Invisible metallic mesh

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A solid material possessing identical electromagnetic properties as air has yet to be found in nature. Such a medium of arbitrary shape would neither reflect nor refract light at any angle of incidence in free space. Here, we introduce nonscattering corrugated metallic wires to construct such a medium. This was accomplished by aligning the dark-state frequencies in multiple scattering channels of a single wire. Analytical solutions, full-wave simulations, and microwave measurement results on 3D printed samples show omnidirectional invisibility in any configuration. This invisible metallic mesh can improve mechanical stability, electrical conduction, and heat dissipation of a system, without disturbing the electromagnetic design. Our approach is simple, robust, and scalable to higher frequencies.

Results

We begin by considering the plane wave scattering by an infinite cylindrical conducting wire with a radius \( r_1 \), as shown in Fig. 1A (when \( r_2 = r_1 \)). When the incident electric field is parallel to the wire, the normalized scattering width \( \sigma_{\text{sc}}/2\pi \) can be derived as \( \sigma_{\text{sc}}/2\pi = (2k_0r_1)\sum_{n=-\infty}^{\infty}J_n(k_0r_1)/|H_n^{(2)}(k_0r_1)|^2 \) by the Mie solution (25) (Derivation of the Scattering Width of the Infinite Corrugated Conducting Wire), where \( J_n \) and \( H_n^{(2)} \) denote a Bessel function and a Hankel function of the second kind, respectively. The normalized scattering width is plotted as the dashed gray line in Fig. 1C. At high frequencies, the scattering width \( \sigma_{\text{sc}} \) approaches a value equal to twice the wire diameter. There is only one resonance occurring at zero (dc) frequency, where the scattering width diverges. This can be explained using the resonance frequency \( \omega_{\text{m}} = 1/(2\pi\sqrt{LC}) \) of inductance \( L \) and capacitance \( C \) of circuit elements. A thin long wire has an effective infinite inductance and capacitance \( (L, C \rightarrow \infty) \) (26, 27), leading to a zero resonance frequency \( \omega_{\text{m}} \rightarrow 0 \).

It is known that a scattering dark state (scattering dip in frequency) universally exists between two resonances (scattering peaks), where the two resonances destructively interfere with equal amplitude but opposite phase in the same scattering and polarization channel (18). We can thus create such a scattering dark state by introducing a second resonance in the wire other than the one at dc. To introduce more resonances, we corrugate the wire by shrinking it periodically along the wire as shown in Fig. 1A. This corrugated wire consists of coaxial cylinders with different radii \( r_1 \) and \( r_2 \) and heights \( d_1 \) and \( d_2 \), where \( d_1 \) and \( d_2 \) are both far smaller than the free-space wavelength. Similar corrugated conducting wires have been proposed for guiding surface plasmon polaritons (28). The open volume can be filled with a low-loss material (dielectric constant \( \varepsilon_r \)) to improve the mechanical strength and increase the working wavelength, so the corrugations are more subwavelength and can be well described by the effective medium theory (29). Each open cylindrical

Significance

We introduce and demonstrate an invisible material—a solid composite possessing identical electromagnetic properties as air so that its arbitrarily shaped object neither reflects nor refracts light at any angle of incidence in free space. Such a material is self-invisible, unlike the cloaks for minimizing the scattering of other items. Invisible materials could provide improved mechanical stability, electrical conduction, and heat dissipation to a system, without disturbing the original electromagnetic design. One immediate application would be toward perfect antenna radomes.

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space inside the wire forms a whispering gallery (WG) resonator; its spectrum and mode profiles are plotted in Fig. S1.

For such an infinite corrugated wire, its normalized scattering width can also be analytically derived, under the effective medium approximation, as (see Derivation of the Scattering Width of the Infinite Corrugated Conducting Wire for detailed derivation)

