Isotope separation of $^{176}\text{Lu}$ a precursor to $^{177}\text{Lu}$ medical isotope using broadband lasers

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A new photoionization scheme accessible by Rhodamine dye lasers is proposed for the isotope separation of $^{176}\text{Lu}$.

$5d6s^2\ 2D_{3/2} (0.0 \text{ cm}^{-1}) \rightarrow 5d6s6p^4F_{3/2} (17427.28 \text{ cm}^{-1}) \rightarrow \text{Autoionization State} \rightarrow \text{Lu}^+$

Optimum conditions for the laser isotope separation have been theoretically computed and compared with the previously reported work. The enrichment of ~ 63% can be obtained with > 22 mg/h production rate even when broadband lasers with bandwidth of 500 MHz are employed for the two step excitation. The simplified system requirements for the photoionization scheme combined with a high production rate of $^{176}\text{Lu}$ than previously reported is expected to reduce the global shortage of $^{176}\text{Lu}$ isotope for medical applications.

Chemistry which is a science of elements, compounds and their separation has immensely helped in the development of mankind. However, separation of isotopes remained a formidable challenge as it is technology intensive; therefore, it is largely limited only to the technologically advanced nations. Some of the isotopes of the low Z-elements could be separated relatively easily by the chemical methods by exploiting the difference in the kinetic isotope effect. Since the kinetic isotope effect for mid-Z and high-Z elements gradually diminishes with increase in Z, it is necessary to switch over to physical methods for the efficient separation of isotopes of these elements. Isotope separation technology which is largely limited to the developed nations has been applied to the separation of actinides (particularly Uranium) due to its applications in the defence and nuclear industry. However the real potential of isotope separation for the separation of rest of the isotopes remains unexploited due to limited access to this technology to most of the nations and the complexity of the systems involved.

Among the several methods of the separation of isotopes, gas centrifugation method has evolved as the most commercially viable method, however, apart from the technological challenges, the primary limitation, remains being highly capital intensive. On the other hand, Atomic Vapor Laser Isotope Separation (AVLIS) has evolved as an alternative technique to the gas centrifugation method. AVLIS has significant advantages over gas centrifugation such as being compact, less capital intensive and yielding in high separation factor. If AVLIS technique is appropriately deployed, these systems open up excellent opportunities for the separation of several isotopes other than Uranium.

In daily life, several radioisotopes are being used for medical applications and in particular for cancer diagnosis and therapy. While many of them are produced as fission products in nuclear reactors, some can be obtained by the irradiation of the precursor (or parent) isotopes. The medical fraternity desires to use nuclear medicine in the highest possible radio-isotopic purity. This demand is normally not met for all the isotopes of nuclear medicine due to challenges in producing the precursor isotopes with a high level of isotopic purity (degree of enrichment) followed by full conversion into the medical isotopes upon irradiation. Among the several isotopes for nuclear medicine, $^{177}\text{Lu}$ isotope has gained lots of attention due to its application in targeted radionuclide therapy (TRT). $^{177}\text{Lu}$ is produced in the nuclear reactor by irradiation of its stable parent isotope $^{176}\text{Lu}$ through the nuclear reaction $^{176}\text{Lu} \rightarrow ^{177}\text{Lu}$. $^{177}\text{Lu}$ is a radioactive isotope having a half-life of 6.65 days, decays to $^{177}\text{Hf}$ emitting $\beta$-particles with energies of 497 keV (76%), 384 keV (9.7%), 176 (12%). $^{177}\text{Hf}$ which is formed in the nuclear excited states decays to the ground state, emitting low energy $\gamma$-radiation with energies of 208 keV and 113 keV. With mean $\beta$-particle penetration depths of 670 $\mu$m, $^{177}\text{Lu}$ is very effective for the treatment of small tumors and metastatic lesions of small size. The low energy $\gamma$-radiation emitted by $^{177}\text{Hf}$ is useful for imaging.
and the studies of bio-distribution and excretion kinetics. Further, large half-life of $^{177}$Lu enables shipment to long distances. For these reasons, $^{177}$Lu has evolved as the most preferred choice for the targeted radionuclide therapy. However, currently, there is a global shortage of $^{177}$Lu isotope primarily due to the shortage of its precursor which is enriched $^{176}$Lu isotope. Therefore, there is a need to enhance the production of this isotope to meet the global demand.

