A Market-Based Framework for Multi-Resource Allocation in Fog Computing

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Abstract—Fog computing is transforming the network edge into an intelligent platform by bringing storage, computing, control, and networking functions closer to end-users, things, and sensors. How to allocate multiple resource types (e.g., CPU, memory, bandwidth) of capacity-limited heterogeneous fog nodes to competing services with diverse requirements and preferences in a fair and efficient manner is a challenging task. To this end, we propose a novel market-based resource allocation framework in which the services act as buyers and fog resources act as divisible goods in the market. The proposed framework aims to compute a market equilibrium (ME) solution at which every service obtains its favorite resource bundle under the budget constraint while the system achieves high resource utilization. This work extends the General Equilibrium literature by considering a practical case of satiated utility functions. Also, we introduce the notions of non-wastefulness and frugality for equilibrium selection, and rigorously demonstrate that all the non-wasteful and frugal ME are the optimal solutions to a convex program. Furthermore, the proposed equilibrium is shown to possess salient fairness properties including envy-freeness, sharing-incentive, and proportionality. Another major contribution of this work is to develop a privacy-preserving distributed algorithm, which is of independent interest, for computing an ME while allowing market participants to obfuscate their private information. Finally, extensive performance evaluation is conducted to verify our theoretical analyses.

Index Terms—General Equilibrium, multi-resource allocation, privacy-preserving distributed optimization, fog computing.

I. INTRODUCTION

Fog Computing (FC), also known as Edge Computing (EC), is an emerging paradigm that complements the cloud to enable a wide range of Internet of Things (IoT) applications, reduce network traffic, and enhance user experience. By distributing storage, computing, control, and networking functions closer to end-users and data sources, FC enjoys many remarkable capabilities, including local data processing and analytics, distributed caching, localization, resource pooling and scaling, enhanced privacy and security, and reliable connectivity [1], [2]. Additionally, FC is the key to meeting the stringent requirements of new systems and low-latency applications such as embedded artificial intelligence, 5G networks, virtual/augmented reality (VR/AR), and tactile Internet.

Despite the tremendous potential, FC is still in its infancy stage and many challenges remain to be addressed. In this paper, we focus on the fog resource allocation problem where a resource pool consisting of multiple Fog Nodes (FN) is shared among different services. Here, we consider an FN as any edge node consisting of one or more computing units (e.g., edge clouds, micro data centers in campus buildings, enterprises, hospitals, malls, and telecom central offices, servers at base stations, and idle PCs in research labs) [1], [3]. Unlike the traditional cloud with virtually infinite capacity, FNPs have limited computational power. They also come with different sizes and configurations. Furthermore, in contrast to a small number of cloud data centers (DC), there are numerous distributed FNs [3]. Due to the heterogeneity of the FNs in terms of location, specifications, reliability, and reputation, the services may have diverse preferences towards them. For example, a service may prefer an FN with powerful hardware and geographically close to it.

Therefore, a primary concern is how to efficiently allocate the limited fog resources to the competing services with diverse characteristics and preferences, considering the service priority and fairness. To address this problem, we propose a new market-based framework whose goal is to harmonize the interests of different market participants so that every service is happy with its allotment while the system maintains high resource utilization. The core idea is to assign different prices to resources of different FNs. Specifically, under-demanded resources are priced low while over-demanded resources have high prices. In our model, each service has a certain budget for resource procurement, which represents the service’s priority level and can be interpreted as the market power of the service [3]. Given the resource prices, each service buys an optimal resource bundle to maximize its utility under the budget constraint. When the market clears, the resulting prices and allocation form a market equilibrium (ME) [4], [5].

The proposed model is motivated by many real-world scenarios [3]. For instance, to divide fog resources among multiple network slices [6], a Telco can grant different budgets to the slices depending on their importance and potential revenue generation (e.g., the total fee paid by the users of each slice). Similarly, using virtual budgets, different labs/departments in a university can fairly share the fog resources located on their campus. Additionally, a service provider (e.g., Uber, Pokemon Go, a sensor network), who owns a set of FNs in a city, may allocate virtual budgets to different groups of users/sensors based on their populations in different areas. Another example inspired by the group buying concept is that several companies (i.e., services) may agree upfront on their individual budgets, then buy/rent a set of FNs together. Instead, these companies can pay subscription fees (i.e., budgets) to use fog resources of a platform that manages a resource pool. This is quite similar to the current postpaid mobile plan models.

For the settings described above, it is necessary to consider both system efficiency and fairness. Therefore, traditional solutions such as welfare maximization, maximin fairness, and auction may not be suitable [3]. For instance, a welfare maximization solution, which is also often the design goal of many auction models [7], typically allocates most of the resources to
agents with high marginal gains while giving very little, even nothing, to agents with low marginal utilities. Thus, it may not be fair to some agents. On the other hands, a maximin fairness allocation often gives too many resources to agents with low marginal utilities, hence, it may not be efficient. Different from these approaches, our proposed market-based solution is both fair and efficient. In particular, we formulate the fog resource allocation problem as a Fisher market where the services act as buyers and the fog resources are divisible goods.

Since there are multiple goods and the purchase decision of each buyer depends on all these goods (both prices and the buyer’s preferences towards them), the proposed problem is inherently a General Equilibrium (GE) problem. The existence of an ME in the GE model was established under some mild conditions (e.g., locally non-satiated utilities) in the seminal work of Arrow and Debreu. Unfortunately, their proof relies on fixed-point theorem and does not give an algorithm to compute an equilibrium. Hence, the algorithmic aspects of the GE theory have recently drawn a lot of interests from theoretical computer scientists. However, there is still no general technique for equilibrium computation.

The strategic behavior of agents in the Fisher market game has also been studied extensively recently. The strategic behavior of agents in the Fisher market game has also been studied extensively recently.

Conventionally, the dual variables associated with the resource capacity constraints are often interpreted as the resource prices. Thus, without budget consideration, popular techniques such as network utility maximization (NUM) can be employed to compute an ME. However, these techniques do not work for the Fisher market. The main challenge stems from the budget constraints and the market clearing condition that couple the allocation decision (i.e., primal variables) and the prices (i.e., dual variables). Fortunately, for a wide class of utility functions, the ME in a Fisher market can be found by solving an Eisenberg-Gale (EG) program.

Unlike the previous works, we focus on a practical setting where the services have utility limits due to their limited demands (e.g., the request rate of a service in a given area is not unlimited). This violates the implicit assumption in the GE literature that requires the utility function is non-satiated (i.e., for any bundle of goods, there is always another bundle of goods that strictly improve the utility).

We theoretically extend the GE literature to consider the practical case where services have utility limits. We show that all non-wasteful and frugal ME exist. Furthermore, the proposed equilibrium is proved to have remarkable fairness properties.

The problem is cast as a Fisher market and we advocate the GE. We are among the first explicitly tackle the MRA problem in the general setting with multiple nodes, each has multiple resource types, and multiple agents with finite demands.

While an ME solution is highly desirable, multiple equilibria may exist. Indeed, when we consider the utility limit, the allocation at some equilibrium can be wasteful and non-Pareto efficient as discussed in [11]. Hence, we impose additional criteria to select an ME with some good properties. Specifically, an allocation should be non-wasteful and frugal. An allocation is non-wasteful if no resources desired by any service are left unallocated and no service receives resources that it has no use for.

The frugality property implies that services want to maximize their utilities while spending less money. Note that the non-wastefulness and frugality concepts do not exist in the traditional Fisher market. The proposed framework allows us to compute a non-wasteful and frugal ME, which implies the allocation is efficient.

We further connect the equilibrium to the fair division
followed by the discussion of related work in Section [VI] and conclusions and future work in Section [VII].

II. SYSTEM MODEL AND PROBLEM FORMULATION

A. System Model

With FC, besides local execution and remote processing at cloud DCs, requests and data from end-devices (e.g., smartphones, PCs, sensors) can be handled by a fog network. Typically, a request first goes to a Point of Aggregation (e.g., switches/routers, base stations, access points), then it will be routed to an FN for processing. In practice, enterprises, factories, organizations (e.g., malls, schools, hospitals), and other third parties (e.g., sensor network owners) can outsource their services and computation to a fog network. Also, content/application/service providers (e.g., Google, Netflix, Uber, AR companies) can proactively place their data and applications onto FNs to serve better their customers.

In this work, we consider a system that consists of multiple services and a set of resource-constrained heterogeneous FNs. The system size may correspond to a metropolitan area network. Each service has a certain budget for resource procurement and wants to offload as many requests as possible to the fog network. The services are assumed to be price-takers, which is reasonable for a large market with multiple services. Each FN provides multiple resource types such as CPU, memory, and bandwidth (BW). We assume that there is a platform lying between the services and the FNs (i.e., the fog resource pool). Based on the information collected from the FNs (e.g., capacities) and the services (e.g., budget, preference, demand), the platform computes an ME solution that assigns a price to every fog resource and allocates an optimal resource bundle to every service. Each service can obtain information about the prices and its allocated resources from the platform. Also, the platform may control and manage the resource pool to implement the allocation decision. Fig. 1 presents the system model.

We assume that the requests (e.g., Apple Siri, Google Voice Search, weather/maps queries, AR, IoT data collection) are independent and each request requires resources in a specific ratio [17], [19], [22]. The amount of resources at an FN needed to handle a request of a service is called the base demand of that service at the FN. For example, if a request of service A needs 0.2 units of CPU and 0.1 units of RAM at FN B, its base demand vector at FN B is (CPU: 0.2, RAM: 0.1). The number of requests of a service can be served is proportional to the amount of resources allocated to it. For instance, if service A receives 0.4 units of CPU and 0.3 units of RAM from FN B, it can process two requests (i.e., \( \min\left\{ \frac{0.4}{0.2}, \frac{0.3}{0.1}\right\}\)). Our model allows a service to specify different base demand vectors at different FNs to express its diverse preferences towards the fog resources.

B. Service Utility Model

We denote the sets of FNs and services by \( M \) and \( N \), respectively. The corresponding numbers of FNs and services are \( M \) and \( N \). Let \( R = \{1, 2, \ldots, R\} \) be the set of \( R \) different types of resources. Define \( i, j, \) and \( r \) as the service index, FN index, and resource type. The capacity of resource type \( r \) at FN \( j \) is \( C_{j,r} \). The base demand vector of service \( i \) for resources of FN \( j \) is \( D^i_j = (a_{i,j,1}, \ldots, a_{i,j,R}) \) where \( a_{i,j,r} \) is the amount of resource type \( r \) at FN \( j \) required to process one request of service \( i \). For simplicity in presentation, we assume that \( a_{i,j,r} > 0 \), \( \forall i, j, r \). Note that all results in this paper still hold if \( a_{i,j,r} = 0 \), for some \( i, j, r \). Due to different hardware specifications and locations of the FNs, the base demand vectors of a service at different FNs can be different. For example, a service may set higher values for bandwidth and computing resources in its base demand vector at an FN far away from it to ensure low latency.

Define \( x_{i,j,r} \) as the amount of resource type \( r \) at FN \( j \) allocated to service \( i \) and \( x_{i,j} = (x_{i,j,1}, \ldots, x_{i,j,R}) \) as the vector of resources allocated to service \( i \) from FN \( j \). The resource bundle that service \( i \) receives from the FNs is \( x_i = (x_{i,1}, x_{i,2}, \ldots, x_{i,M}) \in \mathbb{R}^{M \times R} \). Each bundle \( x_i \) is a matrix where each row represents resources of an FN and each column represents a resource type. As explained in the system model, given \( x_{i,j} \), the number of requests \( y_{i,j} \) of service \( i \) that can be processed by FN \( j \) is [19]–[22].

\[
y_{i,j}(x_{i,j}) = \min \left\{ \frac{x_{i,j,1}}{a_{i,j,1}}, \ldots, \frac{x_{i,j,R}}{a_{i,j,R}} \right\} = \min_r \frac{x_{i,j,r}}{a_{i,j,r}} \quad \forall i, j. \tag{1}
\]

Thus, given resource bundle \( x_i \), the total number of requests \( y_i \) that the fog network can process for service \( i \) is

\[
y_i(x_i) = \sum_j y_{i,j}(x_{i,j}) = \sum_j \min_r \frac{x_{i,j,r}}{a_{i,j,r}} \quad \forall i. \tag{2}
\]

In practice, a service may not have infinite demand (i.e., number of requests over a certain period). Let \( D_i \) be the maximum demand of service \( i \). Then, the actual number of requests \( y_i(x_i) \) of service \( i \) processed by the FNs is

\[
y_i(x_i) = \min \left\{ \sum_j \min_r \frac{x_{i,j,r}}{a_{i,j,r}} \cdot D_j \right\}, \quad \forall i. \tag{3}
\]

The utility function of a service maps each resource bundle to a number quantifying the service’s valuation for that bundle. Define the utility function of service \( i \) as \( u_i(x_i) : \mathbb{R}^{M \times R} \to \mathbb{R} \). Let \( e_i \) be the gain of service \( i \) for each request processed by the fog network. We have:

\[
u_i(x_i) = e_i y_i(x_i) = \min \left\{ e_i \sum_j \min_r \frac{x_{i,j,r}}{a_{i,j,r}}, e_iD_i \right\}, \quad \forall i. \tag{4}
\]
Without loss of generality, we assume $e_i = 1$, $\forall i$. As can be seen later, the proposed allocation scheme is scale-free and independent of $e_i$. Then, we have:

$$\begin{align*}
    u_i(x_i) &= \min \left\{ \sum_j \min_r \frac{x_{i,j,r}}{a_{i,j,r}}, D_i \right\}, \\
    &= \min \left\{ \sum_j u_{i,j}(x_{i,j}) \cdot u_i^{\text{max}}, \forall i, \right\}, \\
\end{align*}$$

(5)

where $u_i^{\text{max}} = D_i$ and $u_{i,j}(x_{i,j}) = \min_r \frac{x_{i,j,r}}{a_{i,j,r}}$. We call $u_i^{\text{max}}$ the utility limit of service $i$.

