Direct laser-driven electron acceleration and energy gain in helical beams

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A detailed study of direct laser-driven electron acceleration in paraxial Laguerre-Gaussian modes corresponding to helical beams LG\(_{m0}\) with azimuthal modes \(m = \{1, 2, 3, 4, 5\}\) is presented. Due to the difference between the ponderomotive force of the fundamental Gaussian beam LG\(_{00}\) and helical beams LG\(_{m0}\) we found that the optimal beam waist leading to the most energetic electrons at full width at half maximum is more than twice smaller for the latter and corresponds to a few wavelengths \(\Delta w_0 = \{6, 11, 19\}\) \(\lambda_0\) for laser powers of \(P_0 = \{0.1, 1, 10\}\) PW. We also found that for azimuthal modes \(m \geq 3\) the optimal waist should be smaller than \(\Delta w_0 < 19\lambda_0\). Using these optimal values we have observed that the average kinetic energy gain of electrons is about an order of magnitude larger in helical beams compared to the fundamental Gaussian beam. This average energy gain increases with the azimuthal index \(m\) leading to collimated electrons of a few 100 MeV energy in the direction of the laser propagation.

I. INTRODUCTION

The well known Laguerre–Gaussian (LG\(_{p,m}\)) modes of paraxial light with integer radial and azimuthal, \(p\) and \(m\), indices have a phase distribution of \(e^{-im\theta}\), where \(\theta\) is the azimuthal angle. Here \(m\) corresponds to the azimuthal or rotational order of the mode with a well defined orbital angular momentum (OAM) of rotational order of the mode with a well defined orbital angular momentum (OAM) of \(m\) per photon \(\hbar\). Such higher order modes lead to twisted light by forming an optical vortex in the direction of propagation associated with the optical phase singularity. The spiral shape of the wavefront is formed by \(|m|\) interconnected helices, hence LG\(_{p,m\neq0}\) beams are called helical beams.

These distinctive properties of higher order Laguerre-Gaussian modes set forth to an ever increasing amount of applications in different areas of optical communications, imaging techniques, quantum information technologies and other topics, see \cite{2,6} and references therein.

In recent years with the development of ultra-intense multi-PW laser technology, Laguerre-Gaussian laser modes have other potential applications in the fields of plasma accelerators, inertial confinement fusion and in the generation of X-rays and \(\gamma\)-rays with OAM \cite{7-14}. In particular, it has been shown that in laser wakefields driven by LG\(_{01}\) helical pulses, the wakefield shows a donut-like structure with a ring-shaped hollow electron beam \cite{15}. Furthermore, for a lower density plasma or a smaller laser spot size, besides the donut-like wakefield, a central bell-like wakefield forms in the center of the donut-like wake. On the other hand by further reducing the plasma density or laser spot size leads to an on-axis electron beam acceleration only.

It was also demonstrated that Laguerre-Gaussian beams transfer a part of their OAM to electrons through the dephasing process similar to the direct electron acceleration (DLA) in Gaussian beams \cite{15}. Furthermore the propagation of optical beams with OAM leads to plasma waves that may also carry OAM which couple to the plasma electrons and involve Landau damping and particle acceleration accompanied with generation of quasistatic axial and azimuthal magnetic fields \cite{16}. When Laguerre-Gaussian plasma waves are subjected to Landau damping, a higher azimuthal mode number leads to a larger OAM transfer to particles traveling close to the phase velocity of the plasma wave \cite{17}. Vacuum-based charge acceleration with Laguerre-Gaussian beams has also been studied very recently \cite{18,19} showing that it is possible to generate GeV high-quality electron bunch with low spread in energy and radial deflection.

Motivated by these interesting results, in this paper we will study direct electron acceleration in vacuum in various helical Laguerre-Gaussian LG\(_{m0}\) laser pulses corresponding to helical modes \(m = \{0, 1, 2, 3, 4, 5\}\). Laser beams with \(p = 0\) and \(m = 0\) define the fundamental Gaussian mode, while beams with \(p = 0\) and \(m \geq 1\) have a hollow ring like transverse intensity profile with zero intensity at the center. The central hollowness corresponds to a potential well which confines/accelerates the electric charges through the transverse/longitudinal ponderomotive forces.

This type of ponderomotive trap is also realized using higher order transverse electromagnetic modes (TEM), i.e., the Hermite-Gaussian modes, such as TEM\(_{1,0}\) or the combination of a TEM\(_{1,0}\) with TEM\(_{0,1}\). This ponderomotive potential is similarly axisymmetric and has a minimum on the axis \cite{20,21}. This is certainly expected since orthogonal Hermite-Gaussian modes may be decomposed into Hermite-Gaussian modes with a phase difference and vice versa, see Refs. \cite{8,22} for the general formulas.

