Abstract

We present the most recent set of world averages for $D^0$-$\bar{D}^0$ mixing and $CP$ violation parameters, as obtained by the Heavy Flavor Averaging Group from a global fit to various measurements. The values obtained for the mixing parameters when allowing for $CP$ violation are $x = (0.98 \pm 0.26)\%$ and $y = (0.83 \pm 0.16)\%$; the significance of mixing is 10.2$\sigma$. There is no evidence for $CP$ violation at the current level of sensitivity.

Introduction

In 2006, the Heavy Flavor Averaging Group (HFAG) [1] convened a new subgroup to calculate world average (WA) values of charm mixing and $CP$ violation ($CPV$) parameters [2]. Since that time, $D^0$-$\bar{D}^0$ mixing has been observed, and a wealth of mixing and $CPV$ results have appeared. The HFAG charm group has calculated several sets of WA values, updating old averages as new results have become available. This paper presents the most recent set of averages, i.e., those based on results that appeared in preprint form by the summer of 2009.

Mixing in the $B^0$ and $B_s^0$ heavy flavor systems is governed by the short-distance box diagram. In the $D^0$ system, however, this diagram is doubly-Cabibbo-suppressed (relative to amplitudes dominating the decay width) and also GIM-suppressed. Thus, the short-distance mixing rate is tiny, and $D^0$-$\bar{D}^0$ mixing is expected to be dominated by long-distance processes. These are difficult to calculate reliably, and theoretical estimates for $D^0$-$\bar{D}^0$ mixing range over 2-3 orders of magnitude [3,4].

The decay rates for $D^0 \to f$ and $\bar{D}^0 \to \bar{f}$ are, respectively,

$$\frac{dN_{D^0}}{dt} \propto e^{-\Gamma t} \left\{ R^+ + \left( \frac{q}{p} \right) \sqrt{R^+} \times \right.$$

$$(y' \cos \phi - x' \sin \phi) (\Gamma t) + \left. \left( \frac{q}{p} \right) \frac{(x'^2 + y'^2)}{4} (\Gamma t)^2 \right\}$$

(1)

$$\frac{dN_{\bar{D}^0}}{dt} \propto e^{-\Gamma t} \left\{ R^- + \left( \frac{p}{q} \right) \sqrt{R^-} \times \right.$$

$$(y' \cos \phi + x' \sin \phi) (\Gamma t) + \left. \left( \frac{p}{q} \right) \frac{(x'^2 + y'^2)}{4} (\Gamma t)^2 \right\}.$$  

(2)

In these expressions, $x' = x \cos \delta + y \sin \delta$ and $y' = y \cos \delta - x \sin \delta$, where $x = (M_2 - M_1)/\Gamma$ and $y = (\Gamma_2 - \Gamma_1)/(2\Gamma)$ are mixing parameters, and $\delta$ is the strong phase difference between amplitudes $A(\bar{D}^0 \to f)$ and $A(D^0 \to \bar{f})$. Parameters $M_1$, $M_2$, $\Gamma_1$, and $\Gamma_2$ are the masses and decay widths of the mass eigenstates $|D_1\rangle \equiv p(D^0) + q(\bar{D}^0)$ and $|D_2\rangle \equiv p(D^0) - q(\bar{D}^0)$, and $\Gamma = (\Gamma_1 + \Gamma_2)/2$. Our convention is $|C| = -|D|$ such that for $q = p$, $D_1$ is $CP$-odd and $D_2$ is $CP$-even.

To obtain WA values of $x, y, \delta, |q/p|$, and $\phi$, we perform a global fit to 28 measured observables. These observables are from measurements of $D^0 \to K^+\ell^+\nu$, $D^0 \to K^+K^-\pi^+\pi^-$, $D^0 \to K^+\pi^-\pi^0$, $D^0 \to K_S^0\pi^+\pi^-$, and $D^0 \to K^+K^-K_S^0$ decays [5], and from double-tagged branching fractions measured in $e^+e^- \to \psi(3770) \to DD$ reactions. To fit these observables, we must include an additional strong phase $\delta_K$ (see below). For $D^0 \to K^+\pi^-$ decays, we combine $R^+$ and $R^-$ into parameters $R_D \equiv (R^+ + R^-)/2$ and $A_D \equiv (R^+ - R^-)/(R^+ + R^-)$. Correlations among observables are accounted for by using covariance matrices provided by the experimental collaborations.

