On the Complexity Reduction of Uplink Sparse Code Multiple Access for Spatial Modulation

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Abstract—Multi-user spatial modulation (SM) assisted by sparse code multiple access (SCMA) has been recently proposed to provide uplink high spectral efficiency transmission. The message passing algorithm (MPA) is employed to detect the transmitted signals, which suffers from high complexity. This paper proposes three low-complexity algorithms for the first time to the SM-SCMA. The first algorithm is referred to as successive user detection (SUD), while the second algorithm is the modified version of SUD, namely modified SUD (MSUD). Then, for the first time, the tree-search of the SM-SCMA is constructed. Based on that tree-search, another variant of the sphere decoder (SD) is proposed for the SM-SCMA, referred to as fixed-complexity SD (FCSD). SUD provides a benchmark for decoding complexity at the expense of bit-error-rate (BER) performance. Further, MSUD slightly increases the complexity of SUD with a significant improvement in BER performance. Finally, FCSD provides a near-optimum BER with a considerable reduction of the complexity compared to the MPA decoder and also supports parallel hardware implementation. The proposed algorithms provide flexible design choices for practical implementation based on system design demands. The complexity analysis and Monte-Carlo simulations of the BER are provided for the proposed algorithms.

Index Terms—Sparse code multiple access (SCMA), spatial modulation (SM), message passing algorithm (MPA), low-complexity algorithms, complexity analysis.

I. INTRODUCTION

Non-Orthogonal multiple access (NOMA) has been recognized as a promising technique for future wireless networks, and has received considerable attention in recent years [1]-[2]. NOMA is composed of two types: power-domain and code-domain. The power and code orthogonality constraints are relaxed for multiple-user access to improve the spectral efficiency and increase the number of served users for power-domain and code-domain NOMA, respectively [3]-[5]. In this paper, sparse code multiple access (SCMA) code-domain NOMA is considered, which was firstly proposed in [6]. In the SCMA scheme, a unique multidimensional codebook is assigned to each user to share the medium with the other users. The SCMA codebooks are sparse (i.e., contain zeros) and carefully designed to provide a good performance [7]-[9]. The sparsity property of the SCMA codebooks makes it feasible to employ the iterative message passing algorithm (MPA) to provide near maximum-likelihood (ML) bit-error-rate (BER) performance at low-complexity detection [10]. The complexity of the MPA is still high for practical implementations; several algorithms have been proposed to tackle this problem [11]-[15]. In [11], the authors proposed a reduced-complexity version of the MPA decoder, whereas the authors in [12]-[15] adopted the concept of the sphere decoder (SD) to reduce the complexity of the SCMA signal detection.

On the other hand, spatial modulation (SM) is a promising technology for single-user communications, which overcomes the inter-channel-interference problem present in multiple-input multiple-output (MIMO) schemes and uses a single radio frequency (RF) chain [16]-[19]. Since most of the existing systems already contain multiple antennas at both transmitter and receiver, the SM system becomes a promising candidate for those applications that cannot afford the aforementioned MIMO drawbacks [19]-[21]. The SM system employs the index of the active antenna to deliver additional information supplementary to the modulated quadrature amplitude modulation (QAM)/phase-shift-keying (PSK) symbol that can be transmitted from that active antenna [20]. At the receiver side, the ML jointly detects the active transmit antenna as well as the transmitted QAM/PSK symbol by implementing an exhaustive search that leads to high decoding complexity. The algorithms in [22]-[23] have been proposed based on the SD and tree-search concepts to significantly reduce the decoding complexity of the SM system while retaining the same BER performance of the ML decoder.

Since the single-user SM has recently been an attractive area of research, it is vital to investigate the multi-user SM scenario using one of the promising multiple access techniques, such as SCMA. Recently, the multi-user SM has been assisted by SCMA (SM-SCMA) to provide a high spectral efficiency transmission for uplink scenario [24]-[31]. The SM-SCMA system requires a high number of transmit antennas to provide high spectral efficiency for all users. To effectively tackle this problem, the rotational generalized SM (RGSM)-SCMA has been proposed in [32]. In the RGSM-SCMA system, the same spectral efficiency of the SM-SCMA can be achieved using a significantly reduced number of transmit antennas at the expense of almost negligible changes to BER performance and decoding complexity, when compared with the SM-SCMA system. For the SM-SCMA and RGSM-SCMA systems, the iterative MPA decoder has been proposed to detect the transmitted signal [31], [32]. The MPA decoder iteratively updates
the users message probabilities until achieving the maximum number of iterations; this leads to an increase in the decoding complexity of both systems. To the best of the authors’ knowledge, the MPA is the only existing decoder for the SM-SCMA system.

In this paper, three low-complexity decoding algorithms for the uplink SM-SCMA system are proposed. The first algorithm is termed successive user detection (SUD). It detects the users messages that share the first orthogonal resource element (ORE), then by using those detected users messages, it successively detects the users messages that share the next OREs. The SUD algorithm detects the user message using only one of the available OREs that carry the signal of that user. The proposed SUD algorithm is considered to be the lower bound of the decoding complexity for the SM-SCMA and RGSM-SCMA systems at the expense of the BER performance. By exploiting all available OREs for each user with some iterative procedure, the modified SUD (MSUD) provides a considerable improvement in the BER performance at the expense of a small increase in the decoding complexity.

The SD and tree-search concepts are carefully designed for the uplink SM-SCMA, referred to as a fixed-complexity SD (FCSD) algorithm. The FCSD algorithm provides almost the same BER performance as that of MPA with a significant reduction in the decoding complexity. The proposed FCSD has a fixed decoding complexity for all values for signal-to-noise ratio (SNR), as well as for its feasibility of parallel hardware implementation, which is proper for practical applications \[33, 36\]. Besides, the FCSD algorithm provides a favorable trade-off between the decoding complexity and BER performance, which fits a wide range of practical applications. In summary, each of the three proposed algorithms enjoys different advantages that can fit a wide range of system specifications. The complexity analysis in terms of the number of real additions and multiplications is derived. The Monte-Carlo simulations for the BER performance of the proposed algorithms are provided to support the paper findings.

The summary of the paper contributions is as follows:

1) Propose a benchmark low-complexity decoder (i.e., SUD), which exhibits the lowest decoding complexity for the SM-SCMA system. This algorithm provides an acceptable BER performance under some practical constraints (i.e., having good link quality or a high number of receive antennas);
2) Propose an enhanced version of the first algorithm (i.e., MSUD), which considerably improves the BER performance with a little increase in the decoding complexity;
3) Form the tree-search decoder for the SM-SCMA system, which is very important for the SD algorithms that can be investigated in the future by researchers;
4) Propose an SD algorithm based on the tree-search concept (i.e., FCSD), which provides a near-optimum BER performance with a significant reduction in the decoding complexity;
5) Provide the mathematical formulation, pseudo-codes, and complexity analysis for all these proposed algorithms;
6) Provide Monte Carlo simulation results to indicate the significant benefits of the proposed algorithms.

