Elastohydrodynamic analysis of couple stress lubricated cylindrical journal bearing

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Abstract. The elastohydrodynamic analysis of plain cylindrical rotor bearing system lubricated with couple stress fluid is performed in this paper. The hydrodynamic pressure is generated entirely by the motion of the journal and depends on the viscosity of the lubricated fluid. In this analysis the static and dynamic characteristics are analyzed against the flexibility of bearing liner. The modified Reynolds equation and three dimensional elasticity equations are solved using an iterative approach. A non-dimensional parameter ‘l’ has been used for couple stress fluid. An increase in couple stress parameter with flexibility causes the enhancement of load carrying capacity. The static and dynamic characteristics of journal bearing system with flexible liner are improved with use of couple stress fluid as lubricant.

1. Introduction

In this present scenario, the heavy loads and high rotating speed with machine compactness is one of the most emerging demand of modern industries. The machine applications like steam turbines, compressors, pumps, internal combustion engines and ship propulsion shafts etc used journal bearing systems to fulfil this demand for better reliability and good service life. Hydrodynamic journal bearings are used and studied from decades to achieve better performances in rotor dynamics. Among these hydrodynamic bearings, plain cylindrical journal bearings are used conventionally. One more factor which effects the performance characteristics of full length cylindrical journal bearing is the flexibility of bearing liner. Under heavy load conditions the elastic deformation of bearing liner has significant effect on performance of rotor bearing system. A comparative study on the hydrodynamic and elastohydrodynamic in cylindrical journal bearings elaborates the significant influence of bearing liner flexibility on bearing performance [1]. The stability margin of journal bearing with deformation coefficient significantly affects the static and dynamic characteristics for a compliant shell journal bearing [2]. The nondimensionalization of dimensionless set of system equations is an important aspect in the analysis for which a straightforward procedure produced [3] and followed its application to the EHL rectangular and elliptical contacts. An another approach [4] explained which combined the boundary element method and finite element method for the analysis of elastic deformation of bearing housing. Conventional journal bearings operate with a single active oil film and the fluid film
geometry is modified due to elastic deformation. Initially the linearized analysis for rotor bearing systems were available. The nonlinear transient study of flexible shell journal bearing was carried out and a comparison of results with rigid bearing was presented [5]. Deformation coefficient factor relating $R_j$, $C$, $t$, $\mu$, $U_0$ and $E_m$, defined [6] to find the performance characteristics of journal bearing with flexible bearing liner. The permeability parameter and/or eccentricity ratio for elastohydrodynamic porous journal bearing systems calculated through an inverse model [7]. As discussed earlier bearings have their wide applications and in the field of automotive sector also. In addition to this the housing stiffness and bearing dimensions have combined effect on the connecting rod of diesel engines [8] and have significant influence of it. It becomes important to consider a well suited mathematical model to predict the pressure distribution behaviour of lubricant film within the clearance space. Keeping in view a numerical algorithm [9] solved the Reylond’s-Koiter model. The suggested model uncoupled the elastic, hydrodynamic and load constraint part to get the good performance results. Classical theories available related to elastohydrodynamic lubrication analysis introduced the lubricant used in journal bearing is Newtonian in nature. For better performances it is required to add some additives in lubricants. Such lubricants with additive inclusion are known as non-Newtonian. One of which mostly considered by several researchers for study is the couple stress fluid. A fluid with long chain polymer additives blended to Newtonian fluid is considered as couple stress fluid. Based on a theory of small micro-continuum [10] the impact of couple-stress in lubricants explained and used by several researchers in their findings. The impact of couple-stress fluids explained by Stokes model [10]. This model is used in many scientific and engineering applications for the analysis fluid phenomenon. By adding the polymer particles in fluid used as lubricants the load-carrying capacity increased whereas the friction reduced [11]. One more research explained that addition of suitable additives to a base oil reduces the friction in elastohydrodynamic contacts [12], resulted the surface damage of mating members in contact pair is going to minimize. However, the change of viscosity with the strain rate is obvious as explained above, and hence the relation of Newtonian and linear shear stress–strain rate does not hold good for longer times. The rheological behaviour for fluids non-Newtonian in nature studied by using this micro-continuum theory. For example, analysis of squeeze film characteristics of long partial journal bearing used the stokes micro-continuum model for a couple-stress lubricated model [13]. The author in another research [14] explore behaviour of short bearing systems by using the same model. The comparison of results explained to use couple stress fluid in place of Newtonian fluid because it improvises the static and dynamic characteristics of the bearing system. Couple stress factor, geometry of bearing and the magnetic parameters are three key factors for governing inlet-outlet film thickness ratio and maximum load carrying capacity in slider bearings [15]. The rotor bearing system running with non-Newtonian fluids behaves more stable compared to the system running with Newtonian fluids in externally pressurized circular step thrust bearings [16] also. In a research it was founded that the effect of surface roughness with couple stress enhanced the load-carrying capacity whereas, the rotor attitude angle and friction reduced [17] significantly. The load carrying capacity in double-layered journal bearings improved by using couple stress fluid as lubricant. It also improves the stability, such that friction also reduced. Some prominent literatures available for non-linear dynamics of rotor-bearing systems explained the behaviour of couple stress lubricated finite line contacts and found that friction and wear near the edges of roller decreased by the use of couple stress fluid as lubricant [19].

