Algebraic renormalization
of twisted $N = 2$ supersymmetry
with $Z = 2$ central extension

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Abstract

We study the renormalizability of (massive) topological QCD based on the algebraic
BRST technique by adopting a non–covariant Landau type gauge and making use
of the full topological superalgebra. The most general local counter terms are deter-
mined and it is shown that in the presence of central charges the BRST cohomology
remains trivial. By imposing an additional set of stability constraints it is proven
that the matter action of topological QCD is perturbatively finite.

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1. Introduction

Topological quantum field theories (TQFT) are characterized by observables depending
only on the topology of the manifold on which these theories are defined [1]. The classical
example is topological Yang–Mills theory (TYM) on a four–manifold as proposed by
Witten [2] whose correlation functions, if computed in the weakly coupled ultraviolet limit,
turn out to be related to the Donaldson invariants [3]. On the other hand, the Donaldson

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invariants of smooth four–manifolds are closely related to $N = 2$ supersymmetric Yang–Mills theories (SYM). In fact, by restricting to flat Euclidean space–time TYM can be reformulated through a twisting of $N = 2$ SYM in Wess-Zumino gauge [2]. The resulting theory has a fermionic symmetry being identified with the topological shift operator $Q$ of the twisted $N = 2$ SUSY algebra. In the cohomological formulation of Labastida–Pernici [4] and Baulieu–Singer [5] TYM is interpreted as gauge–fixed action of the Pontryagin term and $Q$, after identifying the $R$–charge with the ghost number, is interpreted as the BRST operator of the equivariant part of TYM. As a result of that identification the BRST cohomology becomes completely trivial. In the derivation of ultraviolet finiteness properties of various topological models and in the construction of their observables, besides $Q$, still another fermionic symmetry, the so–called vector supersymmetry, plays an important role being generated by the vector charge $\bar{Q}_\mu$ of the twisted $N = 2$ SUSY algebra [6, 7, 8, 9, 10].

Seiberg and Witten have studied also the strongly coupled infrared limit of twisted $N = 2$ SYM which leads to a new class of four–manifold invariants, the so–called Seiberg–Witten invariants [12]. The Seiberg–Witten theory can be considered as the twisted version of $N = 2$ supersymmetric Maxwell theory with one massless hypermultiplet [13]. In a second paper, Seiberg and Witten have studied the strong coupled infrared limit of topological quantum chromodynamic (TQCD), being the twisted version of $N = 2$ SYM coupled to a standard hypermultiplet in the fundamental representation [14]. In Ref. [15] it has been shown that by introducing the $N = 2$ supersymmetric bare mass term to the hypermultiplet the resulting massive TQCD interpolates between the Donaldson and the Seiberg–Witten theory. Such hypermultiplets have a non–vanishing central charge and, therefore, an extension of the $N = 2$ SUSY algebra is required. A useful approach to understand the geometry of the TQCD is based on the Mathai–Quillen formalism [16].

In the present paper we investigate the renormalization properties of twisted $N = 2$ SUSY with two central extensions. In extending the analysis of Ref. [17] we study the ultraviolet behaviour of TQCD in the framework of algebraic renormalization [18, 19] by exploiting both the topological shift symmetry $Q$ and the vector supersymmetry $\bar{Q}_\mu$.

The renormalization properties of twisted $N = 2$ SUSY without central extensions have been widely investigated for particular gauges [6, 7, 17, 20, 21, 22]. In Ref. [21] it has been shown that the cohomological nature of Witten’s topological model is totally insensitive to quantum corrections and, in accord with a previous result [5], that gauge anomalies are absent to all orders of perturbation theory. This analysis has been extended in Ref. [17] by choosing a non–covariant Landau type gauge. This gauge has the merit that the associated vector supersymmetry $\bar{Q}_\mu$ is linearly realized and, therefore, can be employed as a stability constraint in order to improve the ultraviolet finiteness properties of that model.

Here, we study the renormalizability of a slightly more general extension of TQCD which is formed by twisting $N = 2$ SUSY with $Z = 2$ central charges. Let us recall, that for $N = 2$ the twisting procedure is unique. An undesired feature which is displayed by twisting $N = 2$ SYM coupled to a standard massive hypermultiplet [14] is the fact that it fails to be invariant under $R$–symmetry, the latter being broken into $Z_2$. In order to circumvent this problem we consider TQCD in the presence of two central charges $Z$ and $\bar{Z}$, being complex conjugated to each other. It can be shown that this topological model preserves $R$–symmetry and, therefore, the ghost number conservation if one formally ascribes to $Z$ and $\bar{Z}$ the $R$–weights $R(Z) = 2R(Q)$ and $R(\bar{Z}) = 2R(\bar{Q}_\mu)$, respectively.
Furthermore, by adopting the non-covariant Landau type gauge it will be proven that the whole set of stability constraints considered in Ref. [17] can be applied also in that general case – except for the so-called ghost for the ghost equation. Moreover, we shall be able to give an additional set of constraints which ensures the perturbative finiteness of the matter part of TQCD, i.e., the twisted hypermultiplet is not subjected to any renormalization. It is worthwhile to emphasize here that the algebraic proof of that finiteness property extends to all orders of perturbation theory and does not rely on the existence of any regularization scheme.

The paper is organized as follows. Sect. 2 reviews the main features of TQCD. Its purpose is, at first, to introduce our notations and to make this paper reasonably self-contained and, secondly, to bring separately into the play the various elements of $N = 2$ TQCD with two central charges thereby also motivating the choice of (non-covariant) Landau type gauge. In Subsect. 2.1 we introduce the so-called equivariant part of the action of TYM by fixing the topological shift symmetry in an covariant $\xi$-gauge. This procedure closely parallels the construction, in a Feynman type gauge ($\xi = 1$), of the topological action in Ref. [22] but, in contrast to it, we introduce a parameter $\xi$ into the twisted $N = 2$ SUSY algebra for being able also to select the Landau type gauge ($\xi = 0$).

In Subsect. 2.2 we extend the topological Yang-Mills theory by coupling it to a massive hypermultiplet (and its hermitian conjugate) thus obtaining the equivariant part of the action of TQCD. This is achieved by a non-trivial dimensional reduction of a $D = 6$ dimensional $N = 1$ SYM containing a gauge multiplet and a massless hypermultiplet in the adjoint and some (e.g., the fundamental) representation of the gauge algebra $\text{Lie}(G)$, respectively. The main body of that derivation is postponed to Appendix B. In Subsect. 2.3 we introduce the complete action of TQCD by fixing also the remaining gauge symmetry. It turns out that only in the Landau type gauge, $\xi = 0$, the complete action is invariant under both the topological shift symmetry $Q$ and the vector supersymmetry $\bar{Q}_\mu$. In Sect. 3 we construct the action of TQCD in a non-covariant Landau type gauge which allows to realize the vector supersymmetry linearly and, lateron, to employ it as an additional stability constraint. In Sect. 4 we enclose all the symmetry operators of the theory into a single nilpotent BRST operator $s_T$ by associating to each generator of the topological superalgebra a global ghost. We establish the corresponding classical Slavnov-Taylor identity and derive the gauge conditions, the antighost equations and some global constraints, being related to the non-covariant Landau type gauge. In Sect. 5 the problem of renormalizability is treated by standard cohomological methods. It is proven that the BRST cohomology is trivial and the most general local counterterms are determined. Appendix A contains the Euclidean spinor conventions being used throughout this paper. Appendix B gives a detailed derivation of the twisted $N = 2$ SYM coupled to a (massive) hypermultiplet with two central charges by using the method of dimensional reduction [23].

In this paper we use the convention $[T^i, T^j] = f^{ijk}T^k$ and $\text{tr}(T^iT^j) = \delta^{ij}$, the antihermitean generators $T^i$ being a basis of the Lie algebra $\text{Lie}(G)$ of the gauge group $G$, which we assume to be a simple compact Lie group, in some representation $\mathcal{R}$, e.g., a fundamental representation. We also adopt the matrix notation $\varphi = \varphi^i T^i$ and $\delta/\delta \varphi = T^i \delta / \delta \varphi^i$ for any field $\varphi^i$ transforming according to the adjoint representation of $G$. — Furthermore, we use the convention $\bar{\varphi}$ for the hermitian conjugate of some field $\varphi$. —
2. Topological QCD with $Z = 2$ central extensions

2.1. Topological Yang–Mills theory: Equivariant part

One of the possibilities to introduce TYM consists in twisting a set of conventional (spinorial) supercharges $Q_A^a$ and $\bar{Q}_\dot{A}a$ of $N = 2$ SYM in $D = 4$ dimensional Euclidean spacetime [3]. Thereby, the $N = 2$ gauge multiplet consists of a (antihermitian) gauge field $A_\mu$, the $Sp(2)$–doublets of chiral and of anti-chiral 2–spinors, $\lambda_A^a$ and $\bar{\lambda}_{\dot{A}a}$, respectively, and a complex scalar field $\phi$. In order to close the SUSY algebra it is necessary to introduce a (symmetric) auxiliary field $\chi_{ab} = \chi_{ba}$. All the fields of that off-shell gauge multiplet $V = \{A_\mu, \lambda_A^a, \bar{\lambda}_{\dot{A}a}, \phi, \bar{\phi}, \chi_{ab}\}$ are in the adjoint representation and take their values in the Lie algebra $Lie(G)$ of some compact gauge group $G$. The rotation group of Euclidean space–time, $SO(4)$, is locally isomorphic to $SU(2)_L \otimes SU(2)_R$ and the spinor indices in the fundamental representation of $SU(2)_L$ and $SU(2)_R$ will be denoted by $A = 1, 2$ and $\dot{A} = \dot{1}, \dot{2}$, respectively. The global internal symmetry group of $N = 2$ SUSY is $Sp(2) \otimes U(1)_R$ corresponding to symplectic rotations and chiral transformations ($R$–symmetry). The $R$–charges (chiral weights) of $Q_A^a$ and $\bar{Q}_{\dot{A}a}$ are 1 and $-1$, respectively. The internal $Sp(2)$ indices, labelling the different $N = 2$ charges, are denoted by $a = 1, 2$.

In the absence of a central extension, i.e., in a theory without massive fields which will be considered first, the $N = 2$ SUSY algebra in the Wess–Zumino gauge is characterized by the eight spinorial supercharges $Q_A^a$ and $\bar{Q}_{\dot{A}a}$ which, together with the generator $P_\mu \equiv i\partial_\mu$ of space–time translations, obey the relations

$$
\begin{align*}
\{Q_A^a, Q_B^b\} &= -4\epsilon^{ab}\epsilon_{AB}\delta_G(\phi), \\
\{Q_A^a, \bar{Q}_{\dot{B}b}\} &= -2\delta^a_b(\sigma^\mu)_{\dot{A}\dot{B}}(P_\mu + i\delta_G(A_\mu)), \\
\{\bar{Q}_{\dot{A}a}, \bar{Q}_{\dot{B}b}\} &= 4\epsilon_{ab}\epsilon_{AB}\delta_G(\bar{\phi}),
\end{align*}
$$

with $\bar{\phi}$ being the hermitean conjugate of $\phi$ (The conventions of Euclidean spinor algebra are collected in Appendix A; the on–shell version of the algebra [3] is derived in Appendix B). Since in Wess–Zumino gauge the supersymmetry is realized nonlinearly the algebra (3) closes only modulo the field dependent gauge transformations $\delta_G(\omega)$, $\omega = \{A_\mu, \phi, \bar{\phi}\}$, respectively (cf., Eqs. (B.2)).

As explained in Ref. [3], TYM is obtained from $N = 2$ SYM by replacing the group $SU(2)_L \otimes Sp(2)$ through its diagonal subgroup or, in other words, by identifying the internal index $a$ with the spinor index $A$. According to this twisting procedure one constructs from the generators $Q_A^a$ and $\bar{Q}_{\dot{A}a}$ the twisted ones,

$$
\begin{align*}
Q &= \frac{1}{2}\epsilon^{AB}Q_{AB}, & \bar{Q}_\mu &= \frac{1}{2}i(\sigma_\mu)^{AB}\bar{Q}_{AB}, & Q_{\mu\nu} &= \frac{1}{2}(\sigma_{\mu\nu})^{AB}Q_{AB},
\end{align*}
$$

being a scalar $Q$, a vector $\bar{Q}_\mu$ and a self–dual tensor $Q_{\mu\nu}$, respectively. Since it turns out that TYM is already completely specified by the topological shift symmetry $Q$ and the vector supersymmetry $\bar{Q}_\mu$, we actually do not take into account the self–dual tensor supersymmetry $Q_{\mu\nu}$ in the following. In terms of $Q$ and $\bar{Q}_\mu$, from (3) for the twisted (or topological) superalgebra we get [3]

$$
\begin{align*}
\{Q, Q\} &= -2\delta_G(\phi), & \{Q, \bar{Q}_\mu\} &= -iP_\mu + \delta_G(A_\mu), & \{\bar{Q}_\mu, \bar{Q}_\nu\} &= -2\delta_{\mu\nu}\delta_G(\bar{\phi}).
\end{align*}
$$
From the first of these relations it follows that scalar supercharge $Q$ is equivariantly nilpotent, i.e., it squares to gauge transformations $\delta_G(\phi)$ generated by $\phi$, and, applying the Jacobi identity on $\phi$, it follows that the field $\phi$, from which the Donaldson invariant is constructed, must be $Q$–invariant, $Q\phi = 0$.

The second relation, being typical for a topological theory, states that, due to the existence of the vector supercharge $Q_\mu$, the space–time translations $P_\mu$ can be represented as a $Q$–anticommutator modulo the gauge transformation $\delta_G(A_\mu)$. This allows, starting from the Donaldson invariant, $\mathcal{O}$, to construct all the (global) observables of TYM [2] by applying successively the vector supercharge $Q_\mu$ [22] (see also [1(d)]),

$$\mathcal{O} \equiv \int d^4x \, \text{tr} \, \phi^2, \quad \bar{Q}_\mu \mathcal{O}, \quad \bar{Q}_\mu \bar{Q}_\nu \mathcal{O}, \quad \bar{Q}_\mu \bar{Q}_\nu \bar{Q}_\rho \bar{Q}_\sigma \mathcal{O}. \quad (4)$$

(It is worthwhile to note that the same observables can also be recovered through the so–called equivariant cohomology, compare, e.g., Ref. [24]). In this manner one obtains that part of the action of TYM which results by fixing, in a Feynman type gauge, the topological shift symmetry [22]:

$$W_T^{(\xi=1)} = \frac{1}{4} \int d^4x \, \text{tr} \, F_{\mu\nu} \tilde{F}^{\mu\nu} - \frac{1}{24} \epsilon^{\mu\nu\rho\sigma} \bar{Q}_\mu \bar{Q}_\nu \bar{Q}_\rho \bar{Q}_\sigma \int d^4x \, \text{tr} \, \phi^2, \quad (5)$$

where

$$\tilde{F}_{\mu\nu} = \frac{1}{2} \epsilon_{\mu\nu\rho\sigma} F^{\rho\sigma}, \quad F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu + [A_\mu, A_\nu], \quad D_\mu = \partial_\mu + [A_\mu, \cdot];$$

here, $\tilde{F}_{\mu\nu}$ is the dual of the YM field strenght $F_{\mu\nu}$, and $D_\mu$ is the covariant derivative (in adjoint representation). The first term in (5) is the classical action (Pontryagin term) and the second one is the gauge–fixing term which removes the degeneracy of the classical action with respect to the topological shift symmetry $Q$ still leaving out of account the original gauge symmetry. Usually, $W_T^{(\xi=1)}$ is called equivariant part of the topological action being defined by the equivariant cohomology of TYM (prior introduction of the gauge ghost field). Obviously, the Wess-Zumino gauge of supersymmetry immediately leads to the Feynman type gauge of TYM which is mediated by the vector charge $\bar{Q}_\mu$.

Of course, other gauges may be used as well, e.g., those introduced by Eqs. (6) below. For notational simplicity we have set the gauge coupling equal to one, $g = 1$. Let us also remark that, in principle, the Pontryagin term could be multiplied by $\theta$ with some arbitrary parameter $\theta$ which, here, is set equal to one, $\theta = 1$.

From the third relation in (4) it follows that also the operator $Q_\mu$ is equivariantly nilpotent, but now modulo the gauge transformation $\delta_G(\bar{\phi})$, and, for the same reason as before, that the field $\bar{\phi}$ must be $Q_\mu$–invariant, $\bar{Q}_\mu \bar{\phi} = 0$. Let us stress that the second term in (4) is different from zero if and only if $Q_\mu$ does not anticommute with itself, showing the importance of the presence of the gauge transformation on the right–hand side of $\{Q_\mu, Q_\nu\} = -2\delta_{\mu\nu} \delta_G(\bar{\phi})$. On the other hand, that gauge–fixing in (5) is incomplete. In order to obtain the complete action of TYM one still has to add a further gauge–fixing term removing the degeneracy of the classical action with respect to the remaining gauge symmetry (which will be done in Subsect. 2.3). However, that additional term turns out to be invariant under the vector supersymmetry $\bar{Q}_\mu$ if and only if $\{\bar{Q}_\mu, Q_\nu\} = 0$, i.e., if $\bar{Q}_\mu$ is strictly nilpotent. Hence, by choosing a Feynman type gauge the vector charge $\bar{Q}_\mu$ can not be really a symmetry operator of the complete gauge–fixed action of TYM. However,
such a situation can be circumvented by choosing a Landau type gauge, i.e., by modifying the second term of the action (3) appropriately.

