Grapho-analytical modelling of technological chain of logging operations in dynamic natural and production conditions

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Abstract. Different variants of technological chain can be organized in logging process. Water or land types of transport can be used for transportation of timber. The use of timber landings where wood can be processed also increases the number of multiple variants of technological process. Performance of operations in various natural and production environment differs in productivity and material costs. Production efficiency depends on the choice of technology of logging operations in dynamic natural and production environment. The given research suggests the variant of the grapho-analytical model for solution of the set task. Detailed graphic models of transportation of wood from logging unit to consumer, loading/unloading and processing operations at the intermediate and lower timber landing are developed. They show possible options of technological chains of logging process. Mathematical dependences are shown on the basis of the problem of finding the maximum flow of minimum cost in the suggested dynamic network. They define the conditions for solution of the set problem.

1. Introduction

In recent years, the removal of wood from logging units to consumer is characterized by multiple variants of technological process that involves timber landings. Most of them are used to store wood between transportation seasons [1].

Transshipment landings are organized on the borders between cheaper and more expensive transport means. Intermediate landings on the borders between temporary and year-round forest roads are organized to increase traffic capacity of seasonal roads and to use the existing logging truck fleet all year around. Lower shore landings and railway landings are organized for transshipment from road transport to water or rail transport. Landings can be reloading and cutting when wood-processing operations are performed there. Wood in the form of trees and full-length logs is delivered there. But limbing and bucking are performed at the landing.

The applicable technological chain, the machinery and equipment system, the technical, technological and transport accessibility of forest resources depend on the availability and location of transport, landing and processing operations [2].

It is necessary to choose the technological chain of logging operations carefully in order to provide the effective use of temporary forest roads with small traffic capacities [3-5].
It follows from the above that the following engineering tasks with various solutions appear when transportation of wood from logging unit to consumer is being organized: order of wood removal from various logging units; applicability and non-applicability of landings; choice of transport (road, water or railway); choice of technological chain of transport, landing and processing operations; choice of season for performance of certain technological operations; choice of saleable products for consumers to be purchased; choice of consumer.

2. Results and discussion
Considering variety of existing options of technological process of transporting and storing operations it is possible to show them in a form of graph. Figures 1-2 shows the graph that characterizes the technological process of transporting of timber from logging unit to consumer(s) including performance of all loading/unloading and rafting operations. Unloading/loading and wood processing operations at timber landings or booming grounds are shown in Figures 3-4.

![Graphical model of transportation of timber from logging unit to consumer](image_url)

**Figure 1.** Graphical model of transportation of timber from logging unit to consumer (left part).
Figure 2. Graphical model of transportation of timber from logging unit to consumer (right part).

This graphic model allows to implement analytical approach to justification of the technological chain of logging operations of timber landings, booming grounds for loading/unloading operations, processing operations, types of transport (water, land), choice of consumer and type of finished products in dynamic natural and production environmental conditions.

The “time-stretched” dynamic graph [6] that is created through formation of a separate copy of each vertex in each considered period $\theta \in T$ is shown. The number of periods during the analyzed time interval may be different and it depends on volume and quality of the initial information and required accuracy of the obtained results. Therefore, the set of vertices $X_p$ of graph $G_p$ is defined as $X_p = \{(x_i, \theta) : (x_i, \theta) \in X \times T\}$. The set of arcs $\tilde{A}_p$ is shown with arcs emanating from each vertex-time pair $(x_i, \theta) \in X_p$ into each vertex-time pair $(x_j, \theta)$ and $(x_j, \theta + t_{ij}(\theta))$. At the same time, $x_j \in \Gamma(x_i)$, and $\theta + t_{ij}(\theta) \leq p$. Traffic capacities $\tilde{\nu}(x_i, x_j, \theta, \theta + t_{ij}(\theta))$ that connect the vertex-time pairs $(x_i, \theta)$ and $(x_j, \theta + t_{ij}(\theta))$ are equal to $\infty$, and traffic capacity $\tilde{\nu}(x_i, x_j, \theta, \theta)$ that connect
vertex-time pairs \((x_j, \theta)\) and \((x_j, \theta)\) are equal to \(\tilde{V}_{ij}(\theta)\) and can be calculated through analysis of the labor costs \(\tilde{f}_{ij}(\theta)\) (that indicated on the graphs) for performance of individual operations.

