FORBIDDING HAMILTON CYCLES IN UNIFORM HYPERGRAPHS

JIE HAN AND YI ZHAO

ABSTRACT. For $1 \leq d \leq \ell < k$, we give a new lower bound for the minimum $d$-degree threshold that guarantees a Hamilton $\ell$-cycle in $k$-uniform hypergraphs. When $k \geq 4$ and $d < \ell = k - 1$, this bound is larger than the conjectured minimum $d$-degree threshold for perfect matchings and thus disproves a well-known conjecture of Rödl and Ruciński. Our (simple) construction generalizes a construction of Katona and Kierstead and the space barrier for Hamilton cycles.

1. Introduction

The study of Hamilton cycles is an important topic in graph theory. A classical result of Dirac [4] states that every graph on $n \geq 3$ vertices with minimum degree $n/2$ contains a Hamilton cycle. In recent years, researchers have worked on extending this theorem to hypergraphs—see recent surveys [16, 18, 20].

To define Hamilton cycles in hypergraphs, we need the following definitions. Given $k \geq 2$, a $k$-uniform hypergraph (in short, $k$-graph) consists of a vertex set $V$ and an edge set $E \subseteq \binom{V}{k}$, where every edge is a $k$-element subset of $V$. Given a $k$-graph $H$ with a set $S$ of $d$ vertices (where $1 \leq d \leq k - 1$) we define $\text{deg}_H(S)$ to be the number of edges containing $S$ (the subscript $H$ is omitted if it is clear from the context). The minimum $d$-degree $\delta_d(H)$ of $H$ is the minimum of $\text{deg}_H(S)$ over all $d$-vertex sets $S$ in $H$. For $1 \leq \ell \leq k - 1$, a $k$-graph is said to be an $\ell$-cycle if its vertices can be ordered cyclically such that each of its edges consists of $k$ consecutive vertices and every two consecutive edges (in the natural order of the edges) share exactly $\ell$ vertices. In $k$-graphs, a $(k - 1)$-cycle is often called a tight cycle. We say that a $k$-graph contains a Hamilton $\ell$-cycle if it contains an $\ell$-cycle as a spanning subhypergraph. Note that a Hamilton $\ell$-cycle of a $k$-graph on $n$ vertices contains exactly $n/(k - \ell)$ edges, implying that $k - \ell$ divides $n$.

Let $1 \leq d, \ell \leq k - 1$. For $n \in (k - \ell)\mathbb{N}$, we define $h_d^l(k, n)$ to be the smallest integer $h$ such that every $n$-vertex $k$-graph $H$ satisfying $\delta_d(H) \geq h$ contains a Hamilton $\ell$-cycle. Note that whenever we write $h_d^l(k, n)$, we always assume that $1 \leq d \leq k - 1$. Moreover, we often write $h_d(k, n)$ instead of $h_d^{k-1}(k, n)$ for simplicity. Similarly, for $n \in k\mathbb{N}$, we define $m_d(k, n)$ to be the smallest integer $m$ such that every $n$-vertex $k$-graph $H$ satisfying $\delta_d(H) \geq m$ contains a perfect matching. The problem of determining $m_d(k, n)$ has attracted much attention recently and the asymptotic value of $m_d(k, n)$ is conjectured as follows. Note that the $o(1)$ term refers to a function that tends to 0 as $n \to \infty$ throughout the paper.

Conjecture 1.1. [6, 15] For $1 \leq d \leq k - 1$ and $k | n$,

$$m_d(k, n) = \left( \max \left\{ \frac{1}{2}, 1 - \left(1 - \frac{1}{k}\right)^{k-d} \right\} + o(1) \right) \frac{n - d}{k - d}.$$

Conjecture 1.1 has been confirmed [11, 17, 23, 25] for $\min\{k - 4, k/2\} \leq d \leq k - 1$ (the exact values of $m_d(k, n)$ are also known in some cases, e.g., [24, 25]). On the other hand, $h_d^l(k, n)$ has also been extensively studied [2, 9, 10, 11, 12, 13, 14, 19, 20, 22]. In particular, Rödl, Ruciński and Szemerédi [20, 22] showed that $h_{k-1}(k, n) = (1/2 + o(1))n$. The same authors proved in [21] that $m_{k-1}(k, n) = (1/2 + o(1))n$ (later they determined $m_{k-1}(k, n)$ exactly [23]). This suggests that the values of $h_d(k, n)$ and $m_d(k, n)$ are closely related and inspires Rödl and Ruciński to make the following conjecture.