The rest of the paper is organized as follows: In Section II the system model of the uplink SM-SCMA transmitter and receiver is summarized. In Section III the proposed decoding algorithms for the SM-SCMA system are introduced. In Section IV the complexity analysis of the proposed decoding algorithms are derived in terms of the number of real additions and multiplications. The simulation results and conclusions are provided in Sections V and VI, respectively.

II. System Model

In this section, the transmitter and receiver of the uplink SM-SCMA system are discussed. Assume that U users are sharing R OREs, where \( U > R \). Each of these users has an unparalleled multidimensional codebook, \( C^u \in \mathbb{C}^{R \times M}, u = 1, \ldots, U \), with \( c^u_m \in \mathbb{C}^{R \times 1} \), \( m = 1, \ldots, M \) as codewords within the codebook and \( M \) as the number of codewords. Since \( c^u_m \), \( m \), is sparse, the number of non-zero elements for each codeword is denoted by \( d_u \), whereas the number of zero elements is \( R - d_u \). It should be noted that the positions of zero and non-zero elements are fixed for a codebook (i.e., for a user), and vary from codebook to another to provide a fixed number of overlapped users per ORE of \( \forall R \). In this paper, the number of overlapped users per ORE is denoted by \( d_f \).

A. Transmitted and Received Signal

Fig. 1 shows the block diagram of the uplink SM-SCMA system with \( U \) users. Consider an \( N_r \times N_t \) MIMO system for each user, where \( N_t \) and \( N_r \) represent the number of transmit and receive antennas, respectively. For the \( u \)-th user in the SM-SCMA transmitter, the first \( \log_2(N_t) \) of the input bits select the transmit antenna to be activated, while the remaining \( \log_2(M) \) bits are mapped to choose a corresponding codebook, \( c^u_{r,m} \), to be transmitted from that active antenna. Hence, the spectral efficiency of the \( u \)-th user is given by

\[
\eta_u = \log_2(N_t) + \log_2(M),
\]

where \( \eta_u \) is the spectral efficiency of the \( u \)-th user that is measured in bit per channel use (bpcu). It should be noted that the total system spectral efficiency for all users is \( U \eta_u \) bpcu.

At the receiver, the noisy received signal at the \( n_r \)-th receive antenna of the \( r \)-th ORE, \( y_{n_r} \), is

\[
y_{n_r}^r = \sum_{u \in \Lambda_r} \left( h_{n_r}^{r,u} c^u_{r,m} \right) + n_{n_r}^r, \quad r = 1, \ldots, R,
\]

where \( h_{n_r}^{r,u} \) represents the Rayleigh fading channel coefficient between the \( n_r \)-th \( \in \{1, \ldots, N_r\} \) receive antenna and

\( \Theta \) Notations: Boldface lowercase and uppercase letters represent vectors and matrices, respectively. \( CN \) denotes a complex-valued normal random variable. \( \text{diag}(\cdot) \) converts a vector into a diagonal matrix with diagonal elements that are the same as the original vector elements. \( \| \cdot \| \) denotes the Euclidean norm. \( \text{card} \{ \cdot \} \) denotes the cardinality of a set that refers to the number of elements in that set. \( [\cdot]^T \) denotes the matrix or vector transpose. \( \mathbf{E} \{ \cdot \} \) denotes the expectation operation. \( \mathbf{P}(\cdot) \) is the probability of an event. \( f(\cdot) \) denotes the probability density function (pdf) of a random variable. \( \emptyset \) is the empty set.
\( n_u^u \in \{1, \ldots, N_i\} \) transmit antenna of the \( u \)-th user for the \( r \)-th ORE, \( c_m^{ru} \) is the non-zero \( r \)-th element for the \( m \)-th codeword of the \( u \)-th user. Here, \( A_r \) denotes the set of users indices that share the \( r \)-th ORE, and \( n_u^u \sim CN(0, \sigma^2) \) is the complex additive white Gaussian noise (AWGN) with zero-mean and a variance of \( \sigma^2 \) for the \( r \)-th ORE at the \( n_u^u \)-th receive antenna.

For all OREs, the received signal at the \( n_u^u \)-th receive antenna, \( y_n^u \in \mathbb{C}^{R \times 1} = [y_{n_1}, \ldots, y_{n_R}]^T \), is given by

\[
y_{n_u} = \sum_{u=1}^{U} \left( \text{diag} \left( h_{n_u}^u n_t \right) \right) c_{m_u}^u + n_{n_u}, \tag{3}\n\]

where \( h_{n_u}^u n_t \in \mathbb{C}^{R \times 1} = [h_{1,n_t}^u, \ldots, h_{R,n_t}^u]^T \) is the Rayleigh fading channel vector between the \( n_u^u \)-th receive antenna and \( n_t^u \)-th transmit antenna of the \( u \)-th user, and \( n_{n_u} \in \mathbb{C}^{R \times 1} = [n_{1,n_t}^u, \ldots, n_{R,n_t}^u]^T \) is the AWGN vector.

It is worth noting that the relationship between the position of zero/non-zero elements of users codebooks and OREs can be described by a binary indicator matrix, \( F \). In the indicator matrix, the number of rows and columns represents the number of OREs and number of users, respectively. Moreover, the ones in \( F \) show the position of non-zero elements of the user codebooks. In this paper, six users overloaded over four OREs (i.e., \( U = 6 \) and \( R = 4 \)) are considered, with \( F \) given by (6), (7): 

\[
F = \begin{bmatrix}
0 & 1 & 1 & 0 & 1 & 0 \\
1 & 0 & 1 & 0 & 0 & 1 \\
0 & 1 & 0 & 1 & 0 & 1 \\
1 & 0 & 0 & 1 & 1 & 0 
\end{bmatrix}. \tag{4}\n\]

As seen from (4), \( d_v = 2 \) for all users and \( d_f = 3 \) for all OREs. A useful representation for the indicator matrix is

\[
A_r = \{A_r(1), \ldots, A_r(d_f)\}, \tag{5}\n\]

where \( A_r(1) \) denotes the index of the first user that shares the \( r \)-th ORE, and card\(\{A_r\} = d_f \). Thus, \( F \) in (4) yields

\[
\begin{align*}
A_1 &= \{A_1(1), A_1(2), A_1(3)\} = \{2, 3, 5\}, \tag{6a} \\
A_2 &= \{A_2(1), A_2(2), A_2(3)\} = \{1, 3, 6\}, \tag{6b} \\
A_3 &= \{A_3(1), A_3(2), A_3(3)\} = \{2, 4, 6\}, \tag{6c} \\
A_4 &= \{A_4(1), A_4(2), A_4(3)\} = \{1, 4, 5\}. \tag{6d}
\end{align*}\n\]

B. Signal Detection

At the receiver side, the decoder task is to estimate the activated transmit antenna and the mapped codeword for each user (i.e., user message). In this subsection, the ML and MPA decoders are discussed.

