Improved Mathematical Modeling of Six Phase Induction Machines Based on Fractional Calculus

AHMED M. SHATA\textsuperscript{1}, AYMAN S. ABDEL-KHALIK\textsuperscript{2}, (Senior Member, IEEE), RAGI A. HAMDY\textsuperscript{2}, (Senior Member, IEEE), MOHAMED ZAKARIA MOSTAFA\textsuperscript{2}, AND SHEHAB AHMED\textsuperscript{3}, (Senior Member, IEEE)

\textsuperscript{1}Alexandria Higher Institute of Engineering and Technology (AIET), Alexandria 21311, Egypt
\textsuperscript{2}Electrical Engineering Department, Faculty of Engineering, Alexandria University, Alexandria 21544, Egypt
\textsuperscript{3}CEMSE Division, King Abdullah University of Science and Technology, Thuwal 23955, Saudi Arabia

Corresponding author: Ahmed M. Shata (ahmed.shata@aiet.edu.eg)

ABSTRACT Multiphase induction machine modelling represents a crucial research topic for both machine control and performance evaluation purposes. Generally, multiphase induction machines are preferably modelled using the vector space decomposition technique with some assumptions to simplify the mathematical model. However, different sources of non-linearities, including low order harmonics mapped to secondary subspaces, cross-coupling saturation and iron losses result in a notable deviation from the experimentally measured waveforms. Furthermore, considering full symmetry amongst motors phases seems to be a rather idealistic assumption. Fractional order modelling has recently emerged as a promising mathematical technique to model highly nonlinear electrical and mechanical systems. This paper proposes an improved vector space decomposition (VSD)-based fractional order model of an asymmetrical six-phase induction machine under both healthy and open phase fault conditions with different neutral arrangements. The appropriate differentiation orders have been obtained by optimizing the error function between simulated and experimental waveforms. The results are compared with the conventional integral order-based model. Experimental validation has been carried out using a 1.5Hp prototype induction machine.

INDEX TERMS Asymmetrical six phase, induction machine, fractional calculus, fractional order modeling, particle swarm optimization, healthy condition, fault condition, connected neutrals, isolated neutrals.

I. INTRODUCTION

Multiphase machines are used nowadays in a number of industrial applications, such as high-speed elevators, trains, ship propulsion, oil and gas industry, and high-power wind energy conversion systems [1], [2]. This is a result of their advantages over traditional three phase machines, including lower torque ripples, higher efficiency, lower space harmonics content, lower power ratings of semiconductor devices and higher fault tolerance [2], [3]. Asymmetrical six phase machines (A6P) are the most popular multiphase machines in most practical industrial applications, especially in ship propulsion and the oil and gas industry [2]. An asymmetrical six-phase machine comprises two three-phase stator windings having a spatial phase shift angle of 30°, while the two neutrals can either be connected or isolated [5].

Modelling of multiphase induction machines (IMs) incorporates multiple challenges due to the different core nonlinearities. Therefore, some assumptions are always taken into consideration to simplify the mathematical modelling in most available literature, which include: [3], [4]

- Linear magnetic circuit.
- The iron losses are neglected.
- The machine air gap is considered constant, smooth and symmetrical.
- All space harmonics are neglected.

Although modelling of the A6P IM under these assumptions may give acceptable results under steady-state healthy conditions, a notable gap between simulation and experimental results always exists under both transient and fault conditions [4].

The mathematical modelling of the A6P IM has been the objective of many research papers in the available literature [6]–[8]. The A6P IM is commonly modelled using either double \textit{dq} or vector space decomposition (VSD) modelling
modelling using fractional order calculus is to develop more integral and differential orders [19]. The aim behind systematic applications, dynamics of most mechanical and electrical systems since the overall system degrees of freedom are decoupled from mechanical to electrical states from integer to non-integer values. This increases the flexibility of the mathematical model as it can be used to identify the differential orders of different states by minimizing the summation of the absolute values of the motor phase currents.

