Semileptonic Transition of $\Sigma_b \rightarrow \Sigma \mu^+ \mu^-$ in Family
Non-universal $Z'$ Model

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Abstract

Using newly available form factors obtained from light cone QCD sum rules in full theory, we study the flavor changing neutral current transition of $\Sigma_b \rightarrow \Sigma \mu^+ \mu^-$ decay in the family non-universal $Z'$ model. In particular, we evaluate the differential branching ratio, forward-backward asymmetry as well as some related asymmetry parameters and polarizations. We compare the obtained results with the predictions of the standard model and discuss the sensitivity of the observables under consideration to family non-universal $Z'$ gauge boson. The order of differential branching ratio shows that this decay mode can be checked at LHC in near future.

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1 Introduction

The heavy baryons containing a single heavy quark constitute a perfect laboratory to test the non-perturbative aspects of QCD. The $\Lambda_b \rightarrow \Delta \mu^+\mu^-$ transition which is based on the flavor changing neutral current (FCNC) transition of $b \rightarrow s \mu^+\mu^-$ at quark level has been recently observed in CDF collaboration at FermiLab [1]. It is also planned to be checked at LHCb collaboration at CERN [2].

The theoretical studies on the branching ratio of $\Sigma_b \rightarrow \Sigma \ell^+\ell^-$ in standard model (SM) [3] show that this decay mode is also possible to be observed at LHC. It is expected that the experimental studies on the heavy baryons and their decay properties constitute one of the main direction of research program at LHC. Hence, the theoretical calculations can play an essential role in this regard.

Although the SM has predictions in perfect agreement with collider data up to now, there are some problems such as neutrino oscillations, baryon asymmetry, unification, dark matter, strong CP violation and the hierarchy problem, etc. which can not be addressed by the SM and still remain unsolved. To cure these deficiencies, there are a plenty of new physics (NP) models such as different extra dimension models (ED), various supersymmetric (SUSY) scenarios, etc. One of the most important new physics scenarios is Z’ model, appears in many grand unified theories, such as $SU(5)$ or string-inspired $E_6$ models [4–8]. The two Z’ models in agenda are family non-universal Z’ [9, 10] and leptophobic Z’ scenarios [11, 12].

The idea of extra heavy Z boson comes from the extension of gauge group $SU(5)$, predicted by the grand unification theories to larger group $SO(10)$. The $SO(10)$ gauge group is the next important one after $SU(5)$ having one extra rank. Hence, this gauge group requires at least one extra neutral gauge boson [13]. In general, Z’ gauge couplings are family universal [10,14–19], however, due to different constructions of the different families, in string models it is possible to have family non-universal Z’ couplings. In some of them, three generation of leptons and also the first and second generation of quarks have different coupling to Z’ boson when compared to the third families of quarks [10,20,21]. For more information about this model see for instance [9,10,22–26]. The leptophobic Z’ model implies that the new neutral gauge boson does not couple to the ordinary SM charged leptons.

The study of the Z’ phenomenology is an important part of the scientific program of every present and future colliders and due to its heaviness, the Z’ boson may be used to calibrate the future detectors [13]. For constraints on the mass of the Z’ boson and the mixing parameters of the model see for example [27–29]. There are direct searches for $Z’ \rightarrow e^+e^-$ decay [30] at Tevatron, and the possibility to discover this gauge boson is analyzed in [31].

In the present work we investigate the FCNC transition of $\Sigma_b \rightarrow \Sigma \mu^+\mu^-$ in family non-universal Z’ model. In particular, we analyze the differential branching ratio, forward-backward asymmetry as well as some related asymmetry parameters and polarizations and compare the results with the predictions of the SM. Note that the rare baryonic $\Lambda_b \rightarrow \Delta \ell^+\ell^-$ decay within family non-universal Z’ model was analyzed in [32–34] within also family non-universal Z’ model. The implications of non-universal Z’ model on B meson decays were
Z are the Wilson coefficients. When \( \alpha \) where \( B \) investigated in [35–43]. The effects of a family non-universal \( Z' \) gauge boson is also searched for \( B \to \pi \pi \) decays in [47]. Recently, the \( B_s^0 - B_s^0 \) mixing, \( B \to K^*l^+l^- \), \( B_s \to \mu^+\mu^- \), \( B \to K\pi \) and inclusive \( B \to X_u l^+l^- \) decays have been analyzed and the stronger constraints have been put for the family non-universal \( Z' \) model parameters in [48–52] (see also [33, 34]).

The outline of the paper is as follows. In section 2, we present the effective Hamiltonian and transition matrix elements responsible for the \( \Sigma_b \to \Sigma \mu^+\mu^- \) transition. In section 3, we analyze the differential branching ratio, forward backward asymmetry, double lepton polarizations as well as some other related asymmetries in the \( Z' \) model and compare the obtained results with the SM predictions. The last section encompasses our concluding remarks.

2 The \( \Sigma_b \to \Sigma \mu^+\mu^- \) transition in Family Non-universal \( Z' \) Model

2.1 The Effective Hamiltonian

Neglecting terms proportional to \( \frac{V_{ub}V_{us}^*}{V_{tb}V_{ts}^*} \approx O(10^{-2}) \), the effective Hamiltonian of the FCNC transition of \( \Sigma_b \to \Sigma ll \), proceed via quark level \( b \to s l^+l^- \) in the SM, can be written as [53–56]

\[
\mathcal{H}_{\text{eff}}^{Z'} = \frac{G_F \alpha_{em} V_{tb} V_{ts}^*}{2\sqrt{2}\pi} \left[ C_{9}^{\text{eff}} \bar{s} \gamma_\mu (1 - \gamma_5) b \bar{\ell} \gamma_\mu \ell + C_{10}^{\text{eff}} \bar{s} \gamma_\mu (1 - \gamma_5) b \bar{\ell} \gamma^\mu \gamma_5 \ell \right. \\
- 2m_b C_{7}^{\text{eff}} \frac{1}{q^2} \bar{s} \sigma_{\mu\nu} q^\nu (1 + \gamma_5) b \bar{\ell} \gamma^\mu \ell \right],
\]

(2.1)

where \( \alpha_{em} \) is the fine structure constant at Z mass scale, \( G_F \) is the Fermi coupling constant, \( V_{ij} \) are elements of the Cabibbo-Kobayashi-Maskawa (CKM) matrix, and \( C_{7}^{\text{eff}}, C_{9}^{\text{eff}}, C_{10}^{\text{eff}} \) are the Wilson coefficients. When \( Z' \) boson is considered and \( Z - Z' \) mixing is neglected, the extra part which should be added to the above effective Hamiltonian is written as [35, 36]

\[
H_{\text{eff}}^{Z'} = -\frac{2G_F}{\sqrt{2}} V_{tb} V_{ts}^* \left[ \frac{B_{sb}^L B_{t\ell}^L}{V_{tb} V_{ts}^*} \bar{s} \gamma_\mu (1 - \gamma_5) b \bar{\ell} \gamma_\mu (1 - \gamma_5) \ell + \frac{B_{sb}^L B_{t\ell}^R}{V_{tb} V_{ts}^*} \bar{s} \gamma_\mu (1 - \gamma_5) b \bar{\ell} \gamma_\mu (1 + \gamma_5) \ell \right] + h.c.
\]

