I. INTRODUCTION

Wolf discovered how the spatial coherence characteristics of the source affect the spectrum of the radiation in the far zone. In particular the spatial coherence of the source can result either in red or blue shifts in the measured spectrum. His predictions have been verified in a large number of different classes of systems. Wolf and coworkers usually assume a given form of source correlations and study its consequence. In this paper we consider microscopic origin of spatial coherence and radiation from a system of atoms. We discuss how the radiation is different from that produced from an independent system of atoms. We show that the process of radiation itself is responsible for the creation of spatial correlations within the source. We present different features of the spectrum and other statistical properties of the radiation, which show strong dependence on the spatial correlations. We show the existence of a new type of two-photon resonance that arises as a result of such spatial correlations. We further show how the spatial coherence of the field can be used in the context of radiation generated by nonlinear optical processes. We conclude by demonstrating the universality of Wolf shifts and its application in the context of pulse propagation in a dispersive medium.

![Diagram of radiation sources and observation point](image-url)

We start by giving a summary of Wolf’s main results. Consider the radiation produced by two point sources $P_1$ and $P_2$ at the observation point $P$, Fig. 1. Let us consider for simplicity the case of scalar fields $U(P, \omega)$. The spectrum of the field at $P$ is given by

$$S_U(P, \omega) = \langle U^*(P, \omega)U(P, \omega) \rangle,$$

where as the spectrum of the source is defined by

$$S_Q(\omega) = \langle Q^*(P_1, \omega)Q(P_1, \omega) \rangle = \langle Q^*(P_2, \omega)Q(P_2, \omega) \rangle.$$

We assume identical spectra for the two sources. Let $\mu_Q(\omega)$ be the spectral degree of coherence between two sources

$$\mu_Q = \frac{\langle Q^*(P_1, \omega)Q(P_2, \omega) \rangle}{S_Q(\omega)}.$$

---

*Festschrift in honor of Prof. E. Wolf, Edited by T. Jansen, SPIE Publication No (2004)
†email: gsa@prl.ernet.in
This is a measure of correlation between the two sources. For two coherent sources \( \mu = 1 \) whereas for incoherent sources \( \mu = 0 \). The field \( U \) at the point \( P \) can be related to the strength of the sources via

\[
U(P, \omega) = Q(P_1, \omega) \frac{e^{ikR_1}}{R_1} + Q(P_2, \omega) \frac{e^{ikR_2}}{R_2}.
\]

(5)

Here we have ignored unnecessary numerical factors. Using Eq. (5) the spectrum of the field is related to the spectrum of the source and the degree of spatial coherence

\[
S_U(P, \omega) = S_Q(\omega) \left( \frac{1}{R_1^2} + \frac{1}{R_2^2} + \frac{1}{R_1 R_2} \left[ \mu_Q(\omega) e^{ik(R_2 - R_1)} + \text{c.c.} \right] \right).
\]

(6)

Clearly in general, the source spectrum and the spectrum at \( P \) are not equal

\[
S_U(P, \omega) \neq S_Q(\omega).
\]

(7)

Clearly the measured spectral characteristics will also be determined by \( \mu_Q \) and \( S_U(P, \omega) \), in general, would exhibit correlation induced spectral shifts. Wolf used phenomenological model for \( S_Q \) and \( \mu_Q \) to demonstrate a variety of spectral shifts and even the correlation induced splitting of a line into several lines. Clearly it is desirable to understand the origin of source correlations.

II. MICROSCOPIC ORIGIN OF SOURCE CORRELATIONS

We thus examine the question of how the atom radiate. Consider for example an atom in its excited state. It interacts with the modes of quantized electromagnetic field in vacuum state. The atom makes a transition to the ground state by the emission of a photon. The photon can be emitted in any mode of the field. The atom has infinity of available modes. It is known that the spectrum of the emitted radiation has Lorentzian spectrum

\[
S_A(\omega) = \frac{\gamma/\pi}{(\omega - \omega_0)^2 + \gamma^2},
\]

(8)

where \( \omega_0 \) is the frequency of the atomic transition and \( \gamma \) is half the Einstein \( A \) coefficient.

