Radiative Decay of $\Upsilon$ into a Scalar Glueball

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Abstract

We study the radiative decay of $\Upsilon$ into a scalar glueball $\Upsilon \to \gamma G_s$ using QCD factorization. We find that for this process the non-perturbative effects can be factorized into a matrix element well defined in non-relativistic QCD (NRQCD) and the gluon distribution amplitude. The same NRQCD matrix element appears also in leptonic decay of $\Upsilon$ and therefore can be determined from data. In the asymptotic limit the gluon distribution amplitude is known up to a normalization constant. Using a QCD sum-rule calculation for the normalization constant, we obtain $Br(\Upsilon \to \gamma G_s)$ to be in the range $(1 \sim 2) \times 10^{-3}$. We also discuss some of the implications for $\Upsilon \to \gamma f_i$ decays. Near future data from CLEO-III can provide crucial information about scalar glueball properties.

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1. Introduction

The existence of glueballs are natural predictions of QCD. Some of the low lying states are \(0^{++}, 0^{-+}, 1^{++}\) and \(2^{++}\) with the lowest mass eigenstate to be \(0^{++}\) in the range of \(1.5 \sim 1.7\) GeV from theoretical calculations \([1]\). There are indications that \(f_0(1370), f_0(1500)\) and \(f_0(1710)\) contain substantial scalar glueball content. For the search of glueballs, decays of quarkonia are well suited processes because the decays are mediated by gluons. Among these decays, two-body radiative decays are ideal places to study this subject, because there is no complication of interactions between light hadrons. Radiative decays of \(\Upsilon\) have been studied before, in particularly by CLEO \([2,3]\) recently. With the current data sample, there are already several observations of radiative decay of \(\Upsilon\) into mesons. Among them only a few with good precisions, such as the decay \(\Upsilon \rightarrow \gamma f_2(1270), \ Upsilon \rightarrow \gamma f_0(1710) \rightarrow \gamma K\bar{K}\), while the others have large errors \([3]\). About 4 fb\(^{-1}\) \(b\bar{b}\) resonance data are planned to be taken at CLEO-III in the year prior to conversion to low energy operation (CLEO-C) \([4]\). This will produce the largest data sample of \(\Upsilon\) in the world. More radiative decay modes of \(\Upsilon\) may be observed. Combining experimental data in the near future and theoretical results glueball properties can be studied in details.

In this paper we carry out a theoretical study of the radiative decay of \(\Upsilon\) into a scalar glueball by using QCD factorization. We find that the non-perturbative effects can be factorized into a matrix element well defined in non-relativistic QCD (NRQCD), and the gluon distribution amplitude. The NRQCD matrix element can be determined from leptonic \(\Upsilon\) decays. The asymptotic form of the gluon distribution amplitude is known in QCD up to a normalization constant. Using a QCD sum rule calculation for this constant, the branching ratio \(Br(\Upsilon \rightarrow \gamma G_s)\) is predicted to be in the range of \((1 \sim 2) \times 10^{-3}\). Combining this result with experimental data, we find that none of the candidate scalar glueballs \(f_0(1500)\) and \(f_0(1710)\) can be a pure glueball. Existing information on glueball mixing allow us to predict the branching ratios for several radiative decays, such as \(\Upsilon \rightarrow \gamma f_0(1370, 1500, 1710) \rightarrow \gamma K\bar{K}(\pi\pi)\). A mixing pattern suggested in the literature is shown to be in conflict with data. Near future experimental data from CLEO-III will pro-
vide crucial information about scalar glueball properties.

