The project of “quantum spacetime phenomenology” focuses on searching pragmatically for the Planck scale quantum features of spacetime. Among these features is the existence of a characteristic length scale addressed commonly by effective approaches to quantum gravity (QG). This characteristic length scale could be realized, for instance and simply, by generalizing the standard Heisenberg uncertainty principle (HUP) to a “generalized uncertainty principle” (GUP). While usually it is expected that phenomena belonging to the realm of QG are essentially probable solely at the so-called Planck energy, here we show how a GUP proposal containing the most general modification of coordinate representation of the momentum operator could be probed by a “cold atomic ensemble recoil experiment” (CARE) as a low energy quantum system. This proposed atomic interferometer setup has advantages over the conventional architectures owing to the enclosure in a high finesse optical cavity which is supported by a new class of low power consumption integrated devices known as “micro-electro-opto-mechanical systems” (MEOMS). In the framework of a top-down inspired bottom-up QG phenomenological viewpoint and by taking into account the measurement accuracy realized for the fine structure constant (FSC) from the Rubidium ($^{87}$Rb) CARE, we set some constraints as upper bounds on the characteristic parameters of the underlying GUP. In the case of superposition of the possible GUP modification terms, we managed to set a tight constraint as $0.999978 < \lambda_0 < 1.00002$ for the dimensionless characteristic parameter. Our study shows that the best playground to test QG approaches is not merely the high energy physics, but a table-top nanosystem assembly, as well.

PACS numbers: 04.60.-m, 04.60.Bc

I. INTRODUCTION

Despite three well-tested and powerful theories: quantum field theory (QFT), standard model of particles (SM) and general relativity (GR), our theoretical framework for studying the universe is incomplete for two main reasons. The primary reason is that no well-formulated quantum gravity (QG) theory that unifies the micro level universe (subjected to quantum mechanics (QM)) and Einstein’s theory of gravity (relevant to macro level universe) has been constructed properly so far. The secondary reason is the lack of a well-established framework for unification of gravity with three other fundamental interactions. The root of this incomplete issue comes back to substantive differences between QM and GR, which are well discussed in, for instance, Refs. [1, 2]. Nowadays, there is a great curiosity in theoretical physics community about understanding the physics of the Planck scale which is considered to be one of the challenging research areas. Given that QM quantizes any dynamical field as well as its physical content, so QG would be related to the discreteness of geometrical concepts such as, length, area and volume. This implies that the combination of the relativistic and quantum effects results in reshaping the common thought of distance around the natural scale known as "Planck scale" $\ell_{pl} \approx 10^{-35}$ m. Although, a fully functional unified QG theory has remained out of reach for less than a century, however there are some convincing approaches available such as string theory [3, 4], canonical quantum gravity [5, 6], various models of deformed special relativity (DSR) [7, 8], and also a variety of generalized uncertainty principle (GUP) [9, 10], which seem to be potential candidates for converging the study. These approaches implicitly predict a fundamentally quantized space due to the existence of a minimal uncertainty in physical distance $\Delta x_{\min}$ of the order of the Planck length $\ell_{pl}$. In fact Einstein’s perspective on gravity as a property of spacetime, and not merely as a usual force, has led to make an effective framework of QG with the impression that a quantum particle is moving on a spacetime with a geometry equipped with a fundamental, minimal characteristic length. It can be shown easily that the minimal length uncertainty results in a modification of the standard canonical commutation relations (CCR) between position and momentum and can be interpreted as a Lorentz invariant natural cutoff, [11]. In other words, the concept
of minimal physical length appears naturally in QG scenarios by generalization of the ordinary Heisenberg commutator relations between position and momentum in Hilbert space. In this respect, it is important to note that all the above mentioned approaches to QG can be considered as the origin of GUP which allow the study of the effects of a minimal length scale in different areas of physics, see [12] for a detailed review. So, the deformation of Heisenberg’s uncertainty principle (HUP), that is, GUP, is a natural and inevitable consequence of QG proposal. As an additional motivation for emphasizing the theoretical importance of GUP, let’s to mention attempts for resolution of the information loss puzzle in black hole physics by considering GUP, see for instance [11,13,15]. Concerning the black hole physics, in [19] argued that there could be black hole remnant due to GUP. In this direction, the implication of GUP on the complementarity principle proposed for the black hole information loss paradox, investigated in [20]. Also in [21] it is shown that by taking GUP in the context of extra dimensions theories, there is a possibility to agree with the outcomes of LHC which claims no black holes are recognizable around 5 TeV [22]. Since a deformation of Heisenberg uncertainty principle affects various quantum phenomena, some efforts have been made to understand phenomenological aspects of GUP models in the context of astrophysics, cosmology and high energy physics in recent years, see for instance [23,31]. The consequence of GUP on the physics of white dwarf stars also studied in [32,34]. Violation of weak equivalence principle within QM is one of the controversial outcomes of GUP [35,36] which the deviation reported in neutron interferometry experiment [37] confirms it.

