A schematic model for QCD. III:  
Hadronic states.

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The hadronic spectrum obtained in the framework of a QCD-inspired schematic model, is presented. The model is the extension of a previous version, whose basic degrees of freedom are constituent quarks and antiquarks, and gluons. The interaction between quarks and gluons is a phenomenological interaction and its parameters are fixed from data. The classification of the states, in terms of quark and antiquark and gluon configurations is based on symmetry considerations, and it is independent of the chosen interaction. Following this procedure, nucleon and ∆ resonances are identified, as well as various penta- and hepta-quarks states. The lowest pentaquarks state is predicted at 1.5 GeV and it has negative parity, while the lowest hepta-quarks state has positive parity and its energy is of the order of 2.5 GeV.

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I. INTRODUCTION

In previous papers [1, 2, 3], we have proposed a schematic model aimed at the description of the non-perturbative regime of QCD. The main concepts about the model can be found in [1], together with the applications to the meson spectrum of QCD. Along this line, Ref. [2] deals with the appearance of phase transitions and condensates. A preliminary set of results, about the energy and parity of systems of the type $q\bar{q}^3$ (pentaquarks), and $q^3\bar{q}\bar{q}$ (heptaquarks), was presented in [3].

As it was discussed in [1], this model describes reasonable well the main features of the meson spectrum of QCD. This is a nice result since it shows that, also in the conditions of low energy QCD, e.g.; large coupling constants and non-conservation of particle number, the use of simple models may be very useful. The model of [1] belongs to the class of models [4] which may be solved by algebraic, group theory and symmetry-enforcing techniques, and that describe the interaction between fermions and bosons. Some examples of this class of models can be found in [5], [6], and [7]. Concerning the use of symmetry principles, the work of [8] shows how the ordering of levels of many gluon states may be understood. In [8], the predictive power of the model, based on the microscopical treatment of gluons configurations, was tested by means of the spectrum related to exotic quantum numbers.

The purpose of this work is to extend the model of Ref. [1] to describe the main structure of the hadronic spectrum of QCD. The essentials of the model are the following:

a)color, flavor and spin degrees of freedom are taken explicitly into account to built the configurations, both in the quark and gluon sectors of the model;

b)the quarks and antiquarks are placed in a s-state;

c)the interaction of the quarks with the gluons proceeds via gluon pairs coupled to spin and color zero, only. Other possible gluon states are considered as spectators.

The present work is organized as follows. Section II gives a brief description of the model. We shall avoid as much as possible the repetition of details which can be found in the already published papers [1, 2, 3], and concentrate in the aspects which are relevant for the description of the hadronic spectrum. In Section III we discuss the classification of many quark-antiquark states, and the classification of many gluon states [8]. In Section IV we show and discuss the hadronic spectrum predicted by the model, as a natural continuation of the study presented in [3], where the meson sector was analyzed. Conclusions are drawn in Section V.

II. THE MODEL

The basic degrees of freedom of the model are illustrated in the level scheme shown in Figure 1. The quarks occupy two levels, one at negative and the other at positive energy. Antiquarks are depicted as holes in the lower level. If
with \( c^\dagger_{\alpha i} \) and \( c^\alpha_i \) we denote the creation and annihilation operators of quarks, in the upper \((i = 1)\) and lower \((i = 2)\) level, the relation to quark (antiquark) creation and annihilation operators is given by

\[
\begin{align*}
\alpha^\dagger_\alpha &= c^\dagger_{\alpha 1}, \\
\alpha_\alpha &= c_{\alpha 2},
\end{align*}
\]

where the index \( \alpha \) refers to color, flavor and spin quantum numbers. Since three degrees of color, three degrees of flavor (only u, d and s quarks are taken into account) and two degrees of spin are considered, each level is 18-fold degenerate.

The energy of each level is approximated to one third of the nucleon mass and it represents a constituent (effective) mass of the quarks. In this picture, the mass comes mainly from the kinetic energy because quarks are confined to a sphere of radius 1fm. The model describes quarks and antiquarks distributed in an orbital s-level. Orbital excitations can also be accounted for by increasing the number of degrees of freedom. To this system of quarks we add a boson level, which represents pairs of gluons of color and spin zero. Its position is fixed as it was done in Ref. [8]. The coupling between quarks and antiquarks and gluons proceeds via this gluon-pair configurations. Other couplings to different pairs of gluons are neglected, because their contributions, at least for the meson spectrum, were found to be negligible [1]. Nevertheless, gluons of other types are still present in the model but they are considered as spectators.

The basic excitations in the fermion sector are \( q\bar{q} \) pairs, which are described by \( B^{\dagger}_{\lambda,SM} \) and \( B^{\lambda,SM} \) pair operators [1], where \( \lambda = 0,1 \) stands for flavor states \((0,0)\) and \((1,1)\), and the spin \( S \) can be 0 or 1. These pair of fermions, a sub-class of bi-fermion operators [6, 7], can be mapped onto a boson space [9]. We denote the corresponding operators by \( b^{\dagger}_{\lambda,SM} \).

This is the basic structure of the model presented previously [1]. In order to extend it to the description of baryons, and because we work in the boson space of quark-antiquark pairs, one should add three valence quarks via the operators \([D^{(\lambda_0,\mu_0)S_0} \times a^\dagger_{(\lambda,\mu)}]_{f,ms}\), where two ideal quark operators \(a^\dagger_\alpha\) [8] are coupled as a di-quark \(D^{(\lambda_0,\mu_0)S_0} = [a^\dagger \times a^\dagger]_{f_0,m_0}\) with flavor, spin \((\lambda_0, \mu_0)S_0\) either \((0,1)0\) or \((2,0)1\).

