String Dualities at Order $\alpha'^3$

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Abstract

We compute the cosmological reduction of the fourth powers of the Riemann tensor claimed to arise in string theory at order $\alpha'^3$, with overall coefficient proportional to $\zeta(3)$, and show that it is compatible with an $O(9,9)$ symmetry. This confirms the general result in string theory, due to Sen, that classical string theory with $d$-dimensional translation invariance admits an $O(d,d)$ symmetry to all orders in $\alpha'$. 
1 Introduction

String theory continues to be a most promising framework for a consistent theory of quantum gravity. At low energies string theories are described by Einstein’s theory of general relativity coupled to matter fields, which universally include an antisymmetric tensor (B-field) and a scalar (dilaton). Intriguingly, however, even classical string theory modifies general relativity in two important respects: it includes an infinite number of higher-derivative corrections governed by the (inverse) string tension $\alpha'$ \cite{1,2}, and it permits dualities identifying solutions that are drastically different in standard geometry. These general features suggest promising scenarios for cosmology \cite{3-7}, but an immediate obstacle is that the explicit form of the $\alpha'$ corrections is at best known to the first few orders. For Type II string theories not even the first non-trivial higher-derivative corrections, which arise at order $\alpha'^3$, are known completely. Moreover, notwithstanding early important work in \cite{8-10}, the compatibility of $\alpha'$ corrections with string dualities such as T-duality has only in recent years become the focus of attention. The ‘space of duality invariant cosmologies’ has been explored to all orders in $\alpha'$ and shown to permit novel features \cite{11}, but it is not known which points in this theory space actual string theories inhabit. It is thus a matter of some urgency to find efficient methods to deal with $\alpha'$ corrections.

In this letter we investigate T-duality at order $\alpha'^3$. Our goal is to analyze whether the eight-derivative corrections (quartic in the Riemann tensor) with overall coefficient proportional to the transcendental $\zeta(3)$ are compatible with the standard $O(9,9)$ T-duality transformations upon reduction to one dimension (cosmic time). We use the general framework developed in \cite{11,12} that systematically uses field redefinitions to bring both the dimensionally reduced action and the most general $O(d,d)$ invariant action to a form that involves only first order derivatives. Following this procedure one eliminates, or at least hopes to eliminate, all ambiguities resulting from the freedom to perform integrations by part and to use lower-order equations of motion to modify higher-derivative terms. The resulting actions can then be tested for $O(d,d)$ invariance in its standard form. This procedure has been successfully applied to first order in $\alpha'$ in cosmological reductions and, more recently, for general torus compactifications \cite{13}. While the complete higher-derivative corrections at order $\alpha'^3$ are not known, the eight-derivative terms involving only the metric are believed to be known completely. We will see that this is sufficient to show compatibility with $O(d,d)$, which in turn determines the B-field and dilaton couplings that survive upon cosmological reduction.

Apart from potential applications in cosmology, this result is of conceptual interest in view of these eight-derivative corrections being proportional to the transcendental number $\zeta(3)$. The transcendentality implies that these corrections cannot be linked by conventional symmetry transformations to corrections with rational coefficients, as present in bosonic and heterotic string theory starting at first order in $\alpha'$. It is sometimes questioned whether the $\zeta(3)$ couplings are compatible with the continuous $O(d,d,\mathbb{R})$. In a previous version of this letter we arrived at the incorrect conclusion that these couplings are not compatible with $O(d,d,\mathbb{R})$ invariance, but this would have been in quite serious conflict with basic principles of string theory. As shown by Sen, classical (tree-level) string theory truncated to states of zero momentum along $d$ directions admits an $O(d,d,\mathbb{R})$ invariance to all orders in $\alpha'$ \cite{15}. While the original proof was couched in the language of string field theory the argument only relies on the symmetries
of the S-matrix of this consistently truncated sector. At tree-level, holomorphic factorization yields two independent manifest $O(d,\mathbb{R})$ symmetries, and combining this $O(d,\mathbb{R}) \times O(d,\mathbb{R})$ invariance with the $GL(d,\mathbb{R})$ symmetry following from diffeomorphism invariance and constant shifts of the B-field implies $O(d,d,\mathbb{R})$ invariance \cite{16}. Thus, the tree-level corrections at order $\alpha'^3$ proportional to $\zeta(3)$ really ought to be consistent with $O(d,d,\mathbb{R})$, and indeed they are.

