Cosmological Constant Dominated Transit Universe from the Early Deceleration Phase to the Current Acceleration Phase in Bianchi-V Spacetime

YADAV Anil Kumar
Department of Physics, Anand Engineering College, Keetham, Agra-282 007, India

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We present the transition of the universe from the early decelerating phase to the current accelerating phase with viscous fluid and time-dependent cosmological constant $\Lambda$ as a source of matter in Bianchi-V spacetime. To study the transit behaviour of the universe, we assume the scale factor as an increasing function of time, which generates a time-dependent deceleration parameter (DP). The study reveals that the cosmological term does not change its fundamental nature for $\xi = \text{const}$ and $\xi = \xi(t)$, where $\xi$ is the coefficient of bulk viscosity. The $\Lambda(t)$ is found to be positive and is a decreasing function of time. The same behavior was observed during recent supernovae observations. The physical behaviour of the universe is discussed in detail.

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One of the outstanding problems in particle physics and cosmology is the cosmological constant problem: its theoretical expectation values from quantum field theory exceed observational limits by 120 orders of magnitude. [1] Even if such high energy is suppressed by super-symmetry, the electroweak corrections are still 56 orders higher. This problem was further sharpened by recent observation of supernova Ia (SN Ia), which reveals the striking discovery that our universe has lately been in its accelerated expansion phase [2-3]. Cross checks from the cosmic background microwave radiation (CMBR) and large scale structure (LSS), all confirm this unexpected result. [4,5] Numerous dynamical dark energy models have been proposed in the literature, such as quintessence, [6] phantom, [7] k-essence, [8] tachyon, [9] DGP [10] and chaplygin gas. [11] However, the simplest and most theoretically appealing candidate for dark energy is vacuum energy (or the cosmological constant $\Lambda$) with a constant equation of state parameter equal to $-1$.

Experimental study of the isotropy of cosmic microwave background radiation (CMBR) and speculation about the amount of helium formed at the temperature just above neutrino decoupling, matter behaved like a viscous fluid (Klimek [21]) in the early stages of evolution. It has been suggested that in a large class of homogeneous but anisotropic universes, the anisotropy dies away rapidly. The most important mechanism in reducing the anisotropy is neutrino viscosity at temperature just above $10^{10}$ K. It is important to develop a model of dissipative cosmological processes in general, so that one can analyze the overall dynamics of dissipation without getting lost in the detail of complex processes. Coley [22] studied Bianchi-V viscous fluid cosmological models for barotropic fluid distribution. Murphy [23] has investigated the role of viscosity in avoiding the initial big bang singularity. Padmanabhan and Chitre [24] have shown that bulk viscosity leads to an inflationary like solution. Pradhan et al [25,26] and Yadav [27,28] have shown that bulk viscosity leads to an inflationary like solution. Pradhan et al [25,26] and Yadav [27,28] have shown that bulk viscosity leads to an inflationary like solution.
et al.\cite{29} have studied bulk viscous cosmological models with variable $G$ and $A$. In this Letter, we study the transit behaviour of universe with time-dependent $A$ in Bianchi-V spacetime. To study the transit behaviour of the universe, we assume the scale factor as an increasing function of time which generates a time-dependent (deceleration parameter) DP.

We consider the spacetime metric of spatially homogeneous and anisotropic Bianchi-V of the form

$$ds^2 = -dt^2 + A^2 dx^2 + e^{2\alpha x}(B^2 dy^2 + C^2 dz^2),$$

(1)

where $A(t)$, $B(t)$ and $C(t)$ are the scale factors in different spatial directions and $\alpha$ is a constant. We define the average scale factor $a$ of the Bianchi-V model as

$$a = (ABC)^{1/3},$$

(2)

The spatial volume is given by

$$V = a^3 = ABC.$$

(3)

Therefore, the generalized mean Hubble parameter $H$ reads

$$H = \frac{\dot{a}}{a} = \frac{1}{3}(H_1 + H_2 + H_3),$$

(4)

where $H_1 = \dot{A}/A$, $H_2 = \dot{B}/B$ and $H_3 = \dot{C}/C$ are the directional Hubble parameters in the directions of $x$, $y$, and $z$, respectively. An over dot denotes the differentiation with respect to cosmic time $t$.

