Description of superdeformed nuclei in the interacting boson model

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Abstract

The interacting boson model is extended to describe the spectroscopy of superdeformed bands. Microscopic structure of the model in the second minimum is discussed and superdeformed bosons are introduced as the new building blocks. Solutions of a quadrupole Hamiltonian are implemented through the $1/N$ expansion method. Effects of the quadrupole parameters on dynamic moment of inertia and electric quadrupole transition rates are discussed and the results are used in a description of superdeformed bands in the Hg-Pb and Gd-Dy regions.

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I. INTRODUCTION

Spectroscopy in the superdeformed minimum has reached a certain level of maturity to justify a phenomenological analysis of the available data (see [1,2] for recent reviews). Such an approach would be useful in systematizing the data and would also provide a complimentary perspective to the more microscopic theories. For this purpose, we use the interacting boson model (IBM) [3] which has been established as one of the simplest and most versatile collective models. It has been especially successful in correlating spectroscopic data in deformed nuclei in terms of a few parameters of a quadrupole Hamiltonian [4].

Microscopic study of the nucleon pair structure in the superdeformed well [5] indicates that, compared to the deformed nuclei, they have about three times more active pairs of nucleons (bosons), and the $L = 4$ pair ($g$ boson) plays a much more significant role. As numerical diagonalization of an $sdg$-IBM Hamiltonian is not possible for more than 10 bosons, one needs alternative methods of solution to apply the IBM to superdeformed nuclei. Here, we use the angular momentum projected mean field theory which leads to a $1/N$ expansion for all matrix elements of interest [6]. Accurate representation of high-spin states in the $1/N$ expansion formalism requires terms to order $(L/N)^6$ which have been obtained recently using computer algebra [7]. The extended formalism provides an analytical method for analysis of superdeformed states which is both accurate and efficient.

The plan of the paper is as follows. After reviewing the microscopic basis of the IBM for superdeformed nuclei, we introduce the $1/N$ expansion formalism and discuss its recent extension to higher orders. We then use the $1/N$ expansion formulas for a quadrupole Hamiltonian to study systematic features of dynamic moment of inertia and $B(E2)$ values. The results are used in a description the superdeformed bands in the Hg-Pb and Gd-Dy regions.
II. MICROSCOPIC BASIS

In this section, we study the typical structure of strongly deformed states and investigate the relation between the superdeformation and the IBM based on [4]. For this purpose, we use the Nilsson + BCS model with particle number projection. Superdeformed states can be characterized as ground states in a superdeformed potential well which is separated from the normal one by a potential barrier. For such ground-like states which show strongly collective nature, this model seems to work well. Using the experimental deformation parameters and electric transition probabilities (or moments) as input, one can obtain reasonable wave functions. These wave functions are analyzed from the viewpoint of collective nucleon pairs, which leads to a natural extension of the usual IBM.

We briefly summarize the formulation of the Nilsson + particle-number-conserving BCS model. The single particle orbits in a deformed potential are described well by the Nilsson Hamiltonian [8]

\[ H_{\text{Nilsson}} = -\frac{\hbar^2}{2m} \nabla^2 + \frac{m}{2} \omega_0^2 r^2 [1 - \frac{4}{3} \delta P_2(\cos \theta)] - 2\hbar \omega_0 \kappa \mathbf{l} \cdot \mathbf{s} - \hbar \omega_0 \kappa \mu (\mathbf{l}^2 - \langle \mathbf{l}^2 \rangle_N), \]  

(1)

where \( \delta \) is the deformation parameter and \( P_2(\cos \theta) \) denotes the Legendre polynomial. The term \( \langle \mathbf{l}^2 \rangle_N = \frac{1}{2} N(N+3) \) is the expectation value of \( \mathbf{l}^2 \) averaged over one major shell with the principal quantum number \( N = 2n + l \). The value of the oscillator frequency for a mass-\( A \) nucleus is determined from \( \hbar \omega_0 = 41A^{-\frac{1}{3}} \), which is about 7.1 MeV for the superdeformed nuclei in the Hg-Pb region (\( A \sim 190 \)) and 7.7MeV for those in the Dy-Gd region (\( A \sim 150 \)). We use the usual values for the parameters \( \kappa \) and \( \mu \) which are 0.0637, 0.60 for proton orbits and 0.0637, 0.42 for neutron orbits, respectively [9]. In order to include short range correlations, the monopole pairing interaction is added to the Nilsson hamiltonian [10]

\[ H = H_{\text{Nilsson}} + G \mathbf{P} \dagger \mathbf{P}, \]  

