Amplitude analysis and branching fraction measurement of the decay $D_s^+ \rightarrow K^+\pi^+\pi^-$

**E-mail:** besiii-publications@ihep.ac.cn

**ABSTRACT:** Using 6.32 fb$^{-1}$ of $e^+e^-$ collision data collected at the center-of-mass energies between 4.178 and 4.226 GeV with the BESIII detector, we perform an amplitude analysis of the decay $D_s^+ \rightarrow K^+\pi^+\pi^-$ and determine the amplitudes of the various intermediate states. The absolute branching fraction of $D_s^+ \rightarrow K^+\pi^+\pi^-$ is measured to be $(6.11 \pm 0.18_{\text{stat}} \pm 0.11_{\text{syst}}) \times 10^{-3}$. The branching fractions of the dominant intermediate processes $D_s^+ \rightarrow K^+\rho^0, \rho^0 \rightarrow \pi^+\pi^-$ and $D_s^+ \rightarrow K^*(892)^0\pi^+, K^*(892)^0 \rightarrow K^+\pi^-$ are determined to be $(1.96 \pm 0.19_{\text{stat}} \pm 0.23_{\text{syst}}) \times 10^{-3}$ and $(1.85 \pm 0.12_{\text{stat}} \pm 0.13_{\text{syst}}) \times 10^{-3}$, respectively. The intermediate resonances $f_0(500), f_0(980)$, and $f_0(1370)$ are observed for the first time in this channel.

**KEYWORDS:** Branching fraction, Charm Physics, $e^+e^-$ Experiments, Particle and Resonance Production

**ArXiv ePrint:** 2205.08844
1 Introduction

One popular approach for studies of hadronic charm decays involves application of approximate flavor symmetries, such as flavor SU(3)$_F$ [1]. However, the SU(3)$_F$ flavor symmetry breaking effect has been observed in $D^0 \rightarrow K^+K^-$ and $D^0 \rightarrow \pi^+\pi^-$ for the first time, and later in the other singly Cabibbo-Suppressed (SCS) charm decays [2]. The SCS decay $D_s^+ \rightarrow K^+\pi^+\pi^-$, with low contamination from other charm decays, is a promising channel to study the SU(3)$_F$ breaking effect. Furthermore, the measurements of the asymmetries of the branching fractions (BFs) of the charge conjugated decays of charmed mesons aid our understanding of charge-parity violation in the charm sector. To date, there have been a few measurements of charge-parity asymmetries, $A_{CP}$, in the SCS $D_s^\pm$ decay modes [3–5].

Two-body charmed meson decays $D_s^\pm \rightarrow VP$, where $V$ and $P$ denote vector and pseudoscalar mesons, respectively, have been studied in various approaches. The theoretical predictions of the BFs of the $D_s^+ \rightarrow K^+\rho^0$ ($\rho^0$ represents $\rho(770)^0$ throughout this paper) and $D_s^+ \rightarrow K^*(892)^0\pi^+$ processes are listed in table 1. References [6, 7] studied these decay channels taking into account the SU(3)$_F$ flavor symmetry breaking effect, while ref. [8] uses a factorisation-assisted topological-amplitude approach with the $\rho-\omega$ mixing. Information about $D_s^+ \rightarrow K^0\rho^+$, $D_s^+ \rightarrow K^*(892)^0\pi^+$ and $D_s^+ \rightarrow K^*(892)^+\pi^0$ has been
| Channel                                      | PDG [2] | Cheng et. al [6] | Wu et. al [7] | Qin et. al [8] |
|----------------------------------------------|---------|------------------|---------------|---------------|
| $D_s^+ \rightarrow K^+\rho^0$                | 2.5 ± 0.4 | 1.22 ± 0.06      | 1.2           | 2.5           |
| $D_s^+ \rightarrow K^+(892)^0\pi^+$          | 2.13 ± 0.36 | 2.06 ± 0.08      | 3.3           | 2.35          |

Table 1. The experimental measurements and theoretical predictions of the BFs of $D_s^+ \rightarrow K^+\rho^0$ and $D_s^+ \rightarrow K^+(892)^0\pi^+$ ($\times 10^{-3}$).

extracted from the decay $D_s^+ \rightarrow K^0\pi^+\pi^0$ [5], but is inconclusive regarding these models. More measurements are needed to confront the theoretical predictions.

The CLEO collaboration has reported the absolute BF of $D_s^+ \rightarrow K^+\pi^+\pi^-$ to be $(0.654 \pm 0.033_{\text{stat.}} \pm 0.025_{\text{syst.}})\%$ [3], using 600 pb$^{-1}$ of $e^+e^-$ collisions recorded at a center-of-mass energy $\sqrt{s} = 4.17$ GeV. An amplitude analysis of this channel has been performed by the FOCUS collaboration with 567 ± 31 signal events [9]. Using 6.32 fb$^{-1}$ of $e^+e^-$ collision data collected with the BESIII detector at $\sqrt{s} = 4.178 - 4.226$ GeV, we perform an amplitude analysis and BF measurement of the $D_s^+ \rightarrow K^+\pi^+\pi^-$ decay with the world’s best precision. Charge conjugation is implied throughout this paper except when discussing CP violation.

2 Detector and data sets

The BESIII detector is a magnetic spectrometer [10, 11] located at the Beijing Electron Positron Collider (BEPCII) [12]. A helium-based multilayer drift chamber (MDC), a plastic scintillator time-of-flight system (TOF), and a CsI(Tl) electromagnetic calorimeter (EMC) compose the cylindrical core of the BESIII detector, and they are all enclosed in a superconducting solenoidal magnet providing a 1.0 T magnetic field. The solenoid is supported by an octagonal flux-return yoke with resistive plate counter muon identifier modules interleaved with steel. The acceptance of charged particles and photons is 93% over a 4π solid angle. The charged-particle momenta resolution at 1.0 GeV/c is 0.5%, and the specific energy loss ($dE/dx$) resolution is 6% for the electrons from Bhabha scattering. The EMC measures photon energies with a resolution of 2.5%(5%) at 1 GeV in the barrel (end-cap) region. The time resolution of the TOF barrel part is 68 ps, while that of the end-cap part is 110 ps. The end-cap TOF was upgraded in 2015 with multi-gap resistive plate chamber technology, providing a time resolution of 60 ps [13–15]. About 83% of the data used in this paper benefits from this upgrade.

Data samples corresponding to a total integrated luminosity of 6.32 fb$^{-1}$ are used in this analysis. The integrated luminosities of the data samples taken at different energy points are listed in table 2 [16–18]. These samples are classified into three sample groups, 4.178, 4.189–4.219, and 4.226 GeV according to the years of data taking and their running conditions. Since the $D_s^{*\pm}$ decays to $\gamma D_s^\pm$ and $\pi^0 D_s^\pm$ with BFs of (93.5±0.7)% and (5.8±0.7)% [2], respectively, the signal events discussed in this paper are selected from the process $e^+e^- \rightarrow D_s^{*\pm} D_s^{\mp} \rightarrow \gamma D_s^+ D_s^-$. Simulated inclusive Monte Carlo (MC) samples, forty times larger than the data sets, are produced with a GEANT4-based [19] MC simulation package, which includes the
The integrated luminosities ($\mathcal{L}_{\text{int}}$) and the requirements on $M_{\text{rec}}$ for various center-of-mass energies. The first and second uncertainties are statistical and systematic, respectively. The definition of $M_{\text{rec}}$ is given in eq. (3.1).

| $\sqrt{s}$ (GeV) | $\mathcal{L}_{\text{int}}$ (pb$^{-1}$) | $M_{\text{rec}}$ (GeV/c$^2$) |
|------------------|-------------------------------------|-----------------------------|
| 4.178            | 3189.0±0.2±31.9                     | [2.050, 2.180]              |
| 4.189            | 526.7±0.1± 2.2                      | [2.048, 2.190]              |
| 4.199            | 526.0±0.1± 2.1                      | [2.046, 2.200]              |
| 4.209            | 517.1±0.1± 1.8                      | [2.044, 2.210]              |
| 4.219            | 514.6±0.1± 1.8                      | [2.042, 2.220]              |
| 4.226            | 1056.4±0.1± 7.0                     | [2.040, 2.220]              |

Table 2. The integrated luminosities ($\mathcal{L}_{\text{int}}$) and the requirements on $M_{\text{rec}}$ for various center-of-mass energies. The first and second uncertainties are statistical and systematic, respectively. The definition of $M_{\text{rec}}$ is given in eq. (3.1).

3 Event selection

The tag method [27] is employed to select clean signal samples of $e^+e^- \rightarrow D_s^{*\pm}D_s^{\mp} \rightarrow \gamma D_s^{\pm}D_s^{-}$ in the following analyses. In this method, a single-tag (ST) candidate requires a reconstructed $D_s^-$ decay to any of the ten hadronic final states listed in table 3. A double-tag (DT) candidate requires that the $D_s^+$ is reconstructed in the signal mode $D_s^+ \rightarrow K^+\pi^+\pi^-$ in addition to the $D_s^-$ decay to one of the tag modes. The selection criteria described here are the common requirements for both amplitude analysis and BF measurement. Further requirements for amplitude analysis and BF measurement are discussed in section 4.1 and section 5, respectively.

All charged tracks reconstructed in the MDC must satisfy $|\cos \theta| < 0.93$, where $\theta$ is the polar angle with respect to the direction of the positron beam. For charged tracks not originating from $K^0_S$ decays, the closest distance to the interaction point is required to be less than $\pm 10$ cm along the beam direction and less than 1 cm in the plane perpendicular to the beam. Particle identification (PID) for charged tracks combines the measurements of the $dE/dx$ in the MDC and the flight time in the TOF to form probabilities $\mathcal{L}(h)$ ($h = K, \pi$) for each hadron ($h$) hypothesis. The charged tracks are assigned as kaons or pions if their probabilities satisfy one of the two hypotheses, $\mathcal{L}(K) > \mathcal{L}(\pi)$ or $\mathcal{L}(\pi) > \mathcal{L}(K)$, respectively.

