Development of a Modified Newton Iteration Algorithm for massive MIMO systems with precoding and its study in MATLAB environment

E Glushankov, I Boyko, D Kirik and K Korovin
The Bonch-Bruevich St. Petersburg State University of Telecommunications, 22 building 1 Bolshevikoiv Prospekt, St. Petersburg 193232, Russia

Abstract. The aim of this work is to create an iterative precoding algorithm with a higher convergence rate compared to known algorithms. This paper considers the task of antenna system pattern formation using linear precoding for massive MIMO systems, including precoders with approximate matrix inversion methods, such as the use of the Newton Iteration Algorithm. A variant of the Modified Newton Iteration Algorithm with a variable adaptation step is also proposed. A mathematical description of signal processing in the downlink model for massive MIMO systems is given, as well as linear precoding procedures. The convergence of the algorithm has been analyzed from the initial parameters determining the adaptation step, from the number of user equipment served, as well as the angle of user equipment position.

1. Introduction
Massive Multiple Input Multiple Output (MIMO) technology plays an important role in fifth-generation (5G) communications systems. The complexity of massive MIMO systems increases significantly when using a large number of antennas and radio frequency circuits (RF). For such systems, a lot of investigations have been done to find the optimal precoding algorithm with the lowest complexity and the highest convergence rate [1].

Precoding consists of conditioning the signals to the characteristics of the channel by performing a pre-emphasis [2]. For precoding, channel state information (CSI) is used on the transmitting side. If the CSI is known precisely, optimal precoding is performed, but in practice, it is known approximately, so quasi-optimal precoding is performed [3]. In massive MIMO systems with feedback, precoding is a necessary step because it reduces the effect of interference as well as increases the system capacity [4]. In massive MIMO, the CSI estimation is extracted from the uplink pilot signals received from the receiver terminals [5, 6].

The use of iteration algorithms allows obtaining a quite accurate estimate of the precoding matrix and avoiding the time-consuming operation of channel matrix reversal [7].

2. System model
With time division duplex (TDD), the uplink (UL) and downlink (DL) channels operate in the same frequency band but in different time slots. In a massive MIMO scenario, TDD has better performance than frequency division duplex (FDD), because FDD has a satisfying performance only in line-of-sight conditions and at high values of Rice Factor v. At the base station (BS), the structure of the DL precoder
depends on the channel estimate that had been obtained in the previous UL slot. In TDD, channel recoupling is usually assumed [8, 9].

The covariance matrix plays an important role in resource assignment. The covariance matrix is a matrix composed of the pairwise covariances of elements of the input signal vector. In many works on massive MIMO, it is assumed that it is perfectly known, which can lead to incorrect conclusions since the dimensionality of the matrix depends on the number of antennas and other statistics. Large dimensional covariance matrices can be estimated using a sample covariance matrix. The CSI uncertainty can be reduced to the CSI error approximated by a complex symmetric AWGN (Additive White Gaussian Noise) with different power spectral densities, characterizing the magnitude of the corresponding error.

Channel capacity depends on CSI. In [10, 11], the effect of CSI on channel capacity and energy efficiency has been given. Channel energy efficiency is strongly affected by CSI when transmitting over long distances and weakly affected when transmitting over short distances, as in the case of the 5G model.

In the downlink model for massive MIMO, there are $M$ transmitting antennas of one BS that serve $N$ single antennas of user equipment (UE), with $M \geq N$ [12, 13]. Mathematically, the complex channel gains are described by a channel matrix, where the matrix element is the complex gain of the path from the $m$-th antenna of the BS to the $n$-th UE. For TDD, the channel matrix $H$ of dimension $M \times N$ is the same for downlink (DL) and uplink (UL) and is defined as:

$$H = \begin{bmatrix} h_{11} & \cdots & h_{1N} \\ \vdots & \ddots & \vdots \\ h_{M1} & \cdots & h_{MN} \end{bmatrix}, \quad (1)$$

where $h_{mn}$ are the elements of the matrix $H$, which are random variables, having a complex Gaussian distribution, $d = [d_1 \ldots d_N]^T$ is $N$-dimensional vector of input symbols, converted into $M$-dimensional vector of signal $x = [x_1 \ldots x_N]^T$ at the output of the antenna array as a result of precoding operation:

$$x = \sqrt{\rho}Pd, \quad (2)$$

where $P$ is a linear precoding matrix of dimension $M \times N$, $\sqrt{\rho}$ is the average transmitted power. Then the output signal vector is transmitted to DL through the communication channel separately to each of the $N$ UE:

The received vector of signals $y$ at $N$ UE can be expressed as follows:

$$y = H^Tx + n, \quad (3)$$

where $H$ is the channel matrix, $n$ is the $N \times 1$ vector of AWGN.

Assuming that the CSI is known at the BS with the accuracy of the covariance matrices, the precoder can be used to transmit the signal for each of the $N$ UE in the desired direction. As a result of precoding, pre-empahizes are introduced into the vector of the transmitted signal $d$ to compensate for the influence of the channel. Thus, at the input of the UE, it is possible to obtain the received signal vector $y$ with minimal distortion:

$$y = \sqrt{\rho}H^TPd + n. \quad (4)$$

This is possible if the precoding matrix is an inverse channel matrix. The main method of precoding is to perform a matrix inversion operation, which leads to high computational complexity, especially when $M \gg N$.
\[ P = \sqrt{\beta} H^* G^{-1}, \]  

where \( \sqrt{\beta} \) is average gain, \( G = H^T H^* \) is Gramm matrix.

3. Iteration algorithm

A larger number of elements \( M \) of the antenna array at BS in comparison with the number of UE \( N \) leads to the fact that the matrix \( G^{-1} \) tends to a diagonal matrix, in which the diagonal elements are close to \( M \), and the non-diagonal elements turn to zero [14, 15]. Another disadvantage of the algorithms is the operation of matrix inversion, which leads to increased computational complexity. In addition, when estimating a matrix based on training samples, the estimated sample matrix can become ill-conditioned if the number of sample elements is limited [16]. Consequently, it is reasonable to use iteration algorithms to find the inverse Graham matrix \( G^{-1} \) instead of direct inversion of the matrix.

Newton Iteration (NI) algorithm allows us to obtain an approximate but fairly accurate estimate of the inverse matrix \( G^{-1} \). The recurrence formula for this algorithm is given below:

\[
\tilde{G}^{-1} = X_k = X_{k-1} (2I - GX_{k-1}), \quad (6)
\]

\[
G^{-1} = \lim_{k \to \infty} X_k \text{ where } \|I - GX_0\| < 1, \quad (7)
\]

where \( I \) is a unit matrix of dimension \( N \times N \), \( X_0 \) is the initial rough estimate. NI algorithm requires a significant number of iterations for convergence.

Determining the optimal initial conditions, in this case, is complicated and requires additional calculations. Without adopting optimal initial conditions, NI algorithm takes quite a long time to converge.

4. Modified Newton Iteration Algorithm

Let us propose a Modified Newton Iteration (MNI) algorithm to solve the task. To do this, we introduce an adaptation step, that gradually decreases with each iteration, which allows providing a higher rate of convergence of the iteration algorithm without setting optimal initial conditions.

First, let us consider the NI algorithm. For this purpose, the adaptation step \( \lambda \) links the current and previous values of the estimate of the inverse matrix \( X \).

\[
\tilde{G}^{-1} = X_k = X_{k-1} (2I - GX_{k-1}) = (\lambda + 1)X_{k-1} - \lambda X_{k-1} GX_{k-1}, \lambda = 1. \quad (8)
\]

\[
\tilde{G}^{-1} = X_k = X_{k-1} [(\lambda + 1)I - \lambda GX_{k-1}], \lambda = 1. \quad (9)
\]

