Gear tooth action in a plane synthesis based on the criterion of constant curvature

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Abstract. The relevance of the problem of the gear tooth action in a plane synthesis based on the criterion of tooth profile wheel group curvature is justified. Based on a mathematical model of gear tooth action in a plane with a constant angular speed ratio, an algorithm for solving the problem with a given constant curvature is given. The obtained solutions allow us to perform an analysis of the general type of lantern gear in a plane.

1. Problem statement
Cogwheels with a constant curvature of the tooth profiles are mainly structurally performed by the pinion [1]. This makes it possible to increase the machinability, eliminate the relative slip at the contact point and, accordingly, reduce the wear of the profiles, the loss of power to friction and increase the efficiency by an order of magnitude [2-6].

2. Mathematical model and synthesis algorithm
A mathematical model of gear tooth action in a plane with a constant gear ratio can be represented as:

\[ v_j = -a_j \omega_j \sin \alpha + r (\dot{\alpha} - \omega_j) \quad (1) \]

\[ \dot{r} + a_j \omega_j \cos \alpha = 0 \quad (2) \]

\[ k_j = (\dot{\alpha} - \omega_j)/v_j \quad (3) \]

where \( j = 1, 2 \) - the index wheel, \( v_j \) - speed of the contact point along the tooth profile, \( a_j \) – centrode radius, \( \omega_j \) – angular rate, \( \alpha \) – angle of action, \( r \) - distance from the contact point to the pitch point, \( k_j \) - curvature of the tooth profile, a dot above a parameter means its time derivative t.

Let us consider the solution of the problem of gearing synthesis by the criterion of constant normal curvature \( k_j = const \). We write (3) considering (1) in the form:

\[ R_j = \frac{v_j}{\dot{\alpha} - \omega_j} = r - \frac{a_j \omega_j \sin \alpha}{\dot{\alpha} - \omega_j} \]

from which
\[ r = R_j + \frac{a_j \omega_j \sin \alpha}{\dot{\alpha} - \omega_j} \]  

(4)

where \( R_j = 1/k \) – curvature radius and the second term is the line of action for the criteria \( v_j = 0 \). It is given by the formula:

\[ \hat{r}_{|v_j=0} = -a_j \sin \alpha \mp a_j \sqrt{\sin^2 \alpha + c_0} \]

where the parameter \( c_0 \) is:

\[ c_0 = \frac{2R_0}{a_j} \left( \frac{\hat{r}_0}{2a_j} + \sin \alpha_0 \right) \]

Thus,

\[ r = R_j - a_j \sin \alpha \mp a_j \sqrt{\sin^2 \alpha + c_0} \]  

(5)

Here, the choice of the sign is determined by setting the initial conditions. Differentiating equation (5) in time and comparing with equation (2), we find that the function \( \alpha(t) \) satisfies the equation:

\[ \dot{\alpha} = \frac{\omega_j}{1 \mp \sin \alpha \sqrt{\sin^2 \alpha + c_0}} \]  

(6)

Its solution has the form:

\[ \varphi_1 = \alpha - \alpha_0 \mp \left[ \arcsin \frac{\cos \alpha}{\sqrt{c_0 + 1}} - \arcsin \frac{\cos \alpha_0}{\sqrt{c_0 + 1}} \right] \]  

(7)

Having the expressions (5) and (7), it can be written the formulas for the teeth profiles of the wheels in the form:

\[ x_j = a_j \cos \varphi_j + r \sin (\alpha - \varphi_j) \]

\[ y_j = -a_j \sin \varphi_j - r \cos (\alpha - \varphi_j) \]

and if the second link is a translationally moving racking, then

\[ x'_2 = rsina \]

\[ y'_2 = -a_j \varphi_j - r cosa \]

The results obtained correspond to out-of-band lantern gear by introducing an additional geometric connection to the model.

\[ (a_1 + e) \sin \varphi_1 = -(r + \rho) \cos \alpha \]  

(8)

where \( \rho \) – pin radius, \( e \) - offset of the center of the pin relative to the pole.

Let's establish a connection between the constants \( c_0 \) and \( e \). Comparing equations (7) and (8), we get:

\[ e = a_1 (\sqrt{c_0 + 1} - 1) \]  

(9)
The lines of action for the constant curvature of the first link profile for out-of-band pinion engagement with \( a_1 = 50 \text{ mm} \) and the gear ratio \( i_{12} = 2 \) are shown in figure 1. The two curves correspond to the gear sign “±” in equation (5). Line corresponding to the “+” sign, located inside the center line of the pin, and the line corresponding to the “−” sign is on the outside. In both cases, the profile of the pinion is the arc of the circle. In more detail, the profile of the pinion and the profiles of the second link are shown in figure 2.

