Stellar disruption by a supermassive black hole: is the light curve really proportional to $t^{-5/3}$?

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Accepted 2008 October 5. Received 2008 September 29; in original form 2008 August 14

ABSTRACT

In this paper, we revisit the arguments for the basis of the time evolution of the flares expected to arise when a star is disrupted by a supermassive black hole. We present a simple analytic model relating the light curve to the internal density structure of the star. We thus show that the standard light curve proportional to $t^{-5/3}$ only holds at late times. Close to the peak luminosity the light curve is shallower, deviating more strongly from $t^{-5/3}$ for more centrally concentrated (e.g. solar type) stars. We test our model numerically by simulating the tidal disruption of several stellar models, described by simple polytropic spheres with index $\gamma$. The simulations agree with the analytical model given two considerations. First, the stars are somewhat inflated on reaching pericentre because of the effective reduction of gravity in the tidal field of the black hole. This is well described by a homologous expansion by a factor which becomes smaller as the polytropic index becomes larger. Secondly, for large polytropic indices wings appear in the tails of the energy distribution, indicating that some material is pushed further away from parabolic orbits by shocks in the tidal tails. In all our simulations, the $t^{-5/3}$ light curve is achieved only at late stages. In particular, we predict that for solar-type stars, this happens only after the luminosity has dropped by at least 2 mag from the peak. We discuss our results in the light of recent observations of flares in otherwise quiescent galaxies and note the dependence of these results on further parameters, such as the star/hole mass ratio and the stellar orbit.

Key words: black hole physics – hydrodynamics – galaxies: nuclei.

1 INTRODUCTION

X-ray flares from quiescent [non-AGN (active galactic nucleus)] galaxies are often interpreted as arising from the tidal disruption of stars as they get close to a dormant supermassive black hole (SMBH) in the centre of the galaxy (Komossa & Bade 1999). Similar processes also occur on much smaller scales, such as in compact binary systems, where the black hole is only of stellar mass (Rosswog, Ramirez-Ruiz & Hix 2008a; Rosswog et al. 2008b).

The pioneering work by Lacy, Townes & Hollenbach (1982), Rees (1988), Phinney (1989) and, from a numerical point of view, by Evans & Kochanek (1989) has set the theoretical standard for the interpretation of such events. In particular, a distinctive feature of this theory is an apparent prediction for the time dependence of the light curve of such events, in the form of $L(t) \propto t^{-5/3}$ (note that the original paper by Rees 1988 quotes a $t^{-5/2}$ dependence, later corrected to $t^{-5/3}$ by Phinney 1989). Since then, a $t^{-5/3}$ light curve is generally fitted to the observed luminosities of events interpreted as stellar disruptions. In this paper, we revisit the theoretical arguments behind such scaling and we show (as also originally argued by Rees 1988) that the light curve does not need to have this scaling and, in particular, that it critically depends on the internal structure of the star being disrupted. We provide a simple model to calculate the light curve starting from the density profile of the star and we show that more centrally concentrated stars tend to produce shallower light curves. We further supplement our model by a numerical calculation of the process, using smoothed particle hydrodynamics (SPH).

We start by briefly summarizing the main argument of Rees (1988). Let us consider a star, originally in hydrostatic equilibrium at a large distance from the black hole. Since pressure forces and the internal self-gravity of the star are in equilibrium, the only unbalanced force is the gravitational pull of the black hole. The various fluid elements of the star therefore move in essentially Keplerian orbits around the black hole, each one with its own eccentricity, that is initially very close to the eccentricity of the centre of mass of the star (in the following, for simplicity, we make the simple assumption that the centre of mass of the star is in a parabolic orbit around the black hole). Therefore, the distribution of specific mechanical energy within the star is very narrow around the energy...
of the centre of mass. As the star moves closer to the black hole, the various Keplerian orbits tend to be squeezed, perturbing the hydrostatic balance. Pressure forces then redistribute energy inside the star, therefore widening the specific energy distribution. After the encounter, the star is thus characterized by a much wider distribution of internal energies, with part of the fluid having a negative energy (and therefore being bound to the black hole) and part having a positive energy (and therefore remaining unbound). In the picture of Rees (1988), it is this energy distribution (and only this) that determines the light curve of the event. Indeed, after the encounter the fluid elements again move in Keplerian orbits (but with their new energy). The bound elements then come back close to pericentre after a Keplerian period $T$, linked to their (negative) energy $E$ by

$$E = -\frac{1}{2} \left( \frac{2\pi G M_p}{T} \right)^{2/3},$$

where $M_p$ is the black hole mass. The mass distribution with specific energy $dM/dE$ then translates, through equation (1), into a mass distribution of return times $dM/dT$. The next fundamental assumption is that once the bound material has come back to the pericentre it loses its energy and angular momentum on a time-scale much shorter than $T$, thus suddenly accreting on to the SMBH and giving rise to the flare. The mass distribution of return times is therefore effectively the accretion rate of the black hole during the event, from which the luminosity can be easily computed. We thus have

$$\frac{dM}{dT} = \frac{dM}{dE} \frac{dE}{dT} = \left( \frac{2\pi G M_p}{3} \right) \frac{dM}{dE} r^{-5/3}. \tag{2}$$

