Gravitational radiation from highly magnetized nascent neutron stars in supernova remnants

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ABSTRACT
We consider the spin evolution of highly magnetized neutron stars in a hypercritical inflow just after their birth in supernovae. Presence of a strong magnetic field could deform the star and if the symmetry axis of the field is misaligned with that of stellar rotation, the star will be an emitter of gravitational wave. Here we investigate the possibility of gravitational radiation from such a star when there is a hypercritical inflow onto it. For doing this we adopt a simplified model of the system in which the star is approximated as a Newtonian spherical polytrope with index \( N = 1 \). The stellar configuration is slightly deformed away from spherical by the intense magnetic field; the rotational angular frequency of the star is determined by the balance between the accretion torque and the magnetic dipole radiation. We take into account the ‘propeller’ process in which a rotating stellar magnetic field flings away in-falling matter; the inflow is assumed to be a self-similar advection dominated flow. An estimation of the characteristic amplitude of the gravitational radiation from such systems is given. The computation of the signal-to-noise ratio suggests that for the case of an initially rapidly rotating and highly magnetized star (surface field \( 10^{15} \) G) in the Virgo Cluster, its ellipticity would need to be larger than \( 10^{-5} \) in order for the gravitational waves to be observed.

Key words: stars: neutron, magnetic fields, rotation – gravitational waves – supernovae – accretion

1 INTRODUCTION
Recently Watts & Andersson (2002) have considered the possibility of detecting gravitational radiation from a newly born neutron star in a supernova remnant. In their scenario, debris of a supernova explosion (typically \( \sim 0.1 \odot \)) falls back hypercritically (\( M > 10^{-5} \odot \text{yr}^{-1} \)) onto a neutron star with a normal-strength magnetic field (\( B < 10^{13} \) G) and transfers angular momentum to the star. On the other hand, the r-mode instability† which is driven by gravitational radiation, can extract angular momentum from the system quite efficiently. The accretion torque and the torque of gravitational radiation balance to give the star a characteristic rotational period of a few milliseconds. As a result, this system can be an efficient emitter of gravitational waves of nearly constant frequency for several years. Their work indicates that for stars with a normal-strength magnetic field (\( B < 10^{13} \) G), the system can be a promising source for gravitational wave.

On the other hand, for moderately magnetized stars (i.e., \( B \sim 10^{10} - 10^{12} \) G) Rezzolla et al. have argued that the coupling of r-modes with the magnetic field can prevent the instability from developing. Thus the results by Watts & Andersson may not be relevant to the stars with rather strong magnetic fields (\( B > 10^{13} \) G).

As a complementary study to theirs, we study here a similar system but with the neutron star having a much larger magnetic field (\( B \sim 10^{14} - 10^{15} \) G). Magnetic fields of this strength are associated with the so-called magnetars (Duncan & Thompson 1996).

When the magnetic field reaches such large intensities, the equilibrium configuration would be distorted by the magnetic tension considerably. In the case of an axisymmetric magnetic field, the star will be deformed in an axisymmetric way aligned with the axis of the magnetic field (Bonazzola and Gourgoulhon 1996). If this symmetry axis is not aligned with the rotational one, this will produce a time varying quadrupole moment and lead to the emission of gravitational waves.

In this respect, our system is quite different from the one considered by Watts & Andersson (2002), since here gravitational radiation is not produced by the r-mode instability, and has only a passive role in the evolution of the system. The evolution of the system, i.e., of the rotational angular frequency and of the mass, is regulated by the evolution of the accretion torque.

In particular, the so-called ‘propeller’ mechanism (Pringle & Rees 1972, Illarionov & Sunyaev 1975) becomes active rather early. (This mechanism consists of the in-falling matter being flung away from the stellar magnetosphere when the rotational frequency is larger than the local orbital frequency, because the centrifugal...
force on the matter exceeds the gravitational force.) Also the torque from magnetic dipole radiation may be important in the later phases for highly magnetized stars.

The plan of the paper is the following. In section 2, the formulation of our model is outlined. In the following section, the results, (i.e., the evolution of the stellar rotational frequency and the mass accretion rate onto the star, the characteristic amplitudes of gravitational waves from the system, and the signal-to-noise ratio of gravitational wave for laser-interferometric detectors), are presented for several parameter sets. The final section contains the summary and some comments about our model.

2 A SIMPLIFIED MODEL

Our model of the system is rather simplified and follows closely the one presented by Watts & Andersson (2002). Our description of the physical processes is rather idealized but, nevertheless, it allows the basic features of the gravitational wave emission from these systems to be appreciated.

