A vorton gun

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Abstract

In about half of all near-cusp events in superconducting cosmic strings with chiral currents (and probably with general currents as well), the string intersects itself near the cusp. Intercommutation causes the conversion of the string near the cusp into a vorton (in the chiral case) with very high Lorentz boost. We demonstrate how to analyze the cusp shape in a Lorentz frame that makes the motion simple, by the use of a 5-dimensional procedure, and analyze the resulting production of vortons.

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I. INTRODUCTION

Cosmic strings are topological defects that may have been created by symmetry breaking phase transitions in the early universe. (For reviews see [1,2]). Witten [3] discovered that in many theories, cosmic strings could acquire a condensate of zero-mass particles, and thus could behave as superconducting wires carrying charge or current. The existence of charges and currents on the string can substantially modify its dynamics. If the charge and current are large enough, the string may have a stable static configuration known as a vorton [4].

For a non-superconducting string, as long as the string thickness (typically a tiny microscopic quantity) is small as compared to its radius of curvature (typically of cosmological size), the motion of the string is given by the Nambu-Goto equations of motion, which are easily solved. With an appropriate choice of parameter (gauge), the string position can be given by a function $x(\sigma, t)$, where $\sigma$ parameterizes the energy on the string, and $t$ is the ordinary time coordinate. The general motion of the string can then be written

$$x(\sigma, t) = \frac{1}{2} [a(\sigma - t) + b(\sigma + t)]$$

where the functions $a$ and $b$ are arbitrary except that they must obey the constraints

$$|a'(\sigma)|^2 = 1$$

$$|b'(\sigma)|^2 = 1$$

In general, a Nambu-Goto string loop will have two or more points at which $b'(\sigma, t) = -a'(\sigma, t)$. Such a point is called a cusp. It has $x' = (a' + b')/2 = 0$ and $\dot{x} = (b' - a')/2 = b'$. Thus $|\dot{x}| = 1$; a point moves at the speed of light.

For a superconducting string, the equations of motion are much more complicated. However, if we can neglect long-range electromagnetic interactions (either because the condensate is not coupled to the electromagnetic field, or because such corrections are small), the situation is much simpler. If furthermore the current is “chiral” (charge and current equal in magnitude), then the equations can be solved exactly [5–7]. This situation can occur both in strings which can carry currents in only one direction [8,9], but also in more usual superconducting strings. As the string approaches a cusp, all charge-carriers will be accelerated, and left-moving and right-moving charge carriers can scatter with each other and be ejected from the string. This will lead to a predominance of whichever type of charge-carrier was in larger numbers originally, and thus an approach to a chiral state.

In the chiral case, the motion is still given by Eq. (1), but now the constraints are

$$|a'(\sigma)|^2 = 1$$

$$|b'(\sigma)|^2 = 1 - j^2$$

(or the same with $a$ and $b$ exchanged, depending on the direction of current flow), where $j$ is a measure of the magnitude of the current, to be discussed later. From Eqs. (1) and

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1We work in units where $c = 1$. 

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we can see that such a string cannot have a cusp, per se. Since $a'$ and $b'$ have different magnitudes, they cannot cancel exactly, and since $|b'| < 1$, we cannot have $|\dot{x}| = 1$.

However, if $j \ll 0$, then we can have an event that looks very similar to a Nambu-Goto cusp at large distances, but has a different structure near the place that the cusp would have occurred. Thus we will consider a point with $a'$ and $b'$ pointing in opposite directions, even though their magnitudes are different, so $b' = -\sqrt{1 - j^2}a'$. This means that $x'$ has some small but nonzero magnitude. We will show that this can lead to a self-intersection near the cusp and the emission of a vorton.

