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SOME RESULTS ON THE JANOWSKI’S STARLIKE FUNCTIONS OF COMPLEX ORDER

By : YASAR POLATOGLU AND METIN BOLCAL

Department of Mathematics of Faculty of Science and Arts Istanbul Kltr University

ABSTRACT : The purpose of this paper is to give exact value of the radius of starlikeness and distortion theorem, Koebe domain for the class of Janowski’s starlike functions of complex order. We note that the class of Janowski’s starlike functions of complex order contain many interesting subclasses of univalent functions.

INTRODUCTION : Let Ω be the family of functions ω(z) regular in the unit disc $D = \{ z | |z| < 1 \}$ and satisfying the conditions $\omega(0) = 0$, $|\omega(z)| < 1$ for $z \in D$.

Next for arbitrary fixed numbers $A$ and $B$, $-1 < A \leq 1$, $-1 \leq B < A$, denote by $P(A, B)$ the family of functions

$$p(z) = 1 + p_1 z + p_2 z^2 + \cdots$$

regular in $D$ such that $p(z)$ in $P(A, B)$ if and only if

$$p(z) = \frac{1 + A\omega(z)}{1 + B\omega(z)}$$

for some functions $\omega(z) \in \Omega$ and every $z \in D$. (This class was introduced by Janowski [9]).

Moreover let $S^*(A, B, b) \ (b \neq 0, complex)$ denote the family of functions.

$$f(z) = z + a_2 z^2 + a_3 z^3 + \cdots$$

regular in $D$ and such that $f(z) \in S^*(A, B, b)$ if and only if

$$\left[ 1 + \frac{1}{b} \left( \frac{f'(z)}{f(z)} - 1 \right) \right] = p(z) \quad , \quad (b \neq 0, complex)$$

for some $p(z)$ in $P(A, B)$ and all $z$ in $D$.

Finally, we use $p$ denote the class of functions

$$p_1(z) = 1 + c_1 z + c_2 z^2 + \cdots$$
which are analytic in $D$ and have a positive real part in $D$.

It is to be note that special selections $A, B$ and $b$ lead to familiar sets of univalent functions. Therefore the sets of univalent functions are listed by the following

1) $S^*(1,-1,1)$ is the class of starlike functions (well known class) [1,2,6].
2) $S^*(1,-1,b)$ is the class of starlike functions of complex order. Introduced by Wiatarowski [7].
3) $S^*(1,-1,1-\beta),\ 0 \leq \beta < 1$, is the class of starlike functions of order $\beta$. This class was introduced by Robertson [5].
4) $S^*(1,-1,e^{-i\lambda}\cos\lambda),\ |\lambda| < \frac{\pi}{2}$ is the class of $\lambda$-spirallike functions introduced by Spacek [3].
5) $S^*(1,-1,(1-\beta)e^{-i\lambda}\cos\lambda),\ 0 \leq \beta < 1,\ |\lambda| < \frac{\pi}{2}$ is the class of $\lambda$-spirallike functions of order $\beta$. This class was introduced by Libera [8].

The expression \[
\left[1 + \frac{1}{p}(z\frac{f'(z)}{f(z)} - 1)\right]
\]
is denoted by $ST(b)$, then

6) $S^*(1,0,b)$ is the set defined by $|ST(b) - 1| < 1$.
7) $S^*(\beta,0,b)$ is the set defined by $|ST(b) - 1| < \beta,\ 0 \leq \beta < 1$.
8) $S^*(\beta,-\beta,b)$ is the set defined by $\left|\frac{ST(b)-1}{ST(b)+1}\right| < \beta,\ 0 \leq \beta < 1$.
9) $S^*(1,(-1+\frac{1}{M}),b)$ is the set defined by $|ST(b) - M| < M$.
10) $S^*(1-2\beta,-1,b)$ is the set defined by $ReST(b) > \beta,\ 0 \leq \beta < 1$.

II. THE RADIUS OF STARLIKENESS FOR THE CLASS $S^*(A,B,b)$

From the definition of the class of $S^*(A,B,b)$ we easily obtain the following lemma.

