Learning Taxonomy for Text Segmentation by Formal Concept Analysis

Mihaiela Lupea  
"Babeș-Bolyai" University  
Cluj-Napoca, Romania

Doina Tatar  
"Babeș-Bolyai" University  
Cluj-Napoca, Romania

Zsuzsana Marian  
"Babeș-Bolyai" University  
Cluj-Napoca, Romania

Abstract

In this paper the problems of deriving a taxonomy from a text and concept-oriented text segmentation are approached. Formal Concept Analysis (FCA) method is applied to solve both of these linguistic problems. The proposed segmentation method offers a conceptual view for text segmentation, using a context-driven clustering of sentences. The Concept-oriented Clustering Segmentation algorithm (COCS) is based on k-means linear clustering of the sentences. Experimental results obtained using COCS algorithm are presented.

1. Introduction

Formal Concept Analysis (FCA) studies how objects can be hierarchically grouped together when their common attributes are studied in a given context. Linguists often characterize datasets using distinct features, such as semantic components or syntactical and grammatical markers, which can easily be interpreted using FCA. However, linguists argue that formal concepts are quite different from cognitive processes relating to natural language [13]. This is why current FCA applications in linguistics focus more on formal structures than on cognitive linguistic phenomena.

Eventually, in the linguistic domain FCA applications provide a very suitable alternative to statistical methods.

In this paper we address the problem of deriving a taxonomy from a text for text segmentation by concept-driven clustering. This conceptual view of segmentation is useful when different users have quite different needs with regard to way of segmentation.

The needed knowledge in our Concept-oriented Clustering Segmentation algorithm COCS is only the taxonomy derived from text. It is used the k-means algorithm for a linear clustering of the sentences.

The paper is structured as follows: Section 2 introduces the basic notions of ontologies and FCA. Section 3 surveys the related work in taxonomies extraction from a text and in text segmentation. Section 4 introduces CLTE (concept lattice-taxonomy extraction) algorithm and COCS algorithm for text segmentation. In Section 5 experimental results obtained using COCS algorithm are presented. We finish the paper with conclusions and future work directions in Section 6.

2. Abstract Ontologies and FCA

Following [6], an ontology is a formal specification of a shared conceptualization of a domain of interest to a group of users. Formal implies that the ontology should be machine readable, and shared implies it is accepted by a group or community.

Definition 1. An abstract ontology $O$ is a model represented by:

$$O = (C, H, R, A)$$

where:

- $C$ is a set of concepts (concept identifiers);
- $H$ is a taxonomic relation (IS-A) between concepts, $H \subseteq C \times C$, that means it is a partial and transitive order on $C$;
- $R$ is a set of non-taxonomic relations, $R \subseteq C \times C$;
- $A$ is a set of logical axioms (or inference rules).

Mostly approaches focus on the first two elements of an ontology $C$ and $H$, which form the "core ontology" while the researches on the sets $R$ and $A$ are least addressed.

The above definition doesn’t make a distinction between a concept and its lexical expression. Completing $O$ with a lexicon could be addressed the problems of synonymy (a set of lexical expressions represents the same concept) and that of the polysemy (a lexical expression represents a set of concepts).

In the particular case of learning a taxonomy from a text we will present the method used by [4] and our proposed version.
2.1. A short survey of Formal Concept Analysis (FCA)

FCA has been introduced by B. Ganter and R. Wille in 1982 (for a textbook see [6]). During the last years, FCA has grown into an international research community with applications in many different domains as artificial intelligence, linguistics, software engineering, medicine, etc.. Formal concepts in FCA can be seen as a mathematical formalization of what has been called the theory of concepts, which states that a concept is formally defined via its features [13]. From a philosophical point of view, a concept is a unit consisting from two parts: the intension (the set of attributes valid for all these objects) and the extension (the set of objects belonging to this concept) and the intension (the set of attributes valid for all these objects). The frame for defining a set of concepts is the so called Formal Context.

