Correspondence between Ricci and other dark energies

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Purpose of the present paper is to view the correspondence between Ricci and other dark energies. We have considered the Ricci dark energy in presence of dark matter in non-interacting situation. Subsequently, we have derived the pressure and energy density for Ricci dark energy. The equation of state parameter has been generated from these pressure and energy density. Next, we have considered the correspondence between Ricci and other dark energy models, namely tachyonic field, DBI-essence and new agegraphic dark energy without any interaction and investigated possible cosmological consequences.

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I. INTRODUCTION

An exotic form of negative pressure matter called dark energy is used to explain the acceleration of the universe inferred from the the observations of distant type Ia supernovae, cosmic microwave background radiation (CMBR), and Sloan Digital Sky Survey (SDSS) [1]. The simplest candidate of dark energy is the vacuum energy density or cosmological constant \( \Lambda \), whose energy density remains constant with time \( \rho_\Lambda = \Lambda / 8\pi G \) and whose equation of motion is also fixed, \( w = p_\Lambda / \rho_\Lambda = -1 \) (\( p_\Lambda \) is the pressure) during the evolution of the universe. The resulting cosmological model, \( \Lambda CDM \), consists of a mixture of vacuum energy and cold dark matter. Another possibility is QCDM cosmologies based on a mixture of cold dark matter and quintessence (\( -1 < w < 0 \) [2]). To alleviate the ‘fine tuning’ and ‘cosmological coincidence’ problems [3] associated with the \( \Lambda CDM \), various dark energy candidates have been proposed such as quintessence mentioned, k-essence, tachyons, phantoms, ghost condensates and quintom, etc. An important advancement in the studies of black hole theory and string theory is the suggestion of the so-called holographic principle, which may provide some clues for solving these problems. Gao et al [4] has discussed how the holographic dark energy model deals with the two fundamental problems mentioned above. This holographic dark energy model and its interacting versions are successful in fitting the current observations [5]. Inspired by the holographic dark energy models,Gao et al [4] proposed another possibility, where the density is proportional to the Ricci scalar curvature \( R \) and named this dark energy as Ricci dark energy (RDE). The Ricci scalar of FRW universe is given by \( R = -6(\dot{H} + 2H^2 + \frac{k}{a^2}) \), where \( \dot{H} \) denotes a derivative with respect to time \( t \) and \( k \) is the spatial curvature.

The purpose of the present work is to investigate the correspondence between Ricci dark energy and other dark energy candidates namely tachyonic field [6], DBI-essence [7] and new agegraphic dark energy [8]. Although the RDE is a recently introduced dark energy model, a literature survey shows that some significant works have been already done in RDE, which can be regarded as a kind of holographic dark energy with the square root of the inverse Ricci scalar as its infrared cutoff [9]. The RDE is given by its density [4]

\[
\rho_R = 3c^2 \left( \dot{H} + 2H^2 + \frac{k}{a^2} \right)
\]

where \( c \) is a dimensionless parameter which will determine the evolution behavior of RDE. When \( c^2 < 1/2 \), the RDE will exhibit a quintomlike behavior; i.e., its equation of state will evolve across the cosmological-constant boundary \( w = -1 \) [4]. In the work by Zhang under reference [4], \( c^2 \) was taken as 0.3, 0.4, 0.5, and 0.6 and it was observed that for \( c^2 < 1/2 \) the equation of state parameter \( w < -1 \), i.e. behaves like quintom and for

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it has been shown that $w < -1$, i.e. behaves like a quintessence. In flat FRW universe, where $k = 0$, the RDE will be given by

$$\rho_R = 3c^2(\dot{H} + 2H^2)$$

(2)

For a flat FRW universe the Hubble parameter is given by $H = \dot{a}/a$, where $a$, a function of the cosmic time $t$, is the scale factor. Some recent advances in RDE are the references in [10]. In our work, we would consider the RDE with dark matter and would investigate its association with the other dark energies.

