2-Form Gauge Field Theories and “No Go” for Yang-Mills Relativistic Actions.

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Abstract

The transformation properties of a Kalb-Ramond field are those of a gauge potential. However, it is not clear what is the group structure to which these transformations are associated. In this paper, we complete a program started in previous articles in order to clarify this question. Using the spectral theorem, we improve and generalize previous approaches and find the possible group structures underneath the 2-form gauge potential as extensions of Lie groups, when its representations are assumed to act into any tensor (or spinor) space with inner product.

We also obtain a fundamental representation where a two-form field turns out to be a connection on a flat Euclidean basis manifold, with a corresponding canonical curvature. However, we show that these objects are not associated to space-time tensors and, in particular, that a standard Yang-Mills action is not relativistically invariant, except (as expected) in the Abelian case. This is our main result, from the physical point of view.

1 Introduction

The (Abelian) Kalb-Ramond field [1] (KR), $b_{\mu\nu}$, is a two-form field which appears in the low energy limit of String Theory [2], in Quantum Gravity [3] and in several other frameworks in Particle Physics [4]. In particular, many attempts to incorporate mass to gauge fields in four dimensions take this object into account [5, 6]. Its dynamics is governed by an action that is invariant under a symmetry transformation remarkably similar to that of a one-form gauge field [7]:

$$b_{\mu\nu} \rightarrow b_{\mu\nu} + \partial_{[\mu} \beta_{\nu]},$$

where $\beta_{\nu}$ is a 1-form parameter. The question is: how can we associate the parameters $\beta_{\mu}$ to the manifold of some gauge group [8, 9]? This problem was rigorously analyzed in

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ref. [10], for specific spinor representations, and an Abelian connection was constructed with the KR field [11].

¿From the physical point of view, it is essential to ask if a genuine gauge theory may be formulated, i.e., if the 2-form gauge potential may be stated as a connection on any group manifold. This is important because, as is known, this structure would be crucial for the identities which determine the finiteness or not of physical models. In particular, in ref. [12], it has been proved that massive (non-Abelian) gauge models [5, 6], based on the KR field are ill-defined. The final goal of this article is to argue that, in a very general context, there are additional objections for non-Abelian gauge actions with KR fields, that are associated with relativistic invariance. This kind of difficulties have already been indicated along different lines in previous literature [13]. Here, however, we analyze them from a very different perspective.

More recently, other points of view on the problem have been presented, related to minimal coupling with a matter field [14], where expected applications in gravitation with torsion and Kalb-Ramond cosmology are pointed out [14, 15, 16, 17].

On the other hand, it is well-known that anti-symmetric tensor fields, subjected to the gauge transformations (1), are equivalent to massless scalar particles [1, 2, 6, 18]. In a first-order formulation of this tensor gauge theory, it is easy to show that there is an equivalence between these non-Abelian rank-2 anti-symmetric tensor fields, considered as gauge potentials, and torsionless non-linear \( \sigma \)-models [19]. Recent models, that aim phenomenological applications [20], explore this equivalence describing the interactions among vector mesons in terms of the geometrical properties of the target manifold. Spin-2 meson resonances may also be naturally accommodated, whenever the \( \sigma \)-model’s target manifold is non-symmetric.

In this class of models, the rank-2 field plays the role of a torsion and its pull-back to space time allows the description of multiplets of excited spin-1 resonances. Another stream of research adopts the rank-2 gauge potential and explores its non-minimal coupling to charged matter to discuss the appearance of defects, such as cosmic strings [21], vortices [22] and magnetic monopoles [23].

This type of constructions, which defend the gauge character of the KR field, actually enforce the idea of a group structure since a closed algebra of infinitesimal variations is required.

This work is organized as follows: in Section 2, we deal with the conditions to parametrize an Abelian group with an 1-form parameter. In Section 3, we extend this to the non-Abelian case. In Section 4, we recover a 2-form field from the connection and discuss the construction of gauge theories in this framework, pointing out the main conclusion.

## 2 1-Form Group Parameter and Spectral Theorem

Let us assume a four-dimensional flat space-time \((M, \eta_{\mu\nu})\) and some Lie group, denoted by \(G\), whose associated algebra is \(\mathfrak{g}\); \(\tau^a\) are the matrices representing the generators of
the group with $a = 1, \ldots, \dim G$; $\tau_{abc}$ are the structure constants. Let us take the gauge parameter to be an adjoint 1-form that can be expanded as below:
\[
\beta_\mu = \beta_\mu^a T^a.
\] (2)

Consider also a vector space $S = \{\psi\}$ where a representation for $G$ acts. We wish to find the transformation law under a group element parametrized by the object $\beta^a_\mu$. Let us take an infinitesimal transformation parametrized by $\epsilon^a_\mu$. We shall get
\[
\psi'_I = (g_\epsilon I_J (\psi)_J = (I_{IJ} + i(M(\epsilon))_{IJ} + (a^2(\epsilon))_{IJ})\psi_J,
\] (3)

where, for an arbitrary 1-form $\beta_\mu$, $M(\beta)$ must be a linear operator from $S$ into itself.

