Magnetic field amplification in accretion discs around the first stars: implications for the primordial IMF

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ABSTRACT

Magnetic fields play an important role in the dynamics of present-day molecular clouds. Recent work has shown that magnetic fields are equally important for primordial clouds, which form the first stars in the Universe. While the primordial magnetic field strength on cosmic scales is largely unconstrained, theoretical models strongly suggest that a weak seed field existed in the early Universe. We study how the amplification of such a weak field can influence the evolution of accretion discs around the first stars, and thus affect the primordial initial mass function (IMF). We perform a suite of 3D magneto-hydrodynamic (MHD) simulations with different initial field strengths and numerical resolutions. We find that, in simulations with sufficient spatial resolution to resolve the Jeans scale during the collapse, even initially weak magnetic fields grow exponentially to become dynamically important due to both the so-called small-scale turbulent dynamo and the large-scale mean-field dynamo. Capturing the small-scale dynamo action depends primarily on how well we resolve the Jeans length, while capturing the large-scale dynamo depends on the Jeans resolution as well as the maximum absolute resolution. Provided enough resolution, we find that fragmentation does not depend strongly on the initial field strength, because even weak fields grow to become strong. However, fragmentation in runs with magnetic fields differs significantly from those without magnetic fields. We conclude that the development of dynamically strong magnetic fields during the formation of the first stars is likely inevitable, and that these fields had a significant impact on the primordial IMF.

Key words: stars:Population III – stars:formation – turbulence – magnetohydrodynamics – early Universe – ISM:magnetic fields

1 INTRODUCTION

From the formation of molecular clouds to their collapse into protostar-accretion disc systems, turbulence and magnetic fields play several roles in setting the overall direction for a star formation episode. While extensive studies have been carried out to investigate the role of turbulent magnetic fields in present-day star formation (see reviews by Crutcher 2012; Han 2017; Wurster & Li 2018; Hennebelle & Inutsuka 2019; Krumholz & Federrath 2019; Crutcher & Kemball 2019; Zhao et al. 2020), only a handful of 3D simulations have looked at their role in the early Universe, especially during the formation of the first generation of stars (Machida et al. 2008; Sur et al. 2010; Schleicher et al. 2010; Turk et al. 2012; Latif et al. 2013; Machida & Doi 2013; Latif et al. 2014; Liao et al. 2019; Grete et al. 2019). This is primarily due to the lack of solid constraints on the magnetic field strength and topology in the early Universe (Widrow 2002; Giovannini 2004; Widrow et al. 2012; Ryu et al. 2012; Wagstaff et al. 2014; Subramanian 2016). However, there is a growing consensus on the presence of a cosmic-scale primordial field, no matter how weak (Brandenburg et al. 1996; Hammond et al. 2012; Subramanian 2016; Planck Collaboration et al. 2016). This motivates studying magnetic fields that may be amplified from the primordial field during the
collapse of molecular clouds, leading to Population III star formation.

Several studies have conclusively shown that the presence of a turbulent dynamo (Kazantsev 1968; Meneguzzi et al. 1981; Brandenburg & Subramanian 2005; Subramanian 2016) can exponentially amplify any weak seed field to near-saturation values (e.g., Federrath et al. 2011b; Schober et al. 2012; Federrath et al. 2014; Schober et al. 2015; Federrath 2016; Xu & Lazarian 2016; McKee et al. 2020). In the early Universe, the presence of such a turbulent dynamo driven by gravity is expected when baryonic matter starts collapsing towards the centre of dark matter minihaloes (Greif et al. 2008; Wise et al. 2008; Turk et al. 2012; Grete et al. 2019). This infall leads to the creation of over-dense regions that harbour the first molecular clouds where Population III star formation ultimately takes place (see reviews by Bromm 2013; Klessen 2019; Haemmerlé et al. 2020). Apart from the action of the small-scale turbulent dynamo, it is also expected that accretion discs around Population III stars may contain a large-scale mean field component (Liao et al. 2019). This can occur if discs undergo differential rotation and angular momentum transport through viscous stresses, thereby generating a large-scale dynamo from a seed field that can sustain a dynamically strong and coherent mean field component (Ruzmaikin et al. 1988a; Brandenburg et al. 1995; Hawley et al. 1996; Stone et al. 1996). In fact, given that the characteristic diffusion timescale in accretion discs is very short ($10^2 - 10^4$ s) as compared to viscous timescales (order of few yr), dynamically strong magnetic fields that last for the lifetime of the disc can only be generated by a dynamo operating in accretion discs (Ruediger et al. 1995).

The expectation that dynamically-significant magnetic fields might be present during the formation of Population III stars naturally raises the question of how such fields might affect the initial mass function (IMF) of the first stars. In a recent work, Sharda et al. (2020, hereafter, SFK20), we presented the first suite of 3D magneto-hydrodynamical (MHD) simulations of Population III star formation aimed at answering this question. We showed that dynamically strong magnetic fields, if present during the formation of the first stars, suppress fragmentation in primordial clouds, thereby increasing the mean stellar mass and greatly decreasing the prevalence of low-mass Population III stars that could potentially survive to the present day. While radiation feedback is thought to play a dominant role over magnetic fields in setting the present-day stellar IMF (Krumholz et al. 2010; Bate 2012; Myers et al. 2014; Krumholz et al. 2016; Federrath et al. 2017; Guszejnov et al. 2018; Cunningham et al. 2018; Wurster et al. 2019; Krumholz & Federrath 2019), SFK20 argue that this might not be the case for Population III stars because the late onset of radiation feedback due to the absence of dust (Hosokawa et al. 2011, 2012; Sugimura et al. 2020) allows a much longer period when magnetic effects and magnetic pressure can dominate. However, the results of SFK20 do not fully resolve the question of whether magnetic fields significantly influence the first star IMF, because they did not determine the magnetic field strength self-consistently; they only showed that, if fields near dynamo-saturation levels are present, they have a significant effect on the IMF of the first stars. Calculating the field strength self-consistently is a challenging numerical problem, because dynamo amplification is exquisitely sensitive to numerical dissipation, and thus, very high resolution is required to recover even qualitatively correct estimates for the rate of dynamo growth (Federrath et al. 2014; Schober et al. 2015; Federrath 2016; McKee et al. 2020). The simulations of SFK20 only marginally resolve the dynamo action, and thus leave the question of the true magnetic field strength in primordial star-forming regions unsolved.

