**S-TYPE AND P-TYPE HABITABILITY IN STELLAR BINARY SYSTEMS: A COMPREHENSIVE APPROACH. I. METHOD AND APPLICATIONS**

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**ABSTRACT**

A comprehensive approach is provided for the study of both S-type and P-type habitability in stellar binary systems, which in principle can also be expanded to systems of higher order. P-type orbits occur when the planet orbits both binary components, whereas in the case of S-type orbits, the planet orbits only one of the binary components with the second component considered a perturbator. The selected approach encapsulates a variety of different aspects, which include: (1) the consideration of a joint constraint, including orbital stability and a habitable region for a putative system planet through the stellar radiative energy fluxes (“radiative habitable zone”; RHZ), needs to be met; (2) the treatment of conservative, general, and extended zones of habitability for the various systems as defined for the solar system and beyond; (3) the provision of a combined formalism for the assessment of both S-type and P-type habitability; in particular, mathematical criteria are presented for the kind of system in which S-type and P-type habitability is realized; (4) applications of the attained theoretical approach to standard (theoretical) main-sequence stars. In principle, five different cases of habitability are identified, which are S-type and P-type habitability provided by the full extent of the RHZs; habitability, where the RHZs are truncated by the additional constraint of planetary orbital stability (referred to as ST- and PT-type, respectively); and cases of no habitability at all. Regarding the treatment of planetary orbital stability, we utilize the formulae of Holman & Wiegert as also used in previous studies. In this work, we focus on binary systems in circular orbits. Future applications will also consider binary systems in elliptical orbits and provide thorough comparisons to other methods and results given in the literature.

**Key words:** astrobiology – binaries: general – celestial mechanics – planetary systems

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1. **INTRODUCTION**

Starting more than a decade ago, considerable observational evidence indicating that planets are able to exist in stellar binary (and higher order) systems has been obtained; see the results and discussion by, e.g., Patience et al. (2002) and Eggenberger et al. (2004, 2007). These observations are in line with the empirical finding that binary (and higher order) systems occur at a high frequency in the local Galactic neighborhood (Duquennoy & Mayor 1991; Lada 2006; Raghavan et al. 2006; Bonavita & Desidera 2007; Raghavan et al. 2010). For example, Raghavan et al. (2010) presented results of a detailed analysis of companions to solar-type stars, based on a sample size of 454, and concluded that the overall fractions of double and triple systems are about 33% and 8%, respectively, if all confirmed stellar and brown dwarf companions are accounted for. Meanwhile, updated results were provided by Roell et al. (2012). This study showed that 57 exoplanet host stars are identified as having a stellar companion.

The fairly frequent occurrence of planets in binary systems is furthermore consistent with the presence of debris disks in a considerably large number of main-sequence star binary systems (e.g., Trilling et al. 2007). In principle, as discussed by Perryman (2011), planets in binary systems can be identified by two different means: first, binaries or multiple star systems can be surveyed for the presence of planets by utilizing established detection methods. Second, stars with detected planets can be scrutinized afterward to check if they possess one or more widely separated stellar companion(s); in this case, the planet(s) will also be categorized as belonging to a binary (or higher order) system.

From the viewpoint of orbital mechanics, there are two different kinds of possible orbits (notwithstanding positions near the Lagrangian points L4 and L5) for planets in binary systems: S-type and P-type orbits (Dvorak 1982). A P-type orbit is attributed when the planet orbits both binary components, whereas in the case of an S-type orbit, the planet orbits only one of the binary components with the second component behaving as a perturbator. Eggenberger et al. (2004) presented a list of 15 planet-bearing binary systems with all planets in S-type orbits. They constitute mostly wide binaries with separation distances of up to ~6400 AU; however, smaller separation distances on the order of 20 AU or less have also been identified. Systems with planets in P-type orbits have also been identified. Arguably the most prominent case is Kepler-16, as reported by Doyle et al. (2011) and previously suggested by Slawson et al. (2011), containing a Saturnian mass circumbinary planet. Quarles et al. (2012) have subsequently studied this system with regard to the possibility of habitable exoplanets and habitable exomoons. Recently, a transiting circumbinary multi-planet system, i.e., Kepler-47, has also been identified (Orosz et al. 2012).
There is a significant body of literature devoted to the study of habitability in binary systems as well as in multi-planetary systems, which often also encompass stellar evolutionary considerations. Examples include the works by Jones et al. (2001), Noble et al. (2002), Menou & Tabachnik (2003), Asghari et al. (2004), Sandor et al. (2007), Takeda et al. (2008), Dvorak et al. (2010), Jones & Sleep (2010), and Kopparapu & Barnes (2010). An important aspect that has received increased recognition in the literature is that in order for habitability to exist and to be maintained, a joint constraint that includes both orbital stability and a habitable environment for a system planet through the stellar radiative energy fluxes needs to be met. In the framework of this paper, the zone related to this latter requirement will subsequently be referred to as the radiative habitable zone (RHZ), which constitutes a necessary, though often insufficient, condition for the existence of circumstellar habitability.

Previous works, mostly concentrating on the existence of habitability in single star multi-planetary systems, entailed the publication of detailed “stability catalogs” for the habitability zones of extrasolar planetary systems (e.g., Menou & Tabachnik 2003; Sandor et al. 2007; Takeda et al. 2008; Dvorak et al. 2010; Kopparapu & Barnes 2010). For example, Menou & Tabachnik (2003) quantified the dynamical habitability of 85 planetary systems by considering the perturbing influence of giant planets beyond the traditional Hill sphere for close encounters with theoretical terrestrial planets. They concluded that a significant fraction of the identified extrasolar planetary systems are unable to harbor habitable terrestrial planets. A statistical study on the stability of Earth-mass planets orbiting solar-mass stars in the presence of stellar companions, focusing on both the statistical properties of ejection times and the general prospects of planetary habitability, was given by Fatuzzo et al. (2006).

Additional work providing stability assessments for various observed extrasolar planetary systems based on detailed stability maps was provided by Sandor et al. (2007), Takeda et al. (2008) explored the orbital stability of planets in double-planet systems for binaries by supplying an analytic framework based on secular perturbation theory; they also provided dynamical classification categories. Additional stability analyses to assess the habitability of planetary systems based on detailed numerical simulations were provided by Dvorak et al. (2010) and Kopparapu & Barnes (2010); note that the study of Dvorak et al. (2010) also dealt with limited cases of planets in double-star systems in orbit either around one stellar component (S-type) or around both components (P-type).

The study of planetary dynamics and habitable planet formation has meanwhile been described by, e.g., Quintana & Lissauer (2010) and Haghighipour et al. (2010). They showed that Earth-like mass planets are, in principle, able to form in stellar binary systems, although many details of the relevant processes are not fully understood. The overarching conclusion of these investigations is that habitable planets in stellar binary systems (and, as anticipated, in higher-order systems) are, in general, possible, which is a clear motivation for providing a comprehensive study of S-type and P-type habitability in binary systems. The approach adopted in this study will be entirely analytic. Specifically, it will consider both S-type and P-type habitable orbits in view of the joint constraint including orbital stability and a habitable region for a system planet through the appropriate amount of stellar radiative energy fluxes. In an earlier study, Eggl et al. (2012) focused on S-type habitability in binary systems, taking into account both circular and elliptic orbits for the stellar binary components; this latter aspect is, however, beyond the scope of this work as we focus solely on systems in circular orbits. Numerical studies for P-type habitable environments with applications to Kepler-16, Kepler-34, Kepler-35, and Kepler-47 have been provided by Kane & Hinkel (2013).

Our paper is structured as follows. In Section 2, we comment on the adopted main-sequence star parameters and single star habitability. In Section 3, we introduce our theoretical approach, which is suitable for stellar systems of the order of N, although our focus will be on binary systems. In this regard, both S-type and P-type orbits will be examined, and detailed mathematical criteria for the existence of S-type and P-type RHZs will be derived. In Section 4, we consider the additional constraint of planetary orbital stability for establishing circumstellar habitability. Applications regarding S-type and P-type systems are given in Section 5, and the habitability classifications S, P, ST, and PT are introduced in Section 6. Section 7 provides our summary and conclusions.

2. STELLAR PARAMETERS AND SINGLE STAR HABITABILITY

In this study, S-type and P-type habitability is investigated, mostly pertaining to standard (i.e., theoretical) main-sequence stars. The adopted stellar parameters, which are the stellar effective temperatures $T_{\text{eff}}$, stellar radii $R_*$ (which together allow the definition of the stellar luminosities $L_*$), and the stellar masses $M_*$, are mainly based on the work by Gray (2005); see his Table B.1), which assumes detailed photospheric spectral analyses. For stellar spectral types with no available data, the missing data were computed by employing biaxial interpolation.

The exception, however, are data for stars of spectral type K5 V and below. Here, we relied on the results from the spectral models of R. L. Kurucz and collaborators. They took into account hundreds of millions of spectral lines for a large set of atoms and molecules; see Castelli & Kurucz (2004) and Kurucz (2005) for details. The effective temperatures implied by these models are in close similarity to those given by Gray (2005) for most types of stars; however, Gray (2005) reports consistently higher effective temperatures for stars of spectral types late-K and -M; for the latter, the difference amounts to nearly 300 K. Table 1 depicts the stellar parameters adopted in the work presented here.

An alternative approach, which is expected to provide very similar results for either the stellar luminosity or the stellar mass (with the other parameter taken as fixed), is the employment of a mass–luminosity relationship applicable to main-sequence stars. The work by Reid (1987), as well as data from subsequent
CHZ is defined by the first CO$_2$ condensation attained by the onset of the formation of CO$_2$ clouds at a temperature of 273 K; see, e.g., Underwood et al. (2003) and Selsis et al. (2007) for further details. Table 3 shows the results for the HZs for the different types of main-sequence stars of the this study, with the different limits referred to as HZ($s_i$).

For the outer edge of circumstellar habitability, even less stringent limits have been introduced (e.g., Forget & Pierrehumbert 1997; Mischna et al. 2000). They are based on the assumption of relatively thick planetary CO$_2$ atmospheres as well as strong backwarming that may further be enhanced by CO$_2$ crystals and clouds. These limits, which in the case of the Sun correspond to 2.4 AU ($s_1 = s_6$), conform to the extended habitable zone (EHZ), and have also been taken into account in our study, although the significance of the EHZ has been criticized as a result of detailed planetary radiative transfer models (Haley et al. 2009). Moreover, in the framework of the this study, we also consider planetary Earth-equivalent positions defined as $R_{\oplus, \text{equiv}} \simeq \sqrt{L_\ast/M_\odot}$ and labeled as $s_1 = s_6$; see Table 3. It is intended as an intriguing reference distance of habitability, both regarding single stars and stellar binary systems.

