Extracting $|V_{bc}|$, $m_c$ and $m_b$ from Inclusive $D$ and $B$ Decays

Michael Luke$^{ab}$ and Martin J. Savage$^{bc}$

a) Department of Physics, University of Toronto, Toronto, Canada M5S 1A7
b) Department of Physics, University of California, San Diego, La Jolla CA 92037
c) Department of Physics, Carnegie Mellon University, Pittsburgh PA 15213

Abstract

Using recent results for nonperturbative contributions to the $B$ and $D$ meson inclusive semileptonic widths, a model independent extraction of $|V_{bc}|$, $m_c$ and $m_b$ is made from the experimentally measured $B$ and $D$ lifetimes and semileptonic branching ratios. Constraining the parameters of the HQET at $\mathcal{O}(1/m_Q^2)$ by the $D$ semileptonic width, $|V_{bc}|$ is found to lie in the range $0.040 < |V_{bc}| < 0.057$. The $c$ and $b$ quark masses are not well constrained due to uncertainty in the relevant scale of $\alpha_s$. These results assume the validity of perturbative QCD at the low scales relevant to semileptonic charm decay. Without making this assumption, somewhat less stringent bounds on $V_{bc}$ from $B$ decay alone may be obtained.
There has been much recent interest in the application of the techniques of the heavy quark effective theory (HQET) \cite{1}–\cite{3} to inclusive decays of hadrons containing a single c or b quark \cite{3}–\cite{12}. As first made explicit in \cite{3}, the differential decay rate for an inclusive factorisable process such as $B \rightarrow X_c e\nu_e$ or $B \rightarrow X_s \gamma$ may written, using an operator product expansion, in terms of the matrix elements of local operators between $B$ mesons. Using the techniques of HQET, it was shown in \cite{3} that the leading term of the expansion in $1/m_b$ reproduces the parton model, and that the $O(1/m_b)$ corrections vanish by the equations of motion for the heavy quark. More recently, the $O(1/m_b^2)$ corrections to the parton model have been calculated for semileptonic decays \cite{6}–\cite{11}, as well as for the rare decays $B \rightarrow X_s \gamma$ \cite{8}–\cite{12} and $B \rightarrow X_s e^+e^-$. A crucial observation which emerges from the operator product expansion is that the semileptonic width of a heavy meson is proportional to the fifth power of the heavy quark mass, not the meson mass. The heavy quark mass $m_q$ is a well-defined quantity in HQET, and is defined such that the residual mass term in the effective heavy quark Lagrangian vanishes (for a detailed discussion of the definition of $m_q$, see \cite{13}).

In addition to these nonperturbative corrections, there are calculable $O(\alpha_s(m_q))$ corrections to the free quark decay picture coming from real and virtual gluon emission \cite{14}. In this letter, the complete expression to $O(\alpha_s(m_q), 1/m_q^2)$ for the semileptonic width of a heavy meson is used to extract the c quark mass from the measured $D^\pm$ lifetime and semileptonic branching ratio. From this, the b quark mass is determined and the $B$ lifetime predicted in terms of the KM matrix element $|V_{bc}|$. Comparing this with the experimentally measured lifetime allows $|V_{bc}|$ to be determined.

Throughout this letter the mass of a heavy meson will be denoted by $M_Q$ and the mass of the corresponding quark by $m_q$. The meson and quark masses are related by \cite{15}

$$M_Q = m_q + \overline{\Lambda} - \frac{\lambda_1 + 3\lambda_2}{2m_q} + O\left(\frac{1}{m_q^2}\right).$$

(1)

The parameter $\overline{\Lambda}$ in (1) may be interpreted as the energy of the light degrees of freedom of the meson. It corresponds to the constituent mass of the light quark in the meson, and so is expected to be of order a few hundred MeV. From consideration of QCD mass inequalities analogous to those used in the light quark sector to prove that $m_\rho > m_\pi$, Guralnik and Manohar \cite{16} have shown the existence of a rigorous lower bound

$$\overline{\Lambda} > 237 \text{ GeV}.$$  

(2)
We will also take $\Lambda < 800$ MeV in this work when quoting limits on $m_c$, $m_b$ and $V_{bc}$. This bound is consistent with the QCD sum rules estimate \[17\] (see also the second reference in \[18\])

$$\Lambda^{r.s.} = 570 \pm 70 \text{ MeV}$$

and is also consistent with the quark model, in which $\Lambda$ is expected to be a few hundred MeV.

