Suspension of atoms and gravimetry using a pulsed standing wave

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(Dated: February 2, 2009)

Atoms from an otherwise unconfined $^{87}$Rb condensate are shown to be suspended against gravity using repeated reflections from a pulsed optical standing wave. Reflection efficiency was optimized using a triple-pulse sequence that, theoretically, provides accuracies better than 99.9%. Experimentally, up to 100 reflections are observed, leading to dynamical suspension for over 100 ms. The velocity sensitivity of the reflections can be used to determine the local gravitational acceleration. Further, a gravitationally sensitive atom interferometer was implemented using the suspended atoms, with packet coherence maintained for a similar time. These techniques could be useful for the precise measurement of gravity when it is impractical to allow atoms to fall freely over a large distance.

PACS numbers: 03.75.Dg, 37.25.+k, 42.50.Wk

Many of the recent advances in the physics of ultra-cold atoms have relied on the use of magnetic and/or optical fields to confine atoms and suspend them against gravity. Typically, confinement and suspension go together [1], so that if one wishes to study unconfined atoms, they must be in free fall. Achieving long observation times therefore requires either a large drop distance or a microgravity environment [2, 3, 4]. Freely falling atoms are also used when measuring gravity in atom interferometry experiments [5], since any applied trapping forces would spoil the measurement accuracy. The large drop distances that this requires are one limit to the performance and applicability of the technique.

In this Letter, we demonstrate a new method for suspending ultra-cold atoms that imposes negligible confinement and can be sufficiently precise to be used in a gravimetry experiment. The suspension consists of “bouncing” the atoms repeatedly from a pulsed optical standing wave. A similar idea was recently proposed by Impens et al. [6]. In addition to implementing the suspension method, we also demonstrate a gravity-sensitive interferometer using the confined atoms. Although we do not achieve significant precision here, improvements may permit gravimetry with precision comparable to that achieved using falling atoms, but requiring a negligible drop distance.

Atoms have previously been suspended using multiple bounces from evanescent waves [7], magnetized surfaces [8], light sheets [9], and magnetic fields [10]. However, these techniques introduce complex forces that significantly modify the atomic wave function from what it would be in a freely falling frame, and that typically are not characterized well enough to permit accurate gravimetry. Furthermore, these methods have been limited to only a few bounces, while we have observed up to 100 bounces and expect more to be possible. Our method is related to the phenomenon of Bloch oscillations of atoms held in a static standing wave potential [11], which has also been used to measure gravity [12] but does not model freely falling atoms.

The manipulation of atoms by an off-resonant standing-wave laser beam was first used in thermal atomic beams, [13, 14] and has proven useful for cold atoms as well [15, 16]. Through the ac Stark effect, the laser induces a periodic potential that acts as a diffraction grating for the atomic wave function, producing coupling between momentum components that differ by $2\hbar k$ for light with wavenumber $k$. By controlling the intensity and duration of the applied pulse, various beam-splitting and reflecting operations can be achieved. Generally the

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results are sensitive to the initial velocity of the atoms, but the low velocity spread in an ultra-cold sample allows the operations to be quite precise [17].

We define an order-$n$ reflection to be the operation driving $|n\hbar k\rangle \rightarrow |n\hbar k\rangle$, where $|p\rangle$ denotes an atomic state with momentum $p$. Suspension of atoms using such an operation starts with a sample of mass $m$ atoms held in a conventional trap. At time $t = 0$, the trap is switched off, allowing the atoms to fall in the local gravitational field $g$. At $t = t_n = n\hbar k/mg$, the atomic momentum will reach $-n\hbar k$ and a reflection operation is applied using a vertically oriented laser beam, reflecting the atoms upward with $p = +n\hbar k$. They move ballistically for an interval $2t_n$, after which they again have $p = -n\hbar k$ and the reflection can be repeated. We used both order-1 and order-2 reflections, with trajectories illustrated in Fig. 1(a) and (b) respectively.

Losses during the reflection operation will limit the number of bounces that can be achieved. We studied a simple order-2 reflection pulse in Ref. [17] and found it to be limited to a fidelity of 0.94, which would permit roughly 20 bounces. However, by shaping the intensity-time profile of the light, we were able to achieve better performance. We modeled the operation by numerical solution of the Schrödinger equation using the Bloch expansion

$$\psi(z, t) = \sum_n c_n(t) e^{i(2nk+\delta)z}$$

for optical potential $V_L(z, t) = \hbar \beta(t) \cos(2kz)$. Here $\psi$ is the atomic wavefunction, $\delta$ accounts for an initial momentum offset, and $\beta(t)$ is proportional to the light intensity. This yields a set of equations

$$i \frac{dc_n}{dt} = \frac{\hbar}{2m} (2nk + \delta)^2 c_n + \frac{\beta}{2} (c_{n-1} + c_{n+1}),$$

