Performance and Optimization of Reconfigurable Intelligent Surface Aided THz Communications

Hongyang Du, Jiayi Zhang, Ke Guan, Dusit Niyato, Huaying Jiao, Zhiqin Wang, and Thomas Küner

Abstract—TeraHertz (THz) communications can satisfy the high data rate demand with massive bandwidth. However, severe path attenuation and hardware imperfection greatly alleviate its performance. Therefore, we utilize the reconfigurable intelligent surface (RIS) technology and investigate the RIS-aided THz communications. We first prove that the small-scale amplitude fading of THz signals can be accurately modeled by the fluctuating two-ray distribution based on two THz signal measurement experiments conducted in a variety of different scenarios. To optimize the phase-shifts at the RIS elements, we propose a novel swarm intelligence-based method that does not require full channel estimation. We then derive exact statistical characterizations of end-to-end signal-to-noise plus distortion ratio (SNDR) and signal-to-noise ratio (SNR). Moreover, we present asymptotic analysis to obtain more insights when the SNDR or (SNDR) and signal-to-noise ratio (SNR). Moreover, we present asymptotic analysis to obtain more insights when the SNDR or
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Index Terms—Fluctuating two-ray fading, channel fitting, reconfigurable intelligent surface, swarm intelligence optimization, THz communications.

I. INTRODUCTION

In the past few years, wireless data traffic has been witnessing unprecedented expansion. Several technological advances, such as artificial intelligence and massive multiple-input multiple-output (MIMO) systems, have been presented[1] while the wireless world moves towards the sixth generation (6G). Developing new spectrum resources for future 6G communications systems is urgently demanded.

Because of the high spectrum availability, TeraHertz (THz) communications emerge as a key candidate in future generations [2]–[5]. However, THz signals have considerably high sensitivity to blockage, i.e., in urban environments with high obstacle density. Thus, the direct usage of THz-band in wireless access scenarios is challenging. Recently, the reconfigurable intelligent surface (RIS) technology has drawn increasing attention [6], [7] due to its ability to improve both spectrum and energy efficiency. RIS is based on massive integration of low-cost tunable passive elements that can be easily coated on existing infrastructures such as walls of buildings, facilitating low-cost and low-complexity implementation [8]–[11]. By smartly adjusting the scattering properties of its reflecting elements, the RIS can form equally strong beams in any direction for THz signals, which leads to a more controllable wireless environment. Compared to conventional relays, RISs can enhance the transmission performance without being impaired by self-interference and thermal noise. More importantly, RIS can be deployed to leverage the line-of-sight (LoS) components with respect to both the base station (BS) and the users to assist their end-to-end THz communications, which potentially reduce the path attenuation that originates from the high frequencies.

However, how to model the small-scale amplitude fading of THz signals in the RIS-aided system is still an open question. Methods of modeling and evaluating the performance of THz communications have been extensively studied in the literature [12]–[20]. For example, the authors in [19] delivered a simplified molecular absorption loss model that was used in [20] for the quantification of the THz link capacity. However, the THz system performance is overestimated by neglecting small-scale fading. Although the small-scale fading is taken into consideration in [17], [18], the channel modeling was not based on real THz signal measurements. Furthermore, in [21], the envelope of the fading coefficient was proved by the authors experimentally to follow the Nakagami-m distribution under both non-line-of-sight (NLoS) and LoS conditions. Similarly, experimental verification of the existence of shadowing in the 300 GHz band were exploited in [22]. Very recently, a THz measurement campaign was conducted in a train test center with trains, tracks, and lampposts [23]. We fit the measurement data to Gaussian, Rician, Nakagami-m, and fluctuating two-ray (FTR) distributions [24], respectively, and prove that the FTR distribution, which is a generalization of the two-wave with diffuse power fading model and allows
the constant amplitude specular waves of LoS propagation to fluctuate randomly, performs a much better fit than others.

Therefore, with the help of the FTR fading model, we investigate the RIS-aided THz communications. We propose an algorithm to obtain the optimal design of RIS’s reflector array. By designing the phase shift, we can then theoretically derive the maximum of SINR and corresponding probability density function (PDF) and cumulative distribution function (CDF) expressions. The main contributions of this paper are summarized as follows:

- To model the small-scale amplitude fading of THz signals, we prove that the FTR distribution performs a much better fit than those of the other fading models with the help of the THz measurement data [23], [25]. Moreover, we derive exact PDF and CDF expressions of end-to-end SNR and SNDR in terms of the multivariate Fox’s H-function, respectively. The impact of transceiver antenna gains, the level of transceiver hardware imperfections, the transmission distance, the misalignment between the RIS and the destination, the operating frequency, and the environmental conditions including temperature, humidity, and pressure, are all comprehensively considered in our analysis.

- For the practical RIS with a finite number of phase-shifters at each element, we propose a swarm intelligence-based optimization (SIO) algorithm to obtain the optimal phase-shifts. Furthermore, the impact of the number of discrete phase-shift levels on the transmission performance is revealed, and we find that RIS with discrete phase-shifts can achieve performance close to that of the continuous phase-shifts.

- We derive exact closed-form expressions of outage probability for the RIS-aided THz communications system with both ideal and non-ideal radio frequency (RF) chains. Moreover, asymptotic analysis is presented when the SNDR or the number of RIS’s elements is high, which is easier to calculate and obtain insights. For the ergodic capacity, the exact closed-form expression for ideal RF chains is obtained and the tight upper bound expressions for both ideal and non-ideal RF chains are derived. Important physical and engineering insights have been revealed. The derived results show that the propagation loss is more detrimental than the molecular absorption loss, and the performance is improved significantly by increasing the number of reflecting elements equipped in an RIS.

The remainder of the paper is structured as follows. We briefly introduce the RIS-aided THz communications system and signal models in Section II. Section III presents the optimal phase-shifts of the RIS’s reflector array under discrete level constraints. In Section IV, we derive exact PDF and CDF expressions of end-to-end SNR and SNDR of the considered system. Section V presents the asymptotic analysis to obtain several insights. Important performance metrics, such as outage probability and ergodic capacity, are obtained in Section VI. Numerical results accompanied with Monte-Carlo simulations are presented in Section VII. Finally, the conclusions are drawn in Section VIII.

II. SYSTEM AND SIGNAL MODELS

A. System Description

We focus on a single cell of the THz communications system. As shown in Fig. 1, the Source is equipped with $M$ antennas, and the Destination has one antenna. The interference among THz links can be significantly small because the signal in THz communications is highly directional. Thus, the interference from other cell-users is ignored. Because THz signals suffer from high path and penetration losses, an RIS is set up to assist the communications. The RIS is a uniform planar array (UPA) containing $L$ nearly passive reconfigurable reflecting units, where each element is capable of electronically controlling the phase of the incident electromagnetic waves. To establish a practical RIS-aided THz communications model, we consider the following coefficients and give the corresponding reasons:

- **Small-Scale Fading Coefficient $h_{sf}$**: Although some of the existing literature on RIS-aided THz communications considers the utilization of small-scale fading models, classical models such as Rayleigh and Rician fadings are typically used. However, recent real measurements of THz signals show that these classical models cannot fit well the THz signals. Although the authors of [25] claim that $\alpha-\mu$ fading obtains good results and passes the Kolmogorov-Smirnov (K-S) test, we can observe that the $\alpha-\mu$ fading is still not accurate enough in fitting the bimodal characteristics of THz signal’s amplitude fading. Therefore, a more accurate channel model is needed.

- **Misalignment Fading Coefficient $h_{fp}$**: Random building sways, which will be caused by wind loads and thermal expansions, will result in pointing errors at Destination. Because the THz signals are highly directional, we need to take the misalignment into consideration.

- **Path Gain Coefficient $h_{p}$**: Because of the THz signals’ inability to penetrate solid objects, the path gain in the THz communications system concludes the propagation gain and the molecular absorption gain.

- **Hardware Impairments $n_{g}$ and $n_{d}$**: Physical transceivers have hardware impairments that create distortions that degrade the performance of communications systems. The vast majority of technical contributions in the area of relaying neglect hardware impairments and, thus, assume...
ideal hardware. Such approximations make sense in low-rate systems, but can lead to very misleading results when analyzing future high-rate systems. This paper quantifies the impact of hardware impairments on the RIS-aided THz system.

By considering the above coefficients and the characteristics of RIS carefully, we can derive the expression of the received signal, which allows us to further analyze the statistical characteristics of SNR and performance metrics. A more detailed analysis of the above coefficients are given as follows:

1) Small-Scale Fading Measurement and Modeling: We consider the RIS is electrically small and the correlation between the amplitudes of the signals is weak enough to be ignored [26], [27]. Let $g_1 \in \mathbb{C}^{M \times L}$ and $g_2 \in \mathbb{C}^{1 \times L}$ denote the channels from the THz Source to the RIS, and from the RIS to the Destination, respectively, $\Phi = \beta \text{diag}(\varphi)$ denote the diagonal reflection coefficients matrix of the RIS, with $\beta \in (0,1]$ is the phase-shifts matrix. The small-scale fading between BS and RIS is defined as $H_\ell = g_2 \Phi g_1^H$, where $\varphi = (e^{i\varphi_1}, \ldots, e^{i\varphi_L})^T$, $\varphi_\ell$ is the phase-shift of the $\ell$-th element of the RIS, $i = \sqrt{-1}$, $g_2 = (g_2^{(1)}, \ldots, g_2^{(L)}) = (g_2^{(1)}, g_2^{(L)})$, and

$$g_1 = \begin{pmatrix} g_1^{(1,1)} & \ldots & g_1^{(1,L)} \\ \vdots & \ddots & \vdots \\ g_1^{(M,1)} & \ldots & g_1^{(M,L)} \end{pmatrix} = \begin{pmatrix} g_1^{(1,1)} e^{-i\theta_1^{(1,1)}} & \ldots & g_1^{(1,L)} e^{-i\theta_1^{(1,L)}} \\ \vdots & \ddots & \vdots \\ g_1^{(M,1)} e^{-i\theta_1^{(M,1)}} & \ldots & g_1^{(M,L)} e^{-i\theta_1^{(M,L)}} \end{pmatrix}. \quad (1)$$

The elements in $g_1$ and $g_2$ represent the small-scale fading of the THz signals, for which we have to find the appropriate channel fading model. Recently, the FTR fading [24] has received a great amount of research interest, because the FTR model yields a better fit than those of the other conventional distributions, e.g., Rayleigh, Rice, and Nakagami-$m$, and FTR distribution, respectively, which are also compared with the CDF of the measurement data. As shown in Figs. 2, we find that the FTR distribution performs the best fit.

In [25], three sets of experimental measurements, which have been conducted in a shopping mall, an airport check-in area, and an entrance hall of a university towards different time periods, are used to accurately model the THz signal fading distribution. Analysis shows that conventional distributions, e.g., Rayleigh, Rice, and Nakagami-$m$, lack fitting accuracy, whereas the $\alpha-\mu$ distribution has an almost-excellent fit. However, we can observe that the $\alpha-\mu$ distribution is still deficient to show the characterizations of the experimental results: 1) The experimental PDF appears bimodality in many test environments, e.g., Transmitter 4 (TX4) in the entrance hall or the shopping mall (see Fig. 2 in [25]), which cannot be fitted by the PDF of $\alpha-\mu$ distribution. 2) As the variable $x$ increases, the slope of CDF gradually increases and then decreases to 0, but the CDF of $\alpha-\mu$ distribution cannot fit the process of the increasing slope of CDF when $x$ is small. As shown in Fig. 3, we can observe that the FTR fading is better than the $\alpha-\mu$ fading. Furthermore, although both the $\alpha-\mu$ and FTR distribution can pass the K-S test, the values for the FTR distribution is always smaller.

