Student Errors in Mathematics Word Problem: What Can Be Learned?

Adri Nofrianto¹*, Albert D.C. Gulo², Mira Amelia Amri¹, Elfa Rafulta¹

¹ STKIP YDB Lubuk Alung, Padang Pariaman, West Sumatera, 25581, Indonesia
² SMP N 4 Gunung Sitoli, Medan, North Sumatera 22815, Indonesia

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Abstract
This study aims to understand better student errors in solving mathematics word problems. A Word problem is a problem that has a story or arrangement based on sentences. The qualitative research approach was used. The data were collected by giving tests and interviewing two male high achievers senior high school students in mathematics. The student error analysis adopted the Newman error analysis system. The source triangulation was used to ensure the data validity. Based on the collecting data and data analysis, the decoding/reading error happened caused by students' common understanding or unfamiliarity of the mathematical terms used in the problem. The subjects showed inconsistency in interpreting problem sentences and misused mathematical symbols. Furthermore, the subject encountered difficulties with the arithmetic process, especially fractions and their operation. This research identified students' difficulties in solving problem mathematics word problems. Moreover, this research provides evidence that encourages further research on mathematical literacy, mathematical communication skills, students' arithmetic skills, and the evaluation process of the problem-solving activity.

Keywords: mathematics; word problem; student errors; problem-solving

I. Introduction
Indonesia Students’ performance in international assessments such as TIMSS and PISA is considered underrated. According to the result of PISA, Indonesia is placed in the ten lowest performance (Argina et al., 2017; OECD, 2018; PISA, 2016). TIMSS and PISA results become one of the reasons for curriculum transformation. Indonesia has shifted the primary purpose of education from outcome-based evaluation to process-based evaluation and set new goals for the education system. The main goal of education in Indonesia is to make Indonesia people as a human that can adapt and survive in any life challenges (Keputusan Menteri Pendidikan Dan Kebudayanan No 24 Tahun 2016 Tentang Entang Kompetensi Inti Dan Kompetensi Dasar Pelajaran Pada Kurikulum 2013 Pada Pendidikan Dasar Dan Pendidikan Menengah, 2016). Students should have the skills and abilities to integrate their knowledge into solving real problems. Therefore, in corresponding to this requirement, problem-solving learning has become a method suggested to be used in school.

The problem was presented in pictures, videos, and word problems. Word problems have become popular recently with the concept of mathematics literacy. Word problems can familiarize students with real-life problems (Udil et al., 2021). It can be concluded that using world
problems will develop students' skills and abilities in solving real problems. Moreover, Word problem is believed to develop students' logical, critical, and creative thinking (Nofrianto et al., 2016). It can also enhance students' ability to model, analyze, and evaluate their works.

Word problems have been used in many areas of research, such as developing student skills in problem-solving skills and developing student higher-order thinking skills. Research conducted by Handayani (2017) focused on the factor that contributed to students' abilities to solve mathematics word problems and showed that there are two main factors: students' experience and thinking abilities. This research has not discussed how these factors contributed to students' abilities to solve mathematics word problems.

Mutamima & Manoy (2019) tried to gain a deeper understanding of student's ability to solve a mathematics word problem and its correspondence with mathematics communication skills and personalities. The research found that students' personalities and mathematics communication skills were strongly related to students' performance in solving mathematics word problems. Zahrah (2016) attempted to help students enhance their problem-solving skills, especially in mathematics word problems, by using contextual problems. This research has contributed to the understanding and improving students' problem-solving skills. However, it failed to examine the core problem of low student performance in solving mathematics word problems.

More research has been conducted on understanding students' difficulties and errors in solving mathematics word problems. Magfirah et al. (2019) researched finding student errors using Newman's analysis of junior high school students. The problems were mathematics word problems on a solid geometry topic. The same research topic was also conducted by Nurjanatin et al. (2017) with different research subjects. Udil et al. (2021) conducted research and used Newman error analysis to gain information on elementary students in solving mathematics word problems related to numbers operation. Rofi’ah et al. (2019) used Polya's problem-solving stages to analyze student error in solving mathematics word problems. This research focuses on linear equations with triple variables. The research subject was high achiever male senior high school students.

