Experimental Research on the Elastic Deformation Mode of S235JR Rolled Steel Fastened between the Centers of a Universal Lathe

LL Tabacaru¹, E Axinte² and G Musca³
¹,²,³ "Gheorghe Asachi" Technical University of Iasi, Faculty of Machine Manufacturing and Industrial Management, Iasi, Romania

E-mail: ltabacaru@tcm.tuiasi.ro

Abstract. Elastic deformations of the technological system occur during the mechanical treatment of a blank, regardless of the manner in which it is fastened. The elastic deformation of the blank is significant especially when machining shaft-like parts. The purpose of our research is to compare the mathematical model of blank deformation to the experimental model when the blank, which is a part belonging to the shaft class, is fastened between centers.

1. Introduction
The technological system may suffer elastic deformations under the action of exterior forces. These deformations occur on the direction of action of the forces. System rigidity is defined as its capacity to resist deformation. The forces that act in the technological system are the cutting forces. The Py force is usually used to determine system rigidity. The proper approach would be to calculate the resultant of the three forces first and then the rigidity [1].

The rigidity of a system is calculated as the component/deformation ratio

\[ R = \frac{P_y}{\gamma} \text{[daN/mm]} \]

The force/deformation ratio was determined experimentally, by entering different values of the Py cutting force into the technological system and then measuring the deformation triggered by them. Figure 2 shows the deformation law applying to regular machine tools where the \( \gamma_0 \) represents plastic deformation is actually the weight shift in view of the processing of the clearances occurring between the assembly components.

The surface between the curves is the mechanical work meant to resist the friction forces between components on clearance processing, on unevenness processing.

Further to repeated loading on the same component, the surface between the two curves decreases and the mechanical work diminishes. This surface shape is called rigidity hysteresis.

\[ R_{med} = \frac{AB}{OB} = \tan \alpha \text{ - mean rigidity} \]

Further to repeated loading on the same component, the surface between the two curves decreases and the mechanical work diminishes. This surface shape is called rigidity hysteresis:
2. Mathematical model

The work pattern shown in figure 3 [1] is used to determine the processing error caused by the elastic deformation of the technological system when the blank is placed between centers (figure 13).

The following technological system deformations occur under the influence of the Py force:
- headstock shift;
- tailstock shift;
- part shift in section x caused by headstock and tailstock shift
- part arrow in section x;
- tool and tool support shift under the action of $P_y$;

$$ t_{pr} - t_r = t_{rem} = y_1 + y_2 + y_3 ; \quad y_1 = FH + GH = y_{pf} + \frac{x}{l} (y_{pm} - y_{pf}) $$

$$ \frac{\Delta GH}{\Delta EI} = \frac{EI}{CI} \cdot GH = \frac{x}{l} (y_{pm} - y_{pf}) $$
The problem arising here is to determine the extent to which the Py force acts on the tailstock and the extent to which the same force acts on the headstock (figure 4).

\[ P_{yt} = P_y \frac{l-x}{l} \]
\[ P_{ys} = P_y \frac{x}{l} \]
\[ y_{pt} = \frac{P_y}{R_{pf}} \frac{l-x}{l} \]
\[ y_{pm} = \frac{P_y}{R_{pm}} \frac{x}{l} \]
\[ y_1 = \frac{P_y}{R_{pf}} \frac{l-x}{l} + \frac{x}{l} \left( \frac{P_y}{R_{pm}} \frac{x}{l} - \frac{P_y}{R_{pf}} \frac{l-x}{l} \right) \]
\[ y_2 = \frac{P_y}{R_{pf}} \left( \frac{l-x}{l} - \frac{x}{l} \right)^2 + \frac{P_y}{R_{pm}} \left( \frac{x}{l} \right)^2 \]
\[ y_3 = \frac{P_y}{3EI} \frac{x^2(l-x)^2}{l} \]
\[ t_{pr} = t_r + \frac{P_y}{R_{pf}} \left( \frac{l-x}{l} \right)^2 + \frac{P_y}{R_{pm}} \left( \frac{x}{l} \right)^2 + \frac{P_y}{3EI} \frac{x^2(l-x)^2}{l} + \frac{P_y}{R_{sc}} \text{ if } P_y = c \cdot k \cdot t_r \]
\[ t_{pr} = \frac{1}{1+A} \]
\[ t_{rem} = t_{pr} - t_r = t_{pr} \left[ 1 - \frac{1}{1+A} \right] \]

