Bell’s theorem and the experiments: Increasing empirical support to local realism?

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Abstract

It is argued that local realism is a fundamental principle, which might be rejected only if experiments clearly show that it is untenable. A critical review is presented of the derivations of Bell’s inequalities and the performed experiments, with the conclusion that no valid, loophole-free, test exists of local realism vs. quantum mechanics. It is pointed out that, without any essential modification, quantum mechanics might be compatible with local realism. This suggests that the principle may be respected by nature.
I. Introduction

Forty years have elapsed since John Bell discovered his celebrated inequalities. These inequalities, which involve measurable quantities, provide necessary conditions for local realism. Bell also proved that, in some experiments with ideal set-ups, the predictions of quantum mechanics violate the inequalities. During these four decades a lot of papers have been written pointing out quantum-theoretical violations of the inequalities in very many different phenomena, but only a few dozens empirical tests have been actually performed. The results of all performed experiments are compatible with local realism and, with few exceptions, agree with the quantum predictions (see Secs. 5, 6 and 8 below). The logical interpretation of these facts, unbiased by theoretical prejudices, should be that there is no empirical evidence against local realism and that quantum mechanics has been confirmed, the few disagreements with its predictions being of little significance. Nevertheless the standard wisdom is that local realism has been refuted, which is concluded because allegedly plausible extrapolations of the empirical results could violate a Bell inequality.

In my view the current wisdom is misleading and harmful for the progress of science. Misleading because it attempts answering a fundamental scientific question by means of a subjective assessment of plausibility. Harmful because it discourages people from making the necessary effort to perform a real, loophole-free, test.

The long time elapsed without a true disproof of local realism may be compared, for instance, with the discovery of parity non-conservation, which required a few months to go from the theoretical paper by Lee-Yang, in 1957, to the uncontroversial (loophole-free) experiment by Wu et al. I think that the logical conclusion of the long standing unsuccessful effort to disprove local realism is that it is preserved by nature.

In this paper I shall begin analyzing the concept of local realism and its relevance in physics (Sec. 2). Then I shall sketch the derivation of Bell’s inequalities, distinguishing those which follow just from local realism (Sec. 3) from those which require auxiliary assumptions (Sec. 4). After that I shall review the experiments aimed at testing local realism vs. quantum mechanics (Secs. 5 and 6). Then I shall rebut the standard wisdom that quantum mechanics is incompatible with quantum mechanics (Sec. 7). Finally, after a digression on philosophy and sociology of science (Sec. 8) I shall discuss the
II. Local realism and its relevance in natural science

It is not easy to define realism with a few words, as is proved by the existence of whole books devoted to the subject. Here I shall give a simple definition appropriate for physics. Realism is the belief that material bodies have properties independent of any observation, and that the results of any possible measurement depend on these properties. The said properties may be called “elements of reality” and are frequently identified with hidden variables. However I think that the latter correspond rather to the parameters used for the description of the said properties and should not be confused with the former.

Realism alone, as defined above, does not contradict quantum mechanics. In order to clarify the point I shall give an example. If I throw upwards a coin, after a while the coin will collide with, say, a table and will soon become at rest on it, with either the head or the tail upwards. The described experiment consists, as is typical, of the preparation of the state of a system (the coin thrown upwards) followed by the evolution of the system and finishing by the measurement of a quantity on it. Our intuition says that the result (head or tail) is determined by the elements of reality of the coin during the fly. Or maybe, taking into account the unavoidable existence of non-idealities (e.g. friction with the air), the elements of reality just determine the probability of the result. In any case we should carefully distinguish between the observable (head or tail) and the elements of reality (associated to motion of the coin). The relevant lesson of our example is that the result of a measurement depends on both the measured system (the coin) and the measuring apparatus (the table). Sometimes the observable (head-tail in our example) is even devoid of sense without the measuring apparatus (the table). Therefore it is not so strange that quantum mechanics forbids the “simultaneous existence of definite values for some observables”, namely those which cannot be measured together. This is the essential content of the Kochen-Specker theorem forbidding non-contextual hidden variables. Our example shows that the validity of the theorem does not preclude realism.

Some quantum physicists may consider that realism is just a philosophical opinion which may or may not be true, but I disagree. In my view natural science would be impossible without accepting realism as defined
above. Actually, even the most pragmatic quantum physicists would admit that states of physical systems have some “capabilities” of influencing the results of eventual future measurements on the system. It is a rather semantic question whether we name these capabilities “elements of reality”.

Locality is the belief that no influence may be transmitted with a speed
greater than that of light. Thus we might identify locality with relativistic causality. The concept of locality is subtle, however. In fact, quantum mechanics is local in the sense that it forbids the transmission of superluminal signals (say from a human being to another one), but local realism as analyzed here is stronger than that. At a difference with the idea of realism which I consider as an unavoidable requirement for the existence of science, locality derives from our experience at the macroscopic level and might be violated without demolishing the whole building of physics. That is, we might assume that some influences travel at a speed greater than that of light even if this fact does not allow the transmission of superluminal signals. This seemed the position of John Bell.

In spite of this I think that locality is also important, that is local realism is so fundamental a principle of physics that it should not be rejected without extremely strong reasons, an opinion which I believe is quite close to what Einstein maintained until his death. On the other hand the question of local hidden variables is less relevant than the question of local realism. It is true that if local realism is untrue local hidden variables would be impossible, but if local realism is true local hidden variables may still be useless in practice, although possible in principle. Thus I shall refer to local realism, rather than to local hidden variables, in the rest of this article.

