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Critical divergence of the symmetric \((A_{1g})\) nonlinear elastoresistance near the nematic transition in an iron-based superconductor

J. C. Palmstrom\(^1\), A. T. Hristov\(^1\), S. A. Kivelson\(^2\), J.-H. Chu\(^4\) and I. R. Fisher\(^1\)

\(^1\)Geballe Laboratory for Advanced Materials and Department of Applied Physics, Stanford University, Stanford, CA 94305, USA.
\(^2\)Geballe Laboratory for Advanced Materials and Department of Physics, Stanford University, Stanford, CA 94305, USA.
\(^3\)Stanford Institute for Materials and Energy Science, SLAC National Accelerator Laboratory, 2575 Sand Hill Road, Menlo Park, CA 94025, USA.
\(^4\)Department of Physics, University of Washington, Seattle WA 98195, USA.

We report the observation of a nonlinear elastoresistivity response for the prototypical underdoped iron pnictide \(\text{Ba(Fe}_{0.975}\text{Co}_{0.025})_{2}\text{As}_2\). Our measurements reveal a large quadratic term in the isotropic \((A_{1g})\) electronic response that was produced by a purely shear \((B_{2g})\) strain. The divergence of this quantity upon cooling towards the structural phase transition reflects the temperature dependence of the nematic susceptibility. This observation shows that nematic fluctuations play a significant role in determining even the isotropic properties of this family of compounds.

**Usage:**

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I. INTRODUCTION

Nonlinear responses of crystalline materials are described by high rank tensors and can therefore provide valuable information concerning subtle phase transitions and broken symmetries. For example, previous nonlinear measurements of tensor properties have revealed interesting transitions in several strongly correlated materials\(^1\) to \(^3\). Here we demonstrate a new type of nonlinear transport response, associated with changes in the conductivity of a material in response to strain: nonlinear elastoresistivity. This technique allows us to not only look at broken symmetries across a phase transition, but to characterize properties of the disordered state. We perform these measurements for a representative underdoped Fe-based superconductor, \(\text{Ba(Fe}_{0.975}\text{Co}_{0.025})_{2}\text{As}_2\), which has previously been shown to exhibit a large nematic susceptibility for temperatures above a tetragonal-to-orthorhombic structural phase transition\(^4\) to \(^12\). The most remarkable aspect of the current data is that they reveal a diverging nonlinear response in the isotropic elastoresistivity in response to a perfectly antisymmetric (shear) strain. This observation, which is intimately tied to the large nematic susceptibility of the material studied, serves to underscore the role played by nematic fluctuations in determining even the isotropic properties of the Fe-based superconductors.

Elastoresistivity relates changes in the resistivity \((\Delta \rho = \rho(\epsilon) - \rho(\epsilon = 0))\) to strains \((\epsilon)\) experienced by a material:

\[
\frac{\Delta \rho}{\rho_0} = \sum_{\alpha, \alpha'} (m_{\alpha} \epsilon_\alpha + m_{\alpha}^{\alpha'} \epsilon_\alpha \epsilon_{\alpha'} + ...) \tag{1}
\]

where the \(\alpha\)'s represent a complete, orthogonal basis set for the system, \(\epsilon_\alpha\) is the component of the overall strain along a given basis vector, and \(\rho_0\) is an appropriate normalization factor\(^1\): here, the in-plane resistivity of the tetragonal phase. A natural basis to work in is the irreducible representations of the crystallographic point group. In the absence of a magnetic field and in the \(D_{4h}\) point group (appropriate for the material studied here), both strain and \(\Delta \rho/\rho_0\) have six independent components. Of these, four unique combinations correspond to distinct representations: \((\Delta \rho/\rho_0)_{B_{1g}} = \frac{1}{2}((\Delta \rho/\rho_0)_{xx} - (\Delta \rho/\rho_0)_{yy}), (\Delta \rho/\rho_0)_{B_{2g}} = (\Delta \rho/\rho_0)_{xy}, \) and \((\Delta \rho/\rho_0)_{E_{g}} = ([\Delta (\rho/\rho_0)]_{zz}, (\Delta \rho/\rho_0)_{yz}).\) Objects with \(B_{1g}\) and \(B_{2g}\) symmetry are antisymmetric (odd) with respect to a 90° rotation about the \(z\)-axis. There is also a two-dimensional space of components belonging to the \(A_{1g}\) representation, the basis of which is not uniquely defined by symmetry alone\(^5\). Objects with \(A_{1g}\) symmetry are symmetric (even) with respect to a 90° rotation around the \(z\)-axis. In this paper we focus on one (of the two) components with \(A_{1g}\) symmetry reflecting the in-plane changes in resistivity i.e. \((\Delta \rho/\rho_0)_{A_{1g}} = \frac{1}{2}((\Delta \rho/\rho_0)_{xx} + (\Delta \rho/\rho_0)_{yy}).\)

The linear elastoresistivity response is described by a fourth rank tensor, which in the present basis corresponds to \(m_{\alpha}^{\alpha'}.\) As shown previously, \(m_{B_{1g}}\) and \(m_{B_{2g}}\) are proportional to the nematic susceptibility in the corresponding symmetry channels, \(\chi_{B_{1g}}\) and \(\chi_{B_{2g}}\)\(^4\) to \(^12, 14\). To linear order, correctly decomposed symmetry channels cannot mix. For example, for a tetragonal material, antisymmetric strain \((\epsilon_{B_{1g}} + \epsilon_{B_{2g}})\) cannot cause a symmetric resistivity response, i.e. \(m_{A_{1g}}^{B_{1g}} = m_{A_{1g}}^{B_{2g}} = 0.\) However, this is not true when considering the nonlinear response. In the present work, we demonstrate the presence of a large and strongly temperature dependent nonlinear \(A_{1g}\) elastoresistivity in response to antisymmetric \(B_{2g}\) strain (i.e. we show that \(m_{A_{1g}}^{B_{2g}} \gg 1\)). We further show that this behavior reflects the diverging nematic susceptibility of the material.
II. EXPERIMENTAL METHODS