With the first order expression (3), we may build a group element corresponding to non-infinitesimal parameters, $\beta_\mu$, by considering $\beta^a_\mu = N \epsilon^a_\mu$ for a large integer $N$. Using the linearity of $M$, $M(N\epsilon) = NM(\epsilon)$, we obtain $g(\beta) = \exp iM(\beta)$.

For a better understanding of the problem, let us first consider the case of an Abelian and unitary group ($G \sim U(1)$), the representation space $S$ being assumed to be a complex vector space with an inner product. The matrix corresponding to the parameters $\beta_\mu$ in the given representation, $M$, is a linear map from $\Lambda_1$ (the space of Lie algebra valued 1-forms) into the space of linear operators over $S$ and
\[
\delta \psi = iM(\beta)\psi.
\] (4)

By virtue of unitarity, $M$ is Hermitian, so that it can be diagonalized. Therefore, we can invoke the Spectral Theorem (ST), which states that, if $M$ has $K$ different eigenvalues, then there exist linear operators $E_1, \ldots, E_K$ over $S$, such that:

1. $M(\beta) = \sum_{i=1}^{K} c_i(\beta) E_i$, where $c_i$ are the eigenvalues of $M$.
2. $I = \sum_{i=1}^{K} E_i$.
3. $E_i E_j = 0$, $i \neq j$.
4. $E_i E_i = E_i$ ($E_i$ is a projector).
5. $S = \bigoplus_{i=1}^{K} S_i$ (direct orthogonal sum), $E_i$ being the identity on the vector subspace $S_i$.

Notice that, although the eigenvalues $c_i$ are dependent on $\beta$, the projectors $E_i$ are not, because, for an arbitrary pair of parameters $\beta_1, \beta_2$, $[M(\beta_1), M(\beta_2)] = 0$ (due to the Abelian character of the group), what makes them both diagonal in the same basis, thus defining the same set $\{E_i\}$. On the other side, using that the $E_i$’s are $\beta$-independent and satisfy

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$^3$Later, we shall see that unitarity is related to the Euclidean character of space-time. Thus, in order to avoid this kind of question at this point, we assume a Riemannian space-time metric, since this does not represent any conceptual or technical inconvenient in field theories.
1-5, one can easily show that the commutator between $M(\beta_1)$ and $M(\beta_2)$ vanishes, for arbitrary $\beta_1, \beta_2$,

$$[M(\beta_1), M(\beta_2)] = \sum_{i,j} c_i(\beta_1)c_j(\beta_2)[E_i, E_j] = \sum_i c_i(\beta_1)c_i(\beta_2)(E_i - E_i) = 0.$$ \hspace{1cm} (5)

Going on with our construction, as $M$ is a first-order homogeneous function of $\beta$, then $c_i(\beta)$ must be a real linear functional of the 1-form $\beta$. Therefore, by definition, there exist $K$ space-time vectors, $e_i$, such that $c_i = \langle \beta; e_i \rangle$. Then, if we require that the full one form $\beta$ can be recovered from the matrix $M(\beta)$, i.e., that the mapping $\beta \rightarrow M(\beta)$ is invertible, at least $d$ (the space-time dimension) of the $e_i$'s are linearly independent (otherwise it would be impossible to recover $\beta$). So, we can verify that the dimension of the representation, $D$, satisfies $D \geq d$: it is obvious that $D \geq K$; on the other hand, $\beta$ recovery requires that $K \geq d$. Therefore, we obtain the first important conclusion on this group structure: the dimension of the representation has to satisfy the constraint $D \geq d$. We show below that, in fact, $D = d$ constitutes the minimal (i.e. fundamental) representation.

Notice, in addition, that each $E_i$ is associated to a vector $e_i$: let us take $\beta = e_i^4$; then, since $c_i = 1, c_{j \neq i} = 0$, we obtain that the projector must be given in terms of this unit vector.

$$M(e_i) = E_i.$$ \hspace{1cm} (6)

By virtue of this, notice that the set of diagonal matrices $\mathcal{M}$ is a closed algebra which can be decomposed in a direct sum of invariant spaces, each one associated with a single space-time direction:

$$\mathcal{M} = \bigoplus_i \mathcal{M}_i,$$ \hspace{1cm} (7)

which coincides with the main conclusion of ref. [10].

