Dynamical Breakdown of Symmetry in a (2+1) Dimensional Model Containing the Chern-Simons Field

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Abstract

We study the vacuum stability of a model of massless scalar and fermionic fields minimally coupled to a Chern-Simons field. The classical Lagrangian only involves dimensionless parameters, and the model can be thought as a (2+1) dimensional analog of the Coleman-Weinberg model. By calculating the effective potential, we show that dynamical symmetry breakdown occurs in the two-loop approximation. The vacuum becomes asymmetric and mass generation, for the boson and fermion fields takes place. Renormalization group arguments are used to clarify some aspects of the solution.
I. INTRODUCTION

Quantum field theories involving the Chern-Simons (CS) field in (2+1) dimensions present many peculiar and surprising aspects. Fractional spins, exotic statistics and the existence of massive gauge fields are well known examples in this direction. Applications of these models are, for example, planar Aharonov-Bohm effect, fractional quantum Hall effect and surface effects in liquid helium [1].

Physicists working in these theories have a special concern about their phase structure and phase transitions. These properties are investigated through the calculation of the renormalization group functions and the effective potential. So, renormalization and its intermediate step, regularization, are central issues to these studies. As the Lagrangian density of the CS field involves the Levi-Civita tensor, these models are not easily extensible outside (2+1) dimensions, making the dimensional regularization scheme [2], a very cumbersome procedure. In fact, besides the extension out of (2+1) dimensions, dimensional regularization for these models requires the introduction of an extra regularization term in the Lagrangian. Of course, a safe escape would be to avoid the use of regularizations at all, like in the BPHZ renormalization program [3], but this is also a complex procedure due to the zero mass fields involved [4, 5]. For these reasons, a simple regularization method called dimensional reduction, has been largely employed [6, 7, 8] to deal with the CS models. It consists in a simplification of dimensional regularization, in which all tensor contractions appearing in Feynman graphs, are first realized in (2+1) dimensions and only the resulting scalar integrals are extended to $D = 3 - \epsilon$ dimensions. But, as stressed by several authors [9], this method may lead to ambiguities, jeopardizing the invariance of the theory under gauge transformations. Nevertheless, given its great simplicity it may be worth to use dimensional reduction in practical calculations, after validating its consistency through the explicit verification of the Ward identities related to the gauge symmetry.

In this paper we calculate the effective potential for the theory of a massless complex scalar field with a sextuple self-interaction and minimally coupled with a CS field. Dynamical symmetry breakdown and mass generation are explicitly verified not in one loop as in the Coleman-Weinberg (CW) [11] model, but in the two-loop approximation. The Feynman integrals are regularized by dimensional reduction. The verification that this regularization procedure preserves the Ward identities, was carried out in previous works [7, 10].
The model is then extended by including a massless fermion minimally interacting with the CS field, and coupled to the scalar field through an “Yukawa” term \[12\]. Up to two loops, the new graphs that appear in this extended model involve products of two Levi-Civita tensors at most. As discussed in \[9\], possible differences between the expressions calculated using dimensional reduction and dimensional regularization only appear in the product of three and more Levi-Civita tensors. Therefore, up to the order we are dealing no breakdown of the Ward identities takes place. Here again symmetry breakdown occurs due to two-loop radiative corrections. Curiously, for a certain relation between the Yukawa and the scalar gauge couplings the effective potential is insensitive to the fermion gauge coupling. We also analyze some renormalization group aspects of the extended model by determining the modifications in the anomalous dimension of the scalar field and the beta function associated with the sextuple self-interaction.

The organization of the paper is as follows: in Sec. II the effective potential for the scalar/CS model is calculated and the symmetry breakdown is discussed. In Sec. III this study is extended to the scalar/fermion/CS model. In Sec. IV a renormalization group analysis of the model is performed. In the Conclusions the comparison with the literature is carried. Three Appendices present the main integrals necessary in the work and the calculation of the wave function renormalization for the \( \phi \) field.

II. THE BOSONIC MODEL

In this section we consider the model of a massless complex scalar field with a sextuple self-interaction and minimally interacting with a CS field, whose Lagrangian density is

\[
\mathcal{L} = \frac{1}{2} \epsilon_{\mu\nu\rho} A^\mu \partial^\nu A^\rho + (D^\mu \phi)^\dagger (D_\mu \phi) - \frac{\nu}{6} (\phi^\dagger \phi)^3,
\]

(1)

where \( \nu \) is a positive constant and \( D^\mu = \partial^\mu - i e A^\mu \). The metric used is \( g^{\mu\nu} = (1, -1, -1) \) and the fully antisymmetric Levi-Civita tensor \( \epsilon_{\mu\nu\rho} \) is normalized as \( \epsilon_{012} = 1 \). It is worth to stress that all the parameters in this Lagrangian are dimensionless, what makes it a (2+1) dimensional analog of the well known CW model in (3+1) dimensions. If instead of the CS, a Maxwell dynamics for \( A_\mu \) were used, a dimensional parameter would be introduced in the Lagrangian and this analogy would be lost.
It is convenient to decompose the complex scalar field as \( \varphi = (\varphi_1 + i\varphi_2) / \sqrt{2} \), where \( \varphi_1 \) and \( \varphi_2 \) are real fields, so that the Lagrangian (1) becomes:

\[
\mathcal{L} = \frac{1}{2}(\partial^\mu \varphi_i)^2 + e\epsilon^{ij}A_\mu \varphi_j \partial^\mu \varphi_i + \frac{e^2}{2} A^\mu A_\mu \varphi_i \varphi_i - \frac{\nu}{48}(\varphi_i \varphi_i)^3 + \frac{1}{2} \epsilon_{\mu\nu\rho} A^\mu \partial^\nu A^\rho, \tag{2}
\]

where \( \epsilon^{12} = -\epsilon^{21} = 1 \) and the summation convention is implied, with \( i \) and \( j \) running over 1 and 2. We shall work in a 't Hooft gauge, which has the advantage of eliminating non-diagonal terms in the free propagators. The gauge fixing term is taken as

\[
\mathcal{L}_{GF} = -\frac{1}{2\xi}(\partial^\mu A_\mu - \xi e \epsilon^{ij} \varphi_i u_j)^2, \tag{3}
\]

and the corresponding Faddeev-Popov Lagrangian is given by

\[
\mathcal{L}_{FP} = -c^\dagger (\partial^2 + \xi e^2 \epsilon^{ij} \varphi_i u_j) c, \tag{4}
\]

where \( u_i \) is a constant to be chosen later. To calculate the effective potential we follow the functional method [13]. The first step is to consider the action, \( \hat{I}(\varphi_i, \chi; \phi_i) = \int d^3x \hat{\mathcal{L}}(\varphi_i, \chi; \phi_i) \), obtained by shifting the scalar fields by a constant: \( \varphi_i \rightarrow \varphi_i + \phi_i \) in the original Lagrangian (2) and subtracting the terms which are either independent or linear in the fields, i.e.,

\[
\hat{I}(\varphi_i, \chi; \phi_i) = I(\varphi_i + \phi_i, \chi) - I(\phi_i, 0) - \int d^3x \frac{\delta I}{\delta \varphi_i}(\phi_i, 0) \varphi_i - \int d^3x \frac{\delta I}{\delta \chi}(\phi_i, 0) \chi, \tag{5}
\]

where \( \phi \) is the constant expectation value of \( \varphi \) and \( \chi \) represents the fields \( A_\mu, c \) and \( c^\dagger \).