In Figures 1-2, the vertex S is a fictitious source and the vertex T is a fictitious sink. Arcs enters to graphs \(L_{\theta} g\) (the logging units) from the fictitious source. These arcs characterize traffic capacity and they are determined by the volume of timber removal from the corresponding logging unit \(V_{\theta}\). Each analyzed operation of the technological process is represented in the graph by intermediate vertices that are located between the graphs that denote logging unit \(L\) and consumers \(U\). Arcs enter the vertices \(M_{\theta0} N\) from the vertices \(L_{\theta} g\).

Here \(M\) is the name of the object of labor during execution of operation (trees (T), full-length logs (S), assortments (L)); \(h\) is the stage of transportation; \(\theta\) is the number of the analyzed period; \(N\) is the number of the analyzed logging unit. So, for example, the vertices \(T111, ..., T1p1\) characterize the operations of transportation of trees performed at the first stage in each of \(p\) periods of the first analyzed logging unit. The number of logging unit is not indicated in the designation of vertex in the subsequent transportation stages as it does not impact the characteristic of an operation. For example, \(t21, ..., t2p\) characterize the operation of assortment transportation at the second stage (after the intermediate landing) in each of \(p\) periods. At the third stage, after the lower landing, the designation \(W\) may appear in the vertex characteristic and it represents transportation by water or railway. Arcs of the given graph that enter the vertex represent the loading operation. Arcs outgoing from vertex represent timber transportation. These arcs characterize flow traffic capacity and they are determined by the labor costs \(f\) for performance of an operation. Arcs also characterize variable \(C\) and constant \(Z\) costs. Constant costs are taken into account only if timber is stored between periods. Arcs outgoing from the logging units have the characteristic \(V_{\theta0} M_{\theta} N\) that denote the volume of the analyzed object of labor in a certain period \(\theta\) from the logging unit \(N\). Arcs entering vertices of consumers have characteristic \(Q_{AM}\) that represent the maximum output \(M\) which can be purchased by consumer \(A\). Examples of arc designation: arc between the vertex \(L_{\theta} g\) and the vertex \(T11g\), the arc \(fT11g; CT11g; ZT11g\) represents the flow out of logging unit \(g\) with labor costs, variable and constant costs for loading of trees in the \(g\)th logging unit in the first period for transportation in the first stage; the arc \(fS31W, Uk; CS31W, Uk; Qks\) characterizes the flow with labor costs and variable costs for assortment transportation in the third stage (after the lower landing) in the first period by water (railway) transport to the \(k\)th consumer with a limited amount of wood to be purchased in the form of assortments.

The background on the graphs defines the field of operations with a certain subject of labor: diagonal hatch denotes tree processing operations, vertical hatch denotes full-length log processing operations, and horizontal hatch denotes assortment processing operations.

Figure 3 shows an area of the graph for loading/unloading and processing operations that are performed at the intermediate forest landing (1st terminal).

The graph (Figure 3) shows the vertices that characterize four types of operations: unloading of timber (P), limbing (D), bucking (B) and loading to timber trucks (R). The arcs entering the intermediate landing represent the transportation of wood with certain labor costs and variable costs from different logging units. Arcs enter the vertices \(XM0N\). Here \(X\) characterizes the type of the analyzed operation. Vertices that characterize the processing operations (limbing and bucking) are indicated by two symbols \(X\theta\). The vertices that characterize the loading operation are indicated by three symbols \(X\theta0\). Notations of arcs in this graph are similar to notations in the graph for wood transportation from logging unit to consumer.

Figure 4 shows the graph area for loading/unloading and processing operations that are carried out at the shore or railway forest landing (2nd terminal).

The given graph (Figure 4) shows the notation of graph vertices and arcs that connect them similarly to the graph for loading/unloading and processing operations of intermediate landing. Flows that are involved in the graph arrive from both the logging unit and the intermediate landing (1st
terminal). The lower landing has additional vertices for loading to alternative transportation means (water and railway). Vertices and arcs of such operations are denoted with the additional letter W.

Figure 3. Graphical model of loading/unloading and processing operations at the intermediate landing (1st terminal).

