PARTIAL DYNAMICAL SYMMETRY AS AN INTERMEDIATE SYMMETRY STRUCTURE

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We introduce the notion of a partial dynamical symmetry for which a prescribed symmetry is neither exact nor completely broken. We survey the different types of partial dynamical symmetries and present empirical examples in each category.

1 Introduction

Symmetries play an important role in dynamical systems. They provide quantum numbers for the classification of states, determine selection rules and facilitate the calculation of matrix elements. An exact symmetry occurs when the Hamiltonian of the system commutes with all the generators \( g_i \) of the symmetry-group, \([H, g_i] = 0\). In this case, all states have good symmetry and are labeled by the irreducible representations (irreps) of the group. The Hamiltonian admits a block structure so that inequivalent irreps do not mix and all eigenstates in the same irrep are degenerate. In a dynamical symmetry the block structure of the Hamiltonian is retained, the states preserve the good symmetry but in general are no longer degenerate (splitting but no mixing). When the symmetry is completely broken \([H, g_i] \neq 0\), and none of the states have good symmetry. In-between these limiting cases there may exist intermediate symmetry structures, called partial (dynamical) symmetries for which the symmetry is neither exact nor completely broken.

Models based on spectrum generating algebras, such as those developed by F. Iachello and his colleagues, form a convenient framework for discussing these different types of symmetries. In such models the Hamiltonian is written in terms of the generators of a Lie algebra, called the spectrum generating algebra. A dynamical symmetry occurs if the Hamiltonian can be written in terms of the Casimir operators \( \hat{C}_{G_i} \) of a chain of nested algebras

\[
G_1 \supset G_2 \supset \ldots \supset G_n
\]

in which case it has the following properties: (i) solvability: all states are solvable and analytic expressions are available for energies and other observables; (ii) quantum numbers: all states are classified by quantum numbers \( \alpha_1, \alpha_2, \ldots, \alpha_n \), which are the labels of the irreps of the algebras in the chain; (iii) pre-determined structure: the structure of wave functions is completely dictated by symmetry and is independent of the Hamiltonian’s parameters \( a_i \)

\[
H = a_1 \hat{C}_{G_1} + a_2 \hat{C}_{G_2} + \ldots + a_n \hat{C}_{G_n} .
\]

The merits of a dynamical symmetry are self-evident. However, in most applications to realistic systems, the predictions of an exact dynamical symmetry are rarely fulfilled and one is compelled to break it. This is usually done by including in the
Figure 1. Spectra of $^{168}$Er ($N = 16$). Experimental energies (EXP) are compared with IBM calculations in an exact SU(3) dynamical symmetry [SU(3)], in a SU(3) PDS with a Hamiltonian $H + \lambda L \cdot L$, Eq. (4), and parameters $t_0 = 2t_2 = 4$, $\lambda_1 = 13$ keV (PDS), and in a broken SU(3) symmetry (WCD) where an $O(6)$ term is added to an SU(3) Hamiltonian.

Hamiltonian symmetry-breaking terms associated with different sub-algebra chains of the parent spectrum generating algebra ($G_1$). In general, under such circumstances, solvability is lost, there are no remaining non-trivial conserved quantum numbers and all eigenstates are expected to be mixed. A partial dynamical symmetry (PDS) corresponds to a particular symmetry breaking for which some (but not all) of the above mentioned virtues of a dynamical symmetry are retained. It is then possible to identify the following types of partial dynamical symmetries:

- **type I:** partial of the states have all the dynamical symmetry

- **type II:** all the states have partial of the dynamical symmetry

- **type III:** partial of the states have partial of the dynamical symmetry

In what follows we explain each type of partial symmetry and show an empirical example of it. For that purpose we use the interacting boson model (IBM) based on a $U(6)$ spectrum generating algebra. The model describes low-lying quadrupole collective states in even-even nuclei in terms of a system of $N$ monopole ($s$) and quadrupole ($d$) bosons representing valence nucleon pairs.
2 SU(3) PDS (type I)