In order to prepare the frame for a more general covariant gauge let us deform the topological superalgebra (3) without changing its topological character:

\[ \{Q, Q\} = -2\delta_G(\phi), \quad \{Q, Q_\mu^{(\xi)}\} = -iP_\mu + \delta_G(A_\mu), \quad \{Q_\mu^{(\xi)}, Q_\nu^{(\xi)}\} = -2\xi\delta_{\mu\nu}\delta_G(\bar{\phi}), \quad (6) \]

thus making \(Q_\mu^{(\xi)}\) also \(\xi\)-dependent. Here, \(\xi\) is the gauge parameter interpolating between Feynman \((\xi = 1)\) and Landau \((\xi = 0)\) type gauge. In turn, the topological action (5) changes into

\[ W_T^{(\xi)} = \frac{1}{4\xi} \int d^4x \text{tr} \, F_{\mu\nu} \tilde{F}^{\mu\nu} - \frac{1}{24\xi \epsilon^{\mu\nu\rho\sigma}} \tilde{Q}_\mu^{(\xi)} \tilde{Q}_\nu^{(\xi)} \tilde{Q}_\rho^{(\xi)} \tilde{Q}_\sigma^{(\xi)} \int d^4x \text{tr} \, \phi^2, \quad (7) \]

where both terms are rescaled by \(1/\xi\) in order to ensure that the action is well–defined also for \(\xi = 0\) (see Eq. (10) below). From (7) it follows that, by construction, \(W_T^{(\xi)}\) will be left invariant by the twisted operators \(Q\) and \(\bar{Q}_\mu^{(\xi)}\):

\[ QW_T^{(\xi)} = 0, \quad \bar{Q}_\mu^{(\xi)}W_T^{(\xi)} = 0. \]

The symmetry transformations corresponding to \(Q\) and \(\bar{Q}_\mu^{(\xi)}\) can be obtained by applying the previous twisting procedure to the transformation laws generated by \(Q_A^a\) and \(\bar{Q}_{\dot{A}a}\) (see Eqs. (B.6) for the on–shell transformations in the Feynman type gauge). Before showing their explicit form let us briefly introduce the twisted gauge multiplet \(V_T\) of TYM by its relation to the gauge multiplet \(V = \{A_\mu, \lambda^a_A, \bar{\lambda}_{\dot{A}a}, \phi, \bar{\phi}, \chi_{ab}\}\) of \(N = 2\) SYM.

The anti–chiral spinor \(\bar{\lambda}_{\dot{A}a}\), being the hermitean conjugate of \(\lambda^a_A\), is related to a Grassmann–odd vector field \(\psi_\mu\) through

\[ \psi_\mu = -\frac{1}{2}(\sigma_\mu)^{AB}\bar{\lambda}_{\dot{A}B}, \]

which is the topological ghost; \(\phi\) is the ghost of the topological ghost and \(\bar{\phi}\) is the corresponding antighost. The chiral spinor \(\lambda^a_A\) is associated with both a Grassmann–odd antisymmetric self–dual tensor field \(\chi_{\mu\nu} \equiv \chi_{\mu\nu}^+\) and a Grassmann–odd scalar field \(\eta\) according to

\[ \chi_{\mu\nu} = -\frac{1}{2}i(\sigma_{\mu\nu})^{AB}\lambda_{AB}, \quad \eta = -\frac{1}{2}i\epsilon^{AB}\lambda_{AB}, \]

where \(\eta\) plays the role of an auxiliary field. Finally, the symmetric auxiliary field \(\chi_{ab}\) is related to a Grassmann–even antisymmetric self–dual tensor field \(\lambda_{\mu\nu} \equiv \lambda_{\mu\nu}^+\) through

\[ \lambda_{\mu\nu} = \frac{1}{2}(\sigma_{\mu\nu})^{AB}\chi_{AB}, \]

which, again, ensures the closure of the twisted superalgebra (3). Hence, the field content of TYM is given by the following twisted gauge multiplet \(V_T = \{A_\mu, \psi_\mu, \chi_{\mu\nu}, \eta, \phi, \bar{\phi}, \lambda_{\mu\nu}\}\) whose properties, together with those of the topological charges, are displayed in the following Table 1.
Let us now give the transformation law for the twisted gauge multiplet:

(i) The topological shift symmetry $Q$ takes the form

$$QA_\mu = \psi_\mu, \quad Q\psi_\mu = D_\mu \phi, \quad Q\phi = 0,$$

$$Q\bar{\phi} = \eta, \quad Q\eta = [\bar{\phi}, \phi], \quad Q\lambda_{\mu\nu} = \lambda_{\mu\nu}, \quad Q\lambda_{\mu
u} = [\lambda_{\mu\nu}, \phi];$$

(it is independent of the choice of $\xi$. Indeed, after rescaling by $\xi$ the fields $\bar{\phi}$, $\eta$ and $\lambda_{\mu\nu}$,

$\lambda_{\mu\nu}$ — which belong to the non–minimal part (in the sense of BRST symmetry) of the

multiplet $V_T$ — these transformation rules obviously remain unchanged.

(ii) The transformations rules for the vector supersymmetry $\bar{Q}_\mu^{(\xi)}$, are given by

$$\bar{Q}_\mu^{(\xi)} A_\nu = \xi (\delta_{\mu\nu} \eta + \chi_{\mu\nu}), \quad \bar{Q}_\mu^{(\xi)} \psi_\nu = F_{\mu\nu} - \xi (\delta_{\mu\nu} [\bar{\phi}, \phi] + \lambda_{\mu\nu}), \quad \bar{Q}_\mu^{(\xi)} \phi = \psi_\mu,$$

$$\bar{Q}_\mu^{(\xi)} \phi = 0, \quad \bar{Q}_\mu^{(\xi)} \eta = D_\mu \bar{\phi}, \quad \bar{Q}_\mu^{(\xi)} \lambda_{\rho\sigma} = \delta_{\mu\rho} D_\sigma \bar{\phi} - \delta_{\mu\sigma} D_\rho \bar{\phi} + \epsilon_{\mu\rho\sigma\nu} D^\nu \bar{\phi},$$

$$\bar{Q}_\mu^{(\xi)} \lambda_{\rho\sigma} = D_\mu \lambda_{\rho\sigma} + \delta_{\mu\rho} (\lambda_{\sigma\nu} [\bar{\phi}, \psi_\nu] - D_\sigma \eta) - \delta_{\mu\sigma} (\lambda_{\rho\nu} [\bar{\phi}, \psi_\nu] - D_\rho \eta) + \epsilon_{\mu\rho\sigma\nu} ([\bar{\phi}, \psi_\nu] - D^\nu \eta).$$

These transformations sensitively depend on the choice of the gauge parameter $\xi$. Notice,

that for $\xi = 0$ the operator $\bar{Q}_\mu^{(\xi)}$ leaves the gauge field $A_\mu$ inert and, therefore, does not change the classical action. Thus, the vector supersymmetry represents a non–trivial symmetry only with respect the gauge–fixing terms of TYM.

Now, we are in a position to determine the second term in the topological action \( \xi \) explicitly. As a result one gets (remind that $\lambda^{\mu\nu}$ and $\chi^{\mu\nu}$ are self-dual)

$$W_T^{(\xi)} = \int d^4x \text{tr} \left\{ \lambda^{\mu\nu} F_{\mu\nu} - 2\lambda^{\mu\nu} D_\mu \psi_\nu + 2\eta D^\mu \psi_\mu + 2\bar{\phi} \{ \psi_\mu, \psi_\mu \} + 2\bar{\phi} D^2 \phi \right\}$$

$$- \xi \left( \frac{1}{2} \lambda^{\mu\nu} \lambda_{\mu\nu} - \frac{1}{4} \lambda^{\mu\nu} [\chi_{\mu\nu}, \phi] + 2[\bar{\phi}, \phi] \bar{\phi} \phi - 2\eta [\bar{\phi}, \phi] \right)^2 \right\}.$$
2.2. Massive topological QCD: Equivariant part

So far we have implicitly assumed that the complex field $\phi$ does not induce a central extension of the SUSY algebra (1), or equivalently, that its vacuum expectation value is zero [25]. Now we remove that restriction and consider $N = 2$ SYM coupled to a massive hypermultiplet which, after twisting, leads to topological QCD.

In this more general case we are faced with a $N = 2$ SUSY algebra with two central charges $Z$ and $\bar{Z}$,

$$\{Q_A^a, Q_B^b\} = -4\epsilon^{ab}\epsilon_{AB}(Z + \delta_G(\phi)),
\{Q_A^a, \bar{Q}_{Bb}\} = -2\delta^a_6(\sigma^\mu)_{AB}(P_\mu + i\delta_G(A_\mu)),
\{Q_{A\dot{a}}, \bar{Q}_{B\dot{b}}\} = 4\epsilon_{ab}\epsilon_{\dot{A}\dot{B}}(\bar{Z} + \delta_G(\bar{\phi})), \quad (13)$$

$\bar{Z}$ being the complex conjugate of $Z$. Thereby, $Z$ and $\bar{Z}$, together with $P_\mu$, satisfy the condition $4Z\bar{Z} = P^\mu P_\mu$. The eigenvalues of $Z$ and $\bar{Z}$ are $\pm m$ and $\pm \bar{m}$, respectively, where the positive (negative) sign relates to the hypermultiplet $Y$ ($\bar{Y}$). Here, we have introduced two central charges in order to ensure that, lateron, the $R$–symmetry remains unbroken if we formally assign to $Z$ and $\bar{Z}$ the $R$–charges 2 and $-2$, respectively. (By coupling the gauge multiplet to the standard massive hypermultiplet [26], i.e., for only one (real) central charge $Z = \bar{Z}$, the $R$–symmetry is broken into $Z_2$; cf., Ref. [3].)

In order to construct the topological action with central charges $Z$ and $\bar{Z}$ we may consider in a $D = 6$ dimensional space–time [27] a $N = 1$ SYM whose gauge multiplet $\{A_M, \lambda_3\}$ is coupled to a (massless) hypermultiplet $\{\psi, \zeta\}$ in some representation $\mathcal{R}$ of the gauge group $G$ (for a detailed presentation, see, Appendix B). Then, one carries out a non–trivial dimensional reduction onto a torus with respect to the extra spatial dimensions $x^{5}$ and $x^{6}$ [27], thereby, introducing two (real) masses, $m^{5} = m + \bar{m}$ and $m^{6} = i(\bar{m} - m)$, which are the inverse periods of the fields of the hypermultiplet with respect to $x^{5}$ and $x^{6}$. The extra components $P_5$ and $P_6$ of the generator of space–time translation are related to the central charges according to $P_5 = Z + \bar{Z}$ and $P_6 = i(Z - \bar{Z})$, respectively. In the same way, the extra components $A_5$ and $A_6$ of the gauge field, which are assumed to be independent on $x^{5}$ and $x^{6}$, are identified with the complex scalar field according to $A_5 = -i(\bar{\phi} + \phi)$ and $A_6 = \bar{\phi} - \phi$.

After compactification, assuming the only the first non–zero modes are excited, the hypermultiplet $Y$ (and its hermitean conjugate $\bar{Y}$) becomes massive. The resulting hypermultiplet $\tilde{Y}$ consists of two Weyl spinors being the components of the corresponding (Dirac) spinor, $q = (\alpha^A, \overline{\beta}_A)^T$, a $Sp(2)$–doublet of complex scalar fields $\zeta^a$ and, in order to close the superalgebra (13) also off–shell, a $Sp(2)$–doublet of complex auxiliary fields $\chi^a$. After the twisting procedure the scalar fields $\zeta^a$ become the components of a bispinor field $\zeta^A$. The appearance of bispinors after twisting is a new feature of TQCD. Similarly, the auxiliary field $\chi^a$ should be replaced by a bispinor field $\chi^A$. But, a further and somehow surprising feature of the twisting procedure is that, in the presence of central charges, twisting of the superalgebra (13) does not lead to a topological superalgebra which closes off–shell [28] – despite having started with a closed algebra!

However, one can proceed like in the case of twisting $N = 2$ conformal supergravity [29]. Namely, replacing $\chi^a$ instead by $\chi^A$ through another bispinor $\bar{\chi}_A$ the resulting topological superalgebra closes off–shell [16]. Thus, the multiplets from which TQCD is constructed
consist of the (massless) twisted gauge multiplet \( V_T = \{ A_\mu, \psi_\mu, \chi_{\mu\nu}, \eta, \phi, \bar{\phi}, \lambda_{\mu\nu} \} \) in the adjoint representation of \( G \) and two twisted massive hypermultiplets \( Y_T = \{ \alpha^A, \bar{\beta}_\dot{A}, \zeta^A, \bar{\chi}_{\dot{A}} \} \) and \( \bar{Y}_T = \{ \beta_A, \bar{\alpha}^\dot{A}, \bar{\zeta}_A, \chi^\dot{A} \} \) in the representation \( \mathcal{R} \). Their properties are displayed in the following Table 2.

| Field | \( \alpha^A \) | \( \beta_A \) | \( \bar{\alpha}^\dot{A} \) | \( \bar{\beta}_\dot{A} \) | \( \zeta^A \) | \( \bar{\zeta}_A \) | \( \chi^A \) | \( \bar{\chi}_{\dot{A}} \) |
|-------|--------------|------------|----------------|-------------|--------|--------|--------|-------------|
| ghost number | 1 | 1 | -1 | -1 | 0 | 0 | 0 | 0 |
| mass dimension | 3/2 | 3/2 | 3/2 | 3/2 | 1 | 1 | 2 | 2 |
| scale dimension | 1 | 1 | 2 | 2 | 1 | 1 | 2 | 2 |
| \( Z^- \), \( \bar{Z}^- \)-charge | 1 | -1 | -1 | 1 | 1 | -1 | 1 | -1 |
| Grassmann parity | even | even | even | even | odd | odd | odd | odd |

The twisted action for the matter fields, including the terms for the new auxiliary fields \( \chi^A \) and \( \bar{\chi}_{\dot{A}} \), is

\[
W_M^{(\xi)} = \int d^4x \left\{ i\bar{\alpha}^A(\sigma^{\mu})_{AB} \bar{D}_\mu \alpha^B + i\beta_A \bar{D}_\mu (\sigma^{\mu})^{AB} \bar{\beta}_B - 2\bar{m}_A \alpha^A \\
+ i\bar{\alpha}^A(\sigma^{\mu})_{AB} \psi_\mu \zeta^B - i\bar{\zeta}_A(\sigma^{\mu})^{AB} \psi_\mu \bar{\beta}_B - 2\bar{\alpha}^A(\phi + m) \bar{\beta}_\dot{A} \\
- i\bar{\chi}^A(\sigma^{\mu})_{AB} \bar{D}_\mu \zeta^B + i\bar{\zeta}_A(\sigma^{\mu})^{AB} \bar{\chi}_B - 2\bar{\chi}_\dot{A} + 2\bar{m}_A(\phi + m) \zeta^A \\
- \frac{\xi}{4} \alpha_A(\sigma^{\mu\nu})^{AB} \chi_{\mu\nu} \zeta_B + \frac{1}{4} \bar{\zeta}_A(\sigma^{\mu\nu})^{AB} \chi_{\mu\nu} \zeta_B - \frac{1}{4} \bar{\zeta}_A(\sigma^{\mu\nu})^{AB} \chi_{\mu\nu} \alpha_B \\
+ \bar{\zeta}_A \eta \alpha^A - \bar{\zeta}_A \bar{\phi}(\phi + m) \zeta^A + 2\bar{\beta}_\dot{A} \bar{\phi} \alpha^A + \beta_A \eta \zeta^A - \bar{\zeta}_A(\phi + m) \bar{\phi} \zeta^A \right\},
\]

where the covariant derivatives in the \( \mathcal{R} \)-representation are given by

\[
\bar{D}_\mu = \bar{\partial}_\mu + A_\mu, \quad \bar{D}_\mu = \bar{\partial}_\mu - A_\mu.
\]

In the Feynman type gauge (\( \xi = 1 \)) this part of the action is symmetric in \( m \) and \( \bar{m} \). Furthermore, let us recall that in the twisted theory the \( R \)-symmetry is identified with the ghost number conservation. Hence, by inspection of (14), we observe that the mass terms preserve the ghost number if we formally assign to \( m \) and \( \bar{m} \) the ghost number 2 and \(-2\), respectively.

Let us now consider the transformation law of the twisted hypermultiplets.

(i) The topological shift symmetry \( Q \) takes the form

\[
Q \zeta^A = \alpha^A, \quad Q \alpha^A = -(\phi + m) \zeta^A, \quad Q \bar{\beta}_\dot{A} = \bar{\chi}_{\dot{A}}, \quad Q \bar{\chi}_{\dot{A}} = -(\phi + m) \bar{\beta}_\dot{A},
\]

\[
Q \bar{\zeta}_A = \beta_A, \quad Q \beta_A = \bar{\zeta}_A(\phi + m), \quad Q \bar{\alpha}^\dot{A} = \chi^\dot{A}, \quad Q \chi^\dot{A} = \bar{\alpha}^\dot{A}(\phi + m); \quad (15)
\]

again, it is independent of the choice of the gauge parameter \( \xi \). But now, the anticommutator of the operator \( Q \) includes, besides the gauge transformation \( -2\delta_L(\phi) \), for \( m \neq 0 \) also the central charge transformation \( Z \). Together with those of \( \bar{Z} \) they are given by

\[
Z V_T = 0, \quad Z Y_T = m Y_T, \quad Z \bar{Y}_T = -m \bar{Y}_T,
\]

\[
\bar{Z} V_T = 0, \quad \bar{Z} Y_T = \bar{m} Y_T, \quad \bar{Z} \bar{Y}_T = -\bar{m} \bar{Y}_T. \quad (16)
\]
(ii) The transformation rules for the vector supersymmetry $Q^{(\xi)}$ are given by

$$
\begin{align*}
Q^{(\xi)}_\mu \zeta^A &= i(\sigma_\mu)^{AB} \beta^B, \\
Q^{(\xi)}_\mu \beta^A &= i(\sigma_\mu)_{AB}(\xi \phi + \bar{m}) \zeta^B, \\
Q^{(\xi)}_\mu \alpha^A &= -i(\sigma_\mu)^{AB} \bar{\chi}_B + \bar{D}_\mu \zeta^A, \\
Q^{(\xi)}_\mu \bar{\chi}_A &= -i(\sigma_\mu)_{AB}(\xi \phi + \bar{m}) \alpha^B - \xi i(\sigma_\mu)_{AB} \eta \zeta^B + \bar{D}_\mu \beta^A, \\
Q^{(\xi)}_\mu \bar{\alpha}^A &= i(\sigma_\mu)^{AB} \bar{\zeta}_B(\xi \phi + \bar{m}), \\
Q^{(\xi)}_\mu \beta_A &= i(\sigma_\mu)_{AB} \bar{\zeta}_B + \bar{\zeta}_A D_\mu, \\
Q^{(\xi)}_\mu \bar{\zeta}_A &= -i(\sigma_\mu)_{AB}(\xi \phi + \bar{m}) - \xi i(\sigma_\mu)^{AB} \bar{\zeta}_B \eta + \bar{\alpha} \bar{D}_\mu.
\end{align*}
$$

As before, the anticommutator $\{Q^{(\xi)}_\mu, Q^{(\xi)}_\nu\}$ does no longer coincide with the $\xi$–dependent gauge transformation $-2\xi \delta_{\mu\nu} \delta_\phi(\bar{\phi})$ alone but in addition it includes the (complex conjugate) central charge transformation $Z$.

