Spectral functions of a time-periodically driven Falicov-Kimball model: Real-space Floquet dynamical mean-field theory study

Tao Qin and Walter Hofstetter
Institut für Theoretische Physik, Goethe-Universität, 60438 Frankfurt/Main, Germany

We present a systematic study of the spectral functions of a time-periodically driven Falicov-Kimball Hamiltonian. In the high-frequency limit, this system can be effectively described as a Harper-Hofstadter-Falicov-Kimball model. Using real-space Floquet dynamical mean-field theory (DMFT), we take into account the interaction effects and contributions from higher Floquet bands in a non-perturbative way. Our calculations show a high degree of similarity between the interacting driven system and its effective static counterpart with respect to spectral properties. However, as also illustrated by our results, one should bear in mind that Floquet DMFT describes a nonequilibrium steady state, while an effective static Hamiltonian describes an equilibrium state. We further demonstrate the possibility of using real-space Floquet DMFT to study edge states on a cylinder geometry.

PACS numbers: 67.85.-d, 37.10.Jk, 03.65.Vf

I. INTRODUCTION

Time-periodically driven systems are a versatile toolbox for simulating artificial gauge fields in experiments. There has been great recent experimental progress in realizing topologically nontrivial Hamiltonians. Two paradigmatic models, the Harper-Hofstadter (HH) model [1] and the Haldane model [2], have been realized in ultracold atoms by laser-assisted hopping [3, 4] and lattice shaking [5], following seminal proposals [6, 7]. These have triggered high interest in the physics of Floquet systems both from a theoretical [8, 9] and an experimental [10, 11] point of view.

The effective Hamiltonian approach [12–16] works in the high-frequency limit. Different methods, such as the Magnus expansion [14], high-frequency expansion [13, 17], flow equations [12], and Brillouin-Wigner theory [15], can be used to obtain effective Hamiltonians. This approach is in good agreement with experimental measurements [3, 4] in noninteracting cases. Several efforts [8, 18, 19] have been made to include interaction effects within effective Hamiltonians. Even though their validity is limited to high frequencies, they showed the potential of Floquet engineering for obtaining exotic topologically nontrivial phases.

Floquet topological insulators [16, 20–23] are a fascinating application of periodically driven systems. They show clear differences from (static) topological insulators, especially with respect to the bulk-edge correspondence [21]. Because of the periodicity of the unbounded Floquet quasienergy with the driving frequency $\Omega$, Floquet edge states can appear in different quasienergy gaps [20, 21]. To obtain the correct bulk-edge correspondence, in Ref. [21] two approaches were proposed: a winding number based on the time-evolution operator $U(k, t)$, where $k$ is the momentum and $t \in (0, T)$ with the period $T = \frac{2\pi}{\Omega}$, instead of $U(k, T)$, and a formalism in frequency space. The former has been generalized to the time-reversal symmetric case [22]. All studies so far have focused on noninteracting models. Floquet systems are always in a nonequilibrium state, and for the interacting case up to now we lack a way to calculate Floquet topological invariants. The effective topological Hamiltonian approach [24] cannot be directly applied to a Floquet system since one cannot easily define Green’s functions on the imaginary time axis in a nonequilibrium situation. On the other hand, we will in the following demonstrate the strength of real-space dynamical mean-field theory (DMFT) both for the driven system and the effective Hamiltonian, which allows calculating edge states in the presence of interactions $U$.

While the effective Floquet Hamiltonian describes the long-time dynamics of Floquet systems in a nonequilibrium steady state (NESS), one needs to couple a Floquet system to a bath, which absorbs energy from the system, in order to achieve such a NESS [25–28]. One key issue is how to make the physical properties of the NESS most similar to its desired effective static counterpart. Using the quantum master or kinetic equation, Refs. [25–27] have studied the population of quasienergy levels in noninteracting Floquet systems, and found that the NESS can be characterized by a finite density of excitations above the effective Fermi level. They have also shown how to control a NESS by different baths. The situation is even more interesting if we introduce interactions.

