Theory of a double-dot charge detector

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A double quantum dot charge detector, with one dot Coulomb coupled to the electron to be detected and the other modulated by a time-dependent plunger voltage, is analyzed in a minimal model. The signal and noise of the detector are calculated by a standard master equation and MacDonald formula technique. We find a dip in the noise spectrum at the double Rabi frequency of the double dot, defining the bandwidth available for detecting charges in motion.

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Mesoscopically narrow passages of electrons, controlling the flow of electric current by potential barriers just below or above the Fermi level, are obviously very sensitive to external electric fields, which suggests their use as charge detectors. Most often, a mesoscopic trap or "island" carrying the charge is Coulomb coupled to a quantum point contact or a quantum dot - in that arrangement called a single-electron transistor (SET), to offer a charge sensitivity sufficient to detect individual electrons. Similar tools are being explored as readout devices for charge qubits, with some results emerging about the quantification of the amount of information accessible in the readout process.

Most recently, double quantum dots (DQDs) here also called double island single-electron transistors, have been used for charge detection, the main advantage being their relative immunity against noise of various origins. The operation is based on the highly coherent inter-dot tunneling, manifest through Rabi oscillations, providing sharp boundaries on the current-voltage characteristic of DQDs. This new kind of device has been theoretically analyzed by Tanamoto and Hu, through a model mainly focusing on the complications from weak Coulomb blockade, and specific aspects related to quantum information, also discussed in Ref. 10.

In most of the experimental work cited above, the device is operated at radio frequency, drawing on analogous work on radio-frequency SETs efficiently avoiding the band of low frequencies, strongly contaminated by 1/f noise, which is ubiquitous, and hard to analyze theoretically. On the contrary, shot noise, extending to much higher frequencies, is open to analysis by standard tools, which is also our main concern here.

In the present paper we discuss charge detection in DQDs, supplemented by the possibility of fast time control, as sketched in Fig. 1. The island trapping the electron to be detected would be placed next to the first compartment of the DQD, so the electron would detune the tunneling resonance set by the two bias plungers. Time control, as inspired by the experiment of Nakamura et al., could be achieved by a synchronized modulation of the pump voltage sending the electron onto the island, and the second plunger of the DQD, driving the device through a work point. A variety of delay times and plunger pulse shapes would allow a detailed analysis of the dynamics of one-electron detection.

In calculating signal and noise of the DQD detector, we evaluate the reduced density matrix of the coupled DQD+trap system by means of a standard Markovian master equation approach (see, e.g., Refs. 15,16,17), supplemented by the possibility of resolving the dynamics according to the number of achieved tunneling events,15 The latter being slow, their fast screening in the external metallic circuits can be approximately treated by the Ramo-Shockley theorem in its symmetric form. Frequency-dependent stationary noise is evaluated by means of MacDonald’s formula,18 which is a shortcut avoiding the more familiar Wiener-Khinchine analysis of time correlation functions.

The physical time and energy scale of the processes envisaged is set by the Rabi frequency. For a typical DQD formed by metal contacts on top of a GaAs/AlGaAs heterostructure, it is of the order $\Omega \approx 10^{10}$ Hz. To preserve coherence, tunneling rates should be set by gate voltages to the same order of magnitude, thereby assuring time control on the ns scale, easy to follow by external electronics.

Coulomb blockade allows one to consider the low-
temperature dynamics of the DQD + trap system on a truncated basis, stretched by states of 0 or 1 extra electron above a neutral background on each dot and the trap. A further step of truncation is brought about by the interdot Coulomb repulsion which for the geometry of the experiment of Hayashi et al. amounts to \( \approx 10^3 \hbar \Omega \), effectively excluding states for which both dots are occupied.\(^{21,22}\)

