QCD corrections for double charmonia production in $e^+e^-$ annihilation

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Abstract. We discuss $J/\psi \eta_c$ production in $e^+e^-$ annihilation at next-to-leading order of pQCD. We are focusing at virtual $Z_0$ contribution into this process: the interference between virtual photon and $Z_0$-boson is required for careful study. Cross-sections behavior at high energies is studied. At energies $\sim M_{Z_0}$ the NLO contribution enhances cross-sections up to 2 times.

1. Introduction

Associative $J/\psi \eta_c$ production was well studied at $B$-factories in the past. Belle and Babar collaborations measured the cross-sections near the threshold energies. It turned out that theoretically predicted cross-sections were more than 5 times lower than measured ones. This gap launched an intensive study of different corrections and production mechanisms (see for example [1],[2],[3],[4],[5]). Now this process is studied at two loops accuracy [6]. While cross-sections measured in BaBar have been achieved, Belle values are still unexplained by theory. Study of charmonia production in the future is encouraged by two big projects: ILC and FCC. Both of them propose $e^+e^-$ collisions at energies of order of $Z$-boson mass: energy range announced for FCC is $\sqrt{s} = 90 \div 400 \text{ GeV}$ and $\sqrt{s} = 250 \text{ GeV}$ is proposed for ILC.

In this paper we consider double charmonia production in close relation to pair $B_c$ production. At energies of order of $Z$-boson mass the interference through $Z$-boson may become dominant. However it can be comparable with photon-photon production as well. For pair $B_c$ production these two contributions are already calculated in NLO and published (see papers [7] and [8]). For double charmonia production we solve three challenges concurrently: VV, VP and PP cases:

$$\begin{align*}
  e^+e^- & \xrightarrow{Z_0^\ast} \eta_c \eta_c \\
e^+e^- & \xrightarrow{\gamma^\ast Z_0^\ast} J/\psi \eta_c \\
e^+e^- & \xrightarrow{Z_0^\ast} J/\psi J/\psi
\end{align*}$$

For more careful consideration we include virtual $\gamma$ and $Z$ interference in annihilation. According to charge parity conservation photon annihilation is allowed only in $J/\psi \eta_c$ case. $J/\psi J/\psi$ and $\eta_c \eta_c$ production go with $Z$-boson exchange only.
2. Theoretical approach

Complete analysis of QCD corrections involves interference between LO and NLO amplitudes as well as interference between $\gamma^*$ and $Z_0^*$. An important notation should be added: we gain no corrections for real gluon radiation in case of pair production since the bound states are colour singlets. Thereby we calculate these seven terms:

\[
|A|^2 = |A^{LO}_\gamma|^2 + |A^{LO}_Z|^2 + 2\text{Re} \left( A^{LO}_\gamma A^{LO*}_Z \right) + \\
+ 2\text{Re} \left( A^{LO}_\gamma A^{NLO*}_Z \right) + 2\text{Re} \left( A^{LO}_Z A^{NLO*}_\gamma \right) + 2\text{Re} \left( A^{LO}_Z A^{NLO*}_\gamma \right) + \ldots
\]

The approximation we are working in is typical for production of double heavy bound states. Nonrelativistic QCD allows one to convolute the matrix element of hard production of two $c\bar{c}$ pairs with quarkonia wave functions deriving from potential models. For unpolarized S-wave states we can simply multiply matrix element by radial wave functions at origin. Actually the production amplitude can be written as follows:

\[
A^{SJJ_z} = \int T_{cc\bar{c}\bar{c}}^{SJJ_z}(p_i, k(q_1), k(q_2)) \cdot \left( \Psi_{cc\bar{c}\bar{c}}^{JJ_z}(q_1) \Psi_{cc\bar{c}\bar{c}}^{JJ_z}(q_2) \right)^* \cdot C_{sJl}^{J_z} \frac{dq_1}{(2\pi)^3} \frac{dq_2}{(2\pi)^3},
\]

