Comment on “A proposed method for measuring the electric dipole moment of the neutron using acceleration in an electric field gradient and ultracold neutron interferometry”, II

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(December 14, 2017)

Abstract

We discuss the proposal of Freedman, Ringo and Dombeck [1] to search for the neutron electric dipole moment by use of the acceleration of ultracold neutrons in an inhomogeneous electric field followed by amplification of the resulting displacement by several methods involving spin independent interactions (gravity) or reflection from curved (spin independent) mirrors. We show that the proposed technique is inferior to the usual methods based on magnetic resonance.

PACS: 21.20.Ky, 14.20.-c, 03.75.Dg

I. INTRODUCTION

Since the appearance of our original paper [2] there has been some additional discussion [3] and [4], so that it now seems reasonable to publish a revised version with a more detailed exposition of our quantum mechanical treatment (sec. II B.)

Searches for a neutron (or elementary particle) electric dipole moment (edm) are interesting because an observation of an edm would be a demonstration of the violation of time reversal (T) invariance outside the $K^0$ system. Freedman et al. [1] have proposed a new method to search for the neutron edm. The method is claimed to offer the possibility of vastly improved sensitivity due to the amplification of the effects of the interaction of an edm with an electric field by means of subsequent motion in a gravitational field or reflection from a convex mirror. In the present paper we will review this proposal and show that the claimed gain in sensitivity is based on a misunderstanding of a semi-classical model of the processes involved.

According to the proposal, Ultra-cold neutrons (UCN) polarized along the $x$ axis enter a chamber (accelerator) where the electric field directed along $z$ has a gradient in the $x$ direction. Since the incident spin state which is an eigenstate of $\sigma_x$ can be considered as a coherent superposition of the two eigenstates $|\pm\rangle_z$ of $\sigma_z$, we can suppose that neutrons possessing a non-zero edm in one of the eigenstates will suffer an acceleration

$$a_x = \frac{\mu_e}{m} \left( \frac{\partial E_z}{\partial x} \right).$$

(1)
After time $T_1$ the two spin states will be separated by a distance

$$\delta x = \frac{\mu_e}{m} \left( \frac{\partial E_z}{\partial x} \right) T_1^2$$  \hspace{1cm} (2)

After the period of acceleration, the neutrons are allowed to leave the chamber in a vertical direction (+z) rising against gravity. After reflection from a surface inclined at 45°, the horizontal separation $\delta x$ is converted into a vertical separation $\delta x = \delta z$. The neutrons then follow parabolic trajectories under the influence of gravity. Because of the difference in initial heights, $\delta z$, hence a difference in kinetic energy, the two spins will accumulate a phase difference along the two trajectories

$$\Delta \phi = \frac{1}{\hbar} \int \Delta V \, dt = mg\Delta z T_2$$  \hspace{1cm} (3)

where $T_2$ is the time of flight along the parabolic trajectory(ies). This is called a ‘gravitational amplifier’ by the authors [1]. It is then proposed to measure this amplified phase difference between the states $|\pm\rangle_z$ as a precession of the polarization vector in the $x, y$ plane:

$$\Delta \phi = gT_2\mu_e \frac{\partial E_z}{\partial x} T_1^2$$  \hspace{1cm} (4)

Putting in practical values for the parameters the authors expect a sensitivity to the edm of $10^{-28} \text{ e-cm}$, which is superior to that expected for other methods [3]. It is seen that the proposed effect depends crucially on the description of the spin 1/2 system where the phase difference is calculated along trajectories ending at different points while the polarization is calculated by assuming the two states combine coherently at a single point.

The authors also propose a second type of amplifier based on repeated reflections from a curved surface, given by $Z(x)$. Then the two ‘points’ representing the two spin states separated by $\delta x$ due to the edm acceleration [1] will reflect from portions of the surface with slightly different slopes, differing by:

$$\alpha (x) = \frac{\partial^2 Z(x)}{\partial x^2} \delta x$$  \hspace{1cm} (5)

and, after reflection the angle between the trajectories will increase by $2\alpha (x)$. Again, the increased separation is supposed to result in an amplification of the phase difference and hence of the precession in the $x, y$ plane. This has the same feature as the “gravitational amplifier”: trajectories ending at increasingly distant points are used to calculate a phase difference which is then thought to be measured as a precession of the polarization which is calculated by considering that the two states combine coherently at a single point.

