Implementation of an optimization algorithm using modified Lagrange functions on the example of a steel hinge-rod system

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Abstract. An example of the optimal design of a steel truss with the use of modified Lagrange functions is considered. To solve the problem, it is necessary to select the cross-sectional areas of the truss at a given interval by minimizing its volume, provided that the regulatory requirements for strength and rigidity are met. The external diameters of the sections of the truss elements vary. A detailed algorithm for solving the problem is presented. The algorithm is implemented in the mathematical package MathCAD, which allows you to visually trace the sequence of commands, as well as receive graphs reflecting the state of the problem at each iteration.

1. Introduction
In this paper, we propose for consideration an algorithm for the design calculation of flat rod systems, where the condition for the optimality of these systems is put forward, and the normative requirements for their behavior with a given degree of error are formulated.

This formulation can be formalized as a conditionally extreme nonlinear programming problem [1-2].

\[
\text{find } \min f(x), \quad x \in E^n, \quad (1)
\]

with constraints:
\[
 g_j(x) \leq 0, \quad j = 1,2...m, \quad (2)
\]

\[
 x_i^L \leq x_i \leq x_i^U, \quad i = 1,2...nx, \quad (3)
\]

where \( f(X) \) is the objective function of the varied parameters, \( \{X\} \) is the vector of these parameters on the interval \( \{XL\} \) - \( \{XU\} \), \( g(X) \) are the functions of constraints in the form of equalities and inequalities.

Based on the research studied, considering the issue of optimization in the design of rod systems [3-5], and publications describing various variants of methods for solving conditionally extreme problems [6-15], an iterative algorithm for solving the problem was adopted for the numerical solution of the problem (1-3), which operates with the modified Lagrange function \( FP(X, Y) \) [16].

\[
 F_p = k_f F_L + 0.5 \{ g \}^T [k][g] + 0.5k_f \{ Y \}^T \{ (\delta - [I]) \{ \Delta Z \} \}, \quad (4)
\]

This expression \( F_L = f(x) + \{ Y \}^T \{ g \} \), is the standard Lagrange function. \([I]\) – unit matrix; \([k]\)– diagonal matrix of penalty coefficients; \(k_f\) – normalizing multiplier that increases the stability of the algorithm; \(\Delta Z_j\) – value of shift of the \(j\)-th restriction into the admissible area;
[δ] – is a diagonal matrix of Boolean variables, the elements of which are determined from the condition:

\[ \delta_{jj} = \begin{cases} 1, & g_j + \Delta Z_j > 0 \\ 0, & \text{otherwise} \end{cases} \]

The problem is solved in the space of vectors of lines \{X\} and dual variables \{Y\} and is reduced to finding the saddle point of the function \( F_p \) from the condition:

\[ \max_{y \in \mathbb{R}^n} \min_{x \in \mathbb{R}^m} F_p(x, y). \]

We will use a version of this algorithm, when at each iteration \( t \) direct variables are determined from the condition for minimizing the function \( F_P \):

\[ \{X\} \in \text{Arg} \min F_p(X', Y') \]

\[ \{X^L\} \leq \{X\} \leq \{X^U\}, \quad (5) \]

and the dual variables from the equality of condition of stationarity by \( X \) of the Lagrange function, \( FL \) and \( FP \) which, after appropriate transformations, has the form:

\[ y^*_{jj} = \max \left( y_j + \frac{k_j'}{k_j} g \left( u^* \right) \right). \quad (6) \]

This algorithm was studied in [17].

The iterative process ends according to the convergence condition:

\[ \left| X(t) - X(t-1) \right| \leq \varepsilon \left| X(t) \right|, \quad g_j \leq \varepsilon_g, \quad j = 1...m, \quad (7) \]

or when exceeding the specified limit number of iterations \( it \_ lim \).

In expression (8) \( \varepsilon, \varepsilon_g \) – is a given computational error; \( t \) – is the iteration number.

2. Formulation of the problem

Let’s consider the implementation of this algorithm using the example of the design calculation of a 19-bar truss.

The geometry and loading conditions of a nineteen-bar truss are shown in Figure 1.

![Figure 1. Geometry and applied loads of the nineteen-bar truss.](image)

Initial data: \( d = 3 \) m; \( h = 2.6 \) m; \( F = 400 \) kN. Physical characteristics: modulus of elasticity \( E = 20600 \) MPa; design resistance \( R_u = 34.5 \) MPa. Limit value of displacements of nodes \( |\Delta| = \pm 0.014 \) m.