2. QCD factorization of $\Upsilon \rightarrow \gamma G_s$

It is known that properties of $\Upsilon$ can be well described with non-relativistic QCD (NRQCD) \[5\]. The decay of $\Upsilon \rightarrow \gamma G_s$ can be thought of as a free $b\bar{b}$ quark pair first freed from $\Upsilon$ with a probability which is characterized by matrix elements defined in NRQCD, this pair of quarks decays into a photon and gluons, and then the gluons subsequently converted into a scalar glueball. In the heavy quark limit $m_b \rightarrow \infty$, the glueball has a large momentum, this allows a twist-expansion to describe the conversion. Also, the gluons are hard and perturbative QCD can be applied for the decay of the $b\bar{b}$ pair into a photon and gluons. This implies that the decay width can be factorized. In the real world, the $b$-quark mass is 5 GeV and a scalar glueball has a mass around 1.5 GeV as suggested by lattice QCD simulations \[2\]. This may lead to a question if the twist expansion is applicable. For the radiative decay of $\Upsilon$, the glueball has a momentum of order of $m_b$. The twist expansion means a collinear expansion of momenta of gluons in the glueball, components of these momenta have the order of $(\mathcal{O}(k^+), \mathcal{O}((k^-), \mathcal{O}(\Lambda_{QCD}), \mathcal{O}(\Lambda_{QCD}))$, where $k$ is the momentum of the glueball. Here we used the light-cone coordinate system. Hence the expansion parameters are

$$\frac{k^-}{k^+} = \frac{m_G^2}{M_T^2} \sim 0.02, \quad \frac{\Lambda_{QCD}}{k^+} \sim 0.1, \quad (1)$$

where $m_G$ is the mass of the glueball and we have taken $\Lambda_{QCD} \approx 500$MeV. In the above estimation we have used the fact that the probability for the conversion of gluons into a glueball is suppressed if the “+” component of the momentum of a gluon is very small. We see that the relevant expansion parameters are small, therefore twist expansion is expected to be a good one. We note that the same approximation may not be applied to $J/\psi$ system because in this case the relevant expansion parameters are not small. We now provide some details of the calculations.

The leading Feynmann diagrams for $\Upsilon \rightarrow \gamma G_s$ are from $b\bar{b}$ annihilation into two gluons
and a photon. The basic formalism for such calculations have been developed in Ref. [6] and has been used in the case of $\Upsilon \to \gamma \eta(\eta')$ to obtained consistent result with experimental data [7]. With appropriate modifications we can obtain the S-matrix for $\Upsilon \to \gamma G_s$ decay. It is given by

$$\langle \gamma G_s | S | \Upsilon \rangle = -\frac{i}{2}eQ_b g_s^2 \epsilon^* \int d^4x d^4y d^4zd^4x_1 d^4y_1 e^{iq \cdot z} \langle \gamma | G_{\mu}(x) G_{\nu}(y) | 0 \rangle \langle 0 | \bar{b}_j(x_1) b_i(y_1) | \Upsilon \rangle \cdot M_{ij}^{\mu \nu \rho,ab}(x, y, x_1, y_1, z),$$

(2)

where $M_{ij}^{\mu \nu \rho,ab}$ is a known function from evaluation of the Feynmann diagrams, $i$ and $j$ stand for Dirac- and color indices, $a$ and $b$ is the color indices of gluon. $\epsilon^*$ is the polarization vector of the photon and $Q_b = -1/3$ is the $b$ quark electric charge. Since $b$ quark is heavy and moves with small velocity $v$, one can expand the Dirac fields in NRQCD fields:

$$\langle 0 | \bar{b}_j(x) b_i(y) | \Upsilon \rangle = -\frac{1}{6}(P_+ \gamma^I P_-)_{ij} \langle 0 | \chi^I \sigma^I \phi | \Upsilon \rangle e^{-ip \cdot (x+y)} + O(v^2),$$

(3)

where $\chi^I(\psi)$ is the NRQCD field for $\bar{b}(b)$ quark and $P_\pm = (1 \pm \gamma^0)/2$. The $b$ is almost at rest, then $p_\mu = (m_b, 0, 0, 0)$ with $m_b$ being the $b$ quark pole mass.