1) ML Decoder: The ML decoder jointly performs an exhaustive search for all possible combinations between the transmit antennas and codewords for all users (i.e., \((N_iM)^U\) possible combinations). Although the ML provides the optimum BER performance, it has an impractically high decoding complexity. The mathematical formulation of the ML decoder is given by

\[
\{c, \hat{m}_r\} = \arg \min_{j = 1, \ldots, N_i^U} \left\{ \sum_{n_u = 1}^{N_r} \left\| \text{diag} \left( h_{n_u}^u n_t \right) c_{m_u}^u \right\|_2^2 \right\} 
\]

where \( J = \{n_1^1, \ldots, n_6^U\} \) denotes the set of indices of the estimated active transmit antenna for all \( U \) users, with \( \hat{m}_r \) as the estimated index of the active transmit antenna for the \( u \)-th user, \( n_u^u \) is the active transmit antenna index of the \( u \)-th user that corresponds to the \( j \)-th antenna combinations (out of \((N_i)^U\) combinations) of all \( U \) users, \( C \in \mathbb{C}^{R \times U} = [c_1^u, \ldots, c_6^u] \) represents the estimated transmitted codewords of the \( U \) users, with \( \hat{c}_m^u \) as the estimated transmitted codeword of the \( u \)-th user, and \( m(\cdot) \) is the \( m \)-th codeword of the \( u \)-th user that corresponds to the \( l \)-th codeword combinations (out of \((M)^U\) combinations) of all \( U \) users.

2) MPA Decoder: The MPA is an alternative practical decoder to the ML decoder. It iteratively updates the probability of users messages between the function nodes (FNs) that represent the number of OREs, and the variable nodes (VNs) that represents the number of users. It is worth noting that each of the FNs is connected with all VNs that share the same FN based on indicator matrix in (4) to form what is called a factor graph. The factor graph of the MPA decoder used in this paper is shown in Fig. 1 for \( U = 6 \), \( R = 4 \) and \( F \), which is given from (4).

It is assumed that the probability of passing the \( u \)-th user message, \( \{c_m^u, n_t^u\} \), from the \( u \)-th VN to the \( r \)-th FN and vice versa at the \( k \)-th iteration (out of \( K \) iterations) is \( P_{n_u \rightarrow f_r}(\{c_m^u, n_t^u\}) \) and \( P_{f_r \rightarrow v_u}(\{c_m^u, n_t^u\}) \), respectively. Initially, all users messages passing from the VNs to FNs are equiprobable, i.e.,

\[
P_{v_u \rightarrow f_r}(\{c_m^u, n_t^u\}) = \frac{1}{N_iM}, \quad \forall u, \forall r, \forall m. \tag{8}\n\]

The mathematical formulation of updating the messages at the \((k+1)\)-th iteration of the MPA decoder is given by (9):

\[
P_{f_r \rightarrow v_u}(\{c_m^u, n_t^u\}) = \sum_{\psi(i) \in A_r \setminus u} \left\{ \prod_{n_u = 1}^{N_r} P(y_{n_u}, |\psi(i), \psi(u) = \{c_m^u, n_t^u\}) \times \prod_{i \in A_r \setminus u} P_{v_u \rightarrow f_r}(\psi(i)) \right\}, \quad \forall m, \forall r, u \in A_r, \tag{9}\n\]
The set of all estimated users messages using the MP A in (13), \(\hat{\Theta}_{\text{MPA}}\), can be given as

\[
\hat{\Theta}_{\text{MPA}} = \left\{ \{\hat{c}_m^1, \hat{n}_t^1\}^{(K)}, \ldots, \{\hat{c}_m^l, \hat{n}_t^l\}^{(K)} \right\},
\]

where \(\hat{c}_m^l\) and \(\hat{n}_t^l\) are the estimated message for the \(m\)-th user and the ORE index of the \(l\)-th ORE, respectively. The set of all ORE indices that correspond to \(d_v\) non-zero positions for the \(u\)-th user, \(\Omega_u\), \(\forall r \in \Omega_u\) represents the possible messages of all users that \(\forall\) share the \(r\)-th ORE. Consequently, the SUD algorithm considers that the user’s message is given for the rest of the shared OREs (i.e., \(d_v - 1\) OREs). Thus, the SUD algorithm uses only one ORE to detect this message. Then, these estimated users messages are employed to estimate the messages of other users that share the next OREs. Sequentially, the SUD algorithm estimates the undetected users

III. THE PROPOSED DECODING ALGORITHMS

In this section, the three proposed decoding algorithms are introduced. The first two algorithms focus on decoding the signal with very low complexity and acceptable BER performance. The third proposed algorithm employs the SD concept to provide a near-optimum BER performance with low-decoding complexity in addition to other advantages, such as the feasibility of parallel hardware implementation and the flexible trade-off between decoding complexity and BER performance.

A. The SUD Algorithm

The SUD algorithm provides the lowest decoding complexity among the proposed algorithms at the expense of BER performance. It successively detects the users’ messages using only one ORE. Then, the SUD algorithm uses these detected messages as given information in the next OREs to detect the rest of the users’ messages. Consequently, the SUD algorithm does not benefit from the diversity gain (i.e., sharing the information over several OREs) of the users’ codebook. For instance, if a user spreads his message over \(d_v\) OREs (as in Fig. 1), the SUD algorithm uses only one ORE to detect this message. Thus, the SUD algorithm considers that the user’s message is given for the rest of the shared OREs (i.e., \(d_v - 1\) OREs).

At the beginning, the SUD algorithm performs an exhaustive search for all combinations of the users messages that share the first ORE. It starts with the OREs with highest energy, \(E^r\), based on the following

\[
E^r = \sum_{u \in \Lambda_r} \sum_{n_t = 1}^{N_t} \left| h_{r,v_n}^{u,n_t} \right|^2, \quad r = 1, \ldots, R.
\]

Then, these estimated users messages are employed to estimate the messages of other users that share the next OREs. Sequentially, the SUD algorithm estimates the undetected users

\[
\mathcal{P}(y_n, \psi^r) = \frac{1}{\sqrt{2\pi\sigma}} \exp \left( -\frac{(y_n^r - \sum_{u \in \Lambda_r} (h_{r,v_n}^{u,n_t} c_{m}^{r, u}))^2}{2\sigma^2} \right),
\]

where \(\psi^r\) represents the possible messages of all users that share the \(r\)-th ORE.

Now, \(\mathcal{P}^{(k+1)}_{v_n \rightarrow f_c}\left(\{c_m^{r,u}, n_t^{u}\}\right)\) can be calculated as

\[
\mathcal{P}^{(k+1)}_{v_n \rightarrow f_c}\left(\{c_m^{r,u}, n_t^{u}\}\right) = \gamma^{(k+1)}_{u,r} \prod_{j \in \Omega_u \setminus r} \mathcal{P}^{(k+1)}_{v_n \rightarrow f_c}\left(\{c_m^{r,u}, n_t^{u}\}\right), \quad \forall m, \forall u, r \in \Omega_u,
\]

where \(\gamma^{(k+1)}_{u,r}\) is

\[
\gamma^{(k+1)}_{u,r} = \left( \frac{1}{M} \sum_{m=1}^{M} \sum_{n_t=1}^{N_t} \mathcal{P}^{(k)}_{v_n \rightarrow f_c}\left(\{c_m^{r,u}, n_t^{u}\}\right) \right)^{-1}.
\]
messages until they are all estimated based on the descending order of $E^r$ in (15) for $\forall R$.

