Anyonic strings and membranes in AdS space and dual Aharonov-Bohm effects

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Abstract

It is observed that strings in $AdS_5 \times S^5$ and membranes in $AdS_7 \times S^4$ exhibit long range phase interactions. Two well separated membranes dragged around one another in $AdS$ acquire phases of $2\pi/N$. The same phases are acquired by a well separated F and D string dragged around one another. The phases are shown to correspond to both the standard and a novel type of Aharonov-Bohm effect in the dual field theory.
1 Introduction

It is by now well established that point particles in 2+1 dimensions may have fractional, or anyonic, statistics [1, 2, 3, 4]. These are particles that do not obey either Bose-Einstein or Fermi-Dirac statistics. Under exchange of two identical anyons, the wavefunction of the system does not get multiplied by ±1, but may rather change by an arbitrary, fixed, phase. A physical example of anyons are the quasiparticle or quasihole excitations of a fractional quantum hall fluid [5].

Anyons may be physically understood as particles with a long range phase interaction. In 2+1 dimensions a particle may be charged under an abelian Chern-Simons gauge potential. The Chern-Simons interaction attaches a magnetic flux to all electric point charges. Therefore when two such charges are dragged around one another, at arbitrarily large distances, they acquire a phase through the Aharonov-Bohm effect.

The possibility of anyonic phases in two spatial dimensions is directly associated with the topology of the two particle configuration space. In particular, \( H_1(\mathbb{R}^2 \setminus \{0\}) = \mathbb{Z} \), so the path of a particle that is dragged around another is topologically nontrivial. In higher dimensions, these paths are homologically trivial and hence anyonic phases are not possible for point particles. However, higher dimensional objects such as strings and membranes do have nontrivial configuration spaces in higher dimensions. In particular, \( H_2(\mathbb{R}^4 \setminus \mathbb{R}) = H_3(\mathbb{R}^6 \setminus \mathbb{R}^2) = \mathbb{Z} \), which implies that strings in 4+1 dimensions and membranes in 6+1 dimensions can describe topologically nontrivial cycles as they are dragged around one another. Some previous work on the configuration space of strings in 3+1 dimensions may be found in [6, 7]. The 3+1 dimensional analogue of the physics to be considered here would involve a string linking a point source, c.f. [8].

The next observation is that two of the most basic backgrounds of string and M theory, the \( AdS_5 \times S^5 \) solution of type IIB string theory and the \( AdS_7 \times S^4 \) solution of M theory, realise the potential for anyonic behaviour of strings and membranes. The strings/membranes need to be well separated relative to the size of the sphere in the background, so that the physics is effectively five/seven dimensional, respectively. The two ingredients that make nontrivial phases possible are firstly the nonlinear Chern-Simons like terms in the supergravity actions and secondly the presence of a background flux in the solution. The resulting mechanism is essentially identical to that for point charges in 2+1 dimensional Chern-Simons theory.

These \( AdS \) backgrounds are of special interest because they are supergravity duals of conformal field theories [9]. After explicitly exhibiting the advertised long range phase
interactions in these backgrounds below, we will show that, in the case when one of the
strings or membranes reaches the boundary of AdS, then the bulk anyonic behaviour has a
dual interpretation as a nonabelian Aharonov-Bohm effect. If neither of the objets reach the
boundary, we show how the phase may be dually understood as a novel type of Aharonov-
Bohm effect involving intersecting flux tubes.

2 Anyonic membranes in AdS$_7 \times S^4$

Start with the case of a single M2 brane coupled to eleven dimensional supergravity. The
physics we are studying only depends upon the action for the three form $C$ field. Letting
$G = dC$, the required action is

$$S_{\text{field}} = -\frac{1}{4\kappa_{11}^2} \int \left( G \wedge \star G + \frac{1}{3} C \wedge G \wedge G \right) + T_{M2} \int_{M2} C. \quad (1)$$

Using $2\kappa_{11}^2 = (2\pi)^8 l_{11}^8$ and $T_{M2}^{-1} = (2\pi)^2 l_{11}^2$ one obtains the equations of motion

$$d \star G + \frac{1}{2} G \wedge G - (2\pi l_{11})^6 \delta(x) = 0. \quad (2)$$

Here $l_{11}$ is the eleven dimensional Planck length and $\delta(x)$ is the Poincaré dual eight form
to the worldvolume of the membrane. Recall that the Poincaré dual is defined by

$$\int_{M2} C = \int C \wedge \delta(x), \quad (3)$$

for any $C$. The second integral here is over the whole spacetime.