In order to obtain the ‘standard’ $r^{-5/3}$ light curve, we then have to make the second fundamental assumption that the energy distribution is uniform. Note that Rees (1988) did not show that this should be the case, and only assumed it for simplicity. Later, the numerical simulations by Evans & Kochanek (1989) apparently showed a uniform energy distribution, hence suggesting that the light curve is generally proportional to $r^{-5/3}$. In the following, we first show analytically that the energy distribution need not be uniform, but depends on the properties of the star, and in particular on its internal structure. We then show numerically that in fact it is not uniform and does depend on the properties of the stars, in a way that approximately reproduces the analytical results.

Starting from the pioneering work of Carter & Luminet (1982, 1983), numerical simulations of this process have been performed in a variety of studies (Bicknell & Gingold 1983; Evans & Kochanek 1989; Laguna, Miller & Zurek 1993a; Laguna et al. 1993b; Ayal, Livio & Piran 2000; Bogdanović et al. 2004). Most of these studies used SPH (Monaghan 1992), mostly because of its ability of following the system over a wide range of physical scales, where much of the simulated region is essentially ‘empty’. These various attempts have considered the effects of varying the orbital parameters of the encounter (Bicknell & Gingold 1983; Evans & Kochanek 1989), of the inclusion of relativistic terms in the equation of motion (Laguna et al. 1993a,b; Ayal et al. 2000) and have described the expected observational outcome (Bogdanović et al. 2004). However, surprisingly, no attempt has been made at exploring the effect of varying the internal structure of the star. Indeed, all such analyses have considered a simple polytropic model for the star, with $Y$ invariably set to $5/3$ (although Rosswog et al. 2008a,b have consider the encounter of a white dwarf with a stellar mass black hole, which is in the much smaller mass ratio regime).

The paper is organized as follows. In Section 2, we describe our analytical model to derive the energy distribution of the disrupted debris. In Section 3, we describe our numerical code and the set up of our simulations. In Section 4, we describe the results of the simulations. In Section 5, we discuss our results and draw our conclusions.

## 2 THE PROCESS OF TIDAL DISRUPTION OF A STAR BY A SMBH

A simple and instructive way to consider the process is by treating the interaction of the star with the black hole under the impulse approximation, that is assuming that the interaction occurs in a very small time-span as the star gets close to pericentre. This approximation is probably appropriate for highly hyperbolic encounters, but is only approximate for the parabolic case considered here. We therefore expect the details to differ somewhat from what is derived here below, even if the qualitative behaviour is correct. In this approximation, the motion of the star is simply a straight line until it reaches pericentre, at which point it is subject to a short impulse that deflects the various fluid elements, each of them individually conserving their specific energy. Until it reaches pericentre, therefore, the structure of the star is essentially unchanged and it keeps its original radial density profile and its initial radius. The key thing to realize here is that the spread of specific energy the star has reached just before the impulse is simply given by the different depths at which the various fluid elements are within the black hole potential well because they all share the same velocity and therefore the same kinetic energy. This was clearly realized by Lacy et al. (1982) and Evans & Kochanek (1989) who pointed out that ‘The spread in specific energy of the gas... is given by the change in the black hole potential across a stellar radius’. The impulse occurs instantaneously and therefore does not modify the kinetic energy of the fluid elements but imparts some degree of rotation in the star, with $\Omega \approx (2GM_p/R_p)^{1/2}$, where $R_p$ is the pericentre distance (cf. Evans & Kochanek 1989). We consider here the case in which the stellar radius $R_\star$ is much smaller than $R_p$, which is appropriate for the case of stellar disruption by a SMBHs (although not for the case of compact binaries). This allows us to easily estimate the expected energy spread, as

$$\Delta E = \left( \frac{dE}{dr} \right)_{r_\epsilon} \Delta r_{\text{max}} = \frac{GM_p}{R_p^2} R_\star, \tag{3}$$

where $E_p = GM_p/r$ is the potential energy due to the black hole and $\Delta r_{\text{max}} = R_\epsilon$ is the maximum deviation from the pericentre distance (cf. Lacy et al. 1982). We thus expect the energy distribution to extend roughly between $-\Delta E$ and $\Delta E$. This simple result has been obtained in all the early analyses of the problem. However, in the same approximation as before, we are also able to derive the whole energy distribution starting from the density, by calculating what is the fraction of stellar mass at a given $\Delta r$ from the centre. Fig. 1 illustrates the geometry. It can be easily shown that

$$\frac{dM}{d\Delta r} = 2\pi \int_{\Delta r}^{\infty} \rho(r) r^2 \, dr, \tag{4}$$

where $\rho(r)$ is the spherically symmetric mass density of the star. The relation between the distribution of $\Delta r$ and the distribution of energy $E$ is simply given by

$$\frac{dM}{d\Delta E} = \frac{R_\star}{\Delta E} \frac{dM}{d\Delta r} \Delta E, \tag{5}$$

where $\Delta E$ is given by equation (3).

