Cosmic Rays from Cosmic Strings with Condensates

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We re-visit the production of cosmic rays by cusps on cosmic strings. If a scalar field ("Higgs") has a linear interaction with the string world-sheet, such as would occur if there is a bosonic condensate on the string, cusps on string loops emit narrow beams of very high energy Higgses which then decay to give a flux of ultra high energy cosmic rays. The ultra-high energy flux and the gamma to proton ratio agree with observations if the string scale is $\sim 10^{13}$ GeV. The diffuse gamma ray and proton fluxes are well below current bounds. Strings that are lighter and have linear interactions with scalars produce an excess of direct and diffuse cosmic rays and are ruled out by observations, while heavier strings ($\sim 10^{15}$ GeV) are constrained by their gravitational signatures. This leaves a narrow window of parameter space for the existence of cosmic strings with bosonic condensates.

The detection of cosmic topological defects can provide a direct window to fundamental physics and the very early universe (for a review, see [1]). Hence it is no surprise that there has been a concerted effort to examine observational consequences of cosmic topological defects, including gravitational wave emission, gravitational lensing, fluctuations in the cosmic microwave background, and ultra-high energy cosmic rays. If cosmic strings are superconducting, they may also lead to electromagnetic phenomena such as gamma ray and radio bursts.

The gravitational effects of cosmic strings are stronger for heavier strings and current observations of the cosmic microwave background rule out strings above the $\sim 10^9 \times 10^{15}$ GeV energy scale. As lighter strings are considered, gravitational effects become less significant, and other particle physics signatures become relatively important. If the strings are superconducting, the currents on the string can lead to electromagnetic radiation that could be observable. However, if the strings are not superconducting, such signatures will be absent, and one must turn to particle emission from cosmic strings. If string dynamics forces loop production at the smallest possible scales, as suggested in [2], particles can be copiously emitted, leading to strong constraints. However, other studies indicate that string loops are large compared to microscopic length scales [1] and particle emission is suppressed [3]. Even in this case, portions of a string loop may get boosted to very high Lorentz factors, creating a “cusp” on the string (see Fig. 1), and this may potentially provide a burst of particles that could be seen in cosmic ray detectors.

Particle emission from cusps on cosmic strings has been studied by several authors. Srednicki and Theisen [4] considered a quadratic interaction of a scalar field with an idealized (zero thickness) string and came to the conclusion that particle emission is insignificant for astrophysical size string loops. Our analysis for the radiation is similar to that of Refs. [4, 5], though the particular linear interaction of the scalar field (call it $H$) with the string world-sheet, as would occur if $H$ condenses on the string [27], has not been considered before. A linear interaction causes an enhancement of particle production by a factor of $M/m$ where $M$ is the string scale and $m$ is the mass of $H$. If $M$ is the Grand Unification scale, while $m$ is the electroweak scale, this factor can be as large as $10^{13}$.

Particle emission from cusps on thick strings has been considered in Ref. [6, 7]. Now the cusp consists of overlapping, oppositely oriented strings that can annihilate and give off energetic particles. A careful study of this process, including numerical evolution of the field equations, shows that the resulting flux of particles is too small to be of interest [8, 9]. In contrast, the linear interaction mechanism we study is insensitive to the thickness of the string, and occurs over a string length that is much
larger than the length over which cusp annihilation occurs. Thus we can ignore cusp annihilation and work in the zero thickness limit.

There are important observational constraints on the so-called “top down” models for production of ultra-high energy cosmic rays (UHECR), in which a heavy particle decays to give ultra-high energy protons and gamma rays. The constraint arises because there are bounds on the diffuse gamma ray flux in the EGRET window and also on the fraction of photons to protons in the observed UHECR. In previous studies, using cusp annihilation on cosmic strings as the source of Higgs injection, it was found that the EGRET constraint on diffuse gamma ray fluxes implies that the flux of UHECR is uninterestingly low. However, the constraint on diffuse gamma rays is sensitive to the spectral features of the injected Higgs particles and to the particular interaction of the string. The Higgs particle emission that we consider yields a diffuse flux of gamma rays and protons that is below EGRET bounds if the string scale is \( > 10^6 \text{ GeV} \) and an UHECR flux consistent with observations for string scale \( \sim 10^{13} \text{ GeV} \). The ratio of photons to protons in the UHECR flux is also consistent with current bounds. Hence our model can explain the observed UHECR and not run into trouble with the EGRET bounds.