Floquet DMFT is a powerful tool for studying interacting, time-periodically driven systems. It is a nonperturbative method for solving Hubbard-type models with driving, and applicable in the full range of driving frequencies. Similar to equilibrium DMFT [29], it maps a driven, interacting lattice model onto a single driven Anderson impurity model, which is determined self-consistently [30–32]. Every lattice site is coupled to an additional bath in order to dissipate energy and to help the system reach the NESS. References [31–34] introduced the framework of Floquet DMFT and Refs. [33, 34] studied the NESS of the Falicov-Kimball model attached to a free-fermion bath and irradiated by intense light. They calculated the electron occupation above the Fermi
level due to photon-assisted tunneling, and found a clear deviation from the Fermi-Dirac distribution in the effective distribution. However, in the sense of the effective Hamiltonian there is no gauge field induced by ac driving in the model of Refs. [33, 34], where, instead, only a renormalized hopping is present. In this paper, we apply Floquet DMFT to a driven ultracold atomic system, where artificial gauge fields are induced. We generalize the formalism of Floquet DMFT to real-space Floquet DMFT. We find very similar spectral functions in the driven system and its static effective counterpart when the driving frequency is high, and a clear discrepancy for lower frequencies. We furthermore study edge states in a cylinder geometry.

II. THE MODEL

We first give a short review of Floquet’s theorem. For a time-periodically driven system, which is often termed a Floquet system, the Hamiltonian satisfies in the time domain $H(0 + T) = H(t)$. According to Floquet’s theorem [14, 30], there is a solution of the Schrödinger equation of the form $\Psi(t) = e^{-i\Omega t}u_\alpha(t)$, where $\alpha$ labels different energy states and $u_\alpha(t) = u_\alpha(t + T)$, an analog to the Bloch function in space. $u_\alpha(t)$ can be Fourier expanded as $u_\alpha(t) = \sum_{m=-\infty}^{\infty} u_m^{(\alpha)} e^{-i m \Omega t}$. One can show that $\sum_{m,n} H_{mn} u_n^{(\beta)} = (\varepsilon_\alpha + m \Omega) u_m^{(\alpha)}$, where $H_{mn} = \frac{1}{T} \int_0^T dt e^{i (m-n) \Omega t} H(t)$. $\varepsilon_\alpha + m \Omega$ is denoted as the quasienergy and is not bounded. In the non-interacting case, different Floquet bands are separated by $\Omega$. One can thus expect that the effects of higher Floquet bands are negligible when $\Omega \to \infty$. At finite $\Omega$, the effects from higher bands may be relevant. The situation is more involved for interacting systems.

We now present the noninteracting Hamiltonian, which is a fermionic version of the model realized in the experiments [3, 4],

\[
H^{(0)}(t) = H_{\text{kin}} + H_{\text{drive}}(t),
\]

\[
H_{\text{kin}} = -\sum_{ij} \left[ J_x c_{i+1,j}^\dagger c_{ij} + J_y c_{i,j+1}^\dagger c_{ij} + H.c. \right],
\]

\[
H_{\text{drive}}(t) = \sum_{ij} \left[ \frac{V_0}{2} \sin \left( \Omega t - \phi_{ij} + \frac{\phi_{\Box}}{2} \right) + i\Omega \right] n_{ij},
\]

where $i$ and $j$ label the position $R = ie_x + je_y$ of a site in a two-dimensional square lattice, with $e_x$ and $e_y$ the primitive lattice vectors in the $x$ and $y$ directions. We consider hopping terms up to nearest neighbors. $V_0$ is the driving amplitude. $\phi_{\Box}$ is the flux in every primitive unit cell, and $c_{i,j}^\dagger$ is the creation operator for itinerant atoms at site $(i,j)$, with $n_{ij} = c_{i,j}^\dagger c_{i,j}$. For simplicity, we use the Landau gauge $\phi_{ij} = \Phi_{\Box} + \frac{\phi_{\Box}}{2}$ [10]. We set $J_x = J_y = 1$ as our energy unit in the following.