Consider the case \( k_j = \text{const} \), \( v_j = \text{const} \). For the relative velocity, considering equations (5) and (6), it was obtained the following expression:

\[
v_j = -a_j\omega_j\sin\alpha + \left( R_j - a_j\sin\alpha \mp a_j\sqrt{\sin^2\alpha + c_0} \right) \frac{\omega_j}{\frac{1}{\sin\alpha} \mp \frac{a_j\sin\alpha}{\sqrt{\sin^2\alpha + c_0}}} = \frac{R_j}{r - R_j} a_j\omega_j\sin\alpha
\]

Then, for \( v_j = \text{const} \), there must be \( r = R_j + C\sin\alpha \). Substituting this in equation (10), we get the identity:
\[
(C + a_j)^2 - a_j^2 \sin^2 \alpha = a_j^2 c_0 = 0
\]

Figure 3. Lines of action and profile links for the case \(k_1 = \text{const}\) (pole pinion engagement, \(c_0 = 0\)):
1 — line of action; 2 — line centers pin; 3 — profile of the first link (pinwheel); 4 — profile of the second link; 5 — profile rail; \(O_1\) - axis of rotation of the first link; \(O_2\) - axis of rotation of the second link; \(P\) — pitch point.

It holds for any \(\alpha\) only under the condition \(C = -2a_j, c_0 = 0\). From equation (9) it also follows \(e = 0\), i.e. the case of constant curvature and velocity corresponds to the pole pinion engagement. The engagement line has the form:

\[ r = R_j - 2a_j \sin \alpha \tag{11} \]

which is the polar equation of limaon of Pascal.

Line of action and profile links for constant curvature profile of the first link for the pin cycloid gear with \(a_1 = 50\) mm and reduction ratio \(i_{12} = 2\) is shown in figure 3. Line of action is limaon of Pascal, and the profile of the first line is a circle.

Consider the case of \(k_1 = \text{const}, k_2 = \text{const}\). In this case, we obtain together with equations (1)–(3) a redundant system of equations, the solution of which leads to the imposition of an additional connection on the constant parameters of the engagement. We rewrite the relation (4) in the form:

\[ \dot{\alpha} - \omega_j = \frac{a_j \omega_j \sin \alpha}{r - R_j} \tag{12} \]

Hence, when \(R_1 = \text{const}, R_2 = \text{const}\):

\[ \omega_1 + \frac{a_1 \omega_1 \sin \alpha}{r - R_1} = \omega_2 + \frac{a_2 \omega_2 \sin \alpha}{r - R_2} \tag{13} \]

The formula for the engagement line in the case of \(k_1 = 0, k_2 = \text{const}\) is obtained from the equation (13) in the form:

\[ r = R_2 + \frac{a_j \omega_j \sin \alpha}{\omega_1 - \omega_2} \tag{14} \]

This, like equation (11), is the polar equation of Pascal snail. It follows from (12) that:

\[ \dot{\alpha} = \omega_1 \tag{15} \]

Differentiating equation (14) with respect to time \(t\) and using equation (2), considering (15), it was obtained:

\[ \frac{\omega_1 a_j \omega_j \cos \alpha}{\omega_1 - \omega_2} = -a_j \omega_j \cos \alpha \]

Hence \(\frac{\omega_1}{\omega_1 - \omega_2} = -1\). This means that such transfer may be made in the form of a face with internal gear and gear ratio \(i_{12} = 1, 2[7-8]\). The pinwheel is located on the second wheel, and the profiles of the
teeth of the first wheel are straight, parallel, radial direct. In this case, the velocities have expressions, respectively \( v_1 = -a_1 \omega_1 \sin \alpha \), \( v_2 = R_2 (\omega_1 - \omega_2) = \text{const} \).

If \( k_1 = k_2 \), then from equation (4) we have \( \omega_1 = \omega_2 \) and, consequently, \( a_1 = a_2 \), which corresponds to a coupling mechanism with zero center distance with arbitrary but congruent link elements. The same result corresponds to the solution for \( k_1 = k_2 = \text{const} \), \( k_1 = k_2 = \infty \) \((R_1 = R_2 = 0)\).

Figure 4. The line of action for the case \( k_1 = 0 \), \( k_2 = \text{const} \):
\( O_1 \) — the axis of rotation of the first link; \( O_2 \) — the axis of rotation of the second link; \( P \) — pitch point.

Figure 4 shows the engagement line for the case \( k_1 = 0 \), \( k_2 = \text{const} \).

3. Conclusion

The resulting formulas describe the entire set of pole and non-pole pinion gears in a plane with internal, external and rack-and-pinion gearing. As a result of solving the problem, it becomes possible to analyze them for the entire set of basic and derived performance criteria, as well as the ability to choose the optimal ratios of constant parameters — the center distance, gear ratio, the radius of the handguards and the radius of the installation of the handguards.

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