Extraction of Correlated Jet Pair Signals in Relativistic Heavy Ion Collisions

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Multi-particle correlation techniques are frequently used to study jet shapes and yields in hadronic and nuclear collisions. To date, a standard assumption applied in such analyses is that the observed correlations arise from either jets and associated hard scattering phenomena, or from a background component due to combinatorial pairs connected only through whole even correlations. Within this assumption of two essentially independent sources, a fundamental problem centers around determining the relative contributions of each component. We discuss the methods commonly used to establish the background yield in jet correlation analyses, with a full explanation of the absolute background normalization technique which establishes the background yield without assumptions about the shape of jet correlations. This is especially important in relativistic heavy ion collisions where the jet shapes are significantly distorted from the well separated back-to-back di-jets observed in proton-proton collisions.

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I. INTRODUCTION

One of the main goals of high energy nuclear physics experiments is to produce and study the properties of hot and dense nuclear matter in ultra-relativistic heavy ion collisions. Probing this matter with strongly interacting partons produced in hard scattering processes in the initial stages of the collision has been a useful tool for deducing the properties of the created matter [1, 2, 3, 4]. In high energy physics, hard processes are often studied using an algorithm to identify and reconstruct jets composed of partonic fragments to infer properties of the parent parton. In heavy ion collisions, such measurements are extremely difficult due to the large number of total particles compared to the multiplicity within a jet. Instead, one-, two- and three-particle observables have been used to study hard scattering phenomena. The study of particle yields and their correlations, when compared to expectations from p+p collisions, provides quantitative information about the modification of jet production in the presence of hot nuclear matter.

Two particle correlations have been a particularly important tool for describing modification of jets in the nuclear environment. Dramatic modification has been observed in jet yields and shapes in central heavy ion collisions compared to the smaller baseline colliding systems. In p+p collisions, two particle azimuthal correlations have a small angle near-side peak from particles which arise from the same jet and an away-side peak at ∆φ = π from particles which arise from opposing sides of a pair of back-to-back di-jets. In central heavy ion collisions at √s_{NN} = 20-200 GeV the picture is qualitatively different. The away-side peak is shifted aside to ∆φ ≈ 2 rad and is much broader than in p+p collisions [5] while the near-side peak also contains an elongated structure in the longitudinal direction [6, 7]. However, quantification of these results requires removal of the combinatorial background of particle pairs which are only correlated through whole-event properties. This combinatorial background is large and must be removed in order to extract correlations arising from jet processes. The background is largest in central events and at lower p_T, where it can be as much as two orders of magnitude larger than the jet signal.

As the statistical precision of the correlations data from relativistic heavy ion collisions increases, better understanding of the accuracy and limitations of the experimental methods used to measure two particle correlations is needed. To this end we discuss the merits and drawbacks associated with methods commonly used to extract the jet signal in multi-particle angular correlations and specifically discuss a method to determine the combinatorial background that requires no assumptions about the shape of the jet correlations.

II. TWO PARTICLE ANGULAR CORRELATIONS

The angular correlation technique has been used extensively to deduce jet properties in hadronic and nuclear collisions, and is described in detail in several places...
A brief description of the method is provided here. For simplicity, we will focus the following discussion on two-particle azimuthal correlations projected onto the transverse plane.

The angular correlation technique statistically examines the relationship between particles classified as triggers (denoted as type $A$) and partners (denoted as type $B$). When studying jets, trigger particles are typically selected to have larger $p_T$ values than partners and both categories usually have $p_T > 1$ GeV/$c$, though this is necessary.

