Can galileons solve the muon problem?
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(Dated: August 11, 2015)

PACS numbers: 04.50.Kd, 04.80.Cc, 12.60.Rc, 36.10.Ee

I. INTRODUCTION

Several measurements in muons physics\cite{1–5} have varying levels of disagreement with theoretical calculations. This “muon problem” could be a sign of the violation of lepton universality from beyond standard model (BSM) physics. That muons should be more susceptible to new physics is intuitively reasonable from an effective field theory point of view, given its dramatically larger mass compared to the electron. However, this is similar to the enhancement of weak interactions in muonic systems. We devote the work of this paper to investigating the via-

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II. ON RADII

In Ref. [27], the cutoff radius, \( r_i \), was assumed to be approximated by the charge radius of the proton \( \sqrt{\langle r^2 \rangle} \equiv r_p \) which is derived from hydrogen spectroscopy to be \( r_p = 0.8758(77) \) fm. For this value of \( r_i \), \( M = 320 \) MeV was found to explain the \( r_p \) discrepancy. This low value for the coupling scale can be reconciled with other constraints by embedding the disformal scalars in a galileon theory with a screening mechanisms caused by higher-order operators [28]. With this in mind, we will only consider models of disformal scalars that are embedded in galileon models. In this paper, we study how limits on lepton compositeness affect the galileon explanation for the muon problem.

There are two ways to interpret the assumption \( r_i = r_p \). Ref. [27] seems to use the charge radius as a proxy for the size of the nuclei that can be used as a cutoff. This interpretation is troubling because it prevents sensible predictions of the galileon in other bound states. Since nuclei can contain neutrons with a negative \( \langle r^2 \rangle = -0.1149(35) \) fm\(^2\), the charge radius of a nuclei can be reduced while the size of the nuclei could increase. Another troubling part of using the charge radius is that this choice is arbitrary. A charged particle has a number of radii, each one reflecting a different distribution (e.g. electric charge, weak interaction, neutron density, strange quark density, matter density) and as we will see, assuming any two are (nearly) equal has implications for other particles. A further complication of this view is that it provides no explanation how leptonic bound systems can regulate the divergence of Eq. [3].

Another way to understand this assumption is that it expresses a relationship between the underlying distributions. In this paper we investigate whether this relationship can be sustained quantitatively with leptonic bound states. To begin, formally the charge radius of a particle is defined via the electric form factor,

\[
G_E(q^2) = \int d^3 x e^{i q \cdot x} \rho(x) = \int d^3 x \left( 1 + i q \cdot x + \frac{(q \cdot x)^2}{2} + \cdots \right) \rho(x) = Q_{\text{tot}} - \frac{1}{6} |q|^2 \langle r^2 \rangle + \cdots, \tag{4}
\]

where \( G_E \) is the electric form factor, \( \rho(x) \) is the charge density, and \( Q_{\text{tot}} \) is the total charge of the particle. The standard definition of \( \langle r^2 \rangle \) is then

\[
\langle r^2 \rangle = -6 \frac{d G_E}{dq^2} \bigg|_{q^2=0}. \tag{5}
\]

In analog to this, we argue that an appropriate definition for \( r_i \) should be via a disformal form factor, and therefore would be

\[
r_i^2 = -6 \frac{d G_{\text{dis}}}{dq^2} \bigg|_{q^2=0}. \tag{6}
\]

By this definition, we see that \( r_i \) is related to a disformal density \( \rho_{\text{dis}}(x) \) that represents the spatial distribution of matter coupling to the galileons. Therefore the assumption \( r_i = r_p \) isn’t an arbitrary statement, but is tantamount to saying

\[
\int d^3 x |x|^2 \rho(x) = \int d^3 x |x|^2 \rho_{\text{dis}}(x). \tag{7}
\]

Since setting the right hand side of Eq. [7] for leptons to zero is unacceptable due to the divergence in Eq. [3] this implies that leptons must have a charge distribution different from a point particle. If instead \( r_i \) is a property of particles unconnected to their charge radius, then the so far unobserved lepton charge radius would give no constraint. This would be in analogy to how the Zemach radius is a property of charged particles arising from the magnetic field distribution, and therefore has no necessary relation to the charge radius.

