Study of non-canonical scalar field model using various parametrizations of dark energy equation of state

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Abstract

In this present work, we try to build up a cosmological model using a non-canonical scalar field within the framework of a spatially flat FRW space-time. In this context, we have considered four different parametrizations of the equation of state parameter of the non-canonical scalar field. Under this scenario, an analytical solution for the various cosmological parameters have been found out. It has been found that the deceleration parameter shows a smooth transition from a positive value to some negative value which indicates that the universe was undergoing an early deceleration followed by late time acceleration which is essential for the structure formation of the universe. With these four parametrizations, the future evolution of the models are also discussed. We have also shown that the two models mimic as the concordance $\Lambda$CDM in the near future, whereas the other two models diverge due to the future singularity. Finally, we have studied these theoretical models with the Union2.1 SN Ia dataset.

Keywords: Dark energy, Non-canonical scalar, Parametrization, Data analysis

1 Introduction

Recent cosmological observations \cite{1} strongly suggest that our universe is presently accelerating. In literature, there has been a number of theoretical models to explain the origin of this acceleration mechanism. In this context, the most accepted idea is that an exotic component of the matter sector with large negative pressure is responsible for this accelerated expansion of the universe, dubbed as “dark energy” (DE). DE also makes up about 73\% of the total energy budget of the universe at present epoch. However, to understand the origin and nature of DE is still a challenging problem in modern cosmology. A number of models have been proposed phenomenologically as DE models, such as quintessence (canonical scalar field) \cite{2}, phantom \cite{3}, k-essence \cite{4}, Chaplygin gas \cite{5, 6, 7}, $f(R)$-gravity models \cite{8} and so on. The simplest theoretically candidate of DE is the vacuum energy with a constant equation of state (EoS) parameter $\omega = -1$, but it suffers from cosmological constant problem \cite{9}. The dynamical nature of dark energy also

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introduces a new cosmological problem, namely, “coincidence” problem [10]. One alternative to the coincidence problem is coupled dark energy models where DE interchanges energy with the dark matter (DM) by means of a coupling term [11] or other unknown matters. Though a number of theoretical models have been constructed, none of them provides a satisfactory solution to the problems. Hence, there is still a need of an appropriate model to explain current observations.

On the other hand, a question often arises whether the EoS parameter of DE is evolving with time or whether it is a constant. To address this question, many authors are interested to parametrize the effective EoS parameter to describe DE models in both the theoretical and observational aspects. In particular, a large number of parametrizations of DE equation of state have been proposed so far [21, 22, 23, 24, 25, 26, 27, 28, 29, 30]. But, most of these analysis has been carried out only for canonical scalar field models of DE. Recently, non-canonical scalar field models have appeared in theoretical cosmological studies where the role of inflation is played by the non-canonical scalar field (for review on this topic, see [12, 13, 14, 15, 16, 17]). In our recent work [20], we have studied an interacting non-canonical scalar field model with a constant EoS parameter for the scalar field. The model was however restricted in the sense that the EoS parameter should not be a constant in general. Motivated by the above facts, in this present work, we wish to test four popular DE parametrizations for a non-canonical scalar field model to explain the late-time scenario of the universe, those are Chevallier-Polarski-Linder parametrization [29, 30], Jassal-Bagla-Padmanabhan parametrization [32], Barboza-Alcaniz parametrization [25] and Generalized Chaplygin Gas parametrization [5, 6, 7].

The features of these various parametrizations have been discussed in detail in the next section. We have obtained the expressions for different relevant cosmological parameters, such as the deceleration parameter, density parameters of the scalar field and matter field for each model. We have then shown that such non-canonical scalar field could be responsible for the accelerated expansion of the universe at the present stage. We have also compared the results with standard canonical scalar field models considering these parametrizations. Furthermore, in this paper, we have discussed about the future evolution of the universe by considering these EoS parametrizations and we have found that only two parametrizations mimic as the standard ΛCDM in the near future, while the other two parametrizations fail to provide the future evolution of the universe. Finally, we have also studied the constraint on the EoS parameter coming from the SN Ia data [33].

The paper is organized as follows. In section 2, we have described the basic theoretical framework for the non-canonical scalar field model of a flat FRW universe. We have then solved the governing dynamical equations for this toy model using four different types of DE parametrizations of the EoS parameter in this scenario. It has been found that the resulting cosmological scenarios are in good agreement with the current observations in each case. In section 3, we have obtained the observational constraints on this model parameters using SNIa data. Finally, some conclusions are presented in the last section.
2 Field equations and their solutions

The action for this present model (with $8\pi G = c = 1$) is given by

$$S = \int \sqrt{-g} dx^4 \left[ \frac{R}{2} + \mathcal{L}(\phi, X) \right] + S_m$$

(1)

where $R$ is the Ricci scalar curvature, $\mathcal{L}(\phi, X)$ is called Lagrangian density which is an arbitrary function of the scalar field $\phi$ and its kinetic term $X$. The kinetic term $X$ is defined as $X = \frac{1}{2} \partial_{\mu} \phi \partial^{\mu} \phi = \frac{1}{2} \dot{\phi}^2$ for a spatially homogeneous scalar field and $S_m$ represents the action of the background matter field.

