On Dynamics of Correlations in Quantum Many-Particle Systems

V.I. Gerasimenko

Institute of Mathematics of NAS of Ukraine,
3, Tereshchenkivs'ka Str.,
01601, Kyiv, Ukraine

Abstract. The paper deals with the problem of the rigorous description of the evolution of states of large particle quantum systems by means of correlation operators. A nonperturbative solution of the Cauchy problem of the hierarchy of nonlinear evolution equations for a sequence of marginal correlation operators is constructed. For initial states specified in terms of a one-particle density operator and correlation operators we also develop an approach to the description of the processes of the creation and the propagation of correlations within the framework of a one-particle density operator. Moreover, the mean field asymptotic behavior of constructed marginal correlation operators is established.

Key words: correlation operator; group of nonlinear operators; nonlinear BBGKY hierarchy; quantum Vlasov-type kinetic equation; mean field scaling limit

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E-mail: gerasym@imath.kiev.ua
1 Introduction: on the propagation of initial correlations

In papers [1], [2] it was developed two new approaches to the description of the propagation of initial correlations of large particle quantum systems in the mean field scaling limit. We note that initial states specified by correlations are typical for the condensed states of particle systems in contrast to their gaseous state [3], [4].

In paper [1] the property of the propagation of initial correlations was proved within the framework of the description of the evolution by means of marginal observables and in paper [2] this property was established by another method in terms of a one-particle (marginal) density operator governed by the generalized quantum kinetic equation [5]. It was proved that for a system of a non-fixed (i.e., arbitrary but finite) number of identical particles, obeying the Maxwell – Boltzmann statistics, in case of initial states specified by a sequence of the following marginal correlation operators (we use notations accepted in review [6])

\[
g^{(c)} = (I, f_1^0(1), g_2(1, 2) \prod_{i=1}^2 f_1^0(i), \ldots, g_n(1, \ldots, n) \prod_{i=1}^n f_1^0(i), \ldots),
\]

in the mean field scaling limit the evolution of all possible correlations is described by the following sequence of the limit marginal correlation operators (we use notations accepted in review [6])

\[
g_n(t, 1, \ldots, n) = \prod_{i_1=1}^n \mathcal{G}^*_1(t, i_1) g_n(1, \ldots, n) \prod_{i_2=1}^n \left(\mathcal{G}^*_1\right)^{-1}(t, i_2) \prod_{j=1}^n f_1(t, j), \quad n \geq 2,
\]

(1)
where the one-particle density operator \( f_1(t) \) is a solution of the Cauchy problem of the quantum Vlasov-type kinetic equation with initial correlations:

\[
\frac{\partial}{\partial t} f_1(t, 1) = \mathcal{N}^*(1) f_1(t, 1) + \text{Tr}_2 \mathcal{N}^\text{int}_1(1, 2) \prod_{i_1=1}^2 \mathcal{G}^1(t, i_1)(1 + g_2(1, 2)) \prod_{i_2=1}^2 (\mathcal{G}^1)^{-1}(t, i_2) f_1(t, 1) f_1(t, 2),
\]

\[
f_1(t)|_{t=0} = f^0_1.
\]

In expressions (1) and kinetic equation (2) it was used: the units where \( h = 2\pi\hbar = 1 \) is a Planck constant, \( m = 1 \) is the mass of particles and the group of operators \( \mathcal{G}^1(t, j) \) of the free motion of \( j \)th particle defined on the space of trace class operators \( f_1 \):

\[
\mathbb{R} \ni t \mapsto \mathcal{G}^1(t) f_1 = e^{-itK(j)} f_1 e^{itK(j)},
\]

where the operator \( K(j) \) is the kinetic energy operator of the \( j \)th particle. The inverse group to the group \( \mathcal{G}^1(t) \) we denote by \( \mathcal{G}^1(t)^{-1} = \mathcal{G}^1(-t) \). On its domain of definition the von Neumann operator \( \mathcal{N}^*(1) \) of the free motion is defined as follows: \( \mathcal{N}^*(1) f_1 = -i (K(1) f_1 - f_1 K(1)) \), and the operator \( \mathcal{N}^\text{int}_1(1, 2) \) is defined by the operator of a two-body interaction potential \( \Phi \), respectively, \( \mathcal{N}^\text{int}_1(1, 2) f_2 = -i (\Phi(1, 2) f_2 - f_2 \Phi(1, 2)) \).

Thus, it was established that mean field dynamics does not create new correlations except of those that generating by initial correlations (1).

We remark that the conventional approach to the problem of the description of the propagation of initial chaos, i.e. in case of initial states specified by a one-particle density operator without correlation operators, is based on the consideration of an asymptotic behavior of a solution of the quantum BBGKY hierarchy for marginal density operators constructed within the framework of the perturbation theory [7]–[11] (for collisional dynamics of hard spheres see also [12]–[15]).

In this paper we consider the problem of the rigorous description of the evolution of states of large particle quantum systems within the framework of marginal correlation operators. In next section 2, we construct a nonperturbative solution of the Cauchy problem of the hierarchy of nonlinear evolution equations for marginal correlation operators. Then in section 3, we consider an approach to the description of correlations by means of a one-particle (marginal) density operator. In section 4, we establish a mean field asymptotic behavior of the constructed correlation operators. Finally, in section 5, we conclude with some advances of the developed approaches to the description of quantum correlations.

## 2 A nonperturbative solution of the hierarchy of evolution equations for marginal correlation operators

It is known [6], that the evolution of states of large particle quantum systems can be described within the framework of marginal (s-particle) density operators as well as in terms of marginal correlation operators. Traditionally marginal correlation operators are introduced by means of the cluster expansions of marginal density operators. The physical interpretation of marginal correlation operators is that the macroscopic characteristics of fluctuations of mean values of
observables are determined by them on the microscopic level. In this section we construct a solution of the Cauchy problem of the fundamental evolution equations for marginal correlation operators.

2.1 Preliminaries: dynamics of correlations

Let the space $\mathcal{H}$ be a one-particle Hilbert space, then the $n$-particle space $\mathcal{H}_n = \mathcal{H}^\otimes n$ is a tensor product of $n$ Hilbert spaces $\mathcal{H}$. We adopt the usual convention that $\mathcal{H}^\otimes 0 = \mathbb{C}$. The Fock space over the Hilbert space $\mathcal{H}$ we denote by $\mathcal{F}_\mathcal{H} = \bigoplus_{n=0}^\infty \mathcal{H}_n$. A self-adjoint operator $f_n$ defined on the $n$-particle Hilbert space $\mathcal{H}_n = \mathcal{H}^\otimes n$ will be also denoted by the symbol $f_n(1, \ldots, n)$.

