Growth Rates of the Upper-hybrid Waves for Power-law and Kappa Distributions with a Loss-cone Anisotropy

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Abstract Fine structures of radio bursts play an important role in diagnostics of the solar flare plasma. Among them the zebras, which are prevalently assumed to be generated by the double plasma resonance instability, belong to the most important. In this paper we compute the growth rate of this instability for two types of the electron distribution: a) for the power-law distribution and b) for the kappa distribution, in the both cases with the loss-cone type anisotropy. We found that the growth rate of the upper-hybrid waves for the power-law momentum distribution strongly depends on the pitch-angle boundary. The maximum growth rate was found for the pitch-angle $\theta_c \approx 50^\circ$. For small angles the growth rate profile is very flat and for high pitch-angles the wave absorption occurs. Furthermore, analyzing the growth rate of the upper hybrid waves for the kappa momentum distribution we found that a decrease of the characteristic momentum $p_c$ shifts the maximum of the growth rate to lower values of the ratio of the electron-plasma and electron-cyclotron frequencies, and the frequency widths of the growth rate peaks are very broad. But, if we consider the kappa distribution which is isotropic up to some large momentum $p_m$ and anisotropic with loss-cone above this momentum then distinct peaks of the growth rate appear and thus distinct zebra stripes can be generated. It means that the restriction for small momenta for the anisotropic part of distributions is of principal importance for the zebra stripes generation. Finally, for the 1 August 2010 zebra stripes, the growth rates in dependence on radio frequency were computed. It was shown that in this case the growth rate peaks are more

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distinct than in usually presented dependencies of growth rates on the ratio of the plasma and cyclotron frequencies.

**Keywords:** Sun: corona — Sun: flares — Sun: radio radiation

1. Introduction

Zebra structure is a fine structure of Type IV radio bursts observed during solar flares in the decimetric, metric and centimetric wavelength ranges (Slottje, 1972; Chernov et al., 2012; Tan et al., 2012, 2014). There are many models of this fine structure (Rosenberg and Tarmstrom, 1972; Zheleznyakov and Zlotnik, 1973; Kuipers, 1973; Chernov, 1974, 1994; LaBelle et al., 2003; Bara and Karlicky, 2006; Lednev, Yan, and Fu, 2006; Kuznetsov and Tsap, 2007; Laptukhov and Chernov, 2008; Tan, 2010; Karlicky, 2013), see also reviews by Chernov (2010); Chernov, Yan, and Fu (2014); Zheleznyakov et al. (2016). Among these models the most commonly used model is based on the double-plasma resonance (DPR) instability, see e.g. the review by Zheleznyakov et al. (2016).

The process of the double plasma resonance, which generates the upper-hybrid waves, is the most effective in the flare loop regions, where the condition $\omega_p \simeq s\omega_B$ is fulfilled ($\omega_p$ and $\omega_B$ means the electron plasma and electron gyro frequency, $s$ is the gyro-harmonic number). However, this process strongly depends on distributions of accelerated electrons. In many papers (Zheleznyakov and Zlotnik, 1973; Winglee and Dulk, 1986; Yasnov and Karlicky, 2004; Benáˇcek, Karlický, and Yasnov, 2017; Yasnov, Benáˇcek, and Karlický, 2017; Benáˇcek and Karlický, 2018) studying zebra stripes the distribution of accelerated electrons were described by the Dory-Guest-Harris (DGH) type function (Dory, Guest, and Harris, 1965). However, this distribution has not a clear physical foundation. The distributions that are a result of processes of accelerations and reflections of electrons in magnetic mirrors in closed magnetic loops are physically more acceptable (Stepanov, 1974; Kuijpers, 1974; White, Melrose, and Dulk, 1983). Therefore Winglee and Dulk (1986) considered the loss-cone distribution with the exponential function of the momentum of electrons. Furthermore, in the paper by Kuznetsov and Tsap (2007), the authors considered the loss-cone distribution with the power-law function of the momentum of electrons. This distribution is more realistic because it is used in a power-law fitting of hard X-ray spectra of solar flares. But in this distribution the low-energy cut-off needs to be defined, which is difficult to estimate from observations (Holman et al., 2003; Saint-Hilaire and Benz, 2007; Kontar, Dickson, and Kašparová, 2008). Therefore, in recent years an interest about the kappa distribution is increasing. This distribution has no low-energy cut-off and is close to Maxwellian distribution at low energies and at high energies is similar to the power-law one. Kappa distributions are supported by theoretical considerations of particle acceleration in collisional plasmas (Bian et al., 2014). Furthermore, the X-ray spectra of coronal X-ray sources are well fitted using kappa distributions (Kašparová and Karlický, 2009; Oka et al., 2013, 2015).

