Remark about Non-BPS D-Brane in Type IIA Theory

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ABSTRACT: In this paper we would like to show simple mechanisms how from the action for non-BPS Dp-brane we can obtain action describing BPS D(p-1)-brane in IIA theory.

KEYWORDS: D-branes.
1. Introduction

In the last year many new results about D-branes in string theories have emerged. In the remarkable series of paper by Sen [5, 6, 11, 12, 13], the problem of nonsupersymmetric configuration in string theories has been studied. It is clear that non-BPS D-branes in the IIA, IIB theories are as important as supersymmetric ones. On the other hand it is known that non BPS D-branes are not stable, so that they can decay into supersymmetric string vacuum. Instability of this system is a consequence of tachyon field that lives on world-volume of non-BPS D-brane. But presence of non-BPS brane in the string theory is important from one reason. We can construct kink solution of tachyonic field on the world-volume of non-BPS D-brane, which forms D-brane of dimension smaller than original non-BPS D-brane. It can be shown on base of topological arguments that this solution is stable (for a review of this subject, see [4, 2]). Witten generalised this construction and showed that all branes in IIB theory can be constructed as topological defects in space-time filling world-volume of D9 branes and D9 antibranes [7]. Hořava extended this construction to the case of IIA theory and showed that all D-branes in IIA theory can emerge as topological solutions in space-time filling non-BPS D9 branes. Hořava also proposed an intriguing conjecture about Matrix theory and construction of D0 branes in K theory [8, 14]. For review of the subject D-branes and K theory, see [10], where many references can be found.

In recent paper Sen [1] proposed an supersymmetric invariant action for non-BPS D-branes. Because non-BPS branes break all supersymmetries, it seems to be strange to construct supersymmetric action describing this brane. However, although there is no manifest supersymmetry of world-volume theory, we still expect world-volume theory to be supersymmetric, with the supersymmetry realised as a spontaneously...
broken symmetry. From these arguments Sen showed that the action has to contain the full number of fermionic zero modes (32), because they are fermionic Goldstone modes of completely broken supersymmetry, while BPS D-brane contains 16 zero modes, because breaks one half of supersymmetry. Sen showed that the DBI action for non-BPS D-brane (without presence of tachyon) is the same as the supersymmetric action describing BPS D-brane. This action is manifestly symmetric under all space-time supersymmetries. Sen argued that ordinary action for BPS D-brane contain DBI term and WZ term, which are invariant under supersymmetry but only when they both are present in the action for D-brane, the action is invariant under local symmetry on brane $\kappa$ symmetry that is needed for gauging away one half of fermionic degrees of freedom, so that on BPS D-brane only 16 physical fermionic fields live, as should be for object breaking 16 bulk supersymmetries. Sen showed that the DBI term for non-BPS D-brane is exactly the same as DBI term in action of BPS D-brane (when we suppose, that other massive fields are integrated out, including tachyon) that is invariant under supersymmetric transformations, but has no a $\kappa$ symmetry, so that number of fermionic degrees of freedom is 32 which is a appropriate number of fermionic Goldstone modes for object braking bulk supersymmetry completely.

Sen also showed how we could include the tachyonic field into action. Because mass of tachyon is of order of string scale, there is not any systematic way to construct this effective action for tachyon, but on grounds of invariance under supersymmetry and general covariance Sen proposed the form of this term expressing interaction between tachyon and other light fields on world-volume of non-BPS D-brane. This term has a useful property that for constant tachyon field is zero, so that the action for non-BPS vanishes identically.

In present paper we would like to extend the analysis in paper [1]. We propose the form of the term containing the tachyon and we show that condition of invariance under supersymmetric transformation places strong constraints on the form of this term. Then we will show that tachyon condensation in the form of kink solution leads to the DBI action for BPS D-brane of codimension one with gauged local $\kappa$ symmetry.

In conclusion, we will discuss other problems with non-BPS D-brane and relation of this construction to the K-theory.

