Properties of doubly charmed baryons in the quark-diquark model

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Abstract

Baryons containing two heavy quarks are important and interesting systems to study the quark-diquark structure of baryons and to understand the dynamics of QCD at hadronic scale. The Selex Collaboration has recently reported the discovery of Ξ^cc with a mass of 3.519 GeV. However, other groups such as Babar, Belle and Focus have all failed to confirm this state. So there is a demand to review this state both theoretically and experimentally. We report here the spectra and magnetic moments of ccq(q ∈ u, d, s) systems using coulomb plus Martin potential. Here the two heavy quarks are considered for the diquark states. The same potential form is used for the diquark interaction as well as for the quark-diquark interaction. The chromomagnetic one gluon exchange interaction are perturbatively treated here to get the masses of \( J^P = \frac{1}{2}^+ \) and \( J^P = \frac{3}{2}^+ \) states. Accordingly, we obtain Ξ^++ (3519, 3555), Ξ^+ (3527, 3562) and Ω^+ (3648,3688) states of the ccq (\( \frac{1}{2}^+, \frac{3}{2}^+ \)) combinations. Our predictions are in accordance with the selex result for Ξ^++ (3519) and other model predictions. Predictions of other properties of these ccq systems will be presented in detail.

Introduction

The first observation of \( B_c^+ \) meson by the CDF collaboration [1] opens a new direction in the physics containing heavy quarks. This particle completes the list of heavy \( Q\bar{Q} \) mesons accessible for the experimental investigations. It begins another list of hadrons containing two heavy quarks. The investigation of properties of hadrons containing heavy quarks is of great interest in understanding the dynamics of QCD at the hadronic scale. Though the experimental and theoretical data on the properties of heavy flavour mesons are available plenty in literature, the masses of most of the heavy baryons have not been measured yet experimentally [2]. Thus the recent predictions about the heavy baryon mass spectrum have become a subject of renewed interest due to the experimental facilities at Belle, BABAR, DELPHI, CLEO, CDF etc [3, 4, 5, 6, 7, 8]. These experimental groups have been successful in discovering heavy baryonic

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states along with other heavy flavour mesonic states and it is expected that more heavy flavour baryon states will be detected in near future. Most of the new states are within the heavy flavour sector with one or more heavy flavour content and some of them are far from most of the theoretical predictions. Though there are consensus among the theoretical predictions on the ground state masses [9, 10], there seemed to have little agreement among the model predictions of the properties like spin-hyperfine splitting among the $J^P = \frac{1}{2}^+$ and $J^P = \frac{3}{2}^+$ baryonic states, the form factors [9], magnetic moments etc [11]. All these reasons make the study of the heavy flavour spectroscopy extremely rich and interesting.

Recently, the SELEX Collaboration [12] has reported the discovery of $\Xi^{++}_{cc}$ with a mass of 3.519 GeV. However, other groups such as BABAR [13], BELLE [14] and FOCUS [15] have all failed to confirm this state. So there is a demand to review this state both theoretically and experimentally. Also, there is renewed interest in the static properties of heavy flavour baryons such as its masses and magnetic moments [16-18, 20-24]. Baryons containing two heavy quarks are important and interesting systems to study the quark-diquark structure of baryons and to understand the dynamics of light quark in the vicinity of heavy quarks. Many of these narrow hadron resonances expected to be observed experimentally, might bring many surprises in QCD spectroscopy [25].

We report here the mass spectra and magnetic moments of $ccq (q \in u, d, s)$ systems using coulomb plus martin potential within the quark-diquark model of the Baryon. Here the two heavy quarks are considered for the diquark states.

**Theoretical framework**

The notion of diquark is as old as the quark model itself. Gell-Mann [26] mentioned the possibility of diquarks in his original paper on quarks. Soon afterwards, Ida and Kobayashi [27] and Lichtenberg and Tassie [28] introduced effective degrees of freedom of diquarks in order to describe baryons as composed of a constituent diquark and quark. Since its introduction, many articles have been written on this subject [29] up to the most recent ones [30]. The presence of a coherent diquark structure within baryons simplifies baryonic calculations by considering diquark as antiquark. By treating diquark as antiquark, the problem of three-body quark excitations within baryons reduces to that of a two-body quark-diquark interaction. In the Present Study of doubly charm baryons, the two charm quarks are considered as the diquark states.

