Generating Primordial Black Holes Via Hilltop-Type Inflation Models

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It has been shown that black holes would have formed in the early Universe if, on any given scale, the spectral amplitude of the Cosmic Microwave Background (CMB) exceeds $P_\zeta \sim 10^{-4}$. This value is within the bounds allowed by astrophysical phenomena for the small scale spectrum of the CMB, corresponding to scales which exit the horizon at the end of slow-roll inflation. Previous work by Kohri et al. (2007) showed that for black holes to form from a single field model of inflation, the slope of the potential at the end of inflation must be flatter than it was at horizon exit. In this work we show that a phenomenological Hilltop model of inflation, satisfying the Kohri et al. criteria, could lead to the production of black holes, if the power of the inflaton self-interaction is less than or equal to 3, with a reasonable number or $e$-folds. We extend our analysis to the running mass model, and confirm that this model results in the production of black holes, and by using the latest WMAP year 5 bounds on the running of the spectral index, and the black hole constraint we update the results of Leach et al. (2000) excluding more of parameter space.

PACS numbers:

I. INTRODUCTION

The temperature anisotropies of the Cosmic Microwave Background (CMB) have been measured by WMAP on angular scales down to $\theta \sim 0.3^\circ$, whereas they have yet to be measured to an effectual degree of accuracy on scales $\theta < 0.3^\circ$ [1]. In fact, CMB data naively allows for a very large spectrum on these scales, i.e. a spectrum a few orders of magnitude above $P_\zeta \sim 10^{-1}$. This value is within the bounds allowed by astrophysical phenomena for the small scale spectrum of the CMB, corresponding to scales which exit the horizon at the end of slow-roll inflation. Previous work by Kohri et al. (2007) showed that for black holes to form from a single field model of inflation, the slope of the potential at the end of inflation must be flatter than it was at horizon exit. In this work we show that a phenomenological Hilltop model of inflation, satisfying the Kohri et al. criteria, could lead to the production of black holes, if the power of the inflaton self-interaction is less than or equal to 3, with a reasonable number or $e$-folds. We extend our analysis to the running mass model, and confirm that this model results in the production of black holes, and by using the latest WMAP year 5 bounds on the running of the spectral index, and the black hole constraint we update the results of Leach et al. (2000) excluding more of parameter space.

The $\theta \geq 0.3^\circ$ spectrum has been used extensively in discriminating between models of inflation (c.f. [19, 20, 21, 22, 23, 24, 25]). These analyses are based on the assumption that the anisotropies in the CMB, and hence the origin of large scale structure, are sourced by quantum fluctuations in a scalar field during or straight after inflation. As such, we assume that the fluctuations which

sourced the PBHs are also generated during inflation, specifically towards the end of inflation, as it is during this epoch that the small scale fluctuations exit the horizon. Measuring the $\theta < 0.3^\circ$ spectrum will therefore not only probe generic signatures of inflation [2, 20] but also act as an indicator for the shape of the inflationary potential [18, 27]. In this paper we aim to exploit the latter purpose of measuring the small scale spectrum, and investigate whether single field models of inflation can lead to the formation of PBHs.

To be specific, the primordial black hole (PBH) condition [6, 14, 17, 40, 41] can be expressed as:

$$P_\zeta^{1/2} \approx 0.03 = 10^{3}P_\zeta^{1/2}$$

(1.1)

where the subscripts $e$ and $*$ refer to the end of inflation and horizon exit respectively. Assuming a constant spectral index then:

$$n_s - 1 \approx \frac{d \ln P_\zeta}{dN} \approx \frac{\Delta \ln P_\zeta}{\Delta N} \approx \frac{14}{\Delta N}$$

(1.2)

where we used (1.1) in the last semi-equality. $\Delta N \sim N$ refers to the number of $e$-folds that elapse from the time when scales of cosmological interest leave the horizon till the end of inflation.