In the present article, firstly, we follow the study of Kuznetsov and Tsap (2007). We extend their analysis in order to show changes of the growth rate...
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Figure 1. Maximum growth rate of the upper-hybrid waves for the power-law momentum distribution in dependence on the ratio $\omega_p/\omega_B$ and the loss-cone angle $10^\circ$ (black line), $30^\circ$ (red line), $50^\circ$ (blue line), $65^\circ$ (green line) and $80^\circ$ (violet line). The power-law index of the power-law distribution is $\delta = 5$, the gyro-harmonic number is $s = 16$ and the minimum electron momentum $p_m$ corresponds to the velocity $0.3 \, c$, i.e. to the low-energy cut-off $\approx 30 \, \text{keV}$.

for the power-law momentum distribution in dependence on the low-energy cut-off and loss-cone angle. Then we present growth rates for the anisotropic kappa distribution and kappa distribution which is isotropic up to some large momentum and anisotropic above this momentum. Finally, for the first time, for the zebra stripes observed at 1 August 2010, we compute the growth rates in dependence on radio frequency.

2. Growth Rates for Power-law Distributions

Let us briefly describe the growth rate calculation. We follow the method according to Kuznetsov and Tsap [2007]. We consider a plasma with two components: a) the background Maxwellian plasma and b) hot non-equilibrium plasma component with the plasma densities $n_0$ and $n_h$, respectively, where $n_0 \gg n_h$. The
Figure 2. Maximum growth rate of the upper-hybrid waves for the power-law momentum distribution in dependence on the ratio $\omega_p/\omega_B$ and the minimum electron momentum $p_m$ for the gyro-harmonic number $s = 15$ (red line) and $s = 16$ (blue line). Plot a) is for $p_m$ corresponding to the velocity $0.5 \, c$, b) for $p_m$ corresponding to $0.3 \, c$, c) for $p_m$ corresponding to $10 \, v_T$, d) for $p_m$ corresponding to $5 \, v_T$, e) for $p_m$ corresponding to $3 \, v_T$, and f) for $p_m$ corresponding $v_T$. The power-law index is $\delta = 5$ and the loss-cone is $\theta_c = 50^\circ$. Note that the scale on the y-axis increases from plot a) to f).
Figure 3. Maximum growth rate of the upper-hybrid waves for the power-law momentum distribution in dependence on the ratio $\omega_p/\omega_B$ and the loss-cone angle $10^\circ$ (black line), $30^\circ$ (red line), $50^\circ$ (blue line), $65^\circ$ (green line) and $80^\circ$ (violet line). The power-law index of the power-law distribution is $\delta = 10$, the gyro-harmonic number is $s = 16$ and the minimum electron momentum $p_m$ corresponds to the velocity $0.3 \, c$, i.e. to the low-energy cut-off $\approx 30 \, \text{keV}$.

Electron distribution function of this hot non-equilibrium component is taken as

$$f(p, \theta) = \varphi(p) \begin{cases} 0, & \theta \leq \theta_c - \Delta\theta_c, \\
\frac{\theta - \theta_c + \Delta\theta_c}{\Delta\theta_c}, & \theta_c - \Delta\theta_c < \theta < \theta_c, \\
1, & \theta > \theta_c. \end{cases} \quad (1)$$

This function describes the distribution with the loss-cone having the pitch angle boundary $\theta_c$ and the boundary width $\Delta\theta_c \ll 1$. The function $\varphi(p)$ describes the distribution in dependence on the electron momentum.

