Dynamic constitutive model of penetrating jointed rock mass based on ZTW model

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Abstract. According to the characteristics of stress-strain curves of soft rock with through joints under dynamic loading and the related research results of existing dynamic constitutive models of soft rock, a dynamic uniaxial compression constitutive model for soft rock with through joints is established. The model is based on Zhu-Wang-Tang model, in which damage body, joint plane closure element and shear deformation element are connected in series to simulate the static compressive mechanical properties of jointed rock mass, and Maxwell composite element is connected in series to describe the strain rate effect of jointed rock mass. First, the micro-damage model is described assuming that the integrity of the intact rock mass follows the tensile strain criterion and the statistical damage of the micro-intensity and damage satisfies the Weibull distribution; Secondly, the Mohr-Coulomb strength criterion is introduced into the strength damage variable to modify the macroscopic damage variable according to the shear failure characteristics of the through-jointed rock mass along the Plane-straight joint plane under uniaxial compressive load. Thirdly, based on the Lemaitre equivalent strain hypothesis, a coupled damage variable (tensor) considering the macroscopic and microscopic effects is established, and then the damage viscoelastic dynamic constitutive model of the jointed soft rock is established. Finally, the model is used to discuss the effects of joint cohesion, joint internal friction angle and joint dip angle on the dynamic mechanical properties of rock mass. The results show that the uniaxial compression dynamic peak strength of jointed rock mass increases with the joint cohesion and the internal friction angle of the joint, U-shaped with the increase of the joint inclination angle, and the minimum value is obtained at 60°.

1. Introduction

With the rapid development of national economy, the mining depth of underground mine is increasing, and the occurrence conditions and stress environment of rock mass are becoming more and more complex. High-impact rock pressure and gas outburst are more prominent under the dynamic disturbances such as tunneling and rock slab fracture. In order to study the failure mechanism of rock mass under disturbance, it is necessary to fully grasp the dynamic mechanical response of rock mass under dynamic loading, especially the relationship among strain rate, strain and stress of rock mass under dynamic loading, which is the necessary prerequisite for the study of mine safety and stability design. Therefore, it is of great theoretical significance and engineering application value to establish reasonable dynamic constitutive relation of rock mass.

At present, there are many studies on the dynamic constitutive model of rock materials at home and abroad, but the main influencing factors are the over-stress model proposed by U.S. Lindholm and its modified model. However, viscoelastic continuous damage constitutive model based on over-
stress model usually assumes that viscous components can be damaged, which makes the physical concept of this kind of model more ambiguous. Therefore, many scholars have established dynamic constitutive models based on ZWT model for soft rock materials.

Initially, Tang Zhiping[6] proposed a non-linear viscoelastic constitutive equation for the first time, namely the Zhu Tangwang constitutive equation (simply called ZWT), which can be used to describe the viscoelastic materials of polymers from quasi-static to impact loads. Then, scholars [7-9] established a damage-type viscoelastic dynamic constitutive model equation that based on ZWT model, according to the dynamic characteristics of strain hardening and plastic flow of soft rock and concrete under dynamic load. Shan Renliang et al.[7] expressed the plastic flow of anthracite with a viscoelastic model with one spring and two Maxwell bodies in parallel. Zhao Guangming et al.[8] considered the damage softening effect of soft rock materials, and replaced the inelastic springs in the ZWT model with the damage body to establish a viscoelastic statistical damage model. Based on the Weibull distribution of damage evolution of cement mortar under thermal damage, Tao Junlin et al. [9] proposed a thermal viscoelasticity damage constitutive model of cement mortar to describe the dynamic characteristics of cement mortar under both high temperature and impact loads.

The above improved ZWT model considers that the adhesive pot component is deformable and damaged, which is inconsistent with the fact that the clay adhesive element will not be damaged, making the physical concept of the improved damage ZWT model obscure. For this reason, Xie Menxiang et al.[10] considering that the damage of solid materials is guided by micro-cracks and holes, a new constitutive model is established by replacing linear and non-linear elastic elements in Zhu Tangwang's model with damaged bodies. However, the dynamic constitutive model can only reflect the mesoscopic damage of the material and cannot solve the influence of the coupling effect of the macroscopic and meso-fractures in the actual engineering rock mass on the damage characteristics of the fractured rock mass. On this basis, based on ZWT model and damage and fracture mechanics, the macro and micro damage variables of jointed rock mass are derived. Secondly, according to Lemaitre's equivalent hypothesis, a damage variable considering macro-micro coupling is established, and then a dynamic constitutive model of jointed rock mass is established. Finally, its rationality is preliminarily verified.

