Small neutrino masses and gauge coupling unification

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The physics responsible for gauge coupling unification may also be responsible for providing small neutrino masses. We present a model based on the $SU(3)_C \otimes SU(3)_L \otimes U(1)_X$ (3-3-1) gauge group in which neutrinos are massless at tree-level. Gauge mediated radiative corrections generate calculable neutrino masses. The three gauge coupling constants unify for a 3-3-1 scale of order TeV, making the model directly testable at the LHC.

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Preliminaries

The fact that gauge coupling unification is a “near-miss” within the Standard Model (SM) provides an indication in favor of the idea of unification [1]. Likewise, the existence of neutrino masses, required to account for neutrino oscillation data [2], also provides another motivation towards unified or GUT1-like extensions of the SM. However, the most characteristic feature of GUT-type unification, namely matter instability, has so far defied experimental confirmation [3]. On the other hand, neither the generation of neutrino masses nor the tilting in the evolution of the gauge couplings evolution require unification in the conventional sense. For instance, it is well known that the gauge couplings merge in the minimal supersymmetric extension the SM, provided that supersymmetric states lie around the TeV scale [3, 4]. So far, though, there has been no trace of such states in the LHC data [5].

Here we consider an alternative approach in which new states present at the TeV scale realize an extended gauge structure where baryon-number is perturbatively conserved. For definiteness we consider the $SU(3)_C \otimes SU(3)_L \otimes U(1)_X$ [6, 7] (3-3-1) framework, which implies that the number of generations equals the number of colors, in order to cancel anomalies. This scheme has attracted attention recently also in connection with B physics [8–10], or flavor symmetries [11]. In this letter we present a model in which the gauge couplings can naturally unify at some accessible energy, and where small calculable neutrino masses are induced by new gauge boson exchange, in the absence of supersymmetry. Neutrino masses arise at the TeV scale [12] instead of the conventional high-scale seesaw mechanism [13] We first recall that, by adding three gauge singlet fermions $S_i$, the light neutrinos acquire mass only at one-loop order, through the exchange of new gauge bosons [14]. Unfortunately, however, unification does not occur, as can be seen in figure 1. This is mostly due to fact that the new gauge bosons make $\alpha_L$ weaker at high energies, while the new colored particles strengthen $\alpha_C$.

Here we contemplate the possibility of unifying the gauge couplings in such a scheme. The idea is to promote the three fermion singlets to three octets of the enlarged electroweak $SU(3)_L$ symmetry factor group. We propose a new variant of the model in [14], which opens the possibility of reconciling the radiative mechanism of neutrino masses generation with the idea of gauge coupling unification in the framework of the extended $SU(3)_L \otimes U(1)_X$ electroweak gauge group. This group is broken down to the standard $SU(2)_L \otimes U(1)_Y$ model at some scale $M_{331}$ characterizing the new gauge boson masses. This scale is found to lie in the $1 - 10 \text{ TeV}$ range, with a plethora of new states expected to be directly accessible to LHC searches, making the phenomenology of the model especially rich and predictive.

The model

We consider a simple variant of the model introduced in [14], where the fermion singlet is promoted to an octet.
representation of $SU(3)_L$. The model is based on the same $SU(3)_C \otimes SU(3)_L \otimes U(1)_X$ gauge symmetry, extended with a global $U(1)_C$, which is necessary in order to consistently define lepton number, and an auxiliary parity symmetry whose purpose will be made clear below.

| SU$(3)_C$ | SU$(3)_L$ | U$(1)_X$ | $\phi_1$ | $\phi_2$ | $\phi_3$ |
|-----------|-----------|-----------|-----------|-----------|-----------|
| $\psi_{L,i}$ | $\ell^R_{L,i}$ | $Q_{L,i}^U$ | $Q_{L,i}^L$ | $u^R_{L,i}$ | $\Omega^c$ |

| Generations | 3 | 3 | 2 | 1 | 4 | 5 | 3 | 1 | 1 | 1 |

TABLE I: Field content of the model.