2. Action for non-BPS D-brane in IIA theory

We start this section with recapitulating basic facts about non-BPS D-branes in IIA theory, following [1]. Let $\sigma_\mu, \mu = 0, \ldots, p$ are world-volume coordinates on D-brane. Fields living on this D-brane arise as the lightest states from spectrum of open string ending on this D-brane. These open strings have two CP sectors [3]: first, with unit $2 \times 2$ matrix, which correspond to the states of open string with usual GSO
projection \((-1)^F |\psi\rangle = |\psi\rangle\), where \(F\) is world-sheet fermion number and \(|\psi\rangle\) is state from Hilbert space of open string living on Dp-brane. The second CP sector has CP matrix \(\sigma_1\) and contains states having opposite GSO projection \((-1)^F |\psi\rangle = - |\psi\rangle\). The massless fields living on Dp-brane are ten components of \(X^M(\sigma), M = 0, \ldots, 9\); \(U(1)\) gauge field \(A(\sigma)_\mu\) and fermionic field \(\theta\) with 32 real components transforming as Majorana spinor under transverse Lorenz group \(SO(9, 1)\). We can write \(\theta\) as sum of left handed Majorana-Weyl spinor and right handed Majorana-Weyl spinor:

\[
\theta = \theta_L + \theta_R, \quad \Gamma_{11} \theta_L = \theta_L, \quad \Gamma_{11} \theta_R = - \theta_R
\]

All fields except \(\theta_R\) come from CP sector with identity matrix, while \(\theta_R\) comes from sector with \(\sigma_1\) matrix. \(^1\)

As Sen \([1]\) argued, action for non BPS D-brane (without tachyon) should go to the action for BPS D-brane, when we set \(\theta_R = 0\) (we have opposite convention that \([1]\)). From this reason, action for non-BPS D-brane in \([1]\) has been constructed as supersymmetric DBI action, which is manifestly supersymmetric invariant, but has not \(\kappa\) symmetry, so we cannot gauge away one half of fermionic degrees of freedom, so that this action describes non-BPS D-brane.

The next thing is to include the effect of tachyon. In order to get some relation between tachyon condensation and supersymmetric D-branes, we would like to have an effective action for massless field and tachyon living on world-volume of non-BPS D-brane. This effective action should appear after integrating out all massive modes of open string ending on Dp-brane. Because tachyon mass is of order of string scale, there is no systematic way to obtain effective action for this field, but we can still study some general properties of this action. Following ref.\([1]\), the effective action for non-BPS Dp-brane with tachyonic field on its world-volume should has a form:

\[
S = - \int d^{p+1}\sigma \sqrt{- \det(\mathcal{G}_{\mu\nu})} F(T, \partial T, \theta_L, \theta_R, \mathcal{G}, \ldots)
\]

\[
\Pi^M_\mu = \partial_\mu X^M - \bar{\theta} \Gamma^M \partial_\mu \theta, \quad \mathcal{G}_{\mu\nu} = \eta_{MN} \Pi^M_\mu \Pi^N_\nu
\]

and

\[
\mathcal{F}_{\mu\nu} = F_{\mu\nu} - [\bar{\theta} \Gamma_{11} \Gamma_M \partial_\mu \theta (\partial_\nu X^M - \frac{1}{2} \bar{\theta} \Gamma^M \partial_\nu \theta) - (\mu \leftrightarrow \nu)]
\]

The constant \(C = \sqrt{2} T_p = \sqrt{2} \pi g\) was included in the function \(F\) in (2.2).

We must say few words about function \(F\), which expresses the presence of tachyon on world-volume of unstable non-BPS D-brane. We know that this function must

\(^1\)Our conventions are following. \(\Gamma^M\) are \(32 \times 32\) Dirac matrices appropriate to 10d with relation \(\{\Gamma^M, \Gamma^N\} = 2\eta^{MN}\) with \(\eta^{MN} = (-1, 0, \ldots, 0)\). For this choice of gamma matrices the massive Dirac equation is \((\Gamma^M \partial_\mu - M) \Psi = 0\). We also introduce \(\Gamma_{11} = \Gamma_0 \ldots \Gamma_9, (\Gamma_{11})^2 = 1\). We also work in units \(4\pi^2\alpha' = 1\). Then tension of BPS Dp-brane is equal to \(T_p = \frac{2\pi g}{\alpha'}\) where \(g\) is a string coupling constant.
be invariant under Poincare symmetry and supersymmetry. We also expect, that this function must express interaction between light massless fields living on world-volume of non-BPS D-brane and tachyon. We will also suppose in construction of this function, that other massive fields were integrated out. And finally, following [4] we demand, that this function is zero for tachyon equal to its vacuum expectation value $T_0$ and for $T = 0$ is equal to $T_p$ tension of BPS D-brane, which corresponds to $(-1)^{F_L}$ projection, which projects out tachyon field and also fermionic field from $\sigma_1$ sector, so that resulting D-brane is BPS Dp-brane in Type IIB theory [2]:

$$F(T = T_0) = 0, \quad F(T = 0) = \frac{2\pi}{g}$$

(2.5)

Now we propose the form of this function. Firstly, it must contain kinetic term for tachyon, which should be written in manifest supersymmetry invariant way:

$$I_{KT} = \tilde{G}^{\mu\nu} \partial_\mu T \partial_\nu T$$

(2.6)

where $\tilde{G}$ denotes the matrix inverse of $G + \frac{1}{2\pi}F$ and $\tilde{G}_S$ means the symmetric part of the matrix. As was argued in ref. [1], the choice of this metric was motivated by work [15], where was argued that in constant background $B$ field the natural metric for open strings is $\tilde{G}_S$ and ordinary products becomes noncommutative products. On the other hand, in [16, 17] it was shown, that natural noncommutative parameter is gauge invariant combination of $F - B$, where $F$ is a constant background field strength of gauge field living on world-volume Dp-brane. From these reasons, we take the same metric as in [4], but due to the vanishing of $B$ field and background $F$ field strength we expect that noncommutative effects do not appear in our case and hence we will consider ordinary products between any functions. We will see that the function $F$ is really invariant under all symmetries presented above.

There should be potential term for tachyon in function $F$. We suppose form of this term as:

$$V(T) = -m^2 T^2 + \lambda T^4 + \frac{m^4}{4\lambda}$$

(2.7)

This potential has a vacuum value equal to

$$\frac{dV}{dT} = 0 \Rightarrow T_0^2 = \frac{m^2}{2\lambda}, \quad V(T_0) = 0$$

(2.8)

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2 In fact, Sen argued, that in case of constant $T$, $F$ reduces to the potential for tachyon and as a consequence of general form of potential for tachyon, this term is zero for $T = T_0$. In this article, we slightly change behaviour of this function, because we only demand that in point on world-volume, where the tachyon is equal to its vacuum value $T_0$, we should recover supersymmetric vacuum, so that there are no fields living on non-BPS D-brane, so we have (2.3). Sen also argued that for $T = 0$ function $F$ should be equal to $C$, but we think that this function should rather be equal to $T_p$ from reasons explained above.
We also expect that some interaction terms between tachyon and $X$ fields and gauge fields will be presented in $F$. In fact the interaction between $T$ and $X,A$ is presented in kinetic term for tachyon, which can be seen from the form of $\tilde{G}_S$. We must also stress that tachyon is not charged with respect to gauge field because transforms in adjoin representation of gauge group and $U(1)$ has no adjoin representation. From this reason there are no covariant derivatives in the action.

As a last thing, we will consider the interaction term between tachyon and fermionic fields $\theta_L, \theta_R$. We propose this term in the form:

$$I_{TF} = g(T)(\theta_R^T \Gamma^0 \Gamma_M \Pi^M_\mu \tilde{G}_S^\mu \partial_\nu \theta_R + \theta_L^T \Gamma^0 \Gamma_M \Pi^M_\mu \tilde{G}_S^\mu \partial_\nu \theta_L) + f(T)\tilde{G}_S^\mu \partial_\mu \theta_R^T \Gamma^0 \partial_\nu \theta_L$$  \hspace{1cm} (2.9)

where $g(T)$ is even function of $T$ and $f(T)$ is odd function of $T$, which comes from the fact that in perturbative diagrams in string theory $T$ comes with CP factor $\sigma_1$ and $\theta_L$ with CP matrix $\mathbf{1}$ and finally $\theta_R$ with CP factor $\sigma_1$. Then it is clear that $\theta_R \theta_L$ gives $\sigma_1$ so in order to have nonzero trace over CP factors, we must have $f(T)$ with odd powers of $T$, which gives factor $\sigma_1$. In the same way it can be shown that $g(T)$ must be even function. We have also used the important properties of Majorana-Weyl spinors which say that expression of two Weyl spinors of the same chirality with odd number of gamma matrices is zero and expression of two Weyl spinors of opposite chirality with even number of gamma matrices is zero, which can be seen from following simple arguments:

$$\theta_L^T (\Gamma_i ... \Gamma_k) \theta_L = \theta_L^T \Gamma_{11} (\Gamma_i ... \Gamma_k) \Gamma_{11} \theta_L = -\theta_L^T (\Gamma_i ... \Gamma_k) \theta_L$$  \hspace{1cm} (2.10)

where we have used the fact, that $\Gamma_{11}$ commutes with even number of gamma matrices and anticommutes with odd numbers of gamma matrices.

In summary, we expect that $F$ has a form:

$$F = \frac{2\pi \sqrt{2}}{g}(\tilde{G}_S^\mu \partial_\mu T + V(T) + I_{TF})$$  \hspace{1cm} (2.11)

We know that $F$ must be invariant under supersymmetric transformations as well as under Lorenz transformations and translations. We will see that requirement of supersymmetric invariance place important conditions on various terms in the action.

Under space-time translation, which has a form

$$\delta_\xi X^M = \xi^M, \delta_\xi \theta_{L,R} = 0, \delta_\xi T = 0$$  \hspace{1cm} (2.12)

we have

$$\delta_\xi \Pi^M_\mu = 0 \Rightarrow \delta_\xi G = 0, \delta_\xi F = 0$$  \hspace{1cm} (2.13)

so that $F$ is invariant.

Under $SO(1,9)$ Lorenz symmetry various fields transform as

$$X'^M = \Lambda^M_N X^N, \theta' = R(\Lambda) \theta, \bar{\theta}' = \bar{\theta} R(\Lambda)^{-1}, T' = T$$  \hspace{1cm} (2.14)
We then obtain
\[ \Pi^M_\mu = \Lambda^M_N \partial_\mu X^N - \overline{\theta} R(\Lambda)^{-1} \Gamma^M R(\Lambda) \theta = \Lambda^M_N \Pi^N_\mu \] (2.15)
where we have used: \( R(\Lambda)^{-1} \Gamma^M R(\Lambda) = \Lambda^M_N \Gamma^N \). Then
\[ \mathcal{G}'_{\mu\nu} = \eta_{MN} \Lambda^M_K \Pi^K_\mu \Lambda^L_N \Pi^L_\nu = \mathcal{G}_{\mu\nu} \] (2.16)
with using \( \Lambda^M_K \eta_{MN} \Lambda^L_N = \eta_{KL} \). Then
\[ \mathcal{G}'_{\mu\nu} = \eta_{MN} \Lambda^M_K \Pi^K_\mu \Lambda^L_N \Pi^L_\nu = \mathcal{G}_{\mu\nu} \] (2.16)
with using \( \Lambda^M_K \eta_{MN} \Lambda^L_N = \eta_{KL} \). Then
\[ G'_{\mu\nu} = \eta_{MN} \Lambda^M_K \Pi^K_\mu \Lambda^L_N \Pi^L_\nu = G_{\mu\nu} \] (2.16)
with using \( \Lambda^M_K \eta_{MN} \Lambda^L_N = \eta_{KL} \). Then
\[ \delta \Lambda \mathcal{F} = 0 \] and consequently
\[ \delta \Lambda \tilde{\mathcal{G}}_S = 0 \] (2.17)
To prove Lorenz invariance of fermionic terms we use
\[ \theta_R = \frac{1}{2} (1 + \Gamma_{11}) \theta, \theta_L = \frac{1}{2} (1 - \Gamma_{11}) \theta \] (2.18)
as well as basic properties of \( \Gamma_{11} \) matrix: \( \Gamma^T_{11} = \Gamma_{11} \), \( R(\Lambda)^{-1} \Gamma_{11} R(\Lambda) = \Gamma_{11} \) to rewrite the expression \( \partial_\mu \theta_R \Gamma^0 \partial_\nu \theta_L \) as
\[ \frac{1}{4} \partial_\mu \theta^T (1 + \Gamma_{11}) \Gamma^0 (1 - \Gamma_{11}) \partial_\nu \theta = \frac{1}{2} \partial_\mu \overline{\theta} (1 - \Gamma_{11}) \partial_\nu \theta \] (2.19)
which transforms under Lorenz transformation as
\[ \frac{1}{2} \partial_\mu \overline{\theta} (1 - \Gamma_{11}) \partial_\nu \theta' = \frac{1}{2} \partial_\mu \overline{\theta} R(\Lambda)^{-1} (1 - \Gamma_{11}) R(\Lambda) \partial_\nu \theta = \frac{1}{2} \partial_\mu \overline{\theta} \partial_\nu \theta \] (2.20)
which prove the Lorenz invariance of this term. In the same way we can prove invariance of expression
\[ \theta^T_R \Gamma^0 \Gamma_M \Pi^M_\mu \partial_\nu \theta_R = \frac{1}{2} \overline{\theta} \Gamma_M (1 + \Gamma_{11}) \Pi^M_\mu \partial_\nu \theta \] (2.21)
This term transforms under Lorenz transformation as
\[ \frac{1}{2} \overline{\theta} R^{-1} \Gamma_M (1 + \Gamma_{11}) \Lambda^K_M \Pi^K_\mu \partial_\nu \theta = \frac{1}{2} \overline{\theta} \Lambda^K_M \Gamma_L (1 + \Gamma_{11}) \Lambda^K_M \Pi^K_\mu \partial_\nu \theta = \frac{1}{2} \overline{\theta} \Gamma_M (1 + \Gamma_{11}) \Pi^M_\mu \partial_\nu \theta \] (2.22)
We see that the all integration terms between fermions and tachyon are Lorenz invariant.