The $K^0_S$ candidates are selected from all pairs of tracks with opposite charges whose distances to the interaction point along the beam direction are less than 20 cm. The selected tracks are assigned as $\pi^\pm$ and no further PID requirements are applied. A primary vertex
Tag mode | Mass window (GeV/c$^2$)  
--- | ---  
$D_s^- \to K_S^0 K^-$ | [1.948, 1.991]  
$D_s^- \to K^- K^+ \pi^-$ | [1.950, 1.986]  
$D_s^- \to K_S^0 K^- \pi^0$ | [1.946, 1.987]  
$D_s^- \to K^+ K^- \pi^- \pi^0$ | [1.947, 1.982]  
$D_s^- \to K_S^0 K^- \pi^- \pi^+$ | [1.958, 1.980]  
$D_s^- \to K_S^0 K^+ \pi^- \pi^-$ | [1.953, 1.983]  
$D_s^- \to \pi^- \pi^- \pi^+$ | [1.950, 1.987]  
$D_s^- \to \pi^- \eta$ | [1.930, 2.000]  
$D_s^- \to \pi^- \pi^0 \eta$ | [1.920, 2.000]  
$D_s^- \to \pi^- \eta'$ | [1.938, 1.997]  

Table 3. Requirements on $M_{\text{tag}}$ for the ten tag modes.

and a secondary vertex are reconstructed, and the decay length between the two vertexes is required to be greater than twice its uncertainty. The $K_S^0$ candidates are required to have a $\pi^+ \pi^-$ invariant mass ($M_{\pi^+ \pi^-}$) in the range [0.487, 0.511] GeV/c$^2$.

The photon candidates are selected using the EMC showers. The minimum deposited energy of each shower in the barrel region ($|\cos \theta| < 0.80$) and in the end-cap region ($0.86 < |\cos \theta| < 0.92$) must be greater than 25 MeV and 50 MeV, respectively. The opening angle between the location of each shower in the EMC and the extrapolated position of the closest charged track must be greater than 10 degrees to reject showers originating from charged tracks. The shower is required to start within [0, 700] ns from the event time to suppress electronic noise and showers unrelated to the event.

The $\pi^0$ and $\eta$ candidates are reconstructed from photon pairs with invariant masses in the ranges [0.115, 0.150] GeV/c$^2$ and [0.500, 0.570] GeV/c$^2$, respectively, which correspond to about three standard deviations of the invariant mass resolutions. To improve their invariant mass resolutions, we require that at least one photon comes from the barrel region of the EMC. A kinematic fit constraining the $\gamma \gamma$ invariant mass to the $\pi^0$ or $\eta$ known mass [2] is performed. The $\chi^2$ of the kinematic fit is required to be less than 30. The $\eta'$ candidates are formed from $\pi^+ \pi^- \eta$ combinations with an invariant mass within the range of [0.946, 0.970] GeV/c$^2$.

Ten tag modes are reconstructed and the corresponding mass windows on the tag $D_s^-$ mass ($M_{\text{tag}}$) are listed in table 3 [5]. The quantity $M_{\text{rec}}$ is defined as

$$M_{\text{rec}} = \sqrt{E_{\text{cm}}^2 - (|\vec{p}_{D_s^-}|^2 + m_{D_s^-}^2)} - |\vec{p}_{D_s^-}|^2, \quad (3.1)$$

where $E_{\text{cm}}$ is the energy of the initial state measured from the beam energy, $|\vec{p}_{D_s^-}|$ is the three-momentum of the $D_s^-$ candidate in the $e^+e^-$ center-of-mass frame and $m_{D_s^-}$ is the nominal $D_s^-$ mass [2]. The mass windows of $M_{\text{rec}}$ for $D_s^-$ candidates at each center-of-mass
energy are listed in table 2 and help to suppress backgrounds that come from non-$D_s^{*\pm}D_s^\mp$ processes [28].

4 Amplitude analysis

4.1 Event selection

Further selection criteria used to improve the signal purity for the amplitude analysis are described next. These criteria will not be used in the BF measurement.

An extra photon candidate for the process $D_s^{*\pm} \rightarrow \gamma D_s^\pm$ (satisfying the requirements in section 3) is selected in order to perform a six-constraint (6C) kinematic fit. This fit constrains the $D_s^-$ decay to the utilized tag mode and the $D_s^+$ decay to the signal mode with two hypotheses: the signal $D_s^+$ comes from a $D_s^0$ or the tag $D_s^-$ comes from a $D_s^{*+}$. The total four-momentum is constrained to the initial four-momentum of the $e^+e^-$ system and the invariant masses of tag $D_s^-$ and $D_s^{*\pm}$ candidates are constrained to the corresponding nominal masses. The combination with the minimum $\chi^2$ is chosen. A $\chi^2_{6C} < 200$ requirement is applied to suppress background from other processes. To ensure that all events fall into the phase-space (PHSP) boundary, a mass constraint on the signal $D_s^+$ is added to the 6C kinematic fit. The four-momenta of the final state particles from this new 7C kinematic fit are used in the amplitude analysis.

The $D_s^+ \rightarrow K^0_{S}(\rightarrow \pi^+\pi^-)K^+$ candidates are the dominant background for the $D_s^+ \rightarrow K^+\pi^+\pi^-$ decay. These backgrounds are rejected by requiring $M_{\pi^+\pi^-}$ being outside the mass interval of $[0.4676, 0.5276]$ GeV/c$^2$. The fits to the invariant-mass distributions of the selected signal $D_s^+$ candidates ($M_{\text{sig}}$) for various data samples are shown in figure 1. In the fits, the signal is described by the MC-simulated shape convolved with a Gaussian function describing the data-MC resolution difference, and the background is described with the shape derived from the inclusive MC sample. Requiring $M_{\text{sig}} \in [1.950, 1.986]$ GeV/c$^2$, we obtain 772, 444 and 140 signal candidates with purities of $(95.7 \pm 0.7)\%$, $(95.2 \pm 1.0)\%$, and $(92.6 \pm 2.2)\%$ for the data samples at $\sqrt{s} = 4.178$, 4.189-4.219, and 4.226 GeV, respectively.

4.2 Fit method

The amplitude analysis of $D_s^+ \rightarrow K^+\pi^+\pi^-$ is performed by an unbinned maximum likelihood fit. The likelihood function $\mathcal{L}$ is constructed with a signal-background combined probability density function (PDF). The log-likelihood is written as

$$
\ln \mathcal{L} = \sum_{i=1}^{3} \sum_k N_{D,i} \ln [\omega^i f_S(p^k) + (1 - \omega^i) f_B(p^k)],
$$

where $i$ indicates the data sample groups. The $p^k$ denote the four-momenta of the final state particles $K^+$, $\pi^+$, and $\pi^-$, where $k$ denotes the $k^{th}$ event in data $i$. The $N_{D,i}$ are the number of candidates in data $i$, $f_S(f_B)$ is the signal (background) PDF and the $\omega^i$ are the purities of the signals discussed in section 4.1.
Figure 1. Fits to the $M_{\text{sig}}$ distributions of the accepted signal candidates from the data samples at $\sqrt{s} = (a) 4.178$ GeV, (b) 4.189–4.219 GeV and (c) 4.226 GeV. The black points with error bars are data. The blue solid lines are the total fits. The red dotted and black dashed lines are the fitted signals and backgrounds, respectively. The pairs of red arrows indicate the signal regions.

The signal PDF $f_S(p)$ is given by

$$f_S(p) = \frac{\epsilon(p)|\mathcal{M}(p)|^2R_3(p)}{\int \epsilon(p)|\mathcal{M}(p)|^2R_3(p) \, dp}, \quad (4.2)$$

where $\epsilon(p)$ is the detection efficiency and $R_3(p)$ is the three-body PHSP function. The $R_3(p)$ is defined as

$$R_3(p) = \delta \left( p_{D_s} - \sum_{j=1}^{3} p_j \right) \prod_{j=1}^{3} \delta \left( p_j^2 - m^2 \right) \theta(E_j), \quad (4.3)$$

where $j$ runs over the three daughter particles, $E_j$ is the energy of particle $j$ and $\theta(E_j)$ is the step function. The total amplitude $\mathcal{M}$ is treated with the isobar model, which uses the coherent sum of the amplitudes of the intermediate processes, $\mathcal{M}(p) = \sum c_n A_n(p)$, where $c_n = \rho_n e^{i\phi_n}$ is the corresponding complex coefficient. The magnitude $\rho_n$ and phase $\phi_n$ are the free parameters in the fit. The amplitude of the $n^{\text{th}}$ intermediate state ($A_n$) is

$$A_n(p) = P_n(p) S_n(p) F^r_n(p) F^D_n(p). \quad (4.4)$$

Here, $P_n(p)$ is the propagator of the intermediate resonance, $S_n(p)$ is the spin factor, $F^r_n(p)$ and $F^D_n(p)$ are the Blatt-Weisskopf barrier factors for the intermediate resonance and $D_s^+$, respectively.
The background PDF is given by
\[ f_B(p) = \frac{B_{\epsilon}(p)R_3(p)}{\int \epsilon(p)B_{\epsilon}(p)R_3(p)dp}, \quad (4.5) \]
where \( B_{\epsilon}(p) = B(p)/\epsilon(p) \) is the efficiency-corrected background shape. The shape of the background in data is modeled by the background events in the signal region derived from the inclusive MC samples. The comparisons of the \( M_{K^{+}\pi^{+}} \), \( M_{K^{+}\pi^{-}} \) and \( M_{\pi^{+}\pi^{-}} \) distributions of events outside the \( D_s^{+} \) mass signal region between data and MC simulation validate the description from the inclusive MC samples. We have also examined the distributions of the background events in the inclusive MC samples inside and outside the \( D_s^{+} \) mass signal region. Generally, they are compatible with each other within statistical uncertainties. The background shape \( B(p) \) is modeled using RooNDKeysPDF [29], which is a kernel estimation method [30] implemented in RooFit [29] to model the distribution of an input dataset as a superposition of Gaussian kernels.

In the numerator of eq. (4.2), the \( \epsilon(p) \) and \( R_3(p) \) terms are independent of the fitted variables, so they are regarded as constants in the fit. As a consequence, the log-likelihood becomes
\[ \ln L = \sum_{i=1}^{3} \sum_{k=1}^{N_{MC}} \ln \left[ \omega_i \frac{|M(p^k)|^2}{\int \epsilon(p^k)|M(p^k)|^2R_3(p^k)dp^k} + (1-\omega_i) \frac{B_{\epsilon}(p^k)}{\int \epsilon(p^k)B_{\epsilon}(p^k)R_3(p^k)dp^k} \right]. \quad (4.6) \]
The normalization integrals of signal and background are evaluated by MC integration,
\[ \int \epsilon(p)|M(p)|^2R_3(p)dp \approx \frac{1}{N_{MC}} \sum_{k=1}^{N_{MC}} |M(p^k)|^2 \left| \frac{M_{\text{gen}}(p^k)}{\epsilon_{\text{gen}}(p^k)} \right| \quad (4.7) \]
\[ \int \epsilon(p)B_{\epsilon}(p)R_3(p)dp \approx \frac{1}{N_{MC}} \sum_{k=1}^{N_{MC}} B_{\epsilon}(p^k) \left| \frac{M_{\text{gen}}(p^k)}{\epsilon_{\text{gen}}(p^k)} \right|^2, \]
where \( k \) is the index of the \( k^{th} \) event of the MC sample and \( N_{MC} \) is the number of the selected MC events. The \( M_{\text{gen}}(p) \) is the signal PDF used to generate the MC samples in MC integration.

