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Optimization of Pipeline Network Layout for Multiple Heat Sources Distributed Energy Systems Considering Reliability Evaluation †

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Abstract: Due to the target of carbon neutrality, energy saving has become more important than ever. At the same time, the widespread use of distributed energy systems and the regional utilization of industrial waste heat leads to the existence of multiple heat sources in an area. Therefore, how to design an economical and reliable pipeline network to meet energy-saving demand under multiple heat source conditions becomes a problem. In this work, an optimization method is established to determine the optimal pipeline network topology with minimum total annual cost. In this optimization method, Star tree algorithm, Kruskal algorithm and GeoSteiner algorithm are combined with a linear programming model to establish a distributed energy pipeline network for multiple heat sources. The model incorporates Euclidean Steiner Minimum Tree and Rectilinear Steiner Minimum Tree in the consideration of the topology optimization of Distributed Energy System pipeline networks. Four pipeline network topologies, STAR, Minimum Spanning Tree, Euclidean Steiner Minimum Tree and Rectilinear Steiner Minimum Tree, are evaluated in this paper from economic and reliability perspectives. A case extracted from a real industrial park where steam is the medium is used to prove the validity of the model. The optimization results show that a Euclidean Steiner Minimum Tree pipeline network has a lower total annual cost than three other types of pipeline network and ranks second in reliability. Considering the comprehensive economy and reliability, ESMT is the optimal pipeline network type of distributed energy system with steam as the medium.

Keywords: distributed energy system; pipeline network layout; reliability; GeoSteiner algorithm; Kruskal algorithm; Star tree algorithm

1. Introduction

Based on the proposal of carbon neutrality target, optimization for energy-saving and emission-reduction in energy system field attracts various academic research. As a high efficiency and promising technology [1] for energy conservation, a distributed energy system (DES) can use natural gas and renewable clean energy as primary energy source, including biomass, solar energy and hydrogen, and achieve the step utilization of the energy. Therefore, it has become very popular recently compared with traditional centralized energy systems.

DES can be improved from several aspects and a number of progress have been made. Firstly, DES can be improved by optimizing the process within the station. A number of works focus on applying renewable energy in DES. For instance, Ren et al. [2]
combined solar thermochemical fuel production technology with cogeneration of cold, heat and power to improve energy utilization efficiency. Multi-objective linear programming was used to optimize the operation of DES. Wu et al. [3] established a multi-objective optimization model considering cogeneration and renewable energy. The best combination of relative technologies is determined to reduce total annual cost. Jing et al. [4] proposed a comprehensive framework of multi-objective optimization and multi-criteria evaluation for the optimization of DES. In addition, solar-assisted solid oxide fuel cell technology was introduced into the system. Besides considering renewable energy, some other elements are also considered to improve the performance of DES. For example, Yuan et al. [5] considered thermal energy storage system and proposed a control strategy to improve the performance of the DES. Wang et al. [6] used different waste heat recovery technologies to recover heat with different temperatures for further improving the system efficiency. A MILP model was used to optimize the system. In some other studies, more evaluation criterions are used rather than single economic performance. For example, Buoro et al. [7] developed a multi-objective MILP model to determine the optimal operation mode of DES with the goal of minimizing the emissions and total cost of the system. Wang et al. [8] compared different DES modeling methods under various uncertainties, and optimized the operation of the system by combining Monte Carlo simulation and multi-objective optimization (economic and environmental objectives).

Beyond the work of improving energy generation process, how to effectively transport energy to consumers is also a hot spot of research interests. During transportation, it is desirable to reduce heat loss with an acceptable investment, so Keçebaş et al. [9] optimized the thickness of insulation layer of pipeline with the objective of optimal economy. Salem et al. [10] employed heating degree-days method to optimize the insulation thickness of pipelines. The diameter of pipeline is also a key element that has a significant impact on pipeline investment and power consumption. Li et al. [11] determined the pipeline diameter by considering the variation of cold and thermal loads based on fixed road topology constraint using GA algorithm. Wang et al. [12] considered the siting and sizing of the station considering pipeline network design using a cost-based function model. The topology of pipeline network will also affect the performance of energy transportation process. Currently, tree pipeline network topology is frequently studied, for instance, Zeng et al. [13] established an optimization method to determine the most economic pipeline network based on substation loads. Improved GA is used to find the optimal pipeline diameter for the tree pipeline network topology. Mehleri et al. [14] developed a MILP model to optimize the heating pipeline network considering the storage tank capacity and the size of the tree pipeline network topology. Khir et al. [15] proposed a MIP model to optimize the tree pipeline network size and layout based on the investment and operating costs in a district cooling system. Chan et al. [16] combined GA with local search techniques to optimize tree pipeline network layout of district cooling systems. Carl et al. [17] established a linear model to optimize the pipeline network layout considering the variation in the needs of consumers in different periods. Studies on pipeline network topology can be found in some other area except DES, for example, Sanaye et al. [18] adopted GA to optimize the gas pipeline network layout with MST topology structure. Su et al. [19] considered the structure of tree pipeline network and adopted NSGA-II algorithm to optimize the gas pipeline network. Sokolov et al. [20] proposed a new dynamic programming methodology to optimize the tree-shaped water network. Liu [21] considered the problem of pipeline network layout on the road and proposed a pipeline network optimization method based on Kruskal algorithm to optimize the direction of load access to the pipeline network. However, most of the above-mentioned studies are only based on tree network topology for optimization, with little consideration given to other topologies of the pipeline network.

Reliability is an important aspect in pipeline network design. Some studies have been done in this subject, for instance, Rimkevicius et al. [22] proposed probabilistic mathematical analysis, deterministic hydrothermal force analysis and deterministic probabilistic structural integrity analysis to evaluate the reliability of pipeline network energy
At present, large amount of low degree waste heat is discharged in industries, and such heat can be used in the region to save energy for lowering carbon emission. When industry and DES are combined to provide energy, several heat sources are existed in the region and such situation requires more consideration on how to effectively establish the pipeline network topology, but this point has not been well studied. Moreover, it is important to find the best pipe connection between heat source and heat users to cut down the investment and energy consumption. Aim to solve such problem, the new method is proposed in this work to optimize pipeline network under multiple heat sources situation. The aim of the methodology is to find the optimal pipeline network considering both economic and reliable factors. Moreover, different pipeline network topologies have different performance in economic and reliability, but the research considered reliability in multiple heat sources condition were rare. To study the above-mentioned problems, this work firstly uses clustering algorithm to divide consumers into different heat sources. Then, through coupling clustering algorithm combining Star tree algorithm, Kruskal algorithm, GeoSteiner algorithm and Linear Programming (LP) model, different pipeline network topology and flow rate of each branch pipeline are solved. This work takes the minimum value of total annual cost (TAC) as the objective function, including construction cost, pressure loss cost and heat loss cost of pipeline network. Additionally, then, the reliability of pipeline network is assessed. Finally, both the economy and reliability aspects of different types of pipeline network are evaluated to provide a reference for the optimization of pipeline network layout of DES.