The lack of a direct experimental test of tentative theories of QG is due to the limits, both in measurement and huge energy required in some controllable experimentally phenomena in the vicinity of the Planck scale. However, this attempt has evolved seriously as one of the major challenges of the QG phenomenological studies in recent years, see for instance Refs.[38,39]. In the light of proposed experimental tests, the current path of QG could be more coherent since naturally it is expected that some of the involved proposals may be redundant and even ruled out. Indeed, it is expected that application of some experimental practicality within the different proposals related to QG make them more efficient and realizable depending on the engineered improvements in the current designs. Within the last two decades, technical developments in experiments related to phenomenology of quantum gravity and probing spacetime structure at short distances were focused on the control of physical systems at the Planck scale. These attempts were able to provide promising facilities in order to check correctness of the phenomenological predictions of QG theories [10]. The already conceived idea of QG tempts one to think that the intersection between QM and GR is too difficult to be accessed through laboratory experiments and seems indeed to be on the other side of the accessible knowledge horizon [11]. In other words, it was assumed that firm predictions are impossible without a fully fledged theory of QG, which combines QM and GR in a new set of laws comparable with observations. However, in a new look at QG phenomenology, that has started with the works referred in Refs. [42–45], it is believed that there is the possibility of meeting the theory with experiment even in the absence of a well-established, final theory of QG. The new aspect of QG phenomenology can be defensible from the following two perspectives. Firstly, over the time the empirical techniques have made significant progress that has now made it possible to test the QG effects credibly. Secondly, theoreticians succeeded in handling some proposals and ideas with permissible prospects that could be tested by common experimental techniques. In a generic sense, the foremost purpose of QG phenomenological studies is to try to suggest some ways of deriving potential experimental signatures from numerous approaches. The modifications arising from QG in the form of relevant GUP(s) affect Hamiltonian of systems under study, expected to lead to small modifications in measurable quantities in the bed of some physical experiments. So far, numerous attempts have been made to explore spacetime structure at the Planck scale by tuning the characteristic parameter introduced by the relevant QG proposal with experimental data, see Refs. [46,53]. The upper bounds released so far strongly indicate the fact that one needs to look for even more advanced and special experimental setups to detect QG effects with ideal resolution expected in theory. Of course, the progress made in recent years in coherence between theoretical and experimental aspects of quantum optics, in particular the new approach towards light-matter interactions, has opened a novel direction for testing new areas of physics such as Planck scale physics with excellent resolution [55–57].

One of the most accurate and trustworthy tools that can be used to explore new areas of physics is “atomic interferometry” (AI) which in its advanced setups is exploited “laser cooled atoms”. It is important to note that because of certain sensitivity of AI, it has been used extensively to achieve accurate measurements in physics 1 [61,63]. Also in recent years we faced with impressive application of AI for testing fundamental laws and principles of physics [57,64]. Amelino-Camelia et.al were pioneer to propose the idea that AI [65] can be used for controlling the QG characteristic parameter released in DSR deformed dispersion relation [66]. This precious idea has motivated us in this paper to investigate the possibility of probing the Planck scale spacetime with resolution allowed by the Rubidium cold atom embedded in an AI setup, proposed based on a new class of microsystems technology known as “Micro-opto-electro-mechanical systems” (MOEMS) 2. As one of the most sensitive and accurate measurements performed by AI that plays a central role in our analysis is determination

---

1 AI systems also have some other applications like that of being developed as accelerometers [58] and gyroscopes [59]. It is interesting to mention that recently such systems were used to map the gravimagnetic field of the earth in a circling satellite [60].

2 In a simple term, MOEMS is a subset of microsystems technology which together with Micro-electro-mechanical systems (MEMS) forms the specialized technology fields using miniaturized combinations of optics, electronics and mechanics.
of the “fine structure constant” (FSC) $\alpha$. In more details, AI has managed to obtain a value of $\alpha^{-1} = 137.035999037(91)$ \cite{67} which is the second best value relative to the one concluded from the electron anomaly measurement \cite{68}. This has prompted the investigation that AI can provide the possibility of exploring Planck scale spacetime with improved resolution. In this manuscript, we confront a GUP proposal which contains the most general deformation as proposed in \cite{69} with cold atom recoil measurement. The proposed study predicts the narrowest constraints, considering all modification terms up to the highest possible order of GUP characteristic scale.

II. NATURAL CUTOFFS MODIFIED COMMUATION RELATIONS: THE MOST GENERAL FORM OF MODIFICATION

Common versions of GUP comprise of modification terms containing linear and quadratic momentum operator or their combinations \cite{9} \cite{10}. Here, we plan to introduce a GUP proposal containing a general deformation function which is expected to address the most general form of modification of momentum operator. Our idea explicitly is that by taking combinations \cite{9,10}. Here, we plan to introduce a GUP proposal containing a general deformation function which is compact form as follows

\[ [x^i, p^j] = i\hbar \left( \delta^i_j + f[p]^i_j \right), \]

in which $f[p]^i_j$ denotes a tensorial function so that its functional dependency to momentum strongly depends on the form of deformed coordinate representation of the momentum operator admitted by various GUP models \cite{69}. By employing an inductive method one can obtain such expression as

\[ p_i \to \tilde{p}_i = p_i \left( 1 + \lambda_0 + \lambda_1 (p^j p_j)^{1/2} + \lambda_2 (p^j p_j) + \lambda_3 (p^j p_j)^{3/2} + \lambda_4 (p^j p_j)^2 + \ldots \right), \]

for the most general form of the modified momentum operator. The above expression can also be demonstrated in a more compact form as follows

\[ p_i \to \tilde{p}_i = p_i \left( 1 + \sum \lambda_n (p^j p_j)^{n/2} \right), \quad n = 0, 1, 2, 3, \ldots \]

in which $\lambda_n = \frac{\lambda_0}{(M N c)^{n}}$ is the most general form of GUP characteristic parameter containing dimensionless constant $\lambda_0$. Of course, if we insist on the condition $\lambda_0 > 0$ to be satisfied, the most general form of the modified momentum operator \cite{3}, re-expresses as

