Effect of solid surface in vicinity of multi-bubble array in cryogenic environment

J Mondal\textsuperscript{1,3}, A Mishra\textsuperscript{1}, R Lakkaraju\textsuperscript{2}, M Ashokkumar\textsuperscript{3}, P Ghosh\textsuperscript{1}

\textsuperscript{1} Cryogenic Engineering Centre, IIT Kharagpur, Kharagpur, 721302, India
\textsuperscript{2} Mechanical Engineering Department, IIT Kharagpur, Kharagpur, 721302, India
\textsuperscript{3} School of Chemistry, University of Melbourne, VIC 3010, Australia

Email: joymondal@gmail.com

Abstract. Multiple bubble interactions in initially quiescent liquid are often accompanied by generation of jets, shockwaves and light. At cryogenic temperature (< 123 K) when certain materials (particularly bcc-type) become brittle, such afore-mentioned physical effects can be effective in disintegrating them to smaller fragments. CFD techniques based on direct numerical simulations can help to understand this phenomenon that may benefit nanotechnology-based industries and oil-gas exploration-firms working with air-gun arrays. In this paper, multiple bubble-pairs are simulated in a co-centric manner around a centrally located solid target (5 mm radius). The ambient fluid is liquid nitrogen (77 K) and the bubbles are gaseous nitrogen (87 K). 2D numerical simulation using the VOF method in compressible domain is carried out neglecting the effect of phase change and gravity. The stand-off distance between the solid target and bubble-pairs are varied systematically and its influence on the fluid-dynamic effects (e.g. pressure shockwave & jets) are compared. Initial calculations suggest that for stand-off distance of 0.93 mm, shockwaves measure above 10 times the ambient pressure and liquid jet speeds around 30 m/s in cryogenic environment, at multiple locations very close to the solid target. These consecutive physical impacts can foster ample liquid-hammer pressures, making it promising for solid wear at 77 K when juxtaposed against room-temperature cases.

1. Introduction & Motivation

The presence of a surface near a bubble is known to promote asymmetric collapse of the bubble, triggering high-speed jets. The strength of such effect depends on a few factors e.g. distance of bubble from the surface, type of surface (rigid/porous/elastic), geometry of surface [1] etc. Under such circumstances, the presence of another bubble adds a surface for interaction. This might amplify or diminish the strength of bubble-induced physical effects e.g. jets and shockwaves [2] based on the relative orientation of surfaces.

Such ordered bubble arrangements or bubble-arrays replicate multiple bubbles, the interaction of which is omnipresent in many advanced applications. These include bubble gun-arrays for oil-gas exploration [3], cell sonoporation [4], tumour treatment [5] etc. Under cryogenic temperatures (<123K), where systems are more prone to heat-inleak leading to bubble generation, study of bubble interaction becomes slightly interesting. Therein, the physical effect of bubble-induced jet and shockwave on structurally brittle material, common to liquid cryogen temperature, might be susceptible for easy disintegration [6]. Remote variations of such unconventional fragmentation method include cryogenic grinding, cryomilling etc, which are popular for being environment-friendly methods. But the difficulty in related experimental investigation lie in small spatial scale (\textmu m) and fast temporal duration (\textmu s) of the interaction. Also, issue of cost, safety and inconvenience with
handling cryogens are common. However, these can be solved by using CFD-based direct numerical simulation (DNS) techniques[7].

This paper describes relevant numerical methodology for simulating co-centric multiple bubble arrays from an erosive point-of-view. The bubbles taken are of gaseous nitrogen (GN2) incepted in initially quiescent liquid nitrogen (LN2). The effect of a circular solid (resembling a target) near the bubble-array is examined by systematically varying the radial distance from the centroid of the bubble array. The consequent bubble shape-transition with time and the consequent erosive effects are identified and analysed.