$\lambda_1$ and $\lambda_2$ are defined in terms of the expectation values in the heavy quark effective theory,

$$< H_Q | \overline{h}(iD)^2 h | H_Q > = 2M_Q \lambda_1$$

$$< H_Q | \overline{h} \left( \frac{1}{2} \right) \sigma^\mu\nu G^\mu\nu h | H_Q > = 6M_Q \lambda_2(\mu)$$

(4)

where $H_Q$ is the pseudoscalar heavy meson ($B$ or $D$) and $h$ is the heavy quark field in the effective theory. $\lambda_2$ parameterises the effects of the chromomagnetic moment operator, and may be extracted from the $B^* - B$ mass splitting

$$\lambda_2(m_b) = \frac{m_b}{8} (M_{B^*} - M_B) \approx \frac{1}{4} (M_{B^*}^2 - M_B^2) = 0.12 \text{ GeV}^2$$

(corresponding to $\lambda_2(m_c) = 0.10 \text{ GeV}^2$ \[19\]). $\lambda_1$ parameterises the kinetic energy of the $b$ quark inside the hadron. Since it contributes equally to $m_B$ and $m_{B^*}$, $\lambda_1$ cannot be extracted from the meson masses.

At $\mathcal{O}(1/m_c^2)$, the nonperturbative corrections to the parton model are also parameterised in terms of $\lambda_1$ and $\lambda_2$. Combining the results from \[7\]–\[11\] with the $\alpha_s$ corrections to the free quark model \[14\] gives the complete expression for the semileptonic $D$ width to $\mathcal{O}(1/m_c^2, \alpha_s(\mu_c))$

$$\Gamma(D \to e^+ \nu_e X_q) = \frac{G_F^2 m_c^5}{192 \pi^3} \left\{ |V_{cs}|^2 \left[ \left(1 - \frac{2\alpha_s(\mu_c)}{3\pi} g \left( \frac{m_s}{m_c} \right) + \frac{\lambda_1}{2m_c^2} \right) f_1 \left( \frac{m_s}{m_c} \right) - \frac{9\lambda_2}{2m_c^2} f_2 \left( \frac{m_s}{m_c} \right) \right] \right. \right.$$

$$\left. + |V_{cd}|^2 \left[ 1 - \frac{2\alpha_s(\mu_c)}{3\pi} g(0) + \frac{\lambda_1 - 9\lambda_2}{2m_c^2} \right] \right\} + \mathcal{O} \left( \frac{1}{m_c^2}, \frac{\alpha_s(\mu_c)}{m_c^2} \right).$$

(6)

The functions $f_{1,2}$ arise from the finite mass of the quark in the final state,

$$f_1(x) = 1 - 8x^2 + 8x^6 - x^8 - 24x^4 \log x$$

$$f_2(x) = 1 - \frac{8}{3}x^2 - 8x^4 + 8x^6 + \frac{5}{3}x^8 + 8x^4 \log x,$$

(7)
while the function \( g(x) \) arising from one-gluon graphs is tabulated in Ref. \[14\] (we note that \( g(0) = 3.62 \), \( g(2/1.6) = 3.15 \) and \( g(1.6/4.8) = 2.40 \)). We also take \( m_s = 200 \pm 80 \text{ MeV} \) and \( m_d \simeq 0 \).