In this study, we answer this question by studying in detail how dynamo amplification can occur in first star discs. We find that, given sufficient resolution in the disc, even an initially weak field can be exponentially amplified due to the presence of both the small-scale and the large-scale dynamo; the former primarily amplifies the turbulent component of the field whereas the latter amplifies the mean component. We show that the resulting saturation level of the field is high enough that magnetic effects on the IMF are inevitably significant. The remainder of this paper is organised as follows. In Section 2, we describe our suite of simulations. In Section 3 we present our simulation results and discussions; in Section 4, we comment on how our results can potentially impact the primordial IMF, and we summarise the implications of our findings in Section 5.

## 2 SIMULATION SUITE

The simulations presented here are similar to those described in SFK20, where we motivate in detail the choice of initial conditions and numerical methods. Here, we only summarise the key aspects of the simulation setup and methods. For details, we refer the reader to SFK20.

### 2.1 MHD code and basic initial conditions

We perform 3D MHD simulations of Population III star formation using the adaptive mesh refinement (AMR) code FLASH (Fryxell et al. 2000; Dubey et al. 2008), together with the primordial chemistry network from the astro-chemistry package KROME (Grassi et al. 2014). We use sink particles to represent stars (Federrath et al. 2010b); the density threshold for sink particle formation is $n_{\text{sink}} \sim 10^{13}$ cm$^{-3}$. We start the simulations from a spherical core of mass $M_\odot = 10^3 M_\odot$, with uniform density ($n = 9.05 \times 10^3$ cm$^{-3}$), temperature (265 K) and composition (with mass fractions $\mu_\text{H} = 0.7502$, $\mu_\text{He} = 0.0006$, $\mu_\text{H}_2 = 0.2492$) as appropriate for the formation of the first stars at the centre of dark matter minihaloes at a redshift of 30 (Sharda et al. 2019, and references therein). The simulation box is of size $2.49 cpc$ and the boundary conditions are outflow/inflow for the hydrodynamics and isolated for computing gravitational interactions. The initial conditions also include a driven, mixed mode of turbulence (Federrath et al. 2010a, 2011a) that initially follows a velocity power spectrum $P_k \propto k^{-1.8}$, where $k$ is the wave number that spans $2 \leq k \leq 20$. The initial Mach number is trans-sonic, such that the velocity fluctuations equal the local sound speed at the initial temperature. The maximum resolution of the simulations is $\Delta x = 7.6 au$, equivalent to a maximum effective resolution of 65,536$^3$ grid cells.
2.2 Criteria for resolving dynamo action and initial conditions for the magnetic field

In our previous simulations (SFK20), the refinement criteria were set so as to guarantee that, on all levels at or above the finest, the Jeans length (Federrath et al. 2010b),

\[ \lambda_J = \sqrt{\frac{\pi c_s^2}{G\rho}} \]  

(1)

is resolved by at least 32 cells at all times (here, \(c_s\) is the sound speed). These simulations used three different initially turbulent magnetic field strengths of 1 fG, 9 \(\mu\)G and 30 \(\mu\)G. The latter two of these correspond to plausible scenarios whereby the turbulent dynamo saturates at a ratio of magnetic energy, \(E_{mag}\), to turbulent kinetic energy, \(E_{turb,kin}\), of 0.01 and 0.1, respectively (Federrath et al. 2014; Schober et al. 2015; Federrath 2016). The magnetic power spectrum goes as \(P_{mag} \propto k^{-4.5}\) for \(2 \leq k \leq 20\). We use the first and third sets, that is, runs with a field strength of 1 fG and 30 \(\mu\)G, in this analysis\(^1\). We call these runs weakJ32 and strongJ32, to represent that they start with a weak and strong field, respectively, and the Jeans length is refined with 32 cells at all times. SFK20 provide 50 realisations of each of these cases, which are identical in their mean properties, but differ in the random realisation of the turbulent velocity and magnetic fields. We use half of their suite (25 realisations of each magnetic field strength) in this study.

As we discuss in Section 1, dynamo simulations are extremely sensitive to resolution. We therefore repeat these earlier simulations, but at a higher resolution of 64 cells per Jeans length instead of 32 as used by SFK20. We call these two sets of runs weakJ64 and strongJ64, respectively. Our motivation to go to higher Jeans resolution is to check the operation of the turbulent dynamo in the weak-field case; we expect the strong-field case not to show any small-scale dynamo action, since the initially turbulent field should be close to saturation. Note that a higher Jeans resolution does not mean that we resolve the grid to a smaller cell size; higher Jeans resolution simply implies that the grid creates more cells (of the same size) to better resolve the Jeans length. Thus, the minimum value of \(\Delta x\) remains the same in runs between 32 and 64 cells per Jeans length. However, we also discuss two cases below where we increase the maximum resolution, but these are not part of our main simulation suite, because we are unable to perform a large number of such simulations due to computational expense. Indeed, increasing only the Jeans resolution requires substantially more computational time (Federrath et al. 2011b). For example, runs with 64 cells per Jeans length are up to 8 times more expensive than the respective runs with 32 cells per Jeans length. This increased cost of the simulations precludes us from performing higher-resolution runs for the entire suite of 50 simulations presented in SFK20. However, Figure 7 of SFK20 indicates that 25 realisations constitute a large enough sample to allow us to recover the true statistics of the sink mass distribution with reasonable accuracy. In particular, even 25 realisations are sufficient to show a clear distinction between the distributions of sink particle masses produced in magnetised versus purely hydrodynamic simulations, which is the critical question for us. We summarise the full simulation set we use in this paper in Table 1.

| ID       | B      | \(\Delta x\) | \(N_r\) | Source          |
|----------|--------|--------------|--------|-----------------|
| weakJ32  | 1 fG   | 32           | 7.6 au | 25 SFK20        |
| weakJ64  | 1 fG   | 64           | 7.6 au | 25 This Work    |
| strongJ32| 30 \(\mu\)G | 32       | 7.6 au | 25 SFK20        |
| strongJ64| 30 \(\mu\)G | 64       | 7.6 au | 25 This Work    |

3 RESULTS AND DISCUSSIONS

Following SFK20, we stop the runs at a time when the sink particle has accreted 50 M\(_{\odot}\), corresponding to a parameterized star formation efficiency, \(SFE = \sum M_{sink}/M_\Delta = 0.05\), where \(M_\Delta = 10^3 M_\odot\) is the initial cloud mass. We stop the simulations based on this criterion, because we do not include radiation feedback, which starts to play a dominant role for massive first stars (Hosokawa et al. 2011, 2012, 2016; Sugimura et al. 2020). Note that for all the analysis except for the effects of Jeans resolution on fragmentation, we only use the subset of simulations that forms only a single sink particle (~10 out of the 25 realizations in each case). This is because such simulations have a well-defined accretion disc, enabling a cleaner study of the effects of the magnetic-field amplification in the disc. The simulations where secondary fragmentation takes place form more complex disc-like structures characterised by strong spiral density waves and circum-binary or circum-ternary discs. In such cases, studying the amplification of the small- and/or large-scale dynamo is challenging as it would demand that all the accretion discs be well resolved, and the full simulation be followed to a significantly longer time.