On the basis of researches and literature available, the influence of flexibility of bearing liner and couple stress fluid improves the rotor bearing system stability and enhances the performances characteristics. This paper considers a plain cylindrical rotor bearing system with elastic deformation. The lubricant used is couple stress fluid. The analysis is based on finding the static and dynamic performances characteristics. The equations are solved by using an iterative approach.
2. Analysis
There is a wide applicability of hydrodynamic plain cylindrical journal bearings. The stability analysis of such bearings is necessary from the designer’s point of view. These bearings are lubricated with single oil film in which the wedge form of pressure distribution is generated. It applies forces between the mating surfaces of fluid film and bearing. The geometry of plain cylindrical journal bearing with flexible shell is explained in Fig. 1 below. The system is having a set of equations, which are solved to study and find the performance characteristics of journal bearing systems statically andodynamical point of view.

2.1. Modified Reynolds Equation
According to Stokes micro-continuum theory The Reynolds equation for non-Newtonian fluids using Stokes micro-continuum theory in non-dimensional form is given by,

\[
\frac{\partial}{\partial x} \left[ \frac{G(R, D)}{u} \frac{\partial \bar{p}}{\partial x} \right] + \frac{\partial}{\partial \beta} \left[ \frac{G(R, D)}{u} \frac{\partial \bar{p}}{\partial \beta} \right] = 6 \frac{\partial \bar{h}}{\partial x} + \frac{\partial \bar{h}}{\partial \tau} \tag{1}
\]

where,

\[
G(R, D) = \frac{K}{\alpha} - 12\bar{l}^2\bar{h} + 24\bar{f}^2\tan \frac{K}{2\alpha} \tag{2}
\]

\[
\alpha = \frac{x}{R}, \quad \beta = \frac{y}{R}, \quad \bar{h} = \frac{h}{c}, \quad \bar{p} = \frac{p}{\mu_0R^2}, \quad \bar{\mu} = \frac{\mu}{\mu_0}, \quad \bar{\tau} = \frac{\tau}{\omega_0}, \quad U = R\omega, \bar{I} = \frac{I}{c} \tag{3}
\]

function \(G\) in the above expression is representing the interactions between base lubricant and additives.
The effect of bearing flexibility in oil film thickness is given by,

\[
\bar{h} = 1 - (\bar{x}_j + \bar{R})\cos \theta - (\bar{y}_j + \bar{Z})\sin \theta + \delta \tag{4}
\]

where,

\[
\bar{x}_j = \varepsilon \sin \phi, \quad \bar{y}_j = -\varepsilon \cos \phi \quad \text{and} \quad \bar{R} = \frac{K}{R}, \quad \bar{Z} = \frac{Z}{c} \tag{5}
\]

Term \(\delta\) in equation (4) is the radial deformation component of bearing liner which is a function of deformation coefficient \((C_d)\) and pressure,

\[
\delta = C_d \bar{x} \bar{p} \tag{6}
\]

where, \(C_d = \psi X \gamma_0\) such that \(\psi = \left\{ \frac{\mu_0}{R} \left( \frac{R}{c} \right) \right\}^3\), and \(\gamma_0 = \left( \frac{1+\gamma_1(1-2\gamma)}{1-\gamma} \right) \tag{7}\)