When a batch of parts is processed, the prescribed depth ranges from to with a variation which causes an error on the y direction.

\[ \Delta_y = t_{rem_{max}} - t_{rem_{min}} = \left[ t_{pr_{max}} - t_{pr_{min}} \right] \left[ 1 - \frac{1}{1+A} \right] \]

on the diameter:

\[ \Delta D = 2 \Delta y \]

Assuming that the machine tool is very rigid, which means that tailstock, headstock and tool shifts are absent, y1 and y3 are null and hence the processing error is the result of the elastic deformation of the part. The shape of the part is shown in figure 5.

When the part is very rigid, i.e. it does not undergo any elastic deformation, and assuming that the tool and the support are very rigid, the shape of the part is influenced solely by y1.

The processing error will only be determined by y1(figure 5, 6).

\[ t_{rem} = y_1 = \frac{P_y}{R_{pf}} \left( \frac{l-x}{l} \right)^2 + \frac{P_y}{R_{pm}} \left( \frac{x}{l} \right)^2 \]
Figure 5. Shape of the part when the machine tool is very rigid

Figure 6. Processing error caused solely by $y_1$.

Figure 7. Experimental pattern used to assess the static elastic deformation of various components subject to the $F_y$ force

A shape error also occurs, which is a quadric.

$y_{\text{min}} = y_{\text{max}} - y_{\text{min}}$

- In order to determine $y_{\text{min}}$, we will vanish $t_{\text{rem}} = 0$:

$$t_{\text{rem}} = \frac{P_y}{R_{pf}} \cdot \frac{2x}{l^2} - \frac{P_y}{R_{pf}} \cdot \frac{2l}{l^2} + \frac{P_y}{R_{pm}} \cdot \frac{2x}{l^2} = 0$$

$$\frac{x}{l} \left[ \frac{1}{R_{pf} \cdot l} + \frac{1}{R_{pm} \cdot l} \right] = \frac{1}{R_{pf} \cdot l} \Rightarrow x = \frac{R_{pm}}{R_{pf} + R_{pm}}$$

By replacing in $t_{\text{rem}}$, we determine $y_{\text{min}}$:

$$y_{\text{min}} = \frac{P_y}{R_{pf}} \left[ 1 - \frac{R_{pm}}{R_{pf} + R_{pm}} \right]^2 + \frac{P_y}{R_{pm}} \left( \frac{R_{pm}}{R_{pf} + R_{pm}} \right)^2 \cdot \left[ \frac{R_{pf} + R_{pm} - R_{pm}}{R_{pm} + R_{pm}} \right]^2$$

$$y_{\text{min}} = \frac{P_y}{R_{pf}} \cdot \left( \frac{R_{pf}}{R_{pf} + R_{pm}} \right)^2 + \frac{P_y}{R_{pm}} \cdot \left( \frac{R_{pm}}{R_{pf} + R_{pm}} \right)^2 = \frac{P_y}{R_{pf} + R_{pm}}$$
3. Experimental model

In this paper we aim at determining the processing errors caused by the elastic deformations of the technological system when placing parts between centers. For this experiment we used a 1000 mm long and 40 mm wide S 235 JR metal bar, which observed the SR EN 10025 requirements, and a SN 250 machine tool.

The method consists of the static simulation of the forces generated during the turning process; the elastic deformations of the various components are determined after the force applied on the work piece has been increased, which should provide data on the rigidity of the turning system figure 7.

Indeed, from the viewpoint of the various methods of work piece fastening, it may be interesting to assess the system’s resistance to deformation caused by the cutting forces. Several simplifying hypotheses were considered for the case described in figure 7, namely that the $F_y$ force.