Note that a service $i$ may want to be served by only a subset of FNs $A_i \subseteq M$ that satisfy certain criteria (e.g., in terms of hardware specifications, reliability, and communication distance). For instance, a latency-sensitive service may be interested in only FNs that are geographically close to it. In this case, we have $u_i(x_i) = \min \left\{ \sum_{j \in A_i} u_{i,j}(x_{i,j}) \cdot u_i^{\text{max}}, \forall i, \right\}$ and $x_{i,j,r} = 0$, $\forall r$, $j \in M/\mathcal{A}_i$. For simplicity of notations, we do not consider this constraint and assume $A_i = M$, $\forall i$. Indeed, the subsequent analysis and solution approaches remain unchanged if we consider this constraint.

C. Fog Computing Resource Allocation Problem

We cast the problem as a Fisher market where the services act as buyers and the fog resources act as divisible goods. The market aims to price and allocate the resources to the services effectively. Denote by $p_j = (p_{j,1}, p_{j,2}, \ldots, p_{j,\text{R}})$ the price vector of resources of FN $j$ where $p_{j,r}$ is the price of one unit of resource type $r$ at FN $j$. Let $p = (p_1, p_2, \ldots, p_M) \in \mathbb{R}^{\text{MAF}}$ capture the resource prices of all the FNs. Define $B_i$ as the budget of service $i$. The optimal (i.e., utility-maximizing) resource bundle of service $i$ at prices $p$ can be found by solving the following service utility maximization problem.

$$\begin{align*}
    \text{maximize} \quad & u_i(x_i) \\
    \text{subject to} \quad & \sum_j \sum_r x_{i,j,r} p_{j,r} \leq B_i, \\
    & x_{i,j,r} \geq 0, \quad \forall j, r, \\
\end{align*}$$

(6)

(7)

(8)

where (7) captures the budget limit of service $i$. Note that each service computes its optimal resource bundle based simply on the resource prices and it does not care about the resource capacity constraints of the FNs. From the definition of the service utility function given in (5), we can rewrite the optimization problem (6)-(8) of each service $i$ as follows.

$$\begin{align*}
    \text{maximize} \quad & u_i(x_i) \\
    \text{subject to} \quad & u_i = \min \left\{ \sum_j u_{i,j}(x_{i,j}) \cdot u_i^{\text{max}}, \right\} \\
    & u_{i,j}(x_{i,j}) = \min_r \frac{x_{i,j,r}}{a_{i,j,r}}, \quad \forall j \\
    & \sum_j \sum_r x_{i,j,r} p_{j,r} \leq B_i, \\
    & x_{i,j,r} \geq 0, \quad \forall j, r. \\
\end{align*}$$

(9)

(10)

(11)

(12)

(13)

Problem Statement: The proposed scheme aims to find an ME solution $(p^*, X^*)$ consisting of equilibrium prices $p^* = (p_{1}^*, p_{2}^*, \ldots, p_{N}^*)$ and equilibrium allocation $X^* = (x_{1}^*, x_{2}^*, \ldots, x_{N}^*)$. At the equilibrium, every service receives its optimal resource bundle. In other words, $x_{i}^*$ is an utility-maximizing resource bundle of service $i$ at the equilibrium prices $p^*$. Additionally, we would like to achieve high utilization of the available fog resources. The definition of an ME is formally given as follows [3].

Definition 2.1: $(p^*, X^*)$ is an ME if the two following conditions hold [5].

- **Service Satisfaction Condition**: Given a non-negative price vector $p^*$, every service receives its favorite resource bundle $x_{i}^*$ that maximizes its utility under the budget constraint. In other words, $x_{i}^*$ is an optimal solution to the problem (9)-(13) with $p = p^*$, i.e., we have:

$$\forall i : \quad x_{i}^* \in \left\{\arg\max_{x_i \geq 0} u_i(x_i) \right\}.$$  

(14)

- **Market-Clearing Condition**: Every fog resource is either fully allocated or has zero price, i.e., we have:

$$\forall j, r : \quad \sum_i x_{i,j,r}^* = C_{j,r}, \quad p_{j,r}^* = 0,$$

(15)

These two conditions ensure that the equilibrium allocation maximizes the happiness of every service while maintaining high resource utilization. The second condition presents Walras’ law [4, 5] which states that all fog resources with positive prices ($p_{j,r}^* > 0$) are fully allocated ($\sum_i x_{i,j,r}^* = C_{j,r}$) and the remaining resources (i.e., not fully allocated) have zero prices. Indeed, due to zero prices, these resources can be allocated arbitrarily to the services without violating the budget constraints. Also, this additional allocation does not strictly improve the utility of any service since $x_{i}^*$ is already a utility-maximizing resource bundle of service $i$ among all its affordable bundles (i.e., following the service satisfaction condition). Hence, we can understand that the market clears.

In our problem, the services are players competing for the limited fog resources, while the platform tries to satisfy the market clearing condition. Prices are used to coordinate the market. Without loss of generality, we normalize the resource capacities of the FNs, i.e., we have:

$$C_{j,r} = 1, \quad \forall j, r.$$  

(15)

The prices and the base demand vectors of the services can be scaled accordingly. This normalization is just to follow the standard market equilibrium literature [5, 9], and to simplify expressions and equations in the paper.

It is worth emphasizing that (9)-(13) or equivalently (9)-(13) is the optimization problem of individual services while the market/platform (not the services) needs to ensure the global constraints including the market-clearing condition and the resource capacity constraints as follows:

$$\begin{align*}
    \left( \sum_i x_{i,j,r}^* - 1 \right) p_{j,r}^* &= 0, \quad \forall j, r \quad (16) \\
    \sum_i x_{i,j,r}^* &\leq 1, \quad \forall j, r. \quad (17)
\end{align*}$$

Our design goal is to find an ME $(p^*, X^*)$ that simultaneously maximizes the utility of every service (i.e., $x_{i}^*$ is an optimal solution of (9)-(13) with $p = p^*$, $\forall i$) while satisfying the global constraints (16)-(17).

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1The words “service” and “buyer” are used interchangeably in this paper.
### III. Centralized Solution Approach

This section presents centralized solutions for computing market equilibria of the proposed problem. First, we briefly introduce the Fisher market and the Eisenberg–Gale (EG) program for computing ME when the services’ utility functions are concave and homogeneous of degree one [4, 5]. However, due to the finite demands of the services, their utilities are not homogeneous. We show that the optimal solution of the EG program for services with infinite demands is still an ME of our model for services with finite demands. However, this equilibrium allocation can be wasteful. Then, we rigorously demonstrate that all the non-wasteful and frugal market equilibria of our model can be captured by a convex program which is a natural generalization of the EG program. Furthermore, we show compelling fairness features of the ME solution obtained from this convex program.

#### A. The Eisenberg-Gale program

Consider a Fisher market with $N$ buyers and $K$ divisible goods with unit capacities. Let $i$ and $k$ be the buyer index and goods index. Buyer $i$ is characterized by budget $B_i$ and a utility function $U_i(x_i)$ where $x_i = (x_{i1}, \ldots, x_{iK})$ is the vector of resources allocated to buyer $i$. If the buyers’ utility functions are all concave and homogeneous of degree one, the optimal solution of the following EG program is an exact ME of this Fisher market [8, 13].

\[
\begin{align*}
\text{maximize} & \quad \sum_i B_i \ln U_i(x_i) \\
\text{subject to} & \quad \sum_i x_{ik} \leq 1, \quad \forall k \\
& \quad x_{ik} \geq 0, \quad \forall i, k.
\end{align*}
\]

Specifically, let $(p^*, X^*)$ be an optimal solution of the EG program (18)-(20) where $p^*$ is the dual variables associated with constraint (19). Then, for every buyer $i$, $x^*_i$ maximizes $U_i(x_i)$ under the budget constraint $\sum_k x_{ik} p^*_k \leq B_i$ and $x_{ik} \geq 0, \forall k$. Furthermore, the market-clearing condition is satisfied, i.e., $\left(\sum_i x^*_i - 1\right) p^*_k = 0, \forall k$.

Many well-known functions, such as linear, Cobb-Douglas, and Leontief, satisfy both concavity and homogeneity of degree one [5]. A function $u(x)$ is linear if $u(x) = \sum_k a_k x_k$. A Cobb-Douglas function has the form $u(x) = \prod_k x_k^{a_k}$ where $\sum_k a_k = 1$. Finally, a Leontief function has the form $u(x) = \min_k (x_k/a_k)$. Another popular class of homogeneous utility functions is the Constant Elasticity of Substitution (CES) function [4, 5].

#### B. Centralized Solution

We now return to our resource allocation problem. It can be observed that the service utility function in (5) is a combination of linear and Leontief functions (i.e., sum of Leontief functions $u_{i,j}(x_{i,j})$). Additionally, the utility limit makes this function more complex.\(^2\)

\(^2\)A function $f(x)$ is homogeneous of degree $d$ if $f(\alpha x) = \alpha^d f(x)$, for all $\alpha > 0$.

1) Services with Infinite Demands: First, we consider the case in which services have unlimited demands. Then, from (5), the utility of service $i$ becomes

\[
u_i(x_i) = u_i^{inf}(x_i) = \sum_j u_{i,j}(x_{i,j}) = \sum_j \min_{r} \frac{x_{i,j,r}}{a_{i,j,r}}, \quad \forall i.
\]

Proposition 3.1: If a service has infinite demand, its utility function is concave and homogeneous of degree one.

Proof. Please refer to Appendix A in [46].

Thus, we can directly apply the EG program (18)-(20), where goods $k$ corresponds to resource type $r$ of FN $j$ and each service is a buyer, and have the following corollary.

Corollary 3.2: If all the services have infinite demands, the optimal solution of the following problem

\[
\begin{align*}
\text{maximize} & \quad \sum_i B_i \ln \left(\sum_j \min_{r} \frac{x_{i,j,r}}{a_{i,j,r}}\right) \\
\text{subject to} & \quad \sum_i x_{i,j,r} \leq 1, \quad \forall j, r \\
& \quad x_{i,j,r} \geq 0, \quad \forall i, j, r.
\end{align*}
\]

is an exact ME of our fog resource allocation problem.

2) Services with Finite Demands: When the services have limited demands, which is often the case in practice, it is easy to see that the utility function in (5) is not homogeneous. For example, when a service reaches its utility limit, its utility remains unchanged when we double the amount of resources allocated to it. We show that the optimal solution to (22)-(24) is still an ME of the proposed resource allocation model where services have finite demands (i.e., utility limits). However, this solution may lead to a wasteful allocation because some services may receive too many resources at the equilibrium without improving their utilities. An allocation is non-wasteful if: i) the remaining resources after the allocation cannot improve the utility of any service; ii) no service receives more resources than its need [16, 17].

Proposition 3.3: The optimal solution to the problem (22)-(24) (i.e., without considering utility limit) is an ME of the proposed resource allocation problem for services with finite demands. However, this solution may lead to a wasteful and non-Pareto optimal allocation.

Proof. Let $(p^*, X^*)$ be an optimal solution to (22)-(24) and $p^*_{r,j}$ is the dual variable associated with the capacity constraint (23). From Corollary 3.2, $(p^*, X^*)$ is an ME of our model for services with infinite demands. We will show $(p^*, X^*)$ is also an ME of our model for services with finite demands.

According to Definition 2.1 at prices $p^*$, $x^*_i$ maximizes $\sum_j \min_{r} \frac{x_{i,j,r}}{a_{i,j,r}}, \forall i$, under the service’s budget constraint. For services with utility limits (i.e., finite demands), since $x^*_i$ maximizes $\sum_j \min_{r} \frac{x_{i,j,r}}{a_{i,j,r}}$ at prices $p^*$, $x^*_i$ also maximizes $\min \left\{ \sum_j \min_{r} \frac{x_{i,j,r}}{a_{i,j,r}}, \nu_i^{max} \right\}$ at $p^*$ while respecting the budget constraint. Additionally, $(p^*, X^*)$ still satisfies the market clearing condition. Therefore, $(p^*, X^*)$ is an exact ME of the proposed problem for services with finite demands.

