1. Introduction
Circular motion is one of the central topics in high-school physics curriculum, but most physics students enter university with only a partial understanding of the forces involved (see e.g. [1, 2]). Carousels, swings and roller coasters offer the possibility to experience these forces in our own body, including the varying forces during circular motion in a vertical plane. This study deals with first-year students trying to make sense of the forces in different parts of a vertical loop, in relation to a problem given as part of their written exam, where they were asked to calculate velocities and acceleration and draw arrows representing force and acceleration at the top, bottom and sides of a circular loop, neglecting friction.

In this paper we will begin by presenting the forces acting on riders in roller coaster loops (section 3) and categories of students’ written solutions to the exam question (section 5). Further, section 6 introduces the focus group interview and the following sections (6.1–7.1) presents excerpts on various topics from the focus group interview together with a brief discussion of each topic. Finally, section 7 discusses more general topics emerging from the interview.

2. Background
It has long been known that students struggle with their understanding of mechanics in general, as captured well by Van Heuvelen [2](p.894):
Students, at the end of their conventional study, have little structure to their knowledge. Their understanding consists of random facts and equations that have little conceptual meaning. When given a problem, they identify some structural feature described in the problem (an inclined plane, a rope, a spring, etc.) They then search randomly for and inappropriately use an equation they associate with that feature.

Alternative conceptions can be found both before [3, 4] and after [5] instruction and thus seem very persistent. As we will see below, this also holds for the students in our study. The well-known force concept inventory [1, 6] has repeatedly demonstrated students’ alternative conceptions concerning mechanics, including circular motion (see, for example [5]).

2.1. Research questions

In this paper we are interested in how students at university level address circular motion problems. Based on the above, and our experience from extensive teaching of physics, we aim to address students conceptions of circular motion by considering the following research questions.

(i) How do physics students discuss forces in a roller coaster loop?
(ii) What concepts do students bring to discussions of forces in a roller coaster loop?
(iii) To what extent do students connect the relevant concepts?

In addition, we raise the question if student difficulties may be traced to textbook presentations.

3. The physics behind the story: forces in roller coaster loops

Roller coasters are characterized by conversion between kinetic and potential energy, and small frictional losses, making them suitable examples for learning about force and motion. In well-designed roller coasters with properly banked curves, the force on the rider is essentially perpendicular to the track, in the direction of the spine of the rider, during most of the ride. During parts of the ride the acceleration may be close to the acceleration of gravity \( g \), leading to apparent weightlessness, where no force from the ride on the rider is needed to follow the track. Roller coaster loops, as in figure 1, are often designed for weightlessness at the top. Should the train be slower than required for weightlessness, the rider will hang on the restraint system, while the wheel construction ensures that the train does not fall off the track.

Textbook roller coasters loops are often drawn as circles with the train entering and exiting the loop on horizontal tracks, like in figure 2. This shape would lead to an abrupt change in acceleration (i.e. a large jerk [7, 8]), with the sudden change of radius of curvature. (The lettering in the exam problem was chosen to indicate the train continuing in the loop after C, to steer the students away from considering the entrance and exit.) Today’s roller coasters always have larger radius at the bottom than at the top, as, for example, in the loop shown in figure 1 [7, 9].

The exam problem provided the students with a drawing of a circular loop (similar to figure 2) taken from the students’ textbook [10] where the radius had the value \( r = 12.2\, \text{m} \). (In fact, most real roller coaster loops have a smaller radius.) Students were asked to calculate velocities and draw arrows—to scale —representing force and acceleration at points A, B, C and D for the case that the acceleration is \( g \) at point A. Figure 3 shows a possible solution to this exam problem.

Circular motions in vertical planes are not part of our everyday experiences, but amusement parks offer many real-life examples, which are often well described by textbook mathematics and physics. In this paper we use the simplification of neglecting friction, treating the train as a point mass and describing the loop by a circle. In project work using a numerical treatment students can go beyond these simplifications [11].

3.1. Problem solving strategies

General strategies for solving problems concerning force and motion, as presented in most physics textbooks, include the following points

- Draw a sketch of the situation and identify what is given and what you need to find out
- Draw arrows representing all forces on the body
Students making sense of motion in a vertical roller coaster loop

Choose a suitable coordinate system
Write down physics relations that may be useful
For each body, write a separate equation for each component of Newton’s second law
Do the algebra
Evaluate the result

In many cases, only the direction of the forces is known before the situation is analysed in more detail, but the aim should be to draw all forces in the same scale in the final free-body diagram.

