Heating of trapped atoms near thermal surfaces

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Abstract. – We study the electromagnetic coupling and concomitant heating of a particle in a miniaturized trap close to a solid surface. Two dominant heating mechanisms are identified: proximity fields generated by thermally excited currents in the absorbing solid and time-dependent image potentials due to elastic surface distortions (Rayleigh phonons). Estimates for the lifetime of the trap ground state are given. Ions are particularly sensitive to electric proximity fields: for a silver substrate, we find a lifetime below one second at distances closer than some ten µm to the surface. Neutral atoms may approach the surface more closely: if they have a magnetic moment, a minimum distance of one µm is estimated in tight traps, the heat being transferred via magnetic proximity fields. For spinless atoms, heat is transferred by inelastic scattering of virtual photons off surface phonons. The corresponding lifetime, however, is estimated to be extremely long compared to the timescale of typical experiments.

The last few years have witnessed an increasing interest in tightly confining traps of cold particles. These devices allow to envisage a broad spectrum of applications ranging from single-mode coherent matter wave manipulation and low-dimensional quantum gases [1, 2, 3, 4] to quantum logical registers [5, 6]. Since steep trapping fields exist near surfaces, traps in their vicinity enjoy increasing popularity. This raises the question at what timescale the cold particles in these “surface assisted traps” will be heated up, and how they are coupled to the nearby bulk which is typically at room temperature [1, 2, 3]. The question is of primordial importance for the above-mentioned applications since the heat transfer to the trap inevitably destroys the coherence of the matter waves [1].

In this Letter, we outline simple models that allow to compute the lifetime of the trapped particle which is limited due to its coupling to thermal excitations of the nearby solid. The interaction with thermal blackbody radiation is certainly a candidate for a mechanism of heating and decoherence. Estimates given by Wineland and Dehmelt [7] and others [8, 9] show, however, that this source is negligible for typical trap configurations. This is mainly due to the fact that the trapped particles are most sensitive to the field fluctuations at the

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resonant trap oscillation frequency (a few MHz at most) which is rather low compared to thermal frequencies which are in the THz range. More importantly, the resonant photon wavelengths are at least several meters. This means that the particle is always located in the near field of its macroscopic environment where the electromagnetic field fluctuations differ from the free-space blackbody field \[10, 11\]. The excitations of the solid that give rise to this near-field effect come in two species: fluctuating electric currents related to the dissipation in the solid (finite electric conductivity), and elastic waves (Rayleigh phonons) that propagate along the surface. Current fluctuations generate electric and magnetic fields above the surface ("proximity fields") that couple to the particle’s charge, spin or polarizability. Surface waves, on the other hand, distort the electrostatic image of the particle in the solid and lead to a time-dependent image potential. We find that ions are particularly sensitive to proximity fields and estimate a typical lifetime of less than a second for distances smaller than 10 \(\mu\text{m}\) above a metal surface. Atoms, being neutral particles, are less affected by the presence of the "hot" surface: for a nonzero magnetic moment, they survive several minutes even at distances of a few micrometers. Finally, spinless atoms are completely decoupled from the surface at experimentally relevant time scales.

**Model.** – We consider a particle in the ground state \(|0\rangle\) of a one-dimensional harmonic trap (oscillation frequency \(\omega_t\)) that is oriented along the unit vector \(\mathbf{n}\) (see fig.1). To simplify the calculations, we assume that the trap is located above a flat surface whose distance \(z\) from the trap center is large compared to the size \(a = (\hbar/(2M\omega_t))^1/2\) of the ground state wave function \((M\text{ is the particle’s mass})\). The interaction potential for the particle is then of the form

\[
V(t) = -\mathbf{x} \cdot \mathbf{F}(\mathbf{r}, t)
\]  

where \(\mathbf{x} = x\mathbf{n}\) is the displacement of the particle from the trap center and \(\mathbf{F}(\mathbf{r}, t)\) is a fluctuating force field at the trap position \(\mathbf{r}\). If this force shows fluctuations at the trap frequency, it may resonantly excite the particle to the first excited trap state \(|1\rangle\). According to second-order perturbation theory (Fermi’s Golden Rule), this happens with a rate

\[
\Gamma_{1\rightarrow0}(\mathbf{r}) = \hbar^{-2} |\langle 1|\mathbf{x}|0\rangle|^2 \sum_{ij} n_i n_j S_{ij}^{\text{el}}(\mathbf{r}, \mathbf{r}; -\omega_t)
\]  

where \(\langle 1|\mathbf{x}|0\rangle\) is a “dipole matrix element” (equal to \(a\) for a harmonic trap), and the cross
correlation tensor of the force fluctuations is defined by

$$S^{ij}_{F}(r', r; \omega) = \int_{-\infty}^{+\infty} dt \langle F_i(r', t) F_j(r, 0) \rangle_T e^{i\omega t},$$

(3)

where $i, j$ denote cartesian indices. The average is taken in thermal equilibrium at temperature $T$. Note that the “heating rate” $\Gamma_{1\to0}$ also governs the decay of the coherence between different trap levels, as is easily shown by deriving a master equation for the particle’s density matrix in the usual Born-Markov approximation. Decoherence in ion traps was theoretically investigated by several authors [6, 7, 8, 9, 12, 13], particularly in the wake of a recent experiment by Meekhof et al. [14].