Moreover, Table I shows that The FTR distribution achieves the smallest K-S test values in the fit of the experimental results of each THz signal measurement. Thus, in the following, we use the FTR fading model to illustrate the small-scale amplitude fading of THz communications.

2) Misalignment Fading Coefficient: We consider that Destination has a circular detection beam, and the radius of the beam is $\alpha$. Furthermore, the RIS has a circular beam of radius $\rho$, where $0 \leq \rho \leq \omega_d$, and $\omega_d$ denotes the maximum radius of the beam. Moreover, we assume that $\tau$ is the pointing error. Due to the symmetry of the beam shapes, the misalignment fading coefficient that represents the fraction of the power that $N_T$ is the truncation term. Using the property of PDF, we have $\int_0^\infty f_\gamma(\gamma) = 1$. With the help of [29, eq. (3.351.3)] and [29, eq. (8.339.1)], the $\gamma_T$ should be set to satisfy $\sum_{j=0}^{N_T} d_j \to 1$.

Typically, we can use 40 terms to make the series converge for all cases considered in this paper. Furthermore, the truncation error, which is defined as $1 - \sum_{j=0}^{N_T} d_j$, is less than $10^{-6}$.

To prove that the FTR model can fit the THz signal’s amplitude fading best, we consider the real measurement data from two campaigns [23], [25], and evaluate the fitting of those theoretical fading distributions to the empirical data with the help of K-S goodness-of-fit statistical test [30].

- Very recently, a measurement campaign was conducted in a train test center [23]. The authors use a novel $M$-sequence correlation-based ultra-wideband channel sounder to measure the THz channel. In the measurement, the channel sounder was operated at 304.2 GHz with 8 GHz bandwidth. The small-scale fading was extracted for each channel impulse response based on the data. We have fitted the measurement data to a Gaussian, Rician, Nakagami-$m$, and FTR distribution, respectively, which are also compared with the CDF of the measurement data. As shown in Figs. 2, we find that the FTR distribution performs the best fit.

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collected by Destination, $h_P$, depends only on the radial distance $r$. Thus, $h_P$ can be approximated as [32, eq. (11)]
\[
fh_p(x) = \gamma^2 A_o^2 x^{2-1}, \quad 0 \leq x \leq A_o.
\]
(4)
where $A_o = [\text{erf}(u)]^2$, $u \triangleq \sqrt{\pi}a/(\sqrt{2}w_d)$,
\[
\gamma = \frac{w_d^2}{2\pi}, \quad \text{and} \quad \sigma^2
\]
denotes the variance of the pointing error displacement at Destination, $\omega_{eq}$ is the equivalent beam-width, and $w_d^2 = \omega_{eq}^2 = \omega_{eq}^2 \exp(u^2)\left(2u \exp(-u^2)\right)^{-1}.

3) Path Gain Coefficient: The path gain coefficient can be evaluated as $h_L = h_{FL}h_{AL}$, where $h_{FL}$ is the propagation gain, and $h_{AL}$ is the molecular absorption gain. Using Friis equation, we can express $h_{FL}$ as
\[
h_{FL} = \frac{c^2\sqrt{G_t G_r}}{(4\pi f)^2 (d_1d_2)}.
\]
(5)
where $G_t$ and $G_r$ represent the antenna orientation dependent transmission and reception gains, respectively, $c$ is the speed of light, $f$ stands for the operating frequency, $d_1$ is the distance between Source and RIS, and $d_2$ is the distance between RIS and Destination. Furthermore, $h_{AL}$ can be evaluated as [19]
\[
h_{AL} = \exp\left(-\frac{1}{2}\kappa_\alpha(f)(d_1 + d_2)\right),
\]
(6)
where $\kappa_\alpha(f)$ denotes the absorption coefficient which can be expressed as [19]
\[
\kappa_\alpha(f) = \frac{A(v)}{B(v) + \left(\frac{1}{\text{TROC}} - c_1\right)^2} + \frac{C(v)}{D(v) + \left(\frac{1}{\text{TROC}} - c_2\right)^2} + p_1f^2 + p_2f^2 + p_3f + p_4,
\]
(7)
where $c$ denotes the speed of light, $c_1 = 10.835$ cm$^{-1}$, $c_2 = 12.664$ cm$^{-1}$, $p_1 = 5.54 \times 10^{-37}$ Hz$^{-3}$, $p_2 = -3.94 \times 10^{-25}$ Hz$^2$, $p_3 = 9.06 \times 10^{-14}$ Hz$^{-1}$, $p_4 = -6.36 \times 10^{-3}$, $A(v) = 0.2205v(0.1303v + 0.0294)$, $B(v) = (0.4093v + 0.0925)^2$, $C(v) = 2.014v(0.1702v + 0.0303)$, and $D(v) = (0.537v + 0.0956)^2$ [19]. Moreover, $v$ can be evaluated as
\[
v = \phi(0.06116\rho^{-1} + 2.1148 \times 10^{-7}) \exp\left(\frac{17.5027T}{240.97 + T}\right),
\]
(8)
where $T$ is the temperature, $\phi$ and $\rho$ denote the relative humidity and the atmospheric pressure, respectively. Note that the molecular absorption loss model that we considered here provides an accurate model for the 275 – 400 GHz frequency band. Meanwhile, for frequencies higher than 400 GHz, we can obtain the value of $\kappa_\alpha(f)$ from the high resolution transmission (HITRAN) database [33], [34]. Using (5)-(8), we finally obtain the path gain coefficient of the considered RIS-aided THz communications system.

B. Received THz Signal Expression Analysis

The signal received from Source through RIS for Destination can be expressed as\(^2\)
\[
y = Ph_1h_Ph_Fw^T(x + n_S) + n_D + n,
\]
(9)
\(^2\)Because the THz signals have high susceptibility to blockages [33], we assume that the direct link between the source and destination is negligible due to the severe attenuation.

Fig. 2. (a). PDF of the measurement data [23] and fitted distributions. The parameters of fitted distributions are obtained by fittist(·) function in Matlab [31] and are shown in Table I (b). CDF of the measurement data and fitted distributions with the same parameters as Fig. 2 (a).

Fig. 3. (a). PDF of the measurement data [25] and fitted distributions. The parameters of $\alpha-\mu$ distribution are given in [25, Table 5, TX1], i.e., $\alpha = 8.54867$, $\mu = 0.23377$, and $\beta = 8.40874$. The parameters of FTR distribution are $\sigma = 0.8991$, $m = 30$, $K = 30$, and $\Delta = 0.8$ (b). CDF of the measurement data [25] and fitted distributions. The parameters of $\alpha-\mu$ distribution are given in [25, Table 1, TX1 and TX25]. The parameters of FTR distribution are shown in Table I.
where \( n \sim \mathcal{CN}(0, N_0) \), \( \mathcal{CN}(t_1, t_2) \) denotes the white Gaussian process with mean \( t_1 \) and variance \( t_2 \), \( x \) is the transmitted signal satisfying \( \mathbb{E} \left[ |x|^2 \right] = 1 \), \( \mathbb{E} \left[ |w|^2 \right] = 1 \), \( \| \cdot \| \) denotes the Frobenius norm. \( P \) is the transmit power, the parameters \( n_S \sim \mathcal{CN}(0, \kappa_S^2 P) \) and \( n_D \sim \mathcal{CN}(0, \kappa_D^2 P) \), where \( h_{L,F} \) are distortion noise from hardware impairments in both Source and Destination, respectively [35], and \( \kappa_S \) and \( \kappa_D \) are non-negative parameters that characterize the level of hardware impairments [36] in Source and Destination, respectively. In the RIS-aided communications, the \( P \) and \( \Phi \) are adjustable. Considering the optimal beamforming vector design, e.g., using the maximum ratio transmitting [37], [38], we can define \( w = h_F / \| h_F \| \), and the corresponding small-scale fading coefficient can be expressed as

\[
\Phi = [h_{F,1}, \ldots, h_{F,M}]
\]

The square of the Frobenius norm of \( h_F \) is upper-bounded as

\[
\| h_F \|^2 = \sum_{m=1}^{M} \left| \sum_{l=1}^{L} g_{2}^{(l)} e^{j \varphi_l} g_1^{(m,l)} \right|^2 
\]

where the error between the upper-bound and the exact value of \( \| h_F \|^2 \) decreases as RIS’s phase shift design becomes more accurate. However, the right-hand side of (11) is still hard to be used in the performance analysis. The reason is that \( |g_2^{(l)}| \) and \( |g_1^{(m,l)}| \) follow i.i.d. FTR fading distribution. Deriving the PDF of \( \| h_F \|^2 \) is equivalent to deriving the PDF of square of two-fold sum of product of FTR RVs, which is extremely difficult, if not impossible, to be obtained. Fortunately, the distribution of the sum of FTR random variables (RVs) can be accurately approximated by the single FTR distribution by following the method proposed in [39], i.e.,

\[
\| h_F \|^2 \approx \left( \sum_{m=1}^{M} \left| g_1^{(m)} \right| \right)^2 
\]

Thus, with the optimal phase shift design, we have

\[
\| h_F \|^2 \approx \left( \sum_{m=1}^{M} \left| g_1^{(m)} \right| \right)^2 
\]

where the two-fold summation is reduced to a one-fold summation, thus giving the possibility of further performance analysis. Note that (12) is also the exact small-scale fading expression when considering a single antenna transmitter. In the following, we analyze the SNR expressions in two cases.

1) Non-Ideal RF Chains: With the non-ideal RF chains, the instantaneous SNDR can be expressed as

\[
\gamma_N = \frac{\| h_F \|^2 |h_{L,F}|^2}{\kappa^2 |h_F|^2 |h_{L,F}|^2 + N_0},
\]

where \( \kappa^2 \triangleq \kappa_S^2 + \kappa_D^2 \).

2) Ideal RF Chains: Let us consider the case of ideal RF chains. Using (13) and letting \( \kappa = 0 \), we can express the instantaneous SNR of the received signal as

\[
\gamma_I = \frac{\| h_F \|^2 |h_{L,F}|^2}{N_0}. 
\]

### III. Phase Shift Design Under Discrete Phase Shift Constraints

In this section, we propose an SIO algorithm to obtain the optimal design of RIS’s reflector array, \( \Phi \). With the optimal phase shift design of RIS, we can theoretically obtain the maximum of SNDR. In practical systems, ideal continuous phase-shifts are hard to implement because of hardware limitation. Thus, we consider the discrete phase-shifts design, which is more energy efficient [40].