By a rather lengthy calculation it can be proven that the transformations (15) – (17) together with (8) and (13) satisfy the following topological superalgebra:

$$
\begin{align*}
\{Q, Q\} &= -2(Z + \delta_\phi(\phi)), \\
\{Q, Q^{(\xi)}_\mu\} &= -iP_\mu + \delta_\phi(A_\mu), \\
\{Q^{(\xi)}_\mu, Q^{(\xi)}_\nu\} &= -2\delta_{\mu\nu}(Z + \xi \delta_\phi(\bar{\phi})).
\end{align*}
$$

By making use of the $Q$–transformations (15) it is easy to show that the twisted action $W^{(\xi)}_M$ can be written as an exact $Q$–cocycle,

$$
W^{(\xi)}_M = Q\Psi^{(\xi)}_M \quad \text{with} \quad Q^{(\xi)}_\mu \Psi^{(\xi)}_M = 0, \quad Z\Psi^{(\xi)}_M = 0, \quad \bar{Z}\Psi^{(\xi)}_M = 0, \quad \text{(19)}
$$

where the $\bar{m}$–dependent gauge fermion $\Psi^{(\xi)}_M$ is given by

$$
\Psi^{(\xi)}_M = -\int d^4x \left\{ i\bar{\alpha}^A(\sigma_\mu)_{AB} \bar{D}_\mu \zeta^B - i\bar{\zeta}_A \bar{D}_\mu (\sigma_\mu)^{AB} \beta^B + \bar{m}(\bar{\zeta}_A \alpha^A - \beta_A \zeta^A) + \bar{\alpha} \bar{A}_A + \chi^A \bar{\beta}^A \right\}
+ \xi \left( \bar{\zeta}_A \phi \alpha^A - \beta_A \bar{\phi} \zeta^A + \frac{1}{4} \bar{\zeta}_A (\sigma^{\mu\nu})^{AB} \chi_{\mu\nu} \zeta^B \right). \quad \text{(20)}
$$

Let us stress that the gauge fermion $\Psi^{(\xi)}_M$ is completely specified by the topological shift symmetry $Q$ and the vector supersymmetry $Q^{(\xi)}_\mu$. Furthermore, identifying $Z = \bar{Z}$ and, therefore, $m = \bar{m}$, we obtain for the matter action, Eq. (14), the result of (15).

Hence, the twisted $N = 2$ SYM coupled to the massive hypermultiplets $Y_T$ and $\bar{Y}_T$ is described by the action

$$
W^{(\xi)} = W^{(\xi)}_T + W^{(\xi)}_M = Q(\Psi^{(\xi)}_T + \Psi^{(\xi)}_M). \quad \text{(21)}
$$

This defines completely the equivariant part of TQCD. It is invariant under the topological superalgebra (18) and possesses the crucial property of being an exact $Q$–cocycle.
2.3. Massive topological QCD: Complete action in Landau gauge

Next, we are faced with the problem to construct the complete action of TQCD. This is achieved by adding to the equivariant part a supplementary gauge-fixing term which removes the residual gauge degeneracy of the classical action thereby preserving the invariance under the topological superalgebra \([18]\). The last requirement turns out to be very restrictive and can be fulfilled only when in \([21]\) the Landau type gauge \(\xi = 0\) has been chosen.

To begin with, let us introduce the following prolonged BRST operator

\[ s_Q = s + Q, \]  

which, according to the first of Eqs. \([18]\), is required to be nilpotent modulo the central charge \(Z\), i.e., \(\{s_Q, s_Q\} = -2Z\). In order to identify \(s\) with the (usual) nilpotent BRST operator, let us introduce the gauge ghost \(C\) by assuming, as usual, that the topological ghost \(\phi\) is the exact \(Q\)-cocycle, \(\phi = QC\). Then, applying \(\{Q, Q\} = -2(Z + \delta_G(\phi))\) on \(C\) it follows that \(s\) is defined by \(s = \delta_G(C)\) when acting on the gauge multiplet \(V\) and \(sC = C^2\) when acting on the gauge ghost. The transformation law for the antighost \(\bar{C}\) and the auxiliary field \(B\) will be defined by \(s\bar{C} = B + \{C, \bar{C}\}\) and \(sB = [\bar{C}, \phi] + [C, B]\).

Notice, that \(C, \bar{C}\) and \(B\) take their values in \(\text{Lie}(G)\) and have vanishing central charges; their properties are summarized in the following Table 3.

|          | \(C\) | \(\bar{C}\) | \(B\) |
|----------|------|----------|------|
| ghost number | 1    | -1       | 0    |
| mass dimension | 1/2  | 3/2      | 2    |
| scale dimension | 0    | 2        | 2    |
| \(Z\)-, \(\bar{Z}\)-charge | 0    | 0        | 0    |
| Grassmann parity | odd  | odd      | even |

Together with the action of the topological shift symmetry \(Q\), Eqs. \([8]\), we get the BRST transformation:

\[
\begin{align*}
    s_Q A_\mu &= \psi_\mu - D_\mu C, \\
    s_Q \psi_\mu &= D_\mu \phi + \{C, \psi_\mu\}, \\
    s_Q C &= \phi + C^2, \\
    s_Q \phi &= [C, \phi], \\
    s_Q \bar{\phi} &= \eta + [C, \bar{\phi}], \\
    s_Q \eta &= [\bar{\phi}, \phi] + \{C, \eta\}, \\
    s_Q \chi_{\mu\nu} &= \lambda_{\mu\nu} + \{C, \chi_{\mu\nu}\}, \\
    s_Q \lambda_{\mu\nu} &= [\chi_{\mu\nu}, \phi] + [C, \lambda_{\mu\nu}], \\
    s_Q \bar{C} &= B + \{C, \bar{C}\}, \\
    s_Q B &= [C, \phi] + [C, B].
\end{align*}
\]  

(23)

Now, we are in a position to define the complete action of TYM by adding to the topological action \([11]\) and \([12]\) the remaining gauge fixing part \(W_{YM}\):

\[
W^{(\xi)}_{TYM} = W^{(\xi)}_T + W_{YM} \quad \text{with} \quad W^{(\xi)}_T = s_Q \Psi^{(\xi)}_T, \quad W_{YM} = s_Q \Psi_{YM},
\]  

(24)

11
where $\Psi_{YM}$ is the YM gauge fermion in the Landau gauge,

$$\Psi_{YM} = 2 \int d^4x \text{tr}\{\bar{C} \partial^\mu A_\mu\}. \quad (25)$$

(In principle, also here we could have chosen a gauge fermion $\Psi^{(\xi)}_{YM}$ in an intermediate gauge with another parameter $\xi'$ which, for the Landau gauge, is set equal to zero.) Notice, that $W_{\xi}^{(\xi)}$ can be rewritten in the form $W_{\xi}^{(\xi)} = s_\xi \Psi_{\xi}^{(\xi)}$, since $\Psi_{\xi}^{(\xi)}$ is gauge invariant, i.e., $s_\xi \Psi_{\xi}^{(\xi)} = 0$. Hence, by construction, the action (24) is invariant under the modified BRST transformations, Eqs. (23). For the irreducible part of TYM we obtain

$$W_{YM} = s_\xi \Psi_{YM} = 2 \int d^4x \text{tr}\{B \partial^\mu A_\mu - C \partial^\mu D_\mu \bar{C} - \bar{C} \partial^\mu \psi_\mu\}. \quad (26)$$

Let us now study the invariance properties of $W_{YM}$ under the $\bar{Q}_\mu^{(\xi)}$-transformations. By applying $\{Q, \bar{Q}_\mu^{(\xi)}\} = -i P_\mu + \delta C(A_\mu)$ on $C$ and using $QC = \phi$ we get $\bar{Q}_\mu^{(\xi)}C = -A_\mu$. This allows to cast the previous relation into the form $\{s_\xi, \bar{Q}_\mu^{(\xi)}\} = -i P_\mu$. Then, by making use of this new relation, from the requirements $s_\xi C = B + \{C, C\}$ and $s_\xi B = [\bar{C}, \phi] + [C, B]$ we get $\bar{Q}_\mu^{(\xi)}C = 0$ and $\bar{Q}_\mu^{(\xi)}B = D_\mu \bar{C}$. Now, recalling that $\bar{Q}_\mu^{(\xi)}$ leaves $A_\mu$ inert iff $\xi = 0$, see Eqs. (3), by applying $\{\bar{Q}_\mu^{(\xi)}, \bar{Q}_\mu^{(\xi)}\} = -2 \delta_{\mu\nu}(\bar{Z} + \xi \delta C(\phi))$ on $C$ and using $\bar{Q}_\mu^{(\xi)}C = -A_\mu$ we conclude that the irreducible part $W_{YM}$ can not be invariant under the vector supersymmetry $\bar{Q}_\mu^{(\xi)}$ as long as $\xi \neq 0$.

For that reason, from now on we shall choose the Landau type gauge, $\xi = 0$, for the topological action. In that gauge the transformations of the complete gauge multiplet under the vector supersymmetry $\bar{Q}_\mu^{(0)}$ read (cf., Eqs. (9)):

$$\bar{Q}_\mu^{(0)}A_\nu = 0, \quad \bar{Q}_\mu^{(0)}\psi_\nu = F_{\mu\nu},$$

$$\bar{Q}_\mu^{(0)}C = -A_\mu, \quad \bar{Q}_\mu^{(0)}\phi = \psi_\mu, \quad \bar{Q}_\mu^{(0)}B = D_\mu \bar{C}, \quad \bar{Q}_\mu^{(0)}\bar{C} = 0,$$

and for the massive hypermultiplets we obtain (cf., Eqs. (17))

$$\bar{Q}_\mu^{(0)}\zeta^A = i(\sigma_\mu)^{AB} \bar{\beta}_B^A, \quad \bar{Q}_\mu^{(0)}\bar{\beta}_A^B = i m(\sigma_\mu)^{AB} \zeta^B,$$

$$\bar{Q}_\mu^{(0)}\alpha^A = -i(\sigma_\mu)^{AB} \bar{\chi}_B^A + \bar{D}_\mu \zeta^A, \quad \bar{Q}_\mu^{(0)}\bar{\chi}_A^\dagger = -i m(\sigma_\mu)^{AB} \alpha_B + \bar{D}_\mu \bar{\beta}_A^\dagger,$$

The corresponding BRST transformations of the hypermultiplet read (cf., Eqs. (13))

$$s_\xi \zeta^A = \alpha^A + C\zeta^A, \quad s_\xi \alpha^A = -(\phi + m)\zeta^A + C\alpha^A,$$

$$s_\xi \bar{\beta}_A^B = \bar{\chi}_A^B + C\bar{\beta}_A^B, \quad s_\xi \bar{\chi}_A^B = -(\phi + m)\bar{\beta}_A^B + C\bar{\chi}_A^B,$$

$$s_\xi \beta_A = \bar{\zeta}_A - \bar{\zeta}_A C, \quad s_\xi \bar{\zeta}_A = \bar{\zeta}_A (\phi + m) + \beta_A C,$$

$$s_\xi \alpha_A^\dagger = \chi_A^\dagger + \alpha_A^\dagger C, \quad s_\xi \chi_A^\dagger = \bar{\alpha}_A (\phi + m) - \chi_A^\dagger C. \quad (29)$$
By a tedious but straightforward calculation it can be proven that the transformations (13), (23) and (27) together with (28) and (29) satisfy the following superalgebra:

$$\{s_Q, s_Q\} = -2Z, \quad \{s_Q, Q^{(0)}_\mu\} = -iP_\mu, \quad \{Q^{(0)}_\mu, Q^{(0)}_\nu\} = -2\delta_{\mu\nu}Z,$$

which is obtained from (18) after substituting $Q$ by $s_Q$ and putting $\xi$ equal to zero.

From (10) for the topological action in the Landau type gauge we obtain

$$W^{(0)}_T = \int d^4x \text{tr} \left\{ \chi^{\mu\nu} F_{\mu\nu} - 2\chi^{\mu\nu} D_\mu \psi_\nu + 2\phi D^2 \phi + 2\phi \{\psi^\mu, \psi_\mu\} \right\},$$

and from (14) for the matter action we immediately get

$$W^{(0)}_M = \int d^4x \left\{ -i\alpha^+ (\sigma^\mu)_{AB} \tilde{D}_\mu \alpha^B + i\beta^- (\sigma^\mu)_{AB} \tilde{D}_\mu \beta^B - 2\bar{m}\beta_A \alpha^A \\
+ i\alpha^+ (\sigma^\mu)_{AB} \psi^B - i\beta^- (\sigma^\mu)_{AB} \psi^B \beta^B + 2\bar{m}\beta_A (\phi + m) \zeta^A \\
- i\chi^{\mu\nu} D_\mu \psi_\nu + i\beta^\mu (\sigma^\mu)_{AB} \bar{D}_\mu \zeta^B - 2\chi^{\mu\nu} D_\mu \psi_\nu + 2\phi \{\psi^\mu, \psi_\mu\} \right\}.$$
ghost for the ghost equation [17], there is an additional set of stability constraints which improve the finiteness properties displayed by this model.

To begin with, let us first express the BRST transformations (23) by redefining the fields \( \eta, \lambda^{\mu\nu} \) and \( B \), being only auxiliary ones, according to

\[
\eta \rightarrow \eta - [C, \bar{\phi}], \quad \lambda^{\mu\nu} \rightarrow \lambda^{\mu\nu} - \{C, \chi^{\mu\nu}\}, \quad B \rightarrow B - \{C, \bar{C}\},
\]

in such a way that all the fields belonging to the non–minimal sector occur as trivial BRST–doublets,

\[
s_Q A_\mu = \psi_\mu - D_\mu C, \quad s_Q \psi_\mu = D_\mu \phi + \{C, \psi_\mu\}, \quad s_Q C = \phi + C^2, \quad s_Q \phi = [C, \phi],
\]

\[
s_Q \bar{\phi} = \eta, \quad s_Q \eta = 0, \quad s_Q \chi^{\mu\nu} = \lambda^{\mu\nu}, \quad s_Q \lambda^{\mu\nu} = 0, \quad s_Q \bar{C} = B, \quad s_Q B = 0. \tag{36}
\]

Next, let us adopt the preceding construction of TQCD in the covariant Landau type gauge as a guiding principle for the gauge–fixed action of TYM in the non–covariant Landau type gauge [17], namely requiring

\[
W_{TYM} = s_Q \Psi_{TYM}, \quad Q_\mu \Psi_{TYM} = 0, \tag{37}
\]

with the following modification of the gauge fermion (cf., Eqs. (25) and (34))

\[
\Psi_{TYM} = \int d^4x \text{tr} \left\{ \lambda^{\mu\nu} F_{\mu\nu} + 2 \bar{\phi} \partial^\mu \psi_\mu + 2 \bar{C} \partial^\mu A_\mu \right\}, \tag{38}
\]

which consists in choosing a linear gauge condition not only for the ordinary gauge symmetry but also for the topological shift symmetry. Then, by making use of (36), one gets

\[
W_{TYM} = \int d^4x \text{tr} \left\{ \left( \lambda^{\mu\nu} - \{C, \chi^{\mu\nu}\} \right) F_{\mu\nu} - 2 \chi^{\mu\nu} D_\mu \psi_\nu + 2 \eta \partial^\mu \psi_\mu \right.
\]

\[
+ 2 \bar{\phi} \partial^\mu \left( D_\mu \phi + \{C, \psi_\mu\} \right) + 2 B \partial^\mu A_\mu - 2 \bar{C} \partial^\mu \left( \psi_\mu - D_\mu C \right) \right\}. \tag{39}
\]

Moreover, it is easily seen that this action exhibits, besides the BRST symmetry (36), invariance under the following linear vector supersymmetry [17]:

\[
\bar{Q}_\mu A_\nu = 0, \quad \bar{Q}_\mu \psi_\nu = \partial_\mu A_\nu,
\]

\[
\bar{Q}_\mu C = 0, \quad \bar{Q}_\mu \phi = \partial_\mu C, \quad \bar{Q}_\mu \bar{\phi} = 0, \quad \bar{Q}_\mu \eta = \partial_\mu \bar{\phi},
\]

\[
\bar{Q}_\mu \chi_{\rho\sigma} = 0, \quad \bar{Q}_\mu \lambda_{\rho\sigma} = \partial_\mu \chi_{\rho\sigma}, \quad \bar{Q}_\mu \bar{C} = \partial_\mu \bar{\phi}, \quad \bar{Q}_\mu B = \partial_\mu \bar{C} - \partial_\mu \eta. \tag{40}
\]

These transformations are determined by the second of the Eqs. (37) and the requirement, together with (36), to obey the topological superalgebra (30) without central extensions.