The Hamiltonian of the baryon, in the diquark model, can be written in terms of diquark Hamiltonian plus quark-diquark Hamiltonian as [31]

$$H = H_{jk} + H_{i,jk}$$

(1)
The internal motion of the diquark \((jk)\) is described by
\[
H_d = H_{jk} = \frac{p^2}{2m_{jk}} + V_{jk}(r_{jk})
\]  
where, \(p\) is the relative momentum of the quarks within the diquark. The Hamiltonian of the relative motion of the diquark \((jk)\) and the third quark \((i)\) is
\[
H_{i,d} = H_{i,jk} = \frac{q^2}{2m_{i,jk}} + V_{i,jk}(r_{id})
\]
Here,
\[
m_{jk} = \frac{m_j m_k}{m_j + m_k}, m_{i,jk} = \frac{m_i (m_j + m_k)}{m_1 + m_2 + m_3}
\]
The diquark potential can be written as,
\[
V_{jk} = -\frac{2}{3} \alpha_s \frac{1}{r_{jk}} + b r_{jk}^\nu
\]
and the quark-diquark potential
\[
V_{i,jk} = -\frac{4}{3} \alpha_s \frac{1}{r_{id}} + b r_{id}^\nu
\]
where, \(r_{id}\) is the quark-diquark separation distance, \(\nu\) is a general power corresponding to the confining part of the potential, such that \(\nu = 0.1\) in Eqn (5) represents the coulomb plus Martin potential, \(b\) is model parameter corresponding to the confining part of the potential, which is assumed to be same for the di-quark interaction as well as between the quark-diquark interaction. The running strong coupling constant \((\alpha_s(\mu))\) is computed using the relation
\[
\alpha_s(\mu) = \frac{\alpha_s(\mu_0)}{1 + \frac{33 - 2n_f}{12\pi} \alpha_s(\mu_0) \ln(\frac{\mu}{\mu_0})}
\]
where, \(\alpha_s(\mu_0 = 1GeV) \approx 0.7\) has been used [32]. For the present study, we employ the variational method and the hydrogenic wavefunction given by
\[
R_{nl}(r) = \left(\frac{\mu^3(n-l-1)!}{2n(n+l)!}\right)^{1/2} (\mu r)^l e^{-\mu r} \frac{L_{2l+1}}{L_{n-l-1}} (\mu r)
\]
has been used as the trial wavefunction. Here, \(n, l\) are the quantum numbers, \(\mu\) is variational parameter and \(L_{2l+1}^{n-l-1}(\mu r)\) is Laguerre polynomial. The expectation value of the Hamiltonian described by Eqn (7) provides the binding energy of the baryon as
\[
E(\mu) = \langle H_{jk} \rangle + \langle H_{i,jk} \rangle
\]
Table 1: The quark model parameters

| Quark Masses         | m_u = 0.322 (in GeV) |
|----------------------|-----------------------|
|                      | m_d = 0.336 (in GeV)  |
|                      | m_s = 0.510 (in GeV)  |
|                      | m_c = 1.422 (in GeV)  |

| Model Parameter      | b = 0.197 GeV^{(\nu+1)} |

The spin average mass of baryonic system (i.e. without spin contribution) is then obtained as

\[ M_{QQq} = \Sigma m_i + E(\bar{\mu}) \] (10)

The quark mass parameters and the potential parameters of the model employed in our calculations are listed in Table 1. The spin average masses of \( \Xi_{cc}^{++} \), \( \Xi_{cc}^+ \) and \( \Omega_{cc}^+ \) baryons for different quark-diquark states are also listed in Table 2.

The degeneracy of the states are removed by introducing the spin dependent interaction potential among the diquark (d) as well as among the light quark (l)-diquark \((l - d)\) system given by [33]

\[
\begin{align*}
V_{SD}^{(d)}(r_{jk}) &= \frac{1}{2} \left( \frac{L_d \cdot S_d}{2m_c^2} \right) \left( -\frac{dV(r_{jk})}{r_{jk}dr_{jk}} + \frac{8}{3}\alpha_s \frac{1}{r_{jk}^3} \right) \\
&+ \frac{2}{3}\alpha_s \frac{1}{m_c^2} \frac{L_d \cdot S_d}{r_{jk}^3} \\
&+ \frac{4}{3}\alpha_s \frac{1}{3m_c^2} S_{c1} \cdot S_{c2} [4\pi\delta(r_{jk})] \\
\end{align*}
\] (11)

for the diquark states, and

\[
\begin{align*}
V_{SD}^{(l)}(r) &= \frac{1}{2} \left( \frac{L \cdot S_d}{2m_c^2} + \frac{2L \cdot S_l}{2m_l^2} \right) \left( -\frac{dV(r)}{rdr} + \frac{8}{3}\alpha_s \frac{1}{r^3} \right) \\
&+ \frac{1}{3}\alpha_s \frac{1}{m_c m_l} \frac{L \cdot S_d + 2L \cdot S_l}{r^3} \\
&+ \frac{4}{3}\alpha_s \frac{1}{3m_c^2} (S_d + L_d) \cdot S_l [4\pi\delta(r)] \\
\end{align*}
\] (12)

for the \((l - d)\) system. Where, \( r \) is the relative coordinate of \( l - d \) \((i, jk)\), \( L \) is the relative angular momentum of the \( l - d \) system and \( S_l \) and \( S_d \) are the light quark and diquark spins, respectively. The first term in both expressions takes into account the relativistic corrections to the potential \( V(r) \). The second and third terms are the relativistic corrections coming from the one-gluon exchange between the quarks. The computed masses of \( \Xi_{cc}^{++}, \Xi_{cc}^+ \) and \( \Omega_{cc}^+ \) Baryons with
Table 2: Spin average masses of $\Xi^{++}_{cc}$, $\Xi^{+}_{cc}$ and $\Omega^{+}_{cc}$ baryons.