Measuring the elasto-resistance in the $A_{1_g}$ symmetry channel presents several technical challenges. In order to precisely decompose the elasto-resistance response into the isotropic and antisymmetric components, the resistivity in two orthogonal directions must be measured simultaneously for identical strain conditions; otherwise, the $B_{2g}$ elasto-resistance (which for these materials is much larger than the $A_{1g}$ elasto-resistance response) gets admixed. A second important consideration is that to confidently extract the linear and quadratic $A_{1g}$ elasto-resistance coefficients, the sample must be close to conditions of neutral anisotropic strain ($\epsilon_{x'x'} - \epsilon_{y'y'} \approx 0$; here the primed coordinate frame refers to the normal strain frame\cite{14}). As we demonstrate, a modified Montgomery technique is especially suitable for both purposes\cite{3}.

The crystals are cut into thin square plates with the electrical contacts made at the four corners, enabling measurement of $\rho_{x'x'}$ and $\rho_{y'y'}$ simultaneously while the crystal is held under a measured set of strain conditions. The $B_{2g}$ neutral strain point is determined by the condition of $\rho_{x'x'} = \rho_{y'y'}$, since for a crystal with tetragonal symmetry the in-plane resistivity is isotropic if there is zero anisotropic strain.

In our experimental setup, we apply biaxial stress to the samples by affixing them to a lead-zirconate-titanate (PZT) stack (Part No.: PST150/5x5/7 cryo 1, from Piezomechanik GmbH). When positive voltage is applied to the PZT stack, it expands along its poling axis (the $y'$ axis) and contracts along the perpendicular axis (the $x'$ axis). For thin samples, the crystal deforms with the PZT stack. The ratio of the strain experienced by the sample along the $y'$ and $x'$ axes is dictated by the in-plane Poisson ratio, $\nu_P$, of the PZT stack ($\epsilon_{y'y'} = -\nu_P \epsilon_{x'x'}$). This is a weakly temperature dependent quantity, with an average value for our PZT stacks of $\sim 2.3$. Since the magnitude of strains along the $x'$ and $y'$ directions are not equal, the strain can be decomposed into two parts: a part that is even with respect to rotation by $90^\circ$ about the $z$-axis (in-plane $A_{1g}$ symmetry; $\epsilon_{A_{1g}} = \frac{1}{2}(\epsilon_{x'x'} + \epsilon_{y'y'})$), and an odd part ($B_{1g/2g}$ symmetry; $\epsilon_{B_{1g/2g}} = \frac{1}{2}(\epsilon_{x'x'} - \epsilon_{y'y'})$). As shown in the inset of Fig. 1, by aligning the samples square edges along either the tetragonal [100] or tetragonal [110] direction, we selectively cause the material to experience $A_{1g} + B_{1g}$ symmetry strain (pink) or $A_{1g} + B_{2g}$ symmetry strain (blue). Further details about the sample preparation, experimental protocol, and characterization of the strain transmission can be found in Appendices A and B.

III. RESULTS AND DISCUSSION

There is a qualitative difference in the strain-dependence of the elasto-resistivity between samples that experience $B_{1g}$ and $B_{2g}$ symmetry strain. Fig. 1 shows representative data for Ba(Fe$_{0.975}$Co$_{0.025}$)$_2$As$_2$ above the structural phase transition. Multiple samples of both orientations have been measured and are in good agreement with the representative data shown here. The sam-

![FIG. 1. Representative data showing the resistivity response to strain of Ba(Fe$_{0.975}$Co$_{0.025}$)$_2$As$_2$ at 116 K. The left-hand column (a) shows data for a crystal oriented with the crystal axes parallel to the normal strain frame (represented by the schematic pink-colored crystal in the inset to panel (a)(i)), such that the crystal experiences an admixture of $A_{1g}$ and $B_{1g}$ symmetry strain. The right-hand column (b) shows data for a crystal with the axes oriented at 45 degrees to the normal strain frame (shown schematically for the blue crystal in the inset to panel (b)(i)), such that the crystal experiences an admixture of $A_{1g}$ and $B_{2g}$ symmetry strain. The top graph (i) in each column shows the resistive response of the sample along the $x'$ and $y'$ axes due to the strain, where the $x'$ and $y'$ axes are defined by the normal strain frame (inset). The zero antisymmetric strain condition is marked by a vertical line in panel (b). The middle graph (ii) shows the antisymmetric response, given by the difference $\frac{1}{2}[(\Delta \rho/\rho_0)_{x'x'} - (\Delta \rho/\rho_0)_{y'y'}] = \frac{1}{\nu_P}\epsilon_{B_{1g/2g}}$. For both crystal orientations, the antisymmetric response is linear (black lines show linear fits). The bottom graph (iii) shows the symmetric $(A_{1g})$ response, given by the sum $\frac{1}{2}[(\Delta \rho/\rho_0)_{x'x'} + (\Delta \rho/\rho_0)_{y'y'}] = \frac{1}{\nu_P}\epsilon_{A_{1g}}$. This response is found to be always linear for samples that experience $A_{1g} + B_{1g}$ symmetry strain (black line shows linear fit), while that of the samples that experience $A_{1g} + B_{2g}$ symmetry strain is clearly nonlinear and is fit by a second order polynomial (black line).]
ple that experiences $B_{1g}$ strain exhibits a linear change in $\rho_{xx'}$ and $\rho_{yy'}$ under strain. Consequently, both the antisymmetric response ($[(\Delta \rho/\rho_0)_{B_{2g}}]$) and the symmetric response ($[(\Delta \rho/\rho_0)_{A_{1g}}]$) are also linear in strain. In contrast, the sample that experiences $B_{2g}$ strain exhibits a clear nonlinearity in both $\rho_{xx'}$ and $\rho_{yy'}$ as the strain is varied. The antisymmetric ($B_{2g}$) response is perfectly linear (black line in Fig. 1(b)(ii)) and comparatively large, whereas the symmetric ($A_{1g}$) response exhibits a striking nonlinearity and is well fit by a quadratic function (black line in Fig. 1(b)(iii)). The minimum of the quadratic function does not occur at the same strain as the neutral $B_{2g}$ strain point (vertical line in Fig. 1(b)), indicating the presence of a linear term in addition to the quadratic coefficient.