The mathematical formulation of the SUD algorithm is given by

$$
\left\{ \hat{C}^r, \hat{j}^r \right\} = \arg \min_{j = 1, \ldots, N_U^r} \sum_{u \in A_r, \Lambda_r} \sum_{n_t = 1}^N y_n^r - \sum_{u \in A_r, \Lambda_r} h_{n_r, n_t}^r(j) e_{m}^r e_{m}(l),
$$

where $A_r$ is the set of users indices that share the $r$-th ORE in which their messages are already estimated previously, $A_r \setminus A_r$ is $A_r$ except $A_r$, or it is the set of users indices that share the $r$-th ORE and their messages need to be estimated, $\hat{U}^r = \arg \min \{ |A_r \cap \hat{A}_r| \} \leq d_l$ is the number of users whose messages need to be estimated at the $r$-th ORE, $\hat{j}^r$ represents the set of users that are currently estimated active transmit antennas for all $\hat{U}^r$ users at the $r$-th ORE, and $C^r$ denotes the estimated transmitted codewords of the $\hat{U}^r$ users at the $r$-th ORE. Here, $\text{Term 1}$ and $\text{Term 2}$ represent the users messages that have already been estimated from previous OREs and that need to be estimated at the $r$-th ORE, respectively. It is worth noting that $\text{Term 1}$ equals zero at the first ORE used by the SUD algorithm (i.e., $A_1 = \phi$). After estimating all users messages from certain OREs, the set of complete estimated users messages using the SUD algorithm in (16), $\hat{\Theta}_{\text{SUD}}$, can be written as

$$
\hat{\Theta}_{\text{SUD}} = \left\{ \{e_{m}^{1}, n_t^{1}\}, \ldots, \{e_{m}^{U}, n_t^{U}\} \right\}.
$$

Consequently, the SUD algorithm detects users messages using a single ORE. Then, these detected messages are used as given messages to detect the others that share the rest of the $d_u - 1$ OREs. It should be noted that the SUD algorithm may not use all received signals on OREs if all users messages are already estimated using certain OREs. The SUD algorithm is summarized in Algorithm 1.

### B. The MSUD Algorithm

As mentioned in the SUD algorithm, the user’s message is detected using a single ORE; however, the $(d_u - 1)$ non-zero OREs for each user are not included in the decoding process with the aim of reducing the decoding complexity. This leads to a significant deterioration in the BER performance (i.e., losing the diversity gain). Unlike the SUD algorithm, the MSUD algorithm considers all OREs that carry the user’s message for detection, to improve the BER performance.

The MSUD is an iterative algorithm that estimates the user message by considering only one user message unknown at a time. In contrast, the rest of the users messages are considered to be known from the previous iteration. Moreover, the MSUD algorithm considers the received signals from all $d_u$ non-zero OREs for each user in the detection process to improve the BER performance. It is important to mention that the initial values of the users messages used in the MSUD algorithm are estimated using the SUD algorithm. In other words, the MSUD algorithm performs the SUD algorithm first. Then, $K$ iterations are performed to improve the BER performance.

To formulate the MSUD algorithm, user messages are first estimated from the SUD algorithm (i.e., $\hat{\Theta}_{\text{SUD}}$, in (17)) and are subsequently used as input to/initialization of the iteration stage of the MSUD algorithm. At the $(k+1)$-th iteration, the estimated $u$-th user message, $\{e_{m}^{u}, n_t^{u}\}^{(k+1)}$, is given by

$$
\left\{ e_{m}^{u}, n_t^{u} \right\}^{(k+1)} = \arg \min_{j = 1, \ldots, N_U} \sum_{u \in A_r, \Lambda_r} \sum_{n_t = 1}^N y_n^r - \sum_{u \in A_r, \Lambda_r} h_{n_r, n_t}^r(j) e_{m}^r e_{m}(l),
$$

where $\text{Term 3}$ and $\text{Term 4}$ represent the given estimated users messages that share the same ORE with the $u$-th user and the desired user message of the $u$-th user to be estimated, respectively. The MSUD algorithm uses all $d_u$ non-zero OREs for each user in the detection, which can be seen from $\sum_{r \in \Omega_u}$ in (18). The estimation process using (13) is performed for all $U$ users for each iteration. After $K$ iterations, the set of estimated messages for all $U$ users, $\hat{\Theta}_{\text{MSUD}}$, is

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**Algorithm 1** The proposed SUD algorithm pseudo-code.

- **Store** codebooks for all users;
- **Input** channel matrices for all users;
- **Define** $\Theta_{\text{SUD}}$ and $\Lambda$ as the set of estimated users messages and set of users indices corresponding to the estimated messages in $\Theta_{\text{SUD}}$, respectively;
- **Initialize** $\Theta_{\text{SUD}} = \{\}$ and $\Lambda = \{\}$;
- **Order** the OREs which should be visited based on (15):

```plaintext
1: While $r \leq R$, do
2: Set $\hat{A}_r \leftarrow \{A \cap A_r\}$;
3: Assign $\hat{y}_{n_r}^r \leftarrow y_{n_r}^r - \sum_{u \in \hat{A}_r} h_{n_r, n_t}^r e_{m}^r e_{m}(l)$;
4: Find $\{C^r, \hat{j}^r\}$ that solves the following:

$$
\arg \min_{\hat{r}, \hat{j}} \left\{ \sum_{n_r = 1}^N \hat{y}_{n_r}^r - \sum_{u \in \hat{A}_r} h_{n_r, n_t}^r(j) e_{m}^r e_{m}(l) \right\}^2,
$$

s.t. $j = 1, \ldots, N_U^r$ and $l = 1, \ldots, M^r$;
5: Update $\Theta_{\text{SUD}}$ based on $\{C^r, \hat{j}^r\}$;
6: Update $\Lambda$ based on $\Theta_{\text{SUD}}$;
7: if $\text{card}\{A\} = U$ break and end the algorithm;
8: end if;
9: Set $r \leftarrow r + 1$;
10: end While;
11: Output $\hat{\Theta}_{\text{SUD}}$.
```
Algorithm 2 The proposed MSUD algorithm pseudo-code.