Partial dynamical symmetry of the first type corresponds to a situation for which part of the states preserve all the dynamical symmetry. In this case the properties of solvability, good quantum numbers, and symmetry-dictated structure are fulfilled exactly, but by only a subset of states. As an example we consider the IBM chain

\[ U(6) \supset SU(3) \supset O(3) \]

\[ [N] \quad (\lambda, \mu) \quad K \quad L \]

applicable to axially deformed nuclei. A rotational-invariant IBM Hamiltonian with partial SU(3) symmetry has the form

\[ H = t_0 \Gamma_0^\dagger \Gamma_0 + t_2 \Gamma_2^\dagger \cdot \tilde{\Gamma}_2 \]
Table 1. \( B(E2) \) branching ratios from states in the \( \gamma \) band in \(^{168}\text{Er} \). The column EXP is the experimental ratios, WCD is the broken \( SU(3) \) calculation and PDS is the reported \( SU(3) \) partial dynamical symmetry calculation.

| \( L^+_I \) | \( L^+_F \) | EXP | PDS | WCD |
|---|---|---|---|---|
| \( 2^+_\gamma \) | \( 0^+_\gamma \) | 54.0 | 64.27 | 66.0 |
| \( 2^+_\gamma \) | \( 0^+_\gamma \) | 100.0 | 100.0 | 100.0 |
| \( 4^+_\gamma \) | \( 2^+_\gamma \) | 6.8 | 6.26 | 6.0 |
| \( 3^+_\gamma \) | \( 2^+_\gamma \) | 2.6 | 2.70 | 2.7 |
| \( 4^+_\gamma \) | \( 2^+_\gamma \) | 1.7 | 1.33 | 1.3 |
| \( 2^+_\gamma \) | \( 0^+_\gamma \) | 100.0 | 100.0 | 100.0 |
| \( 4^+_\gamma \) | \( 2^+_\gamma \) | 1.6 | 2.39 | 2.5 |
| \( 6^+_\gamma \) | \( 4^+_\gamma \) | 1.1 | 1.07 | 1.0 |
| \( 2^+_\gamma \) | \( 0^+_\gamma \) | 100.0 | 100.0 | 100.0 |
| \( 5^+_\gamma \) | \( 4^+_\gamma \) | 2.91 | 4.15 | 4.3 |
| \( 6^+_\gamma \) | \( 4^+_\gamma \) | 3.6 | 3.31 | 3.1 |
| \( 3^+_\gamma \) | \( 0^+_\gamma \) | 100.0 | 100.0 | 100.0 |
| \( 4^+_\gamma \) | \( 0^+_\gamma \) | 122.0 | 98.22 | 98.5 |

It consists of boson-pairs

\[
\Gamma_0 = d^+ \cdot d^+ - 2 (s^+)^2 \quad , \quad \Gamma_{2,\mu}^+ = 2 s^+ d^+_\mu + \sqrt{7}(d^+ d^+)_\mu^{(2)}
\]

which are \( SU(3) \) tensors with \((\lambda, \mu) = (0, 2) \) and \( L = 0 \). For \( t_0 = t_2 \) the above Hamiltonian is related to the Casimir operator of \( SU(3) \), hence has an exact \( SU(3) \) symmetry. For \( t_0 \neq t_2 \), \( H \) involves a mixture of \( SU(3) \) tensors with \((\lambda, \mu) = (0, 0) \oplus (2, 2) \) and although it is not an \( SU(3) \) scalar, it has a subset of solvable states with good \( SU(3) \) symmetry. This arises from the fact that the boson pairs of Eq. \( (3) \) satisfy \( \Gamma^*_{L,\mu} |c; N\rangle = 0 \), where

\[
|c; N\rangle = (N!)^{-1/2} (b^+_c)^N |0\rangle \quad , \quad b^+_c = (\sqrt{2} d^+_0 + s^+) / \sqrt{3}
\]