Tracking and PID differences between data and MC simulation are corrected for by multiplying the weight of the MC event by a factor \( \gamma_{\epsilon} \), which is calculated as
\[ \gamma_{\epsilon}(p) = \prod_{n} \frac{\epsilon_{n,\text{data}}(p)}{\epsilon_{n,\text{MC}}(p)}, \quad (4.8) \]
where \( n \) refers to tracking or PID, \( \epsilon_{n,\text{data}}(p) \) and \( \epsilon_{n,\text{MC}}(p) \) are the tracking or PID efficiency as a function of the momenta of the daughter particles for data and MC simulation, respectively. The tracking and PID efficiencies are studied using clean samples of
\[ e^+e^- \rightarrow K^+K^-K^+K^-, \quad e^+e^- \rightarrow K^+K^-\pi^+\pi^-, \quad e^+e^- \rightarrow K^+K^-\pi^+\pi^0, \quad e^+e^- \rightarrow \pi^+\pi^-\pi^+\pi^- \quad \text{and} \quad e^+e^- \rightarrow \pi^+\pi^-\pi^+\pi^-\pi^0 \]
processes. By weighting each signal MC event with \( \gamma_{\epsilon} \), the MC integration is given by
\[ \int \epsilon(p)|M(p)|^2R_3(p)dp \approx \frac{1}{N_{MC}} \sum_{k=1}^{N_{MC}} \gamma_{\epsilon}(p^k_{\text{MC}})|M(p^k)|^2 \left| \frac{M_{\text{gen}}(p^k)}{\epsilon_{\text{gen}}(p^k)} \right|^2. \quad (4.9) \]
4.2.1 Propagator

The intermediate resonances $f_0(1370)$, $K^*(892)$ and $K^*(1410)^0$ are parameterized with the relativistic Breit-Wigner (RBW) formulas,

$$P(m) = \frac{1}{(m_0^2 - m^2) - im_0 \Gamma(m)},$$

$$\Gamma(m) = \Gamma_0 \left( \frac{q}{q_0} \right)^{2L+1} \left( \frac{m_0}{m} \right) \left( \frac{X_L(q)}{X_L(q_0)} \right)^2,$$  \begin{equation} \tag{4.10} \end{equation}

where $m^2$ is the invariant mass squared of the daughter particles of the intermediate resonances, $m_0$ and $\Gamma_0$ are the mass and width of the intermediate resonance, which are fixed to 1350 MeV/$c^2$ and 265 MeV [31], respectively, for $f_0(1370)$, and to the PDG values for the other resonances. In a process $a \to bc$, the variable $q$ is defined as

$$q = \sqrt{(s_a + s_b - s_c)^2 - s_a},$$  \begin{equation} \tag{4.11} \end{equation}

where $s_a, s_b,$ and $s_c$ are the invariant-mass squared of particles $a$, $b$ and $c$, respectively. The value of $q_0$ in eq. (4.10) is that of $q$ when $s_a = m_a^2$, where $m_a$ is the mass of particle $a$.

The $\rho^0$ and $\rho(1450)$ mesons are parameterized as the Gounaris-Sakurai (GS) line shape [32], which is given by

$$P_{ GS}(m) = \frac{1 + d f_{m_0}^2}{m_0^2 - m^2} + f(m) - im_0 \Gamma(m),$$  \begin{equation} \tag{4.12} \end{equation}

where

$$f(m) = \Gamma_0 \frac{m_0^2}{q_0^2} \left[ q^2(h(m) - h(m_0)) + (m_0^2 - m^2)q_0^2 \left| \frac{dh}{dm^2} \right|_{m^2=m_0^2} \right],$$  \begin{equation} \tag{4.13} \end{equation}

and the function $h(m)$ is defined as

$$h(m) = \frac{2}{\pi} \ln \left( \frac{m + 2q}{2m_\pi} \right),$$  \begin{equation} \tag{4.14} \end{equation}

with

$$\left. \frac{dh}{dm^2} \right|_{m^2=m_0^2} = h(m_0) [(8q_0^2)^{-1} - (2m_0^2)^{-1}] + (2\pi m_0^2)^{-1},$$  \begin{equation} \tag{4.15} \end{equation}

where $m_\pi$ is the mass of $\pi$, and the normalization condition at $P_{ GS}(0)$ fixes the parameter $d = f_{(0)}^{(0)}/m_0$. It is found to be

$$d = \frac{3}{\pi} \frac{m_\pi^2}{q_0^2} \ln \left( \frac{m_0 + 2q_0}{2m_\pi} \right) + \frac{m_0}{2q_0} - \frac{m_\pi^2m_0}{\pi q_0^2}. $$  \begin{equation} \tag{4.16} \end{equation}

The $f_0(980)$ is parameterized with the Flatté formula [31]:

$$P_{ f_0(980)} = \frac{1}{M_{ f_0(980)}^2 - m^2 - i(\alpha \rho_{\pi\pi}(m^2) + \alpha_{K\bar{K}} \rho_{K\bar{K}}(m^2))},$$  \begin{equation} \tag{4.17} \end{equation}

where $g_{\pi\pi,K\bar{K}}$ are the constants coupling to individual final states. The parameters are fixed to be $g_{\pi\pi} = (0.165 \pm 0.010 \pm 0.015) \text{GeV}^2/c^4$, $g_{K\bar{K}} = (4.21 \pm 0.25 \pm 0.21) g_{\pi\pi}$ and $M =
where the parameters with 

\[ \rho = \frac{2}{3} \sqrt{1 - \frac{4m_{\pi}^2}{m^2}} + \frac{1}{3} \sqrt{1 - \frac{4m_{\pi}^2}{m^2}}, \]

\[ \rho_{K\bar{K}} = \frac{1}{2} \sqrt{1 - \frac{4m_{K\bar{K}}^2}{m^2}} + \frac{1}{2} \sqrt{1 - \frac{4m_{K\bar{K}}^2}{m^2}}. \]  

(4.18)

The resonance \( f_0(500) \) is parameterized with the formula given in ref. [33]:

\[ P_{f_0(500)} = \frac{1}{m_0^2 - m^2 - i\alpha_0 \Gamma_{tot}(m)}, \]

(4.19)

where \( \Gamma_{tot}(m) \) is decomposed into two parts:

\[ \Gamma_{tot}(m) = g_{\pi\pi} \rho_{\pi\pi}(m) + g_{\pi\pi} \rho_{4\pi}(m) \]

(4.20)

and

\[ g_{\pi\pi} = (b_1 + b_2 m^2) \left( \frac{m^2 - m_{\pi}^2/2}{m_{\pi}^2 - m_{\pi}^2/2} \right) e^{(m_{\pi}^2 - m_{\pi}^2)/a}, \]

(4.21)

where \( \rho_{\pi\pi} \) is the PHSP of the \( \pi^+\pi^- \) system and \( \rho_{4\pi} \) is the PHSP of the \( 4\pi \) system and is approximated by

\[ \rho_{4\pi} = \sqrt{(1 - 16m_{\pi}^2/m^2)} \]

(4.22)

with the parameters fixed to the values given in ref. [34].

The \( K_0^*(1430) \) is parameterized with the Flatté formula:

\[ P_{K_0^*(1430)} = \frac{1}{M_{K_0^*(1430)}^2 - m^2 - i\Gamma_{K_0^*(1430)}(m^2) [g_{K\pi\rho_{K\pi}(m^2)} + g_{\rho_{K\pi\rho_{K\pi}}}(m^2)]}, \]

(4.23)

where \( \rho_{K\pi}(m^2) \) and \( \rho_{\rho_{K\pi}}(m^2) \) are the Lorentz invariant PHSP factors, and \( g_{K^+\pi^-\rho_{K\pi}} \) are the constants coupling to individual final states. The parameters of the \( K_0^*(1430) \) are fixed to \( M = 1471.2 \text{ MeV}/c^2 \), \( g_{K^+\pi^-} = 546.8 \text{ MeV}/c^2 \), and \( g_{\rho_{K\pi}} = 230 \text{ MeV}/c^2 \), from CLEO [35].

The \( K\pi \) S-wave modeled by the LASS parameterization [36] is described by a \( K_0^*(1430) \) Breit-Wigner together with an effective range non-resonant component with a phase shift. It is given by

\[ A(m) = F\sin\delta_F e^{i\delta_F} + R\sin\delta_R e^{i\delta_R} e^{2i\delta_F}, \]

(4.24)

with

\[ \delta_F = \phi_F + \cot^{-1} \left[ \frac{1}{aq} + \frac{rq}{2} \right], \]

\[ \delta_R = \phi_R - \tan^{-1} \left[ \frac{M\Gamma(m_{K^*})}{M^2 - m_{K^*}^2} \right], \]

(4.25)

where the parameters \( F(\phi_F) \) and \( R(\phi_R) \) are the magnitudes (phases) for non-resonant state and resonance terms, respectively. The parameters \( a \) and \( r \) are the scattering length and effective interaction length, respectively. We fix these parameters \( (M, \Gamma, F, \phi_F, R, \phi_R, a, r) \) to the results obtained from the amplitude analysis to a sample of \( D^0 \to K^0 \pi^+\pi^- \) by the BABAR and Belle experiments [37]; these parameters are summarised in table 4.
Table 4. The $K\pi$ S-wave parameters, obtained from the amplitude analysis of $D^0 \rightarrow K^0 S \pi^+ \pi^-$ by the BABAR and Belle experiments [37]. Uncertainties are statistical only.

| Parameter | Value |
|-----------|-------|
| $M$ (GeV/c$^2$) | $1.441 \pm 0.002$ |
| $\Gamma$ (GeV) | $0.193 \pm 0.004$ |
| $F$ | $0.96 \pm 0.07$ |
| $\phi_F$ (°) | $0.1 \pm 0.3$ |
| $R$ | 1 (fixed) |
| $\phi_R$ (°) | $-109.7 \pm 2.6$ |
| $a$ (GeV/c$^{-1}$) | $0.113 \pm 0.006$ |
| $r$ (GeV/c$^{-1}$) | $-33.8 \pm 1.8$ |

4.2.2 Spin factors

The spin-projection operators are defined as [38]

$$P^0(a) = 1,$$

$$P^{(1)}_{\mu\nu}(a) = -g_{\mu\nu} + \frac{p_a^\mu p_a^\nu}{p_a^2}, \quad (S \text{ wave})$$

$$P^{(2)}_{\mu_1\nu_2\mu_2\nu_1}(a) = \frac{1}{2} \left( P^{(1)}_{\mu_1\nu_1} P^{(1)}_{\mu_2\nu_2} + P^{(1)}_{\mu_1\nu_2} P^{(1)}_{\mu_2\nu_1} \right) - \frac{1}{3} P^{(1)}_{\mu_1\mu_2} P^{(1)}_{\nu_1\nu_2}, \quad (D \text{ wave})$$

The spin factors for $S, P,$ and $D$ wave decays are

$$S_n = 1, \quad (S \text{ wave})$$

$$S_n = \tilde{T}^{(1)\mu}(D_s) \tilde{T}^{(1)}_{\mu}(a), \quad (P \text{ wave})$$

$$S_n = \tilde{T}^{(2)\mu\nu}(D_s) \tilde{T}^{(2)}_{\mu\nu}(a), \quad (D \text{ wave})$$

where $\tilde{T}$ has the same definition as $\tilde{t}$. The tensor describing the $D^+_s$ decays is denoted by $\tilde{T}$ and that of $a$ decays is denoted by $\tilde{t}$.