II. SELF INTERSECTION

Because the two-dimensional world sheet current is conserved, we can describe it as the curl of an auxiliary scalar field $\phi$. The world sheet current is then

$$J^a = q \frac{1}{\sqrt{-\gamma}} \epsilon^{ab} \phi_{,b}$$  \hspace{1cm} (4)

where $q$ is the electric charge multiplied by the integral of the field density across the string core, and renormalized if necessary, and $\gamma^{ab}$ is the induced metric in the world sheet. The physical current is

$$J^\mu(x) = \int d\sigma dt \sqrt{-\gamma} J^a x_a^{\mu} \delta^{(4)}(x - x(\sigma, t)) = q \int d\sigma \epsilon^{ab} \phi_{,a} x_b^{\mu} \delta^{(3)}(x - x(\sigma, t))$$  \hspace{1cm} (5)

In the chiral case, we can assume without loss of generality that the charge carriers move in the negative $\sigma$ direction, so that $\phi$ is a function only of $\sigma + t$, and thus $\phi' = \dot{\phi}$. We will define $j = 2\dot{\phi}' / \sqrt{\mu}$, which gives the constraints of Eqs. (3). The current $j$ is dimensionless and gives a measure of the ratio of the energy in the current to the energy in the string. With $j = 1$ we would have $b' = 0$ which gives a static vorton solution.

We will consider a cusp in a string loop with $j \ll 1$, and we will define $\Delta = |b'| = \sqrt{1 - j^2}$ and $\epsilon = 1 - \Delta$, and let subscript 0 denotes quantities at the time and place of the cusp. We have $|a'_0| = 1$, $b'_0 = -\Delta a'_0$, and so

$$x'_0 = (a'_0 + b'_0)/2 = (\epsilon/2)a'_0$$ \hspace{1cm} (6a)

$$\dot{x}_0 = (b'_0 - a'_0)/2 = -\frac{1 + \Delta}{2}a'_0$$ \hspace{1cm} (6b)

In [10] we showed that, for an ordinary string cusp, one can choose a Lorentz frame in which $\dot{x}_0$ and $x''_0$ are parallel. In the appendix, we extend this result to chiral superconducting strings, and show that we can work in a frame with $x'_0$ and $x'''_0$ parallel. In that frame, we can expand the position of the string at the time of the cusp. We take the cusp to occur at $\sigma = t = 0$ and its position to be $x = 0$. Thus

$$x(\sigma) = x'_0 \sigma + x''_0 \frac{\sigma^2}{2} + x'''_0 \frac{\sigma^3}{6} + \cdots$$  \hspace{1cm} (7)

If $x'_0$ and $x'''_0$ point in opposite directions, which one would expect roughly half the time, then the string will intersect itself at $\sigma = \pm \sigma_1$, where
\[
\sigma_1 = \sqrt{\frac{|x_0'\prime|}{6|x_0''|}}
\]  

(8)

This self-intersection will lead to an intercommutation and the emission of a loop containing the original string from \(-\sigma_1\) to \(\sigma_1\).

III. THE EMITTED LOOP

After the intercommutation, the main string will have kinks where the cusp would have been, and there will be a small loop (also with kinks) that has been produced by intercommutation. The effect on the main string is very similar to emission of string due to overlap of the core, as discussed in [11]. Here we will study the small loop that is emitted.

First of all, what amount of the string is emitted? The magnitude of \(x_0'\) is \(\epsilon/2\), while \(x_0''\) could be expected to be of order \(L^{-2}\), where \(L\) is the size of the original loop, or the typical feature size in the case of a long string. Thus we expect \(\sigma_1 = O(\sqrt{\epsilon L}) = O(jL)\). If we start out, for example, with 1% of the vorton current, then roughly 1% of the original loop will be chopped off.

We would like to compute the velocity and rest-frame properties of the emitted loop. The energy-momentum tensor is given by \([6]\)

\[
T^{\mu\nu} = \int d\sigma d\tau \sqrt{-\gamma} \left( \gamma^{ab} + \theta^{ab} \right) x^\mu_a x^\nu_b \delta^4(x - x(\sigma, \tau)) = \mu \int d\sigma \eta^{ab} x^\mu_a x^\nu_b \delta^3[x - x(\sigma, t)]
\]  

(9)

and thus the total energy-momentum in a region is

\[
P^\mu = \int d^3x T^{\mu 0} = \mu \int d\sigma \eta^{ab} x^\mu_a x^0_b = \mu \int d\sigma \dot{x}^\mu
\]  

(10)

Since \(x^0 = t\), we have \(\dot{x}^0 = 1\), and so the emitted energy is just \(E = P^0 = 2\mu \sigma_1\), which is just the energy in a segment of string of length \(2\sigma_1\). We can expand \(\dot{x}\) in a Taylor series,

\[
\dot{x} = \dot{x}_0 + \sigma \ddot{x}_0' + \frac{1}{2} \sigma^2 \dddot{x}_0''
\]  