Unless explicitly stated otherwise, we calculate all quantities of interest in the frame of reference of the disc once it is formed, averaging over a cylindrical region centred on the sink particle, with the symmetry axis of the cylinder aligned with the angular momentum vector of the mass within 500 au of the sink particle. We define the usual cylindrical basis vectors (\(\hat{r}, \hat{\phi}, \hat{z}\)) to denote position within this analysis region. We find that using an analysis region of radius 500 au and half-height 50 au ensures that the resulting volume covers the entire disc in all our realisations. We have also verified that our results are relatively insensitive to the exact choice of radius and height for our analysis region (provided it is large enough to cover most of the mass of the disc), since we calculate mass-weighted quantities, which means that the low-density material does not contribute significantly to our quantitative analyses of the disc material.

\(^1\) The statistical outcomes of the runs with an initial field strength of 9 and 30 \(\mu\)G are very similar, so we use only the latter for simplicity.

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3.1 Qualitative outcomes

Figure 1 shows the projections of density-weighted number density ($n$), temperature ($T$) and magnetic field strength ($B$) for a representative run from each of the four different suites listed in Table 1. All the snapshots are centred at the single sink particle that forms in the simulations (noting again that for this part of the analysis we select simulations that only form one star), and show the time at which the simulation reaches SFE = 5 percent. $T_s$ denotes the time elapsed since the formation of the sink particle. It is straightforward to notice that the morphology of the system varies significantly in the run weakJ32 as compared to the other three runs. In runs weakJ64, strongJ32 and strongJ64, the snapshots reveal the presence of a hot, spherical bubble that expands radially outwards with time (see movies M1 and M2 attached as online material with this paper for reference) such that there are higher temperatures inside the bubble that lead to more dissociation of $H_2$. A similar resolution-dependent effect has been noted by Turk et al. (2012) during the formation and collapse of dark matter minihaloes in their cosmological simulations with a seed magnetic field. However, this phenomenon does not occur primarily due to magnetic fields. We show in Appendix A that the qualitative difference in the outcome is a result of how well we resolve the length and timescales for chemical evolution and radiative cooling across shock fronts. However, the effect is not related to dynamo amplification, and has little impact on the overall results because the thermal pressure is dynamically-unimportant in all cases. For this reason, we do not discuss it further in the main text.

The difference that is of the greatest interest to us is in the magnetic fields (see bottom panel of Figure 1). Strikingly, we see that the magnetic field strength and morphology of the weakJ64 run is much closer to the results we find for strongJ32 or strongJ64 than to weakJ32. Despite having started from identical initial conditions, the field in weakJ64 is $\sim 3$ orders of magnitude stronger than in weakJ32. To explore this difference further, in Figure 2 we plot mass-weighted, azimuthally-averaged radial profiles of the different components of the magnetic fields for the simulations shown in Figure 1. We define the turbulent component of the field, $B_{\text{turb}}$, as,

$$B_{\text{turb}} = (B_r - \langle B_r \rangle) \hat{r} + (B_\theta - \langle B_\theta \rangle) \hat{\theta} + (B_z - \langle B_z \rangle) \hat{z},$$

(2)

where $B_r$, $B_\theta$, and $B_z$ are the cylindrical components of the total magnetic field, and angle brackets indicate the azimuthal average of a given quantity; we denote the magnitude of the turbulent field as $B_{\text{turb}} \equiv |B_{\text{turb}}|$. In line with the morphological differences between weakJ32 and the other runs, we find that all the components of the field are substantially lower in weakJ32 compared to the other runs. We also see that while the initial magnetic field we imposed is completely random, in all cases except weakJ32, a substantially mean toroidal field develops in the disc, as is clear from the radial profile of $\langle B_\phi \rangle$ in Figure 2. This component is comparable in strength to the turbulent component.

By looking at the time evolution of the magnetic field profiles (available as movie M3 in the supplementary material), we find that initially, when the sink forms, all the three components of the magnetic field are of the same strength. As the disc around the sink starts to grow and expand
outwards to conserve angular momentum, a strong toroidal component of velocity ($v_\phi$) is generated, which winds up the magnetic field in the $\phi$ direction, thus giving rise to a strong $B_\phi$ component. This happens through the development of the $\Omega$ effect that results from shear instabilities (Babcock 1961). We explore the $\Omega$ effect further in Section 3.2.2.

3.2 Magnetic field amplification

We have seen that in our weakJ64 simulations starting from an initially weak field, the simulations eventually develop both strong turbulent and mean fields. This suggests the operation of both the small-scale turbulent and the large-scale mean-field dynamo in the disc. In the next two subsections, we quantify the action of these dynamos in accretion discs around the sink particles in our simulations.

3.2.1 Small-Scale dynamo

Traditionally, the presence of a small-scale dynamo is verified by an exponential increase in the ratio,

$$Q_{ss} = \frac{\langle B_{\text{turb}} \rangle_{\text{rms}}}{\rho^{2/3}},$$

over the lifetime of the simulation (e.g., Sur et al. 2010; Federrath et al. 2011b; Turk et al. 2012; Schober et al. 2012; Latif et al. 2013; Schober et al. 2015; Federrath 2016); here, $\langle B_{\text{turb}} \rangle_{\text{rms}}$ is the root-mean-square strength of the turbulent component of the magnetic field, averaged over some region of interest (see below). The motivation for the normalisation by $\rho^{2/3}$ in the definition of $Q_{ss}$ is to remove the effects of flux-freezing: even in the absence of dynamo action, a collapse that increases the gas density will also increase the strength of the frozen-in field. The fastest growth occurs for the spherical collapse of a region with a dynamically-unimportant, tangled field, in which case $B \propto \rho^{2/3}$ (Banerjee & Pudritz 2006; Crutcher et al. 2010); stronger fields that force anisotropic collapse produce scalings closer to $B \propto \rho^{1/2}$ (Ames 1973; Crutcher 1999; Desch & Mouschovias 2001; Li et al. 2004; Machida et al. 2006; Mocz et al. 2017; Hennebelle & Inutsuka 2019). Thus, $Q_{ss}$ is either a conserved or decreasing quantity in the absence of dynamo action, and an increase in $Q_{ss}$ indicates that the small-scale dynamo is operating.

