Low-energy excitations of spin-Peierls chains with modified bond-impurities

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The introduction of modified bond-defects in spin-Peierls systems is investigated in a model of antiferromagnetic Heisenberg spin chains coupled to adiabatic phonons. Generically, new low-energy magnetic or non-magnetic excitations appear below the bulk spin gap energy. When two adjacent bonds are modified, these excitations can be interpreted in terms of bound states of a soliton with the localized spin-1/2 located on the impurity site. It is shown that the confining potential occurs even in the case of isolated chains.

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I. INTRODUCTION

Impurity doping in quasi-one-dimensional spin-Peierls systems has recently sparked renewed both experimental and theoretical attention in this field. Experimental studies in CuGeO\textsubscript{3} include so far in-chain magnetic (spin-1 Ni) or non-magnetic (Zn, Mg) impurity doping and off-chain (Si substituting Ge) doping. A rapid reduction of the spin-Peierls temperature has been generically observed as impurity concentration increases.\textsuperscript{1,2} Furthermore, impurity doping favors a new phase at low temperature, which has been characterized as a three-dimensional antiferromagnetic (AF) ordering by specific heat measurements,\textsuperscript{3} neutron scattering,\textsuperscript{4} or nuclear magnetic resonance.\textsuperscript{5,6} More surprisingly, the AF phase has been shown to coexist with the spin-Peierls phase for low impurity concentrations.\textsuperscript{1,7} For a theoretical understanding of these effects, the starting point is the realization that the elementary excitations of these quasi-one-dimensional systems are objects which appear on chains and which have been characterized as topological defects called solitons.\textsuperscript{8,9} These solitons do not interact with non-magnetic impurities in a strictly one-dimensional (1D) model, but are bound to spin-1 impurities. However, impurities and solitons are confined into bound states by the three-dimensional character of phonons.\textsuperscript{10}

Most theoretical studies have so far been concentrated on the effects of in-chain impurities in which a Cu ion is substituted by another ion. In this paper, we theoretically study a new class of impurity systems, namely chains where some bonds have been modified. This could physically correspond to the substitution of a Cu ion by another spin-1/2 ion thus changing the values of the exchange couplings connecting the impurity site with its two nearest neighbors (NN) sites. If next nearest neighbors (NNN) couplings are included in the model, they would be modified as well. The case of the spin-1/2 impurity could also be thought as coming from defects in the crystal structure (structural disorder, radiation damage, grain boundary in polycrystalline samples, etc.) which change locally the values of the couplings. These modified couplings are then described by an effective spin-1/2 impurity (see e.g. Ref. 12). Similar kind of defects could also be due to the substitution of the out-of-chain Ge ions by Si (see e.g., Ref. 13) then changing only one NN and two NNN bonds (“bond-centered impurity”). Since, as it was pointed out above, the elementary excitations in these quasi-1D systems are objects which exist already in strictly 1D systems, the first step is to characterize these excitations by using purely 1D models and this is the purpose of the present work. A second step would be then to understand how these excitations and their interactions are modified by interchain (magnetic and elastic) interactions present in CuGeO\textsubscript{3}. The first study of the effects of bond impurities has been done on isolated Heisenberg chains in Ref. 13 using essentially field-theoretical techniques. We will compare our results with the ones obtained in that study in Section 1. For spin-Peierls systems, analytical studies on the same model we will consider (see Eq. 4 below) have been done using bosonization techniques.\textsuperscript{14} We think, however that in those studies there is not a proper understanding of the elementary interactions at a microscopic level.

In practice, we introduce these bond impurities in our model by redefining the exchange coupling constants on two bonds connected at the impurity site (spin-1/2 or “site-centered impurity”) or a single bond (bond-centered impurity) as $J_{imp} = x J$. In the first case, from a purely theoretical point of view, this allows us to interpolate continuously between a periodic chain with even number of sites and an open chain with odd number of sites plus a spin disconnected to the chain. We find that there are also some interesting features in the region $x > 1$. By assuming that lattice distortions are adiabatic, we are thus led to the following Hamiltonian:
\[ H = J \sum_i (1 + \delta_i) \mathbf{S}_i \cdot \mathbf{S}_{i+1} + \alpha J \sum_i \mathbf{S}_i \cdot \mathbf{S}_{i+2} + \frac{K}{2} \sum_l \delta_l^2 + J_{\text{imp}} \sum_j (1 + \delta_j) \mathbf{S}_j \cdot \mathbf{S}_{j+1} + J'_{\text{imp}} \sum_j \mathbf{S}_j \cdot \mathbf{S}_{j+2}, \] (1)