**Definition 2.** A Formal Context is a triple:

$$K = (G, M, I)$$

where $G$ is the set of objects, $M$ is the set of attributes, and $I$ is a binary relation between $G$ and $M$ ($I \subseteq G \times M$), representing the incidence relation. The pair $(g, m) \in I$ is read as "the object $g$ has the attribute $m$".

Usually a Formal Context is given by an incidence matrix, where a star "*" on the line of $g$ and the column of $m$ means that the object $g$ has the attribute $m$.

For a set $A \subseteq G$, the set of all attributes shared by the objects from $A$, called the "derivative" of $A$ and denoted by $A'$ is defined as:

$$A' = \{m \in M \mid \forall g \in A, (g, m) \in I\}$$

Dually, for a set $B \subseteq M$, the set of all objects which share the attributes from $B$, called the "derivative" of $B$ and denoted by $B'$ is defined as:

$$B' = \{g \in G \mid \forall m \in B, (g, m) \in I\}$$

**Definition 3.** A Formal Concept of the Formal Context $K = (G, M, I)$ is a pair $(A, B)$, with $A \subseteq G$, $B \subseteq M$ and satisfying the relations:

$$A' = B \quad \text{and} \quad B' = A$$

The set $A$ is called the extent of the Formal Concept $(A, B)$ and the set $B$ is called the intent of the same Formal Concept.

Between the Formal Concepts the relation $\leq$ of subconcept-superconcept is defined as below:

$$(A_1, B_1) \leq (A_2, B_2) \text{ if and only if } A_1 \subseteq A_2 \text{ or equivalently } (A_1, B_1) \leq (A_2, B_2) \text{ if and only if } B_2 \subseteq B_1.$$
Let us remark that the lattice is not clarified because the set of objects: \{excursion, trip\} have the same intent: \{bookable, joinable\}. This is the reason in the node C5 the “object” label is formed by the set \{bookable, joinable\}.

**Example 1.** Consider the concept \(C_1 = (A_1, B_1)\) from the previous lattice. Here the extent \(A_1\) is formed by all the objects situated on paths starting with \(C_1\). \(A_1 = G = \{\text{apartment, car, motor-bike, excursion, trip, hotel}\}\). The intent is \(B_1 = \{\text{bookable}\}\). The relations \(A_1' = B_1\) and \(B_1' = A_1\) are verified.

For the concept \(C_2 = (A_2, B_2)\) the extent is \(A_2 = \{\text{apartment, car, motor – bike}\}\) and the intent is \(B_2 = \{\text{bookable, rentable}\}\). Again, the relations \(A_2' = B_2\) and \(B_2' = A_2\) are verified. The **Concept lattice** relation \(C_2 \leq C_1\) is valid, because \(A_2 \subseteq A_1\) (and, equivalently, \(B_1 \subseteq B_2\)).

From the **Concept lattice** of the tourism example the following taxonomy is obtained [4]:

\[
\begin{align*}
\text{bookable} & \quad \downarrow \quad \text{rentable} \\
\downarrow & \quad \downarrow \\
\text{excursion} & \quad \text{apartment} \\
\downarrow & \quad \downarrow \\
\text{trip} & \quad \text{driveable} \\
\downarrow & \quad \downarrow \\
\text{hotel} & \quad \text{motor-bike}
\end{align*}
\]

**Remark:** In this kind of taxonomy the name of verbs could be replaced by the name of corresponding noun: for example **joinable** could be replaced by **join** or **driveable** by **vehicle** to improve the readability of the taxonomy.

As we already have mentioned above, in [4] the **Formal Context** is obtained selecting as \(M\) the set of transitive verbs from a text and as \(G\) the set of nouns playing the role of (direct) complement for the verbs from \(M\).

For the selected domain: \(M = \{\text{bookable, rentable, driveable, rideable, joinable}\}\), \(G = \{\text{apartment, car, excursion, motor-bike, trip, hotel}\}\) and the relation \(I\) is given by the incidence matrix (Table 1).

