Onset of radial flow in p+p collisions

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Abstract

It has been debated for decades whether hadrons emerging from p+p collisions exhibit collective expansion. The answer is hindered by low multiplicity and large fluctuation in p+p collisions. Tsallis Blast-Wave (TBW) model is a thermodynamic approach, introduced to handle the overwhelming correlation and fluctuation in the hadronic processes. We have systematically studied the identified particle spectra in p+p collisions from RHIC to LHC using TBW and found no appreciable radial flow in p+p collisions below $\sqrt{s} = 900$ GeV. At LHC higher energy of 7 TeV in p+p collisions, the radial flow velocity achieves an average value of $\langle \beta \rangle = 0.337 \pm 0.006$. This flow velocity is comparable to that in peripheral (40-60\%) Au+Au collisions at RHIC.

Keywords: Spectra, Radial flow, Tsallis statistics, Blast-wave model

1. Introduction

The searches for a Quark-Gluon Plasma (QGP) have been conducted in hadron collisions in all collision energies and species. Many have argued that some features observed in p+p collisions at high multiplicity and/or high energy resemble a QGP. The most acclaimed evidence has been the observation of a collective expansion \cite{1,2,3}. However, what constitutes a collective expansion when the particles reach our detectors are free streaming by nature? While it is seemingly trivial to argue that flow is a mass effect and therefore a systematic enhancement of heavier particles at higher momentum \cite{4,5,6,7} would be a signature of flow, large fluctuation in temperature and/or the creation of mini-jets in semi-hard processes can produce similar qualitative features \cite{8,9,10}. Hydrodynamic simulation with small viscous correction has been successful in interpreting many phenomena observed in heavy-ion collisions. However, its applicability to p+p collisions with large fluctuation and viscosity is not obvious.

With increasing colliding energy in p+p collisions, two possible phenomena emerge: color glass condensate (CGC) and black hole radiation \cite{11}. At LHC energies, model incorporating CGC \cite{12} correctly describes the CMS data on di-hadron correlation \cite{13} without flow while the argument from black hole radiation predicts large radial flow in p+p collisions at high multiplicity. Recently, on-going debates focus on whether hydrodynamics are applicable to small system when such a system has large shear viscous effect by design. It is therefore problematic for the elliptic flow to be quantitatively interpreted in a hydrodynamic evolution for p+p collisions. However, radial flow is expected to be less affected by the viscous correction. Anisotropic flow is by definition a relative quantity while radial flow velocity is an absolute velocity. Extracting this radial velocity has been at qualitative level and is model dependent in both p+p and A+A collisions. The main reason of the failure is that radial flow is not the dominant feature in identified particle spectra in p+p collisions and to a progressively lesser degree in A+A collisions.

Although it is known that fragmentation from hard processes and hadronization in QCD contribute significantly to the particle production at low momentum, it has been a subject of investigation to find an elegant approach to incorporate these phenomena in a thermodynamic or statistical approach. The framework allows application of hydrodynamic-inspired blast-wave model \cite{14} to extract flow velocity while being able to correctly fit the available data with very good $\chi^2$ per degree of freedom (ndf) in a large transverse momentum range. This is the philosophy presented in this paper. We have used a non-extensive thermodynamic model, Tsallis statistics \cite{15}, incorporated into the blast-wave expansion to describe data and extract flow velocity and other thermodynamic parameters \cite{16}. The model can be vetted by its simplicity in interpreting physics phenomena and by achieving best $\chi^2$ description of data. We emphasize that this is not to replace the more fundamental QCD theory or hydrodynamic simulation. On the contrary, the method resembles an “experimental” approach to extract physical quantities from data, which can then be concisely used to compare with elaborated theories.