\[ p_i \to \tilde{p}_i = p_i \left( 1 \pm \sum \lambda_n (p^j p_j)^{n/2} \right), \quad n = 0, 1, 2, 3, \ldots \]

where in which follows we focus on this version. Here, should be stress that although in the presentation \cite{3}, both cases $\lambda_0 > 0$ and $\lambda < 0$ are possible but due to the form of \cite{1}, we deal with only with positive values. Although from the perspective of phenomenology, it is believed that the GUP parameter should be positive, however in \cite{23} \cite{70} the possibility of a negative\cite{3} one also discussed. Theoretically, exploring quantum spacetime cannot be fully achieved unless in the case of $\lambda_0 = 1$. This brings the notion that on deriving upper bounds close to unity, the quantum spacetime becomes more accessible which is a step forward in the direction of testing relevant QG predictions. In Ref. \cite{69}, further details can be found on the GUP proposal at hand.

III. EXPERIMENTAL SCHEME: INBUILT ATOM INTERFEROMETER MEOMS BASED OPTICAL FINESSE CAVITY

The basic principle of this assembly is that the cold ensemble of atoms, when is launched inside an optical cavity, splits into two clouds which both traverse different paths based on the laser input. This generates a velocity-based selection of atoms based on their excitation in the hyperfine states, allowing them to recoil backwards or be defined still by the states $|g\rangle$ and $|e\rangle$. The difference between recoiling atoms and steady atoms sets a number of Bloch oscillations which transmits the recoiling velocities to the slowed atoms and ensures that the internal state of atom does not change. The atomic velocity increases by $2u_r$ where $u_r$ is the recoil velocity of the atom when absorbing the momentum of the photon. The final velocity at this stage is the sum of the initial velocity distribution and the $N$ Bloch oscillations. Meanwhile, another

\footnote{According to literatures the possibility of negative sign for GUP parameter was first raised in \cite{71} to formulate it on a crystal lattice.}
Raman pulse is applied to the final velocity transferring the atoms back to the $|g\rangle$. The operation of the proposed system comprises of the engineered Fabry-Perot optical finesse cavity which conventionally constitutes of two confocal mirrors supporting the free space propagation of light, while in this design one of the confocal mirrors is replaced by vibrating micromechanical oscillator. The role of optical finesse cavity has been broadly discussed in several proposed experiments for probing non-commutative theories and the possible realization of QG phenomena using the current technology. The design of optical cavity coupled with nano and micro-mechanical elements have been reported in previous studies, see Refs. [56, 57]. However, in the present study we propose a novel confirmation of the architecture based on the same optical coupling with electromechanical systems, but now by introducing the atomic ensemble in the cavity and replacing one of the spherical confocal mirrors with micro mechanical oscillator. We propose the introduction of MEOMS component which besides electro-mechanical coupling provides for the integration of optical component as well for exploring the QG phenomena. This setup is schematically displayed in Fig. 1. The Fabry-Perot optical cavity amplifies the light on each pass which creates a laser gain medium in the cavity and limits the frequency noises adding to its sensitivity towards wavelength of the light. The resonant enhancement in the cavity, increases the transitions due to minimum laser power which otherwise consumes a multi watt power system. The AI enclosed in the optical cavity limits the wavefront distortions found in conventional system which increases its sensitivity and efficiency. One of the pivotal components in the assembly generally described in Fig. 1 is a miniature MEOMS based oscillator which also acts as a complimentary confocal mirror for the cavity and has been designed in such a way that it acts as a transducing element to pick up the optical signal for the atomic ensemble. The oscillator keeps moving so as to arrest the ensemble during its “Raman transition” between the hyperfine levels. The transition should be coherent to the Bloch’s oscillation provided by the probe laser creating about $N$ Bloch oscillations in each run and the velocity of the atoms could be effectively measured. The periodic motion of the vibrational MEOMS based oscillator is capable of limiting the spatial variations to reduce the interferometric contrast. The MEOMS based oscillator is proposed to be made of a free standing monolayer of Molybdenum disulphide (MoS₂)
which is known to have an optical bandgap of about 1.6 $-$ 2 eV and shows strong photoluminescence near bandgap due to circularly polarized optical pumping. The changes due to atom recoil on the monolayer MoS$_2$, are detected by the low power consumption photodetector and causes minimum scattering and loss [72]. In this design, the oscillator is attached to the photodetector which monitors the recoil velocity of the atomic ensemble without affecting the internal state of the atoms. The initial leg of the experiment starts with a 2D-MOT (two dimensional-magneto-optical trap) molasses which is expected to load the atoms into the system by slow atomic beam into hyperfine level $|g\rangle$. Then using a standard 3D-MOT molasses it is brought into the vicinity of the optical cavity. The advantage of this architecture is that it allows the combination of a high atomic flux and controlled atomic velocities which were not precisely provided by conventional use of Zeeman slower or thermal beam devices. As a result, when the cold atoms are pumped into the cavity by the 3D molasses activation, it provides a precise control of the velocity in the optical cavity [73]. The velocity adjustments contribute to the flat parabolic atomic trajectories compensating for the default gravitational acceleration effect. This velocity selection of atoms in the cavity provides for more flexible time dependent studies like Raman transitions between two hyperfine ground states and a complimentary Bloch oscillation which is schematically displayed in Fig. 2. It is worthy of mention that in this setup the time dependent noise sources also will limit making it potentially more efficient. The atomic source is the Rubidium $^{87}$Rb and the ensemble is induced into the optical cavity through the MOT arrangements. The atoms exist in between two hyperfine ground states $|g\rangle$ and $|e\rangle$. The atomic ensemble in the optical cavity detects maximum possible recoils due to Raman transition when two counter propagating laser beams excite the atoms that are trapped in it after the cooling. The Raman frequencies of $\omega_1$ and $\omega_2$ and wave vectors $\vec{k}_1$ and $\vec{k}_2$ create coherent atomic beams in the $|e\rangle$ state where the remaining atoms of $|g\rangle$ state are tuned to single photon transition. The reception of these transitions in hyperfine states give rise to the well defined $N$ Bloch oscillations which are well received by the MEOM oscillator.