III. The Bell inequalities

From what we have said it might appear that local realism is a purely philosophical concept. But a physical necessary condition for local realism was introduced by John Bell as follows: Any correlation between measurements performed at different places should derive from events which happened in the intersection of the past light cones of the measurements. In order to give an empirical content to the statement Bell considered a generic experiment consisting of the preparation of a pair of particles (or, more generally, physical systems) which are let to evolve in such a way that the two particles go to macroscopically distant regions ( the argument that follows has been
exposed in more detail elsewhere. Thus Bell searched for the probability, \( p(A, a; B, b) \), of getting the result \( a \) in the measurement of an observable \( A \) of the first particle and the result \( b \) in the measurement of the observable \( B \) of the second particle. He proposed that, if local realism holds true, the probability could be written

\[
p(A, a; B, b) = \int \rho(\lambda) P_1(\lambda; A, a) P_2(\lambda; B, b) d\lambda,
\]

where \( \lambda \) is one or several parameters which contain all relevant information about the intersection of the past light cones of the two measurements. An expression similar to (1) for the total probability \( p(A, a) \), of getting the result \( a \) in the measurement of the observable \( A \) on the first particle, follows at once from the fact that it is unity the sum of probabilities associated to particle 2. That is

\[
p(A, a) = \sum_b \int \rho(\lambda) P_1(\lambda; A, a) P_2(\lambda; B, b) d\lambda = \int \rho(\lambda) P_1(\lambda; A, a) d\lambda.
\]  

From now on we shall consider only dichotomic observables, so that the result of the measurement of the observable \( A \) may be only 1 (yes) or 0 (not). Thus we shall simplify the notation writing \( P_1(\lambda, A) \) (or \( P_2(\lambda, B) \)) for \( P_1(\lambda; A, a) \) (or \( P_2(\lambda; B, b) \)), and \( p(A, B) \) (or \( p(A) \)) for the left side of (1) (2) so that eqs. (1) and (2) will be written

\[
p(A) = \int \rho(\lambda) P_1(\lambda, A) d\lambda, \quad p(A, B) = \int \rho(\lambda) P_1(\lambda, A) P_2(\lambda, B) d\lambda.
\]

The functions \( P \) and \( \rho \) in the formula fulfil the conditions required for probabilities and probability densities, respectively. That is

\[
\rho(\lambda) \geq 0, \quad \int \rho(\lambda) d\lambda = 1,
\]

\[
P_1(\lambda, A), P_2(\lambda, B) \geq 0,
\]

\[
P_1(\lambda, A), P_2(\lambda, B) \leq 1.
\]

It is important to stress that the value of \( P_1(\lambda, A) \) is assumed to be independent of \( B \), that is independent on what measurement is performed on the second particle, which is Bell’s condition of locality. This independence has
been called “parameter independence”, which is compatible with a possible “outcome dependence”, that is the results of the measurements of A and B may be correlated. Hence, using the notation $A'$, $B'$ for the result 0 in the measurement of A and B respectively, we obtain a similar independence for the measurable probabilities

$$p(A) = p(A, B) + p(A, B') = p(A, D) + p(A, D') = ...$$

Parameter independence holds true also in quantum mechanics and it guarantees that superluminal communication is not possible.

From the conditions (3) to (6) it is possible to derive inequalities involving only measurable probabilities. We consider an experiment in which we prepare once and again, say 4N times ($N \gg 1$), a pair of particles in a given state, the same for all preparations. Here, the same means that the parameters which may be controlled in the preparation have the same values. After N preparations, chosen at random amongst the 4N made, we measure the dichotomic observables A and B of the two particles. After another N preparations, also chosen at random, we measure the dichotomic observables C and D. Similarly C with B are measured N times, and A with D also N times. We assume that the result of the measurement of any of the observables may be either 0 or 1, and call $p(A, B)$ the probability of getting the result 1 for both observables, A and B (the frequencies measured in the experiment should approach the probabilities if $N$ is large enough). Similarly we may define the probabilities $p(A, D)$, $p(C, B)$ and $p(C, D)$, and also the probability $p(A)$ corresponds to getting the value 1 in the measurement of A and any value (1 or 0) in the measurement of B, or D, performed on the partner particle, and similar for $p(B)$. It is an easy task to derive, from (1) to (6), inequalities involving measurable probabilities. For instance

$$p(A, B) + p(A, D) + p(C, B) - p(C, D) \leq p(A) + p(B).$$

This inequality may be related to the existence of a “metric” in the set of propositions associated to the results “yes”, “no” in the four measurements. In fact we may define a formal (not measurable) joint probability distribution on the observables $\{A, B, C, D\}$ by means of expressions similar to (3) applied to the four observables, the six pairs $\{AB, AC, AD, BC, BD, CD\}$ and the four triples $\{ABC, ABD, ACD, BCD\}$, in spite of some of them not being actually measurable (e. g. $p(A,C)$ cannot be got empirically because A and C
correspond to alternative, incompatible, measurements on the same particle). Now the mere possibility of defining a formal joint probability implies the existence of a metric in the set of propositions (yes-no experiments) and the essential property of the metric is the fulfillment of triangle inequalities, which are closely related to the inequality (7). But I shall not pursue the subject here (details may be seen elsewhere).

IV. Bell’s vs. tested inequalities. The CHSH case.

Soon after Bell’s discovery in 1964, it was realized that no performed experiment had shown a violation of local realism. Furthermore, no simple experiment could do the job. In my view, the difficulty is a proof that it is wrong the wisdom according to which quantum mechanics predicts “highly non-local effects”. The truth is that non-local effects, if any, are extremely weak and difficult to observe.

In 1969 Clauser, Horne, Shimony and Holt (CHSH) made the first serious proposal for an empirical test of Bell’s inequality. They suggested the measurement of the polarization correlation of optical photon pairs. By optical we mean that the corresponding frequencies are in the visible, the near ultraviolet or the near infrared parts of the spectrum. The mentioned authors derived the Bell inequality

$$S \equiv E(A, B) + E(A, D) + E(C, B) - E(C, D) \leq 2,$$

where \( \{ A, C \} \) correspond to two possible positions of a polarization analyzer for the first photon and \( \{ B, D \} \) for the second. The correlations are defined by

$$E(X, Y) = p_{++}(X, Y) + p_{--}(X, Y) - p_{+-}(X, Y) - p_{-+}(X, Y),$$

with \( X = A \) or \( C \), \( Y = B \) or \( D \), \( p_{++}(X, Y) \) being the probability that the polarization of the first photon is found in the plane \( X \), and that of the second in the plane \( Y \), \( p_{+-}(X, Y) \) the probability that the polarization of the first photon is found in the plane \( X \) and that of the second is in the plane perpendicular to \( B \), etc.