is the lowest weight state in the \( SU(3) \) irrep \((\lambda, \mu) = (2N, 0) \). In addition, \([\Gamma^*_{L,\mu}, \Gamma^+_2] |c; N\rangle \propto \delta_{L,2} \delta_{\mu,2} |c; N\rangle \) and \([\Gamma^*_L, \Gamma^+_{2,2}] \propto \delta_{L,2} \delta_{\mu,2} \Gamma^+_{2,2} \), from which it follows that the sequence of states \(|k\rangle = (\Gamma^*_{2,2})^k |c; N - 2k\rangle \) are eigenstates of \( H \) with good \( SU(3) \) symmetry \((\lambda, \mu) = (2N - 4k, 2k) \). The states \(|k\rangle \) are deformed and serve as intrinsic states representing the ground band \((k = 0) \) and \( \gamma^k \) bands with angular momentum projection \( K = 2k \) along the symmetry axis. Since the Hamiltonian \( H \) of Eq. \( (3) \) is an \( O(3) \) scalar, the rotational states projected from these intrinsic states are also solvable eigenstates of \( H \) with good \( SU(3) \) symmetry. States in other bands are mixed. Adding to \( H \) \( O(3) \) rotation terms produces an \( L(L + 1) \) splitting and lead to a \( SU(3) \) PDS of type I. The corresponding spectrum is shown in Fig. 1 in comparison with \(^{168}\text{Er} \), and the \( SU(3) \) decomposition of the lowest bands is given in Fig. 2. The ground \((K = 0_1) \) and \( \gamma \) \((K = 2_1) \) bands are solvable with good \( SU(3) \) symmetry \((\lambda, \mu) = (2N, 0) \) and \((2N - 4, 2) \) respectively. Unlike the case of an exact dynamical symmetry, the first \( K = 0_2 \) band is no longer degenerate with the \( \gamma \)-band, in agreement with the empirical situation in most deformed nuclei. Furthermore, the \( K = 0_2 \) band involves a mixture of \( SU(3) \) irreps.
(2N − 4, 2) ⊕ (2N − 8, 4) ⊕ (2N − 6, 0) or equivalently a mixture of a single-phonon
(87.5% β) and double-phonon (12.4% γ2K=0 and 0.1% β2) components 4.

Electromagnetic transitions provide a sensitive test for the structure of states. As shown in Table 1, the SU(3) PDS E2 rates for transitions originating from the γ band are found to be in excellent agreement with experiment. The calculated values are obtained by using the general IBM E2 operator

\[ T^{(2)} = α Q^{(2)} + θ (d^\dagger s + s^\dagger d). \]

\[ Q^{(2)} = d^\dagger s + s^\dagger d - (√7/2)(d^\dagger d)^{(2)}. \]

is an SU(3) generator, hence cannot connect the ground and γ bands which have different SU(3) character. This property combined with the fact that the corresponding wave functions of these solvable bands are determined solely by symmetry, imply that the \( B(E2) \) ratios for \( γ \rightarrow g \) transitions quoted in Table 1 do not depend on parameters of the E2 operator nor of the Hamiltonian and therefore are parameter-free predictions of SU(3) PDS. The agreement between these predictions and the data confirms the relevance of SU(3) PDS to the spectroscopy of \( ^{168}\text{Er} \).