We show the value of $Q_{ss}$ versus time for all our non-fragmenting runs in the top panel of Figure 3. For the purpose of this plot, we calculate $Q_{ss}$ in a spherical region of radius 0.01 pc centred on the point of maximum density before the sink particle forms, and then shift to a cylindrical geometry that represents the accretion disc around the sink. However, our results are quite insensitive to these choices, as long as the volume over which we compute $Q_{ss}$ is large enough to capture the entire disc. In Figure 3, the solid lines are the mean values averaged over the ~10 non-fragmenting simulations in each category, and the colored bands denote the 5th and the 95th percentiles.

The percentiles requested can be outside the range that can be computed given the limited input sample size in our work. To take this into account, we use the `numpy` percentile function with the linear interpolation option such that if the request percentile is

Figure 2. Azimuthally-averaged, mass-weighted radial profiles of different components of the magnetic field in the accretion discs around the central star in our four sets of simulations, shown at the end of the simulation when SFE = 5 percent. $B_{\text{turb}}$ is defined as in equation 2. Note that $\langle B_\phi \rangle$ is the largest component, indicating a large-scale mean field in the toroidal direction. There is also a strong turbulent component, $\langle B_{\text{turb}} \rangle$, indicating the presence of the small-scale dynamo.
Figure 3. The evolution of the small-scale dynamo ratio, $Q_{ss}$, as a function of time in the core before the formation of the sink at time $T_s$, and as a function of star formation efficiency (SFE) in the disc around the sink after its formation (SFE = 0.05 implies that the sink particle has accreted 50 $M_\odot$). We calculate $Q_{ss}$ using equation 3, averaging over a spherical volume of radius 0.01 pc before the collapse, and a cylindrical region of radius 500 au and half-height 50 au, oriented to lie in the same plane as the accretion disc, afterwards. The solid lines represent the mean averaged over the non-fragmenting ($N_f \sim 10$) realizations in each case. The colored bands represent the 5th and the 95th percentiles. The bottom panel is identical, except that it shows the ratio of magnetic to turbulent kinetic energy, which quantifies the growth and saturation of the small-scale dynamo.
The initial amplification in the pre-sink phase \( (T_s < 0) \) is similar to that observed in Sur et al. (2010) and Federrath et al. (2011b), and is not due to the dynamo, as the ratio of the magnetic to the turbulent kinetic energy, \( E_{\text{mag}}/E_{\text{turb,kin}} \), remains constant. There is a small plateau close to the sink formation time, \( T_s = 0 \), which results because the evolution is so fast that the snapshots we use (which are taken every 50 timesteps) do not resolve the time frames that we parameterize by the SFE.

Turning now to the phase of the simulation after sink formation, the plot shows that, on average, the weak J64 runs show a substantial small-scale dynamo amplification. The value of \( Q_{\alpha} \) asymptotically approaches the value found in the strong-field runs. However, there is a large scatter, so the amount of dynamo amplification varies significantly with the random seed for the initial turbulent velocity and magnetic field. On the other hand, runs with an initially strong magnetic field do not show any amplification in \( Q_{\alpha} \), independent of resolution. This is in accordance with the expectations laid out in section 2 of SFK20, namely that the strong-field runs correspond to an initially saturated magnetic field that cannot be further amplified.

The bottom panel of Figure 3 shows the ratio \( E_{\text{mag}}/E_{\text{turb,kin}} \). Consistent with our discussion of \( Q_{\alpha} \), we see that this ratio is nearly constant in the strong-field runs, further implying that the field is saturated. The saturation level is close to 0.1, in very good agreement with that expected from isothermal MHD turbulence simulations with similar Mach number (Federrath et al. 2014; Federrath 2016), but here with realistic chemistry and cooling. Most interestingly, in the weak J64 case, the ratio of energies increases from \( \sim 10^{-7} \) for our initial state to \( \sim 10^{-3} \) to \( \sim 2 \) by the time the SFE has reached 4–5 percent. However, there is a great deal of scatter about this result, with some runs showing no increase in magnetic energy density at all, and others reaching a ratio of almost 0.1.

While it may seem from Figure 3 that the small-scale dynamo action is not resolved with 32 cells per Jeans length, this is not strictly the case. In fact, field amplification is only delayed, not suppressed entirely. To illustrate this point, we have continued one realisation of a weak J32 run to an SFE of 12 percent; we show \( Q_{\alpha} \) and \( E_{\text{mag}}/E_{\text{turb,kin}} \) for this run in Figure 4. As the green curve in the top panel of Figure 4 shows, small-scale dynamo amplification does occur, but not until after SFE = 5 percent. Thus, the small-scale dynamo is active even at a J=32 Jeans resolution; however the time at which amplification begins seems to be both stochastic and resolution-dependent. This observation confirms that J \( \sim 30 \) is a threshold for dynamo amplification (Sur et al. 2010; Federrath et al. 2011b) even in the presence of primordial chemistry and cooling.

We also use this realisation to test for the effects of increasing the maximum resolution, as opposed to changing the number of cells per Jeans length. To this end, we repeat the weak-field case with 32 and 64 cells per Jeans length but at a higher absolute resolution, such that \( \Delta x = 3.8 \text{ au} \) on the finest AMR level (instead of the \( \Delta x = 7.6 \text{ au} \) for all the other simulations). It is clear from Figure 4 that the runs with higher absolute resolution produce results that are very similar to the ones at our standard absolute resolution. While we are unable to repeat these higher-resolution tests in more cases due to the computational expense, the experiment we have performed suggests that absolute resolution is less important for capturing small-scale dynamo effects than resolving the Jeans length by a sufficiently large number of cells. Further, we also find that the onset of the small-scale dynamo action depends on the degree of smoothness and circularity in the disc. We show this in the movie M4, by comparing the evolution of magnetic field strength in two realizations of the weak J64 runs that show no and high amplification, respectively. This demands a detailed analysis of the interaction of disc dynamos with disc instabilities, which is beyond the scope of this work since the inner disc is not well resolved, as we discuss in Section 3.2.2.

