Spin selective transmission through a multi-terminal Rashba ring with AAH modulation

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Abstract. For the first time we explore spin selective electron transmission through a multi-terminal nano ring subjected to Rashba spin-orbit coupling and magnetic flux, in presence of cosine modulation. The modulation is taken in the form of well known Aubry-Andre or Harper model which is associated with a phase factor that can be tuned externally with a suitable setup. Describing the quantum systems within a tight-binding framework, we compute the results based on the Green’s function formalism. Our results suggest that a large degree of spin polarization along with a phase reversal can be achieved by regulating the physical parameters associated with the system. The present analysis provides several important features that may be utilized to investigate spin dependent transport in other multi-terminal geometries with similar kind of cosine modulation.

1. Introduction

‘Spintronics’-Spin Electronics, an emerging branch in condensed matter physics where spin of electron is manipulated with suitable operators. No doubt an enormous amount of work has already been done [1-8] in last few years though many open questions still persist and several opportunities are still there to probe along this line of work. For instance, the most fundamental question is that can we establish a suitable setup that on one hand can produce a large degree of spin polarization, and on the other hand the phase can be regulated ‘externally’. Earlier, ferromagnetic materials were used as functional elements though their usability gradually decreases with time due to many reasons, which include resistivity mismatch, poor injection efficiency and smaller phase coherence length. The tuning of spin polarization is of course another important issue, since for it we need to apply suitable magnetic field, which in some cases becomes quite difficult to implement due to small scale.

All these facts quite circumvented with the use of Spin-Orbit (SO) coupled systems, which are considered as role models of future spintronic devices. In solid state materials usually two types of SO couplings are taken into account depending on their sources. One is originated due to asymmetry in the confining potential which is referred to as Rashba SO coupling [9], while the other, known as Dresselhaus SO coupling [10], is generated due to the asymmetry in the bulk. Among these two,
Rashba SO coupling draws significant attention as its strength can be regulated by an external gate voltage, while the kind of SO coupling cannot be tuned externally [11-12] as it is a sample specific property.

The enormous possibilities of Rashba SO coupling have already been revealed in different bridge systems like 2-terminal, 3-terminal and even more, exploring many interesting features [13-20]. Here we propose a new prescription of getting controlled spin polarization in a spin-orbit coupled system by introducing a Cosine type modulation in site energies of a tight-binding (TB) ring. The most popular model involving this kind of modulation is known as Aubre-Andre or Harper (AAH) model, a classic example of quasi-crystals [21-24]. Due to the peculiar gapped energy spectrum, AAH model always provides non-trivial signatures over the conventional lattices. Moreover, the phase factor associated with gives a possible tuning of spin polarization. In this work we investigate in detail the effect of AAH modulation on spin polarization considering a three-terminal junction setup (see Fig.1).

The work is framed as follows. In Sec. 2 we present the nanojunction and give a brief outline of the theoretical formulation for all the calculations. The results are thoroughly discussed in Sec. 3. Finally, in Sec. 4 we summarize our essential findings.

2. Quantum system and Theoretical framework

Let us start with the spin polarized set up as shown in Fig.1, where a SO coupled ring, subjected to AAH kind of modulations coupled to three electrodes. Electrode 1 is used for injecting the unpolarized electrons, while they get transmitted through the two outgoing electrodes 2 and 3.

Figure 1. Spin polarized setup where a SO coupled ring, subjected to a magnetic flux $\phi$, is directly coupled to three leads, where lead-1 acts as the input lead, while the other two are used for the output leads. Lead-1 is coupled site 1, while the other two leads are attached to sites $p$ and $q$ (those are variable) of the ring.

The purpose of this setup is to get two different kinds of spin polarization (up and down) from the two outgoing electrodes simultaneously which are no longer possible in a two-terminal junction. The ring is further threaded by a magnetic flux $\phi$, which is commonly known as Aharonov-Bohm flux, and it plays the role of an additional tuning parameter in our analysis. The AB effect always plays a key role in transport phenomena as it directly modulates the phenomenon of quantum interference among the electronic waves [25-26].
We describe the quantum system within a tight-binding framework with nearest-neighbor hopping (NNH) approximation. The TB Hamiltonian of the full junction setup is written as,

\[ H = H_{\text{ring}} + H_{\text{elec}} + H_{\text{nn}} \]  

where the terms describe the Hamiltonians for the ring, the side attached electrodes and the coupling between the ring and the electrodes, respectively.

