Counting statistics for entangled electrons

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The counting statistics (CS) for charges passing through a coherent conductor is the most general quantity that characterizes electronic transport. CS not only depends on the transport properties of the conductor but also depends on the correlations among particles which compose the incident beam. In this paper we present general results for the CS of entangled electron pairs traversing a beam splitter and we show that the probability that Q charges have passed is not binomial, as in the uncorrelated case, but rather it is symmetric with respect to the average transferred charge. We furthermore consider the joint probability for transmitted charges of a given spin and we show that the signature of entanglement distinctly appears in a correlation which is not present for the non-entangled case.

I. INTRODUCTION

Probably one of the most striking feature of quantum mechanics is entanglement [1] which refers to the nonlocal correlations existing, even in the absence of interaction, between two (spatially separated) parts of a given quantum system. Besides the fundamental interest in its generation and detection, a great deal of interest has been brought forth by its role in quantum information which is attracting a vast effort due to the very important impact of its potential applications, ranging from quantum computation to quantum teleportation [2]. Entanglement is one of the most promising tools for achieving a speed-up in quantum computation and communication.

Counting statistics [3] for entangled electrons was discussed in Ref. [6] where it has been shown that the presence of spatially separated pairs of entangled electrons can be revealed by using a beam splitter, as in Fig. 1, and by measuring the correlations of the current fluctuation (noise) at the exiting terminals (labeled by 3 and 4 in the figure). Provided that the electrons injected into leads 1 and 2 are in an entangled state:

\[ |\psi\rangle = \frac{1}{\sqrt{2}} \left( a_{21}^\dagger a_{11}^\dagger \pm a_{21}^\dagger a_{11}^\dagger \right) |0\rangle, \]

bunching and anti-bunching behaviours are found depending on whether state \( |\psi\rangle \) is a spin singlet (lower sign) or a spin triplet (upper sign). More precisely current noise is enhanced by a factor of 2 with respect to non-entangled states in the former case and suppressed to zero in the latter. Note that while this allows to detect a singlet entangled state, it does not discriminate between entangled and non-entangled triplets. Given the general set-up, in order to find the signatures of entanglement in the noise spectrum one needs a physical realization of both the entangler (that enables the pair production) and the beam splitter. As the entangler one can resort to the phenomenon of Andreev reflection in hybrid normal-superconducting systems as discussed in Refs. [6-8].

Besides electrons, it is possible to produce entangled states with Cooper pairs in superconducting nanocircuits [11] or, by coupling a mesoscopic Josephson junctions with superconducting resonators [12,13], between Cooper pairs and the resonator mode.

In this paper we consider the same approach as in Ref. [6] and give for granted the existence of an entangler. We address the question whether the study of the full statistics of charge transport [6] at the exit terminals 3 and 4 of such system can provide more information (as compared to the noise) on the correlation of the injected particles in terminals 1 and 2. The main result of this paper is that not only the value of the noise characterizes the entangled singlet state with respect to uncorrelated states (as shown in [6]), but also the whole probability distribution for the transfer of charges is qualitatively modified. More precisely, we show that the probability distribution relative to incident particles in the entangled singlet state is not binomial, in contrast to the case of uncorrelated injected states, and moreover it is symmetric around its average value. In addition, we show that the use of spin-sensitive electron counters, on the one hand, provides a more stringent tool for detecting entangled states which is based on general properties of the probability distribution. On the other, it allows to distinguish between entangled and non-entangled triplets states.

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In the second quantization formalism, the field operator for the transmission of a particle from lead \(i\) to \(j\) is
\[
\hat{a}_{ij}^\dagger(E) = \delta_{ij} \delta(E - E') .
\]
In the case of two and three dimensional leads one can separate longitudinal and transverse particle motion. Since the transverse motion is quantized, the wave function relative to the plane perpendicular to the direction of transport is characterized by a set of quantum numbers which identifies the channels of the lead. Such channels are referred to as open when the corresponding longitudinal wave vectors are real, since they correspond to propagating modes. Note that the case of a single open channel corresponds to a one dimensional lead.

