Hidden quartic symmetry in $N = 2$ supersymmetry

Rutwig Campoamor-Stursberg†
† I.M.I-U.C.M, Plaza de Ciencias 3, E-28040 Madrid, Spain
E-mail: rutwig@pdi.ucm.es

Michel Rausch de Traubenberg‡
‡IPHC-DRS, UdS, CNRS, IN2P3, 23 rue du Loess, F-67037 Strasbourg Cedex, France
E-mail: Michel.Rausch@IReS.in2p3.fr

Abstract. It is shown that for $N = 2$ supersymmetry a hidden symmetry arises from the hybrid structure of a quartic algebra. The implications for invariant Lagrangians and multiplets are explored.

1. Introduction

The consideration of quadratic algebras — Lie (super-)algebras — largely dominates the algebraic formalism underlying theoretical physics, although non-quadratic structures emerge, in more or less natural form, in various descriptions of physical phenomena. For instance, generalized algebraic approaches are given by the $n$-linear algebras in Quantum Mechanics [1], ternary structures in the description of multiple $M_2$-branes [2, 3] or higher order extensions of the Poincaré algebra [4].

The notion of Lie algebras of order $F > 2$ was first considered and later intensively studied in [4, 5, 6, 7], motivated by the fact that these structures, beyond their formal mathematical properties, constitute a cornerstone for the construction of higher order extensions of the Poincaré algebra. In this context, a specific cubic extension in arbitrary space-time dimension was shown to be of interest in the frame of Quantum Field Theory [8, 9, 10]. Finally, it was realised that it was possible to associate a group [6] and an adapted superspace associated to these structures [11].

Probably, the main technical difficulty related to Lie algebras of order $F$ is their hybrid structure: the “algebra” is partially quadratic and partially of order $F$. In this work, we would like to show that to graded Lie superalgebras of certain form, one can naturally associate a quartic algebra. It is shown that along the lines of this construction, one can associate to $N = 2$ supersymmetry a quartic extension of the Poincaré algebra. This construction indicates that some kind of hidden quartic symmetry appears in usual supersymmetry, which further means that invariant Lagrangians constructed so far are also invariant under the induced quartic structure.

We illustrate the fact that on the top of the representations of supersymmetry, a hierarchy of representations can be constructed. The work presented in this note was obtained in [12] in more detail.
2. Lie algebras of order four – quartic extensions of the Poincaré algebra

There are various types of extensions of Lie algebras that can be considered. The case under inspection here, enabling us to construct non-trivial extensions of the Poincaré algebra, are related to the quartic case, for which we recall the main properties.

The vector space \( \mathfrak{g} = \mathfrak{g}_0 \oplus \mathfrak{g}_1 \) with basis \( \{ X_i, i = 1, \ldots, \dim \mathfrak{g}_0 \}, \{ Y_a, a = 1, \ldots, \dim \mathfrak{g}_1 \} \) is called an elementary Lie algebra of order four if it satisfies the following brackets [4]

\[
\begin{align*}
[X_i, X_j] &= f_{ij}^k X_k, \\
[Y_{a_1}, Y_{a_2}, Y_{a_3}, Y_{a_4}] &= \sum_{\sigma \in S_4} Y_{\sigma(a_1)} Y_{\sigma(a_2)} Y_{\sigma(a_3)} Y_{\sigma(a_4)} = Q_{a_1 a_2 a_3 a_4} i X_i,
\end{align*}
\]

(1)

\( S_4 \) being the permutation group with four elements. In addition, we have also the following generalised Jacobi identities:

\[
\begin{align*}
[Y_{a_1}, \{ Y_{a_2}, Y_{a_3}, Y_{a_4}, Y_{a_5} \}] + [Y_{a_2}, \{ Y_{a_3}, Y_{a_4}, Y_{a_5}, Y_{a_1} \}] + [Y_{a_3}, \{ Y_{a_4}, Y_{a_5}, Y_{a_1}, Y_{a_2} \}] + [Y_{a_4}, \{ Y_{a_5}, Y_{a_1}, Y_{a_2}, Y_{a_3} \}] + [Y_{a_5}, \{ Y_{a_1}, Y_{a_2}, Y_{a_3}, Y_{a_4} \}] &= 0.
\end{align*}
\]