2. Methodology

2.1. Problem Statement

For the energy supply problem in a region with industries, it is necessary to connect multiple heat sources with a number of consumers scattered in different parts of the region to form an energy transportation pipeline network. How to construct a pipeline network topology with the low cost and the high reliability should be considered.

2.1.1. Assumption

(1) The pipeline length is the distance between the two vertices that are connected.
(2) The steam in the pipeline network system is an incompressible fluid.
(3) The temperature of the fluid in each pipeline is constant.
(4) The flow rate of the fluid in each pipeline is constant.

2.1.2. Given

(1) The number of consumers and heat sources.
(2) The coordinates and heat demand of consumers.
(3) The coordinates and the heat supply of heat source.

2.1.3. Determine

(1) The pipeline network topology of multiple heat sources DES.
(2) Total annual cost and reliability of multiple heat sources DES.

As shown in Figure 1, this work first classifies the consumers into clusters and applies the Star tree algorithm, Kruskal algorithm or GeoSteiner algorithm to obtain different
pipeline network topologies. LP model is then applied to find the optimal flow rate of each branch pipeline. After that, parameters such as the length and flow rate of branch pipelines are embedded in the pipeline network system, and the economic model with minimum TAC as the objective function is solved. Finally, reliability assessment model is solved.

**Figure 1.** Schematic diagram of proposed methodology.

Four different types of pipeline network topologies are STAR, Minimum Spanning Tree-MST, Rectilinear Steiner Minimum Tree-RSMT and Euclidean Steiner Minimum Tree-ESMT. Among them, STAR is solved by Star tree algorithm, MST is solved by Kruskal algorithm, and RSMT and ESMT are solved by GeoSteiner algorithm.

Figure 2 presents the optimization schematic diagram of a DES pipeline network layout consisting of 3 heat sources and 14 consumers. The optimal design of the pipeline network layout consists of four parts. Firstly, clustering algorithm (see Section 3.1 for details) is applied to cluster all consumers, which is able to avoid too many pipelines with high flow rates to reduce investment costs, as shown in Figure 2a. Secondly, each heat source is connected to obtain a tree (see Sections 3.2–3.4 for details) to achieve the energy supply–demand balance between heat source, as shown in Figure 2b. Thirdly, the consumers within each cluster are connected to obtain a tree (see Sections 3.2–3.4 for details). Figure 2c takes the ESMT pipeline network as an example, and Steiner Point can be obtained by GeoSteiner algorithm in Section 3.4. Finally, the complete pipeline network layout of the multiple heat sources DES is obtained, as shown in Figure 2d.

**Figure 2.** Schematic of optimization of multiple heat sources DES network system: (a) cluster consumers; (b) generate heat source–heat source tree; (c) generate heat source–consumers tree; (d) obtain the layout of multiple heat sources DES pipeline network.
2.2. Mathematical Model

2.2.1. Objective Function

The objective function in this paper is to minimize the total annual cost \( TAC(Y \cdot a^{-1}) \), including construction cost \( C_{\text{Pipeline}}(Y \cdot a^{-1}) \), pressure loss cost \( C_{\text{Pressure Loss}}(Y \cdot a^{-1}) \) and heat loss cost \( C_{\text{Heat Loss}}(Y \cdot a^{-1}) \), of the pipeline network, as shown Equation (1). It is noted that steam is used as heat transfer medium in this work, as the studied DES providing steam to industry consumer. For the DES providing hot water, the method can be easily adopted by changing properties from steam to hot water.

\[
\begin{align*}
\min TAC &= C_{\text{Pipeline}} + C_{\text{Pressure Loss}} + C_{\text{Heat Loss}} \\
C_{\text{Pipeline}} &= \sum_{j=1}^{N_{\text{Cluster}}} \sum_{i=1}^{N_{\text{Cluster}}^j} C_{i,j}^{\text{Pipeline}} \\
C_{\text{Pressure Loss}} &= \sum_{j=1}^{N_{\text{Cluster}}} \sum_{i=1}^{N_{\text{Cluster}}^j} C_{i,j}^{\text{Pressure Loss}} \\
C_{\text{Heat Loss}} &= \sum_{j=1}^{N_{\text{Cluster}}} \sum_{i=1}^{N_{\text{Cluster}}^j} C_{i,j}^{\text{Heat Loss}} 
\end{align*}
\]  

(1)

where \( N_{\text{Cluster}} \) is the number of clusters of pipeline network system and \( N_{\text{Cluster}}^j \) is the number of branch pipelines in cluster \( j \). For instance, in Figure 2, the number of clusters of pipeline network system \( N_{\text{Cluster}} \) = 3, and the number of branch pipelines within each cluster is \( N_{\text{Cluster}}^1 = 7 \), \( N_{\text{Cluster}}^2 = 3 \), \( N_{\text{Cluster}}^3 = 4 \).

2.2.2. Linear Programming Model

Besides the length of pipeline, the diameter of pipeline also has a great impact on the total cost of pipeline network. To consider diameter of pipeline, flowrate in each branch of pipeline network should be optimized.

