Improving the security of multiparty quantum secret sharing against Trojan horse attack

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We analyzed the security of the multiparty quantum secret sharing (MQSS) protocol recently proposed by Zhang, Li and Man [Phys. Rev. A 71, 044301 (2005)] and found that this protocol is secure for any other eavesdropper except for the agent Bob who prepares the quantum signals as he can attack the quantum communication with a Trojan horse. That is, Bob replaces the single-photon signal with a multi-photon one and the other agent Charlie cannot find this cheating as she does not measure the photons before they runs back from the boss Alice, which reveals that this MQSS protocol is not secure for Bob. Finally, we present a possible improvement of the MQSS protocol security with two single-photon measurements and six unitary operations.

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In classical secret sharing,1 the boss, say Alice divides her secret message into two pieces and sends them to her two agents, Bob and Charlie who are at remote place, respectively, for her business. Bob and Charlie can reconstruct the secret if and only if they collaborate. That is $M_A = M_B \oplus M_C$, where $M_A$, $M_B$ and $M_C$ are the messages hold by Alice, Bob and Charlie, respectively. The advantage of secret sharing is that one of the two agents can keep the other one from doing any damage when they both appear in the process for the business. As classical signal is in one of the eigenstates of an operator, say $\sigma_z$, it can be copied freely and fully without leaving a track. Quantum mechanics provides some novel ways for message transmitting securely, such as quantum key distribution2,3,4,5,6,7,8,9,10, quantum secure direct communication11,12, quantum dense coding11,12, and so on.

Quantum secret sharing (QSS) is an important branch of quantum cryptography2 and it is the generalization of classical secret sharing into quantum scenario13,14. Since a pioneering QSS scheme was proposed by Hillery, Bužek and Berthiaume in 1999 by using a three-particle or a four-particle entangled Greenberger-Horne-Zeilinger (GHZ) state for sharing a classical information, called HBB99 customarily for short, there has been a lot of works focused on QSS in both theoretical15,16,17,18,19,20,21,22,23,24,25,26,27 and experimental28,29 aspects. Almost all the existing QSS protocols can be attributed to the two types according to their goals. One is used to distribute a common key among some users13,14,15,16,17,18,19,20,21,22,23,24,25,26,27, and the other is used to split a secret including a classical one13,14,15,16,17 or a quantum one24,25,26,27.

Recently, Zhang, Li and Man23 proposed a multiparty quantum secret sharing (MQSS) protocol for splitting a classical secret message among three parties, say Alice, Bob and Charlie with single photons following some ideas from the Ref.3. In this paper, we will show that the Zhang-Li-Man MQSS protocol can be eavesdropped by the agent Bob who prepares the quantum signals with a Trojan horse attack strategy as he can steal almost all the information encoded by the other agent Charlie if he replaces the single photon with a multi-photon quantum signal. As Charlie does not measure the photons for eavesdropping check before the quantum signal runs back from the boss Alice, he cannot find this cheating, which is different in essence to the quantum secure direct communication protocol3 and the quantum key distribution protocol with the practical faint laser pulses4. Moreover, we present a possible improvement of the Zhang-Li-Man MQSS protocol security with two single-photon measurements and four unitary operations.