The model contains three generations of lepton $SU(3)_L$ anti-triplets, two generations of quark triplets and one of anti-triplets (quarks and charged leptons are accompanied with their, $SU(3)_L$ singlet, right-handed partners), three generations of fermion octets, and finally three scalar boson anti-triplets. We summarize the particle content of the model in table I. The allowed lepton interactions compatible with the quantum number assignments given in table I are the following:

$$\mathcal{L}_{\text{leptons}} = (y_{ij}^c)^* \psi^T_{L,i} C \ell^R_{R,j} \phi_1 + (y_{ij}^c)^* \psi^T_{L,i} C \Omega^c \phi_2^c + \frac{1}{2} (M_{ij})^c \phi_1^T C \Omega^c \phi_2 + \text{h.c.}$$

(1)

The components of $\psi_L$ and the $\phi_j$ can be written as follows:

$$\psi_L, i = \begin{pmatrix} \ell_L \\ -\nu_L \\ N^c \end{pmatrix}, \quad \phi_1 = \begin{pmatrix} \phi_1^0 \\ -\phi_1^+ \\ \phi_1^- \end{pmatrix}, \quad \phi_{2,3} = \begin{pmatrix} \phi_{2,3}^0 \\ -\phi_{2,3}^+ \\ \phi_{2,3}^- \end{pmatrix}. \quad (2)$$

The scalars take vevs in the directions $(\phi_1) = (k_1, 0, 0)$ and $(\phi_{2,3}) = (0, -k_{2,3}, n_{2,3})$. As for the octets, one can write

$$\Omega_i^c = \begin{pmatrix} -\frac{1}{\sqrt{2}} T^0 + \frac{1}{\sqrt{6}} N^c & -T^+ & \ell_L \\ -T^- & \frac{1}{\sqrt{2}} T^0 + \frac{1}{\sqrt{6}} N^c & -\nu_L \\ \frac{1}{2} \sqrt{2} T^0 + \frac{1}{\sqrt{6}} N^c & -\bar{\nu}_L & -\frac{2}{\sqrt{6}} N^c \end{pmatrix} \quad (3)$$

such that $\Omega_i^c$ is transformed into $U \Omega_i^c U^\dagger$ under an $SU(3)_L$ gauge transformation, where $U$ is the transformation matrix of the triplet representation.

Under $SU(2)_L \otimes U(1)_Y$, each $\Omega_i^c$ breaks into the representations $(3, 0) \equiv (T^+, T^0, T^-)_i$, $(2, -\frac{1}{2}) \equiv (\ell_L, -\bar{\nu}_L)_i$, $(2, \frac{1}{2}) \equiv (\bar{\nu}_L, \ell_L)_i$, $(1, 0) \equiv (\nu_L, T^0)_i$, $(1, 0) \equiv (\ell_L, -\bar{\nu}_L)_i$.

Note also that the electric charge and lepton number assignments of the particles of the model follow from

$$Q = T_3 + \frac{1}{\sqrt{3}} T_8 + X, \quad (4)$$

$$L = \frac{4}{\sqrt{3}} T_8 + \mathcal{L}, \quad (5)$$

where $T_3$ and $T_8$ are the diagonal generators of $SU(3)_L$.

**Gauge couplings unification**

The one-loop renormalization group equation of the $\alpha_i \equiv g_i^2/4\pi$ is given by [17, 18]:

$$\frac{d\alpha_i^{-1}}{dt} = -\frac{b_i}{2\pi}, \quad (6)$$

This term is required in order to provide an adequately large mass to the new charged leptons.

2 Since $T_8$ is a gauge generator, there is no physical Goldstone boson associated with spontaneous lepton number violation [15, 10].
where $t$ is the logarithm of the energy scale, and the $b_i$ coefficients are functions of the Casimir of the gauge group, $C(G_i)$, and of the Dynkin index of the scalar and (Weyl) fermion representations, $T(S)$ and $T(F)$, respectively:

$$b_i = -\frac{11}{3} C(G_i) + \frac{2}{3} \sum F T(F) + \frac{1}{3} \sum S T(S).$$  (7)

