KINETICS OF PARTON-ANTIPARTON PLASMA VACUUM CREATION IN THE TIME-DEPENDENT CHROMO-ELECTRIC FIELDS OF ARBITRARY POLARIZATION

A.V. Filatov, S.A. Smolyansky, A.V. Tarakanov

Physical Department of Saratov State University, 410026, Saratov, Russia
E-mail: smol@sgu.ru

Abstract

The kinetic equation of non-Markovian type for description of the vacuum creation of parton-antiparton pairs under action of a space homogeneous time-dependent chromo-electric field of the arbitrary polarization is obtained on the strict non-perturbative foundation in the framework of the oscillator representation. A comparison of the effectiveness of vacuum creation with the case of linear polarization one is fulfilled.

1 Introduction

The Schwinger effect [1] of the vacuum production of electron-positron pairs (EPP’s) under the action of electromagnetic fields is one from a few QED effects, that has not up to now an accurate experimental test. It is stipulated by the huge electric fields $E \sim E_c = 1.3 \cdot 10^{16} V/cm$ for the electrons that is necessary for observation of this effect in a constant field. Such field strength is unachievable for static fields therefore main attention was involved the theoretical study of pair creation by time-varying electric fields ([2]-[6]). The detailed description was obtained for the case of linearly polarized spatially homogeneous time dependent electric field. The sufficiently strong electric field can be achieved nowadays with laser beams only. The estimations made before ([2]-[8]) showed that pair creation by a single laser pulse with $E \ll E_c$ could be hardly observed. The more optimistic results have been obtained for a planning X-ray free electron lasers ([9]-[11]) and for the counter-propagating laser beams of optical range ([12]-[14]).

In the present work we make step on the way of theoretical research of the parton-antiparton vacuum creation in the nonstationary chromo-electric field of arbitrary polarization. The corresponding kinetic equation (KE) will be derived below on the strict non-perturbative dynamics basis. We will restrict ourself here by consideration of the nonstationary Schwinger effect in vacuum only leaving in a site the analysis of this effect in some plasma-similar medium (see, e.g., [15]). We use the oscillator representation (OR) for the construction of the kinetic theory (initially, this representation was be suggested in the scalar QED [16]). OR leads in the shortest way to quasipartical (QP) representation, in which all dynamical operators of observable quantities have diagonal form [5]. On this basis, the Heisenberg-like equations of motion for the creation and annihilation operators will be obtained in the spinor QED (Sect.2) that corresponds to a large N in QCD. The corresponding kinetic theory will be constructed in Sect. 3. The preliminary communication about these results has made on the conference [17]. In general case, the obtained KE of the non-Markovian type is rather complicated because of spin effects. The case of rather weak of the chromo-electric external field is considered in the Sect. 4. The short conclusions are summarized in Sect. 5.

We use the metric $g^{\mu\nu} = diag(1, -1, -1, -1)$ and the natural units $\hbar = c = 1$. 
2 Oscillator representation

Let us consider the QED system in the presence of an external quasi-classical spatially homogeneous time-dependent electric field of arbitrary polarization with the 4-potential (in the Hamilton gauge) $A^\mu (t) = (0, A(t))$ and the corresponding field strength $E(t) = -\dot{A}(t)$ (the overdots denote the time derivative). Such a field can be considered either as an external field, or as a result of the mean field approximation [18]. The Lagrange function is

$$\mathcal{L} = \frac{i}{2} \{ \bar{\psi} \gamma^\mu D_\mu \psi - (D_\mu^* \bar{\psi}) \gamma^\mu \Psi \} - m \bar{\psi} \psi,$$

where $D_\mu = \partial_\mu + ieA_\mu (t)$ and $-e$ is the electron charge. The equations of motion are

$$\begin{align*}
(i \gamma^\mu D_\mu - m) \psi &= 0, \\
\bar{\psi} (i \gamma^\mu D_\mu^* + m) &= 0,
\end{align*}$$

where $\bar{\psi} = \psi^+ \gamma^0$. The fields $\psi$ and $\psi^+$ compose the pair of canonical conjugated variables. The corresponding Hamiltonian is $(k=1,2,3)$

$$H(t) = i \int d^3 x \bar{\psi}^+ \psi = \int d^3 x \bar{\psi} \{-i \gamma^k D_k + m\} \psi.$$