It is to be noted that cosmic ray production by strings increases as the string scale decreases. Hence strings on scales less than \( \sim 10^{13} \text{ GeV} \), and with these interactions, are excluded by the observed flux of UHECR. This is especially interesting since heavier strings are constrained by their gravitational signatures. Hence there is a narrow window between, say, \( 10^{13} - 10^{15} \text{ GeV} \) for the mass scale of cosmic strings having linear interactions with a scalar field.

To summarize, the novelty in the present work is that we are considering a new, generic, interaction of cosmic strings with scalar fields that leads to high energy particle emission. This interaction seems to have been missed in the literature. Also, we have been careful to take the beamed nature of the emission into account. We have focused on deriving approximate analytical estimates so as to keep the physical aspects of the problem as apparent as possible. More detailed predictions will require numerical evaluation of the production and propagation.

We begin in Sec. I by describing the field theory interactions under consideration. We then evaluate the rate at which a single cusp emits Higgs particles in Sec. II. In Sec. III we use the results of Sec. II to determine the cosmological Higgs injection function. The diffuse gamma ray and proton fluxes are calculated in Secs. IV and V respectively. These cosmic rays originate in strings that are relatively far away from the Earth. Higgs emission from strings that are closer to us and pointed at us can give UHECR. The direct flux of UHECR is calculated in Sec. VI where we also discuss the ratio of photons to protons in UHECR. We conclude in Sec. VII. In Appendix A we summarize some known facts about cosmic string loop dynamics.

I. FIELD THEORY

The interaction we consider is

\[
S_{\text{int}} = -\kappa M \int d^2\sigma \sqrt{-\gamma} h \tag{1}
\]

where \( \kappa \) is a coupling constant assumed to be \( \sim 1 \), \( M \) is the string energy scale assumed to be at the Grand Unified scale, \( \gamma_{ab} \) the string world-sheet metric, and \( h \) a scalar field.

A linear interaction can be considered quite generally. It can also arise if there is a bosonic condensate on the string. To see this explicitly, consider the model

\[
S = S_0[\Phi, H, \ldots] + \kappa \int d^4x (\Phi^\dagger \Phi - M^2) H^\dagger H \tag{2}
\]

where \( S_0 \) is a field theory action that yields cosmic string solutions when \( \Phi \) gets a vacuum expectation value (VEV). \( H \) is a scalar field that we can take to be the electroweak Higgs for concreteness.

At energy scales above the Grand Unified scale, the VEVs of \( \Phi \) and \( H \) are both zero. At lower energy scales, but still above the electroweak scale, \( \Phi \) gets a VEV so that \( |\Phi|^2 = M^2 \) but \( |H|^2 = 0 \). At this stage, we also have string solutions whose tension is \( \mu \sim M^2 \) and width is \( \sim M^{-1} \). Inside the core of the string, where \( \Phi^\dagger \Phi \) can become small, it may be favorable for \( H \) not to vanish since the coefficient of the \( H^\dagger H \) term in Eq. (2) becomes negative. As in the case of bosonic superconducting strings [10], there can be an \( H \) condensate in the core of the string. Since \( M \) is the only mass scale in the problem at this stage, the magnitude of \( H \) is of order \( M \) within the core of the string. At a lower energy scale, \( H \) too gets a VEV. For example, if \( H \) is the electroweak Higgs, this scale is \( m \sim 100 \text{ GeV} \). We will assume \( m \ll M \) and hence the VEV of \( H \) within the string is unaffected by the lower scale (electroweak) symmetry breaking.

