Constraints on Cosmological Parameters from Existing CMB Data

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We use current cosmic microwave background data to constrain cosmological parameters. The results are presented as confidence regions in the $(\Omega - H_o)$ plane for both open and flat geometries, and are based on a $\chi^2$ minimization to the power spectrum. Although such an approach does not strictly apply to power spectrum estimates, the results should nonetheless be indicative. We find that purely open, low-$\Omega$ models are strongly disfavored, due to the position of the “Doppler Peak” at high $l$; flat models are less strongly constrained. The parameter space explored is the largest to date, covering $\Omega$, $H_o$, $n$, $Q$ and $\Omega_B h^2$.

1 Introduction

It is already possible, with existing data on the fluctuations of the cosmic microwave background (CMB), to constrain some cosmological parameters (Rocha & Hancock 1996; Bond & Jaffe 1996; Lineweaver et al. 1997). Of course, the actual data is far from providing the kind of constraints eagerly awaited from the next generation of experiments (Jungman et al. 1996; Bond et al. 1997), but there are nevertheless more than 10 different experiments detecting fluctuations over a large range of angular scales. To place our present work in context, consider the general problem of analyzing the results (say, a map) of a CMB experiment. If the true sky fluctuations and, as is fortunately often the case, the instrumental noise are gaussian, then the probability
of obtaining any particular set of $N$ pixel values is a multivariate gaussian in the pixel vector $\vec{d}$ (a one-dimensional listing of the possible pixel values):

$$L(\vec{p}) = P(\vec{d}/\vec{p}) = \frac{1}{(2\pi)^{N/2}|C|^{1/2}}e^{-\frac{1}{2}(\vec{d}-C^{-1}\cdot\vec{p})^T C^{-1}(\vec{d})/2}$$

The cosmological model is completely embodied (as well as the noise) in the covariance matrix of the pixels, $C \equiv C^T(\vec{p}) + C(noise)$, where we denote the underlying model parameters by the vector $\vec{p}$. Thus, we have a likelihood function for $\vec{p}$, given a particular $\vec{d}$, i.e., a map of the sky. It is important to note that even if the pixels are gaussian, the likelihood function $L(\vec{p})$ is NOT so, in general, because the parameters enter into the covariance matrix and not as linear combinations of the pixel values themselves. Maximization of this likelihood function is one way to estimate cosmological parameters.

Another approach is to work in Fourier space with estimates of the power spectrum, i.e., the $C_l$. These are nothing more than the variances of the individual spherical harmonic coefficients of the temperature fluctuations. We are really doing the same thing as before, just working with the covariances of Fourier coefficients instead of the measured pixel values. A practical reason for using this method is that band-power estimates and their uncertainties are readily available in the literature, this being the most common way of concisely reporting the results of an experiment.

In the present contribution, we assemble various band-power estimates (e.g., $C_l$ intergrated over experimental window functions) and constrain certain cosmological parameters by fitting to these power spectrum data points. We must mention several caveats applying to the results given here: Firstly, the data set is certainly not homogeneous, if for no other reason than for the different ways that the so-called cosmic variance has been estimated and included in the quoted error bars. Secondly, we have not yet fully accounted for the experimental window functions. A third important point is that the $C_l$ are NOT gaussian random variables, unlike the individual pixel values (or Fourier coefficients). This is clear – the $C_l$ represent the variance of gaussian random variables, and thus are themselves distributed according to a $\chi^2$ distribution. We have, for the work presented in this contribution, effectively assumed gaussianity in the sense that we apply a $\chi^2$ minimization for the fitting. In principle, this could lead to biased best-estimates and incorrect confidence intervals. We are currently working on improving these aspects of our calculations. The results presented here should therefore be taken as indicative, but perhaps relatively good indications, all the same, given the current status of affairs (Jaffe & Bond 1997).

| parameter | Open Models | Flat Models |
|-----------|-------------|-------------|
| $\Omega$  | 0 – 0.95    | 0 – 1       |
| $H_0$     | 15 – 100 km/s/Mpc | 15 – 100 km/s/Mpc |
| $n$       | 0.5 – 1.5   | 0.55 – 1.45 |
| $Q$       | 14 – 20 $\mu$K | 14 – 20 $\mu$K |
| $\Omega_B h^2$ | 0.015 | 0.006 – 0.030 |
2 The models

