Pseudo-active actuators: A concept analysis

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Abstract
The superior performance of active vibration control systems largely depends on the four-quadrant controllable execution capability in the available force-velocity diagram of active actuators. Although semi-active vibration control systems have the advantages of low energy consumption, simple structure, and high reliability, the system performance is not comparable to active control systems, due to the partial capability in only the first and third quadrants. On the basis of the comprehensive advantages of active and semi-active actuators, to reform the design philosophy of semi-active actuators to realize pseudo-active actuators that have both mechanical properties of active actuators and energy consumption advantages of semi-active actuators, that is, new semi-active actuators with four-quadrant controllable execution capability, will very likely cause a revolution in the related fields of mechanical design and system control. The basic design principle of pseudo-active actuators that use semi-active controllable actuators to achieve active actuator performance in the way of conceptual analysis is proposed. The proposed pseudo-active actuators should consist of two half-four-quadrant actuators, that is, one is for the first and third quadrants and the other one for the second and fourth quadrants. This study employs two semi-active controllable damping actuators and one mechanical compensation mechanism. One of the actuators provides the damping force in the first and third quadrants, and the other one combining with the mechanical compensation mechanism is for the second and fourth quadrants. A global mathematical model of the proposed actuator is established to describe the four different operational modes of the proposed actuator. It is proved that the two operational modes of the proposed actuator can realize active vibration control, and a case study of realizing active control is presented. The other two operational modes are compared with the conventional two-degree-of-freedom model. More specifically, the application cases of the pseudo-active operational mode of the proposed actuator in the quarter-car/body-powertrain suspension system are given, a pseudo-active suspension named dual-hook automobile suspension is presented. Furthermore, an equivalent expression of the electrical network is given for the mechanical network under different operational modes of the proposed actuator.
1 | INTRODUCTION

Efficient suppression or utilization of mechanical vibration can avoid unwanted results, such as fatigue of the structural parts of the system and reduced comfort/safety performance.1–5 Although the active vibration control system has superior performance, it encounters coldness in the process of large-scale promotion due to its “congenital defects,” that is, huge energy consumption, bulky system, and high cost. In contrast, the semi-active vibration control system receives very high expectations due to its low energy consumption, simple system, and high reliability.6

However, typical semi-active shock absorbers with adjustable damping/stiffness/inertial capacity, for example, electromagnetic/magnetorheological shock absorbers, regenerative shock absorbers, controllable valve-type shock absorbers, air springs, and inverters cannot provide negative forces as active actuators do. On the one hand, the reason is that active actuators can achieve mechanical control within the four quadrants of the available force-velocity diagram, while semi-active actuators can only achieve a limited region in the first and third quadrants of the available force-velocity diagram,7,8 as shown in Figure 1. More essentially, from the energy point of view, the active control method and the passive control method have different energy flow directions. The output mechanical energy of active actuators in the control loops could be from other forms of energy, such as electrical energy9,10 and hydraulic energy.11 In contrast, in the semi-active control systems, the semi-active actuators are the same as the passive actuators, which can only accept mechanical energy.12 Other forms of energy input to semi-active actuators for control13,14 are not converted into external mechanical energy. Therefore, in the full frequency range, taking the automobile ride comfort indices as an example, there is a nonnegligible theoretical gap between the semi-active suspensions and the active suspensions. The root-mean-square value of the former is lower than about 20% of the latter.15,16 Figure 1 also presents the inability of the semi-active actuators for control systems. The existing active/semi-active vibration control technology has failed to change the state of “stalling” in the large-scale promotion of intelligent suspensions, and the reason would be the “congenital defects” of the active/semi-active suspensions.

The boundary between the concepts of active actuators and semi-active actuators is clear, but the boundary between the corresponding control strategies is ambiguous. Semi-active control systems based on active control strategies and semi-active actuators show similar characteristics of control effect as active actuators. It can be inferred that the realization of active control does not necessarily require active actuators. It is an important scientific concept and idea to study how to use semi-active actuators, e.g., controllable stiffness/damping/inertance actuators, with new topological relationships to achieve pseudo-active actuators with active control effects, that is, semi-active actuators with four-quadrant controllable execution capabilities. Again, taking automotive suspensions as an example, it is conceivable to realize a new type of intelligent suspension by using pseudo-active actuators, which not only has the performance of active suspension but also avoids its “defects” of high energy consumption and high cost. It also has the advantages of simplicity, reliability, and low energy consumption of the semi-active suspensions and can avoid the lack of performance in the high-or low-frequency range. It may be an important way to promote a new round of research and promotion for automotive intelligent suspensions as well as other vibration/shock control.

Negative stiffness, with the reactive force to assist deformation progress instead of resisting,17–19 is one of the most wonderful elements to implement the output force transition. A schematic of the force transition of a controllable damping actuator via a linear

![Figure 1](https://i.imgur.com/123456.png)  
**Figure 1** Available force-velocity areas of actuators: (A) semi-active actuators, first and third quadrants and (B) active actuators, all four quadrants.
negative stiffness component is presented in Figure 2. It can be observed from Figure 2 that partial output force in the first and third quadrants (i.e., Q1 and Q3) can be translated to the second and fourth quadrants (i.e., Q2 and Q4) in the available force-velocity diagram. Then a specific pseudo-active actuator, the negative stiffness damper, is realized. Such a new concept of actuators with the combination of (variable) damping and (variable) negative stiffness was presented by Lemura and Pradono, Weber and his associates, Shi and Zhu, and later Li's group. All the theoretical results of their studies on the applications indicated that the new actuator could provide significant improvement on vibration attenuation, which is comparable to the system using an active actuator. Prototype realization of the new actuators with "controllable damping + negative stiffness" was only provided by Shi and Zhu and Yang et al. A permanent magnet or an electromagnetic device was used for the negative stiffness capacity for such actuators. Because of the pseudo-active mechanical property of the developed actuators, that is, four-quadrant execution capability, as seen from Figure 2, the experimental results are following what is expected and are comparable to the active systems.

On the basis of what was reviewed and analyzed above, we may infer further, it is possible and reasonable to do the structural design of the pseudo-active actuators with four-quadrant execution capability according to the first principle—combinations of mechanisms. This study proposes the concept and design principle of pseudo-active actuators (i.e., a four-quadrant semi-active actuator). Original innovation for the existing vibration control technologies and new ideas for the research of existing active/semi-active vehicle suspensions are presented and investigated, which are of significance to academic research and potential engineering applications. Specifically, the main technical contributions are as follows: (i) A variable-structure pseudo-active actuator with four operational modes is proposed. The proposed pseudo-active actuator uses two semi-active actuators and a rack-pinion motion pair to construct a variable degree-of-freedom (DOF) mechanism. Four different topological relations between the two rigid bodies and the two semi-active actuators are derived, according to different connection relations between the two rigid bodies with DOFs and the ground. (ii) The possibility and feasibility of the variable-structure pseudo-active actuator using semi-active actuators in the single-DOF operational mode to achieve pseudo-active output force are analyzed. One step further, the potential application areas of the proposed pseudo-active actuator in 2DOF operational mode with a further presented dual-hook suspension as an example are studied. (iii) The dynamic principles and system zeros and poles of the variable-structure pseudo-active actuator under different operational modes are analyzed, and a completely equivalent electrical network is established for subsequent related theoretical study.