- **Store** codebooks for all users;
- **Input** channel matrices for all users;
- **Perform** Algorithm 1 to obtain $\Theta_{SUD}$;
- **Initialize** $\Theta_{MSUD} = \Theta_{SUD}$;

1: For $k = 1 : K$, do
2: For $u = 1 : U$, do
3: Assign $\tilde{y}_{nu} \leftarrow y_{nu} - \sum_{i \in A_{nu}} \{ h_{nu}^{r,u} e_{mi} \}^{k}$;
4: Find \{$c_{nu}^{k}, \tilde{n}_{tu}^{k} \}$ that solves the following:
   \[
   \arg\min_{j \in \{1, \ldots, N_{t}\}} \{ \sum_{u \in A_{nu}} | y_{nu}^{r} - h_{nu,n_{t}^{(j)}} e_{mi}^{r,u} |^{2} \}
   \]
   s.t. $j = 1, \ldots, N_{t}$ and $l = 1, \ldots, M$;
5: Update $\Theta_{MSUD}$ based on \{$c_{nu}^{k}, \tilde{n}_{tu}^{k} \}$;
6: end For
7: end For
8: **Output** $\hat{\Theta}_{MSUD}$.

$\hat{\Theta}_{MSUD} = \{ \{ c_{nu}^{1}, \tilde{n}_{tu}^{1} \}^{(K)}, \ldots, \{ c_{nu}^{U}, \tilde{n}_{tu}^{U} \}^{(K)} \}$. \hspace{1cm} (19)

For example, assume that we need to detect the message of the second user (i.e., $u = 2$) using the MSUD algorithm. From (6a) and (6c), the received signal of the first and third OREs will be considered in the detection, while the rest of the overlapped users’ messages (i.e., $u = 3, 5, 4$ and 6) will be given from the previous iteration. Algorithm 2 shows the summary of the MSUD algorithm.

**C. The FCSD Algorithm**

The MPA decoder has a limited support to the parallel hardware implementation, where all users messages are detected together after iterative sequential stages, as seen from (5) and (11). In practice, this kind of hardware implementation is not preferable. Besides, the MPA decoder provides a limited trade-off between decoding complexity and BER performance, which limits its practicality for applications with specific requirements.

The FCSD algorithm supports the parallel hardware implementation and also provides a flexible trade-off between decoding complexity and BER performance. To clearly understand the concept of the FCSD algorithm, a tree-search for the SM-SCMA should be constructed first.

1) **SM-SCMA Tree-search:** The ML decoder of the SM-SCMA in (1) can be represented as a multi-level tree-search, as in Fig. 2. Each of the tree-search levels corresponds to an ORE (i.e., the number of levels equals $R$). At each level, there is a certain number of nodes representing the distance metric between the received signal at the $r$-th ORE and possible combinations of the users messages that share this ORE. Each node at the $r$-th level is expanded into child nodes at the next level.

The mathematical formulation of the $i$-th node at the $r$-th level, $d_{i}^{r}$, is

\[
d_{i}^{r} = d_{i}^{r-1} + e_{i}^{r}, \hspace{1cm} r = 1, \ldots, R,
\]

where $d_{i}^{0}$ is the mother node of $d_{i}^{r}$ and $e_{i}^{r}$ is given by

\[
e_{i}^{r} = \sum_{m_{r}=1}^{N_{t}} \left| y_{nu}^{r} - \sum_{u \in \Lambda_{nu}} h_{nu,n_{t}^{(m)}} e_{mi}^{r,u} - \sum_{u \in \Lambda_{nu}} h_{nu,n_{t}^{(m)}} e_{mi}^{r,u} \right|^{2}.
\]

At the first level (i.e., $r = 1$), $d_{0}^{0} = 0$ in calculating $d_{i}^{0}$ and $i = 1, \ldots, (M N_{t})^{d_{f}}$. From (20) and (21), it should be noted that a node is an accumulation of the distance metric of all preceding nodes in the same branch and that the value of $e_{i}^{r}$ increases as $r$ increases, respectively.

Unlike the construction of the SM tree-search in (5) and MIMO tree-search (14)-(16), the number of expanded nodes for each mother node of the SM-SCMA tree-search, as seen in Fig. 2, gradually reduces across the OREs, reaching a limit of one. Consequently, the SM-SCMA tree-search consists of two stages. The upper stage in which each of the mother nodes is fully expanded to multiple child nodes, is referred to as fully expanded stage (FES). The lower stage is called single expanded stage (SES), in which each of the mother nodes is expanded to only one node. Typically, each level of FES has at least one or more users messages that have not been estimated from the previous OREs; the number of these users messages gradually decreases as the ORE increases,

\[
d_{f} \geq \hat{U}^{2} \geq \cdots \geq \hat{U}^{r} > 1, \hspace{1cm} r \in \text{FES}.
\]

It is worth noting that the number of nodes at the first level is $(N_{t} M)^{d_{f}}$ since there are $d_{f}$ users sharing ORE 1 and no users messages have been estimated previously.

2) **The FCSD Algorithm:** In the tree-search provided in Fig. 2, the ML solution in (7) (close to the MPA solution) can be achieved by visiting all nodes, which is extremely high in terms of decoding complexity. The basic concept of the FCSD algorithm is to reduce the decoding complexity of the SM-SCMA system by reducing the search space inside the tree-search based on a predetermined pruned radius (i.e., threshold). For that, at each level, the nodes that have values smaller than a certain threshold (i.e., pruned radius) are the only ones which are expanded at the next level. It is worth noting that the tree-search levels of Fig. 2 can be ordered based on (15) before performing the FCSD algorithm.
Let us consider that the pruned radius is denoted by $\gamma \in \mathbb{R}^{R-1} = [\gamma_1, \ldots, \gamma_{R-1}]$ and keeps $[\rho_1 \ldots \rho_r \ldots \rho_{R-1}]$ survived nodes, where $\gamma_r$ is the pruned radius and $\rho_r$ is the number of survived nodes at the $r$-th level. At the final level (i.e., the $R$-th level), the minimum node is chosen to be the solution of the algorithm. Consequently, $\rho_r$ for the upper $R-1$ levels is given by

$$\rho_r = \left\{d_{r,i}^2 \leq \gamma_r | i = 1, \ldots, \rho_{r-1}(N_rM)^{U^r}\right\},$$

$$0 \leq \hat{U}^r \leq d_f, \quad 1 \leq r \leq R-1,$$  

(23)

where $\hat{U}^r = 0$ at $r \in \text{SES}$, $0 < \hat{U}^r \leq d_f$ at $r \in \text{FES}$, and $\rho_0 = 1$ at the first ORE (i.e., $r = 1$). At the last level (i.e., $r = R$), the number of nodes is $\rho_{R-1}$, since there are only $\rho_{R-1}$ survived nodes from the $R-1$-th level. Thus, the FCSD algorithm declares the argument of the minimum node at the last level as the solution, which can be represented as

$$\{\hat{C}, \hat{j}\} = \arg \min_{i = 1, \ldots, \rho_{R-1}} \{d_i^R\}.$$  

(24)