Varying this action with respect to the metric $g^{\mu\nu}$ gives the Einstein field equations as

$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = \frac{\partial \mathcal{L}}{\partial X} \partial_\mu \phi \partial_\nu \phi - g_{\mu\nu} \mathcal{L} + T^m_{\mu\nu}$$

(2)

where $T^m_{\mu\nu}$ represents the energy-momentum tensor of the matter field which is modeled in the form of an “ideal perfect fluid” and defined as

$$T^m_{\mu\nu} = (\rho_m + p_m) u_\mu u_\nu + p g_{\mu\nu}$$

(3)

where $\rho_m$ is the energy density, $p_m$ is pressure of the matter field respectively and $u_\mu$ is the four-velocity of the fluid. Secondly, variation of the action with respect to the scalar field $\phi$ gives the equation of motion for $\phi$ as

$$\ddot{\phi} \left( \frac{\partial \mathcal{L}}{\partial X} + 2X \frac{\partial^2 \mathcal{L}}{\partial X^2} \right) + \left( 3H \frac{\partial \mathcal{L}}{\partial X} + \dot{\phi} \frac{\partial^2 \mathcal{L}}{\partial X \partial \phi} \right) \dot{\phi} - \frac{\partial \mathcal{L}}{\partial \phi} = 0$$

(4)

The energy density ($\rho_{\phi}$) and the pressure ($p_{\phi}$) of such a field is given by

$$\rho_{\phi} = \left( \frac{\partial \mathcal{L}}{\partial X} \right) 2X - \mathcal{L}, \quad p_{\phi} = \mathcal{L}$$

(5)

In this context, the non-canonical scalar field Lagrangian density is given by Fang et al. [15],

$$\mathcal{L}(\phi, X) = F(X) - V(\phi)$$

(6)

where $V(\phi)$ is a self-interacting potential for the scalar field $\phi$, $F(X)$ is an arbitrary function of $X$, which is defined as $X = \frac{1}{2} \dot{\phi}^2$. Motivated by Refs. [16, 18, 19, 20], in this paper, we have considered a Lagrangian density of the following form

$$\mathcal{L}(\phi, X) = X^2 - V(\phi), \quad X = \frac{1}{2} \dot{\phi}^2$$

(7)

which is also considered in an earlier model studied by us [20]. In this case, the energy density and pressure associated with this Lagrangian density can be obtained from equations (5) and (7) as

$$\rho_{\phi} = \frac{3}{4} \dot{\phi}^4 + V(\phi)$$

(8)
\[ p_\phi = \frac{1}{4} \dot{\phi}^4 - V(\phi) \]  \tag{9}

The metric for a homogeneous, isotropic and spatially flat FRW model of the universe is characterized by the following line element

\[ ds^2 = dt^2 - a^2(t)[dr^2 + r^2 d\theta^2 + r^2 \sin^2 \theta d\phi^2] \]  \tag{10}

where \( a(t) \) is the scale factor, normalized so that at present \( a(t)|_{t=t_0} = 1 \) and \( t \) is the cosmic time. The Einstein field equations for the space-time given by equation (10) with matter in the form of pressureless perfect fluid takes the form,

\[ 3H^2 = \rho_m + \frac{3}{4} \dot{\phi}^4 + V(\phi) \]  \tag{11}

\[ 2\dot{H} + 3H^2 = -\frac{1}{4} \dot{\phi}^4 + V(\phi) \]  \tag{12}

\[ \dot{\rho}_\phi + 3H(\rho_\phi + p_\phi) = 0 \]  \tag{13}

\[ \dot{\rho}_m + 3H\rho_m = 0 \]  \tag{14}

Here an overdot indicates differentiation with respect to the cosmic time \( t \). Among the above four equations (equations (11)-(14)), only three are independent equations with four unknown parameters \( H, \rho_m, \phi \) and \( V(\phi) \). So we still have freedom to choose one parameter to close the above system of equations.

It is well known that the parametrization of DE equation of state plays an crucial role in understanding the nature of DE component. So, we need to parametrize EoS parameter as,

\[ \omega_\phi(z) = \frac{p_\phi}{\rho_\phi} = \omega_0 + \omega_1 f(z) \]  \tag{15}

where \( \omega_0, \omega_1 \) are real numbers and \( f(z) \) is a function of redshift \( z \). It may be noted that the standard flat \( \Lambda \)CDM model is represented by this parametrization with the choice of \( \omega_0 = -1 \) and \( \omega_1 = 0 \). In fact, many functional forms of \( f(z) \) have been considered in literature [21, 22, 23, 24, 25, 26, 27, 28, 29, 30]. In this present work, we shall use four popular parametrizations of \( \omega_\phi(z) \) to study the behavior of the deceleration parameter \( q(z) \) of this non-canonical scalar field model.

### 2.1 Chevallier-Polarski-Linder (CPL) parametrization

Among various parametrizations, the CPL parametrization (for details, see Refs.[29, 30]) is one of the most popular one and is given by

\[ \omega_\phi(z) = \omega_0 + \omega_1 (1 - a) = \omega_0 + \omega_1 \left( \frac{z}{1 + z} \right) \]  \tag{16}

where \( z = \frac{1}{a} - 1 \) is the redshift, \( \omega_0 \) represents the current value of \( \omega_\phi(z) \) and the second term represents variation of the EoS parameter with respect to redshift. In this present model, we have considered CPL parametrization of EoS parameter because this parametrization has the
advantage of giving finite $\omega_\phi$ in the entire range, $0 < z < \infty$.

The solution for $\rho_\phi$ from equation (13) is obtained as

$$\rho_\phi(z) = \rho_{\phi 0}(1 + z)^{3 \alpha_1} e\left(-\frac{3 \alpha_1 z}{1 + z}\right)$$

(17)

where, $\alpha_1 = (1 + \omega_0 + \omega_1)$ and $\rho_{\phi 0}$ is an integrating constant. From equation (14), we have the expression for energy density of matter as

$$\rho_m(z) = \rho_{m 0}(1 + z)^{3}$$

(18)

where, $\rho_{m 0}$ is an integrating constant. From equation (11), the Hubble expansion rate can also be written as

$$H^2(z) = H_0^2 \left[ \Omega_{m 0}(1 + z)^3 + \Omega_{\phi 0}(1 + z)^{3 \alpha_1} e\left(-\frac{3 \alpha_1 z}{1 + z}\right) \right]$$

(19)

Here, $H_0$ is the Hubble parameter at the present epoch, $\Omega_{m 0} = \frac{\rho_{m 0}}{3H_0^2}$ and $\Omega_{\phi 0} = \frac{\rho_{\phi 0}}{3H_0^2}$ are the density parameters at the present epoch of the matter and scalar field respectively.