where \( i \) (\( j, l \)) indicate bulk bonds (impurity bonds, all bonds, respectively). We assume for simplicity that the spring constants \( K \) are not modified by the defects. Moreover, we assume also that the two NNN couplings at the impurity site also involve the same frustration parameter \( \alpha \), i.e. \( J'_{\text{imp}} = \alpha J_{\text{imp}} \). The variable \( x \) will be extended to negative values in which case the impurity couplings are ferromagnetic, thus including the possibility of studying a spin-3/2 impurity when two adjacent bonds are modified (e.g. a Co ion replacing a Cu ion in the chain, see Ref. [13] or a spin-1 Ni impurity if only one bond is changed (see Ref. [10]). We have solved this Hamiltonian numerically on finite chains by the self-consistent procedure described in Ref. [17] using Exact Diagonalization (ED) and Quantum Monte Carlo (QMC). The relevance of this kind of calculations to describe experimental results on \( \text{CuGeO}_3 \), in particular the use of the adiabatic approximation which somehow interpolates between the full quantum spin-lattice coupling and the simplest models with fixed dimerization, was emphasized in a number of previous studies. In particular, recent calculations within this formalism, using parameters realistic for \( \text{CuGeO}_3 \), have given results that describe Raman experiments not only at a qualitative but at a quantitative level, in spite of the fact that for these experiments dynamical lattice effects are in principle important.

II. CHAINS WITH TWO ADJACENT MODIFIED BONDS

We start our study with the case of a “site-centered” impurity. A frustrating exchange \( J'_{\text{imp}} = \alpha J_{\text{imp}} = \alpha x J \) is also included between the impurity site and its two next nearest neighbors. We first consider even chains in the region \( 0 \leq x \leq 1 \). In the case of \( \alpha = 0 \), the \( x = 0 \) point is equivalent to an open odd chain with a disconnected spin-1/2. It has been previously shown that, within this 1D model, no binding occurs between the spin-1/2 soliton released in the chain and its edges (which here correspond to the location of the impurities). On the other hand, for \( x = 1 \), we recover the pure system and there are of course no magnetoelastic solitons in the ground state (GS). One can then consider that the soliton is tightly bound to the spin-1/2 impurity, forming a spin singlet. As \( x \) is reduced from 1 to 0, this bound state is weakened and the soliton eventually moves away from the impurity site. This behavior can be qualitatively seen from the QMC data at \( T = 0.05J \) for \( L = 40 \), \( K = 1 \), as shown in Fig. 1. In (a), \( x = 0.9 \), the distortion pattern is almost indistinguishable from that of a pure chain with periodic boundary conditions. In (b), \( x = 0.3 \), the soliton has only moved one lattice spacing away from the impurity. In contrast, as shown in (c) corresponding to \( x = 0.1 \), the bound state has disappeared and the soliton moves freely in a region around the center of the chain segment.

FIG. 1. QMC results at \( T = 0.05J \) for the distortion \( \delta_i \), on a 40 site chain with PBC, \( K = 2 \), and for various values of \( x \); (a) \( x = 0.9 \), (b) \( x = 0.3 \), (c) \( x = 0.1 \), (d) \( x = -2.5 \). The spin-1/2 impurity is located at site number 40 at the extreme right of the plots.

