Jet production in electron-proton collisions at HERA provides a unique testing ground for Quantum Chromodynamics (QCD). Apart from the determination of the strong coupling constant $\alpha_s$, $ep$ jet data may especially be used to gain insight into the dynamics of the exchanged parton cascade, whose structure is probed by the high-$E_T$ dijet system; thus information on the parton content of the proton and (quasi-)real and virtual photons is obtained. This report touches some of these aspects revealed in recent jet data from the HERA experiments which are testing perturbative QCD at the limits of applicability.

1 Introduction

Multi-jet production in deep inelastic $ep$-scattering (DIS) provides special sensitivity to the mechanisms of the strong interaction and can hence be used to test the predictions of perturbative Quantum Chromodynamics (pQCD) in a rather inimitable way. This is especially true since the HERA experiments deliver data over a large range of both the four-momentum transfer, $Q^2$, of the exchanged photon and the transverse energy of the observed jets, $E_T$.

In lowest order, multi-jet production in DIS is described by the QCD-Compton and the boson-gluon fusion processes (Figure 1), with the momentum distributions of the incoming partons taking part in the hard interaction given by the parton density functions (PDFs) of the proton; the $Q^2$-evolution of the latter is described by the DGLAP equations which in lowest order are equivalent to the assumption of exchanging a strongly $k_t$-ordered parton cascade, $k_t$ being the transverse momentum of the partons within the cascade. However, as illustrated by the generic diagram in Figure 1c, to calculate $ep$ jet cross sections requires additional terms, such that in certain parts of phase space, where $k_t$-ordered parton emission is no longer manifest, the standard DGLAP approach has to be extended e.g. with the concept of photon structure, which preserves the perturbative ansatz of the DGLAP evolution scheme but results in an artificial violation of the $k_t$-ordering. Within this concept, high $E_T$ jets can be produced not only by direct processes in which the virtual photon interacts as a point-like particle with a parton out of the proton, but also by resolved processes where the photon interacts hadronically. In the photoproduction limit ($Q^2 \to 0$), the cross section is thus sensitive to the...
Figure 1. Schematic Feynman diagrams for the dijet production mechanism with (a) the QCD-Compton and (b) the boson-gluon fusion process; (c) generic picture for dijet production via photon-parton fusion where the actual interaction mechanism is unknown and to be resolved by the dijet system. The two scales involved in these processes are indicated as the four-momentum transfer $Q^2$ and the transverse jet energy $E_T$.

parton distributions of both the photon and the proton at a scale set by the transverse energy of the jets, $E_T$.

The two scales involved in $ep$ jet production, $\sqrt{Q^2}$ and $E_T$, can now be used to resolve the structure of the photon-parton interaction in general. Several kinematic regions can be distinguished, depending on the absolute and relative sizes of these two scales. In the regime where $Q^2$ and $E_T^2$ are both large, perturbative methods are clearly justified and the DGLAP ansatz can be used to extract information on the proton structure and the strong coupling $\alpha_s$. Typically, leading-order (LO) Monte Carlo models approximate higher order pQCD contributions in this regime by parton showers. If $Q^2 \ll E_T^2$ the jet cross sections become sensitive to the hadronic structure of the photon. The region where $Q^2 \approx E_T^2$ is of special interest, since in this regime the DGLAP ansatz is expected to break down for small values of the variable $x_B$ (Bjorken-$x$) revealing effects of non-$k_t$-ordered parton cascades. Characteristics expected from other evolution schemes like BFKL or CCFM could be observable in this phase space regime.

Any deviations from next-to-leading order (NLO) predictions may, however, simply indicate that fixed order calculations beyond NLO are needed. These may also help to resolve other ambiguities associated with high $E_T$ jet production in DIS: where $Q^2$ and $E_T^2$ both are substantially large ($\gg \Lambda_{QCD}^2$) there is ambiguity in the choice of the renormalization scale. It is important to know where these ambiguities have a significant effect on the cross sections.

2 Testing pQCD at Large Scales

Jet production at large $Q^2$ and large $E_T$ has been studied by both the H1 and the ZEUS collaborations. In this regime the choice of the renormalization scale $\bar{a}_s$ is of special interest, since in this regime the DGLAP ansatz is expected to break down for small values of the variable $x_B$ (Bjorken-$x$) revealing effects of non-$k_t$-ordered parton cascades. Characteristics expected from other evolution schemes like BFKL or CCFM could be observable in this phase space regime.