$$\sigma_{inc}/2r_1 = \frac{2}{k_0r_1} \sum_{n=0}^{\infty} |c_n|^2$$

where $c_n = (J_n(k_{eff}r_1)Y_n(k_{eff}r_2) - Y_n(k_{eff}r_2)J_n(k_{eff}r_1)) - \eta_{eff}(k_{eff}r_1)J_n(k_{eff}r_2)Y_n(k_{eff}r_1) / k_{eff}r_2$ and $\eta_{eff}(k_{eff}r_1)J_n(k_{eff}r_2)Y_n(k_{eff}r_1) / k_{eff}r_2 - J_n(k_{eff}r_2)Y_n(k_{eff}r_1)$ denotes the expanded $n$th-order scattering coefficient in Bessel ($J_n$ and $Y_n$), Hankel functions ($H_n^{(2)}$), and their derivatives. Here $k_{eff} = \omega\sqrt{\mu\varepsilon} = \omega\sqrt{\mu}\sqrt{\varepsilon\mu}$, $\eta_{eff} = \sqrt{\varepsilon_{eff}}/\sqrt{\mu_{eff}} = \sqrt{\mu}/\sqrt{\varepsilon}$, and $\mu_{eff} = \sqrt{\mu_{eff}}/\sqrt{\mu_{eff}} = \mu$ are the effective permittivity and permeability of the corrugated volume ($r_2 \leq r \leq r_1$) derived in Derivation of the Scattering Width of the Infinite Corrugated Conducting Wire. We note that $J_n(k_{eff}r_1)$ approaches zero when $k_{eff}r_1$ is a small number for a thin wire. So, the nodal frequency of $c_0$ has almost no dependence on $\eta_{eff}$, consequently independent of $d_1$ and $d_2$. Although the total scattering width $\sigma_{inc}/2r_1$ is the sum of all of the scattering coefficients $|c_n|^2$, $|c_n|^2$ is negligibly small when $n$ is larger than $k_{eff}r_1$ (0.3 in our case) (30). As shown in Fig. 1F, $|c_n|^2$ is as small as $10^{-8}$. So, the electromagnetic properties can mostly be determined by the first two scattering coefficients $c_0$ and $c_1$.

Shown as a white stripe in Fig. 1B, there will always exist a scattering dark state whose frequency $\omega_1$ lies between those of the dc resonance and the first WG resonance ($\omega_0 \approx 0.29$) of the corrugated wire. In this plot, we vary the ratio of $c_{1\eta}$ while fixing $d_1/d_2 = 1$ and $\varepsilon = 6$. In Fig. 1C, we decompose the total scattering width into individual orders and find that each order has its own zero-scattering frequency. Because the zeroth order is dominant, the dip in the total scattering corresponds to the nodal frequency of the zeroth scattering order $c_0$. The same mechanism enabled previous studies on transparent (cloaking) wires (3–9), invisible particles (10–18), or scattering dark states (31–34).

However, the vanishing of $c_0 = 0$ is not enough to make a collection of these wires invisible; the total scattering amplitude is not small enough when $c_1 \neq 0$. In Fig. 1D, we show the obvious distortion of a scattered wave by a set of these wires packed closely. For a better understanding, we also show the effective constitutive parameters of an array of such wires at the bottom of Fig. 1D, using a homogenization approach (35). At $\omega_1$, $\varepsilon_{eff} = 1$ and $\mu_{eff} = 0.65$. Although the effective permittivity of the material $\varepsilon_{eff}$ is 1 (the same as that of free space), $\mu_{eff} \neq 1$. By solving the Mie scattering solutions for a homogeneous dielectric thin wire (Scattering of a Homogeneous Infinite Dielectric Cylinder with
Radius $r_i$, we show that $c_0 = 0$ requires $\varepsilon = 1$ and $c_1 = 0$ requires $\mu = 1$. Because the nodal frequencies of $c_0$ and $c_1$ in general occur at different frequencies for a single element, $\varepsilon = \mu = 1$ cannot be satisfied simultaneously for an assembly of them. This is why no transparently invisible metamaterial has been reported to date.

Now, we tune the nodal frequency of $c_1$ to coincide with that of $c_0$ for an individual wire (i.e., $c_0 = c_1 = 0$ at the same frequency), which results in a further decrease of the total scattering amplitude by 5 orders of magnitude to a negligible value. Consequently, $\varepsilon_{\text{eff}} \approx \mu_{\text{eff}} \approx 1$ for an arbitrary assembly of such wires. We achieve this by tuning the geometry of the corrugation. We have seen that $c_0$ is almost independent of $d_2/d_1$, whereas the $c_1$ ($i > 0$) have a strong dependence on $d_2/d_1$. For example, the white line (nodal frequency of $c_0$) in Fig. 1E is almost a straight vertical line that does not change with $d_2/d_1$. So, by varying $d_2/d_1$, we can freely tune the nodal frequency of $c_1$ toward that of $c_0$.