However, $X^*$ can be a wasteful allocation since some services with limited utilities may get redundant resources at the equilibrium (i.e., $\sum_j \min_{r} \frac{x_{i,j,r}}{a_{i,j,r}} > \nu_i^{max}$). These resources can be reallocated to other services that have not reached utility limit to improve their utilities. Thus, $X^*$ may not be
Pareto-optimal since some service can get strictly better utility without affecting the utility of other services. This issue is illustrated in the simulation (e.g., see Fig. 2).

In addition to the result presented in Fig. 2 in the simulation, interested readers can refer to Appendix F and Appendix G for some toy examples of the wasteful equilibrium allocation phenomenon. Since multiple market equilibria can exist, we need to impose some criteria to select a good ME. First, we are interested in finding a non-wasteful ME, which implies the equilibrium allocation is efficient. Assume $X^*$ is a non-wasteful allocation and $(p^*, X^*)$ is a non-wasteful ME. The market clearing condition of an ME ensures that no resources desired by any service are left unallocated. Specifically, resources that are not fully allocated have zero prices. As explained in the paragraph following the Definition 2.1, these resources can be given to any service without increasing the utility of the service. Thus, an ME always satisfies the first requirement of a non-wasteful allocation (defined before Proposition 3.3). To satisfy the second requirement that no service receives resources that it has no use for, i.e., we need:

$$\sum_j \min_i \frac{x_{i,j,r}^*}{a_{i,j,r}} \leq u_i^{\max}, \forall i. \quad (25)$$

Additionally, resources allocated to a service from an FN must be proportional to the service’s base demand vector (i.e., $x_{i,j,r}^* = \beta a_{i,j,r}$ for all $i$, $j$, $r$, and some constant $\beta$).

The second criterion is frugality. It is natural to assume that the services want to maximize their utilities while spending as least money as possible. Hence, a service would prefer an FN with the cheapest price for its base demand vector. Let $p$ be the resource prices. The price for the base demand vector $D_i$ of service $i$ at FN $j$ is

$$q_{i,j} = \sum_r p_{j,r} a_{i,j,r}, \forall i, j. \quad (26)$$

Define $q_{i,j}^{\min} = \min_j q_{i,j}, \forall i$. The frugality property states that service $i$ will buy resources only from the set of FNs $j$ such that $q_{i,j} = q_{i,j}^{\min}$. We also define $\alpha_i = \frac{1}{q_{i,j}^{\min}}$ as maximum bang-per-buck (MBB) of service $i$, which generalizes the notion of MBB for linear utility in [9].

Before presenting our main result, it is necessary to understand the buying strategy (i.e., action) of the services at a non-wasteful and frugal ME $(p^*, X^*)$. Consider service $i$. To maximize its utility as well as to satisfy the frugality condition, service $i$ will buy resources from the cheapest FNs only (i.e., the FNs offer it MBB). In other words, service $i$ buys resources only from the set of FNs $j'$ such that $q_{i,j'} = q_{i,j'}^{\min} = \min_j \sum_r p_{j,r} a_{i,j,r}$. Furthermore, service $i$ either spends full budget to buy resources from the cheapest FNs or spends money until reaching its utility limit. Intuitively, this buying strategy maximizes the utility of the service and satisfies the frugality condition.

Our main finding is that the following optimization problem, which is a natural extension of the EG program, allows us to find a non-wasteful and frugal ME.

$$\max_{x,u} \sum_i B_i \ln \sum_j u_{i,j} \quad (27)$$

subject to

$$u_{i,j} a_{i,j,r} = x_{i,j,r}, \forall i, j, r \quad (28)$$

$$\sum_j u_{i,j} \leq u_i^{\max}, \forall i \quad (29)$$

$$\sum_i x_{i,j,r} \leq 1, \forall j, r \quad (30)$$

$$x_{i,j,r} \geq 0, \forall i, j, r. \quad (31)$$

While the idea of adding utility limit constraints is natural, the following theorem and its proof are not trivial since the existing results on ME in the Fisher market (and the GE theory in general) rely heavily on the critical assumptions of non-satiated utility functions [5], [15], which is not the case in our model. Furthermore, the non-wastefulness and frugality notions do not exist in the traditional Fisher model. Theorem 3.4 includes two main parts. The first part states that the optimal solutions to the problem (27)-(31) are non-wasteful and frugal market equilibria of our problem. More importantly, the second part states that the set of all the non-wasteful and frugal market equilibria are exactly the set of optimal solutions to the problem (27)-(31).

Based on the analysis of the optimal buying strategy of the services, to prove the first part, we need to show: i) the optimal solutions to (27)-(31) satisfy the market-clearing condition in Definition 2.4; ii) the services buy resources from the cheapest FNs only (i.e., FNs offer them MBB); iii) every service either reaches its utility limit or spends full budget. Note that constraints (28)-(29) imply that any optimal solution $X^*$ to this problem is a non-wasteful allocation. To prove the second part of the theorem, we need to show that from the conditions of an ME, non-wastefulness, and frugality, we can reconstruct the problem (27)-(31).

**Theorem 3.4:** The optimal solutions to the problem (27)-(31) are exactly non-wasteful and frugal market equilibria. At the equilibria, every service either spends all budget or attains its utility limit. The ME always exists and the utilities are unique across all such equilibria. Finally, all non-wasteful and frugal market equilibria are captured by the problem (27)-(31).

**Proof.** Notice that the problem (27)-(31) always has an interior feasible solution by simply setting $x_{i,j,r}$ positive and sufficiently small, $\forall i, j, r$, so that all the constraints (29)-(31) are satisfied with strict inequalities. Hence, Slater’s condition holds and the Karush–Kuhn–Tucker (KKT) conditions are necessary and sufficient for optimality [13]. Define $\lambda_{i,j,r}, \mu_i, \gamma_{i,j,r}$ as the dual variables associated with (28), (29), (30), and (31), respectively. From the KKT conditions, we can infer

$$\forall i, j : \frac{B_i}{u_{i,j}} - \sum_r \lambda_{i,j,r} a_{i,j,r} - \mu_i = 0 \quad (32)$$

$$\forall i, j, r : \lambda_{i,j,r} \leq p_{j,r} \quad (33)$$

$$\forall i, j, r : \text{if } x_{i,j,r} > 0 \Rightarrow \lambda_{i,j,r} = p_{j,r} \quad (34)$$

$$\forall j, r : \sum_i x_{i,j,r} - 1 = 0 \quad (35)$$

$$\forall i, j \text{ if } \mu_i > 0 \Rightarrow u_i = \sum_j u_{i,j} = u_i^{\max}. \quad (36)$$

3 Please refer to Appendix B in [46] for the detailed derivation.
Indeed, (35) is exactly the market clearing condition. Also, from (33), if $p_{j,r} > 0$, then $\sum_i x_{i,j,r} = 1$, $\forall j, r$, which means all resources with positive prices are fully allocated. Additionally, if $\sum_i x_{i,j,r} < 1$, then $p_{j,r} = 0$, $\forall j, r$, which means all non-fully allocated resources have zero prices. From (32)-(34) and the definition of $q_{i,j}$ in (26), we have:
\[
\forall i, j : q_{i,j} = \sum_r p_{j,r} a_{i,j,r} \geq \sum_r \lambda_{i,j,r} a_{i,j,r} = B_i / u_i - \mu_i \quad (37)
\]
\[
\forall i, j : \text{if } x_{i,j,r} > 0 \Rightarrow q_{i,j} = \sum_r p_{j,r} a_{i,j,r} = B_i / u_i - \mu_i. \quad (38)
\]
The inequality in (37) is due to (33). The last equality in (37) is from (32). From (34), if service $i$ buys resource type $r$ at FN $j$ (i.e., $x_{i,j,r} > 0$), then $\lambda_{i,j,r} = p_{j,r}$, $\forall i, j, r$. Thus, from (34) and (37), we can obtain (33). Denote $q^{\min}_i = B_i / u_i - \mu_i$, $\forall i$. From (37) and (38), we have $q_{i,j} \geq q^{\min}_i$, $\forall i, j$, and if $x_{i,j,r} > 0$, then $q_{i,j} = q^{\min}_i$. Hence, the services buy resources only from the cheapest FNs. Furthermore, from (22), we have:
\[
B_i - \mu_i u_i = \left( \sum_r \lambda_{i,j,r} a_{i,j,r} \right) u_i = \sum_r \lambda_{i,j,r} a_{i,j,r} \sum_j u_{i,j} = \sum_j \sum_r \lambda_{i,j,r} \left( a_{i,j,r} u_{i,j} \right) = \sum_j \sum_r \lambda_{i,j,r} x_{i,j,r} = \sum_j \sum_r p_{j,r} x_{i,j,r}. \quad (39)
\]
The fourth equality in (39) is from the feasibility constraint (28). The last equality in (39) is due to (34) and $x_{i,j,r} \geq 0$, $\forall i, j, r$. Thus, from (39), we have:
\[
\mu_i u_i = B_i - \sum_j \sum_r p_{j,r} x_{i,j,r}, \quad \forall i. \quad (40)
\]
Since $\sum_j \sum_r p_{j,r} x_{i,j,r}$ is the total money spent by service $i$ for procuring fog resources, $\mu_i u_i$ can be inferred as the budget surplus of service $i$ after purchasing the resources. Consequently, if $\mu_i = 0$, we have $B_i = \sum_j \sum_r p_{j,r} x_{i,j,r}$.

In other words, if $\mu_i = 0$, service $i$ spends all of its budget to buy fog resources. Also, from (46), if $\mu_i > 0$, then $u_i = u_i^{\max}$. Hence, a service either spends all money at the equilibrium or reaches its utility limit. Furthermore, we showed that the services buy resources from the cheapest FNs only. Additionally, from (40), the services do not overspend since $\mu_i, u_i \geq 0$, $\forall i$. Therefore, the optimal solution $X$ to the problem (27)-(31) maximizes the utility of every service under the budget constraint.

We have shown that all the conditions of a non-wasteful and frugal ME can be inferred from the KKT conditions of (27)-(31). As a result, the optimal solutions to (27)-(31) are indeed non-wasteful and frugal market equilibria.

Now, we show that the utilities are unique across all such equilibria. Obviously, the problem (27)-(31) is equivalent to:
\[
\begin{align*}
\text{maximize} \quad & \sum_i B_i \ln u_i \\
\text{subject to} \quad & u_i = \sum_j u_{i,j}, \quad \forall i; \quad (28) - (31).
\end{align*}
\]
Since this problem has a strictly concave objective function, it has a unique optimal utility vector $u^*$.

Finally, we show that any non-wasteful and frugal ME is an optimal solution to the problem (27)-(31). Let $(p, X)$ be a non-wasteful and frugal ME in our resource allocation problem with the service utility function given in (5). By definition, $x_i$ is an optimal solution of the service maximization problem (9)-(13) at the price vector $p$. We will show that $X$ is an optimal solution to the problem (27)-(31).

Specifically, we prove that $X$ is feasible (i.e., satisfying (28)-(31)) and $(p, X)$ satisfies the KKT conditions (32)-(34).

Since the $X$ is non-wasteful, it does not over-allocate any fog resource, i.e., $\sum_i x_{i,j,r} \leq 1, \forall j, r$. Also, from (25), we have: $\sum_j \min_r \frac{p_{j,r}}{a_{i,j,r}} \leq u_i^{\max}, \forall i$. Because a non-wasteful allocation allocates resources of an FN to a service proportional to the service’s base demand vector, we have: $\frac{\sum_j p_{j,r} x_{i,j,r}}{\sum_j a_{i,j,r}} = \frac{\sum_j p_{j,r} x_{i,j,r}}{\sum_j \min_r \frac{p_{j,r}}{a_{i,j,r}}}, \forall i, j, r$. Hence, we can define $u_{i,j} = \min_r \frac{p_{j,r}}{a_{i,j,r}}, \forall i, j, r$. Then, $\sum_i u_{i,j} \leq u_i^{\max}, \forall i, j$. Thus, $X$ is a feasible solution to the problem (27)-(31).

