Quark Model Calculations of Symmetry
Breaking in Parton Distributions

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ABSTRACT

Using a quark model, we calculate symmetry breaking effects in the valence quark distributions of the nucleon. In particular, we examine the breaking of the quark model SU(4) symmetry by color magnetic effects, and find that color magnetism provides an explanation for deviation of the ratio $d_V(x)/u_V(x)$ from $1/2$. Additionally, we calculate the effect of charge symmetry breaking in the valence quark distributions of the proton and neutron and find, in contrast to other authors, that the effect is too small to be seen experimentally.

1. Introduction

Symmetries have traditionally played a central role in our understanding of hadrons. When the symmetry is unbroken, we use it to make predictions without reference to any model for the underlying wavefunctions. Better still, when the symmetry is broken we can often use it as a filter with which to study the wavefunction itself, and thus are provided with a sensitive probe of the underlying dynamics.

In this talk, we shall operate in the second of these modes by examining the effect of symmetry breaking on the valence quark distributions of the nucleon. To begin, we give a brief description of the rationale and method used to relate the phenomenological wavefunctions of a quark model to the parton distributions measured in high energy scattering experiments. The following section describes the application of our method to breaking of the quark model SU(4) spin-isospin symmetry by color magnetic interaction, and how this symmetry breaking manifests itself in the well known difference between the $u$ and $d$ valence quark distributions in the nucleon. Finally, we look at the case of charge symmetry breaking by quark mass differences and by electromagnetic effects. This effect has not yet been looked for experimentally, but
may play a small role in the determination of $\sin^2 \theta_W$ in $\nu$-nucleon scattering. Although it has been suggested by some authors that (relatively) large effects are to be expected, we do not find this to be the case.

2. Quark Model Valence Distributions

We begin with a statement of the rationale that allows us to use quark models in the study of parton distributions. Clearly, any such attempt cannot consist of a simple evaluation of the relevant matrix elements in terms of quark model wavefunctions, since the only degrees of freedom in those models are the valence quarks and (sometimes) a phenomenological representation of the confining interaction. This picture clashes miserably with the diverse parton distributions required by high energy experiments, which receive large contributions from both gluons and sea quarks. How can these two very different pictures of a hadron be reconciled?

A possible answer lies in the renormalization group approach of Jaffe and Ross. They argue that at large momentum scales, a hadron is, as the data indicates, a very complicated object. But as the renormalization scale is decreased, most or all of the glue/sea found in the hadron is reabsorbed into the valence quarks until, at very low momentum scales, the picture changes to one in which only a relatively few degrees of freedom are required to describe the hadron. It is this simplified picture which one may reasonably hope to represent by a quark model.

In the calculations described here, we shall adopt this prescription, and proceed by evaluating the twist two contribution to the quark distributions using quark model wavefunctions. The resulting distributions will then be interpreted as the twist two contribution to $q_V(x)$ evaluated at a very low renormalization scale $\mu^2_{\text{bag}}$, and next to leading order QCD perturbation theory will be used to evolve the distributions to high $Q^2$, where they can be compared to experiment.

The matrix elements that determine the shape of the valence quark distributions are given by

\[
q(x) = \frac{1}{4\pi} \int d\xi^- e^{iq^+\xi^-} < N|\bar{\psi}(\xi^-)\gamma^+\psi(0)|N > |_{LC}
\]

\[
\bar{q}(x) = -\frac{1}{4\pi} \int d\xi^- e^{iq^+\xi^-} < N|\bar{\psi}(0)\gamma^+\psi(\xi^-)|N > |_{LC}
\]

where $N$ denotes the nucleon wavefunction, $q^+ = -Mx/\sqrt{(2)}$ with $x$ the Bjorken scaling variable, and $LC$ denotes the light cone condition on $\xi$, $\xi^+ = \xi_+ = 0$. The procedure we use is a relatively straightforward evaluation of the matrix elements in a Peierls-Yoccoz projected momentum eigenstate, the details of which may be found in Refs. 8-10. Alternatives to this procedure may be found in Ref. 11.

In Fig. 1, the SU(4) symmetric contribution the bag scale valence quark distributions in the nucleon are shown for the MIT Bag, the Los Alamos Potential Model.
Fig. 1. SU(4) Symmetric Valence Quark Distributions at the bag scale.

and the soliton bag. The most striking difference between these models is the area under the graphs, which gives the momentum fraction carried by a single valence quark. For the MIT Bag the area under the curve is nearly 1/3, indicative of the non-existence in the model of any degree of freedom to represent the confining forces, while for the soliton bag, where the confining degree of freedom is fully included in the calculation of the valence wavefunctions, each valence quark carries 1/4 of the momentum. The LAMP model, in which confinement is implemented via a linear scalar potential, yields a result that lies in between the others. As a practical matter, it is advantageous to work with a model in which a significant fraction of the hadron’s momentum is carried by non-valence degrees of freedom so that the perturbative evolution from the bag scale to the scale at which data are taken is more trustworthy. As a compromise, we shall carry out the symmetry breaking calculations to come in the LAMP model, where the sharing of momentum is complemented by the relative ease of calculation of wavefunctions.

