The backtracking uncertainty bounding algorithm for chlorine sensor fault detection

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Abstract

Due to parameter uncertainties, such as errors in water flow estimation and unknown chlorine decay rate, there is typically significant uncertainty in the estimated chlorine concentration in a Drinking Water Distribution Network (DWDN). For certain applications, such as control, monitoring and event detection, it is crucial to have available not only the estimated chlorine concentration but also the possible interval of concentration values. This paper presents a novel methodology, the Backtracking Uncertainty Bounding Algorithm (BUBA), for calculating bounds on chlorine concentration at specific locations in DWDNs. The utility of this algorithm is demonstrated in a chlorine sensor fault detection scheme.

Keywords: Water Distribution Networks; Water Quality Monitoring; Chlorine Sensor Faults

1. Introduction

A drinking water distribution network (DWDN) is an interconnected collection of pipes, water sources and hydraulic control elements such as pumps, valves and tanks, that delivers to consumers water at the demanded quantity and pressure. Drinking water delivered to consumers should contain a small disinfectant residual in order to reduce the risk of human exposure to pathogens. A number of water utilities use chlorine for disinfection because it is inexpensive and effectively controls a number of disease-causing organisms. The regulation of chlorine concentration in drinking water, also referred to as water quality, requires chlorine injection stations. For monitoring chlorine concentration, chlorine sensors are used and regulation is performed either by a human operator, or automatically using real time sensor measurements and feedback control algorithms. Water quality monitoring and control is an important issue since customer complaints can occur if the disinfectant applied to the water is not regulated properly, but most importantly, in the case of a contamination event, thousands of people can be affected if not detected in time.

In order for water quality monitoring and control to be applied, reliable water quality sensors are needed. In some cases, these sensors may become unreliable if they suffer hardware failure, battery failure, component deterioration.

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or communication problems. Additionally, like many sensors, they require frequent calibration. False sensor readings can result in poor regulation of chlorine concentration. Due to the cost of such sensors, redundancy through the placement of additional sensors may not always be an option. Analytical redundancy could be added, through the development of sensor fault detection algorithms that make it possible to identify and replace faulty water quality sensors.

Sensor fault detection methods are classified into process history and model-based methods. [1] Process history methods do not have a priori knowledge about the system model but they assume a large amount of historical data. Model-based fault detection methods rely in general on an explicit model of the monitored plant which is used to generate inconsistencies between the actual and expected behaviour. Such inconsistencies are called residuals. The use of residuals for fault diagnosis is the essence of analytical redundancy in systems. The generated residuals are utilized to form a decision rule for detection. The decision rule is in many cases formulated with the use of a threshold for the residual, which when crossed, a fault is detected. The threshold can be constant or it can change, adapting to changing input excitation, varying system parameters or modelling uncertainties. In this case it is called an adaptive threshold [2].

The problem of chlorine sensor fault detection in DWDN has not yet been directly addressed. This is due mainly to the intertwined nature of chlorine sensor faults and contamination detection in DWDN, with the later being the main subject of interest in water quality research, as discussed below. When a contaminant enters the water, the chemical reaction of it with chlorine will alter the chlorine concentration in the infected areas, which will result in unexpected levels of chlorine concentration at sensor locations during normal sensor operating conditions. The discrimination between sensor faults and contamination can be performed mainly from correlations between measurements from many sensors and it is a subject of research.

Contamination detection can either use chlorine concentration as the signal used for detection, or other water parameters such as Oxidation Reduction Potential (ORP), total organic carbon, turbidity, as well as changes in water pH and conductivity [3]. In past research, various works have investigated the contamination detection problem using approaches that involve either multiple water quality parameters, either solely chlorine concentration as an indicator. In the CANARY event detection tool of the US EPA, time series analysis of multiple water quality parameters can be performed. An estimation error is then computed which is compared to an adaptive threshold. The threshold depends on the standard deviation of a moving window of measurements [4]. In [5] contaminant reaction models with water quality parameters commonly measured in DWDN were used for real-time event detection.

Model-based approaches for contaminant event detection have also been proposed. In [6] the effect of contaminants on certain monitored parameters was used for contaminant detection. This was achieved using the model of a benchmark DWDN in EPANET-MSX software [7]. The use of chlorine sensors specifically for contamination detection was proposed in [8]. In most cases the actual chlorine reaction dynamics with contaminants is not known, so empirical models are proposed [9].