which were truncated at $n = \pm 6$. For $\beta(t)$, we considered a symmetric sequence of three square pulses with durations ($T_1, T_2, T_1$) and intensities ($\beta_1, \beta_2, \beta_1$). An optimization algorithm was used to determine the best values of the $T$’s and $\beta$’s. For the order-1 reflection, we found ($T_1, T_2$) = ($0.355, 0.592$)$\omega_r^{-1}$, and ($\beta_1, \beta_2$) = (1.73, 3.45)$\omega_r$, where $\omega_r = \hbar k^2/(2m) \approx 2.36 \times 10^4$ s$^{-1}$ is the atomic recoil frequency. The order-2 reflection was optimized at ($T_1, T_2$) = (0.256, 1.46)$\omega_r^{-1}$ and ($\beta_1, \beta_2$) = (2.28, 4.59)$\omega_r$. In both cases, the calculated fidelity exceeded 0.9998, but the model neglected losses due to spontaneous emission. Maintaining an error below $5 \times 10^{-3}$ required $\delta/k$ to be less than 0.05 for the order-1 operation and 0.02 for the order-2 operation.

Bouncing was implemented using approximately $10^4$ $^{87}$Rb atoms from a Bose-Einstein condensate. The atoms were prepared in the $|F = 2, m_F = 2\rangle$ hyperfine state in a magnetic trap with oscillation frequencies ($\omega_x, \omega_y, \omega_z$) = $2\pi\times(7.4, 0.8, 4.3)$ Hz, for $z$ vertical. For $^{87}$Rb, the photon recoil velocity $v_r = \hbar k/m$ is 5.88 mm/s and the fall time $t_1 = v_r/g$ is 0.6 ms. We drop the atoms by turning off the trap current. The current decay is non-exponential, but reaches $1/e$ of its initial value after 160 $\mu$s with repeatability better than 1 $\mu$s. Due to the finite turn-off time, the atoms take longer than time $t_1$ to reach a momentum of $-\hbar k$, and we compensate for this by delaying the first reflection pulse.

The standing wave was produced by a home-built diode laser with a wavelength of 780.193 nm, 27 GHz blue of the $5S_{1/2} \leftrightarrow 5P_{3/2}$ laser cooling transition. An acousto-optic modulator was used to control the optical intensity. The light was then coupled into a single-mode fiber, which provided spatial filtering and pointing stability. The output from the fiber passed vertically through the vacuum cell and was retro-reflected from an external mirror to produce the standing wave. At this detuning, the expected loss due to spontaneous emission is $7 \times 10^{-4}$ for the order-1 reflection and $2 \times 10^{-3}$ for the order-2 reflection. The beam was approximately Gaussian with a waist of 1 mm.

We investigated suspension by releasing the atoms and then applying a sequence of reflection operations. The time before the first pulse and the time between pulses were varied to maximize the number of atoms remaining; the final number of atoms and their momentum state was monitored using time-of-flight absorption imaging with a resonant probe traveling along the horizontal $y$-axis. The results for both order-1 and order-2 bouncing are shown in Fig. 2. In both cases, suspension times exceeding 100 ms were observed.

We observe a non-exponential decay of the atom number, indicating that atom loss is larger for the later operations. The form of this fall-off varied from day to day, but the time scale was consistent. The reason for the decay is not yet clear, but a few possibilities can be...
suggested. For instance, if the reflection timing is incorrect, then the momentum error \( h\delta \) will increase over time, leading to a reduction in operation fidelity. Also, the condensate expands considerably in this time, and if the standing-wave intensity is insufficiently uniform, spatially-dependent errors in \( \beta \) will develop. We plan to investigate these issues further, since according to our model, thousands of bounces should be possible.

The bouncing experiment already provides a measurement of gravity, since the optimum timing depends on the value of \( g \). From the bouncing experiments shown, we obtain \( t_1 = 603.0 \pm 0.5 \) \( \mu s \), where the uncertainty is determined as the variation sufficient to reduce the atom number by roughly a factor of two. This gives \( g = (\hbar k)/(m t_1) = 9.759 \pm 0.008 \) m/s\(^2\), noticeably different from the expected value of 9.81 m/s\(^2\). Since the atoms are in a state with non-zero magnetic moment, the discrepancy can be explained by a modest ambient magnetic field gradient. Measurements outside the vacuum cell indicated a vertical gradient of \( B' = 7 \pm 2 \) G/m. We also measured the gradient at the position of the atoms by modifying the trap turn-off procedure to be non-adiabatic for the atomic spins, so that multiple hyperfine states were populated. As the atoms bounced, the magnetic force caused the states to separate, as in a Stern-Gerlach experiment. The separation could be observed in the absorption images, and from it we obtain a more accurate value of \( B' = 8.6 \pm 0.1 \) G/m. This gives a corrected value for \( g \) of 9.814 \pm 0.008 m/s\(^2\). We note that well-localized Stern-Gerlach measurements with long interaction times are already an interesting application for the suspension technique.