II. DISCUSSION OF THE PROPOSAL ACCORDING TO DIFFERENT MODELS

As the present proposal is based on a mixing of models we will present a discussion of the proposal from several view points. It is important that the models be well defined and applied in a consistent manner. Mixing different models leads to errors and confusion.

A detailed discussion of the relation between classical, semi-classical and quantum mechanical descriptions of similar situations has been given in [3] with a more complete quantum mechanical discussion in [7]. We will review these ideas with emphasis on the application to the present model.
A. Classical Model

As an edm interacting with an electric field behaves identically to a magnetic moment interacting with a magnetic field we choose to discuss the problem in terms of magnetic moments in a constant magnetic field as this is probably more familiar. We will see later that the introduction of field gradients in the author’s proposal is neither essential nor desirable.

A classical magnetic moment $\vec{\mu}$ coupled to an angular moment $\vec{j}$, $\vec{\mu} = \gamma \vec{j}$ in an external field $\vec{B}$, (of course the same discussion will apply to an edm in an electric field) obeys the equation of motion

$$\frac{d \vec{j}}{dt} = \vec{\mu} \times \vec{B} \quad (6)$$

whose solution is seen to be a precession of $\vec{j}$ around $\vec{B}$ with the Larmor frequency $\omega_L = \gamma B$ independent of the angle between $\vec{j}$ and $\vec{B}$ and which is a constant of the motion. If the field exists in a region of length $L$, then the particle will cross the field in a time $t = L/v$ during which time the spin will precess through an angle

$$\varphi_L = \omega_L t = \omega_L L/v = \gamma B L/v \quad (7)$$

The spin components, $\sigma_{x,y}$, averaged over the velocity spectrum $f(v)$ of the beam, will then be given by

$$\langle \sigma_x \rangle = \int \frac{dv f(v)}{v} \cos \left( \frac{\omega_L L}{v} \right)$$

$$\langle \sigma_y \rangle = \int \frac{dv f(v)}{v} \sin \left( \frac{\omega_L L}{v} \right)$$

This is the basis of a technique that is often used to measure velocity distributions.

Note that as $\omega_L L$ increases enough the polarization $\langle |\sigma| \rangle$ is expected to decrease. In a classical model the separation of states in space does not appear. As the gravitational interaction is spin-independent there is no gravitational amplification in this model. Moments entering the field directed perpendicular to it undergo no energy change since $\vec{\mu} \cdot \vec{B} = 0$.

The classical model is expected to give accurate results so long as the separation between trajectories associated with different states is small compared to the correlation (coherence) lengths of the wave function. Thus the classical model breaks down in the Stern-Gerlach effect but gives a very good description of Larmor precession. Therefore one could conclude on this basis that the proposed amplification does not exist but we are aware that this argument would not be seen as compelling.

B. Quantum mechanical treatment

Since the semi-classical model is in some ways the most difficult, primarily because it is prone to misinterpretation, we will first discuss the quantum mechanical model. Note that
to our knowledge no quantum mechanical demonstration of the alleged amplification effect has been presented, [1], [3], [4].

We start with some general, elementary remarks. A wave function $\psi(x)$ and its Fourier transform $\Psi(k)$ are respectively, the probability amplitudes for finding the particle in a given region of position or momentum space. Then a displacement of the particle is equivalent to a phase shift (linear in momentum) of the momentum wave function:

$$\psi(x + \vec{\delta}x) = \int d^3k e^{i\vec{k} \cdot \vec{x}} \Psi(k) e^{-i\vec{k} \cdot \vec{x}}$$

(9)

A more general phase shift

$$\Psi(k) = \Psi(k_0) \left| e^{i\varphi(k)} \right|$$

(10)

can be considered as leading to a displacement, $\vec{\delta}x$, such that

$$\varphi(k) = \varphi(k_0) + \vec{\kappa} \cdot \vec{\nabla}_k \varphi(k) = \varphi(k_0) + \vec{\kappa} \cdot \vec{\delta}x$$

