Skew Information for a Single Cooper Pair Box Interacting with a Single Cavity Field

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Abstract The dynamics of the skew information (SI) is investigated for a single Cooper Pair Box (CPB) interacting with a single cavity field. By suitably choosing the system parameters and precisely controlling the dynamics, novel connection is found between the SI and entanglement generation. It is shown that SI can be increased and reach its maximum value either by increasing the number of photons inside the cavity or considering the far off-resonant case. The number of oscillations of SI is increased by decreasing this ratio between the Josephson junction capacity and the gate capacity. This leads to significant improvement of the travelling time between the maximum and minimum values.

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1 Introduction

Cooper pairs represent one of the most important promising candidates in the context of quantum information filed. These pairs are classified as a physical realization of the solid states qubit.[1] In quantum teleportation, the generated entangled state between the Cooper Pair Box (CPB) and the cavity mode is used as a quantum channel to perform the original[2] quantum teleportation protocol.[3] In the presence of noise these channels are employed to implement the original quantum teleportation protocol.[4] The quantum computational speed of a single Cooper Pair Box is investigated by evaluation the speed of orthogonality.[5] Moreover, the generated channel between a single CPB and a cavity mode is used to send coded information in the presence of perfect and imperfect operation during the coding process.[6] The dynamics of the purity, entropy and the coherent vector of a single Cooper pair interacting with a single cavity mode is investigated in Ref. [7].

Recently, skew information has been used as a test for quantum entanglement, where the Bell inequality is obtained by means of skew information. This new inequality provides an exact test to distinguish entangled from non-entangled pure states of two qubits.[8] Quantum state transformation and general design scheme on teleportation protocols have been discussed.[9] Also, different properties of stationary quantum discord and entanglement between a qubit and cavity field have been discussed.[9–18] Pezzè et al. have introduced a measure of entanglement between multi-particles in terms of Fisher information.[24] Luo[25] has found a mathematical relation between the Fisher information, which has many applications in quantum information, and the skew information. This motivates us to investigate the skew information for a single CPB interacting with a single cavity mode.

In this paper we investigate the dynamics of the skew information for a single Cooper Pair Box interacting with a single cavity field. The effect of the cavity field and the Cooper Pair Box’s parameters on the dynamics of the skew information is discussed. We show that the skew information increases faster for larger values of photons inside the cavity or by increasing the detuning parameter between the cavity and the CPB. Also, the effect of the relative ratio of Josephson junction capacity to the gate capacity on the behavior of the skew information is investigated.

The remainder of the paper is structured as follows. In Sec. 2 we describe the system we address and describe the interaction of a single Cooper Pair Box with a single cavity field. Section 3 is devoted to the analysis of the skew information, where we review its definition in the context of quantum information and its relation to the degree of entanglement. Also, we discuss the dynamics of the skew information for different values of the CPB and cavity’s parameters. Finally, we draw our conclusions in Sec. 4.

2 The Suggested Model

A single Cooper Pair Box (CPB) is an example of a qubit with states $|0\rangle$ and $|1\rangle$.[26] It consists of a small superconducting island connected to the outside by Josephson junction $E_J$ and a gate capacitor $C_J$. A gate voltage $V_{g}$ is coupled to the superconducting island through the gate capacity $C_g$.[27–28] This system can be described by two levels system with Hamiltonian

$$H_s = 4E_c(n - n_g)^2 - E_J \cos \phi,$$

where, $E_c = e^2(C_J + C_g)/2$ is the charging energy, $E_J = (h/e)I_c/2$ is the Josephson coupling energy, $e$ is the charge
of the electron, \( n_g = (1/2)(V_g/e)C_g \) is the dimensionless gate charge, \( n \) is the number operators of excess Cooper pair on the island and \( \phi \) is the phase operator.\(^1\) The Hamiltonian of the system (1) reduces to

\[
H_s = -\frac{1}{2}B_z\sigma_z - \frac{1}{2}B_x\sigma_x ,
\]