Conjecture 1.2. [18, Conjecture 2.18] Let $k \geq 3$ and $1 \leq d \leq k - 2$. Then

$$h_d(k, n) = m_d(k, n) + o(n^{k-d}).$$

© 2016. This manuscript version is made available under the Elsevier user license http://www.elsevier.com/open-access/userlicense/1.0/
By using the value of $m_d(k,n)$ from Conjecture 1.1, Kühn and Osthus stated this conjecture explicitly for the case $d = 1$.

**Conjecture 1.3.** [16] Conjecture 5.3 Let $k \geq 3$. Then

$$h_1(k,n) = \left(1 - \left(1 - \frac{1}{k}\right)^{k-1} + o(1)\right)\binom{n-1}{k-1}.$$ 

In this note we provide new lower bounds for $h_d'(k,n)$ when $d \leq \ell$.

**Theorem 1.4.** Let $1 \leq d \leq k-1$ and $t = k-d$, then

$$h_d(k,n) \geq \left(1 - \left(\frac{t}{\lfloor t/2 \rfloor}\right)^{[\frac{t}{2}]^{[\lfloor t/2 \rfloor] + 1}} + o(1)\right)\binom{n}{t}.$$

**Theorem 1.5.** Let $1 \leq d \leq \ell \leq k-1$ and $t = k-d$. Then

$$h_d'(k,n) \geq \left(1 - b_{t,k-\ell}2^{-t} + o(1)\right)\binom{n}{t},$$

where $b_{t,k-\ell}$ equals to the largest sum of $k - \ell$ consecutive binomial coefficients taken from $\{\binom{k}{0}, \ldots, \binom{k}{d}\}$.

Theorem 1.4 disproves both Conjectures 1.2 and 1.3.

**Corollary 1.6.** For all $k$,

$$h_{k-2}(k,n) \geq \left(\frac{5}{9} + o(1)\right)\binom{n}{2}, \quad h_{k-3}(k,n) \geq \left(\frac{5}{8} + o(1)\right)\binom{n}{3}, \quad h_{k-4}(k,n) \geq \left(\frac{409}{625} + o(1)\right)\binom{n}{4}$$

and in general, for any $1 \leq d \leq k-1$,

$$(1.1) \quad h_d(k,n) \geq \left(1 - \frac{1}{\sqrt{3(k-d)/2 + 1}}\right)\binom{n}{k-d}.$$ 

These bounds imply that Conjecture 1.2 is false when $k \geq 4$ and $\min\{k-4, k/2\} \leq d \leq k-2$, and Conjecture 1.3 is false whenever $k \geq 4$.

We will prove Theorem 1.4, Theorem 1.5, and Corollary 1.6 in the next section.

We believe that Conjecture 1.2 is false whenever $k \geq 4$ but due to our limited knowledge on $m_d(k,n)$, we can only disprove Conjecture 1.2 for the cases when $m_d(k,n)$ is known.

This bound $h_{k-2}(k,n) \geq \left(\frac{5}{9} + o(1)\right)\binom{n}{2}$ coincides with the value of $m_1(3,n)$ it was shown in [6] that $m_1(3,n) = (5/9 + o(1))\binom{n}{2}$, and it was widely believed that $h_1(3,n) = (5/9 + o(1))\binom{n}{2}$, e.g., see [19]. On the other hand, it is known [17] that $m_2(4,n) = \left(\frac{5}{8} + o(1)\right)\binom{n}{4}$, which is smaller than $\frac{5}{9}\binom{n}{2}$. Therefore $k = 4$ and $d = 2$ is the smallest case when Theorem 1.4 disproves Conjecture 1.2. More importantly, (1.1) shows that $h_d(k,n)/(\binom{n}{k-d})$ tends to one as $k-d$ tends to $\infty$. For example, as $k$ becomes sufficiently large, $h_{k-\ell}(k,n)$ is close to $\binom{n}{k-d}$, the trivial upper bound. In contrast, Conjecture 1.1 suggests that there exists $c > 0$ independent of $k$ and $d$ ($c = 1/e$, where $e = 2.718\ldots$, if Conjecture 1.1 is true) such that $m_d(k,n) \leq (1-c)^{\binom{n-d}{k-d}}$.

