System for dynamic synthesis of differential equations of mobile machines motion

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Abstract. The process of creating the calculation model structure of mobile machines of arbitrary configuration is formalized. An algorithm for dynamic synthesis of motion equations according to the calculation model is created, which enables to automate the development of differential equations of motion without the participation of a researcher. The method of displaying simulation results using a database of graphic elements based on raster images is substantiated. Based on the object approach, all stages from the development of the flowchart to the obtaining of the final simulation result are combined into a complete algorithm.

1. Introduction
Mathematical modeling is becoming an effective tool in the practice of developing new designs for ground mobile vehicles [1].

The level of development of computer technology provides today the technical possibility of using methods of mathematical modeling in problems of modeling mobile machines and optimizing their parameters [2]. At the same time, the absence of generalized models for describing the dynamics of mobile machines restrains the process of automating their research and justifying the parameters during design. The methods of classical mechanics provide a general basis for solving such a problem, but are associated with the need for analytical transformations that cannot be automated. From these positions, it is advisable, based on the specifics of the configuration of mechanical systems that make up mobile machines, to concretize a number of aspects of the equations of classical mechanics in relation to the area under consideration in order to bring the synthesis process of a mathematical model of a mobile machine to the level of a formal algorithm and implement this algorithm programmatically.

If we consider the kinematics of mobile machines for various purposes, it can be noted that the main feature is a finite number of links with kinematic two-way connections. At the same time, it is easy to establish that these connections can be considered as hinged [3]. Therefore, without losing the generality of reasoning, any mobile machine can be represented in a mathematical model as a hinged mechanical system of solids under the action of external forces, such as the reaction of the soil to support wheels, working bodies, etc. Thus, it is justified in relation to the mechanical model to consider mobile machines as hinged multi-link machines. In this formulation, it becomes possible to concretize equations of motion of the links and the mathematical description of external influences.

Mobile machines can be considered as multi-link hinged structures under the influence of external forces. The uncertainty of the problem of the synthesis of differential equations is associated with a
variable number of links, hinges, and, therefore, with a variable number of degrees of freedom of this mechanical system, depending on the configuration. This does not allow us to formulate a unified system of differential equations for the general case and leads to the need to synthesize differential equations of motion depending on the configuration of the mobile machine. General methods for constructing equations, such as using the Lagrange – Appel equations and others, are poorly automated because they require analytical transformations. The way out is to apply the law of motion of the center of the mass of a mechanical system in combination with the law of change in the angular momentum recorded for each of the branches of the hinged machine.

2. Discussion

Let us consider a pivotally connected mechanical system including \( n \) links connected by an \((n-1)\) hinge. Such a system has \((3n + 3)\) degrees of freedom in three-dimensional space. According to the principle of release from links, it is permissible to decompose this system along any of the hinges if we add equivalent link reactions to the system of external forces, that is, consider an isolated mechanical system, representing any of the branches of the original, taking into account the equivalent reactions of hinges. A feature of the hinge joint is the presence of the resultant reaction of the bonds with the center coinciding with the center of the hinge. Thus, if, with such a decomposition, the law of change in the angular momentum for the selected subsystem (branch) is written relative to the center of the hinge, the moments of the reactions of the connections in the dependencies are zeroed, that is, it is realistic to compose the equations of motion for each of the branches of a hinged machine, considering them isolated.

For each of the \((n-1)\) subbranches, we write down the equations for changing the angular momentum. In vector form, they can be written as follows:

\[
\frac{d\vec{K}_i}{dt} = \vec{M}_i, \quad i = 1, \ldots (n - 1),
\]

where \( \vec{K}_i \) - vector of the angular momentum of the \( i \)-th branch relative to the \( i \)-th hinge;

\( \vec{M}_i \) - vector of the moment of external forces applied to the \( i \)-th branch of the articulated machine relative to the \( i \)-th hinge.

Let us add to this system equations of motion of the center of mass and the equations for changing the angular momentum for the entire hinged machine:

\[
m \frac{d\vec{V}_c}{dt} = \sum_i \vec{F}_i, \quad \frac{d\vec{K}}{dt} = \vec{M},
\]

where \( \vec{V}_c \) - velocity vector of the mass center;

\( \vec{F}_i \) - \( i \)-th external force applied to the mobile machine;

\( \vec{K}, \vec{M} \) - respectively, the angular momentum vector and the main moment of external forces for a single center of reference;

\( m \) - weight of the mobile machine

As a result, a system of \((3n + 3)\) scalar equations is formed that establishes the law of motion of a mobile machine.

