New mean field theory with the parity and charge mixing for the pion in nuclei

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We give a brief review on the important role of the pion-exchange interaction characterized by the strong tensor force. In order to illustrate the role of the pion, we have proposed a new mean field theory with the charge and parity mixing, by which the pion is treated explicitly in the mean field framework. We introduce the essence of the new mean field theory and illustrate the calculated results of the pion (tensor force) correlation by taking the alpha particle as an example.

1. Introduction

We propose a mean field theory to describe nuclei by treating the pion explicitly. In the non-relativistic framework, the pion-exchange interaction appears mainly as the tensor force, which is much stronger than the central force. To make the important role of the pion in nuclei clear, we briefly discuss the role of the tensor force in the reaction matrix theory, showing the density dependence of the central component of the $G$-matrix, together with some discussion on the results of the variational calculations for light nuclei based on the realistic nuclear force. The importance of the pion demonstrated through those studies, leads us to develop a theoretical framework based on the mean field theory which enables us to treat the pion-exchange interaction on the same footing as other meson-exchange interactions. A new mean field theory has been formulated by breaking the charge and parity symmetries of the single-particle orbits and further by recovering both the symmetries of the total wave function. Thus we call it the charge- and parity-projected Hartree-Fock (CPPHF) scheme, with which we study the correlation caused by the pion (or tensor force). We discuss the CPPHF scheme in the latter half of this paper.

2. Standard model for nuclei

The reaction matrix theory initiated by Brückner has been developed with the intention of understanding the nuclear structure through the multiple scattering correlation based on the two-particle scattering in nuclei starting from the realistic nuclear force. The most basic element in the theory is the reaction matrix ($G$-matrix). The introduction of the
$G$-matrix is necessitated by the singular nature of the two-body forces in the short range region, that is, the repulsive core and the strong non-central components.

![Diagram](image)

**Figure 1.** The $G$-matrix for two particles in a nucleus, which includes the effects of the hard core and the tensor force.

The elementary process of the multiple scattering is the two-nucleon scattering within a nucleus which is described by “the reaction matrix” given by $G = V + V\{Q/e\}G$ with $e = \omega - QTQ$, where $V$, $T$, $\omega$, and $Q$ express the two-body nuclear potential, the kinetic energy operator, the starting energy of the scattering two nucleons, and the Pauli exclusion operator, respectively. The reaction matrix $G$ is a sort of “effective interaction” in nuclei in contrast to the original nuclear force $V$. Because the two-body scattering correlation is known to be dominant compared with higher multiple scattering correlations, the reaction matrix $G$ can be a good basic element for investigating the nuclear structure with the standard models (the shell model and the mean field model).

### 3. Renormalization of the tensor force effect in the effective interactions

According to the definition of $G$, the repulsive core of $V$ appears simply in $G$ as a fairly weakened core. However, the tensor force causes the significant modifications in the characteristics of the “effective interaction $G$,” because the effect of the tensor force arises first as the second order effect $V_T\{Q/e\}V_T$ and produces a very large contribution to the central component of $G$. The many-body effects included in the $G$-matrix arises only through the Green function $Q/e$. These are the Pauli effect due to $Q$ and the dispersion effect due to $1/e$, which bring about various kinds of the structure dependence reflecting the characteristics of the structure assumed initially. Thus the contribution of the tensor force to the central component of $G$ is very sensitive to the nuclear structure itself and to the circumstance of the surrounding nuclear medium [1]. We discuss here the density dependence of $G$ as a typical example of the structure dependence.