2 | CONCEPT AND PRINCIPLE

The concept "pseudo-active actuator" is defined by the actuators that use semi-active controllable actuators to achieve active actuator performance. The pseudo-active actuators are then expected to possess both advantages of semi-active actuators—low energy consumption, simple structure, and high reliability, and of active actuators—four-quadrant execution capability as well as superior system control performance. Inspired by the performance analysis of a negative stiffness damper, as seen in Figure 2, we propose a basic design principle of the pseudo-active actuators in Figure 3. Observing Figure 3, the proposed pseudo-active actuator should consist of two half-four-quadrants, which may be implemented via two mechanisms. One is for the first and third quadrants and the other one is for the second and fourth. As seen from Figure 3, this study employs two semi-active controllable damping actuators as an example and one mechanical compensation mechanism. One of the actuators provides the damping force in the first and third quadrants, and the other combining with the mechanical compensation mechanism is for the second and fourth quadrants. As reasonable, it is inferred that pseudo-active actuators are also possibly realized via any combination of positive controllable stiffness/damping/inertia and mechanical compensation mechanism. Here it is noted that the combination of the mechanical compensation mechanism and another (controllable) stiffness/damping/inertia is used to provide (controllable) negative stiffness/damping/inertia in the second and fourth quadrants. For sure, if the second- and fourth-quadrant available outputs can be realized by specific actuators, such as permanent magnet, electromagnetic device, or others, then the mechanical compensation mechanism is not needed anymore.
According to the design principle presented in Figure 3, Figures 4A,B show the principle and equivalent dynamic model of a proposed pseudo-active actuator, respectively. This study takes the controllable damping for the pseudo-active actuator as an example, and any combined realization of controllable damping/controllable stiffness/controllable inerterance could be implemented under the proposed principal frame. According to Figure 4A, the pseudo-active actuator is composed of two semi-active actuators and a rack-pinion motion pair. Four different structural forms between the two rigid bodies and the two semi-active actuators can be completed with application requirements. In Figure 4A, \( y_o \), \( y_g \), and \( y_u \) are, respectively, the displacements of the output end, the ground end, and the input end, and these three components of this pseudo-active actuator are connected to the outside. The input end is connected to the excitation. The masses of the output end and the ground end are \( m_o \) and \( m_g \), respectively. \( k_l \) and \( k_r \) are the stiffnesses on the two sides. \( c_l \) and \( c_r \) are the damping coefficients of the dampers on the two sides. \( z_l \) and \( z_r \) are the racks on the left and right sides, and the mass is assumed to be negligible to simplify the analysis since their actual weight is significantly lighter compared to \( m_o \) and \( m_g \) in practical use. \( z_g \) is the grounded gear, and the moment of inertia is assumed to be negligible for its low rotation rate in practical use. The index circle radius is \( r \). The grounded gear and the left and right racks form a mechanical compensation mechanism to ensure that the racks on the two sides are equal and opposite to the center speed of the rotation pair of the grounded gear. \( F_l \) and \( F_r \) are the forces on the left and right racks, respectively. The compression direction is defined as the positive direction.
According to Figure 4A and the d’Alembert principle, the output end is given by

\[ m_2 \ddot{y}_o + (c_1 + c_2)(\ddot{y}_o - \ddot{y}_c) + c_1(\dot{y}_o - \dot{y}_c) + c_2(\ddot{y}_o - \ddot{y}_c) + k_1(\dot{y}_o - \dot{y}_c) + k_2(\ddot{y}_o - \ddot{y}_c) = 0 \]  

(1)

and can be rewritten as

\[ m_2 \ddot{y}_o + 2c_1(\ddot{y}_o - \ddot{y}_c) + c_1(\dot{y}_o - \dot{y}_c) + c_2(\ddot{y}_o - \ddot{y}_c) + 2k_1(\dot{y}_o - \dot{y}_c) + k_1(\ddot{y}_o - \ddot{y}_c) = 0, \]  

(2)

where \( c_1 \) and \( c_2 \) are the damping coefficients with the same absolute value while opposite mathematical sign; \( k_1 \) and \( k_2 \) are the stiffness difference coefficients with the same absolute value while opposite mathematical sign.

According to the d’Alembert principle, the force analysis of the grounded gear \( z_g \) in Figure 4 can be expressed as

\[ m_0 \ddot{y}_g + 2r F_l = 0. \]  

(3)

For the left rack, its force \( F_l \) is

\[ F_l = c_1(\ddot{y}_g - \ddot{y}_o) + k_1(\dot{y}_g - \dot{y}_o) + c_2(\ddot{y}_g - \ddot{y}_o) + k_2(\dot{y}_g - \dot{y}_o). \]  

(4)

Combining Equations (3) and (4) yields

\[ m_0 \ddot{y}_g + 2c_1(\ddot{y}_g - \ddot{y}_o) + 2k_1(\dot{y}_g - \dot{y}_o) + 2c_2(\ddot{y}_g - \ddot{y}_o) + 2k_2(\dot{y}_g - \dot{y}_o) = 0. \]  

(5)

According to Equations (2) and (5), the equivalent dynamic model of the pseudo-active actuator can be drawn in reverse as shown in Figure 4B. Further, to control the relative motion relationship between the components in the mechanism and thereby change the topological relationship between the components in the equivalent dynamic model, switches \( S_0 \) and \( S_1 \) are added to the figure. The switches are connected between the ground end on the structure and the actual ground, and the ground end and the output end on the structure, and the two components connected to the switch have no relative displacement after being turned on.

According to the “on” and “off” states of the switches \( S_0 \) and \( S_1 \), the pseudo-active actuator can be summarized into four operational modes. (i) Mode I: when both \( S_0 \) and \( S_1 \) are on, \( m_0 \) and \( m_g \) are both connected to the ground. In this mode, the actuator has zero DOF and realizes the basic mechanical performance of the semi-active actuator. (ii) Mode II: when \( S_0 \) is on and \( S_1 \) off, \( m_g \) is connected to the ground. In this mode, the actuator has a single DOF. Negative damping is added to the positive damping of the semi-active actuator via the mechanical compensation mechanism. That is, the execution capability of the active actuator is realized then. Essence and definition of pseudo-active actuator: the input end characteristic is a semi-active actuator while the output end characteristic is an active actuator, are introduced. (iii) Mode III: when both \( S_0 \) and \( S_1 \) are off, the actuator has two DOFs. In this operational mode, relationships among the input end and the two masses are constructed. The connection is different from the conventional quarter-car model and dynamic vibration absorber model, which is called a dual-hook automobile suspension structure or a pseudo-active suspension. (iv) Mode IV: when \( S_0 \) is off and \( S_1 \) on, the actuator has a single DOF. In this mode, there is no relative movement between the two masses. Regardless of the internal parameters of the mechanism system, it shows a damped single DOF to the outside. This study will not make a specific analysis of its characteristics.