### Table 1

| Sp. Type | $T_{\text{eff}}$ | $R_\ast$ | $L_\ast$ | $s_{\text{rel,} \ast}$ | $M_\ast$ |
|---------|----------------|---------|---------|-----------------|----------|
| F0      | 7178           | 1.62    | 6.255   | 1.145           | 1.60     |
| F2      | 6909           | 1.48    | 4.481   | 1.113           | 1.52     |
| F5      | 6528           | 1.40    | 3.196   | 1.072           | 1.40     |
| F8      | 6160           | 1.20    | 1.862   | 1.037           | 1.19     |
| G0      | 5943           | 1.12    | 1.405   | 1.019           | 1.05     |
| G2      | 5811           | 1.08    | 1.194   | 1.009           | 0.99     |
| G5      | 5657           | 0.95    | 0.830   | 0.997           | 0.91     |
| G8      | 5486           | 0.91    | 0.673   | 0.985           | 0.84     |
| K0      | 5282           | 0.83    | 0.481   | 0.971           | 0.79     |
| K2      | 5055           | 0.75    | 0.330   | 0.957           | 0.74     |
| K5      | 4487           | 0.64    | 0.149   | 0.926           | 0.67     |
| K8      | 4006           | 0.53    | 0.066   | 0.905           | 0.58     |
| M0      | 3850           | 0.48    | 0.045   | 0.900           | 0.51     |

Notes. $S_{\text{rel,} \ast}$ is calculated for $s_1 = s_3$.

### Table 2

| $\ell$ | $s_1$ | HZ Limit | Description                       |
|-------|-------|----------|-----------------------------------|
| 1     | 0.84  | GHZ/CHZ  | Runaway greenhouse effect         |
| 2     | 0.95  | CHZ      | Start of water loss               |
| 3     | 1.00  |         | Earth-equivalent position         |
| 4     | 1.37  | CHZ      | First CO$_2$ condensation         |
| 5     | 1.67  | GHZ      | Maximum greenhouse effect, no clouds |
| 6     | 2.40  | EHZ      | Maximum greenhouse effect, 100% clouds |

Notes. See text for references.

### Table 3

| Sp. Type | HZ($s_1$) | HZ($s_2$) | HZ($s_3$) | HZ($s_4$) | HZ($s_5$) | HZ($s_6$) |
|---------|-----------|-----------|-----------|-----------|-----------|-----------|
| F0      | 1.98      | 2.25      | 2.33      | 2.91      | 3.66      | 5.49      |
| F2      | 1.70      | 1.93      | 2.00      | 2.54      | 3.18      | 4.72      |
| F5      | 1.46      | 1.65      | 1.72      | 2.24      | 2.78      | 4.08      |
| F8      | 1.13      | 1.28      | 1.34      | 1.78      | 2.19      | 3.19      |
| G0      | 0.99      | 1.12      | 1.17      | 1.58      | 1.94      | 2.80      |
| G2      | 0.91      | 1.03      | 1.09      | 1.48      | 1.81      | 2.61      |
| G5      | 0.77      | 0.87      | 0.91      | 1.25      | 1.53      | 2.19      |
| G8      | 0.69      | 0.78      | 0.83      | 1.15      | 1.39      | 1.99      |
| K0      | 0.59      | 0.67      | 0.71      | 0.99      | 1.20      | 1.71      |
| K2      | 0.49      | 0.55      | 0.59      | 0.84      | 1.01      | 1.43      |
| K5      | 0.34      | 0.38      | 0.40      | 0.59      | 0.71      | 0.99      |
| K8      | 0.22      | 0.25      | 0.27      | 0.41      | 0.49      | 0.67      |
| M0      | 0.19      | 0.21      | 0.23      | 0.35      | 0.41      | 0.56      |

3. THEORETICAL APPROACH

#### 3.1. Basic Equations

Next, we introduce the governing equations for investigating the RHZs of binary systems pertaining to both $S$-type and $P$-type orbits. This approach targets the requirement of providing a habitable region for a system planet based on the radiative energy fluxes of the stellar components. The requirement of planetary orbital stability will be disregarded for now; it will be revisited in Section 4. The importance of orbital stability for allowing circumstellar habitability in stellar binaries will, however, be considered in an appropriate and consistent manner in the main body of the study.

For a star of luminosity $L_\ast$, given in units of solar luminosity $L_\odot$, the distance $d_\ell$ of the habitability limit $s_\ell$ as identified for the Sun, which may constitute either an inner or outer limit of habitability (except $\ell = 3$), is given as

$$d_\ell = s_\ell \sqrt{\frac{L_\ell}{S_{\text{rel,} \ell} L_\odot}}. \quad (2)$$

The physical significance of the various kinds of HZs obtained by Kasting et al. (1993) can be summarized as follows: the GHZ is defined as bordered by the runaway greenhouse effect (inner limit) and the maximum greenhouse effect (outer limit). Concerning the latter, it is assumed that a cloud-free CO$_2$ atmosphere is still able to provide a surface temperature of 273 K. The inner limit of the CHZ is defined by the onset of water loss. In this case, a wet stratosphere is assumed to exist where water is lost by photodissociation and subsequent hydrogen escape to space. Furthermore, the outer limit of the...
In the case of a multiple star system of the order of N with distances \( d_i \), the limit of habitability related to \( s_\ell \) is given as

\[
\sum_{i=1}^{N} \frac{L_i}{S_{\text{rel},i}d_i^2} = \frac{L_\odot}{s_\ell^2},
\]

(3)

In Equations (2) and (3), \( S_{\text{rel},i} = S_{\text{rel},i}(T_{\text{eff}}) \) (see Table 1) describes the stellar flux in units of the solar constant, which is a function of the stellar effective temperature \( T_{\text{eff}} \) (e.g., Kasting et al. 1993; Underwood et al. 2003). Specifically, using the formalism by Selsis et al. (2007), we find that

\[
S_{\text{rel},i} = \left( \frac{s_\ell}{s_\ell - a_i T_e - b_i T_e^2} \right)^2
\]

(4)

with \( s_\ell, a_i, \) and \( b_i \) in AU, and \( T_e = T_{\text{eff}} - 5700 \) in K. Selsis et al. (2007) also found that for \( s_\ell < 1 \), corresponding to inner limits of habitability, the fitting parameters are given as \( a_i = 2.7619 \times 10^{-5} \) and \( b_i = 3.8095 \times 10^{-9} \), whereas for \( s_\ell > 1 \), corresponding to the outer limits of habitability, they are given as \( a_i = 1.3786 \times 10^{-4} \) and \( b_i = 1.4286 \times 10^{-9} \); note that \( s_\ell \equiv 1 \) corresponds to the customary notion of Earth-equivalent positions. Appropriate values for \( s_\ell \) are given in Table 2.

In the following, we will focus on the case of binary systems, i.e., \( N = 2 \). In this case, Equation (3) reads

\[
\frac{L_1}{S_{\text{rel},1}d_1^2} + \frac{L_2}{S_{\text{rel},2}d_2^2} = \frac{L_\odot}{s_\ell^2}
\]

(5)

with

\[
d_1^2 = a^2 + z^2 + 2az \cos \varphi
\]

(6a)

\[
d_2^2 = a^2 + z^2 - 2az \cos \varphi.
\]

(6b)

Here \( a \) denotes the semidistance of binary separation, \( z \) the distance of a position at the habitability limit contour (which later on will be referred to as the “radiative habitable limit” (RHL), see below), and \( \varphi \) the associated angle; see Figure 1 for information on the coordinate setup for both \( S \)-type and \( P \)-type orbits. We will also assume \( L_1 \geq L_2 \) without loss of generality.

With \( L'_{\ell,i} \) defined as

\[
L'_{\ell,i} = \frac{L_i}{L_\odot S_{\text{rel},i}},
\]

(7)

henceforth referred to as recast stellar luminosity (see Table 4),

\footnote{This equation is analogous to an equivalent equation of electostatics relating a general distribution of charges to the resulting electrostatic potential in free space (Jackson 1999, see p. 40, Equation (1.48)); a modified version of Equation (3) has previously been considered by, e.g., Eggl et al. (2012).}

\footnote{\( S_{\text{rel},i} \) represents the normalized stellar flux in units of the solar constant, 1368 W m\(^{-2}\), given by the stellar spectral energy distribution. Therefore, ordinarily, no \( s_\ell \) dependence for \( S_{\text{rel},i} \) should exist. However, the formulae by Selsis et al. (2007) utilize previous results by Kasting et al. (1993) who provided numerical values for limits of habitability for different types of stars considering various limit definitions (i.e., \( s_\ell \) values identified for the Sun). However, Kasting et al. (1993) used an unusually low value of 5700 K for the solar effective temperature instead of 5777 K as currently accepted (e.g., Stix 2004). Hence, transforming the polynomial fit based on the work by Kasting et al., aimed at considering the correct solar effective temperature, renders a weak dependence on \( s_\ell \) for the \( S_{\text{rel},i} \) values. In contrast, the method by Underwood et al. (2003) provides a polynomial fit for \( S_{\text{rel},i} \) without considering the solar temperature revision. An alternative method has been used by Cuntz & Yeager (2009) and subsequent works. In this approach, the polynomial fit by Underwood et al. (2003) is corrected via a triangular function based on data for stars of spectral type F0 V, G0 V, and K0 V. As a result, the corresponding \( S_{\text{rel},i} \) values also do not depend on \( s_\ell \).}



\[
z(\varphi) \text{ is given as}
\]

\[
z^4 + A_2 z^2 + A_1 z + A_0 = 0
\]

(8)

with

\[
A_2 = 2a^2 (1 - 2 \cos^2 \varphi) - \frac{s_\ell^2}{2} (L'_{1,\ell} + L'_{2,\ell})
\]

(9a)

\[
A_1 = 2a s_\ell^2 \cos \varphi (L'_{1,\ell} - L'_{2,\ell})
\]

(9b)

\[
A_0 = a^4 - a^2 s_\ell^2 (L'_{1,\ell} + L'_{2,\ell}).
\]

(9c)

Equation (8) constitutes a fourth-order algebraic equation that is known to possess four possible solutions (Bronshtein & Semendyayev 1997), although some (or all) of them may constitute unphysical solutions, i.e., \( z(\varphi) \) having a complex or imaginary value. The adopted coordinate system constitutes, in essence, a polar coordinate system except that negative values for \( z \) are permitted; in this case, the position of \( z \) is found on the opposite side of the angle \( \varphi \).

In principle, it is possible to consider that Equation (8) only has solutions for \( z(\varphi) \) given as \( z \geq 0 \); in this case, the entire interval for \( z(\varphi) \), which is \( 0 \leq \varphi < 2\pi \), needs to be examined. The following types of solutions are identified. For \( S \)-type orbits, two solutions exist in the intervals centered at \( \varphi = 0 \) and at \( \varphi = \pi \) (or one coinciding solution at each tangential point); see Figure 1. However, there will be no solution in the typically
relatively large intervals containing \( \varphi = \pi /2 \) and \( \varphi = 3\pi /2 \). Clearly, the size of any of those intervals critically depends on the system parameters \( a, s, L_{1\ell}, \) and \( L_{2\ell}, \) as expected. For \( P\)-type orbits, on the other hand, there will be one solution for each value of \( \varphi \) in the range of \( 0 \leq \varphi < 2\pi \).