Here we specifically focus our study on the energy gain in femtosecond lasers of low to very high intensity, thereby gaining valuable estimates about relevant parameters of lasers operating at ELI-NP \cite{26}. Our results are based on the 3-dimensional numerical solution of the relativistic equations of motion for free electrons in paraxial laser fields. Henceforth, similarly as in very low density plasmas, in DLA we also observe collimated and intertwining electron beams in the direction of the laser prop-
agation, while in addition we also show that the average energy gain increases with the azimuthal mode index $m$.

Furthermore, we found that the optimal beam waist leading to the most energetic electrons is more than twice smaller in case of higher order Laguerre-Gaussian beams than in case of the fundamental Gaussian beam that was presented in [27]. For lasers of $P_0 = \{0.1, 1, 10\}$ PW power, the optimal beam waists are $\Delta w_0 = \{6, 11, 19\} \lambda_0$ and correspond to $I_0 \approx 10^{12} \times \{0.76, 2.2, 7.6\} \text{W/cm}^2$ peak intensities, while taking into account the intensity profiles of the LG$_{0m}$ beams these values are reduced by $I_0 \approx \{0.37, 0.27, 0.23, 0.19, 0.17\}$ for $m = \{1, 2, 3, 4, 5\}$. Using these optimal values we have observed that for the same laser power the average kinetic energy gain of electrons is about an order of magnitude larger in helical beams compared to the fundamental Gaussian beam.

The paper is organized as follows. In Sect. II we present some of the characteristic properties of helical beams, the equations of motion for electrons and the initial conditions corresponding to our study. In Sect. III A we have estimated the optimal values of beam waist leading to the most energetic electrons for given laser power. Applying these optimal values we present and discuss the electron dynamics and energy gains in linearly polarized (LP) and circularly polarized (CP) helical beams Sect. III B. The conclusions are summarized in Sect. IV.

II. DIRECT LASER-DRIVEN ELECTRON ACCELERATION IN HELICAL BEAMS

A. Laguerre-Gaussian beams

A well known solution [28, 29] to the paraxial wave equation is obtained in cylindrical coordinates $(r, \theta, z)$, with the help of generalized Laguerre polynomials. These solutions are cylindrically symmetric around the axis of propagation $z$, with radius $r = \sqrt{x^2 + y^2}$ and azimuth $\theta = \arctan(x/y)$, expressed in Cartesian coordinates. These are the Laguerre-Gaussian (LG$_{pm}$) beams [31, 32, 33], with radial index $p$ and azimuthal index $m$. The general expression for the electric field distribution of a monochromatic LG$_{pm}$ pulse is,

$$E_{T, pm}(r, \theta, z) = C_{pm} E_0 \exp \left[ -i w_0 z + i \phi_0 \right] \times \frac{w_0}{w(z)} \exp \left[ -\frac{r^2}{w^2(z)} - i \frac{r^2}{Z_R w^2(z)} + i \arctan \left( \frac{z}{Z_R} \right) \right] \times \frac{\sqrt{2r}}{w(z)} \exp \left[ i \left( 2p + |m| \right) \arctan \left( \frac{z}{Z_R} \right) \right] \times L_p^{[|m|]} \left( \frac{2r^2}{w^2(z)} \right) \exp \left[ -im\theta \right],$$  \hspace{1cm} (1)

where $k_0 = \omega_0 / c$ is the wavenumber, $c = 1/\sqrt{\epsilon_0 \mu_0}$ is the speed of light in vacuum, $\omega_0$ is the angular frequency, $E_0$ is the amplitude of the electric field, and $\phi_0$ is the initial phase. Furthermore, $Z_R = w_0^2 k_0 / 2$ is the Rayleigh range, $R_c(z) = (z^2 + Z_R^2) / z$ is the radius of curvature, and $\phi_G(z) = \arctan(z / Z_R)$ is the Gouy phase. The beam waist is defined as,

$$w(z) = w_0 \sqrt{1 + \left( \frac{z}{Z_R} \right)^2},$$ \hspace{1cm} (2)

where the beam waist radius at focus is $w_0 \equiv w(z = 0)$.

The Laguerre polynomials are denoted by $L^m_p$, and the normalization constant, $C_{pm} = \sqrt{\frac{p!}{(p + m)!}}$, follows from the orthonormality of the Laguerre polynomials [34]. The fundamental Gaussian beam LG$_{00}$ is obtained for $p = m = 0$, where $C_{00} = L_0^0 = 1$.

The components of the electric and magnetic fields are

$$E_{x, pm} = \alpha_x E_{T, pm}, \quad E_{y, pm} = -i \alpha_y E_{T, pm},$$ \hspace{1cm} (3)

$$B_{x, pm} = -\frac{1}{c} E_{y, pm}, \quad B_{y, pm} = \frac{1}{c} E_{x, pm},$$ \hspace{1cm} (4)

where $\alpha_x = \sqrt{(1 + \alpha_p)^2}$ and $\alpha_y = \sqrt{(1 - \alpha_p)^2}$ such that $\alpha_p = 1$ or $-1$ in case of linear polarization along the $x$-axis or $y$-axis respectively, and $\alpha_p = 0$ for circular polarization, while elliptic polarization otherwise.

