Constitutive model of confined concrete with stirrups by acid rain erosion

ZHENG Yue1,2*, ZHENG Shansuo1,2, LIU Xiaohang1,2
1 School of Civil Engineering, Xi’an University of Architecture and Technology, Xi’an, 710055, China
2 Key Lab of Structural Engineering and Earthquake Resistance, Ministry of Education(XAUAT), Xi’an, 710055, China
*Corresponding author’s e-mail: yanglu@xauat.edu.cn

Abstract: In order to study the influence of stirrup corrosion level on the peak stress, peak strain, ultimate strain and shape of stress-strain curve of the confined concrete, 15 reinforced concrete prism specimens were subjected to acid rain erosion by artificial climate simulation technique followed by axial pressure tests. Based on Mander’s model and the existing research results, the calculation formulas of the peak stress $f_{cc}^0$, peak strain $\varepsilon_{cc}^0$, ultimate strain $\varepsilon_{cu0}$ and shape factor $r$ of the uncorroded specimens are determined. The factor calculation formulas for peak stress, peak strain, ultimate strain and shape factor of corroded specimens is developed by regression analysis of test data, respectively, and then the constitutive model of confined concrete by acid rain erosion is established. By comparing the simulation results with the experimental data, it can be found that all the peak stress, peak strain, ultimate strain and stress-strain curves shape of the specimens obtained by the proposed method are in good agreement with the experimental data. Thus the constitutive mode for confined concrete established in this paper can accurately reflect the mechanical performance of RC prism specimen by acid rain erosion, indicating its adaptiveness for estimating the residual bearing capacity and the seismic performance of RC structure under the acid rain environment.

1. Introduction
Concrete carbonation and steel corrosion caused by acid rain erosion are one of the main reasons for the deterioration of the seismic performance of RC structures [1]. The stirrups configured in the RC component can restrain the transverse deformation of the concrete and make it in a three-dimensional stress state, thereby improving the bearing capacity and deformation capacity of the concrete in the restraint area. Christopher et al. [2] have shown that the corrosion of stirrups will significantly reduce the restraint effect on the concrete in the constrained area, causing the bearing capacity and ductility of components to degrade to varying degrees. Therefore, it is necessary to study the influence of stirrup corrosion caused by acid rain on the mechanics and seismic performance of RC structures.

The constitutive model of concrete confined by stirrups eroded by acid rain is the basis for the study of elastoplastic analysis, residual bearing capacity and seismic performance of in-service RC structures under acid rain environment. In recent years, there have been a large number of studies on axial compression of concrete confined by stirrups at home and abroad [3], but there are few studies on the constitutive model of confined concrete considering the influence of stirrup corrosion, such as: Vu [4], Liu Lei [5] et al. used electrochemical methods to corrode RC prism specimens, based on the
results of axial compression tests, they established a constrained concrete constitutive model considering the effect of stirrup corrosion. Due to the difference between electrochemical corrosion and natural environment corrosion, whether the proposed constitutive model can be applied to RC structures in acid rain environment remains to be verified.

In view of the reasons above, to be closer to reality and easy to apply, this paper used artificial climate environment accelerated corrosion technology to simulate acid rain environment, and 15 groups of RC prismatic specimens were subjected to accelerated corrosion, and then the axial pressure test was carried out on these corroded specimens to study the degradation law of the mechanical properties of the specimens under different design parameters. Finally, a constitutive model of concrete confined by stirrups by acid rain erosion was established to provide theoretical support for the evaluation of residual bearing capacity and seismic performance of RC structures under acid rain environment.

2. Test

2.1. Specimen design

In the experiment, 15 sets of RC prism specimens were designed and manufactured based on the degree of stirrup corrosion and the volumetric stirrup ratio. The design parameters of the test piece are as follows: the size of the test piece is 150 mm× 150 mm× 450 mm, the thickness of the concrete protective layer is 12 mm, the longitudinal reinforcement is HRB335 steel bar, the stirrup is HPB300 steel bar, and the stirrup forms are ϕ6@80, ϕ6@60, ϕ8@80. The geometric dimensions and reinforcement of the specimen are shown in Figure 1, and the other design parameters are shown in Table 1. Among them, the degree of stirrup corrosion is controlled by the number of acid rain erosion spray cycles.