Now we come to the crucial question of supersymmetry transformation, which have a form
\[ \delta_\epsilon \theta = \epsilon, \delta_\epsilon X^M = \overline{\epsilon} \Gamma^M \theta \]
It is well known, that these transformations leave \( \Pi^M_\mu \) invariant, consequently \( \mathcal{G} \) as well. It can be also shown \[19\] that \( \mathcal{F} \) is invariant as well. As a result we have
\[ \delta_\epsilon \tilde{\mathcal{G}}_S = 0 \] (2.23)
Now we are ready to prove invariance of the term
\[ f(T) \tilde{G}^{\mu \nu}_{S} \partial_{\mu} \theta_{R}^{T} \Gamma^{0} \partial_{\nu} \theta_{L} = \frac{1}{2} f(T) \tilde{G}^{\mu \nu}_{S} \partial_{\mu} \overline{\theta}(1 - \Gamma_{11}) \partial_{\nu} \theta \] (2.24)

This term is clearly invariant under supersymmetry transformations due to the presence of partial derivative. On the other hand, the term
\[ \frac{1}{2} g(T) \tilde{G}^{\mu \nu}_{S} \overline{\theta} \Gamma_{M} (1 + \Gamma_{11}) \Pi^{M}_{\mu} \partial_{\nu} \theta \] (2.25)
leads after supersymmetric transformation to the variation of the action
\[ \delta S = \int D \pi \Gamma_{M} (1 + \Gamma_{11}) \Pi^{M}_{\mu} \partial_{\nu} \theta = - \int \partial_{\nu}(D \Pi^{M}_{\mu}) \pi \Gamma_{M} (1 + \Gamma_{11}) \theta \] (2.26)
where we have used
\[ D = \sqrt{- \text{det}(G + \frac{1}{2 \pi} F)} \frac{1}{2} g(T) \tilde{G}^{\mu \nu}_{S} \] (2.27)

We see that requirement of invariance under transformation of supersymmetry leads to conclusion that term \( g(T)... \) should not be present in the action for non-BPS D-brane, so that we consider interaction term between fermions and tachyon in the form:
\[ f(T) \tilde{G}^{\mu \nu}_{S} \partial_{\mu} \theta_{R}^{T} \Gamma^{0} \partial_{\nu} \theta_{L} \] (2.28)