As far as charge transport is concerned, quantities which are most frequently considered are the conductance and the noise, the latter arising due to the discrete nature of the charge carriers, even at zero temperature. However it is more general to consider the probability distribution for the transfer of charges \(Q\) of which conductance and noise are the first and second moments, respectively. Following Refs. 1718, within the scattering approach the characteristic function of the probability distribution for the transfer of particles in a structure attached to \(n\) leads at a given energy \(E\) can be written as
\[
\chi_E(\vec{\lambda}) := \left\langle \prod_{j=1,n} e^{i\lambda_j \hat{N}^\dagger_j + \hat{N}_j} \prod_{j=1,n} e^{-i\lambda_j \hat{N}^\dagger_j + \hat{N}_j} \right\rangle ,
\]
where the brackets \(\langle \ldots \rangle\) stand for the quantum statistical average in thermal equilibrium. Assuming a single channel per lead, \(\hat{N}_j^\dagger = \hat{a}_{ij}^\dagger \hat{a}_{ij}\) is the number operator for incoming (outgoing) particles with spin \(\sigma\) in lead \(j\) and \(\vec{\lambda}\) is a vector of \(n\) real numbers, one for each open channel. Number operators can be written in terms of the above operators as \(\hat{N}_j = \hat{a}_{ij}^\dagger \hat{a}_{ij}\) and \(\hat{N}_j^\dagger = \hat{\phi}_{ij}^\dagger \hat{\phi}_{ij}\). Note that (\ref{eq:7}) is simply a generalization of the spinless, single-channel case for which it is easy to show that
\[
\chi_E(\lambda) := \sum_{m,n=0}^1 P_E(m,n) e^{i\lambda m} e^{-i\lambda n} = \langle e^{i\lambda \hat{N}_i} e^{-i\lambda \hat{N}_O} \rangle .
\]
Here \(P_E(m,n)\) is the joint probability for \(m\) particles to propagate to the right and \(n\) particles to propagate to the left, with energy \(E\).

For long measurement times \(t\) [13], the total characteristic function \(\chi\) is the product of contributions from different energies, so that
\[
\chi(\vec{\lambda}) = e^{\frac{\vec{\lambda}^T \vec{S} \vec{\lambda}}{2}} \int dE \log \chi_E(\vec{\lambda})
\]
and the joint probability distribution for transferring \(Q_1\) electronic charges in lead 1, \(Q_2\) in lead 2, etc. is given by:
\[ P(Q_1, Q_2, \ldots) = \frac{1}{(2\pi)^n} \int_{-\pi}^{\pi} d\lambda_1 d\lambda_2 \ldots \chi(\vec{\lambda}) e^{i\vec{\lambda} \cdot \vec{Q}}. \]  

(10)

In Refs. [20][21] it was first proved that in a quantum conductor with a single open channel the distribution probability is binomial, in contrast to the classical case where the distribution is Poissonian. In Ref. [22] the characteristic function was generalized to many open channels and an explicit expression for its cumulants was obtained. This allowed to prove that the probability distribution for a tunnel barrier with very small transmission recovers the Poissonian distribution. Counting statistics has been so far studied for several systems including hybrid normal-metal/superconductor structure [18][23][24], metallic diffusive wires [22][25] and chaotic cavities [26].

In the rest of the paper we specialize to the beam splitter

\[ \chi_E(\lambda_3, \lambda_4) = \langle e^{i\lambda_3 (\hat{N}_{\uparrow}^3 + \hat{N}_{\downarrow}^3)} e^{i\lambda_4 (\hat{N}_{\uparrow}^4 + \hat{N}_{\downarrow}^4)} \rangle e^{-i\lambda_3 (\hat{N}_{\uparrow}^3 + \hat{N}_{\downarrow}^3)} e^{-i\lambda_4 (\hat{N}_{\uparrow}^4 + \hat{N}_{\downarrow}^4)}. \]  