This paper is organized as follows: we present the analysis method of all the identified particle spectra in p+p collisions at $\sqrt{s} = 200$, 340, 900 and 7000 GeV. A two-particle correlation function is also introduced in this paper based on TBW model. The results from the TBW fits to the data are presented in subsequent section. The result provides an onset of beam energy where radial flow has been developed in minimum-bias p+p collisions. At the end, possible improvement and more data collection and analyses are discussed.
2. Analysis Method

Similar to what presented in the literatures, we have used the TBW model to extract thermodynamic and hydrodynamic quantities from data. The single-particle spectrum can be written as

\[
\frac{d^2N}{2\pi m_{r} dm_{r} dy} = A \int_{y_{m}}^{y_{0}} e^{-\frac{m_{r}}{T}} \cos(y) dy \int_{0}^{R} rdr \times \int_{-\pi}^{\pi} \left[ 1 + \frac{q - 1}{T} E_{T} \right]^{-1/(q-1)} d\phi. \tag{1}
\]

Where

\[
m_{r} = \sqrt{p_{T}^{2} + m_{N}^{2}}, \tag{2}
\]

\[
y_{b} = \ln \left( \frac{\sqrt{s}}{m_{N}} \right), \tag{3}
\]

\[
E_{T} = m_{r} \cos(y) \cosh(\rho) - p_{T} \sinh(\rho) \cos(\phi). \tag{4}
\]

\(A\) is a normalization factor, \(m\) is the mass of the particle, \(m_{N}\) is the mass of the colliding nucleon, \(y_{b}\) is the beam rapidity and \(\phi\) is the azimuthal angle between the flow velocity and the emitted particle velocity in the rest frame of the emitting source. The emitting source are boosted with the boost angle

\[
\rho = \tanh^{-1} \left[ \beta_{S} \left( \frac{R}{R} \right) \right]. \tag{5}
\]

Where \(r\) is the radius of the emitting source, \(\beta_{S}\) is the velocity of the source at the outermost radius (\(r = R\)), \(n = 1\) determines the source velocity profile.

One of the significant advantages of TBW in comparison to the Boltzmann-Gibbs-Blast-Wave (BGBW) is the capability of producing large fluctuation and correlation. Based on the nonextensive Tsallis statistics, the temperature distribution of the nonequilibrium system is characterized by the parameters \(q\) and \(T\), where \(T\) is related to the average of the inverse temperature and \(q\) can be interpreted as its fluctuation [13,18,19]. The free parameters required to predict the \(p_{T}\) spectra of a given particle species are \(\beta_{S}, T, q\) and \(A\). If only the shape is concerned, the normalization factor \(A\) is not needed.

In recent theory development, the correlations originated from initial gluon scattering could be enhanced by the radial pressure from bulk flow [20,21]. K. Dusling and R. Venugopalan [22] presented a schematic description of the enhancement. It has been argued that significant radial flow has been ruled out by the di-hadron correlation from CMS [13]. It is therefore imperative to study the correlation effect in the present of radial flow in \(p+p\) collisions. To implement such effect in TBW, we have introduced an anisotropic emission of particles from the source. The anisotropic emission is described as

\[
\frac{dN}{d\phi} \propto 1 + 2p_{T} \cos(2\phi). \tag{6}
\]

The TBW formula becomes

\[
\frac{d^2N}{2\pi m_{r} dm_{r} dy} = A \int_{y_{m}}^{y_{0}} e^{-\frac{m_{r}}{T}} \cos(y) dy \int_{0}^{R} rdr \times \int_{-\pi}^{\pi} \left[ 1 + 2p_{T} \cos(2\phi) \right] \times \left[ 1 + \frac{q - 1}{T} E_{T} \right]^{-1/(q-1)} d\phi. \tag{7}
\]

The azimuthal anisotropic coefficient \(v_{2}\) can be obtained through

\[
v_{2}(p_{T}) = \langle \cos(2\phi) \rangle. \tag{8}
\]

The correlated distribution is on top of a large isotropic underlying event background. Taking this contribution into account, \(v_{2}\) becomes

\[
v_{2}(p_{T}) = s_{2} \langle \cos(2\phi) \rangle, \tag{9}
\]

where \(s_{2}\) depicts the fraction of the anisotropic emitting source (0 \(\leq s_{2} \leq 1\)). The di-hadron correlation can be obtained from the \(v_{2}\) of hadrons through

\[
\frac{dN^{\text{Assoc}}}{N^{\text{Assoc}} d\Delta\phi} = \frac{N^{\text{Assoc}}}{2\pi} \left[ 1 + 2v_{2}^{\text{diag}}(p_{T}^{\text{diag}}) v_{2}^{\text{Assoc}}(p_{T}^{\text{Assoc}}) \cos(2\Delta\phi) \right]. \tag{10}
\]

3. Results and discussion

STAR and PHENIX at RHIC, UA1, UA2 and UA5 at SppS, E735 at FermiLab, and CMS and ALICE at LHC have published a comprehensive collection of identified particle spectra in \(p+p\) collisions at 200, 540, 900 GeV and 7 TeV. Table 1 lists the available data from each reference from the collaborations.