### 3.2.2 Large-Scale dynamo

The kinetic helicity, \( F = \int \mathbf{v} \cdot \mathbf{dW} \) \( (\mathbf{W} = \nabla \times \mathbf{v}) \) is the vorticity is finite and non-zero in our simulations, thus suggesting the presence of helical turbulence (e.g., Kulsrud 1999; Brandenburg & Subramanian 2005; Brandenburg et al. 2019). It is well known that helical turbulence in the presence of a vertical density gradient (stratification) and differential rotation in discs can lead to the generation of a large-scale magnetic field through the \( \alpha \Omega \) dynamo (Pudritz 1981a,b). While the small-scale dynamo generates field structures on smaller scales, it cannot lead to the production of a coherent field on large scales. The presence of the mean toroidal field as we observe in our simulation implies the presence of a large-scale dynamo\(^4\). This happens due to winding-up of the magnetic field in the toroidal direction by shearing motions (\( \Omega \) effect, see Babcock 1961). However, the \( \Omega \) effect alone cannot explain the strong poloidal component that we observe in addition to the toroidal field, which implies an additional amplification mechanism at work, likely the \( \alpha \) effect (Steenbeck et al. 1966). This phenomenon is well-known as the \( \alpha \Omega \) large-scale dynamo (Brandenburg & Subramanian 2005). In our simulations, we speculate that the \( \alpha \Omega \) dynamo acts to amplify the small-scale field produced by the small-scale dynamo (provided the resolution is high enough), and that this transforms the small-scale field into the large-scale one that we observe. While it is generally believed that the small-scale dynamo can quench the action of the mean-field dynamo (Kulsrud & Anderson 1992; Subramanian 1999; Schekochihin et al. 2004; Brandenburg & Subramanian 2005; Brandenburg et al. 2012), recent high-resolution simulations find that a large-scale mean field can co-exist with a small-scale field of comparable strength, if both shear and helical turbulence are present (Bhat et al. 2012).

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\(^{4}\) As Brandenburg & Subramanian (2005) note, the differentiation between a small-scale and a large-scale dynamo is artificial, and in reality, the two regimes are connected.
Figure 4. Same as Figure 3, but for one particular realization, including runs with the weak field at a higher absolute resolution. In this plot, the strongJ32, strongJ64, and weakJ64 cases are all run with the standard resolution. We run the weakJ32 case shown with the standard resolution as well, but allow the run to continue to SFE = 12 percent rather than 5 percent. Finally, for the two runs (weakJ32, high-res) and (weakJ64, low-res), we use the same initial conditions and refinement criteria as weakJ32 and weakJ64, but add an extra level of refinement, so the maximum resolution is $\Delta x = 3.8$ au rather than 7.6 au. The main conclusion from this is that higher Jeans resolution is more critical for resolving dynamo amplification than absolute maximum resolution.

2016; Singh et al. 2017; Bhat et al. 2019), due to the unified action of the two dynamos.

The operation of the $\alpha$ effect depends on the competition between how efficiently the field is regenerated as compared to how quickly is it dissipated (by turbulence) in the poloidal direction. Similarly, the operation of the $\Omega$ effect depends on how efficiently the field is amplified as compared to how quickly is it dissipated in the toroidal direction. Thus, the two effects can be quantified under the assumption of axisymmetric accretion discs (Raeder 1986) by taking the ratio of field amplification rate to its dissipation rate (Pudritz 1981b; Ruzmaikin et al. 1988a; Stepinski & Levy 1990; Brandenburg & Subramanian 2005),

$$R_\alpha = \frac{\alpha h}{\eta T} \quad \text{and} \quad R_\Omega = \frac{S h^2}{\eta T}$$

where $h$ is the disc scale height at some radius $r$ and $S$ is the radial shear caused by differential rotation, $S = \frac{\partial \Omega}{\partial r}$. Further, $\alpha$ is a pseudo-scalar$^5$ that represents the transport coefficient responsible for the $\alpha$ effect ($\alpha = 0$ if the turbulence is not helical), and $\eta T$ is the second transport coefficient, given as the sum of microscopic and turbulent magnetic diffusivity (Moffatt 1978; Krause & Raedler 1980; Ruzmaikin et al. 1988a; Brandenburg 2018). Theoretically, the operation of the large-scale dynamo requires that the large-scale dynamo number,

$$D_{\alpha \Omega} = R_\alpha R_\Omega,$$

be larger than unity, implying that the amplification of the field by the two effects is more rapid than dissipation$^6$.

In order to verify that a large-scale $\alpha\Omega$ dynamo is operating in our simulations, we must estimate $\alpha$ and $\eta T$, so that we may compute $R_\alpha$ and $R_\Omega$, and hence $D_{\alpha \Omega}$ (equation 5). For accretion discs, the microscopic diffusivity is much less than the turbulent magnetic diffusivity as the discs are highly conducting (e.g., Krause & Roberts 1976; Pudritz 1981a; Hartmann et al. 1998). In a simulation such as ours, which does not include explicit resistivity and where the physical scale of magnetic diffusion is unresolved, the magnetic diffusivity is dictated solely by the finite resolution of the grid on which we discretise the MHD equations (Kowal et al. 2009; Santos-Lima et al. 2012; McKee et al. 2020). We can estimate the diffusivity by noting that, in the absence of explicit viscosity or resistivity, the dissipation scale is always of order the cell size $\Delta x$, and thus the fluid and magnetic Reynolds numbers $Re$ and $Rm$ must be close to unity for length scales $\sim \Delta x$ (e.g., Haugen et al. 2004; Schekochihin et al. 2004; Balsara et al. 2004). Thus, $\eta T \sim c_s \Delta x \sim 10^{20}$ cm$^2$ s$^{-1}$.

To calculate $\alpha$, we make use of the fact that, in the presence of helical turbulence, the induction equation for the mean field has an additional term, $\chi$, that depends on the turbulent velocity and magnetic field (Subramanian 2016, see their equation 151). Assuming spatially isotropic turbulence and a finite scale separation between small and large scales (Blackman & Field 2002), $\chi$ can be expressed under a first-order smoothing approximation (neglecting quadratic terms) in the kinematic regime as,

$$\chi = (\nabla \times B_{\text{turb}}) = \alpha (B) - \eta T \nabla \times (B).$$

Note that equation 6 can only be used if: (1) $Rm$ is small (Cattaneo & Hughes 2009), and (2) $B_{\text{turb}}$ is small compared to $B$. The latter assumption is violated in our simulations, since $B_{\text{turb}} \sim \langle B \rangle$. However, direct numerical simulations report that equation 6 holds approximately even when

$^5$ The pseudo-scalar, $\alpha$, is actually a compressed version of the symmetric part of the $\alpha$ tensor, obtained under the assumption that the turbulent field is isotropic (invariant under rotation) and homogeneous (see equation 7.15 in Moffatt 1978). Certain simulations have calculated the different components of the $\alpha$ tensor (e.g., Schrinner et al. 2007; Warnecke et al. 2018; Viviani et al. 2019; Bendre et al. 2020), however, as we explain in the main text, this is not within the scope of this work.