As shown in figure 7, a dynamometer is placed between the tool holder and the work piece in order to read at any time the force entered into the system, by the rotation of one of the dynamometer nuts. We used dial gauges placed every 170 mm and at the ends of the blank, in order to read the elastic deformation of the work piece subject to different $F_y$ load forces, which materialize various cutting forces. The dynamometer touched the work piece and a parallelepiped piece replacing the turning tool, by means of two metal balls, with the aim of ensuring the best possible transmission of the radial force.

| Exp. no. | Force $F_y$[daN] (increments) applied to $x=170$ mm | Elastic deformation [mm] |
|----------|-----------------------------------------------|------------------------|
|          | Rotating center - headstock | $x=170$ | $x=340$ | $x=510$ | $x=680$ | Rotating center - tailstock |
| 1        | 0 (0)                          | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 |
| 2        | 18.18 (20)                     | 0.0 | 0.0 | 0.05 | 0.11 | 0.09 | 0.2 | 0.09 | 0.16 | 0.06 | 0.07 |
| 3        | 36.36 (40)                     | 0.3 | 0.3 | 0.12 | 0.18 | 0.21 | 0.3 | 0.18 | 0.27 | 0.14 | 0.05 |
| 4        | 54.54 (60)                     | 0.5 | 0.4 | 0.25 | 0.24 | 0.30 | 0.4 | 0.27 | 0.36 | 0.21 | 0.12 |
| 5        | 73.05 (80)                     | 0.6 | 0.6 | 0.33 | 0.31 | 0.40 | 0.5 | 0.38 | 0.46 | 0.28 | 0.19 |
| 6        | 91.57 (100)                    | 0.8 | 0.7 | 0.41 | 0.38 | 0.50 | 0.5 | 0.48 | 0.54 | 0.36 | 0.26 |
| 7        | 109.19 (120)                   | 0.9 | 0.9 | 0.50 | 0.45 | 0.60 | 0.6 | 0.58 | 0.64 | 0.39 | 0.34 |
| 8        | 127.27 (140)                   | 1.1 | 1.1 | 0.57 | 0.58 | 0.75 | 0.7 | 0.71 | 0.73 | 0.41 | 0.42 |
| 9        | 145.45 (160)                   | 1.1 | 1.1 | 0.60 | 0.60 | 0.86 | 0.8 | 0.80 | 0.80 | 0.41 | 0.41 |

In order to determine the highest value of the $F_y$ experimental force that may be used during the experimental research, we considered hard turning using a regular cutting tool. The work piece was
Table 2. Elastic deformation read in various points on the part for various $F_y$ force values applied at $x = 340$ mm.

| Exp. no. | Force $F_y$[daN] applied at $x=340$ mm | Elastic deformation [mm] |
|----------|--------------------------------------|-------------------------|
|          | Rotating center - headstock          | x=170 | x=340 | x=510 | x=680 | Rotating center - tailstock |
|          |                                      | C | D | C | D | C | D | C | D | C | D | C | D |
| 1        | 0                                    | 0.0 | 0.0 | 0.0 | 0.00 | 0.00 | 0.0 | 0.00 | 0.0 | 0.00 | 0.0 | 0.0 |
| 2        | 18.18                                | 0.1 | 0.2 | 0.0 | 0.00 | 0.13 | 0.04 | 0.21 | 0.01 | 0.15 | 0.06 | 0.0 |
| 3        | 36.36                                | 0.3 | 0.4 | 0.1 | 0.00 | 0.23 | 0.03 | 0.37 | 0.28 | 0.15 | 0.12 | 0.0 |
| 4        | 54.54                                | 0.5 | 0.6 | 0.2 | 0.00 | 0.32 | 0.04 | 0.54 | 0.39 | 0.15 | 0.21 | 0.0 |
| 5        | 73.05                                | 0.7 | 0.8 | 0.4 | 0.00 | 0.43 | 0.06 | 0.71 | 0.52 | 0.25 | 0.28 | 0.0 |
| 6        | 91.57                                | 0.9 | 1.0 | 0.5 | 0.00 | 0.52 | 0.08 | 0.87 | 0.64 | 0.31 | 0.34 | 0.0 |
| 7        | 109.19                               | 1.1 | 1.2 | 0.6 | 0.00 | 0.63 | 0.09 | 1.02 | 0.77 | 0.38 | 0.40 | 0.0 |
| 8        | 127.27                               | 1.3 | 1.4 | 0.7 | 0.00 | 0.70 | 1.16 | 1.18 | 0.89 | 0.44 | 0.47 | 0.0 |
| 9        | 145.45                               | 1.5 | 1.6 | 0.7 | 0.00 | 0.79 | 1.33 | 1.33 | 1.01 | 0.55 | 0.55 | 0.0 |