### 3 | PERFORMANCE ANALYSIS

#### 3.1 | Proof of feasibility of active control

As shown in Figure 4B, when the switches \( S_0 \) and \( S_1 \) are both on, the force \( F_{m_0} \) on \( m_0 \) is given by

\[ F_{m_0} = (c_r - c_l) \dot{y}_u + (k_r - k_l) y_u. \]  

(6a)

When \( S_0 \) is on and \( S_1 \) off, the force \( F_{m_0} \) on \( m_0 \) is

\[ F'_{m_0} = (c_r - c_l) \dot{y}_u + (k_r - k_l) y_u + 2c_1 \ddot{y}_o + 2k_1 \dot{y}_o. \]  

(6b)

The input force \( F_{\text{input}} \) at the input end is

\[ F_{\text{input}} = (c_r + c_l) \dot{y}_u + (k_r + k_l) y_u. \]  

(7)

According to Equation (6a), when \( S_0 \) and \( S_1 \) are both on, the force \( F_{m_0} \) on \( m_0 \) is determined by four parameters, that is, the damping coefficient difference \( c_r - c_l \), the stiffness difference \( k_r - k_l \), the input velocity \( \dot{y}_u \), and the input displacement \( y_u \). The force \( F_{m_0} \) on \( m_0 \) is the output as the desired force \( F_{\text{desired}} \), and the desired force output can be achieved by changing the four parameters. Among the four parameters, only the damping coefficient difference \( c_r - c_l \) can be tuned arbitrarily, and the stiffness difference \( k_r - k_l \) is set as a known quantity. Mathematically, for any desired force \( F_{\text{desired}} \) at any time, as long as the inputs \( y_u \) and \( \dot{y}_u \) are given, the required damping coefficient difference \( c_r - c_l \) can be obtained. It is given by

\[ c_r - c_l = \frac{F_{\text{desired}} - (k_r - k_l) \dot{y}_u}{\dot{y}_u}. \]  

(8)

Smith defines passive elements as

\[ \int_0^\infty f(t) \nu(t) \, dt \geq 0, \]  

(9)

where \( t \) is the continuous time; \( f(t) \) is the force on the two ends of the element, and the positive direction is the compression direction of the element; \( \nu(t) \) is the velocity at the two ends of the element, and the positive direction is the direction in which the two ends of the element approach each other.
According to Equation (9), for passive elements, mechanical energy can only flow into the element from the outside, but the element cannot output mechanical energy to the outside. For semi-active elements, it is essentially passive elements with adjustable parameters/performances. So Equation (9) will also describe the characteristics of the semi-active elements. Further for passive damping elements, Equation (9) is strictly greater than zero. The value range of Equation (9) is an entire real number range for active elements. Figures 1A,B are the working ranges of semi-active (passive) actuators and active actuators, respectively. That is, the working range of active actuators is the entire plane, while the working range of semi-active (passive) actuators is only partial first and third quadrants, that is, Q1 and Q3.

If S0 is on and S1 off, and the mass m0 is set to zero, the input end y0 and the ground end y2 are regarded as the two ends of a certain component, and the ground end y2 and the output end y6 as two parts of a certain component, these two components meet the requirements of the components in the mechanical network. It means that the masses can be ignored, the two ends have relative displacement, and the performance is not affected by the spatial position. The ground end y2 is the common port of these two components. Solution of the input mechanical energy of the two components can be obtained via integration of Equations (6b) and (7) over the time t:

\[
W_{\text{inpt}} = \int_0^t [(c_1 + c_2)\dot{y}_2 + (k_1 + k_2)y_2]\dot{y}_2 \, dt \\
+ \int_0^t [(2c_1 + 2k_1 + 2k_2 + 2c_2)\dot{y}_2^2 + k_2y_2^2] \, dt + ky_2^2.
\]

(10a)

\[
W_{\text{outpt}} = \int_0^t [(c_1 + c_2)\dot{y}_2 + (k_1 + k_2)y_2]\dot{y}_2 \, dt \\
= (c_1 + c_2)\int_0^t \dot{y}_2^2 \, dt + \frac{1}{2}(k_1 + k_2)y_2^2.
\]

(10b)

where \(W_{\text{inpt}}\) is the input mechanical energy of the component between the ground end and the output end; \(W_{\text{outpt}}\) is the input mechanical energy of the component between the ground end and the input end. When the output velocity \(\dot{y}_2\) and the output displacement \(y_2\) are small, the value range of \(W_{\text{outpt}}\) is the entire real number range, while the value range of \(W_{\text{inpt}}\) can only be nonnegative real numbers. It shows that the components between the ground end and the output end have the characteristics of an active actuator, while the components between the ground end and the input end have the characteristics of a passive element.

On the basis of the above analysis, it can be concluded that the mechanical performance of the active actuator as shown in Figure 1B can be realized in theory by using the semi-active actuators with a mechanical compensation mechanism.

### 3.2 Case study of active control

Setting S0 on, S1 off, and m0 nonzero, the operational mode of the pseudo-actuator turns to be a case of using semi-active actuators to achieve active control. On the basis of the d’Alembert principle, we have the expression for \(m_0\):

\[
m_0\ddot{y}_0 + (c_1 + c_2)\dot{y}_0 + (k_1 + k_2)y_0 = (c_1 - c_2)\dot{y}_0 + (k_1 - k_2)y_0.
\]

(11)

The Laplace transform of transmissibility \(y_0/y_u\) of the controlled rigid body to the input displacement is given by

\[
y_0/y_u = \frac{(c_1 - c_2)s + (k_1 - k_2)}{m_0s^2 + (c_1 + c_2)s + (k_1 + k_2)}.
\]

(12)

The theoretical proof presented in Section 3.1 is simulated and verified: assuming \(m_0 = 1\) kg, \(k_1 = k_2 = 0.5\) N/m, and setting constraint \(c_1 + c_2 = 1.4\) Ns/m to ensure that a unique solution can be obtained when calculating the control variables \(c_1\) and \(c_2\). The active control target is that the amplitude of output force equals 1 N and the frequency is an integer multiple of the input excitation \(y_u\), so that \(m_0\) vibrates at an integer multiple of the input excitation frequency of \(y_u\).

