Quantum many-body systems out of equilibrium

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How do closed quantum many-body systems driven out of equilibrium eventually achieve equilibration? And how do these systems thermalize, given that they comprise so many degrees of freedom? Progress in answering these—and related—questions has accelerated in recent years—a trend that can be partially attributed to success with experiments performing quantum simulations using ultracold atoms and trapped ions. Here we provide an overview of this progress, specifically in studies probing dynamical equilibration and thermalization of systems driven out of equilibrium by quenches, ramps and periodic driving. In doing so, we also address topics such as the eigenstate thermalization hypothesis, typicality, transport, many-body localization and universality near phase transitions, as well as future prospects for quantum simulation.

The physical systems considered here are local quantum many-body systems out of equilibrium, later sections will also touch on continuous models and open quantum systems.

Equilibration after quenches

The question of how thermodynamics emerges from microscopic quantum dynamics already intrigued the founding fathers of quantum theory, but has also been a key topic in recent years. A very clean setting for non-equilibrium dynamics is the one emerging from a sudden global quench. Initially, the system is in a state \( \rho_0 \), which could be the ground state of a local Hamiltonian. Then one instantaneously alters the system’s parameters globally and considers the many-body unitary time evolution under some local Hamiltonian \( H \). Of specific interest are expectation values of observables \( A \) at later times

\[
\langle A(t) \rangle = \text{tr}(e^{-iHt} \rho_0 e^{iHt} A)
\]

A main question is to what extent and for what times the situation can be described by a suitable equilibrium ensemble. Notably, the dynamics is time-reversal invariant and for finite systems recurrent, so, a priori, it seems far from clear how and in what sense equilibrium can be reached dynamically.

A first important insight is that such systems generically indeed relax and equilibrate in the following sense: even though the dynamics is entirely unitary, following transient non-equilibrium dynamics, expectation values \( \langle A(t) \rangle \) of many observables equilibrate. This is specifically true for local observables, which are supported only on a small number of sites. The apparent long-time equilibrium state has to be equal to the time average

\[
\omega = \lim_{T \to \infty} \frac{1}{T} \int_0^T dt \ e^{-itH} \rho_0 e^{itH}
\]

This state is the maximum entropy state, holding all constants of motion fixed\(^\text{27}\). In the case of non-degenerate eigenvalues of the Hamiltonian, this ensemble is also called the diagonal ensemble\(^\text{18}\). This feature is reminiscent of a dynamical Jaynes’ principle: a many-body system is pushed out of equilibrium and follows unitary

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dynamics. Yet, for most times, the system seems as if it had equilibrated to a maximum entropy state.

The general expectation that many-body systems equilibrate can be made rigorous in a number of ways. For some free models, specifically for the case of the integrable non-interacting limit of the Bose–Hubbard model (that is, $U \to 0$ in equation (1)), equilibration is provably true in the strong sense of equilibration during intervals. In this setting, one can show for local observables $A$ that $|\langle A(t) \rangle - \text{tr}(\omega A)|$ is arbitrarily small after a known initial relaxation time, and remains so until a recurrence time that grows linearly with the system size $t_{0}$17. For such a statement to hold, one merely has to assume that the otherwise arbitrary initial state has suitably decaying correlations. At the heart of such a rigorous argument are non-commutative central limit theorems as well as Lieb–Robinson bounds, elaborated upon later. Such bounds lead to an effective light cone and give an intuitive explanation for the linear scaling of the recurrence time, as it is the time that excitations need to travel through the whole system and be reflected at the boundary.

If no locality structure is available or is made use of, one can still show equilibration on average for all Hamiltonians that have non-degenerate energy gaps: one can bound the time average $E_{\text{cfl}}(\infty, \omega)|\langle A(t) \rangle - \text{tr}(\omega A)|$ by a quantity that usually is expected to scale as one over the square root of the Hilbert space dimension of the total system $N$. This means that such out-of-equilibrium quantum systems seem to relax for most times, even if by that statement alone no information on timescales can be deduced. More refined bounds have been derived, also providing some information about the relevant timescales of equilibration18,22, but they are far from those observed in numerical simulations7,11,16 and experiments14,24,25. These findings are also related to further efforts to describe the size of the time fluctuations after relaxation14,27.