From the above we obtain the decay amplitude for $\Upsilon \to \gamma G_s$ as

$$\mathcal{T} = \frac{eQ_b g_s^2}{6} \langle 0 | \chi^I \cdot \sigma \psi | \Upsilon \rangle \int_0^1 dz \frac{1}{z(1-z)} \mathcal{F}_s(z),$$

(4)

the decay width then reads:

$$\Gamma = \frac{2}{9m_b^2} \pi Q_b^2 a_s^2 \langle \Upsilon | O_1(\bar{3}S_1) | \Upsilon \rangle \left| \int_0^1 dz \frac{1}{z(1-z)} \mathcal{F}_s(z) \right|^2.$$  

(5)

In the above $\mathcal{F}_s$ is the gluon distribution amplitude of $G_s$ and is given by

$$\mathcal{F}_s(z) = \frac{1}{2\pi k^+} \int dx^- e^{-izk^+ x^-} \langle G_s(k) | G_{\alpha, \mu}(x^-) G_{\alpha, + \mu}(0) | 0 \rangle.$$

(6)

Here we have used a gauge with $G^+ = 0$ such that the gauge link between the field strength operators vanish. This distribution characterizes basically how two gluons are converted into $G_s$, where one of the two gluons has the momentum $(zk^+, 0, 0)$.  

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In the above equations, the matrix element \( \langle \Upsilon | O_1 (^3S_1) | \Upsilon \rangle \) is defined in NRQCD contains the bound state effect of b-quarks in \( \Upsilon \) and can be extracted from leptonic \( \Upsilon \rightarrow l^+ l^- \) decay. A prediction can be made for \( \Upsilon \rightarrow \gamma G_s \) if the distribution amplitude is known.

The distribution amplitude can be written as

\[
F_s(z) = f_s f(z), \quad \text{with } \int_0^1 dz f(z) = 1.
\] (7)

where \( f(z) \) is a dimensionless function and its asymptotic form is:

\[
f(z) = 30 z^2 (1 - z)^2.
\] (8)

With the asymptotic form in Eq.(6) we have:

\[
R_s = \frac{\Gamma(\Upsilon \rightarrow \gamma + G_s)}{\Gamma(\Upsilon \rightarrow \ell^+ \ell^-)} = \frac{25 \pi \alpha_s^2}{3\alpha} \cdot \frac{|f_s|^2}{m_b^2}.
\] (9)

In the above we have used the fact that both \( \Upsilon \rightarrow \gamma G_s \) and \( \Upsilon \rightarrow l^+ l^- \) are proportional to \( \langle \Upsilon | O_1 (^3S_1) | \Upsilon \rangle \).

The use of the asymptotic form for \( f(z) \) may introduce some errors, because the scale \( \mu \) here is actually \( m_b \), not \( \mu \rightarrow \infty \). However, with a large \( m_b \) one may expect that it can provide a good order of magnitude estimate with the asymptotic form. We will use Eq. (9) later for our numerical discussions.

We note that at this stage the state \( G_s \) can be any particle with the same quantum number as \( G^{a+} \mu G_{\mu}^{a+} \), i.e., \( J^{PC} = 0^{++} \). The normalization constant \( f_s \) depends on the properties of the specific particle. In order to obtain the branching ratio of \( G_s \) as a scalar glueball, we have to evaluate \( f_s \) with \( G_s \) specified to be the scalar glueball. In the following we provide an estimate based on QCD sum rule calculation.

3. QCD sum rule calculation of the normalization constant

The constant \( f_s \) has dimension one in mass and is related to the the product of local operator:

\[
\langle G_s(k) | G^{\mu \rho} G_{\mu \rho} | 0 \rangle = f_0 m^2_B g^{\mu \nu} + f_s k^\mu k^\nu.
\] (10)
The fact that the same $f_s$ appears in Eq.(9) and Eq.(12) can be checked by integrating over $z$ on the both sides of Eq.(6).