where it is assumed that the temperature is low enough and Josephson coupling energy \( E_J \) is much smaller than the charging energy i.e., \( E_J < E_C \) and \( B_z = -(2n-1)E_C \), where \( E_C \) is the electric energy, \( B_x = E_J \) and \( \sigma_x, \sigma_y, \) and \( \sigma_z \) are the Pauli matrices.\(^2\) If the single Cooper Pair Box, CPB is placed inside a single-mode microwave cavity, then the Hamiltonian of the system can be written as\(^3\)

\[
\mathcal{H} = \omega a^\dagger a + \omega_c\sigma_z - g(\mu - \cos \theta \sigma_z + \sqrt{1-\nu^2}\sigma_x(a^\dagger + a)),
\]

where \( \omega \) is the cavity resonance frequency, \( \omega_c = \sqrt{E_J^2 + 16E_C^2(2ng - 1)^2} \) is the transition frequency of the cooper pair qubit,

\[
g = \frac{\sqrt{C_I}}{C_g + C_J}\frac{\sqrt{\frac{\omega}{2\hbar}}}{e^2}
\]

is coupling strength of resonator to the Cooper Pair Box, \( \mu = 1 - n_g \), and \( \theta = -\arctan(1/E_C)(E_J/2n_g - 1) \) is the mixing angle.

Let us assume that a single Cooper pair, prepared initially in a superposition state as \( |\psi_i(0)\rangle = (1/\sqrt{2})(|e\rangle + |g\rangle) \) interacts with a single cavity mode, prepared initially in a number state, \( |\psi_f\rangle = |n\rangle \). In this case, the initial state of the total system is defined by \( |\psi_s(0)\rangle = (1/\sqrt{2})(|e, n\rangle + |g, n\rangle) \). The time evolution of the initial state vector is given by,

\[
|\psi_s(t)\rangle = \mathcal{U}(t)|\psi_s(0)\rangle ,
\]

where \( \mathcal{U}(t) \) is a unitary operator defined by

\[
\mathcal{U}(t) = B_1|e\rangle\langle e| + B_2|e\rangle\langle g| + B_3|g\rangle\langle e| + B_4|g\rangle\langle g| ,
\]

where

\[
B_1 = C_{n+1} - i\delta S_{n+1} , \quad B_2 = -i S_n a , \quad B_3 = i S_n a^\dagger , \quad B_4 = c_{n+1} + i\delta S_{n+1} ,
\]

\[
C_n = \cos \Omega \tau \sqrt{\Delta^2 + n} , \quad S_n = \frac{2\lambda}{\sqrt{\Delta^2 + 4g^2n}} \sin \Omega \tau \sqrt{\Delta^2 + n} , \quad \Omega = \frac{\sqrt{C_I}}{C_J + C_g} , \quad T = \sqrt{\frac{\omega}{2\hbar}} , \quad \Delta = \frac{\delta}{2g} .
\]

Using Eq. (5) then the state vector Eq. (4) becomes \( |\psi_s(t)\rangle = A_1|e, n\rangle + A_2|g, n+1\rangle + A_3|e, n-1\rangle + A_4|g, n\rangle \),

where

\[
A_1 = \frac{B_1}{\sqrt{2}} , \quad A_2 = \frac{B_1}{\sqrt{2}} \sqrt{n+1} ,
\]

\[
A_3 = \frac{B_1}{\sqrt{2}} \sqrt{n} , \quad A_4 = \frac{B_1}{\sqrt{2}} .
\]

Since the final state of the initial state vector is obtained, we can investigate all the classical and quantum phenomena associated with this quantum state vector. In this context, we are interested in investigating the dynamics of the skew information.