However, in general, negative values for the solutions of \( z(\varphi) \) also exist. If taken into account, it will be sufficient to restrict the evaluation of Equation (8) to the range \( 0 \leq \varphi \leq \pi \). In this case, for \( S\)-type orbits, there will be four solutions in the intervals with the endpoints \( \varphi = 0 \) and \( \varphi = \pi \), as well as two solutions (if \( L_{1\ell} \neq L_{2\ell} \)) in a more extended interval containing these points. Also, a pair of solutions will become one coinciding solution at each tangential point. However, again, there will be no solution in the interval containing \( \varphi = \pi /2 \). In the case of \( P\)-type orbits, there will be two solutions for any value of \( \varphi \) in the range \( 0 \leq \varphi \leq \pi \). We will revisit this assessment in conjunction with the algebraic method for attaining the solution \( z_1 \); additionally, detailed mathematical criteria will be given for the existence of \( \text{RHZ} \) for \( S\)-type and \( P\)-type orbits.

Next, we will focus on equal-star binary systems. Detailed solutions for general binary systems (i.e., systems of stellar components with by default unequal masses, luminosities, and effective temperatures) pertaining to both \( S\)-type and \( P\)-type orbits will be given in Section 3.3. Both subsections will be aimed at deriving \( \text{RHZs} \); see, e.g., Williams & Pollard (2002) for general discussion on the role of \( \text{RHZs} \) for the attainment of habitability in star–planet systems. However, strictly speaking, they will deal with identifying \( \text{RHLs} \) connected to a distinct value of \( s_\ell \), noting that manifesting a \( \text{RHZ} \) requires that the \( \text{RHL} \) for \( s_{\ell,\text{in}} \) is located completely outside the \( \text{RHL} \) for \( s_{\ell,\text{in}} \) with \( s_{\ell,\text{out}} \) appropriately paired. A summary regarding the existence and structure of the \( \text{RHZs} \), encompassing the radiative \( \text{CHZs} \), \( \text{GHZs} \), and \( \text{EHZs} \), will be given in Section 3.4; this subsection will also describe cases where no \( \text{RHZs} \) exist, due to the behavior of the \( \text{RHLs} \) owing to the choices of \( s_{\ell,\text{in}} \) and \( s_{\ell,\text{out}} \).

### 3.2. Equal-star Binary Systems

Now we focus on the special case of equal-star binary systems, i.e., stars of identical recast luminosities, i.e., \( L_{1\ell} = L_{2\ell} = L'_{\ell} \). For theoretical main-sequence stars, this assumption also implies \( S_{\ell,\text{rel}} = S_{\ell,\text{rel}} \text{ and } M_1 = M_2 \); this latter assumption regarding the stellar masses is relevant for the orbital stability constraint of system planets. With \( A_1 = 0 \), Equation (8) now constitutes a biquadratic equation that can be solved in a straightforward manner. The other coefficients are given as

\[
A_2 = 2a^2(1 - 2\cos^2 \varphi) - 2s_\ell^2 L'_{\ell} \quad (10a)
\]

\[
A_0 = a^4 - 2a^2 s_\ell^2 L'_{\ell} \quad (10b)
\]

Thus, the solution of Equation (8) is given as

\[
z = \pm \sqrt{-a^2(1 - 2\cos^2 \varphi) + s_\ell^2 L'_{\ell}^2} \pm \sqrt{D_2} \quad (11)
\]

with

\[
D_2 = s_\ell^4 L'_{\ell}^2 + 4a^4 \cos^4 \varphi - 4a^2 \cos^2 \varphi(a^2 - s_\ell^2 L'_{\ell}) \quad (12)
\]

With known system parameters, which are \( a, s_\ell, \) and \( L'_{\ell}, \) the function \( z(\varphi) \), describing the habitability limits for the binary system associated with inner limit and outer limit values \( s_\ell \) derived for the Sun (see Section 2 for details), can be obtained in a straightforward manner.

Owing to system symmetry, the existence of \( S\)-type and \( P\)-type \( \text{RHLs} \) can be identified by obtaining the solutions of Equation (11) for \( \varphi = 0 \) and \( \varphi = \pi /2 \). First we examine the solutions of Equation (11) for \( \varphi = 0 \), i.e., \( \cos \varphi = 1 \), which are given as

\[
z = \pm \sqrt{a^2 + s_\ell^2 L'_{\ell}^2 \pm s_\ell^2 L'_{\ell}^2 + 4a^2 L'_{\ell}} \quad (13)
\]

This allows us to explore the existence of \( S\)-type \( \text{RHLs} \). The total number of solutions for \( z_1 \) (if it exists) is four as expected, which can be ordered as \( z_1 < z_2 < z_3 < z_4 \). Due to symmetry it is found that \( z_3 \geq 0 \), which implies that

\[
a^2 + s_\ell^2 L'_{\ell}^2 - s_\ell^2 L'_{\ell}^2 \geq 0 \quad (14)
\]

Thus, the condition for the existence of \( S\)-type \( \text{RHLs} \) is given as

\[
a \geq s_\ell \sqrt{2L'_{\ell}} \quad (15)
\]

Next we examine the solutions of Equation (11) for \( \varphi = \pi /2 \), i.e., \( \cos \varphi = 0 \). This allows us to explore the existence of \( P\)-type \( \text{RHLs} \); the latter implies two solutions for Equation (11) regardless of the value for \( \varphi \). If the positive root of \( D_2 \) (see Equation (12)) is considered, the solution is given as

\[
z = \pm \sqrt{-a^2 + 2s_\ell^2 L'_{\ell}} \quad (16)
\]

Thus, the condition for the existence of \( P\)-type \( \text{RHLs} \) is given as

\[
a \leq s_\ell \sqrt{2L'_{\ell}} \quad (17)
\]

Therefore, Equations (15) and (17) allow the identification of the conditions for \( S\)-type and \( P\)-type \( \text{RHLs} \), respectively, for equal-star binary systems, which depend on the system parameters \( a, s_\ell, \) and \( L'_{\ell} \); note that the equals signs in these equations carry little relevance. Comparing Equations (15) and (17) also implies that the joint existence of \( S\)-type and \( P\)-type habitability in equal-star binary systems in circular orbits is not possible, irrespective of the system parameters and the planetary orbital stability requirement (see Section 4), noting that the latter imposes an additional constraint on habitability, even when the \( \text{RHZ-related conditions are met. Figure 2 depicts the borders of the } S\text{-type and } P\text{-type } \text{RHLs, for different values of } s_\ell \text{ with regard to } a \text{ and } L'_{\ell}.\)
there is only one appropriate choice for \( y \) solution is available (see Section 3.2). The solutions that for equal-star binaries, a more straightforward method of case of non-equal-star binaries assumed in the following. Note
\[ P = \frac{1}{2} \hat{a}_1 - \frac{1}{9} \hat{a}_2^2 \]
and with \( Q \) and \( R \) given as
\[ Q = \frac{1}{3} \hat{a}_1 - \frac{1}{9} \hat{a}_2^2 \]
while noting that
\[ \hat{a}_0 = 4A_0 A_2 - A_1^2 \]
\[ \hat{a}_1 = -4A_0 \]
\[ \hat{a}_2 = -A_2. \]

These sets of equations can be solved and appropriate values for \( z(\varphi) \) can be obtained. The results will depend on the system parameters \( a, s_\ell, L_{1\ell}, \) and \( L_{2\ell}, \) as expected.

Next, we describe the solutions for \( S \)-type and \( P \)-type RHLs in more detail. It is important to recognize that a priori choices about the existence of \( S \)-type and \( P \)-type RHLs are neither necessary nor possible, as the existence of any of those RHLs is determined by the fulfillment of well-defined mathematical conditions; they will also be given in the following.

### 3.3.2. \( S \)-type Orbits

An analysis of the possible solutions for Equations (18a) to (18d) shows that for \( S \)-type RHLs valid solutions are obtained based on
\[ S = \sqrt{R^2 + K^2} \left( \cos \xi + i \sin \xi \right) \]
\[ T = \sqrt{R^2 + K^2} \left( \cos \xi - i \sin \xi \right) \]
with
\[ K = \sqrt{|D_3|} \]
\[ \xi = \frac{1}{3} \arctan \left( \frac{K}{R} \right) \]

with \( R \) given by Equation (23b). Therefore, the solution of the resolvent cubic equation, Equation (20), is given as
\[ y_1 = -\frac{1}{3} \hat{a}_2 + 2\sqrt{R^2 + K^2} \cos \xi. \]

For the values of \( z_i \), it is found that for the interval centered at \( \varphi = 0 \), \( z_1 \) and \( z_2 \) exhibit negative values, whereas \( z_3 \) and \( z_4 \) exhibit positive values with \( z_1 < z_2 < z_3 < z_4 \). Thus, \( z_1 \) and \( z_2 \) describe the RHL regarding star S1, whereas \( z_3 \) and \( z_4 \) describe the RHL regarding star S2 (see Figure 1). Conversely, for the interval centered at \( \varphi = \pi \), \( z_1 \) and \( z_2 \) again exhibit negative values and \( z_3 \) and \( z_4 \) exhibit positive values. In this case, \( z_1 \) and \( z_2 \) describe the RHL for star S2, whereas \( z_3 \) and \( z_4 \) describe the RHL for star S1. No solutions are obtained in the vicinity of \( \varphi = \pi/2 \) and \( \varphi = 3\pi/2 \), as expected. Thus, for each angle \( \varphi \) in the range \( 0 \leq \varphi < 2\pi \), the appropriate number of solutions is obtained to describe \( S \)-type RHLs. However, due to symmetry, solutions are only needed for \( 0 \leq \varphi \leq \pi \).

The existence of \( S \)-type RHLs requires that \( z_2 \leq z_3 \) because otherwise the two distinct \( S \)-type RHL contours about the two binary components would not be separated, which corresponds to the condition
\[ C \geq \frac{1}{2} \left( D + \varepsilon \right). \]
Equation (28) can be rewritten to provide an expression based on the system parameters \( A_0, A_1, \) and \( A_2 \) defined through Equations (9a)–(9c). It is found that

\[
2y_1 (A_2 - y_1)^2 - A_1^2 \geq 0 \tag{29}
\]

with \( y_1 = y_1 (A_0, A_1, A_2) \). The expression for \( y_1 \) is highly complicated; however, it can be obtained based on Equations (21)–(24c) by using, e.g., MATHEMATICA® in a straightforward manner.