Now, we are faced with the problem to construct an action of TQCD which, on the one hand, bears in mind the non–covariant gauge choice \( \partial_\mu \psi^\mu = 0 \) and, on the other hand, satisfies the topological superalgebra (30) with the central charges \( Z \) and \( \bar{Z} \). This problem amounts to look for a linear vector supersymmetry \( \bar{Q}_\mu \) of the matter fields, too, which together with (29), (36) and (40) obeys the superalgebra (30). It is not difficult to
convince oneself that this requirement is indeed fulfilled by defining the action of $\hat{Q}_\mu$ on the matter fields as

\[
\hat{Q}_\mu \zeta^A = i(\sigma_\mu)^{AB} \bar{\beta}_B, \quad \hat{Q}_\mu \bar{\beta}_A = i\tilde{m}(\sigma_\mu)_A^{\;\;B} \zeta^B, \\
\hat{Q}_\mu \alpha^A = -i(\sigma_\mu)^{AB} \bar{\chi}_B + \partial_\mu \zeta^A, \quad \hat{Q}_\mu \bar{\chi}_A = -i\tilde{m}(\sigma_\mu)_A^{\;\;B} \alpha^B + \partial_\mu \bar{\beta}_A, \\
\hat{Q}_\mu \bar{\zeta}_A = -i(\sigma_\mu)_A^{\;\;B} \alpha^B, \quad \hat{Q}_\mu \bar{\alpha}^A = i\tilde{m}(\sigma_\mu)^{AB} \bar{\zeta}_B, \\
\hat{Q}_\mu \beta_A = i(\sigma_\mu)_A^{\;\;B} \chi^B + \partial_\mu \bar{\zeta}_A, \quad \hat{Q}_\mu \bar{\chi}^A = -i\tilde{m}(\sigma_\mu)^{AB} \beta_B + \partial_\mu \bar{\alpha}^A. \tag{41}
\]

The transformation rules (41) are obtained from (28) by simply replacing the gauge covariant derivative through the ordinary partial derivative.

Since $\hat{Q}_\mu$ leaves the gauge field $A_\mu$ inert, from (41) one infers that now the above replacement procedure simply can be repeated in order to get the action of TQCD:

\[
W_{\text{TQCD}} = W_{\text{TYM}} + W_M, \quad W_M = s_Q \Psi_M, \quad \hat{Q}_\mu \Psi_M = 0, \tag{42}
\]

with the following linearized gauge fermion (see Eq. (20))

\[
\Psi_M = -\int d^4x \left\{ i\bar{\alpha}^A(\sigma^\mu)_A^{\;\;B} \partial_\mu \zeta^B - i(\partial_\mu \bar{\zeta}_A)(\sigma^\mu)^{AB} \bar{\beta}_B + \tilde{m}(\bar{\zeta}_A \alpha^A - \beta_A \zeta^A) + \bar{\alpha}^A \bar{\chi}_A + \chi^A \bar{\beta}_A \right\}, \tag{43}
\]

and, by making use of (29), the action corresponding to the matter fields reads:

\[
W_M = \int d^4x \left\{ i\bar{\alpha}^A(\sigma^\mu)_A^{\;\;B} \partial_\mu \alpha^B + i(\partial_\mu \beta_A)(\sigma^\mu)^{AB} \bar{\beta}_B - 2\tilde{m}\beta_A \alpha^A \right. \tag{44}
\]

\[+ i\bar{\alpha}^A(\sigma^\mu)_A^{\;\;B} (\partial_\mu C) \zeta^B - i\bar{\zeta}_A(\sigma^\mu)^{AB} (\partial_\mu C) \bar{\beta}_B + 2\tilde{m}\zeta_A(\phi + m) \zeta^A \]

\[- i\chi^A(\sigma^\mu)_A^{\;\;B} \partial_\mu \zeta^B + i(\partial_\mu \bar{\zeta}_A)(\sigma^\mu)^{AB} \bar{\chi}_B - 2\chi^A \bar{\chi}_A - 2\bar{\alpha}^A(\phi + m) \bar{\beta}_A \right\}.
\]

Finally, let us notice that the ghost $C$ enters into the action (39) only as derivative $\partial_\mu C$ as well as through the combinations $\eta = [C, \bar{\phi}]$, $\lambda^{\mu\nu} = \{C, \chi^{\mu\nu}\}$ and $B = \{C, \bar{C}\}$. These global constraints can be expressed by the so-called ghost equation (30), usually valid in the Landau type gauge. There exists still another set of global constraints, the so-called ghost for the ghost equation (17), being related to the non-covariant Landau type gauge. It expresses the fact that the ghost for the ghost field $\phi$ enters into the action (39) only as derivative $\partial_\mu \phi$ as well as through the combination $B - [\bar{\phi}, \phi]$. On the other hand, by inspection of the action (44) one observes that only the first of the above-mentioned ghost equation can be imposed as global constraint, whereas the second one leads to a non-linear breaking term.

4. Slavnov–Taylor identity and stability constraints

In the previous section we have constructed the complete gauge-fixed action (12) of TQCD being invariant under the whole set of symmetry operators $s_Q$, $\hat{Q}_\mu$, $P_\mu$, $Z$ and $\bar{Z}$ obeying the topological superalgebra (30). Our aim is now to collect all the symmetry properties.
of that action into a unique Ward identity – the Slavnov–Taylor identity. This could be achieved in the conventional Batalin–Vilkovisky (BV) approach [31] where for every field a corresponding antifield with opposite Grassmann parity is introduced and then the symmetry properties are compactly formulated by some master equation. Here, however, we adopt the completely equivalent method of Ref. [32], where the antifields (sources) are introduced only for the fields belonging to the minimal sector, and where the symmetry operators \( s_Q, \bar{Q}_\mu, P_\mu, Z \) and \( \bar{Z} \) are collected into a unique nilpotent BRST operator \( s_T \) by associating to each of the generators \( \bar{Q}_\mu, P_\mu, Z \) and \( \bar{Z} \) a global ghost \( \rho^\mu, \xi^\mu, \bar{\xi} \) and \( \xi \), respectively.

In this context let us recall that identifying the \( R \)-charge with the ghost number has the meaning of setting the global ghost \( \rho \) associated to the generator \( Q \) equal to one, i.e., \( \rho = 1 \). This already has been done by defining the operator \( s_Q \) according to \( s_Q = s + Q \) and regarding \( Q \) as a BRST–like operator. Then, the aforementioned Slavnov–Taylor identity can be derived in the usual manner by coupling the antifields to the non–linear parts of field transformations generated by \( s_Q \). In addition, the action (42) obeys a set of gauge–fixing and antighost conditions as well as global constraints being related to the non–covariant Landau type gauge.

4.1. Introduction of antifields

Let us begin by introducing a set of global ghosts \( \rho^\mu, \xi^\mu, \bar{\xi} \) and \( \xi \), associated, respectively, to the generators \( \bar{Q}_\mu, P_\mu, Z \) and \( \bar{Z} \), and defining, in this way, the total BRST operator

\[
s_T = s_Q + \rho^\mu \bar{Q}_\mu - i \xi^\mu P_\mu + \bar{\xi} Z + \xi \bar{Z},
\]

with \( s_T \rho^\mu = 0, \quad s_T \xi^\mu = -\rho^\mu, \quad s_T \bar{\xi} = 1, \quad s_T \xi = -\rho^\mu \rho_\mu. \)

Here, the transformation rules of the global ghosts are chosen in such a way that \( s_T \) is strictly nilpotent. Then, it holds

\[
s_T W_{\text{TQCD}} = 0, \quad \{s_T, s_T\} = 0,
\]

where all the relevant features of the superalgebra \( [30] \) are now encoded in the nilpotency of the unique BRST operator \( s_T \). The properties of the global ghosts are displayed in the following Table 4.

|                | \( \rho^\mu \) | \( \xi^\mu \) | \( \bar{\xi} \) | \( \xi \) |
|----------------|----------------|---------------|---------------|--------|
| ghost number   | 2              | 1             | −1            | 3      |
| mass dimension | 0              | −1/2          | −1/2          | −1/2   |
| Grassmann parity | even          | odd           | odd           | odd    |

Let us now introduce the antifields of the minimal sector, \( V_T^* = \{ A^*_\mu, \psi^*_\mu, C^*, \phi^* \} \), transforming according to

\[
s_Q \psi^*_\mu = A^*_\mu, \quad s_Q A^*_\mu = 0, \quad s_Q \phi^* = C^*, \quad s_Q C^* = 0
\]

(46)
and
\[ \bar{Q}_\mu \psi_\mu^* = 0, \quad \bar{Q}_\mu A_\mu^* = \partial_\mu \psi_\mu^*, \quad \bar{Q}_\mu \phi^* = 0, \quad \bar{Q}_\mu C^* = \partial_\mu \phi^*, \]

so that the superalgebra (30) is satisfied. In particular, from Eqs. (46) it is obvious that the antifields are grouped in BRST–doublets.

Next, we extend the action (39) by adding a pure BRST invariant term \( s_Q \Upsilon_T \),
\[ S_{TYM} = W_{TYM} + s_Q \Upsilon_T, \]

with
\[ \Upsilon_T = - \int d^4 x \text{tr} \left\{ A_\mu^*(A^\mu + \psi^*_\mu \psi^\mu - C^* C + \phi^* \phi) \right\} \]
\[ s_Q \Upsilon_T = \int d^4 x \text{tr} \left\{ A_\mu^*(s_Q A^\mu - \psi^\mu) - \psi^*_\mu(s_Q \psi^\mu) + C^*(s_Q C - \phi) + \phi^*(s_Q \phi) \right\}. \]

Notice, that \( \Upsilon_T \), by virtue of (40), does not violate the vector supersymmetry \( \bar{Q}_\mu \), i.e., it holds \( \bar{Q}_\mu \Upsilon_T = 0 \). Then, by making use of (36), for the antifield dependent terms we obtain
\[ s_Q \Upsilon_T = \int d^4 x \text{tr} \left\{ - A_\mu^* D^\mu C - \psi^*_\mu (D^\mu \phi + \{ C, \psi^\mu \}) + C^* C^2 + \phi^*[C, \phi] \right\}. \]

Here, it is worthwhile to draw the attention to a particular feature of the operator \( s_Q \). First, since the antifields in the transformation law (46) appear in BRST–doublets they do not contribute to the BRST cohomology [33]. Second, the antifields in (48) do not couple to the linear parts \( \psi_\mu \) and \( \phi \) of the field transformations \( s_Q A_\mu \) and \( s_Q C \) (cf., Eqs. (36)).

Furthermore, as a useful hint we remark that also the fields of the minimal sector could be cast into BRST–doublets as well, namely
\[ s_Q A_\mu = \psi_\mu, \quad s_Q \psi_\mu = 0, \quad s_Q C = \phi, \quad s_Q \phi = 0. \]

This might be achieved by redefining \( \psi_\mu \) and \( \phi \) according to the replacements
\[ \psi_\mu \to \psi_\mu + D_\mu C \quad \text{and} \quad \phi \to \phi - C^2. \]

For that reason, one expects that the cohomology of the operator \( s_Q \) is completely trivial [5, 24]. However, needless to say, such redefinitions can be performed only at the lowest order of perturbation theory.

Finally, let us display the properties of the antifields of the gauge multiplet in Table 5.

|          | \( A_\mu^* \) | \( \psi^*_\mu \) | \( C^* \) | \( \phi^* \) |
|----------|--------------|----------------|----------|----------|
| ghost number | -1          | -2            | -2       | -3       |
| mass dimension | 3           | 5/2           | 5/2      | 3        |
| scale dimension | 3           | 3             | 4        | 4        |
| \( Z^- \), \( \bar{Z}^- \)-charge | 0           | 0             | 0        | 0        |
| Grassmann parity | odd       | even          | even     | odd      |
Proceeding in the same manner as before, let us introduce for each matter field a corresponding antifield, \( Y_T^* = \{ \alpha_A^*, \bar{\beta}_A^*, \zeta_A^*, \bar{\chi}_A^* \} \) and \( \bar{Y}_T^* = \{ \beta_A^*, \bar{\alpha}_A^*, \bar{\zeta}_A^*, \bar{\lambda}_A^* \} \), transforming according to

\[
\begin{align*}
s_Q \alpha_A^* &= \zeta_A^*, & s_Q \zeta_A^* &= -m \alpha_A^*, & s_Q \bar{\chi}_A^* &= \bar{\beta}_A^*, & s_Q \bar{\beta}_A^* &= -m \bar{\chi}_A^*, \\
& & s_Q \bar{\alpha}_A^* &= \bar{\zeta}_A^*,
\end{align*}
\]

and

\[
\begin{align*}
\bar{Q}_\mu \bar{\chi}_A^* &= i(\sigma_\mu)_{\bar{A}B} \alpha_B^*, \\
& & \bar{Q}_\mu \alpha_A^* &= -i \bar{\mu}(\sigma_\mu)_{AB} \bar{\chi}^B, \\
Q_\mu \bar{\beta}_A^* &= -i(\sigma_\mu)_{\bar{A}B} \bar{\zeta}_B^* + \partial_\mu \bar{\chi}_A^*, \\
& & Q_\mu \zeta_A^* &= i \mu(\sigma_\mu)_{AB} \beta_B^* + \partial_\mu \alpha_A^*, \\
\bar{Q}_\mu \bar{\chi}_A^* &= (\sigma_\mu)_{\bar{A}B} \beta_B^*, \\
& & \bar{Q}_\mu \beta_A^* &= (\sigma_\mu)_{AB} \alpha_B^* + (\sigma_\mu)_{\bar{A}B} \bar{\alpha}_B^* + \partial_\mu \beta_A^*,
\end{align*}
\]

which together with the central charge transformations

\[
\begin{align*}
ZV_T^* &= 0, & ZY_T^* &= -m Y_T^*, & Z\bar{Y}_T^* &= m \bar{Y}_T^*, \\
\overline{ZV}_T^* &= 0, & \overline{ZY}_T^* &= -\bar{m} Y_T^*, & \overline{Z\bar{Y}}_T^* &= \bar{m} \bar{Y}_T^*,
\end{align*}
\]

obey the superalgebra (30).

Next, let us add also to the matter action (44) a pure BRST invariant term \( s_Q \Upsilon_M \),

\[
S_M = W_M + s_Q \Upsilon_M,
\]

with

\[
\Upsilon_M = - \int d^4x \left\{ \alpha_A^* \alpha^A + \zeta_A^* \zeta^A + \bar{\chi}_A^* \bar{\chi}_A - \bar{\beta}_A^* \bar{\beta}_A \\
&\quad + \beta_A^* \bar{\beta}_A + \bar{\zeta}_A^* \beta_A + \bar{\chi}_A^* \lambda^A - \bar{\alpha}_A^* \bar{\alpha}^A \right\}
\]

\[
s_Q \Upsilon_M = \int d^4x \left\{ - \alpha_A^* (s_Q \alpha^A - m \zeta^A) + \zeta_A^* (s_Q \zeta^A - \alpha^A) \\
&\quad + \bar{\chi}_A^* (s_Q \bar{\chi}_A - m \bar{\beta}_A) + \bar{\beta}_A^* (s_Q \bar{\beta}_A - \bar{\chi}_A) \\
&\quad - \beta_A^* (s_Q \beta_A + m \bar{\zeta}_A) + \bar{\zeta}_A^* (s_Q \bar{\zeta}_A - \beta_A) \\
&\quad + \chi_A^* (s_Q \chi^A + m \alpha_A) + \bar{\alpha}_A^* (s_Q \bar{\alpha}^A - \chi_A^A) \right\}.
\]

As before, by virtue of (44), it can be verified that \( \Upsilon_M \) does not spoil the vector supersymmetry \( \bar{Q}_\mu \), i.e., it holds \( \bar{Q}_\mu \Upsilon_M = 0 \). Thus, by making use of (28), for the antifield dependent terms we get

\[
s_Q \Upsilon_M = \int d^4x \left\{ \alpha_A^* (\phi \zeta^A - C \alpha^A) + \zeta_A^* \phi \zeta^A - \bar{\chi}_A^* (\phi \bar{\beta}_A - C \bar{\chi}_A) + \bar{\beta}_A^* C \beta_A \\
&\quad - \beta_A^* (\bar{\zeta}_A \phi + \beta_A C) - \bar{\zeta}_A^* \bar{\zeta}_A C - \chi^*_A (\bar{\alpha}_A \phi - \chi^*_A C) + \bar{\alpha}_A^* \bar{\alpha}^*_A \right\}.
\]

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Again, here one observes that when carrying out in (29) the replacements
\[\alpha^A \rightarrow \alpha^A - C\zeta^A, \quad \beta^A \rightarrow \beta^A + \bar{\zeta}_A C, \quad \bar{\chi}_A \rightarrow \bar{\chi}_A - C\bar{\beta}_A, \quad \chi^\dagger \rightarrow \chi^\dagger - \bar{\alpha}^\dagger C,\]
a doublet–like structure of the matter fields would follow,
\[s_Q\zeta^A = \alpha^A, \quad s_Q\alpha^A = m\zeta^A, \quad s_Q\bar{\beta}_A = \bar{\chi}_A, \quad s_Q\bar{\chi}_A = m\bar{\beta}_A,\]
showing that the cohomology of the operator \(s_Q\), when acting on the space of integrated local polynomials of the fields and antifields with vanishing central charge, would be trivial, too.

The properties of the matter–antifields are displayed in Table 6.

|                  | \(\alpha^*_A\) | \(\beta^{A*}\) | \(\bar{\alpha}^*_A\) | \(\bar{\beta}^{A*}\) | \(\zeta^{A*}\) | \(\bar{\zeta}^{A*}\) | \(\chi^{A*}\) | \(\bar{\chi}^{A*}\) |
|------------------|----------------|----------------|----------------------|----------------------|----------------|----------------------|----------------|----------------------|
| ghost number     | -2             | -2             | -2                   | -2                   | -1             | -1                   | -1             | -1                   |
| mass dimension   | 5/2            | 5/2            | 5/2                  | 5/2                  | 3              | 3                    | 2              | 2                    |
| scale dimension  | 3              | 3              | 2                    | 2                    | 3              | 3                    | 1              | 1                    |
| \(Z, \bar{Z}\)–charge | 1              | -1             | -1                   | 1                    | 1              | -1                   | -1             | 1                    |
| Grassmann parity | odd            | odd            | odd                  | odd                  | even           | even                 | even           | even                 |

Finally, putting together (58) and (54) we get the extended antifield-dependent action of TQCD,
\[S_{\text{TQCD}} = S_{\text{TYM}} + S_M, \quad s_T S_{\text{TQCD}} = 0, \quad (57)\]
where any of its symmetry properties is encoded in the single operator \(s_T\).