(11)

only under the condition that $\Psi(k)$ is narrow enough that higher order terms in the expansion of $\varphi(k)$ can be neglected. Also $\varphi(k_0)$ should not play an important role. In the case of neutron spin echo [6] (or an edm accelerator with constant electric field) where $\varphi \propto 1/k$ the condition for this is $(\kappa/k_0 \ll 1)$ where $\kappa$ is the width of the wave in momentum space centered around $k_0$. In the spin echo case we also have $\delta x \gg 1/\kappa$ so that the concept of a displacement between the states has physical meaning and, as will be seen below, no polarization can be observed under these conditions. It is necessary to cancel the phase shift (this is called obtaining the echo), and thus eliminate the displacement $\delta$, in order to observe the polarization.

The reason for belaboring these rather self-evident points is that in the work in question there is a tendency to make a distinction between the displacement and the precession methods of searching for an edm whereas we have seen that the two concepts are only different ways of looking at the same phenomenon. There is only one phase for each quantum state and attempts to separate this phase into various components can often lead to confusion. This applies to the present case where a term linear in $k$ is singled out as a ‘displacement’ or in attempts at separating the phase into ‘geometric’ and ‘dynamic’ parts. This latter separation is strongly dependent on the coordinate system used for the calculation - only the total phase remains unchanged [3].

If we consider a wave function $\Psi(k)$ for a state where the spins are initially polarized in the $x$ direction

$$|\Psi(t = 0)\rangle = \Psi(k) \begin{pmatrix} 1 \\ 1 \end{pmatrix}$$

(12)

and, as a result of a spin dependent interaction, the spin states pick up an additional phase $\varphi_\pm$ so that at some time $t$ we have

$$|\Psi(k, t)\rangle = \Psi(k) \begin{pmatrix} e^{i\varphi_+^k(t)} \\ e^{i\varphi_-^k(t)} \end{pmatrix}$$

(13)
then the expectation value of $\mathcal{F}$ will be

$$
\langle \sigma_x + i\sigma_y \rangle = \langle \Psi (t) | \sigma_x + i\sigma_y | \Psi (t) \rangle = \int |\Psi \left( \frac{\mathbf{r}}{k} \right) |^2 e^{i(\varphi_k^* - \varphi_k)} d^3k
$$

(14)

In the case where the spin dependent interaction, $V_\pm = \pm V_o$, is small compared to the kinetic energy of the beam particles (this is certainly the case for any interaction involving a particle edm and holds for the spin echo case as well [8]) and slowly varying we can use the WKB approximation to write

$$
\varphi_\pm = \int dx \sqrt{\frac{k^2 - 2mV_\pm}{\hbar^2}} \approx kx \mp \int dx \frac{mV_o (x)}{\hbar^2 k}
$$

(15)

For $V_o (x) = \text{const} = \mu B$ or $\mu_e E$ we can write, taking $\alpha = \omega L m/2\hbar$, with $\omega_L = 2\mu B/\hbar$ or $2\mu_e E/\hbar$

$$
\varphi^\pm_k = \pm \frac{\alpha}{k} = \pm (\varphi_o - \kappa \delta x)
$$

(16)

where we have expanded around $k_o$, $(k = k_o + \kappa)$ for a narrow spectrum centered on $k_o$ using $\delta x = \alpha/k_o^2$; see (11), $(\varphi_o = \alpha/k_o)$. The case where $V_o (x)$ has a constant gradient, $V_o = \mu_e x (\partial \mathbf{E} / \partial x)$ is seen not to introduce any significant differences into (15). Then

$$
\langle \sigma_x + i\sigma_y \rangle = \langle \psi^* (x - \delta x, t) \psi (x + \delta x, t) e^{i2\varphi_o} \rangle = \langle \psi^* (x, t) \psi_- (x, t) \rangle \equiv I(t)
$$

(17)