The longitudinal components of the electric and magnetic fields are calculated from Maxwell’s equations, $\nabla \cdot \vec{E} = 0$, $\nabla \cdot \vec{B} = 0$, thus in the paraxial approximation,

$$E_{z, pm} \equiv \frac{i}{k_0 w^4(z)} [x E_{x, pm} + y E_{y, pm}],$$ \hspace{1cm} (5)

$$B_{z, pm} \equiv \frac{i}{c k_0} \left( \frac{\partial E_{x, pm}}{\partial y} - \frac{\partial E_{y, pm}}{\partial x} \right),$$ \hspace{1cm} (6)

Therefore the electromagnetic field of Laguerre-Gaussian pulses are given by

$$\vec{E}_{pm}(t, r, \theta, z) = \text{Re} \left[ \vec{E}_{pm}(r, \theta, z) g(t, z) \right],$$ \hspace{1cm} (7)

$$\vec{B}_{pm}(t, r, \theta, z) = \text{Re} \left[ \vec{B}_{pm}(r, \theta, z) g(t, z) \right],$$ \hspace{1cm} (8)

where $\text{Re}$ denotes the real part of a complex expression, and $g(t, z) = \exp \left[ i \omega_0 t - \left( \frac{z - z_R}{\tau_0} \right)^2 \right]$. \hspace{1cm} (9)
Note that there are more appropriate choices for the temporal profile such as the hyperbolic secant \( g(t, z) \sim 1/\cosh \left( \frac{t - z - x_0}{c} \right) \), see Refs. 32-34 for more details. Using this profile we found that the energy gain may be reduced by as much as 30% compared to the Gaussian.

**B. Helical beams**

The Laguerre-Gaussian beams, \( \text{LG}_{0m} \), with non-zero azimuthal modes \( |m| \neq 0 \) contain a phase change given by \( e^{-im\theta} \). Note however that for all modes the generalized Laguerre polynomials have a contribution equal to one, i.e., \( L_0^0(x) = L_0(x) = 1 \). These type of Laguerre-Gaussian beams, \( \text{LG}_{0m} \), are the helical beams associated with the nonzero OAM of light [1, 35].

![FIG. 1. The relative intensity profiles at constant power and beam waist as a function of the radius of an LP laser. The fundamental Gaussian beam \( \text{LG}_{00} \) with dotted black line and the helical beams \( \text{LG}_{01}, \text{LG}_{02}, \text{LG}_{03}, \text{LG}_{04} \) and \( \text{LG}_{05} \) with green, magenta, red, blue and black lines correspondingly.](image)

In Fig. 1 the normalized intensity profiles of a linearly polarized fundamental Gaussian and different helical beams are shown as a function of radial distance, in units of beam waist radius \( w_0 \). These plots represent \( I_{0m} = |E_{0m}|^2/E_0^2 \) using Eq. 14, for \( t = z = 0 \), \( k_0 = \omega_0 = w_0 = 1 \), while the intensity is, \( I_{0m} \sim E_{0m}^2 \).

The intensity of \( \text{LG}_{0m} \)-modes is largest for \( m = 0 \) corresponding to the fundamental Gaussian beam, and it is decreasing with increasing number of azimuthal modes. Due to the \( \left( \frac{\sqrt{\pi}}{w_0(z)} \right)^{|m|} \) factor, all modes with \( |m| \geq 1 \) have zero intensity at the center, \( r = 0 \), that is known as an optical vortex or phase singularity on the axis.

The intensity profile of Gaussian beams are concave functions on the whole interval. The intensity of helical beams is independent of \( \theta \), and has a convex part with maxima at \( r_m = w_0 \sqrt{|m|/2} \) after which the functions change from convex to concave. The width of the convex part widens with increasing azimuthal index \( m \), while using the positions of maxima the intensity peaks of \( \text{LG}_{0m} \)-modes relative to the fundamental Gaussian leads, \( I_{0m}(r_m) = C_{0m}|m|^{|m|}e^{-|m|} \), as shown in Fig. 1.

**C. Electron acceleration in electromagnetic field**

The motion of electrically charged particles in an external electromagnetic field is governed by the Lorentz force, \( \vec{F}_L \equiv \frac{d\vec{p}}{dt} = q \left( \vec{E} + \vec{v} \times \vec{B} \right) \), and leads the following set of non-linear differential equations [36].

\[
\frac{d\vec{x}}{dt} = c\vec{\beta},
\]

\[
\frac{d\vec{\beta}}{dt} = -\frac{e}{\gamma m_c} \left[ -\vec{\beta} \cdot \vec{E} + \vec{E} + c\vec{\beta} \times \vec{B} \right],
\]

Here \( q = -e \) is the electron’s charge, the Cartesian coordinates and normalized velocity are denoted by \( \vec{x} \) and \( \vec{\beta} \equiv \vec{v}/c \), while the Lorentz factor is \( \gamma = 1/\sqrt{1 - \beta^2} \).