Table 3. Elastic deformation read in various points on the part for various $F_y$ force values applied at $x = 510$ mm.

| Exp. no. | Force $F_y$[daN] applied at $x=510$ mm | Elastic deformation [mm] |
|----------|--------------------------------------|-------------------------|
|          | Rotating center - headstock          | x=170 | x=340 | x=510 | x=680 | Rotating center - tailstock |
|          |                                      | C | D | C | D | C | D | C | D | C | D | C | D |
| 1        | 0                                    | 0.0 | 0.0 | 0.0 | 0.00 | 0.00 | 0.0 | 0.00 | 0.0 | 0.00 | 0.0 | 0.0 |
| 2        | 18.18                                | 0.1 | 0.2 | 0.1 | 0.00 | 0.14 | 0.04 | 0.19 | 0.05 | 0.17 | 0.08 | 0.1 |
| 3        | 36.36                                | 0.2 | 0.4 | 0.2 | 0.00 | 0.24 | 0.03 | 0.43 | 0.29 | 0.34 | 0.18 | 0.2 |
| 4        | 54.54                                | 0.4 | 0.6 | 0.3 | 0.00 | 0.32 | 0.04 | 0.54 | 0.44 | 0.48 | 0.26 | 0.2 |
| 5        | 73.05                                | 0.6 | 0.8 | 0.4 | 0.00 | 0.42 | 0.06 | 0.69 | 0.60 | 0.65 | 0.35 | 0.3 |
| 6        | 91.57                                | 0.8 | 1.0 | 0.5 | 0.00 | 0.59 | 0.08 | 0.86 | 0.75 | 0.80 | 0.42 | 0.4 |

IOP Publishing
20th Innovative Manufacturing Engineering and Energy Conference (IManEE 2016)
IOP Conf. Series: Materials Science and Engineering 161 (2016) 012050
doi:10.1088/1757-899X/161/1/012050
made of carbon steel and used the following mode of operation: cutting depth, \( t = 4 \text{ mm} \) and \( s = 0.4 \text{ mm/rot} \). In this case, the \( F_y \) force may be calculated using the following relation \([1, 2, 3]\):

\[
F_y = C_{Fy} t^x s^y HB^n
\]

where \( C_{Fy} \) is a coefficient the value of which depends on the cutting tool type, on the material the cutting tool and the workpiece are made of, \( C_{Fy} = 0.027 \) and HB is workpiece material hardness HB=150- S 235 JR. The exponents are \( x = 0.9 \), \( y = 0.75 \), \( n = 2.0 \). When the cutting depth is \( t = 4 \text{ mm} \) and the cutting feed is \( s = 0.4 \text{ mm/rot} \), \( F_y = 0.027 \times 4^{0.9} \times 0.4^{0.75} \times 150^2 = 105 \text{ daN} \) for S 235 JR.

On the other hand, the forces generated in the workpiece during the experimental research should not exceed the maximum elastic deformation values applying to the workpiece material. A \( F_y \) force of variable value, ranging from 0 to 145.45 \text{ daN}, was entered into the system by means of the dynamometer, and then the \( F_y \) force was decreased to 0 \text{ daN}; the deformation values shown in tables 1-5 were read by the dial gauges for each \( F_y \) force value \([4, 5]\).