The use of water quality models in chlorine residual estimation does not produce accurate results because of their time varying nature. The time-varying nature is due to the variability of water demands, which is the greatest source of uncertainty in water quality models [10]. Since an accurate real-time estimate of water demands is difficult to be obtained, there have been some efforts to estimate water quality considering the uncertainty on water demands. In [11], a chlorine concentration interval estimator was implemented which calculates an interval in which chlorine residual should remain throughout the network, considering uncertainty in water flows. In [12], a chlorine concentration bound estimator is implemented for contamination event detection, using varying water flows in the network. The bounds are calculated in real-time using EPANET software through Monte-Carlo simulations.

Contrary to contamination detection in DWDN, sensor fault detection does not require a full state estimation of chlorine concentration in the network, only estimation at sensor nodes. Since chlorine sensors are not only used for monitoring, but also for contamination event detection, the ability to detect when a fault has occurred in one of them becomes of critical importance. In this work we investigate the use of a standard model-based sensor fault detection technique, for detecting faults on chlorine sensors monitoring DWDN. This is achieved by using the proposed Backtracking Uncertainty Bounding Algorithm (BUBA) in order to calculate in real-time the upper and lower bound on chlorine concentration at specific locations in the network. At this stage we assume that no other faults occur in the system, including faults classified as contamination events.
Fig. 1: General diagram of water quality sensor fault detection scheme using the proposed Backtracking Uncertainty Bounding Algorithm (BUBA)

This paper is organized as follows: The water network description and the problem formulation are described in Section 2. The design of the sensor fault detection methodology is described in Section 3. In Section 4 the proposed Backtracking Uncertainty Bounding Algorithm (BUBA) is presented. Simulation results of the application of the BUBA in the described sensor fault detection framework on an example network are provided in Section 5 followed by some concluding remarks in Section 6.

2. Problem Formulation

A water distribution network’s topology is described by a directed graph denoted as $G = (N, P)$. The vertices $N = \{N_1, N_2, \cdots, N_n\}$ correspond to network nodes, where the order of the graph, $|N| = n$, is the number of nodes. Nodes represent junctions of pipes. Consumer water demands, water sources and water tanks are assumed to be located at nodes. The edges $P = \{p_{ij} : i \neq j, N_i \in N, N_j \in N\}$ represent network pipes, where the graph’s size, $|P| = m$, is the number of pipes. Each pipe $p_{ij}$ is characterized by pipe length $L_{ij}$, pipe cross section area $A_{ij}$ and pipe wall reaction rate $K_{w_{ij}}$. The edges can also represent water pumps and pipe valves, with these being the main hydraulic control elements in a water network. The dynamics of a water network can be divided into two subsystems: 1) the hydraulic dynamics which describe the flow and pressure and 2) the quality dynamics, which for the purpose of this work describe the chlorine concentration. The hydraulic dynamics affect the quality dynamics, but not vice versa.

2.1. Hydraulic dynamics

The hydraulic dynamics are governed by the physical laws of fluids (in this case water) moving in a distribution network, and their form depends on the topology of $G = (N, P)$. The dynamic equations describing hydraulic dynamics are solved in discrete time, using a hydraulic time step $\Delta t_h$, with $k$ being the discrete time step that corresponds to the continuous time instant $k\Delta t_h$. When solving these equations, all the quantities are considered constant between hydraulic steps.

The hydraulic dynamics are driven by the consumer water demands, denoted by $d(k) = [d_1(k) \ d_2(k) \ \cdots \ \ d_n(k)]^\top$, where $d_i(k)$ is the consumer demand at node $N_i$ at time $k\Delta t_h$. Hydraulic quantities can be changed using certain hydraulic control actions in the network, denoted by $v(k) = [v_1(k) \ v_2(k) \ \cdots \ \ v_a(k)]^\top$ where $a$ is the number of control inputs. These control actions correspond to water pumps which are used to add energy to the system, or valves that remove energy or regulate the flow of water in a pipe, depending on their type.