The global fit determines central values and errors for $x, y, \delta, R_D, A_D, |q/p|, \phi$, and $\delta_K$ using a $\chi^2$ statistic. Parameters $x$ and $y$ govern mixing, and parameters $A_D, |q/p|$, and $\phi$ govern $CPV$. The parameter $\delta_K$ is the strong phase difference between amplitudes $A(\bar{D}^0 \to K^+\pi^-\pi^0)$ and $A(D^0 \to K^+\pi^-\pi^0)$ evaluated at $M_{K^+\pi^-\pi^0} = M_{K^-}(890)$.

Input Observables

The global fit determines central values and errors for $x, y, \delta, R_D, A_D, |q/p|, \phi$, and $\delta_K$ using a $\chi^2$ statistic. Parameters $x$ and $y$ govern mixing, and parameters $A_D, |q/p|$, and $\phi$ govern $CPV$. The parameter $\delta_K$ is the strong phase difference between amplitudes $A(\bar{D}^0 \to K^+\pi^-\pi^0)$ and $A(D^0 \to K^+\pi^-\pi^0)$ evaluated at $M_{K^+\pi^-\pi^0} = M_{K^-}(890)$.

All input values are listed in Table 1. The values for observables $R_M = (x' + y')/2$ [6], $y_{CP}$ [7], and $A_\Gamma$ [7] are HFAG WA values [8]. They are calculated as weighted averages of measurements, taking into account correlations.
among systematic errors and sometimes also statistical errors. As an example, the weighted average for $\eta_{CP}$ is shown in Fig. 1. The values of observables from $D^0 \to K^+K^-$ decays [9] for no-CPV are HFAG WA values [8], but for the CPV-allowed case only Belle measurements are available. The $D^0 \to K^+\pi^-$ results used [10] are from Belle, Babar, and CDF, as these results have much greater precision than earlier ones. The $D^0 \to K^+\pi^-\pi^0$ results are from Babar [11], and the $\psi(3770) \to D\bar{D}$ results are from CLEOc [12].

The relationships between the observables and the fitted parameters are listed in Table 2. For each set of correlated observables, we construct a difference vector $\vec{V}$. For example, $V = (\Delta x, \Delta y, \Delta |q/p|, \Delta \phi)$ for $D^0 \to K^0_S \pi^+\pi^-$ decays, where $\Delta$ represents the difference between the measured value and the fitted parameter value. The contribution of a set of observables to the $\chi^2$ is calculated as $\vec{V} \cdot (M^{-1}) \cdot \vec{V}^T$, where $M^{-1}$ is the inverse of the covariance matrix for the measurement. All covariance matrices used are listed in Table 1.

**Fit results**

The global fit uses MINUIT with the MIGRAD minimizer, and all errors are obtained from MINOS. Three separate fits are performed: (a) assuming CP conservation ($A_D$ and $\phi$ are fixed to zero, $|q/p|$ is fixed to one); (b) assuming no direct CPV ($A_D$ is fixed to zero); and (c) allowing full CPV (all parameters floated). Results from the first and last fits are listed in Table 3. For the CPV-allowed fit, individual contributions to the $\chi^2$ are listed in Table 4. The total $\chi^2$ is 26.3 for $28 - 8 = 20$ degrees of freedom; this corresponds to a confidence level of 0.16.

Confidence contours in the two dimensions ($x,y$) and ($|q/p|, \phi$) are obtained by letting, for any point in the two-dimensional plane, all other fitted parameters take their preferred values. The resulting 1-$\sigma$-5-$\sigma$ contours are shown in Fig. 2 for the CP-conserving case, and in Fig. 3 for the CPV-allowed case. The contours are determined from the increase of the $\chi^2$ above the minimum value ($\chi^2_{\text{min}}$). One observes that the ($x,y$) contours for no-CPV and for CPV-allowed are almost identical. In the latter case, the $\chi^2$ at the no-mixing point $(x,y) = (0,0)$ is 110 units above the minimum value; this difference corresponds to a confidence level of 10.2$\sigma$. Thus, no mixing is excluded at this high level. In the ($|q/p|, \phi$) plot, the no-CPV point (1,0) is within the 1$\sigma$ contour; thus the data is consistent with CP conservation.

