The ‘dipole instability’ in complex plasmas and its role in plasma crystal melting

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Abstract. Using the dipole model for binary dust–dust interactions, the instability of wave perturbations in a one-dimensional particle string oriented along the ion flow (it highlights the major aspect of the multi-layer crystals) is considered. Longitudinal short wavelength perturbations are found to exist below a certain threshold value of gas pressure. They may act as a possible precursor of the melting transition observed in multi-layer plasma crystals. For different discharge conditions, the threshold of this ‘dipole instability’ is found to be in the range 10–150 Pa.

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So-called ‘plasma crystals’ have attracted much interest in fundamental physics, plasma science as well as technological applications (see e.g. [1]–[4]). In ground-based experiments, these strongly coupled formations are observed in the sheath of the lower electrode, where the upward-directed electrostatic force balances the gravitational force on microparticles [5]–[9]. The particles mostly arrange themselves in a flat crystal with a diameter of a few hundred interparticle distances and a thickness of up to a few tens of layers. The flatter systems exhibit the thermodynamically expected two-dimensional hexagonal order. In the vertical direction, the microparticles are found to be aligned, forming vertical chains [5]–[10]. The typical side-view of such a plasma crystal is shown in figure 1. Note that if only repulsive interparticle forces dominate, externally confined grains should organize themselves in a close-packed structure (the minimum energy state for repulsive forces), not in aligned chains. As a result, the attraction force due to the formation of a positive space-charge region underneath the suspended microparticles (‘ion wake’), has been discussed [11]–[15]. This force was also invoked for the interpretation of the particle ‘pairing’ in a plasma sheath [12, 16, 17]. The particle–wake interactions introduce an anisotropy in dust–dust interactions, which also affects collective wave modes in monolayers. In particular, the wake effect can cause a resonance instability of dust-lattice (DL) modes developing at low gas pressures and linear mode coupling [19]–[21].

Observations of the plasma crystals, which reach equilibrium very rapidly and can be easily turned between their ordering and disordering states, stimulate theoretical investigations of the processes underlying the solid-to-liquid phase transition. In this context, the instability of the horizontal oscillations in adjacent horizontal layers due to the asymmetry in the particle–wake interactions has been given great attention in the past, but which so far has only been investigated quantitatively for double layer crystals, not multi-layers [10, 11, 18]. The onset of oscillations interpreted as a precursor of the melting transition in the double-layer horizontal structure was attributed to the attractive force on the lower particle provided by the ion cloud of the upper grain. The equivalent attractive force for the upper particle does not exist, because the wake charge is created by the upper grain and is always located below it. This asymmetry is the key point in the onset of the instability of the double-layer particle arrangement.

Up to now all the studies of wave perturbations have dealt with horizontal particle layers. However, real experiments often exhibit vertical structures, oriented along the ion flows (see figure 1). Moreover, observations of multi-layer crystals from the top and the sides (figure 2) suggest that pre-melting particle displacements and oscillations are isotropic (within the statistical limits). This implies that compressive instabilities are important. In this paper, we therefore consider the stability of the vertical particle chain (it highlights the major aspect of the multi-layer crystals) in the presence of perturbations along and transverse to the direction of the ion flow. We introduce a ‘dipole’ type of particle interactions due to the formation of the ion wake charges. It turns out that the dipole attractive forces can overcome the Coulomb repulsion even for small effective wake charges. They can also be responsible for the development of a dipole instability below a certain threshold value of gas pressure (i.e. gas damping). Such an instability could trigger phase transition in plasma crystals from solid to liquid state when the gas pressure is reduced. Therefore, the dipole instability could be the critical process for multi-layer plasma crystals which so far had not been identified. Moreover, the scenario developed in this paper could become a ‘kinetic blueprint’ for other systems involving anisotropic interactions (e.g. magnetized matter, electromagnetically induced dipole–dipole interactions of fermionic gas or atomic Bose–Einstein condensate [22, 23]) and thus assume a relevance of generic proportions.
1. Binary particle–wake interactions