2. Literature Review

Bubble interaction in cryogenic environment is both an interesting as well as complex phenomena. Amamoto et al. [8] mentions that cryogens offer less explored test cases having lower values of density, viscosity and enthalpy of vaporisation. Sreedhar et al. outlined more such thermo-physical properties that govern bubble interaction e.g. jets, shockwaves etc. [9], whose typical magnitudes are found to vary considerably with temperature, as shown in Figure 1. Fluids where these properties vary are likely to exhibit slightly different bubble phenomena as is observed from Rayleigh-Plesset equation (all symbols are explained in Index) which analytically captures bubble growth and collapse in incompressible liquid.

\[
\frac{\ddot{R}}{R} + \frac{3}{2} \frac{\dot{R}^2}{R} = \frac{1}{\rho} \left( P_0 - \frac{2\sigma}{R} \left( \frac{R_0}{R} \right)^3 \gamma - 4\mu \frac{\dot{R}}{R} - \frac{2\sigma}{R} \right) \tag{1}
\]

![Figure 1: Typical values of transport properties compared between room (air bubbles in water at 300 K, 0.1 MPa) and cryogenic environment (GN2 bubbles in LN2 at 77 K, 0.296 MPa)](image)

In fact experimentally, high-speed directional micro-jets [10] can be generated using bubble-pairs with magnitudes around 100 m/s. Such high speeds are governed by parameters (Eq 2-5) e.g. inter-bubble distance (d), time lag of inception, maximum bubble-size (R\text{max}). In fact, when one bubble starts to grow while the other begins to collapse, certain “catapult”-type of phenomena trigger jetting through one of the bubbles [11,12]. For example, high speed jet (~120 m/s) was found to occur for ψ=0.77, Δθ=0.76 and S=1[13] as defined in Equations 2-5. This configuration is employed for analysis of our bubble-pair array (see configuration in Figure 2).

Non-dimensional inter-bubble distance:

\[
\gamma = \frac{d}{2R_{\text{max}}} = \frac{d}{2R_{\text{max}}} \tag{2}
\]

Non-dimensional phase difference:

\[
\Delta \theta = \frac{2\Delta t}{T_{\text{osc}}} \tag{3}
\]

Non-dimensional size ratio:

\[
S = \frac{R_{\text{max, s}}}{R_{\text{max, l}}} \tag{4}
\]

Non-dimensional standoff distance:

\[
\gamma_s = \frac{d'}{R_{\text{max}}} \tag{5}
\]
In case of regular-shaped multiple-bubble arrays, the peripheral bubbles tend to collapse first, gradually inducing a more violent collapse towards the centre [14,15]. Suitably placed bubble-pairs can similarly propel directional jets towards centroid of arrangement [7], making that location prospective for material erosion. Influence of solid wall near to a bubble seem to be governed also by the relative location of the bubble with reference to the wall, as is defined by standoff distance, $\gamma_s$ in Equation 5. Numerical simulation of centrally-placed circular target near a bubble array captures the overall physical effects, that can become particularly damaging at liquid nitrogen temperature (77 K).

The effect of introducing a solid body within a bubble array at cryogenic temperature lacks proper comparison with its room temperature counterpart and demands deeper understanding of the consequent bubble interaction. Hence, this paper attempts to outline the qualitative and quantitative differences due to the presence of a solid boundary near GN2 bubbles at cryogenic temperature (LN2).

3. Methodology

3.1. Numerical model description

The volume-of-fluid (VOF) method uses a single fluid, homogeneous multiphase approach based on direct numerical simulation that is convenient for including topological changes unlike the common BEM (boundary element methods) [16]. Effect of gravity is neglected due to the very small size ($\mu$m) of the vapor bubbles compared to the containing liquid domain. This justifies that fact that bubbles are most likely to oscillate instead of exhibiting buoyancy-related displacement [12]. The surface tension and viscosity forces are incorporated nonetheless. Following are respectively the governing equations for mass conservation, momentum conservation and liquid-vapor phase transition at the liquid-bubble interface:

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho U) = 0$$

$$\frac{\partial (\rho U)}{\partial t} + \nabla \cdot (\rho U \otimes U) = -\nabla p + \nabla \cdot \tau + f_{ST}$$

$$\frac{\partial \alpha_l}{\partial t} + \nabla \cdot (\alpha_l U) + \nabla \cdot (\alpha_l \alpha_v U_r) = \alpha_l \alpha_v \left( \frac{\psi_l - \psi_v}{\rho_l - \rho_v} \right) \frac{Dp}{Dt}$$