Since the total money $\sum_j \sum_r p_{j,r} x_{i,j,r}$ spent by service $i$ is constrained by its budget (i.e., constraint (12)), we have:
\[
\begin{align*}
B_i & \geq \sum_j \sum_r p_{j,r} x_{i,j,r} = \sum_j \sum_r \left( a_{i,j,r} \frac{x_{i,j,r}}{u_{i,j}} \right) = \sum_j \sum_r p_{j,r} a_{i,j,r} u_{i,j} = \sum_j u_{i,j} \left( p_{j,r} a_{i,j,r} \right).
\end{align*}
\]
Note that $u_{i,j} = \frac{\sum_j \min_r \frac{p_{j,r}}{a_{i,j,r}}}{\sum_j a_{i,j,r}} = \frac{\sum_j \min_r \frac{p_{j,r}}{a_{i,j,r}}}{\sum_j \min_r \frac{p_{j,r}}{a_{i,j,r}}}, \forall i, j, r$. Also, the frugality condition imposes: $x_{i,j} > 0$ only if FN $j$ satisfies $\sum_r p_{j,r} a_{i,j,r} = q_i^{\min}$. If $\sum_r p_{j,r} a_{i,j,r} > q_i^{\min}$, then $x_{i,j} = 0$ and $u_{i,j} = 0$. Hence:
\[
\begin{align*}
B_i & \geq \sum_j u_{i,j} \left( \sum_r p_{j,r} a_{i,j,r} \right) = \sum_j u_{i,j} q_i^{\min} = u_i q_i^{\min}. \quad (43)
\end{align*}
\]
Let $\mu_i = \frac{B_i}{u_i} - q_i^{\min}$. Then, $\mu_i \geq 0, \forall i$. Also, we have $q_{i,j} = \sum_r p_{j,r} x_{i,j,r} \geq q_i^{\min} = \frac{B_i}{u_i} - \mu_i, \forall i, j$. Again, by the frugality property, if $x_{i,j} > 0$, then $q_i = \sum_r p_{j,r} x_{i,j,r} = q_i^{\min} = \frac{B_i}{u_i} - \mu_i$. Hence, (37) and (35) hold. Consequently, by properly setting $\lambda_{i,j,r}$, we obtain (32)-(34).

By Definition (2.7) prices $p$ are non-negative. Also, if $p_{j,r} > 0$, resource type $r$ at FN $j$ is fully allocated (i.e., $\sum_i x_{i,j,r} = 1$). Thus, $\sum_i x_{i,j,r} = 1 \Rightarrow p_{j,r} > 0, \forall i, j, r$, which is exactly (35). The final step is to show (36).

Assume $\mu_i > 0$. Then, $\frac{B_i}{u_i} - q_i^{\min} > 0$. Hence:
\[
\begin{align*}
B_i & > q_i^{\min} u_i = q_i^{\min} \sum_j u_{i,j} = \sum_j q_{i,j} u_{i,j} \quad (44)
\end{align*}
\]
where the last equality is because $u_{i,j} = \frac{\sum_j \min_r \frac{p_{j,r}}{a_{i,j,r}}}{\sum_j a_{i,j,r}}, \forall i, j, r$. The second equality is from the frugality property of $X$, which imposes: $u_{i,j} = 0$ if $q_{i,j} > q_i^{\min}$; and $u_{i,j} > 0$ only if $q_{i,j} = q_i^{\min}$. Thus, if $\mu_i > 0$, the total money spent by service $i$ is strictly less than its budget $B_i$. Define $\phi_i = \frac{B_i}{\sum_j \sum_r q_{i,j,r}}, 1 \geq 1$. We will prove (36) by contradiction. Assume $u_i = \sum_j u_{i,j} < u_i^{\max}$. Since bundle $(\phi_i x_i)$ is affordable to service $i$, the service can strictly improve its utility by purchasing bundle $(\phi_i x_i)$ because, obviously, $u_i(\phi_i x_i) > u_i(x_i)$. Thus, $X$ does not satisfy
the service satisfaction condition, which contradicts to the assumption that \((p, X)\) is an ME. Therefore, if \(\mu_i > 0\), then \(u_i = \sum_j u_{i,j} = u_i^{\text{max}}, \forall i\), which is exactly (30).

To understand the importance and impact of non-wastefulness and frugality as well as utility limit, we have several remarks as follows.

**Remark 1**: we again emphasize that from Proposition 3.3, it seems to be natural to add the utility limit constraint to achieve non-wasteful ME. Hence, one may think that “every non-wasteful ME is an optimal solution to the problem (27)-(31)”\(^ \) However, this statement may not hold. A non-wasteful ME may not necessarily be a solution to (27)-(31). In other words, this convex program may not necessarily capture the set of all non-wasteful market equilibria. Indeed, you can observe that the proof of Theorem 3.4 relies heavily on the frugality property.

**Remark 2**: For a system with one FN only (i.e., buyers with Leontief utility functions), we may not need the frugality criteria since a buyer has only one node to buy resources (i.e., goods) from. The non-wasteful properties may be sufficient since it enforces every buyer to either fully exhaust its budget or reach his utility limit. However, when there are multiple FNs (i.e., the hybrid linear-Leontief utility function in our model), at the equilibrium prices (or any price vector), a buyer who can reach his utility limit may have multiple options to buy resources. Note that in the Fisher market, money has no intrinsic value to the buyers. Hence, he may not need to buy resources from the cheapest FNs to obtain his utility limit. Therefore, frugality is important for our result. Note that if a buyer does not fully spend his budget, the surplus budget can be accumulated/redistributed into later time slots to give more priority to him in the later slots. The detailed policy and implementation are flexible and not the focus of this work. Our work focuses on one time slot only when the system parameters (e.g., budget, limit, preferences, capacities) are given.

**Remark 3**: Without utility limit, obviously, every buyer will spend full budget to buy more resources to improve his utility. Consequently, every ME is non-wasteful. However, it is not true when we consider utility limit. As shown in the theorem above, some buyers do not fully exhaust their budgets at the equilibrium.

**Remark 4**: As shown in Proposition 3.1., without utility limit, every ME is an optimal solution to the EG program (5). Furthermore, the utilities of each buyer at such equilibria are unique (5). However, when we consider utility limit, there may exist multiple market equilibria. The utilities of each buyer at the equilibria may not be unique (e.g., wasteful ME in Proposition 3.3 and non-wasteful ME in Theorem 3.4). It is also worth noting that there can be other market equilibria which are not the optimal solutions to either the problem (22)-(24) or the problem (27)-(31).

Based on the proof above, we can further show that the market equilibria obtained from (27)-(31) are Pareto-optimal. An allocation is Pareto-optimal if there is no other allocation that would make some service better off without making some other service worse off \(^4\), which means there is no strictly “better” allocation. It is worth noting that the famous first fundamental theorem of welfare economics \(^4\), \(^8\) states that under some mild conditions, an ME is always Pareto-optimal. However, its proof relies on the crucial assumption of locally non-satiated utility functions (e.g., see Proposition 16.C.1 in \(^4\)). Without considering the utility limit, the service utilities become locally non-satiated, hence, every ME is Pareto-efficient. However, due to the utility limit, the utility functions in our problem are locally satiated. Hence, not every equilibrium allocation in our model is Pareto-optimal (e.g., the equilibria mentioned in Proposition 3.3. Therefore, our result on Pareto-optimality is interesting since the utility functions in our problem are locally satiated due to the utility limit.

**Lemma 3.5**: The market equilibria obtained from the problem (27)-(31) are Pareto-optimal.

**Proof**: We show this by contradiction. Let \((p^*, X^*)\) be an optimal solution to (27)-(31). Assume \(X^*\) is a strictly better allocation than \(X^\). Hence, \(u_i(x^*_i) \geq u_i(x^*_i), \forall i\) and strict inequality holds for some \(i\). Let \(h\) be the index of the service that has strictly better utility at \(X^*\). Obviously, if service \(h\) reaches its utility limit at \(X^*\), it cannot strictly increase its utility. Hence, at \(X^*\), service \(h\) has not reached its utility limit. Thus, from the results of Theorem 3.4, service \(h\) spends full budget at \(X^*\). Furthermore, we proved that every service buys resources only from cheapest FNs. Hence, at prices \(p^*\), service \(h\) cannot improve its utility and \(X^*\) is Pareto-optimal. \(\square\)

In summary, a non-wasteful and frugal ME \((p^*, X^*)\) is an optimal solution to (27)-(31). Since this is a convex optimization problem, efficient techniques such as the interior-point method can be used to solve it effectively \(^13\), \(^52\). The computational aspect of this convex program is further discussed in Appendix E.

### C. Fairness Properties of the ME

In this section, we show that the proposed ME, which is an optimal solution to (27)-(31), possesses appealing fairness properties that encourage the services to participate in the proposed scheme. An allocation is envy-free if no service envies with the allocation of any other service (i.e., if every service weakly prefers its allocation to the allocation of any other service). In other words, every service is happy with its allocation and does not want to swap its allocation with that of another service. When the budgets are equal, an envy-free allocation \(X\) implies \(u_i(x_i) \geq u_i(x_{i'}), \forall i, i' \in N\) \(^18\). It is well-known that every competitive equilibrium from equal income (CEEI) is envy-free \(^53\). For the Fisher market, equal income means the buyers have equal budgets. If the budgets are not equal, it is easy to see that a market equilibrium can be not envy-free (e.g., two agents with the same utility function and one good, the agent with lower budget may envy with the allocation of the agent with higher budget). It is because different budgets results in different sets of affordable resource bundles of different agents at the equilibrium prices. Since the services may have different budgets, we need to extend the classical definition of envy-freeness. An allocation \(X\) is envy-free if \(u_i(x_i) \geq u_i(B_{i'} x_{i'}), \forall i, i' \in N\).

**Sharing-incentive** is another popular fairness criterion. An allocation satisfies this property if it gives every service a better utility than its utility in the proportional sharing scheme. Proportional sharing is an intuitive way to share resources
fairly (in terms of resource quantity), which allocates every resource to the services proportional to their budgets. Let \( \hat{x} \) be the allocation in which every service received resources from the FNs proportional to its budget, i.e., \( \hat{x}_{i,j,r} = \frac{B_i}{\sum_i B_i} \), \( \forall i, j \). Indeed, \( \hat{x} \) can be interpreted as a resource-fair allocation. The sharing-incentive property implies \( u_i(x_i) \geq u_i(\hat{x}_i) \), \( \forall i \).

Additionally, in the fog federation setting where each service \( i \) contributes a portion \( \hat{x}_{i,j,r} \) of every resource to FN \( j \) in a resource pool that consists of all the FNs, sharing-incentive ensures that every service prefers the equilibrium allocation to its initial resources contributed to the pool. This property can be interpreted as resource-fairness.

Finally, proportionality is a well-known fairness notion in Economics [18]. Let \( C \in \mathbb{R}^{M \times R} \) be the set of all the fog resources. Obviously, \( u_i(C) \) is the maximum utility that service \( i \) can achieve by being allocated all the available resources (i.e., \( C_{j,r} = 1, \forall j, r \)). It is natural for service \( i \) to expect to obtain a utility of at least \( \frac{B_i}{\sum_i B_i} u_i(C) \). The proportionality property guarantees that the utility of every service at the equilibrium is at least proportional to its budget. In other words, an allocation \( X \) satisfies proportionality if \( u_i(x_i) \geq \frac{B_i}{\sum_i B_i} u_i(C) \), \( \forall i \). Thus, this property can be interpreted as the utility-fairness (i.e., every service feels fair in terms of the achieved utility).

Theorem 3.6: The proposed ME is envy-free, and satisfies the sharing-incentive and proportionality properties.

Proof. Please refer to Appendix C in [16]. \( \square \)

IV. DECENTRALIZED SOLUTION

In this section, we develop a privacy-preserving distributed algorithm for computing a non-wasteful and frugal ME that is an optimal solution to the problem (27)-(31). The developed algorithm converges to the ME while preserving the services’ privacy. Specifically, no service needs to reveal any private information to other parties including the other services and the platform, which would significantly mitigate the strategic behavior of the market participants. For example, if a service can learn private information of some other services, it may be able to guess the utility functions of these services and submit strategic bids to the platform to gain benefits. Our scheme prevents this issue. The proposed scheme is also robust to common attacks such as collusion and eavesdropping.

We assume that all the services are semi-honest, often called honest-but-curious, which is a common security model in security, privacy, and cryptography research [23], [24]. In the honest-but-curious setting, all parties follow the protocol (i.e., the distributed algorithm) honestly, but they are still curious about the information of other parties. To achieve the design goal of the privacy-preserving distributed algorithm, we first convert (27)-(31) to a standard ADMM form such that the distributed algorithm can be conducted in parallel by the services. This algorithm requires only the average of the optimal demand vectors of the services at each iteration. Thus, our task is to derive an effective procedure for calculating the exact average vector in a privacy-preserving manner.

It is worth noting that if there is only one FN (i.e., a small DC), the ME can be found in a distributed manner using the dual decomposition method [13]. Since the objective function in the problem (27)-(31) is not strictly concave in \( u_i \), we leverage the ADMM technique, which works for non-strictly convex functions. Due to space limitation, here we do not study the single FN case.