Let us start with the brief description of the Zhang-Li-Man MQSS protocol23. We discuss the simple case in which there are three parties, the Boss Alice, two agents, Bob and Charlie, shown in Fig.1. The principle for other case is the same as this simple one with just a little modification or not. In the Zhang-Li-Man MQSS protocol23, the agent Bob prepares a batch of $N$ single photons with choosing one of the two measuring bases (MBs), namely, the rectilinear basis, $\sigma_z$, i.e., $|H\rangle = |0\rangle$, $|V\rangle = |1\rangle$ and the diagonal basis, $\sigma_x$, i.e., $|u\rangle = \frac{1}{\sqrt{2}}(|0\rangle + |1\rangle)$, $|d\rangle = \frac{1}{\sqrt{2}}(|0\rangle - |1\rangle)$ randomly, similar to the Ref.3. Bob sends the single photons to Charlie, and Charlie first takes one of the three unitary
operations $I, U, H$ on each photon randomly and then sends them to Alice. Here $I$ is the identity operation, $U = iσ_y = |0⟩⟨1| - |1⟩⟨0|$ whose nice feature is that it flips the state in both measuring bases $I, U, H$, and $H = \frac{1}{\sqrt{2}}(|0⟩⟨0| + |1⟩⟨1|)$ is the Hadamard operation which can realize the transformation between the two MBs $23$. Charlie’s operations are equal to the encryptions on the states of the single photons. After receiving the single photons, Alice stores most of them and picks out a subset of the photons as the samples for eavesdropping check. For each sample, Alice performs Bob publish the initial state first and then Charlie tell her the encrypting operation, or vice versa. She performs single-photon measurements on the samples by choosing one of the two measuring bases $σ_z$ and $σ_x$ according to the information published by Bob and Charlie, and analyzes the security of the transmission of the photons between Bob and Alice. If the error rate is lower than the threshold, Alice performs the identity operation $I$ on the state of the single photon if she want to encode a bit 0, otherwise she performs the operation $U$ on the photon. Alice sends the single photons to Charlie, and Charlie will read out the message with the help of Bob’s. That is, Bob tell Charlie the initial states of the photons, and then gets the operations done by Alice.

As pointed out in the Refs. $13, 14$, if the dishonest one in the agents in an MQSS cannot eavesdrop the quantum communication without disturbing the quantum system, any eavesdropper can be found out if he wants to steal the information. In this way, the main goal for the security of an MQSS is simplified to prevent the dishonest agent from eavesdropping the information. The Zhang-Li-Man MQSS protocol is secure if the eavesdropper is Charlie or not the two agents as the eavesdropping will disturb the quantum system and leave a trick in the results of the measurements on the sample photons before the message is encoded on the other photons. Moreover, the parties can perform a quantum privacy amplification $11$ on the batch of the polarized single photons for improving its security in a noise channel, similar to that in the Ref. $2, 10$. However the Zhang-Li-Man MQSS protocol is not secure if the agent Bob who prepares the quantum signal is the dishonest one as he can eavesdrop the information freely with Trojan horse attack $2$ even though there are no losses and noise in the quantum channel. We limit our discussion in this attack below.

For the attack, Bob need only read out the operations that Charlie encodes on the photons for encrypting their states, and then he can get the information transmitted by the boss Alice freely. The Trojan horse attack can be implemented as following: (1) Bob replaces the single-photon quantum signal with a multi-photon quantum signal, and Charlie cannot find out this cheating as he does not measure the photons before they runs back from Alice, which is different in essence to the quantum secure direct communication protocol $3$. (2) Bob measures the photons with some photon number splitters (PNSs: 50/50) and some detectors. As Bob has the information about the initial states of the photons, he can read out the operations done by Charlie with a large probability if there are several photons in each quantum signal.

To present the Trojan horse attack clearly, we use a four-photon quantum signal as the fake signal with which Bob eavesdrops Charlie’s operations for the sake of simplicity. Bob prepares the four photons in the same state, say $|H⟩ = |0⟩$ and sends them to Charlie. As Charlie takes his operation on each signal, the four photons will be performed a same unitary operation and then there are four copies of the states encoded by Charlie with the same unitary operation. Bob uses three PNSs to split the fake signal and performs $σ_z$ measurement on each photon, shown in Fig.2. If Charlie takes the operation $I$ or $U$, the outcomes of the measurements on the four photons with $σ_z$ are the same one with the probability 100%. On the contrast, Bob gets the same outcome with the probability $(\frac{1}{2})^4 = \frac{1}{16}$ if Charlie operates the signal with $H$ operation as Bob obtains the outcome "0" or "1" with the probability 50% for each photon in the state $|u⟩ = \frac{1}{\sqrt{2}}(|0⟩ + |1⟩)$. If Charlie performs the operations $\{I, U, H\}$ on each signal with the same probability, the probability $P_e$ that
Charlie cannot determine which operation is chosen by Charlie on the signal is $\frac{1}{3} \times \frac{1}{2} = 2.1\%$. If Bob replaces the single-photon signal with an N-photon signal, then $P_e = \frac{1}{3} \times \frac{1}{2}^N$. When $N = 10$, $P_e = 3.26 \times 10^{-4}$.

