A Sigma-Point Batch Filter for Spacecraft Orbit Estimation using the Geomagnetic Field Measurements

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1. Introduction

The low-accuracy orbit estimation problem using the geomagnetic field measurements has been studied by many researchers because of its advantages in providing a relatively simple and low-cost device. Magnetometers can also give continuous observations and be easily modeled using geomagnetic models such as the International Geomagnetic Reference Field (IGRF). These properties make the magnetometer-based spacecraft orbit estimation problem a secondary orbit estimation method for low-cost missions that usually use the two line element set from the North American Aerospace Defense Command (NORAD).

Most of the previous literature has focused on the verification of the filter’s performance and application to a specific orbital inclination; i.e., polar orbit. However, the tests were limited to the polar orbit and the along-track for an equatorial orbit. This phenomenon is mainly caused by the dipole property of the geomagnetic field. Additionally, the proposed unscented batch filter is compared with the Bayesian batch filter to investigate the filter’s performance. The comparison results show that the unscented batch filter has strength in convergence speed over the Bayesian filter. The achieved position accuracies are similar in the both filters; i.e., approximately 1–2 km depending on the magnetometer noise level. However, the unscented batch filter can estimate the orbit more rapidly than the Bayesian filter when nonlinearity is strengthened. It shows the unscented batch filter has strength beyond that of the Bayesian one in highly nonlinear situations.

Key Words: Sigma-Point Filtering, Orbit Determination, Geomagnetic Field
divergence. Efforts to overcome these linearization shortcomings are usually dedicated for sequential filters like the UKF based on UT, particle filters, and the exact nonlinear recursive filter. Specially, the UKF has received great attention in the nonlinear system estimation problem for a couple of reasons. It calculates a mean and covariance more accurately from carefully chosen samples or so-called “sigma points” considering the second order Taylor expansion of the true a posteriori probability distribution. In addition, it needs no Jacobian matrix calculation for the system and measurement functions during implementation. Due to these advantages, the UKF has been extensively investigated for its use for various dynamic systems including determining a spacecraft’s attitude, and the orbit estimation problem based on range and angular observations. Both studies indicated that the UBF has the same advantage as the UKF does. Namely, the UBF converges in fewer iterations than the traditional batch filters such as the least square and Bayesian filter in highly nonlinear cases.

Finally, the orbit estimation tests are conducted using the actual magnetometer data of CHAMP as well as the simulated one based on the truth model. The test using real-flight data make the proposed algorithm more reliable than the truth model simulation that assumes Gaussian distribution of measurement noise. Through the current study, the magnetometer-based orbit estimation strategies combined with the UBF can be an alternative low-acuracy satellite tracking method. The rest of this paper consists of four parts; i.e., the derivation of the UBF algorithm along with the introduction to the UT, the definition of the magnetometer-based orbit estimation problem, the presentation of the test results using the simulated and the real-flight data, and lastly the conclusion of the study is presented.

2. Filtering Scheme

In this section, the algorithm of the UBF is derived. Firstly, the algorithm of the batch filter based on the Bayesian rule is introduced for comparison. Then, the UBF algorithm is explained in the next section. There are three representative methods used for batch processing. The least square method estimates the parameters as the values that minimize the sum of the squares of the observation residuals. Therefore, the optimization theory is usually applied to find the solution. However, the least square method does not include any statistical information about measurements and state variables to be estimated. The minimum variance estimate is one method for removing this limitation. This method finds the linear, unbiased minimum variance estimate of the state vector when the state and measurement equation is given in a linear form. The maximum likelihood estimation is used to estimate the state that maximizes the likelihood function which is defined as the joint density function or conditional density function.

\[ p(x_0|y_0, y_1, \cdots y_N) = p(x_0|\tilde{y}) \]  \hspace{1cm} (1)

where, \( \tilde{y} \) means a batch measurement. The Bayesian estimate method; i.e., the Bayesian batch filter (BBF) is identical to the maximum likelihood estimate for a Gaussian density function. In this study, the BBF is used for verification and comparison of the proposed algorithm (i.e., the UBF).