The choice of the appropriate scale \( \mu_c \) in (3) is a potentially troublesome one. The scale is set roughly by \( m_c \); however, since the final state hadron typically carries off only a fraction of the available energy, the appropriate scale may be somewhat lower. Formally this choice of scale is a higher order effect; however, in practice there is a considerable difference between, for example, \( \alpha_s(m_c) \) and \( \alpha_s(m_c/3) \). In particular, at scales much less than \( m_c \), QCD perturbation theory cannot be trusted. Since we have not done the two-loop calculation, we cannot resolve this scale ambiguity. In this letter we will take \( \mu_c \simeq m_c \), and the sensitivity of the results to higher order terms in \( \alpha_s \) will be estimated by formally varying \( \mu_c \) between \( m_c \) and \( m_c/3 \). Taking \( \Lambda_{QCD}^{(4)} = 250 \text{ MeV} \) and \( \Lambda_{QCD}^{(3)} = 300 \text{ MeV} \), and using the two-loop expression for \( \alpha_s(\mu) \) gives \( \alpha_s(m_c) = 0.09 \pi \) and \( \alpha_s(m_c/3) \simeq 0.4 \pi \). We stress that we are not claiming that QCD perturbation theory is valid at \( \mu \simeq m_c/3 \), but rather that if perturbation theory is meaningful for \( D \) decays, we expect this approach to give a reasonable estimate of the uncertainty due to higher order QCD corrections.† Fortunately, the determination of \( V_{bc} \) is relatively insensitive to the uncertainty in \( m_c \) extracted from the \( D \) width.

From the observed semileptonic branching ratio, lifetime and mass of the \( D^\pm \) [21]

\[
\begin{align*}
\text{Br}(D^\pm \rightarrow e^\pm + \text{anything}) &= (17.2 \pm 1.9)\% \\
\tau_{D^\pm} &= 10.66 \pm 0.23 \times 10^{-13} \text{ s} \\
M_{D^\pm} &= 1869.3 \pm 0.3 \text{ MeV}
\end{align*}
\]

(8)

it is a simple matter to extract \( m_c \) and \( \lambda_1 \) from Eqs. (3) and (4) for any value of \( \Lambda \). (We have checked that the \( D^0 \) gives results consistent with the \( D^\pm \); the \( D^\pm \) meson is used, as the uncertainty in its semileptonic branching fraction is somewhat smaller than for the \( D^0 \).) The results are shown in fig. 4 (a) and (b). In each figure the narrow band corresponds to \( \mu_c = m_c \), while the broader band correspond to \( m_c/3 < \mu_c < m_c \). The uncertainty corresponding to 1σ errors on all experimental input parameters is included in the width of each band. From fig. 4, the Guralnik-Manohar bound (2) on \( \Lambda \) translates to limit

\[
\lambda_1 > -0.5 \text{ GeV} \ (1 \sigma).
\]

(9)

† Note that the energies encountered in this problem are similar to those encountered in hadronic \( \tau \) decays, for which the corrections to \( O(\alpha_s(m_\tau)^4) \) have been calculated, and for which the perturbation series appears to be converging [20].
Because of the uncertainty in $\mu_c$, the $c$ quark mass is not well constrained from the data, although taking $\overline{\Lambda} \lesssim 800$ MeV suggests $m_c \gtrsim 1460$ MeV. Since the upper limit of the error bar in fig. 1(a) is both very sensitive to the value of $\alpha_s$ at low scales and corresponds to $m_c > m_D$, a useful upper bound on $m_c$ cannot be extracted.

For $B$ mesons, formulas analogous to (4) and (3) hold, with the replacements $M_D \rightarrow M_B$, $m_c \rightarrow m_b$, $V_{cs} \rightarrow V_{bc}$, $V_{cd} \rightarrow V_{bu}$ and $m_s \rightarrow m_c$. Since $\lambda_1$ is now determined from $D$ decays for each value of $\overline{\Lambda}$, the $b$ quark mass $m_b$ may be solved for as a function only of $\overline{\Lambda}$. Using $M_B = 5.278 \pm 0.002$ GeV gives the results shown in fig. 1(c). Since $M_B$ is less sensitive to $\lambda_1$ than $M_D$, it is not surprising that $m_b$ is roughly a linear function of $\overline{\Lambda}$, with smaller error bars than $m_c$. $m_b$ is found to lie in the range $5140$ MeV $< m_b < 4600$ MeV.