The qualitative behavior shown in Fig. 1 is characteristic of both crystal orientations for the range of measured temperatures. Data of the elastoresistance response at different temperatures are shown in Fig. 2 for the sample that was oriented to experience $B_{2g}$ symmetry strain. For this crystal orientation, the antisymmetric response is linear for all temperatures measured, with a slope that grows larger as temperature decreases. Similarly, the symmetric ($A_{1g}$) response exhibits a strong temperature dependence, with a clear increase in the coefficient of the quadratic term as temperature is reduced towards the structural transition. In contrast, the sample that experiences $B_{1g}$ symmetry strain exhibits only a weak temperature dependence in the linear response for both symmetry channels, as shown in Fig 1(a), and never exhibits any measurable nonlinearity.

We first consider the linear response to antisymmetric strains, $m_{B_{1g}}^{B_{1g}}$ and $m_{B_{2g}}^{B_{2g}}$, shown in Fig. 3(a). As found previously, $m_{B_{1g}}^{B_{1g}}$ is small and exhibits almost no temperature dependence. In contrast, $m_{B_{2g}}^{B_{2g}}$ is large and can be well fit by a Curie-Weiss temperature dependence with a Weiss temperature $\Theta = 75.8 \pm 0.6$ K (adjusted R-squared, $R_{adj}^2 = 0.9995$), bearing witness to the divergent nematic susceptibility in this material. The coupled nematic/structural phase transition occurs at a higher temperature $T_s = 98 \pm 2$ K due to bilinear coupling between the nematic order parameter and lattice strain with the same symmetry.

The linear response to $A_{1g}$ strain, $m_{A_{1g}}^{A_{1g}}$, is small and only weakly temperature-dependent (Figure 3(b)). Moreover, values of $m_{A_{1g}}^{A_{1g}}$ determined from both crystal orientations agree (as they must, since by symmetry both $\epsilon_{A_{1g}}$ and $(\Delta \rho/\rho_0)_{A_{1g}}$ are invariant to rotations about the $z$-axis), providing additional confidence that the $B_{2g}$ neutral strain point has been accurately identified. For further discussion of errors associated with identification of the neutral strain point, see Appendix C.

From a symmetry perspective, non-linear contributions to $(\Delta \rho/\rho_0)_{A_{1g}}$ are possible due to all three strains considered. To quadratic order,
The isotropic response to anisotropic strain. The temperature-dependence of this non-linear coefficient, \( m_{A1g} \), for \( A1g + B_{1g}/2g \) symmetry strains, respectively. The temperature dependence of these weighted sums, obtained from quadratic fits to the data shown in Fig. 2(b) with appropriate transformation of the strain axis, are plotted in Fig. 3(c). Evidently, \( m_{A1g,A1g} \) and \( m_{B1g,B1g} \) (the weighted sum of which is shown by the pink data) are vanishingly small. Hence, the striking non-linear response seen in Fig. 2(b) derives solely from \( m_{B2g,B2g} \); that is, the non-linear symmetric response derives solely from purely antisymmetric \( (B_{2g}) \) strain.

The Curie-Weiss temperature dependence of \( m_{B2g} \) directly attests to the presence of an electronic degree of freedom (the nematic order parameter \( \phi_{B_{2g}} \)) that is separated from, though bi-linearly coupled to, anisotropic strain \( \epsilon_{B_{2g}} \); \( \phi_{B_{2g}} = \chi_{B_{2g}} B_{2g} \propto m_{B_{2g}} B_{2g} \). From the same perspective, in addition to a bare contribution to \( (\Delta \rho/\rho_0)_{A1g} \) that is directly proportional to \( [\epsilon_{B_{2g}}]^2 \), there should be additional induced terms proportional to \( \phi_{B_{2g}} \phi_{B_{2g}} \) and \( [\phi_{B_{2g}}]^2 \). All these terms are allowed by symmetry, and since \( \phi_{B_{2g}} = \chi_{B_{2g}} \epsilon_{B_{2g}} \), the latter two contributions should be increasingly strong with decreasing temperature, so that:

\[
m_{B_{2g},B_{2g}} = \frac{a}{(T - \Theta)^2} + \frac{b}{T - \Theta} + c \tag{3}
\]

where \( a, b, \) and \( c \) are coefficients to be determined. The Weiss temperature \( \Theta \), which is independently determined from the temperature dependence of \( m_{B_{2g}} \), is not a fit parameter. The black line in Fig. 3(c) shows the best fit to this functional form, with \( \sqrt{a} = 4 \pm 1 \times 10^3 \) K and \( b = 7 \pm 1 \times 10^5 \) K; both terms are important and necessary to fully fit the response (see Appendix D for more details on the fitting). This fit is in excellent agreement with the data \( (R_{ad} = 0.99655) \) and confirms our understanding of the contributing symmetry terms and the underlying physics. The quality of fit also implies that the proportionality constant relating \( \chi_{B_{2g}} \) and the elastoresistivity coefficients have negligible temperature dependence over the fit range.

