Higgs Potential from Wick Rotation in Conformal BSM

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Abstract

It is well known that in order to make the path integral of general relativity converge, one has to perform the Wick rotation over the conformal factor in addition to the more familiar Wick rotation of the time axis to pass to the space-time with Euclidean signature. In this article, we will apply this technique to a scalar field in the conformally invariant scalar-tensor gravity with a conformally invariant beyond-standard-model (BSM). It is then shown that a potential term in the conformally invariant potential, which corresponds to the Higgs mass term in the Higgs potential of the standard model (SM), can have a negative coefficient. The change of sign of the potential term naturally induces spontaneous symmetry breakdown of the electroweak gauge symmetry after symmetry breaking of conformal symmetry (local scale symmetry) via the Coleman-Weinberg mechanism around the Planck scale. The present study might shed light on the fact that the existence of a stable vacuum in quantum gravity is relevant to that in the SM.

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The discovery [1, 2] of a relatively light Higgs particle with properties consistent with the standard model (SM) has marked a significant milestone in the history of particle physics. The SM of particle physics, which describes the electro-magnetic, weak and strong interactions in a concise manner, has passed a series of stringent tests so far. It is remarkable that as the parameters in the model have been measured precisely by many of experiments, points of disagreement existed in the past have completely faded away and the SM has been put on the more sound ground.

From the viewpoint of the SM, it appears that the Planck scale $M_{Pl}$ is a special point in the sense that

1. Scalar self-coupling is zero: $\lambda(M_{Pl}) = 0$, 
2. Its beta function is zero: $\frac{d\lambda}{dt}|_{M_{Pl}} \equiv \beta(M_{Pl}) = 0$, 
3. Higgs bare mass is zero [4]: $m^2(M_{Pl}) = 0$.

These facts suggest that the SM might secretly know the physics at the Planck scale even if it does not involve gravity. Here it is of interest to reverse this viewpoint and suppose that the physics at the Planck scale, which we call quantum gravity, might give us some useful information on the SM or the construction of a theory beyond the standard model (BSM). Under such a situation, there might be some aspects of the SM that do not involve gravity directly but nevertheless require some information from quantum gravity. As such an example, in this article, we shall shed light on the sign problem of the tachyonic mass term in the Higgs potential of the SM.

The above observations also suggest that it would be conceivable that the SM is the low-energy limit of a distinct special theory with a global scale symmetry at the Planck scale. However, as stressed in our previous work [5], both no-hair theorem of quantum black holes and the fact that in string theory any additive global symmetries are either gauge symmetries or explicitly violated in a tacit way seem to insist that a global scale symmetry must be promoted to a local scale symmetry, which we call conformal symmetry in this article.3

As far as experiments based on accelerators are concerned, the SM does a rather excellent job of accounting for various kinds of particle phenomena. The objection to the opinion that the SM is a complete theory mainly comes from a theoretical side. In particular, the SM has a number of arbitrary parameters which cannot be explained theoretically but are fixed only by measurements. For instance, the renormalizability of the SM requires that the Higgs potential takes the simple form

$$V(H) = m^2(H\dagger H) + \frac{\lambda_H}{2}(H\dagger H)^2,$$  \hspace{1cm} (1)

With the top mass, $m_t = 173\text{GeV}$, the renormalization group equation for the Higgs self-coupling constant $\lambda$ implies that $\lambda$ becomes negative around $10^{11}\text{GeV}$ [3], whereas with the current uncertainty of experiments, the top quark might have the lighter value, $m_t = 170\text{GeV}$, and then $\lambda$ becomes zero around the Planck mass scale.

We have already constructed such models with scale symmetries at the classical level [6]-[9].
up to radiative corrections. For spontaneous symmetry breakdown to occur, the renormalized value of the mass parameter $m^2$ must be negative. But the parameter $m^2$ could have either sign; there is no logic that we prefer one sign to the other. We should therefore answer the question why the mass parameter $m^2$ is negative in order to understand the Higgs mechanism completely [10].