By no means are sudden global quenches the only relevant setting of quantum many-body systems out of equilibrium. Similarly important are scenarios of local quenches28–36, where the entire system is not uniformly modified, but rather the system is locally suddenly driven out of equilibrium by a change limited to a bounded number of lattice sites. Ramps or sweeps37,38, during which the Hamiltonian is changed according to some schedule, are of interest as well. In this framework, specifically pump–probe type settings39 generate an out-of-equilibrium situation that can be used as a diagnostic tool in the condensed-matter context, giving rise to an interesting research topic in its own right.

Thermalization

The success of thermodynamics in describing large-scale systems indicates that often the equilibrium values can be described by a few parameters, such as the global temperature and particle number. We use the term thermalization to refer to equilibration towards a state that is in a suitable sense close to being indistinguishable from a thermal equilibrium state proportional to $e^{-\beta H}$ for some inverse temperature $\beta > 0$. There are several mechanisms that can lead to thermalization in this sense: the first is based on the eigenstate thermalization hypothesis (ETH; ref. 8). It conjectures that sufficiently complex quantum systems have eigenstates that—for physically relevant observables such as local variables—are practically indistinguishable from thermal states with the same average energy. The ETH and its breakdown39,40 in integrable systems have recently been investigated extensively, mostly with numerical methods. Supporting evidence for this has been collected in a plethora of models41,42.

Without invoking the ETH one can still rigorously show thermalization in weakly interacting systems under stronger conditions on the initial state43. Furthermore, dynamical thermalization of local expectation values to those in the global thermal state44 has been shown under stronger conditions, most notably translation invariance and the existence of a unique Gibbs state. Effective thermalization under unitary quantum dynamics can also be made plausible under conditions of classical chaos45. The notion of relative thermalization46 focuses on the decoupling of a subsystem from a reference system. A subsystem $S$ is called approximately thermal relative to a reference system $R$, if the joint state $\rho_{S\otimes R}$ is close in trace norm to a state of the form $\rho_{S} \otimes \rho_{R}$, with $\rho_{S}$ being a suitable microcanonical state. Yet, to summarize, there is so far no compelling proof that globally quenched non-integrable systems dynamically thermalize under local unitary many-body dynamics.

These dynamical approaches are complemented by works based on typicality arguments. Here, rather than following the dynamical evolution of a system, one tries to justify the applicability of ensembles by showing that most states drawn randomly according to some measure have the same physical properties as some appropriate ensemble. For example, the overwhelming majority of pure states drawn uniformly at random from a high-dimensional subspace look, for local observables, almost exactly like the microcanonical state on that subspace47,48.

If a quantum system is integrable49, in the sense that it has sufficiently many local conserved quantities, one should not expect the system to thermalize: These constants of motion prohibit full thermalization to the canonical ensemble. One can still expect the system to equilibrate to the maximum entropy state given these locally conserved quantities, a so-called generalized Gibbs ensemble (GGE; refs 5,6,15,17,18,50). If this is the case, the equilibrium state is close to a state proportional to $e^{-\beta \sum_{k \neq k'} A_{k} \rho_{k} / \lambda_{k}}$, with $A_{k}$ corresponding to the (known, usually linearly many) conserved quantities of the system and $\rho_{k}$ to the respective real Lagrange multipliers. However, the explicit construction of the GGE for general interacting integrable models remains an open problem, and there are non-integrable systems where the GGE, taking into account all known conserved quantities, fails to correctly describe the equilibrium state51,52.