It is useful to introduce dimensionless quantities. We then define $\epsilon = -E/\Delta E$ (where we have also included a minus sign because
we are interested in material with negative specific energy) as our dimensionless energy, \( x = \Delta r / R_\star \) as our radial coordinate within the star, \( x_p = R_p / R_\star \gg 1 \) as our dimensionless pericentre distance and \( m = M / M_\star \) as our dimensionless mass. We also introduce a fiducial time unit \( T_0 = 2 \pi (R_\star / G M_\star)^{1/2} \) and a dimensionless time \( t = T / T_0 \), as well as a fiducial density \( \rho_0 = M_\star / R_\star^3 \) and a dimensionless density \( \hat{\rho} = \rho / \rho_0 \). The outcome of the disruption event depends on the ‘penetration factor’ \( \beta = R_p / R_\star \), that is the ratio of the pericentre distance to the tidal radius \( R_t = q^{1/3} R_\star \), where \( q = M_h / M_\star \) is the mass ratio between the black hole and the star. In order to tidally disrupt the star, we require \( \beta \lesssim 1 \). For example, for a mass ratio \( q = 10^3 \), we have \( \beta = 1 \) for \( R_p = 100 R_\star \). To give an idea of the numbers involved, we note that for \( R_p = 100 R_\odot \), \( M_h = 10^6 M_\odot \), \( M_\star = 1 M_\odot \) and \( R_\star = R_\odot \), we have \( T_0 \approx 3.18 \times 10^4 \) yr \( \approx 0.11 \) d, while the unit for the accretion rate is \( M_\odot / T_0 \approx 3.1 \times 10^3 M_\odot \) yr\(^{-1}\). In these units, equations (1), (2), (4) and (5) become simply:

\[
\epsilon = \frac{x_p}{2}^{-2/3},
\]

while the unit for the accretion rate on to the black hole as a function of the internal energy is:

\[
\frac{dm}{d\tau} = \frac{x_p}{3} \frac{dm}{d\epsilon} \tau^{-5/3},
\]

\[
\frac{dm}{dx} = 2\pi \int_x^1 \hat{\rho}(x') x' dx',
\]

\[
\frac{dm}{d\epsilon} = \frac{dm}{dx}.
\]

The above simple set of equations therefore allows us to calculate the accretion rate on to the black hole as a function of the internal stellar structure. In general, we expect the density to show a peak at small radii \( x \) and therefore a peak at small specific energies \( \epsilon \). Since material at lower energies contributes to the accretion at later times, we can already predict what relative changes do we expect with respect to the standard \( t^{-5/3} \) light curve. In particular, we expect that if the star is more centrally condensed the flare should start with a relatively longer delay (less matter at large energies – small return time) and should have a shallower light curve (more matter at small energies – large return time). However, unless the density is strongly diverging at small radii, we expect \( dm/dx = dm/d\epsilon \) to flatten at the lowest energies and therefore the light curve to approach a \( t^{-5/3} \) profile at late times.

As an example, we can use the above analytical formulae to calculate the specific energy distribution and the accretion rate as a function of time predicted for some simple stellar models with known density profiles. We have thus considered simple polytropic spheres with different indices \( \gamma = 5/3, 1.4 \) and \( 4/3 \). We have first solved numerically the Lane–Emden equation for the three cases and have then computed the various relevant quantities using equations (6)–(9) above, assuming \( x_p = 100 \). The results are shown in Fig. 2.