(2.2)

where \( B_{sb}^L = |B_{sb}^L| e^{i\phi_s^L} \) and \( B_{t\ell}^{L,R} \) correspond to the chiral \( Z' \) couplings to quarks and leptons, respectively. Considering the running effects from \( m_W \) to \( m_b \) scale [57], to get the effective Hamiltonian for the transition under consideration in \( Z' \) model, we need to make the following replacements in Eq. (2.1) to include \( Z' \) boson contributions besides the \( Z \) boson:

\[
C_{9}^{\text{eff}} \rightarrow C_{9}^{\text{eff}} = C_{9}^{\text{eff}} - \frac{4\pi}{\alpha_s}(28.82) \frac{B_{sb}^L}{V_{tb} V_{ts}^*} (B_{t\ell}^L + B_{t\ell}^R),
\]

2
\begin{equation}
C_{10} \rightarrow C'_{10} = C_{10} + \frac{4\pi}{\alpha_s} (28.82) \frac{B_{5b}^L}{V_{tb}V_{ts}^*} (B_{6\ell}^L - B_{6\ell}^R),
\end{equation}

where \( \alpha_s \) is the strong coupling constant. Here we should mention that the Wilson coefficient \( C_7^{\text{eff}} \) remains unchanged.

In SM, the Wilson coefficient \( C_7^{\text{eff}} \) in leading logarithm approximation is written as (see [58])

\begin{equation}
C_7^{\text{eff}}(\mu_b) = \eta_{\text{314}} C_7(\mu_W) + \frac{8}{3} \left( \eta_{\text{2314}} - \eta_{\text{314}} \right) C_8(\mu_W) + C_2(\mu_W) \sum_{i=1}^8 h_i \eta^a_i,
\end{equation}

where

\begin{equation}
C_2(\mu_W) = 1, \quad C_7(\mu_W) = -\frac{1}{2} D_0(x_t), \quad C_8(\mu_W) = -\frac{1}{2} E_0(x_t).
\end{equation}

The functions, \( D_0(x_t) \) and \( E_0(x_t) \) are defined as

\begin{align}
D_0(x_t) &= -\frac{(8x_t^3 + 5x_t^2 - 7x_t)}{12(1-x_t)^3} + \frac{x_t^2(2 - 3x_t)}{2(1-x_t)^4} \ln x_t, \\
E_0(x_t) &= -\frac{x_t(x_t^2 - 5x_t - 2)}{4(1-x_t)^3} + \frac{3x_t^2}{2(1-x_t)^2} \ln x_t,
\end{align}

where \( x_t = \frac{m_t^2}{M_W^2} \) with \( m_t \) and \( M_W \) being the top quark and W boson masses, respectively. The coefficients \( a_i \) and \( h_i \) are given as [59, 60]

\begin{align}
a_i &= ( \frac{14}{23}, \frac{16}{23}, \frac{6}{23}, \frac{-12}{23}, 0.4086, -0.4230, -0.8994, 0.1456 ), \\
h_i &= ( 2.2996, -1.0880, -\frac{3}{7}, -\frac{1}{14}, -0.6494, -0.0380, -0.0186, -0.0057 ).
\end{align}

The parameter \( \eta \) in Eq. (2.4) is also defined as

\begin{equation}
\eta = \frac{\alpha_s(\mu_W)}{\alpha_s(\mu_b)},
\end{equation}

with

\begin{equation}
\alpha_s(x) = \frac{\alpha_s(m_Z)}{1 - \beta_0 \frac{\alpha_s(m_Z)}{2\pi} \ln(m_Z^2)},
\end{equation}

where \( \alpha_s(m_Z) = 0.118 \) and \( \beta_0 = \frac{23}{3} \).

The Wilson coefficient \( C_9^{\text{eff}}(\hat{s}') \) is written as [59, 60]:

\begin{align}
C_9^{\text{eff}}(\hat{s}') &= C_9^{\text{NDR}}(\hat{s}') + h(z, \hat{s}') (3C_1 + C_2 + 3C_3 + C_4 + 3C_5 + C_6) \\
&\quad - \frac{1}{2} h(1, \hat{s}') (4C_3 + 4C_4 + 3C_5 + C_6) \\
&\quad - \frac{1}{2} h(0, \hat{s}') (C_3 + 3C_4) + \frac{2}{9} (3C_3 + C_4 + 3C_5 + C_6),
\end{align}

\( 3 \)
where \( s' = \frac{q^2}{m_b^2} \) with \( 4m_b^2 \leq q^2 \leq (m_{Z_b} - m_{Z})^2 \), and

\[
C_9^{NDR} = P_0^{NDR} + \frac{Y_0(x_t)}{\sin^2 \theta_W} - 4Z_0(x_t) + P_E E(x_t),
\]

(2.12)

here, NDR stands for the naive dimensional regularization scheme. We ignore the last term in this equation due to the negligible value of \( P_E \). The \( P_0^{NDR} = 2.60 \pm 0.25 \) [59, 60] and the functions \( Y_0(x_t) \) and \( Z_0(x_t) \) are defined in the following form:

\[
Y_0(x_t) = \frac{x_t}{8} \left[ \frac{x_t - 4}{x_t - 1} + \frac{3x_t}{(x_t - 1)^2} \ln x_t \right],
\]

(2.13)

and

\[
Z_0(x_t) = \frac{18x_t^4 - 163x_t^3 + 259x_t^2 - 108x_t}{144(x_t - 1)^3} + \left[ \frac{32x_t^4 - 38x_t^3 - 15x_t^2 + 18x_t}{72(x_t - 1)^4} - \frac{1}{9} \right] \ln x_t.
\]

(2.14)

In Eq.(2.11), the \( \eta(s') \) is given as

\[
\eta(s') = 1 + \frac{\alpha_s(\mu_b)}{\pi} \omega(s'),
\]

(2.15)

with

\[
\omega(s') = -\frac{2}{9} \pi^2 - \frac{4}{3} \ln \frac{s'}{s} \ln(1 - s') - \frac{5 + 4s'}{3(1 + 2s')} \ln(1 - s') - \frac{2s'(1 + s')(1 - 2s')}{3(1 - s')^2(1 + 2s')} \ln s' + \frac{5 + 9s' - 6s'^2}{6(1 - s')(1 + 2s')}.
\]

(2.16)

At \( \mu_b \) scale, for the coefficients \( C_j \ (j = 1, \ldots, 6) \) we have

\[
C_j = \sum_{i=1}^{8} k_{ji} \eta^{a_i},
\]

(2.17)

where \( k_{ji} \) are given as:

\[
\begin{align*}
  k_{1i} &= (0, 0, 0, \frac{1}{2}, -\frac{1}{2}, 0, 0, 0, 0), \\
  k_{2i} &= (0, 0, 0, \frac{1}{2}, \frac{1}{2}, 0, 0, 0, 0), \\
  k_{3i} &= (0, 0, 0, -\frac{1}{14}, \frac{1}{6}, 0.0510, -0.1403, -0.0113, 0.0054), \\
  k_{4i} &= (0, 0, -\frac{1}{14}, -\frac{1}{6}, 0.0984, 0.1214, 0.0156, 0.0026), \\
  k_{5i} &= (0, 0, 0, 0, -0.0397, 0.0117, -0.0025, 0.0304), \\
  k_{6i} &= (0, 0, 0, 0, 0.0335, 0.0239, -0.0462, -0.0112).
\end{align*}
\]

(2.18)