In general, the wake effect is difficult to model, because we lack a complete plasma transport description in the sheath region of a plasma discharge. Numerical simulations show that the ion focusing charge is mostly formed within the screening length behind the particle [13, 14]. An attractive idea is to introduce a positive point-like effective wake charge $q$ located at some small distance $l$ beneath the particle within the Debye sphere. In this approximation, a system of the charged particle with its ion density enhancement is then equivalent to the uncompensated residual particle charge $Q$, plus an electric dipole $P = ql$, which accounts for the anisotropy of the plasma due to the ion flow. Accordingly, the force describing dust–dust interactions (between any pair $j$ and $n$ grains) can be represented as a combination of the electrostatic force due to the repulsion of the residual like particle charges $F^{(Q)}_{jn} = -Q_j (\partial \phi_n / \partial r)_{r=r_j}$ and a dipole force $F^{(p)}_{jn}$.
due to the streaming ions, so that

\[ F_{jn} = -Q_j (\partial \varphi_n / \partial r)_{r=r_j} + F_{jn}^{(p)}. \]  \hfill (1)

The electrostatic potential of each particle is taken as a screened Debye potential

\[ \varphi_n (r) = Q_n \exp \left( -\frac{|r - r_n|}{\lambda_D} \right). \]  \hfill (2)

Here, \( Q_n \) and \( \lambda_D \) refer to the \( n \)th particle charge (residual), and the screening length, respectively, while \( |r - r_n| \) is the distance measured from the \( n \)th particle.

The electric dipole moment is determined by

\[ P_n = q_n l, \]

where \( q_n \) denotes the effective wake charge, and \( l = l z^0 \) with \( l \) a characteristic length scale of the ion focusing \((l \lesssim \lambda_D)\) and \( z^0 \) the unit vector in the direction of the ion flow. The electric dipole field, \( E_n \), is

\[ E_n (r) = \frac{3((r - r_n) \cdot P_j)(r - r_n)}{|r - r_n|^5} - \frac{P_j}{|r - r_n|^3}, \]  \hfill (3)

and the dipole force \( F_{jn}^{(p)} \) exerted on the particle \( j \), by the grain \( n \), is

\[ F_{jn}^{(p)} = (P_j \nabla)E_n. \]  \hfill (4)

The use of model (3) and (4) implies that the dipole field (3) is unshielded. The latter can be justified by the calculations of a test particle potential in the case of anisotropic (in velocity space) plasmas [24, 25]. These results showed that the effective potential falls off, in general, as inverse third power of the distance from the test charge and thus support the model ‘unshielded’ dipole interactions at large distances. Therefore, the approximation (1)–(4) is valid when the interparticle distances are at least \( |r_j - r_n| \gtrsim \lambda_D \).

Introducing the hybrid type interactions through (1) immediately modifies the character of binary particle interactions, making them strongly asymmetric. Indeed, the mutual interactions between two identical particles are now determined by a force, which can be attractive or repulsive, depending on the plasma parameters and relative positions of the two dipoles. In particular, the force component along the radius vector between two particles is given by

\[ F = \frac{Q^2}{\lambda_D^2} \left[ \frac{(1 + \kappa) \exp (-\kappa)}{\kappa^2} + 3\zeta \frac{(1 - 3 \sin^2 \chi)}{\kappa^4} \right]. \]  \hfill (5)

where \( \chi \) is the angle between the horizontal direction and radius vector connecting two particles, \( \kappa = \Delta / \lambda_D \) is the ‘lattice parameter’, and \( \Delta \) is the interparticle distance. In the following, we assume that the introduced dimensionless coefficient \( \zeta = q^2 l^2 / (Q \lambda_D)^2 \lesssim 1 \) specifies the value of the electric dipole moment. When \( (1 - 3 \sin^2 \chi) > 0 \), so that \( |\chi| < 0.62 \), the particle are repelled since both forces act in the same direction, while for \( |\chi| > 0.62 \), the electrostatic and dipole forces compete with each other and the resulting force corresponds to either attraction or repulsion.

