Where have all the large Representations gone?

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Gauge theories describe the interactions of the fundamental building blocks of nature with great success. The Standard Model achieves a partial unification of the electromagnetic and weak interactions, and it also accommodates the strong interactions. The known quarks and leptons appear in the fundamental representations (or singlets) of the $SU(3)_c \times SU(2)_L \times U(1)_Y$ gauge symmetry. However, larger representations (EW triplets, color sextes, etc.) could also occur in principle. Bounds on such exotic states based on electroweak precision tests, unitarity, perturbativity and collider searches, indicate that they should be very heavy or may be non-existent. But why only small representations occur in nature? Several ideas that could give some light into this problem are discussed here, including the approach of Nielsen et al, as well as the possible compositeness of quarks and leptons. Then, we discuss the problem within the context of grand unified theories, where a principle of "minimal complexity" is proposed to restrict the size of large representations, when they are required to form unified multiplets.

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I. INTRODUCTION

The success of the Standard Model (SM) relies on a plethora of experimental tests, which include now the discovery of the Higgs boson at the LHC collider experiments \textsuperscript{1,2}. The Higgs boson mass ($m_h = 125$ GeV) is consistent with Electro-Weak (EW) precision measurements, which were tested previously by the detection of the top quark, also with a mass lying in range expected from those precision tests \textsuperscript{3}. These results seems to confirm that the fundamental building blocks of nature and their interactions are well described within the framework of quantum field theory (QFT).

However, the merits of the SM are somehow obscured by its arbitrary number of free parameters (19, without considering neutrino masses and mixing), which seem to be large for a theory that us supposed to be a fundamental one. Besides the gauge couplings, we have the Higgs parameters which are associated with the spontaneous breaking of the electroweak symmetry \textsuperscript{4}, while the Yukawa parameters trigger the generation of fermion masses. Although the gauge couplings could be unified at the GUT mass scale, with some degree of success, there remains the $\theta$ parameter, which in itself poses a problem \textsuperscript{5}. The origin of these SM parameters is not known, despite the theoretical efforts from the last 30 years, such as supersymmetry, extra dimensions and composite models \textsuperscript{6,7}. Other problems that suggest the need for some extension of the SM include: the lack of explanations for dark matter, baryogenesis, inflation, which can not be explained within the minimal SM \textsuperscript{8}.

However, the searches for direct signals of new physics have resulted into bounds on the corresponding mass scale that are now entering into the multi-TeV range. Although we need to wait for future LHC stages in order to get stronger bounds on the new physics scale, and to reach a more solid conclusion on what comes after the SM, the fact that the LHC has not found evidence of new physics is perhaps telling us something important. May be we should start thinking at a deeper level and to scrutinize the SM structure, in order to find clues that may allow us to understand those open problems of the SM.

One such regularities of the SM, but for which we do not have an explanation, is the appearance of matter in representation of small size. Namely, after decades of experimental guide, ranging from beta-decay to the discovery of top quark, we know that matter appears only in certain representations of the gauge symmetries, namely the known quarks and leptons are either singlets or appear in the fundamental representations (doublets and/or triplets) of $SU(2)_L$ and $SU(3)_c$, respectively. Furthermore, the chiral fermions must have different properties, in order to fit the maximal parity violation observed in the weak interactions. Is this whole arrangement just something given? or is it telling us somethin deep? Is it an issue to ask where have all the large representations gone?

Furthermore, one can classify the fundamental constituents of the visible universe by tabulating the spin ($S$) and weak isospin ($T$) that could occur in a generic QFT, as shown in table 1. It seems remarkable that the whole (visible) universe is built out of some very few entries of this table. Although some of these states are realized in nature, there are other possibilities that have not been detected yet, despite having some strong theoretical motivations; such as
the spin-2 ($T = 0$) graviton, which appears in case the gravitational force were quantized as the other fundamental interactions of nature. Considerations based on supersymmetry provide motivations for the existence of its spin-3/2 superpartner (also with $T = 0$), the gravitino [9].

In this paper, we shall assume that the appearance of small representations within the SM needs some explanation, and shall discuss several aspects of the problem. First, in section 2 we discuss the known theoretical and experimental bounds on the mass scale of exotic representations, including electroweak precision tests, unitarity, perturbativity and collider searches. Then, in section 3 we discuss some ideas aimed to solve this problem, which we label as effective selection rules, here we include the work of H. Nielsen et al. [14], as well as the use of spacetime discrete symmetries; we close the section with a discussion of the implications on the representation problem from a possible sub-structure of quarks and leptons. Then, in section 4, we discuss how to restrict the matter representations following arguments based on the unification paradigm. We postulate a "Principle of Minimal Complexity" which could help to discriminate the allowed representations in nature, including the Higgs multiplets, whenever they are required to forma unified multiplets. Finally, section 5 contains our conclusions and outlook.

