Novel design of electrostatic micro cantilever for enhanced travel range

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Abstract. Electrostatic micro actuators are commonly deployed micro electro mechanical system (MEMS) devices due to their unpretentious construction and well-matched micro fabrication processes. The phenomenon of pull-in instability puts substantial restrictions on the execution of electrostatically driven MEMS beam type actuators by restraining the range of travel. A larger working range is desirable for a wide variety of tuning applications. In this paper, mechanism of pull-in instability and means to extend the useful working range of the microactuator by changing the design is presented. It shows drastic improvement in the results. Important conclusions are drawn from the results.

1. Introduction

Micro-electro-mechanical systems (MEMS) are progressively becoming well accepted in a variety of industrial applications such as aerospace, automotive and biomedical industries [1]. A conventional MEMS device is the combination of mechanical elements, actuators, sensors and electronics on a common silicon substrate. Popular and industrial MEMS devices comprise digital micro mirror device (DMD), automotive crash sensors, catheter tip pressure sensors, ink jet printer nozzles etc.\cite{2}. Nowadays, national and international vigor in the MEMS's field reveal the ever growing scientific interest in this field. Sensors and actuators are the two very basic features of the MEMS structure. A sensor collects the information from the environment by utilizing various properties such as mechanical, biological, optical and magnetic \cite{3}\cite{4}. MEMS devices are classified ap per their actuation mechanisms, which will vary depending on the suitability. The commonly used actuation mechanisms are piezoelectric, thermal, electrostatic and pneumatic and .Out of all the mentioned methods electrostatic is the most popular method. Selection of electrostatic actuation in MEMS compared to other actuators is due to the following reasons \cite{5}.

Design and microfabrication of electrostatic devices are to some extent, in a more mature stage than some other MEMS devices. Fabrication of electrostatically driven systems is compatible with the well-established and well-explored silicon fabrication technology. Power consumption is low as compared to the other actuation techniques. Use of electrostatic force at the micro level can be easily justified with the help of scaling laws of force fields. The dominance of electrostatic force over other forces like electromagnetic force makes it a favorable choice for micro actuation. Higher energy densities can be obtained in electrostatic actuation.
Examples of devices in which electrostatic actuation is used include; microrelays, capacitive pressure sensors, accelerometers, micromotors, Digital Micromirrors etc. A very useful application developed by Texas Instruments is the Digital Micromirror Device (DMD). Currently, the display devices include Cathode Ray Tubes (CRT), or Liquid Crystal Displays (LCD). DMD is considered to be the next candidate in the race. Essentially, DMD is a projection system based on a very large array of micro machined mirrors, which finds its application in the Adaptive Optics (AO) systems. In the present paper a novel design of electrostatically driven cantilever beam type microactuator is proposed which can enhance the travel range and improve the performance of the device for various tuning applications.

2. Mechanics of pull-in instability

The major disadvantage for the use of electrostatic actortor is the pull-in instability [6][7]. Though used in the cases such as, material properties extraction [8] or binary operations for digital applications, the peculiarity of pull-in discourages an efficient use of these actuators in many applications that require higher travel range. The root of the pull-in instability remains in the interface of nonlinear electrostatic force; which varies as per the inverse of the square and the linear restoring force. The present work deals with the means to understand the mechanics of the pull-in instability in a practical microactuator system and methods to gain some control over the same. When a DC voltage is applied gradually beween fixed and movable electrode, movable electrode starts deforming towards fixed electrode. When gradually applied DC voltage crosses a critical value, linear elastic restoring force of the canilever beam is not able to sustain nonlinear electrostatic force, heading to nonfunctional state of the device [9]. The value of the voltage at this bistable condition is known as static pull-in voltage and the corresponding displacement is called static pull-in displacement, collectively called as static pull-in parameters. The distance traveled by the tip of the cantilever by applying voltage is known as static travel range.