It is not difficult to see that the (8) inequality is equivalent to (7) provided that the sum of the four probabilities involved is unity, that is

$$p_{++}(X, Y) + p_{--}(X, Y) + p_{+-}(X, Y) + p_{-+}(X, Y) = 1.$$
In fact in this case it is easy to go from (8) to (7), or vice versa, by repeated use of relations like

\[
p_{++}(X,Y) = p(X,Y), \quad p_{+-}(X,Y) = p(X) - p(X,Y),
\]
\[
p_{--}(X,Y) = 1 - p(Y) - p_{+-}(X,Y).
\] (11)

With respect to the empirical tests, however, the two inequalities look rather different, and only the inequality (7) may be easily adjusted to actual experiments. In fact, in the experiments either eq. (10) is not true, thus (8) not being a true Bell inequality (it cannot be derived from local realism alone) or the quantities E(X,Y) are no longer correlations, as we explain in the following.

In typical experiments there are two arms in the apparatus, each one consisting of a lens system followed by a polarization analyzer (polarizer, for short) and a detector (for the moment we do not consider the case of two-channel analyzers, but see below). Thus we may interpret p(X,Y) as the probability that both photons are detected, after crossing the appropriate polarizers, and p(X) the probability that the “red” photon of the pair is detected, with independence of what happens to the “green” photon (for clarity of exposition we attach fictitious colours, red and green, to the photons of a pair). However, if this interpretation is carried upon the quantities E(X,Y), via the relations (11), such quantities would be correlations only in the case that both photons of every pair arrive at the polarizers and every photon is detected (with 100% efficiency) whenever it has crossed the corresponding polarizer. But this idealized situation never happens.

The current practice in recent experiments is to use two-channel polarizers, with a detector after each outgoing channel. Attaching the labels + or - to the detectors after the first or second outgoing channel of a polarizer, respectively, it is possible to define p_{++} as the probability that both photons are detected in detectors with label +, p_{+-} the probability that the red photon is detected in a detector with label + and the green photon in a detector with label -, etc. With this interpretation the quantities E(X,Y) of (9) are indeed true correlations and the inequality (8) is never violated in actual experiments, because all probabilities p_{++}, p_{+-}, etc. are much smaller than unity due to the low collection-detection efficiency (i.e. for most photon pairs only one photon, or none, is detected). The “solution” proposed for this problem has been to renormalize the probabilities defining
the correlations by 

\[ E^*(X, Y) = \frac{p_{++}(X, Y) + p_{--}(X, Y) - p_{+-}(X, Y) - p_{-+}(X, Y)}{p_{++}(X, Y) + p_{--}(X, Y) + p_{+-}(X, Y) + p_{-+}(X, Y)}. \]  

(12)

Thus people use the inequality (compare with (8))

\[ S^* \equiv E^*(A, B) + E^*(A, D) + E^*(C, B) - E^*(C, D) \leq 2, \]

(13)

in the empirical tests. Indeed, this is the inequality violated in most of the recent experiments. The inequality, however, cannot be derived from eqs. (3) to (6) alone (without additional assumptions) and therefore it is not a genuine Bell inequality.

V. Experiments using optical photons

The first experimental test using optical photons was made by Freedman and Clauser. They used photon pairs produced in the decay of excited calcium atoms via a 0-1-0 cascade. That is, the initial and final atomic states had 0 total angular momentum, so that the two emitted photons were entangled in polarization. The dichotomic observables measured were detection or non-detection of a photon, after it passed through a polarizer. The labels A and C are associated to two different positions of the polarizer for the “red” photon and similarly B and D for the “green” one. The authors were aware that the inequality (7) could not be violated with the technology of the moment because the detection efficiencies of the available detectors were too small (less than 10%). As the left hand side of the inequality (7) is proportional to the efficiency squared, whilst the right side is proportional to the efficiency, the latter is more than ten times the former, so that the inequality is very well fulfilled.

More specifically, the prediction of quantum mechanics for the experiment may be summarized as follows, with some simplifications for the sake of clarity. The measurable quantities in the experiment are the single rates, \( R_1 \) and \( R_2 \), and coincidence rate, \( R_{12}(\phi) \), the latter being a function of the angle, \( \phi \), between the polarizer’s planes X and Y. In terms of the production rate, \( R_0 \), in the source they are given by

\[ R_1(A) = R_2(B) = \frac{1}{2}R_0\eta, \quad R_{12}(X, Y) = \frac{1}{4}R_0\eta^2\alpha(1 + V\cos(2\phi)). \]  

(14)
Here $\alpha$ is an angular correlation parameter and $\eta$ is the overall detection efficiency of a photon, which includes collection efficiency and quantum efficiency of the detectors (for simplicity we put the same efficiency $\eta$ for the red and the green photons, which is approximately true in practice, but the generalization would be rather trivial). In actual experiments the quantum prediction (14) is confirmed, except for small deviations which are not considered significant.

The probabilities needed to test the inequality (7) are just the ratios

$$p(A) = \frac{R_1}{R_0}, \quad p(B) = \frac{R_2}{R_0}, \quad p(X, Y) = \frac{R_{12}(\phi)}{R_0}.$$ 

The production rate, $R_0$, is not measured but it is not difficult to show that, if we insert (14) into (7), $R_0$ cancels out and the inequality becomes

$$\alpha \eta \left[1 + \frac{1}{2} V \left(\sum_1^3 \cos(2\phi_j) - \cos(2\phi_4)\right)\right] \leq 2,$$

where $\{\phi_j\}$ are the angles between the polarization planes of the analyzers, that is between A and B, A and D, C and B, C and D, respectively. These angles fulfil $\phi_1 + \phi_4 = \phi_2 + \phi_3$ and the maximum of $\sum_1^3 \cos(2\phi_j) - \cos(2\phi_4)$ with that constraint is $2 \sqrt{2}$. Thus the Bell inequality (15) holds true, for any choice of polarizers positions, whenever

$$\alpha \eta \left(1 + \sqrt{2} V\right) \leq 2. \quad (15)$$

In the actual experiment $V \simeq 0.85$, but $\eta \simeq 0.0001$, and $\alpha \simeq 1$, so that the inequality was safely fulfilled ($\eta$ is the product of the quantum efficiency, $\zeta$, of a detector times the collection efficiency of the apertures, see below eq.(19)).