Equation (7) takes into account the interface motion with $U_r$ as the relative velocity between vapour and liquid interfaces [17]. The liquids in both room temperature and cryogenic environments are kept compressible by following the TAIT equation of state as described in Equation 8. The vapour bubble is modelled to follow ideal gas equation (Equation 9) with adiabatic expansion-compression behaviour.

$$\frac{p + B}{p_\infty + B} = \left( \frac{\rho}{\rho_\infty} \right)^n$$

$$p = \rho RT$$

$n=7.15; B=304.6$ MPa for water [18]; $n=18.9; B=37$ MPa for liquid nitrogen [19].

The implicit Euler scheme is chosen for the discretization of transient terms. Maximum Courant number is fixed at 0.25 to avoid distortion of interface and facilitate smoother convergence. The time-step size is taken as 0.1 $\mu$s to effectively compute and capture micro-scale interaction. A 2D numerical domain of $100 \times 100$ mm$^2$ (shown in Fig 2) is selected to model the bubble arrangement (dimensions in Table 1). The initial bubble conditions are calculated using the RP equation (Eq 1) and each bubble-pair i.e. bubble 1 and 2 share the configuration of $\gamma=0.77$, $\Delta \theta=0.76$ and $S=1$. These values correspond to two bubbles whose centres are separated by 4.774 mm, created with interval of 221 $\mu$s between their respective inceptions and each can independently grow up to same maximum radii ($R_{max}$...
3.1 mm). The walls of the domain are made reflective to imitate far-field boundary conditions. The fluid-dynamic properties employed for both the room and cryogenic temperature cases are detailed in [7].

![Diagram](image)

**Figure 2:** Diagrammatic representation and numerical model showing 2 cases- a) without solid target, b) with solid target; “+” indicates the point where measurements are taken for each case; 1 & 2 represent a bubble-pair.

The initial conditions shown in Table 1 pertain to typical spark-generated bubble configurations, but the bubble interaction would qualitatively remain unchanged whether bubbles are generated by spark, laser or even by acoustic means. The use of dimensionless parameters (Eq. 2-5) corroborate the universal nature of this bubble-study.

| Parameter       | Bubble 1 | Bubble 2 | Fluid |
|-----------------|----------|----------|-------|
| Radius (mm)     | 1.17     | 3        | -     |
| Pressure (kPa)  | 30000    | 58.4     | 296   |

3.2. Validation of numerical model

The grid convergence test for the described numerical model is elucidated in [20]. Different meshes have been checked and the respectively simulated time-histories of bubble radii have been compared to arrive at optimum value as described in Table 2.

| Parameter       | Bubble 1 | Bubble 2 | Fluid |
|-----------------|----------|----------|-------|
| Domain type     | a) Without solid target | b) With solid target (hole) |
| Number of points| 1203202  | 882400   |
| Number of cells | 600000   | 440000   |
| Type of cells   | hexahedra | hexahedra |

4. Result & discussion

The above numerical setups can supply the full time-history and exhaustive flow-field information throughout the bubble interaction. Bubble-pair interaction had been previously investigated by Bing Han [13,21], Rui Han [22] etc. to an extent by simulating a fraction of the domain. Full 2D simulation can be a further improvement yet informative for overall erosion-based analysis, which is the aim of this study.

The interaction mechanism of bubble-pairs has previously been experimentally analysed by Chen et al. [23,24]. The underlying concept here is that when one of the bubbles grows into the other bubble, the intercepting liquid layer between the bubbles are propelled by the transition shapes of the interacting bubble-pair. This leads to focussing of liquid-layer [25], ultimately growing into a high-speed jet which pierces through one of the bubbles and impinges the solid surface in the vicinity. The same can be observed through the snapshots taken over time and shown in Figure 3.