**II. RICCI DARK ENERGY MODEL WITH DARK MATTER**

In this section, we include the dark matter without interaction with RDE. The conservation equations for RDE and dark matter are written as

$$\dot{\rho}_R + 3H(\rho_R + p_R) = 0; \quad \dot{\rho}_m + 3H(\rho_m + p_m) = 0$$

(3)

Solving the conservation equation for RDE we get

$$p_R = -c^2\left(\frac{\ddot{H}}{H} + 7\dot{H} + 6H^2\right)$$

(4)

and for dark matter we introduce a constant equation of state parameter $w_m$ so that

$$p_m = w_m\rho_m$$

(5)

As we are considering a mixture of the RDE and dark matter we write

$$\rho_1 = \rho_R + \rho_m$$

$$p_1 = p_R + p_m$$

(6)

For the mixture of RDE and dark matter without interaction, the conservation equation would take the form

$$\dot{\rho}_1 + 3H(1 + w_1)\rho_1 = 0$$

(7)

where, $w_1 = \frac{\rho_R + \rho_m}{\rho_R + \rho_m}$.

Using the conservation equation for dark matter and the equation (5) we get

$$\rho_m = \rho_{om}a^{-(1 + w_m)}$$

(8)

where the subscript 0 denotes the value of $\rho_m$ at present time (zero redshift). Using the expression for $w_1$ in the conservation equation (7) we can rewrite the Friedman equation as follows

$$H^2 = \frac{1}{3(2 + (w_m - 4)c^2)}\left[3a^{\frac{c^2}{2}}(w_m - 4)c^2 + 2a^{-w_m}c^2\rho_{om}\right]$$

(9)

$$\rho_1 = \frac{1}{3(2 + (w_m - 4)c^2)}\left[18a^{\frac{c^2}{2}}(w_m - 4)c^2 + 12a^4\rho_{om}a^{-w_m} + 18\left(6a^{\frac{c^2}{2}}c2(1 - 2c^2)^2(2 + (w_m - 4)) + c^6a^{-w_m}w_m\rho_{om}\right)\right]$$

(10)
III. CORRESPONDECE BETWEEN RICCI DARK ENERGY MODEL AND TACHYONIC FIELD

The present section aims to investigate the conditions under which there is a correspondence between RDE model and the tachyonic field, in the flat FRW Universe. That is, to determine an appropriate potential for tachyonic field which makes the two dark energies to coincide with each other. Let us first consider the energy density $\rho$ and the pressure $p$ for the tachyonic field [12]

$$\rho_T = \frac{V(\phi)}{\sqrt{1 - \dot{\phi}^2}}$$

(12)

and

$$p_T = -V(\phi)\sqrt{1 - \dot{\phi}^2}$$

(13)

for which the equation of state reads as

$$w_T = -(1 - \dot{\phi}^2)$$

(14)

Equation $w_T$ with the RDE equation of state $w_1$ we reconstruct the scalar field and potentials for the tachyonic field as
Figs. 2 and 3 show the variation of $\phi$ against $a$ and $V$ against $\phi$ when a correspondence between RDE and tachyonic field is considered. Here, $c^2 = 0.6$, $\rho_{om} = 0.01$, and $w_m = 0.01$.

$$
\dot{\phi}^2 = \frac{3a^2 - \frac{2}{c^2}(c^2(w_m - 4) + 2)}{6c^2 + c^2(3c_1 w_m - 12c_1 + 2\rho_{om} a^{-4+w_m})} \left[ 1 + (3(c_1 c^2 - 2)(c^2(w_m - 4) + 2) - \frac{1}{3} a^4 e^8(w_m - 4)(w_m - 3)\rho_{om}) \right] \\
\times \left( 18a^{2+w_m} c^4 c_1 (c^2(w_m - 4) + 2) + 12a^4 e^8 \rho_{om} 3(6a^{2+w_m} c^2 c_1 (c^2(w_m - 4) + 2) + a^4 e^6 w_m^2 \rho_{om}) \times \right.
\left. \sqrt{c_1 a^{-4+w_m} + \frac{2c^2 \rho_{om} a^{-4+w_m}}{3(c^2(w_m-4)+2)}} \right]^{-1}
$$

and

$$
V = \frac{3c_1 (c^2(w_m - 4) + 2)a^{-4+w_m + \frac{2}{c^2}(c^2(w_m - 4) + 2)^2}}{c^2(w_m-4)+2} \left[ 1 - 3a^2 + w_m (c^2(w_m - 4) + 2) + \\
\times \left( \frac{9a^{2+w_m} c^4 c_1 (c^2(w_m - 4) + 2)^2}{18a^{2+w_m} c^4 c_1 (c^2(w_m - 4) + 2) + 12a^4 e^6 \rho_{om} + 3(6a^{2+w_m} c^2 c_1 (c^2(w_m - 4) + 2) + a^4 e^6 w_m^2 \rho_{om}) \times} \right. \right.
\left. \times \sqrt{c_1 a^{-4+w_m} + \frac{2c^2 \rho_{om} a^{-4+w_m}}{3(c^2(w_m-4)+2)}} \right]^{\frac{1}{2}}
$$