Separation of $^{177}$Lu (natural abundance = 2.59%) from the other natural major isotope $^{175}$Lu (natural abundance = 97.41%) through gas centrifugation is not possible as it has no volatile compounds. Seemingly, AVLIS remains as the only available option for the enrichment of $^{177}$Lu. Kurchatov Institute $^{5-10}$, Russia has extensively worked on the isotope separation of Lu isotopes. The experimental facilities required are technologically most demanding, particularly, with respect to the bandwidth of the excitation lasers, which has to be limited to < 100 MHz level. Such systems cannot be developed by the nations which are not technologically advanced, thus hampering large scale production of enriched $^{177}$Lu for medical applications. Therefore, search for suitable photoionization schemes with less stringent requirements and ability to produce at higher production rates is currently on.

In the present work, a new photoionization pathway is proposed for the isotope separation of $^{176}$Lu from natural Lu. The proposed photoionization scheme has been theoretically evaluated by computing ionization efficiency and isotope selectivity under various experimental conditions using density matrix formalism.

Results and discussion

The primary limitation in direct irradiation of natural Lu for the production of $^{177}$Lu is the low natural abundance of its precursor $^{176}$Lu (2.59%). Upon direct irradiation of natural Lu in a medium to high flux reactors (neutron flux $10^{14-10^{15}}$ neutrons/cm$^2$-s), the content of $^{177}$Lu varies between 0.3 and 1.2% in the irradiated sample (Fig. 1a). When 50% enriched $^{176}$Lu is irradiated, it produces $^{177}$Lu whose content varies between 6 and 24% (Fig. 1b) in the irradiated sample. Further enrichment of $^{176}$Lu does not result in significant improvement in $^{177}$Lu content upon irradiation. For example, 100% enriched $^{176}$Lu produces $^{177}$Lu whose content varies between 12 and 47% (Fig. 1c). Therefore, enrichment of $^{176}$Lu to the degree of > 50% can be considered as optimum.

Density matrix formalism provides most accurate description of the laser-atom interaction $^{11}$. Lineshapes arising in double-resonance $^{12}$ and triple resonance ionization $^{13}$ of atoms have been investigated in detail. Recently, investigation of the following photoionization pathway has been carried out for the ionization efficiency and isotope selectivity under various experimental conditions using density matrix formalism.

540–535 nm scheme.

$$5d6s^2 \cdot 2D_{3/2}(0.0 \text{ cm}^{-1}) \xrightarrow{540.4068 \text{ nm}} 5d6s6p^4P_{5/2}(18504.58 \text{ cm}^{-1}) \xrightarrow{535.0626 \text{ nm}}$$

$$5d6s7s^4D_{3/2}(37193.98 \text{ cm}^{-1}) \xrightarrow{618.0061 \text{ nm}} 5p3375 \text{ cm}^{-1} \text{ Autoionization State} \rightarrow Lu^+$$

The results obtained were in good agreement with the experimental results $^{14}$. The optimum conditions for the isotope selective photoionization of $^{176}$Lu have been derived. The primary limitation of the scheme for the production of enriched $^{176}$Lu is the utilization of narrowband lasers with bandwidth < 100 MHz. Moreover, incorporation of additional apertures along the atomic beam axis is required to limit the Doppler broadening of the atomic beam. This also causes reduction in the throughput of the enriched isotope.

From the energy levels of Lu, it is possible to formulate several photoionization pathways, among them; the following photoionization pathway seems to be particularly promising.

573–560 nm scheme.

$$5d6s^2 \cdot 2D_{3/2}(0.0 \text{ cm}^{-1}) \xrightarrow{573.8130 \text{ nm}} 5d6s6p^4P_{5/2}(17427.28 \text{ cm}^{-1}) \xrightarrow{560.3114 \text{ nm}}$$

$$6s6p^2 \cdot 2^4P_{5/2}(3527.45 \text{ cm}^{-1}) \rightarrow \text{ Autoionization State} \rightarrow Lu^+$$

This photoionization pathway has a number of advantages over the photoionization scheme reported previously $^{5-10}$. They are,

1. The wavelengths of the excitation lasers are easily accessible by high conversion efficiency Rhodamine dye lasers.
2. The large separation between hyperfine components of the Lu isotopes than previously reported enables better enrichment.
3. Possibility of employing relatively broadband lasers for the isotope separation minimises the stringent bandwidth requirements of the excitation lasers.

