Finite-time Passive Analysis and Passification for Nonlinear Neutral Stochastic Systems with Uncertainties and Time-varying Delays

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Abstract. The finite-time passive analysis and passification problems of neutral stochastic systems (NNSSs) with nonlinear uncertainties and time-varying delays are investigated in this paper. To guarantee the finite-time passification of NNSSs with nonlinear uncertainties and time-varying delays, a state feedback controller is designed respectively, which are delay independent and delay dependent, respectively. By constructing proper Lyapunov-Krasovski functionals, several sufficient conditions are obtained in terms of linear matrix inequalities (LMIs), in which the free-weighting matrix and sector conditions are involved. Finally, a numerical example with simulations is given to demonstrate the correctness of the derived results and the effectiveness of the proposed methods.

1. Introduction
Over the last few decades, neutral systems have been greatly investigated in many significant fields[1,2]. Due to their wide applications in the world, many attention has been paid to the delay-dependent stability and controlled problems of neutral stochastic systems [3,4]. By the utilization of linear matrix inequalities (LMIs) technology, some sufficient conditions for delay dependent exponential stability of neutral stochastic systems are presented in [5,6]. Cheng et al. investigated the stability and stabilization of neutral stochastic delay Markovian jump systems in [7]. Moreover, nonlinear neutral stochastic systems have also attracted sizable attention [8]. The stability of the neutral stochastic switching time-delay system was studied using the average dwell time method in [9]. Based on the free-weighting matrix approach, the stability of a class of neutral time-delay systems with nonlinear disturbances was studied[10].

Passivity is part of the generalized dissipative theory. The main idea of the passive theory is that the passivity of the system can keep the stability inside the system. The reason for this is that passivity is closely linked to finite time stability, which is also important in control theory. The finite-time stability was mainly focused on qualitative transient property in a given certain finite time. For instance, finite time tracking control of rigid spacecraft is considered [11]. And the finite-time stability for linear systems is discussed in [12]. It should be pointed out that passivity is closely related to circuit analysis. Therefore, it has attracted tremendous attention from the control community since 1970s [13]. It has provided a mighty instrument for system stability analysis [14] and has been obtained applications in many fields, for instance signal processing, synchronization and fuzzy
control and so on [15,16,17]. In addition, the research on passivity of nonlinear system mainly focuses on the establishment of passivity and the design of controller based on passivity theory. The controller is designed to make the closed-loop system satisfy the passive condition and achieve the stabilization of the closed-loop system. The finite time passification of Markov switched systems with stochastic time delay are studied [18,19,20], in addition, by combining with Jensen’s inequality, the passivity of Markovian jump systems are presented, [21,22].

However, as far as the authors know, the problem of the finite-time passification for NNSSs with nonlinear uncertainties and time-varying delay is rarely addressed in the current literatures. Motivated by the mentioned discussions above, the finite-time passification of NNSSs is investigated in this paper. The systems investigated in this paper are also very complicated, which completely discuss the influence of time-delays, neutral term, uncertainties and stochastic disturbance to the systems. Correspondingly, a state feedback controller is designed respectively, which depend on not only instantaneous state information but also delayed state information. Meanwhile, a proper L-K functional is constructed in this paper, the matrices are transformed by the free-weighting matrix, so the conservativeness be reduced.

2. Preliminaries
A class of NNSSs with nonlinear uncertainties and time-varying delay are considered as follow:

\[
\begin{aligned}
\dot{x}(t) &= A(t)x(t) + A_2 x(t) + f(x(t)), \\
f_u(x(t)) &= u(t) + B(t)v(t) dt + G(t, x(t), x(t)), w(t)) dt, \\
z(t) &= E_1 x(t) + E_2 u(t), \\
x(t) &= \varphi(t), \forall t \in [-\tau, 0],
\end{aligned}
\]

where \( x(t) \in \mathbb{R}^n \) is the state; \( v(t) \in \mathbb{R}^n \) is the disturbance input of the system which belongs to \( L_2(0, +\infty) \), \( u(t) \in \mathbb{R}^n \) is the control input; \( z(t) \in \mathbb{R}^q \) is the control output; \( w(t) \) is an \( m \)-dimensional Brownian motion satisfying \( E[\text{d}w(t)] = 0 \) and \( E[\text{d}w^2(t)] = \text{d}t \), \( \varphi(t) \) is a real-valued initial function on \([-\tau, 0]\), \( \tau(t) \) is the time-varying delay.

