Transport phenomenology for a holon-spinon fluid

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We propose that the normal-state transport in the cuprate superconductors can be understood in terms of a two-fluid model of spinons and holons. In our scenario, the resistivity is determined by the properties of the holons while magnetotransport involves the recombination of holons and spinons to form physical electrons. Our model implies that the Hall transport time is a measure of the electron lifetime, which is shorter than the longitudinal transport time. This agrees with our analysis of the normal-state data. We predict a strong increase in linewidth with increasing temperature in photoemission. Our model also suggests that the AC Hall effect is controlled by the transport time.

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The normal state of the cuprate superconductor exhibits anomalous transport properties [1]. In this paper, we discuss the implications of the experimental results for theoretical ideas based on spin-charge separation in this system. We believe that the longitudinal transport may be described by a boson-only theory for the charge degree of freedom [2], while transport in a magnetic field is controlled by spin-charge recombination.

We review here the experimental results which provide severe constraints on possible theories. We first focus on the case of optimal doping where the superconducting transition temperature $T_c$ is highest. The in-plane resistivity is linear in temperature $T$. The relaxation rate, measured from a Drude-like peak in the optical conductivity, appears to be universal [3, 4]:

$$\hbar/\tau_{tr} \simeq 2k_B T. \tag{1}$$

The spectral weight under the Drude-like peak (or derived from the London penetration depth) is proportional to the hole doping $x$ and can be written as $e^2x/ma^2$, where $a$ is the lattice constant and $m$ is found to be close to twice the free electron mass. In a tight-binding model, this mass corresponds to a hopping integral of 1540K. This is close to the antiferromagnetic exchange $J$ but can also be interpreted as $t/3$. The latter interpretation is consistent with recent studies of the $t$-$J$ model [5].

The Hall coefficient, on the other hand, is found to be suppressed from the classical value $1/\tau_{tr}$ [3, 6]. Using the mass extracted from the optical spectral weight, we obtain $W_H = (\hbar/2e^2)(k_B D)^{1/2}/J \simeq 65K$ for 90K YBCO. The Hall time $\tau_H$ is therefore shorter than the longitudinal time $\tau_{tr}$ above 130K, i.e. in the whole of the normal state except for the region close to $T_c$. Under the assumption of a single mass, the Hall coefficient is $R_H \simeq (1/\xi_{neu})\tau_H/\tau_{tr}$ so that its reduction from the classical value is direct evidence that $\tau_H < \tau_{tr}$. We note that this analysis is different from the original analysis of Refs. [7, 8], which assumes that $\tau_H$ is controlled by the decay of a long-lived quasiparticle ($W_H \sim J$) so that it is longer than the transport time $\tau_{tr}$. As recognized by these authors, this leads one to deduce a carrier mass 20 times larger than the one used above.

The dynamical time for the decay of Hall currents can be obtained in the AC Hall effect. For 90K YBCO, Ref. [9] gives a ratio between 2 and 4 for $\tau_{H}^6/\tau_{tr}$ at 95K while Ref. [10] gives a lower limit of order unity. The proximity to $T_c$ makes one worry whether this is characteristic of the normal state. Measurements at higher temperatures would be desirable.

The discussion so far has been concerned with the optimally-doped cuprates. In the underdoped regime, the physics is complicated by the existence of a spin gap. This pseudogap causes a reduction in the resistivity and $1/\tau_{tr}$ by roughly a factor of two, but $\cot \theta_H$ remains quadratic in $T$ with a small increase in $W_H$ [11]. In overdoped samples, both the Hall angle and the resistivity...
have quadratic temperature dependences [14]. Although a scattering rate of $T^2/W$ is in accordance with Fermi-liquid theory, we observe that the temperature scale $W$ is much smaller than the bandwidth $J$. To estimate $W$, we note that the resistivities of overdoped Tl$_2$Ba$_2$CuO$_{6+\delta}$ and optimally doped YBCO cross at room temperature [4]. Optical data [13] indicate that the spectral weight of the overdoped Tl compound is similar to that of the optimally doped materials, so that the resistivity ratio between the two compounds is a good indication of the lifetime ratio. We therefore estimate that $W \approx 150K$ in the overdoped compound. Similarly, $\cot \theta_H$ for Tl is 60% smaller than for YBCO. This yields an estimate of $W_H \approx 110K$. Both $W$ and $W_H$ are much smaller than the bandwidth, indicating that, even in overdoped materials, the scattering mechanism is not the conventional screened Coulomb interaction between electrons.