The hyperfine structure constants of Lu isotopes were measured by Brenner et al $^{15}$ and Kuhnert et al $^{16}$ for the 0.0 cm$^{-1}$ and 17,427.28 cm$^{-1}$ levels. Vergès and Wyart $^{17}$ have measured hyperfine structure constants of $^{176}$Lu isotope for the 35,274.5 cm$^{-1}$ level using Fourier Transform Infrared Spectrometry, whereas the HFS constants for the $^{176}$Lu have not been reported so far. In the absence of hyperfine anomalies $^{15,18}$, the relationship between the hyperfine structure constants can be expressed as...
Figure 1. Percentage abundance of $^{177}\text{Lu}$ after irradiation in a reactor with different nuclear fluxes for the cases of (a) natural Lu, (b) 50% enriched $^{176}\text{Lu}$ and (c) 100% enriched $^{176}\text{Lu}$. 
numbered as 3 and 6 closely pass through the 17/2–17/2–19/2 hyperfine pathway of 176Lu (Fig. 2a). The two-photon sum frequency of 17/2–17/2–19/2 hyperfine pathway of 176Lu is −8198.1 MHz + 29,358.1 MHz = 21,160 MHz (Table 1). The two-photon sum frequencies of the hyperfine excitation pathways of 175Lu numbered 3 and 500 MHz for obtaining the degree of enrichment of ~66%.

Two-dimensional contour plots of the ionization efficiency of Lu isotopes are plotted in Fig. 2. The resonance frequency positions of all the possible hyperfine excitation pathways are numbered as per the Table 2 along with their stepwise excitation scheme each denoted by a serial number and have been listed in Table 2 along with their density of excitation and ionization lasers for the Doppler free case and optimum conditions for the enrichment of 176Lu can be accomplished by ionization through the 17/2–17/2–19/2 hyperfine excitation pathway.

Enhanced ionization observed at the intersection of horizontal and vertical ridges correspond either to the diagonal ridge can be observed when the detuning (|Δ|) of the laser frequency from its resonance transition. For the strong transitions, as in the present case, the diagonal ridge can be observed even for large detuning (Fig. 2). The resonance position of the 17/2–17/2–19/2 of 176Lu has been shown as a “filled circle” in the 175Lu two-dimensional contour plot in Fig. 2a. The diagonal ridges correspond to the coherent two photon ionization pathway which is observed when the sum total energy of the two photons is equal to the energy of the upper level. In stepwise excitation schemes, coherent two photon ionization can be observed for the detuning (from the intermediate level) values matching the condition Δ1 = −Δ2. The probability of coherent two photon ionization decreases with the increase in the detuning (|Δ|) of the laser frequency from its resonance transition. For the strong transitions, as in the present case, the diagonal ridge can be observed even for large detuning (Fig. 2).

Diagonal ridges corresponding to the coherent two photon ionization of 175Lu hyperfine excitation pathways numbered as 3 and 6 closely pass through the 17/2–17/2–19/2 hyperfine pathway of 176Lu (Fig. 2a). The two photon sum frequency of 17/2–17/2–19/2 hyperfine pathway of 176Lu is −8198.1 MHz + 29,358.1 MHz = 21,160 MHz (Table 2). Two photon sum frequencies of the hyperfine excitation pathways of 175Lu numbered 3 and 6 ranges between 14,894.1 MHz to 16,945.4 MHz thus have considerable impact on the degree of enrichment of 176Lu.

Table 1. HFS coupling constants of lutetium energy levels. * — Value in mK.

| Energy level | 176Lu | 176Lu |
|-------------|-------|-------|
|              | A (MHz) | B (MHz) | A (MHz) | B (MHz) | References |
| 5d6s2 3D3/2 (0.0 cm−1) | 194.331617 | 1511.398650 | 137.920537 | 2132.296936 | (5) |
| 5d6s6p 3P3/2 (17,427.28 cm−1) | −2509.26 | 1372.9 (calculated) | −651.47 | 2494.2 | (5) |
| 6s6p 3P0 (35,274.5 cm−1) | 64.5# | 1933.66 | −83.70# | −2509.26 | (17) |

Degree of enrichment after laser interaction is calculated using the expression

\[ \text{Degree of enrichment} = \left( \frac{176\text{Lu}}{175\text{Lu}} \right) \times 100 \] (5)

Two dimensional contour plots of the ionization efficiency of Lu isotopes are plotted in Fig. 2. The resonance frequency positions of all the possible hyperfine excitation pathways are numbered as per the Table 2. Horizontal ridges correspond to the first step excitation and vertical ridges correspond to the second step excitation. Strong horizontal and vertical ridges rise due to the strong transitions of the first and second excitation steps. Enhanced ionization observed at the intersection of horizontal and vertical ridges correspond either to the hyperfine pathway (numbered as per the Table 2) or crossovers. Origin of such crossovers has been discussed in detail in a recent article. The resonance position of the 17/2–17/2–19/2 of 176Lu has been shown as a “filled circle” in the 175Lu two-dimensional contour plot in Fig. 2a.