One-dimensional confidence curves for individual parameters are obtained by letting, for any value of the pa-
Table 1: Input values used for the global fit, from Refs. [6, 7, 9, 10, 11, 12].

| Observable | Value | Comment |
|------------|-------|---------|
| $y_{CP}$   | $(1.107 \pm 0.217\%)$ | WA $D^0 \rightarrow K^+K^-/\pi^+\pi^-$ and $D^0 \rightarrow K^+K^-K^0_S$ results [8] |
| $A_{CP}$   | $(0.123 \pm 0.248\%)$ | |
| $x$ (no CPV) | $(0.811 \pm 0.334\%)$ | No CPV: |
| $y$ (no CPV) | $(0.309 \pm 0.281\%)$ | |
| $|q/p|$ (no direct CPV) | $0.95 \pm 0.22 \pm 0.10$ | WA $D^0 \rightarrow K^0_S \pi^+\pi^-$ results [8] |
| $\phi$ (no direct CPV) | $(-0.035 \pm 0.19 \pm 0.09)$ rad | |

$CPV$-allowed:

Belle $D^0 \rightarrow K^0_S \pi^+\pi^-$ results. Correlation coefficients:

$$
\begin{pmatrix}
1 & 0.007 & -0.255\alpha & 0.216 \\
-0.007 & 1 & -0.019\alpha & -0.280 \\
-0.255\alpha & -0.019\alpha & 1 & -0.128\alpha \\
0.216 & -0.280 & -0.128\alpha & 1
\end{pmatrix}
$$

Note: $\alpha = (|q/p| + 1)^2/2$ is a variable transformation factor.

$R_M$ (0.0130 $\pm$ 0.0269\%) | WA $D^0 \rightarrow K^+\ell^-\nu$ results [8] |

$z''$ (2.61 $\pm$ 0.57 $\pm$ 0.39\%) | Babar $D^0 \rightarrow K^+\pi^-\pi^0$ result. Correlation coefficient = $-0.75$. |

$y''$ (0.06 $\pm$ 0.56 $\pm$ 0.34\%) | Note: $z'' \equiv x \cos \delta_{K^{+}\pi^{-}\pi^{0}} + y \sin \delta_{K^{+}\pi^{-}\pi^{0}}; y'' \equiv x \cos \delta_{K^{+}\pi^{-}\pi^{0}} - x \sin \delta_{K^{+}\pi^{-}\pi^{0}}$. |

CLEO results from "double-tagged" branching fractions measured in $\psi(3770) \rightarrow DD$ decays. Correlation coefficients:

$$
\begin{pmatrix}
1 & -0.064 & 0.0072 & 0.0607 \\
-0.064 & 1 & -0.3172 & -0.8331 \\
0.0072 & -0.3172 & 1 & 0.3893 \\
0.0607 & -0.8331 & 0.3893 & 1
\end{pmatrix}
$$

Note: the only external input to these fit results are branching fractions.

$R_D$ (0.303 $\pm$ 0.0189\%) | Babar $D^0 \rightarrow K^+\pi^-$ results. Correlation coefficients: |

$z^{2+}$ (0.024 $\pm$ 0.052\%) | 1 \quad 0.77 \quad -0.87 \\
$y^{2+}$ (0.98 $\pm$ 0.78\%) | 0.77 \quad 1 \quad -0.94 \\
$y^{2-}$ (0.96 $\pm$ 0.75\%) | -0.87 \quad -0.94 \quad 1 \\

$A_D$ (2.3 $\pm$ 4.7\%) | Babar $D^0 \rightarrow K^+\pi^-$ results; correlation coefficients same as above. |

$z^{2-}$ (0.020 $\pm$ 0.050\%) | 0.364 $\pm$ 0.018\% | Belle $D^0 \rightarrow K^+\pi^-$ results. Correlation coefficients: |

$y^{2+}$ (0.032 $\pm$ 0.037\%) | 0.655 \quad 1 \quad -0.909 \\
$y^{2-}$ (0.12 $\pm$ 0.58\%) | -0.834 \quad -0.909 \quad 1 \\

$A_D$ (2.3 $\pm$ 4.7\%) | Belle $D^0 \rightarrow K^+\pi^-$ results; correlation coefficients same as above. |

$z^{2-}$ (0.006 $\pm$ 0.034\%) | 0.304 $\pm$ 0.055\% | CDF $D^0 \rightarrow K^+\pi^-$ results. Correlation coefficients: |

$y^{2+}$ (0.012 $\pm$ 0.038\%) | 0.923 \quad 1 \quad -0.971 \\
$y^{2-}$ (0.20 $\pm$ 0.54\%) | 0.923 \quad 1 \quad -0.984 \\

$y'$ (0.85 $\pm$ 0.76\%) | -0.971 \quad -0.984 \quad 1 |
Table 2: Left: decay modes used to determine the parameters $x$, $y$, $\delta$, $R_D$, $A_D$, $|q/p|$, $\phi$, and $\delta_{K\pi\pi}$. Middle: observables measured for each decay mode. Right: the relationships between the observables and the fitted parameters.