Consider the cantilever microbeam as shown in Figure1 having following dimensional and material terms: length is termed as $L$, width as $b$ and thickness as $t$ working as a deformable electrode. The fixed electrode and movable electrode are parted by an initial gap $g_0$. When DC voltage $V$ is applied between the two electrodes, the cantilever deplaces due to electrostatic force. The deformed shape of the cantilever is shown by the curves1. It is assumed that, the deflection of the cantilever is defined by the equation 1,

$$y(x) = a_0 + a_1x + a_2x^2 + a_3x^3 + a_4x^4$$  \hspace{1cm} (1)

where, $x$ is the co-ordinate along the length of the movable electrode. Applying the kinematic boundary conditions for the micro cantilever, the deformed shape can be written by the equation 2,

$$y(x) = a_4(6L^2x^2 - 4Lx^3 + x^4) = a_4[F(x)].$$  \hspace{1cm} (2)

The equation of potential energy of the cantilever microbeam can be written as,

$$\Pi=\left(\int_0^L \frac{EI}{2} [\ddot{y}(x)]^2 dx\right) - \left(\frac{\varepsilon_0 bV^2}{2} \int_0^L \frac{dx}{g_0 - y(x)}\right)$$  \hspace{1cm} (3)

Since the deflection is only the function of $a_4$, we can differentiate the expression for the total potential energy with respect to $a_4$ and equate it to zero, in order to find the stationary value. This follows the principle of minimum potential energy. Hence,

$$\frac{d\Pi}{da_4} = \left(\frac{EI}{2} \int_0^L \frac{dx}{2} [\ddot{y}(x)] \frac{d\ddot{y}}{da_4} dx\right) - \left(\frac{\varepsilon_0 bV^2}{2} \int_0^L \frac{dx}{[g_0 - y(x)]^2} \right) = 0$$  \hspace{1cm} (4)
we have,
\[
\frac{d^2y}{da_4} = 12 \left( L^2 - 2Lx + x^2 \right) \tag{5}
\]
\[
\frac{dy}{da_4} = F(x) = \left( 6L^2x^2 - 4Lx^3 + x^4 \right) \tag{6}
\]
Equation 4 reduces to
\[
\frac{d\Pi}{da_4} = \left( \frac{EI}{2} \int_0^L 2a_4 (12)^2 (L-x)^4 dx \right) - \left( \frac{\varepsilon_0 bV^2}{2g_0^2} \int_0^L \frac{F(x)}{g_0 - a_4F(x)}dx \right)^2 = 0 \tag{7}
\]
which can be expressed as
\[
\frac{d\Pi}{da_4} = \left( \frac{144L^5 a_4 EI}{5} \right) - \left( \frac{\varepsilon_0 bV^2}{2g_0^2} \int_0^L \frac{F(x)}{1 - a_4 F(x)}dx \right)^2 = 0 \tag{8}
\]
where,
\[
A = \frac{144EI L^5}{5}, \quad B = \frac{\varepsilon_0 bV^2}{2g_0^2}, \quad C = F(x), \quad D = \frac{F(x)}{g_0} \tag{10}
\]
Using the constants from above equations, we can calculate the static pull-in parameters.

2.1. Pull-in parameters of variable width cantilever microbeam

By varying the width of the microbeam, we had done a preliminary analysis to investigate the influence of geometry on the static pull-in parameters of a cantilever microbeam. The following width function was suggested, which smoothly varied the width of the cantilever microbeam,
\[
b(x) = \alpha b_0 [1 - f_1(x)^n]^m \tag{11}
\]
The representative variable width geometries were analyzed by the numerical scheme and the results indicated that reducing the width of the cantilever microbeam at its free end increases the travel range. But, this modification to the device geometry also calls for a need of an increased actuation voltage requirement. We have now analyzed the static response of the variable width electrostatic microactuators and the corresponding analysis is presented below. The application of the Rayleigh-Ritz energy technique, which make it even more accurate and time efficient. Using this scheme, estimation of the static behavior of the prismatic cantilever was performed and found the values of static pull-in parameters. The governing differential equation of a cantilever is expressed by,
\[
\frac{d^4u(x)}{dx^4} = \frac{\varepsilon_0 AV^2}{2(g_p - u(x))^2} \tag{12}
\]
where, \(u(x)\) is the displacement of the cantilever towards the fixed one, \(A\) is the area of overlap between two electrodes, \(I\) indicates the moment of inertia. In the begining, both the electrodes are separated by a gap \(g_p\) and the permittivity of the free space is \(\varepsilon\). The Young's modulus is termed as \(E\) of the cantilver material. The length co-ordinate is designated by \(x\) and it ranges from 0 to \(L\). An electrostatically actuated MEMS cantilever microbeam is schematically shown in Figure1 and Figure2.
It is required to calculate the deflection of the microbeam for a particular voltage by solving the coupled nonlinear differential equation described in Eq. 12. Also, the voltage at which the pull-in takes place and the corresponding value of the deflection are the parameters of concern.