(11)

and integrate to find the momentum,

\[
P = 2\mu \sigma_1 \dot{x}_0 + \frac{\sigma^2}{3} \mu \dddot{x}_0'' = E \left( \dot{x}_0 + \frac{\sigma^2}{6} \dddot{x}_0'' \right) = E \left( -a_0' + \frac{\epsilon}{2} a_0' + \frac{\sigma^2}{6} \dddot{x}_0'' \right).
\]  

(12)

The magnitude of \(a_0'\) is 1, whereas the magnitude of the second term is \(O(\epsilon)\), and the third is likewise \(O(\epsilon)\), since \(\sigma^2 = O(\epsilon L^2)\) and \(\dddot{x}_0'' = O(L^{-2})\). Furthermore, the second and third terms cannot cancel, because \(\dddot{x}_0'' \cdot a_0' = (b_0'' - a_0'') \cdot a_0' / 2 = O(||b_0'||^2 + ||a_0'||^2) > 0\), so the two vectors cannot point in opposite directions. Thus the magnitude of \(P\) is

\[
P = E[1 - O(\epsilon)].
\]  

(13)

The rest-frame energy of the loop is \(\sqrt{E^2 - P^2}\), so the length of the loop in its rest frame is
\[\tilde{\sigma}_1 = \mu^{-1} \sqrt{E^2 - P^2} = \mu^{-1} E O(\sqrt{\epsilon}) = 2 \sigma_1 O(j) = O(\epsilon L)\] (14)

Since \(\phi\) is a scalar, it is unaffected by the boost. The current \(j\) is proportional to \(\phi' = d\phi/d\sigma\) and thus is enhanced by the ratio of \(\sigma\) in the moving frame to \(\sigma\) in the rest frame, which is \(O(1/j)\). The new value is \(\tilde{j} = O(1)\), so we have essentially a vorton.

What is the boost vector \(\beta\) that brings the loop to rest? It must point in the direction of \(-\dot{x}_0\) and should give a Lorentz factor \(\gamma = \sigma_1/\tilde{\sigma}_1 = O(\epsilon^{-1/2}) = O(j^{-1})\). Thus the boost should have magnitude \(\beta = \sqrt{1 - \gamma^{-2}} = 1 - O(\epsilon)\), so \(\beta \sim -b_0'\). Then from Eq. (A14b), \(f_B \sim 1/(\gamma(1 - \beta^2)) = \gamma \sim 1/j_0\). From Eq. (A13) the current at the cusp transforms as

\[\tilde{j}_0 = f_B j_0 \sim 1,\] (15)

in accord with the above, and from Eq. (A18) the derivative of the current is

\[\tilde{j}_0' = f_B^2 j_0' + f_B f_B, B j_0 \sim \frac{j_0'}{j_0^2} + \frac{\beta \cdot b_0''}{j_0^2}.\] (16)

Now \(\beta \cdot b_0'' = -j_0' j_0\), so the second term is of the same order as the first, and \(\tilde{j}_0 \sim j_0'/j_0^2 \sim 1/(j_0 L)\). This means that the change in current over the loop is

\[\tilde{j}(\tilde{\sigma}_1) - \tilde{j}_0 \sim (1/(j_0 L)) \cdot j_0^2 L = j_0 \ll \tilde{j}_0 \sim 1\] (17)

and thus the current is essentially constant over the loop.

**IV. DISCUSSION**

We have shown that cusps (or, perhaps more properly, “cusp-like events”) in chiral strings come in two forms. If \(x_0''\) points in the same direction as \(x_0'\), in the frame in which these are parallel, then the cusp is merely smoothed out, as shown by the dashed line in Fig. I. However, if \(x_0''\) points in the opposite direction to \(x_0'\), then there is a self intersection, as shown by the solid line in Fig. I. We thus expect that half of all cusps in strings with chiral currents will have self intersections. When the string does intersect itself, intercommutation emits a loop of string with large Lorentz factor which is approximately in a vorton state already, and could be expected to become a stable vorton.

What about currents which are not chiral? We do not have an exact solution in this case, but we expect similar behavior. The effect of the current is still to prevent the string from doubling back on itself, so there will be a finite value of \(x_0\) where the point of the cusp would have been. This vector controls the shape very close to this point. Further away, however, the effect of \(x_0''\) becomes more important, and if these vectors point in opposite directions, we might expect a self intersection.