The left-hand panel shows the prediction for the energy distribution, where the solid line indicates the relatively non-compact case \( \gamma = 5/3 \), the short-dashed line indicates \( \gamma = 1.4 \) and the long-dashed case shows the most compact case \( \gamma = 4/3 \). As can be seen, the energy distributions do extend up to \( \epsilon \sim 1 \), but are not flat except at very low energies, the effect becoming more pronounced for the more compact cases. The middle panel shows the predicted evolution of the mass accretion rate \( \dot{m} = dm/d\tau \), for the three values of \( \gamma \) with the same line styles as the left-hand panel. The red line shows for comparison a simple power law with index \( -5/3 \). It can be seen that indeed the light curves are slightly shallower that \( t^{-5/3} \) and

\[
\Delta r
\]

**Figure 1.** Schematic view of the geometry of the system. The radius of the star is \( R_\star \). The SMBH is in the right-hand side, at a distance \( R_p \gg R_\star \).

\[
\text{To BH}
\]

\[
\Delta r
\]

**Figure 2.** Left-hand side: distribution of internal energy for polytropic stars with different indices. Solid line: \( \gamma = 5/3 \), long-dashed line: \( \gamma = 1.4 \), short-dashed line: \( \gamma = 4/3 \). Centre: corresponding evolution of the accretion rate for the same three cases. The red line indicates for comparison a simple \( t^{-5/3} \) power law. Right-hand side: time evolution of the power-law index \( n = d \ln \dot{m} / d \ln t \). As can be seen the value \( n = -5/3 \) is only approached at late times.
approach it only at late times. This is even more evident in the right-hand panel, where we plot the power-law index $n = d\ln m/d\ln r$ for the three cases. If we want to put some numbers on the estimates above, note that for our standard numerical values described above a time of 1 yr corresponds to roughly $\tau \approx 3000$. We then see that the power-law index after 1 yr of the flare is $n \approx -1.5$ for $\gamma = 5/3$, which is reasonably close to the expected $-5/3$. However, such stellar model is probably unrealistic for a solar-type star, whose structure is rather more similar to a $\gamma = 4/3$ polytrope, in which case, after 1 yr of the flare the power-law index is still $n \approx -0.8$.

3 NUMERICAL SIMULATIONS

The model described in the previous section is only approximate in that it treats the interaction between the star and the black hole as instantaneous. In particular, the distribution of specific energy of the disrupted stellar material, and consequently the resultant light curve, has been computed by assuming that the stellar structure is essentially unchanged until it reaches pericentre. Still, it highlights some important features of the stellar disruption process: the expected energy distribution is in general not flat, and it tends to become progressively more peaked towards lower energies as the stellar structure model gets more centrally concentrated (i.e., as the polytropic index $\gamma$ becomes smaller). In order to gain a better understanding of the process, we have therefore compared the analytical expectations with the results of numerical hydrodynamical simulations of the process.

3.1 Numerical setup

In the case where the encounter is parabolic, as mentioned above, the two relevant dynamical parameters are the mass ratio between the star and the black hole, $q = M_\star / M_\bullet$, and the penetration factor $\beta = R_p / R_\star$. In this work, we have considered the case where $\beta = 1$ and $q = 10^4$, which imply that the pericentre distance is equal to 100 times the radius of the star.

Following the several investigations summarized in the Introduction, we have also used a non-relativistic SPH code to simulate the encounter. Our code uses individual particle time-steps (Bate, Bonnell & Price 1995), it evolves the smoothing length by keeping a fixed mass within a smoothing sphere (equivalent to roughly 60 particles) and includes the relevant terms needed to ensure energy conservation when the smoothing length is variable (see Price 2005 for a recent review). We also adopt a standard SPH artificial viscosity (Monaghan 1992) with viscosity parameters $\alpha_{\text{sph}} = 1$ and $\beta_{\text{sph}} = 2$.

In order to describe the basic dynamics of the encounter, we do not require to use an extremely large number of particles in order to reach a satisfactory resolution. Indeed, Evans & Kochanek (1989) have shown that their results were numerically converged with a number of particles $N$ equal to a few $10^4$. Even recent calculations have only used a relatively small number of particles, of the order of $10^7$ (Ayal et al. 2000) up to $2 \times 10^4$ (Bogdanović et al. 2004). In this work, we have run all our simulations at the two resolution of $N = 10^4$ and $10^5$ and have noticed no appreciable difference in the results, thereby confirming the numerical convergence of the results. In the following, we only show the higher resolution results.

We initialize our simulations by placing the SPH particles to form the structure of a polytropic star of given index $\gamma$ (we have considered the four cases $\gamma = 1.4, 1.5, 5/3$ and 1.8). This is done by initially placing the particles using close sphere packing and then differentially stretching their radial position to achieve the desired density profile. This method minimizes the statistical noise associated with random placing of the particles (we thank Walter Denhen for providing this setup routine). We then relax the structure of the star by evolving it in isolation until its internal properties settle down.

We have considered four different values of $\gamma = 1.4, 1.5, 5/3$ and 1.8. In this way, we encompass the expected range for different kinds of stars, from radiative to convective ones. Indeed, a solar-type star has a density profile close to a $\gamma = 4/3$ polytrope (it is actually best described by $\gamma \approx 1.3$). Unfortunately, a $\gamma = 4/3$ polytrope is difficult to simulate, as it has zero binding energy. The lowest value of $\gamma$ that we use is then 1.4. Red giants and low-mass stars can be described by a $\gamma = 5/3$ polytrope, while neutron stars have a structure which is probably closer to a $\gamma = 2.5$ polytrope.