We next derive the effective Hamiltonian. Usually, $\Omega$ is a large energy scale in a time-periodically driven system. We perform a unitary transformation to rotate the Hamiltonian to a frame in which there are no terms of order $\Omega$. With the unitary rotation $\psi(t) \to V(t)\psi(t)$ where $V(t) = e^{i \sum_{ij} \left( -\frac{V_0}{2\Omega} \cos (\Omega t - \phi_{ij} + \frac{\phi_{\Box}}{2}) \right) + i\Omega} n_{ij}$, we have $\tilde{H}(t) = VH V^\dagger + i V \frac{dV^\dagger}{dt}$. Explicitly,

\[
\tilde{H}^{(0)}(t) = -\sum_{ij} \left( g(t) c_{i,j}^\dagger c_{i+1,j} + f(t) c_{i,j}^\dagger c_{i,j+1} + H.c. \right),
\]

where $g(t) = e^{iA_{\Box} \sin (\Omega t - \phi_{ij}) - \Omega t}$ and $f(t) = e^{iA_{\Box} \sin (\Omega t - \phi_{ij})}$, and we define $A \equiv \frac{V_0}{\Omega} \sin \frac{\phi_{\Box}}{2} \equiv \frac{V_0}{\Omega} \sin (\pi\alpha)$ with $\Phi_{\Box} = 2\pi\alpha$. Using the Magnus expansion [14], the effective Hamiltonian up to zeroth order in $\frac{1}{\Omega}$ is

\[
\tilde{H}^{(0)}_{\text{eff}} = \frac{1}{T} \int_0^T dt \tilde{H}^{(0)}(t) = -\sum_{ij} \left[ J_l(A) e^{-i \phi_{ij}} c_{i,j}^\dagger c_{i+1,j} + J_0(A) c_{i,j}^\dagger c_{i,j+1} + H.c. \right],
\]

where $J_l(A)$ is the $l$-th order Bessel function of the first kind. This effective Hamiltonian is exact in the limit $\Omega \to \infty$ while higher orders contribute for finite $\Omega$. The Hamiltonian (2) is slightly different from the usual HH model, since the hopping amplitudes $J_l(A)$ depend on the flux $\alpha$ through $A$.

We are now in a position to present Hamiltonian (1) in Floquet space. Using Floquet’s theorem [35] and an extended Hilbert space [36], we can write $c_{i,j}(t) = \sum_{n=-\infty}^{\infty} c_{i,j,n} e^{-i n \Omega t}$. The Hamiltonian $\tilde{H}^{(0)}(t)$ in the Heisenberg picture can be transformed to Floquet space. Its matrix element $\tilde{H}_{mn}^{(0)}$ is given by

\[
\tilde{H}_{mn}^{(0)} = -\sum_{ij} \left[ J_{n-m+1}(A) e^{i(m-n-1)\phi_{ij}} c_{i,j,m}^\dagger c_{i,j+1,n} + J_{m-n-1}(A) e^{i(m-n+1)\phi_{ij}} c_{i,j+1,m}^\dagger c_{i,j,n} + J_{m-n}(A) e^{i(m-n)\phi_{ij}} c_{i,j,m}^\dagger c_{i,j+1,n} + J_{n-m}(A) e^{i(m+n)\phi_{ij}} c_{i,j+1,m}^\dagger c_{i,j,n} \right],
\]

which in physical terms corresponds to the stimulated emission ($m > n$) or absorption ($m < n$) of photons [33]. The diagonal terms correspond to the strongest hopping, while the off-diagonal parts are higher-order corrections. In principle, the dimensionality of $\tilde{H}^{(0)}$ is infinite. In reality, we can keep a finite matrix, with a size inversely proportional to the driving frequency $\Omega$. We also make the important observation that Hamiltonian (3) recovers the effective Hamiltonian (2) if we set the dimension of Floquet space to 1.

III. FLOQUET DMFT FORMALISM AND ITS REAL-SPACE GENERALIZATION

We are mostly interested in the interaction effects in the driven system. We turn on an interaction of the
observe six peaks in the spectrum for different fluxes and driving frequencies. (i) We refer to the Falicov-Kimball model, which we calculate for an effective HH-Falicov-Kimball model, where the latter is already realized in experiments [3, 10]. In this work, we go one step further by generalizing Floquet DMFT calculations to inhomogeneous systems (see the Appendix Sec. IX A).