4.2. Slavnov–Taylor identity

We translate now the symmetry property (57) into the Slavnov–Taylor identity,
\[S_T(S) = 0, \quad (58)\]
of the classical action \(S_{\text{TQCD}}\) which, for notational simplicity, has been denoted by \(S\). Here, \(S_T(S)\) displays the following expansion with respect to the global ghosts,
\[S_T(S) \equiv S_Q(S) + \rho^\mu \bar{Q}_\mu S - i\xi^\mu P_\mu S + \xi Z S + \bar{\zeta} \bar{Z} S - \rho^\mu \frac{\partial S}{\partial \xi^\mu} + \frac{\partial S}{\partial \xi} - \rho^\mu \rho_\mu \frac{\partial S}{\partial \xi}, \quad (59)\]
where \(Q_\mu, P_\mu, Z\) and \(\bar{Z}\) denote the Ward operators of vector supersymmetry transformations, space–time translations and central charge transformations in the space of fields and antifields, respectively. Furthermore, we represent the nilpotent BRST operator \(s_T\) by a linear nilpotent functional differential operator:
\[S_T = S_Q + \rho^\mu \bar{Q}_\mu - i\xi^\mu P_\mu + \bar{\zeta} \bar{Z} + \xi Z - \rho^\mu \frac{\partial}{\partial \xi^\mu} + \frac{\partial}{\partial \xi} - \rho^\mu \rho_\mu \frac{\partial}{\partial \xi}, \quad (60)\]
where $\bar{Q}_\mu$, $P_\mu$, $Z$ and $\bar{Z}$, together with $S_Q$, are required to obey the topological superalgebra

$$\{S_Q, S_Q\} = -2Z, \quad \{S_Q, \bar{Q}_\mu\} = -iP_\mu, \quad \{\bar{Q}_\mu, \bar{Q}_\nu\} = -2\delta_{\mu\nu}\bar{Z}. \quad (61)$$

From Eq. (31) it follows that $S_Q$ becomes a nilpotent operator when acting on the space of integrated local polynomials with vanishing central charge (and being independent of the global ghosts). Finally, we obtain $\{S_T, S_T\} = 0$.

In (30) the operator $S_Q(S)$ lumps together both the linear and non–linear parts of the BRST transformations, the latter ones being expressed by derivatives with respect to the antifields (see Eqs. (14), (15), (16), (17) and (18)). It takes the form

$$S_Q(S) = \int d^4x \left\{ \left( \frac{\delta S}{\delta A_\mu^*} + \psi^* \right) \frac{\delta S}{\delta A_\mu} + \left( \frac{\delta S}{\delta C} + \phi \right) \frac{\delta S}{\delta C} \right\}$$

$$+ \int d^4x \left\{ \left( \frac{\delta S}{\delta \alpha_A} + \alpha^A \right) \frac{\delta S}{\delta \bar{\alpha}_A} + m\alpha^A \frac{\delta S}{\delta \bar{\alpha}_A} \right\}$$

From $S_Q(S)$ one reads off the linearized Slavnov–Taylor operator

$$S_Q = \int d^4x \left\{ \left( \frac{\delta S}{\delta A_\mu^*} + \psi^* \right) \frac{\delta S}{\delta A_\mu} + \left( \frac{\delta S}{\delta C} + \phi \right) \frac{\delta S}{\delta C} \right\}$$

$$+ \int d^4x \left\{ \left( \frac{\delta S}{\delta \alpha_A} + \alpha^A \right) \frac{\delta S}{\delta \bar{\alpha}_A} + m\alpha^A \frac{\delta S}{\delta \bar{\alpha}_A} \right\}$$

$$+ \int d^4x \left\{ \left( \frac{\delta S}{\delta \alpha_A} + \alpha^A \right) \frac{\delta S}{\delta \bar{\alpha}_A} + m\alpha^A \frac{\delta S}{\delta \bar{\alpha}_A} \right\}.$$
If the action \( S \) is a solution of \( S_Q(S) = 0 \) then, by a tedious but straightforward calculation, one can show that \( S_Q \) obeys the relation \( \{ S_Q, S_Q \} = -2Z \), where the central charge operators \( Z \) and \( \bar{Z} \) are given by (cf., Eqs. (13) and (53))

\[
\frac{Z}{m} = \frac{\bar{Z}}{\bar{m}} = \int d^4x \left\{ -\alpha^A \delta_{\delta\alpha^A} - \beta_A \delta_{\delta\beta_A} - \zeta^A \delta_{\delta\zeta^A} - \chi^A \delta_{\delta\chi^A} \\
+ \beta_A \delta_{\delta\beta_A} + \alpha^A \delta_{\delta\alpha^A} + \zeta_A \delta_{\delta\zeta_A} + \chi^A \delta_{\delta\chi^A} \\
+ \alpha_A^* \delta_{\delta\alpha_A^*} + \beta_A^* \delta_{\delta\beta_A^*} + \zeta^*_{\delta\zeta_A^*} + \chi^*_{\delta\chi_A^*} \\
- \beta_A^* \delta_{\delta\beta_A^*} - \alpha^*_{\delta\alpha_A^*} - \zeta^*_{\delta\zeta_A^*} - \chi^*_{\delta\chi_A^*} \right\}.
\]

Furthermore, if \( S \) is also a solution of \( \bar{Q}_\mu S = 0 \), where \( \bar{Q}_\mu \) is given by (cf., Eqs. (40), (41), (42), (47) and (62))

\[
\bar{Q}_\mu = \int d^4x \text{tr} \left\{ \partial_\mu A_\nu \delta_{\delta A_\nu} + \partial_\mu \psi_\nu \delta_{\delta A_\nu} + \partial_\mu C \delta_{\delta\phi} + \partial_\mu \phi^* \delta_{\delta C^*} \\
+ \partial_\mu \bar{\phi} \delta_{\delta C} + (\partial_\mu C - \partial_\mu \eta) \delta_{\delta B} + \partial_\mu \bar{\phi} \delta_{\delta B} + \frac{1}{2} \partial_\mu \chi_{\rho\sigma} \delta_{\delta \chi_{\rho\sigma}} \right\} \\
+ \int d^4x \left\{ (\partial_\mu \bar{\zeta} + i(\sigma_\mu)_{AB} \chi^B) \frac{\delta}{\delta \beta_A} + (\partial_\mu \zeta^A - i(\sigma_\mu)_{AB} \bar{\chi}^B) \frac{\delta}{\delta \alpha_A^*} \\
+ (\partial_\mu \beta_A^* - i\bar{m}(\sigma_\mu)_{AB} \bar{\alpha}_B^*) \frac{\delta}{\delta \zeta^A} + (\partial_\mu \alpha_A^* + i\bar{m}(\sigma_\mu)_{AB} \beta_B) \frac{\delta}{\delta \zeta^A} \\
- i(\sigma_\mu)_{AB} \bar{\beta}_B \frac{\delta}{\delta \zeta^A} + i\bar{m}(\sigma_\mu)_{AB} \bar{\zeta}_B \frac{\delta}{\delta \beta_A^*} + (\partial_\mu \bar{\alpha}^A - i\bar{m}(\sigma_\mu)_{AB} \beta_B) \frac{\delta}{\delta \chi^A} \\
+ i(\sigma_\mu)_{AB} \bar{\alpha}_B \frac{\delta}{\delta \chi^A} + i\bar{m}(\sigma_\mu)_{AB} \chi_B \frac{\delta}{\delta \beta_A^*} + (\partial_\mu \bar{\beta}^A - i(\sigma_\mu)_{AB} \bar{\zeta}_B^*) \frac{\delta}{\delta \alpha_A^*} \\
+ i(\sigma_\mu)_{AB} \bar{\alpha}_B \frac{\delta}{\delta \alpha_A^*} - i\bar{m}(\sigma_\mu)_{AB} \bar{\chi}^B \frac{\delta}{\delta \alpha_A^*} + (\partial_\mu \bar{\chi}^A - i(\sigma_\mu)_{AB} \bar{\zeta}_B^*) \frac{\delta}{\delta \beta_A^*} \right\},
\]

then one can verify that the vector supersymmetry \( \bar{Q}_\mu \) allows to decompose the translation operator \( P_\mu \) as \( \{ \bar{Q}_\mu, S_Q \} = -iP_\mu \). Moreover, one can establish, in accordance with the superalgebra (61), that it holds \( \{ \bar{Q}_\mu, \bar{Q}_\nu \} = -2\delta_{\mu\nu}Z \).

### 4.3. Equations of motion and global constraints

Besides obeying the Slavnov–Taylor identity (58), the complete action \( S \) turns out to be characterized by further constraints, namely both Landau gauge-fixing conditions (see Eqs. (39) and (50))

\[
\frac{\delta S}{\delta B} = 2\partial_\mu A^\mu, \quad \frac{\delta S}{\delta \eta} = 2\partial_\mu \psi^\mu, \quad (64)
\]

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and the corresponding antighost equations of motion
\[ \frac{\delta S}{\delta \bar{C}} + 2 \partial_\mu \frac{\delta S}{\delta A_\mu} = -2 \partial_\mu \psi^\mu, \quad \frac{\delta S}{\delta \phi} + 2 \partial_\mu \frac{\delta S}{\delta \bar{\psi}_\mu} = 0, \] (65)
where the terms on the right-hand side, being linear in the quantum fields, are classical breakings, i.e., they are not subjected to any specific renormalization.

As already emphasized in Sect. 3 the essential reason for imposing the non-covariant Landau type gauge relies on the fact that in such a case the action \( S \) exhibits much larger symmetries than in a covariant one. In particular, it can be easily verified that in this gauge the dependence of \( S \) on the whole set of matter fields is completely fixed by the following linearly broken Ward identities (see Eqs. (44) and (56)),

\[
\frac{\delta S}{\delta \chi^A} = -i(\sigma^a)_{AB} \partial_\mu \zeta^B - 2 \bar{\chi}_A - C \chi^*_A,
\frac{\delta S}{\delta \tilde{\chi}^A} = i(\sigma^a)_{AB} \partial_\mu \beta^B + 2 \bar{\chi}^A - \bar{\chi}^*_A C,
\frac{\delta S}{\delta \tilde{\beta}^A} = -i(\sigma^a)_{AB} \partial_\mu \tilde{\beta}^B + 2 \bar{\beta}^A - \tilde{\beta}^*_A C,
\frac{\delta S}{\delta \alpha^A} = -i(\sigma^a)_{AB} \partial_\mu \tilde{\alpha}_B - 2 \bar{\alpha}^A - \alpha^*_A C,
\frac{\delta S}{\delta \zeta^A} = -i(\sigma^a)_{AB} \partial_\mu \zeta^B + 2 \bar{\zeta}_A - \zeta^*_A C,
\frac{\delta S}{\delta \beta^A} = -i(\sigma^a)_{AB} \partial_\mu \beta^B + 2 \bar{\beta}^A - \beta^*_A C,
\frac{\delta S}{\delta \bar{\alpha}_A} = -i(\sigma^a)_{AB} \partial_\mu \bar{\alpha}_B - 2 \bar{\bar{\alpha}}_A - \bar{\alpha}^*_A C,
\frac{\delta S}{\delta \bar{\zeta}_A} = -i(\sigma^a)_{AB} \partial_\mu \bar{\zeta}_B + 2 \bar{\bar{\zeta}}_A - \bar{\zeta}^*_A C.
\] (66)
The stability constraints (64) – (66), which drastically reduce the number of independent invariant counterterms, can be established to all orders of perturbation theory by using the renormalized quantum action principles [19].

Moreover, there exists a set of global constraints, usually valid in the Landau type gauge. The first one is the so-called ghost equation [30], which in the present case reads

\[
G S = \int d^4 x \left\{ \left[ A^*_\mu, A^\mu \right] + [\psi^*_\mu, \psi^\mu] - [C^*, C] + [\phi^*, \phi] - T^i \left( \zeta_A^i T^i \zeta_A^* + \alpha_A^i T^i \alpha_A^* \right.ight.
\left.\left. - \tilde{\beta}^*_A T^i \tilde{\beta}_A + \tilde{\chi}^*_A T^i \tilde{\chi}_A - \tilde{\zeta}^*_A \tilde{\chi}^i T^i - \tilde{\beta}^*_A \tilde{\beta}^A T^i + \tilde{\alpha}_A^i \tilde{\alpha}_A^* T^i - \chi_A^i \tilde{\chi}^*_A T^i \right) \right\},
\] (67)
with
\[
G = \int d^4 x \left\{ \frac{\delta}{\delta C} + \left[ \bar{C}, \frac{\delta}{\delta B} \right] + \left[ \bar{\phi}, \frac{\delta}{\delta \eta} \right] + \frac{1}{2} \left[ \chi^\mu, \frac{\delta}{\delta \lambda^\mu} \right] \right\},
\]
where the terms on the right-hand side of that equation are linear classical breakings, i.e., they will not get radiative corrections.
As usual, commuting the ghost equation (67) with the ST identity (58) one gets a further global constraint fulfilled by the action $S$, namely the Ward identity for the rigid gauge invariance,

$$ RS = 0, $$

(68)

where $R = \{S_T, G\}$ denotes the Ward operator for rigid gauge transformations in the space of fields and antifields, expressing the fact that all the (anti)fields belong to either the adjoint or the $R$–representation of the gauge group $G$.

It is known [19] that the structure of the invariant counterterms entirely will be governed by a set of classical stability constraints, provided they can be extended to all orders of perturbation theory. The proof, that the constraints (67) and (68) are extendable to any order of perturbation theory, can be appreciably simplified by adopting again the strategy of Ref. [32]. One associates to each operator $G$ and $R$ a global ghost, $\gamma$ and $\tau$, respectively, which take their values in $\operatorname{Lie}(G)$, and introduces the following operator:

$$ O_T = S_T + \text{tr} \left\{ \gamma G + \tau R + [\tau, \gamma] \frac{\partial}{\partial \gamma} - \gamma \frac{\partial}{\partial \tau} + \tau^2 \frac{\partial}{\partial \tau} \right\}. $$

(69)

One easily verifies that $O_T$ is nilpotent,

$$ \{O_T, O_T\} = 0. $$

Furthermore, if it can be proven that the integrated cohomology of $O_T$ turns out to be empty, then the integrated cohomology of $S_T$ is empty as well and, besides the ST identity (58), the constraints (67) and (68) can be employed to single out invariant counterterms. (The proof, that the integrated cohomology of $O_T$ is indeed empty, will be given in Sect. 5.) The properties of these additional global ghosts are displayed in Table 7.

|                   | $\gamma$ | $\tau$ |
|-------------------|----------|--------|
| ghost number      | 2        | 1      |
| mass dimension    | 0        | 0      |
| parity            | even     | odd    |

5. BRST cohomology: Anomalies and invariant counterterms

Let us discuss now the renormalizability of topological QCD in the framework of the algebraic BRST technique [19] which allows for a systematic study of the quantum extension of the BRST symmetry. In that framework the proof of renormalizability is related to the characterization of some cohomology classes of the linearized ST–operator $S_T$, Eq. (50), which turns out to be essential for the (possible absence of) anomalies and the construction of the invariant counterterms. Let us recall that both the anomalies $\Delta_A$ and the invariant counterterms $\Delta_C$ of the (quantum) action $S$ are integrated local polynomials in
the (anti)fields with (mass) dimension four and ghost number, respectively, one and zero. In addition, they are constrained by the following consistency conditions

\[ \mathbf{S}_T \Delta_A = 0, \quad \Delta_A \neq \mathbf{S}_T \hat{\Delta}_A, \quad \text{gh}(\Delta_A) = 1, \quad (70) \]

and

\[ \mathbf{S}_Q \Delta_C = 0, \quad \frac{\partial \Delta_C}{\partial \rho^\mu} = 0, \quad \frac{\partial \Delta_C}{\partial \xi^\mu} = 0, \quad \frac{\partial \Delta_C}{\partial \bar{\xi}} = 0, \quad \frac{\partial \Delta_C}{\partial \bar{m}} = 0, \quad \text{gh}(\Delta_C) = 0. \quad (71) \]

Therefore, the cohomological relevant solutions of Eqs. (70) and (71) are the non–trivial cocycles of the integrated cohomology of \( \mathbf{S}_T \) and \( \mathbf{S}_Q \), respectively. Let us mention that both, \( \Delta_A \) and \( \Delta_C \), by virtue of \( [\mathbf{Z}, \mathbf{S}_T] = 0 \), must have vanishing central charge, i.e., \( \mathbf{Z} \Delta_A = 0 \) and \( \mathbf{Z} \Delta_C = 0 \); analogously for \( \bar{\mathbf{Z}} \).

In order to characterize the integrated cohomology of \( \mathbf{S}_T \) we introduce the filtration

\[ \mathbf{F}_T = 2\rho^\mu \frac{\partial}{\partial \rho^\mu} + \xi^\mu \frac{\partial}{\partial \xi^\mu} - \bar{\xi} \frac{\partial}{\partial \bar{\xi}} + m \frac{\partial}{\partial \bar{m}} + m \frac{\partial}{\partial m}, \]

which obviously, by virtue of \( [m \partial/\partial m, \mathbf{Z}] = \mathbf{Z} \), induces a separation of \( \mathbf{S}_T \) according to

\[ \mathbf{S}_T = \sum_{n=0}^{\infty} \mathbf{S}_T^{(n)}, \quad [\mathbf{F}_T, \mathbf{S}_T^{(n)}] = n \mathbf{S}_T^{(n)}, \]

\( \mathbf{S}_T^{(0)} \equiv \mathbf{S}_Q^{(m=0)} \) being the \( m^- \) (and \( \bar{m}^- \)) independent part of \( \mathbf{S}_Q \), Eq. (63). Since for \( m = 0 \) and \( \bar{m} = 0 \) the central charges vanish the operator \( \mathbf{S}_T^{(0)} \) is strictly nilpotent,

\[ \{ \mathbf{S}_T^{(0)}, \mathbf{S}_T^{(0)} \} = 0. \]

According to the general results on cohomology [34], the integrated cohomology of \( \mathbf{S}_T \) is isomorphic to a subspace of the integrated cohomology of \( \mathbf{S}_T^{(0)} \).