Several quantum many-body systems exhibit some form of pre-thermalization: this term refers to any apparent equilibration to a meta-stable state on a short timescale, before—a long timescale—the system relaxes to a state indistinguishable from a genuine thermal state53. Examples are almost-integrable systems54,55 and continuous systems of coupled Bose–Einstein condensates56,57. Specifically, in a close-to-free model, there is an initial build-up of correlations where quasi-particles are formed, whereas actual thermalization occurs on much longer timescales58.

‘Light cones’ and entanglement dynamics

An important stepping stone for a better understanding of the non-equilibrium dynamics of local Hamiltonian models is the insight that Lieb–Robinson bounds57–60 limit the speed of information propagation in such systems. There are several ways of stating such bounds: one is to say that for any two observables $A$ and $B$

$$\|\langle A(t), B \rangle \| \leq c \| A \| \| B \| \min(|A|, |B|) e^{-\gamma t (d(A, B) - v t)}$$

where $d(A, B)$ is the distance between the support of the observables, $|A|$ and $|B|$ the size of their supports, $v \geq 0$ takes the role of a group velocity, and $c$ and $\mu$ are positive constants. That is to say, information propagation outside a ‘light cone’ is exponentially suppressed. It immediately follows that correlation functions can grow significantly in time only inside the light cone59. Although these bounds are not necessarily tight, the picture that the local dynamics leads to excitations travelling through the
The light-cone-like propagation of information has implications for entanglement dynamics. Many-body states are said to satisfy an area law for the entanglement entropy, if for a subset of sites the latter scales only like the boundary ‘area’ of the subset rather than its ‘volume’ [65,66]. It has been shown that if initially such a law holds, this will remain true for longer times [65,67]. Yet, at the same time, the pre-factor of this area law is expected to grow exponentially [66]. This is the ultimate reason why numerical tensor network methods [67,68], such as the density-matrix renormalization group method [69], can simulate out-of-equilibrium dynamics efficiently for short times on a classical computer, whereas long times are not accessible.

These numerical techniques are complemented by other powerful approaches, such as those based on Monte Carlo [70] and dynamical mean-field theory (DMFT; ref. 71). Although Monte Carlo DMFT is well established for static properties from which phase diagrams can be explored, it can also be applied to time evolution [72] as long as no sign problem occurs [72]. However, all numerical tools share the feature that they are limited to short times, which is in line with arguments using complexity theory, which provide evidence that generic quantum long-time dynamics cannot be efficiently classically simulated [73]. If interactions are not strictly local, modified Lieb–Robinson bounds still hold [73], giving rise to a rich phenomenology, which has been experimentally explored in systems of trapped ions exhibiting long-range order [74,75].

Transport
Understanding electronic transport is one of the main motivations that started the field of condensed-matter physics and gave rise to a large body of work trying to accurately capture the conductivity properties of these systems. Most relevant for the purpose of this review are the transport properties of simple paradigmatic models such as the Heisenberg spin chain or Hubbard-type models. In these models, transport is expected to lead to equilibration, as it allows the system to locally forget its precise initial configuration (manuscript in preparation). In spin chains, transport typically refers to a spreading of magnetized domains [75]. In optical lattices one commonly studies the transport of particles starting from an initially trapped situation [76,77], or of quasi-particles [78,79].

Two of the key aims are to distinguish between diffusive and ballistic transport and to understand the limitations of linear response theory [79]. Whereas simple, integrable models can be solved exactly and thus provide an excellent benchmark, our understanding of transport in realistic set-ups relies heavily on numerical tools. Tools explicitly storing the full wavefunction in the memory, such as exact diagonalization, are limited to comparably small systems. Tensor network tools, such as t-DMRG, TEBD, and variants thereof [80,89], allow one to parametrize the state more efficiently and are accurate up to machine precision as long as entanglement entropies are sufficiently small [80]. Thus, for short and intermediate times these tools allow an investigation of transport and equilibration in systems with several hundred sites [14,16,75,77].