Enhanced ionization observed at the intersection of horizontal and vertical ridges correspond either to the hyperfine pathways of the interfering 175Lu isotope. Isotope separation of 176Lu can be accomplished by ionization through the 17/2–17/2–19/2 hyperfine excitation pathway.

In an AVLIS process, when the first and second excitation lasers are tuned to the frequencies \( \nu_1 \) and \( \nu_2 \) respectively, isotope selectivity is defined as

\[ \text{Isotope Selectivity } S (\nu_1, \nu_2) = \left( \frac{\eta_{176\text{Lu}}}{\eta_{175\text{Lu}}} \right) \] (3)

where \( \eta \) is the ionization efficiency of the isotope.

For the case, wherein the abundance of the isotopes is not equal, one need to normalize the selectivity with the abundance of the constituent isotopes; thus, isotope ratio enhancement factor is defined as

\[ \text{Isotope Ratio Enhancement Factor } \text{IRE} (\nu_1, \nu_2) = \left( \frac{\eta_{176\text{Lu}}}{\eta_{175\text{Lu}}} \right) \times \frac{A_{176\text{Lu}}}{A_{175\text{Lu}}} \] (4)

where \( A, B \) are the magnetic dipole and electric quadrupole constants; \( \mu \) is the magnetic moment, \( I \) is the nuclear spin and \( Q \) is the quadrupole moment. The hyperfine structure constants for the 176Lu were calculated to be \( A = 1372.9 \text{ MHz} \) and \( B = −3538.06 \text{ MHz} \) for the 35,274.5 \text{ cm}^{-1} \text{ level} (Table 1).

