Radar Signal Sorting Algorithm Based on PRI for Large Jitter

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Abstract. The traditional PRI transform method has high estimation accuracy for the pulse repetition interval, but its anti-jitter capability is much worse. The modified PRI transform algorithm combines shifting time origins and overlapped PRI bins methods to improve the traditional PRI transform algorithm. Although it has a certain anti-jitter capability after the improvement, it still cannot be obtained in the case of large jitter. In this paper, the least squares method is introduced to segment the jitter signal to reduce the jitter effect, and then combine the traditional PRI transform method to sort the jitter signal. The simulation results show that the signal sorting method has a good effect in the case of large jitter.

1. Introduction
Radar signal sorting is a key process in radar countermeasure technology and reconnaissance systems. Through radar signal sorting, various radars and their parameters in the space are accurately identified and placed in the radar library for subsequent positioning, tracking and interference processing [1-4]. The pulse repetition interval (PRI) parameter has many characteristics such as working pattern, large parameter range and fast change. It is difficult to get radar with the same PRI in a small range, so PRI is a good parameter for signal sorting. The parameter of PRI has always been increasingly popular in the field of radar signal sorting, and there are all kinds of detection methods for PRI parameters [5, 6].

In the 1980s, researchers led by Rogers began to use pulse arrival time (TOA) correlation PRI parameter to achieve deinterleaving of pulse sequences. After continuous development, the dynamic extended association method appears. The method continuously searches for the pulse backwards through the selected reference pulse and the obtained PRI window. When the number of consecutively searched pulses exceeds the threshold, the PRI is considered to be the correct value. When the frequency of a certain parameter value exceeds the detection threshold, the parameter is considered to be a potential PRI. Based on the principle of statistical histogram, Mardia combines sequence search method to propose cumulative difference histogram algorithm (CDIF) [7]. The CDIF algorithm is not sensitive to pulse loss and impulse interference, but the complexity of computation is larger. Milojevic improved the CDIF algorithm in 1992 and proposed the sequence difference histogram (SDIF) algorithm [8]. The SDIF needs to calculate a lower order, which greatly reduces the complexity of the algorithm, leading to a great improvement in the operation speed. The SDIF algorithm has been widely concerned by many scholars. With the continuous research on this algorithm, many improved algorithms based on SDIF have appeared.

The algorithms introduced above are based on the autocorrelation function of the received pulse sequence, so the subharmonics problem will inevitably occur. In order to solve this problem, the PRI transform method has been proposed [9]. This algorithm suppresses the subharmonics problem well,
but the detection effect on signals such as jitter and slip is worse. Later, scholar Amari proposes a modified PRI transformation method, which combines shifting time origins and overlapped PRI bins method, and makes the algorithm applied more widely [10]. As a key signal sorting parameter, PRI still has high research value. Especially when the battlefield environment changes and the original algorithm gradually fails, domestic and foreign scholars have begun to conduct in-depth research on PRI parameters. Studies have shown that these traditional sorting algorithms are unable to complete the correct sorting of jitter signals. The modified PRI transform algorithm can sort the signal with less jitter. When the amount of jitter is more than 20%, the correct sorting of the signal cannot be finished. In view of the situation above, this paper uses the combination of least squares method and traditional PRI transform algorithm to solve the problem that cannot be sorted under large jitter.

2. Sorting method based on PRI transform

2.1 PRI transform

Setting the pulses leading edge time to indicate the pulses arrival time. Let \( t_n \) \((n = 0,1,2...,N-1)\) be the pulse arrival time, Where \( N\) is the number of sampling pulses. If only the TOA parameter is considered, the sampling pulse train can be modeled as the sum of unit impulses functions, which can be expressed as

\[
g(t) = \sum_{n=0}^{N-1} \sigma(t - t_n)
\]  

(1)