The model Hamiltonian is written as in Ref. [1], with the inclusion of new terms which are needed for the description of baryons, namely:

\[
H = 2\omega f \mathbf{p}_f + \omega_b \mathbf{p}_b + ...
\]

FIG. 1: Schematic representation of the model space. The fermion levels are indicated by their energies \( \pm \omega_f \). The gluon-pairs are represented by the level at the energy \( \omega_b \).

[1] [2] [3] [4] [5] [6] [7] [8] [9]
were adjusted to the mesonic spectrum and their values are:

\[
\sum_{\lambda S} V_{\lambda S} \left\{ \left( (b_{\lambda S}^\dagger)^2 + 2 b_{\lambda S}^\dagger b_{\lambda S} + (b_{\lambda S})^2 \right) (1 - \frac{n_f}{2\Omega})b^+ \right\}
\]

\[b^1(1 - \frac{n_f}{2\Omega}) \left( (b_{\lambda S}^\dagger)^2 + 2 b_{\lambda S}^\dagger b_{\lambda S} + (b_{\lambda S})^2 \right) \right\} +
\]

\[n_{(0,1)} \left( D_1 n_b + D_2 (b^1 + b) \right) + n_{(2,0)} \left( E_1 n_b + E_2 (b^1 + b) \right) \right),
\]

where \((b_{\lambda S}^\dagger)^2 = (b_{\lambda S}^\dagger \cdot b_{\lambda S})\) is a short hand notation for the scalar product. The factor \((1 - \frac{n_f}{2\Omega})\) simulates the effect of the terms which would appear in the exact boson mapping of the quark-antiquark pairs. The operators \(b^1\) and \(b\) are boson creation and annihilation operators of the gluon pairs with spin and color zero.

The interaction describes scattering and vacuum fluctuation terms of fermion and gluon pairs. The strength \(V_{\lambda S}\) is the same for each allowed value of \(\lambda\) and \(S\), due to symmetry reasons, as shown in \([1]\). These parameters were determined in \([1]\) and they are taken as fixed values in the present calculations. The matrix elements of the meson part of the Hamiltonian are calculated in a seniority basis. For details, please see Ref. \([1]\). The operators \(n_{(\lambda_0,\mu_0)S_0}\) are number operators of the di-quarks coupled to flavor-spin \((\lambda_0, \mu_0)S_0\). The interaction does not contain terms which connect states with different hypercharge and isospin. It does not contain flavor mixing terms, either. Therefore, in the comparison with data, the energies should be corrected by the mass formula given in Ref. \([10]\), with parameters deduced as in \([11]\) and turning off flavor mixing terms. The procedure is explained in details in \([1]\).

The parameters \(D_k\) are adjusted to the nucleon resonances, while the parameters \(E_k\) are adjusted to the \(\Delta\) resonances. The two last terms in (2) are needed to describe the interaction between the valence quarks and the meson-like states. The parameters \(\omega_f\) and \(\omega_v\) were fixed at 0.33 GeV and 1.6 GeV, respectively. The parameters \(V_{\lambda S}\) were adjusted to the mesonic spectrum and their values are: \(V_{00}=0.0337\ \text{GeV},\ V_{01}=0.0422\ \text{GeV},\ V_{10}=0.1573\ \text{GeV}\) and \(V_{11}=0.0177\ \text{GeV}\), respectively. This shows the strong effects of the \(V_{10}\) interaction, which acts on pairs coupled to flavor \((1,1)\) and spin \(0\) (pion-like states). The parameters \(D_k\) and \(E_k\) \((k=1,2)\) are adjusted to the nucleon resonances and \(\Delta\) resonances respectively. Their values are \(D_1=-1.442\ \text{GeV},\ D_2=-0.439\ \text{GeV},\ E_1=-1.187\ \text{GeV}\) and \(E_2=-0.362\ \text{GeV}\).

### III. THE MODEL SPACE

The complete classification of the many quark-antiquark system was given in Ref. \([1]\). Here we will summarize it and discuss the aspects which are relevant for the present calculations. The main idea about the classification of the many quark-antiquark system is to treat quarks in the two level system, as depicted in Fig. 1. The ground state (vacuum), for the case of no interaction, is given by 18 quarks occupying the lowest level. A \(n\) particle-hole excitation of this ground state represents \(n\) physical quarks and antiquarks. Thus, baryons are represented by three physical quarks in the upper level. Particles with a structure of the form \(q^n(q\bar{q})^n\) are then described by three valence quarks plus \(n\) particle-hole excitations. The group chain, describing these states, is given by

\[
\begin{align*}
[1^N] & \cup \ C & \cup \ [h^T] \\
U(4\Omega) & \cup U(\frac{1}{2}) & \cup U(12) \\
\cup & (\lambda_C, \mu_C) SU(3) & \cup \ U(4) \otimes SU(3) (\lambda_f, \mu_f) \\
\cup & SU(2) S, M & \\
\end{align*}
\]