3. Spin-Isospin Symmetry Breaking

We begin the discussion of symmetry breaking effects by considering the SU(4) spin-isospin symmetry that appears in nearly all quark models as a result of the spin independence of the confining degrees of freedom and the near degeneracy of the u and d quark masses. In the case of perfect SU(4) symmetry, the potential cannot distinguish between quarks of different spin or flavor, and all the valence quark wavefunctions and energies are the same. Hence, when we calculate the valence quark distribution of the proton in models with only a spin-independent interaction, the only difference between the valence distributions $u_V(x)$ and $d_V(x)$ is the relative number of each type of quark, and we obtain

$$u_V(x) = 2d_V(x)$$

for all values of $x$. Neither this, nor the corresponding prediction that the nucleon and Δ masses are degenerate is manifest experimentally. In particular, the nucleon-
$\Delta$ mass splitting can be understood in terms of SU(4) symmetry breaking by either virtual pion emission or by color magnetic interactions. In the present context, we shall ignore the contribution due to pions, which are more likely to contribute to sea quark distributions, and concentrate instead on the role of color magnetism.

Omitting the details, which may be found in Ref. 9, the central idea is that the color magnetic interaction introduces a dependence of the quark wavefunctions on the spin state of the other quarks in the nucleon. Since the naive SU(4) symmetric wavefunction of the nucleon contains correlations between the spin and flavor of quarks required by the Pauli principle, the spin dependence of the quark wavefunctions is transmuted into a flavor dependence of the spin averaged wavefunctions, which in turn results in a flavor dependence of quark momentum distributions. Neglecting color electric effects, the corrections to the SU(4) quark distributions can be written in perturbation theory as

$$\delta q^\alpha(x) = \sum_{\alpha \neq \beta} \sigma^\alpha \cdot \sigma^\beta T^{2b}(x) + \sum_{\alpha \neq \beta \neq \epsilon} \sigma^\epsilon \cdot \sigma^\beta T^{3b}(x),$$  \hspace{1cm} (3)

where $\alpha$ denotes the struck quark, $\beta$ and $\epsilon$ the spectator valence quarks, and $T^{2b}(x)$ and $T^{3b}(x)$ are functions of the quark wavefunctions corresponding to processes in which the gluon is exchanged between the struck quark and a spectator, and between the spectators, respectively. As advertised, the average over the quark spins is flavor dependent, so that

$$\delta u(x) = -2T^{2b}(x) - 4T^{3b}(x)$$

$$\delta d(x) = -4T^{2b}(x) + T^{3b}(x).$$  \hspace{1cm} (4)

Physically, $T^{2b}(x)$ is an interaction between the two spectators, so it cannot alter the shape of the struck quark’s momentum distribution. As a result of this, the $\delta d(x)$ quark is roughly four times larger than $\frac{\delta u(x)}{u(x)}$. Since the net effect of the gluonic interaction is to allow the valence quarks to lose momentum to gluons, the d quark distribution is suppressed relative to the u quark and both distributions carry less momentum than they would in the SU(4) symmetric limit.

Fixing the value of $\alpha_s$ to that required to reproduce the nucleon-$\Delta$ mass splitting, we obtain the ratio $d_V(x)/u_V(x)$ at the bag scale shown in Fig. 2. Also shown is the result for the ratio at 15 GeV$^2$, assuming $\mu^2_{BAG} = 0.4$ GeV$^2$, and $\nu$ data taken at the same scale. While the qualitative trend (enhancement of $d_V(x)$ relative to $u_V(x)$ at low $x$, rapidly decreasing with $x$), is unmistakeable, the quantitative agreement with data is less satisfactory. This situation is expected to improve in models in which the SU(4) symmetric valence distributions carry less momentum. In light of this, we conclude that color magnetic interactions provide a natural mechanism for producing
4. Charge Symmetry Breaking

Of more recent interest is the as yet unobserved breaking of charge symmetry between the $u$ and $d$ quark distributions in the nucleon. The symmetry, which is broken by $u - d$ quark mass difference and by electromagnetic effects, is expected to be good to a few per cent. A measure of the size of charge symmetry breaking (CSB) effects on valence quark distributions is given by the ratios

$$
R_{\text{min}}(x) = \frac{2d_{pV}(x) - u_{nV}(x)}{d_{pV}(x) + u_{nV}(x)}
$$

$$
R_{\text{maj}}(x) = \frac{2u_{pV}(x) - d_{nV}(x)}{u_{pV}(x) + d_{nV}(x)},
$$

where $d_{pV}(x)$ is the minority valence quark distribution in the proton, $d_{nV}(x)$ the majority valence quark distribution in the neutron, and so on.

We have calculated these ratios in the LAMP model, incorporating a $u - d$ mass difference of 4 MeV and Coulomb effects in both the quark energy eigenvalues and wavefunctions. In Fig. 3, we plot the result for $R_{\text{min}}(x)$, after evolution from a bag scale of 0.4 GeV$^2$ to 10 GeV$^2$. Also shown is the result of incorporating charge symmetry breaking effects into the Altarelli-Parisi equation. For $x < 0.7$, we find that no individual contribution to CSB is greater than about 2%, and that for the minority quarks, the total CSB effect is less than 4%. For the majority quark distribution, the total CSB effect is less than 2%, due to partial cancellation between the effect of the neutron-proton mass difference and the change in the quark eigenvalues.

These results agree with the qualitative results obtained in Ref. 3, where a model independent analysis of some of the corrections discussed here was performed. They contrast markedly, however, with the results of Ref. 4, where CSB effects on the order 10-15% were found for $x$ near 0.7. This huge result comes about as a result of extreme sensitivity of the quark distribution to the diquark mass parameter introduced in the
Fig. 3. Minority quark ratio, $R_{\text{min}}(x)$, for the LAMP at 10 GeV$^2$.

Adelaide group’s method for extracting valence distributions from quark models. The question of which of these procedures is correct cannot be resolved at the level of model building, but may be resolved experimentally in Drell-Yan experiments. The result of these experiments will undoubtedly give us new insight into the internal dynamics of the strong interaction.

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