In most available VSD models, the secondary subspaces are treated as non-flux/torque producing subspaces that are simply modelled using the stator impedance [5]. While this assumption could only be reasonable under healthy conditions, the low order harmonics mapped to secondary subspaces introduce a notable effect on machine dynamics under transient and fault conditions. Therefore, an improved low-order space harmonic model that takes into consideration the effect of these parasitic harmonic components has been introduced in [7]. The study showed that the neutral arrangement, namely, isolated neutrals and connected neutrals, as shown in Fig. 1, will highly affect the starting behavior of an A6P IM under open phase fault (OPF) conditions. The dominant low-order harmonics, namely, 3rd, 5th and 7th order air gap space harmonics, commonly mapped to the zero and x-y subspaces, respectively, have been included to better estimate the machine dynamic performance under healthy as well as healthy conditions. The proposed model was capable of exploring the effect of these low order harmonics which introduces synchronous speed points in the torque-speed curve. Nevertheless, some notable deviations were observed in the dynamic response of the motor phase currents.

Fractional order calculus has long been a purely mathematical tool until it was successfully utilized to provide explanations for several physical phenomena [18]. Fractional calculus expands the conventional integral order calculus from integer to non-integer values. A more accurate mathematical model could be accomplished by fractional calculus since the overall system degrees of freedom are increased [19], [20]. It is fair to say that in the utmost realistic applications, dynamics of most mechanical and electrical systems are inherently characterized to have non-integer integral and differential orders [19]. The aim behind system modelling using fractional order calculus is to develop more accurate dynamic system models to facilitate improved comparison with experimental results [21].

Recently, a body of research has been proposed to express electrical and mechanical systems based on fractional calculus. In [21], a comparative study is done between the mutual inductance behavior expressed in fractional order and integral order-based models. While, the work done in [22] and [23] proposes a mathematical fractional order modelling of Voltage Source Converters (VSC). In a related context, the work in [24] has introduced a fractional order approach to analyze boost DC-DC converters. It has been proven that the inductance seems to have an inherent fractional order behavior. The work in [25] describes a fractional order buck converter using the fractional definition of the Riemann-Liouville. A fractional order circuit-based model for lithium-ion batteries for electric vehicles was introduced in [26]–[28]. It was obvious that utilizing fractional order models has enhanced system dynamics. Other researches focused on clarifying the fractional order originality of electric circuit components. For instance, the study introduced in [29] give a brief survey on fractional order circuits. Another research has focused on investigating the fractional order capacitance [30]–[32] and supercapacitance [33]. Fractional order modeling has also given more accurate modelling and identification for piezoelectric actuators, as discussed in [34]. Moreover, an advanced analysis and effective implementation of fractional order circuits has been covered in [35], [36]. Despite the notable interest from the research community to adopt this mathematical modelling technique in different electrical systems, no prominent effort has been dedicated, to the best of the authors’ knowledge, to improve the mathematical modelling of electric machines so far.

This paper introduces an improved VSD-based model for A6P IM based on fractional calculus. The work done herein extends the study given in [7] to model the A6P IM by assuming that the differential orders of all mechanical and electrical states have non-integer values. This increases the modelling degrees of freedom and improves the accuracy of estimating system dynamics. The study is carried out under both healthy and open phase conditions and the neutral arrangement is also taken into consideration. The main aim behind this work is to improve the estimation accuracy of the dynamical behavior and reduce the gap between simulation and experimental results. An optimization technique is utilized to identify the differential orders of different states by minimizing the summation of the absolute values of the motor speed and phase current errors. The accuracy of the proposed fractional order models (FOM) is validated and compared with a conventional integral order model (IOM), having the same machine parameters, using a 1.5Hp prototype machine.

II. PROPOSED VSD BASED FRACTIONAL ORDER MODEL
In this section, the proposed FOM of an A6P IM is introduced. The proposed model extends the IOM given in [4] by simply replacing the integration order of all electrical and mechanical states from integer to non-integer values.

FIGURE 1. Possible neutral configurations for asymmetrical six-phase stator winding. (a) Isolated (2N) arrangement. (b) Connected (1N) arrangement.
Furthermore, the method used to identify the differentiation orders of different system states is explained.