2. Damage model of through joint rock mass considering macroscopic and microscopic defect coupling

2.1. Establishment of uniaxial compression damage model for jointed rock mass

Tang Zhiping[6] proposed the ZTW model to describe the mechanical behavior of the material from the strain rate of \(10^{-4}\text{s}^{-1}\) to \(10^{3}\text{s}^{-1}\) when studying the mechanical behavior of the viscoelastic material. The model structure is shown in Fig.1. The model uses a non-linear spring element to reflect the material's non-linear response, a high frequency Maxwell body to reflect the material's viscoelasticity at high strain rate, and a low frequency Maxwell body to reflect the material's viscoelasticity at low strain rate. The constitutive equation of the model is as follows:

\[
\sigma(t) = E_0\dot{\varepsilon} + \alpha\dot{\varepsilon}^2 + \beta\dot{\varepsilon}^3 + E_1\int_0^t \dot{\varepsilon} \exp\left(-\frac{t-\tau}{\phi_1}\right) d\tau + E_2\int_0^t \dot{\varepsilon} \exp\left(-\frac{t-\tau}{\phi_2}\right) d\tau \tag{1}
\]

In the formula: \(E_0, \alpha\) and \(\beta\) are all elastic constants of non-linear elastomers; \(E_1, \phi_1\) and \(E_2, \phi_2\) correspond to the elastic constants and relaxation time of Maxwell bodies respectively; \(t\) indicates time.

![Figure 1. Z-W-T model](image)
The model can better reflect the viscoelasticity of materials at different strain rates. Then many scholars improved this model to study the dynamic mechanical behavior of complete concrete and rock materials[7-10]. In addition, Liu Hongyan et al.[11] divided the axial static deformation characteristics of the jointed rock mass into the axial deformation of the intact rock mass, the compression deformation of the joint surface and the shear deformation. Therefore, this paper draws on the above research ideas to propose a dynamic mechanical model of the jointed rock mass at high strain rate as shown in Fig. 2. In this model, the damage body $D_1$ is used to describe the natural meso-damage evolution of rock blocks under static loading, and $D_2$ is used to describe the deformation of joints under compression load, which consists of joint-plane closing elements and shear-deformation elements in series. The $D_1$ and $D_2$ series are named conjoined I to describe the static deformation behavior of jointed rock mass; the Maxwell body is named conjoined II to reflect the viscoelastic properties of jointed rock mass at high strain rate. A dynamic mechanical model for uniaxial compression of jointed rock mass is presented by parallel connection of conjoined I and II. In order to facilitate the solution, this paper puts forward the damage body suit from Hooke's law. If the total stress of series I is $\sigma_1$, where the stress and strain of the damaged body $D_1$ and $D_2$ are respectively $\sigma_{11}$, $\sigma_{12}$, $\varepsilon_{11}$ and $\varepsilon_{12}$, then according to the series rule $\sigma=\sigma_{11}=\sigma_{12}$, $\varepsilon=\varepsilon_{11}=\varepsilon_{12}$; the total stress of series II is $\sigma_2$.

![Figure 2. Dynamic damage model of through jointed rock mass](image_url)

### 2.2. Static mechanical model of through-joint rock mass based on macro and micro damage

Rock mass is the product of long-term geological action, and there are lots of random geological defects, such as pores, micro-cracks and so on. Under the action of external force, these mesoscopic elements produce rupture evolution along a certain direction, which is manifested by the plastic characteristics of rock deformation and failure process[12]. At present, statistical damage mechanics is widely used to study the fracture evolution of these meso elements. According to the properties of rock materials, it is usually assumed that rock microelements obey strength criteria, such as Mohr-Columb criterion, tensile strain strength criterion, Hoek-Brown criterion, etc. Meanwhile, it is assumed that the strength of the infinitesimal element obeys the law of statistical distribution, such as Weibull distribution, power function distribution, logarithmic orthogonal distribution, etc.[13]. To reduce the model counting parameters, this paper assumes that the micro-element strength obeys the maximum tensile strain failure criterion, and the intensity distribution obeys the Weibull distribution with the parameter $(m, F)$. Then the constitutive model of the damaged body $D_1$ is[14-15]

$$P(\varepsilon) = \frac{m}{F} (\frac{\varepsilon}{F})^{m-1} \exp \left( -\left( \frac{\varepsilon}{F} \right)^m \right)$$

(2)

In the formula: $P(\varepsilon)$ is the strength distribution function of the element; $\varepsilon$ is the strain variable of the random distribution of the strength of the element; $m$ and $F$ are Weibull distribution parameters, which are related to the properties and shape of the material.

