Abstract

Estimation of the information content in a neural network model can be prohibitive, because of difficulty in finding an optimal codelength of the model. We propose to use a surrogate measure to bypass directly estimating model information. The proposed Information Transfer ($L_{IT}$) is a measure of model information based on prequential coding. $L_{IT}$ is theoretically connected to model information, and is consistently correlated with model information in experiments. We show that $L_{IT}$ can be used as a measure of generalizable knowledge in a model or a dataset. Therefore, $L_{IT}$ can serve as an analytical tool in deep learning. We apply $L_{IT}$ to compare and dissect information in datasets, evaluate representation models in transfer learning, and analyze catastrophic forgetting and continual learning algorithms. $L_{IT}$ provides an informational perspective which helps us discover new insights into neural network learning.

1 Introduction

In machine learning, learning a model can be understood as a process of gaining information about the model parameters from the data [1]. The learned model is able to generalize because the information gained captures regularities in the data. For deep learning models, the information, or “knowledge” learned from a large amount of data is crucial for its effectiveness in a wide range of tasks, for example in computer vision [2] and NLP [3]. Life-long machine learning [4] even identifies the ability to accumulate and reuse information an indispensable aspect of AI. A natural question to ask is how to measure the information content in a model, or the amount of information transferred from a certain dataset to the model during the process of training.

The pioneering work of Minimum Message Length (MML) [5] and Minimum Description Length (MDL) [6] view learning as data compression, and they give a framework and guidelines on finding the optimal codelength of model and data. In algorithmic information theory, the minimum codelength (or description length) can be used as a measure of the amount of information.

For measuring the description length of a neural network model together with data, it has been shown that prequential codes in practice achieve much better codelength than variational or Bayesian codes [7]. It is also showed [6] that the prequential codelength of a target dataset is linked to the generalizability of a transfer learning model. However, as the prequential code is a one-part code [6] which encodes both the model and the data, one cannot easily measure the codelength of the model, nor the information content in the model itself.
Although the codelength of a model can be given by a two-part code, current implementations of two-part codes for neural network models give impractical codes [9]. We take an approach based on prequential codes which are proved to work well with neural networks. Instead of finding a theoretical bound of codelength or a practical code, we insist on finding a useful measure of information content of a neural network model. The measure ideally should have good correlation with the amount of information stored in the model, relative to the model prior.

A “surrogate” measure of information will not give any bounds to the true codelength, but as we will show in this paper, it is useful in many scenarios for the analysis and understanding of the learning process, from the perspective of information.

The main contributions of this paper are summarized as follows:

- We propose a practical measure that correlates with the information content in a neural network model, which we termed Information Transfer.
- We demonstrate the ability of Information Transfer to measure the information gained in a neural network model after training. We can also use it to measure the information content of a task or a dataset.
- We perform analysis of transfer learning and continual learning from an informational perspective using Information Transfer, which brings new insight into the nature of transfer learning and continual learning.

2 The Information Transfer Measure

Information content can be measured using the length of the shortest code that can reproduce the data, with the idea of Kolmogorov Complexity [10]. To compress the data, one could learn a model to predict the data, and only encode the residues of prediction. Using prequential codes (or online codes) is a model-agnostic way to encode data together with a model.

The basic idea of prequential coding is illustrated in the following example. Imagine Alice wants to send the labels $y_1$ to $y_n$ of a dataset to Bob, and they both agree on an initial model $θ_0$. Each time Alice sends one example, she first uses her model to predict the label $y_i$ and encodes $y_i$ using the output distribution of $θ_{i-1}$. Then she optimizes and updates her model to $θ_i$ with example $i$. Bob also uses his model to make predictions about $y_i$ and then recover the true $y_i$ with the help of the code he receives. Afterward he updates his model in the same fashion. The length of such a code is:

$$L_{\text{preq}}(y_1:y_n|x_1:y_n) := -\sum_{i=1}^{n} \log p_{\hat{θ}}(y_i|x_1:i, y_1:i-1)$$

(1)

$$= \log K - \sum_{i=2}^{n} \log p_{\hat{θ}_{i-1}}(y_i|x_i)$$

(2)

where $\hat{θ}_i$ is the parameter of the model learned on data samples 1 to $i$. After Bob receives the whole dataset, he also ends up with the same model that Alice has. In this sense, the prequential code encodes the model together with the data without explicitly encoding the model.

Because the prequential code also transmitted the model, the codelength of the model $L_{\text{model}}$ alone must be strictly smaller than $L_{\text{preq}}$. To approach $L_{\text{model}}$, one could subtract from $L_{\text{preq}}$ the codelength of the incompressible noise in the dataset: $\hat{L}_{\text{model}} = L_{\text{preq}} - nE(Y|X)$, where $E(Y|X)$ is the conditional entropy of the label given the input. However, one still needs to estimate $E(Y|X)$, and there is no principled way of validating how far the estimation $\hat{L}_{\text{model}}$ is from the real $L_{\text{model}}$.

Instead of directly estimating $L_{\text{model}}$, we next introduce a surrogate measure $L_{IT}$, that correlates with $L_{\text{model}}$ in a meaningful way.

2.1 Definition of $L_{IT}$

The information transfer measure is defined as the difference of two prequential codelengths:

$$L_{IT}(n, k) = L_{\text{preq}}(y_1:k|x_1:k) - L_{\text{preq}}(y_{n+1:n+k}|x_{n+1:n+k})$$

(3)
By $L_{IT}(n, k)$, we intend to measure the amount of information in the model $\theta_n$ after training on a dataset $\{x_{1:n}, y_{1:n}\}$ of $n$ examples. Intuitively, it is measured by comparing the codelength of encoding $k$ examples with the model before ($\theta_0$) and after training ($\theta_n$). The more information the model gains about the task from training, the larger the reduction of codelength would be.