The deceleration parameter is defined as, $q = -\frac{\ddot{a}}{aH^2} = -(1 + \frac{\ddot{H}}{H^2})$. For this model, $q$ takes the following form

$$q(z) = \frac{1}{2} + \frac{3}{2} \left[ \frac{\omega_0 + \omega_1 \left(\frac{\kappa}{1 + z}\right)}{1 + \kappa(1 + z)(3 - 3 \alpha_1)} e\left(-\frac{3 \alpha_1 z}{1 + z}\right) \right]$$

(20)

where, $\kappa = \frac{\rho_m}{\rho_\phi} = \frac{\Omega_{m 0}}{\Omega_{\phi 0}}$. If $q < 0$ the model accelerates while $q > 0$ indicates deceleration of the universe. From the figure (1a), we have seen that $q$ decreases from positive to negative value by suitable choices of model parameters.

![Figure 1: a) Plot of $q$ as a function of $z$ (left panel) and b) Plot of $\Omega_m$ (dashed curve) and $\Omega_\phi$ (solid curve) as a function of $z$ (right panel). This is for $\kappa = \frac{\Omega_{m 0}}{\Omega_{\phi 0}} = 0.27$, $\omega_0 = -1$ and $\omega_1 = 0.2$. Here, $\alpha_1 = (1 + \omega_0 + \omega_1)$.](image)

For this model, the evolution of the density parameters of the matter and scalar field are obtained respectively as,

$$\Omega_m(z) = \frac{1}{1 + \kappa(1 + z)^{3 \alpha_1} e\left(-\frac{3 \alpha_1 z}{1 + z}\right)}$$

(21)
\[ \Omega_\phi(z) = \frac{1}{1 + \kappa(1 + z)^{3-3\omega_1}e^{\frac{3\omega_1 z^2}{2(1+z)^2}}} \]  

which further yields, \( \Omega_m(z) + \Omega_\phi(z) = 1 \). Figure (1b) shows the plot of density parameters for the scalar and the matter field as a function of \( z \). This graph shows that \( \Omega_\phi \) starts dominating over \( \Omega_m \) at around \( z \sim 0.5 \). This result is compatible with the observational results [31].

However, the model presented here is restricted because \( \omega_\phi(z) \) diverges when \( z \to -1 \) i.e., this model cannot predict about the future evolution. So, the model can nicely describe the evolution history of the universe in the past and near future upto \( z \geq -1 \) but can not predict about the evolution beyond that limit.

### 2.2 Jassal-Bagla-Padmanabhan (JBP) parametrization

Recently, Jassal et al. [32] extended the above parametrization to a more general case:

\[ \omega_\phi(z) = \omega_0 + \omega_1 \frac{z}{(1+z)^p} \]  

(23)

For the present model, we choose \( p = 2 \). It must be noted that the EoS parameter \( \omega_\phi \sim \omega_0 \) at both high and low redshifts for \( p = 2 \). Also, one can obtain the widely used CPL parametrization of EoS from equation (23) for \( p = 1 \). For the JBP parametrization, using equation (13), the expression for \( \rho_\phi \) can be obtained as

\[ \rho_\phi(z) = \rho_{\phi0}(1+z)^{3(1+\omega_0)}e^{\frac{3\omega_1 z^2}{2(1+z)^2}} \]  

(24)

where \( \rho_{\phi0} \) is an integrating constant and represents the present value of the scalar field density. The Hubble parameter for this model takes the following form

\[ H^2(z) = H_0^2 \left[ \Omega_{m0}(1+z)^3 + \Omega_{\phi0}(1+z)^{3(1+\omega_0)}e^{\frac{3\omega_1 z^2}{2(1+z)^2}} \right] \]  

(25)

In this model, we express deceleration parameter \( q \) as

\[ q(z) = \frac{1}{2} + \frac{3}{2} \left[ \frac{\omega_0 + \omega_1 \frac{z}{(1+z)^2}}{1 + \kappa(1+z)^{-3\omega_0}e^{-\frac{3\omega_1 z^2}{2(1+z)^2}}} \right] \]  

(26)

The corresponding density parameters are now given by

\[ \Omega_m(z) = \frac{1}{1 + \frac{1}{\kappa(1+z)^{3\omega_0}e^{\frac{3\omega_1 z^2}{2(1+z)^2}}}} \]  

(27)

\[ \Omega_\phi(z) = \frac{1}{1 + \kappa(1+z)^{-3\omega_0}e^{-\frac{3\omega_1 z^2}{2(1+z)^2}}} \]  

(28)

Figure (2a) shows the plot of \( q(z) \) as a function of \( z \). This plot clearly shows the transition of \( q \) from the decelerating to the accelerating regime at \( z \sim 0.8 \). The evolutions of \( \Omega_m \) and \( \Omega_\phi \) against \( z \) are shown in figure (2b). The plots are for \( \omega_0 = -1 \) and \( \omega_1 = 0.2 \). In both the graphs, the resulting cosmological scenarios are in good agreement with observations. For the JBP model also, \( \omega_\phi(z) \) diverges as \( z \to -1 \) and thus future evolution can not be predicted.
Barboza-Alcaniz (BA) parametrization

The next parametrization considered in this paper was proposed by Barboza et al. [25], which has the following functional form

$$\omega_{\phi}(z) = \omega_0 + \omega_1 \frac{z(1 + z)}{1 + z^2}$$  \hspace{1cm} (29)

where $\omega_{\phi}(z = 0) = \omega_0$ (the present value of the EoS parameter), $\omega_1 = \frac{d\omega_{\phi}}{dz}|_{z=0}$ (which measures the variation of the EoS parameter with $z$), $\omega_{\phi}(z = \infty) = \omega_0 + \omega_1$ and the EoS parameter reduces to $\omega_{\phi}(z) = \omega_0 + \omega_1 z$ at the low redshift ($z << 1$). It is remarkable that the BA parametrization does not diverge like CPL model when $z \to -1$ and it remains bounded within the entire range $z \in [-1, \infty)$.