2 Cosmological Reduction

We now review the leading corrections in Type II string theory, and compute the minimal form of the one-dimensional effective action obtained after a cosmological reduction. The $\alpha'$ corrections in Type II string theory begin at $\alpha'^3$. The couplings for the gravitational sector were originally computed from four-point scattering amplitudes \cite{II}, and later from the sigma-model $\beta$-function \cite{17,19}. They take the compact form

$$J(c) = t_8 t_8 R^4 + \frac{c}{8} \epsilon_{10} \epsilon_{10} R^4,$$  \hspace{1cm} (2.1)

where $c = 1$ has been determined in the literature, but here we keep it more general in order to see whether this is fixed by duality arguments. The first term in (2.1) is

$$t_8 t_8 R^4 = t^{\mu_1 \ldots \mu_8} t_{\nu_1 \ldots \nu_8} R_{\mu_1 \mu_2}^{\nu_1 \nu_2} R_{\mu_3 \mu_4}^{\nu_3 \nu_4} R_{\mu_5 \mu_6}^{\nu_5 \nu_6} R_{\mu_7 \mu_8}^{\nu_7 \nu_8}$$

$$= 3 \cdot 2^7 \left[ R_{\alpha \beta \mu \nu} R_{\beta \gamma \nu \rho} R_{\gamma \delta \rho \sigma} + \frac{1}{2} R_{\alpha \beta \mu \nu} R_{\beta \gamma \nu \rho} R_{\gamma \delta \rho \sigma} R_{\delta \alpha \sigma \mu} - \frac{1}{2} R_{\alpha \beta \mu \nu} R_{\gamma \delta \rho \sigma} R_{\delta \alpha \rho \sigma} R_{\gamma \beta \sigma \mu} + \frac{1}{4} R_{\alpha \beta \mu \nu} R_{\alpha \beta \mu \nu} R_{\gamma \delta \rho \sigma} R_{\delta \alpha \rho \sigma} R_{\gamma \beta \sigma \mu} + \frac{1}{32} R_{\alpha \beta \mu \nu} R_{\alpha \beta \mu \nu} R_{\gamma \delta \rho \sigma} R_{\delta \alpha \rho \sigma} R_{\gamma \beta \sigma \mu} + \ldots \right],$$  \hspace{1cm} (2.2)

where the $t_8$ tensor can be defined by its action over generic matrices \cite{20,21}

$$t^{\alpha \beta \gamma \delta \mu \nu \rho \sigma} M_1^{\alpha \beta} M_2^{\gamma \delta} M_3^{\mu \nu} M_4^{\rho \sigma} = 8 \text{Tr} \{ M_1 M_2 M_3 M_4 + M_1 M_3 M_2 M_4 + M_1 M_3 M_4 M_2 \} - 2 \left( \text{Tr} \{ M_1 M_2 \} \text{Tr} \{ M_3 M_4 \} + \text{Tr} \{ M_1 M_3 \} \text{Tr} \{ M_2 M_4 \} + \text{Tr} \{ M_1 M_4 \} \text{Tr} \{ M_2 M_3 \} \right).$$  \hspace{1cm} (2.3)