To obtain the acceleration at the different points in figure 2, students need to understand that the value for the acceleration at the top is an indirect way to provide the speed—at the top, there is no tangential acceleration. The total acceleration, \( g \), is then given by the centripetal acceleration, \( a_c = \frac{v^2}{r} \), and the speed can then be obtained from the relation \( v_A^2 = gr \).

To find the centripetal acceleration in the other points, an expression is needed to relate the speed in different points. Since the problem formulation stated that friction can be neglected, the relation for conservation of mechanical energy, which most students remember from high school, can...
be applied, i.e. \( mv_1^2/2 + mgh_1 = mv_2^2/2 + mgh_2 \), giving
\[
v_2^2 - v_1^2 = 2g(h_1 - h_2) = -2g\Delta h. \tag{1}
\]
Insertion in the expression for centripetal acceleration then gives \( \Delta a_c = \Delta(v^2/r) = -2g\Delta h/r \). For the points to the side we find \( h_B = h_D = h_A - r \) giving \( a_c,B = a_c,D = a_c,A + 2g \). Similarly, the centripetal acceleration in the lowest point of the circular loop is given by \( a_c,C = a_c,A + 4g \).

In addition, students need to understand the vector character of force and acceleration and the relation between them—also at the points when all forces are in the vertical direction. In the highest point (A) when the acceleration is given as the acceleration of gravity there is no force between ride and rider. In the points to the side of the loop (B and D), the ride needs to provide the force for the centripetal acceleration, while in the lowest point (C) it must also compensate for the force of gravity. Thus, the solution strategies in the different points require different subsets of a more general strategy.

3.2. Free-body diagrams and acceleration

Newton’s second law relates the acceleration of the body to a sum of all forces, which can be illustrated graphically as in figure 4 for three of the points in the loop. Inserting these forces and accelerations into the roller coaster loop leads to the result in figure 3.

Textbooks vary in their recommendations on how to treat the acceleration with respect to the arrows representing forces. The book used by the students [10], recommends against including it, since it is not a force, as do many other university textbooks (e.g. [12]). This is also how this group of students were taught.

The recent textbook by Mazur [13] (p200) states explicitly: ‘draw a vector representing the object’s acceleration next to the centre of mass that represents the object. Check that the vector sum of your force vectors points in the direction of the acceleration’. (emphasis in original)

Engineering mechanics books (e.g. [14, 15]) are more likely to recommend that acceleration (or rather \( \mathbf{ma} \)) be included as a dashed line and/or with different colour in the diagram summing all forces. Figure 5 gives an example for the roller coaster loop problem studied in this paper.
students got to see their exam solutions. During the later part of the interview, students were also shown an animation of a car driving inside a vertical loop.

The students knew each other well, having studied together for nearly a year, and the atmosphere during the interview was friendly and reflecting. Three of the authors (AMP, LO and KS) took part in the interview which was video recorded and the dialogue was transcribed. One of the other authors (UE) then checked the transcription for consistency. The discussions covered a number of different topics which are presented below.

5. Results: student written solutions to the exam problem

The loop problem in this paper was part of a written examination taken by around 60 students. Eight of the students solved the problem correctly, with force and acceleration drawings consistent with their calculations. Forces were drawn in the correct direction by most students attempting to solve the problem. However, in spite of the explicit requirement in the exam problem that forces should be drawn to scale, many students drew all forces with the same size, as shown in the force diagram in figure 6—even if they had worked out numerical values of the accelerations in points B, C and D that would require much larger forces. A number of typical student acceleration drawings are shown in figure 6. Eleven of the students drew acceleration diagrams similar to alternative (a), which is consistent with the forces (but not with the problem text). At least 15 of the 60 students confused acceleration and velocity, with diagrams similar to figure 6(f), although four of them using centripetal acceleration. Twelve other student responses were distributed over the other alternatives in figure 6.

One student commented that ‘the normal force causes centripetal acceleration towards the centre of the loop’. It is worth noting that, although

the centripetal acceleration in points B and D is caused solely by the normal force, gravity is part of the net radial force at the top and bottom. Dealing with the vector relation between normal force and acceleration in the lowest and highest points seems to have caused more problem than on the sides when the different axes gave a more clear hint of the need for vector addition. Indeed, in general, the students applied Pythagoras theorem correctly to combine the centripetal acceleration with the acceleration of gravity in the points B and D, where the track is vertical.