The statistical damage variable $D_1$ is defined as the ratio of $N_c$ to the total $N$ of the infinitesimal, i.e. $D_1=N_c/N$. $NP(x)dx$ is generated within the interval $[F,F+dF]$. When the load is loaded to $F$, the total number of damaged microelements is

$$N_c = \int_0^F NP(x)dx = N \left[ 1 - \exp \left( -\left( \frac{F}{F_0} \right)^m \right) \right]$$

(3)
Then the damage coefficient $D_1$ is

$$D_1 = \frac{N_c}{N} = 1 - \exp\left(\frac{F}{F_0}\right)^m$$

(4)

In the formula: $D_1$ is the statistical damage variable; $N_c$ is the number of micro-destructions destroyed at a certain load level; $N$ is the number of micro-elements before destruction.

The stress-strain relationship of the damaged body $D_1$ is

$$\sigma_{11} = \sigma_1 = E_0\epsilon_{11}(1 - D_1) = E_0\epsilon_{11}\exp\left[-\left(\frac{\epsilon_{11}}{\epsilon_0}\right)^m\right]$$

(6)

In the formula: $E_0$ is the elastic modulus before damage.

According to the definition of damage variable by damage mechanics, the macroscopic damage variable $D_2$ is defined as the yield stress as:

$$D_2 = 1 - \frac{\sigma_c}{\sigma_0}$$

(7)

In the formula: $\sigma_c$ and $\sigma_0$ are the strengths of the damaged medium and the non-damaged medium, respectively.

According to the single structural plane theory proposed by Jeager[16], as shown in Figures 3 and 4. Under the three-dimensional stress, the normal stress and the shear stress on the joint surface AB are:

$$\sigma = \frac{\sigma_1 + \sigma_3}{2} + \frac{\sigma_1 - \sigma_3}{2}\cos(2\beta)$$

(8)

$$\tau = \frac{\sigma_1 - \sigma_3}{2}\sin(2\beta)$$

(9)

Figure 3. Force diagram of rock mass with single joint surface

Figure 4. Mohr-Coulomb diagram of rock mass with monoclinic joints
In this paper, the joints used in rock mass are smooth joints, and the influence of joint roughness on the strength of the joints is not considered. Therefore, Mohr-Coulomb strength criterion is chosen to study the shear failure of rock mass on the joints.

\[ \tau = c_j + \sigma \tan \phi_j \]  

(10)

By substituting equations (8) and (9) into equations (10), the shear failure conditions along the joint surface AB are obtained:

\[ \sigma_{12} = \sigma_1 = \sigma + \frac{2(c_j + \sigma_1 \tan \phi_j)}{2(1 - \tan \phi_j \cot \beta) \sin 2\beta} \]  

(11)

As shown in Fig. 4, when the joint dip angle A > B, the rock mass will shear failure along the structural plane, that is,

\[ \beta_1 = \frac{\phi_j}{2} + \frac{1}{2} \arcsin \left[ \frac{(\sigma_1' + \sigma_3' + 2c_j \cot \phi_j) \sin \phi_j}{\sigma_1' - \sigma_3'} \right] \]  

(12)

\[ \beta_2 = \frac{\pi}{2} + \frac{\phi_j}{2} - \frac{1}{2} \arcsin \left[ \frac{(\sigma_1' + \sigma_3' + 2c_j \cot \phi_j) \sin \phi_j}{\sigma_1' - \sigma_3'} \right] \]  

(13)

When the rock mass is only subjected to axial stress, it can be obtained by substituting \( \sigma_3' = 0 \) into equation (11)

\[ \sigma_1' = \frac{2c_j}{\sin 2\beta - \cos 2\beta \tan \phi_j} \]  