2.1. Bayesian batch filter

The algorithms of the filters being explained here start with the definition of the state \( (x_i) \) and measurement \( (y_i) \) vectors, and their dynamic and measurement equations as follows:

For dynamic equation

\[ \dot{x}_i = f(x_i, t_k) + w_k \]  \hspace{1cm} (2)

For measurement equation

\[ y_k = h(x_k, t_k) + v_k \]  \hspace{1cm} (3)

where, \( w \) and \( v \) are the process and measurement noise vectors, which are assumed as zero-mean white Gaussian noise with covariance of \( Q_k \) and \( R_k \) respectively. The subscript \( k \) means the value at \( t_k \). The Bayesian batch filter estimates the state vector \( x_0 \) that maximizes the joint probability density function given all observations. The measurement equation for \( \tilde{y} \) can be defined as the column sum of Eq. (3), namely,

\[ \tilde{y} = \begin{bmatrix} y_0 \\ \vdots \\ y_N \end{bmatrix} = \begin{bmatrix} h(x_0) \\ \vdots \\ h(x_N) \end{bmatrix} + \begin{bmatrix} v_0 \\ \vdots \\ v_N \end{bmatrix} = \tilde{h}(x_0) + \tilde{v} \]  \hspace{1cm} (4)

In Eq. (4), the function \( \tilde{h} \) includes state propagation from \( t_0 \) to every epoch \( (t_k, k = 0, \ldots, N) \) using the dynamic model of Eq. (2) as well as the measurement model of Eq. (3). Here, the state vector to be estimated is assigned at the epoch \( t_0 \) just for convenience. However, it can be at any epoch between \( t_0 \) and \( t_N \).

The posteriori estimate of the BBF assumes that the system model is a linear system. Therefore, the linearized approximation for Eq. (4) with respect to the state error vector \( \Delta x_0 \) is needed as follows.

\[ \Delta \tilde{y} = \tilde{H} \Delta x_0 + \tilde{\nu} \]  \hspace{1cm} (5)

Equation (5) can be thought to include the linear approximations for the dynamic and measurement equations; i.e., Eqs. (2) and (3).

Resultantly, the best estimate for the state error vector \( \Delta \tilde{x}_0 \) and its covariance \( \tilde{P}_0 \) are calculated as follows:

\[ \Delta \tilde{x}_0 = \left( \tilde{H}^T \tilde{R} \tilde{H} + \tilde{P}_0^{-1} \right)^{-1} \left( \tilde{H}^T \tilde{R} \Delta \tilde{y} + \tilde{P}_0^{-1} \Delta \tilde{x}_0 \right) \]  \hspace{1cm} (6)

\[ \tilde{P}_0 = \left( \tilde{H}^T \tilde{R} \tilde{H} + \tilde{P}_0^{-1} \right)^{-1} \]  \hspace{1cm} (7)

where, \( \Delta \tilde{x}_0 \) and \( \tilde{P}_0 \) are the priori estimates of the state error and covariance respectively, and \( \tilde{R} \) is covariance matrix of the total observation noise vector, \( E(\tilde{v} \tilde{v}^T) \).

In most cases, the filter is iterated until the root mean square (RMS) of the observation residuals is less than a given tolerance, namely.
where, \( \varepsilon \) stands for tolerance and is generally set to \( 1 \times 10^{-3} \). The RMS is computed from
\[
\text{RMS} = \left\{ \frac{\mathbf{\Delta y}^T \mathbf{R}_i^{-1} \mathbf{\Delta y}}{n_y} \right\} \quad (9)
\]
where, \( n_y \) is the total number of measurements.

In summary, the Bayesian batch filter needs linear approximation in order to apply to a nonlinear system and this assumption has to be numerically implemented for the system and measurement models, Eqs. (2) and (3). This linear approximation and implementation are usually conducted using the Taylor’s expansion ignoring higher order terms.

A batch filter often yields accuracy superior to that of the EKF,\(^{10}\) making it generally used to find more precise estimates than that from a sequential filter. However, the resultant linear dynamic and measurement model are only valid for calculating the posteriori estimate. Thus, the algorithm of the UBF consists of three parts—the calculation of the sigma points, the prediction of measurements, and the update of the state and covariance.