The other functions in Eq. (2.11) are also given as:

\[
h(y, s') = \frac{8}{9} \ln \frac{m_b}{\mu_b} - \frac{8}{9} \ln y + \frac{8}{27} + \frac{4}{9} x - \frac{2}{9} (2 + x) |1 - x|^{1/2} \begin{cases} \left( \ln \frac{\sqrt{x} + 1}{\sqrt{1 - x}} - i\pi \right), & \text{for } x \equiv \frac{4x^2}{s'} < 1 \\ 2 \arctan \frac{1}{\sqrt{x - 1}}, & \text{for } x \equiv \frac{4x^2}{s'} > 1, \end{cases}
\]

(2.19)
where \( y = 1 \) or \( y = z = \frac{m_c}{m_b} \) and,

\[
h(0, s') = \frac{8}{27} - \frac{8}{9} \ln \frac{m_b}{\mu_b} - \frac{4}{9} \ln s' + \frac{4}{9} i \pi. \tag{2.20}
\]

We also consider the long distance contribution \( Y_{LD} \) coming from \( J/\psi \) family resonances to \( C_y^{eff} \) and parameterized using Breit-Wigner ansatz \([53]\) as

\[
Y_{LD} = \frac{3\pi}{\sqrt{\alpha_{em}^2}} C(0)^6 \sum_{i=1}^{6} \Gamma(V_i \to l^+ l^-) m_{V_i} \frac{1}{m_{V_i}^2 - q^2 - im_{V_i} \Gamma_{V_i}}, \tag{2.21}
\]

where \( C(0) = 0.362 \), and we consider only two lowest resonances \( J/\psi(1S) \) and \( \psi(2S) \) and choose the corresponding phenomenological factors \( \kappa_1 = 1 \) and \( \kappa_2 = 2 \). We use the experimental results on the masses and total decay rates of dilepton decays of the considered vector charmonium states \([61]\). For more details about the calculation of long distance contributions, see for instance \([62, 63]\).

Finally, the explicit expression for \( C_{10} \) is given as:

\[
C_{10} = -\frac{Y_0(x_t)}{\sin^2 \theta_W}, \tag{2.22}
\]

where, \( \sin^2 \theta_W = 0.23 \).

### 2.2 Transition Matrix Elements and Form Factors

The \( \Sigma_b \to \Sigma \mu^+ \mu^- \) decay amplitude is obtained by sandwiching the effective Hamiltonian between the initial and final states

\[
\mathcal{M} = \langle \Sigma(p) | H_{\text{eff}} | \Sigma_b(p + q) \rangle, \tag{2.23}
\]

which reads

\[
\mathcal{M} = \frac{G_F \alpha_{em} V_{tb} V_{ts}^*}{2\sqrt{2} \pi} \left\{ C_{y}^{\text{eff}} \langle \Sigma(p) | \bar{s} \gamma_{\mu}(1 - \gamma_5)b | \Sigma_b(p + q) \rangle \bar{\gamma}^\mu l + C'_{10} \langle \Sigma(p) | \bar{s} \gamma_{\mu}(1 - \gamma_5)b | \Sigma_b(p + q) \rangle \bar{l} \gamma_{\mu} \gamma_5 l - 2m_b C_{7}^{\text{eff}} \frac{1}{q^2} \langle \Sigma(p) | \bar{s} i \sigma_{\mu\nu} q''(1 + \gamma_5)b | \Sigma_b(p + q) \rangle \bar{l} \gamma_{\mu} l \right\}. \tag{2.24}
\]

To calculate the amplitude, we need to parameterize the transition matrix elements \( \langle \Sigma(p) | \bar{s} \gamma_{\mu}(1 - \gamma_5)b | \Sigma_b(p + q) \rangle \) and \( \langle \Sigma(p) | \bar{s} \sigma_{\mu\nu} q''(1 + \gamma_5)b | \Sigma_b(p + q) \rangle \) in terms of twelve form factors \( f_i, g_i, f_{1T}^T \) and \( g_{iT}^T \) (\( i = 1, 2, 3 \)) as follows:

\[
\langle \Sigma(p) | \bar{s} \gamma_{\mu}(1 - \gamma_5)b | \Sigma_b(p + q) \rangle = \bar{u}_{\Sigma}(p) \left[ \gamma_{\mu} f_1(q^2) + i\sigma_{\mu\nu} q'' f_2(q^2) + q'' f_3(q^2) \right] u_{\Sigma_b}(p + q),
\]

\[
\langle \Sigma(p) | \bar{s} i \sigma_{\mu\nu} q''(1 + \gamma_5)b | \Sigma_b(p + q) \rangle = \bar{u}_{\Sigma}(p) \left[ \gamma_{\mu} f_{1T}^T(q^2) + i\sigma_{\mu\nu} q'' f_{2T}^T(q^2) + q'' f_{3T}^T(q^2) \right] u_{\Sigma_b}(p + q), \tag{2.25}
\]
where $u_{\Sigma_b}$ and $u_{\Sigma}$ are spinors of $\Sigma_b$ and $\Sigma$ baryons, respectively.

The form factors as the main inputs in the analysis of the $\Sigma_b \rightarrow \Sigma \ell^+ \ell^-$ have been recently calculated in full theory via light cone QCD sum rules in [3] (for details about the light cone QCD sum rules see for instance [64–66]). The fit function of transition form factors is given as

$$f_i^{(T)}(q^2)[g_i^{(T)}(q^2)] = \frac{a}{(1 - \frac{q^2}{m_{fit}^2})} + \frac{b}{(1 - \frac{q^2}{m_{fit}^2})^2},$$

(2.26)

where the fit parameters $a$, $b$, and $m_{fit}$ are presented in Table 1.

|   | a                | b                | $m_{fit}$  | $q^2 = 0$        |
|---|------------------|------------------|------------|-----------------|
| $f_1$ | $-(0.035 \pm 0.006)$ | $0.130 \pm 0.023$ | $5.1 \pm 1.0$ | $0.095 \pm 0.017$ |
| $f_2$ | $0.026 \pm 0.006$ | $-(0.081 \pm 0.018)$ | $5.2 \pm 1.0$ | $-0.055 \pm 0.012$ |
| $f_3$ | $0.013 \pm 0.004$ | $-(0.065 \pm 0.020)$ | $5.3 \pm 1.1$ | $-0.052 \pm 0.016$ |
| $g_1$ | $-(0.031 \pm 0.008)$ | $0.151 \pm 0.038$ | $5.3 \pm 1.1$ | $0.121 \pm 0.031$ |
| $g_2$ | $0.015 \pm 0.005$ | $-(0.040 \pm 0.013)$ | $5.3 \pm 1.1$ | $-0.025 \pm 0.008$ |
| $g_3$ | $0.012 \pm 0.003$ | $-(0.047 \pm 0.012)$ | $5.4 \pm 1.1$ | $-0.035 \pm 0.009$ |
| $f_1^{T}$ | $1.0 \pm 0.0$ | $-(1.0 \pm 0.0)$ | $5.4 \pm 1.1$ | $0.0 \pm 0.0$ |
| $f_2^{T}$ | $-(0.290 \pm 0.089)$ | $0.421 \pm 0.129$ | $5.4 \pm 1.1$ | $0.131 \pm 0.041$ |
| $f_3^{T}$ | $-(0.240 \pm 0.071)$ | $0.412 \pm 0.122$ | $5.4 \pm 1.1$ | $0.172 \pm 0.051$ |
| $g_1^{T}$ | $0.450 \pm 0.135$ | $-(0.460 \pm 0.138)$ | $5.4 \pm 1.1$ | $-0.010 \pm 0.003$ |
| $g_2^{T}$ | $0.031 \pm 0.009$ | $0.055 \pm 0.015$ | $5.4 \pm 1.1$ | $0.086 \pm 0.024$ |
| $g_3^{T}$ | $-(0.011 \pm 0.003)$ | $-(0.180 \pm 0.057)$ | $5.4 \pm 1.1$ | $-0.190 \pm 0.060$ |