It is worth noting that a higher value of the pruned radius may lead to expanding unnecessary nodes, which increases the decoding complexity. On the other hand, a smaller value of the pruned radius may cause an early dropping of the optimum solution, which deteriorates the BER performance. Thus, the appropriate choice of the pruned radius is a crucial process in the FCSD algorithm. For more clarifications, the accumulated node, $d_{r,i}^2$ in (20) is a non-central chi-squared random variable with $2N_r$ degrees of freedom and its pdf is given by [37]

$$f_{d_{r,i}^2}(d_{r,i}^2) = \frac{1}{\sigma^2} \left(\frac{d_{r,i}^2}{\alpha_{r,i}^2}\right)^{(rN_r-1)/2} \times \exp\left(-\frac{\alpha_{r,i}^2 + d_{r,i}^2}{\sigma^2}\right) I_{rN_r-1}\left(\frac{\sqrt{d_{r,i}^2 \alpha_{r,i}^2}}{\sigma^2 / 2}\right),$$

(25)

where $I_{rN_r-1}(\cdot)$ is the first kind modified Bessel function with order $(rN_r - 1)$ and the non-centrality parameter $\alpha_{r,i}^2$ is

$$\alpha_{r,i}^2 = \sum_{n_r=1}^{N_r} \sum_{n_i=1}^{r} \sum_{m \in A_r} \left(h_{n_r,n_i}^{r,u} \bar{c}_m^{r,u}\right)^2 - \sum_{m \in A_r \setminus \hat{A}_r} h_{n_r,n_i}^{r,u} \bar{c}_m^{r,u} - \sum_{m \in \hat{A}_r} h_{n_r,n_i}^{r,u(i)} \bar{c}_m^{r,u}.$$  

(26)

Since $d_{r,i}^2$ has an even degrees of freedom value, the probability of not dropping the optimum solution early, $\mathcal{P}(d_{r,i}^2\text{opt} \leq \gamma_r)$, can be calculated as [37] (Ch. 2)

$$\mathcal{P}(d_{r,i}^2\text{opt} \leq \gamma_r) = 1 - Q_{rN_r} \left(\frac{\alpha_{r,i}}{\sigma/\sqrt{2}}, \frac{\sqrt{\gamma_r}}{\sigma/\sqrt{2}}\right),$$

(27)

where $Q_{rN_r}(\cdot, \cdot)$ is the generalized Marcum function of order $rN_r$. As seen from (27), by increasing the value of $\gamma_r$, the value of $\mathcal{P}(d_{r,i}^2\text{opt} \leq \gamma_r)$ becomes closer to unity.

In the FCSD algorithm, the value of $\gamma_r$ is empirically selected to choose a fixed number of nodes from each level to increase the probability of including the optimal solution based on (27). Accordingly, at each level, the value of $\rho_r$ in the FCSD algorithm is fixed for $1 \leq r \leq R - 1$. Finally, the FCSD algorithm selects the minimum node among all expanded nodes at the last level to be declared as a solution. Thus, the set of estimated messages for all $U$ users, $\hat{\Omega}_{\text{FCSD}}$, is

$$\hat{\Omega}_{\text{FCSD}} = \left\{\hat{C}_m^{1,1}(R), \ldots, \hat{C}_m^{U,1}(R)\right\},$$

(28)

where $\hat{\Omega}_{\text{FCSD}} = \left\{\hat{C}_m^{U,1}\right\}$ is the estimated message of the $u$-th user corresponding to the minimum node at the $R$-th level. Algorithm 3 summarizes the procedure of the FCSD algorithm.

### IV. Complexity Analysis

In this section, the decoding complexities of the conventional MPA and the proposed algorithms for the SM-SCMA system are discussed. In this paper, the decoding complexity is measured by the number of real additions and multiplications required to perform a particular algorithm. For the conventional MPA decoder of the SM-SCMA system, the required number of real additions and multiplications, Add(MPA) and Mul(MPA), respectively, are given by [32]

$$\text{Add}(\text{MPA}) = Rd_f(N_rM)^{df} (2N_r(2d_f+1) - 1) + KRd_f(N_rM)^{df} - 1,$$

(29)

and

Algorithm 3 The proposed FCSD algorithm pseudo-code.

- **Input**
  - Channel matrices for all users.
  - Input $\rho = [\rho_1 \ldots \rho_r \ldots \rho_{R-1}] \in \mathbb{R}^{R-1}$.
  - Order the OREs which should be visited based on (15).
  - Assign $\nabla_r$ as an empty vector that contains the distance metric nodes at the $r$-th level.

- **Define** $\ell^r$ as the total number of nodes in the $r$-th level.

1. **While** $r \leq R - 1$, **do**
   2. For $i = 1 : \ell^r$, **do**
      3. Compute $d_i^r$ from (20) and (21).
      4. Store $d_i^r$ in $\nabla_r$.
   5. **end For**
   6. Keep the smallest $\rho_r$ nodes from $\nabla_r$.
   7. Expand the survived $\rho_r$ nodes from Line #6 into $\nabla_{r+1}$.
   8. Set $r \leftarrow r + 1$.
   9. **end While**
10. **Find** the minimum node in $\nabla_R$.

- **Output** $\hat{\Omega}_{\text{FCSD}}$ as the messages corresponding to the argument of the minimum node in Line #10.
\[ \text{Mul}^{(MPA)} = R_d f (N_r M)^{d_f} (2N_r (2d_f + 1) + K_d f + 1) \\
+ N_t M (d_v - 1)(K R d_f + U). \quad (30) \]

A. The SUD Algorithm

In the SUD algorithm, the cost of \((15)\) is \(R(2N_r d_f - 1)\) real additions and \(2RN_r d_f\) real multiplications. The cost of one possible combination of \(j\) and \(l\) in \((16)\) for \(N_r\) receive antennas is \(N_r(4d_f + 2) - 1\) real additions and \(N_r(4d_f + 2)\) real multiplications. The number of possible combinations between \(j\) and \(l\) in \((16)\) varies from one ORE to another based on the system indicator matrix. Thus, the required number of real additions and multiplications, \(\text{Add}^{(SUD)}\) and \(\text{Mul}^{(SUD)}\), respectively, of the SUD algorithm can be written as

\[ \text{Add}^{(SUD)} = R (2N_r d_f - 1) \]
\[ + (N_r (4d_f + 2) - 1) \sum_{r = 1}^{R} (M N_t)^{U_r}, \quad (31) \]

and

\[ \text{Mul}^{(SUD)} = 2RN_r d_f + N_r (4d_f + 2) \sum_{r = 1}^{R} (M N_t)^{U_r}. \quad (32) \]

The summation term in \((31)\) and \((32)\) depends on the indicator matrix of the system.