Due to the back-to-back production of partons by a hard scattering process, the distribution of relative azimuthal angles $\Delta \phi = \phi^A - \phi^B$ is expected to peak at $\Delta \phi = 0$ and $\pi$. However, the measured shape of this distribution may be significantly distorted when measured in a real detector due to non-uniform angular coverage and dead or inefficient areas. The shape of the detector acceptance in $\Delta \phi$ can be determined by pairing triggers and partners from different events through a process of event mixing. A mixed pair contains correlations due to the detector acceptance but the physical correlations are eliminated. The ratio of the same event pair distribution to that of mixed pairs allows pair acceptance effects to cancel, leaving a distribution reflecting only physical correlations. This ratio is conventionally defined as the correlation function, $C(\Delta \phi)$:

$$C(\Delta \phi) = \frac{d\langle n_{AB}^{mix} \rangle}{d\Delta \phi} / \frac{d\langle n_{same} \rangle}{d\Delta \phi} = \frac{d\langle n_{AB}^{mix} \rangle}{d\Delta \phi} / \frac{d\langle n_{same} \rangle}{d\Delta \phi}$$

where $n_{AB}$ is the number of pairs per event for either same or mixed pairs and $\langle \rangle$ indicates averaging over many events within some centrality selection.

Under the assumption that observed correlations arise from two independent and separable sources, the $d\langle n_{same}^{AB} \rangle/d\Delta \phi$ distribution consists of $AB$ pairs that are correlated with each other by only whole event correlations (which we call “background” pairs), and those where the particles carry additional spatial correlation. The source of these additional correlations is generally thought of as an association to a particular hard scattering, thus these pairs are labelled “jet” pairs [17]. Single particles are assumed to be either from jets or some non-jet source. Background pairs contain pairs where 1) one particle is from a jet and the other is not, 2) pairs where both particles are not from jets, and 3) pairs where both particles are from jets and not associated with the same hard scattering.

$$\frac{d\langle n_{same}^{AB} \rangle}{d\Delta \phi} = \frac{d\langle n_{jet}^{AB} \rangle}{d\Delta \phi} + \frac{d\langle n_{bg}^{AB} \rangle}{d\Delta \phi}$$

The two source model can be written as:

$$C(\Delta \phi) = J(\Delta \phi) + b_0 (1 + 2v_2^A v_2^B \cos(2\Delta \phi)).$$

where $v_2$ is the quadrupole anisotropy coefficient. The quadrupole anisotropy coefficients $v_2^{A,B}$ are taken as inputs from independent measurements of type $A$ and $B$ particles. Higher order terms of the anisotropy expansion are smaller and often neglected. In Eq 3, the approximation is made that the background pair anisotropy is equivalent to the product of the single-particle anisotropy coefficients:

$$\langle v_2^A v_2^B \rangle = \langle v_2^A \rangle \langle v_2^B \rangle.$$

Within the assumption that the azimuthal modulation of the background is independent of the jet signal, the fundamental problem of decomposing the correlation function into jet and background components amounts to the determination of the background level, $b_0$, as shown in Fig 1. By equating the final background terms in Eq 2 and 3 and integrating over $\Delta \phi$, using the definition of $C(\Delta \phi)$ in Eq 1, the background level can be expressed in terms of per-event pair multiplicities as:

$$b_0 = \frac{\langle n_{bg}^{AB} \rangle}{\langle n_{same}^{AB} \rangle}.$$

In practice $b_0$ and thereby $\langle n_{bg}^{AB} \rangle$ has been calculated using two approaches: the first involves scaling $b_0$ so that some portion of the angular distribution is zero, while the second uses a calculation to obtain the quantities in Eq 5. We follow the conventional nomenclature and refer to the former class of methods as the Zero Yield At Minimum (ZYAM) [10] approach, and the latter as the Absolute Background Subtraction (ABS) [5, 9, 11] technique. These methods are discussed in detail in the following sections.