\((\Omega = 9)\) is the degeneracy corresponding to 3 degrees of freedom in the color sector and three degrees of freedom in flavor \([1]\), where the irreducible representation (irrep) of \(U(4\Omega)\) is the completely antisymmetric one and \(N\) is the number of particles involved. Similar group chains can be constructed for larger values of \(\Omega\). The upper index in \([h^T]\) refers to the transposed Young diagram of \([h]\), where the columns and rows are interchanged \([13]\). Due to the antisymmetric irreps \([1\Omega]\) of \(U(4\Omega)\) the irreps of \(U(\Omega/3)\) and \(U(12)\) are complementary and the irrep of \(U(\Omega/3)\), which for \(\Omega = 9\) is the color group, has maximally three rows \([13]\). In the group chain \([3]\) no multiplicity labels are indicated. There is a multiplicity \(\rho_f\) for \((\lambda_f, \mu_f)\) and \(\rho_S\) for the spin \(S\). The color labels \((\lambda_C, \mu_C)\) are related to the \(h_i\) via \(\lambda_C = h_1 - h_2\) and \(\mu_C = h_2 - h_3\). The irrep of the \(U(4)\) group is given by a Young diagram with four rows, i.e. \([v_1, v_2, v_3, v_4]\). The complete state is given by

\[
[N, [v_1 v_2 v_3 v_4], (\lambda_C, \mu_C), \rho_f(\lambda_f, \mu_f) Y T T_z, \rho_S S M] > ,
\]

where \(Y\) is the hypercharge, \(T\) is the isospin and \(T_z\) its third component. For meson-like states, the color quantum numbers to be considered are \((\lambda_C, \mu_C) = (0,0)\).
multidimensional harmonic oscillators [1]. For each harmonic oscillator the basis states are given by
while the irrep $S$ configurations with since all of these contain at least two quark-antiquark pairs. The irreps $(3,0)$ and $(0,3)$ have total spin 1 while the
To extend the meson-like configurations discussed in Ref. [1], we shall include the flavor irreps $(3,0)$, $(0,3)$ and $(2,2)$, as indicated in (3). For $\Omega = 9$ and color zero the meson irrep of $U(12)$ is given by $[\bar{3}^60^6]$ and for baryons it is $[3^70^9]$.

As an example, in Table I we list the partial $U(4) \times U_f(3)$ content for meson states of the $U(12)$ irrep $[\bar{3}^60^6]$ and in Table II the partial content of the $U(4)$ irreps listed in Table I. The irreps which are shown contain up to two quark-antiquarks pairs, only [23]. The $U(4)$ irreps have in general four rows while the $U_f(3)$ of the flavor group have three rows $[p_1 p_2 p_3]$. The adopted notation for flavor is $(\lambda, \mu)$ where $\lambda = p_1 - p_2$ and $\mu = p_2 - p_3$. The minimal number of quarks in an $U(4)$ irrep is given by the sum of the two last rows of the $U(4)$ Young diagram while the number of antiquarks is given by the difference between $2\Omega = 18$ and the sum of the two first rows of the $U(4)$ Young diagram. In the spin representation for quarks $|q_1 q_2 \rangle$, the sum $n_q = q_1 + q_2$ gives the number of quarks and $S_\bar{q} = (q_1 - q_2)/2$ is the spin carried by the quarks. Similarly, for antiquarks $|\bar{q}_1 \bar{q}_2 \rangle$ $S_q = (q_1 - q_2)/2$ and $2\Omega - n_q = \bar{q}_1 + \bar{q}_2$. This is related to the fact that the unperturbed ground state, of the two level model space, contains 18 quarks, as it was defined before. The total spin $S$ of the system can be obtained via the condition $|S_q - S_\bar{q}| \leq S \leq S_q + S_\bar{q}$. In Tables III and IV the classification of the baryon states is partly given. In Table III only $U(4) \otimes U_f(3)$ states are considered which contain five quarks and two antiquarks. In Table IV only lowest states which contain unusual flavor are listed. The $U(12)$ irrep is given by $[3^70^9]$.

To extend the meson-like configurations discussed in Ref. [3], we shall include the flavor irreps $(3,0)$, $(0,3)$ and $(2,2)$, since all of these contain at least two quark-antiquark pairs. The irreps $(3,0)$ and $(0,3)$ have total spin 1 while the irrep $(2,2)$ may have total spin values 0, 1, and 2. Within our boson representation of the quark-antiquark pairs, the $(3,0)$ and $(0,3)$ irreps with spin 1 can be obtained via the coupling scheme (e.g. for $(3,0)$)$[b^\dagger_{(1,1)0} \times b^\dagger_{(1,1)1}]_{f_{0}m_{0}=1}^{(3,0)} |0 >$
while the irrep $(2,2)S = 0, 1, 2$ comes from $[b^\dagger_{(1,1)1} \times b^\dagger_{(1,1)1}]_{f_{0}m_{0}=S}^{(2,2)S=0} |0 >$.
$[b^\dagger_{(1,1)0} \times b^\dagger_{(1,1)0}]_{f_{0}m_{0}=S}^{(2,2)S=1} |0 >$, where $f_0$ denotes the flavor state with maximum weight.