The basic idea of the QCD sum rule calculation for our estimate is to consider the two point correlator

$$\Pi_{\mu\nu,\mu'\nu'}(Q^2) = \int d^4x e^{iqx} \langle 0| T G_{\mu\alpha} G^\alpha_{\nu'}(x), G_{\mu'\beta} G^\beta_{\nu'}(0) | 0 \rangle$$

$$= T_{\mu\nu\mu'\nu'} \Pi_T(Q^2) + V_{\mu\nu\mu'\nu'} \Pi_V(Q^2) + S_{\mu\nu\mu'\nu'}^{1} \Pi_S(Q^2)$$

$$+ S_{\mu\nu\mu'\nu'}^{2} \Pi_{S_2}(Q^2) + S_{\mu\nu\mu'\nu'}^{3} \Pi_{S_3}(Q^2), \quad (11)$$

for a region of $Q$ in which one can incorporate the asymptotic freedom property of QCD via the operator product expansion (OPE), and then relate it to the hadronic matrix elements via the dispersion relation. The tensors in Eq. (11) are defined as

$$T_{\mu\nu\mu'\nu'} = g^i_{\mu\nu} g^i_{\mu'\nu'} + g^i_{\mu\nu'} g^i_{\mu'\nu} - \frac{2}{3} g^i_{\mu\nu} g^i_{\mu'\nu'}$$

$$V_{\mu\nu\mu'\nu'} = g^i_{\mu\nu} q_{\mu'} q_{\nu'} + g^i_{\mu\nu'} q_{\mu'} q_{\nu'} + g^i_{\mu\nu'} q_{\mu} q_{\nu'} + g^i_{\mu\nu} q_{\mu} q_{\nu'}$$

$$S_{\mu\nu\mu'\nu'}^{1} = g^i_{\mu\nu} g^i_{\mu'\nu'} + S_{\mu\nu\mu'\nu'}^{2} = g^i_{\mu\nu} q_{\mu'} q_{\nu'} + g^i_{\mu\nu'} q_{\mu} q_{\nu'}, \quad S_{\mu\nu\mu'\nu'}^{3} = q_{\mu} q_{\nu} q_{\mu'} q_{\nu'}, \quad (12)$$

where $g^i_{\mu\nu} = g_{\mu\nu} - q_{\mu} q_{\nu}/q^2$. The corresponding terms $\Pi_T(Q^2), \Pi_V(Q^2), \Pi_S(Q^2), \Pi_{S_2}(Q^2)$ and $\Pi_{S_3}(Q^2)$ are from the contributions of $2^{++}, 1^{-+}$ and $0^{++}$ states respectively.

In a deep Euclidean region $Q^2 = -q^2 >> \Lambda_{QCD}$, they can be expanded as

$$\Pi_i(Q^2) = C^0_i(Q^2) I + C^1_i(Q^2) \alpha_s \langle G_{\mu\nu} G^{\mu\nu} \rangle + C^2_i(Q^2) \langle g_s f_{abc} G^a_{\alpha} G^b_{\beta} G^c_{\beta} \rangle + \cdots, \quad (13)$$

where $C^j_i$ are Wilson coefficients which need to be determined later.

On the other hand, the correlator in Eq.(11) can be saturated by all possible resonances and continuum. We have

$$\text{Im} \Pi_{\mu\nu,\mu'\nu'}(Q^2) = \sum_R \langle 0| G_{\mu\alpha} G^\alpha_{\nu'}|R \rangle \langle R| G_{\mu'\beta} G^\beta_{\nu'}|0 \rangle \pi \delta(Q^2 + m_R^2) + \text{continuum}, \quad (14)$$