In this model, $\rho_{\phi}(z)$ becomes

$$\rho_{\phi}(z) = \rho_{\phi 0}(1 + z)^3(1 + \omega_0)(1 + z^2)^{\frac{3\omega_1}{2}}$$  \hspace{1cm} (30)

Now the equation (11) can be written as

$$H^2(z) = H_0^2 \left[ \Omega_{m 0}(1 + z)^3 + \Omega_{\phi 0}(1 + z)^3(1 + \omega_0)(1 + z^2)^{\frac{3\omega_1}{2}} \right]$$  \hspace{1cm} (31)

In this case, the deceleration parameter $q(z)$ can be expressed as

$$q(z) = \frac{1}{2} + \frac{3}{2} \left[ \frac{\omega_0 + \omega_1 \frac{z(1 + z)}{1 + z^2}}{1 + \kappa(1 + z)^{-3\omega_0}(1 + z^2)^{-\frac{3\omega_1}{2}}} \right]$$  \hspace{1cm} (32)

Furthermore, one can express the density parameters of the matter and scalar field respectively as

$$\Omega_m(z) = \frac{1}{1 + \frac{1}{\kappa}(1 + z)^{3\omega_0}(1 + z^2)^{\frac{3\omega_1}{2}}}$$  \hspace{1cm} (33)
\[ \Omega_\phi(z) = \frac{1}{1 + \kappa(1 + z)^{-3\omega_0}(1 + z^2)^{-\frac{\omega_1}{2}}} \]  

(34)

Figure (3a) shows the evolution of the deceleration parameter with redshift \( z \). We have observed from figure (3a) that the evolution of the universe is in accelerating phase \( (q < 0) \) at present epoch. Figure (3b) shows that the density parameters \( \Omega_m \) increases with \( z \), whereas \( \Omega_\phi \) decreases with \( z \). This features of \( q(z) \), \( \Omega_m \), and \( \Omega_\phi \) are consistent with the present day observations.

2.4 Generalized Chaplygin Gas (GCG) parametrization

It is well known that the generalized chaplygin gas (see Refs. [5, 6, 7]) behaves like dark matter in the past and it behaves like cosmological constant at present. Motivated by this idea, in this paper, we are interested to describe the late-time dynamics of the universe produced by the GCG. For this purpose, we have assumed that the universe contains both the dark matter and the GCG. Additionally, we have also considered another interesting possibility where the non-canonical scalar field \( \phi \) plays the role of GCG to explore the late time cosmic scenarios. The GCG equation of state is described by [5, 6, 7]

\[ p_\phi = -\frac{A}{\rho_\phi^\alpha} \]  

(35)

where \( A \) is a positive constant and \( \alpha \) is another constant in the range \( 0 < \alpha \leq 1 \). The original chaplygin gas corresponds to the case \( \alpha = 1 \) [5]. By inserting equation (35) into the energy conservation equation (13), one finds that the density of the scalar field \( \phi \) evolves as

\[ \rho_\phi(z) = \left[A + B(1 + z)^{3(1+\alpha)}\right]^{\frac{1}{1+\alpha}} \]  

(36)

where, \( B \) is an integration constant. Equation (36) can be re-written in the following form

\[ \rho_\phi(z) = \rho_{\phi0}\left[A_s + (1 - A_s)(1 + z)^{3(1+\alpha)}\right]^{\frac{1}{1+\alpha}} \]  

(37)
where, for simplicity, we have defined $A_s = \frac{A}{A+B}$ and $\rho_{\phi 0} = (A+B)^{-\frac{1}{1+\alpha}}$ is the present value of the energy density of the GCG. To ensure the finite and positive value of $\rho_{\phi}$ we need $-1 < \alpha \leq 1$ and $0 \leq A_s \leq 1$.

In this case, the Hubble parameter is given by

$$H^2 = H_0^2 \left[ \Omega_m (1+z)^3 + \Omega_{\phi 0} \left( A_s + (1-A_s)(1+z)^3(1+\alpha) \right) \right]^{\frac{1}{1+\alpha}}$$  \hspace{1cm} (38)

The corresponding expression for the EoS parameter is given by

$$\omega_{\phi}(z) = -\frac{A_s}{A_s + (1-A_s)(1+z)^3(1+\alpha)}$$  \hspace{1cm} (39)

Like earlier mentioned three models, the EoS parameter of the GCG also depends on two independent model parameters ($A_s$ and $\alpha$) along with redshift $z$. At present epoch, the above EoS parameter becomes, $\omega_{\phi}(z = 0) = -A_s$. In this case, it is very interesting to notice that the GCG will behave like pure cosmological constant when we put $A_s = 1$.

The deceleration parameter $q$ can be written as

$$q(z) = \frac{1}{2} + \frac{3}{2} \left[ -\frac{A_s}{A_s + (1-A_s)(1+z)^3(1+\alpha)} \right] \left[ \frac{\kappa(1+z)^3}{[A_s + (1-A_s)(1+z)^3(1+\alpha)]^{1+\alpha}} \right]$$  \hspace{1cm} (40)

where $\kappa = \frac{\rho_{m 0}}{\rho_{\phi 0}} = \frac{\Omega_m}{\Omega_{\phi 0}}$.