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\footnote{Analogously such ambiguity exists also for the factorization scale. However, since the effects due to the choice of the renormalization scale are generally larger, this ambiguity is not explicitly mentioned in the context of this paper.}
tion scale \( \mu_r = \sqrt{Q^2}, E_T \) is of minor importance since scale dependencies of the cross sections are generally small if \( \mu_r \) is large. Hence NLO order calculations are expected to provide a good description of the data as can be seen from Figure 2 which shows the inclusive jet cross section \( d^2\sigma / dE_T dQ^2 \) measured by H1. Jets have been defined using the inclusive \( k_\perp \) algorithm in the Breit frame where the photon collides head-on with the incoming proton. Over the whole analyzed phase space the DISENT NLO calculation, using the CTEQ5M1 PDFs as input and corrected for hadronization effects (<10%), agrees perfectly well with the data. The size of the scale uncertainties together with the error contribution from hadronization corrections can be inferred from Figure 2 (right). From these data, H1 extracts the strong coupling constant \( \alpha_s \) in bins of \( E_T \), chosen to be the renormalization scale. The QCD predictions are fitted to the jet cross sections using the CTEQ5M1 parameterization of the PDFs and the strong coupling constant as the single free parameter. The result is shown in Figure 3 and clearly indicates the running of \( \alpha_s \) according to the renormalization group equation. A combined fit to all 16 data points shown in Figure 2 results in

\[ \alpha_s(M_Z) = 0.1186 \pm 0.0030 \text{ (exp.)}^{+0.0039}_{-0.0045} \text{ (theo.)}^{+0.0033}_{-0.0023} \text{ (pdf)} \ (\mu_r = E_T), \]

where the largest contributions to the experimental error come from uncertainties of the hadronic energy scale. The theoretical error is dominated by
the uncertainty in the hadronization corrections and the dependence on the choice of the renormalization scale. The uncertainty in the knowledge of the parton density functions has been estimated using the correlated errors provided by a recent QCD analysis together with its global PDF-fits. This error contribution is substantially reduced when using the dijet rate $R_{2+1}$ to extract $\alpha_s$ as done by the ZEUS collaboration, leading to the preliminary result

$$\alpha_s(M_Z) = 0.1166^{+0.0039}_{-0.0047} \text{(exp.)}^{+0.0055}_{-0.0042} \text{(theo.)}^{+0.0012}_{-0.0011} \text{(pdf)} \left(\mu_r = \sqrt{Q^2}\right).$$

Both measurements of $\alpha_s$ depend on the knowledge of the parton content of the proton. Vice versa, by using the best knowledge on $\alpha_s$ one can use (multi-)jet cross section measurements to extract the quark and gluon densities in the proton. A test of pQCD independent of data from other experiments, has been made by the H1 collaboration in performing a simultaneous determination of both quantities. This is done in a fit to the inclusive jet and dijet cross sections together with the inclusive DIS cross section, where the latter is restricted to the kinematic range $150 \leq Q^2 \leq 1000$ GeV$^2$ and only constrains the quark densities in the proton. The results of this fit are shown in Figure 4 as a correlation plot between $\alpha_s(M_Z)$ and the gluon density $xg(x)$ evaluated at four different values of the gluon momentum fraction $x = 0.01$, $x = 0.02$, $x = 0.04$, and $x = 0.1$.
0.02, 0.04 and 0.1. While the present data do not allow a simultaneous determination of both parameters with competitive precision the sensitivity to the product \( \alpha_s \cdot x g(x) \) can clearly be seen.

3 Scale Ambiguities

The relevance of the choice of the renormalization scale at smaller \( Q^2 \) can be seen from Figure 5, showing the dijet cross section as a function of \( \log Q^2 \) as measured by the ZEUS collaboration. As for the inclusive jet cross section, pQCD in NLO describes the data down to values of \( Q^2 \approx 150 \text{ GeV}^2 \). However, at \( Q^2 \) values below 150 GeV\(^2 \) scale uncertainties become large and the choice of the renormalization scale is no longer irrelevant: while NLO calculations using \( \mu_r = \sqrt{Q^2} \) describe the dijet cross section down to \( Q^2 \approx 10 \text{ GeV}^2 \), this is no longer the case for \( \mu_r = E_T \).