Let $\mathfrak{L}^1(\mathcal{H}_n)$ be the space of trace class operators $f_n \equiv f_n(1, \ldots, n) \in \mathfrak{L}^1(\mathcal{H}_n)$ that satisfy the symmetry condition: $f_n(1, \ldots, n) = f_n(i_1, \ldots, i_n)$ for arbitrary $(i_1, \ldots, i_n) \in (1, \ldots, n)$, and equipped with the norm: $\|f_n\|_{\mathfrak{L}^1(\mathcal{H}_n)} = \text{Tr}_{1,\ldots,n}|f_n(1, \ldots, n)|$, where $\text{Tr}_{1,\ldots,n}$ are partial traces over $1, \ldots, n$ particles. We denote by $\mathfrak{L}^1_0(\mathcal{H}_n)$ the everywhere dense set of finite sequences of degenerate operators with infinitely differentiable kernels with compact supports.

On the space of trace class operators $\mathfrak{L}^1(\mathcal{H}_n)$ it is defined the one-parameter mapping $\mathcal{G}^*_n(t)$

$$\mathbb{R}^1 \ni t \mapsto \mathcal{G}^*_n(t)f_n \equiv e^{-itH_n}f_ne^{itH_n},$$

where the operator $H_n$ is the Hamiltonian of a set of $n$ particles, obeying Maxwell – Boltzmann statistics, and we use units where $\hbar = 2\pi h = 1$ is a Planck constant and $m = 1$ is the mass of particles. The inverse group to the group ($\mathcal{G}^*_n(t)$) we denote by ($\mathcal{G}^*_n(−t)$). On its domain of the definition the infinitesimal generator $\mathcal{N}^*_n$ of the group of operators $\mathcal{G}^*_n$ is determined in the sense of the strong convergence of the space $\mathfrak{L}^1(\mathcal{H}_n)$ by the operator

$$\lim_{t \to 0} \frac{1}{t}(\mathcal{G}^*_n(t)f_n − f_n) = −i(H_nf_n − f_nH_n) \doteq \mathcal{N}^*_nf_n,$$

that has the following structure: $\mathcal{N}^*_n = \sum_{j=1}^n \mathcal{N}^*_j + \epsilon \sum_{j_1 < j_2 = 1} \mathcal{N}^*_\text{int}(j_1, j_2)$, where the operator $\mathcal{N}^*_j$ is a free motion generator of the von Neumann equation $[6]$, the operator $\mathcal{N}^*_\text{int}$ is defined by means of the operator of a two-body interaction potential $\Phi$ by the formula: $\mathcal{N}^*_\text{int}(j_1, j_2)f_n = −i(\Phi(j_1, j_2)f_n − f_n\Phi(j_1, j_2))$, a scaling parameter we denote by $\epsilon > 0$.

On the space $\mathfrak{L}^1(\mathcal{F}_\mathcal{H}) = \bigoplus_{n=0}^\infty \mathfrak{L}^1(\mathcal{H}_n)$ of sequences $f = (f_0, f_1, \ldots, f_n, \ldots)$ of trace class operators $f_n \in \mathfrak{L}^1(\mathcal{H}_n)$ and $f_0 \in \mathbb{C}$ the following nonlinear one-parameter mapping is defined:

$$\mathcal{G}(t; 1, \ldots, s | f) \doteq \sum_{\mathbb{P}: (1, \ldots, s) = \bigcup_j X_j} \mathfrak{A}_{|\mathbb{P}|}(t; \{X_1\}, \ldots, \{X_{|\mathbb{P}|}\}) \prod_{X_j \subset \mathbb{P}} f_{|X_j|}(X_j), \quad s \geq 1,$$

where the symbol $\sum_{\mathbb{P}: (1, \ldots, s) = \bigcup_j X_j}$ means the sum over all possible partitions $\mathbb{P}$ of the set $(1, \ldots, s)$ into $|\mathbb{P}|$ nonempty mutually disjoint subsets $X_j \subset Y \equiv (1, \ldots, s)$, the set $(\{X_1\}, \ldots, \{X_{|\mathbb{P}|}\})$ consists from elements which are subsets $X_j \subset Y$, i.e. $|\{X_1\}, \ldots, \{X_{|\mathbb{P}|}\}| = |\mathbb{P}|$. The generating operator $\mathfrak{A}_{|\mathbb{P}|}(t)$ of expansion $[7]$ is the $|\mathbb{P}|$th-order cumulant of groups of operators $[5]$ defined by the following expansion:

$$\mathfrak{A}_{|\mathbb{P}|}(t; \{X_1\}, \ldots, \{X_{|\mathbb{P}|}\}) \doteq \sum_{\mathbb{P}': (\{X_1\}, \ldots, \{X_{|\mathbb{P}|}\}) = \bigcup_k Z_k} (-1)^{|\mathbb{P}'|−1(|\mathbb{P}'|−1)!} \prod_{Z_k \subset \mathbb{P}'} \mathcal{G}^*_{|\mathbb{P}'|}(t, \theta(Z_k)),$$
where the declusterization mapping $\theta$ is defined as follows: $\theta(\{X_1\}, \ldots, \{X_\nu\}) = (1, \ldots, s)$.

Below we adduce the examples of mapping expansion (7):

$$G(t; 1 \mid f) = A_1(t, 1)f_1(1),$$
$$G(t; 1, 2 \mid f) = A_1(t, \{1, 2\})f_2(1, 2) + A_{1+1}(t, 1, 2)f_1(1)f_1(2),$$
$$G(t; 1, 2, 3 \mid f) = A_1(t, \{1, 2, 3\})f_3(1, 2, 3) + A_{1+1}(t, 1, \{2, 3\})f_1(1)f_2(2, 3) + A_{1+1}(t, 2, \{1, 3\})f_1(2)f_2(1, 3) + A_{1+1}(t, 2, \{1, 2\})f_1(3)f_2(1, 2) + A_3(t, 1, 2, 3)f_1(1)f_2(1, 2)f_1(3),$$

On operators $f_s \in L^1(\mathcal{H}_s)$, $s \geq 1$, the mapping $G(t; Y \mid f)$ is defined and, according to the inequality for cumulant (8) of groups of operators (5)

$$\|A_{|P|}(t, \{X_1\}, \ldots, \{X_\nu\})f_s\|_{L^1(\mathcal{H}_s)} \leq |P| \|e\|_p \|f_s\|_{L^1(\mathcal{H}_s)},$$

the following estimate is true:

$$\|G(t; 1, \ldots, s \mid f)\|_{L^1(\mathcal{H}_s)} \leq s! e^{2s}e^\epsilon, \tag{9}$$

where $e \equiv e^3 \max(1, \max_{P : Y = \bigcup X_i} \|f_{|X_i|}\|_{L^1(\mathcal{H}_{|X_i|})})$.

On the space $L^1(\mathcal{F}_\mathcal{H})$ one-parameter mapping (7) is a bounded strong continuous group of nonlinear operators and it is determined a solution of the Cauchy problem for the von Neumann hierarchy for correlation operators [16].