Here the $\varphi(p)$ function is taken in the power-law function form:

$$\varphi(p) = \begin{cases} \frac{\delta-3}{2\pi(\pi-\theta_c)p_m} \left( \frac{p}{p_m} \right)^{-\delta}, & p \geq p_m, \\
0, & p < p_m, \end{cases} \quad (2)$$

where $p_m$ is the low-momentum cut-off, and $\delta$ is the power-law index. Note that this distribution is normalized to one.
Generally, for the growth rate of the upper-hybrid waves we can write

$$\gamma = -\frac{\text{Im} \epsilon_{\parallel}}{\omega},$$

(3)

$$\frac{\partial \text{Re} \epsilon_{\parallel}}{\partial \omega} \bigg|_{\epsilon_1=0} \simeq \frac{2}{\omega} \left(2 - \frac{\omega_p^2}{\omega^2} \right),$$

(4)

where $\epsilon_{\parallel}$ is dielectric permeability and $\omega$ is the frequency of the upper-hybrid waves.

For the term $\text{Im} \epsilon_{\parallel}$ we use the relation (17) from the paper by Kuznetsov and Tsap [2007]

$$\text{Im} \epsilon_{\parallel}^{(s)} \simeq -2\pi^2 m^4 c^2 \frac{\omega_p^2}{k^2} \frac{n_h}{n_0} \Gamma_s^4 \frac{J_s^2}{m \omega_B} \times$$

$$\left[ \frac{\partial \phi(p)}{\partial p} + \frac{\phi(p) \tan \theta_c}{p \Delta \theta_c} \left(\frac{s \omega_B}{\Gamma_s \sin^2 \theta_c} - 1\right) \right] \frac{\Delta p_z}{p_0},$$

(5)

where $J_s$ is the s-th order Bessel function, $\omega_B$ is the electron cyclotron frequency, $p = (p_\perp, p_z) = (p_0 \sin \theta_c, p_0 \cos \theta_c)$ is the electron momentum, $m$ is the electron mass, $c$ is the speed of light, $k$ is the wave number. The distance in momentum space $\Delta p_z$ between intersection points with straight line for small parameter $\Delta \theta_c \ll 1$ is

$$\Delta p_z = 2p_0 \frac{\omega}{s \omega_B} \sqrt{2\Delta \theta_c \tan \theta_c}.$$

(6)

Then the normalized growth rate can be expressed in agreement with the paper by Kuznetsov and Tsap [2007] as

$$\gamma_n = \frac{\gamma}{\omega} \frac{n_0}{n_h} \sqrt{\Delta \theta_c} = \frac{4}{\sqrt{2}} \pi^2 m^4 c^2 \frac{\omega_p^2}{k^2} \frac{n_h}{n_0} \Gamma_s^4 \frac{J_s^2}{m \omega_B} \times$$

$$\left[ \frac{\partial \phi(p)}{\partial p} + \frac{\phi(p) \tan \theta_c}{p \Delta \theta_c} \left(\frac{s \omega_B}{\Gamma_s \sin^2 \theta_c} - 1\right) \right] \frac{1}{p_0 \left(2 - \frac{\omega_p^2}{\omega^2} \right)} \left(\frac{1}{\Gamma_s^2 \sin^2 \theta_c} - 1\right),$$

(7)

where $p_0$ is the lower boundary for hot electron momentum

$$p_0 = \frac{mc\sqrt{\omega^2 - s^2 \omega_B^2}}{s \omega_B},$$

(8)

$k = (k_z, k_\perp)$ is the wave vector with the components along and in the perpendicular direction to magnetic field

$$k^2 = k_z^2 + k_\perp^2 = \frac{\omega^2 - s^2 \omega_B^2}{c^2 \cos^2 \theta_c} + \frac{\omega^4 - \omega_p^2 \omega_B^2 - \omega_B^2 \omega_p^2}{3c^4 \omega_B^2 \omega_p^2},$$

(9)

$\Gamma_r$ is the relativistic factor

$$\Gamma_r = \frac{\omega}{s \omega_B}.$$

(10)
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and $v_T$ is the thermal velocity of the background plasma.

