QUARK-GLUON-PLASMA FIREBALL EVOLUTION WITH ONE LOOP CORRECTION IN THE PESHIER POTENTIAL

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The study of free energy evolution of quark-gluon plasma (QGP) with one loop correction factor in the peshier potential is discussed. The energy evolution with the effect of the correction factor in peshier potential shows the transition temperature obtained in the range of temperature $T = 180 - 250$ MeV. The transition temperature is also affected with the decrease of dynamical flow parameter of quark and gluon used in the potential and it shows the observable QGP droplets of the stable size of fermi radius viz $2.5 - 4.5$ fm.

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I. INTRODUCTION

The study of phase transition [1] from a confined matter of hadrons to a deconfined matter has become an interesting topic in last two decades. During the early stages of the universe formation, there was matter of deconfined quarks and gluons, and in due process of cooling it leads to matter of confined hadrons. The process of such early stage of the universe is indeed explained as a complicated phenomena by the heavy-ion collider experiments. So, the study of quark-gluon plasma (QGP) fireball in Ultra Relativistic Heavy-Ion Collisions becomes an exciting field in the present day of heavy ion collider physics [2]. In this brief paper, we focus on the QGP evolution through the free energy expansion of the system. To evolve the free energy we use the peshier potential with one loop correction to construct the density of states of particles in the system. Thus the free energy evolution is obtained through this density of state. Due to the correction factor in the potential through coupling value [3-5], there is a lot of changes in the free energy expansion of QGP fireball and stability of droplet with the variation of dynamical quark and gluon flow parameters.

In brief, the paper is organized as the construction of density of states with one loop correction in the potential and study the free energy evolution effected by the loop. In conclusion, we give the details of evolution of QGP fireball with differently flow parametrization values of quark and gluon.

II. DENSITY OF STATES FOR QGP WITH ONE LOOP CORRECTION

The potential $V_{\text{conf}}(q)$ is now modified with inclusion of one loop correction factor from simple confining potential. The modified potential is therefore obtained through the expansion of strong coupling constants of one loop factor within the perturbation theory as [6,7]:

$$V_{\text{conf}}(q) = \frac{2\pi}{q} \gamma \alpha_s(q) T^2 [1 + \frac{\alpha(q)}{4\pi} a_1] - \frac{m_0^2}{2q},$$

where

$$\gamma = \sqrt{2} \times \sqrt{(1/\gamma_q)^2 + (1/\gamma_q)^2},$$

called the effective rms value of parametrization factor with $\gamma_q = 1/8$ and $\gamma_q = (8 - 10) \gamma_q$. These factors determine the dynamics of QGP flow and subsequent transformation to hadrons. $\alpha_s(q)$ is the coupling value of quark and gluon with degree of freedom $n_f$, as

$$\alpha_s(q) = \frac{4\pi}{(33 - 2n_f) \ln(1 + q^2/\Lambda^2)}.$$
in which \( \Lambda \) QCD parameter is taken equal to 0.15 GeV. The coefficient \( a_1 \) in the confining potential is the correction factor of one loop connection in their interactions and it is given as [8]:

\[
 a_1 = 2.5833 - 0.2778 \ n_f,
\]

(4)

where \( n_f \) is considered with the number of light quark elements [8,9].

Now the density of states in phase space with loop correction in the peshier potential is obtained through Thomas and Fermi model as [10]:

\[
\int \rho_{q,g} dq = \nu/\pi^2 [-V_{conf}(q)]^2 dV_{conf}/dq,
\]

or,

\[
\rho_{q,g}(q) = \nu/\pi^2 \left[ \frac{\gamma_3 g^3 T^2}{2} \right] g^6(q) A,
\]

(6)

where

\[
A = \left[ 1 + \frac{\alpha_s g a_1}{\pi} \right]^2 \left[ (1 + \frac{\alpha_s g a_1}{\pi}) \right] + \frac{2(1 + 2 \alpha_s g a_1/\pi)}{q^2(q^2 + \Lambda^2) \ln(1 + \frac{q^2}{\Lambda^2})}
\]

(7)

and \( \nu \) is the volume occupied by the QGP and \( q \) is the relativistic four-momentum in natural units and \( g^2(q) = 4\pi \alpha_s(q) \).

### III. THE FREE ENERGY EVOLUTION

The free energy of quarks and gluons is defined in the following with the density of states as [11]:

\[
F_i = \mp T g_i \int dq \rho_{q,g}(q) \ln(1 \pm e^{-(\sqrt{m_i^2 + q^2})/T})
\]

(8)

with low energy cut off as:

\[
V(q_{\text{min}}) = (\gamma_{q,g} N^{1/2} T^2 A^4/2)^{1/4}
\]

(9)

where \( N = (4/3)[12\pi/(33 - 2n_f)] \).