The four momentum of the electron is \( p^\mu \equiv (\gamma v^0, \vec{p}) = m_c\gamma c \left( 1, \vec{\beta} \right) \) where \( m_c \equiv \sqrt{p_0^2/c^2} = 0.511 \text{ MeV}/c^2 \) is its invariant rest mass. The relativistic energy \( \mathcal{E} \) and momentum \( \vec{p} \) of electrons are expressed as

\[
p^0 \equiv \frac{\mathcal{E}}{c} = m_c c, \quad \vec{p} = m_c \vec{v}.
\]

Laser-driven electron acceleration in vacuum is the consequence of the direct interaction of the laser pulse with electrons [37-46]. At any given time, Eqs. (10, 11), are input for the electromagnetic field of the laser pulse, i.e., Eqs. (7, 8), hence the trajectory of the propagating electric charge dynamically maps the laser pulse.

The 3-dimensional solutions to the electron trajectories and velocities are obtained by solving these coupled differential equations numerically by an adaptive time-step Runge-Kutta method with an accuracy and numerical precision up to 12-digits.

**D. Initial conditions**

Unless stated otherwise, initially all electrons are at rest, i.e., \( \vec{\beta}(t_0) = \vec{\beta}_{0, i} \equiv 0 \) and \( \gamma_{0, i} = 1 \). These electrons at \( x_{0, i} = 0 \) coordinates are uniformly distributed in the orthogonal plane, \( (x_{0, i}, y_{0, i}) \), on a disk with a radius that is three times the beam waist radius, i.e., \( r_0 = 3w_0 \). Thus, initially over 99% of the laser’s energy is contained within this disk. The initial position of the peak of the laser pulse is located on the longitudinal axis at \( z_F = -5\tau_0 c \) behind the electrons, i.e., full-pulse interaction, while all electrons are independent from each other and only interact with the laser pulse [27].

For current purposes we have fixed the laser wavelength to \( \lambda_0 = 800 \text{ nm} \). The laser pulse duration at Full Width at Half Maximum (FWHM), \( \Delta \tau_0 = 25 \text{ fs} \), corresponds to \( \tau_0 = \Delta \tau_0/2\sqrt{\ln 2} \). Similarly the beam waist at FWHM \( \Delta w_0 \) leads to \( w_0 = \Delta w_0/2\sqrt{\ln 2} \) waist radius.
The peak intensity and peak power for a monochromatic LP Gaussian laser are \( I_0 \equiv a_0^2 \left( \frac{\hbar c}{2m_e} \right)^2 \frac{E_0}{w_0} \) and \( P_0 \equiv I_0 \frac{\pi w_0^2}{2} \), where the normalized electric field amplitude is \( a_0 = \frac{e E_0}{m_e c w_0} \). For \( P_0 = \{0.1, 1, 10\} \) PW power and beam waists of \( \Delta w_0 = \{6, 11, 19\} \lambda_0 \), these values are listed in Table I. For helical beams the field intensities at the local maximum are \( a_0^{\text{hm}}(r_m) = a_0 \sqrt{C_{0m} |m| |m| e^{-|m|}} \), since the total power of the beam is constant, \( P_0 \equiv \frac{E_0^2}{m} \int_0^\infty 2 \pi r \left( \sqrt{2} r / w_0 \right)^{2|m|} \exp \left(-2r^2 / w_0^2\right) dr = E_0^2 w_0^2 \pi / 2 \).

The energy gain of electrons interacting with an \( LG_{0m} \) laser pulse is a function of the initial location of electrons, the laser spot size and azimuthal mode \( m \). To estimate the value of the beam waist that correspondingly leads to maximum energy gains for a given laser power we have varied the initial position of electrons uniformly \( x_{0,i} = y_{0,i} \equiv \{0.05, 0.1, 0.15, \cdots, 2.5\} w_0 \) at \( z_{0,i} = 0 \), and calculated the respective energy gain of a single electron for each position,

\[
\Delta \mathcal{E}_i(t) \equiv \mathcal{E}_i(t) - \mathcal{E}_i(t_0) = (\gamma_i - 1) m_e c^2, \tag{13}
\]

where \( \mathcal{E}_i(t_0) = m_e c^2 \). Furthermore, for all discrete values of beam waist \( \Delta w_0 = \{1, 2, \cdots, 200\} \lambda_0 \), we have also calculated the average of these energy gains, \( \Delta \mathcal{E} = \frac{\sum_i \Delta \mathcal{E}_i / N} {N} \), where \( N = 50 \) corresponds to the number of the initial positions of electrons in the transverse plane.