#### A. Optimal Phase Shift Design of RIS’s Reflecting Elements

1) Motivations for proposing the SIO algorithm: There are several challenges for the existing RIS phase shift design algorithms, which can be summarized as follows:

- When the number of reflecting elements at the RIS, \( L \), is large, most of the existing algorithms [7], [40]–[42] require a lot time to converge, which motivates us to propose an effective algorithm for practical RIS-aided systems. The running time should not be greatly effected by \( L \).
- Eliminating the channel phases is difficult because the RIS is assumed to be passive. Although a novel and simple algorithm based on the binary search tree was proposed to obtain \( \Phi_{\text{opt}} \) without the channel phases information [41], the algorithm proposed in [41] is based on the assumption that the phase-shifts is continuous at

| Measurement | Channel Model | Parameters | K-S Test Values |
|-------------|---------------|------------|----------------|
| [23]        | FTR Fading    | \( \sigma = 4.84 \times 10^{-4} \), \( m = 0.63 \), \( K = 2.9 \), \( \Delta = 0.3 \) | 0.0383 (✓) |
| [23]        | Gaussian Fading | \( \mu_G = 1.83 \times 10^{-4} \), \( \sigma_G = 2.01 \times 10^{-4} \) | 0.1870 (✗) |
| [23]        | Rician Fading  | \( s_G = 7.78 \times 10^{-6} \), \( \sigma_G = 1.92 \times 10^{-4} \) | 0.3687 (✗) |
| [23]        | Nakagami-m Fading | \( \mu_N = 0.324 \), \( \omega_N = 7.35 \times 10^{-8} \) | 0.1058 (✓) |
| [25]        | TX1 Fading     | \( \sigma = 2.7729 \), \( m = 6 \), \( K = 7 \), \( \Delta = 0.2 \) | 0.0175 (✓) |
| [25]        | TX2 Fading     | \( \alpha = 3.35467 \), \( \mu = 1.11365 \), \( \beta = 11.58104 \) | 0.1838 (✓) |
| [25]        | TX15 Fading    | \( \sigma = 2.382 \), \( m = 3 \), \( K = 6 \), \( \Delta = 0.04 \) | 0.0203 (✓) |
| [25]        | TX15 Fading    | \( \alpha = 3.11872 \), \( \mu = 0.91962 \), \( \beta = 9.83341 \) | 0.0566 (✓) |

**TABLE I**

Kolmogorov–Smirnov Goodness of Fit Tests.
In a real RIS-aided THz communications system, the value of the dimensional search space to optimize a fitness function [43]. Swarm Optimization (PSO) and evolution algorithms, without the channel phases information, inspired from Particle Swarm Optimization, is applied in RIS systems. The velocity of the particles is updated as follows:

\[
v_{id}(k + 1) = \omega v_{id}(k) + c_1 r_1 (p_{id}(k) - x_{id}(k)) + c_2 r_2 (p_{gd}(k) - x_{id}(k)),
\]

where \( i = 1, 2, \ldots, m \), \( d = 1, 2, \ldots, L \), \( \omega \) is the inertia weight, \( c_1 \) and \( c_2 \) are the acceleration constants, and \( R_1 \) and \( R_2 \) are uniformly distributed random numbers in \([0, 1]\). \( P_i = (p_{i1, 1}, p_{i1, 2}, \ldots, p_{i1,L}) \) is the best previous position of the \( i_{th} \) particle, and \( P_g = (p_{g1, 1}, p_{g1, 2}, \ldots, p_{g1,L}) \) is the best position among all the particles.

3) Design of the SIO algorithm: The main disadvantage of the PSO algorithm is that it is difficult to balance local and global search and keep the diversity of the population. Thus, by combining the modified PSO algorithm [44], [45] with evolution algorithms, we present the SIO Algorithm 1, which maintains the optimal balance between exploration and exploitation, and reduces computation time, to find \( \Phi_{opt} \). In the following, we introduce the SIO’s working mechanism and the important parameters:

- Each particle is given a periodic value, \( P_e \), whose initial value is 0. When the particle updates to a better state, the periodic value resets to 0, and otherwise, the value increases by 1. We set the size of swarm reaches \( n_{\text{begin}} \). As the searching time increases, the particle that has the largest periodic value will be deleted, until the size of the swarm reaches \( n_{\text{end}} \). We can set \( n_{\text{begin}} = 30 \) and \( n_{\text{end}} = 10 \) to reduce the computational resources and keep the diversity of the population.
- The inertia weight \( \omega \) can significantly affect the global and local search-ability. By linearly decreasing \( \omega \) from a relatively larger value to a smaller value, the algorithm’s global search-ability at the beginning and the local search-ability near the end of the search procedure can be efficiently enhanced [46]. In experimental work (i.e., in Figs. 4-6), \( \omega \) is kept in between 0.9 to 0.4.
- Adding an exploitative component allows the algorithm to utilize local information to guide the search towards better regions in the search space. The accelerate constants, i.e., \( c_1 \) and \( c_2 \), represent the particle stochastic acceleration weights toward the best positions of the local, \( i_{th} \) particle and among all the particles. The system designer can set larger accelerate constants for fast search, or smaller accelerate constants to reduce oscillations of particles.

In contrast to the continuous phase-shifts, we consider the practical case where the RIS only has a finite number of discrete phase-shifts that are equally spaced in \([0, 2\pi]\). The set of phase-shifts at each element is given by \( \theta = \{0, \Delta \theta, \ldots, \Delta \theta(K-1)\} \), where \( K = \lfloor \frac{2\pi}{\Delta \theta} \rfloor \). If we use \( b \) bits to represent each of the levels, we can also express \( \theta \) as \( \theta = \{\theta_1, \theta_2, \ldots, \theta_P\} \), after finding the three best values, i.e., \( \theta_1, \theta_2, \) and \( \theta_P \), the velocities and positions are updated with the following formulas:

\[
v_{id}(k + 1) = \lceil [\omega v_{id}(k) + \Delta v(k)] / \Delta \theta \rceil \times \Delta \theta,
\]

\[
\Delta v(k) = c_1 R_1 (p_{id}(k) - x_{id}(k)) + c_2 R_2 (p_{gd}(k) - x_{id}(k))
\]

\[
+ c_3 R_3 (p_{gd}(k) - x_{id}(k)),
\]

\[
x_{id}(k + 1) = x_{id}(k) + v_{id}(k + 1),
\]

where \( \lceil \cdot \rceil \) is the rounding function. From (17)-(19), we can observe that, in practical communications systems, we only need to measure the \( E_{\text{rec}} \) in the time domain and adjust the RIS’s phase shift with the help of Algorithm 1, and does not need any channel estimation to obtain \( \Phi_{opt} \).

B. Convergence and Effectiveness Analysis

1) Convergence Analysis of the SIO: To prove the convergence of our proposed SIO, we first derive the location iteration function. However, when \( \Delta \theta \neq 0 \), it is difficult to substitute (17) into (19) because of the rounding function in (17). To solve this problem, we rewrite (17) as

\[
v_{id}(k + 1) = \omega v_{id}(k) + \Delta v(k) + c(k),
\]

where \( c(k) \) denotes the mismatch between \( \omega v_{id}(k) + \Delta v(k) \) and RIS’s practical discrete phases. We can obtain the value of \( c(k) \) by

\[
c(k) = (\lfloor [\omega v_{id}(k) + \Delta v(k)] / \Delta \theta \rfloor \times \Delta \theta - \omega v_{id}(k) + \Delta v(k)).
\]

Thus, we have \(- \frac{1}{2} \Delta \theta < c(k) < \frac{1}{2} \Delta \theta\). The velocity of each particle is updated as

\[
v_{id}(k + 1) = x_{id}(k + 1) - x_{id}(k).\]

Let \( T_i \triangleq c_i R_i \). Substituting (18) and (20) into (22), we can derive the location iteration function as

\[
x_{id}(k + 1) - (1 - T_1 - T_2 - T_3 + \omega) x_{id}(k) + \omega x_{id}(k - 1)
\]

\[
= T_1 p_{id}(k) + T_2 p_{gd}(k) + T_3 p_{gd}(k) + c(k).
\]

When the RIS’s elements can adjust the phase-shifts continuously, we have \( \Delta \theta = 0 \) and \( c(k) = 0 \). For the discrete case, the \( c(k) = 0 \) can be regarded as a bounded noise. Note that the recurrence equation of SIO is linear non-homogeneous. To calculate the characteristic roots, we consider the secular equation as

\[
x^2 - (1 - T_1 - T_2 - T_3 + \omega) x + \omega = 0.
\]

The discrete linear system that has such a secular equation has been well studied [47]. Because the latent roots of (24) can be derived easily, we can obtain the expression of \( x(k) \) and prove that the SIO algorithm can converge to the optimal solution. Moreover, SIO suffers from a linear increase in the
Algorithm 1 The swarm intelligence-based optimization algorithm to find $\Phi_{\text{opt}}$ for the RIS with finite-level phase-shifters.

**Input:** Input the number of elements on RIS: $L$, the minimum value of phase-shifts: $\Delta \theta$, maximum iterations: Max, size of swarm: $n_{\text{begin}}$ and $n_{\text{end}}$, inertia weights: $\omega_{\text{begin}}$ and $\omega_{\text{end}}$, acceleration constants: $c_1$, $c_2$ and $c_3$, maximum of velocity: $v_{\text{max}}$

**Output:** The phase-shift variable $\varphi_{\ell}$ ($\ell = 1, \ldots, L$) for each RIS element should be set.

1: Set iteration index $k = 0$. Initialize the velocities and positions of $n$ particles, $\omega \leftarrow \omega_{\text{begin}} - k/\text{Max} \ast (\omega_{\text{begin}} - \omega_{\text{end}})$.
2: for Every particle in swarm do
3: Obtain corresponding expectation of the amplitude of the received signal, $E_{\text{re}}$.
4: Update personal best position of each particle, global best position and corresponding values.
5: while $k < \text{Max}$ do
6: for Every particle do
7: Obtain corresponding $E_{\text{re}}$.
8: if Current position is personal best position for this particle then
9: Update its personal best position.
10: end
11: if Current value of $E_{\text{re}}$ is smaller than the last value then
12: Increase the periodic value: $P_e \leftarrow P_e + 1$
13: end
14: Calculate the number of particles that will be deleted: $D \leftarrow n - \lfloor n_{\text{begin}} - k/\text{Max} \ast (n_{\text{begin}} - n_{\text{end}}) \rfloor$
15: if $D > 0$ then
16: Delete $D$ particles which have the largest periodic value, $P_e$
17: end
18: for Every particle do
19: Obtain $v$ with the help of (17) and (18)
20: if $v > v_{\text{max}}$ then
21: end
22: Update $x$ with the help of (19)
23: Update the inertia weight: $\omega \leftarrow \omega_{\text{begin}} - k/\text{Max} \ast (\omega_{\text{begin}} - \omega_{\text{end}})$
24: end
25: end
26: end
27: return In $L$-dimensional search space, the global best position is $\varphi_{\ell}$ ($\ell = 1, \ldots, L$)

search complexity with an increase in the maximum iteration number, Max. In addition, $E_{\text{re}}$ is obtained $n$ times in each iteration. Thus, the complexity of SIO is given as $O(n \text{Max})$.

2) Effectiveness Analysis of the SIO: Let $E_{\text{re}} \triangleq \mathbb{E}[|h_{L}|^2 | h_{P}| | h_{R}|]$ and $E_{\text{opt}} \triangleq |h_{L}| \mathbb{E}[|h_{P}|] \mathbb{E}\left[\sum_{j=0}^{L} g_1(j) g_2(j)\right]$, where $E_{\text{re}}$ is the fitness function in our SIO algorithm which can be easily measured in the time domain, and $E_{\text{opt}}$ the maximum achievable $E_{\text{re}}$, $\mathbb{E}[\cdot]$ denotes the mathematical expectation.

Fig. 4. The number of iterations (until $E_{\text{re}}/E_{\text{opt}} \geq 90\%$) versus the number of RIS’s elements, with $\Delta \theta = \pi/10$.