We plot in Fig. 3 the initial density profile of our four models as predicted from the solution of the Lane–Emden equation (left-hand panel) and as realized after the initial conditions have been allowed to relax (right-hand side). As can be seen, the four models differ in their central concentration, such that the $\gamma = 1.4$ model is the most concentrated and the $\gamma = 1.8$ is the least. It might be worth also to recall that models with larger $\gamma$ are less compressible than models with lower $\gamma$.

Finally, we introduce the black hole as a point mass at the origin and we displace the star so as to place its centre of mass on the required parabolic orbit (since the star is an extended object, this actually means that the total mechanical energy of the star is slightly negative, amounting to roughly $-0.005$ in our units). The initial distance from the black hole is 3 times the pericentre distance (in other simulations not described here, we have also used a larger initial distance and found no significant difference). Our code units are $R_\bullet$ for length and $M_\star$ for mass, which ensure that our results are described in the same dimensionless variables as described in Section 2. The black hole is modelled as a sink on to which SPH particles can be accreted if they come closer to the black hole that a distance 0.25 in code units. However, in practice, given that our pericentre is very large and that we do not follow the evolution of the debris long after the interaction, no particles are actually accreted during the course of our simulations.

4 RESULTS

4.1 The $\gamma = 5/3$ case

Before comparing the results obtained with various polytropic indices, we start by describing the results that we have obtained in the $\gamma = 5/3$, which is directly comparable to the simulations discussed in previous papers. In particular, this simulation is essentially a higher resolution version of the one initially discussed in Evans & Kochanek (1989).

Two snapshots of the projected density of the star are shown in the lower left-hand panels of Figs 4(a) and (b), at two different times, that is when the star is at pericentre and when it is at roughly 2 times the pericentre distance, after the encounter. The overall structure of the star looks qualitatively similar to the one shown in Evans & Kochanek (1989). It is interesting to notice that at pericentre the star is already quite distorted with respect to its initial configuration and in particular it has expanded somewhat (recall that its initial radius is 1 in code units). This occurs because, in isolation, the star is in hydrostatic equilibrium between its pressure and its self-gravity. The star approaches the black hole, the tidal field effectively acts as to reduce the stellar gravity, making pressure forces unbalanced and therefore ‘inflating’ the star. This effect is expected to be more
Figure 3. Radial density profiles for the four models considered here. In the left-hand panel, we show the density of four solutions of the Lane–Emden equation with (from the highest to the lowest central density) $\gamma = 1.4, 1.5, 5/3$ and $1.8$. In the right-hand panel, we show the corresponding SPH density estimates for the initial conditions of our simulations.

Figure 4. (a) Projected density of the of the star at pericentre. The black hole is outside the image, at the origin of the coordinate system. The four panels refer to different values of $\gamma = 1.4$ (upper left-hand side), 1.5 (upper right-hand side), 5/3 (lower left-hand side) and 1.8 (lower right-hand side). (b) Same as panel (a), but after the encounter, when the star is located at roughly 2 times the pericentre distance.

significant for small than for large $\gamma$. This reflects the fact that the radius of a polytrope with small $\gamma$ is more sensitive to the effective gravity.

A more quantitative comparison can be done by looking at the distribution of specific energies of the disrupted star. This is shown in Fig. 5 at four different times during the simulations: at $t = 0$ (upper left-hand panel), at pericentre (upper right-hand side), and after the encounter, when the star is roughly at 4 times the pericentre distance (lower left-hand side) and 10 times the pericentre distance (lower right-hand side). For ease of comparison with Evans & Kochanek (1989), only for this plot we have used a logarithmic scale for the distribution. It can be seen that initially the distribution is very narrow and centred at $\epsilon = 0$, which just reflect the fact that the whole star is initially on a parabolic orbit. As the star approaches the black hole, the distribution becomes wider and indeed approaches the width predicted by the simple analysis of Section 2 (which is equal to unity in the units adopted here). The lower left-hand panel, in particular, showing the distribution at 4 times the pericentre, compares almost exactly with the distribution shown by Evans & Kochanek (1989) (their fig. 3), confirming that indeed our simulations replicate accurately their results. However, one can see that the density distribution keeps evolving until the star is at roughly 10 pericentre distances, where it finally settles down in the configuration shown in the lower right-hand panel of Fig. 5. We thus see that the distribution is characterized by a central peak at lower energies, followed by two ‘wings’ at larger energies. The presence of a central peak is expected based on the analytical model described above. The wings, on the other hand, refer to the stellar material at the surface of the star, which at pericentre is somewhat distorted from its initially spherical shape (as can be
profile does approximately match the shape of the distribution at the peak, except for the presence of the wings, indicating the presence of more material at extreme energies than predicted by the model. Note that, obviously, the light curve produced in this case does not show the standard $r^{-5/3}$ decline, especially at early times. More details on the resultant light curves are given in the next section, where we compare the results obtained with different values of the polytropic index.

