Compton and double Compton scattering processes at colliding electron-photon beams

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Radiative corrections (RC) to the Compton scattering cross section are calculated in the leading and next-to leading logarithmic approximation to the case of colliding high energy photon-electron beams.

RC to the double Compton scattering cross section in the same experimental set-up are calculated in the leading logarithmic approximation.

We consider the case when no pairs are created in the final state. We show that the differential cross section can be written in the form of the Drell-Yan process cross-section. Numerical values of the $K$-factor and the leading order distribution on the scattered electron energy fraction and scattering angle are presented.

I. INTRODUCTION

The Compton scattering process

$$\gamma(k_1) + e^-(p_1) \rightarrow \gamma(k_2) + e^-(p_2),$$

$$k_1^2 = k_2^2 = 0, \quad p_1^2 = p_2^2 = m^2,$$

$$\kappa_1 = 2p_1 k_1 = 4\epsilon_1 \omega_1, \quad \kappa_1' = 2p_2 k_1 = 2\epsilon_2 \omega_1(1 - c), \quad s_1 = 2p_1 p_2 = 2\epsilon_1 \epsilon_2(1 + c),$$

$$\kappa_1 \sim \kappa_1' \sim s_1 \gg m^2, \quad \epsilon_2 = \frac{2\epsilon_1 \omega_1}{\omega_1(1 - c) + \epsilon_1(1 + c)},$$

(with $\epsilon_{1,2}, \omega_1$-are the energies of initial and scattered electrons and the initial photon; $c = \cos \theta$, $\theta$ is the angle between $p_2, \vec{k}_1$)- plays an important role as a perspective calibration process at high-energy photon-electron colliders [1]. To obtain a radiative corrected cross
section of this process is the motivation of this paper. Modern methods based on the renormalization group approach in combination with the lowest order radiative corrections (RC) permit us to obtain a differential cross section with the leading 
\[ (((\alpha/\pi)L)^n \sim 1, \text{with \ "large logarithm\" } L = \ln(s_1/m^2)) \] and the next-to leading approximation (i.e. keeping the terms of the order \((\alpha/\pi)^n L^{-1}\)). So the accuracy of the formulae given below is determined by the terms of the order (which are systematically omitted)

\[ \frac{m^2}{\kappa_1}, \frac{\alpha^2}{\pi^2} L, \frac{\alpha}{M_Z^2}, \] (2)

compared with the terms of order of unity and is at the level of per-mille for typical experimental conditions \[ \theta \sim 1, \kappa_1 \sim 10\text{GeV}^2. \] We consider the energies of initial particles to be much less than the Z-boson mass \(M_Z\) and, therefore, the weak corrections to the Compton effect in our consideration are beyond our accuracy.

The first papers devoted to cancellation of radiative corrections to Compton scattering were published in 1952 by Brown and Feynman (virtual and soft real photon emission contribution), and Mandl and Skyrme (emission of an additional hard photon).

In the work of H. Veltman, the lowest order radiative corrections to the polarized Compton scattering were calculated in non-relativistic kinematics. This case of kinematics was also considered in the paper of M. Swartz.

In the papers of A. Denner and S. Dittmaier, the lowest order radiative corrections in the framework of Standard Model was calculated for the case of polarized electron and photon.

In this paper, we consider the case of high-energy electron and photon Compton scattering (cms energy supposed to be much higher than the electron mass but much less than the Z-boson mass). We found that the cross-section with radiative corrections of all orders of PT taken into account could be written down in the form of the Drell-Yan process. Both leading and next-to-leading contributions are derived explicitly.

We imply the kinematics when the initial photon and electron move along the \(z\) axis in the opposite directions. The energy of the scattered electron will be the function of its scattering angle:

\[ z_0 = \frac{\varepsilon_2}{\omega_1} = \frac{2\rho}{a}, \quad a = a(c, \rho) = 1 - c + \rho(1 + c), \quad \rho = \frac{\varepsilon_1}{\omega_1}. \] (3)