![Figure 1. Dimensions and reinforcements of specimens(unit: mm)](image)

| No. | Concrete strength | Stirrup | Stirrup rate /% | Corrosion cycles/time | Stirrup corrosion rate /% | Corrosion rate of longitudinal bars /% |
|-----|------------------|---------|-----------------|-----------------------|--------------------------|---------------------------------------|
| L1  | C40              | A6@80   | 1.12            | 0                     | 0                        | 0                                     |
| L2  | C40              | A6@80   | 1.12            | 120                   | 4.3                      | 0                                     |
| L3  | C40              | A6@80   | 1.12            | 240                   | 9.4                      | 4.1                                   |
| L4  | C40              | A6@80   | 1.12            | 320                   | 16.7                     | 6.4                                   |
| L5  | C40              | A6@80   | 1.12            | 360                   | 18.6                     | 7.2                                   |
| L6  | C40              | A6@60   | 1.50            | 0                     | 0                        | 0                                     |
| L7  | C40              | A6@60   | 1.50            | 120                   | 4.2                      | 0                                     |
| L8  | C40              | A6@60   | 1.50            | 240                   | 8.8                      | 4.0                                   |
| L9  | C40              | A6@60   | 1.50            | 320                   | 16.0                     | 6.2                                   |
| L10 | C40              | A6@60   | 1.50            | 360                   | 18.7                     | 7.2                                   |
| L11 | C40              | A8@80   | 1.99            | 0                     | 0                        | 0                                     |
| L12 | C40              | A8@80   | 1.99            | 120                   | 3.2                      | 0                                     |
| L13 | C40              | A8@80   | 1.99            | 240                   | 8.6                      | 3.9                                   |
| L14 | C40              | A8@80   | 1.99            | 320                   | 15.5                     | 5.9                                   |
| L15 | C40              | A8@80   | 1.99            | 360                   | 18.3                     | 7.0                                   |
This test used C40 P.O 42.5R cement concrete formulation, with the ratio of: cement: water: sand: gravel = 390: 120: 885: 890, which was used to make RC prism specimens. Through material performance test, the concrete cube compressive strength $f_{cu}=42.2$ MPa, axial compressive strength $f_c=32.1$ MPa, elastic modulus $E_c=3.25\times10^4$ MPa, the steel material property test results are shown in Table 2.

| Rebar diameter | Yield Strength $f_y$/MPa | Tensile strength $f_u$/MPa | Elastic modulus $E_s$/MPa |
|---------------|--------------------------|---------------------------|------------------------|
| A6            | 319                      | 420                       | 2.1\times10^5          |
| A8            | 313                      | 418                       | 2.1\times10^5          |
| Φ12           | 350                      | 458                       | 2.0\times10^5          |

2.2. Test scheme

The artificial climate laboratory parameters are set to simulate the acid rain environment in the test. In this paper, the periodic spray corrosion test scheme adopted in studies of Zheng et al. [6] was used to accelerate the corrosion of RC prisms, and used CO$_2$ to simulate concrete carbonation in the actual environment. Among them, the preparation plan of the corrosion solution is that in order to reflect the characteristics of sulfuric acid rain in China, first adding a sulfuric acid (H$_2$SO$_4$) solution with a concentration of $\rho=1.84$ g/cm$^3$ in the water until the sulfate ion concentration reaches 0.06 mol/L; then adding a nitric acid (HNO$_3$) solution with a concentration of $\rho=1.42$ g/cm$^3$ into the corrosion solution to adjust the PH to 2.0. The specific corrosion process of the test piece is as follows: 1) Adjusting the laboratory temperature to 25±5 °C and spraying the corrosive solution for 240 min; 2) Increasing the temperature of the laboratory to 65±5 °C to accelerate the corrosion rate of the corrosive medium; 3) Cooling down to 25±5 °C, starting the next corrosion cycle. The duration of a single corrosion cycle is 6 h, and the accelerated corrosion simulation test and cycle process are shown in Figure 2.