4.2.3 Blatt-Weisskopf barriers

For a decay process $a \rightarrow b c$, the Blatt-Weisskopf barriers factors [39] depend on the angular momenta $L$ and the momentum $q$ of the final-state particle $b$ or $c$ in the rest system of $a$. 

[4.28]
Figure 2. The Dalitz plots of $M_{K^+\pi^-}^2$ versus $M_{\pi^+\pi^-}^2$ of the selected DT candidates from (a) the data sample and (b) the signal MC sample generated based on the amplitude analysis results at $\sqrt{s} = 4.178 - 4.226$ GeV. The black lines indicate the physical border.

They are taken as

$$X_{L=0}(q) = 1,$$

$$X_{L=1}(q) = \sqrt{\frac{z_0^2 + 1}{z^2 + 1}},$$

$$X_{L=2}(q) = \sqrt{\frac{4z_0^2 + 3z_0^2 + 9}{z^4 + 3z^2 + 9}},$$

where $z = qR$ and $z_0 = q_0R$ with $q_0$ defined in section 4.2.1. The effective radius of barrier $R$ is fixed to be $3.0 \text{ GeV}^{-1}$ for the intermediate resonances and $5.0 \text{ GeV}^{-1}$ for the $D_s^+$ meson.

4.3 Fit results

Figures 2(a) and 2(b) show the Dalitz plots of $M_{K^+\pi^-}^2$ versus $M_{\pi^+\pi^-}^2$ of the selected DT candidates from the data samples and the signal MC samples generated based on the results of the amplitude analysis, respectively. In the fit, the magnitude and phase of the reference amplitude $D_s^+ \to K^+ \rho^0$ are fixed to 1.0 and 0.0, respectively, while those of the other amplitudes are left floating. The $\omega^i$ values are fixed to the purities given in section 4.1.

In addition to the dominant amplitudes of $D_s^+ \to K^+ \rho^0$ and $D_s^+ \to K^+(892)^0 \pi^+$, we have also tested other possible intermediate resonances, including $K^+(1410)^0$, $K_0^*(1430)^0$, $f_0(500)$, $f_0(980)$, $f_0(1370)$, $\rho(1450)^0$, $(K^+\pi^-)_{S\text{-wave}}$ (using LASS parameterization [36] or K-matrix [40]), etc. Finally, the amplitudes of $D_s^+ \to K^+ \rho^0$, $D_s^+ \to K^+ \rho(1450)^0$, $D_s^+ \to K_f(500)$, $D_s^+ \to K_f(980)$, $D_s^+ \to K^+ f_0(1370)$, $D_s^+ \to K^* f_0(892)$, $D_s^+ \to K^*(892)^0 \pi^+$, $D_s^+ \to K^* (1410)^0 \pi^+$, and $D_s^+ \to K_0^*(1430)^0 \pi^+$, which have statistical significances greater than five standard deviations, are retained in the nominal fit. The statistical significances are determined from the changes in log-likelihood and the numbers of degrees of freedom (NDOF) between the fits with a given amplitude included or excluded.

The PHSP MC truth information without detector acceptance and resolution effects is used to calculate the fit fractions (FFs) for individual amplitudes. The FF for the $n^{th}$
The phases, FFs and statistical significances for various amplitudes in the nominal fit. The first and second uncertainties of the phases and FFs are statistical and systematic, respectively. The total FF is 130.1%.

\begin{table}[h]
\centering
\begin{tabular}{|c|c|c|c|}
\hline
Amplitude & Phase \(\phi_n\) (rad) & FF(\%) & Statistical significance(\(\sigma\)) \\
\hline
\(D_s^+ \rightarrow K^+\rho^0\) & 0.0 (fixed) & 32.1 \(\pm\) 3.7 \(\pm\) 3.7 & >10 \\
\(D_s^+ \rightarrow K^+\rho(1450)^0\) & 2.74 \(\pm\) 0.14 \(\pm\) 0.24 & 13.1 \(\pm\) 3.1 \(\pm\) 2.9 & >10 \\
\(D_s^+ \rightarrow K^+f_0(500)\) & 1.01 \(\pm\) 0.17 \(\pm\) 0.28 & 7.2 \(\pm\) 2.1 \(\pm\) 4.4 & 6.8 \\
\(D_s^+ \rightarrow K^+f_0(980)\) & 5.05 \(\pm\) 0.15 \(\pm\) 0.17 & 4.5 \(\pm\) 1.3 \(\pm\) 1.2 & 6.9 \\
\(D_s^+ \rightarrow K^+f_0(1370)\) & 6.04 \(\pm\) 0.14 \(\pm\) 0.26 & 19.9 \(\pm\) 2.9 \(\pm\) 9.3 & >10 \\
\(D_s^+ \rightarrow K^*(892)^0\pi^+\) & 3.03 \(\pm\) 0.08 \(\pm\) 0.04 & 30.2 \(\pm\) 1.8 \(\pm\) 2.0 & >10 \\
\(D_s^+ \rightarrow K^*(1410)^0\pi^+\) & 5.60 \(\pm\) 0.14 \(\pm\) 0.09 & 4.5 \(\pm\) 2.1 \(\pm\) 2.5 & 5.2 \\
\(D_s^+ \rightarrow K_0^*(1430)^0\pi^+\) & 1.90 \(\pm\) 0.19 \(\pm\) 0.20 & 18.5 \(\pm\) 2.5 \(\pm\) 2.6 & 8.6 \\
\hline
\end{tabular}
\caption{The phases, FFs and statistical significances for various amplitudes in the nominal fit.}
\end{table}

The interference between amplitudes is listed in table 6. The statistical significances for amplitudes tested but not included in the nominal fit are listed in table 7.

The mass projections of the nominal fit for the amplitude analysis are shown in figure 3. Their systematic uncertainties will be discussed in next section. The sum of the FFs is not unity due to interferences among amplitudes.

### 4.4 Systematic uncertainties for the amplitude analysis

The systematic uncertainties for the amplitude analysis are summarized in table 8, and are described below.

- Fixed parameters in the amplitudes. The masses and widths of \(K^*(892)\) and \(K^*(1410)^0\) are shifted by their corresponding uncertainties [2]. The mass and width of \(f_0(1370)\) are shifted according to the uncertainties from ref. [31]. The masses and coupling constants of the \(f_0(980)\) and \(K^*(1430)\) Flatté formulas are varied according to ref. [31] and ref. [35], respectively. The uncertainties of the lineshapes of \(\rho^+\) and \(\rho(1450)^0\)
Table 6. Interference between amplitudes, in unit of % of total amplitude. I denotes $D_s^+ \rightarrow K^+ \rho^0$, II $D_s^+ \rightarrow K^+ \rho(1450)^0$, III $D_s^+ \rightarrow K^+ f_0(500)$, IV $D_s^+ \rightarrow K^+ f_0(980)$, V $D_s^+ \rightarrow K^+ f_0(1370)$, VI $D_s^+ \rightarrow K^*(892)^0\pi^+$, VII $D_s^+ \rightarrow K^*(1410)^0\pi^+$, and VIII $D_s^+ \rightarrow K_0^*(1430)^0\pi^+$. The uncertainties are statistical only.

|     | I     | II    | III   | IV   | V     | VI    | VII   | VIII  |
|-----|-------|-------|-------|------|-------|-------|-------|-------|
| I   | 32.1 ± 3.7 | 1.8 ± 3.0 | 0.0 ± 0.0 | 0.0 ± 0.0 | 0.0 ± 0.0 | -6.5 ± 0.4 | 1.1 ± 0.8 | -7.2 ± 0.6 |
| II  | 13.1 ± 3.1 | 0.0 ± 0.0 | 0.0 ± 0.0 | 0.0 ± 0.0 | -5.4 ± 1.3 | -4.8 ± 1.0 | 3.9 ± 2.7 |
| III | 7.2 ± 2.1  | -4.8 ± 1.4 | -6.1 ± 2.5 | 2.9 ± 0.5 | -1.3 ± 0.8 | 4.5 ± 1.1 |
| IV  | 4.5 ± 1.3  | 10.7 ± 1.6 | -2.3 ± 0.4 | 0.3 ± 0.4 | -3.9 ± 0.7 |
| V   | 19.9 ± 2.9 | -8.4 ± 0.8 | 0.5 ± 1.1 | -9.4 ± 2.0 |
| VI  | 30.2 ± 1.8 | 4.4 ± 0.9  | 0.0 ± 0.0 |
| VII | 4.5 ± 2.1  | 0.0 ± 0.0 |
| VIII| 18.5 ± 2.5 |

Table 7. Statistical significances for amplitudes tested, but not included in the nominal fit.

| Amplitude                        | Statistical significance(σ) |
|----------------------------------|-----------------------------|
| $D_s^+ \rightarrow K^+ f_2(1270)$ | 2.3                         |
| $D_s^+ \rightarrow K^+ f_0(1500)$ | 3.1                         |
| $D_s^+ \rightarrow K^+ f_0'(1525)$ | 2.3                         |
| $D_s^+ \rightarrow K^*(1680)^0\pi^+$ | 3.3                         |
| $D_s^+ \rightarrow (K^+\pi^-)_{S-wave}\pi^+$ | <1                           |

are estimated by replacing the GS with the RBW formula. The uncertainties of the lineshape of $f_0(500)$ are estimated by replacing the propagator with a RBW function with the mass and width fixed at 526 MeV and 534 MeV, respectively [34]. The changes of the phases $\phi$ and FFs are assigned as the associated systematic uncertainties.