A quantity that is frequently of interest is the per-trigger jet pair yield, which describes the conditional jet pair multiplicity as a function of relative azimuthal angle. The term “conditional” refers to the coincidence of a trigger-partner pair within some angular region divided by the production rate of triggers in the same centrality category. It can be shown that the per-trigger yield of total pairs (i.e. including jets and background) is related to that of mixed pairs allows pair acceptance effects to cancel, leaving a distribution reflecting only physical correlations. This ratio is conventionally defined as the correlation function, $C(\Delta \phi)$:

$$C(\Delta \phi) = \frac{d\langle n_{AB}^{mix} \rangle}{d\Delta \phi} / \frac{d\langle n_{same} \rangle}{d\Delta \phi} = \frac{d\langle n_{AB}^{mix} \rangle}{d\Delta \phi} / \frac{d\langle n_{same} \rangle}{d\Delta \phi}$$

where $n_{AB}$ is the number of pairs per event for either same or mixed pairs and $\langle \rangle$ indicates averaging over many events within some centrality selection. [16].

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$$C(\Delta \phi) = J(\Delta \phi) + b_0 (1 + 2v_2^A v_2^B \cos(2\Delta \phi)).$$

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$$b_0 = \frac{\langle n_{bg}^{AB} \rangle}{\langle n_{same}^{AB} \rangle}.$$
to the correlation function in a simple way:

\[
\frac{1}{\langle n^A \rangle} \frac{d\langle n^A_{AB} \rangle}{d\Delta \phi} = \frac{\langle n^A_{same} \rangle}{\langle n^A \rangle} \frac{C(\Delta \phi)}{C(\Delta \phi')}d\Delta \phi'
\]  

(6)

where \(\langle n^A \rangle\) is the mean number of triggers per event. Since \(J(\Delta \phi)\) is the fraction of the correlation function from jets, the per-trigger yield of the jet pairs can be calculated as in:

\[
\frac{1}{\langle n^A \rangle} \frac{d\langle n^A_{jet} \rangle}{d\Delta \phi} = \frac{\langle n^A_{same} \rangle}{\langle n^A \rangle} \frac{J(\Delta \phi)}{C(\Delta \phi')d\Delta \phi'}.
\]  

(7)

III. THE ABS METHOD

The absolute background normalization method is based on the assumption that the background is combinatorial in nature and that hard scattering results in large correlations between the production rates of jet particles. The combinatorial background carries only eventwise correlations. The background pair production rate is given by the product of the single particle production rates: \(\langle n^A_{bg} \rangle = \langle n^A \rangle \langle n^B \rangle\). Thus the true combinatorial background level in an ideal case is simply:

\[
\xi_{ideal} = \frac{\langle n^A \rangle \langle n^B \rangle}{\langle n^A_{same} \rangle}.
\]  

(8)

The values of \(\langle n^A \rangle\) and \(\langle n^B \rangle\) are measurable and, in the absence of other correlations, an accurate knowledge of these quantities is sufficient to determine the background level. However, \(n^A\) and \(n^B\) are both dependent on the event centrality, and this dependence gives rise to a multiplicity correlation. More central events typically have both larger \(n^A\) and \(n^B\). Because of the correlation between \(n^A\) and \(n^B\) when events are grouped into centrality bins, the number of measured background pairs is larger than that expected from Eq 8, \(\langle n^A_{bg} \rangle > \langle n^A \rangle \langle n^B \rangle\). An additional correction is needed to account for this effect when calculations are made for data selections that span a finite centrality range.

A. Centrality Bias Correction

We define a scale factor correction, \(\xi\), for the production rate product that accounts for the covariance effects arising from the centrality bias; \(\xi\) is defined as:

\[
\xi = \frac{\langle n^A n^B \rangle}{\langle n^A \rangle \langle n^B \rangle}.
\]  

(9)

The diagram shown in Fig 2 depicts the basic features of the procedure to calculate the centrality correction. The single-particle production rates, \(n^A\) and \(n^B\), are a function of some global property of the collision related to particle production, such as the number of nucleons participating in the collisions \(N_{part}\) or the number of binary nucleon-nucleon collisions \(N_{coll}\). The variations of \(\langle n^A \rangle\) and \(\langle n^B \rangle\) are measured over specific intervals in these parameters, which are specified by the width of the centrality bins used for event mixing. Typically centrality ranges are approximately 5%. The values of \(N_{part}\) or \(N_{coll}\) are based on the results of a Glauber Monte Carlo simulation [12]. For the purpose of narration, we discuss the calculation using \(N_{part}\), though \(N_{coll}\) is of equivalent utility, and calculations using this parametrization are also made in parallel. In practice, the variation in \(\xi\) from using the two different parametrizations provides a gauge of the systematic uncertainty inherent in the method: the two parameterizations bracket the expected scalings of hard and soft production that may both contribute to the background pair production. Interpolations between measurements of \(\langle n^A \rangle\) and \(\langle n^B \rangle\) in narrow centrality bins are used to estimate the average production of single particles at any particular value of \(N_{part}\).