These weighted averages for different laser powers are shown in Figs. II for both LP and CP pulses. Here the average energy gains corresponding to laser powers of \( P_0 = \{0.1, 1, 10\} \) PW are plotted with red, blue and black, as a function of initial spot size. The full and dashed lines correspond to CP and LP lasers respectively.

First we discuss these results qualitatively. For a given laser power, the amplitude of the electron oscillations along the polarization direction increases with intensity. Increasing the beam waist decreases the intensity and the scattering of electrons decreases hence they remain confined in the pulse being able to gain more energy from the laser, until the oscillations become larger than the

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### Table I

| \( P_0 \) [PW] | \( w_0 = 3.6 \lambda_0 \) | \( w_0 = 6.6 \lambda_0 \) | \( w_0 = 11.4 \lambda_0 \) |
|----------------|----------------|----------------|----------------|
| 0.1            | 2a_0 = 18.9   | 2a_0 = 10.3   | 2a_0 = 5.9    |
| 1              | 2a_0 = 59.8   | 2a_0 = 32.6   | 2a_0 = 18.9   |
| 10             | 2a_0 = 199.3  | 2a_0 = 103.2  | 2a_0 = 59.7   |

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### III. RESULTS

#### A. Optimal beam waist

First of all we are interested in the values of the beam waist for different laser powers that lead to maximal energy gains, in case of the Gaussian beam \( LG_{00} \), and the helical beams \( LG_{01}, LG_{02}, LG_{03}, LG_{04} \) and \( LG_{05} \).
waist and the electron scatters out from the pulse.

On the other hand increasing the waist of the beam also decreases the longitudinal components of the electric field and the Lorentz force, i.e., \( E_z \) and \((i \times \vec{B})_z\), and therefore reduces the net kinetic energy gain of electrons. The electrons are accelerated to larger and larger velocities in the front part of the pulse and thus the electron trajectories are elongated in the direction of the laser propagation while the deceleration in the back part of the pulse becomes less efficient.

This means that the beam waist corresponding to the highest average energy gain, i.e., the highest peaks of the averages in Figs. \( \text{FIG. 2} \) represent the optimal waist for the given laser power and polarization. For larger power lasers a wider initial beam waist is more optimal to ensure that the electrons remain confined inside the pulse to gain more energy.

Therefore using Figs. \( \text{FIG. 2} \) we can approximate of the beam waists corresponding to the peaks in net energy gain. In case of the fundamental Gaussian beam, the optimal beam waist at FWHM correspond to few tens of wavelengths, \( \Delta w_0 = \{6, 11, 19\} \lambda_0 \), for laser powers of \( P_0 = \{0.1, 1, 10\} \) PW, such that the net energy gain increases about \( \Delta \epsilon \approx \sqrt{10} \) for every order of magnitude increase in laser power. Even with those optimal values the average energy gain of only a few MeV was observed in LG00 beams, see Ref. \( \text{[27]} \) for more details. Note also that in case of the Gaussian pulse the outcome is independent on the polarization, and the averaged results overlap. This behavior is similar for the LG01 helical beam, but for higher modes the distinction between CP and LP pulses becomes more apparent.

Now, averaging the energy gains \( \Delta \epsilon \) of the helical beams LG01 and LG02 for a circularly polarized laser from Fig. \( \text{FIG. 2} \) we have approximated the optimal beam waists at FWHM. These optimal beam waists are \( \Delta w_0 = \{6, 11, 19\} \lambda_0 \) corresponding to increasing laser power of \( P_0 = \{0.1, 1, 10\} \) PW. These approximated values represent the chosen optimal values for direct electron acceleration in all helical beams of interest, LG01, \( \cdots \), LG05.

Note however that the optimal beam waist also reduces slightly as the mode number increases, see Fig. \( \text{FIG. 2} \). Therefore the previously chosen beam waists are suboptimal for LG04 and LG05 modes where even smaller beam waist would be more favorable for larger energy gains.

The most important observation in case of CP beams is that the optimal spot sizes of helical beams are more than twice smaller than for the fundamental Gaussian beam. In case of LP beams this difference further increases with the mode index \(|m| \geq 1\). Furthermore for these relatively tight initial waists, of a few wavelengths, the helical beams might lead to almost an order of magnitude larger net energy gains compared to the fundamental Gaussian beam at high laser power.