For the second term in (2.1) we have the following convention for the Levi-Civita tensor

$$\epsilon_{10} \epsilon_{10} R^4 = \epsilon^{\alpha \beta \mu_1 \ldots \mu_4} \epsilon_{\alpha \beta \nu_1 \ldots \nu_4} R_{\mu_1 \mu_2}^{\nu_1 \nu_2} R_{\mu_3 \mu_4}^{\nu_3 \nu_4} R_{\mu_5 \mu_6}^{\nu_5 \nu_6} R_{\mu_7 \mu_8}^{\nu_7 \nu_8}$$

$$= - 2 \cdot 8! \epsilon^{\alpha \beta \mu_1 \ldots \mu_4} \epsilon_{\alpha \beta \nu_1 \ldots \nu_4} R_{\mu_1 \mu_2}^{\nu_1 \nu_2} R_{\mu_3 \mu_4}^{\nu_3 \nu_4} R_{\mu_5 \mu_6}^{\nu_5 \nu_6} R_{\mu_7 \mu_8}^{\nu_7 \nu_8}$$

$$= 3.2^{10} \left[ R^{\alpha \beta \gamma \delta \mu \nu \rho \sigma} M_1^{\alpha \beta} M_2^{\gamma \delta} M_3^{\mu \nu} M_4^{\rho \sigma} + R^{\gamma \delta \alpha \beta \mu \nu \rho \sigma} M_1^{\gamma \delta} M_2^{\alpha \beta} M_3^{\mu \nu} M_4^{\rho \sigma} + \frac{1}{2} R_{\alpha \beta \mu \nu} R_{\gamma \delta \rho \sigma} R_{\delta \alpha \rho \sigma} R_{\gamma \beta \sigma \mu} + \frac{1}{4} R_{\alpha \beta \mu \nu} R_{\alpha \beta \mu \nu} R_{\gamma \delta \rho \sigma} R_{\delta \alpha \rho \sigma} R_{\gamma \beta \sigma \mu} + \ldots \right],$$  \hspace{1cm} (2.4)

where the dots stand for terms containing Ricci tensors and scalars, which can be eliminated by using field redefinitions, at the expense of introducing dilaton couplings that we will ignore at the moment.
The couplings given by $t_8 t_8$ have nonzero contribution at four-graviton level \[1\], while the $\epsilon_{10} \epsilon_{10}$ interactions have nonzero contributions starting only at five-graviton level \[22\]. The presence of this term in the tree-level effective action was inferred by the $\beta$-function approach in \[17–19\], predicting $c = 1$. This prediction was confirmed in \[23\] through sphere-level scattering amplitudes of five gravitons. The literature also suggests that this value for $c$ is required by supersymmetry \[24, 25\] and the emergence of T-duality symmetry in a circle compactification \[26,27\]. For the specific value $c = 1$ it can be shown using Bianchi identities that the corrections are given by only two terms \[18\]

$$J(1) = -3 \cdot 2^6 \left[ R^\alpha \beta \mu \nu R_{\mu \nu} \gamma \delta R_{\alpha \gamma \rho \sigma} R_{\rho \sigma \beta \delta} - 4 R_{\alpha \beta \gamma \delta} R_{\delta \mu \alpha \nu} R_{\nu \rho \beta \sigma} R_{\sigma \gamma \mu \rho} \right]. \quad (2.5)$$

The consensus is that these are the unique purely gravitational terms appearing in the leading $\alpha'$ corrections in Type II string theory.

The simplest way to test for $O(d, d)$ invariance is by performing a cosmological reduction in which the $D = 10$ dimensional target space splits into a single temporal external direction and $d = 9$ internal ones. The fields only depend on time and we use the following ansatz

$$G_{\mu \nu} = \text{diag} \left(-n^2, g_{ij}\right), \quad \phi = \frac{1}{2} \Phi + \frac{1}{2} \log(\sqrt{g}), \quad B_{\mu \nu} = \text{diag} \left(0, b_{ij}\right), \quad (2.6)$$

where $\mu, \nu$ are $D = 10$ indices and $i, j$ are $d = 9$ indices. All partial derivatives but $\partial_0 \Psi = \partial_t \Psi \equiv \dot{\Psi}$ are set to zero. After the reduction, the effective one-dimensional action can be cast in terms of the following quantities

$$L_{ij} \equiv g_{\imath \kappa} \dot{b}_{\kappa j}, \quad M_{ij} \equiv g_{\imath \kappa} \dot{b}_{\kappa j}, \quad (2.7)$$

plus the lapse function $n$, the lower-dimensional dilaton $\Phi$ and their time derivatives.