In the considered case, the system is space homogeneous and nonstationary. Therefore the transition in the Fock space can be realized on the basis functions $\phi = \exp (\pm i kx)$ and creation and annihilation operators become the time dependent one, generally speaking. Hence, we have the following decompositions of the field functions in the discrete momentum space $(V = L^3$ and $p_i = (2\pi/L)n_i$ with an integer $n_i$ for each $i = 1,2,3)$:

$$\begin{align*}
\psi (x) &= \frac{1}{\sqrt{V}} \sum_k \sum_{\alpha=1,2} \left\{ e^{i k x} a_\alpha (k, t) u_\alpha (k, t) + e^{-i k x} b_\alpha^+ (k, t) v_\alpha (k, t) \right\}, \\
\bar{\psi} (x) &= \frac{1}{\sqrt{V}} \sum_k \sum_{\alpha=1,2} \left\{ e^{-i k x} a_\alpha^+ (k, t) \bar{u}_\alpha (k, t) + e^{i k x} b_\alpha (k, t) \bar{v}_\alpha (k, t) \right\}.
\end{align*}$$

The nearest aim is derivation of the equations of motion for the creation and annihilation operators on the basis of the primary equations [2] and the use of the free $u,v$-spinors as the basic functions with the natural substitution of the canonical momentum with the corresponding kinematic one (that corresponds to the basic OR idea). It is necessary to take into account, that the electron and positron states are different by sings of the charges and hence their kinematic momentum are $p = k - eA$ for electrons and $p^c = k + eA$ for positrons. Thus, the following “free-like” equations for the spinors are postulated in OR:

$$\begin{align*}
[\gamma p - m] u (k, t) &= 0, \\
[\gamma p^c + m] v (k, t) &= 0,
\end{align*}$$

where $p^0 = \omega (p) = \sqrt{m^2 + p^2}$. These equations have the orthogonal solutions which is convenient to normalize on unit [4,19]

$$\begin{align*}
u^+_\alpha (k, t) v_\beta (-k, t) &= 0, \\
u^+_\alpha (k, t) u_\beta (k, t) &= v^+_\alpha (-k, t) v_\beta (-k, t) = \delta_{\alpha\beta}, \\
u_\alpha (k, t) u_\beta (k, t) &= \frac{m}{\omega (k, t)} \delta_{\alpha\beta}, \\
u_\alpha (k, t) v_\beta (k, t) &= -\frac{m}{\omega (k, t)} \delta_{\alpha\beta}.
\end{align*}$$
The decompositions (14) and the relation (6) lead to the diagonal form of the Hamiltonian (3) at once (before second quantization)

\[ H(t) = \sum_{k,\alpha} \omega(k, t) \left[ a_{\alpha}^+(k, t)a_{\alpha}(k, t) - b_{\alpha}(-k, t)b_{\alpha}^+(k, t) \right]. \]  

(7)

Such form of the Hamiltonian is necessary for interpretation of the time dependent operators \( a^+, a \) (and \( b^+, b \)) as the operators of creation and annihilation of quasi-particles (anti-quasi-particles). Thus, this way results to QP representation at once.

Now, in order to get the equations of motion for the creation and annihilation operators in the OR, let us substitute the decomposition (14) in the Eq.(2) and use the relations (6). Then we obtain as the intermediate result the following closed system of equations of motion in the matrix form:

\[ \dot{a}(k, t) + U_{(1)}(k, t)a(k, t) + U_{(2)}(k, t)b^+(-k, t) = -i\omega(k, t)a(k, t), \]
\[ \dot{b}(-k, t) - b(-k, t)V_{(2)}(k, t) + a^+(k, t)V_{(1)}^+(k, t) = -i\omega(k, t)b(-k, t). \]

(8)