(2)

Let us note that the structure defined by equations (1) and (2) is neither an algebra nor a 4-algebra in the usual sense, but a kind of hybrid structure. Some of the brackets will be quadratic \([\mathfrak{g}_0, \mathfrak{g}_0] \subseteq \mathfrak{g}_0, [\mathfrak{g}_0, \mathfrak{g}_1] \subseteq \mathfrak{g}_1\), while some others will be quartic \(\{ \mathfrak{g}_1, \mathfrak{g}_1, \mathfrak{g}_1, \mathfrak{g}_1 \} \subseteq \mathfrak{g}_0\). This feature obviously generates the question whether from this hybrid structure we can extract some additional properties that cannot be codified either by the binary or quartic structure alone.

In the preceding context, the quartic extensions of the Poincaré algebra in \( D = 4 \) dimensions are realised by means of two Majorana spinors. Using the \( \mathfrak{sl}(2, \mathbb{C}) \cong \mathfrak{so}(1, 3) \) notations of dotted and undotted indices, a left-handed spinor is given by \( \psi^\alpha_L \) and a right-handed spinor by \( \psi_R^\dot{\alpha} \). The spinor conventions to raise/lower indices are the following: \( \psi_L^\alpha = \bar{\epsilon}_{\alpha \beta} \psi_L^\beta, \psi_{R}^{\dot{\alpha}} = \epsilon^{\dot{\alpha} \beta} \psi_{R}^{\beta} \) with \( (\psi_\alpha)^* = \bar{\psi}_\dot{\alpha} \), \( \epsilon_{12} = \epsilon_{1\dot{2}} = 1, \epsilon_{2\dot{1}} = \epsilon_{21} = -1 \). The 4D Dirac matrices, in the Weyl representation, are

\[
\Gamma^\mu = \begin{pmatrix} 0 & \sigma^\mu \\ \bar{\sigma}^\mu & 0 \end{pmatrix},
\]

(3)

with \( \sigma^\mu_{a\dot{a}} = (1, \sigma^i), \bar{\sigma}^\mu_{a\dot{a}} = (1, -\sigma^i), \sigma^i (i = 1, 2, 3) \) being the Pauli matrices. With these notations, we introduce two series of Majorana spinors \( Q_L^i, Q_{\dot{L}}\dot{i} \) satisfying the relation \( (Q_L^i)^\dagger = Q_{\dot{L}}\dot{i} \). The Lie algebra of order four with \( \mathfrak{g}_0 = I \mathfrak{so}(1, 3) \) (the Poincaré algebra) and \( \mathfrak{g}_1 = \langle Q_L^i, Q_{\dot{L}}\dot{i} \rangle \) define the following quartic extension of the Poincaré algebra (we only give the quartic brackets explicitly)

\[
\begin{align*}
\{ Q_{L_1}^{i_1}, Q_{L_2}^{i_2}, Q_{\dot{L}_3}^{i_3}, Q_{\dot{L}_4}^{i_4} \} &= 0, \\
\{ Q_{L_1}^{i_1}, Q_{L_2}^{i_2}, Q_{\dot{L}_3}^{i_3}, Q_{\dot{L}_4}^{i_4} \} &= 2i \left( \delta_{L_1}^{L_2} \epsilon_{L_3}^{L_4} \epsilon_{\alpha_2 \alpha_4} \sigma^\mu_{\alpha_1 \dot{a}_4} + \delta_{L_2}^{L_4} \epsilon_{L_1}^{L_3} \epsilon_{\alpha_3 \alpha_4} \sigma^\mu_{\alpha_2 \dot{a}_4} \right. \\
&\left. + \delta_{L_2}^{L_4} \epsilon_{L_1}^{L_3} \epsilon_{\alpha_1 \alpha_2} \sigma^\mu_{\dot{a}_3 \dot{a}_4} \right) P_\mu, \\
\{ Q_{L_1}^{i_1}, Q_{L_2}^{i_2}, \bar{Q}_{\dot{L}_3}^{\dot{i}_3}, \bar{Q}_{\dot{L}_4}^{\dot{i}_4} \} &= 0,
\end{align*}
\]