III. COMPOSITENESS OF LEPTONS

As stated above, the results of Ref. [27] relied upon \( r_p > 0 \) in order to cut off the divergences. Using the measured value from hydrogen or electron scattering experiments, a novel correction to the Lamb shift of muonic hydrogen can mimic a scenario with a smaller \( r_p \). Since galileons couple to all matter content equally, this interaction should occur in leptonic systems also. But if leptons are truly point like, the potential would lack regularization and would yield unacceptably large corrections to the Lamb shift and 1s - 2s intervals. In order to prevent this effect, we are forced to introduce a composite radius. Here, we investigate the strict constraint that \( \sqrt{\langle r^2 \rangle} = r_i \), i.e., the charge radius is the composite radius, and a more general constraint on a composite radius.

Since we must demand screening mechanisms to evade other bounds, it is non-trivial to construct a composite scale from galileons alone; therefore, we compute several different limits on composite lepton radii.

A. Spectroscopy

Derived constraints from hyperfine splitting (hfs), Lamb shifts, and 1s - 2s intervals are either directly on the \( \langle r^2 \rangle \) or on the Zemach radius, \( \langle r \rangle \). The Zemach radius is approximately linearly related to \( \sqrt{\langle r^2 \rangle} \), with a model-dependent \( O(1) \) coefficient (See Ref. [37] for a discussion). Composite leptons have finite-size contributions similar to those of the proton [28]. For the case of an s-state energy level, the leading finite-size contribution is known,

\[
\delta E = \frac{2}{3n^3} (Ze)^4 \mu^3 \langle r^2 \rangle, \tag{8}
\]
where \( n \) is the principal quantum number, \( Z \) is the charge of the particle, \( \mu \) the reduced mass of the system. At this order, \( p \) states are not affected by \( \langle r^2 \rangle \), so for the Lamb shift, the contribution is

\[
\delta E_{\text{Lamb}} = \frac{1}{12} (Z\alpha)^4 \mu^3 \langle r^2 \rangle. \tag{9}
\]

Furthermore for \( 1s - 2s \) interval, this contribution yields

\[
\delta E_{1s-2s} = \frac{7}{12} (Z\alpha)^4 \mu^3 \langle r^2 \rangle. \tag{10}
\]

We can derive stronger limits from the hyperfine splitting (hfs), where the leading-order effect is given in Ref. [39]:

\[
\delta E_{\text{hfs}} = -2(Z\alpha)\mu\langle r \rangle_{(2)} E_F, \tag{11}
\]

where the Fermi energy is given by

\[
E_F = \frac{8}{3} (Z\alpha)^4 (1 + a_i) \frac{m_j}{m_i} \left( \frac{\mu}{m_j} \right)^3 m_e, \tag{12}
\]

where \( a_i \) is the anomalous magnetic moment of particle \( i \). In addition to the Zemach radius, there is a higher-order contribution directly from \( \langle r^2 \rangle \), which is found in Ref. [10]:

\[
\delta E_{\text{hfs}} = \frac{4}{3} (Z\alpha)^2 \ln (Z\alpha) \mu^2 \langle r^2 \rangle E_F. \tag{13}
\]

Applying these expressions to the measured energy spectrum of positronium and muonium states, we can set limits on \( \langle r \rangle_{(2)} \) and \( \langle r^2 \rangle \). While the Lamb shift and \( 1s - 2s \) constraints in muonium can be applied to either the \( e \) and \( \mu \) component, the shift to the hfs is not mass symmetric, so these limits apply only to muons. Table I is devoted to listing the various constraints, using the experimental and theoretical values found in Table II. We point out that the Zemach radius constraint in muonium give a numerical bound very close to that from \( a_\mu \), and so in Fig. I we only label the Zemach radius.