Combining Equation (6a) and the assumption above yields

\[
F_{\text{des}} = \sin \omega_{\text{oupt}} t = y_0(c_1 - c_2).
\]

Excitation input is expressed as \(y_u = \sin \omega_{\text{inpt}} t\). \(\omega_{\text{inpt}}\) and \(\omega_{\text{oupt}}\) are the excitation frequency and output frequency, respectively. Therefore, the value of the damping coefficient difference \(c_1 - c_2\) can therefore be expressed in the time domain and s domain (Laplace transform) as

\[
c_1 - c_2 = \frac{\sin \omega_{\text{oupt}} t}{\omega_{\text{inpt}}} \frac{\omega_{\text{oupt}}}{\cos \omega_{\text{oupt}} t}.
\]

(13a)

\[
c_1 - c_2 = \frac{1}{s^2 + \omega_{\text{oupt}}^2} \frac{s^2 + \omega_{\text{inpt}}^2}{s^2 + \omega_{\text{oupt}}^2} \omega_{\text{inpt}}.
\]

(13b)

Hence, the damping coefficient difference \(c_1 - c_2\) is chosen as the control variable for the responses of the system described by Equation (11).

Because \(c_1 + c_2 = 1.4\) Ns/m, the limited range of damping coefficient difference is \(c_1 - c_2 \in [-1.4, 1.4]\). Figure 5A shows the simulation results of the active-controlled output and Figure 5B shows those of the output frequency control effect. Excitation input was assumed to be \(y_u = \sin(1t)\) in the simulation.

According to Figure 5A, the pseudo-active actuator can use semi-active actuators to achieve an active-controlled output force in this mode, which is different from the excitation frequency. It can be seen from Figure 5B that the controlled rigid body \(m_0\) is able to move at a frequency different from the excitation frequency. It indicates that the pseudo-active actuator has the potential to achieve more complex active control effects.

### 4 CHARACTERISTICS ANALYSIS OF 2DOF OPERATIONAL MODE

#### 4.1 Mathematical model and nondimensional analysis of 2DOF operational mode

The single-DOF operational mode (Mode II) of the pseudo-active actuator analyzed in Section 3 shows a flaw: the port shared between
two equivalent components must be connected to the ground. After disconnecting the common port from the ground, that is, both $S_0$ and $S_1$ are off, the pseudo-active actuator is in a 2DOF operational mode, i.e., Mode III. As shown in Figure 4B, it is found that the pseudo-active actuator has two elements with negative stiffness $k_l$ and negative damping $c_l$ in the 2DOF operational mode. They are introduced by the mechanical compensation mechanism after the new topological construction. This new 2DOF model is different from the widely accepted conventional 2DOF model in Figure 6, in the topological relationship between the components and the characteristics of the used components. As seen from Figure 6, the upper mass in the conventional 2DOF model does not have a direct connection with excitation, and if any relative movement is disabled among the two masses and the excitation, the same single-DOF model will be derived. Then, it is necessary to compare this new form of the 2DOF model with the widely accepted one. This section will analyze the passive performance of the new 2DOF model to establish the fundamentals for the development of control algorithms in the future.

Nondimensional method is employed to simplify the model so as to conduct the performance analysis of the new 2DOF operational mode based on the pseudo-active actuator. Combining Equations (2) and (5) in the 2DOF operational mode, we obtain

$$\frac{y_s}{y_u} = \frac{a_{n0}s^3 + a_{n1}s^2 + a_{n2}s + a_{n3}}{b_{n4}s^4 + b_{n3}s^3 + b_{n2}s^2 + b_{n1}s + b_{n0}}. \quad (14)$$

$$\frac{y_c}{y_u} = \frac{c_{n3}s^2 + c_{n2}s + c_{n1} + c_{n0}}{b_{n4}s^4 + b_{n3}s^3 + b_{n2}s^2 + b_{n1}s + b_{n0}}. \quad (15)$$

where the subscript “n” represents “new”.

Let

Natural frequency of $m_o$: $\omega_{n2} = \sqrt{\frac{k_l - k}{m_o}}. \quad (16a)$

Damping ratio of $m_o$: $\xi_{n2} = \frac{c_l - c}{2\sqrt{k_l m_o}}. \quad (16b)$

Damping ratio of $m_g$: $\xi_{n1} = \frac{c_l}{\sqrt{k_l m_g}}. \quad (16c)$

Frequency ratio: $\alpha_n = \frac{\omega_{n2}}{\omega_{n1}}. \quad (16d)$

Mass ratio: $\mu_n = \frac{m_o}{m_g}. \quad (16e)$

Unit excitation frequency: $\ddot{s} = \frac{s}{\omega_{n1}}. \quad (16f)$

Incorporating Equations (14) and (15) into Equations (16a)–(16g), we obtain

$$\frac{y_s}{y_u} = \frac{a_{n0}s^3 + a_{n2}s^2 + a_{n1}s + a_{n0}}{b_{n4}s^4 + b_{n3}s^3 + b_{n2}s^2 + b_{n1}s + b_{n0}}. \quad (17)$$
\[
\frac{y_2}{y_u} = \frac{c_{10}s^3 + c_{12}s^2 + c_{11}s + c_{10}}{b_{44}s^4 + b_{13}s^3 + b_{13}s^2 + b_{13}s + b_{10}}.
\] (18)

The dimensional and nondimensional parameters of Equations (14), (15), (17), and (18) are listed in Table 1.

The governing dynamic equation of the conventional 2DOF model is given by

\[
\frac{y_2}{y_u} = \frac{a_{12}s^2 + a_{11}s + a_{10}}{b_{44}s^4 + b_{13}s^3 + b_{13}s^2 + b_{13}s + b_{10}},
\] (21a)

\[
\frac{y_1}{y_u} = \frac{c_{12}s^2 + c_{11}s + c_{10}}{b_{44}s^4 + b_{13}s^3 + b_{13}s^2 + b_{13}s + b_{10}}.
\] (21b)

where the subscript “c” represents “conventional”.

Similarly, the nondimensional method is used to simplify the calculation:

Natural frequency of \(m_2\) : \(\omega_{c,2} = \sqrt{\frac{k_2}{m_2}}\). (21c)

Damping ratio of \(m_2\) : \(\xi_{c,2} = \frac{c_2}{2\sqrt{k_2}m_2}\). (21d)

Natural frequency ratio : \(c_n = \frac{\omega_2}{\omega_1}\). (21e)

Mass ratio : \(\mu_c = \frac{m_2}{m_1}\). (21f)

Unit excitation frequency : \(\bar{s} = \frac{s}{\omega_{c,1}}\). (21g)

Incorporating Equations (21a)–(21g) into Equations (19) and (20), we have

\[
\frac{y_2}{y_u} = \frac{a_{12}s^2 + a_{11}s + a_{10}}{b_{44}s^4 + b_{13}s^3 + b_{13}s^2 + b_{13}s + b_{10}}.
\] (22)

\[
\frac{y_3}{y_u} = \frac{c_{12}s^2 + c_{11}s + c_{10}}{b_{44}s^4 + b_{13}s^3 + b_{13}s^2 + b_{13}s + b_{10}}.
\] (23)

The dimensional and nondimensional parameters of Equations (19), (20), (22), and (23) are listed in Table 2.