### 3 Skew Information

Skew information \( (S_I) \) represents a measure in information content of the density operator with respect to a self adjoint operator.\(^4\) Mathematically for a density operator \( \rho \) and a self-adjoint operator \( \mathcal{H} \), the skew information is defined by

\[
S_I = \frac{1}{2} \text{tr} \{\sqrt{\rho \mathcal{H}} - \sqrt{\mathcal{H} \rho} \} ^2 .
\]

Therefore one can say that \( S_I \) measures the non-commutativity between \( \rho \) and \( \mathcal{H} \).\(^5\) Moreover, it has been shown that the skew information can be used to detect entanglement, where the Bell inequality is proposed in terms of \( S_I \) and, it is proved that the inequality provides an exact test to distinguish entangled from separable pure states of two qubits.\(^6\) In an equivalent form of Eq. (8), the skew information can be written as,

\[
S_I = \text{tr} \{\rho \mathcal{H}^2 \} - \text{tr} \{\sqrt{\rho \mathcal{H}} \sqrt{\mathcal{H} \rho} \} .
\]

To investigate the skew information for the Cooper Pair Box interacting with a cavity mode initially prepared in a number state, we consider a set of spin 1/2 operators for the CPB and the cavity as \( \sigma_i \) and \( \tau_i, i = x, y, z \) respectively. Now, the skew information for the CPB and the cavity is defined as

\[
S_I = \sum_i \{\Delta t_i^2 + \Delta _i^2 \} ,
\]

where \( \mathcal{C} \) is the concurrence which quantifies the degree of entanglement between the CPB and the field, where for two qubits, the concurrence is calculated in terms of the eigenvalues \( \eta_1, \eta_2, \eta_3, \eta_4 \) of the matrix \( \mathcal{R} = \rho \sigma_y \otimes \sigma_y \rho ^* \sigma_y \otimes \sigma_y \). It is given by \( \mathcal{C} = \max(0, \eta_1 - \eta_2 - \eta_3 - \eta_4) \), where \( \eta_1 \geq \eta_2 \geq \eta_3 \geq \eta_4 \). For maximally entangled states concurrence is 1 while for separable states it is zero.\(^7\)

The dynamics of the skew information for different values of the detuning parameter is displayed in Fig. 1, where it is assumed that the other parameters have fixed values. It is clear that, for the resonant case, i.e., \( \Delta = 0 \), the skew information \( S_I \) increases as \( T \) increases to reach its maximum value for the first time at \( T \approx 7 \). However, for larger \( \tau \) the skew information decreases smoothly to reach its minimum value for the first time at \( \tau \approx 12.5 \). This behavior is repeated as the scaled time increases. For non-resonant case, as example for \( (\Delta = 0.3) \), the behavior of the skew information is similar to that predicted in
the non-resonant case. However, $S_I$ increases faster and reaches its maximum value for the first time at $T \approx 3$. As the interaction increases the skew information reaches its minimum values faster than that depicted for the resonant case.

![Fig. 1](image1.png)

**Fig. 1** The skew information $S_I$ between the Cooper pair and the cavity mode for a system initially prepared in $|n,e\rangle$. The solid, dotted and dash dotted curves are evaluated for $\Delta = 0.0, 0.3$, and 0.9 respectively. The ratio $\gamma = C_j/C_g = 1/4$ and the number of photon inside the cavity $n = 1$.

![Fig. 2](image2.png)

**Fig. 2** The same as Fig. 1, but for different values of photons inside the cavity. The solid, dotted and dash dotted curves are evaluated for $n = 2, 5,$ and 8 respectively. The ratio $\gamma = C_j/C_g = 1/4$ and the detuning parameter $\Delta = 0.0$.

Figure 2 displays the effect of different numbers of photons inside the cavity. In this figure, we consider the resonant case, i.e. $\Delta = 0.0$, and fix the ratio $\gamma = C_j/C_g = 1/4$. The dynamics of the $S_I$ is similar to what has been shown in Fig. 1. However, the number of oscillations depends on the number of photons inside the cavity. It is clear that, for larger values of $n$, the skew information reaches its maximum value faster than that evaluated for smaller number of photons inside the cavity and consequently, $S_I$ vanishes earlier for larger values of $n$. Comparing Fig. 1 (solid curve) and its corresponding one in Fig. 2, we can see that for $n = 1$, the skew information reaches its minimum value at $T = 12.5$, while for $n = 2$ (solid curve in Fig. 2), the skew information reaches its minimum value at $T = 10$.