Most students who attempted to solve the exam problem invoked energy methods to work out the relation between the speeds at the different parts of the loop using equation (1). However, many of them failed to use the information that the acceleration was given as \(g\) to obtain the velocity in the highest point. Instead they stated or assumed that the velocity would be zero as in the following excerpt: ‘assuming point A to be the highest point from the ground, \(v_A\) will be assumed to be zero in the rest of this exercise’. One student even started by doing a correct calculation at the top, but then realized that energy methods would be useful to get speeds for the other points, adds a comment that there is ‘only gravitational energy at point A’, and concluded that \(v = 0\). In textbook problems involving energy conservation, the velocity is, indeed, often zero in the highest point. However, that is not the case for this problem.

Another concept getting in the way for a couple of the students is instantaneous velocity: ‘at the top of the loop the velocity of the roller coaster is instantaneously zero, so \(v_A = 0\)’.

A few students stated explicitly that they took the tangential speed or angular velocity to be constant (‘We recognize this system to be in uniform circular motion’), which would imply a constant centripetal acceleration. The question about velocity in the other points was then simply answered by drawing the direction of the motion. None of the students assuming uniform circular

![Figure 6. A typical incorrect force diagram drawn by students as solutions to the exam problem, together with a number of different types of accompanying acceleration diagrams. Only the drawing in a) is consistent with the force diagram, but none is consistent with the higher speeds in the lower points.](image)
motion drew force and acceleration diagrams consistent with this assumption.

Below, we present results from a focus group interview, aiming to gain a deeper understanding of student strategies and understanding of the problem.

6. Results: student interviews

This chapter presents different themes from the group discussion with four students about the loop problem in figure 2.

6.1. Acceleration, velocity and forces at the top of the loop

During their written exam a few weeks before the interview, none of the four students participating in the group interview had used the information that the acceleration is \( g \) at the top to obtain a value for the speed. Student 1, thinking first about ‘circular motion, uniform circular motion and energy conservation’, described that ‘the biggest challenge in my mind was realizing what the text actually meant’.

One student (2), noted that ‘It did not occur to me, especially because—I was reading it fast, I was reading during the exam, and there’s an acceleration... at point A, and it is g, and of course gravity acts everywhere. My brain, when I read it—well it is not necessarily the _only_ acceleration’. This student then chose to let the train enter at the bottom of the loop with unknown velocity, and then calculate the velocities in the other points, using energy conservation.

The other three students in the interview had instead assumed that the velocity was zero at the top, and obtained the velocities in the other points from energy conservation, one of them observing that the text stated ‘that friction could be neglected’. Student 1 discussed the concern about the riders falling straight down if the acceleration would be \( g \), and that they ‘probably would not survive that. So when it came to write down the forces acting on the loop in point A, I wrote down an arrow pointing up with the same magnitude as mg. And I wrote that N is a theoretical force given by the rails to the cart at that specific point’. (During the interview this was also referred to—with a smile—as ‘this magic force’.) As the students discussed the problem between themselves immediately after the exam, ‘someone came up and said \( g \) is the only acceleration [...] I immediately understood that A could not be zero velocity, it made no sense’, concerned that the train would then never get away from point A. The insight that \( v \) did not need to be zero did not come until after the exam, when the student noted that ‘because we have some relationship which we can actually write the velocity of the cart from there’. Another student (3) had thought about ‘an example from the book, where you throw a baseball in the air, and it hits its peak [...] Instantaneously the speed is zero’.

Student 4 realized directly that it was non-uniform circular motion and remembered a problem from class, with a ball swinging around on string, and tried to reconstruct the forces from that situation. Still, the velocity at the top ended up being zero in the exam solution. A possible explanation for the train staying on track was then provided: ‘it could be the wheel, it could be the rail holding it up so that if it is zero velocity it just would not fall off’.

Student 3 described experiences from real roller coasters: ‘there are instants when you have the feeling that the velocity is zero, but what you feel is not like what is the actual velocity’. The experience was elaborated: ‘when you are at the top, you have the feeling that—I think what happens if you have a shift of weight’. Here, the body was used to demonstrate a shift, first sideways then forward. ‘For a short instance you have a feeling that there is no movement. You are at the same place, but it is only for a second or two, and then you carry on going. But that’s just what you feel. The train or the roller coaster itself might still be moving even though it is maybe like only a millimeter distance’. This was one of very few references to the experience of the body in relation to the loop problem.