![Information Transfer Measure](image)

Figure 1: An illustration of the information transfer measure $L_{IT}$. $L_{IT}(n, k)$ is equal to the difference between the area of the left shaded region and the right shaded region below the coding curve.

An illustration of $L_{IT}$ is given in Figure 1. Next we examine theoretical connections between $L_{IT}(n, k)$ and $L_{model}$.

### 2.2 Properties of $L_{IT}$

In the definition of $L_{IT}$, $k$ is introduced as an auxiliary variable that helps one calculate $L_{IT}$ using prequential codes. For all possible $k$, the smallest is $k = 1$:

$$L_{IT}(n, 1) = - \log p_{\theta_0}(y_1|x_1) - \log p_{\theta_n}(y_{n+1}|x_{n+1})$$  \hfill (4)

$$\mathbb{E}[L_{IT}(n, 1)] = L_{val}(\theta_0) - L_{val}(\theta_n)$$  \hfill (5)

When the loss function $\mathcal{L}$ is cross-entropy, the expectation of $L_{IT}(n, 1)$ over all possible data is equal to the reduction of validation loss.

On the other hand, if $k \to \infty$: (Proof of (6) and (7) is given in Appendix A)

$$L_{IT}(n, \infty) = \sum_{i=1}^{n} \log p_{\theta_{i-1}}(y_i|x_i) - \sum_{i=1}^{n} \log p_{\theta_{oracle}}(y_i|x_i)$$  \hfill (6)

$L_{IT}(n, \infty)$ is the reduction of codelength of $n$ examples, comparing the initial model $\theta_0$ with an oracle model $\theta_{oracle}$ (which is the best model in this model family).

It turns out that the true model codelength $L_{model}$ falls between the two extremes:

$$\mathbb{E}[L_{IT}(n, 1)] \leq L_{model}(\theta_n) \leq L_{IT}(n, \infty)$$  \hfill (7)

The second inequality becomes apparent by moving the $p_{\theta_{oracle}}$ term in $L_{IT}(n, \infty)$ to the left-hand side. With the above relationship, we can hope to find an appropriate $k$ that makes $L_{IT}(n, k)$ correlate well with $L_{model}(\theta_n)$, which can then serve as a surrogate measure of the codelength of model $\theta_n$.

Next we investigate the behavior of $L_{IT}$ with varying $n$. Intuitively, the codelength of the model $\theta_n$ grows as the number of training examples $n$ increases. In Figure 2 we plot $L_{IT}$ as a function of $n$ for a number of datasets. A common pattern is observed: After a short initiation phase, $L_{IT}$ grows roughly linearly with $\log n$, and then finally saturates. The linear growing phase is where learning mostly takes place, and saturation happens when the model is close to converging, as the model stops gaining new information from more data.

One finds a resemblance to the linear relationship of $L_{IT}$ with $\log n$ in the connection between prequential codes and MDL. Under regularity conditions on model family $\mathcal{M}$ and data $x$, the following equation states that the prequential code is equivalent to a two-part code, whose “model part” length is proportional to $\log n$:

$$L_{\theta}^{\text{pred}}(x_{1:n}) = - \sum_{i=1}^{n} \log_2 p_{\theta_n}(x_i) + \frac{d}{2} \log n + O(1)$$  \hfill (8)

What makes the resemblance interesting is general neural models do not satisfy conditions of (8).
3 Measuring Information Transfer in Neural Networks

In this section, we apply the information transfer $L_{IT}$ in experiments to see whether it can serve as a good measure of model information. The following observations show that $L_{IT}$ measures the generalizable information a model gains from a given dataset. Furthermore, $L_{IT}$ can be used to measure generalizable information in a dataset, which is lower-bounded by model information. This makes $L_{IT}$ a useful analytic tool to understand models and datasets.

An important property of $L_{IT}$ is that it only measures generalizable knowledge. Information gain from fitting (or remembering) particular examples does not contribute to $L_{IT}$ (see Appendix A.4). This can be easily seen from the fact that simply remembering labels of past examples $y_{1:n}$ does not reduce the codelength of future examples $y_{n+1}$:

$\sum_{n=1}^{\infty} p(y_{n+1} | y_{1:n}) \log p(y_{n+1} | y_{1:n})$.

For choice of parameter $k$ in $L_{IT}$, we let $k = 5000$ for small datasets and $k = 10000$ for large datasets throughout our experiments. More experiment details that are not elaborated here can be found in the appendix.

3.1 Synthetic Dataset

First we verify the correlation between $L_{IT}$ and $L_{model}$ using synthetic datasets. We generate text corpus using 2-gram language models, then we train LSTM language models on them. The amount of generalizable information $L_{model}$ can be controlled by varying the number of independently sampled 2-grams in the model generating the corpus. We measure $L_{IT}$ of converged LSTM language models.

In Figure 3 we observe a roughly linear relationship between $L_{IT}$ and $L_{model}$ ($\rho \approx 0.98$).

Figure 3: Relationship between $L_{IT}$ and $L_{model}$ on synthetic corpus.

3.2 Measuring Information in MNIST-variants

Next we apply $L_{IT}$ to a ResNet-18 model on the commonly used MNIST dataset and some of its variants. Figure 4 illustrates $L_{IT}$ of a model trained on each dataset. One is able to tell that a model classifying 10 Japanese characters (KMNIST [11]) contains significantly more information than a model classifying 10 digits (MNIST [12], EMNIST [13]). Because Japanese characters are more complex than digits, it requires more knowledge to classify the former. A model classifying the
English alphabet (upper and lower-case, 47 classes [13]) contains even more information, as the task is, again, more difficult.

