Hydrological frequency analysis of large-ensemble climate simulation data using control density as a statistical control

Daiwei Cheng¹, Keita Shimizu¹* and Tomohito J. Yamada²
¹Graduate School of Engineering, Hokkaido University, Japan
²Faculty of Engineering, Hokkaido University, Japan

Abstract:
Uncertainty in hydrological statistics estimated with finite observations, such as design rainfall, can be quantified as a confidence interval using statistical theory. Ensemble climate data also enables derivation of a confidence interval. Recently, the database for policy decision making for future climate change (d4PDF) was developed in Japan, which contains dozens of simulated extreme rainfall events for the past and 60 years into the future, allowing the uncertainty of design rainfall to be quantified as a confidence interval. This study applies an order statistics distribution to evaluate uncertainty in the order statistics of extreme rainfall from the perspective of mathematical theory, while a confidence interval is used for uncertainty evaluation in the probability distribution itself. An advantage of the introduction of an order statistics distribution is that it can be used to quantify the goodness-of-fit between observation and ensemble climate data under the condition that the extreme value distribution estimated from observations is a true distribution. The order statistics distribution is called the control density distribution, which is derived from characteristics that order statistics from standard uniform distribution follows beta distribution. The overlap ratio of the control density distribution and frequency distributions derived from ensemble climate data is utilized for evaluation of the degree of goodness-of-fit for both data.

KEYWORDS climate change; control density distribution; ensemble climate data; hydrological frequency analysis; probability limit method

INTRODUCTION

The fifth report of the Intergovernmental Panel on Climate Change (IPCC) stated that there is no doubt that there is warming of the climate system and predicted that extreme rainfall in mid-latitude land areas would increase by the end of this century (IPCC, 2014). In light of this situation, research related to adaptation to climate change is increasing (e.g. Koninklijk Nederlands Meteorologisch Instituut (KNMI), 2020; Headquarters U.S. Army Corps of Engineers (USACE), 2011). In Japan, design rainfall used for flood-control management has been estimated from rainfall data, which are obtained from observations over several decades (Ministry of Land, Infrastructure, Transport and Tourism (MLIT), 2004). To make adaptation measures for torrential rainfall associated with climate change, a flood risk evaluation introducing risk-based approach has been proposed (e.g. Berkhourt et al., 2014; Yamada et al., 2018; Yamada, 2019). The risk-based approach suggested by Yamada et al. (2018) and Yamada (2019) introducing mutual uncertainty evaluation based on huge ensemble climate data and mathematical statistics has characteristics that allow evaluation of extreme phenomenon that are hard to predict with observations. Also, ensemble climate databases (Mizuta et al., 2017; Yamada et al., 2018; Sasaki et al., 2008, 2011) developed in Japan are utilized to set change ratios of rainfall with design level in each region, and flood-control management based on basin scale follows (MLIT, 2021).

In this study, we utilized the Database for Policy Decision Making for Future Climate Change (d4PDF) (Mizuta et al., 2017) which consists of simulations from the atmospheric general circulation model (AGCM) called the Meteorological Research Institute AGCM, version 3.2 (MRI-AGCM3.2) (Mizuta et al., 2012) with a horizontal resolution of 60 km (d4PDF-60 km), and dynamical downscaling (DDS) using the Non hydrostatic Regional Climate Model (NHRCM) (Sasaki et al., 2008) of the d4PDF-60 km to a horizontal resolution of 20 km targeted over Japan (d4PDF-20 km). The series of analyses presented in this study were performed using rainfall data developed by Yamada et al. (2018) and Yamada (2019) which were downscaled from d4PDF-20 km to a horizontal resolution of 5 km (d4PDF-5 km) over the Hokkaido region, northmost Japan, and its surrounding area.