6.2. Force and acceleration at the bottom

The roller coaster train reaches its maximum speed and maximum centripetal acceleration at the bottom of the loop, which can be obtained from energy considerations. In this way, the maximum centripetal acceleration is found to be 5\( g \) (upwards) at the bottom of a circular loop, if it is \( g \) downwards in the highest point.
Students making sense of motion in a vertical roller coaster loop

Student 3, who had used energy conservation was asked to go to the whiteboard to draw the forces, starting with the situations at the top and bottom, getting the centripetal acceleration at the bottom to 4\(g\), since the velocity had been assumed to be zero at the top.

Student 4 worked instead directly with forces at the bottom during the exam and wrote the net force at the bottom as \[ \frac{mv^2}{r} - mg = ma \] and then \('I replaced a with g and rearranged the formula and the mass cancels out, and I solved for the velocity'. Initially during the exam, the velocity had come out as zero, but then ‘I just switched the sign’. Later in the interview, this sign change was mentioned again: ‘so then i started to switch these around, even though it does not make too much sense to me’.

Student 1 also confessed to having claimed that acceleration at the bottom is zero ‘since at this point there is a force upward given by \(\frac{v^2}{r}\) and then there is \(mg\) pointing downwards’. When asked what the motion would look like after point C if the acceleration were zero, student 1 used the hand to indicate that the train would continue along the horizontal straight track, which was part of the drawing in the problem, rather than continuing along the loop for a second turn.

In principle, the question about the acceleration in point C is ambiguous, changing from 5\(g\) to 0\(g\), if the train continues along a straight track, rather than into the loop. There was no indication in the the written exam solutions that this had led to confusion, nor any explicit mention during the interview. Still, it could be preferable to avoid this unintended ambiguity by replacing the horizontal tracks with slightly inclined tracks as in figure 7

![Figure 7. Inclined tracks leading into a circular loop. This combination was used in the mid-19th century loop ‘Chemin de Centrifuge’ [17].](image)

6.3. The normal force

None of the students in the interview had any difficulty concluding that the acceleration at the sides was a vector sum of the centripetal acceleration towards the centre of the loop and the acceleration of gravity, whereas the discussion of forces at the bottom brought out miscellaneous sign combinations of \(mg\), \(N\) and \(\frac{mv^2}{r}\) in the equation.

Student 2 summarized the situation at the bottom: ‘in this case we have to have a net force upwards, to maintain our roller coaster in our circular orbit. And that force, the centripetal force, has to be equal to the gravitational force and the normal force that pushes up from the railing. So the net force is the centripetal force’. Student 1 tried to clarify ‘If I get this correctly. Are we saying that \(mg - N = \frac{mv^2}{r}\)? Is this what we are saying?’ Student 1 was then asked to go to the whiteboard and draw the forces as vectors to ensure that all agree on the directions. During the following brief discussion, a number of different sign combinations appeared in the equation, but without any graphical representation (such as figure 4) explicitly showing the relation between different vectors involved.

Many teachers may recognize the common student approach to ‘fix’ the sign in similar situations, and then moving on to the next problem, repeating the mistake, as ‘only a sign problem’, even if they have, of course, been taught a correct treatment. Often students are able to get the forces at the top and bottom right without using a
consistent strategy, but by invoking their everyday experiences.

6.4. Simple harmonic motion as model for a loop

The motion in pendula and roller coasters are both examples of transformation between kinetic and potential energy which is sometimes used in textbooks to calculate forces at the bottom of a swing or the speed at different points of a roller coaster.

In the interview, student 1 discussed: ‘If it works like a pendulum, and we see this as simple harmonic motion—when we have the pendulum at $\theta = 0$, is not the tension provided by the string the same as the $mg$ pushing the mass down? That’s why you call it equilibrium position in the first place. When it is passing through the equilibrium point it has...maximum velocity’.

One of the interviewers took a spiral toy to demonstrate the difference in length between the toy at rest and at the bottom of a circular motion [18], as the motion changes from going down to going up. Student 2 concludes ‘I see. So the tension is actually higher than $mg$ when it passes the lowest point’. Student 1 asked ‘So my improvised idea about simple harmonic motion is...’ and the interviewer points out that you typically look only in the tangential acceleration in discussing harmonic motion. Student 2 added that ‘in that direction it is zero’ and student 1 happily exclaims that ‘It is the one in the same direction’, using the hand to show a circular motion and continues ‘That’s how I approached it also on the exam. I was like—this could easily be represented like a pendulum. Just part of the motion of course. But of course tension. It is the tangential acceleration which is zero’.