### 4.2 | Influence of nondimensional parameters on the steady-state performance of the system

Both the new and the conventional 2DOF models have four nondimensional parameters, that is, the natural frequency ratios \(a_n\) and \(c_n\); the mass ratios \(\mu_a\) and \(\mu_c\); the damping ratios \(\xi_{c,1}, \xi_{c,2}, \xi_a,\) and \(\xi_c\). To make the analysis results more intuitive, the natural frequencies of the two 2DOF models are set differently. The default natural frequency ratios \(a_n\) and \(\alpha_c\) are set to 5 and their variation range is 0–10. We will only analyze the response of the new 2DOF model when \(\xi_{c,2} > 0\). The default value and variation range of the other nondimensional parameters are set as follows: \(\mu_n = \mu_c = 1, \mu_a = \mu_c \in [0, 10], \xi_{c,1} = \xi_{c,2} = \xi_a = 0.7, \) and \(\xi_c \in [0, 1].\) The influence of the natural frequency ratio \(\alpha_n\) on the frequency responses of the new 2DOF model is shown in Figures 7A,B, and \(\alpha_c\) of the conventional 2DOF model in Figures 8A,B.

According to Figure 7, for the new 2DOF model, both the frequency responses of \(m_2\) and \(m_3\) show only one natural frequency, which is similar to the response of a low-pass filter or a single-DOF model. In Figures 7A,B, \(\alpha_n\) only influences significantly the cut-off

| TABLE 1 Dimensional and nondimensional parameters of the new 2DOF model |
|--------------------------|--------------------------|
| Dimensional parameters   | Nondimensional parameters |
| \(a_{10} = m_b c_i - m_c c_i\) | \(a_{10}' = 8a_l \mu C_{d1}\) |
| \(a_{12} = 4c_i c_j - m_b k_j + m_c k_j\) | \(a_{12}' = 16a_l \mu C_{d1} \xi_{d2} + 4a_l^2 \mu C_{d1}\) |
| \(a_{11} = 4k_j c_i + 4k_j c_i\) | \(a_{11}' = 8a_l \mu C_{d1} + 8a_l^2 \mu C_{d1}\) |
| \(b_{44} = m_b m_b\) | \(b_{44}' = 4a_l \mu + 1\) |
| \(b_{14} = m_b m_c\) | \(b_{14}' = 4a_l \mu + 1\) |
| \(b_{30} = m_b c_i + m_c c_i + 4m_c c_i\) | \(b_{30}' = 8a_l \mu C_{d1} + 8a_l^2 \mu C_{d1}\) |
| \(b_{12} = m_b k_j + m_c k_j + 4c_i c_j + 4m_j k_j\) | \(b_{12}' = 16a_l \mu C_{d1} \xi_{d2} + 4a_l^2 \mu \mu + 4a_l^2 \mu\) |
| \(b_{11} = 4k_j c_i + 4k_j c_i\) | \(b_{11}' = 8a_l \mu C_{d1} + 8a_l^2 \mu C_{d1}\) |
| \(c_{13} = 2m_c c_i\) | \(c_{13}' = 4a_l \mu C_{d1}\) |
| \(c_{12} = 2m_c k_j + 4c_i c_i\) | \(c_{12}' = 2a_l \mu C_{d1} + 16a_l \mu C_{d1} \xi_{d2} + 4a_l^2 \mu\) |
| \(c_{11} = 4k_j c_i + 4k_j c_i\) | \(c_{11}' = 8a_l \mu C_{d1} + 8a_l^2 \mu C_{d1}\) |
| \(c_{10} = 4k_j c_i\) | \(c_{10}' = 4a_l \mu \mu + 1\) |

Abbreviation: 2DOF, two degrees-of-freedom.
The dimensional and nondimensional parameters of the conventional 2DOF model are shown in Table 2.

| Dimensional parameters | Nondimensional parameters |
|------------------------|---------------------------|
| $a_{22} = c_1 c_2$    | $a_{22} = 4 a_c \xi_1 \xi_2$ |
| $a_{11} = k_1 c_2 + k_2 c_1$ | $a_{11} = 2 a_c \xi_2 + 2 a_c^2 \xi_1$ |
| $a_{10} = k_1 k_2$    | $a_{10} = a_c^2$ |
| $b_{22} = m_1 m_2$    | $b_{22} = 1$ |
| $b_{11} = c_1 m_2 + c_2 m_1$ | $b_{11} = 2 \xi_2 + 2 \mu_c + 2 a_c \xi_2$ |
| $b_{12} = c_1 c_2 + k_1 m_2 + k_2 m_1$ | $b_{12} = 4 a_c \xi_1 \xi_2 + 1 + a_c^2 \mu_c + a_c^2$ |
| $b_{10} = k_1 c_2 + k_2 c_1$ | $b_{10} = 2 a_c \xi_2 + 2 a_c^2 \xi_1$ |
| $c_{10} = k_1 k_2$    | $c_{10} = a_c^2$ |

Abbreviation: 2DOF, two degrees-of-freedom.

Furthermore, since the two controllable damping actuators in the two 2DOF models are connected to $m_1$ (or $m_2$), both the damping ratios $\xi_1$ (or $\xi_1$) and $\xi_2$ (or $\xi_2$) influence the response of $m_1$ (or $m_2$). Thus analysis of the two 2DOF models is conducted with the combined considered of the damping ratios $\xi_1$ (or $\xi_1$) and $\xi_2$ (or $\xi_2$).

Figure 11 shows the influence of the damping ratios $\xi_1$ and $\xi_2$ on the frequency response of the new 2DOF model. The influence of the damping ratios $\xi_1$ and $\xi_2$ on the gain at the two natural frequencies of the new 2DOF model is shown in Figure 12. Again, for comparison and analysis, the influences of the damping ratios $\xi_1$ and $\xi_2$ on the frequency response and the gains at the natural frequencies of the conventional 2DOF model are presented in Figures 13 and 14, respectively.