Given $m_b$ and $\lambda_1$, the semileptonic width of the $B$ may now be predicted, allowing the KM matrix element $|V_{bc}|$ to be determined as a function of $\overline{\Lambda}$. The inclusive branching ratio of the $B$ to electrons is measured to be $Br(B \rightarrow e + \text{anything}) = 10.7 \pm 0.5\%$ (10) (where the charge of the $B$ is not determined). Averaging the recent measurements of the $B$ lifetime by the DELPHI collaboration [22] and by the CDF collaboration [23] gives a $B$ lifetime of

$$\tau_B = 1.32 \pm 0.09 \times 10^{-12} \text{ s}$$ (11)

(statistical and systematic errors added in quadrature). This gives $|V_{bc}|$ as a function of $\overline{\Lambda}$, as shown in fig. 2. The two bands in the figure correspond to the two choices of scale $\mu_b = m_b$, $\mu_c = m_c$ and $m_c/3 < \mu_c < m_c$, $m_b/3 < \mu_b < m_b$ (with $\mu_c/\mu_b$ fixed). Over the range of values for $\overline{\Lambda}$ shown in fig. 2, $|V_{bc}|$ is found to lie in the range $0.040 < |V_{bc}| < 0.057$. The lower bound is set by the rigorous lower bound on $\overline{\Lambda}$, while the upper corresponds to $\overline{\Lambda} = 800$ MeV. This extraction is consistent with the current value of $|V_{bc}|(1.32 \text{ ps}/\tau_B)^{1/2} = 0.051 \pm 0.008$ extracted from the exclusive decay $B \rightarrow D^* e \nu_e$ using HQET [18]. We note that choosing a lower scale for $\mu_c$ and $\mu_b$ tends to raise the preferred values of $|V_{bc}|$.

For a given value of $\overline{\Lambda}$ the effects of higher order terms in $1/m_q$ and $\alpha_s$ on this extraction of $|V_{bc}|$ may be estimated. These terms may be parametrized by $\delta_{b,c}$ and $\epsilon_{b,c}$, defined by

$$M_D = m_c \left( 1 + \frac{\overline{\Lambda}}{m_c} - \frac{\lambda_1 + 3\lambda_2}{2m_c^2} + \delta_c \right)$$

$$M_B = m_b \left( 1 + \frac{\overline{\Lambda}}{m_b} - \frac{\lambda_1 + 3\lambda_2}{2m_b^2} + \delta_b \right)$$

(12)
and

$$\Gamma(D \to X_q e^+ \nu_e) \propto m_c^5 (1 + \epsilon_c)$$

$$\Gamma(B \to X_q e^+ \nu_e) \propto m_b^5 (1 + \epsilon_b).$$

They correspond to a change in $|V_{bc}|$ of

$$\frac{\Delta |V_{bc}|}{|V_{bc}|} \approx \frac{1}{2} \left( \frac{m_c}{m_b}(\epsilon_c + 5\delta_c) - (\epsilon_b + 5\delta_b) \right) \approx \frac{1}{6} (\epsilon_c - 3\epsilon_b + 5(\delta_c - 3\delta_b)).$$

The leading contributions to $\delta_c$ are of order $(\Lambda/m_c)^3$ and $\alpha_s(m_c)/\pi \lambda_2/m_c^2$, which we estimate to be at most a few percent, while $\delta_b$ is expected to be at least an order of magnitude smaller. We have attempted to take the leading contributions to $\epsilon_{c,b}$ arising from higher order QCD corrections into account by varying the scales $\mu_{c,b}$ between $m_{b,c}$ and $m_{b,c}/3$. From (14), we can understand why our results were relatively insensitive to the choice of scale: an uncertainty in the charm width only gives $\sim 1/6$ this uncertainty in $|V_{bc}|$. Furthermore, the uncertainty in scale tends to cancel between $\epsilon_c$ and $\epsilon_b$.