Finally, we note that \( m_{A1g} \) is positive. This implies that the average resistance is expected to be larger in the anisotropic nematic phase than an extrapolation of the in-plane resistivity determined from the isotropic tetragonal state. Since this is a second order effect, we expect the resistivity increase to scale as the square of the on-setting nematic order parameter, i.e. to have a T-linear temperature dependence, for temperatures close to \( T_s \). This is consistent with the observation\(^{18}\) that the resistivity of twinned Ba\((\mathrm{Fe}_{0.975}\mathrm{Co}_{0.025})_2\mathrm{As}_2\) samples linearly increases upon cooling through the structural transition\(^{19}\).
IV. SUMMARY

In summary, we have shown a diverging nonlinear $A_{1g}$ elastoelastoresistivity response to $B_{2g}$ symmetry strain. The most remarkable aspect of this measurement is not that $B_{2g}B_{2g} \neq 0$, since this is allowed by symmetry, but how large this quantity is. Indeed, close to the structural transition the nonlinear response of $(\Delta \rho/\rho_0)_{A_{1g}}$ to $\epsilon_{B_{2g}}$ is an order of magnitude larger than the linear response to $\epsilon_{A_{1g}}$ for the range of strain considered here. Furthermore the temperature dependence of this coefficient directly reveals that the effect is driven by the large nematic susceptibility of the material, meaning that even the isotropic properties of the Fe-based superconductors (in this case $(\Delta \rho/\rho_0)_{A_{1g}}$) are strongly affected by the nematic character of the material. These observations demonstrate a new means to witness the divergent nematic susceptibility in these materials based on the measurement of the isotropic response to anisotropic strain. They also provide a new point of comparison for microscopic models of the transport properties of Fe-based superconductors.

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Appendix A: Further Details of Sample Preparation and Experimental Protocol

Single crystals of Ba(Fe$_{0.975}$Co$_{0.025}$)$_2$As$_2$ were grown using the FeAs self flux technique as described elsewhere$^{18}$. The crystals were cleaved into thin plates and cut into approximately square rectilinear tablets with typical side lengths of 400-750 $\mu$m and thicknesses of 10-30 $\mu$m. The $B_{2g}$ sample shown in the main text has dimensions $30 \mu m \times 730 \mu m \times 700 \mu m$ and the $B_{1g}$ sample has dimensions $10 \mu m \times 540 \mu m \times 530 \mu m$. The modified Montgomery method assumes square isotropic equivalent samples, however deviations from square, up to roughly side length ratios of 4:1, introduce errors significantly smaller than the errors from Taylor series truncations (~4%) used in the method$^{20}$. The roughly square geometry does ensure equal strain relaxation in both the $x'$ and $y'$ directions. The samples are contacted on the corners of their top surface by gold wires affixed with an air-dry silver epoxy (Dupont 4929N) on sputtered gold pads. The samples are glued to a PZT stack (Part No.: PST150/5x5/7 cryo 1, from Piezomechanik GmbH) with either Devcon 5-minute epoxy or Master Bond EP21TCHT-1. A photograph of a typical sample and a diagram of the PZT setup can be seen in Fig. 4. Stress was applied to the sample by step wise cycling the voltage from -150 V to 150 V (below 150 K) and -50 V to 150 V (above 150 K) on the PZT stack at a fixed temperature. Three to four voltage sweeps were performed at each temperature, with typical voltage ramp rates between 8-15 V/s. Using the modified Montgomery method$^{4,20}$, $\rho_{x'x'}$ and $\rho_{y'y'}$ were measured simultaneously at each voltage step.

The strain of the PZT stack is measured by a strain gauge (Part No.: WK-06-062TT-350 from VPG) glued to the back of the PZT stack. Typically only one direction of strain is measured and the orthogonal strain is calculated using the measured Poisson ratio of the PZT stack $(\epsilon_{y'y'} = -\nu \epsilon_{x'x'})^2$. For measurements done here we assume perfect strain transmission through the glue and sample such that the strain experienced by the sample is the same as the strain of the PZT stack. Imperfect strain transmission would scale the resistive response in all symmetry channels, but would neither change our symmetry decomposition nor affect our main conclusions. This is discussed in detail in Appendix B.

Both the glue and the PZT stack have differing coefficients of thermal expansion, neither of which are matched to the puctide sample. In addition, the glue will contract as it dries during the mounting of the sample which can introduce uncontrolled strains. All of these factors contribute to a temperature dependence of the voltage required to have the sample experience zero antisymmetric strain. There does not appear to be a common trend between samples in the evolution of this neutral point as

FIG. 4. Left) Photograph of a representative Ba(Fe$_{0.975}$Co$_{0.025}$)$_2$As$_2$ sample prepared for an elastoresistance measurement using the modified Montgomery method. The sample is cut into a square, affixed with electrical contacts, and glued onto the PZT stack. This sample is aligned with its crystallographic axes rotated 45 degrees with respect to the normal strain frame. Right) Schematic diagram showing the PZT stack prepared for an elastoresistance measurement, seen from the side. The sample is glued to the top face of the PZT. Strain is measured via a strain gauge glued to the back of the PZT stack.
a function of temperature. For example, the evolution is often non-monotonic and net changes are evenly split between shifts to positive voltages and negative voltages amongst the six samples measured. This indicates that this effect is not solely due to differential thermal contractions of the sample on PZT, implying that the epoxy plays a significant role in determine the bias strain experienced by the sample when zero volts are applied to the PZT stack.