\[ \delta_{\text{sel}} = \Delta + \frac{\hbar (\vec{k}_1 - \vec{k}_2)^2 + m\vec{u}_i^2}{2m} \cdot \vec{u}_i. \]  

FIG. 2: Schematic representation of Raman transition between hyperfine levels $|g\rangle$ and $|e\rangle$ along with Bloch oscillations.

### A. Formulation

Now we formulated the above description mathematically. The energy conservation law imposes the following equality at resonance

\[ \hbar (\omega_1 - \omega_2 - \omega_{\text{HFS}}) = \hbar \Delta + \frac{[\hbar (\vec{k}_1 - \vec{k}_2) + m\vec{u}_i]^2 - m^2 \vec{u}_i^2}{2m}. \]  

(5)

Here, quantities $m$, $\Delta$ and $\hbar \omega_{\text{HFS}} = E_{g_1} - E_{g_2}$ refer to the mass of atoms, the single photon setting of the atomic levels and the energy difference between the hyperfine levels $|g\rangle$, $|e\rangle$, respectively. By defining $\delta_{\text{sel}} \equiv \omega_1 - \omega_2 - \omega_{\text{HFS}}$ as a Raman transition setting, the above conservation equation takes the following form

\[ \delta_{\text{sel}} = \Delta + \frac{\hbar}{2m} (\vec{k}_1 - \vec{k}_2)^2 + (\vec{k}_1 - \vec{k}_2) \cdot \vec{u}_i. \]  

(6)

In order to transfer the high recoil velocities to atoms in a short time, so that the relevant hyperfine internal state of atoms does not change after process of velocity selection, number of $N$ Bloch oscillations comes into display. Every Bloch
oscillations separately increase the atomic velocity by factor of $2\tilde{u}_r$, where $\tilde{u}_r = \frac{\hbar k_B}{m}$ refers to the recoil velocity of the atom while absorbing a photon with relevant momentum $\hbar k_B$. As a result, the final velocity attributed to the atoms can be denoted by $\tilde{u}_f = \tilde{u}_i + 2N\tilde{u}_r$. In the second pair of Raman $\pi$ pulses, the final velocity distribution is centered on $\tilde{u}_f$.

Here also by applying the energy conservation law, the Raman regulating for the velocity measurement reads off as

$$\delta_{\text{meas}} = \Delta + \frac{\hbar}{2m} (\tilde{k}_1 - \tilde{k}_2)^2 + (\tilde{k}_1 - \tilde{k}_2) \cdot \tilde{u}_f,$$

(7)

where by subtracting it from Eq. (6), one gets

$$|\delta_{\text{sel}} - \delta_{\text{meas}}| = (k_1 + k_2)|u_f - u_i|.$$

(8)

Now the ratio of Planck constant to the mass of the atoms i.e. $\frac{h}{m}$, is found by

$$\frac{h}{m} = \frac{2\pi|\delta_{\text{sel}} - \delta_{\text{meas}}|}{2Nk_B(k_1 + k_2)},$$

(9)

which addresses the FSC $\alpha$ via the following well known relation

$$\alpha^2 = \frac{2R_{\infty}}{c} \frac{m}{m_e} \frac{h}{m}.$$

(10)

Here, $R_{\infty}$, $m_e$ and also $c$ are the Rydberg constant, the electron mass and the speed of light, respectively. In Ref. [67], by the consideration of the Rubidium $^{87}$Rb atoms the authors performed an excellent measurement of $\frac{h}{m}$ which after an approximate margin of error, has released a value as $\frac{h}{m} = 4.5913592729(57) \times 10^{-9} m^2 s^{-1}$. In this measurement the role of the internal hyperfine levels $|g\rangle$ and $|e\rangle$ lies in $5S_{\frac{1}{2}} |F=2, m_F=0\rangle$ and $5S_{\frac{1}{2}} |F=1, m_F=0\rangle$ states of $^{87}$Rb atoms, respectively. As a necessary information in this measurement, an atomic beam with an initial velocity $u_i = 20 m/s$, was prepared in the $F = 2$ hyperfine level. The Ti sapphire laser with relevant wavelength $\lambda_B = \frac{2\pi}{k_B} = 532$ nm was produced which could produce $N = 500$ Bloch oscillations for the application on $^{87}$Rb atoms in each run.

IV. EXPLICIT CONSTRAINTS FOR PLANCK SCALE CHARACTERISTIC PARAMETER

A. The case without superposition of the GUP modified terms

Here we address the GUP modifications in terms of different values of $n$ separately. In a lucid term, we assume that there is no superposition of GUP modification term which perceives each GUP modification term to be considered separately in the absence of other possible terms.

