effects. In fact, it will not be possible for us to completely rule out the breakdown of either SU(3) or nonet symmetry by studying Cabibbo-allowed decays alone because there are simply too many free parameters. However, if nature cooperates, we may be able to confirm explicitly the breakdown of nonet symmetry in the vector sector. The prospect of seeing this in the VV channels relies on the overall improvement of data and the observation of two channels [relations (8) and (9)] both having the $\omega$ in their final states.

Finally, there is also the possibility of contributions from higher partial waves. In particular, $p$-wave contributions will lead to new SU(3) amplitudes so that there will be as many independent amplitudes in the VV channels as there are in the PV channels. Though unlikely to be the case, this is an issue that can be resolved experimentally. So far, it has not been supported by data [9].

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