After determining the connection of the pipeline network, the sum of the mass flow of steam in all branch pipelines of the steam pipeline network \( W^S \) can be obtained by solving the LP model shown in Equation (5).

\[
\begin{align*}
\min W^S &= \sum_{z}^{N_{\text{VT}}} \sum_{x}^{N_{\text{VT}}} \sum_{y}^{N_{\text{VT}}} b_{x,y,z}^{\text{Binary}} \times w_{x,y,z} \\
\sum_{x}^{N_{\text{VT}}} \sum_{y}^{N_{\text{VT}}} \left( (-1)^T \times b_{x,y,z}^{\text{Binary}} \times w_{x,y,z} + (-1)^T \times b_{y,x,z}^{\text{Binary}} \right) &= W^S 
\end{align*}
\]  

(5)

The constraints of the LP model are shown in Equations (6)–(10).

\[
\begin{align*}
\sum_{x}^{N_{\text{VT}}} \sum_{y}^{N_{\text{VT}}} \left( (-1)^T \times b_{x,y,z}^{\text{Binary}} \times w_{x,y,z} + (-1)^T \times b_{y,x,z}^{\text{Binary}} \right) &= W^S \\
b_{x,y,z}^{\text{Binary}} &= \begin{cases} 
1, & (x, y) \in E_z^T \\
0, & \text{otherwise} \end{cases} \\
D &= \begin{cases} 1, & x < y \\
0, & x \geq y \end{cases} \\
w_{x,y,z} &\geq 0 \\
w_{y,x,z} &\geq 0
\end{align*}
\]  

(6)\( \cdots \) (10)

where \( N_{\text{VT}} \) is the number of vertices in the pipeline network topology. The binary variables \( b_{x,y,z}^{\text{Binary}} \) are obtained from Equation (7) based on the result of the pipeline network topology.
\( b_{x,y,z} \) indicates whether there is an edge \((x, y)\), which belongs to the connected tree, is directly connected to vertex \(z\). Equation (6) accounts for the material balance for each vertex in the pipeline network topology. \( w_{x,y,z} \) (kg \cdot s\(^{-1}\)) is the mass flow rate of steam in the edge \((x, y)\) connected to vertex \(z\) when the material balance is performed on the vertex \(z\). \( D \) is an exponent, which is determined by Equation (8). \( W_{z} \) (kg \cdot s\(^{-1}\)) is the mass flow rate at vertex \(z\). If vertex \(z\) produces steam, \( W_{z} \) is negative, otherwise, \( W_{z} \) is positive. The flow in each branch line has two directions from \(x\) to \(y\) and from \(y\) to \(x\). In Equation (7), for each vertex \(z\), there is a set of edges \( E_{z}^{t} \) connected to it, each set \( E_{z}^{t} \) is a subset of the connected tree, and the number of edge sets is \( N^{t} \), so that one of the two vertices corresponding to each edge in \( E_{z}^{t} \) must be \(z\). If the edge \((x, y)\) belongs to \( E_{z}^{t} \), \( b_{x,y,z}^{Binary} \) is equal to 1, otherwise, it is equal to 0.

In Equation (6), edge \((x, y)\) calculates the mass flow rate twice from \(x\) to \(y\) (\(w_{x,y,z}\)) and from \(y\) to \(x\) (\(w_{y,x,z}\)). The two mass flow rates are independent with each other due to the constraints (Equations (9) and (10)). After solving the optimization, one of \(w_{x,y,z}\) and \(w_{y,x,z}\) must be 0, and the other is a positive number. The aim of solving Equation (5) is to find the value of \(w_{x,y,z}\) and \(w_{y,x,z}\) and let one of them to be 0.

2.2.3. Pipeline Cost Model

The construction cost [28] of the pipeline \(C_{ij}^{Pipeline} \) (\(¥ \cdot m^{-1}\)) and the unit price of the pipeline \(a_{i,j} \) (\(¥ \cdot m^{-1}\)) are obtained by Equations (11) and (12).

\[
C_{ij}^{Pipeline} = a_{i,j}L_{ij} \frac{I(1 + I)^{N^{year}}}{(1 + I)^{N^{year}} - 1} \tag{11}
\]

\[
a_{i,j} = 5.74W_{t_{ij}} + 1295(D_{ij}^{outer,pipeline})^{0.48} + 47.6 + 2065D_{ij}^{outer,pipeline} \tag{12}
\]

\[
W_{t_{ij}} = 644.3(D_{ij}^{inner,pipeline})^{2} + 72.5D_{ij}^{inner,pipeline} + 0.4611 \tag{13}
\]

\[
D_{ij}^{outer,pipeline} = 1.052D_{ij}^{inner,pipeline} + 0.005251 \tag{14}
\]

\[
D_{ij}^{inner,pipeline} = \sqrt{\frac{4W_{t_{ij}}}{\pi \cdot u}} \tag{15}
\]

where, \(N^{year}(a)\) is the life cycle of the pipeline network system, \(I\) is the annual interest rate, and \(L_{ij}(m)\) is the length of the branch pipeline.

The weight per unit length of the pipeline \(W_{t_{ij}} \) (kg \cdot m\(^{-1}\)), the outer diameter \(D_{ij}^{outer,pipeline} \) (m) of the pipeline and the inner diameter \(D_{ij}^{inner,pipeline} \) (m) of the pipeline are obtained from Equations (13)–(15), where \(W_{t_{ij}} \) (kg \cdot s\(^{-1}\)) is the mass flow rate of steam in the pipeline, \(\rho \) (kg \cdot m\(^{-3}\)) is the density of the fluid, and \(u \) (m \cdot s\(^{-1}\)) is the velocity of the fluid in the pipeline.

2.2.4. Pressure Loss Cost Model

The pressure loss cost \(C_{ij}^{Pressure Loss} \) (\(¥ \cdot a^{-1}\)) of the pipeline network, the shaft power \(N_{ij} \) (W) to overcome the resistance during steam transportation and the effective power \(Ne_{ij} \) (W) of the transportation equipment are calculated by Equations (16)–(19). To consider the local resistance, it is assumed an average of four standard elbows (90\(^{\circ}\)) per 100 m, then \(\zeta_{Elbow} \) can be calculated by Equation (20).

\[
c_{ij}^{Pressure Loss} = \frac{a^{E_{t}^{Operating}}N_{ij}}{1000} \tag{16}
\]

\[
N_{ij} = \frac{Ne_{ij}}{\eta} \tag{17}
\]
Figure 2. Schematic of optimization of multiple heat sources DES network system: (a) generate heat source–heat source tree; (b) generate heat source–consumers tree; (c) generate consumers–consumers tree. (3) \[ C_{\text{up}}(\text{m}) = \frac{C_{\text{up}}}{2} \]

(4) \[ \zeta_{\text{Elbow}} = \frac{L_{ij}}{2S_{\text{inner,pipeline}}} \]

(20) \[ \zeta_{\text{Elbow}} = \left\{ \begin{array}{ll}
0, & [i = 0] \lor \left[ S_{\text{inner,pipeline}}^i = S_{\text{inner,pipeline}}^j \right]
\end{array} \right. \]

(21) \[ S_{\text{inner,pipeline}}^i = \frac{\pi}{4} \left( D_{\text{inner,pipeline}}^i \right)^2 \]

where \( a^E \left( \text{Y} \cdot \text{kW}^{-1} \cdot \text{h}^{-1} \right) \) is the power cost, \( T_{\text{Operating}} \) (h) is the number of annual operating hours of the device, and \( \eta \) is the efficiency of the conveying equipment. The head loss \( H_{\text{ij}}^f \) (m) caused by the resistance along the pipeline is calculated by Equation (19), where \( \zeta_{\text{ij}} \) is the local resistance coefficient when the cross section suddenly increases or decreases, \( \zeta_{\text{Elbow}}^E \) is the local resistance coefficient at the elbow, \( S_{\text{inner,pipeline}} \) (m\(^2\)) is the cross-sectional area of the branch pipeline, and \( \sigma \) is the pipeline friction coefficient.