3 O(6) PDS (type I)

It is possible to apply a similar procedure to construct a Hamiltonian with a partial symmetry for the chain

\[ U(6) \supset O(6) \supset O(5) \supset O(3) \]

\[ [N] \quad (0, σ, 0) \quad (τ, 0) \quad L \]

The O(6) intrinsic state for the ground band

\[ |c; N⟩ = (N!)^{-1/2}(b^\dagger_c)^N|0⟩, \quad b^\dagger_c = (d^\dagger_0 + s^\dagger)/√2 \]

has \( σ = N \) and the boson pair which annihilates it, \( P_0|c; N⟩ = 0 \), has the form

\[ P^\dagger_0 = d^\dagger \cdot d^\dagger - (s^\dagger)^2. \]

The resulting Hamiltonian, \( H_{O(6)} = A P^\dagger_0 P_0 \) is related to the Casimir operator of O(6), hence has an exact O(6) symmetry. Adding to it the O(5) and O(3) Casimir operators induces \( τ(τ + 3) \) and \( L(L + 1) \) splitting and lead to an O(6) dynamical symmetry. The latter has been used \(^7\) to describe the structure of the \( γ \)-unstable deformed nucleus \( ^{196}\text{Pt} \). The agreement is excellent for properties of the ground band (\( σ = N \)), yet the resulting fit for the observed anharmonicity of excited bands is quite poor. In the dynamical symmetry limit the lowest bands have \( σ = N, N − 2, N − 4 \) and the eigenvalues \( A(N − σ)(N + σ + 4) \) of \( H_{O(6)} \) imply a fixed anharmonicity: \( 2[1 − \frac{N + 2}{N + 4}] \). For \( ^{196}\text{Pt} \) with \( N = 6 \), the predicted anharmonicity is 1.71 compared to the empirical value 1.30. One is therefore motivated to search for a Hamiltonian which will improve the fit to the intrinsic spectrum without destroying the good O(6) description for the ground band. This can be accomplished \(^8\) by the following Hamiltonian with an O(6) PDS of type II

\[ H = r_0 R^\dagger_0 R_0 + r_2 R^\dagger_2 \cdot R_2. \]

The boson-triplets

\[ R^\dagger_0 = s^\dagger P^\dagger_0, \quad R^\dagger_2,μ = d^\dagger_μ P^\dagger_0. \]
are $O(6)$ tensors with $\sigma = 1$. For $r_0 = r_2$, the Hamiltonian $H$ is proportional to $H_{O(6)}$ hence has an exact $O(6)$ symmetry. For $r_0 \neq r_2$ it involves a mixture of $O(6)$ tensors with $(\sigma = 0) \oplus (\sigma = 2)$. In general, although $H$ is not an $O(6)$ scalar, it satisfies by construction $H|c; N\rangle = 0$, and therefore has an exactly solvable ground band with good $O(6)$ symmetry $\sigma = N$. Since $H$ is an $O(5)$ scalar, states of good $O(5)$ symmetry $\tau$ and good angular momentum $L$ projected from $|c; N\rangle$ are also eigenstates of $H$ and form a ground band endowed with good $O(6)$ dynamical symmetry. In contrast, states in excited bands mix several $\sigma$ irreps. Clearly, the Hamiltonian [10] with added $O(5)$ and $O(3)$ rotational terms exhibits $O(6)$ PDS of type I. Preliminary calculations [8] indicate that such Hamiltonian preserves the good $O(6)$ description for the ground band and is able reproduce the empirical anharmonicity of excited bands in $^{196}$Pt.

It is also possible to consider a partial dynamical symmetry with respect to the third IBM chain: $U(6) \supset U(5) \supset O(5) \supset O(3)$ with quantum numbers $N, n_d, \tau, L$ respectively. A three-body Hamiltonian with a $U(5)$ PDS of type I was presented by Talmi [9]. A general algorithm how to construct Hamiltonians with partial dynamical symmetry of type I for any semi-simple group is available [10].