The star is assumed to be a Newtonian polytrope with polytropic index \( N = 1 \), having radius \( R \) and the mass \( M \). Debris of the supernova falls back onto the star according to a simple power law of time, \( \dot{M}_{\text{fall}} \sim t^{-3/2} \) (Mineshige et al. 1997; Menou et al. 1999). Note that this in general should be distinguished from the mass accretion rate onto the star \( \dot{M} \), especially when the propeller mechanism is active. We assume that,

\[
\dot{M} = \begin{cases} \dot{M}_{\text{fall}} & \text{(propeller is inactive)} \\ 0 & \text{(propeller is active)} \end{cases}
\]

The magnetospheric radius, at which the ram pressure of the in-falling matter balances the magnetic pressure, is defined by,

\[
r_m = \left( \frac{B^2 R^6}{2M_{\text{fall}} \sqrt{2GM}} \right)^{2/7},
\]

where \( B \) is the surface polar field. The in-falling matter is stopped at around this radius.

When \( B < 2^{3/4} \pi^{1/4} M^{1/4} \dot{M}_{\text{fall}} R^{-5/4} \), the magnetospheric radius defined by Eq. (2) is below the stellar surface and the effect of the magnetic field on the accreting matter flow is suppressed. The accretion flow is then assumed to be advection dominated (we here adopt the description of Narayan & Yi 1994,1995) with an accretion torque,

\[
Q_a = \dot{M} R \left( \frac{2GM}{r} \right)^{1/2}.
\]

We should note that the behaviour of the in-falling matter is rather uncertain especially in the hypercritical regime in the presence of a strong magnetic field which possibly affects the motion of the matter. The magnetic field may produce a strong outflow or jet, and/or destroy the innermost structure of the in-falling flow and change it drastically. Modeling these flows is beyond the scope of this paper and hereafter we will assume that the flow is described by that of a simple, advection-dominated disk.

When \( r_m \) exceeds \( R \), \( Q_a \) takes the form (Watts & Andersson 2002),

\[
Q_a = 2r_m^2 \left[ \Omega_k(r_m) - \Omega \right] \dot{M}_{\text{fall}},
\]

where \( \Omega \) is the rotational angular frequency of the star, and \( \Omega_k(r) \) is the Keplerian orbital frequency around the star at a radial distance \( r \).

When the radius of the light cylinder \( r_L = c/\Omega \) of the star is smaller than \( r_m \), we also include the torque given by the magnetic dipole radiation,

\[
Q_b = -\frac{2B^2 R^6 \Omega^3}{3c^3}.
\]

We adopt the condition for the appearance of the propeller effect, \( r_m > R \) and \( \Omega > \Omega_b(r_m) \).

The evolution of the system is described by the time-variation of \( M \) and \( \Omega \) (note that \( R \) is constant for this polytrope). The evolution equations are then,

\[
d\dot{M} = \frac{d^2 \Omega}{dt^2} = \frac{Q_a + Q_b(r_m - r_L)}{\kappa MR^2} - \frac{3 \dot{M} R}{2M} \Omega.
\]

Here \( \theta(x) \) is the Helmholtz step function and \( \kappa \) is a dimensionless constant in terms of which the moment of inertia is given as \( I = \kappa MR^2 \). For an \( N = 1 \) polytrope \( \kappa = 0.261 \).

Following Watts & Andersson (2002), we start integrating Eqs. (1) and (2) from \( t = t_i \) after the bounce of the collapsing core, when a recognizable central compact object is thought to appear. We set the total amount of in-falling debris as \( \Delta M_{\text{fall}} \) (typically \( \sim 0.1M_\odot \)). The integration is terminated if \( M_{\text{fall}} < 10^{-4}M_\odot \) yr\(^{-1} \), when the hypercritical advection dominated accretion ceases (Chevalier 1989, Brown et al. 2000).

3 RESULTS

3.1 Evolution of rotational frequency and mass

The parameters defining the model are the following: 1) the initial time \( t_i \), at which the time integration is started which we choose to be 100s (cf. Watts & Andersson 2002); 2) the total mass of in-falling matter, \( \Delta M_{\text{fall}} \); 3) the strength of the surface polar magnetic field \( B \). 4) the initial rotational frequency of the star \( f_0 = \Omega(0)/2\pi \). In what follows, we show the evolution of the mass and of the rotational frequency of systems with typical parameter sets.