A method to bring the effective action to a minimal form that makes it systematic to assess its $O(9, 9)$ invariance was introduced in \[11,12\]. The idea is that the lower dimensional equations of motion (where we have gauge fixed $n = 1$ after varying the action)

$$\ddot{L} = M^2 + \dot{\Phi} L,$$
$$\ddot{M} = ML + \dot{\Phi} M,$$
$$\ddot{\Phi} = \frac{1}{2} \left( \dot{\Phi}^2 + \frac{1}{4} \text{Tr} \left( L^2 - M^2 \right) \right), \quad (2.8)$$

can be combined with integrations by part to remove all higher derivative terms containing dilatons, and also allow one to remove the derivatives from $L$ and $M$, leaving a final minimal form containing only powers of $L$ and $M$. It was then shown which of these interactions can be cast in terms of the generalized metric

$$S \equiv \mathcal{H} \eta^{-1} = \begin{pmatrix} b g^{-1} & g - b g^{-1} b \\ g^{-1} & -g^{-1} b \end{pmatrix}, \quad (2.9)$$

so as to make the $O(d, d)$ symmetry manifest, if present. We refer to \[11,12\] for details on this procedure.
In the two-derivative case, the parent action

\[ S_0 = \int d^D x \sqrt{-G} e^{-2\phi} \left[ R + 4 (\nabla \phi)^2 - \frac{1}{12} H^2 \right], \tag{2.10} \]

compactifies to an action where the \( O(9,9) \) symmetry is manifest \[9,11,12\]

\[ S_0 = \int dt e^{-\Phi} \left[ -\dot{\Phi}^2 - \frac{1}{8} \text{Tr}(\dot{\mathcal{S}}^2) \right], \tag{2.11} \]

where we used that \( \text{Tr}(\dot{\mathcal{S}}^2) = 2 \text{Tr} \left( M^2 - L^2 \right) \).

In the following we will simplify the problem by setting the B-field to zero, which is sufficient in order to display the cosmological effective action in a duality invariant form. In this case, the zeroth order equations of motion \[2.8\] allow for the redefinitions

\[ \dot{L} \rightarrow \dot{\Phi} L, \quad \dot{\Phi} \rightarrow \frac{1}{2} \left( \dot{\Phi}^2 + \frac{1}{4} (L^2) \right), \quad \dot{\Phi}^2 \rightarrow \frac{1}{4} (L^2), \tag{2.12} \]

where from now on we will denote the traces of \( d \times d \) matrices by parenthesis, i.e.,

\[ L_{ii} = \text{Tr} (L), \quad (L^2)_{ii} = (L^2), \quad \ldots \tag{2.13} \]

but we will keep the \( \text{Tr} \) notation for the duality covariant \( 2d \times 2d \) matrix \( \mathcal{S} \). Indices are raised and lowered with \( g \), namely \( L_{ij} = g_{ik} L^{k}j = g_{ij} \).

In the simplified case with vanishing two-form, the generalized metric is related to \( L \) by (with \( M, N \) denoting doubled internal indices)

\[ S^M_N = \begin{pmatrix} 0 & g_{ij} \\ g^{ij} & 0 \end{pmatrix}, \quad (\dot{\mathcal{S}}^{2m})^M_N = \begin{pmatrix} (-1)^m (L^{2m})^i_j & 0 \\ 0 & (-1)^m (L^{2m})^i_j \end{pmatrix}, \tag{2.14} \]

and so

\[ \text{Tr}(\dot{\mathcal{S}}^{2m}) = 2 (-1)^m (L^{2m}), \quad \text{Tr}(\dot{\mathcal{S}}^{2m-1}) = 0, \quad m \in \mathbb{N}. \tag{2.15} \]

This shows that only traces containing even powers of \( L \) can be written in terms of the generalized metric; those involving odd powers do not admit a duality covariant expression.