In the process of design rainfall determination, goodness-of-fit testing is generally applied to evaluate validity over an assumed distribution of extreme rainfall. The Kolmogorov-Smirnov test is one such test (Kolmogorov, 1933; Smirnov, 1939). When this test is applied to frequency analysis of extreme hydrological events (e.g. World Meteorological Organization (WMO) 1989; USACE, 1994; Japan Meteorological Agency (JMA), 2021), the results can be used to determine the range of uncertainty in extreme rainfall under an assumed distribution. This test can also be applied to evaluate the goodness-of-fit between...
ensemble climate data and observations (e.g. Tanaka et al., 2019; Shimizu et al., 2020). However, the power of the Kolmogorov-Smirnov test becomes weak at the tail of an assumed distribution. The acceptance region of this test is quite wide at the tail of the assumed probability distribution, which leads to the existence of infinite values of extremes under the Kolmogorov-Smirnov test. In the light of weak power of the Kolmogorov-Smirnov test, a probability limit method was proposed by Moriguti (1995a), which has strong power at the tail of an assumed probability distribution. Shimizu et al. (2018, 2020) introduced the probability limit method into the construction of confidence intervals. The confidence interval based on the probability limit method test quantifies the threshold of the range estimated probability distribution from finite observation data with high accuracy. Also, the validity of confidence interval derived from resampling to d4PDF-5 km was supported by its consistency with the confidence interval derived based on the probability limit method (Shimizu et al., 2018, 2020).

Uncertainty in estimated probability distributions is quantified by the above confidence interval. Thus, to evaluate uncertainty in order statistics of extreme rainfall we adopted the concept of a control band, which was first described by Gumbel (1958). Here, the control band expresses the range of the i-th order statistics following an assumed distribution. Specifically, we applied an i-th order statistics for extreme rainfall, which is constructed using characteristics of beta distribution described later; we call this function the control density distribution. The control density distribution can be applied to evaluate the goodness-of-fit between ensemble climate data and observed data under the condition that the probability distribution used for construction of the control density distribution is true. Here, the confidence interval proposed by Shimizu et al. (2020) is based on a hypothesis test, the probability limit method, and diagnostics of rejection of an assumed distribution. On the other hand, control density distribution is not based on the hypothesis theory, and test statistics do not exist. Thus, the control density distribution is not used for diagnosis on the rejection of an assumed distribution. We describe our method, analyze the properties of the control density distribution and its relation to sample size, verify it using the Monte Carlo method, and apply the control density distribution method to d4PDF-5 km. The novelty of this study is the introduction of a control density distribution for the goodness-of-fit evaluation between ensemble climate data and observations.

CONCEPT OF THE CONTROL DENSITY DISTRIBUTION

In Gumbel’s Statistics of Extremes (1958), he discussed the theoretical distribution of the i-th order statistics of the Gumbel distribution. The control band can be derived from this theoretical distribution. The lower boundary \(x_{\text{lower}}^{(i)}\) and upper boundary \(x_{\text{upper}}^{(i)}\) of the control band of the i-th order statistics are derived from the following equations:

\[
\int_{-\infty}^{0_{\text{lower}}} f_{G}(x)dx = \frac{\alpha}{2} \int_{0_{\text{upper}}}^{\infty} f_{G}(x)dx = 1 - \frac{\alpha}{2}, \tag{1}
\]

where \(x^{(i)}\) is the i-th order statistics, \(f_{G}(x)\) is its theoretical probability distribution function (PDF), and \(\alpha\) is the significance level. Connecting \(x_{\text{lower}}^{(i)}\) and \(x_{\text{upper}}^{(i)}\) for different i gives the control curve of the assumed distribution (Gumbel, 1958).

The control band and control curve are useful for statistical control, but \(f_{G}(x)\) is not always known for distributions other than the Gumbel distribution. However, it provides more information than the control band and control curve. Thus, we suggest the following method to determine the control bands for an arbitrary distribution. Then, using those control bands, the PDF \(f_{G}(x)\) can be derived. The derived distribution is referred to as the control density distribution in this study.

Theoretical method of finding the control band for an arbitrary distribution

A control band of order statistics was employed using the following procedure.