(2)

where \( G \) denotes the pairing strength parameter, and
\[ P^\dagger = \sum_{k>0} c_k^\dagger c_k^\dagger, \]  

is a pair creation operator. Here \( c_k^\dagger \) stands for the creation operator of a nucleon in the spherical single particle orbit \( k \), and \( \bar{k} \) denotes the time reversed state of \( k \). This Hamiltonian is solved by the variation using a BCS wave function

\[ | \Psi \rangle = \prod_{\alpha>0} \left( u_\alpha + v_\alpha a^\dagger_\alpha a^\dagger_\bar{\alpha} \right) | 0 \rangle, \]  

where \( a^\dagger_\alpha \) denotes the creation operator for a nucleon in the deformed canonical (Nilsson) orbit labeled by \( \alpha \). The particle number conservation has been found to be important in the case of weak pairing correlations and also for moments of inertia of high spin states in the cranking calculation of the superdeformed states [11,12]. Thus we carry out the particle number projection before variation according to the method given in Ref. [13]. The solution corresponds to the minimum of the number projected energy

\[ E^P[\Psi] = \frac{\langle \Psi | H PN | \Psi \rangle}{\langle \Psi | P^N | \Psi \rangle}, \]  

where \( P^N \) denotes the particle number projection operator.

The deformation parameters of the superdeformed states in the Hg-Pb (Dy-Gd) region are given by \( \delta \sim 0.40 \) \((0.50)\), which is equivalent to the axis ratio of \(5:3\) \((2:1)\). Because of this strong deformation, it is insufficient to take only one active major shell and take into account the corrections due to the core-polarization effect through renormalization in one major shell. Thus we first seek a suitable model space for description of superdeformed states. For simplicity, we turn off the pairing force which is not important for this purpose. We take \(^{194}\)Hg \((N=114, \ Z=80)\) and \(^{152}\)Dy \((N=86, \ Z=66)\) as examples of the Hg-Pb and the Dy-Gd regions, respectively.

In order to define the model space necessary for description of superdeformed states, we utilize the intrinsic quadrupole moment \( Q_0 \). The intrinsic quadrupole moment is calculated in the space of all spherical orbits up to the principal quantum number \( N = N_{\text{max}} \). Then \( N_{\text{max}} \) is increased until the value of \( Q_0 \) is saturated to a good extent. From this procedure we obtain \( N_{\text{max}} = 12 \). The corresponding values of \( Q_0 \) for proton and neutron orbits are
19\(b\) and 29\(b\) for \(^{194}\text{Hg}\), and 19\(b\) and 27\(b\) for \(^{152}\text{Dy}\), respectively. Note that the experimental values of \(Q_0\) are 18 ± 3 \(eb\) for \(^{152}\text{Dy}\) \(^{1}\)[14] and 17 ± 2 \(eb\) for \(^{194}\text{Hg}\) \(^{1}\)[15], which are consistent with the present results if we take the bare charges, \(e_p = 1\) and \(e_n = 0\).

We now consider the inert core of superdeformed states. The Nilsson wave function is obtained by putting all nucleons in the Nilsson orbits from the bottom. One Nilsson orbit can be expanded as a linear combination of many spherical harmonic oscillator orbits, and the square of expansion coefficients gives the occupation probability of each spherical orbit. We expand all the occupied Nilsson orbits and sum up all the occupation probabilities which belong to the same spherical orbits, to obtain the total occupation probability for a given spherical harmonic oscillator basis. Due to the strong quadrupole field, one Nilsson orbit spreads over many spherical orbits. Thus the orbits with very high single particle energy can gain some finite occupation probabilities, while the occupation of the orbits with small single particle energy may become incomplete. Nevertheless several lower spherical orbits are occupied almost completely and can be considered as a new inert spherical core for the superdeformed states. Note that we do not take the usual “hole” picture as it is meaningful only for states whose configuration are well described within one major shell.