Step I. Calculation of the sigma points

The UBF algorithm starts by selecting sigma points, which consist of the reference point and the set of points around the reference point. The sigma points are calculated by adding and subtracting the square root of the scaled covariance matrix to/from the reference state vector. Generally, the reference point is defined as augmented state vector including the state and process noise vector. However, there is no time-propagation of the state and process noise in the batch filter and the process noise is evidently additive form. Therefore, the non-augmented UT is used here and it reduces computational burden by sampling smaller sigma points. When the priori estimate of the state and covariance is \( (\bar{x}_0, \bar{P}_0) \) as in the Bayesian batch filter, the sigma points are defined as follows,\(^{14}\)

\[
\begin{align*}
\bar{x}_0 &= \bar{x}_0 \\
\bar{x}_i &= \bar{x}_0 + \left( \sqrt{(L+\lambda)} \bar{P}_0 \right)_i, \quad i = 1, \ldots, L \\
\bar{x}_i &= \bar{x}_0 - \left( \sqrt{(L+\lambda)} \bar{P}_0 \right)_i, \quad i = L + 1, \ldots, 2L
\end{align*}
\] (10)

where, \( \left( \sqrt{(L+\lambda)} \bar{P}_0 \right)_i \) denotes the \( i \)-th column of the matrix square root and \( \lambda \) is the dimension of the state vector. The scaling parameter \( \lambda = \alpha^2 (L + \kappa) - L \) determines the extent of the sigma points along with the first scaling parameter, \( \alpha \), which is usually set to a small positive value less than one. The second scaling parameter \( \kappa \) is generally set to \( 3 - L \) (see Ref. 9 for details). The performance of the unscented filter is dependent on the size of the selected sigma points; namely, the first scaling parameter. The effects of \( \alpha \) on the performance of the UBF are analyzed in the next section.

Step II. Prediction of measurements

The next step is the prediction of the measurement vector and covariance. For each sigma point, \( \bar{x}_i \), the whole measurements vector, \( \mathbf{\tilde{y}}_i \), is calculated using the measurement function of Eq. (4); i.e., \( \mathbf{\tilde{h}}(\bar{x}_i) \). The predicted measurement vector \( \mathbf{\tilde{y}}_i \) and covariance \( \mathbf{\tilde{P}}^{\tilde{y}}_i \) are calculated as follows,\(^{14,15}\)

\[
\begin{align*}
\mathbf{\tilde{y}}_i &= \sum_{i=0}^{2L} W_i^{(m)} \mathbf{\tilde{y}}_i \\
\mathbf{\tilde{P}}^{\tilde{y}}_i &= \sum_{i=0}^{2L} W_i^{(c)} (\mathbf{\tilde{y}}_i - \mathbf{\tilde{y}})(\mathbf{\tilde{y}}_i - \mathbf{\tilde{y}})^T + \mathbf{R}
\end{align*}
\] (12)

Note that the dimensions of \( \mathbf{\tilde{y}}_i \) and \( \mathbf{\tilde{P}}^{\tilde{y}}_i \) are \( (n_y \times 1) \) and \( (n_y \times n_y) \), respectively. The cross-correlation matrix \( \mathbf{P}^{\tilde{y}0} \) of \( \mathbf{x}_0 \) and \( \mathbf{y} \) is calculated as follows.

\[
\mathbf{P}^{\tilde{y}0} = \sum_{i=0}^{2L} W_i^{(c)} (\bar{x}_i - \mathbf{x}_0)(\mathbf{\tilde{y}}_i - \mathbf{\tilde{y}}) \] (13)

The weighing factors \( \left( W_i^{(m)}, W_i^{(c)} \right) \) for the measurement and covariance are calculated as\(^{15}\)

\[
W_i^{(m)} = \begin{cases} 
\frac{\lambda}{(L+\lambda)} & \text{when, } i = 0 \\
\frac{1}{2(L+\lambda)} & \text{when, } i = 1, \ldots, 2L
\end{cases}
\] (14)

\[
W_i^{(c)} = \begin{cases} 
\frac{\lambda}{(L+\lambda)} + 1 - \alpha^2 + \beta & \text{when, } i = 0 \\
\frac{1}{2(L+\lambda)} & \text{when, } i = 1, \ldots, 2L
\end{cases}
\] (15)

where, the third scaling parameter \( \beta \) in Eq. (15) plays a role in incorporating prior knowledge of the distribution of \( \mathbf{x} \), while \( \beta = 2 \) is known to be optimal for a Gaussian distribution.\(^{12}\)