For simplicity of the investigation, we give some assumptions as follows.

**Assumption 1** The nonlinear function \( F(\cdot, \cdot, \cdot, \cdot) : R_+ \times R^n \times R^n \times R^n \times R^p \rightarrow R^n \) is a Borel measurable function, \( A, A_1, B \) are known real matrices with appropriate dimensions. \( f(x) \) and \( f_u(x(t)) \) are nonlinear vector-valued functions, which satisfies the following sector condition[23,24].

\[
\begin{align*}
[f(x) - R_1 x(t)] \leq 0, \\
[f_u(x(t)) - R_2 x(t)] \leq 0,
\end{align*}
\]

where \( R_1, R_2, R_3, R_4 \in R^{n \times n} \) are known real constant matrices with appropriate dimensions. We assume that:

\[
f(0) = 0, f_u(0) = 0.
\]
where $E_1, E_2$ are known real matrices with appropriate dimensions.

**Assumption 2** $G(\cdot,\cdot,\cdot,\cdot):\mathbb{R}^n \times \mathbb{R}^n \times \mathbb{R}^p \to \mathbb{R}^n$ is a nonlinear vector function which satisfies the following condition:

$$
G(t,x(t),x(t-\tau(t)),v(t))^T G(t,x(t),x(t-\tau(t)),v(t)) \leq x^T(t)H^t(t)Hx(t)
$$

$$
+ x^T(t-\tau(t))H^t_d(t)H_d x(t-\tau(t)) + v^T(t)E^T E v(t),
$$

where $H, H_d, E$ are known constant matrices with appropriate dimensions.

**Assumption 3** Without loss of generality, for the neutral term $\mu(x(t-\tau(t)))$, it satisfies the following condition:

$$
\| \mu(t-\tau(t)) \| \leq \kappa \| x(t-\tau(t)) \|, \kappa \in (0,1).
$$

In fact, the above inequality is equivalent to

$$
\kappa^2 x^T(t-\tau(t))x(t-\tau(t))-\mu^T(x(t-\tau(t)))\mu(x(t-\tau(t))) \leq 0.
$$

**Assumption 4** In the NNSSs(5), $\tau(t)$ is the time-varying delay function, which is differentiable, and holds the following conditions:

$$
0 \leq \tau_{min} \leq \tau(t) \leq \tau_{max}, \tau(t) \leq h < 1,
$$

where $\tau_{min}, \tau_{max}, h$ are known nonzero constants.

**Assumption 5** For a given positive parameter $\delta$, the exogenous disturbance input $v(t)$ is time-varying and satisfies

$$
\int_0^t v^T(s)v(s)ds \leq \delta, \delta \geq 0.
$$

**Definition 1** For some given constants $c_2 > c_1 \geq 0, T > 0, \delta > 0$ and symmetric matrix $\bar{R} > 0$, such that

$$
\mathbb{E}\{x^T(t_0)\bar{R} x(t_0)\} \leq c_1 \Rightarrow \mathbb{E}\{x^T(t)\bar{R} x(t)\} \leq c_2, \forall t_0 \in [-h,0], t \in [0,T].
$$

Then, the closed-loop systems (5) are said to be finite-time stochastically stabilized (FTSS) with respect to $(c_1, c_2, \delta, T, \bar{R})$.

**Definition 2** ([14]) The system (5) is called passive if there exists a scalar $\beta > 0$ such that

$$
2\mathbb{E}\int_0^t z^T(s)v(s)ds \geq -\beta \mathbb{E}\int_0^t v^T(s)v(s)ds, \forall t \in [0,T].
$$

3. Main results

In this section, which is divided into two parts to discuss as time-delay independent and time-delay dependent. Different feedback controllers are designed in every part. The passivity conditions are derived for NNSSs(5), and the sufficient criteria are obtained for the NNSSs (5) by using different Lyapunov functions.