IV. HOFSTADTER BUTTERFLY

We first present a real-space Floquet DMFT calculation for the local spectral function \( A_{ij}(\omega') = -\frac{1}{\pi} \text{Im} G_{ij,nn}^R(\omega) \) on site \((i, j)\) of a driven Falicov-Kimball model at half filling with \( w_0 = w_1 = \frac{1}{2} \). \( G_{ij,nn}^R(\omega) \) is the \((n, n)\) Floquet component of the nonequilibrium retarded Green’s function on site \((i, j)\) (see the Appendix Sec. IX A), \( \omega \in \left(-\frac{\Omega}{2}, \frac{\Omega}{2}\right) \), and \( \omega' \equiv \omega + n\Omega \) is in the full range of the frequency spectrum. The spectral function is shown for the center site of a 15×15 square lattice, which is in the bulk and preserves the symmetry of the system. We have chosen values for the interaction \( U \) which are significantly lower than the driving frequency \( \Omega = 7 \). It is therefore expected that the effective HH Hamiltonian can capture the features of the driven Falicov-Kimball model. Indeed, in Fig. 1, we clearly observe a Hofstadter butterfly structure. Furthermore, we see that increasing interaction smears out the fine structure of the butterfly, consistent with DMFT calculations for the HH-Falicov-Kimball model [40]. These results clearly demonstrate the possibility to observe the Hofstadter butterfly in a Floquet system.

We next have a close look at the properties of spectral functions for typical fluxes \( \alpha = \frac{1}{6} \) and \( \alpha = \frac{1}{3} \), where the latter is already realized in experiments [3, 10]. To this end, we compare results from real-space DMFT calculations for an effective HH-Falicov-Kimball model, and real-space Floquet DMFT calculations for a driven Falicov-Kimball model, which we refer to, respectively, as the static and driven cases from now on.

In Fig. 2, we show the static and driven spectral functions for different fluxes and driving frequencies. (i) We observe six peaks in the spectrum for \( \alpha = \frac{1}{6} \) and four peaks for \( \alpha = \frac{1}{3} \), when \( U = 0 \). With increasing interaction \( U \), the peaks decay, and the fine structure of the butterfly is smeared out. Only two Mott-insulator bands are left for \( U = 3 \). (ii) We observe good consistency of the spectral functions in the driven case at high frequency and the static case. This similarity is very important and justifies the use of the NESS for simulating the equilibrium state. On the other hand, we see a clear discrepancy between the static and driven cases at an intermediate driving frequency \( \Omega = 3.3 \). (iii) Note that the spectral functions for the driven system are not as symmetric as those for the static case. This is due to contributions from higher Floquet bands, since the Floquet Hamiltonian reduces to the effective HH-Falicov-Kimball Hamiltonian if we set the dimension of the Floquet matrix to 1. (iv) The static and driven cases are essentially different, which is shown by the effective distribution \( f_{ij}(\omega') = N_{ij}(\omega')/A_{ij}(\omega') \) with \( N_{ij}(\omega') = \frac{1}{2\pi} \text{Im} G_{ij,nn}^R(\omega) \) in Fig. 2. While for the static case the distribution is naturally of a Fermi-Dirac type, it is clearly different in the NESS of the driven system. The effective distribution thus qualitatively describes how far the NESS is from an equilibrium state.

We need to point out that in these plots of spectral functions we choose the amplitude \( V_0 \) to satisfy the condition \( A = 1.435 \), i.e., \( V_0 = 1.435 \frac{\Omega}{\sin(\alpha \pi)} \), where \( J_0(A) = J_1(A) \). In this way, the effective distribution amplitudes in \( x \) and \( y \) are the same [41], which makes the effective Hamiltonian exactly equivalent to the standard HH model. To illustrate this point, in Fig. 3 we show the difference between \( A = 1, 1.435, \) and 2 for \( \Omega = 7 \) and \( \alpha = \frac{1}{6} \). The choices \( A = 1 \) and 2 lead to additional peaks in the spectrum. It is therefore of experimental relevance to choose the special driving amplitude satisfying \( A = 1.435 \). To observe a Hofstadter butterfly, the temperature should be smaller than the gaps between subbands [42]. Additional peaks for a nonoptimal choice of \( A \) would imply even smaller gaps and increase the experimental challenge.
V. EDGE STATE ON A CYLINDER STRUCTURE