(4)

the remaining brackets involving three \( Q \) and one \( Q \) or four \( Q \) being obtained immediately (the tensor \( \epsilon^{ij} \) is defined by \( -\epsilon^{12} = \epsilon^{21} = \epsilon_{12} = -\epsilon_{21} = 1 \)).
As noted previously, the quartic extension of the Poincaré algebra obtained is neither an algebra nor a four-algebra. This feature represents one of the difficulties to handle with these algebraic structures. Consequently one natural question we should address concerns the possibility to associate appropriate quadratic structures to Lie algebras of order four. Its has to be mentioned that a similar analysis has been performed for Lie algebras of order three, were it has been shown that, no possibility to construct an associated quartic structure exists.

3. Quartic structures associated to Lie superalgebras

As mentioned earlier, higher order extensions are no fully satisfactory, in spite of various interesting results derived for them [8, 9, 10]). Thus one may wonder whether or not some quadratic structure should be related to the algebra (4). This question is partially motivated by the fact that some ternary algebras [14] of the Filippov type considered in the Bagger-Lambert-Gustavsson model are equivalent to certain Lie (super-)algebras [2, 13, 14].

We consider the \( \mathbb{Z}_2 \times \mathbb{Z}_2 \)-graded Lie superalgebra

\[
\mathfrak{g} = (\mathfrak{g}(0,0) \oplus \mathfrak{g}(1,1)) \oplus (\mathfrak{g}(1,0) \oplus \mathfrak{g}(0,1)),
\]

where \((a, b) \in \mathbb{Z}_2 \times \mathbb{Z}_2\) and \(\mathfrak{g}(a, b)\) is even (resp. odd) when \(a + b = 0\) mod 2 (resp. \(a + b = 1\) mod 2). Introduce the corresponding bases for the grading blocks:

\[
\begin{align*}
\mathfrak{g}(1,1) &= \{B_i, i = 1, \cdots, \dim \mathfrak{g}(0,0)\}, & \mathfrak{g}(0,0) &= \{Z\}, \\
\mathfrak{g}(1,0) &= \{F^+_a, a = 1, \cdots, \dim \mathfrak{g}(0,1)\}, & \mathfrak{g}(0,1) &= \{F^-_a, a = 1, \cdots, \dim \mathfrak{g}(0,1)\},
\end{align*}
\]

the corresponding commutation relations are

\[
\begin{align*}
[B_i, B_j] &= f_{ij}^{\quad k} B_k, & [B_i, Z] &= 0, \\
[B_i, F^+_a] &= R^+_a B_i F^+_a, & [Z, F^+_a] &= 0, \\
\{F^+_i, F^-_j\} &= Q_{ij}^a B_a, & \{F^+_i, F^+_j\} &= g^+_i Z.
\end{align*}
\]

We mention that, the superalgebra defined by (7) satisfies also appropriate Jacobi identities (that we do not recall here since they will not be relevant for our purpose). It is important to notice that \(\mathfrak{g}(0,0)\) commutes with all remaining factors, in other words, that \(Z\) acts like a central charge.

We now show that to the algebra (7) one can naturally and simply associate a quartic structure which share some similarities with the algebra (1). Indeed, using the obvious relation,

\[
\{A_1, A_2, A_3, A_4\} = \{\{A_1, A_2\}, \{A_3, A_4\}\} + \{\{A_1, A_3\}, \{A_2, A_4\}\} + \{\{A_1, A_4\}, \{A_2, A_3\}\},
\]

the relations

\[
\begin{align*}
\{F^+_1, F^+_2, F^+_3, A_4\} &= \left(g^+_1 a_2 g^+_3 a_4 + g^+_1 a_3 g^+_2 a_4 + g^+_1 a_4 g^+_2 a_3\right) Z^2, \\
\{F^+_1, F^+_2, F^+_3, F^-_4\} &= 2 Z\left(g^+_1 a_2 Q_{a_3 a_4} + g^+_1 a_3 Q_{a_2 a_4} + g^+_1 a_4 Q_{a_3 a_2}\right) B_i, \\
\{F^+_1, F^+_2, F^-_3, F^-_4\} &= \left(Q_{a_1 a_3} Q_{a_2 a_4} + Q_{a_1 a_4} Q_{a_2 a_3}\right) \{B_i, B_j\} + 2 g^+_1 (g^-_{12} g^-_{34} Z^2),
\end{align*}
\]