Since the extraction of $\lambda_1$ as a function of $\overline{\Lambda}$ depends on the validity of perturbation theory for QCD at a low ($\mu \lesssim m_c$) scale, it is instructive to compare this extraction of $|V_{bc}|$ with that obtained by ignoring the $D$ decay data altogether and simply varying $\lambda_1$ between $\pm 1$ GeV$^2$ for each given value of $\overline{\Lambda}$. This amounts to working at $O(1/m_q)$ in the heavy quark expansion with a reliable estimate of the uncertainty from $1/m_q^2$ terms. As before, the scale $\mu_b$ is also varied between $\mu_b = m_b/3$ and $\mu_b = m_b$. The results are shown in fig. 3. The shaded area on each graph corresponds to both the experimental errors on the input parameters and the variations in $\lambda_1$ and $\mu_b$. As expected, the uncertainty is $|V_{bc}|$ is somewhat larger than when the $D$ decay data are included, particularly at large values of $\overline{\Lambda}$; however, $|V_{bc}|$ is still well constrained since the prediction for the $B$ width is relatively insensitive to $\lambda_1$. For $237$ MeV $< \overline{\Lambda} < 800$ MeV, $|V_{bc}|$ is found to lie in the range $0.04 < |V_{bc}| < 0.06$.

It is encouraging that the extraction of $|V_{bc}|$ from inclusive decays is consistent with that obtained from exclusive decays over the full allowed range of $\overline{\Lambda}$. Furthermore, the extraction of $|V_{bc}|$ from inclusive decays has some theoretical and experimental advantages over exclusive decays. No exclusive hadronic final state needs to be identified, and the extrapolation of form factors to zero recoil is not required. The method is limited by

\[\text{† The operator } h(iD)^2h \text{ receives no strong interaction corrections to its coefficient, by reparameterisation invariance [24].}\]
the uncertainties in $\bar{\Lambda}$ and the relevant hadronic scales $\mu_b$ and $\mu_c$. Additional input, such as a lattice measurement of $\bar{\Lambda}$ or $\lambda_1$, or a two-loop calculation of the semileptonic decay rate, would further constrain $|V_{bc}|$. For example, from fig. 1(a), the QCD sum rules estimate (3) of $\bar{\Lambda} = 570 \pm 70$ MeV corresponds to $0.1 \text{ GeV}^2 < \lambda_1 \lesssim 1.5 \text{ GeV}^2$ and $0.044 < |V_{bc}| < 0.054$ (although the upper bound on $\lambda_1$ should not be taken too seriously, since both the $1/m_c$ expansion and QCD perturbation theory are unreliable at this point). On the other hand, the nonrelativistic quark model suggests that $\bar{\Lambda} \sim 350$ MeV, the mass of the light constituent quark, and that $\lambda_1 \sim -(350$ MeV$)^2 \sim -0.1 \text{ GeV}^2$ (note that $\lambda_1 < 0$ corresponds to a positive mass shift from the kinetic energy of the heavy quark). From fig. 1(a), these values of $\lambda_1$ and $\bar{\Lambda}$ are consistent with one another, and are consistent with somewhat lower values of $|V_{bc}|$.

We thank M. B. Wise for useful discussions, particularly concerning the relevant scales $\mu_c$ and $\mu_b$, and the possibilities of extracting $V_{bc}$ without using the $D$ decay data. We are also grateful to A. Falk, B. Holdom, A. Manohar and L. Wolfenstein for useful comments and criticisms. MJS acknowledges the support of a Superconducting Supercollider National Fellowship from the Texas National Research Laboratory Commission under grant FCFY9219. This research was supported in part by TNRLC grant RGFY93-206 and by the Department of Energy under contracts DE–FG03–90ER40546 (UC San Diego) and DE–FG02–91ER40682 (CMU).
References

