New resonances along with cyclotron resonance in heterostructures: A new radiation.

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The usual cyclotron resonance occurs at \( \omega_c = eB/m^*c \). The new resonances occur at \( \omega_{c\pm} = \frac{1}{2}g_\pm eB/m^*c \) where \( \frac{1}{2}g_\pm = (l + \frac{1}{2} \pm s)/(2l + 1) \). The energy in the centre of two resonance frequencies varies as the square root of the two-dimensional density of the electrons due to spin access in the Gaussian model. The frequencies \( \omega_{c\pm} \) are linearly proportional to the magnetic field except near crossing point where the linear combination of wave functions must be made, i.e., \( \frac{1}{\sqrt{2}}(|n, l, \uparrow> \pm |n, l, \downarrow>) \).

PACS numbers: 76.40.+b, 73.20.Mf

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1. Introduction

When an external magnetic field is applied to the electrons they go into cyclotron orbits. When energy is swept, there is a resonance at \( \omega_c = eB/m^*c \). Since \( e \) and \( c \) are already known and \( B \) can be measured accurately, a measurement of \( \omega_c \) leads to a measurement of the effective mass of the electron. Here \( \omega_c \) is called the cyclotron frequency, \( e \) the charge of the electron, \( c \) the velocity of light, \( m^* \) the effective mass of the electron and \( B \) the external magnetic field[1].

In this paper, we propose that there must exist new resonances at \( \omega_{c+} = \frac{1}{2}g_+ eB/m^*c \) and at \( \omega_{c-} = \frac{1}{2}g_- eB/m^*c \). The \( g_\pm \) are related by Kramers time reversed states and the energy at the centre of these states is proportional to the square root of the two-dimensional number density of electrons. We have found[2-7] the factor before the charge during our study of the quantum Hall effect where it is used to describe the effective fractional charge of the quasiparticles. It has recently been noted that the fractional charge which were not understood in the beginning are due to electron clusters where spin 1/2 is not sufficient. In these clusters the spin may be larger than 1/2 such as 3/2, 5/2, 7/2, etc. The repulsive Coulomb interactions align the electron spins ferromagnetically so that the spin of a cluster depends on the number of electrons[5]. Since usually a large magnetic field is present, the spins align parallel to the field although some may also be directed opposite to the magnetic field. So the electrons align even though there is no ferromagnetism. Our report of the new resonances makes use of the experimental measurements carried out by Syed et al[6].

2. Theory.
The cyclotron resonance consists of a single resonance line at,

\[ \omega_c = \frac{eB}{m^*c}. \] (1)

We predict new resonance lines at,

\[ \omega_{c+} = \frac{1}{2}g_+ \frac{eB}{m^*c} \] (2)

and at,

\[ \omega_{c-} = \frac{1}{2}g_- \frac{eB}{m^*c} \] (3)

where

\[ \frac{1}{2}g_\pm = \frac{l + \frac{1}{2} \pm s}{2l + 1} \] (4)

as described in ref. 2. The spin is not restricted to \( s = 1/2 \) only. When there are electron clusters, it may be larger value also. The \( s = +1/2 \) is the Kramers time reversed state of \( s = -1/2 \). Therefore, \( \frac{1}{2}g_\pm \) is having two values. When we reverse one spin from the \( N_\uparrow \) state and put it in the \( N_\downarrow \) state, the spin of the system changes by \( 2s \). This is a textbook problem which shows that,

\[ \frac{N_\uparrow - N_\downarrow}{2} = s \text{ proportional to } N^{1/2} \] (5)

as given by Kittel and Kroemer[7] for ordinary Gaussian distribution. Since, the energy in the centre of two Kramers conjugate states will be proportional to \( N_\uparrow - N_\downarrow = 2s \), we expect that it varies as the square root of 2-dimensional electron density. Thus we have two new resonances at \( \omega_{c+} \) and \( \omega_{c-} \) with Kramers symmetry and Gaussian distribution for the central energy.