In Fig. 4 we present spectral functions for the static and driven cases in a cylinder geometry, which is periodic in the $x$ direction and finite in the $y$ direction, using real-space (Floquet) DMFT. We show the total spectrum containing bulk and edge states. We compare properties of the driven system for different driving frequencies with the static case at the same $U$. The driven case at high frequency ($\Omega = 7$) is very similar to the static one, even though there is a slight difference in the symmetry of the spectrum, which is, again, due to contributions from higher Floquet bands. For intermediate frequency $\Omega = 3.3$, higher Floquet bands become visible at $\omega \approx \pm 2.5$. With increasing $U$, we observe that fine structures are washed out, and finally a gap opens. For $U = 4$ and $\Omega = 3.3$, we observe a clear difference compared to the high-frequency case. It may be due to photon-assisted tunneling inside the Floquet band around $\omega = 0$. When $U = 0$, edge states are present for all driving frequencies. Overall, we see a pronounced similarity between $\Omega = 7$ and the effective static case ($\Omega = \infty$). This justifies the idea to engineer nontrivial topological states by driving. Because the spectrum is washed out with increasing interactions, edge states become invisible at large $U$. Even though we do not observe edge states...
induced by strong interactions in this simple model, it may be possible to observe this effect in other models. We emphasize that it is crucial to study edge states in an interacting Floquet system, since currently there is no approach for calculating the Chern number or related topological indices of interacting non-equilibrium states. Therefore, edge states provide important signatures for topological transitions.

VI. EXPERIMENTAL REALIZATION

The set up and results we have presented are accessible for experimental measurements. The spectral functions presented above can be detected using momentum-resolved radio-frequency (rf) spectroscopy [43, 44], a counterpart to angle-resolved photoemission spectroscopy (ARPES) used for electronic materials. We have some remarks on the bath. The role of the free-fermion bath is to allow a driven system to reach a NESS. It has been proposed [45] that atoms in an optical lattice can be cooled by immersion in a Bose-Einstein condensate (BEC), which also serves as a bath.

Reference [46] demonstrates a novel approach, which has the potential to realize a Falicov-Kimball model, where different species (hyperfine states) can be subjected to different driving amplitudes. For a system of two species, one species can be “dynamically localized” by tuning the renormalized hopping amplitude to zero. Therefore, experimental techniques for realizing both the Falicov-Kimball model and the HH model are available.

VII. CONCLUSION

Time-periodically driven ultracold atoms are a promising platform for simulating topologically nontrivial band structures. In the presence of interactions, these are even more intriguing and interesting. Using real-space (Floquet) DMFT, we have systematically studied the spectral function of the driven Falicov-Kimball Hamiltonian, and of its effective Hamiltonian in the high-frequency limit, both for open boundary conditions and for a cylinder geometry. For a large driving frequency we observed similar spectra of the driven system and the effective Hamiltonian. This demonstrates that topologically nontrivial bands can be simulated by a realistic driven system. Nevertheless, as we have shown, the NESS of the driven Hamiltonian is essentially different from the equilibrium state of an effective Hamiltonian. Our work also highlights the possibility of studying edge states and topological properties of an interacting system by using real-space Floquet DMFT.

VIII. ACKNOWLEDGMENTS

The authors acknowledge useful discussions and communication with M. Eckstein, K. Le Hur, and N. Tsuji. This work was supported by the Deutsche Forschungsgemeinschaft via DFG FOR 2414 and the high-performance computing center LOEWE-CSC.

IX. APPENDIX

A. Floquet DMFT and its real-space generalization

While the equilibrium real-space DMFT formalism has been detailed in Ref. [47], here we give a short introduction to Floquet DMFT and its real-space generalization. We first highlight two important aspects. (i) Floquet DMFT addresses the NESS in a non-equilibrium system [30–32], and is based on the Keldysh Floquet Green’s function [33, 37]. Because of driving, we need to consider the two-time Green’s function $G(t, t') = G(t + T, t' + T) \neq G(t - t')$, where $t$ and $t'$ are defined on a two-branch contour, ranging from $-\infty$ to $+\infty$, and from $+\infty$ to $-\infty$, respectively. The Green’s function in frequency space is $G_n(\omega) = \int_{-\infty}^{\infty} dt_{rel} \frac{1}{\beta} \int_0^T dt_{av} G(t_{rel}, t_{av}) e^{i\omega t_{rel} + i\hbar\omega t_{av}}$, where $t_{rel} = t - t'$ and $t_{av} = \frac{t + t'}{2}$. Generally, one can calculate the noninteracting Green’s function analytically and obtain the interacting Green’s function using the Dyson equation. To keep a structure of the Dyson equation that is convenient for calculations, one needs to use the Floquet Green’s function. It is defined by the map $G_{mn}(\omega) = G_{m-n}(\omega + \frac{m+n}{2} \Omega)$, where $\omega \in (-\frac{\Omega}{2}, \frac{\Omega}{2})$. (ii) To achieve a NESS, every lattice site is coupled to a bath, which extracts energy from the driven lattice. The bath can be fermionic [30, 34, 37] or bosonic [39]. We consider a free-fermion bath in our calculations. The effect of the bath is equivalent to a correction to the self-energy, namely [30, 34, 37],