According to the method for obtaining the **Concept Lattice** ([6]), the set of all **Formal Concepts** are represented in Table 2. Applying the definition of the *subconcept* relation, the following **Concept lattice**, is obtained:

| Extent of concept | Intent of concept | Concept |
|-------------------|------------------|---------|
| \{apartment, car, motor – bike, trip, excursion, hotel\} | \{bookable\} | C1 |
| \{apartment, car, motor – bike\} | \{bookable, rentable\} | C2 |
| \{car, motor – bike\} | \{bookable, rentable, driveable\} | C3 |
| \{motor – bike\} | \{bookable, rentable, rideable\} | C4 |
| \{excursion, trip\} | \{bookable, joinable\} | C5 |
| \(\Phi\) | \{bookable, rentable, driveable, rideable, joinable\} | C6 |

**Table 1. The incidence matrix for tourism example**

| Extent of concept | Intent of concept | Concept |
|-------------------|------------------|---------|
| \{apartment, car, motor – bike, trip, excursion, hotel\} | \{bookable\} | C1 |
| \{apartment, car, motor – bike\} | \{bookable, rentable\} | C2 |
| \{car, motor – bike\} | \{bookable, rentable, driveable\} | C3 |
| \{motor – bike\} | \{bookable, rentable, driveable, rideable\} | C4 |
| \{excursion, trip\} | \{bookable, joinable\} | C5 |
| \(\Phi\) | \{bookable, rentable, driveable, rideable, joinable\} | C6 |

**Table 2. The Formal Concepts for tourism example**
A discourse segment consists of a sequence of sentences that display local coherence. Text segmentation is the automatic identification of boundaries between segments. The need for discourse segmentation derives from its applicability in many fields as for example:

- **Information Retrieval (IR).** Many authors, like [8] and [15], showed that segmenting into distinct topics is useful as IR needs to find relevant portions of text that match with a given query;
- **Anaphora resolution (AR).** Mining the text only in some segments for finding the antecedents for some referential expressions could improve the quality of AR ([12]);
- **Text summarization.** Segmentation as a preprocessing step in automatic summarization (as in this paper) could improve the quality of summaries [3].

While the need for segmentation of discourse is almost universally agreed upon, there is no consensus on how the segmentation could be accomplished [1].

A final classification of segmentation methods is into cohesion based methods (as for example lexical chains) and coherence based methods (as in RST theory and Hobbs’s coherence relations theory).

### 4. This paper proposal

#### 4.1. Obtaining the Concept Lattice and the Concept Hierarchy from a text

FCA is used to build the Concept Lattice and then to extract the Concept Hierarchy from a text using as attributes the transitive verbs and as objects the corresponding nouns with the role of direct complements from the studied corpus. We propose the Concept Lattice - Taxonomy Extraction (CLTE) algorithm which introduces specific rules for deriving the taxonomy as a quasi-tree from the Concept Lattice.
Concept Lattice - Taxonomy Extraction algorithm (CLTE):

**Input:** Text - a text document.

**Output:** K-the formal context, L-the concept lattice, T- the taxonomy based on the concept lattice.

**Step1:** Text-Pos = Pos-tagging(Text).

**Step2:** Pairs = {{verb, noun-direct-complement}};
                     = extract-pairs(Text-Pos).

**Step3:** Pairs-lemma=lemmatize-verbs-nouns(Pairs).

**Step4:** M = frequent-verbs(Pairs-lemma);

G = frequent-nouns(Pairs-lemma).

**Step5:** Build the formal context: K = (G, M, I)
where (n, v) ∈ I, if (v, n) ∈ Pairs-lemma.

**Step6:** Build the concept lattice L=B(K).

**Step7:** Build the taxonomy T, represented as a quasi-tree, based on the concept lattice L.

**Remarks:**

- The POS annotation is enough and no parsing is needed for the initial text corpus. Rules for determining the dependency verb - noun as a direct complement must be used.
- Generally the taxonomy, derived from a concept lattice, cannot be represented as a tree like in Cimiano’s example, but using a special data structure, called a quasi-tree (a node may have more parents and two internal nodes may have the same label), T = (X, E), with the following properties:

  - X = G ∪ M and E, the set of edges, is obtained from de subconcept relation of the Concept lattice according to special rules.
  - The most general concept (the top of the lattice) is the root of the quasi-tree.
  - The leaves of the quasi-tree T are labeled with nouns (objects) from G and the internal nodes are labeled with verbs (attributes) from M.
  - Let C^a,o → C^{a', o'} be an edge in the Concept lattice, where the node C^a,o introduces the object o and the attribute a. There are 16 cases (a, o, a', o' can be equal or not equal with ∅), some of them impossible cases. The most used rules for adding nodes and edges in the taxonomy, represented as a quasi-tree, are the following:

    * if a ∉ ∅, o = ∅, a' ≠ ∅ then (a, o') ∈ E;
    * if a = ∅, o ≠ ∅, a' = ∅ then (a, o') ∈ E;
    * if a ∉ ∅, o ≠ ∅, a' = ∅ then (a, o') ∈ E, o is a leaf node;
    * if a = ∅, o ≠ ∅, a' ≠ ∅ then (a, o') ∈ E, (a, o) ∈ E, o is a leaf node;
    * if a ≠ ∅, o = ∅, a' ≠ ∅ then (a, o') ∈ E, (a, o) ∈ E, o is a leaf node;
    * if a ≠ ∅, a' = ∅, o' ≠ ∅ then (a, o') ∈ E, o' is a leaf node;
    * if a ≠ ∅, a' ≠ ∅, o' ≠ ∅ then (a, o') ∈ E, (a, o) ∈ E, (a, o') ∈ E.

- A path from the root to a leaf node provides a hierarchy regarding the concept terms (verbs and nouns) on that path.

**4.2. Concept-oriented segmentation by clustering**

The process of segmentation is seen as an objective method, which provides one clearly defined result. However, different users have quite different needs with regard to a segmentation because they view the same text from completely different, subjective, perspectives. Segmenting a text must be associated with an explanation of why a given set of segments is produced. All these could be realized by viewing the process of segmentation as a clustering process of the sentences of a text [16].

When the cluster Cl = \{Si_1, \ldots, Si_m\} is one of the set of obtained clusters, and \(t_1 \leq i_2 \leq \cdots \leq i_m\), then the linear segmentation is: \([Si_1, Si_{i_1-1}], [Si_{i_1}, Si_{i_2}], \ldots, [Si_{i_{m-1}}, Si_{i_m}], [Si_{i_m}, Si_n]\). The concept terms which are “specific” to this cluster Cl (concept terms specific to the center of cluster Cl) explain the reason of the segmentation.

Let us remark that usually clustering texts means selecting of the most important (by frequency) words (terms) as features of clustering ([3]). In our method we choose as words the transitive verbs and complement nouns which form the concepts in the FCA approach ([4]). In what follows we refer to these words (terms) as concept terms, namely concept attribute terms, M, and concept object terms, G.

A sentence is represented as a vector of concept terms: an entry of each vector specifies the frequency that a concept term occurs in the text, including the frequency of subconcept terms.

The following algorithm is an improvement of an own algorithm introduced in [16].

**Concept-oriented Clustering Segmentation algorithm COCS:**

**Input:** Text = \{S_1, \ldots, S_n\} of n sentences,
- the output of the CLTE algorithm;
K- the formal context, L- the concept lattice;
T- the taxonomy based on L.

**Output:** Different segmentations of the text Text, according to different sets of concepts.

- **Step1:** Calculate the frequency \(f(i, t)\) of the concept term \(t \in G \cup M\) in the sentence \(S_i\).
The used clustering method is **K-means** which we survey below.

**K-means algorithm**[11]:

**Input:** Text = \{S_1, \cdots, S_n\} of n sentences, the corresponding vectors \{V(1), \cdots, V(n)\} obtained at Step4 of COCS algorithm.