| Decay Mode | Observables | Relationship |
|------------|-------------|--------------|
| $D^0 \to K^+ K^- /\pi^+ \pi^- /K^+ K^- K^0_S$ | $y_{\text{CP}}$ | $2y_{\text{CP}} = ((|q/p| + |p/q|) y \cos \phi - (|q/p| - |p/q|) x \sin \phi$ |
| | $A_\phi$ | $2A_\phi = ((|q/p| - |p/q|) y \cos \phi - (|q/p| + |p/q|) x \sin \phi$ |
| $D^0 \to K^0_S \pi^+ \pi^-$ | $x$ | $x = \frac{y - 2y_{\text{CP}}}{4}$ |
| | $y$ | $y = \frac{2A_\phi - 2y_{\text{CP}}}{4}$ |
| | $\phi$ | $\phi = \arctan \frac{y}{x}$ |
| $D^0 \to K^+ e^- \nu$ | $R_M$ | $R_M = (x^2 + y^2)/2$ |
| (Dalitz plot analysis) | $x''$ | $x'' = x \cos \delta_{K\pi\pi} + y \sin \delta_{K\pi\pi}$ |
| | $y''$ | $y'' = y \cos \delta_{K\pi\pi} - x \sin \delta_{K\pi\pi}$ |
| “Double-tagged” branching fractions measured in $\psi(3770) \to DD$ decays | $R_M$ | $R_M = (x^2 + y^2)/2$ |
| | $y$ | $y = \frac{2A_\phi - 2y_{\text{CP}}}{4}$ |
| | $R_D$ | $R_D = (R^+ + R^-)/2$ |
| | $A_D = (R^+ - R^-)/(R^+ + R^-)$ | $x' = x \cos \delta + y \sin \delta$ |
| | | $y' = y \cos \delta - x \sin \delta$ |
| | | $A_M \equiv (|q/p|^4 - 1)/(|q/p|^4 + 1)$ |
| | | $x^{\pm} = \left((1 \pm A_M)/(1 \mp A_M)\right)^{1/4}(x' \cos \phi \mp y' \sin \phi)$ |
| | | $y^{\pm} = \left((1 \pm A_M)/(1 \mp A_M)\right)^{1/4}(y' \cos \phi \mp x' \sin \phi)$ |

Figure 2: Two-dimensional contours for mixing parameters $(x, y)$, for no CPV.

Table 3: Left: decay modes used to determine the parameters $x$, $y$, $\delta$, $R_D$, $A_D$, $|q/p|$, $\phi$, and $\delta_{K\pi\pi}$. Middle: observables measured for each decay mode. Right: the relationships between the observables and the fitted parameters.

Summary

We summarize the fit results listed in Table 2 and shown in Figs. 3 and 4 as follows:

- The experimental data consistently indicate that $D^0$ mesons undergo mixing. The no-mixing point $(x, y) = (0, 0)$ is excluded at 10.2σ. The parameter $x$ differs from zero by 3.2σ, and the parameter $y$ differs from zero by 4.8σ. The effect is presumably dominated by long-distance processes, which are difficult to calculate.

- Since $y_{\text{CP}}$ is positive, the CP-even state is shorter-lived, as in the $K^0 - \overline{K^0}$ system. However, since $x$ is also positive, the CP-even state is heavier, unlike in the $K^0 - \overline{K^0}$ system.

- The strong phase difference $\delta$ is probably not small: the fitted value is $(26.4 \pm 9.9)^\circ$.

- There is no evidence yet for CPV in the $D^0 - \overline{D^0}$ system. Observing CPV at the current level of sensitivity would indicate new physics.

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Figure 4: The function $\Delta \chi^2 = \chi^2 - \chi^2_{\text{min}}$ for parameters $x$, $y$, $\delta$, $\delta_{K\pi\pi}$, $|q/p|$, and $\phi$. The points where $\Delta \chi^2 = 3.84$ (denoted by the dashed horizontal line) determine a 95% C.L. interval.
References

[1] See http://www.slac.stanford.edu/xorg/hfag/

[2] The group has since expanded its activities to include calculating WA values of hadronic branching fractions, semileptonic form factors, properties of excited \( D_{s}^{*+} \) states, and the \( D_{s}^{+} \) decay constant \( f_{D_{s}} \).

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[5] Charge-conjugate modes are implicitly included unless noted otherwise.

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