This paper is concerned with the problems of the finite-time passification for NNNSSs with uncertainties and time-varying delay(5), our interest is focused on designing a memoryless state feedback controller:

$$
u(t) = Kx(t),
$$

for NNSSs(5) such that the following closed-loop system is passive.
\[
\begin{aligned}
\left[ d\left[ x(t) - \mu(x(t - \tau(t))) \right] \right]
&= [(A + K)x(t) + A_d x(t - \tau(t)) + \\
&f(t) + f_d ((x(t - \tau(t))) + Bv(t)] dt + G(t, x(t), x(t - \tau(t)), v(t)) dw(t), \\
z(t) &= (E_1 + E_2K)x(t), \\
x(t) &= \varphi(t), \forall t \in [-h, 0].
\end{aligned}
\]

**Theorem 1** If there exists a positive scalar $\beta > 0$ and two Symmetric positive matrices $P > 0, Q > 0$ such that the follows LMI holds

\[
\begin{bmatrix}
\Theta_1 \\
* \Theta_2 \\
* * \Theta_3
\end{bmatrix} < 0.
\]

Where

\[
\Theta_1 = \begin{bmatrix}
\Phi_{11} & A_d & \Phi_{13} \\
* & \Phi_{22} & 0 \\
* * & \lambda_d I - \beta I
\end{bmatrix},
\Theta_3 = \begin{bmatrix}
* & -2I & -I \\
* & -2I & -I \\
* * & * & -Z_4
\end{bmatrix},
\]

\[
\Theta_2 = \begin{bmatrix}
I + XR^T + XR^T_2 & I \\
0 & -X \lambda^T - Y^T_1 \\
0 & 0 & -B^T
\end{bmatrix},
\]

\[
Z_1 = XQX, Z_2 = \lambda_{d} XH^T HX, Z_3 = 2XR^T R_2 X, Z_4 = X^T X,
\]

\[
\Phi_{11} = AX + Y_1 + X^T A^T + Z_1 + Z_2 - Z_3, \Phi_{13} = B - XE^T_1 - Y^T_1 E_2,
\]

\[
\Phi_{22} = \kappa^2 I - (1 - h)Q + \lambda_d H_d^T H \_2 - 2R^T_1 R_4, \ Y_1 = KX.
\]

Then, the system (14) is passive in the sense of Definition 1, and the state feedback controller can be designed as $u(t) = Kx(t) = X X^T x(t)$.

**Proof** For the passification analysis of the system (14), we construct the following Lyapunov-Krasovskii functional:

\[
V(x, t) = (x(t) - \mu(x(t - \tau(t))))^T P(x(t) - \mu(x(t - \tau(t)))) + \int_{t-\tau}^t x^T(s)Qx(s)ds.
\]

By Ito formula, the stochastic derivative of $V(x, t)$ along system (14) can be obtained as:

\[
dV(x, t) = \mathcal{L}V(x, t) dt + 2(x(t) - \mu(x(t - \tau(t))))^T P \times G(t, x(t), x(t - \tau(t)), v(t)) dw(t).
\]

Define a new function as follow:

\[
\Omega(x, t) = \mathcal{L}V(x, t) - \beta V(x, t) - \beta^T v(t) v(t).
\]

Considering equation (16), equation (17), equation (18) and Schur complement, it follows that

\[
\mathcal{L}V(x, t) - \beta V(x, t) < \beta^T v(t) v(t).
\]

Multiplying both sides of the above inequality (19) by $e^{-\beta t}$ taking mathematical expectation and integrating it from 0 to $t$, we can get

\[
\mathbb{E}V(x, t) \leq e^{-\beta t} \left[ c_1 (\lambda_{max}(\mathcal{P})) + \tau(0) \lambda_{max}(\mathcal{Q}) + \delta(1 - e^{-\beta t}) \right],
\]

where $\mathcal{P} = R \_2 R \_2^T, \mathcal{Q} = R \_2 Q R \_2^T$.

From equation (20), it is clearly that
\[
x^T(t)Px(t) < c_1 \lambda_{max}(\bar{P}) + \tau(0) \lambda_{max}(\bar{Q}) + \delta(1 - e^{-\beta \tau}).
\]

(21)

Condition equation (21) implies that for \( \forall t \in [0, T] \), we can get \( x^T(t)Px(t) < c_2 \). Hence, the system (14) is FTSS.