In this case the density parameters have the following form

Figure 4: a) Plot of $q$ as a function of $z$ (left panel) and b) Plot of $\Omega_m$ (dashed curve) and $\Omega_{\phi}$ (solid curve) as a function of $z$ (right panel). Both the plots are for $A_s = 0.9$, $\alpha = -0.5$ and $\kappa = \frac{0.27}{0.73}$. 
\[ \Omega_m(z) = \frac{1}{1 + \left[ A_s + (1 - A_s) (1 + z)^3 (1 + \alpha) \right]^{1/(1+\alpha)}} \]

\[ \Omega_\phi(z) = \frac{1}{1 + \left[ A_s + (1 - A_s) (1 + z)^3 (1 + \alpha) \right]^{1/(1+\alpha)}} \]

Figure (4a) shows the evolution of \( q(z) \) vs. redshift \( z \) for \( A_s = 0.9 \) and \( \alpha = -0.5 \). In fact, at low redshift, the transition of \( q(z) \) from decelerating to accelerating regime depends upon the choice of the parameters \( A_s \) and \( \alpha \). Also, the evolutions of \( \Omega_m \) and \( \Omega_\phi \) against \( z \) are shown in figure (4b) for the earlier mentioned same chosen values of \( A_s \) and \( \alpha \).

2.5 Comparison between canonical and non-canonical scalar field models for the above parametrizations:

For all these models, the relevant potential for the scalar field \( \phi \) in terms of redshift \( z \) can be written as (from equations (8) and (9))

\[ V(z) = \frac{1}{4} (1 - 3\omega_\phi(z)) \rho_\phi(z) \]

which immediately gives

\[ V_{\text{CPL}}(z) = V_0 \left( 1 - 3\omega_0 - \frac{3\omega_1 z}{1 + z} \right) (1 + z)^{3\alpha_1} e^{\frac{3\omega_1 z}{1 + z}} \]

\[ V_{\text{JBP}}(z) = V_0 \left( 1 - 3\omega_0 - \frac{3\omega_1 z}{(1 + z)^2} \right) (1 + z)^{3(1+\omega_0)} e^{\frac{3\omega_1 z^2}{2(1 + z)^2}} \]

\[ V_{\text{BA}}(z) = V_0 \left( 1 - 3\omega_0 - 3\omega_1 \frac{z(1 + z)}{1 + z^2} \right) (1 + z)^{3(1+\omega_0)} \left( 1 + z^2 \right)^{\frac{3\omega_1 z^2}{2}} \]

\[ V_{\text{GCG}}(z) = V_0 \left( 4A_s + (1 - A_s)(1 + z)^{3(1+\alpha)} \right) \left[ A_s + (1 - A_s)(1 + z)^{3(1+\alpha)} \right]^{-\frac{\phi_0}{1+\alpha}} \]

where \( V_0 = \frac{3\Omega_{\phi0}H_0^2}{4} \). \( V_{\text{CPL}}, V_{\text{JBP}}, V_{\text{BA}} \) and \( V_{\text{GCG}} \) are the potential for the CPL, JBP, BA and GCG models respectively. Here, \( H_0 \) and \( \Omega_{\phi0} \) represent the present day values for the Hubble parameter and the dark energy density parameter respectively.

Adding equations (8), (9) and replacing \( \dot{\phi} = aH \frac{d\phi}{da} \), one can obtain the general expression for the non-canonical scalar field \( \phi \) as,

\[ \phi(z) = \phi_0 + \int_0^z \left[ \frac{(1 + \omega_\phi(z')) \rho_\phi(z')}{1 + z'} H(z') \right]^{\frac{1}{1+\alpha}} dz' \]

where \( \phi_0 \) is an arbitrary integration constant.

Now, we will focus on the extensively studied canonical scalar field case, in which the Lagrangian density is obtained from equation (6) as

\[ \mathcal{L}(\phi_{\text{cano}}, X) = X - V(\phi) \]
Figure 5: This figure shows the variation of the potential $V$ with $\phi$ by assuming $\omega_0 = -1$, $\omega_1 = 0.2$ for CPL (thick curve), JBP (dashed curve) and BA (dotted curve) parametrizations and $A_s = 0.9$, $\alpha = -0.5$ for GCG (thin curve) parametrization. All the plots are for the parameter choices $\Omega_\phi = 0.73$, $\phi = 0.1$ and $H_0 = 72 \text{ km s}^{-1} \text{ Mpc}^{-1}$.

The energy density and pressure for the scalar field ($\phi_{\text{cano}}$) are given by

$$\rho_{\text{cano}} = \frac{1}{2} \dot{\phi}_{\text{cano}}^2 + V_{\text{cano}}(\phi_{\text{cano}}), \quad p_{\text{cano}} = \frac{1}{2} \dot{\phi}_{\text{cano}}^2 - V_{\text{cano}}(\phi_{\text{cano}})$$

where, $V_{\text{cano}}$ is the potential of the canonical scalar field. The EoS parameter $\omega_{\text{cano}} = \frac{p_{\text{cano}}}{\rho_{\text{cano}}}$ is a dynamical variable which gives the continuity equation (13) in an integrated form

$$\rho_{\text{cano}} = \rho_0 \exp \left[ 3 \int_0^z \frac{1 + \omega_{\text{cano}}(z')} {1 + z'} dz' \right]$$

where $\rho_0$ is an integration constant. The Friedmann equation then becomes

$$H^2_{\text{cano}}(z) = H_0^2 \left[ \Omega_m(1 + z)^3 + (1 - \Omega_m)\exp \left( 3 \int_0^z \frac{1 + \omega_{\text{cano}}(z')} {1 + z'} dz' \right) \right]$$

In this case, the expressions for $V_{\text{cano}}$ and $\phi_{\text{cano}}$ can be written as

$$V_{\text{cano}}(z) = \frac{1}{2} (1 - \omega_{\text{cano}}(z)) \rho_{\text{cano}}(z)$$

and

$$\phi_{\text{cano}}(z) = \phi_0 + \int_0^z \frac{\left[ (1 + \omega_{\text{cano}}(z')) \rho_{\text{cano}}(z') \right]^\frac{1}{2}} {(1 + z') H_{\text{cano}}(z')} dz'$$

In the previous subsection, we have discussed various parametrizations of the EoS parameter for the non-canonical scalar field. Now, we will consider these parametrizations for canonical scalar field models to compare their behavior with the non-canonical scalar field models. However, it is interesting to note that one obtains the same expressions of $\rho_{\text{cano}}(z)$ and $H_{\text{cano}}(z)$ for both canonical and non-canonical scalar field models. But, the expressions for potential associated
with the scalar field \( \phi \) will be different for canonical and non-canonical scalar field models (see equations (43), (48), (53) and (54)).