Anderson has emphasized that the appearance of $\tau_H$ may be a signature of spin-charge separation. There have been attempts to derive (3) based on the Boltzmann transport of a single carrier with unusual scattering mechanisms [14,15]. For example, Coleman et al. [17] introduce a mechanism which does not conserve particle number. Kotliar et al. [18] use a skew scattering rate which diverges at low temperatures, and $\tau_H$ appears not as a physical rate but as a ratio of two rates so that its behavior with impurity is difficult to rationalize. In this paper, we abandon the notion of a single carrier, and explore a phenomenology based on spin-charge separation.

We review first the picture of spin-charge separation in the $t$-$J$ model which, we believe, describes the low-energy physics of the cuprates. In the slave-boson treatment [19,20], the introduction of a physical hole (of spin $\sigma$ at site $i$) away from half-filling is represented as the creation of a charged hard-core boson (holon) and the destruction of a neutral spin-half fermion (spinon): $\epsilon^e_{i\sigma} = b^\dagger_{i\sigma} f_{i\sigma}$, with a single-occupancy constraint: $b^\dagger_i b_i + \sum_\sigma f^\dagger_{i\sigma} f_{i\sigma} = 1$. For a doping of $x$ holes per site, the holon and spinon densities are $n_b = x$ and $n_f = 1 - x$ respectively. In the uniform resonating-valence-bond ansatz, short-range antiferromagnetic correlations are incorporated into the model by assuming that $\sum_\sigma \langle f^\dagger_{i\sigma} f_{j\sigma} \rangle = \xi e^{i\alpha_{ij}}$. At the mean-field level, there is no net gauge flux ($a_{ij} = 0$) so that the holons have a bandwidth controlled by the hopping integral $t$ of the original electrons while the spinon bandwidth is controlled by the antiferromagnetic exchange $J$. In this paper, we will focus on the cuprates near optimal doping where this slave-boson scheme is believed to apply. For instance, it gives rise to a large Fermi surface, as observed in photoemission experiments.

The fluctuations in the gauge field $a_{ij}$ are strong. For temperatures above the experimental $T_c$, the transverse part of the fluctuations corresponds to a magnetic field with a root-mean-square value of the order of a flux quantum per plaquette [3]. These fluctuations arise because the single-occupancy constraint requires that the spinon and holon number currents cancel each other:

$$J_f + J_b = 0 \quad . \quad (3)$$

At sufficiently low temperatures, the bosons become phase-coherent, leading to well-defined physical electron quasiparticles, i.e. spinon-holon confinement or the breakdown of spin-charge separation. We believe that this confinement occurs at $T_c$, consistent with the fact that the electronic quasiparticles are long-lived in the superconducting state [12].

Consider now the effect of the gauge field on the transport properties of the system. Longitudinal transport should be dominated by the dynamics of the charged holons. The holons are strongly scattered by the internal magnetic fields which can be regarded as quasistatic disorder at long wavelengths and low temperatures. We have shown in a quantum Monte Carlo study [2] that this gives rise a holon scattering rate equal to $2k_B T$ which should also be the scattering rate relevant to longitudinal transport. This is consistent with the relaxation rate deduced from the optical conductivity.

The picture that emerges from our study is that, in the normal state, the boson de Broglie wavelength is much larger than the interparticle spacing so that the bosons undergo strong exchange and should be viewed as a quantum liquid rather than single particles. The strong gauge field forces the boson world lines to retrace each other, and prevents the development of a superfluid density. This Bose liquid is insensitive to magnetic fields because retracing paths do not detect any Aharonov-Bohm phase. The holon fluid therefore has negligible Hall effect and magnetoconductivity. In the random-phase approximation, the total Hall coefficient of the spinon-holon fluid is given by the Ioffe-Larkin rule [19,20]:

$$R_H = \frac{\chi_f R_{H,b} + \chi_b R_{H,f}}{\chi_f + \chi_b} , \quad (4)$$

where $R_{H,b}$ and $\chi_b$ are the holon Hall coefficient and orbital susceptibility and $R_{H,f}$ and $\chi_f$ are the corresponding quantities for the spinons. We therefore see that the spinon contribution to the Hall response is also small, since the orbital susceptibility of the bosons is suppressed by the gauge fluctuations for the same reason that their Hall response is suppressed.