**Table 4. Elastic deformation read in various points on the part for various \( F_y \) force values applied at \( x = 680 \text{ mm} \).**

| Exp. no. | Force \( F_y \)[daN] applied at \( x = 680 \text{ mm} \) | Elastic deformation [mm] |
|----------|--------------------------------|------------------------|
|          | Rotating center - headstock | x=170 | x=340 | x=510 | x=680 | Rotating center - tailstock |
|          | C | D | C | D | C | D | C | D | C | D | C | D |
| 1        | 0 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| 2        | 18.18 | 0.12 | 0.04 | 0.01 | 0.18 | 0.12 | 0.04 | 0.01 | 0.18 | 0.12 | 0.04 | 0.01 |
| 3        | 36.36 | 0.15 | 0.04 | 0.01 | 0.25 | 0.20 | 0.04 | 0.01 | 0.25 | 0.20 | 0.04 | 0.01 |
| 4        | 54.54 | 0.35 | 0.04 | 0.01 | 0.42 | 0.37 | 0.04 | 0.01 | 0.42 | 0.37 | 0.04 | 0.01 |
| 5        | 73.05 | 0.48 | 0.04 | 0.01 | 0.59 | 0.54 | 0.04 | 0.01 | 0.59 | 0.54 | 0.04 | 0.01 |
| 6        | 91.57 | 0.37 | 0.04 | 0.01 | 0.76 | 0.73 | 0.04 | 0.01 | 0.76 | 0.73 | 0.04 | 0.01 |
| 7        | 109.19 | 0.42 | 0.04 | 0.01 | 0.93 | 0.91 | 0.04 | 0.01 | 0.93 | 0.91 | 0.04 | 0.01 |
| 8        | 127.27 | 0.53 | 0.04 | 0.01 | 1.10 | 1.08 | 0.04 | 0.01 | 1.10 | 1.08 | 0.04 | 0.01 |
| 9        | 145.45 | 0.55 | 0.04 | 0.01 | 1.24 | 1.22 | 0.04 | 0.01 | 1.24 | 1.22 | 0.04 | 0.01 |
Table 5. Elastic deformation read in various points on the part for various Fy force values applied at x = 850 mm.

| Exp. no. | Force Fy[daN] applied at x=850 mm | Elastic deformation [mm] |
|----------|-----------------------------------|--------------------------|
|          | Rotating center - headstock | x=170 | x=340 | x=510 | x=680 | Rotating center - tailstock |
|          | C | D | C | D | C | D | C | D | C | D |
| 1        | 0 | 0.00 | 5 | 0.00 | 4 | 0 | 0.00 | 3 | 0 | 0.00 | 1 | 0 | 0.00 | 2 |
| 2        | 18.18 | 0.1 | 0.20 | 5 | 0.00 | 0.19 | 9 | 0.15 | 3 | 0.15 | 0.00 | 1 | 0.03 | 2 |
| 3        | 36.36 | 0.1 | 0.34 | 9 | 0.29 | 0.30 | 0.15 | 0.24 | 3 | 0.24 | 0.15 | 0.24 | 0.2 | 1 |
| 4        | 54.54 | 0.2 | 0.40 | 4 | 0.60 | 0.62 | 0.21 | 0.32 | 4 | 0.32 | 0.21 | 0.32 | 0.2 | 1 |
| 5        | 73.05 | 0.3 | 0.48 | 1 | 0.82 | 0.84 | 0.28 | 0.40 | 5 | 0.28 | 0.28 | 0.40 | 0.3 | 9 |
| 6        | 91.57 | 0.6 | 0.54 | 6 | 0.83 | 0.85 | 0.35 | 0.48 | 4 | 0.35 | 0.35 | 0.48 | 0.4 | 9 |
| 7        | 109.19 | 0.4 | 0.59 | 2 | 0.84 | 0.86 | 0.42 | 0.56 | 5 | 0.42 | 0.42 | 0.56 | 0.5 | 9 |
| 8        | 127.27 | 0.5 | 0.60 | 1 | 0.85 | 0.88 | 0.51 | 0.62 | 4 | 0.51 | 0.51 | 0.62 | 0.6 | 9 |
| 9        | 145.45 | 0.6 | 0.64 | 4 | 0.90 | 0.90 | 0.63 | 0.87 | 3 | 0.63 | 0.63 | 0.87 | 0.8 | 9 |