To investigate the effectiveness of the proposed phase optimization method, we first derive $E_{\text{opt}}$ in the following theorem

**Theorem 1.** The maximum achievable expectation of the amplitude of the received signal, $E_{\text{opt}}$, can be derived as

$$E_{\text{opt}} = \frac{|h_L|^2 A_w}{\gamma^2 + 1} \sum_{\ell=1}^{L} \frac{m_{\ell}}{m_{\ell-1}} \sum_{j=0}^{N_{\ell}} \frac{K_{\ell}^j d_\ell (2\pi^2)^j}{\Gamma(j+1)} \Gamma \left( \frac{3}{2} + j \right).$$

(25)

**Proof:** Please refer to Appendix A. $\blacksquare$

We use $E_{\text{re}}/E_{\text{opt}}$ to characterize the expectation accuracy and verify the convergence and effectiveness of the proposed algorithm. In Figs. 4 and 5, we verify the convergence and effectiveness of the proposed algorithm for different iterations, by setting $P = 40$ dBW, $m_{\ell,1} = 3$, $m_{\ell,2} = 4$, $2\sigma_{\ell,1} (1 + K_{\ell,1}) = 20$ dB, $2\sigma_{\ell,2} (1 + K_{\ell,2}) = 10$ dB, $f = 300$ GHz, $d_1 = 30$ m, $d_2 = 20$ m, $N_0 = 1$ dBW, $K_{\ell,1} = 4$, $K_{\ell,2} = 5$, $\Delta_{\ell,1} = 0.7$, $\Delta_{\ell,2} = 0.5$, $\kappa_S = \kappa_D = 0.2$, $n_{\text{begin}} = 30$, $n_{\text{end}} = 10$, $c_1 = 1$, $c_2 = 2$, $c_3 = 2$, $\omega_{\text{begin}} = 0.9$, and $\omega_{\text{end}} = 0.4$. In Fig. 4, the proposed SIO is compared with traditional PSO under the different number of reflecting
elements, \( L \). As it can be observed, although the numbers of iterations for both PSO and SIO increase linearly with the number of RIS elements, the increase rate when SIO is used is reduced by about 34\% compared to that when PSO is used, which means that the SIO is always superior to the traditional PSO at finding optimal phase-shifts. Moreover, the red and blue shaded ranges in Fig. 4 indicate the uncertainty in the actual operation of the algorithms. We can see that the fluctuation range of the number of iterations of the SIO (typically less than 25) is smaller than that of the PSO. Figure 5 depicts \( E_{\text{rec}}/E_{\text{opt}} \) versus the amount of reflecting elements on the RIS, with different choices of \( \Delta \theta \). Although the error of expectation increases as \( \Delta \theta \) is increased, we can observe that using the RIS with large \( \Delta \theta \), e.g., \( \pi/4 \), can achieve the accuracy close to that when using RIS with continuous phase-shifts. Furthermore, Fig. 6 shows that the proposed SIO algorithm is better than the binary search tree-based algorithm proposed in [41]. Although both the proposed SIO algorithm and algorithm proposed in [41] need to measure the \( E_{\text{rec}} \), the number of reflecting elements only affects the number of measurements in the algorithm proposed in [41]. Thus, especially when the RIS is equipped with a large number of elements, the SIO algorithm can achieve a higher \( E_{\text{rec}}/E_{\text{opt}} \) than that of the algorithm proposed in [41] with the same number of measurements.

IV. EXACT STATISTICS OF THE END-TO-END SNR AND SNDR

Substituting \( R = \sqrt{\gamma} \) into (2), we can easily obtain the PDF expression of FTR RVs. Let \( Y = \sum_{i=1}^{L} \prod_{\ell=1}^{N} R_{i,\ell} \) be the sum of product of \( N \) FTR RVs, where \( R_{i,\ell} \sim FTR \left( K_{i,\ell}, m_{i,\ell}, \Delta_{i,\ell}, \sigma_{i,\ell}^2 \right) (i = 1, \ldots, L \text{ and } \ell = 1, \ldots, N) \), the exact PDF and CDF of \( Y \) have been respectively derived in multivariate Fox’s H-function [48, eq. (A.1)] as [41, eq. 11] and [41, eq. 12].

In Figs. 4-6, we verify that Algorithm 1 can be used to obtain the phase-shifts of RIS’s elements which enable the received power tightly close to the optimal received power. Then, we present the instantaneous optimal SNR and SNDR for the case of ideal and non-ideal RF chains, respectively.

A. Non-Ideal RF Chains

To derive a generic analytical expression for the CDF of the instantaneous SNDR of non-ideal RF chains, we first present the following lemma with the help of (13).

Lemma 1. Considering hardware impairments, we derive an integral representation for the PDF and CDF of \( \gamma_N \) in (13) assuming arbitrarily distributed \( |h_F| \) and \( |h_P| \) as

\[
\begin{align*}
    f_{\gamma_N}(x) &= \int_{0}^{\Delta} \frac{1}{y} f_{|h_F|}(y) \left( \frac{1}{\|h_L\|} x N_o \right) \frac{1}{|h_F|} f_{|h_P|}(y) \, dy \\
        & \times 2 \sqrt{x(1 - x\kappa^2)} |h_L|^2 P(1 - x\kappa^2) \frac{1}{N_o} \nu_{h_F,\ell}
    \end{align*}
\]

(26)

\[
F_{\gamma_N}(x) = \int_{0}^{\Delta} F_{|h_F|}(y) \left( \frac{1}{\|h_L\|} x N_o \right) \frac{1}{|h_F|} f_{|h_P|}(y) \, dy.
\]

(27)

Proof: Please refer to Appendix B.

With the help of Lemma 1, for special distributed \( |h_F| \) and \( |h_P| \) in this paper, the statistics of instantaneous SNDR can be obtained as the following theorem.

Theorem 1. The PDF and CDF of \( \gamma_N \) can be derived as (28) and (29), shown at the bottom of the next page, where \( \{\alpha_n\}^N_1 = (\alpha_1, \ldots, \alpha_N) \) and \( \nu_{h_F,\ell} = \{(-j, n, 0.5)\}^L_1, (1 - \gamma^2, 1) (\ell = 1, \ldots, L) \).

Proof: Please refer to Appendix C.

Remark 1. The multivariate Fox’s H-function cannot be evaluated directly by popular mathematical packages such as MATLAB or Mathematica. However, many implementations have been proposed [49], [50]. In the following, we use an efficient and accurate Python implementation [50] which can provide accurate results of multivariate Fox’s H-function quickly by estimating the multivariate integrals with simple rectangle quadrature. For example, the execution time for calculating ternary Fox’s H-function is less than a few seconds.

B. Ideal RF Chains

Theorem 2. The PDF and CDF of \( \gamma_I \) can be derived as (30) and (31), shown at the bottom of the next page.

Proof: Substituting \( \kappa^2 = 0 \) into (28) and (29), we obtain the PDF and CDF of \( \gamma_N \) after performing some mathematical manipulations, which completes the proof.

The statistics of \( \gamma_I \) and \( \gamma_I \) that we derived in Theorem 2 and Theorem 3 can be used to obtain exact closed-form expressions for key performance metrics.

V. ASYMPTOTIC ANALYSIS

Although the previously derived analytical results are obtained in closed-form, they may not provide many insights as to the factors affecting system performance. In the following, the asymptotic performance analysis that becomes tight in high-SNDR and \( L \) regimes is presented.
A. Asymptotic analysis in the high SNDR regime

Proposition 1. A simple approximation of the CDF of $\gamma_N$ in the high-SNDR regime can be derived as

$$F_{\gamma_N}^{HS}(x) = \sum_{j_1, \ldots, j_L=0}^{N_T} \sum_{j_{1, \ldots, j_L}=0}^{N_T} \prod_{l=1}^{L} \prod_{t=1}^{2} K_{j_{1, l}, t, j_{2, t}} m_{1, l} m_{2, t} \gamma^2$$

$$\times \lim_{\varpi \to \varpi_0} (\varpi + \varpi_0) \prod_{l=1}^{L} \Gamma(1 + j_{1, l} - \frac{1}{2} \varpi) \Gamma(\varpi) \Gamma(\gamma^2 - \varpi)$$

$$\times \left( A_0 \left| \frac{|h_L|}{x N_o} \right|^2 \prod_{l=1}^{L} \sqrt{2 \sigma_{2, l}} \right)^{-\varpi_0},$$

where $\varpi_0 = \min \{ \gamma^2, 2 + 2j_{1, l}, 2 + 2j_{2, t} \}$, and

$$\lim_{\varpi \to \varpi_0} (\varpi + \varpi_0) \prod_{l=1}^{L} \Gamma(1 + j_{1, l} - \frac{1}{2} \varpi) \Gamma(\varpi) \Gamma(\gamma^2 - \varpi)$$

$$= \prod_{l=1}^{L} \Gamma(1 + j_{1, l} - \frac{1}{2} \varpi) \Gamma(\gamma^2), \varpi_0 = \gamma^2$$

Proof: With the help of the definition of multivariate Fox’s $H$-function [48, eq. (A-1)], eq. (29) can be expressed in terms of multiple Mellin-Barnes type contour integrals. When $SNDR \to \infty$, we have $P \to \infty$. The Mellin-Barnes integrals can be approximated by evaluating the residue at the minimum pole on the left-hand side of $d\varpi_0$ as [48, Theorem 1.11].

Proportionality 1.

$$f_{\gamma_N}(x) = \sum_{j_1, \ldots, j_L=0}^{N_T} \sum_{j_{1, \ldots, j_L}=0}^{N_T} \prod_{l=1}^{L} \prod_{t=1}^{2} K_{j_{1, l}, t, j_{2, t}} m_{1, l} m_{2, t} \gamma^2$$

$$\times P_{0, 1, 3, \ldots, 3}^{0, 1, 3, \ldots, 3} \prod_{l=1}^{L} \left| \frac{|h_L|^2}{x N_o} \right|^2 \prod_{l=1}^{L} \sqrt{2 \sigma_{2, l, t}}$$

$$\times \left( A_0 \left| \frac{|h_L|^2}{x N_o} \right|^2 \prod_{l=1}^{L} \sqrt{2 \sigma_{2, l, t}} \right)^{-\varpi_0},$$

(28)

Proportionality 1.

$$f_{\gamma_L}(x) = \sum_{j_1, \ldots, j_L=0}^{N_T} \sum_{j_{1, \ldots, j_L}=0}^{N_T} \prod_{l=1}^{L} \prod_{t=1}^{2} K_{j_{1, l}, t, j_{2, t}} m_{1, l} m_{2, t} \gamma^2$$

$$\times H_{0, 1, 3, \ldots, 3}^{0, 1, 3, \ldots, 3} \prod_{l=1}^{L} \left| \frac{|h_L|^2}{x N_o} \right|^2 \prod_{l=1}^{L} \sqrt{2 \sigma_{2, l, t}}$$

$$\times \left( A_0 \left| \frac{|h_L|^2}{x N_o} \right|^2 \prod_{l=1}^{L} \sqrt{2 \sigma_{2, l, t}} \right)^{-\varpi_0},$$

(29)

Proportionality 1.

$$f_{\gamma_L}(x) = \sum_{j_1, \ldots, j_L=0}^{N_T} \sum_{j_{1, \ldots, j_L}=0}^{N_T} \prod_{l=1}^{L} \prod_{t=1}^{2} K_{j_{1, l}, t, j_{2, t}} m_{1, l} m_{2, t} \gamma^2$$

$$\times H_{0, 1, 3, \ldots, 3}^{0, 1, 3, \ldots, 3} \prod_{l=1}^{L} \left| \frac{|h_L|^2}{x N_o} \right|^2 \prod_{l=1}^{L} \sqrt{2 \sigma_{2, l, t}}$$

$$\times \left( A_0 \left| \frac{|h_L|^2}{x N_o} \right|^2 \prod_{l=1}^{L} \sqrt{2 \sigma_{2, l, t}} \right)^{-\varpi_0},$$

(30)

Proportionality 1.

$$F_{\gamma_N}(x) = \sum_{j_1, \ldots, j_L=0}^{N_T} \sum_{j_{1, \ldots, j_L}=0}^{N_T} \prod_{l=1}^{L} \prod_{t=1}^{2} K_{j_{1, l}, t, j_{2, t}} m_{1, l} m_{2, t} \gamma^2$$

$$\times H_{0, 1, 3, \ldots, 3}^{0, 1, 3, \ldots, 3} \prod_{l=1}^{L} \left| \frac{|h_L|^2}{x N_o} \right|^2 \prod_{l=1}^{L} \sqrt{2 \sigma_{2, l, t}}$$

$$\times \left( A_0 \left| \frac{|h_L|^2}{x N_o} \right|^2 \prod_{l=1}^{L} \sqrt{2 \sigma_{2, l, t}} \right)^{-\varpi_0},$$

(31)
Therefore, the expression in (32) is derived, which completes the proof.