We wish to consider the effect of a single membrane in the AdS$_7 \times S^4$ background. This
background has flux

$$G^{(0)} = 3\pi N l_{11}^3 \text{vol}_{S^4}, \quad (4)$$

where $\text{vol}_{S^4}$ is the volume form on a unit radius four sphere. The effect of the membrane
is to shift this background flux by a small amount. Let us set $G \rightarrow G^{(0)} + G$ and work to linearised order in $G$. The equation of motion becomes

$$d \star G + N(2\pi l_{11})^3 \frac{3}{8\pi^2} \text{vol}_{S^4} \wedge G - (2\pi l_{11})^6 \delta(x) = 0. \quad (5)$$

Place this membrane in the AdS$_7$ part of the geometry.

Now consider a second membrane in the same background. Let us adiabatically drag this
membrane along a closed spatial curve that is within AdS$_7$ and encircles the first membrane,
ending up back where it started. This is well defined as the first membrane is occupying
two spatial dimensions and is therefore a point in the transverse four spatial dimensions
Figure 1: $M2$ denotes the first membrane and $\partial \Sigma$ is the worldvolume traced out by the second membrane.

in $AdS_7$, that can be encircled by a three volume. Denote by $\partial \Sigma$ the worldvolume of this second membrane. This process is illustrated in figure 1.

More precisely, a spatial slice of $AdS_7$ has topology $\mathbb{R}^6$. Removing the space occupied by the first membrane gives topology $\mathbb{R}^6 \setminus \mathbb{R}^2$. However $H_3(\mathbb{R}^6 \setminus \mathbb{R}^2) = \mathbb{Z}$, and it is this nontrivial three cycle we are dragging the second membrane around.

The phase acquired by the partition function of the second membrane upon completion of the closed loop is computed as follows

$$\Delta \Phi_M = \frac{2\pi}{(2\pi l_{11})^3} \int_{\partial \Sigma} C$$

$$= \frac{2\pi}{(2\pi l_{11})^3} \int_{\partial \Sigma \times S^4} C \wedge \text{vol}_{S^4} \quad \text{[as } R \to \infty]\right]$$

$$= \frac{2\pi}{(2\pi l_{11})^3} \int_{\Sigma \times S^4} G \wedge \text{vol}_{S^4}$$

$$= -\frac{2\pi}{N (2\pi l_{11})^6} \int_{\Sigma \times S^4} [d \ast G - (2\pi l_{11})^6 \delta(x)]$$

$$= \frac{2\pi}{N} - \frac{2\pi}{N (2\pi l_{11})^6} \int_{\partial \Sigma \times S^4} \ast G. \quad (6)$$

In the second step we have required that the minimum spatial distance between the two membranes, $R$, be large. At large distances, the $C$ field in $AdS_7$ sourced by the first membrane is independent of the coordinates on the $S^4$. This fact permits us to wedge $C$ with $\text{vol}_{S^4}$. The higher harmonics of the $C$ field decay exponentially at spatial distances beyond the size of the $S^4$, which is the $AdS$ scale, due to the Laplacian term $d \ast dC$ in (5). In the appendix we have exhibited this exponential falloff explicitly by solving a toy
model for (5): a point charge in $\mathbb{R}^{1,2} \times S^1$ with analogous Chern-Simons like couplings. This intuition is not confined to flat space. The higher harmonics acquire masses in $AdS_7$, and massive fields on $AdS$ decay exponentially in geodesic distance, see e.g. [10, 11, 12].

In order to evaluate the expression for the phase (6), we should solve the equation of motion (5) for $G$. However, we can immediately note that at long distances from the source membrane, the topological mass term in (5) is important and the field strength dies off exponentially. Hence the flux term in the last line of (6) is negligible. This falloff in $G$ occurs even for the homogeneous mode on $S^4$ (the homogeneous $C$ that survives at long distance has $G = dC = 0$). Thus we obtain the phase

$$\Delta \Phi_M = \frac{2\pi}{N} \quad \text{[as } R \to \infty\text{].}$$

(7)

Rephrasing this result in terms of membrane exchange, in which the second membrane is only taken ‘half way’ around the first, and adding together the phases acquired by the two membranes, we have found that well separated membranes in $AdS_7 \times S^4$ obey anyon statistics with angle $2\pi/N$.