It is important to stress that this term is consistent with requirement, that \( f(T) \) should be odd function of \( T \). This can be seen from the fact that \( \tilde{G}_{S} \) has a CP factor equal to unit matrix. To prove this we expand \( G \) as follows:
\[ G_{\mu \nu} = \eta_{MN} \partial_{\mu} X^{M} \partial_{\nu} X^{N} - 2 \eta_{MN} \partial_{\mu} X^{M} \overline{\theta} \Gamma^{N} \partial_{\nu} \theta + \eta_{MN} (\overline{\theta} \Gamma^{M} \partial_{\mu} \theta)(\overline{\theta} \Gamma^{N} \partial_{\nu} \theta) \] (2.29)

we know that \( X^{M} \) comes from CP sector with unit matrix, so that only one ”dangerous” term is
\[ \overline{\theta} \Gamma^{N} \partial_{\nu} \theta = (\theta_{R} + \theta_{L})^{T} \Gamma^{0} \Gamma^{N} \partial_{\nu} \theta_{R} + \theta_{R}^{T} \Gamma^{0} \Gamma^{N} \partial_{\nu} \theta_{L} \] (2.30)

These two terms give CP factors either \( (11 = 1) \) (for \( \theta_{L} \)) or \( (\sigma_{1} \sigma_{1}) = 1 \) for \( (\theta_{R}) \)
(\( \) In previous part we have used the result: \( \theta_{R}^{T} \Gamma^{0} \Gamma^{M} \partial_{\mu} \theta_{L} = - \theta_{R}^{T} \Gamma^{0} \Gamma^{M} \partial_{\mu} \theta_{L} = - \theta_{R}^{T} \Gamma^{0} \Gamma^{M} \partial_{\mu} \theta_{L} \).

In the same way we can prove that \( F \) comes with unit matrix of CP factors. For example expression \( \overline{\theta} \Gamma_{11} \Gamma_{M} \partial_{\mu} \theta \) is equal to \( \theta_{R}^{T} \Gamma^{0} \Gamma_{11} \Gamma_{M} \partial_{\mu} \theta_{R} + \theta_{L}^{T} \Gamma^{0} \Gamma_{11} \Gamma_{M} \partial_{\mu} \theta_{L} \), where we have used the identity
\[ \theta_{R}^{T} \Gamma^{0} \Gamma_{11} \Gamma_{M} \theta_{R} = - \theta_{L}^{T} \Gamma_{11} \Gamma_{0} \Gamma_{11} \Gamma_{M} \theta_{R} = - \theta_{L}^{T} \Gamma_{0} \Gamma_{11} \Gamma_{M} \theta_{R} \] (2.31)

It seems to us that requirement of supersymmetry places strong constraint on various coupling between fermions and tachyon. In particular, we have seen that fermions must always come with partial derivative. In next section we will shown that tachyon condensation in form of kink solution leads to correct supersymmetric invariant DBI action of BPS D(p-1)-brane.
3. Tachyon condensation on world-volume of non-BPS D-brane

In this section we will consider tachyon condensation on world-volume of non-BPS Dp-brane in the form of kink solution. In order to get clear picture of resulting D-brane, we will consider tachyon kink solution in the form:

\[
T(x) = \begin{cases} 
-T_0, & x < 0 \\
0, & T = 0 \\
T_0, & x > 0 
\end{cases}
\]  
(3.1)

where \( x \) is one particular coordinate on world-volume of non-BPS Dp-brane. We get equation of motion for tachyon from variation of (2.2), which give (we consider dependence of tachyon only on \( x \), say \( p \) coordinate):

\[
d dx \left( \frac{\delta F}{\delta \partial_x T} \right) - dF = 0 
\]  
(3.2)

where \( F \) has a form:

\[
F = \frac{2\pi \sqrt{2}}{g} \left( \tilde{G}^\mu_\nu \partial_\mu T \partial_\nu T + V(T) + f(T)\tilde{G}^\mu_\nu \partial_\mu \theta_R \Gamma^0 \partial_\nu \theta_L \right) 
\]  
(3.3)