This issue is closely related to the gauge hierarchy problem. The SM action is invariant under a global scale transformation except the Higgs mass term, that is, one could say that our world is almost scale invariant. Indeed, Bardeen has advocated the idea that instead of supersymmetry, the global scale symmetry might be a fundamental symmetry and play an important role in the naturalness problem [11]. With scale invariance, the Higgs potential consists of solely the second term in Eq. (1), so the mass correction is only the logarithmic divergence rather than quadratic one, thereby alleviating the gauge hierarchy problem.

In our recent study [5], it has been shown that both the Planck and electroweak scales can be generated from conformal gravity via the Coleman-Weinberg mechanism [12], which explicitly breaks conformal symmetry, and via the conformally invariant potential corrected by the Coleman-Weinberg mechanism. To this end, symmetry breakings must occur at two steps: at the first step, the Planck scale is generated by radiative corrections associated with gravitons, by which conformal symmetry is explicitly broken. At the second step, the electroweak gauge symmetry is spontaneously broken via the conformally invariant Higgs potential modified by the Coleman-Weinberg mechanism. The huge hierarchy between the two scales is explained in terms of a very tiny coupling between the scalar and Higgs fields.

As in many of similar scale-invariant BSM models [13, 14], a tantalizing aspect of this study is that the conformally invariant scalar potential does not give rise to spontaneous symmetry breakdown of the electroweak symmetry unless we assume that a term in the potential, which exactly corresponds to the Higgs mass term in Eq. (1), has a negative coefficient. Since we believe that this sign problem would be clarified in future, we should derive the negative coefficient by some mechanism within the framework of the BSM. Much of the impetus for the present work stemmed from the realization that the scalar field in our conformal BSM is very similar to a conformal factor of the metric perturbation in general relativity and both of them are non-dynamical fields at least at the classical level. Just as the Wick rotation over the conformal factor guarantees that the Euclidean Einstein-Hilbert action is bounded from below and consequently makes the path integral be convergent in general relativity [15, 16, 17], we expect that the Wick rotation over the scalar field might enable us to flip the sign in front of the Higgs mass term from positive to negative, thereby making it possible to trigger the spontaneous symmetry breaking of the electroweak symmetry in a natural way. This possibility was briefly suggested in the previous work [5], but was not investigated in detail. The purpose of the present short article is to pursue this possibility and spell out its detail and result. Incidentally, this procedure cannot be applied to the case of a global scale symmetry since in this case the scalar field is in general a dynamical field even in the classical

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\footnote{It is easy to extend a global scale symmetry to conformal symmetry by introducing the conformally invariant coupling between the Higgs field and the scalar curvature as seen shortly.}
Now let us start with the following conformally invariant Lagrangian density:

\[
\frac{1}{\sqrt{-g}} \mathcal{L}_C = -\frac{1}{2\xi^2} C_{\mu\nu\rho\sigma} C^{\mu\nu\rho\sigma} + \frac{1}{12} \phi^2 R + \frac{1}{2} g^{\mu\nu} \partial_\mu \phi \partial_\nu \phi - \frac{1}{6} (H^\dagger H) R - g^{\mu\nu} (D_\mu H) (D_\nu H) + V(\phi, H) + L_m. \tag{2}
\]

Here \( \xi \) is a dimensionless coupling constant, and the conformal tensor (or Weyl tensor) \( C_{\mu\nu\rho\sigma} \) is defined as

\[
C_{\mu\nu\rho\sigma} = R_{\mu\nu\rho\sigma} - (g_{\mu[\rho} R_{\sigma]\nu - g_{\nu[\rho} R_{\sigma]\mu}) + \frac{1}{3} g_{\mu[\rho} g_{\sigma]\nu} R, \tag{3}
\]

where \( A_{[\mu} B_{\nu]} = \frac{1}{2} (A_{\mu} B_{\nu} - A_{\nu} B_{\mu}), \mu, \nu, \cdots = 0, 1, 2, 3, \) \( R_{\mu\nu\rho\sigma}, R_{\mu\nu} \) and \( R \) are the Riemann tensor, the Ricci tensor and the scalar curvature, respectively.\(^5\) We have introduced two scalar fields, one of which is the Higgs doublet \( H, D_\mu \) is a covariant derivative including the SM gauge fields, and \( L_m \) denotes the remaining Lagrangian density of the SM but the Higgs mass term, which is also conformally invariant. The second and third terms on the RHS of Eq. (2) represent the conformally invariant scalar-tensor gravity with a positive Newton constant and a scalar ghost \( \phi \). And the fourth and fifth terms correspond to the conformally invariant terms for the Higgs field.