The kinetic moment \( \vec{K} \) can be written through the main vector of angular velocity \( \vec{\omega}_i \) of the branch of the link, equal to the geometric sum of the angular velocities of the links of the branch in the form:

\[
\vec{K}_i = J_i \cdot \vec{\omega}_i,
\]

where \( J_i \) - tensor of inertia of the \( i \)-th branch.

Then the change in the angular momentum \( \vec{K}_i \) will be written as follows:
\[ \frac{d\vec{\dot{F}}_i}{dt} = \frac{dJ_i}{dt} \vec{\omega}_i + J_i \frac{d\vec{\omega}_i}{dt} = \frac{dJ_i}{dt} \vec{\omega}_i + J_i \vec{\dot{e}}_i, \]

where \( \vec{\dot{e}}_i \) - the geometric sum of the angular accelerations of the branch links.

The first element of the last sum is of the second order of smallness in comparison with the second element and can be neglected.

Then the equations of motion can be written in the following form:

\[ J_i \vec{\dot{e}}_i = \vec{M}_t, J \vec{\dot{e}} = \vec{\dot{M}}, \vec{m}\vec{\dot{a}}_c = \sum \vec{F}_i, i = 1, \ldots (n - 1) \]

where \( \vec{\dot{a}}_c \) - vector of acceleration of the mass center of the mobile machine.

Thus, for numerical simulation, it is necessary to recalculate the tensors of inertia at each iteration.

Let’s consider the algorithms for this calculation. For this purpose, we will decide on a method for constructing branches.

Let’s renumber all the links of the mobile machine.

Each hinge will be considered as two half-hinges, when connected to form a ball joint.

The position of each half-hinge is tied to the link of the working machine and is characterized by the coordinates of the center of the joint in the coordinate system associated with the link of the mobile machine. We say that the half-hinge \( i \) is set on link \( j \) and has as its coordinates vector \( \vec{X}_{ij} \).

An arbitrary number of half-hinges can be installed on the link. Therefore, for each link \( j \), we introduce into consideration vector \( \vec{S}_j = (S_{j1}, S_{j2}, \ldots, S_{jn}) \), the elements of which are determined by the formula:

\[ S_{jk} = \begin{cases} 1 & \text{if } k-th \text{ half-hinge is installed on the link } \\ 0 & \text{otherwise} \end{cases} \]

We will consider vectors as column vectors.

Let us also introduce into consideration the matrix of joints of the half-hinges \( A = \{a_{ij}\} \), the elements of which are formed by the formula:

\[ a_{ij} = \begin{cases} 1 & \text{if } i-th \text{ half-hinge is connected to } j-th \\ 0 & \text{otherwise} \end{cases} \]

Then

\[ \vec{S}_j^T \cdot A \cdot \vec{S}_i = \begin{cases} 1 & \text{if links } i \text{ and } j \text{ are connected by a hinge} \\ 0 & \text{otherwise} \end{cases} \]

Thus, it is possible to form a matrix for connecting the links of a mobile machine

\[ C = \{C_{ij}\} = \{\vec{S}_i^T \cdot A \cdot \vec{S}_j\} \]

Let us consider an example of a connection diagram for the links of a mobile machine (Figure 1).
The vectors for placing half-hinges will be written as follows:

\[ S_1 = (1,0,0,0,0,0,0,0,0,0,0,0,0,0), \quad S_2 = (0,1,1,0,0,0,0,0,0,0,0,0,0,0) \]
\[ S_3 = (0,0,0,1,0,1,0,0,0,0,0,0,0,0), \quad S_4 = (0,0,0,0,0,1,0,1,1,0,0,0,0,0) \]
\[ S_5 = (0,0,0,0,0,0,0,0,1,0,0,0,0,0), \quad S_6 = (0,0,0,0,0,0,0,0,0,1,0,0,0,0) \]
\[ S_7 = (0,0,0,0,0,0,0,0,0,0,1,0,0,0), \quad S_8 = (0,0,0,0,0,0,0,0,0,0,0,1,0,0) \]

The matrix of joints of half-hinges for the diagram of a mobile machine shown in Figure 1 is as follows:

\[
A = \begin{bmatrix}
1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 1 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 1 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 1 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1
\end{bmatrix}
\]

Then the matrix of link connections will be calculated as follows:
The matrix of connections has a unique property: if you select an \( i \)-row from the matrix and multiply it by matrix \( C \), then in the resulting sequence of vectors the indices of nonzero elements will correspond to the numbers of links connected at least indirectly to link \( i \), i.e. if we uncouple any hinge and obtain zero in the matrix of connections \( A \) instead of any length, then the mechanical system will split into two branches. Using this algorithm, these links included in these branches are distinguished by a formal procedure.