$L_{IT}$ calculates prequential codes using $\theta_n$ relative to $\theta_0$. Therefore it gives the information gain, or “new knowledge” in the dataset compared to what is already in the initial model $\theta_0$. We can use the model pre-trained on MNIST as $\theta_0$ to measure the amount of new information a model can learn from each dataset. From $L_{IT}(M_{init})$ in Figure 4, KMNIST, Fashion-MNIST [14] and EMNIST-alphabet all contain much new information that is not in MNIST. Shuffled-MNIST is a version of MNIST with its labels permuted, therefore it only introduces a little new information. The same is with EMIST-digits, which only differ from MNIST in image pre-processing.

Comparing $L_{IT}(M_{init})$ with $L_{IT}$, knowledge about MNIST significantly reduces new information gained from Japanese characters and English alphabet, signifying these datasets share some common information (for example, strokes). On the other hand, $L_{IT}$ for Fashion-MNIST does not reduce as much, showing there is less information in common between handwritten digits and clothes images.

### 3.3 Dissecting Information in CIFAR-10

Although the absolute value of $L_{IT}$ depends on the choice of $k$, by fixing $k$ to a certain value we are able to compare the information content in different models and in different datasets. In the following example, we use $L_{IT}$ to dissect the knowledge about object classification in CIFAR-10, which also shows the consistency of $L_{IT}$ in measuring information.

The 10 object classes in CIFAR-10 belong to two categories: vehicles (4 classes) and animals (6 classes). We can therefore split CIFAR-10 into three tasks: $T_V$ and $T_A$ for classifying within vehicles and animals respectively, and $T_{V/A}$ for classifying between the two categories. By measuring $L_{IT}$ on learning each subtask, and on transfer learning from one subtask to another, we are able to produce a Venn diagram representing the information content and the relationship of three subtasks (Figure 5).

The information about classifying objects can be learned from the whole CIFAR-10 dataset $T_{full}$ as well as from a combination of subtasks. Figure 6 plotted $L_{IT}$ measured on sequentially training on subtasks; for example, training on $T_V$ first, then on $T_A$ ($T_V \rightarrow T_A$). $L_{IT}$ always adds up roughly to a fixed value (the total information of CIFAR-10), regardless of how one learns from subtasks. This is further evidence that $L_{IT}$ is consistently correlated with the true amount of information.

$L_{IT}$ gives us a lot of information about the task CIFAR-10. For example, classifying animals requires much more information than classifying vehicles. In the knowledge used to classify animals, about $1/4$ can be shared with classifying vehicles, and the remaining $3/4$ is specific to animals. $L_{IT}$ also reveals the dynamics of neural network transfer learning. For example in experiments involving a second transfer, one is able to tell from $L_{IT}$ the amount of information forgotten about the previous task, after training the model on a new task.

### 4 Measuring Information Transfer in Transfer Learning

The success of transfer learning depends on sharing knowledge from previous tasks [15]. Recently we have seen great success with pre-trained representation models in vision and NLP. Models pre-trained on large-scale datasets acquire good general knowledge of image and text, which helps them achieve state-of-the-art performance when adapted to a variety of tasks.
A natural question then is how to measure the quality of pre-trained representation models. Better representations not only achieve better performance on target tasks, but also require fewer training examples to reach a certain level of performance. As pointed out by [8], the ability to generalize rapidly to a new task by using previously acquired knowledge cannot be measured by performance metrics alone. They proposed to use prequential codes as a measure of linguistic intelligence.

From an informational perspective, the quality of representation models can be measured by the quantity of generalizable information in the model, which can be reused in downstream tasks. To facilitate comparison of different models using information transfer, we introduce the Information Advantage of model \( \theta \) over \( \theta_{ref} \):

\[
L_{IA}(k) = L_{\theta_{ref}}^{preq}(y_{1:k}|x_{1:k}) - L_{\theta}^{preq}(y_{1:k}|x_{1:k})
\]  

(9)

We can measure the information already in a model \( \theta \) before training with \( L_{IA} \). This means that if we initialize the model with \( \theta \), we gain \( L_{IA} \) bits more information comparing to \( \theta_{ref} \). We can measure the information already in a model \( \theta \) before training with \( L_{IA} \): Models that are more “knowledgeable” gain less new information from the target task, resulting in a higher \( L_{IA} \).

In Table 1 we measure the model information of pre-trained image classification models and pre-trained language models. The target tasks are CIFAR-10 and MultiNLI. Examining model performance, clearly there is not a single measure that represents the quality of the model: Different models have an advantage at different numbers of training examples. Task-specific knowledge (from pre-training on a similar task) contributes to few-shot effectiveness, while general knowledge helps more in many-shot. Both kinds of knowledge can be measured and are reflected in \( L_{IA} \).

More expressive model and pre-training on more data both contribute to increased information in the model by \( L_{IA} \). Pre-training on similar tasks (e.g., from SNLI to MultiNLI) is also a method to significantly increase model information about the target task.

5 Measuring Information Transfer in Continual Learning

The ability to continually learn is a fundamental element of genuine intelligence. Neural network models, although powerful, tend to suffer from catastrophic forgetting [26, 27] in continual learning settings. Some methods have been proposed (for example, [28, 29]) to prevent the model from forgetting about old tasks. However, recent studies pointed out that these methods often fail to significantly prevent forgetting or being unfeasible in practice [30, 31].

We take a different route to investigate catastrophic forgetting, by estimating model information with \( L_{IA} \): “Forgetting,” after all, means losing information. We found that information tells quite a different story than performance metrics in continual learning.