Freedman and Clauser\textsuperscript{14} found a “solution”, to circumvent the problem of the low detection efficiency, consisting of the replacement of condition (6) by another one, called “no-enhancement”, which they claimed plausible. This assumption states that, for any value of the parameter $\lambda$, the following inequality holds true:

$$P_1(\lambda, A) \leq P_1(\lambda, \infty), \quad P_2(\lambda, B) \leq P_2(\lambda, \infty) \quad (16)$$

where $P_j(\lambda, \infty)$ are the probabilities of detection of the photon with the corresponding polarizer removed. From inequalities (1) to (5) plus (16), the authors\textsuperscript{14} derived the inequality

$$p(A, B) + p(A, D) + p(C, B) - p(C, D) \leq p(A, \infty) + p(\infty, B), \quad (17)$$

10
where \( p(A, \infty) \) ( \( p(\infty, B) \) ) is the probability of coincidence detection with the polarizer corresponding to the red (green) photon removed. The results of the measurement, and the quantum predictions, for these probabilities are

\[
p(A, \infty) = p(\infty, B) = \frac{1}{2} \alpha \eta^2,
\]

and the inequality (17) implies

\[
\left( 1 + \sqrt{2} V \right) \leq 2 \Leftrightarrow V \leq \sqrt{2}/2,
\]

to be compared with (15). This was the inequality tested, and violated, in the commented experiment.

Note that, in sharp contrast with the obvious inequality (9), the inequality (16) is not only empirically untestable, it is counterfactual. In fact, as said above, \( \lambda \) is a set of parameters which contains all relevant information about the intersection of the past light cones of the measurements. But the past light cone of one measurement (with a polarizer in place) is necessarily different from the past light cone of a different measurement (with the polarizer removed). In order to give a meaning to the inequality (16) it is necessary to compare a fact (one of the measurements) with a belief (about what would have happened in a different experiment having the same past light cone). For this reason I say that the inequality is counterfactual. Of course, it may be checked empirically that the average over \( \lambda \) of the left hand side is not greater than the average of the right hand side, that is for any light beam the detection rate does not increase when we insert a polarizer. (However, it might increase if we insert a polarization rotator plus a polarization analyzer when the incoming light is linearly polarized). In summary, the first alleged empirical disproof of local realism rests upon a counterfactual belief qualified as plausible. Therefore, strictly speaking, it did not test local realism. However I do not mean that the experiment was useless because it opened an important new line of experimental research.

In the decade that followed the commented experiment, several similar atomic-cascade experiments were performed\(^{15,16}\). In addition to the requirement of introducing untestable auxiliary assumptions (like (16)), all of them had the problem of being static. That is, the positions of the polarizers were fixed well before the detection events took place. Therefore the experiments could not test locality, in the sense of relativistic causality. In order to solve the problem, Alain Aspect and coworkers\(^{17}\) performed in 1982 a new
atomic-cascade experiment where (in some sense) the polarizers positions were chosen when the photons were already in fly. However the inequality tested was of the type (17) rather than a genuine Bell inequality like (7).

The experiment of Aspect is usually presented as the definite refutation of local realism. One of the reasons is that, during the preparation of the experiment, Aspect was in close contact with Bell, who approved it. Although Bell was aware that there existed a loophole due to the low efficiency of the available photon detectors, he considered acceptable to make a fair sampling assumption. That is, to extrapolate the results actually got in the experiment, with low efficiency detectors, to detectors 100% efficient. This amounts to testing an inequality obtained from (7) by dividing the right hand side by the efficiency, $\eta$, and the left side by $\eta^2$. The inequality so obtained is practically the same as (17). The fair sampling assumption was justified by Bell with the frequently quoted sentence: “It is hard for me to believe that quantum mechanics works so nicely for inefficient practical set-ups and is yet going to fail badly when sufficient refinements are made.” But this sentence cannot be applied to the commented experiments because the predictions of quantum mechanics for any atomic-cascade experiment are compatible with local realism even if the experiment is made with ideal set-up, in particular 100% efficiency detectors, as is shown in the following. Apparently Bell was not aware of this fact before he untimely died in october 1990.

The atomic cascade decay, giving rise to a photon pair, is a three-body problem with the consequence that the angle, $\chi$, between the directions of emission of the two photons is almost uniformly distributed over the sphere. This implies that the angular correlation parameter $\alpha$ (see (14)) is almost independent of the angle $\chi$, that is $\alpha(\chi) \simeq 1$. On the other hand both the overall detection efficiency, $\eta$, and the “visibility”, $V$, of the coincidence curve are functions of the angle, $\theta$, determined by the apertures of the lens system (as seen from the source). The dependence $V(\theta)$ is a loss of polarization correlation when the “red” and “green” photons do not have opposite wavevectors. In the Aspect experiment, as in other atomic-cascade experiments, the predicted functions are

$$\eta = \frac{1}{2} (1 - \cos \theta) \zeta, \ V = 1 - \frac{2}{3} (1 - \cos \theta)^2, \ \alpha \simeq 1,$$

where $\zeta$ is the quantum efficiency of the detectors. Using these expressions it is easy to see that the maximum value of the left hand side of (15) is about 0.74 $\zeta$, and the inequality is safely fulfilled even for ideal detectors (i.e. e.
\(\zeta = 1\). The figure should be multiplied times 2, giving 1.48 \(\zeta\), if we assume that both photons, red and green, may be detected in either detector. But still the inequality \([16]\) holds true for any \(\zeta \leq 1\). In summary taking into account the low angular correlation of the photon pairs produced in atomic cascades, these experiments cannot discriminate between local realism and quantum mechanics. In spite of this fact, the Aspect experiment is quoted everywhere as the definite refutation of local realism.

The problem of the lack of angular correlation might be solved if the recoil atom were detected\([20]\) but that experiment would be extremely difficult. A more simple solution is to use optical photon pairs produced in the process of parametric down conversion, and this has been the source common in all experiments since about 1984. See, for instance, the paper by Kurtsiefer et al.\([21]\) and references therein. At a difference with atomic-cascade experiments, here the photons have a good angular correlation. In fact the parameter \(\alpha\) of \([14]\) as a function of the angle, \(\chi\), between the wavevectors of the two photons is such that the probability of detection of the green photon conditional to the detection of the red one is just the quantum efficiency \(\zeta\) (or close to it.) Thus putting the detectors in appropriate places we may rewrite \([15]\) with \(\zeta\) substituted for \(\alpha\eta\), that is

\[
\zeta \left( 1 + \sqrt{2}V \right) \leq 2. \tag{20}
\]

This inequality might be violated if \(V\) is close to 1 (which is achievable in actual experiments) and \(\zeta > 2 \left( \sqrt{2} - 1 \right) \simeq 0.82\). But such a high value of the detection efficiency has not yet been achieved and the low efficiency of detectors remains as a persistent loophole for the disproof of local realism.