Statement of the problem: it is required to select the diameters of the sections of the truss elements at a given interval by minimizing its volume, provided that the normative requirements for strength and rigidity are met.

The external diameters of sections of elements 1-10 vary. The diameters of elements 11-19 were taken from the symmetry condition of the truss. The thicknesses were specified as part of the diameters from the condition: \( tw = de / nt \) (in this example \( nt = 22 \) (Figure 2). The initial values of the diameters are \( de_0 = 12 \) cm \([1 \div 50 \text{ cm}]\).
The objective function is the volume of the truss.

\[ f(X) = \left( \frac{1}{C} \right) \cdot \left[ L^T \cdot Xe \right]^2 \]

where \( L \) – length of elements, \( Xe \) – diameters of truss elements.

The restrictions are as follows [18-20]:

Strength check in the \( i \)-th truss element:

\[ g_i = k_s \left( \frac{|N|}{\varphi \cdot R_y \cdot A} \right) - \varepsilon_i \leq \varepsilon \], \( i = 1 \ldots 10 \) \hspace{1cm} (9)

Constraint on vertical movement of node 8:

\[ g_{20} = k_s \left( \frac{|Z_{18}|}{\Delta} \right) - 1 \leq \varepsilon \], \hspace{1cm} (10)

where \( Z \) is the full vector of nodal displacements.

The restrictive function numbers were as follows:

- constraints 1-19 – for strength in elements 1-19;
- constraints 20 – on node displacement 8 (\( |\Delta| = 1.4 \text{ cm} \)).

Here, the \( kg \) parameter is introduced to improve the stability of the algorithm.

The \( eg \) value specifies the required computational accuracy.

Thus, the number of varied variables \( nx = 10 \), and the number of constraints \( m = 20 \).

The optimization algorithm parameters are adopted as follows:

- the minimum value of the penalty coefficient (\( k_{\text{min}} \)) – 30;
- maximum shift value (\( \Delta Z_{\text{max}} \)) – 0.03;
- coefficient of normalization of the objective function (\( kf \)) – 1;
- coefficient of normalization of functions of restrictions (\( kg \)) – 30.

3. Algorithm for solving the problem

The solution was carried out in the universal mathematical package \textit{MathCAD}. Here is a sequence of operations with designations that are accepted in the \textit{MathCAD} program:

1. Input of initial data:
   1.1. Assignment of geometric \( (d, h) \) and physical \( (E, R_y) \) parameters of the problem, as well as the tolerance for displacement of nodes \( A_{\text{lim}} \).
   1.2. Setting the truss topology:
      - number of elements \( ne \), number of nodes \( np \), number of support links \( nop \);
      - coordinates of nodes \( xp, yp \);
      - connections of nodes for each element (array \( ni \));
      - zero degrees of freedom, in the direction of the support links (vector \( iop \)).
   1.3. Assignment of the initial parameters of \( X \) variation on the \( X_{\text{min}} \) and \( X_{\text{max}} \) ranges.
1.4. Assignment of the optimization algorithm parameters: dimension of the constraint vector \( m \); normalization coefficients of the objective function and the constraint vector \( k_f, k_g \); the minimum value of the penalty coefficient \( k_{min} \) and the maximum value of the shift value outside the permissible range \( \Delta Z_{max} \).

1.5. Assignment of the vector of dual variables \( y_i \) \((i=1...m)\).

2. Formalization of the objective function \( f(X) \) and the vector of the constraint function \( g(X) \) through the variable parameters:

2.1. Formation of the objective function \( f(X) \) through variable parameters.

2.2. Writing a subroutine to go from the number of variable parameters \( X \) to the number of truss elements \( X_e \).

2.3. Writing a subroutine for determining coefficient of accounting for longitudinal bending \( \phi \).

2.4. Writing a subroutine for calculating the moment and radius of gyration of a tubular section \( I_s \), is.

2.5. Writing a subroutine for calculating the conditional flexibility of the bar \( \lambda \).

2.6. Writing a subroutine FEA \((X)\), where the finite element calculation of the truss is implemented. The output parameters of this subroutine are contained in an array, which includes five vectors - \( X_e \) (number of elements), \( Z \) (nodal displacements), \( Nabs \) (values of forces in absolute value), \( L \) (lengths of elements) \( \phi \) (coefficient of accounting for longitudinal bending).