where $T_{cc\bar{c}\bar{c}}$ is an amplitude of the hard production of two heavy quark pairs; $\Psi_{cc\bar{c}\bar{c}}^{JJ_z}(q_{1,2})$ are wave functions of quarkonia; $J$ and $J_z$ are the total angular momentum and its projection on $z$-axis in $e^+e^-$ center-of-mass frame; $L$ and $l_z$ are the orbital angular momenta of quarkonia and their projections on $z$-axis; $S$ and $s_z$ are the total quarkonia spin and its projection; $C_{sJl}^{J_z}$ are Clebsh-Gordon coefficients; $p_i$ are momenta of $c,\bar{c}$-quarks; $q_{1,2}$ are three momenta of $c$-quarks in the $J/\psi$ and $\eta_c$ rest frames correspondingly (in those frames $k(q_{1,2}) = (0, q_{1,2})$).

Therefore for unpolarized S-wave states we reduce the formula to:

\[
A = \frac{1}{4\pi} R_{J/\psi}(0) R_{\eta_c}(0) \cdot T_{cc\bar{c}\bar{c}}(p_i) \big|_{q_{1,2}=0^+}.
\]

The following projectors onto the bound states are used in our study:

\[
\Pi_{J/\psi}(P, m) = \frac{\hat{P} - m}{2\sqrt{2}} \gamma^\mu \varepsilon^\mu_{J/\psi} \times \frac{1}{\sqrt{3}}, \quad \Pi_{\eta_c}(Q, m) = \frac{\hat{Q} - m}{2\sqrt{2}} \gamma^5 \times \frac{1}{\sqrt{3}}.
\]

In the case of pair $B_c$ production the projectors have quite similar form. At the same time the projection procedure differs a little: $J/\psi \eta_c$ bound states can be formed either from $c$ and $\bar{c}$-quarks of two currents independently or from $c$ and $\bar{c}$-quarks originating from different currents. Forming two $B_c$ bound states involves the latter case only.

It should be clarified that we treat bound states as colour singlets only. We work in so-called $\delta$-approximation (the internal motion of quarks in quarkonium is negligible) and the velocities of $c\bar{c}$ quarks are directly put equal (before the projection onto the bound state $\Psi_{cc\bar{c}\bar{c}}$).

The renormalization procedure is organized by building counter-terms from the leading order diagrams. The renormalization constants are listed in (5)–(7). “On shell” scheme is fixed for mass and spinors renormalization and $\overline{MS}$ scheme is fixed for coupling constant. The singular parts of amplitudes carry poles $O(1/\varepsilon)$ only (no poles with $\varepsilon$ squared).

\[
A^{CT} = Z_2^s A^{LO}_{m\rightarrow Zm, m \rightarrow Zg, g \rightarrow Zg}.
\]
\[ Z_{m}^{OS} = 1 - \frac{\alpha_s}{4\pi} C_F C_e \left[ \frac{3}{\epsilon_{UV}} + 4 \right] + O(\alpha_s^2), \quad (5) \]

\[ Z_{g}^{OS} = 1 - \frac{\alpha_s}{4\pi} C_F C_e \left[ \frac{1}{\epsilon_{UV}} + \frac{2}{\epsilon_{IR}} + 4 \right] + O(\alpha_s^2), \quad (6) \]

\[ Z_{g}^{TTS} = 1 - \frac{\beta_0}{2} \frac{\alpha_s}{4\pi} \left[ \frac{1}{\epsilon_{UV}} - \gamma_E + \ln(4\pi) \right] + O(\alpha_s^2). \quad (7) \]

3. Calculation details

Working with Feynman diagrams one can factorize the annihilation process and firstly consider the Z-boson decay into two $c\bar{c}$ pairs (see figure 1) as well as decay of virtual photon. There are 4 diagrams at leading-order and 86 diagrams with one loop for $Z_0$ decay and the same number of diagrams for $\gamma^*$ decay. The diagrams and the corresponding analytic expressions are generated with FeynArts-package in Mathemtica.