ii $R$ values. The estimation of the systematic uncertainty associated with the $R$ parameters in the Blatt-Weisskopf factors is performed by repeating the fit procedure after varying the radii of the intermediate states and $D_s^+$ mesons by $\pm 1$ GeV$^{-1}$.

iii Fit bias. An ensemble of 600 signal MC samples is generated according to the results of the amplitude analysis. The pull distribution, which is supposed to be a normal distribution, is used to validate the fit performance. The fitted pull values for FFs of $D_s^+ \rightarrow K^+\rho^0$, $D_s^+ \rightarrow K^+ f_0(980)$ and $D_s^+ \rightarrow K_0^*(1430)^0\pi^+$ and the fitted pull values for phases of $D_s^+ \rightarrow K^+ f_0(500)$, $D_s^+ \rightarrow K^+ f_0(980)$ and $D_s^+ \rightarrow K^*(892)^0\pi^+$ deviate from zero by more than three, but less than five, standard deviations. Hence, the differences between input values and average fit results are taken as the systematic uncertainties.

iv Background estimation. The fractions of signal, i.e. $\omega^i$ in eq. (4.1), are varied within their uncertainties and the largest difference from the fits is taken as the uncertainty from the background level. The uncertainty corresponding to the background shape is determined by replacing the input parameters (keeping $M_{K^+\pi^-}^2$ but replacing $M_{\pi^+\pi^-}^2$ with $M_{K^+\pi^+}^2$) and changing the smoothing parameters in RooNDKeysPdf [29].
Figure 3. The projections on (a) $M_{K^+\pi^+}$, (b) $M_{K^+\pi^-}$, and (c) $M_{\pi^+\pi^-}$ of the nominal fit. The data samples at $\sqrt{s} = 4.178 - 4.226$ GeV are represented by points with error bars, the fit results by the solid blue lines, and the background estimated from the inclusive MC samples by the black dashed lines. Colored curves show the components of the fit model. Due to interference effects, the total is not necessarily equal to the linear sum of the components. Pull projections are shown beneath each distribution; if there are less than 10 events in a bin, that bin is merged with the next bin until the number of events is larger than or equal to 10.

v Experimental effects. The systematic uncertainty from knowledge of the $\gamma_c$ factors in eq. (4.9), which correct for data-MC differences in tracking and PID efficiencies, is evaluated by performing the fit after varying the weights according to their uncertainties.

vi Insignificant amplitudes. The intermediate resonances with statistical significances less than $5\sigma$ in table 7 are added to the model one by one. The largest variations from the nominal result are taken as the corresponding systematic uncertainties.
| Amplitude                  | Source | i  | ii | iii | iv | v  | vi | Total |
|----------------------------|--------|----|----|-----|----|----|----|-------|
| $D_s^+ \to K^+ \rho^0$     | FF     | 1.10 | 0.58 | 0.05 | 0.04 | 0.01 | 0.15 | 1.26 |
| $D_s^+ \to K^+ \rho(1450)^0$ | $\phi$ | 1.62 | 0.63 | 0.13 | 0.07 | 0.06 | 0.14 | 1.75 |
| $D_s^+ \to K^+ f_0(500)$   | FF     | 1.77 | 0.43 | 0.12 | 0.00 | 0.33 | 0.83 | 2.04 |
| $D_s^+ \to K^+ f_0(980)$   | $\phi$ | 1.82 | 0.07 | 0.25 | 0.07 | 0.00 | 0.47 | 1.11 |
| $D_s^+ \to K^+ f_0(1370)$  | $\phi$ | 0.99 | 0.13 | 0.19 | 0.07 | 0.00 | 0.47 | 1.11 |
| $D_s^+ \to K^* (892)^0 \pi^+$ | $\phi$ | 0.45 | 0.13 | 0.05 | 0.13 | 0.00 | 0.13 | 0.50 |
| $D_s^+ \to K^* (1410)^0 \pi^+$ | $\phi$ | 0.56 | 0.31 | 0.14 | 0.07 | 0.08 | 0.07 | 0.67 |
| $D_s^+ \to K^*_0 (1430)^0 \pi^+$ | $\phi$ | 0.90 | 0.20 | 0.05 | 0.10 | 0.00 | 0.53 | 1.07 |

**Table 8.** Systematic uncertainties on the $\phi$ and FF for each amplitude in units of the corresponding statistical uncertainty. The sources are: (i) fixed parameters in the amplitudes, (ii) $R$ values, (iii) fit bias, (iv) background estimation, (v) experiment effects, (vi) insignificant amplitudes.

5 BF measurement

The BF measurements are based on the following equations:

$$N_{\text{tag}}^{\text{ST}} = 2N_{D_s^+ D_s^-} B_{\text{tag}}^{\text{ST}},$$  \hspace{1cm} (5.1)

$$N_{\text{tag,sig}}^{\text{DT}} = 2N_{D_s^+ D_s^-} B_{\text{tag}}^{\text{DT}},$$  \hspace{1cm} (5.2)

where $N_{\text{tag}}^{\text{ST}}$ is the ST yield for the tag mode, $N_{\text{tag,sig}}^{\text{DT}}$ is the DT yield, $N_{D_s^+ D_s^-}$ is the total number of $D_s^+ D_s^-$ pairs produced from the $e^+ e^-$ collisions, $B_{\text{tag}}$ and $B_{\text{sig}}$ are the BFs of the tag and signal modes, respectively. The $\epsilon_{\text{tag}}^{\text{ST}}$ is the efficiency to reconstruct the tag mode alone and $\epsilon_{\text{tag,sig}}^{\text{DT}}$ is the efficiency to reconstruct both the tag and signal modes. In the case of more than one tag mode and energy group,

$$N_{\text{total}}^{\text{DT}} = \sum_{\alpha,i} N_{\alpha,i}^{\text{DT}} = B_{\text{sig}} \sum_{\alpha,i} 2N_{D_s^+ D_s^-} B_{\alpha} \epsilon_{\alpha,sig,i}^{\text{DT}},$$  \hspace{1cm} (5.3)

where $\alpha$ represents tag modes in the $i^{th}$ energy group. Solving for $B_{\text{sig}}$,

$$B_{\text{sig}} = \frac{N_{\text{total}}^{\text{DT}}}{\sum_{\alpha,i} N_{\alpha,i}^{\text{DT}} \epsilon_{\alpha,sig,i}^{\text{ST}}},$$  \hspace{1cm} (5.4)
Table 9. The ST yields for the samples collected at \( \sqrt{s} = (I) \) 4.178 GeV, (II) 4.189–4.219 GeV, and (III) 4.226 GeV. The uncertainties are statistical.

| Tag mode       | (I) \( N_{\text{ST}} \) | (II) \( N_{\text{ST}} \) | (III) \( N_{\text{ST}} \) |
|----------------|--------------------------|---------------------------|--------------------------|
| \( D_s^- \to K_S^0 K^- \) | 31941 ± 312              | 18559 ± 261               | 6582 ± 160               |
| \( D_s^- \to K^+ K^- \pi^- \) | 137240 ± 614             | 81286 ± 505               | 28439 ± 327              |
| \( D_s^- \to K_S^0 K^- \pi^0 \) | 11385 ± 529              | 6832 ± 457                | 2227 ± 220               |
| \( D_s^- \to K^+ K^- \pi^- \pi^0 \) | 39306 ± 799              | 23311 ± 659               | 7785 ± 453               |
| \( D_s^- \to K_S^0 K^- \pi^- \pi^+ \) | 8093 ± 326               | 5269 ± 282                | 1662 ± 217               |
| \( D_s^- \to K_S^0 K^- \pi^- \pi^- \) | 15719 ± 289              | 8948 ± 231                | 3263 ± 172               |
| \( D_s^- \to \pi^- \pi^- \pi^+ \) | 37977 ± 859              | 21909 ± 776               | 7511 ± 393               |
| \( D_s^- \to \pi^- \pi^- \eta \) | 17940 ± 402              | 10025 ± 339               | 3725 ± 252               |
| \( D_s^- \to \pi^- \pi^- \pi^0 \) | 42618 ± 1397             | 26067 ± 1196              | 10513 ± 1920             |
| \( D_s^- \to \pi^- \pi^- \eta' \) | 7759 ± 141               | 4428 ± 111                | 1648 ± 74                |

where \( N_{\alpha,i}^{\text{ST}} \) and \( \epsilon_{\alpha,i}^{\text{ST}} \) are obtained from the data and inclusive MC samples, respectively. The \( \epsilon_{\alpha,\text{sig},i}^{\text{DT}} \) is determined with signal MC samples in which \( D_s^+ \to K^+ \pi^+ \pi^- \) events are generated according to the baseline model of the amplitude analysis.

In order to ensure that the DT sample is a subset of the ST sample in the BF measurement, the ST candidates are selected ahead of the selection of DT candidates. In addition to the selection criteria for final-state particles described in section 3, the requirement \( p(\pi) > 100 \text{ MeV}/c \) is applied to all pions in order to exclude transition pions from \( D^+ \) decays. If there are multiple ST candidates, the combination with the \( M_{\text{rec}} \) closest to the known mass of \( D_s^\pm \) [2] is kept. The yields for various tag modes are obtained by fitting the corresponding \( M_{\text{tag}} \) distributions and listed in table 9. As an example, the fits to the \( M_{\text{tag}} \) distributions of the selected ST candidates from the data sample at \( \sqrt{s} = 4.178 \text{ GeV} \) are shown in figure 4. In the fits, the signal is modeled by an MC-simulated shape convolved with a Gaussian function to take into account the data-MC resolution difference. The background is described by a second-order Chebyshev polynomial. For the tag modes \( D_s^- \to K_S^0 K^- \) and \( D_s^- \to \pi^- \eta' \), there are peaking background contributions coming from \( D^- \to K_S^0 \pi^- \) and \( D_s^- \to \eta \pi^+ \pi^- \pi^- \) decays, respectively. The \( D^- \to K_S^0 \pi^- \) background are estimated to be 1724 ± 34 and 89 ± 5 events according to the BFs given by PDG [2] and ref. [41], corresponding to about 0.3% and less than 0.1% of the total ST yields, respectively.

Once a tag mode is identified, we attempt to reconstruct the signal decay \( D_s^+ \to K^+ \pi^+ \pi^- \). If there are multiple candidates, the DT candidate with the average mass, \( (M_{\text{sig}} + M_{\text{tag}})/2 \), closest to the \( D_s^\pm \) known mass is retained. A 6C kinematic fit is also performed for the BF measurement, and the same \( K_S^0 \) veto and \( \chi_{6C}^2 \) requirements as in section 4.1 are applied to suppress the background.