In the boson representation the states are given by the direct product of the eigenstates of one-, three-, eight- and 24-dimensional harmonic oscillators [1]. For each harmonic oscillator the basis states are given by

$$N_{N_{p},m_{p}}(b^\dagger_{p};b^\dagger_{p})_{f_{m_{0}}=0}^{N_{p}S_{0}} |0 >.$$  (5)
where $N_{\lambda S}$ is the number of bosons of type $[\lambda, S]$, $\nu_{\lambda S}$ is the seniority and $N'_{\lambda S\nu_{\lambda S}}$ is a normalization constant. The seniority is the number of uncoupled $b_{\lambda S}$-bosons. The quantity $\alpha_{\lambda S}$ contains all other quantum numbers needed to specify a particular harmonic oscillator. The state (5) can be viewed as the superposition of a certain number of quark-antiquark pairs coupled to a given flavor-spin combination. This number of quark-antiquark pairs will be used to denote non-trivial configurations.

The Hamiltonian changes only the number of trivial boson pairs (of the type $(b_{\lambda S}^\dagger \cdot b_{\lambda S}^\dagger)$), and it leaves the seniority invariant. The number of these boson pairs is not conserved, i.e. a general state (5) contains a given seniority, which

| $U(4)$ | $[q_1 q_2]$ | $n_q$ | $S_q$ | $[q_1 q_2]$ | $n_q$ | $S_q$ | $S$ |
|--------|-------------|-------|-------|-------------|-------|-------|-----|
| [8811] | [11]        | 2     | 0     | [88]        | 2     | 0     | 0   |
| [9711] | [11]        | 2     | 0     | [97]        | 2     | 1     | 1   |
| [8820] | [20]        | 2     | 1     | [88]        | 2     | 0     | 1   |
| [9720] | [20]        | 2     | 1     | [97]        | 2     | 1     | 0, 1, 2 |
| [9810] | [10]        | 1     | $\frac{1}{2}$ | [98] | 1 | $\frac{1}{2}$ | 0, 1 |
| [9810] | [11]        | 2     | 0     | [97]        | 2     | 1     | 1   |
| [9810] | [11]        | 2     | 0     | [88]        | 2     | 0     | 0   |
| [9810] | [20]        | 2     | 1     | [97]        | 2     | 1     | 0, 1, 2 |
| [9900] | [00]        | 0     | 0     | [99]        | 0     | 0     | 0   |
| [9900] | [10]        | 1     | $\frac{1}{2}$ | [98] | 1 | $\frac{1}{2}$ | 0, 1 |
| [9900] | [20]        | 2     | 1     | [97]        | 2     | 1     | 0, 1, 2 |

| $U(4)$ | $U_f(3)$ | $(\lambda, \mu)$ | mult |
|--------|----------|-------------------|------|
| [8832] | [1074]   | (3,3)             | 1    |
| [8841] | [1074]   | (3,3)             | 1    |
| [9732] | [1074]   | (3,3)             | 1    |
| [9822] | [777]    | (0,0)             | 1    |
| [9822] | [876]    | (1,1)             | 1    |
| [9822] | [966]    | (3,0)             | 1    |
| [9822] | [975]    | (1,1)             | 1    |
| [9831] | [777]    | (0,0)             | 1    |
| [9831] | [876]    | (1,1)             | 2    |
| [9831] | [966]    | (3,0)             | 1    |
| [9831] | [885]    | (0,3)             | 1    |
| [9831] | [975]    | (2,2)             | 1    |
| [9831] | [1065]   | (4,1)             | 1    |
| [8850] | [777]    | (0,0)             | 1    |
| [8850] | [876]    | (1,1)             | 1    |
| [8850] | [885]    | (0,3)             | 1    |
| [8850] | [975]    | (2,2)             | 1    |
| [9840] | [876]    | (1,1)             | 1    |
| [9840] | [975]    | (2,2)             | 1    |
| [9821] | [876]    | (1,1)             | 1    |
| [9930] | [966]    | (3,0)             | 1    |

TABLE II: Quark-antiquark content of some $U(4)$ meson irreps. The table shows the irreps which contain at most two quarks and two antiquarks, only.

TABLE III: Partial $U(4) \times U_f(3)$ content of the $U(12)$ irrep $[3^7 0^5]$ (color zero, baryon like states). The flavor irreps $(0,0)$, $(1,1)$, $(3,3)$, $(2,2)$, $(4,1)$ describe the flavor singlet, octet, decuplet, anti-decuplet, 27-plet, and 35-plet, respectively. The multiplicity of the $U(4) \times U_f(3)$ content in the $U(1)$ irrep $[3^7 0^5]$ is given in the last column. The first three rows give some higher lying $U(4)$ irreps which are associated with a flavor irrep $(3,3)$. The double line separates some high lying irreps from the low lying ones.
interaction have to be added. Otherwise, the parameters of the Hamiltonian should be adjusted to reproduce the

concerning baryon states. Since the Hamiltonian conserves flavor, spin and parity, all states belonging to the same

angular momentum, \( \pi \) and maximally six effective gluons. The lowest glue-ball state is the first excited

and the model Hamiltonian used is discussed in Ref. [8]. In Table V we show all states of glue-balls with color zero

In the model [8] the basic building blocks are effective gluons with color (1,1) and spin 1. The detailed group structure

degeneration of different flavor-spin irreps. As an example, if the configuration \( (1 \) which the three valence quarks can be coupled, to describe nucleon and \( \Delta \) resonances, respectively. This implies a

spectrum obtained for a given flavor-spin combination and a given seniority combination will be identical to another

one with a different spin-flavor combination but with the same seniority distribution. This leads to the appearance

of a degeneration in the spectrum.