Amalia et al. (2018) found several factors related to students' difficulties in solving mathematics word problems, such as reading ability or literacy, accuracy, haste, and problem-solving procedures. Rahmawati & Permata (2018) utilized the Newman error analysis system in observing students' errors in solving a mathematics word problem. They found that the most errors of the students are comprehension errors, which is 81.6%, process skills errors 56.7%, transformation errors 30.0%, and reading errors 23.3%. This research showed inconsistencies in students' errors in solving mathematics word problems. One showed that most errors happen in understanding the terms, words, and sentences used on the problem. Meanwhile, others found that most students conducted an error on comprehension, which is the next step in Newman error analysis. Therefore, further research is needed to understand student error more. The most suitable way to progress in understanding an error occurs is by utilizing a qualitative approach.

This exciting fact showed that we still have a long way to go in finding out the real problems encountered by the student when solving mathematics word problems. Furthermore, Utami & Zukarnaen (2019) provide different results regarding student errors in solving mathematics word problems. They found that the highest error was in process skills errors, followed by the encoding of transformation errors—the process error caused by students' low performance in algorithmic skills. Meanwhile, encoding errors are caused by students' unfamiliarity with conducting the evaluation process in problem-solving activities. This result showed students' low performance in solving
mathematics word problems due to students' behaviors and habits in problem-solving activities.

Most research on student errors when solving mathematics problems focus on general indicators such as reading, comprehending, transforming, processing, and encoding. Moreover, previous research focused on quantitative data. It only provides general information about students' errors. Therefore, this research utilizes qualitative data and sub-indicators of student errors. It provides more details and rigorous data on students' errors in solving mathematics word problems. It brings new insight to help students solve their weaknesses.

Based on preliminary research that was conducted in analyzing students' errors in solving a mathematics word problem, we found that most students encountered difficulties, including high achieving students. Students were given a set of word problems consisting of 5 questions on the linear equation and in-equation with three variables. Most students cannot answer the given problems; even high achieving students can only answer 2 out of the problems. This is the main reason for conducting a more thorough analysis to find students' errors in solving a mathematics word problem.

II. Research Method

The research used a descriptive qualitative approach which described all the qualitative data that was gathered and the data discussed. The subjects are two male high achievers in mathematics at the senior high school level. The subject was selected by using purposive sampling. The subject was selected from 81 students of class X senior high school students. The students were categorized based on their mathematical performance. The students were given the mathematics word problem and time to solve it. Two male high achieving students were selected to be interviewed.

The data was collected by using a problem-solving test and interview. The problem-solving test was given at first and followed by the interview. The data was validated by using source triangulation. The triangulation compared test results and interviews and also compared the data from subject 1 and subject 2.

The problem that was used related to three variable linear equation. The proposed problem was described as follows:

Problem:

*There are three numbers. The mean of the numbers is 24. The second number multiplied by 2 will equal the sum of other numbers. The third number equals the sum of other numbers subtracted by 18. Determine each number!*

The student's error was analyzed by adopting Newman's error analysis. The errors that were analyzed were reading errors, comprehending errors, transforming errors, processing errors, and encoding errors (Centre, 2020). The indicators are described in the following table.

Table 1. Newman's error analysis indicators

| No | Types of Error | Error’s indicators                                      |
|----|----------------|--------------------------------------------------------|
| 1  | Reading        | a. No engagement with the task                         |
|    |                | b. Obvious misreading                                  |
|    |                | c. Unfamiliarity with the technical term                |
| 2  | Comprehending  | a. Response showing only Superficial engagement with the task |
|    |                | b. Responses consist of a different question from the one being asked |
| 3  | Transforming   | The numbers that are used are correct, but the wrong operation |
| 4  | Processing     | a. Arithmetic errors                                   |
|    |                | b. Procedural errors                                   |
|    |                | c. Incomplete solution                                 |
| 5  | Encoding       | a. Incomplete solution                                 |
|    |                | b. Responses required some mathematical skill but did not answer the question asked |

Source: PAT Teaching Resource Centre.
III. Result and Discussion

Reading/Decoding Errors

Based on the student's results in answering the problem and the interview, student errors are in understanding the mathematics terms. The second subject conducted an error in understanding the term "mean" of the three numbers. The subject's interpretation of the word "mean" was equal to the sum of the three given numbers. The subject work can be seen in figure 1 below.

The student's answer showed that one of the errors that are commonly made by the problem solver is the understanding of the mathematics term. The student's interpretation of the word problem indicated that he did not realize that there was a misinterpretation of the word means. The subject claims he did not understand the concept of means; to be exact, he forgot about the term. It showed that problem solver understanding of all mathematical terms used in the given problem becomes one of the contributing factors to problem-solving activities errors.