Taking a standard value of $N \approx 60$, it then follows from (1.1) that $n_s \approx 1.3$. This is beyond the upper limit of 0.993 allowed by the recent WMAP data, at 95% confidence limit with no tensor modes or running. Therefore we consider the variation in the spectrum up to second order, and assume that the spectral index depends on scale, i.e. $P \propto k^{n_s(k)-1}$, then:

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1 For multi-field or multi-stage inflation models to produce PBHs, see Refs. 24, 25, 31, 32, 33, 34, 35, 36, 37, 38, 39 and references therein.
\[
\frac{d \ln \mathcal{P}}{d \ln k} = (n_s - 1) + n_s' \ln k
\]

\[
\ln \left( \frac{P_\zeta}{P_\zeta(N)} \right) = N(n_s - 1) + \frac{1}{2} n_s' N^2 \tag{1.3}
\]

\[
n = 14 \tag{1.4}
\]

where we used \( \ln k = \ln(aH) = N \) in the second step. Taking \( n_s \approx 0.95 \) and \( N = 60 \) then requires \( n_s' \approx 0.0061 \) to satisfy the WMAP bounds and produce Primordial Black Holes.

We now wish to rewrite the Primordial Black Hole Condition \((1.1)\) in terms of the slow roll parameters. Recalling that the spectrum can be written in terms of the potential \( V \) and the slow roll parameter \( \epsilon = m_{Pl}^2 \left( \frac{V'}{V} \right)^2/2 \), then \[12\]:

\[
P_\zeta = \frac{1}{24 \pi^2 m_{Pl}^2} \frac{V}{\epsilon} \tag{1.5}
\]

Defining a new parameter \( \mathcal{B} = \epsilon / \epsilon_s \), and combining equations \((1.3)\) and \((1.5)\) we find that the condition for PBH formation, without violating the aforementioned astrophysical and cosmological bounds is:

\[
\mathcal{B} \approx 10^{-6} \tag{1.6}
\]

An ‘absolute’ upper bound on the spectrum is given by \[10\] as \( P_{\zeta,c}^{1/2} \approx 10^{-1} \), which translates to a lower bound \( \mathcal{B} > 10^{-8} \).

Equation \((1.6)\) tells us that for an inflationary potential to lead to the production of PBHs, its’ slope must flatten towards the end of inflation. This shape is satisfied by a phenomenological model akin to the one analysed in \[43\], and also the running mass model, first introduced in \[44\]. In these types of models, we require that the inflaton initially be sitting at around the top of the hill, that is near a local maxima. This condition can be considered natural \[43\] from the viewpoint of eternal inflation \[43\], and can be understood as follows: via some mechanism, be it quantum tunnelling or an inhomegenous pre-inflationary universe, the inflaton will somewhere, at some time find itself sitting at the top of the potential, at which point the universe will start to inflate. As long as the inflaton is undisturbed, the universe will inflate indefinitely, and can end up volumetrically dominating the universe. Since this process can lead to an indefinitely large volume, even if there was the smallest probability that inflation were to start, it would \[40\] \[47\]. Within this patch, quantum fluctuations in some sub-patches displace the inflaton from its vestige causing it to roll either to the left or the right, and ending inflation in those regions, while overall, the patch continues to inflate \[43\] \[48\]. We introduce these models in sections \((11)\) and \((11)\) respectively. We present our results in section \((14)\) and discuss them in section \((14)\).

A. The Number of \( e \)-folds

In this paper we use the duration of inflation, otherwise known as the number of \( e \)-folds \( N \), as a discriminator. It is defined as the ratio of the scale factor at the end of inflation to \( a \) at the ‘beginning’ of inflation:

\[
N = \ln \left( \frac{a_e}{a_s} \right) \approx m_{Pl}^{-1} \int_{\phi_s}^{\phi_e} \frac{d\phi}{\sqrt{2\epsilon}} \tag{1.7}
\]

where the final semi-equality comes from the slow roll approximation.