(11)

We assume, as usual, that the incoming particles are independent and originate from reservoirs. Therefore we set the chemical potentials of reservoirs connected to leads 3 and 4 to zero and chemical potentials of reservoirs connected to leads 1 and 2 either to zero or to \( eV \). At zero temperature, the statistical average over the Fermi distribution function in Eq. (11) simplifies to the expectation value onto the following state

\[ |\psi\rangle = \begin{cases} \hat{a}_1^\dagger \hat{a}_1^\dagger \hat{a}_2^{\dagger} \hat{a}_2 \hat{a}_{3\downarrow} \hat{a}_{3\downarrow} \hat{a}_{4\uparrow} \hat{a}_{4\uparrow} |0\rangle & \text{for } E < 0 \\ \hat{a}_1^\dagger \hat{a}_1^{\dagger} |0\rangle & \text{for } 0 < E < eV \\ 0 & \text{for } E > eV \end{cases}, \]  

(12)

in the case where only reservoir 1 is at finite chemical potential \( eV \). The new situation we are interested in corresponds to the propagation of entangled incident states from branches 1 and 2, as if originating from an "entangler". We describe this device by replacing in (11) the state \( \hat{a}_1^\dagger \hat{a}_1^\dagger |0\rangle \) with the state

\[ \frac{1}{\sqrt{2}} \left( \hat{a}_{2\uparrow}^\dagger \hat{a}_{1\uparrow}^\dagger \pm \hat{a}_{2\downarrow}^\dagger \hat{a}_{1\downarrow}^\dagger \right) |0\rangle, \]  

(13)

for \( 0 < E < eV \) and, to ensure no net transfer of charges, we leave unchanged the state relative to \( E < 0 \) and \( E > eV \). In (13) the minus sign refers to the spin singlet and the plus sign to the spin triplet. It is easy to show that for \( E < 0 \) and \( E > eV \) one gets \( \chi_E(\lambda_3, \lambda_4) = 1 \), whereas, for \( 0 < E < eV \), Eq. (13) reduces to

\[ \chi_E(\lambda_3, \lambda_4) = \langle e^{-i\lambda_3 (\hat{N}_{\uparrow}^3 + \hat{N}_{\downarrow}^3)} \rangle e^{-i\lambda_4 (\hat{N}_{\uparrow}^4 + \hat{N}_{\downarrow}^4)}, \]  

(14)

since states (12) and (13) do not contain incoming particles from leads 3 and 4. By using the identity

\[ e^{-i\lambda_j \hat{N}_{\sigma}^{ij}} = \left[ 1 + (e^{-i\lambda_j} - 1) \hat{N}_{\sigma}^{ij} \right] \]  

(15)

with upper sign referring to the triplet state and lower sign referring to the singlet state. \( R^2 = |r_{31}^2|^2 = |r_{42}^2|^2 \) and \( T^2 = |t_{32}^2|^2 = |t_{41}^2|^2 \) are reflection and transmission probabilities, respectively. Note that the second equalities in the above relationships are completely general in the case of no back-scattering. For comparison, in the case of uncorrelated incoming particles described by the state in Eq. (12) the characteristic function is given by

III. CHARACTERISTIC FUNCTION FOR ENTANGLED ELECTRONS

We concentrate in the calculation of the probability distribution for the transfer of particles in leads 3 and 4 when particles are injected from leads 1 and 2. Since we are not interested in counting the particles passing through the entering leads 1 and 2, we set \( \lambda_1 = \lambda_2 = 0 \), so that Eq. (11) becomes
\[ \chi_E(\lambda_3, \lambda_4) = \prod_{\sigma = \uparrow, \downarrow} (R^\sigma e^{-i\lambda_3} + T^\sigma e^{-i\lambda_4}) . \]  

(19)