2.2.5. Heat Loss Cost Model

The heat loss cost \( C_{\text{ij}}^{\text{Heat Loss}} \left( \text{Y} \cdot \text{a}^{-1} \right) \) can be calculated by Equation (23). Considering the insulation, the heat loss \( Q_{\text{ij}} \) (kJ \cdot m\(^{-1}\) \cdot s\(^{-1}\)) of the steam pipeline network is calculated by Equation (24).

\[ C_{\text{ij}}^{\text{Heat Loss}} = \frac{a^S L_{\text{ij}} T_{\text{Operating}}}{q} \]  

(23)

\[ Q_{\text{ij}} = \frac{\epsilon \pi (T - T_0)}{1000 \left( \frac{1}{\ln \frac{D_{\text{outer,insulation}}}{D_{\text{inner,insulation}}} + \frac{2}{\epsilon D_{\text{outer,insulation}}} \right)} \]  

(24)

where \( a^S \left( \text{Y} \cdot \text{kg}^{-1} \right) \) is the unit price of steam, \( q \) (kJ \cdot kg\(^{-1}\)) is the latent heat of steam, \( T \) (°C) is the temperature of the fluid medium in the pipeline, \( T_0 \) (°C) is the ambient temperature, \( D_{\text{inner,insulation}} \) (m) and \( D_{\text{outer,insulation}} \) (m) are the inner diameter and outer diameter of the insulation layer, \( \lambda \) (W \cdot m\(^{-1}\) \cdot K\(^{-1}\)) is the thermal conductivity of the insulation material, at the average temperature, \( \epsilon \) (W \cdot m\(^{-2}\) \cdot K\(^{-1}\)) is the heat transfer coefficient between the outer surface of the insulation layer and the surroundings. The inner diameter of the insulation layer \( D_{\text{inner,insulation}} \) is taken as the outer diameter of the steam pipeline \( D_{\text{outer,pipe}} \). Due to the difference in fluid temperatures and pipeline diameters, the thickness of the insulation layer is different. In this work, the thickness of insulation layer is selected according to Table 1.
Table 1. Table for selecting glass wool insulation layer thickness.

| DN (mm) | Pipeline Surface Temperature (°C) | Insulation Layer Thickness (mm) |
|---------|-----------------------------------|--------------------------------|
|         | ≤60                               | 15 30 30 40 50 50  |
|         | ≤150                              | 20 30 30 40 50 50  |
|         | ≤250                              | 25 30 30 50 50 60  |
|         | ≤300                              | 30 30 50 50 60 70  |
|         | ≤350                              | 35 30 50 50 60 70  |

2.2.6. Reliability Assessment Model

To calculate the reliability of the pipeline network system $R_{Reliability}$, the connected probability between the consumer $\alpha$ and the heat source $P_{\alpha}^{Probability}$ should be known first. Connected probability is used to describe when some part of the pipeline does not work, the probability that the consumer can still connect with the heat source. Reliability $R_{Reliability}$ and connected probability $P_{\alpha}^{Probability}$ can be calculated by the Equations (25) and (26). $P_{\alpha}^{Probability}$ is the product value of the probability of connectivity of each pipeline between consumer $\alpha$ to the heat source.

$$R_{Reliability} = \frac{\sum_{\alpha=1}^{N_{VT}} (P_{\alpha}^{Probability} \times Q_{\alpha}^{Heat Demand})}{\sum_{\alpha=1}^{N_{VT}} Q_{\alpha}^{Heat Demand}}$$  \hspace{1cm} (25)$$

$$P_{\alpha}^{Probability} = \prod_{\beta} (p_{\alpha,\beta}^{Path}) \frac{L_{\alpha,\beta}}{1000}$$  \hspace{1cm} (26)$$

where $Q_{\alpha}^{Heat Demand}$ is the heat demand of consumer $\alpha$, $P_{\alpha,\beta}^{Path}$ (km⁻¹) is the probability of connecting the $\beta$th pipeline in the path between consumer $\alpha$ and the heat source, and the size of the array $p_{\alpha,\beta}^{Path}$ is not exactly the same for different consumers, and $L_{\alpha,\beta}$ (m) is the length of the $\beta$th pipeline between consumer $\alpha$ and the heat source.

3. Algorithm

This method uses the concept of graph theory. In graph theory, each graph consists of some preset points and the lines connecting them. Such a graph is used to describe some certain relation between some things. The concept of graph theory is very suitable to solve the problem raised in this work. The algorithms chose in this work are all well accepted algorithms to solve such problems. This work aims to find the best connection
between heat sources and heat users. The computational complexity is the combination of clustering, LP, and different graph theory algorithms.

### 3.1. Clustering Algorithm

The clustering algorithm is used in this paper to obtain a set of clusters \( C = (C_1, C_2, \ldots, C_{N_{C_{Cluster}}}) \) by assigning different consumers to the heat source with the shortest Euclidean distance. The specific process of the clustering algorithm is shown in Algorithm 1.

#### Algorithm 1: Clustering Algorithm

**Input:** The coordinates of consumers \([x, y] = \{(x_1, y_1), (x_2, y_2), \ldots, (x_{N_{Consumer}}, y_{N_{Consumer}})\}\), the number of clusters \( N_{Cluster} \), and the coordinates of heat sources \([\mu, \lambda] = \{(\mu_1, \lambda_1), (\mu_2, \lambda_2), \ldots, (\mu_{N_{Consumer}}, \lambda_{N_{Consumer}})\}\).