Table 1: Parameters appearing in the fit function of the form factors, $f_1$, $f_2$, $f_3$, $g_1$, $g_2$, $g_3$, $f_1^{T}$, $f_2^{T}$, $f_3^{T}$, $g_1^{T}$, $g_2^{T}$ and $g_3^{T}$ in full theory for $\Sigma_b \rightarrow \Sigma \ell^+ \ell^-$ together with the values of the form factors at $q^2 = 0$ [3].
3 Observables Related to the $\Sigma_b \to \Sigma \ell^+ \ell^-$ Transition

3.1 Branching Ratio

Using the amplitude mentioned above, the differential decay rate is found as (see also [67])

\[
\frac{d\Gamma(z, \hat{s})}{d\hat{s} \ dz} = \frac{G_F^2 m_{\Sigma_b}^2}{16384\pi^5} |V_{tb} V_{ts}^*|^2 v \sqrt{\lambda} \left[ \mathcal{T}_0(\hat{s}) + \mathcal{T}_1(\hat{s}) z + \mathcal{T}_2(\hat{s}) z^2 \right],
\]

where $\hat{s} = \frac{q^2}{m_{\Sigma_b}^2}$ and $z = \cos \theta$ with $\theta$ being the angle between the momenta of $\Sigma_b$ and $\ell^+$ in the center of mass of leptons, $\lambda = \lambda(1, r, \hat{s}) = 1 + r^2 + \hat{s}^2 - 2r - 2\hat{s} - 2r\hat{s}$, $r = m_{\Sigma_b}^2/m_{\Sigma_b}^2$ and $v = \sqrt{1 - \frac{4m_{\ell}^2}{q^2}}$. The functions, $\mathcal{T}_0(\hat{s})$, $\mathcal{T}_1(\hat{s})$ and $\mathcal{T}_2(\hat{s})$ are given as

\[
\mathcal{T}_0(\hat{s}) = 32m_t^2m_{\Sigma_b}^4\hat{s}(1+r-\hat{s}) \left( |D_3|^2 + |E_3|^2 \right) \\
+ 64m_t^2m_{\Sigma_b}^3(1-r-\hat{s}) \Re[D_1^*E_3 + D_3E_1^*] \\
+ 64m_{\Sigma_b}^2\sqrt{r}(6m_t^2 - m_{\Sigma_b}^2) \Re[D_1^*E_1] \\
+ 64m_{\Sigma_b}^2\sqrt{r}(2m_{\Sigma_b}^2 \Re[D_3^*E_3] + (1-r+\hat{s}) \Re[D_1^*D_3 + E_1^*E_3]) \\
+ 32m_{\Sigma_b}^2(2m_t^2 + m_{\Sigma_b}^2) \left( (1+r-\hat{s})m_{\Sigma_b}^2\sqrt{r} \Re[A_1^*A_2 + B_1^*B_2] \\
- m_{\Sigma_b}^2(1-r-\hat{s}) \Re[A_1^*B_2 + A_2^*B_1] - 2\sqrt{r} \Re[A_1^*B_1] + m_{\Sigma_b}^2 \Re[A_1^*B_2] \right) \\
+ 8m_{\Sigma_b}^2 \left\{ 4m_t^2 \left( 1 + \lambda - 2\hat{s} \right) \right\} \left( |A_1|^2 + |B_1|^2 \right) \\
+ 8m_{\Sigma_b}^2 \left\{ 4m_t^2 \left[ \lambda(1+r-\hat{s})\hat{s} + m_{\Sigma_b}^2 \hat{s} \left( (1-r)^2 - \hat{s}^2 \right) \right] \right\} \left( |A_2|^2 + |B_2|^2 \right) \\
- 8m_{\Sigma_b}^2 \left\{ 4m_t^2 \left( 1 + r - \hat{s} \right) - m_{\Sigma_b}^2 \left( (1-r)^2 - \hat{s}^2 \right) \right\} \left( |D_1|^2 + |E_1|^2 \right) \\
+ 8m_{\Sigma_b}^5 \hat{s} \left\{ -8m_{\Sigma_b}^2 \sqrt{r} \Re[D_3^*E_2] + 4(1-r+\hat{s}) \sqrt{r} \Re[D_1^*D_2 + E_1^*E_2] \\
- 4(1-r-\hat{s}) \Re[D_1^*E_2 + D_2^*E_1] + m_{\Sigma_b}^2 \left( (1-r)^2 - \hat{s}^2 \right) \right\} \left( |D_2|^2 + |E_2|^2 \right),
\]

\[
\mathcal{T}_1(\hat{s}, 1/R) = -16m_{\Sigma_b}^4\hat{s}v\sqrt{\lambda} \left\{ 2\Re(A_1^*D_1) - 2\Re(B_1^*E_1) \right\} \\
+ 2m_{\Sigma_b} \Re(B_1^*D_2 - B_2^*D_1 + A_1^*E_1 - A_1^*E_2) \\
+ 32m_{\Sigma_b}^2 \hat{s} v\sqrt{\lambda} \left\{ m_{\Sigma_b}^2(1-r) \Re(A_2^*D_2 - B_2^*E_2) \\
+ \sqrt{r} \Re(A_2^*D_1 + A_1^*D_2 - B_2^*E_1 - B_1^*E_2) \right\},
\]

\[
\mathcal{T}_2(\hat{s}, 1/R) = -8m_{\Sigma_b}^4v^2 \lambda \left( |A_1|^2 + |B_1|^2 + |D_1|^2 + |E_1|^2 \right) \\
+ 8m_{\Sigma_b}^6 \hat{s}v^2 \lambda \left( |A_2|^2 + |B_2|^2 + |D_2|^2 + |E_2|^2 \right),
\]

where

\[
A_1 = \frac{-2m_{\Sigma_b}}{q^2} C_7^{sf} \left( f_1^T + g_1^T \right) + C_9^{sf} (f_1 - g_1).
\]
\[
A_2 = A_1 (1 \rightarrow 2),
A_3 = A_1 (1 \rightarrow 3),
B_i = A_i \left( g_i \rightarrow -g_i; \ g_i^T \rightarrow -g_i^T \right),
D_1 = C_{10} (f_1 - g_1),
D_2 = D_1 (1 \rightarrow 2),
D_3 = D_1 (1 \rightarrow 3),
E_i = D_i (g_i \rightarrow -g_i). \tag{3.31}
\]

To numerically analyze the differential branching ratio with respect to \( q^2 \), we perform integral over \( z \) in the interval \( z \in [-1, 1] \) in Eq. (3.27) and take the values of input parameters as \( m_t = (173.5 \pm 0.6 \pm 0.8) \text{ GeV} \), \( m_W = (80.385 \pm 0.015) \text{ GeV} \), \( m_Z = (91.1876 \pm 0.0021) \text{ GeV} \), \( m_{\Sigma} = (1192.642 \pm 0.024) \text{ MeV} \), \( m_{\Sigma_b} = (5815.5 \pm 1.8) \text{ MeV} \) and \( m_{\mu} = (105.6583715 \pm 0.0000035) \text{ MeV} \) [61], \( m_b = (4.8 \pm 0.1) \text{ GeV} \), \( m_c = (1.46 \pm 0.05) \text{ GeV} \) [68], \( |V_{tb}V_{ts}^*| = 0.041 \), \( G_F = 1.17 \times 10^{-5} \text{ GeV}^{-2} \) and \( \alpha_{em} = \frac{1}{129} \).