Experiments are performed in two scenarios: image classification, where we split the 200-class Tiny-ImageNet into four 50-class classification tasks, and language modeling, where we extracted 4 topics from the 1-billion-word-benchmark corpus [32] as four tasks. Algorithms employed are plain transfer, L2 regularization, Elastic Weight Consolidation [28], and Incremental Moment Matching.

\[\text{from hereafter, } L_{IT} \text{ and } L_{IA} \text{ are measured in k-nats}\]
Table 1: Information transfer in transfer learning. AlexNet and BERT-base are used as the reference models in $L_{IA}$. De-CIFAR stands for a color-distorted version of CIFAR-10. Red color marks the top-2 best performances. **Bold** marks the top-2 in model information.

| CIFAR-10 | Information | Performance (Accuracy %) | Pretraining | $L_{IT}$ | $L_{IA}$ | Zero-shot | Few-shot | Many-shot |
|----------|-------------|--------------------------|-------------|---------|---------|-----------|---------|-----------|
| AlexNet  | ImageNet [18] | 4.93 | 0 | 10.0 | 47.1 | 86.7 |
| VGG11    | ImageNet     | 4.39 | 0.55 | 10.0 | 48.7 | 87.9 |
| ResNet-18 | CIFAR-100 [16] | 5.42 | -0.49 | 10.0 | 46.6 | 82.6 |
| ResNet-18 | De-CIFAR     | 3.59 | 1.33 | 70.8 | 71.9 | 84.6 |
| ResNet-18 | ImageNet     | 2.50 | 2.44 | 10.0 | 58.9 | 91.8 |
| ResNet-18 | ImageNet + De-CIFAR | 0.88 | 4.05 | 86.6 | 87.9 | 93.3 |
| ResNet-18 | ImageNet | 1.87 | **3.07** | 10.0 | 62.5 | 93.1 |

| MultiNLI | Information | Performance (Accuracy %) | Pretraining | $L_{IT}$ | $L_{IA}$ | Zero-shot | Few-shot | Many-shot |
|----------|-------------|--------------------------|-------------|---------|---------|-----------|---------|-----------|
| BERT-base | Unsup.      | 3.81 | 0 | 33.3 | 50.9 | 76.4 |
| BERT-base | Unsup. + SNLI [23] | 2.37 | 1.44 | 65.9 | 69.6 | 78.2 |
| BERT-large| Unsup.      | 3.02 | 0.79 | 33.3 | 61.5 | 79.8 |
| XLMR-large | Unsup.      | 1.51 | **2.30** | 33.3 | 76.9 | 84.7 |
| RoBERTa-large | Unsup. | 1.91 | 1.90 | 33.3 | 69.2 | 86.0 |
| RoBERTa-large | Unsup. + SNLI | 0.30 | 3.51 | 78.7 | 81.6 | 85.8 |

(weight-transfer and L2-transfer) [29]. Single-task and multi-task training are used as baselines. Models used are ResNet-56 for image classification and 2-layer LSTM for language modeling.

In Table 2, we compare performance metrics as well as $L_{IA}$. $L_{IA}$ measures the amount of information the final model has about each task. Under **All past** is given the performance and information on all four past tasks combined. In **Future** we transfer the final model to a larger task to examine the representation learned throughout continual learning. We have the following observations:

Table 2: Information transfer in continual learning. *acc.* and *ppl.* stand for accuracy and perplexity. **Red** color marks the top-2 best performance. **Bold** marks the top-2 model information.

| Tiny-ImageNet | Task 0 | Task 1 | Task 2 | Task 3 | All past | Future |
|---------------|--------|--------|--------|--------|---------|--------|
| **Method**    | acc. $L_{IA}$ | acc. $L_{IA}$ | acc. $L_{IA}$ | acc. $L_{IA}$ | acc. $L_{IA}$ | acc. $L_{IA}$ |
| Plain         | 7.1    | 8.4    | 17.8   | 9.1    | 25.5    | 10.5   | 61.1    | 13.9   | 27.9    | 33.9   | 7.6 |
| L2            | 44.2   | 11.3   | 38.2   | 9.5    | 37.8    | 9.1    | 45.6    | 9.0    | 41.5    | 38.8   | 30.7   | 6.0 |
| EWC           | 47.9   | 11.5   | 43.5   | 9.2    | 41.6    | 9.1    | 44.8    | 8.9    | 44.5    | 38.6   | 29.7   | 5.4 |
| IMM-mean (wt) | 27.9   | 10.7   | 45.0   | 11.9   | 41.5    | 11.9   | 41.2    | 10.3   | 38.9    | 44.8   | 32.8   | 7.7 |
| IMM-mode (wt) | 12.1   | 9.3    | 25.2   | 11.3   | 27.3    | 11.0   | 21.7    | 9.9    | 21.6    | 41.4   | 32.4   | 6.8 |
| IMM-mean (l2) | 57.7   | 12.0   | 50.6   | 9.7    | 49.1    | 9.7    | 48.7    | 9.5    | 51.5    | 40.9   | 28.9   | 5.6 |
| IMM-mode (l2) | 57.4   | 12.1   | 52.3   | 9.6    | 49.6    | 9.3    | 47.9    | 9.1    | 51.8    | 40.1   | 28.6   | 5.5 |
| Single-task   | 56.6   | 0.0    | 60.7   | 0.0    | 56.3    | 0.0    | 53.9    | 0.0    | -       | -      | -      | - |
| Multi-task    | 62.8   | 14.7   | 63.6   | 14.4   | 61.1    | 14.3   | 61.1    | 13.9   | 62.2    | 57.3   | 38.6   | 9.8 |