The integral formula of (1) is:

\[
D(\tau) = \int_{-\infty}^{+\infty} g(t)g(t + \tau)\exp(2\pi it / \tau)dt
\]

where \( \tau > 0 \). Equation (2) is the PRI transform, which gives a kind of PRI spectrums with peaks appearing where the PRI of the frame is represented. Substituting (2) into (3) gives:

\[
D(\tau) = \sum_{n=0}^{N-1} \sigma(t - t_n + t_m) \cdot \exp[2\pi it / (t_n - t_m)]
\]

(3)

Let \([\tau_{\min}, \tau_{\max}]\) be the range of PRI and divide it into K cells (i.e. PRI bins). The width of each PRI bin is \( b = (\tau_{\max} - \tau_{\min}) / K \), then the \( kth \) PRI bin is \( \tau_k = [(k - 1) / K] (\tau_{\max} - \tau_{\min}) + \tau_{\min}, k = 1,2,..., K \). PRI statistics are performed based on equation (3).

\[
D_k = \int_{\tau_k-b/2}^{\tau_k+b/2} D(\tau)d\tau = \sum_{\tau_k-b/2 < t_n < \tau_k+b/2} \exp[2\pi it / (t_n - t_m)]
\]

(4)

\(|D_k|\) is the number of PRI, and the corresponding PRI exceeding the threshold is the possible PRI.

2.2 Modification of PRI transform

The PRI transform method has a good sorting effect on suppressing harmonics and pulse loss. However, when there is a jitter in the PRI of the pulses, the true PRI is overwhelmed by noise when the PRI is estimated by the PRI transform method, so that the correct PRI cannot be estimated. Analyzing the situation above, that is mainly because the phase error of the phase factor \( \exp[2\pi it / (t_n - t_m)] \) also increases as the pulse arrival time is gradually away from the starting time. And because of the influence of PRI jitter, the pulses that should be in the same PRI bin falls into several bines near the PRI center value, which makes the true PRI submerged by the noise. In view of these two shortcomings of the PRI transform method, the following two aspects are improved.

For one thing, shifting time origins are used to reduce the phase difference of the phase factor. By
shifting time origins, the phase error caused by the TOA away from the starting point of time is reduced. By continuously moving the shifting time origins, the phase error of the phase factor cannot be accumulated and the error of the phase factor can be reduced. For another thing, Overlapping PRI bins are used to increase the width of the PRI bin to overcome the contradiction among PRI bin width, PRI jitter width and PRI estimation accuracy.

Compared with the traditional PRI transform method, the modified PRI transform method can overcome a certain degree of jitter effect. However, due to the continuous shifting time origins and the introduction of the overlapping bins, the calculation complexity is increased, and the sorting time is increased.

3. The combination algorithm of least squares theory and PRI transform

Least squares is a mathematical optimization technique, thorough finding the best function match for the data by minimizing the sum of the squares of the errors. Using the least squares method, the unknown data can be easily obtained, and the sum of the squares of the errors between the obtained data and the actual data is minimized. The least squares method can also be used for curve fitting. The traditional PRI modification algorithm also introduces an overlapping bins to reduce the effect of jitter on the signal. In this paper, to reduce the influence of jitter on the signal, the signal is segmented by the gradient size, and then the signal is fitted on each segment, thus reducing the impact of jitter on the signal.

For the unary linear regression model, it is assumed that \( n \) sets of observations \((X_1, Y_1), (X_2, Y_2), \ldots, (X_n, Y_n)\) are obtained from the population. For these \( n \) points in the plane, the sample regression function is required to fit this set of values as well as possible in the infinite number of fit curves. In a word, this line at the center of the sample data is the most reasonable. The criteria for selecting the best fitting curve can be determined as: the total fitting error (i.e., the total residual) is minimized with the sum of the residual, the sum of the absolute values of the residual and the sum of the squares of the residuals, thought of as the selection criteria.