The above considerations justify the use of a minimal model in which the retained basis vectors are denoted as \( |a\rangle = |000\rangle, |b\rangle = |100\rangle, |c\rangle = |010\rangle, |d\rangle = |001\rangle, |e\rangle = |101\rangle, |f\rangle = |011\rangle \); the three occupation numbers belonging to left dot, right dot, and trap respectively. In addition, each of those three parts is coupled to a separate metallic contact, acting as a fermion reservoir.\(^{23}\) The dynamics of the whole is generated by the Hamiltonian \( \dot{H} = \dot{H}_{DQD} + \dot{H}_T + \dot{H}_{int} \), with

\[
\begin{align*}
\dot{H}_{DQD}/\hbar &= \epsilon_1 a_1^\dagger a_1 + \epsilon_2 a_2^\dagger a_2 + \Omega (a_1^\dagger a_2 + a_2^\dagger a_1) + \sum_l w_l b_l^\dagger b_l + \sum_r w_r b_r^\dagger b_r + \sum_l \lambda_l^\dagger b_l^\dagger a_1 + h.c. \\
&+ \sum_r \lambda_r^\dagger b_r^\dagger a_2 + h.c., \quad (1a)
\end{align*}
\]

\[
\begin{align*}
\dot{H}_T/\hbar &= \sum_p w_p d_p^\dagger d_p + \epsilon_T c^\dagger c + \sum_p \lambda_p^\dagger d_p^\dagger c + h.c., \quad (1b)
\end{align*}
\]

\[
\dot{H}_{int}/\hbar = \nu a_1^\dagger a_1 c^\dagger c. \quad (1c)
\]

Reduced dynamics of the DQD + trap subsystem are obtained by carrying out thermal averaging over the states of the contacts in the initial state. In the above formulas, \( h\epsilon_i \) (\( i = 1, 2 \)) is the energy of an electron occupying the single state allowed by Coulomb blockade in the \( i \)th quantum dot, \( \hbar \Omega \) is the amplitude of tunneling between the two dots, \( h\lambda_l \) and \( h\lambda_r \) are the respective tunneling amplitudes from left reservoir to left dot and from right dot to right reservoir; \( h\nu \) is the Coulomb interaction matrix element characterizing the coupling of the trap to the first dot, \( h\nu_l \) is the tunneling amplitude between pump and trap, and \( h\nu_T \) is the energy level of the trap; \( h\nu_l \) and \( h\nu_r \) are the one-electron energies in the left and right contacts, and \( h\nu_p \) is the pump. \( a_i \) is the one-dot electron annihilation operator, \( b_l \) and \( b_r \) are those for the respective contacts, \( c \) is that of the electron trap, and \( d_p \) that in the \( p \)th one-electron state of the pump.

The one-dot levels \( \epsilon_i \) can be modulated in time through plunger voltages \( V_p \), to scan near-resonance conditions. Introducing the notation \( \delta = \epsilon_1 - \epsilon_2 \),

\[
\delta + \nu = 0 \quad (2)
\]

is the condition of resonance, assuring that if the trap is occupied by one electron, the DQD detector exhibits maximum dc transmission, i.e., maximum signal with respect to the empty trap case.

As mentioned above, in conjunction with a Ramo-Shockley treatment of screening\(^{24}\) it is convenient to decompose the density matrix according to the total number \( N \) of tunneling events occurring at both external contacts of the detector, which immediately furnishes the Ramo-Shockley screened current as\(^{15}\)

\[
\frac{1}{e} I(t) = \frac{1}{2} \dot{N} = \frac{1}{2} \sum_N N \dot{p}_N(t), \quad (3)
\]

where

\[
p_N(t) = \sum_i \rho_i^{[N]}(t) = \nu \cdot [\underline{x}_N(t) + \underline{y}_N(t)] \quad (4)
\]

with the notations

\[
\underline{x}_N \equiv (\rho_{aa}^{[N]}, \rho_{bb}^{[N]}, \rho_{cc}^{[N]}, \rho_{dd}^{[N]}, \rho_{ee}^{[N]}, \rho_{ff}^{[N]})^T, \quad (5a)
\]

\[
\underline{y}_N \equiv (\rho_{da}^{[N]}, \rho_{ec}^{[N]}, \rho_{ef}^{[N]}, \rho_{fd}^{[N]}, \rho_{fe}^{[N]}, \rho_{ff}^{[N]})^T, \quad (5b)
\]

and

\[
\nu = (1, 1, 0, 0, 1)^T. \quad (6)
\]

The vectors \( \underline{x}_N(t) \) and \( \underline{y}_N(t) \) are determined by the solution of a system of master equations, with the appropriate initial conditions, which are by no means trivial.\(^{24}\)

