Hexapod robot fault tolerant gait on slope and simulation verification

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Abstract. Currently, hexapod robot has been widely used in the field of outdoor detection and nuclear power disaster relief because of its adaptability to terrain, excellent movement performance and redundant fault tolerance. In this paper, fault-tolerant gait and motion planning method on the slope are researched. Firstly, the kinematics model of the robot on the slope is established, and the Jacobian matrix mapping the foot end velocity and joint velocity is derived, which is the basis of the robot movement. Then the phase sequence of the robot with one leg fault is planned to solve the problem of leg phase coordination. After then, the adjustment strategy of the centre of gravity on the slope is planned, which can effectively improve the stability of the robot. Finally, a simulation is demonstrated to verify the research.

1. Introduction
As a kind of special equipment that can walk on unstructured terrain, legged robots have more forms of motion. Hexapod robot is an excellent solution to the problem of moving in the complex unstructured environment. However, due to the complexity of mechanical structure and other factors, leg failure may happen to the hexapod robot, resulting in one or two legs cannot work normally.

Some scholars have made researches on these situations. Yang took a hexapod robot with a rectangular body and symmetrical legs as the research object, and uses the planning strategy of static stable region to propose a triangular fault-tolerant gait [1]. He also proposed a crab gait, and through the analysis of the fan-shaped workspace at the end of the robot, he proposed an optimization method [2]. Later, he extended the fault-tolerant gait to uneven ground [3]. Ding discussed the stability, fault tolerance, turning ability, and terrain adaptability of rectangular hexapod and symmetric hexapod robot [4]. Further, F. Gao deduced residual mobility using the screw theory in the case of partial actuator fault, and a motion planning method based on fault tolerance Jacobian matrix was proposed [5,6]. As for the movement on the slope, a simple body regulation plan has been designed based on the local slope of the terrain in [7]. [8] describes a hexapod robot which uses CPG method to generate different gaits, to inspect the gradeability of the robot when climbing in uneven slopes.

More attention has been focused on the research on even terrain, however, it is scarcely possible to avoid the slope. The movement on the slope will lead to the change of the position of centre of gravity (COG), which will bring more instability compared to the flat road. This paper studies the walking gait of the hexapod robot on the slope, and solves the problem of the movement of the hexapod robot with leg failure on the slope.
2. Kinematic model
Based on standard D-H notations \[9,10\], the kinematic model of the robot is established. To describe the robot, four kinds of coordinate systems are needed, which are world coordinate system \(\Sigma O_w\), body coordinate system \(\Sigma O_b\), reference coordinate systems \(\Sigma O_{ia}\), and joint coordinate systems \(\Sigma O_{ij}(j=1,2,3,4)\). The subscript \(i\) represents the number of six legs of the robot. The left three legs are 1, 2, 3 from front to rear, and the right three legs are 4, 5, 6. \(j\) represents the coordinate system of coxa joint, femur joint, tibia joint, and foot end of each leg, respectively. As shown in the Figure 1.

Because the legs of the robot have the same mechanical structure, the coordinate systems of the legs are exactly the same. Therefore, the transformation matrix from the foot end coordinate system to the reference coordinate system of each leg is the same. The homogeneous transformation matrix is:

\[
\begin{bmatrix}
c \beta_i & c \beta_{23} & s \beta_i & s \beta_{23} & (l_1 + l_2 + l_3 - \beta_{23}) c \beta_i \\
s \beta_i & s \beta_{23} & c \beta_i & c \beta_{23} & (l_1 + l_2 + l_3 - \beta_{23}) s \beta_i \\
0 & 0 & 0 & 1 & 0 \\
0 & 0 & 0 & 1 & 0
\end{bmatrix}
\]

(1)

where \(\beta_{23} = \beta_2 + \beta_3\). \(\beta_1\), \(\beta_2\), \(\beta_3\) is the joint angle respectively, \(l_1, l_2, l_3\) is the length of the leg link.

Based on the forward kinematics analysis, the conversion relationship between the reference coordinate system and the body coordinate system is obtained, which can be expressed as follows:

\[
\begin{bmatrix}
c \psi_i & -s \psi_i & 0 & b_{P_0x} \\
s \psi_i & c \psi_i & 0 & b_{P_0y} \\
0 & 0 & 1 & b_{P_0z} \\
0 & 0 & 0 & 1
\end{bmatrix}
\]

(2)

where \(\psi_i\) is the yaw angle from body coordinate to reference coordinate of leg \(i\).