To get a proper handle on how long inflation lasted from the time of horizon exit, one needs a complete history of the Universe. At present though, we do not have an agreed upon mechanism of reheating. Therefore, one assumes an instant transition from inflation to a radiation dominated universe, and gets the bounds \[30\]:

\[
10 \lesssim N \lesssim 110 \tag{1.8}
\]

The lower bound comes from the assumption that Nucleosynthesis is well bounded, and the upper bound assumes that the universe underwent a few bouts of ‘fast’ roll inflation. We do note that these are extreme bounds, and that the limits \( N = 54 \pm 7 \) are more widely acceptable (c.f. \[ 20 \] \[ 42 \] \[ 49 \]).

B. Slow Roll and Cosmological Parameters

The lowest order slow order parameters are given by:

\[
\epsilon = \frac{m_{Pl}^2}{2} \left( \frac{V'}{V} \right)^2 \tag{1.9}
\]

\[
\eta = m_{Pl}^2 \frac{V''}{V} \tag{1.10}
\]

\[
\xi^2 = \frac{m_{Pl}^4}{2} \frac{V'V''}{V^2} \tag{1.11}
\]

which we then use to compute the spectral index and the running:

\[
n_s = 1 + 2\eta - 6\epsilon \tag{1.13}
\]

\[
n_s' = \frac{dn}{d\ln k} = 16\epsilon - 24\epsilon^2 - 2\xi^2 \tag{1.14}
\]

We use the bounds on cosmological parameters given by the WMAP year 5, Baryon Acoustic Oscillations and Supernovae data sets \[50\]:

\[
0.939 < n_s < 1.109 \tag{1.15}
\]

\[-0.0728 < n_s' < 0.0087 \tag{1.16}\]

where a zero tensor mode was assumed in the prior. This prior is reasonable since we are considering small field
models, characterised by a field variation that is smaller than the Planck mass $\Delta \phi < m_{p_1}$. In these models the gravitational wave signature will be small [51], and by small we mean well below the sensitivity of WMAP5 parameter estimation, so we do not calculate the associated parameter.

II. THE TREE LEVEL POTENTIAL

We consider the potential of the form:

$$V = V_0 \left(1 + \eta_p \left(\frac{\phi}{m_{p_1}}\right)^p - \eta_q \left(\frac{\phi}{m_{p_1}}\right)^q\right) \quad (2.1)$$

where $p \geq 2$ and $q > p$, plotted in Fig. 1. The case $p = 2$ and $q \geq 4$ can be generated from a flat direction in the Minimal Supersymmetric Model (MSSM) [52, 53, 54, 55, 56, 57]. In this case $q$ is the non-renormalisable operator that depends on the flat directions. One can also motivate the parameter range $p = 2$ and $q \geq 3$ in [57, 58, 59], in this scenario the inflaton higher order terms are not Planck suppressed, and one gets a lower energy scale inflation, on the order of the TeV – GUT scale. We also maintain $\phi < m_{p_1}$, a realistic bound from an effective particle physics perspective, which demands that one not consider mass scales larger than the largest naturally occurring scale, in this case the Planck mass. Then due to the Lyth bound, the gravitational wave contribution of this model is negligible, regardless how long inflation lasts.

In this setup we require that the potential at the end of inflation be flatter than it was at the time of horizon exit, so the inflaton must roll towards the origin. We denote the inflaton value at the maximum of the potential as $\phi_m$, at horizon exit as $\phi_*$ and at the end of inflation as $\phi_e$. We impose the conditions:

$$\phi_* < \phi_m \quad (2.2)$$
$$0 < \phi_e < \phi_o \quad (2.3)$$

where $\phi_o$ is the inflection point. $\phi_m$ and $\phi_o$ are then given by (2.1):

$$\frac{\phi_m}{m_{p_1}} = \left(\frac{m_{p_1}}{\eta q}\right)^{1/(q-p)} \quad (2.4)$$
$$\phi_o = \phi_m \left(\frac{p-1}{q-1}\right)^{1/(q-p)} \quad (2.5)$$

