Uncorrelated velocity and size residuals across galaxy rotation curves

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1 INTRODUCTION

Dynamically, to first order, a galaxy is described by a mass, \( M \), a size, \( R \), and a characteristic velocity, \( V \). Understanding the relations between these properties as a function of cosmic time is a major goal of galaxy astrophysics, with ramifications not only for the connection between galaxies of various types and their host dark matter haloes, but also for the processes that drive galaxy formation and evolution.

The \( M–R–V \) relation of early-type galaxies forms a Fundamental Plane (FP; Djorgovski & Davis 1987; Dressler et al. 1987), implying a single constraint between these variables. In contrast, late-type galaxies follow two separate relations, the Tully–Fisher relation (TFR) \( M–V \) (Tully & Fisher 1977) and mass–size relation (MSR). These are decoupled, in that their residuals \( \Delta V = \log (V) - \langle \log (V) \rangle \langle \log (M) \rangle \) and \( \Delta R = \log (R) - \langle \log (R) \rangle \langle \log (M) \rangle \) are uncorrelated (McGaugh 2005; Pizagno et al. 2007; Reyes et al. 2011; Lelli, McGaugh & Schombert 2016a). The FP may be understood as arising from virial equilibrium [for suitable choices of stellar initial mass function (IMF), dark matter fraction, and radial orbit anisotropy; Dutton et al. 2013; Desmond & Wechsler 2017], the independence of the TFR and MSR has been used to argue for an additional constraint between the properties of late-type galaxies that reduces the dimensionality of the effective parameter space (e.g. Famaey & McGaugh 2012).

It may be surprising that \( \Delta V \) and \( \Delta R \) are independent because a larger galaxy at fixed mass has a less concentrated baryonic mass profile and should therefore rotate more slowly by Kepler’s laws. The baryon-only prediction \( \Delta V \propto -0.5 \Delta R \) is amply ruled out by the data (e.g. McGaugh 2005). However, this neglects the effect of both the shape of the halo velocity profile (which larger galaxies sample at larger radii) and the dependence of the galaxy–halo connection on galaxy size. These effects have been modelled in different ways generating disagreement among literature studies, whose conclusions range from assertions of incompatibility between the observations and predictions of standard galaxy formation models (e.g. McGaugh 2015; Lelli et al. 2016a) to assertions of complete compatibility (e.g. Courteau & Rix 1998; Dutton et al. 2007). If the independence of the TFR and the MSR is not due to an additional constraint, then it must arise ‘by chance’ from the interrelation of the density profiles of baryonic and dark matter: our work explores how this may come about.

**Key words:** galaxies: formation – galaxies: fundamental parameters – galaxies: haloes – galaxies: kinematics and dynamics – galaxies: statistics – dark matter.
There is confusion in the literature for three reasons.

(i) Summarizing the rotation curve (RC) of a galaxy by a single velocity, $V$, introduces a degree of arbitrariness since measuring $V$ at different radii may be expected to yield different results. Thus, while standard galaxy formation naturally predicts negligible $\Delta V - \Delta R$ correlation where RCs plateau far beyond where most of the baryonic mass resides (Desmond 2017b), it may fail to do so if measured within the stellar disc (Desmond & Wechsler 2015, hereafter DW15). Different Tully–Fisher studies tend to use various definitions of radii to measure $V$ (see Yegorova & Salucci 2007 for a comparison of different choices). The relation between these different definitions of radii to measure $V$ is well known from abundance matching (AM) studies, as well as more recent results from hydrodynamical simulations of galaxy formation (Desmond 2017a,b and here). Given the specific angular momenta of galaxies and haloes are proportional to their mass and halo properties at fixed $M_*$, it is common that high-quality photometry is available. In particular, the baryonic mass resides (Desmond 2017b), $\Delta V - \Delta R$ correlation is easily measured at small radii at which the mock and real observations are not made in the same way. Populating more massive or more concentrated haloes with larger galaxies at fixed baryonic mass will clearly induce a positive $\Delta V - \Delta R$ correlation. Most authors impose no correlation between $R$ and halo properties at fixed $M_*$ (e.g. Dutton et al. 2011, 2013; Di Cintio & Lelli 2016), a strong assumption that neglects any potential correlation between the galaxy and halo angular momentum. Other authors impose an anticorrelation between $R$ and $c$ by assuming that the specific angular momenta of galaxies and haloes are proportional (e.g. Mo, Mao & White 1998; DW15). One can also use results from hydrodynamical simulations of galaxy formation (Desmond et al. 2017c), apply prescriptions for converting gas to stars as a function of baryonic surface density (e.g. Dutton et al. 2007), or employ tuneable toy models (Desmond 2017a,b and here). Given the sensitivity of the $\Delta V - \Delta R$ relation to correlation between $R$ and halo properties, it is not surprising that these studies reach apparently contradictory conclusions.

