Optimization of Heat Transfer Model Based on Greedy Algorithm

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Abstract. Aiming at the problem of furnace temperature curve, this paper uses Fourier’s law of heat conduction and Newton’s law of cooling to establish a heat transfer model to obtain the temperature changing law of the welding area and the furnace temperature curve; uses the single-objective optimization model to establish the maximum conveyor belt passing speed and find out the optimal temperature for each temperature zone; the greedy algorithm and traversal search are used to obtain the undetermined parameters of the dual-objective model through local optimization. In response to question 1, it is required to study the temperature change in the center of the welding area. First, we set the heat transfer model from heat conduction law and convection heat transfer law. The interface of the reflow furnace is two-dimensional: the conveyor belt direction is the x-axis and the vertical direction of the reflow furnace is the y-axis. The energy balance method is used to find the relationship between the x and y-axis heat conduction to establish a differential equation, after then we use Newton Cooling Law and the temperature distribution in the reflow furnace and establish the difference equation of the welding zone temperature. This paper uses the numerical calculation to solve the differential equation sets, obtaining the statistics of the temperatures at intervals of 0.5s. The theoretical furnace temperature curve is drawn as well. The second problem requires the maximum furnace passing speed of the conveyor belt after the temperature of each zone was determined and the constraints of the process limit must be met. A single-objective optimization model was established in this paper and the maximum conveyor belt speed was taken as the objective function. Without exceeding the constraints of the problem, the maximum conveyor belt passing speed was 92.6 cm/min after the computer finished the programme. For problem three, it is required to find out the suitable temperature of each zone and the belt-speed, making the area minimized which is covered by the temperature curve exceeding 217°C and under the peak point. This paper establishes a single-objective optimization model, where the objective function is the minimum coverage area; the constraints are the same as the five constraints above in question 2. We use the programming software to search the feasible solution: the temperature of the preheating zone is 167 °C, the temperature of the constant temperature zone 6 is 185.1°C, the temperature of the constant temperature zone 7 is 225°C, the temperature of the reflow zone is 259.2°C, and the passing speed of the conveyor belt is 91.2cm /min. For question four, it is required to make the furnace temperature curve as symmetrical to the peak value as possible while considering the coverage area. This paper defines the symmetry coefficient to quantify the peak symmetry. On this basis, a dual-objective optimization model is first established, and then the coverage area in the objective is converted into constraint conditions, which is transformed into a single-objective optimization problem, and the unknown parameters are finally determined. The full traversal search needs to determine 5 parameters so the calculation is time-consuming. Therefore, we first use the greedy algorithm.
to solve some of the local optimal parameters, and then use the full traversal search to simplify the calculation. The final result: the temperature of the preheating zone is 169.5° C, the temperature of the constant temperature zone 6 is 185° C, the temperature of the constant temperature zone 7 is 225° C, the temperature of the reflow zone is 258.7° C, the furnace passing speed of the conveyor belt is 90.4 cm/min. Finally, the model established in this paper is discussed and analyzed, the model established is comprehensively evaluated, and the conclusion is made.

**Keywords**: Heat Transfer Model, Single Objective Optimization, Ergodic Search, Greedy Algorithm.

1. Introduction

1.1. Problem Background

In the production of electronic products such as integrated circuit boards, it is necessary to place a printed circuit board with various electronic components in a reflow furnace, and to automatically solder the electronic components to the circuit board by heating. During this production process, keeping all the components of the reflow oven at the temperature required for the process is very important for product quality. At present, many works in this field can be controlled and adjusted through experimental tests. The purpose of this question is to analyze and research through the mechanism model.

There are several small temperature zones in the reflow furnace, which can be divided into 4 large temperature zones functionally: preheating zone, constant temperature zone, reflow zone and cooling zone. Both sides of the circuit board are placed on a conveyor belt and enter the furnace at a constant speed for heating and soldering.

![Reflow furnace](image)

**Figure 1.** Reflow furnace

Nowadays, with regard to the setting of the temperature distribution of the reflow soldering temperature field, experimental testing methods are still used to formulate the required process temperature of each part of the reflow furnace, which has caused a huge waste of cost. Therefore, it is particularly important to establish a related mechanism model to study and analyze the temperature field distribution of the reflow oven. The question has the following structure and related parameter settings:

1. There are 11 small temperature zones in the reflow furnace and the front (rear) area of the furnace. The length of each small temperature zone is 30.5 cm and there is a gap of 5 cm between the
small temperature zones. The length of the front (rear) area of the furnace is all 25cm. The thickness of the welding area is 0.15mm.

(2) The temperature of the production workshop is maintained at 25°C.

(3) On the basis of the experiment temperature setting of this question, the temperature setting of the small temperature zone 1~8 can be adjusted within the range of ±10°C, and the temperature setting in the small temperature zone 1~5, 8~9 is maintained Unanimous. In addition, the temperature in the small temperature zone 10-11 is kept at 25°C.

(4) The furnace passing speed of the conveyor belt can be adjusted within the range of 65~100cm/min.

1.2. Restatement of the Problem

Based on the above background, we need to establish relevant mathematical models to solve the following problems:

Question 1: According to the relevant parameters and working principles of each part of the reflow oven, on the basis of a given conveyor belt passing speed, comprehensively consider various heat transfer methods, establish a mathematical model to analyze and explore the law of temperature change in the welding area. And on the basis of the set value of each temperature zone, find the temperature change in the center of the welding zone over time.

Question 2: On the basis of question 1, change the temperature setting value of each temperature zone, take the restriction conditions in the conveying process as constraints, establish a single-decision variable single-objective optimization model of the maximum conveyor belt passing speed, and realize the best design.

Question 3: During the welding process, the time for the temperature in the center of the welding area to exceed 217°C should not be too long, and the peak temperature should not be too high. The ideal furnace temperature curve should minimize the area covered by the peak temperature above 217°C. Please determine the optimal furnace temperature curve under this requirement, as well as the set temperature of each temperature zone and the furnace passing speed of the conveyor belt, and give the corresponding area.