III. THE RUNNING MASS MODEL

With the exception of $p = 2$ and integral values $3 \leq q \leq 9$, the previous model is a phenomenological one. However the shape of potential does appear in a more theoretically motivated setup, the running mass model [44, 60, 61, 62, 63, 64, 65, 66, 67] which has the potential:

$$V = V_0 \left[1 - \frac{1}{2} \mu^2 \frac{\phi^2}{m_{p_1}^2}\right] \quad (3.1)$$

where the mass of the inflaton is scale dependent and can be expressed as:

$$\mu^2(\phi) = \mu_0^2 + A_0 \left[1 - \frac{1}{(1 + \alpha \ln(\phi/m_{p_1}))^2}\right] \quad (3.2)$$

where $\mu_0^2$ is the mass of the inflaton squared, $A_0$ is the gaugino mass squared in units of $m_{p_1}$, and $\alpha$ is related to the gauge coupling.

The potential can then be written as:

$$V = V_0 \left[1 - \frac{1}{2} B_0^2 \left(\frac{\phi}{m_{p_1}}\right)^2 + \frac{A_0}{2(1 + \alpha \ln(\phi/m_{p_1}))^2} \left(\frac{\phi}{m_{p_1}}\right)^2\right] \quad (3.3)$$

and $B_0 = \mu_0^2 + A_0$, with inflation occurring in the regime $\phi \ll m_{p_1}$. This potential has the shape in Fig. 2 and the parameters have theoretically motivated constraints [61]:

$$1 \lesssim \mu_0^2 \lesssim O(10)$$
$$0 \lesssim A_0 \lesssim O(10)$$
$$10^{-3} \lesssim \alpha \lesssim 10^{-1} \quad (3.4)$$

A. Linear Approximation

The linear approximation of the running mass potential is given by [27, 62]:

$$\frac{V}{V_0} = 1 - \frac{\phi^2}{2}(\mu_0^2 + c \ln(\phi/\phi_m)) \quad (3.5)$$

which is basically equation (3.3) expanded about the maximum of the potential. The terms $c$ and $\mu_0^2$ are related to the theoretical parameters $A_0, \mu_0^2$ and $\alpha$ by:

$$\mu_0^2 = \mu_0^2 + A_0 \left[1 - \frac{1}{(1 + \alpha \ln(\phi_m))^2}\right]$$
$$c = \frac{2A_0}{(1 + \alpha \ln(\phi_m))^3} \quad (3.6)$$

Since $\phi_m$ defines the maximum of the potential then $\mu_0^2 = -c/2$, this allows us to write:

$$N = -\frac{1}{c} \ln \left[\frac{\ln(\phi_m/\phi_e)}{\ln(\phi_m/\phi_*)}\right] \quad (3.7)$$
$$\frac{n-1}{2} = -c \left[\ln \left(\frac{\phi}{\phi_m}\right) + 1\right] \quad (3.8)$$
$$= \sigma e^{-\sigma N} - c \quad (3.9)$$
\[ \sigma = -c \ln(\phi_e / \phi_m). \]

This approximation is only valid near the maximum. We neither expect nor get reasonable values of \( \phi_e \) using this estimate. However given \( \phi_* \) and \( \phi_e \), we found that the linear approximation for \( N \) in (3.7) appears consistent with the numerical calculation.

IV. RESULTS

A. Hilltop

For the tree level potential, Fig. 3 depicts the dependence of \( N \) on \( p \) and \( q \), we note that \( N \) increases sharply as \( p \) increases, as expected. We then searched for the range of \( p \) and \( q \) parameters that satisfy the bounds \( 10 < N < 110 \) and found that the condition for PBH formation within this range is \( 2 \leq p < 3 \) and \( p < q < 4 \). Larger values of \( p \) lead to \( N \gg 110 \) while larger values of \( q/p \) do not satisfy the WMAP bounds, since they steepen the potential and result in an increased \( n' \). We also found that \( p \approx 2 \) and \( q < 3 \) (note \( q \neq 3 \)) places \( N \) in the range \( N = 54 \pm 7 \).