First consider the case of \(^{194}\text{Hg}\). In Fig. \(^{1}\)[11] the occupation probability of each spherical harmonic oscillator orbit is shown for neutrons (a) and protons (b). The orbits are ordered according to their single particle energy at \(\delta = 0\) as \(1s_{1/2}, 1p_{3/2}, \cdots\). The case of \(\delta = -0.13\) which simulates the deformation of normal oblate states is also shown for comparison. For normal deformation, it is seen from Fig. \(^{1}\)[11]-a that the occupation of the proton orbits is almost complete at \(2d_{5/2} (Z = 64)\), while the occupation probability is almost vanished for orbits above \(Z = 82\). These results suggest that we can consider the \(Z = 64\) subshell as an inert core and three valence orbits (\(2d_{3/2}, 3s_{1/2}\) and \(1h_{11/2}\)) as active. This gives the valence proton number as \(Z_v = 16\). In the case of \(\delta = 0.40\), the proton orbits are almost completely occupied up to \(Z = 50\). Above \(Z = 50\), the occupation probability drops suddenly though it remains about 10% over many orbits. Clearly, one should incorporate the contributions of these high energy orbits. Thus it is reasonable to take the \(Z = 50\) spherical inert core
and include quite many orbits above there as active valence orbits. In this case the valence proton number becomes $Z_v = 30$.

In the same way, it can be seen from Fig. 1-b that the spherical inert core for neutron orbits are $N = 100$ ($N_v = 14$) and 82 ($N_v = 32$) for $\delta = -0.13$ and 0.40, respectively. The active valence orbits for $\delta = -0.13$ are $2f_{5/2}$, $3p_{3/2}$, $3p_{1/2}$ and $1i_{13/2}$, while for $\delta = 0.40$ it is still insufficient to include only two or three major shells.

We can see a similar behaviour of occupation probabilities in the wavefunctions of $^{152}$Dy, which is shown in Figs. 1-c for proton and 1-d for neutron orbits. In these figures two cases of $\delta = 0.50$ (superdeformed state) and 0.25 (normal prolate state) are compared. It is clear that $Z = 50$ and $N = 82$ inert cores are good for normal states, while $Z = 40$ and $N = 50$ cores are suitable for superdeformed states. Thus the inert core of superdeformed states becomes much smaller than that of normal states in both the Hg-Pb and Dy-Gd regions.

From the viewpoint of the IBM, the number of bosons is determined by half of the number of valence nucleons. Because of the small inert core the number of bosons increases significantly for superdeformed states in comparison with that in the usual IBM. In fact in the case of $^{194}$Hg, the boson number in the usual IBM is $N_{\text{normal}} = (82 - 80)/2 + (128 - 114)/2 = 8$ by taking the usual hole picture, and $N_{\text{normal}} = (80 - 64)/2 + (114 - 100)/2 = 15$ with the particle picture mentioned above. On the other hand, the number of bosons for superdeformation becomes $N_{\text{super}} = (80 - 50)/2 + (114 - 82)/2 = 31$. Similarly, in the case of $^{152}$Dy, $N_{\text{normal}} = (66 - 50)/2 + (86 - 82)/2 = 10$ while $N_{\text{super}} = (66 - 40)/2 + (86 - 50)/2 = 31$. In general, $N_{\text{super}}$ is about three times larger than $N_{\text{normal}}$. Note that the number of proton bosons and neutron bosons are close in these two cases, and this approximate equality seems to be a general tendency of the superdeformed states. This result can be naturally understood since equal numbers of valence protons and neutrons maximizes the attractive proton-neutron interaction.

Next we consider the effects of pairing correlations on the structure of wave functions of superdeformed states. The strength parameter $G$ of the pairing interaction should be chosen depending on the model space. Since the value of $G$ for such a large space is not known
empirically, we first describe normal states within the extended valence space, and determine
the value of $G$ by requiring that the pairing gap $\Delta$ takes a reasonable value. For $^{194}\text{Hg}$ the
value of $G$ has turned out to be 0.06 MeV which gives $\Delta \sim 1$ MeV. Using this value, we
investigate the effect of the pairing correlations on the structure of valence wave functions.
For this value of $G$, the gap for superdeformed states becomes about $\Delta = 0.5$ MeV for both
proton and neutron orbits. In contrast to normal deformed states, which are sensitive to
changes in values of $G$, the superdeformed states are almost insensitive to $G$ values (the
intrinsic quadrupole moment and the occupation probabilities change very little). Thus the
following discussion about the structure of valence wave function of superdeformation is
almost independent of pairing correlations.