The non-dimensional form of a Reynolds equation for non-Newtonian fluid as described by using Stokes theory of micro-continuum can be expressed as

\[
\frac{\partial}{\partial x} \left[ \frac{G(R, D)}{u} \frac{\partial \bar{p}}{\partial x} \right] + \frac{\partial}{\partial \beta} \left[ \frac{G(R, D)}{u} \frac{\partial \bar{p}}{\partial \beta} \right] = 6 \frac{\partial \bar{h}}{\partial x} + \frac{\partial \bar{h}}{\partial \tau} \tag{8}
\]

where,

\[
G(R, D) = \frac{K}{\alpha} - 12\bar{l}^2\bar{h} + 24\bar{f}^2\tan \frac{K}{2\alpha} \tag{9}
\]

\[
\alpha = \frac{x}{R}, \quad \beta = \frac{y}{R}, \quad \bar{h} = \frac{h}{c}, \quad \bar{p} = \frac{p}{\mu_0R^2}, \quad \bar{\mu} = \frac{\mu}{\mu_0}, \quad \bar{\tau} = \frac{\tau}{\omega_0}, \quad U = R\omega, \bar{I} = \frac{I}{c} \tag{10}
\]

function \(G\) in the above expression is representing the interactions between base lubricant and additives.
The effect of bearing flexibility is given by,
\[ \bar{h} = 1 - \left( \bar{x}_j + \bar{x} \right) \cos \theta - \left( \bar{z}_j + \bar{Z} \right) \sin \theta + \delta \]  

(11)

where \( \bar{x}_j = \varepsilon \sin \phi; \bar{z}_j = -\varepsilon \cos \phi \) and \( \bar{x} = \frac{X}{c}, \bar{Z} = \frac{Z}{c} \)  

(12)

Term \( \delta \) in equation (4) is the radial deformation component of bearing liner which is a function of deformation coefficient \( (C_d) \) and pressure,  

\[ \delta = C_d X \bar{p} \]  

(13)

where, \( C_d = \psi X \gamma_0; \) such that \( \psi = \left( \frac{\mu \omega}{E} \right) \left( \frac{2k}{R} \right) \left( \frac{K_c}{c} \right)^3 \), and \( \gamma_0 = \frac{(1+\nu)(1-2\nu)}{(1-\nu)} \)  

(14)

2.2 Elastic Deformation

The distribution of pressure along the fluid film thickness is changes due to forces applied on liner surface. This variation of deformation in bearing liner is calculated by using an elasticity equation (three dimensional) which is given by,

\[ [K] \{\delta\} = \psi [F] \]  

(15)

where \([K]\) stands for stiffness matrix, \( \{\delta\} \) for deformation matrix and \( \{F\} \) for force matrix of flexibility model.

3. Solution Procedure

The static performance characteristics for the journal bearing system assumed is obtained by solving the system equations.

The solution procedure adopted to obtain the performance characteristics of the system is explained in following paragraphs.

By using an iterative approach for the method of Finite Element Method. Primarily initialize the system by defining input parameters of bearing geometry and journal positions. An initial value of pressure is also considered to solve the system equations. The model is discretized by generating mesh and this discretized model is solved to calculate the pressure profile in clearance space of journal bearing operating with couple stress fluid. The model is solved repeatedly for pressure calculation till the pressure is converged and the error is reduced to \( 10^{-5} \).

After pressure convergence the trailing edge is finalized and established which satisfies the boundary conditions. As the trailing edge is established a domain for finding the fluid film thickness is decided. So over this domain of trailing edge the fluid film thickness is obtained by solving the set of system equations which are used to obtain the final pressure profile for the selected domain.

Using these values if fluid film thickness and pressure profile, the system is solved and the static performance characteristics are obtained and presented in results and discussions.

Boundary conditions used are as follows

\[ \frac{\partial \bar{h}}{\partial \bar{r}} \bigg|_{r_0} = 0, \]  

at the film trailing edge,  

\[ \left( \frac{P_{\text{avg}} - P_{E-1}}{P_{E-1}} \right) < \left( \frac{1}{10^5} \right), \]  

criteria for pressure convergence,  

where \( k = 1, 2, \ldots, n \) and \( i \) is iteration index

4. Results and Discussions

In this paper a cylindrical hydrodynamic journal bearing is studied from statical and dynamical point of view. For the acceptability of the model and approach adopted for analysis, the results presented are compared with the results available in literature as shown in table 1.