Hereafter we imply the kinematic case \(\rho < 1\). The case \(\rho > 1\) will be considered in section VII.
The differential cross section in the Born approximation will be
\[
\frac{d\sigma_B}{dc}(p_1, \theta) = \frac{\pi \alpha^2 U_0}{\omega_1^2 a^2}, \quad U_0 = \frac{a}{1-c} + \frac{1-c}{a},
\]
When taking into account RC of higher orders (arising from both emission of virtual and real photons) a simple relation between the scattered electron energy and the scattering angle changes, so the differential cross section is in general dependent on the energy fraction \( z \) of the scattered electron. Accepting the Drell-Yan form a of cross section, we can put it in the form
\[
\frac{d\sigma}{dcdz}(p_1, \theta, z) = \int_0^1 dx D(x, L) \int_0^{z_0} dt D(\frac{z}{t}, L) \frac{d\sigma_h}{dcdt}(xp_1, \theta, t)(1 + \frac{\alpha}{\pi}K),
\]
where the structure function \( D(x, L) \) (specified below) describes the probability to find the electron (considered as a parton) inside the electron, \( K \) is the so-called K-factor which can be calculated from the lowest RC orders, and the "hard" cross section is
\[
\frac{d\sigma_B(xp_1, \theta)}{dc} = \frac{\alpha^2}{\omega_1^2} \frac{1}{(1-c + \rho x(1+c))^2} \left( \frac{1-c}{1-c + \rho x(1+c)} + \frac{1-c + \rho x(1+c)}{1-c} \right),
\]
with \( K \) specified below (see (8,19,26)).

The cross section written in the Drell-Yan form explicitly satisfies the Kinoshita-Lee-Nauenberg theorem \[7\]. Really, being integrated on the scattered electron energy fraction \( z \), the structure function corresponding to the scattered electron turns to unity due to its property
\[
\int_0^1 dz \int_0^{z_0} \frac{dt}{t} D(\frac{z}{t}, L)f(t) = \int_0^1 df(t).
\]
Mass singularities associated with the initial lepton structure function remain.

So our master formula for the cross section with RC taken into account is
\[
\frac{d\sigma}{dzdc}(p_1, p_2) = \int_0^{\frac{z}{t(x)}} \frac{dx}{t(x)} D(x, L) \frac{d\sigma_B}{dc}(xp_1, \theta)D(\frac{z}{t(x)}, L) + \frac{\alpha}{\pi} \frac{d\sigma_B(p_1, \theta)}{dc} \left[ K_{SV}\delta(z - z_0) + K_h \right],
\]

\[
z = \frac{\epsilon_2^2}{\omega_1^2} < z_0, \quad x_0 = \frac{z(1-c)}{\rho(2 - z(1+c))}, \quad L = \ln \frac{2\omega_1^2 z_0 \rho(1+c)}{m^2},
\]
with the nonsinglet structure function $D$ defined as

$$D(z, L) = \delta(1 - z) + \sum_{k=1}^{\infty} \frac{1}{k!} \left( \frac{\alpha L}{2\pi} \right)^k P_1(z) \otimes^k,$$

$$P_1(z) \otimes^k = P_1(z) \otimes \cdots \otimes P_1(z), \quad P_1(z) \otimes P_1(z) = \int_{z}^{1} P_1(t)P_1 \left( \frac{z}{t} \right) \frac{dt}{t},$$

$$P_1(z) = \frac{1 + z^2}{1 - z} \theta(1 - z - \Delta) + \delta(1 - z) \left( 2 \ln \Delta + \frac{3}{2} \right), \quad \Delta \ll 1.$$  

In Conclusion (see 30) we put the so-called ”smoothed” form of a structure function.

The second term in rhs of (8) collects all the nonleading contributions from virtual, soft, and hard photons emission, with $K_{SV}$ given in section III where the virtual and soft real contributions are considered. In sections III IV we consider the contribution from an additional hard photon emission and introduce an auxiliary parameter $\theta_0$ to distinguish the collinear and noncollinear kinematics of photon emission. Also, we put the expression for the hard photon contribution $K_h$. The results of numerical estimation for K-factor and leading contributions are given in section V. In Section VI (App. A), we demonstrate the explicit cancellation of $\theta_0$ dependence. In section VII (App. B), we consider the kinematic case $\varepsilon_1 > \omega_1$.