Starting with the configuration in Fig. 1B where $r_2/r_1$ is fixed at 0.01, we tune the ratio of $d_2/d_1$ from 1 to $\sim$6.4 in Fig. 1E. The nodal frequencies of $c_0$ and $c_1$ coincide and the total scattering width decreases by 5 orders of magnitude to a record-low scattering width of $3.5 \times 10^{-8}$ (which eventually will be limited by material losses in experiments). At the same time, $\mu_{\text{eff}}$ increases from 0.65 to 1.0006. Consequently, the wave experiences no distortion after impinging on closely arranged wires in Fig. 1G, compared with Fig. 1D. (More results are provided in Fig. S2 for different arrangements of the wires.) This means arbitrary composites of such wires should be practically invisible. We emphasize that such an alignment of nodal frequencies can robustly occur at any frequency by tuning $r_2/r_1$ and $d_2/d_1$ (Fig. S3).

For ease of fabrication, we modify the cylinders in the wire into cubes in Fig. 2A. We connect the cubes with thin square-shaped rods symmetrical in the $x$-, $y$-, and $z$ directions. This makes the original wire structure cubic symmetric, which removes the previous constraint that the field polarization has to be vertical. This conducting skeleton is embedded in a low-loss dielectric. Such a modified construction, still being subwavelength, has no qualitative change in its scattering properties from the corrugated wires studied analytically in Fig. 1. We performed full-wave simulations on this rectangular wire structure using CST Microwave Studio.

The dimensions are $d_1 = 4$ mm, $d_2 = 3$ mm, and $d_3 = 0.6$ mm. The conductor is copper with a conductance of $5.986 \times 10^5$ S/m, and the dielectric is polysulfone (PSU) with dielectric constant of 3 and loss tangent of 0.0013. Shown in Fig. 2C, the invisible frequency occurs at around 10 GHz with a normalized scattering width as low as $5 \times 10^{-5}$. The scattered electric field (difference between fields with and without the wire) is almost all localized inside the wire, consistent with the near-zero scattering width. The opposite phase at different sections along the wire leads to the cancellation of the outgoing waves in the far field. We note that this structure has a low loss at the invisible frequency that is spectrally far away from the resonances.
When the wires are packed into a single 2D plane as in Fig. 2 (Inset), the reflection spectra off the mesh sheet show hardly any dispersion in either the polarization direction or incident angle, as long as the incident electric field is parallel to the sample plane (S-polarized). The reflection is lower than $-45$ dB for normal incidence and remains below $-30$ dB for the incident angle of 80°. The performance is also independent of the polarization angle as shown in Fig. 2C, a result of its in-plane geometry. Again, we show the effective constitutive parameters of this layer of wires in Fig. 2D (Top). At 10 GHz, $e_{\text{eff}} = 0.9999 + 0.006i$ and $\mu_{\text{eff}} = 1$. Accordingly, the real part of its effective refractive index is almost unity and it is nearly independent of the number of layers as shown in Fig. 2D (Bottom). So, we can conclude that arbitrary arrangements of this mesh will be invisible as long as the electric field is parallel to the metallic wire within the beam width. To further illustrate this unique air-like material, we performed full-wave simulations on a network of wires with selected wires to represent the words “invisible material” shown in Fig. 2E. Under an oblique incidence of plane wave at 10 GHz, the steady-state total electric fields in air stay undisturbed, showing a perfect invisibility. Animation of the electric field propagation can be found in Movies S1–S4.

As shown in Fig. 3A, the sample was fabricated by sandwiching the copper-connected cubes between two pieces of PSU covers. The dimensions of the samples are 217 × 252 × 7 mm$^3$ (31 × 36 × 1 in periods). To make the copper structure, we first 3D-print a plastic array of the connected cubes using stereolithography (material: Accura 60). Then, a metal sputtering process was used to coat the surfaces of the plastic array with 50 μm of copper film that is well above the skin depth (0.64 μm) at 10 GHz. The two PSU
cover layers were machined with grooves and square openings so the copper structure could be embedded tightly inside.