This difficulty has led to the use of the modified CHSH inequality \([13]\) as the standard inequality tested in practically all recent experiments with optical photons. These experiments use two-channel polarizers, and the prediction of quantum mechanics for them may be summarized in terms of four coincidence detection rates as follows

\[
R_{++} (\phi) = R_{--} (\phi) = \frac{1}{2} \eta R_0 \left[ 1 + V \cos (2\phi) \right],
\]

\[
R_{+-} (\phi) = R_{-+} (\phi) = R_{++} \left( \phi + \frac{\pi}{2} \right), \tag{21}
\]

whilst the single rates are usually not measured (or, at least, not reported as relevant). In the actual experiments there are small departures from \([21]\)
which are not considered significant, but may be relevant for the reasons (see end of this section.) As said above the inequality tested is (13), and the probabilities involved may be obtained from (21) as ratios between the measured coincidence rates and the production rate. That is, putting (21) into (12) we get

$$E^* = V \cos (2\phi).$$

(22)

If this is used in (13), steps similar to those leading to (18) give

$$S^* = 2\sqrt{2}V \leq 2 \Leftrightarrow V \leq \sqrt{2}/2.$$  

(23)

This inequality looks the same as (18), but here V is obtained from measurements using two-channel polarizers. In practice V may be got by at least three different procedures:

1) From the best fit of the measured correlation, $E(\phi)$, to the theoretical curve (9), where the probabilities, $p_{++}(\phi)$, etc., are the ratios of measured rates, $R_{++}(\phi)$, etc., to the production rate, $R_0$. It is easy to see that the fitting does not require the measurement of $R_0$. We shall label just V the quantity so obtained.

2) As half the “visibility” of the empirical curve $E(\phi)$, that is the difference between the maximum and the minimum values divided by the sum. Again the value of $R_0$ is not required. I shall label $V_A$ this quantity.

3) From the value of $S^*$ measured for the angles $\phi_1 = \phi_2 = \phi_3 = \pi/8, \phi_4 = 3\pi/8$, using the first equality (23). These angles provide the maximum value of $S^*$ if the empirical data agree with (21). This value will be labelled $V_B$ and it is the quantity commonly used in the test of the inequality (23). Indeed in recent times it has become standard practice to claim that local realism is refuted whenever $V_B > \sqrt{2}/2$.

According to quantum predictions the equality $V = V_A = V_B$ should hold true, but in actual experiments there are small differences between them. On the other hand some natural families of local realistic models predict inequalities involving the quantities $V_A$ and $V_B$. One of these inequalities has already been tested with the result that it is fulfilled, whilst the equality predicted by quantum mechanics seems to be violated.

A procedure to circumvent the low efficiency loophole in experiments with optical photons has been proposed recently using homodyne detection instead of photon counting.
VI. Other experiments aimed at testing local realism

In his pioneer work Bell used the example of an Einstein-Podolsky-Rosen-Bohm system, that is a pair of spin-1/2 particles with zero total spin. Thus it is not strange that some experiments have been proposed consisting of the measurement of the spin correlation of two spin-1/2 particles. The use of massive particles has the advantage that they may be quite reliably detected, so that such experiments do not suffer from the detection loophole. The proposed experiments use non-relativistic particles. As far as I know, no experiment of this kind has been proposed using relativistic particles. The reason is probably the difficulty for producing a pair with zero total spin if we take into account that spin of relativistic particles is not strictly conserved (only the total angular momentum of a free particle is strictly conserved in Dirac’s theory).

The non-relativistic particles present the problem that it is difficult to guarantee the space-like separation of the measurements. As an example, we may consider the experiment proposed by Lo and Shimony. It consists of the dissociation of molecules with two sodium atoms followed by the measurement of their spins by means of a Stern-Gerlach apparatus. The typical velocity of the sodium atoms, after dissociation, is about 3000 m/s and the length of the measuring magnets 0.25 m. This giving a measurement time about $10^{-4}$ s. Thus, in order that the measurements were space-like separated, the Stern-Gerlach apparatused should be distant by more than 30 km. It is rather obvious that such experiment could not be a practical test of local realism as defined above. Similar problems appear in the proposed experiment by Adelberger and Jones using neutron pairs. The neutrons should collide at low energy in order to insure a pure S-wave scattering so that, by Pauli’s principle, the total spin should be zero. Again the distance between the spin measurements (by scattering with magnetized material) should be extremely large in order to be possible the violation of locality (relativistic causality).

In addition there are fundamental constraints, derived from Heisenberg uncertainty principle, on experiments using non-relativistic particles. For instance, let us assume that the particle detectors are static and placed on opposite sides and at a distance $L$ from the source each, so that the distance between detectors is $2L$. If the particles have mass $m$ (the same for both, for simplicity) and travel at a velocity $v$, then the initial position and velocity are uncertain by, at least, $\Delta x \Delta v = \hbar/2m$. Thus the arrival time at the de-
tectors will be uncertain by, at least, $\sqrt{2L/(mv^3)}$. We may be sure that the measurements are space-like separated only if this quantity is smaller than $2L/c$, which leads to the constraint $L \geq 2hc^2/(mv^3)$, a macroscopic quantity (for instance, in the experiment proposed by Lo and Shimony, this gives about 1 m.) The quantity is not so big as to put unsurmountable practical difficulties, but it shows that a Bell test using non-relativistic particles requires measurements made at quite macroscopic distances.