The evolution of all possible states of quantum systems of a non-fixed (i.e., arbitrary but finite) number of identical particles, obeying the Maxwell–Boltzmann statistics, can be described by means of the sequence $g(t) = (g_0, g_1(t), \ldots, g_s(t), \ldots) \in L^1(\mathcal{F}_\mathcal{H})$ of the correlation operators $g_s(t) \equiv g_s(t, 1, \ldots, s)$, $s \geq 1$, governed by the Cauchy problem of the von Neumann hierarchy [17]:

$$\frac{\partial}{\partial t}g_s(t, 1, \ldots, s) = \mathcal{N}(1, \ldots, s \mid g(t)), \tag{10}$$
$$g_s(t)|_{t=0} = g_s^0, \quad s \geq 1, \tag{11}$$

where $\epsilon > 0$ is a scaling parameter. The generator of the hierarchy of nonlinear evolution equations (10) has the structure

$$\mathcal{N}(1, \ldots, s \mid g(t)) = \mathcal{N}_s^* g_s(t, 1, \ldots, s) + \epsilon \sum_{P: (1, \ldots, s) = X_1 \cup X_2} \sum_{i_1 \in X_1} \sum_{i_2 \in X_2} \mathcal{N}_{int}(i_1, i_2)g_{|X_1|}(t, X_1)g_{|X_2|}(t, X_2), \tag{12}$$

where the symbol $\sum_{P: (1, \ldots, s) = X_1 \cup X_2}$ means the sum over all possible partitions $P$ of the set $Y = (1, \ldots, s)$ into two nonempty mutually disjoint subsets $X_1 \subset Y$ and $X_2 \subset Y$, and the operator $\mathcal{N}_s^*$ is the von Neumann operator defined by formula (6) on the subspace $L^1_0(\mathcal{H}_s) \subset L^1(\mathcal{H}_s)$.

We remark that correlation operators are introduced by means of the cluster expansions of the density operators (the kernel of a density operator is known as a density matrix) governed by the von Neumann equations, and it is to enable to describe of the evolution of states by the equivalent method in comparison with the density operators [6].
A nonperturbative solution of the Cauchy problem of the von Neumann hierarchy (10), (11) for correlation operators is determined by the group of nonlinear operators (7), i.e.

\[ g(t, 1, \ldots, s) = \mathcal{G}(t; 1, \ldots, s \mid g(0)), \quad s \geq 1, \]

(13)

where \( g(0) = (g_0, g_1^{0,\epsilon}, \ldots, g_n^{0,\epsilon}, \ldots) \) is a sequence of initial correlation operators.

We note that in case of the absence of correlations between particles at initial time, i.e. initial data (11), satisfying a chaos condition, the sequence of initial correlation operators has the form

\[ g(0) = (0, g_0^{0,\epsilon}, 0, \ldots, 0, \ldots). \]

Then solution (13) of the Cauchy problem of the von Neumann hierarchy (10), (11) is represented by the following expansions:

\[ g_s(t, 1, \ldots, s) = \mathfrak{A}_s(t, 1, \ldots, s) \prod_{i=1}^{s} g_1^{0,\epsilon}(i), \quad s \geq 1, \]

where the operator \( \mathfrak{A}_s(t) \) is the \( s \)th-order cumulant of groups of operators (5) determined by the expansion

\[ \mathfrak{A}_s(t, 1, \ldots, s) = \sum_{P: (1, \ldots, s) = \bigcup_i X_i} (-1)^{|P| - 1} |P|! \prod_{X_i \subset P} G_{|X_i|}(t, X_i), \]

(14)

and we used notations accepted in formula (7).

### 2.2 A nonperturbative solution of the nonlinear BBGKY hierarchy for marginal correlation operators

We introduce the marginal correlation operators by means of the macroscopic characteristics of fluctuations of mean values of observables are determined.

The marginal correlation operators are defined within the framework of a solution of the Cauchy problem of the von Neumann hierarchy (10), (11) by the following series expansions:

\[ G_s(t, 1, \ldots, s) = \sum_{n=0}^{\infty} \frac{1}{n!} \text{Tr}_{s+1,\ldots,s+n} \mathcal{G}(t; 1, \ldots, s + n \mid g(0)), \quad s \geq 1. \]

(15)

According to estimate (9), series (15) exists and the following estimate holds: \( \|G_s(t)\|_{\mathcal{L}^1(\mathcal{H}_s)} \leq s! (2e^2)^s c^s \sum_{n=0}^{\infty} (2e^2)^n c^n \).

The evolution of all possible states of quantum large particle systems, obeying the Maxwell - Boltzmann statistics, can be described by means of the sequence \( G(t) = (I, G_1(t), G_2(t), \ldots, G_s(t), \ldots) \in \mathcal{L}^1(\mathcal{F}_\mathcal{H}) \) of marginal correlation operators governed by the Cauchy problem of the following hierarchy of nonlinear evolution equations (the nonlinear quantum BBGKY hierarchy):

\[ \frac{\partial}{\partial t} G_s(t, Y) = \mathcal{N}(Y \mid G(t)) + \text{Tr}_{s+1} \sum_{i \in Y} \mathcal{N}^\ast_{\text{int}}(i, s + 1) \left( G_{s+1}(t, Y, s + 1) + \epsilon \sum_{P: (Y, s + 1) = X_1 \cup X_2, i \in X_1; s + 1 \in X_2} G_{|X_1|}(t, X_1) G_{|X_2|}(t, X_2) \right), \]

\[ G_s(t)\big|_{t=0} = G_s^{0,\epsilon}, \quad s \geq 1. \]

(16)
In evolution equations (16) it was used notations accepted above, in particular, \((1, \ldots, s) \equiv Y\), the operators \(\mathcal{N}(1, \ldots, s \mid G(t)) \equiv \mathcal{N}(Y \mid G(t))\), \(s \geq 1\), are generators (12) of the von Neumann hierarchy (10) and \(\epsilon > 0\) is a scaling parameter.

The rigorous derivation of the hierarchy of evolution equations for marginal correlation operators (16), according to definition (15), consists in its derivation from the von Neumann hierarchy for correlation operators (10) (for marginal density operators see paper [20]).