**II. CONTRIBUTION OF VIRTUAL AND SOFT REAL PHOTONS**

To obtain the explicit form of the $K$ factor, we reproduce the lowest order RC calculation. It consists of virtual photon emission contribution and the contribution from the real (soft and hard) photon emission taken into account. The virtual and soft photon emission contribution was first calculated in the famous paper of the 1952 year by Laura Brown and Richard Feynman[2]. The result has the form

$$\frac{d\sigma_{\text{virt}}}{d\sigma_B} = -\frac{\alpha}{\pi} \frac{U_1}{U_0},$$

with (see 2, kinematic case II):

$$\frac{U_1}{U_0} = (1 - L) \left( \frac{3}{2} + 2 \ln \frac{\lambda}{m} \right) + \frac{1}{2} L^2 - \frac{\pi^2}{6} - K_V, \quad U_0 = \frac{\kappa_2}{\kappa_1} + \frac{\kappa_1}{\kappa_2}.$$

$\kappa_1$ and $\kappa_2$ are the kinematic variables.
with $K_V$ (virtual photon contribution to the $K$-factor):

$$K_V = -\frac{1}{U_0} \left[ (1 - \frac{\kappa_2}{2\kappa_1} - \frac{\kappa_1}{\kappa_2}) \left( \ln^2 \frac{s_1}{\kappa_1} - \ln \frac{s_1}{\kappa_1} + 2 \ln \frac{\kappa_2}{\kappa_1} \right) 
+ (1 - \frac{\kappa_1}{2\kappa_2} - \frac{\kappa_2}{\kappa_1}) \left( \ln^2 \frac{s_1}{\kappa_2} - \ln \frac{s_1}{\kappa_2} - \ln \frac{\kappa_1}{\kappa_2} + \pi^2 \right) \right],$$

and

$$\frac{\kappa_2}{\kappa_1} = \frac{z_0(1 - c)}{2\rho}, \quad \frac{s_1}{\kappa_1} = \frac{z_0(1 + c)}{2}, \quad \frac{s_1}{\kappa_2} = \frac{\rho(1 + c)}{1 - c}.$$  

The soft photon emission for our kinematics has the form

$$\frac{d\sigma_{soft}}{d\sigma_B} = -\frac{4\pi\alpha}{16\pi^3} \int \frac{d^3k}{\omega} \left( \frac{p_1}{p_1k} - \frac{p_2}{p_2k} \right)^2 \delta(\omega = \sqrt{k^2 + \lambda^2} < \Delta \epsilon < \epsilon_1 \sim \epsilon_2).$$

Standard calculations lead to the result

$$\frac{d\sigma_{soft}}{d\sigma_B} = \frac{\alpha}{\pi} \left[ (L - 1) \ln \left( \frac{m^2 \Delta \epsilon^2}{\lambda^2 \epsilon_1 \epsilon_2} \right) + \frac{1}{2} \ln^2 \frac{\epsilon_1}{\epsilon_2} - \frac{\pi^2}{3} + \text{Li}_2 \left( \frac{1 - c}{2} \right) \right].$$

The resulting contribution to the cross section from virtual and soft real photons does not depend on the fictitious "photon mass" $\lambda$ as well as the terms of $L^2$ type. It can be written in the form

$$\left( \frac{d\sigma}{dzdc} \right)_{sv} = \frac{d\sigma_{virt}}{dc} + \frac{d\sigma_{soft}}{dc} \delta(z - z_0)$$

$$= \frac{\alpha}{2\pi} \frac{d\sigma_B(p_1, \theta)}{dc} \left[ (L - 1)(P_{1\Delta} + P_{2\Delta}) + 2K_{SV} \right] \delta(z - z_0),$$

where we have introduced the notation

$$P_{1\Delta} = \frac{3}{2} + 2 \ln \frac{\Delta \epsilon}{\epsilon_1}, \quad P_{2\Delta} = \frac{3}{2} + 2 \ln \frac{\Delta \epsilon}{\epsilon_2}.$$  

We can see that the terms proportional to the "large" logarithm $L$ have the form conforming with the RG prescription of the structure function. The contribution of nonleading terms $K_{SV}$ is

$$K_{SV} = -\frac{\pi^2}{6} + \text{Li}_2 \left( \frac{1 - c}{2} \right) - \frac{1}{2} \ln \frac{z_0}{\rho} + K_V.$$  

**III. HARD COLLINEAR REAL PHOTON EMISSION CONTRIBUTION**

The dependence on the auxiliary parameter $\Delta \epsilon$ will be eliminated when taking into account the emission of real additional hard photon with 4-momentum $k$ and the energy $\omega$ exceeding $\Delta \epsilon$. 
It is convenient to consider the kinematics when this additional photon moves within the narrow cone of the angular size $m/\epsilon_1 \ll \theta_0 \ll 1$ along the directions of the initial or scattered electrons. The contribution of these kinematic regions can be obtained by using the ”Quasi Real Electron Method” \[10\] instead of using a general (rather cumbersome) expression for the cross section of the double Compton (DB) scattering process \[3\].