Now using the relation (7) we computed the growth rate for the following parameters: The lower limit for the momentum of electrons $p_m$ is taken as corresponding to the minimum energy $E_m \approx 30$ keV, i.e., to the minimum velocity of electrons $v_m/c = 0.3$, the power-law index is $\delta = 5$ and the gyro-harmonic number is $s = 16$. The pitch-angle boundary varies as $\theta_c = 10^\circ, 30^\circ, 50^\circ, 65^\circ$ and $80^\circ$ and the temperature of the background plasma is $T_0 = 3 \times 10^6$ K.

In computations we only varied the magnetic field $B$, while the plasma frequency was kept constant ($f_p = \omega_p/2\pi = 1$ GHz). For each value of $\omega_p/\omega_B$ the growth rate $\gamma_n$ was computed in the frequency interval $\sqrt{\omega_p^2 + \omega_B^2} < \omega \leq \omega_{max}$. The frequency $\omega_{max}$ was taken by an experimental way in order to find the maximum value of $\gamma_n$ in this interval. The results of these computations are shown in Figure 1. As seen here the growth rate strongly depends on the value of the pitch-angle boundary. The maximum peak is for $\theta_c \approx 50^\circ$. For small angles no distinct peak is visible, and for high angles the absorption appears.

Now, let us analyze an effect of variation of $p_m$ (i.e., the low-velocity limit of electrons) on the growth rate. The result for $\delta = 5$ and $\theta_c = 50^\circ$ is shown in Figure 2. The value of $p_m$ varies in correspondence with the minimum electron velocity $v_m \in (0.5 \ c - v_T)$, $v_T = 6.75 \times 10^6$ m s$^{-1}$. Figure 2 shows that the maximum contrast between peaks is for the growth rates with the minimum electron velocities in the $0.3 \ c - 10 \ v_T$ range, see also Table 1. For velocities greater than $0.5 \ c$ the contrast of peaks decreases and for the velocities $\leq 5 \ v_T$ the peaks are shifted to much lower ratio of $\omega_p/\omega_B$. If we accept that the growth rate profiles correspond to the intensity of zebra stripes, it means that for the low $p_m$ no zebra stripes can be generated.

The same computations were made also for the power-law distribution with the power-law index $\delta = 10$. The computed growth rates for this power-law distribution with different pitch angles are shown in Figure 3. Similarly as in the case with the power-law distribution with the power-law index $\delta = 5$, we can see a strong dependence on the pitch angle, however, the maximum is one again for the pitch angle $\theta_c \approx 50^\circ$. On the other hand, the growth rates are higher and narrower in frequency comparing with the previous case.

Summarizing all these results, in Table 1 we present the ratio $(\omega_p/\omega_B)_{max}$, where the growth rate has maximum and the peak width $\Delta(\omega_p/\omega_B)$ (taken at half of the maximum) in dependence on the minimum electron velocity $v_m$ (minimum of $p_m$) for the gyro-harmonic number $s = 16$ and for the power-law index $\delta = 5$ and 10. To see separate zebra stripes the peak width needs to be smaller than 0.5. While for the power-law index $\delta = 5$ the zebra structure can be formed only in limited interval of $v_m$ around the velocity $10 \ v_T = 6.75 \times 10^7$ m s$^{-1}$, in the case with $\delta = 10$ the width of the growth rates are two time smaller and thus more favorable for the zebra pattern generation. Positions of the growth rate maxima in both the cases are approximately the same.
Table 1. Frequency ratio of the growth rate maximum \( \frac{\omega_p}{\omega_B} \) and the bandwidth of the growth rate peak \( \Delta \frac{\omega_p}{\omega_B} \) for the harmonic number \( s = 16 \) in dependence on the minimum velocity \( v_m \) (corresponding to \( p_m \)) of the power-law distribution for two power-law indices (\( \delta = 5 \) and 10).