4 $O(5)$ PDS (type II)

The second type of PDS corresponds to a situation for which all the states preserve part of the dynamical symmetry. In this case there are no analytic solutions, yet selected quantum numbers (of the conserved symmetries) are retained. This occurs, for example, when the Hamiltonian contains interaction terms from two different chains with a common symmetry subalgebra, as in the following IBM chains [11]

$$U(6) \supset U(5) \supset O(6) \supset O(5) \supset O(3).$$

A realization of such an $O(5)$ PDS of type II, is given by the following Hamiltonian, typical for the $U(5)$ (spherical) to $O(6)$ (deformed $\gamma$-unstable) transition region

$$H = \epsilon \hat{n}_d + A P_0^1 P_0.$$

Here $\hat{n}_d$ is the $d$-boson number operator which is a Casimir operator of $U(5)$ and the $A$-term is the $O(6)$ pairing term mentioned in Eq. (11). In this case, all eigenstates of $H$ have good $O(5)$ symmetry but none of them have good $U(5)$ nor good $O(6)$ symmetries and hence only part of the dynamical symmetry of each chain in Eq. (12) is observed. The $E(5)$ critical point of the second order shape-phase transition, considered recently by Iachello [12], correspond to the Hamiltonian of Eq. (13) with $\epsilon = (N - 1) A$, and falls into the present PDS category.

5 $O(6)$ PDS (type II)

An alternative situation where PDS of type II can occur is when the Hamiltonian preserves only some of the symmetries $G_i$ in the chain [11], and only their irreps are
Figure 3. Experimental spectra (EXP) of $^{162}$Dy compared with calculated spectra of $H_1 + \lambda_1 L \cdot L$, Eq. (14), and $H_2 + \lambda_2 L \cdot L$, Eq. (15), with parameters (in keV) $\kappa_0 = 8$, $\kappa_2 = 1.364$, $\lambda_1 = 8$ and $h_0 = 28.5$, $h_2 = 6.3$, $\lambda_2 = 13.45$ and boson number $N = 15$.

unmixed. Such a scenario was recently considered by Van Isacker in relation to the $O(6)$ chain of Eq. (7), using the following Hamiltonian

$$H_1 = \kappa_0 P_0^4 + \kappa_2 (\Pi^{(2)} \times \Pi^{(2)})^{(2)} \cdot \Pi^{(2)}.$$

(14)

The $\kappa_0$ term is the $O(6)$ pairing term mentioned in Eq. (10). The $\kappa_2$ term is constructed only from the $O(6)$ generator, $\Pi^{(2)} = d^\dagger s + s^\dagger d$, which is not a generator of $O(5)$. Therefore, it cannot connect states in different $O(6)$ irreps but can induce $O(5)$ mixing subject to $\Delta \tau = \pm 1, \pm 3$. Consequently, $H_1$ preserves the $U(6)$, $O(6)$, and $O(3)$ symmetries (with good quantum numbers $N, \sigma, L$) but not the $O(5)$ sym-
metry (and hence leads to $\tau$ admixtures). These are the necessary ingredients of an $O(6)$ PDS of type II associated with the chain in Eq. (7).

In Fig. 3 we show the experimental spectrum of $^{162}$Dy and compare with the calculated spectra of $H_1$. The spectra display rotational bands of an axially-deformed nucleus, in particular, a ground band ($K = 0$) and excited $K = 2$ bands. As shown in the upper portion of Fig. 4, all bands of $H_1$ are pure with respect to $O(6)$. Specifically, the $K = 0, 1, 2$ bands have $\sigma = N$ and the $K = 0_2$ band has $\sigma = N - 2$. In this case the diagonal $\kappa_0$-term in Eq. (14) simply shifts each band as a whole in accord with its $\sigma$ assignment. On the other hand, the $\kappa_2$-term in Eq. (14) is an $O(5)$ tensor with $\tau = 3$ and, therefore, all eigenstates of $H_1$ are mixed with respect to $O(5)$. This mixing is demonstrated in the upper portion of Fig. 5 for the $L = 0, 2$ members of the ground band.