For the SM, the $b_i$ are $b_i^{SM} = \{-7, -\frac{19}{6}, \frac{11}{10}\}$, while in the $SU(3)_C \otimes SU(3)_L \otimes U(1)_X$ phase they read $b_i^{331} = \{-5, -\frac{13}{2} + 2n, \frac{13}{2}\}$, for $n$ active fermion octets $\Omega_i$. It should be noted that while we do not speculate here about the possible embedding of $SU(3)_C \otimes SU(3)_L \otimes U(1)_X$ into some bigger group (for example $SU(5)$), it can be shown on very general grounds that the $U(1)_X$ charge normalization should be $X_{\text{canonical}} = \sqrt{3}/2X$. Given the relation between $X$ and the SM hypercharge indicated by equation 4, it follows that $\alpha_X^{-1} = \frac{1}{5} (\alpha_{\text{SM}}^{-1} + 4\alpha_{\text{X}}^{-1})$ at the 3-3-1 breaking scale.

Figure 2 illustrates the running of the gauge coupling constants in our model, with the 3-3-1 scale fixed at 1 TeV and the three octets $\Omega_i$ integrated out at 3 TeV. Given that the $b_i^{SM}$ coefficients are not very different from $b_i^{331}$ with the three octets, unification is sensitive mostly to the ratio $M_8/M_{331}$, and not to the 3-3-1 scale per se. The effect of changing this latter scale is shown in figure 3; allowing for threshold and 2-loop effects, one can see that unification constrains the $M_8/M_{331}$ to be roughly between 1 and 6.

**Neutrino masses**

The lepton mass matrices after $SU(3)_L \otimes U(1)_X$ is broken to $U(1)_{\text{em}}$ can be computed from the Lagrangian in equation 1:

$$\mathcal{L}_{\text{lepton masses}} = \ell^T C M_8^{\ell_T} \ell + \frac{1}{2} \nu^T C M_8^{\nu} \nu + h.c.$$  (8)

In the basis where $\ell = (\ell_L, \ell_R, T^-)^T$ and $\ell = (\ell_R, \ell_L, T^+)^T$, the charged leptons mass matrix reads

$$M_\ell = \begin{pmatrix} M_\ell & M_{331} & M_8^{\ell_T} \\ 0 & M_8 & 0 \\ 0 & 0 & M_8 \end{pmatrix},$$  (9)

where the entries are given by $(M_\ell)_{ij} = y_{ij}^L k_1$, $(M_{331})_{ij} = y_{ij}^R k_2$, and $(M_8^{\ell_T})_{ij} = y_{ij}^N n_2$.

On the other hand, adopting the eigenbasis $\nu = (\nu_L, \tilde{\nu}_L, \bar{\nu}_L, N^c, \bar{N}^c, T^0)^T$, one finds the neutrino mass matrix given as

$$M_\nu = \begin{pmatrix} 0 & 0 & M_{331} & 0 & 0 & \frac{1}{\sqrt{3}} M_8^{\ell_T} \\ 0 & M_8 & 0 & 0 & -\frac{1}{\sqrt{3}} M_{331} & 0 \\ \frac{1}{\sqrt{3}} M_8^{\ell_T} & 0 & \frac{1}{2} M_8 & 0 & 0 & 0 \\ 0 & 0 & 0 & M_8 & 0 & 0 \\ 0 & 0 & 0 & 0 & M_8 & 0 \end{pmatrix}.$$  (10)

The $\nu_2$ (together with $n_2$) sets the $SU(3)_L \otimes U(1)_X$ breaking scale, that is $n_2 \sim M_{331}$. While the $\nu_2$ and $k_2$ are expected to lie at the electroweak scale since they belong to $SU(2)_L$ doublets after the breaking of $SU(3)_L$. In the one-family approximation, $M_\nu$ has one null eigenvector given as

$$\nu_{\text{light}} = \sum \omega^\alpha \nu_\alpha,$$  (10)

