Calculation methods for reinforcement at the edge of epoxy joints of precast segmental box girders based on STM

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Abstract. To study the failure mechanism caused by diagonal crack near the joints in precast segmental box girder, a finite element model (FEM) of a standard segment of precast segmental box girder under pure bending was established. The stress field distribution for a selected segment was obtained by numerical analysis. Topological optimisation was adopted to determine the load path through the standard segment. Then, a strut-and-tie model (STM) for the standard segment under pure bending was established and verified, and calculation methods and formulae were provided for determining the reinforcement requirement at the edge of epoxy joints and at the middle of the web of the segments. Finally, a standard segment from the Fourth Nanjing Yangtze River Bridge was taken as an example to check the requirement for vertical reinforcement at the edge of the joint and horizontal reinforcement in the web, using the proposed STM. The results show that, using the STM for a standard segment when designing the reinforcement required at the edge of the joints and at the middle of the web, is acceptable, which further validates the feasibility of the proposed method.

1. Introduction

With the rapid increase in application of segmental precast box girders in engineering practice, increasing number of researchers have focused on segmental precast box girder analysis [1-4]. Differing from the ordinary monolithic box girder, ordinary steel bars are broken at the joints, and the segments are bonded together by epoxy resin adhesive. Due to this typical feature, the joint is the weak part of the girder, where the joint is opened. Once the joint is opened, the stresses on girder will be redistributed [5].

J Turmo et al. found that, after the dry joints were opened, diagonal cracks would occur at the lower edge of the compression zone of the girder [6]. During testing, the author also observed that diagonal cracking extended to the loading point at the joint interface, when the epoxy joint was cracked [7-10]. Moreover, J Turmo et al. proposed that the cracks are caused by the spalling stress generated by the self-balancing of the concrete structure on the basis of their experiments and finite element analysis (FEA) [11-12]. The spalling stress refers to the surrounding concrete under the action of concentrated force generating a stress perpendicular to the direction of the concentrated force in the concrete to maintain strain coordination. The resultant stress (usually tensile) is the spalling stress. To prevent failure of the girder by cracking and spalling, stirrups or hanger reinforcement should be arranged at the joint edge [13-14].

In summary, there is little research available pertaining to the design of the reinforcement against typical diagonal cracks at the joint edge of a segmental girder [15], therefore, this work combined the author’s experiments, established a FEM of the experimental segmental box girder, and investigated
the characteristics of the principle stress path and topological optimisation. The mechanism of occurrence of the diagonal crack was revealed, and then a STM was established and verified. Finally, a standard segment of the precast segmental box girder used on the Fourth Nanjing Yangtze River Bridge was assessed using the STM. The reinforcement was checked, and design recommendations are provided.

2. FEA of a segmental precast box girder

2.1 Establishment of the FEM

The span of the experimental girder is 5.5 m. Each segment is 500 mm long, 600 mm deep, with an upper flange plate of 1500 mm in width, a lower flange plate of 700 mm in width, and an 80 mm-thick web. The reinforced concrete in the FEM was simulated by Solid65 elements; the Poisson’s ratio is 0.2; the density is 2600 kg/m³, and the elastic modulus is 32,500 N/mm². The area of concrete in compression on each side of the segment can be calculated based on static equilibrium considerations, and the applied force on both sides of the compression zone of the segment is equal to the product of the design value of the concrete compressive strength and the area of the compression zone. The concrete stress-strain constitutive relationship is given by:

\[
\sigma_c = \sigma_0 \left[ 2 \frac{\epsilon_c}{\epsilon_0} - \left( \frac{\epsilon_c}{\epsilon_0} \right)^2 \right] \quad \epsilon_c \leq \epsilon_0
\]

\[
\sigma_c = \sigma_0 \quad \epsilon_0 < \epsilon_c \leq \epsilon_{cu}
\]

in which, \(\sigma_0\) is peak stress, \(\sigma_0 = 0.85 f'_c\); \(f'_c\) is concrete cylinder compressive strength; \(\epsilon_0\) is strain at peak stress, \(\epsilon_0 = 0.002\); \(\sigma_c\) is concrete stress; \(\epsilon_c\) is concrete compressive strain; \(\epsilon_{cu}\) is ultimate compressive strain in the concrete, \(\epsilon_{cu} = 0.0035\).

