Leslie, Spencer; Lonergan, Gus
Parity sheaves and Smith theory. (English) [Zbl 1478.14033]
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Summary: Let \( p \) be a prime number and let \( X \) be a complex algebraic variety with an action of \( \mathbb{Z}/p\mathbb{Z} \). We develop the theory of parity complexes in a certain 2-periodic localization of the equivariant constructible derived category \( D^b_{\mathbb{Z}/p\mathbb{Z}}(X, \mathbb{Z}_p) \). Under certain assumptions, we use this to define a functor from the category of parity sheaves on \( X \) to the category of parity sheaves on the fixed-point locus \( X^{\mathbb{Z}/p\mathbb{Z}} \). This may be thought of as a categorification of Smith theory. When \( X \) is the affine Grassmannian associated to some complex reductive group, our functor gives a geometric construction of the Frobenius-contraction functor recently defined by M. Gros and M. Kaneda via the geometric Satake equivalence.

MSC:
14F08 Derived categories of sheaves, dg categories, and related constructions in algebraic geometry
13F35 Witt vectors and related rings
14L30 Group actions on varieties or schemes (quotients)
14M15 Grassmannians, Schubert varieties, flag manifolds

Full Text: [DOI](https://doi.org/)
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