$$G_{\mathbf{k}}^{-1}(\omega) = G_{\mathbf{k}\Omega}^{-1}(\omega) - \Sigma(\omega) - \Sigma_{\text{bath}}(\omega).$$

All quantities here have three components, for example, $G_{\mathbf{k}}(\omega) = \begin{pmatrix} G^R_{\mathbf{k}}(\omega) & G^A_{\mathbf{k}}(\omega) \\ G^A_{\mathbf{k}}(\omega) & G^R_{\mathbf{k}}(\omega) \end{pmatrix}$, and every component is a Floquet matrix. Within a flat density of states approximation [30, 37], $\Sigma_{\text{bath}}(\omega) = \frac{i}{\beta} 2iF(\omega)I$ with the $n$th component of $F(\omega)$ given by $F_n(\omega) = \tan^{-1} \frac{\hbar(\omega + \Omega)}{2k_B T}$, where the bath is described by two parameters: damping rate $\Gamma$ and bath temperature $T$. $I$ is the unit matrix. The noninteracting part is $G_{\mathbf{k}\Omega}^{-1}(\omega) = \begin{pmatrix} (G^R_{\mathbf{k}\Omega})^{-1}(\omega) & (G^{A^R}_{\mathbf{k}\Omega})^{-1}(\omega) \\ (G^{A^A}_{\mathbf{k}\Omega})^{-1}(\omega) & (G^{A^R}_{\mathbf{k}\Omega})^{-1}(\omega) \end{pmatrix}$, where $(G^R_{\mathbf{k}\Omega})^{-1}(\omega)$ can be determined from the noninteracting Hamiltonian, and it can be shown from the
fluctuation-dissipation theorem that \((G^{-1}_{K_0} (\omega))^K\) is negligible \([30, 34]\). For a driven Hubbard model we can use iterative perturbation theory \([29, 30]\) as an impurity solver. For the driven Falicov-Kimball model the effective impurity problem can be solved analytically in infinite dimensions.

We next describe the formalism for generalizing Floquet DMFT to a position-dependent self-energy, which is suitable for studying an inhomogeneous system. We have a lattice of driven, effective impurity models, which are coupled via the lattice Dyson equation, which is the real-space version of Eq. (5),

\[
(G^{-1})_{R, mn}(\omega) = \left( (G^{-1}_0)_{R', mn}(\omega) - \Sigma_{R, mn}(\omega) \delta_{R,R'} - (\Sigma_{\text{bath}})_{R, mn}(\omega) \delta_{R,R'} \right)^{-1}.
\]

The noninteracting Floquet Green’s function is

\[
(G^{-1}_0)_{R', mn}(\omega) = \left( (G^{-1}_0)_{R', mn}(\omega) - \Sigma_{R', mn}(\omega) \delta_{R,R'} \right)^{-1} = \omega + n\Omega - \mathcal{H}_{R', mn}^{(0)} \pm i\Gamma, \]

where \(\mathcal{H}_{R', mn}^{(0)}\) is given by Eq. (3) in the main text, with lattice sites \(R = ie_x + je_y\) and \(R' = i'e_x + j'e_y, \ i(i')\) and \(j(j')\) label the \(x\) and \(y\) coordinates of the sites of a finite lattice, and \(m\) and \(n\) are indices of a Floquet matrix. \((G^{-1}_0)_{R', mn}(\omega)^{-1}\) is again negligible. The self-energies \(\Sigma_R\) and \(\Sigma_{\text{bath}, R}\) are diagonal in position space, and \(\Sigma_R\) is determined self-consistently by an impurity solver.