2.2. Numerical Technique
The principle of operation in case an electrostatically actuated microbeam is as follows: When a potential difference in DC voltage form \( V \) is applied across electrodes, the cantilever bends due to the electrostatic force. The force acting on a deformed beam is in proportion to the deflection of the cantilever. Since, the deflection varies along the length of the cantilever, the value of the electrostatic force also changes proportionately. In case of the cantilever, the maximum force is experienced by its tip, since it experiences the maximum deflection.

2.3. The Rayleigh-Ritz Energy Method
Approximation of the deflection of the cantilever by a fourth order polynomial function \( \phi \), is performed here. Mathematical representation is as follows:

\[
 u(x) = a\phi(x) \quad \text{(13)}
\]

where \( a \) represents the amplitude of the shape function. The nondimensional definitions for \( u^* \) and \( x^* \) are stated in the next section. This polynomial approximation function satisfies the mechanical boundary conditions of the cantilever microbeam. The deflection approximations are defined by the following equations,

\[
 \phi = 6x^2 - 4x^3 + x^4 \quad \text{(14)}
\]

It is assumed that the microactuator to be a voltage controlled device, hence, the total potential energy of the cantilever in any deflected position can be expressed as,

\[
 \frac{EI}{2} \int_0^L \left( u'' \right)^2 \, dx - \frac{\varepsilon_0 b V^2}{2} \int_0^L \frac{dx}{\left( g_0 - u \right)} \quad \text{(15)}
\]

3. Applications of Nonprismatic Microbeams
The phenomenon of pull-in instability is detrimental to the device operation in many commercial applications, since it hampers the expedient travel range of the cantilever. However, some applications like material property determination, switching operation make use of the pull-in
phenomenon to accomplish certain tasks[9]. Hence, it is the need of an hour for customized microactuator design that satisfies the requirements of a particular application. In this connection, the static pull-in analysis of nonprismatic cantilever is presented, in which the width of the microbeam is smoothly varied along its length. This alteration to the standard rectangular geometry offers some means to gain a control over the pull-in parameters. In the preceding section, we saw the static pull-in analysis of the prismatic cantilever microbeam. The analysis revealed that the pull-in instability confines the travel range of the cantilever not more than 50% of the gap between the fixed and movable electrodes in the initial stage. In many applications like Digital Micromirror Device [10], polychromator grating [?]; more deflection of the cantilever is desirable. Some of the proposed methods to overcome this issue include current pulse drive, leveraged bending and piecewise linearity [11]. In the recent past, researchers proposed the utility of using variable geometry microbeam actuators in order to gain some control over the pull-in parameters[12][8]. In this design, the shape of the cantilever is optimized, which fulfills the needs of particular application. Optimization affable parametric width function is proposed for the cantilever microbeam. This function efficiently varies the width of the cantilever. The pull-in performance of the nonprismatic microbeams is compared with that of the prismatic microbeams to reach to the conclusions.

3.1. Static Analysis of Nonprismatic cantilever microbeam

We started from prismatic shape (i.e. constant width), a Rayleigh-Ritz-based minimum energy is deployed to calculate the pull-in parameters of any cantilever having variable width [13]. Consider the cantilever as shown in figure 2.

The beam is modeled as Euler-Bernoulli beam, the governing differential equation is [13]

$$\frac{d^4u}{dx^4} + \frac{E}{EI} \left( \frac{d^2b}{dx^2} \right)^2 = \frac{V^2}{2(g_0 - \hat{u}(x))^2}$$

where, $\hat{u}(\hat{x})$ is the displacement of the cantilever, $b(\hat{x})$ is the width at co-ordinate $\hat{x}$.

The total potential energy of the nonprismatic electrostatic-cantilever microbeam can be defined using equation 17,

$$\hat{\Pi} = \frac{Eb_0^3}{24} \int_0^L b(\hat{x}) \left( \frac{d^2\hat{u}(\hat{x})}{d\hat{x}^2} \right)^2 d\hat{x} - \frac{\hat{V}^2}{2} \int_0^L \frac{b(\hat{x}) d\hat{x}}{\hat{g}_0 - \hat{u}(\hat{x})}$$

The following normalized terms are defined to generalize the analysis.