We restrict ourselves to the zero-temperature case; then the Fermi levels of the contacts, which control the detector and the pumping of the trap, respectively, do not appear explicitly in the calculation, apart from distinguishing pumping-in and pumping-out periods, according to whether the pump electrode Fermi level is above or below the trap level \( h\nu_T \). Starting with Hamiltonian \((1)\), and following any of the equivalent standard procedures used, e.g., in the papers listed under Refs.\(^{15,16,17}\), one invariably arrives at a system of Markovian master equations for the components of the density matrix, which can be written in the form

\[
\dot{\underline{x}}_N = \mathbf{A} \underline{x}_N + \mathbf{B} \underline{x}_{N-1} + \Gamma_T \underline{x}_N, \quad (7a)
\]

\[
\dot{\underline{y}}_N = (\mathbf{A} + \mathbf{C}) \underline{y}_N + \mathbf{B} \underline{y}_{N-1} - \Gamma_T \underline{x}_N \quad (7b)
\]

where

\[
\underline{x}_N = \begin{cases} 
-\underline{x}_N & \text{for pumping-in,} \\
\underline{y}_N & \text{for pumping-out,}
\end{cases} \quad (8)
\]

and we have introduced the matrices

\[
\mathbf{A} = \begin{pmatrix} 
-\Gamma_L & 0 & 0 & 0 & 0 \\
0 & \nu_T & -i\Omega & -i\Omega & 0 \\
0 & -i\Omega & -\delta - \Gamma_R / 2 & 0 & -i\Omega \\
0 & i\Omega & 0 & i\Omega & -\Gamma_R \\
0 & 0 & i\Omega & 0 & \Gamma_R
\end{pmatrix}, \quad (9a)
\]

\[
\mathbf{B} = \begin{pmatrix} 
0 & 0 & 0 & 0 & 0 \\
\Gamma_L & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0
\end{pmatrix}, \quad \mathbf{C} = \begin{pmatrix} 
0 & 0 & 0 & 0 & 0 \\
0 & 0 & -i\nu & 0 & 0 \\
0 & 0 & 0 & i\nu & 0 \\
0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0
\end{pmatrix}. \quad (9b)
\]
Here $\Gamma_L$ and $\Gamma_R$ are the familiar Fermi golden rule rates of dissipative tunneling transitions from left contact to left dot and from right dot to right contact, respectively. $\Gamma_T$ is the rate of transitions from pumping contact to trap and vice versa. Damping and dephasing of coherent processes in the DQD detector are controlled by these tunneling rates, microscopically rather ill defined because of their high sensitivity to defects and impurities, but - as already mentioned - easy to control experimentally by gate voltages.

Trivially, the same system of master equations is obeyed by the vectors $\mathbf{a}, \mathbf{y}$ composed of the unresolved density matrix elements $\rho_{ij} = \sum_N \rho_{ij}^{[N]}$. It is easy to find their steady-state solution $\rho_{ij}^{stac}$; then through Eq. \ref{eq:10} we get the steady-state current\

$$I_{stac} = \frac{e}{1 + \alpha^2} \left[ \frac{\Gamma/3}{1 - \alpha^2} + \frac{\Gamma}{12} \left( \frac{\delta}{\Omega} \right)^2 + \frac{1/3}{1 - \alpha} \left( \frac{\delta}{\Omega} \right) \right].$$

(10)

where

$$\tilde{\delta} = \begin{cases} \delta & \text{for empty trap}, \\ \delta + \nu & \text{for filled trap}; \end{cases}$$

(11)

in addition, we have introduced the asymmetry parameter $\alpha = (\Gamma_L - \Gamma_R)/(\Gamma_L + \Gamma_R)$ and the mean tunneling rate $\Gamma = (\Gamma_L + \Gamma_R)/2$.

The zero-frequency signal of the detector is defined by the difference between the current with filled and empty trap, the detector being active all the time. Large asymmetries (i.e., $\alpha$ close to unity) cut down the current, which is the signal of our detector. Although all the subsequent analysis can be carried through for arbitrary values of $\alpha$, the gain thereby is negligible, therefore in what follows, we restrict ourselves to the symmetric case $\alpha = 0$.