Similarly, by Theorem 1.5 if $k-\ell = o(\sqrt{t})$, $h_d'(k,n)/(\binom{n}{t})$ tends to one as $t$ tends to $\infty$ because

$$1 - b_{t,k-\ell}2^{-t} \geq 1 - \frac{k-\ell - t}{2t}\left(\frac{t}{\lfloor t/2 \rfloor}\right) \approx 1 - \frac{o(\sqrt{t})}{\sqrt{\pi t}/2}.$$

Theorem 1.5 also implies the following special case: suppose $k$ is odd and $\ell = d = k-2$. Then $t = 2$ and $b_{t,k-\ell} = b_{2,2} = 3$, and consequently $h_{k-2}(k,n) \geq \left(\frac{1}{3} + o(1)\right)\binom{n}{2}$. Previously it was only known that $h_{k-2}(k,n) \geq 1 - \frac{(k-1)^2}{2\ell + 1} + o(1)\binom{n}{2}$ by (2.1) with $a = \lfloor k/(k-\ell) \rfloor = (k+1)/2$ from Section 2. When $k$ is large, the bound provided by Theorem 1.5 is much better.

Finally, we do not know if Theorems 1.4 and 1.5 are best possible. Glebov, Person, and Weps [5] gave a general upper bound (far away from our lower bounds)

$$h_d'(k,n) \leq \left(1 - \frac{1}{ck^{3k-3}}\right)\binom{n-d}{k-d},$$

where $c$ is a constant independent of $d, \ell, k, n$. 
2. The proofs

Before proving our results, it is instructive to recall the so-called space barrier. Throughout the paper, we write $X \cup Y$ for $X \cup Y$ when sets $X$, $Y$ are disjoint.

**Proposition 2.1.** [13] Let $H = (V, E)$ be an $n$-vertex $k$-graph such that $V = X \cup Y$ and $E = \{ e \in \binom{Y}{2} : e \cap X \neq \emptyset \}$. Suppose $|X| < \frac{1}{a(k-\ell)} n$, where $a := [k/(k-\ell)]$, then $H$ does not contain a Hamilton $\ell$-cycle.

A proof of Proposition 2.1 can be found in [13 Proposition 2.2] and is actually included in our proof of Proposition 2.2 below. It is not hard to see that Proposition 2.1 shows that $\max_{X} \lambda_{1}(\hat{H}) = j$ (when $j$ Hamilton actually contains no $|V|\leq j$). By the definition of $H$, $\lambda_{1}(\hat{H}) = 3$ appears in [19, Construction 2].

In the proofs of Theorems 1.4 and 1.5, we will consider binomial coefficients $\binom{p}{q}$ with $q < 0$ in this case $\binom{p}{q} = 0$. We will conveniently write $|X| = xn$, where $0 < x < 1$, instead of $|X| = [xn]$ – this does not affect our calculations as $n$ is sufficiently large.

In the proofs of Theorem 1.4 we apply Proposition 2.2 with $j$ and $x$ such that $\frac{j-1}{k} < x < \frac{j+1}{k}$. Note that $(j-1)/(k-1) \leq x < \frac{j+1}{k}$ implies that $\frac{j-1}{t+1} < x < \frac{j+1}{t+1}$, in particular,