where the sum on $R$ are for all possible resonances. The term $\langle 0| G_{\mu\alpha} G^\alpha_{\nu'}|R \rangle \langle R| G_{\mu'\beta} G^\beta_{\nu'}|0 \rangle$ in the above equation contains the information of $f_0$ and $f_2$ when $R$ is the scalar glueball.
The $T$ and $V$ tensors are not related to $f_s$. They are irrelevant to our calculations. The functions $\Pi_{S_1, S_2, S_3}$ contain linear combinations of $f_0$ and $f_s$. QCD sum rule calculations for $\langle G_s | G_{\mu \nu} G_{\mu \nu} | 0 \rangle = (4f_0 + f_s)m_G^2$ has been carried out before [9]. Therefore if one of the $\Pi_{S_1, S_2, S_3}$ is known, one can obtain $f_s$. From eq. (10) and the tensor structure of eq. (11), we find that $\Pi_{S_3}$ is proportional to $(f_0 + f_s)^2$. Therefore the study of $\Pi_{S_3}$ is sufficient for our purpose of determining $f_s$. $\Pi_{S_1, 2}$ also contain information about $f_0$ and $f_s$. However the non-perturbative contributions for them begin at the level of dimension-8 operators. The results obtained are not as reliable as the one from $\Pi_{S_3}$ which has a lower dimension. We now concentrate on $\Pi_{S_3}$.

There may be several bound states with the same quantum numbers to include in the QCD sum rule calculation, such as a pure scalar glueball, quark bound states and higher excited states. The contributions from higher excited states are suppressed upon the use of Borel transformation which is discussed in the below. For the quark bound states, OZI rule implies that the conversion of bound quark state into a scalar glueball is suppressed compared with the conversion of two gluon into a scalar glueball [12], perturbatively suppressed by powers in $\alpha_s$. If this is indeed true, the corresponding $f_s$ parameters for quark bound states will be smaller than pure glueball state. We will work with this approximation in the following discussions. To be consistent with our previous expansion, we again work to order $\alpha_s$. To this order, using the method in Ref. [11], we find

$$\Pi_{S_3}(Q^2) = \frac{1}{8\pi^2} \ln \frac{\mu^2}{Q^2} + \frac{1}{2Q^4} (\langle G_{\mu \nu} G^{\mu \nu} \rangle + \frac{2g_s}{Q^6} \langle f^{abc} G_a^\alpha G_b^\beta G_c^\gamma \rangle).$$  \hspace{1cm} (15)

The correlator in Eq. (15) obtained by using OPE is related to Eq. (14) via the standard dispersion relation

$$\Pi_{\mu \nu, \mu' \nu'}(Q^2) = \frac{1}{\pi} \int_0^\infty ds \frac{Im \Pi_{\mu \nu, \mu' \nu'}(-s)}{s + Q^2}.$$  \hspace{1cm} (16)

In practice one may only include ground states in the calculation. In order to reduce uncertainty due to higher excited states and also continuum states, we apply Borel transformation and obtain
\[
\hat{B}\Pi S_3(Q^2) = \frac{1}{M^2} \int_0^{s_0} ds e^{-s/M^2} \rho_{S_3}(s),
\]
where \(\rho_{S_3}(s) = (1/\pi) Im\Pi S_3(-s)\), and
\[
\hat{B}\Pi(Q^2) = \lim_{Q^2, n \to \infty} \frac{1}{(n-1)!}(Q^2)^n (-\frac{d}{dQ^2})^n \Pi(Q^2),
\]
Here one also needs to have the limit \(Q^2/n = M^2 = \text{constant}\).

In our numerical calculation we have varied \(s_0\) in the range of \(3 - 6 GeV^2\), and found that the uncertainty is around 10 percent. The parameters determined are reasonably stable.