Figure 11 shows that the frequency responses of $m_1$ and $m_2$ are similar to those in a single-DOF system. It can be seen from Figure 11A that $\xi_1$ has no obvious effect on the gain at any frequency of $m_0$. In Figure 11B, for the new 2DOF model, $\xi_1$ only affects the gain of $m_1$ at its natural frequency. In Figure 11C, $\xi_2$ has a significant effect on the gain near the natural frequency of $m_0$, but little influence on the gain at the first-order natural frequency of $m_1$. As $\xi_2$ increases, the gain at the second-order natural frequency of $m_1$ decreases. But when $\xi_2 > 0.2$, the response of $m_1$ changes little, as shown in Figure 11D. It is seen that the responses of $\xi_1$ and $\xi_2$ to $m_1$ and $m_2$ are decoupled, according to the analysis of the influence of $\xi_1$ and $\xi_2$ on the gains at the first- and second-order natural frequencies of $m_1$ and $m_2$.

In Figures 12A,B, the gradient lines are almost parallel to the coordinate axis. It means that $\xi_2$ is the corresponding dominant factor that affects $m_2$ at 5 rad/s, while having no obvious effect on the gain of $m_1$ at 1 rad/s. Similarly, $\xi_1$ is the dominant parameter that affects the gain of $m_2$ at 1 rad/s. It further confirms the decoupling of the responses of $\xi_1$ and $\xi_2$ to $m_1$ and $m_0$.

For the conventional 2DOF model, it can be seen from Figure 13 that both $\xi_1$ and $\xi_2$ affect the gains of $m_1$ and $m_2$ at their first- and second-order natural frequencies. But they are synchronous effects, and the cases for $m_1$ and $m_2$ cannot be distinguished from the frequencies. It means that the effects of $\xi_1$ and $\xi_2$ on the responses of $m_1$ and $m_2$ are coupled.

In Figures 14A,B, the angles between the gradient lines and the coordinate axis are large, and the effects of $\xi_1$ and $\xi_2$ are coupled to the responses of $m_1$ and $m_2$. It is worth noting that the response of $m_2$ in Figure 14A is negative, while the Z-axis coordinate scale is also different from those in Figures 12 and 14B.

### 4.3 Influence of nondimensional parameters on the transient performance of the system

As seen in the parameters listed in Tables 1 and 2, the four nondimensional parameters contain higher-order terms, and the root locus expression cannot be obtained for more intuitive demonstration. In this study, we use the parameter-system pole distribution diagram instead, with assumption $\mu_n = \mu_c = 1$, $a_n = a_c = 5$, and $\xi_n = \xi_2 = \xi_1 = \xi_2 = 0.7$. Each parameter takes a value from...
FIGURE 7  Influence of natural frequency ratio $\alpha_n$ on frequency responses of the new 2DOF model: (A) $m_o$ and (B) $m_g$. 2DOF, two degrees-of-freedom

FIGURE 8  Influence of natural frequency ratio $\alpha_c$ on frequency responses of the conventional 2DOF model: (A) $m_2$ and (B) $m_1$. 2DOF, two degrees-of-freedom

FIGURE 9  Influence of mass ratio $\mu_n$ on frequency responses of the new 2DOF model: (A) $m_o$ and (B) $m_g$. 2DOF, two degrees-of-freedom
FIGURE 10  Influence of mass ratio $\mu_c$ on frequency responses of the conventional 2DOF model: (A) $m_2$ and (B) $m_1$. 2DOF, two degrees-of-freedom.

FIGURE 11  Frequency responses of the new 2DOF model with single changing damping ratio: (A) $\xi_{n1}$ on $m_o$, (B) $\xi_{n1}$ on $m_g$, (C) $\xi_{n2}$ on $m_o$, and (D) $\xi_{n2}$ on $m_g$. 2DOF, two degrees-of-freedom.
FIGURE 12  Frequency responses of the new 2DOF model with two changing damping ratios: (A) $m_2$ amplitude at 5 rad/s (natural frequency) and its contour and (B) $m_2$ amplitude at 1 rad/s (natural frequency) and its contour. 2DOF, two degrees-of-freedom.

FIGURE 13  Frequency responses of the conventional 2DOF model with single changing damping ratio: (A) $\xi_1$ on $m_2$, (B) $\xi_1$ on $m_1$, (C) $\xi_2$ on $m_2$, and (D) $\xi_2$ on $m_1$. 2DOF, two degrees-of-freedom.
1e−3 to 1e3, whose scope is sufficient to cover most engineering applications. The parameter-system pole diagram is shown in Figure 15.

It is known that the pole closest to the imaginary axis and near-zero is the dominant pole of the system. It has a large time constant and a slower settling time, which plays a leading role in the time domain response of the system. Therefore, the stability of the system should be compared with the real part of the pole closest to the imaginary axis. In Figure 15A, the new 2DOF model has only imaginary complex roots for the root locus of the changes of $\alpha_n$ and $\alpha_c$, while the conventional 2DOF model has real roots. In contrast, in Figure 15B, the new 2DOF model has real roots for the root locus of the changes of $\mu_n$ and $\mu_c$, while the conventional one has only imaginary complex roots except for the zero poles. As observed in Figure 15C, when $\xi_{22} > 0$, the stability of the new 2DOF model is greater than that of the conventional one near the default parameters. The new 2DOF model is more stable than the conventional one. When $\xi_{22} < 0$, as shown in Figure 15D, the roots of the new 2DOF model are mostly distributed on the right half plane. The system is unstable. For the conventional 2DOF model, no matter how the nondimensional parameters change, the roots of the system are distributed on the negative half plane. It indicates that the system is stable. Hence a stable parameter set of the new 2DOF model system is given to ensure system stability in practical applications.

### 4.4 Conditions for system stability

To ensure the stability of the new 2DOF model in practical applications, it is of significance to find the corresponding parameter set. The Routh–Hurwitz criterion is a necessary and sufficient condition for system stability. Conditions for the Routh–Hurwitz criterion of the fourth-order system are

\[ b_i > 0, \quad i \in \{1, 2, 3, 4\}, \quad (24a) \]

\[ b_2 b_3 - b_1 b_4 > 0, \quad (24b) \]

\[ b_1 b_2 b_3 - b_1^2 b_4 - b_3 b_5^2 > 0. \quad (24c) \]

The adopted nondimensional model requires that $\omega_{n2} = \sqrt{\frac{k_n - k_i}{m_0}}$ is a real number. It implies that $k_n - k_i > 0$. Therefore, dimensional parameters are used for calculation. Incorporating the parameters in Table 1 into Equations (24a)–(24c), the stability conditions for the new 2DOF model are

\[ m_2 c_1 [c_i (k_2 + k_i^2) + (k_n c_i - k_i c_i)^2] + 4m_n c_i c_i (k_n c_i + k_i c_i) (c_i + c_i) + 16m_n c_i c_i (k_n c_i - k_i c_i) + 16m_2 k_2 c_i c_i + 8m_n m_2 c_i c_i (k_n - k_i) > 0. \quad (25) \]

According to Equation (25), the system stability can be guaranteed when the equation is true, regardless of the values of the other two-dimensional parameters.