In Figs. 3 and 4, the evolutionary curves of the mass (upper panel) and the rotational frequency (lower panels) are plotted. As the in-fall rate decreases, the propeller effect becomes active, and the matter cannot accrete onto the star. This occurs earlier for stronger surface magnetic fields (compare the upper panels of Fig. 3 and 4). Also, the torque produced by the magnetic dipole radiation is stronger for larger surface magnetic fields. As a result, the rotational frequency decreases faster with stronger fields, for the same \( \Delta M_{\text{fall}} \). Also note that the asymptotic behaviour of the rotational evolution is rather insensitive to \( \Delta M_{\text{fall}} \). In the later phases of the evolution, the in-fall rate is so low that the angular frequency evolution is mainly driven by the magnetic dipole radiation term (Eq. (2)). Thus the dependence on the in-fall rate can be neglected. Furthermore, in the later phases of evolution the frequency converges to the same value, irrespective of the initial rotational frequency \( f_0 \). This is also because in this phase the mass accretion rate is so small that the evolution of the frequency is solely determined by the magnetic dipole radiation. From Eqs. (1) and (2), it is easy to see that the frequency behaves as:

\[
\frac{d\Omega}{dt} = \frac{Q_a + Q_b(r_m - r_L)}{\kappa MR^2} - \frac{3 \dot{M} R}{2M} \Omega.
\]

\[
\frac{d\dot{M}}{dt} = \frac{d^2 \Omega}{dt^2} = \frac{Q_a + Q_b(r_m - r_L)}{\kappa MR^2} - \frac{3 \dot{M} R}{2M} \Omega.
\]

In this case, we simply replace \( r_m \) with \( r_L \).
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3.2 Gravitational wave strain

Gravitational waves from a star deformed by a magnetic field and having its rotation axis misaligned with respect to its magnetic axis, have been discussed by Bonazzola & Gourgoulhon (1996). We here use their formula to estimate the dimensionless strain of gravitational waves.

The strain of the produced gravitational waves observed on the earth depends not only on the misalignment angle of the rotational and magnetic axes, but also on the inclination angle of the system with respect to the line of sight to the observer (see Eqs.(20) and (21) of Bonazzola & Gourgoulhon 1996).

The factor which does not depend on these angles is found to be (cf. Eq.(25) of Bonazzola & Gourgoulhon 1996):

$$h_0 = \frac{4G I \epsilon f^2}{c^2 D}$$

$$= 1.67 \times 10^{-25} \left[ \frac{f}{1 \text{kHz}} \right]^2 \left[ \frac{D}{1 \text{Mpc}} \right]^{-1} \left[ \frac{I}{10^{45} \text{g cm}^2} \right] \times \left[ \frac{\epsilon}{10^{-6}} \right] .$$

where $f$ is the rotational frequency, $D$ is the distance to the source, $I$ is the moment of inertia and $\epsilon$ is the ellipticity measuring the deformation of the star.

Thus far we have almost no information about the ellipticity produced by the magnetic field, except for the upper bounds on normal pulsars (Bonazzola & Gourgoulhon 1996). On the theoretical side, this is mostly due to the fact that we know hardly anything about the configuration of the magnetic field inside the star, although there have been works estimating the ellipticity (Bonazzola & Gourgoulhon 1996; Konno et al. 2000). Bonazzola & Gourgoulhon (1996) have parametrized the ellipticity as,

$$\epsilon = \beta \mu_0 \frac{M^2 R^2}{G I^2} ,$$

where $\mu_0$ is the permeability of the vacuum and $M$ is the magnetic dipole moment of the star. The parameter $\beta$ depends on the configuration of the magnetic field in the star as well as on the compactness of the star. For a Newtonian uniform density star with a uniform distribution of magnetic field, $\beta = 1/5$. Bonazzola & Goulgourhon (1996) obtained values of $\beta$ up to $10^3$. On the other hand, Konno et al. (2000) studied the effect of the compactness of the star assuming a dipolar field inside it, and found that the ellipticity can be enhanced by general relativistic effects.

Hereafter we assume $\beta = 1$ as a ‘moderate’ case for the ellipticity. As can be seen in Eq.(9), the results here can easily be scaled to other choices for this parameter.