So, we can explicitly verify that a vectorial representation realizes this structure for $D = d = 4$ and then constitutes a minimal representation. Let us take as $\mathcal{S}$, the space of complex vectors tangent to the space-time base manifold, so that the matrices $M$ are tensors of type $(0, 2)$. Due to the existence of a flat (Euclidean) inner product, $\langle ; ; \rangle$, we do not distinguish between vector and one-form, in our notation and, in what follows, we will not assume summation over repeated tensor indices.

Given an ordered orthonormal space-time basis $\{e_\mu\}_{\mu=1}^4$, the projectors are

$$E_\mu = e_\mu \otimes e_\mu,$$ \hspace{1cm} (8)

and

$$c_\mu(\beta) = \langle \beta; e_\mu \rangle.$$ \hspace{1cm} (9)

Finally, we have the manifest (diagonal) form of the matrices representing the Lie Algebra

$$M(\beta) = \sum_{\mu=1}^4 < \beta; e_\mu > e_\mu \otimes e_\mu.$$ \hspace{1cm} (10)

\footnote{Rigorously speaking, one should say: the 1-form (dual) corresponding to the vector $e_i$. However, we are relaxing, since we are considering an Euclidean metric for the space-time that is flat.}
and, according to (6), $M(e_\mu) = E_\mu$.

For a generic algebra valued 1-form parameter $\beta = \beta^a \tau^a$ (for an unitary group) the first part of the ST holds (1-5) but the commutation relations do not vanish. Then, we cannot see in this way that $E_\mu$ are independent of $\beta$. We shall discuss the non-Abelian generalization later on.

Notice that if one takes another orthonormal space-time basis $\{e'_\mu\}_{\mu=1}^4$ related to the previous one by a Lorentz transformation\(^5\) $e'_\mu = \sum_\nu \lambda_\mu^\nu e_\nu$, then we must have, according to (6):

$$M(e'_\mu) = E'_\mu.$$  \hspace{1cm} (11)

On the other hand, the coefficients, referred to the previous basis are:

$$c_\mu(e'_\nu) := <e'_\nu; e_\mu> = \sum_\alpha \lambda_\nu^\alpha <e_\alpha; e_\mu> = \lambda_\nu^\mu,$$  \hspace{1cm} (12)

where we have used the orthonormality of the basis. Therefore, we get

$$M(e'_\mu) = \sum_\nu \lambda_\mu^\nu E_\nu.$$  \hspace{1cm} (13)

Comparing (11) and (13), we obtain that the projectors in two different space-time cartesian coordinates are related by:

$$E'_\mu = \sum_\nu \lambda_\mu^\nu E_\nu.$$  \hspace{1cm} (14)

### 3 Non-Abelian Extension.

We can extend this structure to non-Abelian groups in a natural way, simply by considering that the 1-form parameter takes values in the algebra $\mathcal{G}$. So, the gauge parameter reads $\beta_\mu = \beta^a_\mu \tau^a$ and the matrix corresponding to it is $M(\beta) = \sum_\mu \beta_\mu E_\mu$, where $E_\mu$ are projectors ($E_\mu E_\nu = \delta_{\mu\nu} E_\nu$) satisfying the properties 2-5 of the spectral theorem. Thus, the commutation relation between two arbitrary algebra elements reads

$$[M(\beta), M(\beta')] = \sum_{\mu, \nu, a, b} \beta_\mu^a \beta_\nu^b [E_\mu \tau^a, E_\nu \tau^b] = \sum_{\mu, \nu, a, b} \beta_\mu^a \beta_\nu^b \delta_{\mu\nu} E_\nu [\tau^a, \tau^b] =$$

$$= \sum_{\mu, a, b, c} \beta_\mu^a \beta_\mu^b E_\mu \tau^{abc} \tau^c = \sum_{\mu} [\beta_\mu, \beta'_\mu] E_\mu.$$  \hspace{1cm} (15)

It is straightforward to verify that the tensor products, $E_\mu \tau^a$, generate a well-defined algebra, that is an extension of the original Lie algebra (defined by the relations (18) below). They obey the commutation relations:

$$[E_\mu \tau^a, E_\nu \tau^b] = \delta_{\mu\nu} E_\nu [\tau^a, \tau^b]$$  \hspace{1cm} (16)

\(^5\)For us, an element of $SO(4)$, since we are assuming an Euclidean space-time metric.
\[ [E_\mu \tau^a, \tau^b] = E_\mu [\tau^a, \tau^b] \]  
(17)

and, by definition,

\[ [\tau^a, \tau^b] = \tau^{abc} \tau^c. \]  
(18)

They clearly satisfy Jacobi’s identities (this can be seen by noticing that they are satisfied for the subalgebra (18) and that the \( E_\mu \) are projectors). After that, the construction of the gauge field theory (connection and field strength) can be pursued systematically.

