Modelling of Crack Development Trajectories in Continuous Thin-Walled Steel-Reinforced Concrete Beams under Moving Load

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Abstract. The scheme and its testing for calculating continuous thin-walled steel-reinforced concrete beams for constant and temporary load taking into account crack opening are described. The moving load is set in the form of a concentrated force applied with eccentricity with respect to the axis of the bar. For a bar experiencing bending and torsion deformations, analytical expressions for forces and displacements depending on the position of the temporary load are obtained, enveloping bending moment and bimoment diagrams are constructed. Based on the diagrams, the largest normal tension stresses were calculated at the lower points of the main beams and at the points of the reinforced concrete slab. An algorithm for calculating a continuous steel-reinforced concrete beam for a moving load is described taking into account crack opening. Trajectories of crack opening in a steel-reinforced concrete slab under a moving load are constructed.

1. Introduction

It is known that the design diagrams of various elements of engineering structures can be reduced to thin-walled bar, plate, shell or volumetric systems, arbitrarily fixed and loaded.

In the practice of bridge building, design diagrams in the form of thin-walled bars are also used. These are economical continuous reinforced concrete bridge spans of rectilinear and curvilinear structure. Such bridges are considered “narrow”, since their width is much less than the length of the smallest span. When the load moves along the bridge, along with the bend of the span, its twisting occurs. Therefore, the design diagram of spans can be represented by a thin-walled bar of open profile with a rectilinear or curvilinear structure [1, 2, 3, 4].

2. Problem statement

Consider the steel-reinforced concrete span of a continuous bridge with spans scheme of 63.0 m + 63.0 m + 63.0 m. The span consists of two steel beams joined at the top by a reinforced concrete slab, and below by lateral bracing. The design, geometric characteristics of the cross section and the mechanical characteristics of the span material are shown in figure 1. The width of the carriageway of the bridge is 9.0 m.
The bar is loaded with a constant load of intensity $q = 5.77 \text{kN/m}$ symmetrical about the axis. As a moving load, a concentrated force $P = 400 \text{kN}$ is adopted, which is applied with an eccentricity $e = 2m$ with respect to the longitudinal axis of the bar.

We apply the concentrated force $P$ to the point of the axis of the rod, but at the same time add a couple of forces $m_x = P \cdot e$. As a result, we arrive at two design diagrams for a continuous thin-walled bar.

Figure 2 shows the design diagram of a bending bar: $q$ - constant load intensity, $P$ - concentrated force, the point of application of which is fixed by a coordinate $\xi$.

Figure 3 shows the design diagram of a torsion bar: $m_x$ - a couple of forces whose application point is fixed by a coordinate $\xi$.
3. Results

Continuous bar (figure 2) is divided into four sections length: \( \xi_i; l - \xi_i; l; l \). In each section, we write down four differential equations of the first order

\[
\frac{d}{dx} Q_i(x) = -q, \quad \frac{d}{dx} M_i(x) = Q_i(x), \quad EJ \frac{d}{dx} \varphi_i(x) = -M_i(x), \quad \frac{d}{dx} V_i(x) = \varphi_i(x),
\]

\((i = 1, 2, 3, 4)\) .

The following notation is introduced in equations (1): \( V_i(x) \) - deflection of the bar in the section with the coordinate \( x \) in the section \( i \); \( \varphi_i(x) \) - the angle of rotation of the cross section; \( M_i(x) \) - bending moment; \( Q_i(x) \) - shear force; \( EJ \) - bending stiffness; \( q \) - the intensity of the distributed load.

We supplement the system of equations (1) with boundary conditions at the ends of the bar and conjugation conditions at the boundaries of the sections

\[
V_1(0) = 0, M_1(0) = 0, \\
V_1(\xi_1) = V_2(\xi_1), \varphi_1(\xi_1) = \varphi_2(\xi_1), M_1(\xi_1) = M_2(\xi_1), Q_1(\xi_1) - Q_2(\xi_1) = P,
\]

\[
V_2(l) = 0, V_3(l) = 0, \varphi_2(l) = \varphi_3(l), M_2(l) = M_3(l), \\
V_3(l) = 0, V_4(l) = 0, \varphi_3(l) = \varphi_4(l), M_3(l) = M_4(2l), \\
V_4(3l) = 0, M_4(3l) = 0.
\]

In equations (2) \( P \) is the concentrated force applied at a point \( \xi_1 \).

Below is an analytical solution of the differential problem (1) - (2) obtained in the Maple computer mathematics system.

Figure 4 shows bending moments diagrams in sections of a continuous bar when the force \( P \) is moved in the first span (red color). Bending moments diagrams when moving force \( P \) in the second span are shown in blue, green when moving force \( P \) in the third span.

Figure 5 presents enveloping diagrams of bending moments \( M^{+\max} \) and \( M^{-\max} \) from the combined action of constant and temporary loads.
Consider the torsion of a continuous thin-walled rod loaded with a couple of forces $m_x = P \cdot e$ at a point $\xi$ (figure 3). Continuous bar is divided into four sections length: $\xi, l - \xi, l, l$. In each section, we write down four differential equations of the first order

$$\frac{d}{dx} M_{\omega_j}(x) + GJ_d \left( \frac{d}{dx} \theta_j(x) \right) = 0, \quad \frac{d}{dx} B_{\omega_j}(x) = M_{\omega_j}(x) \cdot EJ_{\omega_j} \left( \frac{d}{dx} \theta_j(x) \right) = -B_{\omega_j}(x), \quad (j=1,2,3,4). \tag{3}$$

The following designations are introduced in equations (3): $\phi_j(x)$ - angle of twisting of the section with coordinate $x$ in the section $j; \theta_j(x)$ - relative angle of twist; $B_{\omega_j}(x)$ - bimoment, $M_{\omega_j}(x)$ - bending torque; $GJ_d, EJ_{\omega_j}$ - torsional and sectorial stiffness; $G, E$ - shear modulus and elastic modulus of the bar material.

