Model of the transducer based on Murali-Lakshmanan-Chua non-autonomous chaotic oscillator for discrete sensors intended for monitoring production processes

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Abstract. The article discusses the model of the detector of periodic signals under the background noise intended for discrete type control devices. It is proposed to construct the detector based on the non-autonomous chaotic Murali – Lakshmanan – Chua oscillator. To detect, the parameters of a chaotic oscillator must ensure proximity to the bifurcation boundary between the periodic and chaotic modes. In the absence of the detectable signal at the input of a chaotic oscillator, chaotic motion is observed in it, and if present, a bifurcation occurs and a periodic mode is established. The numerical model that allows solving the Cauchy problem for the system of differential equations of the chaotic oscillator Murali – Lakshmanan – Chua has compiled in Matlab / Simulink. The output informative parameter of the detector is the number of chaotic bursts during the detection time. The proposed model allowed us to obtain empirical dependencies for the number of bursts. On the basis of the obtained dependencies, the choice of the optimal parameters of the system is made. The study as a whole led to the conclusion about the possibility of using the proposed detector as a node of devices that receive the measuring signal under the action of non-stationary interference.

1. Introduction

The method of detecting periodic signals under the background noise using bifurcation in a non-autonomous chaotic oscillator has been actively studied over the past 20 years [1-5]. The basis of the method is a chaotic oscillator, the parameters of which are set in such a way that there is a chaotic mode with intermittent behavior. Intermittent behavior is a sign of proximity to the boundary with a tangential bifurcation. The mixture of the detectable signal and noise is fed into the chaotic oscillator and under the action of the detected signal a bifurcation occurs, and the chaotic mode is replaced by a periodic mode. For coherent-on-receive it is necessary that the chaotic oscillator has its own source of a periodic signal - the reference oscillator. The most well-known detector model includes the Duffing – Holmes oscillator in a numerical model, and the detection of the steady state is performed by calculating the largest Lyapunov exponent.

In miniature sensors [6] there is a need for a simple detector without the use of numerical models. It is proposed to use the Murali – Lakshmanan – Chua oscillator [7] as a chaotic oscillator for the detector, which has a simpler realization in the form of an electrical circuit than the Duffing – Holmes oscillator. It is proposed to use the number of chaotic bursts during the detection time as output parameter of the
measuring transducer. The decision on the presence of a detectable signal is made by comparing the obtained number of bursts with the established threshold.

2. Basic principle

The proposed signal detector is described by the system of equations of the chaotic oscillator Murali – Lakshmanan – Chua under the influence of an external signal. In equation (1) and further, all variables are presented in a dimensionless form:

\[
\begin{align*}
\dot{x} &= y - m(x) \\
\dot{y} &= -\beta y - \beta x + A_0 \sin \omega \tau + \eta(\tau)
\end{align*}
\]

where \( x \) and \( y \) are dynamical variables, \( \beta \) is a bifurcation parameter of the system, \( A_0 \) is an amplitude of the sinusoidal source, \( \omega \) is a frequency of the sinusoidal source, \( \tau \) is a time, \( \eta(\tau) \) is received signal, \( m(x) \) is a function, describing a non-linear element also known as Chua’s diode:

\[
m(x) = bx + \frac{1}{2}(a - b)(|x + 1| - |x - 1|),
\]

where \( a = -1.02 \) and \( b = -0.55 \) are constant coefficients.

The tangent bifurcation in this system is observed at \( \beta = 0.9, \omega = 0.4 \) and the critical amplitude of the sinusoidal source \( A_{cr} \approx 0.0825 \). In the vicinity of the tangential bifurcation, an intermittent motion is observed in the system. It consists of long laminar phases separated by short chaotic bursts. The appearance of chaotic bursts is random in nature. Thus, the amplitude of the reference oscillator must be set slightly below the bifurcation boundary, and a bifurcation should occur upon receipt of a detectable signal. Above the bifurcation boundary, the chaotic bursts cease.

For the signal, supplied to detect \( \eta(\tau) \), we introduce the following notation:

\[
\eta(\tau) = d \cdot \sin \omega \tau + \sigma_{in} \cdot n(\tau),
\]

where \( d \) is the amplitude of the detected signal \( \sin \omega \tau \), \( \sigma_{in} \) - the effective value of the noise, \( n(\tau) \) - a narrow-band Gaussian random process with zero mean and variance equal to one. Let us denote \( A \) as the sum of the amplitudes of the sinusoidal source and the detected signals. In order for a bifurcation to occur, inequality must be fulfilled:

\[
A = A_0 + d > A_{cr}.
\]

It is known [8] that in a chaotic oscillator, even with \( A > A_{cr} \), under the influence of noise, the occurrence of chaotic bursts is possible. The decision on the presence or absence of a detectable signal is made by comparing the number of chaotic bursts \( N \) during the detection time \( \tau_d \) with a certain threshold \( h' \), but to set this threshold, it is necessary to know the dependencies for this number on the signal-to-noise ratio.

3. Model description

The model was created in Matlab / Simulink according to the scheme presented in Figure 1. The RandomNumber pseudo-random number generator with the BandpassFilter filter is a source of narrowband noise with relative bandwidth \( \gamma = 0.1 \) and center frequency \( \omega \). For each numerical experiment 100 launches were performed with a different Seed value for the source RandomNumber. The block of the binary phase detector and counter is designated in the model as Subsystem. The time interval \( \tau_d \) corresponds to \( Z = 500 \) periods of the sinusoidal source SineWave. The calculation was performed by the Adams method with an adaptive step. The obtained number \( N \) of chaotic bursts was saved to a file for further processing.