First, we rewrite the problem (27)-(31) as follows:

\[
\begin{align*}
\text{maximize} & \quad \sum_i B_i \ln \sum_j \frac{x_{i,j,1}}{a_{i,j,1}} \\
\text{subject to} & \quad \sum_j a_{i,j,1} \leq u_i^{\max}, \quad \forall i \\
& \quad \sum_i x_{i,j,1} \leq 1, \quad \forall j, r \\
& \quad x_{i,j,1} \geq 0, \quad \forall i, j, r.
\end{align*}
\]

The traditional ADMM approach [25] cannot be directly applied to tackle this problem since it can only solve optimization problems without inequality constraints. More importantly, the problem (45) has multi-block variables (i.e., \( x_i \)'s) while the traditional ADMM approach can only solve problems with two blocks of variables. An intuitive solution is to extend the traditional ADMM to multi-block variables and update \( x_i \) sequentially. However, such Gauss-Seidel ADMM may not converge for \( N \geq 3 \) [26]. Furthermore, in the traditional ADMM, the blocks are updated one after another, hence, it is not amenable for parallelization. Another solution is to convert the multi-block problem into an equivalent two-block problem via variable splitting [25]. However, this method substantially increases the number of variables and constraints in the problem, especially when \( N \) is large. In [27], the authors propose the Jacobi-Proximal ADMM approach, which is suitable for parallel and distributed optimization, by adding additional proximal terms to the variable-update step. Our approach does not utilize proximal terms. Finally, the existing approaches require the agents to share their private information with each other in every iteration. Thus, they are not suitable for our design goal.

To make use of ADMM, we first clone all variables \( x_i \)'s using auxiliary variables \( z_i = (z_{i,1}, \ldots, z_{i,M}) \) and obtain the following problem that is equivalent to the problem (45).

\[
\begin{align*}
\text{maximize} & \quad \sum_i B_i \ln \sum_j \frac{x_{i,j,1}}{a_{i,j,1}} \\
\text{subject to} & \quad x_{i,j,1} - z_{i,j,1} = 0, \quad \forall i, j, r \\
& \quad \sum_j a_{i,j,1} \leq u_i^{\max}, \quad \forall i \\
& \quad \sum_i z_{i,j,1} \leq 1, \quad \forall j, r \\
& \quad x_{i,j,1} \geq 0, \quad \forall i, j, r.
\end{align*}
\]
To convert this problem into ADMM form, we define:
\[
\begin{align*}
  f_i(x_i) &:= v_i(x_i) + h_i(x_i), \quad \forall i \\
  v_i(x_i) &:= -B_i \ln \sum_j x_{i,j,1} a_{i,j,1}, \quad \forall i \\
  h_i(x_i) &:= \begin{cases} 
    0 & \text{if } x_i \in X_i \\
    +\infty & \text{otherwise}
  \end{cases} \\
  g(z_i) &:= \begin{cases} 
    0 & \text{if } z_i \in Z \\
    +\infty & \text{otherwise}
  \end{cases} \\
  X_i &:= \left\{ x_i \bigg| \sum_j x_{i,j,1} a_{i,j,1} \leq u_i^{\text{max}}, \quad \forall i \right\} \\
  Z &:= \{ z \bigg| \sum_z z_{i,j,r} \leq 1, \quad \forall j, r \},
\end{align*}
\]
where \( h_i(\cdot) \) and \( g(\cdot) \) are indicator functions. Then, the problem (46) can be written in ADMM form as follows:
\[
\begin{align*}
  \text{minimize}_{x,z} & \quad \sum_i f_i(x_i) + g(\sum_i z_i) \\
  \text{subject to} & \quad x_{i,j,r} - z_{i,j,r} = 0, \quad \forall i, j, r,
\end{align*}
\]
with variables \( x_i, z_i \in \mathbb{R}^{M \times K}, \forall i \). To simplify the notation, we consider each resource type \( r \) at FN \( j \) as an item \( k \) and we have a total of \( K = M \times R \) items. Then, \( x_i, z_i \in \mathbb{R}^K, \forall i \), and the problem (47) can be expressed as below:
\[
\begin{align*}
  \text{minimize}_{x,z} & \quad \sum_i f_i(x_i) + g(\sum_i z_i) \\
  \text{subject to} & \quad x_i - z_i = 0, \quad \forall i.
\end{align*}
\]

We have eliminated the coupling constraints between variables \( x_i \)’s. Furthermore, the variables now can be divided into \( 2 \) blocks (i.e., one contains all \( x_i \)’s, and the other contains all \( z_i \)’s). Hence, we are ready to use the ADMM approach to construct a parallel and distributed algorithm for computing the ME. First, define \( \overline{x}, \overline{z} \in \mathbb{R}^K \) as follows
\[
\overline{x} = \frac{1}{N} \sum_i x_i; \quad \overline{z} = \frac{1}{N} \sum_i z_i.
\]
Also, let \( p_i = (p_{i,1}, p_{i,2}, \ldots, p_{i,K}) \in \mathbb{R}^K, \forall i \), be the dual variables associated with constraint (49). Indeed, we can show that \( p_i = p, \forall i \) (see Appendix D). The ADMM-based decentralized implementation is summarized in Algorithm 1. Due to the space limitation, we do not present the construction of this algorithm. Please refer to Appendix D in [46] or in the supplementary material for the detailed derivation.

In this algorithm, \( r_{\text{primal},t+1} \) and \( r_{\text{dual},t+1} \) are the primal and dual residuals at iteration \( t + 1 \), respectively. As defined in [25], we have \( ||r_{\text{primal},t+1}||_2 = \sqrt{\sum_{i=1}^{N} ||x_i - x_{t+1}^i||^2} \) and \( ||r_{\text{dual},t+1}||_2 = \rho \cdot ||z_{t+1} - z||_2 \). At each iteration \( t \), given \( \overline{x} \) and price signal \( p \), each service \( i \) solves a convex optimization problem to find its optimal resource bundle \( x_{t+1}^i \) and sends it to the platform. After collecting the optimal demands of all the services, the platform updates \( \overline{x} \) by solving (51). Based on the updated values of \( \overline{x} \) and \( \overline{z} \), the resource prices can be updated by the platform or the FN’s. The algorithm terminates when the primal and dual residuals are sufficiently small [25] or the number of iterations becomes sufficiently large. Otherwise, the new values of \( \overline{x}, \overline{z} \), and \( \rho \) are sent to every service. The steps described above repeat until the algorithm converges. The convergence properties of Algorithm 1 follows directly the standard ADMM method [25].

Remark: It is worth noting that the \( z \)-update and dual-update steps can be carried out by every service and the services only need to exchange messages to update \( \overline{x} \). By this way, the algorithm can be implemented in a fully distributed manner by the services (i.e., without the platform).

Algorithm 1 DISTRIBUTED IMPLEMENTATION

1: Initialization: set \( \{x_1^t\}, \{z_1^t\}, \{p_1^t\}, \rho, \gamma_1, \gamma_2, t = 0 \).
2: repeat
3: \( \overline{x} \)-update: each service \( i \) solves the following sub-problem to compute \( x_{t+1}^i \)
\[
\min_{x_i} v_i(x_i) + \frac{\rho}{2} \left\| x_i - x_{t+1} + \overline{x} - \overline{z} - \frac{1}{\rho} p_i \right\|_2^2 \text{ s.t. } x_i \in X_i.
\]
4: \( \overline{z} \)-update: to compute \( \overline{z}_{t+1} \), the platform solves
\[
\min_{\overline{z}} \left\| \overline{x} - \overline{z} - 1 \right\|_2, \text{ s.t. } N \overline{z} \in Z.
\]
5: until \( \rho_{t+1} := p_{t+1} + \rho \cdot \rho_{t+1} = \rho \cdot \rho_{t+1} \) \forall k.

Now we consider the privacy-preserving issue. It can be observed that in Algorithm 1, the utility function of a service is only known to that service. However, \( \overline{x}_{t+1} \) is required for the \( z \)-update step. A conventional approach to obtain \( \overline{x}_{t+1} \) is to ask every the services to report \( x_{t+1}^i \) to the platform and \( \overline{x}_{t+1} = \frac{1}{N} \sum_i x_{t+1}^i \). However, this approach reveals \( x_{t+1}^i \), which is undesirable. For instance, by learning a sequence of \( x_i \), which is the optimal resource bundle of service \( i \) given the price signal \( p \), an attacker (e.g., the platform or another service) may be able to guess the utility function of service \( i \). A privacy-preserving protocol should not disclose any private information of the services. If we can find a method to obtain \( \overline{x}_{t+1} \) without asking the exact value of \( x_{t+1}^i \) from every service \( i \), we can turn Algorithm 1 into a privacy-preserving distributed algorithm. This procedure is executed right before the \( z \)-update step and does not affect the properties (e.g., convergence rate) of the ADMM-based distributed algorithm.

Conventional cryptographic techniques for average computation are quite sophisticated [23, 24, 28]. On the other hands, differential privacy approaches do not produce the exact average value [29]. Recently, [30] proposes a novel privacy-preserving average consensus method. This method runs in an iterative manner to find the average value, thus, may not be suitable for our iterative optimization problem. Here, we introduce a simple and efficient technique for privacy-preserving average computation. Specifically, after the \( x \)-update step, each service \( i \) holds a vector \( x_i = (x_{i,1}, \ldots, x_{i,K}) \). The iteration index is removed for simplicity. We need to compute the average vector \( \overline{x} = (\overline{x}_1, \ldots, \overline{x}_K) \) while keeping each vector \( x_i \) known to service \( i \) only. The proposed privacy-preserving average computation algorithm works as follows. First, each service \( i \) selects a random set \( N_i \) of \( n_i \) other services. Then, it generates a set of \( n_i + 1 \) random numbers which includes \( y_i^0 \) and \( y_i^l \) for all \( l \in N_i \) such that sum of these numbers is zero, i.e., \( y_i^0 + \sum_{l \in N_i} y_i^l = 0, \forall i \). Also, \( y_i^l = 0, \forall l \notin N_i \).
Each service $i$ keeps $y_i^0$ and sends $y_i^1$ to the corresponding service $l$ in $N_i$. To enhance security, the message containing $y_i^1$ between service $i$ and $l$ can be encrypted to prevent attackers from eavesdropping. In the next step, each service $i$ waits to receive all the messages from the other services who selected $i$. If the message sent from service $l$ to service $i$ is encrypted, service $i$ has to decrypt the message using the shared key between them to extract $y_i^1$. Then, service $i$ computes $z_i = x_i + y_i^0 + \sum_{l \in N_i} y_l^1$ and sends the resulting vector $z_i$ to the platform. Obviously, service $i$ can also encrypt $z_i$ before sending it to the platform. It can be observed that

$$
\sum_{i} z_i = \sum_{i} \left( x_i + y_i^0 + \sum_{l \in N_i} y_l^1 \right) = \sum_{i} x_i + \sum_{i} y_i^0 + \sum_{l \in N_i} y_l^1 + \sum_{l \in N_i} y_l^1 = \sum_{i} x_i.
$$

Thus, the platform can take the average of all the vectors $z_i$’s that it receives, which is precisely $\bar{x}$. The steps above are summarized in Algorithm 2, which allows us to find the exact average vector $\bar{x}$ without disclosing $x_i$. It is worth noting that the generated noises $y_i^0$, $\forall i$, $l$, cannot be reused. In other words, we need to rerun the privacy-preserving average computation algorithm at every iteration in Algorithm 1 to prevent an attacker to learn $x_i^{t+1} - x_i^t$, which is private information of service $i$. In particular, if the attack can overhear $z_i$ information, she can infer $z_i^{t+1} - z_i^t = (x_i^{t+1} + y_i^0 + \sum_{l \in N_i} y_l^1) - (x_i^t + y_i^0 + \sum_{l \in N_i} y_l^1) = x_i^{t+1} - x_i^t$. Finally, for the fully distributed version of Algorithm 1 (i.e., without the platform), Algorithm 2 can be slightly modified by selecting a service to act as the platform to compute $\bar{x}$. Then, this service will broadcast the $\bar{x}$ to the other services.

Algorithm 2: Privacy-Preserving Averaging

1: Input: $N$ services. Each service $i$ keeps a vector $x_i = (x_{i1}, \ldots, x_{iK})$.
2: Step 1: each service $i$ picks a set $N_i$ of $n_i$ other services.
3: Step 2: each service $i$ locally generates $n_i + 1$ random numbers including $y_i^0$ and $y_i^1$, $\forall l \in N_i$, such that $y_i^0 + \sum_{l \in N_i} y_l^1 = 0$. For every service $l \notin N_i$, $y_l^0 = 0$. Also, service $i$ keeps $y_i^0$ for herself.
4: Step 3: each service $i$ sends $y_i^0$ to service $l \in N_i$ and does not send anything to any service $l \notin N_i$.
5: Step 4: after receiving all $y_i^0$ from other services, service $i$ computes $z_i = x_i + y_i^0 + \sum_{l \in N_i} y_l^1$ and sends $z_i$ to the platform.
6: Step 5: the platform computes $\bar{x} = \frac{1}{N} \sum_i z_i$.
7: Output: The average vector $\bar{x}$.