4.2 Varying the polytropic index

We now discuss the effects of varying the polytropic index $\gamma$ on the structure of the disrupted star. A first comparison can be obtained by looking at Fig. 4, where the various panels show the projected density of the star at pericentre (left-hand side) and at 2 times the pericentre (right-hand side) for the four cases considered (from top left-hand side to bottom right-hand side: $\gamma = 1.4$, 1.5, 5/3 and 1.8). Several interesting differences can be already seen from these images. First of all, note that the overall expansion of the star is similar in all cases. However, for larger values of $\gamma$ the density structure of the star is much more uniform. This is particularly evident in the right-hand panel, which refers to well after pericentre passage. In the case where $\gamma = 1.4$ the high-density core is compact, with the density in the ‘puffy’ tidal tails gently declining. In contrast, at the opposite extreme of $\gamma = 1.8$ the high-density region is more extended and the edge of the tidal tails is more clearly defined, revealing a sharper density cut-off at the edge. It is also interesting to note the different degree of internal rotation induced in the star by the tidal interaction, with the elongated core being more aligned with the line joining the star and the black hole (at the origin of the coordinate system) for smaller $\gamma$ than for larger ones.

Let us now look at the distribution of specific energies within the star for the four different simulations. This is shown in Fig. 7, where the solid lines refer to the simulations, averaged over 10 time units when the star has reached a distance of roughly 20 times the pericentre. Note that in each simulation, as mentioned above, the stars are somewhat inflated once they reach pericentre. In order to compare the numerical results with the analytical predictions, we therefore have to take into account this expansion. We have thus simply taken the initial equilibrium density as a function of radius within the star and rescaled the radius by a constant factor $\xi$, thus effectively applying a homologous expansion to the stellar structure. We have then calculated the expected energy distribution based on equations (8) and (9) for this ‘inflated’ profile. The resulting analytical predictions are then shown in Fig. 7 with a dashed line. The expansion factor to match the numerical data is $\xi = 2.5$, 2.1, 1.63 and 1.6 for the four cases $\gamma = 1.4$, 1.5, 5/3 and 1.8, respectively. It is interesting to see that this expansion parameter decreases as $\gamma$ increases, reflecting the reduced response to variations in the gravity field as $\gamma$ gets larger. It can be seen that for $\gamma = 1.4$ our inflated polytropic model describes very accurately the outcome of the simulation. However, as $\gamma$ increases the results of the simulations start to deviate from the model, in particular in the appearance of wings in the tail of the distribution. These wings become progressively more prominent as $\gamma$ gets larger. In the previous section, we have already shown that the core of the distribution for $\gamma = 5/3$ is well described by a non-inflated model. Essentially, what is happening here is that as $\gamma$ increases the expansion of the star becomes progressively less homologous, with more material being pushed to higher energies (in absolute value). Since the expansion velocity is significantly supersonic, the only way to transfer energy within the star is through shocks, occurring in the tidal tails. As $\gamma$ increases,
the density profile of the star becomes shallower, and more material undergoes shocks in the outer layers of the star, hence increasing the appearance of the wings. We estimate quantitatively the importance of shocks in our simulations in the following way. For each particle \( i \) in our simulations we compute the quantity

\[
q_i = \begin{cases} 
\frac{h_i (\nabla \cdot \mathbf{u})}{c_{s,i}} & \text{when } (\nabla \cdot \mathbf{u}) < 0 \\
0 & \text{otherwise,}
\end{cases}
\]

(10)

where \( h_i \) is the particle’s smoothing length, \( (\nabla \cdot \mathbf{u}) \) is the local divergence of flow velocity and \( c_{s,i} \) is the local sound speed. The quantity \( q \) is therefore non-zero and negative in regions of convergent flow and shocks occur where \( |q| \geq 1 \). Fig. 8 shows the structure of the disrupted star for the \( \gamma = 5/3 \) case at \( t = 16.75 \). The top panel shows the projected density, while the bottom panel shows a vertical cross-section of \( q \). It can be seen that most of the disrupted star is expanding, except for the tip of the tidal tails, where there is a strong convergent flow, which has indeed \( |q| > 1 \) and therefore undergoes a shock.

To see how does the effect of shocks change as the polytropic index is varied, we also compute the quantity \( \delta m_{\text{shock}} \), that we define as the total mass of particles that have \( |q| > 1 \). Fig. 9 shows the time evolution of \( \delta m_{\text{shock}} \) for the four simulations with \( \gamma = 1.4 \) (solid line), \( \gamma = 1.5 \) (short-dashed line), \( \gamma = 5/3 \) (long-dashed line) and \( \gamma = 1.8 \) (dot-dashed line). This plot shows a few interesting features.