Moreover, the new potential \( V(\phi, H) \) beyond the SM, which is conformally invariant as well, is added and has the form

\[
V(\phi, H) = \frac{\lambda_\phi}{4!} \phi^4 + \lambda_{H\phi} (H^\dagger H) \phi^2 + \frac{\lambda_H}{2} (H^\dagger H)^2, \tag{4}
\]

where all the coupling constants \( \lambda_i (i = \phi, H\phi, H) \) are dimensionless. Note that the requirement of conformal invariance, gauge invariance and renormalizability uniquely fixes the form of the potential. (The removal of the requirement of renormalizability would allow the presence of non-polynomial terms such as \( (H^\dagger H)^3 \).) In order to obtain a stable ground state, we require that for arbitrary positive scalar fields, the potential is positive, \( V(\phi, H) > 0 \). This constraint on the potential implies that all the coupling constants must obey the relation

\[
\lambda_i > 0. \tag{5}
\]

Next, it is straightforward to prove that the action, \( S_c \equiv \int d^4x \mathcal{L}_C, \) is invariant under conformal transformation:

\[
g_{\mu\nu} \rightarrow \Omega^2(x) g_{\mu\nu}, \quad \phi \rightarrow \Omega^{-1}(x) \phi, \quad H \rightarrow \Omega^{-1}(x) H, \quad A_\mu \rightarrow A_\mu. \tag{6}
\]

Then, let us notice that the new composite metric

\[
\hat{g}_{\mu\nu} \equiv \frac{1}{6M^2_{Pl}} \phi^2 g_{\mu\nu}, \tag{7}
\]

\(^5\)We will follow the conventions and notation by Misner et al. [18].
is invariant under the conformal transformation (6). The factor $\frac{1}{6M_p^2}$ is inserted for the
dimensional alignment and later convenience. With this new metric $\hat{g}_{\mu\nu}$, the Lagrangian
density for the conformally invariant scalar-tensor gravity can be rewritten as the Einstein-
Hilbert form

$$\mathcal{L}_{CST} = \sqrt{-g} \left( \frac{1}{12} \phi^2 R + \frac{1}{2} g^{\mu\nu} \partial_\mu \phi \partial_\nu \phi \right)$$

$$= \frac{1}{16\pi G} \sqrt{-\hat{g}} \hat{R},$$

(8)

where we have set $\frac{M_p^2}{2} \equiv \frac{1}{16\pi G}$. To prove this equation, we have used the following properties
under the conformal transformation (6):

$$\sqrt{-g} \to \Omega^4(x) \sqrt{-g}, \quad R \to \Omega^{-2}(x) \left( R - 6\Omega^{-1}(x) \nabla^2 \Omega(x) \right).$$

(9)

Furthermore, because of conformal symmetry, the total classical Lagrangian density $\mathcal{L}_C$ in
Eq. (2) can be rewritten as the same form as before except $\mathcal{L}_{CST}$ when expressed in terms of
the metric tensor $\hat{g}_{\mu\nu}$

$$\frac{1}{\sqrt{-g}} \mathcal{L}_C = -\frac{1}{2\xi^2} \hat{C}_{\mu\rho\sigma} \hat{C}^{\mu\rho\sigma} + \frac{1}{16\pi G} \hat{R} - \frac{1}{6} (\hat{H}^\dagger \hat{H}) \hat{R} - \hat{g}^{\mu\nu} (D_\mu \hat{H})^\dagger (D_\nu \hat{H})$$

$$+ V(\hat{\phi}, \hat{H}) + L_m,$$

(10)

where we have used $\hat{D}_\mu = D_\mu$ owing to $\hat{A}_\mu = A_\mu$.