For the four-point loaded segmental box girder under pure bending, when the girder was loaded and the joint was opened, the upper flange of the segmental box girder was compressed, therefore, symmetrical loads and vertical constraints were applied to both sides of the interface of the upper flange segment as load and boundary conditions. According to the author’s experimental results, the height of the compression zone of the segmental box girder is between 50 and 150 mm [10]. To analyse the stress distribution across such a segmental box girder, the height of the compression zone is taken as 100 mm.

2.2 Analysis of stress trajectory

As shown in figure 1, the contour diagrams of the first principal stress and the third principal stress on the segmental box girder were obtained through FEA. A large tensile stress was generated at the edge of the segment after the joint had opened, and the stress at the bottom of the segmental box girder is almost zero. Moreover, it can be seen from figure 1(b) that the compression zone gradually spread downwards from the loaded area to the centre of the segment, and protruded toward the web side of the girder.

![Figure 1](image1.png)

(a)First principal stress (b)Third principal stress

Figure 1. Segmental principal stress clouds

The principal compressive stress vector and the principal tensile stress vector of the segmental box girder are shown in figure 2: the principal compressive stress is mainly concentrated in the roof area and slightly expanded downward; the principal tensile stress mainly appears near the edge of the joint edge and the lower edge of the compression zone.
This analysis shows that the tensile stress at the edge of segment under pure bending is mainly caused by the spalling force when the segment is loaded at four points.

2.3. Topological optimisation

To investigate how the load is transferred, a topological optimisation method and the STM are introduced. The topological optimisation takes the minimum strain energy as the objective function, so the strain energy in the STM is mostly concentrated in the tie bars, and the strain energy is given by:

\[
\Pi = \sum_{i=1}^{n} \frac{T_i^2 L_i}{2E_i A_i}
\]

in which, \(\Pi\) is strain energy in the STM; \(T_i\) is internal force in the \(i^{th}\) tie bar; \(L_i\) is length of the \(i^{th}\) tie bar; \(E_i\) is modulus of elasticity of the \(i^{th}\) tie bar; \(A_i\) is area of the \(i^{th}\) tie bar.

The principle of topological optimisation is such that: under the condition that the constraint is satisfied, units with the lowest efficiency are continuously removed from the continuum, and finally the main load transfer skeleton for the structure is generated. The model is most reasonable when the strain energy of STM is minimised [\(\min(\Pi)\)].

The FEM is subjected to multiple iterations to obtain a topological optimisation model with different optimisation rates. The results are shown in figure 3.

According to the topological optimisation process, when the optimisation rate \(\rho\) is 30%, the bottom area is removed first, which indicates that the concrete strength of the bottom slab was almost zero after the joint had opened; when \(\rho\) is 45%, there was a significant tensile stress zone in the joint edge and the middle of the web, which is consistent with the stress vectors.

3. Reinforcement design for the joint edge of the box girder

3.1. STM for reinforcement design of a joint edge under pure bending

The STM is a useful tool when determining the optimal reinforcement layout required in D-regions [16-18]. Combining the stress contour diagram, stress vector diagram, and topological optimisation analysis results, the tie bar is arranged in the tensile zone of the principle stress vector diagram of the segment, and the compressive strut is arranged in the compressive zone.

In figure 4: \(I, J, K, L, M, N\) are number of nodes per element; \(L\) is the length of the calculated segment; \(h_w\) is the height of the box girder web; \(h_f\) is the cantilever root height of box girder upper
flange; $\theta$ is the angle between the 1\# compressive strut and the horizontal; $\alpha$ is the angle between the 4\# compressive strut and the horizontal.

3.2. Verification of STM for reinforcement design at a joint edge

J Turmo et al. also conducted a finite element simulation of an actual girder. Figure 5 illustrates the principal tensile stress vector of the whole girder after the joint was opened under bending [12]. If the tie bar is arranged in the principal tensile stress zone, and the compression strut is arranged in the principal compressive stress zone, the STM is as shown in figure 5. The FEA results presented here are consistent with the principle tensile stress vectors in a segmental girder under bending after the joint had opened (J Turmo et al.), so the proposed STM matched J Turmo’s FEA results, which further verified the applicability of the proposed STM.

3.3. Calculation of reinforcement layout at the joint edge of a box girder

Combined with figure 4, the calculation steps to determine the requirement for reinforcement area at the joint edge and the web of a box girder using the proposed STM are as follows:

- Step 1: determine the direction of the 1\# compressive strut.
  