Observe that $z_i$ is the only message of service $i$ that contains vector $x_i$. However, even if an attacker can overhear and decrypt this message to obtain $z_i$, it is still difficult for her to infer $x_i$. To recover $x_i$, the attacker needs to know $y_i^0$ and $y_i^1$, $\forall l \in N_i$. Note that $y_i^0$ depends on $y_i^1$, $\forall l \in N_i$.

Thus, $x_i$ is revealed only if all the communication channels between service $i$ and the platform as well as the services in $\mathcal{L}_i$ are compromised. Define $P_i$ as the probability that the communication channel between service $i$ and another service (or the platform) is cracked. Let $Q_i(m)$ be the probability that the number of services in set $\mathcal{L}_i \setminus N_i$ is $m$. Then, given $P_i$, the probability that $x_i$ is disclosed can be approximated by $P_i^{n_i+1} \sum_{m=0}^N Q_i(m) P_m$ if the platform is not corrupt, and by $P_i^{n_i} \sum_{m=0}^N Q_i(m) P_m$ if the platform is corrupt.

In summary, by embedding Algorithm 2 into Algorithm 1, we achieve a fully parallel and distributed privacy-preserving algorithm. The additional step of running the privacy-preserving averaging protocol (i.e., Algorithm 2) is independent and does not affect the convergence property of Algorithm 1, which follows directly the standard ADMM algorithm. Obviously, the added privacy-preserving feature comes at a cost, which is mainly the communication cost of exchanging messages among the services in Algorithm 2. This cost decreases as the size of set $N_i$ decreases. For simplicity, assume $n_i = |N_i| = b$, $\forall i$. Then, the communication cost of Algorithm 2 reduces significantly as $b$ decreases. On the other hand, the proposed algorithm becomes more robust as $b$ increases. Thus, there is a tradeoff between the robustness of the privacy-preserving feature and the communication cost. Depending on the specific system and design goal, the platform can decide a suitable value of $b$ to balance between the privacy and communication overhead.

V. Performance Evaluation

A. Simulation Setting

We generate data for a system that consists of 100 FNs and 40 services (e.g., in a metropolitan area network). Each FN is chosen randomly from the set of M4 and M5 Amazon EC2 instance types. There are three types of resources (e.g., CPU, RAM, and bandwidth) associated with each FN. For example, an M4 Amazon EC2 machine of type m4.xlarge has 16 vCPUs, 64 GiB of memory, and 2000 Mbps of bandwidth. It is assumed that each service has the same base demand vector at different FNs. The base demand vectors of different services can be different. For each service, the amount of resource type 1 (vCPU), type 2 (RAM), and type 3 (bandwidth) in its base demand vector is generated randomly in the ranges of [0, 0.5] vCPUs, [0.4, 2.0] GiB, and [10, 50] Mbps, respectively.

The maximum demand of every service is set to be 600 (i.e., maximum number of requests over a certain time period). Hence, all the services have the same utility limit of 600 (i.e., $u_{\text{max}} = u_{\text{max}} = 600, \forall i$). From the generated data, we can normalize the resource capacity of the FNs and compute $a_{i,j,r}$ accordingly, $\forall i, j, r$. By normalization, every resource type at an FN has a capacity of one unit (i.e., $C_{j,r} = 1, \forall j, r$). Then, note that one memory unit of FN 1 may correspond to 64 GiB (e.g., FN 1 is an m4.xlarge instance) while one memory unit of FN 2 corresponds to 160 GiB (e.g., FN 2 is an m4.10xlarge instance). Thus, one unit of the same resource type at different FNs may have different values to a service.

For the sake of clarity in the figures and analysis, in the base case, we consider a small system with 40 FNs and 8 services

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4https://aws.amazon.com/ec2/instance-types/
produce by EG and GEG are the same. The simulations served to illustrate our theoretical results only. To reproduce our test cases and results, interested readers can refer to our code using Matlab and CVX/MOSEK. The simulation was implemented on a Macbook Air 2017 1.8GHz dual-core Intel Core i5 and 8GB of RAM. We utilize the same seed (i.e., same dataset) to run the simulations. Our code can be used to create arbitrary datasets/settings. We have run the simulations using different datasets and observed similar trends and conclusions for all the figures.

**B. Numerical Results**

We consider the following five allocation schemes.

- **Generalized Eisenberg-Gale scheme (GEG):** This is the proposed scheme where the allocation is an optimal solution to the generalized EG program (27)-(31).
- **Eisenberg-Gale scheme (EG):** In this scheme, the resource allocation is an optimal solution to (22)-(24).
- **Proportional sharing scheme (PROP):** Every buyer is allocated a portion of every resource proportional to her budget (i.e., buyer $i$ receives $\frac{B_i}{B} \cdot b_i$ of every resource).
- **Social welfare maximization scheme (SWM):** In this scheme, the platform determines an optimal allocation that maximizes the total utility of all the buyers (i.e., $\sum_i u_i(x_i)$) subject to the resource capacity constraints of the FNs. The budget constraint is not considered.
- **Maximin fairness scheme (MM):** The system tries to maximize the utility of the buyer with the lowest utility (i.e., $\max \min_i u_i(x_i)$) under the FNs’ capacity constraints. This scheme also ignores the budget constraint.

Note that GEG and EG are the two allocation schemes studied in this paper. The others are considered as benchmark schemes. In all the five schemes, the resulting utilities of the buyers are truncated by their corresponding utility limits. From Proposition 3.3 and Theorem 3.4, the solutions in both EG and GEG schemes are market equilibria in our resource allocation problem for buyers with utility limits. Obviously, if all the buyers have infinite demands (i.e., $\ell = \infty$), the solutions produced by EG and GEG are the same.

![Fig. 2: Comparison between EG and GEG schemes](image)

Fig. 2 compares the ME solutions in the EG and GEG schemes. The bars labeled “No Limit” present the buyers’ utilities at the market equilibrium in our resource allocation problem where buyers have no utility limits. Indeed, the buyers’ utilities in the EG scheme are their utilities in the “No Limit” setting truncated by the utility limits. We can observe that when the utility limit is considered, the optimal utilities in the GEG scheme tend to dominate those in the EG scheme. This is because some buyers in the EG scheme receive too many resources while they already reach their utility limits (e.g., buyers 1, 6, and 7), which reduces the resources available to other buyers. Since the GEG scheme takes this issue into account by redistributing the wasteful resources to other buyers, it outperforms the EG scheme. In the EG scheme, if the redundant resources of buyers 1, 6, and 7 are reallocated to the other buyers, it can strictly improve the utilities of some buyers. Thus, the ME produced by the EG scheme is not Pareto-efficient. These results align with our statements in Proposition 3.3.

![Fig. 3: Individual utility comparison](image)

Figs. 3(a)–3(b) present the performance comparison among the five schemes with and without the utility limit. We can see that GEG balances well the tradeoffs between the system efficiency and fairness. Specifically, Figs. 3(a)–3(b) show that the utility of every buyer in GEG is always greater or equal to that in the PROP scheme. Additionally, the total utility of all the buyers in GEG is significantly higher than that in PROP as illustrated in Figs. 3(a)–3(b). These observations imply the sharing-incentive property of the proposed GEG scheme.

Although the SWM scheme gives high utilities for some buyers, it may result in unfair allocations. For example, some buyers have very low or even zero utilities as shown in Figs. 4(a)–4(b). Finally, the MM scheme inherently generates a fair allocation in terms of utility but it may lead to low system efficiency. Also, MM may not be fair in terms of resource quantity allocated to the buyers since some buyers receive too many resources to compensate for their low marginal utilities. In Fig. 4(b), the total utility of the buyers becomes saturated as number of FNs increases. It is because when resources are abundant, all the buyers can reach their utility limits.

![Fig. 4: Comparison of the total utility (N = 8)](image)

Figs. 5(a)–5(b) show the envy-freeness indices of the five schemes. The envy-free index (EF) of an allocation $X$ is 

$$EF(X) = \min_{i \neq i'} \frac{u_i(x_i)}{u_i(b_i, x_i')}, \forall i, i' \in N.$$
An allocation $X$ is envy-free if $\text{EF}(X) = 1$. The higher $\text{EF}$ is, the better an allocation is in terms of envy-freeness. Obviously, PROP is envy-free by definition. We can observe that both EG and GEG are also envy-free, which confirms the envy-freeness property of the proposed GEG scheme as theoretically proved. These figures also reveal that both SWM and MM are not envy-free. Noticeably, SWM may produce a very unfair allocation (e.g., in Fig. 5(b), $\text{EF} = 0$ as $M \leq 40$) when the number of FNs is small (i.e., when the fog resource is scarce). Similarly, given the same amount of fog resources, the EF index of SWM decreases as the number of buyers increases. This result is not shown here due to space limitation. Thus, our proposed ME solution significantly outperforms SW and MM schemes in terms of envy-free fairness.

Figs. 6(a)–6(b) illustrate the proportionality fairness property of the proposed GEG scheme. In particular, $\text{PR}$ of a buyer is defined as the ratio between her actual utility and her maximum possible utility by receiving all the resources (i.e., $\text{PR}_i = u_i(x_i)/u_i(C)$). When there are eight buyers (i.e., $N = 8$) with the same budget, an allocation satisfies the proportionality property if the $\text{PR}$ of every buyer is greater or equal to $B_i/\sum_{k'} B_{k'} = 1/8$. Hence, these figures support our claim that the equilibrium allocation produced by GEG satisfies the proportionality property.

In the following, we study the proposed GEG scheme only. The impact of budget on the utilities of the buyers is presented in Fig. 7 where we vary the budget of buyer 2 and fix the budgets of other buyers to be one (i.e., $B_1 = B_3 = B_4 = B_5 = B_6 = B_7 = B_8 = 1$). It can be seen that with and without considering the utility limit, when we double the budget of buyer 2, her utility increases significantly while the utilities of the other buyers tend to decrease. Thus, the proposed scheme is effective in capturing the service priority in making allocation decision.

Figs. 8(a)–8(b) depict the utilization of different resource types at several FNs. As we can see, the proposed scheme produces an ME with high resource utilization. Additionally, at least one resource type at every FN is fully utilized at the equilibrium. Thus, the remaining resources cannot improve the utility of any buyer and the allocation is efficient and non-wasteful. Note that non-fully utilized resources have zero prices at the equilibrium. Also, as the number of buyers $N$ increases, the resource utilization tends to increase.

Finally, the convergence of Algorithm 1 is illustrated in Figs. 9(a)–9(d) for different system sizes. Here, we set the initial values of $x_{i,k}$ and $z_c$ to be $\frac{1}{N}$ as in the proportional sharing scheme, and $p_k^i = 1$, $\forall k$. As we can see, the algorithm converges to the optimal solution within a reasonable number of iterations, which implies the effectiveness and practicality of the proposed decentralized implementation. For instance, for a system with 100 FNs and 20 services, it takes about 130 iterations for convergence (less than 1s for each iteration).

VI. RELATED WORK

FC and related concepts like EC and cloudlet have drawn a lot of attention recently. In [31], A. Ceselli et al. present a comprehensive mobile edge-cloud framework considering various design factors including cloudlet placement, user mobility, and service-level agreement. In [32], the authors jointly optimize task image placement and task scheduling in a fog...
network with dedicated storage and computing servers to minimize the task completion time. By exploiting the Priced Timed Petri Nets concept, [33] proposes a dynamic resource allocation strategy that assists users to select suitable fog resources from a group of pre-allocated resources. In [34], the authors formulate a workload allocation problem in a hybrid fog-cloud system, which aims to minimize the energy cost under latency constraints. The online primal-dual approach is employed in [35] to examine the edge resource crowdsourcing problem. Another major line of research considers the joint allocation of communication and computational resources for task offloading in wireless networks [2]. Different from the existing literature, we study FC from the market design and algorithmic game theory perspectives [5], with a specific focus on pricing design and resource allocation in a multi-FN multi-buyer environment.

Indeed, cloud resource allocation and pricing have been studied extensively in the literature [7]. In [36], a dynamic pricing scheme is introduced to maximize the cloud provider’s revenue. The resource and profit sharing problem among providers in a cloud federation is investigated in [37]. In [38], the interaction between cloud providers and multiple services is modeled as a generalized Nash game. Reference [39] formulates the single-cloud multi-service resource provisioning and pricing problem as a Stackelberg game that aims to maximize the provider’s revenue while minimizing the services’ costs. Additionally, various auction models have been proposed to study cloud resource allocation [40]–[42] with the goal of maximizing the social welfare or the cloud provider’s profit. Typically, only winners in an auction receive resources. Also, most of the existing auction models do not consider flexible demands [41]. For instance, bidders are often assumed to be single-minded, who are only interested in a specific bundle and have zero value for other bundles. Unlike the existing works on cloud economics and resource allocation, we examine a market with multiple nodes and budget-constrained buyers. This model captures practical aspects such as requests can be served at different nodes and the buyers’ demands can be defined flexibly. More importantly, the new ME solution concept, which optimizes both the buyers’ utilities and the resource utilization, is the salient feature of our work.