In order to characterize the integrated cohomology of \( \mathbf{S}_T^{(0)} \) we introduce the filtration

\[ \mathbf{F}_Q = \int d^4x \text{tr} \left\{ A^\mu \frac{\delta}{\delta A^\mu} + \psi^\mu \frac{\delta}{\delta \psi^\mu} + \frac{1}{2} \chi^{\mu\nu} \frac{\delta}{\delta \chi^{\mu\nu}} + \frac{1}{2} \lambda^{\mu\nu} \frac{\delta}{\delta \lambda^{\mu\nu}} \right. \]
\[ + \bar{C} \frac{\delta}{\delta \bar{C}} + B \frac{\delta}{\delta B} + \bar{\phi} \frac{\delta}{\delta \bar{\phi}} + \eta \frac{\delta}{\delta \eta} + 2C \frac{\delta}{\delta C} + 2\phi \frac{\delta}{\delta \phi} \}
\[ + \int d^4x \left\{ \alpha^A \frac{\delta}{\delta \alpha^A} + \hat{\beta}_A \frac{\delta}{\delta \hat{\beta}_A} + \bar{\zeta}^A \frac{\delta}{\delta \bar{\zeta}^A} + \chi_\bar{A} \frac{\delta}{\delta \chi_\bar{A}} \right. \]
\[ + \beta_\bar{A} \frac{\delta}{\delta \beta_\bar{A}} + \bar{\alpha}^A \frac{\delta}{\delta \bar{\alpha}^A} + \zeta_A \frac{\delta}{\delta \zeta_A} + \chi_A \frac{\delta}{\delta \chi_A} \}, \]

which has the structure of a counting operator. Therefore, \( \mathbf{F}_Q \) possesses the property of decomposing \( \mathbf{S}_T^{(0)} \) as follows:

\[ \mathbf{S}_T^{(0)} = \sum_{n=0}^{\infty} \mathbf{S}_T^{(n,0)}, \quad [\mathbf{F}_Q, \mathbf{S}_T^{(n,0)}] = n \mathbf{S}_T^{(n,0)}, \]
where \( S_{T}^{(0,0)} \) is just that linear, \( m \)– and \( \bar{m} \)–independent part of the operator \( S_Q \) which also does not depend on \( S \):

\[
S_{T}^{(0,0)} = \int d^4x \text{tr}\left\{ \psi^{\mu} \frac{\delta}{\delta A^\mu} + A^*_{\mu} \frac{\delta}{\delta \phi^\mu} + \phi \frac{\delta}{\delta C} + C^* \frac{\delta}{\delta \phi^*} + B \frac{\delta}{\delta \phi} + \frac{1}{2} \lambda^{\mu\nu} \frac{\delta}{\delta \chi^{\mu\nu}} \right\} \\
+ \int d^4x \left\{ \alpha A \frac{\delta}{\delta \bar{\zeta} A} + \zeta^{*} \frac{\delta}{\delta \bar{\alpha} A} + \beta A \frac{\delta}{\delta \bar{\chi} A} + \bar{\chi} \frac{\delta}{\delta \bar{\alpha} A} + \bar{\alpha} \frac{\delta}{\delta \bar{\chi} A} \right\}. 
\]

Since in \( S_{T}^{(0,0)} \) all the (anti)fields appear in BRST–doublets, one concludes that the integrated cohomology of \( S_{T}^{(0,0)} \) is empty \([33, 34]\). Thus, the integrated cohomology of the operator \( S_T \) is empty as well, due to the fact that it is, in turn, isomorphic to a subspace of the integrated cohomology of \( S_{T}^{(0,0)} \). This result implies that the general solutions of the consistency conditions (70) and (71) are given by

\[
\Delta_A = 0, \\
\Delta_C = S_Q \Delta_C, \quad \text{gh}(\Delta_C) = -1, \tag{72}
\]

where \( \Delta_C \) is the most general integrated local polynomial in the (anti)fields, with dimension 7/2 and ghost number minus one. Hence, the ST identity \([58]\) is anomaly free and the invariant counterterms are trivial cocycles of the integrated cohomology of \( S_Q \).

The absence of anomalies of the ST identity, \( S_Q(S) = 0 \), as well as of the Ward identities \( Q_{\mu} S = 0, P_{\mu} S = 0, Z = 0 \) and \( \bar{Z} S = 0 \), with \( S \) being independent of \( \rho^\mu, \xi^\mu, \bar{\xi} \) and \( \xi \), implies that \( \Delta_C \), besides (71), is subjected also to the following consistency conditions,

\[
Q_{\mu} \Delta_C = 0, \quad P_{\mu} \Delta_C = 0, \quad Z \Delta_C = 0, \quad \bar{Z} \Delta_C = 0. \tag{73}
\]

Furthermore, from the ghost equation (67) and the Ward identity (68), which can be imposed to any order of perturbation theory, it follows that \( \Delta_C \) is required to obey the following constraints, too,

\[
G \Delta_C = 0, \quad R \Delta_C = 0. \tag{74}
\]

In order to prove that the stability constraints (67) and (68) can be extended to the quantum level it is sufficient to verify that the integrated cohomology of the nilpotent operator \( O_T \), Eq. (69), is empty. For that purpose, let us introduce the filtration

\[
N_T = F_T + \text{tr}\left\{ 2\gamma \frac{\partial}{\partial \gamma} + 2\tau \frac{\partial}{\partial \tau} \right\}
\]

which clearly induces a separation of \( O_T \), namely

\[
O_T = \sum_{n=0} O_T^{(n)}, \quad [N_T, O_T^{(n)}] = nO_T^{(n)}, \tag{75}
\]
with
\[
O_T^{(0)} = S_T^{(0)} - \text{tr} \left\{ \gamma \frac{\partial}{\partial T} \right\}.
\]  
(76)

Since the integrated cohomology of \( S_Q(0) \) is empty and because the extra term in (70) appears as BRST–doublet the integrated cohomology of \( O_T^{(0)} \) is empty as well and, therefore, with the same reasoning as before, the integrated cohomology of \( O_T \) is empty, too.

From the stability constraints (74) – (76) for \( \Delta_C \) we get a further set of restrictions:
\[
\frac{\delta \Delta_C}{\delta B} = 0, \quad \frac{\delta \Delta_C}{\delta \bar{C}} + 2 \partial_\mu \frac{\delta \Delta_C}{\delta A_\mu} = 0, \quad \frac{\delta \Delta_C}{\delta \eta} = 0, \quad \frac{\delta \Delta_C}{\delta \phi} + 2 \partial_\mu \frac{\delta \Delta_C}{\delta \psi_\mu} = 0 
\]  
(77)

and
\[
\frac{\delta \Delta_C}{\delta \chi^A} = 0, \quad \frac{\delta \Delta_C}{\delta \bar{\chi}^A} = 0, \quad \frac{\delta \Delta_C}{\delta \alpha^A} = 0, \quad \frac{\delta \Delta_C}{\delta \bar{\alpha}^A} = 0, \quad \frac{\delta \Delta_C}{\delta \beta^A} = 0, \quad \frac{\delta \Delta_C}{\delta \bar{\beta}^A} = 0, \quad \frac{\delta \Delta_C}{\delta \gamma} = 0, \quad \frac{\delta \Delta_C}{\delta \bar{\gamma}^A} = 0.
\]  
(78)

The constraints (73) imply that \( \Delta_C \) is required to be invariant under vector supersymmetry transformations, space–time translations and central charge transformations. From (74) one infers that the ghost \( C \) enters into \( \Delta_C \) either as derivative \( \partial_\mu C \) or through the combinations
\[
B - \{ C, \bar{C} \}, \quad \eta - [ C, \bar{\phi} ] \quad \text{and} \quad \chi^{\mu v} - \{ C, \chi^{\mu v} \},
\]
respectively, and that \( \Delta_C \) can be taken to be rigid gauge invariant.

Concerning the constraints (77) it follows that the auxiliary fields \( B \) and \( \eta \) cannot appear in \( \Delta_C \) and that the antighosts \( \bar{C} \) and \( \bar{\phi} \) can enter only through the combinations
\[
A^*_\mu - 2 \partial_\mu \bar{C} \quad \text{and} \quad \psi^*_\mu - 2 \partial_\mu \bar{\phi},
\]
respectively. Furthermore, from (78) it follows that the matter fields \( \bar{\chi}^A, \chi^A, \alpha^A \) and \( \beta^A \) cannot appear in \( \Delta_C \) and that \( \bar{\alpha}^A, \bar{\beta}^A, \bar{\gamma}^A \) and \( \gamma^A \) can enter only through the following combinations:
\[
\zeta^A - i (\sigma^\mu)^A_B \partial_\mu \bar{\gamma}^B, \quad \bar{\gamma}^A + 2 \bar{\alpha}^A, \quad C^* + \bar{\chi}^A \alpha^A, \\
\bar{\zeta}^A - i (\sigma^\mu)^A_B \partial_\mu \bar{\beta}^B, \quad \chi^A - 2 \bar{\beta}^A, \quad C^* - \bar{\beta} \bar{\chi}^A, \\
\bar{\alpha}^A + i (\sigma^\mu)^A_B \partial_\mu \zeta^B, \quad \beta^A + 2 \bar{\gamma}^A, \quad C^* + \zeta^A \bar{\alpha}^A, \\
\bar{\beta}^A + i (\sigma^\mu)^A_B \partial_\mu \bar{\zeta}^B, \quad \alpha^A + 2 \bar{\gamma}, \quad C^* - \beta^A \bar{\zeta}^A.
\]

Let us now turn to the computation of \( \hat{\Delta}_C \). First of all we point out that the whole set of constraints is stable under the action of \( S_Q \) and, therefore, \( \Delta_C = S_Q \Delta_C \) satisfies these constraints if \( \hat{\Delta}_C \) will do. Although, generally, \( \Delta_C \) does not have to obey them it may
be chosen to do so, except for the constraint $G \Delta_C = 0$ \cite{17}. Since the constraints above are very restrictive they will give rise to a rather small set of independent counterterms. Indeed, the only possible invariant counterterms which are compatible with the constraints (73), (74) and (77) are obtained from $\Delta_C = S_Q \Delta_C$ by choosing for $\Delta_C$ the following seven combinations of fields and antifields,

$$
\Delta_C = \int d^4x \text{tr}\left\{ z_1(A^*_\mu - 2\partial_\mu \bar{C})A^\mu + z_1(\psi^*_\mu - 2\partial_\mu \bar{\phi})\psi^\mu \\
+ z_2(\psi^*_\mu - 2\partial_\mu \bar{\phi})\partial^\mu C + z_3\chi^{\mu\nu}\partial_\mu A_\nu + z_4\chi^{\mu\nu}[A_\mu, A_\nu] \right\}
$$

where $z_1, z_2, \ldots, z_7$ are arbitrary coefficients. This set of independent counterterms is reduced further by applying the last constraints (78), leading to some relations among the renormalization factors, namely $z_5 = z_7$, $z_6 = z_7$ and $z_7 = 0$, respectively. Hence, one ends up with only four possible invariant counterterms,

$$
\Delta_C = \int d^4x \text{tr}\left\{ z_1(A^*_\mu - 2\partial_\mu \bar{C})A^\mu + z_1(\psi^*_\mu - 2\partial_\mu \bar{\phi})\psi^\mu \\
+ z_2(\psi^*_\mu - 2\partial_\mu \bar{\phi})\partial^\mu C + z_3\chi^{\mu\nu}\partial_\mu A_\nu + z_4\chi^{\mu\nu}[A_\mu, A_\nu] \right\}.
$$

(79)

Here, some remarks are in order. First, since all the quantum corrections to the action $S$ are trivial $S_Q$–cocycles there appears no physical coupling parameter in TQCD and therefore, the coefficients $z_1, z_2, z_3$ and $z_4$ are anomalous dimensions redefining the fields and antifields. Second, an unexpected feature of the non–covariant Landau type gauge adopted here is the fact that the independent counterterms of TQCD agree with those of TYM. This means, on the one hand, that the matter action of TQCD is finite, i.e., it does not receive any radiative corrections from higher loops, and, on the other hand, that the requirement of the ghost for the ghost equation is not actually necessary. Third, as already pointed out in Ref. \cite{21, 17}, as a consequence of the fact that in the Landau type gauge, $\xi = 0$, is stable under renormalization (i.e., that only in this gauge $\xi$ receives no renormalization), the counterterm

$$
\Delta_C \sim \int d^4x \text{tr}\left\{ F_{\mu\nu}^+ F^{\mu\nu+} \right\}.
$$

disappears. This result is essential in preserving the topological nature of the model. Furthermore, the absence of that counterterm guarantees that the $\beta$–function vanishes. This is in accordance with an one–loop computation carried out in Refs. \cite{2, 1}.
6. Concluding remarks

In this paper we have studied the renormalizability of twisted $N = 2$ supersymmetry with $Z = 2$ central charges. By coupling the gauge multiplet to the standard massive hypermultiplet, i.e., with only one central charge, the $R$–symmetry is broken into $Z_2$. In such a case we are faced with the situation that the ghost number of the gauge–fixed action and, in consequence of this, the cohomology classes of the counter terms and anomalies are not uniquely characterized. Here, it has been shown that this problem can be avoided by introducing two central charges $Z$ and $\bar{Z}$, being complex conjugate to each other, and formally ascribing to them, as well as to their eigenvalues $\pm m$ and $\pm \bar{m}$, the $R$–weights (ghost numbers) $R(Z) = R(m) = 2R(Q)$ and $R(\bar{Z}) = R(\bar{m}) = 2R(\bar{Q})$.

By adopting the non–covariant Landau type gauge and by making use of both the topological shift symmetry $Q$ and the vector supersymmetry $\bar{Q}$ it has been proven that the twisted hypermultiplets are not subjected to any renormalization, i.e., the matter action of TQCD is perturbatively finite. Thus, in that particular gauge, which should be as acceptable as any other gauge choice, TQCD is renormalizable with the same counter terms as TYM. Since the invariant counterterms are trivial BRST–cocycles no physical parameters appear in that model.

In this paper we have not analyzed the question whether the choice of a non–covariant gauge, which significantly differs from the original one of TYM, may eventually change the construction of the topological observables of that model.

Furthermore, no attention has been paid to a possible nontrivial $\theta$ term in the topological action. The question whether this term, without loss of generality, can be dropped at the beginning or whether the $\theta$ angle does receive radiation corrections from higher loops will be studied elsewhere.

Appendix A: Two–spinor notations in Euclidean space–time

The Weyl 2–spinor conventions in Euclidean space–time adopted in this paper are those of App. E in Ref. [35]. The matrices $(\sigma_\mu)^{AB}$ and $(\sigma_\mu)_{\dot{A}\dot{B}}$, being invariant numerical tensors of $SL(2,C)$ if $\mu$ transforms according to the vector representation of $SO(4)$, are defined by

$$(\sigma_\mu)^{AB} = (\sigma_\alpha, iI_2)^{\dot{A}\dot{B}}, \quad (\sigma_\mu)_{\dot{A}\dot{B}} = (\sigma_\alpha, -iI_2)_{\dot{A}\dot{B}},$$

with $\sigma_\alpha (\alpha = 1,2,3)$ being the Pauli matrices.

Another set of invariant tensors are the antisymmetric matrices $\epsilon_{AB}$ and $\epsilon_{\dot{A}\dot{B}}$,

$$\epsilon_{AB} = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} = \epsilon^{AB}, \quad \epsilon_{\dot{A}\dot{B}} = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix} = \epsilon^{\dot{A}\dot{B}},$$

with $\epsilon^{AB}$ and $\epsilon^{\dot{A}\dot{B}}$ being defined by $\epsilon^{AC}\epsilon_{CB} = -\delta^A_B$ and $\epsilon^{\dot{A}\dot{C}}\epsilon_{\dot{C}\dot{B}} = -\delta^{\dot{A}}_{\dot{B}}$. These tensors raise and lower the spinor indices according to $\psi_A = \psi_B \epsilon_{BA}$, $\psi^A = \epsilon^{AB}\psi_B$ and $\psi_{\dot{A}} = \psi_{\dot{B}} \epsilon_{\dot{B}\dot{A}}$, $\psi^{\dot{A}} = \epsilon^{\dot{A}\dot{B}}\psi_{\dot{B}}$, respectively.
The matrices $(\sigma_\mu)^{AB}$ and $(\sigma_\mu)_{AB}$ satisfy the Clifford algebra
\[
(\sigma_\mu)^{AC}(\sigma_\nu)_{CB} + (\sigma_\nu)^{AC}(\sigma_\mu)_{CB} = 2\delta_{\mu\nu}\delta^A_B,
\]
\[
(\sigma_\mu)^{\dot{A}C}(\sigma_\nu)_{\dot{B}C} + (\sigma_\nu)^{\dot{A}C}(\sigma_\mu)_{\dot{B}C} = 2\delta_{\mu\nu}\delta^{\dot{A}}_{\dot{B}},
\]
and, in addition, the completeness relations
\[
(\sigma_\mu)^{AB}(\sigma_\nu)^{B\dot{A}} = 2\delta_\nu^\mu,
\]
\[
(\sigma_\mu)^{AB}(\sigma_\nu)^{C\dot{D}} = 2\varepsilon_{\dot{A}\dot{C}}\varepsilon_{BD},
\]
\[
(\sigma_\mu)^{\dot{A}B}(\sigma_\nu)^{C\dot{D}} = 2\varepsilon^{\dot{A}\dot{C}}\varepsilon_{BD},
\]
\[
(\sigma_\mu)^{AB}(\sigma_\nu)^{C\dot{D}} = 2\varepsilon^{AC}\varepsilon^{B\dot{D}}.
\]
Since $(\sigma_\mu)^{AB}$ is the hermitean conjugate of $(\sigma_\mu)_{AB}$, it holds $(\sigma_\mu)^{AB} \equiv \varepsilon^{AC}\varepsilon^{B_D}(\sigma_\mu)_{DC} = (\sigma_\mu)^{\dot{B}A}$ and, lowering its indices, $(\sigma_\mu)^{AB} = (\sigma_\mu)_{BA}$. Hence, $(\sigma_\mu)^{AB}$ and $(\sigma_\mu)_{AB}$ are symmetric in their spinor indices.