Figure 1(c) shows the \(m_{T}\) spectra of \(\pi^{+}, \pi^{-}, K^{+}, K^{-}, p, \bar{p}, \Lambda(\Lambda), \Xi(\Xi)\) and inclusive charged hadrons in \(p+p\) collisions at \(\sqrt{s} = 900\) GeV. The \(p_{T}\) spectra of these particles are fit simultaneously with TBW (Eq. 1). There fit parameters and the best \(\chi^{2}\) per fitting degree of freedom (ndf) are listed in Tab. 2. The parameters \((\beta) = 2\beta_{S}/3\) and \(T = \text{common to all of the particle species. The parameters } q_{M} \text{ and } q_{B} \text{ are common to all of the mesons and baryons, respectively. In addition to these 4 common parameters, each particle species has its own normalization factor } A. \text{ The fit function for the inclusive charged hadron is the sum of that for } \pi^{+}, K^{+}, p \text{ and } \bar{p}. \text{ We performed a least-squares fit of the 11 } p_{T} \text{ spectra simultaneously with the TBW functions controlled by the } 4 + 11 \text{ parameters. Then the } p_{T} \text{ spectra are converted to } m_{T} \text{ spectra and rescaled to have the same value at } m_{T} = 2 \text{ GeV}/c^{2} \text{ as } \pi^{+}, \text{ as shown in Fig. 1(c).}\]

The pion mass is applied for inclusive charged hadron when we do the \(p_{T}\) to \(m_{T}\) spectra conversion. The data and fit curve have the same rescale factor. Figure 1(d) and 1(a, b) show the rescaled identified hadron and inclusive charged hadron \(m_{T}\) spectra in \(p+p\) collisions at \(\sqrt{s} = 7\) TeV, 200 and 540 GeV. The TBW fit curves are shown for all the particles as well.
Table 2: Summary of the parameters.

| √s (GeV) | ⟨β⟩ | T (MeV) | qM − 1 | qB − 1 | χ²/ndf |
|----------|-----|---------|--------|--------|--------|
| 7 TeV    | 0.337 ± 0.006 | 67.6 ± 0.8 | 0.1316 ± 0.0004 | 0.1019 ± 0.0011 | 276/250 |
| 900 GeV  | 0.258 ± 0.009 | 75.8 ± 0.9 | 0.1134 ± 0.0004 | 0.0837 ± 0.0013 | 182/220 |
| 540 GeV  | 0.000 +0.105 | 81.8 ± 0.6 | 0.1158 ± 0.0007 | 0.0841 ± 0.0036 | 205/168 |
| 200 GeV  | 0.000 ±0.000 | 92.3 ± 0.3 | 0.0946 ± 0.0006 | 0.0743 ± 0.0015 | 268/268 |

Figure 1: Identified particle $m_T$ spectra in $p+p$ collisions at √s = 200 GeV (a), 540 GeV (b), 0.9 TeV (c) and 7 TeV (d). The symbols represent experimental measurements and the curves represent TBW fit results. At each energy, all of the $m_T$ spectra are rescaled to have the same value at $m_T = 2$ GeV/c² as $π^+$. The references of the experimental measurement are summarized in Tab. 1.
In all the energies, all the spectra display power-law behavior at high $m_T$ with grouping of baryons and mesons. The TBW describes the shape of the $m_T$ spectra of more than 10 particles over a broad $m_T$ range (0-10 GeV/$c^2$) at each energy, with only 4 quantities, as listed in Tab. 2. The quality of the fits are very good, the ratio of $\chi^2$/ndf are between 0.83 and 1.22. At LHC energy, the radial flow velocity achieved an average value of $\langle \beta \rangle = 0.337 \pm 0.006$ and $0.258 \pm 0.009$ in $p+p$ collisions at 7 TeV and 900 GeV, respectively. The velocity is comparable to that in peripheral (40-60%) Au+Au collisions at $\sqrt{s_{NN}} = 200$ GeV at RHIC (0.282 $\pm$ 0.017 [16]). While at $\sqrt{s} = 540$ GeV and 200 GeV, the velocity in $p+p$ collisions is consistent with zero ($\langle \beta \rangle = 0.000^{+0.105}_{-0.006}$ and $0.000^{+0.124}_{-0.100}$, respectively). The parameter $q$ is found to increase with increasing beam energy, and it is significantly higher for meson than for baryon at all of the energies. $T$ shows a reverse dependence on beam energy.