Note that the CDF of $\gamma_j$ in the high-SNDR regime can be derived by substituting $\kappa^2 = 0$ into (32).

**B. Asymptotic analysis in the high L regime**

Considering the RIS typically has hundreds of elements, we next derive closed-form statistics of the RV $h_F = \sum_{i=1}^{L} \prod_{k=1}^{K} R_{i,k}$ in high-SNDR and L regimes. Employing central limit theorem (CLT) as [51], we have $h_F \sim N(\mathbb{E}[h_F], \mathbb{V}[h_F])$, where $\mathbb{V}[\cdot]$ denotes the variance of an RV. Considering that $\mathbb{V}[\cdot] = \mathbb{E}[h_F^2] - \mathbb{E}[h_F]^2$, we can obtain the values of $\mathbb{E}[h_F]$ and $\mathbb{V}[h_F]$ with the help of [41, eq. (6)]. Therefore, the CDF of $h_F$ in the high L regime can be expressed as [52]

$$F_{h_F}(x) = \frac{1}{2} \left( 1 + \text{erf} \left( \frac{x - \mathbb{E}[h_F]}{\sqrt{2\mathbb{V}[h_F]}} \right) \right) = \frac{1}{2} \text{erfc} \left( \frac{x - \mathbb{E}[h_F]}{\sqrt{2\mathbb{V}[h_F]}} \right),$$

where $\text{erf}(\cdot)$ is the error function [29, eq. (8.250.1)] and $\text{erfc}(\cdot)$ is the complementary error function [29, eq. (8.250.4)]. When $L \to \infty$, the CDF of $\gamma_N$ can be derived with the help of (34), by following the similar method as shown in Appendix C. However, note that $\text{erf}(\cdot)$ or $\text{erfc}(\cdot)$ in (34) is a non-elementary function, which is defined in the form of a definite integral. Thus, it is hard to obtain new insights. Therefore, we use a five-first-moment (5FM) based method to obtain the approximate CDF expression.

**Proposition 2.** An approximation of the CDF of $\gamma_N$ in the high-L regime can be expressed as

$$F_{\gamma_N}(x) \approx a_1 a_2 x^2 G_{3,4}^{1,3} \left( \frac{\mathcal{Y}}{A_{\gamma} a_2} \middle| \begin{array}{c} 1, a_3 + 1, \gamma^2 + 1, a_4 + 2, a_5, 1, a_6 \end{array} \right),$$

where $\mathcal{Y} = \sqrt{\frac{\pi}{\mathbb{L}}} \mathbb{P}(1-x^2)$, $G_{3,4}^{1,3}$ is the Meijer’s $G$ function [29, eq. (9.30)], $\Omega_i$ denotes the $i_{th}$ moment of $h_F$, $\varphi_i = \Omega_i/\Omega_{i-1}$, $a_3 = (4\varphi_2 - 9\varphi_3 + 6\varphi_2 - \Omega_1)/\Omega_2 - 3\varphi_3 - 3\varphi_2 + \Omega_1$, $a_2 = \frac{\varphi_2}{\varphi_3} - 2\varphi_3 + 2\varphi_2$, $a_4 = 2\varphi_2 - 3\varphi_3 + 2\varphi_2$, $a_6 = -1$ $a_7 = (a_6 + a_7)/2$, and $a_1 = \Gamma(a_3 + 1) / (a_2\Gamma(a_4 + 1) \Gamma(a_5 + 1))$.

**Proof:** Please refer to Appendix D.

By substituting $\kappa^2 = 0$ in (35), after performing some mathematical manipulations, we can obtain the CDF of $\gamma_j$ in the high L regime.

**C. Verification and Insights**

With the help of the approximate CDF expressions in Propositions 1 and 2, the difficulty of deriving performance metrics for RIS-aided THz communications systems can be avoided. For example, the outage probability $P$ can be defined as the probability that the received SNR per signal falls below a given threshold $\gamma_\text{th}$, which can be directly obtained using the derived CDF expressions. Figure 7 shows the outage probability versus the transmit power. We can observe that the analytical results ((29) in Theorem 1) match well with the high-L approximate results obtained by the two methods, i.e., CLT and 5FM, in Proposition 2 with $L = 20$, which means that the high-L approximate expressions are useful because the RIS can be equipped with hundreds of reflecting elements in the practical system. Note that when the number of reflecting elements is small, the CLT method is not accurate, but the 5FM method can be used because the parameter $\Omega_i$, which denotes the $i_{th}$ moment of small-scale fading coefficient, contains the impact of the number of elements. Furthermore, the accuracy of the approximate CDF expressions derived in Proposition 1 is verified in Fig. 8 in the following numerical analysis section. We can observe that at high-transmitting power regime, (32) approximates well the exact analytical expression.

In addition to simplifying the performance analysis, we can observe several insights from the approximate expressions. For example, similar to the diversity order [53], the high-SNR slope is a key performance indicator that can explicitly capture the impact of channel parameters on OP performance, which is defined as [54]

$$G_d = -\lim_{P \to \infty} \frac{\log(\text{OP}_{\text{app}})}{\log(P)}.$$

With the help of the accurate 5FM-based approximation (35), we derive the simple approximate OP in high-SNDR regime as

$$\text{OP}_{\text{app}}(x) \approx \frac{x \to a_{\min} \left( a_{\min} - x \right) \Gamma^{-1} \left( \gamma^2 + 1 - a_{\min} \right) \left( \frac{\mathcal{Y}}{A_{\gamma} a_2} \right)^{a_{\min}} \left( a_{3} + 1 - a_{\min} + 1 \right) \Gamma(a_{3} + 1)}{\Gamma(a_{3} + 1) \Gamma(a_{4} + 1 - a_{\min})} \times a_1 a_2 \gamma^2 \Gamma(a_4 + 1 - s) \Gamma(a_5 + 1 - s) \Gamma(\gamma^2 - s) \Gamma(s),$$

where $a_{\min} = \min \{ a_5 + 1, a_4 + 1, \gamma^2 \}$ and

$$\lim_{x \to a_{\min}} \left( a_{\min} - x \right) \Gamma(a_4 + 1 - s) \Gamma(a_5 + 1 - s) \Gamma(\gamma^2 - s) \Gamma(s)$$

$$\approx \left( \Gamma(a_5 - a_4) \Gamma(\gamma^2 - a_4 - 1) \Gamma(a_4 + 1), a_{\min} = a_4 + 1 \right)$$

$$\approx \left( \Gamma(a_5 - a_4) \Gamma(\gamma^2 - a_4 - 1) \Gamma(a_5 + 1), a_{\min} = a_5 + 1 \right)$$

$$\approx \left( \Gamma(a_5 - a_4) \Gamma(\gamma^2 - a_4 - 1) \Gamma(\gamma^2 - a_5 - 1) \Gamma(a_5 + 1), a_{\min} = a_5 + 1 \right)$$

$$\approx \left( \Gamma(a_5 + 1 - \gamma^2) \Gamma(\gamma^2 - a_5 - 1) \Gamma(\gamma^2 - a_5 - 1) \Gamma(a_5 + 1), a_{\min} = \gamma^2 \right).$$

**Proof:** In the high-SNDR regime, e.g., when the transmit power is high, we have $\mathcal{Y} \to 0$. The Mellin-Barnes integral
over \( \mathcal{L}_s \) can be approximated by evaluating the residue at the minimum pole on the left-hand side of \( s \) as [48, Theorem 1.11]. With the help of [29, eq. (8.331.1)] and [29, eq. (9.113)], we can obtain (37), which completes the proof. 

From (37) and (36), we can obtain that 
\[
G_d = \frac{a_{\min}}{2} = \min \left\{ a_5 + 1, a_4 + 1, \gamma^2 \right\}. 
\]
Recall that \( a_4 \) and \( a_5 \) are determined by the moments of the small-scale fading coefficient, and \( \gamma^2 \) is defined as the ratio of THz signal’s equivalent bandwidth and the variance of the pointing error displacement at Destination. Considering that \( G_d \) indicates how much the OP decreases with the increase of the transmit power, we can conclude that when the pointing error is large, the key to improve \( G_d \) lies in enhancing the beamforming to increase \( \gamma \) when the pointing error is small, \( G_d \) is mainly determined by the small-scale fading. This observation is verified in Fig. 7: the blue and pink lines, i.e., case 1 and case 2, respectively, use the same value of \( G_d \), so they have the same slope in the high SNDR region. However, in the simulation of the red line, i.e., case 3, the \( G_d \) is increased from 0.5 to 0.8, so that the maximum slope is achieved.

VI. PERFORMANCE ANALYSIS

A. Outage Probability

The outage probability of considered THz communications can be obtained as \( P_{\text{out}} = P(\gamma_{\ell} < \gamma_{\text{th}}) = F_{\gamma_{\ell}}(\gamma_{\text{th}}) \), where \( \ell \in N, I \). For the case of non-ideal RF chains, the outage probability of the RIS-aided system can be directly obtained by using (29). Furthermore, we can obtain the outage probability for the case of ideal RF chains with the help of (31). Moreover, in high SNDR and \( L \) regimes, the outage probability can be derived with the help of (32) and (35), respectively. Thus, we can observe that an RIS equipped with more reflecting elements will also achieve a low outage probability.

B. Ergodic Capacity

The ergodic capacity under optimal rate adaptation with constant transmit power is given by [55, eq. (29)] as
\[
C = \mathbb{E} \left[ \log_2 \left( 1 + \gamma_{\ell} \right) \right], \quad \ell \in N, I.
\]

1) Non-Ideal RF Chains: For the case of non-ideal RF chains, an exact expression of the ergodic capacity is hard to be derived due to the complexity of (28). However, the following proposition provides a closed-form upper bound of the ergodic capacity, which is more generic, and practically more useful and reliable. Moreover, in Fig. 10, we show that the derived capacity upper bound is close to the exact simulation results.

**Proposition 3.** The effective capacity of the RIS-aided THz communications for the case of non-ideal RF chains is bounded by
\[
C_N \leq \log_2 \left( 1 + \frac{P|h_L|^2 \mathbb{E} \left[ |h_{FP}|^2 \right]}{P \kappa^2 |h_L|^2 \mathbb{E} \left[ |h_{FP}|^2 \right] + N_0} \right), \quad (39)
\]
where \( \mathbb{E} \left[ |h_{FP}|^2 \right] \) is derived as (40), shown at the bottom of the next page.