6 O(6) PDS (type III)

The third type of partial symmetries has a hybrid character, for which part of the states of the system under study preserve part of the dynamical symmetry. Such a generalized partial symmetry associated with the $O(6)$ chain of Eq. (7), can be realized by the Hamiltonian

$$H_2 = \hbar_0 P_0^\dagger P_0 + \hbar_2 P_2^\dagger \cdot \tilde{P}_2.$$  \hspace{1cm} (15)

Here $P_0^\dagger$ is the $\sigma = 0$ pair of Eq. (14) and the second boson-pair

$$P_{2,\mu}^\dagger = \sqrt{2} s^\dagger d^\dagger_\mu + \sqrt{7}(d^\dagger d^\dagger)^{(2)}_\mu$$  \hspace{1cm} (16)
is an $O(6)$ tensor with $\sigma = 2$. For $h_0 \neq h_2$ the Hamiltonian $H_2$ is neither an $O(6)$-scalar nor an $O(5)$-scalar hence can induce both $O(6)$ and $O(5)$ mixing subject to $\Delta \sigma = 0, \pm 2$ and $\Delta \tau = \pm 1, \pm 3$. Although $H_2$ is not invariant under $O(6)$, it still has an exactly solvable ground band with good $O(6)$ symmetry. This arises from the fact that the boson pairs of Eqs. (9) and (16) annihilate the state $|1; N\rangle$ of Eq. (5), which is the $O(6)$ intrinsic state for the ground band with $\sigma = N$. Since $H_2$ is rotational invariant, states of good angular momentum $L$ projected from $|c; N\rangle$ are also eigenstates of $H_2$ with good $O(6)$ symmetry and form its ground band. These projected states do not have good $O(5)$ symmetry and their known wave functions contain a mixture of components with different $\tau$. It follows that $H_2$ has a subset of solvable states with good $O(6)$ symmetry ($\sigma = N$), which is not preserved by other states. All eigenstates of $H_2$ break the $O(5)$ symmetry but preserve the $O(3)$ symmetry. These are precisely the required features of type III $O(6)$ PDS.

The spectra of $H_2$ is shown in Fig. 3, while the $O(6)$ and $O(5)$ decomposition of selected states are shown in the lower portion of Fig. 4 and Fig. 5 respectively. For $H_2$, the solvable $K = 0_1$ ground band has $\sigma = N$ and all eigenstates are mixed with respect to $O(5)$. However, in contrast to $H_1$ of Eq. (14), excited bands of $H_2$ can have components with different $O(6)$ character. For example, the $K = 0_2$ band of $H_2$ has components with $\sigma = N$ (85.50%), $\sigma = N - 2$ (14.45%), and $\sigma = N - 4$ (0.05%). These $\sigma$-admixtures can in turn be interpreted in terms of multi-phonon excitations. Specifically, we find that the $K = 0_2$ band is composed of 36.29% $\beta$, 63.68% $\gamma^2 = 0$, and 0.03% $\beta^2$ modes, i.e., it is dominantly a double-gamma phonon excitation with significant single- $\beta$ phonon admixture. The $K = 2_1$ band has only a small $O(6)$ impurity and is an almost pure single-gamma phonon band. The combined results of Figs. 4 and 5 constitute a direct proof that $H_2$ possesses a type III $O(6)$ PDS which is distinct from the type II $O(6)$ PDS of $H_1$.

In Table 2 we compare the presently known experimental $B(E2)$ values for tran-

### Table 2. Calculated and observed $B(E2)$ values (in $10^{-2}e^2b^2$) for $^{162}$Dy. The $E2$ operator is $T(2) = e_B(d^1s^1d^1\chi(d^1d^2))$ with parameters $e_B = 0.138 [0.127]$ $eb$ and $\chi = -0.235 [-0.557]$ for $H_1$ and $H_2$.

