Optical-Mechanical System for Stabilizing an Inverted Pendulum

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Abstract. Controlling open-loop unstable systems is a common benchmark for designing the algorithms to maintain the equilibrium state of anthropomorphous technical devices used within control theory. In this connection, considerable attention is currently being focused on the problem of stabilizing the inverted pendulum system. In this work, the execution of swinging-up the pendulum and, subsequently, maintaining its upward equilibrium state is presented with the help of the laboratory bench TP-802 by Festo Didactics and the movement control device. The configuration of dynamic system for stabilizing the inverted pendulum is offered. The algorithms to swing-up the pendulum and balance it around its upright position are offered as well.

1. Introduction

The problem of controlling the representatives of a pendulum type objects is fundamental for a range of scientific faculties. Its solution is reflected in the investigation of control techniques [1-3] used while modeling flying machines, rockets, anthropomorphous robots, segways etc [4].

A physical pendulum is one of the most popular physical models which consists of an oscillating massive object (pendulum bob) hung by an extensible weightless string or a thin vertical rod.

A special case of this type of systems is an inverted pendulum – an unstable physical object [5] with two equilibrium positions: in the highest and the lowest points, wherein an arbitrary small disturbance of any kind will accelerate it from the highest point down to the lowest point. Different kinds of feedback devices providing with pendulum movement control [6] may be added to the system so that the pendulum is stabilized at its upward equilibrium state.

The scientific works [7-11] are addressed to stabilizing the inverted pendulum.

A linear model of the system motion is described by two differential equations [12, 13], which represent the dependence of the vertical pendulum angle on force applied to the cart and the horizontal displacement of the cart.

\[
\dot{\theta} = \frac{1}{(I+mI^2)} \left( mglx \sin \theta - \dot{x} ml \cos \theta - c \dot{x} \right), \\
\dot{x} = \frac{1}{M+m} \left( F - \dot{\theta} m l \cos \theta + \dot{\theta}^2 m l x \sin \theta - k \dot{x} \right),
\]

considering \( m \) is a mass of the pendulum, \( M \) is a mass of the cart, \( l \) is a length of the pendulum rod hinged on the cart, \( \theta \) is a vertical pendulum angle, \( F \) is a horizontal force exerted on the cart, \( x \) is a
displacement of the cart, \( I \) is a second moment of inertia, \( k \) is a viscous friction coefficient of the cart, \( c \) is a viscous friction coefficient of the pendulum.

2. Elaboration of balancing the inverted pendulum system
The task assigned in this work was to simulate and to implement the process of swinging-up the pendulum succeeded by stabilizing it in this position with the use of the laboratory bench TP-802 by Festo Didactics as a setup adjuster of the upward equilibrium state of the pendulum along with other stabilizing components.

Therefore, a physical control object is a pendulum inverted after a certain number of electromechanical drive carrier movements, operated by a step motor MTR-ST with PC-master by way of applying coordinate positioning controller SPC-200 [14].

The stabilization system startup follows swinging-up the pendulum. In order to complete this task, considering [15, 16], the algorithm represented in Fig.1 and the application program for positioning the cart were elaborated. The following assumptions have been made: (1) a cart moves along the X-axis between the points \( X = 0 \) mm and \( X = 300 \) mm; (2) the reference position of the cart is \( X = 150 \) mm; (3) \( N \) is a displacement value of the cart from the reference point; (4) \( K \) is a preset displacement value increment.

The code of the program for swinging-up the pendulum elaborated in software Festo WinPisa 4.41 [17] is presented below.

```plaintext
N000 #LR0 = 0
N001 G01 X150 FX10
N002 G00 G91 X-25
N003 G00 G91 X+50
N004 G00 G91 X-75
N005 G00 G91 X+100
N006 G00 G91 X-125
N007 G00 G91 X+150
N008 G00 G91 X-175
N009 G00 G91 X+200
N010 G00 G91 X-250
N011 G00 G91 X+30
N012 G00 G91 X-30
N013 M02
```
The pendulum is in the upward point?

Start

$N = 25$

Negative travel direction set

Reference travel of the cart

$N$ mm displacement of the cart in the preset direction of motion

$N = N + K$

Reverse the travel direction of the cart

End

Figure 1. Algorithm of the swing-up procedure
After the program run the cart reference travel is executed. During running the strings 3-11 (in the code presented as N002...N010) of the program divergent oscillations appear, succeeded by two movements of the cart after which the pendulum is stabilized in the upright equilibrium position for a short period of time.

Immediately after the pendulum achieves the highest point the elaborated stabilization system starts controlling the cart with the pendulum hinged on the top of it. One of the key components of this system which structure is presented at the Fig.2 is an optical movement control device described in the paper [18].

![Figure 2. System structure](image)

The cart 3 moves along the X-axis placed on a stable basement 1. A pendulum 7 consists of an oscillating massive object 8, hung by a thin vertical rod. At the top of the pendulum, there is a light emitter 9. A cart is rigidly bounded to the step motor 4 through the linear electromechanical drive 2. The step motor 4 is operated by positioning controller 6 with the help of a step motor controller 5. Computer 14 controls the light emitter 9 which include current translator mechanism [19]. The signals of the light sensors 10, 11, 12 are the input of the multiplexer switch 13 also controlled by the computer 14. The outputs of the light emitter and the multiplexer switch are connected to the computer 14. Moreover, the central light sensor 10 forms an output signal only in case the inverted pendulum is in vertical state.

The system works as follows: the pendulum 7 is swung-up into the upward unstable equilibrium state after a certain amount of moves of the cart operated by a step motor 5. Furthermore, the travel distance of the cart is 300 mm. The light emitter 9 attached to the pendulum bob 8 is switched on during the swinging-up. After a signal corresponding to the upward equilibrium state is produced (when the pendulum 7 is inverted) it flows through the multiplexer switch 13 to the computer 14 where it is finally captured.
There is no possibility the pendulum keeps the upward equilibrium state as there are physical forces applied to it. When the pendulum tilts with some angle, another light sensor captures the light emitter signal because of the displacement of the pendulum. The coordinates of the pendulum (the angular displacement of the vertical state) and the displacement direction are determined according to the index number of light sensor, which detected the light emitter signal the first. The number of light sensors and distance between each of them depends on the accuracy of measurements demanded. Measurement data flows to the computer where it is processed according to the program being executed. Consequently, a control input for positioning controller is formed, and a cart moves certain number of steps, which depends on the angular displacement of the pendulum.

Thus, an elaborated system stabilizes the upward equilibrium state of the inverted pendulum.

The algorithm of the system execution is presented at Fig.3. After the system startup an initialization of its components is carried out. The initialization includes microprocessor adjustment, positioning controller and step motor controller startup, forming switch-on emitter signal. Then the pendulum is inverted and the moment when the pendulum achieves the upward position is captured. The algorithm of this procedure is presented at Fig.1. After that, the direction and the amount of displacement from the vertical state are determined. Considering these, the control input is formed. The cart moves in the direction of the pendulum slope and the pendulum is eventually stabilized. The system is checked for the angular displacement presence again, and so on.

![Figure 3. Algorithm of system execution](image-url)
3. Conclusion
Thus, in this paper the application program of swinging-up the pendulum is elaborated, the algorithm of stabilization the pendulum in its upper equilibrium position is offered, the structure of optical-mechanical system for stabilizing the inverted pendulum consisting of computer with multiplexer switch, laboratory bench TP-802 by Festo Didactics and an optical movement control device is offered as well.

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