$$j - d \leq [x(t+1)] \leq j.$$  

**Proof of Theorem 1.4.** Let $x = [t/2]/(t+1)$ and $j = [t/2]+1$. Since $\sum_{i=1}^{j-1}(\frac{j-1}{k}, \frac{j+1}{k}) = (0, 1)$ and $1/3 \leq \frac{[t/2]}{t+1} \leq 1/2$, there exists an integer $i \in [k-1]$ such that $\frac{i-1}{k} < \frac{i+1}{k} < \frac{i+1}{k}$. Let $H = (V, E)$ be an $n$-vertex $k$-graph such that $V = X \cup Y$, $|X| = xn$ and $E = \{ e \in \binom{Y}{2} : e \cap X \neq \emptyset \}$. Since $\frac{j-1}{k} n < |X| < \frac{j+1}{k} n$, $H$ contains no tight Hamilton cycle by Proposition 2.2 with $\ell = k-1$.

Now let us compute $\delta_{q}(H)$. For $0 \leq i \leq d$, let $S_{i}$ be any $d$-vertex subset of $V$ that contains exactly $i$ vertices in $X$. By the definition of $H$,

$$\deg_{H}(S_{i}) = \binom{n-d}{t} - \binom{|X|-i}{j-i} \binom{Y-(d-i)}{t-j+i} = \binom{n-d}{t} - \binom{|X|-i}{j-i} \binom{|Y|-(d-i)}{t-j+i};$$

1It can be shown that among all $0 < x < 1$ and $0 \leq j \leq k$ such that $\frac{i-1}{k} < x < \frac{i+1}{k}$, our choice of $x$ and $j$ minimizes max$_{j \leq i \leq j}$ $\binom{p}{q} x(1-x)^{j-i}$, and thus maximizes $\delta_{q}(H)$ in (2.2).
That this holds for \( i > j \) or \( i < j - t \) trivially. So we have
\[
\delta_d(H) = \min_{0 \leq i \leq d} \left\{ \binom{n-d}{t} - \binom{|X|-i}{t-j-i} \left( \binom{|Y|-(d-i)}{t-j+i} \right) \right\} = \binom{n}{t} - \max_{j-d \leq i' \leq j} \left\{ \binom{|X|}{t-i'} \left( \binom{|Y|}{t-i'} \right) \right\} + o(n^t).
\]
Write \( |X| = xn \) and \( |Y| = yn \). When \( 0 \leq i' \leq t \), we have
\[
\binom{|X|}{i'} \left( \binom{|Y|}{t-i'} \right) = \frac{(xn)^{i'}(yn)^{t-i'}}{i'! (t-i')!} + o(n^t) = \binom{t}{i'} x^{i'} y^{t-i'} \frac{n^t}{t} + o(n^t).
\]
When \( i' < 0 \) or \( i' > t \), we have \( \binom{|X|}{i'} \left( \binom{|Y|}{t-i'} \right) = 0 = \binom{t}{i'} x^{i'} y^{t-i'} \frac{n^t}{t} \). In all cases, we have
\[
(2.3) \quad \delta_d(H) = \binom{n}{t} - \max_{j-d \leq i' \leq j} \left\{ \binom{t}{i'} x^{i'} y^{t-i'} \frac{n^t}{t} \right\} + o(n^t).
\]
Let \( a_i := \binom{t}{i} x^i y^{t-i} \). Since \( x = \lceil t/2 \rceil/(t+1) \) and \( y = 1 - x \), it is easy to see that \( \max_{0 \leq i \leq t} a_i = a_{\lceil t/2 \rceil} \) (e.g., by observing \( \frac{a_i}{a_{i+1}} = \frac{t-i}{i+1} \) for \( 0 \leq i < t \)). Moreover, by (2.2), we have \( j-d \leq \lceil t/2 \rceil \leq j \). Therefore,
\[
\max_{j-d \leq i' \leq j} \left\{ a_i \right\} = a_{\lceil t/2 \rceil} = \left( \frac{t}{\lceil t/2 \rceil} \right) \frac{\lceil t/2 \rceil}{(t+1)^t},
\]
and we complete the proof by substituting it into (2.3). \( \square \)

Now we turn to the proof of Theorem 1.5, in which we assume that \( |X| = n/2 \), though a further improvement of the lower bound may be possible by considering other values of \( |X| \).

