Deep Learning Based Pilot Design for Multi-user Distributed Massive MIMO Systems

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Abstract—This letter proposes a deep learning based pilot design scheme to minimize the sum mean square error (MSE) of channel estimation for multi-user distributed massive multiple-input multiple-output (MIMO) systems. The pilot signal of each user is expressed as a weighted superposition of orthonormal pilot sequence basis, where the power assigned to each pilot sequence is the corresponding weight. A multi-layer fully connected deep neural network (DNN) is designed to optimize the power allocated to each pilot sequence to minimize the sum MSE, which takes the channel large-scale fading coefficients as input and outputs the pilot power allocation vector. The loss function of the DNN is defined as the sum MSE, and we leverage the unsupervised learning strategy to train the DNN. Simulation results show that the proposed scheme achieves better sum MSE performance than other methods with low complexity.

Index Terms—Pilot design, deep learning, deep neural network (DNN), multi-user distributed massive MIMO.

I. INTRODUCTION

Distributed massive multiple-input multiple-output (MIMO) systems have emerged as a promising antenna topology in 5G wireless communication, due to its advantage in improving spectral and energy efficiency, as well as lowering cutoff rate [1]. Channel state information (CSI) is essential to most of signal processing in wireless communication, such as signal detection and beamforming. It is usually acquired by transmitting pilot sequences. However, due to limited time and frequency resources, some users have to reuse the same orthogonal pilot sequences, which leads to pilot contamination. Hence, more and more people devote themselves to the research of alleviating pilot contamination in distributed massive MIMO systems.

To the best of our knowledge, the most effective method to alleviate pilot contamination is optimizing pilot assignment schemes among users. The optimal pilot assignment scheme can be obtained by exhaustively searching all possible cases. However, the exponential complexity of exhaustive search makes it impractical. To achieve a tradeoff between the performance and complexity, some heuristic algorithms were proposed to allocate pilot sequences based on the channel covariance matrix [2], [3]. Unfortunately, most of heuristic algorithms can not achieve a satisfying tradeoff in a practical system. Recently, [4] proposed a novel pilot design where the pilot signals are a weighted superposition of orthonormal pilot basis vectors, and treated the associated pilot power coefficients as continuous variables, which can be optimized by utilizing optimization theory. However, the complexity of above-mentioned methods are still very high.

With the rapid development of deep learning, it has gradually become a promising tool in solving difficult wireless communication problems due to its excellent performance and low complexity, such as resource allocation [5], channel decoding [6] and channel estimation [7], [8], [7] designed a supervised learning based pilot assignment scheme for a massive MIMO system, in which the network is trained with the optimal pilot assignment results of exhaustive search serving as the ground truth. However, this method is applicable only when the search space is small because of the exponential complexity of exhaustive search.

In this letter, we propose a pilot design problem, which optimizes the power allocated to each pilot sequence for each user, to minimize the sum mean square error (MSE) of channel estimation for a distributed massive MIMO system. It is a NP-hard problem, and we propose an unsupervised learning based pilot power allocation scheme to solve it. Specifically, a deep neural network (DNN) is designed to train the mapping from the input (channel large-scale fading coefficients) to the output (pilot power allocation vector). In the training process, we select the sum MSE as the loss function, and continually train the network parameters to adjust the output, further minimizing the sum MSE. Compared to the supervised learning scheme adopted in [7], the ground truth is not necessary in unsupervised learning, thus the proposed pilot power allocation scheme can be applied to the case with large number of users. Simulation results show that the proposed scheme achieves better performance than traditional pilot assignment scheme with lower complexity.

The notations used in this paper are conformed to the following convention. Boldface letters stand for matrices (upper case) or vectors (lower case). The transpose and conjugate transpose are denoted by $(\cdot)^T$ and $(\cdot)^H$ respectively. $\mathbf{I}_M$ stands for the $M \times M$ identity matrix, and $\mathcal{N}(-\mu, \sigma^2)$ denotes the circularly symmetric complex Gaussian distribution with mean $\mu$ and variance $\sigma^2$.