As an extreme case, we consider two vertically aligned particles \( (\chi = \pi/2) \). Figure 3 shows the normalized force \( f = F \lambda_D^2 / Q^2 \) as a function of \( \kappa \) for different coefficients \( \zeta \). It can be easily
verified that the particle attraction due to the short-ranged dipole force $\alpha -1/\Delta^4 (f<0)$ changes to repulsion ($f>0$) only if $\zeta<\zeta_{ct}=\kappa^2(2\kappa+\kappa^2+2)\exp(-\kappa)/24$. The latter gives $\zeta_{ct}\lesssim0.3$ for typical values of $\kappa\sim1/4$. For such $\zeta$, the dipole term prevails at either small interparticle distances ($\zeta\lesssim1/2.5$) or large distances (e.g. $\kappa\gtrsim3.5/6$) (the latter is due to screening of Coulomb interactions of the particle charges). Therefore, in this model, the interactions are repulsive only for a rather narrow region of $\kappa$, which is determined by specific values of $\zeta$ (e.g. $\kappa\sim1.5/4$ for $\zeta\sim0.3,0.2$). The obtained values of $\kappa$ are comparable to those observed in the experiments on particle ‘pairing’: transition from the vertical aligned position to the horizontal one (attraction–repulsion) usually occurred at $\kappa\lesssim1$ [16, 17]. This can be considered as an indirect indication that the coefficient $\zeta$ indeed can be of the order of 0.1–0.3.

2. Derivation of the ‘dipole instability’

2.1. Dispersion relations

We now examine the stability of vertical particle structures. In this situation, it is important to note that any given grain is attracted more strongly to the upstream neighbour than to the downstream particle, because of the greater proximity of the wake field from the upstream particle. To study the stability of such aligned complex plasma structures, we employ the simplest model of an infinite one-dimensional (1D) particle string, used previously in theoretical considerations of the DL waves [10], [19]–[21], [26] for both longitudinal and transverse motion. The 1D string, oriented along the ion stream coordinate ($z$-axis), is formed by identical particles of mass $M$, charge $Q$ separated by an average distance $\Delta$. Once again the wake is modelled by a point-like effective charge $q$, located at some distance $l$ beneath the particle, leading to a dipole moment parallel to the $z$-axis, $P_n(0,0,q_nl)$ and thus the value $Q$ relates to the residual particle charge. Finally, we assume that there is a harmonic potential well in the vertical and horizontal directions with eigenfrequencies $\Omega_v$, and $\Omega_H$, related to the confinement potential via $U_{conf} = -M(\Omega_v^2z^2 + \Omega_H^2x^2)/2$. 

**Figure 3.** The qualitative behaviour of the interacting force, $f = F_d^2/Q^2$, between two vertically aligned particle–wake structures ($\chi = \pi/2$) as a function of $\kappa = d/\lambda_D$ for different contributions of the dipole term, namely $\zeta = 0$—pure Coulomb interactions (thin solid line), dust–dust interactions accounting for wake effect $\zeta \lesssim 0.3$ (solid line), $\zeta \gtrsim 0.3$ (dashed curve).
Assuming interaction only between the closest neighbours (a realistic assumption if the interparticle distance exceeds the screening length: $\Delta / \lambda_D \sim 1.5 \div 2.5$), the equation of motion for the $n$th particle in the string can be written as

$$\ddot{r}_n + 2\gamma \dot{r}_n = M^{-1}(F_{n,n+1} + F_{n,n-1} + F_{n,\text{conf}}), \quad (6)$$

