Characterizations of Mapping via $N_{nc}$ Z-open Sets

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Abstract. The aim of this paper we introduce $N_{nc}$Z-irresolute, $N_{nc}$Z-open, $N_{nc}$Z-closed, pre $N_{nc}$Z-open and pre $N_{nc}$Z-closed mappings and investigate properties and characterizations of these new types of mappings.

Keywords and phrases: $N_{nc}$Z-irresolute mapping, $N_{nc}$Z-open, $N_{nc}$Z-closed mapping, Pre $N_{nc}$Z-open, Pre $N_{nc}$Z-closed mappings.

1. Introduction

Smarandache's neutrosophic framework have wide scope of constant applications for the fields of Computer Science, Information Systems, Applied Mathematics, Artificial Intelligence, Mechanics, dynamic, Medicine, Electrical & Electronic, and Management Science and so forth [1, 2, 3, 4, 20, 21]. Topology is an classical subject, as a generalization topological spaces numerous kinds of topological spaces presented throughout the year. Smarandache [13] characterized the Neutrosophic set on three segment Neutrosophic sets (T-Truth, F-Falsehood, I-Indeterminacy). Neutrosophic topological spaces (nts’s) presented by Salama and Alblowi [10]. Lellies Thivagar et.al. [8] was given the geometric existence of $N$ topology, which is a non-empty set equipped with $N$ arbitrary topologies. Lellis Thivagar et al. [9] introduced the notion of $N_{nc}$-open (closed) sets in $N$ neutrosophic crisp topological spaces. Al-Hamido et al. [5] investigate the chance of extending the idea of neutrosophic crisp topological spaces into $N$-neutrosophic crisp topological spaces and examine a portion of their essential properties. In 2008, Ekici [6] introduced the notion of $e$-open sets in topology. In 2020, [14, 17] introduced $N$-neutrosophic $\delta$-open, $N$-neutrosophic $\delta$-semiopen, $N$-neutrosophic $\delta$-preopen and $N$-neutrosophic $Z$-open sets are introduced. We continue to explore further properties and characterizations of $N_{nc}$Z-irresolute and $N_{nc}$Z-open mappings. We also introduce and study properties and characterizations of $N_{nc}$Z-closed, pre $N_{nc}$Z-open and pre $N_{nc}$Z-closed mappings.

2. Preliminaries

Salama and Smarandache [12] presented the idea of a neutrosophic crisp set in a set $P$ and defined the inclusion between two neutrosophic crisp sets, the intersection (union) of two neutrosophic...
crisp sets, the complement of a neutrosophic crisp set, neutrosophic crisp empty (resp., whole) set as more then two types. And they studied some properties related to neutrosophic crisp set operations. However, by selecting only one type, we define the inclusion, the intersection (union), and neutrosophic crisp empty (resp., whole) set again and discover a few properties.

**Definition 2.1** Let $P$ be a non-empty set. Then $H$ is called a neutrosophic crisp set (in short, *ncs*) in $P$ if $H$ has the form $H = (H_1, H_2, H_3)$, where $H_1, H_2,$ and $H_3$ are subsets of $P$.

The neutrosophic crisp empty (resp., whole) set, denoted by $\phi_n$ (resp., $P_n$) is an *ncs* in $P$ defined by $\phi_n = (\phi, \phi, P)$ (resp. $P_n = (P, P, \phi)$). We will denote the set of all *ncs*’s in $P$ as $ncS(P)$. In particular, Salama and Smarandache [11] classified a neutrosophic crisp set as the followings.

A neutrosophic crisp set $H = (H_1, H_2, H_3)$ in $P$ is called a neutrosophic crisp set of Type 1 (resp. 2 & 3) (in short, *ncs*-Type 1 (resp. 2 & 3)) , if it satisfies $H_1 \cap H_2 = H_2 \cap H_3 = H_3 \cap H_1 = \phi$ (resp. $H_1 \cap H_2 = H_2 \cap H_3 = H_3 \cap H_1 = \phi$ and $H_1 \cup H_2 \cup H_3 = P$ & $H_1 \cap H_2 \cap H_3 = \phi$ and $H_1 \cup H_2 \cup H_3 = P$).

$ncS_1(P)$ ($ncS_2(P)$ and $ncS_3(P)$) means set of all *ncs* Type 1 (resp. 2 and 3).

**Definition 2.2** Let $H = (H_1, H_2, H_3), M = (M_1, M_2, M_3) \in ncS(P)$. Then $H$ is said to be contained in (resp. equal to) $M$, denoted by $H \subseteq M$ (resp. $H = M$), if $H_1 \subseteq M_1, H_2 \subseteq M_2$ and $H_3 \subseteq M_3$ (resp. $H \subseteq M$ and $M \subseteq H$), $H^c = (H_3, H_2^c, H_1), H \cap M = (H_1 \cap M_1, H_2 \cap M_2, H_3 \cap M_3), H \cup M = (H_1 \cup M_1, H_2 \cup M_2, H_3 \cap M_3)$. Let $(H_j)_{j \in J} \subseteq ncS(P)$, where $H_j = (H_{j_1}, H_{j_2}, H_{j_3})$. Then $\bigcap_{j \in J} H_j$ (simply $\bigcap H_j = (\bigcap H_{j_1}, \bigcap H_{j_2}, \bigcap H_{j_3})$; $\bigcup_{j \in J} H_j$ (simply $\bigcup H_j = (\bigcup H_{j_1}, \bigcup H_{j_2}, \bigcap H_{j_3})$).

The following are the quick consequence of Definition 2.2.

**Proposition 2.1** [7] Let $L, M, O \in ncS(P)$. Then

(i) $\phi_n \subseteq L \subseteq P_n$,
(ii) if $L \subseteq M$ and $M \subseteq O$, then $L \subseteq O$,
(iii) $L \cap M \subseteq L$ and $L \cap M \subseteq M$,
(iv) $L \subseteq L \cup M$ and $M \subseteq L \cup M$,
(v) $L \subseteq M$ iff $L \cap M = L$,
(vi) $L \subseteq M$ iff $L \cup M = M$.

Likewise the following are the quick consequence of Definition 2.2.

**Proposition 2.2** [7] Let $L, M, O \in ncS(P)$. Then

(i) $L \cup L = L$, $L \cap L = L$ (Idempotent laws),
(ii) $L \cup M = M \cup L$, $L \cap M = M \cap L$ (Commutative laws),
(iii) (HSSociative laws) : $L \cup (M \cup O) = (L \cup M) \cup O$, $L \cap (M \cap O) = (L \cap M) \cap O$,
(iv) (Distributive laws) : $L \cup (M \cap O) = (L \cup M) \cap (L \cup O)$, $L \cap (M \cup O) = (L \cap M) \cup (L \cap O)$,
(v) (Hbsorption laws) : $L \cup (L \cap M) = L$, $L \cap (L \cup M) = L$,
(vi) (DeMorgan’s laws) : $(L \cup M)^c = L^c \cap M^c$, $(L \cap M)^c = L^c \cup M^c$,
(vii) $(L^c)^c = L$,
(viii) (a) $L \cup \phi_n = L, L \cap \phi_n = \phi_n$,
(b) $L \cup P_n = P_n$, $L \cap P_n = L$,
(c) $P_n^c = \phi, \phi_n^c = P_n$,
(d) in general, $L \cup L^c \neq P_n$, $L \cap L^c \neq \phi_n$.

**Proposition 2.3** [7] Let $L \in ncS(P)$ and let $(L_j)_{j \in J} \subseteq ncS(P)$. Then
(i) \(( \bigcap L_j )^c = \bigcup L_j^c\), \(( \bigcup L_j )^c = \bigcap L_j^c\).
(ii) \( L \cap ( \bigcup L_j ) = (L \cap L_j) \), \( L \cup ( \bigcap L_j ) = (L \cup L_j)\).

**Definition 2.3** [11] A neutrosophic crisp topology (briefly, *nc-sets*) on a non-empty set \( P \) is a family \( \tau \) of *nc* subsets of \( P \) satisfying the following axioms

(i) \( \phi_n, P_n \in \tau \).
(ii) \( H_1 \cap H_2 \in \tau \) \( \forall H_1 \& H_2 \in \tau \).
(iii) \( \bigcup_{a \in P} H_a \in \tau \), for any \( \{H_a : a \in J\} \subseteq \tau \).

Then \((P, \tau)\) is a neutrosophic crisp topological space (briefly, *ncsts*) in \( P \). The \( \tau \) elements are called neutrosophic crisp open sets (briefly, *ncos*) in \( P \). A *ncs* \( C \) is closed set (briefly, *nccls*) iff its complement \( C^c \) is *ncos*.

**Definition 2.4** [5] Let \( P \) be a non-empty set. Then \( ncs\tau_1, ncs\tau_2, \ldots, ncs\tau_N \) are *N*-arbitrary crisp topologies defined on \( P \) and the collection \( N_{nc\tau} = \{H \subseteq P : H = (\bigcup_{j=1}^N H_j) \cup (\bigcap_{j=1}^N L_j)\} \), \( H_j, L_j \in ncs\tau_j \) is called *nc-tops* on \( P \) if the axioms are satisfied:

(i) \( \phi_n, P_n \in N_{nc\tau} \).
(ii) \( \bigcup_{j=1}^\infty H_j \in N_{nc\tau} \forall \{H_j\}_j^\infty \in N_{nc\tau} \).
(iii) \( \bigcap_{j=1}^n H_j \in N_{nc\tau} \forall \{H_j\}_j^n \in N_{nc\tau} \).