Water demands at each node of a DWDN are typically unknown, but can be estimated using flow measurements from the network. We denote these measurements as $z(k) = \Gamma q(k)$, where $\Gamma \in \mathbb{R}^{r \times m}$ a constant matrix. By using the
physical laws that describe the flow relationships in pipes, there exists a nonlinear function $f_h : \mathbb{R}^r \times \mathbb{R}^a \times G \mapsto \mathbb{R}^m$, e.g. as described in EPANET (Rossman, 2000), for estimating the steady state water flows in each pipe of the water network, denoted by $\hat{q}(k) = [\hat{q}_1(k) \ \hat{q}_2(k) \cdots \hat{q}_m(k)]^T$. This function can be posed as follows:

$$\hat{q}(k) = f_h(z(k), v(k); G). \quad (1)$$

As this is an estimator, the estimated water flows may not be the same as the actual water flows in the network, denoted by $q(k)$. This is because of modelling uncertainty and insufficient knowledge of water demands $d(k)$. The relationship between the real water flows in the network and the estimated water flows is expressed as:

$$q(k) = \hat{q}(k) + e_q(k), \quad (2)$$

where $e_q(k)$ is defined as the water flow estimation error. The main assumption made in this work related to the hydraulics in a water network is:

**Assumption 1:** There exists a hydraulic estimator that uses the nonlinear function $f_h(\cdot)$ of (1) to give an estimate of all water flows in the water network. The estimation error $e_q(k)$ of this estimator is unknown but uniformly bounded, with a known bound denoted as $\bar{e}_q(k)$, i.e. $|e_q(k)| \leq \bar{e}_q(k)$. Based on the estimation error bound, a lower and upper bound for the water flow estimate is calculated denoted by $\underline{q}(k)$ and $\bar{q}(k)$ respectively. These bounds will define an interval of possible values for $\hat{q}(k)$ that will ensure that $q(k) \in [\underline{q}(k), \bar{q}(k)]$.

### 2.2. Water Quality Dynamics

Water quality dynamics are governed by the chemical reactions of different substances in the water. In this work, we refer to water quality as the chlorine concentration in water. Water quality dynamics depend on the water flows $q(k)$ in each pipe of the network and the water network topology given by the graph $G$. The dynamic equations describing quality dynamics are solved in discrete time, usually using a time step smaller than the one the hydraulic estimator uses. In this work we will consider a common time step for the hydraulic and quality estimators, denoted as $\Delta t$, with $k$ being the discrete time step that corresponds to the continuous time instant $k\Delta t$. The quality can be regulated through chlorine injection stations at specific nodes of the network. The chlorine concentration forced at injection nodes is the quality control input and it is denoted by $u(k) = [u_1(k) \ u_2(k) \cdots \ u_b(k)]^T$, where $b$ the number of chlorine injection points or booster stations. The chlorine concentration $u(k)$ forced at chlorine injection locations is known for all $k$.

By using the physical and chemical laws that describe chlorine transport in water networks, there exists a nonlinear function $f_y : \mathbb{R}^{n \times D} \times \mathbb{R}^{b \times D} \times G \mapsto \mathbb{R}^s$, e.g. as described in [13,14], which produces an estimate of the chlorine concentration at a number of $s$ monitored nodes where chlorine sensors are installed. This estimate is denoted by $\hat{y}(k) = [\hat{y}_1(k) \ \hat{y}_2(k) \cdots \hat{y}_s(k)]^T$ and it can be calculated as follows:

$$\hat{y}(k) = f_y(\hat{q}(k), \hat{q}(k-1), ..., \hat{q}(k-D), u(k), u(k-1), ..., u(k-D); G). \quad (3)$$

where $D$ is the memory of function $f_y(\cdot)$. As this is an estimator, the estimated quality will differ from the actual water quality at monitored nodes, denoted by $y(k)$. This is due to modelling uncertainty, but also due to the use of estimated water flows in the function, which include estimation errors. The relationship between the real chlorine concentration at monitored nodes $y(k)$ and the estimated one, is given by:

$$y(k) = y(k) + e_y(k), \quad (4)$$

where $e_y(k)$ is defined as the water quality estimation error which is non-zero due to the error in water flows estimate used and due to water quality modelling uncertainty.