2. Analysis of data

We will show that all of the above discussed properties can be extracted from the experimental work of Syed et al[6] and the new resonances at \( \omega_{c\pm} \) can be identified from the data. The far infrared transmission data of a two-dimensional electron gas (2DEG) of density \( 1.14 \times 10^{12} cm^{-2} \) in AlGaN/GaN at 12.5 T shows a strong resonance at 6.9 meV and a weaker one to 5.1 meV. We assign 6.9 meV to \( g_+ \mu_B 12.5 \times 10^4 \) and 5.1 meV to \( g_- \mu_B 12.5 \times 10^4 \). The value of \( g_+ \) is obtained as follows.

\[ g_+ = 9.274 \times 10^{-21} \times 12.5 \times 10^4 = 6.9 \times 10^{-3} \times 1.602 \times 10^{-12} \] (6)

where the value of the Bohr magneton is \( \mu_B = 9.274 \times 10^{-21} \text{ erg/Gauss} \) and the magnetic field is \( 12.5 \times 10^4 \text{ Gauss} \). The resonance occurs at \( 6.9 \times 10^{-3} \text{ eV} \) and we multiply it by \( 1.602 \times 10^{-12} \) to obtain erg units. This gives,

\[ g_+ = 9.5353 \] (7)
Similarly using the resonance at 5.1 meV, we obtain,

\[ g_- = 7.047 \]  

(8)

From the above two values we obtain

\[ \frac{1}{2} g_+ + \frac{1}{2} g_- = 0.5749 \]  

(9)

and

\[ \frac{1}{2} g_+ - \frac{1}{2} g_- = 0.4250 \]  

(10)

The sum of the above two numbers is 0.9999. According to one of our theorems \( \nu_+ + \nu_- = 1 \). Therefore 0.9999 is just what we expected. Therefore the interpretation of resonances at 6.9 meV and at 5.1 meV in terms of \( g_+ \) and \( g_- \) is correct. Thus the new radiation at \( \omega_{c+} \) and at \( \omega_{c-} \) is discovered. It shows that the usual cyclotron resonance occurring at \( \omega_c \) is flanked by two new resonances at \( \omega_{c\pm} \). Sometimes, the prefactors may be zero or one, in which case \( \omega_{c\pm} \) will occur in such a way that \( \omega_c \) will not occur.

The spin of \( g_+ \) is + and the spin of \( g_- \) is -, so when one spin is removed from \( N_\uparrow \) and put in \( N_\downarrow \), the spin excess is 2s. For Gaussian distribution, the centre of two new resonances varies as the square root of the number density of two-dimensional electrons. Indeed, the variation of this point is already plotted in ref.6 and the measured value agrees with the predicted square root of the number density.

The resonance condition varies linearly with magnetic field. However, there is a crossing point or the centre of the two lines at \( \omega_{c\pm} \). The energy levels at \( \frac{1}{2} g_- \omega_c(n + \frac{1}{2}) \) are narrowly spaced. When the magnetic field is increased, these narrowly spaced levels separate out until their separation can become equal to those of \( \frac{1}{2} g_+ \omega_c(n + \frac{1}{2}) \). Thus there is a crossing point. The states are characterized by \( l \) and \( s \) and the Landau level number \( n \). Thus the states are of the form \( |n, l, \uparrow> \) and \( |n, l, \downarrow> \). Near the crossing point, the states get mixed so that the proper way of writing the wave function becomes,

\[ \frac{1}{\sqrt{2}} (|n, l, \uparrow> \pm |n', l', \downarrow>) \]  

(11)

This is the reason why energy as a function of magnetic field bends near the crossing point. The resonances above 6 meV are \( g_+ \) type and below 6 meV are \( g_- \) type. When energy is plotted as a function of magnetic field, bending occurs just as predicted. The prediction of new radiation at \( \omega_{c\pm} \) is thus confirmed by the experiments.

3. Conclusions.

We predict new resonances at \( \omega_{c\pm} \). Their frequency locks the spin as given in the expressions. The centre of these lines varies as the square root of the number density of two-dimensional electron gas. The energy bends near the central point due to mixing of states. This represents fundamentally different resonance than the cyclotron resonance, known since 1953, because the value of the spin enters in the frequency whereas the frequency of the cyclotron resonance is independent of the same. The cyclotron resonance
is only one frequency, whereas many different values can be detected in the new resonances because of the many different values of \( l \) and \( s \). Some of the details of the new resonance frequencies can be derived from the results given in ref.2.

4. References

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