(18)

where $\psi (x)$ is the Fourier transform of $\Psi (k)$ and equ. (17) is the result of the Wiener-Khintchin theorem applied to (13). We see that $\langle \sigma_x + i\sigma_y \rangle$ in equ. (14) as a function of $\varphi$ represents the Fourier transform of the beam velocity spectrum in agreement with the classical calculation, equ. (8). From equ. (17) it follows that this is a decreasing function of $\varphi_k = \varphi^+_k - \varphi^-_k$ or $\delta x$ for large enough $\varphi_k$ so that increasing $\varphi_k$ or $\delta x$ far enough will result in a reduction of the net polarization. As $\delta x$ increases so much that the correlation function in (17) approaches zero we approach the case of the Stern-Gerlach effect where the spin states can be considered as truly separated and one can talk about a displacement without any confusion. Thus we have seen that the discussion can be carried out equally either in terms of $\varphi_k$ or $\delta x$, they both represent the same physical situation.

The main thrust of the proposal under discussion [1] is that after a period in the electric field region the $x$ velocity of the UCN is changed into the $z$ direction by reflection from a mirror at 45$^\circ$. Subsequent motion in the gravitational field is supposed to significantly increase the phase difference $\varphi$, according to the (incorrect) semi-classical argument presented in [1] and outlined in the introduction.

The key point is that no spin-independent interaction can influence the phase shift between the two spin states. At the entrance to the gravitational field drift region (called the "amplifier" by the authors) the wave function will be of the form given by (13) and (16); taking the Fourier transform

$$
|\psi \left( \frac{\mathbf{x}}{\kappa}, t \right) |^2 = \begin{pmatrix}
    e^{i\varphi_o} \psi_o \left( \frac{\mathbf{x}}{\kappa} + \frac{\delta \mathbf{x}}{\kappa}, t \right) \\
    e^{-i\varphi_o} \psi_o \left( \frac{\mathbf{x}}{\kappa} - \frac{\delta \mathbf{x}}{\kappa}, t \right)
\end{pmatrix} = \begin{pmatrix}
    \psi^+_o \left( \frac{\mathbf{x}}{\kappa}, t \right) \\
    \psi^-_o \left( \frac{\mathbf{x}}{\kappa}, t \right)
\end{pmatrix}
$$

(19)
for a narrow spectrum. The superscript $^o$ refers to the incoming beam at the entrance orifice of the amplifier region.

We now calculate the wave function at the exit of this region. It is sufficient to consider only the motion in the $z$ direction and confine ourselves to the steady state situation. Then using the WKB method for the case of the slowly varying gravitational potential, which is not necessarily small compared to the particle kinetic energy, we can write the wave function at the output (in the case of zero electric field in the accelerator)

$$
\psi(z) = \sum_k A(k) e^{i \int_0^z \sqrt{(k^2 + K_z^2)} dz} = \sum_k A(k) e^{i \frac{\hbar^2}{3m^2g} (k^2 + K_z^2)^{3/2}} e^{-i \frac{\hbar^2}{3m^2g} k^3} \tag{20}
$$

where $K_z^2 \equiv 2m^2 g z / \hbar^2 = m^2 v_z^2 / \hbar^2$. $v_z$ is the velocity of a particle that falls a distance $z$, starting from rest. In order to investigate the relation between phase shifts and displacements we wish to study the behavior of $\psi$ in a small region located around $z = Z$, so we set $z = Z + \epsilon$, $\epsilon \ll Z$ and expand the exponential in (20) around $Z$.

$$
\psi(z = Z + \epsilon) = \sum_k A(k) \exp i \left[ \frac{\hbar^2 k_o}{3m^2g} \left( k^2 + K_Z^2 \right)^{3/2} + \left( k^2 + K_Z^2 \right)^{1/2} \epsilon \right] e^{-i \frac{\hbar^2}{3m^2g} k^3} \tag{21}
$$

