Boundary revision of a beam model for a thin-walled waveguide at bending vibration

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Abstract. The paper discusses the problem of choosing a model type for a straight thin-walled waveguide with a rectangular cross-section during its vibrations. To do this, calculations were made of the first natural frequency of vibration for waveguides of different geometric sizes. The restraints of the waveguide were accepted by a cantilever, hinged, and fixed supports. The difference between the values of the first natural frequency of vibration for the beam and shell waveguide models was estimated. Calculations were carried out analytically on the theory of beam vibration and by the numerical method of finite elements. The results show that the boundary known in static calculations of the applicability of the beam model $R_{\text{max}}/L = 0.1$ requires clarification at bending vibrations for the small thickness of the section wall. With small wall thicknesses, the error of calculating the first natural frequency of vibrations will increase sharply due to the effect of a beam section warping.

1. Introduction

There are three main construction models in mechanics: beam, shell, and massive. The main difference between models is in the dimension of the geometric object and the corresponding complexity of the mathematical description and solution of the problem. Calculating massive bodies is the most difficult task, and this model is rarely used in calculations.

The shell model very accurately models thin-walled structures, which in particular is a waveguide. However, the rectangular cross-sectional shape of the waveguide makes it difficult to use shell theory due to the right angles of the section. The solution is achieved only using numerical methods, which limits its interpretation and use in practice.

The beam model is a very simplified model of the object and therefore has many limitations on use to obtain the correct calculation results [1]. However, the ease of setting the problem and the resulting analytical solutions allow you to conduct different research and analysis of the results. In many cases, this is a decisive factor in favor of choosing a beam model.

One of the most important criteria for the applicability of beam theory to the studied structure is the relationship of its dimensions to each other. One of the geometric parameters, length, must be many times larger than the other two dimensions (width and height). The known literature does not accurately determine the ratio of dimensions, it is considered sufficient if, in static loading, the length exceeds the maximum cross-sectional dimension of the structure by 10 times.

The beam theory [2-22] is applicable for the calculation of any extended structures that meet the condition of the ratio of their length to the maximum transverse cross-section size.
$\frac{R_{\text{max}}}{l} < 0.1,$ \hspace{1cm} (1)

Where $l$ is the length; $R_{\text{max}}$ – maximum cross-sectional size.

The dimensions of most real waveguides satisfy this requirement. However, it is not clear whether such a boundary to the applicability of the beam model is correct for its vibrations.

In this work comparative calculation of the first natural frequency of vibration for beam and shell models of a thin-walled waveguide with rectangular cross-section is carried out. Taking the results of the shell model as a reference, the amount of deviation in the results for the beam model was determined with different restraints and dimensions.

2. Problem statement
A straight section of the waveguide is studied, which has three options for restraint: cantilever, hinges, and fixed supports at the ends (figure 1).

![Figure 1. Restraint methods and the first mode of beam vibration.](image)

Three types (figure 2) of the waveguide cross-section were examined: $hxb$ 9x18, 15x35, and 20x60 mm. The wall thickness varied from 0.2mm to 1.2 mm, the length changed from 0.1m to 0.7m. Waveguide material: aluminum alloy with $E = 7 \times 10^{10}$ Pa, density $2770 \text{ kg/m}^3$.

2.1. Analytical solution on beam theory
According to the accepted beam model of the waveguide and its design scheme (figure 1), the free vibration equation has the form [2-22]:

$$EJ_{\text{min}} \frac{\partial^4 y}{\partial x^4} + m \frac{\partial^2 y}{\partial t^2} = 0,$$

where $y=y(x)$ is the beam deflection; $E$ is the elastic modulus of the beam material; $J_{\text{min}}$ is the minimum moment of inertia of the beam cross-section; $m$ is the mass per beam length.

To solve equation (2), you must specify four boundary conditions that conform to the beam fixation conditions in the supports. Substituting the deflection equation into the free vibration equation (2) and taking into account the boundary conditions, we obtain a system of linear equations. The first eigenvalue of this system will determine the value for the first vibration frequency of the beam, which has the form [2-22]:

\begin{align*} &EJ_{\text{min}} \frac{\partial^4 y}{\partial x^4} + m \frac{\partial^2 y}{\partial t^2} = 0, \\
\end{align*}
\[ f_i = \frac{\alpha^2}{2\pi^2} \cdot \sqrt{\frac{EI_{\text{min}}}{m}}, \]

Where \( \alpha \) is the support factor, which takes into account the method of beam fixation at vibrations.

The values of this factor for various beam support systems (figure 1) can be found in the reference books or calculated from equation (3) taking into account the boundary conditions of a beam [23,24].

2.2. Numerical solution for a shell model
The shell model of the waveguide was created in the Ansys software and its calculation was carried out by the numerical method of finite elements [25,26]. The finite element model of the waveguide consisted of a large number of finite elements so that this obviously did not affect the accuracy of the results. To do this, the influence of the number of FEs on the results of the modal analysis was previously checked. It was found that an increase in the number of FEs from 7000 to 35000 results in a change in the value of the first natural frequency by no more than 0.5%. With this in mind, the finite element grid consisted of 10-20 thousand finite elements of type Shell128 depending on the type of section.

3. Solution and results
In the results of the calculations, the value of the first frequency for the bending shape of the vibration of the entire waveguide was monitored. The obtained data were processed so as to obtain the dependence of the difference \( \Delta f \) in the values of the first natural frequencies of vibration for the beam and shell models with respect to the dimensionless relation (2), which characterizes the relative length of the waveguides. The vibration frequency of the shell model was taken as the reference.

Figure 3 shows the typical dependencies obtained for the medium waveguide size of 15x35 at different wall thicknesses and restraint conditions, which are indicated in the caption to the figure.

![Graph](image)

a) cantilever beam.
Similar results were obtained for other cross-sectional dimensions considered.

4. Discussion
Analysis of the obtained results shows that to simulate and calculate the waveguide according to the theory of beam, then the error of calculations for wall thicknesses $t = 0.8...1.2\text{mm}$ will be less than 10% which is quite consistent with the assumptions of this theory (1) itself. For smaller wall thicknesses, the error can reach already 15% and even much more, which is caused by the significant influence of local deformations of thin section walls (figure 3).

With smaller waveguide lengths, that is, at $B/L > 0.1$, the difference in the calculation results increases sharply. This is due to the fact that the first vibration frequencies will no longer be bending for the structure as a whole, but are associated with the vibrations of the individual plates that are part of the waveguide section, which is not considered here (figure 4).
a) bending first mode at \( B/L < 0.1 \)
b) non-bending first mode at \( B/L > 0.1 \)

**Figure 4.** Example of waveguide modal analysis results.

This phenomenon is especially pronounced at very small wall thicknesses, 0.6 mm or less. The study is too limited to derive a general analytical expression that would define the wall thickness requirement. The known condition [22] for wall thickness in our case was not correct enough:

\[
\frac{t}{R_{\text{max}}} > 0.1.
\]

Thus, the theory of the beam vibration is quite correct for extended structures that meet the condition of their extension (1) and the wall thickness is sufficient to maintain the shape of the cross-section. In this case, the error in determining the value of the first vibration frequency will not exceed 10%.

5. Conclusion

The results of the calculation of the first natural frequency for beam and shell models of a straight waveguide with a thin-walled rectangular cross-section were investigated. The size of the cross-section and the length of the waveguide varied. The results of the comparison showed that the beam model of the waveguide at vibration is correct according to the accepted assumptions, but requires providing a certain wall thickness in order to avoid influencing the results of the cross-section warping. The results obtained are well consistent with the known data from the literature and can be considered as an addition to them.

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