The interaction term in Eq. (2) can now be written as

\[
S_{\text{int}} = \kappa \int d^2\sigma \int d^2x \sqrt{-\gamma} (\Phi^\dagger \Phi - M^2) H^\dagger H
\]

\[
= \kappa \int d^2\sigma \sqrt{-\gamma} \int d^2x (\Phi^\dagger \Phi - M^2) \times
\]
II. HIGGS EMISSION

The equation of motion for the Higgs field is

\[ (\Box + m^2) h = j \]

where

\[ j(x) = -\kappa M \int d\tau \, d\sigma \sqrt{-\gamma} \, \delta^4(x - X(\sigma, \tau)) \]

Then the number of Higgs particles with momentum \( k \) produced due to a source is

\[ dN_k = |\tilde{j}(\omega_k, k)|^2 \frac{d^3k}{2\omega_k} \]

where the Fourier transform of the source \( j(x) \) is given by

\[ \tilde{j}(\omega_k, k) = -\kappa M \int d\tau d\sigma \sqrt{-\gamma} e^{-i[\omega_k \tau - k \cdot X(\sigma, \tau)]} \]

where \( X^\mu(\sigma, t) \) denotes the string world sheet and \( \omega_k = \sqrt{k^2 + m^2} \).

The dominant contribution to \( \tilde{j} \) comes from the region around the cusp where \( |\omega_k \tau - k \cdot X(\sigma, \tau)| < 1 \). Choosing world-sheet coordinates so that the cusp occurs at \( \sigma = 0 \) and \( \tau = 0 \), this occurs for a range \( |\tau|, |\sigma| < L/(kL)^{1/3} \) provided \( k > m\sqrt{mL} \) (see Appendix A). Also,

\[ \sqrt{-\gamma} = 1 - \tilde{x}^2 \sim \frac{|\sigma|^2}{L^2} \sim (kL)^{-2/3} \]

Therefore

\[ \tilde{j}(\omega_k, k) \sim \kappa M |\sigma|^4 \frac{L^2}{L} \sim \frac{\kappa M L^2}{(kL)^{4/3}} \]

The angular width of the beam of particles emitted from the cusp can also be estimated by evaluating the integral in Eq. (7) in the stationary phase approximation, or as in Ref. [11]. The result is

\[ \theta \sim \frac{1}{(kL)^{1/3}} \]

Then Eq. (6) gives,

\[ dN_k \sim |\tilde{j}|^2 \theta^2 kd\kappa \sim \kappa^2 M^2 L^{2/3} \frac{dk}{k^{1/3}} \]

The estimate applies for

\[ k \in (m\sqrt{mL}, M\sqrt{ML}) \].

The upper cut-off on \( k \) arises because the wavelength of the emitted particles should be larger than the string width in the rest frame of the cusp; \( \lambda > M^{-1} \). Boosting to the rest frame of the loop, this yields \( k < M\sqrt{1 - \tilde{x}^2} \) and together with the estimate in Eq. (8) gives the upper cut-off. The lower cut-off comes from the requirement that \( |k \cdot X| < 1 \) (see Appendix A). For \( k < m\sqrt{mL} \), \( |\sigma| \sim k/m^2 \) and the spectrum is a rapidly increasing function of \( k \)

\[ dN_k \sim k^2 M^2 L^{25/3} \frac{dk}{m^{16} L^{14/3}} \]

Hence \( dN_k \) goes to zero very fast as \( k \to 0 \). In the following sections, we will ignore this part of the spectrum and only consider \( k > m\sqrt{mL} \).

At this stage we can also compare Higgs emission from cusps to the process of cusp annihilation (see Fig. 2). Higgs emission at momentum \( k \) occurs over a length \( L/(kL)^{1/3} \). With the upper bound, \( k = k_{\text{max}} = M\sqrt{ML} \), this length is \( \sqrt{L/M} \) and coincides with the cusp annihilation length [12]. Hence, cusp annihilation does not affect our estimates of Higgs emission for \( k < k_{\text{max}} \).

A caveat to this statement is that the presence of the condensate should not significantly change the string dynamics. Also, we have been considering cusp annihilation, but the condensate itself has some width which is larger than the string width and there could, in principle, be “condensate annihilation” even where there is no cusp annihilation [13].

