A damage-coupled viscoplastic model for compressed asphalt concrete

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Abstract. The damage theory is employed to investigate the mechanical strength deterioration caused by micro defects. Introducing the effective stress, this paper presents a damage-coupled viscoplastic model by applying the Schwartz model with an elastic component. The model is calibrated and validated by asphalt concrete creep test under various stresses. With the help of recursive numerical method, the model parameters include viscoplastic and damage functions for different stresses are determined. The result shows the model not only characterize the three stage creep deformation behaviour at a range of stress, but also can predict the damage evolution during the increased creep strain.

1. Introduction

Asphalt concrete (AC) is widely used in pavement, airport and hydraulic engineering[1]. There are complex component of asphalt concrete including mastic, aggregate and micro defects[2]. Due to microstructural heterogeneities and randomly voids, the mechanical properties of asphalt concrete are so complex that the mechanical behaviour prediction of asphalt concrete is a main challenge for the researchers.

The mechanical behaviour of asphalt concrete can be well-predicted using linear viscoelastic model such as Burgers and Maxwell model when the strain not exceed 10^-4 and temperature not exceed 5℃. However, due to the evolution of micro-cracks and micro-voids under high stress and temperature, the mechanical property of asphalt mixture probably demonstrates nonlinear[3]. Hence, in order to provide the relationship between the creep deformation and damage parameter, the damage-coupled viscoplastic model for asphalt concrete is significant for the pavement deformation behavior research field.

In order to characterize the viscoplastic deformation of asphalt concrete, Perzyna’ theory or Schwartz model has been widely used by researchers. However, the initiated and propagated of micro crack, namely damage are neglected in the above viscoplastic model. Damage is the mainly reason of the degradation of mechanical properties of asphalt concrete[4]. Hence, many researchers introduced the damage theory to the viscoelastic model to establish the damage coupled model for investigate the changes in the material’s microstructure.

The present paper is aim to present a viscoplastic-viscodamage model for asphalt concrete and corresponding damage evolution under various stress loading. The Schwartz model is combined with the effective stress concept, and then the numerical algorithms of the proposed model is implemented in modified Euler method code using the MATLAB software. The model parameters are determined by the creep test results and then validated by the similar test result under different stresses. Whilst the
comparison between the model prediction and experiment are present, the damage evolution is given to suggest the effect of stress on the failure mechanism.

2. Constitutive model

2.1. Model equations

The creep strain is decomposed into two parts, including recoverable part and irrecoverable part[5]:

\[ \varepsilon = \varepsilon_e + \varepsilon_p \]  (1)

where \( \varepsilon_e \) and \( \varepsilon_p \) are the elastic and viscoplastic potential function, respectively.

\[ \varepsilon_e = \frac{d\varepsilon}{E_0} = D_0 \varepsilon_e = D_0 (\sigma_j - \sigma_{j-1}) \]  (2)

The Schwartz viscoplastic constitutive model is used to describe irrecoverable strain[6], the relationship between the stress and strain is expressed as

\[ \frac{d\varepsilon_p}{dt} = \frac{1}{\alpha_{vp}} \frac{\partial F}{\partial \sigma_i} \]  (3)

\[ \varepsilon_p = \frac{B\sigma^q}{A(\varepsilon_{vp} + \delta)^p} = \frac{C\sigma^q}{(\varepsilon_{vp} + \delta)^p} \]  (4)

where \( A, B, \) and \( q \) are the material parameters, \( C = \frac{B}{A}, \sigma_u = 1 \text{MPa}, \) namely stress unity to normalize the stress function. \( \delta \) is introduced to eliminate the singularity. It is noteworthy that the instantaneous plastic strain is contained within the viscoplastic strain.

In order to accurately describe the degradation in strength and stiffness, the concept of effective stress in continuum damage mechanics (CDM) is used

\[ \sigma_e = \frac{\sigma}{(1 - \phi)^2} \]  (5)

where \( \phi \) is the damage variable, \( \phi = 0 \) indicates intact, and \( \phi = 1 \) indicates total rupture.

The damage evolution equation is established by using the exponential form of strain

\[ \dot{\phi} = \Gamma_0 \exp(k\varepsilon) \]  (6)

where \( \dot{\phi} \) is the damage evolution rate, \( \Gamma_0 \) is damage viscosity parameter and \( k \) is constant parameter.