Chlorine decay in the water of a distribution network depends mainly on the organic substances present in the water and the pipe in which the water is in. The rate of decay of chlorine due to organic substances in the water is called bulk reaction rate and is denoted by $K^b$. The decay due to reaction of chlorine with pipe walls is characterized by the pipe wall coefficient of each pipe, denoted by $K^p_{ij}$. The overall decay rate in a pipe is found by combining both bulk and wall reaction rates, is denoted by $K_{ij}$ and an estimate is usually available by manual sampling [15].
Assumption 2: In this work, water quality modelling uncertainty corresponds to unknown chlorine decay rate. A known estimate of the chlorine decay rate $\hat{K}_{ij}$ is available for every pipe $p_{ij}$ of the network. The modelling uncertainty is bounded with a known bound. This bound is calculated using a known constant $\beta$ which defines an upper and lower bound of the real chlorine decay rate $K_{ij}$ for each pipe $p_{ij}$, as follows:

$$\tilde{K}_{ij} = \hat{K}_{ij} + \beta \hat{\hat{K}}_{ij},$$

$$\check{K}_{ij} = \hat{K}_{ij} - \beta \hat{\hat{K}}_{ij}.$$  

(5)

The water quality model used is an algorithm that implements the function $f_i(\cdot)$ described in (3) to give an estimate $\hat{y}(k)$ of the chlorine concentration at monitored nodes. This model takes as input an estimate $\hat{q}(k)$ of the water flows and the chlorine injection $u(k)$. The water quality model used makes some assumptions about the actual water quality dynamics which are: 1) chlorine decay is characterized by a first order chlorine reaction model and 2) there is instant and complete mixing of chlorine concentrations in pipe junctions [15]. It also assumes that the initial chlorine concentration in the network is known.

The chlorine sensors that monitor water quality are placed on a specific subset of the network nodes, denoted as $N_s \subset N$, after solving the optimal sensor placement problem described in [16,17]. The sensor readings are indicated as $\{y_i(k) \mid i \in S_i\}$, where the measurement $y_i(k)$ corresponds to sensor $S_i$ at time instant $k$ with $i \in [1 \cdots n]$. The relationship between sensor readings and the actual chlorine concentration is given by:

$$y_i(k) = y_i(k) + n_i(k) + f_i(k),$$

(6)

where $n_i(k)$ is the measurement noise of sensor $S_i$ and $f_i(k) = B_i(k - \tau_0)\phi_i(k)$ is a possible additive fault on sensor $S_i$, with $B_i$ and $\tau_0$ being the time profile and the time instant of occurrence of the $i$th fault respectively, and $\phi_i$ being the fault signature. The time profile in abrupt and permanent sensor faults is described as $B_i(k) = 0$, if $k < 0$ and $B_i(k) = 1$ if $k > 0$. The fault signature $\phi_i$ in this work will be a proportion of the sensor output without a fault.

Assumption 3: The noise corrupting the measurements of sensor $S_i$ is unknown but uniformly bounded, i.e. there exists a known constant bound $\bar{n}_i(k)$ for which $|n_i(k)| \leq \bar{n}_i(k), \forall k$ with $i \in [1 \cdots s]$.

The aim of this work is to use the BUBA to calculate an upper bound $\tilde{e}_i(k)$ for the water quality estimation error and use this information in order to detect when a fault $f_i(k)$ has occurred.

3. Sensor Fault Detection Module

The Fault Detection Module (FDM) contains the detection logic which utilizes the model information provided, as well as the sensor measurements, to decide if a fault has occurred. As seen in Fig. 1, the FDM receives as input the chlorine concentration bounds $\tilde{\gamma}(k)$ and $y(k)$ and the sensor readings $y_i(k)$. The bounds are generated by the proposed BUBA and accommodate the modelling uncertainty and inaccurate knowledge of water flows and at any given moment these bounds should guarantee that the real chlorine concentration $y(k) \in [\tilde{\gamma}(k), \check{\gamma}(k)]$. Based on this bounds, a chlorine concentration estimate is defined as:

$$\hat{y}_i(k) = \left(\tilde{\gamma}_i(k) + \check{\gamma}_i(k)\right)/2.$$  

(7)

The maximum estimation error for each monitored node is then defined as:

$$\tilde{e}_{y_i}(k) = \left(\tilde{\gamma}_i(k) - \check{\gamma}_i(k)\right)/2.$$  

(8)

The FDM then gives as output a vector of binary indicators which correspond to a fault occurring at the corresponding sensor. The output of FDM is denoted by $\tilde{f}^d(k) = \left[f^d_1(k) f^d_2(k) \cdots f^d_s(k)\right]$ where $f^d_i(k) = 0$ when sensor $S_i$ is healthy and $f^d_i(k) = 1$ when a fault has been detected on sensor $S_i$.