By setting $n = 0$, Eq. (4) reads off as

$$p_i \rightarrow \tilde{p}_i = p_i (1 \pm \lambda_0).$$

(11)

In this new representation, Hamiltonian related to a one-dimensional quantum system can be expanded as

$$H_{\text{GUP}(n=0)} = (1 + \lambda_0^2) \frac{\tilde{p}^2}{2m} + V(x).$$

(12)

By applying the above GUP-modified Hamiltonian to the cold $^{87}$Rb atoms, the relevant GUP term modifies the kinetic energy of $^{87}$Rb atoms as follows

$$E_{K-\text{GUP}(n=0)} = (1 + \lambda_0^2) \frac{\tilde{p}^2}{2m_{Rb}}.$$

(13)

According to the prior pattern, here the GUP counterpart of Eq. (8) takes the following form

$$|\delta_{\text{sel}} - \delta_{\text{meas}}|_{\text{GUP}(n=0)} = (1 \pm \lambda_0)^2 (k_1 + k_2)|u_f - u_i|,$$

(14)

where by putting it into Eqs. (9) and (10) finally we will arrive at

$$\alpha_{\text{GUP}(n=0)}^{-1} = \left(\frac{2R_{\infty} m}{c m_e} \frac{h}{m_{Rb}} (1 \pm \lambda_0)^2\right)^{-\frac{1}{2}}.$$

(15)
Given that the setup at hand is a low energy quantum system, by estimating the GUP effect on the FSC via the following relation, one can derive an explicit upper bound for \( \lambda_0 \)

\[
\frac{|\alpha_{GU P}^{-1} - \alpha^{-1}|}{\alpha^{-1}} < \text{measurement accuracy of } \alpha^{-1}
\]  

(16)

This relation expresses the fact that by taking into the account the measurement of \( \alpha^{-1} \) with a precision order of magnitude of \( 2.5 \times 10^{-10} \) \text{[ME3]}, one can put constraint on the value of \( \lambda_0 \), so that it still will be valid to the measurement of \( \alpha^{-1} \) in the presence of QG effects. This is known as a bottom-up QG phenomenological approach in which our analysis in this study is based on it. Indeed, an interesting idea transposed into this approach is that relying on data due to low energy phenomena, there is the possibility of probing Planck scale spacetime with different resolutions depending on the relevant precision measurements. Now by setting values of \( R_\infty = 1097373.565839(55)m^{-1}, m_{Rb} = 86.909180535(10)m, m_e = 5.4857990946(22) \times 10^{-4}m, M_P \sim 1.3 \times 10^{19}m, c = 3 \times 10^8 m/s, \) the following results are obtained for bounds on \( \lambda_0 \)

\[-2 < \lambda_0 < -1
\]  

(17)

and

\[1 < \lambda_0 < 2,
\]  

(18)

for two cases of positive and negative signs, respectively. As we have mentioned previously, the condition \( \lambda_0 > 0 \) should be satisfied. So, the first constraint is rejected.

By setting \( n = 1 \), Eq. \([4]\) in one dimension reads off as

\[
p \rightarrow \tilde{p} = p(1 \pm \lambda_1 p), \quad \lambda_2 = \frac{\lambda_0}{(M_Pc)}. 
\]  

(19)

In this new representation, by keeping all powers of \( \lambda_0 \) the relevant GUP modified kinetic energy of \(^{87}\text{Rb} \) atoms can also be written as

\[
E_{K-GUP(n=1)} = \frac{\tilde{p}^2}{2m_{Rb}} + \lambda_0^2 \frac{\tilde{p}^4}{2m_{Rb}(M_Pc)^2} \pm \lambda_0 \frac{\tilde{p}^3}{m_{Rb}(M_Pc)}. 
\]  

(20)

Here the GUP counterpart of the Eq. \([8]\) takes the following form

\[
|\delta_{\text{set}} - \delta_{\text{meas}}|_{GUP(n=1)} = (k_1 + k_2)|u_f - u_i| + \frac{2m_{Rb}^2 \lambda_0^2(k_1 + k_2)(u_f^3 - u_i^3)}{(M_Pc)^2} \pm \frac{3m_{Rb}\lambda_0(k_1 + k_2)}{(M_Pc)}(u_f^2 - u_i^2),
\]  

(21)

where by putting it into Eqs. \([9]\) and \([10]\) finally we will arrive at

\[
\alpha_{GU P(n=1)}^{-1} = \left[ \frac{2R_\infty}{c} \frac{m_u}{m_e} \frac{k}{m_{Rb}} \left( 1 + \frac{2(m_{Rb})^2 \lambda_0^2}{(M_Pc)^2}(u_f^2 + u_i u_f + u_i^2) \pm \frac{3m_{Rb}\lambda_0}{(M_Pc)}(u_f + u_i) \right) \right]^{-\frac{1}{2}}.
\]  

(22)

Using the same method described as above, we obtain the following upper bounds on \( \lambda_0 \) as

\[1 \leq \lambda_0 < 1.5 \times 10^{14}
\]  

(23)

and

\[1 \leq \lambda_0 < 4 \times 10^{23}
\]  

(24)

for two cases of positive and negative signs, respectively.