In the measurements, three sets of sample configurations were studied. In the first configuration (Fig. 3B), one layer of the assembled slab was placed on a rotation stage spinning around a small monopole antenna with a separation distance of d (20 mm) from the slab, shown in Fig. 3 B, I. We choose this subwavelength separation to observe a strong redistribution of fields due to the sample. A wideband signal was fed into the monopole, and a wideband receiving lens antenna was placed at the other side to detect the far-field radiation patterns by measuring transmission amplitudes $S_{21}$ ($S$ parameter) using an Agilent E8361A network analyzer. All transmission amplitudes are normalized by the reference transmission signal when the sample is removed. For comparison, a reference measurement was performed by replacing the sample with a PSU slab of the same size. Shown in Fig. 3 B, II, the radiation pattern of the PSU sample is strongly directional for all frequencies. But, for the designed sample shown in Fig. 3 B, III, there exists a frequency range around 10.4 GHz where the radiation pattern is almost a circle within a 7.2% relative bandwidth where the scattering amplitudes are less than $\pm 1$ dB. For a direct comparison in Fig. 3 B, IV, we plot the transmission amplitudes in an angular polar coordinates for both the sample and the reference at 10.4 GHz, validating the omnidirectional invisibility of the sample. Although fabrication imperfections inevitably degrade the performance and shift the operating frequency from 10 to 10.4 GHz, the measurement results agree with our analytic and numerical results.

In the second configuration (Fig. 3C), two slab samples were stacked together as a thicker one. Equivalent sets of measurements were performed as those for the first configuration. In the third configuration (Fig. 3D), the two slab samples were separated on the two sides of the source antenna. In both configurations, similar results were obtained as those of the first configuration. The above results confirm that the fabricated sample is omnidirectionally invisible regardless of its geometry.

**Discussion**

In conclusion, we have demonstrated the ability to construct invisible microwave materials out of corrugated wires with record-low scattering width. Our analytical analyses, numerical simulations, and experimental measurements are all consistent. We hope this work will inspire new technological applications, one important example being the construction of perfect antenna radomes. Objects can also be cloaked inside the metallic cubes. The proposed approach is simple, robust, and scalable to higher frequencies using low-loss metals. Based on the general ability to control the frequency dispersions by multiple resonant structures (36), it should be possible to design wider-bandwidth materials invisible to both polarizations using our approach.