Question 4: In the welding process, in addition to meeting the process limits, it is also hoped that the furnace temperature curve that exceeds 217°C on both sides of the peak temperature as the center line should be as symmetric as possible. Please combine with Question 3 to further give the optimal furnace temperature curve, the set temperature of each temperature zone and the conveyor belt passing speed, and give the corresponding index value.

2. Problem analysis

Question one: For the first question, the Fourier heat transfer law in thermodynamics and the Newton cooling law are used to calculate the convective heat transfer, and the heat transfer model in the reflow furnace is obtained.

First determine the heat transfer mode in the furnace: heat conduction and convection heat transfer to establish a heat transfer model. The interface of the reflow furnace is two-dimensional, the conveyor belt direction is the x-axis and the vertical direction of the reflow furnace is the y-axis to establish a plane rectangular coordinate system; the energy balance method is used to find the relationship between the x and y-axis heat conduction to establish a differential equation, and then use Newton The cooling law and the temperature distribution of air along the x-axis, establish the difference equation of the welding area temperature, use the numerical method of the difference equation and the differential equation to solve the equations, obtain the data of the temperature time interval of the center of the welding area as 0.5s, and plot it Corresponding furnace temperature curve.

Use the temperature zone data in Annex 1 to test the model and compare and calculate the error.

Question two: The second problem is to fix the specific temperature of each small temperature zone and set the maximum transmission speed as the solution target. Its purpose is to increase the production capacity of the reflow oven by increasing the conveying speed of the conveyor belt in a
clean production environment. When solving this problem, the passing speed of the furnace must be increased under the premise of ensuring product quality. Therefore, the second problem can be classified as a single objective optimization problem. The guarantee of product quality is actually the process limit in the problem, and the optimal target value can be solved by the traversal search algorithm.

Question three: Different from question 1 and question 2, question 3 does not give the specific temperature of each temperature zone. It hopes to get the temperature and conveyor belt passing speed under the ideal furnace temperature curve, and calculate the area covered by the furnace temperature curve from the liquidus line to the peak. This is a multi-decision variable single-objective optimization problem. It should be noted that the focus of this question is the optimization of the furnace temperature curve, that is, when the process limit is met, the time and peak temperature of the curve exceeding 217°C are adjusted to achieve the minimum required area.

Question four: Question 4 is based on the combination of question 3. It is hoped that the furnace temperature curve exceeding 217°C should be centered on the peak temperature and try to maintain symmetry on both sides. So as to get a better furnace temperature control curve. This can add the curve symmetry requirement as an optimization goal to Problem 3, turn the problem into a multi-decision variable dual-objective optimization model, and then convert the dual-objective optimization model into a single-objective optimization model for solution.

3. General Assumption
For these problems we have the following assumptions:

Assumption I: The attached data are all original data, and the source is true and reliable
Assumption II: The heat radiation energy in the reflow furnace is very low and can be ignored
Assumption III: The influence of the gap in the temperature zone of the reflow furnace is negligible
Assumption IV: The influence of the gap in the temperature zone of the reflow furnace is negligible
Assumption V: The reflow furnace only exchanges energy with the outside world, and there is no material exchange.

4. Symbol Description

| Symbol | Description |
|--------|-------------|
| \(h\)  | Convection heat transfer coefficient |
| \(k\)  | Thermal conductivity |
| \(L\)  | Distance between object and heat source |
| \(\rho\)  | Fluid density |
| \(\mu\)  | Fluid viscosity coefficient |
| \(k'\)  | Positive proportional coefficient of thermal conductivity and temperature |
| \(A\)  | Contact area between fluid and object |
| \(v\)  | Conveyor belt passing speed |
| \(T(x)\)  | The temperature of the small temperature zone at \(x\) |
| \(T_\infty\)  | Gas temperature on channel in reflow furnace |
| \(\delta\)  | Welding thickness |
| \(Q\)  | Thermodynamically conducted energy |
| \(h_\varepsilon\)  | Local convective heat transfer coefficient |
| \(C_\varepsilon\)  | Hysteresis coefficient, the degree of hysteresis of the temperature of the welding piece relative to the temperature change of the welding space |
5. Models and Results

5.1. Question 1

Problem analysis. This question uses the energy balance method to find the relationship between the x-axis and the y-axis heat conduction to establish a differential equation, and then uses Newton’s law of cooling and the temperature distribution of the air along the x-axis to establish the differential equation for the temperature of the welding area. The method of numerical solution of two equations of differential equation and difference equation is used to obtain the furnace temperature curve.

5.1.1. Question 1 model preparation. Consider the establishment of a thermodynamic model in this question. There are three main forms of heat transfer: heat conduction, convection heat transfer and radiation.

a. Heat Conduction

Heat conduction relies on the thermal movement of microscopic particles inside an object, thereby generating heat transfer; when there is a temperature gradient in a static medium, whether it is a solid or a fluid, the medium will transfer heat and the object with high temperature always transfers heat to the temperature. The rate equation of heat conduction, also called Fourier's law, the expression:

\[ q'' = -k \frac{dT}{dx} \]  

In the formula, \( q'' \) is the heat flux density; the negative sign is because the direction of the heat flux is opposite to the direction of the temperature gradient, that is, heat is always transferred from a place with a higher temperature to a place with a lower temperature.

b. Convective heat transfer

Convection is the difference in temperature between fluids, and the resulting macroscopic movement causes heat to be transferred. The convective heat transfer mode is maintained by the random movement of molecules and the overall movement of the fluid in the boundary layer. In the vicinity of the surface, the velocity of the fluid is low, and the random movement (diffusion) of molecules and molecules play a major role.