For simplicity, we have neglected the deformation introduced by rotation. If the rotation and magnetic field are weak sufficiently

\[ \text{Figure 1. Evolution of the rotational frequency (lower panel) and the mass (upper panel) of the model with } B = 10^{14} \text{G and initial mass } M = 1.4M_{\odot}. \text{ The solid lines are for } f_0 = 0 \text{Hz. Thin and thick lines correspond to the values of } \Delta M_f \text{ all equal to } 0.01M_{\odot} \text{ and } 0.1M_{\odot}, \text{ respectively. The dotted lines are for } f_0 = 217 \text{Hz with the difference between the thick and thin lines being the same as above. The dashed lines are for } f_0 = 1301 \text{Hz.} \]

\[ \text{Figure 2. Same as Fig.1, but with } B = 10^{15} \text{G. The correspondence between line-types and parameter values are the same as in Fig. 1. In the upper panel, the thick dashed, thin dotted and the thin dashed lines degenerated to a single line at the bottom.} \]
to be treated as a small perturbations to the force balance of the stellar matter (a back-of-envelope calculation shows that the smallest magnetic field affecting the stellar configuration more seriously is, \( B \sim 10^{17} \) G), the perturbation to \( I \) can be regarded as a linear superposition of these two contributions. As long as the stellar rotation rate is not near to the mass shedding limit, the deformation by rotation only slightly changes the moment of inertia \( I \) and has a negligible effect on the deformation which is not aligned with the rotation axis and which produces the gravitational radiation.

The timescale of the evolution of the rotational frequency is much longer than the rotational periods of the stars, so that the evolution can be regarded as secular. In this case, the characteristic amplitude of the signal is defined by (Owen et al. 1998),

\[
  h_c = h_0 \sqrt{\frac{f^2}{\Delta f}} \tag{11}
\]

where the frequency \( f \) is a secularly changing function of time. In Fig. 2, we plot \( h_c \) for several parameters of our model. The distance to the source is assumed to be 20Mpc. The curves labeled as A#B#C# are for the initial rotational frequency \( f_0 = 0, 217, 1301 \) Hz, respectively. The number after the letters distinguishes the strength of the magnetic field, i.e., “15” corresponds to \( B = 10^{15} \) G and “14” to \( B = 10^{14} \) G. The total in-falling mass \( \Delta M_{\text{fall}} \) is fixed to be \( 0.1 M_\odot \). For comparison, the noise curves of several laser interferometric detectors (LIGO, VIRGO, GEO600) are also plotted.

For all of the curves apart from (C15), the curves consist of two parts with increasing and decreasing amplitude \( h_c \) (the lower and the upper branches of the curves respectively). The reason for there sometimes being two values of \( h_c \) corresponding to a single value of \( f \) is that we are here calculating \( h_c \) as a function of time (using Eq. (11)) and not integrating over all observed times.

In general, the evolution of the system starts with a phase of increasing stellar rotation frequency, which is followed by a plateau of frequency and then the frequency decreases (see eg. the lower panel of Fig. 1). To begin with, both \( h_c \) and \( f \) increase; \( h_c \) then rises to the maximum value and then decreases again (with an associated decrease in \( f \)) tracing out the upper branch of the curves from right to left.

On the other hand, for (C15), only one branch exists. This is because in this case, the magnetic field and the initial spin frequency are so high that the propeller mechanism is active all the time.

In Fig. 3, the sensitivity curves of LIGO detectors (long-dashed lines; reproduced by the fitting formula by Owen et al. (1998)), that of VIRGO detector (short- and long-dashed line; fitting formula is obtained by J.Y. Vinet (http://www.virgo.infn.it/)), and that of GEO600 (dot and short-dashed line; fitting formula are found in Damour et al. 2001) are plotted.

In Fig 4, the value of computed \( S/N \) is plotted as a function of the initial rotation frequency for a star at a distance of 20Mpc. As expected, the \( S/N \) becomes larger with increases of the stellar magnetic field, the total mass of the in-falling matter and the initial rotation frequency of the star. With the parameters adopted here, we conclude that sources at this distance cannot be observed by LIGO III or VIRGO. However, as mentioned before, the parameter \( \beta \) which measures the deformation of the star by the magnetic field is quite uncertain. We should note again that we have here adopted a moderate choice for this parameter, \( \beta = 1 \). For instance, if this value goes up by one order of magnitude, an initially rapidly rotating \( (f \sim 500 \) Hz), highly magnetized \( (B \sim 10^{15}) \) G star can be observed at \( S/N \sim 3 \).