The detection logic is the following: using the estimate of water quality $\hat{y}(k)$ and the sensor readings $y^*(k)$, a residual for each sensor $S_i$ is generated:

$$e_i(k) = y^*_i(k) - \hat{y}_i(k).$$

(9)
By substituting (6) into (9), the residual becomes: $\epsilon_i(k) = (y_i(k) - \hat{y}_i(k)) + n_i(k) + f_i(k)$. We assume an upper bound on measurement noise $\bar{n}_i(k)$. Using the measurement noise bound along with the estimation error bound, a threshold $\bar{\epsilon}_i(k)$ can be generated which is given by:

$$\bar{\epsilon}_i(k) = \bar{\epsilon}_{y_i}(k) + \bar{n}_i(k). \quad (10)$$

When the sensors operate in healthy conditions and no sensor is experiencing a fault, the following Analytical Redundancy Relation (ARR) is satisfied:

$$\bar{\epsilon}_i(k) \geq |\epsilon_i(k)|. \quad (11)$$

When a fault occurs on sensor $S_i$, the residual corresponding to that sensor will become:

$$\epsilon_i(k) = e_{y_i}(k) + n_i(k) + f_i(k), \quad (12)$$

with $f_i(k) \neq 0$. Assuming that the fault occurs at time instant $k_d$, the fault will be detected at the first discrete time instant $k_d$ when the ARR corresponding to that sensor is not satisfied, i.e. when $f_i(k)$ is large enough so that $\bar{\epsilon}_i(k_d) < |\epsilon_i(k_d)|$.

4. The Backtracking Uncertainty Bounding Algorithm

To calculate the chlorine concentration bounds $\bar{y}(k)$ and $y(k)$ needed for the FDM module, the proposed Backtracking Uncertainty Bounding Algorithm (BUBA) is implemented. This uses the backtracking approach [13] and takes as inputs the maximum estimation error of water flows $\bar{\epsilon}_i(k)$ and an upper and lower bound of chlorine decay rate $\bar{K}$ and $K$, as seen in Fig. 1. The algorithm gives as output the upper and lower bound of chlorine concentration $\bar{y}(k)$ and $y(k)$ at monitored nodes.

The water networks on which the BUBA is applied is a class of networks that do not have storage tanks and where no flow reversal occurs, i.e. $q(k) \geq 0 \forall k$. This assumption is made because the algorithm does not describe the behaviour of chlorine when it goes through storage tanks or when flow reverses. This limitation will be removed in future work. The BUBA consists of three algorithms formulated as functions that recursively call each other. A description of each algorithm is given below. The calculation of the maximum chlorine concentration is omitted since it’s trivial when the procedure for finding the minimum is elaborated.

Algorithm 1 takes as input the network node which we currently investigate, which we will call the active node and denote as $N_a \in \mathcal{N}$. It also takes as input the active pipe $p_{aj} \in \mathcal{P}$ which is defined as a pipe that it is connected and brings water into the active node at the current time step $k$. The output of the algorithm is the upstream node of the active node through the active pipe which we will denote as $N_j \in \mathcal{N}$. Additionally the algorithm returns the minimum and maximum time the water has spent into the pipe (detention time), denoted as $\bar{\tau}_{p_{aj}}(k)$ and $\tau_{p_{aj}}(k)$ respectively. These are calculated based on the bounds on water flows.

Algorithm 2 calculates the minimum and maximum chlorine concentration that the active pipe can contribute to the active node, denoted as $c_{\tau_{p_{aj}}}(k)$ and $\bar{c}_{\tau_{p_{aj}}}(k)$. It achieves this by calling Algorithm 1 to acquire the detention times in the pipe and the upstream node and by using the bounds on chlorine decay rate. If the upstream node is a chlorine injection node, the concentration is exactly known for all time steps and the minimum and maximum concentration that the pipe contributes can be calculated using only the detention times. If the upstream node is not an input node, the minimum and maximum concentration of the upstream node is unknown and it must be calculated. This is achieved by calling Algorithm 3 which returns the minimum and maximum concentration of the upstream node.