Figure 4. Graphical model of loading/unloading and processing operations at the shore and rail landing (2nd terminal).
The motion of flow along the graph arcs is characterized by traffic capacity of arcs that represent the capacity of machinery and equipment that are directly dependent on both the labor costs for performance of each operation and variable costs. The variable material costs that are fixed while the flow is carried between the operations of technological process, can include fuel costs, piecework payments for workers, etc. The costs of transmission $C(x_i, x_j, \theta, \theta)$ of flow unit along the graph arc that connect the vertex-time pair $(x_i, \theta)$ and $(x_j, \theta)$ are equal to $C_{ij}(\theta)$.

It is possible to account both constant and variable material costs for performance of operations analyzed in the graph. Arcs between the vertices that represent operations of the same notation but in different periods characterize available raw materials that did not pass next stage of technological process during the periods considered earlier. So, for example, constant costs that are fixed during transmission of flow from one period to the next one include costs connected with amortization expenses, planned maintenance and repair costs for equipment in use, time rates for workers, land rental payments for landings, etc. The costs of transmission $Z(x_i, x_j, \theta, \theta + \tau_{ij}(\theta))$ of flow unit along the graph arc that connect the vertex-time pair $(x_i, \theta)$ and $(x_j, \theta + \tau_{ij}(\theta))$ are equal to $Z_{ij}(\theta)$.

The task of finding the maximum flow of minimum cost [7–9] in the suggested dynamic network of operations of technological wood processing during its delivery from the logging units to consumers can be defined with the following mathematical dependencies [10]:

1) It is required to determine the minimum route for transmission of specified timber flow along graphs of dynamic operation network over a suggested number of periods.

$$
\sum_{\theta=1}^{P} \sum_{(x_i, x_j) \in A} \left( \hat{c}_{ij} \cdot \hat{\xi}_{ij}(\theta) + Z_{ij} \cdot \min\{1; \hat{\xi}_{ij}(\theta)\} \right) \rightarrow \min
$$

2) The maximum flow volume $\bar{\nu}$ over $p$ periods is equal to the flow outgoing from the source over $p$ periods.

$$
\sum_{\theta=1}^{P} \sum_{x_i \in X} \left[ \hat{\xi}_{ij}(\theta) - \bar{\nu}_s \left( \theta - \tau_{ij}(\theta) \right) \right] - \bar{\nu}(p) = 0
$$

3) The value of the flow $\hat{\xi}_{ij}$ entering the vertex $x_i$ in the moment $(\theta - \tau_{ij})$ is equal to the number of flow units $\hat{\xi}_{ij}$ outgoing from the vertex $x_i$ in the moment $\theta$.

$$
\sum_{x_j \in X} \left[ \hat{\xi}_{ij}(\theta) - \hat{\xi}_{ij}(\theta - \tau_{ij}(\theta)) \right] = 0, \ x_i \neq s, t; \ \theta \in T.
$$

This condition is be satisfied for each vertex $x_i$ without taking into account the fictitious vertices of source and sink.

4) The maximum value of flow $\bar{\nu}$ that passed along the graph arcs over $p$ periods is equal to the flow that enters the sink over the same period.

$$
\sum_{\theta=1}^{P} \sum_{x_j \in X} \left[ \hat{\xi}_{lj}(\theta) - \hat{\xi}_{lj}(\theta - \tau_{lj}(\theta)) \right] - \bar{\nu}(p) = 0.
$$

5) When justifying the value of flow that passes along the graph arcs, it is necessary to consider that total labor costs for performing the same technological operations of each $(\theta)$ period are not to exceed the maximum operating time $(m)$ of this period. Therefore, the value of flow $\hat{\xi}_{i=b}^{X}(\theta)$ that passes along the arc $(x_{i=b}, x_j)$ in the analyzed period $(\theta)$ is to comply with the following inequation:

$$
\bar{\nu} \leq \hat{\xi}_{i=b}^{X}(\theta) \leq m(\theta) - \sum_{i \in [1, b] \cup [b, g]} \sum_{x_j \in X} \hat{\xi}_{ij}(\theta) \cdot \hat{\xi}_{i=b}^{X}(\theta)
$$

This condition is to be satisfied for all periods and graph vertices except for source and arcs that connect different periods between each other.
\forall(x_i,x_j) \in \hat{A}(\theta); \theta \in T; x_i \neq s; (x_i,x_j) \neq (x_{x_i},\xi^{\theta};x_{x_j}(\theta+1)),  
\tag{6}

where \(b\) is the order number of analyzed arc \((1 \leq b \leq g)\); \(m\) is the maximum duration of the operation time of the analyzed period, \(h\); \(\hat{\xi}_{ij}^{X}(\theta)\) is the labor costs for processing or transportation of \(\xi\) when \(X\) (the technological process operation) is performed over the \(\theta\) period, \(\text{m}^{3}\); \(\hat{\xi}_{ij}^{X}(\theta)\) is the value of flow passing from the vertex \(i\) over \(\theta\) (period) when \(X\) (the technological process operation) is performed, \(\text{m}^{3}\).