In the case when the collinear photon is emitted along the initial electron the result has the form:

\[
\frac{d\sigma}{dz \, dc} |_{\vec{p}_1} = \frac{\alpha}{2\pi} \int_0^{1 - \Delta_1 / \epsilon_1} dx \frac{d\sigma_B (xp_1, \theta)}{dc} \frac{1 + x^2}{1 - x} (L_1 - 1) + 1 - x) \delta(z - t(x)),
\]

\[
L_1 = \ln \frac{\theta_0^2 \epsilon_1^2}{m^2} = \ln \frac{\theta_0^2 \rho}{2z_0 (1 + c)}.
\]

When the photon is emitted along the scattered electron we have:

\[
\frac{d\sigma}{dz \, dc} |_{\vec{p}_2} = \frac{\alpha}{2\pi} \frac{d\sigma_B (p_1, \theta)}{dc} \int_{z(1 + \Delta_2)}^{z_0} \frac{dt}{t} \delta(t - z_0) \frac{1 + z^2}{1 - \frac{z}{t}^2} (L_2 - 1) + 1 - \frac{z}{t}),
\]

\[
L_2 = \ln \frac{\epsilon_2^2 \theta_0^2}{m^2} = \ln \frac{\theta_0^2 \rho}{2 \rho (1 + c) z_0},
\]

where $z = \frac{\epsilon_2'}{\omega_1} < z_0$ is the energy fraction of the scattered electron (after emission of the collinear photon).

It is convenient to write down the contribution of the collinear kinematics in the form

\[
\frac{d\sigma}{dz \, dc} |_{\text{coll}} = \frac{\alpha}{2\pi} (L - 1) \left[ \int_0^1 dx \frac{1 + x^2}{1 - x} \theta(1 - x - \Delta_1) \frac{d\sigma_B (xp_1, \theta)}{dc} \delta(z - t(x)) \right.
\]

\[
+ \left. \int_{z(1 + \Delta_2)}^{z_0} \frac{dt}{t} \frac{1 + (\frac{z}{t})^2}{1 - \frac{z}{t}^2} \theta(1 - \frac{z}{t} - \Delta_2) \frac{d\sigma_B (p_1, \theta)}{dc} \delta(t - z_0) \right] + \frac{df^{(1)}}{dz \, dc} + \frac{df^{(2)}}{dz \, dc},
\]

with

\[
\frac{df^{(1)}}{dz \, dc} = \frac{\alpha^3}{4 \rho (1 - c) \omega_1^2} \left( \frac{2 - z(1 + c)}{2} + \frac{2}{2 - z(1 + c)} \right)
\]

\[
\times \left[ \frac{1 + x^2}{1 - x} \ln \frac{\rho \theta_0^2}{2z_0 (1 + c)} + 1 - x \right]_{x = x_0} \theta(1 - x - \Delta_1),
\]

\[
\frac{df^{(2)}}{dz \, dc} = \frac{\alpha^3}{4 \rho a \omega_1^2} \left( \frac{1 - c}{a} + \frac{a}{1 - c} \right)
\]

\[
\times \left[ \frac{1 + \frac{z^2}{t^2}}{1 - \frac{z}{t}} \ln \frac{z^2 \theta_0^2}{2 \rho (1 + c) z_0} + 1 - \frac{z}{t} \right]_{t = z_0} \theta(1 - \frac{z}{t} - \Delta_2), \quad \Delta_{1,2} = \frac{\Delta \epsilon}{\epsilon_{1,2}}.
\]
We use here the relation $\delta(z - t(x)) = \frac{2x^2_0\rho}{(z^2(1 - c))}\delta(x - x_0)$.

Again we can see that the terms containing large logarithm $L$ have the form conforming with the structure function. So our ansatz (5) is confirmed.

The auxiliary parameter $\theta_0$ dependence vanishes when taking into account the contribution of noncollinear kinematics of the additional hard photon emission (see Section VII).