(1) Probability-representing function

Generally, a random variable’s distribution is represented by a PDF or a cumulative distribution function (CDF). Moriguti (1995b) proposed the following probability-representing function (PRF):

\[
G(y) = \text{Inverse}(F(x)), \tag{2}
\]

where \(F(x)\) is a cumulative distribution function, and its inverse function, \(G(y)\), is defined as the PRF. Because the domain of \(G(y)\) is the range of \(F(x)\), it becomes \([0, 1]\). Using the PRF, the uniform distribution on \([0, 1]\) can be transformed to any type of distribution. If a random variable \(Y\) follows a uniform distribution on \([0, 1]\), the cumulative distribution function of the random variable \(X = G(Y)\) will be \(F(x)\). In this way, the characteristics of the uniform distribution on \([0, 1]\) can be generalized to any distribution; this is discussed below.

(2) Characteristics of the i-th order statistics of the uniform distribution on \([0, 1]\)

Because the properties of the uniform distribution on \([0, 1]\) can be extended to any type of distribution using the PRF, the characteristics of the statistics of the uniform distribution on \([0, 1]\) should be examined. The i-th order statistics \(x^{(i)}\) of the uniform distribution on \([0, 1]\) follows a beta distribution, for which the cumulative distribution function is as follows:

\[
F(x; \alpha, \beta) = \frac{B(x; \alpha, \beta)}{B(1; \alpha, \beta)}, \quad B(x; \alpha, \beta) = \int_0^x t^{\alpha-1}(1-t)^{\beta}dt, \tag{3}
\]

where \(B(x; \alpha, \beta)\) is incomplete beta function, \(\alpha\) and \(\beta\) are parameters of the beta distribution, and for the i-th order statistics in a sample of size \(n\), \(\alpha = i, \beta = n - i + 1\).

(3) Finding the arbitrary distribution of the i-th order statistics

For the results shown in Figure 1, we assumed a sample size of 60 and plotted the PDF of the 1st, 10th, 20th, 30th, 40th, 50th, and 60th order statistics derived from a uniform distribution. Here, the function form of order statistics
derived from the uniform distribution is equivalent to the beta distribution. By projecting the control band, \( x^{(i) - \text{uniform}}_{\text{lower}} \) and \( x^{(i) - \text{uniform}}_{\text{upper}} \), of the \( i \)-th order statistics derived from the uniform distribution to the PRF of the Gumbel distribution (or any type of distribution), we can obtain the control band of the \( i \)-th order statistics of the Gumbel distribution:

\[
\begin{align*}
    x^{(i) - \text{Gumbel}}_{\text{lower}} &= G_{\text{Gumbel}}(x^{(i) - \text{uniform}}_{\text{lower}}), \\
    x^{(i) - \text{Gumbel}}_{\text{upper}} &= G_{\text{Gumbel}}(x^{(i) - \text{uniform}}_{\text{upper}}).
\end{align*}
\]

(4)

**Deriving the control density distribution from the control band**

Using the control band, the PDF of the \( i \)-th order statistics can be reverse derived. The process is shown in Figure 2. To ensure that the CDF of the Gumbel distribution follows a straight line, the Gumbel probability paper is applied in this figure. The projection points of the \( i \)-th order statistics from the uniform distribution to the Gumbel distribution are plotted in Figure 2(a). The probability between two consecutive projection points was fixed to 0.5%. Thus, connecting the projection points gives the control curve of the Gumbel distribution. The control curves representing the 99%, 80%, 60%, 40%, and 20% control bands are plotted in Figure 2(b).

Figure 2(c) represents our concept of the control density distribution. For each order statistics, the density is calculated by dividing the fixed probability of 0.5% by the distance between two consecutive projection points. For example, the probability density between the lower boundary of the 99% control band and the lower boundary of the 98% control band is

\[
f\left(\frac{x^{99\%}_{\text{lower}} + x^{98\%}_{\text{lower}}}{2}\right) = \frac{0.5\%}{x^{98\%}_{\text{lower}} - x^{99\%}_{\text{lower}}}.
\]

(5)

Next, we checked the reliability of the control density distribution using the Monte Carlo method. The test procedure can be summarized as follows: The true distribution is the Gumbel distribution with location parameter \( \mu = 0 \), and scale parameter \( \beta = 1 \), and the sample size is 60. Then, \( 60 \times 10,000 \) samples are generated to obtain the probability densities in each order, and compare them to control density distribution. The results are shown in Figure 3. It is apparent that the frequency distributions of order statistics, which are generated from the Gumbel distribution accord with the control density distribution quite well. Figure 3(c) shows that the histogram of order statistics is almost identical to the control density distribution, indicating that the reliability of the control density distribution is high.