| \( \delta \) | \( v_m \) | 0.5 c | 0.3 c | 10 \( v_T \) | 5 \( v_T \) | 5 \( v_T \) |
|---|---|---|---|---|---|---|
| 5 | \( \frac{\omega_p}{\omega_B} \) | 18.41 | 16.60 | 16.16 | 14.42 | 5.98 |
| 5 | \( \Delta \frac{\omega_p}{\omega_B} \) | 0.73 | 0.56 | 0.46 | 0.84 | 1.90 |
| 10 | \( \frac{\omega_p}{\omega_B} \) | 18.43 | 16.50 | 15.95 | 14.41 | 6.00 |
| 10 | \( \Delta \frac{\omega_p}{\omega_B} \) | 0.40 | 0.26 | 0.25 | 0.61 | 1.19 |

3. Growth Rates for Kappa Distributions

Now, we consider a plasma with the hot component having the kappa distribution with the loss-cone anisotropy for all electron momentums. The kappa distribution is taken as (Bian et al., 2014)

\[
f_\kappa(v) = \frac{n_\kappa \Gamma(\kappa + 1)}{\pi^{\frac{3}{2}} \theta_\kappa^{\frac{3}{2}} \Gamma(\kappa - \frac{1}{2})} \left( 1 + \frac{v^2}{\kappa \theta_\kappa^2} \right)^{-\kappa - 1}, \tag{11}
\]

where \( \kappa \) is the kappa index, \( n_\kappa = \int f d^3v \),

\[
\theta_\kappa^2 = \frac{2k_B T_\kappa}{m} \frac{\kappa - \frac{4}{3}}{\kappa}, \tag{12}
\]

is the characteristic velocity, \( m \) is the electron mass, \( k_B \) is the Boltzmann constant, \( T_\kappa \) is the mean kinetic temperature and \( \Gamma \) is Gamma function.

Similarly as in the calculation of the growth rate for the power-law distribution we assume the loss-cone type distribution according to the relation (11). However in this case the function \( \varphi(p) \), derived from the relation (11) has the form

\[
\varphi(p) = \frac{2 \Gamma(\kappa + 1)}{\pi^{\frac{3}{2}} \theta_\kappa^{\frac{3}{2}} \Gamma(\kappa - \frac{1}{2})(\pi - \theta_\kappa)} \left( 1 + \frac{p^2}{\kappa p_\kappa^2} \right)^{-\kappa - 1}. \tag{13}
\]

where \( p \) is the electron momentum and \( p_\kappa = m \theta_\kappa \).

Using these relations we computed the maximum growth rates for the kappa momentum distributions with the loss-cone anisotropy. The results are shown in Figures 4, 5 and 6. Figure 4 presents the growth rate in dependence on the ratio \( \omega_p/\omega_B \) for the kappa distribution with the kappa index \( \kappa = 1.5 \) (it corresponds to \( \delta = 5 \) for the power law distribution) for the gyro-harmonic numbers \( s = 15 \) (red solid line) and 16 (red dashed line), \( \theta_c = 30^\circ \), and \( p_\kappa \) corresponding to the velocity 0.3 c). The growth rate for the same parameters, but for \( \kappa = 4 \) are
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Figure 4. Maximum growth rate of the upper-hybrid waves for the kappa momentum distribution in dependence on the ratio $\omega_p/\omega_B$ for the kappa index $\kappa = 1.5$ and the gyro-harmonic number $s = 15$ (red solid line), $k = 1.5$ and $s = 16$ (red dotted line), $k = 4$ and $s = 15$ (blue solid line), and $\kappa = 4$ and $s = 16$ (blue dotted line). The loss-cone is $\theta_c = 30^\circ$, and $p_\kappa$ corresponds to the velocity $0.3 \, c$.