Appendix B: Strain Transmission

The strain transmission through the crystal will depend on geometric factors; for example, the thicker the crystal is compared with the in-plane dimensions, the more the strain will relax along the $z$-axis of the crystal. In order to quantify the strain transmission we compare a strain gauge mounted on top of a large undoped BaFe$_2$As$_2$ sample prepared as described in Appendix A and a strain gauge glued directly to the back of the PZT stack. For this experiment we measure the range of strain along the $y'$ direction for the sample mounted strain gauge ($\Delta \epsilon^{y'}_{B2g}$) and for the PZT mounted strain gauge ($\Delta \epsilon^{y'}_{PZT}$) for fixed temperature voltage sweeps. The range of strain along the $x'$ direction is estimated based on the Poisson ratio of the PZT stack. From this we can calculate the range of antisymmetric strain experienced by the sample strain gauge ($\Delta \epsilon^{s}_{B2g} = \frac{1}{2}(\Delta \epsilon^{x}_{B2g} - \Delta \epsilon^{y}_{B2g})$) and PZT strain gauge ($\Delta \epsilon^{s}_{PZT} = \frac{1}{2}(\Delta \epsilon^{x}_{PZT} - \Delta \epsilon^{y}_{PZT})$). The temperature dependence of the ratio $\Delta \epsilon^{s}_{B2g}/\Delta \epsilon^{s}_{PZT}$ is shown in Fig. 5(a), a ratio of one implies perfect strain transmission through the sample. At 270K there is a sharp increase in strain transmission which we attribute to a freezing transition of the glue. Below 250K the strain transmission is $\geq 80\%$ and has only a weak temperature dependence. This temperature dependence is small compared with the temperature dependence of the elastoresistance response which is demonstrated in Fig. 5(b) where $m_{B2g}^{sg}$ is calculated twice, once using the measured strain of the strain gauge mounted on the sample and once using the measured strain of the strain gauge mounted on the PZT stack. The two calculations are in good agreement below 250K.

The majority of samples are too small to accommodate a strain gauge on their surface. To quantify the strain transmission as a function of sample size, three undoped BaFe$_2$As$_2$ samples: small (280 $\mu$m $\times$ 300 $\mu$m $\times$ 20 $\mu$m), medium (760 $\mu$m $\times$ 750 $\mu$m $\times$ 40 $\mu$m), and large (3140 $\mu$m $\times$ 3330 $\mu$m $\times$ 50 $\mu$m), this sample is large enough to have a strain gauge on its surface and is the sample measured for the data shown in Fig. 5) were measured. The extracted $m_{B2g}^{sg}$ responses are shown in Fig. 6(a).

In these calculations of $m_{B2g}^{sg}$ strain was measured by a strain gauge glued to the back of the PZT stack and the strain was assumed to be fully transmitted, i.e. the sample experiences the same strain as the PZT stack. The large and medium samples have the same temperature dependence and magnitude of response, indicating that both samples have similar strain transmission ($\geq 80\%$). While the magnitude of the response of the small sample is significantly reduced, likely due to an overestimation of the strain experienced by the sample. This implies that for the small sample either the crystal is positioned on the PZT stack such that it experiences a smaller strain or that there is a strain gradient along the $z$ crystallo-

FIG. 5. Strain transmission through a large (3140 $\mu$m $\times$ 3330 $\mu$m $\times$ 50 $\mu$m) BaFe$_2$As$_2$ crystal under $A_{1g}$ + $B_{2g}$ symmetry strains. A strain gauge is glued on top of the sample ($\Delta \epsilon^{s}_{B2g}$) and a second strain gauge is affixed directly to the back of the PZT stack ($\Delta \epsilon^{s}_{PZT}$). For this particular test, $\Delta \epsilon^{s}_{B2g}$ is estimated based on the measured Poisson ratio of the PZT, allowing for the estimation of the antisymmetric strains $\Delta \epsilon^{s}_{B2g}$ and $\Delta \epsilon^{s}_{PZT}$, $\frac{1}{2}(\Delta \epsilon^{x}_{PZT} - \Delta \epsilon^{y}_{PZT})$ and $\frac{1}{2}(\Delta \epsilon^{x}_{B2g} - \Delta \epsilon^{y}_{B2g})$ respectively. (a) The temperature dependence of the ratio of the range of antisymmetric strain experienced by the two strain gauges during fixed temperature voltage sweeps. Below 250K the strain transmission through the samples is $\geq 80\%$ and only has a weak temperature dependence. (b) The extracted $m_{B2g}^{sg}$ elastoresistivity response calculated from both strain gauges. The two traces are in good agreement below 250K, indicating that the temperature dependence of the response is dominated by the intrinsic temperature dependence of the electronic sample properties over the temperature dependence of the strain transmission.
demonstrates that imperfect strain transmission results in good agreement below 250K, which is shown in Fig. 6(b). The two normal-

dependence of the small and large samples can be com-

comparison. This is shown in Fig. 6(b). The two normalized responses are in good agreement below 250K, which demonstrates that imperfect strain transmission results in only a simple scaling of the magnitude of the elastoresistance response.

This allows us to use the magnitude of the $m_{B_{2g}}$ response as an approximate measure of strain transmission, with the assumption that samples with in-plane dimensions $\sim 750 \mu m$ or larger have $> 80\%$ strain transmission. For samples oriented to experience $B_{1g}$ strain estimating the overall strain transmission is more challenging. Rough estimates can be made based off of their relative size compared to samples that experience $B_{2g}$ strain. The $B_{2g}$ sample shown in the main text $(30 \mu m \times 730 \mu m \times 700 \mu m)$ has $> 80\%$ strain transmission while the $B_{1g}$ sample $(10 \mu m \times 540 \mu m \times 530 \mu m)$ has $\geq 60 - 80\%$ strain transmission.

Appendix C: Errors in Extracting the Linear and Quadratic Response from $\rho_{A_{1g}}$ Arising from Uncertainty in Identifying the Neutral Strain Point

As shown in the main text, the main finding of the current work is that for the strain ranges we employ the elastoresistance of Ba(Fe$_{0.975}$Co$_{0.025}$)$_2$As$_2$ is linear with the exception of a large nonlinear $m_{B_{2g}B_{2g}}$ term. However, due to differences in the thermal expansion of the PZT, sample, and the glue holding the sample in place and the volume contraction of the glue as it dries when the sample is attached to the PZT stack, the sample once mounted and cooled may experience external strains even when no voltage is applied to the PZT stack. The neutral $B_{2g}$ and $A_{1g}$ strain points may even be offset from each other. Since the PZT applies a fixed ratio of symmetric and antisymmetric strains, if there is an offset in the neutral points at best we can tune through one neutral point at a time (i.e. $\epsilon_{A_{1g}} = 0$ or $\epsilon_{B_{2g}} = 0$).