c. Radiant heat transfer

When the temperature of an object is higher than absolute zero, it will emit energy to the outside world in the form of electromagnetic waves. Regardless of the state of the substance, this type of emission is caused by changes in the arrangement of electrons in the atoms or molecules that make up the substance. The energy of the radiation field is transmitted by electromagnetic waves. Relying on heat conduction or convection to transfer energy requires a material medium, but radiation does not. In the process of radiation transmission, the radiation emitted from the surface of an object originates from the internal energy of the internal matter bounded by the surface. Given by Stephen Boltzmann’s law:

\[ E = \sigma T_w^4 \]

In the formula, \( T_w \) is the surface temperature of the object, \( \sigma \) is the Steven-Boltzmann constant, \( \sigma = 5.67 \times 10^{-8} W/(m^2 \cdot K^4) \)

By searching the literature, we know that the reflow oven in this article is a reflow oven, and the only heating method is to rely on heat conduction and convection heat transfer; considering the relationship between radiant energy and temperature, the temperature of the reflow oven in this question is not high, so the radiation energy is ignored.
5.1.2. Model establishment of problem 1. First of all, the interface of the reflow furnace is two-dimensional, the conveyor belt direction is the x-axis and the vertical direction of the reflow furnace is the y-axis to establish a rectangular coordinate system; the energy balance method is used to find the relationship between the x and y-axis heat conduction to establish a differential equation, and then Using Newton's law of cooling and the temperature distribution of air along the x-axis, the differential equation of the welding area temperature is established, and the equations are solved by the numerical method of the differential equation and the differential equation.

Draw the flowchart

Figure 2. Heat conduction model flow chart

(1) Heat conduction equation (Fourier's law)
In the temperature field:

$$\vec{q} = -k \nabla T = -k \left( \frac{\partial T}{\partial x} \hat{i} + \frac{\partial T}{\partial y} \hat{j} + \frac{\partial T}{\partial z} \hat{k} \right)$$  (3)
For the two-dimensional steady-state condition where there is no internal heat source and the physical properties are constant, the heat conduction equation is:

$$\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} = 0$$  \hspace{1cm} (4)

If convection occurs, although the difference equation is still satisfied, $q$ is first-order undifferentiable and discontinuous.

(2) The x-axis and y-axis are respectively approximated to construct the heat conduction equation. The direction of the conveyor belt is the x-axis, the vertical direction of the reflow furnace is the y-axis, and the fixed point in the front area of the furnace is the starting point to establish a plane rectangular coordinate system.

The plane rectangular coordinate system is shown in the figure:

![Figure 3. Build a department in the reflow furnace](image)

Suppose the temperature of a point on the x-axis is $T(x_i)$. The corresponding point $x_i$ on the x-axis with the small temperature zone corresponds to the temperature $T'(x_i)$.

Usually have

$$T'(x_i) \neq T(x_i)$$  \hspace{1cm} (5)

That is, the temperature on the conveyor belt is inconsistent with the temperature in the small temperature zone.

(3) Determination of local thermal conductivity and average thermal conductivity

From the above convection heat transfer, the principle of heat conduction can be obtained:

$$q = \begin{cases} 
  h(T_{\infty} - T_s) & \text{Convection} \\
  k \frac{\Delta T}{L} & \text{Conduction} 
\end{cases}$$  \hspace{1cm} (6)
Among them, \( L \) is the distance between the object and the heat source, \( k \) is the thermal conductivity, and \( h \) is the convective heat transfer coefficient.

It can always be assumed that the thermal conductivity \( k \) is a constant under solid conditions, but the gas is different. Looking up the data, the influence of temperature, pressure and chemical composition on gas thermal conductivity can be explained by gas molecular motion theory. According to this theory, the thermal conductivity \( k \) is proportional to the gas density \( \rho \), the average molecular velocity \( \bar{c} \) and the mean free path \( \lambda_{mf} \), namely:

\[
k = \frac{1}{3} c_e \rho \bar{c} \lambda_{mf} \tag{7}
\]

However, since \( \rho \) and \( \lambda_{mf} \) are respectively proportional and inversely proportional to the gas pressure, the thermal conductivity of the gas has nothing to do with the pressure except in extreme cases. It can be seen from this: \( k \propto T \). Let

\[
k = k' T \tag{8}
\]

In the formula, \( k' \) is the positive proportionality coefficient.

Assuming that the temperature changes approximately linearly in the \( y \) direction, there is

\[
k = \int_s^b kdx \left( \frac{T_a + T_b}{2} \right) \cdot k' \tag{9}
\]

(4) Determine the convection coefficient \( h \)

In any convection problem, the local and average convection coefficients are extremely important. Because these coefficients not only depend on many properties, such as density, viscosity, thermal conductivity, specific heat capacity and other fluid properties, but also related to the surface geometry and flow state. The diversity of this independent variable is due to the convection transfer and the surface formation. The boundary layer is related.

![Figure 4. Thermal boundary layer on isothermal plates](image)

There is no fluid movement on the surface, and the transfer of energy can only be done through conduction:
Substituting Newton’s law of cooling:

\[ q_s = -k_f \frac{\partial T}{\partial y}\bigg|_{y=0} \]  \hspace{1cm} (9)

Substituting Newton’s law of cooling:

\[ h = \frac{-k_i \partial T/\partial y\big|_{y=0}}{T_s - T_\infty} \]  \hspace{1cm} (10)

Use boundary layer similarity to introduce dimensionless independent variables

\[
\begin{align*}
    x^* &= \frac{x}{L},
    y^* &= \frac{y}{L},
    u^* &= \frac{u}{v'},
    v^* &= \frac{v}{v'},
    T^* &= \frac{T - T_s}{T_\infty - T_s}
\end{align*}
\]  \hspace{1cm} (11)

Boundary layer conservation equation

\[ u^* \frac{\partial T^*}{\partial x^*} + v^* \frac{\partial T^*}{\partial y^*} = \frac{1}{Re_L Pr} \times \frac{\partial^2 T^*}{\partial y^*} \]  \hspace{1cm} (12)

