Finite element discretizations of problems in computational physics often rely on adaptive mesh refinement (AMR) to preferentially resolve regions containing important features during simulation. However, these spatial refinement strategies are often heuristic and rely on domain-specific knowledge or trial-and-error. We treat the process of adaptive mesh refinement as a local, sequential decision-making problem under incomplete information, formulating AMR as a partially observable Markov decision process. Using a deep reinforcement learning approach, we train policy networks for AMR strategy directly from numerical simulation. The training process does not require an exact solution or a high-fidelity ground truth to the partial differential equation at hand, nor does it require a pre-computed training dataset. The local nature of our reinforcement learning formulation allows the policy network to be trained inexpensively on much smaller problems than those on which they are deployed. The methodology is not specific to any particular partial differential equation, problem dimension, or numerical discretization, and can flexibly incorporate diverse problem physics. To that end, we apply the approach to a diverse set of partial differential equations, using a variety of high-order discontinuous Galerkin and hybridizable discontinuous Galerkin finite element discretizations. We show that the resultant deep reinforcement learning policies are competitive with common AMR heuristics, generalize well across problem classes, and strike a favorable balance between accuracy and cost such that they often lead to a higher accuracy per problem degree of freedom.

Keywords: Reinforcement learning, adaptive mesh refinement, finite element methods, computational fluid dynamics.

1 Introduction

In recent decades, the finite element community has developed principled, efficient techniques for solving partial differential equations (PDEs). Not only are these techniques extremely general methods that may be applied to almost any PDE, they also provide guarantees such as stability, consistency, and convergence [13, 31]. On the other hand, the machine learning community has developed a broad set of methods to learn latent patterns from large datasets in the absence of a model [9, 36, 44]. However, for problems in computational physics, it is often the case that the PDE is an excellent model of the underlying physical phenomena, often down to the molecular level, where the continuum assumption begins to lose validity. Attempts to use machine learning to learn solutions to PDEs directly have shown promise for relatively trivial problems, but are as of yet subject to several failure modes in terms of generalization to even moderately more complicated problems [40]. Rather than ignoring the extensive body of work in either field, we propose combining techniques from numerical mathematics and machine learning in order to preserve mathematically hard-earned guarantees while improving accuracy and efficiency by applying machine learning to the peripheral aspects of numerical methods which lack a model and rely purely on heuristics. Adaptive mesh refinement is one such aspect.