We obtain the range for \(f_s\) as
\[
f_s = (100 \sim 130)\text{MeV},
\]
with \(f_0 = 190\) MeV and \(f_s = 100\) MeV for \(m_{0^{++}} = 1.5\) GeV, and \(f_0 = 130\) MeV and \(f_s = 130\) for \(m_{0^{++}} = 1.7\) GeV. In obtaining the above result, we have re-evaluated \(f_0\) also using the same parameters. The input parameters used are \([10]\) \(\alpha_s(\mu) = 4\pi/9 \ln(\mu^2/\Lambda_{QCD}^2)\), \(\Lambda_{QCD} = 0.25\text{GeV}\), \(\mu = M\), \(\langle\alpha_s G_{\mu\nu}^a G_{\mu\nu}^a\rangle = 0.06 \pm 0.02\text{GeV}^4\), and \(g_s \langle f^{abc} G^a G^b G^c \rangle = 0.27\text{GeV}^2 \langle\alpha_s G_{\mu\nu} G_{\mu\nu}\rangle\).

For consistency, we also calculated the glueball masses. We find that for the \(0^{++}\) state the mass is \(1.5 \sim 1.7\) GeV, and for \(2^{++}\) the mass is \(2.0 \sim 2.2\) GeV. These are in agreement with other calculations \([9]\).

If the scalar glueball is a pure one, using the above results we obtain the branching ratio for \(\Upsilon \to \gamma G_s\) to be in the range
\[
Br(\Upsilon \to \gamma G_s) = (1 \sim 2) \times 10^{-3},
\]
with a larger branching ratio for a larger glueball mass up to 1.7 GeV. Here we have used \(\alpha_s = 0.18\) which is the typical value for \(\alpha_s\) in the energy range of the decay. We obtain a large branching ratio for \(\Upsilon \to \gamma G_s\). We would like to point out that considering several uncertainties, the assumptions of factorization and single pure glueball state in the QCD sum rule calculation, the above numbers should be used as an order of magnitude estimate.
4. Discussions of phenomenological implications

Experimental measurement of $\Upsilon \to \gamma G_s$ may be non-trivial. One has to rely on the decay products of glueballs. There are several ways the glueball can decay with reasonably large branching ratios, $G_s \to K\bar{K}$ or $G_s$ to multi-pions. As mentioned earlier that there are several candidates for scalar glueball, the $f(1370)$, $f_0(1500)$ and $f_0(1710)$. Decays of $\Upsilon \to \gamma f_0(i) \to \gamma(K\bar{K}$ or multi-pions) can provide important information.

Experimentally there is only an upper bound $[3]$ of $\text{Br}(\Upsilon \to \gamma f_0(1710) \to \gamma K\bar{K}) < 2.6 \times 10^{-4}$ at 90% C.L.. If $f_0(1710)$ is a pure glueball, experimental measurement $[3]$ of $\text{Br}(f_0(1710) \to K\bar{K}) = 0.38_{-0.19}^{+0.09}$ $[3]$ would imply $\text{Br}(\Upsilon \to \gamma f_0(1710) \to \gamma K\bar{K})$ to be in the range of $(0.4 \sim 1.0) \times 10^{-3}$ which seems to indicate that $f_0(1710)$ may not be a pure glueball. At present it can not rule out the possibility that one of the $f_0(1370)$ or $f_0(1500)$ being a pure glueball state. Data also allow certain mixing among glueball state and other quark bound states.

Theoretical calculation of the mixings among glueball and quark bound states is a very difficult task. There is no reliable theoretical calculation. Lattice calculations may eventually give accurate predictions for the mixing parameters. At present there are some phenomenological studies of glueball mixings. We now study some implications of the branching ratio for the radiative decay of a $\Upsilon$ into a pure scalar glueball obtained in the previous section on a mixing pattern suggested in Ref. [13].