The spinor constructions was introduced here

\[ U_{(1)}^{\alpha\beta}(k, t) = u_{\alpha}^+(k, t)\dot{u}_{\beta}(k, t), \quad U_{(1)}^+ = -U_{(1)}, \]
\[ U_{(2)}^{\alpha\beta}(k, t) = u_{\alpha}^+(k, t)\dot{v}_{\beta}(-k, t), \quad U_{(2)}^+ = -V_{(1)}, \]
\[ V_{(2)}^{\alpha\beta}(k, t) = v_{\alpha}^+(k, t)\dot{v}_{\beta}(-k, t), \quad V_{(2)}^+ = -V_{(2)}. \]

(9)

The matrices \( U_{(2)} \) and \( V_{(1)} \) describe transitions between states with the positive and negative energies and different spin while the matrixes \( U_{(1)} \) and \( V_{(2)} \) show the spin rotations in the external field \( A^k(t) \).

The equations (8) are compatible with the standard anti-commutation relations because the matrix \( U_{(1)} \) is anti-hermitian:

\[ \{a_\alpha(k, t), a_{\beta}^+(k', t)\} = \{b_\alpha(k, t), b_{\beta}^+(k', t)\} = \delta_{k'k}\delta_{\alpha\beta}. \]

(10)

Let us write the \( u, v \)-spinors in the explicit form using the corresponding free spinors [20]:

\[ u_1^+(k, t) = A(p) \left[ \omega_+ + p^3, p_-, \right], \quad u_2^+(k, t) = A(p) \left[ 0, \omega_+, p_+, -p^3 \right], \]
\[ v_1^+(k, t) = A(p) \left[ -p^3, -p_-, \omega_+, \right], \quad v_2^+(k, t) = A(p) \left[ -p_+, p^3, 0, \omega_+ \right], \]

(11)

where \( p^\pm = p^1 \pm ip^2, \omega_+ = \omega + m \) and \( A(p) = [2\omega\omega_+]^{-1/2} \). In this representation \( U_{(1)} = V_{(2)} \) and \( U_{(2)} = -V_{(1)} \) so a sufficient set is

\[ U_{(1)}(k, t) = i\omega a[pE]\sigma, \quad U_{(2)}(k, t) = q\sigma, \]

(12)

where \( \sigma^k \) are the Pauli matrices, \( q = a[p(pE) - E\omega\omega_+] \) and \( a = e/2\omega^2\omega_+ \).

The operator equations of motion (8) become more simple:

\[ \dot{a}(k, t) = -U_{(1)}(k, t)a(k, t) - U_{(2)}b^+(-k, t) - i\omega(k, t)a(k, t), \]
\[ \dot{b}(-k, t) = b(-k, t)U_{(1)}(k, t) + a^+(k, t)U_{(2)}(k, t) - i\omega(k, t)b(-k, t). \]

(13)
3 Kinetic equation (the general case)

In order to get KE for time dependent electric fields of arbitrary polarization, let us introduce

\[ f_{\alpha\beta}(\mathbf{k}, t) = < a^+_{\beta}(\mathbf{k}, t) a_{\alpha}(\mathbf{k}, t) >, \]
\[ f^c_{\alpha\beta}(\mathbf{k}, t) = < b_{\beta}(-\mathbf{k}, t) b^+_{\alpha}(-\mathbf{k}, t) >, \]

where the averaging procedure is performed over the in-vacuum state \[5\]. The diagonal parts of these correlators are connected with relations

\[ \sum_{k,\alpha} (f_{\alpha\alpha}(\mathbf{k}, t) + f^c_{\alpha\alpha}(\mathbf{k}, t)) = Q, \]

where \( Q - \) total electric charge of the system. Differentiation over time leads to equations

\[ \dot{f} = [f, U(1)] - (U(2) f^{(+)} + f^{(-)} U(2)), \]
\[ \dot{f}^c = [f^c, U(1)] + (f^{(+)} U(2) + U(2) f^{(-)}), \]

where the auxiliary correlation functions was introduced

\[ f^{(+)}_{\alpha\beta}(\mathbf{k}, t) = < a^+_{\beta}(\mathbf{k}, t) b^+_{\alpha}(-\mathbf{k}, t) >, \]
\[ f^{(-)}_{\alpha\beta}(\mathbf{k}, t) = < b_{\beta}(-\mathbf{k}, t) a_{\alpha}(\mathbf{k}, t) >. \]

The equations of motion for these functions can be obtained similarly:

\[ \dot{f}^{(+)} = [f^{(+)}, U(1)] + (U(2) f - f^c U(2)) + 2i\omega f^{(+)}, \]
\[ \dot{f}^{(-)} = [f^{(-)}, U(1)] + (f U(2) - U(2) f^c) - 2i\omega f^{(-)} \]

with the connection \( f^{(+)} = f^{(-)}. \) In general case, the Eqs.(16) and(18) represent the closed system of 16 ordinary differential equations.

