Magnetic warm inflation

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In this work we explore the effects that a possible primordial magnetic field can have on the inflaton effective potential, taking as the underlying model a warm inflation scenario, based on global supersymmetry with a new-inflation-type potential. The decay scheme for the inflaton field is a two-step process of radiation production, where the inflaton couples to heavy intermediate superfields, which in turn interact with light particles. In this context, we consider that both sectors, heavy and light, are charged and work in the strong magnetic field approximation for the light fields. We find an analytical expression for the one-loop effective potential, for an arbitrary magnetic field strength, and show that the trend of the magnetic contribution is to make the potential flatter, preserving the conditions for a successful inflationary process.

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I. INTRODUCTION

Magnetic fields are present at all scales in the Universe [1–3], and their origin is currently uncertain. Two basic scenarios have largely been explored in the literature: magnetic fields could be either of astrophysical or primordial origin. The largest coherence scales, as in voids [4], seem to point to a primordial origin. One attractive -although not problem free- possibility, is inflationary magnetogenesis. Besides requiring breaking the conformal invariance of electromagnetism to produce long wave magnetic fields via excitation of the vacuum fluctuations [5], this scenario typically faces a backreaction problem which can put strong constraints on the parameter space of the model [6]. It has been shown [7, 8] that the backreaction, which becomes important when the energy density of the electromagnetic field -in particular, of the electric one- approaches the inflaton energy density, changes the dynamics of the inflaton, making the genesis of magnetic fields difficult, but not necessarily spoiling the inflationary process.

The generation of the presently observed large scale magnetic fields is certainly far from being successfully explained but what seems quite probable is that these fields should be present, at all times, during the universe evolution. In this sense, it is important to take into account the magnetic field effect when addressing some early universe events [9–11]. In particular, in the case of the inflationary process, if the magnetic field has to play any role, dissipative processes have to be considered. This could happen in models where the inflaton is coupled to gauge fields [8]; to supersymmetric light fields, as in models of trapped inflation [12]; or to heavy ones, as in warm inflation [13]. We are interested here in the warm inflation scenario, where the inflaton is assumed to interact with other fields during the whole inflationary process. Early models of inflation -dubbed super-cooled models (see e.g. [14] for a review)- assumed very little interaction of the inflaton with all other fields until the reheating process, at the end of inflation. With the proposal of warm inflation [13], this picture changed: the inflaton is now assumed to interact with other fields, both during the inflationary expansion as well as at reheating, in a continuous and more natural way. It is a model where (near) thermal equilibrium conditions are maintained during the inflationary expansion, with no need for very flat potentials, nor for a tiny coupling constant. The model does require a dissipative component $\Gamma$ of sizable strength as compared to the expansion rate of the universe. This is opposed to the standard inflationary scenario where the damping term comes only from the universe’s expansion. A successful implementation of this model is embedded in the framework of supersymmetry, in order to ensure the cancellation of quantum fluctuations from fermions and bosons, protecting the flatness of the potential. Besides, it rests on a two-step process of radiation production, $\phi \to \chi \to y \bar{y}$, where $\phi$ represents the inflaton, $\chi$ an intermediary heavy field and $y$ the light sector, composed of fermions $\psi_y$ and scalars $y$. In this way, the contribution from thermal corrections to the inflaton mass coming from heavy sector loops is Boltzmann suppressed. These $X$ fields are too heavy to be produced on shell and only appear as virtual $\chi$ (bosons) and $\bar{\psi}_\chi$ (fermions) pairs, that decay into the light fields, that thermalize. At the end, it is assumed that there is a soft SUSY breaking in the heavy sector. In this context, it has been shown that the quantum and radiative corrections do not spoil the slow-roll conditions required for inflation [15, 16].

On the observational side, Planck’s results establish a series of constraints on the abundant family of inflationary potentials proposed up to now [17, 18]. It is interesting to note that some potentials that are essentially ruled out by Planck in the context of cold inflation, are completely in agreement with observations when inflation evolves in a thermal bath [19, 20]. Among single-field slow-roll inflationary models, warm inflation, although constrained...
remains viable, the key feature being that the additional damping term introduced in this model can lower the tensor-to-scalar ratio $r$ for a given potential [18].

In a previous work [11], we explored the effect of a weak magnetic field on the warm inflation effective potential, up to one loop, for neutral heavy bosons interacting with the charged light sector, showing that the magnetic field makes the potential flatter, retarding the transition, and works as an additional SUSY breaking scale. Here, we broaden the scenario, allowing for magnetic fields of arbitrary strength and charged heavy fields.

The paper is organized as follows: In Sec. II we present a supersymmetric model we work with, in which all particles, except the inflaton field, are charged. In Sec. III we calculate the analytical expression for the inflaton one-loop effective potential for an arbitrary magnetic field strength within the model described in the previous section. In order to account for the magnetic field effect on the heavy particle masses, in Sec. IV we compute the one-loop heavy field self-energy, coming from the light fields, in the strong field limit. In Sec. V we show our results through a plot of the effective potential for different values of the magnetic field. Finally, in Sec. VI we present our conclusions.

II. MODEL

Let us start by considering the equation of motion of the inflaton field that accounts a dissipation term that comes from the interaction with a thermal bath and the universe expansion

$$\ddot{\phi} + (3H + \Gamma)\dot{\phi} + V_{T,\phi} = 0,$$

where $H$ is the Hubble parameter, and $V_{T,\phi}$ is the derivative with respect to $\phi$ of the inflaton effective potential (usually taken as the finite temperature one-loop Coleman-Weinberg potential). Warm inflation requires $\Gamma > 3H$.