**IV. NONCOLLINEAR KINEMATICS CONTRIBUTION, DOUBLE COMPTON SCATTERING PROCESS**

The general expression for the cross section of the DC scattering process

$$\gamma(k_1) + e^-(p_1) \rightarrow \gamma(k_2) + \gamma(k) + e^-(p_2),$$

$$\kappa = 2kp_1, \quad \kappa' = 2kp_2, \quad \kappa_2 = 2k_2 p_1, \quad \kappa'_2 = 2k_2 p_2,$$

was obtained years ago by Mandl and Skyrme [3]. The expression for the cross section presented in this paper is exact but unfortunately, too complicated. Instead, we use the expression for the differential cross section calculated (by the methods of chiral amplitudes [11]) with the assumption that all kinematic invariants are large compared with the electron mass squared $\kappa \sim \kappa' \sim \kappa_i \sim \kappa'_i \gg m^2$

$$\frac{\varepsilon_2 d\sigma_{DC}^{\theta_0}}{d^3p_2} = \frac{1}{2!} \frac{\alpha^3}{2\pi^2\kappa_1} Rd\Phi, \quad d\Phi = \frac{d^3k_2 d^3k}{\omega^2} \delta^4(p_1 + k_1 - p_2 - k_2 - k),$$

$$R = \frac{\kappa\kappa'\kappa_2\kappa'_2}{\kappa\kappa'_1\kappa_1\kappa'_2}(\kappa_2^2 + \kappa'_2^2 + \kappa_1^2 + \kappa'_1^2 + \kappa_2 \kappa'_2),$$

The explicit expression for the contribution to the $K$ factor from hard photon emission $K_h$ is

$$\frac{\alpha d\sigma_B}{\pi dc} K_h = \frac{d\sigma_{DC}^{\theta_0}}{dzdc} + df^{(1)} + df^{(2)}.$$

with

$$\frac{d\sigma_{DC}^{\theta_0}}{dzdc} = \frac{\alpha^3 z}{2!4\pi\rho} \int Rd\Phi,$$

and the phase volume $d\Phi$ is restricted by the conditions $\omega, \omega_2 > \Delta \epsilon$ and the requirement that the angles between 3-vectors $\vec{k}_2, \vec{k}$ and 3-vectors $\vec{p}_1, \vec{p}_2$ exceed $\theta_0$.

The values of $K_h$ calculated numerically are given in Tables 1,7. We show numerically and analytically (see Appendix A) the independence of $K_h$ on the auxiliary parameters $\theta_0, \Delta \epsilon$. 
The cross section of the DC scattering process in an inclusive experimental set-up with the leading logarithmic approximation in terms of structure functions has the form

\[ d\sigma^{DC}(p_1, k_1; p_2, k, k_2) = \frac{1}{x} \int_0^1 dx D(x, L) \int_0^1 \frac{dt}{t} d\sigma_0^{DC}(xp_1, k_1; tp_2/z, k, k_2), \]

with the structure functions given above and

\[ d\sigma_0^{DC}(p_1, k_1; p_2, k, k_2) = \frac{\alpha^3}{4\pi^2k_1} R \frac{d^3k_2d^3kd^3p_2}{\omega_2\omega c_2} \delta^4(p_1 + k_1 - p_2 - k_2 - k). \]

V. CONCLUSION

The characteristic form "reverse radiative tail" (see Tables 2,4) of the differential cross section on the energy fraction \( z \) can be reproduced if one uses the "smoothed" expression for nonsinglet structure functions which includes the virtual electron pair production \[ D(x, L) = \frac{\beta^2}{2}(1-x)^{\beta/2-1}(1 + \frac{3}{8}\beta) - \frac{\beta}{4}(1 + x) + O(\beta^2), \quad \beta = \frac{2\alpha}{\pi}(L - 1), \]

\[ O(\beta^2) = \frac{\beta^2}{2}(1-x)^{\beta/2-1}(-\frac{1}{48}\beta^2(1 + \frac{3}{8}L + \pi^2 - \frac{47}{8})) + \frac{1}{32}\beta^2(-4(1 + x)\log(1 - x) - \frac{1 + 3x^2}{1 - x} \log x - 5 - x). \]

In Figure 1 we put the magnitude of RC in the leading approximation

\[ R(\theta) = \left(\frac{d\sigma_B}{dc}\right)^{-1} \left( \int dz \frac{d\sigma}{dzdc} - \frac{d\sigma_B}{dc} \right). \]

The results cited above imply the experimental set-up without additional \( e^+e^- \), \( \mu^+\mu^- \), \( \pi^+\pi^- \) real pairs in the final state.

The accuracy of the formulae given above is determined by the order of magnitude of the terms omitted (see (2)) compared to the terms of order of unity, i.e., is of the order of 0.1% for typical experimental conditions. In particular, it is the reason why we omit the evolution effect of the \( K \)-factor terms.