Proof of Theorem 1.5. The proof is similar to the one of Theorem 1.4. Let \( H = (V,E) \) be an \( n \)-vertex \( k \)-graph such that \( V = X \cup Y \), \( |X| = n/2 \) and \( E = \{ e \in \binom{Y}{k} : e \cap X \geq \{ \lceil \ell/2 \rceil \ldots, \ell/2 + k - \ell - 1 \} \} \). Note that
\[
a'(k-\ell) = \left[ k \begin{array}{c} k-\ell \end{array} \right] (k-\ell) \geq k - (k-\ell-1) = \ell + 1 > 2(\lceil \ell/2 \rceil - 1) \text{, and}
\]
\[
a(k-\ell) = \left[ k \begin{array}{c} k-\ell \end{array} \right] (k-\ell) \leq k + (k-\ell-1) < 2(k-\lceil \ell/2 \rceil) = 2(\lceil \ell/2 \rceil + k - \ell).
\]
So we have
\[
\frac{\lceil \ell/2 \rceil - 1}{a'(k-\ell)} n < |X| = \frac{n}{2} < \frac{\lceil \ell/2 \rceil + k - \ell}{a(k-\ell)} n.
\]
Thus, \( H \) contains no Hamilton \( \ell \)-cycle by Proposition 2.2.

Fix \( 1 \leq d \leq k-1 \) and let \( t = k-d \). Now we compute \( \delta_d(H) \). For \( 0 \leq i \leq d \), let \( S_i \) be any \( d \)-vertex subset of \( V \) that contains exactly \( i \) vertices in \( X \). It is easy to see that
\[
\deg_H(S_i) = \binom{n}{t} - \sum_{p=i'}^{i'+k-\ell-1} \binom{|X|}{p} \binom{|Y|}{t-p} + o(n^t),
\]
where \( i' = \lceil \ell/2 \rceil - i \). Using \( |X| = |Y| = n/2 \) and the similar calculations in the proof of Theorem 1.4 we get
\[
\deg_H(S_i) = \binom{n}{t} - \sum_{p=i'}^{i'+k-\ell-1} \binom{t}{p} \frac{1}{2^t} \binom{n}{t} + o(n^t).
\]
By the definition of \( b_{t,k-\ell} \), we have
\[
\delta_d(H) = \min_{0 \leq i \leq d} \deg_H(S_i) \geq \binom{n}{t} - b_{t,k-\ell} 2^{-t} \binom{n}{t} + o(n^t). \quad \square
\]

Corollary 1.6 follows from Theorem 1.4 via simple calculations.
Proof of Corollary 1.6.  Let $t = k - d$ and

$$f(t) := \left( \frac{t}{t/2} \right) \frac{[t/2]^{[t/2]}([t/2]+1)^{[t/2]}}{(t+1)^t}.$$  

Theorem 1.4 states that $h_{k-1}(k, n) \geq (1 - f(t) + o(1)) \binom{n}{t}$ for any $1 \leq t \leq k - 1$. Since

$$f(2) = \frac{4}{9}, \quad f(3) = \frac{3}{8}, \quad \text{and} \quad f(4) = \frac{216}{625},$$

the bounds for $h_{k-1}(k, n)$, $t = 2, 3, 4$, are immediate. To see (1.1), it suffices to show that for $t \geq 1$,

$$(2.4) \quad 1 - f(t) > 1 - \frac{1}{\sqrt{3t/2 + 1}}. $$

When $t$ is odd, $\frac{[t/2]^{[t/2]}([t/2]+1)^{[t/2]}}{(t+1)^t} = 1/2^t$; when $t$ is even, $\frac{[t/2]^{[t/2]}([t/2]+1)^{[t/2]}}{(t+1)^t} < (t+1)/2^t$. Thus, for all $t$, we have

$$f(t) \leq \left( \frac{t}{[t/2]} \right) \frac{1}{2^t},$$

where a strict inequality holds for all even $t$. Now we use the fact $2m/\binom{n}{m} \leq 2^{m}/\sqrt{3m+1}$, which holds for all integers $m \geq 1$. Thus, for all even $t$, we have $f(t) \leq 1/\sqrt{3t/2 + 1}$; for all odd $t$,

$$f(t) \leq \left( \frac{t}{[t/2]} \right) \frac{1}{2^t} = \frac{1}{2} \left( \frac{t + 1}{[t/2] + 1} \right) \frac{1}{2^t} \leq \frac{1}{\sqrt{3(t+1)/2 + 1}} < \frac{1}{\sqrt{3t/2 + 1}}.$$  