The $m_T$ spectra of identified hadrons was found to have a universal behavior in high-energy $p+p$ collisions, as shown as $m_T$ scaling. Equation [15] show that if there is a non-zero radial flow, the shape of the $m_T$ spectra depends not only on $m_T$, but also on $p_T$. This means the $m_T$ scaling will be broken if there is a non-zero radial flow. To have a closer look at the effects on the $m_T$ spectra induced by the non-zero radial flow, we tested the $m_T$ scaling behavior of the identified particle spectra in $p+p$ collisions as shown in Fig. 2. To illustrate the effect in linear scale, all of the data points and fit curves (shown in Fig. 1) for mesons (baryons), the data points represent the ratio of rescaled $m_T$ spectra shown in Fig. 1 to the corresponding TBW curve of $\pi^+$ ($p$).

Figure 2: $m_T$ scaling behavior of the identified particle spectra in $p+p$ collisions at $\sqrt{s} = 200$ GeV (a), 540 GeV (b), 0.9 TeV (c) and 7 TeV (d). For mesons (baryons), the data points represent the ratio of rescaled $m_T$ spectra shown in Fig. 1 to the corresponding TBW curve of $\pi^+$ ($p$).
well. At lower energy, all the spectra still follow the \( m_T \) scaling, as shown in Fig. 3(a, b).

To illustrate how radial flow boosts the particle collinear emission and enhances a pre-existing angular correlation, we assume that there is an existing correlation originating from initial condition and manifesting itself as anisotropic emission from its source at rest with \( p_{Tz} \), and only a fraction of all emission source \( (s_2) \) possesses this characteristics and been driven by the later stage bulk radial flow. The scenarios are independent of hadron \( p_T \) and source location, and only serve for illustration purpose and are likely not realistic. Figure 3 shows \( v_2 \) as a function of \( p_T \) for pion, kaon and proton, predicted by TBW according to Eq. [9]. The parameters \( p_2 \) and \( s_2 \) are assumed to be 10%. This means the fraction of initial anisotropic source is 10%, and the particles emitted from the anisotropic source has \( v_2 = 10\% \). The parameters \( T \), \( q_M \) and \( q_B \) are fixed to the values obtained from the fit to the \( p_T \) spectra at 7 TeV. The radial flow velocity \( \beta_z \) varies from 0.0 to 1.0 (from bottom to top). When there is no radial flow \((\beta_z = 0)\), \( v_2 \) is a constant of 10\% \( \times \) 10\% = 1\%. Once there is a non-zero radial flow, \( v_2 \) is enhanced depending of the magnitude of radial flow velocity and \( p_T \). It increases rapidly at low-\( p_T \) \( (p_T \lesssim 1 \text{ GeV}/c) \) and then tend to saturate. The mass ordering at low-\( p_T \) and baryon/meson grouping at intermediate- and high-\( p_T \) range is reproduced. In the whole \( p_T \) range, the predicted \( v_2 \) increases with increasing radial flow velocity. For the radial flow velocity of what we extracted from the 7 TeV data \((\beta_z = 0.337)\), the saturated \( v_2 \) at \( p_T \gtrsim 2 \text{ GeV}/c \) is predicted to be about 4.7\% and 5.2\% for light mesons and baryons, respectively. As a consequence, the associated particle yield from the di-hadron correlation is predicted to be enhanced by a factor of \( \sim 25 \) at this \( p_T \) range. The enhancement could be even larger if we take into account the “blue shift” of \( p_T \) spectra induced by radial flow.