We can identify the neutral antisymmetric strain point above the tetragonal to orthorhombic structural transition (98K) because, for a tetragonal material at the neutral antisymmetric strain point, $\rho_{xx} = \rho_{yy}$. The modified Montgomery method is well suited to identify the antisymmetric strain neutral point since it simultaneously measures $\rho_{x'x'}$ and $\rho_{y'y'}$ under identical strain conditions in a single sample. This is one advantage of the modified Montgomery method over the previously used differential technique$^{12}$. It is more challenging to identify the symmetric strain neutral point and it is not done in this work. Below is a detailed calculation of the effects of the misidentification of strain offsets on the calculated elastoresistivity tensor components. For simplicity we look at an offset in $\epsilon_{x'y'}$ which still allows for tuning to the neutral point of both $A_{1g}$ and $B_{1g/2g}$ symmetry strains, but the results hold even if the neutral points are separated. The main conclusions are that, for this material, the correct identification of the antisymmetric neutral point is required to accurately estimate $m_{A_{1g}}$ for samples that experience $A_{1g}$ and $B_{2g}$ symmetry strain, however neither $m_{B_{1g/2g}}$ nor $m_{A_{1g}}$ are dependent on the identifi-
cations of the neutral point. All results are robust to the determination of the symmetric strain neutral point.

We start with the simple case of the linear antisymmetric response, assuming no offset between the neutral $A_{1g}$ and $B_{1g}/2g$ strain points and that $\epsilon_{A_{1g}}, \epsilon_{B_{1g}/2g}$, and $\epsilon_{x'x'}$ are all measured relative to the neutral point where $\epsilon_{A_{1g}} = \epsilon_{B_{1g}/2g} = \epsilon_{x'x'} = 0$. Then the change in antisymmetric resistivity to $\epsilon_{x'x'}$ is described by,

$$\frac{(\Delta \rho/\rho_0)_{B_{1g}/2g}}{\Delta \rho/\rho_0}_{B_{1g}/2g} = \frac{m_{B_{1g}/2g} \epsilon_{B_{1g}/2g}}{m_{B_{1g}/2g}} (1 + \nu \rho_{meas} \frac{1}{2}\epsilon_{x'x'}')$$

(C1)

If the neutral strain point is misidentified by an amount $\Delta \epsilon_{x'x'}$ such that $\epsilon_{x'x'}^{true} = \epsilon_{x'x'}^{meas} + \Delta \epsilon_{x'x'}$ then there will be an offset in both the symmetric and antisymmetric neutral points (i.e. $\Delta \epsilon_{A_{1g}} = \frac{1-\nu \rho_{meas}}{2} \Delta \epsilon_{x'x'}$ and $\Delta \epsilon_{B_{2g}} = \frac{1+\nu \rho_{meas}}{2} \Delta \epsilon_{x'x'}$). Then the antisymmetric response becomes,

$$\frac{(\Delta \rho/\rho_0)_{B_{1g}/2g}}{\Delta \rho/\rho_0}_{B_{1g}/2g} = \frac{m_{B_{1g}/2g} \epsilon_{B_{1g}/2g}}{m_{B_{1g}/2g}} (1 + \nu \rho_{meas} \frac{1}{2}\epsilon_{x'x'}')$$

(C2)

The linear antisymmetric elastoresistivity coefficient is extracted from the slope of the linear fit of $(\Delta \rho/\rho_0)_{B_{1g}/2g}$ vs $\frac{1-\nu \rho_{meas}}{2} \epsilon_{x'x'}'$ ($\epsilon_{x'x'}^{meas}$). In this case the extracted slope is the true elastoresistivity coefficient, $m_{B_{1g}/2g}$, independent of the error in the identification of the strain neutral point $\Delta \epsilon_{x'x'}$.

For the isotropic resistivity response, we again start by assuming no offset between the neutral symmetric and antisymmetric strain points and that all strains are measured relative to the neutral point where $\epsilon_{A_{1g}} = \epsilon_{B_{1g}/2g} = \epsilon_{x'x'} = 0$. For simplicity we will perform these calculations for a sample that experiences $A_{1g}$ and $B_{2g}$ symmetry strain (the same calculation can be done for a sample that experiences $A_{1g}$ and $B_{1g}$ symmetry strains by simply replacing all references to $B_{2g}$ with $B_{1g}$). The isotropic resistivity response is then described by,

$$\frac{(\Delta \rho/\rho_0)_{A_{1g}}}{\Delta \rho/\rho_0}_{A_{1g}} = m_{A_{1g}} \epsilon_{A_{1g}} + m_{A_{1g}} \epsilon_{A_{1g}} [\epsilon_{A_{1g}}] \times m_{A_{1g}} \epsilon_{A_{1g}} \left[ \frac{1-\nu \rho_{meas}}{2} \epsilon_{x'x'}' \right]$$

(C3)

Now we introduce a misidentification of the neutral strain point by an amount $\Delta \epsilon_{x'x'}$ ($\epsilon_{x'x'}^{true} = \epsilon_{x'x'}^{meas} + \Delta \epsilon_{x'x'}$). The isotropic resistivity response then becomes,

$$\frac{(\Delta \rho/\rho_0)_{A_{1g}}}{\Delta \rho/\rho_0}_{A_{1g}} = m_{A_{1g}} \epsilon_{A_{1g}} + m_{A_{1g}} \epsilon_{A_{1g}} [\epsilon_{A_{1g}}] \times m_{A_{1g}} \epsilon_{A_{1g}} \left[ \frac{1-\nu \rho_{meas}}{2} \epsilon_{x'x'}' \right]$$