A straightforward explanation can be formulated in terms of the ponderomotive force, \( F_P^{\text{om}} = - \nabla \Phi^{\text{om}} \), where the ponderomotive potential, the cycle-averaged oscillation energy, is directly proportional to the intensity, \( \Phi^{\text{om}} \sim c^2 |E_{\text{om}}|^2 / (4 \pi \omega_0^2) \sim I_0^{\text{om}} \), shown in Fig. \( \text{FIG. 1} \). The ponderomotive force causes the charges oscillating in an inhomogeneous electric field to drift from where the electric field is larger to where it is smaller. Therefore the immediate consequence of the transverse ponderomotive force is the scattering of charges from “regions” of higher

**FIG. 3.** All figures represent CP lasers of \( P_0 = 1 \) PW power, with \( \Delta \tau_0 = 25 \) fs and \( \Delta w_0 = 11 \lambda_0 \). From left to right: (a0), (b0) and (c0) show the electric fields seen by the electrons \( E_{z, i}, E_{y, i}, E_{x, i} \) in units of \( a_0 \) as function of time, corresponding to the LG00 beam. Similarly, (a1), (b1) and (c1), are for the helical beam LG01.
to lower electric field intensity. Due to the fact that the intensity of the helical beams have a wide convex region the charges found this region are naturally driven to the beam center with zero intensity.

In other words, the transverse ponderomotive force in Gaussian beams is always positive, hence the electrons are scattered outwards from the pulse. In helical beams the transverse ponderomotive force of the convex region is negative, and hence the electrons are effectively trapped inside the "hollow" pulse, see Fig. 1. This leads to less spread, better focusing and collimated electron trajectories confined near the axis of propagation. These captured electrons are accelerated further by the longitudinal ponderomotive force while continue to gain more energy through the phase synchronization process leading to larger energy gains, see Ref. 11 for more details.

B. Energy gain for optimal beam waists

Using the previously given initial conditions together with the optimal values listed in Table I for both CP and LP lasers of \( P_0 = \{0.1, 1, 10\}\) PW power and initial beam waist of \( \Delta w_0 = \{6, 11, 19\}\) \( \lambda_0\), we have numerically calculated the direct laser driven electron acceleration corresponding to the fundamental Gaussian LG_{00} and LG_{0m} helical CP and LP beams. The 3rd row: Similar as previous rows but now the figures, (a4), (b4), and (a5), (b5), correspond to LG_{04} and LG_{05} helical CP and LP beams.

FIG. 4. All figures show the energy gained from laser pulses with \( \Delta \tau_0 = 25\) fs and \( \Delta w_0 = 11\lambda_0\) of \( P_0 = \{0.1, 1, 10\}\) PW power with red, blue and black correspondingly, as a function of the same initial radial position of electrons. The 1st row from left to right: The 1st and 2nd figures, (a0) and (b0), show the energy gained from a Gaussian laser pulse corresponding to CP and LP pulses, which are plotted with "o" and "x" respectively. The 3rd and 4th figures in the 1st row, (a1) and (b1), show the energy gained as a function of the initial radial position in LG_{01} helical CP and LP beams. The 2nd row: Similar as before but now the figures, (a2), (b2), and (a3), (b3), correspond to LG_{02} and LG_{03} helical CP and LP beams. The 3rd row: Similar as previous rows but now the figures, (a4), (b4), and (a5), (b5), correspond to LG_{04} and LG_{05} helical CP and LP beams.
FIG. 5. The figures show the logarithmic scale histogram of net energy gained $\Delta \mathcal{E}_i$ and the polar angle histogram $\phi_i$ of electrons after interaction with a circularly polarized laser pulse with $\Delta r_0 = 25$ fs, $\Delta w_0 = 11\lambda_0$ beam waist and $P_0 = 10$ PW power. The 1st row from left to right: The 1st and 2nd figures, (a0) and (b0), show the energy gained form a Gaussian pulse and the corresponding angular distribution of electrons. Similarly as before, the 3rd and 4th figures in the 1st row, (a1) and (b1), show the energy gain and angular distribution in case of the LG$^{01}_{01}$ helical beam. The 2nd row: Similar as the 1st row, but now figures (a2), (b2), and (a3), (b3), are for the helical beams LG$^{02}_{02}$ and LG$^{03}_{03}$. The 3rd row: Similar as previous rows, but now figures (a4), (b4), and (a5), (b5), are for the helical beams LG$^{04}_{04}$ and LG$^{05}_{05}$.

Boundary of range. In helical beams, e.g., LG$^{01}_{01}$ and LG$^{05}_{05}$, the available ”peak” intensities are about 2.5– and 5.5–times smaller than in LG$^{00}_{00}$. However these intensity maxima are located further away from the midpoint, i.e., $r = 0$, the location of maximum intensity of the Gaussian beam and zero intensity for the helical beams, see Fig. 1. Therefore the electrons found in the convex part of the pulse are captured and accelerated for a much longer time, being clearly visible on Figs. 3. Here we observe that the net-energy gain as a function of the initial radial distance, $r_0,i = \sqrt{x_{0,i}^2 + y_{0,i}^2}$, at the origin $z_{i,0} = 0$, for varying laser power. Here all figures correspond to an initial beam waist of $\Delta w_0 = 11\lambda_0$, while CP and LP pulses are plotted with ”o” and ”x” respectively. Similarly to the previously presented Figures, the red, blue and black correspond to $P_0 = \{0.1, 1, 10\}$ PW laser power. In the 1st row of Figs. 4 the 1st and 2nd figures correspond to Gaussian beams, while the 3rd and 4th are for helical beams LG$^{01}_{01}$. Similarly in the 2nd and 3rd rows: the helical beams, LG$^{02}_{02}$, LG$^{03}_{03}$ and LG$^{04}_{04}$, LG$^{05}_{05}$, with different polarizations are shown.