Combining the Assumption 2, we have

\[
G^T(t, x(t), x(t - \tau(t), v(t)))PG(t, x(t), x(t - \tau(t), v(t)))
\]

\[
\leq \lambda_0(x^T(t)H^THx(t) + x^T(t - \tau(t))H_d^TH_d x(t - \tau(t)) + v^T(t)v(t)).
\]

(22)

Where \( \lambda_0 = \lambda_{max}(\bar{\lambda}) \).

It follows from equation (12) that

\[
J(t) = \mathbb{E}[dV(t) - 2z^T(t)v(t)dt - \beta v^T(t)v(t)dt] \leq \mathbb{E}[\Xi_t \bar{Y}_1 \Xi_t^T],
\]

(23)

where

\[
\Xi_t = \{x^T(t), x^T(t - \tau(t)), v^T(t), f^T(t), f_d^T(x(t - \tau(t))), \mu^T(x(t - \tau(t)))\}
\]

\[
Y_1 = \begin{bmatrix}
\Phi_{11} & PA_d & \Phi_{13} & P + R_1^T + R_2^T & P & -(A + K)^TP \\
* & \Phi_{22} & 0 & 0 & R_4^T + R_5^T & -A_d P \\
* & * & \lambda_0 I - \beta I & 0 & 0 & -B^TP \\
* & * & * & -2I & 0 & -P \\
* & * & * & * & -2I & 0 \\
* & * & * & * & * & -I \\
\end{bmatrix} < 0,
\]

\[
\Phi_{11} = P^T(A + K) + (A + K)^TP + Q + \lambda_0 H^TH - 2R_4^T R_2,
\]

\[
\Phi_{22} = \kappa^2 I - (1 - \mu)Q + \lambda_0 H_d^TH_d - 2R_3^T R_d. \Phi_{13} = PB - E_1^T - K^T E_2^T.
\]

By using Schur complement lemma, the LMI (15) implies \( \bar{Y}_1 < 0 \). Together with equation (23), we have

\[
J(t) < 0.
\]

(24)

Therefore, the passification for the system holds. The proof is completed.

Similar to the Theorem 1, we can get the delay-dependent passification theorem as follows.

**Theorem 2** The system (14) is passive in the sense of Definition 1, if there exists positive scalar \( \beta > 0, \epsilon_i > 0 \) and some positive definite matrices \( P > 0, Q_i > 0, i = 1, 2, 3, 4, 5, 6 \), such that the following LMI holds

\[
\begin{bmatrix}
\tilde{\Theta}_{11} & \tilde{\Theta}_{12} \\
* & \tilde{\Theta}_{22} \\
\end{bmatrix} < 0.
\]

(25)

Where
Then, the state feedback controller can be designed as $u(t) = Kx(t) = Y_1^T X^{-1} x(t)$. 

### 4. Numerical example

In order to prove the effectiveness of the method and the correctness of the conclusion, the following example and the parameters are given as follows:

$$A = \begin{bmatrix} 25 & 0 \\ 0.2 & 1 \end{bmatrix}, \quad A_d = \begin{bmatrix} 1 & 0 \\ -1 & 0 \end{bmatrix}, \quad B = \begin{bmatrix} -1 & 0 \\ -2 & 0 \end{bmatrix}, \quad H = \begin{bmatrix} -1 & 1 \\ 2 & 3 \end{bmatrix}, \quad H_d = \begin{bmatrix} -0.5 & 0 \\ -0.6 & 0.3 \end{bmatrix},$$

$$R_1 = \begin{bmatrix} 1 & 0.6 \\ 0.6 & 1.5 \end{bmatrix}, \quad R_2 = \begin{bmatrix} -0.4 & 0 \\ 0 & 0.3 \end{bmatrix}, \quad R_3 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, \quad R_4 = \begin{bmatrix} 0.5 & 0 \\ 0 & 0.5 \end{bmatrix},$$

$$E = \begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix}, \quad E_1 = \begin{bmatrix} -1 & 1 \\ 1 & 1 \end{bmatrix}, \quad E_2 = \begin{bmatrix} -1 & 1 \\ 1 & 1 \end{bmatrix}, \quad Q_0 = \begin{bmatrix} 160 & 0 \\ 0 & 160 \end{bmatrix}.$$ 

Here, we choose the values as $\tau_{\text{max}} = 0.8, \tau_{\text{min}} = 0.4, \kappa = 0.1, \lambda_0 = 1, \epsilon_1 = 0.1, h = 0.1$.