**Lemma 2.1.** Let $f(z) \in S^*(A,B,b)$, then

\[
\left|\frac{f'(z)}{f(z)} - \frac{1 - \left[B^2 - b(AB - B^2)\right]r^2}{1 - B^2r^2}\right| \leq \frac{|b|(A-B)r}{1 - B^2r^2}.
\]

**Proof:** Let $p(z) \in P(A,B)$ then

\[
(2.1) \quad \left|p(z) - \frac{1 - ABr^2}{1 - B^2r^2}\right| \leq (A-B)r \frac{1 - B^2r^2}{1 - B^2r^2}.
\]

The relation (2.1) was proved by Janowski [9]. Therefore from the definition of the class
$S^*(A, B, b)$ we can write

\[ (2.2) \quad \left| \left[ 1 + \frac{1}{b} \left( z \frac{f'(z)}{f(z)} - 1 \right) \right] - \frac{1 - ABr^2}{1 - B^2r^2} \right| \leq \frac{(A - B)r}{1 - B^2r^2}. \]

After the simple calculations from the relation (2.2) we obtain the desired result of the lemma.

**THEOREM 2.1.** The radius of starlikeness of the class $S^*(A, B, b)$ is

\[ r_s = \frac{2}{|b|(A - B) + \sqrt{|b|^2(A - B)^2 + 4(B^2 + (AB - B^2)Re b)}}. \]

This radius is sharp. Because the extremal function is

\[ f_*(z) = \begin{cases} z(1+Bz) \frac{b(A-B)}{b} & B \neq 0 \\ e^{bAz} & B = 0 \end{cases} \]

\[ z = \frac{r \left( r - \frac{B}{b} \right)^{1/2}}{1 - r \left( \frac{B}{b} \right)^{1/2}} \]

**Proof:** From the Lemma 2.1. we have

\[ (2.3) \quad Re z \frac{f'(z)}{f(z)} \geq \frac{1 - |b|(A - B)r - |B^2 + (AB - B^2)|r^2}{1 - B^2r^2}. \]

Hence for $r < r_s$ the first side of the preceding inequality is positive, this implies that

\[ r_s = \frac{2}{|b|(A - B) + \sqrt{|b|^2(A - B)^2 + 4(B^2 + (AB - B^2)Re b)}}. \]

Also note that the inequality (2.3) become an equality for the function $f_*(z)$.

**I.** For $A = 1$ , $B = -1$,

\[ r_s = \frac{1}{|b| + \sqrt{|b|^2 - 2Re b + 1}}. \]

This is the radius of starlikeness for the class of starlike functions of complex order was obtained by M.A.Nasr and M.K.Aouf [4].
In this case;
If we give the special values to \( b \), we obtain the radius of starlikeness for the corresponding classes. These radius had been found the authors [1],[3],[5],[8],[9].

II. For \( A = 1 \), \( B = 0 \),
\[
r_s = \frac{1}{|b|}.
\]

In this case;
Under the conditions \(|\lambda| < \frac{\pi}{2} \), \( 0 \leq \alpha < 1 \).
(i) \( b = 1 \) \( r_s = 1 \),  
(ii) \( b = 1 - \alpha \) \( r_s = \frac{1}{1 - \alpha} \),  
(iii) \( b = e^{-i\lambda} \cdot \cos \lambda \) \( r_s = \frac{1}{\cos \lambda} \),  
(iv) \( b = (1 - \alpha) e^{-i\lambda} \cdot \cos \lambda \) \( r_s = \frac{1}{(1 - \alpha) \cos \lambda} \)

III. For \( A = \beta \), \( B = 0 \), \( 0 \leq \beta < 1 \).
\[
r_s = \frac{1}{|\beta| |b|}.
\]

IV. For \( A = \beta \), \( B = -1 \), \( 0 \leq \beta < 1 \).
\[
r_s = \frac{1}{\beta |b| + \sqrt{|b|^2 - 2 \text{Re} \ b + 1}}.
\]

In this case;
Under the conditions \(|\lambda| < \frac{\pi}{2} \), \( 0 \leq \alpha < 1 \).
(i) \( b = 1 \) \( r_s = \frac{1}{\beta} \),  
(ii) \( b = (1 - \alpha) \) \( r_s = \frac{1}{\beta(1 - \alpha)} \),  
(iii) \( b = e^{-i\lambda} \cdot \cos \lambda \) \( r_s = \frac{1}{\beta \cos \lambda} \),  
(iv) \( b = (1 - \alpha) e^{-i\lambda} \cdot \cos \lambda \) \( r_s = \frac{1}{\beta(1 - \alpha) \cos \lambda} \)