So, the cut off in the model leads to finite integrals by avoiding the infra-red divergence, taking the consideration of the magnitude of \( \Lambda \) and \( T \) as same order of the lattice QCD as the characteristic feature of the free energy assume to be in the same order/similar direction of other models. \( g_i \) is degeneracy factor (color and particle-antiparticle degeneracy) which is 6 for quarks and 8 for gluons and 3 for pions. The interfacial energy obtained through a scalar Weyl-surface in Ramanathan et al. [4,12] with suitable modification to take care of the hydrodynamic effects is given as:

\[
F_{\text{interface}} = \frac{1}{4} \gamma R^2 T^3.
\]

(10)

This energy replaces the bag energy of MIT model as the MIT model produces drawback in the numerical calculations of pressure and energy density. The pion free energy is [13]

\[
F_\pi = (3T/2\nu^2) \nu \int_0^\infty q^2 dq \ln(1 - e^{-\sqrt{m_i^2 + q^2}/T}).
\]

(11)

Thus to calculate all these corresponding energies the particle masses are taken as: quark masses \( m_u = m_d = 0 \text{ MeV} \) and \( m_s = 0.15 \text{ GeV} \) just as taken in Ref.[11] and pion mass as \( m_\pi = 0.14 \text{ GeV} \).

We can thus compute the total modified free energy \( F_{\text{total}} \) as,

\[
F_{\text{total}} = \sum_i F_i + F_{\text{interface}} + F_\pi,
\]

(12)

where \( i \) stands for \( u, d \) and \( s \) quark and gluon.

### IV. RESULTS:

The free energy of the constituent particles of QGP fireball with the one loop correction factor in the peshier potential is numerically calculated. The evolution of QGP-hadron fireball with the modification in the density of states of each particle is explained in the figures. The free energies of the individual particles are shown in figure (1) at a particular temperature \( T = 152 \text{ MeV} \) for the parametrization \( \gamma_q = 1/8, \gamma_g = 12\gamma_0 \) and the energy shows the behavior of QGP-hadron transition.
droplet formation. It means that with the inclusion of one loop correction in the Peshier potential, the free energy of the system is modified with the change in the amplitude and stability of the droplet formation. So the flow parameters have to change its value to reproduce the earlier results [4] and the change in the parameter causes to increase the interaction between the constituent particles showing the decrease in the amplitude of the free energy with these smaller flow parameters.

The plots of various droplet formation for the various flow parameters ranging from $4\gamma_q \leq \gamma_g \leq 8\gamma_q$ for various values of temperature.

FIG. 1: Individual free energy contribution $F_i$ vs. $R$ at $\gamma_q = 1/8$, $\gamma_g = 12\gamma_q$ at the particular temperature $T = 152\ MeV$.

FIG. 2: The free energy vs. $R$ at $\gamma_q = 1/8$, $\gamma_g = 4\gamma_q$ for various values of temperature.

FIG. 3: The free energy vs. $R$ at $\gamma_q = 1/8$, $\gamma_g = 6\gamma_q$ for the various values of temperature.

FIG. 4: The free energy vs. $R$ at $\gamma_q = 1/8$, $\gamma_g = 8\gamma_q$ for the various values of temperature.
The free energy vs. $R$ at $\gamma_q = 1/8$, $\gamma_g = 9\gamma_q$ for the various values of temperature.

The free energy vs. $R$ at $\gamma_q = 1/8$, $\gamma_g = 10\gamma_q$ for various values of temperature.

$\gamma_g \leq 10\gamma_q$ are shown in the figures (2 – 6). In fig. (2), there exists phase transition at the temperatures $T = 190$ MeV with flow parameters $\gamma_q = 1/8$, $\gamma_g = 4\gamma_q$ and the changes of transition temperature is also found up to the temperature $T = 250$ MeV with the increase of gluon parameter $\gamma_g < 8\gamma_q$. In Fig. (3), it shows the unstable droplet formation with the increase of $\gamma_g = 6\gamma_q$ with the decrease of amplitude with temperature. The formation of droplets in this range is also highly unstable till the stable droplet formation. In Figs. (4 – 6) we can easily observe the stability of droplet formation with the flow parameters of $\gamma_q = 1/8$ and $8\gamma_q \leq \gamma_g \leq 10\gamma_q$. The stability is obtained with the different size of droplet and the stable droplets are found in the range $2.5 – 4.5$ fm and its size decreases as the value of the gluon parameter increases. This is obtained with the decrease in the quark and gluon flow parameters.

V. CONCLUSION:

We can conclude from these results that due to the presence of loop correction in the potential the system increase the stability of droplet formation and decrease in the amplitude of free energy with the new flow parameters. So, we can further study the surface tension and thermodynamic properties of QGP on the basis of these smaller droplet size. This means that the results with one loop correction in the peshier potential make the ad-hoc choice of flow parameters most appropriate and will be in agreement with lattice gauge expectation values. This is good outputs in the free energy with smaller quark and gluon parameters over free energy of earlier results.

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