(plus similar relations involving either three \(F^-\) and one \(F^+\) or four \(F^-\) follow at once.)

Since we are constructing an analogue of the four-Lie algebra (1), we also assume that the algebra is partially quadratic and partially quartic. This means that in addition to the brackets (8), we have also to define the quadratic brackets \([\mathfrak{g}_0, \mathfrak{g}_0] \subseteq \mathfrak{g}_0, [\mathfrak{g}_0, \mathfrak{g}_1] \subseteq \mathfrak{g}_1\). We simply assume that these brackets are the same of the corresponding brackets of the Lie superalgebra. As the quartic brackets are concerned, we observe that \(\{\mathfrak{g}_1, \mathfrak{g}_1, \mathfrak{g}_1, \mathfrak{g}_1\}\) close quadratically in \(\mathfrak{g}_0\). The next
step in the construction is to impose the Jacobi identities (2). This is an extra condition. Indeed, one can show that the Jacobi identity of the Lie superalgebras do not reproduce the generalised Jacobi identity (2). This will not lead to any contradiction since it happens that if we have a finite dimensional representation of (8), the identities (2) are trivially satisfied. Moreover, for the case under inspection in this work this will not be a constraint, since the generalised Jacobi identity will be trivially satisfied as well. This happens because the four-brackets \{g_1, g_1, g_1, g_1\} close upon \(P_\mu\) or \(Z\) (see below) thus we automatically have \([\{g_1, g_1, g_1, g_1\}, g_1] = 0\).

Finally since by construction the relations (8) are just a consequence of (7), this means that the Lie algebras of the form (7) present some hidden quartic symmetry. In some sense we could say that the algebra (8) is the “square” of the algebra (6). Furthermore, the fact that quadratic relations imply quartic relations means that any representation of the Lie superalgebra (6) will also be a (non-faithful) representation of the quartic algebra (8). Of course, the converse is not necessarily true.

We now focus on the relevant case of the \(N = 2\) supersymmetric extension of the Poincaré algebra with central charge, and show that it is of the form (7). Indeed, for the even part, we define \(g_1(1,1) = f_{00}(1,3) = (L_\mu, P_\mu)\) to be the Poincaré algebra in four-dimensions and \(g_1(0,0) = (Z)\) the central charge. Although, for the odd part we introduce two series of Majorana spinors \(Q^I, \bar{Q}_{I\dot{a}}, I = 1, 2\) such that \(g_1(0,0) = \langle Q^I \rangle\) and \(g_1(0,1) = \langle \bar{Q}_{I\dot{a}} \rangle\). Since the \(N = 2\) supersymmetric extension of the Poincaré algebra takes the form

\[
\begin{align*}
\{Q^I, \bar{Q}_{I\dot{a}}\} &= -2i\delta^I_{\dot{a}} \varepsilon^{\mu\nu\rho\sigma} P_{\mu} \\
\{Q^I, Q^J\} &= 2Z \varepsilon_{IJ} \varepsilon_{\alpha\beta} \\
\{\bar{Q}_{I\dot{a}}, \bar{Q}_{J\dot{b}}\} &= -2Z \varepsilon_{IJ} \varepsilon_{\dot{a}\dot{b}},
\end{align*}
\]

which is analogous to (7), the results of the previous section give rise to the four order quartic extension of the Poincaré algebra