In fact, there is a ride, the Larson loop [19], which can be described as a hybrid between a roller coaster and a pendulum ride, although the construction with the relatively long train is mounted on a moving circle, makes the acceleration at the bottom lower than it would be for a point particle or a short train.

6.5. Forces on a pendulum

As a follow-up to the discussion about the pendulum motion, the students were asked about the acceleration at the turning point of the pendulum. Student 1 started by suggesting to look at the components of the forces on the body, but student 3 argued that ‘the acceleration is zero, because it hits its maximum displacement. The acceleration would be zero for an instant and then it would change direction’. The other students looked bewildered and student 2 came to the rescue, as student 3 realized that he was confusing acceleration and velocity. ‘So the only forces acting here are the tension and gravity and take the vector sum’. One of the interviewers draws the arrows on the whiteboard, explicitly making the arrow representing the tension shorter than the arrow for $mg$. Student 1 gave support with a hand gesture as student 2 noted that ‘It would be our tangent acceleration which is at a maximum there, and I wonder if—since the velocity there is zero, the radial acceleration should be zero’. As the interviewer drew the acceleration arrow in a different color, student 2 was delighted to note ‘Oh, the vector sum is actually tangential’, and student 1 adds ‘Because the other components cancel out’.

Most textbook descriptions of pendulum motion focus only on the simple harmonic motion relating to the angle and omit discussions of the tension in the string. One exception is ‘Matter and interaction’ [20], which opens a chapter with a question about why the vine breaks when Tarzan attempts to swing across a river, even if he had tried it for strength before the attempt. This situation is then dealt with in several contexts in the next couple of chapters, and the harmonic motion aspect is shifted to an exercise for the students, with several sub-questions. Another exception can be found in [21].

7. Discussion

7.1. Car or roller coaster train: affordances from different problem formulations and illustrations

One of the important aspects to discern in circular motion is in fact motion. As an attempt to help the students focus their attention on the motion as well as the changing speed, the students were shown an animation of a car driving in a vertical loop, towards the end of the group interview.
the animation, the car moved slowly at the highest point and a couple of the students exclaimed that it looked like the car had zero velocity at the top. If a roller coaster train would have zero velocity as it reaches the top of a loop, the train would not continue along the track, as noted by student 2, who had rejected that possibility in his solution: ‘If the velocity at the top is truly zero, then it cannot move, and then it would stay up there for ever’. Student 1 reflected that it would have ‘helped me understand that there has to be an acc—a velocity because otherwise there is no reason the car would keep moving. Once it reaches (the top) and there is zero velocity, there is no reason at all for the car to keep moving. I guess it could have helped me to make it click’.

Student 4 noted that ‘A roller coaster can be upside down without falling. If it was a car I would have panicked a bit more. If the exam question had shown a car, then I would have approached differently and got the correct answer,

Although a roller coaster train and an animation of a car (figures 2 and 8), are two different representations involving the same physics, they trigger different starting points for student 4, who did not discern the motion of the car in the highest point without the animation.

The discernment of non-zero velocity is mediated through associations with the students prior knowledge and experiences. Roller-coaster carts can traverse loops and even get stuck upside down, whereas a car getting stuck upside-down in a loop is unheard of and would create sufficient suspicion to make the student reassess the assumption.

7.2. When the normal force gets in the way

As seen in section 6.1, two of the students in the interview introduced an implicit normal force at the top, which would prevent the train from falling down, without being recognized as a force. This, of course, is exactly what normal forces do—prevent bodies from going through various surfaces, but by exerting a force, which is just as large as needed for the body to remain in place or follow a particular path. However, a common confusion is to assume that the normal force always exactly counteracts the force of gravity.

When asked to draw forces at the lowest point of the loop, student 1 stated ‘There is a normal force but there is also... $mv^2/r$’. The question ‘What is the normal force’, prompted a hesitating response: ‘is not it the force from Newton 3—you have mg pointing down...’ One of the interviewers pointed out that normal means orthogonal to the surface, and the students nodded.