Step III. Posteriori estimate of the state and covariance

The final step of the proposed UBF is the update of priori estimate of the state and covariance using the standard Kalman filter equations as the UKF does. The posteriori estimate of the state \( \mathbf{\tilde{x}}_i \) and its covariance \( \mathbf{\tilde{P}}_i \) and the gain matrix \( \mathbf{K} \) are calculated as follows.

\[
\begin{align*}
\mathbf{\tilde{x}}_i &= \mathbf{\tilde{x}}_0 + \mathbf{K} (\mathbf{\tilde{y}} - \mathbf{\tilde{y}}) \\
\mathbf{\tilde{P}}_i &= \mathbf{\tilde{P}}_0 - \mathbf{K} \mathbf{\tilde{P}}_0^{\tilde{y}} \mathbf{K}^T \\
\mathbf{K} &= \mathbf{\tilde{P}}_0^{\tilde{y}} \left( \mathbf{\tilde{P}}^{\tilde{y}}_i \right)^{-1}
\end{align*}
\] (18)
The dimensions of $\mathbf{P}_0^{ij}$ and $\mathbf{K}$ in Eqs. (13) to (18) are $n_x \times n_y$, where $n_x$ is the dimension of the state vector. Therefore, in the UBF the most time-consuming part is the calculation of the gain matrix, because it includes the inverse of a large dimension matrix; i.e., $\mathbf{P}_0^{ij}$.

The above three steps in the UBF are iterated until the RMS converges under the specified tolerance as the Bayesian batch filter does. The advantage of the UBF is that it is relatively simple to implement mainly due to eliminating the linearization process. One thing that should be noted is that the covariance value may not be updated for the first few iterations in order to speed up convergence, especially when a large initial error is given.

3. Orbit and Measurement Model

3.1. Orbit dynamic model

In the orbit estimation problem, the accuracy of the equation of motion should be modeled accounting for the fidelity of the measurement model and the accuracy of the observation. Over the last few decades, with help of the success of the geomagnetic field exploration mission, the accuracy of the IGRF model has improved significantly from a standard error of several hundreds of nT in the 1960s to a few tens of nT in the 1980s. Currently, the latest version, the IGRF-11 starts to provide the model with the sub nT precision. Roh et al.\(^1\) recommended that the best dynamic model should include the geopotential up to degree and order of $4 \times 4$ and the atmospheric drag perturbation and estimate the ballistic coefficient along with orbital position. As suggested by Roh et al.\(^1\) the state vector $(\mathbf{x})$ is set to $[\mathbf{r}^T, \mathbf{v}^T, \mathbf{B}^T]^T$; i.e., position and velocity in the inertial frame and the inverse of the ballistic coefficient in this study. The inverse of the ballistic coefficient is defined by multiplication of the drag coefficient and the area-to-mass ratio. Thus, the dynamic equation corresponds to Eq. (2) is defined as follows.

$$
\mathbf{\dot{x}} = \begin{bmatrix} \mathbf{\dot{r}} \\ \mathbf{\dot{v}} \\ \mathbf{\dot{B}} \end{bmatrix} = \begin{bmatrix} \mathbf{v} \\ \mathbf{a}_{\text{geo}}(\mathbf{r}) + \mathbf{a}_{\text{drag}}(\mathbf{r}, \mathbf{v}, \mathbf{B}^*) \end{bmatrix} \quad (19)
$$

For the geopotential perturbation, the JGM-3 gravity model is applied and the Harris-Priester model\(^18\) is used for the atmospheric density calculation. In Eq. (19), the orbital motion is described in the J2000 inertia frame based on the referenced time system of Terrestrial Time (TT). Since the geopotential acceleration is a function of the position vector in the International Terrestrial Reference Frame (ITRF), the coordinate transformation between the ITRF and J2000 is performed using SOFA library\(^19\) and the Earth Rotation Parameters (ERP) from the International Earth Rotation and Reference Systems Service.\(^20\) The reason for using the high-accuracy ERP model is to remove the errors caused by the outside of the geomagnetic observation as much as possible. The same coordinate transformation applies the IGRF geomagnetic field model.