  Take node I for internal force equilibrium analysis, where
  
  $$ F = F_1 \cos(\theta) $$
  $$ T_2 = F_1 \sin(\theta) $$
  
  in which, $F_1$ is the internal force on the 1\# compressive strut; $T_2$ is the internal force on the 2\# tie bar; $F$ is the resultant force on the concrete compressive zone in the upper part of the segmental box girder, which can be obtained from the internal force equilibrium condition of the segment [8], and can be expressed as:
  
  $$ A_{p,e} f_{p,e} + A_{p,i} f_{p,i} = A_{p}' \left( f_{pd}' - \sigma_{p0}' \right) + F $$
  
  in which, $A_{p,e}$ is area of the external prestressed tendon in the tensile zone; $f_{p,e}$ is design yield strength of the external prestressed tendon in the tensile zone; $A_{p,i}$ is area of the internal prestressed tendon in the tensile zone; $f_{p,i}$ is design yield strength of the internal prestressed tendon in the tensile zone; $A_{p}'$ is prestressed tendon area in the compressive zone; $f_{pd}'$ is design yield strength of the prestressed tendon in the compressive zone; $p_{0}'$ is stress of the prestressed tendon when the concrete in the compressive zone is destressed.

  $T_2$ can be obtained from the tensile stress integral, as given by:
  
  $$ T_2 = \int_0^r \sigma \, \text{d}x $$
  
  in which, $r$ is width of the web.

  Substituting equation (6) into equation (4), the angles of the 1\# compressive strut in the three experimental girders are calculated to be 2.233°, 2.051°, and 2.019° respectively, and the average value is 2.097°. Therefore, the angles in the STM are recommended to be set to 2°.

- Step 2: determine the internal force of the 1\# compressive strut and the 2\# tie bar.
Establish a balance equation at the “I” joint of the segmental box girder, to determine $F$, and substitute $\theta = 2^\circ$ into equation (4) to obtain the internal force in 1st compressive strut and 2nd tie bar, which can be expressed as:

$$
\begin{align*}
F_1 &= F \cos(\theta) \\
T_2 &= F \sin(\theta) / \cos(\theta)
\end{align*}
$$

\quad \text{(7)}

- **Step 3:** determine the reinforcement at the joint edge.

The required vertical reinforcement area $A_{sv}$ at the edge of the joint of the box girder is given by:

$$
A_{sv} \geq T_2 / (\Phi f_y)
$$

\quad \text{(8)}

in which, $\Phi$ is the strength reduction factor of the steel bar (the specification \[19\] recommends a value of 0.75); $f_y$ is the yield strength of the steel stirrup bar.

- **Step 4:** determine the location of node $M$.

It can be seen from figure 6 that the principal tensile stress of the web mainly appears in the central part of the web. Therefore, for convenience of calculation, the position of node $M$ is set to the mid-height $h_w$, that is $h_w / 2$.

- **Step 5:** determine the location of node $L$.

The location of node $L$ must be determined based on the results of topological optimisation. The area of the $i$th tie bar ($A_i$) is:

$$
A_i \geq T_i / (\Phi f_y)
$$

\quad \text{(9)}

in which, $T_i$ is the internal force in the $i$th tie bar.

Then substitute equation (9) into equation (3) to obtain

$$
\begin{align*}
\Pi &= \sum_{i=1}^{n} \frac{T_i^2 l_i}{2E A_i} = \frac{\Phi f_y}{2E} \sum_t L_t
\end{align*}
$$

\quad \text{(10)}

Let $\eta = \tan(\alpha)$. To minimise equation (10), it must satisfy

$$
\frac{\partial \Pi}{\partial \eta} = 0
$$

\quad \text{(11)}

Solving equation (11) gives

$$
\eta = \tan(\alpha) = \frac{h_w + h_l - L_t \tan(2^\circ)}{2L_t}
$$

\quad \text{(12)}

- **Step 6:** determine the horizontal reinforcement in the web of the box girder.

The horizontal reinforcement in the web of the segmental box girder can be determined by the horizontal projection of the 4th tie bar, which can be expressed as

$$
A_{sh} \geq T_1 \cos(\alpha) / (\Phi f_y) = \mu \Pi / (\Phi f_y)
$$

\quad \text{(13)}

$$
\mu = \cos(\alpha) \left[ \cos(\alpha) \left( \frac{h_l + h_w}{L_t} - \tan(\alpha) \tan(2^\circ) \right) + \sin(\alpha) \right]
$$

\quad \text{(14)}

in which, $A_{sh}$ is area of the horizontal reinforcement in the web of the box girder; $\mu$ is coefficient.