**Output:** A set of clusters \( C = (C_1, C_2, \ldots, C_{N_{C_{Cluster}}}) \).

1. Let \( C_j = \{\text{Heat source}_j\}, j = 1, 2, \ldots, N_{Cluster} \)
2. for all \( i = 1, 2, \ldots, N_{Consumer} \) do
3. \( d_{ij} = \sqrt{(x_i - \mu_j)^2 + (y_i - \lambda_j)^2} \), \( j = 1, 2, \ldots, N_{Cluster} \)
4. \( \xi_i = \text{argmin}_{d_{ij}, j = 1, 2, \ldots, N_{Cluster}} \)
5. \( C_{\xi_i} = C_{\xi_i} \cup \{(x_i, y_i)\} \)
6. end for
7. return \( C = (C_1, C_2, \ldots, C_{N_{C_{Cluster}}}) \)

### 3.2. Star Tree Algorithm

Star tree algorithm [29] is a graph theory algorithm that directly connects the consumer to a selected center. The specific flow of the algorithm is shown in Algorithm 2.

#### Algorithm 2: Star tree algorithm

**Input:** A graph \( G = (V, E) \).

**Output:** A Star tree \( T = (V^T, E^T) \).

1. Let \( E^T = \Phi \)
2. choose the heat source as the center of all consumers
3. for all \( i = 1, 2, \ldots, N^V \) do
4. generate \( e_i^T \) by connecting consumer \( i \) and the center straightly
5. \( E^T = E^T \cup \{e_i^T\} \)
6. end for
7. return \( T = (V^T, E^T) \)

### 3.3. Kruskal Algorithm

Kruskal algorithm [30] is a classical algorithm for solving MST of weighted connected graphs, which gradually transforms the connected components into an MST connecting all vertices by finding the edges with minimum weights. The specific process of Kruskal algorithm is shown in Algorithm 3.
Algorithm 3: Kruskal algorithm

Input: A connected graph $G = (V, E, W)$.
Output: A minimum spanning tree $T = (V^T, E^T, W^T)$.

1: Let $E^T = \emptyset$
2: Sort the edges such that $W(e_1) \leq W(e_2) \leq \cdots \leq W(e_m)$
3: for all $i = 1, 2, \ldots, N^E$ do
4: $E^T = E^T \cup \{e^T_i\}$
5: if cycle is generated in $T$ then
6: delete $e^T_i$ from the $E^T$
7: else
8: maintain the $E^T$ unchanged
9: end if
10: if $N^{E^T} = N^V - 1$ then
11: break
12: else
13: continue
14: end if
15: end for
16: return $T = (V^T, E^T, W^T)$

3.4. GeoSteiner Algorithm

GeoSteiner algorithm is a fast and accurate algorithm for solving ESMT and RSMT. It has been improved through the work of Zachariasen [31] and Warme [32]. The geometric Steiner tree problem is known to be NP-hard for the Euclidean metric, and NP-complete for the rectilinear metric.

For the problem of generating Steiner trees, GeoSteiner algorithm includes two phases: First, a small but sufficient set of full Steiner trees (FSTs) is generated and then a Steiner minimum tree (SMT) is constructed from this set. These phases are called FSTs generation and FSTs concatenation. Before generating FSTs, a preprocessing phase is required to reduce the complexity of the calculation process.

The input to the FSTs generation algorithm is the set of terminal points, and the output is an embedded hypergraph. The embedding of each hyperedge (or FSTs) is the geometric tree structure of the FSTs. FSTs generation in GeoSteiner is performed by enumerating branch trees, and only the results after passing distance tests, long-leg segment tests, branch tree tests and FST tests will be retained to the final concatenation phase.

The FSTs concatenation problem can be modelled as an instance of the MST in hypergraph (MSTHG) problem [33]: given a hypergraph $H = (V^H, E^H)$, and a weight function $w^H : E^H \rightarrow \mathbb{R}$ find a subset $T^H \subseteq E^H$ such that $T^H$ is a spanning tree of $H$ that minimizes $w^H(T^H) = \sum_{e \in T^H} w^H(e)$. The MSTHG problem can be solved by integer programming, as shown in Equations (27)–(31):

Minimize $\sum_{e \in E^H} w^H(e) x_e$ \hspace{1cm} (27)

Subject to

$\sum_{e \in E^H} (|e| - 1) x_e = |V^H| - 1$ \hspace{1cm} (28)
\[
\sum_{e \in \mathcal{E}_H} \max(|e \cap S| - 1, 0)x_e \leq |S| - 1, \forall S \subset \mathcal{V}_H, |S| \geq 2
\]  
(29)
\[
0 \leq x_e \leq 1, \forall e \in \mathcal{E}_H
\]  
(30)
\[
x_e \in \mathbb{Z}, \forall e \in \mathcal{E}_H
\]  
(31)

where \(w^H \in \mathbb{R}^{\mathcal{E}_H}\) is the cost vector for all \(e \in \mathcal{E}_H\), \(x_e\) is the solution vector, and \(x \in \mathbb{R}^{\mathcal{E}_H}\). The constraint demonstrated in Equation (28) ensures that the spanning tree has the right number and sizes of hyperedges, while the subtour constraint demonstrated in Equation (29) ensures that the tree has no cycles.

For the Euclidean metric, the edges at a Steiner point in ESMT must meet exactly at an angle of 120°, and all other edges must meet at an angle of 120° or more, otherwise the tree can be easily shortened. Additionally, all edges are straight line segments directly connected by terminals and/or Steiner points. Figure 3 shows a schematic topology consisting of 3 terminals, where Terminal-1, Terminal-2, and Terminal-3 are the three vertices of the unit square triangle. From Figure 3, it can be calculated that the total length of MST is 2 and the total length of ESMT is \(\sqrt{3}\). In terms of total length, ESMT is shorter than MST.

![Figure 3. A schematic diagram of the topology consisted of 3 terminals.](image)

For the rectilinear metric [33], all line segments meet at a common corner point \(w = (u_x, v_y)\), which is obtained by intersecting the vertical line from point \(u\) and the horizontal line from point \(v\). Additionally, the backbone, for Hwang topologies [34], \((u, v)\) is consisted of line segments \(uw\) and \(vw\). The points \(u, v, w\) may be any combination of terminals, Steiner points, and corner points.

4. Case Study

4.1. Data Acquisition

In this case, the number of heat sources is 3. By clustering, consumers are assigned to these heat sources. The whole optimization process is realized by C++ programming. This case is simplified through a real industrial park, all the energy consumers require steam rather than hot water. The input data of consumers and heat sources is shown in Table 2, and if the heat demand is positive, then it is a consumer, on the contrary it is a heat source. The required parameters in the mathematical model are shown in Table 3.