II. CONSTRAINTS ON LARGE REPRESENTATIONS

The fact that only a restricted set of representations of the gauge symmetry is realized in nature, suggest that some fundamental principle is missing in our current formulation of the SM; something that will tell us why nature only uses a few entries of the Table 1. The study of the properties and phenomenology of the missing large representations, provide some limits on its mass scale. Here, we shall consider the constraints derived from the following arguments:

1. Electroweak Precision tests (EWPT) The indirect effects of heavy quanta on physical observables, is conveniently represented by the Peskin-Takeuchi parameters ($S, T, U$), which has been treated in many works and applied to several cases. In particular, Ref. [10] contains the evaluation of the effects from higher-dimensional fermion representations on the $S, T, U$ parameters. It is found that in order to keep $T$ UV finite one needs to impose a sum rule for the states with isospin $j, j_3 = l$ and corresponding mass $m_j^2$, i.e.:

$$\sum_l (j^2 + j - 3l^2)m_j^2 = 0,$$

(1)

This relation is satisfied automatically for a weak doublet ($T = 1/2$), but for general isospin it imposes constraints on the mass of the multiplet components, which look somehow ad hoc. In fact, this result could be already used to look at the large representations with suspicious eye.

2. Unitarity bounds. Invoking the unitarity bound has been a useful tool in order to analyze the high-energy behavior of a given QFT. Here, one requires that the zeroth partial wave amplitude ($a_0$) satisfies the tree-level partial wave unitarity, i.e. $|Re(a_0)| \leq 1/2$. The amplitudes are evaluated using perturbative methods from the corresponding lagrangian.

The authors of Ref. [11], find that in order to respect the unitarity argument the weak isospin of a complex scalar multiplet must satisfy the bound: $T \leq 7/2$. They have also combined the EWPT with the unitary bounds for particular models that include an stable DM candidate, to further constrain the allowed parameter space for the large representations. In particular, from their figure 1, one reads that small mass difference between the stable component of the mutiplet and the first charged one, are allowed; for instance, taking $\Delta m = 5$ GeV and $M_{dm} \approx 5$ TeV, implies that values of $j > 7/2$ are already excluded.

3. Perturbativity. In Ref. [19], the authors discuss bounds on the weak isospin, obtained from analyzing the contribution of such large representations to the RG evolution of the $SU(2)_L$ gauge coupling. They find that
a real multiplet must have $T \leq 3$ in order to avoid $g_2$ entering into the non-perturbative domain, below the Planck scale. For a complex multiplet, they find that the corresponding limit becomes $T \leq 5/2$.

4. Collider searches. The LHC has presented results from dedicated searches for exotic particles. In some cases, they present model-independent results for specific states (such as leptoquarks), while in other cases the searched particles are part of well motivated theories. For instance the search for gluinos within the MSSM can also be considered as the search for a colored fermion octet. Overall, the LHC bound for such states is of order $M > O(1)$ TeV.

Thus, we conclude that large representations, beyond the SM ones, are disfavored by present data. Although those results do not constitute a proof to forbid its existence, it certainly motivates the need for an explanation of the absence of such exotic representations. If such states are not detected in the future LHC runs, it will become more pressing to look for an explanation of this aspect of the SM.

III. EFFECTIVE SELECTION RULES AND COMPOSITENESS

A. What do we know? Effective selection rules.

For some time, H. Nielsen and collaborators [14], have studied how to make sense on the peculiarities of the SM structure, using Group Theory and some physical arguments. We shall call their approach as "effective selection rules", in the sense that some effective explanation is sought, which may be written using only the SM degrees of freedom, which are assumed provisionally to be fundamental ones, although they may actually be of some emergent nature, at a deeper level. Nielsen and collaborators have emphasized that some aspects of the SM structure, e.g. charge quantization among others, can be better understood by distinguishing between the Lie algebra of the SM, i.e. $u(1)_Y \times su(2) \times su(3)$, and the Lie group, which happens to be $S(U(2) \times U(3))$, thanks to the structure of the corresponding covering group, and the relationship between the hypercharge and the duality/triality of the fermion representations. Furthermore, they also show that the SM gauge group is rather special, as it shows a minimal degree of "skeweness", namely that it has the smallest number of automorphisms [12].