At short distances the full eleven dimensional geometry must be considered and there is no effective linking of membranes. Furthermore, the derivative term in (5) will be important. It may seem surprising that the phase depends on the distance between the membranes. However, the same behaviour occurs in Maxwell-Chern-Simons theory in three dimensions [13]. The effect of the Maxwell term is to give the magnetic flux tubes attached to the anyons a finite width. When the charges are close enough together for these to overlap, the fractional statistics is lost. The characteristic feature of anyons in any case is the long (infinite) range phase interaction, independently of short distance physics.

To gain intuition for this system, it is useful to solve the equation for a topologically massive four form field strength on $\mathbb{R}^{1,6}$. This is done in the appendix. The exponential decay of the flux at large distances may be seen very explicitly. The propagator for a topologically massive form in $AdS$ has been computed in [14] and exhibits exponential decay with geodesic distance.

3 Long range phase interaction for strings in $AdS_5 \times S^5$

Consider a single D or F string coupled to type IIB supergravity. We are interested in an effect due to the two form potentials in the theory. The action for these forms, in Einstein
frame, is
\[ S_{B_2,C_2} = -\frac{1}{4\kappa_{10}^2} \int \left( e^{-\Phi} H_3 \wedge \star H_3 + e^\Phi \tilde{F}_3 \wedge \star \tilde{F}_3 + \frac{1}{2} \tilde{F}_5 \wedge \star \tilde{F}_5 + C_4 \wedge H_3 \wedge F_3 \right) \]
\[ - T_{F1} \int_{F1} B_2 - \mu_1 \int_{D1} (C_2 - CB_2) , \]
(8)
where \( H_3 = dB_2, F_3 = dC_2, \tilde{F}_3 = F_3 - CH_3 \) and \( \tilde{F}_5 = F_5 - C_2 \wedge H_3 \). The self duality condition \( \tilde{F}_5 = \star \tilde{F}_5 \) is imposed on the equations of motion. We have included contributions from both an F and D string to the action. To start with, we are interested in one or the other.

The equations of motion for the two form potentials \( C_2 \) and \( B_2 \) are found to be
\[ d \star \left( e^\Phi \tilde{F}_3 \right) - F_5 \wedge H_3 - 2\kappa_{10}^2 \mu_1 \delta(x_D1) = 0 , \]
(9)
and
\[ d \star \left( e^{-\Phi} H_3 - C e^\Phi \tilde{F}_3 \right) + F_5 \wedge F_3 - 2\kappa_{10}^2 T_{F1} \delta(x_{F1}) + 2\kappa_{10}^2 \mu_1 C \delta(x_D1) = 0 . \]
(10)
In these expressions, \( \delta(x) \) denotes the Poincaré dual eight form to the string worldvolume.

We want to consider the effect of a single D or F string in the \( AdS_5 \times S^5 \) background. This background has five form flux
\[ F_5^{(0)} = 16\pi N l_s^4 \left( \text{vol}_{S^5} + \star \text{vol}_{S^5} \right) , \]
(11)
and furthermore has constant dilaton and axion: \( e^\Phi = g \) and \( C = \theta/2\pi \). Here \( \text{vol}_{S^5} \) is the volume form on an \( S^5 \) of unit radius and \( l_s \) is the string length. The string source introduces a small amount of three form flux which we describe by linearising the equations of motion above. Using \( T_{F1} = \mu_1 = 1/(2\pi l_s^2) \) and \( 2\kappa_{10}^2 = (2\pi)^7 l_s^8 \), and writing the equations in a more symmetric way, gives
\[ g d \star \tilde{F}_3 - \frac{N(2\pi l_s)^4}{\pi^3} (1 + \star) \text{vol}_{S^5} \wedge H_3 - (2\pi l_s)^6 \delta(x_D1) = 0 , \]
(12)
and
\[ \frac{1}{g} d \star H_3 + \frac{N(2\pi l_s)^4}{\pi^3} (1 + \star) \text{vol}_{S^5} \wedge \tilde{F}_3 - (2\pi l_s)^6 \delta(x_{F1}) = 0 . \]
(13)
Note that here
\[ \tilde{F}_3 = F_3 - \frac{\theta}{2\pi} H_3 . \]
(14)

