LEVEL DENSITY AND RADIATIVE STRENGTH FUNCTIONS IN LIGHT NUCLEI: $^{60}$Co AS AN EXAMPLE OF THE METHOD FOR DETERMINATION AND THEIR RELIABILITY VERIFICATION

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Introduction

The question about the reliability of observables is very important for every experimental work.

Level density and radiative strength functions of cascade gamma-transitions below the neutron binding energy $B_n$ are typical example of two problems which should be solved in order to provide high reliability of the observables. It is necessary:

1. to develop appropriate method for extraction of the desired parameters from experimental spectra when they cannot be observed directly;  
2. to get maximum possible statistics of the experimental data which would provide uniqueness of the determined parameters and possibility for their additional independent verification.

Up to now level density $\rho$ was obtained from the experimental data on:

(a) neutron evaporation spectra (see, for example, [1]);  
(b) intensities of the two-step gamma-cascades [2]  

\[ I_{\gamma\gamma} = \sum_{\lambda,f} \sum_l \frac{\Gamma_{\lambda l}}{\Gamma_{\lambda}} \frac{\Gamma_{lf}}{\Gamma_{l}} = \sum_{\lambda,f} \frac{\Gamma_{\lambda}}{<\Gamma_{\lambda}>} \frac{n_{\lambda l}}{<m_{\lambda l}>} \frac{\Gamma_{lf}}{<\Gamma_{lf}>} m_{lf}, \]  

(1)

connecting compound state (neutron resonance) with the group of low-lying levels of the studied nucleus and determined according to [3] for all possible energy intervals of their primary transitions;  

(c) gamma-ray spectra following de-population of levels with the excitation energy $E_{ex}$ [4] in nuclear reactions like (d,p) and ($^3$He,α) [5].

In the methods [2] and [5], $\rho$ is estimated simultaneously with the radiative strength functions of cascade gamma-transitions  

\[ k = f \times A^{2/3} = \frac{\Gamma_{\lambda}}{(E_{\gamma}^3 \times A^{2/3} \times D_{\lambda})}. \]  

(2)

Here $A$ is the mass of a nucleus, $E_{\gamma}$ is the energy of gamma-transition, $D_{\lambda}$ is the mean spacing between decaying levels $\lambda$, $\Gamma_{\lambda}$ is the mean partial width of transitions between the compound state $\lambda$ and some set of levels $l$.

It is obvious that the criterion of reliability of determination of $\rho$ and $k$ is the agreement between the results of different experiments or, at least, possibility to explain origins of existing discrepancy.
1 Main sources of systematical uncertainty in determination of \( \rho \) within different methods

The presence of serious enough discrepancy between level densities determined in the frameworks of different methods [1,5] and [2] requires one to estimate reliability of these results.

In methods [1,5], the main problem for extraction of, for example, \( \rho \) is the proportionality of intensity of the registered spectra to product of number of levels \( m = \rho \times \Delta E \) in a given excitation energy interval \( \Delta E \) and emission probability \( T \) of the reaction products. In addition to infinite number of functional dependences \( \rho = f(E_{ex}) \) and \( T = \phi(E_{ex}) \) reproducing corresponding spectra, the values of level density and emission probability of reaction product \( T \) can vary in interval from \( -\infty \) to \( +\infty \). I.e., the kind of spectra measured in methods [1,5] leads to low confidence level of the obtained results.

Besides, the value of \( \rho \) is determined in [1] from experimental data with the use of the model calculated penetrability \( T \) of nuclear surface for evaporated nucleon (light nucleus). It is to be calculated to a precision of some tens of percent with accounting for possible high-frequent sign-variable variation of \( T \) with respect to its average value. This effect is directly observed at extraction of \( \rho \) and \( k \) from intensities of the two-step cascades.

As a result, the confidence of the results like that given in [1] cannot be estimated impartially. For example, there are no reasons to exclude the possibility of compensation of decrease (increase) in level density in some excitation energy interval by increase (decrease) in emission probability of neutron in evaporation spectra or gamma-quantum in the primary transition spectra depopulating levels in the vicinity of the excitation energy \( E_{ex} \). Such situation is directly observed [6] for \( \rho \) and \( k \) derived from the intensities \( I_{\gamma\gamma} \). Optical potentials used in methods like [1], most probably, cannot provide calculation of \( T \) in required details and guaranteed error in determination of \( \rho \) at the level achieved in [6].

All results in [5] were obtained in the methodical variant containing several sources of systematical errors that lead to unknown resulting error for both \( \rho \) and \( k \). Main part of them, however, can be [7] removed or noticeably reduced by suitable modification of the methods of obtaining spectra of primary transitions [8] and determination [9] from them of reliable values of the parameters \( \rho \) and \( k \). The difference of energy dependence of the radiative strength functions of gamma-transitions with equal energy but de-populating levels with different energy \( E_{ex} \) must be taken into account in [5], as well. According to [10], strong difference between strength functions of primary and secondary transitions manifests itself in different nuclei from the mass region \( 40 \leq A \leq 200 \).