By choosing \( u = \phi \), bilinear terms in \( A_\mu \) and \( \varphi_i \) are eliminated from the resulting \( \hat{\mathcal{L}} \), which becomes

\[
\hat{\mathcal{L}} = \frac{1}{2} A_\mu [\epsilon_{\mu \nu \rho} \partial_\nu + m_3 g^{\mu \nu} + \frac{1}{\xi} \partial^\mu \partial^\nu] A_\nu + c^\dagger (-\partial^2 - m_3 \xi) c
+ \frac{1}{2} \varphi_i [\delta_{ij}(-\partial^2 - m_1^2 - m_3 \xi) + (m_3 \xi - 4 m_1^2) \hat{\phi}_i \hat{\phi}_j] \varphi_j +
- e A^\mu \epsilon_{ij} \partial_\mu \varphi_j + e^2 \phi \cdot \varphi A^2 - e^2 \xi \phi \cdot \varphi c^\dagger c
- \frac{\nu}{6}(\phi \cdot \varphi)^3 - \frac{\nu}{4} \phi^2 \phi \cdot \varphi^2 + \frac{e^2}{2} A^2 \varphi^2 - \frac{\nu}{16} \phi^2 \varphi^4
- \frac{\nu}{4}(\phi \cdot \varphi)^2 \varphi^2 - \frac{\nu}{8} \phi \cdot \varphi^4 - \frac{\nu}{48} \varphi^6. \tag{6}
\]
where \( m_1^2 = \nu \phi^4 / 8 \), \( m_3 = e^2 \phi^2 \), \( \hat{\phi}_i = \phi_i / \sqrt{\phi^2} \) and \( \phi \cdot \varphi, \phi^2 \) and \( \varphi^2 \) stand respectively for \( \phi_i \varphi_i, \hat{\phi}_i \hat{\phi}_i \) and \( \varphi_i \varphi_i \). For later use we will also define \( m_2^2 = 5m_1^2 \).

The effective potential is given by

\[
V = \nu \phi^6 - \frac{i}{2} \int \frac{d^3 k}{(2\pi)^3} \ln[\det (i\Delta^{-1}_{\alpha\beta}(k,\phi_i))] + i < 0| T \exp i \int d^3 x \hat{\mathcal{L}}_{\text{int}} | 0 >. \tag{7}
\]

The first term in (7) is the tree approximation as can be read from (2). The second term is the one-loop correction and the \( i\Delta^{-1}_{\alpha\beta}(k,\phi_i) \) is the coefficient matrix of the quadratic terms of \( \mathcal{L} \). The third term is the sum of the two and more loops vacuum diagrams, calculated from \( \mathcal{L} \); it summarizes the infinity sum of all higher order loop diagrams with any number of external scalar lines \( \varphi = \phi \), gotten from the original Lagrangian \( \mathcal{L} \|11\).

The one-loop effective potential is obtained using the inverse propagators from (6), that are

\[
i\Delta_{ij}^{-1}(k) = \delta_{ij}(k^2 - m_i^2 - \xi m_3) + \hat{\phi}_i \hat{\phi}_j (-4m_i^2 + \xi m_3), \tag{8}
\]

\[
i\Delta_{\mu\nu}^{-1}(k) = -i\epsilon_{\mu\rho\nu}k^\rho + m_3 g_{\mu\nu} - \frac{1}{\xi} k_{\mu} k_{\nu}, \tag{9}
\]

\[
i\Delta_{\text{ghost}}^{-1}(k) = k^2 - \xi m_3, \tag{10}
\]

for the scalar, gauge and ghost fields respectively. As said in the Introduction, we employ the dimensional reduction method in which the integrals are promoted to \( D = 3 - \epsilon \) and a mass scale \( \mu \) is introduced to keep the dimensions of the relevant quantities unchanged (\( \int d^3k \rightarrow \mu^\epsilon \int d^3\epsilon k \)). Thus, using [8]-[10], we obtain that the one-loop contribution is:

\[
V^{B(1)} = -\frac{1}{12\pi} [m_1^3(1 + 5^2) + m_3^3] = -\frac{1}{12\pi} [\nu^{3/2} \frac{1 + 5^{3/2}}{16\sqrt{2}} + e^6] \phi^6. \tag{11}
\]

Here and in what follows we are retaining only the contributions that do not vanish as \( \epsilon \) tends to zero. We have also chosen to work in the Landau limit (\( \xi \rightarrow 0 \)) where the ghosts decouple and do not contribute to the potential.

As a consequence of the dimensional reduction regularization in (2+1) dimensions, in one loop, no infinity appears. Up to one loop, the effective potential is given by:

\[
V(\phi) = \frac{\nu}{48} \phi^6 +
\]
$V^{B(1)}(\phi) + \frac{C}{4\phi^4}\phi^6$, where $C$ is a convenient finite counterterm. It does not have any non trivial $\phi \neq 0$ minimum, and dynamical breakdown of symmetry does not occur. The symmetry breakdown in the CW model in (3+1) dimensions is made possible by the induction of a term of the form $\phi^4 \ln \phi$. In the analogous calculation in (2+1) dimensions, the dimension of the phase space precludes the induction of a similar term (actually $\phi^6 \ln \phi$). The only effect, of the one-loop calculation, is the change in the coefficient of the $\phi^6$ term of the classical potential, and so symmetry breakdown does not happen. Thus, to pursue a possible symmetry breakdown we will study the two-loop approximation.