4.2. Optimal Results and Analysis

The results are summarized in Table 4. Figure 4 shows the topological schemes of four types of pipeline network. The red line segment in Figure 4 represents the pipeline between the heat sources, and the other line segments represent the pipeline between the consumers and the heat sources. The thickness of the line segment in Figure 4 indicates the diameter of the branch pipeline.

From the perspective of economy, the TAC of STAR is the highest while that of ESMT pipeline network is the lowest, as shown in Table 4. The following analysis is carried out according to the pipeline construction cost, pressure loss cost and heat loss cost in the objective function.
In terms of pipeline construction cost, as a result of the existence of multiple heat sources, as shown in Figure 4, a single heat source no longer covers all the consumers, so the pipeline length is reduced, especially for those long, thick pipelines. This is beneficial for reducing pipeline costs.

Table 2. The table of input data of consumers and heat sources.

| Name       | Coordinate X (m) | Coordinate Y (m) | Heat Demand (t h\(^{-1}\) Steam) |
|------------|------------------|------------------|----------------------------------|
| Consumer-01| 7097             | 9542             | 15.0                             |
| Consumer-02| 8800             | 4024             | 6.0                              |
| Consumer-03| 9602             | 5124             | 0.5                              |
| Consumer-04| 12,013           | 7072             | 4.0                              |
| Consumer-05| 13,392           | 11,430           | 3.0                              |
| Consumer-06| 13,949           | 14,857           | 5.0                              |
| Consumer-07| 3384             | 24,093           | 10.0                             |
| Consumer-08| 25,483           | 0                | 25.0                             |
| Consumer-09| 11,914           | 2235             | 3.0                              |
| Consumer-10| 11,369           | 3893             | 4.0                              |
| Consumer-11| 6561             | 6546             | 5.0                              |
| Consumer-12| 12,437           | 10,805           | 12.0                             |
| Consumer-13| 12,454           | 11,538           | 0.3                              |
| Consumer-14| 16,227           | 13,461           | 10.0                             |
| Consumer-15| 18,914           | 14,341           | 1.2                              |
| Consumer-16| 7171             | 12,650           | 0.3                              |
| Consumer-17| 13,255           | 20,390           | 4.0                              |
| Consumer-18| 15,489           | 16,736           | 7.0                              |
| Consumer-19| 17,153           | 17,937           | 1.0                              |
| Consumer-20| 22,789           | 24,165           | 2.0                              |
| Consumer-21| 24,416           | 24,203           | 1.2                              |
| Consumer-22| 6206             | 20,012           | 10.0                             |
| Consumer-23| 5818             | 10,120           | 4.0                              |
| Consumer-24| 3465             | 24,915           | 10.0                             |
| Heat source-01| 0               | 3144            |                                   |
| Heat source-02| 12,200         | 21,944           |                                   |
| Heat source-03| 19,200         | 7544             |                                   |

Table 3. The table of mathematical model parameters.

| Parameter | Number | Unit          | Parameter | Number | Unit          |
|-----------|--------|---------------|-----------|--------|---------------|
| N\(year\) | 10     | a             | \(a^5\)   | 0.1945 | Y·kg\(^{-1}\) |
| I         | 0.02   | q             | \(T^u\)   | 1999.9 | kJ·kg\(^{-1}\) |
| \(\rho\)  | 0.60   | kg·m\(^{-3}\) | \(\lambda\) | 3.5   | °C            |
| \(u\)     | 30.00  | m·s\(^{-1}\)  | \(v\)     | 0.06  | W·m\(^{-1}\)·K\(^{-1}\) |
| \(a^E\)   | 0.21   | Y·kW·h\(^{-1}\) | \(\varepsilon\) | 3.50  | m·s\(^{-1}\) |
| \(\bar{T}_{\text{Operating}}\) | 8760 | h             | \(\mu_{a,b}\) | 11.63 | W·m\(^{-2}\)·K\(^{-1}\) |
| \(\eta\)  | 0.8    | \(\mu_{\text{Path}}\) | 0.98      |       | km\(^{-1}\)   |
| \(\sigma\) | 0.015  |               |           |        |               |

Table 4. The table of results of optimization cases.

| Topology Type of Pipeline Networks | STAR | MST | RSMT | ESMT |
|-----------------------------------|------|-----|------|------|
| Total pipeline cost (\(\times 10^7\) Y·a\(^{-1}\)) | 2.706 (100%) | 2.035 (75.2%) | 2.152 (79.5%) | 1.962 (72.5%) |
| Pressure loss cost (\(\times 10^7\) Y·a\(^{-1}\)) | 0.668 (100%) | 0.504 (75.4%) | 0.533 (79.8%) | 0.486 (72.8%) |
| Heat loss cost (\(\times 10^7\) Y·a\(^{-1}\)) | 0.990 (100%) | 0.623 (62.9%) | 0.654 (66.0%) | 0.605 (61.1%) |
| Total annual cost (\(\times 10^7\) Y·a\(^{-1}\)) | 4.364 (100%) | 3.162 (72.5%) | 3.339 (76.5%) | 3.053 (70.0%) |
| \(R_{\text{Reliability}}\) | 0.848 (100%) | 0.815 (96.1%) | 0.802 (94.6%) | 0.822 (96.9%) |
Table 4. The table of results of optimization cases.

| Topology  | Type of Pipeline Networks | STAR | MST | RSMT | ESMT |
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(R)௘௞௜௘௜௧௬ 0.848 (100%) | 0.815 (96.1%) | 0.802 (94.6%) | 0.822 (96.9%)

(a) STAR. (b) MST. (c) RSMT. (d) ESMT.

Figure 4. Pipeline network layouts.

On the basis of multiple heat sources, different pipeline network topologies affect pipeline costs through pipeline length, as shown in Figure 5. For the STAR pipeline network, since each consumer needs to be connected to the heat source in the region, there is no shared pipeline, leading to a significant increase in the pipeline length. As a result, this type is not applicable in large-scale projects. The MST pipeline network takes into account the situation of shared pipelines to reduce the pipeline length. Compared with STAR and MST, RSMT and ESMT pipeline networks introduce Steiner points to optimize the pipeline network connection mode. When the limitation of the actual path is considered, the RSMT pipeline network can better adapt to the actual road layout through rectangular connection, as shown in Figure 4c. However, this rectangular connection increases the total length of the network to a certain extent. Compared with RSMT pipeline network, ESMT pipeline network is not limited by the actual path, and the linear connection between two points makes its pipeline length the shortest.