$$u(x) = \frac{\hat{u}(\hat{x})}{g_0}, x = \frac{\hat{x}}{L}, V^2 = \frac{\epsilon b_0 L^4 \hat{V}^2}{2E \hat{g}_0^3}, I = \frac{b_0 h_0^3}{12}$$

where $I$ is the area moment of inertia of the prismatic microbeamcantilever. Using these entities, the normalized total potential energy is expressed as,

$$\Pi = \frac{12\hat{V} L^3}{Eb_0 h_0^3}$$

Following the principle of minimum potential energy and the clause for instability, the pull-in parameters of the cantilever can be calculated using,

$$\frac{d^2\Pi}{da^2} = 0, \quad \frac{d\Pi}{da} = 0$$
The parametric function for variable width of the cantilever microbeam can be described by the equation [13],

\[ b(x) = \alpha b_0 [1 - f_1(x)^n]^m \] (21)

where \(\alpha\), \(f_1\), \(m\) and \(n\) are the design variables and \(b_0\) is the width of a prismatic beam. The representative variable width geometries were analyzed by the numerical scheme and the results indicated that reducing the width of the cantilever microbeam at its free end increases the travel range.

### 3.2. Optimization of design parameters

The objective here is to find the optimal parameters, \(\alpha\), \(f_1\), \(m\) and \(n\) of the parametric width function, which maximize the pull-in displacement. The optimization problem can be defined as:

\[
\text{Minimize } G(\alpha, m, n, f) = 1 - u_p + P_1 (C_{\text{area}}) + P_2 (C_{b\text{min}} + C_{b\text{max}} + C_{f1} + C_{mn})
\] (22)

Subject to:

\[
\frac{b_{\text{min}}}{L} \leq \frac{b(x)}{L} \leq \frac{b_{\text{max}}}{L}, \quad \int_0^1 b_0 dx = \int_0^1 b(x) dx \]

\[ 0 \leq f_1 \leq 1, \quad 0 \leq m \leq \infty, \quad 0 \leq n \leq \infty \] (24)

where \(u_p\) is the static tip deflection of the cantilever. \(b_{\text{min}}\) is the lower bound of the width and \(b_{\text{max}}\) is the upper bound of the width in the cantilever microbeam. \(P_1\) and \(P_2\) are the penalties deployed for violating the constraints. The constant area constraint is deployed to avoid trivial solutions and to examine the influence of shape, than size, on the pull-in behavior of the cantilever [13]. The constraint related terms, which are multiplied by the penalties are described here,

- The term \(C_{\text{area}}\) defines the severity of the constant area constraint violation

\[
C_{\text{area}} = \left( 1 - \int_0^1 b(x) dx \right)^2
\]

- The term \(C_{b\text{min}}\) defines the severity of the violation of constraint - minimum width

\[
C_{b\text{min}} = \left[ \frac{\min(0, b_{\text{min}}^i - b_{\text{min}})}{b_{\text{min}}} \right]^2
\]

where, \(b_{\text{min}}^i\) indicates the minimum width of the cantilever achieved at \(i^{th}\) iteration.

- The term \(C_{b\text{max}}\) quantifies the severity of violation of the maximum width constraint.

\[
C_{b\text{max}} = \left[ \frac{\min(0, b_{\text{max}}^i - b_{\text{max}})}{b_{\text{max}}} \right]^2
\]

where, \(b_{\text{max}}^i\) defines the maximum width of the cantilever that is achieved at \(i^{th}\) iteration.

- The parameter \(f1\) lies between 0 and 1.

\[
C_f = \left[ \max(0, (f1 - 1))^2 + \min(0, f1) \right]^2
\]

- On parallel lines, the terms \(m\) and \(n\) should not be negative. This is accommodated in the term \(C_{mn}\), which is,

\[
C_{mn} = \left[ \min(0, n)^2 + \min(0, m) \right]^2
\]
The penalty $P_1 = 100$ is decided for the desecration of the area constraint. The penalty $P_2 = 10000$ is set on the violation on the width constraints. As a substitute of numerical solution of the partial derivatives of $u_{ps}$ with respect to design parameters $\alpha, f1, m, n$ and then using derivative based algorithms, it is suggested to apply the derivative free algorithms for optimization of the function values.

4. Travel range optimization using simulated annealing technique

For the present case, begining from an initial value of the design parameters $\alpha, f1, m$ and $n$, the Simulated Annealing algorithm changes the guess to decrease the value of function $G$ in every iteration. Simulated annealing (SA) is a random-search method which develops an analogy between the metal cooling and a minimum energy crystals (the annealing process); it formulates the foundation of an optimization method for combinatorial problems[14].