The DT yield is determined from the fit to the \( M_{\text{sig}} \) distribution. The fit result is shown in figure 5, where the signal shape is modeled by an MC-simulated shape convolved
Figure 4. Fits to the $M_{\text{tag}}$ distributions of the ST candidates from the data sample at $\sqrt{s} = 4.178$ GeV. The points with error bars are data, the blue solid lines are the total fits, and the black dashed lines are the fitted backgrounds. The pairs of red arrows denote the signal regions.
with Gaussian function, while the background shape is described with the shape derived from the inclusive MC sample. The DT yield obtained is $1415 \pm 42$. Based on this, we determine the BF to be $B(D_s^+ \rightarrow K^+\pi^+\pi^-) = (6.11 \pm 0.18_{\text{stat.}} \pm 0.11_{\text{syst.}}) \times 10^{-3}$ taking into account the differences in $K^+$ and $\pi^\pm$ tracking and PID efficiencies between data and MC simulation.

The BFs for the charge-conjugated modes are measured separately. The BFs of $D_s^+ \rightarrow K^+\pi^+\pi^-$ and $D_s^- \rightarrow K^-\pi^-\pi^+$, denoted as $B(D_s^+)$ and $B(D_s^-)$, are measured to be $(5.88 \pm 0.25_{\text{stat.}} \pm 0.11_{\text{syst.}}) \times 10^{-3}$ and $(6.28 \pm 0.26_{\text{stat.}} \pm 0.11_{\text{syst.}}) \times 10^{-3}$, respectively. The asymmetry of the two BFs is determined to be $
abla = B(D_s^+) - B(D_s^-) = 3.3 \pm 3.0_{\text{stat.}} \pm 1.3_{\text{syst.}}$%. The systematic uncertainties of tracking and PID have been canceled in the $A_{CP}$ calculation. The result is consistent with the hypothesis of $CP$ symmetry [8].

The systematic uncertainties in the BF measurement are discussed below.

- ST yield. The uncertainty of the total yield of the ST $D_s^-$ mesons is determined to be 0.5% by taking into account the background fluctuation in the fit, and examining the changes of the fit yields when varying the background shape.

- Background shape. To estimate the uncertainty due to the background shape of the signal $D_s^+$ invariant mass distribution, a second-order Chebychev polynomial is used to replace the MC-simulated shape, and an uncertainty of 0.6% is obtained.

- Tracking and PID. The processes $e^+e^- \rightarrow K^+K^-K^+K^-, K^+K^-\pi^+\pi^-(\pi^0)$ and $\pi^+\pi^-\pi^+\pi^-\pi^-(\pi^0)$ are used to study the tracking and PID efficiencies of $K^+$ and $\pi^\pm$. The data-MC tracking and PID efficiencies ratios of $\pi^+(\pi^-)$ are $0.998 \pm 0.003$ ($1.002 \pm 0.003$) and $1.002 \pm 0.002$ ($1.003 \pm 0.002$), respectively. The data-MC tracking and PID efficiencies ratios of $K^+(K^-)$ are $0.998 \pm 0.003$ ($0.997 \pm 0.003$) and $1.003 \pm 0.002$ ($1.003 \pm 0.002$), respectively. Finally, the systematic uncertainties associated with tracking and PID efficiencies for each charged particle are estimated to be 0.3% and 0.2%, respectively.
Table 10. Systematic uncertainties in the BF measurement.

| Source                      | Uncertainty (%) |
|-----------------------------|-----------------|
| ST yield                    | 0.5             |
| Background shape            | 0.6             |
| Tracking                    | 0.9             |
| PID                         | 0.6             |
| MC sample size              | 0.4             |
| Amplitude model             | 0.5             |
| $\chi^2_{6C}$ requirement   | 1.0             |
| **Total**                   | **1.8**         |

- **MC sample size.** The uncertainty of the MC sample size is given by $\sqrt{\sum_{\alpha} (f_{\alpha} \epsilon_{\alpha})^2}$, where $f_{\alpha}$ is the tag yield fraction and $\epsilon_{\alpha}$ is the average DT efficiency of tag mode $\alpha$. The corresponding uncertainty is determined to be 0.4%.

- **Amplitude model.** The uncertainty from the amplitude model is determined by varying the amplitude model parameters based on their error matrix 600 times. A Gaussian function is used to fit the distribution of 600 DT efficiencies and the fitted width divided by the mean value is taken as an uncertainty. The related uncertainty is 0.5%.

- **$\chi^2_{6C}$ requirement.** The uncertainty of the $\chi^2_{6C}$ requirement is assigned to be the difference between the data and MC efficiencies of the $D_s^+ \rightarrow K^+K^0\pi^+$ candidates. The data and MC simulation control samples of $D_s^+ \rightarrow K^+K^0\pi^+$ including over 99% signal events are selected. Then, the efficiency corresponding to the $\chi^2_{6C}$ requirement is obtained, and the uncertainty is calculated by $1 - \frac{\epsilon_{\text{data}}}{\epsilon_{\text{MC}}}$, where $\epsilon_{\text{data}}$ and $\epsilon_{\text{MC}}$ are the selection efficiencies of data and MC simulation, respectively. The associated systematic uncertainty is assigned to be 1.0%.

- **$K_S^0$ rejection.** The uncertainty of $K_S^0$ rejection has been included in the uncertainty of the amplitude model, in which the inconsistency of the structure of $\pi^+\pi^-$ spectrum between data and MC is estimated by varying the amplitude model parameters by 600 times. On the other hand, the remained $D_s^+ \rightarrow K_S^0 K^+\pi^+$ events are less than 0.1% after $K_S^0$ rejection, thus the systematic uncertainty of $K_S^0$ rejection can be negligible.

All of the systematic uncertainties are summarized in table 10. Adding them in quadrature results in a total systematic uncertainty of 1.8% in the BF measurement.

6 Summary

Using $e^+e^-$ collision data equivalent to an integrated luminosity of 6.32 fb$^{-1}$ recorded with the BESIII detector at the center-of-mass energies between 4.178 and 4.226 GeV, an amplitude analysis of the decay $D_s^+ \rightarrow K^+\pi^+\pi^-$ has been performed. The results for the
FFs and phases of the different intermediate processes are listed in table 5. The BF for the decay $D_s^+ \to K^+\pi^+\pi^−$ is measured to be $(6.11 \pm 0.18_{\text{stat.}} \pm 0.11_{\text{syst.}}) \times 10^{-3}$, which is improved by about a factor of 2 compared to the world average value [2]. The BFs for the intermediate processes calculated with $B_i = \text{FF}_i \times B(D_s^+ \to K^+\pi^+\pi^-)$ in this analysis and from the PDG [2] are listed in table 11. The BFs of $D_s^+ \to K^+ f_0(500)$, $D_s^+ \to K^+ f_0(980)$, and $D_s^+ \to K^+ f_0(1370)$ are determined for the first time. The asymmetry of the BFs of $D_s^+ \to K^+\pi^+\pi^−$ and $D_s^- \to K^-\pi^−\pi^+$ is determined to be $(3.3 \pm 3.0_{\text{stat.}} \pm 1.3_{\text{syst.}})\%$. No indication of CP violation is found.

The obtained BF of $D_s^+ \to K^+ \rho^0$ is in good agreement with the predictions in ref. [8], and the measured BF of $D_s^+ \to K^* (892)^0 \pi^+$ is consistent with the prediction in ref. [6]. Meanwhile, our result deviates from the predictions of $D_s^+ \to K^+ \rho^0$ in refs. [6, 7] and $D_s^+ \to K^* (892)^0 \pi^+$ in refs. [7, 8] over two standard deviations. Moreover, ref. [8] predicts the ratio of BF of $D_s^+ \to K^+ \rho^0$ to that of $D_s^+ \to K^+ \omega$ is far greater than one, while ref. [6] calculates that it should be close to one. The ratio is determined to be about two by taking the results in this analysis and in ref. [42]. More precise theoretical predictions are desirable to understand the $D_s^\pm \to VP$ processes and SU(3)$_F$ flavor symmetry breaking effect.

Table 11. The BFs for various intermediate processes decaying into the final state $K^+\pi^+\pi^−$ in this analysis and from the PDG [2], where the first and second uncertainties are statistical and systematic, respectively.

| Intermediate process | BF($10^{-3}$) | PDG($10^{-3}$) |
|----------------------|---------------|----------------|
| $D_s^+ \to K^+ \rho^0$ | $1.96 \pm 0.19 \pm 0.23$ | $2.5 \pm 0.4$ |
| $D_s^+ \to K^+ \rho(1450)^0$ | $0.80 \pm 0.19 \pm 0.18$ | $0.69 \pm 0.64$ |
| $D_s^+ \to K^* (892)^0 \pi^+$ | $1.85 \pm 0.12 \pm 0.13$ | $1.41 \pm 0.24$ |
| $D_s^+ \to K^* (1410)^0 \pi^+$ | $0.27 \pm 0.13 \pm 0.15$ | $1.23 \pm 0.28$ |
| $D_s^+ \to K^*_0 (1430)^0 \pi^+$ | $1.13 \pm 0.16 \pm 0.16$ | $0.50 \pm 0.35$ |
| $D_s^+ \to K^+ f_0(500)$ | $0.44 \pm 0.13 \pm 0.27$ | — |
| $D_s^+ \to K^+ f_0(980)$ | $0.27 \pm 0.08 \pm 0.07$ | — |
| $D_s^+ \to K^+ f_0(1370)$ | $1.22 \pm 0.18 \pm 0.57$ | — |
| $D_s^+ \to (K^+\pi^+\pi^-)_{NR}$ | — | $1.03 \pm 0.34$ |

Acknowledgments

The BESIII collaboration thanks the staff of BEPCII and the IHEP computing center for their strong support. This work is supported in part by National Key Research and Development Program of China under Contracts Nos. 2020YFA0406400, 2020YFA0406300; National Natural Science Foundation of China (NSFC) under Contracts Nos. 11625523, 11635010, 11735014, 11775027, 11822506, 11835012, 11875054, 11935015, 11935016, 11935018, 11961141012, 12192260, 12192261, 12192262, 12192263, 12192264, 12192265; the Chinese Academy of Sciences (CAS) Large-Scale Scientific Facility Program; Joint Large-Scale Scientific Facility Funds of the NSFC and CAS under...
References

[1] A. Ryd and A.A. Petrov, *Hadronic D and D(s) Meson Decays*, Rev. Mod. Phys. 84 (2012) 65 [arXiv:0910.1265] [insPIRE].

[2] Particle Data Group collaboration, *Review of Particle Physics*, PTEP 2020 (2020) 083C01 [insPIRE].