In conclusion, for \( S \)-type RHLs to exist for the system parameters \( a, s, L_1, \) and \( L_2 \), it is necessary that the relations (28) and (29), which are equivalent, must be fulfilled for any angle of \( \phi \), though the evaluation can be limited to \( \phi = 0 \). Furthermore, through analytical transformations it can be shown that the condition depicted as Equations (28) and (29) requires

\[
6912A_0^3 - 3456A_0^2 A_2^2 + 432A_0 A_2^4 - 729A_1^4 \leq 0. \tag{30}
\]

In the limiting case of equal-star binary systems, attained as \( A_1 \rightarrow 0 \), Equation (30) can be simplified as

\[
(4A_0 - A_2^2)^2 \geq 0; \tag{31}
\]

this relationship is fulfilled in a trivial manner.

### 3.3.3. \( P \)-type Orbits

An analysis of the possible solutions for Equations (18a)–(18d) also shows that for \( P \)-type RHLs valid solutions require

\[
D_3 \geq 0; \tag{32}
\]

see Equation (22c). The detailed evaluation of this condition requires the evaluation of various sets of equations denoted as Equations (9a)–(9c), (22a)–(22c), and (23a)–(23b); see Sections 3.1 and 3.3.1.

In terms of the solutions for \( P \)-type RHLs, it is found that for \( 0 \leq \varphi < \pi/2 \) and \( 3\pi/2 < \varphi < \pi \), \( z_1 \) exhibit negative values and \( z_2 \) exhibit positive values, whereas \( z_3 \) and \( z_4 \) are undefined; they are also not needed for outlining \( P \)-type RHLs. Moreover, for the range \( \pi/2 < \varphi < 3\pi/2 \), \( z_3 \) exhibit negative values and \( z_4 \) exhibit positive values, noting that \( z_1 \) and \( z_2 \) remain undefined. For \( \pi/2 \) and \( 3\pi/2 \), removable singularities are identified, which can easily be fixed through interpolation, taking values of \( z \) for neighboring angles of \( \varphi \). In summary, for each angle \( \varphi \) in the range \( 0 \leq \varphi < 2\pi \), two values of \( z_i \) (i.e., one positive and one negative value) are identified allowing the determination of \( P \)-type RHLs. However, due to symmetry, solutions are only needed for \( 0 \leq \varphi \leq \pi \).

Moreover, it can be shown through analytical transformations that the condition depicted as Equation (32) can be rewritten as

\[
16A_0 A_1^2 A_2 - 128A_0^2 A_2^2 + 256A_0^3 - 4A_1 A_2^2 - 27A_1^4 \leq 0. \tag{33}
\]

with \( A_0, A_1, \) and \( A_2 \) defined through Equations (9a)–(9c) with the left hand side of Equation (33) representing \(-10A \cdot D_3\). In conclusion, for \( P \)-type RHLs to exist for the system parameters \( a, s, L_1, \) and \( L_2 \), it is necessary that the relations (32) and (33), which are equivalent, must be fulfilled for any angle of \( \varphi \), though the evaluation can be limited to \( \varphi = \pi/2 \). In the limiting case of equal-star binary systems, attained as \( A_1 \rightarrow 0 \), Equation (33) can be simplified as

\[
(4A_0 - A_2^2)^2 \geq 0; \tag{34}
\]

this relationship, already given as Equation (31), is fulfilled in a trivial manner.

### 3.4. Calculation of RHZs for Binary Systems

The identification of the RHZs in binary systems requires the calculation of the limits of HZs, i.e., RHLs, as pointed out in Section 3.1. The RHZs need to be established for values of \( s_i \) with \( i = 1, 2, 4, 5, \) and 6 (see Section 2 and Table 2), which are informed by the model-dependent physical limits of habitability for the solar environment (e.g., Kasting et al. 1993). As part of the process, the parameters of \( s_i \) need to be appropriately paired in terms of the inner and outer limits of habitability. For the CHZ, the parameters \( (s_{i,\text{in}}, s_{i,\text{out}}) \) need to be paired as \((s_2, s_4)\), whereas for the GHZ, they need to paired as \((s_1, s_3)\). For the EHZ, the parameters \( s_i \) need to paired as \((s_1, s_0)\), considering that both the CHZ and the GHZ are to be viewed as subdomains of the EHZ.

For \( S \)-type and \( P \)-type orbits, the representation of the RHZ(\( z \)), which constitutes a circular region (annulus) around each star S1 and S2 (\( S \)-type) or both stars (\( P \)-type), can be determined as

\[
\text{RHZ}(z) = \min(\mathcal{R}(z, \alpha)|_{s_{i,\text{out}}} - \max(\mathcal{R}(z, \alpha)|_{s_{i,\text{in}}})\]

and

\[
\text{RHZ}(z) = \min(\mathcal{R}(z, \varphi)|_{s_{i,\text{out}}} - \max(\mathcal{R}(z, \varphi)|_{s_{i,\text{in}}})\]

respectively; see Figure 1 for coordinate information. Here \( \mathcal{R}(z, \alpha) \) and \( \mathcal{R}(z, \varphi) \) describe the areas bordered by the RHLs defined by \( s_{i,\text{in}} \) and \( s_{i,\text{out}} \). The calculation of the extremum is applied to the angles \( \alpha \) and \( \varphi \) for the intervals \( 0 \leq \alpha \leq \pi \) and \( 0 \leq \varphi \leq \pi/2 \), respectively; note that we assume \( L_1 \geq L_2 \) without loss of generality. In the \( S \)-type case, the calculation of the extremum pertaining to the RHZ values is based on the angular coordinate \( \alpha \) instead of \( \varphi \); however, the angular coordinate \( \varphi \) is still needed for the calculation of \( z_i \) as part of the overall approach toward identifying \( S \)-type and \( P \)-type habitability.

Figure 3 depicts examples of RHLs and RHZs for different types of systems. In the \( S \)-type case, the RHZs are bent toward the center of the system, whereas in the \( P \)-type case, they are of notable elliptical shape. The RHZs always constitute circular annuli obtained through inspecting the appropriate minima and maxima of the RHLs. The examples as depicted include \( S \)-type and \( P \)-type systems with separation distances \( 2a \) of 0.5 AU and 5.0 AU, respectively. Cases of both equal-star and non-equal-star binaries are selected. The focus of this figure is the identification of the appropriate circular region (i.e., annulus) for each case. The figure also indicates the portions within the \( \mathcal{R}(z, \alpha) \) and \( \mathcal{R}(z, \varphi) \) domains that are not part of the RHZ(\( z \)) annuli.

Next, we determine the values for the extremum pertaining to RHZ(\( z \)) following Equations (35) and (36) based on the solutions of Equation (8) given as Equations (18a)–(18d). In cases where four solutions \( z_i \) exist, it is found that they are ordered as \( z_1 < z_2 < z_3 < z_4 \) with \( z_1 \) and \( z_2 \) constituting negative values, and \( z_3 \) and \( z_4 \) constituting positive values. If the negative solutions for \( z_i \) are permitted, it is sufficient for \( S \)-type orbits, both for star S1 and S2, to only consider solutions for \( \varphi = 0 \). For \( P \)-type orbits, a more detailed assessment is required (see below). For \( S \)-type orbits, regarding star S1, the extremum are obtained as

\[
\text{RHZ}_{\text{in}} = \max(\mathcal{R}(z, \alpha)|_{s_{i,\text{in}}} = |a + z_2(0)|_{s_{i,\text{in}}} \tag{37a}
\]

\[
\text{RHZ}_{\text{out}} = \min(\mathcal{R}(z, \alpha)|_{s_{i,\text{out}}} = |a + z_1(0)|_{s_{i,\text{out}}} \tag{37b}
\]

and for star S2, they are obtained as

\[
\text{RHZ}_{\text{in}} = \max(\mathcal{R}(z, \alpha)|_{s_{i,\text{in}}} = |a - z_3(0)|_{s_{i,\text{in}}} \tag{38a}
\]
a numerical solution may be preferred.

\[ \text{RHZ}_{\text{out}} = \min(\mathcal{R}(z, \alpha))|_{s_{\text{L}}}, = |a - z_4(0)| |s_{\text{L}}| \]  \hspace{1cm} (38b)

The size of each annulus \( \Delta_{\text{RHZ}} \) for pairs \((s_{\text{L}, \text{in}}, s_{\text{L}, \text{out}})\) is given as \( \Delta_{\text{RHZ}} = \text{RHZ}_{\text{out}} - \text{RHZ}_{\text{in}} \). \text{RHZ}_{\text{in}} also constitutes a generalization of \( \text{HZ}(s_{\ell}) \) with \( \ell = 1, 2 \) previously defined for single stars (see Section 2 and Table 3). Likewise, \( \text{RHZ}_{\text{out}} \) constitutes a generalization of \( \text{HZ}(s_{\ell}) \) with \( \ell = 4, 5, \) and 6.

For \( P \)-type orbits, the extrema are given as

\[ \text{RHZ}_{\text{in}} = \max(\mathcal{R}(z, \varphi))|_{s_{\text{L}, \text{in}}} = |z_1(0)| |s_{\text{L}, \text{in}}| \]  \hspace{1cm} (39a)
\[ \text{RHZ}_{\text{out}} = \min(\mathcal{R}(z, \varphi))|_{s_{\text{L}, \text{out}}} = |z_1(\varphi_{\text{out}})| |s_{\text{L}, \text{out}}| \]  \hspace{1cm} (39b)

We note that for the angle \( \varphi_{\text{out}} \) for \( \text{RHZ}_{\text{out}} \), no straightforward expression exists; it is located in the interval \( 0 < \varphi_{\text{out}} \leq \pi/2 \).

It can be found numerically as it is given by the angle where the minimum of \( |z(\varphi)| \) occurs. In the special case of \( L_{1, \ell}^{\prime}/L_{1, \ell} \ll 1 \), with \( L_{1, \ell}^{\prime} \) and \( L_{2, \ell}^{\prime} \) denoting the stellar primary and secondary, respectively, it is found that \( \varphi_{\text{out}} \to 0 \), whereas for \( L_{1, \ell}^{\prime} = L_{2, \ell}^{\prime} \) it is found that \( \varphi_{\text{out}} = \pi/2 \) (see Section 3.2).