Since in warm inflation radiation is produced during the whole epoch of inflation, light fields, associated to this radiation, must be present in the Lagrangian. In these models, the inflaton interacts all the time with other fields, but its direct interaction with the light fields brings up some inconsistencies. Since the inflaton has a large expectation value, fields that interact directly with it acquire large masses. This fact is inconsistent with the radiation-like nature of such fields. Alternatively, one could limit the value of the coupling between the inflation and the light fields. However this would require an extremely low upper bound for the coupling, which in practice would make the interaction negligible. In view of these observations, a heavy field is introduced. This field acts as a mediator for the inflaton decay into light fields. This mechanisms results in a two step process of radiation production, $\phi \rightarrow \chi \rightarrow \tilde{y}\tilde{y}$, where $\phi$ represents the inflaton, $\chi$ the intermediary field and $\tilde{y}$ the light sector, composed of fermions $\Phi_\nu$ and scalars $y$. Also, in order to keep the flateness of the potential, one can resort to work within the framework of supersymmetry, since with such scenario, quantum fluctuations from fermions and bosons cancel out, which is a welcome feature to avoid spoiling the slow-roll conditions that are necessary for the flatness of the potential.

We start by extending the supersymmetric model used in Ref. [15] to a model where all particles are charged, except the inflaton field. With this in mind, the superpotential is

$$W = -g\Phi X^2 - hXY_1Y_2,$$

where $\Phi$, $X$ and $Y_{1,2}$ are chiral superfields, and the second and the latter represent the heavy and light sector, respectively. The last term in the superpotential accounts for the interaction between light and heavy sectors.

The scalar interaction terms are derived from the superpotential in Eq. (2) as

$$\mathcal{L}_S = -|\partial_\phi W|^2 - |\partial_X W|^2 - |\partial Y_1 W|^2 - |\partial Y_2 W|^2.$$

Defining $\phi = \sqrt{2} \text{Re}(\phi)$, the inflaton potential, up to one loop correction is

$$V(\phi) = \frac{1}{2}g^2 M_s^2 \left[ \phi^2 \ln \left( \frac{\phi^2}{\phi_0^2} \right) + \phi_0^2 - \phi^2 \right],$$

where $\phi_0$ is the vev of the inflaton field.

The Yukawa sector is added to represent the interaction between the scalar and the fermionic sector:

$$\mathcal{L}_{\text{Yukawa}} = -\frac{1}{2} \frac{\partial^2 W}{\partial \phi_n \partial \phi_m} \bar{\psi}_n P_L \psi_m - \frac{1}{2} \frac{\partial^2 W^*}{\partial \phi_n^* \partial \phi_m^*} \bar{\psi}_n P_R \psi_m,$$

where $\phi_m$ is a superfield and

$$P_{L,R} = (1 \mp \gamma_5)/2.$$
Since fermion-boson cancellation takes place thanks to SUSY, the quantum corrections to the inflaton potential are shown to be small. Furthermore, the contribution from thermal corrections to the inflaton mass coming from heavy sector loops is Boltzmann suppressed. These $X$ fields are too heavy to be produced on shell and only appear as virtual $\chi$ (bosons) and $\psi$ (fermions) pairs, that decay into the light fields. We neglect the heavy fields decay sub-leading process through $\chi \rightarrow y_1 y_2 \phi$ compared to $\chi \rightarrow y_1 \tilde{y}_2$. It is also assumed that there is a soft SUSY breaking in the heavy sector and that light radiation thermalizes.

The set of interactions that involve the inflaton and the $\chi$ field can be obtained from the scalar ($\mathcal{L}_s$) and fermion ($\mathcal{L}_f$) sectors of the Lagrangian, given by

$$
\mathcal{L}_s = g^2 |\chi|^4 + 4g^2 |\varphi|^2 |\chi|^2 + h^2 (|y_1|^2 + |y_1|^2) |\chi|^2 + h^2 |y_2|^2 y_2^2 + 2gh(y_1 y_2 \varphi^\dagger \chi^\dagger + y_1 y_2^\dagger \varphi \chi)
$$

$$
\mathcal{L}_f = g(\varphi \bar{\psi}_x P_L \psi_x + \varphi^\dagger \bar{\psi}_x P_R \psi_x) + 2g(\chi \bar{\psi}_x P_L \psi_x + \chi^\dagger \bar{\psi}_x P_R \psi_x)
$$

$$
+ h \chi (\bar{\psi}_y P_L \psi_{y_2} + \bar{\psi}_y P_L \psi_{y_1}) + h \chi^\dagger (\bar{\psi}_y P_R \psi_{y_2} + \bar{\psi}_y P_R \psi_{y_1})
$$

$$
+ 2h(y_1 \bar{\psi}_{y_2} P_L \psi_x + y_2 \bar{\psi}_{y_1} P_L \psi_x) + 2h(y_1^\dagger \bar{\psi}_{y_2} P_R \psi_x + y_2^\dagger \bar{\psi}_{y_1} P_R \psi_x),
$$

(7)

where $\varphi$, $\chi$ and $y_{1,2}$ are the scalar field component of the chiral superfields $\Phi$, $X$ and $Y_{1,2}$, respectively. $\psi_i$ denotes the fermion fields coming from the different sectors and $g$ and $h$ are coupling constants, whose values, limited by the slow-roll conditions and the constraints from density perturbations, are $O(0.1)$ [15].

The effect of a magnetic field on the inflationary process can be accounted for in two ways: through heavy fields effective potential and their quantum fluctuations due to the interaction with the light sector. With these ideas in mind, we shall calculate the magnetic contributions to the heavy fields one-loop potential in vacuum, since these fields are so heavy that their contribution from thermal corrections to the inflaton mass, coming from this sector loops, is Boltzmann suppressed. By doing this, we are considering that the magnetic field strength can be of the same order of magnitude as the heavy masses. On another hand, since the magnetic field is the highest scale, the thermal contributions on the heavy sector masses coming from the interaction with the light particles are also neglected.

III. EFFECTIVE POTENTIAL

Taking into account that the heavy fields are fermionic and bosonic, the contribution to the inflaton effective potential coming from this sector, up to one loop, reads

$$
V^1(\phi) = V^1_\chi + V^1_\psi,
$$

(8)

which, in absence of an external magnetic field, have the form

$$
V^1_{\chi,0} = \frac{4}{2} \int \frac{d^4 p}{(2\pi)^4} \ln(p^2 - m^2_\chi),
$$

(9)

$$
V^1_{\psi,0} = \frac{4}{2} \int \frac{d^4 p}{(2\pi)^4} \ln(p^2 - m^2_\psi),
$$

(10)

with $m_\chi$ and $m_\psi$ the boson and fermion masses, respectively; fermions are Weyl spinors with two degrees of freedom, and the coefficients account for the charge and spins. The subscript “0” emphasizes the absence of the external magnetic field.