| 1b-word-benchmark | Task 0 | Task 1 | Task 2 | Task 3 | All past | Future |
|-------------------|--------|--------|--------|--------|---------|--------|
| **Method**        | ppl. $L_{IA}$ | ppl. $L_{IA}$ | ppl. $L_{IA}$ | ppl. $L_{IA}$ | ppl. $L_{IA}$ | ppl. $L_{IA}$ |
| Plain             | 283    | 20.6   | 395    | 19.8   | 370     | 24.8   | 193     | 29.5   | 300     | 94.6   | 256    | 23.8 |
| l2                | 213    | 23.7   | 298    | 20.5   | 381     | 20.1   | 251     | 24.0   | 279     | 88.2   | 281    | 22.7 |
| EWC               | 197    | 23.2   | 233    | 22.4   | 300     | 22.7   | 262     | 23.3   | 244     | 91.6   | 304    | 21.0 |
| IMM-mean (wt)     | 437    | 22.8   | 399    | 24.4   | 687     | 25.7   | 1752    | 20.0   | 666     | 92.9   | 258    | 24.1 |
| IMM-mode (wt)     | 288    | 23.2   | 341    | 24.2   | 772     | 23.0   | 1854    | 17.1   | 605     | 87.5   | 289    | 20.3 |
| Single-task       | 111    | 0.0    | 108    | 0.0    | 152     | 0.0    | 232     | 0.0    | -       | -      | -      | - |
| Multi-task        | 90     | 31.2   | 90     | 31.7   | 126     | 32.8   | 203     | 29.4   | 119     | 125    | 202    | 29.7 |
$L_{IA}$ measures forgetting while performance alone does not. We found that performance on past tasks is largely irrelevant to the performance on future tasks. Model information (as measured by $L_{IA}$), however, is correlated with both performance and model information on future tasks. This indicates that $L_{IA}$ is a reasonable measure of generalizable knowledge in the model: Models with larger $L_{IA}$ preserve more information about past tasks, thus transfer more effectively to new tasks.

$L_{IA}$ can also help us understand why measuring forgetting with performance alone is flawed. Table 3 listed three common strategies to deal with the final output layer in continual learning. Using EWC on the three designs yields very different results in performance (Table 3). Only “separate” is able to perform well, because it keeps the final layer of past tasks intact. However, from $L_{IA}$, one is able to tell that the “separate” and the “union” strategies are very similar in the ability to preserve information. “Reuse” is less effective because the shared final layer impedes learning.

Table 3: Continual learning with EWC on the three strategies

| Method | Task 0 | Task 1 | Task 2 | Task 3 | All past |
|--------|--------|--------|--------|--------|----------|
|        | acc.   | $L_{IA}$ acc. | $L_{IA}$ acc. | $L_{IA}$ acc. | $L_{IA}$ acc. |
| Separate | 47.9   | 11.5   | 43.5   | 9.2    | 41.6    | 9.1    | 44.8    | 8.8    | 44.5    | 38.6    |
| Union   | 0.0    | 10.4   | 0.0    | 8.9    | 0.0     | 8.8    | 45.6    | 9.4    | 11.4    | 37.6    |
| Reuse   | 4.3    | 9.9    | 3.4    | 6.8    | 3.1     | 5.8    | 41.3    | 6.4    | 13.0    | 28.8    |

Catastrophic forgetting in neural networks is not really catastrophic. From the above discussion, we argue that forgetting needs to be understood at a deeper level. Forgetting involves two distinct factors: preserving knowledge and preserving the exact decision boundary. We use $L_{IA}$ to measure the former factor. While plain transfer fails by performance metrics, it does a decent job of preserving knowledge, keeping over 70% of total information in past tasks. This is also confirmed by its performance on future tasks (which is not affected by the exact decision boundary of past tasks).

Information transfer provides a new perspective on the effectiveness of continual learning. Continual learning strives to make a model learn like a human and to become more intelligent over time. Therefore, there are two main goals of continual learning: 1) perform well on past tasks (not forgetting), and 2) adapt quickly and perform well on new tasks (learn better representations). Previous works mainly focus on the first goal. Either goal requires the ability to preserve information, while the second goal is more directly determined by it.

In comparing the effectiveness of preserving information in Table 2, no continual learning method is significantly more effective than plain transfer learning. Figure 7 plots the ratio of preserved information on each task, and we can see that each method achieves a different balance of information: Regularization-based methods (L2, EWC) preserve more information from the first and the last tasks, while model averaging (IMM) more favors information from tasks in the middle. Plain transfer clearly remembers most about the last task. It is an interesting phenomenon that no method is able...
to cover the significant gap between continual learning and multi-task learning, given that neural networks are often over-parameterized \[33\] and have more than enough capacity.

6 Conclusion

In this paper, we have shown in multiple situations that model information and dataset information are important and useful metrics for analyzing deep learning. Measuring information in a neural network is a non-trivial task, but a surrogate information measure such as the proposed information transfer can have good correlation with true model information and is easy to calculate. We hope information transfer serves as a tool to motivate further investigation of neural models from an informational perspective. It is also a relevant future direction to develop learning algorithms to maximize generalizable model knowledge.

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Appendix

A The Information Transfer Measure

A.1 Proof of Equation (6)

From the definition of $L_{IT}$, when $k > n$

$$L_{IT}(n, k) = L_{\theta_0}^{\text{preq}}(y_{1:k} \mid x_{1:k}) - L_{\theta_n}^{\text{preq}}(y_{n+1:n+k} \mid x_{n+1:n+k})$$

Assume model converges to $\theta_{\text{oracle}}$ given a sufficiently large dataset, as $k \to \infty$ we have $\theta_k \to \theta_{\text{oracle}}$ and

$$L_{\theta_k}^{\text{preq}}(y_{n+1:n+k} \mid x_{k+1:n+k}) \to L_{\theta_{\text{oracle}}}^{\text{preq}}(y_{1:n} \mid x_{1:n})$$

therefore,

$$L_{IT}(n, k) = \sum_{i=1}^{n} \log p_{\theta_{i-1}}(y_i \mid x_i) - \sum_{i=1}^{n} \log p_{\theta_{\text{oracle}}}(y_i \mid x_i)$$

as $k \to \infty$.