3.2. Measurement model

The scalar magnitude of the geomagnetic field measure-
ments is used as the measurement model. Therefore, the magnetometer-based orbit estimation can be performed without the spacecraft attitude information. The geomagnetic field is modeled using the IGRF model in this study. The IGRF model calculates the geomagnetic field vector of the down, east and north components at a given location and time. Therefore, the resultant measurement model has the following form.

$$
y_k = h(\mathbf{x}_k, t) = \sqrt{(B_{\text{down}}^2 + B_{\text{east}}^2 + B_{\text{north}}^2)} \quad (20)
$$

As the IGRF model is a function of the position vector in ITRF and the time in UTC, the coordinate and time transformation used in the geopotential calculation is also applied here.

The latest released IGRF model is IGRF-11 that consists of the Definitive Geomagnetic Reference Field (DGRF) coefficient sets for the epoch up to 2005, the provisional coefficient for epoch 2010 and their average secular variation for the extrapolation from 2010 to 2015. These coefficients are updated every five years. In order to take into account the time gap between the geomagnetic model and the actual observation, the DGRF model is used to simulate observation for the truth model test. The orbit estimation is performed using the IGRF. The test using the real-flight data of the CHAMP also uses the IGRF coefficient in the measurement equation. In the measurement model, we consider only the white Gaussian noises. There could be biases in the measurements due to the sensor itself or a spacecraft’s electric system. These biases are usually removed during the preprocess step and also can be estimated by setting one as the state vector.\(^1\) In this study, these system biases are assumed to be preprocessed because the magnetometer data of the CHAMP tested in this study are generally distributed after preprocessing their biases.

4. Numerical Tests

The geomagnetic field’s distribution roughly resembles a bar magnet aligned with the line connecting the north and south magnetic poles. Thus, the simplest geomagnetic field is modeled as a dipole; i.e., the strongest around the poles and the weakest near the equator. It means that orbital inclination is the dominant factor to determine the geomagnetic field’s distribution. In this section, firstly, the orbit estimation is performed using the simulated observations based on the truth model. In order to ensure the geomagnetic field based orbit estimation as a practical method, the orbit estimation results are analyzed for different orbital inclinations. The proper measurement duration is also investigated using various numerical tests. Secondly, the real-flight data of the CHAMP is used for the evaluation of the simulated tests. Lastly, the performance of the two batch filters (i.e., the UBF and the Bayesian filter) are also compared under different levels of nonlinearity.

4.1. Truth model simulation

In this subsection, the orbit estimation processes are conducted using the simulated observations. The truth orbit is
generated using the Satellite Tool Kit’s High Precision Orbit Propagator (HPOP) \( ^{21} \) with an altitude of 600 km and a different inclination from 0° to 80° with 20° interval. For each orbit, the observation data are simulated using the DGRF coefficients with predefined noise level. The epoch of this simulation is set to 2007-Jul-1 12:00 h arbitrary, so the geomagnetic field coefficients are calculated as a linear interpolation using DGRF-2005 and 2010. In the HPOP Module, the Gauss-Jackson method for the numerical integration of the equations of motion, the EGM96 model (up to a degree and order of 70-by-70) for perturbation due to nonsymmetric geopotential, Jacchia79 model for atmospheric density, and the DE403 JPL coefficient for the lunar/solar ephemeris are used. Perturbations due to atmospheric drag, lunar/solar gravitational attraction, and solar radiation pressure are included. The coefficients of atmospheric drag (\( C_d \)) and the solar radiation pressure (\( C_s \)) are set to 2.0 and 1.0 respectively, and the constant area-to-mass ratio set to 0.02. The measurement noises are applied to each component of the magnetometer measurement as white Gaussian distribution with a standard deviation of 20 nT.