In addition, in this model, pipeline construction cost is not only related to pipeline length, but also related to pipeline diameter. Since the diameter of pipeline is directly related to flow rate, the total flow rate of the pipeline network is defined as the sum of the flow rate of each piecewise pipeline. The total flow rate can reflect the overall diameter of the pipeline network and to further reflect the economy of the pipeline network. Figure 6 also demonstrates the relationship between TAC and the total flow rate.
MST, RSMT and ESMT pipeline networks introduce Steiner points to optimize the pipeline length. Compared with STAR and the situation of shared pipelines to reduce the pipeline length. Compared with STAR and the actual road layout through rectangular connection mode. When the limitation of the actual path is considered, the linear connection between two points makes its pipeline length the shortest. ESMT pipeline network is not limited by the actual path, and the linear connection between two points makes its pipeline length the shortest. This rectangular connection increases the total length of the network to a certain extent. Compared with RSMT pipeline network, this rectangular connection increases the total length of the network to a certain extent. Compared with RSMT pipeline network, this rectangular connection increases the total length of the network to a certain extent. Compared with RSMT pipeline network, this rectangular connection increases the total length of the network to a certain extent. Compared with RSMT pipeline network, this rectangular connection increases the total length of the network to a certain extent. Compared with RSMT pipeline network, this rectangular connection increases the total length of the network to a certain extent.

Figure 5. The relationship between the pipeline length and each cost of the four topologies.

Figure 6. The relationship between the flow rate, the total pipeline cost and TAC of the four topologies.

On the basis of multiple heat sources, a comprehensive analysis of the influence of pipeline length and pipeline diameter on pipeline cost shows that, when the main pipeline length is almost constant, the increase of total flow in pipeline network will lead to the increase of total cost, as shown in Figures 5 and 6 (MST and RSMT). This indicates that increasing the flow of the pipeline and making the pipeline diameter larger will result in increased costs. For the STAR and MST networks, the overall cost of the MST is significantly reduced, indicating that pipeline length is a more sensitive factor compared with flow rate. For ESMT network and RSMT network, where the total flow is similar, the economy of ESMT is better because the former has shorter pipeline length.

The STAR pipeline network also has the highest cost in terms of pressure loss and heat loss. This is because the cost of pressure loss and heat loss are related to the pipeline length. As shown in Figure 5, the longer pipeline length of STAR pipeline network causes more pressure loss and heat loss. On the other hand, for ESMT pipelines, the pressure loss and heat loss are significantly reduced due to shorter pipeline.

From the perspective of reliability, the reliability of Star is the highest, while that of RSMT is the lowest, as shown in Table 4. In this model, the reliability of the pipeline network is related to the connection mode between the consumers and the heat sources as well as the length of the main pipeline. The existence of multiple heat sources can shorten the long-distance transportation and reduce the number of shared pipelines, so that the system reliability can be improved.

In this case, the STAR pipeline network is more reliable than the other three structures because it does not contain shared pipelines. This means that the consumers are inde-
dependent from each other. In the process of energy transportation, the damage of a single pipeline will not cause a large-scale failure of the pipeline network system. In order to meet the actual road layout, some consumers are connected by vertical/horizontal pipelines in RSMT structure, leading to a longer pipeline compared to MST, so that the reliability of RSMT is lower than that of MST. Compared to RSMT and MST, ESMT is the shortest, so that the reliability is higher, making ESMT superior to the other two in terms of reliability.

It can be found from the results that the STAR pipeline network has the best reliability, but at the same time it is the most expensive one. MST pipeline network, as a common pipeline network topology, has better economy compared with the STAR structure. Compared with MST, the economy and reliability of RSMT becomes worse, but it may fit the actual road layout better. The performance of ESMT pipeline network is the best one in economic aspect and the second best one in reliability.

4.3. Analysis of Small-Scale DES

From the analysis above, it can be found that the distance is a key factor in the problem. The case used in last section is a large-scale problem. To identify the performance of the four types in small scale problem, this section will discuss DES that are scaled down in equal proportions, where the heat sources are more geographically concentrated with the consumers.

By comparing the data of Tables 4 and 5, it can be seen that in terms of the economics of DES, the TAC of STAR compared with that of ESMT changed from 142.9% in the large-scale case to 137.5% in the small-scale case, and the gap narrowed by 5.4%; when the pipeline network topologies are MST and RSMT, none of the gap changes by more than 1% in this ratio, and the TAC of ESMT is still the smallest. In terms of the reliability of DES, STAR has the highest reliability and ESMT has the second highest reliability, they are also in accordance with large-scale case.

| Topology Type of Pipeline Networks          | STAR      | MST       | RSMT      | ESMT      |
|--------------------------------------------|-----------|-----------|-----------|-----------|
| Total pipeline length (×10⁴ m)             | 4.603     | 2.421     | 2.433     | 2.345     |
| Total pipeline cost (×10⁶ Y·a⁻¹)           | 5.412     | 4.069     | 4.304     | 3.925     |
| Pressure loss cost (×10⁸ Y·a⁻¹)            | 1.313     | 1.002     | 1.062     | 0.967     |
| Heat loss cost (×10⁶ Y·a⁻¹)                | 1.313     | 1.002     | 1.062     | 0.967     |
| Total annual cost (×10⁶ Y·a⁻¹)             | 6.725     | 5.072     | 5.366     | 4.892     |
| Reliability                               | 0.967     | 0.959     | 0.956     | 0.961     |

From the analysis, it can be concluded that the scale of the region does not have a great impact on the relative performance of the four structures.

5. Conclusions

In this paper, the layout of the DES pipeline network is optimized for a multiple heat sources scenario to save energy during the energy transportation process. In this work, a topology optimization model for the pipeline network of DES containing multiple heat sources is developed by coupling clustering algorithm, Star tree algorithm, Kruskal algorithm, GeoSteiner algorithm and LP model. In the context of multiple heat sources, this paper investigates the effects of four different topologies (STAR, MST, ESMT, RSMT) on the pipeline network system in terms of both economy and reliability. The following conclusions are obtained based on the case studies.

1. Compared with the single heat source scenario, the multiple heat sources system will reduce the long-distance and high-flow pipelines in the system, so that both economy and reliability of the pipeline network system is improved.
2. Compared with the traditional pipeline network obtained using MST, an ESMT pipeline network can reduce the total annual cost by 3% and increase reliability by 1%.
3. When considering the actual path constraints, the RSMT pipeline network can be better adapted to the road layout.
4. The geographically scale of the problem does not have a great impact on the relative performance of the four structures.
5. By using the proposed method, both economic and reliability can be improved for the DES system.