Noticeably more favorable situation is with the achieved confidence of \( \rho \) and \( k \) values obtained with the use of [2] and possibility to improve it. It is obvious that level densities and strength functions derived from the experimental cascade intensities \( I_{\gamma\gamma} \) contain both ordinary statistic and specific systematic errors.

Probable value of ordinary errors can be easily obtained from estimation of the top uncertainty of the observed cascade intensity distributions by means of the standard formula of error transfer. For example, comparison of intensities of the cascade primary transitions from [11] and [12] used for normalization of \( I_{\gamma\gamma} = F(E_{1}) \) and application of the results of extrapolation [13] of distributions of random cascade intensities with the
energies of their intermediate levels $E_{ex} < 0.5B_n$ to the zero detection threshold provides realistic estimation of systematic errors of both amplitude and form of function $F(E_1)$.

Variation of its values as input data [2] permits one easily enough and reliably to estimate ordinary systematic errors of $\rho \ k$ at the presence of non-linear relation between $\delta F(E_1)$, $\delta \rho$ and $\delta k$.

As it was obtained in [14], $\delta \rho$ and $\delta k$ estimated in this way cannot explain step-like structures in energy dependence of $\rho$ determined within method [2] and discrepancy of these results with conventional ideas of “smooth” energy dependence of level density.

2 Specific of determination of $\rho$ and $k$ from $I_{\gamma\gamma}$

The problems in determination of $\rho$ and $k$ and possibility of their solution can be most easily analysed on the example well studied nucleus with relatively low level density but complicated enough that the level density and radiative strength functions in it can be presented as “smooth” functions. Between the nuclei studied by us, $^{60}\text{Co}$ well satisfies these conditions. Total intensity $\sum i_1$ of all observed intensities $i_1$ of the cascade primary transitions in this nucleus exceeds 76% and the parameter $d = \sum i_\gamma E_\gamma/B_n = 95\%$ [12], respectively. This means that the low energy primary transitions are practically absent in the data [12]. Because they form continuous component in spectra of the two-step cascades, one can assume that joint analysis of the data on two-step cascades and single gamma-transitions will allow appearing of main behavior of the gamma-decay process of this compound nucleus. In the other words, this provides obtaining of rather reliable data on $\rho$ and $k$ and possibility to test them. Total intensity of two-step cascades $I_{\gamma\gamma}$ with the sum energy $E_1 + E_2 = B_n - E_f$ in $^{60}\text{Co}$ equals 63.2(9) % of decays for $E_f \leq 1068$ keV. Well established decay scheme allows us to allocate all the observed two-step gamma-cascades. This information is necessary for determination of quanta ordering in cascades whose intermediate levels are depopulated by the only transition. This is needed for determination [3] of dependence $I_{\gamma\gamma} = F(E_1)$ (Fig. 1) and population of the cascade intermediate levels, as well.

Energy dependence of level density derived from these data has clearly expressed “steplike” structure as practically all nuclei studied by the method [2].

3 Reduction of systematic errors

$I_{\gamma\gamma}$ from Eq. (1) is determined by values of three unknown parameters: total density of the cascade intermediate levels in a given energy interval and sums of radiative strength functions of the primary and secondary dipole gamma-transitions. Strong anti-correlation of these parameters results in rather narrow interval of variations of sums of level density with different parity and spin (spin interval is rather unambiguously determined by multipolarity selection rule) and sum of strength functions of $E1$ and $M1$ transitions which precisely ($\chi^2/f << 1$) reproduce $I_{\gamma\gamma}$. It is obvious that interval of variations separately for levels with $\pi = +$, $\pi = -$ and $k(E1), k(M1)$ is noticeably wider than that for their sums. This statement is true only in case when relation between partial widths of primary and secondary transitions is set over whole interval of their variations on the grounds of some information. At the lack of this information, the only possibility to determine $\rho$ and $k$ in
the frameworks of method [2] is to take an assumption about equal form of their energy dependence. Although partial compensation of this incorrect approach in [2] is provided, for example, in case of sign-variable deviation of \( k \) for secondary transitions from that for primary transitions at different gamma-transition energies. I.e., relative change in the total radiative width \( \Gamma_l \) of the cascade intermediate level can be considerably less than relative change in \( k \).