Based on the spectroscopic selection rules, a total of 30 hyperfine excitation pathways are possible in the stepwise excitation scheme each denoted by a serial number and have been listed in Table 2 along with their resonance frequencies. Among them, the hyperfine excitation pathway 17/2–17/2–19/2 of 176Lu (listed in serial no 3 in Table 2) lies far away from the hyperfine pathways of the interfering 175Lu isotope. Isotope separation of 176Lu can be accomplished by ionization through the 17/2–17/2–19/2 hyperfine excitation pathway.
| S no. | F1–F2–F3 | Frequency Position of the first hyperfine transition (MHz) | Normalised Intensity | Frequency Position of the second hyperfine transition (MHz) | Normalised intensity | Sum two photon frequency Position of the second hyperfine transition (MHz) | Normalised intensity | 176Lu |
|-------|-----------|----------------------------------------------------------|---------------------|----------------------------------------------------------|---------------------|-------------------------------------------------|---------------------|-------|
| 1     | 5–5–6     | 5418.3                                                   | 100.0              | 815.6                                                    | 3.2                 | -4996.1                                         | 17/2–17/2–15/2      | -8198.1 |
| 2     | 5–5–4     | 5418.3                                                   | 100.0              | 10,394.3                                                 | 23.7                | 4582.6                                          | 17/2–17/2–17/2      | -8198.1 |
| 3     | 5–5–5     | 5418.3                                                   | 100.0              | 20,705.8                                                 | 100.0               | 14,894.1                                        | 17/2–17/2–19/2      | -8198.1 |
| 4     | 4–5–4     | 3367.1                                                   | 38.9               | 815.6                                                    | 3.2                 | -2944.8                                         | 15/2–17/2–15/2      | -5731.2 |
| 5     | 4–5–5     | 3367.1                                                   | 38.9               | 10,394.3                                                 | 23.7                | 6633.9                                          | 15/2–17/2–15/2      | -5731.2 |
| 6     | 4–5–6     | 3367.1                                                   | 38.9               | 20,705.8                                                 | 100.0               | 16,945.4                                        | 15/2–17/2–17/2      | -5731.2 |
| 7     | 5–4–3     | 2056.2                                                   | 38.9               | -10,854.6                                                | 8.7                 | -13,304.2                                       | 17/2–15/2–13/2      | -4175.0 |
| 8     | 5–4–4     | 2056.2                                                   | 38.9               | -2546.5                                                  | 34.3                | -4996.1                                         | 17/2–15/2–15/2      | -4175.0 |
| 9     | 5–4–5     | 2056.2                                                   | 38.9               | 7032.2                                                   | 60.9                | 4582.6                                          | 17/2–15/2–17/2      | 4175.0   |
| 10    | 4–4–3     | 5.0                                                      | 24.2               | -10,854.6                                                | 8.7                 | -11,253.0                                       | 15/2–15/2–13/2      | 1708.0   |
| 11    | 4–4–4     | 5.0                                                      | 24.2               | -2546.5                                                  | 34.3                | -2944.8                                         | 15/2–15/2–15/2      | 1708.0   |
| 12    | 4–4–5     | 5.0                                                      | 24.2               | 7032.2                                                   | 60.9                | 6633.9                                          | 15/2–15/2–17/2      | 1708.0   |
| 13    | 3–4–3     | 340.5                                                    | 50.5               | -10,854.6                                                | 8.7                 | -10,907.5                                       | 13/2–15/2–13/2      | 937.2    |
| 14    | 3–4–4     | 340.5                                                    | 50.5               | -2546.5                                                  | 34.3                | -2599.3                                         | 13/2–15/2–15/2      | 937.2    |
| 15    | 3–4–5     | 340.5                                                    | 50.5               | 7032.2                                                   | 60.9                | 6979.4                                          | 13/2–15/2–17/2      | 937.2    |
| 16    | 2–3–2     | 4048.0                                                   | 37.9               | -21,662.2                                                | 15.4                | -18,011.6                                       | 15/2–13/2–11/2      | 3486.3   |
| 17    | 2–3–3     | 4048.0                                                   | 37.9               | -15,058.7                                                | 33.7                | -11,404.1                                       | 15/2–13/2–13/2      | 3486.3   |
| 18    | 2–3–4     | 4048.0                                                   | 37.9               | -6750.5                                                  | 31.7                | -5095.9                                         | 15/2–13/2–15/2      | 3486.3   |
| 19    | 4–3–2     | 4199.1                                                   | 50.5               | -21,662.2                                                | 15.4                | -17,860.5                                       | 11/2–11/2–11/2      | 3859.0   |
| 20    | 4–3–3     | 4199.1                                                   | 50.5               | -15,058.7                                                | 33.7                | -11,253.0                                       | 11/2–11/2–13/2      | 3859.0   |
| 21    | 4–3–4     | 4199.1                                                   | 50.5               | -6750.5                                                  | 31.7                | -2944.8                                         | 11/2–11/2–13/2      | 3859.0   |
| 22    | 3–3–2     | 4544.6                                                   | 0.0                | -21,662.2                                                | 15.4                | -17,515.0                                       | 11/2–13/2–11/2      | 4257.1   |
| 23    | 3–3–3     | 4544.6                                                   | 0.0                | -15,058.7                                                | 33.7                | -10,907.5                                       | 13/2–13/2–13/2      | 4257.1   |
| 24    | 3–3–4     | 4544.6                                                   | 0.0                | -6750.5                                                  | 31.7                | -2599.3                                         | 13/2–13/2–15/2      | 4257.1   |
| 25    | 2–2–1     | 8084.5                                                   | 25.3               | -30,287.0                                                | 23.1                | -22,099.3                                       | 11/2–11/2–9/2       | 8067.9   |
| 26    | 2–2–2     | 8084.5                                                   | 25.3               | -25,702.7                                                | 23.1                | -17,515.0                                       | 11/2–11/2–11/2      | 6067.9   |
| 27    | 2–2–3     | 8084.5                                                   | 25.3               | -19,095.2                                                | 11.5                | -11,404.1                                       | 11/2–11/2–13/2      | 6067.9   |
| 28    | 3–2–1     | 8581.1                                                   | 37.9               | -30,287.0                                                | 23.1                | -22,099.3                                       | 13/2–11/2–9/2       | 1,006.0  |
| 29    | 3–2–2     | 8581.1                                                   | 37.9               | -25,702.7                                                | 23.1                | -17,515.0                                       | 13/2–11/2–11/2      | 1,006.0  |
| 30    | 3–2–3     | 8581.1                                                   | 37.9               | -19,095.2                                                | 11.5                | -10,907.5                                       | 13/2–11/2–13/2      | 1,006.0  |