**Characteristics of the control density distribution**

In frequency analysis of hydrological events, the size of the sample represents the period that the hydrological data cover (in years). For Japan, observations over about 60 years are considered for flood-control management. It is very difficult to estimate rainfall with a return period that is significantly longer than the period of observation. On the other hand, the control density distribution estimates ranges of order statistics following an assumed probability distribution.

Figure 4 shows the 99% control band of the Gumbel distribution for various sample sizes, which were set to 10, 50, 100, 200 and 500. Some conclusions can be drawn. First, for a sample of size \( n \), the return period that the control band can cover ranges up to \( n \). Therefore, extrapolation is necessary for estimating extreme rainfall with a return period longer than the sample size. Second, by connecting the upper and lower limits of the control band of the different sample sizes, we obtain a straight line that is parallel to the true Gumbel distribution (Figure 4a). The same situation of the control band is derived for different significance levels, which estimates the maximum values of the different sample sizes following the Gumbel distribution with the same parameter as in the true Gumbel distribution. Hence,
it is reasonable to extrapolate the control band with a straight line that parallels the true distribution. Third, for a specific return period, a larger sample size results in a narrower control band.

APPLYING CONTROL DENSITY DISTRIBUTION TO ENSEMBLE CLIMATE DATA

In this section, the control band and control density distribution are applied to a goodness-of-fit evaluation using
In conventional frequency analysis, the following steps are conducted (e.g. Stedinger et al., 1993; Takara, 1998). First, a distribution type is found for the observations. Second, the best-fitting parameter is determined for the observations. Third, hypothesis testing is used to determine whether the distribution type should be changed. Fourth, if the best-fitting distribution passes the hypothesis test, the distribution is considered the assumed distribution and applied to estimate the rainfall for a return period larger than the sample size.

In extreme value theory (Fisher and Tippett, 1928; Gnedenko, 1943), even when the sample follows the assumed distribution and uses the best-fitting curve, uncertainty is present. In this study, the uncertainty in one sample can be quantified using the control band and control density distribution.

To use the ensemble climate dataset to help consideration on the assumed distribution, we propose two ideas:

1. Continue to use the best-fitting distribution of the observations as the assumed distribution because there is no model bias in the observations and the results can easily be compared to those of traditional frequency analysis.

2. Use the average of the best-fitting distribution of the ensemble climate dataset as the assumed distribution. With this method, the estimated distribution is affected by the climate model; however, it includes large ensemble members and can thus complement idea (1).

Consider the best-fitting distribution of the observation data as the assumed distribution

Figure 5 shows the analysis of idea (1). We considered an annual maximum 3-day rainfall. The Kolmogorov-Smirnov test and the probability limit method were both applied. As shown in the figure, for the same 5% significance level, the probability limit method had a smaller acceptance region at the tail ends of the estimated distribution compared to the Kolmogorov-Smirnov test. This means that the probability limit method had a stronger power for testing in these areas, in line with Shimizu et al. (2020). This result is important for frequency analyses of extreme hydrological events, which often focus on long return periods and would fall into the tail ends of the distribution. Using hypothesis tests for stochastic control, all of the observations were in the acceptance region for both tests; hence, no observations should be considered outliers and the estimated distribution should not be rejected at the 5% significance level.

The 99% control band in the figure is for the 100-year sample, the same period as the observation. For comparisons with d4PDF-5 km, we also needed to construct a control density distribution with the same sample size as d4PDF-5 km, which is 60 years. The results are shown in Figure 5(b). We can assess the uncertainty in the rainfall for a certain return period using the control bands. For example, Figure 5(c) shows the risks of exceeding the observations for return periods of 200, 100, 50, and 20 years are 7%, 7%, 21%, and 39%, respectively. For conventional frequency analysis, the largest observation value has a return period that is much longer than 100 years; however, with the present method, it has the same return period as the length of the observation, i.e. 100 years, and we know that the risk of exceeding that value is 7%.