The results available for Newtonian fluid for an eccentricity of 0.4 and flexibility 0.25 at load, \( W = 2 \) and \( C_d = 0.25 \), resembles with literature [20]. A slight difference between the values are due to the difference in approach used for calculation and analysis.
Table 1. Comparison of stiffness and damping characteristics at W = 2 and C_d = 0.25

| Validation | Kxx/w | Kzx/w | Kxz/w | Kzz/w | Cxx/w | Czx/w | Cxz/w | Czz/w |
|------------|-------|-------|-------|-------|-------|-------|-------|-------|
| Ref.()     | 1.98  | -3.99 | 2.05  | 1.65  | 4.5   | -1.7  | -1.7  | 8.15  |
| Present    | 1.904 | -3.998| 2.066 | 1.706 | 4.543 | -1.634| -1.634| 8.185 |
| % diff.    | 3.850285 | 0.18953 | 0.76959 | 3.39665 | 0.96454 | 0.979375 | 3.879375 | 0.43387 |

Figure 2. Load w.r.t. C_d for ε = 0.4

Figure 3. Stiffness coefficients (Kxx) w.r.t. C_d for ε = 0.4

Figure 4. Stiffness coefficients (Kzx) w.r.t. C_d for ε = 0.4

Figure 5. Stiffness coefficients (Kxz) w.r.t. C_d for ε = 0.4

Figure 6. Stiffness coefficients (Kzz) w.r.t. C_d for ε = 0.4

Figure 7. Damping coefficients (Cxx) w.r.t. C_d for ε = 0.4
The flexible bearing used couple stress fluid as lubricant is analysed for finding the static and damping characteristics and compared for the eccentricity values of 0.4 and 0.5. After analysing from Fig. 2 and 11 it seems that load bearing capacity increases with couple stress factor. With increasing the flexibility factor, the load carrying capacity is decreased for a particular value of eccentricity. But the load bearing capacity increased with couple stress factor and eccentricity for C_d 0 to 1.
From Fig. 3-6 it is clear that for eccentricity 0.4 the direct stiffness coefficient increased with increasing couple stress parameter for \( C_d \) 0 to 1, whereas the cross coupled stiffness is reduced with increase of couple stress parameter. Similarly, for an eccentricity of 0.5 and \( C_d \) 0 to 1, Fig. 12-15 the cross coupled stiffness is reduced whereas the direct stiffness increased with an increase in couple stress parameter. From Fig. 7-9 for eccentricity 0.4 and 16-18 for eccentricity 0.5 the direct damping is reduced whereas the cross coupled damping is increased with couple stress factor for flexibility parameter 0 to 1. The value of maximum pressure is increased on increasing the eccentricity and couple stress factor for flexibility range 0 to 1.

5. Conclusion
The deformation of bearing liner effects the performance of a plain cylindrical rotor bearing. It has been found that by using couple stress fluid (non-Newtonian) in place of Newtonian fluid the performance of fluid film bearing is enhanced. Following conclusions have been made on the basis of above analysis:

- An increase in load bearing capacity with the increase of couple stress factor.
- With increasing the flexibility factor, the load bearing capacity is decreased for any particular value of eccentricity. The load bearing capacity increases with \( C_d \) eccentricity and couple stress factor.
- From Fig. 3-6 it is clear that for eccentricity 0.4 the direct stiffness coefficient increased with increasing couple stress parameter for \( C_d \) 0 to 1, whereas the cross coupled stiffness is reduced with increase of couple stress parameter.
- Similarly, for an eccentricity of 0.5 and \( C_d \) 0 to 1, Fig. 12-15 the cross coupled stiffness is reduced whereas the direct stiffness increased with an increase in couple stress parameter.
From Fig. 7-9 for eccentricity 0.4 and 16-18 for eccentricity 0.5 the direct damping is reduced whereas the cross coupled damping is increased with couple stress factor for flexibility parameter 0 to 1.

The value of maximum pressure is increased with eccentricity and couple stress factor for flexibility range 0 to 1.

The journal bearings are mostly commonly used in turbomachinery for smooth movement of two relatively rotating bodies. Practically the elastic deformation of bearing linear effects the performance characteristics. So this study will help the designers to design the journal bearing for different applications with using couple stress fluid as lubricant so that the efficiency and performance of system is improved. The researches available are for static studies of journal bearing. Present study also helps the researchers and designers for practical applicability of journal bearing.

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