A nonperturbative solution of the Cauchy problem (16), (17) is represented by a sequence of the following self-adjoint operators:

\[
G_s(t, Y) = \sum_{n=0}^{\infty} \frac{1}{n!} \text{Tr}_{s+1, \ldots, s+n} \mathfrak{A}_{1+n}(t; \{Y\}, s+1, \ldots, s+n \mid G(0)), \quad s \geq 1,
\]

where a sequence of initial marginal correlation operators we denote by \(G(0) = (I, G_1^{0,\epsilon}(1), \ldots, G_s^{0,\epsilon}(1, \ldots, s), \ldots)\). The generating operators of series expansion (18), in particular, the operator \(\mathfrak{A}_{1+n}(t; \{Y\}, s+1, \ldots, s+n \mid G(0))\) is the \((1+n)th\)-order cumulant of groups of nonlinear operators (7) of the von Neumann hierarchy for correlation operators

\[
\mathfrak{A}_{1+n}(t; \{Y\}, s+1, \ldots, s+n \mid G(0)) = \sum_{P: ((Y), s+1, \ldots, s+n) = \bigcup_k X_k} (-1)^{|P|-1}(|P| - 1) G(t; \theta(X_1) \mid \ldots G(t; \theta(X_{|P|}) \mid G(0)) \ldots),
\]

where the composition of mappings (10) of corresponding noninteracting groups of particles is denoted by \(G(t; \theta(X_1) \mid \ldots G(t; \theta(X_{|P|}) \mid G(0)) \ldots)\), for example,

\[
G(t; 1 \mid G(t; 2 \mid f)) = \mathfrak{A}_1(t, 1) \mathfrak{A}_2(t, 2) f_2(1, 2),
\]

\[
G(t; 1, 2 \mid G(t; 3 \mid f)) = \mathfrak{A}_1(t, \{1, 2\}) \mathfrak{A}_3(t, 3) f_3(1, 2, 3) + \mathfrak{A}_2(t, 1, 2) \mathfrak{A}_1(t, 3) f_3(1, 2, 3) + f_1(1) f_2(2, 3) + f_1(2) f_2(1, 3).
\]

Below we adduce the examples of expansions (19). The first order cumulant of the groups of nonlinear operators (7) is the same group of nonlinear operators, i.e.

\[
\mathfrak{A}_1(t; \{1, \ldots, s\} \mid G(0)) = G(t; 1, \ldots, s \mid G(0)),
\]

in case of \(s = 2\) the second order cumulant of groups of nonlinear operators (7) represents by the following expansion:

\[
\mathfrak{A}_{1+1}(t; \{1, 2\}, 3 \mid G(0)) = G(t; 1, 2, 3 \mid G(0)) - G(t; 1, 2 \mid G(t; 3 \mid G(0))) = \mathfrak{A}_{1+1}(t, \{1, 2\}, 3) G_3^{0,\epsilon}(1, 2, 3) + (\mathfrak{A}_{1+1}(t, \{1, 2\}, 3) - \mathfrak{A}_{1+1}(t, 2, 3) \mathfrak{A}_1(t, 1)) G_1^{0,\epsilon}(1) G_2^{0,\epsilon}(2, 3) + (\mathfrak{A}_{1+1}(t, \{1, 2\}, 3) - \mathfrak{A}_{1+1}(t, 1, 3) \mathfrak{A}_1(t, 2)) G_1^{0,\epsilon}(2) G_2^{0,\epsilon}(1, 3) + \mathfrak{A}_{1+1}(t, \{1, 2\}, 3) G_3^{0,\epsilon}(3) G_2^{0,\epsilon}(1, 2) + \mathfrak{A}_3(t, 1, 2, 3) G_1^{0,\epsilon}(1) G_2^{0,\epsilon}(2) G_3^{0,\epsilon}(3),
\]

where the operator \(\mathfrak{A}_3(t, 1, 2, 3) = \mathfrak{A}_{1+1}(t, \{1, 2\}, 3) - \mathfrak{A}_{1+1}(t, 2, 3) \mathfrak{A}_1(t, 1) - \mathfrak{A}_{1+1}(t, 1, 3) \mathfrak{A}_1(t, 2)\) is the third order cumulant of groups of operators (7).

In case of initial data specified by the sequence of marginal correlation operators

\[
G^{(c)} = (0, G_1^{0,\epsilon}, 0, \ldots, 0, \ldots),
\]

(20)
where the generating operator $A$ of series (18) as cluster expansions

$$G_s(t, 1, \ldots, s) = \sum_{n=0}^{\infty} \frac{1}{n!} \text{Tr}_{s+1, \ldots, s+n} \mathcal{A}_{s+n}(t; 1, \ldots, s + n) \prod_{i=1}^{s+n} G_1^{0,\epsilon}(i), \quad s \geq 1, \quad (21)$$

where the generating operator $\mathcal{A}_{s+n}(t)$ of this series is the $(s+n)$th-order cumulant (14) of groups of operators (5).

We remark that within the framework of marginal density operators defined by means of the cluster expansions of marginal correlation operators

$$F_s^{0,\epsilon}(1, \ldots, s) = \sum_{P : (1, \ldots, s) = \bigcup_i X_i} \prod_{X_i \subset P} G_s^{0,\epsilon}(X_i), \quad s \geq 1,$$

initial state similar to (20) is specified by the sequence $F^{(c)} = (I, F^{0,\epsilon}_1(1), \ldots, \prod_{i=1}^{n} F^{0,\epsilon}_1(i), \ldots)$, and in case of sequence (21) the marginal density operators are represented by the following series expansions (a nonperturbative solution of the quantum BBGKY hierarchy [6]):

$$F_s(t, 1, \ldots, s) = \sum_{n=0}^{\infty} \frac{1}{n!} \text{Tr}_{s+1, \ldots, s+n} \mathcal{A}_{1+n}(t; \{Y\}, s + 1, \ldots, s + n) \prod_{i=1}^{s+n} F^{0,\epsilon}_1(i), \quad s \geq 1,$$

where the generating operator $\mathcal{A}_{1+n}(t)$ is the $(1+n)$th-order cumulant of groups of operators (5).

One of the methods to derive the series expansion (18) for marginal correlation operators consists on the application of the cluster expansions of groups of nonlinear operators (7) over cumulants (19) in the definition of marginal correlation operators (13) and the sequence of initial correlation operators $g(0) = (I, g^{0,\epsilon}_1(1), \ldots, g^{0,\epsilon}_n(1, \ldots, n), \ldots)$ determined by means of the marginal correlation operators:

$$g_s^{0,\epsilon}(1, \ldots, s) = \sum_{n=0}^{\infty} (-1)^n \frac{1}{n!} \text{Tr}_{s+1, \ldots, s+n} G_{s+n}^{0,\epsilon}(1, \ldots, s + n), \quad s \geq 1. \quad (22)$$

Indeed, developing the generating operators of series (18) as cluster expansions

$$G(t; 1, \ldots, s+n \mid f) = \sum_{P : (1, \ldots, s+n) = \bigcup_i X_i} \mathcal{A}_{|X_1|}(t; X_1 \mid \ldots \mathcal{A}_{|X_P|}(t; X_P \mid f) \ldots), \quad (23)$$

according to definition (22), we derive expressions (18). The solutions of recursive relations (23) is represented by expansions (19).