Then \((P, N_{nc\tau})\) is called a *nc* topological space (briefly, *ncsts*) on \( P \). The \( N_{nc\tau} \) elements are called *nc-open sets* (*ncos*) on \( P \) and its complement is called *nc-*closed sets (*nccls*) on \( P \). The elements of \( P \) are known as *nc-sets* (*ncs*) on \( P \).

**Definition 2.5** [5] Let \((P, N_{nc\tau})\) be *ncsts* on \( P \) and \( H \) be an *ncs* on \( P \), then the *nc* interior of \( H \) (briefly, \( N_{nc}inter(H) \)) and *nc* closure of \( H \) (briefly, \( N_{nc}cl(H) \)) are defined as

(i) \( N_{nc\tau}inter(H) = \bigcup\{C : H \subseteq C \& C \in N_{nc\tau}\} \).
(ii) \( N_{nc\tau}cl(H) = \cap\{C : H \subseteq C \& C \in N_{nc\tau}\} \).

The complement of an *ncos* (resp. *ncSos*, *ncPos*, *ncacos*, *ncbos* & *ncgos*) is called an *nc-regular* (resp. *ncsemi*, *ncpre*, *ncalpha*, *ncbeta* & *ncgamma*) closed set (briefly, *nccls* (resp. *ncScs*, *ncPcs*, *ncaccs*, *ncbcs* & *ncgos*)) in \( P \).

The family of all *ncos* (resp. *ncSos*, *ncPos*, *ncPcs*, *ncSos*, *ncScs*, *ncacos*, *ncbos*, *ncbcs*, *ncgos*) of \( P \) is denoted by \( N_{nc\text{ROS}}(P) \) (resp. \( N_{nc\text{RCS}}(P) \), \( N_{nc\text{POS}}(P) \), \( N_{nc\text{PCS}}(P) \), \( N_{nc\text{SOS}}(P) \), \( N_{nc\text{SCS}}(P) \), \( N_{nc\text{OOS}}(P) \), \( N_{nc\text{CAS}}(P) \), \( N_{nc\text{BOS}}(P) \), \( N_{nc\text{BSCS}}(P) \), \( N_{nc\text{OGOS}}(P) \& N_{nc\text{CS}}(P) \)).

**Definition 2.6** [17] A set \( H \) is said to be a

(i) *ncdelta* interior of \( H \) (briefly, \( N_{nc\delta}(H) \)) is defined by \( N_{nc\delta}(H) = \bigcup\{H : H \subseteq H \& H \text{ is a } N_{nc\tau}\}.\)
(ii) \(N_{nc}\delta\) closure of \(H\) (briefly, \(N_{nc}\delta\text{cl}(H)\)) is defined by \(N_{nc}\delta\text{cl}(H) = \bigcup\{x \in P : N_{nc}\text{int}(N_{nc}\text{cl}(L)) \cap H \neq \emptyset, \ x \in L \land L \text{ is a } N_{nc}\text{os}\}.

Definition 2.7 [17] A set \(H\) is said to be a

(i) \(N_{nc}\delta\)-open set (briefly, \(N_{nc}\delta\text{o}\)) if \(H = N_{nc}\delta\text{int}(H)\).
(ii) \(N_{nc}\delta\)-pre open set (briefly, \(N_{nc}\delta\text{P}o\)) if \(H \subseteq N_{nc}\text{int}(N_{nc}\delta\text{cl}(H))\).
(iii) \(N_{nc}\delta\)-semi open set (briefly, \(N_{nc}\delta\text{Sos}\)) if \(H \subseteq N_{nc}\text{cl}(N_{nc}\delta\text{int}(H))\).

The complement of an \(N_{nc}\delta\text{o}\) set (resp. \(N_{nc}\delta\text{P}o\) set) is called an \(N_{nc}\delta\) set (resp. \(N_{nc}\delta\)-pre & \(N_{nc}\delta\)-semi closed set (briefly, \(N_{nc}\delta\text{cs}\) set)).

Definition 2.8 [18] Let \(H\) be an \(N_{nc}\text{o}\) set on a \(N_{nc}\text{ts}\) \(P\). Then \(H\) is said to be a

(i) \(N_{nc}\text{e}-\text{open}\) set (briefly, \(N_{nc}\text{e}o\)) if \(H \subseteq N_{nc}\text{cl}(N_{nc}\text{e}\text{nt}(H)) \cup N_{nc}\text{int}(N_{nc}\delta\text{cl}(H))\).
(ii) \(N_{nc}\text{e}-\text{closed}\) set (briefly, \(N_{nc}\text{e}c\)) if \(N_{nc}\text{cl}(N_{nc}\text{e}\text{nt}(H)) \cap N_{nc}\text{int}(N_{nc}\delta\text{cl}(H)) \subseteq H\).

The complement of an \(N_{nc}\text{e}o\) set is called an \(N_{nc}\text{e}\) closed set (briefly, \(N_{nc}\text{e}c\)) in \(P\). The family of all \(N_{nc}\text{e}o\) set of \(P\) is denoted by \(N_{nc}\text{e}\text{OS}(P)\) (resp. \(N_{nc}\text{e}\text{CS}(P)\)).

Definition 2.9 [19] A function \(h : (P, N_{nc}\tau) \rightarrow (Q, N_{nc}\sigma)\) is called \(N_{nc}\) precontinuous (resp. \(N_{nc}\delta\)-semicontinuous, \(N_{nc}\gamma\)-continuous, \(N_{nc}\text{e}-\text{continuous}\) ) (briefly, \(N_{nc}\text{P}Cts\) (resp. \(N_{nc}\delta\text{CS}Cts, N_{nc}\gamma\text{Cts} & N_{nc}\text{e}Cts\))) if \(h^{-1}(V)\) is \(N_{nc}\text{Po}\) (resp. \(N_{nc}\delta\text{s}o, N_{nc}\gamma\text{O} & N_{nc}\text{e}o\) ) for each \(V \in \sigma\).

Definition 2.10 A mapping \(h : (P, N_{nc}\tau) \rightarrow (Q, N_{nc}\sigma)\) is called \(N_{nc}\text{Z}\)-continuous (briefly, \(N_{nc}\text{Z}Cts\)) if the inverse image of each \(N_{nc}\text{o}\) set of \((Q, N_{nc}\sigma)\) is \(N_{nc}\text{Zo}\) in \((P, N_{nc}\tau)\).

Definition 2.11 [14] Let \((P, N_{nc}\tau)\) be a \(N_{nc}\text{ts}\). Let \(H\) be an \(N_{nc}\text{o}\) in \((P, N_{nc}\tau)\). Then \(H\) is said to be a

(i) \(N_{nc}\text{Z}-\text{open}\) set (briefly, \(N_{nc}\text{Z}o\)) if \(H \subseteq N_{nc}\text{cl}(N_{nc}\text{Z}\text{nt}(H)) \cup N_{nc}\text{int}(N_{nc}\text{cl}(H))\).
(ii) \(N_{nc}\text{Z}-\text{closed}\) set (briefly, \(N_{nc}\text{Z}c\)) if \(N_{nc}\text{cl}(N_{nc}\text{Z}\text{nt}(H)) \cap N_{nc}\text{cl}(N_{nc}\text{int}(H)) \subseteq H\).

The family of all \(N_{nc}\text{Zo}\) set of \(P\) containing \(H\) is called the \(N_{nc}\text{Z}\)-closure of \(H\) and is denoted by \(N_{nc}\text{Zcl}(H)\).

Definition 2.12 [14] Let \((P, N_{nc}\tau)\) be a \(N_{nc}\text{ts}\). Then:

(i) The union of all \(N_{nc}\text{Zo}\) sets of \(P\) contained in \(H\) is called the \(N_{nc}\text{Z}\) interior of \(H\) and is denoted by \(N_{nc}\text{Z}\text{nt}(H)\).
(ii) The intersection of all \(N_{nc}\text{Zc}\) sets of \(P\) containing \(H\) is called the \(N_{nc}\text{Z}\)-closure of \(H\) and is denoted by \(N_{nc}\text{Zcl}(H)\).

Definition 2.13 [14] Let \((P, N_{nc}\tau)\) be a \(N_{nc}\text{ts}\) and \(H \subseteq P\). Then the \(N_{nc}\text{Z}\)-boundary of \(H\) (briefly, \(N_{nc}\text{Z}b(H)\)) is defined by \(N_{nc}\text{Z}b(H) = N_{nc}\text{Zcl}(H) \cap N_{nc}\text{Zcl}(P \setminus H)\).