II. SYSTEM MODEL

This paper considers a multi-user (MU) distributed massive MIMO system. There are $M$ remote antenna units (RAUs) equipped with $N$ antennas and $K$ single-antenna users that share the same bandwidth in the cell. We let $\mathcal{M} = \{1, \ldots, M\}$ and $\mathcal{K} = \{1, \ldots, K\}$ represent the set of RAUs and users respectively.
A. Channel Model

This paper considers a block flat-fading channel, and the channel vector from user \( k \) to all RAUs can be modeled as
\[
g_k = \Lambda_k^{1/2} h_k \in \mathbb{C}^{MN \times 1},
\]
with
\[
\Lambda_k = \text{diag}(\{\lambda_{k,1} \cdots \lambda_{k,M}\}^T) \otimes I_N \in \mathbb{C}^{MN \times MN}
\]
where \( \lambda_{km} \) represents the large-scale fading between user \( k \) and RAU \( m \); \( d_{km} \) is the distance from user \( k \) to RAU \( m \); \( \zeta \) is the path loss exponent; \( s_{km} \) is a log-normal shadow fading variable. In addition, \( h_k \in \mathbb{C}^{MN \times 1} \) models small-scale fast fading, which follows \( \mathcal{N}(0,1) \). As widely recognized in most of literature, we also assume that the large-scale fading \( \Lambda_{km} \) is known to RAU \( m \), and the small-scale fading \( h_{km} \) needs to be estimated at RAU \( m \).

B. Uplink Channel Estimation

It is assumed that there are \( \tau \) orthogonal pilot sequences to be used for uplink channel estimation with \( \tau < K \), which are denoted by the mutually orthonormal basis vectors \( \{s_1 \cdots s_{\tau}\} \), where \( s_b \in \mathbb{C}^\tau \) is a vector whose \( b \)th element is one, while all other elements are zero. Using the pilot design in [4], the pilot vector of user \( k \) can span arbitrarily over the above \( \tau \) basis vectors with different power coefficients, which can be shown as
\[
\phi_k \in \mathbb{C}^{\tau \times 1} = \sum_{b=1}^{\tau} p_b^k s_b, \forall k,
\]
where \( p_b^k \) is the power that user \( k \) assigns to the \( b \)th pilot basis. The received signal \( Y \in \mathbb{C}^{MN \times \tau} \) at all RAUs is given by
\[
Y = \sum_{k=1}^{K} g_k \phi_k^H + N,
\]
where \( N \in \mathbb{C}^{MN \times \tau} \) is spatially independent additive white gaussian noise (AWGN) with zero mean and variance \( \sigma_n^2 \).

Correlating \( Y \) with the pilot signal \( \phi_k \) of user \( k \), we can obtain
\[
y_k = Y \phi_k = \sum_{j=1}^{K} \sqrt{p_b^k} g_j + N \phi_k.
\]

Considering minimum mean-square error (MMSE) channel estimator, we have
\[
\hat{g}_k = \sum_{b=1}^{\tau} p_b^k \Lambda_k Q_k^{-1} y_k,
\]
where
\[
Q_k = \sum_{j=1}^{K} \left( \sum_{b=1}^{\tau} \sqrt{p_b^k p_j^b} \right)^2 \Lambda_j + \sigma_n^2 \sum_{b=1}^{\tau} p_b^k I_{MN}
\]
Furthermore, the mean square error (MSE) of \( \hat{g}_k \) is
\[
\text{MSE}_k = \frac{\sum_{m=1}^{M} \tau_{km}}{\sum_{m=1}^{M} \tau_{km}} \sum_{b=1}^{\tau} p_b^k \sum_{j=1}^{K} \left( \sum_{b=1}^{\tau} \sqrt{p_b^k p_j^b} \right)^2 \Lambda_j + \sigma_n^2 \sum_{b=1}^{\tau} p_b^k I_{MN},
\]
where \( \tau_{km} \) is one, while \( \tau_{km} \) needs to be estimated at RAU \( m \), and the small-scale fading \( h_{km} \) needs to be estimated at RAU \( m \).

C. Proposed Problem

In this paper, we want to optimize the pilot power allocation \( \{p_b^k\} \) to minimize the sum MSE of channel estimation. With the assumption that the total pilot transmit power for each user is a predefined constant, the sum MSE minimization problem can be stated as
\[
P_0 : \min \{\sum_{k=1}^{K} \sum_{m=1}^{M} \tau_{km} \mid p_b^k\} \quad \text{s.t.} \quad \sum_{b=1}^{\tau} p_b^k = p_{tot}^k, \forall k,
\]
where \( p_{tot}^k \) is the total pilot transmit power for user \( k \). The considered problem is NP-hard because of the nonconvexity of the objective. In next section, we propose to design a DNN to solve the problem in an end-to-end fashion. In the proposed scheme, the nonlinear mapping from the channel large-scale fading coefficients \( \{\lambda_{km}\} \) to the optimal pilot power allocation \( \{p_b^k\} \) is learned by training the DNN with the sum MSE serving as the loss function.