| $n_d$(diquark)-$n_l$(light-quark) | Present | Others  |
|-----------------------------------|---------|---------|
|                                   |         | [33]    | [10]    |
| 1S 1s                             | 3.544   | 3.560   | 3.673   |
| 1P 1s                             | 3.614   | 3.790   | 3.898   |
| 2S 1s                             | 3.618   | 3.900   | 3.968   |
| 1S 1p                             | 3.648   | 4.030   | 4.077   |
| 1S 2s                             | 3.652   | –       | –       |
| 1P 1p                             | 3.718   | 4.250   | 4.166   |
| 1P 2s                             | 3.722   | –       | –       |
| 2S 1p                             | 3.722   | 4.360   | –       |
| 1S 1s                             | 3.551   | 3.560   | 3.673   |
| 1P 1s                             | 3.621   | 3.790   | 3.898   |
| 2S 1s                             | 3.625   | 3.900   | 3.968   |
| 1S 1p                             | 3.657   | 4.030   | 4.077   |
| 1S 2s                             | 3.661   | –       | –       |
| 1P 1p                             | 3.727   | 4.250   | 4.166   |
| 1P 2s                             | 3.731   | –       | –       |
| 2S 1p                             | 3.731   | 4.360   | –       |
| 1S 1s                             | 3.675   | –       | 3.825   |
| 1P 1s                             | 3.745   | –       | 4.052   |
| 2S 1s                             | 3.749   | –       | 4.124   |
| 1S 1p                             | 3.788   | –       | –       |
| 1S 2s                             | 3.803   | –       | –       |
| 1P 1p                             | 3.858   | –       | –       |
| 1P 2s                             | 3.862   | –       | –       |
| 2S 1p                             | 3.873   | –       | –       |
Table 3: The mass spectrum of Ξ_{cc}^{++}, Ξ_{cc}^{+} and Ω_{cc}^{+} Baryons (Masses in GeV).

| (n_d q_n l)J^P | Present | Others |
|---------------|---------|--------|
|               |         | [33]   | [10]   |
| (Ξ_{cc}^{++}) |         |        |        |
| (1S 1s)1/2^+  | 3.519   | 3.478  | 3.620  |
| (1S 1s)3/2^+  | 3.555   | 3.610  | 3.727  |
| (1P 1s)1/2^-  | 3.586   | 3.702  | 3.838  |
| (2S 1s)1/2^+  | 3.596   | 3.812  | 3.910  |
| (1S 1p)1/2^-  | 3.626   | 3.927  | 4.053  |
| (1P 1s)3/2^-  | 3.628   | 3.834  | 3.959  |
| (2S 1s)3/2^+  | 3.629   | 3.944  | 4.027  |
| (1S 2s)1/2^+  | 3.633   | —      | —      |
| (1S 1p)3/2^-  | 3.635   | 4.039  | 4.101  |
| (1S 1p)1/2'^- | 3.639   | 4.052  | 4.136  |
| (1S 1p)3/2'^- | 3.648   | 4.034  | 4.196  |
| (1S 1p)5/2'^- | 3.659   | 4.047  | 4.155  |
| (1S 2s)3/2^+  | 3.660   | —      | —      |
| (1P 2s)1/2^-  | 3.700   | —      | —      |
| (1P 2s)3/2^-  | 3.733   | —      | —      |
| (Ξ_{cc}^{+})  |         |        |        |
| (1S 1s)1/2^+  | 3.527   | 3.478  | 3.620  |
| (1S 1s)3/2^+  | 3.562   | 3.610  | 3.727  |
| (1P 1s)1/2^-  | 3.594   | 3.702  | 3.838  |
| (2S 1s)1/2^+  | 3.604   | 3.812  | 3.910  |
| (1P 1s)3/2^-  | 3.635   | 3.834  | 3.959  |
| (2S 1s)3/2^+  | 3.636   | 3.944  | 4.027  |
| (1S 1p)1/2^-  | 3.639   | 3.927  | 4.053  |
| (1S 2s)1/2^+  | 3.642   | —      | —      |
| (1S 1p)3/2^-  | 3.644   | 4.039  | 4.101  |
| (1S 1p)1/2'^- | 3.648   | 4.052  | 4.136  |
| (1S 2s)3/2^+  | 3.669   | —      | —      |
| (1P 2s)1/2^-  | 3.709   | —      | —      |
| (1S 1p)3/2'^- | 3.710   | 4.034  | 4.196  |
| (1S 1p)5/2'^- | 3.723   | 4.047  | 4.155  |
| (1P 2s)3/2^-  | 3.742   | —      | —      |
| $(n_d n_d n_d) J^P$ | Present | Others |
|---------------------|---------|---------|
| $(\Omega^{++}_{cc})$ |         |         |
| $(1S 1s) 1/2^+$     | 3.648   | 3.778   |
| $(1S 1s) 3/2^+$     | 3.688   | 3.872   |
| $(1P 1s) 1/2^-$     | 3.715   | 4.002   |
| $(2S 1s) 1/2^+$     | 3.725   | 4.075   |
| $(1P 1s) 3/2^-$     | 3.761   | 4.102   |
| $(2S 1s) 3/2^+$     | 3.762   | 4.174   |
| $(1S 1p) 1/2^-$     | 3.772   | 4.208   |
| $(1S 1p) 3/2^-$     | 3.777   | 4.252   |
| $(1S 1p) 1/2^-$     | 3.781   | 4.271   |
| $(1S 2s) 1/2^+$     | 3.785   |         |
| $(1S 1p) 3/2^-$     | 3.787   | 4.303   |
| $(1S 1p) 5/2^-$     | 3.795   | 4.325   |
| $(1S 2s) 3/2^+$     | 3.811   |         |
| $(1P 2s) 1/2^-$     | 3.852   |         |
| $(1P 2s) 3/2^-$     | 3.884   |         |