(ii) Velocity and size residuals are sensitive to the baryonic mass distributions of galaxies and the radii at which the RCs are sampled. It is difficult to ascertain the bias introduced by comparing mock galaxies to observed ones with different mass profiles, or in cases where the mock and real observations are not made in the same way. While the use of simplistic functional forms for mass components is common, baryonic density profiles may be matched exactly between real and mock galaxies where high-quality photometry is available.

We construct a semi-empirical Λ cold dark matter (ΛCDM) model for galaxies in the Spitzer Photometry and Accurate Rotation Curves (SPARC; Lelli, McGaugh & Schombert 2016b) sample by adapting and expanding previous work in Desmond (2017b). Our particular interest is in comparing predicted and observed $\Delta V - \Delta R$ correlations when $V$ is measured at a range of radii across the RCs of galaxies. Besides clarifying the relative importance of various factors in setting the agreement of data and theory, we will show that this information brings new constraining power to the dependence of galaxy size on halo properties and the inner density profiles of haloes. We identify models in approximate agreement with the measured $\Delta V - \Delta R$ relation for all velocity choices, and hence show how the decoupling of the TFR and MSR may come about in ΛCDM.

2 METHODS

Our method is based on that of Desmond (2017b). We begin with a sample of 153 galaxies from the SPARC data base (Lelli et al. 2016b), requiring inclination $i \geq 30^\circ$ and quality flag $Q < 3$. We use the stellar mass of each galaxy (assuming a mass-to-light ratio of 0.5 at $3.6 \mu m$ for the disc and 0.7 for the bulge; Lelli et al. 2016a) to assign a halo from the DARKSKY-400 simulation (Skillman et al. 2014) by the AM prescription of Lehmann et al. (2017). We then combine the halo parameters output by ROCKSTAR (Behroozi, Wechsler & Wu 2013) with the measured baryon profile to create a model RC for each SPARC galaxy. We sample this RC at the same radii as the real data, and include observational uncertainty by scattering the model velocities by the quoted SPARC uncertainties. As the model is probabilistic, we adopt a Monte Carlo approach to error propagation and sample variance by analysing 200 independent realizations.

We also investigate the result of using the halo profile fits to the RCs from Katz et al. (2017). In that work, both a Navarro–Frenk–White (NFW) profile, derived from $N$-body simulations, and a DC14 profile (Di Cintio et al. 2014), derived from hydrodynamical simulations, were fit to the SPARC RCs using a Markov chain Monte Carlo (MCMC) algorithm to map out the posterior distributions of halo parameters. It was found that the DC14 profile, which can be cuspy or cored at the centre depending on $M_*/M_{\text{halo}}$, could better reproduce the RCs while satisfying the stellar mass–halo mass relation from AM and the mass–concentration relation from dark matter-only simulations.

For a given radius $r$ at which $V$ is measured (see below), we calculate the velocity and radius residuals as

$$\Delta V_r \equiv \log(V(r)) - \langle \log(V(r)) \rangle \log(M_b),$$

$$\Delta R_r \equiv \log(R_{\text{eff}}) - \langle \log(R_{\text{eff}}) \rangle \log(M_b),$$

where $M_b$ is total (cold) baryonic mass, $\log(M_b) + 1.33 \log(R_{\text{eff}})$ is 3.6 $\mu m$ half-light radius, and angular brackets denote the expectation of a third-order fit to the TFR or MSR in log space, fitting also for intrinsic scatter. The subscript $r$ highlights the dependence of $V$ on the radius at which it is measured. We then calculate $\rho_{\text{rel}}(r)$ as the Spearman’s rank coefficient of the $\Delta V_r - \Delta R_r$ correlation, and repeat the analysis for each model realization.