Our task now is to consider the gravitational path integral on the basis of the metric
tensor $\hat{g}_{\mu\nu}$. We shall perform a Wick rotation and henceforth work on the manifold with
the Euclidean signature metric $(+, +, +, +)$. With the Euclidean signature, the Lagrangian
density (10) can be cast to the form

$$\frac{1}{\sqrt{-g}} \mathcal{L}_{C}^{(E)} = -\frac{1}{2\xi^2} \hat{C}_{\mu\rho\sigma} \hat{C}^{\mu\rho\sigma} - \frac{1}{16\pi G} \hat{R} + \frac{1}{6} (\hat{H}^\dagger \hat{H}) \hat{R} - \hat{g}^{\mu\nu} (D_\mu \hat{H})^\dagger (D_\nu \hat{H})$$

$$+ V(\hat{\phi}, \hat{H}) + L_m.$$

(11)

With respect to this Lagrangian density, it is worthwhile to emphasize that the square of con-
formal tensor is certainly positive definite so that the action of conformal gravity is bounded
from below. However, this positiveness of the conformal gravity action does not mean the
positiveness of the total Lagrangian density (11) since the conformal gravity does not include
the conformal factor due to conformal invariance. To put differently, the total action corre-
sponding to the Lagrangian density (11) is not bounded from below owing to the presence of

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6Since the concept of time has no physical meaning in general relativity because of diffeomorphisms, defining
its Euclidean continuation as an analytic continuation of the time coordinate is not a natural prescription.
A more reasonable prescription is to regard the Wick rotation as an analytic continuation of not the time
coordinate but metric tensor.
the conformal factor which is involved only in the Einstein-Hilbert term. Thus, even in the present situation we have to rely on the Wick rotation over the conformal factor to make the path integral converge as in general relativity. It is known that the Euclidean action for matter fields is positive semi-definite so it is free from the conformal factor problem [15], which will be explained later. Since we are interested in the conformal factor problem, we shall henceforth pay attention to only the Einstein-Hilbert action and ignore the other actions.

At a more elementary level, we wish to define the gravitational path integral by the perturbation theory. To do that, we make use of the background field method and therefore split as

\[
g_{\mu\nu} = \bar{g}_{\mu\nu} + h_{\mu\nu}, \quad \phi = \bar{\phi} + \varphi,
\]

where we have defined \( \bar{\phi} = \sqrt{6} M_{Pl} \). Then, the conformally invariant new metric takes the form

\[
\hat{g}_{\mu\nu} = \bar{g}_{\mu\nu} + \hat{h}_{\mu\nu} + \mathcal{O}(\hat{h}^2),
\]

with \( \hat{h}_{\mu\nu} \) being defined as

\[
\hat{h}_{\mu\nu} = h_{\mu\nu} + \sqrt{\frac{2}{3}} \frac{1}{M_{Pl}} \bar{g}_{\mu\nu} \varphi.
\]

In this article, we shall work with a perturbation theory where it is assumed that

\[
|h_{\mu\nu}| \ll 1, \quad \varphi \ll M_{Pl}, \quad |\hat{h}_{\mu\nu}| \ll 1.
\]

In a curved space-time, it is more convenient to introduce the York decomposition [19, 17]:

\[
\hat{h}_{\mu\nu} = \hat{h}_{\mu\nu}^{TT} + \bar{\nabla}_\mu \xi_\nu + \bar{\nabla}_\nu \xi_\mu + \bar{\nabla}_\mu \bar{\nabla}_\nu \sigma - \frac{1}{4} \bar{g}_{\mu\nu} \bar{\nabla}^2 \sigma + \frac{1}{4} \bar{g}_{\mu\nu} \hat{h}.
\]

Here, our conventions are the following: all indices are raised and lowered with the background metrics \( \bar{g}^{\mu\nu} \) and \( \bar{g}_{\mu\nu} \), and the trace is defined as \( \hat{h} \equiv \bar{g}^{\mu\nu} \hat{h}_{\mu\nu} \). Moreover, \( \bar{\nabla}_\mu \) denotes the Levi-Civita connection of the background metric \( \bar{g}_{\mu\nu} \), \( \hat{h}_{\mu\nu}^{TT} \) is transverse and traceless, and \( \xi_\mu \) is transverse:

\[
\bar{\nabla}^\mu \hat{h}_{\mu\nu}^{TT} = 0, \quad \bar{g}^{\mu\nu} \hat{h}_{\mu\nu}^{TT} = 0, \quad \bar{\nabla}^\mu \xi_\mu = 0.
\]