This work is also closely related to the MRA literature, which has received considerable attention recently. The state-of-the-art research on MRA for data centers is surveyed in [17]. Noticeably, Dominant Resource Fairness (DRF) [19] is the most prominent approach to MRA fairness in data centers. In a nutshell, DRF is a generalization of max-min fairness for multiple resources, which aims to maximize the minimum dominant resource share across all the users. For the simplistic setting where all resources are pooled together in a single location and with some strong assumptions [19], DRF offers many compelling properties. This approach has been extended in the follow-up works [16], [20]–[22] to address more realistic assumptions such as user demands are finite and resources are distributed over multiple nodes. Both of these aspects are captured in our work. To the best of our knowledge, this is the first comprehensive work that employs the Fisher market model and competitive equilibrium to address the MRA problem in a general setting with multiple nodes, each of which has multiple resource types, and multiple users with finite demands. While similar market-based approaches have been examined in [16], [43], they consider simpler settings (e.g., a single resource pool, a single resource type, and/or infinite demands).

Finally, although numerous distributed optimization techniques [13], [14], [25]–[27] are available in the literature, most of them require agents to exchange and reveal their estimates explicitly to neighboring agents in every iteration to reach consensus on the final optimal solution. We are among the first to propose a privacy-preserving parallel and distributed algorithm, which converges to the optimal solution without asking the agents to reveal any private data. Unlike differential privacy based methods [29] that add carefully-designed noises to cover sensitive information and are subject to a trade-off between privacy and accuracy, our proposed scheme enables privacy preservation without sacrificing accuracy.

Another approach to enable data privacy is to employ cryptographic techniques such as Garble Circuit (GC) [23] and Fully Homomorphic Encryption (HE) [28], which are often used in secure multi-party computation [24]. However, HE is computationally expensive due to public key operations and GC has expensive communication costs. Hence, these cryptographic approaches are not suitable for distributed optimization which typically needs many iterations to converge. Indeed, [44] is the only work that we are aware of studying privacy-preserving decentralized optimization. Different from their proposed algorithm that based on ADMM and partially homomorphic cryptography, our construction of the privacy-preserving part is non-cryptographic and simpler.

VII. Conclusion

In this work, we introduced a new solution approach for allocating multiple resource types of different FNs to competing services with diverse priorities and preferences. The proposed solution produces a non-wasteful and frugal ME that makes every service happy with the allocation decision while maximizing the resource utilization efficiency. Also, we showed that the equilibrium has appealing fairness properties including envy-freeness, sharing-incentive, and proportionality, which encourages the services to engage in the proposed scheme. Furthermore, a privacy-preserving distributed algorithm was developed to compute the ME while obfuscating the private information of the services, which significantly limits the strategic capabilities of the market participants. The proposed framework has potential to be applied in other settings such as allocating virtual machines in data centers to cloud users and allocating virtual network functions and middleboxes to different network slices (e.g., consider each network function at a network node as a resource type and each slice as a buyer).

REFERENCES

[1] M. Chiang and T. Zhang, “Fog and IoT: an overview of research opportunities,” IEEE Internet Things J., vol. 3, no. 6, pp. 854–864, Dec. 2016.
[2] Y. Mao, C. You, J. Zhang, K. Huang, and K.B. Letaief, “A survey on mobile edge computing: the communication perspective,” IEEE Commun. Surv. Tut., vol. 19, no. 4, pp. 2322–2358, Fourthquarter 2017.
[3] D.T. Nguyen, L.B. Le, and V. Bhargava, "Price-based resource allocation for edge computing: a market equilibrium approach", IEEE Trans. Cloud Comput., to appear.
APPENDIX

A. Proof of Proposition 3.1

The utility function $u_i(x_i)$ of service $i$ with unlimited demand is given in (21). It is easy to verify the concavity of $u_i(x_i)$ by checking the definition of a concave function [13]. Furthermore, for every $\alpha > 0$, we have

$$u_i(\alpha x_i) = \sum_j \min_r \frac{x_{i,j,r}}{a_{i,j,r}} - \alpha \sum_j \sum_r x_{i,j,r} = u_i(x_i),$$

which confirms $u_i(x_i)$ is homogeneous of degree one.

B. The KKT Conditions

Consider the problem (27)-(31). The Lagrangian is

$$L(X, u, \lambda, \mu, p, \gamma) = \sum_i B_i \ln \sum_j u_{i,j},$$

$$+ \sum_j \mu_j (\sum_i u_{i,j} - \sum \mu_i x_{i,j,r}) + \sum_j p_{j,r} (1 - \sum_i x_{i,j,r})$$

$$+ \sum_{i,j,r} \lambda_{i,j,r} (x_{i,j,r} - u_{i,j} a_{i,j,r}) + \sum_{i,j,r} x_{i,j,r} \gamma_{i,j,r}. \quad (53)$$

The KKT conditions of the problem (27)-(31) include

$$\frac{\partial L}{\partial u_{i,j}} = \frac{B_i}{\sum_j u_{i,j}} - \sum_r \lambda_{i,j,r} a_{i,j,r} - \mu_j = 0, \forall i,j \quad (54)$$

$$\frac{\partial L}{\partial x_{i,j,r}} = \lambda_{i,j,r} + \gamma_{i,j,r} - p_{j,r} = 0, \forall i,j,r \quad (55)$$

$$u_{i,j} a_{i,j,r} = x_{i,j,r}, \forall i,j,r; \quad \mu_j (u_1^{\text{max}} - \sum_j u_{i,j}) = 0, \forall i \quad (56)$$

and primal feasibility conditions.

Since $u_i = \sum_j u_{i,j}$, from (54), we have

$$\forall i,j : \frac{B_i}{u_i} - \sum_r \lambda_{i,j,r} a_{i,j,r} - \mu_j = 0. \quad (59)$$

From (55), we have:

$$\lambda_{i,j,r} = p_{j,r} - \gamma_{i,j,r}, \forall i,j,r. \quad (60)$$

Since $\gamma_{i,j,r} \geq 0$, $\forall i,j,r$, we have

$$\forall i,j,r : \lambda_{i,j,r} \leq p_{j,r}. \quad (61)$$

Also, from the second equality in (57), if $x_{i,j,r} > 0$, then $\gamma_{i,j,r} = 0$. Combined with (60), we have

$$\forall i,j,r : \lambda_{i,j,r} = p_{j,r} \quad (62)$$

The first equality in (57) implies

$$\forall j,r : p_{j,r} > 0 \Rightarrow \sum_i x_{i,j,r} = 1 \quad (63)$$

$$\forall j,r : \sum_i x_{i,j,r} < 1 \Rightarrow p_{j,r} = 0 \quad (64)$$

Finally, from the second equality in (56), we have

$$\forall i : \mu_i > 0 \Rightarrow u_i = \sum_j u_{i,j} = u_i^{\text{max}}. \quad (65)$$

The equations above, including (59), (61)-(62), (65) and the first equation in (57), are exactly equations (32)-(36) in the paper. Therefore, we can infer (32)-(36) from the KKT conditions of the problem (27)-(31).

C. Proof of Theorem 3.6

- Envy-freeness: To prove that the allocation $X^*$ is envy-free, we need to show that $u_i(x_i^*) \geq u_i(\frac{B_i}{B_{i'}} x_{i'}^*)$, $\forall i, i'$. Since $x_i^*$ is the optimal resource bundle of service $i$ at the equilibrium, service $i'$ can afford to buy bundle $x_i^*$ at prices $p^*$. Hence, we have:

$$\sum_j \sum_r p_{j,r} \left( \frac{B_i}{B_{i'}} x_{i,j,r}^* \right) \leq B_i, \quad \forall i. \quad (66)$$

Therefore, bundle $\left( \frac{B_i}{B_{i'}} x_{i,j,r}^* \right)$ is affordable to service $i$ at the equilibrium prices $p^*$. However, $x_i^*$ is the favorite bundle of service $i$ at the equilibrium. Hence, we have: $u_i(x_i^*) \geq u_i(\frac{B_i}{B_{i'}} x_{i,j,r}^*)$, $\forall i, i'$.

- Sharing-incentive: At the equilibrium, no service spends more than its budget, i.e., $\sum_j \sum_r p_{j,r} x_{i,j,r}^* \leq B_i, \forall i$. Hence,

$$\sum_i \sum_j \sum_r p_{j,r} x_{i,j,r}^* \leq \sum_i B_i. \quad \text{(In other words, we have $\sum_i \sum_j \sum_r p_{j,r} x_{i,j,r}^* \leq \sum_i B_i$. As shown before, if $\sum_i x_{i,j,r}^* < 1$, then $p_{j,r} = 0$. Also, if $p_{j,r} > 0$, then $\sum_i x_{i,j,r}^* = 1$. Therefore, we have)}$$

$$\sum_i \sum_j \sum_r p_{j,r} x_{i,j,r}^* \leq \sum_i B_i.$$

Consequently, the resource bundle $x_i^*$ costs service $i$:

$$\sum_j \sum_r x_{i,j,r} p_{j,r}^* = \sum_j \sum_r \frac{B_i}{B_{i'}} p_{j,r}$$

$$= \frac{B_i}{\sum_i B_i} \sum_j \sum_r p_{j,r} \leq \frac{B_i}{\sum_i B_i} \sum_i B_i = B_i, \forall i.$$ 

Therefore, service $i$ can afford to buy bundle $x_i^*$ at prices $p^*$. However, given the set of affordable (i.e., feasible) resource bundles to service $i$, her favorite (i.e., utility-maximizing) one is $x_i^*$. Thus, $u_i(x_i^*) \geq u_i(x_i)$, $\forall i$.

- Proportionality: By definition

$$\frac{B_i}{\sum_i B_i} u_i(C) = \frac{B_i}{\sum_i B_i} \min \left\{ \sum_j \min_r \frac{1}{a_{i,j,r}} x_{i,j,r}^{\text{max}} \right\}$$

$$= \min \left\{ \sum_j \min_r \frac{B_i}{a_{i,j,r}} x_{i,j,r}^{\text{max}} \right\}.$$ 

From the sharing-incentive property, we have

$$u_i(x_i^*) \geq u_i(\frac{B_i}{B_{i'}} C)$$

$$= \min \left\{ \sum_j \min_r \frac{B_i}{a_{i,j,r}} x_{i,j,r}^{\text{max}} \right\}$$

$$\geq \min \left\{ \sum_j \min_r \frac{B_i}{a_{i,j,r}} x_{i,j,r}^{\text{max}} \right\}.$$ 

Hence, $u_i(x_i^*) \geq \frac{B_i}{\sum_i B_i} u_i(C)$. Consequently, the equilibrium allocation satisfies the proportionality property.
D. Derivation of Algorithm 1

The augmented Lagrangian of the problem (48)-(49) is

\[
L(x, z, p) = \sum_i f_i(x_i) + g(\sum_i z_i) + \sum_i \left( p_i^T (x_i - z_i) + (\rho/2)||x_i - z_i||^2_2 \right).
\]

The ADMM updates are

\[
x_i^{t+1} := \arg\min_{x_i} \left( f_i(x_i) + (\rho/2)||x_i - z_i^t + w_i^t||^2_2 \right)
\]

\[
z_i^{t+1} := \arg\min_{z_i} \left( g(\sum_i z_i) - \sum_i (p_i^t)^T z_i + \sum_i (\rho/2)||z_i - x_i^{t+1}||^2_2 \right)
\]

\[
p_i^{t+1} := p_i^t + \rho(x_i^{t+1} - z_i^{t+1}),
\]

where \(t\) is the iteration index and \(z = (z_1, z_2, \ldots, z_N)\). Define \(w_i = p_i/\rho\), which is called scaled dual variable, we have the following scaled form of ADMM

\[
x_i^{t+1} := \arg\min_{x_i} \left( f_i(x_i) + (\rho/2)||x_i - z_i^t + w_i^t||^2_2 \right)
\]

\[
z_i^{t+1} := \arg\min_{z_i} \left( g(\sum_i z_i) + (\rho/2)||z_i - x_i^{t+1} - w_i^t||^2_2 \right)
\]

\[
w_i^{t+1} := w_i^t + x_i^{t+1} - z_i^{t+1}.
\]

From the definition of \(f_i(x_i)\), we can write

\[
x_i^{t+1} := \arg\min_{x_i} \left( v_i(x_i) + (\rho/2)||x_i - x_i^t + \bar{x}^t - \bar{z}^t + w^t||^2_2 \right)
\]

subject to \(x_i \in \mathcal{X}_i\).