The remaining parameters are related to the family non-universal \( Z' \) model. The modifications on the Wilson coefficients in our case are described by the four parameters, \( |B_{sb}^L|, \varphi_s^L, B_{\ell \ell}^L, B_{\ell \ell}^R \). Constraints to \( |B_{sb}^L|, \varphi_s^L \) are put fitting the results of the \( B_s^0 - \bar{B}_s^0 \) mixing observables to recent measurements performed at Tevatron and LHC [48]. These parameters are chosen as \( |B_{sb}^L| = (0.4 \pm 0.1) \times 10^2 \), \( \varphi_s^L = (\pm 150) \times 100 \) to maximize the effects of the additional \( Z' \) gauge boson [33, 34]. Comparing also the theoretical predictions and observational results for the decay channels of \( B \rightarrow X_s\mu^+\mu^- \) [69, 70], \( B \rightarrow K^*\mu^+\mu^- \) [71, 72] and \( B \rightarrow \mu^+\mu^- \) [51], the parameters \( B_{\ell \ell}^L = -9.0 \times 10^{-3} \) and \( B_{\ell \ell}^R = 1.7 \times 10^{-2} \) are obtained [33, 34].

We plot the differential branching ratio, forward backward asymmetry, other asymmetry parameters and polarizations with respect to \( q^2 \) to check the sensitivity of these observables to the model parameters mentioned above at muon channel. We compare predictions of the \( Z' \) and SM models when the uncertainties of the form factors are taken into account. For this aim, first we plot the variations of the differential branching ratio in terms of \( q^2 \) in two models in Figure 1. In this figure, the brown-yellow band surrounded by green lines refers to the family non-universal \( Z' \) model while the blue band surrounded by the red lines denotes the SM results. From this figure, we obtain the following results:

- the SM and \( Z' \) bands intersect each other, the discrepancy between the differential branching ratios obtained from two models is small. Hence the differential branching ratio has not essential sensitivity to \( Z' \) gauge boson in \( \Sigma_b \rightarrow \Sigma\mu^+\mu^- \) channel. Similar result is obtained in [33] for \( \Lambda_b \rightarrow \Lambda\mu^+\mu^- \) decay channel.

- The order of differential branching ratio indicates a possibility for this decay mode to be checked at LHC in near future.
Figure 1: The $q^2$ dependence of the differential branching ratio for the $\Sigma_b \to \Sigma \mu^+ \mu^-$ decay in family non-universal $Z'$ model as well as in the $SM$.

3.2 Forward-backward Asymmetry

One of the most promising tools in detecting the NP effects is the forward-backward asymmetry of leptons which is defined as

$$A_{FB} = \frac{N_f - N_b}{N_f + N_b},$$

(3.32)

where $N_f$ is the number of events that particle is moving "forward" with respect to any chosen direction, and $N_b$ is the number of events that particle moves to "backward" direction. In technique language, the forward-backward asymmetry $A_{FB}(\hat{s})$ is defined in terms of the differential decay rate as

$$A_{FB}(\hat{s}) = \frac{\int_{0}^{1} \frac{d\Gamma}{d\hat{s}dz}(z, \hat{s}) dz - \int_{-1}^{0} \frac{d\Gamma}{d\hat{s}dz}(z, \hat{s}) dz}{\int_{0}^{1} \frac{d\Gamma}{d\hat{s}dz}(z, \hat{s}) dz + \int_{-1}^{0} \frac{d\Gamma}{d\hat{s}dz}(z, \hat{s}) dz},$$

(3.33)

We depict the dependence of $A_{FB}$ on $q^2$ in Figure 2 considering the uncertainties of the form factors. From this figure we conclude that the effects of $Z'$ model’s parameters to the forward backward asymmetry are considerable compared to the differential branching ratio. Although the errors of the form factors partially kill the discrepancies between two models predictions in some regions, they do not sweep the same areas especially at higher values of $q^2$. Similar results have been obtained in $\Lambda_b \to \Lambda \mu^+ \mu^-$ channel for higher values of $q^2$ [33]. However small values of $q^2$ the two baryonic channels show different behaviours when we compare the $Z'$ model predictions with those of the SM.

3.3 Baryon polarization asymmetry parameter - $\alpha_\Sigma$

Asymmetry parameters characterize the angular dependence of differential decay width for the cascade decay $\Sigma_b \to \Sigma (\to a + b)V^* (\to l^+l^-)$ with polarized and unpolarized heavy baryons. In this part, sensitivity of the baryon polarization asymmetry parameter to new
Wilson coefficients is analyzed. The helicity amplitudes for the \( \Sigma_b \to \Sigma \ell^+\ell^- \) decay are obtained by analyzing quasi two body decay \( \Sigma_b \to \Sigma V^* \), followed by the leptonic decay of \( V^* \to \ell^+\ell^- \) in [73]. These amplitudes are obtained using helicity amplitude formalism and polarization density matrix method, demonstrated in [74, 75]. Considering

\[
\Sigma^{1/2+}_b \to \Sigma^{1/2+} (\to a + b) + V^*(\to \ell^+\ell^-),
\] (3.34)

with \( V^* \) being off-shell \( \gamma \) or \( Z \) boson, the normalized joint angular decay distribution for the two cascade decay is written as [74, 76–80]

\[
\frac{d\Gamma(q^2)}{dq^2 d\cos\theta d\cos\theta_\Sigma} = \left| \frac{G_F \alpha_{em}}{8\sqrt{2\pi}} V_{tb} V_{ts}^* \right|^2 \frac{\sqrt[4]{\lambda(m_{\Sigma}^2, m_{V}^2, q^2)} \sqrt[8]{\lambda(m_{\Sigma}^2, m_a^2, m_b^2)}}{1024\pi^3 m_{\Sigma_b} m_{V}^2} v_B(\Sigma_b \to a + b) |\mathcal{M}|^2,
\] (3.35)

where \( \mathcal{M} \) is calculated in [73]. For the leptonic part, in the rest frame of the intermediate boson, the angle of the anti-lepton with respect to its helicity axes is shown by \( \theta \). For the hadronic decay, in the rest frame of the \( \Sigma_b \), \( \theta_\Sigma \) is the angle of the \( a \) momentum with respect to its helicity axes.