Moreover, Bob’s eavesdropping does not introduce errors in the results of the measurements done by Charlie in the final process if he just measures $N-1$ photons in the multi-photon signal and sends the other one to Alice.

In the Ref. 6, as any eavesdropper does not know the initial state of the quantum signal and the parties take measurements on a subset of photons chosen randomly for eavesdropping check of each transmission, the eavesdropper cannot steal the information with a Trojan horse attack strategy. In the Zhang-Li-Man MQSS protocol 23, there are not those two features for the agent Charlie. So any eavesdropper can get the operations done by Charlie with a Trojan horse attack strategy, including the agent Bob. Certainly, this eavesdropping process is useful just to Bob, not to other eavesdroppers as they will be detected by the three legitimate parties if they want to steal the information about the operations done by Alice or the initial states prepared by Bob.

For improving the security of the Zhang-Li-Man MQSS protocol 23, the three parties must have the capability to hold back an eavesdropper to attack the quantum communication with a Trojan horse. This MQSS protocol is secure if Charlie and Alice can forbid Bob to eavesdrop the quantum line. Charlie must have the ability to distinguish whether each quantum signal is a single-photon one or a multi-photon one before he encrypts the signals with the unitary operations. For this end, Charlie can store the quantum signals and chooses randomly a subset of the quantum signals, similar to Alice. He splits the sample signals with a photon number splitter (PNS), similar to the Trojan horse attack done by Bob (see Fig. 2) and then measures the two signals with the measuring bases $\sigma_x$ and $\sigma_z$ randomly. Just this modification, Charlie can prevent Bob from eavesdropping without being detected as both the measurements will have an outcome if the quantum signal is a multi-photon one. Charlie uses the probability $P_m$ that there are many photons in each quantum signal sent by Bob to determine whether Bob is honest. If the $P_m$ is very low, Bob is a honest one. Certainly, Charlie can improve the security with three or more PNSs largely, same as that in Fig.2.

For the symmetry, Charlie can exploit the fourth unitary operation $H = \sqrt{2}(|0\rangle\langle0| - |1\rangle\langle1|)$ for the encryption. That is, Charlie chooses randomly one of the four unitary operations $\{I, U, H, \overline{H}\}$ to encrypt the state of each photon, which will reduce the probability that Bob obtains the information about the operations done by Charlie with Trojan horse attack in particular in the case with a noise quantum channel. That is, the probability that Bob distinguishes the operations $\{I, U\}$ from the operations $\{H, \overline{H}\}$ will decrease.

For the integrity, let us describe the modified Zhang-Li-Man MQSS protocol as follows in brief.

(a) The agent Bob prepares a batch of $N$ single photons $S$ randomly in one of the four polarization states $\{|H\rangle, |V\rangle, |\uparrow\rangle, |\downarrow\rangle\}$ randomly, similar to the Ref. 2, 6. He sends $S$ to the agent Charlie.

(b) After receiving $S$, Charlie chooses a sufficiently large subset of photons in $S$, and splits each signal with a PNS. He measures the two signals after the PNS with choosing one of the two MBs $\sigma_x$ and $\sigma_z$ randomly.