We remark that on the space $\mathcal{A}^1(F\mathcal{H})$ the generating operator (19) of series expansion (18) can be represented as the $(1+n)$th-order reduced cumulant of groups of nonlinear operators (7) of the von Neumann hierarchy [19]

$$U_{1+n}(t; \{1, \ldots, s\}, s+1, \ldots, s+n \mid G(0)) \doteq \sum_{k=0}^{n} (-1)^k \frac{n!}{k!(n-k)!} \sum_{P : \theta(1, \ldots, s), s+1, \ldots, s+n-k = \bigcup_i X_i} \mathcal{A}_{|P|}(t; \{X_1\}, \ldots, \{X_{|P|}\}) \times \prod_{k_1=0}^{k} \frac{k!}{k_1!(k-k_1)!} \ldots \prod_{k_{|P|-1}=0}^{|P|-2} \frac{k_{|P|-2}!}{k_{|P|-1}!(k_{|P|-2} - k_{|P|-1})!} G_{|X_1|+k-k_1}(X_1),$$

$$s + n - k + 1, \ldots, s + n - k_1 \ldots G_{|X_{|P|}+k_{|P|-1}|}(X_{|P|}, s + n - k_{|P|-1} + 1, \ldots, s + n). \quad (24)$$
We adduce simplest examples of reduced cumulants (24) of groups of nonlinear operators (7):

\[ U_1(t; \{1, \ldots, s\} | G(0)) = G(t; 1, \ldots, s | G(0)) = \sum_{P: (1, \ldots, s) = \bigcup_j X_i} \mathcal{A}_{|P|}(t, \{X_1\}, \ldots, \{X_{|P|}\}) \prod_{X_i \in P} G_{|X_i|}(X_i), \]

\[ U_{1+1}(t; \{1, \ldots, s\}, s + 1 | G(0)) = \sum_{P: (1, \ldots, s+1) = \bigcup_j X_i} \mathcal{A}_{|P|}(t, \{X_1\}, \ldots, \{X_{|P|}\}) \prod_{X_i \in P} G_{|X_i|}(X_i) - \sum_{P: (1, \ldots, s) = \bigcup_j X_i} \mathcal{A}_{|P|}(t, \{X_1\}, \ldots, \{X_{|P|}\}) \sum_{j=1}^{P} G_{|X_j|+1}(X_j, s + 1) \prod_{X_i \in P, X_i \neq X_j} G_{|X_i|}(X_i). \]

We note also that nonperturbative solution of the nonlinear quantum BBGKY hierarchy (18) or in the form of series expansions (24) can be transformed to the perturbation (iteration) series as a result of the application of analogs of the Duhamel equation to cumulants (8) of groups of operators (5).

The following statement is true.

**Theorem 1.** If \( \max_{n \geq 1} \|G_n^0\|_{L^1(\mathcal{H}_n)} < (2e^3)^{-1} \), then in case of bounded interaction potentials for \( t \in \mathbb{R} \) a solution of the Cauchy problem of the nonlinear quantum BBGKY hierarchy (10), (17) is determined by a sequence of marginal correlation operators represented by series expansions (18). If \( G_n^0 \in L^1_0(\mathcal{H}_n) \subset L^1(\mathcal{H}_n) \), it is a strong solution and for arbitrary initial data \( G_n^0 \in L^1(\mathcal{H}_n) \) it is a weak solution.

The proof of the existence theorem is similar to the case of the reduced representation of a nonperturbative solution of the nonlinear quantum BBGKY hierarchy (19).

### 3 On the representation of marginal correlation operators by means of a one-particle density operator

In case of initial states specified in terms of a one-particle (marginal) density operator and correlation operators the evolution of all possible states of quantum large particle systems can be described within the framework of a one-particle density operator governed by the kinetic equation without any approximations. In this section we consider an approach to the description of the processes of the creation correlations and the propagation of initial correlations by means of a one-particle density operator that is a solution of the generalized quantum kinetic equation with initial correlations.

#### 3.1 Marginal correlation functionals

If initial states specified in terms of a one-particle density operator (20), then the evolution of states given in the framework of the sequence \( G(t) = (I, G_1(t), \ldots, G_s(t), \ldots) \) of marginal correlation operators (18) can be described by the sequence \( G(t | G_1(t)) = (I, G_1(t), G_2(t | G_1(t)), \ldots, G_s(t | G_1(t)), \ldots) \) of marginal correlation functionals \( G_s(t, 1, \ldots, s | G_1(t)) \), \( s \geq 2 \), with respect to the one-particle density (correlation) operator \( G_1(t) \).
In case of initial states \((20)\) the marginal correlation functionals \(G_s(t \mid G_1(t)), s \geq 2,\) are represented by the series expansions:

\[
G_s(t, 1, \ldots, s \mid G_1(t)) = \sum_{n=0}^{\infty} \frac{1}{n!} \text{Tr}_{s+1,\ldots,s+n} \Psi_{s+n}(t, \theta(\{Y\}), s + 1, \ldots, s + n) \prod_{i=1}^{s+n} G_1(t, i), \tag{25}
\]

In this formula it is used the notion of the declusterization mapping: \(\theta(\{Y\}) = Y \equiv (1, \ldots, s),\) and the one-particle (marginal) correlation operator is determined by the series expansion

\[
G_1(t, 1) = \sum_{n=0}^{\infty} \frac{1}{n!} \text{Tr}_{2,\ldots,1+n} \mathfrak{A}_{1+n}(t) \prod_{i=1}^{n+1} G_1^{0,i}(i),
\]

where the generating operator \(\mathfrak{A}_{1+n}(t) \equiv \mathfrak{A}_{1+n}(t, 1, \ldots, n + 1)\) is the \((1 + n)th\)-order cumulant \((14)\) of groups of operators \((5)\). We adduce simplest examples of the generating operators of series expansion \((25)\):

\[
\begin{align*}
\Psi_s(t, \theta(\{Y\})) &= \hat{\mathfrak{A}}_s(t, \theta(\{Y\})) \equiv \mathfrak{A}_s(t, \theta(\{Y\})) \prod_{i=1}^{s} \mathfrak{A}_1^{-1}(t, i), \\
\Psi_{s+1}(t, \theta(\{Y\}), s + 1) &= \hat{\mathfrak{A}}_{s+1}(t, \theta(\{Y\}), s + 1) - \hat{\mathfrak{A}}_s(t, \theta(\{Y\})) \sum_{i=1}^{s} \mathfrak{A}_2(t, i, s + 1),
\end{align*}
\]

where the operator \(\mathfrak{A}_1^{-1}(t)\) is inverse to the operator \(\mathfrak{A}_1(t),\) and in particular case \(s = 2\) we have

\[
\Psi_2(t, \theta(\{1, 2\})) = \hat{G}_2(t, 1, 2) - I,
\]

where \(\hat{G}_2(t, 1, 2) \equiv G_2^s(t, 1, 2)(G_1^s)^{-1}(t, 1)(G_1^s)^{-1}(t, 2)\) is the scattering operator.

A method of the construction of marginal correlation functionals \((25)\) is based on the application of kinetic cluster expansions \((6)\) to the generating operators of series \((21)\). The structure of generating operators of series \((25)\) we shall define below for more general case of initial states.

