A simple method for amplitude/phase calibration of tri-axial accelerometers in the low frequency range

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Abstract. A simple method for amplitude/phase calibration of tri-axial accelerometers in the low-frequency range is proposed, based on a linear slide, used to excite all the axes of the accelerometer at the same time, and a Laser Doppler Vibrometer (LDV) as a reference. For the phase evaluation a cross-correlation analysis has been carried out. The procedure includes many validation actions in order to verify the correctness of data processing and to reduce the calibration uncertainty.

Results show that the phase is a critical aspect to consider, more than the amplitude, and the comparison with the theoretical model is useful to verify the hypotheses. Different behaviors result depending on the elements of the measurement chain and not only on the type of accelerometer.

1. Introduction
Due to the wide use of accelerometers in the most varied applications, many kinds of accelerometers have been designed, using different physical principles, and having different characteristics, which are often conveniently integrated in practical applications.

Among all, the capacitive (and in particular MEMS capacitive) and the piezoelectric accelerometers are the most widespread kinds of accelerometers.

Micro-electro-mechanical system (MEMS) accelerometers have been receiving particular attention because of their low cost and small size [1]. MEMS capacitive accelerometers have the benefits of low power consumption, low sensitivity to temperature changes, low cost, and they are suitable for measuring low-frequency acceleration, but suffer from a limited bandwidth and a low signal-to-noise ratio.

Piezoelectric accelerometers are characterized by low noise output, and wide frequency range, but their cost is significantly higher and they are unable to measure static and quasi-static acceleration [4].

For this reason, they could be in crisis in the lower side of their frequency range, and the information of data sheets is not always comprehensive in this regard.

In literature, the integration of this kinds of accelerometers is proposed, to combine the best features of both [4].

In the field of the civil structures monitoring, the combination of these kinds of sensors could be useful. In fact, the need of covering wide areas through a multi-sensor network, in order to include
different buildings, sets the requirement of low-cost solutions suitable in the low-frequency range [5-6], like capacitive MEMS accelerometers. Piezoelectric accelerometers are reliable and stable sensors and could be used as a reference for the MEMS sensors.

Knowing the behavior of these sensors in the low-frequency range, in terms of amplitude and phase, could be a useful information for the design of a distributed sensors network.

However, many other applications arise in different fields of engineering, where a single or a network of accelerometers are used, to monitor the system of interest in the low frequency range as, for instance, energy production [7], geotechnical applications [8], human vibration and bio-dynamics [9]. For this reason, micro accelerometers [10-11] and calibration methods [12-13] have been specifically designed for the low frequency range, but they are limited to the amplitude evaluation.

Furthermore, it should be considered that for piezoelectric accelerometers, the low-frequency response of the measuring set-up depends primarily on the amplifier used in the measuring system. The output of a piezoelectric accelerometer should be corrected by eliminating gain and phase shift caused by the amplifier [14].

Alternatively, the behavior of the entire measurement chain could be experimentally investigated, taking into account the contribution of all its elements.

Based on the above considerations MEMS accelerometers, in comparison to high-performance piezo-electric transducers, exhibit lower accuracy and, for this reason, a calibration of both amplitude and phase is particularly important for a reliable vibration analysis [2, 3].

The goal of this paper is to investigate the behavior of tri-axial piezoelectric and capacitive MEMS accelerometers in the lower limit of the operating frequency range, from the point of view of sensitivities and phases, considering the whole measuring chains.

For this purpose, a calibration method is proposed, based on a linear slide, used to excite the accelerometer being tested in the low frequency range, and a Laser Doppler Vibrometer (LDV), used as a reference. The low frequency behavior of the LDV is also a key-aspect to be considered and it is studied in [15]. The proposed method promises to be interesting, being simpler than the standard one, to be used for in-line and on-line calibration [12-13].

2. Materials and methods

This paper aims to investigate the behavior, in terms of sensitivity and phase, of capacitive MEMS and IEPE (Integrated Electronics Piezo-electric) accelerometers in the frequency range 0.5 – 12 Hz, considering the whole measuring chains.

For this purpose, a calibration bench is designed, based on a LDV VS 100, by Ometron, as a reference (Figure 1), and a simple procedure is developed, requiring a single set of tests to excite all the axes at the same time, and based on a data processing algorithm implemented in Matlab environment.

![Figure 1. Picture of the calibration bench.](image-url)
The test bench used is a vibrating table with a horizontal linear slide, the APS 113 ELECTROSEIS shaker. It is a long-stroke, electro-dynamic force generator specifically suitable for low-frequency vibration testing. The slide is moved according to a sinusoidal law.