A more precise determination of \( g \) can be made by implementing an atom interferometer using the suspended atoms. One way to achieve this is illustrated in Fig. 1(c). The atoms are dropped as before, but at time \( t_1 \) when \( p = -\hbar k \), the beam-splitting operation \( |(-\hbar k)\rangle - \frac{1}{\sqrt{2}} (|\hbar k\rangle - i |+\hbar k\rangle) \) is applied. The two resulting wave packets can then be independently suspended using alternating order-1 and order-2 reflections, as shown. (Note that the order-2 reflection has no effect on the momentum of atoms with \( p = 0 \).) After many reflections, the packets can be recombined by another beam-splitting operation, with a result that depends on their phase difference. Since one packet is always above the other, it is clear that the phase difference will be sensitive to \( g \).

We calculate the phase difference between the packets using the quantum-mechanical solution for a falling plane wave state. A packet initially described by a wave function \( \psi(z,t=0) = \exp(i q z) \) will at subsequent times become \( \psi(z,t) = \exp[i(q - \gamma t)z] \exp[i \Theta(q,t)] \) for \( \gamma \equiv mg/h \) and

\[
\Theta(q,t) = \frac{\hbar}{2m} \left( q^2 t - q\gamma t^2 + \frac{1}{3} \gamma^2 t^3 \right).
\]

This can be verified by substitution into Schrodinger’s equation. The phase difference developed during one cycle of the interferometer can therefore be expressed as

\( \Phi = \Theta(k+\delta,2\tau) - \Theta(-k+\delta,\tau) - \Theta(3k-\gamma \tau+\delta,\tau) + \phi_{r2} + \phi_{v1} \)  

where \( \hbar \delta \) is the momentum offset at the start of the cycle, \( 2\tau \) is the cycle duration, and \( \phi_{r\nu} \) is the phase difference imparted by an order-\( \nu \) reflection. In addition, the momentum offset changes from \( \delta \) to \( \delta - 2k + 2\gamma \tau \). Evaluation of (4) yields

\[
\Phi = \frac{\hbar k}{m} (-4k\tau + 2\gamma \tau^2) + \phi_{v1} + \phi_{v2},
\]

Thus, after \( N \) cycles, the wavefunction will be

\[
\psi = \frac{h}{h} - \frac{\hbar}{h} (k + \delta N) - i e^{i \Phi} |h(-k + \delta N)\rangle
\]

for final momentum offset \( h\delta N \). The beam-splitting operation is then applied with shifted phase \( \phi_s \), resulting in a fraction of atoms \( f_+ = \sin^2[(N\Phi + \phi_s)/2] \) with momentum +\( \hbar k \). We vary \( \phi_s \) by shifting the frequency of the standing-wave laser before the final beam-splitter, as in [18]. By plotting \( f_+ \) vs \( \phi_s \), the phase \( N\Phi \) is determined for various numbers of cycles, as seen in Fig. 3. We find \( \Phi = 2\pi j - 0.035 \pm 0.003 \), where \( j \) is an integer. The value of \( g \) obtained from the bouncing experiments fixes \( j = -9 \).

The accuracy of the data is limited by deviations from the expected linear dependence on \( N \), as seen in the residuals in Fig. 3(b). We attribute the oscillating structure to...
residual oscillations in the magnetic field after turning off the trap. The signal corresponds to a decaying gradient with initial amplitude 10 G/m, about 0.3% of the original trap field gradient. Eliminating this field might prove difficult, but its effects could be significantly reduced by transferring the atoms to an $m = 0$ hyperfine state before turning off the trap, since their magnetic moment would then be nearly zero.

To analyze the measured phase, the reflection phases $\phi_{r1}$ and $\phi_{r2}$ must be determined from the model. To do so accurately, the effect of gravity during the pulse should be included. This is accomplished by working in the interaction picture with respect to the gravitational interaction $mgz$. The calculation proceeds as in Eq. [2], but using the interaction Hamiltonian

$$H_I(t) = U_0^t \delta(t) V_z(z,t) U_0(t)$$

(7)

where $U_0(t) |q\rangle = \exp[i\Theta(q,t)] |q - \gamma t\rangle$. We reference $t$ to the beginning of the pulse sequence for the initial beam-splitter operation, to the center of the reflection sequences, and to the end of the final recombination sequence. The cycle time $\tau$ is defined accordingly. We obtain $\phi_{r1} = (1 \pm 1) \times 10^{-2}$ and $\phi_{r2} = 0.56 \pm 0.16$. These uncertainties are the dominant source of error in the experiment. They arise primarily from a sensitivity of the phase to the intensity of the standing wave, which is difficult to control precisely. We obtain a value for $g$ of 9.745 $\pm$ 0.027 m/s², actually less precise than that obtained from bouncing. However, if the reflection phase errors were eliminated, the fractional uncertainty would be reduced to $5 \times 10^{-5}$.