Although the state \( h \) in the thermal boundary layer depends on the physical properties of the fluid \((k, e_p, \mu, \rho)\), the fluid velocity \( v' \), the length scale \( L \), and the surface geometry, this dependence can be simplified by combining these variables into the Reynolds number. Specifically, and the solution of the above equation is simplified into the following form:

\[ T^* = f\left(x^*, y^*, Re_L, Pr, \frac{dp^*}{dx^*}\right) \]  \hspace{1cm} (13)

Substitute and simplify to get:

\[ h = \frac{k_f L}{\mu} \times \frac{\partial T^*}{\partial y^*}\bigg|_{y^*=0} \]  \hspace{1cm} (14)

The Reynolds coefficient \( Re \) is related to the fluid velocity, density, and viscosity coefficient, and because it is the same gas, the comparison parameters remain unchanged.

\[ Re = \frac{\rho v'd}{\mu} \]  \hspace{1cm} (15)

Where \( v' \) is the fluid velocity; \( \rho \) is the density of the fluid; \( \mu \) is the viscosity coefficient of the fluid; \( d \) is the characteristic length. Assuming that each air outlet of the reflow furnace has the same accuracy, the Reynolds coefficient remains unchanged in each temperature zone.
Experiments show that the local convective heat transfer coefficient $h_x(x)$ satisfies:

$$h_x(x) = ax^{-0.1} \tag{16}$$

Among them, $a$ is the coefficient $[W/(m^{1.9} \cdot k)]$, and $x(m)$ is the distance calculated from the front edge of the plate.

$$h = \int_{A_s} h \, dA_s \tag{17}$$

$$\bar{h}_x = \bar{h}_x(x) = \frac{1}{x} \int_0^x h_x(x) \, dx \tag{18}$$

Substitution:

$$\bar{h}_x = 1.1h_x \tag{19}$$

Similar to the application of thermal boundary layer, each temperature zone $h$ is approximately constant and constant.

Decompose the plane into the $x$-direction and $y$-direction according to the coordinate axis, and still assume that the depth is 1, and all heat conduction is carried out only through the channel.

(5) Heat transfer in the $y$ direction

Without internal heat source, direction $y$:

$$q_y = h_i (T'(x) - T(x)) + k \frac{T'(x) - T(x)}{L} \tag{20}$$

Substitute the air thermal conductivity coefficient $k$:

$$q_y = (T'(x) - T(x)) \cdot \left[ h_i + \frac{k'}{2L} (T'(x) - T(x)) \right] \tag{21}$$

There should be system stability conditions when reaching the steady state:

$$\begin{cases}
\frac{dq_y}{dt} = 0 \\
\frac{dT(y)}{dt} = Const
\end{cases} \tag{22}$$

(6) Heat transfer in the $x$ direction $q_x$

The temperature in the $x$-direction is only thermally conducted, and the continuous derivative is obtained
There should be system stability conditions when reaching the steady state:

\[
\begin{align*}
\frac{dq_x}{dt} &= 0 \\
\frac{dT(x)}{dt} &= \text{Const}
\end{align*}
\]

(24)

Figure 5. Heat conduction from a neighboring node to an internal node

That is, for the node \((m, n)\), the incoming heat is equal to the outgoing heat, plus the heat output of the internal heat source

\[
E_{in} + E_{out} = 0
\]

(25)

\[
\sum_{i=1}^{4} q(i) \rightarrow (m, n) + q \left( \Delta x \cdot \Delta y \cdot 1 \right) = 0
\]

(26)

Among them, \(i\) represents adjacent nodes, \(q(i) \rightarrow (m, n)\) is the heat conduction rate between nodes, and the depth is assumed to be 1 in this formula; \(\Delta y \cdot 1\) is the heat transfer area.

In the two-dimensional condition, there are only four adjacent nodes and no internal heat source. The formula is transformed into:

\[
\sum_{i=1}^{4} q(i) \rightarrow (m, n) = 0
\]

(27)
Conditions: The system is an unopened system, and there is no mass exchange with the outside world, and the anti-energy exchange reaches a stable heat transfer state in thermodynamics. Therefore, when the circuit board is not put in, the channel is regarded as a gas without convection state, only heat conduction.

\[
\begin{align*}
q_y &= h_1(T'(x_i) - T(x)) + k'(x_i) - T(x) \times T'(x_i) - T(x) \times \\
q_x &= kT(x) \frac{dT(x)}{dt} \\
dq_x &= dq_y = 0 \\
\frac{d}{dt} &= \text{Const}
\end{align*}
\]

(28)

Let \( T(x) = y \), then we have

\[
q_y = (T'(x_i) - y) \left[ h_1 + \frac{k'}{2} (T'(x_i) + y) \right]
\]

(29)

Obtain the functional formula of the air temperature change in the reflow furnace:

\[
y' = 2 \frac{k'}{k} (T'(x_i) - y) \cdot \left[ h_1 + \frac{k'}{2} (T'(x_i) + y) \right]
\]

(30)

(8) The relative motion principle gives the temperature difference formula of the welding area

Using the principle of relative displacement, it can be regarded as the heat input that the circuit board is blown by the hot wind with speed \( \nu \):

\[
q = h(T_s - T_\infty)
\]

(31)

Among them, \( T_\infty \) is the x-axis gas temperature on the channel. Assuming that the welding area is short enough, \( T_\infty \) does not change over a short distance.