The polar angle distribution of \( \Sigma \to a + b \) decay is obtained by integrating Eq. (3.35) with respect to \( \theta \) and it gives the differential decay rate in terms of \( q^2 \) and \( \theta_\Sigma \) [73]

\[
\frac{d\Gamma(q^2)}{dq^2 d\cos\theta_\Sigma} \sim 1 + \alpha(q^2) \alpha_\Sigma(q^2) \cos\theta_\Sigma(q^2),
\] (3.36)

where the baryon polarization asymmetry parameter \( \alpha_\Sigma \) is given as

\[
\alpha_\Sigma(q^2) = \frac{8}{3\Delta(q^2)} \left\{ 4m_\ell^2 |A_{+1/2,+1}|^2 + 2q^2 \left( |A_{+1/2,+1}|^2 + v^2 |B_{+1/2,+1}|^2 \right) \\
- 2m_\ell^2 |A_{+1/2,0}|^2 - q^2 \left( |A_{+1/2,0}|^2 + v^2 |B_{+1/2,0}|^2 \right) - 6m_\ell^2 |B_{+1/2,0}|^2 \\
- 4m_\ell^2 |A_{-1/2,-1}|^2 - 2q^2 \left( |A_{-1/2,-1}|^2 + v^2 |B_{-1/2,-1}|^2 \right) \\
+ 2m_\ell^2 |A_{-1/2,0}|^2 + q^2 \left( |A_{-1/2,0}|^2 + v^2 |B_{-1/2,0}|^2 \right) + 6m_\ell^2 |B_{-1/2,0}|^2 \right\},
\] (3.37)
and the asymmetry parameter $\alpha$ is obtained as

$$
\alpha(q^2) = \frac{8}{3\Delta(q^2)} \left\{ 4m_\ell^2 \left| A_{1/2,1} \right|^2 + 2q^2 \left( \left| A_{1/2,1} \right|^2 + v^2 \left| B_{1/2,1} \right|^2 \right) 
+ 2m_\ell^2 \left| A_{1/2,0} \right|^2 + 6m_\ell^2 \left| B_{1/2,t} \right|^2 + q^2 \left( \left| A_{1/2,0} \right|^2 + v^2 \left| B_{1/2,0} \right|^2 \right) 
- 4m_\ell^2 \left| A_{-1/2,-1} \right|^2 - 2q^2 \left( \left| A_{-1/2,-1} \right|^2 + v^2 \left| B_{-1/2,-1} \right|^2 \right) 
- q^2 \left( \left| A_{-1/2,0} \right|^2 + v^2 \left| B_{-1/2,0} \right|^2 \right) - 2m_\ell^2 \left| A_{-1/2,0} \right|^2 - 6m_\ell^2 \left| B_{-1/2,t} \right|^2 \right\}, \quad (3.38)
$$

with

$$
\Delta(q^2) = \frac{8}{3} \left( 4m_\ell^2 \left| A_{1/2,1} \right|^2 + 2q^2 \left( \left| A_{1/2,1} \right|^2 + v^2 \left| B_{1/2,1} \right|^2 \right) 
+ 2m_\ell^2 \left| A_{1/2,0} \right|^2 + 6m_\ell^2 \left| B_{1/2,t} \right|^2 + q^2 \left( \left| A_{1/2,0} \right|^2 + v^2 \left| B_{1/2,0} \right|^2 \right) 
+ 4m_\ell^2 \left| A_{-1/2,-1} \right|^2 + 2q^2 \left( \left| A_{-1/2,-1} \right|^2 + v^2 \left| B_{-1/2,-1} \right|^2 \right) 
+ q^2 \left( \left| A_{-1/2,0} \right|^2 + v^2 \left| B_{-1/2,0} \right|^2 \right) + 2m_\ell^2 \left| A_{-1/2,0} \right|^2 + 6m_\ell^2 \left| B_{-1/2,t} \right|^2 \right\}. \quad (3.39)
$$

The definitions for $A_{\lambda_i,\lambda_V}$ and $B_{\lambda_i,\lambda_V}$ are given in [73] where $\lambda_i$ and $\lambda_V$ are helicities of lepton pairs and vector boson, respectively. We plot the dependence of the baryon asymmetry parameter $\alpha_\Sigma$ on $q^2$ in Figure 3.

From this figure, it is clear that the family non-universal $Z'$ model’s prediction deviates significantly from the SM prediction such that the errors of the form factors can not kill the discrepancies between two model predictions for the baryon asymmetry parameter $\alpha_\Sigma$.

![Figure 3: The $q^2$ dependence of the asymmetry parameter $\alpha_\Sigma$.](image)

**3.4 Polar angle distribution parameters - $\alpha_\theta$ and $\beta_\theta$**

The polar angle distribution of $V^* \rightarrow l^+l^-$ decay is obtained by integrating Eq. (3.35) with respect to $\theta$, which gives the differential decay rate in terms of $q^2$ and $\theta$ 

$$
\frac{d\Gamma(q^2)}{dq^2 d \cos \theta} \sim 1 + 2\alpha_\theta \cos \theta + \beta_\theta \cos^2 \theta, \quad (3.40)
$$

1
where
\[ \alpha_\theta(q^2) = \frac{1}{\Delta_1(q^2)} 2 v q^2 \text{Re} [A_{+1/2,+1} B_{+1/2,+1}^* - A_{-1/2,-1} B_{-1/2,-1}^*], \quad (3.41) \]

and
\[
\beta_\theta(q^2) = \frac{1}{\Delta_1(q^2)} \left\{ -4m_\ell^2 |A_{+1/2,+1}|^2 + q^2 \left( |A_{+1/2,+1}|^2 + v^2 |B_{+1/2,+1}|^2 \right) \\
+ 4m_\ell^2 |A_{+1/2,0}|^2 - q^2 \left( |A_{+1/2,0}|^2 + v^2 |B_{+1/2,0}|^2 \right) \\
- 4m_\ell^2 |A_{-1/2,-1}|^2 + q^2 \left( |A_{-1/2,-1}|^2 + v^2 |B_{-1/2,-1}|^2 \right) \\
+ 4m_\ell^2 |A_{-1/2,0}|^2 - q^2 \left( |A_{-1/2,0}|^2 + v^2 |B_{-1/2,0}|^2 \right) \right\}, \quad (3.42)
\]

with
\[
\Delta_1(q^2) = 4m_\ell^2 |A_{+1/2,+1}|^2 + q^2 \left( |A_{+1/2,+1}|^2 + v^2 |B_{+1/2,+1}|^2 \right) \\
+ 4m_\ell^2 |B_{+1/2,0}|^2 + q^2 \left( |A_{+1/2,0}|^2 + v^2 |B_{+1/2,0}|^2 \right) \\
+ 4m_\ell^2 |A_{-1/2,-1}|^2 + q^2 \left( |A_{-1/2,-1}|^2 + v^2 |B_{-1/2,-1}|^2 \right) \\
+ 4m_\ell^2 |B_{-1/2,0}|^2 + q^2 \left( |A_{-1/2,0}|^2 + v^2 |B_{-1/2,0}|^2 \right). \quad (3.43)
\]

We plot the dependence of the polar angle distribution parameters \( \alpha_\theta \) and \( \beta_\theta \) on \( q^2 \) in Figures 4 and 5, respectively. From these figures, we read that

![Graph](image)

Figure 4: The \( q^2 \) dependence of the asymmetry parameter \( \alpha_\theta \).