In a computational algorithm, \(\xi\) can be calculated by throwing an \(N_{part}\) value, according to the distribution of events within the centrality selection, \(w^{glaub}\), as taken from the Glauber Monte Carlo calculation. The average number of type \(A\) and \(B\) particles is determined from the interpolated centrality dependence and their product gives the number of combinatorial pairs in the event. Events are created in this manner until \(\xi\) is numerically stable.

The production of \(A\) and \(B\) particles at a given \(N_{part}\) is typically modelled with a Poisson distribution. However, the details of this functional form do not affect the calculation so long as the displacement from the average value is independent between triggers and partners. To demonstrate this, \(\xi\) has been calculated for a delta function, a step function spanning \(\pm 25\%\) the average, and a pair of asymmetric delta functions where the production at \(-25\%\) was twice that at \(+50\%\) the average. The \(\xi\) at 50-60\% centrality using these distributions was found to be 1.1012(1), 1.1012(2), and 1.1013(2), respectively, where the Poisson form gives 1.1010(6). This agreement is within the statistical precision of the computational tests. Thus, even though Poisson distributions are of-

![FIG. 2: (Color online) Schematic of the basic features of the ξ calculation.](image-url)
ten used, they are not in general required so long as the deviations from average are independent.

Using the insight that only the average value is relevant, ξ can be calculated equivalently from the Glauber distributions and the yield interpolations by summing over all \( N_{\text{part}} \) values. The correction for centrality selection becomes a simple matter of finding the event-weighted averages of the three functions \( (n^A, n^B, n^A n^B) \) for the centrality bin in question. The expression for ξ is re-written as:

\[
ξ = \frac{\sum_i n_i^A n_i^B w_i^{\text{glaub}}}{\sum_i n_i^A w_i^{\text{glaub}} \sum_i n_i^B w_i^{\text{glaub}} \sum_i w_i^{\text{glaub}}} (10)
\]

where \( i \) indexes sequentially over all \( N_{\text{part}} \) values from 2 to \( N_{\text{part}}^{\text{max}} \) and \( n_i^A = n^A (N_i) \).

For trends of \( n^A \) and \( n^B \) that rise (or fall) in concert, the value of ξ will be always larger than 1. If either \( n^A \) or \( n^B \) is independent of centrality, the correction is precisely 1. If for some reason, one trend rises and the other falls, ξ will be less than one. In practice, the trends of trigger and partner production rates with centrality are in the same direction and ξ is an upward correction on the production rate product. The magnitude of the correction depends on how strongly the trends vary across the centrality bin compared to the yield of the bin. Since particle production rises most quickly in peripheral events, the magnitude of ξ is largest in this region. For the same reason wider centrality bins require larger corrections than more narrow bins.

We now provide an example of calculating ξ by using the charged hadron yields published in Ref. [13]. In practice, uncorrected yields should be used to determine ξ in order to properly take into account the multiplicity dependence of the reconstruction efficiency. Under all but extreme cases, the physical centrality dependence dominates the value of ξ as detector efficiency usually varies only slowly with centrality. Therefore the ξ trends produced here contain the general features of a typical calculation.