Accounting of charge symmetry (in consequence of that \( f^c = 1 - f \) allows to reduce this number up to 12. If to express the anomalous correlators (17) via the original functions (14) with help of Eqs.(16), it can obtain the closed KE in the integro-differential form [17]. Let us write this KE of non-Markovian type in the following matrix form:

\[ \dot{f}(t) = [f(t), U(1)] - U(2)(t) S(t) \int_{t_0}^{t} dt' S^+(t') [U(2)(t') f(t') - f^c(t') U(2)(t')] S(t') S^+(t') e^{2i\theta(t,t')}, \]

\[ - S(t) \int_{t_0}^{t} dt' S^+(t') [f(t') U(2)(t') - U(2)(t') f^c(t')] S(t') S^+(t') U(2)(t) e^{-2i\theta(t,t')}, \]

where the evolution operator of the spin rotations \( S(\mathbf{k}, t) \) is defined by equation

\[ \dot{S} = -U(1)(t) S(t) \]

with the initial condition \( S(t_0) = 1 \) (\( t_0 \) is some initial time) and \( \theta(t, t') = \theta(t) - \theta(t') \),

\[ \theta(t) = \int_{t_0}^{t} dt' \omega(\mathbf{k}, t'). \]
In comparison with the KE for the known case of the linear polarized field

$$A(t) = \{0, 0, A^3(t) = A(t)\}, \quad (22)$$

KE (19) has more complicated form because nontrivial spin effects. In general case, KE (19) is not allow simplification because of $[U_{(1)}, U_{(2)}] \neq 0$.

4 Perturbation theory

Let us write the source term (the right hand side) of KE (19) in the leading (second) order of the perturbative theory with respect to weak external field, $E_m/E_c \ll 1$. The adiabatic parameter $[8] \gamma = \frac{m\nu}{eE_m}$ is arbitrary (here $E_m$ is amplitude of external electric field, $\nu$ is it characteristic frequency). In according to the relations (12), $U_{(1)} \sim U_{(2)} \sim E_m$ in the leading approximation. Then in the leading order it is necessary to put $S \rightarrow S_0 = 1$ according to Eq. (20).

We take into account also electroneutrality of the system and relation (10), so $f^e = 1 - f$. In the considered leading approximation, the diagonal terms of the correlation functions (12) if small in comparison with unit, $f_{\alpha\alpha}$, and the non-diagonal terms $f_{\alpha\beta} \sim E^2$ for $\alpha \neq \beta$, that allows to omit the corresponding contribution in the source term

$$\dot{f}(t) = \int_{t_0}^{t} S p\{U_{(2)}(t)U_{(2)}(t')\} \cos 2\theta(t, t'). \quad (23)$$

As it follows from Eq. (12) ($\omega_+ = \omega_0$),

$$2Sp\{U_{(2)}(t)U_{(2)}(t')\} = \frac{e^2}{2\omega^2\omega_0^2} \{E(t)E(t')\omega_0 - (E(t)p)(E(t')p)\} = \Phi(p|t, t'). \quad (24)$$

If at the initial time before switch-on of an electric field the electrons and positrons are absent, we can write the total density of quasiparticles

$$n(t) = \frac{1}{4\pi^3} \int d^3p \int_{t_0}^{t} dt_1 \int_{t_0}^{t_1} dt_2 \Phi(p|t_1, t_2) \cos [2\theta(t_1, t_2)]. \quad (25)$$

In the case of the linear polarization (22), from Eqs. (24) and (25) it follows the well known result [13, 14]:

$$n(t) = \frac{1}{4\pi^3} \int d^3p \left| \int_{t_0}^{t} dt' \lambda(t') \exp (2i\theta(t, t')) \right|^2, \quad (26)$$

where $\lambda(p, t) = e(E(t)e\perp/2\omega^2$ and $\varepsilon\perp^2 = m^2 + p^2\perp$, $p\perp$ is the transversal momentum relatively of the vector $E(t)$.