The data acquisition system (DAQ) is the Compact RIO 9004 by National Instruments, together with the module NI 9234 for both IEPE and non-IEPE sensors, and the analog input module NI 9215.

The accelerometers tested and their measuring chains are shown in Table 1. The first two accelerometers are of the same type, but they have different sensitivities.

It must be noticed that the LDV output is acquired by the same module of the accelerometer under test to guarantee the synchronism of the acquisitions.

The accelerometer being tested is mounted on an inclined plate at an angle of 45° with respect to the horizontal plane on which the motion is realized. Furthermore, the accelerometer is rotated on the clamp surface with an angle of 45°, in order to simultaneously excite the three axes, with a single horizontal sinusoidal acceleration (Figure 2).

Table 1. Accelerometers under test and their measuring chains.

| Type of accelerometer | Acquisition module | Acquisition mode (accelerometer) | Acquisition mode (LDV) |
|-----------------------|--------------------|----------------------------------|------------------------|
| 1 Tri-axial IEPE accelerometer | IEPE module | IEPE AC coupling | AC coupling |
| 2 Tri-axial IEPE accelerometer | IEPE module | IEPE AC coupling | AC coupling |
| 3 Tri-axial MEMS capacitive accelerometer | Analog input module | DC coupling | DC coupling |
|  | IEPE module | AC coupling | AC coupling |
|  | IEPE module | DC coupling | DC coupling |

Figure 2. Scheme of the measuring set-up.

2.1. Sensitivity evaluation
Considering that the LDV output is proportional to the velocity, it must be differentiated to obtain the acceleration signal.

This acceleration, indicated as $a_{ref}$ in Figure 3, has to be projected along the directions of the axes. So, the reference accelerations along the three axes are obtained as follows:

$$a_x = a_{ref} \cdot \cos(45°) \cdot \cos(45°) \quad (1)$$

$$a_y = -a_{ref} \cdot \cos(45°) \cdot \cos(45°) \quad (2)$$

$$a_z = a_{ref} \cdot \sin(45°) \quad (3)$$
The sensitivities are estimated by dividing the amplitudes of the x, y and z outputs, by the amplitudes of the reference signals, all evaluated by an FFT analysis:

$$S_{xx} = \frac{v_x}{a_x}, \quad S_{yy} = \frac{v_y}{a_y}, \quad S_{zz} = \frac{v_z}{a_z}$$

(4)

The transversal sensitivities are neglected in this analysis, in the hypothesis that they are less than 5%, but their effects are considered in the uncertainty budget.

2.2. Phase evaluation

With regard to the phase evaluation, the reference signals and the outputs of the accelerometer under test (Figure 3) have been compared by a cross correlation analysis, executed in Matlab environment by the “finddelay” function.

![Figure 3. Example of delay between the reference acceleration and the output signal of the accelerometer under test.](image)

In this way, the phase delay of the whole measuring chain with respect to the reference one is tested.

In the phase evaluation, the phase of the reference is a critical aspect to consider [15]. As a first approximation, the LDV phase behavior has been considered ideal (i.e. negligible phase effect of the frequency of vibration). That hypothesis will be discussed in the Result section.

2.3. Validation

Validation actions are necessary to confirm the correctness of the procedure and of data processing [16-17]. In particular, the following controls have been carried out:

- Check of the delay between the same signals acquired by different channels, both in DC (Direct Coupling) or AC (Capacitive Coupling) mode; in this case, the “finddelay” function provides a null delay.
- Check of the delays evaluated by the “finddelay” function, for several digitally built signals with different delays; the “finddelay” algorithm provides the digital set delay.
- Cross comparisons among different sensors and channels; the relative delays provide consistent results.
- Agreement of results with the indications of Lissajous figures.
- Systematic comparison between experimental results and theoretical models.
2.4. Uncertainty evaluation

In Table 2 the uncertainty budget for the sensitivity estimation is shown. It can be seen that the major contributions are represented by the uncertainty of the positioning angles of the accelerometers and the transversal sensitivities, whose uncertainty is in the order of 2% in the hypothesis that they are less than 5% with respect to the main sensitivities. The method is not suitable for applications where transversal sensitivity effects could be relevant.