It is possible that the self-retracing approximation breaks down due to gauge-field dynamics or a reduction in gauge amplitude so that the response to a magnetic field is gradually restored at low temperatures. In this paper, we explore another possibility. We suggest that the retracing picture remains valid down to $T_c$ so that the magnetic response is beyond the scope of a holon-only model. Instead, we propose that the magnetic response could be understood in terms of the incipient recombination of the holons and spinons. This is based on the observation that the physical hole does not experience any
fictitious gauge fields so that its magnetic response should not be suppressed. In other words, the Aharonov-Bohm phases of the holons and the spinons due to the internal gauge field now cancel each other, and the physical hole is not self-retracing. As already mentioned, the Hall coefficient of the cuprates indeed approaches the classical value as one approaches the confinement regime, and physical holes do not provide an additional mechanism to dissipate momentum from the holon-spinon system. Note that this argument requires only the total drift current to be conserved during recombination and decay. One might ask how a holon (which carries the charge) might have memory of its pre-recombination velocity when it re-emitted upon the decay of the physical hole. From the view of the holon which carries a small momentum, it would appear that its momentum is strongly affected

We therefore see that the Hall effect is reduced from the Fermi-liquid result \( \tau_H = \tau_{tr} \) by a fraction of \( \tau_{hole}/\tau_b \). One can also see in this picture that the x-component of the Lorentz force gives rise to a negative magnetoconductivity proportional to \( (\omega_c/\tau_H)^2 \).

More concisely, we have a model where the drift velocity \( \mathbf{v} \) obeys the following dynamics:

\[
m\dot{\mathbf{v}} + \frac{m\mathbf{v}}{\tau_{tr}} = e\mathbf{E} + \frac{e}{c} \eta(t) \mathbf{v} \times \mathbf{B}.
\]  

The random function \( \eta(t) \) is zero except for spikes of value unity and duration \( \tau_{hole} \). These spikes occur with a time spacing of the order of \( \tau_{tr} \). Therefore, at time scales greater than \( \tau_{tr} \), the system sees an effective reduction in the magnetic field by a factor of \( \eta \mathbf{B} \), where \( \eta = \tau_{hole}/\tau_b \) is the time-averaged value of \( \eta \). It should be noted that the simple model \( \eta \mathbf{B} \) does not involve separate decay rates for the longitudinal and transverse drift velocities so that the width \( 1/\tau_H^2 \) of the AC Hall angle \( \theta_H(\omega) \) is given by the transport relaxation time \( 1/\tau_{tr} \) rather than \( 1/\tau_H \). This provides an important test of our hypothesis. As mentioned above, current experimental data give \( \tau_H^2 \) to be of the same order of magnitude as \( \tau_{tr} \) just above the superconducting transition. To settle this issue, it would be necessary to measure the full temperature dependence of the AC Hall relaxation rate.

In a regime of spin-charge separation, spin-charge recombination is rare and the electron lifetime is short, i.e., \( \tau_{hole} \ll \tau_b \), so that \( n_{hole} \ll n_f, n_b \), and \( n_{hole} \approx x \tau_{hole}/\tau_b \). As one approaches the confinement regime, \( \tau_{hole} \) becomes larger than \( \tau_b \), and \( n_{hole} \approx n_b \).

We will now discuss the implications of this scenario for transport properties. As mentioned above, the charge carrier responds to external magnetic fields only as a physical hole. Consider a simple classical model where the response of the charge carriers to an external magnetic field is switched on for a duration of \( \tau_{hole} \) and switches off for a duration of \( \tau_b \gg \tau_{hole} \). Corresponding to the deconfined and confined states respectively. An electric field \( \mathbf{E} \) in the \( x \)-direction accelerates a particle for a duration of \( \tau_{tr} \) before the particle velocity is randomized. Thus, the drift velocity of the system is \( v_x \sim e\mathbf{E}\tau_{tr}/m \), and \( \sigma_{xx} = ne^2\tau_{tr}/m \) where \( m \) is the holon mass in the spin-charge-separated regime. In this time interval, a particle also receives on average \( \tau_{tr}/\tau_b \) impulses of \( ev_yB\tau_{hole}/c \) in the \( y \)-direction due to the Lorentz force. The transverse drift momentum is \( mv_y \sim (ev_yB/c)(\tau_{tr}/\tau_b)\tau_{hole} = e\mathbf{E}\omega_c\tau_{tr}^2\tau_{hole}/\tau_b \). From this, we deduce a Hall angle of \( \theta_H \approx v_y/v_x = \omega_c\tau_H \) where

\[
\tau_H \approx \frac{\tau_{hole}}{\tau_b} \tau_{tr}.
\]  

\[g\]FIG. 1. Schematic picture of recombination and decay between spinons (solid line) and holons (dashed line) and physical holes (box). A hole lives for a time \( \tau_{hole} \), shorter than the holon lifetime \( \tau_b \) and the spinon lifetime \( \tau_{tr}/x \). Only the physical hole experiences an external magnetic field.