In order to quantify the problem of the absence of the large representations, Nielsen et al. have built an argument to show that quarks and leptons appear not only in the smallest representations of the non-abelian $SU(2)$ and $SU(3)$ gauge groups, but that also the abelian charges are of minimal size. Their arguments involves a combined use of Han-Nambu charges and anomaly cancellations. More recently, they have argued that not only the matter representations fall into the fundamental representations, but also the gauge groups has associated some quantity that gets maximized for the SM gauge group [13]. This quantity is defined as a modification of the ratio of the quadratic Casimir invariant for the adjoint representation and that for the smallest faithful representation. This quantity gets maximized and singles out the $S(U(2) \times U(3)$ gauge group, which is singled out. Furthermore, this quantity also helps to single out the fundamental representation for the quarks, leptons and even the Higgs doublets.

Another line of reasoning has been presented in ref. [15], which uses discrete symmetries as a way to constrain the size of the allowed representations. First, it is argued that only some representations allow to use consistently the same definition for the discrete symmetries. Then, by using invariance under change of basis restricts possible representations, it is concluded that only fundamental representations of $SU(N)$ are allowed. In this regard, we should also mention that Nielsen et al. have extended their work to look for a related quantity that can be applied to the dimension of spacetime ($D$), and they find that such quantity gets maximized for $D = 4$.

We could also mention that a similar discussion holds for the allowed spin that can be described consistently within QFT. In that case, some difficulties are associated with the propagation of higher spin fields, when they interact with an external c-number field, which may include problems with causality, the presence of unphysical states and violation of unitarity. These results indicate that fundamental particles with spin $S \leq 2$ have difficulties to be treated in QFT [16]. If somehow the isospin were related with the spin, it may be possible to argue that only the related small values of isospin for fundamental particles could be included in QFT. One such possibility could be the extra dimensional scenarios, where the interactions have a geometrical origin.

B. Small representations from compositeness

Based on the successive sub-structure layers that appear when going from molecules to atoms, and from there to nuclei and hadrons, we could also argue that something similar could happen at the quarks and leptons level. This would explain why we only observe small representations, due to a dynamical reason. Namely, if the known quarks
and leptons were composite, the dynamics could operate in such a way that it does not allow the appearance of larger representations.

It could be useful to review the similar situation in nuclear and hadron physics, to illustrate our point and to gain some insight into the problem. Namely, we can recall that only nuclei with atomic number $Z \lesssim 120$ are observed in nature. We know that this happens because in this limit the nuclear attractive force between nucleons ($p & n$) dominates over the electromagnetic repulsion. Namely, when

$$F_{em} \sim k \frac{Z^2 e^2}{r^2} \sim F_{nucl} \approx e^{-mr/r},$$

(2)

Then one finds that this implies: $Z \simeq 1/\alpha_{em} \simeq 137$.

Similarly, it could happen that quarks and leptons are composite, but in such a way that their quantum numbers only allow doublets of $SU(2)_L$ and triplets of $SU(3)_c$, because there exist a repulsive force among the constituents that makes impossible to form bound states with large values of isospin or color. If this were the case, it is possible that some imprint would be left on the SM fermion properties by the composite dynamics, for instance on the anomalous magnetic moments. In ref. [17], several models were proposed to treat the anomalous magnetic moment of a composite fermion, with an scale $\Lambda \simeq 1 \text{ TeV}$.

**IV. SM REPRESENTATIONS AND GRAND UNIFICATION.**

Another possible argument to explain the appearance in nature of matter fermions in small representation, could arise within the unification paradigm. Namely, by assuming that nature only accepts the known SM multiplets or at most the addition of minimal extensions, we could ask what are the allowed dimensions for such representations, when they are assumed to form unified multiplets under some unified gauge group.

**A. GUTs and the principle of "Minimal Complexity"**

Let us discuss our point within the context of minimal $SU(5)$ GUT. Here one finds the remarkable property that the known quarks and leptons, being color triplets or weak doublets (singlets), can fit into an (almost) small unified representation, e.g. 5 and 10 of $SU(5)$. Both of these representations can also be accommodated into the spinorial 16 dimensional representation of $SO(10)$ GUT, provided that a right-handed singlet neutrino is added to the SM matter fields.