As it appears from Eq. (17) and Eq. (19), the characteristic function relative to entangled pairs of incident particles, Eq. (17), possesses a different structure with respect to the one relative to the ordinary situation of independent particles, Eq. (19). In particular, while Eq. (19) only depends on probability coefficients, the characteristic function for entangled electrons depends directly on the scattering amplitudes. Furthermore, unlike Eq. (17), Eq. (19) can be factorized into spin-up and spin-down contributions, reflecting the fact that, in the ordinary situation, electrons with different spin undergo independent scattering processes. In the simplest case of spin-independent transport, such that \( r_{ij}^\uparrow = r_{ij}^\downarrow \) and \( t_{ij}^\uparrow = t_{ij}^\downarrow \), the constant in Eq. (18) takes the value \( A = \frac{1}{2}(|t|^2 - |r|^2)^2 \) for the entangled singlet and \( A = 1/2 \) for the entangled triplet. This implies that pairs of particles in an entangled triplet state show the same characteristic function as for non-entangled triplets (of the form \( |\psi\rangle = a_1^\uparrow a_2^\downarrow \)), namely

\[ \chi_E(\lambda_3, \lambda_4) = e^{-i(\lambda_3 + \lambda_4)} . \]  

(20)

Note, moreover, that the result given in Eq. (20) for non-entangled triplets does not depend on transport amplitudes.

It is worthwhile noting that if we allow for spin-polarized transport, for example using ferromagnetic metals for terminals 3 and 4, the characteristic functions for all the cases will be distinguished from each other. The constant \( A \) in Eq. (17), in fact, will take the value

\[ A = \frac{1}{2}(t^\uparrow t^\downarrow \pm r^\uparrow r^\downarrow)(t^\uparrow t^\uparrow \pm r^\uparrow r^\uparrow) \]  

(21)

in the case of a symmetric beam splitter (where \( r_{31}^\sigma = r_{32}^\sigma = r \) and \( t_{31}^\sigma = t_{32}^\sigma = t \)). This makes the characteristic function of the entangled spin triplet to differ from the one relative to non-entangled triplets, since in the latter case \( \chi_E \) is again given by Eq. (20), independent of scattering amplitudes.

A. Counting statistics on a single terminal

Let us now turn the attention to the probability distributions for the transfer of particles. As already mentioned in section II, it can be easily computed by a Fourier transform of the total characteristic function \( \chi \), so that the probability for transferring a number of \( Q_\alpha \) electronic charges, regardless their spin, into lead \( \alpha \) is given by

\[ P(Q_\alpha) = \frac{1}{2\pi} \int_{-\pi}^{+\pi} d\lambda_\alpha \chi(\lambda_\alpha) e^{i\lambda_\alpha Q_\alpha} . \]  

(22)

Note that \( \chi(\lambda_\alpha) \) is obtained from the complete \( \chi(\vec{\lambda}) \) by setting to zero every \( \lambda_\beta \) with \( \beta \neq \alpha \). In the limit of small bias voltage \( V \) and zero temperature, the total characteristic function \( \chi \), can be reduced to \( \chi(\vec{\lambda}) = [\chi_0(\vec{\lambda})]^M \) with \( M = \frac{Q}{Q_3} \), in such a way that one only needs to calculate the characteristic function at zero energy. In the case of entangled incident particles, state (13), we find that

\[ P(Q_3) = \sum_{k=|Q_3-M|}^{M} \binom{M}{k} \left( \frac{1}{2} - A \right)^k \left( 2A \right)^{M-k} \left( \frac{k}{Q_3 - 2M + k} \right) , \]  

(23)

with the sum restricted to values of \( k \) such that \( (Q_3 - M + k) \) is an even number. It is easy to show that the distribution (23) is symmetrical with respect to the position of the maximum \( (Q_3 = M) \), independently of the scattering amplitudes. This result is in contrast with the ordinary situation of independently injected particles where, as expected, the distribution is binomial:

\[ P(Q_3) = \binom{2M}{Q_3} R^{Q_3} (1 - R)^{2M - Q_3} \]  