**Proof:** Please refer to Appendix E.

2) Ideal RF Chains: Exact ergodic capacity can be derived for the case of ideal RF chains.

**Proposition 4.** Effective capacity of the RIS-aid system for the case of ideal RF chains can be derived as (41), shown at the bottom of the next page.

**Proof:** Please refer to Appendix F.

**Proposition 5.** Effective capacity of the RIS-aid system for the case of ideal RF chains are bounded by
\[
C_I \leq \log_2 \left( 1 + \mathbb{E} \left[ |\gamma_{\ell}| \right] \right). \quad (42)
\]

**Proof:** With the help of Jensen’s inequality [56], the ergodic capacity can be upper-bounded as
\[
C_I \leq \log_2 \left( 1 + \mathbb{E} \left[ |\gamma_{\ell}| \right] \right). \quad (42)
\]

From (39), (41), and (42), as expected, we can observe that the multipath parameters have a positive effect on the ergodic capacity of RIS-aid THz communications.

VII. NUMERICAL RESULTS AND DISCUSSIONS

In this section, we present numerical results to investigate the joint impacts of misalignment, multipath fading, channel conditions, and transceivers hardware imperfections on the outage and ergodic capacity performance of the RIS-aided THz communications. Following the practical parameters in [2], [19], [57]–[59], we set the parameters as follows: \( T = 27 \) °C, \( p = 101325 \) Pa, \( c = 3 \times 10^8 \) m/s, \( u_d/a = 6 \), \( G_i = G_r = 40 \) dBi, \( \sigma_3 = 0.01 \) m, and the number of Monte Carlo iterations is \( 10^6 \).

Figure 8 illustrates the outage probability performance versus the transmit power with \( d_2 = 20 \) m, \( N_0 = 0 \) dBW, \( L = 40, m_{\ell, 1} = 5, m_{\ell, 2} = 7, K_{\ell, 1} = 5, K_{\ell, 2} = 6, \Delta_{\ell, 1} = 0.6, \Delta_{\ell, 2} = 0.4, k_G = k_D = 0.1, \) and \( 2\kappa_{\ell, 1} (1 + K_{\ell, 1}) = 2\kappa_{\ell, 2} (1 + K_{\ell, 2}) = 20 \) dB (\( \ell = 1, \ldots, L \)). It can be observed that the approximate expressions match the exact results well in the high-SNDR regime. For a given transmission distance, we can observe that the outage probability increases as the relative humidity increases. Furthermore, the increase in the transmission distance of the signal can cause a significant increase in the OP. Comparing the impact of the propagation loss and the molecular absorption loss, we observe that the propagation loss has a more adverse impact on the system performance than that of the molecular absorption loss. The reason is that the outage probability degradation caused by increasing \( d_1 \) is severer than the corresponding degradation caused by increasing \( \phi_1 \). Moreover, the outage probability increases as the frequency increases. When the frequency increases from 340 GHz to 380 GHz, the outage probability decreases by 58%, which is faster than that when the frequency increases from 300 GHz to 340 GHz (34%). The reason is that a high molecular absorption exists in 380 GHz because 380 GHz is one of the water vapor resonance frequencies [60]. Thus, it is significant for the THz communications system designers to consider the variation of the environmental conditions and the molecular absorption phenomenon.
Figure 9 plots the outage probability versus the threshold, γ_{th}, with P = 30 dBW, m_{ℓ,1} = m_1, m_{ℓ,2} = m_2, 2σ_{ℓ,1} (1 + K_{ℓ,1}) = 2σ_{ℓ,2} (1 + K_{ℓ,2}) = 20 dB, f = 300 GHz, d_1 = 10 m, d_2 = 20 m, N_0 = 1 dBW, K_{ℓ,1} = 5, K_{ℓ,2} = 6, ∆_{ℓ,1} = 0.6, ∆_{ℓ,2} = 0.4, and κ_S = κ_D = 0.1. We can observe that the outage probability increases by 72% when γ_{th} increases by 3 dB. Moreover, Fig. 9 shows the effects of different channel conditions. The outage probability decreases as shaping parameters m_1 and m_2 increase because of the better channel conditions. Furthermore, the number of RIS’s reflecting elements, L, has a significant impact on the performance of the RIS-aided THz communications. For example, when L increases from 40 to 60, the outage probability decreases by 97%. The reason is that the RIS can effectively change the phase of reflected signals to enhance the received signal quality, enabling the desired THz wireless channel to exhibit a perfect line-of-sight.

Figure 10 depicts the ergodic capacity versus the transmit power with m_{ℓ,1} = 5, m_{ℓ,2} = 7, f = 360 GHz, d_1 = 8 m, d_2 = 6 m, N_0 = 1 dBW, K_{ℓ,1} = 5, K_{ℓ,2} = 6, ∆_{ℓ,1} = 0.6, ∆_{ℓ,2} = 0.4, L = 40, and 2σ_{ℓ,1} (1 + K_{ℓ,1}) = 2σ_{ℓ,2} (1 + K_{ℓ,2}) = 25 (ℓ = 1, . . . , L) for different κ_S = κ_D = κ. We can observe that the ergodic capacity increases when the transmission power increases. Thus, increasing the transmission power can be regarded as a direct measure to reduce the impact of hardware imperfections. Furthermore, Fig. 10 reveals that the hardware imperfections have a small effect on the ergodic capacity in the low transmit power regime, while having a more adverse impact in the high transmit power regime. Therefore, the efficient method to increase the ergodic capacity is to decrease the hardware imperfections when the transmit power is high. In addition, we observe that the derived capacity upper bound is close to the Monte Carlo simulation results.

Figure 11 illustrates the ergodic capacity versus the number of RIS’s reflecting elements with 2σ_{ℓ,1} (1 + K_{ℓ,1}) = 2σ_{ℓ,2} (1 + K_{ℓ,2}) = 2σ (1 + K), κ_S = κ_D = 0, m_{ℓ,1} = 5, m_{ℓ,2} = 7, d_1 = 8 m, d_2 = 6 m, N_0 = 1 dBW, K_{ℓ,1} = 5, K_{ℓ,2} = 6, ∆_{ℓ,1} = 0.6, and ∆_{ℓ,2} = 0.4 for different f and 2σ (1 + K). From Fig. 11, we can observe that the ergodic capacity increases by 25% when the number of reflecting elements increases from 40 to 80. This is due to the reason that more elements let RIS have superior capability in manipulating electromagnetic waves. Thus, equipping RIS with more reflecting elements can significantly improve the performance of wireless communications systems. Furthermore, the ergodic capacity decreases when the operating frequency is changed from 300 GHz to 380 GHz. Thus, it becomes evident that the system’s performance will be significantly influenced by the selection of operating frequency, which also verifies the conclusion obtained from Fig. 8.

VIII. CONCLUSIONS

We investigated an RIS-aided THz communications system and presented exact expressions of the end-to-end SNR and SNDR. Based on two THz signal measurement experiments conducted in a variety of different scenarios, we proved that the FTR distribution performs a good fit for modeling the small-scale amplitude fading of THz signals. Furthermore, an SIO algorithm was proposed to find the optimal phase-shifts design at RIS under discrete phase-shift constraints. The impact of the different number of discrete phase-shift levels was investigated. Moreover, important performance metrics, including the outage probability and ergodic capacity, are derived in terms of the multivariate Fox’s H-function. To obtain more insights, the asymptotic analysis was presented when the SNDR or the number of RIS’s elements is large. The tight upper bound expressions of ergodic capacity for ideal and non-ideal RF chains are also derived. The numerical results confirmed that the propagation loss is more detrimental to the performance of the RIS-aided THz communications system than the molecular absorption loss, and the RIS can bring a significant performance gain for the proposed THz communications.

\[
\mathbb{E} [ | h_{FP} |^2 ] = \sum_{j_1, \ldots, j_L = 0}^{N_T} \sum_{k_1, \ldots, k_L = 0}^{N_T} \frac{K_j \ell_{j,1} d_j \ell_{j,1} m_j \ell_{m,j} \gamma^2}{j! \ell! \Gamma(m_j) \Gamma(j+1)} \\
\times H^{0,1,1,3,1,3,2,0,3,2,3,2,1}_{0,1,1,3,1,3,2,0,3,2,3,2,1} \left( \begin{array}{c} 1; -0.5, \ldots, -0.5, -1; 0; 1, \ldots, 1; 0; v_{FP,j}; \ldots; v_{FP,L}; (0, 1) \\
\ldots \\
1; -0.5, \ldots, -0.5, -1; 0; 1, \ldots, 1; 0; (0, 1), (-\gamma^2, 1); \ldots; (0, 1), (-\gamma^2, 1) \\
\end{array} \right) 
\]

\[
C = \sum_{j_1, \ldots, j_L = 0}^{N_T} \sum_{k_1, \ldots, k_L = 0}^{N_T} \frac{K_j \ell_{j,1} d_j \ell_{j,1} m_j \ell_{m,j} \gamma^2}{j! \ell! \Gamma(m_j) \Gamma(j+1)} \\
\times H^{0,1,1,3,1,3,2,1,2,0,3,2,3,2,1}_{0,1,1,3,1,3,2,1,2,0,3,2,3,2,1} \left( \begin{array}{c} 1; -0.5, \ldots, -0.5, -1; 0; 1, \ldots, 1, 0; v_{FP,j}; \ldots; v_{FP,L}; (0, 1) \\
\ldots \\
1; -0.5, \ldots, -0.5, -1; 0; 1, \ldots, 1; 0; (0, 1), (-\gamma^2, 1); \ldots; (0, 1), (-\gamma^2, 1) \\
\end{array} \right) 
\]
Outage Probability

Ergodic Capacity (bit/s/Hz)

Fig. 8. Outage probability versus the threshold, \( \gamma_{th} \), with \( P = 30 \) dBW. \( m_{\ell,1} = m_{1}, m_{\ell,2} = m_{2}, 2\sigma_{\ell,1} (1 + K_{\ell,1}) = 20 \) dB, \( f = 300 \) GHz, \( d_1 = 10 \) m, \( d_2 = 10 \) m, \( N_0 = 1 \) dBW, \( K_{\ell,1} = 5, K_{\ell,2} = 6, \Delta_{\ell,1} = 0.6, \Delta_{\ell,2} = 0.4, \kappa_S = \kappa_D = 0.1, \) and \( 2\sigma_{\ell,1} (1 + K_{\ell,1}) = 2\sigma_{\ell,2} (1 + K_{\ell,2}) = 20 \) dB.

Fig. 9. OP versus the threshold, \( \gamma_{th} \), with \( P = 30 \) dBW. \( m_{\ell,1} = m_{1}, m_{\ell,2} = m_{2}, 2\sigma_{\ell,1} (1 + K_{\ell,1}) = 20 \) dB, \( f = 300 \) GHz, \( d_1 = 10 \) m, \( d_2 = 10 \) m, \( N_0 = 1 \) dBW, \( K_{\ell,1} = 5, K_{\ell,2} = 6, \Delta_{\ell,1} = 0.6, \Delta_{\ell,2} = 0.4, \) and \( \kappa_S = \kappa_D = 0.1. \)

Fig. 10. Ergodic capacity versus the transmit power with \( m_{\ell,1} = 5, m_{\ell,2} = 7, f = 300 \) GHz, \( d_1 = 8 \) m, \( d_2 = 6 \) m, \( N_0 = 0 \) dBW, \( K_{\ell,1} = 5, K_{\ell,2} = 6, \Delta_{\ell,1} = 0.6, \Delta_{\ell,2} = 0.4, L = 40, \) and \( 2\sigma_{\ell,1} (1 + K_{\ell,1}) = 2\sigma_{\ell,2} (1 + K_{\ell,2}) = 25 \) dB (\( \ell = 1, \ldots, L \)).