To put these statements on a more quantitative basis, let us start by analyzing the energy per site as a function of \( x \), \( e(x) = E(x)/L \). Since for \( x = 0 \) there is no soliton-impurity binding, a value of the energy lower than \( e(x = 0) \) could be considered an indication of the presence of a bound state in the system. The QMC results for \( L = 40 \), \( K = 1, 2 \) and 3, obtained for \( T = 0.05 \) which can be taken as the ground state energies for the system considered here, except very close to \( x = 0 \) as discussed below, are shown in Fig. 2. For all the values of \( K \) studied here it can be seen that the \( e(x) \) is maximum at \( x = 0 \). This might indicate that there is an impurity-soliton bound state for \( x > 0 \) and that the binding energy decreases as \( x \) is reduced from \( x = 1 \), consistently with Fig. 1(a), (b). The fact that \( e(x) \) is lower than \( e(0) \) for \( x < 0 \) can be interpreted by assuming that an effective spin-3/2 impurity starts to form and hence the soliton is again attracted to this impurity. For the particular value of \( K = 2 \), we have studied the size dependence of the results by computing the energy for \( L = 20 \) and \( L = 80 \) in addition to \( L = 40 \). The energy is always maximum at \( x = 0 \) and, as expected, it tends to become constant as \( L \) increases. The inset shows the energy of the lowest state in the \( S^z = 1 \) sector together with the energy of the excited state containing one extra soliton-antisoliton \((s - \bar{s})\) pair. It can be seen that for \( x < 0 \) the ground state of the system has \( S = 1 \). We will discuss these features later.
Peierls chains including lattice relaxation
tons and non-magnetic impurities do not bind in spin-
indicated in the figure. The insert shows the first two
when
on a 40 sites chain (except otherwise stated) with PBC and
finite
states for
x=0
K=2, L=40

The deconfinement of the impurity-soliton bound state
when x → 0 (x > 0) is a rather subtle issue. The above
results obtained at finite temperature $T = 0.05J$ and
x = 0.1 shown in Fig. 2(c) indicate that there is no
soliton-impurity binding in that case. However, for a
temperature larger than the (zero temperature) binding
energy, one expects that, due to thermal fluctuations,
the soliton can escape from the small confining potential. In
order to confirm this scenario and to determine if the
above bound state survive in the thermodynamic limit at
zero temperature, we have computed the soliton-impurity
(T = 0) binding energy which is rigorously defined as

$$ E_B(L) = (E_{\text{imp}}(L) - E_0(L)) - e_{\text{imp}} - e_{\text{sol}}, $$

where $e_{\text{sol}} = \lim_{L \to \infty} [E_0(L + 1) - E_0^*(L + 1)]$, $E_0(L)$
$(E_{\text{imp}}(L))$ is the GS energy of a pure (bond-modified) L-
site chain ($L = 2p$ even), $E_0^*(L + 1) = (E_0(L) + E_0(L + 2))/2$, and $e_{\text{imp}} = \lim_{L \to \infty} [E_{\text{imp}}(L + 1) - E_0^*(L + 1)]$.
These quantities have been computed within an ED
treatment. An impurity-soliton bound state corresponds to $E_B(\infty) < 0$.

Some results for $E_B(L)$ are shown in Fig. 3. For $x = 0$, by using extrapolations of the form $a + b/L \exp(-L/L_0)$,
we have obtained a vanishing binding energy (within an
estimated error of $\sim 0.001J$) as it should be, as soli-
tons and non-magnetic impurities do not bind in spin-
Peierls chains including lattice relaxation. For $x = 1$, the
binding energy is the energy necessary to free one spin
which is part of a singlet, i.e. the spin gap. We obtained
$E_B = -0.39J = -\Delta_{\text{spin}}$ which implies that there is no
$\hat{s} - \hat{s}$ binding in this model. For $x = 0.1$, the situation,
as depicted also in Fig. 3 is more intricate. In this case,
there is a cross-over between the free and bound regimes
near $L = 14$ as signaled by a change of the curvature of
the scaling form of the binding energy. The exponential
fit is of course no more valid but one can crudely esti-
mate that in the bulk limit $E_B < -0.01J$. This result
suggests that, for fixed positive $x$, binding sets up only
for sufficiently large chain lengths. Alternatively, this
implies the existence of a finite critical value of $x$, $x_{\text{cr}}(L)$,
below which there is no impurity-soliton bound state for
a given chain size $L$. This critical value $x_{\text{cr}}(L)$ has been
evaluated by ED for various lattice sizes and results are
shown in Fig. 3. Note that this crossover disappears for
$x > 0.3$ i.e. the soliton is bound for all sizes we consid-
ered and the extrapolated binding energy is clearly non
zero (negative). The extrapolation of the $x_{\text{cr}}(L)$ curve
when $L \to \infty$ is consistent with a finite binding energy
in the thermodynamic limit for any finite value of $x$.