III. HIGGS INJECTION FUNCTION

Let \( d\Phi_H(k, t) \) be the number of Higgs produced by string cusps with energy \( (k, k + dk) \) at time \( t \) per unit volume per unit time. \( d\Phi_H \) is called the “Higgs injection function”. Then

\[ d\Phi_H(k, t) = dk \int_{L_{\text{min}}}^{L_{\text{max}}} dL \frac{dN_H}{dk} \frac{dn_L}{dL} \frac{dn_c}{dt} \]

where the first factor in the integrand is the number of Higgses with energy \( k \) produced by a cusp on a loop of length \( L \). The second factor is the number density of loops of length \( L \) at time \( t \). The third factor is the number of cusps per unit time on a loop of length \( L \). The
integration is over all loops of length from $L_{\text{min}}$ to $L_{\text{max}}$. The smallest loop can have length $\sim M^{-1}$ but most of the ultra-high energy cosmic ray signal will come from longer loops. $L_{\text{max}}$ is clearly bounded by the cosmic horizon size $\sim t$ but, for a fixed value of $k$, it is also bounded due to the constraints in Eq. (12).

Let us deal with the last factor in (14) first. Since the motion of the loop is periodic or quasi-periodic

$$\frac{dN_c}{dt} = \frac{f_c}{L}$$

(15)

where $f_c$ is a parameter that gives the average number of cusps on a loop per oscillation period. For loops that aren’t too complicated, we expect $f_c \sim 1$.

The rate of Higgs production from a single cusp – the first factor in the integral in (13) – is given in Eq. (11). Now we need to determine the second factor – the number density of loops – in the integrand in Eq. (14).

The number distribution of cosmic string loops is currently under discussion [2, 14, 15, 16, 17, 18]. In [2] the density of loops – in the integrand in Eq. (14) – is given in Eq. (11). The differential equation can be solved but it is simpler to approximate it as

$$\frac{dy}{dx} = -1 - \frac{1}{\sqrt{y}}, \quad y(x = 0) = y_i$$

(19)

The differential equation can be solved but it is simpler to approximate it as

$$\frac{dy}{dx} = -1, \quad y > 1$$

$$\frac{dy}{dx} = -\frac{1}{\sqrt{y}}, \quad y < 1$$

(20)

The Higgs injection function in Eq. (14) can now be written as

$$d\Phi_H(k,t) = \frac{\mu f_c}{t^2} \alpha^{4/3} \frac{dk}{k^{7/2}} I(Y,x)$$

(21)

where $Y \equiv \alpha L_{\text{max}}$ with

$$\alpha \equiv \left( \frac{\Gamma_g G\mu}{\Gamma_h} \right)^2 m$$

(22)
and

\[ I(Y,x) = \int_0^Y \frac{dy}{y^{1/3} y_i^3} \quad (23) \]

Note that we have set the minimum length to 0 since the integration will be dominated by the upper limit. The maximum length (needed to determine \( Y \)) cannot be more than the Hubble size but it is also restricted by the value of \( k \) since \( k > k_{\text{min}} = m\sqrt{mL} \). Therefore

\[ Y = \alpha \min \left( \frac{k^2}{m^3}, t \right) \quad (24) \]

To determine \( I(Y,x) \) we need to write \( y_i \) in terms of \( y \) and \( x \) in Eq. (23). For this we need to solve Eq. (20) for all \( y_i \), to obtain \( y_i(y(x),x) \). Then this function should be inverted to get \( y_i^{-1}(y(x),x) \) which can then be inserted in Eq. (23) (see Fig. 3). We shall do these steps approximately but analytically.

If \( Y \) is very small compared to \( x \), the integration over \( y \) is for \( y \ll x \) and then \( y_i \approx x \) up to some numerical factor of order 1. Therefore

\[ I(Y,x) \approx \frac{Y^{2/3}}{x^2}, \quad Y \ll x \quad (25) \]

In the opposite limit, \( Y \gg x \), the integral needs to be split into two pieces, one for \( y \) from 0 to \( x \) and the other from \( x \) to \( Y \). The first piece is approximated as for the \( Y \ll x \) case. The second piece is different since here \( y_i \sim y \) more appropriate. More explicitly, for \( Y \gg x \),

\[ I(Y,x) = \int_0^Y \frac{dy}{y^{1/3} y_i^3} \approx \frac{x^{2/3}}{x^2} + \int_x^Y \frac{dy}{y^{7/3}} \approx \frac{1}{x^{1/3}} \quad (26) \]

Note that \( I(Y,x) \) does not depend on \( Y \) and, since \( Y \) is \( k \) dependent (Eq. (21)), neither does \( I(Y,x) \) depend on \( k \) in this limit.