(C4)

For this material the only non-negligible quadratic response is $m_{A_{1g}} m_{A_{1g}} \approx m_{A_{1g}} m_{A_{1g}} \approx 0$). This further simplifies the equation,

$$\frac{(\Delta \rho/\rho_0)_{A_{1g}}}{\Delta \rho/\rho_0}_{A_{1g}} m_{B_{2g}} m_{B_{2g}} \left[ \frac{1+\nu \rho_{meas}}{2} \epsilon_{x'x'}' \right]$$

$$m_{A_{1g}} \epsilon_{A_{1g}} \left[ \frac{1-\nu \rho_{meas}}{2} \epsilon_{x'x'}' \right]$$

(C5)

Fits to the linear $(\Delta \rho/\rho_0)_{A_{1g}}$ response vs $\frac{1-\nu \rho_{meas}}{2} \epsilon_{x'x'}'$ ($\epsilon_{x'x'}^{meas}$) incorrectly identify the slope, the effective measured $m_{A_{1g}}$, as $m_{A_{1g}} = m_{A_{1g}} + \frac{1+\nu \rho_{meas}}{2} m_{A_{1g}} \epsilon_{x'x'}' = m_{A_{1g}} + 2(1+\nu \rho_{meas}) m_{A_{1g}} \epsilon_{x'x'}'$, so to accurately measure this quantity the neutral $B_{2g}$ strain point must be correctly identified. If a similar procedure is followed for a sample experiencing $A_{1g}$ and $B_{1g}$ symmetry strain there is no error introduced to the measured $m_{A_{1g}}$ for misidentification of the neutral strain point or for offsets between the $B_{1g}$ and $A_{1g}$ neutral points since there is no contribution from the quadratic response. Thus estimates of $m_{A_{1g}}$ extracted from samples that experience $A_{1g}$ and $B_{1g}$ symmetry strains are robust. Fits to the quadratic $(\Delta \rho/\rho_0)_{A_{1g}}$ response vs $\frac{1-\nu \rho_{meas}}{2} \epsilon_{x'x'}'$ ($\epsilon_{x'x'}^{meas}$) correctly extract the quadratic coefficient, $m_{B_{2g}} m_{B_{2g}}$, independent of the neutral strain.

Two experimental observations confirm that we can correctly identify the neutral $B_{2g}$ strain point. First the estimates of $m_{A_{1g}}$ (shown in Fig. 3(b) of the main text) are the same for crystals oriented such that they exhibit $A_{1g} + B_{2g}$ and $A_{1g} + B_{1g}$ strains. Secondly,
misidentification of the $B_{2g}$ neutral point would admix some amount of $m_{A_{1g}}^{B_{2g}}$ into the nominal measurement of $m_{A_{1g}}$, which would introduce a strong temperature dependence—this is not observed.

Appendix D: Fitting $m_{B_{2g}}$ and $m_{A_{1g}}^{B_{2g}}$

The linear antisymmetric response to $B_{2g}$ symmetry strain, $m_{B_{2g}}$, is extracted from a first order fit of $(\Delta \rho/\rho_0)_{B_{2g}}$ versus $\epsilon_{B_{2g}}$. The temperature dependence of $m_{B_{2g}}$ can then be fit to a Curie-Weiss functional form, $m_{B_{2g}} = \lambda \alpha \left( \frac{1}{T-\Theta} \right) + m_{B_{2g},0}$. The antisymmetric response deviates from a true Curie-Weiss behavior at high temperatures where the epoxy softens and at low temperatures due to the structural transition and disorder. The optimal temperature range to extract the best Curie-Weiss fit is chosen following the procedure outlined in reference, except that the reduced $\chi^2$ error was minimized as opposed to the standard deviation. For the sample shown in the main text the best fit temperature range was found to be 104K-181K. Extracted fit parameters are $\lambda \alpha = -2980 \pm 71$ K, $\Theta = 75.8 \pm 0.6$ K, and $m_{B_{2g},0} = 17.3 \pm 0.7$ with $R^2_{Adj} = 0.9995$. The nonlinear symmetric response to antisymmetric $B_{2g}$ strain, $m_{A_{1g}}^{B_{2g}}$, was extracted from the quadratic coefficient of a second order fit of $(\Delta \rho/\rho_0)_{A_{1g}}$ versus $\epsilon_{B_{2g}}$ using the same temperature range, 104K-181K. As described in the main text, by symmetry the temperature dependence of $m_{A_{1g}}^{B_{2g}}$ is allowed to include the terms $\{(a \alpha^{B_{2g}}/T-\Theta), (b \beta^{B_{2g}}/T-\Theta)\}$. The Weiss temperature, $\Theta$, is fixed to be 75.8K from the Curie-Weiss fit of $m_{B_{2g}}$. The fitted coefficients are $a = (2.0 \pm 0.2) \times 10^7$ K$^2$, $b = (7.4 \pm 1) \times 10^5$ K, and $c = (6.7 \pm 0.9) \times 10^3$ with $R^2_{Adj} = 0.99655$. We can compare the magnitude of the contributions from individual terms in the fit. Close to the Weiss temperature (i.e. $T = \Theta$), we expect the quadratic term $(a \alpha^{B_{2g}}/T-\Theta)$ to dominate; however, at high temperatures the Curie-Weiss term $(b \beta^{B_{2g}}/T-\Theta)$ is largest. The cross over point is roughly 23K above the Weiss temperature, so for the accessible range of temperatures considered here (i.e. above $T_s = 98$K) the Curie-Weiss term is equal in magnitude or larger than the $(a \alpha^{B_{2g}}/T-\Theta)$ component. It is not significantly larger over the whole temperature range and both terms are necessary to fully fit the response.