[3] CLEO collaboration, *Improved Measurement of Absolute Hadronic Branching Fractions of the D^+_s Meson*, Phys. Rev. D 88 (2013) 032009 [arXiv:1306.5363] [insPIRE].

[4] CLEO collaboration, *Measurements of D Meson Decays to Two Pseudoscalar Mesons*, Phys. Rev. D 81 (2010) 052013 [arXiv:0906.3198] [insPIRE].

[5] BESIII collaboration, *Amplitude analysis and branching-fraction measurement of D^+_s \to K^{0}_S \pi^+\pi^0*, JHEP 06 (2021) 181 [arXiv:2103.15098] [insPIRE].

[6] H.-Y. Cheng and C.-W. Chiang, *CP violation in quasi-two-body D \to VP decays and three-body D decays mediated by vector resonances*, Phys. Rev. D 104 (2021) 073003 [arXiv:2104.13548] [insPIRE].

[7] Y.-L. Wu, M. Zhong and Y.-F. Zhou, *Exploring final state hadron structure and SU(3) flavor symmetry breaking effects in D \to PP and D \to PV decays*, Eur. Phys. J. C 42 (2005) 391 [hep-ph/0405080] [insPIRE].

[8] Q. Qin, H.-n. Li, C.-D. Lü and F.-S. Yu, *Branching ratios and direct CP asymmetries in D \to PV decays*, Phys. Rev. D 89 (2014) 054006 [arXiv:1305.7021] [insPIRE].

[9] FOCUS collaboration, *Study of the doubly and singly Cabibbo suppressed decays D^+ \to K^+\pi^+\pi^- and D_{s}^{+} \to K^{+}\pi^{+}\pi^{-}*, Phys. Lett. B 601 (2004) 10 [hep-ex/0407014] [insPIRE].
[10] BESIII collaboration, Design and Construction of the BESIII Detector, *Nucl. Instrum. Meth. A* **614** (2010) 345 [arXiv:0911.4960] [inSPIRE].

[11] BESIII collaboration, Future Physics Programme of BESIII, *Chin. Phys. C* **44** (2020) 040001 [arXiv:1912.05983] [inSPIRE].

[12] C. Yu et al., BEPCII Performance and beam dynamics studies on luminosity, in Proc. of International Particle Accelerator Conference (IPAC’16), Busan, Korea, May 8–13, 2016, no. 7 in International Particle Accelerator Conference, Geneva, Switzerland, pp. 1014–1018, JACoW, June, 2016, DOI.

[13] X. Li et al., Study of MRPC technology for BESIII endcap-TOF upgrade, *Radiat. Detect. Technol. Methods* **1** (2017) 13.

[14] Y.-X. Guo et al., The study of time calibration for upgraded end cap TOF of BESIII, *Radiat. Detect. Technol. Methods* **1** (2017) 15.

[15] P. Cao et al., Design and construction of the new BESIII endcap Time-of-Flight system with MRPC Technology, *Nucl. Instrum. Meth. A* **953** (2020) 163053.

[16] BESIII collaboration, Measurement of the integrated luminosities at BESIII for data samples at collision energies around 4 GeV, arXiv:2203.03133 [inSPIRE].

[17] BESIII collaboration, Precision measurement of the integrated luminosity of the data taken by BESIII at center of mass energies between 3.810 GeV and 4.600 GeV, *Chin. Phys. C* **39** (2015) 093001 [arXiv:1503.03408] [inSPIRE].

[18] BESIII collaboration, Measurement of the center-of-mass energies at BESIII via the di-muon process, *Chin. Phys. C* **40** (2016) 063001 [arXiv:1510.08654] [inSPIRE].

[19] GEANT4 collaboration, GEANT4 — a simulation toolkit, *Nucl. Instrum. Meth. A* **506** (2003) 250 [inSPIRE].

[20] S. Jadach, B.F.L. Ward and Z. Was, Coherent exclusive exponentiation for precision Monte Carlo calculations, *Phys. Rev. D* **63** (2001) 113009 [hep-ph/0006359] [inSPIRE].

[21] S. Jadach, B.F.L. Ward and Z. Was, The Precision Monte Carlo event generator KK for two fermion final states in $e^+e^-$ collisions, *Comput. Phys. Commun.* **130** (2000) 260 [hep-ph/9912214] [inSPIRE].

[22] D.J. Lange, The EvtGen particle decay simulation package, *Nucl. Instrum. Meth. A* **462** (2001) 152 [inSPIRE].

[23] R.-G. Ping, Event generators at BESIII, *Chin. Phys. C* **32** (2008) 599 [inSPIRE].

[24] J.C. Chen, G.S. Huang, X.R. Qi, D.H. Zhang and Y.S. Zhu, Event generator for $J/\psi$ and $\psi(2S)$ decay, *Phys. Rev. D* **62** (2000) 034003 [inSPIRE].

[25] R.-L. Yang, R.-G. Ping and H. Chen, Tuning and Validation of the Lundcharm Model with $J/\psi$ Decays, *Chin. Phys. Lett.** 31** (2014) 061301 [inSPIRE].

[26] E. Richter-Was, QED bremsstrahlung in semileptonic $B$ and leptonic $\tau$ decays, *Phys. Lett. B* **303** (1993) 163 [inSPIRE].

[27] MARK-III collaboration, Direct Measurements of Charmed d Meson Hadronic Branching Fractions, *Phys. Rev. Lett.* **56** (1986) 2140 [inSPIRE].

[28] BESIII collaboration, Measurement of the absolute branching fractions for purely leptonic $D_s^+$ decays, *Phys. Rev. D* **104** (2021) 052009 [arXiv:2102.11734] [inSPIRE].
[29] W. Verkerke and D.P. Kirkby, *Roofit users manual v2.91*, https://root.cern/download/doc/RooFit_Users_Manual_2.91-33.pdf (2019).

[30] K.S. Cranmer, *Kernel estimation in high-energy physics*, Comput. Phys. Commun. **136** (2001) 198 [hep-ex/0011057] [inspire].

[31] BES collaboration, *Resonances in J/ψ → φπ⁺π⁻ and φK⁺K⁻*, Phys. Lett. B **607** (2005) 243 [hep-ex/0411001] [inspire].

[32] G.J. Gounaris and J.J. Sakurai, *Finite width corrections to the vector meson dominance prediction for ρ → e⁺e⁻*, Phys. Rev. Lett. **21** (1968) 244 [inspire].

[33] BES collaboration, *Resonances in J/ψ → φπ⁺π⁻ and φK⁺K⁻*, Nucl. Phys. B **607** (2005) 243 [hep-ex/0411001] [inspire].

[34] J.R. Pelaez, *From controversy to precision on the sigma meson: a review on the status of the non-ordinary f₀(500) resonance*, Phys. Rept. **658** (2016) 1 [arXiv:1510.00653] [inspire].

[35] CLEO collaboration, *Dalitz plot analysis of the D⁺ → K⁻π⁺π⁺ decay*, Phys. Rev. D **78** (2008) 052001 [arXiv:0802.4214] [inspire].

[36] D. Aston et al., *A study of K⁻π⁺ scattering in the reaction K⁻ p → K⁻π⁺n at 11 GeV/c*, Nucl. Phys. B **296** (1988) 493 [inspire].

[37] BABAR and Belle collaborations, *Measurement of cos2β in B⁰ → D(*)h⁰ with D → K₂⁰π⁺π⁻ decays by a combined time-dependent Dalitz plot analysis of BaBar and Belle data*, Phys. Rev. D **98** (2018) 112012 [arXiv:1804.06153] [inspire].

[38] B.S. Zou and D.V. Bugg, *Covariant tensor formalism for partial-wave analyses of ψ decay to mesons*, Eur. Phys. J. A **16** (2003) 537 [hep-ph/0211457] [inspire].

[39] BESIII collaboration, *Amplitude analysis and branching fraction measurement of the decay D⁺ s → K⁺K⁻π⁺, Phys. Rev. D** 104** (2021) 012016 [arXiv:2106.13536] [inspire].

[40] A.V. Anisovich and A.V. Sarantsev, *K-matrix analysis of the Kπ S-wave in the mass region 900–2100 MeV and nonet classification of scalar q̅q states*, Phys. Lett. B **413** (1997) 137 [hep-ph/9705401] [inspire].

[41] BESIII collaboration, *Study of the Decay D⁺ s → π⁺π⁺π⁻η and Observation of the W-annihilation Decay D⁺ s → a₀(980)+ρ⁰, Phys. Rev. D** 104** (2021) 071101 [arXiv:2106.13536] [inspire].

[42] BESIII collaboration, *Amplitude analysis and branching fraction measurement of the decay D⁺ s → K⁺π⁺π⁻π⁰*, arXiv:2205.13759 [inspire].
The BESIII collaboration