If a system of strings can produce vortons, then it is very likely that the vortons will contribute to the matter density of the universe in amounts excess of observation. Thus most such theories are ruled out [12,13]. For a string to become a vorton, the current must somehow rise to the necessary level to oppose the tension in the string. We have shown here a new mechanism by which strings with small currents can nevertheless produce stable vortons.
FIG. 1. Cusp-like events in a chiral superconducting string. Dashed line: $x'_0$ and $x''_0$ parallel. Solid line: $x'_0$ and $x''_0$ antiparallel.

This mechanism may lead to vorton production in more theories than would otherwise have this problem, and thus restrict further the range of allowable superconducting string theories.

It should be noted that, in the case of the string with chiral current and no coupling to the electromagnetic field, the motion will be strictly periodic [6]. After a single cusp has formed at a particular point in the oscillation of the string loop, and a vorton has been emitted, there will be a kink, rather than a cusp, and no further emission of vortons. However, long strings, currents that are not exactly chiral, and coupling to the electromagnetic field produce motion which is not exactly periodic, and thus perhaps lead to ongoing vorton emission. Smoothing of the kink may also produce future cusps, as discussed in [11].

APPENDIX A: LORENTZ TRANSFORMATIONS

In this appendix we extend our results [10] for Lorentz boosting of ordinary string cusps to superconducting strings with chiral currents.

The motion of a chiral superconducting string is given by Eq. (1) with the constraints of Eqs. (3). Following the technique of [13,14], we can treat the current in the same way as the physical position by considering a 4+1-dimensional spacetime where the fourth spatial component of $b$ is $b_4 = 2\phi/\sqrt{\mu}$, so that the fourth spatial component of $b'$ is $j$. Since the current travels only in one direction, the new components of $a$ and its derivatives are all 0.

In this appendix, we will let $a_3$ and $b_3$ denote the 3-space vectors previously called $a$ and $b$, let $a$ and $b$ denote the new 4-space vectors, and introduce 5-dimensional spacetime vectors $A$ and $B$. 
In general, we have

\[ a' = (a'_3, 0) \]  \hspace{1cm} (A1a)
\[ b' = (b'_3, j) \]  \hspace{1cm} (A1b)
\[ a'' = (a''_3, 0) \]  \hspace{1cm} (A1c)
\[ b'' = (b''_3, j') \]  \hspace{1cm} (A1d)

and so on. At the cusp

\[ b' = (-\Delta a'_3, j). \]  \hspace{1cm} (A2)

Each 4-vector has the same relationship with its own derivatives as usual,

\[ a'' \cdot a' = 0 \]  \hspace{1cm} (A3a)
\[ a''' \cdot a' = -|a''| \]  \hspace{1cm} (A3b)

and likewise for \( b \).

At the cusp, we can take \( b'_3 \), which is parallel to \(-a'\), as the “direction of the cusp”. The vector \( a'' \) is perpendicular to this vector, but \( b''_3 \) will not be perpendicular. Instead we have, at the cusp,

\[ 0 = b'' \cdot b' = -\Delta b'' \cdot a' + jj' \]  \hspace{1cm} (A4)

so

\[ b''_3 \cdot a'_3 = b'' \cdot a' = \frac{jj'}{\Delta}. \]  \hspace{1cm} (A5)

We can define a new vector \( c = b''_3 - (jj'/\Delta)a' \), so that

\[ c \cdot a' = 0. \]  \hspace{1cm} (A6)

Now we have two vectors, \( a'' \) and \( c \), in general independent, which are both perpendicular to \( x'_3 \). Both have vanishing component in the current direction. Thus if we can find a frame where \( x'''_3 \) is perpendicular to both \( a'' \) and \( c \), then \( x'''_3 \) must be parallel to \( x'_3 \).