$^6$ In practice, the critical dynamo number above which the dynamo operation is sustained is a function of the disc aspect ratio (Bera et al. 2019, see their Figure 2), however, it is generally of the order of 1 - 10 in astrophysical systems (Ruzmaikin et al. 1988a,b).
If $Rm \leq 1$, the first-order smoothing approximation estimates have to be scaled by $Rm$.

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**Figure 5.** Azimuthally-averaged radial profile of the large-scale dynamo number, $D_{\alpha \Omega}$ (see equation (5)), in the disc for different runs at SFE = 5 percent. This mean-field dynamo operates due to the $\alpha \Omega$ effect in the disc, requiring a critical $D_{\alpha \Omega} > 1$. It does not act in the inner disc due to coarser resolution there, and for $\log_10 (\ell/\text{au}) \geq 1.5$, the weak-field models have $D_{\alpha \Omega} > 1$ and all models have $D_{\alpha \Omega} \gg 1$ further out in the disc ($r \gtrsim 100 \text{au}$), demonstrating the effectiveness of the $\alpha \Omega$ dynamo.

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Dynamo amplification in first stars

While the presence or absence of a dynamo in primordial accretion discs is interesting in itself, the main astrophysical question in which we are interested is how any resulting magnetic fields might affect the IMF of the first stars. This is something that does have at least potentially observable consequences. To investigate this question, we collect information on the sink mass distribution of all the four simulation categories: weakJ32, weakJ64, strongJ32, strongJ64, as well as the control case from SFK20, which did not include a magnetic field and the Jeans length was resolved by 32 cells; we refer to this as the HDJ32 case. The total number of sink particles (used as a proxy for stars) formed in weakJ32, weakJ64, strongJ32 and strongJ64, over the 25 realisations, are 121, 175, 70 and 130, respectively. This implies that higher Jeans resolution leads to more fragmentation in the MHD runs, by as high as a factor of 2. It is not easy to pin-point the cause of this finding, because the simulations are highly chaotic and non-linear. However, broadly speaking, we can attribute this effect to the fact that the accretion discs around the primary sink, and thus disc instabilities and sub-structure, are better resolved in J64 runs as compared to J32 runs. Given this result, we compare the sink mass distributions for the runs with 32 and 64 cells per Jeans length separately, so that we can disentangle the effects of magnetic fields and resolution. While this approach means that we are not necessarily capturing the true amount of fragmentation, since simulations are not fully converged, it does allow us to test with confidence how magnetic fields and dynamo amplification shift the IMF.

The left panel of Figure 6 shows the sink mass distri-
Figure 6. *Left panel:* The mass distribution (top) and the cumulative distribution (bottom) of sink particles that form till SFE = 5 percent in 25 realizations in the weak- and strong-field runs with 32 cells per Jeans length. We also show the distribution for HDJ32 (without magnetic fields), adopted from SFK20. *Right panel:* the same distributions resulting from runs with 64 cells per Jeans length.

bution for simulations with 32 cells per Jeans length. It is straightforward to see that the sink mass distribution of the strongJ32 runs is different from the other two, while the weakJ32 and HDJ32 runs are very similar, at least for \( M \lesssim 10 \, M_\odot \). To confirm this visual impression quantitatively, we apply the Kolmogorov-Smirnoff (KS) test for each pair of the runs shown in this panel. This test returns a \( p \)-value that describes the confidence level with which we can rule out the null hypothesis that the masses in each pair of runs were drawn from the same underlying distribution. Following Sharda et al. (2019) and SFK20, we classify two distributions to be significantly different, if the \( p \)-value is < 0.01. The \( p \)-values for the pairs HDJ32–weakJ32, HDJ32–strongJ32 and weakJ32–strongJ32 come out to be 0.55, \( 5 \times 10^{-5} \) and \( 8 \times 10^{-4} \), respectively. Thus, the sink mass distribution produced by the strong magnetic field runs has a different origin than that produced by the weak field and HD runs. This finding is consistent with that of SFK20. However, we note that the mass distributions for \( M \gtrsim 10 \, M_\odot \) are much more similar between weakJ32 and strongJ32, both showing a significantly higher number of massive stars than HDJ32.

The right panel of Figure 6 shows the same distributions for the runs with 64 cells per Jeans length. Visually, the weakJ64 and strongJ64 distributions are much closer to one another than are the weakJ32 and strongJ32 cases. The \( p \)-value for the pair weakJ64–strongJ64 is 0.12, implying no statistically significant difference in fragmentation between the weak- and the strong-field runs at higher Jeans resolution. This is not entirely unexpected, given that weakJ64 runs show significant field amplification. Thus, we find that first star cores with an initial field that falls below equipartition by a factor of \( \sim 10^7 \) produce an IMF that is significantly different from those that start near equipartition when we do not resolve dynamo amplification, but that this difference greatly diminishes, to the point of statistical undetectability, when we do capture dynamo growth. As further evidence of this effect, we note that, while we do not have a set of non-magnetic simulations at 64 cells per Jeans length to enable a direct comparison, the weakJ64 run shows less fragmentation, and higher mean masses, than the HDJ32 case, despite having higher resolution, which tends to favour more fragmentation. Thus, the effect of the dynamo-amplified magnetic field in suppressing fragmentation outweighs the effect of increasing the resolution.

Our results confirm the suggestion made by SFK20 that the weak-field case is physically implausible; even if a weak magnetic seed field is present in primordial clouds, it will be quickly driven to saturation and becomes dynamically
strong during Population III star formation. This is important because it means that (1) strong magnetic fields were likely present during Population III star formation, and (2) they had a significant impact on the primordial IMF.

5 CONCLUSIONS

In this work, we study how magnetic fields can be amplified through a dynamo mechanism both on small and large scales in the accretion discs around Population III stars. There is a growing consensus that seeds of primordial magnetic fields, no matter how weak, were present in the early Universe (Widrow et al. 2012; Subramanian 2016; Planck Collaboration et al. 2016), and that they can be exponentially amplified during the collapse of minihaloes at \( z \approx 20 \rightarrow 30 \) (Turk et al. 2012). Recent analysis has also shown that if dynamically strong magnetic fields were present during Population III star formation, they will significantly reduce fragmentation, thereby changing the IMF of the first stars (Sharda et al. 2020). However, previous work has left unresolved the question of how strong can magnetic fields grow during first star formation, and thus of how strong magnetic effects on the IMF are likely to be. This uncertainty is largely a function of numerical limitations: resolving the amplification of magnetic fields by dynamo action requires far higher resolution than is traditionally used in simulations of gravitational collapse and fragmentation.