Algorithm 3 calculates the minimum and maximum concentration of the active node, denoted as $c_{N_a}(k)$ and $\bar{c}_{N_a}(k)$. It is the algorithm called when we want to find the chlorine concentration bounds of a monitored node. The algorithm finds the set of pipes $\mathcal{P}_{N_a}$ which is defined as the set of pipes that bring water into the active node. It then calculates each pipe’s minimum and maximum chlorine concentration contribution by calling Algorithm 2. By assuming instantaneous and complete mixing of chlorine in pipe junctions, the optimization problem of (14) is formulated and solved, returning the minimum and maximum concentration of the active node.
The concept of instant and complete mixing of chlorine at pipe junctions, mentioned in subsection 2.2, is expressed mathematically by equation (13):

\[
c_{N_j}(k) = \frac{\sum_{p_{aj} \in P_{N_j}} q_{p_{aj}}(k) c_{p_{aj}}(k)}{\sum_{p_{aj} \in P_{N_j}} q_{p_{aj}}(k)} ,
\]

(13)

where \(c_{N_j}(k)\) the chlorine concentration at the junction node, \(p_{aj}\) is a pipe bringing water into node \(N_a\) from node \(N_j\), \(P_{N_j}\) the set of all pipes that bring water into \(N_a\), \(q_{p_{aj}}(k)\) the water flow of pipe \(p_{aj}\) at time instant \(k\) and \(c_{p_{aj}}(k)\) the chlorine concentration contribution of pipe \(p_{aj}\) at time instant \(k\).

In this work, water flows are considered uncertain but within a predefined range, which transforms the calculation of (13) into an optimization problem with constraints, with the flows \(q(k)\) being the variable which is subject to constraints. The minimum value of the objective function \(J(q(k))\) is the minimum concentration at the active node. Additionally, we must consider that the chlorine contribution of each pipe is again uncertain but within a predefined range. This means that the coefficients \(c_{p_{aj}}(k)\) of variable \(q_{p_{aj}}(k)\) will not be constants, but an interval, with \(c_{p_{aj}}(k) \in [\underline{c}_{p_{aj}}(k), \overline{c}_{p_{aj}}(k)]\). The optimization procedure is finally formulated as a linear fractional problem with interval coefficients \([18]\) given by:

\[
c_{N_j}(k) = \min_{q_{p_{aj}}(k)} \frac{\sum_{p_{aj} \in P_{N_j}} q_{p_{aj}}(k) \underline{c}_{p_{aj}}(k)}{\sum_{p_{aj} \in P_{N_j}} q_{p_{aj}}(k)} \text{ s.t. } q_{p_{aj}}(k) < q_{p_{aj}}(k) < \overline{q}_{p_{aj}}(k) \forall p_{aj} \in P_{N_j}, \quad (14)
\]

### Algorithm 1 Water detention time in a pipe

**Input**: active node \(N_a\), active pipe \(p_{aj}\), time step \(k\)  
**Output**: upstream node \(N_j\), minimum and maximum water detention time \(\underline{\tau}_{p_{aj}}(k)\) and \(\overline{\tau}_{p_{aj}}(k)\)

**begin**
1: Find upstream node \(N_j\) of active node \(N_a\) through pipe \(p_{aj}\) using the network graph \(G = (N, P)\)
2: Find \(\overline{\tau}_{p_{aj}}(k)\) as a function of \(q\)
3: Find \(\underline{\tau}_{p_{aj}}(k)\) as a function of \(\overline{q}\)
**return** \(N_j, \overline{\tau}_{p_{aj}}(k), \underline{\tau}_{p_{aj}}(k)\)

### Algorithm 2 Pipe minimum concentration contribution

**Input**: active node \(N_a\), active pipe \(p_{aj}\), current time step \(k\)  
**Output**: pipe minimum concentration \(c_{p_{aj}}(k)\)