In the presence of an external and uniform magnetic field $B$, which defines the $z$-direction, the above expressions become

$$
V^1_\chi = \frac{4}{2} \int \frac{d^4 p}{(2\pi)^4} \ln(D_B^{-1}(p)),
$$

(11)

$$
V^1_\psi = \frac{1}{2} \int \frac{d^4 p}{(2\pi)^4} \ln(Det(S_B^{-1}(p))),
$$

(12)

where $D$ and $S$ are the propagators of boson and fermion, respectively, and are given by

$$
D_B(p) = \int_0^\infty \frac{ds}{\cos q \chi BS} \exp\left\{is(p^2 - p_1^2 \tan q \chi BS - m^2_\chi + i\epsilon)\right\},
$$

(13)

$$
S_B(p) = \int_0^\infty \frac{ds}{\cos q \chi BS} \exp\left\{is(p^2 - p_1^2 \tan q \chi BS - m^2_\psi + i\epsilon)\right\} \left[(m_\psi + p_1) e^{q \chi BS \Sigma_3} - \frac{p_\perp}{\cos q \chi BS}\right],
$$

(14)
with $s$ the Schwinger’s proper time parameter, $(a \cdot b)_{\parallel} = a_{0}b_{0} - a_{3}b_{3}$, $(a \cdot b)_{\perp} = a_{1}b_{1} + a_{2}b_{2}$ and $\Sigma_{a} = i\gamma^{1}\gamma^{2}$. $q_{ \chi}$ denotes the charge associated to the heavy superfield fermion or boson components.

Once the integration over the momentum is carried out, and all divergent terms are isolated, the effective potential can be rewritten as

$$\mathcal{V}_{1}(\phi, B) = \mathcal{V}_{01} + \mathcal{V}_{\chi B}^{2} + \mathcal{V}_{df}^{1},$$

with

$$\mathcal{V}_{01}^{1} = \frac{1}{8\pi^{2}} \int_{0}^{\infty} \frac{ds}{s^{3}} \left\{ e^{-s m_{\chi}^{2}} - e^{-s m_{\chi}^{2}} \right\},$$

$$\mathcal{V}_{\chi B}^{2} = -\frac{1}{8\pi^{2}} \int_{0}^{\infty} \frac{ds}{s} \left\{ e^{-s m_{\chi}^{2}} + 2e^{-s m_{\chi}^{2}} \right\} \frac{(q_{\chi} B)^{2}}{6},$$

$$\mathcal{V}_{df}^{1} = \frac{1}{8\pi^{2}} \int_{0}^{\infty} \frac{ds}{s^{3}} \left\{ e^{-s m_{\chi}^{2}} \left[ -q_{\chi} B s \sinh(q_{\chi} B s) - 1 + \frac{1}{6} (q_{\chi} B s)^{2} \right] - e^{-s m_{\chi}^{2}} \left[ q_{\chi} B s \coth(q_{\chi} B s) - 1 - \frac{1}{3} (q_{\chi} B s)^{2} \right] \right\},$$

where the masses $m_{\chi}$ and $m_{\psi}$ keep track of the bosonic and fermionic sectors, respectively. Notice that the effective potential in Eq. (18) is divergence free ($df$) [22], meanwhile, the first two contributions to the effective potential are divergent when SUSY is broken. However, since we are considering the external magnetic field as a classical field, then the energy of the quantum fluctuations cannot go beyond a certain scale $\Lambda$. Thus, in these two integrals we have to introduce this ultraviolet cutoff. By doing this, it is not difficult to show that each expression goes like

$$\mathcal{V}_{01}^{1} = -\frac{1}{32\pi^{2}} \left\{ m_{\chi}^{4} \ln \left( \frac{m_{\chi}^{2}}{\Lambda^{2}} \right) - m_{\chi}^{4} \ln \left( \frac{m_{\chi}^{2}}{\Lambda^{2}} \right) \right\} + C,$$

$$\mathcal{V}_{\chi B}^{2} = -\frac{1}{8\pi^{2}} \frac{(q_{\chi} B)^{2}}{6} \left\{ -\gamma_{E} - \ln \left( \frac{m_{\chi}^{2}}{\Lambda^{2}} \right) \right\} + 2 \left\{ -\gamma_{E} - \ln \left( \frac{m_{\chi}^{2}}{\Lambda^{2}} \right) \right\},$$

where $C$ is a constant which can be determined from the renormalization conditions. Note that the main divergences cancel out and the remaining ones are due to the soft SUSY breaking term which we have defined as the slight difference between the fermion and boson masses, that is

$$m_{\chi}^{2}(B) = 2q^{2}\phi^{2} + m_{\chi}^{2}(B) + M_{s}^{2},$$

$$m_{\psi}^{2}(B) = 2q^{2}\phi^{2} + m_{\psi}^{2}(B),$$

where $m_{\chi}^{2}(B)$ and $m_{\psi}^{2}(B)$ are the magnetic one-loop self-energy corrections to the fermion and boson masses, respectively.

The integral over the proper time in Eq. (18) can be exactly done, obtaining

$$\mathcal{V}_{df}^{1} = \frac{(q_{\chi} B)^{2}}{8\pi^{2}} \left\{ \left[ 4\zeta^{(1,0)} \left( -1, \frac{m_{\chi}^{2}}{2q_{\chi} B} \right) - 2\zeta^{(1,0)} \left( -1, \frac{m_{\chi}^{2}}{q_{\chi} B} \right) + \frac{m_{\chi}^{4} \left( 2 \ln \left( \frac{m_{\chi}^{2}}{2q_{\chi} B} \right) - 1 \right)}{4(q_{\chi} B)^{2}} - \frac{m_{\chi}^{4} \ln(4)}{2q_{\chi} B} - \frac{1}{6} \ln \left( \frac{m_{\chi}^{2}}{4q_{\chi} B} \right) - \frac{1}{6} \right] \right\}$$

$$- \left\{ 4\zeta^{(1,0)} \left( -1, \frac{m_{\psi}^{2}}{2q_{\chi} B} \right) + m_{\psi}^{4} \left( 2 \ln \left( \frac{m_{\psi}^{2}}{2q_{\chi} B} \right) - 1 \right) - \frac{m_{\psi}^{2} \ln(4)}{q_{\chi} B} + \frac{1}{3} \ln \left( \frac{m_{\psi}^{2}}{2q_{\chi} B} \right) + \frac{1}{3} \right\}.$$