For the interaction in Eq. (4) in the main text, we adopt the following solver for each one of the effective impurity problems, i.e., for the impurity at site \(R\) the Floquet Green’s function is

\[
G_R(\omega) = w_0 G_{0,R}(\omega) + w_1 \left[ G_{-1, R}(\omega) - U \right]^{-1},
\]

where \(G_{0,R}(\omega)\) is the Weiss function and determined self-consistently, \(w_1\) is the filling of the localized \(f\) atoms, and \(w_0 = 1 - w_1\). Equations (6), (7), and the impurity Dyson equation

\[
G_{-1, R}(\omega) = G^{-1}_R(\omega) + \Sigma_R(\omega)
\]

form the set of self-consistency equations of real-space Floquet DMFT (Fig. 5).

\[B. \ \text{Bath effects}\]

Let us also comment on the free-fermion bath coupled to the lattice. For a flat density of states, the bath is described by two parameters: the dissipation \(\Gamma\) and the temperature \(T\). In principle, one can tune the parameters of the bath to change the final NESS. For the simple bath used here, we can show that the effect of the bath is not very pronounced. In Fig. 6, we show spectral functions for the driving frequency \(\Omega = 7\) and \(\alpha = \frac{1}{5}\), with different bath parameters. For \(U = 0\), there is a clear difference because if it affects the imaginary part of \((G_0^{R(A)})_{R', mn}^{-1}(\omega)^{-1}\) directly. For finite \(U\), there is only a minor difference.

\[\text{Figure 5: Flow chart of real-space Floquet DMFT. All Green's functions are defined on the Keldysh contour and in real and Floquet space.}\]

\[\text{Figure 6: Spectral functions } A_{ij} (\omega + n\Omega) = -\frac{1}{2} \text{Im} G_{ij, mn}^R(\omega) \text{ at the center site of a } 15 \times 15 \text{ square lattice with different baths for } \Omega = 7 \text{ and } \alpha = \frac{1}{5}. \quad \text{“Bath 0” with } \Gamma = 0.05 \text{ and } T = 0.05 \text{ is what we used for all the results in the main text. For “Bath 1”, } \Gamma = 0.025 \text{ and } T = 0.05, \text{ and for “Bath 2”, } \Gamma = 0.05 \text{ and } T = 0.025.\]