When the field $\phi$ is plotted against the scale factor $a$, it is observed that the field is increasing with increase in $a$. The plot is presented in figure 2. When the potential $V$ is plotted against the scalar field $\phi$, it is found that the potential is decreasing with increase in the field $\phi$ as seen in figure 3. In figure 4, we have seen that the potential of the tachyonic field is decreasing with increase in the scale factor $a$. This indicates that as the universe evolves, the potential of the tachyonic field decreases when it is considered as Ricci dark energy.

IV. CORRESPONDENCE BETWEEN RICCI DARK ENERGY MODEL AND DBI ESSENCE

The density and pressure for DBI-essence are read as [13].
Fig. 4 shows the variation of the potential $V$ against $a$ when a correspondence between RDE and tachyonic field is considered.

$$\rho = (\gamma - 1)T(\phi) + V(\phi) \tag{17}$$

$$p = \left(\frac{\gamma - 1}{\gamma}\right)T(\phi) - V(\phi) \tag{18}$$

where, $T(\phi)$ is the tension, $V(\phi)$ is the potential, $\phi$ denotes the scalar field for DBI-essence and the quantity $\gamma$ is reminiscent of the usual Lorentz factor given by

$$\gamma = \frac{1}{\sqrt{1 - \frac{\dot{\phi}^2}{T(\phi)}}} \tag{19}$$

We assume that $T = n\dot{\phi}^2$, and we find that

$$\dot{\phi}^2 = 2 \left(\frac{n - 1}{n}\right) \left(\frac{(2c^2 - 1)c_1 a^{-4 + \frac{2}{n}}}{c^2} + \frac{c^2 w_m \rho_{om} a^{-w_m}}{3(c^2(w_m - 4) + 2)}\right) \tag{20}$$

$$V = 2 \left(\frac{\sqrt{n(n - 1) - w_m}}{\sqrt{n - 1}}\left(\frac{n - 1}{n}\right) \left(\frac{(2c^2 - 1)c_1 a^{-4 + \frac{2}{n}}}{c^2} + \frac{c^2 w_m \rho_{om} a^{-w_m}}{3(c^2(w_m - 4) + 2)}\right) \tag{21}$$

Where, $w_1$ is given by equation (13). It is observed in figure 5 that the scalar field of DBI essence is increasing with scale factor $a$ when DBI essence is considered as Ricci dark energy. In figure 6 it is seen that the potential of DBI essence in such case is decreasing with increase in the scalar field $\phi$. The decrease of the DBI essence potential with evolution of the universe is apparent in figure 7.

\section{V. Correspondence between Ricci Dark Energy Model and New Agegraphic Dark Energy}

Quantum mechanics together with general relativity leads to the Karolyhazy relation and a corresponding energy density of quantum fluctuations of space-time. Based on this energy density, Cai [14] proposed a dark
Figs. 5 and 6 show the variation of $\phi$ and $V$ against $a$ when DBI essence is considered as a Ricci dark energy.

Fig. 7 shows the variation of the potential $V$ against $a$ when DBI essence is considered as a Ricci dark energy.

energy model, the so-called agegraphic dark energy model, in which the age of the universe is introduced as the length measure. The corresponding energy density is given by

$$\rho_q = \frac{3n^2m_p^2}{T^2} \quad (22)$$

where,
Figs. 8 and 9 show the evolution of $\eta$ and $w$ with increase in the scale factor $a$ when new agegraphic dark energy is considered as a Ricci dark energy.

\[ T = \int \frac{da}{Ha} \]  

(23)

The agegraphic dark energy was constrained by using some old high redshift objects and type Ia supernovae. A new agegraphic dark energy model was proposed in [15], where the time scale is chosen as the conformal time $\eta$ instead of the age of the universe. For this new agegraphic dark energy, the energy density $\rho_A$ is given as

\[ \rho_A = \frac{3n^2m_p^2}{\eta^2} \]  

(24)

where

\[ \eta = \int \frac{dt}{a} \]  

(25)