Fig. 11. Ergodic capacity versus the number of RIS’s reflecting elements with \( \kappa_S = \kappa_D = 0, m_{\ell,1} = 5, m_{\ell,2} = 7, d_1 = 8 \) m, \( d_2 = 6 \) m, \( N_0 = 1 \) dBW, \( K_{\ell,1} = 5, K_{\ell,2} = 6, \Delta_{\ell,1} = 0.6, \) and \( \Delta_{\ell,2} = 0.4. \)

APPENDIX A

PROOF OF THEOREM 1

With the help of [41, eq. (6)], \( \mathbb{E} \left[ \sum_{\ell=1}^{L} \sum_{t=1}^{m_{\ell}} g_1^{(t)} g_2^{(t)} \right] \) can be expressed as

\[
\mathbb{E} \left[ \sum_{\ell=1}^{L} \sum_{t=1}^{m_{\ell}} g_1^{(t)} g_2^{(t)} \right] = \sum_{\ell=1}^{L} \sum_{t=1}^{m_{\ell}} \frac{m_{\ell,t}}{\Gamma(m_{\ell})} \sum_{j=0}^{\infty} \left( K_{\ell,1} \right)^j \sigma_1^2 \frac{1}{\sqrt{j!}} \Gamma(j+1) \Gamma \left( \frac{3}{2} + j \right).
\]

(A-1)

The \( s \)-th moment of \( |h_P| \) is defined as \( \mathbb{E} [|h_P|^s] = \int_0^\infty x^s f_{h_P}(x) dx \). With the help of (4), we obtain

\[
\mathbb{E} [|h_P|^s] = \frac{\sigma_1^2}{A_0} \int_0^{\infty} x^{s+2-1} dx = \frac{\sigma_1^2 s}{2 s+4} A_0^s.
\]

(A-2)

By substituting \( s = 1 \) into (A-2), \( \mathbb{E} [|h_P|] \) is derived. Thus, we obtain (25) which completes the proof.

APPENDIX B

PROOF OF LEMMA 1

A. Proof of PDF

With the help of (13), we obtain \( \frac{1}{7N} = \kappa^2 + \frac{N_0}{|h_L|^2 P |h_P|^2} \), where \( |h_P| \triangleq |h_F| \times |h_P| \). Let us define that \( V \triangleq \frac{1}{|h_P|^2} \), \( W \triangleq \frac{N_0}{|h_L|^2 P} \), \( V = \frac{1}{7N} - \kappa^2 \). The PDF of \( W \) can be respectively expressed as

\[
f_W(w) = \frac{|h_L|^2 P}{N_0} f_V \left( \frac{|h_L|^2 P}{N_0} w \right) = \frac{\sigma_1^2}{2 \sigma_1^2 \sqrt{|h_L|^2 P}} f_{h_P} \left( \sqrt{\frac{N_0}{|h_L|^2 P} w} \right).
\]

(B-1)
By performing the change of variables $\gamma_N = \frac{1}{W+\kappa z}$, $f_{\gamma_N}$ can be rewritten as

$$f_{\gamma_N}(x) = \frac{1}{2(1-x)^2} \frac{1}{N_o} f_{h|FP|} \left( \frac{x}{N_o \gamma_N} \right).$$

(B-2)

Moreover, with the help of [61, eq. (62)], we obtain the PDF of $|h_{FP}|$ as

$$f_{h|FP|}(x) = \int_0^{\frac{A_o}{y_0}} f_{h|FP|} \left( \frac{x}{y} \right) f_{h|FP|}(y) dy.$$

(B-3)

Substituting (B-3) into (B-2), we derive the integral representation for the PDF of $|h_{FP}|$ as (26).

B. Proof of CDF

Following similar procedures as Appendix B-A (i.e., the proof of Lemma 1), we can derive the CDF of $V$ and $W$ as $F_V(v) = 1 - F_{|h_{FP}|} \left( \frac{v}{\sqrt{2}} \right)$ and $F_W(w) = F_V \left( \frac{|h_{FP}|}{\sqrt{2}} \right)$. Making the change of variable $\gamma_N = \frac{1}{W+\kappa z}$, we can derive the CDF of $\gamma_N$ as $F_{\gamma_N}(x) = F_{|h_{FP}|} \left( \frac{x}{N_o \gamma_N} \right) \frac{\gamma_N}{|h_{FP}|^2} \frac{\gamma_N}{1-x^2}.$

With the help of [61, eq. (65)], the CDF of $|h_{FP}|$ can be expressed as

$$F_{|h_{FP}|}(x) = \int_0^{\frac{A_o}{y_0}} F_{|h|} \left( \frac{x}{y} \right) f_{h|FP|}(y) dy.$$

(B-4)

Substituting (B-4) into $F_{\gamma_N}(x)$, the integral representation for the CDF of $|h_{FP}|$ as (27) can be derived to complete the proof.

APPENDIX C

PROOF OF THEOREM 1

We first derive the PDF and CDF of $|h_{FP}| \sim |h_F| \times |h_P|$. 

A. PDF of $|h_{FP}|$

Using the definition of multivariate Fox’s $H$-function [48, eq. (A-1)], we can express [41, eq. 11] as

$$f_{|h_{FP}|}(y) = \sum_{j=1}^{N_T} \sum_{L=1}^{N_T} \prod_{l=1}^{L} \frac{K_{e_1,\ldots,e_L}|y|^{e_1+\ldots+e_L}}{\Gamma(m_{e_1})\cdots\Gamma(m_{e_L})}$$

$$\times \frac{1}{2\pi i} \int_{\gamma} \frac{1}{\Gamma(-\frac{1}{2}+\sum_{l=1}^{L} e_l)} \left( 2\pi \right)^{\frac{1}{2}} \left( \frac{2\pi}{y} \right)^{\frac{1}{2}} \left( 2\pi \right)^{\frac{1}{2}} \left( \frac{2\pi}{y} \right)^{\frac{1}{2}} \Gamma(-\frac{1}{2}+\sum_{l=1}^{L} e_l)$$

where the integration path of $\gamma \ell$ ($\ell = 1, 2, \ldots, L$) goes from $a_L - \infty j$ to $a_L + \infty j$ and $a_L \in \mathbb{R}$. Substituting (C-1) and (4) into (B-3), with the help of Fubini’s theorem and [29, eq. (8.331.1)), we obtain

$$f_{|h_{FP}|}(x) = \sum_{j=1}^{N_T} \sum_{L=1}^{N_T} \prod_{l=1}^{L} \frac{K_{e_1,\ldots,e_L}|y|^{e_1+\ldots+e_L}}{\Gamma(m_{e_1})\cdots\Gamma(m_{e_L})}$$

$$\times \frac{1}{2\pi i} \int_{\gamma} \frac{1}{\Gamma(-\frac{1}{2}+\sum_{l=1}^{L} e_l)} \left( 2\pi \right)^{\frac{1}{2}} \left( \frac{2\pi}{y} \right)^{\frac{1}{2}} \left( 2\pi \right)^{\frac{1}{2}} \left( \frac{2\pi}{y} \right)^{\frac{1}{2}} \Gamma(-\frac{1}{2}+\sum_{l=1}^{L} e_l)$$

where

$$I_A = \int_0^{A_o} y^{\gamma+\gamma_1-1} dy = \frac{1}{\gamma+\gamma_1} A_o^{\gamma+\gamma_1} = \frac{\Gamma(\gamma+\gamma_1)}{\Gamma(\gamma+\gamma_1+1)} A_o^{\gamma+\gamma_1}.$$

(C-3)

Substituting $I_A$ into (C-2) and using [48, eq. (A-1)], we can obtain $f_{h|FP|}(x)$.

B. CDF of $|h_{FP}|$

Substituting [41, eq. 12] and (4) into (B-4), we can express the CDF of $|h_{FP}|$ as

$$F_{|h_{FP}|}(x) = \sum_{j=1}^{N_T} \sum_{L=1}^{N_T} \prod_{l=1}^{L} \frac{K_{e_1,\ldots,e_L}|y|^{e_1+\ldots+e_L}}{\Gamma(m_{e_1})\cdots\Gamma(m_{e_L})}$$

$$\times \frac{1}{2\pi i} \int_{\gamma} \frac{1}{\Gamma(-\frac{1}{2}+\sum_{l=1}^{L} e_l)} \left( 2\pi \right)^{\frac{1}{2}} \left( \frac{2\pi}{y} \right)^{\frac{1}{2}} \left( 2\pi \right)^{\frac{1}{2}} \left( \frac{2\pi}{y} \right)^{\frac{1}{2}} \Gamma(-\frac{1}{2}+\sum_{l=1}^{L} e_l)$$

where $I_A$ has been solved in (C-3). Substituting $I_A$ into (C-4), the CDF of $|h_{FP}|$ can be expressed in terms of multivariate Fox’s $H$-function. Therefore, with the help of (26), (27), (C-2), and $F_{h|FP|}(x)$, the PDF and CDF of $\gamma_N$ can be derived as (28) and (29), respectively.

APPENDIX D

PROOF OF PROPOSITION 2

To derive the approximate PDF of $\gamma_N$, we need first to get the PDF of $|h_F|$, where $|h_F| = \sum_{l=1}^{N_T} \left| g_l(t) \right|$. With the help of a 5FM based method proposed in [62] and the definition of Meijer’s $G$ function [29, eq. (9.30)], we can obtain the approximate PDF of $|h_F|$ as

$$F_{|h_F|} = \frac{1}{2\pi i} \int_{\gamma} \frac{1}{\Gamma(-\frac{1}{2}+\sum_{l=1}^{L} e_l)} \left( 2\pi \right)^{\frac{1}{2}} \left( \frac{2\pi}{y} \right)^{\frac{1}{2}} \Gamma(-\frac{1}{2}+\sum_{l=1}^{L} e_l)$$

(D-1)

where $I_D$ can be solved with the help of [29, eq. (8.331.1)] as

$$I_D = \int_0^{A_o} y^{\gamma+\gamma_1-2} dy = A_o^{\gamma+\gamma_1-2} = \frac{\Gamma(\gamma+\gamma_1)}{\Gamma(\gamma+\gamma_1+1)} A_o^{\gamma+\gamma_1}.$$

(D-3)

Substituting $I_D$ into (D-2) and using [29, eq. (9.30)], we can obtain (35) to complete the proof.

APPENDIX E

PROOF OF PROPOSITION 3

With the help of Jensen’s inequality [56], the upper-bound of the ergodic capacity can be expressed as $C_N \leq \log_2 (1 + \mathbb{E} [\gamma_N])$, where

$$\mathbb{E} [\gamma_N] = \mathbb{E} \left[ \frac{|h_{FP}|^2|h_L|^2 P}{\sqrt{2}|h_{FP}|^2|h_L|^2 P + N_o} \right] = \frac{P|h_L|^2 \mathbb{E} [h_{FP}]^2}{\sqrt{2}|h_{FP}|^2|h_L|^2 P + N_o}.$$

(E-1)
\[ L \text{ s.th. moment of } |h_{FP}| \text{ is defined as } \mathbb{E}[|h_{FP}|^s] = \int_{0}^{\infty} x^s f_{h_{FP}}(x) \, dx. \]

With the help of (C-2), we can express \( \mathbb{E}[|h_{FP}|^s] \) as shown at the bottom of the next page, where \( I_C = \int_{0}^{\infty} x \sum_{i=1}^{L} \xi_i^{-1} \, dx. \)

Let \( L^{-1}[p(t)] = P(x) \). With the help of the property of Laplace transform, it follows that \( L \{ \int_{0}^{t} p(z) \, dz \} = \frac{P(x)}{e^x} \).