For a short while there was a confusion in the discussion about whether the centripetal force is the same as the normal force or the net force. Many students have learned in high-school state that the ‘centripetal force is always a resultant’, which holds for uniform circular motion, but not for all parts of the loop: at points B and D, the normal force must be sufficiently large to provide the force needed for the centripetal acceleration whereas the force of gravity leads to a tangential acceleration: the normal force is the net radial force at B and D. For the lowest point in the loop, the normal force must combine with $mg$ to give the net radial force required for the centripetal acceleration.

7.3. Similarities to other examples

When the students described during the interview how they approached the problem, some of them described how they tried to invoke similarities with problems or examples from the book or discussed in class. Examples given included the
conversion between potential and kinetic energy in simple harmonic motion, a ball thrown in the air or attached to a string, and roller coasters starting from an unknown elevation of the highest point of the loop. One of the students also mentioned energies in an LC circuit and a diagram showing bars representing the interchange between different types of energy of a pendulum or baseball thrown straight up into the air.

In some of these examples the velocity is, indeed, zero in the highest point. It can be noted that most textbook discussions of acceleration start from velocity zero, whether in free fall or a car accelerating on the road. A few students explicitly referred to ‘energy conservation’ in their exam solutions, concluding that ‘velocity is zero in the highest point’.

The students’ common use of earlier problems solved during the course points to the usefulness of well-selected problems, with rich physics content, and sufficient variation to train students to discern the relevant physics aspects of a given situation. We hope that the problem discussed here can serve as a useful resource for the teaching of force and motion.

7.4. How long is an instant?

One of Zeno’s paradoxes [22] deals with the impossibilities of motion, suggesting for example that ‘If everything is motionless at every instant, and time is entirely composed of instants, then motion is impossible’. This paradox seems to have been tempting for a few of the students solving this problem.

‘How long is an instant?’ This question was brought up in different situations by student 3, in comments such as ‘Instantaneous. It could be microseconds, it could be milliseconds. It is a very broad definition’ later referring to experiences from roller coasters where ‘there are instants when you have the feeling that the velocity is zero’, but noting that ‘the train or the roller coaster itself might still be moving even though if it is maybe like only millimeter distance’.

Student 2 refers to a common difficulty of distinguishing zero velocity from zero acceleration: ‘It was hard for me to separate acceleration and velocity because you always think that when acceleration occurs, velocity is a result of that. Velocity will come out if you are accelerating. Whether you are increasing velocity or you are decreasing velocity it is hard not to associate to a car, when you press the pedal you are accelerating and you will speed up. But when you […] only look at the moment when you press the pedal but you are not moving, but you are about to move, that’s a difficult kind of change in your mind, that you have to think about being about to move, and actually moving’.

The discussions presented above point to the importance of discerning the motion of the train, and understanding ‘that it moves, it has a velocity. Velocity is not zero. It is not a static situation up here. It is not static down here’.

7.5. Relevance structures

One of the students realized the relevance of pendulum motion to understand a roller coasters loop: both situations involve the same physics concepts and students often forget the centripetal acceleration at the bottom, as documented for pendulum motion, e.g. in [23, 24]. The maximum speed is instead taken as an indication that the acceleration is zero, possibly with the everyday meaning of acceleration as increase in speed taking precedence over the full physics concept. Dandare [23] used a questionnaire on pendulum motion and found that only three of 29 pre-service physics teachers, provided correct answers—in spite of at least 18 of them having physics in their undergraduate degree.

However, as seen from the interviews in this study, students who had omitted the centripetal acceleration at the bottom, did not need much prompting to see their mistake. During the interviews, all the students had shown that they knew Newton’s second law, that acceleration is a vector, and the formula for centripetal acceleration, but somehow they failed to connect them when considering the exam problem.

One of us recalls a discussion in a small group of engineering physics students in their first month of studies, where one student claimed (like student 1 in this study) that the acceleration must be zero at the bottom of a swing, since the velocity has a maximum, and another student responding that the acceleration must be non-zero since you feel heavier at the bottom, but they could not on their own resolve the apparent paradox. Later cohorts of students have been asked to comment
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... on this dialogue in an introductory questionnaire, where most have been able to recall the relevance of the centripetal acceleration whereas some students forgot about Newton’s second law, as they responded in ways like, for example, that ‘The acceleration is zero, but there is still a force’. We have also encountered teachers who look surprised when a pendulum is referred to as an example of circular motion.

It is rare for textbooks to relate acceleration to the experience of the human body, and students are not used to making this connection, as seen e.g. in [27]. Using accelerometers, e.g. in smartphones [28–30], the experience of the body can also be measured quantitatively, providing an additional semiotic resource to help students develop a deeper understanding of the forces required for acceleration in pendula [28, 31], loops [11] and more irregular motions [8, 27, 32].