Definition 2.14 [14] Let \(H\) is a \(N_{nc}\) set of a \(N_{nc}\text{ts} \ (P, N_{nc}\tau)\). Then a point \(p \in P\) is called a \(N_{nc}\) \(N_{nc}\text{Z}\)-limit point of a set \(H \subseteq P\) if every \(N_{nc}\text{Zo} \ (G \subseteq P\) containing \(p\) contains a point of \(H\) other than \(p\). The set of all \(N_{nc}\text{Z}\)-limit points of \(H\) is called a \(N_{nc}\text{Z}\) derived set of \(H\) and is denoted by \(N_{nc}\text{Zd}(H)\).
3. $N_{nc}Z$-irresolve mapping

**Definition 3.1** A mapping $h : (P, N_{nc}τ) \rightarrow (Q, N_{nc}σ)$ is called $N_{nc}Z$-irresolve (briefly, $N_{nc}ZIrr$) if

$$h^{-1}(O) \in N_{nc}ZOS(P)$$

for each $O \in N_{nc}ZOS(P)$.

**Theorem 3.1** Let $h : (P, N_{nc}τ) \rightarrow (Q, N_{nc}σ)$ be a mapping, then the followings are equivalent:

(i) $h$ is $N_{nc}ZIrr$,

(ii) The inverse image of each $N_{nc}Zc$ in $(Q, N_{nc}σ)$ is $N_{nc}Zc$ in $(P, N_{nc}τ)$,

(iii) $N_{nc}Zcl(h^{-1}(M)) \subseteq h^{-1}(N_{nc}Zcl(M)) \subseteq h^{-1}(N_{nc}cl(M))$, for each $M \subseteq Q$,

(iv) $h(N_{nc}Zcl(h(H)) \subseteq N_{nc}Zcl(h(H)) \subseteq N_{nc}cl(h(H))$, for each $H \subseteq P$,

(v) $h^{-1}(N_{nc}Zint(M)) \subseteq N_{nc}Zint(h^{-1}(M))$, for each $M \subseteq Q$,

(vi) $N_{nc}ZBd(h^{-1}(M)) \subseteq h^{-1}(N_{nc}ZBd(M))$, for each $M \subseteq Q$,

(vii) $N_{nc}Zb(h^{-1}(M)) \subseteq h^{-1}(N_{nc}Zb(M))$, for each $M \subseteq Q$,

(viii) $h(N_{nc}Zb(h(H)) \subseteq N_{nc}Zb(h(H))$, for each $H \subseteq P$,

(ix) $h(N_{nc}Zd(h(H))) \subseteq N_{nc}Zcl(h(H))$, for each $H \subseteq P$.

**Proof.**

(i) $\Rightarrow$ (ii). Obvious.

(ii) $\Rightarrow$ (iii). Let $M \subseteq Q$ and $M \subseteq N_{nc}Zcl(M) \subseteq N_{nc}cl(M)$. Then by (ii) $N_{nc}Zcl(h^{-1}(M)) \subseteq N_{nc}Zcl(h^{-1}(N_{nc}Zcl(M))) = h^{-1}(N_{nc}Zcl(M)) \subseteq h^{-1}(N_{nc}cl(M))$.

(iii) $\Rightarrow$ (iv). Immediately by replacing $M$ by $h(H)$ in (iii).

(iv) $\Rightarrow$ (i). Let $W \in N_{nc}ZO(Q)$ and $F = Q \setminus W \in N_{nc}ZC(Q)$. Then by (iv).

$$h(N_{nc}Zcl(h^{-1}(F))) \subseteq N_{nc}Zcl(h(h^{-1}(F))) \subseteq N_{nc}Zcl(F) = F.$$

So $N_{nc}Zcl(h^{-1}(F)) \subseteq h^{-1}(F)$ and hence, $h^{-1}(F) = P \setminus h^{-1}(W) \in N_{nc}ZCS(P)$, thus $h^{-1}(W) \in N_{nc}ZOS(P)$. Therefore $h$ is $N_{nc}ZIrr$.

(i) $\Rightarrow$ (v). Let $M \subseteq Q$. Then $N_{nc}Zint(M)$ is $N_{nc}Zo$ in $Q$. By (i), $h^{-1}(N_{nc}Zint(M))$ is $N_{nc}Zo$ in $P$. Hence $h^{-1}(N_{nc}Zint(M)) = N_{nc}Zint(h^{-1}(N_{nc}Zint(M))) \subseteq N_{nc}Zint(h^{-1}(M))$.

(v) $\Rightarrow$ (vi). Let $M \subseteq Q$. Then by (v), $h^{-1}(N_{nc}Zint(M)) \subseteq N_{nc}Zint(h^{-1}(M))$ we have $h^{-1}(M) \setminus N_{nc}Zint(h^{-1}(M)) \subseteq h^{-1}(M) \setminus h^{-1}(N_{nc}Zint(M))$. Therefore, $N_{nc}ZBd(h^{-1}(M)) \subseteq h^{-1}(N_{nc}ZBd(M))$.

(vi) $\Rightarrow$ (v). Let $M \subseteq Q$. Then by (v), $N_{nc}ZBd(h^{-1}(M)) = h^{-1}(M) \setminus N_{nc}Zint(h^{-1}(M)) \subseteq h^{-1}(N_{nc}ZBd(M)) = h^{-1}(M)\setminus N_{nc}Zint(h^{-1}(M)) = h^{-1}(M)\setminus N_{nc}Zint(M)$.

(v) $\Rightarrow$ (i). Let $M \subseteq Q$ be $N_{nc}Zo$. Then $M = N_{nc}Zint(M)$. Hence by (v) we have $h^{-1}(M) = h^{-1}(N_{nc}Zint(M)) \subseteq N_{nc}Zint(h^{-1}(M))$. Thus $h^{-1}(M)$ is $N_{nc}Zo$ in $P$. So, $h$ is $N_{nc}ZIrr$.

(i) $\Rightarrow$ (vii). Let $M \subseteq Q$, by (iii), we have $N_{nc}Zb(h^{-1}(M)) = N_{nc}Zcl(h^{-1}(M))\setminus N_{nc}Zint(h^{-1}(M)) \subseteq h^{-1}(N_{nc}Zcl(h^{-1}(M))) \setminus h^{-1}(N_{nc}Zb(M) \cup N_{nc}Zint(M))$.

(vii) $\Rightarrow$ (i). Let $M \in N_{nc}ZCS(Q)$ and $N_{nc}Zb(h^{-1}(M)) \subseteq h^{-1}(N_{nc}Zb(M))$. Then, $N_{nc}Zb(h^{-1}(M)) \subseteq h^{-1}(N_{nc}Zcl(M)) \setminus N_{nc}cl(M)$. Hence $h^{-1}(M) = h^{-1}(M)\setminus N_{nc}Zint(M) = h^{-1}(N_{nc}ZBd(M)) \subseteq h^{-1}(M)$ by Theorem 4.2 [14], we have, $h^{-1}(M) \in N_{nc}ZCS(P)$. Therefore $h$ is $N_{nc}ZIrr$.

(vii) $\Rightarrow$ (viii). Follows by replacing $h(H)$ instead of $M$ in (vii).
(vii) ⇒ (viii). Let $M \subseteq Q$, by (viii), we have $h(N_{nc}Zb(h^{-1}(M))) \subseteq N_{nc}Zb(h^{-1}(M))) \subseteq N_{nc}Zb(M)$ and therefore $N_{nc}Zb(h^{-1}(M)) \subseteq h^{-1}(N_{nc}Zb(M))$.

(i) ⇒ (ix). Let $H \subseteq P$. Then by (iv), $h(N_{nc}Zd(H)) \subseteq h(N_{nc}Zcl(H)) \subseteq N_{nc}Zcl(h(H))$.

(ix) ⇒ (i). Let $F$ be a $N_{nc}Zc$ set in $Q$, by (vii), $h(N_{nc}Zd(h^{-1}(F))) \subseteq N_{nc}Zcl(h(h^{-1}(F))) \subseteq N_{nc}Zcl(F) = F$, then $N_{nc}Zd(h^{-1}(F)) \subseteq h^{-1}(F)$ by Theorem 3.4 [14], we have $h^{-1}(F) \in N_{nc}ZCS(P)$. Therefore $h$ is $N_{nc}ZIrr$.

**Theorem 3.2** A mapping $h : (P, N_{nc}) \to (Q, N_{nc})$ is $N_{nc}ZIrr$ iff for each $x$ in $P$, the inverse image of every $N_{nc}Znbd$ of $h(x)$ is a $N_{nc}Znbd$ of $x$.

**Proof.** Necessity. Let $x \in P$ and let $M$ be $N_{nc}Znbd$ of $h(x)$. Then there exists $O \in N_{nc}ZOS(Q)$ such that $h(x) \in O \subseteq M$. This implies that $x \in h^{-1}(O) \subseteq h^{-1}(M)$. Since $h$ is $N_{nc}ZIrr$, so $h^{-1}(O) \in N_{nc}ZOS(P)$. Hence $h^{-1}(M)$ is a $N_{nc}Znbd$ of $x$.