The propagators (in the Landau limit) for the $A^\mu$ and $\varphi$ fields obtained from the shifted Lagrangian are:

$$\Delta_{\mu\nu}(k) = -\frac{\epsilon_{\mu\nu\rho}k^\rho}{k^2 - m^2} - i\frac{m_3}{k^2 - m_3}(g_{\mu\nu} - \frac{k_\mu k_\nu}{k^2}),$$

(12)

$$\Delta_{ij}(k) = i\left[\frac{1}{k^2 - m_i^2}(\delta_{ij} - \hat{\varphi}_i \hat{\varphi}_j) + \frac{1}{k^2 - m_j^2}\hat{\varphi}_i \hat{\varphi}_j\right].$$

(13)

The interaction vertices are given by (we consider only those vertices which contribute to the two-loop calculation):

- Quadrilinear $A_\mu A_\nu \varphi_i \varphi_j$ vertex $\leftrightarrow \frac{i}{2}e^2\delta_{ij}g_{\mu\nu}$
- Quadrilinear $\varphi_i \varphi_j \varphi_k \varphi_l$ vertex $\leftrightarrow -i\nu \phi^2 \left(\frac{1}{2}\delta_{ij} + 2\hat{\varphi}_i \hat{\varphi}_j\right)\delta_{kl}$
- Trilinear $\varphi_i \varphi_k \varphi_j$ vertex $\leftrightarrow -i\nu \phi^3 \hat{\varphi}_i (\delta_{jk} + \frac{2}{3}\hat{\varphi}_j \hat{\varphi}_k)$
- Trilinear $\varphi_i A_\mu A_\nu$ vertex $\leftrightarrow i e^2 \hat{\varphi}_i g_{\mu\nu}$
- Trilinear $\varphi_i \varphi_j A_\mu$ vertex $\leftrightarrow -\frac{e}{2}\epsilon_{ij}(p + q)_\mu$

(14)  
(15)  
(16)  
(17)  
(18)

The two-loop contributions to the effective potential are drawn in Fig. 1. The corresponding analytic expressions are listed in the Appendix B. In the dimensional reduction approach the Lorentz indices are contracted in three dimensions, and only after all the tensor simplifications are done, the resulting scalar integrals are promoted to $D = 3 - \epsilon$ dimensions. The calculations, tedious but straightforward, are carried by using the formulas presented in the Appendices. $V^{B(2)}_{\Pi}$ and $V^{B(2)}_{\Pi}$, which correspond to the diagrams in Figs. 11 and 12.
respectively, are convergent, since they are given by products of non overlapping one-loop integrals. The other three diagrams have divergences proportional to $\phi^6$, in accordance with the renormalizability of the model. Summarizing, the results are

\[ V_{\text{B}(2)}^{\text{l}} = \frac{\nu^2}{2^2 \pi^2} \phi^2 \left( \frac{3}{2} m_1^2 + 3 m_1 m_2 + \frac{11}{2} m_2^2 \right), \]

\[ V_{\text{B}(2)}^{\text{l}} = \frac{e^2}{2^4 \pi^2} (m_1 m_3^2 + m_2 m_3^2), \]

\[ V_{\text{B}(2)}^{\text{l}} = -\frac{1}{2^2} \nu^2 \left[ 2 I(m_1, m_2, m_1) + \frac{50}{3} I(m_2, m_2, m_2) \right] \phi^6, \]

\[ V_{\text{B}(2)}^{\text{l}} = -\frac{1}{2^2} 2 e^2 m_3 \left[ \frac{1}{2^2 \pi^2} (m_1 m_3 + m_2 m_3) + 2(m_1^2 + m_2^2) I(m_2, m_1, m_3) \right. \\
\left. - \frac{2}{m_3^2} [K(m_2, m_1, m_3) + K(m_1, m_2, m_3) - K(m_2, m_1, 0) - K(m_1, m_2, 0)] \right]. \]

and

\[ V_{\text{B}(2)}^{\text{l}} = e^4 \phi^2 \left[ \frac{3}{2^4 \pi^2} m_3 m_2 - 2 J(m_3, m_3, m_2) + J(0, m_3, m_2) \right. \\
\left. - 6 m_3^2 I(m_3, m_3, m_2) + 3 m_3^2 I(0, m_3, m_2) - \frac{1}{m_3^2} [K(m_3, m_3, m_2) \right. \\
\left. - K(m_3, 0, m_2) - K(0, m_3, m_2) + K(0, 0, m_2)] \right]. \]

After using the results of the Appendix A where the functions $I$, $J$ and $K$ are defined, and for convenience introducing $I_{\text{div}} = \frac{1}{\epsilon} - \gamma + \ln 4\pi + 1$, we have

\[ V_{\text{B}(2)}^{\text{l}} = \frac{1}{2^2 \pi^2} \nu^2 (29 + 3 \sqrt{5}) \phi^6, \]

\[ V_{\text{B}(2)}^{\text{l}} = \frac{1}{2^5 \sqrt{2} \pi^2} e^6 \sqrt{\nu} (1 + \sqrt{5}) \phi^6, \]
\[ V^{B(2)} = \frac{1}{32\pi^2} \nu^2 \phi^6 \left[ 6 \ln(2 + \sqrt{5}) + 56 \ln(\sqrt{\frac{\nu}{8}}) + 50 \ln 3 + 25 \ln 5 \right] 
\quad + \frac{7}{32\pi^2} \nu^2 \phi^6 \left[ \ln(\frac{\phi^2}{\mu}) - \frac{1}{2} I_{\text{div}} \right], \] (26)

\[ V^{B(2)} = \frac{1}{25\pi^2} \phi^6 \left[ - \sqrt{\frac{\nu}{8}} \nu e^6 - \sqrt{\frac{5\nu}{8}} \nu e^6 + (3e^8 - \frac{3}{2} \nu e^4 + \frac{1}{4} \nu^2) \ln(\sqrt{\frac{\nu}{8}} + \sqrt{\frac{5\nu}{8}} + e^2) \right. 
\quad + \left. \sqrt{\frac{\nu}{8}} \nu e^4 - \frac{1}{4} \sqrt{\frac{5\nu}{8}} \nu^{3/2} e^2 + \frac{1}{4\sqrt{2}} \nu^{3/2} e^2 - \frac{1}{4} \nu^2 \ln(1 + \sqrt{5}) - \frac{1}{4} \nu^2 \ln(\sqrt{\frac{\nu}{8}}) \right. 
\quad + \left. 2 \pi^2 \nu^2 \right] + \frac{1}{25\pi^2} \phi^6 \left[ \ln(\frac{\phi^2}{\mu}) - \frac{1}{2} I_{\text{div}} \right] (-\frac{3}{2} \nu e^4 + e^8) \right] \] (27)

and

\[ V^{B(2)} = -\frac{1}{25\pi^2} \phi^6 \left[ -20 e^6 \sqrt{\frac{5\nu}{8}} + 10 \nu \ln(2 e^2 + \sqrt{\frac{5\nu}{8}}) e^4 + 12 e^8 \right. 
\quad + 20 \ln(e^2 + \sqrt{\frac{5\nu}{8}}) e^8 - 5 \ln(e^2 + \sqrt{\frac{5\nu}{8}}) e^4 \nu - 48 e^8 \ln(2 e^2 + \sqrt{\frac{5\nu}{8}}) \right. 
\quad - \frac{5}{4} \frac{2^5 \nu^2 \ln(5) - 5^2 2 \nu^2 \ln(\sqrt{\frac{\nu}{8}})}{25} \phi^4 \ln(\frac{\phi^2}{\mu}) - \frac{1}{25\pi^2} \phi^6 \left[ \ln(\frac{\phi^2}{\mu}) - \frac{1}{2} I_{\text{div}} \right] (-\frac{5}{4} \nu e^4 + 7 e^8). \right] \] (28)