(24)

and centered around the value \( Q_3 = 2MR \), for spin-independent transport (the factor 2 comes from the spin degeneracy). Note that the width of (24) for spin singlet is double with respect to the ordinary case of (24) and zero for the triplet. In particular, for the entangled spin triplet we have

\[ P(Q_3) = \delta_{Q_3,M} , \]  

(25)

equal to the non-entangled triplet states.
in the case of entangled incident particles from lead 1 and 2. It is worthwhile noting that for either \( \lambda_3 = 0 \) or \( \lambda_4 = 0 \), the function \((31)\) is independent of \( A \) and, in particular, it is equal for singlet and triplet states. This results in the following expression for the probability of separately counting \( Q_3 \) spin-up charges in terminal 3 and \( Q_4 \) spin-down charges in terminal 4:

\[
P^\uparrow(Q_3) = \left( \frac{M}{Q_3} \right) \frac{1}{2^M} , \tag{32}
\]

\[
P^\downarrow(Q_4) = \left( \frac{M}{Q_4} \right) \frac{1}{2^M} . \tag{33}
\]

For completeness, we mention that the characteristic function in the ordinary case of independent incident particles reads:

\[
\chi_E(\lambda_3, \lambda_4) = (T^\uparrow + R^\downarrow e^{-i\lambda_3})(R^\uparrow + T^\downarrow e^{-i\lambda_4}) , \tag{34}
\]

which gives the following binomial probability distribution:

\[
P^\uparrow(Q_3) = \left( \frac{M}{Q_3} \right) (R^\uparrow)^{M-Q_3} (1 - R^\uparrow)^{Q_3} . \tag{35}
\]

For the non-entangled spin-triplets we have

\[
\chi_E(\lambda_3, \lambda_4) = e^{-i\lambda_3} , \tag{36}
\]

which yields

\[
P^\uparrow(Q_3) = \delta_{Q_3,M} . \tag{37}
\]

B. Counting statistics on both terminals: joint probability

Let us now consider the joint probability for transferring a number of \( Q_\alpha \) and \( Q_\beta \) electronic charges into, respectively, lead \( \alpha \) and \( \beta \), given by

\[
P(Q\alpha, Q\beta) = \int_{-\pi}^{+\pi} d\lambda_\alpha \int_{-\pi}^{+\pi} d\lambda_\beta \chi(\lambda_\alpha, \lambda_\beta) e^{i\lambda_\alpha Q_\alpha + i\lambda_\beta Q_\beta} . \tag{38}
\]

We can distinguish between the two situations: i) spin-insensitive counters with \( \chi_E \) given by \((14)\); ii) spin-sensitive counters with \( \chi_E \) given by \((31)\). In case i) it is easy to show that the following relationship holds:

\[
P(Q_3, Q_4) = P(Q_3) \delta_{2M-Q_3+Q_4} = P(Q_4) \delta_{2M-Q_3+Q_4} . \tag{39}
\]

which merely expresses the conservation of particles. Being \( 2M \) the total number of particles injected from leads 1 and 2 over the time \( t \) and \( Q_3 \) the number of particles exiting lead 3, \( Q_4 = 2M - Q_3 \) will be the number of particles recorded by counter in 4. \( P(Q_3, Q_4) \), therefore, expresses the correlations due to particles conservation. This makes explicit the fact that a measure
of the joint probability distribution on terminal 3 and 4 does not give more information than a measure of the probability distribution on a single terminal. The picture changes completely when the constraint of conservation of particles being counted is lifted, for example, by using spin-selective counters. This can be realized when a spin-up electron counter is placed on terminal 3 and a

\[ P^{\uparrow\downarrow}(Q_3, Q_4) = \min[Q_3 + Q_4, 2M - (Q_3 + Q_4)] \sum_{k=|Q_3 - Q_4|}^{M} \binom{M}{k} \left( \frac{1}{2} - A \right)^k A^{M-k} \left( \frac{k}{Q_3 + Q_4 + k} \right) \left( \frac{M-k}{Q_3 + Q_4 - k} \right), \tag{40} \]