Sufficiency. Let $M \in N_{nc}ZOS(Q)$. Put $H = h^{-1}(M)$. Let $x \in H$. Then $h(x) \in M$. But $M$ being $N_{nc}Zo$ set is a $N_{nc}Znbd$ of $h(x)$. So by hypothesis, $H = h^{-1}(M)$ is a $N_{nc}Znbd$ of $x$. Hence there exists $H_x \in N_{nc}ZOS(P)$ such that $x \in H_x \subseteq H$. Thus $H = \bigcup\{H_x : x \in H\}$. Therefore $h$ is $N_{nc}ZIrr$.

**Theorem 3.3** A function $h : (P, N_{nc}) \to (Q, N_{nc})$ is $N_{nc}ZIrr$ iff $h(N_{nc}Zd(H)) \subseteq h(H) \cup N_{nc}Zd(h(H))$, for each $H \subseteq P$.

**Proof.** Necessity. Let $h : P \to Q$ be $N_{nc}ZIrr$. Let $H \subseteq P$ and $a_0 \in N_{nc}Zd(H)$. Assume that $h(a_0) \notin h(H)$ and let $V$ denote a $N_{nc}Znbd$ of $h(a_0)$. Since $h$ is $N_{nc}ZIrr$, so by Theorem 3.2, there exists a $N_{nc}Znbd$ $O$ of $a_0$ such that $h(O) \subseteq V$. From $a_0 \in N_{nc}Zd(H)$, it follows that $O \cap H \neq \phi$, therefore, at least one element $a \in O \cap H$ such that $h(a) \in h(H)$ and $h(a) \in V$. Since $h(a_0) \notin h(H)$, we have $h(a) \neq h(a_0)$. Thus every $N_{nc}Znbd$ of $h(a_0)$ contains an element of $h(H)$ different from $h(a_0)$, consequently, $h(a_0) \in N_{nc}Zd(h(H))$. This proves necessity of the condition.

Sufficiency. Assume that $h$ is not $N_{nc}ZIrr$. Then by Theorem 3.2, there exists $a_0 \in P$ and a $N_{nc}Znbd$ $V$ of $h(a_0)$ such that every $N_{nc}Znbd$ $O$ of $a_0$ contains at least one element $a \in O$ for which $h(a) \notin V$. Put $H = \{a \in P : h(a) \notin V\}$. Then $a_0 \notin H$ since $h(a_0) \in V$, and therefore $h(a_0) \notin h(H)$; also $h(a_0) \notin N_{nc}Zd(h(H))$, since $h(H) \cap (V \setminus \{h(a_0)\}) = \phi$. It follows that $h(a_0) \in h(N_{nc}Zd(H)) \setminus (h(H) \cup N_{nc}Zd(h(H))) \neq \phi$, which is a contradiction to the given condition.

**Theorem 3.4** Let $h : (P, N_{nc}) \to (Q, N_{nc})$ be a $1 - 1$ mapping. Then $h$ is $N_{nc}ZIrr$ iff $h(N_{nc}Zd(H)) \subseteq N_{nc}Zd(h(H))$, for each $H \subseteq P$.

**Proof.** Necessity. Let $h$ be $N_{nc}ZIrr$. Let $H \subseteq P$, $a_0 \in N_{nc}Zd(H)$ and $V$ be a $N_{nc}Znbd$ of $h(a_0)$. Since $h$ is $N_{nc}ZIrr$, so by Theorem 3.2, there exists a $N_{nc}Znbd$ $O$ of $a_0$ such that $h(O) \subseteq V$. But $a_0 \in N_{nc}Zd(H)$, hence there exists an element $a \in O \cap H$ such that $H \neq a_0$, then $h(a) \in h(H)$ and, since $h$ is $1 - 1$, $h(a) \neq h(a_0)$. Thus every $N_{nc}Znbd$ $V$ of $h(a_0)$ contains an element of $h(H)$ different from $h(a_0)$, consequently $h(a_0) \in N_{nc}Zd(h(H))$. We have $h(N_{nc}Zd(H)) \subseteq N_{nc}Zd(h(H))$.

Sufficiency. It follows from Theorem 3.3.

**Theorem 3.5** Let $h : (P, N_{nc}) \to (Q, N_{nc})$ be a mapping, then the following statements holds:

(i) $g \circ h$ is $N_{nc}ZIrr$ if both $h$ and $g$ are $N_{nc}ZIrr$.

(ii) $g \circ h$ is $N_{nc}ZCts$ if $h$ is $N_{nc}ZIrr$ and $g$ is $N_{nc}ZCts$.

**Definition 3.2** A space $(P, N_{nc})$ is called:
sets in $P$

Suppose that $P, N \in \mathbb{Z}$ are hold:

(i) $N_{nc}Z-T_1$-space if for any pair of distinct points $x, y \in P$, there is a $N_{nc}Zo$ set $O \subseteq P$ such that $x \in O$ and $y \notin O$ and there is a $N_{nc}Zo$ set $V \subseteq P$ such that $y \in V$ and $x \notin V$.

(ii) $N_{nc}Z-T_2$-space if for each two distinct points $x, y \in P$, there exists two disjoint $N_{nc}Zo$ sets $O, V$ with $x \in O, y \notin V$.

**Theorem 3.6** Let $h : (P, N_{nc}\tau) \rightarrow (Q, N_{nc}\sigma)$ be injective and $N_{nc}ZIrr$ mapping. Then the followings are hold:

(i) If $Q$ is $N_{nc}Z-T_1$-space, then $P$ is $N_{nc}Z-T_1$-space,

(ii) If $Q$ is $N_{nc}Z-T_2$-space, then $P$ is $N_{nc}Z-T_2$-space.

**Proof.** (i) Let $x, y$ be any distinct points in $P$. Since $h$ is injective and $Q$ is a $N_{nc}Z-T_1$-space, there exists two $N_{nc}Zo$ sets $O$ and $V$ in $Q$ such that $h(x) \in O, h(y) \notin O$ or $h(y) \in V, h(x) \notin V$ with $h(x) \neq h(y)$. By using $N_{nc}Z$-irresoluteness of $h$, then $h^{-1}(O)$ and $h^{-1}(V)$ are $N_{nc}Zo$ sets in $P$ such that $x \subseteq h^{-1}(O), y \notin h^{-1}(O)$ or $x \notin h^{-1}(V), y \in h^{-1}(V)$. Therefore, $P$ is a $N_{nc}Z-T_1$-space.

(ii) similar to (i).

**Definition 3.3** A space $(P, N_{nc}\tau)$ is said to be $N_{nc}Z$-compact (resp. $N_{nc}Z$-Lindelöf) if every $N_{nc}Zo$ cover of $P$ has a finite (resp. countable) subcover.

**Theorem 3.7** Let $h : (P, N_{nc}\tau) \rightarrow (Q, N_{nc}\sigma)$ be surjection and $N_{nc}ZIrr$ mapping. Then the followings are hold:

(i) If $P$ is $N_{nc}Z$-compact, then $Q$ is $N_{nc}Z$-compact.

(ii) If $P$ is $N_{nc}Z$-Lindelöf, then $Q$ is $N_{nc}Z$-Lindelöf.

**Proof.** (i) Let $h : (P, N_{nc}\tau) \rightarrow (Q, N_{nc}\sigma)$ be surjection and $N_{nc}ZIrr$ mapping and let $O = \{O_i : i \in I\}$ be a cover of $Q$ by $O_i \in N_{nc}ZO(Q, N_{nc}\sigma)$, for each $i \in I$. Then $O = \{h^{-1}(O_i) : i \in I\}$ is a cover of $P$. Since $h$ is $N_{nc}ZIrr$, then $O$ is a $N_{nc}Zo$ cover of $P$ which is $N_{nc}Z$-compact. Hence, there exists a finite subset $I_0$ of $I$ such that $P = \bigcup\{h^{-1}(O_i) : i \in I_0\}$ which implies $P = h^{-1}(\bigcup I_0)$. and therefore $Q = \bigcup\{O_i : i \in I_0\}$. This shows that $Q$ is $N_{nc}Z$-compact.

(ii) similar to (i).

**Definition 3.4** A space $(P, N_{nc}\tau)$ is said to be $N_{nc}Z$-connected if it cannot be written as a union of two non-empty disjoint $N_{nc}Zo$ sets.

**Theorem 3.8** Let $h : (P, N_{nc}\tau) \rightarrow (Q, N_{nc}\sigma)$ be $N_{nc}ZIrr$ and $P$ is $N_{nc}Z$-connected. Then $Q$ is $N_{nc}Z$-connected.

**Proof.** Suppose that $Q$ is not $N_{nc}$ connected. Then there exist two non-empty disjoint $N_{nc}Zo$ sets $O$ and $V$ in $P$ such that $Q = O \cup V$. Then $h^{-1}(O)$ and $h^{-1}(V)$ are non-empty disjoint $N_{nc}Zo$ sets in $P$ with $P = h^{-1}(O) \cup h^{-1}(V)$ which contradicts the fact that $P$ is $N_{nc}Z$-connected.

4. $N_{nc}Z$-open and $N_{nc}Z$-closed mappings

**Definition 4.1** A mapping $h : (P, N_{nc}\tau) \rightarrow (Q, N_{nc}\sigma)$ is said to be:

(i) $N_{nc}Z$-open (briefly, $N_{nc}ZO$) if the image of each $N_{nc}O$ set in $(P, N_{nc}\tau)$ is $N_{nc}Zo$ sets in $(Q, N_{nc}\sigma)$.