4.1. Procedure to solve optimization problem by SA

- Let the $f(X)$ is to be optimized for $Xi(l) <= Xi <= Xi(u)$
- Begin with some start point $X_0$ and calculate $f(X_0)$
- At the current state $i$, of the solid, with energy $E_i = f(X_0)$, next set $i+1$ is generated by perturbation process which converts the present state into a subsequent state by a small distortion. The energy of the next state is $E_{i+1}$.
- The probability of accepting new point $i+1$ is based on Metropolis criteria
- If $E_{i+1} - E_i <= 0$, $i+1$ is a current state but if $E_{i+1} - E_i > 0$, $i$ is accepted with the probability $P(\Delta f) = \exp(E_{i+1} - E_i)/K_bT$ here, $K_b$ is a Boltzman constant

$\Delta f = f(X_{i+1}) - f(X_i)$
- Repeat the above steps till temperature $T$ becomes very small.

5. Results and discussions

Table 1. Input data for static test cases of cantilever

| $b_{min}/L \downarrow b_{max}/L \rightarrow$ | 0.2 | 0.3 | 0.4 |
|------------------|-----|-----|-----|
| 0.02             | C1  | C2  | C3  |
| 0.04             | C4  | C5  | C6  |
| 0.06             | C7  | C8  | C9  |

We have used 9 test cases as shown in Table1 and number of iterations $N = 35000$ Balling cooling schedule. Start point for all the design variables are as follows: $m = 0, n = 1, \alpha = 1, f1 = 1$. These values refer to the rectangular geometry. Balling cooling schedule: According to Balling [15]

$$F = \frac{(\log P_s)}{(\frac{1}{(N-1)}) (\log P_f)}$$

where $P_s$ and $P_f$ is the probability of accepting worst case in the starting and at the end respectively. $P_s = 0.99999$, $P_f = 0.00001$. Using Balling cooling schedule, the values of the parameters are: $N = 35000,k = 1/(N - 1), F = (\log(Ps)/\log(Pf))^k$ $C = 1, M = 0$, $T = -1/(\log(Ps))$

Results obtained using Balling cooling schedule is shown in the Table 2.
Table 2. Optimized design variables using Simulated Annealing for cantilever

| Case No | m      | n      | alpha  | H      | G      | ups    |
|---------|--------|--------|--------|--------|--------|--------|
| 1       | 10.250298 | 4.236121 | 1.322676 | 0.19379 | 0.477144 | 0.523135 |
| 2       | 2.578837  | 1.331798 | 1.958812 | 0.617247 | 0.478117 | 0.525621 |
| 3       | 7.668786  | 1.757603 | 1.905888 | 0.289006 | 0.463205 | 0.536894 |
| 4       | 2.968883  | 3.4914  | 1.26476  | 0.407891 | 0.504876 | 0.499075 |
| 5       | 11.584822 | 1.827703 | 1.629923 | 0.144241 | 0.497128 | 0.504795 |
| 6       | 7.639875  | 1.776183 | 1.657084 | 0.212944 | 0.496958 | 0.505416 |
| 7       | 5.853314  | 3.061333 | 1.254242 | 0.174651 | 0.517039 | 0.484518 |
| 8       | 4.310797  | 3.593793 | 1.207178 | 0.224494 | 0.516431 | 0.483571 |
| 9       | 2.55539   | 1.645953 | 1.45439  | 0.397253 | 0.515542 | 0.485762 |

5.1. Comparison of Results of Simulated Annealing with Nelder Mead algorithm

The results obtained using Simulated Annealing are we have compared with the Nelder Mead algorithm for optimization[13] and [14]. The comparison % of travel range is mentioned in the Table 3.

Table 3. Comparison of travel range of SA with Nelder-Mead algorithm

| Case no | Nelder Mied | SA Balling |
|---------|-------------|------------|
| 1       | 52.72       | 52.3135    |
| 2       | 53.78       | 52.5621    |
| 3       | 54.06       | 53.6894    |
| 4       | 49.97       | 49.9075    |
| 5       | 50.47       | 50.4795    |
| 6       | 50.48       | 50.5416    |
| 7       | 48.46       | 48.4518    |
| 8       | 48.61       | 48.3571    |
| 9       | 48.61       | 48.5762    |

5.2. Optimized shapes of cantilever microbeam

Figure 3. optimized shape of the microcantilever case 3

Figure 4. optimized shape of the microcantilever case 7

After optimizing the pull-in parameters of cantilever microbeam, shapes obtained is shown in the figure 3 and figure 4. For the representation of the shape, case 3 and case 7 are considered and shape using two different cooling schedules are displayed.
6. Proposed design of the cantilever microbeam
Using the concept of variable width function [13], results indicate that the pull-in displacement can be further improved in the design of microactuator. We propose novel design for extending the travel range of microactuator that is extend the travel range by increasing the length of the actuator.