The problem in this paper involves several steps, and Sherwood notes that students often see mechanics ‘as a disconnected jumble of many special purpose formulas’ [25] and raises the important question ‘Do students fail to start from a physics principle, or do they fail to complete the multistep reasoning chain’ [26].

7.6. Problem solving strategies and reality

Towards the end of the interview, one of the students (2) reflected on how solving textbook problems somehow trains them to ‘Look for what is zero. What can you cancel out? What can you take away? It sometimes takes precedence to actually taking a holistic perspective [...] we are not really trained to look at a problem, where we have actually more realistic and more complex things happening, where acceleration and velocity are a bit all over the place and you cannot really cancel out’.

Three of the students chose to use energy conservation and also discussed their motivations: student 1 used that ‘the problem stated that we can neglect the effect of friction’ as an indication of energy being conserved. Student 2 recalled an example from the book with a baseball thrown into the air and the interplay between potential and kinetic energy. Student 3 started from an initial velocity at the lowest point but ‘quickly realized that it would not be as easy as I thought. And I knew there would have been a possibility for many ways where it could go wrong. That’s why I chose to use an energy method, based on [...] the energy at point C’.

Reflecting on real-life roller coaster impressions of nearly zero velocity, student 3 added that this had been used during the exam, although without relating it to the relevant physics, continuing ‘you are like buckled down so you do not—your notion of motion is a bit influenced and you do not have the same—you do not feel it’. According to Newton’s first law, you do not feel velocity, only acceleration. Student 1 added that ‘you do feel air resistance’.

7.7. Students non-discoveries of contradictions

The discussions and many of the written exam solutions reveal inconsistencies in different parts of the student solutions.

If the velocity would have been zero at the top, as assumed in many of the solutions, the roller coaster train would have stopped in the highest point—zero velocity is also inconsistent with the problem text.

In figure 6, acceleration and force diagrams agree only for alternative (a). However, figure 6(a) involves an acceleration of 2g at the top which is inconsistent with the original exam text, stating that the acceleration was g at the top. It is also inconsistent with the calculated larger speeds at the bottom.

One student commented explicitly that ‘We recognize this system to be in uniform circular motion’, whereas many other quietly assumed constant angular velocity or speed. However, this assertion was never accompanied by consistent force and acceleration diagrams.

Many students had problem ensuring that their responses for acceleration and force in the highest and lowest points were in agreement with Newton’s second law. Brief teacher interventions in small-group discussions can be essential to help students recognize contradictions in their argumentation. Invoking the experiences of the body in a real life situation can sometimes help to make such contradictions visible.

We were happy to note that students in the interview offered some reflection over unrealistic results and were willing to discuss consequences of their statements, indicating that they had also been trained to reflect on their results.
8. Conclusion

The question about forces and acceleration in the idealized roller coaster loop in the exam problem was found to reveal many partial student conceptions about force and motion and contradictions in their responses. In a tutorial session, teachers may ask follow-up questions to help students discern the inconsistencies between different responses, or with the situation given in the problem text. We hope that this paper may prepare teachers to challenge common inconsistencies in student solutions.

The roller coaster loop problem in this paper is more realistic than many textbook problems, but is still an idealization of real roller coasters, which are never completely friction-less. The center of mass in the train moves with a smaller radius than the radius of the track and the position in the train influences the forces on the rider. In addition, modern roller coaster loops are not circular [7, 9].

For teachers who like to use this problem in their teaching, we suggest that the horizontal tracks in the textbook and exam problem be replaced with sloping tracks as in figure 7, which was also the shape of earlier 20th century loops [7] (www.parkworld-online.com/in-the-loop-a-brief-history-of-upside-down-coasters/). In summarizing discussions, teachers may address the abrupt change in acceleration (‘jerk’, [8]) as the track changes from a straight line to a circular arc and discuss the motivation for a larger radius at the bottom that can be seen in photos or movies of real looping roller coasters.

Acknowledgments

We thank the students for taking part and sharing their experiences and the teacher of the course for including the loop problem on the exam.