(ii) $N_{nc}Z$-closed (briefly, $N_{nc}ZC$) if the image of each $N_{nc}C$ set in $(P, N_{nc}\tau)$ is $N_{nc}Ze$ sets in $(Q, N_{nc}\sigma)$. 
Theorem 4.1 For a $N_{nc}ZO$ (resp. $N_{nc}ZC$) mapping. If $W \subseteq Q$ and $F \subseteq P$ is a $N_{nc}c$ (resp. $N_{nc}o$) set containing $h^{-1}(W)$, then there exists $N_{nc}Zc$ (resp. $N_{nc}Zo$) set $H \subseteq Q$ containing $W$ such that $h^{-1}(H) \subseteq F$.

**Proof.** Let $H = Q \setminus h(P \setminus F)$. Since $h^{-1}(W) \subseteq F$ which is a $N_{nc}c$ set and $W \subseteq H$, $P \setminus F$ is an $N_{nc}o$ set. Since $h$ is $N_{nc}ZO$ mapping, then $h(P \setminus F)$ is $N_{nc}Zo$ set. Therefore $H$ is $N_{nc}Zc$ and $h^{-1}(H) = P \setminus h^{-1}(P \setminus F) \subseteq F$. While the second side of the theorem can be proved in the same manner.

Theorem 4.2 Let $h : (P, N_{nc}\tau) \to (Q, N_{nc}\sigma)$ be $N_{nc}Zo$ and let $M \subseteq Q$. Then $h^{-1}(N_{nc}Zc!)\{N_{nc}Zc!(N_{nc}Zc!(M))) \subseteq N_{nc}c!(h^{-1}(M))$.

**Proof.** Since $N_{nc}c!(h^{-1}(M))$ is $N_{nc}c$ in $P$ containing $h^{-1}(M)$, then by Theorem 4.1 there exists a $N_{nc}Zc$ set $M \subseteq H \subseteq Q$, such that $h^{-1}(H) \subseteq N_{nc}c!(h^{-1}(M))$. Thus, $h^{-1}(N_{nc}Zc!(N_{nc}Zc!(M))) \subseteq h^{-1}(N_{nc}Zc!(N_{nc}Zc!(M))) \subseteq h^{-1}(H) \subseteq N_{nc}c!(h^{-1}(M))$.

Theorem 4.3 For a mapping $h : (P, N_{nc}\tau) \to (Q, N_{nc}\sigma)$ the following statements are equivalent:

1. $h$ is $N_{nc}Zo$,
2. For each $x \in P$ and each $N_{nc}nbd O$ of $P$, there exists $W \subseteq N_{nc}ZO!\{P\}!\{P\}$ containing $h(x)$ such that $W \subseteq h(O)$,
3. $h^{-1}(N_{nc}int(N_{nc}c!(M))) \cap h^{-1}(N_{nc}c!(N_{nc}c!(M))) \subseteq N_{nc}c!(h^{-1}(M))$, for each $M \subseteq Q$,
4. If $h$ is bijective, then $N_{nc}c!(N_{nc}c!(h(H))) \cap N_{nc}c!(N_{nc}c!(h(H))) \subseteq h(N_{nc}c!(H))$, for each $H \subseteq P$.

**Proof.** (i) $\iff$ (ii). is immediately,

(i) $\implies$ (iii). Let $M \subseteq Q$ and $h$ is $N_{nc}ZO$ mapping. Then by Theorem 4.1, there exists $N_{nc}Zc$ set $V \subseteq Q$ containing $M$ such that $N_{nc}c!(h^{-1}(M)) \supseteq h^{-1}(M) \supseteq h^{-1}(N_{nc}int(N_{nc}c!(V))) \cap h^{-1}(N_{nc}int(N_{nc}c!(V))) \supseteq h^{-1}(N_{nc}int(N_{nc}c!(M))) \cap h^{-1}(N_{nc}int(N_{nc}c!(M)))$ and therefore $h^{-1}(N_{nc}c!(N_{nc}c!(M))) \cap h^{-1}(N_{nc}c!(N_{nc}c!(M))) \subseteq N_{nc}c!(h^{-1}(M)))$.

(iii) $\implies$ (iv). Let $h$ be a bijective mapping and $h(H) \subseteq Q$ by (ii) $h^{-1}(N_{nc}int(N_{nc}c!(h(H)))) \cap h^{-1}(N_{nc}c!(N_{nc}int(h(H)))) \subseteq N_{nc}c!(h^{-1}(h(H))) = N_{nc}c!(H)$. Hence $N_{nc}c!(N_{nc}c!(h(H))) \cap N_{nc}c!(N_{nc}int(h(H))) \subseteq h(N_{nc}c!(H))$.

(iv) $\implies$ (i). Let $V \in N_{nc}\tau$, by (iv), $h(N_{nc}c!(P \setminus V)) = h(P \setminus V) \supseteq N_{nc}c!(N_{nc}c!(h(P \setminus V))) \cap N_{nc}c!(N_{nc}int(h(P \setminus V)))$. By bijection $h$, we have $h(V) \subseteq N_{nc}c!(N_{nc}c!(h(V))) \cup N_{nc}c!(N_{nc}int(h(V)))$ and so $h$ is $N_{nc}ZO$.

Remark 4.1 The bijection condition in Theorem 4.3 (iv) is necessary as shown by the following example.

**Example 4.1** Let $P = \{a, b, c, d\}, n_{c1} = \{\phi, P, A, B, C\}, n_{c2} = \{\phi, P\}$. $A = \{\{a\}, \{\phi\}, \{b, c, d\}\}, B = \{\{b\}, \{\phi\}, \{a, d\}\}, C = \{\{a, b, c\}, \{\phi\}, \{d\}\}$. Let $\sigma$ be an identity mapping and $H = \{\{a\}, \{\phi\}, \{b, c, d\}\}$, then $2_{n_{c1}} = \{\phi, P, A, B, C\}$. Let $f$ be an identity mapping and $H = \{\{a\}, \{\phi\}, \{b, c, d\}\}$, then $2_{n_{c1}} = \{\phi, P, A, B, C\}$. Let $\sigma$ be an identity mapping and $H = \{\{a\}, \{\phi\}, \{b, c, d\}\}$, then $2_{n_{c1}} = \{\phi, P, A, B, C\}$. Let $f$ be an identity mapping and $H = \{\{a\}, \{\phi\}, \{b, c, d\}\}$, then $2_{n_{c1}} = \{\phi, P, A, B, C\}$.

Theorem 4.4 A mapping $h : (P, N_{nc}\tau) \to (Q, N_{nc}\sigma)$ is $N_{nc}ZO$ if $h(N_{nc}int(H)) \subseteq N_{nc}Zint(h(H))$, for each $H \subseteq P$. 


Proof. (i) Let \( h \) be a \( N_{nc}ZO \) mapping and \( H \subseteq P \), then \( N_{nc}Zint(h(N_{nc}int(H))) = h(N_{nc}int(H)) \) \( \subseteq N_{nc}ZOS(Q) \). Therefore \( N_{nc}Zint(h(N_{nc}int(H))) = f(N_{nc}int(H)) \subseteq N_{nc}Zint(h(H)) \).

Conversely, let \( O \in N_{nc}τ \), and \( h(O) = h(N_{nc}int(O)) \subseteq N_{nc}Zint(h(O)) \). Then \( h(O) = N_{nc}Zint(h(O)) \). Thus, \( h(O) \) is \( N_{nc}Zo \) in \( Q \). Therefore, \( h \) is \( N_{nc}Zo \).

Example 4.2 In Example 4.1, let \( f \) is an identity map and \( H = \langle \{a, b\}, \{\phi\}, \{c, d\} \rangle \), then \( h(2_{nc}int(H)) = \langle \{a\}, \{\phi\}, \{b, c, d\} \rangle \) and \( 2_{nc}Zint(h(H)) = \langle \{a, b\}, \{\phi\}, \{c, d\} \rangle \). Hence, \( h(2_{nc}int(H)) \neq 2_{nc}Zint(h(H)) \).

Theorem 4.5 A mapping \( h : (P, N_{nc}τ) \to (Q, N_{nc}σ) \) is \( N_{nc}ZO \) iff \( N_{nc}int(h^{-1}(M)) \subseteq h^{-1}(N_{nc}Zint(M)) \), for each \( M \subseteq Q \).

Proof. Necessity. Let \( M \subseteq Q \). Since \( N_{nc}int(h^{-1}(M)) \) is \( N_{nc}o \) in \( P \) and \( h \) is \( N_{nc}Zo \), then \( h(N_{nc}int(h^{-1}(M))) \) is \( N_{nc}Zo \) in \( Q \). Also, we have \( h(N_{nc}int(h^{-1}(M))) \subseteq h^{-1}(M) \subseteq M \). Hence, \( h(N_{nc}int(h^{-1}(M))) \subseteq N_{nc}Zint(M) \). Therefore, \( N_{nc}int(h^{-1}(M)) \subseteq h^{-1}(N_{nc}Zint(M)) \).