Remark 1: Tradition pilot assignment problem assumes that the total pilot transmit power for each user is fully assigned to one pilot sequence, which is a special case of the considered problem \( P_0 \).

III. UNSUPERVISED LEARNING BASED PILOT POWER ALLOCATION

A. Network Design

A fully connected DNN is exploited to address the pilot power allocation problem \( P_0 \). The network structure is illustrated in Fig. 1. Specifically, the network consists of one input layer of \( KM \) nodes, one output layer of \( K \tau \) nodes and \( L-1 \) fully connected hidden layers. Let \( \mathcal{L} = \{0, \cdots, L\} \) represent the set of layers, where \( l = 0 \) and \( l = L \) denote the input layer and output layer respectively. The number of nodes of each layer \( l \in \mathcal{L} \) is denoted by \( n_l \), and we have \( n_0 = K M \) and \( n_L = K \tau \).

The input of the network is formed by aligning the channel large-scale fading coefficients \( \{\lambda_{km}\} \) as a column vector, denoted as \( q = [q_1^T, \cdots, q_K^T]^T \), where \( q_k = [\lambda_{k,1}, \cdots, \lambda_{k,M}]^T \). The output of the network is the pilot power allocation vector \( p = [p_1^T, \cdots, p_K^T]^T \), where \( p_k = [p_1^k, \cdots, p_{\tau}^k]^T \). For each hidden layer \( l \), the output of the network is calculated as follows
\[
o_l = \begin{cases} \text{ReLU} (\text{BN} (W_l q)), & l = 1, \\
\text{ReLU} (\text{BN} (W_l o_{l-1} + b_l)), & l \in \{2, \cdots, L-1\},
\end{cases}
\]
It can be easily proved that \( \sum_{j=1}^{\tau} \sigma(t_k)_j = 1 \). According to (11), we have \( \sum_{b=1}^{\tau} p_{k_b} = p_{k}^{\text{tot}} \), which is just the constraint (7b) in \( P_0 \).
huge volumes of channel large-scale fading coefficients data for training and testing, where the path loss exponent $\zeta$ is 3, and the variance of shadow fading is 6dB. For the sake of simplicity, the total pilot transmit power for all users $P_k^{\text{tot}}$ are 6 Watts, and the noise power is set as $10^{-8}$ Watts. For comparison, we consider the following three methods.

- **Average Pilot Power Allocation (APPA):** The total pilot transmit power for each user is apportioned equally among $\tau$ orthogonal pilot sequences.
- **Exhaustive Search based Pilot Assignment (ESPA):** The total pilot transmit power for each user is fully assigned to one pilot sequence. Actually, it is just the traditional pilot assignments problem. We exhaustively search all possible pilot assignment schemes to find the optimal one.
- **Random Pilot Assignment (RPA):** Similar to ESPA method, each user can only be assigned one pilot sequence, and we randomly select a pilot assignment scheme.

Fig. 3 and Fig. 4 respectively show the sum MSE performance and elapsed time of the proposed scheme and three comparative methods with $K = 12$ and $\tau = 4$. The input size is $KM = 48$, and the output size is $K\tau = 48$. The first and the last hidden layer contain 64 neurons, and other three hidden layers have 128 neurons. The mini-batch size $s = 1000$, and a total of 1000 iterations are executed to train the network. The cumulative distribution function (CDF) curves in Fig. 3 and the heights of red bars in Fig. 4 are from a testing dataset with 2000 samples. For comparison, all methods are implemented on a computer with intel core i7-8700CPU@3.20GHz and 16GB RAM.

It can be seen from Fig. 3 that the proposed scheme achieves the best sum MSE performance, followed by ESPA, RPA and APPA. Correspondingly, the elapsed time of all methods are showed in Fig. 4. Here, the RPA and APPA have the lowest elapsed time because of $O(1)$ complexity. The ESPA requires 10559s to find the optimal pilot assignment with $O(r^K)$ complexity. The proposed scheme only requires 1.4567s to solve the considered problem $P_0$ for 2000 channel samples. Although it takes 968.1423s to train the DNN, network training can be performed at a much longer scale than the channel block duration.

**V. Conclusion**

In this letter, we proposed a deep learning based pilot design scheme to alleviate pilot contamination for multi-user distributed massive MIMO systems. By leveraging the unsupervised learning strategy, the trained DNN can be used to solve the sum MSE minimization problem online. Simulation results show that the proposed scheme achieves the best sum MSE performance with low complexity compared to other methods.

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