Kinematic model of a cow milking manipulator with simultaneous positioning at four given points

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Abstract. During the study, the authors have solved the problems associated with the kinematics of a manipulator intended for the automatic care of cattle. The working unit of the manipulator is a system positioning all four milking cups in the individual location of the udder teats. A significant advantage of this manipulator kinematics is the ability to simultaneously connect milking cups to the teats of a cow's udder to provide physiologically more correct milking and reduce the total milking time.

1. Introduction

Industrial robots are widely used in manufacturing and in science. As a rule, industrial robots are represented by automatic programmable manipulators that perform complex operations of spatial moving [1-4]. These robots are used in hazardous or high-precision manufacturing. They are used in light and heavy industry, and the list of robotic systems used in agriculture has been recently expanding. For example, large dairy farming companies are widely using milking robots, self-propelled robots for distributing and transporting feeds [5-10]. The most technologically advanced solutions are milking robots, as they perform a number of operations, such as locating the teats, their pre-cleaning with brushes or washing in the milking cup, feeding, weighing an animal, one-by-one fitting teat cups over cow’s teats, individual milking each quarter of the udder, assessing qualitative indicators of milk, and successive disengaging teat cups. This robotic system contributes to complete replacement of human labor and improved product quality, as well as ensures preventive protection against teat diseases resulting from dry milking [11-14]. The most expensive and complex part of the milking robot is the system for manipulating the working units, and a feedback system for spatial orientation required for manipulations [15]. But even if this part is the most complex and expensive, it does not provide the maximum completeness of milking, since the cups are connected to the teats in turn, which weakens the response to the irritation phase of the teat receptors to stimulate the maximum return of alveolar milk, which reduces complete emptying of the alveoli. Delays during the attachment of teat cups also reduce the ability to fully utilize the neurohumoral phase of milk transfer, since oxytocin is secreted only once and enters the udder 35-50 seconds after the first teat has been stimulated, by this time existing milking robots are only finishing fitting the last teat cup over the teat still containing a portion of cistern milk [16,17].
2. Development of robotic manipulator
To solve the problem of simultaneous fitting teat cups over a cow’s teats, the authors have developed a robotic manipulator shown in figure 1.

![Figure 1. View of the robotic manipulator of a milking unit.](image)

The design presented above was developed based on the kinematics of the previously designed manipulator [18], supplemented with a module that provides the ability to control milking cups in a horizontal plane shown in figure 1 B. And then the authors solved the direct and inverse kinematics problems and tested them in the boundary conditions.

3. Direct kinematic problem
The core difficulty of finding a solution to kinematic problems lies in the manipulator links, which are located in the 0xy plane, the kinematic diagram of which is shown in figure 2. Solving the direct kinematic problem involves finding the positioning coordinates of the working units. In the kinematic diagram, the working units are indicated by the points K(x₁,y₁), L(x₂,y₂), M(x₃,y₃) and N(x₄,y₄).

![Figure 2. Kinematic diagram of the manipulator robot of a milking machine, the links of which are located in the 0XY plane.](image)

A system of equations was compiled to find a solution to the direct kinematic problem, presented below:
The system of equations (1), consisting of 8 equations and 8 unknowns, describes the direct kinematic problem.

4. Inverse kinematic problem

The inverse kinematic problem involves finding the angles $\alpha_1 - \alpha_4$ and distances $S_1 - S_5$ from known positioning points – K(x1,y1), L(x2,y2), M(x3,y3) and N(x4,y4) (see figure 2).

There can be numerous solutions to the inverse kinematic problem, which must be limited by boundary conditions. Below is the derivation of a system of equations that analytically describes the kinematic diagram shown in figure 2 and is intended to search for a unique solution according to given boundary conditions.

5. Boundary condition for solving the inverse kinematic problem

Figure 3 shows the working unit of the manipulator.

The boundary condition for finding a solution to the inverse kinematic problem is that the distance $S_1 = S_2$ and $S_3 = S_4$. In other words, the point C will be equidistant from the points K(x1,y1) and L(x2,y2) in the same way as the point D will be equidistant from M(x3,y3) and N(x4,y4). Based on the above, the following system of equations is valid:

\[
\begin{align*}
\begin{cases}
x_1 = |AA_1| + |BB_1| + |CC_1| + |KK_1|, \\
y_1 = |AA_1| + |AB_1| + |BC_1| + |CK_1|, \\
x_2 = |AA_2| + |BB_2| + |CC_2| + |LL_2|, \\
y_2 = |AA_2| + |AB_2| + |BC_2| + |CL_2|, \\
x_3 = |AA_3| + |BB_3| + |DD_3| + |MM_3|, \\
y_3 = |AA_3| + |AB_3| + |BD_3| + |DM_3|, \\
x_4 = |AA_4| + |BB_4| + |DD_4| + |NN_4|, \\
y_4 = |AA_4| + |AB_4| + |BD_4| + |DN_4|,
\end{cases}
\end{align*}
\] (1)
\[
\begin{align*}
\alpha_1 &= \frac{\pi}{2} - \arccos \left[ \frac{L_1^2 + x_c^2 + y_c^2 - (L_2 + S_1)^2}{2 \cdot L_1 \cdot (x_c^2 + y_c^2)^{1/2}} \right] - \arccos \left[ \frac{x_c}{(x_c^2 + y_c^2)^{1/2}} \right], \\
\alpha_2 &= \arccos \left[ \frac{L_2^2 + (L_2 + S_1)^2 - x_c^2 - y_c^2}{2 \cdot L_2 \cdot (L_2 + S_1)} \right], \\
\alpha_3 &= \frac{\pi}{2} - \arccos \left( \frac{S_2^2 + (L_3 - S_2)^2 - (x_d - x_i)^2 - (y_d - y_i)^2}{2 \cdot S_2 \cdot (L_3 - S_2)} \right), \\
\alpha_4 &= \frac{\pi}{2} - \arccos \left[ \frac{S_2^2 + (L_3 - S_2)^2 - (x_c - x_i)^2 - (y_c - y_i)^2}{2 \cdot S_2 \cdot (L_3 - S_2)} \right].
\end{align*}
\] (2)