The remaining inner products are given by

\[ -|a''|^2 = a''' \cdot a' = -\frac{a''' \cdot b'}{\Delta} \]  \hspace{1cm} (A7a)
\[ -|b''|^2 = b''' \cdot b' = -\Delta b''' \cdot a' + jj' \]  \hspace{1cm} (A7b)

so

\[ a' \cdot b' = -\Delta \]  \hspace{1cm} (A8a)
\[ a'' \cdot b' = 0 \]  \hspace{1cm} (A8b)
\[ a''' \cdot b' = \Delta |a''|^2 \]  \hspace{1cm} (A8c)
\[ b''' \cdot a' = \frac{|b''|^2}{\Delta} + jj'' \]  \hspace{1cm} (A8d)
\[ b''' \cdot a' = \frac{|b''|^2}{\Delta} + jj'' \]  \hspace{1cm} (A8e)
As in [10], we have two null spacetime vectors, here 5-dimensional,

\[ A^\mu = (1, -a') \]  
\[ B^\mu = (1, b') \]  

We define

\[ B_2^\nu = B^\mu \partial_\mu B^\nu = (0, \dot{b}' + b'') = (0, 2b'') \]  
\[ A_2^\nu = A^\mu \partial_\mu A^\nu = (0, -a' + a'') = (0, 2a'') \]  
\[ A_3^\nu = A^\mu \partial_\mu (A'\partial_\nu A^\rho) = (0, -4a'') \]  
\[ B_3^\rho = B^\mu \partial_\mu (B'\partial_\nu B^\rho) = (0, 4b'') \]

and we have inner products at the cusp,

\[ g(A, B) = \epsilon \]  
\[ g(A_2, A) = g(B_2, B) = 0 \]  
\[ g(A_2, B) = 0 \]  
\[ g(B_2, A) = \frac{2jj'}{\Delta} \]  
\[ g(A_3, A) = -4|a''|^2 \]  
\[ g(B_3, B) = -4|b''|^2 \]  
\[ g(B_3, A) = \frac{4|b''|^2}{\Delta} + \frac{4jj''}{\Delta} \]  
\[ g(A_3, B) = 4\Delta|a''|^2 \]  

As before, a coordinate transformation yields

\[ A^{\tilde{\mu}} = (A^{\tilde{I}}, A) \]  
\[ B^{\tilde{\mu}} = (B^{\tilde{I}}, B) \]  

and

\[ \tilde{A}^{\tilde{\mu}} = (1, -\tilde{a}') = A/A^{\tilde{I}} \]  
\[ \tilde{B}^{\tilde{\mu}} = (1, \tilde{b}') = B/B^{\tilde{I}} \]

We will consider only boosts which do not involve the fifth direction. The above then says that \( b'_3 \) is boosted and rescaled by the transformation, while \( j \) is merely rescaled, because there is no boost in that direction.

We will let our new coordinate system move with velocity \(-\beta\), so that a particle at rest in the original system is moving with velocity \( \beta \) with respect to the new coordinates. The vector \( \beta \) is a space 4-vector, but has zero current component. The Lorentz transformation then gives \( A^{\tilde{I}} = \gamma(1 - a' \cdot \beta) \) and \( B^{\tilde{I}} = \gamma(1 + b' \cdot \beta) \), where \( \gamma = 1/\sqrt{1 - \beta^2} \). We will define

\[ f_A = \frac{1}{A^{\tilde{I}}} = \frac{1}{\gamma(1 - a' \cdot \beta)} \]  
\[ f_B = \frac{1}{B^{\tilde{I}}} = \frac{1}{\gamma(1 + b' \cdot \beta)} \]
exactly as before. From Eq. (A14b) and (A13b) we see that the current transforms as

\[ j = f_B j. \]  

(A15)

We find the transformation laws

\[
\begin{align*}
\tilde{A}_2 &= f_A^2 A_2 + f_A f_{A,A} A \\
\tilde{B}_2 &= f_B^2 B_2 + f_B f_{B,B} B \\
\tilde{A}_3 &= f_A^3 A_3 + 3 f_A^2 f_{A,A} A_2 + (f_A f_{A,A}^2 + f_A^3 f_{A,A,A}) A \\
\tilde{B}_3 &= f_B^3 B_3 + 3 f_B^2 f_{B,B} B_2 + (f_B f_{B,B}^2 + f_B^3 f_{B,B,B}) B
\end{align*}
\]  

(A16)

with \( f_{A,A} = A^\mu \partial_\mu f_A \) and \( f_{B,B} = B^\mu \partial_\mu f_B \). We also have

\[
\begin{align*}
f_{A,A} &= -2 \gamma f_A^2 \beta \cdot a'' \\
f_{B,B} &= -2 \gamma f_B^2 \beta \cdot b''.
\end{align*}
\]

(A17)

Applying Eq. (A16b) to the derivative of the current, we find in particular,

\[ j' = f_B^2 j' + f_B f_{B,B} j. \]  

(A18)

Every result from [10] that is not specific to the point of the cusp goes through unchanged. Thus

\[
\begin{align*}
|\tilde{a}''| &= f_A^2 |a''| \\
|\tilde{b}''| &= f_B^2 |b''|
\end{align*}
\]

(A19)

as before. Note, however, that \( b'' \) includes \( b''_4 = j' \).