\[ \begin{align*}
Z_{m}^{OS} & = 1 - \frac{\alpha_s}{4\pi} C_F C_e \left[ \frac{3}{\epsilon_{UV}} + 4 \right] + O(\alpha_s^2), \\
Z_{g}^{OS} & = 1 - \frac{\alpha_s}{4\pi} C_F C_e \left[ \frac{1}{\epsilon_{UV}} + \frac{2}{\epsilon_{IR}} + 4 \right] + O(\alpha_s^2), \\
Z_{g}^{TTS} & = 1 - \frac{\beta_0}{2} \frac{\alpha_s}{4\pi} \left[ \frac{1}{\epsilon_{UV}} - \gamma_E + \ln(4\pi) \right] + O(\alpha_s^2).
\end{align*} \]

Figure 1. Some sample diagrams for $Z \rightarrow c\bar{c}c\bar{c}$ process at next-to-leading-order.

The computation strategy is based on the following tollchain: \texttt{FeynArts $\rightarrow$ FeynCalc} \[9\] (\texttt{FeynCalcFormLink, TIDL}) $\rightarrow$ \texttt{Apart $\rightarrow$ FIRE} \[10\] $\rightarrow$ \texttt{X-package} \[11\]. The amplitudes are moved from \texttt{FeynArts} to \texttt{FeynCalc} which provides algebraic calculations with Dirac and colour matrices. Taking traces is established in both \texttt{FeynCalc} and \texttt{FORM}. We don’t run \texttt{FORM} code directly rather work with package \texttt{FeynCalFormLink}, it embegges \texttt{FORM} into Mathemtica and allows one to save time a lot. The Passarino-Veltman reduction is conducted with \texttt{TIDL} library which also appears in \texttt{FeynCalc}. We keep only scalar expressions by loop momentum $k$ after this reduction. \texttt{Apart} function does the extra simplification by partial fractioning for IR-divergent
integrals. At last FIRE package provides the complete reduction to master integrals. The master integrals are evaluated by substitution of their analytical expressions with the help of X-package. In this challenge we treat only 1,2,3-point integrals $A_0, B_0, C_0$ after FIRE. It is worth to clarify that all the calculations are analytical. The masses and other parameters are set at last step to obtain the numerical values.

To present the regularization technique it should be mentioned that we work in conventional dimensional regularization CDR where all the momenta live in $D$ dimensions: loop momentum as well as external momenta. The Dirac matrices also live in $D$ dimensions. However the problem of $\gamma^5$ interpretation arises as it is badly determined in $D$-dimensions. We use the so-called naive interpretation when $\gamma^5$ anticommutates with all other matrices and therefore disappears in traces with an even number of $\gamma^5$. For traces with an odd number of $\gamma^5$ the following expression is inserted:

$$\gamma^5 = \frac{-i}{24} \varepsilon_{\alpha\beta\sigma\rho} \gamma^\alpha \gamma^\beta \gamma^\sigma \gamma^\rho,$$

where $\varepsilon_{\alpha\beta\sigma\rho}$ is either 4- or $D$-dimensional.

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**Figure 2.** $\sigma_{LO}$ and $\sigma_{NLO}$ dependence on $\sqrt{s}$.

**Figure 3.** $\sigma_{LO}$ and $\sigma_{NLO}$ dependence on $\sqrt{s}$.

**Figure 4.** $\sigma_{LO}$ and $\sigma_{NLO}$ dependence on $\sqrt{s}$. Peak at $\sqrt{s} \approx 90$ GeV corresponds to the natural width of $Z_0$-boson.

**Figure 5.** $\sigma_{NLO}/\sigma_{LO}$ ratio dependence on $\sqrt{s}$: annihilation with $\gamma^*$ only and full annihilation with $\gamma^*$ and $Z^*$ interference.
4. Results
Cross-sections behaviour with energy is shown in the figures 2–4, where both $\gamma^*$ and $Z^*$ contribute to annihilation. In these plots $\mu$ is fixed at $\mu = \sqrt{s}$. It can be seen that NLO contribution significantly enhances the LO values. Near the threshold energies virtual photon exchange plays the dominant role. Maximum in the figure 2 corresponds to energy $6 \div 7$ GeV. At energies close to $Z$-boson mass virtual $Z$-boson exchange becomes dominant (see figure 4 where the peak shows the natural width of $Z$-boson). The asymptotic behaviour of cross-sections with center-of-mass energy is established: cross-sections decrease with energy as $O(s^{-3})$ at LO as well as at NLO.