A similar classification can be given for the baryon states. Within the boson representation, the ideal valence quarks are added to describe baryons. One has to couple the meson-like states with the states \( (1 \), \( 0 \)) or with \( (3 \), \( 0 \)) to which the three valence quarks can be coupled, to describe nucleon and \( \Delta \) resonances, respectively. This implies a degeneration of different flavor-spin irreps. As an example, if the configuration \( (1 \), \( 1 \)\) of the three valence quarks is coupled to a meson irrep \( (1 \), \( 1 \)\) then the resulting allowed flavor-spin irreps are \( (0 \), \( 0 \)) \( (1 \), \( 1 \)) \( (3 \), \( 0 \)) \( (0 \), \( 3 \)) \( (2 \), \( 2 \)) and \( (2 \), \( 2 \))

Another ingredient of the model is the non-conservation of the particle number (quarks, antiquarks and gluons). This is a fundamental property of a relativistic theory with large interaction constants. It means that physical particles, in general, do not contain a fixed number of quarks, antiquarks and/or gluons but rather an average number of them. In this picture, the nucleons will not contain, mainly, three valence quarks but also an average number of quark-antiquark and gluon pairs. The mixing of particle number turned out to be essential in order to remove the multiplicity of states at low energy, a feature which any constituent quark model exhibits when the number of quarks, antiquarks and gluons is fixed.

Herewith we summarize, for completeness, the configurations which may appear in the gluon sector of the theory. In the model the basic building blocks are effective gluons with color (1,1) and spin 1. The detailed group structure and the model Hamiltonian used is discussed in Ref. [8]. In Table V we show all states of glue-balls with color zero and maximally six effective gluons. The lowest glue-ball state is the first excited \( J^{PC} = 0^{++} \) state, where \( J \) is the angular momentum, \( \pi \) is the parity and \( C \) is the charge conjugation. For details, see Ref. [8].

### IV. APPLICATIONS

In this section we shall show and discuss the results of the model introduced in the previous section, particularly, concerning baryon states. Since the Hamiltonian conserves flavor, spin and parity, all states belonging to the same flavor irrep are degenerate. In order to introduce a splitting in the isospin and hypercharge the Gel’man-Okubo interaction [14] have to be added. Otherwise, the parameters of the Hamiltonian should be adjusted to reproduce the

| \( U(4) \) | \( q_1 \) | \( q_2 \) | \( m \) | \( S \) | \( q_1 \) | \( q_2 \) | \( S \) |
|-------|-------|-------|-------|-------|-------|-------|-------|
| [8832] | [32]  | 5     | 1     | [88]  | 2     | 0     | \( \frac{1}{2} \) |
| [8841] | [41]  | 5     | 1     | [88]  | 2     | 0     | \( \frac{1}{2} \) |
| [9732] | [32]  | 5     | 1     | [97]  | 2     | 1     | \( \frac{1}{2} \) |
| [9831] | [41]  | 5     | 1     | [97]  | 2     | 1     | \( \frac{1}{2} \) |
| [8850] | [50]  | 5     | 1     | [88]  | 2     | 0     | \( \frac{1}{2} \) |
| [9840] | [50]  | 5     | 1     | [97]  | 2     | 1     | \( \frac{1}{2} \) |
| [9840] | [41]  | 5     | 1     | [98]  | 1     | 0     | \( \frac{1}{2} \) |
| [9840] | [31]  | 4     | 1     | [98]  | 1     | 0     | \( \frac{1}{2} \) |
| [9921] | [40]  | 4     | 1     | [98]  | 1     | 0     | \( \frac{1}{2} \) |
| [9930] | [30]  | 3     | 1     | [0]   | 0     | 0     | \( \frac{1}{2} \) |

**TABLE IV:** Quark-antiquark content of some \( U(4) \) baryon irreps. Only those irreps are shown which contain either the nucleon and \( \Delta \) resonances (last two rows) or penta- and heptaquark states with unusual flavor. There are more irreps at lower energy but with a flavor which can also be reached by a three quark system. The first three rows shows the content of those irreps which are associated with flavor irreps \( (3,3) \) (see first three rows in Table III). The double line separates some high lying irreps from the low lying ones.
TABLE V: The classification of many-gluon states, with color (0,0). The multiplicity of the configuration (0,0) is one up to six gluons. For the notation of the groups, which appear in the many-gluons states, and their irreps see Ref. [8], where the general classification, i.e. with open color, can be deduced. $P$ and $C$ refer to the parity and charge conjugation of the gluon states respectively.