To measure the strength of the $\Delta V_r - \Delta R_r$ correlation, we calculate $V$ at either $r = xR_{\text{eff}}$ or $r = xR_p$, where $x$ is a universal variable in the range $[0.2, 6]$ and $R_p$ is the radius at which the baryonic component of the RC peaks (McGaugh 2005). As the median $R_{\text{eff}}$ across the sample is 3.1 kpc and the median $R_p$ is 5.4 kpc, this probes the RCs in the range $\sim 0.6$–$30$ kpc. We remove galaxies with RCs that do not extend to $xR_{\text{eff}}$ or $xR_p$, which is $\sim 50$ per cent of the sample for $x < 0.4$ and $\sim 70$ per cent for $x > 5$. The use of $R_{\text{eff}}$ versus $R_p$ corresponds to different relative $r$ values between the galaxies at fixed $x$, according to their baryon mass distributions. In each case we calculate the median $\rho_{\text{rel}}(r)$ values across the model realizations and the standard deviation among them.

We compare the data to a series of models of increasing complexity. In our fiducial model, the halo profile is unaffected by galaxy formation, AM is described by the best-fitting parameters of Lehmann et al. (2017) ($\sigma_{\text{AM}} = 0.6$ and $\sigma_{\text{AM}} = 0.16$ dex) and galaxy size is uncorrelated with halo properties at fixed $M_b$. By varying these assumptions we exhibit the sensitivity of the $\rho_{\text{rel}}(r)$ relation to them; we will show in particular that this relation brings important

\[\text{http://astroweb.cwru.edu/SPARC/}\]
new constraining power to the relation between galaxy size and halo mass or concentration.

3 RESULTS

We begin with $V(xR_{\text{eff}})$. As the dashed black line of Fig. 1(a), we show $\rho_\text{bg}(r)$ for the SPARC galaxies as $x$ is varied. We find the $|\rho_\text{bg}|$ values to be fairly small for all $x$, indicating that $\Delta V$ and $\Delta R$ are always insignificantly correlated, although there is a moderate anticorrelation for $r \ll R_{\text{eff}}$. The prediction of the fiducial model is depicted as the solid magenta line, which also shows a statistically insignificant correlation but with a $\rho_\text{bg}$ value somewhat larger than in the data. The dependence of $\rho_\text{bg}$ on $r$ in the model is set by a combination of two factors.

(i) As $r$ decreases, the baryonic contribution to $V$ increases and hence the more $V$ is reduced when the galaxy is made larger at fixed $M_h$. This induces a negative $\rho_\text{bg}$. This is the dominant effect for $r \lesssim R_{\text{eff}} (x \lesssim 1)$ where the greater part of $V$ is set by the baryons. At very small radii ($r \lesssim 0.2R_{\text{eff}}$) $\rho_\text{bg}$ tends to the baryon-only result, which we show below to be $\sim -0.6$.

(ii) Since the radius at which the velocity is measured scales with the size of a galaxy, smaller galaxies sample the halo RC at smaller radius. If halo properties do not depend on $R_{\text{eff}}$ at fixed $M_h$ – as is the case in our fiducial model – this simply reduces $V$ if the halo RC is rising, inducing a positive $\rho_\text{bg}$. This is the dominant effect at large $x$ where the halo RC is still rising but the baryonic contribution to $V$ is small. At very large $x$ (beyond the halo scale radius) the halo RC starts to decrease, and hence $\rho_\text{bg}$ does also. This is difficult to trace out because few SPARC galaxies have velocity measurements at such large galactocentric radii, although there is a slight indication of it in the model for $x \gtrsim 5$.