Given the York decomposition (16), let us define the conformal factor in the metric \( \hat{g}_{\mu\nu} \) by

\[
\hat{s} \equiv \hat{h} - \bar{\nabla}^2 \sigma = h - \bar{\nabla}^2 \sigma + 4 \sqrt{\frac{2}{3}} M_{Pl} \varphi.
\]

For simplicity, in what follows, we will assume the background metric \( \bar{g}_{\mu\nu} \) to belong to the Einstein space.\(^7\) The Einstein space, which is defined as the space satisfying the Einstein equation with a cosmological constant \( \Lambda \)

\[
\bar{R}_{\mu\nu} = \Lambda \bar{g}_{\mu\nu},
\]

\(^7\)In order to consider the Einstein space and make the argument clear, we add the cosmological term to the action. It is easy to see that this modification does not change the final result.
is a classical solution to the field equation of the Einstein-Hilbert action.

Under these conditions, the quadratic part of the Euclidean Einstein-Hilbert action can be evaluated to be

$$S^{(2)}_{EH}(\hat{h}; \bar{g}) = \frac{1}{32\pi G} \int d^4x \sqrt{\bar{g}} \left[ \frac{1}{2} \hat{h}^{TT} (\hat{\Delta}_L^2 - 2\Lambda) \hat{h}^{TT} - \frac{3}{16} \hat{s} \left( -\nabla^2 - \frac{4}{3}\Lambda \right) \hat{s} \right], \quad (20)$$

where the operator $\Delta_L^2$ represents the Lichnerowicz Laplacian acting on generic second-rank symmetric tensors $T_{\mu\nu}$ which is concretely defined as

$$(\hat{\Delta}_L^2 T)_{\mu\nu} = -\nabla^2 T_{\mu\nu} + \bar{R}_\mu{}^\rho T_{\rho\nu} + \bar{R}_\nu{}^\rho T_{\rho\mu} - 2\bar{R}_\mu{}^\rho \nu{}^\sigma T_{\rho\sigma}. \quad (21)$$

Now it is clear what the conformal factor problem in general relativity is. The first term in Eq. (20) is positive whereas the second one is not so. A negative kinetic term in Euclidean signature usually means negative energy in Lorentzian signature, but there is no such pathology in general relativity. This issue is not restricted to perturbation theory and has a root that the full Euclidean Einstein-Hilbert action is unbounded from below [15]. Actually, under the conformal transformation (6), the integrand of the Einstein-Hilbert action is transformed as

$$\sqrt{-\bar{g}} R \rightarrow \sqrt{-g} \Omega^2(x) \left( R - 6\Omega^{-1}(x)\nabla^2 \Omega(x) \right), \quad (22)$$

where we have used Eq. (9). One sees that this quantity can be as negative as one wants by selecting a rapidly varying conformal factor $\Omega(x)$.

There could be several resolutions to this problem. A well-known resolution is that the integration over the conformal factor is rotated in the complex plane in order to make the integrand converge [15]. A more detailed analysis has been done at least at the one-loop level in perturbation theory in [16], which we shall follow in this paper. At first sight, the conformal factor $\hat{s}$, which is a scalar field invariant under diffeomorphisms, is a dynamical, propagating mode, but it is an illusion. Indeed, the Jacobian associated with the change of variables turns out to include a determinant [16, 17]

$$\sqrt{\det(-\nabla^2 - \frac{4}{3}\Lambda)} / \sqrt{\det(-\nabla^2)}, \quad (23)$$

whose numerator precisely cancels the determinant coming from the Gaussian integration over the conformal factor $\hat{s}$ in Eq. (20). Thus, the Wick rotation over $\hat{s}$ is then justified at least at one-loop order due to the fact that $\hat{s}$ is not a dynamical field [16].