Thus, the indices 1, 2, 3, 4, 5 that occur when accessing this sub-program indicate which of these five arrays is being calculated. Nodal displacements and forces are functions of variable parameters \( X \) (section diameters of elements), therefore, they are defined as \( Z(X) \) and \( Nabs(X) \). The length of the elements \( L \) does not change in the process of optimization, therefore, it is calculated once.

2.7. Formation of the constraint vector \( g_1(X) \), which contains 19 checks on the strength of each element of the truss, multiplied by the normalization factor \( k_g \).

2.8. Formation of the stiffness constraint \( g_2(X) \), which contains a check for vertical displacement of the truss node 8 (this displacement has an index of 16 in the general vector of nodal displacements \( Z \)). The constraint is also multiplied by the normalization factor \( k_g \).

2.9. Formation of the complete constraint vector \( g(X) \) by joining the vectors \( g_1(X) \) and \( g_2(X) \). This vector has 20 elements, where 19 corresponds to the strength constraints, and the 20th corresponds to the stiffness constraint.

3. Organization of the iterative cycle of the search optimization algorithm:

3.1. Formation of diagonal elements of the matrix of penalty coefficients \( k_{ij} \) \((i=1..m)\).

3.2. Formation of elements of the vector \( \Delta Z_i \) \((i=1..m)\).

3.3. Formation of the elements of the matrix of Boolean variables \( \delta_{ij} \).

3.4. Formation of the Lagrange function \( FL(X) \), as well as the modified Lagrange function \( FP(X) \).

3.5. Finding the minimum of the function \( FP(X) \) by \( X \) on the interval \( X_{min} \) and \( X_{max} \). This task is solved by means of the MathCAD program. It should be noted that with each call to the calculation of the function \( FP \), there is a call to the problem of finite element analysis of the truss.

3.6. Entering the maximum value in the vector of constraints on iterations \( it \) into the vector \( G_i \).

3.7. Calculation of the elements of the vector of dual variables \( y_i \) \((i=1..m)\).

4. Output of results:

4.1. The values of the vectors of change in the target and maximum values of the restrictive functions at iterations.

4.2. The values of the optimal diameters of the truss elements \( X_e \).

4.3. Values of residuals of constraints without taking into account the coefficient \( k_g \).

4.4. Graphs of changes in the target and restrictive functions at iterations.

4.5. Rounding of section diameters to the required digit capacity.

MathCAD program listings are shown in Figures 3-6.
Figure 3. Implementation of numerical methods in the MathCAD package: initial data of the problem.
Figure 4. Optimization result.
Figure 5. The optimal solution subject to rounding.

Figure 6. Graphs of changes in the objective function and maximum residuals of constraints on iterations.

4. Evaluation of the results of the algorithm implementation

As you can see from the graphs, a solution close to the optimal was obtained already at the 5th iteration of the search process. The following restrictions were active: $-3.721 \cdot 10^{-10}$ by strength in elements 5 and 15; $-3.721 \cdot 10^{-10}$ by strength in elements 6 and 16; $-3.721 \cdot 10^{-10}$ by rigidity for vertical displacement at node 8. The calculation accuracy was estimated by discrepancy of these constraints and amounted to $10^{-8}$ to $10^{-10}$.

The main difficulties in the implementation of the task were associated with setting up the algorithm by setting such parameters as the coefficients of normalization of the objective and restrictive functions, the penalty coefficient. It is revealed that the algorithm gives the most stable convergence if the values of the objective function have the same order as the penalty additions.

5. Conclusion

The solution of the optimization problem for a flat rod system has shown the effectiveness of the presented algorithm. At the same time, its peculiarity associated with tuning for stable convergence was noted. For this, the corresponding penalty and normalizing coefficients are introduced.

The algorithm is presented in the mathematical package MathCAD, which makes it possible to visualize its commands and allows you to use this algorithm to solve similar problems in project or educational activities.
In software implementation of practical optimization problems for complex rod systems with a large number of parameters, it is more expedient to perform the problem of finding an unconditional extremum using zero-order methods, such as the deformable polyhedron method, or based on metaheuristic algorithms.

Further development of the direction of optimization of planar and spatial rod systems can be obtained by building optimization algorithms based on neural networks.

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