![Figure 1. Numerical model of the detector in Matlab / Simulink.](image-url)
4. Simulation results

In a numerical experiment, the statistics of the number of chaotic bursts \( N \) during the detection time at a different amplitude of a sinusoidal source \( A \) and a different effective value of the noise \( \sigma_{in} \) was investigated. For each experiment, the average number of bursts \( N_m \), the standard deviation of the number of bursts \( \sigma_N \) were obtained and a normalized histogram was constructed. In the course of the study, it was assumed to obtain a two-parameter diagram on the parameter plane \( A, \sigma_{in} \) for the number of chaotic bursts during the detection time. Figures 2 and 3 show diagrams for \( N_m \) and for \( \sigma_N \), the time is defined by \( Z = 500 \).

In all numeric experiments, the mean number of bursts was less than the variance, i.e. \( N_m < \sigma_N^2 \), which means that it is impossible to use the model based on the Poisson distribution. Since it is necessary to independently set the mean value and variance, a model based on a negative binomial distribution can be used to describe a random variable at the output of the measuring transducer.

![Graphs](image)

**Figure 2.** The mean number of chaotic bursts \( N_m \) with \( Z = 500 \), \( \gamma = 0.1 \), depending on the amplitude of the reference oscillator \( A \) and the effective value of the random noise \( \sigma_{in} \).
Figure 3. Standard deviation $\sigma_N$ of the number of chaotic bursts with $Z = 500$, $\gamma = 0.1$, depending on the amplitude of the reference oscillator $A$ and the effective value of the random noise $\sigma_{in}$.

The lines shown in Figure 2, showing the average number of chaotic bursts, diverge like a fan. This makes it possible to determine the optimal amplitude of the reference oscillator. The amplitude of the reference oscillator is chosen such that, with only interference with different dispersion, the statistics of the number of chaotic bursts during the detection time would remain unchanged. Selection of the optimal value for the amplitude of the reference oscillator allowed us to obtain the value of $A_0 = 0.08226$, which is 0.36% less than $A_{cr}$.

It should be noted that if the noise variance varies within three orders of magnitude ($\sigma_{in}^2 = 10^{-6} \div 10^{-3}$), then there is a statistical similarity in the number of bursts at the detector output. This makes it possible to create sensors with a constant level of false alarm in non-stationary noise conditions. For the number of chaotic bursts at $A = A_0$, i.e. in the absence of a detectable signal at the detector input, an approximate formula was obtained:

$$N_m(Z) \approx \zeta_1 \cdot Z, \quad \sigma_N(Z) \approx \zeta_2 \sqrt{Z},$$  \hspace{1cm} (3)

where $\zeta_1 = 0.092$, $\zeta_2 = 0.41$. Using the diagram presented in Figure 2, approximate empirical formulas were obtained, allowing establishing the mean number of chaotic bursts during the observation interval at $A > A_{cr}$:

$$N_m(\sigma_{in}, A, Z) \approx \zeta_3(\sigma_{in} \cdot \zeta_1(A) + \zeta_2(A)) \frac{Z}{500}$$  \hspace{1cm} (4)

where $\zeta_1(A) = \frac{30}{1.05 \cdot A - 0.08}$, $\zeta_2(A) = 3^{-150} - 11$, $\zeta_3(x) = \begin{cases} x, & x > 0 \\ 0, & x \leq 0 \end{cases}$.

Using the diagram presented in Figure 3, approximate empirical formulas were obtained, allowing establishing the standard deviation of the number of chaotic bursts during the observation interval (Figure 4):

$$\sigma_N(\sigma_{in}, A, Z) \approx \left[ \zeta_1 - \frac{\zeta_1}{1 + e^{\sigma_{in} \cdot \zeta_2 - \zeta_3(A)}} \right] \sqrt{Z},$$  \hspace{1cm} (5)

where $\zeta_1 = 0.358$, $\zeta_2 = 10^3$, $\zeta_3(A) = 1.942 + \frac{A - A_{cr}}{0.006} \cdot 2.16$, $\zeta_4(A) = 2.277 + \frac{A - A_{cr}}{0.006} \cdot 2.8.$
We note, using formulas (3) - (5), that the average number of bursts increases linearly with the duration of the detection time, and the standard deviation as square root, which coincides with properties of correlation detector.

![Figure 4](image)

**Figure 4.** Empirical dependences for the number of chaotic bursts with $Z = 500$, depending on the effective value of the random noise $\sigma_{\text{in}}$ at different amplitudes of the sinusoidal source: a – the mean value $N_m$; b – the standard deviation $\sigma_N$.

5. Conclusion

As a result of the study, the optimal value of the amplitude of the reference oscillator was established, which makes it possible to ensure a constant probability of false positives under the background noise with varying dispersion. Empirical dependences are obtained for the mean and standard deviation of the number of chaotic bursts during the detection time. Using these dependencies, the parameters of a random process at the output of the measuring transducer can be obtained for different signal-to-noise ratios. Using the inverse function of the negative binomial distribution, it becomes possible to determine the decision threshold by the Neumann-Pearson criterion. This work shows the possibility of using a chaotic oscillator to detect a signal in control devices of production processes, when the received signal is supplied with non-stationary interference.

References

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