We then plot the dependence of \( N \) on \( \eta_p \) and \( \eta_q \) in Fig. 4. As expected \( N \) increases for decreasing \( \eta_p \) and vice versa for \( \eta_q \). Once we have filtered out the reasonable values of \( N \), we find that within the range \( \{ \eta_p, \eta_q \} = \{0, 1\} \) PBH formation will occur. It seems that the stronger constraints for PBH formation with \( 10 < N < 110 \) come from \( p \) and \( q \).

Next we consider the case of defining \( \phi_* \) by the condition that \( n(\phi_*) = 0.95 \) and \( \phi_e \) by \( N = 60 \) and \( N = 100 \). Fig. 5 shows that for both \( N = 60 \) and \( N = 100 \), the parameter ranges satisfy the WMAP bounds and the PBH constraints. However, for \( N = 60 \) integral values of \( p \) and \( q \) do not lead to the formation of PBHs, while for
we find that in order to avoid the overproduction of primordial black holes, \( n > 1 \) would be ruled out for \( N = 45 \), and this bound is strengthened to ruling out \( n > 0.95 \) for \( N = 60 \). These bounds are slightly stronger than those found by Ref. 67, who rule out \( n \gtrsim 1.1 \) for \( N = 45 \).

However, there is a discrepancy between our spectral index contour lines and Ref. 67, which we found was resolved by evolving our system an extra \( \sim 7 \epsilon \)-folds. Via a process of elimination, we think this may be due to the fact that Ref. 67 solved the background equations numerically without resorting to the slow roll approximation, while using the extended slow roll formalism of Ref. 68 to solve for the perturbations.

Either way, as our method underestimates the allowed parameter range for each \( n \), then using ref. 67 method would strengthen our conclusions i.e. our bounds are conservative.

V. DISCUSSION AND CONCLUSIONS

In this paper we utilised the spectrum on scales \( \theta \lesssim 0.3 \degree \), corresponding to the end of inflation, to further the field of inflation model discrimination. The spectrum on these scales has yet to be measured, but future CMB surveys such as the PLANCK mission may constrain its value. Astrophysical phenomena determines the upper bound to be \( P_c \sim 10^{-4} \), corresponding to the criteria for the formation of primordial black holes (PBHs). In terms of the \( \epsilon \) slow roll parameter, this means that the value of \( \epsilon \) at the end of inflation must be much smaller than its value at horizon exit (Ref. 60). The models of inflation that satisfy this condition must therefore exhibit the unique property of having a flatter slope towards the end of inflation. So far, we only know of generic Hilltop models of inflation that fulfill this criteria.

We have investigated whether these generic models of Hilltop inflation would lead to the production of primordial black holes with a spectrum \( P_c \sim 10^{-4} \). We found that within the range of parameters allowed by the latest WMAP data, the Hilltop model (24) would lead to the formation of PBHs without violating astrophysical bounds for \( p < 2.5 \) and \( q \leq 3 \) if \( N > 60 \), and for \( p \sim 2 \), \( 2 < q < 3 \) if \( N > 40 \). Integral values of \( p \) and \( q \), which have some theoretical motivation, only lead to PBH formation within the bound (1.9) for \( p = 2 \) and \( q = 3 \), with \( 60 \ll N < 100 \). In all cases it seems that near maximal running is required. If, however we were to allow \( N \gg 110 \) the range of parameters that would lead to PBH formation would be extended.