4.1. Qualitative aspect of bubble-array interaction

Figure 3 shows the time-history of bubble-array interaction along with the transitioning bubble shapes. This interaction is exhibited by initial formation and dissipation of shockwaves, followed later
by impingement of liquid jet and finally shearing of the liquid layer (observed only in presence the solid). The pressure ratio, \( P_r \), as defined in Equation 11 gives an idea of the shockwave emanating from bubbles towards the centroid of arrangement. The velocity is indicative of the thin jet that hits the solid target [20].

\[
P_r = \frac{P}{P_{amb}}
\] (11)

It is well-known that presence of solid boundary near a solitary bubble can trigger asymmetric collapse, that can induce jets. Also, bubble-pair with phase differences (\( \Delta \theta \)) can generate jets. In complex cases, where both bubble-pair and solid surface are present, the jet configurations are influenced by both these factors. For this paper, the bubble-pair configuration has been kept fixed (\( \gamma = 0.77, \Delta \theta = 0.76 \) and \( S = 1 \)), only the wall to bubble-centre distance \( d' \) (non-dimensionalized by \( \gamma_S \)) has been varied systematically. The magnitude of the shockwave-impact and jet-hammering have been compared with \( \gamma_S \). The values of \( \gamma_S \) used here are 0.3, 0.8 and 1.2 which correspond to \( d' \) values of 0.93, 2.48 & 3.72 mm respectively. Additional curves are also shown for comparing the case at room temperature in presence of solid surface.

---

**Figure 3**: Pressure ratio and velocity contours of the bubble-interaction with left-hand section indicating velocity magnitude \( |U| \) while right-hand sections show pressure-ratio, \( P_r \). Black outline indicates bubble interface with ambient liquid. Circular target is coloured as brown.
4.2. Effect of shockwave with varying distances

Figure 4a clearly shows the shockwaves developing few times higher than the ambient pressure in the first few seconds of bubble-interaction. This is due to the fast growth of bubble 1, pushing the liquid outwards from it. The inset graph compares the possible values of this expansion shockwaves, measured at target points (indicated by ‘+’ in Figure 2b) for different $d'$. In case of room temperature, shockwave effect is most pronounced when bubble 1 is 2.48 mm ($\gamma_S =0.8$) radially away from the wall. But such optimum case is absent for cryogenic conditions, both of which show monotonous trends irrespective of the presence of solid. The closer the bubble 1 is to the wall, the stronger is the shockwave experienced. However, it is to be kept in mind that in case of absence of solid wall, the net pressure wave is overlap of all 3 shockwaves emitted by the 3 bubble-pairs. Another distinct observation is the considerable difference in shockwave values between the room and cryogenic temperature cases. This may be attributed to the significant difference in the viscosity values (shown in Figure 1) that affects the damping of pressure waves at both temperatures. In LN2 this shockwave measures beyond 2.96 MPa i.e. 10 times the ambient pressure of 0.296 MPa, though it sustains for around 10 $\mu$s (Figure 4a). This is immediately followed by several short spurs of shockwaves but of much lesser magnitude owing to the fluid disturbance created in the vicinity by secondary waves.

![Figure 4: (a) Expansion shockwaves for different $\gamma_S$ in LN2 without solid target, inset shows the variation of pressure ratio with $\gamma_S$. (b) Velocity of impact, (only Y component) variation with $\gamma_S$ for different cases plotted to compare the jet-hammering effects.](image)

4.3. Effect of jet-impact with varying distances

The jet impact velocities are compared in Figure 4b. The presence of solid target is found to enhance the jet velocity hitting the solid surface which is prospective for target-based applications using bubbles. Also, $\gamma_S$ varying from 0.3 to 1.2 seems to have an inverse effect on the liquid jet velocity unlike the case of jetting for single bubble interaction with flat wall where an optimum $\gamma_S$ leads to a local maximum jet velocity. The reason may be the significant influence of the bubble-pair ($\gamma, \Delta \theta$) compared to that of wall ($\gamma_S$) on the jet formation. However, larger number of closely-placed data-points could help to predict a much clearer trend. In the same figure, an additional comparison with room temperature is also shown. These values are found to exceed the critical velocity values for causing wear in cases of jet hammering (e.g. 30 m/s velocity of water jet to dent Cu at room temperature) [26]. As shown in Figure 4b for $\gamma_S = 0.3$, the LN2 jet velocities are between 20 m/s to 40 m/s while that for water reaches 100 m/s. This verifies that cryogenic velocities are 1/2~1/3 of their room temperature counterparts, as experimentally reported by Watanabe et. al. [27]. The same trend is followed for different $\gamma_S$, irrespective of the solid target.