Thus, $\dot{\eta} = 1/a$. The corresponding equation of state is

\[ w = -1 + \frac{2}{3n} \sqrt{\Omega_A a} \]  

(26)

where,

\[ \Omega_A = \frac{n^2}{H^2\eta^2} \]  

(27)

Taking the form of the Hubble parameter $H$ as in equation (9) and $w$ as $w_1 = p_1/\rho_1$ we get

\[ \eta = \frac{3aH}{2}(1 + rw_1) \]  

(28)

Evolution of $\eta$ and $w$ are plotted against $a$ in figures 8 and 9 respectively. It is observed that $\eta$ is decreasing with increase in the scale factor $a$. However, the equation of state parameter of the new agegraphic dark energy is increasing with the evolution of the universe when it is considered as Ricci dark energy.
VI. CONCLUSION

In this work we first studied the evolution of the equation of state parameter for the Ricci dark energy in presence of dark matter in non-interaction. We observed that depending on the value of $c^2$ it evolves across the cosmological-constant boundary. Subsequently, we studied the correspondence between Ricci dark energy and tachyonic field. In a non-interacting situation, we reconstructed the potential and scalar field for the tachyonic field considering it as Ricci dark energy. Plotting the potential against the scale factor $a$ and the scalar field $\phi$ we observed that as the universe evolves, the potential of the tachyonic field decreases when it is considered as Ricci dark energy. Next we reconstructed the potential and scalar field of the DBI-essence considering it as Ricci dark energy. In this case, increase of the scalar field and decrease of the potential was observed with the evolution of universe. Considering the new agegraphic dark energy as Ricci dark energy we reconstructed the equation of state parameter of new agegraphic dark energy and it has been observed that the equation of state parameter of the new agegraphic dark energy is increasing with the evolution of the universe when it is considered as Ricci dark energy.

References:

[1] A.G. Riess, et al., Astron. J. 116 1009 (1998); S. Perlmutter, et al., Astrophys. J. 517 565 (1999); D.N. Spergel, et al., Astrophys. J. Suppl. 170 377(2007); J.K. Adelman-McCarthy, et al., arXiv: 0707.3413.
[2] R. R. Caldwell, R. Dave, and P. J. Steinhardt, Phys. Rev. Lett. 80 1582 (1998).
[3] S. Weinberg, in Sources and Detection of Dark Matter and Dark Energy in the Universe, edited by D. B. Cline (Springer, New York, 2001), pp. 1826 (2000).
[4] C. Gao, F. Wu, X. Chen and Y-G Shen, Phys. Rev. D 79 043511 (2009)arXiv:0712.1394v4 [astro-ph]; X. Zhang, Phys. Rev. D 79 103509 (2009).
[5] S. Chattopadhyay and U. Debnath, Astrophys Space Sci 319 183 (2009); J. Zhang, X. Zhang, and H. Liu, Eur. Phys. C 52 693 (2007); M. R. Setare, J. Zhang, and X. Zhang, J. Cosmol. Astropart. Phys. 03 (2007) 007.
[6] A. Sen, JHEP 065 0207.
[7] J. Martin and M. Yamaguchi, Phys. Rev. D D 77 123508 (2008).
[8] M. Setare, Astrophys. Space Sci DOI: 10.1007/s10509-009-0214-4 (2009).
[9] C. Feng and X. Li, Phys. Lett. B 680 355 (2009).
[10] C.J. Feng, arXiv:0806.0673 [hep-th]; C.J. Feng, Phys. Lett. B 670 231 (2008) ; C.J. Feng, Phys. Lett. B 672 94 (2009) ; C.J. Feng, Phys. Lett. B 676 168 (2009); C.J. Feng, X. Zhang, arXiv:0904.0045 [gr-qc]; C.J. Feng, X.-Z. Li, arXiv:0904.2972 [hep-th]; L.N. Granda, A. Oliveros, Phys. Lett. B 671 199 (2009) .
[11] M. R. Setare and J. Sadeghi, Int J Theor Phys 47 3219 (2008).
[12] T. Padmanabhan, Current Science 88 1057 (2005).
[13] J. Martin and M. Yamaguchi, Phys. Rev. D 77 123508 (2008).
[14] R. G. Cai, Phys. Lett. B 657 228 (2007).
[15] H. Wei and R. G. Cai, Phys. Lett. B 660 113 (2008).