**Output:** The set of clusters \( C = \{C_1, C_2, \ldots, C_k\} \)

Begin

Select \( k \) initial centroids:

\[ \{f_1, f_2, \ldots, f_k\} \subseteq \{V(1), \cdots, V(n)\} \]

While the stopping criterion is not true Do

For \( j = 1 \) to \( k \) Do

\[ C_j = \{V(i) | \forall f_j, d(V(i), f_j) \leq d(V(i), f_i), \frac{1}{\cosine(V(i), V(j))}\} \]

End-For

For \( j = 1 \) to \( k \) Do

\[ f_j = \sum_{i \in C_j} \frac{x_i}{|C_j|} \]

End-For

End-While

End-algorithm

The **K-means algorithm** begins with a set of initial cluster centers, selected such that they are as least similar as possible. At each while-iteration, each vector is assigned to the cluster whose center is closest and then the centroids of the modified clusters are recomputed as a mean of its members. The distance between two vectors is computed as the inverse of the similarity of the vectors. The stopping criterion can be the condition that the diameters of all clusters are smaller than a threshold value or that there are no changes in \( C \) from the previous iteration. A diameter of a cluster is the distance between the least similar elements in the cluster.

### 5. Experimental results

The algorithms proposed in the previous sections were implemented and tested on texts from different domains as art, music, law.

Considerations for implementation:
- For POS-tagging and lemmatization of verbs and nouns we have used Online CST tools which incorporate a tokenizer, name recognizer, Brill-POS-tagger (an error-driven transformation-based tagger), lemmatiser, NP recognizer and other tools ([http://conexp.sourceforge.net/index.html](http://conexp.sourceforge.net/index.html)).
- The pairs (transitive verb, noun as a direct complement) were obtained using our specific rules for determining this type of dependency.
- The most frequent verbs and nouns were chosen such that they appear twice in the set of selected pairs.
- The construction from the concept lattice of the quasi-tree representing the taxonomy of the concept terms is based on the rules proposed in Subsection 4.1.
- The implementation of COCS-algorithm follows the described above steps.

As experimental results we describe an example of a text, consisting of 320 sentences, from the law domain. An extract of 30 sentences occurs in the Figure 2. The Concept lattice is computed with the CLTE algorithm and visualized in Figure 1, using ConExp. This is a software tool aimed for handling the tasks involved in the study of lattice theory, mainly formal concepts. (More information is available at [http://conexp.sourceforge.net/index.html](http://conexp.sourceforge.net/index.html)).

The taxonomy is too complex to be depicted, but we present some paths in the corresponding quasi-tree representing hierarchies of concept terms:

- **inform → support → progress**
- **continue → represent → tradition**
- **have → influence → law → system**
- **codify → make → law**
- **develop → reject → principle**

The **COCS** algorithm was applied only to the first 102 sentences of the initial text. At Step2, the frequency of a concept term \( t \) in a sentence is obtained as the sum of its own frequency and the frequencies of the direct descendents of \( t \) in the taxonomy. For example: 

\[
Total_S(14, \text{concern}) = f(14, \text{concern}) + f(14, \text{justice}) + f(14, \text{system}) = 1 + 1 + 0 = 2.
\]
There are 21 terms (representing the value of $m$ in Step 4 of COCS algorithm): \{concern, have, kill, law, own, offenders, include, write, boy, condone, preserve, eat, hold, do, create, make, govern, provide, buy, shape, jewel\} used as features for clustering. After the clustering process 4 clusters were obtained.

The cluster $C_1 = \{S_8, S_{19}, S_{27}, S_{31}, S_{37}, S_{40}, S_{60}, S_{63}\}$ is characterized by the concept terms: \{have, offenders, write, condone, do, govern\}, meaning that these terms appear in the sentences of the cluster.

The corresponding linear segmentation of the text is: $[S_1, S_2], [S_8, S_{18}], [S_{19}, S_{26}], [S_{27}, S_{30}], [S_{41}, S_{30}], [S_{37}, S_{39}], [S_{40}, S_{59}], [S_{60}, S_{62}], [S_{63}, S_{102}]$.