FIG. 2. QMC results for the energy per site as a function of
x on a 40 sites chain (except otherwise stated) with PBC and
one spin1/2 impurity, $T = 0.05$ and various values of $K$ as
indicated in the figure. The insert shows the first two $S^z = 1$
states for $K = 2$ and $L = 40$.

At finite temperature, a similar cross-over should also
appear. However, since for sufficiently large chains the
thermal length becomes the only relevant length scale,
one expects that $x_{\text{cr}}(L)$ extrapolates, when $L \to \infty$, to a
(small) finite value consistently with the above QMC
results of Fig. 3(c) and in contrast with the $T=0$ case.
The schematic behavior of $x_{\text{cr}}(L)$ at finite temperature
is shown in Fig. 4 for comparison.

Let us now consider negative values of $x$, i.e. the case
of a ferromagnetic bond. The limit $|x| >> 1$ is simple to
understand. In this case, a spin-3/2 effective spin forms
around the impurity leaving a spin-1/2 in the rest of the
even chain. This behavior is qualitatively shown in (d) for
$x = -2.5$. Since, as we shall discuss more quantitatively
later on, the soliton is weakly antiferromagnetically
coupled to the spin-3/2 impurity, we expect the GS to be
degenerate. On the contrary, for $|x| << 1$, the small
ferromagnetic couplings connected to the impurity site
produce an effective interaction that weakly binds the
localized spin-1/2 to the spin-1/2 soliton into a triplet

FIG. 3. Impurity-soliton bound state energy as a function
of the inverse lattice size 1/L for $\alpha = 0$, $K = J$ and $x=0.0$
(•) or $x=0.1$ (□) obtained by ED. An exponential fit is also
plotted for the case $x = 0$. The arrow points out the crossover
between two scaling behaviors.
bound state.

FIG. 4. Evaluation of critical parameter $x_{cr}$ as a function of the inverse lattice size $1/L$ by ED. Parameters such as $\alpha = 0$ and $K = J$ have been used. An hypothetic curve at finite $T$ is also plotted.

In Fig. 5 we show the zero temperature binding energy in the thermodynamic limit as a function of $x$ together with the estimate of the error (resulting from the finite size scaling procedure). These results are consistent with the above qualitative discussion and confirm the existence of a soliton-impurity bound state in the bulk limit, except for $x = 0$. In the $x < 0$ part, a spin-3/2 is located on the impurity site and its two neighbors, and a finite binding is also observed. In the $x > 1$ region, the soliton is very strongly bound to the impurity, and it becomes impossible to deconfine the two objects. In other words, trying to pull out the two components of the singlet pair from each other will result into the spontaneous creation of a soliton-antisoliton pair out of the “vacuum”. Consequently, the binding energy, as it is defined, becomes identical to the spin gap i.e. the energy to create such $s - \bar{s}$ pair.

To complete the overall picture, we have computed the singlet-triplet spin gap defined as $\Delta^{01} = E(S = 1, \{\delta\}_1) - E(S^z = 0, \{\delta\}_0)$ where $\{\delta\}_0$, $\{\delta\}_1$ are the set of bond distortion determined for $S = 0$ and $S = 1$ respectively. It is striking that the overall behavior of $\Delta^{01}$ vs $x$ is totally similar to that of the binding energy shown in Fig. 5. This strongly suggests that around $x = 0$ the lowest singlet ($x < 0$) or triplet ($x > 0$) excitation is made of a deconfined soliton. When $x > 1$, the spin gap is constant as the soliton remains bound to the impurity, and bulk $s - \bar{s}$ excitations are energetically more favorable.

FIG. 5. Impurity-soliton binding energy obtained by ED in the bulk limit as a function of $x$. Parameters $\alpha = 0$ and $K = J$ have been used.

FIG. 6. Energy difference between the $S = 1$ and $S = 0$ sectors obtained by ED in the bulk limit as a function of $x$ for $\alpha = 0$ and $K = J$.

It is instructive to relate the above results to the analysis of Eggert and Affleck for a spin-1/2 Heisenberg chain (without coupling to the lattice) in the presence of impurities. In this work, they show that when $x > 0$, in the renormalization group language, the open chain will be unstable and ultimately flow to the stable periodic chain with the impurity site included (“healing” effect). This is consistent with our result that there is a bound state for all $x > 0$ in the bulk limit. Now, for $x < 0$, the system would have a marginal flow (as $L \to \infty$) towards the open chain with a decoupled spin, i.e. no impurity soliton-binding in the bulk limit. This result is the opposite to our result shown in Fig. 5. This indicates that the Luttinger liquid approach of Ref. 13 cannot be directly extended to the case when a coupling to the lattice is present.