Figure 5. Left: Maximum growth rate of the upper-hybrid waves for the kappa momentum distribution in dependence on the $\omega_p/\omega_B$. The kappa index is $\kappa = 1.5$, the gyro-harmonic number is $s = 16$ and the loss-cone is $\theta_c = 30^\circ$. The red line is for $p_\kappa$ corresponding to $0.3 \, v_T$, blue line for $p_\kappa$ corresponding to $10 \, v_T$, green line for $p_\kappa$ corresponding to $5 \, v_T$, and violet line for $p_\kappa$ corresponding to $3 \, v_T$. Right: The same, but for $\theta_c = 50^\circ$. 
Figure 6. Maximum growth rate of the upper-hybrid waves for the kappa momentum distribution in dependence on the $\omega_p/\omega_B$. The kappa index is $\kappa = \infty$, the gyro-harmonic number is $s = 16$ and the loss-cone is $\theta_c = 30^\circ$. The red line for $p_\kappa$ corresponding to 0.3 $v_T$, blue line for $p_\kappa$ corresponding to 10 $v_T$, green line for $p_\kappa$ corresponding to 5 $v_T$, and violet line for $p_\kappa$ corresponding to 3 $v_T$.

Expressed by the blue solid line for $s = 15$ and by the blue dashed line for $s = 16$. As seen in this figure when the kappa index increases, i.e. the kappa distribution becomes more closer to Maxwellian one, the bandwidth of the growth rate peaks are broader. Furthermore, while the values of growth rates for $s = 15$ and 16 and $\kappa = 1.5$ are similar, the value of the growth rate for $s = 16$ and $\kappa = 4$ is much smaller comparing to that with $s = 15$.

Furthermore, in Figure 5 we show the dependence of the growth rate for the kappa distribution in dependence on the ratio $\omega_p/\omega_B$ and $p_\kappa$ for two values of the loss-cone angle $\theta_c = 30^\circ$ and $50^\circ$. As seen in both these figures when we decrease $p_\kappa$ the maximum of the growth rate increases and shifts to lower values of the ratio $\omega_p/\omega_B$. The growth rates for $\theta_c = 50^\circ$ are greater than those for $\theta_c = 30^\circ$.

Finally, for comparison with Figure 5 left, in Figure 6 we added the growth rates for the kappa distribution in dependence on the ratio $\omega_p/\omega_B$ for the kappa index $\kappa = \infty$. The plots of the growth rates are similar, but the values of the growth rates for $\kappa = \infty$ are higher.

In both Figures 5 and 6 in all cases the peaks of the growth rates are very broad. If we assume that the growth rate profiles correspond to radio emission, it means that in the case with the kappa momentum distribution distinct zebra stripes cannot be generated.

Therefore now we consider more realistic case, namely, the kappa distribution in all momentums ($p > 0$), but isotropic up to some large momentum $p_m$ and
Figure 7. Maximum growth rate of the upper-hybrid waves for the kappa momentum distribution (14) in dependence on the $\omega_p/\omega_B$. The kappa index is $\kappa = 1.5$, the gyro-harmonic number $s = 15$ (red line) and $s = 16$ (blue line), the loss-cone is $\theta_c = 50^\circ$, $p_\kappa = p_m$ corresponds to 0.3 c.

anisotropic above this momentum $p_m$. Now the isotropic part of the kappa distribution plays a role of the dense background plasma and the anisotropic part with the kappa momentum distribution and loss-cone anisotropy plays a role of the low density hot component. Such a division of the distribution is possible due to the fact that only the anisotropic part of this distribution is important for the growth rate of the upper-hybrid waves. The isotropic part of the kappa distribution does not contribute to the growth rate. Therefore for the following computations only the anisotropic part with momentums above $p_m$ needs to be expressed. For it we take the distribution, which is normalized to 1, as follows.

$$\varphi(p) = \begin{cases} 
\frac{(2\kappa-1)(\frac{p^2}{p_m^2} + \frac{\kappa^2}{p_m^2})^{-\kappa-1}}{2\pi(\pi-\theta_c)p_m^3} _2F_1\left(\kappa - \frac{1}{2}, \kappa + 1; \kappa + \frac{1}{2}; -\frac{\kappa^2}{p_m^2}\right), & p > p_m, \\
0, & p \leq p_m,
\end{cases} \quad (14)$$

where $_2F_1\left(\kappa - \frac{1}{2}, \kappa + 1; \kappa + \frac{1}{2}; -\kappa\right)$ is the hypergeometric function.