with the sum restricted to values of \( k \) such that \([Q_3 \pm (Q_4 - k)]\) is an even number. We see immediately that in the present case Eq. \((\text{39})\) does not hold and, in particular, \( P^{\uparrow\downarrow}(Q_3, Q_4) \) cannot be expressed in terms of \( P^\uparrow(Q_3) \) and \( P^\downarrow(Q_4) \). This means, in contrast to case i), that a measure of \( P^{\uparrow\downarrow}(Q_3, Q_4) \) provides more information than \( P^\uparrow(Q_3) \) or \( P^\downarrow(Q_4) \) alone and reflects the fact that particles counted in terminals 3 and 4 are correlated in a non-trivial way. On the contrary, in the ordinary situation of independent incident particles coming from terminal 1 with \( \chi_E \) given by \((\text{34})\) we have that

\[ P^\uparrow(Q_3) P^\downarrow(Q_4) = P^\uparrow(Q_3) P^\downarrow(Q_4). \tag{42} \]

Eq. \((\text{41})\) confirms that the transfer of spin-up charges into lead 3 and spin-down charges into lead 4 are independent processes, since the joint probability is equal to the product of probabilities on individual terminals. For completeness, we note that when \( A = 1/2 \) in Eq. \((\text{40})\), \( i.e. \) the injected particles are in the entangled triplet states, we have

\[ P^{\uparrow\downarrow}(Q_3, Q_4) = \binom{M}{Q_3} \frac{1}{2^M} \delta_{Q_3, Q_4}, \tag{43} \]

and, when the triplets are non-entangled,

\[ P^{\uparrow\downarrow}(Q_3, Q_4) = \delta_{Q_3, M} \delta_{Q_4, 0}. \tag{44} \]

Remarkably the two expressions above are different even for spin-independent transport.

The net result is that the relationship between joint probability, on one side, and single-terminal probabilities, on the other, depends on the specific incident particle state. For entangled singlet electrons, in particular, such a relationship does not exist and furthermore \( P^{\uparrow\downarrow}(Q_3, Q_4) \) depends on the scattering amplitudes, while \( P^\uparrow(Q_3) \) does not. The relevant consequence is that a measure of such a spin-sensitive counting statistics can provide an unambiguous mean of detecting entangled singlet, triplet or non-entangled states, since it relies on properties of the characteristic function rather than on the value of quantities like shot noise. In practice one should separately measure \( P^\uparrow(Q_3) \), \( P^\downarrow(Q_4) \) and finally \( P^{\uparrow\downarrow}(Q_3, Q_4) \) and compute the ratio

\[ r = \frac{P^{\uparrow\downarrow}(Q_3, Q_4)}{P^\uparrow(Q_3) P^\downarrow(Q_4)}. \tag{45} \]

If \( r = 1 \) independently of \( Q_3 \) and \( Q_4 \), we are in the ordinary situation of independent particles injected either from lead 1 or 2. If \( r = 1 \) only in the point \((M, 0)\) of the \((Q_3, Q_4)\) plane and zero everywhere else, then we are in the presence of non-entangled triplets. If \( r \neq 1 \), but different from zero only along the direction \( Q_3 = Q_4 \), we are in the presence of triplet entangled states. Finally, if \( r \neq 1 \) and finite independently of \( Q_3 \) and \( Q_4 \) we are in the presence of a singlet entangled state. As an example we plot in Figs. \(3\) and \(4\) the distribution \((\text{40})\) and the ratio \( r \), respectively, for a singlet entangled state injected in a spin-independent beam splitter characterized by \( T = 0.7 \) and \( M = 50 \). Fig. \(3\) shows that \( P^{\uparrow\downarrow} \) possesses an elongated shape along the direction \( Q_4 = M - Q_3 \), which gets sharper as \( T \) goes toward 1/2. Fig. \(4\) shows that \( r \) varies very much in the \((Q_3, Q_4)\) plane: this allows an easy distinction between different injected particles states. As a final remark we note that the cross-terminal shot noise in the case of independent injected particles is zero, whereas in the entangled case is