To proceed further, let us notice that as seen in Eq. (18), the Wick rotation over the conformal factor $\hat{s}$ in the conformally invariant metric $\hat{g}_{\mu\nu}$ means the simultaneous Wick rotation over the conformal factor in the original metric $g_{\mu\nu}$, $s \equiv h - \nabla^2 \sigma$, and the scalar field $\varphi$:

$$\hat{s} \rightarrow is \iff s \rightarrow is, \quad \varphi \rightarrow i\varphi. \quad (24)$$
Since, with the Euclidean signature metric, the conformally invariant scalar-tensor gravity (8) reads

\[ L^{(E)}_{\text{CST}} = \sqrt{g} \left( -\frac{1}{12} \phi^2 R + \frac{1}{2} g^{\mu\nu} \partial_\mu \phi \partial_\nu \phi \right) = -\frac{1}{16\pi G} \sqrt{\hat{g}} \hat{R}, \]  

(25)

performing the Wick rotation over \( \phi \) and using Eq. (12) lead to the expression

\[ L^{(E)}_{\text{CST}} = \sqrt{g} \left[ +\frac{1}{12} (\varphi - i \bar{\varphi})^2 R - \frac{1}{2} g^{\mu\nu} \partial_\mu \varphi \partial_\nu \varphi \right] = \sqrt{g} \left[ +\frac{1}{12} \varphi^2 R - \frac{1}{2} g^{\mu\nu} \partial_\mu \varphi \partial_\nu \varphi \right], \]  

(26)

where at the last step we have performed the shift of variables, \( \varphi \to \varphi + i \bar{\varphi} \), in the functional measure \( D\varphi \) of the path integral.\(^8\) It is more convenient to go back to Lorentzian signature

\[ L_{\text{CST}} = -\sqrt{-g} \left[ \frac{1}{12} \varphi^2 R + \frac{1}{2} g^{\mu\nu} \partial_\mu \varphi \partial_\nu \varphi \right], \]  

(27)

which implies that compared with Eq. (8), we have now obtained the conformally invariant scalar-tensor gravity with a negative Newton constant and a normal scalar field. However, this is also an illusion. Actually it is strange that via the Wick rotation over the scalar field we can obtain the conformally invariant scalar-tensor gravity with the opposite properties since the Wick rotation does not change the physical contents of the theory at all. The source of misunderstanding can be found in Eq. (7), which becomes after the Wick rotation and the shift of variables

\[ \hat{g}_{\mu\nu} = -\frac{1}{6M_P^2} \varphi^2 g_{\mu\nu}. \]  

(28)

This relation shows that the overall sign in the metric has been changed, which is not allowed in the conformal transformation (6). The proper procedure is first to make a transformation \( g_{\mu\nu} \to -g_{\mu\nu} \) and then perform the conformal transformation to reach (28). It is easy to see that the transformation \( g_{\mu\nu} \to -g_{\mu\nu} \) changes Eq. (27) to be the form

\[ L_{\text{CST}} = \sqrt{-g} \left[ \frac{1}{12} \varphi^2 R + \frac{1}{2} g^{\mu\nu} \partial_\mu \varphi \partial_\nu \varphi \right], \]  

(29)

which is nothing but the conformally invariant scalar-tensor gravity with a positive Newton constant and a scalar ghost as in (8). Incidentally, as seen shortly, with the symmetry breaking

\(^8\)Of course, it is possible to perform this shift of variables since the integrand in the path integral is in general complex with the analytic continuation. Since the present integral is of the Gaussian form with respect to \( \varphi \), a suggestive formula would be

\[ 1 = \sqrt{\frac{2}{\pi}} \int_{-\infty}^{+\infty} dx e^{-ax^2} = \sqrt{\frac{2}{\pi}} \int_{-\infty}^{+\infty} dx e^{-a(x^2-ib)^2} \]  

where \( a, b \) are some real numbers.
of conformal symmetry, $\phi = \langle \phi \rangle$, the first term in Eq. (29) yields the conventional Einstein-Hilbert term with the positive Newton constant, which is also bounded from below by the Wick rotation $s \to is$ in Eq. (24) in case of the Euclidean signature metric.

The most appealing point in this article is that as the result of the above Wick rotation and the shift of variables over $\phi$, the potential term beyond the SM, Eq. (4), can take the form

$$V(\phi, H) = \frac{\lambda_\phi}{4!} \phi^4 - \lambda_{H\phi} (H^\dagger H) \phi^2 + \frac{\lambda_H}{2} (H^\dagger H)^2,$$

where the sign in front of the second term on the RHS has flipped from positive to negative, which would provide us with a natural symmetry breaking mechanism for the electroweak gauge symmetry. It is of interest that the gravitational physics provides the important effect for the generalized Higgs potential of the BSM.