An experiment using the scattering of non-relativistic protons was performed by Lamehi-Rachti and Mittig in 1976. The spin components of the protons were measured by scattering on carbon foils. The experimental results agreed with quantum predictions, but the auxiliary assumptions needed for the experiment to be a test of a Bell inequality were stronger than in experiments with optical photons. An experiment has been recently performed using two $^9\text{Be}^+$ ions in a trap, each of which behaves as a two-state system. It has been claimed, and widely commented, that the experiment "has closed the detection loophole" because the atoms may be detected with 100% efficiency. However the distance between ions in the trap, $3\mu\text{m}$, was very small. Although this distance is about 100 times the size of an ion wavepacket, it is $10^6$ times smaller than the wavelength of the photons involved in the atomic transitions between the two levels (compare with the fundamental constraints commented in the previous paragraph). In these conditions the experiment cannot test locality in the sense of relativistic causality.

A loophole-free experiment involving spin measurements of atoms has also been proposed. It consists of the disociation of mercury molecules followed by the measurement of nuclear spin correlation of the atoms. In order to make the measurement time very short, the idea was to use a polarized pulse of laser light, which would induce selectively the ionization of the atom when it is in one specific spin state (say up) but not in the other possible state (down). After several years of preparation, the detailed proposal of the experiment was published in 1995, but nine years later no results have been reported. (In the Oviedo Conference, held in July 2002, Fry reported that important difficulties had been found. Fry’s talk was not published in the proceedings.)

Many other experimental tests of a Bell inequality have been performed or proposed, each one suffering from loopholes. For instance, several experiments have been performed measuring the polarization correlation of gamma
rays produced in the decay of positronium, one of the experiments violating
the quantum prediction. These experiments have the difficulty that the
polarization cannot be measured with high enough precision.

There have been also proposals using high energy particles. For instance,
the strangeness oscillations of pairs $K^0 - \bar{K}^0$ have been the subject of many
papers, but no loophole-free violation of a Bell inequality seems possible
in this case due to the small decay time of the short $K^0$ in comparison with
the oscillation period. Also an experiment has been recently performed using
$B^0$ mesons, but here also the damping made impossible the violation of a
Bell inequality, and only a normalization of the correlation function to the
undecayed pair leads to the violation of $\text{(13)}$, not a genuine Bell inequality.

In recent years a lot of effort has been devoted to the so-called “tests
without inequalities”. The idea is to prepare a system in some state and
perform a measurement such that the quantum prediction is definite (say
“yes”) but the prediction of any local realistic model is the opposite (“no”).
For a proof of the incompatibility between local realism and quantum me-
chanics, in ideal experiments, the proposal is very appealing but from a
practical point of view the possible experiments are less reliable than those
resting upon Bell’s inequalities. In particular they require an extreme con-

trol of the purity of the prepared state, which is not the case in the Bell tests
(see section 4). An experimental test of local realism resting upon the idea
has been performed but the experiment is not conclusive, as is shown be
the existence of a local model reproducing the results.

In summary, no performed experiment has been able to test a genuine Bell
inequality with the condition that the measurements are performed at space-
like separation. And, as far as I know, only a detailed proposal for a loophole-
free experiment with available technology exists but this experiment seems
to present unsurmountable difficulties. In consequence local realism has not
been refuted. Furthermore it is the case that, strictly speaking, local realism
has not yet been tested against quantum mechanics. That is no experiment
has been performed able to discriminate between local realism and quantum
mechanics.
VII. Is quantum mechanics truly incompatible with local realism?

The standard wisdom of the community of quantum physicists is that local realism does not hold true in nature. Certainly this opinion does not follow from just the results of the empirical tests of Bell’s inequalities because, as commented in the two previous sections, there are loopholes in all performed experiments. Actually the current wisdom derives from the theoretical argument that the validity of local realism would imply that quantum mechanics is false (Bell’s theorem). And, for good reasons, nobody is willing to accept that quantum mechanics is wrong. Thus the scientific community dismisses the mentioned loopholes as irrelevant (see, e. g., the relatively recent article by Laloe; excellent in most other respects.) However a violation of local realism is no more acceptable than a violation of quantum mechanics, for the reasons explained in section 2. Consequently there exists a real problem whose only solution seems to me the compatibility of local realism with quantum mechanics, or some “small” modification of this theory. But, is it possible to modify quantum mechanics without destroying its formal beauty and its impressive agreement with experiments?. In the following I argue for this possibility.

According to the traditional formulation, quantum mechanics consists of two quite different ingredients: the formalism (including the equations) and the theory of measurement, both of which are postulated independently. (Actually the two ingredients are to some extent contradictory, because the quantum evolution is continuous and deterministic except during the measurement, where the “collapse of the wavefuction” is discontinuous and stochastic. Thus the modern approach tends to remove any postulated theory of measurement, see below). We must assume that the quantum equations are correct, because the extremely accurate agreement between the predicted and the measured, for instance in quantum electrodynamics, cannot be explained otherwise. In contrast, only a small part of the quantum theory of measurement is really used in most experiments, which suggests that it might be substantially weakened. For instance, the postulate about position measurements, i. e. Born’s rule, is enough for the interpretation of all scattering experiments.

The point is that standard proofs of “Bell’s theorem” rest upon the theory of measurement (and preparation of states). In fact, in a typical proof
it it assumed that: 1) A pure spin zero state of a system of two spin-1/2 particles may be manufactured in the laboratory with the two particles able to fly, maintaining the same joint spin state, up to macroscopic distances, b) The spin projection of each particle, along any freely chosen direction, may be measured with arbitrary small error. Both these assumptions might be false without any danger for the formalism and the basic equations (Dirac’s, Maxwell’s, etc.) of the theory. Consequently I guess that a weakening of the standard measurement theory, without touching the formalism, might make quantum mechanics compatible with local realism. For instance, the weakening of the preparation and measurement assumptions might be as follows.

It is frequently assumed that there is an one-to-one correspondence between the possible states of a given physical system and the vectors in the Hilbert space, except for superselection rules. That assumption is called superposition principle. But the unrestricted superposition principle also implies an one-to-one correspondence between self-adjoint operators and observables. This is because any self-adjoint operator may be written as a linear combination of projectors onto its eigenstates (i.e. subspaces in the Hilbert space) and each projector should be an observable if all states are physically realizable and distinguishable. However the one-to-one correspondence between self-adjoint operators and observables is far too strong from a physical point of view because, how could we measure an observable like $x^m p^n + p^n x^m$ with very large integers $m$ and $n$? Thus it seems more appropriate to assume that only some self-adjoint operators represent observables and only some vectors represent states. We might go a step further and assume that only some density matrices represent physical realizable states. (For instance we might restrict the states to density matrices fulfilling an inequality like $\text{Tr}(\rho^2) \leq k < 1$ which, for some $k$, might be sufficient to prevent the violation of Bell’s inequalities. But I mention this possibility just as an illustrative example.)