III. CHAINS WITH A SINGLE MODIFIED BOND

We now turn to the case where a single bond is modified, i.e., on one NN bond, $J \to J_{\text{imp}}$, and its two overlapping NNN bonds, $\alpha J \to J'_{\text{imp}}$. This configuration corresponds to a “bond-centered” rather than a “site-centered” impurity.

In the present situation, even in the case $x = 0$, the number of sites in the chain remains even and so there is
no extra spin carrying the soliton as in the previous case. Hence, QMC results for
the distortion \( \delta_i \), for \( 0 \leq x \leq 1 \), for \( K = 2 \) and \( \alpha = 0 \) look very similar to those depicted
in Fig. 9. As it was mentioned in the Introduction, in the limit \( x \to -\infty \), the impurity bond leads to an
effective \( S = 1 \) impurity. On the other hand, for \( x \to \infty \),
a strong dimer is formed at the modified bond and it will
be relatively decoupled from the remainder of the chain.
However, for intermediate values of \( x \), it is possible that
a \( s - \bar{s} \) pair can be bound to the impurity bond and, in
fact, QMC results for \( \delta_i \) indicate that this may be the
case for \( x \leq 0 \).

If the energies of large systems are examined (Fig. 8),
it can be seen that the highest energy is again located at
\( x = 0 \). Notice that for \( x < 0 \) the system is frustrated and
classically a higher energy than for \( x = 0 \) is expected.
For all \( x \), the GS is a singlet and the \( S = 1 \) states are
well separated from it, as the insert of Fig. 7 shows. Also
as this insert indicates, there is a crossover at \( x = 1 \) be-
tween two \( S^z = 0 \) states. The ground state for \( x > 1 \) corresponds to the above mentioned rela-
tively decoupled dimer which leads to an energy decreasing
roughly linearly with \( x \). It is then clear that the local
“impurity” potential lifts the degeneracy of the two-fold
degenerate GS occurring exactly at \( x = 1 \) immediately as
one moves away from this particular point. Indeed, the
two states crossing at \( x = 1 \) exhibit opposite bulk dimer-
ization far away from the impurity site. This can be
qualitatively understood from the fact that, for \( x < 0 \),
a singlet is formed around the impurity which involves
three bonds weakly connected to the rest (a triplet on
the center bond bound to two solitons) compared to only
one when \( x > 1 \). In the limit \( x \to -\infty \) one recovers
the case of a spin-1 impurity (corresponding to a Ni \( \to \)
Cu substitution) previously studied. Precisely in Ref.
23 it was emphasized the role of the three-site subsystem
composed by the \( S = 1 \) site and its two \( S = 1/2 \) NN sites.
We have also computed the related spin gap for this
type of impurity. Results obtained by both ED and QMC
are shown in Fig. 8. The extrapolation to the bulk limit
using the law \( a \exp[-L/L_0]/L^n \), is also shown. It can
be seen that the gap is roughly constant (within error
bars) in the interval \(-2 \leq x \leq 2 \), in striking contrast with
the behavior of the site-centered impurity as discussed
in the previous Section. It is also interesting to notice
that two different triplet excitations also cross exactly
at \( x = 1 \). Since the energy separation from the GS is,
to a good accuracy, independent of \( x \) and equal to the
bulk spin gap, these states can be interpreted as a bulk
(deconfined) \( s - \bar{s} \) pair excitation.