[1] M. B. Voloshin and M. A. Shifman, Yad. Phys. 45 (1987) 463 [Sov. J. Nucl. Phys. 45 (1987) 292]; Yad. Fiz. 47 (1988) 801 [Sov. J. Nucl. Phys. 47 (1988) 511].
[2] N. Isgur and M. B. Wise, Phys. Lett. 232B (1989) 113, Phys. Lett. 237B (1990) 527.
[3] H. Georgi, Phys. Lett. 240B (1990) 247.
[4] B. Grinstein, Nucl. Phys. B339 (1990) 253.
[5] E. Eichten and B. Hill, Phys. Lett. 243B (1990) 427.
[6] J. Chay, H. Georgi and B. Grinstein, Phys. Lett. 247B (1990) 399.
[7] I. I. Bigi, N. G. Uraltsev and A. I. Vainshtein, Phys. Lett. 293B (1992) 430.
[8] I. I. Bigi, B. Blok, M. Shifman, N. G. Uraltsev and A. Vainshtein, TPI-MINN-92/67-T (1992); Phys. Rev. Lett. 71 (1993) 496.
[9] B. Blok, L. Koyrakh, M. Shifman and A. Vainshtein, NSF-ITP-93-68, hep-ph/9307247.
[10] A. V. Manohar and M. B. Wise, UCSD-PTH 93-14, hep-ph/9308246.
[11] T. Mannel, IKDA 93/16, hep-ph/9308262.
[12] A. F. Falk, M. Luke and M. J. Savage, UCSD-PTH 93-23.
[13] A. F. Falk, M. Luke and M. Neubert, Nucl. Phys. B388 (1992) 363.
[14] N. Cabibbo and L. Maiani, Phys. Lett. 79B (1978) 109.
[15] A. F. Falk and M. Neubert, Phys. Rev. D47 (1993) 2965.
[16] Z. Guralnik and A.V. Manohar, Phys. Lett. 302B (1993) 103.
[17] M. Neubert, Phys. Rev. D46 (1992) 1076.
[18] M. Neubert, Phys. Lett. 264B (1991) 455; SLAC-PUB-6263 (1993) (to appear in Physics Reports).
[19] A. F. Falk, B. Grinstein and M. Luke, Nucl. Phys. B357 (1991) 185; E. Eichten and B. Hill, Phys. Lett. 243B (1990) 427.
[20] E. Braaten, S. Narison and A. Pich, Nucl. Phys. B373 (1992) 581, and references therein.
[21] Particle Data Group, Phys. Rev. D45 (1993) 1.
[22] P. Abreu, et. al., (DELPHI Collaboration), CERN-PPE/93-80 (1993).
[23] M.L. Mangano (CDF Collaboration), FERMILAB-Pub-93/139-E (1993).
[24] M. Luke and A. V. Manohar, Phys. Lett. 286B (1992) 348.
Figure Captions

Fig. 1. $\lambda_1, m_c$ and $m_b$ as functions of $\Lambda$. In each graph, the vertical line corresponds to the Guralnik-Manohar bound $\Lambda > 237$ GeV. The shaded regions correspond to the experimental error in the input parameters and, for the broader band, the variation in $\mu_c$.

Fig. 2. The weak mixing angle $|V_{bc}|$ as a function of $\Lambda$ for $\tau_B = 1.32 \pm 0.09$ ps. The vertical line corresponds to the constraint imposed by the bound on $\Lambda$. The error bar on the left indicates the current value of $|V_{bc}|$ extracted from the exclusive decay $B \to D^* e^- \nu_e$ and its associated uncertainty. The shaded region corresponds to the experimental error in the input parameters and, for the broader band, the variations in $\mu_c$ and $\mu_b$.

Fig. 3. The weak mixing angle $|V_{bc}|$ as a function of $\Lambda$ for $\tau_B = 1.32 \pm 0.09$ ps, without imposing constraints on $\lambda_1$ from $D$ decay. The vertical line corresponds to the lower bound on $\Lambda$, and the shaded regions correspond to the experimental error in the input parameters, varying $\lambda_1$ from $-1$ GeV to 1 GeV and $\mu_b$ from $m_b$ to $m_b/3$. The error bar on the left indicates the current value of $|V_{bc}|$ extracted from the exclusive decay $B \to D^* e^- \nu_e$ and its associated uncertainty.
Figure 1 (a)

Figure 1 (b)
Figure 3