IV. For \( A = \beta \), \( B = -\beta \), \( 0 \leq \beta < 1 \).
\[
r_s = \frac{1}{\beta |b| + \sqrt{|b|^2 - 2 \text{Re} \ b + 1}}.
\]

In this case ;
(i) \( b = 1 \) \( r_s = \frac{1}{\beta} \),  
(ii) \( b = (1 - \alpha) \) \( r_s = \frac{1}{\beta(1 + 2\alpha)} \),  
(iii) \( b = e^{-i\lambda} \cdot \cos \lambda \) \( r_s = \frac{1}{\beta[\cos \lambda + |\sin \lambda|]} \),  
(iv) \( b = (1 - \alpha) e^{-i\lambda} \cdot \cos \lambda \) \( r_s = \frac{1}{\beta[(1 - \alpha) \cos \lambda + \sqrt{1 - (1 - \alpha^2) \cos \lambda}]} \).

V. For \( A = 1 - 2\beta \), \( B = -1 \).
\[
r_s = \frac{1}{(1 - \beta)|b| + \sqrt{1 + 2(1 - \beta) \text{Re} \ b + |b|^2 \beta^2}}.
\]
In this case :

(i) \( b = 1 \) \( r_s = \frac{1}{(1-\beta)+\sqrt{\beta^2-2\beta+3}} \), (ii) \( b = (1-\alpha) \) \( r_s = \frac{1}{2(1-\alpha)(1-\beta)+1} \),

(iii) \( b = e^{-i\lambda} \cdot \cos \lambda \) \( r_s = \frac{1}{(1-\beta)\cos \lambda + \sqrt{1+(1-\alpha)^2(1-\beta)^2+2(1-\alpha)(1-\beta)\cos^2 \lambda}} \),

(iv) \( b = (1-\alpha)e^{-i\lambda} \cdot \cos \lambda \) \( r_s = \frac{1}{(1-\beta)(1-\alpha)\cos ^2 \lambda + \sqrt{1+(1-\alpha)^2(1-\beta)^2+2(1-\alpha)(1-\beta)\cos^2 \lambda}} \).

VI. For \( A = -1, B = (\frac{1}{M} - 1) \).

\[ r_s = \frac{1}{|b|(2 - \frac{1}{M}) + \sqrt{|b|^2(2 - \frac{1}{M})^2 + 4\frac{1}{M}(\frac{1}{M} - 1)\Re b + 1}} \]

In this case :

(i) \( b = 1 \) \( r_s = \frac{1}{(2-\frac{1}{M})^2 + \frac{1}{M^2} - \frac{1}{M} + 1} \)

(ii) \( b = (1-\alpha) \) \( r_s = \frac{1}{(1-\alpha)(2 - \frac{1}{M}) + \sqrt{(1-\alpha)^2(2 - \frac{1}{M})^2 + 4\frac{1}{M}(\frac{1}{M} - 1)(1-\alpha) + 1}} \)

(iii) \( b = e^{-i\lambda} \cdot \cos \lambda \) \( r_s = \frac{1}{(2 - \frac{1}{M})(1-\alpha)\cos \lambda + \sqrt{(\frac{1}{M^2} - \frac{8}{M} + 4)\cos ^2 \lambda + 1}} \)

(iv) \( b = (1-\alpha)e^{-i\lambda} \cdot \cos \lambda \) \( r_s = \frac{1}{(2 - \frac{1}{M})(1-\alpha)\cos \lambda + \sqrt{(\frac{1}{M^2} - \frac{8}{M} + 4)(1-\alpha)^2\cos ^2 \lambda + 1}} \)

III. THE ESTIMATION OF \( |f(z)| \) IN \( S^*(A, B, b) \)

In this section we shall give the estimation of \( |f(z)| \) and the Koebe domain for the class of \( S^*(A, B, -b) \).