Hence $f(t) \leq 1/\sqrt{3t/2 + 1}$ for all $t \geq 1$. Moreover, by the computation above, regardless of the parity of $t$, the strict inequality always holds and thus (2.4) is proved.

We next show that whenever $k \geq 4$ and $2 \leq t \leq k - 1$,

$$1 - f(t) > \max \left\{ \frac{1}{2}, 1 - \left( 1 - \frac{1}{k} \right)^t \right\}.$$  

This implies that Conjecture 1.3 fails for $k \geq 4$, and Conjecture 1.2 fails for $k \geq 4$ and $\min\{k - 4, k/2\} \leq d \leq k - 2$ (because $m_d(k, n)/(k/n) = \max \left\{ \frac{1}{2}, 1 - (1 - \frac{1}{k})^{k-d} \right\} + o(1)$ in this case). It suffices to show that for $k \geq 4$ and $2 \leq t \leq k - 1$,

$$f(t) < 1/2 \quad \text{and} \quad f(t) \leq \left( 1 - \frac{1}{k} \right)^t.$$  

The first inequality immediately follows from (2.4) and $1/\sqrt{3t/2 + 1} \leq 1/2$. For the second inequality, note that

$$f(t) < \frac{1}{\sqrt{3t/2 + 1}} \leq \frac{1}{e} < \left( 1 - \frac{1}{k} \right)^{k-1} \leq \left( 1 - \frac{1}{k} \right)^t$$

for all $t \geq 5$. For $t = 2, 3$ and all $k \geq 4$, one can verify $f(t) < (3/4)^t \leq (1 - \frac{1}{k})^t$ easily. Also, for $t = 4$ and all $k \geq 5$, we have $f(4) < (4/5)^4 \leq (1 - \frac{1}{k})^4$.

\[ \square \]

Acknowledgement

We thank two referees for their valuable comments.