### A. MACHINE DYNAMIC EQUATIONS

The conventional VSD modeling approach for A6P machines decomposes the machine phase quantities, $Q_{\alpha\beta}$, into three orthogonal subspaces, namely, $\alpha - \beta, x - y$ and the $0^+ - 0^-$ subspaces, using (1) [1]-[3].

$$T_{\text{VSD}} = \frac{1}{\sqrt{3}}$$

The model proposed in this study employs the same IOM given in [7] and simply replaces the integral differential operator $p$ by $p^{\text{sub}}$ where $n_{\text{sub}} \in \{n_{\alpha\beta}, n_{xy}, n_{0}\}$, representing the differential orders for the three subspaces, respectively. The differential order of the mechanical equation is represented by $n_m$. In the case of the IOM, the value of all differential orders is simply one.

Hence, the equations of the $\alpha\beta$ subspace are represented using (2)-(5) [7].

$$v_{\alpha\beta} = R_i i_{\alpha\beta} + p^{\text{sub}} \lambda_{\alpha\beta}$$

$$v_{\alpha\beta} = R_i i_{\alpha\beta} + p^{\text{sub}} \lambda_{\alpha\beta}$$

$$0 = R_i i_{\alpha\beta} + p^{\text{sub}} \lambda_{\alpha\beta} + \omega_r \lambda_{\alpha\beta}$$

$$0 = R_i i_{\alpha\beta} + p^{\text{sub}} \lambda_{\alpha\beta} - \omega_r \lambda_{\alpha\beta}$$

Similarly, the flux linkage equations for this subspace are given by (6)-(9) [4].

$$\lambda_{\alpha\beta} = (l_{i1} + L_{m1}) i_{\alpha\beta} + L_{m1} i_{\alpha\beta}$$

$$\lambda_{\alpha\beta} = (l_{i1} + L_{m1}) i_{\alpha\beta} + L_{m1} i_{\alpha\beta}$$

$$\lambda_{\alpha\beta} = (l_{i1} + L_{m1}) i_{\alpha\beta} + (l_{r1} + L_{m1}) i_{\alpha\beta}$$

$$\lambda_{\alpha\beta} = (l_{i1} + L_{m1}) i_{\alpha\beta} + (l_{r1} + L_{m1}) i_{\alpha\beta}$$

The flux linkage equations for this subspace are expressed using (16)-(21) [7].

$$\lambda_{\alpha\beta} = (l_{i1} + L_{m1}) i_{\alpha\beta} + L_{m1} i_{\alpha\beta} + L_{m1} i_{\alpha\beta} + L_{m1} i_{\alpha\beta}$$

$$\lambda_{\alpha\beta} = (l_{i1} + L_{m1}) i_{\alpha\beta} + L_{m1} i_{\alpha\beta} + L_{m1} i_{\alpha\beta}$$

$$\lambda_{\alpha\beta} = (l_{i1} + L_{m1}) i_{\alpha\beta} + L_{m1} i_{\alpha\beta} + L_{m1} i_{\alpha\beta}$$

$$\lambda_{\alpha\beta} = (l_{i1} + L_{m1}) i_{\alpha\beta} + L_{m1} i_{\alpha\beta} + L_{m1} i_{\alpha\beta}$$

$$\lambda_{\alpha\beta} = (l_{i1} + L_{m1}) i_{\alpha\beta} + L_{m1} i_{\alpha\beta} + L_{m1} i_{\alpha\beta}$$

Finally, the voltage equations of the bidirectional zero-sequence subspace are represented using (22)-(25). The dominant low order harmonic of this subspace is the third order harmonic flux component.

$$v_{0+} = R_i i_{0+} + p^{\text{sub}} \lambda_{0+}$$

$$v_{0-} = R_i i_{0-} + p^{\text{sub}} \lambda_{0-}$$

$$0 = R_i i_{0+} + p^{\text{sub}} \lambda_{0+} + 3\omega_r \lambda_{0-}$$

$$0 = R_i i_{0-} + p^{\text{sub}} \lambda_{0-} - 3\omega_r \lambda_{0+}$$

The corresponding flux linkage equations are shown in (26)-(29).

$$\lambda_{0+} = (l_{i3} + L_{m3}) i_{0+} + L_{m3} i_{0-}$$

$$\lambda_{0-} = (l_{i3} + L_{m3}) i_{0-} + L_{m3} i_{0+}$$

$$\lambda_{0+} = (l_{i3} + L_{m3}) i_{0+} + L_{m3} i_{0-}$$

$$\lambda_{0-} = (l_{i3} + L_{m3}) i_{0-} + L_{m3} i_{0+}$$

The torque components corresponding to the three subspaces are determined from (30)-(32), respectively.