where $\gamma$ is the damping rate due to neutral gas friction [27], and $F_{n,\text{conf}} = -\partial U_{\text{conf}} / \partial r_n$ is the force due to the external confinement. The forces between the (nodal) $n$th and $n \pm 1$ particles, $F_{n,n\pm 1}$, are determined by equation (1). Introducing the particle displacements in the horizontal and vertical directions of the nodal $n$th particle $(x_n, z_n)$ and similarly for the vertically aligned neighbour grains $(x_{n \pm 1}, z_{n \pm 1})$, the interparticle distances can be written as $\Delta Q_{\pm} = \sqrt{(\Delta + \delta z_{n \pm 1})^2 + \delta x_{n \pm 1}^2}$. The distances between the nodal $(n)$ particle and the effective wake charges of the adjacent $(n \pm 1)$ grains, $\Delta q_{\pm}$, are not symmetric: $\Delta q_{\pm} = \sqrt{(\Delta \mp l + \delta z_{n \pm 1})^2 + \delta x_{n \pm 1}^2}$. Here, $\delta z_{n \pm 1}$ and $\delta x_{n \pm 1}$ denote the relative displacements with respect to the upper particle, $\delta z_{n+1} = z_{n+1} - z_n$, $\delta x_{n+1} = x_n - x_{n+1}$ and to the lower one, $\delta z_{n-1} = z_n - z_{n-1}$, $\delta x_{n-1} = x_n - x_{n-1}$. These displacements are assumed to be small, so that $\delta x_{n+1}, \delta z_{n+1} \ll \Delta, l$.

Linearizing the equation of motion (6) and considering wave perturbations $x_n, z_n \propto \exp[-i(\omega t - nk\Delta)]$ (the wavenumber $k$ obeys $-\pi \leq k\Delta \leq \pi$), we obtain two independent dispersion relations: for compressional longitudinal $(||)$—or vertical—perturbations, propagating along the $z$-axis and plane polarized transverse $(\perp)$—or horizontal—modes, propagating along the $x$-axis, namely

$$\omega^2 + 2i\gamma\omega = \Omega_H^2 - 4\Omega_\perp^2 \sin^2 \frac{k\Delta}{2} - 2i\Omega_\perp^2 \sin k\Delta, \quad (7)$$

$$\omega^2 + 2i\gamma\omega = \Omega_v^2 + 4\Omega_\parallel^2 \sin^2 \frac{k\Delta}{2} + 2i\Omega_\parallel^2 \sin k\Delta. \quad (8)$$

Here, we have used $\tilde{l} = l/\Delta \simeq \kappa^{-1} < 1$, which is usually relevant for strongly coupled complex plasmas. The wave frequencies involved are defined as

$$\Omega_\perp^2 = \Omega_0^2 - \Omega_1^2 \quad \text{and} \quad \Omega_\parallel^2 = \Psi(\kappa) \Omega_0^2 - 2\Omega_1^2 \quad (9)$$

with the characteristic frequencies $\Omega_0$ and $\Omega_1$, resulting from electrostatic and dipole interactions, respectively

$$\Omega_0^2 = \frac{Q^2}{MA_0^3} \varphi(\kappa) \exp(-\kappa) \quad \text{and} \quad \Omega_1^2 = \frac{12\zeta Q^2 \lambda_D^2}{MA_0^3}, \quad (10)$$

where $\varphi(\kappa) = (1 + \kappa)$ and $\Psi(\kappa) = \varphi(\kappa) + \varphi^{-1}(\kappa)$. Finally, the wave frequencies, $\Omega_{\perp}^2$ and $\Omega_{\parallel}^2$, arising from the asymmetry in the particle–wake interactions, are given by $\Omega_{\perp}^2 = 5\tilde{l}\Omega_1^2 \simeq 5\Omega_1^2 / \kappa$ and $\Omega_{\parallel}^2 = 10\tilde{l}\Omega_1^2 \simeq 10\Omega_1^2 / \kappa$. Note that $\Omega_{\perp}^2$ and $\Omega_{\parallel}^2$ are comparable with one of the main characteristic frequencies of the system $\Omega_1^2$. 