Fig 3 shows the Glauber event distributions for various centrality bin selections in both \( N_{\text{part}} \) and \( N_{\text{coll}} \) [12]. Invariant yields as a function of \( N_{\text{part}} \) and \( N_{\text{coll}} \) for partners, \( p_T^B = 2.9 \text{ GeV}/c \), and triggers, \( p_T^A = 5.0 \text{ GeV}/c \), are shown in Fig 4. The data, \( n^A \) and \( n^B \), are fit with the following two functional forms, chosen for their smoothness and well-controlled behavior for large \( N \). The inverse tangent, Eq 11, function is referred to as Fit 1 and the exponential function, Eq 12, is denoted as Fit 2.

\[
n^{(A,B)} = γ \arctan(βN^α) \quad (11)
\]

\[
n^{(A,B)} = γ(1 - e^{-βN^α}) \quad (12)
\]

where \( N \) is either \( N_{\text{part}} \) or \( N_{\text{coll}} \). Sensitivity to the fit functional form is assessed by comparison of the resulting ξ values from use of the two fits.

FIG. 3: (Color online) \( N_{\text{part}} \) (top) and \( N_{\text{coll}} \) (bottom) distributions from the Glauber Monte Carlo.

The calculated values of ξ from Eq 10 for these trigger-partner selections are shown in Table I. For central collisions ξ is a small correction to the background level, however since the background level is large compared to the jet signal, the effect of including the centrality correlations on the extracted jet signal is substantial. The centrality correction uncertainty is estimated from the spread in calculated values using the \( N_{\text{part}} \) vs. \( N_{\text{coll}} \) description and from using the two choices of functional forms.

TABLE I: ξ values for charged hadrons pairs between 5.0 GeV/c triggers and 2.9 GeV/c partners.

| Centrality (%) | \( N_{\text{part}} \) (Fit 1) | \( N_{\text{part}} \) (Fit 2) | \( N_{\text{coll}} \) (Fit 1) | \( N_{\text{coll}} \) (Fit 2) |
|---------------|----------------|----------------|----------------|----------------|
| 0-10          | 1.0041         | 1.0048         | 1.0041         | 1.0030         |
| 10-20         | 1.0097         | 1.0107         | 1.0082         | 1.0089         |
| 20-30         | 1.0205         | 1.0205         | 1.0150         | 1.0149         |
| 30-40         | 1.0369         | 1.0353         | 1.0246         | 1.0236         |
| 40-50         | 1.0606         | 1.0582         | 1.0405         | 1.0392         |
| 50-60         | 1.1012         | 1.1005         | 1.0757         | 1.0753         |
| 60-70         | 1.1825         | 1.1873         | 1.1694         | 1.1639         |
| 70-80         | 1.2918         | 1.3065         | 1.2966         | 1.3091         |
| 80-90         | 1.3952         | 1.4224         | 1.4419         | 1.4678         |

B. Other Correlations

The factorization of pair quantities into singles products appears often in pair analyses and the degree to which the factorizations hold should be examined in each
FIG. 4: (Color online) Invariant yield of charged hadrons as a function of \( N_{\text{part}} \) and \( N_{\text{coll}} \) at \( p_T = 2.9 \) GeV/c and \( p_T = 5.0 \) GeV/c. Data are from Ref. \[13\] and the errors shown are statistical and centrality dependent systematic uncertainties. Fits are to Eq 11 and Eq 12.
cut survival probability via \[18\]:
\[
\langle n_{bg}^{AB} \rangle = \xi \kappa \langle n^A \rangle \langle n^B \rangle.
\]

\[13\]

### D. Working Equation

Thus, fully corrected ABS background levels in realistic scenarios may be calculated in the manner described above as:
\[
b_0 = \xi \kappa \frac{\langle n^A \rangle \langle n^B \rangle}{\langle n_{same}^{AB} \rangle}.
\]
\[14\]

### E. Limits of the Method

The single particle production rate, \(n^A\), can be written as \(n^A = j^A + b^A\) where \(j^A\) are particles from jets and \(b^A\) are particles from non-jet sources. A similar decomposition can be made for type \(B\) particles. Using this notation, all pairs in the event can be expanded and factorized as:
\[
\langle n^A n^B \rangle = \langle j^A + b^A \rangle \langle j^B + b^B \rangle
\]
\[
= \langle j^A j^B \rangle + \langle j^A b^B \rangle + \langle j^B b^A \rangle + \langle b^A b^B \rangle
\]
\[
= \langle j^A j^B \rangle + \xi \kappa \left[ \langle j^A \rangle \langle b^B \rangle + \langle b^A \rangle \langle b^B \rangle \right].
\]
\[15\]