$$T_d = p L_m (i_{\alpha\beta} i_{\alpha\beta} - i_{\alpha\beta} i_{\alpha\beta})$$

$$T_d = 5 p L_m (i_{\alpha\beta} i_{\alpha\beta} - i_{\alpha\beta} i_{\alpha\beta}) + 7 p L_m (i_{\alpha\beta} i_{\alpha\beta} - i_{\alpha\beta} i_{\alpha\beta})$$

$$T_d = 3 p L_m (i_{\alpha\beta} i_{\alpha\beta} - i_{\alpha\beta} i_{\alpha\beta})$$

The total torque is then given by (33) and the equation of motion is represented by (34) [1].

$$T_d = T_d + T_d + T_d$$

$$T_d = T_d + T_d + T_d$$

$$T_d = T_d + T_d + T_d$$

where, $\omega$ is the moment of inertia and $F$ represents the friction coefficient.

### B. IDENTIFICATION OF FOM DIFFERENTIAL ORDERS

The main challenge of the proposed FOM is to identify the required differential orders for different states that minimize the error between simulated and practical motor behavior. This case, therefore, involves an optimization problem to minimize the total error of all system states. To this end, the particle swarm optimization technique (PSO) will be employed in this study. This method is relatively simple in comparison with other heuristic algorithms and is shown to be efficient for both linear and nonlinear systems [37]-[39]. The objective function is defined as the total absolute error.
between the experimental and simulation results. There are two types of error signals, one for mechanical speed response and the other for the six-phase currents. The error in the motor speed response is expressed as $e_m(t)$, while the current errors...
are defined by $e_{a\rightarrow f}(t)$. Hence, the total error is expressed by,

$$e_t(t) = |e_m(t)| + \sum_{i=a\rightarrow f} |e_i(t)| \quad (35)$$

The objective function is chosen to be in the form of an Integral Absolute Error (IAE) because it yields better results compared to a simple error integration. Therefore, the objective function is expressed as in (36) [27];

$$\text{IAE} = \int_0^{\tau_s} e_t(t) \, dt \quad (36)$$

where $\tau_s$ is the motor settling time.

The machine model is built in the MATLAB/Simulink platform. The model is run, and the simulated machine variables are used to estimate the total error according to (35). The PSO algorithm is used to update the machine inertia and all the integration orders. This process is repeated until (36) is minimized. The flow chart for the optimization algorithm is shown in Fig. 2.

III. MODEL VERIFICATION

In this section, a comparative study between the experimental results and the proposed FOM is briefly discussed. The experimental setup used to validate the proposed model is first described. In the subsequent subsections, the optimized differential orders are obtained for three cases, namely, healthy condition, OPF condition with isolated neutral arrangement, and OPF condition with connected neutral arrangement.

A. EXPERIMENTAL SETUP

The prototype A6P machine is built by rewinding a 1.5Hp, 4-pole, 380V, 50Hz standard three-phase IM with two separate three-phase windings displaced by a spatial angle of $30^\circ$ with the same conductor cross sectional area and same rated current of the original machine. Table 1 shows the new machine specification after the rewinding process. The machine is fed from two three-phase inverters connected to the same dc-link and controlled using conventional sinusoidal pulse width modulation (SPWM) at a 5kHz switching frequency. The prototype system is shown in Fig. 3. The machine parameters are determined based on the technique introduced in [22] and given in Table 2.

It is worth mentioning that the main deviation between the simulation and experiential results takes place during the transient periods and under fault conditions. The best way to carry out this test is to run the machine using open loop-control under free running mode [7]. To avoid saturating the current measuring boards during the starting period, the machine is operated at reduced voltage level of 50V and rated frequency. For the cases shown, the optimizer is run for both the IOM and FOM. For the IOM, the machine inertia
is the only optimized variable, while the integration orders for all states are assumed unity. On the other hand, for the FOM, the optimized variables are the motor inertia as well as the integration orders for both mechanical and electrical states.

**B. MODEL VALIDATION**

In this subsection, the machine response is simulated under healthy and open-phase conditions. Under healthy conditions, all phases are connected to the supply. While for the open phase case, phase \( a \) is disconnected. By running the
optimization algorithm under different cases, the optimized values for the machine inertia as well as the integration orders for different subspaces are given in Table 3.