For example, in the previous scheme, we will release from the connection connecting links 2 and 3 (we will disconnect half-hinges 3 and 5) by putting elements \( a_{35}=a_{53}=0 \) in matrix \( A \). Then matrix \( C \) will take the following form:

\[
C^* = \begin{bmatrix}
1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\
1 & 1 & 1 & 1 & 0 & 0 & 0 & 0 \\
0 & 1 & 1 & 0 & 1 & 1 & 0 & 0 \\
0 & 1 & 0 & 1 & 0 & 0 & 1 & 1 \\
0 & 0 & 1 & 0 & 1 & 0 & 0 & 0 \\
0 & 0 & 0 & 1 & 0 & 0 & 1 & 0 \\
0 & 0 & 0 & 1 & 0 & 0 & 0 & 1
\end{bmatrix}
\]

Let’s choose, for example, the fourth line

\( S_4 = (0,1,0,1,0,0,1,1) \)

Then

\[
S_4 \cdot A = (1,2,0,4,0,0,2,2)
\]
\[
S_4 \cdot A \cdot A = (3,7,0,10,0,0,6,6)
\]

The iterative process has ended at the very beginning. With a less successful choice of the matrix row, the number of iterations can be large. For example, let’s select the first line:

\( S_1 = (1,1,0,0,0,0,0,0) \)

Then

\[
S_1 \cdot A = (2,2,0,1,0,0,0,0)
\]
\[
S_1 \cdot A \cdot A = (4,5,0,3,0,0,1,1)
\]
\[
S_1 \cdot A \cdot A \cdot A = (9,12,0,10,0,0,4,4)
\]

The process has converged. In both cases, the procedure enabled us to distinguish two sets of numbers of links that make up the associated mechanical systems.

\( N_1 = \{1,2,4,7,8\} \)
\( N_2 = \{3,5,6\} \)

After separating the links of the branch, it is easy to calculate the tensor of inertia of the resulting system of material bodies relative to the center of the hinge, along which the release from the bond was made.
Let the Cartesian coordinate system drawn through the center of the hinge (having it as its origin) is connected with the moving coordinate system associated with the link by matrix $D$ and displacement vector of the origin of coordinates $\vec{R}$. Any vector $\vec{X}$, specified in the coordinate system associated with the link, will be written in the coordinate system associated with the hinge, in the following form:

$$\vec{x} = D \cdot \vec{X} + \vec{R}$$

Then the tensor of inertia in the new coordinate system takes the form

$$J^* = D \cdot J \cdot D^T + m \cdot \vec{R} \cdot \vec{R}^T$$

where $m$ - link mass.

Having numbered all the links of the $i$-th selected branch, we obtain the current tensor of inertia

$$J_i = \sum_{k \in N_i} (D_k \cdot J_k \cdot D_k^T + m_k \cdot \vec{R}_k \cdot \vec{R}_k^T)$$

At each iteration of the simulation, transformation matrix $D_k$ and vector $\vec{R}_k$ are determined based on the current values of the generalized coordinates, taking into account the matrix of connections of links $A$ by the usual kinematic methods.

The next important point in the dynamic synthesis of equations is the selection of external forces for each branch of the system and the calculation of the main vector and the main moment of this system of forces when reduced to the corresponding hinge. In the presence of a scheme of the selected branches of links in the form of sets $N_i$, for example, links and matrix $C$ of their connection, it is sufficient to know the main vector and the main moment of external forces acting on each link when bringing this link to the mass center. Then main vector $F$ of the system of forces is defined as the geometric sum of the main vectors of forces of the system $\vec{T}_i$ taking into account transition to the coordinate system associated with the hinge:

$$F = \sum_i D_i \vec{F}_i D_i^T$$

The main point is determined by the formula

$$\vec{M} = \sum_i \vec{M}_i + \sum_i \vec{R}_i \times (D_i \cdot \vec{F}_i \cdot D_i^T)$$