Reading error is one of the significant common factors in solving mathematical word problems (Phonapichat et al., 2014). This statement is consistent with research conducted by Fuchs et al. (2008). They stated that most of the student's difficulties in solving mathematical word problems were related to reading ability. Furthermore, Helwig et al. (1999) student reading errors in solving mathematical word problems associated with the word count, numbers of verbs, and word familiarity. Word familiarity can be defined as students' understanding of the word or terms used in the problem. This research showed that students' understanding of mathematics affects their understanding of the given problem. This leaded to student error in solving the given problem. Therefore, the teachers must check students' understanding of the mathematical term.

Comprehending Errors

The comprehending error was the student's inconsistency in interpreting the problem. There are considerable differences in what was written with what he thought or spoke while explaining the problem with his own words or sentences. This happened when interpreting the second and the third sentences in the problem. In the second sentence, "The second number multiplied by 2 will equal with the sum of other numbers," the second number multiplied by two was equal to the first and the third.

The transcript of can be seen below:

Interviewer : Can you explain the problem base on your understanding?
Subject 1 : Yes, I can. The problem clearly stated that 2 times the second number equals the sum of two other numbers. It means that if there are two numbers. For example, if x and y are combined, it will equal z.
Subject 2 : The second number multiplied by two was equal to the first and the third.

Based the subject 1 and subject 2, interpretations of the sentences "The second number multiplied by 2 will equal with the sum of other numbers" were different in meaning. Although both interpretations were different, they showed consistency in the misinterpretation of the sentence. Student error in understanding and interpreting the sentence displayed limited communication skills. In other words, students have problems with mathematical communication skills.

The same error also happened in interpreting the third sentence. In the third sentence, "The third number is equal to the sum of
other numbers subtracted by 18 (if the numbers were symbolized as $x, y, z$ respectively as the first, second, and third number, the interpretation of the sentences would be $z = x + y - 18$)’’ was interpreted by the subject as $z - 18 = x + y$. After reading the problems a few times and writing the known problem, he realized that he had misinterpreted the sentences and corrected himself. He claimed that he was not careful, was in a hurry, and was feeling anxious at the moment. Admitting that the error was only made by subject 2, the consistency of the error in interpreting the problem statement became one of the errors found in student activities in solving a word problem.

This error is categorized as a verbal or natural language register error (Bossé, Michael, and Chandler, 2014). These errors included the error in understanding and interpreting the words, sentences, punctuation, and symbols.

Student errors in comprehending the given problem are consistent with the research result of Maulyda et al. (2020). They found that students have difficulties communicating mathematical ideas and errors in converting problem sentences into mathematical models. This indicates that students struggle to understand the problem sentences and convert them into mathematical languages. It also represents problems related to student literacy and mathematical communication skills. Indonesian students’ mathematical communication skills are still low (Nofrianto et al., 2017). This research suggests that more attention is needed to enhance Indonesian students’ mathematical communication skills.

Looking back, Riduan (2010) found that 52.91% of students’ errors in solving mathematical word problems were related to students’ understanding of the problem. Student Understanding of the problem is strongly related to reading and comprehending skills. Moreover, recent research conducted by Arifin et al. (2021) also consistently found that student understanding of mathematical concepts becomes an obstacle to solving a mathematics problem. This research added a deeper understanding that sentences and how words were used in the sentences affect student comprehension of the problem. Moreover, it showed that the subject still does not have adequate mathematics communication skills, especially in transforming words or sentences into mathematics equations.

**Transforming Errors**

The error in transforming was misused of the equation symbols. The subject used the equation symbols to explain that the two equations are identical. The equation symbols were used in transforming the equation into a simple form. The misused of the equation symbols can be seen in figure 2.

![Figure 2. Subject 1 result showed a misused equation symbol to represent equivalency](image)

Based on figure 2, we can see that the students used the identical symbols as equivalence. This error can be categorized algebraic symbolic register error (Bossé et al., 2009). This error is related to students' understanding of the mathematics symbols and operations. Hadi et al., (2018) found that 15.59% of the participants had transformation errors. These numbers can be considered a high number of problem-solving errors. Research subject failure on transforming the mix-fraction form into ordinary form fraction showed that student is still struggling with mathematics subject related to fraction operation. It proves that fraction is still a difficult concept that needs to be mastered by students. Therefore, it is necessary to conduct further research on understanding how fractions become a difficult subject for students even after they are in high school.