At this stage, it is worth reviewing our previous work [5] where it was shown that both the Planck and electroweak mass scales can be generated by starting with the present formulation of the BSM. We can envision the process of symmetry breaking as two independent steps. At the first step, around the Planck scale, conformal symmetry is explicitly broken via the Coleman-Weinberg mechanism, thereby generating the Planck scale and general relativity. Next, at the second step, around the electroweak scale, the electroweak gauge symmetry is spontaneously broken by the potential in Eq. (4) which is modified by the Coleman-Weinberg mechanism at the first step of the symmetry breaking. The key observation is that at the second step of the symmetry breaking, in order to trigger spontaneous symmetry breakdown of the electroweak symmetry, it was necessary to replace $\lambda_{H\phi}$ with $-\lambda_{H\phi}$ in Eq. (4) in an ad hoc manner, which is assumed in many of scale-invariant models as well [13, 14]. By contrast, in the formulation at hand, the ad hoc replacement is naturally derived in terms of the Wick rotation over the scalar field $\phi$ which is allowed since the scalar field is a non-dynamical field [16]. This fact is also understood from the fact that the scalar field $\phi$ in the conformally invariant scalar-tensor gravity is a gauge freedom associated with conformal symmetry. This situation is very similar to that of general relativity in the sense that the conformal factor is a gauge freedom associated with diffeomorphisms.

Now we are ready to present the second step of the symmetry breaking of the electroweak symmetry since the first step is the same as that in our previous work [5]. Taking account of the Coleman-Weinberg mechanism of conformal symmetry and the Wick rotation, the potential (4) is modified to be the following effective potential at the one-loop level:

$$V_{\text{eff}}(\phi, H) = \frac{5}{9216 \pi^2} \xi^4 \langle \phi \rangle^4 \left( \log \langle \phi \rangle^2 - \frac{1}{2} \right) - \lambda_{H\phi} (H^\dagger H) \langle \phi \rangle^2 + \frac{\lambda_H}{2} (H^\dagger H)^2.$$

Inserting the minimum $\phi = \langle \phi \rangle$ and completing the square, the effective potential reads

$$V_{\text{eff}}(\langle \phi \rangle, H) = \frac{\lambda_H}{2} \left( H^\dagger H - \frac{\lambda_{H\phi}}{\lambda_H} \langle \phi \rangle^2 \right)^2 - \frac{1}{2} \left( \frac{\lambda_{H\phi}^2}{\lambda_H^2} + \frac{5}{9216 \pi^2} \xi^4 \right) \langle \phi \rangle^4.$$
It is obvious that this effective potential has a minimum at $H^\dagger H = \frac{\lambda_{H\phi}}{\lambda_H} \langle \varphi \rangle^2$ due to $\lambda_H > 0$ and $\lambda_{H\phi} > 0$. Taking the unitary gauge $H^T = \frac{1}{\sqrt{2}} (0, v + h)$, this fact implies that the square of the vacuum expectation value of the Higgs field, $v^2$, and the square of Higgs mass, $m_h^2$, are respectively given by

$$v^2 = \frac{2\lambda_{H\phi}}{\lambda_H} \langle \varphi \rangle^2, \quad m_h^2 = \lambda_H v^2. \quad (33)$$

Using the relation $M_{Pl}^2 = \frac{1}{6} \langle \varphi \rangle^2$, which is obtained at the first step of the symmetry breaking [5], the magnitude of the coupling constant $\lambda_{H\phi}$ is given by

$$\lambda_{H\phi} = \frac{1}{12} \left( \frac{m_h}{M_{Pl}} \right)^2 \sim \mathcal{O}(10^{-33}). \quad (34)$$

This relation makes it clear that in order to account for the big hierarchy between the electroweak scale and the Planck scale one needs to take a very tiny value of the coupling constant $\lambda_{H\phi}$.

To summarize, through the Wick rotation over a scalar field existing in the gravitational sector, we have clarified why a potential term of the BSM, which corresponds to the Higgs mass term in the Higgs potential of the SM, possesses the negative coefficient. This phenomenon provides us with an example that the gravitational physics essentially defined around the Planck scale gives rise to useful information on the SM around the electroweak scale. It appears that the existence of a stable vacuum in quantum gravity is relevant to that in the SM.

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