There is also another complication in the identification of \( \text{RHZ} \) \( P \)-type orbits. Generally, it is required that the RHL of \( s_{\ell, \text{out}} \) is located completely outside the RHL of \( s_{\ell, \text{in}} \), i.e.,

\[ \min(\mathcal{R}(z, \varphi))|_{s_{\text{L}, \text{out}}} \geq \max(\mathcal{R}(z, \varphi))|_{s_{\text{L}, \text{in}}} \]  \hspace{1cm} (40)

This condition is, however, violated in some models, especially for relatively large values of \( a \) as well as relatively small ratios of \( L_{2, \ell}^{\prime}/L_{1, \ell}^{\prime} \). In this case, the \( \text{RHZ} \) for \((s_{\ell, \text{in}}, s_{\ell, \text{out}})\) is nullified, a behavior that may occur for the pairings \((s_2, s_4), (s_1, s_5)\), and \((s_1, s_5)\), corresponding to the CHZ, GHZ, and EHZ, respectively. In this regard, the existence of the CHZ is in the most jeopardy for the smallest bracket among the various kinds of HZs (see Table 2). Also note that if the GHZ is nullified, the CHZ will be nullified as well, considering that the CHZ (if it exists) is entirely located within the GHZ. Likewise, if the EHZ is nullified, the existence of both the CHZ and the GHZ will

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\( z_{\text{L}} \): Due to the nature of the underlying equations for \( z_{\text{L}} \), an analytic expression for \( \varphi_{\text{out}} \) is deemed possible. However, it would be highly complicated and thus a numerical solution may be preferred.
be nullified. Detailed examples will be given in the application segment of this paper; see Section 5.2 for details. However, this type of phenomenon does not occur for RHZs pertaining to S-type orbits.

For equal-star binary systems, with the property of \( L_{1\ell} = L_{2\ell} = L_{\ell}' \), expressions for RHZ\(_{\text{in}}\) and RHZ\(_{\text{out}}\) for S-type and P-type orbits can be obtained based on Equations (13) and (16). For S-type orbits we find

\[
\text{RHZ}_{\text{in}} = a - \sqrt{a^2 + s_{\ell,\text{in}}^2 L_{\ell,\text{in}}' + s_{\ell,\text{in}} \sqrt{s_{\ell,\text{in}}^2 L_{\ell,\text{in}}'' + 4a^2 L_{\ell,\text{in}}'}}
\]

\[
\text{RHZ}_{\text{out}} = \left| a - \sqrt{a^2 + s_{\ell,\text{out}}^2 L_{\ell,\text{out}}' - s_{\ell,\text{out}} \sqrt{s_{\ell,\text{out}}^2 L_{\ell,\text{out}}'' + 4a^2 L_{\ell,\text{out}}'}} \right|.
\]

It is also intriguing to explore the limits \( a \gg s_{\ell,\text{in}} \sqrt{L_{\ell,\text{in}}'} \) and \( a \gg s_{\ell,\text{out}} \sqrt{L_{\ell,\text{out}}'} \). If these limits are met, it is found that

\[
\text{RHZ}_{\text{in}} = \frac{1}{2a} \left( s_{\ell,\text{in}}^2 L_{\ell,\text{in}}' + s_{\ell,\text{in}} \sqrt{s_{\ell,\text{in}}^2 L_{\ell,\text{in}}'' + 4a^2 L_{\ell,\text{in}}'} \right)
\]

\[
\text{RHZ}_{\text{out}} = \left| \frac{1}{2a} \left( s_{\ell,\text{out}}^2 L_{\ell,\text{out}}' - s_{\ell,\text{out}} \sqrt{s_{\ell,\text{out}}^2 L_{\ell,\text{out}}'' + 4a^2 L_{\ell,\text{out}}'} \right) \right|.
\]

Moreover, in the limit of \( a \rightarrow \infty \), the expressions for single star HZs regarding RHZ\(_{\text{in}}\) and RHZ\(_{\text{out}}\) are recovered, as expected. They are in agreement with the expressions previously obtained by Kasting et al. (1993), Underwood et al. (2003), Selvs et al. (2007), and others.

Results for P-type orbits can be obtained considering

\[
\text{RHZ}_{\text{in}} = \max \left( R(z, 0), R \left( z, \frac{\pi}{2} \right) \right) \bigg|_{s_{\ell,\text{in}}} = R(z, 0)|_{s_{\ell,\text{in}}}
\]

\[
\text{RHZ}_{\text{out}} = \min \left( R(z, 0), R \left( z, \frac{\pi}{2} \right) \right) \bigg|_{s_{\ell,\text{out}}} = R \left( z, \frac{\pi}{2} \right) \bigg|_{s_{\ell,\text{out}}}.
\]

In this case, we find

\[
\text{RHZ}_{\text{in}} = \sqrt{a^2 + s_{\ell,\text{in}}^2 L_{\ell,\text{in}}'} + s_{\ell,\text{in}} \sqrt{s_{\ell,\text{in}}^2 L_{\ell,\text{in}}'' + 4a^2 L_{\ell,\text{in}}'}
\]

\[
\text{RHZ}_{\text{out}} = \sqrt{-a^2 + 2s_{\ell,\text{out}}^2 L_{\ell,\text{out}}'}
\]

based on the system parameters \( a, s_{\ell,\text{in}}, s_{\ell,\text{out}}, L_{\ell,\text{in}}', \) and \( L_{\ell,\text{out}}' \). Additionally, the requirement to avoid the RHL for \( s_{\ell,\text{out}} \) being partially or completely located inside the RHL for \( s_{\ell,\text{in}} \), see Equation (40), entails

\[
a^2 + \frac{1}{2} s_{\ell,\text{in}}^2 L_{\ell,\text{in}}' + s_{\ell,\text{in}} \sqrt{a^2 L_{\ell,\text{in}}' + \frac{1}{4} s_{\ell,\text{in}}^2 L_{\ell,\text{in}}'' - s_{\ell,\text{out}}^2 L_{\ell,\text{out}}'} \leq 0
\]

\[(45)\]

which allows setting constraints on the separation distance \( 2a \) of the binary system, noting that the values of \( s_{\ell,\text{in}}, s_{\ell,\text{out}}, L_{\ell,\text{in}}', \) and \( L_{\ell,\text{out}}' \) are subject to distinct restrictions, particularly in the case of main-sequence stars (see Tables 1 and 2). Depictions of the condition (45) for equal-star binaries for the pairings \((s_2, s_4)\), \((s_1, s_5)\), and \((s_1, s_9)\), corresponding to the CHZ, GHZ, and EHZ, respectively, are given in Figure 4. A similar expression is expected to hold for non-equal-star binaries, albeit it will be highly complicated. Hence, for those systems a numerical assessment of RHZ\(_{\text{in}}\) and RHZ\(_{\text{out}}\) (see Equations (39a) and (39b)) may be preferred to accommodate condition (40); see Section 5.2.2 for additional information and data.

4. CONSTRAINTS ON HABITABILITY DUE TO PLANETARY ORBITAL STABILITY

A primary constraint on planetary habitability is that planets are required to exist in the HZ for a sufficient amount of time allowing basic forms of life to emerge and develop. In order to adhere to this criterion, planetary orbital stability is required. There is a significant body of literature devoted to this topic, including studies of binary and multi-planetary systems, which often also consider aspects of stellar evolution (e.g., Jones et al. 2001; Noble et al. 2002; Menou & Tabachnick 2003; Sándor et al. 2007; Takeda et al. 2008; Dvorak et al. 2010; Haghighipour et al. 2010; Kopparapu & Barnes 2010).

Early studies of planetary orbital stability pertaining to planets in both S-type and P-type orbits demonstrated that planets can exist in systems of binary stars for 3000 binary periods Dvorak (1984, 1986). Although these investigations considered relatively short integration times, Dvorak determined upper and lower bounds for planetary orbital stability, considering the orbital elements of semimajor axis and eccentricity, for the proposed binary stars. Since this pioneering work, many additional studies have been performed. The foremost investigation extended the original study by a factor of 10 in integration times and an extended range of orbital elements (Holman & Wiegert 1999). In addition, the nature of the bounding formula was derived and discussed using a more statistical framework. Holman & Wiegert developed fitting formulae for both S-type and P-type planets in binary systems given as

\[
\frac{a_{cr}}{a} = 0.464 - 0.38 \mu + \mathcal{F}_S(\mu, e_h)
\]

and

\[
\frac{a_{cr}}{a} = 1.60 + 4.12 \mu + \mathcal{F}_P(\mu, e_h),
\]

respectively.
These equations give the critical semimajor axis \( a_{\text{cr}} \) in units of the semimajor axis \( a \) in the case of \( S \)-type and \( P \)-type orbits. For an \( S \)-type orbit, the ratio \( a_{\text{cr}}/a \), see Equation (46), conveys the upper limit of planetary orbital stability, whereas for a \( P \)-type orbit, the ratio \( a_{\text{cr}}/a \), see Equation (47), conveys the lower limit of planetary orbital stability. Moreover, \( \mu \) denotes the stellar mass ratio given as \( \mu = M_2/(M_1 + M_2) \), where \( M_1 \) and \( M_2 \) constitute the two masses of the binary components with \( M_2 \leq M_1 \). Equations (46) and (47) also contain the parameter functions \( F_S(\mu, e_b) \) and \( F_P(\mu, e_b) \), which depend on the aforementioned mass ratio \( \mu \) and the eccentricity of the stellar binary, \( e_b \). Considering that this paper is solely aimed at stellar binaries in circular orbits (i.e., \( e_b = 0 \)), it is found that \( F_S = F_P = 0 \).

Planetary orbital stability has been investigated by many authors using chaos indicators, such as maximal Lyapunov exponents (MLEs), the fast Lyapunov indicator, and the mean exponential growth factor of nearby orbits, to name those most commonly used; see, e.g., Satyal et al. (2013) for details, recent applications, and references. These methods have also been used to characterize the transition from stable to unstable orbits within the framework of the circular and elliptical three-body problems, see, e.g., Cuntz et al. (2007), Eberle et al. (2008), and Szenkovits & Makó (2008) for details.

Previously, Musielak et al. (2005) studied the stability of both \( S \)-type and \( P \)-type orbits in stellar binary systems, and deduced orbital stability limits for planets. These limits were found to depend on the mass ratio between the stellar components and the distance ratio between planetary and binary semimajor axes. This topic was revisited by Eberle et al. (2008), who used the concept of Jacobi’s integral and Jacobi’s constant to deduce stringent criteria for the stability of planetary orbits in binary systems for the special case of the coplanar circular-restricted three-body problem. Recently, planetary orbital stability was studied through the perspective of a chaos indicator, the MLE, by, e.g., Quarles et al. (2011). From the use of a chaos indicator, a cutoff value for the maximum Lyapunov exponent was determined as an additional stability criterion for \( S \)-type planets in the circular restricted three-body problem.

5. CASE STUDIES

5.1. \( S \)-type Habitability in Binary Systems

Next, we investigate \( S \)-type habitability for selected binary systems, including systems of equal and non-equal masses (see Table 5). Our main intent is to demonstrate the functionality of the method-as-proposed5; an extensive parameter study will be given in Section 6. Figure 5 allows comparative insight into \( S \)-type habitability for selected binary systems, i.e., systems with masses of \( M_1 = M_2 = 1.0 \, M_\odot \) and \( M_1 = 1.5 \, M_\odot \), \( M_2 = 1.0 \, M_\odot \); the binary separation distances are chosen as 10 AU and 20 AU. For single stars of 1.0 \( M_\odot \), the radiative CHZ extends from 1.049 to 1.498 AU, and the radiative GHZ extends from 0.927 to 1.831 AU; these values are slightly higher than those for G2 V stars given in Table 3 owing to a minuscule difference in mass (i.e., 0.99 versus 1.0 \( M_\odot \)).