In order to determine the $\Lambda$ and $C$ values, we impose that at $B = 0$, the effective potential lower value be zero at $\phi = \phi_{0}$, with $\phi_{0}$ the inflaton vev, that is

$$\mathcal{V}_{1}(\phi, B)|_{\phi = \phi_{0}} = 0$$

and

$$\frac{\partial}{\partial \phi} \mathcal{V}_{1}(\phi, B)|_{\phi = \phi_{0}} = 0.$$
getting
\[ \ln \left( \frac{\Lambda^2}{2g^2\phi_0^2 + M_s^2} \right) = \frac{1}{2} + \frac{2g^2\phi_0^2}{M_s^2} \ln \left( \frac{2g^2\phi_0^2 + M_s^2}{2g^2\phi_0^2} \right) \]  
(25)

and
\[ C = \frac{g^2\phi_0^2 M_s^2}{32\pi^2} \left[ \frac{4g^2\phi_0^2 + M_s^2}{2g^2\phi_0^2} + 2 \frac{2g^2\phi_0^2 + M_s^2}{M_s^2} \ln \left( \frac{2g^2\phi_0^2 + M_s^2}{2g^2\phi_0^2} \right) \right]. \]  
(26)

In the next section we shall calculate the magnetic contribution to the heavy particle masses through the light quantum fluctuations.

**IV. MAGNETIC MASSES**

According to Eq.(7), there are four Feynman diagrams in which the heavy fields interact with light particles. In what follows we calculate each one of them in the presence of an uniform magnetic field, bearing in mind that this external magnetic field is the highest physical scale compared with the light particle parameters, as temperature and masses.

**A. Scalar self-energy dressed with magnetic field effects**

In the case of the heavy scalar field there are two different vertices that couple them with the light fields. Let us start with the simplest interaction, displayed in Fig. 1a, representing the interaction between two heavy and two light scalar fields. In the configuration space, it has associated the following mathematical expression
\[ -i\Sigma(w,w) = \frac{-i4h^2}{2} \sum_i D_{y_i}(w,w), \]  
(27)

where the sum over \( i \) accounts for the two light boson species and the symmetry factor has been accounted for. In the momentum space, the above equation reads
\[ \Sigma(p) = 2h^2 \sum_i \int \frac{d^4k}{(2\pi)^4} D_{y_i}(k), \]  
(28)

with \( D_{y_i}(k) \) the scalar propagator given in Eq. (13). This propagator can be rewritten in terms of the Landau levels by doing a deformation in the integration contour over the proper time \( s \), as shown in [23].

Since, we are working with a strong magnetic field, the light particles can be considered as constrained into the Lowest Landau Level (LLL), where the scalar propagator takes the form
\[ D_{y_i}(k) = \frac{2ie^{-k^2/2q_iB}}{k^2 - q_iB - m_i^2 + i\epsilon}. \]  
(29)

Replacing this propagator in Eq. (28) and once the integration over the transverse momentum is carried out, we get
\[ \Sigma(p) = \frac{i}{2} \sum_i \frac{h^2 q_i B}{2\pi^2} \int_0^{\sqrt{q_iB}/k^2} \frac{k_{||}dk_{||}}{k_{||} - q_iB - m_i^2}, \]  
(30)

where the integral upper limit refers to the highest energy scale in this approach.

Performing the integration over the parallel momentum in Eq. (30) by a standard procedure, we obtain
\[ \Sigma(p) = \frac{h^2 q_i B}{4\pi^2} \ln \left( \frac{2q_i B + m_i^2}{q_i B + m_i^2} \right) \approx \frac{h^2(q_1 + q_2)B}{4\pi^2} \ln(2). \]  
(31)
Next, the other contribution to the scalar self-energy, depicted in Fig. 1b, comes from the interaction between one heavy scalar and two light fermion fields. In configuration space, it has the form

\[ -i \Sigma^B(w, v) = \hbar^2 Tr[S^B_1(w, v)S^B_2(v, w) - \gamma^5 S^B_1(w, v)\gamma^5 S^B_2(v, w)]. \] (32)

where

\[ S^B(x, z) = \Omega(x, z)\tilde{S}^B(x - z) \] (33)

with \( \Omega(x, z) = \exp[-iq \int_x^z d\xi \cdot A(\xi)] \), the Schwinger phase, and

\[ \tilde{S}^B(x - z) = \int \frac{d^4 p}{(2\pi)^4} e^{ip \cdot (x - z)} S^B(p) \] (34)

the fermion propagator symmetric part in the momentum space given in Eq.(14).

Note that, as the heavy field is charged, the following relation holds

\[ q_\chi = q_1 - q_2, \] (35)

with \( q_i > 0 \) the charge of each light particle.

Using the above relations in Eq. (32), we get

\[ \Sigma^B(w, v) = i\hbar^2 \Omega_1(w, v)\Omega_2(v, w)Tr[\tilde{S}^B_1(w - v)\tilde{S}^B_2(v - w) - \gamma^5 S^B_1(w - v)\gamma^5 S^B_2(v - w)], \] (36)

which, in momentum space, becomes

\[ \Sigma^B(k, l) = i\hbar^2 \int d^4 w d^4 v e^{i k \cdot w - i l \cdot v} \Omega_1(w, v)\Omega_2(v, w) \times \int \frac{d^4 p d^4 j}{(2\pi)^8} e^{-ip \cdot (w - v)} e^{-ij \cdot (v - w)} Tr[S^B_1(p)S^B_2(j) - \gamma^5 S^B_1(p)\gamma^5 S^B_2(j)]. \] (37)