The combinatorial background as estimated in ABS and expanded in terms of \(j\) and \(b\) becomes:
\[
\langle n^A \rangle \langle n^B \rangle = \langle j^A + b^A \rangle \langle j^B + b^B \rangle
\]
\[
= \langle j^A \rangle \langle j^B \rangle + \langle j^A \rangle \langle b^B \rangle + \langle j^B \rangle \langle b^A \rangle + \langle b^A \rangle \langle b^B \rangle
\]
\[16\]

Note that unlike the last three terms, the first term, \(\langle j^A \rangle \langle j^B \rangle\), is not part of the background. So the ABS subtraction produces an extra term beyond the jet signal, \(\langle j^A j^B \rangle\), such that:
\[
\langle n^A n^B \rangle - \xi \kappa \langle n^A \rangle \langle n^B \rangle = \langle j^A j^B \rangle - \xi \kappa \langle j^A \rangle \langle j^B \rangle
\]
\[17\]

For the background subtraction to work without substantial over subtraction, this extra term is required to be small with respect to the jet signal.
\[
\xi \kappa \langle j^A \rangle \langle j^B \rangle \ll \langle j^A j^B \rangle
\]
\[18\]

Since hard scattering produces particles at rates determined by the characteristics of the scattering itself, like momentum transfer, the jet particle production rates for \(A\) and \(B\) particles will be highly correlated with one another. The presence of jet triggers increases the likelihood of production of jet partner particles within the same event.

### IV. THE ZYAM METHOD

The Zero Yield At Minimum (ZYAM) methodology sets the normalization of the background contribution through an assumption that the jet contribution falls to zero yield at some point or region in \(\Delta \phi\).

In addition to the assumption that the jet and background correlations are from essentially independent sources and thus separable, the validity of the ZYAM method is conditioned upon the existence of (a) one or more points with vanishing yield in the actual jet contribution, and (b) a sufficiently well-sampled correlation function that enables a stable and precise determination of the minimum value.

In heavy ion collisions at sufficiently high transverse momenta (\(p_T^{(A,B)} \gtrsim 4\,\text{GeV}/c\)) or in \(p+p\) collisions, the jet contribution to the correlation function consists of well separated near-side and away-side peaks [14]. In these cases due to the relative narrowness of the jet peaks, there is a broad region over which the background contribution dominates and can be determined with little bias under the ZYAM method. In the case of modified shapes in the jet contribution such as those found at intermediate \(p_T\) in central heavy ion collisions, the ZYAM assumption could be broken by jet broadening or other modifications creating yield between the locations of the near- and away-side peaks. Without independent verification of the ZYAM assumption, the method can potentially produce unreliable results due to over-subtraction in these cases.

In the simplest, and least reliable, application of the ZYAM procedure, the level of the background contribution is adjusted (with the harmonic amplitude remaining fixed at its measured value) until one measured bin in the jet function is zeroed. Clearly, small bins relative to features in the jet contribution are needed to limit jet contamination of the ZYAM bin. However, division of a fixed sample size into smaller and smaller \(\Delta \phi\) bins increases statistical scattering, and hence the degree to which the lowest \(\Delta \phi\) bin is influenced by downward fluctuations. A slightly more sophisticated method uses the average of three neighboring bins in place of a single bin. The moving average of neighboring bins attempts to balance the effects of jet contamination and statistical fluctuations (however, depending on the width of the bins and the physics of interest, this broader ZYAM region could make the assumption of a zero yield region much less valid). The most stable determination of background is to fit the correlation function and raise the background contribution to touch the fit at the minimum value. Assuming a reliable interpolation can be found (which requires sufficient statistics or outside assumptions), this method affords the best reliability against downward statistical fluctuations.