\[ s_{34}^{\uparrow\downarrow} = \frac{2e^3V}{h} \left( A - \frac{1}{4} \right), \tag{46} \]

non-zero even for triplets. This is in contrast with case i) where conservation of counted particles implies that cross-terminal shot noise is always equal in magnitude (with opposite sign) to same-terminal shot noise.
In this paper we have studied the counting statistics of a beam splitter when pairs of entangled electrons are injected from the entering terminals 1 and 2. First we considered the situation in which spin-insensitive electron counters are placed on terminals 3 and 4. We found, on the one hand, that the single-terminal probability distribution relative to singlet entangled electrons qualitatively differs from the one relative to uncorrelated electrons. In the former case, in fact, the distribution is not binomial, in contrast to the latter case, and furthermore it is symmetric with respect to the average number of transmitted charges. On the other hand, we found that the distributions relative to the triplet states, both entangled and non-entangled, are equal and given by unity when the charge transferred is $M$ and zero otherwise. Triplet states can be distinguished, however, when the transport is spin-polarized, for example when ferromagnetic terminals are used. If this is the case, the single-terminal counting statistics for the entangled triplet broadens to a finite width, while the non-entangled triplets remains as before. Interestingly we also noticed that the joint probability for counting $Q_3$ electrons arrived in lead 3 and $Q_4$ electrons arrived in lead 4 does not contain more information than single-terminal probabilities because of the conservation of particles. Such a constraint can be lifted by using spin-sensitive electron counters, for example placing a spin-up counter on terminal 3 and a spin-down counter on terminal 4. In this case the joint probability unambiguously characterizes the state of the incident electrons. In particular we found that, unlike in the uncorrelated case, in the presence of entanglement the joint probability cannot be expressed as a product of single-terminal probabilities. In addition, triplet states exhibit distinguished joint probability depending on whether they are entangled or not. Note that the single-terminal counting statistics for the entangled states is also binomial as for the uncorrelated case, but with probability of the two outcome equal to 1/2, therefore independent of scattering amplitudes and total angular momentum of the pair. Operatively, we concluded by showing that the ratio defined in (13) can serve as a tool for discerning among the differently correlated incident electron states. As shown in paragraph III B, a plot of such a ratio as a function of the number of transferred charges provides an easy and definite way of identifying entangled singlet, triplet and non-entangled incident states. We believe that these results can be used for detecting the presence of entanglement in electronic systems and provide an additional mean for studying and understanding the production and manipulation of entangled electrons.

IV. CONCLUSIONS

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FIG. 1. The prototype setup consists of an “entangler” connected to a beam splitter. The “entangler” produces pairs of entangled electrons from a source of uncorrelated particles entering from terminals 1’ and 2’. In the beam splitter, the entangled electrons injected in terminals 1 and 2 are transmitted and reflected into terminals 3 and 4 by the semi-transparent mirror (dashed line). No back-scattering into leads 1 and 2 is allowed.

FIG. 2. Single-terminal counting statistics $P(Q_3)$ for a spin-insensitive electron counter. Dashed line is relative to uncorrelated electrons, thin line and bold line are relative to entangled singlet and triplet electrons, respectively. The spin-dependent beam splitter is characterized by $R^{\uparrow} = 0.2$, $R^{\downarrow} = 0.1$ and $M = 50$.

FIG. 3. Joint probability $P^{\uparrow \downarrow}(Q_3, Q_4)$ for a spin-up electron counter placed on lead 3 and a spin-down electron counter placed on lead 4. The 3D-plot is relative to entangled singlet electrons injected from leads 1 and 2. Beam splitter characterized by $T = 0.7$ and $M = 50$.

FIG. 4. 3D-plot of the ratio $r(Q_3, Q_4)$ defined in Eq.(45) relative to entangled singlet particles injected in lead 1 and 2. Beam splitter characterized by $T = 0.7$ and $M = 50$. 