Coherent perfect absorption in a weakly coupled atom-cavity system

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We study coherent perfect absorption (CPA) theoretically based on a weakly coupled atom-cavity system with an optically pumped second-order nonlinear crystal (SOC) embedded in the cavity. Our system does not require a strong coupling, which is often needed for CPA in previous studies but is challenging to implement experimentally in some systems. The role of the SOC is to introduce a tunable effective decay rate of the cavity, which can lead to CPA in the weak coupling regime. The proposed system exhibits bistable behaviors, with bistable patterns switchable between conventional and unconventional shapes. By varying the properties of the SOC, the operation point of CPA can be tuned to be inside or outside the bistable regime. It can also be located at the upper or the lower stable branch or even the unstable branch of the bistable hysteresis loop. It is however robust against the parameters of the SOC for any fixed effective decay rate. Our system can potentially be applied to realize optical devices such as optical switches in the weakly coupled regime.

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I. INTRODUCTION

It is well known that an optical cavity with gain can produce outgoing optical fields with a definite frequency and phase relationship, termed laser [1]. Applying time reversal symmetry, a cavity illuminated by two coherent incoming waves has the gain medium replaced by an absorbing medium. Once the coherent incoming waves are completely absorbed, coherent perfect absorption (CPA) [2] occurs. Hence, CPA can be regarded as the time reversed process of lasing at the threshold [2, 3]. The underlying physics of CPA is a joint action of the system dissipation and the destructive interference between the transmitted and reflected fields [2, 3]. Due to wide potential applications [5–7] in optical communications and photonic devices such as transducers, modulators, optical switches and transistors, CPA has attracted considerable interest [2–4, 8–24].

Previous studies [18–20, 25] have shown that strong coupling between light and matter is necessary for realization of CPA and the conditions under which CPA occurs cannot be fine-tuned. However, achieving the strong coupling regime is still a challenge for relevant quantum systems such as atom/spin-cavity systems, and a tunable system is always desirable in quantum computation and quantum information processing. Motivated by these, we put forward a proposal of a system exhibiting controllable CPA in the weak coupling regime. The proposed system consists of a two-level atom coupled to a cavity containing a second-order nonlinear crystal (SOC). The SOC gives rise to tunable effective decay rate of the cavity. This extends the CPA conditions in [18–20] to the weak coupling regime. In addition, we show that the proposed system under the new CPA conditions exhibits bistable behaviors. The bistable property can be switched from conventional to unconventional forms via the tuning of system parameters. Moreover, the location of the CPA point can be fine-tuned to be inside or outside the bistable region, and it can locate at an arbitrary branch of the bistable hysteresis loop. We also show that the location of the CPA point is robust against the parameters of the SOC for a given effective decay rate.

The paper is organized as follows. In Sec. II, the model is formulated theoretically and the Hamiltonian is explained. Then we apply quantum Langevin equations to derive steady-state properties of the system in Sec. III. In Sec. IV, the CPA criterion is given and impacts of the system parameters are discussed. In Sec. V, we give a numerical study on the CPA behaviors. Finally, a conclusion is given in Sec. VI.

II. MODEL AND HAMILTONIAN

We consider a quantum system consisting of a two-level atom coupled to an optical cavity containing a SOC. The cavity is formed by two partially transmitting (reflecting) mirrors. Each mirror is exerted by an external driving field [see Fig. 1]. The system Hamiltonian can be written as (setting $\hbar = 1$)

$$H_{\text{sys}} = H_0 + H_I + H_{\text{nl}} + H_d.$$  (1)
Here, $H_0 = \omega_c c^\dagger c + \omega_{\text{TLS}} \sigma_z$ is the total free energy of the cavity and the two-level atom, where $\omega_c$ is the angular frequency of the cavity mode and $\omega_{\text{a}}$ is the transition frequency between the ground state $|g\rangle$ and the first excited state $|e\rangle$ of the two-level atom. Also, $c^\dagger (c)$ is the creation (annihilation) operator of the cavity field, and $\sigma_z = \frac{1}{2} (\sigma^+_z + \sigma^-_z)$ is the Pauli-$z$ operator, where $\sigma^+_z = |e\rangle\langle g|$ ( $\sigma^-_z = |g\rangle\langle e|$) is the raising (lowering) operator of the atom.

The term $H_1$ in Eq. (1) represents the interaction Hamiltonian between the cavity and the two-level atom. Under the rotating wave approximation (i.e., neglecting the fast-oscillating terms $c^\dagger \sigma^+_z + c \sigma^-_z$), $H_1$ reads

$$H_1 = g (c^\dagger \sigma^-_z + c \sigma^+_z),$$

where $g = d\sqrt{\omega_c/2\pi\epsilon_0V_0}$ is the coupling strength with $d$ being the dipole momentum, $\epsilon_0$ the vacuum permittivity, and $V_0$ the cavity mode volume.

