D-branes in pp-wave spacetime with nonconstant NS-NS flux

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ABSTRACT

We find classical solutions of $D$-branes in pp-wave spacetime with nonconstant $NS – NS$ flux. We also present $Dp – Dp'$ bound state solutions in this background. We further analyze the supersymmetric properties of these brane solutions by solving the type IIB killing spinor equations explicitly.

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1 Introduction

Study of string theory in pp-wave background has been a subject of much interest for theoretical physicists recently. It is known that pp-wave spacetime yields exact classical backgrounds for string theory, since all curvature invariants and therefore all $\alpha'$ corrections, vanish [1,2]. Hence pp-wave spacetimes correspond to exact conformal field theory backgrounds. These backgrounds are shown to be exactly solvable in light cone gauge [3–5]. Many of these are obtained from $AdS_p \times S^q$ type geometries in Penrose limit and are also maximally supersymmetric [6,7]. Strings in pp-wave background are also investigated to establish the duality, between the supergravity modes and the gauge theory operators in the large R-sector of the gauge theory [8].

PP-wave backgrounds with nonconstant flux are also studied recently [9–13]. The worldsheet theory is described by nonlinear sigma model. These theories are the non-trivial examples of interacting theories in light cone gauge. The sigma model with $RR$ five form flux is supersymmetric and one can have linearly realized ‘supernumary’ supersymmetries in these backgrounds [14]. The corresponding sigma model with 3-form NS-NS and RR flux is non-supersymmetric in general [10], unless there exists some target space isometry and corresponding Killing vector fields, which ensure the worldsheet supersymmetry [13]. Also the supernumary supersymmetries are absent in this case due to structure of gamma matrices. The bosonic part is same in both cases and the two models share many properties, e.g. both are exact string theory backgrounds and integrability structure is same etc. The classical solutions of $D$-branes in pp-wave background with constant NS-NS and RR flux are already discussed in literature [15–22]. $Dp$-branes from the worldsheet point of view are constructed in [23]. Supersymmetric properties of $D$-branes in these backgrounds are analyzed both from supergravity and worldsheet point of view. Recently, supersymmetric properties of $D$-branes in these backgrounds are realized more rigorously [24].

In earlier work, we found some supersymmetric solutions of $D$-branes along with its open string spectrum in pp-wave background, arising from $AdS_3 \times S^3$ geometry, with constant three form flux [20]. Keeping in view of the importance of $D$-branes in understanding the nonperturbative as well as duality aspects of string theory, it is useful to study them in more general backgrounds with flux being turned on. In this paper, we study the classical solutions of $D$-branes in pp-wave spacetimes with nonconstant three form $NS – NS$ flux.

The plan of the paper is as follows. In next section, we present classical solutions of $Dp$ as well as $Dp – Dp'$ bound states. Section-3 is devoted to supersymmetric properties of the $D$-brane solutions constructed in section-2. We conclude in section-4 with some remarks.
2 Supergravity solutions

In this section we present classical solutions of $Dp$ as well as $Dp-Dp'$-branes with nonconstant NS-NS three form flux transverse to brane worldvolume. We start by writing down the supergravity solutions of $D$-string in pp-wave background with nonconstant NS-NS three form flux. The metric, dilaton and the field strengths are given by:

\[
\begin{align*}
    ds^2 &= f_1^{-\frac{1}{2}} \left( 2dx^+dx^- + K(x_i)(dx^+)^2 \right) \\
        &+ f_1^\frac{1}{2} \sum_{m=1}^{8} (dx^m)^2, \quad (i = 1, ..., 4), \\
    H &= \partial_1 b_2(x_i) \; dx^+ \wedge dx^1 \wedge dx^2 + \partial_3 b_4(x_i) \; dx^+ \wedge dx^3 \wedge dx^4, \\
    e^{2\phi} &= f_1, \quad F_{+ - n} = \partial_n f_1^{-1},
\end{align*}
\]

with $b(x_i)$ and $K(x_i)$ satisfying the equations $\Box b(x_i) = 0$ and $\Box K(x_i) = -(\partial_i b_j)^2$ respectively and $f_1 = 1 + \tfrac{Q_1}{r^6}$ is the harmonic function in the transverse space. We have checked that the above solution satisfies type IIB field equations. For constant three form flux this solution reduces to that of ref [20]. All other $Dp$-brane ($p = 2, ..., 5$) solutions can be found out by applying $T$-duality along $x^5, ..., x^8$ directions. For example: the classical solution for a system of $D3$-brane in such a background is given by:

\[
\begin{align*}
    ds^2 &= f_3^{-\frac{1}{2}} \left( 2dx^+dx^- + K(x_i)(dx^+)^2 + (dx^7)^2 + (dx^8)^2 \right) \\
        &+ f_3^\frac{1}{2} \sum_{m=1}^{6} (dx^m)^2, \quad (i = 1, ..., 4), \\
    H &= \partial_1 b_2(x_i) \; dx^+ \wedge dx^1 \wedge dx^2 + \partial_3 b_4(x_i) \; dx^+ \wedge dx^3 \wedge dx^4, \\
    e^{2\phi} &= 1, \quad F_{+ - 78n} = \partial_n f_3^{-1},
\end{align*}
\]

with $b(x_i)$ and $K(x_i)$ satisfying the equations $\Box b(x_i) = 0$ and $\Box K(x_i) = -(\partial_i b_j)^2$ respectively and $f_3 = 1 + \tfrac{Q_3}{r^4}$ is the harmonic function satisfying the Green function equation in the transverse space.

Now we present classical solution of $D1-D5$ system as an example of $p-p'$ bound state in these background. The supergravity solution for a such system is given by:

\[
\begin{align*}
    ds^2 &= (f_1 f_5)^{-\frac{1}{2}} \left( 2dx^+dx^- + K(x_i)(dx^+)^2 \right) + \left( \frac{f_1}{f_5} \right)^{\frac{1}{2}} \sum_{m=5}^{8} (dx^m)^2
\end{align*}
\]
\[ + (f_1 f_5)^4 \sum_{i=1}^{4} (dx^i)^2, \]
\[ e^{2\phi} = \frac{f_1}{f_5}, \]
\[ H = \partial_1 b_2(x_i) \, dx^+ \wedge dx^1 \wedge dx^2 + \partial_3 b_4(x_i) \, dx^+ \wedge dx^3 \wedge dx^4, \]
\[ F_{+i} = \partial_i f_1^{-1}, \quad F_{ijk} = \epsilon_{ijkl} \partial_l f_5, \quad (2.3) \]

with \( b(y_j) \) and \( K(x_i) \) satisfying the equations \( \Box b(x_i) = 0 \) and \( \Box K(x_i) = - (\partial_i b_j)^2 \) respectively and \( f_1 = 1 + \frac{Q_1}{r^2} \) and \( f_5 = 1 + \frac{Q_5}{r^2} \) are the harmonic functions of \( D1 \) and \( D5 \)-branes in common transverse space. One can check that the above ansatz do satisfy type IIB field equations.

3 Supersymmetry Analysis

In this section we present the supersymmetry of the solutions described earlier in section (2).

The supersymmetry variation of dilatino and gravitino fields of type IIB supergravity in ten dimension, in string frame, is given by \([25, 26]\):

\[ \delta \lambda_\pm = \frac{1}{2} (\Gamma^\mu \partial_\mu \phi \mp \frac{1}{12} \Gamma^{\mu \nu \rho} H_{\mu \nu \rho}) \epsilon_\mp + \frac{1}{2} e^{\phi}(\pm \Gamma^M F^{(1)}_M + \frac{1}{12} \Gamma^{\mu \nu \rho} F^{(3)}_{\mu \nu \rho}) \epsilon_\mp, \quad (3.1) \]
\[ \delta \Psi^\pm_\mu = \left[ \partial_\mu + \frac{1}{4} (w^\mu_{\hat{a} \hat{b}} \mp \frac{1}{2} H^\mu_{\hat{a} \hat{b}}) \Gamma^\hat{a} \hat{b} \right] \epsilon_\pm \]
\[ + \frac{1}{8} e^{\phi} \left[ \mp \Gamma^\mu F^{(1)}_\mu \mp \frac{1}{3} \Gamma^{\mu \nu \rho} F^{(3)}_{\mu \nu \rho} \mp \frac{1}{2.5} \Gamma^{\mu \nu \rho \sigma} F^{(5)}_{\mu \nu \rho \sigma} \right] \Gamma_\mu \epsilon_\mp, \quad (3.2) \]

where we have used \((\mu, \nu, \rho)\) to describe the ten dimensional space-time indices, and hat’s represent the corresponding tangent space indices. Solving the above two equations for D-string solution as given in (2.1), we get several conditions on the spinors. First, the dilatino variation gives:

\[ \frac{f_{1, \hat{a}}}{f_1} \Gamma^{\hat{a}} \epsilon_\mp \mp f_1^{-1} (\partial_i b_j) \Gamma^{+\hat{i} \hat{j}} \epsilon_\mp - \frac{f_{1, \hat{a}}}{f_1} \Gamma^{+\hat{a}} \epsilon_\mp = 0, \quad (3.3) \]