**THEOREM 3.1.** If \( f(z) \in S^*(A, B, b) \), then

\[ F(r; -A, -B, |b|) \leq |f(z)| \leq F(r; A, B, |b|) \]

where

\[ F(r; A, B, |b|) = \begin{cases} \frac{r(1+Br)\frac{|b|(A-B)}{B}}{\Re |b|^2 Ar} & \text{if } B \neq 0 ; \\ \Re |b|^2 Ar & \text{if } B = 0. \end{cases} \]

This bound are sharp. Because the extremal function is

\[ f_*(z) = \begin{cases} z(1+Bz)\frac{b(A-B)}{B} & \text{if } B \neq 0 ; \\ z & \text{if } B = 0. \end{cases} \]

**Proof :** Since \( f(z) \in S^*(A, B, b) \) we have

\[ 1 + \frac{1}{b}(z\frac{f'(z)}{f(z)} - 1) = p(z) , \quad p(z) \in P(A, B) \]
and simple calculations from the equality (3.1) we obtain.

\[ f(z) = z \exp \left( \int_0^z \frac{b(p(\xi) - 1)}{z} \, d\xi \right) \]

Therefore

\[ |f(z)| = |z| \exp \left( \text{Re} \left( \int_0^z \frac{b(p(\xi) - 1)}{\xi} \, d\xi \right) \right) \tag{3.3} \]

substituting \( \xi = zt \), we obtain

\[ |f(z)| = |z| \exp \left( \text{Re} \left( \int_0^1 \frac{b(p(zt) - 1)}{t} \, dt \right) \right) \tag{3.4} \]

On the other hand from lemma 2.1 it follows that

\[ \max_{|zt| = rt} \left( \frac{b(p(zt) - 1)}{t} \right) = \frac{|b|(A - B)r}{1 + Br}; \tag{3.5} \]

then after integration we obtain the upper bounds in (3.1) similarly obtain the bounds on the left-hand side of (3.1), which shows that the proof of the theorem is complete.

\[ \text{(I)} \text{For } A = 1 \quad B = -1 \]

\[ \frac{r}{(1 + r)^2|b|} \leq |f(z)| \leq \frac{r}{(1 - r)^2|b|}. \]

This is the distortion for the class of starlike functions of complex order.

\[ \text{In this case;} \]

\[ \text{(i) For } b = 1; \]

\[ \frac{r}{(1 + r)^2} \leq |f(z)| \leq \frac{r}{(1 - r)^2}. \]

this is the distortion for the class of starlike functions \( \text{ (This is well known result, A.W.Goodman, univalent functions vol1,page 140) } \)

\[ \text{(ii) For } b = (1 - \alpha), \quad 0 \leq \alpha < 1; \]

\[ \frac{r}{(1 + r)^2(1-\alpha)} \leq |f(z)| \leq \frac{r}{(1 - r)^2(1-\alpha)}. \]
This result is the distortion for the class of starlike function of order \( \alpha \). This result was obtained by M.S. Robertson [5].

(iii) For \( b = e^{-i\lambda}\cos \lambda, \quad |\lambda| < \frac{\pi}{2} \):

\[
\frac{r}{(1 + r)^2 \cos \lambda} \leq |f(z)| \leq \frac{r}{(1 - r)^2 \cos \lambda}.
\]

This is the distortion for the class of \( \lambda \)-spirallike functions.

(iv) For \( b = (1 - \alpha)e^{-i\lambda}\cos \lambda, \quad |\lambda| < \frac{\pi}{2}, \quad 0 \leq \alpha < 1 \):

\[
\frac{r}{(1 + r)^2(1 - \alpha) \cos \lambda} \leq |f(z)| \leq \frac{r}{(1 - r)^2(1 - \alpha) \cos \lambda}.
\]

This is the distortion for the class of \( \lambda \)-spirallike functions of order \( \alpha \).

(II) For \( A = \beta, \quad B = -\beta \):

\[
\frac{r}{(1 + \beta r)^2 |b|} \leq |f(z)| \leq \frac{r}{(1 - \beta r)^2 |b|}.
\]

In this case:

(i) For \( b = 1 \):

\[
\frac{r}{(1 + \beta r)^2} \leq |f(z)| \leq \frac{r}{(1 - \beta r)^2}.
\]

(ii) For \( b = (1 - \alpha), \quad 0 \leq \alpha < 1 \):

\[
\frac{r}{(1 + \beta r)^2(1 - \alpha)} \leq |f(z)| \leq \frac{r}{(1 - \beta r)^2(1 - \alpha)}.
\]