**begin**
1: Find \(N_j, \overline{\tau}_{p_{aj}}(k), \underline{\tau}_{p_{aj}}(k)\) by calling Algorithm 1
2: if \((N_j\) is a booster station) then
3: \(\text{for } \tau = \overline{\tau}_{p_{aj}}(k) \text{ to } \underline{\tau}_{p_{aj}}(k) \text{ do}\)
4: \(c_{p_{aj}} = \exp\left(\frac{K_{p_{aj}}}{\Delta \tau} (\Delta \tau)^k k - \tau\right)\)
5: \(\underline{c}_{p_{aj}}(k) = \min\left(c_{p_{aj}}, \underline{c}_{p_{aj}}(k)\right)\)
6: \(\text{end for}\)
7: \(\text{return } \underline{c}_{p_{aj}}(k)\)
8: \(\text{end if}\)
9:
10: \(c_{p_{aj}}(k) = \infty\)
11: \(\text{for } \tau = (k - \overline{\tau}_{p_{aj}}(k)) \text{ to } (k - \underline{\tau}_{p_{aj}}(k)) \text{ do}\)
12: \(\text{Find } c_{N_j}(\tau)\) by calling Algorithm 3
13: \(c_{p_{aj}} = \exp\left(\frac{K_{p_{aj}}}{\Delta \tau} (\Delta \tau)^k \right) c_{N_j}(\tau)\)
14: \(\underline{c}_{p_{aj}}(k) = \min\left(c_{p_{aj}}, \underline{c}_{p_{aj}}(k)\right)\)
15: \(\text{end for}\)
**return** \(\underline{c}_{p_{aj}}(k)\)
5. Simulation Example

The BUBA in combination with the sensor fault framework described were tested using the example network seen in Fig. 2, which is a modified version without storage tanks of a benchmark network introduced by [19]. The network was simulated using the water network simulator EPANET and the results are used as the real network behaviour in order to test the proposed algorithm.

The parameters used for the simulation are the following: a hydraulic time step of 30 minutes and a quality time step of 3 minutes were chosen. The network was simulated for a time period of 3 days. A chlorine injection station is regulating the chlorine concentration at node $N_1$ at a varying set-point of $u_1(k) = u_1(k-1) + \sin(0.1k) + \cos(0.2k)$ mg/L. The initial chlorine concentration in all nodes is 0.5 mg/L. The monitored node in which the chlorine sensor is installed, is node $N_9$. The water quality bounds were calculated using an uncertainty in water flows of 20% of the estimated value and an uncertainty of chlorine decay coefficient of 10% of the estimated value. We note that the bounds are able to be calculated after $D = 1375$ minutes of the simulation has passed, which is the minimum time period at which the input affects the output through all the paths that connect the two. After the water quality bounds were calculated, a series of 170 Monte-Carlo simulations were performed in which the water flows and chlorine decay rates in each pipe varied with a uniform distribution within the predefined bounds. The results are shown in Fig. 3. We can observe that the chlorine concentration at node $N_9$ remains within the calculated bounds during every simulation.

Using the calculated bounds in a fault detection framework, a simulation of the same network was performed in which the bounds on flows and decay rate are the same. The sensor in node $N_9$ has random added noise of which the upper bound is equal to 0.05. In this simulation a fault occurs at time instant $\tau_0 = 2515$ minutes which causes the sensor to give a reading which is 20% lower than the reading in healthy operation. As it can be seen in the results shown in Fig. 4, the ARRs stop being satisfied at time instant $k_d = 2784$ at which the fault is detected, 269 minutes after it has occurred.

6. Conclusion

In this work, we presented a model-based chlorine sensor fault detection methodology for detection and isolation in a DWDN. To achieve this, we used the Backtracking Uncertainty Bounding Algorithm (BUBA) which calculates chlorine concentration bounds that were verified using the results of Monte-Carlo simulations obtained by EPANET. The calculated bounds were used in a sensor fault detection framework and successfully detected a fault imposed
on a chlorine sensor in the network. In designing the proposed methodology, some issues have arisen which will be addressed in future work. These include the following:

- A modification of the proposed algorithm is needed in order to work on networks that include storage tanks and flow reversals occur.
• In the case of a contamination event in the water network, the algorithm will not be able to discriminate between a sensor fault and a contaminant affecting the sensor. The fault decision logic should be improved, ideally by utilizing contamination detection techniques available in the literature.
• The detection thresholds can be improved by introducing a learning scheme into the algorithm that will learn some uncertain parameters of the water quality model, resulting in less conservative bounds on chlorine concentration.
• The uncertainty on water flows can be modelled more efficiently, by designing a hydraulic estimator which will calculate in real time an estimate of flows as well as the maximum estimation error. The maximum estimation error can vary and can be minimized using available measurements from the network and historic data.

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