To increase the convergence rate of the algorithm, the adaptation step can be increased, but this leads to an increase in the estimation error in the later iteration steps. To reduce the estimation error of the inverse matrix, it is necessary that the adaptation step gradually decreases with each iteration to provide the best approximation of the estimate to the real value. Hence, it is necessary to use a variable adaptation step \( \lambda_k \) in the NI algorithm:

\[
\tilde{G}^{-1} = X_k = X_{k-1} [(\lambda_k + 1)I - \lambda_k GX_{k-1}], \quad (10)
\]

where \( \lambda_k \) gradually decreases with each iteration:

\[
\lambda_k = \lambda_{k-1} (1 - \epsilon), \quad (11)
\]

where \( \epsilon \) is the step reduction factor, determining the reduction rate of the adaptation step \( \lambda_k \).
The limit value will be \( \lambda_0 = 0 \), when the values are not updated (after convergence of the algorithm). Thus, the matrix convergence in the adaptation algorithm is realized in a finite number of iterations.

5. Simulation results

In the proposed Modified Newton Iteration algorithm the convergence rate is determined by two parameters: the initial value of the adaptation step \( \lambda_0 \) and the step reduction factor \( \varepsilon \). The initial value of the adaptation step determines the value of the convergence step, the larger \( \lambda_0 \), is, the faster the algorithm converges, but at a very large value of \( \lambda_0 \), the convergence of the algorithm is not ensured. The reduction factor \( \varepsilon \) determines the rate of the adaptation step reduction. The greater \( \varepsilon \), the longer the convergence lasts (the longer the tail of the convergence characteristic). Figures 1-2 shows the results of analysis of the convergence of the precoding matrix of the modified Newton algorithm at different initial values of the adaptation step \( \lambda_0 \) and coefficient \( \varepsilon \) at \( M = 8, N = 3 \).

Plots of the convergence of the precoding matrix and the convergence of the input signal vector \( \mathbf{y} \) to the transmitted signal vector \( \mathbf{d} \) of the algorithms from the user signal arrival angles are shown in Figures 3 – 6.

To simplify the proposed algorithm, the optimal value of the coefficient \( \varepsilon \) providing the fastest convergence of the algorithm can be estimated as \( 1/\lambda_0 \). In this case, the convergence of the algorithm depends on only one parameter – \( \lambda_0 \).

For an antenna array consisting of 8 elements, we obtain the results of the convergence of the precoding matrix with the maximum possible value of \( \lambda_0 \) at different numbers of users (Figure 7). With a larger value of the adaptation step, the convergence of the algorithm is not confirmed.

![Figure 1](image1.png)

**Figure 1.** Results of convergence analysis of the MNI algorithm at different initial values of the adaptation step \( \lambda_0 \) and a fixed value of reduction factor \( \varepsilon \)

![Figure 2](image2.png)

**Figure 2.** Results of convergence analysis of the MNI algorithm at different values of reduction factor \( \varepsilon \) and a fixed initial value of the adaptation step \( \lambda_0 \)

![Figure 3](image3.png)

**Figure 3.** Convergence results of the precoding matrix at UE angles \([-45; 0; 45]\)

![Figure 4](image4.png)

**Figure 4.** Convergence results of the precoding matrix at UE angles \([-30; 0; 30]\)

![Figure 5](image5.png)

**Figure 5.** Convergence results of the precoding matrix at UE angles \([-10; 0; 10]\)

![Figure 6](image6.png)

**Figure 6.** Convergence results of the precoding matrix at UE angles \([-5; 0; 5]\)
Figure 7. Convergence results of the precoding matrix with the maximum possible value of $\lambda_0$ for different numbers of UE.

To avoid divergence of the algorithm, it is more appropriate to use the adaptation step value equal to the maximum possible value at the minimum number of the served UE. Thus, the convergence of the algorithm does not depend on the number of UE. Figure 8 shows the results of the convergence of the modified algorithm in the case of one served user at different values of the adaptation step corresponding to the maximum possible values at 1, 5, and 7 users.

Figure 8. Convergence results of the precoding matrix for one UE with the different values of $\lambda_0$.

The convergence results of the algorithm for one served UE with different initial values of the adaptation step corresponding to the maximum possible, with 1, 5 and 7 users, differ only by one iteration. Thus, reducing the initial value of the adaptation step to the maximum possible value with the maximum number of UE has little effect on the convergence of the algorithm with the minimum number of UE.

