The mechanism of cumulative proton production in the collision of $^{12}$C and $^9$Be nuclei at the energies of 0.6, 0.95 and 2 GeV/nucleon

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Abstract. The collision of carbon nuclei with beryllium nuclei was simulated in the framework of Liège Intranuclear-Cascade model at the initial kinetic energies of the carbon nuclei 0.6, 0.95, 2.0 GeV/nucleon. The invariant cross sections of inclusive proton production at an angle of 3.5° are obtained. It is shown that the dependence of experimental invariant cross sections of proton production on the cumulative variable $x$ in the range $0.9 < x < 2.4$ can be explained on the basis of Fermi motion of nucleons in the nuclei, multiple scattering processes and delta resonance formation. A comparison was made with results obtained within the framework of the quark cluster model and experimental data.

1. Introduction

Since the discovery of the cumulative particles in the seventies [1, 2], the production mechanism of such particles remains open to this day.

By the cumulative reaction in this report we mean the production of particles in the kinematic region, which is forbidden by the energy-momentum conservation laws for free nucleon-nucleon collisions (here by "free" we mean collisions of nucleons in a vacuum) [1, 2].

To characterize the cumulative particles a dimensionless quantity is introduced. This variable is called cumulative order $x$ (or cumulative variable) [3]. There are various ways of determining such variable [1, 2]. In this paper, the cumulative variable $x$ is the ratio of the momentum of detected proton $p$ to the momentum per nucleon of the carbon nucleus $p_0$ in the laboratory frame:

$$x = \frac{p}{p_0}$$

There are two fundamentally different sources of cumulative processes distinguished by the distances over which the process of cumulative particle formation occurs.

The first source, characterized by large distances, is multiple scattering inside the nucleus, when the projectile or the products of its fragmentation experience several collisions with the nucleons of the nucleus [1]. As a result, it becomes possible to produce a particle in a region that is kinematically forbidden in free nucleon-nucleon scattering.

Another source of cumulative particles are processes that occur at short distances - much smaller than the characteristic nuclear distances.

Among the models based on processes occurring at short distances, two classes can be distinguished: flucton [4] models and the model of short-range correlations SRC [5, 6].
The flucton models are divided into two classes: "cold" models and "hot" ones. "Cold" models assume that the fluctons always exist even in the initial nucleus [7, 1, 8, 9], according to the "hot" models, the fluctons are formed only in the process of collision [10].

In the FRAGM experiment in the heavy-ion accelerator complex TVN-ITEP [11], proton yields were measured at an angle of 3.5° with energies 0.6, 0.95, 2.0 GeV/nucleon on a beryllium target. The data are presented as dependences of the invariant proton cross section on the cumulative variable \( x \) in the range \( 0.9 < x < 2.4 \).

In [7], experimental data [11] has been analysed in the framework of the quark cluster model [8]. Within the framework of this model, clusters in the nucleus are consisting of \( 3k \) (\( k = 1, 2, 3, ... \)) valence quarks. The value of \( k = 1 \) corresponds to the normal nucleons of the nucleus.

However, the contribution of multiple scattering, Fermi motion of nucleons in the nucleus and delta resonance formation to the invariant proton cross section, was not considered in [7].

The purpose of this paper is to calculate the cross sections of cumulative protons production in inclusive reaction

\[
^{12}\text{C} + ^{9}\text{Be} = \text{p} + \text{X} \tag{1}
\]

at initial kinetic energies of carbon ions 0.6, 0.95, 2.0 GeV/nucleon, considering processes of multiple scattering, Fermi motion and delta resonance formation, without invoking the hypothesis of quark clusters.

2. Simulation

To evaluate the contribution of multiple scattering processes, Fermi motion and delta resonance formation to the cross section for proton production in reaction (1), we used the extended model of the Liège Intranuclear Cascade Model [12].

Total cross sections for nucleon-nucleon scattering were calculated using formulas (2), (3) [13, 14]. Here \( p_{lab} \) is momentum in the laboratory frame in units of GeV/c, cross sections are given in mb.

\[
\sigma_{tot,pp} = 34 \left( \frac{p_{lab}}{0.4} \right)^{-2.104} \begin{cases} p_{lab} < 0.44 \\ = 23.5 + 1000(p_{lab} - 0.7)^4 & 0.44 < p_{lab} < 0.8 \\ = 23.5 + \frac{24.6}{1 + \exp\left(\frac{p_{lab} - 1.2}{0.1}\right)} & 0.8 < p_{lab} < 1.5 \\ = 41 + 60(p_{lab} - 0.9)\exp(-1.2p_{lab}) & 1.5 < p_{lab} < 3 \\ = 45.6 - 219p_{lab}^{-4.23} + 0.41 \log^2(p_{lab}) - 3.41 \log(p_{lab}) & 3 < p_{lab} \end{cases} \tag{2}
\]

\[
\sigma_{tot,np} = \begin{cases} 6/3555\exp(-3.2481\log(p_{lab}) - 0.377\log^2(p_{lab})) & p_{lab} < 0.446 \\ = 33 + 196(p_{lab} - 0.95)^{2.5} & 0.446 < p_{lab} < 1 \\ = 24.2 + 8.9p_{lab} & 1 < p_{lab} < 1.924 \\ = 48.9 - 33.7p_{lab}^{3.08} + 0.619 \log^2(p_{lab}) - 5.12 \log (p_{lab}) & 1.924 < p_{lab} \end{cases} \tag{3}
\]