Therefore, the relationship between the position of the foot end in the body coordinate system and the joint angle is:
By inverse kinematics, the joint angle can be obtained to realize the control of the robot:

\[
\beta'_i = \tan^{-1}\left(\frac{\frac{b_{P_4}^i - b_{P_0}^i}{(b_{P_4}^i - b_{P_0}^i)}\psi_i}{\sqrt{(\xi + l_{2}^2)^2 + 4l_3^2l_{2}^2 - (\xi - l_{2}^2)^2}}\right) - \tan^{-1}\left(\frac{\xi + l_{2}^2}{2l_3l_{2}}\right)
\]

\[
\beta'_i = \cos^{-1}\left(\frac{\xi - l_{2}^2 - l_{2}^2}{2l_3l_{2}}\right)
\]

where \(\xi = \sqrt{\left(\frac{b_{P_4}^i - b_{P_0}^i}{(b_{P_4}^i - b_{P_0}^i)}\right)^2 + \left(\frac{b_{P_3}^i - b_{P_0}^i}{(b_{P_3}^i - b_{P_0}^i)}\right)^2 - l_3^2}\).

The Jacobian matrix defines the relationship between the velocity of the foot end and the angular velocity of the joint. It can be deduced by calculating the partial derivative of the equation (3):

\[
J = \begin{bmatrix}
(l_1 + l_2 + l_3 + l_4 + l_5 + l_6)\sin\beta'_1 & (l_1 + l_2 + l_3 + l_4 + l_5 + l_6)\cos\beta'_1 & 0 \\
(l_1 + l_2 + l_3 + l_4 + l_5 + l_6)\sin\beta'_2 & (l_1 + l_2 + l_3 + l_4 + l_5 + l_6)\cos\beta'_2 & 0 \\
0 & 0 & l_1 + l_2 + l_3 + l_4 + l_5 + l_6 \\
\end{bmatrix}
\]

If the velocity of the foot in the body coordinate is known, the angular velocity of the joint angle in the joint space can be obtained according to the Jacobian matrix:

\[
\dot{\beta} = J^{-1}V
\]

By measuring the attitude change of the body, the homogeneous transformation matrix from the body coordinate system to the world coordinate system is expressed as:

\[
^wT_B = \begin{bmatrix}
c\theta_2 & -c\theta_2 & -c\theta_2 & c\theta_1 & c\theta_1 & c\theta_1 & s\theta_1 & s\theta_1 & s\theta_1 & s\theta_1 & s\theta_1 & s\theta_1 & s\theta_1 & s\theta_1 & s\theta_1 \\
c\theta_2 & c\theta_2 & c\theta_2 & c\theta_1 & c\theta_1 & c\theta_1 & c\theta_1 & c\theta_1 & c\theta_1 & c\theta_1 & c\theta_1 & c\theta_1 & c\theta_1 & c\theta_1 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
\end{bmatrix}
\]

where \(\theta_1, \theta_2, \theta_3\) is the attitude angles of the body, roll angle, pitch angle, and yaw angle respectively.

### 3. The gait planning method

#### 3.1. Gait phase sequence planning

Gait planning is to coordinate the leg phase relationship and plan the leg phase sequence. The leg fault here refers to that one whole leg is locked in a certain position and will not affect the movement of other legs. Because the hexapod robot needs at least three legs to support the body to ensure the stability, the number of fault legs is up to 2. In this paper, the problem of walking with one leg failure is studied.

First of all, a kind of leg swing motion sequence is planned according to the theory of stability margin. The most versatile gait of a hexapod robot is tripod gait, with legs 1, 3, and 5 in a group, and legs 2, 4, and 6 in a group, respectively. Define that if a leg breaks down suddenly, it is denoted \(T_f\). For the convenience of representation, a set of symbols is defined at first, as shown in the Table 1. The finite state of hexapod with one leg fault in locomotion is described by state sets \(S = \{S_0, S_1, \ldots, S_n\}\).
movement process. The hexapod transform during the states in order, as shown in the Figure 2. And the definition of all states is described in Table 2.