Formation of a system of external forces acting on a separate link depends on the specifics of the link itself and should be laid down when adapting the software package to a specific area of research of mobile machines. Common is the binding of the system of forces to the center of mass of the link and the coordinate system associated with this center of mass. Since external forces, depending on their nature, can depend on generalized coordinates and generalized speeds of the link, the base of force models should be included in the system of properties of the program object of the link along with the interface for accessing this base. For basic objects for which there is a developed mathematical description of the system of forces (such as pneumatic wheels, shock absorbers, elastic elements, etc.), these interfaces are implemented in the model synthesis subsystem. At the same time, for the flexibility of the system, it is necessary to have free interfaces implemented for specific links directly by researchers.

Let’s dwell on the object representation of links. Each link, as a software object, in the dynamic synthesis system has the following set of properties:

1. Weight.
2. Tensor of inertia.
3. Container of objects that implement half-hinges.
4. Vector of generalized coordinates.
5. Vector of generalized velocities.
6. Pointer to the coordinate system relative to which the generalized coordinates and
   generalized speeds of the link are specified.
7. Matrix of transformation of coordinates to the base coordinate system.
8. Vector of coordinates of the center of mass relative to the base coordinate system.
9. Container of graphic resources to display presentation.
10. Accelerator on display context.
11. Link type.
12. Link name.

The link, as a software object, implements the following methods:
1. Formation of a container of semi-hinges and registration of each of them in the matrix of
   connections A.
2. Mapping the graphical view to the context passed from the calling software environment.
3. Transformations of own generalized coordinates and generalized velocities to the external
   coordinate system.
4. Calculation of the main vector and the main moment of external forces acting on the link and
   their transformation to the external coordinate system.
5. Calculation of its tensor of inertia in the transition to the external coordinate system.

The events to be handled by the program object representing the link include the following:
1. Changing the external context of the image. When this event is processed, its own view is
   drawn in the changed context.
2. Change of external properties. Recalculation of own mutable properties and rendering of a
   new view in the display context is conducted.
3. Change of internal properties. Requires redrawing of the new display into the context.
   Graphic resources for displaying objects must provide 3 types of displays:
   1. Schematic representation for presentation in the calculation model.
   2. Representation in the form of an icon for display on the dashboard of the synthesis program
      of the mobile machine.
   3. Detailed presentation.

The schematic and detailed view is scalable to be linked to other links on the mobile machine. In
addition to scalability, it is required to rotate the display about some axis to be able to represent it in
the modeling process. In this case, it is necessary to decide what type of graphics - raster or vector, it
is advisable to use to form the display. With all the advantages of vector graphics, the complexity of
its formation makes the vector format unacceptable for real use. A raster image, when making
rotations, requires a significant amount of time to recalculate the coordinates of the graphics, since it
requires recalculation of all points. The way out of this is formation of a database of graphic images of
various links of mobile machines. With sufficient accuracy in this way it is possible to generate tables
of symbols, for which the key fields would be the scale and angles of rotation of the image relative to
the coordinate axes. The structure of the database tables is as shown in Figure 2.
Indexing of all key fields will provide quick access to the desired raster image, and positioning accuracy should be ensured by the correspondence of the imaging grid in the database to the real display device.

3. Results
Thus, the procedure for the synthesis and functioning of the mobile machine model has been fully formalized. The subsystem of synthesis of the model structure allows in the graphic window of the designer to arrange in the desired configuration and the required number of schematic images of the links of the mobile machine, to set the diagram of their connections with the corresponding half-hinges and to set the parameters of the links. In this case, objects of links are formed and the values of their properties are set. Next, the modeling subsystem comes into play, which consists in solving by numerical methods a dynamically generated system of differential equations. Formation of this system at each iteration consists in the selection of branches in the scheme having one of the hinges as their vertex, determination of the inertia tensor of the main vector and the main moment of the system of external forces acting on the branch when reduced to the center of the hinge. At the end of the iteration, events for updating internal and external properties are triggered for all objects that represent the links of the system, which leads to the formation of a display and fixation of simulation results.

The research results can be used in the study of dynamics of the movement of multi-link hinged mobile machines, as well as in the development of computer-aided design systems.

References
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