Collecting all these two-loop contributions we obtain:

\[ V^{B(2)} = X_b(e, \nu, \epsilon) \phi^6 + Z_b(e, \nu) \phi^6 \ln(\frac{\phi}{\sqrt{\mu}}), \] (29)

with \( X_b(e, \nu, \epsilon) \) standing for the sum of all coefficients of \( \phi^6 \) in Eqs. (24-28) and

\[ Z_b(e, \nu) = \frac{1}{8\pi^2} (4e^8 - \frac{11}{8} e^4 \nu + \frac{7}{12} \nu^2). \] (30)

From (7), (11) and (29) we can write the regularized effective potential up to two loops (with a counterterm \( C \) included) as:

\[ V^{B}_{\text{reg}} = Z_b \phi^6 \ln(\frac{\phi e^{\gamma_b} Z_b}{\sqrt{\mu}}). \] (31)
where $Y_b(e, \nu)$ is the constant

$$Y_b(e, \nu, \epsilon) = \frac{\nu}{48} - \frac{1 + 5\sqrt{5}}{32^{3/2}\pi} \nu^{3/2} - \frac{e^6}{12\pi} + X_b(e, \nu, \epsilon) + \frac{C}{48}. \quad (32)$$

The parameters $\mu$ and $C$ can be eliminated by imposing the condition

$$\frac{dV_{\text{ren}}^B}{d\phi} \bigg|_{\phi = v} = 0, \quad (33)$$

where $v$ is an arbitrary non null parameter. The resulting potential is

$$V_{\text{ren}}^B = Z_b \phi^6 \left( \ln \frac{\phi}{v} - \frac{1}{6} \right) \quad (34)$$

and $\phi = v$ is a local minimum (vacuum) of this effective potential, if the generated squared mass of the scalar field

$$m_\phi^2 = \left. \frac{d^2V_{\text{ren}}^B}{d\phi^2} \right|_{\phi = v} = 6 Z_b v^4, \quad (35)$$

is positive, what means $Z_b > 0$. We will choose $Z_b$ through the condition

$$\left. \frac{d^6V_{\text{ren}}^B}{d\phi^6} \right|_{\phi = v} = 15\nu \equiv \left. \frac{d^6V_{\text{tree}}^B}{d\phi^6} \right|_{\phi = v}, \quad (36)$$

what implies in

$$\nu = \frac{548}{5} Z_b = \frac{137}{10\pi^2} \left( 4e^8 - \frac{11}{8} e^4 \nu + \frac{7}{12} \nu^2 \right) \approx 1.39 \left( 4e^8 - \frac{11}{8} e^4 \nu + \frac{7}{12} \nu^2 \right). \quad (37)$$

The solution of this equation for both $\nu$ and $e^2$ in the perturbative regime ($\nu$ and $e^2 \ll 1$) is given by

$$\nu \approx \frac{274}{5\pi^2} e^8 + \mathcal{O}(e^{12}), \quad (38)$$

and in the leading approximation the effective potential and the generated squared mass result to be

$$V_{\text{ren}}^B(\phi) = \frac{e^8}{2\pi^2} \phi^6 \left( \ln \frac{\phi}{v} - \frac{1}{6} \right), \quad (39)$$
\[ m^2_\phi = \frac{d^2 V_{ren}}{d\phi^2} \bigg|_{\phi=v} = \frac{3}{\pi^2} e^4 v^4 > 0. \]  

The ratio of this mass to the induced squared mass of \( A_\mu \), that is \( m^2_A \approx m^2_3(\phi = v) = e^4 v^4 \), gives: \( m^2_\phi / m^2_A \approx 3 e^4 / \pi^2 \).

Summarizing, the CW mechanism is operative due to the two-loop radiative corrections, that is, in this approximation a non trivial vacuum is induced and masses for the boson and CS fields are generated. Similarly to what happens in four dimensions, no symmetry breakdown occurs in the model of a single scalar field in self-interaction. Indeed, by making \( e = 0 \) in (37) we get \( \nu = 120\pi^2 / 959 \approx 1.23 \) and so, out of the perturbative regime of validity of the calculations.

The two-loop effective potential for the scalar/CS model was previously calculated in [14], using dimensional regularization. As shown by the authors, in that regularization scheme, the extension out of (2+1) dimensions is not enough to regularize the model; it becomes also necessary, to introduce in the Lagrangian, an extra regularization term, a Maxwell like term: \(-\frac{a}{4} F^{\mu\nu} F_{\mu\nu}\), depending on a parameter \( a \). No trouble is found when making the regulators \( \epsilon \) and \( a \) to go to zero after the renormalization, in the calculation of the effective potential. This is not true in the calculation of the renormalization group parameters (see discussion in Sec. IV).

III. ADDING FERMIONS TO THE MODEL

Many interesting phenomena in planar physics, which have the CS model as an effective theory, also involve fermion particles. Since the advent of the CS field theory [15], a vast literature (see [16] and the references therein) on the subject has appeared, but the effect of fermions on the effective potential for a model like (2), does not seem to have been studied. In this Section we will extend the model (2), by including a Dirac fermionic field interacting with all the other fields. This is done by adding to (2) the following Lagrangian density:

\[ \mathcal{L}_{Dirac} = i \bar{\psi} \gamma^\mu (\partial_\mu - iq A_\mu) \psi - \frac{\alpha}{2} \varphi_i \varphi_i \bar{\psi} \psi, \]  

where \( \psi \) is a two-component massless Dirac field that represents a particle and its antiparticle with the same spin projection. The \( \gamma^\mu \) matrices were chosen as \( (\gamma^0, \gamma^1, \gamma^2) = (\sigma^3, i\sigma^1, i\sigma^2) \),
where $\sigma^a$ are the Pauli matrices. Besides the minimal interaction with the CS field, the fermion (charge $q$) also couples to the scalar field through an “Yukawa” term with the coupling constant $\alpha$. As the parameters of the purely bosonic model, these new coupling constants, $\alpha$ and $q$, are also dimensionless.