For an equal-mass binary system of 1.0 \( M_\odot \) with a separation distance \( 2a \) of 10 AU, the radiative CHZ and GHZ extend from 1.056 to 1.511 AU, and from 0.932 to 1.853 AU, respectively, for each component. Furthermore, the outer limit of the radiative EHZ is altered from 2.64 to 2.70 AU. However, there is now an upper orbital stability limit of 1.37 AU imposed on each star. Consequently, significant portions of the radiative CHZ and GHZ are unavailable as circumstellar habitable regions. If the second star is placed at a distance of 20 AU, the alteration of the radiative CHZ and GHZ relative to single stars is very minor. Specifically, for binary separations of 10 AU and 20 AU, the sizes of the radiative GHZ increase by 1.9% and 0.6% relative to the case of single stars; Moreover, for the system with a separation distance of 20 AU, the imposed orbital stability limit is found at 2.74 AU; consequently, the full extents of the radiative CHZ, GHZ, and EHZ are now available for planetary habitability.

Figure 5 also shows results for the pairs \( M_1 = 1.5 \, M_\odot \) and \( M_2 = 1.0 \, M_\odot \). In the case of a single 1.5 \( M_\odot \) mass star, the radiative CHZ and GHZ extend from 1.88 to 2.49 AU, and from 1.65 to 3.11 AU, respectively, whereas the radiative EHZ extends up to 4.61 AU. In this type of system, a secondary star of 1.0 \( M_\odot \) placed at a separation distance of 10 AU again modifies the extents of the radiative CHZ and GHZ, which now extend from 1.89 to 2.49 AU and from 1.66 to 3.14 AU, respectively; however, the planetary orbital stability limit now occurs at 1.56 AU. Therefore, the entire domains of the radiative CHZ, GHZ, and EHZ of the primary are unavailable as circumstellar habitable regions. If the secondary star is placed at a separation distance of 20 AU, the radiative CHZ, GHZ, and EHZ of the primary star are again similar to those of a 1.5 \( M_\odot \) mass star. However, the orbital stability limit is now found at a distance of 3.12 AU from the primary; therefore, the entire supplement of the radiative EHZ, given by the bracket \( (s_6 - s_5) \), is now considered habitable.

In summary, for potentially habitable \( S \)-type binaries, owing to the implied requirement of the relatively large separations of the stellar components, the effect of the stellar secondary on the extents of the RHZs is often minor, i.e., about a few percent or less, with the biggest impact occurring in \( F \)-type systems. For most systems, the secondary’s main influence on circumstellar habitability thus consists in limiting planetary orbital stability rather than offering significant augmentations of the RHZs, a feature most pronounced in close binaries.

5.2. \( P \)-type Habitability in Binary Systems

5.2.1. Case Studies

Various sets of models have been pursued to examine \( P \)-type habitability (see Figure 6). As examples, we considered systems with masses of \( M_1 = M_2 = 1.0 \, M_\odot \) and \( M_1 = 1.5 \, M_\odot \) and \( M_2 = 0.5 \, M_\odot \); additionally, we also focused on models of \( M_1 = 1.25 \, M_\odot \) and \( M_2 = 0.75 \, M_\odot \) (see Tables 5–8 for details). The separation distances \( 2a \) were chosen as 0.5, 1.0, and 2.0 AU,
respectively. Our approach consists again of two steps. First, we explore the existence and extent of the radiative CHZs, GHZs, and EHZs. Subsequently, we consider the additional constraint of planetary orbital stability, which in the case of \( P \)-type orbits constitutes a lower limit (see Section 4). Our results can be summarized as follows.

For systems with masses of \( M_1 = M_2 = 1.0 \, M_\odot \), the following behavior is found. For separation distances \( 2a \) of 0.5 AU, the inner limit (i.e., RHL; see Section 3.4) of the radiative CHZ varies between 1.46 and 1.54 AU as a function of polar angle \( \varphi \) with 1.54 AU to be considered as an acceptable inner limit; see Equation \( (39a) \). Furthermore, the outer limit of the radiative CHZ varies between 2.10 and 2.16 AU with 2.10 AU as an acceptable outer limit; see Equation \( (39b) \). In consideration of the orbital stability limit at 0.92 AU (see Equation \( (45) \)), constituting an inner limit of orbital stability, the entire extent of the radiative CHZ is available as a circumbinary habitable region. The acceptable inner limit of the radiative GHZ is given as 1.38 AU, whereas the acceptable outer limit occurs at 2.58 AU; hence, the entire radiative GHZ is again identified as habitable.

For separation distances of 1.0 AU, the orbital stability limit is given at 1.83 AU, which falls inside the domain of the radiative CHZ ranging from 1.69 to 2.06 AU; therefore, only about half of the radiative CHZ is available for circumbinary habitability, whereas the other half is not. Since the radiative CHZ is fully embedded into the radiative GHZ, only a fraction of the radiative GHZ offers circumbinary habitability. However, the full extent of the supplementary radiative EHZ, given by the bracket \((s_6 - s_5)\), with an acceptable outer limit of 3.83 AU, offers habitability. We also considered models with binary separations of 2.0 AU. In this case, the orbital stability limit is found at 3.66 AU. Therefore, both the radiative CHZ and GHZ are unavailable for providing habitability; the latter has an outer limit that varies between 3.60 and 4.09 AU, with 3.60 AU as an acceptable limit. The outer limit of the radiative EHZ varies between 3.60 and 4.09 AU, with 3.60 AU to be ruled acceptable as the conservatively selected (i.e., inner) limit of the radiative EHZ. Hence, the entire radiative EHZ is also not considered available for providing circumbinary habitability.

Most significantly, we also pursued case studies for systems of unequal distributions of mass and, by implication, of unequal distributions of luminosity as, for example, the system \( M_1 = 1.5 \, M_\odot \) and \( M_2 = 0.5 \, M_\odot \). According to the mass–luminosity relationship for main-sequence stars, it is found that a 1.5 \( M_\odot \) star possesses a luminosity \( L_* \) about

Figure 5. Domains of \( S \)-type habitability for main-sequence star binary systems. The solutions are given in polar coordinates with the radial coordinate depicted in units of AU. The left column depicts systems with masses of \( M_1 = M_2 = 1.0 \, M_\odot \), whereas the right column depicts systems with masses of \( M_1 = 1.5 \, M_\odot \) and \( M_2 = 1.0 \, M_\odot \). The separation distances \( 2a \) are given as 10 AU (top) and 20 AU (bottom). Results for the conservative, general, and extended RHZs are given as dark gray, medium gray, and light gray areas, respectively. The orbital stability limits are indicated by red dashed lines.

(A color version of this figure is available in the online journal.)
Figure 6. Domains of P-type habitability for main-sequence star binary systems. The solutions are given in polar coordinates with the radial coordinate depicted in units of AU. The left column depicts systems with masses of $M_1 = M_2 = 1.0 \, M_\odot$, whereas the right column depicts systems with masses of $M_1 = 1.5 \, M_\odot$ and $M_2 = 0.5 \, M_\odot$. The separation distances $2a$ are given as 0.5 AU (top), 1.0 AU (middle), and 2.0 AU (bottom). Results for the conservative, general, and extended RHZs are given as dark gray, medium gray, and light gray areas, respectively. The orbital stability limits are indicated by red dashed lines.

(A color version of this figure is available in the online journal.)

3.5 times higher than a 1.0 $M_\odot$ star; a similar factor of difference exists for the recast stellar luminosity $L'_i\ell$ (see Tables 4 and 5). Thus, the combined luminosity of the (1.5 $M_\odot$, 0.5 $M_\odot$) system is considerably higher than the combined luminosity of the (1.0 $M_\odot$, 1.0 $M_\odot$) system, as expected. On the other hand, following the work by Dvorak (1986) and Holman & Wiegert (1999), an unequal distribution of stellar mass, i.e., a smaller value of $\mu$ (see Section 4), entails a smaller orbital stability limit. Since it constitutes a lower limit, i.e., positioned more closely to the stellar system, it offers a larger “window of opportunity” for planets in the RHZs (if they exist) to be orbitally stable.

Results for separation distances $2a$ of 0.5, 1.0, and 2.0 AU are given in Figure 6. For a binary separation of 0.5 AU, it is found
that both the radiative CHZ and GHZ exist, and habitability in these domains is fully permitted according to the planetary orbital stability constraint, although the width of the CHZ is relatively small. The CHZ extends from 2.14 to 2.28 AU, whereas the GHZ extents from 1.91 to 2.90 AU; the orbital stability limit is given at 0.66 AU. In this type of system, there are extreme variations for the inner and outer limits of both the radiative CHZ and GHZ. For example, the inner RHL for the CHZ varies between 1.65 and 2.14 AU, whereas its outer RHL varies between 2.28 and 2.76 AU as a function of polar angle $\varphi$. There is also a considerably large domain of the supplementary portion of the radiative EHZ, which has an outer limit that varies between 4.40 and 4.89 AU. Detailed depictions of the variations of the inner and outer limits of the RHZs for the various systems are given in Figure 7. This figure indicates relatively small bars of variation for equal-mass systems such as $M_1 = M_2 = 1.0 M_\odot$ with small separation distances as, e.g., $2a = 0.5$ AU. However, large bars of variation are obtained for non-equal-mass binaries or for equal-mass binaries with large separation distances as, e.g., $2a = 2.0$ AU.

In systems with a binary separation of 1.0 AU, the radiative CHZ is nullified; note that the orbital stability limit in this system is given at 1.32 AU. The reason for the disallowance of the CHZ is that the RHL for $s_1$, which is at 2.38 AU, is located inside the RHL for $s_2$, given as 2.05 AU. The same criterion (see Equation (40)) also leads to a relatively small width of the radiative GHZ, which extends between 2.16 and 2.66 AU. At a binary separation of 2.0 AU, the situation is even more drastic as both the radiative CHZ and GHZ are disallowed. The only type of circumbinary habitable region remaining is that provided by the relatively large supplementary portion of the radiative EHZ given by the bracket $(s_1 - s_2)$. In this zone, habitable planets are expected to be possible as their existence would be consistent with the planetary orbital stability constraint.