For a given set of data points \(\{(X_i, y_i)\} \mid i = 0, 1, 2\ldots m\}\) in the given function class \(\psi\), \(p(X) \in \psi\) is solved so that the sum of the squares of the errors \(E^2\) is the smallest \(E^2 = \sum [p(X_i) - y_i]^2\). From the geometric meaning, it is to find the curve of \(y = p(x)\) with the smallest squared sum of the distances of given point set \(\{(X_i, y_i)\} \mid i = 0, 1, 2\ldots m\}\). The function \(p(x)\) is called the fitting function or the least squares solution. The method of fitting the function \(p(x)\) is called the least squares method of curve fitting.

The matrix model of the least squares method is

\[Ax = b\]  \hspace{1cm} (5)

Where \(A\) is a matrix of \(n \times k\), \(x\) is a column vector of \(k \times 1\), and \(b\) is a column vector of \(n \times 1\). Usually \(\min \|Ax = b\|\) is calculated and solved for \(x\).

The flow of algorithm is as follows:
1. Firstly, the gradient analysis of the received pulse sequence is carried out, and the gradient threshold \(\varepsilon\) is set. The point where the gradient is large is the point where the signal jitter is relatively large. When the gradient threshold is set as \(\varepsilon > 10\) in the algorithm, the jitter is larger.
2. The pulse sequence is segmented by nodes with large gradient changes, and the least squares method described above is fitted in each segment to reduce the influence of jitter on the signal.
3. PRI transformation is applied to the processed signal, and the estimated PRI is analyzed and identified for subsequent processing.

The specific flow is show in Figure.1.
4. Simulation and results

Assume that there are 3 radars in the signal stream to be sorted, and complex radar signals without jitter (PRI1=1, PRI2=\(\sqrt{2}\), PRI3=\(\sqrt{5}\)). Firstly, as we can see in figure 2(a), the original PRI transform algorithm is used for sorting, and the PRIs of the three radars in the pulse train can be correctly estimated. As we can see in figure 2(b), 10% jitter is added to the three radar pulse PRIs, and then the PRI transform method is used for sorting. The traditional modified PRI transform algorithm is used for signal sorting, as shown in figure 3 (a). When the amount of jitter is 20%, the conventional modified PRI transform algorithm is used for sorting, and the result is shown in figure 2 (b).

It can be seen from the simulation results that the original PRI transform algorithm can perform good sorting of the signal without jitter. When the jitter is added, the PRI is completely submerged in the noise. The traditional modified PRI transform algorithm can better sort the jitter signal when the jitter is less than 10%. In the case of large jitter, if the jitter reaches 20%, the signal cannot be sorted, as shown in figure 3 (b). The simulation using the algorithm in this paper, as shown in figure 4 and figure 5. Furthermore it can be seen that the pulse number error of the PRI estimated by the new algorithm is relatively small, as shown in table 1.
The simulation results show that the original PRI transform algorithm can accurately estimate the PRI under the condition of no jitter, and the error rate of the number of sorted pulses is low, but the PRI will be submerged in the noise when sorting the jitter signal. The traditionally modified PRI transform algorithm can sort the jitter signal, but only the amount of jitter is below 20%, and the error rate of sorting the number of pulses is large. The algorithm in this paper can meet the jitter signal sorting requirements of jitter below 30%, and the error rate of the number of sorted signal is low.

5. Conclusions
The research on radar signal sorting under large jitter is of great significance. Firstly, the paper proposes a piecewise fitting of the jitter signal according to the gradient change and then segmentally performs the PRI transform algorithm to solve the problem that the PRI transform algorithm cannot achieve signal sorting under the jitter condition. Through simulation, compared with the PRI modified algorithm, it can be seen that the traditional PRI modified algorithm has better sorting of signals with lower jitter, and there is a certain error for the number of sorting pulses, and in the case of 20% of the jitter, the jitter signal cannot be correctly sorted. However, the algorithm of this paper can meet the jitter signal sorting requirements of jitter below 30%, and the error of the number of sorted pulse signals is low.

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7. References

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