After shifting the scalar field: $\varphi \rightarrow \varphi + \phi$, as in Section II we get

$$\hat{L}_{\text{Dirac}} = i \bar{\psi} \gamma^\mu \partial_\mu \psi + q \bar{\psi} \gamma^\mu A_\mu \psi - \frac{\alpha \phi^2}{2} \bar{\psi} \psi - \frac{\alpha}{2} \varphi_i \bar{\psi} \psi - \alpha \cdot \varphi \bar{\psi} \psi.$$  (42)

The fermion propagator is $S(p) = i/(\gamma^\mu p_\mu - m_4)$ with $m_4 = \alpha \phi^2/2$, and the interaction vertices are given by

- Trilinear $A_\mu \bar{\psi} \psi$ vertex $\leftrightarrow i q \gamma_\mu$ (43)
- Trilinear $\varphi_i \bar{\psi} \psi$ vertex $\leftrightarrow -i \alpha \phi \hat{\phi}_i$ (44)
- Quadrilinear $\varphi_i \varphi_j \bar{\psi} \psi$ vertex $\leftrightarrow -i \frac{\alpha}{2} \delta_{ij}$ (45)

The one-loop contribution to the effective potential is

$$V^{F(1)} = i \int \frac{d^3 k}{(2 \pi)^3} \ln \det(\gamma^\mu p_\mu - m_4) = \frac{1}{6 \pi} m_4^3 = \frac{\alpha^3}{48 \pi} \phi^6,$$  (46)

and the new two-loop contributions are represented by the diagrams in Fig. 2. The results are (see Appendix B)

$$V^{F(2)}_{2a} = -\frac{\alpha}{4 \pi^2} \left( m_4^2 m_1 + m_4^2 m_2 \right),$$  (47)

$$V^{F(2)}_{2b} = \frac{\phi^2 \alpha^2}{2} \left( -\frac{1}{8 \pi^2} m_4 m_2 + J(m_4, m_2, m_4) + 4m_4^2 I(m_4, m_2, m_4) \right),$$  (48)

and

$$V^{F(2)}_{2c} = \frac{2q^2}{16 \pi^2} \left[ -\frac{1}{m_3} \left( \frac{1}{16 \pi^2} m_4^3 + (m_3^2 m_4 - m_4^2 m_3) I(m_4, m_3, m_4) + \frac{m_3}{2} J(m_4, m_3, m_4) - \frac{1}{m_3} K(m_4, 0, m_4) \right) \right].$$  (49)
With the aid of the formulas in the Appendices we have

\[ V_{2\alpha}^{F(2)} = -\frac{\alpha^3}{16\pi^2} \phi^6 \left( \sqrt{\frac{\nu}{8}} + \sqrt{\frac{5\nu}{8}} \right), \quad (50) \]

\[ V_{2b}^{F(2)} = -\frac{\alpha^2}{128\pi^2} \phi^6 \left[ 4\alpha \sqrt{\frac{5\nu}{8}} - \frac{20\nu}{8} \ln(\alpha + \sqrt{\frac{5\nu}{8}}) - \alpha^2 + 4\alpha^2 \ln(\alpha + \sqrt{\frac{5\nu}{8}}) \right] \]

\[-\frac{1}{32\pi^2} \phi^6 \left[ \ln\left(\frac{\phi^2}{\mu}\right) - \frac{1}{2} I_{\text{div}} \right] \left( -\frac{5\nu}{8} \alpha^2 + \alpha^4 \right), \quad (51)\]

and

\[ V_{2c}^{F(2)} = -\frac{1}{128\pi^2} q^2 \phi^6 \left[ -2\alpha^3 + 4e^6 \ln(\alpha + e^2) + e^2 \alpha^2 - 8e^4 \alpha \ln(\alpha + e^2) + 4e^2 \alpha^2 \ln(\alpha + e^2) \right. \]

\[-4\alpha e^4] + \frac{q^2}{32\pi^2} \phi^6 \left[ \ln\left(\frac{\phi^2}{\mu}\right) - \frac{1}{2} I_{\text{div}} \right] (e^6 - 2e^4 \alpha + e^2 \alpha^2). \quad (52)\]

Thus the fermion sector contributes to the effective potential with

\[ V^F = X_f \phi^6 + Z_f \phi^6 \ln\left(\frac{\phi}{\sqrt{\mu}}\right), \quad (53)\]

where \( X_f \) is the sum of the coefficients of the terms \( \phi^6 \) in the previous expressions and \( Z_f \) is the sum of the coefficients of the terms \( \phi^6 \ln(\phi/\sqrt{\mu}) \) which results to be

\[ Z_f(q, \alpha, \nu) = \frac{1}{16\pi^2} \left( \frac{5\nu}{8} \alpha^2 - \alpha^4 + q^2 e^6 - 2q^2 e^4 \alpha + q^2 e^2 \alpha^2. \right. \quad (54)\]

The complete effective potential is the sum of (31) and (53) and is given by

\[ V_{\text{reg}} = Z \phi^6 \ln\left(\frac{\phi e^{Y/Z}}{\sqrt{\mu}}\right), \quad (55)\]

where \( Y = Y_b + X_f \) and

\[ Z = Z_b + Z_f = \frac{1}{8\pi^2}[4e^8 - \frac{11}{8} e^4 \nu + \frac{7}{12} \nu^2 + \frac{5}{16} \nu \alpha^2 - \frac{1}{2} \alpha^4 + \frac{1}{2} q^2 e^2 (e^2 - \alpha)^2]. \quad (56)\]

After using the vanishing of the first derivative of \( V(\phi) \) in \( \phi = v \), it can be written as

\[ V_{\text{ren}}(\phi) = Z \phi^6 \ln\left(\frac{\phi}{v}\right) - \frac{1}{6}. \quad (57)\]

As in the pure bosonic model, the positivity of the induced squared mass requires that \( Z > 0 \), and as there, we choose to fix it through the condition (36), what leads to an
equation similar to (37) for $Z$. In the perturbative regime (that is, $\nu, e, q$ and $\alpha << 1$), this equation implies that the second, the third and the fourth terms on the right hand side of (56) are infinitesimals of higher order and can be dropped, leaving the equation

$$\nu = \frac{548}{5} Z \approx \frac{137}{10\pi^2} \left[4e^8 + \frac{1}{2}q^2 e^2 (e^2 - \alpha)^2 - \frac{1}{2}\alpha^4 \right].$$

(58)

Dynamical symmetry breakdown and mass generation occur if the condition $4e^8 + \frac{1}{2}q^2 e^2 (e^2 - \alpha)^2 > \frac{1}{2}\alpha^4$ is satisfied, what is true for a continuum of values of the coupling constants, their magnitudes chosen as $\nu \sim e^8, q^8, \alpha^4 << 1$. Some particular cases are worth being mentioned: 1. choosing $\alpha = e^2$, we still have the solution $\nu \approx \frac{137}{10\pi^2}(\frac{7}{2}e^8)$, but the result becomes independent of the charge of the fermion field. 2. dropping the Yukawa interaction (by making $\alpha = 0$), we get $\nu \approx \frac{137}{10\pi^2}(4e^8 + \frac{q^2 e^6}{2})$, showing that the fermion indirect interaction with the scalar field (mediated by its interaction with the CS field) reinforces the symmetry breakdown. 3. for $e = q = 0$, that is, for the model of a boson in self-interaction, and interacting with a fermion field through an Yukawa term, we see from (58) that no symmetry breakdown is possible; this result is in agreement with the conclusions of [12] in which (besides other possibilities) a similar model with a Dirac two-component fermionic field and a real scalar field was considered. 4. symmetry breakdown does not occur if we only take $e = 0$ leaving the scalar field indirectly interact with the CS through its coupling with the fermion field.