Once we perform the integration over \( w \) and \( v \) \[24\], we get

\[ \Sigma^B(k, l) = i\hbar^2 4(2\pi)^6 \int d^4 p d^4 j \delta^{(2)}(k - p + j)\delta^{(2)}(-l + p - j) e^{-\frac{2\pi}{\hbar\epsilon_{ij}(k_i - p_i + j_i)(-l_i + p_i - j_i)}} \times Tr[S^B_1(p)S^B_2(j) - \gamma^5 S^B_1(p)\gamma^5 S^B_2(j)], \] (38)

where \( \epsilon_{ij} \) is the Levi-Civita tensor in a 2-dimensional space associated with the transverse components.
Following the same procedure as for the scalar propagator, the fermion propagator in the LLL, has the form

\[
\Sigma(k, l) = \frac{i}{(q\chi B)^2} \delta^{(2)}(k - l) \int \frac{d^4 p\parallel}{(2\pi)^4} e^{\frac{-i(q\chi B)}{2}(k - l)^2} e^{i\frac{q}{\chi B}\epsilon_{k,l}}\epsilon_{k,l} i \Gamma \left[ S^B_{1B}(p)S^B_{2B}((p - l)\parallel, j\perp) - \gamma^5 \Sigma^B_S((p - l)\parallel, j\perp) \right].
\]

Following the same procedure as for the scalar propagator, the fermion propagator in the LLL, has the form [25]

\[
S^B(p) = i \frac{e^{-\frac{p^2}{2m^2}}}{|\mu\parallel - m^2 + i\epsilon} (m + \not{p}\parallel)(1 - i\gamma_1\gamma_2).
\]

Using this propagator in Eq. (39), and once the integration over transverse momentum and the trace over the Dirac gamma matrices are performed, we get

\[
\Sigma(k, l) = i 16 \frac{h^2 q_1 q_2}{q\chi^2} (2\pi)^2 \delta^{(2)}(k - l) e^{-\frac{(q_1 + q_2)}{q\chi^2} (k - l)^2} e^{\frac{-i q_1}{q\chi^2}\epsilon_{k,l}}
\]

\[
\times \int \frac{d^2 p\parallel}{(2\pi)^2} \frac{p\perp}{[p\parallel^2 - m^2 + i\epsilon][(p - l)^2 - m^2 + i\epsilon]}.
\]

The remaining integral over the parallel momenta can be easily done resorting to the standard procedure in QFT, obtaining

\[
\Sigma(k, l) = i 16 \frac{h^2 q_1 q_2}{q\chi^2} (2\pi)^2 \delta^{(2)}(k - l) e^{-\frac{(q_1 + q_2)}{q\chi^2} (k - l)^2} e^{\frac{-i q_1}{q\chi^2}\epsilon_{k,l}}
\]

\[
\times i \left\{ \ln \left\{ \frac{m_1 m_2}{\mu^2} \right\} + \frac{l^2 - m_1^2 - m_2^2}{2 \sqrt{(l^2 - (m_1 - m_2)^2)((l^2 - (m_1 + m_2)^2)}} \right. \]

\[
\left. \times \ln \left( \frac{l^2 - m_1^2 - m_2^2 + \sqrt{(l^2 - (m_1 - m_2)^2)((l^2 - (m_1 + m_2)^2)}}{(l^2 - m_1^2 - m_2^2) - \sqrt{(l^2 - (m_1 - m_2)^2)((l^2 - (m_1 + m_2)^2)}} \right) \right\}. \]

In order to match with the standard dimensions for the self-energy, which in this case are \([E^2]\), we need to invoke momentum conservation (see Appendix A) as follows

\[
\Sigma^B(k, l) = (2\pi)^4 \delta^{(2)}(k - l) \left( \frac{1}{q\chi B} \right) \tilde{\Sigma}(l, k\perp), \]

where the factor that involves the magnetic field emphasises the momentum conservation in the transverse direction. In such a way, from Eq. (42), we get

\[
\tilde{\Sigma}(k, l\perp) = \frac{4 h^2 q_1 q_2 q\chi B}{q\chi^2 \pi^3} e^{-\frac{(q_1 + q_2)}{q\chi^2} (k - l)^2} e^{\frac{-i q_1}{q\chi^2}\epsilon_{k,l}}
\]

\[
\times \left\{ \ln \left( \frac{m_1 m_2}{\mu^2} \right) + \frac{l^2 - m_1^2 - m_2^2}{2 \sqrt{(l^2 - (m_1 - m_2)^2)((l^2 - (m_1 + m_2)^2)}} \right. \]

\[
\left. \times \ln \left( \frac{l^2 - m_1^2 - m_2^2 + \sqrt{(l^2 - (m_1 - m_2)^2)((l^2 - (m_1 + m_2)^2)}}{(l^2 - m_1^2 - m_2^2) - \sqrt{(l^2 - (m_1 - m_2)^2)((l^2 - (m_1 + m_2)^2)}} \right) \right\}. \]

In the above equation, strictly speaking, we do not have a Dirac delta function that accounts for the transverse momentum conservation of the heavy field, nevertheless, as both \(l\perp\) and \(k\perp\) are weighted by the magnetic field, they are of the same order of magnitude, which is negligible within this approximation. With this in mind, we hereafter
Once we perform the integration over momentum is carried out, we arrive at

\[
\Sigma(l) = \frac{4}{\pi^3} \frac{\hbar^2 q_1 q_2 B}{q_x} \times \left\{ \ln \left( \frac{m_1 m_2}{\mu^2} \right) + \frac{l_{\parallel}^2 - m_1^2 - m_2^2}{2 \sqrt{(l_{\parallel}^2 - (m_1 - m_2)^2)(l_{\parallel}^2 - (m_1 + m_2)^2)}} \right. \\
\left. \times \ln \left( \frac{l_{\parallel}^2 - m_1^2 - m_2^2 + \sqrt{(l_{\parallel}^2 - (m_1 - m_2)^2)(l_{\parallel}^2 - (m_1 + m_2)^2)}}{l_{\parallel}^2 - m_1^2 - m_2^2 - \sqrt{(l_{\parallel}^2 - (m_1 - m_2)^2)(l_{\parallel}^2 - (m_1 + m_2)^2)}} \right) \right\},
\]

(45)

where \( \mu \) is an ultraviolet cutoff, that can be related with the magnetic field \( (\mu^2 \propto q_1 B) \). As the main contribution to the mass comes from the region \( l_{\parallel}^2 \lesssim q_1 B \), and Eq. \( (45) \) becomes

\[
\Sigma(l) = -\frac{4}{\pi^3} \frac{\hbar^2 q_1 q_2 B}{q_x} \ln \left( \frac{q_1 B}{m_1 m_2} \right).
\]

(46)

The interaction between one heavy and two light scalar fields, Feynman diagram in Fig. 1c, does not contribute to the scalar self-energy as shown in [15], in this way we ignore it.