The allowed parameter range for the production of primordial black holes with \( P_c \lesssim 10^{-4} \) in the running mass model is again dependent on \( N \) as can be seen from Fig. 6. We find that for \( \alpha = 0.01 \) and \( \mu_0^2 \gtrsim 1.1 \), black holes could form after \( N < 47 \epsilon \)-folds, and therefore before what can be considered a ‘reasonable’ end to inflation. This is problematic on two counts, if we assume that the PBHs formed prior to the end of inflation, then this could lead to the overclosure of the universe (c.f. 68). On the same note, we know that on CMB scales the spectrum is too small to support PBH production. On the second count, assuming that the formation of the PBHs coincided with the end of inflation, then the arguments we presented in section 4A apply. Thus, using \( N \) as a discriminator we rule out \( A_0 > 3 \) and \( \mu_0^2 > 1.1 \) for \( \alpha = 0.01 \), \( A_0 > 6 \) and...
\( \alpha^2 > 2.4 \) for \( \alpha = 0.005 \), and \( A_0 > 5 \) and \( \mu_0^2 > 8.75 \) for \( \alpha = 0.001 \). As we mentioned in the text \( \alpha = 0.1 \) is ruled out on WMAP consistency grounds.

This model has also been analysed by [70], in which the authors use neutrino and \( \gamma - \) ray background data to constrain the PBH mass spectrum, which determines the spectral index on small scales \( k \sim 15 \text{ Mpc}^{-1} \). Then assuming that the running mass model is correct they reconstruct the power spectrum, finding that on these small scales the spectrum is highly sensitive to the running of the spectral index. Combining these two pieces of information they get bounds on \( n_s \) and \( n'_s \), which turn out to be inclusive of the WMAP limits on these parameters. That is, by using a different approach to ours [70] conclude that the running mass model is consistent with WMAP while avoiding PBH over-production, congruously with our findings.

Finally, we note that our generic results are consistent with the findings of [71]. They tackle the question of PBH production utilising the arising constraints to derive bounds on the cosmological parameters, and conclude that the PBH constraint is ‘strongly’ dependent on \( N \) and the spectrum at the end of inflation. Characteristics exhibited by our specific models.

VI. ACKNOWLEDGEMENTS

We thank Andrew Liddle, James Lidsey, David Lyth, Karim Malik, Hiranya Peiris, and David Seery for useful comments and discussion. LA is supported by the Science and Technologies Facilities Council (STFC) under Grant PP/E001440/1. K.K. is supported in part by STFC grant, PP/D000394/1, EU grant MRTN-CT-2006-035863, and the European Union through the Marie Curie Research and Training Network “UniverseNet”.

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FIG. 5: Plot of $\log(B)$ versus $\log(n')$ for the Hilltop model with $N = 60$ (figure on left), $N = 100$ (figure on right) and $n_s = 0.95$. The hatched region is excluded, representing $\log(n') > -2$ and $\log(B) < -8$. The region $\log(B) > -6$ does not lead to the formation of PBHs, and is represented by the tan colour in the figure. PBHs can form in the region $-8 \leq \log(B) \leq -6$ without violating astrophysical or cosmological bounds, and is represented by the light orange region. The yellow dots correspond to $\{p, q\} = \{3, 4\}$, the green dots to $\{p, q\} = \{2, 3\}$ and the blue dots to $\{p, q\} = \{2, 2.5\}$.

FIG. 6: Contour plots of the number of $e$–folds produced in the running mass model for three values of the gauge coupling $\alpha = [0.001, 0.005, 0.01]$. We found that $\alpha = 0.1$ did not satisfy the WMAP bounds on $n_s$ and $n'$. We have filtered out the allowed parameter space for $10 \leq N \leq 110$, and coloured it in shades of green. The dashed regions in each plot correspond to the more ‘reasonable’ bound $N = 54 \pm 7$.

FIG. 7: In these plots we fix $N$ and $\alpha$, plotting contour lines of the spectral index (dashed) and $\log(B)$ (solid). Note that the contour lines do not exactly match Ref. [67], an anomaly that we discuss in the text. Parameter space below $\log(B) \sim -8$ is excluded.