\[
\begin{align*}
(x_2 - x_1)^2 + (y_2 - y_1)^2 &= 4 \cdot S_1^2, \\
(x_1 - x_1)^2 + (y_1 - y_1)^2 &= 4 \cdot S_1^2, \\
(x_4 - x_3)^2 + (y_4 - y_3)^2 &= 4 \cdot S_1^2, \\
(x_1 - x_1)^2 + (y_1 - y_1)^2 &= 4 \cdot S_1^2, \\
4 \cdot (L_3 - S_2)^2 &= (x_1 + x_2 - x_i - x_4)^2 + (y_1 + y_2 - y_i - y_4)^2.
\end{align*}
\] (3)

To determine the desired quantities \( S_1 - S_5 \), it is necessary to solve the system of equations:

\[
\begin{align*}
(x_c - x_1)^2 + (y_c - y_1)^2 &= S_1^2, \\
(x_c - x_2)^2 + (y_c - y_2)^2 &= S_2^2, \\
(x_d - x_3)^2 + (y_d - y_3)^2 &= S_3^2, \\
(x_1 - x_3)^2 + (y_1 - y_3)^2 &= S_4^2, \\
4 \cdot (L_3 - S_2)^2 &= (x_1 + x_2 - x_i - x_4)^2 + (y_1 + y_2 - y_i - y_4)^2.
\end{align*}
\]

To solve the system of nonlinear equations (2) we can use the approximate Newton method.

6. Checking kinematics by boundary conditions

To verify the direct and inverse kinematic problems, studies on boundary conditions were carried out:

A. \( \alpha_1 = \pi/2, \alpha_2 = \pi, \alpha_3 = \alpha_4 = 0, L_1 = L_2 = 0.5 \text{ m}, L_3 = 0.4 \text{ m}, S_1 = S_2 = S_3 = S_4 = S_5 = 0.1 \text{ m} \). In this case, the manipulator will be operating along the abscissa axis over its full length equal to the total length of all the links \( L_1 + L_2 + L_3 \). Figure 4A shows a graphical representation of the manipulator in the \( 0XY \) plane.

B. \( \alpha_1 = 0, \alpha_2 = \pi, \alpha_3 = \alpha_4 = 0, L_1 = L_2 = 0.5 \text{ m}, L_3 = 0.4 \text{ m}, S_1 = S_2 = S_3 = S_4 = S_5 = 0.1 \text{ m} \). In this case, the manipulator will be operating along the ordinate axis over its full length equal to the total length of all the links \( L_1 + L_2 + L_3 \). Figure 4B shows a graphical representation of the manipulator in the \( 0XY \) plane.

C. \( \alpha_1 = 0, \alpha_2 = \pi/2, \alpha_3 = \alpha_4 = 0, L_1 = L_2 = 0.5 \text{ m}, L_3 = 0.4 \text{ m}, S_1 = S_2 = S_3 = S_4 = S_5 = 0.1 \text{ m} \). In this case, the manipulator will be operating along the ordinate axis by the length \( L_1 \) and along the abscissa axis by the length \( L_2 + L_3 \). Figure 4C shows a graphical representation of the manipulator in the \( 0XY \) plane.

D. \( \alpha_1 = \pi/4, \alpha_2 = \pi, \alpha_3 = \alpha_4 = 0, L_1 = L_2 = 0.5 \text{ m}, L_3 = 0.4 \text{ m}, S_1 = S_2 = S_3 = S_4 = S_5 = 0.1 \text{ m} \). In this case, the manipulator will be operating at an angle of 45 degrees to the abscissa axis and ordinate axis to its entire length equal to the total length of all the links \( L_1 + L_2 + L_3 \). Figure 4D shows a graphical representation of the manipulator in the \( 0XY \) plane.
Figure 4. Graphical display of the manipulator in the OXY plane for performing a direct task.

7. Conclusions

The study has offered solutions to the direct and inverse kinematic problems. When solving the inverse kinematic problem by the Newton method to achieve a correction of 0.001 m, the number of approximations did not exceed 10. The obtained reachability zone of the manipulator is symmetrical with respect to the second link.

The issues discussed in this paper will significantly simplify the search for control algorithms for the manipulator of the milking unit. With the help of the developed program, it is possible to study the kinematics of similar design schemes with various dimensions.

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