IV. SUMMARY AND CONCLUSIONS

Based on the previous QMC and ED results, a
schematic representation of the behavior of the low-
energy states with \( x \) can be drawn. A summary of the
structure of the low-energy spectrum in the case of a
“site-centered” impurity is schematically represented in
Fig. 9. For \( x > 1 \), the GS includes an impurity region
consisting of two adjacent strongly dimerized bonds car-
rying a spin-1/2 and separating two dimerized regions
with opposite \( q = \pi \) lattice order parameters. On an even
chain, this defect binds a soliton. Since, on a closed chain,
the soliton can be on either side of the defect the GS is
then two-fold degenerate as shown in Fig. 9. The lowest
excitation corresponds to a deconfined soliton-antisoliton pair decoupled from the impurity (which could be in singlet or triplet degenerate states) as represented by the dotted line in Fig. 9. However, for 0 < x < 1, the lowest excitation is a quite different state involving a spin flip of the soliton spin linked to the spin-1/2 defect at the impurity site. This qualitative change manifests itself as a level crossing of the first excitations at x = 1. At exactly x = 0, we expect additional level crossings of both GS and excited states as shown in Fig. 9. Indeed, in this case, the first excited state in a chain with an even number of sites (including the impurity site) is highly degenerate and corresponds to the 3 possible spin configurations (S = 0, 1, 2) of the three free spin-1/2 of the two mobile solitons and the localized spin-1/2. Arbitrary small ferromagnetic or antiferromagnetic couplings on the two impurity bonds are expected to lift this degeneracy into several levels as seen in Fig. 9 in agreement with the previous numerical calculations.

![Fig. 9](image-url)

**Fig. 9.** Schematic representation of the low-energy spectrum of an even chain with a “site-centered” defect as a function of x. The spin sectors as well as the nature of the states are indicated in the figure.

A similar plot of the low-energy spectrum is shown in Fig. 10 in the case of a “bond-centered” impurity. Contrary to the previous case, such a defect favors one of the two bulk dimerization patterns so that the GS is non-degenerate for all x, except at x = 1. For x > 1 the dimerization pattern with a singlet located on the modified (strongly dimerized) bond is selected while, for x < 1, strong bonds occur next to the modified one. These two orthogonal GS naturally cross at x = 1 as seen in Fig. 10. For all x, the lowest triplet excitation corresponds to the creation of a bulk (deconfined) s – ñ pair.

We finish this work by discussing the implications of the above results to experimental systems of doped spin-Peierls chains. Impurity doping should lead to important effects in magnetic properties (i.e. in inelastic neutron scattering, Raman scattering, etc...) when the spin excitation gap is reduced, $\Delta_{\text{spin}}^{01} < \Delta_{\text{spin}}$ (0 < x < 1) or when the GS carries a finite spin (x < 0). For defects centered on a site affecting the values of the exchange couplings on each side, low-energy magnetic excitations are predicted by our model as seen in Fig. 9. Such states seen in inelastic neutron scattering of Zn-doped CuGeO$_3$ materials (x = 0) should also survive for more general kinds of defects such as substitution of Cu by a different spin-1/2 impurity.

![Fig. 10](image-url)

**Fig. 10.** Schematic representation of the low-energy spectrum of an even chain with a “bond-centered” impurity as a function of x. The spin sectors as well as the nature of the states are indicated in the figure.

The substitution of a Ge ion by a Si one would produce a decrease of the neighbor Cu-O-Cu angle and at the same time presumably a shortening of the corresponding Cu-Cu distance. The outcome of these competing effects on the value of the effective NN Cu-Cu exchange coupling is difficult to predict but it is quite likely that x = 1 + $\Delta_{x}$ with $|\Delta_{x}| < 1$. For both signs of $\Delta_{x}$ there is a common behavior: the impurity bond forces the chain to pick one of the two possible dimerization patterns. This selected pattern may or may not correspond to the one determined for the whole plane due to interchain magnetic and elastic interactions. In the former case, there is a cost in energy which is proportional to the length of the “wrong” segment of the chain (assumed finite for a finite defect density). Then, this energy cost may be high enough to allow the formation of a soliton in an odd chain or a s – ñ pair on an even chain. The solitons will be bound to the defect and at this point the analysis follows the one previously developed for the case of non-magnetic impurities. Thus, we have for Si-doped CuGeO$_3$ the main behavior seen for in-chain non-magnetic impurities, namely a transition for the spin-Peierls phase to an AF one. Moreover, in the vicinity of x = 1, we predict a new low-energy singlet excitation below the spin gap as seen in Fig. 10. Such an excitation should not be affected by interchain interactions. It is tempting to relate this low-energy singlet excitation to the ones seen in Raman scattering experiments. However, a more quantitative study is necessary to uniquely characterize this excitation. Besides, other changes detected in these experiments, like the decrease of the continuum intensity with increasing doping were rather attributed to an en-
larged NNN interaction along the chains, an effect which was not included in our study.

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