[1] D. R. Hofstadter, Phys. Rev. B 14, 2239 (1976).
[2] F. D. M. Haldane, Phys. Rev. Lett. 61, 2015 (1988).
[3] M. Aidelsburger, M. Atala, M. Lohse, J. T. Barreiro, B. Paredes, and I. Bloch, Phys. Rev. Lett. 111, 185301 (2013).
[4] H. Miyake, G. A. Siviloglou, C. J. Kennedy, W. C. Burton, and W. Ketterle, Phys. Rev. Lett. 111, 185302 (2013).
[5] G. Jotzu, M. Messer, R. Desbuquois, M. Lebrat, T. Uehlinger, D. Greif, and T. Esslinger, Nature (London) 515, 237 (2014).
[6] D. Jaksch and P. Zoller, New J. Phys. 5, 56 (2003).
[7] T. Oka and H. Aoki, Phys. Rev. B 79, 081406 (2009).
[8] M. Bukov and A. Polkovnikov, Phys. Rev. A 90, 043613 (2014).
[9] N. Goldman, J. Dalibard, M. Aidelsburger, and N. R. Cooper, Phys. Rev. A 91, 033632 (2015).
[10] M. Aidelsburger, M. Lohse, C. Schweizer, M. Atala, J. T. Barreiro, S. Nascimbene, N. R. Cooper, I. Bloch, and N. Goldman, Nat. Phys. 11, 162 (2015).
[11] C. J. Kennedy, W. C. Burton, W. C. Chung, and W. Ketterle, Nat. Phys. 11, 859 (2015).
[12] A. Verdeny, A. Mielke, and F. Mintert, Phys. Rev. Lett. 111, 175301 (2013).
[13] N. Goldman and J. Dalibard, Phys. Rev. X 4, 031027 (2014).
[14] M. Bukov, L. D’Alessio, and A. Polkovnikov, Adv. Phys. 64, 139 (2015).
[15] T. Mikami, S. Kitamura, K. Yasuda, N. Tsuji, T. Oka, and H. Aoki, Phys. Rev. B 93, 144307 (2016).
[16] A. Eckardt, Rev. Mod. Phys. 89, 011004 (2017).
[17] S. Rahav, I. Gilary, and S. Fishman, Phys. Rev. A 68, 013820 (2003).
[18] M. Račiunas, G. Žlabys, A. Eckardt, and E. Anisimovas, Phys. Rev. A 93, 043618 (2016).
[19] K. Plekhanov, G. Roux, and K. Le Hur, Phys. Rev. B 95, 045102 (2017).
[20] T. Kitagawa, T. Oka, A. Brataas, L. Fu, and E. Demler, Phys. Rev. B 84, 235108 (2011).
[21] M. S. Rudner, N. H. Lindner, E. Berg, and M. Levin, Phys. Rev. X 3, 031005 (2013).
[22] D. Carpentier, P. Delplace, M. Fruchart, and K. Gawędzki, Phys. Rev. Lett. 114, 106806 (2015).
[23] F. Nathan and M. S. Rudner, New J. Phys. 17, 125014 (2015).
[24] Z. Wang and S.-C. Zhang, Phys. Rev. X 2, 031008 (2012).
[25] T. Iadecola, T. Neupert, and C. Chamon, Phys. Rev. B 91, 235133 (2015).
[26] H. Dehghani, T. Oka, and A. Mitra, Phys. Rev. B 90, 195429 (2014).
[27] K. I. Seetharam, C.-E. Bardyn, N. H. Lindner, M. S. Rudner, and G. Refael, Phys. Rev. X 5, 041050 (2015).
[28] D. Vorberg, W. Wustmann, R. Ketzermei, and A. Eckardt, Phys. Rev. Lett. 111, 240405 (2013).
[29] A. Georges, G. Kotliar, W. Krauth, and M. J. Rozenberg, Rev. Mod. Phys. 68, 13 (1996).
[30] H. Aoki, N. Tsuji, M. Eckstein, M. Kollar, T. Oka, and P. Werner, Rev. Mod. Phys. 86, 779 (2014).
[31] J. K. Freericks and A. V. Joura, “Nonequilibrium density of states and distribution functions for strongly correlated materials across the Mott transition,” in Electron Transport in Nanosystems, edited by J. Bonča and S. Kruchinin (Springer, Dordrecht, 2008), pp. 219–236.
[32] A. V. Joura, J. K. Freericks, and T. Pruschke, Phys. Rev. Lett. 101, 196401 (2008).
[33] N. Tsuji, T. Oka, and H. Aoki, Phys. Rev. B 78, 235124 (2008).
[34] N. Tsuji, T. Oka, and H. Aoki, Phys. Rev. Lett. 103, 047403 (2009).
[35] A. Gómez-León and G. Platero, Phys. Rev. Lett. 110, 200403 (2013).
[36] H. Sambe, Phys. Rev. A 7, 2203 (1973).
[37] N. Tsuji, Theoretical study of nonequilibrium correlated fermions driven by ac fields, Ph.D. thesis, Department of Physics, University of Tokyo, 2010.
[38] A. Lubatsch and J. Kroha, Ann. Phys. (Berlin) 18, 863 (2009).
[39] W.-R. Lee and K. Park, Phys. Rev. B 89, 205126 (2014).
[40] M.-T. Tran, Phys. Rev. B 81, 115119 (2010).
[41] We notice that the isotropic case has been achieved in the experiments of Refs. [4] and [10] with fluxes $\frac{1}{2}$ and $\frac{1}{4}$, respectively.
[42] C. Chin and E. J. Mueller, Physics 6, 118 (2013).
[43] P. Törmä, Phys. Scr. 91, 043006 (2016).
[44] J. T. Stewart, J. P. Gaebler, and D. S. Jin, Nature (London) 454, 744 (2008).
[45] A. Griessner, A. J. Daley, S. R. Clark, D. Jaksch, and P. Zoller, Phys. Rev. Lett. 97, 220403 (2006).
[46] G. Jotzu, M. Messer, F. Görg, D. Greif, R. Desbuquois, and T. Esslinger, Phys. Rev. Lett. 115, 073002 (2015).
[47] M. Snoek, I. Titvinidze, C. Töke, K. Byczuk, and W. Hofstetter, New J. Phys. 10, 093008 (2008).