6. Summary

In this paper, the main iteration linear precoding algorithms are considered, and a new Modified Newton Iteration algorithm is proposed, which allows increasing the algorithm's convergence rate and reducing its calculation complexity. The introduction of a variable adaptation step in Newton Iteration algorithm makes it possible to increase the algorithm's convergence rate. The size of the adaptation step is determined by two parameters: the initial value of the step, and the reduction factor.

By changing the parameters of the modified Newton algorithm, it is possible to adjust the convergence characteristic of the algorithm. The greater the initial value of the adaptation step $\lambda_0$, the lower the error level at the first iteration steps, however, if the value is large enough, the stability of the algorithm can be disturbed. The larger the value of coefficient $\epsilon$, the longer the convergence time of the algorithm. Using the optimal coefficient estimation $\epsilon = 1/\lambda_0$, the convergence of the modified algorithm will depend only on the initial value of the adaptation step $\lambda_0$.

It is recommended to choose the initial value of the adaptation step equal to the maximum possible value with one serviced user equipment.
Acknowledgments
This research was funded by the Ministry of Digital Development, Communications and Mass Media of the Russian Federation, contract number P33-1-26/7 (Moscow, Russia).

References
[1] Albreem M A, Habbash A H A, Abu-Hudrouss A M and Ikki S S 2021 Overview of precoding techniques for massive MIMO IEEE Access 9 60764–801
[2] Oesges C and Clerckx B 2013 MIMO wireless communications. Channels, techniques and standards for multi-antenna, multi-user and multi-cell systems p 776 (Oxford: Academic Press)
[3] Kreindelin V, Smirnov A and Rejeb T B 2018 Effective precoding and demodulation techniques for 5G communication systems Moscow 2018 Systems of Signals Generating and Processing in the Field of on Board Communications 14–15 1–6
[4] Luo F L and Zhang C J 2016 Signal Processing for 5G: Algorithms and Implementations 577–81 (John Wiley & Sons)
[5] Renzo D M, Haas H, Ghrayeb A, Sugiura S and Hanzo L 2014 Spatial modulation for generalized MIMO: challenges, opportunities, and implementation Proceedings of the IEEE 102 56-103
[6] Elijah O, Leow C Y, Rahman T A, Nunoo S and Iliya S Z 2016 A comprehensive survey of pilot contamination in massive MIMO–5G System Communications Surveys & Tutorials vol 18 no 2 pp 905-23
[7] Kreindelin V B, Pankratov D Y and Stepanova A G 2018 Analysis of iterative demodulation algorithm for mimo system with different number of antennas WECONF 1–4
[8] Albreem M A M 2015 5G wireless communication systems: Vision and challenges I4CT 493–97
[9] Pillai S S, Dhanya S and Jeemon B K 2017 Performance comparison of multicast MIMO systems employing spatial modulation and coded spatial modulation in fading channels ICICICT 1529–33
[10] Hu F, Wang K, Huo H and Jin L 2018 An adaptive energy consumption optimization method based on channel correlation information in massive mimo systems ICEIEC 158–61
[11] Rosas F and Oberli C 2015 Impact of the channel state information on the energy-efficiency of MIMO communications IEEE Transactions on Wireless Communications 14 4156–69
[12] McCloud M L, Scharf L L and Varanasi M K 2003 Beamforming, diversity, and interference rejection for multiuser communication over fading channels with a receive antenna array IEEE Transactions on Communications 51 116-24
[13] Kim S 2020 Transmit antenna selection for precoding-aided spatial modulation IEEE Access 8 40723-31
[14] Grigoriev V A 2016 Adaptive antenna arrays. Part 2 81-170 (St. Petersburg: ITMO University)
[15] Brown A D 2017 Electronically scanned arrays MATLAB® modeling and simulation (CRC Press)
[16] Mendlovic D, and Lohmann A W 1997 Space–bandwidth product adaptation and its application to superresolution: fundamentals JOSA A 14.3 558-62