Since metric (1) is completely characterized by the average scale factor, let us consider that the average scale factor is an increasing function of time as follows:

$$a = (t^n e^t)^{1/m},$$

(5)

where $m > 0$ and $n \geq 0$ are constant. Such a type of ansatz for the scale factor has already been considered by Yadav,\cite{30} which generalized the one proposed by Pradhan and Amirhashchi.\cite{31} The proposed law (5) yields a time-dependent DP which describes the transition of the universe from the early decelerating phase to the current accelerating phase.

The value of the DP, $q$, for model (1) is found to be

$$q = -\frac{\ddot{a}}{a^2} = -1 + \frac{mn}{(n + 1)^2}.$$  

(6)

From Eq. (6), it is clear that the DP $q$ is time-dependent. Also, the transition redshift from deceleration expansion to accelerated expansion is about 0.5. Now for a universe which was decelerating in the past and accelerating at the present time, the DP must show signature flipping.\cite{32,33} It is however possible to have $n = 0$ in Eq. (5) for which we would have inflationary universe. The sign of $q$ indicates whether the model inflates or not. A positive sign of $q$, i.e. $t < \sqrt{mn} - n$ corresponds to the standard decelerating model whereas the negative sign $-1 \leq q < 0$ indicates inflation. It may be noted that the current observation of SN Ia and CMBR favour accelerating models ($q < 0$), but they do not altogether rule out the existence of a decelerating phase in the early history of our universe.\cite{34}

For the bulk viscous fluid, the energy momentum tensor is given by

$$T^i_j = (\rho + \bar{p})v^i v_j - p g^i_j,$$

(7)

where $\rho$ is the energy density, $\bar{p}$ is the effective pressure of the fluid, and $v^i$ is the fluid four velocity vector. In the comoving system of co-ordinates, we have $v^i = (1, 0, 0, 0)$. The effective pressure $\bar{p}$ is related to the equilibrium pressure $p$ by\cite{35}

$$\bar{p} = p - 3\xi H,$$

(8)

where $\xi$ is the coefficient of bulk viscosity that determines the magnitude of viscous stress relative to expansion.

Einstein’s field equations with cosmological constant (in gravitational units $c = 1, 8\pi G = 1$) read

$$R^i_j - \frac{1}{2}g^i_j R = -T^i_j + A g^i_j,$$

(9)

The Einstein field Eqs. (9) for the line-element (1) leads to the following system of equations

$$\frac{\ddot{A}}{A} + \frac{\ddot{B}}{B} + \frac{\ddot{C}}{C} = -\frac{\ddot{\rho} + \ddot{p}}{3},$$

(10)

$$\frac{\dot{A}}{A} + \frac{\dot{B}}{B} + \frac{\dot{C}}{C} = \frac{\ddot{\rho}}{3} - \frac{\ddot{p}}{3} + \frac{\ddot{\rho}}{3},$$

(11)

$$\frac{\dot{A}}{A} + \frac{\dot{B}}{B} + \frac{\dot{C}}{C} = \frac{\ddot{\rho}}{3} - \frac{\ddot{p}}{3} + \frac{\ddot{\rho}}{3},$$

(12)

$$\frac{\ddot{A}B + \ddot{A}C + \ddot{B}C}{AB + AC + BC} = \frac{3\alpha^2}{\alpha^2} = \frac{\rho}{3} + A,$$

(13)

$$\frac{2\dot{A}}{A} - \frac{\dot{B}}{B} - \frac{\dot{C}}{C} = 0.$$  

(14)

Combining Eqs. (10)–(13), one can easily obtain the continuity equation as

$$\dot{\rho} + (\rho + \bar{p})\left(\frac{\dot{A}}{A} + \frac{\dot{B}}{B} + \frac{\dot{C}}{C}\right) = 0.$$  

(15)

Integrating Eq. (14) and absorbing the constant of integration into $B$ or $C$, we obtain

$$A^2 = BC.$$  

(16)

From Eqs. (10)–(12) and taking second integral of each, we obtain the following three relations, respectively.

$$\frac{A}{B} = b_1 \exp \left( x_1 \int a^{-3} dt \right),$$

(17)

$$\frac{A}{C} = b_2 \exp \left( x_2 \int a^{-3} dt \right),$$

(18)

$$\frac{B}{C} = b_3 \exp \left( x_3 \int a^{-3} dt \right),$$

(19)
where $b_1$, $b_2$, $b_3$, $x_1$, $x_2$ and $x_3$ are constants of integrations.