The sensitivity to the standing wave intensity comes from a lack of symmetry between the two arms, since the lower arm undergoes an order-2 reflection while the upper arm does not, and gravity acts in the opposite sense from a lack of symmetry between the two arms, since the lower arm undergoes an order-2 reflection while the upper arm does not, and gravity acts in the opposite sense.

In conclusion, we have demonstrated the ability to suspend otherwise unconfined atoms in gravity for over 100 ms, using repeated reflections from a standing wave laser beam. We note that free atoms would fall a distance of 5 cm in this time. To achieve this, we developed high-precision reflection operations, for which our model predicts that a factor of ten greater suspension time should be possible. We further demonstrated a gravitationally sensitive atom interferometer using the suspended atoms. With some improvements, this technique might be able to achieve gravimetric precision similar to that of free-falling atom interferometers, but with a much reduced space requirement.

We are grateful for helpful conversations with B. Deissler, V. Ivanov, and G. Tino. We thank J. Tiamsuphat and K. Saenboonruang for contributions to the work. This research was sponsored by the Defense Advanced Research Projects Agency (award No. 51925-PR-DRP) and by the National Science Foundation (award No. PHY-0244871).
1. C. A. Sackett, Phys. Rev. A 73, 013626 (2006).
2. P. Bouyer and A. Bresson, eds., Quantum Mechanics for Space Application: From Quantum Optics to Atom Optics and General Relativity (Special issue), vol. 84 of Appl. Phys. B (2006).
3. S. Dimopoulos, P. W. Graham, J. M. Hogan, and M. A. Kasevich, Phys. Rev. Lett. 98, 111102 (2007).
4. A. Vogel, M. Schmidt, K. Sengstock, K. Bongs, W. Lewoczko, T. Schultd, A. Peters, T. V. Zoest, W. Ertmer, E. Rasel, et al., App. Phys. B 84, 663 (2006).
5. P. R. Berman, ed., Atom Interferometry (Academic Press, San Diego, 1997).
6. F. Impens, P. Bouyer, and C. J. Bordé, App. Phys. B 84, 603 (2006).
7. C. G. Aminoff, A. M. Steane, P. Bouyer, P. Desbiolles, J. Dalibard, and C. Cohen-Tannoudji, Phys. Rev. Lett. 71, 3083 (1993).
8. T. M. Roach, H. Abele, M. G. Boshier, H. L. Grossman, K. P. Zetie, and E. A. Hinds, Phys. Rev. Lett. 75, 629 (1995).
9. K. Bongs, S. Burger, G. Birkl, K. Sengstock, K. Rzazewski, A. Sanpera, and M. Lewewinstein, Phys. Rev. Lett. 83, 3577 (1999).
10. A. S. Arnold, C. MacCormick, and M. G. Boshier, Phys. Rev. A 65, 031601(R) (2002).
11. M. Raizen, C. Salomon, and Q. Niu, Phys. Today 50, 30 (1997).
12. G. Ferrari, N. Poli, F. Sorrentino, and G. M. Tino, Phys. Rev. Lett. 97, 060402 (2006).
13. D. M. Giltner, R. W. McGowan, and S. A. Lee, Phys. Rev. A 52, 3966 (1995).
14. P. J. Martin, B. G. Oldaker, A. H. Miklich, and D. E. Prichard, Phys. Rev. Lett. 60, 515 (1988).
15. M. Kozuma, L. Deng, E. W. Hagley, J. Wen, R. Lutwak, K. Helmerson, S. L. Rolston, and W. D. Phillips, Phys. Rev. Lett. 82, 871 (1999).
16. S. Wu, E. J. Su, and M. Prentiss, Euro. Phys. J. D 35, 111 (2005).
17. K. J. Hughes, B. Deissler, J. H. T. Burke, and C. A. Sackett, Phys. Rev. A 76, 035601 (2007).
18. O. Garcia, B. Deissler, K. J. Hughes, J. M. Reeves, and C. A. Sackett, Phys. Rev. A 74, 031601(R) (2006).
19. H. Müller, S. W. Chiow, Q. Long, S. Herrmann, and S. Chu, Phys. Rev. Lett. 100, 180405 (2008).
20. V. V. Ivanov, A. Alberti, M. Schioppo, G. Ferrari, M. Artoni, M. L. Chiofalo, and G. M. Tino, Phys. Rev. Lett. 100, 043602 (2008).