References

1. N. Alon, P. Frankl, H. Huang, V. Rödl, A. Ruciński, and B. Sudakov, Large matchings in uniform hypergraphs and the conjecture of Erdős and Samuels, J. Combin. Theory Ser. A 119 (2012), n. 6, 1200–1215. MR 2915641
2. E. Buß, H. Hán, and M. Schacht, Minimum vertex degree conditions for loose Hamilton cycles in $3$-uniform hypergraphs, J. Combin. Theory Ser. B 103 (2013), no. 6, 658–678. MR 3127586
3. A. Czygrinow and T. Molla, Tight codegree condition for the existence of loose Hamilton cycles in $3$-graphs, SIAM J. Discrete Math. 28 (2014), no. 1, 67–76. MR 3150175
4. G. A. Dirac, Some theorems on abstract graphs, Proc. London Math. Soc. (3) 2 (1952), 69–81. MR 0047308 (13,856e)
5. R. Glebov, Y. Person, and W. Weps, On extremal hypergraphs for Hamiltonian cycles, European J. Combin. 33 (2012), no. 4, 544–555. MR 2864440
6. H. Hán, Y. Person, and M. Schacht, On perfect matchings in uniform hypergraphs with large minimum vertex degree, SIAM J. Discrete Math 23 (2009), 732–748.
7. H. Hán and M. Schacht, Dirac-type results for loose Hamilton cycles in uniform hypergraphs, Journal of Combinatorial Theory. Series B 100 (2010), 332–346.
8. J. Han and Y. Zhao, Minimum degree conditions for Hamilton \((k/2)\)-cycles in \(k\)-uniform hypergraphs, manuscript.
9. , Minimum codegree threshold for Hamilton \(\ell\)-cycles in \(k\)-uniform hypergraphs, Journal of Combinatorial Theory, Series A 132 (2015), no. 0, 194 – 223.
10. , Minimum vertex degree threshold for loose hamilton cycles in 3-uniform hypergraphs, Journal of Combinatorial Theory; Series B 114 (2015), 70 – 96.
11. G. Katona and H. Kierstead, Hamiltonian chains in hypergraphs, Journal of Graph Theory 30 (1999), no. 2, 205–212.
12. P. Keevash, D. Kühn, R. Mycroft, and D. Osthus, Loose Hamilton cycles in hypergraphs, Discrete Mathematics 311 (2011), no. 7, 544–559.
13. D. Kühn, R. Mycroft, and D. Osthus, Hamilton \(\ell\)-cycles in uniform hypergraphs, Journal of Combinatorial Theory. Series A 117 (2010), no. 7, 910–927.
14. D. Kühn and D. Osthus, Loose Hamilton cycles in 3-uniform hypergraphs of high minimum degree, Journal of Combinatorial Theory. Series B 96 (2006), no. 6, 767–821.
15. D. Kühn and D. Osthus, Embedding large subgraphs into dense graphs, Surveys in combinatorics 2009, London Math. Soc. Lecture Note Ser., vol. 365, Cambridge Univ. Press, Cambridge, 2009, pp. 137–167. MR 2588541 (2011c:05275)
16. , Hamilton cycles in graphs and hypergraphs: an extremal perspective, Proceedings of the International Congress of Mathematicians 2014, Seoul, Korea Vol 4 (2014), 381–406.
17. O. Pikhurko, Perfect matchings and \(K^4_3\)-tilings in hypergraphs of large codegree, Graphs Combin. 24 (2008), no. 4, 391–404. MR 2438870 (2009e:05214)
18. V. Rödl and A. Ruciński, Dirac-type questions for hypergraphs — a survey (or more problems for endre to solve), An Irregular Mind Bolyai Soc. Math. Studies 21 (2010), 561–590.
19. V. Rödl and A. Ruciński, Families of triples with high minimum degree are Hamiltonian, Discuss. Math. Graph Theory 34 (2014), no. 2, 361–381. MR 3194042
20. V. Rödl, A. Ruciński, and E. Szemerédi, \(K_4\) and \(K^4_3\) -tilings in hypergraphs, Combinatorics, Probability and Computing 15 (2006), no. 1-2, 229–251.
21. V. Rödl, A. Ruciński, and E. Szemerédi, Perfect matchings in uniform hypergraphs with large minimum degree, European J. Combin. 27 (2006), no. 8, 1333–1349. MR 2260124 (2007g:05153)
22. V. Rödl, A. Ruciński, and E. Szemerédi, An approximate Dirac-type theorem for \(k\)-uniform hypergraphs, Combinatorica 28 (2008), no. 2, 229–260.
23. V. Rödl, A. Ruciński, and E. Szemerédi, Perfect matchings in large uniform hypergraphs with large minimum collective degree, J. Combin. Theory Ser. A 116 (2009), no. 3, 613–636. MR 2500161 (2010d:05124)
24. V. Rödl, A. Ruciński, and E. Szemerédi, Dirac-type conditions for Hamiltonian paths and cycles in \(3\)-uniform hypergraphs, Advances in Mathematics 227 (2011), no. 3, 1225–1299.
25. A. Treglown and Y. Zhao, Exact minimum degree thresholds for perfect matchings in uniform hypergraphs II, J. Combin. Theory Ser. A 120 (2013), no. 7, 1463–1482. MR 3092677
26. Y. Zhao, Recent advances on dirac-type problems for hypergraphs, Recent Trends in Combinatorics, the IMA Volumes in Mathematics and its Applications, vol. 159, Springer, New York, 2015.

Instituto de Matemática e Estatística, Universidade de São Paulo, Rua do Matão 1010, 05508-090, São Paulo, Brazil.

E-mail address, Jie Han: jhanime.usp.br

Department of Mathematics and Statistics, Georgia State University, Atlanta, GA 30303, USA

E-mail address, Yi Zhao: yzhao6@gsu.edu