The self–dual and antiself–dual $SO(4)$ generators $(\sigma_\mu)^{AB}$ and $(\sigma_\mu)_{AB}$, respectively, being antisymmetric in their vector indices and symmetric in their spinor indices, are defined by
\[
(\sigma_\mu)^{AB} = (\sigma_\mu)^{AC}(\sigma_\nu)_{CB}^B + \delta_{\mu\nu}\varepsilon^{AB}, \quad (\sigma_\mu)^{AB} = + (\sigma_\mu)^{AB}
\]
\[
(\sigma_\mu)^{\dot{A}B} = (\sigma_\mu)^{AC}(\sigma_\nu)_{\dot{B}C}^B - \delta_{\mu\nu}\varepsilon^{\dot{A}B}, \quad (\sigma_\mu)^{\dot{A}B} = - (\sigma_\mu)^{\dot{A}B}.
\]

A vector $V_\mu$ and an antisymmetric tensor $T_{\mu\nu}$ are represented by
\[
V_\mu = -\frac{1}{2}(\sigma_\mu)^{AB}V_{AB}, \quad V_{\dot{A}\dot{B}} = (\sigma_\mu)^{\dot{A}B}V_\mu,
\]
and
\[
T_{\mu\nu} = \frac{1}{4}(\sigma_\mu)^{AB}(\sigma_\nu)^{C\dot{D}}T_{ABCD} = T^{+\mu} + T^{-\mu}, \quad T_{ABCD} = \varepsilon_{BD}T^{+}_{AC} + \varepsilon_{AC}T^{-}_{BD},
\]
respectively, where $T^{\pm}_{\mu\nu} = \pm (\sigma_\mu)^{AB}T^{+}_{AB}$ is the (anti)self–dual part of $T_{\mu\nu}$.

Finally, some often used identities are
\[
(\sigma_\rho)^{AC}(\sigma_\mu)^{\dot{B}}_{\dot{C}} = \delta_{\rho\mu}(\sigma_\nu)^{AB} - \delta_{\rho\mu}(\sigma_\nu)^{\dot{A}\dot{B}} - \varepsilon_{\mu\nu\rho\sigma}(\sigma_\sigma)^{A\dot{B}},
\]
\[
(\sigma_\rho)^{AC}(\sigma_\mu)^{C}_{\dot{B}} = \delta_{\rho\mu}(\sigma_\nu)^{AB} - \delta_{\rho\mu}(\sigma_\nu)^{\dot{A}B} + \varepsilon_{\mu\nu\rho\sigma}(\sigma_\sigma)^{\dot{A}B},
\]
\[
(\sigma_\rho)^{\dot{A}C}(\sigma_\mu)^{B}_{\dot{C}} = 2(\delta_{\rho\mu}\varepsilon^{\nu\sigma} - \delta_{\nu\sigma}\varepsilon^{\mu\rho} - \varepsilon_{\mu\nu\rho\sigma})\varepsilon^{A\dot{B}} - \delta_{\rho\mu}(\sigma_\nu)^{AB} + \delta_{\nu\sigma}(\sigma_\mu)^{\dot{A}\dot{B}} + \delta_{\mu\sigma}(\sigma_\mu)^{\dot{A}\dot{B}},
\]
\[
(\sigma_\rho)^{AC}(\sigma_\mu)^{\dot{C}}_{\dot{B}} = 2(\delta_{\rho\mu}\varepsilon^{\nu\sigma} - \delta_{\nu\sigma}\varepsilon^{\mu\rho} + \varepsilon_{\mu\nu\rho\sigma})\varepsilon^{AB} - \delta_{\rho\mu}(\sigma_\nu)^{AB} + \delta_{\nu\sigma}(\sigma_\mu)^{\dot{A}\dot{B}} + \delta_{\mu\sigma}(\sigma_\nu)^{\dot{A}\dot{B}}.
\]
and
\[
(\sigma_\mu)^{\dot{A}B}(\sigma_\rho)^{\dot{C}}_{\dot{B}} = 2(\delta_{\rho\mu}\varepsilon^{\nu\sigma} - \delta_{\nu\sigma}\varepsilon^{\mu\rho} - \varepsilon_{\mu\nu\rho\sigma}), \quad (\sigma_\mu)^{\dot{A}B}(\sigma_\rho)^{\dot{C}}_{\dot{B}} = 8\varepsilon^{AC}\varepsilon^{B\dot{D}} - 4\varepsilon^{A\dot{B}}\varepsilon^{\dot{C}\dot{D}},
\]
\[
(\sigma_\mu)^{\dot{A}B}(\sigma_\rho)^{\dot{C}}_{\dot{B}} = 2(\delta_{\rho\mu}\varepsilon^{\nu\sigma} - \delta_{\nu\sigma}\varepsilon^{\mu\rho} + \varepsilon_{\mu\nu\rho\sigma}), \quad (\sigma_\mu)^{\dot{A}B}(\sigma_\rho)^{\dot{C}}_{\dot{B}} = 8\varepsilon_{AC}\varepsilon_{BD} - 4\varepsilon_{AB}\varepsilon_{CD};
\]
here, the antisymmetric tensor $\epsilon_{\mu\nu\rho\sigma}$ is normalized according to $\epsilon_{1234} = 1$.  

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APPENDIX B: Twisting of \( N = 2 \) supersymmetric theories
with two central charges \( Z \) and \( \bar{Z} \)

In this Appendix we use the method of dimensional reduction in order to include central charges in the superalgebra of \( N = 2 \) SYM coupled to two (massive) hypermultiplets (in the fundamental and its conjugate representation of \( SL(2, C) \)).

As is well known there exists a close relationship between extended \( N = 2 \) SYM in \( D = 4 \) dimensions and simple \( N = 1 \) SYM with gauge multiplet \( (A_M, \lambda), M = 1, \cdots , 6, \) in \( D = 6 \) dimensions. The transition from the latter to the former is achieved by a trivial dimensional reduction, namely by demanding that the gauge potential \( A_M \) and the chiral spinors \( \lambda, \bar{\lambda} \) is independent of the extra dimensions \( x^5 \) and \( x^6 \). After that dimensional reduction the extra components of \( A_M \) simply become complex scalar fields, \( A_5 = -i(\phi + \bar{\phi}) \) and \( A_6 = \bar{\phi} - \phi \), and the rotation group in \( D = 6 \) dimensional Euclidean space–time is broken down according to \( SO(6) \supset SO(4) \otimes SO(2) \). After reduction the chiral fields transform under the spinor representation of the universal covering group \( SL(2, C) \otimes U_R(1) \). (Since \( \lambda \) and \( \bar{\lambda} \) are not subjected to a symplectic reality condition the \( Sp(2) \) internal symmetry of the \( N = 2 \) SYM is not accounted for in Ref. [24]. This problem can be circumvented by reformulating \( N = 1 \) SYM in such manner that the \( Sp(2) \) symmetry will be manifest (see, below.).)

There exist also non–trivial dimensional reductions which allow to generate also central charges in both the massive matter multiplets [36] and the massive ghost excitations [37]. The central charge \( Z \) in the standard massive hypermultiplet [26] occurs by only compactifying the sixth dimension \( x^6 \) into a circle [27] and reducing the fifth dimension trivially. In order to get two central charges \( Z \) and \( \bar{Z} \), being complex conjugate to each other, one has to compactify the extra dimensions \( x^5 \) and \( x^6 \) into a torus and to assume that the complex scalar fields \( \zeta, \bar{\zeta} \) and the anti-chiral Dirac spinor \( \psi \) of which the (originally massless) hypermultiplet consists are periodic in \( x^5 \) and \( x^6 \) with the (inverse) periods \( m_5 = m + \bar{m} \) and \( m_6 = i(\bar{m} - m) \), respectively [27]. Thereby, the central charges are identified with the extra components of the generator of space–time translations according to \( P_5 = Z + \bar{Z} \) and \( P_6 = i(\bar{Z} - Z) \), respectively.

In order to implement the central charges \( Z \) and \( \bar{Z} \) into the matter multiplet let us start from \( N = 1 \) SYM coupled to a (massless) hypermultiplet in \( D = 6 \) dimensional Euclidean space–time in Wess-Zumino gauge:

\[
W^{(D=6)} = W^{(N=1)}_{\text{SYM}} + W^{(Z=0)}_M,
\]

which is built from an antihermitean vector field \( A_M (M = 1, \cdots , 6) \) and a \( Sp(2) \)–doublet of chiral (symplectic) Majorana spinors [38] \( \lambda_a \ (a = 1, 2) \) in the adjoint representation of the gauge group,

\[
W^{(N=1)}_{\text{SYM}} = \int d^6x \text{tr} \left\{ \frac{1}{4} F^{MN} F_{MN} - \frac{1}{2} i \bar{\lambda}^a \Gamma^M D_M \lambda_a \right\},
\]

and from a \( Sp(2) \)–doublet of complex scalar fields \( \zeta_a, \bar{\zeta}^a \equiv \zeta_a^\dagger \), and an anti-chiral Dirac spinor \( \psi \) in some representation \( \mathcal{R} \) (with generators \( T^i \)), e.g., the fundamental representation, of the gauge group,

\[
W^{(Z=0)}_M = \int d^6x \left\{ i \bar{\psi} \Gamma^M D_M \psi - i \bar{\psi} \lambda_a \zeta^a + i \bar{\zeta}^a \bar{\lambda}^a \psi - \frac{1}{2} (\bar{\zeta}_a D^2_M \zeta^a) - \frac{1}{4} (\zeta_a T^i \zeta^b)(\bar{\zeta}_b T^i \zeta^a) \right\}.
\]
with
\[ D_M = \partial_M + A^i_M T^i. \]

Here, the 8–dimensional Dirac matrices \( \Gamma_M \) and \( \Gamma_7 \) are represented as follows:
\[
\Gamma_\mu = \gamma_\mu \otimes I_2, \quad \gamma_\mu = \begin{pmatrix} 0_2 & -(\sigma_\mu)^{AB} \\ (\sigma_\mu)^{BA} & 0_2 \end{pmatrix}, \quad \mu = 1, 2, 3, 4,
\]
\[
\Gamma_{4+\alpha} = \gamma_5 \otimes \sigma_\alpha, \quad \gamma_5 = -i \begin{pmatrix} \delta^A_B & 0_2 \\ 0_2 & -\delta_A^B \end{pmatrix}, \quad \alpha = 1, 2, 3,
\]
where \( \gamma_\mu \) and \( \gamma_5 \) are the (usual) 4–dimensional Dirac matrices and \( \sigma_\alpha \) are the Pauli matrices. They obey the relations
\[
\{\Gamma_M, \Gamma_N\} = -2\delta_{MN}I_8, \quad \Sigma_{MN} = -\frac{1}{2}[\Gamma_M, \Gamma_N],
\]
with
\[
\Sigma_{\mu\nu} = \sigma_{\mu\nu} \otimes I_2, \quad \sigma_{\mu\nu} = \begin{pmatrix} (\sigma_{\mu\nu})^A_B & 0_2 \\ 0_2 & (\sigma_{\mu\nu})^{BA} \end{pmatrix}, \quad \mu,\nu = 1, 2, 3, 4,
\]
\[
\Sigma_{\mu4+\alpha} = -\gamma_5 \gamma_{4+\alpha} \otimes \sigma_\alpha, \quad \Sigma_{4+\alpha,4+\beta} = I_4 \otimes \frac{1}{2}[\sigma_\alpha,\sigma_\beta], \quad \alpha,\beta = 1, 2, 3.
\]

In order to ensure that the action (B.1) is manifestly invariant under the internal symmetry group \( Sp(2,R) \cong SU(2) \) the chiral 8–spinors \( \lambda_\alpha \) and \( \bar{\lambda}^a \) are required to obey both the Weyl condition \( \lambda_\alpha = i \Gamma_7 \lambda_\alpha \) (chirality condition) and the \( Sp(2) \) covariant Majorana condition \( \lambda_\alpha = -CT_5 \bar{\lambda}_a^T \) (symplectic reality condition) [38], where \( C \) is the charge conjugation matrix. These conditions on the 8–spinors \( \lambda_\alpha \) and \( \bar{\lambda}^a \) restrict them to be of the form
\[
\lambda_\alpha = \begin{pmatrix} i\lambda^A_a \\ 0 \\ 0 \\ \bar{\lambda}_{\dot{a}} \end{pmatrix}, \quad \bar{\lambda}^a = (0, -i\bar{\lambda}^A_a, \lambda_A^a, 0), \quad C = \begin{pmatrix} \epsilon^{AB} & 0_2 \\ 0_2 & \epsilon_{AB} \end{pmatrix} \otimes I_2,
\]
with the chiral and anti-chiral 2–spinors \( \lambda_A^a \) and \( \bar{\lambda}_{\dot{a}} \) (\( \lambda_{\dot{a}}^A \))\(^\dagger \), respectively. The \( Sp(2) \) index \( a \) is raised and lowered as follows, \( \lambda_{\dot{a}}^A = \lambda_{\dot{a}b}^A \epsilon_{ba} \) and \( \lambda^A_a = \epsilon^{ab} \lambda_{ab} \) where \( \epsilon_{ab} \) is the invariant tensor of \( Sp(2) \), \( \epsilon^{ac}\epsilon_{cb} = -\delta^a_b \) (analogously for \( \bar{\lambda}_{\dot{a}a} \)).

The anti-chiral 8–spinors \( \psi \) with \( \psi = -i\Gamma_7 \bar{\psi} \) and \( \bar{\psi} = -i\bar{\psi} \Gamma_4 \) are of the form
\[
\begin{pmatrix} 0 \\ i\bar{\beta}_{\dot{a}}^A \\ \alpha^A \\ 0 \end{pmatrix}, \quad \bar{\psi} = (-i\bar{\psi}_{\dot{a}}, 0, 0, \bar{\alpha}^A),
\]
where the Weyl 2–spinors \( \alpha^A \), \( \bar{\beta}_{\dot{a}}^A \) and \( \bar{\alpha}^A \), \( \beta_{\dot{a}} \) transform according to the fundamental and its hermitean conjugate representation of \( SL(2,C) \), respectively.

The action (B.1) is invariant under the gauge transformations \( \delta_G(\omega) \) with \( \omega \equiv \omega^i T^i \),
\[
\delta_G(\omega) A_\mu = -D_\mu \omega, \quad \delta_G(\omega) \lambda_\alpha = [\omega, \lambda_\alpha], \quad \delta_G(\omega) \bar{\lambda}^a = [\omega, \bar{\lambda}^a], \quad \delta_G(\omega) \psi = \omega \psi, \quad \delta_G(\omega) \bar{\psi} = -\bar{\psi} \omega, \quad \delta_G(\omega) \zeta^a = \omega \zeta^a, \quad \delta_G(\omega) \bar{\zeta}_a = -\bar{\zeta}_a \omega,
\]
\[ (B.2) \]
and the following rigid on–shell supersymmetry transformations \( \delta Q = \bar{\rho}^a \bar{Q}_A a - \rho^a A a \) with the constant chiral symplectic Majorana spinors \( \rho_a \) and \( \bar{\rho}^a \),

\[
\begin{align*}
\delta Q A_{\mu} &= \frac{1}{2} \bar{\rho}^a \Gamma^\mu \lambda_a - \frac{1}{2} \lambda^a \Gamma_\mu \rho_a, \\
\delta_Q \lambda_a &= - \frac{1}{2} i \Gamma^{\mu\nu} F_{\mu
u} \rho_a + i T^i (\bar{\zeta}_a T^i \zeta_b) \rho_b, \\
\delta_Q \zeta^a &= 2 \bar{\rho}^a \psi, \\
\delta_Q \psi &= i \Gamma^\mu D_\mu \zeta^a \rho_a, \\
\delta_Q \bar{\psi} &= = i \bar{\rho}^a \bar{\zeta}_a D_\mu \Gamma_\mu.
\end{align*}
\]

The corresponding 8–component spinorial supercharges

\[
Q^a = \begin{pmatrix} i Q^{A a} \\ 0_2 \\ 0_2 \\ \bar{Q}^A_a \end{pmatrix}, \quad \bar{Q}_a = (0, -i \bar{Q}_A, Q_{A a}, 0),
\]

obey, together with the generators \( P_M \ (M = 1, \ldots , 6) \) of space–time translations, the \( N = 2 \) supersymmetry algebra,

\[
Q^a \otimes \bar{Q}_b + \bar{Q}_b \otimes Q^a = - \delta_b^a (I_8 + i \Gamma_7) \Gamma^M (P_M + i \delta_G (A_M)),
\]

where the symbol \( \doteq \) means that the algebra is satisfied only on–shell, i.e., by taking into account the equations of motion. This algebra can also be closed off–shell at the expense of introducing two sets of auxiliary fields, namely the \( Sp(2) \)–vector field \( \chi_{ab} = \chi_{ba} \) and the two conjugate \( Sp(2) \) 2–spinor fields \( \chi_a \) and \( \bar{\chi}^a \equiv \chi_a^\dagger \).