For this distribution function we computed the maximum growth rates of the upper-hybrid waves in dependence on the ratio $\omega_p/\omega_B$ for $\theta_c = 50^\circ$, $\kappa = 1.5$, $s = 16$ and $s = 15$, and $p_\kappa = p_m$ corresponding to 0.3 c.

Figure 7 shows that the kappa distribution, bounded at small momentum values, also yields peaks in the spectrum of the growth rate which are distinctly isolated similarly as the peaks for the power-law distribution (compare with Figure 2 b).
Figure 8. Dependence of the growth rate of the upper-hybrid waves on the frequency $f$ for the distribution (14), with $\kappa = 1.5$, $\theta_c = 50^\circ$, $p_m$ corresponds to the velocity $0.3\,c$. The solid line is for $s = 25$, the dashed line for $s = 26$ and the dot-dashed line for $s = 27$.

4. Frequency Spectrum of Growth Rates

Zebras are observed in the spectrum in dependence on the radio frequency, not in the spectrum depending on the ratio $\omega_p/\omega_B$. Therefore, it is of interest to compute such a frequency spectrum. For it we need to take the plasma frequency from some observed zebra stripes. In the following computations, we take them for three stripes of the zebra ($s = 25$, 26 and 27) observed in the 1 August flare (Yasnov, Karlický, and Stupishin, 2016): $f_p = 1.344 \times 10^9 \,\text{Hz}$ for $s = 25$, $f_p = 1.323 \times 10^9 \,\text{Hz}$ for $s = 26$ and $f_p = 1.301 \times 10^9 \,\text{Hz}$ for $s = 27$. To obtain the frequency spectrum, it is necessary to integrate the above derived growth rates with respect to $\omega_p/\omega_B$, setting for each band its own plasma frequency

$$\bar{\gamma}_n = \int \gamma_n \, d\left(\frac{\omega_p}{\omega_B}\right). \quad (15)$$

Figure 8 shows the dependence of the growth rate of the upper-hybrid waves on the frequency $f$ for the distribution (14), with $\kappa = 1.5$, $\theta_c = 50^\circ$, $p_m$ corresponds to the velocity $0.3\,c$. Here and in the following, the value of $p_\kappa$ is taken as corresponding to the temperature of the thermal plasma ($T_e = 3 \times 10^6 \,\text{K}$).

As seen here the spectrum shows distinct isolated peaks giving in the spectrum distinct stripes. An analogous result was also obtained for the power distribution, not shown here.

Now, let us now check if the conclusion about the significant influence of the value of the pitch-angle boundary on the frequency spectra is valid. Figure 9 shows the frequency spectrum of the growth rate of the upper-hybrid waves for the distributions (14) with $\theta_c = 80^\circ$, $\kappa = 1.5$ and $p_m$ corresponding to velocity $0.3\,c$. As seen here, similarly as in the case with the power-law distribution, the growth rates are negative. Note that the negative spectral peaks are well separated from each other.

Figure 10 shows the frequency spectrum of the growth rate of the upper-hybrid waves for the distribution (14) with $\theta_c = 10^\circ$, $\kappa = 1.5$ and $p_m$ corresponding to velocity $0.3\,c$. Now the spectral bands are very broad. As a result, these bands
merge and thus not forming isolated peaks which are necessary for zebra stripes generation.