The other lines in Fig. 1(a) show the result of introducing a correlation between the residuals of galaxy size and halo concentration, of the form $\Delta c = m_\Delta R_{\text{eff}}$ with 0.1 dex scatter in $c$ at fixed $M_h$ (cf. equation 1; Desmond 2017a,b). For $m < 0$ this puts larger galaxies in less concentrated haloes at fixed $M_h$, causing a reduction in $\Delta V$ as $\Delta R$ is increased. Overall agreement with the data is maximized for $-0.8 \lesssim m \lesssim -0.4$, which agrees well with the constraint of Desmond (2017a), where $m$ was inferred from the correlation of residuals of the mass discrepancy–acceleration (or radial acceleration) relation with galaxy size, and of Desmond (2017b) which examined the $\Delta V$–$\Delta R$ correlation with $V$ measured at $R_{\text{eff}}$. Although $m < 0$ could be inferred using any radius choice within the range we consider, the full $\Delta V$–$\Delta R$ relation demonstrates that a moderate anticorrelation of $\Delta R_{\text{eff}}$ with $\Delta c$ improves agreement with the observations regardless of the choice of radius. Note also that halo mass could have been used rather than concentration: the relevant quantity is the dark matter mass within $r$, which is a function of both. We use concentration because it has a larger scatter than $M_{\text{vir}}$ at fixed $M_h$ due to a weaker correlation with the AM proxy.

Finally, the purple star in Fig. 1(a) shows the result of setting galaxy and halo specific angular momentum proportional to one another (Mo et al. 1998), specifically from the model of DW15 who find $(\rho_\text{bg}) = -0.49$ when $V$ is measured at $R_{\text{eff}} \approx 1.8R_{\text{eff}}$. This model is strongly disfavoured by the data; we discuss this result further in Section 4.

In Fig. 1(b), we show the results of varying other model parameters around the $m = -0.4$ line of Fig. 1(a), including the AM proxy $\sigma_{\text{AM}}$ and scatter $\sigma_{\text{AM}}$ and the response of the halo to galaxy formation. The latter is quantified by $\nu$ (Dutton et al. 2007; DW15; Desmond & Wechsler 2017): $\nu > 0$ corresponds to contraction from a primordial NFW form, and $\nu < 0$ to expansion. If $\nu > 0$ the halo velocity profile rises more steeply in the inner regions and less steeply further out. This causes $\rho_\text{bg}$ to rise at small $x$ and fall at large $x$, and vice versa if the halo expands. However, this effect is small and largely degenerate with $m$. The best fit to the data is given by the a priori plausible case $m = -0.5, \nu = 1$ (Dutton et al. 2007; DW15; Desmond & Wechsler 2017). Adiabatic contraction ($\nu = 1$) is however disfavoured by other kinematical measurements (purple), so that a more likely solution is a lower $\nu$ and slightly lower $m$.

Figs 1(a) and (b) show the average $\rho_\text{bg}$ from many realizations of the model. However, to assess the compatibility of the $\Delta V$–$\Delta R$ measurements and model predictions one must ask whether the
measurements could plausibly have been drawn from the model, which requires knowledge of the sample variance among mock data sets. In Fig. 1(c), we show the results for 15 randomly chosen individual realizations of the model with $m = -0.6$. This spread is sufficient to render the data not unlikely were the model true. We quantify this in Fig. 2, where we plot the discrepancy between the measured $ρ_{\text{b}}$ values and the expectations from various models as a function of $x$. We define the discrepancy as

$$δ_{\text{b}} = (ρ_{\text{b,obs}} - (ρ_{\text{b}}))/\sigma(ρ_{\text{b}}),$$

where $ρ_{\text{b,obs}}$ is the value from the SPARC data, $\bar{ρ}$ denotes the median average of the model realizations, and $\sigma$ the standard deviation between them. The fiducial model in which $R_{\text{eff}}$ is independent of halo properties at fixed $M_0$ has a maximum $δ_{\text{b}} = 4.7\sigma$ discrepancy with the data at $x \approx 4$, indicating statistically significant evidence for an anticorrelation of $ΔR_{\text{eff}}$ and $Δc$ or $ΔM_{\text{vir}}$. The model with $c = −0.4 ΔR_{\text{eff}}$ has a maximum discrepancy of $δ_{\text{b}} = 2.6\sigma$. In principle even stronger evidence for $m < 0$ could be acquired by combining the correlated information across the full range of $x$.