In fact, jet-impacts of 30 m/s can roughly foster 19 MPa of liquid-hammer pressures [7,26] at cryogenic temperature. Such range of values are comparable to the ultimate tensile strengths of brittle
metals (particularly bcc types) at 77 K, making them susceptible to easy wear. Nevertheless, an experimental verification using acoustically-manipulated bubbles can help validate above results and prove cost-effective alternative to laser and spark-based experiments.

5. Conclusion
The several physical effects obtained at several locations for ordered bubble-pair interactions have been identified and numerically analysed at cryogenic condition (LN2 temperature of 77 K) through time-history of bubble interaction in the absence and presence of a solid central target. Our results indicate that presence of solid has little or no effect on the radiating shockwaves, but jet speeds are enhanced. The effect of distance of bubble-pair from the location of jet impact i.e. $\gamma_S$ has been systematically varied to note monotonous trends for both shockwave and jet-impact speeds at 77 K. However, more data points can help predict a clearer trend. Typical magnitudes at 77 K for $\gamma_S=0.3$ include shockwaves measuring above 3 MPa (~10 times the ambient pressure) and liquid jet-impact speeds around 30 m/s facilitating liquid-hammer pressures of 19 MPa. Such fundamental study might pave way for further examination and future applications e.g. particle fragmentation at cryogenic temperatures where certain material (e.g. bcc) resistance is generally less.

6. Index

| Symbol | Description |
|--------|-------------|
| bcc    | Body centered cubic |
| B      | TAIT pressure |
| $C_p$  | Specific heat |
| $\rho$ | density |
| $\tau$ | viscous forces |
| $d$    | distance between twin bubble centres |
| $d'$   | radial distance between target surface & nearest bubble centre |
| $\Delta \theta$ | phase difference |
| $\gamma$ | dimensionless inter-bubble distance |
| $\gamma_S$ | dimensionless stand-off distance from surface |
| $f_{st}$ | Surface tension forces |
| $\Delta t$ | Time difference between bubble nucleation |
| $t$    | time |
| $\sigma$ | Surface tension coefficient |

Subscript

| Subscript | Description |
|-----------|-------------|
| amb       | ambient |
| $i$       | initial |
| $\max$   | maximum |
| $\infty$ | at infinite field |
| $l$       | liquid state |
| $v$       | vapour state |