Then, we could ask whether the addition of extra multiplets, charged under the SM gauged Lie Algebra, could still be unified into the gauge group under consideration, without adding much complications. It turns out that this is not an easy task, because as soon as one include some larger weak-color representations, one needs to invoke larger GUT multiplets.

As we do not like that, we shall look for an argument to keep the dimension of the unified multiplets as small as possible. Thus, in order to achieve that, we present a principle designed to restrict the dimensions of the representations allowed in some given GUT, which we shall call "Principle of Minimal Complexity", which says the following:

"When a multiplet of certain dimension is added to the SM, no representation of larger dimension should be allowed in order to have a complete multiplet under some GUT gauge group."

We can apply this principle to discuss which representations of larger dimensions (beyond doublets of $SU(2)_L$ and triplets of $SU(3)_c$) can have a natural place into the different theories. For instance, electroweak triplets have been considered in the literature for several purposes, such as generating neutrino masses, getting the right Baryon asymmetry of the Universe and also to study Dark Matter candidates. The question is how natural is the inclusion of such triplets in different GUT’s? What is the prize that should be paid in terms of extra multiplets?

For the case of $SU(5)$ GUT group, we show in table 2 the branching rules for the representations of dimension $N = 5, 10, 15, 24, 35, 45$. We can see from this table that adding a weak triplet forces the addition of extra multiplets. For instance, we find that the triplet can be unified into 15-dim. repr., but then besides the weak triplet, one needs to add a color sextet (6). Similarly, when we consider $SO(10)$ GUT, breaking into the Pati-Salam model $SU(2)_L \times SU(2)_R \times SU(4)$, the SM multiplets fit well into the 16-dim representation. But then the branching rules dictate that the inclusion of extra triplets would require invoking the 45 or 54 dimensional representations, which come with extra sextets too. Thus, if we just want to add extra fermions, the safest way to form unified multiplets, within $SU(5)$, is to add fermions that belong to the $N = 5, 10$ dimensional representations, which would then form the 16 under $SO(10)$. 
Representations of SU(5) | SU(2) | SU(3)
---|---|---
5 | 2 | 1
 | 1 | 3
10 | 1 | 1
 | 3 | 2
 | 3 | 3
15 | 3 | 1
 | 2 | 3
 | 1 | 6
24 | 1 | 1
 | 3 | 1
 | 2 | 3
 | 2 | 8
 | 1 | 8
35 | 4 | 1
 | 3 | 3
 | 2 | 6
 | 1 | 10
 | 2 | 1
45 | 1 | 3
 | 3 | 3
 | 1 | 6
 | 1 | 2
 | 8

On the other hand, adding extra Weak fermions can also be considered within trinified models $SU(3)_c \times SU(3)_L \times SU(3)_R$. The SM fermions can appear within the fundamental representation $(3, 3, 1)$ [18], but with extra fermions. Weak triplets can be accommodated within an octet of $SU(3)$, which decomposes under $SU(2) \times U(1)$ as: $8 = 3 + 1 + 2 + 2$. In this case the multiplet does not involve representations larger than what has been included. Thus, adding extra fermions seem to be more natural within this trinified model.

A more ambitious unification program would include the horizontal (family) symmetry. In such case, it could be interesting to look at the constraints that some gauge-family unification could impose to restrict the size of the unified representations. In this regard, it is worth mentioning the work of [22], which looked at the properties of orthogonal Lie groups; their aim was to look for some explanation of the SM structure [21]. Starting from a great gauge group $G$ that contains a single representation $R$ that accommodates the three families, one looks for its breaking into some GUT gauge group $G \rightarrow G'$, in such a way that the representation also break as: $R \rightarrow R + R + R + \ldots$, with $R$ being one family. It turns out that the group $SO(2(n + m))$ enjoy the property that spinor representations break into $2^m$ representations of $SO(2n)$. For $n = 5, m = 4$, the group $SO(18)$ breaks by giving 8 families of $SO(10)$ (V-A) + mirror fermions (V+A). It could be interesting to search weather there exists some group $G'$ that it only allows a unique representation, but so far we have not found a satisfactory answer.

B. Higgs Representations

So far we have discussed the absence of large representations in the matter (fermionic) sector. But a similar discussion could arise in the Higgs sector. There are plenty of motivations for enlarging the scalar sector beyond the SM Higgs doublet, such as generating small neutrino masses, hierarchy problem, dark matter, BAU, which include extra scalar multiplets. The extensions of the Higgs sector could include extra singlets, doublets and triplets. Models with $T = 3/2$ for neutrino masses have been considered in the literature, whereas models with $T \geq 2$ have been shown to provide stable candidates for dark matter [19].