| $U(8)\ (U(3))$ | $O(8)$ | $SO(3)$ | $(J)$ | $P$ | $C$ |
|----------------|--------|---------|-------|-----|-----|
| $[2]$          | (0000) | 0,2     | +1    | +1  |     |
| $[4]$          | (0000) | 0,2,4   | +1    | +1  |     |
| $[2^2]$        | (0000) | 0       | +1    | +1  |     |
| $[6]$          | (0000) | 0,2,4,6 | +1    | +1  |     |
| $[42]$         | (0000) | 0,2,3,4 | +1    | +1  |     |
| $[2^3]$        | (0000) | 0       | +1    | +1  |     |
| $[3]$          | (3000) | 1,3     | -1    | -1  |     |
| $[5]$          | (3000) | 1,3,5   | -1    | -1  |     |
| $[41]$         | (3000) | 1,2,3,4 | -1    | -1  |     |
| $[32]$         | (3000) | 1,2,3   | -1    | -1  |     |
| $[1^3]$        | (1110) | 0       | -1    | +1  |     |
| $[31^2]$       | (1110) | 0,2     | -1    | +1  |     |
| $[21^2]$       | (2110) | 1       | +1    | -1  |     |
| $[41^2]$       | (2110) | 1,3     | +1    | -1  |     |
| $[321]$        | (2110) | 1,2     | +1    | -1  |     |
| $[2^2]$        | (2200) | 0,2     | +1    | +1  |     |
| $[42]$         | (2200) | 0,2,3,4 | +1    | +1  |     |
| $[321]$        | (2200) | 1,2     | +1    | +1  |     |
| $[2^1]$        | (2200) | 0       | +1    | +1  |     |
| $[3^1]$        | (2200) | 0       | -1    | -1  |     |
| $[6]$          | (6000) | 0,2,4,6 | +1    | +1  |     |
| $[42]$         | (4200) | 0,2,3,4 | +1    | +1  |     |
| $[41^2]$       | (4110) | 1,3     | +1    | -1  |     |
| $[3^2]$        | (3300) | 1,3     | +1    | -1  |     |
| $[2^2]$        | (2220) | 0       | +1    | +1  |     |

corrected experimental masses. This is the procedure which we have adopted in the calculations. Next Subsection IV A is devoted to the results concerning mesonic states, others than the ones reported in [1], like those with flavor (2,2). These states illustrate the role of some additional degeneracies of the model, and their possible physical implications. In Subsection IV B we present the results for nucleon resonances, as well as some results about more exotic states, like the predicted energy and parity of the lowest penta- and hepta-quarks states. Finally, in Subsection IV C we analyze the case of $\Delta$ resonances. The experimental data are taken from Ref. [15].

A. Meson States

In Table VI we list the quark-antiquark and gluon contents of some selected mesonic states. The quantity $n_{\lambda S}$ is the average number of $q\bar{q}$ pairs coupled to flavor $(\lambda, \mu)$ and spin $S$. The quantity $n_g$ is the average number of gluon pairs coupled to color singlet and spin zero. The seniority content is given by the value $(\nu) = (v_{00}, v_{01}, v_{10}, v_{11})$, where $v_{\lambda S}$ refers to the seniority in the channel $(\lambda, \lambda)S$. Some of the states shown in this table belongs to flavor irreps which contain exotic combinations of hypercharge and isospin which are not allowed in the simplest $q\bar{q}$ system. From the seniority content one sees that these states contain at least two $q\bar{q}$ pairs.

While the states with flavor (3,0) and (0,3) have energies of the order of 1.5 GeV, the one with flavor (2,2) has a very low energy, nearly the same energy of the $f_0(980)$ state. Experimentally, the $f_0$ state lies at approximately twice the energy of the $\eta(541)$ state. Their dominant decay into $K^+K^-$ leads to the interpretation of these states as $K^+K^-$ molecules. Within our model the (2,2)0+ state is built upon the seniority $\nu = 2$ state $|b_{2(1,1)}^1 \times b_{2(1,1)}^1 \rangle^{(2,2)0}|0\rangle$. 


quarks, 2 antiquarks ($q\bar{q}$) be difficult to obtain in other models, except in the constituent quark model of Ref. [17]. In average there are two
theoretical Roper resonance comes near to the experimental energy of 1.44 GeV. This is a nice result, which may
implies a 59% quark content and a 41% gluon content, a result which is expected from previous evidences. The
nucleon. The number of gluon pairs is approximately 1.4, i.e. in average there are nearly three gluons present. This
pairs is about 0.5, i.e. including the three valence quarks there are in average 3.5 quarks and 0.5 antiquarks in the
considered. Like for the case of mesonic states, the Hamiltonian commutes with spin, parity, color and flavor. All
parameters given in [11], instead.

In order to remove the degeneracy one may still add the
states belonging to the same flavor irrep are degenerate. In order to remove the degeneracy one may still add the
$|\lambda_f, \mu_f, \sigma_f\rangle = (v_{00} v_{01} v_{011})$, expectation value of the boson pairs in the channels (0,0) $0^−$, (0,0) $1^−$, (1,1) $0^−$ and (1,1) $1^−$ and the total number of gluon pairs ($n_g$) with spin 0. In the
last three rows some states, not reported in [1], are listed which contain exotic combinations of hypercharge and isospin. The experimental data are taken from [15]. The $X(797)$ is interpreted as a molecular state. Note that, for the particles in the first (0,0), (1,1) $0^−$ and (0,0), (1,1) $1^−$ irreps, we are listing the value of the masses without flavor mixing (they are marked by an asterisk, see also Ref. [1] for further explanation).

The highest weight state is given by $b^1_{(1,1), f_0, 0} b^1_{(1,1), f_0, 0} |0\rangle$, which is nothing but the direct product of two pairs of the type $(1,1)0^−$. This can be interpreted, within the present model, as a configuration of two $q\bar{q}$-like mesons. The difference between the mass of the two pairs and the mass of the $(2,2)0^+$ state is about 0.4 GeV, which is larger than the observed value. One has to keep in mind that the present model is rather schematic and that it does not include a flavor mixing interaction which could mix $(1,1)0^−$ states with $(2,2)0^+$ states, and may induce a coupling from, e.g., the $f_0$ state to molecular states. The model predicts also states of the type $(q\bar{q})^2$ which can be coupled to flavor irreps $(3,0)$ and $(0,3)$ (see Tables [II] and [III]). The distribution of seniorities is $(0011)$ and the states lie at about 1.44 GeV. The irreps $(3,0)$ and $(0,3)$ contain isospin-hypercharge configurations which cannot be reached by a single quark-antiquark pair.