4.1.1 Finding the proper measurement duration

Firstly, the orbit estimation is conducted for different measurement durations from 100 to 1,400 min to find the proper measurement duration for a stable orbit estimation process. Because the orbital altitude is set to 600 km, the measurement duration of 100 min corresponds roughly to one orbital period. In the tests, the measurement interval is set to 60 s and the initial orbit position error of 10 km is applied to each axis. The initial covariance for position, velocity and \( B^o \) are set to \((1 \text{ km})^2\), \((1 \text{ m/s})^2\) and \((0.001)^2\) respectively. The orbit estimation results are plotted in Fig. 1 altogether for the different measurement durations from 100 to 1,400 min and for the different inclinations from 0° to 80°. The mean measurement residual is chosen for the y-axis of Fig. 1 instead of the position RMS, because the position accuracy is dependent on the orbital inclination but the measurement residual is not. Successfully estimated orbit would give the 20 nT of the measurement residual that is the predefined value for the measurement simulation. As shown in Fig. 1, approximately 1,000 min of the measurement duration give the best measurement residuals for all the inclination cases. In some cases in Fig. 1, more than 1,000 measurements give worse residuals. This phenomenon is mainly caused by the fact that the batch-based orbit estimation using the geomagnetic field measurements can be thought of as curve fitting with dense data with 1 min intervals. Additionally, the orbit dynamic model in the filtering scheme is quite a simplified one compared with that of the truth model which is for the measurements generation. These two facts can possibly cause worse RMS results when too many observations are used rather than a properly chosen number of observations. Resultantly, the most probable number of orbits can be concluded to be approximately 10 regardless of orbital inclination. Based on the results the following tests are performed with 1,000 min of measurement duration.

4.1.2 Effect of an orbital inclination and comparison with the Bayesian batch filter

The numerical tests are also performed for different orbital inclinations in order to investigate how the geomagnetic field’s distribution affects the achieved orbital accuracy as well as the possibility of the orbit estimation itself. In this test, the measurement duration is set to 1,000 min with 60 s intervals. The initial position error and its covariance are set as the same as those of the previous one. The simulation results are summarized in Table 1 along with the results from the BBF. In this test, the first scaling parameter (\( \alpha \)) is set to 1.0, because the effect of \( \alpha \) that determines the extent of the sigma points is not noticeable in all inclination cases. In both the UBF and the BBF, the measurement residuals show that all cases converge to approximately 20 nT. However, the achieved position accuracy is dependent on the average rate of change of the measurements. The cases with higher inclinations (i.e., 40°, 60° and 80°) that have larger average rate of change of the geomagnetic field are able to estimate the orbit more accurately than the lower cases (i.e., 0° and 20°). Incidentally, the UBF is advantageous over the BBF in the number of iterations, though the computation time is not a critical concern in the batch process. Except in the 60° inclination case, the UBF converges with less iterations. A more important property of the UBF over the BBF can be seen in Fig. 2 that plots the histories of position errors and iterations for both the filters in all inclination cases. After the first iteration, the achieved position errors from the UBF are more accurate than the BBF in all cases. It clearly shows that the unscented transform has an advantage in a nonlinear system over the traditional batch filter based on linear assumption.

The decomposed position errors in the radial, along and cross-track directions are depicted in Fig. 3. For all inclinations, the radial and cross-track errors have a similar pattern with better accuracy in higher inclination. However, the along track error in lower inclinations is relatively poorer than the higher cases. This phenomenon is mainly caused by the difference of the gradient of the geomagnetic field between inclinations. In the zero inclination case, the mean
gradient of the geomagnetic field in the along and cross-track direction (1.1 and 8.9 nT) is opposite to the 80° inclination case (5.6 and 1.3 nT). Namely, the lower along-track gradient in the equatorial orbit causes poor position accuracy in the corresponding direction. In the same way, the lower cross-track gradient in the polar orbit is the reason of the larger cross-track error than for the other directions. From these results, the analysis given by Psiaki et al. is only valid for a polar orbit not an equatorial one.

4.2. Real-flight data of CHAMP

The batch algorithm proposed in the current study is also tested using the real-flight data of the CHAMP that is a dedicated mission to research the geomagnetic field. For this main goal, the CHAMP installed the magnetometer at the end of a boom outside of the spacecraft main bus in order to remove noise from the electric system of the spacecraft. The measurement files used in this test are the level-2 vector magnetometer data. The difference between the measurements of the CHAMP and the IGRF model for the tested dates (five days from 1 Jan. 2010) is plotted in Fig. 4 and it shows there is no explicit bias. The mean and the standard deviation of the measurement residual are 1.29 and 13.73 nT, respectively.