Since $A(k)$ represents a fairly narrow wave packet centered around $k_o$ we can write $k = k_o + \kappa$, $\kappa \ll k_o \ll K_Z$ (the latter condition is necessary for the 'amplification' to be significant, see below) and then we obtain from (21), keeping only the terms in $\kappa$ and $\epsilon$

$$
\psi(z = Z + \epsilon) \approx \sum_\kappa A(k_o + \kappa) \exp i \left[ \frac{\hbar^2 k_o}{m^2g} (K_Z - k_o) \kappa + K_Z \epsilon + \frac{k_o^2}{2K_Z} \epsilon + \frac{k_o \epsilon}{K_Z} \right] \tag{22}
$$

The terms in (22) can be interpreted as follows: The first term represents a displacement in position by $v_o t_Z$, the distance the particle with the initial velocity, $v_o$, would go in the time $t_Z$ that the particle takes to fall to $Z$ and an additional displacement $E_o/mg$ associated with the boundary condition at $z = 0$. The next two terms represent the change in wavelength due to the acceleration during the fall. Since $\sum_{\kappa} A(k_o + \kappa) e^{i \epsilon \kappa}$ represents the envelope of the wave function at $z = 0$, the last term in (22) shows that the envelope is spread by the factor $\eta = K_Z/k_o \gg 1$ due to the energy dependence of the index of refraction (dispersion) for the Schrödinger wave.

When the electric field is switched on we have (using equ. (16).)

$$
\psi_{\pm}(z = Z + \epsilon) \approx \sum_\kappa A(k_o + \kappa) e^{i \frac{\hbar}{k_o K_Z} \epsilon} e^{i \kappa^2 \epsilon} \left[ e^{i (\phi_o - \kappa \delta z)} \right] \tag{23}
$$

displaying only the relevant terms. Now if we calculate the center of the wave packet, $\epsilon_o$, according to $\partial \phi / \partial \kappa |_{\epsilon_o} = 0$, we find the center of the packet is indeed shifted by

$$
\delta \epsilon = \eta \delta x \tag{24}
$$

in agreement with the ideas of ref. [1]. The wave packet spreading by the factor $\eta$, effects the electric field induced displacement by the same factor during the transit through the
amplifier region. This is the quantum equivalent of the semi-classical argument given in [1]. However we see immediately from (23) that the $\kappa \delta x$ term in the phase shift is a small part of the total edm induced, spin dependent phase shift and neither term in the phase shift is altered by travel through the 'amplifier' region. Thus from equ. (23) we have

$$\psi^* + \psi^- \bigg|_{(z=Z+\epsilon)} \approx \sum_{\kappa, \kappa'} A^* (k_o + \kappa') A (k_o + \kappa) e^{i k_o (\kappa - \kappa') \delta x} e^{i (2 \varphi_o - (\kappa - \kappa') \delta x)}$$

We see that the edm induced phase shift at the output is exactly the same as at the input to the 'amplifier'. Thus any measurement performed subsequently, whether using a polarization analyzer or interferometer with magnetic beam splitters or any other scheme, will yield the phase shift as it was at the input and there is no amplification. Any attempts to measure the increased displacement directly as a displacement are easily seen to be completely impractical as is evidently recognized by the authors of [1].

In the next section we show that a correct semi-classical argument leads to the same conclusion.

In response to the circulation of an earlier version of this paper, [2], Peshkin [3] introduces what he calls the "No-Go" theorem. Since $\psi_{\pm} (\vec{x}, t)$ satisfy the same Schroedinger equation in the amplifier region (the Hamiltonian is spin independent) the quantity, $I (t)$, (see equ. (18))

$$I (t) = \int d^3x \left[ \psi^*_{+} (\vec{x}, t) \psi_{-} (\vec{x}, t) \right]$$

is independent of time due to unitarity.

However this leaves open the possibility that the integrand over some limited region of space might be time dependent, thus allowing the amplifier to work without violating the theorem. Thus Peshkin states that measuring the polarization "in a small range of $x$ ..instead of over all $x$ at one time , to avoid the integral over all space in $I (t)$ ..avoids the No-Go theorem in principle". He also claims that the No-Go theorem does not address interferometer experiments as "there the phase shift shows up as an overlap integral between two partial wave packets in one emergent beam only, not as the conserved integral over all space. For the same reason, the theorem also does not speak usefully to versions of the proposal [1] in which the phase shift is measured with a Mach-Zender interferometer instead of a polarimeter". The same argument is presented in a recent Letter to the Editor of Nuc. Instr. and Meth. in Physics Research, by Dombeck and Ringo, [4]. The idea is that at the output of an interferometer (with the spin states separated by a magnetic mirror and travelling through the different arms of the interferometer) the output (considering that one spin state is flipped inside the interferometer) would be given by