By setting \( n = 2 \), Eq. \([4]\) in one dimension reads off as

\[
p \rightarrow \tilde{p} = p(1 \pm \lambda_2 p^2), \quad \lambda_2 = \frac{\lambda_0}{(M_Pc)^2}.
\]  

(25)

Here also by keeping all powers of \( \lambda_0 \), the relevant GUP modified kinetic energy of \(^{87}\text{Rb} \) atoms is presented as follows

\[
E_{K-GUP(n=2)} = \frac{\tilde{p}^2}{2m_{Rb}} + \lambda_0^2 \frac{\tilde{p}^4}{2m_{Rb}(M_Pc)^2} \pm \lambda_0 \frac{\tilde{p}^3}{m_{Rb}(M_Pc)^2}
\]  

(26)
where results in the following modified expression

\[
|\delta_{sel} - \delta_{meas}|_{GUP(n=2)} = (k_1 + k_2)|u_f - u_i| + \frac{3m_R^4\lambda_0^2(k_1 + k_2)}{(M_p c)^4}(u_j^5 - u_i^5) + \frac{4m_R\lambda_0(k_1 + k_2)}{(M_p c)^2}(u_j^3 - u_i^3),
\]

for Eq. (8). Putting it into Eqs. (9) and (10), we arrive at

\[
\alpha_{GUP(n=2)}^{-1} = \left[ \frac{2R_{\infty}}{c} \frac{m_u}{m_c} \frac{h}{m_R} \left( 1 + \frac{3(m_R c^3 \lambda_0^2) u_j^5 - u_i^5}{(M_p c)^4} \frac{u_j^3 - u_i^3}{u_j^3 + u_i^3} \pm \frac{4(m_R \lambda_0) u_j^3 - u_i^3}{(M_p c)^2} \right) \right]^{-\frac{1}{2}}
\]

Using the method described previously, we obtain the following upper bounds on \( \lambda_0 \)

\[
1 \leq \lambda_0 < 1.4 \times 10^{38},
\]
and

\[
1 \leq \lambda_0 < 3.2 \times 10^{37},
\]

for two cases of positive and negative sign, respectively.

By setting \( n = 3 \), Eq. (4) in one dimension reads of as

\[
p \to \hat{p} = p(1 \pm \lambda_3 p^3), \quad \lambda_3 = \frac{\lambda_0}{(M_p c)^3}
\]

In the same way, for the relevant GUP modified kinetic energy of \(^{87}\)Rb atoms, we have

\[
E_{K-GUP(n=3)} = \frac{p^2}{m_R} + \frac{\lambda_0^2}{2m_Rc(M_p c)^6} \pm \frac{\lambda_0^2}{m_Rc(M_p c)^3} p^5.
\]

In the presence of the relevant GUP modification, Eq. (8) is rewritten as

\[
|\delta_{sel} - \delta_{meas}|_{GUP(n=3)} = (k_1 + k_2)|u_f - u_i| + \frac{8m_R^6\lambda_0^2(k_1 + k_2)}{(2M_p c)^6}(u_j^5 - u_i^5) \pm \frac{5m_R^6\lambda_0(k_1 + k_2)}{(M_p c)^3}(u_j^3 - u_i^3),
\]

where by putting it into Eqs. (9) and (10), we come to

\[
\alpha_{GUP(n=3)}^{-1} = \left[ \frac{2R_{\infty}}{c} \frac{m_u}{m_c} \frac{h}{m_R} \left( 1 + \frac{4(m_R c^3 \lambda_0^2) u_j^5 - u_i^5}{(M_p c)^6} \frac{u_j^3 - u_i^3}{u_j^3 + u_i^3} \pm \frac{5(m_R \lambda_0) u_j^3 - u_i^3}{(M_p c)^3} \right) \right]^{-\frac{1}{2}}.
\]

Once again via the method described above, we obtain other upper bounds on \( \lambda_0 \) as follows

\[
1 \leq \lambda_0 < 1.5 \times 10^{62},
\]
and

\[
1 \leq \lambda_0 < 3.3 \times 10^{71},
\]

for two cases of positive and negative signs, respectively.

Comparing these obtained constraints for different values of \( n \) shows obviously that by considering higher order powers of momentum in the deformed coordinate representation and neglecting all smaller order terms in momentum in each step, the possibility for accurate resolution of spacetime structure in Planck scale reduces for higher order terms.

B. The case with superposition of the GUP modified terms

Here, unlike the previous subsection, we treat a general form of GUP as a superposition of all possible modification terms (we restrict ourselves to \( n = 0, 1, 2, 3 \)). In this regard, for the one-dimensional system Eq. (4) takes the following form

\[
p \to \hat{p} = p \left( 1 \pm \lambda_0 \pm \lambda_1 p \pm \lambda_2 p^2 \pm \lambda_3 p^3 \pm \ldots \right).
\]
In this case, involving quantities $E_K$, $|\delta_{sel} - \delta_{meas}|$ and $\alpha^{-1}$ can be modified in the following forms

$$E_{K-GUP(\pm)} = \frac{1}{2m_{Rb}} \left[ (1 \pm \lambda_0)^2 p^2 + (2\lambda_0 \lambda_1 \pm 2\lambda_1)p^3 + (\lambda_1^2 + 2\lambda_0 \lambda_2 \pm 2\lambda_2)p^4 \right.$$

$$+ (2\lambda_0 \lambda_3 \pm 2\lambda_1 \lambda_2 \pm 2\lambda_3)p^5 + (\lambda_2^2 + 2\lambda_1 \lambda_3)p^6 + 2\lambda_2 \lambda_3 p^7 + \lambda_3^2 p^8 \left], \right. \quad (38)$$

and

$$|\delta_{sel} - \delta_{meas}|_{GUP(\pm)} = (1 \pm \lambda_0)^2 (k_1 + k_2)|u_f - u_i| + 6m_{Rb}(\lambda_0 \lambda_1 \pm \lambda_1)(k_1 + k_2)(u_f^2 - u_i^2) + 2m_{Rb}^2(\lambda_1^2 + 2\lambda_0 \lambda_2 \pm 2\lambda_2)(k_1 + k_2)(u_i^2 - u_f^2) + 9m_{Rb}^4(2\lambda_1 \lambda_3 + \lambda_2^2)(k_1 + k_2)(u_f^5 - u_i^5) + 14m_{Rb}^5 \lambda_2 \lambda_3(k_1 + k_2)(u_f^6 - u_i^6)4m_{Rb}^6 \lambda_2 \lambda_3(k_1 + k_2)(u_f^7 - u_i^7) \right. \quad (39)$$