Figure 8. Graphical representation of the elastic deformation variation depending on different loading and unloading values of the technological system at the x = 170 mm distance

Figure 9. Graphical representation of the elastic deformation variation depending on different loading and unloading values of the technological system at the distance 340 mm
4. Conclusion

Further to the analysis of the tables containing the experimental values and of the graphical representations in figure 8-12, we may conclude that:

1. a cutting force of \( F_y = 145.45 \) daN applied at the \( x=680 \) mm distance from the left end of the work piece causes a maximum deformation value in the tailstock area of \( y = 2.98 \) mm. When the same cutting force value is applied in the same work piece area at a \( x=510 \) mm distance, the following work piece deformation value is read: \( y = 2.58 \) mm;

2. in figure 11, the unloading curve read in the tailstock area for the \( F_y \) force applied at the \( x=510 \) mm distance leads of a maximum plastic deformation value of \( y=0.3 \) mm; this value proves that the technological system components shifted to resist the friction forces occurring in the system and to absorb the system clearances;

3. our research confirmed that, among the components in direct contact with the work piece, the universal support had the highest static rigidity;

4. the elastic deviation of the work piece is one of the factors that may impede upon the accuracy of the processing during the turning process. The extent of this deviation significantly depends on the static rigidity of the components of the processing systems;

5. our research fully confirms the mathematical model, when the blank is fastened between centers.
Appendix 1. Dynamometer calibration

|       | Pull                     |          |       | Compression |          |
|-------|--------------------------|----------|-------|-------------|----------|
|       | F                        | Gauge value | E  | F          | Gauge value | E  |
|       | daN | ind. | ind./daN | daN | ind. | ind./daN |
| 50    | 55.0 | 1.100 |        | 50 | 54.9 | 1.100 |
| 60    | 66.0 | 1.100 |        | 60 | 66.0 | 1.105 |
| 80    | 88.0 | 1.095 |        | 80 | 88.1 | 1.095 |
| 100   | 109.9 | 1.092 |        | 100 | 110.0 | 1.116 |
| 125   | 137.2 | 1.104 |        | 125 | 137.9 | 1.092 |
| 150   | 164.8 | 1.100 |        | 150 | 165.2 | 1.106 |
| 200   | 219.8 | 1.104 |        | 200 | 220.5 | 1.106 |
| 250   | 275.0 | 1.098 |        | 250 | 275.8 | 1.104 |
| 300   | 329.9 | 1.102 |        | 300 | 331.0 | 1.106 |
| 350   | 385.0 | 1.100 |        | 350 | 386.3 | 1.114 |
| 400   | 440.0 | 1.106 |        | 400 | 442.0 | 1.118 |
| 450   | 495.3 | 1.112 |        | 450 | 497.9 | 1.114 |
| 500   | 550.9 |        |        | 500 | 553.6 |        |

References
[1] Benardos P G Mosialos S Vosniakos G C 2006 Prediction of workpiece elastic deflection under cutting forces in turning Robotics and Computer-Integrated Manufacturing 22 ISSN 0736-5845 pp 55-514
[2] Picos C Pruteanu O Bohosevici C and others 1992 Proiectarea tehnologiilor de prelucrare mecanică prin strunjire Manual Design vol 1 University Publishing ISBN 5-362-00970-2 (Chişinău)
[3] Tabacaru L and Pruteanu OV 2007 Conceptia si managementul tehnologiilor de fabricatie Junimea Publishing ISBN 978-973-37-1210-7 (Iasi)
[4] Tabacaru L and Pruteanu O V 2010 Managementul tehnologiilor de fabricatie Politehnium Publishing ISBN 978-973-621-294-9 (Iasi)
[5] Topal E S and Cogun C 2005 A cutting force induced error elimination method for turning operations Official Journal of Technological Processes of the materials vol 170 no 1-2 ISSN 0924-0136 pp 190-203