The expressions for the potential \( V(z) \) and \( \phi(z) \) are very complicated and it is very difficult to express \( V \) in terms of \( \phi \). So, we have solved equations (48) and (54) numerically and have plotted \( V \) as a function of \( \phi \) (see Figures 5 and 6). In figure 6, for each panel, the dashed curve shows the evolution of the potential \( V_{\text{cano}}(\phi_{\text{cano}}) \) for each parametrization whereas the solid lines represents the evolution for corresponding non-canonical case. It has been found that for canonical case, the slope of the potential is quite flat and \( \phi_{\text{cano}} \) is almost constant (\( \dot{\phi}_{\text{cano}}^2 \ll V_{\text{cano}} \)) throughout the evolution, which yields \( \rho_{\text{cano}} \approx V_{\text{cano}} = \) constant, however, the case is different for a non-canonical scalar field model. Figure 6 also shows for each panel, that for non-canonical case, the trajectory of the potential \( V(\phi) \) changes very slowly at early epoch, but it starts increasing with \( \phi \) in such way that the non-canonical kinetic term (\( \dot{\phi}^4 \)) becomes negligible compared with the potential term at late times independent of initial conditions. So, the non-canonical scalar field exhibits an interesting property of the potential, which can provide a possible solution to the coincidence problem. This new feature occurs due to the non-canonical kinetic term present in the Lagrangian (7).

Figure 6: This figure shows the variation of the potential \( V \) with \( \phi \) by assuming \( \omega_0 = -1, \omega_1 = 0.2 \) for CPL, JBP and BA parametrizations and \( A_s = 0.9, \alpha = -0.5 \) for GCG parametrization. The dashed curve represents the trajectory of the potentials for the canonical scalar field, as shown in each panel. All the plots are for the parameter choices \( \Omega_{\phi 0} = 0.73, \Omega_{m0} = 0.27, \phi_0 = 0.1 \) and \( H_0 = 72 \text{ km} \text{s}^{-1} \text{Mpc}^{-1} \).
3 Observational Constraints on $\omega_\phi(z)$

In this section, we shall use these four parametrized theoretical models to fit observational data. To obtain the observational constraints on the model parameters we have used the latest observational dataset of type Ia supernova (SNIa) [33] of 580 data points. Although the procedure for calculation of individual $\chi^2$ function is quite well-known, we briefly mention the same for completeness.

In order to calculate $\chi^2_{SN}$ for SNIa data we follow the procedure described in Ref. [34, 35]. We fit the observed distance modulus as

$$\mu^{\text{obs}}(z_i) = m^{\text{obs}}(z_i) - M$$  \hspace{1cm} (55)

where $m$ is the apparent magnitude, $z_i$ is the redshift corresponding to the $i^{th}$ data and $M$ is the absolute magnitude (which is believed to be constant for all supernovae of type-Ia) with the theoretical distance modulus,

$$\mu^{\text{th}}(z_i) = m^{\text{th}}(z_i) - M = 5\log_{10}\left(\frac{H_0 d_L}{\text{Mpc}}\right) + \mu_0$$  \hspace{1cm} (56)

The parameter $\mu_0 = 25 - 5\log_{10}(H_0)$, is a nuisance parameter that should be marginalized and the luminosity distance (in units of Mpc) is defined as,

$$d_L(z) = \frac{1 + z}{H_0} \int_0^z \frac{dz'}{h(z')}$$  \hspace{1cm} (57)

where $h(z) = \frac{H(z)}{H_0}$ is the normalized Hubble parameter. For SNIa dataset, the $\chi^2$ function is constructed as

$$\chi^2_{SN} = \sum_{i=1}^{580} \frac{[\mu^{\text{obs}}(z_i) - \mu^{\text{th}}(z_i)]^2}{\sigma_i^2}$$  \hspace{1cm} (58)

where $\sigma_i^2$ are the standard errors on the data. Finally, marginalizing $\mu_0$ and following [34] we obtain

$$\chi^2_{SN} = A - \frac{B^2}{C}$$  \hspace{1cm} (59)

where,

$$A = \sum_{i=1}^{580} \frac{[\mu^{\text{obs}} - 5\log_{10}(d_L)]^2}{\sigma_i^2}$$  \hspace{1cm} (60)

$$B = \sum_{i=1}^{580} \frac{[\mu^{\text{obs}} - 5\log_{10}(d_L)]}{\sigma_i^2}$$  \hspace{1cm} (61)

and

$$C = \sum_{i=1}^{580} \frac{1}{\sigma_i^2}$$  \hspace{1cm} (62)

The CPL, JBP and BA model have three free parameters, namely, $\Omega_{m0}$ (or $\Omega_{\phi0} = 1 - \Omega_{m0}$), $\omega_0$ and $\omega_1$. In this case, the confidence region ellipses in the $\omega_0 - \omega_1$ parameter space can be drawn by
fixing $\Omega_{m0}$ to some constant value. So, we have done $\chi^2$ analysis by fixing $\Omega_{m0}$ (the present value of the density parameter of the matter field) to 0.26, 0.27 and 0.28 for this dataset. Hence, we can now deal with only two free parameters ($\omega_0$, $\omega_1$) and will obtain the observational bounds on this parameter from $\chi^2$ analysis of SN Ia data. With this we have plotted 1$\sigma$ (68.3%) and 2$\sigma$ (95.4%) confidence contours on $\omega_0 - \omega_1$ parameter space for various DE parametrizations. Similarly, the GCG model has three free parameters, namely, $\Omega_{m0}$, $A_s$ and $\alpha$. In this case, we have plotted 1$\sigma$ and 2$\sigma$ confidence contours on $A_s - \Omega_{m0}$ parameter space by considering $\alpha = -0.5$. The best-fit values of the parameters are obtained by minimizing the $\chi^2$ test. In figure (7) and (8), the large dots represent the best fit values of the model parameters and the small dots represent the chosen values of these parameters in our analytical models (as mentioned in previous section).