We can also see that $L_{IT}(n, k)$ is upper-bounded for a given $n$:

$$L_{IT}(n, k) \leq L_{IT}(n, \infty)$$

A.2 Proof of Equation (7)

If we encode only the first label $y$ of a previously unknown dataset, assuming a model $\theta_0$ is at hand, the best codelength is $\min(-\log p_{\theta_0}(y \mid x), -\log K)$. It follows that if we use a two-part code instead, on average

$$\mathbb{E}[L_{\text{model}}(\theta')] - \log p_{\theta}(y \mid x) \geq \mathbb{E}[\min(-\log p_{\theta_0}(y \mid x), -\log K)]$$

if we assume $\theta_0$ is no worse than predicting a uniform distribution over all possible labels (this is roughly satisfied by for example, a random initialized neural network), and letting $\theta'$ be $\theta_n$

$$\mathbb{E}[L_{\text{model}}(\theta_n) - \log p_{\theta_n}(y \mid x)] = L_{\text{model}}(\theta_n) + \mathbb{E}[-\log p_{\theta_n}(y \mid x)] \geq \mathbb{E}[-\log p_{\theta_0}(y \mid x)]$$

then we have

$$L_{\text{model}}(\theta_n) \geq \mathbb{E}[-\log p_{\theta_0}(y \mid x)] - \mathbb{E}[-\log p_{\theta_n}(y \mid x)]$$

$$= \mathbb{E}[-\log p_{\theta_0}(y_1 \mid x_1)] - \mathbb{E}[-\log p_{\theta_n}(y_{n+1} \mid x_{n+1})]$$

$$= \mathbb{E}[-\log p_{\theta_0}(y_1 \mid x_1)] + \mathbb{E}[-\log p_{\theta_n}(y_{n+1} \mid x_{n+1})]$$

$$= \mathbb{E}[L_{IT}(n, 1)]$$

For the second inequality in (7), note that prequential code $L_{\theta_0}^{\text{preq}}(y_{1:n} \mid x_{1:n})$ transmits labels $y_{1:n}$ as well as model $\theta_n$, therefore

$$L_{\text{model}}(\theta_n) \leq L_{\theta_0}^{\text{preq}}(y_{1:n} \mid x_{1:n})$$

the prequential code must also include the incompressible noise in $y_{1:n}$ under model family $\mathcal{M}$, which is $L_{\theta_0}^{\text{preq}}$:

$$L_{\theta_0}^{\text{preq}}(y_{1:n} \mid x_{1:n}) \leq L_{\theta_0}^{\text{preq}}(y_{1:n} \mid x_{1:n})$$

Furthermore, because $L_{\text{model}}(\theta_n)$ is the length of the generalizable information in $\theta_n$, it is independent of the noise part $L_{\theta_{\text{oracle}}}^{\text{preq}}$, the codelength adds up and is still no larger than $L_{\theta_0}^{\text{preq}}$:

$$L_{\text{model}}(\theta_n) + L_{\theta_{\text{oracle}}}^{\text{preq}}(y_{1:n} \mid x_{1:n}) \leq L_{\theta_0}^{\text{preq}}(y_{1:n} \mid x_{1:n})$$

moving $L_{\theta_{\text{oracle}}}^{\text{preq}}$ to the right-hand side results in

$$L_{\text{model}}(\theta_n) \leq L_{IT}(n, \infty)$$
A.3 Connection Between $L_{IT}$ and Performance Metrics

For classification task with cross-entropy loss function, the prequential coding curve ("codelength - example id n") is very close to the "validation loss - dataset size" curve (Figure 8):

![Figure 8](image_url)

Figure 8: The prequential coding curve and the "validation loss - dataset size" curve on CIFAR-10

Therefore, $L_{preq}$ can be interpreted as an integration of the "validation loss - dataset size" curve. Especially, $L_{IA}$ can be interpreted as "how fast" the validation loss deceases as we increase dataset size $n$. Faster decrease of the validation loss symbolizes the model has more prior knowledge about the task.

![Figure 9](image_url)

Figure 9: The prequential coding curve of two different models on CIFAR-10, $L_{IA}(\theta_B)=3.07$

Compare the coding curve of two initial models $\theta_A$ and $\theta_B$ on CIFAR-10 in Figure 9, we see that both model converge to roughly the same performance, but $\theta_B$ requires much less examples to reach a certain level of performance. This indicates that $\theta_B$ has more information about the task than $\theta_A$ (in fact, in this example $\theta_B$ has been pretrained on a subtask of CIFAR-10). This is also given by the information advantage $L_{IA}$ of $\theta_B$ over $\theta_A$.

A.4 Remembering v.s. Generalization

For a randomly-labeled dataset, although the model could achieve high accuracy on the training set, $L_{IT}$ remains close to zero for any $n$ (Figure 10):

![Figure 10](image_url)

This is because the length of generalizable information in model $\theta_n$, $L_{model}(\theta_n)$ is zero. This again shows that information transfer measure $L_{IT}(n, k)$ is closely correlated with $L_{model}(\theta_n)$. 
B Measuring Information Transfer in Neural Networks

B.1 Calculating prequential codes

We follow the method used in [7] to calculate prequential codelengths. Using the definition of $L^{preq}$ in [7] requires training the model $n$ times, which is overly time consuming. An approximation is used in [7] that first partitions the dataset, and only re-train the model on each partition. The prequential codelength becomes:

$$L^{preq}(y_{1:n}|x_{1:n}) = t_1 \log K + \sum_{s=0}^{S-1} - \log p_{\hat{\theta}_s}(y_{t_s+1:t_{s+1}}|x_{t_s+1:t_{s+1}})$$

where $1 = t_0 < t_1 < \ldots < t_S = n$ is a partition of $n$ examples into $S$ sets. Because the log likelihood changes slower as the encoded example increases, we make the number of examples in each set to be 1.5 times that of the previous set. This is to make the calculation of prequential codes more time-efficient while minimizing approximation errors.

B.2 Experiment Settings

The models we used in experiments throughout this paper are ResNet-56 for image classification tasks and 2-layer LSTM of dimension 200 for language modeling tasks. Training models on the full or a subset of datasets uses early-stopping on the validation set.