The numerical value of \( K_h \), leading contributions, and the Born cross-section for different kinematic regions are presented as a functions of \( z, c \) in Tables 1-5,7.
Table 1: The value of $K_h$ as a function of $z$, $\cos \theta$ (calculated for $\rho = 0.4$).

| $z \backslash \cos \theta$ | -0.8 | -0.6 | -0.4 | -0.2 | 0.0 | 0.2 | 0.4 | 0.6 | 0.8 |
|---------------------------|------|------|------|------|-----|-----|-----|-----|-----|
| 0.1                       | -2.82| -2.61| -2.39| -2.19| -2.09| -1.89| -1.87| -2.06| -2.75|
| 0.2                       | -2.77| -2.47| -2.17| -1.90| -1.65| -1.46| -1.39| -1.56| -2.30|
| 0.3                       | -3.43| -2.98| -2.55| -2.14| -1.77| -1.47| -1.30| -1.38| -2.13|
| 0.4                       | -4.96| -3.87| -3.23| -2.65| -2.13| -1.67| -1.34| -1.30| -2.02|

Table 2: The value of $\omega_1^2/\alpha^2 d\sigma/(dcdz)$ (leading contribution, first term in the right-hand side of the master formula (8)) as a function of $z$, $\cos \theta$ (calculated for $\rho = 0.4$, $\omega_1 = 5$ GeV).

| $z \backslash \cos \theta$ | -0.8 | -0.6 | -0.4 | -0.2 | 0.0 | 0.2 | 0.4 | 0.6 | 0.8 |
|---------------------------|------|------|------|------|-----|-----|-----|-----|-----|
| 0.1                       | 0.211| 0.237| 0.265| 0.299| 0.345| 0.413| 0.526| 0.754| 1.450|
| 0.2                       | 0.337| 0.357| 0.378| 0.405| 0.445| 0.508| 0.618| 0.850| 1.576|
| 0.3                       | 0.703| 0.669| 0.643| 0.634| 0.644| 0.685| 0.782| 1.013| 1.784|
| 0.4                       | 3.883| 2.153| 1.554| 1.264| 1.113| 1.054| 1.090| 1.296| 2.122|

Table 3: Born cross section (without factor $\alpha^2/\omega_1^2$) for $\rho = 0.4$.

| $\cos \theta$ | -0.8 | -0.6 | -0.4 | -0.2 | 0.0 | 0.2 | 0.4 | 0.6 | 0.8 |
|---------------|------|------|------|------|-----|-----|-----|-----|-----|
| $\omega_1^2 \frac{d\sigma}{d\omega}$ | 1.779| 2.038| 2.365| 2.796| 3.389| 4.266| 5.721| 8.669| 17.881|

VI. APPENDIX A

Performing the integration over $k_2$ of the phase volume

$$d\Phi = \frac{d^3 k}{\omega} \frac{d^3 k_2}{\omega_2} \delta^4(Q - k - k_2), \quad Q = p_1 + k_1 - p_2,$$

we can put it in the form

$$d\Phi = \frac{\omega d\omega 2dc_1dc_2}{\omega_1^2 \sqrt{D}} \delta[2\rho - \rho z(1 + c) - z(1 - c) - \frac{\omega}{\omega_1} (\rho(1 - c_1) - z(1 - c_2) + 1 + c_1)],$$

with $D = 1 - c_1^2 - c_2^2 - c^2 - 2cc_1c_2$; $c_1, c_2$ are the cosines of the angles between $\vec{k}$ and $\vec{p}_1, \vec{p}_2$ respectively.
For collinear kinematics the following relation can be useful:

1. \( k \approx (1 - x)p_1 \)

\[
R_1 = R_{\vec{k}||\vec{p}_1} = \left( \frac{2x\rho}{z(1 - c)} + \frac{z(1 - c)}{2x\rho} \right) \frac{1}{(1 - x)^2} \frac{1}{2\rho^2(1 - c_1)x\omega^2_1},
\]
\[
d\Phi_1 = d\Phi_{\vec{k}||\vec{p}_1} = \frac{2d^3k}{\omega} \delta((xp_1 + k_1 - p_2)^2) = 2\pi \frac{\rho(1 - x)dx dc_1}{2 - z(1 + c)} \delta(x - x_0),
\]
\[
\frac{d\sigma_1}{dz dc} = \frac{\alpha^3 z}{2!4\pi \rho} \int R_1 d\Phi_1 = \frac{\alpha^3}{4\rho \omega^2_1(1 - c)} \frac{1 + x_0^2}{1 - x_0} \left( \frac{2x_0 \rho}{z(1 - c)} + \frac{z(1 - c)}{2x_0 \rho} \right) \ln\left( \frac{4}{\theta_0^2} \right).
\]