Notice that, even in this non-Abelian generalization, the algebra obeys the structure shown by equation (7), being a direct sum of copies of the same group algebra for each space-time dimension. Therefore, the resulting group is the direct product of the copies of \( G, G \sim \bigotimes \mu G(\mu) \), as previously noticed in [11].

We can anticipate our final result, by observing that problems with relativistic invariance are already present in this non-abelian generalization of the algebra. If we notice that Lorentz transformations are to be included in the automorphisms of this algebra, then the commutation relations (16) must be preserved:

\[ [E'_\mu \tau^a, E'_\nu \tau^b] = \delta_{\mu\nu} E'_\nu [\tau^a, \tau^b], \]  
(19)

where the generators \( E'_\mu \) and \( E_\mu \) are related by (14). Plugging (14) into (19), and using (16), we get

\[ \sum_{\alpha} \lambda_\mu^\alpha \lambda_\nu^\alpha E_\alpha [\tau^a, \tau^b] = \delta_{\mu\nu} \sum_{\alpha} \lambda_\mu^\alpha E_\alpha [\tau^a, \tau^b]. \]  
(20)

If \([\tau^a, \tau^b] \neq 0\) this implies that \( \lambda_\mu^\alpha \lambda_\nu^\alpha = \delta_{\mu\nu} \lambda_\nu^\alpha \), which contradicts the fact that \( \lambda \) is a generic homogeneous coordinate transformation. So, we conclude that, in the non-Abelian case, the set of algebra automorphisms does not contain the Lorentz group. Consequently, one has different group structures for two arbitrary frames, which actually means that the group structure stated here is not relativistically invariant, except in the Abelian case.

Moreover, we are going to show explicitly that these problems remain at the level of objects with physical interpretation.

4 Connection, Curvature and Gauge Theories

Now, because of (5), we have a well-defined group element acting on a complex space-time vector, \( V^\alpha = v^\alpha + i w^\alpha \) \( v, w \in T_p M \), which reads as

\[ g(\beta) = \exp i M(\beta). \]  
(21)

Therefore, using that \( E_\mu \) are projectors \((E_\mu^n = E_\mu \) and \( E_\mu E_\nu = 0 \), for \( \mu \neq \nu \)), a group element may be decomposed as

\[ g(\beta) = \exp i M(\beta) = \sum_{\mu} \left( \sum_{n} \frac{i^n}{n!} (\beta_\mu)^n \right) E_\mu = \sum_{\mu} g_\mu(\beta) E_\mu \]  
(22)
where \( g_\mu(\beta) = \exp i\beta_\mu \) and its inverse \( (g_\mu(\beta))^{-1} = \exp (-i\beta_\mu) \) are well defined in a given Cartesian space-time basis. Obviously, \( \beta_\mu \) means the \( \mu \)-component of \( \beta \), which is a real number.

The covariant derivative acting on a vector field \( V \) is written as \( \nabla_\mu \equiv \partial_\mu - iB_\mu \). Thus, assuming that, under a gauge transformation \( V \to V' = g(\beta)V = e^{iM(\beta)}V \), its covariant derivative transforms as \( \nabla'_\mu V' = g(\beta)\nabla_\mu V \), and \( \nabla'_\mu = \partial_\mu - iB'_\mu \), we see that the connection must change according to

\[
B'_\mu = e^{iM(\beta)}(B_\mu + i\partial_\mu M(\beta))e^{-iM(\beta)}. \tag{23}
\]

Thanks to this property, we obtain that the infinitesimal transformation law for the connection reduces to the familiar expression:

\[
B'_\mu - B_\mu = \partial_\mu M(\beta) + i[M(\beta), B_\mu] + O(\beta^2), \tag{24}
\]

where \( \partial_\mu M(\beta) = \sum_\nu (\partial_\mu \beta_\nu) E_\nu \), as expected. This means that the manifest form of this connection is

\[
B_\mu = \sum_\nu B_{\mu\nu} E_\nu, \tag{25}
\]

a \((0,2)\) type tensor, since it must be an object whose nature is preserved under gauge transformations.