![FIG. 2.](image)

Next consider two atoms located at \( \vec{r}_A \) and \( \vec{r}_B \). Let each atom be initially in its excited state. The question is whether the atoms radiate independently of each other i.e. whether the spectrum of the emitted photons factorizes

\[
S(\omega_1, \omega_2) = S_A(\omega_1) S_B(\omega_2)
\]

(9)

or not. The correlations between the two atoms\(^6,7,8 \) would invalidate (9) and it would in general also imply that

\[
S_A(\omega_1) \neq \int S(\omega_1, \omega_2) d\omega_2,
\]

(10)

![FIG. 3.](image)
i.e., the spectrum of the emitted radiation would be different from the one if the other atom was absent. Note that both atoms interact with a common quantized electromagnetic field. This interaction with a common field results in an effective interaction between two atoms even if the atoms do not interact. This can also be understood by considering, say, the net field on the atom $B(A)$ which would consist of the vacuum field and field radiated by the atom $A(B)$. Let us denote by $\chi_{ij}(\vec{r}_A, \vec{r}_B, \omega)$ as the $i-th$ component of the field at position $\vec{r}_A$ due to a unit dipole oriented in the direction $j$ at the position $\vec{r}_B$.

This field$^9,10,11$ is well known from the solution of Maxwell equations

$$\chi_{ij}(\vec{r}_A, \vec{r}_B, \omega) = \left(\frac{\omega^2}{c^2} \delta_{ij} + \frac{\partial^2}{\partial r_A \partial r_B} \right) \frac{\exp(i|\vec{r}_A - \vec{r}_B|\omega/c)}{|\vec{r}_A - \vec{r}_B|}. \quad (11)$$

This function has close connection with the spatial coherence of the vacuum of the electromagnetic field. Let us write the electric field operator in terms of its positive and negative frequency parts

$$E = E^{(+)} + E^{(-)}. \quad (12)$$

It is well known in quantum optics that $E^{(+)}$($E^{(-)}$) corresponds to the absorption (emission) of photons. Further $E^{(+)}$ is an analytical signal. Let us consider second order coherence function of the electromagnetic field

$$S_{\alpha\beta}^A(\vec{r}_1, \vec{r}_2, \tau) = \langle E^{(+)}_{\alpha}(\vec{r}_1, t + \tau) E^{(-)}_{\beta}(\vec{r}_2) \rangle \quad (13)$$

which is non-vanishing even-though the field is in vacuum state. Its Fourier transform is given by$^9$

$$\int d\tau e^{i\omega \tau} S_{\alpha\beta}^A(\vec{r}_1, \vec{r}_2, \tau) = 2\hbar \text{Im} \chi_{ij}(\vec{r}_1, \vec{r}_2, \omega) \quad \text{if} \quad \omega > 0$$

$$= 0 \quad \text{if} \quad \omega < 0. \quad (14)$$

We thus conclude that the vacuum of the electromagnetic field has spatial coherence which extends over the dimensions of wavelength. Therefore the correlation between atoms would extend over at least distances of the order of wavelength. Clearly in a macroscopic sample these correlation could build up over much larger distances. Explicit results for two atoms can be found in Refs. [6],[7],[8].

### III. SOURCE CORRELATION INDUCED TWO PHOTON RESONANCE
We next discuss several other situations where atom-atom correlations play an important role. Consider first the case of two unidentical atoms with transition frequencies $\omega_A$ and $\omega_B$ and which are located within a wavelength of each other. Let both the atoms start in ground state and let these interact with a laser field of frequency $\omega_l$. We now study the total intensity $I(\omega_l)$ of the emitted radiation as a function of $\omega_l$. Clearly $I(\omega_l)$ will exhibit single photon resonance at $\omega_l = \omega_{A\text{eg}}, \omega_{B\text{eg}}$. In principle there is also the possibility of two photon resonance $2\omega_l = \omega_{A\text{eg}} + \omega_{B\text{eg}}$. It turns out that in the absence of source correlations, the two photon resonance does not occur as the two paths

$$|g_A, g_B\rangle \rightarrow |e_A, g_B\rangle \rightarrow |e_A, e_B\rangle, \quad \text{and} \quad |g_A, g_B\rangle \rightarrow |g_A, e_B\rangle \rightarrow |e_A, e_B\rangle$$

interfere destructively. Thus the source correlations are the key to the two photon resonance. In an earlier work the effect of source correlations on such a two photon resonance was studied in great detail and recently it has been observed in experiments involving single molecules further very recently we show how the source correlation arise in a cavity.