Therefore

\[ d\Phi_H(k,t) \approx \frac{\mu A_f e_m^4}{k_{\text{max}}^4} \frac{dk}{k^2} \frac{Y^{2/3}}{x^2}, \quad Y \ll x \quad (27) \]

and

\[ d\Phi_H(k,t) \approx \frac{\mu A_f e_m^4}{k_{\text{max}}^4} \frac{dk}{k^2} \frac{1}{x^{3/2}}, \quad Y \gg x \quad (28) \]

A more rigorous derivation will lead to a smooth interpolation between these asymptotic forms. For the purpose of our estimates, it is sufficient to extend the above asymptotic forms so that they connect continuously at \( Y = x \).

These equations can be written more neatly in terms of

\[ k_* = m\sqrt{\Gamma g m t_0} \quad (29) \]

Then

\[ d\Phi_H(k,t) = \frac{\mu A_f e_m^4}{k_*^4} \frac{dk}{k^2} \quad k \leq k_* \quad (30) \]

\[ d\Phi_H(k,t) = \frac{\mu A_f e_m^4}{k_*^4} \frac{dk}{k^2} \quad K > k \geq k_* \quad (31) \]

where \( K \equiv m\sqrt{mL} \) and the bound \( k < K \) ensures that \( Y = \alpha k^2/m^3 \) i.e. the loops are less than the horizon size. However, the \( k^{-7/3} \) fall off is also valid for \( k > K \) so the restriction \( K > k \) in Eq. (31) may be dropped. Eq. (31) is our estimate for the Higgs injection function.

An important feature of the injection function in Eq. (31) is that it is inversely proportional to the string scale. Thus lighter strings inject more Higgses than heavier strings. This feature was also noted in Ref. 7 and can be explained by the greater longevity of light string loops.

## IV. DIFFUSE PHOTON FLUX

Once Higgses are injected into the cosmological medium, they will decay, lose energy and eventually cascade into gammas in the EGRET energy range. The energy in gammas in the EGRET window is estimated as a fraction of the total injected energy using 10

\[ \omega_{\text{cas}} \equiv \frac{f_c}{2} \int_0^t dt \int dk \frac{d\Phi_H}{dk} \frac{1}{(1+z)^4} \quad (32) \]

where \( f_c \) is the fraction of energy of the Higgses that goes into pions and \( z \) is the cosmological redshift. The 1/2 accounts for how much energy goes into gammas. The diffuse gamma ray background measured by EGRET is on the order of \( \omega_{\text{cas,obs}} \sim 10^{-6} \text{ eV/cm}^3 \) in the energy range 10 MeV-100 GeV 20.

With the Higgs injection function found in the previous section, we have

\[ \int dk \frac{d\Phi_H}{dk} \approx \int_0^{k_*} dk \frac{\mu A_f e_m^4}{k^2} \frac{1}{(\Gamma g G m t_0)^{3/2} t^{1/2}} \quad (33) \]

The integration is dominated by the upper limit \( (k_*) \). The integration for \( k \in (k_*, \infty) \) will be dominated by its lower limit because of the faster \( (k^{-4/3}) \) fall of and will give a comparable result.

Now with \( 1+z = a(t_0)/a(t) \) where \( a(t) \) is the scale factor in a matter dominated universe the time integration can be done and gives

\[ \omega_{\text{cas}} \sim \frac{3 A_f e_c}{(\Gamma g G m)^{3/2}} \frac{\mu}{m^2} \left( m t_0 \right)^{1/2} \frac{m}{t_0} \quad (34) \]

The time integral is dominated by the present cosmic time and Higgs production during the radiation dominated era has been ignored. As noted at the end of the
previous section, the energy cascade into gamma rays is larger for lighter strings. To get a feel for the numbers, we use $m = 10^2 m_2$ GeV, $\mu = (10^{13} \eta_{13} \text{GeV})^2$ (therefore $G_{\mu} = 10^{-12} \eta_{13}^2$), $t_0 = 10^{28}$ cm. Also we take $A \sim 10$, $\Gamma_g \sim 100$ and we assume $f_\pi \sim 1$. Then

$$\omega_{\text{cas}} \sim 10^{-13} \eta_{13}^{-1} m_2^{-1/2} \text{ eV cm}^{-3}$$

This is quite a bit smaller than the EGRET observation unless the string scale is less than $\sim 10^6$ GeV. Lighter strings with the interactions in Eq. (2) are ruled out, though with the caveat that the dynamics of much lighter strings ($M < 1$ TeV) can be dominated by friction until the present cosmic epoch and so the network properties and loop dynamics can be quite different.