In distinction to the present work, refs. [12, 13] focused mainly on the decay of off-diagonal density matrix elements.

**Ion heating.** – The simplest case is that of a trapped ion. The force in (1) is given in terms of the electric field as $F = qE$. To compute the electric field fluctuations, we use the fluctuation–dissipation theorem that relates their spectral density to the field’s Green tensor $G_{ij}(r', r; \omega)$, i.e., the field created at $r'$ by an oscillating point dipole located at $r$ [10]:

$$S^{ij}_{E}(r, r; \omega) = \frac{2\hbar}{1 - e^{-\omega/T}} \text{Im} G_{ij}(r, r; \omega).$$

(4)

(In our units, $k_B = 1$.) The Green tensor contains a free-space term $\text{Im} G_{ij}^{(bb)}(r, r; \omega) = \omega^2/(6\pi\varepsilon_0 c^3)\delta_{ij}$ that describes the black-body field, and a surface-dependent term due to the reflection at the surface. For typical trap frequencies, the electromagnetic wavelength is much larger than $z$ so that we may use the quasi-static approximation for the Green tensor and get (in SI units)

$$\text{Im} G_{ij}^{(s)}(r, r; \omega) = \frac{s_{ij}}{16\pi\varepsilon_0 z^3} \text{Im} \frac{\varepsilon(\omega) - 1}{\varepsilon(\omega) + 1}$$

(5)

where $s_{ij}$ is a diagonal tensor with $s_{xx} = s_{yy} = \frac{1}{2}$, $s_{zz} = 1$. A reasonable model for the dielectric function $\varepsilon(\omega)$ in (5) is that of a Drude metal [15]. For frequencies below the electronic damping rate, the electrostatic reflection coefficient in (5) becomes

$$\text{Im} \frac{\varepsilon(\omega) - 1}{\varepsilon(\omega) + 1} = 2\omega\varrho\varepsilon_0$$

(6)

where $\varrho$ is the metal’s specific resistance at the trap frequency. Finally, we may also take the high-temperature limit of the Bose-Einstein factor in (4). This gives a surface-induced heating rate for the trapped ion that follows a power law as a function of distance $z$

$$\Gamma_{1\to0}^{(s)}(z) = \frac{q^2T \varrho}{h\omega_1 m} \frac{1 + n_e^2}{16\pi z^3}.$$
For distances well below the transition wavelength, the decay is dominated by nonradiative transfer from the dipole’s near field to the metal, as described by eq. (5). At low frequencies or, equivalently, high temperatures, the decay rate $1 \to 0$ is dominated by stimulated emission and hence proportional to the temperature, see eq. (7). On the other hand, the principle of detailed balance implies that the inverse transition $0 \to 1$ that we are presently interested in, occurs at the same rate in this regime. The ion is hence heated up because thermal energy is transferred from the metal surface to the ion’s near field in a nonradiative way. (A similar reasoning has been presented by Anglin, Paz, and Zurek [17], though they focussed on the energy transfer from a fast-moving ion into a solid and the concomitant decoherence.)

We may also compare the heating rate (7) to the calculations of Wineland and Dehmelt [7], Lamoreaux [8], and James [9] who studied the heating due to thermal voltage fluctuations across the endcaps of a Paul trap. These authors’ results are recovered (up to a geometrical factor of order unity) if we replace in (7) the quantity $g/z$ with the electric resistance $R$ of the endcap circuitry, and interpret $z$ as the endcap distance: the quantity $RT/z^2$ then gives the power spectrum of the thermal electric field between the endcaps. The present model shows, however, that even in front of a single surface, electric proximity fields leak out of the solid whose power spectrum increases even stronger with decreasing distance.