7. References
[1] Tomita Y, Robinson PB, Tong RP, Blake JR. 2002 Growth and collapse of cavitation bubbles near a curved rigid boundary J. Fluid Mech. 466 259-283
[2] Obreschkow D, Tinguely M, Dorsaz N, Kobel P, De Bosset A, Farhat M. 2013 The quest for the most spherical bubble: Experimental setup and data overview. Exp. Fluids 54 1503-9
[3] Huang X, Zhang AM, Liu YL. 2016 Investigation on the dynamics of air-gun array bubbles based on the dual fast multipole boundary element method. Ocean Eng. 124 157–67
[4] Sankin GN, Yuan F, Zhong P. 2010 Pulsating Tandem Microbubble for Localized and Directional Single-Cell Membrane Poration Phys. Rev. Lett. 105 078101
[5] Kennedy JE. 2005 High-intensity focused ultrasound in the treatment of solid tumours. Nat Rev Cancer 5 321–7
[6] Monastyrsky G. 2015 Nanoparticles formation mechanisms through the spark erosion of alloys in cryogenic liquids. Nanoscale Res. Lett. 10 1–8
[7] Mondal J, Mishra A, Lakkaraju R, Ghosh P. 2018 Numerical Examination of Jets Induced by Multi-Bubble Interactions. Proc. ASME 2018 Int. Mech. Eng. Congr. Expo. 1–8.
[8] Yamamoto Y, Nakajima S, Watanabe S, Ota M, Maeno K. 2009 Single Bubble Behavior Induced by Nd : YAG Laser Focusing near Solid Wall in Liquid Nitrogen Trans. JSASS Space Tech. Japan 7 65–70
[9] Sreedhar BK, Albert SK, Pandit AB. 2017 Cavitation damage: Theory and measurements – A review Wear 372–373 177–96
[10] Ohi C-D. 2010 Aiming with bubbles. Physics 3 65
[11] Chew LW, Klaseboer E, Ohi SW, Khoo BC. 2011 Interaction of two differently sized oscillating bubbles in a free field. Phys. Rev. E. 84
[12] Fong SW, Adhikari D, Klaseboer E, Khoo BC. 2009 Interactions of multiple spark-generated bubbles with phase differences. Exp. Fluids 46 705–24
[13] Han B, Köhler K, Jungnickel K, Mettin R. 2015 Dynamics of laser-induced bubble pairs J. Fluid Mech. 771 706–42
[14] Ochiai N, Ishimoto J. 2017 Numerical investigation of multiple-bubble behaviour and induced pressure in a megasonic field. J. Fluid Mech. 818 562–594
[15] Bremond N, Arora M, Ohi C, Lohse D. 2006 Controlled Multibubble Surface Cavitation. Phys. Rev. Lett. 96
[16] Kalumuck KM, Duraipwami R, Chahine GL. 1995 Bubble dynamics fluid-structure interaction simulation by coupling fluid bem and structural fem codes. J Fluids Struct. 9 861–83.
[17] Li T, Wang S, Li S, Zhang A. 2018 Numerical investigation of an underwater explosion bubble based on FVM and VOF Phys. Procedia 74 49–58
[18] Koch M, Lechner C, Reuter F, Köhler K, Mettin R, Lauterborn W. 2016 Numerical modeling of laser generated cavitation bubbles with the finite volume and volume of fluid method, using OpenFOAM. Comput. Fluids 126 71–90.
[19] Tomita Y, Tsubota M, An-Naka N. 2003 Energy evaluation of cavitation bubble generation and shock wave emission by laser focusing in liquid nitrogen. J Appl Phys 93 3039–48.
[20] Mondal J, Mishra A, Lakkaraju R, Ghosh P. 2019 Application of multiple interacting bubbles for particle fragmentation at cryogenic temperature. submitted to 27th National Symposium on Cryogenics and Superconductivity (NSCS 27), IIT, Bombay, 2019
[21] Han B, Pan YX, Xue YL, Chen J, Shen ZH, Lu J, et al. 2011 Mechanical effects of laser-induced cavitation bubble on different geometrical confinements for laser propulsion in water. Opt. Lasers Eng. 49 428–33
[22] Han R, Zhang A, Liu Y. 2015 Numerical investigation on the dynamics of two bubbles. Ocean Eng. 110 325–38
[23] Chen Y, Chu H, Lin I. 2006 Interaction and Fragmentation of Pulsed Laser Induced Microbubbles in a Narrow fluid bem and structural fem codes. J Fluids Struct. 9 861–83.
[24] Chen Y, Lin I. 2008 Dynamics of impacting a bubble by another pulsed-laser-induced bubble : Jetting , fragmentation , and entanglement Phys. Rev. E. 77 1–12
[25] Tagawa Y, Oudalov N, Visser CW, Peters IR, Meer D Van Der, Sun C, et al. 2012 Highly Focused Supersonic Microjets Phys. Rev. X 2 031002
[26] Thiruvengadam 1967 Erosion by cavitation or impingement
[27] Watanabe, S., Sirato, T., Ota, M., & Maeno, K. 2007 Bubble dynamics induced by YAG laser focusing in liquid nitrogen and cryogenic laser processing for particles. XVI International Symposium on Gas Flow, Chemical Lasers, and High-Power Lasers. 6346

Acknowledgments

Special mention of the Computational Mechanics Group, IIT Kharagpur for providing necessary numerical tools and devices.