According to the final value theorem, we have

\[
\lim_{t \to \infty} \left( \int_{0}^{t} p(z) \, dz \right) = e \frac{P(e)}{e} = P(e),
\]

where \( e \) is a number close to zero (e.g., \( e = 10^{-6} \)). Thus, \( I_C \) can be solved as \( I_C = (\frac{1}{2})^L \sum_{i=1}^{L} \Gamma \left( s - \frac{L}{2} \sum_{i=1}^{\xi_i} \right) \). Substituting \( I_C \) into (E-2) and using [48, eq. (A-1)], we can rewrite \( \mathbb{E}[|h_{FP}|^s] \) in terms of multivariate Fox’s \( H \)-function. With the help of (E-1), letting \( s = 2 \), we obtain (39) to complete the proof.

\[ E[|h_{FP}|^s] = \sum_{j_1, \ldots, j_L = 0}^{N_T} \sum_{j_{1,1}, \ldots, j_{L,1} = 0}^{N_T} \prod_{l=1}^{L} \frac{K_{j_{l,1}, \ldots, j_{l,L}} m_{j_{l,1}, \ldots, j_{l,L}}}{\Gamma(m_{j_{l,1}, \ldots, j_{l,L}})} \Gamma(j_{l,1} + 1) \]

\[ \times \prod_{l=1}^{L} \frac{1}{2\pi i} \int_{L+1}^{\infty} \Gamma \left( 1 + j_{l,1} + \frac{1}{2} \xi_i \right) \Gamma(- \xi_i) \Gamma(\xi_i + \gamma^2) \]

\[ \times \left( A_0 \sqrt{\frac{|h_{L,2}|^2}{N_o} \prod_{l=1}^{2} \left( \sqrt{2\sigma_i} \right) } \right)^\xi_i I_{D_1} d\xi_i,
\]

Substituting \( I_{D_1} \) and \( I_{D_2} \) into (F-1), we obtain

\[
C_I = \sum_{j_{1,1}, \ldots, j_{L,1} = 0}^{N_T} \sum_{j_{1,2}, \ldots, j_{L,2} = 0}^{N_T} \prod_{l=1}^{L} \frac{K_{j_{l,1}, \ldots, j_{l,L}} m_{j_{l,1}, \ldots, j_{l,L}}}{\Gamma(m_{j_{l,1}, \ldots, j_{l,L}})} \Gamma(j_{l,1} + 1) \]

\[ \times \prod_{l=1}^{L} \frac{1}{2\pi i} \int_{L+1}^{\infty} \Gamma \left( 1 + j_{l,1} + \frac{1}{2} \xi_i \right) \Gamma(- \xi_i) \Gamma(\xi_i + \gamma^2) \]

\[ \times \left( A_0 \sqrt{\frac{|h_{L,2}|^2}{N_o} \prod_{l=1}^{2} \left( \sqrt{2\sigma_i} \right) } \right)^\xi_i I_{D_1} d\xi_i + \frac{1}{2\pi i} \int_{L+1}^{\infty} \Gamma \left( 1 + \xi_i \right) \Gamma(- \xi_i) \Gamma(\xi_i + \gamma^2) \]

\[ \times \prod_{l=1}^{L} \frac{1}{2\pi i} \int_{L+1}^{\infty} \Gamma \left( 1 + j_{l,1} + \frac{1}{2} \xi_i \right) \Gamma(- \xi_i) \Gamma(\xi_i + \gamma^2) \]

\[ \times \left( A_0 \sqrt{\frac{|h_{L,2}|^2}{N_o} \prod_{l=1}^{2} \left( \sqrt{2\sigma_i} \right) } \right)^\xi_i I_{D_1} d\xi_i.
\]

With the help of [48, eq. (A-1)], we can rewrite (F-4) as (41) that completes the proof.

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Hongyang Du received the B.Sc. degree from Beijing Jiaotong University, Beijing, China, in 2021. He is currently pursuing the Ph.D. degree with the School of Computer Science and Engineering, the Energy Research Institute @ NTU, Interdisciplinary Graduate Program, Nanyang Technological University, Singapore. His research interests include Metaverse, reconfigurable intelligent surface, and communication theory.

Ke Guan (S’10-M’13-SM’19) received B.E. degree and Ph.D. degree from Beijing Jiaotong University in 2006 and 2014, respectively. He is a Professor in State Key Laboratory of Rail Traffic Control and Safety & School of Electronic and Information Engineering, Beijing Jiaotong University. In 2015, he has been awarded a Humboldt Research Fellowship for Postdoctoral Researchers. In 2009, he was a Visiting Scholar with Universidad Politécnica de Madrid, Spain. From 2011 to 2013, he was a Research Scholar with the Institut für Nachrichtentechnik (IN) at Technische Universität Braunschweig, Germany. From September 2013 to January 2014, he was invited to conduct joint research in Universidad Politècnica de Madrid, Spain. He has authored/coauthored two books and one book chapter, more than 300 journal and conference papers, and 10 patents. His current research interests include measurement and modeling of wireless propagation channels, high-speed railway communications, vehicle-to-x channel characterization, and indoor channel characterization for high-speed short-range systems including future terahertz communication systems. Dr. Guan is the pole leader of EURNEX (European Railway Research Network of Excellence). He was the recipient of a 2014 International Union of Radio Science (URSI) Young Scientist Award. His papers received eight Best Paper Awards, including IEEE vehicular technology society 2019 Neal Shepherd memorial best propagation paper award. He is an Editor of the IEEE Vehicular Technology Magazine, the IEEE ACCESS, the IET Microwave, Antenna and Propagation, and the Physical Communication. He serves as a Publicity Chair in PIMRC 2016, the Publicity Co-Chair in ITST 2018, the Track Co-Chair in EuCNC 2019, the Convened Sessions Co-Chair of EuCAP 2021, the Session Convenor of EuCAP 2015-2021, and a TPC Member for many IEEE conferences, such as Globecom, ICC and VTC. He has been a delegate in 3GPP and a member of the ICI1004 and CA15104 initiatives.

Dusit Niyato (M’09-SM’15-F’17) is currently a professor in the School of Computer Science and Engineering, at Nanyang Technological University, Singapore. He received B.Eng. from King Mongkuts Institute of Technology Ladkrabang (KMITL), Thailand in 1999 and Ph.D. in Electrical and Computer Engineering from the University of Manitoba, Canada in 2008. His research interests are in the areas of Internet of Things (IoT), machine learning, and incentive mechanism design.

Huiying Jiao is the Professor-level senior engineer of the China Academy of Information and Communications Technology (CAICT). She has contributed to the design, standardization, and development of 4G (TD-LTE), and 5G mobile communication systems. She has authored or co-authored more than 20 research papers/articles, and over 50 patents in this area. Her achievements have received the First Prize of China Communications Standards Association, and the Second Prize of Chinese Institute of Electronics. Her current research interests include the theoretical research and standardization for 5G-Advanced and 6G.
Zhiqin Wang is the Vice President of the China Academy of Information and Communications Technology (CAICT), the Professor-level senior engineer. She is currently the Chair of the Wireless Technical Committee of China Communications Standards Association (CCSA) and the Director of the Wireless and Mobile Technical Committee of China Institute of Communications (CIC). She is also serving as the Chair of the IMT-2020 (5G) Promotion Group and Chair of the IMT-2030 (6G) Promotion Group in China. She has contributed to the design, standardization, and development of 3G (TD-SCDMA), 4G (TD-LTE), and 5G mobile communication systems. She has authored or co-authored more than 60 research papers/articles, and over 20 patents in this area. Her achievements have received multiple top awards and honors by China central government, including the Grand Prize of the National Award for Scientific and Technological Progress in 2016 (the highest Prize in China), the First Prize of the National Award for Scientific and Technological Progress once, the Second Prize of the National Award for Scientific and Technological Progress twice, the National Innovation Award (once), and the Ministerial Science and Technology Awards many times. Her current research interests include the theoretical research, standardization, and industry development for 5G-Advanced and 6G.

Thomas Kürner (S’91-M’94-SM’01-F’19) received his Dipl.-Ing. degree in Electrical Engineering in 1990, and his Dr.-Ing. degree in 1993, both from University of Karlsruhe (Germany). From 1990 to 1994 he was with the Institut für Höchstfrequenztechnik und Elektronik (IHE) at the University of Karlsruhe working on wave propagation modelling, radio channel characterisation and radio network planning. From 1994 to 2003, he was with the radio network planning department at the headquarters of the GSM 1800 and UMTS operator E-Plus Mobilfunk GmbH & Co KG, Düsseldorf, where he was team manager radio network planning support responsible for radio network planning tools, algorithms, processes and parameters from 1999 to 2003. Since 2003 he is Full University Professor for Mobile Radio Systems at the Technische Universität Braunschweig. His working areas are indoor channel characterisation and system simulations for high-speed short-range systems including future terahertz communication system, propagation, traffic and mobility models for automatic planning and self-organization of mobile radio networks, car-to-car communications as well as accuracy of satellite navigation systems. He has been engaged in several international bodies such as ITU-R SG 3, UMTS Forum Spectrum Aspects Group, COST 231/273/259 where he chaired the working group ‘Network Aspects’, COST 2100 and COST IC 1004. He participated in the European projects FP-5-IST-MOMENTUM on methods for ‘Automatic Planning of large-scale Radio Networks’, ICT-FP7-SOCRATES on ‘Self-Organisation in Wireless Networks’, FP7-SME-GreenNets on ‘Power consumption and CO2 footprint reduction in mobile networks by advanced automated network management approaches’, FP7-SEMAOUR (‘Self-management for unified heterogeneous radio access networks’), Medea+Qstream (‘Ultra-high Data Rate Wireless Communication’) and H2020-iBroW (‘Innovative ultra-BROADband ubiquitous Wireless communications through terahertz transceivers’). Prof. Kürner represents Technische Universität Braunschweig at the NGMN (Next Generation Mobile Networks) alliance as an advisory member. He has actively contributed to the channel modelling document supporting the standardization of IEEE 802.11ad. Currently he is a voting member of IEEE 802.15 and is chairing the IEEE 802.15 IG THz and the IEEE 802.15.3d TG 100G. He is the project coordinator of the TERAPAN project (‘Terahertz communications for future personal area networks’) funded by the German Ministry of Research and Development. Prof. Kürner is a member of the Board of Directors of the European Association on Antennas and Propagation (EuCAP) and chairs the EuCAP WG Propagation. He served as Vice-Chair Propagation at the European Conference on Antennas and Propagation (EuCAP) in 2007, 2009 and 2014. He is a founding member and co-organizer of all six editions of the International Workshop on Self-Organizing Networks (IWSWN). In 2012 he was a guest lecturer at Dublin City University within the Telecommunications Graduate Initiative in Ireland. Since 2008, he has been an Associate Editor of the IEEE TRANSACTIONS ON VEHICULAR TECHNOLOGY. Since 2016, he has also been an Associate Editor of the IEEE TRANSACTIONS ON ANTENNAS AND PROPAGATION.