We conclude only that the generating operator of the \(nth\) term of series expansion \((25)\) of marginal correlation functionals is the \((s + n)th\)-order evolution operator of cumulants of scattering operators.

It should be noted that marginal correlation functionals \((25)\) describe the correlations created by dynamics of large particle quantum systems by means of a one-particle correlation (density) operator.

Now we consider the case of initial states specified by a one-particle marginal density operator with correlations, namely, initial states specified by the following sequence of marginal correlation operators:

\[
G^{(cc)}(t) = (I, G_1^{0,1}(1), G_2^0(1, 2) \prod_{i=1}^{2} G_1^{0,i}(i), \ldots, g_n^0(1, \ldots, n) \prod_{i=1}^{n} G_1^{0,i}(i), \ldots), \tag{26}
\]

where the operators \(g_n^0(1, \ldots, n) \equiv g_n^0 \in \Omega_0^1(\mathcal{H}_n), n \geq 2,\) are specified the initial correlations. We remark that such assumption about initial states is intrinsic for the kinetic description of many-particle systems. On the other hand, initial data \((26)\) is typical for the condensed states of large
particle quantum systems, for example, the equilibrium state of the Bose condensate satisfies the weakening of correlation condition with the correlations which characterize the condensed state \cite{3}.

In this case the marginal correlation functionals \( G_s(t \mid G_1(t)) \), \( s \geq 2 \), are defined with respect to the one-particle (marginal) correlation operator

\[
G_1(t, 1) = \sum_{n=0}^{\infty} \frac{1}{n!} \text{Tr}_{2, \ldots, 1+n} \mathfrak{A}_{1+n}(t) g_{n+1}^{\varepsilon}(1, \ldots, n + 1) \prod_{i=1}^{n+1} G_1^{0, \varepsilon}(i),
\]

where the generating operator \( \mathfrak{A}_{1+n}(t) \equiv \mathfrak{A}_{1+n}(t, 1, \ldots, n+1) \) is the \((1+n)\text{th-order cumulant}\) of groups of operators \cite{14}, and they are represented by the following series expansions:

\[
G_s(t, Y \mid G_1(t)) = \sum_{n=0}^{\infty} \frac{1}{n!} \text{Tr}_{s+1, \ldots, s+n} \mathfrak{G}_{s+n}(t, \theta(\{Y\}), X \setminus Y) \prod_{i=1}^{s+n} G_1(t, i), \quad s \geq 2,
\]

where the \((s+n)\text{th-order generating operator}\) \( \mathfrak{G}_{s+n}(t) \), \( n \geq 0 \), of this series is determined by the following expansion:

\[
\mathfrak{G}_{s+n}(t, \theta(\{Y\}), X \setminus Y) \doteq n! \sum_{k=0}^{n} (-1)^k \sum_{n_1=1}^{n} \cdots \sum_{n_{k-1}=1}^{n-k+1} \frac{1}{(n-n_1 - \ldots - n_{k-1})!} \times
\]

\[
\prod_{j=1}^{k} \sum_{\substack{|D_j| \leq s+n-n_1 - \ldots - n_{j-1} \\text{or} \; |D_j| = s+n-n_1 - \ldots - n_j \\text{and} \; |D_j| \leq s+n-n_1 - \ldots - n_{j-1}}} \frac{1}{|D_j|!} \sum_{i_1 \neq \ldots \neq i_{|D_j|}} X_{i_j} \subset D_j \prod_{i_1 \neq \ldots \neq i_{|D_j|}} 1 \times
\]

\[
\mathfrak{A}_{s+n-n_1 - \ldots - n_{j}}(t, \theta(\{Y\}), s+1, \ldots, s+n-n_1 - \ldots - n_j).
\]

In formula \cite{29} we denote: \( Y \equiv (1, \ldots, s), X \setminus Y \equiv (s+1, \ldots, s+n) \), the sum over all possible dissections of the linearly ordered set \( Z_j \equiv (s+n-n_1 - \ldots - n_j, \ldots, s+n-n_1 - \ldots - n_{j-1}) \) on no more than \( s+n-n_1 - \ldots - n_j \) linearly ordered subsets we denote by \( \sum_{D_j; Z_j = \bigcup_{i_j} X_{i_j}} \) and the \((s+n)\text{th-order scattering cumulant}\) is defined by the formula

\[
\mathfrak{A}_{s+n}(t, \theta(\{Y\}), X \setminus Y) \doteq \mathfrak{A}_{s+n}(t, \theta(\{Y\}), X \setminus Y) g_{s+n}^{\varepsilon}(\theta(\{Y\}), X \setminus Y) \prod_{i=1}^{s+n} \mathfrak{A}_1^{s}(t, i),
\]

where the operator \( g_{s+n}^{\varepsilon}(\theta(\{Y\}), X \setminus Y) \) is specified initial correlations \cite{26}, and notations accepted above were used. We give examples of the scattering cumulants

\[
\mathfrak{G}_s(t, \theta(\{Y\})) = \mathfrak{A}_s(t, \theta(\{Y\})) \doteq \mathfrak{A}_s(t, \theta(\{Y\})) g_s^{\varepsilon}(\theta(\{Y\})) \prod_{i=1}^{s} \mathfrak{A}_1^{s}(t, i),
\]

\[
\mathfrak{G}_{s+1}(t, \theta(\{Y\}), s+1) = \mathfrak{A}_{s+1}(t, \theta(\{Y\}), s+1) g_{s+1}^{\varepsilon}(\theta(\{Y\}), s+1) \prod_{i=1}^{s+1} \mathfrak{A}_1^{s+1}(t, i) -
\]

\[
\mathfrak{A}_s(t, \theta(\{Y\})) g_s^{\varepsilon}(\theta(\{Y\})) \prod_{i=1}^{s} \mathfrak{A}_1^{s}(t, i) \sum_{j=1}^{s} \mathfrak{A}_2(t, j, s+1) g_2^{\varepsilon}(j, s+1) \mathfrak{A}_1^{s}(t, j) \mathfrak{A}_1^{s+1}(t, s+1).
\]
Dynamics of correlations

If \( \| G_1(t) \|_{L^1(\mathcal{H})} < e^{-(3s+2)} \), then for arbitrary \( t \in \mathbb{R} \) series expansion (28) converges in the norm of the space \( L^1(\mathcal{H}_s) \) [6].

We emphasize that marginal correlation functionals (28) describe the processes of the creation and the propagation of correlations generated by dynamics of large particle quantum systems in the presence of initial correlations by means of a one-particle density operator.

3.2 A quantum kinetic equation with initial correlations

We establish the evolution equation for one-particle marginal correlation operator (27).