Then \( T_\infty = T(x) = T(\nu t) \)

\[
\bar{h} = 1.11h(x) = 1.11ax_0^{-0.1}
\]

(32)

Heat absorption per unit time:

\[
Q = \int_A qdA_s = \bar{h}A_s(T_s(t) - T_\infty(t)) = x_0 \cdot d \cdot 1.11ax_0^{-0.1} \cdot (T_s(t) - T(\nu t))
\]

(33)
Assuming that the solder layer is thin enough, the overall temperature can be equivalent to the surface temperature in a short enough time.

\[ Q = V \cdot C_v \cdot \Delta T \]  \hspace{1cm} (34)

In the formula, the volume \( V = x_0 \cdot d \cdot \delta \); \( x_0 \), \( d \), \( \delta \) are the length, width and thickness of the weldment respectively.

\[ 1.11ax^{-0.1} \left( T_{s(t)} - T_{s(d)} \right) = \delta \cdot C_v \left( T_{s(t)} - T_{s(d)} \right) \]  \hspace{1cm} (35)

Obtain the difference formula of the temperature at the center of the welding zone over time:

\[ T_{s(t)} - T_{s(d)} = C_1 \cdot \left( T_{s(t)} - T_{s(d)} \right) \]  \hspace{1cm} (36)

Among them, \( C_1 = \frac{1.11ax^{-0.1}}{\delta \cdot C_v} \) is called the hysteresis coefficient, which reflects the degree of hysteresis of the temperature of the soldering sheet relative to the temperature change of the welding space.

When \( v \) is different, \( C_1 \) takes a different value, and \( C_1 = \frac{v_0}{v} C_0 \) can be obtained by modifying the shape similarity coefficient (Reynold’s coefficient).

5.1.3. Problem One Model Solving. From (30) the functional formula of the air temperature change in the reflow furnace:

\[ y' = \frac{2}{k'y} \left( T'(x) - y \right) \cdot \left[ h_1 + \frac{k'}{2} \left( T'(x) + y \right) \right] \]

Study the trend of the space temperature curve, use the skills of phase space, analyze the area where the air temperature change rate \( y' > 0 \) or \( y' < 0 \) is on the \( x, y \) plane, and find that \( k' \) and \( h_1 \) are normal numbers, then \( h_1 + \frac{k'}{2} \left( T'(x) + y \right) > 0 \) is always true.

(1) When \( T'(x) - y \left[ h_1 + \frac{k'}{2} \left( T'(x) + y \right) \right] > 0 \), there is only \( T'(x) - y > 0 \), \( y' > 0 \);

(2) When \( T'(x) < y \), and \( y \) is always greater than 0, then \( y' < 0 \).

The reduction in \( (x, x + \Delta x) \) is obtained from the Taylor expansion

\[ T(x + \Delta x) - T(x) = \frac{dT(x)}{dx} \cdot \Delta x + o(\Delta x) \]  \hspace{1cm} (37)

The higher-order terms are ignored.
Use matlab programming software to iterate the equation numerically, with a step length of 0.005m:

\[
\lim_{\Delta x \to 0} T(x + \Delta x) = \lim_{\Delta x \to 0} T(x) + \frac{dT}{dx} \cdot \Delta x
\]

(38)

Use matlab programming software to iterate the equation numerically, with a step length of 0.005m:

Get the data of the temperature change in the center of the welding area with time:

**Table 2. Temperature data at the center of part of the welding area**

| Time (s) | 0 | 0.5 | 1 | 1.5 | 2 | 2.5 | 3 | 3.5 | 4 |
|----------|---|-----|---|-----|---|-----|---|-----|---|
| Temperature (°C) | 25 | 25 | 25.01 | 25.04 | 25.08 | 25.13 | 25.19 | 25.26 | 25.34 |
| Time (s) | 4.5 | 5 | 5.5 | 6 | ... | 25 | 25.5 | 26 | 26.5 |
| Temperature (°C) | 25.43 | 25.53 | 25.63 | 25.75 | ... | 36.16 | 37 | 37.88 | 38.79 |
| Time (s) | 27 | 27.5 | 28 | 28.5 | 29 | 29.5 | 30 | 30.5 | ... |
| Temperature (°C) | 39.73 | 40.7 | 41.7 | 42.72 | 43.77 | 44.83 | 45.92 | 47.02 | ... |
| Time (s) | 331 | 331.5 | 332 | 332.5 | 333 | 333.5 | 334 | 334.5 | 335 |
| Temperature (°C) | 145.48 | 144.8 | 144.13 | 143.45 | 142.78 | 142.11 | 141.45 | 140.78 | 140.13 |

Use the data obtained by the model to draw the furnace temperature curve

![Furnace temperature curve](image)

**Figure 6. Furnace temperature curve**

From the overall trend, it conforms to the change law of the standard furnace temperature curve.

5.1.4. **Problem One Model Checking.** Use the temperature of the accessory temperature zone to verify the model, substitute the accessory temperature into the heat transfer model of this question, and compare and analyze the center temperature data of the welding area provided by the accessory to verify the accuracy of the model.
Figure 7. Error comparison curve

Through comparative analysis, it can be seen that the model in this question has a high degree of fit with the actual data, and the accuracy of the model is very good. Perform error analysis:

\[
Z = \left| \frac{T_{\text{model data}} - T_{\text{actual data}}}{T_{\text{actual data}}} \right| \cdot 100\% \tag{39}
\]

Get:

|                | Heat transfer model |
|----------------|--------------------|
| Average error rate /% | 3.10               |
| Maximum error rate /%  | 14.22              |

The average error rate of the model is 3.10%, and the maximum error rate of 14.22 is mainly due to the numerical difference at the beginning of the curve, and the temperature is small, all the differences appear to be large. In general, the average error rate is small and the model accuracy is excellent.

5.2. Model establishment and solution of problem two

5.2.1. Problem two model preparation. In the production of reflow furnace circuit board welding, the furnace temperature curve should meet certain requirements, which is called the process limit (see the table below)

| Boundary name                                      | Lowest value | Highest value | Unit   |
|----------------------------------------------------|--------------|---------------|--------|
| Temperature rise slope                             | 0            | 3             | °C/s   |
| Temperature drop slope                             | -3           | 0             | °C/s   |
| 150°C–190°C time during the temperature rise       | 60           | 120           | s      |
| Time when the temperature is greater than 217°C    | 40           | 90            | s      |
| Peak temperature                                   | 240          | 250           | °C     |
(1) Excessive solvent needs to be volatilized in the preheating stage, but the heating slope needs to be controlled. Excessive heating speed will cause thermal stress impact of the component, damage the component and reduce the service life and performance of the component. In the same way, the slope of the component cooling down also needs to be well controlled.