and

$$\alpha^{-1}_{GUP(\pm)} = \left[ \frac{2Rb}{c} \frac{m_u}{m_u} \frac{h}{m_{Rb}} \left( 1 + (1 \pm \lambda_0)^2 + 6 \frac{m_{Rb}}{m_u}(\lambda_0 \lambda_1 \pm \lambda_1)(u_f + u_i) + 2(\frac{m_{Rb}}{m_u})^2(\lambda_1^2 + 2\lambda_0 \lambda_2 \pm 2\lambda_2)(u_f^2 + u_i u_f + u_i^2) \right. \right.$$

$$+ 5(\frac{m_{Rb}}{m_u})^3(\lambda_0 \lambda_3 + \lambda_1 \lambda_2 \pm \lambda_3)(u_f^3 - u_i^3) + 9(\frac{m_{Rb}}{m_u})^4(\lambda_2^3 + 2\lambda_1 \lambda_3)(u_f^5 - u_i^5)$$

$$+ 14(\frac{m_{Rb}}{m_u})^5 \lambda_2 \lambda_3(u_f^6 - u_i^6) + 4(\frac{m_{Rb}}{m_u})^6 \lambda_2 \lambda_3(u_f^7 - u_i^7) \right)^{-\frac{1}{2}} \right. \quad (40)$$

respectively. By setting $\lambda_2 = \lambda_1^2$ and $\lambda_3 = \lambda_1^3$, the following upper bounds on $\lambda_0$ are achieved

$$-1.00002 < \lambda_0 < -0.999978 \quad (41)$$

and

$$0.999978 < \lambda_0 < 1.00002 \quad (42)$$

for the cases of positive and negative signs, respectively. The case with negative $\lambda_0$ is not acceptable as we have discussed previously. The condition $[42]$ is very tight bound on quantum gravity parameter, $\lambda$. So, in this proposed setup one is able to probe the quantum spacetime structure even in low energy regime. This is so important in testing QG proposal in laboratory in energy scales accessible today.

To summarize, in this work, via an accessible low energy technology, we found possibility of probing the quantum spacetime structure with a favorable theoretical resolution. Concerning the GUP proposal containing the most general modification of coordinate representation of the momentum operator, we have studied how to use the outcomes from $^{87}$Rb CARE (specially the measurement of FSC) schematically designed in Figs. [1] and [2] to constrain the relevant dimensionless, QG characteristic parameter $\lambda_0$. By enclosure the underlying AI setup within an high finesse optical cavity, which is supported by a new class of low power consumption integrated devices known as MEOMS, it has been distinguished from the conventional architectures. MEOMS belong to the family of microsystems technology which along with electro mechanical energy, harness the low dimension optics. As a distinct factor in this setup, it should be pointed out the presence of a micro mechanical oscillator instead of spherical confocal mirrors as one of the components of high finesse optical cavity.

Altogether, this study has been done in two fashion: with and without superposition of separate modification terms in momentum. The first step was based on the assumption that there is no superposition of modification terms so that each term is considered individually in the absence of other possible terms. In this way, the best constraint on $\lambda_0$ is extracted as $1 \leq \lambda_0 < 2$, when in the deformed coordinate representation, makes the momentum operator to appear as a re-scaled quantity. However, the constraints explicitly reflect the fact that by increasing the power of momentum in deformed coordinate representation the resolution for exploring the spacetime structure in Planck’s scale reduces and becomes much weaker.

Contrary to the first step, in the next step we have considered a general form of GUP as a superposition of all possible modification terms (we preferred to consider just four first terms with $n = 0, 1, 2, 3$). We have obtained a strict and tight constraint as $0.999978 < \lambda_0 < 1.00002$ which is very favorable from the theoretical perspective. This constraint obviously challenges the long-lived belief in QG phenomenology project that probing quantum spacetime structure is possible just by having full access to Planck energy. In other words, this setup paves a novel way for probing Planck scale spacetime procured by the accessible low energy technology, in particular the proposed opto-atomic $^{87}$Rb interferometric arrangement. So, one may argue that the best playground to test QG is not just the high energy physics; a table-top nanosystem assembly can act as well.
V. ACKNOWLEDGMENTS

The work of K. Nozari has been financially supported by Research Institute for Astronomy and Astrophysics of Maragha (RIAAM) under research project No. 1/5440-67.