For a \( N \)-site ring with Rashba SO coupling and AB flux, the Hamiltonian reads as

\[ H_{\text{ring}} = \sum_{n} \tilde{c}_{n}^{\dagger} \tilde{c}_{n} + \sum_{n} \left( \tilde{c}_{n,n+1}^{\dagger} \tilde{c}_{n+1} + h.c. \right) \]  

Here, \( \tilde{c}_{n}^{\dagger} = \left( c_{n,n+1}^{\dagger}, c_{n,n-1}^{\dagger} \right) \), \( \tilde{c}_{n} = \left( c_{n,n+1}, c_{n,n-1} \right) \), \( t_{n,n+1} = t_{n,n+1}^{0} + t_{n,n+1}^{R} \), \( t_{n,n+1}^{0} = \begin{pmatrix} t & 0 \\ 0 & t \end{pmatrix} \)

and, \( t_{n,n+1}^{0} = -i \alpha \left( \cos \frac{\varphi_{n} + \varphi_{n+1}}{2} \sigma_{x} + \sin \frac{\varphi_{n} + \varphi_{n+1}}{2} \sigma_{y} \right) \).

The factor \( \alpha \) describes the strength of SO coupling in the ring, \( \varphi_{n} \) represents the azimuthal angle for \( n \)-th site, and mathematically it can be expressed as [27]

\[ \varphi_{n} = \frac{2\pi (n-1)}{N}. \]

A similar kind of TB Hamiltonian, apart from Rashba spin dependent scattering term is used to describe the side attached electrodes. They are parameterized with on-site potential \( \varepsilon_{0} \) and NNH integrals \( t_{0} \). There electrodes are directly coupled to the ring, where the coupling strengths are defined by the parameter \( \tau_{p} \) (\( p = 1, 2, 3 \)).

To study spin selective electron transmission through the nanojunction the first and the foremost thing that needs to be evaluated is the spin dependent transmission probabilities. We compute them following the well-known Green’s function formalism. In this prescription, the effects of the contact leads are incorporated through self-energy correction. The effective Green’s function is defined as [28]

\[ G^{\sigma} = \left( E - H_{\text{ring}} - \Sigma_{1} - \Sigma_{2} - \Sigma_{3} \right)^{-1} \]  

where, \( \Sigma_{p} \) (\( p = 1, 2, 3 \)) is the self energy due to lead-\( p \).

From the Green’s function, we calculate spin dependent transmission probabilities from lead \( p \) with spin \( \sigma \) to lead \( q \) with spin \( \sigma' \) using the following relation [28]

\[ T_{\sigma \sigma'}^{pq} = \text{Tr} \left[ \Gamma_{\sigma}^{p} G^{\sigma} G^{\sigma'} \Gamma_{\sigma'}^{q} \right] \]  

where, \( \Gamma_{\sigma}^{p} \)’s are the coupling matrices and they are defined as [28] \( \Gamma_{\sigma} = -2 \text{Im} \left( \Sigma_{p} \right) \).

Depending on \( \sigma \) and \( \sigma' \), we get pure (\( \sigma = \sigma' \)) or spin-flip (\( \sigma \neq \sigma' \)) transmissions. Using \( T_{\sigma \sigma'}^{pq} \) we define the net up and down spin transmission probabilities through any outgoing lead \( q \) as,

\[ T_{\uparrow \downarrow}^{q} = T_{\uparrow \uparrow}^{q} + T_{\downarrow \downarrow}^{q} \]

\[ T_{\downarrow \downarrow}^{q} = T_{\uparrow \downarrow}^{q} + T_{\downarrow \uparrow}^{q} \]
Finally we define the spin polarization coefficients by the relation

$$P_q = \frac{T^q_u - T^q_d}{T^q_u + T^q_d}$$

(6)

$$P_q \rightarrow 0$$ means no polarization in the output current, while, $$P_q \rightarrow \pm 1$$ denotes 100% up (or down) spin polarization.