(iii) For \( b = e^{-i\lambda}\cos \lambda, \quad |\lambda| < \frac{\pi}{2} \):

\[
\frac{r}{(1 + \beta r)^2 \cos \lambda} \leq |f(z)| \leq \frac{r}{(1 - \beta r)^2 \cos \lambda}.
\]

(iv) For \( b = (1 - \alpha)e^{-i\lambda}\cos \lambda, \quad |\lambda| < \frac{\pi}{2}, \quad 0 \leq \alpha < 1 \):

\[
\frac{r}{(1 + \beta r)^2(1 - \alpha) \cos \lambda} \leq |f(z)| \leq \frac{r}{(1 - \beta r)^2(1 - \alpha) \cos \lambda}.
\]

(III) For \( A = 1, \quad B = 0 \):

\[
\frac{r}{e^{|b|r}} \leq |f(z)| \leq re^{|b|r}.
\]
In this case;

(i) For $b = 1$;
\[ \frac{r}{e^r} \leq |f(z)| \leq re^r \]

(ii) For $b = (1 - \alpha)$;
\[ \frac{r}{e^{(1-\alpha)r}} \leq |f(z)| \leq re^{(1-\alpha)r} \]

(iii) For $b = e^{-i\lambda}\cos \lambda$, $|\lambda| < \frac{\pi}{2}$;
\[ \frac{r}{e^{r\cos \lambda}} \leq |f(z)| \leq re^{r\cos \lambda} \]

(iv) For $b = (1 - \alpha)e^{-i\lambda}\cos \lambda$, $0 \leq \alpha < 1$, $|\lambda| < \frac{\pi}{2}$;
\[ \frac{r}{e^{(1-\alpha)r\cos \lambda}} \leq |f(z)| \leq re^{(1-\alpha)r\cos \lambda} \]

(IV) For $A = \beta$, $B = 0$
\[ re^{-(b|\beta| r)} \leq |f(z)| \leq re^{b|\beta| r} \]

In this case;

(i) For $b = 1$;
\[ re^{-\beta r} \leq |f(z)| \leq re^{\beta r} \]

(ii) For $b = (1 - \alpha)$, $0 \leq \alpha < 1$;
\[ re^{-(1-\alpha)\beta r} \leq |f(z)| \leq re^{(1-\alpha)\beta r} \]

(iii) For $b = e^{-i\lambda}\cos \lambda$, $|\lambda| < \frac{\pi}{2}$;
\[ re^{-(\beta \cos \lambda) r} \leq |f(z)| \leq re^{(\beta \cos \lambda) r} \]

(iv) For $b = (1 - \alpha)e^{-i\lambda}\cos \lambda$, $0 \leq \alpha < 1$, $|\lambda| < \frac{\pi}{2}$;
\[ re^{-(\beta(1-\alpha)\cos \lambda) r} \leq |f(z)| \leq re^{(\beta(1-\alpha)\cos \lambda) r} \]
(V) For $A = 1 - 2\beta$, $B = -1$

$$\frac{r}{(1 + r)^{2|b|\beta}} \leq |f(z)| \leq \frac{r}{(1 - r)^{2|b|\beta}}.$$ 

In this case;

(i) For $b = 1$;

$$\frac{r}{(1 + r)^{2(1 - \beta)}} \leq |f(z)| \leq \frac{r}{(1 - r)^{2(1 - \beta)}}.$$ 

(ii) For $b = (1 - \alpha)$;

$$\frac{r}{(1 + r)^{2(1 - \alpha)(1 - \beta)}} \leq |f(z)| \leq \frac{r}{(1 - r)^{2(1 - \alpha)(1 - \beta)}}.$$ 

(iii) For $b = e^{-i\lambda}\cos\lambda$;

$$\frac{r}{(1 + r)^{2(1 - \beta)\cos\lambda}} \leq |f(z)| \leq \frac{r}{(1 - r)^{2(1 - \beta)\cos\lambda}}.$$ 

(iv) For $b = (1 - \alpha)e^{-i\lambda}\cos\lambda$;

$$\frac{r}{(1 + r)^{(1 - \alpha)(1 - \beta)\cos\lambda}} \leq |f(z)| \leq \frac{r}{(1 - r)^{(1 - \alpha)(1 - \beta)\cos\lambda}}.$$ 