$$|\psi_{+} (\vec{x}, t) + \psi_{-} (\vec{x}, t)|^2$$

the cross terms giving the integral $I(t)$, equ. (23) but with the integral taken only over one of the emergent beams. However the partial beams $\psi_{\pm} (\vec{x}, t)$ at the output of the interferometer share all the same phase properties of the beams at the input of the interferometer and the theorem will apply equally to both situations. Be that as it may we propose a more stringent version of the 'No-Go' theorem, the 'Never-Go' theorem, i.e. the motion of the
spin is unaffected by a gravitational field due to the spin independence of the Hamiltonian as we have shown above (eqns. 23 and 25).

We have shown that while an amplification of the ‘displacement’ between the two spin states does occur in the ‘gravitational amplifier’ due to the wavelength dependence of the index of refraction, this is virtually unobservable; the phase shift between the two spin states remains unchanged on traversing the region as does the direction of the neutron polarization.

As we have not integrated over space or time our result shows that attempts to avoid the No-Go theorem by confining the measurements to limited regions will not work. The inapplicability of the theorem is not a sufficient condition for the existence of an amplification effect.

C. Semi-classical model

This model employs the geometrical optics approach to quantum mechanics, where each spin eigenstate is represented by a different trajectory. In a sense this is the most difficult model as it is prone to misunderstanding. In [1], at least three errors are made in application of the semi-classical model to the spin amplification process. Let us first consider amplification by vertical displacements, as discussed before equ. (3) above.

First, assuming that the concept of a displacement of the two spin eigenstates is correct, we can calculate the gravitational effect on the spin precession angle. The first error in [1] occurs when the assumption that the phase between the two wave functions is simply the phase difference between the two eigenstates evaluated at the respective maxima of the wave function envelopes. This is a quantum mechanically incorrect procedure because the phase determination does not commute with the determination of the wave function center at a given time; in other words, it makes no more sense to compare the phases between the two eigenfunctions at two distinct spatial points than it does to compare the phases at two different times.

The correct procedure is to make a point by point comparison between the two wave functions, then average over the two envelopes. This is the procedure normally used when calculating the interference between two scalar or vector fields as is commonly done in electrodynamics (see, for example, [3], Sec. 7.2). That this is the correct procedure can also be seen from the fact that detection occurs at a single point in space-time (for example, the neutron is absorbed on a $^3$He nucleus thereby “collapsing” the wave function to a single space-time point; the spin direction is given by the two wave function phases at that point in space-time).

We can now properly calculate the phase difference between the two eigenstates at a fixed point in the final polarimeter/detector; it is the phase difference between two classical trajectories that meet at the same space-time point, initially separated a distance $\delta x$ perpendicular to the momentum $\vec{k}$. The change in phase is simply the change in action along the two trajectories, and this can be easily calculated to first order by use of a theorem, which is of crucial importance to interferometry (but universally ignored) due to Chiu and Stodolsky [10]. This theorem states that a change in the action when one of the endpoints of a classical trajectory is displaced is given by

$$\delta S = P^D_\mu \delta x^D_\mu - P^0_\mu \delta x^0_\mu$$

(27)
with summation notation over spatial coordinates implied \((\mu = x, y, z)\), and where \(S\) is 
the action (equal to the quantum mechanical phase up to a factor of \(\hbar\)), \(P_\mu\) refers to 
the momentum, \(x_\mu\) the path endpoint coordinates, \(0\) refers to the trajectory beginning, 
and \(D\) the trajectory end (at the polarization analyzer/detector). As discussed already, \(\delta x_\mu^D\), 
that is, the relative displacement of the path at the detector, is identically zero because a neutron 
is detected at a single point (e.g., a polarized \(^3\)He nucleus). The displacement of the path 
starting point is given by \(\delta x_\mu \equiv \delta x\) as given by equ. (2) above. However, it is assumed in \([1]\) 
that \(\delta x\) is perpendicular to the neutron momentum; therefore, the change in action is exactly 
zero, as given by the Chiu and Stodolsky theorem, and there is no gravitational acceleration 
effect, in essence, by definition. Any other effects that could change \(S\), particularly those 
relating to the electric field gradient or gravitational acceleration in the specific geometry 
given in \([1]\), enter only in second or higher order.