### 5 CASE STUDY OF 2DOF OPERATIONAL MODE—A DUAL-HOOK SUSPENSION

According to the analysis in Section 4, for the new 2DOF model of the pseudo-active actuator: (i) the change of the mass ratio of the 2DOF affects little on the response of the model; and (ii) the two damping ratios $\xi_{11}$ and $\xi_{22}$ decouple the 2DOF separately. Therefore, the possible applications of the 2DOF model would be: the mass ratio changes are large and the 2DOF has different requirements for support capacity and vibration isolation performance. Applications such as monocoque-powertrain systems (engine over body) and truck cabin-frame systems (cab over frame) are called dual-hook suspension. That is also a potential scenario for the implementation of the pseudo-active suspension.
Figure 16 shows a monocoque-powertrain front axle system as an example. In Figure 16, \( m_p \) is the mass of the powertrain; \( m_b \) is the mass of the body; \( \Delta m \) is the mass of occupants and cargo; \( k_f \) and \( c_f \) are the stiffness and damping of the front suspension, respectively; \( k_r \) and \( c_r \) are the stiffness and damping of the rear suspension, respectively; \( k_e \) and \( c_e \) are the vertical stiffness and damping of the powertrain suspension, respectively; \( y_c \), \( y_b \), and \( y_w \) are the vertical displacements of the powertrain, the hinge point on the front suspension, and the front wheel center, respectively; \( a \) and \( b \) are the distances from the front wheel and rear wheel to the center of mass of the body, respectively; \( \rho \) is the inertia radius of the body relative to the Y-axis; and \( m_f \) is the mass of the body, occupants, and cargo on the front axle. It is given by

\[
m_f = (\Delta m + m_b) \frac{b}{a + b} \frac{\rho^2}{ab}.
\]  

(26)

The pitch and roll of the body are ignored, only the vertical movement of the body and the powertrain are considered. Further, the tire stiffness is ignored, and the body is assumed to be symmetrical with respect to the Y plane (society of automotive engineers) and the suspension mass distribution coefficient.
$p^2/ab = 1$. Then the system is simplified as a 2DOF system. $m_i$ and $m_e$ are the lower and upper DOF masses, respectively, which are consistent with those in Figure 6. The new 2DOF model is applied to the system as long as: (i) the wheel center is connected to the input end; (ii) the ground end is connected to the body; and (iii) the output end is connected to the powertrain. It is consistent with the model shown in Figure 4. Since the system can apply two 2DOF models at the same time, that is, the new and conventional 2DOF models, the passive performances of the system using the two 2DOF models are compared, and the influences of body mass changes on the system response are also compared.

The parameters of the linearized vehicle model are set as follows: $m_e = 150 \text{ kg}$, zero load front axle portion of body weight $m_{0e} = 600 \text{ kg}$, full load front axle portion of body weight $m_{fe} = 750 \text{ kg}$, front suspension stiffness $k_i = 900 \text{ N/m}$, and front suspension damping $c_i = 900 \text{ Ns/m}$. The dynamic stiffness of the powertrain mount $k_l = 4 \times 10^5 \text{ N/m}$, and the equivalent vertical damping coefficient $c_e = 5 \times 10^3 \text{ Ns/m}$.

To apply the new 2DOF model to the system, a method of equivalent stiffness and damping is employed. First, there must be sufficient stiffness between the suspension and the body to meet the support requirements of the suspension. Second, the stiffness between the frame and the powertrain should also be equivalent to meet the support requirements of the suspension. The expression of the requirements is given by

$$k_i = k_i + k_e, \quad (27a)$$

$$\frac{k_i + k_e}{2k_i(k_i - k_e)} + 2k_e = k_e. \quad (27b)$$

Equation (27a) is derived by assuming no displacement between powertrain and body while the two models have the same stiffness between wheel and body. Equation (27b) is derived using a similar method.

Combining Equations (27a) and (27b),

$$k_i^2 - \frac{k_i + k_e}{2}k_i^2 + \frac{k_i k_e}{4}k_i - \frac{k_i}{8} = 0. \quad (28)$$

Observing the cubic polynomial Equation (28), there are two forms of the roots, namely, three real roots, a real root, and a pair of conjugate complex roots. But whatever the form of the root is, there must be a real root greater than zero. So $k_l$ is taken as the largest real root of Equation (28). According to Equations (26) and (28), $k_l = k_i = 2 \times 10^5 \text{ N/m}$ is obtained. The damping is equivalent in the same way, and it is calculated as $c = c_r = 2500 \text{ Ns/m}$.

For the system under the conditions of full load and zero load conditions, the frequency response of the body and the powertrain is shown via a Log-log scale plot in Figure 17. Observing the frequency response of the two 2DOF models in Figure 17A, the gain of the new 2DOF model in the low-frequency range of 0–10 rad/s is much smaller than that of the conventional one. The new 2DOF model can reduce the transmissibility of road excitation to the body, and thus improve ride comfort. The conventional 2DOF model shows superiority for the microscopic fluctuations of the road surface at 10–40 rad/s in the middle and low frequency and 70–100 rad/s in the high frequency. As seen from Figure 17B, the new 2DOF model can only achieve a smaller gain than the conventional ones in the low-frequency range of 0.2–0.5 rad/s. At higher frequencies, the gain is much greater than that of the conventional one. But, it is worth noting that this study only uses a very simple parameter equivalent method and compares passive performances. The performance of the new 2DOF model featuring the pseudo-actuator with the four-quadrant output performance will certainly be much improved if a more theoretically based parameter equivalent method and semi-active control algorithm are used.

6 | MECHANICAL–ELECTRICAL ANALOGY

Similarities are found in the mechanical and electrical networks. Usually, the inertial element in the mechanical network is equivalent to the capacitor in the electrical network, the elastic element is equivalent to the inductor, and the damping element is equivalent to...
the resistor. The force in the mechanical network is equivalent to current, and the velocity is equivalent to voltage.30 According to the equivalent method, the mechanical network shown in Figure 6 is equivalent to the electrical network presented in Figure 18. Such a method will be of convenience to implement topology optimization and derivation of the pseudo-active actuator principle. According to Figure 18, the impedances of the network can be expressed as

\[
Z_{o8}(s) = \frac{s}{2s c_1 + 2k_1},
\]

\[
Z_{o9}(s) = \frac{s}{s (c_2 - c_1) + (k_2 - k_1)},
\]

\[
Z_{o10}(s) = \frac{s}{2s c_1 + 2k_1},
\]

\[
Z_{o11}(s) = \frac{s}{m_o s^2},
\]

\[
Z_{o12}(s) = \frac{s}{m_1 s^2},
\]

where \(E_o\) is the equivalent variable to the input velocity \(y_2\); \(E_g\) the velocity \(y_5\) of the mass \(m_2\); \(E_s\) the velocity \(y_6\) of the mass \(m_3\); \(i_{m_2}\) the interaction force between \(m_2\) and \(m_3\); \(i_{m_3}\) the interaction force between \(m_3\) and \(m_5\); \(i_1\) the external force on \(m_2\); \(i_2\) the external force on \(m_3\); \(i_{o1}\) the pseudo-active force exerted on mass \(m_5\) in Mode I; the three impedances \(Z_{o8}(s)\), \(Z_{o11}(s)\), and \(Z_{o12}(s)\) are all regarded as the inductance and resistance in parallel; when \(S_2\) and \(S_3\) are on, \(Z_o\) and \(Z_g\) are short-circuited to the ground, which is corresponding to the condition of no displacement from \(m_2\) and \(m_3\) to the ground. The force between \(m_3\) and \(m_5\) can be equivalent to the current passing through \(S_1\). Substituting Equations (29a)–(29e), it is found that the expression of \(i_{m2}/E_o\) is identical with Equation (6a).