where $\omega = \frac{1}{\sqrt{2}} \left(1, -x, x^{\alpha^2}, -\alpha, -\sqrt{\frac{3}{2}} x^{\alpha}, -\sqrt{\frac{1}{2}} x^{\alpha}\right)^T$. In the expression for $\omega$, $x \equiv M_{331}/M_8$ and $\alpha \equiv M_8/M_{331} = k_2/n_2$ are small parameters and $N$ is some normalization factor. For Yukawa couplings $y_{ij}^{\ell}$ of $O(1)$, $x \approx 1 \Rightarrow M_{331}/M_8$, so that we can apply the constraint derived from unification which bounds the ratio $M_8/M_{331}$ to lie between 1 and 6 approximately. The parameter $\alpha$ is in principle unconstrained as $k_2$ can be arbitrarily small, however, one would naturally expect that it is of the order of the electroweak scale over the 3-3-1 scale, $M_W/M_{331}$. As such, the expression above implies that the observed neutrino states are mostly a mixture of $\nu_L$ and $\tilde{\nu}_L$ which are both in the $(2, -\frac{1}{2})$ representation of the $SU(2)_L \otimes U(1)_Y$ group. The admixture of the remaining neutrino states are suppressed by at least a factor $\alpha$. We have verified that
in the multi-generation case $M_{\rho}$ has a null eigenvector associated to each of the three generations of leptons. As a result neutrinos are protected to be massless in the tree approximation. This property forms the basis of the radiative mechanism discussed below.

As for the charged leptons, there is only a pair which is light in each generation, namely:
\begin{align}
\ell_{\text{light}} &\propto \ell_L - x \ell_R - x_a T^- , \\
\ell^c_{\text{light}} &\propto \ell^c_R - \frac{M_{\ell}}{M_{331}^2} x^2 - \frac{1}{1 + x^2} \ell_L .
\end{align}

These two 2-component states form the standard Dirac charged lepton which now has a squared mass given by $(M_{\ell})^2 \frac{1}{1 + x^2}$, an expression that differs from the SM one. Notice that once more the presence of states which do not come from the $(2, -\frac{1}{2})$ electroweak representation is $\alpha$-suppressed. The two remaining pairs of charged leptons are heavy, with octet-scale masses.

Turning back to neutrinos, at the loop level, the exchange of gauge bosons will give rise to a small neutrino mass which can be calculated rigorously in the gauge and fermion mass eigenbasis. If we then expand the result in powers of the squared masses of the internal gauge bosons and the fermions around some central values $\langle m_{\Lambda}^2 \rangle$ and $\langle m_{\Phi}^2 \rangle$ we obtain:
\begin{equation}
m_{\nu} \approx \frac{1}{(4\pi)^2} \frac{1}{54} g_L^4 (3g_L^2 + 4g_X^2) n_2 (k_3 n_2 + k_2 n_3) \frac{MeV y^2}{\langle m^2_W \rangle^3} \left( 1 - x_i^2 \right)^4 ,
\end{equation}
where we also used the approximation $\langle m_{\Lambda}^2 \rangle / \langle m_{\Phi}^2 \rangle \approx M_{331}^2 / M_8^2 = x_i^2$.

This result should be seen as a mass insertion approximation, where three insertions are made in the fermion line and two in the gauge boson line. Indeed, by calculating the full symmetric matrix $m (\nu_\alpha \nu_\beta)$ where $\nu_{\alpha, \beta}$ runs over all neutrino states in the electroweak basis, one obtains
\begin{equation}
m_{\nu} = \sum_{\alpha, \beta} m (\nu_\alpha \nu_\beta) \omega_\alpha \omega_\beta .
\end{equation}

The vector $\omega$, which was given earlier, describes the composition of the light neutrino states in terms of the 6 electroweak $\nu_\alpha$. Figure 4 provides two representative diagrams which contributes to $m (\nu_L \nu_L)$.

**Summary and outlook**

In this letter we have proposed an electroweak $SU(3)_C \otimes SU(3)_L \otimes U(1)_X$ gauge extension of the SM in which neutrinos are massless at tree-level. Even though the neutral fermion mass matrix has a seesaw structure, the messengers only provide mass at the loop level, thanks to the symmetry protection. However, gauge mediated radiative corrections generate small calculable neutrino masses. The physics responsible for providing small neutrino masses is also responsible for gauge coupling unification, which can be achieved at a characteristic scale of order TeV in the absence of supersymmetry and of GUT-like interactions. A plethora of new states such as new gauge bosons and fermions makes the model directly testable at the LHC, with a non-trivial interplay between the quark sector and the lepton sector. The presence of such new features is presently under investigation.

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