(2) The constant temperature stage has two important functions. One is to make the entire circuit board reach a uniform temperature to reduce the impact of thermal stress into the reflow zone; the other is that the flux in the solder paste starts to react, which can remove the solder. The oxides and impurities on the surface of the parts increase the wettability of the surface of the weldment, so the constant temperature time and temperature must be well controlled.

(3) The reflow zone needs to consider the reflow time and higher temperature. If the reflow time is too long and the maximum temperature is too high, it will cause the intermetallic compound layer to thicken and affect the long-term reliability of the solder joints.

5.2.2. Model establishment of problem two. This question needs to meet the requirements of the process limit under a given temperature zone, and determine the maximum allowable furnace speed for the conveyor belt.

In the first problem, the expression of the circuit board welding center temperature is obtained. In order to obtain the maximum conveyor belt passing speed that meets the reflow conditions, a single-objective optimization model is established on the basis of the first problem.

(1) Objective function:

$$\max \{v\}$$

(40)

(2) Constraints:

a. Slope of temperature curve $K$ (°C/s)

$$-3 \leq K \leq 3$$

b. $150°C \sim 190°C$ time during temperature rise $t_1$ (s)

$$60 \leq t_1 \leq 120$$

c. Time $t_2$ (s) when the temperature is greater than $217°C$

$$40 \leq t_2 \leq 90$$

d. Peak temperature $T_{\text{max}}$ (°C)

$$240 \leq T_{\text{max}} \leq 250$$

e. Adjusting range of furnace passing speed $v$ (cm/min)

$$65 \leq v \leq 100$$

(3) The overall model of single objective optimization:
\[
\max \{ v \}
\begin{align*}
-3 & \leq K \leq 3 \\
60 & \leq t_1 \leq 120 \\
40 & \leq t_a \leq 90 \\
240 & \leq T_{\text{max}} \leq 250 \\
65 & \leq v \leq 100
\end{align*}
\]

(41)

5.2.3. Model solution for problem two. Use matlab to traverse the search to solve the model:
\[
v_{\text{max}} = 92.6\, \text{cm/min}
\]

Plot the furnace temperature curve at this speed:

![Temperature curve at a speed of 92.6 cm/min](image)

**Figure 8.** Temperature curve at a speed of 92.6 cm/min

Comparing the furnace temperature curve in the appendix, the overall trend is consistent.

**Table 5.** Data under maximum speed lower

| Boundary name                              | Numerical value | Unit  |
|--------------------------------------------|-----------------|-------|
| Maximum temperature rise slope             | 2.998           | ℃/s   |
| Maximum temperature drop slope             | -1.998          | ℃/s   |
| The time when the temperature is at 150–190℃ during the rising process | 83.35 | s     |
| Time when the temperature is greater than 217℃ | 54.50 | s     |
| Peak temperature                           | 240.84          | ℃     |

It is observed that the temperature rise slope is the main factor that limits the conveyor belt passing speed; and the remaining four limits can basically be met.
5.3. Establishment and solution of problem three model

The idea of solving this problem is similar to that of problem two, in order to find that the area covered by the furnace temperature curve from exceeding 217°C to the peak temperature is the smallest. In this way, the set temperature of each temperature zone and the furnace passing speed of the conveyor belt are determined, and the optimal furnace temperature curve is drawn, and the corresponding area is calculated.

5.3.1. Establishment of Problem Three Model. Introduce the shadow area S:

![Figure 9. Shaded area](image)

The shaded area in the figure is the area covered by the furnace temperature curve exceeding 217°C to the peak temperature. When the rising temperature reaches 217°C, the intersection point with the x-axis is n; when the peak temperature is reached, the intersection point is m.

\[ S = \left[ \int_{n}^{m} f(x) \, dx - 217 \cdot (m - n) \right] \cdot t_{bc} \]  \hspace{1cm} (42)

Among them, \( f(x) \) is the relational expression of the furnace temperature curve, and \( t_{bc} \) is the time it takes to walk one grid.

Establish a single-objective optimization model:

Objective function:

\[ \min \{ S \} \]  \hspace{1cm} (43)

(2) Constraints:

a. Slope of temperature curve \( K (\degree C/s) \)
\[ -3 \leq K \leq 3 \]

b. 150°C～190°C time \( t_{1} \) (s) during the temperature rise
60 ≤ t₁ ≤ 120
c. Time t₂ (s) when the temperature is greater than 217℃
40 ≤ t₃ ≤ 90
d. Peak temperature $T_{\text{max}}$ (℃)
240 ≤ $T_{\text{max}}$ ≤ 250
e. Adjusting range of furnace passing speed $v$ (cm/min)
65 ≤ v ≤ 100

Single objective optimization total model:

$$\min \{S\}$$

\[
\begin{align*}
-3 & \leq K \leq 3 \\
60 & \leq t_1 \leq 120 \\
40 & \leq t_3 \leq 90 \\
240 & \leq T_{\text{max}} \leq 250 \\
65 & \leq v \leq 100
\end{align*}
\]

\[
y' = \frac{2}{k'y} (T'(x_i) - y) \cdot \left[ h_1 + \frac{k'}{2} (T'(x_i) + y) \right]
\]

\[
T_{s(t)} - T_{s(t)} = C \cdot (T_{a(t)} - T_{a(t)})
\]

5.3.2. Problem three model solution. The same model is used to solve similar problems in the second, using ergodic search to solve, but here in addition to the conveyor belt speed, it is also necessary to determine the set temperature of each temperature zone. With search criteria:

1. The set temperature of each small temperature zone can be adjusted within the range of ±10℃;
2. During adjustment, the temperature in small temperature zones 1 to 5 should be kept consistent, the temperature in small temperature zones 8 and 9 should be the same, and the temperature in small temperature zones 10 and 11 should be kept at 25℃;
3. The adjustment range of the furnace passing speed of the conveyor belt is 65 ~ 100 cm/min.