The statistical uncertainty propagated from the ZYAM method can be calculated with a simple Monte Carlo algorithm. The procedure generates simulated correlation functions by sampling, bin by bin in \(\Delta \phi\), a new point.
In order to extract reliable per-trigger jet pair yield estimates, there is a strong downward tendency using a known distribution under various levels of statistical sampling. There is a strong downward tendency due to statistical fluctuations for both back-to-back and modified jet shapes.

The choice of functional form does not significantly alter the resulting under subtraction for the two cases tested. The jet to background ratio was not varied in these tests. The single-bin implementation of ZYAM is shown to be extremely sensitive to catching sizable statistical fluctuations at low statistics. The three-bin method is more robust against fluctuations, but also fails badly. The functional fit ZYAM method works best, but is not completely robust against failure without the addition of sensible constrains on the fits to the correlation functions. No constraints, such as reasonable jet widths, were required in these tests. Unreasonably narrow jet widths (less than one $\Delta\phi$ bin) are responsible for the failures in these fits at low statistics.

To summarize, the ZYAM method may fail in two cases. The method may over subtract if there is a significant amount of jet yield at the ZYAM point. The method also has problems in some implementations with under subtraction when applied at low statistics. Given the sensitivity of the ZYAM method to low statistics or extremely modified jets, it is good practice to confirm the results independently with the ABS method in these cases.

V. THE UNDERLYING EVENT

The ABS background levels in the most peripheral bins have been found to lie below the ZYAM background [5]. Qualitatively, this is expected because the ZYAM assumption puts an upper limit on the background level. However, it is possible that measurements are also sensitive to the underlying event, as seen in $p+p$ collisions [15]. The underlying event is thought to be composed of initial and final state radiation as well as soft parton interactions besides the one that created the observed jet. These multi-parton interactions are not entirely uncorrelated with the jet. Furthermore, as the background in a small system is the result of very few soft parton interactions, the multiplicity resulting from a single soft interaction to both trigger and partner may become an important effect to model. Thus these effects may introduce additional correlations in the background beyond the centrality correlations which are removed by $\xi$.

In large systems, where the background multiplicity is almost entirely driven by impact parameter, these variations in the combinatorial background play a much smaller role in the average background multiplicities. Here the difference between ZYAM and the ABS background can bracket the uncertainty on the background subtraction. The ABS method will underestimate the background by not including any underlying event and ZYAM will overestimate the background, possibly removing some jet signal. In the small systems at higher momenta, even this extreme in physics assumptions translates into a small uncertainty on the extracted conditional yields. However, small systems at lower momenta fair less well and subtractions may produce significant uncertainties in the extracted conditional yields.

![Crosschecks on ZYAM under subtraction for both back-to-back and modified jet shapes](image-url)
VI. SUMMARY

The separation of pairs originating in the underlying event from those associated with jet production is of great importance for two particle correlations in heavy ion collisions. We have provided the first complete description of how to calculate the effect of centrality correlations, which enables the combinatorial background to be subtracted without assumptions about the shape of the jet correlations. This method has the additional advantage of having small uncertainties, especially in the case of statistics limited probes where the uncertainty on the ZYAM background will be large. This method could trivially be generalized to three or more particle correlations. Proper, consistent treatment of the background is crucial to the quantitative extraction of properties of the hot dense nuclear matter and its geometry from multiparticle correlation measurements.

We have described a method for subtraction of the combinatorial background under the assumption that there are two independent sources of pairs. Reality, however is likely more complicated. The discussion of the underlying event in Section V contains one example of additional correlations which could be important. Currently, the two source assumption has proven useful in the interpretation of two particle correlation measurements. More precise measurements and more complicated experimental questions could necessitate a re-evaluation of the two source model. In the meantime, it is recommended that publications include full correlation functions in order to make available the data containing no assumptions about jet-background decomposition should these considerations prove necessary.

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