5.2.2. Additional Analyses

Next, we explore the existence of $P$-type RHZs, both for equal-mass and non-equal-mass binaries, in a more systematic manner through the means of numerical experiments. Specifically, we pursue sets of model calculations with the binary

| Table 6 | $P$-type Stellar Habitability; $2a = 0.5$ AU |
|---------|-------------------------------------------|
| $\ell$ | $s_\ell$ | RHZ$_{in}$ | RHZ$_{out}$ | $a_{xt}$ |
| 1      | 0.84    | 1.29      | 1.38      | 0.92   |
| 2      | 0.95    | 1.46      | 1.54      | 0.92   |
| 3      | 1.00    | 1.54      | 1.62      | 0.92   |
| 4      | 1.37    | 2.19      | 2.16      | 0.92   |
| 5      | 1.67    | 2.58      | 2.62      | 0.92   |
| 6      | 2.40    | 3.72      | 3.76      | 0.92   |

| Table 7 | $P$-type Stellar Habitability; $2a = 1.0$ AU |
|---------|-------------------------------------------|
| $\ell$ | $s_\ell$ | RHZ$_{in}$ | RHZ$_{out}$ | $a_{xt}$ |
| 1      | 0.84    | 1.42      | 1.91      | 0.66   |
| 2      | 0.95    | 1.65      | 2.14      | 0.66   |
| 3      | 1.00    | 1.73      | 2.22      | 0.66   |
| 4      | 1.37    | 2.28      | 2.76      | 0.66   |
| 5      | 1.67    | 2.90      | 3.38      | 0.66   |
| 6      | 2.40    | 4.40      | 4.89      | 0.66   |

| Table 8 | $P$-type Stellar Habitability; $2a = 2.0$ AU |
|---------|-------------------------------------------|
| $\ell$ | $s_\ell$ | RHZ$_{in}$ | RHZ$_{out}$ | $a_{xt}$ |
| 1      | 0.84    | 0.85      | 1.98      | 3.66   |
| 2      | 0.95    | 1.10      | 2.11      | 3.66   |
| 3      | 1.00    | 1.20      | 2.18      | 3.66   |
| 4      | 1.37    | 1.87      | 2.64      | 3.66   |
| 5      | 1.67    | 2.39      | 3.05      | 3.66   |
| 6      | 2.40    | 3.60      | 4.09      | 3.66   |

| Model: $M_1 = 1.0 M_\odot$, $M_2 = 1.0 M_\odot$ |
| Model: $M_1 = 1.25 M_\odot$, $M_2 = 0.75 M_\odot$ |
| Model: $M_1 = 1.5 M_\odot$, $M_2 = 0.5 M_\odot$ |

| Model: $M_1 = 1.0 M_\odot$, $M_2 = 1.0 M_\odot$ |
| Model: $M_1 = 1.25 M_\odot$, $M_2 = 0.75 M_\odot$ |
| Model: $M_1 = 1.5 M_\odot$, $M_2 = 0.5 M_\odot$ |

| Model: $M_1 = 1.0 M_\odot$, $M_2 = 1.0 M_\odot$ |
| Model: $M_1 = 1.25 M_\odot$, $M_2 = 0.75 M_\odot$ |
| Model: $M_1 = 1.5 M_\odot$, $M_2 = 0.5 M_\odot$ |
separation distance $2a$ considered as an independent variable; see Table 9 for results. The stellar masses are altered between $0.5 \, M_\odot$ and $1.5 \, M_\odot$ in increments of $0.25 \, M_\odot$ (see Table 5). Results are given for the pairings $(s_2, s_4)$ (CHZ), $(s_1, s_5)$ (GHZ), and $(s_1, s_6)$ (EHZ). Note that for equal-mass binaries, it is sufficient to solve Equation (45), whereas for general binary systems a more thorough assessment is needed to satisfy relation (40). This approach allows us to explore the maximum binary separation distances, which are the upper limits for permitting RHZs for each case (i.e., a combination of binary masses and a choice of CHZ, GHZ, or EHZ).

Generally, it is found that for any binary system, the greatest permissible binary separation distance is attained for the EHZ, and furthermore that the value-as-attained is greater for the GHZ than for the CHZ; these findings are as expected. For example, for the system $M_1 = M_2 = 1.0 \, M_\odot$, the expiration distance for the radiative EHZ is given as $4.25 \, AU$, whereas for the radiative GHZ and CHZ, the distances are given as $2.57$ and $1.64 \, AU$, respectively (see Table 9). As another example, the system $M_1 = 1.25 \, M_\odot$ and $M_2 = 0.75 \, M_\odot$, the expiration distances for the radiative EHZ, GHZ, and CHZ are given as $3.80$, $1.93$, and $0.96 \, AU$, respectively.

It is also intriguing to compare results for stellar pairs such as, e.g., $M_1 = M_2 = 1.0 \, M_\odot$ to stellar pairs such as $M_1 = 1.5 \, M_\odot$ and $M_2 = 0.5 \, M_\odot$. For the system of $M_1 = M_2 = 1.0 \, M_\odot$, it is found that the radiative CHZ and GHZ are nullified—as defined by the limit of validity of Equation (45)—at binary

![Figure 7](image-url)
separation distances of 1.64 and 2.57 AU, respectively (see Table 9). At those binary separations, the distances of the vanishing RHZ–CHZ and RHZ–GHZ (as measured from the geometrical center of the system; see Figure 1) are given as 1.95 and 2.25 AU, respectively. In comparison, the limits of planetary orbital stability (to be interpreted as lower limits) are identified as 3.00 and 4.71 AU, respectively. Thus, we conclude that for equal-mass binary systems such as $M_1 = M_2 = 1.0 \, M_\odot$, habitability for widely spaced binaries is lost due to the lack of orbital stability already at binary separations where the circumbinary CHZ–RHZs and GHZ–RHZs are still in place.

The same type of study has been pursued for systems with highly unequal intrabinary distributions of masses and, by implication, stellar luminosities such as, e.g., $M_1 = 1.5 \, M_\odot$ and $M_2 = 0.5 \, M_\odot$. In this case, it is found that the radiative CHZ and GHZ vanish at binary separation distances $2a$ of 0.65 and 1.55 AU, respectively. Furthermore, the distances of the vanishing radiative CHZ and GHZ (as measured from the system center, see Figure 1) are given as 2.21 and 2.43 AU, respectively (see Table 9). The respective limits of planetary orbital stability are identified as 0.85 and 2.04 AU. Thus, for this type of system it is found that habitability is lost due to the vanishing RHZs, even though circumstellar habitability would still be permitted according to the planetary orbital stability criterion. The fact that circumstellar habitability is already lost for systems of relatively small binary separations is a consequence of the extreme radiative imbalance caused by the highly unequal distribution of stellar luminosities, which determine the circumbinary RHLs.

Radiative imbalance within binary systems may cause the RHL for $s_{\text{out}}$ to be partially or completely located inside of the RHL for $s_{\text{in}}$; see Section 3.4. In fact, when the pairs $M_1 = M_2 = 1.0 \, M_\odot$ and $M_1 = 1.5 \, M_\odot$ and $M_2 = 0.5 \, M_\odot$, compared to one another, it is found that although the unequal-mass binary system has almost twice the combined stellar luminosity of the equal-mass binary system (i.e., 3.85 versus 2.0 $L_\odot$), it still possesses much narrower CHZ, GHZ, and EHZ RHZs. In fact, it is found that the condition expressed as Equation (40) is most readily met in cases of equal-mass binary systems of relatively small separation distances, and mostly violated in systems of relatively large separation distances and/or unequal distributions of masses and, by implication, luminosities. Various examples have been depicted in Figure 7; see discussion in Section 5.1.1.

In summary, although an unequal distribution of stellar masses within binary systems is identified as advantageous for facilitating planetary orbital stability, in consideration of the fact that lower stability limits for $P$-type orbits occur for smaller mass ratios $\mu$ (see Section 4), the situation for the existence of the RHZs is much less ideal, even for systems where the stellar primary is highly luminous owing to the behavior of the RHLs. In this regard, the radiative CHZ is most in jeopardy as $(s_4 - s_2)$ constitutes the smallest bracket among the various kinds of HZs (see Table 2). More fortunate scenarios are expected to occur for the radiative GHZ and EHZ, with the brackets given as $(s_5 - s_1)$ and $(s_6 - s_1)$, respectively; they are characterized by considerably larger widths, especially in the case of equal-mass systems of stars with relatively high luminosities.

6. PROPOSED HABITABILITY CLASSIFICATION: HABITABILITY TYPES S, P, ST, AND PT

Another aspect of this study is to provide an appropriate classification of habitability applicable to general binary systems. Previously, Dvorak (1982) introduced the terminology of $S$-type and $P$-type orbits for system planets, which is now widely used by the orbital stability, planetary, and the astrobiology science communities. Evidently, besides the assessment of orbital stability behaviors, these terms are also appropriate for classifying binary system RHZs, if they exist. However, following previous investigations (e.g., Dvorak et al. 2010; Haghhighipour et al. 2010; Eggel et al. 2012), as well as the results of this work, the spatial domain of $S$-type and $P$-type RHZs for the manifestation of habitability is truncated owing to the additional constraint of planetary orbital stability, these zones shall be referred to as $ST$-type and $PT$-type, respectively, in the following.

Detailed results are given in Table 10, which provides an extensive summary of $P$-, $PT$-, $ST$-, and $S$-type habitability for both equal-mass and non-equal-mass binary systems. The stellar masses are varied between 0.5 $M_\odot$ and 1.5 $M_\odot$ in increments of 0.25 $M_\odot$ amounting to a total of 15 combinations. Table 10 features the results for the pairings $(s_1, s_5)$ (CHZ), $(s_1, s_3)$ (GHZ), and $(s_1, s_6)$ (EHZ). In principle, it is found that—with the secondary taken as fixed—the higher the mass and, by implication, the luminosity of the stellar primary, the larger the values obtained for $P$-, $PT$-, $ST$-, and $S$-type habitability. Additionally, larger values for the limits of $P$-, $PT$-, $ST$-, and $S$-type habitability are obtained regarding the GHZ relative to the CHZ, as expected. The largest values are obtained for $PT$- and $S$-type habitability for the EHZ; in this regard, there is no change for $P$- and $ST$-type habitability relative to the GHZ since both types of HZs are based on the same inner bracket value of $s_1$ (see above).

The results of Table 10 are in line with the previously discussed findings regarding highly unequal intrabinary distributions of masses and, by implication, stellar luminosities as, e.g., $M_1 = 1.25 \, M_\odot$ and $M_2 = 0.75 \, M_\odot$ or $M_1 = 1.5 \, M_\odot$ and $M_2 = 0.5 \, M_\odot$ compared to the case of $M_1 = M_2 = 1.0 \, M_\odot$. For systems of highly unequal mass distributions, the domains of $P$-type and $PT$-type habitability are typically relatively small as the RHL for $s_{\text{in}}$ crosses the RHL for $s_{\text{in}}$ in relatively close proximity to the primary, thus allowing only small distance ranges to exhibit $P/PT$-type habitability. It is also found that in seven cases for the CHZ, as well as two cases for the GHZ, the RHZs expire prior to the truncation of habitability due to the planetary orbital stability requirement. In those cases, only $P$-type habitability exists; no $PT$-type habitability is found as the orbital stability constraint bears no relevance.