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2.2. Stability analysis

It turns out that the contribution of the wake interactions in the total wave frequencies $\Omega_\parallel$ and $\Omega_\perp$ can be of the same order of magnitude or even larger than $\Omega_0$ even for rather small particle–wake interaction coefficient $\zeta \sim q^2/Q^2$. It is easy to verify that when the wake effect is significant ($\zeta \sim 0.2$–0.3 or equivalently $q/Q \sim 0.45$–0.55), $\Omega_0^2/\Omega_0^2 > 1$, while $2\Omega_0^2/(\Psi(\kappa)\Omega_0^2) \lesssim 1$ for the typical range of $\kappa \sim 1.5$–3.5. This may lead to a negative transverse frequency $\Omega_\perp$, while the longitudinal one is significantly reduced, but still positive, $\Omega_\parallel^2 > 0$. Only in the case of sufficiently weak–wake interactions ($\zeta \sim 0.1$ or equivalently $q/Q \sim 0.3$), can $\Omega_\parallel^2$ become less than $\Omega_0^2$, and both eigenfrequencies $\Omega_\parallel^2$ and $\Omega_\perp^2$ remain positive. Modifications of the wave frequencies immediately influence the stability of wave perturbations.

We start with the horizontal mode, assuming that the damping is relatively weak, $\gamma^2 \ll \Omega_H^2, |\Omega_\perp^2|$. For strong wake effects ($\zeta \sim 0.3$–0.2), equation (7) yields

\[ (\text{Re } \omega)^2 \simeq \Omega_H^2 + 4|\Omega_\perp^2| \sin^2(k\Delta/2), \]  

\[ \text{Im } \omega \simeq -\gamma - \frac{\Omega_\perp^2 \sin k\Delta}{\sqrt{\Omega_H^2 + 4|\Omega_\perp^2| \sin^2(k\Delta/2)}}. \]  

We see that the wake effect changes the dispersion of the transverse mode, and never leads to its instability, but increases its damping. In the case of weak particle–wake interactions ($\zeta \lesssim 0.1$), $\Omega_\perp^2 > 0$, and the transverse wave reveals its usual optic-like dispersion—this is described by the same equations (11) and (12), where $|\Omega_\perp^2|$ is replaced by $-\Omega_\perp^2$. If we exclude the case of weak horizontal confinement, $\Omega_H^2 < 4\Omega_\perp^2$ (an aperiodic instability), the transverse vibrations are always stable and do not reveal any instability due to the ion wake effect.

The dispersion relation for the longitudinal perturbations (8) is simplified to

\[ (\text{Re } \omega)^2 \simeq \Omega_v^2 + 4\Omega_\parallel^2 \sin^2(k\Delta/2), \]  

\[ \text{Im } \omega = \frac{\Omega_\parallel^2 \sin k\Delta}{\sqrt{\Omega_v^2 + 4\Omega_{\parallel}^2 \sin^2(k\Delta/2)}} - \gamma. \]  

For the wave to be unstable due to particle–wake interactions, the term $\text{Im } \omega$ has to be positive. This determines the value of the critical damping coefficient

\[ \gamma_{cr} = \frac{\Omega_{\parallel}^2 \sin k\Delta}{\sqrt{\Omega_v^2 + 4\Omega_{\parallel}^2 \sin^2(k\Delta/2)}}, \]  

above which small oscillations are stable and do not destroy the vertical structure of the particle chain. The critical damping coefficient goes to zero for long wavelengths and hence there is no
growing solution for \( k \to 0 \). The upper limit of the right-hand side of equation (15) is close to \( \Omega_{q||}^2 / \Omega_v \), hence unstable modes occur whenever

\[ \gamma < \gamma_{cr} \lesssim \Omega_{q||}^2 / \Omega_v. \]  

(16)

The maximal growth rate corresponds to the dimensionless wavenumber \( k_0 \Delta = \arccos \left[ \frac{2 + s - 2\sqrt{(1 + s)}}{s} \right] \) with \( s \) being the ratio \( \Omega_v^2 / 4 \Omega_{q||}^2 \). The value \( k_0 \Delta \) is close to \( \pi/2 \) for \( s \ll 1 \).