Under healthy conditions, the only functional subspace is the \( \alpha \beta \) plane. Hence, the optimal integration order for both \( xy \) and zero subspaces are simply set to unity. The optimizer under this case was run only for the electrical states of the fundamental subspace, \( n_{\alpha \beta} \), and the mechanical equation (\( n_m, J \)).

Based on the obtained optimized values given in Table 3, the comparison between the simulation and experimental data is shown in Figure 9. The figures display the open loop free running mode OPF-1N phase currents response for different cases, with the optimized values for the machine inertia and integration orders provided in Table 3.

**FIGURE 9.** Open loop free running mode OPF-1N phase currents response (a) IOM (b) FOM.
respectively. Under this case, the speed and phase currents are shown in Figs. 6 and 7, respectively, and the simulation results be nullified. Therefore, only the integration order will be added to the optimized variables. The simulation results for speed responses for the 2N and 1N arrangements are shown in Figs. 6 and 7, respectively, and the machine start-up response, especially at low speeds. Hence, the integration order of this subspace, \( n_0 \), will be added to the optimization problem. The simulation results for this case are shown in Figs. 8 and 9. Again, the FOM corresponds to a much better estimation of the motor dynamic response than the IOM, which was also concluded for the previous cases.

The machine is also simulated under OPF conditions, where the nonfundamental subspaces introduce extra low order harmonic components. The optimizer will also search for the optimal integration orders for the nonfundamental subspaces.

Under open-phase conditions, the stator may be configured with any of the two neutral arrangements shown in Fig. 1. The simulation results for speed responses for the 2N and 1N arrangements are shown in Figs. 6 and 7, respectively, and the responses for RMS phase currents are given in Figs. 8 and 9, respectively.

Under the 2N arrangement, the zero-sequence current component is zero, and hence, the effect of this subspace will be nullified. Therefore, only the integration order \( n_{xy} \) will be added to the optimized variables. The simulation results for the speed and phase currents are shown in Figs. 6 and 7, respectively. Under this case, the 5th and 7th space harmonics mapped to the xy subspace introduce a sub-synchronous speed point and a torque dip in the torque speed curve. The motor settling time will, therefore, be longer than that of the healthy case due to motor torque reduction. It is also fair to conclude that the FOM-based simulation results are much closer to the experimental results. The deviation between simulation and experimental results is more obvious as the machine approaches the steady-state operating point.

IV. CONCLUSION

This paper introduced an improved VSD-based mathematical model for asymmetrical six phase induction machines based on fractional calculus. The model has been validated using a 1.5Hp prototype machine. The simulation results of the proposed model have also been compared with the conventional integral order-based model. The fractional order-based model introduces additional degrees of freedom and can better simulate the actual behaviour of the motor dynamics. A particle swarm optimization technique was applied to identify the optimal differential orders of the different subspaces as well as the rotor inertia such that the error between simulation and experimental results was minimized. The simulation results showed that the proposed model corresponds to a closer dynamic behaviour to the experimentally measured responses especially under open phase conditions, where the effect of the nonfundamental subspaces is more pronounced.


table_1=
| SPECIFICATION     | Value |
|-------------------|-------|
| Rated phase voltage (V) | 110   |
| Rated Power (Hp)    | 1.5   |
| Rated phase current (A) | 3.4   |
| Rated frequency (Hz) | 50    |
| Rated speed (rpm)   | 1420  |
| Pole number         | 4     |

table_2=
| SUBSPACE    | \( r_{sk} \) (\( \Omega \)) | \( l_{sk} \) (mH) | \( r_{xy} \) (\( \Omega \)) | \( l_{xy} \) (mH) | \( n_{xy} \) |
|-------------|-------------------------------|------------------|-----------------|------------------|-------------|
| \( a\beta \) | 1.14                          | 1.95             | 12.9            | 161              | 2           |
| \( 0_+0_- \) | 7.8                           | 0.995            | 6.58            | 14.4             | 3           |
| \( xy \)     | 1.46                          | 0.195            | 1.29            | 0.934            | 5           |
| \( k = 7 \)  | 1.46                          | 0.39             | 2.58            | 0.242            | 7           |

table_3=
| Var. | Case | Healthy | OPF 2N | OPF 1N |
|------|------|---------|--------|-------|
| \( \omega \) | IOM  | FOM     | IOM    | FOM   |
| 0.0185 | 0.0195 | 0.0195  | 0.0199 | 0.02  | 0.021 |
| \( \omega_{ag} \) | 1    | 0.99    | 1      | 0.99  |
| \( \omega_{xy} \) | -    | -       | 1      | 0.99  |
| \( n_{0} \)  | -    | -       | -      | 1     | 0.98  |
| \( n_{m} \)  | 1    | 0.96    | 1      | 0.94  | 1     | 0.89  |