One of the main parameters that limits arc traffic capacity is the performance \(P(\theta)\). In this case this indicator describes the volume of work that can be performed before the end of analyzed period. In calculation due to transmission of flow along the graph arcs of one or another cargo traffic the reduction of time \(m^{*}(\theta)\) that is remained until the completion of the stage is carried out.

\[m^{*}(\theta) = m(\theta) - \sum_{i=1}^{g} \sum_{j \in \xi^{X}} \hat{f}_{ij}^{X}(\theta) \cdot \hat{\xi}_{ij}^{X}(\theta)\]  
\tag{7}

In this, the labor costs for performance of one or another operation of technological woodprocessing and the capacity of machines and mechanisms are related to each other through the following mathematical dependence:

\[f_{ij}^{X}(\theta) = \frac{m^{*}(\theta)}{\hat{f}_{ij}^{X}(\theta)}\]  
\tag{8}

and conversely

\[\hat{f}_{ij}^{X}(\theta) = \frac{m^{*}(\theta)}{f_{ij}^{X}(\theta)}\]  
\tag{9}

6) When justifying the value of flow that passes along graph arcs it is necessary to consider that the sum of volumes of all types of transported wood removed from the logging unit \(L_{N}\) over the whole period of its development is not to exceed the maximum volume \(V_{N}\) of the cut wood.

\[V_{N} \geq V_{wN} + V_{SN} + V_{tN} \tag{10}\]

Therefore, the value of flow \(\left(\hat{\xi}_{L_{N}(j=b)}^{X}(\theta)\right)\) that passes along the arc \((L_{N},x_{j=b})\) from the vertex \(L_{N}\) over the analyzed period \(\theta\) is to meet the following inequation:

\[\hat{0} \leq \hat{\xi}_{L_{N}(j=b)}^{X}(\theta) \leq V_{N} - \sum_{\theta=1}^{p} \left( \sum_{j \in [1:b] \cup [b:g]} \hat{\xi}_{L_{N}j}(\theta) - \hat{\xi}_{L_{Nb}}(\theta - \tau_{L_{Nb}}(\theta)) - \hat{\xi}_{L_{Nb}}(\theta + \tau_{L_{Nb}}(\theta)) \right) \tag{11}\]

1) The sum of volumes of each type of wood sold to the consumer \(U\) over all the periods of logging unit development is not to exceed the maximum required volume of this type of product purchased by consumer. Therefore, the value of flow \(\left(\hat{\xi}^{Y}_{(i=b)(j=U)}(\theta)\right)\) that passes along the arc \((x_{(i=b)},x_{j=U})\) out of the vertex \((i=b)\) over the analyzed period \(\theta\) is to comply with the following inequation \((y\) is the product sold to the consumer \(U)\) :

\[\hat{0} \leq \hat{\xi}^{Y}_{(i=b)(j=U)}(\theta) \leq Q_{U}^{Y} - \sum_{\theta=1}^{p} \left( \sum_{i \in [1:b] \cup [b:g]} \hat{\xi}^{Y}_{iU}(\theta) - \hat{\xi}^{Y}_{bU}(\theta - \tau_{bU}(\theta)) - \hat{\xi}^{Y}_{bU}(\theta + \tau_{bU}(\theta)) \right) \tag{12}\]

3. Conclusion

The suggested graphical model will allow to implement algorithmic approach to justification of the order of timber transportation from different logging units, justification of use of timber landings in woodprocessing, booming grounds for loading/unloading and processing operations, type of transport (water or land), choice of consumer and type of final saleable products under fuzzy dynamic natural production environmental conditions.
A rational variant of technological woodprocessing works in dynamic natural and production environment can be justified in resolving the set task.

Acknowledgement
The reported study was funded by Russian Foundation for Basic Research, Government of Krasnoyarsk Territory, Krasnoyarsk Regional Fund of Science, to the research project: «Research and modeling of economic development of the forest industry in the region in the context of climatic conditions and resource potential», grant № 18-410-24003.

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