![Figure 2. The state set of fault tolerance gait.](image)

**Table 1. Definitions of symbols.**

| Symbol | Definition |
|--------|------------|
| $L_f$  | The fault leg. |
| $L_r$  | The ipsilateral anterior leg of the fault leg. |
| $L_l$  | The ipsilateral posterior leg of the fault leg. |
| $L_m$  | The contralateral anterior leg of the fault leg. |
| $L_r$  | The contralateral middle leg of the fault leg. |
| $L_r$  | The contralateral posterior leg of the fault leg. |

**Table 2. Definitions of state sets.**

| State | Definition |
|-------|------------|
| $S_0$ | All the legs shall be adjusted to initial position and supported on the ground except the faulted leg which needs to lift. |
| $S_1$ | $L_f$ and $L_r$ are in the swing phase, $L_r$, $L_l$ and $L_m$ are in the support phase. |
| $S_2$ | $L_l$ and $L_m$ are in the swing phase, $L_f$, $L_r$ and $L_l$ are in the support phase. |
| $S_3$ | $L_f$ are in the swing phase, $L_m$, $L_r$, $L_l$ and $L_r$ are in the support phase. |
| $S_4$ | All the legs shall be supported on the ground except the faulted leg, and push the body forward. |

3.2. **Trajectory planning**

In the slope motion, there appears the component force of robot gravity along the tangent direction, which makes the robot easier to turn over. To provide an effective evaluation index for the locomotion of robot, static stability margin is versatile when the motion velocity is not high. which is represented by the minimum vertical distance between the projection point of the COG on the foot support plane and the boundary of the support polygon. However, the static stability margin on the slope is different from that when the robot moves on the flat ground [11], as shown in Figure 3, which makes the robot body produce more turning torque, especially in the case of a leg failure.
In order to ensure the stability, it is necessary to keep the COG inside the projection support polygon, so the COG needs to be adjusted before the movement. Since the position of the COG is a constant before the state of $S_4$ during a period, based on the leg swing sequence, the support polygon at each time is known, and the overlapping area is a subset of any support polygon. Use the geometric method to calculate the center of the subset as the target position for COG adjustment, as shown in the Figure 4.

For the foot end trajectory, many research achievements have been put forward, such as compound cycloid and polynomial curve. In this paper, to reduce the collision, the swing trajectory put forward in [12] is adopted. The foot end trajectory is established using the following formulas:

$$y = s \left( \frac{t}{T_s} - \frac{1}{2\pi} \sin \left( 2\pi \frac{t}{T_s} \right) \right)$$  \hspace{1cm} (8)

$$z = \begin{cases} 
2H \left( \frac{t}{T_s} - \frac{1}{4\pi} \sin \left( 4\pi \frac{t}{T_s} \right) \right) & 0 \leq t < \frac{T_s}{2} \\
2H \left( 1 - \frac{t}{T_s} + \frac{1}{4\pi} \sin \left( 4\pi \frac{t}{T_s} \right) \right) & \frac{T_s}{2} \leq t \leq T_s 
\end{cases}$$  \hspace{1cm} (9)

where $T_s$ is the period of swing phase, $H$ is the stride height, $s$ is the stride length.

### 4. Simulation research

In order to verify the effectiveness of the proposed method, the kinematics simulation is carried out in the software ADAMS. A physical prototype is built in SolidWorks, and then imported into ADAMS through the software interface. The parameters of the simulation model are shown in Table 3.

| Parameters       | Values | Parameters | Values |
|------------------|--------|------------|--------|
| Slope angle      | 10°    | Stride length | 60 mm |
| Model weight     | 2300 g | Stride height | 10 mm |

In the initial stage, the robot is supported by five legs, and the COG is adjusted to the correct position, it is the initial attitude. Next, the $L_f$ and $L_r$ legs are in the swing phase. Then, the $L_f$ and $L_r$ legs transform to the support phase, and the $T_f$ and $L_w$ legs swing in the aerial, after then, the $T_f$ is in the swing phase. Finally, the body is moved forward by all the support legs. The locomotion process is shown in Figure 5.
Some simulation results are given below. Figure 6 shows the position change of the robot in the world coordinate. It can be seen that the robot moves forward and generate displacement in the Y axis in the $S_1$ state. Due to the influence of slope, the displacement of robot in Z direction rises slightly, but there is no displacement in X direction. Figure 7 shows the attitude angle of robot COG. It can be seen that the change of roll, pitch and yaw angle is minute. In order to observe the rotation of the joints, the angle and angular velocity of the three joints of the leg 5 are exhibited, as shown in Figure 8 and Figure 9. Due to the existence of slope, the femur joint changes greatly, but within a reasonable range.

5. Conclusions
In this paper, a fault-tolerant gait planning method on the slope is proposed. Through the verification of that in the simulation environment, this method has been proved to be effective in solving the problem of robot walking on the slope with one single leg.

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Figure 8. Joint angular of leg 5.

Figure 9. Joint angular velocity of leg 5.

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