In Table I we have used \( x \) as a stand-in for the percent uncertainty in a measurement of energy shifts in true muonium. We see that, for even a 10% measurement of the Lamb shift or hyperfine splitting, true muonium would give competitive limits on muon's size.

| Atom | \( \sqrt{\langle r^2 \rangle_{1s-2s}} \) (m) | \( \sqrt{\langle r^2 \rangle_{1s-2s}} \) (m) | \( \sqrt{\langle r^2 \rangle_{\text{hfs}}} \) (m) | \( \langle r \rangle_{2s} \) (m) |
|------|-------------------------------|-------------------------------|-------------------------------|-------------------------------|
| Mu   | \( 4 \times 10^{-15} \) | \( 1 \times 10^{-14} \) | \( 4 \times 10^{-18} \) | \( 1 \times 10^{-18} \) |
| Ps   | \( 7 \times 10^{-15} \) | \( 8 \times 10^{-15} \) | \( 2 \times 10^{-16} \) | \( 2 \times 10^{-15} \) |
| TM   | \( 4\sqrt{x} \times 10^{-13} \) | \( 8\sqrt{x} \times 10^{-15} \) | \( \sqrt{\langle r^2 \rangle_{\text{hfs}}} \times 10^{-18} \) | \( x \times 10^{-17} \) |

TABLE I. Constraints on the charge radius and Zemach radius for leptonic systems. For true muonium (TM), \( x \) corresponds to the percent precision of a future measurement.

B. Anomalous magnetic moments

In order to solve the muon problem, considering constraints from the anomalous magnetic moments is critical. From the formalism of Brodsky and Drell [42], one can use the precisely measured \( a_i \) to limit possible substructure in the leptons. Models of composite leptons can generically give corrections \( \Delta a_i \propto m_i/M^* \) where \( M^* \) is the scale of new physics. As pointed out in Ref. [42], and again argue that the corresponding mass scales can be interpreted as limits on radius via \( R \approx \hbar/M^*c \), we find flavor-dependent limits on the composite radius of

\[
R_e \lesssim 4 \times 10^{-13} \text{ m},
\]

\[
R_\mu \approx 1 \times 10^{-18} \text{ m},
\]

where, because of the discrepancy between theory and experiment, the results for \( \mu \) are a preferred scale as opposed to a limit. The upcoming \((g-2)_\mu \) experiment [14] anticipates a factor of 4 improvement in the measurement of \( a_\mu \), which could improve our limit by a factor of 2. These limits lack a clear relation to the lepton \( \langle r^2 \rangle \), but are strong limits on compositeness. If we seek to explain the complete muon problem with galileons, a precision constraint from \( a_\mu \) would be essential to derive.

IV. LIMITS ON GALILEONS

It was shown in Ref. [27] that the leading correction to the Lamb shift of an atomic system due to a galileon is

\[
\delta E_{\text{Lamb}} = \frac{3}{248\pi^3} \left( \frac{Z}{a_0} \right)^3 \frac{m_e m_j}{M^* r_i^2} \left[ 1 - \frac{1}{6} \left( \frac{Z}{a_0} \right)^2 r_i^2 \right], \tag{16}
\]

where \( a_0 \) is the Bohr radius of the atom and \( r_i \) is the radius of particle \( i \) at which the divergence is cut off. To derive constraints from this equation, we compute the 1\( \sigma \) value of \( \delta E_{\text{Lamb}} \) for each leptonic system found in Table II. Our procedure for this is to first combine the errors in quadrature, and then sum this with the observed value of \( \delta E_{\text{Lamb}} \). Equating these results to Eq. [16], parts of the parameter space of the \( r_i \) and \( M \) below the lines in Fig. I are excluded at 1\( \sigma \). We find for any fixed value of \( r_i \), hydrogenic systems place stronger limits than leptonic ones on the value of \( M \). While this result at first
seems disappointing, it is important to remember that in hydrogenic systems one previously assumed that \( r_i = r_p \).

In contrast, lepton systems can take on any values of \( r_i \) because no composite scale has been measured.