We can investigate the relation between the superdeformation and the IBM by analyzing
valence wave functions from the viewpoint of collective nucleon pairs. Since the bosons in
the IBM are understood as images of these pairs, such an analysis is essential in establishing
a microscopic basis for the super IBM. We consider $^{194}\text{Hg}$ as an example.

The Nilsson + particle-number-conserving BCS wave function can be expressed as the
condensed state of coherent Cooper-pairs in the deformed potential

$$
P^N | \Psi \rangle \propto (A^\dagger_\pi)^{N_\pi} (A^\dagger_\nu)^{N_\nu} | 0 \rangle, \tag{6}
$$

acting on the inert core $| 0 \rangle$. In this expression, $A^\dagger_\pi$ ($A^\dagger_\nu$) denotes the creation operator of
a Cooper-pair in proton (neutron) orbits and $N_\pi$ ($N_\nu$) means half of the valence proton
(neutron) number. These $A$-pairs can be decomposed into a linear combination of collective
nucleon pairs with good angular momenta

$$
A^\dagger = x_0 S^\dagger + x_2 D^\dagger_0 + x_4 G^\dagger_0 + \cdots, \tag{7}
$$

where $S^\dagger$, $D^\dagger$, $G^\dagger$, $\cdots$ denote the collective nucleon pairs with spin-parity $J^{\pi} = 0^+, 2^+, 4^+$,
$\cdots$ and the $x_j$’s are amplitudes. The probability of each pair in the $A$-pair is given by the
square of each amplitude, and is listed in Table I for two cases of $\delta = -0.13$ and $\delta = 0.40$.
It is well known that in the case of normal deformation the dominant components are the
In fact, these two components account for 100% probability in the case of \( \delta = -0.13 \). In the case of \( \delta = 0.40 \), the total probability of the \( S- \) and \( D- \) pairs is about 80% and we can conclude that these pairs are still dominant in the \( \Lambda \)-pair. However, the probability of the \( G \)-pair is now sizable, and it can no longer be neglected in a detailed description of high-spin states. It should be noted that the ratio of the \( S \)-pair to the other pairs is quite similar to that of \( s \)-boson to the other bosons in the SU(3) limit of the IBM, which are shown in the same table. This suggests that the SU(3) limit of the \( sdg \)-IBM could provide a reasonable phenomenological framework for superdeformed states.

To summarize the microscopic results, we emphasize two important points for the description of superdeformed bands in the IBM: One is the significant increase in the boson number, and the other is the importance of \( g \)-bosons. In addition, it has been found that the bosons for superdeformed states carry the collectivity over many major-shells and that the SU(3) limit is a reasonable starting point.

### III. 1/N EXPANSION FOR SUPER IBM

A simultaneous description of the spectroscopy of normal and superdeformed states requires rather complicated wave functions, therefore we focus on the latter here and leave the complete picture for future work. We introduce the superbosons \( s, d, g \) as the boson images of the \( S, D, G \) collective nucleon pairs in the superdeformed well (bold face notation is used for super bosons to distinguish them from the normal ones). The quadrupole Hamiltonian for this system of bosons has the form

\[
H = -\kappa \mathbf{Q} \cdot \mathbf{Q},
\]

where the quadrupole operator is defined as

\[
\mathbf{Q} = [s^\dagger \tilde{d} + d^\dagger \tilde{s}]^{(2)} + q_{22}[d^\dagger \tilde{d}]^{(2)} + q_{24}[d^\dagger \tilde{g} + g^\dagger \tilde{d}]^{(2)} + q_{44}[g^\dagger \tilde{g}]^{(2)}.
\]