Of course, this connection may be decomposed in its symmetric and anti-symmetric parts, as \( b_{\mu\nu} \equiv B_{[\mu\nu]} \) and \( G_{\mu\nu} \equiv B_{(\mu\nu)} \). We identify the (Abelian) Kalb-Ramond gauge field with \( b_{\mu\nu} \).

Now, we define the field strength for the \( B \)-connection as usual:

\[
[\nabla_\mu, \nabla_\nu]V = -iF_{\mu\nu}V, \tag{26}
\]

which results in

\[
F_{\mu\nu} = \sum_\rho H_{\mu\rho\nu} E_\rho = 2\partial_{[\mu}B_{\nu]} + i[B_{\mu}, B_{\nu}] = \sum_\rho (2\partial_{[\mu}B_{\nu]}_{\rho} + i[B_{\mu\rho}, B_{\nu\rho}])E_\rho. \tag{27}
\]

Using the decomposition (22), it is straightforward to verify the gauge covariance of the curvature tensor,

\[
F' = g(\beta)Fg(-\beta). \tag{28}
\]

With this object we shall construct gauge invariant actions. In doing this, let us consider

\[
FF = \sum_\rho E_\rho H_{\mu\rho\alpha\beta} H_{\alpha\beta\rho}. \tag{29}
\]

which is also gauge covariant \( F'F' = g(\beta)FFg(-\beta) \).

Notice that this curvature is not a space-time tensor (except in the Abelian case) since we cannot associate a tensor to both terms simultaneously.

Consider a generic (diagonal) matrix \( C = \sum_\mu c_\mu E_\mu \), such that \( c \) is any \((0,p+1)\)-tensor and each component \( c_\mu \), with respect to the basis \( \{e_\mu\} \), denotes a generic \((0,p)\)-tensor.
It may be thought as a map \( C : \mathcal{S} \times \mathcal{S}^* \to \Pi_p \) (\( \Pi_p \) denotes the space of tensors of type \((0,p)\))\(^6\). Then we construct the basis dependent vector \( e \equiv \sum_{\mu} e_{\mu} \in \mathcal{S} \). In this way, we can recover the full tensor \( c \), by multiplying \( e \) and \( C \),

\[
e C = c ,
\]  

which is a \((0, p+1)\)-tensor. Therefore, it is easy to verify that, multiplying the curvature by \( e \), we obtain:

\[
e F = 2 \partial_{[\mu} B_{\nu]} + i \sum_{\rho} [B_{\mu\rho}, B_{\nu\rho}] .
\]  

In the above expression, we can express the second term as

\[
e F = 2 \partial_{[\mu} B_{\nu]} + i \sum_{\alpha\beta} [B_{\mu\alpha}, B_{\nu\beta}] \delta_{\alpha\beta} .
\]  

We manifestly observe that the tensorial type of both terms are different (a \((0,3)\)-tensor plus a \((0,2)\)-tensor). Then, clearly, not only this is not a tensor but also there is no algebraic function of it which is. In particular, if we define a Yang-Mills action as any (gauge invariant) algebraic functional of the curvature tensor, we conclude that it does not exist. This is the central argument of the “no go” statement claimed above.

A kind of Yang-Mills action on an Euclidean (four dimensional) space-time should be

\[
S_{YM} \equiv \int d^4 x g^{\mu\nu\alpha\beta} \text{Tr}_{\text{(algebra)}} (F_{\mu\alpha} F_{\nu\beta} ) ,
\]  

where \( g^{\mu\nu\alpha\beta} \) is some object that does not depend on the gauge fields. In particular, the standard Yang-Mills model corresponds (in Euclidean space-time) to \( g^{\mu\nu\alpha\beta} = \delta^{\mu\nu} \delta^{\alpha\beta} \).

Again, the price of keeping Lorentz invariance of the theory is that Yang-Mills-like gauge actions cannot be defined in this framework.

In the Abelian case, however, this is a well-defined Lorentz invariant gauge theory, whose underneath group structure has been established here. So, in the fundamental (vector) representation \( \text{tr} E_{\mu} = \langle e_{\mu}; e_{\mu} \rangle = 1 \) and this may be expressed as:

\[
S_{\text{Abelian}}[B_{\mu\nu}] \equiv \int d^4 x \text{tr}(F F),
\]  

which is a gauge invariant Abelian action.

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\(^6\)Since we are considering an Euclidean flat metric, there is a trivial identification between \( \mathcal{S} \) and its dual \( \mathcal{S}^* \).
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