V. DIFFUSE PROTON FLUX

We shall follow Berezinsky et al. and use a power-law fragmentation function to obtain the diffuse proton flux, $I_p(E)$, as (see Eq. (A13) of 23)

$$I_p(E) = \frac{(2-p)f_N}{4\pi p} \frac{\dot{n}_H}{E_H} \left( \frac{E}{E_H} \right)^{-p} R_p(E)$$

Here $p = 1.9$ and $f_N$ is the fraction of energy transferred to nucleons when a Higgs decays which, for order of magnitude estimates, we will take to be 1. $\dot{n}_H$ is the rate at which Higgses are being produced per unit volume and can be found from our result for $d\Phi_H$ above. $E_H$ is the energy at which the Higgses are produced and $R_p(E)$ is the proton attenuation length at energy $E$ due to scattering off the CMB. At energies of $10^{19}$ eV i.e. below the GZK cut-off, $R_p \sim t_0 \sim 10^{28}$ cm.

To estimate $I_p(E)$ we take

$E_H = k_\pi \approx 10^{19} \eta_{13} m_2^{3/2}$ GeV

and

$$\dot{n}_H(k_\pi, t_0) = \frac{\mu A f_\mu}{k_\pi^4} \approx 10^{-58} m_2^{-2} \eta_{13}^{-2} \text{ cm}^{-3} \text{s}^{-1} \text{sr}^{-1}$$

This gives

$$E^3 I_p(E) \approx 10^{18} \left( \frac{E}{E_{19}} \right)^{1.1} m_2^{-0.65} \eta_{13}^{-1.1} \frac{\text{eV}^2}{m^2 - s - sr}$$

where $E_{19} = 10^{19}$ eV. This is to be compared with the observed flux 23

$$[E^3 I_p(E_{19})]_{\text{obs}} \approx 10^{24} \frac{\text{eV}^2}{m^2 - s - sr}$$

Hence the proton flux is much less than the observed flux unless $\eta_{13} \sim 10^{-6}$, in which case both the diffuse gamma flux and the diffuse proton flux are comparable to observations.

VI. DIRECT FLUX

String loops that are relatively close-by to the Milky Way may beam Higgses directly at the Earth and these would be seen as ultra-high energy cosmic rays. However, as the particles propagate from the cusp to the Earth, they lose energy due to scattering with various components of the cosmological medium. The energy of a particle drops exponentially with distance from the cusp. If $k'$ is the energy of a particle at the cusp, $k$ is the energy at Earth, and $r$ is the distance from the loop to the Earth, we have

$$k = k' e^{-r/R}$$

As the initially emitted Higgs particle decays and loses energy due to interactions, the products spread out over a wider beam angle, $\theta_b$. The total energy in all the particles is proportional to $k \theta_b^2$ and we assume that this remains roughly constant along the length of the beam. Therefore the angular spread of the beam at Earth is

$$\theta_b \approx \frac{\theta_b'}{e^{r/2R}}$$

where the superscripts denote Earth $(E)$ and the loop $(l)$. The number of particles at Earth with energy $k$ produced from a loop of length $L$, at distance $r$ follows from Eq. (21)

$$d\Phi^E(k, L, r) = \frac{\mu A f_\mu \alpha^{4/3}}{k^{7/2}} \frac{dk'}{k'} \frac{dy}{y} \frac{dV}{d\Omega_r}$$

Note that the left-hand side is the flux at Earth and hence is at energy $k$, while the right-hand side contains the injected flux at the location of the loop and hence is at $k'$. The beaming solid angle is

$$d\Omega_r \sim \pi (\theta_b^2)^2 = \pi (\theta_b^2)^2 e^{r/2R} = \frac{\pi e^{r/2R}}{(k'L)^{2/3}}$$

where we have used Eq. (10). Using Eq. (41) and the rescalings in (18) and integrating over loop lengths and
The integration in the $ry$-plane in Eq. 15 can be split up into four integrations over the ranges $(0, r_1)$, $(r_1, r_2)$, $(r_2, r_3)$ and $(r_3, r_4)$. For $y$ values less than $x$ we can use $y_i \sim x$, as argued in Sec. III. For $y$ values greater than $x$, we use $y_i \sim y$. While the full integration can be done, it is unnecessary because the dominant contribution comes from the interval $(0, r_1)$ i.e. the closest strings. The other integrals are suppressed by powers of $k/k_e$ (see Eq. 29).