Noting that $\delta$ is a “soft” experimental parameter, easy to adjust as desired, we observe that the sensitivity of the current to the presence or absence of the coupling term $\nu$ essentially depends on whether that coupling is strong ($\nu \gg \Delta$) or weak ($\nu \ll \Delta$) with respect to

$$\Delta = \sqrt{3\Omega^2 + \Gamma^2}/4.$$ 

(12)

The conclusion for the experimenter is this: to achieve maximum signal, strive at strong coupling in the above sense, and set the bias to $\delta = -\nu$. Taking again the Hayashi et al. experiment\textsuperscript{28} for reference, in all practical cases we are in the strong-coupling limit; we note that the actual Coulomb coupling strongly depends on the geometry and can be modulated by shifting the island.

Detector performance depends on noise as well; in particular, on shot noise, caused by the discreteness of electron transport under the joint control of tunneling and Coulomb blockade.\textsuperscript{22} Time-dependent noise analysis, with optimized filtering of short and noisy time-gated pulses responding to time-dependent charging of the trap, is a difficult issue of signal processing. For the present paper, we neglect all those complications implicit in Eqs. \ref{eq:7}, and restrict ourselves to the case of constant charge on the trap. Then it is straightforward to calculate the frequency-dependent steady-state noise spectrum of the DQD detector, using the now standard\textsuperscript{28,29} technique based on MacDonald’s formula\textsuperscript{19,20}

$$S(\omega) = 2\omega \int_0^\infty d\tau \left( \frac{d}{d\tau} Q^2(\tau) \right) \sin(\omega \tau).$$ 

(13)

Here $Q(\tau)$ is the total charge carried until time $\tau$, determined through the time integral of Eq. \ref{eq:14}, which gives

$$\frac{d}{dt} Q^2(t) = \frac{e^2}{4} \sum_{t=0}^\infty N^2 \dot{p}_N(t) - 2 I_{stac}^2 t$$

(14)

in which the long-time linear time dependence of the first term on the right-hand side is canceled by the last term. Then the desired noise spectrum is evaluated in the form

$$S(\omega) = \omega \frac{e^2}{4i} [M(-i\omega) - M(i\omega)],$$

(15)

with

$$M(z) = \int_0^\infty dt \ e^{-zt} \sum_{t=0}^\infty N^2 \dot{p}_N(t),$$

(16)

to which, as expected, the stationary-current term of Eq. \ref{eq:14} gives no contribution\textsuperscript{20}. Accordingly, we proceed through the Laplace transform solution of Eqs. \ref{eq:16}, using the initial condition\textsuperscript{24}

$$\rho_{ij}^{[N]}(0) = \rho_{ij}^{stac} \delta_{N0}.$$ 

(17)

Having taken the Laplace transform, Eqs. \ref{eq:7} furnish an iteration in $N$ from 0 to $\infty$, presenting the Laplace

![FIG. 2: Current signal of the detector as function of time, dimensionless scales being set by interdot tunneling matrix element $\Omega$ and electron charge $e$. Parameters $\Gamma/\Omega = 2/3, \Gamma_T/\Omega = 1$ are tunable by plunger voltages. For coupling of relative strength $\nu/\Omega = 10/3$ [moderately strong coupling; see Eq. \ref{eq:12}] which is fixed by construction, the DQD is biased to $\delta = -11/3$ to achieve high charge sensitivity. Charge is pumped onto the island at $\Omega t = 20$ and pumped out at $\Omega t = 32$; the detector is shut down at $\Omega t = 50$ by shifting bias $\delta$ to -8.](image-url)
transform of Eq. \(\text{Eq. (14)}\) as a matrix series, which can be summed up in a closed form to give

\[
M(z) = z^\nu \cdot \frac{\mathbf{K} + \mathbf{K}^2}{(\mathbf{I} - \mathbf{K})} \left( \mathbf{zI} - \mathbf{A} \right)^{-1} \mathbf{z}_0(0)
\]

with \(\mathbf{K} = (\mathbf{zI} - \mathbf{A})^{-1} \mathbf{B}\), the parameter \(\delta\) in matrix \(\mathbf{A}\) [see Eq. \(\text{Eq. (9)}\)] being replaced by \(\tilde{\delta}\) [Eq. \(\text{Eq. (11)}\)] to cover both cases of empty and filled traps. The matrix inverses can be evaluated analytically, to finally give the scaled formula