We show the central component of $G$ for the triplet-even state, $G^{[3E]}_C$ with the two-body interaction $V$ in which the tensor force $V_T$ is included, and $\tilde{G}^{[3E]}_C$ with $\tilde{V} = V - V_T$ in which
Figure 2. $r^2G_C^{[3E]}$ for $k_F=1.4, 1.2, 1.0, \text{and } 0.8 \text{ fm}^{-1}$, where the tensor force $V_T$ is included. The attraction in the intermediate and long range ($r > 1 \text{ fm}$) increases with decreasing the Fermi momentum.

tensor force is not included, which are calculated by Y. Akaishi. Figure 2 clearly shows the strong $k_F$ dependence of $G_C^{[3E]}$. On the contrary, there is little difference between two cases of $G_C^{[3E]}$ for $k_F=1.4$ and 0.8 fm$^{-1}$ as shown in Fig. 3 by the solid curves. It is easily understood from Fig. 3 that the density ($k_F$) dependence of $G_C^{[3E]}$ is almost entirely caused by the tensor force and that the tensor contribution $V_T\{Q/e\}V_T$ is estimated by the difference ($G_C^{[3E]} - \tilde{G}_C^{[3E]}$). Figure 3 shows that the ratio ($G_C^{[3E]} - \tilde{G}_C^{[3E]}$)/$\tilde{G}_C^{[3E]} = 1.5 \sim 2.0$ at $r=1.0 \text{ fm}$. These features tell us that the tensor force plays the very important role on the nuclear structures.

4. Variational calculation in the real space

Recent variational calculations of light nuclei based on the realistic nuclear force in the real space are carried out by the Argonne-Illinois Group up to $A=10 \ [2]$. First of all we have been strongly impressed by their successful reproduction of the binding energy of the ground and excited states for all these nuclei. Secondly, we are surprised by their results that the contribution of the one-pion-exchange interaction $V^\pi$ amounts to 70-80% of the strength of the two-body interaction part in the $3\leq A\leq 8$ systems, and the pion is also important for the three-body interaction originating mostly from the pion exchange. Thirdly, we notice that the two-alpha cluster structure is found in $^8\text{Be}$: According to their expression [2], “these results obtained from the VMC wave functions, suggest that the $0^+, 2^+, 4^+$ wave functions for $^8\text{Be}$ have the structure of a deformed rotor consisting of two $\alpha$’s.”

With regard to the importance of the pion, we note here the pioneering work of the variational calculation of the $\alpha$ particle by ATMS [3]. Table 1 exhibits the contributions of various interactions to the binding energy (in the unit of MeV), from which we can
see almost a half of the attraction is due to the tensor force and hence the pion-exchange interaction is responsible in alpha particle.

| Table 1 |
| --- |
| The breakdown of energies for the α particle in the ATMS calculations [3]. |
| Energy | KE | PE | $^3E_C$ | $^1E_C$ | $^{1}O_C+^{3}O_C$ | $^3E_T$ | $^3O_T$ | LS+QLS | P(D) |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| −20.6 | 131.1 | −157.7 | −51.3 | −26.2 | −0.4 | −69.7 | −0.5 | −3.6 | 12.8% |

5. New mean field theory with the parity and charge mixing

In this section, we construct a new framework of the mean field model under the assumption that if we introduce a suitably wide space spanned by a set of single-particle basis, we can describe the tensor correlation with fairly high-momentum components. We also assumed that the short-range correlation caused by the repulsive core can be treated, separately from the tensor correlation, by the $G$-matrix theory with the wide model space mentioned above. We assume the effective interaction in the following form:

$$ G(r) = \tilde{G}(r) + \Omega(r)V_T(r), $$

where $\tilde{G}$ is the $G$-matrix by adopting the two-body interaction $\tilde{V}(= V - V_T)$ and $\Omega(r)$ is a correlation function acting between $S$-wave and $D$-wave states induced by the tensor force.

Let us consider that a virtual pion is emitted and absorbed by a nucleon in a good parity orbit. Then the relevant nucleon makes a jump from one single-particle state to another with the opposite parity, accompanied by a spin-flip due to the pion-nucleon coupling. Therefore, to incorporate the effect of the correlation caused by the pion-exchange interaction in the parity-conserved single-particle space, we must treat the higher configurations, like $2p-2h$ (two-particle–two-hole) states, crossing over major shells. If we want to treat the pion in the framework of a mean field theory, the parity symmetry of single-particle orbits in nuclei should be necessarily broken. Similarly, the emission and absorption of a virtual pion with charge require the change of the charge of a nucleon single-particle orbit in nuclei.