To further elucidate these issues, in Figs. 4 we have plotted the net energy gain of electrons as a function of the initial radial distance, $r_0,i = \sqrt{x_{0,i}^2 + y_{0,i}^2}$, at the origin $z_{i,0} = 0$, for varying laser power. Here all figures correspond to an initial beam waist of $\Delta w_0 = 11\lambda_0$, while CP and LP pulses are plotted with ”o” and ”x” respectively. Similarly to the previously presented Figures, the red, blue and black correspond to $P_0 = \{0.1, 1, 10\}$ PW laser power. In the 1st row of Figs. 4 the 1st and 2nd figures correspond to Gaussian beams, while the 3rd and 4th are for helical beams LG$^{01}_{01}$. Similarly in the 2nd and 3rd rows: the helical beams, LG$^{02}_{02}$, LG$^{03}_{03}$ and LG$^{04}_{04}$, LG$^{05}_{05}$, with different polarizations are shown.

Here we observe that the net-energy gain as a function of the initial radial distance also reflects the initial intensity profiles shown on Fig. 1. The intensity of Gaussian beam falls off exponentially as a function of the radius, hence the electric charges found further away form the center gain less and less energy. In the case of the helical
beams the largest acceleration happens within the convex part of the intensity curves. Furthermore as the distance between the center and the intensity peak widens with the azimuthal index, the peaks in intensity of the LG helical beams is also decreasing, see Fig. 1. This explains why there is less and less net energy gained around the middle of the helical beams with increasing mode index \( m \), see Fig. 1.

In Figs. 5 the histograms of the energy \( \Delta E \) and the polar angle \( \phi \) of electrons is shown, after the interaction with the pulse. Here the 3-dimensional polar angle is, \( \phi = \arccos \left( \frac{z_i}{R_i} \right) \), where \( R_i = \sqrt{x_i^2 + y_i^2 + z_i^2} \) is the radial distance from the origin. All histograms correspond to a CP laser of \( P_0 = 10 \) PW power and \( \Delta w_0 = 11 \lambda_0 \) spot. In Figs. 5 the 1st row from left to right, (a0), (b0) and (a1), (b1), shows the histograms of energy and polar angle corresponding to LG00 and LG01 beams. Similarly, the 2nd and 3rd rows: figures (a2), (b2), (a3), (b3), and (a4), (b4), (a5), (b5), show the outcome from the helical beams LG02, LG03, and LG04, LG05.

These histograms once again reflect the difference in the ponderomotive force and its influence on the energy gain as well as the angular distribution of electrons. Due to the interaction with pulse the electrons have scattered out with \( \phi \) polar angle. In case of the fundamental Gaussian beam this polar angle is predominantly in the dihedral angle orthogonal to the direction of laser propagation, i.e., \( \phi_i \geq 80 \) degrees. For helical beams with increasing azimuthal mode we observe an increasing number of electrons that are scattered parallel to the longitudinal axis, at polar angles of \( 0 \leq \phi_i < 15 \) degree, and at the same time fewer electrons in the orthogonal directions.
This obviously means that helical beams lead to a larger number of collimated electrons than fundamental Gaussian beams, while their number is also increasing with increasing mode index.

Furthermore, the distribution of energy gain is also different in helical beams. The electrons are distributed following a multimodal distribution, leading to at least a second peak at finite energy at about 150 MeV energy for the helical beams, LG03, LG04 and LG05. Although here we have only shown the results for $P_0 = 10$ PW laser, a very similar behavior is observed for the $P_0 = 1$ PW laser, with a peak at about 50 MeV energy. For a $P_0 = 0.1$ PW laser, the intensities of the helical beams are so weak, that these striking differences disappear.

In Figs. (a) and (b), corresponding to initial beam waists of $\Delta w_0 = 6\lambda_0$, $\Delta w_0 = 11\lambda_0$ and $\Delta w_0 = 19\lambda_0$, we have plotted the average net energy and the largest energy gained, $\Delta E_{\text{m}}$, from circularly and linearly polarized LG$_{0m}$ pulses. Here the average energy is calculated from $\Delta E = \frac{1}{N_e} \sum_{i=1}^{N_e} \Delta E_i$, where $N_e = 6000$ is the number of electrons. These averages are shown with full lines for circularly polarized and with dotted lines for a linearly polarized lasers. Similarly as before, red, blue and black, respectively represent the power of the laser $P_0 = \{0.1, 1, 10\}$ PW. In the 1st row of Figs. (a) and (b), the 1st and 2nd figures, (a0) and (b0), show the average energy gained within the fundamental Gaussian beams while the 3rd and 4th figures, (a1) and (b1), show the same for the helical beams LG01 with given waist. In the 2nd row: the 1st and 2nd figures, (a2) and (b2), correspond to LG02, while the 3rd and 4th figures, (a3) and (b3), correspond to LG03. Similarly, the 3rd row: the figures (a4), (b4) and (a5), (b5), presents the outcome for LG04 and LG05 helical beams.