[1] S. Hossenfeldera et al., Collider signatures in the Planck regime, Phys. Lett. B 575 (2003) 85
[2] R. Edrich, Quantum Gravity: Has Spacetime Quantum Properties?, [arXiv:0902.0190 [gr-qc]]
[3] D. J. Gross and P. F. Mendle, String Theory Beyond the Planck Scale, Nucl. Phys. B 303 (1988) 407
[4] K. Konishi, G. Paffuti and P. Provero, Minimum Physical Length and the Generalized Uncertainty Principle in String Theory, Phys. Lett. B 234 (1990) 276
[5] C. Rovelli and L. Smolin, Discreteness of area and volume in quantum gravity, Nucl. Phys. B 442 (1995) 593
[6] A. Ashtekar and J. Lewandowski, Quantum theory of geometry, 1: Area operators, Class. Quantum. Grav. 14 (1997) A55
[7] G. Amelino-Camelia, Doubly special relativity: First results and key open problems, Int. J. Mod. Phys. D 11 (2002) 1643
[8] J. Magueijo and L. Smolin, Lorentz invariance with an invariant energy scale, Phys. Rev. Lett. 88 (2002) 190403
[9] A. F. Ali, S. Das and E. C. Vagenas, Discreteness of Space from the Generalized Uncertainty Principle, Phys. Lett. B 678 (2009) 497
[10] S. Das, E. C. Vagenas and A. F. Ali, Discreteness of Space from GUP II: Relativistic Wave Equations, Phys. Lett. B 690 (2010) 407
[11] A. Kempf, G. Mangano, R. B. Mann, Hilbert space representation of the minimal length uncertainty relation, Phys. Rev. D 52 (1995) 1108
[12] S. Hossenfelder, Minimal Length Scale Scenarios for Quantum Gravity, Living Rev. Rel 16 (2013) 2
[13] A. Kempf, G. Mangano, Minimal length uncertainty relation and ultraviolet regularization, Phys. Rev. D 55 (1997) 7909
[14] A. Kempf, Nonpointlike particles in harmonic oscillators, J. Phys. A 30 (1997) 2003
[15] D. Amati, M. Ciafaloni, G. Veneziano, Can Space-Time Be Probed Below the String Size?, Phys. Lett. B 216 (1989) 41
[16] M. Maggiore, A Generalized uncertainty principle in quantum gravity, Phys. Lett. B 304 (1993) 65
[17] M. Maggiore, The Algebraic structure of the generalized uncertainty principle, Phys. Lett. B 319 (1993) 83
[18] M. Maggiore, Quantum groups, gravity and the generalized uncertainty principle, Phys. Rev. D 49 (1994) 5182
[19] Ronald J. Adler, Pin Chen, David I. Santiago, The Generalized Uncertainty Principle and Black Hole Remnants, Gen. Rel. Grav. 33 (2001) 2101
[20] P. Chen, Y. Chin Ong, D. Yeom, Generalized Uncertainty Principle: Implications for Black Hole Complementarity, JHEP 12 (2014) 021
[21] S. Das and A. F. Ali, No Existence of Black Holes at LHC Due to Minimal Length in Quantum Gravity, JHEP 1209 (2012) 067
[22] CMS Collaboration Collaboration, V. Khachatryan et al., Search for Microscopic Black Hole Signatures at the Large Hadron Collider, Phys. Lett. B 697 (2011) 434
[23] Y. Sabri and K. Nouicer, Phase transitions of a GUP-corrected Schwarzschild black hole within isothermal cavities, Class. Quant. Grav. 29 (2012) 215015
[24] M. Khodadi, K. Nozari, A. Hajizadeh, Some Astrophysical Aspects of a Schwarzschild Geometry Equipped with a Minimal Measurable Length, Phys. Lett. B 770 (2017) 556
[25] N. Khosravi and H. R. Sepangi, The Cosmological implications of a fundamental length: A DSR inspired de-Sitter spacetime, JCAP 0804 (2008) 011
[26] N. Khosravi and H. R. Sepangi, A Fundamental length as a candidate for dark energy: A DSR inspired FRW spacetime, Phys. Lett. A 372 (2008) 3356
[27] A. Tawfik, H. Magdy and A. Farag Ali, Effects of quantum gravity on the inflationary parameters and thermodynamics of the early universe, Gen. Rel. Grav. 45 (2013) 1227
[28] M. A. Gorji, Late time cosmic acceleration from natural infrared cutoff, Phys. Lett. B 760 (2016) 769
[29] M. Khodadi, K. Nozari and H. R. Sepangi, More on the initial singularity problem in gravity’s rainbow cosmology, Gen. Rel. Grav. 48 (2016) 166
[30] M. Khodadi, K. Nozari, Some Features of Scattering Problem in a $\kappa$-Deformed Minkowski Spacetime, Annalen der Physik 528 (2016) 785
[31] M. Khodadi, K. Nozari, E. N. Saridakis, Emergent universe in theories with natural UV cutoffs, Class. Quant. Grav. 35 (2018) 015010
[32] R. Rashidi, Generalized Uncertainty Principle Removes The Chandrasekhar Limit, Annals Phys. 374 (2016) 434
[33] Y. C. Ong, Generalized Uncertainty Principle, Black Holes, and White Dwarfs: A Tale of Two Infinities, JCAP 09 (2018) 015
[34] D. K. Mishra, N. Chandra, Quantum gravity corrections to white dwarf dynamics, [arXiv:1803.06640 [gr-qc]]
[35] V. M. Tkachuk, Deformed Heisenberg algebra with minimal length and equivalence principle, Phys. Rev. A 86 (2012) 062112
[36] S. Ghosh, Quantum Gravity Effects in Geodesic Motion and Predictions of Equivalence Principle Violation, Class. Quant. Grav. 35 (2018) 055025
[37] L. Zhou et al., Test of Equivalence Principle at 10$^{-8}$ Level by a Dual-species Double-diffraction Raman Atom Interferometer, Phys. Rev. Lett. 115 (2015) 013004
[38] G. Amelino-Camelia, Quantum-Spacetime Phenomenology, Living Rev. Rel. 16 (2013) 5
[39] S. Hossenfelder, Experimental Search for Quantum Gravity, [arXiv:1010.3420 [gr-qc]]
[40] J. Stachel, “Early history of quantum gravity, Black Holes, Gravitational Radiation and the Universe”, edited by B. Iyer and...