Absence of thermalization and many-body localization
Crucial for transport properties of materials is the influence of irregularities such as defects. The impact of random potentials on particle mobility has thus been the subject of intensive study over the past century, culminating in the famous result of Anderson [79,81], showing full localization both of the eigenfunctions as well as of the dynamics for single-particle disordered models. In the case of non-interacting fermions in one dimension or, equivalently, via the Jordan–Wigner transformation, for free spin chains, Anderson’s result gives strong localization of the fermionic modes [82,83]. These systems thus fail to serve as their own heat bath, and transport is strongly suppressed.

For interacting many-body systems, the effects of disorder are still much less clear, despite great efforts of research in many-body localization [73,84–87]. Treating disorder as a perturbation, and starting from the Fock space of Slater determinants, a key step in the understanding was to develop an intuition of localization in Fock space [84]. Interestingly, for out-of-equilibrium situations, both the support of time-evolved local observables as well as the entanglement entropy are numerically found to grow logarithmically in the interacting case, and are thus unbounded [86]. Dynamical aspects of many-body localization are expected to go along with localization of the eigenstates. In several recent studies, localization effects due to disorder have been associated with a lack of entanglement in many energy eigenstates [87,88] or alternatively only in those below a so-called mobility edge [84,88]. This could lead to a phase transition between a disordered insulator with exactly zero conductivity and a conducting phase [88]. The localization of eigenstates leads to a clear violation of the ETH and connects to an absence of thermalization effects for localizing many-body systems and the existence of local constants of motion in these models [73,86]. Although these signatures of many-body localization have been extensively explored, a comprehensive definition of many-body localization is still lacking.

Dynamics of quantum phase transitions
Exploring the dynamical signatures of phase transitions is an interesting problem in its own right. It connects to the important question of how slow an experimental ramp needs to be to avoid the creation of defects when preparing a quantum phase and whether this is at all possible in the thermodynamic limit [31]. A key theoretical model for these transitions, based on the adiabatic theorem, has been provided by Kibble and Zurek [31] for the important case of moving from a gapped phase into criticality. Based solely on the critical exponents of the model, this strikingly simple formalism predicts the number of introduced defects as a function of the velocity of the ramp. It was originally developed for thermal transitions, where it has also been experimentally tested [90,92].

For quantum phase transitions such a scaling can be derived in the limit of infinitely slow ramps, based on universality arguments together with adiabatic perturbation theory [93], and seems to be capable of accurately describing the dynamics of quantum phase transitions when crossing a single critical point in some models [31]. For strongly correlated systems in realistic experimental settings, the dynamics of quantum phase transitions is more elusive and there are indications that the complexity of the dynamics cannot fully be captured by employing simple scaling arguments based only on the critical exponents of the model [92,95]. The transition out of criticality has also been investigated, both numerically [94] and experimentally [31]. Despite these first promising results, the dynamics of quantum phase transitions and how to capture them in terms of simple scaling laws is still a largely open problem.

Quantum simulations
When investigating the non-equilibrium behaviour of large-scale many-body systems, analytical and numerical tools quickly reach their limits, especially for lattice models in more than one dimension. A particularly exciting perspective arises from the idea that experimentally controlled quantum many-body systems constitute instances of analogue quantum simulators [95] overcoming these limitations. These devices mimic natural interacting quantum many-body systems by reconstructing their Hamiltonian, but now under precisely controlled conditions. This is in contrast to digital quantum simulators, anticipated devices that approximate quantum
chains can also be simulated using bosonic systems that are otherwise inaccessible. Using tilted lattices, effective spin altering the system’s parameters. In this way, bosonic ultracold phase transitions and slow quenches degenerate one-dimensional (1D) Bose gases have been explored. The dynamics of quantum phase transitions and slow quenches can be probed by slowly altering the system’s parameters. In this way, bosonic ultracold atoms give rise to simulators of static and dynamical properties that are otherwise inaccessible. Using tilted lattices, effective spin chains can also be simulated using bosonic systems. To build confidence in the correctness of the results, Monte Carlo studies as well as tensor network methods can be used to partially validate such simulators.