In Fig. 3, we show the analogue of Fig. 1 when $V$ is measured at a multiple of the radius $R_p$ at which the baryonic RC peaks, rather than $R_{\text{eff}}$. Although the dependence of $ρ_{\text{b}}$ on $r$ is different owing to variations among the galaxies in the radii relative to $R_{\text{eff}}$ at which $R_p$ is achieved, the best models of Fig. 1 again provide a good fit to the data. This demonstrates that our results are not very sensitive to the particular choice of normalization for the radius definition.

We have checked that neither our data nor model relations are sensitive to outliers by excising galaxies lying $>2\sigma$ from the fitted $M_0–R_{\text{eff}}$ and $M_0–V(xR_{\text{eff}})$ relations. We find that $ρ_{\text{b}}$ changes by at most $\sim 0.05$, and the outlier fraction is <10 per cent. We have also checked that bootstrap and jackknife resamples of the SPARC data set have qualitatively similar $ρ_{\text{b}}$ relations.

Finally, in Fig. 4 we show the results from fitting the RCs with the DC1-4 profile, using either $V(xR_{\text{eff}})$ (Fig. 4a) or $V(xR_p)$ (Fig. 4b). The red line shows the median over 200 model realizations drawn from the joint posterior on $M_{\left<10^{11}\right>}$ and host halo mass and concentration for each galaxy (Katz et al. 2017). The thin lines show 15 example realizations. The model matches the $ΔV_r–ΔR$ relation well, with only a small range of $r$ lying outside the model realizations. Note, however, that this is not a forward model but uses information from the RCs themselves, and is therefore tuned to match the data: the red and dashed black lines are not independent. Fig. 4 simply shows that halo profiles known to reproduce some aspects of galaxy kinematics are also able to decouple the TFR and MSR.

We also show in that figure the results using the RCs generated by only the baryons (green) or dark matter (magenta). As expected, the baryons provide a strongly negative contribution to $ρ_{\text{b}}$: a larger galaxy at fixed $M_0$ produces a smaller rotation velocity because its baryonic mass is more diffuse. The halo however provides a positive $ρ_{\text{b}}$, since larger galaxies sample the halo RC at larger $r$ for fixed $x$ and $M_0$. This effect is less pronounced at larger $r$ where the halo RC is less steeply rising: far enough out the halo RC would begin to decline, causing $ρ_{\text{b}} < 0$. The combination of these components in the total RC produces the $|\rho_{\text{b}}|$ values near 0 that are measured.

This provides a new perspective on the ‘disc–halo conspiracy’ (van Albada & Sancisi 1986): not only must the relative distributions of baryons and dark matter conspire to make RCs flat, they must also decouple the sizes of galaxies from their velocities across the RCs.

### 4 Discussion

We have shown that realistic models for the galaxy–halo connection produce statistically insignificant correlations between the residuals of the TFR and MSR. These models illustrate the ‘conspiracy’ between the distributions of baryons and dark matter required to decouple the characteristic velocities and sizes of galaxies, no matter where those velocities are measured. This does not require fine tuning but rather holds for a range of model assumptions. Here we discuss our result in the context of related studies in the literature.

In DW15, $V$ was measured at the radius enclosing 80 per cent of the $i$-band light, which is $1.8R_{\text{eff}}$ for an exponential disc. The DW15 model gives a significantly stronger $ΔV_r–ΔR$ anticorrelation than any of the models we investigate here ($ρ_{\text{b}} ≃ −0.5$; see their
with similar models (Dutton et al. 2007). Nevertheless, these works to halo spin significantly anticorrelate \( \Delta 1 \) never as strongly anticorrelated as predicted by that model outside that we do not restrict ourselves to a single radius, and more measured at 2.2 disc scale lengths to infer the relative amount of largely scale-invariant anticorrelation found in the data.

As Figs 1 and 3, but for the DC14 halo profile fit to the SPARC RCs. Panel (a) is for \( r = xR_{\text{eff}} \) and panel (b) is for \( r = xR_{p} \). The dashed black lines show the observations, the red lines the median total RCs from the halo model over 200 Monte Carlo realizations, the thin cyan lines the 15 example realizations, and the green and magenta lines the medians using only the baryonic and dark matter parts of the RC, respectively. While the baryons alone would produce a significantly negative \( \Delta V_{r} - \Delta R \) correlation in each case, and the halo a moderately positive correlation, the combination produces the very weak and largely scale-invariant anticorrelation found in the data.