In this language, the reduced Riemann tensor reads

\[ R_{ijkl} = \frac{1}{2} L_{ij} L_{k\ell} j, \quad R_{i0j0} = -\frac{1}{2} \dot{L}_{ij} - \frac{1}{4} (L^2)_{ij}. \tag{2.16} \]

The reduced action takes the form

\[ S = \int dt e^{-\Phi} \left( -\dot{\Phi}^2 - \frac{1}{8} \text{Tr}(\dot{\mathcal{S}}^2) + \alpha^3 \frac{\zeta(3)}{3 \cdot 2^{14}} J(c) \right), \tag{2.17} \]

where we use the normalization of \[2\]. Here \( J(c) \), which was defined in \[2.1\], is evaluated using \[2.16\]. We now briefly explain how this computation is performed following the general procedure of \[11,12\]. Inserting \[2.16\] into \[2.1\] one obtains terms involving traces of products of \( L \) and \( \dot{L} \). One may then systematically eliminate all terms that contain \( \dot{L} \) and \( (L^2) \) as follows: First, one uses the EOM \[2.12\] to replace \( \dot{\Phi} \rightarrow \dot{\Phi} L \), leaving terms involving traces of products of \( L \) and powers of \( \Phi \). Even powers of \( \dot{\Phi} \) can then be eliminated by use of the third equation in \[2.12\], \( \dot{\Phi}^2 \rightarrow \frac{1}{4} (L^2) \). Those containing odd powers of \( \dot{\Phi} \) vanish. (To see this use repeatedly the
substitution $\Phi^2 \rightarrow \frac{1}{4}(L^2)$ to arrive at terms with a single $\Phi$, then integrate it by parts to get

$$\int dt e^{-\Phi} X(L) = \int dt e^{-\Phi} X(L).$$

Now using $L \rightarrow \Phi L$ gives the first term back with a different coefficient, thus proving that these terms vanish.) At this point we are left with traces of powers of $L$, and we now argue that terms containing $(L^2)$ vanish,

$$\int dt e^{-\Phi} (L^2) X(L) = 0.$$

(2.18)

This is proved as follows. One uses the lapse EOM to replace $(L^2)$ by $4\Phi^2$, after which one integrates by part one $\dot{\Phi}$ factor, using $e^{-\Phi} \Phi = -\frac{d}{dt}(e^{-\Phi})$. This creates terms with $\Phi$ and $\dot{\Phi}$, for which one uses again the EOMs (2.12) to write the result in terms of $\dot{\Phi}$ and $\dot{\Phi}$. Finally, one replaces $\dot{\Phi}$ by $\frac{1}{4}(L^2)$, using the lapse equation. This reproduces the original integral, but with a different coefficient, hence proving that it is zero.

The upshot of the above procedure is that we may ignore from the beginning all $\dot{\Phi}$ terms, setting $R_{ij0} = -\frac{1}{4}(L^2)_{ij}$, and then eliminate all $(L^2)$ contributions at the end of the computation. For the two contributions to $J(c)$ one then finds

$$t_{8s8} R^4 \simeq \frac{9}{4}(L^8) + \frac{51}{16}(L^4)^2 - 6(L^3)(L^5),$$

$$e_{10} e_{16} R^4 \simeq -90(L^8) + \frac{45}{2}(L^4)^2 + 48(L^3)(L^5),$$

(2.19)

where the symbol $\simeq$ indicates that these equalities hold up to EOMs and integration by parts inside the integral $\int dt e^{-\Phi}$. We then find that the reduction of (2.14) is given by

$$J(c) \simeq \frac{1}{4}(9 - 45c)(L^8) + \frac{1}{16}(51 + 45c)(L^4)^2 - 6(1 - c)(L^3)(L^5).$$

(2.20)

As explained, while the first two terms can be written in a $O(9,9)$ invariant form using the identities (2.15), the last one involves traces of odd powers of $L$ and is then non-invariant. These contributions come from the second term in the second line of (2.14) and the first term in the fourth line of (2.14), respectively.