For a certain return period, such as 100 years, we now have not only the expected values of extreme rainfall events but also their whole distribution for the 100-year return period. Hence, we can compare them to the d4PDF-5 km (Figure 5(c)). We can see that the shape of the control density distribution agrees well with the shape of...
the histogram; however, the histogram appears to be shifted to the left compared to the control density distribution. The reason for this shift may be that the assumed distribution was determined according to the best-fitting Gumbel distribution of the observation. Also, the overlap ratio of the control density distribution and frequency distributions of each order, which are derived from ensemble climate data, can be applied as a goodness-of-fit evaluation index for ensemble climate data against observations. When the overlap ratio is higher, the goodness-of-fit is considered high under the condition that the assumed distribution is regarded as the true distribution.

Consider the average of the best-fitting distribution of the ensemble climate dataset to be the assumed distribution

To evaluate validity of control density distribution, using the average of the best-fitting distribution of the ensemble climate data as the assumed distribution is introduced. The results are shown in Figure 6. First, we can compare the best-fitting Gumbel distribution of the observations to the average of the best-fitting distributions of the d4PDF-5 km. The plot shifts to the right, which agrees with the previous results. Furthermore, comparing Figures 5(c) and 6(c), shows that, in the current case, not only the shape but also the position of the control density distribution agrees better with the histogram than do the results of the previous case. In the current case, the risks of exceeding the observations for return periods of 200, 100, 50, and 20 years become 14%, 14%, 34%, and 57%, respectively. Here, in Figures 5(c) and 6(c), the black line corresponding to the 200-year return period represents the value derived in the following manner. The observed maximum value corresponding to the 100-year return period is extrapolated to obtain a 200-year return period with the same slope as the Gumbel distribution estimated from the observed data and d4PDF-5 km. In Figure 5, the Gumbel distribution estimated from the observation (solid red line) and, in Figure 6, that obtained from d4PDF-5 km (solid green line), are used to estimate observations with a 200-year return period. Then, an extrapolated value corresponding to a 200-year return period can be obtained. To verify the validity of the proposed method, frequency distributions with arbitrary return periods derived from each member of d4PDF-5 km are compared to the control density distribution, which is the Gumbel distribution fitted to a sample constructed of the average values of d4PDF-5 km in Figure 6. That figure confirms that the two distributions accord with each other well, supporting the validity of the proposed control density distribution. A comparison with the values of the previous case reveals some implications.

### SUMMARY

Observational data which we use for various decision making such as flood control management are essentially limited compared to enormous degrees of freedom in the climate system. Therefore, it is necessary to incorporate confidence intervals into the discussion in order to understand statistics of extremes. To quantify uncertainty in statistics of extremes, we adopted the probability limit method, which has high power of test at the tail of an assumed distribution for derivation of confidence interval. In the probability limit method, samples of order statistics from the uniform distribution on [0, 1] are generated by the
Monte Carlo method, and the occurrence probability of the order statistics located near the end most of tail of the beta distribution in each sample is extracted. The distribution of the occurrence probability of these order statistics is defined as the distribution of the test statistic. This enables an analytic derivation of the distribution of possible thresholds for each order at a given significance level and achieves high test power at the tail of the distribution. Therefore, confidence intervals based on the probability limit method, which were proposed in our previous studies (Shimizu et al., 2020), quantifies the uncertainty in the estimated probability distribution with high accuracy, while the control density distribution quantifies the probability distribution of order statistics with arbitrary return periods. In addition, we constructed a method for evaluating the goodness-of-fit between observation and ensemble climate data under the assumption that the population distribution is the probability distribution of extreme rainfall estimated from the observation. Moreover, quantification of the degree of goodness-of-fit between observation and the ensemble climate data is possible through calculation of the overlap ratio between the frequency distribution calculated from the ensemble climate data and the control density distribution for arbitrary return periods. The goodness of fit evaluation based on the control density distribution is not applied to decide rejection of an assumed distribution, because the derivation process of control density distribution does not include the theoretical test statistics distribution. Therefore, for threshold estimation of extreme rainfall and possibility of rejection of an assumed distribution, a confidence interval based on the probability limit method should be utilized. On the other hand, quantification of consistency, in which order statistics in ensemble climate data against observations is required, control density distribution might be useful.