\[
\begin{align*}
\{Q^I_{a_1}, Q^I_{a_2}, Q^I_{\dot{a}_3}, Q^I_{\dot{a}_4}\} &= 2Z^2 \left( \varepsilon_{a_1a_2} \varepsilon_{a_3a_4} \varepsilon_{I1} \varepsilon_{I2} \varepsilon_{I3} \varepsilon_{I4} \\
&\quad + \varepsilon_{a_1a_2} \varepsilon_{a_3a_4} \varepsilon_{I1} \varepsilon_{I2} \varepsilon_{I3} + \varepsilon_{a_1a_4} \varepsilon_{a_2a_3} \varepsilon_{I1} \varepsilon_{I2} \varepsilon_{I4} \right),
\end{align*}
\]

\[
\begin{align*}
\{Q^I_{a_1}, Q^I_{a_2}, Q^I_{\dot{a}_3}, \bar{Q}_{\dot{a}_4}\} &= -2iZ \left( \delta^{I1}_{I3} \varepsilon_{I2} \varepsilon_{I4} \varepsilon_{a_3a_4} \sigma_{a_1a_4} + \delta^{I1}_{I4} \varepsilon_{I2} \varepsilon_{I3} \varepsilon_{a_1a_3} \sigma_{a_2a_4} \\
&\quad + \delta^{I1}_{I2} \varepsilon_{I3} \varepsilon_{I4} \varepsilon_{a_1a_3} \sigma_{a_2a_4} \right) P_{\mu},
\end{align*}
\]

\[
\begin{align*}
\{Q^I_{a_1}, Q^I_{a_2}, \bar{Q}_{I\dot{a}_3}, \bar{Q}_{I\dot{a}_4}\} &= 2 \left( \delta^{I1}_{I3} \delta^{I2}_{I4} \delta^{I1}_{I3} \sigma_{a_1a_3} \sigma_{a_2a_4} + \delta^{I1}_{I3} \delta^{I2}_{I4} \varepsilon_{a_1a_3} \sigma^\nu_{a_2a_4} + \delta^{I1}_{I3} \delta^{I2}_{I4} \varepsilon_{a_1a_3} \sigma^\nu_{a_2a_4} \right) P_{\mu} P_{\nu} \\
&\quad + 2Z^2 \varepsilon_{a_1a_2} \varepsilon_{a_3a_4} \varepsilon_{I1} \varepsilon_{I2} \varepsilon_{I3} \varepsilon_{I4}.
\end{align*}
\]

A representation of the super-Poincaré algebra will automatically be a representation of the induced quartic algebra, as shown in the general case. This fact provides us with an interesting consequence, namely, that the invariant \(N = 2\) Lagrangians constructed so far are moreover invariant with respect to the transformations induced by the quartic algebra. Thus, the corresponding \(N = 2\) supermultiplets and their associated transformations laws will automatically be invariant multiplets of the corresponding quartic structure with the same transformation properties. This construction can thus be interpreted, in some sense, as a possibility to circumvent the constraints of the Haag-Lopuszanski-Sohnius theorem [16]. This analogy should however not be pushed too far: the construction of quartic algebras executed in this work depends essentially on the supersymmetric algebra formalism and the associated constraints.
4. Representation of quartic extensions of the Poincaré algebra

As we have seen, the ansatz linking algebras of order four to Lie superalgebras has remarkable consequences concerning their respective representation theories, in the sense that superalgebra representations automatically induce representations of the order four structures. We point out that the converse of this statement is not true. Consider for instance massive representations. The little algebra is generated by $P^0 = -im$ and $Q^{I\alpha}, Q_{I\dot{\alpha}}$ and the four-brackets take the form

$$\{Q_{a_1}, Q_{a_2}, Q_{a_3}, Q_{a_4}\} = 2Z^2(\varepsilon_{a_1a_2}\varepsilon_{a_3a_4}\varepsilon_{I_1I_2}\varepsilon_{I_3I_4}$$
$$+\varepsilon_{a_1a_3}\varepsilon_{a_2a_4}\varepsilon_{I_1I_3}\varepsilon_{I_2I_4} + \varepsilon_{a_2a_3}\varepsilon_{a_1a_4}\varepsilon_{I_1I_4}\varepsilon_{I_2I_3}),$$

