Dynamics of secondary vortices in turbulent channel flow

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Abstract.
Secondary vortical structures in a straight channel of rectangular cross-section are studied in details experimentally. Special attention is devoted to theirs dynamics. The stereo time-resolved PIV method is used to acquire time series of the 3-component vector fields in plane perpendicular to the mean flow. Dynamics of the vortical structures in the vector fields is studied using Proper Orthogonal Decomposition method. The dynamical modes are shown.

1. Introduction

The problem of secondary flows developing in ducts of a cross section with corners has attracted much attention after the initial observation by Nikuradse (1926) that large-scale secondary motions significantly alter the mean velocity contours in the cross section of the duct. The problem has been studied from many points of view experimentally (see e.g. Brundett & Baines (1964), Hoagland (1960), Gessner (1973)), simulations of the Reynolds-averaged equations (see e.g. Demuren & Rodi (1984)), large eddy simulations (see e.g. Madabhushi & Vanka (1991)), and direct numerical simulations (e.g. Gavrilakis (1992)). The picture that emerges from these studies is that, from averaging over time and/or streamwise distance, a coherent motion of small amplitude exists, formed by eight vortices of very weak streamwise vorticity in the cross section, symmetric about the duct diagonals and the bisection lines. Those structures are called secondary flow of 2nd kind. Such vortices differ from those usually observed near walls in turbulent boundary layers, because they are large-scale and are locked near the corners by the imposed geometric constraints. The observed structure is the cross-section aspect ratio dependent. Recent series of publications (see e.g. Galletti & Bottaro (2004), Bottaro, Soueid & Galletti (2006), Biau, Soueid & Bottaro (2008)) clarify some aspects of the secondary flow origin working on stability theory.

The longitudinal secondary vortices are considered to be stationary - see e.g. Hoagland (1960). The references given above contain a lot of information on the steady mean structure of the secondary flow, however there is lack of information concerning the vortical structures dynamics. The presented paper is an contribution in this field.
2. Description of Experiment and Procedures

The experiments have been carried out in the blow-down facility in the IT. The channel with cross-section $250 \times 100 \text{mm}^2$ and 3 meters in length has been used. Velocity of the air-flow was about $4.6 \text{m/s}$ in the channel inlet, top-hat profile with intensity of fluctuations less than 0.1% and deviations of the mean velocity were less than 1% throughout the cross-section. The flow is subjected to experimental investigation in planes perpendicular to the mean flow. In the top-right corner the 3-components of velocity are evaluated using the stereo-PIV. The coordinate system and area of interest is depicted in Fig. 1.

![Figure 1. System of coordinates.](image)

The stereo time-resolved PIV technique has been used to acquire data in the plane perpendicular to the channel axis. The DANTEC system consists of laser with cylindrical optics and 2 CMOS cameras and software Dynamics Studio 3.14. Laser New Wave Pegasus Nd:YLF, double head, wavelength $527 \text{nm}$, maximal frequency $10 \text{kHz}$, a shot energy is $10 \text{mJ}$ for $1 \text{kHz}$ (corresponding power $10 \text{W}$ per head). Cameras NanoSense MkIII, maximal resolution $1280 \times 1024 \text{pixels}$ and corresponding maximal frequency 500 double-snaps per second. For particle generation the fog-generator SAFEX is used.

For analysis of the acquired dynamically changing velocity field we have used the Proper Orthogonal Decomposition (POD), identifying energetic structures in the flow-field - see Lumley (1967). The dynamics of the identified structures - modes are performed with help the POD modification Bi-Orthogonal Decomposition (BOD) defining both spatial modes (topos) and corresponding temporal modes (chronos). By truncation of low-energy modes we could study dynamics of the flow using low-dimensional dynamical system methods in phase space. The BOD method was proposed by Aubry, Guyonnet & Lima (1991).

The proposed measurement procedure have been tested on the case of longitudinal vortical structures in the wake behind Ahmed body - see Uruba (2010).

3. Results

The experiments were carried out in distance $2.5 \text{m}$ from inlet. The 1600 doublesnaps with frequency $500 \text{Hz}$ were acquired forming the time series of $3.2 \text{s}$.

First, the mean vector field was calculated and subtracted from all instantaneous pictures resulting in time series of fluctuations. In Fig. 2 the mean velocity distribution is shown, the in-plane velocity components $U$ and $V$ are represented by vectors, while the main-flow component
W perpendicular to the measuring plane is shown as color. The coordinates are non-dimensioned using the channel width 100 mm.

**Figure 2.** Mean velocity distribution.  

**Figure 3.** The total fluctuating energy.

The mean flow structure corresponds to that described in literature as secondary structures of the 2nd kind.

In Fig. 3 the kinetic energy distribution in the cross-section is shown. The energy maxims are located in certain distance from the wall, while in the vicinity of the corner axis minimal fluctuating activity is detected. The small perturbation near wall for small $x$ are connected with technical problems close to the wall (reflections).

**Figure 4.** POD cumulative modes energy.  

**Figure 5.** Spectra of first 10 chronoses.

The sequence of 1600 velocity fluctuations snapshots was subjected to standard POD analysis resulting into the same number of energetic modes ordered according to the kinetic energy content. In Fig. 4 the cumulative kinetic energy of the evaluated modes is shown. The first mode contains about 7.7% of total fluctuating energy, while the first 5 modes in sum contain
about 26%, 10 modes 38%, 50 modes 73%, 100 modes 86%, 200 modes 95% and 400 modes almost 99% of the total fluctuating kinetic energy.

Generally, the low-order high-energy modes containing the most of kinetic energy exhibit the low-frequency dynamics. This was proved by analysis of corresponding chronoses. However the spectra of all chronoses are continuous of wide-band type. In Figure 5 the spectra of the first 10 chronoses are depicted. A few frequency peaks could be detected. The first 3 modes have maximal energy between 5 and 10$Hz$. The higher order modes exhibit considerably lower energy on low frequencies, however the high-frequency content ($40Hz$ and higher) is more or less the same.

Now a few toposes will be shown. In Figs. 6 to 13 the typical examples of toposes are shown. The color denoting $w$ velocity component fluctuation is red for positive and blue for negative values. The typical dynamical structures could be recognized. Please note that the toposes are normalized to be orthogonal, however their energy differs (see Fig. 4).

**Figure 6.** The 1st topos.

**Figure 7.** The 2nd topos.

**Figure 8.** The 3rd topos.

**Figure 9.** The 5th topos.
The structures in energetic modes (toposes) are to be analyzed. The structures of velocity fluctuations in longitudinal direction $w$ form spots of pulsations, as the time modes (chronoses) represent positive and negative fluctuations with zero mean value. To see better the structures formed by vector field the vector lines are added.

The vector fields reveal the dynamic structures forming the secondary flows. We could recognize several types of structures, but surprisingly not many spirals corresponding to vortices. Typical structures for lower order modes are saddles and nodes and improper nodes forming curves as defined in Kenwright (1998). The first vortical structure could be recognized in 12th topos - see Fig. 11.

The higher order mode, the smaller energy content and also the smaller topological structures in correspondent topos.
4. Conclusion

The dynamical structures in secondary flow of 2nd kind in a channel of rectangular cross-section have been studied using stereo time-resolved PIV technique. The BOD method shows both time and space energetic modes. The 3 most energetic chronoses contain maximum energy for frequencies between 5 and 10 Hz. In toposes the pulsating spots of longitudinal velocity could be recognized. In-plane structures are typically saddles and nodes, while spirals (corresponding to vortices) are not very frequent.

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