First, we see that as the index \( \gamma \) increases, the amount of shocked mass increases as well, confirming our expectation that more mass is involved with the shocks in the tidal tails. In particular, the two simulations with the largest \( \gamma \), which are the ones displaying the more pronounced “wings” in the energy distribution, are also the two in which more mass undergoes shocks. Secondly, we see that shocks appear to occur in a sequence of peaks. The first one, common to all simulations, occurs at \( t \approx 4 \), which corresponds to pericentre passage. For the largest values of \( \gamma \), we then see a number of other peaks, which can be interpreted as the manifestation of strongly non-linear stellar pulsations induced by the tidal interaction (see also Ivanov & Novikov 2001). The period of these oscillation decreases with increasing \( \gamma \), consistent with the expectation that the period of the fundamental mode of stellar pulsations should vary as \((3\gamma - 4)^{-1/2}\) (e.g. Cox 1980).

As mentioned in Section 2, the fact that for larger \( \gamma \) the energy distribution becomes relatively flatter implies that the distribution of return times should become steeper and more rapidly approach the \( r^{-5/3} \) profile expected for an exactly flat distribution. The resultant accretion rate for the four simulations is shown in Fig. 10 (left-hand panel), where the solid line refers to \( \gamma = 1.4 \), the short-dashed line to \( \gamma = 1.5 \), the dot-dashed line to \( \gamma = 5/3 \) and the long-dashed line to \( \gamma = 1.8 \). To give an idea of the numbers involved we have plotted the results in physical units, assuming \( M_\star = 1 \, M_\odot \) and \( R_\star = R_\odot \) (note that if the disrupted star is a giant, the time...
where we plot the instantaneous power-law index \( n \) (i.e., the logarithmic time derivative of the accretion rate) associated with the light curve for the cases \( \gamma = 1.4 \) (squares) and \( \gamma = 5/3 \) (triangles) as a function of magnitude drop from the peak (we only plot these two cases for simplicity: the two other cases follow essentially the same behaviour). The dashed line at the bottom indicates \( n = -5/3 \). This plot illustrates quite clearly that the \( t^{-5/3} \) regime is only approached at late times, after the luminosity has dropped \( \sim 2 \) mag from the peak for \( \gamma = 1.4 \). In the case \( \gamma = 5/3 \), the asymptotic regime is approached more quickly, after only a luminosity drop of approximately 1 mag.

### 5 Discussion and Conclusions

Candidate tidal disruption events of a star by a dormant black hole are usually associated with luminous flares in the nucleus of an otherwise normal galaxy. These can be detected in X-rays, for example with *Chandra* and *ROSAT* (Halpern, Gezari & Komossa 2004) or with *XMM–Newton* (Esquej et al. 2008) or in the optical/UV (Gezari et al. 2008). X-ray data generally observe the flare evolving down from the peak by a few orders of magnitude, and in the best studied case, NGC 5905, the decline appears to be consistent with a \( t^{-5/3} \) fall-off (Halpern et al. 2004). However, in some other cases (Gezari et al. 2008), the observations only span a relatively small drop in luminosity from the peak. In these cases, the light curve appears to be shallower than \( t^{-5/3} \), and the best fit of Gezari et al. (2008) to their UV data indicates a value of \( n \approx -1.1 \) in one case and \( n \approx -0.82 \) in another. These results are consistent with our prediction that initially the light curve should be shallow, approaching a \( t^{-5/3} \) profile only after the luminosity has dropped by 2–3 mag from the peak.

To summarize, in this paper, we have revisited the arguments at the basis of the expected light curve produced by the tidal disruption of a star in a parabolic orbit close to a SMBHs. The \( t^{-5/3} \) profile originally proposed by Rees (1988) and Phinney (1989) only holds in the case where the energy distribution \( dm/d\epsilon \) of the remnant is flat, which we have shown is not the case, in general. We have proposed a simple analytical model that relates the resultant energy distribution to the density structure of the star. This model predicts that more centrally concentrated (solar-type) stars should produce flares with a light curve shallower than \( t^{-5/3} \), approaching it only at late stages. We have tested the model with numerical simulations and found that it does reproduce the simulated behaviour, with the following two corrections. First, we have to account for the inflation of the star from its initial structure due to the effective reduction of gravity as it moves in the tidal field of the black hole. This is well described by a homologous expansion by a factor which becomes smaller as the polytropic index becomes larger. Secondly, for large polytropic indices, we see the appearance of wings in the tails of the energy distribution, indicating that some material has been put further away from parabolic orbits as a result of shocks in the tidal tails.