Using the constraints on the various radii derived in Sec. [II], we see that leptonic systems require the scale of \( M \) to be higher than that preferred by \( \mu H \). Currently, positronium energy shifts alone are not well enough known to competitively limit the charge or Zemach radius, so we have not plotted them. But combining positronium constraints from Eq. [16] with the \( a_e \), we obtain the bound \( M_a > 1.2 \) GeV. For muonium, all limits on \( r_i \) are superior bounds on \( M \) than those from the muonic hydrogen which require \( r_i = r_p \). These bounds varying from \( M \sqrt{r^2} > 0.67 \) GeV to \( M_a > 1.33 \) GeV.

Further, we notice that a fiducial measurement of the Lamb shift in true muonium with only 50% precision would improve our limit on the scale \( M \) by a factor of 2. In fact, due to the weak \( M \propto E^{-1/8} \) scaling of our constraints, a mere measurement of the existence of a Lamb shift in true muonium would likely provide the strongest limit on galileons from leptonic physics.

Having seen that requiring \( \sqrt{\langle r^2 \rangle_{em}} = r_i \) for leptons would rule out the preferred value of \( M \) from \( (\mu H) \), we can ask whether this assumption is necessary. It is perhaps not surprising that these two scales must be different. \( \sqrt{\langle r^2 \rangle_{em}} \) is a property of particles determined by their charge distribution, while \( r_i \) should in principle be defined in a similar way by the field distribution that couples to galileons. Since the later couples not only to quarks and leptons, but photons, gluons, W’s and Z’s, the two distributions should differ. This is analogous to how the magnetic distribution of the proton results in the Zemach radius being different than the charge radius.

Allowing \( r_i \neq r_p \) for the proton allows the muon problem to be solved for any value of along the \( (\mu H) \) line in \( (r_i, M) \) space not excluded by limits on \( M \) from other sources, and since these are currently the strong compositeness constraints, an unobserved lepton \( r_i \) is acceptable.

### A. Limits from Perturbativity

The condition that the disformal scalars avoid other constraints required us to embed it in a model with chameleon\([67, 68]\) and galileon properties. In Ref. [27], the full action was given by

\[
S = \int d^4x \sqrt{-g} \left( \frac{R}{16\pi G_N} - \frac{1}{2} (\partial\phi)^2 - \frac{1}{\Lambda^4} \partial^2\phi (\partial\phi)^2 - V(\phi) + \frac{1}{M^4} \partial_\mu\phi \partial_\nu\phi T^{\mu\nu}_{\mu} \right) + S_m(\psi_i, A(\phi) g_{\mu\nu}),
\]

(17)

where we must introduce the suppression scale \( \Lambda \) to control the self-interactions of the galileons. Within this model, a upper limit on \( \Lambda \) can be made by requiring perturbative unitarity to have not been violated up currently observed energies. Ref. [27] finds that using LEP data, which constraints unitarity violations up to 200 GeV,

\[
\Lambda \leq 6 \text{ keV} \left( \frac{M}{320 \text{ MeV}} \right)^{16/9}.
\]

(19)

Additionally, we have assumed that the galileon is well approximated by a free scalar field at least down to the scale of the composite radius. In order to justify this assumption, it was found in Ref. [27] that

\[
\Lambda \geq \frac{1}{r_i} \left( \frac{\beta m_i}{m_{Pl}} \right)^{1/3}.
\]

(20)

Putting Eqs. [19] and [20] together, we can obtain additional constraints on the space of \( (r_i, M) \)

\[
\frac{1}{r_i} \left( \frac{\beta m_i}{m_{Pl}} \right)^{1/3} \leq 6 \text{ keV} \left( \frac{M}{320 \text{ MeV}} \right)^{16/9}.
\]

(21)
of the absolute energy level of each state would give a limit on $r_i$ alone. Another possibility is that if one could construct additional galileon observable effects in $a_i$ or the hfs a similar difference in dependence might arise, allowing us to leverage those precision experiments further. One could consider improving the experimental precision, but since $M \propto r_i^{-1/2} \delta E^{-1/8}$, many orders of magnitude improvement in experiments and theory will be required to improve these limits. The exception is the potential measurement of the spectrum in true muonium. Any Lamb shift measurement in true muonium will improve these limits, and an ambitious part-per-million level measurement could completely exclude galileons. Finally, we note that in positronium and true muonium, the existence of an annihilation channel $ll \rightarrow \phi \rightarrow ll$ allows for potentially stronger limits from energy shifts or decay-rate limits from processes like $(\mu^+\mu^-) \rightarrow e^+e^-$. 