Table 1. Position error and measurement residual for various inclination cases.

| Incl. (°) | UBF Iteration # | Position error (km, rms) | Measurement residual (nT) | BBF Iteration # | Position error (km, rms) | Measurement residual (nT) | Average rate of change of measurement (nT/s) |
|----------|-----------------|--------------------------|---------------------------|-----------------|--------------------------|---------------------------|-------------------------------------------|
| 80       | 5               | 0.55                     | 20.40                     | 6               | 0.59                     | 20.40                     | 75.41                                     |
| 60       | 6               | 0.41                     | 20.32                     | 5               | 0.64                     | 20.30                     | 68.09                                     |
| 40       | 6               | 0.51                     | 20.16                     | 7               | 0.45                     | 20.15                     | 68.09                                     |
| 20       | 4               | 1.72                     | 20.22                     | 8               | 1.79                     | 20.10                     | 34.65                                     |
| 0        | 3               | 2.53                     | 20.43                     | 6               | 2.31                     | 20.43                     | 30.00                                     |

Fig. 2. Convergence histories of the position error along with iterations.

Fig. 3. Decomposed position error in radial, along and cross track directions (solid: radial; dotted: along; dashed: cross).
altitude and inclination of the CHAMP at the tested dates are approximately 300 km and 87°/C14, respectively.

The orbit estimation of the CHAMP is conducted under the same condition with that of the truth model test. The initial position error is set to 10 km for each axis and the scale factor, $\alpha$, is set to 1.0 as in the truth model test. Additionally, the different measurement intervals from 1 to 4 min are used in the orbit estimation to compare the UBF and the BBF under different levels of nonlinearity. Namely, lengthening the measurement interval can be thought of as strengthening the nonlinearity of the system. Since the total measurement span is fixed to 1,000 min, the number of measurement is decreased from 1,000 to 500, 333 and 250 as the interval increases from 1 to 4 min. The estimated orbit results from both filters; i.e., the UBF and the BBF are compared with the Rapid Science Orbit of the CHAMP that is known to have approximately 4–5 cm accuracy.22) In Table 2, the achieved position and measurement RMS for the tested days are summarized. The achieved position RMS is highly dependent on the measurement RMS. For example, the best position RMS (0.86 km) is obtained from the minimum measurement RMS (8.76 nT) at 1 Jan. 2010. On the contrary, the worst case (2.06 km) is from the maximum (17.90 nT) measurement RMS at 2 Jan. 2010.

Though there is a small difference in the measurement RMS within the same day depending on the measurement interval, this difference is acceptable considering that the real data has many outliers and is not a perfect Gaussian distribution.

For the best and worst measurement RMS cases (i.e., 1–2 Jan. 2010) the radial, along and cross-track position errors are plotted in Fig. 5 and it shows that the error increases mainly in the cross-track direction. This phenomenon is already indicated at the truth model test. Namely, the cross-track error is dominant in a polar orbit and the along-track error in an equatorial one, and these phenomena come from the geomagnetic field’s dipole property.

Comparison results with the BBF indicate that the most important advantage of the UBF over the BBF is the number of iterations and convergence speed. In terms of the final achieved accuracy, the both filters converge to the similar measurement RMS and resultantly to the similar position

![Measurement error between the CHAMP measurement and the IGRF model.](image.png)