This issue has also been studied by the H1 collaboration investigating dijet event rates, \( R_2 \), at low values of the Bjorken-\( x \) variable \( 10^{-4} < x_B < 10^{-2} \) and low \( Q^2 \), \( 5 < Q^2 < 100 \text{ GeV}^2 \). For different requirements on the transverse energies \( E_{T,(1,2)} \) of the final state jets, NLO QCD calculations are confronted with the data using two different choices of the renormalization scale \( \mu_r = \sqrt{Q^2} \) and \( \mu_r = \sqrt{Q^2 + E_T^2} \). Figure 6 summarizes the preliminary result of this analysis in two representative \((x_B, Q^2)\)-bins. At large \( Q^2 \) \((Q^2 = 71 \text{ GeV}^2)\) and large \( x_B \) \((x_B = 4.7 \cdot 10^{-3})\) the theoretical calculation is...
Figure 6. Dijet rate $R_2$ at two values of $x_B$, $Q^2$ as a function of the transverse energy requirement $E_{T,1} > (5 + \Delta)$ GeV on the jet with larger $E_T$; for the second jet the transverse energy $E_{T,2}$ must be above 5 GeV.

rather insensitive to the choice of the scale (Fig. 3, right), and — in agreement with the results from the dijet cross section measurement at high $Q^2$ — gives a good description of the data. In contrast, one obtains no safe theoretical prediction at low $x_B$, $Q^2$ (Fig. 3, left): although data and NLO QCD calculation do agree for $\mu_r = \sqrt{Q^2}$, the large scale uncertainties lead to almost vanishing predictive power of the theory; choosing, however, a larger scale $\mu_r = \sqrt{Q^2 + E_T^2}$ such that scale uncertainties become small, leads to clear disagreement between the measured event rate $R_2$ and the next-to-leading order QCD calculation, independent of the requirement on the transverse energy of the jets $E_{T,1} > (5 + \Delta)$ GeV. This indicates the necessity to include higher-order contributions by introducing, for example, the concept of photon structure. Choosing the same scale $\mu_r = \sqrt{Q^2 + E_T^2}$, comparison of the data with the NLO calculations by JetViP including a resolved photon contribution, indeed shows better, although not perfect agreement.

4 Probing Parton Dynamics

In order to further investigate the interplay of the two scales $\sqrt{Q^2}$ and $E_T$, the ZEUS collaboration has studied the cross section for jets produced at large pseudo rapidities as a function of the ratio $E_T^2/Q^2$. When compared to LO Monte Carlo models (Figure 7) the data are well described for $E_T^2 \ll Q^2$. However, only those models which include non-$k_t$-ordered parton emission (ARIADNE, RAPGAP) reproduce the $E_T^2/Q^2 \approx 1$ region; the full range can be described solely by models which include a resolved photon contribution as the RAPGAP Monte Carlo and a NLO calculation by JetViP. A

This is true unless a symmetric transverse energy cut is applied where the NLO QCD calculation becomes infrared sensitive. In this region resummed calculations are needed but not yet available. Thus the presented measurement also provides an important reference for improved theoretical predictions.
reasonable description of these data is also found in a recent comparison\textsuperscript{3} with Monte Carlo predictions based on the CCFM\textsuperscript{3} evolution equation which, by means of angular-ordered parton emission, is equivalent to the BFKL approach for $x \to 0$ while reproducing the DGLAP equations at large $x$.

To explore a possible signature of BFKL dynamics, the $Q^2 \approx E_T^2$ regime has been analyzed in more detail studying the $x_B$-dependence of forward particle production. Figure 8 shows the $x_B$-dependence of the forward-jet and forward-$\pi^0$ cross sections in this region\textsuperscript{1} in comparison to Monte Carlo models with and without a resolved photon contribution and — in case of the $\pi^0$ cross sections — a LO BFKL calculation. The need for an additional contribution to the direct $\gamma p$-interaction can clearly be seen from these figures as the data are rather well described by models including a resolved photon component. Moreover, the $\pi^0$ measurement, which accesses very small polar angles, is actually best described by a BFKL-based leading-order calculation\textsuperscript{19} when taking hadronization corrections into account. However, large theoretical scale uncertainties diminish the significance of these comparisons.