**Processing Errors**

Several errors were found in the student’s work solving the given problem. The first error can be categorized as an arithmetic error. The error was in the process of equation
simplification. The detailed student error can be seen in the process of solving the given problem in figure 3.

Figure 3. Subject 1 arithmetics error

Sari & Eric (2016) found that the subject encountered difficulties with the concept of percentage and multiplication. It led to student confusion in deciding the given operation and the number. The concept of percentage is related to fractions and their operation. This result is consistent with our findings. Research conducted by Made (2018); Aminah & Ayu Kurniawati (2018); and Nasiruudin & Hayati (2019) shows consistent results that students encountered difficulties in understanding the concept of operation of a fraction. Zalima et al. (2020) point out that most students have difficulties with fraction operations, especially fractions with different enumerators. This research is consistent with the previous research. It even provides evidence that students in senior high school students encounter difficulties in applying the concept of fraction operation. It also showed that students who could not grasp the concept of fractions when they were on the elementary school level would bring their inability to the senior high school level.

These results are consistency with our findings. The subjects encountered difficulties and conducted an error when transforming the original forms \( \frac{x+y+z}{3} = 24 \), which are in the form of fractions, into simple forms. He failed in conducting the fraction multiplication operation. These facts showed that the subject struggles with fraction concepts and their operations. It can be concluded that Students’ difficulties in understanding the concept of fractions affect their performance in solving related problems.

Despite the error discussed and the student's failure to transform the equation, no noticeable error was found in the student's answer and response. Both subjects can perform the procedural process accurately. They can correctly use the elimination method, substitution, and mix of both methods in solving linear equations.

Even though both students' answers were incorrect mathematically, they were not caused by their errors in the procedural process. It is mainly caused by student reading, comprehending, transforming, and arithmetics errors. The answer that was provided by each of the subjects was completed.

**Encoding Errors**

The encoding error cannot be investigated thoroughly in this research. Both subjects provided the complete solution to the problem. The main reason the respondent could not provide correct answers was that the subjects did not conduct the evaluation process in the problem-solving activities. They did not carry out the evaluation process of solving the problem. They did not confirm the answer again. They just believed that they had already solved the problem correctly.

Considering the indicators for encoding errors are the completeness of the solution, it can be concluded that both participants did not make an error in the encoding processes. This is also consistent with research results that conducted by Hadi et al. (2018). The percentage of encoding error is deficient. It is around 1.34 %. It showed that students seldom made mistakes in the final problem-solving process. However, extra effort is needed to convince students to conduct the evaluation process at the end of the problem-solving activity. This process will develop students' evaluation skills.

**IV. Conclusion**

The result and discussion showed that higher achiever students in mathematics still encounter difficulties in solving a mathematics word problem. They made errors, such as an incorrect conclusion about the problem solution. The subjects had an error in almost all of the used indicators. In reading errors, the errors were strongly related to the interpretations of mathematical terms and the interpretation of the problem statements sentences. This has shown us
that even higher achiever students in mathematics have mathematical literacy and communication problems. Transforming error is mainly related to using mathematical symbols to represent or transform the common language into mathematical languages. The misused of the mathematical symbols showed that our students did not have a strong foundation in understanding basic mathematical symbols. The processing error is strongly related to arithmetic errors. The students were confused about the concept of the equality of fractions. It showed the subject concept of fractions and how to conduct the operation on fractions. The encoding errors cannot be observed thoroughly. We can only conclude that the subjects do not conduct evaluation steps in their problem-solving activities.

Further research is needed in the area of students’ mathematical literacy in solving mathematics word problems with the objective of finding the relationship between the level of students’ mathematical literacy, their ability to solve mathematical problems, and how it works. Moreover, research on mathematical communication skills also needs to be addressed to uncover how students’ communication skills will benefit them in problem-solving, especially in understanding mathematical symbols. Students' arithmetic process related explicitly to fractions is also a worth research area that needs to be focused on, especially to gain more understanding on how to make students understand the fraction operation and on how student's inability to use fraction concept is consistent until they already in the higher level of education. Finally, research on encouraging students to do evaluation processes in problem-solving processes will bring interesting and beneficial results to students' problem-solving skills development.

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