(c) Charlie requires Bob to tell him the information about the original states of the sample photons, and he analyzes the probability $P_m$ that there are many photons in each quantum signal sent by Bob. Also he analyzes the error rate $\epsilon_e$ of the sample photons.

(d) If $P_m$ is very low and $\epsilon_e$ is lower than the threshold, Charlie encrypts almost all the remained photons in $S$ with choosing randomly one of the four unitary operations $\{I, U, H, \overline{H}\}$. Also, he chooses some samples, say $S_C$ from the $S$ sequence, and performs them with one of the two operations $\{\sigma_x, \sigma_z\}$ randomly, and continues to the next step. Otherwise he discards the results and repeats the quantum communication from the beginning.

The unitary operations done by Charlie is equivalent to the uniform encryption on the photons. In a noise channel, the error correction and the privacy amplification techniques should be used on those photons for improving the security, same as those in the Ref. 7. The quantum error correction technique is not difficult in principle to be implemented, and a quantum privacy amplification way for the single photons was proposed also 10. Hence, the quantum communication between Bob and Charlie can be made secure.

(e) Charlie sends the remained photons, say $S'$ to Alice. Alice stores most of the single photons and picks out randomly a sufficiently large subset of single photons for eavesdropping check. She tell Bob and Charlie the positions of the sample photons. She requires Bob tell her the original states of the sample photons first, and then Charlie tell her the operations encoded, or vice versa. For the samples done by Charlie with the operations $\sigma_x$ and $\sigma_z$, Charlie tell Alice the positions first and then Alice requires Bob tell her their original states. Charlie publishes the operations for the samples $S_C$.

(f) Alice measures each of the sample photons with a correlated MB, and determines whether there is an eavesdropper monitoring a MB, and determines whether there is an eavesdropper monitoring the quantum line between Bob and Alice, see Fig.1.

(g) If there is no eavesdropper, Alice encodes the photons in $S'$ except for those chosen for eavesdropping check (namely, $S''$), with the two unitary operations $I$ and $U$ which are coded as the bits 0 and 1, respectively. Surely, Alice should select some photons from $S''$ as the samples for eavesdropping check and operate them with $I$ and $U$, randomly, same as 2, 6. Alice sends $S''$ to Charlie.

(h) Charlie reads out the secret message with the help of Bob’s. That is, Bob tell Charlie the original state for each photon, and then Charlie measures each photon with a correlated MB. Of course, they should check the eavesdropping of the transmission between Alice to Charlie before Bob and Charlie cooperate to read out the
In summary, we analyzed the security of the MQSS protocol proposed by Zhang, Li and Man [24] and found that this protocol is secure for any other eavesdropper except for the agent Bob who prepares the quantum signal as he can attack the quantum communication with a Trojan horse, i.e., he replaces the original signal with a multi-photon signal and measures them with some PNSs. Bob’s eavesdropping cannot be detected. Finally, we present a possible improvement of the MQSS protocol security with two single-photon measurements and six unitary operations. With these modifications, the Zhang-Man MQSS protocol is secure not only for distributing a common key among the users in MQSS but also for splitting a secret message, same as the quantum secure direct communication protocol [9] with the quantum privacy amplification [10].

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Addendum- For improving the security of the Zhang-Man-Li MQSS protocol completely, the parties of the communication should have the capability of prevent the agents from eavesdropping. The eavesdropping done by Charlie who does not know the original states of the photons prepared by Bob can be detected in the simple way that Alice requires Charlie tell her the operations first and then Bob publish the original states of the samples chosen randomly. For forbidding Bob eavesdrop, the best way may be that Charlie adds some decoy photons which are prepared by Charlie and randomly in the states $|0\rangle, |1\rangle, |a\rangle = \frac{1}{\sqrt{2}}(|0\rangle + |1\rangle), |d\rangle = \frac{1}{\sqrt{2}}(|0\rangle + |1\rangle)$, in the sequence $S$. In this way, Charlie should have an ideal single-photon source.

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