IV. RENORMALIZATION GROUP ANALYSIS

In this section some renormalization group aspects of the previous solution for the effective potential will be discussed. The regularized effective potential satisfies the renormalization group equation

$$\left[ \mu \frac{\partial}{\partial \mu} + \beta_\nu \frac{\partial}{\partial \nu} + \beta_e \frac{\partial}{\partial e} + \beta_q \frac{\partial}{\partial q} + \beta_\alpha \frac{\partial}{\partial \alpha} - \gamma_\phi \frac{\partial}{\partial \phi} \right] V_{\text{reg}}(\mu, \nu, e, q, \alpha, \phi) = 0,$$

(59)

where

$$\beta_\nu = \mu \frac{d \nu}{d \mu}, \quad \beta_e = \mu \frac{d e}{d \mu}, \quad \beta_q = \mu \frac{d q}{d \mu}, \quad \beta_\alpha = \mu \frac{d \alpha}{d \mu},$$

(60)

and

$$\gamma_\phi = \frac{\mu}{2Z_\phi} \frac{d Z_\phi}{d \mu},$$

(61)
are respectively the coupling constants beta functions and the scalar field anomalous dimension, and \( Z_\phi \) is the wave function renormalization constant of the scalar field.

As mentioned before dimensional reduction in 2+1 dimensions automatically removes the divergences of the one-loop graphs. Thus non-trivial renormalization group parameters are obtained only when the two-loop approximation is considered. By applying the operator inside the bracket of (59) to the regularized expression (55), we get the following result

\[
\left(-\frac{Z}{2} + \frac{\beta^{(2)}_\nu}{48} - \frac{6\nu}{48} \gamma^{(2)}_\phi \right)\phi^6 = 0,
\]

where the superscript (2) indicates the loop order of the corresponding function. Thus

\[
\beta^{(2)}_\nu = 24Z + 6\nu\gamma^{(2)}_\phi,
\]

with \( Z \) given by Eq. (56).

To obtain \( \gamma^{(2)}_\phi \) we have to calculate \( Z_\phi \) up to two loops. This can be done by considering the model in its symmetric phase where all fields are massless. The Feynman diagrams which give non-trivial contributions to \( Z_\phi \) are given in Fig. 3. Three of them were computed in ref. [7] and the results are quoted in Appendix C where the calculation of the remaining ones is also summarized. From (C5) we get

\[
\gamma^{(2)}_\phi = -\frac{1}{48\pi^2} [7e^4 + 2(q^2e^2 - \alpha^2)].
\]

By replacing this expression in Eq. (63) we finally have

\[
\beta^{(2)}_\nu = \frac{1}{\pi^2} \left[ \frac{7}{4}\nu^2 + e^4(-5\nu + 12\alpha^4) + \frac{\alpha^2}{2} \left( \frac{17}{8}\nu - 3\alpha^2 \right) + \frac{q^2e^2}{2} \left( (\alpha - e^2)^2 - \nu \right) \right].
\]

For the bosonic model \( (q = 0 = \alpha) \) this yields:

\[
\gamma^{(2)}_\phi = -\frac{7}{48\pi^2}e^4, \quad \beta^{(2)}_\nu = \frac{7}{4\pi^2} \left( \nu^2 - \frac{20}{7}\nu e^4 + \frac{48}{7}e^8 \right),
\]

confirming the results obtained in [5] by using soft BPHZ [4] and also in [7] by using dimensional reduction. These results are also in qualitative agreement with that of [8], but, differently of what happens with the effective potential, disagree with those of [14] (which in our notation are \( \gamma_\phi = 0 \) and \( \beta_\nu = \frac{7}{4\pi^2}\nu^2 \)) where dimensional regularization with minimal subtraction was used. However, as discussed in [14], dimensional regularization with minimal subtraction is not perturbatively consistent for the pure CS model (some of the \( \beta \)'s
functions diverge when the regulating Maxwell term is removed). On general grounds, the authors of [14] argue that $\gamma_\phi$ and $\beta_\nu$ should explicitly depend on $e^2$, a characteristic present in our results but not in the ones obtained by the use of the dimensional regularization with minimal subtraction scheme.

An interesting property of the $\beta_\nu$ function given in (67) is the fact that it does not vanish for $\nu = 0$. From the knowledge that $e$ does not change with the renormalization scale (in [7] we showed that $\beta_e^{(2)} = 0$) we conclude that $\gamma_\phi$ is a constant and so $\phi(\mu) = (\mu_0/\mu)^{\gamma_\phi} \phi(\mu_0)$ and the equation $\mu d\nu/d\mu = \beta_\nu$ can immediately be integrated Resulting in:

$$\frac{2\nu - ce^4}{e^4 b} = \left[ \frac{2e^4 - ce^4}{e^4 b} + \tan(ae^4 \ln(\mu/\mu_0)) \right],$$

where $a = \sqrt{59}/(2\pi^2) \approx 0.39$, $b = 4\sqrt{59}/7 \approx 4.39$ and $c = 20/7 \approx 2.86$.

The effective potential is invariant under renormalization group transformations, i.e. $V(\phi, \nu, e, \mu) = V(\phi_0, \nu_0, e, \mu_0)$. Therefore, as the above solution for $\nu$ is regular at $\nu_0 = 0$ ($\beta_\nu \neq 0$ for $\nu = 0$), then by conveniently choosing $\mu_0$ we get $V(\phi, \nu, e, \mu) = V(\phi_0, 0, e, \mu_0)$. This means that the effective potential in presence of the sextuple self-interaction can be obtained from the simpler model in which the boson only interacts with the CS field.

From the above expressions for $V$ and $\phi$ we also have

$$\frac{dV}{d\phi}(\phi, \nu, e, \mu) = \left( \frac{\mu}{\mu_0} \right)^{\gamma_\phi} \frac{dV}{d\phi_0}(\phi_0, \nu_0, e, \mu_0),$$

so that starting at $\phi_0 = v$ and $\nu_0 \approx \frac{274}{5\pi^2} e^4$, which implies that $dV/d\phi = 0$ for a certain value of $\mu_0$, one can go to the values of $\nu$ specified by Eq. (68) and $\phi(\mu) = (\mu_0/\mu)^{\gamma_\phi} v$ which due to Eq. (69) also corresponds to $dV/d\phi = 0$. This shows that the condition (38) for dynamical symmetry breakdown can be relaxed; the only restriction to get a symmetry breakdown is, in fact, that all the coupling constants be small.