An analysis combining other experimental data in Ref. [3] showed that the three $0^{++}$ states $f_0(1370)$, $f_0(1500)$ and $f_0(1710)$ all contain substantial glueball content. Ref. [3] obtained a mixing matrix of physical states in terms of pure glueball and other quark bound states to be [3]

\[
\begin{array}{ccc}
 f^G_{11} & f^S_{12} & f^{(N)}_{31} \\
 f_0(1710) & 0.39 \pm 0.03 & 0.91 \pm 0.02 & 0.15 \pm 0.02 \\
 f_0(1500) & -0.65 \pm 0.04 & 0.33 \pm 0.04 & -0.70 \pm 0.07 \\
 f_0(1370) & -0.69 \pm 0.07 & 0.15 \pm 0.01 & 0.70 \pm 0.07 \\
\end{array}
\]
where the states $G_s, S = |s\bar{s}\rangle$ and $N = |u\bar{u} + d\bar{d}/\sqrt{2}$ are the pure glueball and quark bound states. $f_{i1}^{G_s}$ indicate the amplitude of glueball $G_s$ in the three physical $f_0(i)$ states.

Because of the mixing, when applying our calculations to radiative decay of $\Upsilon$ into a physical state which is not a purely gluonic state the parameters will be modified. If the mixing parameter is known one can obtain the $R_s$ ratios for $\Upsilon \to \gamma f_0(1370, 1500, 1710)$ as

$$R_s(\Upsilon \to \gamma f(i)) = \frac{25\pi\alpha_s^2}{3\alpha_s} \cdot \frac{|f_s|^2}{m_b^2 |f_{i1}^{(G_s)}|^2}. \quad (22)$$

$\Upsilon \to \gamma f(i)$ may also result from $\Upsilon$ decays into a $\gamma$ and $S, N$ quark bound states. However, these processes are suppressed by $\alpha_s^2$.

Using the mixing amplitudes in Eq. (21), one obtains the branching ratios of, $\Upsilon \to \gamma f_0(1370, 1500, 1710)$ to be in the ranges, $(4.8 \sim 9.6, 4.2 \sim 8.4, 1.5 \sim 3.0) \times 10^{-4}$. Combining the branching ratios of $f_0(1370, 1500, 1710) \to K\bar{K}(\pi\pi)) = (0.38^{+0.09}_{-0.19}(0.039^{+0.002}_{-0.021}, 0.044^{+0.021}_{-0.021}(0.454^{+0.104}_{-0.104}, 0.35^{+0.13}_{-0.13}(0.26^{+0.06}_{-0.06}))$ [3], we obtain:

- $Br(\Upsilon \to \gamma f_0(1710) \to \gamma K\bar{K}) \approx 0.6 \sim 1.2$;
- $Br(\Upsilon \to \gamma f_0(1710) \to \gamma \pi\pi) \approx 0.06 \sim 0.12$;
- $Br(\Upsilon \to \gamma f_0(1500) \to \gamma K\bar{K}) \approx 0.2 \sim 0.4$;
- $Br(\Upsilon \to \gamma f_0(1500) \to \gamma \pi\pi) \approx 1.9 \sim 3.8$;
- $Br(\Upsilon \to \gamma f_0(1370) \to \gamma K\bar{K}) \approx 1.7 \sim 3.4$;
- $Br(\Upsilon \to \gamma f_0(1370) \to \gamma \pi\pi) \approx 1.2 \sim 2.4$.

In the above the branching ratios are in unit $10^{-4}$. The branching ratios predicted above can provide further test for QCD factorization. Future experimental data from CLEO III will provide us with important information.

To summarize, we have estimated the branching ratio of $\Upsilon \to \gamma + G_s$ with $G_s$ as a glueball. Our result shows that $f_0(1710)$ may not be consistent with the assumption that it is a pure glueball, but can not rule out the possibility that one of the $f(1370)$, and $f_0(1500)$ being a pure glueball state. We also predicted several $\Upsilon \to \gamma K\bar{K}(\pi\pi)$ branching ratios using a phenomenological glueball mixing pattern which can provide further tests for QCD factorization calculations and glueball mixing. To have a better understanding of the situation, we have to rely on future improved experimental data. Fortunately CLEO-III will provide us with more data in the near future. We have a good chance to understand the
properties of scalar glueball. We strongly encourage our experimental colleagues to carry out the study of radiative decay of $\Upsilon$ into a scalar glueball.

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