B.3 Dissecting Information in CIFAR-10

We use $L_{IT}$ to measure the information gain in learning (or transfer learning) on a subtask of CIFAR-10. Here we use $L(T)$ to denote the amount of information transferred from task $T$ to a randomly-initialized model. $L(T \rightarrow T')$ denotes the amount of new information gained in transfer learning from task $T$ to task $T'$. $L(T \rightarrow T' \rightarrow T'')$ denotes the amount of new information gained from transferring the previous model again to a third task $T''$. $L(T \rightarrow T' \rightarrow T)$ is the information gain when transferring the model back to the first task, i.e., it measures the information forgotten about task $T$ after training on task $T'$. For example, “F” in Figure 6 means $L(T_V \rightarrow T_A \rightarrow T_V)$ and $L(T_A \rightarrow T_V \rightarrow T_A)$ respectively. The results are listed in Table 4.

The following relationships of $L_{IT}$ are examined on the subtasks of CIFAR-10:

- $L(T_V) + L(T_V \rightarrow T_{full}) \geq L(T_{full})$
- $L(T_A) + L(T_A \rightarrow T_{full}) \geq L(T_{full})$
- $L(T_{V/A}) + L(T_{V/A} \rightarrow T_{full}) \geq L(T_{full})$
- $L(T_V \rightarrow T_A) + L(T_V \rightarrow T_A \rightarrow T_{full}) - L(T_V \rightarrow T_A \rightarrow T_V) \geq L(T_V \rightarrow T_{full})$
- $L(T_A \rightarrow T_V) + L(T_A \rightarrow T_V \rightarrow T_{full}) - L(T_A \rightarrow T_V \rightarrow T_A) \geq L(T_A \rightarrow T_{full})$
- $L(T_V \rightarrow T_A \rightarrow T_{full}) - L(T_V \rightarrow T_A \rightarrow T_V) \geq L(T_V \rightarrow T_{A/V/A})$
- $L(T_A \rightarrow T_V \rightarrow T_{full}) - L(T_A \rightarrow T_V \rightarrow T_A) \geq L(T_A \rightarrow T_V \rightarrow T_{V/A})$

Figure 10: $L_{IT}(n)$ on CIFAR-10 with real and random labels
Table 4: $L_{IT}$ on subtasks of CIFAR-10, with different initial models.

| Task          | $L_{IT}$ | Task          | $L_{IT}$ | Task          | $L_{IT}$ |
|---------------|----------|---------------|----------|---------------|----------|
| $T_V$         | 1.07     | $T_V \rightarrow T_A$ | 1.45     | $T_V \rightarrow T_A \rightarrow T_V$ | 0.48     |
| $T_A$         | 2.18     | $T_A \rightarrow T_V$ | 0.54     | $T_A \rightarrow T_V \rightarrow T_A$ | 0.80     |
| $T_{V/A}$     | 0.82     | $T_A \rightarrow T_{full}$ | 2.23     | $T_V \rightarrow T_A \rightarrow T_{V/A}$ | 0.40     |
| $T_{full}$    | 3.53     | $T_{V/A} \rightarrow T_{full}$ | 2.46     | $T_V \rightarrow T_A \rightarrow T_{full}$ | 1.11     |
|               |          |               |          | $T_A \rightarrow T_V \rightarrow T_{full}$ | 1.34     |

By $\hat{=}$ we mean the left-hand side approximates the right-hand side. The first three equations represent learning the task $T_{full}$ can be separated into two stages: learn information from a subtask first, then learn the rest information. The next two equations similarly represent staged learning in three stages. The final two equations represent that the information gain from $\{T_V, T_A\}$ to $T_{full}$ is exactly $T_{V/A}$.

C Transfer Learning

When training data for the target task is insufficient, it is also common to use a fixed representation, and only train the final classifier layer. Some additional performance results are listed in Table 5. The performance when only training the final layer has the similar trend as zero-shot performance. If the representation model has been trained on a similar task, performance is significantly better.

Table 5: More performance results in transfer learning experiments. Fixed-rep stands for fine-tuning the final classifier layer only.

| CIFAR-10 | Performance (Accuracy %) |
|----------|--------------------------|
|          | Fixed-rep | Zero-shot | Few-shot($10^2$) | Many-shot($10^4$) |
| AlexNet  | ImageNet  | 82.7      | 10.0            | 47.1             | 86.7       |
| VGG11    | ImageNet  | 83.5      | 10.0            | 48.7             | 87.9       |
| ResNet-18| CIFAR-100 | 78.5      | 10.0            | 46.6             | 82.6       |
| ResNet-18| de-CIFAR-10| 80.6     | 70.8            | 71.9             | 84.6       |
| ResNet-18| ImageNet  | 80.9      | 10.0            | 58.9             | 91.8       |
| ResNet-34| ImageNet  | 92.6      | 86.6            | 87.9             | 93.3       |
| ResNet-50| ImageNet  | 81.4      | 10.0            | 61.7             | 92.9       |
|          |           | 83.3      | 10.0            | 62.5             | 93.1       |

| MultiNLI | Performance (Accuracy %) |
|----------|--------------------------|
|          | Fixed-rep | Zero-shot | Few-shot($10^3$) | Many-shot($10^4$) |
| BERT-base| Unsup.    | 42.4      | 33.3             | 50.9             | 76.4       |
| BERT-base| Unsup. + SNLI| 68.0    | 65.9            | 69.6             | 78.2       |
| BERT-large| Unsup.   | 38.5      | 33.3             | 61.5             | 79.8       |
| XLNet-large| Unsup.  | 48.0      | 33.3             | 76.9             | 84.7       |
| RoBERTa-large| Unsup.   | 47.0      | 33.3             | 69.2             | 86.0       |
| RoBERTa-large| Unsup. + SNLI| 81.4   | 78.7            | 81.6             | 85.8       |

D Continual Learning

D.1 Task Settings

Image classification we split the Tiny-ImageNet dataset into four subsets, each containing examples for 50 classes (500 examples for each class), as tasks 0-3. “Future” tasks is chosen to be the original Tiny-ImageNet (200-class joint classification).