Table 4: Magnetic moments of the $\Xi^{++}_{cc}$, $\Xi^+_{cc}$ and $\Omega^+_{cc}$ Baryons in terms of nuclear magneton $\mu_N$.
different combinations of the quark-diquark states are listed in Table 3.

**Magnetic Moments of the Double Heavy Flavour Baryons**

For the computation of the magnetic moments, we consider the mass of bound quarks inside the baryons as its effective mass taking into account of its binding interactions with other two quarks. The effective mass for each of the constituting quark \( m_{i}^{\text{eff}} \) can be defined as \[11\]

\[
m_{i}^{\text{eff}} = m_{i} \left( 1 + \frac{E(\bar{\mu})}{\sum_{i} m_{i}} \right)
\]

such that the corresponding mass of the baryon is given by

\[
M_{B} = \sum_{i} m_{i} + E(\bar{\mu}) = \sum_{i} m_{i}^{\text{eff}}
\]

Now, the magnetic moment of baryons are obtained in terms of the bound quarks as \[11\]

\[
\mu_{B} = \sum_{i} \langle \phi_{sf} | \mu_{i} \vec{\sigma}_{i} | \phi_{sf} \rangle
\]

where

\[
\mu_{i} = \frac{e_{i}}{2m_{i}^{\text{eff}}}
\]

Here, \( e_{i} \) and \( \sigma_{i} \) represents the charge and the spin of the quark constituting the baryonic state. \( \langle \phi_{sf} \rangle \) represents the spin-flavour wave function of the respective baryonic state \[11\]. By using spin flavour wave function corresponds to \( J^{P} = \frac{1}{2}^{+} \) and \( J^{P} = \frac{3}{2}^{+} \) \[11\], we compute the magnetic moments of the baryons containing double charm quarks. Our results are listed in Table 4. Other theoretical model predictions of the magnetic moments are also listed for comparison.

**Results and Discussions**

We have employed a simple nonrelativistic approach with coulomb plus martin potential to study the masses of the double charm baryons in the quark-diquark model. The model parameters are obtained to get the ground state spin average masses of the ccq systems. The mass spectra of \( \Xi_{cc}^{++} \), \( \Xi_{cc}^{+} \), and \( \Omega_{cc}^{+} \) Baryons are listed in Table 3. The extra states predicted in this picture would require experimental verification in support of the quark-diquark structure of the double heavy flavour baryons. In conclusion, our results of doubly charmed baryons are found to be in accordance with other model predictions.

It is important to note that the predictions of the magnetic moment of double heavy Baryons studied here are with no additional parameters. Our results
on magnetic moments are compared with one of our recent predictions using hypercentral potential \[11\] as well as with relativistic (RQM) and the non-relativistic quark model (NRQM) predictions of \[35\] in Table 4. The special feature of the present study to compute the magnetic moments of double heavy flavour baryons is the consideration of the effective interactions of the bound state quarks by defining an effective bound state mass to the quarks within the baryon, which vary according to different interquark potential as well as with quark compositions.

Experimental measurements of the heavy flavour baryon magnetic moments are difficult and only few experimental groups (BTeV and SELEX Collaborations) are expected to do measurements in near future.

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