Sufficiency. Let \( H \subseteq P \). Then \( h(H) \subseteq Q \). Hence by hypotheses, we obtain \( N_{nc}int(H) \subseteq N_{nc}int(h^{-1}(h(H))) \subseteq h^{-1}(N_{nc}Zint(h(H))) \). Hence, \( h(N_{nc}int(H)) \subseteq N_{nc}Zint(h(H)) \), for each \( H \subseteq P \). Hence by Theorem 4.4, we have \( h \) is \( N_{nc}Zo \).

Theorem 4.6 A mapping \( h : (P, N_{nc}τ) \to (Q, N_{nc}σ) \) is \( N_{nc}ZO \) iff \( h^{-1}(N_{nc}ZBd(h(H))) \subseteq N_{nc}Bd(h^{-1}(H)) \), for each \( H \subseteq P \).

Proof. It follows from Theorem 4.5.

Theorem 4.7 A mapping \( h : (P, N_{nc}τ) \to (Q, N_{nc}σ) \) is \( N_{nc}ZO \) iff

\[
h^{-1}(N_{nc}Zcl(M)) \subseteq N_{nc}cl(h^{-1}(M))
\]

for each \( M \subseteq Q \).

Proof. Necessity. Let \( M \subseteq Q \) and let \( h \) be \( N_{nc}Zo \). Let \( x \in h^{-1}(N_{nc}Zcl(M)) \). Then \( h(x) \in N_{nc}Zcl(M) \). Assume that \( O \in N_{nc}τ \) such that \( x \in O \). Since \( h \) is \( N_{nc}Zo \), then \( h(O) \) is \( N_{nc}Zo \) set in \( Q \). Hence, \( M \cap h(O) \neq \emptyset \). Thus \( O \cap h^{-1}(M) \neq \emptyset \). Therefore \( x \in N_{nc}cl(h^{-1}(M)) \).

\[
h^{-1}(N_{nc}Zcl(M)) \subseteq N_{nc}cl(h^{-1}(M))
\]

Sufficiency. Let \( M \subseteq Q \). Then \( Q \setminus M \subseteq Q \). By hypotheses, \( h^{-1}(N_{nc}Zcl(Q \setminus M)) \subseteq N_{nc}cl(h^{-1}(Q \setminus M)) \) and hence \( P \setminus N_{nc}cl(P \setminus h^{-1}(M)) \subseteq P \setminus h^{-1}(N_{nc}Zcl(Q \setminus M)) = h^{-1}(Q \setminus N_{nc}Zcl(Q \setminus M)) \). Therefore, \( N_{nc}int(h^{-1}(M)) \subseteq h^{-1}(N_{nc}Zint(M)) \). By Theorem 4.5, we have \( h \) is \( N_{nc}ZO \).

Theorem 4.8 A mapping \( h : (P, N_{nc}τ) \to (Q, N_{nc}σ) \) is \( N_{nc}ZC \) iff

\[
N_{nc}Zcl(h(H)) \subseteq h(N_{nc}cl(H))
\]

for each \( H \subseteq P \).

Proof. Necessity. Let \( h \) be \( N_{nc}ZC \) mapping and \( H \subseteq P \). Then \( h(H) \subseteq h(N_{nc}cl(H)) \). But \( h(N_{nc}cl(H)) \) is \( N_{nc}Zc \) in \( Q \). Therefore, \( N_{nc}Zcl(h(H)) \subseteq h(N_{nc}cl(H)) \).

Conversely, suppose that \( N_{nc}Zcl(h(H)) \subseteq h(N_{nc}cl(H)) \), for each \( H \subseteq P \). Let \( H \subseteq P \) be an \( N_{nc}c \). Then \( N_{nc}Zcl(h(H)) \subseteq h(N_{nc}cl(H)) \). Hence \( h(H) \) is \( N_{nc}Zc \) in \( Q \). Therefore, \( h \) is \( N_{nc}ZC \).
Theorem 4.9 Let \( h : (P, N_{nc}) \rightarrow (Q, N_{nc}) \) be \( N_{nc}ZC \). Then

\[
N_{nc}Zint(N_{nc}Zcl(h(H))) \subseteq h(N_{nc}cl(H))
\]

for each \( H \subseteq P \).

Proof. Suppose \( h \) is a \( N_{nc}ZC \) mappings and \( H \subseteq P \). Then \( h(N_{nc}cl(H)) \) is \( N_{nc}Zc \) in \( Q \). Then \( N_{nc}Zint(N_{nc}Zcl(h(N_{nc}cl(H)))) \subseteq h(N_{nc}cl(H)). \) But \( N_{nc}Zint(N_{nc}Zcl(h(H))) \subseteq N_{nc}Zint(N_{nc}Zcl(h(H))). \) Therefore, \( N_{nc}Zint(N_{nc}Zcl(h(H))) \subseteq h(N_{nc}cl(H)). \)

Theorem 4.10 Let \( h : (P, N_{nc}) \rightarrow (Q, N_{nc}) \) be \( N_{nc}ZC \) and \( M, C \subseteq Q \).

(i) If \( O \) is an \( N_{nc}onbd \) of \( h^{-1}(M) \), then there exists a \( N_{nc}Zonbd \) \( V \) of \( M \) such that \( h^{-1}(M) \subseteq h^{-1}(V) \subseteq O \).

(ii) If \( h \) is onto, then \( h^{-1}(M) \) and \( h^{-1}(C) \) have disjoint \( N_{nc}onbd \) so have \( M \) and \( C \).

Proof. (i) Let \( V = Q \setminus h(P \setminus O) \). Then \( V^c = Q \setminus h(O^c) \). Since \( h \) is \( N_{nc}Zc \), so \( V \) is a \( N_{nc}Zo \) set. Since \( h^{-1}(M) \subseteq O \), we have \( V^c = h(O^c) \subseteq h^{-1}(M^c) \subseteq M^c \). Hence, \( M \subseteq V \) and thus \( V \) is a \( N_{nc}Zonbd \) of \( M \). Further \( O^c \subseteq h^{-1}(h(O^c)) = h^{-1}(V^c) = (h^{-1}(V))^c \). Therefore, \( h^{-1}(V) \subseteq O \).

(ii) If \( h^{-1}(M) \) and \( h^{-1}(C) \) have disjoint \( N_{nc}onbd \) \( M \) and \( C \), then by (i), we have \( N_{nc}Zonbd \)'s \( O \) and \( V \) of \( M \) and \( C \) respectively such that \( h^{-1}(M) \subseteq h^{-1}(O) \subseteq N_{nc}Zint(M) \) and \( h^{-1}(C) \subseteq h^{-1}(V) \subseteq N_{nc}Zint(N) \). Since \( M \) and \( N \) are disjoint, so are \( N_{nc}Zint(M) \) and \( N_{nc}Zint(N) \) and hence so \( h^{-1}(O) \) and \( h^{-1}(V) \) are disjoint as well. It follows that \( O \) and \( V \) are disjoint too as \( h \) is onto.

Theorem 4.11 For a bijective mapping \( h : (P, N_{nc}) \rightarrow (Q, N_{nc}) \) the following are equivalent:

(i) \( h^{-1} \) is \( N_{nc}ZCts \),
(ii) \( h \) is \( N_{nc}ZO \),
(iii) \( h \) is \( N_{nc}ZC \).

Proof. (i) \( \Rightarrow \) (ii). Let \( V \in N_{nc}\tau \) and \( h^{-1} \) be \( N_{nc}ZCts \), by bijective of \( h \). Then \( (h^{-1})^{-1}(V) = h(V) \in N_{nc}ZO(Q) \) and therefore \( h \) is \( N_{nc}Zo \) mapping.

(ii) \( \Rightarrow \) (iii). Let \( V \) be \( N_{nc}c \) in \( P \). Then \( P \setminus O \) is \( N_{nc}o \) in \( P \), by (ii), \( h(P \setminus O) \) is \( N_{nc}Zo \) in \( Q \). But \( h(P \setminus O) = h(P) \setminus h(O) = Q \setminus h(O) \). Thus \( h(O) \) is \( N_{nc}Zc \) in \( Q \). Therefore \( h \) is \( N_{nc}Zc \).

(iii) \( \Rightarrow \) (i). Let \( V \in N_{nc}\tau \), by (iii), we have \( h(P \setminus V) \in N_{nc}Zc \) in \( Q \) and hence, \( h(V) = (h^{-1})^{-1}(V) \in N_{nc}ZO(Q) \). Therefore, \( h^{-1} \) is \( N_{nc}ZCts \).

Remark 4.2 The composition of two \( N_{nc}ZO \) (resp. \( N_{nc}ZC \)) mapping may not be \( N_{nc}ZO \) (resp. \( N_{nc}ZC \)). The following example shows this fact.