B. The MSUD Algorithm

The MSUD algorithm iteratively updates the estimated users messages of the SUD algorithm at an extra cost of \(K U M N_t (N_r (4d_f + 2) - 1)\) and \(K U M N_t N_r (4d_f + 2)\) real additions and multiplications, respectively. Thus, the required number of real additions and multiplications, \(\text{Add}^{(MSUD)}\) and \(\text{Mul}^{(MSUD)}\), respectively, of the MSUD algorithm are given by

\[ \text{Add}^{(MSUD)} = R (2N_r d_f - 1) + (N_r (4d_f + 2) - 1) \]
\[ \times \left( K U M N_t + \sum_{r = 1}^{R} (M N_t)^{U_r} \right), \quad (33) \]

and

\[ \text{Mul}^{(MSUD)} = 2RN_r d_f + N_r (4d_f + 2) \]
\[ \times \left( K U M N_t + \sum_{r = 1}^{R} (M N_t)^{U_r} \right). \quad (34) \]

C. The FCSD Algorithm

The FCSD algorithm visits \((M N_t)^{d_f}\) nodes at the first tree-search level, where each node costs \((N_r (4d_f + 2) - R - 2)\) and \(N_r (4d_f + 2)\) real additions and multiplications, respectively. Then, for the rest of \(R - 1\) levels, the FCSD algorithm visits a fixed number of nodes at each level according to \(\rho_r\). Thus, the required number of real additions and multiplications, \(\text{Add}^{(FCSD)}\) and \(\text{Mul}^{(FCSD)}\), respectively, of the FCSD algorithm are given by

\[ \text{Add}^{(FCSD)} = R (2N_r d_f - 1) + (N_r (4d_f + 2) - R - 2) \]
\[ \times \left( (M N_t)^{d_f} + \sum_{r = 2}^{R} \rho_r (M N_t)^{U_r} \right), \quad (35) \]

and

\[ \text{Mul}^{(FCSD)} = 2RN_r d_f + N_r (4d_f + 2) \]
\[ \times \left( (M N_t)^{d_f} + \sum_{r = 2}^{R} \rho_r (M N_t)^{U_r} \right). \quad (36) \]

V. Simulation Results and Discussions

In this section, the proposed decoding algorithms and conventional MPA decoder in \((4)\) are assessed using Monte-Carlo simulations for the SM-SCMA system. The Rayleigh fading channel coefficients between the transmit and receive antennas for all users are considered to be perfectly known at the receiver side. An SM-SCMA system of six users that share four OREs based on \((4)\) or \((6)\) is considered for the assessment (i.e., \(U = 6\), \(R = 4\), \(d_f = 3\) and \(d_v = 2\)). Two user spectral efficiencies based on \((11)\) are considered in the results: \(\eta_u = 3\) bpcu (\(N_t = 4\) and \(M = 2\)) and \(\eta_u = 4\) bpcu (\(N_t = 4\) and \(M = 4\)), and the \(M\)-QAM scheme is used in the simulations.

Three MIMO scenarios are studied for each user spectral efficiency: under-determined MIMO system (e.g., \(N_r = 2\)), determined MIMO system (e.g., \(N_r = 4\)) and over-determined MIMO system (e.g., \(N_r = 6\) and \(N_r = 10\)). Thus, there are six scenarios within the scope of this paper (i.e., three MIMO scenarios for each of the two user spectral efficiencies). It is worth noting that the BER performance of the conventional MPA decoder for the SM-SCMA converges after five iterations (i.e., \(K = 5\)) for the considered six scenarios. The following simulation results are obtained by running at least \(10^5\) independent realizations.

\(^2\)In this paper, the system in \((6)\) is considered. Consequently, the result of the summation term in \((31)\) and \((32)\) becomes \((MN_t)^3 + (MN_t)^2 + (MN_t)^1).
A. The Effect of the FCSD Pruned Radius on BER

In this subsection, the effect of choosing $\gamma_r$ (or $\rho_r$) across the tree-search level on the BER performance for the proposed FCSD algorithm is studied. In other words, we need to know which level of the tree-search has a great effect on the BER performance to increase/decrease the number of visited nodes. As seen from (27), as $\gamma_r$ increases, the probability of not missing the optimal solution (i.e., MPA solution) increases. The question that arises is which level has a significant effect on the probability in (27). To answer this question, let us define the number of misses (NoM) as the number of times that the FCSD algorithm misses the MPA solution. The NoM can be used as an indicator to study the effect of selecting $\gamma_r$ at each level, taking into account that a small value of the NoM reflects an acceptable BER performance and vice versa. Thus, the NoM can be formulated as

$$\text{NoM} = \mathbb{E}\left\{ \sum_{u=1}^{U} P\left( \{\hat{c}_m^u, \hat{c}_t^u\} \mid \text{FCSD} \neq \{c_m^u, c_t^u\} \mid \text{MPA} \right) \right\},$$

(37)

where $P(\cdot)$ in (37) equals 1 or 0 when the estimated messages of the $u$-th user using the FCSD and MPA decoders are different or the same, respectively.

To study the effect of $\gamma_r$, the FCSD algorithm with $[\rho_1 \rho_2 \rho_3] = [15 15 15]$ is assumed to be the baseline of this study for the three MIMO scenarios of $\eta_u = 3$ bpcu. To note the effect of $\rho_r$ on the BER performance of each level, we increase the number of survived nodes of only one level at a time, while the number of survived nodes of the rest of levels is kept the same. As such, to see the effect of the first level (i.e., $\rho_1$) on the BER performance compared to the baseline, we notice the improvement in the NoM when $\rho_1 = 50$ and $\rho_2 = \rho_3 = 15$ (i.e., $[\rho_1 \rho_2 \rho_3] = [50 15 15]$). It should be noted that these numbers are arbitrarily chosen to study the effect of $\rho_r$ on the BER performance. Next, we do the same thing for the second level (i.e., $[\rho_1 \rho_2 \rho_3] = [15 50 15]$) and notice the improvement in the NoM. From Fig. 3, the improvement in NoM for the first level is negligible compared to the improvement in the NoM obtained from increasing the survived nodes in the second level. Consequently, the second level has a greater effect on BER performance compared to the first level.

It should be noted that $\rho_3$ can not be greater than $\rho_2$ since both belong to SES, as in Fig. 2 Therefore, to continue the study for $\rho_1$ and $\rho_3$, let us consider $[\rho_1 \rho_2 \rho_3] = [50 50 15]$ as a new baseline for comparison. Compared to the new baseline: first, the number of survived nodes of the first level is increased to be 50 (i.e., $[\rho_1 \rho_2 \rho_3] = [50 50 15]$), then we do the same thing for the third level (i.e., $[\rho_1 \rho_2 \rho_3] = [15 50 50]$) and notice the improvement in the NoM. As depicted in Fig. 3, the improvement in NoM from the third level is larger than the improvement obtained from the first level, compared to the new baseline.

By taking an in-depth look at Fig. 3, one can observe that the increase in the number of survived nodes at the second level provides better NoM improvements, compared to the increase in the number of survived nodes at any other level. The reason is that only part of users share the upper levels of FES; thus, the distance metric nodes at FES levels do not represent all users. On the other hand, the nodes at SES levels include the distance metrics of all users, which significantly affects the BER performance. Hence, the second level has the highest effect on the BER performance, then the third level, and finally the first level. In essence, increasing the number of survived nodes at the SES levels is more effective than at the FES levels, especially the upper levels of the SES. It should be noted that the conclusion drawn from this study is independent on the structure of $F$ in (4).