An important assumption behind this phenomenological model is that, whereas the average charge current is relaxed by the gauge-field scattering, the binding of a holon with an antispinon and the subsequent decay of the physical hole do not provide an additional mechanism to dissipate momentum from the holon-spinon system. Note that this argument requires only the total drift current to be conserved during recombination and decay. One might ask how a holon (which carries the charge) might have memory of its pre-recombination velocity when it re-emitted upon the decay of the physical hole. From the view of the holon which carries a small momentum, it would appear that its momentum is strongly affected
in each decay and recombination process. However, from the view of the spinon, which carries a large momentum of order $\pi/a$, it is reasonable that it is scattered mainly in the forward direction and that its velocity is preserved. This is indicated in Fig. 1. We can now appeal to the current constraint (3) to argue that, on average, the boson current is also conserved. The constraint is relaxed locally in a spin-charge separated system, but must remain in force on larger length scales. Another way of saying this is that Fig. 1 is misleading in the sense that the holons are strongly overlapping and exchanging and should not be viewed as individual particles.

So far, our phenomenological model contains three time scales whereas there are only two independent time scales to observe. However, as already pointed out, $1/\tau_H$ is a physical scattering rate, rather than a combination of time scales as shown in (7). This forces us to conjecture further that $\tau_0$ is of the order of $\tau_{tr}$. A consequence of this conjecture is that:

$$\tau_H \sim \tau_{\text{hole}}. \quad (9)$$

In other words, the Hall transport time is a measure of the lifetime of the physical hole. This provides another important test of our model. The hole lifetime can be deduced independently from angle-resolved photoemission linewidths which we predict to grow as $T^2/W_H$. The small size of $W_H$ leads to a severe broadening at room temperature which should be amenable to experiments. However, as already pointed out, $1/\tau_H$ leads to a severe broadening at room temperature which should be amenable to experiments.

The assumption that $\tau_0 \sim \tau_{tr}$ is the weakest point of our argument. The only justification we can offer is that, due to the mismatch of the kinematics of the spinon and the holon, the recombination process is perhaps controlled by the same momentum relaxation process which contributes to $\tau_{tr}$. We also need to argue that, in the presence of impurities, $1/\tau_{\text{hole}}$ becomes $1/\tau_{\text{hole}} + 1/\tau_0$ where $1/\tau_0$ is a residual value due to impurity scattering. This is not unreasonable in that a hole may well disintegrate rapidly on encountering an impurity. Finally, we can offer no explanation of why $\tau_{\text{hole}}$ should scale as $1/T^2$. More importantly, we do not understand the origin of the temperature scale $W_H$ which is small and not very sensitive to doping. It is particularly puzzling that the spin gap in the underdoped cuprates has a much smaller effect on $\tau_H$ than on $\tau_{tr}$.

We will now comment briefly on the overdoped regime. We have already pointed out that the resistivity, albeit quadratic in temperature, is much too high compared to Fermi-liquid theory. We propose that $\tau_{\text{hole}}$ increases with doping so that, in the overdoped regime, there is a region of temperature above $T_c$ where the physical hole is more stable and $\tau_{\text{hole}} > \tau_0$. In this case, although the current is dissipated in the holon state, the rate-determining step for current relaxation is the conversion of the physical hole into a holon and an antispinon. We therefore argue that the resistivity is controlled in this regime by $1/\tau_{\text{hole}} \sim 1/\tau_H$ and not the holon scattering rate. Upon further doping, the system eventually crosses over to Fermi-liquid theory when $\tau_{\text{hole}}$ becomes comparable to the Fermi-liquid scattering time.

In summary, we have put forward a hypothesis for understanding transport in the cuprates, based on the idea of spin-charge reconfinement. This model explains naturally the suppression of the Hall response compared to the classical Drude theory. It also links the Hall transport time $\tau_H$ in DC measurements to the lifetime of a physical hole, while the dynamical time extracted from the AC Hall effect is expected to be the longitudinal scattering time $\tau_{tr}$. We hope that our model, while incomplete, will stimulate further experimental work and serve as the basis for further discussion. This work is supported primarily by the NSF MRSEC program (DMR-9400334).

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