From the structure of the equations of motion (12) and (13), we see that we can obtain nontrivial phases for strings in \( AdS_5 \) in the same way as we did for membranes in \( AdS_7 \). This will occur if we drag an F string around a D string, or vice versa. However, there is
no phase associated with dragging an F string about another F string or a D string about another D string. For two D strings for instance, we can compute the phase as follows

\[ \Delta \Phi_D = -\frac{2\pi}{(2\pi l_s)^2} \int_{\partial \Sigma} \left( C_2 - \frac{\theta}{2\pi} B_2 \right) \]
\[ = -\frac{2\pi}{(2\pi l_s)^2} \frac{1}{\pi^3} \int_{\Sigma \times S^5} \tilde{F}_3 \wedge \text{vol}_{S^5} \quad \text{[as \ } R \to \infty] \]
\[ = -\frac{2\pi}{Ng(2\pi l_s)^6} \int_{\partial \Sigma \times S^5} \star H_3. \quad (15) \]

We have used the same notation as previously for membranes. Now \( \partial \Sigma \) is a two cycle surrounding the first D string in \( AdS_5 \). This is well defined as \( H_2(\mathbb{R}^4 \setminus \mathbb{R}) = \mathbb{Z} \). Unlike the previous case of membranes, this flux does not decay exponentially at large distances but is instead identically zero. This can be seen by considering the electric and magnetic parts

\[ H_3 = E \wedge \sqrt{-g} dt + B \quad \text{and} \quad \tilde{F}_3 = \tilde{E} \wedge \sqrt{-g} dt + \tilde{B}, \]

where \( t \) is a static time coordinate for \( AdS \). From the equations of motion we have that

\[ \frac{1}{g} \star_9 E - \frac{N(2\pi l_s)^4}{\pi^3} (1 + \star) \text{vol}_{S^5} \wedge \tilde{B} = 0, \]
\[ gd \star_9 \sqrt{-g} dt \tilde{B} + \frac{N(2\pi l_s)^4}{\pi^3} (1 + \star) \text{vol}_{S^5} \wedge \sqrt{-g} dt E = 0. \quad (16) \]

There are no source terms in these equations, which are therefore solved by \( E = \tilde{B} = 0 \). It follows from (15) that \( \Delta \Phi_D = 0 \).

As for the membranes, more complete intuition for the coupled equations of motion (12) and (13) may be gained from solving the equations for similarly coupled topologically massive three form field strengths on \( \mathbb{R}^{1,4} \). This is done in the appendix.

Considering \( (p, q) \) strings does not alter the situation. The relative minus sign between the topological mass terms in (12) and (13) means that the effects of the F and D charges cancel and there is no long range phase interaction. Alternatively, this follows from \( SL(2, \mathbb{Z}) \) duality of the D string results. Therefore, the various strings in \( AdS_5 \) do not have anyon statistics. However, there is an infinite range phase interaction between F and D strings. In this sense we might call these strings anyonic. The phase shift in the partition function of a D string dragged around an F string is

\[ \Delta \Phi_{F-D} = -\frac{2\pi}{(2\pi l_s)^2} \frac{1}{\pi^3} \int_{\Sigma \times S^5} \tilde{F}_3 \wedge \text{vol}_{S^5} \quad \text{[as \ } R \to \infty] \]
\[ = -\frac{2\pi}{N} - \frac{2\pi}{gN(2\pi l_s)^6} \int_{\partial \Sigma \times S^5} \star H_3, \quad (17) \]

where \( \partial \Sigma \) is as usual the worldvolume swept out by the D string. The electric flux contribution in the previous equation is determined from (16) except that now there is a delta
function source term due to the F string in the background. As for the case of membranes, the topological mass terms imply that the flux decays exponentially at large distances giving

\[
\Delta \Phi_{F-D} = \frac{2\pi}{N} \quad \text{[as } R \to \infty\text{].}
\]  

\[\text{(18)}\]

4 Dual field theory implications

The various membranes and strings we have discussed are dual to specific states (operators) in the dual conformal field theories. The F and D strings are dual to electric and magnetic flux tubes, respectively [15]. The M2 branes are dual to interesting analogous extended field configurations in the six dimensional (2,0) theory.

A simple case to map to the dual theory is when one of the strings or membranes extends to the \(AdS\) boundary. In this case the endpoint of the string or membrane is an external point or string charge, respectively, in the dual theory. Consider the case of an F string that extends to the boundary. Dragging this string around a D string in the bulk is mapped onto an external electric charge that is dragged around a magnetic flux tube on the boundary. Such a charge will acquire a phase due to the (nonabelian) Aharonov-Bohm effect.