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1. Pendry JB, Schurig D, Smith DR (2006) Controlling electromagnetic fields. Science 312(5781):1780–1782.
2. Chen H, et al. (2013) Ray-optics cloaking devices for large objects in incoherent natural light. Nat Commun 4:2652.
3. Irci E, Ertürk VB (2007) Achieving transparency and maximizing scattering with metamaterial-coated conducting cylinders. Phys Rev E Stat Nonlin Soft Matter Phys 76(5 Pt 2):056603.
4. Edwards B, Aliu A, Silveirinha MG, Engheta N (2009) Experimental verification of plasmonic cloaking at microwave frequencies with metamaterials. Phys Rev Lett 103(15):153901.
5. Tretiakov S, Altalito P, Luukkonen O, Simovski C (2009) Broadband electromagnetic cloaking of long cylindrical objects. Phys Rev Lett 103(10):103905.
6. Valanginopoulos CA, Altalito P (2012) Electromagnetic cloaking of cylindrical objects by multilayer or uniform dielectric claddings. Phys Rev B 85(11):115402.
7. Valanginopoulos CA, Altalito P, Tretiakov SA (2014) On the minimal scattering response of PEC cylinders in a dielectric. IEEE Antenn Wirel Pr 13:403–406.
8. Rybin MV, Filonov DS, Belov PA, Kivshar YS, Limonov MF (2015) Switching from invisibility to maximized scattering using subwavelength metallic obstacles and an external mirror. Opt Express 23(16):20391–20397.
9. Aliu A, Engheta N (2005) Achieving transparency with plasmonic and metamaterial coatings. Phys Rev E Stat Nonlin Soft Matter Phys 72(1 Pt 1):016623.
10. Kerker M (1975) Invisible bodies. J Opt Soc Am 65(4):376–379.
11. Chev H, Kerker M (1976) Abnormally low electromagnetic scattering cross-sections. J Opt Soc Am 66(5):445–449.
12. Aliu A, Engheta N (2007) Cloaking and transparency for collections of particles with metamaterial and plasmonic cover layers. Opt Express 15(12):7578–7590.
13. Aliu A, Engheta N (2008) Multifrequency optical invisibility cloak with layered plasmonic shells. Phys Rev Lett 100(11):113901.
14. Evangelou S, Yannopoulos V, Paspalakis E (2012) Transparency and slow light in a four-level quantum system near a plasmonic nanostructure. Phys Rev A 86(5):053811.
15. Aliu A, Engheta N (2007) Plasmonic materials in transparency and cloaking problems: Mechanism, robustness, and physical insights. Opt Express 15(6):3318–3332.
16. Aliu A, Engheta N (2008) Effects of size and frequency dispersion in plasmonic cloaking. Phys Rev E Stat Nonlin Soft Matter Phys 78(4 Pt 2):045602.
17. Hsu CW, Delacy BG, Johnson SG, Joannopoulos JD, Soljačić M (2014) Theoretical criteria for scattering dark states in nanostructured materials. Nano Lett 14(5):2783–2788.
18. Brown J (1953) Artificial dielectrics having refractive indices less than unity. Proc IEEE 100(5):151–154.
19. Landy NI, Sajjyjige S, Mock JJ, Smith DR, Padilla WJ (2008) Perfect metamaterial absorber. Phys Rev Lett 100(20):207402.
20. Ye D, et al. (2012) Towards experimental perfectly-matched layers with ultra-thin metamaterial surfaces. IEEE Trans Antenn Propag 60(11):5164–5172.
21. Pfeiffer C, Grbic A (2013) Metamaterial-Huygens’ surfaces: tailoring wave fronts with reflectionless sheets. Phys Rev Lett 110(19):197401.
22. Decker M, et al. (2015) High-efficiency dielectric Huygens’ surfaces. Adv Opt Mater 3(6):813–820.
23. Stade I, et al. (2013) Tailoring directional scattering through magnetic and electric resonances in subwavelength silicon nanodisks. ACS Nano 7(9):7824–7832.
24. Kong JA (2000) Electromagnetic Wave Theory (EMW Publishing, Cambridge, MA).
25. Maxwell JC (1878) On the electrical capacity of a long narrow cylinder and of a disk of any size. Proc Lond Math Soc 1:249–265.
26. Maxwell JC (1878) On the capacity of a long narrow cylinder and of a disk of any size. Proc Lond Math Soc 1:249–265.
27. Pendry JB, Holden AJ, Stewart WJ, Youngs I (1996) Extremely low frequency plasmons in metallic mesostructures. Phys Rev Lett 76(25):4773–4776.
28. Maier SA, Andrews SR, Martin-Moreno L, Garcia-Vidal FJ (2006) Terahertz surface plasmon-polariton propagation and focusing on periodically corrugated metal wires. Phys Rev Lett 97(17):177405.
29. Bohren DR, Schultz S, Marks P, Soukoulis CM (2002) Determination of effective permittivity and permeability of metamaterials from reflection and transmission coefficients. Phys Rev B 65(19):195104–195108.
30. Bohren C, Huffman DR (1983) Absorption and Scattering of Light by Small Particles (Wiley, New York).
31. Wu X, Gray SK, Pelton M (2010) Quantum-dot-induced transparency in a nanoscale plasmonic resonator. Opt Express 18(23):23633–23645.
32. Zengin G, et al. (2013) Approaching the strong coupling limit in single plasmonic nanorods interacting with J-aggregates. Sci Rep 3:3074.
33. Forestiere C, Dal Negro L, Miano G (2013) Theory of coupled plasmon modes and Fano-like resonances in subwavelength metal structures. Phys Rev B 88(15):155411.
34. Giannini V, Francesco V, Amrania H, Phillips CC, Maier SA (2011) Fano resonances in nanoscale plasmonic systems: A parameter-free modeling approach. Nano Lett 11(7):2835–2840.
35. Chen X, Grzegorczyk TM, Wu BL, Pacheo J, Jr, Kong JA (2004) Robust method to retrieve the constitutive effective parameters of metamaterials. Phys Rev E Stat Nonlin Soft Matter Phys 70(1 Pt 2):016608.
36. Ye D, et al. (2013) Ultrawideband dispersion control of a metamaterial surface for perfectly-matched-layer-like absorption. Phys Rev Lett 111(18):187402.
37. Pendry JB (1999) The transmission characteristics of a corrugated guide. IEEE Trans Antenn Propag 7(5):183–190.
38. Wait JR (1955) Scattering of a plane wave from a circular dielectric cylinder at oblique incidence. Can J Phys 33(5):189–195.