The model equation as the form of the total effective stress is express as

\[ \varepsilon_p = \frac{B\sigma^q}{A(\varepsilon_{vp} + \delta)^p} \]  (7)

2.2 Numerical implementation

Generally, the constitutive equation (7) can only be solved numerically. The time step is set as \( t_1, t_2, \ldots, t_N \) \( (t_i = 0^+, t_{j+1} > t_j, t_N = T) \). Then the responding stresses are expressed as \( \sigma_j = \sigma(t_j) \) \( (j = 1, 2, L, N) \). Finally the responding elastic strain \( \varepsilon_{e,j} \) and viscoplastic strain \( \varepsilon_{vp,j} \) can be solved by the modified Euler method.

when \( j = 1, \)

\[ \varepsilon_{vp,j} = \varepsilon_{vp}(t_1) = 0 \]  (8)

when \( j = 2, 3, L, N \)
\[ \varepsilon_{\text{vp}}^i = \varepsilon_{\text{vp}}^{i-1} + \frac{\Delta t}{2} \left[ \phi \left( \sigma_{\text{vp}}^{i-1}, \varepsilon_{\text{vp}}^{i-1} \right) + \phi \left( \sigma_{\text{vp}}^{i}, \varepsilon_{\text{vp}}^{i} + \Delta t \phi \left( \sigma_{\text{vp}}^{i}, \varepsilon_{\text{vp}}^{i} \right) \right) \right] \]  \hspace{1cm} (9)

where

\[
\begin{align*}
\phi_j &= \phi_{j-1} + \frac{\phi_j}{\Delta t} \\
\dot{\phi}_j &= \Gamma_0 \exp(k \varepsilon) \\
\bar{\sigma}_j &= \frac{\sigma_j}{(1 - \phi_j)^2}
\end{align*}
\]  \hspace{1cm} (10)

add the elastic strain rate

\[ \dot{\varepsilon}_{e} = D_0 |\dot{\varepsilon}_j| \left( \sigma_j - \sigma_{j-1} \right) \]  \hspace{1cm} (11)

The total strain is expressed as

\[ \varepsilon_j = \varepsilon_c^i + \varepsilon_{\text{vp}}^i, \quad i = 1, 2, 3, N \]  \hspace{1cm} (12)

3. Result and discussion

3.1. Model calibration

All the creep test data are obtained from cylindrical specimens of asphalt concrete. The dimension size is a diameter of 100 mm and a height of 100 mm and the aggregate is AC-13C. The specimen and test setup system are shown in Figure 1 and 2. The viscoplastic parameters were determined by the experimental data of 0.28 MPa, 25°C, which the damage can be considered as zero. The obtained model parameters \( C, p, q, \delta, D_0 \) at the reference stress are list in Table 1.

Then the damage parameters can be determined by the same method. Two creep test are needed, 0.70 MPa to determine \( \Gamma_0^v \) and \( k \), and 0.84 MPa to determine the stress dependency. The damage viscosity parameter \( \Gamma_0^v \) can be expressed by

\[ \Gamma^v (\sigma) = \Gamma_0^v \left( \frac{\sigma}{\sigma_{\text{ref}}} \right)^w \]  \hspace{1cm} (13)

The solved damage parameters are listed in Table 2.
Table 1 Elastic and viscoplastic model parameters

| C          | p    | q    | δ      | $D_0$ (MPa$^{-1}$) |
|------------|------|------|--------|---------------------|
| 2.95×10$^{-6}$ | 0.55 | 2.82 | 7.22×10$^{-5}$ | 0.0758              |

Table 2 Damage model parameters

| $\Gamma_0$ (s$^{-1}$) | m | k | $\sigma_{ref}$ |
|------------------------|---|---|----------------|
| 1.88×10$^{-6}$         | 5 | 50 | 0.7            |

3.2. Model prediction
To validate the availability of the model, the model parameters listed in Table 1 and 2 are used to predict the creep deformation. The tests used for validating the model are creep tests under different stress levels, 0.56MPa and 1.4MPa. The comparisons between experiments and model predictions are shown in Figure 3 and 4. It is suggested that there are good agreements between experimental measurements and model predictions.

The evolution of damage versus total strain under 0.56MPa and 1.4MPa are also plotted as blue line in Figure 5 and 6. It can be observed that the damage is close to zero at lower strain and increases rapidly when the higher strain. When the stress is reach to 1.4MPa, the creep strain represent three stages, illustrating that the damage is insignificant in the first and second stage, but is the dominant cause to failure in the third stage, namely the strain exceeded 10%.
4. Conclusions
This study developed a damage-coupled viscoelastic model to investigate the deterioration of strength and stiffness in the compressed asphalt concrete specimen. After the model parameters are determined, the capability of model prediction is validated and the damage mechanism is evaluated. The following conclusions can be concluded:

1) Comparing the experimental data and model predictions reveals that the proposed model works well over a range of stress.
2) The damage initiated in the second creep stage and increases in the three creep stage according to the damage evolution results.

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