Table 2. Frequencies of the growth rate maxima and their frequency differences in dependence on $\theta_c$ and $v_m$ for the gyro-harmonic numbers $s = 25$, 26 and 27.

| $\theta_c$ (°) | $v_m$ c | $f_{max}^{s=25}$ (GHz) | $f_{max}^{s=26}$ (GHz) | $f_{max}^{s=27}$ (GHz) | $\Delta f(s = 25 - 26)$ (GHz) | $\Delta f(s = 26 - 27)$ (GHz) |
|----------------|---------|------------------------|------------------------|------------------------|-----------------------------|-----------------------------|
| 50             | 0.2 c   | 1.381                  | 1.359                  | 1.337                  | 0.022                       | 0.022                       |
| 50             | 0.3 c   | 1.359                  | 1.338                  | 1.317                  | 0.021                       | 0.021                       |
| 50             | 0.4 c   | 1.365                  | 1.347                  | 1.328                  | 0.018                       | 0.019                       |
| 30             | 0.3 c   | 1.379                  | 1.358                  | 1.336                  | 0.021                       | 0.022                       |
| 65             | 0.3 c   | 1.356                  | 1.334                  | 1.313                  | 0.022                       | 0.021                       |
Finally, we computed the growth rate of the upper-hybrid waves for the distribution \((14)\) in dependence on frequency with \(\kappa = 1.5\), for different \(\theta_c\) and \(p_m\) (corresponding to the velocity \(v_m\)). Other values in computations were taken the same as for Figure 8. In Table 2 we show the frequencies of the growth rate maxima and frequency differences of their maxima. It looks that for \(\theta_c = 50^\circ\) the frequency difference slightly decreases with the increase of \(v_m\). For fixed \(v_m\) this difference (within errors of computations corresponding to the last number in \(\Delta f\)) practically does not depend on \(\theta_c\).

5. Discussion and conclusions

It was shown that the growth rate of the upper-hybrid waves for the power-law momentum distribution with the low-momentum cut-off and the loss-cone anisotropy strongly depends on the pitch-angle boundary. The maximum growth rate was found for the pitch-angle \(\theta_c \approx 50^\circ\). For small angles the growth rate is broad and flat and for high pitch-angles even the absorption occurs.

We made computations for two power-law indices for \(\delta = 5\) and 10. While for the power-law index \(\delta = 5\) the zebra structure can be formed only in limited interval of \(v_m\) around the velocity \(10 v_T = 6.75 \times 10^7 \text{ m s}^{-1}\), in the case with \(\delta = 10\) the width of the growth rates are two time smaller and thus more favorable for the zebra pattern generation. Positions of the growth rate maxima in both the cases are approximately the same. These results agree to those of Kuznetsov and Tsap (2007).

An analysis of the growth rate of the upper-hybrid waves for the anisotropic kappa momentum distribution for all electron momenta \((p > 0)\) (which is a contribution to the dense background plasma) shows: a) When we decrease the characteristic momentum \(p_\kappa\) then the maximum of the growth rate is shifted to lower values of \(\omega_p/\omega_B\). b) The growth rates for the kappa-distribution with \(\kappa = 1.5\) and \(\kappa = \infty\) shows a similar behavior, but values of the growth rates for \(\kappa = \infty\) are a little bit higher. It is due to that in this case the distribution with \(\kappa = \infty\) has more electrons for low momentum values than that with \(\kappa = 1.5\). c) The frequency widths of the growth rate maxima are very broad. We also found that the frequency difference between the frequencies of the growth rate maxima slightly decreases with the increase of \(v_m\). For fixed \(v_m\) this difference practically does not depend on \(\theta_c\).

But, if we take a more realistic distribution, namely, the single kappa distribution which is isotropic up to some large momentum \(p_m\) and anisotropic with loss-cone above this momentum then distinct peaks of the growth rate appear and thus distinct zebra stripes can be generated. It means that the restriction for small momenta for the anisotropic part of distributions (power-law or kappa) is of principal importance for the zebra stripes generation.

For the first time, the dependence of the growth rate on the radio frequency was computed. In this case the spectral peaks are much more distinct than in the case of the dependence of the growth rate on the ratio of the plasma and cyclotron frequencies. Thus, analyzing observed radio spectra, we can assume smaller values of the power-law or kappa indices.
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Note that for high values of the pitch angle anisotropy, where the absorption occurs, the inverse zebra stripes can be produced on some radio continua.

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