In our analytical models, we have chosen $\omega_0 = -1$, $\omega_1 = 0.2$ (for CPL, JBP and BA model) and $A_s = 0.9$, $\Omega_{m0} = 0.27$ (for GCG model) as we are interested to study and understand the effect of individual DE parametrizations for some fixed values of model parameters. For SNIa data, the chosen values of $\omega_0$ and $\omega_1$ ($A_s$ and $\alpha$) are found to be well within the 1$\sigma$ confidence contour. The observational bound on these model parameters as well as their best fit values are presented in table 1 & 2. For each model, we notice from table 1 & 2 that the best fit value of the present EoS parameter is very close to $-1$ which is consistent with the recent observations [37, 38]. The standard flat $\Lambda$CDM model ($\omega_0 = -1$ and $\omega_1 = 0$) corresponds to the intersection point between the dashed line as plotted in figure (7). Hence, the $\Lambda$CDM model comes out to be consistent always because the intersection point has been found inside the 1$\sigma$ confidence contour for all these parameterizations.

| Name   | $\Omega_{m0}$ | $\omega_0$   | $\omega_1$   | Constraints on $\omega_0$ and $\omega_1$ (within 1$\sigma$ confidence level) | $\chi^2_{min}$ |
|--------|---------------|--------------|--------------|--------------------------------------------------------------------------------|---------------|
| CPL model | 0.26         | -0.993502   | 0.22579     | $-1.19 \leq \omega_0 \leq -0.78$, $-0.96 \leq \omega_1 \leq 1.39$              | 562.22        |
|        | 0.27         | -1.00243    | 0.131297    | $-1.21 \leq \omega_0 \leq -0.79$, $-1.13 \leq \omega_1 \leq 1.36$              | 562.22        |
|        | 0.28         | -1.01094    | 0.0225095   | $-1.22 \leq \omega_0 \leq -0.79$, $-1.28 \leq \omega_1 \leq 1.30$              | 562.22        |
| JBP model | 0.26         | -0.998934   | 0.334132    | $-1.25 \leq \omega_0 \leq -0.75$, $-1.56 \leq \omega_1 \leq 2.26$              | 562.24        |
|        | 0.27         | -1.00502    | 0.189472    | $-1.25 \leq \omega_0 \leq -0.75$, $-1.79 \leq \omega_1 \leq 2$                 | 562.23        |
|        | 0.28         | -1.00935    | 0.0192596   | $-1.27 \leq \omega_0 \leq -0.74$, $-2.09 \leq \omega_1 \leq 2.06$              | 562.22        |
| BA model | 0.26         | -0.988294   | 0.12722     | $-1.15 \leq \omega_0 \leq -0.82$, $-0.49 \leq \omega_1 \leq 0.74$              | 562.21        |
|        | 0.27         | -1.00018    | 0.0773109   | $-1.17 \leq \omega_0 \leq -0.82$, $-0.57 \leq \omega_1 \leq 0.72$              | 562.21        |
|        | 0.28         | -1.01254    | 0.02335     | $-1.18 \leq \omega_0 \leq -0.83$, $-0.65 \leq \omega_1 \leq 0.7$              | 562.22        |

Table 1: Best fit values of $\omega_0$, $\omega_1$ and the minimum values of $\chi^2$ corresponding to SNIa data with different choices of $\Omega_{m0}$. 
Table 2: Best fit values of $A_s$, $\Omega_{m0}$ and the minimum values of $\chi^2$ corresponding to SNIa data with different values of $\alpha$. Note that $\omega_0(z = 0) = -A_s$ is the present day value of the EoS parameter for the GCG model. Clearly, at present epoch, the $\Lambda$CDM ($A_s = 1$) model is ruled out by our analysis of the SN Ia dataset at 2$\sigma$ confidence level (see also figure 8).

| $\alpha$ | $A_s$ | $\Omega_{m0}$ | Constraints on $A_s$ and $\Omega_{m0}$ (within 1$\sigma$ confidence level) | $\chi^2_{min}$ |
|----------|-------|---------------|------------------------------------------------------------------------------|----------------|
| -0.5     | 0.9   | 0.24          | $0.79 \leq A_s \leq 0.95, 0.20 \leq \Omega_{m0} \leq 0.40$                   | 563.14         |
| -0.55    | 0.9   | 0.2234        | $0.72 \leq A_s \leq 0.94, 0.20 \leq \Omega_{m0} \leq 0.50$                   | 562.40         |
| -0.6     | 0.9   | 0.2253        | $0.74 \leq A_s \leq 0.94, 0.20 \leq \Omega_{m0} \leq 0.49$                   | 562.42         |

Figure 7: Plot of 1$\sigma$ and 2$\sigma$ confidence contours on $\omega_0 - \omega_1$ parameter space for the CPL parametrization (left panel), JBP parametrization (middle panel) and BA parametrization (right panel) respectively. In this graph, $\chi^2_{min}$ indicates the minimum value of $\chi^2$ corresponding to the best fit values of $\omega_0$ and $\omega_1$ for the SN dataset, as indicated in the frames. The fixed value of $\Omega_{m0}$ is also indicated in the frame.