IV. SPATIAL COHERENCE AND EMISSION IN PRESENCE OF A MIRROR

Another class of systems where spatial coherence plays an important role is for example, the emission of radiation in front of a metallic mirror or in a cavity formed by metallic or dielectric mirrors. The spectrum of the emitted radiation depends on the distance of the atom from mirror. As a matter of fact both line width and line shift become $b$-dependent. If the metallic mirror is treated as a perfect conductor, then the calculations show that the line shifts, for example, are determined by the spatial coherence of the field at the location of the atom and its image. Thus the correlation of the vacuum $\langle \vec{E}^{(+)}(\vec{b}, t)\vec{E}^{(-)}(-\vec{b}, t') \rangle$, which is related to $\chi(\vec{b}, -\vec{b}, \omega)$, determines the line shifts and line widths. Explicit results for the $b$ - dependence of shifts and widths can be found in Refs. [10],[14].

V. SPATIAL COHERENCE INDUCED CONTROL OF NONLINEAR GENERATION

We next discuss the effects of spatial coherence in the context of nonlinear optics. We would show that the generation of radiation using nonlinear processes can be controlled by source correlations. Consider for example, the process of second harmonic generation (SHG) with $P = \chi^{(2)} E^2$, $E \sim e^{i\vec{k}.\vec{r}}$. The efficiency of the SHG depends on the phase matching integral

$$f = \frac{1}{V} \int e^{-i\vec{q}\cdot\vec{r}} e^{2i\vec{k}\cdot\vec{r}} d^3r$$

which goes to unity if $\vec{q} = 2\vec{k}$. The function $f$ determines the direction in which second harmonic generation is dominant.
If however the field $E$ is partially coherent, then in place of (16) we need to consider
\[ f = \int \int \int d^3r' d^3r'' e^{-i\vec{q} \cdot \vec{r}'e} e^{2i\vec{k} \cdot \vec{r}''} \langle P(\vec{r}) P^*(\vec{r}^\prime) \rangle. \] (17)

Note that for SHG with coherent radiation
\[ \langle P(\vec{r}) P^*(\vec{r}^\prime) \rangle \equiv \langle P(\vec{r}) \rangle \langle P^*(\vec{r}^\prime) \rangle \equiv e^{2i\vec{k} \cdot (\vec{r} - \vec{r}^\prime)} \] (18)
and then
\[ I \propto (\text{vol})^2. \] (19)

On the other hand for the case of incoherent radiation
\[ \langle P(\vec{r}) P^*(\vec{r}^\prime) \rangle \equiv |\mathcal{P}|^2 \delta(\vec{r} - \vec{r}^\prime), \] (20)
\[ I \rightarrow |\mathcal{P}|^2 (\text{vol}). \] (21)

For the partially coherent radiation
\[ \langle P(\vec{r}) P^*(\vec{r}^\prime) \rangle = |\chi(2)|^2 \langle E^2(\vec{r}) E^*2(\vec{r}^\prime) \rangle, \] (22)
which under the assumption of a Gaussian field will become
\[ \langle P(\vec{r}) P^*(\vec{r}^\prime) \rangle = 2I^2 |\mu(\vec{r} - \vec{r}^\prime)|^2, \] (23)
where $\mu(\vec{r} - \vec{r}^\prime)$ denotes the degree of spatial coherence of the incident field. Thus SHG would now be determined by the integral
\[ |f(\vec{Q})|^2 = \int \int \int d^3r' d^3r'' |\mu(\vec{r} - \vec{r}^\prime)|^2 e^{\vec{Q} \cdot (\vec{r} - \vec{r}^\prime)} \] (24)
\[ \vec{Q} = -\vec{q} + 2\vec{k}. \] (25)

Clearly now the direction of SHG would be determined by the spatial coherence of the field. Thus spatial coherence can serve as a control parameter for the nonlinear generation. Clearly the above ideas should also find interesting applications in other areas of nonlinear optics as well.

VI. UNIVERSALITY OF WOLF SHIFT

Before concluding the paper we also like to make some general remarks for the universality and applicability of Wolf shifts in the context of other systems. We know for instance, that other standard equations of physics (such as those describing vibrations of string, heat transport) admit following relation between the effect $\Phi$ of the source $P$ at the observation point
\[ \Phi(\vec{r}) = \int G(\vec{r}, \vec{r}') P(\vec{r}') d^3r', \] (26)
where $G$ is Green’s function for the underlying equation. The observed quantities are usually quadratic in $\Phi$. Thus observation at the point $\vec{r}$ would depend on the correlations of the source at two points. This is due to the nonlocal nature of the solution (26).
VII. FLUCTUATING PULSES IN A DISPERSIVE MEDIUM