Here brackets denote tensor coupling of the boson operators and \( \tilde{b}_{lm} = (-1)^m b_{l-m} \). The parameters \( q_{jl} \) in Eq. (8) determine strengths of boson interactions relative to the \( s - d \)
coupling. Since the SU(3) limit is used as a reference point in the rest of the paper, we quote the values for the quadrupole parameters in this limit; $q_{22} = -1.242$, $q_{24} = 1.286$, $q_{44} = -1.589$. As stressed in the introduction, numerical diagonalization of this Hamiltonian for $N \sim 30$ bosons is not possible even on a supercomputer. The large number of bosons are, however, an advantage for the analytic $1/N$ expansion technique which we employ here for solving the Hamiltonian Eq. (8). The $1/N$ expansion method has previously been discussed in detail \cite{6} and the recent extensions to higher orders are given in Ref. \cite{7}. Therefore, we give only a short account of the formalism here, focusing mainly on the accuracy of the results for high-spin states. The starting point of the $1/N$ calculations is the boson condensate

$$|N, \mathbf{x}\rangle = (N!)^{-1/2}(b^\dagger)^N|0\rangle, \quad b^\dagger = x_0s^\dagger + x_2d^\dagger_0 + x_4g^\dagger_0,$$  \hspace{1cm} (10)

where $x_l$ are the mean field amplitudes to be determined by variation after projection (VAP) from the energy expression

$$E_L = \langle N, \mathbf{x}|HP_{00}^L|N, \mathbf{x}\rangle/\langle N, \mathbf{x}|P_{00}^L|N, \mathbf{x}\rangle.$$  \hspace{1cm} (11)

Here $P_{00}^L$ denotes the projection operator. The resulting energy expression is a double expansion in $1/N$ and $\bar{L} = L(L + 1)$, and has the generic form

$$E_L = N^2 \sum_{n,m} \frac{e_{nm}}{(aN)^m}(\frac{\bar{L}}{a^2N^2})^n,$$  \hspace{1cm} (12)

where $a = \sum_l \bar{x}_l^2$ and the expansion coefficients $e_{nm}$ involve various quadratic forms of the mean fields $x_l$, e.g., $e_{00} = (\sum_{jl}(j0l0|20)q_{jl}x_jx_l)^2$. The coefficients $e_{nm}$ have recently been derived up to the third order, $(\bar{L}/N^2)^3$, using computer algebra \cite{7}.

Another observable of interest in the study of superdeformed states is the $E2$ transitions. Assuming that the quadrupole transition operator is the same as in the Hamiltonian, i.e. $T(E2) = eQ$ where $e$ is an effective boson charge, the $E2$ matrix elements are given by

$$\langle L \parallel T(E2) \parallel L - 2 \rangle = eN\hat{L}\langle L0\parallel 20\parallel L - 2 \parallel 0\parallel |m_1 + m_2L(L-1)/N^2\rangle$$  \hspace{1cm} (13)
where $\hat{L} = [2L + 1]^{1/2}$ and the coefficients $m_n$ are given in Ref. [7]. The first term in Eq. (13) gives the familiar rigid-rotor result. The second term is negative and is responsible for the boson cutoff effect in $E2$ transitions.

Before applying the $1/N$ expansion results, we compare them with those obtained from an exact diagonalization of the Hamiltonian. Diagonalization is carried out for $N = 10$ which is the maximum possible boson number for this purpose. The quadrupole parameters $q_{22}, q_{24}, q_{44}$ are scaled down from their SU(3) values with $q = 0.7$ which gives an adequate parametrization for the Hg-Pb region. Fig. 2-a compares exact results for the dynamic moment of inertia $J^{(2)}$ (circles) with three different $1/N$ calculations. The solid line shows the third order VAP results which is seen to follow the exact results very accurately. The second order VAP (dotted line) and the third order VBP results (dashed line) break down around spin $L \sim 2N$. Hence for description of high-spin states, the third order $1/N$ expansion formulas with VAP seem to be both necessary and sufficient. In Fig. 2-b, the exact $E2$ transition matrix elements (circles) are compared with those obtained from Eq. (13) (line). The agreement is again very good up to very high-spins. Note that the accuracy of the $1/N$ expansion results improves with increasing $N$, hence in actual applications with $N \sim 30$, one would expect an even better agreement. The test case discussed here indicates that the extended formalism can be applied with confidence in the spin region $L = N-3N$ which covers the presently available data range for superdeformed bands.