From Eqs. (16)–(19) and Eq. (5), the metric functions can be explicitly written as

$$A(t) = \left( t^n e^l \right)^{1/m}, \quad (20)$$

$$B(t) = d \left( t^n e^l \right)^{1/m} \exp \left( \ell \int (t^n e^l)^{-\frac{m}{n}} dt \right), \quad (21)$$

$$C(t) = d^{-1} \left( t^n e^l \right)^{1/m} \exp \left( -\ell \int (t^n e^l)^{-\frac{m}{n}} dt \right), \quad (22)$$

where $d = (b_2 b_3)^{\frac{1}{2}}$, $\ell = \frac{x_2 + x_3}{3}$ with $b_2 = b_1^{-1}$ and $x_2 = -x_1$.

Fig. 1. The plot of DP $q$ vs time $t$.

The physical parameters such as scalar of expansion, $\theta$, spatial volume $V$, anisotropy parameter $A$, shear scalar $\sigma^2$, and directional Hubble’s parameters $H_x, H_y, H_z$ are, respectively, given by

$$\theta = 3H = \frac{3}{m} \left( \frac{n}{l} + 1 \right), \quad (23)$$

$$V = \left( t^n e^l \right)^{\frac{n}{m}}, \quad (24)$$

$$\dot{A} = \frac{2\ell^2 - 2}{3} \left( t^n e^l \right)^{-\frac{n}{m}} \left( \frac{n}{l} + 1 \right)^{-2}, \quad (25)$$

$$\sigma^2 = \frac{2\ell^2}{3} \left( t^n e^l \right)^{-\frac{n}{m}}, \quad (26)$$

$$H_x = \frac{1}{m} \left( \frac{n}{l} + 1 \right), \quad (27)$$

$$H_y = \frac{1}{m} \left( \frac{n}{l} + 1 \right) + \ell \left( t^n e^l \right)^{-\frac{m}{n}}, \quad (28)$$

$$H_z = \frac{1}{m} \left( \frac{n}{l} + 1 \right) - \ell \left( t^n e^l \right)^{-\frac{m}{n}}. \quad (29)$$

It is observed that the spatial volume is zero and the expansion scalar is infinite at $t = 0$, which shows that the universe starts evolving with zero volume at initial epoch at $t = 0$ with an infinite rate of expansion. The scale factor also vanishes at initial moment, hence the model has a point type singularity at $t = 0$. For $t \to \infty$, we obtain $q = -1$ and $dH/dt = 0$, which implies the greatest value of the Hubble parameter and the fastest rate of expansion of the universe. It is evident that negative value of $q$ would accelerate and increase the age of universe. Figure 1 shows the dynamics of the DP versus cosmic time. It is observed that initially the DP evolves with a positive sign, but later on, DP grows with a negative sign. This DP behaviour clearly explains the decelerated expansion in the past and the accelerated expansion of universe at present as observed in recent observation of SN Ia. Thus the derived model can be utilized to describe the dynamics of the late time evolution of the observed universe.

In the derived model, the present value of the DP is estimated by

$$q_0 = -1 + \frac{n}{m H_0^2 t_0^2}, \quad (30)$$

where $H_0$ is the present value of Hubble’s parameter and $t_0$ is the age of universe at the present epoch. If we set $n = 0.27m$ in Eq. (30), we obtain $q_0 = -0.73$, which exactly matches the observed value of the DP at the present epoch. Thus we constrain $m = 3$ and $n = 0.27m$ in the remaining discussions of the model and graphical representations of the physical parameters.

Fig. 2. The plot of anisotropy parameter $A$ vs time $t$.

Figure 2 depicts the variation of anisotropy parameter $A$ versus cosmic time. It is shown that $A$ decreases with time and tends to zero for sufficiently large times. Thus the anisotropic behaviour of universe dies out at later times and the observed isotropy of universe can be achieved in the derived model at the present epoch.

