Gravitational instability of radiative plasma with finite larmor radius corrections

Sachin Kaothekar and R K Chhajlani
School of Studies in Physics, Vikram University, Ujjain -456010, M.P., India.
'Mahakal Institute of Technology and Management, Ujjain-M.P., India.
E-mail: sac_kaothekar@rediffmail.com

Abstract. The effect of radiative heat-loss function and finite ion Larmor radius (FLR) corrections on the gravitational instability of infinite homogeneous viscous plasma has been investigated incorporating the effects of thermal conductivity and finite electrical resistivity. We find that radiative heat-loss function and thermal conductivity modify the Jeans criterion into radiative instability criterion. Numerical calculations have been performed to show the effect of various parameters on the growth of the gravitational instability.

1. Introduction
Plasma instability is one of the most fascinating fields of physics. It is a well established fact that gravitation plays a key role in understanding the process of fragmentation of interstellar medium, formation of stars, planets, asteroids, comets and other astrophysical objects. Shaikh et al. [1] have investigated the gravitational instability of thermally conducting plasma in a variable magnetic field. Recently Dhiman and Dadwal [2] have discussed the gravitational instability of a non-uniform rotating heat-conducting medium. Janaki et al. [3] have investigated the Jeans instability in a viscoelastic gravitating fluid. Along with this, the recent observations from solar corona and interstellar medium have shown that the radiative heat-loss mechanism plays a significant role in the process of molecular clouds condensation and in star formation, in connection with thermal instability. Bora and Talwar [4] have investigated magneto-thermal instability with generalized ohm’s law. More recently Prajapati et al. [5] have investigated the self-gravitational instability of rotating plasma with arbitrary radiative heat-loss function. In addition to this, in many plasma situations such as in magnetic reconnection, solar corona and interstellar plasmas the FLR plays a key role in stability investigations. Vaghela and Chhajlani [6] have investigated the stabilizing effect of FLR on magneto-gravitational stability of resistive plasma with thermal conduction. Recently Devlen and Pekunlu [7] have investigated the effects of FLR on weakly magnetized, dilute plasmas. Thus in the present work, we have examined the effect of radiative heat-loss function, FLR corrections, thermal conductivity and finite resistivity on gravitational instability of plasma. The above work is applicable to structure formation in interstellar medium.

2. Linearized perturbation equations of the problem
Let us consider an infinite homogeneous, gravitating, radiating, thermally conducting, viscous plasma of finite electrical resistivity in the presence of magnetic field \( H(0,0,H) \). The linearized perturbation equations of the problem with these effects are written as

\[
\frac{\partial \mathbf{u}}{\partial t} = -\frac{1}{\rho} \nabla \Phi - \frac{\nabla \cdot \mathbf{P}}{\rho} + \nabla U \delta \mathbf{u} + \frac{1}{4\pi \rho} (\nabla \times \mathbf{h}) \times \mathbf{H} + \nu \left[ \nabla^2 \mathbf{u} - \frac{\mathbf{u}}{K_1} \right],
\]

\[
\frac{\partial \rho}{\partial t} = -\rho \nabla \cdot \mathbf{u},
\]

\[
\frac{1}{\gamma - 1} \frac{\partial \mathbf{p}}{\partial t} - \frac{\gamma - 1}{\gamma - 1} \frac{\partial \rho}{\partial t} + \rho (L_\nu \mathbf{\Phi} + L_\rho \delta \mathbf{U}) - \nabla^2 \delta T = 0,
\]

\[
\frac{\partial \mathbf{h}}{\partial t} = \nabla \times (\mathbf{u} \times \mathbf{H}) + \eta \nabla^2 \mathbf{h},
\]

\[
\nabla \cdot \mathbf{h} = 0,
\]

\[
\nabla^2 \delta \mathbf{U} = -4\pi G \delta \rho.
\]

where \( p, \rho, u, L_\nu, L_\rho, \gamma, T, \eta, v, G \) and \( \gamma \) are the fluid pressure, density, kinematic viscosity, temperature dependent heat-loss function, density dependent heat-loss function, thermal conductivity, temperature, electrical resistivity, sound velocity, universal gas constant and ratio of two specific heats. We seek plain wave solution of the form

\[
e^{i(k_x x + k_z z - \omega t)},
\]

where \( \sigma \) is the frequency of perturbation and \( k_x, k_z \) are the components of the wave vector \( \mathbf{k} \), in \( x, z \) directions so that \( k^2 = k_x^2 + k_z^2 \).