Gravitino variation gives the following conditions on the spinors:

\[ \delta \psi^\pm \equiv \left( \partial_+ + \frac{1}{4} \hat{a} (K f_1^{-\frac{1}{2}}) \Gamma^{+ \hat{a}} \mp \frac{1}{4} (\partial_i b_j) (\Gamma^{ij}) \right) \epsilon_\mp \]
\[ -\frac{1}{8} \Gamma^{+\hat{n}} K f_{1\hat{n}} \frac{f_{5/4}}{f_1} \Gamma^{+} \epsilon_\pm = 0 \] (3.4)

\[ \delta \psi_\pm \equiv \partial_\mp \epsilon_\pm = 0 \] (3.5)

\[ \delta \psi_n \equiv \left( \partial_n - \frac{1}{8} f_{1n} \right) \epsilon_\pm, \quad (n = 5, \ldots, 8) \] (3.6)

\[ \delta \psi_i \equiv \left( \partial_i + \frac{\delta \epsilon_\pm}{4} f_{1i}^2 (\partial_i b_j) \Gamma^{i\hat{j}} - \frac{1}{8} f_{1i} \right) \epsilon_\pm, \quad (i = 1, \ldots, 4). \] (3.7)

In writing the above equations we have used the brane supersymmetry condition:

\[ \Gamma^{+} \epsilon_\pm = \epsilon_\mp. \] (3.8)

Taking derivative of the eqn. (3.7) with respect to \( \partial_k \) and subtracting the derivative of \( \partial_k \) equation with respect to \( \partial_i \), we get

\[ (\partial_k \partial_i b_j) \Gamma^{i\hat{j}} \epsilon_\pm = 0, \] (3.9)

which can be satisfied for nonconstant \( \partial_i b_j \) only if \( \Gamma^{i} \epsilon_\pm = 0 \). Using \( \Gamma^{+} \epsilon_\pm = 0 \), and brane supersymmetry condition (3.3), the dilatino variation (3.3) is satisfied. Using \( \Gamma^{+} \epsilon_\pm = 0 \), the supersymmetry conditions (3.6) and (3.7) are solved by spinors:

\[ \epsilon_\pm = \exp(-\frac{1}{8} \ln f_1) \epsilon^0_\pm, \] with \( \epsilon^0_\pm \) being a function of \( x^+ \) only. Since \( \epsilon^0_\pm \) is independent of \( x^i \) and \( (\partial_i b_j) \) is a function of \( x^i \) only, the gravitino variation gives the following conditions to have nontrivial solutions:

\[ (\partial_i b_j)(\Gamma^{i\hat{j}}) \epsilon^0_\pm = 0 \] (3.10)

and

\[ \partial_+ \epsilon^0_\pm = 0. \] (3.11)

The condition \( \Gamma^{+} \epsilon_\pm = 0 \) breaks sixteen supersymmetries. The number of remaining supersymmetries depend on the existence of constant \( \epsilon^0_\pm \) solutions of the equation (3.10). For the particular case when \( H_{12} = H_{34} \), the equation (3.10) gives the condition:

\[ (1 - \Gamma^{i\hat{i}3\hat{4}}) \epsilon^0_\pm = 0. \] (3.12)

Therefore in this case, the \( D \)-string solution (2.1), preserves 1/8 supersymmetry. Similarly, one can show that the \( D3 \)-brane solution (2.2) also preserves 1/8 supersymmetry.