If any proof of Bell’s theorem resting upon the quantum theory of measurement is invalid, a correct proof should involve a detailed study of how to prepare the state and how to measure the observables able to violate a Bell inequality. But the definite proof that some state and measurements may be really made in the laboratory is to perform the actual experiment. Thus I conclude that Bell’s “theorem” cannot be proved by theoretical arguments, i.e. it is not a theorem. It is just an argument suggesting that some experiments might exist able to discriminate between quantum and local re-
alistic predictions. This conclusion does not mean a low valuation of Bell’s work, which I consider one of the most important achievements in theoretical physics of the last 50 years. In any case Bell never used the word ”theorem” in this context, as far as I know.

In the modern approach, quantum measurement theory is not postulated but an attempt is made at deriving it from the quantum formalism. I shall analyze the results of this approach in the particular example of an optical photon counter. I write “counter” in order to distinguish it from other types of light detector. For instance, in astronomy a typical observational method is to take a photographic plate of some region of the sky. In this case the intensity of the light may be measured with a small error using a long time of exposure, but no count of individual photons is made. I also include the word “optical” because a single high energy photon (e. g. a gamma ray) has a large enough energy to be detected with a probability close to 100%. This is not the case with optical photons. A counter of optical photons consists of a macroscopic object (e. g. a piece of semiconductor) where there are quantum systems (e. g. electrons) in metastable states. Typically the metastable state appears because an external electric field is included which, combined with the potential due to the ions, creates a potential well where the electron is initially confined, separated by a barrier from another deeper well. When a photon arrives at the detector an electron may make a transition, via a state of the continuum, to the region of deeper potential where it starts moving, this giving rise to an electric current which is amplified by the action of the field. The important point is that, if the external electric field is too weak the amplification does not take place but if it is too strong some counts may be produced due to an electron crossing the potential barrier by tunnel effect. That is, a trade-off exists between increasing the efficiency (decreasing the false negative results) and decreasing the dark rate (the false positive counts). I am not in a position to prove that this fundamental trade-off is enough to prevent the existence of the optical photon counters required for loophole-free tests of a Bell inequality. However, it is equally difficult to prove rigorously that there are no fundamental constraints preventing optical photon counters reliable enough to allow loophole-free tests of local realism.

The current wisdom that the difficulties for manufacturing reliable optical photon counters are not fundamental derives from a theoretical prejudice, namely that optical photons are particles like electrons or atoms. If this were true there would be no reason why detectors could not be manufactured
having 100% efficiency and low noise. But I think that it is closer to the truth the assumption that photons are just quanta of the electromagnetic field, but not particles (Willis Lamb has supported strongly this opinion\textsuperscript{42}). There are two arguments, at least, against optical photons being particles similar to electrons or atoms. Firstly there is no position operator for photons in quantum mechanics, and secondly the photon number is usually not well defined. That is, common states of light, like laser light or thermal light, have an indefinite number of photons. A photon is (or should be associated to) a wavepacket in the form of a needle whose length is of the order of the coherence length, which for atomic emissions means centimeters, and several wavelengths in transverse dimensions. This associates a volume bigger than $10^{16}$ atomic volumes to a typical optical photon. In sharp contrast, a gamma ray photon may be associated to a volume smaller than that of an atom. If we take the atomic volume as standard, we are led to say that high energy photons are localized entities (behaving mainly as particles) whilst optical photons are not localized (behaving mainly as waves).

In any test of local realism using photons, it is necessary to measure both, the position of the photon and another quantity like polarization or phase. The former may be called a particle property whilst the latter is a wave property. Thus, if we remember the Bell inequality \textsuperscript{20}, it is natural to associate the parameters $\zeta$ (detection efficiency) and $V$ (visibility of the polarization correlation curve) to those two quantities and conclude that the Bell inequality forbids a photon behaving as a particle and as a wave at the same time. In contrast, the tested inequality \textsuperscript{23} just constrains the “amount of wave behaviour, V”. Thus its violation means that we cannot dismiss the wave character of optical photons. On the other hand, tests using gamma rays do not have any problem with the position measurement (i.e. the efficiency of detection), but there are difficulties for a precise measurement of polarization, as commented in section 6. Thus I propose that, in tests using photons, a trade-off exists between measurability of position and measurability of polarization, trade-off quantified by the Bell inequality \textsuperscript{20}. The “corpuscular” property (position), may be accurately measured only in photons much smaller than atoms, like gammas, the “wave” property (polarization), in those much larger than atoms, like optical photons.

These and other examples suggest that quantum mechanics may be compatible with local realism, the violations predicted deriving from the idealizations used in the standard calculations, like perturbative approximations,
neglect of tunneling, etc. Maybe the reader does not agree, but certainly the problem is open and the sober attitude is to analyze the empirical results without the common bias that the validity of local realism would imply that quantum theory is wrong.

VIII. Digression on philosophy and sociology of science

For the analysis of significance of the results obtained in the performed tests of local realism it is convenient to make a digression on philosophy and sociology of science. The pragmatic approach to quantum mechanics, which is the basis of the Copenhagen interpretation, has led to an “antimetaphysical” attitude, that is the idea that science should not be constrained at all by any philosophical principle. I think that this position is not correct. Of course, the philosophy of the natural world should rest upon knowledge derived from science, and not viceversa, but it is also true that science itself rests upon some philosophical principles.

One of the central principles of the philosophy of science is that there is not symmetry between confirmation and refutation of a theory. In fact, although a single experiment may refute a theory, no theory can ever be absolutely confirmed by experiments, a principle stressed by Karl Popper. Thus the only possibility to increase the degree of confidence in a theory is to perform many experiments able to refute it. If the results of these experiments are compatible with the theory, it becomes increasingly supported. We may apply this philosophy to the tests of Bell’s inequalities. As more time elapses without a loophole-free violation of local realism, greater should be our confidence on the validity of this principle.