A similar analysis for the complete model, i.e. with the inclusion of the fermion fields, would require a lot more calculations ($\beta_q, \beta_\alpha$, etc) and we do not pursue it here. However, some observations are in order: 1. differently from the other couplings, the Yukawa coupling increases the anomalous dimension of $\phi$ (see (64)). 2. as for the bosonic model, $\beta_\nu$ does not vanish for $\nu = 0$, and as above the effective potential can be get from the simpler model with $\nu = 0$. 3. as in the purely bosonic model, it is also expected that the constraint (38) on $\nu$ can be relaxed, the only restriction being that $\nu << 1$ (as it must be for the others coupling constants).
V. CONCLUSIONS

In this paper we calculated the effective potential, up to two loops, for a (2+1) dimensional model composed of an interacting massless scalar field, a massless fermion field interacting with the scalar field through an Yukawa term, and a CS gauge field, minimally coupled to the scalar and the fermionic fields. As the CW model in (3+1) dimensions, it only involves massless parameters, and is classically invariant under scale transformations, what makes it a possible candidate for dynamical symmetry breakdown. As we verified, dynamical symmetry actually takes place, but differently from the original model of CW, in which this effect already manifests in one-loop corrections, here it only shows up starting in two loops. For particular values of the couplings our effective potential coincides with those found in the literature: by discarding the fermion contribution, it agrees with the previous calculations of [14] for the same model without fermions; if instead we drop the contributions involving the CS field, it agrees with the results of [12] for a model without the gauge field.

We also calculated the renormalization group functions $\beta_\nu$ and $\gamma_\phi$ for the extended model. For the pure bosonic (sub) model they agree with our previous calculation using other regularization/renormalization techniques but disagree with the results of [14]. It would be interesting to compute the other renormalization group functions of the extended model.

VI. ACKNOWLEDGMENTS

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\[\text{[1]}\] The Quantum Hall Effect, Graduate texts in contemporary physics, edited by R. E. Prange and S. M. Girvin (Springer-Verlag, 1990)

\[\text{[2]}\] G. ’t Hooft and M. Veltman, Nucl. Phys. \textbf{B44}, 189 (1972)

G. ’t Hooft, Nucl. Phys. \textbf{B61}, 465 (1973).

\[\text{[3]}\] W. Zimmermman, in Lectures on Elementary Particles and Quantum Field Theory, 1970

Brandeis Summer Institute in Theoretical Physics, edited by S. Deser, M. Grisaru, and H.
Pendleton (MIT Press, Cambridge, 1970)

J. H. Lowenstein, in Renormalization Theory, Proceedings of the International School Ettore Majorana, edited by G. Velo and A. S. Wightman (Reidel, Dordrecht, 1976).

[4] M. Gomes and B. Schroer, Phys. Rev. D 10, 3525 (1974)

J. H. Lowenstein, Commun. Math. Phys. 47, 53 (1976)

J. H. Lowenstein and P. K. Mitter, Ann. Phys. (N.Y.) 105, 138 (1977).

[5] L. C. de Albuquerque, M. Gomes, and A. J. da Silva, Phys. Rev. D 62, 085005 (2000).

[6] G.W. Semenoff, P. Sodano, and Y.S. Wu, Phys. Rev. Lett. 62, 715 (1989)

W. Chen, G.W. Semenoff, and Y.S. Wu, Mod. Phys. Lett. A 5, (1990).

W. Chen, Phys. Lett. B 251, 415 (1990).

W. Chen, G.W. Semenoff, and Y.S. Wu, Phys. Rev. D 46, 5521 (1992).

[7] V. S. Alves, M. Gomes, S. L. V. Pinheiro, and A. J. da Silva, Phys. Rev. D 61, 065003 (2000).

[8] L. V. Avdeev, G. V. Grigoryev, and D. I. Kazakov, Nucl. Phys. B382, 561 (1992).

[9] C. P. Martin, Phys. Lett. B 241, 513 (1990).

G. Giavarini, C. P. Martin, and F. Ruiz Ruiz, Nucl. Phys. B381, 222 (1992).

M. Chaichian and W. F. Chen, Phys. Lett. B 457, 118 (1999).

[10] Alex G. Dias, M.Sc. thesis, Instituto de Física da Universidade de São Paulo, 2002.

[11] S. Coleman and E. Weinberg, Phys. Rev. D 7, 1888 (1973).

[12] D. G. C. McKeon and K. Nguyen, Phys. Rev D 60, 085009 (1999).

F. A. Dilkes, D. G. C. McKeon and K. Nguyen, Phys. Rev. D 57, 1159 (1998).

[13] R. Jackiw, Phys. Rev. D 9, 1686 (1974).

[14] P-N. Tan, B. Tekin, and Y. Hosotani, Nucl. Phys. B502, 483 (1997).

P-N. Tan, B. Tekin, and Y. Hosotani, Phys. Lett. B 388, 611 (1996).

[15] J. Schonfeld, Nucl. Phys. B185, 157 (1991).

S. Deser, R. Jackiw, and S. Templeton, Ann. Phys. 140, 372 (1982).

[16] G. V. Dunne, in Topological Aspects of Low-dimensional Systems, Les Houches, 1998; hep-th/9902115.
APPENDIX A: USEFUL INTEGRALS.

At two loops the following integrals (14) appear ($d^Dp \equiv \mu^3 d^3 p$):

$$I(m_1, m_2, m_3) = \int \frac{d^Dp \ d^Dq}{(2\pi)^{2D}} \frac{1}{((p + q)^2 - m_1^2)(q^2 - m_2^2)(p^2 - m_3^2)}$$

$$= \frac{1}{32\pi^2} \left( \frac{1}{\epsilon} - \gamma + \ln 4\pi + 1 \right) - \frac{1}{16\pi^2} \ln \left[ \frac{m_1 + m_2 + m_3}{\mu} \right], \quad (A1)$$

$$K(m_1, m_2, m_3) = \int \frac{d^Dp \ d^Dq}{(2\pi)^{2D}} \frac{(p \cdot q)^2}{((p + q)^2 - m_1^2)(q^2 - m_2^2)(p^2 - m_3^2)} \quad (A2)$$

$$= \frac{1}{64\pi^2} \left[ m_1^2 (m_2^2 + m_3^2 - m_2 m_3) - m_1 (3m_3^2 + 3m_3^2 + m_2^2 m_3) + m_2 m_3^2 + (m_2^2 + m_3^2) m_2 m_3 + \frac{1}{4} (m_1^2 - m_2^2 - m_3^2)^2 I(m_1, m_2, m_3) \right]$$

and

$$J(m_1, m_2, m_3) = \int \frac{d^Dp \ d^Dq}{(2\pi)^{2D}} \frac{2(p \cdot q)}{((p + q)^2 - m_1^2)(q^2 - m_2^2)(p^2 - m_3^2)} \quad (A3)$$

$$= (m_1^2 - m_2^2 - m_3^2) I(m_1, m_2, m_3) - \frac{1}{16\pi^2} \left[ m_2 m_3^2 - m_1 m_2 - m_1 m_3 \right].$$