The Hamiltonian $H_{\text{nl}}$ in Eq. (1) denotes the interaction between the SOCl and the cavity. By pumping an external field onto the SOC, the nonlinear interaction Hamiltonian $H_{\text{nl}}$ can be written as

$$H_{\text{nl}} = i(Gc^\dagger e^{-i\omega_p t} - G^* c e^{i\omega_p t}),$$

where the parameter $G$ is the effective nonlinear coefficient, proportional to both the original nonlinear coefficient of the SOC and the amplitude of the pumping field with frequency $\omega_p$ and $G^*$ is the complex conjugate of $G$. The Hamiltonian in Eq. (3) can be used to study the phenomena of squeezing effects [28] and coupling amplification [29,30].

The last term $H_d$ in Eq. (1) describes the mirrors of the cavity driven by two external fields with frequencies $\omega_l$ and $\omega_r$, respectively (see Fig. 1). The Hamiltonian $H_d$ takes the form of

$$H_d = i(\Omega_l e^{-i\omega_l t} + \Omega_r e^{-i\omega_r t})c^\dagger + \text{H.c.},$$

where $\Omega_l = \sqrt{\kappa_l}$ and $\Omega_r = \sqrt{\kappa_r}$ are the Rabi frequencies of the left and right driving fields, respectively. Here, $\kappa_l = \Gamma_l/\tau$ is the decay rate of the left (right) mirror cavity, with $T_l$ being the left (right) mirror transmission and $\tau$ the photon round trip time inside the cavity. In addition, $c_{l}^{\dagger}$ and $c_{r}^{\dagger}$ are the amplitudes of the left and right driving fields.

In the rotating frame with respect to the frequency $\omega_r$ of the right driving field, the system Hamiltonian in Eq. (1) becomes

$$\hat{H} = U^\dagger H_{\text{sys}} U - iU^\dagger \partial_t U$$

$$= \Delta_c c^\dagger c + \Delta_{\text{TLS}} \sigma_z + g(c^\dagger \sigma^-_z + c \sigma^+_z)$$

$$+ i(Gc^\dagger e^{i\omega_p t} - G^* c e^{-i\omega_p t}) + i(\Omega_d c^\dagger - \Omega_d^* c),$$

where $U = \exp[-i(\omega_r t)]$ is a unitary transformation operator, $\Delta_c = \omega_c - \omega_r$ is the frequency detuning of the cavity field from the right driving field, and $\Delta_{\text{TLS}} = \omega_{\text{TLS}} - \omega_r$ is the frequency detuning of the two-level atom from the right driving field. The coupling parameters are given by $G = G e^{-i(\omega_p - 2\omega_r)t}$ and $\Omega_d = \Omega e^{-i(\omega_l - \omega_r)t} + \Omega_r$. Here we choose the right driving field as the reference field, so for convenience we set the phase of the reference field to be zero.

Furthermore, we consider the situation that two external fields are resonant and the frequency of the pumping field on the SOC is twice of that of the right driving field, i.e., $\omega_l = \omega_r$ and $\omega_p = 2\omega_r$. The latter condition physically means that a pair of degenerate photons with frequency $\omega_r$ can be obtained when the SOC is illuminated by a field with frequency $\omega_p$. These two conditions directly lead to $\Omega_d = \Omega_l + \Omega_r$ and $G = G_l$, respectively. Therefore, the time-dependent Hamiltonian in Eq. (5) reduces to

$$H = \Delta_c c^\dagger c + \Delta_{\text{TLS}} \sigma_z + g(c^\dagger \sigma^-_z + c \sigma^+_z)$$

$$+ i(Gc^\dagger e^{i\omega_p t} - G^* c e^{-i\omega_p t}) + i(\Omega_d c^\dagger - \Omega_d^* c),$$

which is a time-independent Hamiltonian. Note that the above Hamiltonian can be simulated by a superconducting circuit coupled to a nitrogen-vaccancy center in diamond, where the coupling between the cavity and the atom can be amplified exponentially [29].