**Table 6. Traversal range of each temperature zone**

| Parameters to be determined | Parameters to be determined | Ranges       |
|-----------------------------|-----------------------------|--------------|
| $T_1$ °C                    | Preheating zone (1 ~ 5)     | [165, 185]   |
| $T_2$ °C                    | Constant temperature zone 6 | [185, 205]   |
| $T_3$ °C                    | Constant temperature zone 7 | [225, 245]   |
| $T_4$ °C                    | Recirculation zone (8, 9)   | [245, 265]   |
| $v$ cm/min                  | Conveyor speed              | [65, 100]    |

In order to set the temperature of each temperature zone and obtain the conveyor speed through the furnace, a five-fold traversal solution is required:

The temperature in the preheating zone: 167°C; the temperature in the constant temperature zone 6: 185.1°C; the temperature in the constant temperature zone 7: 225°C; the temperature in the reflux zone: 259.2°C; the conveyor speed: 91.2 cm/min.
Table 7. Parameter determination

| Parameters to be determined | $T_1{^\circ C}$ | $T_2{^\circ C}$ | $T_3{^\circ C}$ | $T_4{^\circ C}$ | v cm/min |
|-----------------------------|-----------------|-----------------|-----------------|-----------------|-----------|
| Iterate over the results    | 167.0           | 185.1           | 225.0           | 259.2           | 91.2      |

Figure 10. The optimal furnace temperature curve in question 3

$$S_{\text{min}} = 446.34 \sin \circ C \cdot s = 7.4 \sin \circ C \cdot s$$

5.4. Model establishment and solution of problem four

This question uses the greedy algorithm to find the local optimal method to avoid the time complexity of the full traversal algorithm. The conversion restriction condition is to use the fitting function to avoid the problem that the difference equation and the differential equation are nested and difficult to solve.

5.4.1. Establishment of Problem Four Model. The idea of solving this problem is similar to that of problem three. The question requires that when the area of the problem three is small, control the furnace temperature curve to be as symmetrical as possible around the center line where the peak is located. This can be regarded as a dual-objective optimization model. The first goal is that the curve is as symmetric as possible on the center line of the peak. The goal is the same as in question 3.

First consider the objective function one:

If the furnace temperature curve is completely symmetric before and after the peak, the center line where the peak is located can be called the local symmetry axis. Obviously, the two points on the furnace temperature curve that are symmetric about the symmetry axis have the same temperature and the slopes are opposite to each other, and the sum is zero. However, the actual situation is before and after the peak value. Due to the large temperature difference between the recirculation zone and the cooling zone, it is difficult for the furnace temperature curve to achieve complete symmetry, so the symmetry coefficient $\alpha$ is defined:
When $\alpha = 0$ is constant, that is, the furnace temperature curve is completely symmetrical about the axis of symmetry; when it is not, refer to the $N - \varepsilon$ definition method of the limit, and use the sum of squares of $a - 0$ to describe the approximate degree of the curve on both sides of the peak. The smaller the sum of squares, the more symmetric the curve.

Define objective function two:

$$
\min\left\{\left[\int_{x}^{m} (a - 0)^2 \, dx\right] / (m - n)\right\}
$$

(46)

Consider the second objective function: $\min \{S\}$

Inherited from the above model, and transformed into a restriction:

$$
S = \left[\int_{x}^{m} f(x) \, dx - 217 \cdot (m - n)\right] \cdot t_{bc}
$$

(47)

The constraints are the same as in question three.

Since it is necessary to determine the temperature of 4 temperature zones and the speed of conveyor belt passing the furnace, 5 parameters are to be determined, and the time complexity is large. Consider optimizing the algorithm of the third question, and use the fitting function to find the local optimal conditions.

Using the greedy algorithm, qualitative analysis to obtain the simplified parameters of the local optimal conditions: select the slope and peak temperature as dangerous conditions, for the problem that the peak temperature is difficult to calculate, use the exponential function to determine the relationship between the furnace temperature, the maximum air temperature difference and the speed $v$

Fitting, aiming at the problem that it is difficult to solve the slope of the furnace temperature curve as the difference equation, the power function is used to fit the relationship between the slope difference of the two temperatures and the distance $x$, and the restriction on the furnace temperature is converted to the restriction on the air temperature. Draw out the differential equation to get the local optimal speed and the temperature of the first temperature zone, and then do single-objective optimization, and traverse the search to determine the remaining three parameters. Such as the flow chart:
According to the experience of question three, the rising slope and peak value in the process boundary are dangerous items.

Because the difference equation of the welding center temperature depends on the speed and the temperature distribution in space, it is difficult to directly determine it. Now we plan to use approximate fitting to simplify and convert the dangerous condition of $T_s$ to the dangerous condition of $T$.

\[
\frac{T_s(t + \Delta t) - T_s(t)}{\Delta t} < 3 \\
T_s(t)_{\text{max}} > 240 \, ^\circ C
\]  

(48)

Qualitative analysis:

The analysis expression finds that the rate of change of $T_s$ is related to the hysteresis coefficient $C_1$ and the space temperature $T(x)$, but when the $T(x)$ curve is determined, the rate of change of $T_s$ is only related to $v$.