By replacing \(w\) with \(\frac{1}{\rho}p\), we obtain Algorithm 1.

E. Computational Complexity of the Centralized Solution

In the centralized solution, we need to solve the convex program (27)-(31). Indeed, convex optimization problems can be solved effectively by efficient techniques such as Interior Point Methods [13]. For example, the optimality gap of the solution computed by the barrier method, which is an Interior Point Method, after \(k\) centering steps is smaller or equal to \(m/(\mu k t(0))\) where \(m\) is the number of inequality constraints and \(\mu\) and \(t(0)\) are constants (see [52] for more details about convergence analysis, especially page 18 in the barrier method lecture). In other words, to reach a desired accuracy level of \(\epsilon\), it requires \(\log(m/(\mu k t(0)\epsilon))/\log\mu\) centering steps. Note that the Newton’s method (see [13] or the Newton’s method lecture in [52]) is used in each centering step.

Note that there exist many efficient convex optimization solvers. In this work, we used CVX with MOSEK solver option [45] to solve the formulated convex optimization problem. The computational time under different system sizes (all with three resource types) is reported in Fig. 10. For each system size, we run 10 simulations with different generated data and take the average computational time. As we can observe, the optimal solution can be computed quite efficiently. For example, for a system with 200 buyers (\(N = 200\)) and 100 FNs (\(M = 100\)), it takes less than 4 seconds to obtain the optimal solution. For moderate system sizes, the optimal solution can be found in less than 1 second.

F. Example with Two Buyers and One Single-resource FN

In addition to the example presented in the paper (see Fig. 2), here we present a simple example with two buyers and a single-resource FN (e.g., consider computing resource only) to illustrate the wasteful equilibrium allocation phenomenon. The example is summarized in Fig. 11. In particular, the budgets of the buyers are \(B_1 = B_2 = 1\). The utility limits of the buyers are \(u_1^{max} = 1\) and \(u_2^{max} = 10\). Additionally,
$a_1 = 0.2$ and $a_2 = 0.1$ (i.e., one request of buyer/service 1 needs 0.2 unit of CPU). Hence, we have the utility functions:

\[
\begin{align*}
    u_1(x_1) &= \min\left\{ \frac{x_1}{a_1}, 1 \right\} = \min(5 \times x_1, 1) \\
    u_2(x_2) &= \min\left\{ \frac{x_2}{a_2}, 10 \right\} = \min(10 \times x_2, 10)
\end{align*}
\]

The optimal solution to the problem (22)–(24) (i.e., the EG scheme mentioned in the simulation) gives us the ME: $p = 2$, $x_1 = x_2 = 0.5$, and $u_1 = 1, u_2 = 5$. At the equilibrium, both buyers exhaust their budget since $p \times x_1 = 1 = B_1$ and $p \times x_2 = 1 = B_2$. Obviously, $(p, x)$ where $p = 2$ and $x = (x_1, x_2) = (0.5, 0.5)$ is an ME to our model since both buyers maximize their utility at the equilibrium price, the fog resource is fully allocated to the buyers, and equilibrium price is positive (i.e., $p = 2$). It is easy to observe that buyer 1 receives more resources than he needs (i.e., $5 \times x_1 = 2.5 > 1 = u_1^{\max}$). Thus, if we can reallocate the redundant resource of buyer 1 to buyer 2, it will not affect the utility of buyer 1 while improving the utility of buyer 2.

For instance, the GEG scheme as mentioned in the simulation (i.e., the optimal solution to the problem (27)–(31)) results in a new ME as follows: $p = 1.25, x = (x_1, x_2) = (0.2, 0.8)$. This is obviously an ME since at the positive price $p = 1.25$, the fog resource is fully allocated (i.e., market clearing), and both buyers maximize their respective utilities at this price. At this new ME, the utilities of the buyers become: $u_1 = 1$ and $u_2 = 8$, which are weakly better than the utilities in the previous ME computed by the EG scheme. There is no wasteful resources since $5 \times x_1 = 1 = u_1^{\max}$ and $u_2 = 8 < u_2^{\max}$. It is also worth mentioning that buyer 1 has a surplus budget of $1 - 0.2 \times 1.25 = 0.75$ at this new ME.

Note that there exist other ME in this example, which are not the optimal solutions to both the problem (22)–(24) (i.e., the EG scheme) and (27)–(31) (i.e., the GEG scheme). For instance, $p = 1.6, x = (x_1, x_2) = (0.375, 0.625)$. Clearly, at price $p = 1.6, x_2 = 0.625$ maximizes the utility of buyer 2 subject to the budget constraint of this buyer. Also, at this price, $x_1$ maximizes the utility of buyer 1 and satisfies his budget constraint. At the positive price $p = 1.6$, the fog resource is fully allocated to the buyers (i.e., $x_1 + x_2 = 1$). Therefore, $(p, x) = (1.6, (0.375, 0.625))$ is another ME in this example. At this equilibrium, we have $u_1 = 1$ and $u_2 = 6.75$. Buyer 2 fully spends his budget while buyer 1 has a budget surplus of $1 - 1.6 \times 0.325 = 0.48$ at this equilibrium.

**G. Example with Two Buyers and Two Single-resource FNs**

In the following, we present another toy example to show the wasteful equilibrium allocation issue in Proposition 3.3. Consider a market with two services and two single-resource FNs. Obviously, the utilities of the services become linear functions in the single-resource setting. Assume $u_1(x_1) = \min\{8 \times x_{1,1} + 2 \times x_{1,2}, 1\}, u_2(x_2) = 5 \times x_{2,1} + 2 \times x_{2,2}, B_1 = 3$, and $B_2 = 1$. The optimal solution to the convex program (23)–(24) gives all resource of FN1 to service 1 and all resource of FN2 to service 2 (i.e., $x_{1,1} = 1$ and $x_{2,2} = 1$) and the prices are $p_1 = 3$ and $p_2 = 1$. It is easy to verify that this price vector and allocation form an ME. At the equilibrium, $u_1 = 1$ and $u_2 = 2$. Since $8 \times x_{1,1} + 2 \times x_{1,2} = 8 > 1$, service 1 receives more resources than it needs. Consider another ME where $p = (p_1, p_2) = (0.7843, 0.3137), x_1 = (x_{1,1}, x_{1,2}) = (0.125, 0)$, $x_2 = (x_{2,1}, x_{2,2}) = (0.875, 1)$ (i.e., we reallocate some redundant resources of service 1 in the previous ME to service 2). At the new ME, $u_1 = 1$ and $u_2 = 6.375$. Obviously, this new equilibrium allocation dominates the previous equilibrium allocation.

**H. Discussion of Frugal Market Equilibria**

Frugality is a natural and desirable property for any market mechanism. Clearly, to obtain the same utility, a service prefers to spend as least money as possible. For example, consider two single-resource FNs with prices $p_1 = 10$ and $p_2 = 1$. Assume that service A has the same utility for each FN (e.g., $u_1(x_1) = 2 \times x_{1,1} + 2 \times x_{1,2}$). Obviously, given the prices, service A will naturally buy resource of FN2 since it is cheaper. However, a non-frugal ME may force service A to buy resource of FN1. In this case, service A has to spend more money to obtain the same utility, which may make it not happy.

Here we give a toy example of a non-frugal ME. The market consists of two services and two single-resource FNs with the same setting as in Appendix G. Beside the market equilibria discussed in Appendix G we consider the following price vector and allocation: $p = (p_1, p_2) = (1, 2)$, $x_1 = (x_{1,1}, x_{1,2}) = (0, 1), x_2 = (x_{2,1}, x_{2,2}) = (1, 0)$, and $u = (u_1, u_2) = (1, 5)$. We can observe that service 1 obtains its optimal utility which is equal to its utility limit of 1. Also, service 2 spends full budget $(x_{2,1} p_1 + x_{2,2} p_2 = 1 = B_2)$ to buy resource of the cheapest FN. Note that FN1 is the cheapest FN for service 2 and FN2 for service 1. However, this ME is not frugal since at the equilibrium, service 1 receives full resource of FN2 while the cheapest FN for service 1 is FN1 because $\frac{8}{p_1} > \frac{5}{p_2}$.
I. Discussion of the Privacy Property in Market Design

In an FC market, or in any market in general, it is desirable for market participants to keep their information private. For instance, if the private data of an agent is revealed, other agents can exploit it to gain benefits. To illustrate this issue, consider an example with two services and two single-resource FNs. The services have the same budget of one. The utility functions of the services are $u_1(x_1) = 4 \times x_{1,1} + 1 \times x_{1,2}$, and $u_2(x_2) = 4 \times x_{2,1} + 4 \times x_{2,2}$, respectively. The GEG scheme produces the following ME: $p = (p_1, p_2) = (1, 1), x_1 = (x_{1,1}, x_{1,2}) = (1, 0), x_2 = (x_{2,1}, x_{2,2}) = (0, 1)$, and $u = (u_1, u_2) = (4, 4)$.

Now assume that service 2 knows the utility of service 1 and knows that service 1 prefers FN1 to FN2. Then, service 2 can pretend that its utility is $u_2(x_2) = 12 \times x_{2,1} + 4 \times x_{2,2}$ (i.e., pretend to be more interested in FN1). The new ME computed by the GEG scheme is: $p = (p_1, p_2) = (3, 2), x_1 = (x_{1,1}, x_{1,2}) = (0.75, 0), x_2 = (x_{2,1}, x_{2,2}) = (0.25, 1)$, and $u = (u_1, u_2) = (3, 5)$. We can see that $u_2$ increases while $u_1$ decreases. Service 1 receives less resources because FN1 becomes more expensive. Indeed, service 2 has many options to manipulate its utility to gain benefit. For example, it can state its utility is $u_2(x_2) = 12 \times x_{2,1} + 4 \times x_{2,2}$ (i.e., pretend to be more interested in FN1). The new ME becomes: $p = (p_1, p_2) = (1.5, 0.5), x_1 = (x_{1,1}, x_{1,2}) = (0.6666, 0), x_2 = (x_{2,1}, x_{2,2}) = (0.3333, 1)$, and $u = (u_1, u_2) = (2.6666, 5.3333)$. This new ME significantly increases the utility of service 2 while drastically reducing the utility of service 1.

Therefore, preserving the privacy of the agents is an important and desirable feature of any market mechanism. It is worth noting that the benefit that an agent can gain by playing strategically in a Fisher market when he knows the private information of the other agents has been studied in [30], [54], [55].

J. Statistical Simulation Results over Multiple Runs

For the ease of explanation, in the main paper, we randomly generated the simulation data for one problem instance only. To further demonstrate the efficacy of the proposed resource allocation algorithm, for every experiment, we redo the simulation over 50 runs (i.e., 50 problem instances) to obtain the statistical results including the mean values and/or boxplot statistics.

We can observe that the shapes of the figures and the observations in the Performance Evaluation section remain unchanged. For example, as can be seen from Figs. 12(a)–12(b) (i.e., Fig. 12 in the main paper), the GEG scheme outperforms the EG scheme. The boxplots in Figs. 13(a)–13(b) (i.e., Figs. 5(a)–5(b) in the main paper) show that the GEG and EG schemes are the same when we do not consider the utility limit and the GEG scheme dominates the EG scheme when the utility limit is considered. Furthermore, the utility of every buyer in the PROP scheme is smaller or equal to that in the GEG scheme, which implies the sharing-incentive property of the proposed GEG scheme.

The MM scheme produces a fair allocation in terms of utility at the cost of low system efficiency (i.e., total utility). Figs. 14(a)–14(b) (i.e., Figs. 4(a)–4(b) present total utility comparison among the five schemes. Finally, we can observe that the SWM scheme has the highest total utility and gives high utilities for the buyers. However, it performs poorly in terms of fairness. Figs. 13(a)–13(b) show that the utility of each buyer varies significantly over 50 runs. Indeed, for each problem instance, there always exist some buyers with very low (even zero) utilities. Furthermore, Figs. 5(a)–5(b) show that SWM has bad envy-free index. Similar to Figs. 6(a)–6(b) in the main paper, Figs. 7(a)–8(b) indicate the proportionality of the proposed resource allocation scheme.

The impact of the budget on the equilibrium utilities is
presented in Figs. 19(a)–19(b) (i.e., Fig. 7). We can observe that when the budget of buyer 2 increases, his utility increases significantly while the utilities of the other buyers tend to decrease. Hence, the proposed scheme can effectively capture the service priority in the allocation decision. Finally, Figs. 20(a)–20(d) (i.e., Figs 8(a)–8(b)) reveal that the proposed scheme produces an ME with high resource utilization and the resource utilization tends to increase as the number of buyers N increases. It is worth mentioning that for each problem instance, each FN has one resource type that is fully utilized. However, the fully-utilized resource type of an FN is not necessarily the same for every run, which explains why the mean values of resource utilization of all resource types of some FNs (e.g., FN5 in 20(a)) are less than 1.