Table 2. A table of hyperfine excitation pathways of $^{175}$Lu and $^{176}$Lu of the photoionization (the frequency position is with reference to the centre of gravity of $^{176}$Lu isotope).
Figure 2. Two dimensional contour of the ionization efficiency of (a) $^{175}$Lu and (b) $^{176}$Lu for the Doppler free condition and for a laser peak power density of 10 W/cm$^2$ and a bandwidth of 10 MHz for the excitation lasers. The resonance frequency positions of the hyperfine excitation pathways are numbered as per the Table 2. The resonance frequency position of the $17/2-17/2-19/2$ of the $^{176}$Lu is shown as "filled circle" in (a).
Numerical computations of ionization efficiency and degree of enrichment have been carried out for various experimental parameters such as bandwidth of the excitation lasers, peak power density of excitation and ionization lasers and half angular divergence limit to the atomic beam with a limit to the ionization efficiency set to $\sim 0.2$ and results are tabulated in Table 4. The ionization efficiency of 0.2 corresponds to the ionization of 80% of population available at the ground hyperfine level $17/2$. In order to compare the present results with the previously reported data\textsuperscript{5,14}, production rates have been calculated for the atomic number density of $5 \times 10^{14}$ of Lu corresponding to the number density of $1.3 \times 10^{13}$ for $^{176}$Lu. Further, the laser-atom interaction length is considered as 200 mm with a diameter of 10 mm. It can immediately be observed that for the case of lasers with a bandwidth of 100 MHz, the degree of enrichment reaches to $> 99\%$ for all angular divergence values. While for the case of laser bandwidth of 250 MHz, the degree of enrichment degrades to $\sim 94\%$; for higher angular divergence values of the atomic beam, the degree of enrichment marginally deteriorates to $\sim 93\%$. The increase in the angular divergence of the atomic beam has little impact on the degree of enrichment, which implies that

| Laser bandwidth (MHz) | Peak power density (W/cm$^2$) | Ionization laser $\eta_{176}$ | Selectivity | IRE (natural Lu) | Degree of enrichment % |
|----------------------|-------------------------------|--------------------------------|-------------|-----------------|------------------------|
| 100                  | 573.8130 nm                   | 20                             | 16,000      | 0.294           | 9784                   | 261                   | 99.6                   |
| 250                  | 560.3114 nm                   | 80                             | 18,000      | 0.197           | 653                    | 17                    | 94.4                   |
| 500                  |                               | 100                            | 22,000      | 0.179           | 73                     | 2                     | 65.6                   |

Table 3. Variation in the selectivity, isotope ratio enhancement (IRE) factor and degree of enrichment of $^{176}$Lu (for the Doppler free case) with the increase in the laser bandwidth.

| Laser bandwidth (MHz) | Half angle divergence | Co-propagating excitation laser beams | Counter-propagating excitation laser beams |
|----------------------|----------------------|--------------------------------------|------------------------------------------|
|                      | $\eta_{176}$         | Selectivity | IRE (natural Lu) | Degree of enrichment % | $\eta_{176}$ | Selectivity | IRE (natural Lu) | Degree of enrichment % |
| 100                  | Doppler free (0°)    | 0.294       | 9784              | 261                   | 99.6             | –           | –               | –                      |
|                      | 5°                   | 0.263       | 8749              | 233                   | 99.6             | 263         | 8743            | 233                   | 99.6             | 3.5                   |
|                      | 15°                  | 0.238       | 7984              | 211                   | 99.6             | 239         | 7893            | 211                   | 99.5             | 9.7                   |
|                      | 30°                  | 0.191       | 6284              | 168                   | 99.4             | 193         | 6284            | 168                   | 99.4             | 15.5                  |
|                      | 45°                  | 0.160       | 5240              | 140                   | 99.3             | 160         | 5212            | 139                   | 99.3             | 18.8                  |
|                      | 60°                  | 0.145       | 4697              | 125                   | 99.2             | 145         | 4641            | 124                   | 99.2             | 19.9                  |
|                      | 75°                  | 0.138       | 4446              | 119                   | 99.1             | 138         | 4371            | 117                   | 99.1             | 19.3                  |
|                      | 90°                  | 0.137       | 4430              | 118                   | 99.1             | 137         | 4367            | 117                   | 99.1             | 19.1                  |
| 250                  | Doppler free (0°)    | 0.197       | 653               | 17                    | 94.4             | 197         | 653             | 17                    | 94.4             | 2.7                   |
|                      | 5°                   | 0.197       | 651               | 17                    | 94.4             | 197         | 650             | 17                    | 94.4             | 2.7                   |
|                      | 15°                  | 0.191       | 631               | 17                    | 94.3             | 191         | 630             | 17                    | 94.2             | 7.7                   |
|                      | 30°                  | 0.178       | 584               | 16                    | 93.8             | 178         | 578             | 15                    | 93.8             | 14.3                  |
|                      | 45°                  | 0.165       | 538               | 14                    | 93.3             | 165         | 528             | 14                    | 93.2             | 19.0                  |
|                      | 60°                  | 0.156       | 506               | 14                    | 92.9             | 156         | 493             | 13                    | 92.8             | 21.0                  |
|                      | 75°                  | 0.151       | 490               | 13                    | 92.7             | 151         | 475             | 13                    | 92.5             | 20.8                  |
|                      | 90°                  | 0.151       | 489               | 13                    | 92.7             | 151         | 475             | 13                    | 92.5             | 20.6                  |
| 500                  | Doppler free (0°)    | 0.179       | 73                | 2                     | 65.6             | 179         | 73              | 2                     | 65.6             | –                      |
|                      | 5°                   | 0.179       | 73                | 2                     | 65.6             | 179         | 73              | 2                     | 65.6             | 2.4                   |
|                      | 15°                  | 0.177       | 73                | 2                     | 65.4             | 177         | 72              | 2                     | 65.3             | 7.2                   |
|                      | 30°                  | 0.173       | 71                | 2                     | 64.8             | 173         | 70              | 2                     | 64.6             | 13.9                  |
|                      | 45°                  | 0.169       | 69                | 2                     | 64.1             | 169         | 68              | 2                     | 63.8             | 19.5                  |
|                      | 60°                  | 0.165       | 67                | 2                     | 63.6             | 165         | 66              | 2                     | 63.1             | 22.2                  |
|                      | 75°                  | 0.163       | 66                | 2                     | 63.3             | 163         | 65              | 2                     | 62.6             | 22.3                  |
|                      | 90°                  | 0.163       | 66                | 2                     | 63.3             | 163         | 65              | 2                     | 62.7             | 22.3                  |