To address this question, we perform a series of simulations in which we systematically vary the resolution (32 and 64 cells per Jeans length) and the initial strength of the turbulent magnetic field (1\( \mu \)G and 30\( \mu \)G, see Table 1). The simulations with initially strong magnetic fields are a control case; they do not show any small-scale dynamo operation at either Jeans resolution, implying that the field is already saturated, as expected given our choice of initial field strength. By contrast, in the simulations where the initial magnetic field is weak, we find that the small-scale dynamo acts efficiently in the accretion discs around the sink particles, amplifying the turbulent field strength such that, by the time a few percent of the initial cloud has accreted, the field in the disc reaches saturation values similar to those in the runs where we start with the field already at saturation (see Figure 3). However, we also find that the timing and strength of field amplification is sensitive to resolution: simulations with 64 cells per Jeans length yield earlier and stronger field amplification than their lower-resolution counterparts.

We also find a strong, large-scale mean toroidal component of the field in all the simulations (see Figure 2), which is due to the operation of a large-scale \( \alpha \Omega \)-type dynamo. In this type of large-scale dynamo, the \( \Omega \) effect winds up the field in the toroidal direction due to differential rotation (shear), and the \( \alpha \) effect regenerates and maintains the poloidal field. Figure 5 shows that the \( \alpha \Omega \) dynamo acts efficiently in the outer disc, where we resolve the disc scale height with enough cells to capture its operation. Our findings are consistent with those of Federrath et al. (2011b), who suggest that fully capturing a dynamo process likely requires resolution of \( \sim 30 \) cells per Jeans length. Overall our results suggest a picture in which protostellar cores containing only seed fields with no organised structure and an energy density \( \sim 7 \)\( \times \) orders of magnitude below equipartition experience rapid growth of the field via both the small-scale dynamo, which increases the turbulent field strength to \( \sim 1 \rightarrow 10 \) percent of equipartition, and the \( \alpha \Omega \) dynamo, which moves a significant fraction of the energy stored in the disorganised, small-scale field into an organised, large-scale toroidal component.

The development of magnetic fields at \( 1 \rightarrow 10 \) percent of equipartition even in protostellar cores that begin far below equipartition has profound implications for the IMF of the first stars. Sharda et al. (2020) show that the presence of an initial near-equipartition field strongly reduces the fragmentation of first star discs, leading to an IMF that is significantly more top-heavy, and deficient in stars with mass \( \lesssim 1M_{\odot} \) that might survive to the present day. Our simulations here show that, thanks to dynamo action, this effect operates even in cores where the initial field is many orders of magnitude smaller, and that simulations can capture this effect, if they reach sufficient resolution. Hence, we propose that a scenario where magnetic fields remain weak throughout a Population III star formation episode is likely unphysical: magnetic field effects are always non-negligible.

A more speculative implication from this would be that Population III star formation might be subject to significant magnetic field-induced feedback effects like magnetic bubbles or jets (Tan & Blackman 2004; Machida et al. 2006; Li et al. 2014; Frank et al. 2014; Dyda et al. 2018; see, however, Gerrard et al. 2019; McKee et al. 2020), and that it should be possible to detect these effects in simulations provided the innermost parts of the disc are sufficiently resolved. As the first massive stars explode, the first supernova explosions are likely to bring the magnetic fields into the interstellar medium, while also enriching it with metals (Greif et al. 2007; Sakuma & Susa 2009; Meiksin & Whalen 2013). The metal enrichment is expected to lead to the formation of lower-mass stars due to cooling via metals and dust grains (e.g., Schneider et al. 2003; Bromm & Loeb 2003; Omukai et al. 2005). For these Population II stars, the magnetic fields built up by dynamos around the first stars may become even more dynamically significant, and more important to limiting fragmentation (Latif et al. 2014), due to the diminished role of thermal pressure in gas subject to efficient cooling. The fields may also be further amplified in the haloes where this process takes place, via the same basic dynamo mechanisms we have explored here (Latif et al. 2013; Grete et al. 2019). Self-consistent models of such environments should therefore always aim to incorporate the magnetic fields.

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APPENDIX A: EFFECTS OF JEANS RESOLUTION ON COOLING

The morphological evolution of the weak field runs changes significantly when we use 64 cells per Jeans length instead of 32. As discussed in Section 3.1, in the higher resolution case the simulation develops a near-spherical bubble of gas at temperatures of ≈ 3000 – 6000 K that expands over time; Turk et al. (2012) noticed a similar phenomenon in their highest-resolution simulations. To determine whether this bubble is associated with the presence of a magnetic field, we repeat the run shown in Figure 1 with identical gas initial conditions, but with no magnetic field, at resolutions of 32 (J32) and 64 (J64) cells per Jeans length. Figure A1 shows the density-weighted temperature projections for the J32 and J64 runs. Given that we observe the same phenomenon as in the magnetic field runs, i.e., a hot bubble appears in J64 but not in J32, we conclude that the presence of the bubble is not solely due to magnetic fields.

Instead, we find that the key distinction between runs where we do and do not form bubbles is how well we resolve the temperature jump across the accretion shocks where matter falls onto the disc. To illustrate this point, we focus on a particular location inside the bubble, which we refer to as $p_1$ hereafter, at a radial distance of $r_1 = 400$ au from the star, located in the plane of the disc, as indicated by the ‘+’ symbol in Figure A1. Figure A2 shows profiles of $\rho$, $T$, $P$, $v_r$ and $c_s$ along a radial ray passing through this point, at two times: just before and just after the bubble reaches $p_1$. We refer to the profile measured immediately before the bubble reaches our sample point as the “Pre-Shock” profile (blue in Figure A2), and the one immediately after as the “Post-Shock” profile (orange in Figure A2).

Table A1 lists the properties of the gas at $p_1$ at times corresponding to the pre-shock and post-shock snapshots shown in Figure A2. The ratios of densities, temperatures, and pressures in the pre- and post-shock conditions are as expected from the Rankine-Hugoniot jump conditions for non-radiative shocks.

Using the post-shock values, we can calculate the total volumetric cooling rate via radiation, $\Gamma_{\text{rad}}$, and via chemical reactions, $\Gamma_{\text{chem}}$ (important at high temperature, where endothermic dissociation of H$_2$ is a significant coolant) from KROME. The time it will take for the gas to traverse the width of the shock is 204 yr.