In fact, we can recover precisely the phase we found from the bulk computation. The total flux along the magnetic flux tube is \(2\pi\), which follows from the Dirac quantisation condition. However, this flux is in the (decoupled) diagonal \(U(1)\) of \(U(N)\), so we can write it as \(2\pi/N\) times the identity \(N \times N\) matrix. The external electric charge is in some specific \(U(1)\) of \(SU(N)\) and therefore is only sensitive to one component of the flux. Thus from the Aharonov-Bohm effect we obtain the phase \(2\pi/N\), in agreement with the bulk computation.

The Aharonov-Bohm setup just described is illustrated on the left hand side of figure 2. Note that the string needs to return to the boundary as it has nowhere to end in the \(AdS\) bulk. The figure depicts an F string that extends to the boundary being dragged around a D string in the bulk. Note also that the surface linking the D string is not strictly closed, as it does not close off at the boundary. However, this finite sized ‘hole’ is infinitely far away from the source and no flux escapes through it.

The right hand side of figure 2 illustrates the case in which the F string does not extend to infinity. Projecting the motion onto the boundary, this implies that the magnetic and electric flux tubes intersect twice, as one is dragged through the other. We will now understand why this process picks up a phase.

The first step is to note that an infinitessimally thin electric flux tube dragged adiabatically around a closed loop, to give a surface \(\Sigma\), acquires a phase \(\int_{\Sigma} F\). This can been
Figure 2: Schematic representation of $AdS_5$ together with its conformal boundary. Time and one spatial dimension are suppressed. The dot denotes a D string and its projection onto the boundary. The lines denote an F string dragged around the D string. In (a) the F string extends to the boundary of $AdS$, whereas in (b) it does not.

seen as a consequence of the fact that adding this term to the action results in the Maxwell equation

$$d \star F = d \delta(x).$$

This is solved by $\star F = \delta(x)$, which precisely describes a flux tube at $x$ carrying an electric field. Therefore, $\int F$ is the correct action for an electric flux tube and leads to a phase in the partition function if the flux tube is dragged along some surface $\Sigma$. Similarly, the action for a magnetic flux tube is $2\pi \int \star F$.

A test of the proposed phase is to consider a reverse Aharonov-Bohm effect, in which a magnetic flux tube is dragged around an electric charge. Then we obtain $2\pi \int_{\Sigma} \star F = 2\pi$, recovering the same phase as in the standard Aharonov-Bohm setup, as we should expect.

Given this phase, we can consider the process of interest in figure 2b. The spatial worldsheet $\Sigma$ of the electric flux tube intersects the magnetic flux tube twice. These intersections contribute to the phase $\int_{\Sigma} F$. As the intersections have opposite orientation, the total magnetic flux through $\Sigma$ is zero. However, the F string is located at different radial locations in $AdS$ at the two intersections. This implies that the cross sectional width of the flux tube is different at the two points, being thinner when the F string is nearer the boundary [15]. In particular, if $L_{1,2}^{(e)}$ denotes the width of the electric flux tube at the two intersections and
$L^{(m)}$ denotes the constant width of the magnetic flux tube, then we have

$$L_1^{(e)} \ll L^{(m)} \ll L_2^{(e)},$$

(20)
corresponding to the fact that the dual strings are separated at radial distances much larger than the AdS scale.

The formula for the phase we presented assumes a thin flux tube. For the intersection near the boundary we have $L_1^{(e)} \ll L^{(m)}$: the electric flux tube is much thinner than the magnetic flux passing through it. The formula should be reliable here. However, at the other intersection, $L^{(m)} \ll L_2^{(e)}$ and the electric flux is much more spread out than the magnetic flux. This suggests that we should swap pictures and view this second intersection as a thin magnetic flux tube that is being dragged (in the opposite direction) through electric flux. We need to carefully keep track of relative orientations as we do this, as illustrated in figure 3. The magnetic flux tube will then pick up a phase $\int \ast F$, the electromagnetic dual of the phase for the electric tube. Now recall that under electromagnetic duality $E \rightarrow B$ but $B \rightarrow -E$. Applying this map to the new setup in figure 3, we find that the phase acquired is the same as that from the first intersection. In particular, the sign is the same. The phases from the two intersections add and do not cancel.

**Figure 3:** The top line shows the two intersections of the electric flux tube with magnetic flux. The second line shows how one of these should be reinterpreted as a magnetic flux tube intersected by electric flux.