We have termed this instability the ‘dipole instability’ with reference to its physical origin. Note that the instability of longitudinal oscillations in the presence of friction is possible because of the available free energy. The source of the free energy is provided by the streaming ions, which are focused in the wake downstream and thus create the effective positive charge region. These charges attract a given particle asymmetrically: the wake below the upper grain attracts more effectively than the wake of the lower particle.

It is instructive to investigate the DL mode stability using other types of the wake models. We start with a model, in which the electrostatic potential, \( \phi_n \), is described as a combination of the screened Debye potentials of the particle charge itself and its effective wake charge (see e.g. [19]–[21])

\[ \phi_n (r) = Q_n \left[ \frac{\exp \left( - |r - r_n| / \lambda_D \right)}{|r - r_n|} - \tilde{q}_n \frac{\exp \left( - |r - r_{nq}| / \lambda_D \right)}{|r - r_{nq}|} \right], \]  

(17)

where \( \tilde{q}_n = q_n / Q_n \) is a relative wake charge, \( |r - r_n| \) and \( |r - r_{nq}| \) are distances from the \( n \)th particle and its effective wake charge. Following the standard procedure of the linear stability analysis, one can find that the longitudinal DL mode are described by the same dispersion relation (8), where the characteristic frequency \( \Omega_{q||}^2 \) responsible for the instability is now given by \( \Omega_{q||}^2 = \tilde{q} \Omega_0^2 (\kappa^2 + 6 \kappa + 3 \kappa^2 + 6) / \varphi(\kappa) \).

Finally, we have used a wake potential proposed in [28] which describes the vertical asymmetry in the particle interactions and is only applicable on the line behind the particles. As a result, this does not affect the horizontal DL waves, but can lead to the instability of the DL perturbations propagating along the particle string. One easily obtains the dispersion relation akin to equation (8), where the role of \( \Omega_{q||}^2 \) plays a value \( (\gamma_2 - \gamma_1) / 2 M \) with \( \gamma_1 \) and \( \gamma_2 \) being the second derivatives of the potentials acting on the downstream particle (subscript 1) and upstream grain (subscript 2) \( (\gamma_2 > \gamma_1) \). Therefore, whilst a dipole representation is only an approximation that allows a quantitative treatment analytically, it catches the essential topology and physics quite well.

3. Application to experiments

In laboratory experiments, the typical values of the characteristic frequencies are \( \Omega_0 \sim 50–100 \text{ s}^{-1} \) and \( \Omega_v \sim 100–200 \text{ s}^{-1} \). For realistic \( \kappa \sim 1.5–4 \) and small effective wake charges \( \zeta \lesssim 0.3 \), the ratio \( 2 \Omega_0^2 / (\Psi(\kappa) \Omega_0^2) \) is of the order of \( \sim 1–0.5 \), leading to

\[ \gamma_{cr} \sim \Omega_{q||}^2 / \Omega_v \sim 5 \Omega_0^2 / \Omega_v. \]  

(18)

Another wake model accounting for shielding of particle–wake interactions (17) gives similarly \( \gamma_{cr} \sim \Omega_{q||}^2 / \Omega_v \sim 10 \tilde{q} \Omega_0^2 / \Omega_v \). As can be seen both threshold values are of the same order of
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Figure 4. Boundary for the onset of the dipole instability in the crystal structure in the plane spanned by the squared relative wake charge \( \zeta \simeq q^2/Q^2 \) and the ratio of the vertical interparticle distance to the length of the dipole, \( \kappa = \Delta/\lambda_D \simeq \Delta/l \). Calculations are performed for plasma parameters of [7, 8]. A transition to the instability can be obtained by increasing asymmetry in the particle–wake interactions either due to growth of the relative wake charge or by reducing \( \kappa \) via growth of a dipole length scale \( l \).

magnitude for approximately the same effective wake charges (\( \tilde{q} \simeq 0.5 \) in the case of the model (17) and \( \tilde{q} \lesssim \sqrt{0.3} \) for the unshielded dipole interactions).