1. J. W. Harter, Z. Y. Zhao, J.-Q. Yan, D. G. Mandrus, and D. Hsieh, Science 356, 295 (2017).
2. L. Zhao, C. A. Belvin, R. Liang, D. A. Bonn, W. N. Hardy, N. P. Armitage, and D. Hsieh, Nat. Phys. 13, 250 (2017).
3. M. Fiebig, V. V. Pavlov, and R. V. Pisarev, J. Opt. Soc. Am. B 22, 96 (2005).
4. H.-H. Kuo, J.-H. Chu, J. C. Palmstom, S. A. Kivelson, and I. R. Fisher, Science 352, 958 (2016).
5. A. E. Böhm and C. Meingast, C. R. Physique 17, 90 (2016).
6. M. Yoshizawa, D. Kimura, T. Chiba, S. Simayi, Y. Nakanishi, K. Kihou, C.-H. Lee, A. Iyo, H. Eisaki, M. Nakajima, and S. Uchida, J. Phys. Soc. Jpn. 81, 024604 (2012).
7. A. E. Böhm, P. Burger, F. Hardy, T. Wolf, P. Schweiss, R. Fromknecht, M. Reinecker, W. Schranz, and C. Meingast, Phys. Rev. Lett. 112, 047001 (2014).
8. Y. Gallais, R. M. Fernandes, I. Paul, L. Chauvière, Y.-X. Yang, M.-A. Méasson, M. Cazayous, A. Sacuto, D. Colson, and A. Forget, Phys. Rev. Lett. 111, 267001 (2013).
9. F. Kretzschmar, T. Böhm, U. Karahasanović, B. Muschler, A. Baum, D. Jost, J. Schmalian, S. Caprara, M. Grilli, C. Di Castro, J. G. Analytis, J.-H. Chu, I. R. Fisher, and R. Hackl, Nat. Phys. 12, 560 (2016).
10. J.-H. Chu, H.-H. Kuo, J. G. Analytis, and I. R. Fisher, Science 337, 710 (2012).
11. H.-H. Kuo and I. R. Fisher, Phys. Rev. Lett. 112, 227001 (2014).
12. H.-H. Kuo, M. C. Shapiro, S. C. Riggs, and I. R. Fisher, Phys. Rev. B 88, 085113 (2013).
13. For data shown in this paper, the expansion is done with respect to zero anisotropic strain, $\Delta \rho = \rho(\epsilon) - \rho(\epsilon_{B_{1g}}) = 0$. For more details see Appendix C and Ref. 14.
14. M. C. Shapiro, P. Hlobil, A. T. Hristov, A. V. Maharaj, and I. R. Fisher, Phys. Rev. B 92, 235147 (2015).
15. A standard delineation for the two $A_{1g}$ symmetry terms in $\epsilon$ and $\Delta \rho/\rho_0$, which we also adopt here, is to separate in-plane ($a = A_{1g1}$ (i.e. $x^2 + y^2$)) and out-of-plane responses ($a = A_{1g2}$ (i.e. $z^2$)).
16. The irreducible representation notation can be related to the Cartesian and Voigt notations that have been used previously: $m_{B_{1g}}^{A_{1g}} = m_{xx,xx} - m_{xx,yy} = m_{11} - m_{12}$ and $m_{B_{2g}} = 2m_{xy,xy} = 2m_{66}$.
17. For the $D_{4h}$ point symmetry tensor, there are four independent combinations of terms in the 4th rank elastoresistivity tensor that can contribute to an $A_{1g}$ symmetry elastoresistivity response. Two possible linear combinations of strain that transform like an $A_{1g}$ object are; $\epsilon_{A_{1g1}} = \frac{1}{2}(\epsilon_{x^2} + \epsilon_{y^2})$ and $\epsilon_{A_{1g2}} = \epsilon_{z^2}$ and similarly for $\Delta \rho/\rho_0$: $\rho_{A_{1g1}} = \frac{1}{2}[(\Delta \rho/\rho_0)_{x^2} + (\Delta \rho/\rho_0)_{y^2}]$ and $\rho_{A_{1g2}} = (\Delta \rho/\rho_0)_{z^2}$. For the measurements described in this paper, the effective out of plane Poisson ratio $\nu_z$ of the sample bonded to the PZT stack determines the strain in the $z$ direction ($\varepsilon_z = -\nu_z \varepsilon_{xx}$), which in turn affects the degree to which these combinations of coefficients admix in the measured symmetric response. Hence, $\frac{\Delta \rho}{\rho_0}_{A_{1g1}} = m_{A_{1g1}}^{A_{1g1}} \epsilon_{A_{1g1}} + m_{A_{1g2}}^{A_{1g2}} \epsilon_{A_{1g2}}$ (4)

Since $\epsilon_{x^2} = -\nu_x \epsilon_{x^2}$ and $\epsilon_{y^2} = -\nu_y \epsilon_{x^2}$, we obtain
\[
\left( \frac{\Delta \rho}{\rho_0} \right)_{A_{1g},1} = (m_{A_{1g},1}^A - \frac{2\nu_s}{1 - \nu P} m_{A_{1g},1}^A) \epsilon_{A_{1g},1} = m_{A_{1g}}^{A_{1g}} \epsilon_{A_{1g},1}
\]

(5)

defining the quantity \( m_{A_{1g}}^{A_{1g}} \) used in the main text.

18 J. H. Chu, J. G. Analytis, C. Kucharczyk, and I. R. Fisher, Phys. Rev. B 79, 014506 (2009).

19 While domain wall scattering can also contribute to the average resistivity of twinned samples in this regime, the linear temperature dependence implies that the dominant effect derives from the intrinsic increase in the average resistivity.

20 C. A. M. dos Santos, A. de Campos, M. S. da Luz, B. D. White, J. J. Neumeier, B. S. de Lima, and C. Y. Shigue, J. Appl. Phys. 110, 083703 (2011).