III. STEADY-STATE INTRACAVITY FIELD

Using the Heisenberg-Langevin approach, the quantum dynamics of the considered system as described by the Hamiltonian (6) can be governed by the following quantum Langevin equations within Markov approximation:

$$\frac{dc(t)}{dt} = - \kappa + i\Delta_c c - ig \sigma_z - 2Ge^\dagger + \Omega_d + c_{\text{in}}(t),$$

$$\frac{d\sigma_{\text{z}}(t)}{dt} = - \gamma/2 + i\Delta_{\text{TLS}} \sigma_z + 2igc \sigma_z + \sigma_{\text{in}}^\dagger(t),$$

$$\frac{d\sigma_{\text{in}}(t)}{dt} = - \gamma(\sigma_z + 1/2) + ig(c^\dagger \sigma_z - c \sigma^+_z) + \sigma_{\text{in}}^\dagger(t).$$

Here, $\kappa = \kappa_l + \kappa_r$ is the total decay rate of the cavity mode, where $\kappa_l$ ( $\kappa_r$) is the external decay rate of
the left (right) mirror of the cavity, $\gamma$ is the decay rate of
the two-level atom, while $c_{in}(t)$, $\sigma_{in}(t)$ and $\sigma_{in}^\dagger(t)$ are
quantum input noises. In the long-time limit, the average
values of these input noises and the time-derivatives of the
mean values of the system operators vanish, i.e.,
$\langle c_{in}(t) \rangle = \langle \sigma_{in}(t) \rangle = \langle \sigma_{in}^\dagger(t) \rangle = 0$ and $d(c(t))/dt = d(\sigma(t))/dt = d(\sigma^\dagger(t))/dt = 0$. Then, we have the
following coupled equations for $\langle c(t) \rangle$, $\langle \sigma(t) \rangle$ and $\langle \sigma^\dagger(t) \rangle$:

$$-\kappa/2 + i\Delta c(c) - ig\langle \sigma \rangle + 2G\langle c \rangle + \Omega_d = 0,$$

$$-(\gamma/2 + i\Delta_{TLS}(\sigma \rangle - 2g(\sigma c) = 0,$$

$$-\gamma(\sigma + 1/2) + ig((\sigma^\dagger - \sigma^\dagger c) = 0.$$

Using the mean-field approximation, the terms $\langle \sigma \rangle$,
$\langle c^\dagger \sigma \rangle$ and $\langle \sigma^\dagger \sigma \rangle$ in Eqs. (11) and Eq. (12) can be
written respectively as $\langle \sigma \rangle = \langle c \rangle \langle \sigma \rangle$, $\langle c^\dagger \sigma \rangle = \langle c^\dagger \rangle \langle \sigma \rangle$ and $\langle \sigma^\dagger \sigma \rangle = \langle \sigma^\dagger \rangle \langle \sigma \rangle$. Then the degrees of freedom of
the two-level atom can be eliminated by solving Eq. (10)
and Eq. (11), after applying conjugation. Thus, Eqs. (10) - (12) can be further reduced to

$$-(\kappa_0 + i\Delta_0)\langle c \rangle + 2G\langle c \rangle + \Omega_d = 0,$$

$$-(\kappa_0 - i\Delta_0)\langle c^\dagger \rangle + 2G^\ast \langle c^\dagger \rangle + \Omega_d^\ast = 0,$$

where

$$\kappa_0 = \frac{\kappa}{2} + \frac{g^2\gamma/2}{\gamma^2/4 + \Delta^2_{TLS} + 2g^2n_c},$$

$$\Delta_0 = \Delta_c - \frac{g^2\Delta_{TLS}}{\gamma^2/4 + \Delta^2_{TLS} + 2g^2n_c}.$$

Here, $\kappa_0$ and $\Delta_0$ can be interpreted as atom-induced
effective cavity linewidth and frequency. Obviously, both
depend on the average photon number $n_c = \langle c^\dagger c \rangle$ in
the cavity. From Eqs. (13) and (14), the steady-state solution
of the intracavity field can easily be obtained as

$$\langle c \rangle = \frac{(\kappa_0 - i\Delta_0)\Omega_d + 2G\Omega_d^\ast}{\kappa_0^\ast + \Delta_0^\ast - 4|G|^2}.$$

As the average photon number $n_c$ in Eq. (17) depends nonlinearly on the system parameters, it may exhibit a
bistability as one varies, for example, the amplitudes of the driving fields at the mirrors.