In the last equation we take into account the same contribution from the region \( k_2 \approx (1 - x)p_2 \)

2. For the case \( k \approx (t/z - 1)p_2 \) we obtain

\[
R_2 = R_{\vec{k}||\vec{p}_2} = \left( \frac{2x\rho}{z(1 - c)} + \frac{z(1 - c)}{2x\rho} \right) \frac{1}{(1 - x)^2} \frac{1}{2\rho^2(1 - c_1)x\omega^2_1},
\]
\[
d\Phi_2 = d\Phi_{\vec{k}||\vec{p}_2} = \frac{2d^3k}{\omega} \delta((xp_1 + k_1 - p_2)^2) = 2\pi \frac{\rho(1 - x)dx dc_1}{2 - z(1 + c)} \delta(x - x_0),
\]

So the contribution of the case \( \vec{k}||\vec{p}_2 \) (\( \vec{k}_2||\vec{p}_2 \)) has the form

\[
\frac{d\sigma_2}{dz dc} = \frac{\alpha^3 z}{2!4\rho \omega^2_1} \int R_2 d\Phi_2 = \frac{\alpha^3}{4\rho \omega^2_1} \left( \frac{1 - c}{a} + \frac{a}{1 - c} \right) \frac{1 + y_0^2}{1 - \frac{y}{a}} \ln\left( \frac{4}{\theta_0^2} \right).
\]

Comparing formulae (34, 36) with (23) we can see explicit cancellation of the \( \theta_0 \) dependence.

VII. APPENDIX B

Here we put the different case of kinematic region for \( \rho, z \)

All the above formulae were considered for the case \( \rho < 1 \), and the possible region for the variable \( z \) was determined by the equation \( x_0 < 1 \)

\[
z \leq \frac{2\rho}{1 - c + \rho(1 + c)},
\]

which means that the low boundary of integration in formula (3) is less than 1. In the case of \( \rho > 1 \) it is convenient to put the new variable

\[
\eta = \frac{\omega_1}{\omega_1}, \ y = \frac{\varepsilon_2}{\varepsilon_1}, \ y_0 = \frac{\varepsilon_2}{\varepsilon_1} = \frac{2\eta}{1 + c + \eta(1 - c)}, \ \eta < 1.
\]
The master equation (38) for the case $\rho > 1$ (or $\eta < 1$) reads

\[
\frac{d\tilde{\sigma}}{dydc}(p_1,p_2) = \int_{\tilde{x}_0}^1 \frac{dx}{t(x)} D(x,\tilde{L}) \frac{d\tilde{\sigma}_B(xp_1,\theta)}{dc} D\left(\frac{y}{t(x)},\tilde{L}\right) + \frac{\alpha d\tilde{\sigma}_B(p_1,\theta)}{dc} \left[ \tilde{K}_{SV}\delta(y-y_0) + \tilde{K}_h \right],
\]

with the possible values for energy fraction of the scattered electron $y(\tilde{x}_0 < 1)$: $y \leq y_0$.

The Born cross section (4,6) and formulae for hard photon emission, $\tilde{K}_{SV}$, $\tilde{K}_h$ for the case $\rho > 1$ appear just by appropriate exchange $\rho \rightarrow \eta^{-1}$:

\[
\frac{d\tilde{\sigma}_B(xp_1,\theta)}{dc} = \frac{\pi\alpha^2}{\varepsilon_1^2} \frac{1}{(\eta(1-c) + x(1+c))^2} \left( \frac{\eta(1-c)}{\eta(1-c) + x(1+c)} + \frac{\eta(1-c) + x(1+c)}{\eta(1-c)} \right).
\]

(40)

| $\cos \theta$ | -0.8 | -0.6 | -0.4 | -0.2 | 0.0 | 0.2 | 0.4 | 0.6 |
|---------------|------|------|------|------|-----|-----|-----|-----|
| 0.05          | 9.658| 11.110| 13.626| 17.513| 23.678| 34.116| 53.669| 98.208|
| 0.10          | 11.350| 15.024| 22.633| 39.297| 86.017 |
| 0.15          | 13.839| 23.190| 56.097 |
| 0.20          | 17.735| 45.672 |
| 0.25          | 24.303 |