![Figure 2. Accelerate corrosion simulation test](image1)

After the corrosion test is completed, a microcomputer-controlled electro-hydraulic servo pressure testing machine is used to perform axial compression test on the prism specimen. Before loading, install a dial gauge on each side of the test piece with a gauge length of 200mm. Connect the dial gauge and the wire of the stirrup strain gauge to the data acquisition instrument to record the test data. This test adopts a constant velocity displacement control loading method, and the displacement rate is 0.3mm/min. The test is stopped when the prism specimen is obviously damaged and cannot continue to bear the axial load.

After the axial compression test is completed, the concrete is crushed and all the steel bars are taken out. The mass loss rate is calculated by referring to the method described in previous research [18] to reflect the actual corrosion of the steel bars. The expression is:

$$\eta = \frac{(m_0 - m_1)}{m_0} \times 100\%$$

(1)

In the formula: $\eta$ is the actual corrosion rate of steel bars expressed in terms of mass loss rate, $m_0$ is the quality of uncorroded steel bars, and $m_1$ is the quality of the rebar according to the
specification “The Test Method for Long-Term Performance and Durability of Ordinary Concrete” GB/T50082-2009[7]. The measured results of the actual corrosion rate of the longitudinal bars and stirrups of each specimen are shown in Table 1.

3. Test stress-strain curve
The axial bearing capacity of the specimen measured by the compression testing machine can be regarded as the sum of the bearing capacity of the longitudinal reinforcement, the unconstrained concrete of the protective layer and the confined concrete in the core area. To obtain the stress of the confined concrete, the contribution of longitudinal reinforcement concrete and the protective layer concrete need to be subtracted. Based on the tensile test, it can be determined that the longitudinal bars bear the load. The load borne by the protective layer concrete can be approximated by multiplying the unconstrained concrete axial compressive strength \( f'_{c0} \) by the protective layer area.

The test stress-strain curves of specimens with different corrosion degrees are shown in Figure 4. The degree of corrosion has a greater influence on the shape of the stress-strain curve of the specimen. For the specimens with less corrosion, such as specimens L2, L7, L12, the rising section of the stress-strain curve is almost parallel to the uncorroded specimen. The descending section is also relatively gentle, and the stiffness and ductility of the specimen are not significantly reduced; as the corrosion degree continues to increase, the initial stiffness of the specimen gradually decreases, the peak point of the stress-strain curve gradually shifts to the lower right, and the peak stress decreases significantly. Compared with the uncorroded specimen L1, the peak stresses of specimens L2, L3, L4, and L5 are reduced by about 3.68%, 5.68%, 12.88%, and 22.54%. After the stirrup is slightly corroded, the pore and gap between the stirrup and the concrete are filled with corrosion products, thereby improving the confinement effect between the stirrup and the concrete, and slightly improving the deformation performance of the specimen, so the peak strain is slightly increased. In addition, as the corrosion rate of stirrups increases, the falling section of the stress-strain curve of the specimen gradually becomes steeper, the ultimate strain gradually decreases, and the horizontal extension of the descending section gradually shortens, indicating that that the ductility of specimens decreases with the increase of stirrup corrosion rate.

![Stress-strain curves of specimens](image)

Figure 3. Stress-strain of specimens

4. Constitutive model

4.1. Model establishment
The Mander model[8] uses a unified curve equation for the ascending and descending segments. The model parameters include the shape factor \( r \), peak stress and peak strain, and the expressions are as follows:

\[
f_c = \frac{f'_{c0} \times r}{r - 1 + x^r}, \quad (2)
\]

\[
x = \frac{\varepsilon_p}{\varepsilon_{p0}}. \quad (3)
\]
Where: \( f_c \) and \( \varepsilon_c \) are the stress and strain of confined concrete; \( f'_{cc0} \) and \( \varepsilon'_{cc0} \) are the peak stress and peak strain of confined concrete; \( r \) is the shape factor of the model.