**IV. APPLICATIONS TO SUPERDEFORMED BANDS**

In this section, we apply the $1/N$ expansion formulas first in a systematic study of dynamic moment of inertia and $B(E2)$ values, and then to describe the experimental data on superdeformed bands. Since $\kappa$ is a scale parameter for energies, we need to study the effect of the three quadrupole parameters $q_{22}, q_{24}, q_{44}$. Fig. 3 shows the effect of variations in each $q_{jl}$ on dynamic moment of inertia while the other two are held constant at the SU(3) values. Here $q$ denotes the scaling parameter from the SU(3) values. Thus $q = 1$, corresponds
to the SU(3) limit which exhibits the rigid-rotor behaviour. To describe the variations in $J^{(2)}$, one needs to break the SU(3) limit. From Fig. 3 it is seen that $J^{(2)}$ is most sensitive to $q_{24}$ (note the different scales in the three figures). The other (diagonal) parameters have smaller and opposite effect on $J^{(2)}$. Since the amount of data does not justify use of too many parameters, we prefer to scale all three with the same parameter $q$. The result of this simultaneous scaling is shown in Fig. 4-a which is essentially the same as the one for $q_{24}$ in Fig. 3. An interesting feature of these results is that the quadrupole Hamiltonian has the scope to describe both the increases and decreases in $J^{(2)}$. For $q < 1$, the $s-d$ coupling is relatively stronger than the $d-g$ coupling which results in loss of monopole pairing with increasing spin, and hence increase in $J^{(2)}$. The opposite happens for $q > 1$. Fig. 4-b shows the effect the simultaneous variations in the quadrupole parameters on the $B(E2)$ values. The curving down of lines is due to boson cutoff which is most effective for smaller values of $q$.

In the light of the above systematic studies, we have carried out fits to the available data on superdeformed bands in the Hg-Pb and Gd-Dy regions. The boson number is determined from microscopics, and $\kappa$ and $q$ are fitted to the data. The parameter values are given in the figure captions and the data are taken from the compilation in Ref. [19]. Figs. 5 and 6 compare the experimental dynamic moment of inertia (circles) with the calculated ones (lines) in Hg and Pb isotopes, respectively. In all cases $J^{(2)}$ exhibits a smooth increase which is well reproduced by the calculations. The situation in the Gd-Dy region is not as favorable for our simple collective model as the other region, because there are definite signs indicating the importance of the single particle degree of freedom. For example, in $^{144-146}$Gd there are sudden jumps in $J^{(2)}$ which are probably due to particle alignment effects [20]. In $^{148-150}$Gd, $J^{(2)}$ behaves reasonably smoothly so we have attempted to describe them (see Fig. 7). The average behaviour of $J^{(2)}$ in $^{148}$Gd is reproduced but the model fails in the case of $^{150}$Gd, underscoring the importance of single particle degree of freedom. For a better description of the data, one needs to incorporate particle alignment effects in the present formalism by including two-quasiparticle states in the basis [21]. The dynamic moment of inertia of
superdeformed bands in Dy isotopes exhibit an entirely different behaviour (Fig. 8). They are very close to the rigid-rotor values, and hence the SU(3) limit as reflected in the values of $q \sim 1$.

The $B(E2)$ values provide a complimentary observable to $J^{(2)}$ which could be used as a further test of the model. In Fig. 8, the available $B(E2)$ data in $^{192-194}$Hg (circles) are compared with the calculations. A reasonable description is obtained using boson effective charges $e = 0.12 - 0.14\ eb$ which are typical values used in the normal IBM calculations. Since the $B(E2)$ values are sensitive to the boson number (they vary as $N^2$), this provides a consistency check on the microscopically derived boson numbers. A further $N$ dependence is provided by the boson cutoff term in Eq. (13) which causes a drop in the calculated $B(E2)$ values at high-spins. Least-square fits to the data indeed indicate a drop in the $B(E2)$ values towards the high-spin end. However, the error bars are too large to reach an unambiguous conclusion whether this effect is genuine or not.