Table 4: The value of $\varepsilon_1^2/\alpha^2 d\tilde{\sigma}/(dcdy)$ (leading contribution, first term in the right-hand side of the master formula (39)) as a function of $z$, $\cos \theta$ (calculated for $\omega_1 = 400$ Mev, $\varepsilon_1 = 6$ GeV).

| $\cos \theta$ | -0.8 | -0.6 | -0.4 | -0.2 | 0.0 | 0.2 | 0.4 | 0.6 | 0.8 |
|---------------|------|------|------|------|-----|-----|-----|-----|-----|
| $\frac{\varepsilon_1^2}{\alpha^2} \frac{d\sigma_B}{dc}$ | 93.317| 60.706| 49.428| 44.994| 44.351| 47.084| 54.584| 72.444| 129.944 |

Table 5: Born cross section (40) (without factor $\alpha^2/\omega_1^2$) for $\omega_1 = 400$ Mev, $\varepsilon_1 = 6$ GeV.

Large amounts of the leading contribution near the kinematic bound can be understood as manifestation of the $\delta(y-y_0)$ character of the differential cross section. The $y_0$, $z_0$ dependence is given in Table 6.

| $\cos \theta$ | -0.8 | -0.6 | -0.4 | -0.2 | 0.0 | 0.2 | 0.4 | 0.6 | 0.8 |
|---------------|------|------|------|------|-----|-----|-----|-----|-----|
| $y_0$         | 0.417| 0.263| 0.192| 0.152| 0.125| 0.106| 0.093| 0.082| 0.074|
| $z_0$         | 0.423| 0.455| 0.489| 0.526| 0.571| 0.625| 0.690| 0.769| 0.870|
Fig. 1: The leading order radiative corrections as $\cos \theta$ distribution (see formulae (31)).

Table 6: The value of $y_0$, $z_0$ as a function of $c$ for $\eta = 0.064$ and $\rho = 0.4$.

| $y \setminus \cos \theta$ | -0.8 | -0.6 | -0.4 | -0.2 | 0.0 | 0.2 | 0.4 | 0.6 | 0.8 |
|---------------------------|------|------|------|------|-----|-----|-----|-----|-----|
| 0.05                      | 0.70 | -1.97| -7.41| -15.54| -26.90| -42.70| -65.40| -100.64| -166.21|
| 0.10                      | 0.36 | -3.20| -9.85| -18.38| -18.35|      |      |      |      |
| 0.15                      | 0.03 | -3.38| -1.34|      |      |      |      |      |      |
| 0.20                      | -0.20| 0.29 |      |      |      |      |      |      |      |
| 0.25                      | -0.25|      |      |      |      |      |      |      |      |

Table 7: The value of $\tilde{K}_h$ as a function of $y$, $\cos \theta$ (calculated for $\eta = 0.064$).

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[1] I. Ginzburg, et. al, JETP Lett. 34, 491 (1981).
[2] L.M. Brown, R.P. Feynman, Phys.Rev. 85 231, (1952)

A.I. Akhiezer, V.B. Berestetski "Quantum Electrodinamics" Nauka, Moscow (1981), formulae (5.5.13)

[3] F. Mandl and T. H. R. Skyrme Proc. Roy. Soc A 215 , 497 (1952).

[4] H. Veltman, Phys. Rev. D 40 (1989) 2810; E. ibid. D42 (1990) 1856

[5] M. L. Swartz, Phys.Rev. D 58, 014010 (1998), hep-ph/9711447

[6] A. Denner, S. Dittmaier, Nucl. Phys. B 407, 43 (1993)

S. Dittmaier, Nucl. Phys. B 423, 384 (1994)

A. Denner, S. Dittmaier, Nucl. Phys. B 540, 58 (1999).

[7] T. Kinoshita, J.Math.Phys. 3, 650 (1962)

T.D. Lee, M. Nauenberg, Phys. Rev. B 133, 1549 (1974).

[8] S. Jadach, M. Skrzypek and B. F. Ward, Phys. Rev. D 47, 3733 (1993).

[9] E. A. Kuraev, V.S. Fadin, Yad.Fiz. 41, 733 (1985).

[10] V. N. Baier, V.S. Fadin and V.A. Khoze, Nucl. Phys. B 65, 381 (1973).

[11] F. Berends, et al., Nucl. Phys. B 206, 61 (1982).