The integration over $(0, r_1)$ is

$$I_1 = \int_0^{r_1} dr \ r^2 e^{-r/R} \int_{y_{\min}}^{y_{\max}} dy \ \frac{1}{y \ y_i}$$

where we have used $y_i \approx x$, $y_{\max}/y_{\min} = (M/m)^3$, and assumed $k/k_e \ll 1$.

This leads to

$$k \frac{d\Phi^E}{dk} = 24\pi^2 A_f e \ln \left( \frac{M}{m} \right) \frac{M^2}{(\Gamma H G p)^2 k^2} \frac{R^3}{t^4}$$

Using $t = 10^4$ Mpc = $10^{17}$ s, and dividing by the Earth’s surface area, $10^8$ km$^2$, we get the flux per unit area

$$k \frac{d\Phi^E}{dk} \approx 10^{-3} \left( \frac{10^{20} \text{ eV}}{k} \right)^2 \left( \frac{10^7 \text{ GeV}}{M} \right)^2 \times \left( \frac{R}{5 \text{ Mpc}} \right)^3 \text{ m}^{-2} \text{ s}^{-1}$$

For $M \sim 10^{13}$ GeV, this is comparable to the observed flux of ultra high energy cosmic rays, $10^{-36}$ m$^{-2}$s$^{-1}$sr$^{-1}$eV$^{-1}$ at $10^{20}$eV [23]. Note that $m$ only enters through the logarithm and its precise value does not make much difference to the overall estimate.

The direct photon flux will also be given by Eq. 33, but the attenuation length $R$ will be specific to photons which is less than that for protons at energy $10^{19}$ eV, and the branching ratio for Higgs decay into protons differs from that to photons. Hence the photon to proton ratio is

$$\frac{\gamma}{p} = \frac{N_\gamma}{N_p} \left( \frac{R}{R_p} \right)^3$$

where $N_\gamma$ and $N_p$ are the number of gammas and the number of protons produced by a Higgs. From Figs. 9 and 11 of Ref. [26], and also see Fig. 2 of [23], we find $R/\gamma < 10^{-2}$ at $10^{19}$ eV. If the heavy particle emitted by the cusp is the electroweak Higgs, the decay products contain a pion fraction of $\sim 0.75$ and a nucleon fraction $\sim 0.15$. The pions then decay into gamma photons. As a conservative estimate of the number of photons to protons in the decay products, we take $N_\gamma/N_p \sim 10^2$. Then the gamma to proton fraction at $10^{19}$ eV is $\sim 10^{-4}$, which is consistent with the observed AUGER bound $\gamma/p < 0.02$ [21]. As pointed out in Ref. [23], at higher energies, since $R_p$ falls quite rapidly, we expect the gamma
to proton ratio to be larger. For example, at $10^{20}$ eV, $R_e/R_p \approx 10^{-1}$ and $\gamma/p \approx 10^{-1}$, whereas the observed AGASA and Yakutsk bound is 0.3 \cite{22}.

We would like to point out that we have performed our analysis by taking $R$ to be a constant, as in Eq. (41). This is only an approximation since $R$ depends on the energy of the particle and changes as the particle propagates. A more complete analysis would take the energy loss to be given by

$$\frac{dk}{dr} = -\frac{k}{R(k)}$$

(56)

where $R(k)$ follows by considering the various interactions that a cosmic ray may encounter en route. Our results are valid only if

$$\frac{dR}{dk} \bigg|_{k_*} \ll \frac{R(k_*)}{k_*}$$

(57)

An improved analysis should take the detailed form of $R(k)$ into account.