Taking into account both contributions, Eq. \( (46) \) and Eq. \( (31) \), the total magnetic effect to the heavy boson mass reduces to

\[
m^2_{\chi}(B) \approx M^2 - \frac{4}{\pi^3} \frac{\hbar^2 q_1 q_2 B}{q_x} \ln \left( \frac{q_1 B}{m_1 m_2} \right),
\]

(47)

with \( M^2 = 2g^2 \phi^2 \).

B. Fermion self-energy dressed with magnetic field effects

In configuration space the self-energy of the fermionic heavy field, interacting with one fermion and one scalar light fields, shown in Fig. 1a, has the form

\[
-i\Sigma^B(w, v) = -2\hbar^2 D_y^B(w, v)S_{\psi_y}^B(v, w) = -2\hbar^2 \Omega_y(w, v)\Omega_{\psi_y}(v, w)D_y^B(w - v)\tilde{S}_{\psi_y}^B(v - w),
\]

(48)

where we have decomposed the boson propagator in the same way as the fermion propagator in Eq. \( (33) \).

In momentum space, the above equation becomes

\[
\Sigma^B(k, l) = -2i\hbar^2 \int d^4w d^4v e^{ik\cdot w} e^{-il\cdot v} \Omega_\Omega(w, v) \Omega_\Omega(v, w) \int \frac{d^4p d^4j}{(2\pi)^8} e^{-i p\cdot (w-v)} e^{-i j\cdot (v-w)} D_y^B(p)S_{\psi_y}^B(j).
\]

(49)

Once we perform the integration over \( w \) and \( v \) \([24]\), we get

\[
\Sigma^B(k, l) = -i\frac{2\hbar^2 4(2\pi)^6}{(q_x B)^2} \int \frac{d^4p d^4j}{(2\pi)^8} D_y^B(p)S_{\psi_y}^B(j)\delta^{(2)}(k - p + j)\delta^{(2)}(l/o + p - j)e^{-\frac{2\pi}{\sqrt{3}}\gamma_{ij}(k_i - p_i + j_i)(-l_j + p_j - j_j)}.
\]

(50)

With the delta functions, the integral over one parallel momentum can straightforwardly be done

\[
\Sigma(k, l) = -i\frac{128\pi^4 \hbar^2}{(q_x B)^2} \delta^{(2)}(k - l) \int \frac{d^4p d^4j}{(2\pi)^8} D_y^B(p)S_{\psi_y}^B((p - l)_{||}, j_{||})e^{\frac{2\pi}{\sqrt{3}}\gamma_{ij}(k_i - p_i + j_i)(-l_j + p_j - j_j)}.
\]

(51)

Using the LLL boson and fermion propagators, Eqs. \( (29) \) and \( (40) \), in Eq. \( (51) \), and once the integration over transverse momentum is carried out, we arrive at

\[
\Sigma(k, l) = \frac{64\pi^4 \hbar^2 q_1 q_2}{q_x^2} \delta^{(2)}(k - l)e^{-\frac{4\pi^2 \mu^2}{q_x^2} B^2} e^{i\pi \gamma_{ij} \epsilon_{ij} k i l j} \\
\times \int \frac{d^2p_{||}}{(2\pi)^2} \frac{(p_{||} - l_{||} + m_2)(1 - i\gamma_1 \gamma_2)}{[p_{||}^2 - q_y B - m_1^2 + i\epsilon][(p - l)_{||}^2 - m_2^2 + i\epsilon]}.
\]

(52)
The remaining integral over the parallel momenta can be easily done resorting to the standard procedure on QFT, obtaining

\[ \Sigma(k, l) = -\frac{16\pi\hbar^2 q_1 q_2}{q_X^2} \delta(k - l) e^{-\frac{q_1 q_2}{q_X^2} (k - l)^2} e^{\frac{2q_1 q_2}{q_X^2} \epsilon_{i,j} k_i l_j} \]

\[
\left\{ \frac{\gamma}{2l^2} \ln \left( \frac{m_2^2}{m_1^2 + q_1 B} \right) + \left( m_2 + \frac{\gamma (l_1^2 + m_2^2 - m_1^2)}{2l_1^2} \right) \frac{i}{\sqrt{4l_1^2 m_2^2 - (l_1^2 + m_2^2 - m_1^2 + q_1 B)^2}} \right\}
\times \ln \left( \frac{l_1^2 - m_1^2 - m_2^2 - q_1 B + i\sqrt{4l_1^2 m_2^2 - (l_1^2 + m_2^2 - m_1^2 + q_1 B)^2}}{l_1^2 - m_1^2 - m_2^2 - q_1 B - i\sqrt{4l_1^2 m_2^2 - (l_1^2 + m_2^2 - m_1^2 + q_1 B)^2}} \right) (1 - i\gamma_1 \gamma_2). \tag{53} \]