4. **Analysis of a typical girder segment**

Taking the segmental box girder adopted for the Fourth Nanjing Yangtze River Bridge as an example, the joint edge of the standard segment was analysed, and the STM is shown in figure 6. The length of each standard segment is 4 m, and there are six external prestressed tendons and six internal tendons, the ultimate strength $f_{uk}$ is 1860 MPa. The number of the external tendons is 19Φ15.2, the number of internal tendons is 15Φ15.2, and the resultant force $F$, as calculated by equation (5), is $6.64 \times 10^3$ kN.
Figure 6. STM for standard segments of the Fourth Nanjing Yangtze River Bridge

Figure 7. Size and reinforcement layout in a standard segment

• Step 1: take the inclination angle $\theta$ of the compressive strut $1^a$ as $2^\circ$.
• Step 2: calculate the internal force in the tie bar according to equation (7).

$$T_i = F \sin(\theta) / \cos(\theta) = 6.64 \times 10^3 \times \sin(2^\circ) / \cos(2^\circ) = 231.87 kN$$

• Step 3: check the area of reinforcement at the joint edge of the segment.

$$A_w \geq T_i / (\Phi f_y) = 231.87 \times 10^3 / (0.75 \times 330) = 936.85 mm^2$$

Figure 7(a) shows that the reinforcement of the joint edge section involves four bars with a diameter of $16 mm$ ($844 mm^2$) and a yield strength of $335 MPa$, which does not meet load-bearing requirements. It is recommended to add another two bars with a diameter of $10 mm$ ($961 mm^2$) to meet load-bearing requirements, as shown in figure 7(b). However, if considering the length of diagonal cracks near the joint edge of a segment is close to $h_w/5$ [5-8], the total reinforcement at a distance of $h_w/5$ from the joint edge ($h_w/5 = 470 mm$) section is twelve reinforcing bars, each with a diameter of $16 mm$ ($2413 mm^2$), and a yield strength of $335 MPa$, which meets load-bearing requirements (i.e., no extra reinforcement needed).

• Step 4: Take the intersection of the centre of the segment and mid-height of the web as the position of node $M$.

• Step 5: Substitute $h_w = 2350 mm$, $h_v = 550 mm$, $L_i = 4000 mm$ into equation (12) to obtain:

$$\eta = \tan(\alpha) = \frac{234 + 55 - 400 \times \tan(2^\circ)}{2 \times 400} = 0.35$$

$$\alpha = \arctan(0.35) = 19.29^\circ$$

Therefore, the distance between nodes $L$ and $I$ along the edge of the segment, $d_{LI}$ is

$$d_{LI} = \frac{2350 + 550}{2} - 2000 \times \tan(\alpha) = 750 mm$$

• Step 6: The distance between nodes $L$ and $M$ along the vertical direction, $d_{LM}$ is

$$d_{LM} = 2000 \times \tan(\alpha) = 700 mm$$

Therefore, it is necessary to arrange horizontal reinforcement in the mid-height of the web within $700 mm$, and

$$\mu = \cos(19.29^\circ) \left[ \cos(19.29^\circ) \left( \frac{55 + 235}{400} - \tan(19.29^\circ) - \tan(2^\circ) \right) + \sin(19.29^\circ) \right]^{-1} = 1.45$$

$$A_{sh} \geq 1.45 \times 936.85 = 1357.6 mm^2$$

In figure 7(b), the horizontal reinforcement in the mid-height of the web of a standard segment is ten bars, each with a diameter of $14 mm$ ($1540 mm^2$), which meets load-bearing requirements (i.e., no extra reinforcement is needed).
5. Conclusion
In this study, based on the test results and the topological optimisation analysis, a STM was proposed to determine the reinforcement at the edge of the joint in a segmental box girder. The following conclusions can be drawn from this study:

(1) A STM was proposed for calculating the reinforcement requirement at the edge of an epoxy joint and at the mid-height of the web of the segmental box girder.

(2) A step by step guide for using the aforementioned STM is provided. The related equations and assumptions are presented. The feasibility of the method was verified by use of FEA results.

(3) Using the STM established above, a case study was used to illustrate how to use the proposed method. The vertical reinforcement at the edge of epoxy joints, and the horizontal reinforcement at the mid-height of the web, of the Fourth Nanjing Yangtze River Bridge standard segment were checked. Some recommendations are also provided.

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