But even in this case the main specific uncertainty of method [2] results from application of notion of equal energy dependence of radiative strength functions for primary and secondary gamma-transitions. In principle, this problem can be solved: it is necessary to measure intensity distribution of cascades to maximum possible number of levels \( E_f \) and analyse these data in appropriate manner. There are no technical obstacles for this variant of determination of \( \rho \) and \( k \).

Partial solution of this problem on the base of accumulated information was suggested for the first time in [10]. Total population of levels \( P = i_1 \times i_2 / i_{\gamma\gamma} \) equals the product of summed population of all higher-lying levels and branching ration at their decay. Practically the same data on \( i \) needed for determination of \( P \) for \(^{60}Co \) are listed in [11] and [12], intensities of the energy resolved individual cascades \( i_{\gamma\gamma} \) are given in [15].

As it is seen from Fig. 2, cascade population \( P - i_1 \) summed over small energy interval cannot be reproduced in calculation using existing model ideas of rho [16,17] and \( \rho \) [18]. Although the use of \( \rho \) and \( k \) determined according to [2] gives better result, but complete simultaneous agreement between the experiment and calculation for cascade intensities and population of levels can be achieved only if analysis take into account dependence of \( k \) on the energy of decaying level.

In the variant of accounting for different energy dependence of \( k \) for primary and secondary transitions in form \( k^{sec}(E_{\gamma}, E_{ex}) = k^{prim}(E_{\gamma}) \times h(E_{ex}) \) suggested in [10], the best values for \(^{60}Co \) slightly differ from that obtained according to [2] (figs. 3 and 4). The function \( h \) is shown in figs. 3 and 4, as well.

### 4 Estimation of significance of observed parameters \( \rho \) and \( k \)

An ensemble of the experimental data obtained for \(^{60}Co \) includes the gamma-ray spectrum following thermal neutron radiative capture [19], too. This independent information is suitable for estimation of the confidence level for the obtained parameters \( \rho \) and \( k \). The experimental and calculated spectra for different values of level density and radiative strength functions is performed in Fig. 5.

It confirms conclusion about inapplicability of models like [16-18] for precision description of cascade gamma-decay of heavy nucleus. Besides, this points at the necessity of further analysis of the specific of this process, decrease and more accurate accounting for influence of all sources of systematic errors on the desired parameters. First of all, it should be decrease in error of distribution \( I_{\gamma\gamma} = F(E_l) \) and determination of function \( k = \phi(E_{\gamma}, E_{ex}) \) directly from experiment. Small “bump” in the region \( E_{\gamma} = 3.5 \) MeV can be resulted from the error cascade intensity in Fig. 1 and difference of function \( h(E_{ex}) \) from that used in [10].
5 Conclusion

Analysis of all totality of information obtained at the thermal neutron capture in $^{59}$Co confirms made earlier for other nuclei conclusion about impossibility to achieve agreement between the observed values of functional of the cascade gamma-decay process with those calculated in the frameworks of existing models of level density and radiative strength functions like [16-18] within the limits of experimental error.

In accordance with the model ideas [20], the reason of this phenomenon is breaking of at least one Cooper pair of nucleons in this nucleus that is not taking into account in other models.

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Fig. 1. The total intensity of two-step cascades as a function of the primary transition energy $E_1$ summed over the energy bins of 0.5 MeV for $^{60}\text{Co}$. The histogram represents experimental data with ordinary statistics errors, curves 1 and 2 show calculation by Eq. 1 within models [17,18] and [16,18], respectively.

Fig. 2. The total cascade population of levels in the 200 keV energy bins. Thin line represents calculation within models [17,18], dashed line shows results of calculation using data [2]. Thick line shows results of calculation using level density [2], and corresponding strength functions of secondary transitions were multiplied by function $h$ [10] presented in figs. 3,4.
Fig. 3. The sum of the radiative strength functions (multiplied by $10^9$) for $E1$ and $M1$ transitions (points with errors) allowing precise reproduction of the two-step cascade intensities for $h \neq 1$. Open circles show similar values in case $h = 1$. Upper and lower thin curves show predictions according to models [17] and [16] under assumption $k(M1)=const$, respectively. Thick curve shows an example of function $h$ for minimum possible $E_2$.

Fig. 4. The total number of cascade intermediate levels (points with errors) allowing reproduction of the complete set of the experimental data. Open points show similar values for the case $h = const$. Curve 1 shows predictions according model [18]. Curve 2 shows the function $h(E_{ex})$ in case $E_2=0$. Triangles demonstrate observed number of intermediate levels in resolved cascades.
Fig. 5. The total gamma-ray spectrum following thermal neutron radiative capture. Upper graph: curves 1 and 2 represent results of calculation according to models [17,18] and [16,18], respectively. Lower graph: curve 1 represents calculation using $\rho$ and $k$ from [2], curve 2 shows calculation which accounts for different energy dependence of primary and secondary transitions.