In the work, the model developed can optimize the pipeline network layout under multiple heat sources conditions. However, the study in this paper has not taken into account the uncertainty of customer demand changes, which limits the optimization results to some extent. In the future, this work will be extended to consider reliability and economic under uncertainty factors. In addition, to make the optimization results more practical, in future, more practical factors should be concerned, e.g., the layout of existing road.

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Nomenclature

Abbreviations
DES Distributed Energy System
ESMT Euclidean Steiner Minimum Tree
FSTs Full Steiner trees
GA Genetic algorithm
LP Linear programming
MILP Mixed Integer Linear Programming
MIP Mixed Integer Programming
MST Minimum Spanning Tree
MSTHG The MST in hypergraph
NSGA-II Non-dominated Sorting Genetic Algorithm II
RSMT Rectilinear Steiner Minimum Tree
SMT Steiner Minimum Tree
STAR Star tree
TAC Total annual cost, ¥·a⁻¹

Indices and Sets
C The set of clusters, denoted by index j
D An index, which is used to control the positive and negative of \( w_{x,y,z} \) by \((-1)^D\)
e An index, referring to the edge in \( E^H \)
E The set of edges in a graph
\( E^H \) The set of edges in the hypergraph
\( E^T \) The set of edges in the connected tree, denoted by indices \( i,\beta \)
\( E^T_z \) The set of edges in the connected tree which are connected directly to the vertex \( z \).
The graph of empty set in Star tree algorithm, or the weighted connected graph without direction in Kruskal algorithm

A hypergraph

The set of all vertices in a graph, denoted by indices $x, y, z$

The set of vertices in the hypergraph, denoted by indices $x, y, z$ $V_H \subseteq V$

The set of vertices in MST, denoted by indices $x, y, z, a, V_T \subseteq V$

The set of weights of edges in a graph, denoted by indices $x, y, z$

The set of weights of edges in the connected tree, denoted by indices $x, y, z, W_T \subseteq W$

Variables

Electricity cost, $Y \cdot \text{kW} \cdot \text{h}^{-1}$

The unit price of the $i$th pipeline of the cluster $j$, $Y \cdot \text{m}^{-1}$

The unit price of steam, $Y \cdot \text{kg}^{-1}$

Binary variables indicating whether the edge $(x, y)$ in the connected tree is connected directly to the vertex $z$.

The heat loss cost of pipeline, $Y \cdot \text{a}^{-1}$

The heat loss cost of the $i$th pipeline of the cluster $j$, $Y \cdot \text{a}^{-1}$

The construction cost of pipeline, $Y \cdot \text{a}^{-1}$

The construction cost of the $i$th pipeline of the cluster $j$, $Y \cdot \text{a}^{-1}$

The pressure loss cost of pipeline, $Y \cdot \text{a}^{-1}$

The pressure loss cost of the $i$th pipeline of the cluster $j$, $Y \cdot \text{a}^{-1}$

The Euclidean distance between vertex $i$ and vertex $j$

The inner diameter of the insulation layer of the $i$th pipeline of the cluster $j$, m

The inner diameter of the $i$th pipeline of the cluster $j$, m

The outer diameter of the insulation layer of the $i$th pipeline of the cluster $j$, m

The outer diameter of the $i$th pipeline of the cluster $j$, m

The head loss of the $i$th pipeline of the cluster $j$, m

The annual interest rate

The length of the $i$th pipeline of the cluster $j$, m

The length of the $\beta$th pipeline between consumer $\alpha$ and the heat source in the area where this consumer is located, m

The number of clusters of the pipeline network system

The number of branch pipelines in cluster $j$

The number of consumers of the pipeline network system

The number of vertices in the connected tree

The number of vertices in the connected tree

The life cycle of the pipeline network system, a

The shaft power of the $i$th pipeline of the cluster $j$, W

The effective power of the $i$th pipeline of the cluster $j$, W

The probability of connecting the $\beta$th pipeline in the path connected between the consumer $\alpha$ and the heat source in its area

The connected probability between the consumer $\alpha$ and the heat source of the area where this consumer is located

The latent heat of steam, $kJ \cdot \text{kg}^{-1}$

The heat demand of consumer $\alpha$

The heat loss of the $i$th pipeline of the cluster $j$, $kJ \cdot \text{m}^{-1} \cdot \text{s}^{-1}$

The reliability of the pipeline network system

The heat loss of the $i$th pipeline of the cluster $j$, $m^2$

The outer surface temperature of the pipeline, °C

The ambient temperature, °C

The number of annual operating hours of the device, h
\(\mu\) The flow rate of the steam, m s\(^{-1}\) 
\(W^S\) The sum of the steam mass flow rate in all branch pipelines in the entire pipeline network 
\(W_{ij}\) The mass flow rate of the \(i\)th pipeline of the cluster \(j\), kg s\(^{-1}\) 
\(W^S_i\) The mass flow rate of steam at vertex \(z\), kg s\(^{-1}\) 
\(w^H(e)\) A weight function of the edges in a hypergraph 
\(w^H(\mathbf{T}^H)\) The weight of edge \(e\) in a spanning tree in the hypergraph 
\(w_{x,y,z}\) and \(w_{y,x,z}\) The sum of weights of the edges in a spanning tree in the hypergraph 
\(W_{t,ij}\) The mass flow rate within the branch pipeline connecting vertices \(x\) and \(y\) when using vertex \(z\) as a reference for material accountancy, kg s\(^{-1}\)

Greek letters 
\(\epsilon\) Heat transfer coefficient between the outer surface of the insulation and the atmosphere, W m\(^{-2}\) K\(^{-1}\) 
\(\zeta_{E,ij}\) The local resistance coefficient at the standard elbows (90°) 
\(\zeta_{Elbow,ij}\) The local resistance coefficient of the \(i\)th pipeline of the cluster \(j\) 
\(\zeta_{ij}\) The resistance coefficient of the \(i\)th pipeline of the cluster \(j\) 
\(\eta\) The efficiency of the conveying equipment 
\(\lambda\) The thermal conductivity of insulating material products at average temperature, W m\(^{-1}\) K\(^{-1}\) 
\(\xi_i\) The index used to determine which heat source consumer \(i\) is assigned to 
\(\rho\) The density of the steam, kg m\(^{-3}\) 
\(\sigma\) The friction coefficient of the pipeline 
\(\Phi\) The empty set

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