3. Numerical Results and Discussions

The central focus of our work is to establish a suitable prescription to get (i) high degree of spin polarization and (ii) its possible tuning under different input conditions. The system Hamiltonian contains several important physical parameters that may strongly affects the polarization efficiency, and here we concentrate on all these quantities one by one to make the present communication a self contained study. Spin selective transmission are measured through the two outgoing leads. We refer the lattice sites of the ring as m and n respectively, where the outgoing leads (i.e., lead 2 and lead 3) are connected. Lead-1 is assumed to be connected at the lattice site 1 of the ring throughout the analysis.

For the numerical calculation we choose the common set of parameter values as follows. The site energies and the NNH integrals in the leads are taken as 0 and 1 eV respectively. The coupling strengths of the electrodes with the ring are fixed at 1 eV. In the ring the hopping strength is set at 1 eV and in absence of any kind of Cosine modulation the site energies in the ring are chosen as zero, i.e., $$\epsilon_{m\uparrow} = \epsilon_{n\uparrow} = 0$$ \forall n. All the results are performed at absolute zero temperature. All energies are measured in units of eV.

Let us start our discussion with Fig 2. Where spin polarization coefficient is shown as a function of energy E considering a perfect ring with $$N = 20$$. Three different ring-lead junction configurations are taken into account to explore the effects of quantum interference. In Figs. 2(a) and (b) the leads are attached asymmetrically while in fig. 2(c) there is connected symmetrically. A complete phase takes place in spin polarization coefficients when the outgoing leads are connected symmetrically with respect to the source lead. It can be argued from the symmetry analysis as put forward by Kim et al.

For this junction configuration the up and down spin electrons scatter exactly in opposite directions due to the SO coupling, and thus the effective spin transmissions are fully opposite in nature. With the breaking of the symmetry by connecting the outgoing leads at different distances compared to the
source electrodes, the complete phase reversal disappears, as clearly noted from the spectra given in Figs. 2(a) and (b). An important feature that is observed here is that for the asymmetric junction setup one can get a large degree of spin polarization (for up or down) through anyone of the two outgoing leads even when the polarization becomes zero in the other lead. This phenomenon is no longer possible for the symmetrically connected setup as either the polarization of the output current through both the leads are zero or finite with identical magnitude. This feature is fully due to the quantum interference effect of electronic waves in this multi-connected geometry.

**Figure 3:** Spin polarizations through lead-2 (red line) and lead-3 (light green line) for a 20-site perfect ring as a function of SO coupling strength $\alpha$ with three distinct junction configurations as considered in Fig.2. All the results are computed at $E = -0.25$ and $\phi = 0$

The role of Rashba SO coupling is shown in Fig. 3. The results are computed for the identical ring-lead interface geometries as taken in Fig. 2. A regular oscillatory pattern with reduced amplitude is observed. Similar kind of oscillations by varying Rashba SO coupling has been reported in several other contemporary works. The interesting fact is that, keeping all the physical parameters unchanged one can alter the phase of polarization (positive or negative) simply by tuning the SO coupling strength. The degree of polarization essentially depends on the misalignment of the up and down spin channels. For any typical energy $E$, when any of these two channels dominates we get up or down spin polarization. Now changing $\alpha$ we eventually regulate the misalignment between the up and down spin bands which provides a regular oscillatory pattern in $P-\alpha$ characteristics. Thus, undoubtedly $\alpha$ is an important physical parameter that can provide selective spin transmission. The reflection symmetry between the two colored curves across $P = 0$ line does not exist for the asymmetric junction configuration.

**Figure 4:** Role of AB flux $\phi$ on spin polarization shown. The results are computed at $E = -0.2$ for the identical ring-lead junction configurations as taken in Fig.2. The other physical parameters are $N = 20$ and $\alpha = 0.5$. The green and black curves represent the results for the lead-2 and lead-3 respectively.