In all cases, we do not obtain a \( t^{-5/3} \) light curve, except at late times. Close to the peak of the luminosity, the light curve is very sensitive to the structure of the star, being shallower for stars with polytropic index close to 4/3, expected for solar-type stars. In this case, the \( t^{-5/3} \) profile is reached only after the luminosity has dropped by at least 2 mag. For stars with a relatively flat density profile, such as red giants and low-mass stars, the \( t^{-5/3} \) profiles is reached earlier.

In this paper, we have only investigated a very simple setup, with a given mass ratio between the star and the SMBH, and one given set of orbital parameters. It is expected that the results would be...
Figure 10. Left-hand side: accretion rate as a function of time for the four simulations. Solid line: $\gamma = 1.4$ (internal structure close to a solar-type star); short-dashed line: $\gamma = 1.5$; dot-dashed line: $\gamma = 5/3$ (internal structure appropriate for low-mass, fully convective stars); long-dashed line: $\gamma = 1.8$. The physical units refer to the case where the disrupted star has $M_\star = M_\odot$ and $R_\star = R_\odot$ (note that if the disrupted star is a giant, the time unit is increased by a factor $(R_\text{giant}/R_\odot)^{3/2}$, which can be several hundreds, then suggesting that the rise to peak might be observable, and the decay time prior to reaching the asymptotic $t^{-5/3}$ behaviour can be very long). Right-hand side: instantaneous power-law index $n = d \log \dot{M}/d \log t$ for $\gamma = 1.4$ (squares) and $\gamma = 5/3$ (triangles) as a function of magnitude drop from the peak. The dashed line at the bottom indicates the commonly-invoked power-law index $n = -5/3$ for the light curve.

Further dependent on the such additional parameters, such as the ratio of tidal radius to pericentre distance and the eccentricity of the orbit. We plan to consider these effects in subsequent investigations.

Finally, it should be further emphasized that all these results refer essentially to the return time of the disrupted debris, and only correspond to an actual luminosity under the further assumption that the subsequent accretion is perfectly efficient and occurs on a much shorter time-scale, which may not be the case (see Ayal et al. 2000).

ACKNOWLEDGMENTS

We thank Walter Dehnen for providing us with his setup routine for polytropic spheres. We acknowledge several interesting discussions with Walter Dehnen, Mark Wilkinson, Sergei Nayakshin and Paul O’Brien. We also thank the Referee, Stephan Rosswog, for an insightful report. All the visualization of SPH simulations have been obtained using the SPLASH visualization tool by Dan Price (Price 2007).

REFERENCES

Ayal S., Livio M., Piran T., 2000, ApJ, 545, 772
Bate M. R., Bonnell I. A., Price N. M., 1995, MNRAS, 277, 362
Bicknell G. V., Gingold R. A., 1983, ApJ, 273, 749
Bogdanović T., Eracleous M., Mahadevan S., Sigurdsson S., Laguna P., 2004, ApJ, 610, 707
Carter B., Luminet J. P., 1982, Nat, 296, 211
Carter B., Luminet J.-P., 1983, A&A, 121, 97
Cox J. P., 1980, Theory of Stellar Pulsation. Princeton Univ. Press, Princeton
Esquej P. et al., 2008, A&A, 489, 543
Evans C. R., Kochanek C. S., 1989, ApJ, 346, L13
Gezari S. et al., 2008, ApJ, 676, 944
Halpern J. P., Gezari S., Komossa S., 2004, ApJ, 604, 572
Ivanov P. B., Novikov I. D., 2001, ApJ, 549, 467
Komossa S., Bade N., 1999, A&A, 343, 775
Lacy J. H., Townes C. H., Hollenbach D. J., 1982, ApJ, 262, 120
Laguna P., Miller W. A., Zurek W. H., 1993a, ApJ, 404, 678
Laguna P., Miller W. A., Zurek W. H., Davies M. B., 1993b, ApJ, 410, L83
Monaghan J. J., 1992, ARA&A, 30, 543
Phinney E. S., 1989, in Morris M., ed., The Center of the Galaxy. IAU Symp., Vol. 136, Manifestations of a Massive Black Hole in the Galactic Center. Kluwer, Dordrecht, p. 543
Price D., 2005, PhD Thesis, preprint (ArXiv Astrophysics e-prints, 0507472)
Price D. J., 2007, Publ. Astron. Soc. Aust., 24, 159
Rees M. J., 1988, Nat, 333, 523
Rosswog S., Ramirez-Ruiz E., Hix W. R., 2008, ApJ, 679, 1385
Rosswog S., Ramirez-Ruiz E., Hix W. R., Dan M., 2008, Comput. Phys. Commun., 179, 184

This paper has been typeset from a T\textsc{e}X/\La\textsc{t}e\textsc{x} file prepared by the author.