For corroded RC prism specimens, because the deterioration of its mechanical properties is affected by multiple factors such as the reduction of the cross-sectional area of the steel bar, the reduction of the elastic modulus, and the degradation of the bonding performance between the steel bar and concrete, it is not realistic to establish its constitutive model through theoretical methods. Therefore, to comprehensively consider the influence of the above-mentioned various factors, an experimental fitting method is adopted. This paper firstly normalizes the test stress-strain curve, and uses 1stopt software to fit each curve to obtain the shape factor \( r' \) of the test stress-strain curve of each specimen, and then consider the influence of stirrup corrosion to obtain the correction formula of shape factor \( r \), and the shape coefficient \( r \) of the uncorroded specimen is modified; based on the test results, the calculation formulas for the peak stress, peak strain and ultimate strain of the stress-strain curve of confined concrete considering the influence of the corrosion degree of the stirrup are established, and finally the constitutive model of concrete confined by acid rain stirrup is established.

### 4.1.1. Determination of the shape factor \( r \)

Considering the influence of stirrup corrosion on the shape coefficient of the confined concrete constitutive model, the shape coefficient correction function \( h(\eta_s) \) is defined, and the calculation formula for the shape coefficient \( r_c \) of the corroded stirrup confined concrete is:

\[
r_c = h(\eta_s) \cdot r
\]

In the formula: \( r \) is the shape coefficient of the uncorroded specimen [8].

Dividing the test shape factor \( r' \) of each group of test pieces by the test shape factor of the uncorroded test piece in each group of test pieces to obtain the corresponding correction factor. Taking the stirrup corrosion rate \( \eta_s \) as the abscissa and the correction coefficient as the ordinate, the variation law of the correction coefficient with the stirrup corrosion rate \( \eta_s \) is obtained, as shown in Figure 4.

It can be seen that as the corrosion rate of stirrups increases, the correction coefficient of the shape coefficient \( r \) of the constitutive model of the corrosion specimens continues to increase, and approximately shows a quadratic parabolic change trend, so this paper assumes the shape coefficient correction function \( h(\eta_s) \) as the quadratic function form of the stirrup corrosion rate \( \eta_s \), and considering the boundary conditions, the expression of the shape coefficient correction function is obtained as follows:

\[
h(\eta_s) = a\eta_s^2 + b\eta_s + 1
\]

In the formula: \( a \) and \( b \) are fitting parameters. In this paper, the shape coefficient correction function is fitted by 1stopt software, and the calculation formula and the determination coefficient \( R^2 \) are obtained as equation (6).

\[
h(\eta_s) = 0.002\eta_s^2 + 0.024\eta_s + 1 \quad R^2 = 0.85
\]

### 4.1.2. Determination of peak stress \( f'_{cc} \) and peak strain \( \varepsilon'_{cc} \)

Considering the influence of stirrup corrosion on the peak stress and peak strain of confined concrete, the peak stress reduction function \( f(\eta_s) \) and peak strain reduction function \( g(\eta_s) \) are defined respectively, and the formula for calculating the peak stress and peak strain of the corroded stirrup confined concrete is:

\[
f'_{cc} = f(\eta_s) \cdot f'_{cc0}
\]

\[
\varepsilon'_{cc} = g(\eta_s) \cdot \varepsilon'_{cc0}
\]

In the formula: \( f'_{cc0} \) and \( \varepsilon'_{cc0} \) are the peak stress and peak strain of the uncorroded specimen respectively [8].
The variation of shape factor "r" correction ratio with stirrup corrosion rates

Figure 4. The variation of shape factor “r” correction ratio with stirrup corrosion rates

(a) Stress correction factor

(b) Strain correction factor

Figure 5. The variation of peak stress&strain correction ratio with stirrup corrosion rates

Figure 6. The ultimate strain correction ratio with stirrup corrosion rates

The test peak stress and test peak strain of each group of specimens are divided by the peak stress and peak strain of the uncorroded specimens in each group of specimens to obtain the corresponding correction coefficients. Taking the stirrup corrosion rate η as the abscissa and the correction coefficient as the ordinate, the variation rules of the peak stress and peak strain correction coefficients with the stirrup corrosion rate η are obtained, as shown in Figure 5.