Ultracold fermionic atoms also give rise to an intriguing platform to probe interacting quantum many-body systems. They allow one to probe Fermi–Hubbard physics, the relaxation dynamics of a Fermi gas, and fermionic transport. In general, cold atoms of cold atoms in optical lattices offer great flexibility. By making use of Feshbach resonances, appropriate periodic driving, and superlattices generated by additional optical lattices with half the wavelength, sophisticated quantum simulations are conceivable—for example, the simulation of lattice gauge theories.

Continuous set-ups of cold atoms facilitate direct experimental studies of the coherence dynamics in both isolated and coupled degenerate one-dimensional (1D) Bose gases. They can be implemented using either optical architectures, resulting in 2D arrays of elongated 1D systems, or using atom chips which allow the implementation of single condensates and thus enable measurements of local phase fluctuations. Such systems have been successfully employed to investigate (pre-)thermalization and out-of-equilibrium dynamics not leading to equilibration on relevant timescales.

Another promising architecture—one that for reasons of space we merely touch upon—is provided by ion chains, allowing the precise control of the individual constituents. Owing to the involved Coulomb interaction, these systems are, among many other applications, well suited to explore the validity of Lieb–Robinson bounds and their breakdown for long-range interactions. Photonic architectures, as well as arrays of superconducting qubits, offer further promising platforms.

Although the precise computational complexity of analogue simulators has not yet been identified, and although it is unclear to what extent genuinely reliable analogue simulation is possible in the absence of fully fledged quantum error correction, there is evidence that such analogue simulators have the potential of outperforming classical computers. Notably, quantum many-body dynamics can already be probed in experimental settings under precisely controlled conditions for longer times than can be kept track of using state-of-the-art tensor network methods on modern classical supercomputers.

Periodically driven systems

Periodically driving a well-controlled quantum system opens up new vistas for quantum simulations. Consider a system with a Hamiltonian that satisfies \( H(t) = H(t + \tau) \) for all times \( t \) for some period \( \tau > 0 \). The eigenstates of such systems are usually constantly changing in time, but the long-time dynamics can be inferred from their Floquet operator \( U(\tau) \), which is the time evolution operator over one period of driving. The Floquet operator gives rise to an effective Hamiltonian \( H_{\text{eff}} \) through

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U(\tau) = e^{-iH_{\text{eff}}\tau}
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and the time evolution of the driven system can be seen as a stroboscopic simulation of the time evolution under \( H_{\text{eff}} \). The effective Hamiltonian can be tuned by choosing an appropriate driving schedule with great flexibility concerning the model parameters. This allows the study of topological effects in driven systems and can be used to engineer gauge fields.

This has lead to an experimental realization of the topological Haldane model in a periodically modulated optical honeycomb lattice as well as to experimental set-ups featuring Hofstadter bands and spin–orbit coupling in optical lattice architectures. The synchronization of the motion of periodically driven systems with the drive allows the description of the long-time behaviour by a periodic ensemble. For free systems it turns out to be given by a maximum entropy ensemble given the constants of motion—reminiscent of Jaynes’ principle for systems in equilibrium and the generalized Gibbs ensemble emerging during equilibration after a quench.

Perspectives for closed and open systems

The research field of quantum many-body systems out of equilibrium is developing rapidly and is taking new and unexpected directions at a remarkably high rate. We end this article by discussing some promising directions for further research and stress some exciting perspectives of this field. Here we also go beyond the closed system setting and briefly sketch some perspectives that open up owing to a clever use of engineered dissipation.