Example 4.3 Let \( P = \{a, b, c\} = Z \), \( Q = \{a, b, c, d\} \), \( \tau_1 = \{\phi_N, P_N, A\} \), \( \tau_2 = \{\phi_N, P_N\} \). \( A = \{\{a\}, \{b\}, \{c\}\}, \) then we have \( 2_{nc} = \{\phi_N, P_N, A\} \), \( 2_{nc} = \{\phi_N, P_N\} \). \( B = \{\{a, c\}, \{b\}, \{d\}\}, \) then we have \( 2_{nc} = \{\phi_N, Y_N, B\} \), \( 2_{nc} = \{\phi_N, Y_N\} \). \( C = \{\{a\}, \{b\}, \{c\}\}, \) \( D = \{\{a, b\}, \{c\}\}, \) \( \tau_1 = \{\phi_N, Y_N, B\} \), \( \tau_2 = \{\phi_N, Y_N\} \). \( \mu_1 = \{\phi_N, Y_N, B\} \), \( \mu_2 = \{\phi_N, Y_N\} \).

Let \( h : (P, 2_{nc}) \rightarrow (Q, 2_{nc}) \) be an identity function and \( g : (Q, 2_{nc}) \rightarrow (Z, 2_{nc}) \) defined as \( g(a) = a, g(b) = g(d) = b \) and \( g(c) = c \). It is clear that \( f \) and \( g \) are \( 2_{nc}ZO \) maps but \( g \circ f \) is not \( 2_{nc}ZO \) map.

Theorem 4.12 Let \( h : (P, N_{nc}P) \rightarrow (Q, \tau_P) \) and \( g : (Q, N_{nc}Q) \rightarrow (Z, N_{nc}Z) \) be two mappings. Then the following statements hold:

(i) If \( h \) is surjective \( N_{nc}O \) (resp. \( N_{nc}C \)) and \( g \) is \( N_{nc}ZO \), then \( g \circ h \) is \( N_{nc}ZO \) (resp. \( N_{nc}ZC \))

(ii) If \( g \circ h \) is \( N_{nc}ZO \) (resp. \( N_{nc}ZC \)) and \( h \) is surjective \( N_{nc}Cts \), then \( g \) is \( N_{nc}O \) (resp. \( N_{nc}C \)).
If $g \circ h$ is $N_{nc}O$ (resp. $N_{nc}C$) and $g$ is injective $N_{nc}ZC$ts, then $h$ is $N_{nc}ZO$ (resp. $N_{nc}ZC$).

**Theorem 4.13** Let $h : (P, N_{nc}τ) \rightarrow (Q, N_{nc}σ)$ be a $N_{nc}ZO$ bijection. Then the following are hold

(i) If $P$ is $N_{nc}Z-T_1$-space, then $Q$ is $N_{nc}Z-T_1$-space,
(ii) If $P$ is $N_{nc}Z-T_2$-space, then $Q$ is $N_{nc}Z-T_2$-space.

**Theorem 4.14** Let $h : (P, N_{nc}τ) \rightarrow (Q, N_{nc}σ)$ be a $N_{nc}ZO$ bijection. Then the following are hold

(i) If $Q$ is $N_{nc}Z$-compact, then $P$ is $N_{nc}Z$-compact.
(ii) If $Q$ is $N_{nc}Z$-Lindelöf, then $P$ is $N_{nc}Z$-Lindelöf.

**Theorem 4.15** Let $h : (P, N_{nc}τ) \rightarrow (Q, N_{nc}σ)$ be a $N_{nc}ZO$ surjection and $Q$ is $N_{nc}Z$-connected. Then $P$ is $N_{nc}Z$-connected.

5. Pre $N_{nc}Z$-open and Pre $N_{nc}Z$-closed Mapping

**Definition 5.1** A mapping $h : (P, N_{nc}τ) \rightarrow (Q, N_{nc}σ)$ is said to be pre $N_{nc}Z$-open (resp. pre $N_{nc}Z$-closed ) (briefly, pre $N_{nc}ZO$ (resp. pre $N_{nc}ZC$)) if $h(V) \in N_{nc}ZO(Q, N_{nc}σ)$ ( resp. $N_{nc}ZC(Q, N_{nc}σ)$), for each $V \in N_{nc}ZOS(P, N_{nc}τ)$ (resp. $N_{nc}ZC(P, N_{nc}τ)$).

**Theorem 5.1** A mapping $h : (P, N_{nc}τ) \rightarrow (Q, N_{nc}σ)$ is pre $N_{nc}ZC$ iff for each $S \subseteq Q$ and each $O \subseteq N_{nc}ZOS(P, N_{nc}τ)$ containing $h^{-1}(S)$, there exists $V \in N_{nc}ZO(Q, N_{nc}σ)$ containing $S$ such that $h^{-1}(V) \subseteq O$.

**Proof.** Let $S \subseteq Q$ and $h^{-1}(S) \subseteq O$. Put $V = Q\setminus h(P\setminus O)$, then $V \in N_{nc}ZOS(Q, N_{nc}σ)$. Since $h^{-1}(S) \subseteq O$, then $h(P\setminus O) \subseteq h^{-1}(Q\setminus S) \subseteq Q\setminus S$ and therefore $h^{-1}(V) \subseteq O$.

Conversely, let $F$ be a $N_{nc}Zc$ in $(P, N_{nc}τ)$. For any $y \in Q\setminus h(F)$, then $h^{-1}(y) \in P\setminus F \in N_{nc}ZOS(P, N_{nc}τ)$. Hence there exists $V_y \in N_{nc}ZOS(Q, N_{nc}σ)$ containing $y$ such that $h^{-1}(V_y) \subseteq P\setminus F$, which implies $y \in V_y \subseteq Q\setminus h(F)$. So $Q\setminus h(F) = \bigcup\{V_y : y \in Q\setminus h(F)\}$ and therefore $h(F)$, $F$ is $N_{nc}Zc$ in $(Q, N_{nc}σ)$.

**Theorem 5.2** A mapping $h : (P, N_{nc}τ) \rightarrow (Q, N_{nc}σ)$ is pre $N_{nc}ZO$ iff $h(N_{nc}Zint(H)) \subseteq N_{nc}Zint(h(H))$, for each $H \subseteq P$.

**Proof.** The proof is similar as Theorem 4.4.

We remark that the equality does not hold in Theorem 5.2 as the following example shows.

**Example 5.1** Let $P = \{a, b, c, d\} = Q$, $nc\tau_1 = \{\phi_N, P_N, A, B, C\}$, $nc\tau_2 = \{\phi_N, P_N\}$. $A = \{\{a\}, \{\phi\}, \{b, c, d\}\}$, $B = \{\{b, c\}, \{\phi\}, \{a, d\}\}$, $C = \{\{a, b, c\}, \{\phi\}, \{d\}\}$, then we have $2_{nc}\tau = \{\phi_N, P_N, A, B, C\}$. $nc\sigma_1 = \{\phi_N, Y_N, D, E, F\}$, $nc\sigma_2 = \{\phi_N, Y_N\}$. $D = \{\{d\}, \{\phi\}, \{a, b, c\}\}$, $E = \{\{b, c\}, \{\phi\}, \{a, d\}\}$, $F = \{\{b, c, d\}, \{\phi\}, \{a\}\}$, then we have $2_{nc}\sigma = \{\phi_N, Y_N, D, E, F\}$. Let $f : (P, 2_{nc}\tau) \rightarrow (Q, 2_{nc}\sigma)$ is an identity map and $H = \{\{b, d\}, \{\phi\}, \{a, c\}\}$, then $h(2_{nc}Zint(H)) = \{\{b\}, \{\phi\}, \{a, c, d\}\}$ and $2_{nc}Zint(h(H)) = \{\{b, d\}, \{\phi\}, \{a, c\}\}$. Hence, $h(2_{nc}Zint(H)) \neq 2_{nc}Zint(h(H))$.

**Theorem 5.3** A mapping $h : (P, N_{nc}τ) \rightarrow (Q, N_{nc}σ)$ is pre $N_{nc}ZO$ iff $h(N_{nc}Zint(h^{-1}(M))) \subseteq h^{-1}(N_{nc}Zint(M))$, for all $M \subseteq Q$.

**Proof.** The proof is similar as Theorem 4.5

**Theorem 5.4** A mapping $h : (P, N_{nc}τ) \rightarrow (Q, N_{nc}σ)$ is pre $N_{nc}ZO$ iff $h^{-1}(N_{nc}ZBd(M)) \subseteq N_{nc}ZBd(h^{-1}(M))$, for all $M \subseteq Q$. 
Proof. It follows from Theorem 5.3.

**Theorem 5.5** A mapping \( h : (P, N_{nc}^\tau) \to (Q, N_{nc}\sigma) \) is pre \( N_{nc}ZO \) iff \( h^{-1}(N_{nc}Zcl(M)) \subseteq N_{nc}Zcl(h^{-1}(M)) \), for all \( M \subseteq Q \).

**Proof.** The proof similar as Theorem 4.7.

**Theorem 5.6** Let \( h : (P, N_{nc}^\tau) \to (Q, N_{nc}\sigma) \) be a mapping such that \( h(N_{nc}Zint(H)) \subseteq N_{nc}cl(N_{nc}int_\delta(h(H))) \), for every subset \( H \) of \( P \). Then \( h \) is pre \( N_{nc}ZO \).

**Proof.** Suppose \( H \) is an \( N_{nc}Zo \) set in \( P \). Then by hypothesis, we have \( h(h(N_{nc}Zint(H))) \subseteq N_{nc}cl(N_{nc}int_\delta(h(H))) \).