Once we employ Eq. (43), and bearing in mind the argument above Eq. (45), we get

\[ \tilde{\Sigma}(l) = -\frac{1}{\pi^3} \frac{\hbar^2 q_1 q_2 B}{q_X l_1^2} \left\{ \frac{\gamma}{l_1^2} \ln \left( \frac{m_2^2}{m_1^2 + q_1 B} \right) + \frac{2l_1^2 m_2 + \gamma (l_1^2 + m_2^2 - m_1^2)}{\sqrt{(l_1^2 - (m_2 + \sqrt{m_1^2 + q_1 B}^2)) (l_1^2 - (m_2 - \sqrt{m_1^2 + q_1 B}^2))}} \right\}
\times \ln \left( \frac{l_1^2 - m_1^2 - m_2^2 - q_1 B - \sqrt{(l_1^2 - (m_2 + \sqrt{m_1^2 + q_1 B}^2)) (l_1^2 - (m_2 - \sqrt{m_1^2 + q_1 B}^2))}}{l_1^2 - m_1^2 - m_2^2 - q_1 B + \sqrt{(l_1^2 - (m_2 + \sqrt{m_1^2 + q_1 B}^2)) (l_1^2 - (m_2 - \sqrt{m_1^2 + q_1 B}^2))}} \right) (1 - i\gamma_1 \gamma_2). \tag{54} \]

In order to calculate the magnetic mass contribution, following the procedure sketched in the Appendix B in Ref. [11], let us write the propagator dressed with the magnetic field through the quantum correction as follows

\[ S_B^{-1} = \psi - M - \tilde{\Sigma}^B \]
\[ = \left( \psi - M - \tilde{\Sigma}^B \right) \Delta(+) + \left( \psi - M - \tilde{\Sigma}^B \right) \Delta(-), \tag{55} \]

with \( \Delta(\pm) \equiv \frac{1}{2} (1 \mp \Sigma) \), the spin projectors along the magnetic field direction. Once we replace Eq. (54) in Eq. (55), the above equation can be rewritten as

\[ S_B^{-1} = (\psi - M^+) \Delta(+) + (A - M^-) \Delta(-) \tag{56} \]

where the asymmetry in the two terms rises as consequence of the interaction between the strong magnetic field and
the spin particle. The $A^\mu$ components and the $M^\pm$ are defined as

$$A^0 = l^0 + l^0 \frac{h^2 q_2 B}{\pi^3 q_x} \left\{ \ln \left( \frac{m^2}{m_1^2 + q_1 B} \right) + \frac{(l_{||}^2 + m^2 - m_1^2)}{\sqrt{l_{||}^2 - (m_2 + \sqrt{m_1^2 + q_1 B})^2} (l_{||}^2 - (m_2 - \sqrt{m_1^2 + q_1 B})^2) \right\} \times \ln \left( \frac{l_{||}^2 - m_1^2 - m_2^2 - q_1 B - \sqrt{l_{||}^2 - (m_2 + \sqrt{m_1^2 + q_1 B})^2} (l_{||}^2 - (m_2 - \sqrt{m_1^2 + q_1 B})^2) \right)$$

$$A^1 = l^1,$n

$$A^2 = l^2,$n

$$A^3 = l^3 + l^3 \frac{h^2 q_2 B}{\pi^3 q_x} \left\{ \ln \left( \frac{m^2}{m_1^2 + q_1 B} \right) + \frac{(l_{||}^2 + m^2 - m_1^2)}{\sqrt{l_{||}^2 - (m_2 + \sqrt{m_1^2 + q_1 B})^2} (l_{||}^2 - (m_2 - \sqrt{m_1^2 + q_1 B})^2) \right\} \times \ln \left( \frac{l_{||}^2 - m_1^2 - m_2^2 - q_1 B - \sqrt{l_{||}^2 - (m_2 + \sqrt{m_1^2 + q_1 B})^2} (l_{||}^2 - (m_2 - \sqrt{m_1^2 + q_1 B})^2) \right)$$

$$M^+ = M, \quad M^- = M - \frac{2 m_2 h^2 q_2 B}{\pi^3 q_x} \ln \left( \frac{l_{||}^2 - m_2^2 - m_1^2 - q_1 B - \sqrt{l_{||}^2 - (m_2 + \sqrt{m_1^2 + q_1 B})^2} (l_{||}^2 - (m_2 - \sqrt{m_1^2 + q_1 B})^2) \right)$$

$$- \frac{l_{||}^2 - m_2^2 - m_1^2 - q_1 B + \sqrt{l_{||}^2 - (m_2 + \sqrt{m_1^2 + q_1 B})^2} (l_{||}^2 - (m_2 - \sqrt{m_1^2 + q_1 B})^2) \right)}{\sqrt{l_{||}^2 - (m_2 + \sqrt{m_1^2 + q_1 B})^2} (l_{||}^2 - (m_2 - \sqrt{m_1^2 + q_1 B})^2) \right)}.$$

(57)

Now, replacing Eq. [66] in the effective potential given in Eq. [12], the magnetic correction to the heavy fermion mass can be identified, in the strong field limit, as

$$m_{\phi X}^2 (B) = M^2 + \frac{h^2 q_2}{\pi^3 q_x} \left\{ q_1 B \ln \left( \frac{q_1 B}{m_2^2} \right) + m^2 - (l_{||}^2 - m_1^2 + 2 m_{\phi X} m_2 + m_2^2) \ln \left( \frac{q_1 B}{m_2^2} \right) \right\}, \quad (58)$$

where we have kept terms up to $O(h^2)$, consistent with the perturbative expansion done in this work. Since the main contribution to the mass comes from the $l_{||}^2 \lesssim q_1 B$ region, then the above equation reduces to

$$m_{\phi X}^2 (B) = M^2 + \frac{h^2 q_2 B}{\pi^3 q_x} \ln \left( \frac{q_1 B}{m_2^2} \right). \quad (59)$$

With the above result we conclude the magnetic contribution to the heavy particle masses. By comparing Eqs. (47) and (59), we note that the soft-SUSY breaking term, introduced by $M^2_\phi$ in the boson mass, has an additional source for symmetry breaking coming from the magnetic field.

In the next section we put together all the magnetic contributions to the inflaton effective potential.

V. RESULTS

The complete one-loop effective potential, including thermal and magnetic effects, can be written as

$$V(\phi, T, B) = -\frac{\pi}{90} g^* T^4 + V^1(\phi, B) \quad (60)$$

where $V^1(\phi, B)$ is the potential in Eq. [15] including the magnetic masses.