The exogenous disturbance input is given as: $v(t) = \begin{bmatrix} e^{-t \sin(t)} \\ e^{-0.8t \cos(2t)} \end{bmatrix}$.

The simulation of state is given in Figure 1 without control, we found that the nonlinear system (5) is unstable or passive. Therefore, we need to design the controller (13) for the nonlinear system (5) to satisfy the passive. From (15), the feasible solutions are as follows:

$$X = \begin{bmatrix} 814.4917 & -182.9690 \\ -182.9690 & 662.3239 \end{bmatrix}, \quad Y = \begin{bmatrix} 834.4717 & 211.4006 \\ 211.4006 & -643.4140 \end{bmatrix}, \quad Z_1 = \begin{bmatrix} 1.1891 & 0.0040 \\ 0.0040 & 1.1872 \end{bmatrix},$$

$$Z_2 = \begin{bmatrix} 4.2365 & -0.0399 \\ -0.0399 & 4.2547 \end{bmatrix}, \quad Z_3 = \begin{bmatrix} 1.8153 & -0.0146 \\ -0.0146 & 1.7222 \end{bmatrix}, \quad Z_4 = \begin{bmatrix} 0.0040 & 1.1872 \end{bmatrix}.$$ 

Therefore, the controller gain can be designed as:
With the controller (13), the simulation of the closed-loop nonlinear delayed system (5) is given in Figure 2. Obviously, the system is passive. From this, the controller we designed is effective.

To make the result of the simulation better, we take 100 sets of numbers randomly as the initial values of $x(0)$ and satisfy $x(0) \in ([1,1],[1,1])^T$. Then the corresponding state trajectories are shown in Figure 3. Moreover, when $x(0) \in ([-6,1],[-6,1])^T$, we have state trajectories in Figure 4. when $x(0) \in ([-2,4],[-2,4])^T$, we have state trajectories in Figure 5. Obviously, the system is passive.

$$K = \begin{bmatrix}
-1.0159 & 0.0385 \\
0.0441 & -0.9593
\end{bmatrix}.$$
5. Conclusions

The article considers the finite-time transfer problem of nonlinear neutral stochastic systems with uncertainties and time-varying delays, and divides them into delay-independent analysis and delay-related passive analysis, combined with Lyapunov-Krasovskii function and inequality scale. A state feedback controller is designed, the definition of finite time and sufficient conditions of the system are given, and the conclusion is verified by numerical examples.

References

[1] Song, B., Park, J. H., Wu, Z., Zhang, Y. (2013). New results on delay-dependent stability analysis for neutral stochastic delay systems. Journal of The Franklin Institute-engineering and Applied Mathematics, 350(4), 840-852.

[2] Xiong, L., Cheng, J., Liu, X., Wu, T. (2017). Improved conditions for neutral delay systems with novel inequalities. The Journal of Nonlinear Sciences and Applications, 10(05), 2309-2317.

[3] Wu, M., He, Y., She, J. H. (2004). New delay-dependent stability criteria and stabilizing method for neutral systems. IEEE Transactions on Automatic Control, 49(12), p.2266-2271.

[4] Chen, G., Shen, Y. (2009). Robust filter design for neutral stochastic uncertain systems with time-varying delay. Journal of Mathematical Analysis and Applications, 353(1), 196-204.

[5] Chen, H., Zhu, C. (2012). Delay-dependent exponential stability for uncertain neutral stochastic linear systems with interval time-varying delay. Control Theory and Applications, 6(15), 2409-2418.

[6] Chen, W., Zhang, B., Ma, Q. (2018). Decay-rate-dependent conditions for exponential stability of stochastic neutral systems with Markovian jumping parameters. Applied Mathematics and Computation, 93-105.