Another early study of the \( \Delta V_{r} - \Delta R \) relation was Courteau & Rix (1998), which used the lack of observed anticorrelation with \( V \) measured at 2.2 disc scale lengths to infer the relative amount of dark and visible matter within that radius. Our study is more general in that we do not restrict ourselves to a single radius, and more precise in that we tailor our models to the observational data set in question. We are therefore able to generalize the conclusion that prior-motivated halo models produce dark matter fractions consistent with those required to generate a negligible \( \Delta V_{r} - \Delta R \) correlation, and provide more detailed information on the conditions for maximal agreement with the data.

We have found evidence for an anticorrelation of \( R_{\text{eff}} \) with \( c \) (or \( M_{\text{halo}} \)) at fixed \( M_{*} \). This is also found in Desmond (2017a,b) and produced in the Evolution and Assembly of GaLaxies and their Environments (EAGLE) hydrodynamical simulation (Desmond et al. 2017c). We note however that \( M_{\text{halo}} \) is likely positively correlated with \( R_{\text{eff}} \); not only is this produced in simulations such as EAGLE (Desmond et al. 2017c), it is also measured with weak lensing (Charlton et al. 2017). The correlation that we infer may be understandable in the future through more detailed physical modelling of the relation between galaxy and halo angular momentum.

Our model for the \( \Delta R_{\text{eff}} - \Delta c \) correlation complements other methods in the literature for incorporating size into the galaxy–halo connection. One alternative is to impose a proportionality between galaxy and halo size, which reproduces the shape of the MSR (Kravtsov 2013) and the size dependence of galaxy clustering (Hearin et al. 2017). This approach is motivated by the angular momentum partition model of Mo et al. (1998), although formally independent of it. It is not yet clear what \( R_{\text{eff}} \propto R_{c} \) implies for the relation between \( R_{\text{eff}} \) and halo properties at fixed stellar or baryonic mass, i.e. after factoring out the principal component of the galaxy–halo connection \( M_{\text{halo}} - f(M_{*}, c) \). A third method for connecting size to halo properties is conditional AM (Hearin et al. 2014) where a second AM is performed on size in bins of stellar mass. Although this naturally models size at fixed galaxy mass, making it orthogonal to \( M_{*} - f(M_{*}, c) \), and reproduces by construction the size function conditioned on \( M_{*} \), it is unclear whether it can account for the kind
of dynamical signals investigated here. Future work should aim to integrate these approaches into a single unified model for the role of size in the galaxy–halo connection and draw out the implications for angular momentum transfer.

5 CONCLUSIONS

Using 153 late-type galaxies from the SPARC sample, we investigate the correlation between the residuals of the mass–size and baryonic TFRs with velocities measured at a range of radii. Our main findings are the following.

(i) The correlation between the velocities and sizes of galaxies at fixed baryonic mass is a weak function of the radius \( r \) at which the velocity is measured, with Spearman’s rank coefficient \( \rho_{sr} \) rising from \( \sim -0.5 \) in the baryon-dominated inner regions \( (r \ll R_{\text{eff}}) \) to \( \sim -0.2 \) further out where the dark matter is more important. The full radial dependence of this relation provides new information about the dependence of galaxy size on halo properties.

(ii) Models that set \( M_\ast \) by AM and assume that \( R_{\text{eff}} \) is uncorrelated with halo properties at fixed galaxy mass overpredict the strength of the \( \Delta V_r - \Delta R \) relation by \( 1\sigma-5\sigma \), depending on \( r \). This suggests an anticorrelation of galaxy size with halo concentration (or mass) at fixed baryonic mass, in line with previous inferences from the \( M-R-V \) relations of late-type galaxies. We show agreement within \( 3\sigma \) for all \( r \) using a model in which \( \Delta c \approx -0.4 \Delta R_{\text{eff}} \).

(iii) The \( \rho_{sr} \) relation may also be matched by fitting the RCs with a partly cored DC14 halo profile. We show explicitly the \( \Delta V_r - \Delta R \) correlation produced by the dark matter and anticorrelation produced by the baryons, thus quantifying the “baryon–halo conspiracy” required for no overall correlation at any \( r > R_{\text{eff}} \).