We observe that the mean energy gain of electrons for any given laser power is largest for an optimal waist, and this optimal waist is increasing with increasing laser power. Therefore for a $P_0 = 0.1$ PW laser the optimal waist size is $\Delta w_0 = 6\lambda_0$, compare the mean energies, i.e., the red lines, in Figs. (a) and (b). Similarly by comparing the blue lines, for a $P_0 = 1$ PW laser the waist size that leads to the largest energy gains is $\Delta w_0 = 11\lambda_0$, while for
a $P_0 = 10$ PW laser the optimal waist size is $\Delta w_0 = 19\lambda_0$, in accordance with the results of Sect. III A.

On the other hand we also recognize that the average energy gain in helical beams is increasing with increasing azimuthal index $|m| \geq 1$. This is evident by comparing the blue and black lines on Figs. 2 and Figs. 5. For a larger initial waist there is larger energy gain for higher laser power. However, in Figs. 5 we also observe that for the largest waist and largest laser power, the average energy is increasing up to $|m| \leq 3$ while it is decreasing for higher modes. This was somewhat expected based on Figs. 2 meaning that for the highest power lasers with $|m| \geq 3$ modes the optimal waist should be smaller than $\Delta w_0 \leq 19\lambda_0$.

The effect of laser polarization on energy gain is a delicate matter as already suggested by Figs. 2. The average net energy gain in fundamental Gaussian beams is largely independent on the polarization. However for helical beams the outcome is different in each case depending on the beam waist and mode index and therefore the difference is from a few percent to up to 50%. We have also observed that a larger average energy gain slightly favors the circularly polarized helical beams with mode indices $|m| \geq 3$ for any beam waist and laser power. For linearly polarized helical beams the optimal waist size should be slightly smaller than in the circularly polarized case to obtain similar energy gains, as can be concluded from the results.

The average net energy gain in Gaussian pulses, for the highest laser power and largest waist is less than 2 MeV, while the highest energy electrons are $E_{\text{max}} \approx 150$ MeV, see Fig. 5. As previously discussed, the energy gains in fundamental Gaussian pulses may be a little higher for optimal beam waists Ref. [21], but even so these values are easily surpassed in helical beams leading to an average of about a few and up to 45 MeV, and in some cases with highest energy electrons of $E_{\text{max}} \approx 650$ MeV or more. Such energy gains are still below the so-called ponderomotive limit $\Delta E \approx m_e c^2 a_0^2/2$ [21, 25], where the intensity maxima of higher-order helical beams can be more than 5-times lower than of Gaussian beams.

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**FIG. 8.** Similar to Figs. 2 and Figs. 5. All figures correspond to lasers with $\Delta T_0 = 25$ fs and $\Delta w_0 = 10\lambda_0$ beam waist radius. The full and dotted lines correspond to a CP and LP laser of $P_0 = \{0.1, 1, 10\}$ PW power, with red, blue and black, respectively.
IV. CONCLUSIONS

In this paper we have studied and compared the direct laser-driven electron acceleration in Laguerre-Gaussian beams with azimuthal mode indices, LG\textsubscript{00} corresponding to the fundamental Gaussian beam, and helical beams LG\textsubscript{01}, LG\textsubscript{02}, LG\textsubscript{03}, LG\textsubscript{04} and LG\textsubscript{05}.

We have found that the acceleration of electrons from rest is vastly different in helical beams compared to the fundamental Gaussian beam, mainly due to the difference in ponderomotive forces. Most importantly for relatively tight initial waists, the helical beams lead to at least an order of magnitude larger energy gains compared to the fundamental Gaussian beam, and this energy gain also increases with the azimuthal mode order |m| ≥ 1.

For laser powers of \( P_0 = \{0.1, 1, 10\} \) PW, the optimal waist of helical beams leading to the most energetic electrons is more than twice smaller, \( \Delta w_0 = \{6, 11, 19\} \lambda_0 \) than in case of the fundamental Gaussian beam [27]. This also means that the beam waist that is optimal for helical beams is suboptimal for Gaussian beam and vice versa.

Finally, compared to Gaussian beams the electron trajectories in helical beams are confined in the direction of laser propagation leading to collimated electrons with trajectories that are at least an order of magnitude longer in the direction of the laser propagation. These conclusion hold for both linearly and circularly polarized lasers.

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