Let us now compactify the fifth and sixth dimension by a non–trivial dimensional reduction, demanding that \( A_M \) and \( \lambda_a \) are independent on \( x^5 \) and \( x^6 \) whereas \( \zeta_a \) and \( \psi \) are periodic in \( x^5 \) and \( x^6 \) with the (inverse) periods \( m_5 \) and \( m_6 \), respectively,

\[
\partial_{5,6} A_M = 0, \quad \partial_{5,6} \lambda_a = 0, \quad \partial_{5,6} \zeta^a = im_{5,6} \zeta^a, \quad \partial_{5,6} \psi = im_{5,6} \psi.
\]

(Here it has been assumed that the higher modes are not stimulated.) We further define

\[
A_5 = -i (\phi + \bar{\phi}), \quad A_6 = \bar{\phi} - \phi, \quad m_5 = m + \bar{m}, \quad m_6 = i (\bar{m} - m),
\]

where the independence of the gauge transformations upon \( x_5 \) and \( x_6 \) have made \( A_5 \) and \( A_6 \) into a complex scalar field \( \phi, \bar{\phi} \equiv \phi^\dagger \).

After this procedure the dimensional reduced action in four dimensions becomes

\[
W^{(D=4)} = W^{(N=2)}_{\text{SYM}} + W^{(Z=2)}_M,
\]

with

\[
W^{(N=2)}_{\text{SYM}} = \int d^4 x \left\{ \frac{1}{4} F_{\mu\nu} F^{\mu\nu} - 2 (D^\mu \bar{\phi})(D_\mu \phi) - 2 [\bar{\phi}, \phi] [\bar{\phi}, \phi] - i \bar{\chi}^a (\sigma^\mu) \bar{A}^B D_\mu \lambda_{B a} + \chi^a [\phi, \lambda_{A a}] + \bar{\chi}^a [\bar{\phi}, \bar{\lambda}^a] \right\}
\]

and

\[
W^{(Z=2)}_M = \int d^4 x \left\{ i \bar{\alpha} (\sigma^\mu) \gamma^{AB} \bar{A}_B \alpha^A - 2 \bar{\alpha} (\phi + m) \beta_A - i (\bar{\alpha} \lambda_{A a} + \beta_A \lambda^A a) \zeta^a \\
+ i \beta_A D_\mu (\sigma^\mu) \gamma^{AB} \beta_B - 2 \bar{\beta}_A (\bar{\phi} + \bar{m}) \alpha^A + i \bar{\lambda}_a (\lambda^A a \alpha^A + \bar{\lambda}^A a \bar{\beta}^a) \\
+ \frac{1}{2} (D^\mu \zeta_a) (D_\mu \zeta^a) + \zeta_a [\phi + m, \bar{\phi} + \bar{m}] \zeta^a - \frac{1}{4} (\zeta_a T^i \zeta^b) (\bar{\zeta}_b T^i \zeta^a) \right\}
\]
which is obviously invariant under the internal symmetry group \( Sp(2) \otimes U_R(1) \) if we (formally) ascribe to \( m \) and \( \bar{m} \) the same \( R \)-charges as to \( \phi \) and \( \bar{\phi} \). The corresponding on-shell supersymmetry transformations in the presence of the central charges are

\[
\delta Q A_\mu = \rho^{Aa} (\sigma_\mu)_{AB} A^B = \bar{\lambda}^{Ba} (\sigma_\mu)_{AB} \rho_a^A, \quad \delta Q \phi = i \rho^{Aa} \bar{\lambda}_a, \quad \delta Q \bar{\phi} = -i \lambda_a^A \rho_a^A,
\]

\[
\delta Q \lambda_a^A = -\frac{1}{2} i (\sigma^{\mu\nu})_{AB} \rho_B F_{\mu\nu} + 2i [\bar{\phi}, \phi] \rho_a^A + i T^i (\bar{\zeta}_a T^i \zeta^a) \rho_{AB} + 2 \bar{\rho} B (\sigma_\mu)_{AB} D_{\mu} \bar{\phi},
\]

\[
\delta Q \bar{\lambda}_a = \frac{1}{2} i \bar{\rho} B (\sigma^{\mu\nu})_{AB} F_{\mu\nu} - 2i \bar{\rho} A [\bar{\phi}, \phi] - i \bar{\rho} B (\bar{\zeta}_a T^i \zeta^b) T^i + 2 (\sigma_\mu)_{AB} \rho_B D_{\mu} \phi,
\]

and

\[
\delta Q \zeta^a = 2 \bar{\rho} A \bar{\beta} \zeta^A + 2 \rho_a^A \alpha^A, \quad \delta Q \alpha^A = -i \bar{\rho} B_a (\sigma_\mu)^{AB} \bar{D}_\mu \zeta^a + 2 \rho_a^A (\phi + m) \zeta^a, \quad \delta Q \bar{\beta} \bar{A} = i \rho^a B (\sigma_\mu)^{AB} \bar{D}_\mu \zeta^a + 2 \bar{\rho} \bar{A} (\bar{\phi} + \bar{m}) \zeta^a, \quad \delta Q \bar{\zeta}_a = 2 \beta_a \rho_A^a + 2 \bar{\alpha} A \bar{\beta} \bar{A}, \quad \delta Q \beta_A = -i \bar{\bar{C}}_a D_{\mu} (\sigma_\mu)^{AB} \bar{\rho}_B + 2 \bar{\zeta}_a (\phi + m) \rho_a^A, \quad \delta Q \bar{\alpha} \bar{A} = i \bar{\bar{C}}_a \bar{D}_\mu (\sigma_\mu)^{AB} \rho_B^a + 2 \bar{\bar{C}}_a (\bar{\phi} + \bar{m}) \bar{\rho} A^a.
\]

Identifying the central charges with certain combinations of the space–time translations on the torus, namely \( P_5 = Z + Z \) and \( P_3 = i(Z - Z) \), and reverting to a two–spinor notation the supersymmetry algebra (B.3) can be recast into the form

\[
\{ Q_a^A, Q_b^B \} = -4 \epsilon^{ab} \epsilon_{AB} (Z + \delta G (\phi)),
\]

\[
\{ Q_a^A, \bar{Q}_{\bar{b}}^{\bar{B}} \} = -2 \delta_{\bar{b}}^\bar{a} \epsilon_{AB} (P_\mu + i \delta G (A_\mu)),
\]

\[
\{ \bar{Q}_{\bar{a}}, Q_b^B \} = 4 \epsilon_{ab} \epsilon_{AB} (\bar{Z} + \delta G (\bar{\phi})),
\]

where the central charge transformations are given by

\[
ZV = 0, \quad \bar{Z}V = 0
\]

for the on-shell gauge multiplet \( V = \{ A_\mu, \lambda_a^A, \bar{\lambda}_{\bar{A}a}, \phi, \bar{\phi} \} \) and

\[
ZY = mY, \quad Z \bar{Y} = -m \bar{Y}, \quad \bar{Z} \bar{Y} = -\bar{m} \bar{Y}
\]

for the (massive) on-shell hypermultiplets \( Y = \{ \alpha^A, \bar{\beta} \bar{A}, \zeta^a \} \) and \( \bar{Y} = \{ \bar{\alpha} \bar{A}, \beta_a A, \bar{\zeta}_a \} \) being hermitean conjugate to each other.

In order to derive the twisted actions (11) and (14) we identify in (B.4) and (B.5) the internal index \( a \) with the spinor index \( A \). It is precisely that identification which defines the twisting procedure [2]. In addition, we introduce another set of auxiliary fields, \( \chi_{AB} = \chi_{BA} \) and \( \lambda A, \bar{\lambda} \bar{A} \), in order to get an off–shell realization of the twisted \( N = 2 \) superalgebra. This gives

\[
W_{TSYM} = \int d^4x \text{tr} \left\{ \frac{1}{4} F^{\mu\nu} F_{\mu\nu} - 2 (D^\mu \bar{\phi}) (D_\mu \phi) - 2 [\bar{\phi}, \phi] [\bar{\phi}, \phi] + \chi_{AB} \chi_{AB} \right\}.
\]
\[
W_M = \int d^4x \left\{ i\tilde{\alpha}^\dagger (\sigma^\mu)_{AB} \tilde{D}_\mu \alpha^B - 2\tilde{\alpha}^\dagger (\phi + m) \tilde{\beta}^A - i(\tilde{\alpha}^\dagger \tilde{\lambda}_{AB} + i\beta_A \lambda^A_B) \zeta^B \\
+ i\beta_A \tilde{D}_\mu (\sigma^\mu)^{AB} \tilde{\beta}^B - 2\beta_A (\tilde{\phi} + \tilde{m}) \alpha^A + i\tilde{\zeta}_B (\lambda_A^B \alpha^A + \tilde{\lambda}^{AB} \tilde{\beta}_A) \\
+ \frac{1}{2} D^\mu \tilde{\zeta}_A D_{\mu} \zeta^B + \tilde{\zeta}_A (\phi + m, \tilde{\phi} + \tilde{m}) \zeta^A + \tilde{\zeta}_A \lambda^A_B \zeta^B - 2\chi^A \chi_A \right\},
\]

(B.10)

with the off–shell hypermultiplets \( Y_T = \{ \alpha^A, \tilde{\beta}_A, \zeta^A; \tilde{\chi}_A \} \) and \( \tilde{Y}_T = \{ \tilde{\alpha}^A, \beta_A, \tilde{\zeta}_A; \chi^A \} \).

Now we are able to construct the complete set of \( N = 2 \) twisted generators,

\[
Q = \frac{1}{2} \epsilon^{AB} Q_{AB}, \quad \tilde{Q}_\mu = \frac{1}{2} (\sigma^\mu)^{AB} \tilde{Q}_{AB}, \quad Q_{\mu\nu} = \frac{1}{2} (\sigma_{\mu\nu})^{AB} Q_{AB},
\]

being, respectively, a scalar \( Q \), a vector \( \tilde{Q}_\mu \) and a self–dual tensor \( Q_{\mu\nu} \), by substituting in (B.6) and (B.7) for \( \rho^{Aa} \) and \( \tilde{\rho}^{Aa} \) the following expressions,

\[
\rho^{AB} = \frac{1}{2} \rho \epsilon^{AB}, \quad \tilde{\rho}^{AB} = \frac{1}{2} i \rho^\mu (\sigma^\mu)^{AB}, \quad \rho^{AB} = \frac{1}{2} \rho^{\mu\nu} (\sigma_{\mu\nu})^{AB},
\]

where \( \rho, \rho^\mu \) and \( \rho^{\mu\nu} \) are some new global symmetry parameters associated to \( Q, \tilde{Q}_\mu \) and \( Q_{\mu\nu} \), respectively. Then, again disregarding the generator \( Q_{\mu\nu} \), the twisted actions (B.9) and (B.10) will be separately invariant under the following twisted supersymmetry transformations \( \delta_T = \rho Q + \rho^\mu \tilde{Q}_\mu \),

\[
\delta_T A_{\mu} = \tilde{\rho}^{AC} (\sigma^\mu)_{AB} \chi^C_B - \tilde{\lambda}^{BC} (\sigma^\mu)_{AB} \rho^{A}_C,
\]

\[
\delta_T \phi = i\rho^{AB} \tilde{\lambda}_AB, \quad \delta_T \tilde{\phi} = -i\lambda^B_A \rho^{A}_B,
\]

\[
\delta_T \lambda_{AC} = -\frac{1}{2} i (\sigma^{\mu})_{AB} \rho^\mu_B F_{\mu\nu} + 2i(\tilde{\phi}, \phi) \rho_{AC} - 2i\chi^A_B \rho_{AB} + 2\tilde{\rho}^B_C (\sigma^\mu)_{AB} D_{\mu} \tilde{\phi},
\]

\[
\delta_T \tilde{\lambda}^A_C = \frac{1}{2} i \rho^{BC} (\sigma^{\mu})_{AB} \tilde{F}_{\mu\nu} - 2i(\tilde{\phi}, \phi) \rho_{AC} + 2i\rho_{A}^B \chi^C_B + 2(\sigma^\mu)_{AB} \rho^C_B D_{\mu} \phi,
\]

\[
\delta_T \chi_{AB} = -\frac{1}{2} D_{\mu} \tilde{\lambda}^C_{A} (\sigma^\mu)^{CD} \rho_{DB} + i(\phi, \lambda^A_B) \rho_{CB}
\]

\[-\frac{1}{2} \tilde{\rho}^C_A (\sigma^\mu)^{CD} D_{\mu} \lambda_{DB} + i \tilde{\rho}^{CA} [\tilde{\lambda}^C_B, \tilde{\phi}] + (A \leftrightarrow B)
\]

and

\[
\delta_T \zeta^B = 2\tilde{\rho}^{AB} \tilde{\beta}_A + 2\rho^B_A A^A,
\]

\[
\delta_T A^A = -i\tilde{\rho}^B_{BC} (\sigma^\mu)^{AB} D_{\mu} \chi^C_B + 2\rho^A_B (\phi + m) \zeta^B - 2\tilde{\rho}^{BA} \chi_B,
\]

\[
\delta_T \tilde{\beta}_A = i\rho^B_C (\sigma^\mu)_{AB} \tilde{D}_{\mu} \chi^C + 2\tilde{\rho}^{AC}_B (\tilde{\phi} + \tilde{m}) \zeta^B - \rho^B_C \tilde{\chi}_A,
\]

\[
\delta_T \chi^A = -\frac{1}{2} i \beta_B D_{\mu} (\sigma^\mu)^{AB} \rho^C_B + \tilde{\alpha}^C (\phi + m) \rho^C_B - \frac{1}{2} i \tilde{\zeta}_B \tilde{\lambda}^{AB} \rho^C_B + i\tilde{\alpha}^B \rho^C_B (\sigma^\mu)^{AB}
\]

\[-\frac{1}{2} \tilde{\rho}^C_A (\sigma^\mu)^{CD} D_{\mu} \beta_{DB} + 2\tilde{\rho}^{CD} (\phi + m) \tilde{\beta}_{AB} - i\tilde{\zeta}_B \lambda^C_B \rho^{AB},
\]

\[
\delta_T \tilde{\zeta}_B = 2\beta_B \rho^A_A + 2\tilde{\alpha}^A \tilde{\rho}^{A}_{AB},
\]

\[
\delta_T \tilde{\alpha}^A = i\tilde{\zeta}_B D_{\mu} (\sigma^\mu)^{AB} \tilde{\rho}^B_C + 2\tilde{\phi}_B (\tilde{\phi} + \tilde{m}) \tilde{\alpha}^A - \chi^A \tilde{\rho}_B,
\]

\[
\delta_T \beta_A = -i\tilde{\zeta}_B D_{\mu} (\sigma^\mu)^{AB} \tilde{\rho}^B_C + 2\tilde{\phi}_B (\tilde{\phi} + \tilde{m}) \tilde{\beta}_A + 2\tilde{\chi} B \tilde{\beta}_{BA},
\]

\[
\delta_T \chi^A = \frac{1}{2} i \rho^C_A (\sigma^\mu)^{AB} D_{\mu} \alpha^B - \rho^C_C (\phi + m) \tilde{\beta}_A - \frac{1}{2} i \rho^C_B \tilde{\lambda}^{AB} \chi^B
\]

\[-i\tilde{\rho}^{AC}_B (\sigma^\mu)^{CD} D_{\mu} \beta_{DB} - 2\tilde{\rho}^{AB} (\phi + m) \alpha^B - i\tilde{\rho}^C_B \lambda^B_C \zeta^C.
\]

(B.13)

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In order to show that the action (B.9) and (B.10) is equivalent to the topological action as given by (7) and (13) we carry out the following replacements:

\[ \chi_{AB} \rightarrow \chi_{AB} - \frac{1}{4} (\sigma^{\mu\nu})_{AB} F_{\mu\nu}, \]
\[ \chi^A \rightarrow \chi^A - \frac{1}{2} i \zeta_B D_\mu (\sigma^\mu)^A,B, \]
\[ \bar{\chi}_A \rightarrow \bar{\chi}_A + \frac{1}{2} i (\sigma^\mu)_{AB} D_\mu \zeta^B, \]

and we revert the spinor notation to the more familiar vector notation by introducing the following set of fields:

\[ \psi_\mu = -\frac{1}{2} (\sigma_\mu)_{AB} \bar{\chi}^{AB}, \]
\[ \eta = -\frac{1}{2} i \epsilon_{AB} \chi^{AB}, \]
\[ \chi_{\mu\nu} = -\frac{1}{2} i (\sigma_{\mu\nu})_{AB} \chi^{AB}, \]
\[ \lambda_{\mu\nu} = \frac{1}{2} (\sigma_{\mu\nu})_{AB} \chi^{AB}. \]

Then, one easily verifies that the resulting action is just the topological action (7), (13) for \( \xi = 1 \) with the Pontrjagin term subtracted, i.e., it is determined by the invariant polynomial \( \text{tr} \phi^2 \) through the very attractive form \([22]\),

\[ W_{\text{TSYM}} = W_T - \frac{1}{4} \int d^4x \text{tr} F_{\mu\nu} \tilde{F}_{\mu\nu} = -\frac{1}{24} \epsilon^{\mu\nu\rho\sigma} \bar{Q}_\mu \bar{Q}_\nu \bar{Q}_\rho \bar{Q}_\sigma \int d^4x \text{tr} \phi^2, \]

thus also giving a suggestive idea of the usefulness of the vector operator \( \bar{Q}_\mu \). Performing the same replacements in (B.10) we arrive at the matter action (14) for \( \xi = 1 \).

Finally, in order to establish the relationship between the twisted \( N = 2 \) SYM and TYM one identifies the \( R \)-charge with the ghost number, i.e., one goes over from a conventional QFT to a cohomological one \([1, 3]\). This is achieved by simply setting in (B.11) – (B.13) the ghost \( \rho \) associated to the singlet operator \( Q \) equal to one, i.e., \( \rho = 1 \). Thereby, the ghost numbers of the remaining fields have to be redefined. In this way one recovers the topological shift symmetry and the vector supersymmetry introduced in (8), (9), (13) and (17) for \( \xi = 1 \).

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