IV. CPA CRITERION

Below we focus our interest on the dependence of the
steady-state output fields on the driving fields. Using
standard input-output theory [32], the steady-state output
fields from the two mirrors of the cavity can be expressed as

$$\langle c_{out}^\dagger \rangle = \sqrt{n}_l \langle c \rangle - c_{in}^\dagger,$$

$$\langle c_{out} \rangle = \sqrt{n}_r \langle c \rangle - c_{in}^\dagger.$$

When CPA occurs, the input fields are totally absorbed
by the coupled atom-cavity system so that $\langle c_{out} \rangle = \langle c_{out}^\dagger \rangle = 0$. This directly leads to

$$\frac{c_{in}^\dagger}{c_{in}} = \sqrt{\kappa_l/\kappa_r},$$

It expresses a constraint that the two input fields and the
two decay rates of mirrors must satisfy before CPA
can be realized. Also, Eq. (20) shows that the two input fields
must be in phase. For simplicity, $\kappa_l = \kappa_r = \kappa/2$ is
assumed in the following. This assumption gives rise to

$$\langle c_{in}^\dagger \rangle = \langle c_{in} \rangle = \langle c_{out} \rangle$$

according to Eq. (20) and thus also

$$\Omega_l = \Omega = \Omega_d/2.$$

Note that the condition in Eq. (20) for CPA is necessary
but not sufficient. To derive the necessary conditions, we set $\langle c_{out} \rangle = 0$ in Eq. (18) [or equivalently,
$\langle c_{out}^\dagger \rangle = 0$ in Eq. (19)]. Then

$$\sqrt{\kappa/2}\langle c \rangle = c_{in},$$

i.e.,

$$\kappa\langle c \rangle = \Omega_d.$$  (22)

Eq. (22) further gives $\text{Re}[\langle c \rangle] = \Omega_d/\kappa$ and $\text{Im}[\langle c \rangle] = 0$.
Using Eq. (17), we obtain

$$\beta = \frac{\Delta_c}{\gamma/2\Delta_{TLS}},$$

$$n_c = \frac{1}{4} \frac{\gamma - \sqrt{\gamma^2 + 4\Delta^2_{TLS}}}{\beta G^2},$$

where $\Delta_c = \Delta_c - 2|G|\sin \phi$ and $G = |G|e^{i\phi}$ with $\phi$ being
the relative phase of the pumping field with respect to the
reference field, and

$$\beta = \kappa/2 + 2|G|\cos \phi.$$

Obviously, the parameter $\beta$ is tunable via the strength
$|G|$ and the relative phase $\phi$ of the pumping field. In addition,
conditions in Eqs. (23) and (24) naturally satisfy
Eq. (20) since they are directly deduced from the condition
$\langle c_{in}^\dagger \rangle = 0$ (or $\langle c_{out}^\dagger \rangle = 0$). Therefore, a necessary
condition of CPA is that Eqs. (23) and (24) are simultaneously
valid. By comparing conditions in Eqs. (23) and (24)
with conditions obtained in Ref. [18], the effective CPA
Hamiltonian of the system with an effective cavity
frequency $\Delta_c'$ and a decay rate $\beta$ can be written as

$$H_{eff} = \Delta_c'c^\dagger c + \Delta_{TLS}\sigma_z + g(c\sigma^- + c\sigma^+) + \Omega_d/2.$$

Eq. (24) also gives a constraint on the coupling $g$ between
the two-level system and the cavity for any given value of
the detuning $\Delta_{TLS}$ [see Fig. 2(a)]. Specifically, the mean intracavity photon number is positive, i.e., $n_c > 0$. Hence one requires that

$$g > g_c(\beta, \Delta_{TLS}) \equiv \frac{1}{2} \sqrt{\beta(\gamma + 4\Delta^2_{TLS}/\gamma)}.$$  (27)
atom/spin-cavity systems since $\gamma < \kappa$. Then the condition in Eq. (28) becomes $g^2/\kappa\gamma > 0.01$. CPA can thus occur over a wide parameter range satisfying $g^2/\kappa\gamma < 1$, corresponding to the weak coupling regime. This shows that CPA can indeed occur in the weak coupling regime for our setup. At present, realizing a strong coupling between a single two-level system (e.g., a nitrogen vacancy center in diamond) and a cavity or a superconducting circuit is still a challenge. Therefore, exploring optical phenomena in weakly coupled quantum systems is of great significance.

Eq. (24) does not only limit the coupling strength $g$ for the occurrence of CPA, but also gives a constraint on the detuning $\Delta_{TLS}$. For a given $g$, CPA can only be observed when $|\Delta_{TLS}| < \sqrt{\frac{1}{2} |\frac{g^2}{\beta} - \frac{\gamma^2}{\kappa} - \frac{\Delta_{TLS}^2}{\gamma^2} |}$ [see the light blue region in Fig. 2(b)]. The dashed blue curve represents the critical detuning $\Delta_{TLS}$ against the parameter $\beta$. From Fig. 2(b), we see that $\Delta_{TLS}$ can vary in a broader range than in Ref. [19] for a fixed $\kappa$. This results from the introduction of the controllable parameter $\beta$. As mentioned above, $\beta$ is required to be very small, so a large $\Delta_{TLS}$ is needed, leading to a small mean photon number $n_c$ according to Eq. (24). Therefore, weak input fields are sufficient to achieve CPA with our setup. This greatly simplifies experimental implementations.