First equation in (3.2) gives

\[
\partial_\mu (\tilde{G}^{\mu x}_S \partial_x T(x)) = \partial_\mu (\tilde{G}^{\mu x}) \partial_x T = 0 
\]  
(3.4)

where we have used the fact that tachyon field is only function of \( x \) and the fact that we can look at the solution (3.1) as a extreme limit of ordinary kink solution, when behaviour of tachyon field can be approximated around the point \( x = 0 \) as \( T \sim x \). More precisely, when we take solution which is equal to tachyon vacuum value \( T_0 \) at distance \( x = L \) from the point \( x = 0 \) than we get tachyon field around the point \( x = 0 \) in the form \( T = T_0 x \). It is clear that second derivative of this expression is zero. In order to get (3.1) we must take limit \( L \rightarrow 0 \) for \( x \rightarrow 0 \) in such a way that \( T \rightarrow 0 \) for \( x \rightarrow 0 \).

Outside the point \( x = 0 \) this equation is trivially obeyed because \( T(x) \) is equal to its vacuum value \( T_0 \). In the point \( x = 0 \) we have \( \partial_x T(x) \sim \delta(x) \) so that to obey equation of motion we must pose the condition

\[
\tilde{G}^{\mu x}_S \bigg|_{x=0} = \text{const.} \Rightarrow G_{\mu x} \bigg|_{x=0} = \text{const.}, \ F_{\mu x} \bigg|_{x=0} = \text{const.} 
\]  
(3.5)

When we look at definition of \( G \) we will see, that this condition implies \( X^\mu = kx \) which has a form of static gauge which certainly is not correct for \( \mu \neq x \) so we come to the result \( G_{\mu x} = 0 \) for \( \mu \neq x \) and with appropriate form of scaling we can take \( G^{xx} = G_{xx} = 1 \). For \( F_{\mu x} = F_{\mu x} = \text{const.} \) (where we have used the upper result \( \partial_x X^\mu = \partial_x \theta_L = \partial_x \theta_R = 0, \ \mu \neq x \)) we can take this constant to be equal to zero, because the presence of the constant term do not affect the equation of motion.
The second term in (3.2) gives
\[-\frac{dV}{dT} - \frac{df(T)}{dT} \tilde{G}^{\mu\nu}_S \partial_\mu \theta_R T^0 \partial_\nu \theta_L = 0 \quad (3.6)\]

It easy to see that (3.1) is a solution of equation \(\frac{dV}{dT} = 2m^2(T^2 - T^0_0) = 0\). Outside the point \(x = 0\) the tachyon field is equal to its vacuum value so the expression in the bracket is zero and in the point \(x = 0\), \(T\) is equal to zero. In the point \(x = 0\) the value of potential is equal to \(\frac{m^4}{4\pi}\).

Now we come to the second term in (3.2). We know that \(f(T)\) is odd function of \(T\) so that we can expect that its derivative \(\frac{df(T)}{dT}\) is nonzero. The only possibility to obey equation of motion for tachyon is to pose the condition that \(\partial_\mu \theta_R\) or \(\partial_\mu \theta_L\) is equal to zero. We choose the condition:
\[\partial_\mu \theta_R = 0 \quad (3.7)\]

Strictly speaking we have obtained this condition from behaviour of tachyon outside the point \(x = 0\). In the point \(x = 0\) we expect that derivative of \(f\) is still nonzero but \(\tilde{G}^{\mu x}_S = 0\) so that in the point \(x = 0\) we have condition
\[\partial_\mu \theta_R = 0, \quad \mu = 0, ..., p - 1 \quad (3.8)\]

but with using (3.5) and consequences resulting from it we see that (3.7) must holds in the point \(x = 0\) as well. We have arrived to the important result that \(\theta_R\) is non-dynamical field which can be set to equal to zero. In other words, we have eliminated via tachyon condensation one half of fermionic degrees of freedom with direct parallel of gauging of \(\kappa\) symmetry on world-volume of BPS D-brane.