[7] Zhang, D., Cheng, J., Cao, J., Zhang, D. (2019). Finite-time synchronization control for semi-Markov jump neural networks with mode-dependent stochastic parametric uncertainties. Applied Mathematics and Computation, 230-242.

[8] Chen, A., Shen, M. (2018). A new method to reliable control of nonlinear stochastic systems with actuator faults. International Journal of Fuzzy Systems.

[9] Chen, H., Shi, P., Lim, C. (2017). Stability of neutral stochastic switched time delay systems: An average dwell time approach. International Journal of Robust and Nonlinear Control, 27(3), 512-532.

[10] Pradeep, C., Cao, Y., Murugesu, R., Rakkiyappan, R. (2017). An event-triggered synchronization of semi-Markov jump neural networks with time-varying delays based on generalized free-weighting-matrix approach. Mathematics and Computers in Simulation, 41-56.
[11] Zhao, L., Jia, Y. (2015). Finite-time attitude tracking control for a rigid spacecraft using time-varying terminal sliding mode techniques. International Journal of Control, 88(6), 1150-1162.

[12] Zhang, Z., Zhang, Z., Zhang, H., Zheng, B., & Karimi, H. R. (2014). Finite-time stability analysis and stabilization for linear discrete-time system with time-varying delay. Journal of The Franklin Institute-engineering and Applied Mathematics, 351(6), 3457-3476.

[13] Dai, M., Xia, J., Xia, H., Shen, H. (2019). Event-triggered passive synchronization for Markov jump neural networks subject to randomly occurring gain variations. Neurocomputing, 403-411.

[14] Lin, W., Byrnes, C. I. (1995). Passivity and absolute stabilization of a class of discrete-time nonlinear systems. Automatica, 31(2), 263-267.

[15] Wei F, Chen G, Wang W. (2020). Finite-time synchronization of memristor neural networks via interval matrix method[J]. Neural Networks, DOI: 10.1016/j.neunet.2020.04.003

[16] Cao, Y., Cao, Y., Wen, S., Huang, T., Zeng, Z. (2018). Passivity analysis of coupled neural networks with reaction–diffusion terms and mixed delays. Journal of The Franklin Institute-engineering and Applied Mathematics, 355(17), 8915-8933.

[17] Zhu, S., Shen, Y., Chen, G. (2010). Exponential passivity of neural networks with time-varying delay and uncertainty. Physics Letters A, 375(2), 136-142.

[18] Qi, W., Gao, X., Wang, J. (2016). Finite-Time Passivity and Passification for Stochastic Time-Delayed Markovian Switching Systems with Partly Known Transition Rates. Circuits Systems and Signal Processing, 35(11), 3913-3934.

[19] Wu, L., Su, X., Shi, P., Qiu, J. (2011). A New Approach to Stability Analysis and Stabilization of Discrete-Time T-S Fuzzy Time-Varying Delay Systems. systems man and cybernetics. 41(1): 273-286.

[20] Zuo, Z., Li, H., Wang, Y. (2013). New criterion for finite-time stability of linear discrete-time systems with time-varying delay. Journal of The Franklin Institute-engineering and Applied Mathematics, 350(9), 2745-2756.

[21] Wang, Y., Wang, Q., Zhou, P., Duan, D. (2012). Delay-Dependent Passivity and Passification for Uncertain Singularly Perturbed Markovian Jump Systems with Time-Varying Delay. Circuits Systems and Signal Processing, 31(6), 2179-2194.

[22] Luo, M., Zhong, S. (2012). Passivity analysis and passification of uncertain Markovian jump systems with partially known transition rates and mode-dependent interval time-varying delays. Computers & Mathematics With Applications, 63(7), 1266-1278.

[23] Duan, W., Fu, X., Yang, X. (2016). Further results on the robust stability for neutral-type Lur’e system with mixed delays and sector-bounded nonlinearities. International Journal of Control Automation and Systems, 14(2), 560-568.

[24] Duan, W., Cai, C. (2014). Delay-range-dependent stability criteria for delayed discrete-time Lur’e system with sector-bounded nonlinearities. Nonlinear Dynamics, 78(1), 135-145.