Given that these contributions are unambiguous, meaning that they cannot be modified through EOMs nor integrations by parts, T-duality then fixes the coefficient to its expected value $c = 1$, for which

$$J(1) \simeq -9(L^8) + 6(L^4)^2 = -\frac{9}{2} \text{Tr}(\dot{S}^8) + \frac{3}{2} \left(\text{Tr}(\dot{S}^4)\right)^2.$$

(2.21)

Using this in (2.17) we then finally arrive at the minimal form of the cosmological effective action written in a manifestly $O(9,9)$ invariant way

$$S = \int dt e^{-\Phi} \left\{ -\dot{\Phi}^2 - \frac{1}{8} \text{Tr}(\dot{S}^2) + \alpha^3 \zeta(3) \frac{1}{215} \left[ -3 \text{Tr}(\dot{S}^8) + \text{Tr}(\dot{S}^4) \text{Tr}(\dot{S}^4) \right] \right\}.$$

(2.22)

Let us finally point out that while the above result was computed for vanishing B-field and dilaton, by duality invariance (2.22) must be the complete cosmological action including B-field and dilaton. One might be worried that there could be dilaton couplings in higher dimensions that upon cosmological reduction and field redefinitions contribute to the gravitational terms and change the above coefficients, but one may convince oneself that this cannot happen. Following the steps outlined above it can be seen that a generic term of the form $\nabla_{\mu_1} \nabla_{\mu_2} \ldots \nabla_{\mu_8} \phi X^{\mu_1 \mu_2 \ldots \mu_8}$, where $X$ is a tensor depending on $G$ and $\phi$, compactifies to terms that can be redefined away, up to terms containing traces $(L)$, which violate duality invariance.
In this letter we have analyzed the compatibility of the continuous $O(d,d,\mathbb{R})$ symmetry with the eight-derivative couplings quartic in the Riemann tensor proportional to $\zeta(3)$ that arise in string theory at order $\alpha'^3$. We have shown that demanding $O(9,9,\mathbb{R})$ invariance upon cosmological reduction to one timelike dimension fixes the relative coefficient between the couplings $t_8 t_8 R^4$ and $\epsilon_{10} \epsilon_{10} R^4$ uniquely to the value determined independently by other methods. This result illustrates the strength of duality invariance, for it allows one to reconstruct from the $t_8 t_8 R^4$ term, which was computed by Gross and Witten from the four-point amplitude \cite{2}, the complete gravitational couplings in ten dimensions, in addition to determining the B-field and dilaton couplings that survive upon cosmological reduction. This gives strong constraints on the possible B-field and dilaton couplings also in ten dimensions, but we do not expect these to be determined completely, since there could be couplings that disappear upon reduction.

The results presented here are useful, in particular, in that they determine the first two non-trivial coefficients for Type II string theory in the general cosmological classification to all orders in $\alpha'$ \cite{11}. There have already been a number of papers exploring cosmological consequences of this $\alpha'$-complete cosmology, see \cite{30–34}, and here we have further constrained the ‘space of duality covariant string cosmologies’. Moreover, our results provide a non-trivial test for the core assumption underlying this classification: that there is a field basis for which the $O(d,d,\mathbb{R})$ symmetry, expected to exist in string theory to all orders in $\alpha'$, takes the standard form. This is in contrast to double field theory \cite{29} and conventional dimensional reduction with a generic number of external dimensions \cite{13,14}, where a Green-Schwarz-type mechanism needs to be invoked that can be viewed as $\alpha'$-deforming the $O(d,d)$ transformations.

Note added: Upon completion of this letter we became aware of the results in \cite{35}, which exhibit obstacles for a double field theory formulation of the $\zeta(3)$ couplings in ten dimensions. The first version of this letter also identified an obstacle for the conventional realization of $O(d,d)$ in dimensional reduction, but this was due to a computational mistake, and the corrected results presented here fully confirm the presence of $O(d,d)$. We therefore expect that there is also a double field theory formulation of the $\zeta(3)$ couplings, perhaps involving novel structures.

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