![Figure 6. $\sigma_{NLO}$ dependence on $\sqrt{s}$ for different scales.](image1)

![Figure 7. $\sigma_{NLO}$ dependence on $\sqrt{s}$ for different scales.](image2)

![Figure 8. $\sigma_{NLO}$ dependence on $\sqrt{s}$ for different scales. Peak at $\sqrt{s} \approx 90$ GeV corresponds to the natural width of $Z_0$-boson.](image3)

![Figure 9. $\sigma_{NLO}/\sigma_{LO}$ ratio dependence on $\sqrt{s}$ for different scales.](image4)

Variation of the scale parameter $\mu$ provides the systematic error of the calculation. In this study we set the same $\mu$ for renormalization scale and for coupling scale (see figures 6–8 reproducing $\sigma_{NLO}$ from the previous figures 2–4). Varying $\mu$ in the range $\sqrt{s}/2 < \mu < 2\sqrt{s}$ we estimate the cross-sections values: the shaded area in the figures.

The ratios of NLO cross sections to LO ones are plotted. Proceeding from the scale variation error one can estimate $\sigma_{NLO}/\sigma_{LO} = 1.6 \div 1.8$. Figure 5 demonstrates that $\sigma_{NLO}/\sigma_{LO}$ for full annihilation reproduces the ratio $\sigma_{\gamma^*/\gamma^*}$ with virtual photon only at energies not more than
one quarter of $M(Z_0)$. Then purple line tends to constant value 1.7 while red line increases without limit. This feature demonstrates the importance of $Z_0$ account: at high energies the results without $Z_0$-annihilation look unreliable. Decreasing of $\sigma^\gamma$ with energy seems to be too fast: $\sigma^\gamma_{LO} \sim 1/s^4$ and $O(1/s^4) < \sigma^\gamma_{NLO} < O(1/s^3)$.

![Figure 10. $d\sigma/d\cos\theta$ dependence on $\cos\theta$ with $\mu$ and $\sqrt{s}$ fixed at $\mu = \sqrt{s} = M_Z/2$.](image1)

![Figure 11. $<\cos\theta>$ dependence on $\sqrt{s}$.](image2)

The presence of azimuthal asymmetry induced by weak current is also obtained. It appears to be the most notable at energies nearby $\sqrt{s} = M_Z/2$ and can be described by the averaged quantity $<\cos\theta> = \frac{1}{\pi} \int d\cos\theta \ (\cos\theta \frac{d\sigma}{d\cos\theta})$. However account for NLO correction minimizes effect of parity asymmetry (see figures 10 and 11).

5. Conclusions

In this study we calculate the cross-sections for associative production of $J/\psi \eta_c$ in $e^+e^-$ annihilation at next-to-leading order precision. It is shown that at energies more than $M_{Z_0}/4$ annihilation with photon only becomes insufficient. Therefore cross sections behaviour is presented through interference between virtual $\gamma$ and $Z_0$ both contributing to annihilation. At energies $\sim M_{Z_0}$ contribution of $Z_0$ becomes dominant and establishes the asymptotic dependence on $\sqrt{s}$ as $O(s^{-3})$. It is obtained that QCD corrections $O(\alpha_S)$ significantly enhance the leading-order cross-sections. At low energies the ratio $\sigma_{NLO}/\sigma_{LO} < 3$ and starting from $\sqrt{s} = 60$ GeV this ratio is fixed in the range $\sigma_{NLO}/\sigma_{LO} = 1.6 \div 1.8$. The results performed in the paper might be relevant for future study of charmonia physics at ILC or FCC colliders.

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