![Fig. 3](image3.png)

**Fig. 3** The same as Fig. 1, but for different values of the the ratio $\gamma$. The solid, dotted and dash dotted curves are evaluated for $\gamma = 1/4, 1/6$, and $1/8$ respectively. The number of photons inside the cavity is $n = 2$, and the detuning parameter (a) for resonant case i.e., $\Delta = 0.0$; (b) for non-resonant case with $\Delta = 0.3$.

![Fig. 4](image4.png)

**Fig. 4** The same as Fig. 1, but for different values of the ratio $\gamma$. The solid, dotted and dash dotted curves are evaluated for $\gamma = 4, 6$, and 8, respectively, while the number of photons inside the cavity is $n = 2$, and the detuning parameter $\Delta = 0.0$.

The effect of the ratio, $\gamma$, is displayed in Fig. 3, for resonant and off-resonant cases, where we fix the other parameters. It is clear that for large values of $\gamma$, the skew information reaches its minimum values faster than that shown for smaller values of $\gamma$. Also, for smaller values of $\gamma$, $S_I$ increases gradually but for larger $\gamma$, the skew information increases hastily. This behavior is clearly displayed in Fig. 3(a) for the resonant case. As the detuning is increased by 30%, the behavior of $S_I$ changes dramatically.
The skew information is increased and decreased hastily regardless of the ratio $\gamma$. In this case the minimum values of $S_f$ is reached much earlier, i.e., $T$ reduces by 5%.

Figure 4 describes the dynamics of $S_f$ for $\gamma$ (say $\gamma = 2, 4$, and 6) and the resonant case. It is clear that, the behavior is similar to that shown in Fig. 3(b). This shows that one can control the behavior of the skew information either by controlling the detuning parameter or by changing the ratio $\gamma$.

4 Conclusion

In this contribution, we investigate skew information for a particle of CPB, which is a promising candidate for quantum information and computations. The suggested model consists of a single Cooper Pair Box prepared initially in the excited state interacting with a cavity mode prepared in the number state. The effect of the cavity field and the Cooper Pair Box’s parameters on the dynamics of the skew information is investigated. The resonant as well as the off-resonant cases have been discussed. For resonant case, the skew information increases gradually and takes more time to reach its maximum or minimum values. However, in the far off-resonant case, the skew information is increased faster for larger values of the detuning parameter and consequently the number of oscillations between the maximum and minimum values is increased. The number of photons inside the cavity has a noticeable effect, as one increases the number of photons, the skew information increases faster and the number of oscillations increases. However, if we increase the number of photon by 1%, the oscillations time between the maximum and minimum values is reduced by 2%.

The effect of the relative ratio of the Josephson junction capacity and the gate capacities play an important role on the the dynamics of skew information. For small values of this ratio the number of oscillations of the skew information is increased and consequently the revival time is decreased. On the other hand, the skew information is increased faster for smaller values of $\gamma = C_j/C_g$, and is increased gradually for larger values of this ratio. Finally, it is possible to control the skew information dynamics using the cavity field or the CPB’s parameters. Therefore, one can speed up the skew information to reach its maximum value either by increasing the number of photons inside the cavity or considering far off-resonance cases.