5 Jets in Photoproduction

The production of hard dijet events in photoproduction is dominated by resolved photon processes in which a parton in the photon with momentum fraction $x_\gamma$ is scattered from a parton in the proton. Hence studies of dijet production can be used to investigate the hadronic structure of the photon

\textsuperscript{c}Particles produced at very small polar angels w.r.t. the incoming proton direction.
at $Q^2 \approx 0$. In leading order the cross section is proportional to the photon flux $f_{\gamma/e}$ and maybe approximated by use of effective parton densities for the proton and photon such that $\sigma_{\text{dijet}} \sim f_{\gamma/e} \cdot f_{\text{eff},\gamma} f_{\text{eff},p} \cdot |\mathcal{M}|^2$, where $\mathcal{M}$ is the matrix element of the hard parton-parton scattering process and $f_{\text{eff},\gamma} = f_q/|\gamma,p| + f_{\bar{q}}/|\gamma,p| + 9/4 f_g/|\gamma,p|$. With this relation H1 extracts the gluon density of the photon in leading order pQCD using the knowledge of $f_{\text{eff},p}$ from DIS and $f_{q}/|\gamma,p|$ from the photon structure function $F_2$ as measured in $e^+e^-$ collisions. The result is shown in Figure 9, indicating a strong rise towards low $x_\gamma$.

The determination of the gluon content of the photon is however strongly affected by non-perturbative effects and the treatment of background from soft underlying events; it therefore can only be done using Monte Carlo programs which include phenomenological models to account for these effects but up to now are still restricted to leading order matrix elements. Other dijet analyses from H1 and ZEUS suppress these non-perturbative effects by harder $E_T$ requirements for the jets and thus investigate the parton distributions in the photon at high $x_\gamma (x_\gamma \gtrsim 0.2)$ where quark contributions dominate. A comparison of the NLO prediction with the dijet cross section measured by ZEUS for $E_{T,\text{jet}} > 14$ GeV as a function of the pseudo rapidities of the two most energetic jets, $\eta_1^{\text{jet}}$ and $\eta_2^{\text{jet}}$, is given in Figure 10. While the NLO calculation describes the data at large $x_\gamma \geq 0.75$ rather well,

\[ x_\gamma^{\text{obs}} = \frac{E_{T,1}e^{-\eta_1^{\text{jet}}} + E_{T,2}e^{-\eta_2^{\text{jet}}}}{2\mu E_e} \]

is studied, where $E_e$ is the electron beam energy. In the leading order massless approximation $x_\gamma^{\text{obs}}$ is equivalent to $x_\gamma$.\(^d\)
a clear discrepancy is seen for the full $x_\gamma$-range if both jets are going in the forward direction (positive $\eta_{\text{jet}}$). This is where substantial contributions from resolved processes ($x_\gamma < 0.75$) are expected suggesting that, in the kinematic region of the measurement presented, the available parameterizations of the parton density functions of the photon are insufficient.

6 Conclusion

Measurements of $ep$ jet production rates have been presented in a wide range of the four-momentum transfer $Q^2$ and the transverse jet energy $E_T$ to study the dynamics of the underlying partonic interaction. At large scales $\mu_r = \sqrt{Q^2}$, $E_T$ the data are successfully described by perturbative QCD allowing a determination of the strong coupling constant $\alpha_s$ and the gluon density in the proton. Scale ambiguities have been studied and, as $Q^2$ gets smaller, sensitivity of the NLO predictions to the choice of the renormalization scale is observed. At low $Q^2$ insight into the structure of the photon and effects of non-$k_t$-ordered parton evolution is revealed.

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Figure 10. The dijet cross section for $E_{T,jet} > 14$ GeV as a function of $\eta_{jet}$ in bins of $\eta_{jet}$. The solid data points correspond to the entire $x_{V}^{obs}$ range while the measurement for $x_{V}^{obs} > 0.75$ is shown by the open points. The curves correspond to the NLO predictions and the shaded bands indicate the uncertainties related to the energy scale.

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