As a result of the differentiation over the time variable of the operator represented by series (27) in the sense of the norm convergence of the space \( L^1(\mathcal{H}) \), then due to the application of the kinetic cluster expansions [2] to the generating operators of obtained series expansion, for one-particle (marginal) correlation operator (27) we derive the following identity

\[
\frac{\partial}{\partial t} G_1(t, 1) = N^* (1) G_1(t, 1) + \epsilon \text{Tr}_2 N^*_{\text{int}} (1, 2) G_1(t, 1) G_1(t, 2) +
\]

where the second part of the collision integral in equation (30) is determined in terms of the marginal correlation functional represented by series expansion (28) in case of \( s = 2 \). This identity we treat as the quantum kinetic equation and we refer to this evolution equation as the generalized quantum kinetic equation with initial correlations.

We emphasize that the coefficients in an expansion of the collision integral of the non-Markovian kinetic equation (30) are determined by the operators specified initial correlations (26).

For the generalized quantum kinetic equation with initial correlations (30) on the space \( L^1(\mathcal{H}) \) the following statement is true.

**Theorem 2.** If \( \| G_{1,0}^{0,\epsilon} \|_{L^1(\mathcal{H})} < (e(1 + e^9))^{-1} \), the global in time solution of initial-value problem of kinetic equation (30) is determined by series expansion (27). For initial data \( G_{1,0}^{0,\epsilon} \in L^1(\mathcal{H}) \) it is a strong solution and for an arbitrary initial data it is a weak solution.

The proof of the existence theorem is similar to the case of the generalized quantum kinetic equation [5].

4 A mean field asymptotic behavior of marginal correlation operators

This section deals with the scaling asymptotic behavior of the constructed marginal correlation operators. The processes of the creation and the propagation of correlations will be described in a mean field limit for two cases: initial states satisfying a chaos property and initial states specified by means of a one-particle density operator and correlations.

4.1 On the propagation of initial chaos

In the beginning we give comments on the mean field asymptotic behavior of constructed solution (18) in case of the initial state (20), satisfying a chaos condition [6].
We assume the existence of a mean field limit of the initial marginal correlation operator (or the one-particle density operator) in the following sense

$$\lim_{\epsilon \to 0} \| \epsilon G_{1}^{0,\epsilon} - g_{1}^{0} \|_{L^{1}(H)} = 0. \quad (31)$$

Since \( n \)th term of series expansion (21) for \( s \)-particle marginal correlation operators is determined by the \((s + n)\)th-order cumulants of asymptotically perturbed groups of operators (5) for which the following equality is true

$$\lim_{\epsilon \to 0} \left\| \frac{1}{\epsilon^{n}} A_{s+n}(t, 1, \ldots, s + n)f_{s+n} \right\|_{L^{1}(H_{s+n})} = 0, \quad s \geq 2, \quad (32)$$

then we establish the property of the propagation of initial chaos (20)

$$\lim_{\epsilon \to 0} \| \epsilon^{s} G_{s}(t) \|_{L^{1}(H)} = 0, \quad s \geq 2.$$ 

In case of \( s = 1 \) for series expansion (21) the following equality is true

$$\lim_{\epsilon \to 0} \| \epsilon G_{1}(t) - g_{1}(t) \|_{L^{1}(H)} = 0,$$

where for arbitrary finite time interval the limit one-particle marginal correlation operator \( g_{1}(t, 1) \) is given by the norm convergent series on the space \( L^{1}(H) \)

$$g_{1}(t, 1) = \sum_{n=0}^{\infty} \int_{0}^{t} dt_{1} \ldots \int_{0}^{t_{n-1}} dt_{n} \Tr_{2, \ldots, n+1} G_{1}^{*}(t - t_{1}, 1) N_{\text{int}}^{*}(1, 2) \prod_{j=1}^{2} G_{1}^{*}(t_{1} - t_{2}, j_{1}) \ldots \prod_{i_{n}=1}^{n} G_{1}^{*}(t_{n} - t_{n}, i_{n}) \sum_{k_{n}=1}^{n} N_{\text{int}}^{*}(k_{n}, n + 1) \prod_{j_{n}=1}^{n+1} G_{1}^{*}(t_{n}, j_{n}) \prod_{i=1}^{n+1} g_{1}^{0}(i). \quad (33)$$

In series (33) the operator \( N_{\text{int}}^{*}(j_{1}, j_{2}) \) is defined according to formula (6) and the mapping \( G_{1}^{*}(t) \) is defined by formula (4). For bounded interaction potential series (33) is norm convergent on the space \( L^{1}(H) \) under the condition that: \( t < t_{0} \equiv (2 \| \Phi \|_{L^{2}(H_{2})} g_{1}^{0} \|_{L^{1}(H)})^{-1}. \)

As a result of the differentiation over the time variable of the operator represented by series (33) in the sense of the norm convergence of the space \( L^{1}(H) \), we conclude that limit one-particle marginal correlation operator (33) is governed by the Cauchy problem of the Vlasov quantum kinetic equation

$$\frac{\partial}{\partial t} g_{1}(t, 1) = N^{*}(1) g_{1}(t, 1) + \Tr_{2} N_{\text{int}}^{*}(1, 2) g_{1}(t, 1) g_{1}(t, 2), \quad (34)$$

$$g_{1}(t)_{|_{t=0}} = g_{1}^{0}, \quad (35)$$

and consequently, in case of pure states we derive the Hartree equation [6], i.e. in terms of the kernel \( g_{1}(t, q, q') = \psi(t, q) \psi^{*}(t, q') \) of the operator (33), describing a pure state, equation (34) reduces to the Hartree equation

$$i \frac{\partial}{\partial t} \psi(t, q) = -\frac{1}{2} \Delta_{q} \psi(t, q) + \int dq' \Phi(q - q') |\psi(q')|^{2} \psi(t, q),$$

where the function \( \Phi \) is a two-body interaction potential.
4.2 On the propagation of initial correlations

Further, we establish the mean field asymptotic behavior of constructed marginal correlation operators \([18]\) in case of the initial state specified by the one-particle marginal density operator with correlations \([26]\).

We assume the existence of a mean field limit of initial one-particle marginal correlation operator in the sense of equality \([31]\) and initial correlations as follows:

\[
\lim_{\epsilon \to 0} \|g_n^\epsilon - g_n\|_{\mathcal{L}^1(\mathcal{H}_n)} = 0, \quad n \geq 2.
\]  

(36)

Hence a mean field limit of initial state \([26]\) is specified by the sequence of limit marginal correlation operators defined on the space \(\mathcal{F}_\mathcal{H} = \bigoplus_{n=0}^\infty \mathcal{H}_n\):

\[
g^{(cc)} = (I, g_0^0(1), g_2(1,2) \prod_{i=1}^2 g_1^0(i), \ldots, g_n(1,\ldots,n) \prod_{i=1}^n g_i^0(i), \ldots).
\]

(37)