The cluster $C_2 = \{S_3, S_{14}, S_{20}, S_{53}, S_{54}, S_{68}, S_{71}, S_{74}, S_{84}\}$ is characterized by the concept terms: \{concern, preserve, buy, shape, jewel\} and provides the segmentation: $[S_1, S_2], [S_3, S_{13}], [S_{14}, S_{19}], [S_{20}, S_{52}], [S_{53}, S_{53}], [S_{54}, S_{67}], [S_{68}, S_{70}], [S_{71}, S_{73}], [S_{74}, S_{83}], [S_{84}, S_{102}]$.

6. Conclusions and further work

In this paper we applied the FCA theory to obtain a taxonomy (algorithm CLTE) for concept-oriented segmentation of a text. The COCS algorithm introduced in this paper approaches the process of segmentation as a clustering process of the sentences of a text, using the taxonomy learned from a text. Each cluster provides a segmentation, explained by the concept terms specific for this cluster.

As further work we propose to improve the taxonomy learned from a text considering also as pairs of attribute-object: (verb at the passive, corresponding noun with the role of subject). More experiments with texts from different domains are needed in order to evaluate our approach.

References

[1] J.Allen: "Natural language understanding", Benjamin/Cummings Publ., 2nd ed., 1995.
Law is a system of rules, usually enforced through a set of institutions. It shapes politics, economics and society in numerous ways and serves as a primary social mediator of relations between people. Contract law regulates everything from buying a bus ticket to trading on derivatives markets. Property law defines rights and obligations related to the transfer and title of personal (often referred to as chattel) and real property. Trust law applies to assets held for investment and financial security, while tort law allows claims for compensation if a person’s rights or property are harmed. If the harm is criminalized in a statute, criminal law offers means by which the state can prosecute the perpetrator. Constitutional law provides a framework for the creation of law, the protection of human rights and the election of political representatives. Administrative law is used to review the decisions of government agencies, while international law governs affairs between sovereign nation states in activities ranging from trade to environmental regulation or military action. Writing in 350 BC, the Greek philosopher Aristotle declared, “The rule of law is better than the rule of any individual.” Legal systems elaborate rights and responsibilities in a variety of ways. A general distinction can be made between civil law jurisdictions, which codify their laws, and common law systems, where judge made law is not consolidated. In some countries, religion informs the law. Law provides a rich source of scholarly inquiry, into legal history, philosophy, economic analysis or sociology. Law also raises important and complex issues concerning equality, fairness and justice. In its majestic equality”, said the author Anatole France in 1894, “the law forbids rich and poor alike to sleep under bridges, beg in the streets and steal loaves of bread.” In a typical democracy, the central institutions for interpreting and creating law are the three main branches of government, namely an impartial judiciary, a democratic legislature, and an accountable executive. To implement and enforce the law and provide services to the public, a government’s bureaucracy, the military and police are vital. While all these organs of the state are creatures created and bound by law, an independent legal profession and a vibrant civil society inform and support their progress. Constitutional and administrative law govern the affairs of the state. Constitutional law concerns both the relationships between the executive, legislature and judiciary and the human rights or civil liberties of individuals against the state. Most jurisdictions, like the United States and France, have a single codified constitution, with a Bill of Rights. A few, like the United Kingdom, have no such document. Constitutional and administrative law govern the affairs of the state. Constitutional law concerns both the relationships between the executive, legislature and judiciary and the human rights or civil liberties of individuals against the state. Most jurisdictions, like the United States and France, have a single codified constitution, with a Bill of Rights. A law "constitution" is simply those laws which constitute the body politic, from statute, case law and convention. A case named Entick v Carrington illustrates a constitutional principle deriving from the common law. 25.Mr Entick’s house was searched and ransacked by Sheriff Carrington. 26.When Mr Entick complained in court, Sheriff Carrington argued that a warrant from a Government minister, the Earl of Halifax, was valid authority. However, there was no written statutory provision or court authority. The great end, for which men entered into society, was to secure their property. That right is preserved sacred and incommunicable in all instances, where it has not been taken away or abridged by some public law for the good of the whole ... If no excuse can be found or produced, the silence of the books is an authority against the defendant, and the plaintiff must have judgment.