Take \( M = N_{nc}int_\delta(h(H)) \). Then \( M \) is \( N_{nc}\delta_0 \) in \( Q \). Also it implies that \( M \subseteq h(H) \subseteq N_{nc}cl(M) \). Hence \( h(H) \) is \( N_{nc}\delta_0 \) in \( Q \). Since \( N_{nc}\delta_0O(Q) \subseteq N_{nc}ZO(Q) \). Thus \( h(H) \) is \( N_{nc}ZO \) in \( Q \). This implies that \( h \) is pre \( N_{nc}ZO \).

**Theorem 5.7** A mapping \( h : (P, N_{nc}^\tau) \to (Q, N_{nc}\sigma) \) is pre \( N_{nc}zc \) if \( N_{nc}Zcl(h(H)) \subseteq h(N_{nc}Zcl(H)) \), for all \( H \subseteq P \).

**Proof.** Necessity. Suppose \( h \) is a pre \( N_{nc}ZC \) mapping and \( H \) is an arbitrary subset of \( P \). Then \( h(N_{nc}Zcl(H)) \) is \( N_{nc}Zc \) in \( Q \). Since \( h(H) \subseteq h(N_{nc}Zcl(H)) \), we obtain

\[
N_{nc}Zcl(h(H)) \subseteq h(N_{nc}Zcl(H)).
\]

Sufficiency. Suppose \( F \) is an arbitrary \( N_{nc}Zc \) set in \( P \). By hypothesis, we obtain \( h(F) \subseteq N_{nc}Zcl((h(F))) \subseteq h(N_{nc}Zcl(F)) = h(F) \). Hence \( h(F) = N_{nc}Zcl(h(F)) \). Thus \( h(F) \) is \( N_{nc}Zc \) in \( Q \). It follows that \( h \) is pre \( N_{nc}ZC \).

**Theorem 5.8** Let \( h : (P, N_{nc}^\tau) \to (Q, N_{nc}\sigma) \) be a pre \( N_{nc}ZC \) function, and \( M, C \subseteq Q \).

(i) If \( O \) is a \( N_{nc}Zombd \) of \( h^{-1}(M) \), then there exists a \( N_{nc}Zombd \) \( V \) of \( M \) such that \( h^{-1}(M) \subseteq h^{-1}(V) \subseteq O \).

(ii) If \( h \) is also onto, then if \( h^{-1}(M) \) and \( h^{-1}(C) \) have disjoint \( N_{nc}nbd's \), so have \( M \) and \( C \).

**Proof.** The proof is similar as Theorem 4.9.

**Theorem 5.9** Let \( h : (P, N_{nc}^\tau) \to (Q, N_{nc}\sigma) \) be a bijection. Then the following are equivalent:

(i) \( h \) is pre \( N_{nc}ZC \),

(ii) \( h \) is pre \( N_{nc}ZO \),

(iii) \( h^{-1} \) is \( N_{nc}ZIrr \).

**Proof.** (i) \( \Rightarrow \) (ii). Let \( O \in N_{nc}ZOS(P, N_{nc}^\tau) \). Then \( P \setminus O \) is \( N_{nc}Zc \) in \( P \). By (i), \( h(P \setminus O) = h(P) \setminus h(O) = Q \setminus h(O) \). Thus \( h(O) \) is \( N_{nc}Zo \) in \( Q \).

(ii) \( \Rightarrow \) (iii). Let \( H \subseteq P \). Since \( h \) is pre \( N_{nc}ZO \), so by Theorem 5.2, \( h^{-1}(N_{nc}Zcl(h(H))) \subseteq N_{nc}Zcl(h^{-1}(h(H))) \). It implies that \( N_{nc}Zcl(h(H)) \subseteq h(N_{nc}Zcl(H)) \). Thus \( N_{nc}Zcl((h^{-1})^{-1}(H)) \subseteq (h^{-1})^{-1}(N_{nc}Zcl(H)) \), for all \( H \subseteq P \). Then by Theorem 5.2, it follows that \( h^{-1} \) is \( N_{nc}ZIrr \).

(iii) \( \Rightarrow \) (i). Let \( H \) be an arbitrary \( N_{nc}Zc \) set in \( P \). Then \( P \setminus H \) is \( N_{nc}Zo \) in \( P \). Since \( h^{-1} \) is \( N_{nc}ZIrr \), \( (h^{-1})^{-1}(P \setminus H) \) is \( N_{nc}Zo \) in \( Q \). But \( (h^{-1})^{-1}(P \setminus H) = h(P \setminus H) = Q \setminus h(H) \). Thus \( h(H) \) is \( N_{nc}Zc \) in \( Q \).

**Theorem 5.10** Let \( h : (P, N_{nc}^\tau_P) \to (Q, N_{nc}^\tau_Q) \) and \( g : (Q, N_{nc}^\tau_Q) \to (Z, N_{nc}^\tau_Z) \) be two mappings such that \( g \circ f : (P, N_{nc}^\tau_P) \to (Z, N_{nc}^\tau_Z) \) is \( N_{nc}ZIrr \). Then:
(i) If $g$ is a pre $N_{nc}ZO$ injection, then $h$ is $N_{nc}ZIrr$.

(ii) If $h$ is a pre $N_{nc}ZO$ surjection, then $g$ is $N_{nc}ZIrr$.

**Proof.** (i) Let $O \in N_{nc}ZOS(Q, N_{nc}\tau Q)$. Then $g(O) \in N_{nc}ZOS(Z, N_{nc}\tau Z)$, since $g$ is pre $N_{nc}ZO$. Also $g \circ h$ is $N_{nc}ZIrr$. Therefore, $(g \circ h)^{-1}(g(O)) \in N_{nc}ZOS(P, N_{nc}\tau P)$. Since $g$ is an injection, so we have $(g \circ h)^{-1}(g(O)) = (h^{-1} \circ g^{-1})(g(O)) = h^{-1}(g^{-1}(g(O))) = h^{-1}(O)$. Consequently, $h^{-1}(O)$ is $N_{nc}Zo$ in $P$. This proves that $h$ is $N_{nc}ZIrr$.

(ii) Let $V \in N_{nc}ZOS(Z, N_{nc}\tau Z)$. Then $(g \circ h)^{-1}(V) \in N_{nc}ZOS(P, N_{nc}\tau P)$, since $g \circ h$ is $N_{nc}ZIrr$. Also $h$ is pre $N_{nc}ZO$, $h((g \circ h)^{-1}(V))$ is $N_{nc}ZO$ in $Q$. Since $h$ is surjective, we note that $h((g \circ h)^{-1}(V)) = (h \circ (g \circ h)^{-1})(V) = (h \circ (h^{-1} \circ g^{-1}))(V) = ((h \circ h^{-1}) \circ g^{-1})(V) = g^{-1}(V)$. Hence $g$ is $N_{nc}ZIrr$.

**Theorem 5.11** For a mappings $h : (P, N_{nc}\tau P) \rightarrow (Q, N_{nc}\tau Q)$ and $g : (Q, N_{nc}\tau Q) \rightarrow (Z, N_{nc}\tau Z)$, then

(i) $g \circ h$ is pre $N_{nc}ZO$ (resp. pre $N_{nc}ZC$) if both $h$ and $g$ are pre $N_{nc}ZO$ (resp. pre $N_{nc}ZC$).

(ii) $g \circ h$ is $N_{nc}ZO$ (resp. $N_{nc}ZC$) if $h$ is $N_{nc}ZO$ (resp. $N_{nc}ZC$) and $g$ are pre $N_{nc}ZO$ (resp. pre $N_{nc}ZC$).

(iii) If $h$ is $N_{nc}ZCts$ surjection and $g \circ h$ is pre $N_{nc}ZO$ (resp. pre $N_{nc}ZC$), then $g$ is $N_{nc}ZO$ (resp. $N_{nc}ZC$).

**Proof.** It is clear.

**Theorem 5.12** Let $h : (P, N_{nc}\tau) \rightarrow (Q, N_{nc}\sigma)$ be a pre $N_{nc}ZO$ bijection. Then the following are hold

(i) If $P$ is $N_{nc}Z-T_1$-space, then $Q$ is $N_{nc}Z-T_1$-space,

(ii) If $P$ is $N_{nc}Z-T_2$-space, then $Q$ is $N_{nc}Z-T_2$-space.

**Theorem 5.13** Let $h : (P, N_{nc}\tau) \rightarrow (Q, N_{nc}\sigma)$ be a pre $N_{nc}ZO$ bijection. Then the following are hold

(i) If $Q$ is $N_{nc}Z$-compact, then $P$ is $N_{nc}Z$-compact.

(ii) If $Q$ is $N_{nc}Z$-Lindelöf, then $P$ is $N_{nc}Z$-Lindelöf.

**Theorem 5.14** Let $h : (P, N_{nc}\tau) \rightarrow (Q, N_{nc}\sigma)$ be a pre $N_{nc}ZO$ bijection and $Q$ is $N_{nc}Z$-connected. Then $P$ is $N_{nc}Z$-connected.

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