Another philosophical point which is required in any serious discussion of the present status of local realism is that established theories are protected, a fact stressed by Imre Lakatos. That is, when a new discovery seems to contradict the theory, it is always possible to introduce some auxiliary hypotheses which allow interpreting the new finding within the accepted theory. It is well known the example put by Lakatos on the hypothetical observation of an anomaly in the motion of a planet. It could be explained, without rejecting Newton’s gravitational theory, by the existence of another, unknown, planet. If this is not found by observation in the predicted place, it might be assumed that there are two planets instead of one, etc. Indeed, it
is a historical fact that no theory has been rejected by its contradiction with a single or even several experiments (e.g., Newton’s gravity by the anomaly in the motion of Mercury). The theory survives until a new, superior, theory is available (e.g., Newton’s gravity survived until the appearance of general relativity.) The consequence of this sociological fact is that any argument for a established theory is accepted without too much discussion, but any argument against the theory is carefully analyzed in order to discover a flaw. Thus, even a honest experimentalist will devote much more care searching for possible errors if an experiment contradicts the assumed predictions of quantum mechanics than if it confirms the theory.

A good example of this behaviour has happened in the early, atomic-cascade, tests of Bell’s inequalities. As said in section 5 the first experiment of that kind was performed by Freedman and Clauser and the results agreed with quantum predictions. The second experiment was made by Holt and Pipkin (see, e.g., the reviews by Clauser and Shimony or by Duncan and Kleinpoppen.) The results of the experiment disagreed with quantum predictions but did not violate the inequality tested. The consequence is that the experimental results were never formally published and many people (including the authors) made a careful search for possible sources of error. The Holt-Pipkin experiment had two main differences with the Freedman-Clauser one: 1) the use of a cascade of atomic mercury, instead of calcium, and 2) the use of calcite polarizers, instead of polarizers made of piles of plates. In order to clarify the anomaly, Clauser “repeated” the Holt-Pipkin experiment, that is performed a new experiment using mercury but, again, piles of plates as polarizers. This time the results agreed with quantum predictions and violated the tested inequality. However the use of calcite may be very relevant because it has an extremely good extinction ratio, less than $10^{-4}$ to be compared with 0.02 for typical piles of plates. In contrast calcite possesses bad efficiency for maximum transmission of linear polarized light, about 80% to be compared with 98% for typical piles of plates. But there are arguments supporting the opinion that it is the minimal, and not the maximal, transmission of the polarizer what matters. In spite of this fact, the Holt-Pipkin experiment has never been repeated in the sense of using calcite polarizers.
IX. Present status of local realism at the empirical level

Now we arrive at the crucial question: Is local realism a valid principle of physics? The current wisdom is that it has been definitely refuted by the optical experiments already performed, modulo some loopholes due to nonidealities which, it is added, are quite common in experimental physics. But, as explained in section 5, this is not true for the atomic-cascade experiments (e.g. Aspect’s) because they do not discriminate between local realism and quantum mechanics, not even in the ideal case. We are left with experimental tests involving optical photons produced in the process of parametric down-conversion (e.g. the mentioned experiment by Kurtsiefer et al.\textsuperscript{21}). As discussed in section 5, these experiments cannot test (genuine) Bell inequalities due to the lack of reliable photon counters. If we exclude the down-conversion experiments, the evidence against local realism is meager because all other tests present greater difficulties. It is true that the efficiency loophole has been closed in experiments with atoms,\textsuperscript{31} what has been used as an argument against the validity of local realism.\textsuperscript{47} In my opinion the fact that loopholes appear in every experiment is an argument for it. Indeed, it suggests that nature preserves local realism in every case. Actually experiments like that of Rowe et al.\textsuperscript{31} do not test local realism but non-contextuality (see Sec. 2), something which is not a principle to be maintained.

In any case I claim that local realism is such a fundamental principle that should not be dismissed without extremely strong arguments. It is a fact that there is no direct empirical evidence at all for the violation of local realism. The existing evidence is just that quantum mechanical predictions are confirmed, in general, in tests of (non-genuine Bell) inequalities like (17) or (13). Only when this evidence is combined with theoretical arguments (or prejudices) it might be argued that local realism is refuted. But, in my opinion, this combination is too weak for such a strong conclusion. Thus I propose that no loophole-free experiment is possible which violates local realism.

This proposal remembers other negative statements, derived from failures at the experimental level, which have been extremely important in the history of physics. I shall put two examples. After James Watt made his heat engine in 1765, many people attempted to increase the efficiency, but in some sense
they failed. In fact, nobody was able to make a *perpetuum mobile* (of the second kind), that is an engine able to produce useful work by just cooling a large reservoir like the sea. It took sixty years to be realized, by Sadi Carnot, that the aim was impossible because a (large) part of the extracted heat should necessarily go to a colder reservoir. Carnot’s discovery led soon to the statement of one of the most important principles of physics: the *second law of thermodynamics*. Another example is the question of the absolute motion of the Earth. Several attempts at measuring it failed, the most sophisticated made by Michelson and Morley in 1887. The failure was “explained” less than 20 years later by Einstein with the hypothesis that absolute motion does not exist. Again, a repeated experimental failure led to a fundamental physical law: the *relativity principle*.

Ian Percival has pointed out that, in classical physics, the second law of thermodynamics does not contradict the laws of (Newtonian) mechanics, but nevertheless it restrict the possible evolutions of physical systems. He proposed that a similar physical principle might prevent the violation of local realism without actually contradicting quantum mechanics. In my view this is an interesting observation, because I presume that it is the second law, with quantum noise taken into account, what may prevent the violation of local realism in the quantum domain. I think that a better understanding of the laws of thermodynamics at the quantum level is required. Indeed, the traditional interpretation of the third law (zero entropy at zero Kelvin) seems difficult to be reconciled with the existence of (non-thermal) quantum vacuum fluctuations. In summary, a serious attention to the loopholes in the empirical tests of the Bell inequalities, rather than their uncritical dismissal, may improve our understanding of nature.

In any case the validity of local realism may be either refuted by a single loophole-free experiment or increasingly confirmed by the passage of time without such an experiment. This is the motivation for the title of the present article.
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