### B. Nuclear resonances, penta- and hepta-quarks

In Table [VII] we show the results for some selected nucleon resonances, as they appear in the schematic model. The content of $q\bar{q}$ pairs of the type $(0,0)0^−$ and $(0,0)1^−$ are not listed because they are always small for the cases considered. Like for the case of mesonic states, the Hamiltonian commutes with spin, parity, color and flavor. All states belonging to the same flavor irrep are degenerate. In order to remove the degeneracy one may still add the Gel’man-Okubo term plus a term describing the flavor dependence. We follow the prescription of Ref. [10] with the parameters given in [11], instead.

The quark, antiquark and gluon contents of the nucleon are given in the first row of Table [VII]. The number of $q\bar{q}$ pairs is about 0.5, i.e. including the three valence quarks there are in average 3.5 quarks and 0.5 antiquarks in the nucleon. The number of gluon pairs is approximately 1.4, i.e. in average there are nearly three gluons present. This implies a 59% quark content and a 41% gluon content, a result which is expected from previous evidences. The theoretical Roper resonance comes near to the experimental energy of 1.44 GeV. This is a nice result, which may be difficult to obtain in other models, except in the constituent quark model of Ref. [17]. In average there are two $q\bar{q}$ pairs and 1.9 gluon pairs. Thus, the Roper resonance, in the present model, contains an average number of 5 quarks, 2 antiquarks $(q^2(q\bar{q})^2)$ and 3.8 gluons. This implies a 65% content of quarks and a 35% content of gluons.
TABLE VII: Particle content for selected baryon states. In columns we indicate the theoretical energy $E_{\text{theo}}$, the flavor $(\lambda_m, \mu_m)$, spin $J_m$ and parity $\pi$ for the meson part, the final flavor irrep $(\lambda, \mu)$ and spin $J$ and parity, the seniority content $(\nu)$ expectation value of the boson pairs in the channels $(1,1)^0$, (1,1) $^\pm$, (1,1) $^-$. The first four rows some additional particles are listed which contain unusual combinations of hypercharge and isospin. The $\Theta^+(1540)$ is the reported pentaquarks state. Another pentaquarks state, according to our notation, is called $X$ while $H_6$ $(k=1:2)$ refers to the lowest heptaquarks states with unusual flavor.

The quark-gluon content of the Roper resonance differs from that of the nucleons. It has more particles, a fact which is reflected in its larger collective nature and low energy. The first negative parity resonance appears at about 1.5 GeV, with $(1,1)^{-}$ and the next ones at 1.79 GeV, with degenerate states with $J^* = \frac{1}{2}^-$ and $\frac{3}{2}^-$. The first state contains a large amount of $q\bar{q}$ pairs of the type $(1,1)^0$ while the last states are nearly pure one $q\bar{q}g^2$ states. These would be good candidates for hybrid baryons. In Table VII states with flavor (0,3) and (2,2) are also listed. In fact, these are the same states, concerning their meson content. They are coupled with the three ideal valence quarks, leading to unusual combinations of hypercharge and isospin. These configurations may be associated to pentaquarks states, like $\Theta^+(1540)$, whose position have been predicted in [19]. Other states of the pentaquarks type may exist, which may have in common the same quantum numbers in hypercharge and isospin as other nucleon resonances, and, therefore, may be difficult to identify experimentally. The interpretation of these states as pentaquarks is based on the value of the seniority of the $q\bar{q}$ pair of the type $(1,1)^0$ $(\nu = 1)$ to which the three ideal valence quarks have to be added. In [20] we present a compilation of results about pentaquarks states, as they are predicted by our model. Within the model, the lowest pentaquarks has negative parity in accordance with QCD sum-rules and lattice gauge calculations [21, 22, 23]. If the orbital spin $L$ is included, pentaquarks states with positive parity may exist with $L=1$. However, these states include an orbital excitation and should have higher energies.

The model contains also heptaquarks, which are characterized by two $q\bar{q}$ pairs added to the three ideal valence quarks. The lowest one with unusual flavor, which cannot be described by a plain three-quark system, is at about 2.5 GeV $(H_1(2541))$ (see Table VII). It has a content of 2.87 $q\bar{q}$ pairs of the type $(1,1)^0$ $(\nu_{10})$coupled with three ideal valence quarks to the unusual flavor irrep of the type $(3,0)$ with spin-parity $\frac{1}{2}^-$. This implies a quark content of 78% and a gluon content of 22%. There are several other heptaquarks states near the same energy, or at slightly higher energies, which contain one $q\bar{q}$ pair of the type $(1,1)^0$ and one of the type $(1,1)^-)$. Also their gluon content is higher by about one extra gluon. Within our model, the parity of the heptaquarks state is positive. Because we are working in a boson space, we should be careful about the appearance of unphysical states [24]. For example, the three valence quarks can be coupled with the meson background with flavor $(2,2)$, e.g., to the flavor irreps $(3,3)$, $(4,1)$ and $(1,4)$. However, only the $(4,1)$ irrep appears in the list of allowed states related to low lying $U(4)$ irreps (see Tables III and IV). The others are, therefore, unphysical. There appear $(3,3)$ irreps at higher configurations of $U(4)$ irreps, though, with spins $S = \frac{1}{2}$ and $\frac{3}{2}$. This shows the importance of the complete classification, because only a comparison of the states in the boson space with the list of irreps in the fermion space gives the possible allowed states.