- as far as the \( \alpha_\theta \) is considered we see a considerable discrepancy between the two model’s predictions. In the case of \( \Lambda_b \to \Lambda \mu^+ \mu^- \) this parameter also shows to be very sensitive to \( Z' \) gauge boson. However in our case this sensitivity is relatively small.
- Similar to the \( \Lambda_b \to \Lambda \mu^+ \mu^- \) channel considered in [33], the asymmetry parameter \( \beta_\theta \) in \( Z' \) model gives roughly the same result as the SM.
3.5 Double Lepton Polarizations

To analyze the double lepton polarizations, we need to define the following orthogonal unit vectors $s_{\pm i}^\mu$ with $i = L,T$ or $N$ in the rest frame of double leptons (see for instance [81–83] for detailed information)

\[
\begin{align*}
    s_{L}^\mu &= (0, e_{L}^-) = \left(0, \frac{\vec{p}_-}{|\vec{p}_-|}\right), \\
    s_{N}^\mu &= (0, e_{N}^-) = \left(0, \frac{\vec{p}_\Sigma \times \vec{p}_-}{|\vec{p}_\Sigma \times \vec{p}_-|}\right), \\
    s_{T}^\mu &= (0, e_{T}^-) = \left(0, \frac{\vec{e}_N \times \vec{e}_L}{|\vec{e}_N \times \vec{e}_L|}\right), \\
    s_{L}^\mu &= (0, e_{L}^+) = \left(0, \frac{\vec{p}_+}{|\vec{p}_+|}\right), \\
    s_{N}^\mu &= (0, e_{N}^+) = \left(0, \frac{\vec{p}_\Sigma \times \vec{p}_+}{|\vec{p}_\Sigma \times \vec{p}_+|}\right), \\
    s_{T}^\mu &= (0, e_{T}^+) = \left(0, \frac{\vec{e}_N \times \vec{e}_L}{|\vec{e}_N \times \vec{e}_L|}\right),
\end{align*}
\]  

(3.44)

where $\vec{p}_\pm$ and $\vec{p}_\Sigma$ are the three-momenta of the leptons $\ell^\pm$ and $\Sigma$ baryon. The symbols L, T and N denote longitudinal, transverse, and normal polarizations, respectively. Using the Lorentz boost, these unit vectors are transformed from the rest frame of the leptons into the center of mass (CM) frame of them along the longitudinal direction. As a result, we get

\[
\begin{align*}
    \left(s_{L}^\mu\right)_{CM} &= \left(\frac{|\vec{p}_\pm|}{m_\ell}, \frac{E_\ell}{m_\ell |\vec{p}_|}\right),
\end{align*}
\]  

(3.45)

where $\vec{p}_\pm = -\vec{p}_-$ and $E_\ell$ and $m_\ell$ are the energy and mass of leptons in the CM frame, respectively. Under the above transformation, the other two unit vectors, $s_{N}^\pm$, $s_{T}^\pm$ remain...
unchanged. The double lepton polarizations are given as [81–83]

\[
P_{ij}(\hat{s}) = \left( \frac{d\Gamma(-\tilde{s}_i^-, -\tilde{s}_j^+)}{d\hat{s}} - \frac{d\Gamma(-\tilde{s}_i^-, \tilde{s}_j^+)}{d\hat{s}} \right) - \left( \frac{d\Gamma(-\tilde{s}_i^-, -\tilde{s}_j^+)}{d\hat{s}} - \frac{d\Gamma(-\tilde{s}_i^-, -\tilde{s}_j^+)}{d\hat{s}} \right)
\]

\[
+ \left( \frac{d\Gamma(-\tilde{s}_i^-, -\tilde{s}_j^+)}{d\hat{s}} + \frac{d\Gamma(-\tilde{s}_i^-, -\tilde{s}_j^+)}{d\hat{s}} \right)
\]

(3.46)

The first (second) subindex of \(P_{ij}\) represents polarization of lepton (anti-lepton). Using the above definitions, some double lepton polarizations are obtained as

\[
P_{LN}(\hat{s}) = -P_{NL}(\hat{s}) = \frac{16\pi m_{\lambda e}^4 \hat{m}_\ell \sqrt{\lambda}}{\Delta'\hat{(\hat{s})^2}} \text{Im} \left\{ (1 - \hat{r})(A_1^* D_1 + B_1^* E_1) \right. \\
+ m_{\Sigma_1} \hat{s}(A_1^* E_3 - A_2^* E_1 + B_1^* D_3 - B_2^* D_1) \\
+ m_{\Sigma_1} \sqrt{\hat{r}\hat{s}}(A_1^* D_3 + A_2^* D_1 + B_1^* E_3 + B_2^* E_1) - m_{\Sigma_1}^2 \hat{s}^2 (B_2^* E_3 + A_2^* D_3) \left. \right\},
\]

(3.47)

\[
P_{LT}(\hat{s}) = \frac{16\pi m_{\Sigma_2}^4 \hat{m}_\ell \sqrt{\lambda \nu}}{\Delta'\hat{(\hat{s})^2}} \text{Re} \left\{ (1 - \hat{r})(|D_1|^2 + |E_1|^2) - \hat{s} \left( A_1^* D_1^* - B_1^* E_1^* \right) \right. \\
- m_{\Sigma_2} \hat{s} \left[ B_1^* D_2^* + (A_2^* + D_2 - D_3) E_1^* - A_1^* E_2^* - (B_2 - E_2 + E_3) D_1^* \right] \\
+ m_{\Sigma_2}^2 \hat{s}(1 - \hat{r})(A_2^* D_2^* - B_2^* E_2^*) \\
+ m_{\Sigma_1} \sqrt{\hat{r}\hat{s}} D_1^* (A_1^* D_3^* + A_2^* D_1^* + B_1^* E_3^* + B_2^* E_1^*) - m_{\Sigma_1}^2 \hat{s}^2 (B_2^* E_3^* + A_2^* D_3^*) \right. \left. \right\},
\]

(3.48)

\[
P_{NT}(\hat{s}) = -P_{TN} = \frac{64m_{\Sigma_3}^4 \lambda \nu}{3\Delta'\hat{(\hat{s})^2}} \text{Im} \left\{ (A_1^* D_1^* + B_1^* E_1^*) + m_{\Sigma_3}^2 \hat{s} (A_2^* D_2^* + B_2^* E_2^*) \right. \left. \right\},
\]

(3.49)

\[
P_{NN}(\hat{s}) = \frac{32m_{\Sigma_3}^4}{3\hat{s}\Delta'\hat{(\hat{s})^2}} \text{Re} \left\{ 24 \hat{m}_\ell^2 \sqrt{\hat{r}\hat{s}} (A_1^* D_1^* + D_1^* E_1^*) - 12 m_{\Sigma_3} \hat{m}_\ell^2 \sqrt{\hat{r}\hat{s}} (1 - \hat{r} + \hat{s})(A_1^* A_2^* + B_1^* B_2^*) \\
+ 6 m_{\Sigma_3} \hat{m}_\ell^2 \hat{s} \left[ m_{\Sigma_3} \hat{s}(1 + \hat{r} - \hat{s}) \left( |D_3|^2 + |E_3|^2 \right) + 2 \sqrt{\hat{r}} \left( 1 - \hat{r} + \hat{s} \right) (D_1^* D_3^* + E_1^* E_3^*) \right] \\
+ 12 m_{\Sigma_3} \hat{m}_\ell^2 \hat{s} \left( 1 - \hat{r} - \hat{s} \right) \left( A_1^* B_2^* + A_2^* B_1^* + D_1^* E_3^* + D_3^* E_1^* \right) \\
- \left[ \lambda \hat{s} + 2 \hat{m}_\ell^2 (1 + \hat{r} - 2 \hat{r} + \hat{r} \hat{s} + \hat{s} - 2 \hat{s}^2) \right] \left( |A_1|^2 + |B_1|^2 - |D_1|^2 - |E_1|^2 \right) \\
+ 24 \hat{m}_\ell^2 \hat{s} \left( A_2^* B_3^* + D_3^* E_3^* \right) - m_{\Sigma_3}^2 \lambda \hat{s}^2 v^2 \left( |D_2|^2 + |E_2|^2 \right) \\
+ m_{\Sigma_3}^2 \hat{s} \left( \lambda \hat{s} - 2 \hat{m}_\ell^2 \left[ 2(1 + \hat{r}^2) - \hat{s}(1 + \hat{s}) - \hat{r}(4 + \hat{s}) \right] \right) \left( |A_2|^2 + |B_2|^2 \right) \right. \left. \right\},
\]