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Received 26 June 2019, in final form 6 August 2019
Accepted for publication 28 August 2019
https://doi.org/10.1088/1361-6552/ab3f18

References

[1] Halloun I A and Hestenes D 1985 The initial knowledge state of college physics students Am. J. Phys. 53 1043–55
[2] Van Heuvelen A 1991 Learning to think like a physicist: a review of research-based instructional strategies Am. J. Phys. 59 891–7
[3] Gunstone R 1984 Circular motion: some pre-instruction alternative frameworks Res. Sci. Educ. 14 125–35
[4] Clement J 1982 Students’ preconceptions in introductory mechanics Am. J. Phys. 50 66–71
[5] Gardner P 1984 Circular motion: some post-instructional alternative frameworks Res. Sci. Educ. 14 136–45
[6] Hake R R 1998 Interactive-engagement versus traditional methods: a six-thousand-student survey of mechanics text data for introductory physics courses Am. J. Phys. 66 64–74
[7] Schützmannsky K 2008 Roller Coaster—Der Achterbahn-Designer Werner Stengel (Heidelberg: Kehrer)
[8] Eager D Pendrill A-M and Reistad N 2016 Beyond velocity and acceleration: jerk, snap and higher derivatives Eur. J. Phys. 37 065008
[9] Pendrill A-M 2005 Roller coaster loop shapes Phys. Educ. 40 517
[10] Young H D and Freedman R A 2016 University Physics with Modern Physics, Global Edition 14th edn (Harlow; Pearson)
[11] Pendrill A-M 2013 Student investigations of forces in a roller coaster loop Eur. J. Phys. 34 1379
[12] Wolfson R and Pasachoff J M 1999 Physics with Modern Physics for Scientists and Engineers 3rd edn (Reading, MA: Addison Wesley Longman)
[13] Mazur E, Crouch C H, Dourashkin P A, Pedigo D and Bieniek R J 2015 Principles & Practice of Physics, Global Edition (Boston, MA: Pearson)
[14] Meriam J L and Kraige L G 2006 Engineering Mechanics: Dynamics 6th edn (New York: Wiley)
[15] Hibbeler R C 2004 Engineering Mechanics: Dynamics (Englewood Cliffs, NJ: Prentice-Hall Inc.)
[16] Robson C 2011 Real World Research: a Resource for Users of Social Research Methods in Applied Settings (New York: Wiley)
[17] Park World 2013 In the loop—a brief history of upside-down coasters, (www.parkworld-online.com/in-the-loop-a-brief-history-of-upside-down-coasters/)
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[18] Pendrill A-M and Williams G 2005 Swings and slides Phys. Educ. 40 527
[19] Larsson W F and Novotny J P 2002 Amusement ride without hubs and spikes US Patent US6402624 (see also (http://larsonintl.com/the-22m-giant-loop/))
[20] Chabay R and Sherwood B A 2011 Matter and Interactions 3rd edn (New York: Wiley)
[21] Etkina E, Planinsic G and Van Heuvelen A 2019 College Physics: Explore and Apply 2nd edn (San Francisco, CA: Pearson)
[22] Aristotel 350 BC Physics translated by Hardie R and Gaye R K, available at (http://classics.mit.edu/Aristotle/physics.html)
[23] Dandare K 2018 A study of conceptions of preservice physics teachers in relation to the simple pendulum Phys. Educ. 53 055002
[24] Czudkov K and Jana Musilová J 2000 The pendulum: a stumbling block of secondary school mechanics Phys. Educ. 35 428
[25] Chabay R W and Sherwood B A 2004 Modern mechanics Am. J. Phys. 72 439
[26] Sherwood B A 2015 Tracking Steps in Multistep Problems (talk at a Meeting of the American Association of Physics Teachers (Salt Lake City, August 2005), available from (https://brucesherwood.net/))
[27] Eriksson U and Pendrill A-M 2019 Up and down, light and heavy, fast and slow—but where? Phys. Educ. 54 025017
[28] Pendrill A-M and Rohlén J 2011 Acceleration and rotation in a pendulum ride, measured using an iPhone 4 Phys. Educ. 46 676–81
[29] Vieyra R and Vieyra C 2014 Analyzing forces on amusement park rides with mobile devices Phys. Teach. 52 149
[30] Staacks S, Hütz S, Heinke H and Stampfer C 2018 Advanced tools for smartphone-based experiments: phyphox Phys. Educ. 53 045009
[31] Pendrill A-M and Modig C 2018 Pendulum rides, rotation and the Coriolis effect Phys. Educ. 53 045017
[32] Pendrill A-M and Rödjegård 2005 A rollercoaster viewed through motion tracker data Phys. Educ. 40 522

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