The relations (25) and (26) are convenient for the numerical analysis, that is planned to made in the following work.
5 Conclusion

Thus, it was shown that the oscillator representation may be used for the KE derivation in the rather non-trivial case of the time-dependent chromo-electric field of arbitrary polarization. The obtained KE's can be used for investigation of particle-antiparticle vacuum creation in strong laser fields of optical and X-ray range as well as in the chromo-electric fields acting in the pre-equilibrium stage of QGP evolution. Besides, the used method opens prospects for further generalization (e.g., the account of a constant magnetic field).

References

[1] J. Schwinger, Phys. Rev. 82, 664 (1951); W. Heisenberg and H. Euler, Z. Phys. 98, 714 (1936); F. Sauter, Z. Phys. 69, 742 (1931).
[2] E. Brezin and C. Itzykson, Phys. Rev. D 2, 1191 (1970).
[3] V.S. Popov, Sov. J. Nucl. Phys. 34, 709 (1972); N.B. Narozni and A.I. Nikishov, Sov. Phys. JETP 38, 427 (1974); A.I. Nikishov, Tr. Fiz. Inst. Akad. Nauk SSSR 111, 152 (1979).
[4] W. Greiner, B. Müller and J. Rafelski, Quantum Electrodynamics of Strong Fields, Springer, Berlin, 1985.
[5] A.A. Grib, S.G. Mamaev and V.M. Mostepanenko, Vacuum Quantum Effects in Strong External Fields, Friedmann Laboratory Publishing, St. Petersburg, 1994.
[6] M.S. Marinov and V.S. Popov, Fortschr. Phys. 25, 373 (1977).
[7] F.V. Bunkin and I.I. Tugov, Sov. Phys. Dokl. 14, 678 (1969); B. Richards and E. Wolf, Proc. Roy. Soc. A (London) 253, 358 (1959); C.J. Troup and H.S. Perlman, Phys. Rev. D 6, 2299 (1972).
[8] S.V. Popov, JETP Lett. 74, 133 (2001); Phys. Lett. A 298, 83 (2002).
[9] A. Ringwald, Phys. Lett. B 510, 107 (2001).
[10] R. Alkofer et. al., Phys. Rev. Lett. 87, 193902 (2001).
[11] C.D. Roberts, S.M. Schmidt, and D.V. Vinnik, Phys. Rev. Lett. 89, 153901 (2002).
[12] H. K. Avetissian, A.K. Avetissian, G.F. Mkrtchian, and Kh.V. Sedrakian, Phys. Rev. E 66, 016502 (2002).
[13] A.V. Prozorkevich, A. Reichel, S.A. Smolyansky and A.V. Tarakanov, in Proceeding of SPIE, 5476, 68 (2004).
[14] D.B. Blaschke, A.V. Prozorkevich, S.A. Smolyansky, A.V. Tarakanov, Preprint: physics/0410114.
[15] S.S. Bulanov, A.M. Fedotov and F. Perogaro, hep - ph / 0409301.
[16] V.N. Pervushin, V.V. Skokov, A.V. Reichel, S.A. Smolyansky, and A.V. Prozorkevich, Int. J. Mod. Phys. A 20, 5689 (2005); hep-ph/0307200.
[17] A.V. Filatov, A.V. Prozorkevich and S.A. Smolyansky, Proc. of SPIE, v6165, 616509 (2006).
[18] V.B. Berestetský, E.M. Lifshiz and L.P. Pitaevskii, Quantum Electrodynamics, M. "Nauka", Fiz.Mat, 1980.
[19] N.N. Bogolubov and D.V. Shirkov, Introduction to the Theory of Quantum Fields, 3rd ed., Wiley, 1980.
[20] L.H. Raider, Quantum Field Theory, Cambridge Univ. Press, Cambridge, 1985.