(3.50)

\[
P_{TT}(\hat{s}) = \frac{32m_{\Sigma_3}^4}{3\hat{s}\Delta'\hat{(\hat{s})^2}} \text{Re} \left\{ -24 \hat{m}_\ell^2 \sqrt{\hat{r}\hat{s}} (A_1^* B_1^* + D_1^* E_1^*) - 12 m_{\Sigma_3} \hat{m}_\ell^2 \sqrt{\hat{r}\hat{s}} (1 - \hat{r} + \hat{s})(D_1^* D_3^* + E_1^* E_3^*) \right. \\
- 6 m_{\Sigma_3} \hat{m}_\ell^2 \hat{s} \left[ m_{\Sigma_3} \hat{s}(1 + \hat{r} - \hat{s}) \left( |D_3|^2 + |E_3|^2 \right) + 2 \sqrt{\hat{r}} \left( 1 - \hat{r} + \hat{s} \right) (D_1^* D_3^* + E_1^* E_3^*) \right] \\
+ 12 m_{\Sigma_3} \hat{m}_\ell^2 \hat{s} \left( 1 - \hat{r} - \hat{s} \right) \left( A_1^* B_2^* + A_2^* B_1^* + D_1^* E_3^* + D_3^* E_1^* \right) \\
- \left[ \lambda \hat{s} + 2 \hat{m}_\ell^2 (1 + \hat{r} - 2 \hat{r} + \hat{r} \hat{s} + \hat{s} - 2 \hat{s}^2) \right] \left( |A_1|^2 + |B_1|^2 - |D_1|^2 - |E_1|^2 \right) \\
+ 24 \hat{m}_\ell^2 \hat{s} \left( A_2^* B_3^* + D_3^* E_3^* \right) - m_{\Sigma_3}^2 \lambda \hat{s}^2 v^2 \left( |D_2|^2 + |E_2|^2 \right) \\
+ m_{\Sigma_3}^2 \hat{s} \left( \lambda \hat{s} - 2 \hat{m}_\ell^2 \left[ 2(1 + \hat{r}^2) - \hat{s}(1 + \hat{s}) - \hat{r}(4 + \hat{s}) \right] \right) \left( |A_2|^2 + |B_2|^2 \right) \right. \left. \right\},
\]

(3.51)
\[-24 m_{\Sigma_b}^2 \hat{m}_i^2 \sqrt{\hat{r}\hat{s}}^2 (A_2 B_2^* + D_3 E_3^*) \]
\[-6 m_{\Sigma_b}^2 \hat{m}_i^2 \hat{s} [m_{\Sigma_b} \hat{s} (1 + \hat{r} - \hat{s}) (|D_3|^2 + |E_3|^2) - 2\sqrt{\hat{r}} (1 - \hat{r} + \hat{s}) (A_1 A_2^* + B_1 B_2^*)] \]
\[-12 m_{\Sigma_b}^2 \hat{m}_i^2 \hat{s} (1 - \hat{r} - \hat{s}) (A_1 B_2^* + A_2 B_1^* + D_1 E_3^* + D_3 E_1^*) \]
\[-\left[ \lambda \hat{s} - 2\hat{m}_i^2 (1 + \hat{r}^2 - 2\hat{r} + \hat{s} + 2\hat{s}^2) (|A_1|^2 + |B_1|^2) \right] \]
\[+ m_{\Sigma_b}^2 \hat{s} \{ \lambda \hat{s} + \hat{m}_i^2 [4(1 - \hat{r})^2 - 2\hat{s}(1 + \hat{r}) - 2\hat{s}^2] (|A_2|^2 + |B_2|^2) \]
\[+ \{ \lambda \hat{s} - 2\hat{m}_i^2 [5(1 - \hat{r})^2 - 7\hat{s}(1 + \hat{r}) + 2\hat{s}^2] \} (|D_1|^2 + |E_1|^2) \]
\[\right\}, \quad (3.51) \]

where
\[\Delta'(\hat{s}) = T_0(\hat{s}) + \frac{1}{3} T_2(\hat{s}). \quad (3.52)\]

We plot the dependence of the double lepton polarizations on $q^2$ in Figures 6-10.

Figure 6: The $q^2$ dependence of Longitudinal-normal (LN) polarization of leptons

Figure 7: The same as figure 6, but for LT polarization

From these figures, we observe that
in general, there are considerable discrepancies between the $Z'$ model and the SM predictions on the double lepton polarization asymmetries under consideration except for the $P_{LT}$.

- The big deviations of the $Z'$ model’s results from the SM predictions appear at lower values of $q^2$. For the $P_{NN}$ and $P_{TT}$, the NP parameters are more effective at lower
values of $q^2$.

- The maximum discrepancy between two model’s predictions belongs to the $P_{LN}$ and $P_{TN}$.

- Compared to the $\Lambda_b \to \Lambda \mu^+ \mu^-$ decay channel [34], the double lepton polarization asymmetries are more sensitive to the $Z'$ gauge boson in $\Sigma_b \to \Sigma \mu^+ \mu^-$ decay channel.

- Determination of zero points of the double lepton polarization asymmetries in the experiment and comparison of the obtained results with the theoretical predictions can give valuable information about the existence of the $Z'$ boson.

4 Conclusion

In the present study, we analyzed the $\Sigma_b \to \Sigma \mu^+ \mu^-$ decay channel in both the SM and $Z'$ model considering the errors of form factors. We discussed the sensitivity of the differential branching ratio, forward-backward asymmetry as well as some asymmetry parameters and polarization asymmetries defining the considered decay channel to $Z'$ gauge boson. Our results on the considered physical quantities overall depict considerable discrepancies between the $Z'$ and SM model’s predictions. In the case of the differential branching ratio the discrepancy between two model’s predictions is small such that the uncertainties of the form factors roughly kill the difference. The maximum discrepancies belong to the double lepton polarization asymmetries and asymmetry parameter $\alpha_\Sigma$ which are quite sensitive to $Z'$ gauge boson. The discrepancies between two model’s predictions on some physical observables can be considered as signals for existing the extra $Z'$ gauge boson. The order of the differential branching ratio indicates that this channel can be studied at LHC in near future. Any measurement on the physical quantities considered in the present work and comparison of the obtained data with the theoretical predictions can give useful information not only about the existence of the $Z'$ gauge boson but also about the nature of participating baryons, $\Sigma_b$ and $\Sigma$. We also compared our results with the results of the considered observables for $\Lambda_b \to \Lambda \mu^+ \mu^-$ decay channel [33, 34] which is in agenda of different experiments nowadays. The study of different FCNC channels can provide us with more data which may help us in searching for $Z'$ gauge boson as new physics effect.

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