Table 4. A table of degree of enrichment and production rates of $^{176}$Lu for various experimental parameters. Production rates are calculated for the atomic number density of $5 \times 10^{14}$ of Lu (corresponding to the number density $1.3 \times 10^{13}$ of $^{176}$Lu); length of the laser-atom interaction region 200 mm, laser beam diameter of 10 mm. Peak power density of the first, second excitation lasers and ionization laser for the 100 MHz case are 40, 20, 16,000 W/cm$^2$; for 250 MHz case are 80, 20, 18,000 W/cm$^2$; for 500 MHz case are 100, 40, 22,000 W/cm$^2$ respectively.

Numerical computations of ionization efficiency and degree of enrichment have been carried out for various experimental parameters such as bandwidth of the excitation lasers, peak power density of excitation and ionization lasers and half angular divergence limit to the atomic beam with a limit to the ionization efficiency set to $\sim 0.2$ and results are tabulated in Table 4. The ionization efficiency of 0.2 corresponds to the ionization of 80% of population available at the ground hyperfine level $17/2$. In order to compare the present results with the previously reported data\textsuperscript{5,14}, production rates have been calculated for the atomic number density of $5 \times 10^{14}$ of Lu corresponding to the number density of $1.3 \times 10^{13}$ for $^{176}$Lu. Further, the laser-atom interaction length is considered as 200 mm with a diameter of 10 mm. It can immediately be observed that for the case of lasers with a bandwidth of 100 MHz, the degree of enrichment reaches to $> 99\%$ for all angular divergence values. While for the case of laser bandwidth of 250 MHz, the degree of enrichment degrades to $\sim 94\%$; for higher angular divergence values of the atomic beam, the degree of enrichment marginally deteriorates to $\sim 93\%$. The increase in the angular divergence of the atomic beam has little impact on the degree of enrichment, which implies that
Table 5. Comparison of the photoionization schemes for the degree of enrichment and production rates of \(^{176}\)Lu for various experimental parameters. *See Table 4 for laser peak power density of the lasers.

| Bandwidth (MHz) | Half angle divergence (degrees) | 540–535 nm scheme | 573–560 nm scheme |
|----------------|---------------------------------|-------------------|-------------------|
|                |                                 | D’yachkov et al.\(^5\) | Previous work\(^4\) | Present work |
|                | Production rate (mg/h) | Degree of enrichment (%) | Production rate (mg/h) | Degree of enrichment (%) | Production rate (mg/h) | Degree of enrichment (%) |
| 100            | 90°                           | 22.55             | 30.7              | 22.55             | 33.8–34.7             | 19.1             | 99.1             |
|                | 5°                            | 3.7               | 68.4              | 3.5               | 66.6–66.8             | 3.5               | 99.6             |
| 250            | 90°                           | –                 | –                 | –                 | –                     | 20.6             | 92.5–92.7       |
|                | 5°                            | –                 | –                 | –                 | –                     | 2.4              | 65.6             |
|                | 45°                           | –                 | –                 | –                 | –                     | 19.5             | 63.8–66.1       |
|                | 90°                           | –                 | –                 | –                 | –                     | 22.3             | 62.7–63.3       |