The above arguments can be immediately applied to the curved mirror amplifier. We 
again assume that the trajectory endpoints must meet in order for there to be an interference, 
and we calculated the change in action as above. Again we find that \(\delta S\) is identically zero 
to first order in the electric field gradient.

The final semiclassical misconception in \([1]\) concerns the use of an electric field gradient 
over the storage volume. It seems to us that a larger (at least two times) \(\delta x\) (hence phase 
shift) can be generated by sending the “bipolarized” neutrons from a region of zero electric 
field to a region of constant high electric field. Each eigenfunction will acquire a change 
in energy, hence a change in velocity, as it enters the electric field region; although \(\delta x\) 
only increases linearly in time, for a given storage time \(T_1\), suddenly accelerating the two 
eigenstates to their final velocities would lead to a larger \(\delta x\) than if the two eigenstates were 
subjected to a weaker electric field gradient averaged over the storage volume, but giving 
the same final velocities only after storing for a time \(T_1\). The implication is that the use of 
an electric field gradient is completely pointless and only leads to a dilution of a possible 
edm effect. However, this is not surprising based on the foregoing considerations: to achieve 
the maximum sensitivity to an edm effect, whether that be interpreted as \(\delta x\) or a change 
in phase between the two spin eigenfunctions, the interaction energy given by the usual 
Hamiltonian

\[
H = -\mu_e \vec{j} \cdot \vec{E}
\]

must be as large as possible over the duration of an experiment, for it is the integral of \(H\) over 
time that gives the relative action between the two eigenstates. We thus see immediately 
that if the average magnitude of \(E\) is compromised by the wasting of electric field strength 
toward the establishment of gradients in the system, the final net sensitivity to the edm 
interaction given by \(H\) must also be compromised.

III. CONCLUSION

We have analyzed the proposal for a new type of neutron edm experiment from three 
different perspectives, and in each case, have arrived at the same conclusion: The new 
technique offers no gain in sensitivity as compared to the usual magnetic resonance technique. In
fact, a careful analysis reveals that the new method is inferior to the conventional methods. In [1], a semiclassical approach was improperly used to analyze the proposed technique, and a comparison with the atomic interferometer of Kasevich and Chu [1] was used to justify the approach. However, there really is no point of comparison between the atomic interferometer and the system described in [1]. The Kasevich and Chu “interferometer” is based on a superposition of internal quantum states of the Cs atom, specifically, the ground state hyperfine levels; there is no discussion here of the center of mass wave function, but only of the phase difference between the internal states. This phase difference evolves at the hyperfine frequency (approximately 10 GHz) and the beauty of the system is that the freely falling atom experiences a Doppler shift relative to an oscillator fixed in the laboratory; this relative frequency shift in the accelerating system makes possible, for example, a precise measurement of the Earth’s gravitational field. In a certain sense, the Kasevich and Chu system really isn’t an interferometer (this point is addressed in [12]); the evolving internal quantum state phase difference serves as a clock which can be compared to the stationary laboratory oscillator, and the system is best described by the classical approach given above. We might invoke the quantum or semiclassical model to calculate the result of some force that would cause the two hyperfine levels to spatially separate; the result of this calculation would simply show a diminishing of the internal interference effect because the two hyperfine eigenfunctions no longer fully overlap in space-time; in the limit where the separation is greater than the center of mass wave function coherence length, the concept of a superposition of internal quantum states entirely loses its meaning.

IV. ACKNOWLEDGMENT

We would like to thank L. Stodolsky, who independently arrived at the same conclusions, for sharing his thoughts on this matter with us, and for encouraging us to write this manuscript.
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