As shown in the figure, with the increase of stirrup corrosion rate, the peak stress correction coefficient of the confined concrete constitutive model of the corroded specimen continues to decrease, and the peak strain correction coefficient continues to increase, and both are approximately linear. To ensure that the fitting results have high accuracy and are easy to be used in numerical simulation, in this paper, both the peak stress reduction function \( f(\eta_s) \) and the peak strain reduction function \( g(\eta_s) \) are assumed to be a linear function of the stirrup corrosion rate \( \eta_s \), and the boundary conditions are considered, thus the expressions of peak stress and peak strain correction functions are obtained as follows:

\[
\begin{align*}
  f(\eta_s) &= 1 + k_1 \eta_s, \\
  g(\eta_s) &= 1 + k_2 \eta_s.
\end{align*}
\]  

In the formula: \( k_1 \) and \( k_2 \) are the fitting parameters. This paper used 1stopt software to fit the peak stress and peak strain correction functions, and obtained the calculation formula and the determination coefficient \( R^2 \) in equations (11) ~ (12).

\[
\begin{align*}
  f(\eta_s) &= 1 - 0.0096 \eta_s, \quad R^2 = 0.96 \\
  g(\eta_s) &= 1 + 0.012 \eta_s, \quad R^2 = 0.90
\end{align*}
\]

4.1.3. Determination of ultimate strain \( \varepsilon_{cu} \). Wang et al. [9] put forward a calculation formula for the ultimate strain of concrete confined by stirrups through statistical analysis of many axial compression test data of concrete confined by stirrups.

\[
\varepsilon_{cu0} = 0.0033 \times \left[ 1 + 166 \times \left( \frac{k_e \cdot \rho_s \cdot f_s}{f_{cu0}} \right)^{1.65} \right]
\]

Where: \( \rho_s \) is the volumetric stirrup ratio, and \( f_s \) is the yield strength of the stirrup.

Based on the above formula, this paper calculates the ultimate strain of uncorroded specimens, and considers the influence of stirrup corrosion on the ultimate strain of confined concrete. The ultimate strain correction function \( f(\eta_s) \) is defined, and the formula for calculating the ultimate strain of confined concrete with corroded stirrups is:
The ultimate strain value of each group of specimens is divided by the ultimate strain of the uncorroded specimens in each group of specimens to obtain the corresponding correction coefficients. Taking the stirrup corrosion rate \( \eta \) as the abscissa and the correction coefficient as the ordinate, the variation law of the ultimate strain correction coefficient with the stirrup corrosion rate \( \eta \) is obtained, as shown in Figure 6.

As the corrosion rate of stirrups increases, the ultimate strain of each specimen continues to decrease, and the trend is approximately linear. Therefore, this paper defines the ultimate strain correction function \( l(\eta) \) as a linear function of the stirrups corrosion rate \( \eta \), and considering the boundary conditions, the calculation formula of the ultimate strain correction function is:

\[
l(\eta) = 1 + b\eta
\]  

(15)

In the formula: \( b \) is the fitting parameter. In this paper, the shape coefficient correction function is fitted by 1stopt software, and the calculation formula and the determination coefficient \( R^2 \) are obtained as equation (16).

\[
l(\eta) = 1 - 0.02\eta, \quad R^2 = 0.81
\]  

(16)

4.2. Model verification

To verify the accuracy of the constitutive model of concrete confined by the acid rain erosion of stirrups, the above constitutive model calculation method is used to simulate and analyze some specimens in this article. The comparison between the calculated results and the test results is shown in Figure 7.