Effect of unknown parameters. In atomic vapor laser isotope separation process, all the atomic parameters such as isotope shifts, hyperfine structures, lifetimes, branching ratios, decay to trapped, autoionization states and cross-sections; laser parameters such as peak power density, pulse width, bandwidth, repetition frequency; source parameters such as temperature, angular divergence all influence the overall degree of enrichment and the production rate. For the case of the 573–560 nm scheme, some of the spectroscopic parameters have not been measured so far and hence are not known. The effect of these parameters on the degree of enrichment and the resultant production rates is discussed below.

(a) Isotope shift for the 560 nm transition

Isotope shift for the \(5d6s6p^4P_{1/2} \rightarrow 6s6p^2^2P_{3/2}(35274.5\, \text{cm}^{-1})\) transition has not been reported so far. Isotope shift of a transition comprises of field shift and the mass shift components which can be expressed as\(^{19}\)

\[
\delta \gamma \approx F\delta \langle r^2 \rangle_{175,176} + M_1\delta (M^2_{175} - M^2_{176})
\]

where \(F\), \(M\) are the field shift and mass shift parameters, \(\delta (r^2)_{175,176}\) is the difference in the mean square nuclear charge radii.

For the case of Lutetium isotopes, the difference in the mean square nuclear charge radii \(\delta (r^2)_{175,176} = 0.041\) fm\(^2\) is small\(^{19}\), resulting in small field shift. The mass shift is also small owing to the small value of the factor \((M^2_{175} - M^2_{176})\). As a result, the isotope shift between \(176\) and \(177\)Lu is small i.e., about \(\sim 100\) MHz. Large isotopic selectivity of the present scheme is due to the large spread of the hyperfine spectrum of \(^{176}\)Lu, therefore, the influence of the variation in isotope shift is not of much significance.

(b) Einstein’s coefficient for the 560 nm transition

Einstein’s \(A\) coefficient for the transition has not reported so far. However, the typical values reported for the \(^{4}P\) fine structure level were reported to be in the range of \(3 \times 10^6\) to \(2 \times 10^7\). A mean value of \(1.3 \times 10^7\) has been taken for the computations. Due to this uncertainty in the value of Einstein’s \(A\) coefficient, the optimum peak power density of the second excitation transition may vary between 20 and 97 W/cm\(^2\) which can easily be achieved for the pulsed lasers. Due to the large spread of the hyperfine spectrum of \(^{176}\)Lu isotope and the separation of its \(17/2–17/2–19/2\) hyperfine excitation pathway from the \(^{175}\)Lu spectrum, the power broadening (due to increase in the laser peak power density) varies the ionization efficiency rather than the selectivity.

(c) Autoionization states and cross-sections
The non-availability of data on the autoionization states and their ionization cross-sections is the primary impediment to the proposed 573–560 nm scheme. Most of the previous work\textsuperscript{[4,9,10,11]} carried out have reported the data pertaining to the Rydberg series and autoionization states originating from the 5d6s7s 4D\textsubscript{3/2,5/2}, 5d6s6p 4D\textsubscript{3/2,5/2} and 6s\textsuperscript{2}nd 5D\textsubscript{3/2,5/2}, 6s\textsuperscript{6}ns 5F\textsubscript{5/2} states. Dyachkov et al.\textsuperscript{[12]} have found a strong autoionization level connecting to the 5d6s7s 4D\textsubscript{3/2} upper level of the 540–535 nm scheme. Further experimental studies on the search for suitable autoionization states connecting to the 6s6p 4F\textsubscript{5/2} upper level of the 573–560 nm scheme is necessary.

Conclusion
A new photoionization pathway has been proposed for the isotope separation of \textsuperscript{176}Lu precursor isotope. The optimum conditions for the isotope separation of \textsuperscript{176}Lu have been obtained through density matrix formalism. It has been shown that it is possible to obtain an enrichment of > 99% with high production rates of 19 mg/h when the excitation lasers with bandwidth of 100 MHz are employed. Even when broadband dye lasers of bandwidth of 500 MHz are used, the degree of enrichment of ~63% can be obtained with a production rate of > 22 mg/h. This significantly eases the restrictions on the bandwidth of the excitation lasers and Doppler broadening enabling high production rates than previously reported, thus expected to minimise the global shortage of the \textsuperscript{176}Lu precursor isotope for the production of the \textsuperscript{177}Lu medical isotope.

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