But even in such case, we could again ask what is the limit on the dimension of the allowed Higgs representations. When the neutral component of the Higgs multiplet of weak isospin $T$ and hypercharge $Y$, acquires a vev, it contributes to the $\rho$ parameter (\(\rho = m_W^2 / m_Z^2 \cos^2 \theta_W\)). In order to keep $\rho \approx 1$, as it is observed in nature, one needs to have: $(2T + 1)^2 = 3Y^2 - 1$ [4]. This relation is always satisfied for Higgs doublets, but one needs to impose some tuning of the vevs whenever we consider a more exotic Higgs sector.

An additional problem arise when the Higgs sector is embedded into a SUSY theory, namely the Higgs scalars are accompanied by its fermionic partners, the Higgsinos, which in general provide a non-vanishing contribution to the abelian anomaly. To have anomaly-free models usually a vector-like theory is choosen, i.e. including fermions of both
chirality which could be made very heavy. However, as shown in ref. [20], it is also possible to have theories of chiral type, which also satisfy the anomaly cancellation conditions.

For instance, within $SU(5)$ GUT, we know that the representations $5$ and $10$ have opposite contributions to the gauge abelian anomaly, so together they form an anomaly free representation. As we said before, sets of representations of $SU(5)$ that complete some representation of $SO(10)$ are anomaly free.

But what about large representations, such as the $45$? This case appears in some solutions to the problem of quark-lepton mass relations. In this case we could need extra representations in order to cancel anomalies. The simplest choice is to consider a vector like SUSY Higgs sector, which includes the $45$ and $\overline{45}$. But this is not the only choice; as shown in ref. [20], we could also consider satisfying the anomaly cancellation condition using a combination of lower-dimensional representations. For instance, we can have a combination of multiplets that satisfy:

$$A(45) + c_5 A(\overline{5}) + c_{10} A(\overline{10}) + c_{15} A(\overline{15}) = 0$$

where the anomaly coefficients are: $A(45) = 6$, $A(\overline{5}) = -1$, $A(\overline{10}) = -1$, $A(\overline{15}) = -9$. For instance a solution to the above equations is given by: $c_5 = c_{10} = 3$ and $c_{15} = 0$, plus plenty of other solutions.

But again, the introduction of larger dimensional multiplets usually involve the addition of even large representations in vector-like theories, or a combination of several multiplets of lower dimension. Besides the aesthetic problems, the inclusion of such multiplets make the unified gauge coupling constant to run into the non-perturbative domain, before the Planck scale.

V. CONCLUSIONS

We have studied the problem of the SM representations, namely the fact that the quarks and leptons appear only in the fundamental (or singlet) representations of the $SU(3) \times SU(2) \times U(1)$ gauge symmetry. Larger representations (EW triplets, color sextets, etc.), which in principle could also occur in nature, shine for its absense. Namely, considerations based on EWPT, unitarity, perturbativity and collider bounds, provide bounds on the corresponding mass scale, which are entering into the multi-TeV domain. We have reviewed some approaches to understand the absence of such representations, including: effective selection rules, use of discrete and spacetime symmetries. An explanation for the problem of large representations, based on a possible substructure layer for the known quarks and leptons, was also briefly considered here.

We have also discussed this problem within the grand unification paradigm, here we proposed a principle of "minimal complexity", which sates that when one adds some representations of certain size, one should not permit the addition of even larger ones, in order to form unified multiplets. As an application of this principle, we argued that, for instance, adding EW triplets is not economical within the $SU(5)$ or $SO(10)$ models, but they could arise more naturally within a trinified model $SU(3)_C \times SU(3)_L \times SU(3)_R$. Here one may question the validity of this approach when one does not include the flavor problem, but then the problem gets really complicated.

May be some radical new approach would be needed in order to explain the apparent absence of large representations. Here I like to speculate that maybe it is time to involve number theory to particle physics. For instance, it is amusing to identify the number of generators in the SM Lie algebra, $SU(3)_C \times SU(2)_L \times U(1)_Y$, i.e. the integers $3, 2, 1$, which are the factors of the first perfect number ($6$). In fact, the prime nature of these numbers, appears in the discussion of Nielsen et al. [13], in their attempt to go beyond the usual considerations of the Lie Algebra of the SM, and actually identify the true gauge group of the SM, as a subgroup of the covering group, which is discussed in detail in Ref. [22].

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