We come to the final result. We have seen that (3.1) is equation of motion for tachyon. When we put this function into (3.3) we get
\[F(T = \text{kink}) = \frac{2\pi \sqrt{2}}{g} T^2_0 \delta(x) \quad (3.9)\]

where we have used \(\tilde{G}^{xx}_S = 1\) and \(\delta(x)^2 = \delta(x)\). In previous equation we have also used the fact that the derivation is much larger that the vacuum value of the potential. In order to get correct tension of resulting D-brane we must fix the vacuum value of tachyon to be equal to \(T^2_0 = \frac{1}{\sqrt{2}}\). When we put (3.3) into (2.2) and with using (3.5), (3.7) we get the \(\kappa\) fixed action for D(p-1)-brane in Type IIA theory:
\[S = -\frac{2\pi}{g} \int d^p \sigma \sqrt{-\det(G + \frac{1}{2\pi} F)} \quad (3.10)\]

There is a place when we can discuss some questions about interaction terms between fermions and tachyon. In previous part we have estimated the possible form
of interaction term between fermions and tachyon on grounds of invariance under supersymmetry. We may ask the question, whether other possible terms which were not included into the form of $F$ do not spoil the result of emergence of BPS D-brane from tachyon condensation on world-volume of non-BPS D-brane. For example, we can consider the interaction of the form:

$$\tilde{G}^{\rho\kappa} \partial_{\rho} g(T^{2}) \Pi^{M} \tilde{G}^{\mu\nu} \partial_{\mu} \bar{\theta}_{L} \Gamma_{M} \partial_{\nu} \theta_{L}$$

(3.11)

which is consistent with requirement of invariance under all symmetries presented above as well as with the trace of CP factors. The answer is that this term is zero for tachyon approaching kink solution and consequently do not give any constraint on massless fields.

When we write $g(T) = T^{2} + T^{4}$, than $\partial_{x} g(T) = 2T \partial_{x} T + 4T^{3} \partial_{x} T + \ldots$ and consequently $\frac{\partial_{x} g}{\partial T} = 2T + 4T^{3} + \ldots$ so that the first term in equation of motion for tachyon gives contribution

$$2\partial T + 12T^{2}\partial T + \ldots$$

(3.12)

while the second term is equal to

$$2\partial T + 12T^{2}\partial T + \ldots$$

(3.13)

then we see, that the term $\partial_{x} g(T^{2})$ obeys equation of motion identically and we do not get any additional constraint on massless fields. Moreover, when we put kink solution into this term we will see that this term is identically equal to zero. Outside the core of the soliton $T$ is in its vacuum value $T^{2} = T_{v}^{2}$ so that its derivative is zero and in the point $x = 0$ we can consider this kink solution as a extreme limit of ordinary solution. Than $T \partial T$ is zero in the point $x = 0$ due to the fact that $T$ is zero in the point $x = 0$. When we go to extreme limit, this expression is always equal to zero in this limiting procedure so we can conclude that is zero in extreme limit as well. We than see that this interaction term is identically zero after tachyon condensation and do not give any new information and any new constraint on fermionic field so that we do not include this term into definition of $F$ function in (2.2). We also expect that other possible interaction terms do not give new information and from this reason we will consider the $F$ function in the form given in (3.3).

We must also discuss the situation when $T$ is equal to its vacuum value everywhere. Naively we could expect from the form of (3.3) that for this value of tachyon we do not get supersymmetric vacuum due to the presence of the interaction term between tachyon and fermions. However, tachyon vacuum value must be solution of equation of motion and as we have seen this leads to requirement of constant spinor field $\theta_{R}$. Then interaction term between fermions and tachyon is equal to zero and from the definition $V(T = T_{0}) = 0$, $\partial_{x} T = 0$ and we get the result that the second bracket is equal to zero and consequently the whole action disappears with agreement with ref. [1].
4. Conclusion

In previous parts we have proposed the possible form of supersymmetric DBI action for non-BPS D-brane in Type IIA theory (For IIB theory the situation will be basically the same with difference that both spinors have the same chirality). We have seen that requirement of invariance under supersymmetric transformations place strong constraints on possible form of this action. Then we have studied the kink solution of tachyon on world-volume of non-BPS Dp-brane in IIA theory and we have shown, that this solution really describes BPS D(p-1)-brane in IIA theory, which is in agreement with results in [5, 7, 8] and in some sense can serve as further support of their results.

We would like to outline the possible extension of this work. It would be certainly nice to study situation when we have \(N\) non-BPS D-branes and tachyon condenses in more general configuration. It would be also interesting to study tachyon condensation on system of D9-branes and antibranes in Type IIB theory following [7]. We hope to return to these important question in the future.
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