As another example of this universality we can consider the propagation of pulses in a dispersive medium which is described by the equation

$$\frac{i}{\tau} \frac{\partial E}{\partial z} = \frac{k}{2} \frac{\partial^2 E}{\partial t^2}$$  \hspace{1cm} (27)

The solution of this equation can be given in terms of Green’s function

$$E(z, t) = \int G(z, t; 0, t') E(0, t') dt' ;$$  \hspace{1cm} (28)

$$G = \frac{i}{2 \pi \omega k} \exp \left(-\frac{i}{2 \omega k} (t^2 - 2 t t' + t'^2) \right) .$$  \hspace{1cm} (29)

If the input pulse has fluctuations, then the intensity of the output pulse would be determined by the correlation in pulses on input plane

$$I(L, t) = \int \int dt' dt'' G^*(L, t; 0, t') G(L, t; 0, t'') \langle E(t') E^*(t'') \rangle .$$  \hspace{1cm} (30)

Clearly the intensity of the pulse at the output plane is not completely determined by the intensity of the pulse at input plane.

VIII. CONCLUSIONS

Thus in conclusion we have shown that the vacuum of electromagnetic field has intrinsic partial spatial coherence in frequency domain which effectively extends over regions of the order of wavelength $\lambda$. This spatial coherence leads to a dynamical coupling between atoms and is the cause of source correlations. We showed how such correlations can lead to a new type of two photon resonance and how these are relevant for near field optics. We further showed how the source spatial correlations can lead to new phase matching conditions for nonlinear optical effects leading to the possibility of using spatial coherence to produce tailor made emissions. We also discussed the universality of source correlation effects and as a specific example we treated the case of the propagation of fluctuating pulses in a dispersive medium.

The author thanks E. Wolf for many discussions on the subject of correlation induced shifts.

[1] E. Wolf, ”Invariance of the Spectrum of Light on Propagation,” Phys. Rev. Lett. 56, pp. 1370-1372 (1986); E. Wolf, ”Red shifts and blue shifts of spectral lines emitted by two correlated sources,” ibid. 58, pp. 2646-2648 (1987); E. Wolf, ”Correlation-induced Doppler-type frequency shifts of spectral lines,” ibid. 63, pp. 2220-2223 (1989).
[2] E. Wolf, ”Non-cosmological redshifts of spectral lines,” Nature (London) 326, pp. 363-366 (1987).
[3] E. Wolf and D. F. V. James, ”Correlation-induced spectral changes,” Rep. Prog. Phys. 59, pp. 771-818 (1996).
[4] L. Mandel and E. Wolf, Optical Coherence and Quantum Optics (Cambridge University Press, 1995).
[5] G. S. Agarwal, in Quantum Optics (Springer Tracts in Modern Physics, Vol. 70, 1974).
[6] G. Varada and G. S. Agarwal, ”Microscopic approach to correlation-induced frequency shifts,” Phys. Rev. A 44, pp. 7626-7634 (1991).
[7] G. Varada and G. S. Agarwal, ”Two-photon resonance induced by the dipole-dipole interaction,” Phys. Rev. A 45, pp. 6721-6729 (1992).
[8] D. F. V. James, ”Frequency shifts in spontaneous emission from two interacting emission,” Phys. Rev. A 47, pp. 1336-1346 (1993).
[9] G. S. Agarwal, ”Quantum electrodynamics in the presence of dielectrics and conductors : I. Electromagnetic-field response functions and black-body fluctuations in finite geometries,” Phys. Rev. A 11, pp. 230-242 (1975).
[10] G. S. Agarwal, ”Quantum electrodynamics in the presence of dielectrics and conductors : IV. General Theory of spontaneous emission in finite geometries,” Phys. Rev. A 12, pp. 1475-1497 (1975).
[11] G. S. Agarwal, in Quantum Electrodynamics and Quantum Optics, ed. A. Barut (Plenum, 1983).
[12] C. Hettich, C. Schmitt, J. Zitzmann, S. Kuhn, I. Gerhardt, and V. Sandoghdar, "Nanometer resolution and and coherent optical dipole coupling of two individual molecules," Science 298, pp. 385-389 (2002).
[13] P. K. Pathak and G. S. Agarwal, "Giant two-atom two-photon vacuum Rabi oscillations in a high quality cavity," to be published.
[14] G. S. Agarwal and H. D. Vollmer, "Surface polariton effects in spontaneous emission," Physica Status Solidi B 79, 249 (1977); G. S. Agarwal and Vollmer, "Surface polariton effects in spontaneous emission. II. Effects of spatial dispersion," ibid. 85, 301 (1978).