Under conditions \([31],(36)\) on initial state \([26]\) there exists a mean field limit of marginal correlation operators \([18]\) in the following sense

\[
\lim_{\epsilon \to 0} \|\epsilon^s G_s(t) - g_s(t)\|_{\mathcal{L}^1(\mathcal{H}_s)} = 0, \quad s \geq 1,
\]

where for \(s \geq 2\) the limit marginal \((s\text{-particle})\) correlation operator \(g_s(t)\) is represented by the operator

\[
g_s(t,1,\ldots,s) = \prod_{i_1=1}^s G^*_i(t,i_1) g_s(1,\ldots,s) \prod_{i_2=1}^s (G^*_i)^{-1}(t,i_2) \prod_{j=1}^s g_1(t,j), \quad s \geq 2,
\]

(37)

and, respectively, the limit one-particle correlation operator \(g_1(t)\) is represented by the following series expansion

\[
g_1(t,1) = \sum_{n=0}^{\infty} \int_0^t \cdots \int_0^{t_{n-1}} \text{Tr}_{2^n} \prod_{j=1}^n G^*_1(t-t_{j-1},1) N^*_\text{int}(1,2) \prod_{j=1}^2 G^*_1(t_{j-1},t_j) \cdots
\]

\[
\prod_{i_n=1}^n G^*_1(t_n-t_{n-1},i_n) \sum_{k_n=1}^n N^*_\text{int}(k_n,n+1) \prod_{j_n=1}^{n+1} G^*_1(t_n,j_n) \times
\]

\[
\sum_{P: (1,\ldots,n+1) = \bigcup_i X_i} \prod_{X_i} g_1|_{X_i}(X_i) \prod_{i=1}^{n+1} g^0_i(i).
\]

For bounded interaction potentials series \([38]\) is norm convergent on the space \(\mathcal{L}^1(\mathcal{H})\) under the condition that: \(t < t_0 \equiv (2 \|\Phi\|_{\mathcal{L}(\mathcal{H}_2)} \|g_0^0\|_{\mathcal{L}^1(\mathcal{H})})^{-1}\).

The operator \(g_1(t)\) represented by series \([38]\) is a solution of the Cauchy problem of the Vlasov-type quantum kinetic equation with initial correlations:

\[
\frac{\partial}{\partial t} g_1(t,1) = N^*_\text{int}(1) g_1(t,1) +
\]

\[
\text{Tr}_2 N^*_\text{int}(1,2) \prod_{i_1=1}^2 G^*_1(t,i_1)(g_2(1,2) + 1) \prod_{i_2=1}^2 (G^*_1)^{-1}(t,i_2) g_1(t,1) g_1(t,2),
\]

\[
g_1(t)|_{t=0} = g^0_1,
\]

(39)
where the operators $N^*(1)$ and $N^*_\text{int}(1,2)$ are defined according to formula (6). We point out that derived kinetic equation (39) is non-Markovian quantum kinetic equation.

Thus, mean field dynamics does not create new correlations except of those that generating by the initial correlations.

The proof of stated results (37) and (38) is based on the validity of equality (32) for cumulants of asymptotically perturbed groups of operators (5) and the explicit structure of the generating operators of series expansion (18) of marginal correlation operators, for example, in case of $s = 1$ series expansion (18) takes the form

$$G_1(t,1) = \sum_{n=0}^{\infty} \frac{1}{n!} \text{Tr}_{2,...,1+n} \mathfrak{A}_{1+n}(t) \prod_{P : (1, \ldots, n+1) = \bigcup_i X_i \subset P} \prod_{i=1}^{n+1} g^\text{0,\epsilon}_{|X_i|} (i),$$

where the generating operator $\mathfrak{A}_{1+n}(t) \equiv \mathfrak{A}_{1+n}(t,1,\ldots,n+1)$ is the $(1 + n)$th-order cumulant (14) of groups of operators (14).

We remark that the sequence of limit marginal correlation operators (37) and (38) is a solution of the quantum Vlasov hierarchy of nonlinear evolution equations [19], which is described a mean field limit of marginal correlation operators (18) in case of arbitrary initial states, i.e.

$$\frac{\partial}{\partial t} g_s(t,1,\ldots,s) = \sum_{i \in (1,\ldots,s)} N^*(i) g_s(t,1,\ldots,s) + \text{Tr}_{s+1} \sum_{i \in (1,\ldots,s)} N^*_\text{int}(i,s+1) (g_{s+1}(t,1,\ldots,s+1) + \sum_{P : (1,\ldots,s+1) = X_1 \cup X_2, \ i \in X_1; s+1 \in X_2} g_{|X_1|}(t,X_1) g_{|X_2|}(t,X_2)), \ s \geq 1,$$

where we used notations similar to accepted in kinetic equation (39).

We note also that for marginal correlation functionals (28) the following equalities hold:

$$\lim_{\epsilon \to 0} \left\| \epsilon^s G_s(t,1,\ldots,s \mid G_1(t)) - \prod_{i_1=1}^{s} g_{1}(t,i_1) g_{s}(1,\ldots,s) \prod_{i_2=1}^{s} (G_{1})^{-1}(t,i_2) \prod_{j=1}^{s} g_{1}(t,j) \right\|_{\mathcal{L}^1(H_{\epsilon\lambda})} = 0, \ s \geq 2,$$

where the limit one-particle (marginal) correlation operator $g_1(t)$ is represented by series expansion (38), i.e. it is a solution of the Cauchy problem (39), (40).

## 5 Conclusion

The marginal correlation operators (18) give an equivalent approach to the description of the evolution of states of large particle quantum systems in comparison with the marginal density operators. The macroscopic characteristics of fluctuations of observables are directly determined by marginal correlation operators on the microscopic scale [3], [19].

This paper deals with a quantum system of a non-fixed, i.e. arbitrary but finite, number of identical (spinless) particles obeying Maxwell–Boltzmann statistics. The obtained results can be extended to large particle quantum systems of bosons and fermions [18].
In the paper it was established that a nonperturbative solution of the Cauchy problem of the nonlinear quantum BBGKY hierarchy \((16),(17)\) for a sequence of marginal correlation operators is represented in the form of series expansion \((18)\) over particle subsystems which generating operators are corresponding-order cumulant \((19)\) of groups of nonlinear operators \((7)\). In case of initial states specified by a sequence of the marginal correlation operators that satisfy a chaos property \((20)\) the correlations generated by dynamics of large particle quantum systems \((21)\) are completely determined by cumulants \((8)\) of groups of operators \((5)\) of the von Neumann equations.

The concept of quantum kinetic equations in case of initial states specified in terms of a one-particle density operator and correlation operators \((30)\), for instance, the correlation operators, characterizing the condensed states, was considered. We remark that in case of pure states the quantum Vlasov-type kinetic equation with initial correlations \((39)\) can be reduced to the Gross–Pitaevskii-type kinetic equation.

We also emphasize that the natural Banach spaces for the description of states of large particle quantum systems, for instance, the operator spaces containing the sequence of operators of equilibrium state, are different from the used Banach space \([6]\). In paper \([21]\) it was introduced the space of sequences of bounded translation invariant operators, making it a better choice for the description of quantum correlations.

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