Table 2. Uncertainty budget for the sensitivity evaluation.

| Main sources of uncertainty | Relative standard uncertainty (%) |
|-----------------------------|----------------------------------|
| Repetability                | 0.2                              |
| Reproducibility             | 0.80                             |
| Reference uncertainty       | 0.30                             |
| Angle $\alpha$              | X 1.2                           |
|                             | Y 1.2                           |
|                             | Z 0.0                            |
| Angle $\beta$               | X 0.21                          |
|                             | Y 0.21                          |
|                             | Z 0.42                          |
| Transversal sensitivities   | 2.0                              |

The uncertainty budget for the phase evaluation is shown in Table 3. In this case, the positioning angles do not influence the results. The main contributions are the repeatability, the resolution linked to the sampling rate and the reference uncertainty.

Table 3. Uncertainty budget for the phase evaluation. $f_c$ is the sampling rate, $f$ is the oscillation frequency.

| Main sources of uncertainty | Standard uncertainty [°] |
|-----------------------------|--------------------------|
| Repetability                | 2°                       |
| Resolution                  | $(f/f_c \cdot 360°)/(2 \cdot \text{rad}(3))$ |
| Reference uncertainty       | 2°                       |

3. Results

3.1. Sensitivity evaluation

The results for sensitivities will be represented as percentage differences with respect to the value at 12 Hz. For instance, for the x-axis:

$$DS_{xx}(f) = 100 \cdot \frac{|S_{xx}(f) - S_{xx}(12)|}{S_{xx}(12)} \quad (5)$$
The results are similar, for both the IEPE accelerometers considered. The sensitivities for all axes decrease by 15% compared to the values obtained at 12 Hz, when the frequency is reduced to 0.5 Hz. (Figure 4).

The results are consistent with those obtained by a calibration bench of the National Institute of Metrological Research (INRiM).

![Figure 4. IEPE accelerometer, IEPE Module, IEPE AC coupled mode.](image)

If the capacitive sensor is coupled DC mode, the differences with respect to 12 Hz remains within ± 3% for both Analog input module and IEPE module (Figure 5(a-b)). The sensitivities for all axes decrease by 6% compared to the values obtained at 12 Hz, when the frequency is reduced to 0.5 Hz.

If acquired in AC coupled mode (Figure 6(c)), for frequencies 0.5-1 Hz the sensitivities decrease by 0÷6% compared to the value obtained at 12 Hz.

![Figure 5. MEMS capacitive accelerometer. a) Analog input Module 9215; b) Module 9234, DC Coupled mode; c) Module 9234, AC Coupled mode.](image)

### 3.2. Phase evaluation

As a first approximation, the LDV phase behavior has been considered ideal (i.e. negligible phase effect of the frequency of vibration due for example to internal high-pass filters).
A simple modeling of the whole measuring chain of the LDV allowed us to confirm this approximation, by comparing the LDV output when coupled AC either DC. Differences appears due to only the different coupling mode (cut-off frequency of the AC mode being 0.5 Hz), suggesting that no more phase shift delay due to the LDV itself should be present, almost down to 0.5 Hz. This result is described in Figure (6).

![Figure 6](image-url)  
**Figure 6.** Comparison between theoretical curves of a first order high-pass filter with cut-off frequency in the range 0.3 – 0.7 Hz and experimental data.

In the following results it should be noticed that when the AC coupling is realized on both channels (reference LDV and accelerometer under test) the effect of AC coupling is compensated. The results are similar, for both the IEPE accelerometers considered. The phase difference with respect to the reference, rapidly increases for frequencies < 2 Hz (Figure 7).

The results are consistent with those obtained by a calibration bench of the National Institute of Metrological Research (INRiM).

![Figure 7](image-url)  
**Figure 7.** IEPE accelerometer, IEPE Module, IEPE AC coupled mode.

If the capacitive sensor is coupled DC mode, the differences with respect to LDV are in the range ± 4° up to 5 Hz for all configurations (Figure 8(a-b-c)).

It must be considered that the tested MEMS is a very good quality one. It would be interesting to examine the behavior of other types of MEMS accelerometers, in particular the very low-cost ones.
4. Conclusions
A simple method to calibrate accelerometers in the low-frequency range has been presented.

The evaluation of sensitivity and phase of the whole measuring chains has been carried out by using a single set of tests, realized mounting the accelerometer on an inclined plate, rotated on the clamp surface by a known angle.

Validation actions have been carried out at every stage of data processing.

Results show that different behaviors, not completely predictable, can result depending on the elements of the measurement chain and not only on the type of accelerometer.

In particular, the phase is a critical aspect to consider, that can strongly affect the evaluations in some applications, so a quantitative evaluation could be useful.

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