2https://tiny-imagenet.herokuapp.com
Language modeling we first trained an LDA topic model \cite{34} on the 1-billion-word-benchmark corpus, and identified 4 topics out of 20: politics, economy, medicine, and movie. For each topic we extracted 100,000 sentences as corpus (11MB of text) for the corresponding task. “Future” tasks is chosen to be the original 1-billion-word-benchmark corpus.

To examine the generalizable knowledge in models after continual learning, the future task is chosen to be a more general and more complex task, that not only include knowledge in subtasks but also possess a significant amount of new information.

D.2 Models and Methods

For our continual learning experiments, we use the “separate” design (Table\cite{3}) for dealing with final layers. For L2 and EWC, we optimized a single hyper-parameter: the regularization coefficient $c$, based on average performance. For IMM (weight-transfer) and IMM (l2-transfer), the weighting vector of tasks $\alpha$ is chosen to be a uniform weighting, as preliminary optimization of $\alpha$ did not yield improved results. The difficulty of optimizing $\alpha$ is also discussed in \cite{31}.

Single-task and multi-task learning are used as baselines for comparison. Especially, $L_{IA}$ in Table\cite{2} are all relative to the single task models of corresponding tasks. Table\cite{6} lists the $L_{IT}$ of single task models. The performance on “future” task is measured at 10000 training examples.

Table 6: $L_{IT}$ of single task models in continual learning experiments

| Tiny-ImageNet | Task 0 | Task 1 | Task 2 | Task 3 | Future |
|---------------|--------|--------|--------|--------|--------|
| Method        | acc. $L_{IT}$ | acc. $L_{IT}$ | acc. $L_{IT}$ | acc. $L_{IT}$ | acc. $L_{IT}$ |
| Single-task   | 56.6 | 13.2 | 60.7 | 12.8 | 56.3 | 13.3 | 53.9 | 12.7 | - | - |
| 1-billion-word-benchmark | | | | | | | | | | |
| Method        | ppl. $L_{IT}$ | ppl. $L_{IT}$ | ppl. $L_{IT}$ | ppl. $L_{IT}$ | ppl. $L_{IT}$ |
| Single-task   | 111 | 28.7 | 109 | 30.0 | 152 | 31.1 | 232 | 26.7 | - | - |

D.3 Additional Experiments and Discussion

Another way to inspect forgetting. In general continual learning settings, the model have no access to past datasets. However, if we are interested in examining the forgetting of representations, we can re-train only the final layer of the network on past tasks to see how it performs. The representation and features of the network (all layers except the final layer) are kept frozen to the value after continual learning. The performance on Tiny-ImageNet task is reported in Table\cite{7}.

Table 7: Re-learn the final layers in continual learning. Acc.(ft) stands for model accuracy after re-learning the final layer.

| Tiny-ImageNet | Task 0 | Task 1 | Task 2 | Task 3 | All past |
|---------------|--------|--------|--------|--------|----------|
| Method        | Acc. | Acc.(ft) | Acc. | Acc.(ft) | Acc. | Acc.(ft) | Acc. | Acc.(ft) | Acc. | Acc.(ft) |
| plain         | 7.1 | 53.8 | 17.8 | 53.6 | 25.5 | 55.9 | 61.1 | 63.2 | 27.9 | 56.7 |
| L2            | 44.2 | 58.4 | 38.2 | 52.7 | 37.8 | 50.2 | 45.6 | 50.6 | 41.5 | 53.0 |
| EWC           | 47.9 | 60.4 | 43.5 | 53.6 | 41.6 | 51.6 | 44.8 | 50.3 | 44.5 | 54.0 |
| IMM-mean (wt) | 27.9 | 59.5 | 45.0 | 61.7 | 41.5 | 50.3 | 41.2 | 54.4 | 38.9 | 59.0 |
| IMM-mode (wt) | 12.1 | 59.1 | 25.2 | 61.5 | 27.3 | 59.8 | 21.7 | 53.1 | 21.6 | 58.4 |
| IMM-mean (12) | 57.7 | 62.4 | 50.6 | 55.2 | 49.1 | 50.6 | 48.7 | 50.4 | 51.5 | 54.7 |
| IMM-mode (12) | 57.4 | 62.9 | 52.2 | 55.4 | 49.6 | 50.8 | 47.9 | 49.9 | 51.8 | 54.7 |
| Single-task   | 56.6 | 60.7 | 56.3 | 53.9 | 56.9 |
| Multi-task    | 62.8 | 63.6 | 61.1 | 61.1 | 62.2 |
It is obvious that by only finetuning the final layer (which accounts for less than 2% of the total number of parameters in the network), the performance can be largely recovered. This indicates that the majority of the information is not forgotten by the network, just as discussed in Section 5.

**Why shuffled-MNIST is not a good task for evaluating forgetting.** Shuffled-MNIST is a commonly used task in evaluating continual learning algorithms [28, 29, 35]. In recent study [31], it is found that permutation based tasks (like shuffled-MNIST) fail to exhibit forgetting in models and therefore should not be used to evaluate forgetting. From an informational perspective, we can see why: Shuffled-MNIST introduce little new information compared to MNIST (Section 3.2). Because shuffled-MNIST is just MNIST with labels permuted, the majority of information is shared between the two tasks. The model simply does not have much to forget in transferring from MNIST to shuffled-MNIST, so all models exhibit little forgetting as in [31].

**Raw coding curves.** For language modeling on 1-billion-word-benchmark, we plot the prequential coding curve on each task in Figure 11-15.
Figure 13: Coding curve on task 2

Figure 14: Coding curve on task 3

Figure 15: Coding curve on task “future”