(14)

Substituting equations (12), (13) and (14) into equation (7), \( D_2 \) is obtained

\[ D_2 = \begin{cases} \frac{\sigma_0}{\sin 2\beta - \cos 2\beta \times \tan \phi_j - \tan \phi_j} & (\beta_1 < \beta < \beta_2) \\ 0 & (\beta > \beta_2 \text{ or } \beta < \beta_1) \end{cases} \]  

(15)

H.Y. Liu et al.\cite{17} based on Lemaitre strain equivalence hypothesis\cite{18} obtained macro meso coupled damage parameter \( D_{12} \).

\[ D_{12} = 1 - \frac{(1 - D_1)(1 - D_2)}{1 - D_1 D_2} \]  

(16)

In the formula: \( D_1 \) is a meso fracture damage variable; \( D_2 \) is a macro crack damage variable.

The macroscopic fissure has obvious anisotropy, while the \( D_{12} \) obtained by equation (16) only reflects the damage in the direction of load. Therefore, based on the study by T.Kawamoto et al.\cite{19}, the damage tensor is introduced to reflect the characteristics of the macroscopic fractures.

\[ \mathbb{\Omega} = \mathbb{D} \mathbb{N} \]  

(17)

In the formula: \( \mathbb{N} \) is a second-order symmetric tensor, which can be obtained from equation (18).

\[ \mathbb{N} = \mathbf{n} \otimes \mathbf{n} = \begin{bmatrix} \cos^2 \beta & \sin \beta \cos \beta \\ \cos \beta \sin \beta & \sin^2 \beta \end{bmatrix} \]  

(18)

In the formula: \( \beta \) is the angle between the through crack and the load acting surface; \( \mathbf{n} \) is the unit normal vector of the joint. Then \( \mathbb{\Omega}_2 \) is

\[ \mathbb{\Omega}_2 = \mathbb{D}_2 \mathbb{N} = \mathbb{D}_2 \begin{bmatrix} \sin^2 \beta & \sin \beta \cos \beta \\ \cos \beta \sin \beta & \cos^2 \beta \end{bmatrix} \]  

(19)

Based on the above method, the macroscopic and mesoscopic damage parameter \( D_{12} \) can be obtained after the tensor quantization

\[ \mathbb{\Omega}_{12} = \mathbb{I} - \frac{(1 - D_1)(\mathbb{I} - \mathbb{\Omega}_2)}{\mathbb{I} - D_1 \mathbb{\Omega}_2} \]  

(20)
In order to solve the problem conveniently, it is considered that the damaged body is linear elastic and obeys Hooke's law. Then, in conventional uniaxial compression, the constitutive model of rock mass damage based on macroscopic and meso-fracture coupling is

$$\mathbf{\sigma} = [E_0][I - \alpha \Omega_2] [\mathbf{\epsilon}]$$  \hspace{1cm} (21)

The constitutive equation of the series I under the axial load is

$$\sigma_1 = D_{12} E_0 \dot{\epsilon} = E_0 \dot{\epsilon} \left(1 - D_2 \exp\left(-\frac{\epsilon}{F}\right)^m\right)$$

$$1 - D_2 + D_2 \exp\left(-\frac{\epsilon}{F}\right)^m$$  \hspace{1cm} (22)

2.3. Dynamic mechanical model of jointed rock mass

Maxwell, whose concatenate II is a series of po elements and elastomers, is constitutionally expressed as

$$\sigma_2(t) = E_i \dot{\epsilon}(t) \phi \left(1 - \exp\left(-\frac{\epsilon}{\phi \dot{\epsilon}}\right)\right)$$  \hspace{1cm} (23)

Since the dynamic mechanical model of the through-joint rock mass is obtained by the parallel connection of series I and series II, the properties of series connection and parallel connection can be obtained

$$\sigma = \sigma_1 + \sigma_2$$  \hspace{1cm} (24)