M. Ablilikim, M.N. Achasov, P. Adlarson, M. Albrecht, R. Aliberti, A. Amoroso, M.R. An, Q. An, X.H. Bai, Y. Bai, O. Bakina, R. Baldini Ferroli, I. Balossino, Y. Ban, V. Batozskaya, D. Becker, K. Begzsuren, N. Berger, M. Bertani, D. Bettoni, F. Bianchi, J. Bloms, A. Bortone, I. Boyko, R.A. Briere, A. Brueggemann, H. Cai, A. Calcatera, G.F. Cao, S. Cao, J. Chen, H.S. Chen, M.L. Chen, S.J. Chen, T. Chen, X.R. Chen, X.T. Chen, Y.B. Chen, Z.J. Chen, W.S. Cheng, C. Cibinetto, F. Cossio, J.J. Cui, H.L. Dai, J.P. Dai, A. Dbyessi, R. E. de Boer, D. Dedovich, Z.Y. Denig, I. Denysenko, M. Destefanis, F. De Moura, Y. Ding, J. Dong, L.Y. Dong, M.Y. Dong, X. Dong, S.X. Du, P. Egorov, Y.L. Fan, J. Fang, S.S. Fang, Y. Fang, R. Farinelli, L. Fava, D. Feldhauer, G. Felici, C.Q. Feng, J.H. Feng, K. Fischer, M. Fritsch, C. Fritzsch, C.D. Fu, H. Gao, Y.N. Gao, Y. Gao, S. Garbolino, I. Garzia, P.T. Ge, Z.W. Ge, C. Geng, E.M. Gersabeck, A. Gilman, L. Gong, W.X. Gong, W. Gradl, M. Greco, L.M. Gu, M.H. Gu, Y.T. Gu, C. Y Guan, A.Q. Guo, L.B. Guo, R.P. Guo, Y.P. Guo, A. Guskov, T.T. Han, W.Y. Han, X.Q. Hao, F.A. Harris, K.K. He, K.L. He, F.H. Heinsius, C.H. Heinz, Y.K. Heng, C. Herold, T. Holtmann, G.Y. Hou, Y.R. Hou, Z.L. Hou, H.M. Hu, J.F. Hu, T. Hu, Y. Hu, G.S. Huang, K.X. Huang, L.Q. Huang, X.W. Huang, X.T. Huang, Y.P. Huang, Z. Huang, T. Hussain, N. Hüskens, W. Imoehl, M. Irshad, J. Jackson, S. Jaeger, S. Janchiv, Q. Ji, Q.P. Ji, X.B. Ji, X.L. Ji, Y.Y. Ji, Z.K. Jin, H.B. Jiang, S.S. Jiang, X.S. Jiang, Y. Jiang, J.B. Jiao, Z. Jiao, S. Jin, M.Q. Jing, J. Johansson, N. Kalantar-Nayestanaki, S.S. Kang, R. Kappert, M. Kavatsyuk, B.C. Ke, I.K. Keshk, A. Kouhazad, P. Kiese, R. Kuchel, O.B. Kolev, B. Kopf, M. Kuemmel, M. Kuusniemi, A. Kupsc, W. Kühn, J.I. Lane, J.S. Lange, P. Lariu, A. Lavanić, L. Laveza, Z.H. Lei, H. Leithoff, M. Lellmann, T. Lenz, C. Li, C. Li, C.H. Li, Cheng Li, D.M. Li, F. Li, G. Li, H. Li, H. Li, H.B. Li, H.J. Li, H.N. Li, J.Q. Li, J.S. Li, J.W. Li, Ke Li, L. L. Li, L.K. Li, Lei Li, M.H. Li, P.R. Li, S.X. Li, S.Y. Li, T. Li, W.D. Li, W.G. Li, X.H. Li, X.L. Li, X. Liu, H. Liang, H. Liang, H. Liang, Y. Liang, Y.F. Liang, Y.T. Liang, G.R. Liao, J.Z. Liao, J. Libby, A. Limpihra, C.X. Liu, D.X. Liu, T. Liu, B.J. Liu, C.X. Liu, D. Liu, F.H. Liu, Fang Liu, H. Liu, G.M. Liu, H. Liu, H. Liu, M.H. Liu, Huanhuan Liu, Huihui Liu, J.B. Liu, J.L. Liu, J.Y. Liu, K. Liu, K.Y. Liu, Ke Liu, L. Liu, L. Liu, M.H. Liu, P.L. Liu, Q. Liu, S.B. Liu, T. Liu, W.K. Liu, W.M. Liu, X. Liu, Y. Liu, Y.B. Liu, Z.A. Liu, Z.Q. Liu, X.C. Lou, F.X. Lu, H.J. Lu, J.G. Lu, X.L. Lu, Y. Lu, Y. Lu, Z.H. Lu, C.L. Luo, T. Luo, Y.F. Lyu, F.C. Ma, H.L. Ma, L.L. Ma, M.M. Ma, Q.M. Ma, R.Q. Ma, R.T. Ma, S.Y. Ma, Y.A. Ma, F.E. Maas, M. Maggiora, S. Maldaner, S. Malde, Q.A. Malik, A. Mangoni, Y.J. Mao, Z.P. Mao, S. Marcello, Z.X. Meng, J.G. Messchendorp, M. Mezzadri, H. Miao, T.J. Min, R.E. Mitchell, X.H. Mo, N. Yu, Muchnoi, H. Muramatsu, N. Yefedov, F. Nerling, I.B. Nikolaev, Z. Ning, S. Nisar, Y. Niu, S.L. Olsen, Q. Ouyang, S. Pacetti, X. Pan, R.G. Ping, S. Phara, P. Bogdán, R. Poling, V. Prasad, H. Qi, H.R. Qi, M. Qi, T.Y. Qi, Y. Qin, Z. Qin, C.F. Qiao, J.J. Qin, L.Q. Qin, X.P. Qin, X.S. Qin, Z.H. Qin, J.F. Qiu, S.Q. Qu, K.H. Rashid.
14 Hangzhou Normal University, Hangzhou 310036, People’s Republic of China
15 Helmholtz Institute Mainz, Staudinger Weg 18, D-55099 Mainz, Germany
16 Henan Normal University, Xinxiang 453007, People’s Republic of China
17 Henan University of Science and Technology, Luoyang 471003, People’s Republic of China
18 Henan University of Technology, Zhengzhou 450001, People’s Republic of China
19 Huangshan College, Huangshan 245000, People’s Republic of China
20 Hunan Normal University, Changsha 410081, People’s Republic of China
21 Hunan University, Changsha 410082, People’s Republic of China
22 Indian Institute of Technology Madras, Chennai 600036, India
23 Indiana University, Bloomington, Indiana 47405, U.S.A.
24 INFN Laboratori Nazionali di Frascati, (A)INFN Laboratori Nazionali di Frascati, I-00044, Frascati, Italy; (B)INFN Sezione di Perugia, I-06100, Perugia, Italy; (C)University of Perugia, I-06100, Perugia, Italy
25 INFN Sezione di Ferrara, (A)INFN Sezione di Ferrara, I-44122, Ferrara, Italy; (B)University of Ferrara, I-44122, Ferrara, Italy
26 Institute of Modern Physics, Lanzhou 730000, People’s Republic of China
27 Institute of Physics and Technology, Peace Ave. 54B, Ulaanbaatar 13330, Mongolia
28 Jilin University, Changchun 130012, People’s Republic of China
29 Johannes Gutenberg University of Mainz, Johann-Joachim-Becher-Weg 45, D-55099 Mainz, Germany
30 Joint Institute for Nuclear Research, 141980 Dubna, Moscow region, Russia
31 Justus-Liebig-Universitaet Giessen, II. Physikalisches Institut, Heinrich-Buff-Ring 16, D-35392 Giessen, Germany
32 Lanzhou University, Lanzhou 730000, People’s Republic of China
33 Liaoning Normal University, Dalian 116029, People’s Republic of China
34 Liaoning University, Shenyang 110036, People’s Republic of China
35 Nanjing Normal University, Nanjing 210023, People’s Republic of China
36 Nanjing University, Nanjing 210093, People’s Republic of China
37 Nankai University, Tianjin 300071, People’s Republic of China
38 National Centre for Nuclear Research, Warsaw 02-093, Poland
39 North China Electric Power University, Beijing 102206, People’s Republic of China
40 Peking University, Beijing 100871, People’s Republic of China
41 Qufu Normal University, Qufu 273165, People’s Republic of China
42 Shandong Normal University, Jinan 250014, People’s Republic of China
43 Shandong University, Jinan 250100, People’s Republic of China
44 Shanghai Jiao Tong University, Shanghai 200240, People’s Republic of China
45 Shanxi Normal University, Linfen 041004, People’s Republic of China
46 Shanxi University, Taiyuan 030006, People’s Republic of China
47 Sichuan University, Chengdu 610064, People’s Republic of China
48 Soochow University, Suzhou 215006, People’s Republic of China
49 South China Normal University, Guangzhou 510006, People’s Republic of China
50 Southeast University, Nanjing 211100, People’s Republic of China
51 State Key Laboratory of Particle Detection and Electronics, Beijing 100049, Hefei 230026, People’s Republic of China
52 Sun Yat-Sen University, Guangzhou 510275, People’s Republic of China
53 Suranaree University of Technology, University Avenue 111, Nakhon Ratchasima 30000, Thailand
54 Tsinghua University, Beijing 100084, People’s Republic of China
55 Turkish Accelerator Center Particle Factory Group, (A)Istinye University, 34010, Istanbul, Turkey; (B)Near East University, Nicosia, North Cyprus, Mersin 10, Turkey
56 University of Chinese Academy of Sciences, Beijing 100049, People’s Republic of China
57 University of Groningen, NL-9747 AA Groningen, The Netherlands
58 University of Hawaii, Honolulu, Hawaii 96822, U.S.A.
59 University of Jinan, Jinan 250022, People’s Republic of China
60 University of Manchester, Oxford Road, Manchester, M13 9PL, United Kingdom
University of Minnesota, Minneapolis, Minnesota 55455, U.S.A.

University of Muenster, Wilhelm-Klemm-Str. 9, 48149 Muenster, Germany

University of Oxford, Keble Rd, Oxford OX13RH, U.K.

University of Science and Technology Liaoning, Anshan 114051, People’s Republic of China

University of Science and Technology of China, Hefei 230026, People’s Republic of China

University of South China, Hengyang 421001, People’s Republic of China

University of the Punjab, Lahore-54590, Pakistan

University of Turin and INFN, (A)University of Turin, I-10125, Turin, Italy; (B)University of Eastern Piedmont, I-15121, Alessandria, Italy; (C)INFN, I-10125, Turin, Italy

Uppsala University, Box 516, SE-75120 Uppsala, Sweden

Wuhan University, Wuhan 430072, People’s Republic of China

Xinyang Normal University, Xinyang 464000, People’s Republic of China

Yunnan University, Kunming 650500, People’s Republic of China

Zhejiang University, Hangzhou 310027, People’s Republic of China

Zhengzhou University, Zhengzhou 450001, People’s Republic of China

† Also at the Moscow Institute of Physics and Technology, Moscow 141700, Russia

‡ Also at the Novosibirsk State University, Novosibirsk, 630090, Russia

§ Also at the NRC “Kurchatov Institute”, PNPI, 188300, Gatchina, Russia

‖ Also at Goethe University Frankfurt, 60323 Frankfurt am Main, Germany

¶ Also at Key Laboratory for Particle Physics, Astrophysics and Cosmology, Ministry of Education; Shanghai Key Laboratory for Particle Physics and Cosmology; Institute of Nuclear and Particle Physics, Shanghai 200240, People’s Republic of China

‖ Also at Key Laboratory of Nuclear Physics and Ion-beam Application (MOE) and Institute of Modern Physics, Fudan University, Shanghai 200443, People’s Republic of China

‖ Also at State Key Laboratory of Nuclear Physics and Technology, Peking University, Beijing 100871, People’s Republic of China

‖ Also at School of Physics and Electronics, Hunan University, Changsha 410082, China

∥ Also at Guangdong Provincial Key Laboratory of Nuclear Science, Institute of Quantum Matter, South China Normal University, Guangzhou 510006, China

‡ Also at Frontiers Science Center for Rare Isotopes, Lanzhou University, Lanzhou 730000, People’s Republic of China

‡ Also at Lanzhou Center for Theoretical Physics, Lanzhou University, Lanzhou 730000, People’s Republic of China

‡ Also at the Department of Mathematical Sciences, IBA, Karachi, Pakistan