### 3. Dispersion relation and discussion

The substitution of equation (8) into equations (1) - (7) gives the general dispersion relation.

\[
\left[ \omega^2 + \omega \Omega_\nu + \Omega_\nu^2 \right] \left[ \omega + \Omega_\nu + \frac{V^2 k_x^2}{d} \right] \left[ \omega + \Omega_\nu + \frac{\nabla^2 k_z^2}{d} \right] + \frac{4\nu_0^2 k_x^2 k_z^2}{d^2} \left[ \omega + \Omega_\nu \right] - \frac{2\nu_0^2 k_x^2 k_z^2}{d^2} \Omega_\nu^2 \left[ \omega + \Omega_\nu \right]
\]

\[
+ \frac{V^2 k_z^2}{d} \left[ k_z^2 + 4k_z^2 \right] \left[ \omega + \Omega_\nu \right] + \frac{V^2 k_x^2}{d} \left[ k_x^2 + 4k_x^2 \right] \left[ \omega + \Omega_\nu \right] + \frac{\nabla^2 k_z^2}{d^2} \left[ k_x^2 + 4k_x^2 \right] \left[ \omega + \Omega_\nu \right] + \frac{\nabla^2 k_x^2}{d^2} \left[ k_z^2 + 4k_z^2 \right] \left[ \omega + \Omega_\nu \right] - \frac{4\nu_0^2 k_x^2 k_z^2}{d^2} \frac{V^2}{\Omega_\nu^2} = 0.
\]

In order to examine the effects of various parameters on the condition of gravitational instability we consider two special cases of propagation:

#### 3.1. Longitudinal mode of Propagation (\( k || H \))

In this case the perturbations are taken parallel to the direction of the magnetic field \((i.e. \ k_x = 0, \ k_z = k)\) the dispersion relation becomes

\[
\left[ \omega + \Omega_\nu \right] \left[ \omega + \Omega_\nu + \frac{V^2 k_z^2}{d} \right] + 4\nu_0^2 k_z^2 \left[ \omega + \Omega_\nu + \frac{\Omega_\nu^2}{B + \nu} \right] = 0.
\]

Equation (10) has three independent factors. The first factor \( \left[ \omega + \Omega_\nu \right] = 0 \) represents a stable damped mode modified by the presence of viscosity and permeability of the medium. The second factor gives the
fourth degree dispersion relation in $\omega$, this is an Alfven mode which is modified by the presence of FLR corrections, viscosity, permeability and finite electrical resistivity of the medium.

The third factor gives the third degree dispersion relation in $\omega$, which is modified by the presence of radiative heat-loss function, self-gravitation, thermal conductivity, viscosity and permeability. The condition of instability obtained from the constant term is given as

$$k^2(y-1)\left(T_L - \rho L_p + \frac{\lambda k^2 T}{\rho}\right) - 4\pi G p(y - 1)\left(\frac{T_H L_p}{p} + \frac{\lambda k^2 T}{p}\right) < 0,$$

(11)

From equation (11) we see that the fundamental Jeans criterion of gravitational instability is modified into radiative instability criterion. Thus the dispersion relation and growth rate is modified due to FLR corrections, viscosity, permeability and finite electrical resistivity of the medium.

In absence of thermal conductivity and radiative heat-loss function, self-gravitation, thermal conductivity, viscosity and permeability. The condition of instability obtained from the constant term is given as

$$k^2 = \frac{1}{2} \left[ \left(\frac{4\pi G p}{c^2} + \frac{\rho^2 L_p}{\lambda T} - \frac{\rho L_p}{\lambda}\right) \pm \left(\frac{4\pi G p}{c^2} + \frac{\rho^2 L_p}{\lambda T} - \frac{\rho L_p}{\lambda}\right)^2 + \frac{16\pi G^2 p^2}{\lambda c^2 - L_T}\right]^{1/2}.$$

(12)

The medium is unstable for wave number $k < k_J$. Here it may be noted that the modified critical wave number involves the derivatives of temperature dependent, density dependent heat-loss functions and thermal conductivity of the medium.

In absence of thermal conductivity and radiative heat-loss function ($\lambda = L_T = L_p = 0$) the condition of instability and critical Jeans wave number is given as

$$(c^2 k^2 - 4\pi G p) < 0,$$

(13)

$$k^2_J = \frac{4\pi G p}{c^2}.$$

(14)

The medium is unstable for $k < k_J$, where $k_J$ is critical Jeans wavenumber. On comparing equations (11) and (13) we conclude that inclusion of radiative heat-loss functions and thermal conductivity changes the fundamental criterion of Jeans gravitational instability into radiative instability criterion.