Cross sections for nucleon-nucleon elastic scattering in the extended model are calculated from formulas (4), (5).
\( \sigma_{el,pp} = \) (4) 

\[
= \sigma_{tot,pp} \\
= \frac{1250}{p_{lab}+50} - 4(p_{lab} - 1.3)^2 \\
= \frac{77}{p_{lab}+1.5} \\
= 11.2 - 22.5p_{lab}^{1.12} + 0.151\log^2(p_{lab}) - 1.62\log(p_{lab}) \\
\]

\( \sigma_{el,np} = \) (5) 

\[
= \sigma_{tot,np} \\
= \frac{31}{\sqrt{p_{lab}}} \\
= \frac{77}{p_{lab}+1.5} \\
\]

Cross sections of inelastic processes can be calculated as the difference between total cross section for nucleon-nucleon scattering and elastic scattering cross section.

Simulation results presented as dependence of the invariant proton production cross section on the cumulative variable \( x \) are shown in figure 1. The invariant proton production cross section was calculated from the formula

\[
\sigma_{inv} = \frac{E}{p_0} \frac{d^2\sigma}{dx dp_t^2} \\
\]

Where \( \sigma \) is the total cross section, \( p_0 \) is the momentum per nucleon of the incident nucleus, \( E, p_t \) is the total energy and transverse momentum of the proton in the laboratory frame.

The experimental data are shown by circles, black triangles - results of modelling with process of delta resonance formation, white – without process of delta resonances formation, that is, exclusively due to the multiple scattering processes.

Also in figure 1, a comparison with results obtained within the framework of the quark cluster approach is presented. The contributions of one-, two- and three-nucleon clusters are shown as a dashed, dashed and dash-dotted lines, respectively, the total contribution of the quark clusters is denoted by a solid line.

3. Discussion of the results

Figure 1 shows that the processes of multiple scattering and the formation of delta resonances lead to the formation of cumulative particles in the range \( x > 1 \).
Figure 1. Dependence of invariant cross section of protons (arbitrary units) on cumulative variable $x$ in the reaction $^{12}$C + $^9$Be → f + X at the angle 3.5° and initial kinetic energy 0.6 GeV/nucleon. White dots – experimental data, white squares – simulation, based on Extension of the Liège Intranuclear Cascade Model, without process of delta resonance formation, black triangles – simulation with taking into account process of delta resonance formations. Contributions of one-, two-, three- nucleon clusters are shown by dotted, dashed and dashed-dotted lines. Total contribution of quark clusters is shown by solid line.

Figure 2 shows example of multiple scattering processes, leading to the cumulative particle formation in the simulation, performed in this paper.

Let us consider example of cumulative particle formation with the help of the event depicted in figure 2. In this event a cumulative proton with $x = 1.58$ and $p = 1925$ MeV/c was registered.

The momentum of the nucleons of the incident nucleus can exceed the value $p_0$, as a result of Fermi motion of nucleons in the nucleus. According to the model of the Liège intranuclear cascade, the nucleon momentum in the nucleus obey the Gaussian distribution, with $RMS = \sqrt{2}p_F$, where $p_F = 270$ MeV/c is the Fermi momentum [15].
At the first stage in the event (figure 2), proton with the index "0" and momentum $p = 1603$ MeV/c elastically collides with the neutron "1", as a result momentum of proton "0" decreases to $p = 1337$ MeV/c (proton loses energy as a result of an elastic collision). The second stage is the elastic collision of the proton "3" (momentum $p = 1421$ MeV/c) with proton "0" ($p = 1337$ MeV/c). Due to this collision, momentum of proton "0" increases to $p = 1925$ MeV/c. This proton is registered in this event as cumulative, with the momentum $p = 1925$ MeV/c and corresponding value of the cumulative variable $x = 1.58$.

Figure 2. Carbon nuclei (on the left) and beryllium (on the right). 0, 1, 3 – indices of intranuclear that participate in the formation of the cumulative particle. Circles with solid boundaries means the nucleon, which becomes cumulative in this process, (n) and (p) – proton and neutron, respectively. Beside each nucleon is the value of its momentum in MeV/c (for example $p = 1925$ MeV/c).

By taking into account the processes of delta resonances formation in the region $x > 1.6$, the invariant cross section increases and becomes closer to the experimental values.

Let's compare the obtained results with the results of the work obtained within the framework of the approach of fragmentation of quark clusters (figure 2).

It can be seen that the processes of multiple scattering, Fermi motion and delta resonance formation in the region $x < 1.4$ describe experimental data as well as the approach of quark clusters, but give lower values of invariant cross sections in the region $x > 1.4$.

A similar picture is observed for initial kinetic energies of carbon nucleus 0.95 and 2.0 GeV/nucleon.

4. Conclusion
The dependences of invariant cross sections on the cumulative variable are obtained. Processes of Fermi motion, multiple scattering and delta resonance formation were considered. Comparison with results, obtained in the framework of quark clusters hypothesis, were made.

It is shown that the processes of multiple scattering, Fermi motion and delta resonance formation lead to the cumulative particles production and provide a significant contribution to the cross section for the production of cumulative particles. Results are in consistentn with the experimental data and quark clusters hypothesis is not required.

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