Progress in the field is substantially driven by novel experimental developments in recent years. In optical lattice systems, novel techniques of single-site resolved fluorescence imaging have open up entirely new possibilities for probing out-of-equilibrium dynamics. Settings with periodic driving now allow the study of topologically non-trivial situations and the development of a deeper understanding of particle physics and condensed-matter systems.

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it is necessary to know whether the relevant physics is captured by the long-time equilibrium state or happens in a possibly markedly different non-equilibrium or quasi-stationary regime. It is hoped that future experiments with quantum simulators can help to shed some light on this notoriously difficult problem and allow one to identify realistic settings and physically relevant observables for which fast equilibration is to be expected. The conditions for thermalization are less clear than those for equilibration and it is questionable whether (non-)integrability is able to capture all facets of this phenomenon\cite{12,13}. The problem of many-body localization has recently become a focus of attention\cite{14,15}, and is developing rapidly, even if a unifying framework of the various approaches still seems missing.

This article is focused on closed quantum systems out of equilibrium, but a research area that deserves special attention is quantum systems out of equilibrium allowing us to probe long-standing questions of generic isolated quantum systems. In this mindset, an open-system quantum simulator with engineered local Markovian noise of matter. In fact, such preparations can be facilitated by suitably

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115. Aidelsburger, M. et al. Measuring the Chern number of Hofstadter bands with ultracold bosonic atoms. Preprint at http://arXiv.org/abs/1407.4205 (2014).

116. Struck, J., Simonet, J. & Sengstock, K. Spin–orbit coupling in periodically driven optical lattices. Phys. Rev. A 90, 031603 (2014).

117. Goldman, N. et al. Direct imaging of topological edge states in cold-atom systems. Proc. Natl Acad. Sci. USA 110, 6736–6741 (2013).

118. Lazarides, A., Das, A. & Moessner, R. Periodic thermodynamics of isolated quantum systems. Phys. Rev. Lett. 112, 150401 (2014).

119. Endres, M. et al. Single-site- and single-atom-resolved measurement of correlation functions. Science 334, 200–203 (2011).

120. Sherson, J. F. et al. Single-atom-resolved fluorescence imaging of an atomic Mott insulator. Nature 467, 68–72 (2010).

121. Martín-Martínez, E., Fuentes, I. & Mann, R. B. Using Berry’s phase to detect the Unruh effect at lower accelerations. Phys. Rev. Lett. 107, 131301 (2011).

122. Agarwal, K. et al. Chiral prethermalization in supersonically split condensates. Phys. Rev. Lett. 113, 190401 (2014).

123. Ritsch, H., Domokos, P., Brennecke, F. & Esslinger, T. Cold atoms in cavity-generated dynamical optical potentials. Rev. Mod. Phys. 85, 553–601 (2013).

124. Linden, N., Popescu, S. & Skrzypczyk, P. How small can thermal machines be? the smallest possible refrigerator. Phys. Rev. Lett. 105, 130401 (2010).

125. Gallego, R., Riera, A. & Eisert, J. Correlated thermal machines in the micro-world. New J. Phys. 16, 125009 (2014).

126. Hubeny, V. E. & Rangamani, M. A holographic view on physics out of equilibrium. Adv. High Energy Phys. 2010, 297916 (2010).

127. Kliesch, M., Gogolin, C., Kastoryano, M. J., Riera, A. & Eisert, J. Locality of temperature. Phys. Rev. X 4, 031019 (2014).

128. Diehl, S. et al. Quantum states and phases in driven open quantum systems with cold atoms. Nature Phys. 4, 878–883 (2008).

129. Barreiro, J. T. et al. An open-system quantum simulator with trapped ions. Nature 470, 486–491 (2011).

130. Barontini, G. et al. Controlling the dynamics of an open many-body quantum system with localized dissipation. Phys. Rev. Lett. 110, 035302 (2013).

131. Diehl, S., Rico, E., Banov, M. A. & Zoller, P. Topology by dissipation in atomic quantum wires. Nature Phys. 7, 971–977 (2011).

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