V. CONCLUSIONS

In this paper, we have reviewed a microscopic basis and a practical formulation of the IBM for application to superdeformed nuclei. The availability of analytical formulas owing to the $1/N$ expansion technique means fast and efficient analysis of data. As first examples, we have considered the superdeformed bands in the Hg-Pb and Gd-Dy regions. A good description of data is obtained in the Hg-Pb region confirming the simple quadrupole nature of these superdeformed bands. In the Gd-Dy region, the dynamic moment of inertia exhibits large variations which cannot be accommodated in a simple collective model. Such variations are due to the single particle degree of freedom and require extension of the model for a better description of the data. Finally, the formalism can be used in investigating some other interesting features of superdeformed nuclei such as identical bands and $C_4$ symmetry which will be pursued in future work.
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FIGURES

FIG. 1. Occupation probability of each spherical basis in the Nilsson potential: (a) protons and (b) neutrons of $^{194}$Hg and (c) protons and (d) neutrons of $^{152}$Dy. Two values of the deformation parameter are considered, $\delta = -0.13$ (dashed line) and 0.40 (solid line) for $^{194}$Hg, and $\delta = 0.25$ (dashed line) and 0.50 (solid line) for $^{152}$Dy. The spherical magic numbers are indicated.

FIG. 2. Comparison of the $1/N$ expansion results (lines) with the exact numerical ones (circles) for $N = 10$ bosons. Fig. 2-a shows the dynamic moments of inertia obtained from the third order calculation with VAP (solid line), third order with VBP (dashed line), and second order with VAP (dotted line). The parameters of $H$ are $\kappa = 20$ keV and $q = 0.7$. Fig. 2-b compares the $E2$ matrix elements.

FIG. 3. Systematic behaviour of dynamic moment of inertia $J^{(2)}$ for various values of the quadrupole parameters $q_{22}, q_{24}$ and $q_{44}$. In each figure, two of these three parameters are fixed at the SU(3) value while the other takes 0.6, 0.8, 1.0, 1.2, 1.4 times the SU(3) value. $N = 30$ and $\kappa$ is chosen such that the moment of inertia are all normalized to $100h^2\text{MeV}^{-1}$ at $\omega = 0$.

FIG. 4. Effect of the simultaneous scaling of the quadrupole parameters (a) on dynamic moment of inertia, and (b) on $B(E2)$ values. The dynamic moment of inertia curves are normalized to $100h^2\text{MeV}^{-1}$ at $\omega = 0$ and the $B(E2)$ values are normalized to $B(E2; 2 \rightarrow 0)$.

FIG. 5. Comparison of the experimental dynamic moment of inertia in $^{190−194}$Hg (circles) with the super IBM calculations (solid lines). $N = 29, 30, 31, \kappa = 35, 34, 33$ keV, $q = 0.68, 0.72, 0.72$ are taken for $^{190−194}$Hg, respectively. The data are from [19].

FIG. 6. Same as Fig. 5 but for $^{192−196}$Pb. $N = 30, 31, 32, \kappa = 33, 33, 34$ keV, $q = 0.66, 0.65, 0.67$ are taken for $^{192−196}$Pb, respectively.

FIG. 7. Same as Fig. 5 but for $^{148−150}$Gd. $N = 29, 30, \kappa = 41, 27$ keV, $q = 1.35, 1.73$ are taken for $^{148−150}$Gd, respectively.
FIG. 8. Same as Fig. 5 but for $^{152-154}$Dy. $N = 31, 32$, $\kappa = 42, 43$ keV, $q = 1.07, 1.02$ are taken for $^{152-154}$Dy, respectively.

FIG. 9. Comparison of the experimental $B(E2)$ values in $^{192-194}$Hg (circles) with the super IBM calculations (solid lines). The boson effective charges are $e = 0.140$ and $0.124$ $eb$ for $^{192-194}$Hg, respectively. The data are from [15,22].
TABLE I. Probability (%) of each angular momentum component in the $\Lambda$-pair. The $\Lambda$-pair is obtained from the Nilsson + particle number conserving BCS wave function. Two cases of $\delta = -0.13$ and 0.40 are shown for $^{194}$Hg. The probability of each boson in the intrinsic boson of the IBM in the SU(3) limit is also listed for comparison.

| pair/boson | $\delta = -0.13$ | $\delta = 0.40$ | IBM-SU(3) |
|------------|------------------|------------------|-----------|
|            | neutron | proton | neutron | proton | sd | sdg |
| $S/s$      | 82      | 93     | 25      | 32     | 33 | 20  |
| $D/d$      | 18      | 7      | 52      | 51     | 67 | 57  |
| $G/g$      | 0       | 0      | 18      | 14     | -  | 23  |
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