6.1. Extending the length of cantilever microbeam
Following the concept given by Senturia and Hung[16], pull-in displacement can be increased by increasing the length of the movable electrode. As shown in the figure 5 fixed electrode and movable electrode are offset by a distance $l_1$. Here the length of the movable electrode is $L + l_1$. The length of the beam is $L$ and we have provided an offset of $l_1 \mu$ between fixed and movable electrode. The governing differential equation of the cantilever beam is as follows.

$$\frac{1}{12} EI \frac{d^2}{d\hat{x}^2} \left( \hat{b}(\hat{x}) \frac{d^2 \hat{u}}{d\hat{x}^2} \right) = \frac{\hat{V}^2 \varepsilon A}{2(g_0 - \hat{u})^2} \quad (26)$$

The associated boundary conditions are specified as

$$u(x)|_{x=0} = u'(x)|_{x=0} = 0 \quad (27)$$

$$EIu''(x)|_{x=L} = EIu'''(x)|_{x=L} = 0 \quad (28)$$

![Figure 5. Cantilever beam with extended length](image)

When we increase the length by $l_1$ the deflection at point B in the non dimensional form is given by

$$\hat{u}_B = a \left( 6\hat{x}^2 - 4\hat{x}^3 + \hat{x}^4 \right) \quad (29)$$

Deflection at point C is given by

$$\hat{u}_C = \frac{ag_0}{(L + g_0)} \left( 12\hat{x} - 12\hat{x}^2 + 4\hat{x}^3 \right) l_1 + \hat{u}_B \quad (30)$$
We have to find out the value of constant $a$. It is done using gauss quadrature formulae and using the fibonacci algorithm. % change in the deflection is given by

$$\text{% change} = \frac{\hat{u}_c - \hat{u}_B}{\hat{u}_B} \times 100\%$$

(31)

We have tested the code using sample data as follows Length of the beam $L = 1000\mu m$, thickness of the cantilever is $th = 1\mu m$, initial gap between fixed and movable electrode is $g_0 = 2.75\mu m$ and Young’s modulus $E = 169 GPa$. Results are displayed in Table 4.

**Table 4.** Effect of increasing the length of the cantilever beam

| % increase in length l1 | Normalized displacement % |
|-------------------------|---------------------------|
| 0                       | 53.6894                   |
| 0.1                     | 53.901445                 |
| 0.2                     | 54.08114                  |
| 0.3                     | 54.261373                 |
| 0.4                     | 54.442145                 |
| 0.5                     | 54.623456                 |
| 0.6                     | 54.805308                 |
| 0.7                     | 54.987701                 |
| 0.8                     | 55.170635                 |
| 0.9                     | 55.354112                 |
| 1                       | 55.538132                 |

Effect of increment in the length of the cantilever microbeam on normalized displacement is presented in Table 4. It is clear from the Table 4 that as the length is increased the pull-in displacement also increases. We can conclude from this simulation that length increment can be used to maximize the travel range of an electrostatic actuator for a given gap size.

7. Conclusions
From the present study following conclusions can be drawn. Rayleigh Ritz method is successfully deployed to predict static pull-in parameters of variable width cantilever microbeam. Using Simulated Annealing optimization, cantilever microbeam geometry can be optimized to enhance static pull-in parameters. By increasing the length of movable electrode and offsetting it from fixed electrode, pull-in can be further delayed. Pull-in displacement is increased drastically by making passive change in the design. Maximum pull-in displacement was observed in case 3 which was 53.6894% for Simulated Annealing Optimization. Results of optimization using Simulated Annealing are in close proximity with those using Nelder-Mead algorithm. By extending the length of variable width cantilever microbeam by 1%, pull-in displacement can be enhanced upto 55.538132% which shows drastic improvement in the static pull-in displacement of the cantilever microbeam. Prismatic microbeam has static pull-in displacement as 45%[9] and the proposed design shows 55.538132%. It means static pull-in displacement can be enhanced by 23.4 % using proposed design.

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