In order to quantify the magnetic field effect on the effective potential, let us define

$$\Delta V(\phi, T, B) = \frac{V^{(1)}(\phi, T, B) - V^{(1)}(0, T, B)}{V^{(1)}(0, 0, 0)} + 1$$

$$= \Delta V(\phi, B), \quad (61)$$
where we compare the effective potential depth in the presence of the magnetic field with the case without magnetic field. Note that, in the physical scenario considered in this work, Eq. (61) does not depend on temperature.

In Fig. (2), we plot Eq. (61) as a function of $\phi/\phi_0$ for different magnetic field strengths. Taking into account the limits imposed on the coupling constants by the slow-roll conditions and the constraints from density perturbations $[15]$, the values we have chosen in the effective potential are in concordance with the hierarchy scale used in the calculations: $M_s/\phi_0 = 0.05$, $m_1/\phi_0 = 10^{-3}$, $m_2/\phi_0 = 5 \times 10^{-3}$, $g = 0.1$ and $h = 0.1$, resulting in $M_\chi/\phi_0 = 0.15$, $M_{\tilde{\chi}}/\phi_0 = 0.14$ and $\Lambda/\phi_0 = 0.3$.

As can be seen, the effect of the magnetic field on the effective potential is to make it less steep, preserving the slow-roll conditions.

![Graph showing the effective potential normalized by $V(0,0)$, Eq. (61), for different magnetic field strengths for $M_s/\phi_0 = 0.05$, $m_1/\phi_0 = 10^{-3}$, $m_2/\phi_0 = 5 \times 10^{-3}$, $g = 0.1$ and $h = 0.1$.](image)

**Fig. 2**: Effective potential normalized by $V(0,0)$, Eq. (61), for different magnetic field strengths for $M_s/\phi_0 = 0.05$, $m_1/\phi_0 = 10^{-3}$, $m_2/\phi_0 = 5 \times 10^{-3}$, $g = 0.1$ and $h = 0.1$.

**VI. CONCLUSIONS**

In this paper we have studied the effects that a possible primordial magnetic field can have on the inflation’s potential, taking as the underlying model a warm inflation scenario and considering that all fields interacting with the inflaton field are charged. The model is based on global supersymmetry and a coupling between the inflaton and heavy intermediate superfields which are in turn coupled to light particles. In this context, since we have considered a possible effect on the heavy field sector, then we worked in the strong magnetic field approximation for the light fields. This limit was taken when computing the light particles contribution to the heavy sector self-energy, but the expression for the effective potential, up to one-loop, is exact. We studied its behavior, choosing coupling constants and masses in concordance with the hierarchy scale used in the calculations.

This work is an extension of a previous one in which the weak magnetic field limit was considered and only the light sector was charged. As in our previous study, here we found that the magnetic field effect on the effective potential is to make it less steep as compared with the vacuum case, showing that magnetic fields do not spoil the inflationary process. This result could be relevant in order to continue exploring the role played by magnetic fields on cosmological events, since there are good chances that they were present during the early stages of the universe, where phase transitions provided suitable conditions for their generation.

**Appendix A: On the self-energy physical dimensions**

In this appendix, we carry out a Fourier transform on the configuration space self-energy, in order to isolate the momentum conservation and, thus, identify the expression for the self-energy in momentum space. With this in mind, let us start by performing a Fourier transform on a generic self-energy, $\Sigma(w, v)$, getting

$$\Sigma(k, l) = \int d^4w d^4ve^{ik\cdot w}e^{-il\cdot v}\Sigma(w, v).$$  \hfill (A1)
Since the magnetic field only affects the transverse components and it is expected that the transverse part of the self-energy can be written as the product of the Schwinger phase and a symmetric part (see e.g. [24]), we can perform the following decomposition

$$\Sigma(k, l) = \int d^4w d^4v e^{ik \cdot w} e^{-il \cdot v} \Omega(w_\perp, v_\perp) \tilde{\Sigma}((w - v)_\parallel, (w - v)_\perp), \quad (A2)$$

in a similar fashion as for the propagator.

Performing the change of variables, $R = w + v$ and $r = w - v$, and working in the symmetric gauge, where the phase takes the form

$$\Omega(w_\perp, v_\perp) = \exp\left(-iq \int_\omega d\xi \cdot A(\xi)\right) = \exp\left[-i\frac{qB}{4}(-r_1 R_2 + r_2 R_1)\right], \quad (A3)$$

the parallel part can easily be integrated, getting

$$\Sigma(k, l) = (2\pi)^2 \delta^{(2)}(k - l) \int d^2R_\perp d^2R_\parallel e^{iR_i[(k-l)_i - \frac{qB}{2}\epsilon_{ij}r_j]} e^{i(k+l) \cdot r_\perp} \tilde{\Sigma}(k_\parallel, r_\perp), \quad (A4)$$

where the indexes $i, j = 1, 2$.

The integral over $R_i$ is now straightforward and we get

$$\Sigma(k, l) = (2\pi)^2 \delta^{(2)}(k - l) \frac{4}{qB}\left(\frac{d^2Q_\perp}{2\pi^2}\right) e^{i4Q_1(k-l)_2 - Q_2(k-l)_1}/qBo \tilde{\Sigma}(k_\parallel, Q_\perp), \quad (A5)$$

where, in the last line, we made a Fourier transformation of the self-energy over the transverse coordinates and evaluated the Dirac delta functions.

Reorganizing the above equation in a convenient way, we rewrite it as

$$\Sigma(k, l) = (2\pi)^4 \delta^{(2)}(k - l) \frac{1}{qB} \left\{ e^{-i\frac{\pi}{2}\epsilon_{i,j}k_j} \int \frac{4d^2Q_\perp}{q^2} e^{i4Q_1(k-l)_2 - Q_2(k-l)_1}/qB \tilde{\Sigma}(k_\parallel, Q_\perp) \right\} \equiv (2\pi)^4 \delta^{(2)}(k - l) \frac{1}{qB} \tilde{\Sigma}(k, l_\perp), \quad (A6)$$

in such a way, the factor within curly brackets is dimensionless, $\tilde{\Sigma}(k_\parallel, Q_\perp)$ has the expected dimensions for the self-energy and the remaining factor accounts for the energy-momentum conservation, which in the absence of a magnetic field becomes $(2\pi)^4 \delta^{(4)}(k - l)$.

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