ACKNOWLEDGMENTS

This study was supported by the Ministry of Education, Culture, Sports, Science and Technology of Japan (MEXT), the Integrated Research Program for Advancing Climate Models (TOUGOU), Theme C “Integrated Climate Change Prediction” JPMXD0717935561, the Ministry of Land, Infrastructure, Transport and Tourism, Japan and also JSPS KAKENHI grant No. 19H02241. The Earth simulator was used in this study for dynamical downscaling under the “Strategic Project with Special Support (Grand Challenge)” of the Japan Agency for Marine-Earth Science and Technology (JAMSTEC).

REFERENCES

Berkhout F, van den Hurk B, Bessembrinder J, de Boer J, Bregman B, van Drunen M. 2014. Framing climate uncertainty: socio-economic and climate scenarios in vulnerability and adaptation assessments. Regional Environmental Change 14: 879–893. DOI: 10.1007/s10113-013-0519-2.
Fisher RA, Tippett LHC. 1928. Limiting forms of the frequency distribution of the largest or smallest member of a sample. Mathematical Proceedings of the Cambridge Philosophical Society 24: 180–190. DOI: 10.1017/S0305004100015681.
Gnedenko B. 1943. Sur la distribution limite du terme maximum d’une serie aleatoire. Annals of Mathematics 44: 423–453.
Gumbel EJ. 1958. Statistics of Extremes. Columbia University
Mizuta R, Yoshimura H, Murakami H, Matsueda M, Endo H, Ose
Japan Meteorological Agency (JMA). 2021. Global Warming
Moriguti S. 1995b. Probability Representing Function. Tokyo
Intergovernmental Panel on Climate Change (IPCC). 2014. Fifth
Assessment Report.
Japan Meteorological Agency (JMA). 2021. Global Warming
Forecast Information, Japan; 74.
KNMI, Deltares. 2015. Wat betekenen de nieuwe klimaatsce‐
nario’s voor de rivierafvoeren van Rijn en Maas? 15. https://
cdn.knmi.nl/system/data_center_publications/files/000/069/
858/original/samenvatting_grade_knmi14_definitief2.pdf?
j495622007. Last access August 28, 2021.
Kolmogorov A. 1933. Sulla determinazione empirica di una legge
di distribuzione. 1933.
Ministry of Land, Infrastructure, Transport, and Tourism (MLIT).
2004. The Japanese Ministry of Land, Infrastructure, Trans‐
port and Tourism Technical Criteria for River Works: Practi‐
cal Guide for Planning. http://www.nilim.go.jp/lab/bcg/
siryou/tnm/tn0519pdf/ks0519.pdf. Last access August 28,
2021.
Ministry of Land, Infrastructure, Transport, and Tourism (MLIT).
2021. Technical study group on flood control planning in light of
climate change available. https://www.hkd.mlit.
go.jp/ky/kn/kawa_ken/splatat000001offi.html. Last access
July 23, 2021.
Mizuta R, Yoshimura H, Murakami H, Matsueda M, Endo H, Ose
T, Kamiguchi K, Hosaka M, Sugi M, Yukimoto S, Kusunoki
S, Kitoh A. 2012. Climate simulations using MRI-
AGCM3.2 with 20-km grid. Journal of the Meteorological
Society of Japan. Ser. II 90A: 233–258. DOI: 10.2151/
jmsj.2012-A12.
Mizuta R, Murata A, Ishii M, Shigama H, Hibino K, Mori N,
Arakawa O, Imada Y, Yoshida K, Aoyagi T, Kawase H,
Mori M, Okada Y, Shimura T, Nagatomo T, Ikeda M, Endo
H, Nosaka M, Arai M, Takahashi C, Tanaka K, Takemi T,
Tachikawa Y, Temur K, Kamae Y, Watanabe M, Sasaki H,
Kitoh A, Takayabu I, Nakakita E, Kimoto M. 2017. Over 5000
years of ensemble future climate simulations by 60-km
global and 20-km regional atmospheric models. Bulletin of the
American Meteorological Society 98: 1383–1398. DOI:
10.1175/BAMS-D-16-0099.1.