It can be seen that the calculated skeleton curve of the specimen is in good agreement with the experimental skeleton curve in terms of shape, peak stress, peak strain and ultimate strain, indicating that the calculation accuracy of this model is higher. At the same time, the calculation error \( E_f \) is used to represent the error between the test curve and the simulation curve of the above-mentioned comparison specimens, and the calculation formula is as follows:

\[
E_f = \frac{1}{\max_{i=1,2,\ldots,N} \left| \sigma_i \right|} \sqrt{\frac{1}{N} \sum_{i=1}^{N} (\sigma_i - \sigma_{i})^2}
\]  

(17)

In the formula: \( E_f \) is the calculation error, the subscript \( i \) represents the \( i \)-th data point, \( N \) represents the total number of data points, \( \sigma_i \) and \( \sigma_{i} \) respectively represent the stress test value and the calculated value of the \( i \)-th data point. The calculation errors \( E_f \) of the specimens L1–L13 are 3.22%, 3.96%, 3.00%, 4.18%, 6.42%, 3.82%, and these errors are mostly smaller than 5%, indicating that the calculation accuracy is good.

5. Conclusion

To evaluate the residual bearing capacity and seismic performance of corroded RC structures under
acid rain environment, this paper studied the constitutive model of concrete confined by stirrups corroded by acid rain. The conclusions are as follows:

(1) With the increase of acid rain erosion, the peak stress of the RC prism specimen decreases significantly, and the peak strain increases slightly. The elastic modulus and ultimate strain of the initial section of the stress-strain curve gradually decrease, and the failure is relatively sudden, indicating the ductility of the specimen was gradually getting worse.

(2) Based on the Mander model and existing research results, the shape factor of the stress-strain curve of the uncorroded RC prism specimen and the calculation formulas for characteristic points such as peak stress, peak strain and ultimate strain are determined, and the test data is analyzed and simulated. In addition, through the analysis and fitting of the test data, the calculation formulas for the shape coefficient and the correction coefficient of each characteristic point considering the influence of the degree of stirrup corrosion were proposed, and finally the constitutive model of the concrete confined by the acid rain erosion of the stirrup was established.

(3) The calculation results of the established acid rain erosion constitutive model of stirrup-confined concrete are in good agreement with the test results, indicating that the model can better reflect the mechanical properties and deformation properties of stirrup-confined concrete under acid rain environment, and can be used to evaluate the residual bearing capacity and seismic performance of RC structures in service under this environment.

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References

[1] HU X B. (2008) Analysis for simulation test on acid rain attacking concrete[J]. Journal of The Chinese ceramic society, 36(s1): 147-152.
[2] CHRISTOPHER H, WILLIAM C. (2006) Tests of reinforced concrete beams with corrosion-damaged stirrups [J]. ACI STRUCTURAL JOURNAL, 103(1): 133-141.
[3] SHI Q X, WANG P, TIAN Y, et al. (2014) Experimental study on seismic behavior of high-strength concrete short columns confined with high-strength stirrups[J]. China Civil Engineering Journal, 47(8): 1-8.
[4] NGOC S V, BO Y, BING L. (2017) Stress-strain model for confined concrete with corroded transverse reinforcement.[J]. Engineering Structures, 151(15): 472-487.
[5] LIU L, NIUD T, LI Q, et al. (2018) Stress-strain constitutive relation model of corroded stirrups confined concrete [J]. Journal of Building Materials, 21(5): 811-816.
[6] ZHENG S S, ZHANG Y X, HUANG Y G, et al. (2017) Experimental study on seismic behaviors of reinforced concrete frame beams in simulated acid environment[J]. Journal of Building Structures, 38(9): 20-27.
[7] Standard for test methods of long-term performance and durability of ordinary concrete: GB/T 50082-2009[S] Beijing: China Architecture & Building Press, 2009, 64-67.
[8] MANDER J B, PRIESTLY M J N, PARK R. Theoretical stress-strain model for confined concrete [J]. Journal of Structural Division, ASCE, 1988, 114(8): 1804-1826.
[9] WANG N, SHI Q X, ZHANG W, et al. (2019) A uniaxial compressive model for concrete confined with stirrups[J]. Journal of Building Materials, 22(6): 933-940.