VII. CONCLUSIONS

It is generally difficult to construct astrophysical scenarios that can accelerate protons to high energies sufficient to arrive as ultra-high energy cosmic rays. Top down models involving topological defects can naturally give the requisite energies. Depending on the precise variety of defect, however, they are likely to suffer from an excess of diffuse gamma ray production.

We have re-visited cosmic ray production from cosmic strings, taking into account the possibility of a linear interaction between a scalar field and the string worldsheet. Such an interaction arises, for example, if a scalar field acquires a VEV ("condenses") within the string. Then beams of Higgs particles are emitted from cusps on cosmic string loops. Our analysis shows that such events can be responsible for the production of UHECR within reasonable parameters and also be consistent with measurements of the diffuse backgrounds and the photon to proton fraction in UHECR.

Our results are also interesting because they show that strings with bosonic condensates are allowed in a narrow window of energy scales. If they are much lighter than $\sim 10^{13}$ GeV, they will produce an excess of UHECR and diffuse fluxes, while if they are much heavier, they will cause conflicts with other cosmological constraints that depend on their gravitational effects. However, if the strings are around the Grand Unified scale, they can produce the observed flux of ultra high energy cosmic rays and also be heavy enough to be detected by their gravitational signatures in the near future.

We have focused on deriving analytical estimates of cosmic ray fluxes under various simplifying assumptions. Further work is needed to obtain more detailed estimates that can be compared to observations.

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APPENDIX A: LOOP DYNAMICS AND CUSPS

Here we summarize some known features of cosmic string loops that are used to obtain the radiation estimates in Sec. II.

A cosmic string loop oscillates under its own tension such that its world-sheet can be written as

$$X(\sigma, t) = \frac{1}{2}[a(\sigma - t) + b(\sigma + t)]$$

(41)

where $a$ and $b$ satisfy

$$|a'| = 1 = |b'|$$

(42)

where primes denote derivatives with respect to the argument. Also, since the loop is a closed string,

$$\int_0^L a'd\sigma = 0 = \int_0^L b'd\sigma$$

(43)

Therefore $-a'$ and $b'$ are two closed curves on a unit sphere whose centers of mass coincide with the center of the sphere. These curves generally intersect leading to

$$a'(\sigma - t) = -b'(\sigma + t)$$

(44)

for one or more values of $\sigma$ and $t$. Since the velocity of a point on the loop is

$$v(\sigma, t) = \frac{1}{2}[-a' + b'],$$

(45)

intersection of the $-a'$ and $b'$ curves implies a point on the loop that reaches the speed of light at one instant per oscillation. Such a point on the string is called a "cusp". The ultra-high boost factors at the cusp can be responsible for burst-like events from cosmic string loops.

We choose our string parametrization so that the cusp occurs at $\sigma = 0 = t$, and then expand the functions $a$ and $b$ around a cusp

$$a(\zeta_-) = a^i_0\zeta_- + \frac{1}{2}a''_0\zeta_-^2 + \frac{1}{6}a'''_0\zeta_-^3 + \ldots$$

(46)

$$b(\zeta_+) = b^i_0\zeta_+ + \frac{1}{2}b''_0\zeta_+^2 + \frac{1}{6}b'''_0\zeta_+^3 + \ldots$$

(47)
where \( \zeta_\pm = \sigma \pm t \). The expansion coefficients are constrained by
\[
\begin{align*}
a'_0 &= -b'_0; \quad a''_0 \cdot a''_0 = b''_0 \cdot b''_0 = 0 \\
|a'_0| &= |b'_0| = 1.
\end{align*}
\]
(A8)

These expansions give
\[
\omega_k t - k \cdot X \sim (\sqrt{k^2 + m^2} - k)\zeta + \delta kL^{-2}\zeta^3 + \ldots \quad (A9)
\]

where we have taken \( \zeta_+ \sim \zeta_- \) and denoted them collectively by \( \zeta \). \( \delta \) is a dimensionless constant that depends on the shape of the cusp. We will take \( \delta \sim 1 \). To have \( |\omega_k t - k X| < 1 \), we require that both terms in Eq. (A9) be less than 1. This gives \( \zeta < \min(k/m^2, L(kL)^{-1/3}) \). For \( k > m\sqrt{mL} \), we get \( \zeta < L(kL)^{-1/3} \) and for \( k < m\sqrt{mL} \), we get \( \zeta < k/m^2 \).

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