| Transition | $H_3$ | $H_2$ | Expt. | Transition | $H_3$ | $H_2$ | Expt. |
|------------|-------|-------|-------|------------|-------|-------|-------|
| $2^+_{K=0_1} \rightarrow 0^+_{K=0_1}$ | 107 | 107 | 107(2) | $2^+_{K=2_1} \rightarrow 0^+_{K=0_1}$ | 2.4 | 2.4 | 2.4(1) |
| $4^+_{K=0_1} \rightarrow 2^+_{K=0_1}$ | 151 | 152 | 151(6) | $2^+_{K=2_1} \rightarrow 2^+_{K=0_1}$ | 3.8 | 4.0 | 4.2(2) |
| $6^+_{K=0_1} \rightarrow 4^+_{K=0_1}$ | 163 | 165 | 157(9) | $2^+_{K=2_1} \rightarrow 4^+_{K=0_1}$ | 0.24 | 0.26 | 0.30(2) |
| $8^+_{K=0_1} \rightarrow 6^+_{K=0_1}$ | 166 | 168 | 182(9) | $3^+_{K=2_1} \rightarrow 2^+_{K=0_1}$ | 4.2 | 4.3 | |
| $10^+_{K=0_1} \rightarrow 8^+_{K=0_1}$ | 164 | 167 | 183(12) | $3^+_{K=2_1} \rightarrow 4^+_{K=0_1}$ | 2.2 | 2.3 | |
| $12^+_{K=0_1} \rightarrow 10^+_{K=0_1}$ | 159 | 163 | 168(21) | $4^+_{K=2_1} \rightarrow 2^+_{K=0_1}$ | 1.21 | 1.14 | 0.91(5) |
| $0^+_{K=2_2} \rightarrow 2^+_{K=0_1}$ | 0.16 | 0.23 | | $4^+_{K=2_1} \rightarrow 6^+_{K=0_1}$ | 0.59 | 0.61 | 0.63(4) |
| $0^+_{K=2_2} \rightarrow 2^+_{K=2_1}$ | 0.14 | 17.23 | | $5^+_{K=2_1} \rightarrow 4^+_{K=0_1}$ | 3.4 | 3.3 | 3.3(2) |
| $2^+_{K=2_2} \rightarrow 0^+_{K=0_1}$ | 0.02 | 0.04 | | $5^+_{K=2_1} \rightarrow 6^+_{K=0_1}$ | 2.9 | 3.1 | 4.0(2) |
| $2^+_{K=2_2} \rightarrow 2^+_{K=2_1}$ | 0.04 | 0.05 | | $6^+_{K=2_1} \rightarrow 4^+_{K=0_1}$ | 0.84 | 0.72 | 0.63(4) |
| $2^+_{K=2_2} \rightarrow 2^+_{K=2_1}$ | 0.03 | 3.69 | | $6^+_{K=2_1} \rightarrow 6^+_{K=0_1}$ | 4.5 | 4.7 | 5.0(4) |
sitions in $^{162}$Dy with PDS calculations. The $B(E2)$ values predicted by $H_1$ and $H_2$ for $K = 0_1 \rightarrow K = 0_1$ and $K = 2_1 \rightarrow K = 0_1$ transitions are very similar and agree well with the measured values. On the other hand, their predictions for interband transitions from the $K = 0_2$ band are very different. Future measurements of these transitions will enable one to distinguish which type of partial $O(6)$ symmetry is more suitable for $^{162}$Dy.

7 Summary and Conclusions

In this contribution we have considered departures from complete dynamical symmetry by introducing the notion of a partial dynamical symmetry (PDS). The latter refers to an intermediate symmetry structure for which some (but not all) of the virtues of a dynamical symmetry (e.g. solvability, quantum numbers) are retained. We have presented empirical examples of nuclei in each category of PDS. Although we have focused the discussion to partial symmetries in systems of one type of bosons (IBM-1) relevant to nuclei, there are also examples of PDS in systems of several types of bosons (e.g. proton-neutron bosons in the IBM-2), in bose-fermi systems (IBFM) and in purely fermionic systems. Thus, PDS seem to be a generic feature in dynamical systems with concrete applications to nuclear and molecular spectroscopy. In addition, PDS have been shown to be relevant to the study of mixed systems with coexisting regularity and chaos.

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