Origin of folded bands in metamaterial crystals

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Abstract — Spectra of metamaterial photonic crystals may contain frequency bands which disappear inside the Brillouin zone (folded bands). We observe that the wave equations for such systems are essentially non-Hermitian, but $\mathcal{PT}$ -symmetric. We show that the real-frequency spectra correspond to $\mathcal{PT}$-symmetric solutions of the wave equation. At those momenta in the Brillouin zone where no real solutions exist, there appear pairs of complex-frequency solutions with spontaneously broken $\mathcal{PT}$ symmetry.

Recently, it has been found [1] that two dimensional array of cylinders made from the left-handed material with simultaneously negative permittivity and permeability exhibits unusual frequency spectrum. In particular, the spectrum contains so-called folded bands which disappear in certain region of the Brillouin zone.

We calculate numerically the transmission of plane electromagnetic wave through two dimensional array of left handed cylinders, find the band structure and study how folded bands arises in the frequency spectrum when the radius $R$ of metamaterial cylinders increases (Figure 1). The existence of folded bands indicates that the wave equation for the electromagnetic field in a metamaterial photonic crystal is non-Hermitian. We indeed show [2] that the operator $\mathcal{M}$ which enters the wave equation for the propagating wave

$$\mathcal{M}\vec{H} \equiv \frac{1}{\mu} \text{rot} \left[ \frac{1}{\varepsilon} \text{rot} \right] \vec{H} = \omega^2 \vec{H}$$

Figure 1: Low frequency ($a/\lambda < 1$) band structure of two dimensional periodic array of left-handed cylinders with permittivity $\varepsilon = -1.8$ and permeability $\mu = -5$ and various cylinder radius $R$ (measured in lattice period $a$). The wave vector $q$ is parallel to the $\Gamma X$ direction. Various folded bands appears when the cylinder radius $R$ increases. Note two folded bands which arise when the radius $R$ increases from 0.270 to 0.280.
is essentially non-Hermitian when both permittivity $\varepsilon$ and the permeability $\mu$ are negative inside cylinders. The essential non-Hermiticity means that, in contrast to dielectric photonic crystals [3], it cannot be avoided by any reformulation of the problem.

In spite of this non-Hermiticity, the operator $\mathcal{M}$ has real frequency spectra, $\omega^2 > 0$, if it possesses additional symmetry – the $\mathcal{PT}$ symmetry in the present case. Folded bands appear in the spectrum if this symmetry is broken [4]. This can be explicitly verified for the simplest one-dimensional case where the eigenfunctions, which correspond to complex frequencies, lost the $\mathcal{PT}$ symmetry [2].

![Figure 2: Dispersion relations along the $\Gamma X$ direction for the two-dimensional model with cylinder radii $R = 0.1a$ and $R = 0.3a$. Shaded regions highlight the frequency gap (with complex momentum solutions inside) and the momentum gap (with complex frequency solutions inside) in the left and right panels, respectively.](image)

Folded bands are closely related to the frequency gap in periodic systems. In fact, it is well known that within the frequency gap, there exist only solutions with a complex wave-vector. On the other hand, folded bands imply the existence of a momentum gap, inside which there appear only states with a complex frequency. Thus the momentum gap is a space-time dual of the frequency gap (Figure 2).

We have explained the origin of the recently discovered folded bands in spectra of metamaterial photonic crystals. They could be observed if (1) the boundary-value problem is essentially non-Hermitian and (2) the possible reality of its spectrum has to be guaranteed by additional symmetry, such as the $\mathcal{PT}$ symmetry in the metamaterial case. In systems fulfilling these assumptions, folded bands should be commonplace.

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