**Table 2. Position and measurement RMS and the number of iterations for the CHAMP.**

| $\Delta t$ (min) | UBF | BBF |
|------------------|-----|------|
|                  | # of iteration | Measurement RMS (nT) | Position RMS (km) | # of iteration | Measurement RMS (nT) | Position RMS (km) |
| 1 Jan. 2010      | 1 [1,000]      | 6               | 8.76             | 0.87           | 7               | 8.74             | 0.93           |
|                   | 2 [500]        | 6               | 8.65             | 0.89           | 7               | 8.66             | 0.87           |
|                   | 3 [333]        | 6               | 8.46             | 0.88           | 8               | 8.51             | 0.85           |
|                   | 4 [250]        | 6               | 8.59             | 1.10           | 9               | 8.71             | 0.99           |
| 2 Jan. 2010      | 1 [1,000]      | 6               | 17.90            | 2.06           | 7               | 17.91            | 2.04           |
|                   | 2 [500]        | 6               | 18.17            | 2.00           | 8               | 18.27            | 2.09           |
|                   | 3 [333]        | 6               | 18.09            | 1.78           | 8               | 18.29            | 2.03           |
|                   | 4 [250]        | 6               | 18.41            | 2.16           | 8               | 18.81            | 2.52           |
| 3 Jan. 2010      | 1 [1,000]      | 5               | 15.96            | 1.20           | 5               | 16.00            | 1.24           |
|                   | 2 [500]        | 5               | 15.78            | 1.20           | 5               | 15.92            | 1.39           |
|                   | 3 [333]        | 5               | 16.28            | 1.40           | 6               | 16.63            | 1.78           |
|                   | 4 [250]        | 5               | 16.28            | 1.26           | 7               | 17.04            | 2.09           |
| 4 Jan. 2010      | 1 [1,000]      | 4               | 11.34            | 1.06           | 4               | 11.36            | 1.15           |
|                   | 2 [500]        | 5               | 11.28            | 1.08           | 5               | 11.34            | 1.22           |
|                   | 3 [333]        | 5               | 11.02            | 1.35           | 6               | 11.16            | 1.61           |
|                   | 4 [250]        | 5               | 12.01            | 1.26           | 7               | 12.33            | 1.80           |
| 5 Jan. 2010      | 1 [1,000]      | 6               | 14.44            | 1.92           | 6               | 14.42            | 1.86           |
|                   | 2 [500]        | 6               | 14.07            | 1.89           | 7               | 14.10            | 1.83           |
|                   | 3 [333]        | 6               | 9.07             | 2.13           | 8               | 15.16            | 2.15           |
|                   | 4 [250]        | 6               | 11.85            | 1.38           | 11              | 12.03            | 1.44           |
RMS for all the tested cases. However, the iteration number of the UBF is not increased for all cases except one date—4 Jan. 2010. However, the BBF needs more iteration as the measurement interval increases in all cases. It shows that the UBF is more robust than the BBF as the nonlinearity is enhanced. The convergence histories of the position RMS in Fig. 6 (black-UBF/gray-BBF) also show that the advantage of the UBF is evident at the first iteration as indicated in the truth model test.

5. Conclusion

In this paper, a batch orbit estimation problem based on the geomagnetic field measurement was extensively investigated as a secondary operational tool for a low-cost mission. Batch filters based on the unscented transformation and the Bayesian filter were used as an estimation method and their results were compared and analyzed under various levels of nonlinearity. The correlation between the geomagnetic field’s distribution along the orbit and their achieved position accuracy was investigated through the truth-model simulation. The real-flight data of the CHAMP was also tested to evaluation.

Through the numerical tests, approximately 10 orbital periods was found as the most promising measurement duration. The most important contribution of this study was that the decomposed position errors into radial, along and cross-track directions in the magnetometer-based orbit estimation problem had different characteristics depending on the orbital inclination; i.e., the geomagnetic field distribution. The previous study only indicated that there is the relatively large cross-track component error in polar orbit estimated using the geomagnetic field measurement. In the current study, the reason of this unknown cross-track error was found to be caused by the dipole property of the Earth’s magnetic field. Namely, the mean gradient of the geomagnetic field was stronger in the latitudinal direction than in the longitudinal one. Thus, in lower inclination orbit, the along-track position error was larger than the cross-track component, and in polar orbit the opposition result was achieved.

The performance comparison between the UBF and the BBF was also analyzed as the achieved position RMS was almost the same in both the UBF and the Bayesian filter. However, in most cases the UBF found the best estimate with less iterations than the BBF. In addition, after the first iteration the position RMS calculated from the UBF was more accurate than that of the BBF. Specially, the test using the real-flight data showed that the orbit can be estimated robustly in the UBF cases even when nonlinearity is strengthened through increasing the measurement interval from 1 to 4 min. Unlike the UBF, the BBF needs more iterations to find the solution as the measurement interval is increased. Incidentally, the achieved position RMS for five days from 1 Jan. 2010 ranged from 1 to 2 km depending on the measurement quality. The orbit estimation strategy proposed in this paper was intensively evaluated through the numerical and the real measurement data, and can be thought of as a backup or secondary orbit estimation method.

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