\section{V. SUMMARY AND CONCLUSIONS}

In this paper, we have shown how leptonic systems offer competitive constraints on the scale $M$ of galileons. From the non-observation of a lepton charge or Zemach radius, we can be confident that either $\sqrt{\langle r_i^2 \rangle} \neq r_i$ or the galileon scale must be $M > 1.33$ GeV. This result would be competitive with collider and astrophysical constraints. Going beyond the assumption that the two radii should be related, the galileon model is still viable for solving the muon problem.

Looking forward, the improvement of these atomic constraints is possible. Understanding the true finite-size effects of galileons requires going beyond the crude estimate that $\sqrt{\langle r_i^2 \rangle} = r_i$, and accurately determining their relation. There are a number of ways this could potentially be done. The simplest theoretical, but relatively difficult experimentally, approach is to note that since $s$ and $p$ states have the same dependence on $M$ but a different dependence on $r_i$, measurements of the ratio of the absolute energy level of each state would give a limit on $r_i$ alone. Another possibility is that if one could construct additional galileon observable effects in $a_i$ or the hfs a similar difference in dependence might arise, allowing us to leverage those precision experiments further. One could consider improving the experimental precision, but since $M \propto r_i^{-1/2} \delta E^{-1/8}$, many orders of magnitude improvement in experiments and theory will be required to improve these limits. The exception is the potential measurement of the spectrum in true muonium. Any Lamb shift measurement in true muonium will improve these limits, and an ambitious part-per-million level measurement could completely exclude galileons. Finally, we note that in positronium and true muonium, the existence of an annihilation channel $ll \rightarrow \phi \rightarrow ll$ allows for potentially stronger limits from energy shifts or decay-rate limits from processes like $(\mu^+\mu^-) \rightarrow e^+e^-$. 

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\begin{table}[h]
\begin{tabular}{|l|l|l|l|l|l|}
\hline
Atom & Obs. & $\Delta E_{exp}$(MHz) & Exp. Ref. & $\Delta E_{theory}$(MHz) & Theory Ref. & $\delta E$(MHz) \\
\hline
Mu & Lamb & 1042(23) & [6] & 1047.490(300) & [47-49] & -5.5(230)(0.3) \\
1s – 2s & 2455528941.0(9.8) & [50] & 2455528935.4(14) & [51-53] & 5.6(98)(14) \\
hfs & 4463.302765(53) & [6] & 4463.30288(55) & [55] & -0.000115(53)(55) \\
Ps & Lamb & 13012.42(67)(154) & [56] & 13012.41(9) & [57] & -0.01(67)(154)(9) \\
1s – 2s & 1233607216.4(32) & [58] & 1233607222.6(6) & [59] & -5.8(34)(6) \\
hfs & 203389.10(74) & [59] & 203392.411(60) & [60-64] & -3.31(74)(48)(6) \\
TM & Lamb & ... & ... & 1.35(5)×10^{7} & [66] & 6.8(5)×10^{6} \\
1s – 2s & ... & [66] & 2.55(5)×10^{11} & [66] & 1.27(5)×10^{11} \\
hfs & ... & [66] & 42330577(800)(1200) & [6] & 21165288(800)(1200) \\
\hline
\end{tabular}
\caption{Experimental and theoretical values for the necessary energy shifts in leptonic systems. For the case of true muonium, we have used the representative value of 50\% of the theoretical values for $\delta E$.}
\end{table}