When \(S_2\) is on and \(S_3\) off, \(Z_g\) is still short-circuited to the ground, the ground end \(m_5\) keeps no displacement, and the voltage across the impedance \(Z_o\) is the velocity of the output end \(m_5\). Substituting Equations (26a)–(26e), it can be found that the expression of \(E_2/E_o\) is the same as Equation (12). When \(S_2\) and \(S_3\) are off, voltages are detected on the two sides of impedance \(Z_o\) and \(Z_g\). At this time, the same as the mechanical network, the equivalent circuit also has two DOFs. Substituting Equations (29a)–(29e), the expression of \(E_2/E_o\) is seen to be the same as Equation (14), and \(E_g/E_o\) as Equation (15). The above analysis validates the equivalence of the electrical network and the proposed mechanical network.

As a briefly extended inspiration, the concept of the pseudo-active actuator, especially the concept of zero or negative dampings and the topology analysis method (i.e., mechanical-electrical analogy), might be helpful for the utilization of mechanical vibration, specifically energy harvesting, for vibration mitigation. If the proposed mechanisms of zero damping and negative damping can be added to the energy harvesters, the generated power and also efficiency will be enhanced. This is because the inherent damping of the energy harvesters might be eliminated by using negative damping. The corresponding part of the wasted power to the passive damping will be converted into generated power of the energy harvesters. On the other hand, similarly, the circuits for energy harvesting have the same issue of the inherent damping, which dissipates generated power and the circuit is not optimal. With the extension of the new concept of negative damping using the method of an equivalent electrical network, the design, and optimization of the circuits for energy harvesting, for example, matching the linear and nonlinear components of the circuits to decrease the dampings can be conducted. Combining all the above analyses, we may further realize a much higher efficiency energy harvester employing the principle of the pseudo-active actuator. A zero damping or negative damping of an energy harvesting circuit using the equivalent electrical network of the pseudo-active actuator may also be optimized and/or developed for the extremely energy scavenging.29

7 | CONCLUSIONS

Active actuators in control loops input energy to the systems, while passive/semi-active actuators only dissipate energy from the systems. This is the reason why active control systems provide better performance than passive/semi-active systems. Graphically, active actuators provide four-quadrant controllable execution capability in the force-velocity diagram, while passive/semi-active ones provide only part of the first and third quadrants. In this way, inspiration is reasonably activated: Do we have enough wisdom to use a combination of semi-active actuators to provide four-quadrant controllable execution capability? and how?

Aiming at realizing a new semi-active actuator with mechanical properties of the active actuators, this study proposed and investigated the concept and basic design principle of a pseudo-active actuator. The proposed pseudo-active actuator is composed of two semi-active actuators and one mechanical compensation mechanism. As an example, the two semi-active actuators are using a combination of semi-active controllable damping actuators and passive springs, and the mechanical compensation mechanism comprises two rack-pinion mechanisms using common gear. The mechanical compensation mechanism regulates the two semi-active actuators to move in reverse directions, and then the total output of the actuator with the mechanical compensation mechanism is the subtraction of the two semi-active actuators. The proposed pseudo-active actuator has three pins in the mechanical network. Four operational modes, that is, the zero-DOF operational mode (Mode I), the pseudo-active single-DOF operational mode (Mode II), the 2DOF operational mode (Mode III), and the semi-active single-DOF operational mode (Mode IV), are defined according to the different connections between the pins and the mechanical ground. The first three operational modes are analyzed in detail.
The output force in the zero-DOF operational mode realizes the goal of the active force output by the semi-active actuator, that is, the arbitrary output force is achieved by adjusting the difference between the damping coefficients of the two semi-active actuators. Switching the connection, one DOF is added into the system and the pseudo-active actuator transits its operational mode. The possibility of using semi-active actuators to realize active control is verified from the energy point of view, and a corresponding proof-of-concept study of the pseudo-active actuator in this operational mode is also provided. The 2DOF operational mode of the pseudo-active actuator is realized by disconnecting to the mechanical ground. It is found that compared to the conventional 2DOF model, the two masses of the pseudo-active actuator in the 2DOF operational mode are decoupled in terms of damping ratios. The stability of this operational mode is better than the conventional one under the same nondimensional parameters. As a result, the stable dimensional parameter set for the 2DOF operational mode is given. In addition, using the pseudo-active actuator, the concept of a dual-hook automobile suspension or a pseudo-active suspension is proposed. Preliminarily analysis was conducted via an application of the new 2DOF operational mode in the monocoque-powertrain system. As compared with the conventional 2DOF monocoque-powertrain system, it is found that the new 2DOF model has a smaller gain in the low-frequency range with the controller off (i.e., passive system), but the gain in the high-frequency range is greater than that of the conventional one. Finally, a mechanical-electrical analogy work was implemented for more convenient analysis, topology optimization, and derivation of the proposed concept. The corresponding analysis of the operational modes of the mechanical network and the equivalent electrical network is provided.

On the other hand, the combinations of the (controllable) negative stiffness actuator and (controllable) damping actuator, found in the recent literature, are regarded as a specific case of the pseudo-active actuator. The (controllable) negative stiffness provides the capacity for translating the first and third quadrants output to second and fourth quadrants, and the four-quadrant controllable execution capability is thus realized. Experimental validations on the significant improvement of the control systems support all the anticipated advantages of the pseudo-active actuators.

As a result, the proposed concept and principle of the pseudo-active actuator might be of help and significance for the wide performance-centered applications, such as vehicle suspensions and energy harvesters and their corresponding circuits, for providing an operable approach on zero-damping or even negative damping. Thereby great improvement of the vibration attenuation or energy scavenging efficiency might be obtained.

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CONFLICT OF INTEREST
The authors declare that there are no conflict of interest.

DATA AVAILABILITY STATEMENT
The data that support the findings of this study are available from the corresponding author upon reasonable request.

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