The system of equations (4) is supplemented by boundary conditions at the ends of the bar and conjugation conditions at the boundaries of the sections

$$\phi_1(0) = 0, B_{\omega_1}(0) = 0,$$

$$\phi_1(\xi_1) = \phi_2(\xi_2), \quad \beta_1(\xi_1) = \beta_2(\xi_2), \quad B_{\omega_2}(\xi_2) = B_{\omega_2}(\xi_2), \quad M_{\omega_2}(\xi_2) = M_{\omega_2}(\xi_2) = \frac{m_x}{2}, \quad (j=1,2,3,4). \tag{4}$$

Equations (4) denote a couple of forces $m_x = P \cdot e$ applied at a point $\xi$.

Below is an analytical solution to the differential problem (3) - (4) obtained in the Maple computer mathematics system.

Figure 6 shows the diagrams of bimoments in sections of a continuous bar when moving a couple of forces $m_x = P \cdot e$ in the first span (red color). In blue, the diagrams of bimoments are shown when moving a couple $m_x$ in the second span, in green - when moving a couple $m_x$ in the third span.

Figure 7 presents the enveloping diagrams of bimoments $B_{\omega_{\text{max}}}^{+}$ and $B_{\omega_{\text{max}}}$ under the combined action of constant and temporary loads.
Normal stresses from bending and torsion at the points of the cross section of a thin-walled bar are determined by the formula

\[ \sigma_x = \frac{M}{J_z} y + \frac{B\omega}{J_{\omega}} \omega, \]

where \( y \) is the coordinate of the cross-sectional point relative to the principal axes of inertia, \( \omega \) is the main sectorial coordinate of the cross-sectional point.

Figure 8 shows a graph of changes in normal tensile stresses at the lower points of the shelves of the main steel beams of the cross section from bending and torsion. The stress values presented on the graph are used in calculating the beam for a moving load, taking into account the behaviour of the material in the elastoplastic stage.

Figure 9 shows a graph of changes in tensile stresses at the extreme points of a reinforced concrete slab (coordinates \( \omega_1 = -3.32m^2, \omega_2 = 3.32m^2 \)).
An essential feature of the operation of continuous steel-reinforced concrete spans is the alternate switching on and off of the reinforced concrete slab of the carriageway from cooperation with metal beams. This circumstance makes it necessary to consider spans as a structurally non-linear system with bending stiffness varying in length.

The calculation of the continuous steel-reinforced concrete beam for a moving load, taking into account the opening of cracks, was carried out according to the following algorithm.

1. We set the intensity of the constant load $q = 5.772 \text{kN/m}$ and the magnitude of the displaced force $F = 400 \text{kN}$, the limiting value of tensile stresses in the reinforced concrete slab $\sigma_{np} = 2.25 \text{MPa}$ and the reduction coefficient of bending and sectorial stiffnesses $\varphi = 0.7$.

2. We divide the beam into sections of length $h = L/n$ with points with coordinates $x_i = x_{i-1} + h; (i = 1, \ldots, n = 189)$.

3. We apply the load $F$ to the beam nodes $x_i$ in increments $h$. At each position of the force $F$, we calculate the normal stresses in all sections with coordinates $x_i$ (at points of the cross section of the reinforced concrete slab);

4. At each load position, we compare the calculated stresses with the voltage limit value $\sigma_{np}$ (figure 9). If the voltage $\sigma(x_k)$ exceeds the limit value, we weaken the corresponding section, changing its bending and sectorial stiffnesses ($EJ_k = \varphi \cdot EJ_k$) (the forces corrected in the stiffness section in the new position remain unchanged);

5. We determine the coordinates of the forces positions at which concrete is turned off from the work, and the coordinates of the weakened sections corresponding to this position of the forces, and we construct graphs (trajectories of crack opening).

Figure 10 shows the trajectories of crack opening. Each segment in the figure is marked with a footnote of two numbers, for example, 29 - 73. The first number indicates the coordinate $x = 29 \text{ m}$ of force $F$ application, and the second indicates the coordinate $x = 73 \text{ m}$ of the plane of weakness.

![Figure 10. The trajectories of crack opening along the length of the beam ($\sigma_{np} = 2.25 \text{MPa}$).](image_url)
zone occurs at cargo positions mainly in the first span, while cracking of the second support zone occurs at cargo positions in both the second and third spans.

4. Conclusion
A comparative analysis of the stress and strain state of a continuous beam system was carried out, taking into account and without taking into account the opening of cracks in the reinforced concrete slab of the roadway. As a result, a quantitative estimate of the deflections and bending moments is obtained. Taking into account constructive nonlinearity leads to a redistribution of the maximum values of bending moments along the length of the beam (a decrease in the reference sections of 7-10%, an increase in the middle of the spans of 3-5%) and an increase in the deflections in the middle of the spans of the beam (in the middle of the first span - 5.5%, in the middle of the second span - 8.5%).

5. References
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