![Figure A1. Density-weighted projections of temperature for the J32 and J64 runs at the end of the simulation, when the SFE has reached 5 percent. The ‘+’ marker denotes the sample point $p_1$ where we calculate the cooling length as the shock front travels through it earlier in the simulation.](image)

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![Figure A2. Profiles of density, temperature, pressure and radial velocity along a radial ray passing through our sample point $p_1$ (Figure A1) at two times, just before (labelled “Pre-Shock”) and just after (labelled “Post-Shock”) the edge of the hot bubble reaches $p_1$, at a distance $r_1 = 400$ au from the central star (indicated by the dashed vertical line). The time it takes for the gas to traverse the width of the shock is 204 yr.](image)
Figure A3. The cooling and H\textsubscript{2} dissociation timescales as a function of temperature, for the fixed post-shock chemical composition and density as listed in Table A1. At lower temperatures, \(t_{\text{diss,H}_2}\) is infinity since there is no net dissociation of H\textsubscript{2}. At higher temperatures, the molecular gas dissociates faster than it can cool.

Table A1. Pre-shock properties at point \(p_1\) as obtained from Figure A2 in the J64 run. The quantity \(x_q\) is the mass fraction of species \(q\).

| Property               | Pre-Shock | Post-Shock |
|------------------------|-----------|------------|
| \(n\) \((\text{cm}^{-3})\) | \(2.6 \times 10^9\) | \(6.2 \times 10^9\) |
| \(T\) \((\text{K})\)       | 1350      | 3110       |
| \(x_{\text{H}}\)         | 0.76      | 0.76       |
| \(x_{\text{H}_2}\)       | \(2.9 \times 10^{-3}\) | \(1.2 \times 10^{-3}\) |
| \(x_{\text{D}}\)         | \(4.6 \times 10^{-5}\) | \(4.6 \times 10^{-5}\) |
| \(x_{\text{H}_2^+}\)     | \(4.0 \times 10^{-7}\) | \(1.5 \times 10^{-7}\) |
| \(x_{\text{D}^+}\)       | \(1 \times 10^{-8}\) | \(4 \times 10^{-8}\) |
| \(\Gamma_{\text{rad}}\) \((\text{erg/cm}^2/\text{s})\) | \(4.8 \times 10^{-14}\) | \(8.9 \times 10^{-17}\) |
| \(\Gamma_{\text{diss}}\) \((\text{erg/cm}^2/\text{s})\) | NA        | \(4.3 \times 10^{-15}\) |
| \(E_T\) \((\text{erg/cm}^3)\) | \(7.3 \times 10^{-4}\) | \(3.8 \times 10^{-3}\) |
| \(t_{\text{cool}}\) \((\text{yr})\)     | 477       | 27428      |
| \(t_{\text{diss,H}_2}\) \((\text{yr})\) | \(\infty\) | 55         |

Similarly, the time it takes for H\textsubscript{2} to dissociate can be given by,

\[
t_{\text{diss,H}_2} = \frac{x_{\text{H}_2}}{x_{\text{H}_2^+}}
\]

where \(x_{\text{H}_2}\) is the H\textsubscript{2} mass fraction, and \(x_{\text{H}_2^+}\) is the rate of change in the H\textsubscript{2} mass fraction; by convention, if \(x_{\text{H}_2} \geq 0\), we take \(t_{\text{diss,H}_2} = \infty\). We see that the pre-shock conditions are characterised by rapid cooling \((t_{\text{cool}} \sim 500 \text{ yr})\) and no dissociation, while the post-shock conditions are characterised by much slower cooling \((t_{\text{cool}} \sim 27,000 \text{ yr})\) and rapid dissociation \((t_{\text{diss,H}_2} \sim 50 \text{ yr})\). The reason for the much longer cooling time is the fact that, at the \(\approx 3000 \text{ K}\) temperature found in the post-shock region, most collisions between H\textsubscript{2} molecules and H atoms lead to collisional dissociation rather than to excitation followed by radiative de-excitation.

In order to understand why resolution matters, it is helpful to consider how the cooling and dissociation times depend on temperature. Figure A3 shows these quantities as a function of temperature for the post-shock chemical composition and density. The key feature to notice is that the thermal and chemical regime changes sharply at \(\approx 2000 \text{ K}\). Now, consider how material on the low-temperature side of this jump evolves as it encounters a shock. In the limit of infinite resolution, the shock has a width of the order of the particle mean free path. Given \(n \approx 10^9 \text{ cm}^{-3}\) and a typical cross-section for neutral species \(\sim 10^{-16} \text{ cm}^2\), the shock width is \(\sim 10^7 \text{ cm}\). The time to traverse this distance at \(\sim 1 \text{ km s}^{-1}\) is \(\sim 100 \text{ s}\), which is tiny as compared to any radiative or chemical timescale. Thus, if this gas crosses a strong shock, its temperature increases by the usual factor \((\gamma + 1)/(\gamma - 1)\), without time for any radiative cooling to occur. If the gas is initially at 1300 K, as is the case for our pre-shock sample point, this causes it to jump from the left to the right side of the 2000 K discontinuity in Figure A3. At that point, H\textsubscript{2} dissociates faster than the gas is able to cool, and we get into the high-temperature, slow-cooling regime that characterises our post-shock region. Thus, the gas never cools.

Now, consider the case where the shock is broadened to a size \(\sim 4 \Delta x\), a typical shock width imposed by artificial viscosity (e.g., Creasey et al. 2011; Hubber et al. 2013). If the resolution inside the region is 23 au, as is the case in the J32 run, then the time required to traverse the shock region is greatly increased to \(\sim 92 \text{ au}/(1 \text{ km s}^{-1}) = 436 \text{ yr}\). Interestingly, this is comparable to the pre-shock cooling time. The net effect is that the gas cools at the same time it is traversing the broadened shock, and thus never crosses over to the right side in Figure A3. It remains cool and with a significant fraction of H\textsubscript{2}, exactly as we observe in the J32 run. On the other hand, if we double the Jeans resolution, then the time to traverse the shock is halved, and we are in the regime where the hydrodynamic time to cross the shock is smaller than the cooling time. Thus the temperature goes up, and we get to the right side of the jump at 2000 K in Figure A3, where \(t_{\text{diss,H}_2} < t_{\text{cool}}\). Once in this regime, the gas does not have enough time to cool before it dissociates, leading to the formation of a hot, H\textsubscript{2}-poor bubble as we observe in the J64 run. This discussion also explains why a magnetic field, though not critical to the phenomenon we have identified, can nonetheless influence it: magnetic pressure helps mediate the shock (e.g., Fragile et al. 2005; Li et al. 2019), and thus changes the rate at which gas heats or cools as it passes the shock front.

Thus, while our motivation to use a higher Jeans resolution was to better resolve the action of the small-scale dynamo, this result, along with earlier findings of Turk et al. (2012), implies that a higher Jeans resolution is also critical for capturing the thermal and chemical changes that occur across shocks.

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