Fig. 4 shows the effect of $\rho_r$ on the BER performance in terms of NoM for $\eta_u = 4$ bpcu. In these scenarios, the FCSD algorithm with $[\rho_1 \rho_2 \rho_3] = [30 30 30]$ is considered. As discussed for $\eta_u = 3$ bpcu, $\rho_2$ provides significant improvements.
in the NoM. On the other hand, \( \rho_1 \) and \( \rho_3 \) provide almost the same improvements for the three scenarios depicted in Fig. 4. In other words, there is no preference for increasing the number of survived nodes at these two levels from the NoM perspective. However, it is preferable to increase \( \rho_5 \) rather than \( \rho_1 \) from the decoding complexity point of view, as seen from [35] and [36]. This means that increasing \( \rho_5 \) results in a lower increase in the decoding complexity compared with the increase of \( \rho_1 \).

Finally, increasing the number of survived nodes at the lower tree-search levels has a better effect on the BER performance or/and decoding complexity. It is worth noting that the number of survived nodes at the first levels should be empirically chosen to avoid the early dropping of the MPA solution. Empirically, the FCSD algorithm with [35 70 50] and [110 320 300] provides near MPA BER performances (i.e., NoM close to zero) for \( \eta_u = 3 \text{ bpcu} \) and \( \eta_u = 4 \text{ bpcu} \), respectively.

\section*{B. BER Performance Assessment}

In this subsection, the BER performance of the proposed decoders is compared with the conventional MPA versus different values of SNR for all six scenarios. The proposed MSUD algorithm and conventional MPA converges at four and five iterations, respectively (i.e., \( K = 4 \) for MSUD and \( K = 5 \) for MPA). Moreover, \( K = 1 \) is provided for the MSUD and MPA to highlight the improvement in the BER performance when using the value of \( K \) at the convergence for both algorithms.

Fig. 5 depicts the BER performance of the proposed and MPA decoders for \( \eta_u = 3 \text{ bpcu} \) in different three MIMO scenarios (i.e., \( N_r = 2, 4, 6, \) and \( 10 \)). As mentioned in Subsection V-A and as seen from this figure, the proposed FCSD algorithm with \( [\rho_1 \rho_2 \rho_3] = [35 70 50] \) provides a very similar BER performance as MPA. The FCSD algorithm with \( [\rho_1 \rho_2 \rho_3] = [5 10 8] \) is depicted in Fig. 5 to show that the FCSD can provide a flexible trade-off between the BER performance and decoding complexity. It is also shown that the proposed SUD provides an acceptable BER performance with a considerable degradation in the BER performance of the MPA. The MSUD with \( K = 1 \) and \( K = 4 \) both provide a considerable improvement in the SUD BER performance.

Fig. 6 shows the BER performance of the proposed and MPA decoders for \( \eta_u = 4 \text{ bpcu} \) in three different MIMO scenarios (i.e., \( N_r = 2, 4 \) and \( 6 \)). Here, the value of \( [\rho_1 \rho_2 \rho_3] \) of the proposed FCSD algorithm is modified to be \([110 320 300]\) to provide a very similar BER performance as MPA. Same as the findings of Fig. 5, the SUD algorithm yields an acceptable BER performance, while the MSUD algorithm significantly improves the BER performance of the SUD algorithm, as seen in Fig. 4.

\section*{C. Decoding Complexity Assessment}

In this subsection, the decoding complexity of the proposed and MPA decoders are compared in terms of the required number of real additions and multiplications, based on the deduced equations mentioned in Section IV.

Figs. 7 and 8 show the required number of real additions and multiplications, respectively, for \( \eta_u = 3 \text{ bpcu} \) for the three MIMO scenarios. On the other hand, Figs. 9 and 10 depict the required number of real additions and multiplications, respectively, for \( \eta_u = 4 \text{ bpcu} \) for the three MIMO scenarios (i.e., \( N_r = 2, 4, 6, \) and \( 10 \)). It can be inferred from all these figures that the proposed SUD algorithm provides the lowest decoding complexity and is significantly low when compared with the MPA and FCSD algorithms. The proposed MSUD algorithm slightly increases the decoding complexity compared with the SUD algorithm; however, its decoding complexity is still very low when compared with the MPA. Finally, although the complexity of the FCSD algorithm is higher when compared with the SUD and MSUD algorithms, it is still significantly lower when compared with MPA.

\section*{D. Discussions}

The proposed SUD and MSUD algorithms provide more than a 90\% reduction in the decoding complexity compared to MPA at the expense of BER performance loss. However, this BER performance loss decreases as \( N_r \) increases. For example, the BER loss is around 10 dB and 5.5 dB for \( N_r = 6 \) and 10, respectively, as seen from Figs. 5(c), 5(d), 6(c) and 6(d). Moreover, for good quality links (i.e., moderate and high SNR), these algorithms provide a good and acceptable BER performance. For instance, the BER performance is around \( 10^{-5} \) at less than 20 dB and 15 dB for \( N_r = 6 \) and 10, respectively, which is an acceptable value.

Thus, the proposed SUD and MSUD algorithms are important, as they exhibit a significant reduction in the decoding complexity while providing an acceptable BER performance in the case of a good quality link and/or with a higher number of receive antennas (which is feasible in the uplink scenario). Furthermore, the proposed FCSD algorithm provides a near-optimum BER performance with reduced decoding complexity. This variety of proposed decoders may fit a wide range of possible candidate applications in practice.

Furthermore, the three proposed decoder concepts may be adapted to decode the RGSM-SCMA signals. However, the formulation and settings of the algorithms need further investigation. In other words, since the RGSM-SCMA system activates more than one antenna associated with rotational angles, a suitable mathematical reformulation for the proposed algorithms is needed.

\section*{VI. Conclusions}

This paper proposes three different low-complexity decoding algorithms, for the first time, for the uplink SM-SCMA system. The proposed SUD algorithm is a non-iterative algorithm that provides a benchmark for the decoding complexity at the expense of the BER performance which is still acceptable for some possible practical applications under certain environments and settings. The degradation of its BER performance comes from using only some of the available OREs in estimating the users messages. The proposed MSUD algorithm is an iterative algorithm that considerably improves the BER performance of the SUD algorithm, with the cost...
of a slight increase in the decoding complexity. The MSUD algorithm uses all available OREs to decode the users messages. The proposed FCSD algorithm provides a close BER performance as MPA with a considerable reduction in the decoding complexity. Unlike the MPA, the proposed FCSD algorithm supports parallel hardware implementation. These proposed algorithms can fit a wide range of possible practical applications with specific requirements for both operation and hardware implementation. The mathematical formulation, complexity analysis for all proposed algorithms, and simulation results are provided to support these findings. As a potential direction, the proposed algorithm may be extended to decode the RGS-M-SCMA signals.

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Fig. 7: Real additions comparison of different SM-SCMA decoders for $\eta_u = 3$ bpcu.

Fig. 8: Real multiplications comparison of different SM-SCMA decoders for $\eta_u = 3$ bpcu.

Fig. 9: Real additions comparison of different SM-SCMA decoders for $\eta_u = 4$ bpcu.

Fig. 10: Real multiplications comparison of different SM-SCMA decoders for $\eta_u = 4$ bpcu.
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