3.2 Transverse mode of propagation ($k \parallel H$)

In this case the perturbations are taken perpendicular to the direction of the magnetic field (i.e. $k_x = k, k_y = 0$). The dispersion relation becomes

$$\left\{\omega + \Omega_x \right\}^2 + \left\{\omega + \Omega_y \right\}^2 + \omega \left(\frac{\Omega_x^2 + \omega \Omega_y^2}{B + \omega} + \omega N_x^2 k^2 \right) + \omega \omega_N k^4 = 0.$$

(15)

Equation (15) has two independent factors. The first factor is discussed earlier. The second factor gives the fifth degree dispersion relation in $\omega$, which is modified by the presence of radiative heat-loss function, FLR corrections, self-gravitation, thermal conductivity, finite electrical resistivity, viscosity, permeability and magnetic field. The condition of instability obtained from the constant term of the dispersion relation is same as discussed in equation (11).

In absence of viscosity ($\nu = 0$) the condition of instability is given as

$$\left(\nu_N^2 k^4(y-1)\left(\frac{T_H L_p}{p} + \frac{\lambda k^2 T}{p}\right) + k^2(y-1)\left(T_L - \rho L_p + \frac{\lambda k^2 T}{\rho}\right) - 4\pi G p(y - 1)\left(\frac{T_H L_p}{p} + \frac{\lambda k^2 T}{p}\right) < 0.$$

(16)

From equation (16) we conclude that FLR correction tries to stabilize the system. Also we see that inclusion of viscosity removes the effects of FLR correction from condition of instability.

In absence of viscosity and radiative heat-loss function ($\nu = L_T = L_p = 0$) the condition of instability is given as

$$\left(\nu_N^2 k^4 + c^2 k^2 - 4\pi G p\right) < 0,$$

(17)
from equation (17) we conclude that FLR have stabilizing effect on the instability of the system. On comparing equations (16) and (17) we conclude that condition of instability given by Vaghela and Chhajlani [6] is modified by inclusion of radiative effects.

In absence of viscosity and finite resistivity \( (\nu = \eta = 0) \) the condition of instability is given as

\[
\left( \nu^* k^4 + V^2 k^2 \left( \frac{T_p L_T}{\rho} + \frac{\lambda k^2 T}{\rho} \right) \right) + k^2 \left( T L_T - \rho L_p + \frac{\lambda k^2 T}{\rho} \right) - 4\pi G\rho \left( \frac{T_p L_T}{\rho} + \frac{\lambda k^2 T}{\rho} \right) < 0. \tag{18}
\]

From equation (18) we conclude that FLR correction and magnetic field try to stabilize the system. Thus from equation (18) we see that condition of instability given by Bora and Talwar [4] excluding electron inertia in that case is modified by inclusion of finite Larmor radius corrections in our case. Thus the present results are the improvement of Bora and Talwar [4]. Numerical calculations were performed for equation (15) by making it dimensionless in terms of self-gravitation to determine the roots of \( \omega^* \), as a function of wave number \( k^* \) taking \( \gamma = 5/3 \). Out of the five modes only one mode is unstable for which the calculations are presented in Figures 1-4, where the growth rate \( \omega^* \) (positive real value of \( \omega^* \)) has been plotted against the wave number \( k^* \) to show the dependence of the growth rate on the different physical parameters.

It is clear from figure 1 and figure 3 that growth rate decreases with increasing temperature dependent heat-loss function and thermal conductivity. Thus the effect of temperature dependent heat-loss function and thermal conductivity is stabilizing, where as from figure 2 we conclude that growth rate increases with increasing density dependent heat-loss function. Thus the effect of density dependent heat-loss function is destabilizing. One can observe from figure 4 that the growth rate decreases with increase in FLR corrections. Thus the effect of FLR correction is stabilizing.

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**Figure 1.** The normalized growth rate v/s normalized wavenumber with variation in temperature dependent heat-loss function \( L_T^* \).

**Figure 2.** The normalized growth rate v/s normalized wavenumber with variation in density dependent heat-loss function \( L_p^* \).

**Figure 3.** The normalized growth rate v/s normalized wavenumber with variation in thermal conductivity \( \lambda^* \).

**Figure 4.** The normalized growth rate v/s normalized wavenumber with variation in FLR corrections \( \nu^* \):

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\( \nu^* = 1.0 : (1) \nu_0^* = 0, \ (2) \nu_0^* = 2 \ (3) \nu_0^* = 5. \)

\( \nu^* = 0 : (4) \nu_0^* = 0, \ (5) \nu_0^* = 2 \ (6) \nu_0^* = 5. \)
4. Conclusion
Thus in the present paper, the effects of radiative heat-loss function, FLR corrections, thermal conductivity, finite electrical resistivity and viscosity on gravitational instability of plasma is investigated. We find that Jeans criterion remains valid and gets modified because of radiative heat-loss function, FLR corrections, thermal conductivity and magnetic field. The effect of viscosity is found to stabilize the system in both the longitudinal and transverse mode of propagation.

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