We can be more quantitative, and recover the phase found from the bulk computation. We have seen that the two intersections give the same phase, so focus on one of them. The
phase acquires a $2\pi$ as previously from the relative normalisation of electric and magnetic flux. A $1/N$ is acquired due to two factors of $1/N$ coming from the distribution of the flux in each flux tube amongst $N$ diagonal entries of the $U(N)$ matrix, as above, together with a factor of $N$ coming from summing over the contributions from each colour. Thus each intersection gives $2\pi/N$ and the total phase is $4\pi/N$. This is also the total phase obtained in the bulk, where there is a $2\pi/N$ phase for the partition function of each string. An assumption underlying this matching is the following: given that at each intersection one of the tubes was very spread out and hence thought of as background flux, the total phase acquired at each intersection is $2\pi/N$, as opposed to this phase being acquired by each of the tubes separately (which would lead to a total phase of $8\pi/N$).

The case of membranes works identically. For instance, the string boundary of a membrane ending on the M theory fivebrane is self dual under the two form potential of the fivebrane theory. There is thus only one basic flux tube in the theory, and an external self dual string source dragged around such a flux tube will acquire an Aharanov-Bohm phase.

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**A Topologically massive forms in flat space**

**A.1 Particle source in $\mathbb{R}^{1,2} \times S^1$**

Write the metric as

$$ds^2 = -dt^2 + dr^2 + r^2d\phi^2 + d\theta^2.$$  \hspace{1cm} (21)

Take $\theta$ to have period $2\pi$. Place the point source at $r = \theta = 0$. We want to solve the following equation for the one form potential $A$:

$$d \ast F + d\theta \wedge F + 4\pi^2 \delta(x) = 0,$$  \hspace{1cm} (22)

where $F = dA$. The equation may be straightforwardly solved in terms of Bessel functions

$$A = (1 - K_1(r)r) \, d\phi + K_0(r) \, dt$$

$$+ \sum_{m \geq 1} C_{m \pm} e^{im \theta} \left[ \pm K_1 \left( \sqrt{m_{\pm}^2 + \frac{1}{4} \pm \frac{1}{2}}r \right) r \, d\phi + K_0 \left( \sqrt{m_{\pm}^2 + \frac{1}{4} \pm \frac{1}{2}}r \right) \right] dt \hspace{1cm} (23)$$
The coefficients of the inhomogeneous modes on the $S^1$ are determined by imposing the correct boundary conditions at the delta function source. At large $r$ all these modes fall off exponentially, leaving the homogeneous magnetic term $d\phi$. Note that $dF = 0$ everywhere.

**A.2 Membrane source in $\mathbb{R}^{1,6}$**

Write the metric as

$$ds^2 = dx^2_{1,2} + dr^2 + r^2d\Omega^2_{S^3}.$$  \hfill (24)

Place the source membrane along the $x$ directions. We want to solve the following equation for the four form $G$

$$d \ast G + G + 2\pi^2 \delta(x) = 0.$$  \hfill (25)

Splitting $G$ into an electric and magnetic part

$$G = A dt \wedge dx \wedge dy \wedge dr + B dr \wedge r^3 \text{vol}_{S^3},$$  \hfill (26)

it is easy to solve the equation in terms of Bessel functions

$$A = \frac{K_1(r)}{r^2} + \frac{K_0(r)}{2r},$$

$$B = -\frac{K_1(r)}{2r}.$$  \hfill (27)

Both of these functions decay exponentially at large $r$. At small $r$ there is electric flux due to the ‘Maxwell’ term. The phase $\int_{B^4} G$ goes from 0 as $r \to 0$ to $-2\pi^2$ as $r \to \infty$. Note that $dG = 0$ everywhere.

**A.3 D string source in $\mathbb{R}^{1,4}$**

Write the metric

$$ds^2 = dx^2_{1,1} + dr^2 + r^2d\Omega^2_{S^2},$$  \hfill (28)

and place the D string source along the $x$ directions. We need to solve the following coupled equations

$$d \ast H_3 + \tilde{F}_3 = 0,$$

$$d \ast \tilde{F}_3 - H_3 + 4\pi \delta(x) = 0.$$  \hfill (29)

It is straightforward to obtain the solution

$$\tilde{F}_3 = \frac{(1+r)e^{-r}}{r^2} dt \wedge dx \wedge dr,$$

$$H_3 = \frac{e^{-r}}{r} dr \wedge r^2 \text{vol}_{S^2}.$$  \hfill (30)
Again we find an exponential decay at large $r$. The D string has sourced $\tilde{F}_3$ electrically as usual, but has also sourced $H_3$ magnetically.

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