For the mixed product, however, things are more complicated. We have, at the cusp,

\[
\tilde{a}'' \cdot \tilde{b}'' = f_A^2 f_B^2 a'' \cdot b'' + f_B f_{A,A} j j' + \frac{\epsilon}{2\Delta} f_A f_{A,A} f_B f_{B,B}.
\]

(A20)

The same situation applies to the third derivatives. For all A’s or B’s, everything is the same, and we get

\[
\begin{align*}
\tilde{a}''' \cdot \tilde{a}''' &= f_A^5 a''' \cdot a''' - f_A^4 f_{A,A} a''^2 = f_A^5 (a''' \cdot a''' + 2 \gamma f_A |a'''|^2 \beta \cdot a''') \\
\tilde{b}''' \cdot \tilde{b}''' &= f_B^5 b''' \cdot b''' + f_B^4 f_{B,B} b''^2 = f_B^5 (b''^2 - 2 \gamma f_B |b''|^2 \beta \cdot b''')
\end{align*}
\]

(A21)

For cross terms, we have

\[
-8 \tilde{a}''' \cdot \tilde{b}''' = g(\tilde{A}_3, \tilde{B}_2) = f_A^3 f_B [f_B g(A_3, B_2) + f_B g(A_3, B)] + 3 f_A^2 f_{A,A} f_B [f_B g(A_2, B_2) + f_B g(A_2, B)]
\]

(A22a)

\[
= f_A^3 f_B [-8 f_B a''' \cdot b'' + 4 f_{B,B} \Delta |a'''|^2] + 12 f_A^2 f_{A,A} f_B [2 f_B j' j' + f_B \epsilon]
\]

(A22b)

\[
8 \tilde{b}''' \cdot \tilde{a}''' = g(\tilde{B}_3, \tilde{A}_2) = f_B^3 f_A [f_A g(B_3, A_2) + f_A g(B_3, A)] + 3 f_B^2 f_{B,B} f_A [f_A g(B_2, A_2) + f_A g(B_2, A)]
\]

(A22c)

\[
= f_B^3 f_A [8 f_B a''' \cdot a'' + 4 \Delta^{-1} f_A |b''|^2 + 4 \Delta^{-1} j' j']
\]

(A22d)
Now we consider the case that $\epsilon \ll 1$, and so $\Delta = 1 + O(\epsilon)$ and $j^2 = O(\epsilon)$. We will look for a Lorentz transformation where $f_B = 1$, so the current is unchanged by the boost. This means that $1 - \Delta \beta_\parallel = 1 - \beta_\parallel + \epsilon \beta_\parallel = 1/\gamma$, and so

$$f_A = \frac{1 - \Delta \beta_\parallel}{1 - \beta_\parallel} = 1 + \frac{\epsilon \beta_\parallel}{1 - \beta_\parallel} = 1 + O(\epsilon).$$  \hspace{1cm} (A23)

There are three parameters in $\beta$, but we have fixed one of them by demanding $f_B = 1$. This leaves two parameters of the Lorentz transformation, which we can vary in an effort to solve the two simultaneous equations $\tilde{x}_{''} \cdot \tilde{a}_{''} = 0$ and $\tilde{x}_{''} \cdot \tilde{c} = 0$. These are the same equations that we solved in [10] for the ordinary string case, with additional perturbations of order $\epsilon \ll 1$. For generic values of the cusp parameters, the solution will be stable and thus the perturbed equations will still be solvable and the effect of the perturbation will be to change the solution by terms of order $\epsilon$. Thus in the present case we will be able to find a frame in which $x_{3''}$ and $x_{3'}$ are parallel, and, as in [10], the required boost will not be particularly large.
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