V. NUMERICAL RESULTS

We now numerically study CPA based on our setup in the weak coupling regime. We put $g = \gamma$, $\kappa = 20\gamma$, and $\beta = 0.02\gamma$, so that $g^2/\kappa\gamma < 1$, $g^2/\beta\gamma > 1$ and $\Delta_{TLS} \approx 4.975\gamma$. This indicates that CPA can only be observed for $|\Delta_{TLS}| \in [0, 4.975]$. As CPA can only occur when both Eqs. (23) and (24) are simultaneously satisfied, the parameter $\Delta_{c}$ can be solved in particular using Eq. (23). Adopting these parameters, we numerically solve Eqs. (17) to (20) to obtain the average photon number $n_c$, which is then substituted into Eqs. (18) and (19) to obtain the output intensity $|c_{\text{out}}|^2$. Fig. 3 plots the output intensity as a function of the input intensity $|c_{\text{in}}|^2$ for frequency detuning $\Delta_{TLS} = 4.5\gamma$ and $1.5\gamma$ under various conditions.

Fig. 3(a) shows results for $|G| = 9.98\gamma$ and $\phi = 2/3\pi$. We observe that the output intensity exhibits a bistability with respect to the input intensity. By varying the parameter $\Delta_{TLS}$, the bistable pattern can be changed from a conventional [inset in Fig. 3(a)] to an unconventional [blue curve in Fig. 3(a)] shape. When the average intracavity photon number satisfies Eq. (24) corresponding to the green dots $A_1$ and $A_2$, CPA is predicted. They are both located at the upper branch of bistable pattern. Therefore, the CPA conditions are outside the bistable region.

By vary the relative phase $\phi$ from $2/3\pi$ to $4/3\pi$ as shown in Fig. 3(b), the bistable pattern of the output intensity with respect to the input intensity becomes ro-
FIG. 3: (Color online) The output intensity as a function of the input intensity with $\Delta_{\text{TLS}} = 4.5\gamma$ and $1.5\gamma$. The relative phase is (a) $\phi = 2/3\pi$, (b) $\phi = 4/3\pi$, and (c) $\phi = \pi$.

We further study the case of $\phi = \pi$ and results are shown in Fig. 3(c). To ensure $\beta = 0.02\gamma$, we have considered an decreased amplitude $|G| = 4.99\gamma$ of the pumping field on the SOC. The output intensity also exhibits a bistable behavior with respect to the input intensity with conventional bistable patterns for both values of $\Delta_{\text{TLS}}$ studied. In addition, the CPA points are located inside the bistable regime in the stable branches [see points $A_1$ and $B_1$ in inset of Fig. 3(c)].

Fig. 3 shows that the CPA points are blue-shifted upon decreasing the frequency detuning of the TLS. Also, their locations are unaffected by the parameters $|G|$ and $\phi$ for fixed $\Delta_{\text{TLS}}$ [see red and blue curves in Fig. 3]. Hence, the location of CPA point is robust against the parameters of the SOC. These results also follow directly from Eq. (24).

We emphasize differences between our system for realizing CPA from the previously studied ones [18–20, 25]. First, CPA can occur in our case in the weak coupling regime and the system exhibits bistable behaviors. Second, we have shown that the bistable pattern can be changed from the conventional to unconventional shape and CPA point can be tuned to appear at the upper or lower stable branch or even at the unstable branch. Third, the location of CPA point is robust against the parameters of the SOC when the effective decay rate $\beta$ is fixed and it can be either inside or outside the bistable region.

VI. CONCLUSION

In summary, we have given a detailed study on CPA in a two-level atom weakly coupled to a cavity embedded with a SOC. Under CPA conditions, the system behaves as a two-level system coupled to a cavity with a tunable effective bandwidth. The output field intensity exhibits a bistability. By tuning system parameters such as the frequency detuning of two-level atom from the input driving field, the coefficient of the nonlinearity crystal and its relative phase, the bistable pattern can be switched from conventional to unconventional sharps or vice versa. Due to the effect of the SOC on the effective frequency $\Delta'_c$, the operation point of CPA can be switched between two stable branches. However, the location of CPA point is robust against the parameters of the SOC when the effective decay rate $\beta$ is fixed. Our study provides a novel way to realize future optical devices utilizing CPA in the weak coupling regime.

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