The effect of magnetic flux $\phi$ on spin polarization is illustrated in Fig.4 considering an ordered ring. The identical junction configurations are also taken into account like above. In presence of $\phi$, the propagating electronic waves acquire a phase referred as AB phase and quantum interference gets modified which affects spin polarization. From all the spectra the dependence of $\phi$ on $P$ is clearly reflected. Moreover, in some cases, just the inclusion of $\phi$ provides a large change in $P$ compared to the zero flux case. This is first of all very helpful since the inclusion of large magnetic flux in a small
ring is itself a very challenging task. The other aspect is that, for the symmetric junction configuration
the polarization coefficients in the two outgoing leads get completely reflected with each other about
an imaginary vertical line passing through $\phi = 0$ axis.

Figure 5: Spin polarization coefficients as a function of $E$ for a symmetrically connected junction ($m = 8$ and $n = 14$) in
presence of AAH modulation where (a) $W = 0.25$, (b) $W = 0.5$ and (c) $W = 0.75$. The AAH phase is set zero. The other
parameters are: $N = 20$, $\alpha = 0.5$ and $\phi = 0$. The two distinct colored curves correspond to the same meaning as described
in Fig. 2.

The results studied so far are computed for ordered rings. Now we include AAH modulations in site
energies. Mathematically we can express the site energies as
\[ \epsilon_n^{\uparrow} = \epsilon_n^{\downarrow} = W \cos(2\pi bn + \phi_e), \]
where $W$ measures the AAH modulation strength and $\phi_e$ represents the AAH phase [21]. With
suitable setup the phase $\phi_e$ can be regulated ‘externally’. This is key feature of the present modulated
system.

Figure 6: Regulation of $P$ by means of the AAH phase $\phi_e$. Here we take the same junction configuration as used in Fig. 5.
The results are worked out at $E = 0$ considering $N = 20$, $W = 1$, $\phi = 0$ and $\alpha = 0.5$. The green and blue lines represent the
polarizations through lead-2 and lead-3, respectively.

The factor is an incommensurate number and we set is as Golden Mean [21] i.e., $b = (1 + \sqrt{5})/2$,
though any other irrational number can also be considered. As the site energies are deterministic in
nature, we do not need to take any configuration averaging. For this disordered case, the symmetry
between the upper and lower arms gets destroyed even though the outgoing leads are coupled at equal
distances from the source lead, and accordingly, the symmetry around $P = 0$ line vanishes. It is well
known that with increasing impurity strength, transmission probability gets reduced, but for our case
$P$ remains reasonably large even we take $W = 0.75$. This is due to the fact that $P$ is determined from
the ratio of transmission probabilities. So, even if they are small the ratio can be sufficiently large.
What emerges from the result is that, even for disordered systems we can have a finite possibility to
get moderate spin polarization.
Finally, we focus on the possible tuning of spin polarization by means of the AAH phase $\phi_r$ when all other physical parameters are kept constant. The results are shown in Fig. 6. Interestingly, we find that the spin polarization can be changed in a wide range along with its phase reversal by regulating $\phi_r$. The effect of $\phi_r$ becomes significant when $N$ is relatively small, as in this case, the change in site energies with $\phi_r$ is quite large which modifies the energy channels accordingly.

While for large $N$ the differences in site energies are too small with the alteration of $\phi_r$ and therefore it is hard to realize the role of $\phi_r$ in the polarization coefficients.

4. Closing Remarks

To summarize, in the present work we have investigated spin selective transmission through a multi-terminal spin-orbit coupled system in presence of cosine modulation and Aharonov-Bohm flux. The quantum system has been described by a TB framework, where all the calculations have been worked out using the well known Green’s function formalism. Several interesting results have been achieved those are mentioned as follows.

i. Results are highly sensitive to the ring-lead interface geometry.
ii. Spin polarization oscillates following a regular pattern with Rashba SO coupling.
iii. Inclusion of even a very small AB flux can produce a large degree of spin polarization.
iv. AAH modulation does not suppress the spin polarization significantly even for moderate $W$.
v. Finally, a large tuning of $P$ can be made by suitably adjusting the AAH phase $\phi_r$.

Before we end we would like to point out that though the results are computed for a particular ring size and some specific set of parameter values, all the physical pictures studied here remain unchanged for other system sizes as well (within the mesoscopic range) and for other values of the physical parameters.

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