APPENDIX B: TWO-LOOP DIAGRAMS

The analytic expressions for the two-loop vacuum diagrams contributing to the effective potential shown in Figs. 1 and 2 are:

$$V^{B(2)}_B = \frac{9\phi^2}{8} \frac{\delta_{ij}}{2} + 2\hat{\phi}_i \hat{\phi}_j \delta_{kl} \int \frac{d^Dp \ d^Dq}{(2\pi)^{2D}}$$

$$\left\{ \Delta_{ij}(p) \Delta_{kl}(q) + \Delta_{ji}(p) \Delta_{lk}(q) + \Delta_{jk}(p) \Delta_{li}(q) \right\} \quad (B1)$$

$$V^{B(2)}_B = -\frac{e^2}{2} \delta_{ij} g_{\mu \nu} \int \frac{d^Dp \ d^Dq}{(2\pi)^{2D}} \Delta_{ij}(p) \Delta_{\mu \nu}(q) \quad (B2)$$
\[ V_{B}^{(2)}(2) = -i\frac{\nu^{2}\phi^{6}}{32} \hat{\phi}_{l}(\delta_{jk} + \frac{2}{3} \hat{\phi}_{j} \hat{\phi}_{k}) \hat{\phi}_{l}(\delta_{mn} + \frac{2}{3} \hat{\phi}_{m} \hat{\phi}_{n}) \times \int \frac{d^{p}p}{(2\pi)^{2D}} \frac{d^{p}q}{(2\pi)^{2D}} \left\{ \Delta_{il}(q)[\Delta_{jm}(p)\Delta_{kn}(p+q) + \Delta_{jn}(p)\Delta_{km}(p+q)] \\
+ \Delta_{im}(q)[\Delta_{jl}(p)\Delta_{kn}(p+q) + \Delta_{jn}(p)\Delta_{kl}(p+q)] \\
+ \Delta_{in}(q)[\Delta_{jm}(p)\Delta_{kl}(p+q) + \Delta_{jl}(p)\Delta_{km}(p+q)] \right\} \] (B3)

\[ V_{B}^{(2)} = i\frac{\nu^{2}e^{2}}{4} \int \frac{d^{p}p}{(2\pi)^{2D}} \frac{d^{p}q}{(2\pi)^{2D}} (2q+p)^{\mu}(2q+p)^{\nu} e^{ij} e^{kl} \Delta_{\nu\mu}(p) \left[ \Delta_{il}(q)\Delta_{jk}(p+q) - \Delta_{ki}(q)\Delta_{jl}(p+q) \right] \] (B4)

\[ V_{B}^{(2)} = -ie^{4}\phi^{2} \int \frac{d^{p}p}{(2\pi)^{2D}} \frac{d^{p}q}{(2\pi)^{2D}} \hat{\phi}_{i} \hat{\phi}_{j} g_{\mu\nu} g_{\sigma\rho} \Delta^{\sigma\mu}(p+q) \Delta^{\nu\rho}(p) \] (B5)

\[ V_{F}^{(2)}(2) = -\frac{\alpha}{2} \int \frac{d^{p}p}{(2\pi)^{2D}} \frac{d^{p}q}{(2\pi)^{2D}} tr[S(p)\Delta_{ii}(q)] \] (B6)

\[ V_{F}^{(2)} = i\frac{(\alpha\phi)^{2}}{2} \hat{\phi}_{i} \hat{\phi}_{j} \int \frac{d^{p}p}{(2\pi)^{2D}} \frac{d^{p}q}{(2\pi)^{2D}} tr[S(p)S(p+q)\Delta_{ij}(q)] \] (B7)

\[ V_{F}^{(2)} = i\frac{g^{2}}{2} \int \frac{d^{p}p}{(2\pi)^{2D}} \frac{d^{p}q}{(2\pi)^{2D}} tr[S(p)\gamma^{\mu}S(p+q)\gamma^{\nu}\Delta_{\mu\nu}(q)] \] (B8)

**APPENDIX C: WAVE FUNCTION RENORMALIZATION OF THE \( \phi \) FIELD UP TO TWO LOOPS**

As known in three dimensions the use of dimensional reduction removes the divergences of one-loop graphs. Therefore the \( \phi \) field renormalization constant only receives nontrivial contributions starting in two loops.

In two loops the nonvanishing contributions to \( Z_{\phi} \) come from the graphs depicted in Fig. 3. The first three graphs, Figs. 3(a–c) were calculated in [7] and here we just quote the result.
\[ Z_{\phi(a-c)}^{(2)} = \frac{7e^4}{48\pi^2} \frac{1}{\epsilon}. \] \hspace{1cm} (C1)

For the remaining graphs, a direct calculation furnishes:

Graph in Fig. 3l = \(-i4e^2\epsilon^{\mu\rho\lambda\sigma} \int \frac{d^Dk}{(2\pi)^D} \frac{k_\mu k_\nu p_\rho p_\sigma \pi_{\mu\beta}(k)}{(k^2)^2(p+k)^2} = -i\frac{e^2q^2p^2}{24\pi^2} \frac{1}{\epsilon} + \text{finite terms}, \) \hspace{1cm} (C2)

where \( \pi_{\mu\beta}, \) given by the upper loop in Fig. 3l, is the fermion contribution to the polarization tensor

\[ \pi_{\mu\beta}(k) = -\frac{q^2}{16}(g_{\mu\beta}k^2 - k_\mu k_\beta) \frac{1}{(k^2)^{2-D/2}} \] \hspace{1cm} (C3)

and

Graph in Fig. 3\( l \) = \( \alpha^2 \int \frac{d^Dk d^Dq}{(2\pi)^{2D}} \text{tr}[S(q)S(p+q+k)] \Delta(k) \)

\[ = -2i\alpha^2 \int \frac{d^Dk d^Dq}{(2\pi)^{2D}} \frac{q^2 + q \cdot (k + p)}{q^2k^2(p+k+q)^2} \]

\[ = i\frac{\alpha^2p^2}{96\pi^2} \frac{1}{\epsilon} + \text{finite terms}. \] \hspace{1cm} (C4)

Thus up to two loops the total wave function renormalization of the \( \phi \) field is

\[ Z_\phi = 1 + \frac{1}{24\pi^2} \left( \frac{7e^4}{2} + e^2q^2 - \frac{\alpha^2}{4} \right) \frac{1}{\epsilon}. \] \hspace{1cm} (C5)
FIG. 1: Diagrams contributing to the two-loop approximation for the effective potential. Continuous and wavy lines represent respectively the $\varphi$ and $A_\mu$ field propagators.

FIG. 2: Additional diagrams contributing to the two-loop approximation to the effective potential when fermions are present. Double lines represents the fermion field propagator.
FIG. 3: Divergent diagrams contributing to the two-loop approximation to the two point function of the scalar field.