REFERENCES

[1] M. Slunjiski, O. Djordjevic, M. Jones, and E. Levi, “Symmetrical/asymmetrical winding reconfiguration in multiphase machines,” IEEE Access, vol. 8, pp. 12835–12844, 2020.
[2] E. Levi, “Multiphase electric machines for variable-speed applications,” IEEE Trans. Ind. Electron., vol. 55, no. 5, pp. 1893–1909, May 2008.
[3] J. Paredes, B. Prieto, M. Satrústegui, I. Elósegui, and P. González, “Improving the performance of a 1-MW induction machine by optimally shifting from a three-phase to a six-phase machine design by rearranging the coil connections,” IEEE Trans. Ind. Electron., vol. 68, no. 2, pp. 1035–1045, Feb. 2021.
[4] Y. Wang, J. Yang, R. Deng, and G. Yang, “Parameters estimation for multiphase induction machine with concentrated windings through finite element method,” IET Electric Power Appl., vol. 14, no. 10, pp. 1807–1817, Oct. 2020.
[5] T. S. de Souza, R. R. Bastos, and B. J. C. Filho, “Synchronous-frame modeling and dq current control of an unbalanced nine-phase induction motor due to open phases,” IEEE Trans. Ind. Appl., vol. 56, no. 2, pp. 2097–2106, Mar./Apr. 2020.
[6] M. J. Duran, E. Levi, and F. Barrero, “Multiphase electric drives: Introduction,” in Wiley Encyclopedia of Electrical and Electronics Engineering, 2017.
[7] A. S. Abdel-Khalik, R. A. Handy, A. M. Massoud, and S. Ahmed, “Low-order space harmonic modeling of asymmetrical six-phase induction machines,” IEEE Access, vol. 7, pp. 6866–6876, 2019.
[8] E. Levi, R. Bojoi, F. Profumo, H. A. Toliyat, and S. Williamson, “Multiphase induction motor drives—a technology status review,” IET Electr. Power Appl., vol. 1, no. 4, pp. 489–516, 2007.
RAGI A. HAMDY (Senior Member, IEEE) received the B.Sc. and M.Sc. degrees from Alexandria University, Alexandria, Egypt, in 1991 and 1992, respectively, and the Ph.D. degree from Heriot-Watt University, U.K., in 1999. He is currently a Professor with the Electrical Engineering Department, Faculty of Engineering, Alexandria University. His current research interests include electric machines, electric drives, and power electronics.

MOHAMED ZAKARIA MOSTAFA received the B.Sc. degree in communications and electronics and the M.Sc. and Ph.D. degrees in electrical engineering from the Faculty of Engineering, Alexandria University, Alexandria, Egypt. He is currently a Professor with the Faculty of Engineering, Alexandria University. He taught many technical courses in electrical engineering system design and implementation, taught up to 20 undergraduate and postgraduate subjects, supervising more than 50 theses, and published more than 110 papers in different international conferences, forums, and journals.

SHEHAB AHMED (Senior Member, IEEE) received the B.Sc. degree in electrical engineering from Alexandria University, Alexandria, Egypt, in 1999, and the M.Sc. and Ph.D. degrees from the Department of Electrical and Computer Engineering, Texas A&M University, College Station, TX, USA, in 2000 and 2007, respectively. He was with Schlumberger Technology Corporation, Houston, TX, from 2001 to 2007, developing downhole mechatronic systems for oilfield service products. He was with Texas A&M University at Qatar, from 2007 to 2018. He is currently a Professor of electrical engineering with the CEMSE Division, King Abdullah University of Science and Technology (KAUST), Saudi Arabia. His research interests include mechatronics, solid-state power conversion, and electric machines.

***