The effective pressure $\bar{p}$ and energy density $\rho$ of the model read

$$\bar{p} = \alpha^2 \left( t^n e^l \right)^{-\frac{m}{n}} - 2 \left[ \frac{1}{m} \left( \frac{n}{l} + 1 \right) + \ell \left( t^n e^l \right)^{-\frac{m}{n}} \right]^2$$

$$- \left[ \frac{1}{m^2} \left( \frac{n}{l} + 1 \right)^2 - \ell^2 \left( t^n e^l \right)^{-\frac{m}{n}} \right] + \Lambda, \quad (31)$$

$$\rho = \frac{3}{m^2} \left( \frac{n}{l} + 1 \right)^2 - \ell^2 \left( t^n e^l \right)^{-\frac{m}{n}}$$

$$- 3\alpha^2 \left( t^n e^l \right)^{-\frac{m}{n}} - \Lambda. \quad (32)$$

Equations (8) and (31) lead to

$$p = 3\xi H = \alpha^2 \left( t^n e^l \right)^{-\frac{m}{n}} - 2 \left[ \frac{1}{m} \left( \frac{n}{l} + 1 \right) + \ell \left( t^n e^l \right)^{-\frac{m}{n}} \right]^2$$

$$- \left[ \frac{1}{m^2} \left( \frac{n}{l} + 1 \right)^2 - \ell^2 \left( t^n e^l \right)^{-\frac{m}{n}} \right] + \Lambda. \quad (33)$$
For the specification of $\xi$, we assume that the fluid obeys the equation of state of the form
\[ p = \gamma \rho, \]  
(34)
where ($0 \leq \gamma \leq 1$) is constant. Thus, we can solve the cosmological parameters by taking a different physical assumption of $\xi(t)$.

We assume that
\[ \xi(t) = \xi_0 = \text{const}. \]

Now, Eq. (33), with use of Eqs. (32) and (34) reduces to
\[ \rho = \frac{3\xi_0}{m(1 + \gamma)} \left( \frac{n}{l} + 1 \right) + \frac{2}{m^2(1 + \gamma)} \left( \frac{n}{l} + 1 \right)^2 \]
\[ - \frac{2}{(1 + \gamma)} \left[ \frac{1}{m} \left( \frac{n}{l} + 1 \right) + \ell(t^n e^t)^{-\frac{2}{\gamma}} \right]^2 \]
\[ - 2\alpha^2 (t^n e^t)^{-\frac{2}{\gamma}}. \]  
(35)
Eliminating $\rho(t)$ between Eqs. (32) and (35), we obtain
\[ A = \frac{3(\gamma + 1)}{m^2(\gamma + 1)} \left( \frac{n}{l} + 1 \right)^2 + 2 \left[ \frac{1}{m} \left( \frac{n}{l} + 1 \right) \right. \]
\[ + \ell^2(t^n e^t)^{-\frac{2}{\gamma}} \left. \right] - \frac{3\xi_0}{\gamma + 1} \left( \frac{n}{l} + 1 \right) - \alpha^2 (t^n e^t)^{-\frac{2}{\gamma}} \]
\[ - \ell^2(t^n e^t)^{-\frac{2}{\gamma}}. \]  
(36)
From Eq. (36), we observe that the cosmological constant is a decreasing function of time and it approaches a small positive value as time progresses (i.e., the present epoch).

![Fig. 3. The plot of cosmological constant $A$ vs time $t$.](image)

Further we assume that
\[ \xi = \xi_0 \rho. \]  
(37)
Finally, Murphy\cite{23} has constructed a class of viscous cosmological models with $\xi = \xi_0 \rho$, which possesses an interesting feature that the big bang type singularity of infinite spacetime curvature does not occurs at finite past. Later on, Pradhan et al.\cite{25,26} presented bulk viscous cosmological models for $\xi = \xi_0 \rho$ in harmony with SN Ia observations.