Moriguti S. 1995a. Testing hypothesis on probability representing
function. Kolmogorov-Smirnov test reconsidered. Journal of the
Japan Statistical Society 25: 233–244 (in Japanese with
English summary).
Moriguti S. 1995b. Probability Representing Function. Tokyo
University Press, Tokyo, Japan.
Sasaki H, Kurihara K, Takayabu I, Uchiyama T. 2008. Prelimi‐
nary experiments of reproducing the present climate using the
non-hydrostatic regional climate model. SOLA 4: 25–28.
DOI: 10.2151/sola.2008-007.
Sasaki H, Murata A, Hanafusa M, Oh’izumi M, Kurihara K. 2011
Reproducibility of present climate in a non-hydrostatic
regional climate model nested within an atmosphere general
circulation model. SOLA 7: 173–176. DOI: 10.2151/
sola.2011-044.
Shimizu K, Yamada T, Yamada TJ. 2018. New hydrological fre‐
quency analysis introducing confidence interval of probabil‐
ity distribution models based on probability limit method
test. Journal of Japan Society of Civil Engineers. Ser. B1
(Hydraulic Engineering) 74: 1.331–1.336.
Shimizu K, Yamada T, Yamada TJ. 2020. Uncertainty evaluation
in hydrological frequency analysis based on confidence
interval and prediction interval. Water 12: 2554.
Smirnov N. 1939. Ob uklonenijah empiriceskoi krivoi rasprede‐
enija. Recueil Mathematique (Matematiceskii Sbornik), N.
S. 6: 3–26.
Stedinger JR, Vogel RM, Foufoula-Georgiou E. 1993. Frequency
analysis of extreme events. In Hand-book of Hydrology,
Maidment D (ed). McGraw-Hill Professional, 18: 1–66.
Takara K. 1998. Recent advancement and perspectives of hydro‐
logic frequency analysis. Journal of Japan Society of
Hydrology and Water Resources 11: 740–756.
Tanaka T, Kawai Y, Tachikawa Y. 2019. Evaluating reproducibil‐
ity of annual maximum basin-averaged rainfall of d4PDF in
all class-A rivers in Japan. Journal of Japan Society of Civil
Engineers, Ser. B1 (Hydraulic Engineering) 75: 1.1135–
1.1140. DOI: 10.2208/jscejhe.75.2_1_1135.
United States. Corps of Engineers, United States. 1994.
Engineering and Design: Flood-Runoff Analysis (Engineer
Manual 1110-2-1417). United States. Army. Bookshop.
https://www.publications.usace.army.mil/Portals/76/Publications/
EngineerManuals/EM_1110-2-1417.pdf?ver=2013-09-04-
070759-920. Last access August 29, 2021.
United States. Army. Corps of Engineers, United States. 2011.
Office of the Assistant Secretary of the Army (Civil Works).
2011. Annual Report of Fiscal Year 2011 of the Secretary of
the Army on Civil Works Activities (1 October 2010–30
September 2011). https://cdm16021.contentdm.oclc.org/
digital/collection/p16021coll6/id/2125. Last access August
29, 2021.
World Meteorological Organization. 1989. Statistical distributions
for flood frequency analysis. Operational Hydrology Report
32: 42–51.
Yamada TJ. 2019. Adaptation Measures for Extreme Floods
Using Huge Ensemble of High-Resolution Climate Model
Simulation in Japan. Summary Report on the Eleventh
Meeting of the Research Dialogue 2019, 28–30, UNFCCC
Bonn Climate Change Conference, Bonn, Germany (19 June
2019). https://unfccc.int/documents/197307. Last access
May 5, 2021.
Yamada TJ, Hoshino T, Masuya S, Uemura F, Yoshida T, Omura
N, Yamamoto T, Chiba M, Tomura S, Tokioka S, Sasaki H,
Hamada Y, Nakatsugawa M. 2018. The influence of climate
change on flood risk in Hokkaido. Advances in River
Engineering 24: 391–396.