4. Summary

In summary, we have applied the Tsallis Blast-Wave (TBW) model to all the identified particle spectra in \( p+p \) collisions at \( \sqrt{s} = 200, 540, 900, 7000 \text{ GeV} \). The TBW function fits the data quite well over a broad transverse momentum range (0-10 GeV/c). The average radial flow velocity extracted from the fit is consistent with zero in \( p+p \) collisions at \( \sqrt{s} = 200, 540 \text{ GeV} \) and increases to \( 0.258 \pm 0.009 \) at \( \sqrt{s} = 900 \text{ GeV} \) and \( 0.337 \pm 0.006 \) at 7 TeV. We have also tested the \( m_T \) scaling behavior of the particle spectra. The particle spectra was found to obey \( m_T \) scaling at 200 GeV and 540 GeV, but significantly deviate from \( m_T \) scaling at beam energy above 900 GeV. The breaking of the \( m_T \) scaling at high-energy \( p+p \) collisions may be attributed to radial flow. This is suggestive of an onset of radial flow at certain beam energy where sufficient energy density could generate collective motion to be observed in minimum bias \( p+p \) collisions.

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References

[1] T. Alexopoulos, et al., Mass identified particle production in \( pp \) collisions at \( \sqrt{s} = 300 \text{ GeV}, 540 \text{ GeV}, 1000 \text{ GeV}, \) and 1800 GeV, Phys. Rev. D48 (1993) 984–997. doi:10.1103/PhysRevD.48.984

[2] P. Lévai, B. Müller, Transverse baryon flow as possible evidence for a quark-gluon-plasma phase, Phys. Rev. Lett. 67 (1991) 1519. doi:10.1103/PhysRevLett.67.1519

[3] M. J. Tannenbaum, R. M. Weiner, Comments on “Observation of Long-Range, Near-Side Angular Correlations in Proton-Proton Collisions at the LHC” by the CMS collaboration, arXiv:1010.0964

[4] P. Braun-Munzinger, J. Stachel, J. Wessels, N. Xu, Thermal equilibration and expansion in nucleus-nucleus collisions at the AGS, Phys. Lett. B344 (1995) 43. arXiv:nucl-th/9410026 doi:10.1016/0370-2693(94)01534-J

[5] B. I. Abelev, et al., Systematic Measurements of Identified Particle Spectra in \( pp, d^* \) \( Au \) and \( Au+Au \) Collisions from STAR, Phys. Rev. C79 (2009) 034909. doi:10.1103/PhysRevC.79.034909

[6] S. Chatrchyan, et al., Study of the inclusive production of charged pions, kaons, and protons in \( pp \) collisions at \( \sqrt{s} = 0.9, 2.76, \) and 7 TeV, Eur. Phys. J. C72 (2012) 2164. arXiv:1207.4724 doi:10.1140/epjc/s10052-012-2164-1

[7] S. Chatrchyan, et al., Spectra of charged pions, kaons, protons in pp collisions at \( \sqrt{s} = 0.9, 2.76, 7 \text{ TeV} \) identified via tracker energy loss, CMS-PAS-FSQ-12-014 (2012).

[8] X.-N. Wang, R. C. Hwa, The Effect of Jet Production on the Multiplicity Dependence of Average Transverse Momentum, Phys. Rev. D39 (1989) 187. doi:10.1103/PhysRevD.39.187

[9] X.-N. Wang, M. Gyulassy, A Systematic study of particle production in \( p + p \) (anti-p) collisions via the HIJING model, Phys. Rev. D45 (1992) 844. doi:10.1103/PhysRevD.45.844

[10] X.-N. Wang, M. Gyulassy, Transverse flow due to mini-jets in \( pp \) collisions at \( S^{1/2} = 1.8 \text{ TeV} \), Phys. Lett. B282 (1992) 466. doi:10.1016/0370-2693(92)90670-Y

[11] E. Shuryak, I. Zahed, High Multiplicity pp and PA Collisions: Hydrodynamics at its Edge and Stringy Black Hole, Phys. Rev. C88 (2013) 044915. arXiv:1301.3470 doi:10.1103/PhysRevC.88.044915