1. Test the relationship between $T(x)_{\text{max}}$ and $T_s(x)_{\text{max}}$ when the speed $v$ changes in $60 \sim 100 \text{ cm/min}$

In order to reduce the error, set the relative temperature difference coefficient as:
Linear function approximation is commonly used in engineering, but analysis of numerator shows that $f_1$ should not be a negative number, and natural logarithm is used for approximation:

$$f_1(v) = ae^{bv} \quad (50)$$

$$T(x)_{\text{max}} = \frac{T_s(x)_{\text{max}}}{1 - a_1 e^{b_1 v}} \quad (51)$$

$a_1, b_1$ is the fitting coefficient. When $T_s(x)_{\text{max}}$, $T(x)_{\text{max}}$ is determined, the furnace speed $v$ can be solved.

$T(x)_{\text{max}}$ is affected by many factors. When $v$ and $T_s(x)_{\text{max}}$ are close to a stable value, $T(x)_{\text{max}}$ approximately obeys a normal distribution. The mean value of $T(x)_{\text{max}}$ can be used as the calculated value in the formula.

(2) When the speed is constant, draw the derivative of two temperature changes with $x$.

**Figure 12.** Fitting diagram of relative error coefficient

**Figure 13.** The slope of the temperature between the temperature and the temperature of the solder joint
In the figure, it can be judged that the air temperature and the slope change of the center of the weldment are consistent, and the extreme values are taken at the same point. Now select the preheating zone as the dangerous zone for analysis and fitting.

Construct the slope difference comparison function $f_2$:

$$f_2 = \frac{dT(x)}{dx} - \frac{dT_s(x)}{dx}$$  \hspace{1cm} (52)

Observe the graph and select the preheating zone as the slope check zone of the heating section.

Reasonable inference: When the temperature change of each temperature zone is small, its influence on the slope difference is a secondary factor.

$$f_2 = f_2(x, v, T'(x))$$  \hspace{1cm} (53)

Control variables: conveyor belt passing speed $v$ and temperature $T'$ in each temperature zone.

When $v = 70, T' = T'_0 = [50 \ 175 \ 195 \ 235 \ 255 \ 25]$

The graph of $f_2 = f_2(x)$ is as follows:

![Figure 14. Slope difference fitting diagram](image)

Choose the power function as the fitting function:

$$f_2 = f_2(x) = a_2 \cdot x^{b_2}$$  \hspace{1cm} (54)

$$y' = \frac{dT(x)}{d(x)} < f_2 + 3$$  \hspace{1cm} (55)

Let $H(x) = y' - (f_2 + 3) \times v$; $\forall x \in [50, 389]$, $H(x) < 0$, analyze $H(x)$, we can see:

$$H(x)_{\text{max}} = H(50)$$  \hspace{1cm} (56)

At this time, the temperature in front of the furnace is determined, and there is $y(x = 50) = 35^\circ C$.

It is concluded that $T'(x_{\text{max}}) = U$ and $U$ are constants.
5.4.2. The solution of problem four model.

\[ T(x)_{\text{max}} = 244.5, \ a_1 = 0.004201, \ b_1 = 0.01634 \]

Substitution:

\[ v = \frac{1}{b_1} \ln \left( \frac{T(x)_{\text{max}} - T_2(x)_{\text{max}}}{T(x)_{\text{max}} - a_1} \right) = 90.4 \]

\[ a_2 = 4.637 \times 10^7, \ b_2 = -4.609 : f_2 = 0.68499 \]

\[ H(50) = y' - (f_2 + 3) \times v = 0 \]

The calculated temperature in the preheating zone: 169.5°C; the temperature in the constant temperature zone 6: 185°C; the temperature in the constant temperature zone 7: 225°C; the temperature in the reflux zone: 258.7°C; the conveyor speed: 90.4 cm/min.

| Parameters | \( T_1 \) °C | \( T_2 \) °C | \( T_3 \) °C | \( T_4 \) °C | v cm/min |
|------------|-------------|-------------|-------------|-------------|-----------|
| result     | 169.5       | 185         | 225.0       | 258.7       | 90.4      |

Use data to draw furnace temperature curve:

![Furnace temperature curve of problem four](image)

**Figure 15.** Furnace temperature curve of problem four

And get the data of each indicator

| Boundary                                                                 | Data   | Unit  |
|-------------------------------------------------------------------------|--------|-------|
| Maximum temperature rise slope                                          | 2.69   | °C/s  |
| Maximum temperature drop slope                                          | -1.95  | °C/s  |
| The time when the temperature is at 150~190°C during the rising process | 87.71  | s     |
| Time when the temperature is greater than 217°C                         | 26.85  | s     |
| Peak temperature                                                        | 240.01 | °C    |
| Shadow area                                                             | 453.38 | °C*s  |
6. Model Assessments and Promotion

This paper comprehensively considers the influence of various parameters on the change of the furnace temperature curve, establishes the heat transfer model of the welding zone, and applies it to the optimal control and design of the furnace temperature curve. The model and algorithm established in this paper have the following advantages and disadvantages.

6.1. Strengths
1. Use matlab software to process the data and make a line chart, which is simple, intuitive and fast;
2. The heat transfer model established in this article is closely related to reality, and fully considers the heat transfer principle of the actual situation, so that the model is closer to reality and has strong versatility;
3. Use ergodic search to solve the single-objective optimization model, and the feasible solution obtained is more accurate;
4. The greedy algorithm solves the local optimum, avoids global search traversal, and reduces the amount of calculation.

6.2. Weaknesses
1. Approximate calculations and errors exist in the heat transfer model;
2. The greedy algorithm solves the local optimum, but there is a deviation from the overall actual situation.

6.3. Conclusion
In the solution of the reflow furnace problem in this paper, a heat transfer model is established; the heat transfer process of reflow soldering is very complicated, and there are heat conduction, heat convection and heat radiation. This type of model can easily describe the temperature change law in the reflow furnace, and play an auxiliary role for the high-tech, high-precision reflow welding process. This type of model can be widely used in various thermal fields to calculate the energy interaction among them. Such as: in the design and production of protective clothing, the principle of induction cooker heating, the heating and protection of floor heating.

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