In the mean field theory with the pion, a single-particle orbit consists of the four components with different charge-parity of $[p,+], [p,-], [n,+], \text{and} [n,-]$:

$$ \psi_{\alpha_i}(i) = \psi_{\alpha_i}^{[p,+]}(i) + \psi_{\alpha_i}^{[p,-]}(i) + \psi_{\alpha_i}^{[n,+]}(i) + \psi_{\alpha_i}^{[n,-]}(i), $$

The intrinsic wave function is defined by a Slater determinant of a set $\{\psi_{\alpha_i}\}$:

$$ \psi_{\text{intr}} = \frac{1}{\sqrt{A!}} \prod_{i=1}^{A} \psi_{\alpha_i}, $$

which is a trial wave function with the parity and charge mixing. Generally it is necessary to recover the parity and charge symmetries of the total wave function since the nuclear
state has a definite charge and parity. By using the following projection operators, we can obtain the total wave function with a good charge \((Z)\) and parity \((+\) or \(-\)).

\[
\psi_{[Z,\pm]} = \hat{P}_c(Z)\hat{P}_p(\pm)\psi_{\text{intr}},
\]

\[
\hat{P}_c(Z) = \frac{1}{2\pi} \int_0^{2\pi} d\theta e^{i(\hat{Z}-Z)\theta}, \quad \hat{Z} = \sum_{i=1}^A \frac{1 + \tau_i^3}{2},
\]

\[
\hat{P}_p(\pm) = \frac{1 \pm \hat{P}}{2}, \quad \hat{P} = \prod_{i=1}^A \hat{p}_i.
\]

Here, we illustrate the characteristic of the parity-projected wave functions for the doubly-closed-shell nuclei;

\[
\hat{P}^p(+)\psi_{\text{intr}} = |[0p-0h]\rangle + |[2p-2h]\rangle + |[4p-4h]\rangle + \cdots,
\]

\[
\hat{P}^p(-)\psi_{\text{intr}} = |[1p-1h]\rangle + |[3p-3h]\rangle + \cdots.
\]

The positive-parity state consists of even number of 1p–1h pairs with \(0^-\). This means that the positive-parity projection provides 2p–2h states as the major correction terms. Hence, the correlated wave function after the parity projection provides the 2p–2h admixture due to the pion-exchange interaction. This admixture corresponds to the \(D\)-state admixture in the \(\alpha\) particle. The \(D\)-state probability in the \(\alpha\) particle is known to be around 10–15%.

The negative-parity state has odd number of 1p–1h pairs with \(0^-\) spin-parity.

The first attempt of the study with the new mean field theory with the pion was made in the framework of the relativistic mean field theory by solving the equation obtained from the variation \(\delta\langle\psi_{\text{intr}}|H|\psi_{\text{intr}}\rangle = 0\), where the Hamiltonian \(H\) is obtained from the Lagrangian including the pion terms \([4]\). We obtain the solution of a parity-mixed self-consistent Hartree field with the finite pion mean field \((\langle\pi\rangle \neq 0)\) by taking the free-space pion-nucleon coupling constant \((g_\pi^0)\).

In Fig. 4 the calculated pion energy per nucleon for various nuclei with the closed-shell configuration is plotted as a function of the pion-nucleon coupling constant \((g_\pi^0)\). The definition of the critical coupling constant \(g_\pi^{cr}\) is one at which there arises a finite pion mean field for each nucleus. The values of \(g_\pi^{cr}\) for various nuclei except for \(^{12}\text{C}\) are distributed in the region satisfying

\[
0.92g_\pi^0 \leq g_\pi^{cr} \leq 0.97g_\pi^0.
\]
This fact indicates that “the parity breaking mean field is fragile” in the sense that various effects not taken into account in the present study could influence the realization of the finite pion mean field.