Now, Eq. (33) with use of Eqs. (32), (34) and (37) reduces to
\[ \left[ 1 + \gamma - \frac{3\xi_0}{m} \left( \frac{n}{l} + 1 \right) \right] \rho = \frac{2}{m^2} \left( \frac{n}{l} + 1 \right)^2 \]
\[ - 2 \left[ \frac{1}{m} \left( \frac{n}{l} + 1 \right) + \ell(t^n e^t)^{-\frac{2}{\gamma}} \right]^2 - 2\alpha^2 (t^n e^t)^{-\frac{2}{\gamma}}. \]  
(38)
Eliminating $\rho(t)$ between Eqs. (32) and (38), we obtain
\[ A = \frac{3}{m^2} \left( \frac{n}{l} + 1 \right)^2 - \ell^2(t^n e^t)^{-\frac{2}{\gamma}} - 3\alpha^2 (t^n e^t)^{-\frac{2}{\gamma}} \]
\[ - \frac{2}{m^2} \left( \frac{n}{l} + 1 \right)^2 + \frac{1}{1 + \gamma - \frac{3\xi_0}{m} \left( \frac{n}{l} + 1 \right)} \times \]
\[ \left[ 2\alpha^2 (t^n e^t)^{-\frac{2}{\gamma}} + \frac{2}{m} \left( \frac{n}{l} + 1 \right) + \ell(t^n e^t)^{-\frac{2}{\gamma}} \right]^2. \]  
(39)
From Eq. (39), it is observed that the cosmological term is positive and a decreasing function of time (i.e., the present epoch), which supports the result obtained from the recent type-Ia supernova observations\cite{2,37}.

The behaviour of the cosmological constant is clearly shown in Fig. 3. The models have a non-vanishing cosmological constant and energy density as $t \to \infty$.

It is well known that with the expansion of universe, i.e., with the increase of time $t$, the energy density decreases and becomes so small that it can be ignored.

We can express Eqs. (10)–(13) in terms of $H, q$ and $\sigma$ as
\[ \bar{p} - A = (2q - 1)H^2 - \sigma^2 + \frac{\alpha^2}{\sigma^2}, \]  
(40)
\[ \rho + A = 3H^2 - \sigma^2 - \frac{3\alpha^2}{\sigma^2}. \]  
(41)
From Eqs. (40) and (41), we obtain
\[ \bar{a} = \frac{A}{3} + \frac{1}{2} \xi \theta - \frac{1}{6} (\rho + 3p) - \frac{2}{3} \sigma^2, \]  
(42)
which is Raychaudhuri’s equation for a given distribution. Equation (42) shows that for $\rho + 3p = 0$, acceleration is initiated by bulk viscosity and the $A$ term. In the absence of bulk viscosity, only $A$ contributes the acceleration that seems to relate $A$ with dark energy.

It also shows that for a positive $A$ the universe may accelerate under the condition $\rho + 3p \leq 0$, i.e., $p$ is negative for positive energy density $\rho$ with a definite contribution of $A$ in the acceleration. In the observational front, the data set coming from the Supernova Legacy Survey (SNLS) shows that dark energy behaves in the same manner as $A$.

In summary, we have presented the generalized law for the scale factor in homogeneous and anisotropic Bianchi-V spacetime that yields the time-dependent DP, representing a model which generates a transition of the universe from an early decelerating phase to a recent accelerating phase. The spatial scale factors
and volume scalar vanish at $t = 0$. The energy density and pressure are infinite at this initial epoch. As $t \to \infty$, the scale factor diverges, and $\rho$ and $p$ both tend to zero. $\Lambda$ and $\sigma^2$ are very large at the initial moment but decrease with cosmic time and vanish at $t \to \infty$.

The model shows an isotropic state at a later time of its evolution. Also we observe that $\bar{p} = -\rho$ as $t \to \infty$.

For $n \neq 0$, all matter and radiation is concentrated at the big bang epoch and the cosmic expansion is driven by the big bang impulse. The model has a point type singularity at the initial moment as the scale factors and volume vanish at $t = 0$. For $n = 0$, the model has no real singularity and energy density becomes finite. Thus the universe has a non-singular origin and the cosmic expansion is driven by the creation of matter particles. It has been observed that $\lim_{t \to 0} \frac{d}{dt}$ turns out to be constant. Thus the model approaches homogeneity and matter is dynamically negligible near the origin.

The cosmological constants for $\xi = \xi_0$ and $\xi = \xi(t)$ are decreasing functions of time and they approach a small positive value as time increases (i.e., the present epoch). The values of the cosmological constant for these models are supported by the results from recent supernovae observations recently obtained by the high-$Z$ supernovae team and the Supernovae Cosmological Project.[2,4,37,38] A positive cosmological constant resists the attractive gravity of matter due to its negative pressure. For most universes, the positive cosmological constant eventually dominates over the attraction of matter and drives the universe to expand exponentially. Thus, with our approach, we obtain a physically relevant decay law for the cosmological constant, unlike other studies where ad hoc laws were used to arrive at a mathematical expressions for decaying $\Lambda$. Thus the derived models are more general than those studied earlier.

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