References

[1] Yu.A. Pashkin, T. Yamamoto, O. Astafiev, Y. Nakamura, D. Averin, T. Tilma, F. Nori, and J.S. Tsai, Physica C 426-431 (2005) 1552.
[2] C.H. Bennett, G. Brassard, C. Crepeau, R. Jozsa, A. Peres, and W.K. Wootters, Phys. Rev. Lett. 70 (1993) 1895.
[3] N. Metwally and A.A.A. El-Amin, Physica E 41 (2009) 718.
[4] A.H. M. Ahmed, M.N. Zakaria, and N. Metwally, Appl. Math. Inf. Sci. 6 (2012) 781.
[5] A.S. Obada, N. Metwally, D.M. Abo-Kahlia, and M. Abdel-Aty, Physica E 43 (2011) 1792.
[6] N. Metwally, J. Opt. Soc. Am. B 3 (2012) 389.
[7] A.H.M. Ahmed, L. Cheong, N. Zakaria1, and N. Metwally, Int. J. Theor. Phys. (2012) DOI 10.1007/s10773-012-1399-9.
[8] Z. Chen, Phys. Rev. A 71 (2005) 052302.
[9] M. Jiang, X. Huang, L. Zhou, and L. Zhou, Chin. Sci. Bull. 57 (2012) 2247; Chen Chang-Yong and Sun Qing, Sci. Chin. Phys. Mech. Astron. 54 (2011) 930.
[10] A.B.A. Mohamed, Quant. Inf. Rev. 1 (2013) 1.
[11] Qian Yi, Zhang Ye-Qi, and Xu Jing-Bo, Chin. Sci. Bull. 57 (2012) 1637.
[12] J.S. Zhang and Ai-Xi Chen, Quant. Phys. Lett. 1 (2012) 69.
[13] N. Metwally, Int. J. Theor. Phys. 49 (2110) 1571; N. Metwally, Int. J. Theor. Phys. 47 (2008) 623.
[14] A.-El Allati, N. Metwally, and Y. Hassouni, Opt. Commun. 284 (2011) 519.
[15] H. Rabitz, Quant. Phys. Lett. 1 (2012) 1.
[16] N. Metwally, M. Abdelaty, and A.S.F. Obada, Opt. Commun. 250 (2005) 148; M.S. Abdalla, A.F. Obada, and M. Abdel-Aty, Ann. Phys. 318 (2005) 266.
[17] M. Sebawe Abdalla and L. Thabet, Appl. Math. Inf. Sci. 5 (2011) 570.
[18] M. Abdel-Aty, J. Phys. A: Math. Ge. 38 (2005) 8589.
[19] A. El-Barakaty, M. Darwish, and A.S.F. Obada, Appl. Math. Inf. Sci. 5 (2011) 122.
[20] Z. Ficek, Appl. Math. Inf. Sci. 3 (2009) 375.
[21] Li-Hui Sun, Gao-Xiang Li, and Z. Ficek, Appl. Math. Inf. Sci. 4 (2010) 315.
[22] H. Eleuch, Appl. Math. Inf. Sci. 3 (2009) 185.
[23] Gui-Long Gao, Gen-Chang Cai, and Shou-Sheng Huang, Commun. Theor. Phys. 57 (2012) 205; Ye Cao, Hui Li, and Gui-Lu Long, Chin. Sci. Bull. 58 (2013) 48.
[24] L. Pezzè and A. Smerzi, Phys. Rev. Lett. 102 (2009) 100401.
[25] S. Luo, Proceeding of Am Math. Soc. 132 (2003) 885.
[26] S. Bose and G.S. Agarwal, New Journal of Physics B 8 (2006) 34.
[27] Y. Nakamura, Yu.A. Pashkin, and J.S. Tsai, Nature (London) 398 (1999) 798.
[28] J.S. Tsai and Y. Nakamura, Physica C 367 (2002) 191.
[29] R. Migliore, A. Messina, and A. Napoli, Eur. Phys. J. B 13 (2000) 585.
[30] M. Zhang, J. Zou, and B. Shao, Int. J. Mod. Phys. 16 (2002) 4767.
[31] E.P. Winger and M.M. Yanase, Proc. Nat. Ascd. Sci. USA 49 (1963) 910.
[32] Sun Hong-Gui, Liu Wan-Fang, and Li Chun-Jie, Chin. Phys. B 20 (2011) 090301.
[33] W.K. Wootters, Phys. Rev. Lett. 80 (1998) 2245.