By substituting formulas (15), (22) and (23) for formulas (24), the dynamic damage constitutive equation of rock mass with through joints under axial dynamic load can be obtained as follows:

$$\left\{ \begin{array}{l}
\sigma = E_0 \dot{\epsilon} \\
1 - D_2 + D_2 \exp\left(-\frac{\epsilon}{F}\right)^m + E_i \dot{\epsilon}(t) \phi \left(1 - \exp\left(-\frac{\epsilon}{\phi \dot{\epsilon}}\right)\right)
\end{array} \right.$$  \hspace{1cm} (25)

$$D_2 = \begin{cases}
\frac{2c_j}{(\sin 2\beta - \cos 2\beta \times \tan \phi_j)} & (\beta_1 < \beta < \beta_2) \\
0 & (\beta > \beta_2 \text{or} \beta < \beta_1)
\end{cases}$$

3. Model parameter analysis

3.1. Effect of Joint Cohesion on Mechanical Properties of Rock Mass

In this paper, based on the author's study of the complete rock mass dynamic constitutive[21], the relevant parameters are selected and taken separately $E_0$=65.34GPa, $E_i$=3.44GPa, $\phi_i$=0.52s, $\dot{\epsilon}$=134.2s$^{-1}$. Weibull damage body parameter F=0.00758, $m$=1.39. Under this strain rate, the dynamic compressive strength of intact rock is 182.3 MPa.

In the calculation of the model, the internal friction angle and the joint inclination angle are $\phi_j = 30^\circ$, $\beta = 45^\circ$, and the joint cohesion $C_j$ is 1, 2 and 5 MPa. The model calculation results are shown in Fig. 5. As can be seen from the figure, with the increase of cohesive force, the dynamic stress-strain slope of jointed rock mass also increases, that is, the elastic modulus is positively correlated with cohesive force. Meanwhile, with the increase of joint cohesion, the dynamic peak strength of jointed rock mass also increases obviously. This indicates that when the joint inclination angle is larger than the joint internal friction angle, the shear failure weight of the rock mass is larger than that of the rock mass damage. That is, when the joint inclination angle is larger than the internal friction angle and less than 90°, the dynamic mechanical properties of the jointed rock mass are larger than the macroscopic damage of the joint surface than the microscopic damage of the rock mass.
3.2. Influence of internal friction angle on mechanical properties of rock mass
The calculation model is still shown in Figures 3 and 4. The cohesion and joint inclination are $C_j=5$, $b=45$, and the internal friction angle is 10°, 15° and 30°, and the other parameters are the same as 2.1.

Figure 6. Dynamic stress-strain curves of jointed rock mass with different internal friction angles
The calculated results of the model are shown in Fig. 6. It can be seen from the figure that with the increase of internal friction angle of the joint, the peak strength of the joint rock mass sample gradually increases, which is 36MPa, 52MPa and 70MPa, respectively. It shows that with the increase of internal friction angle of joint, the shear strength of rock mass increases, and the macroscopic damage coefficient is reduced accordingly, which reduces the macroscopic and mesoscopic coupling damage, thus improving the strength of jointed rock mass. At the same time, as the internal friction angle increases, the slope of the stress-strain curve of the rock increases, which is basically the same as that of Liu Hongyan et al. [11].

3.3. Influence of joint inclination on mechanical properties of rock mass
The calculation models are shown in Figs. 3 and 4. The inclination angles of joints are 0°, 30°, 40°, 60°, 75° and 90°, with $\varepsilon=68.3s$, $c_j=5$MPa, $\phi_1=30°$. The other calculation parameters are the same as those in section 3.1. The calculation results of the model are shown in Fig. 7.
Figure 7. Dynamic stress-strain curves of jointed rock mass with different joint dip angles

Figure 7 is a uniaxial compression dynamic stress-strain curve with different jointed dip angles. It can be seen from the figure that the stress-strain curves of different joint dip angles have similar trends, and the stress-strain curves of the models calculated at 0°, 30° and 90° are completely coincident. When the joint inclination angle is from 30° to 90°, the slope of the front curve of the uniaxial compression curve of the rock mass changes U-shaped. This is a change in the failure mechanism of the rock mass controlled by the joint dip angle.

4. Conclusion

1) Based on the ZWT model and the view of macroscopic and microscopic defect coupling, a damage-type viscoelastic dynamic constitutive model of the jointed soft rock considering the macroscopic and microscopic defects is established.

2) The influence of joint strength parameters and joint inclination on rock mass strength is discussed by using the established constitutive model. The results show that the peak strength of uniaxial compression of jointed rock mass is positively correlated with the cohesion of joints and the angle of internal friction of joints, and U-shaped with the increase of the inclination of joints, and the minimum value at 60°.

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