Second attempt of our study in the new mean field theory is the charge- and parity-projected Hartree-Fock study with the tensor force of light nuclei. An essential improvement made in this study is that we adopt the charge- and parity-projected wave function giving in Eq. (4) as a variational wave function and then we intend to solve the finite mean field based on the variation after the charge and parity projections:

\[
\delta \frac{\langle \Psi[Z,±] | H | \Psi[Z,±] \rangle}{\langle \Psi[Z,±] | \Psi[Z,±] \rangle} = 0. \tag{10}
\]

The variation with respect to each single-particle orbit leads to the charge- and parity-projected Hartree-Fock (CPPHF) equation. When we compare the CPPHF equation with the ordinary Hartree-Fock (HF) equation, the CPPHF equation has a much more complicated form but its structure remains similar to the HF equation.

We solve the CPPHF equation for the \(\alpha\) particle, which is the most simple doubly-closed-shell nucleus. Here, the calculated results for the ground \((0^+)\) state are exhibited in Table 2 for the purpose to examine the responsibility of the CPPHF scheme to describe the tensor correlation. We compare them with the results for the cases of the Hartree-Fock (HF) scheme with the non-projected variational wave function and for the case of the parity-projected Hartree-Fock (PPHF) scheme with the parity-projected variational wave function. Although we make the tensor force 1.5 times stronger than the usual one, we cannot obtain the correlation energy due to the tensor force by the HF scheme but a fairly large correlation energy is obtained by the PPHF scheme. Finally the CPPHF scheme produces a large amount of the correlation energy, which is almost three times larger than that for the PPHF case. (See the detail results and discussions in Ref. 5.)
Table 2
Results for the ground \((0^+\rangle\) state of the alpha particle in various cases. The potential energy \((\langle \hat{v}\rangle \text{ in MeV})\), the kinetic energy \((\langle \hat{T}\rangle \text{ in MeV})\), the total energy \((E \text{ in MeV})\), the root-mean-square matter radius \((R_m \text{ in fm})\) and the probability of the \(p\)-state component \((P(-))\) are given. \(\langle \hat{v}_C\rangle\), \(\langle \hat{v}_T\rangle\), \(\langle \hat{v}_{LS}\rangle\), and \(\langle \hat{v}_{\text{Coul}}\rangle\) are the expectation values for the central, the tensor, the LS, and the Coulomb potentials, respectively (in MeV).

| HF   | PPHF  | CPPHF |
|------|-------|-------|
| \(\langle \hat{v}_C\rangle\) | -56.85 | -61.31 | -64.75 |
| \(\langle \hat{v}_T\rangle\)  | 0.00  | -10.91 | -30.59 |
| \(\langle \hat{v}_{LS}\rangle\) | 0.00  | 0.67   | 1.91  |
| \(\langle \hat{v}_{\text{Coul}}\rangle\) | 0.76  | 0.78   | 0.85  |
| \(\langle \hat{v}\rangle\)    | -56.10 | -70.76 | -92.58 |
| \(\langle \hat{T}\rangle\)    | 39.98 | 49.67  | 64.39  |
| \(E\)                         | -16.12 | -21.09 | -28.19 |
| \(R_m\)                      | 1.63  | 1.50   | 1.37   |
| \(P(-)\)                     | 0.00  | 0.08   | 0.16   |

6. Conclusion

The studies in the model space based on the \(G\)-matrix theory and in the realistic functional space based on the realistic nuclear force, exhibit clearly the importance of the tensor force (the one-pion-exchange force). A new framework is introduced to treat the pionic correlation in the extended model space by introducing the single-particle orbits with the parity and charge mixing and by recovering the parity and charge symmetries by the projection. We consider that the new theoretical framework will provide a powerful method to investigate the mechanism of forming diverse structures of nuclei (shell structure, cluster structure, halo structure and others), which are explored by the experiments.

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