We consider two broad class of inflation–based models (e.g., gaussian fluctuations): Open and flat with a non–zero cosmological constant. For the open models, we explored the four–dimensional parameter space of $\Omega, H_o, n, Q$, where $n$ is the spectral index of an assumed power–law power spectrum and $Q$ is the normalization expressed as the CMB quadrupole; the baryon density was fixed at its nucleosynthesis predicted value of $\Omega_b h^2 = 0.015$. For the flat models, we varied the baryon density in addition to these four parameters, thereby exploring a five–dimensional space. This is all summarized in the Table, where we also provide the individual step sizes on each parameter and the range covered. The calculations were performed with CMBFAST (Seljak & Zaldarriaga 1996). To cover the parameter space indicated, the slower open model calculations required a couple of months of uninterrupted computing time.

3 Results

Our results in the $(\Omega, H_o)$–plane for both types of model are shown in Figure 1. The light solid contours are defined by $\chi^2_{\text{min}} + 2.3$ and $+6.17$, which would correspond to $1\sigma$ and $2\sigma$ limits in this plane for a gaussian likelihood function (of the parameters), which, as we have mentioned, is not really the case. Nevertheless, this gives some idea of the region favored by the data: in general, we find that it corresponds to high $\Omega$ and low values of $H_o$. The dashed lines show the contours whose projection onto the axes give the confidence range on each parameter individually. Finally, for comparison, the dark solid lines (outer contours) are contours of constant goodness–of–fit for, if all were normal, 68% and 95% probability.

The most interesting aspect of these results concerns the open models. We see from Figure 1 that the contours place a lower limit on $\Omega$; values as low as $\Omega = 0.2$ are strongly disfavored for any $H_o$. The reason for this is clear from the left–hand–side of Figure 2. The “Doppler Peak” is displaced too far to the right, due to the focusing of light rays in an open geometry (Hu & White 1996). Thus we find that a “comfortable” model with $\Omega = 0.2$ and $H_o = 60 \text{ km/s/Mpc}$ has very serious difficulty with the CMB data.

How about the cosmological constant? Here, the constraints on $\Omega$ are much less stringent, as can be seen from the contours shown in the right–hand panel of Figure 1. In contrast to the purely open model with $\Omega = 0.2$, the corresponding flat model, with $\Lambda = 0.8$, is acceptable and provides a good–looking fit to the power spectrum (right–hand–side of Figure 2).

4 Conclusions

It is a little early to draw detailed conclusions from the CMB concerning the cosmological parameters, but it is worth noting that some conclusions are already possible with existing data; and the next round of data releases will not be long in coming. Despite the various caveats of the method used here, it seems that one result should remain rather robust - that low–$\Omega$, purely open models are strongly disfavored, for the clear and simple reason that the “Doppler Peak” is just not in the correct place due to the large angular distance to the surface of last scattering in such models.
Figure 1: LEFT: Contours in the \((\Omega, H_o)\) – plane for the **Open** models. The light colored solid lines define confidence contours corresponding, if everything were gaussian, to 68\% \((\chi^2_{\text{min}} + 2\cdot3)\) and 95\% \((\chi^2_{\text{min}} + 6\cdot17)\). The dashed lines are confidence ellipses whose projection onto the axes provides the estimated range on each individual parameter. The darker solid lines are contours of constant goodness–of–fit at 68\% and 95\% probability, respectively, if the statistics were gaussian. RIGHT: Contours in the \((\Omega_v, H_o)\) – plane for the **Flat** models, where \(\Omega_v = 1 - \Omega\) is the vacuum density. The contours are defined as in the left–hand panel.

Figure 2: LEFT: Power spectrum of an **Open** model, with the parameters given at the top of the figure, compared to the data. The relation between \(\eta_{10}\) and the baryon density is: \(\Omega_B h^2 = 0.0036 \eta_{10}\), for a present–day CMB temperature of \(T = 2.726\) K. RIGHT: Power spectrum of the corresponding **Flat** model.
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