The instability condition (18) gives a range of the critical damping coefficient \( \sim 50–300 \text{ s}^{-1} \), and correspondingly the vertically aligned structures formed by micron-sized particles should remain stable above a neutral gas pressure 10–40 Pa in krypton, 15–50 Pa in argon, 20–90 Pa in neon and 30–160 Pa in helium. These estimates correspond well to typical experimental parameters.

To obtain more precise estimates of the threshold value of the dipole instability, we now use parameters for particular plasma-crystal structures observed in a Krypton discharge (figures 1 and 2) [7, 8]: \( Q \simeq 1.7 \times 10^4 e \), \( M \simeq 3 \times 10^{-10} \text{ g} \), \( \Delta \simeq 2 \times 10^2 \mu \text{m} \), \( \kappa \simeq 3–3.5 \). These yield \( \gamma_{\text{cr}} \simeq 60–80 \text{ s}^{-1} \) or equivalently a neutral gas pressure 25–35 Pa. Note that this threshold is in good agreement with the experimental limit \( p_{\text{cr}} \simeq 32–29 \text{ Pa} \), where crystal melting was observed [7, 8]. It is instructive to consider the boundary for the onset of the unstable DL perturbations obtained from equation (15) in the \((\zeta, \kappa)\) plane, where the parameter \( \kappa \) measures the ratio of the interparticle distance to the characteristic length scale of the wake through \( \kappa = \Delta/\lambda_D \simeq \Delta/l \). The results are shown in figure 4. Since the transition to the instability occurs at \( \kappa \sim 3–3.5 \), one can estimate the relative effective wake charge as \( q/Q = \sqrt{\zeta} \sim 0.3 \) (at gas pressures \( p \sim 30 \text{ Pa} \)).

4. Summary and discussion

We have shown that ion streaming around dust grains can significantly modify binary dust–dust interaction creating regions of attractions and repulsion for grains oriented along the ion flows. It turns out that the attractive forces can overcome the Coulomb repulsion of particle charges even for rather small effective wake charges \( (q/Q \lesssim 0.3) \). Moreover, a simple 1D model of a particle chain oriented along the ion flow predicts an instability (termed ‘dipole instability’) of
longitudinal perturbations due to the asymmetry in the particle–wake interactions downstream. Neutral gas damping may quench this instability. Using realistic experimental conditions, the damping threshold for the dipole instability is found to be in the range of gas pressures 10–150 Pa. The instability mainly develops at wavenumber $k_0 \Delta \simeq \pi/2$. It is quite possible that the dipole instability could trigger (and thus be responsible for) phase transitions in plasma crystals from solid to liquid when the gas pressure is reduced. Available experimental data are compatible with this suggestion.

Note that the presented theory of particle–wake interactions is a simplified model which has an advantage of bringing out the qualitative features of the physical processes. To prove that our analysis is not only of limited value because of that dipole approximation, we have investigated the DL mode stability using other types of the wake models. In fact any of the non-symmetric potentials will result in instability of the longitudinal DL mode akin to the considered one.

Finally, we briefly discuss the dipole instability in relation to the ion-induced instability trigger for plasma crystal melting [10, 11, 18]. The estimates of the dipole instability threshold are of the same order of magnitude (or even higher) than those obtained for the instability leading to the onset of the horizontal (transverse) oscillations in horizontal layers of plasma crystals also due to particle–wake interactions. This means that the dipole instability, which produces vertical perturbations in aligned structures, could be as important as a precursor (or initiator) of the melting the crystal structures as the onset of the horizontal oscillations, which have been given great attention in this context in the past, but which so far have only been investigated quantitatively for double layer crystals, not multi-layers [10, 11, 18]. Observations of multi-layer crystals from the top and the side suggest that pre-melting particles displacements and oscillations are isotropic. This suggests that compressive instabilities are important. The dipole instability could be the critical process for multi-layer crystals which so far had not been identified.

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