C. $\Delta$ Resonances

In a similar manner, as it was discussed in the section of nucleon resonances, we can treat $\Delta$ resonances. In Table VIII we list the results of our calculations.
TABLE VIII: Particle content for selected Δ states. In columns we indicate the theoretical energy $E_{\text{theo}}$, the flavor ($\lambda_m, \mu_m$), spin $J_m$ and parity $\pi_m$ for the meson part, the final flavor irrep ($\lambda, \mu$) and spin $J$ and parity, the seniority content ($v_{00}, v_{01}, v_{10}, v_{11}$), expectation value of the boson pairs in the channels (1,1) 0− ($n_{10}$) and (1,1) 1− ($n_{11}$) and the total number of gluon pairs ($n_g$) with spin 0. In the last two rows some additional particles are listed which contain unusual combinations of hypercharge and isospin. The $X$ and $H$ states are predicted particles with exotic hypercharge-isospin combinations. The state in the last row can be interpreted as a heptaquarks state. The asterix indicates that the particular state is obtained via a combination with three valence quarks coupled to (1,1) $\frac{3}{2}^+$. 

\begin{table}[h]
\centering
\begin{tabular}{|c|c|c|c|c|c|c|c|c|}
\hline
particle & $E_{\text{theo}}$ & $J_m^\pi$ & $J_m^\pi$ & ($\lambda, \mu$) & $J^\pi$ & ($v$) & $n_{10}$ & $n_{11}$ & $n_g$ \\
\hline
$\Delta$(1232) & 1.248 & (0,0) & 0+ & (3.0) $\frac{3}{2}^+$ & (0000) & 0.33 & 0.03 & 0.67 \\
$\Delta$(1600) & 1.57 & (0,0) & 0+ & (3.0) $\frac{1}{2}^+$ & (0000) & 1.93 & 0.04 & 1.60 \\
$\Delta$(1620)* & 1.51 & (1,1) & 0− & (3.0) $\frac{1}{2}^−$ & (0010) & 3.14 & 0.03 & 2.81 \\
$\Delta$(1700)* & 1.79 & (1,1) & 1− & (3.0), $\frac{1}{2}^+, \frac{3}{2}^−$ & (0001) & 0.38 & 1.00 & 1.04 \\
$\Delta$(1750)* & 2.49 & (1,1) & 0− & (3.0) $\frac{3}{2}^+$ & (1010) & 2.85 & 0.03 & 2.19 \\
$X$(1640) & 1.64 & (1,1) & 0− & (4.1), (2.2), $\frac{3}{2}^−$ & (0010) & 3.00 & 0.03 & 2.18 \\
$H$(2530) & 2.53 & (2,2) & 0+ & (4.1) $\frac{3}{2}^+$ & (0020) & 2.67 & 0.02 & 0.97 \\
\hline
\end{tabular}
\end{table}

FIG. 2: Nucleon resonances (first group of levels), $\Delta$ resonances (second group), pentaquarks (third group) and heptaquarks (fourth group). On the right side of each level are given the assigned spin and parity ($J^\pi$), and the total quark and antiquark ($n_q + n_{\bar{q}}$) and gluon ($n_g$) contents (see the text) The list is not complete. Only some selected states, of physical interest, are listed.

To conclude with this Section, we show in Figure 2 a compilation of the results commented upon previously.
V. CONCLUSIONS

We have extended the schematic model of [1] to the description of meson states, nucleon resonances and \( \Delta \) resonances. The schematic model does not conserve the number of quarks, antiquarks and gluons, rather an average number of them can be extracted from the calculations. It reflects the fact that the interaction is strong and the particle number is not conserved. It is found that the nucleon has nearly 50% of gluon content, as expected. The degeneracy of states of the type \( q^n\bar{q}^n \) is removed (see [1]), but there is still a degeneracy related to the seniority. The Hilbert space is divided into subspaces with a given seniority of the different \( q \bar{q} \) pairs involved. Spaces with the same distribution of seniority but different flavor and spin are degenerate. This lead to the prediction of unusual flavor-spin combinations which cannot be obtained by one \( q \bar{q} \) pair or three quarks alone. We gave a complete classification for the quarks, antiquarks and gluons states. The quarks interact with pairs of gluons coupled to spin zero. Other types of gluons are present as spectators [8]. The global agreement with data is reasonable and the model gives an insight on the detailed structure of the states. The lowest pentaquarks state is predicted at 1.51 GeV [3]. It has an unusual combination of flavor \( (0^+, 3^-, 1^+, 2^-, 2^-) \), and it has negative parity. Also heptaquarks states are predicted, with positive parity at an energy of about 2.5 GeV, for the lowest lying state. The same was done for \( \Delta \) resonances.

These are features which cannot be obtained by working alone with three quarks or with one quark-antiquark pair. We think that the present results support the notion that the degrees of freedom included in the model may indeed be the relevant ones to describe the low-energy spectrum of QCD, within the limitations posed by the schematic nature of the proposed Hamiltonian.

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