Figures 8 and 9 show various combinations of equal-mass and non-equal-mass binary systems; they all show numerous similarities, though the spatial scales are noticeably different as they are defined through the stellar luminosities. If equal-mass binary systems are considered, taking $M_1 = M_2 = 1.0 \, M_\odot$ and $M_1 = M_2 = 0.5 \, M_\odot$, as examples, the extents of both $P$- and $PT$-type habitability increase with increasing stellar mass or luminosity. The distances for $P$- and $PT$-type habitability are found to almost coincide, indicating that the orbital stability constraint affects the inner and outer limit of $P$-type habitability in about the same manner. Furthermore, the inner and outer limits of both $S$- and $ST$-type habitability are shifted to larger distances from each stellar component for stars of higher luminosity, as expected. Moreover, for stars of higher luminosity, there is a larger spatial domain where $S$-type habitability is truncated due to the additional constraint of planetary orbital
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Figure 8. Range of habitability in equal-star binary systems with the stellar components given as 1.0 and 0.5 \( M_\odot \), respectively. Results are obtained as function of the binary separation distance \( 2a \) pertaining to the GHZ. The two red lines indicate the limits of the \( P \)-type RHZ (i.e., RHZ\textsubscript{in} and RHZ\textsubscript{out}; see Equations (44a) and (44b)), whereas the two blue lines indicate the limits of the \( S \)-type RHZ (see Equations (41a) and (41b)). The available \( S \)-type and \( P \)-type RHZs are depicted as grayish areas. The cyan dashed line indicates the \( P \)-type orbital stability limit, whereas the violet dashed line indicates the \( S \)-type orbital stability limit; see Equations (47) and (46), respectively. Note that the \( P \)-type orbital stability limit constitutes a lower limit, whereas the \( S \)-type orbital stability limit constitutes an upper limit; thus, the available ranges of habitability within the RHZs are indicated as red-hatched and blue-hatched areas, respectively. Hence, \( P \)-type habitability is attained in the range beneath the \( P \) intersection point, \( PT \)-type habitability between the intersection points \( P \) and \( PT \), \( ST \)-type habitability between the intersection points \( ST \) and \( S \), and \( S \)-type habitability beyond the \( S \) intersection point. No habitability is found between the intersection points \( PT \) and \( ST \).

(A color version of this figure is available in the online journal.)

Figure 9. Same as Figure 8, but now for non-equal-star binary systems. The primary is chosen as 1.0 \( M_\odot \), whereas the secondary is chosen as 0.75 and 0.5 \( M_\odot \), respectively. Note that the algebraic solution for \( P \)-type RHZ\textsubscript{in} outside the scope of relevance may be ill-defined.

(A color version of this figure is available in the online journal.)

to exist. The CHZs, GHZs, and EHZs can be either \( S \)- or \( ST \)-type, on one hand, or \( P \)- or \( PT \)-type, on the other hand, to qualify for depiction. The results are given as a function of stellar spectral type, for stars between spectral types F0 to M0. The figure shows that \( P/PT \)-type habitable regions are able to exist for a relatively large range of separation distances in the case of relatively luminous stars (i.e., spectral type F), but only for a relatively small range of separation distances for less luminous stars (i.e., spectral types K and M). Regarding \( S/ST \)-type habitable regions, the situation is reversed. Figure 10 also indicates a notable domain of binary separations where no habitable regions are found owing to the lack of RHZs, the lack of planetary orbital stability, or both. Moreover, no domain of binary separation distances is identified where \( S/ST \)-type and \( P/PT \)-type habitable regions overlap.

7. SUMMARY AND CONCLUSIONS

In this study, we present a new method regarding a comprehensive assessment of \( S \)-type and \( P \)-type habitability in stellar binary systems. \( P \)-type orbits occur when the planet orbits both binary components, whereas in the case of \( S \)-type orbits, the planet orbits only one of the binary components with
the second component considered a perturbator. An important characteristic of the new method is that it combines the orbital stability constraint for a system planet with the necessity that a habitable region given by the stellar radiative energy fluxes ("RHZ") must exist. The requirement to combine these two properties has also been recognized in previous studies (e.g., Eggl et al. 2012; Kane & Hinkel 2013).

Another element of this study is to introduce a habitability classification regarding stellar binary systems, consisting of habitability types $S$, $P$, $ST$, and $PT$. This type of classification also considers whether or not $S$-type and $P$-type RHZs are reduced in size due to the additional constraint of planetary orbital stability. In summary, five different cases were identified, which are $S$-type and $P$-type habitability provided by the full extent of the RHZ; habitability where the RHZ is truncated by the additional constraint of planetary orbital stability (labeled as $ST$- and $PT$-type, respectively); and cases of no habitability at all. This classification scheme can be applied to both equal-mass and non-equal-mass binary systems, as well as to systems with binaries in elliptical orbits, which will be the focus of the forthcoming Paper II of this series. As part of this study, a significant array of results are given for a notable range of main-sequence stars, which are of both observational and theoretical interest.

A key aspect of the proposed method is the introduction of a combined algebraic formalism for the assessment of both $S$-type and $P$-type habitability; in particular, mathematical criteria are presented allowing the determination of for which systems $S$-type and $P$-type RHZs are realized. In this regard, a priori choices regarding the presence of $S$-type and $P$-type RHZs are neither necessary nor possible as the existence of $S$-type as well as $P$-type RHZs is proliferated through well-defined mathematical conditions pertaining to the underlying fourth-order algebraic equation. The coefficients of the polynomial are given by the binary separation distance ($2a$), the solar system-based parameter for the limit of habitability ($s_2$), and the modified values for the luminosities ($L'_{1/2}$, $L'_{2/2}$) of the stellar binary components, referred to as recast stellar luminosities. Regarding the binary system habitable zone, we consider conservative, general, and extended zones of habitability, noting that their inner and outer limits are informed by previous solar system investigations (e.g., Kasting et al. 1993; Underwood et al. 2003; Selsis et al. 2007).

In our segment of applications, we examined the existence of habitable $S$-type orbits for selected examples. We found that regarding the RHZs, owing to the typically relatively large separation of the stellar components, the effect of the stellar secondary on the extents of the RHZs is usually very minor. The secondary’s main influence on circumstellar habitability consists in imposing restrictions regarding planetary orbital stability implemented as an upper stability limit around each stellar component, which often truncates or nullifies $S$-type planetary habitability. In the framework of our study, we specifically considered the radiative EHZ, which is most outwardly extended (i.e., up to 2.4 AU in case of the Sun). It was found that this kind of zone is most affected by the limitation of planetary orbital stability as it is located closest to the secondary stellar component.

Furthermore, we also examined the existence of habitable $P$-type orbits. In this case, relatively complicated scenarios emerge. In general, it was found that the best prospects for circumbinary habitability emerge for (1) systems with stellar components of relatively high luminosities (no surprise here!), (2) systems where the stellar luminosities are relatively similar (for main-sequence stars, as implied by their stellar masses), and (3) systems of relatively small binary separations. If conditions (2) or (3) are not met, it may happen that the outer RHL is located inside the inner RHL, thus nullifying the RHZ irrespective of planetary orbital stability considerations. On the other hand, an unequal intrabinary distribution of masses entails a lower limit of planetary orbital stability (i.e., positioned closer to the binary system) thus implying an enhanced opportunity for circumbinary habitability. However, this aspect is of lesser significance for most systems, compared to the restrictions for the RHZs due to the imbalance given by the stellar luminosities.

Various applications in this study concern stars of masses between 0.75 and 1.5 $M_\odot$. This approach is motivated to unequivocally demonstrate the effects of stellar binarity on the extent and structure of circumstellar habitability, which is most pronounced for massive, i.e., highly luminous stars. Nonetheless, most stars in binaries are expected to be low-mass stars, i.e., stars of spectral types K and M, owing to the skewness of the Galactic initial mass function (e.g., Kroupa 2001, 2002; Chabrier 2003). For example, we compared pairs of systems given by (1.0 $M_\odot$, 1.0 $M_\odot$) and (1.5 $M_\odot$, 0.5 $M_\odot$). Obviously, the overall luminosity is by far greatest in the (1.5 $M_\odot$, 0.5 $M_\odot$) system following the mass–luminosity relationship, i.e., $L_\star \propto M_\star^4$ (e.g., Reid 1987). However, this system is found to be highly unfavorable for the facilitation of circumbinary habitability. Particularly, it is found that the $P$-type GHZ in the (1.0 $M_\odot$, 1.0 $M_\odot$) system extends to 0.91 AU, whereas it extends only to 0.65 AU in the (1.5 $M_\odot$, 0.5 $M_\odot$) system. Furthermore, smaller spatial extents are identified for $P$-type CHZs, as this type of HZ is in the highest jeopardy owing to the relatively small ($s_2 - s_1$) bracket compared to the ($s_3 - s_1$) bracket for GHZs (see Table 2). In fact, a considerable number of systems do not offer CHZs at all, which again is a consequence of the radiative imbalance in those systems. Also, the nullification of CHZs in

![Figure 10. Depiction of S/ST-type and P/PT-type habitability based on the joint constraint of planetary orbital stability and the availability of a habitable region provided by the stellar radiative energy fluxes. The domains of the CHZs, GHZs, and EHZs are depicted as dark gray, medium gray, and light gray areas, respectively. The results refer to the particular case of equal-mass binaries, i.e., main-sequence stars of identical spectral types. Note that no separate S/ST-type contour is attained for the EHZ, owing to the fact that the inner boundary of the EHZ (i.e., $s_1$; see Table 2) agrees with the inner boundary of the GHZ, thus rendering the same mathematical criterion for the inner limit of habitability. For the F0 V, F2 V, and F5 V stars, the S/ST-type contours extend beyond the figure frame. For the GHZ and EHZ, they are given as 15.9, 13.6, and 11.7 AU, and for the CHZ, they are given as 18.1, 15.5, and 13.3 AU, respectively.](image-url)
binary systems is most likely to occur in systems of relatively large separation distance. In contrast, the best opportunities for facilitating circumbinary habitability are given in the context of EHZs, as expected.

Future work will deal with a significant augmentation of our method to other systems, including systems with binary components in elliptical orbits (see Paper II). This will allow us to compare applications of our method, including results for individual systems, to other findings in the literature. We also expect our method to be applicable to general binary systems with main-sequence stars as well as to systems containing evolved stars; this latter effort is motivated by observational evidence and supporting theoretical efforts indicating that planets are also able to exist around stars that have left the main-sequence (e.g., Sato et al.; 2003; Ramm et al.; 2009; Eberle & Cuntz; 2010; Doyle et al.; 2010; Sato et al.; 2013). Particularly, it is highly desirable to augment our method to systems of higher order, as motivated by the steady progress in theory as well as ongoing and future observational discoveries of exosolar planetary systems.

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