Next, we will analyze the supersymmetry properties of \((D1 - D5)\) system that is described in eqn. (2.3) of the previous section.
The dilatino variation gives the following conditions on the spinors:

\[
\frac{f_{1\dot{i}}}{f_1} \left( \Gamma^i \epsilon_+ - \Gamma^{\dot{i} \dot{j}} \epsilon_+ \right) + (f_1 f_5)^{\frac{1}{4}} (\partial_i b_j) \Gamma^i \epsilon_+ - \frac{f_{5\dot{i}}}{f_5} \left( \Gamma^i \epsilon_+ + \frac{1}{3!} \epsilon_{ijkl} \Gamma^{jkl} \epsilon_+ \right) = 0 \tag{3.13}
\]

On the other hand, the gravitino variation gives:

\[
\delta \psi_+^{\pm} \equiv \partial_+ \epsilon_+ + \frac{1}{4} \partial_\alpha (K(f_1 f_5)^{-\frac{1}{4}}) \Gamma^\alpha \epsilon_+ + \frac{1}{4} (\partial_i b_j) \Gamma^{ij} \epsilon_+ \\
- \frac{1}{8} (f_1 f_5)^{-\frac{1}{4}} \left( \Gamma^{\dot{i} \dot{j}} \Gamma_{\dot{i} \dot{j}} \epsilon_+ \right) + \frac{1}{8} \left[ \Gamma^i \epsilon_+ + \frac{f_1 f_5}{f_1 + f_5} \right] \epsilon_+ = 0 \tag{3.14}
\]

\[
\delta \psi_-^{\pm} \equiv \partial_- \epsilon_+ = 0, \quad \delta \psi_m^{\pm} \equiv \partial_m \epsilon_+ = 0, \tag{3.15}
\]

\[
\delta \psi_i^{\pm} \equiv \partial_i \epsilon_+ - \frac{\delta_\alpha}{4} (f_1 f_5)^{\frac{1}{4}} (\partial_i b_j) \Gamma^{ij} \epsilon_+ + \frac{1}{8} \left[ \Gamma^i \epsilon_+ + \frac{f_1 f_5}{f_1 + f_5} \right] \epsilon_+ = 0. \tag{3.16}
\]

In writing down the above gravitino variations we have once again made use of the brane conditions:

\[
\Gamma^{i} \epsilon_+ - \Gamma^{\dot{i} \dot{j}} \epsilon_+ = 0, \tag{3.17}
\]

and

\[
\Gamma^i \epsilon_+ + \frac{1}{3!} \epsilon_{ijkl} \Gamma^{jkl} \epsilon_+ = 0. \tag{3.18}
\]

Taking derivative of the eqn. (3.16) with respect to \( \partial_k \) and subtracting the derivative of \( \partial_k \) equation with respect to \( \partial_i \), we get

\[
(\partial_k \partial_i b_j) \Gamma^{ij} \epsilon_+ = 0, \tag{3.19}
\]

which can be satisfied for nonconstant \( \partial_i b_j \) only if \( \Gamma^i \epsilon_+ = 0 \).

Using \( \Gamma^i \epsilon_+ = 0 \) and brane supersymmetry conditions (3.17) and (3.18), the dilatino condition (3.13) is satisfied. Using \( \Gamma^i \epsilon_+ = 0 \), the supersymmetry condition (3.16) is solved by spinors: \( \epsilon_+ = \exp(-\frac{1}{2} \ln(f_1 f_5)) \epsilon_0^+ \), with \( \epsilon_0^+ \) being a function of \( x^+ \) only. Since \( \epsilon_0^+ \) is independent of \( x^i \) and \( (\partial_i b_j) \) is a function of \( x^i \) only, the gravitino variation gives the following conditions to have nontrivial solutions:

\[
(\partial_i b_j) (\Gamma^{ij}) \epsilon_0^+ = 0 \tag{3.20}
\]

and

\[
\partial_+ \epsilon_0^+ = 0. \tag{3.21}
\]

Once again, the number of supersymmetries depend on the existence of solutions of equation (3.20). For the particular case when \( H_{+12} = H_{+34} \), the \( D1 - D5 \) bound state solution (2.3) also preserves 1/8 supersymmetry.
4 Conclusion

In this paper we have constructed the supergravity solutions of $Dp$ as well as $Dp-Dp'$ branes in pp-wave background with nonconstant $NS-NS$ flux. The supersymmetric properties of these solutions are also verified by analyzing the type IIB killing spinor equations explicitly. All the solutions presented here are shown to preserve 1/8 supersymmetry and the supernumary supersymmetry is absent for the background presented in this paper. $D$-brane solutions with nonconstant $RR$ flux can be found out by applying $S$-duality transformation on the solutions presented here, which will be generalization of those given in [20]. The $D$-brane solutions presented here have the interpretation of $D$-branes in nonsupersymmetric sigma model of [10]. It is also desirable to analyze them from the worldvolume point of view following the procedure of [11].

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