$$\{Q^{I_1}, Q^{I_2}, Q^{I_3}, Q^{I_4}\} = 2mZ(\delta_{I_1I_2}\varepsilon_{I_3I_4}\varepsilon_{a_2a_3}\sigma^0_{0a_1a_4} + \delta_{I_2I_4}\varepsilon_{I_1I_3}\varepsilon_{a_1a_3}\sigma^0_{a_2a_4}$$
$$+ \delta_{I_3I_4}\varepsilon_{I_1I_2}\varepsilon_{a_1a_2}\sigma^0_{0a_3a_4}),$$

$$\{Q^{I_1}, Q^{I_2}, Q^{I_3}, Q^{I_4}\} = 2m^2(\delta_{I_1I_3}\delta_{I_2I_4}\sigma^0_{a_1a_3}\sigma^0_{0a_2a_4} + \delta_{I_1I_4}\delta_{I_2I_3}\sigma^0_{a_1a_4}\sigma^0_{0a_2a_3})$$
$$+ 2Z^2\varepsilon_{a_1a_3}\varepsilon_{a_2a_4}\varepsilon_{I_1I_2}\varepsilon_{I_3I_4}.$$

If we now make the following substitutions (analogous to the corresponding substitution for the $N = 2$ supersymmetric extension with central charge):

$$a^1 = Q^1 - Q^{21}, \quad a^3 = Q^1 + Q^{21},$$
$$a^2 = Q^2 + Q_{21}, \quad a^4 = Q^2 - Q_{21},$$

(10)

one observes that $a^1, \ldots, a^4, a^1\dagger, \ldots, a^4\dagger$ generate the Clifford algebra of the polynomial

$$P^2(x_1, \ldots, x_4, y^1, \ldots, y^4) = \left(2(m + Z)x_1y^1 + 2(2m + Z)x_2y^2 + 2(2m - Z)x_3y^3$$
$$+ 2(2m - Z)x_4y^4\right)^2$$

in the sense that

$$\left(x_Ia^I + y_I^a a^I\dagger\right)^4 = P^2(x_1, \ldots, x_4, y^1, \ldots, y^4).$$

(11)

The representations of the $N = 2$ supersymmetric algebra in four dimensions are obtained from the study of representations of the Clifford algebra, i.e. when the $a^I$'s satisfy the quadratic relation,

$$\left(x_Ia^I + y_I^a a^I\dagger\right)^2 = P(x_1, \ldots, x_4, y^1, \ldots, y^4),$$

(12)

which is obviously compatible with (11). On the contrary, one can construct representations of (11) such that the condition (12) is not satisfied.\(^1\) It can be shown that to any polynomial $f$, a Clifford algebra $C_f$ can be associated to it, and that a matrix representation can be obtained [18]. Uniqueness is however lost for degree higher than two, which complicates considerably the analysis of representations (see, for instance, [19]). Being still an unsolved problem, some structural results have been already obtained in the general frame [20].

In the context that occupies us, this procedure may give new representations corresponding to interesting quartic extensions of the Poincaré algebra. The hierarchy of representations on the top of the standard representations obtained in supersymmetric theories might share some similarities with the parafermionic extension of the Poincaré algebra considered in [21].

\(^1\) The algebra (11) is called the Clifford algebra of the polynomial $P^2$ [17].
5. Concluding remarks

We have pointed out the existence of a formal way to associate a quartic algebra (which closes with fully symmetric quartic brackets) to a graded Lie superalgebra of a certain type, their fundamental interest being their application to standard supersymmetric theories. This specifically alludes to the fact that any representation of $N = 2$ supersymmetric algebras shares a hidden symmetry arising from the quartic structure. Further, for massive representations, it turns out that the role of central charge is essential to the argument.

Clearly these results can be generalised to other space-time dimensions [12]. For example, it turns out that that the quartic extensions in ten space-time dimensions is exceptional and related to type IIA supersymmetry. Although we have focused on the case where $g(0,0)$ is one dimensional, there is no reason for restricting to only one central charge. The straightforward generalisation to a higher number of charges is however subjected to finding the appropriate algebraic structures having a physical significance.

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