(iv) The \( \rho_{sr} \) relation further evidence against the hypothesis that galaxy and halo specific angular momentum are proportional. We conclude that, under standard assumptions for halo density profiles and the galaxy–halo connection, this putative proportionality cannot be responsible for setting galaxy size at low redshift.

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REFERENCES

Behroozi P. S., Wechsler R. H., Wu H.-Y., 2013, ApJ, 762, 109
Charlton P. J. L., Hudson M. J., Balogh M. L., Khatri S., 2017, MNRAS, 472, 2367
Courteau S., Rix H.-W., 1998, in Zaritsky D., ed., ASP Conf. Ser. Vol. 136, Galactic Halos: A UC Santa Cruz Workshop. Astron. Soc. Pac., San Francisco, p. 196
Desmond H., 2017a, MNRAS, 464, 4160
Desmond H., 2017b, MNRAS, 472, L35
Desmond H., Wechsler R. H., 2015, MNRAS, 454, 322 (DW15)
Desmond H., Wechsler R. H., 2017, MNRAS, 465, 820
Desmond H., Mao Y.-Y., Wechsler R. H., Crain R. A., Schaye J., 2017, MNRAS, 471, L11
Di Cintio A., Lelli F., 2016, MNRAS, 456, L127
Di Cintio A., Brook C. B., Dutton A. A., Macciò A. V., Stinson G. S., Knebe A., 2014, MNRAS, 441, 2986
Djorgovski S., Davis M., 1987, ApJ, 313, 59
Dressler A., Lynden-Bell D., Burstein D., Davies R., Faber S., Terlevich R., Wegner G., 1987, ApJ, 313, 42
Dutton A. A., van den Bosch F. C., Dekel A., Courteau S., 2007, ApJ, 654, 27
Dutton A. A. et al., 2011, MNRAS, 416, 322
Dutton A. A., Macciò A. V., Mendel J. T., Simard L., 2013, MNRAS, 432, 2496
Famaey B., McGaugh S. S., 2012, Living Rev. Relativ., 15, 10
Gnedin O. Y., Ceverino D., Gnedin N. Y., Klypin A. A., Kravtsov A. V., Levine R., Nagai D., Yepes G., 2011, preprint (arXiv:1108.5736)
Hearin A. P., Watson D. F., Becker M. R., Reyes R., Berlind A. A., Zentner A. R., 2014, MNRAS, 444, 729
Hearin A., Behroozi P., Kravtsov A., Moster B., 2017, preprint (arXiv:1711.10500)
Katz H., Lelli F., McGaugh S. S., Di Cintio A., Brook C. B., Schombert J. M., 2017, MNRAS, 466, 1648
Kravtsov A. V., 2013, ApJ, 764, L31
Lehmann B. V., Mao Y.-Y., Becker M. R., Skillman S. W., Wechsler R. H., 2017, ApJ, 834, 37
Lelli F., McGaugh S. S., Schombert J. M., 2016a, ApJ, 816, L14
Lelli F., McGaugh S. S., Schombert J. M., 2016b, AJ, 152, 157
Macciò A. V., Dutton A. A., van den Bosch F. C., Moore B., Potter D., Stadel J., 2007, MNRAS, 378, 55
McGaugh S. S., 2005, Phys. Rev. Lett., 95, 171302
McGaugh S. S., 2015, Can. J. Phys., 93, 250
Mo H. J., Mao S., White S. D. M., 1998, MNRAS, 295, 319
Pizagno J. et al., 2007, AJ, 134, 945
Reyes R., Mandelbaum R., Gunn J. E., 1998, MNRAS, 295, 319
Skillman S. W., Warren M. S., Turk M. J., Wechsler R. H., Holz D. E., Sutter P. M., 2014, preprint (arXiv:1407.2600)
Tully R. B., Fisher J. R., 1977, A&A, 54, 661
van Albada T. S., Sancisi R., 1986, Philos. Trans. R. Soc. Lond. A., 320, 447
Wechsler R. H., Tinker J. L., 2018, ARA&A, 56, 435
Yegorova I. A., Salucci P., 2007, MNRAS, 377, 507

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