Figure 8: Plot of 1$\sigma$ and 2$\sigma$ confidence contours on $A_s - \Omega_{m0}$ parameter space for the GCG parametrization. In this graph, $\chi^2_{min}$ indicates the minimum value of $\chi^2$ corresponding to the best fit values of $A_s$ and $\Omega_{m0}$ for the SNIa dataset, as indicated in the frame. This is for $\alpha = -0.5$. 
In figure (9), we have shown the behavior of the EoS parameter \( \omega_\phi(z) = \frac{\dot{\phi}^4 - V(\phi)}{\frac{4}{3} \dot{\phi}^4 + V(\phi)} \), for four different parametrizations using the best fit values of the model parameters (as listed in Table 1 & 2) for this dataset. It may be noted here that the forms of EoS parameter (or potential) are different for different models. But, \( \omega_\phi(z) \) approaches \(-1\) at present epoch \((z = 0\) or \(a = 1\)) for CPL, JBP & BA parametrization model, whereas for GCG model, \( \omega_\phi(z) \) is around \(-0.9\) at present but approach \(-1\) value as evident from figure (9). Consequently, this feature suggests that \( \dot{\phi}^4 \ll V(\phi) \) i.e., the potential term dominates over the non-canonical kinetic term to accelerate the cosmic expansion at present. It is also evident from figure (9) that the CPL, JBP and BA models behave like phantom dark energy \((\omega_\phi < -1, [3])\) today. Interestingly, it has been found that for the BA model, the EoS parameter does not deviate much from \(-1\) in near future \((z = -1)\) and approaches \(-1\) rapidly in the near future \((at z = -1)\) and it remains less than \(-1\) in remote future \((z < -1)\). For CPL & JBP models however, the analysis is valid upto \(z > -1\). However, the GCG behaves like dark energy \((\omega_\phi = -0.9 > -1)\) at late time and its equation of state also settles to \(-1\) in the near future. So, unlike the GCG model, the BA model will avoid the finite-time future singularity \([43, 44]\). It has been also noticed for the CPL and JBP models that the EoS parameter changes rapidly (below the \(\omega_\phi = -1\) boundary) and it diverges in the finite future \((z = -1)\).

In the limit, \( a = \frac{1}{1+z} \rightarrow \infty \), we have \( \rho_\phi(a) \rightarrow \infty \) and \( p_\phi(a) \rightarrow \infty \) for CPL as well as JBP models. This implies that the universe (according to the CPL and JBP models) will end up in Big Rip singularity \([43]\), where the phantom energy density becomes very large in finite time and overcomes the gravitational repulsion.

Figure 9: The left figure represents the plot of the EoS parameter \( \omega_\phi \) vs. redshift using the best fit values of \((\omega_0, \omega_1)\) and \(\Omega_{m0} = 0.27\) (as given in Table 1). The thick, dashed and dotted curves represent CPL model, JBP model and BA model respectively. The right panel corresponds to the evolution of \(\omega_\phi\) for the GCG model. This plot is for the best fit values of \(A_s\) and different values of \(\alpha\) (as given in Table 2); \(\alpha = -0.5\) (thick curve), \(\alpha = -0.55\) (dashed curve) and \(\alpha = -0.6\) (thin curve).
4 Conclusion

It is well known that the late-time accelerating expansion of the universe can be described by a scalar field. For this reason, in this present work, we have discussed about four different types of non-canonical scalar field models with varying dark energy EoS for understanding the observed cosmic expansion. As a time-dependent EoS plays an important role for understanding the nature of DE, so we have considered four phenomenological parametrizations of dark energy EoS. The dynamical features of each models are analyzed, such as the evolutions of the deceleration parameter $q(z)$ and the density parameters ($\Omega_{\phi}$ and $\Omega_m$). The resulting cosmological behavior is found to be very interesting.

For all the toy models, it has been found that the deceleration parameter $q(z)$ indicates an early deceleration followed by a late time acceleration of the universe (see figures 1a, 2a, 3a, 4a). We have also shown the evolution of density parameters and it is found that the results are in good agreement with recent observations [31].

We have also compared our theoretical models with the observational data coming out of the latest Union2.1 SNIa measurements. For this purpose, we have written the Hubble parameter $H(z)$ in terms of observable parameters ($z$, $H_0$ & $\Omega_{m0}$) and other parameters for each DE parametrizations. We have obtained the best fit values of the parameters $\omega_0$ and $\omega_1$ by fixing the value of $\Omega_{m0}$ to 0.26, 0.27 and 0.28 (shown in Table 1). It may be important to mention here that the values of parameters of the model which were chosen for analytical results are well fitted in the $1\sigma$ and $2\sigma$ confidence contours for each parametrizations. We have found that the $\omega_{\phi} = -1$ crossing feature is also allowed by the SNIa dataset for the CPL, JBP and BA models with its present best-fit EoS parameter, $\omega_0 < -1$ (as presented in Table 1). This is consistent with the results obtained by several authors [32, 45, 46, 47], in which the analysis was performed by the SN Ia dataset and their combinations with other observational datasets. On the other hand, we have also found that the standard $\Lambda$CDM is still compatible at the $1\sigma$ confidence level for these models. It should be noted that the GCG model has been studied by many authors for the parameter range, $0 \leq \alpha \leq 1$ [6, 7]. For the GCG model, in this paper, we have obtained the best fit values of the parameter $A_s$ and $\Omega_{m0}$ by fixing the value of other parameter within the range, $-1 < \alpha < 0$ (shown in Table 2). However, the case $\alpha < 0$, is in good agreement with the work of Sen and Scherrer [39], Gong et al. [40] and Hazra et al. [28]. It has been also noticed from table 2 that the range of the allowed values of $\Omega_{m0}$ match well with the previous results obtained by Riess et al. [41] and Sahni et al. [42].

However, as discussed in the previous section, the CPL and JBP models lose their prediction capability regarding the future evolution of our universe. In this present work, we would also like to mention that it is very difficult to distinguish the GCG and BA models from a $\Lambda$CDM in the near future and hence we need more investigations to constrain dark energy models more tightly. Obviously, we can not yet say which model is better as compare to other models by the analysis of the Union2.1 SN Ia dataset. We hope that the next generation observational data (including the combination of SN Ia data with other cosmological observations) can provide more tight constraints on EoS parameter to explore greatly our understanding regarding the nature of dark energy.
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