\section*{4 SUMMARY AND COMMENTS ON ASSUMPTIONS}

Using a simple model, we have computed the evolution of highly magnetized neutron stars with hypercritically in-falling matter just after their birth. The presence of a strong magnetic field would introduce a deformation of the star. If the axis of deformation were misaligned with the rotation axis, the object would emit gravitational waves with frequencies equal to the rotational one and its overtone. We have computed the characteristic amplitude of gravitational waves produced in this way. The results obtained indicate
that if a strong magnetic field \((B \sim 10^{15} \text{G})\) efficiently deforms the star \((\beta \sim 10)\), we could observe the resulting signal from a source in the Virgo Cluster, when matched-filtering techniques are used in up-coming detectors.

Our model is built on several assumptions. Two of them in particular should be commented on.

Firstly, we assume that the accretion flow can be well-described by a simple advection dominated disk. In the present context, the accretion rate is such that an accretion shock is developed far outside the stellar surface (Armitage & Livio 2000; see eq.(17) of Brown et al. 2000). Internal to the shock, the accretion flow can be again hypercritical. However, it is possible that the system might have a jet or an explosive outflow to remove the matter in-flowing onto the central objects (Armitage & Livio 2000). In that case, the accretion rate onto the object would be significantly reduced. At present the mechanism of formation of an outflow from an accretion disk is unknown. For instance, the standard magnetically driven outflow (Blandford 1976; Lovelace 1976) may not work because the ram pressure of the inflow here is comparable with or exceeding that of the magnetic field of the central star.

Secondly we assume that the magnetic field inside and at the surface of the star does not change in the course of the evolution of the system. The decay of neutron star magnetic fields has been an important but unsolved issue which is related to the observational data for radio pulsars and low mass X-ray binaries (Possenti 1999). To explain the different strengths of the magnetic field in ‘younger’ systems (ordinary radio pulsars) and ‘older’ ones (low mass X-ray binaries or millisecond pulsars), some kind of mechanism for field decay seem to be needed. The ‘accretion driven decay’ model and the ‘spin driven decay’ model (Possenti 1999) might both be relevant for our model system here although their applicability is not clear since they have much stronger field. Also notice that the time scale of the decay in these scenarios should be much longer than the time scale of the evolution of the system (see also Zanotti & Rezzolla 2001).

ACKNOWLEDGEMENT

The author thanks M. Colpi, K. Konno, J. Miller, L. Rezzolla, B.S. Sathyaprakash and A. Watts for useful suggestions and discussions. This research has been supported by MIUR and the EU Programme ‘Improving the Human Research Potential and the Socio-Economic Knowledge Base’ (Research Training Network Contract HPRN-CT-2000-00137).

REFERENCES

Andersson N., Kokkotas K.D., 2001, Int. J. Mod. Phys., D10, 381
Armitage P.J., Livio M., 2000, ApJ, 532, 540
Bonazzola S., Gourgoulhon E., 1996, A&A, 312, 675
Blandford R.D., 1976, MNRAS, 176, 465
Brown G.E., Lee C.-H., Bethe H.A., 2000, ApJ, 541, 918
Danour T., Iyer B.R., Sathyaprakash B.S., 2001, Phys. Rev. D63, 044023
Duncan R.C., Thompson C., 1996, in High Velocity Neutron Stars, Eds. R.E. Rothschild and R.E. Lingenfelter, New York: American Institute of Physics Press, AIP Conference Proceedings, Vol. 366
Illarionov A.F., Sunyaev R.A., 1975, A&A, 39, 185
Konno K., Obata T., Kojima Y., 2000, A&A, 356, 234
Lovelace R.V.E., 1976, Nature, 262, 649
Mineshige S., Nomura H., Hirose M., Nomoto K., Suzuki T., 1997, ApJ, 489, 227
Menou K., Esin A.A., Narayan R., Garcia M.R., Lasota J.-P., McClintock J.E., 1999, ApJ, 520, 276
Narayan R., Yi I., 1994, ApJ, 428, L13
Narayan R., Yi I., 1995, ApJ, 444, 231
Owen B.J., Lindblom L., Cutler C., Schutz B.F., Vecchio A., Andersson N., 1998, Phys. Rev., D58, 084020
Possenti A., 1999, PhD Thesis of Università degli Studi di Bologna
Pringle J.E., Rees M.J., 1972, A&A, 21, 1
Rezzolla L., Lamb F.K., Shapiro S.L., 2000, Phys. Rev. D64, 1014
Watts A.L., Andersson N., 2002, MNRAS, 333, 943
Yoshida S., Eriuchi Y., 2000, MNRAS, 316, 917
Zanotti O., Rezzolla L., 2002, MNRAS, 331, 376