\[
S(\omega|\Gamma, \tilde{\delta}, \Omega) = e^{\Delta} \Omega \cdot s \left( \frac{\omega}{\Omega} | \frac{\Gamma}{\Omega}, \frac{\tilde{\delta}}{\Omega} \right)
\]

where \(s(x|y,z) = u(x,y,z)/v(x,y,z)\), with

\[
u(x,y,z) = 4y \left( 16x^8 + 8x^6 \left( 7y^2 - 4 \left( 4 + z^2 \right) \right) \right.
\]
\[+ x^4 \left( 57y^4 + 16 \left( 4 + z^2 \right)^2 - 8y^2 \left( 46 + 11z^2 \right) \right) \]
\[+ 2y^6 \left( 4 + 8y^2 \left( -1 + z^2 \right) + 16 \left( 5 + 6z^2 + z^4 \right) \right) \]
\[+ x^2 y^2 \left( 19y^4 - 4y^2 \left( 37 + 10z^2 \right) \right) \]
\[+ 16 \left( 44 + 23z^2 + 3z^4 \right) \right) ,
\]

and

\[
u(x,y,z) = 16x^8 + y^4 \left( \left( y^2 + 4 \left( 3 + z^2 \right) \right)^2 \right) + 8x^6 \left( 5y^2 \right. \]
\[\left. - 4 \left( 4 + z^2 \right) \right) + x^4 \left( 33y^4 + 16 \left( 4 + z^2 \right)^2 - 8y^2 \right) \]
\[\left( 32 + 7z^2 \right) + 2x^2 y^2 \left( 5y^4 + y^2 \left( 20 - 8z^2 \right) \right) \]
\[+ 16 \left( 20 + 9z^2 + z^4 \right) \right) .
\]

The results are displayed in Fig. 3. The most conspicuous feature is a dip appearing at frequencies \(\omega\) slightly below \(2\Omega\), the frequency of undamped Rabi oscillations. The dip is visible under near-resonance conditions \(\delta \approx 0\), corresponding to peak signal. We think the qualitative reason is this. Shot noise is generated when an electron enters or leaves the DQD at an external contact, enforcing partition of the current. One oscillation period after entering from the left, the electron is just back to the left dot, so it cannot leave right. That kind of history is quantified by the solution of the system of master equations (2).

In conclusion, our “minimal model” offers some insight into the potentialities of charge detection using a double quantum dot. That refers particularly to the noise characteristics: \(1/f\) noise being excluded by radio-frequency operation, intensive shot noise is expected to appear above the Rabi frequency \(2\Omega\), and if not cut by the \(RC\) time in the external electronics, it can be efficiently reduced by low-pass filtering. That sets the intrinsic speed of detection to the time scale \(\Omega^{-1}\), which in turn depends on geometry and gating voltages; according to Ref. 7, the DQD detector itself can be good down to the nanosecond scale. The practical limitation in reaching that performance is typically the external circuit.

It is worth mentioning that such arrangements might serve as tools for studying fundamental issues related to the dynamics of the quantum measurement process.22-24 Indeed, measuring two-detector correlations between detectors differently gated in time may give a new chance to experimentally approach the “collapse of the wave vector,” which has so far notoriously resisted being resolved as a physical process with its own dynamics. Although a disadvantage in the qubit read-out issue, in this context it may turn into an advantage that the operation of fast single-electron detectors is still slower than that of their single-photon counterparts, therefore there is more real chance to achieve the necessary time resolution for the purpose. Deterministic single-electron sources, based on various pump or turn-stile constructions have existed for some time, offering the possibility to time-lock detectors to them. What one may happen to see is nonstandard precursor fluctuations in the detector current, as the random choice inherent in the quantum measurement process develops in time.

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If the trap is part of a coherent charge qubit, through the interaction it gets entangled with the charge detector; our theory can be used to analyze that situation too [J. Zs. Bernád (unpublished)]. Here, as usual, the measured object and DQD, as a composite system, are described by standard linear quantum mechanics, implicitly assuming that “wave function reduction” would take place only in the external circuit measuring the current. For a discussion of that point, see T. Geszti, Phys. Rev. A **69**, 032110 (2004).