(VI) For $A = 1$, $B = \frac{1}{M} - 1$

$$\frac{r}{\left[1 + (1 - \frac{1}{M})r\right]^{\alpha(2 - \alpha)\frac{1}{M}}} \leq |f(z)| \leq \frac{r}{\left[1 - (1 - \frac{1}{M})r\right]^{\alpha(2 - \alpha)\frac{1}{M}}}.$$ 

In this case;

(i) For $b = 1$;

$$\frac{r}{\left[1 + (1 - \frac{1}{M})r\right]^{(2 - \alpha)\frac{1}{M}}} \leq |f(z)| \leq \frac{r}{\left[1 - (1 - \frac{1}{M})r\right]^{(2 - \alpha)\frac{1}{M}}}.$$ 

(ii) For $b = (1 - \alpha)$;

$$\frac{r}{\left[1 + (1 - \frac{1}{M})r\right]^{(1 - \alpha)(2 - \alpha)\frac{1}{M}}} \leq |f(z)| \leq \frac{r}{\left[1 - (1 - \frac{1}{M})r\right]^{(1 - \alpha)(2 - \alpha)\frac{1}{M}}}.$$
For $b = e^{-i\lambda}\cos\lambda$;
\[
\frac{r}{1 + (1 - \frac{1}{M})r} \leq |f(z)| \leq \frac{r(2 - \frac{1}{M})}{1 - (1 - \frac{1}{M})r} \cos\lambda.
\]

For $b = (1 - \alpha)e^{-i\lambda}\cos\lambda$;
\[
\frac{r}{1 + (1 - \frac{1}{M})r} \cos\lambda \leq |f(z)| \leq \frac{r(2 - \frac{1}{M})(1 - \alpha)}{1 + (1 - \frac{1}{M})r} \cos\lambda.
\]

**IV. KOEBE DOMAIN FOR THE CLASS $S^*(A, B, b)$**

In this section we shall give the Koebe domain for the class $S^*(A, B, b)$ under the condition of the definition of Koebe domain.[see 1. page 113-114].

From the theorem 3.1. we have;
\[
|f(z)| \geq r(1 - Br)^\frac{|b|(A - B)}{A} \quad \text{for} \quad B \neq 0
\]
\[
|f(z)| \geq re^{-|b|Ar} \quad \text{for} \quad B \neq 0
\]
then from the definition of Koebe domain we obtain

\[
R = \lim_{r \to 1^-} r(1 - Br)^\frac{|b|(A - B)}{A} = (1 - B)^\frac{|b|(A - B)}{A} \quad \text{for} \quad B \neq 0
\]
\[
R = \lim_{r \to 1^-} re^{-|b|Ar} = e^{-|b|A} \quad \text{for} \quad B = 0
\]

(I) For $A = 1, B = -1 \quad R = \frac{1}{4|b|}$

In this case;

(i) $b = 1 \quad R = \frac{1}{4}$. This is well known result (Koebe-1/4 theorem [1,page 115]).

(ii) $b = 1 - \alpha \quad R = \frac{1}{4(1 - \alpha)}$. This result was obtained by M.S.Robertson [1, page 115,[5]].

(iii) $b = e^{-i\lambda}\cos\lambda \quad R = \frac{1}{4e^{i\lambda}}$. This is the Koebe domain for the class of $\lambda$-spirallike function.
(iv) \( b = (1 - \alpha)e^{-i\lambda}\cos\lambda \quad R = \frac{1}{4(1 - \alpha)e^{\pi}}. \) This is the Koebe domain for the class of \( \lambda \)-spirallike functions of order \( \alpha \).

(II) For \( A = 1 \), \( B = 0 \) \( R = \frac{1}{e^{\|b\|}}. \)

(III) For \( A = \beta \), \( B = 0 \) \( R = \frac{1}{e^{\|b\|\beta}}. \)

(IV) For \( A = \beta \), \( B = -\beta \) \( R = \frac{1}{(1-\beta)\|b\|\beta}. \)

(V) For \( A = 1 \), \( B = -1 + \frac{1}{M} \) \( R = \frac{1}{\|b\|/\beta} \cdot \left(2 - \frac{1}{M}\right)^{1 - \frac{1}{M}}. \)

(VI). For \( A = 1 - 2\beta \), \( B = -1 \) \( R = \frac{1}{\|b\|/\beta}. \)

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