**Magnetic proximity fields.**—Trapped ions and atoms frequently have a magnetic moment $\mu$ and are therefore heated by time-dependent magnetic fields. The force derives from the Zeeman interaction

$$V(\mathbf{r}, t) = -\mu \cdot \mathbf{B}(\mathbf{r}, t).$$

As mentioned in the introduction, magnetic fields are created by fluctuating currents in the solid. Following the seminal work of Lifshitz [20], the spectral density of these currents is proportional to the imaginary part of the dielectric function $\varepsilon(\mathbf{r}; \omega)$. It has recently been proven that the introduction of fluctuating currents also provides a consistent framework to quantize the electromagnetic field in absorbing and dispersive dielectrics [20, 21]. We use the following
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Fig. 3. – Transition rate $0 \rightarrow 1$ (‘heating rate’) vs. trap distance $z$ for a trapped particle with mass $M = 40$ amu. The trap is oriented perpendicular to the surface and has frequency $\omega_t/2\pi = 100$ kHz. Solid line: coupling to magnetic proximity fields above a silver substrate (magnetic moment $\mu/(2\pi \hbar) = 1.4$ MHz/G = 1 Bohr magneton, spin $\frac{1}{2}$, specific resistance $\rho = 1.6 \times 10^{-6}$ Ωcm [13]). Dashed line: same above a glass substrate ($\rho = 10^{11}$ Ωcm). Dash-dotted line: coupling to surface phonons via the van-der-Waals interaction ($c_3/(2\pi \hbar) = 1$ kHz µm$^3$, Ag substrate with $\hbar \omega_0 = 225$ K, $M_s = 108$ amu, $\eta = 0.75$ [22]). The heating rate due to blackbody magnetic fields (not shown) is of the order of $10^{-39}$ s$^{-1}$.

This quantity is represented in fig.3 for a trap close to an Ag substrate, and one observes a relatively large heating rate of the order of $10^{-2}$ s$^{-1}$ at a distance of 1 µm. A glass substrate gives a much smaller heating rate (dashed line) because its resistance is larger. We conclude that one has to avoid either metal surfaces or particles with spin if one wants to store atoms coherently over timescales longer than, say, a few minutes.

**Heating of neutral, spinless particles.** – We now turn to the heating of a neutral, spinless particle. One would expect it to be less sensitive to fluctuating electric fields, although these
couple to the atomic polarizability $\alpha$ via the Stark shift $V = \frac{4}{3}E^2$. We have computed the corresponding heating rate along similar lines as for the trapped ion and found extremely small values (below $10^{-12}\text{s}^{-1}$ even at distances around 100 nm above a Ag substrate). In fact, atom heating is dominated by a different effect: the distortion of the surface by thermal oscillations leading to a time-dependent image potential. The corresponding force may be easily computed using the following effective interaction (strictly valid for a rarefied solid and in the quasi-static limit): we integrate a $1/r^6$ dipole-dipole interaction between the atom and the half-space filled by the solid. For a flat interface, one finds the usual van-der-Waals potential $-c_3/z^3$. If the interface is corrugated, but with an amplitude small compared to the distance $z$, one gets a first-order correction $V^{(1)}(r, t)$ of the form

$$V^{(1)}(r, t) = \sum_Q g(Q; z) u_Q(t) \exp iQ \cdot R.$$  

In this expression, $Q$ is a two-dimensional ‘lateral’ wave vector (in the $xy$-plane parallel to the non-excited surface), $R$ are the lateral coordinates of the trap center, the $u_Q(t)$ are the elastic displacements of the surface at wave vector $Q$, and, finally, the coupling coefficients are given by $g(Q; z) = -(3c_3Q^2/(2z^2))K_2(Qz)$ with $K_2$ a modified Bessel function.

For simplicity, we restrict ourselves to Rayleigh waves whose displacement amplitudes are confined to the vicinity of the surface. Their amplitudes may be written in terms of boson operators

$$u_Q(t) = \sqrt{\frac{\hbar \eta \pi}{N_s M_s \omega D}} \left( a_Q(t) + a_Q^\dagger(t) \right),$$

where the dimensionless parameter $\eta$ specifies the decay of the surface wave into the bulk (it depends on the ratio between the bulk and Rayleigh wave sound velocities), $N_s$ is the number of surface atoms per quantization area, $M_s$ is their mass, and $\omega_D$ is the Debye frequency. Using thermal Bose-Einstein statistics for the Rayleigh waves, we can compute the spectrum of the surface oscillations at the trap frequency. In the result, the magnitude of the Rayleigh wave vector is fixed to $Q = \omega_D/v_R$ where $v_R$ is the Rayleigh wave sound velocity. Typically, the sound wavelength is much longer than the trap distance, and the coupling constant $g(Q; z)$ may thus be approximated by its asymptotic form for $Qz \ll 1$. The heating rate then equals

$$\Gamma_{1\leftrightarrow 0} = \frac{\hbar \omega_D^3}{h \omega_1 \omega_D^3 M M_s} \frac{72\pi^3 \eta (1 + \eta^2)}{z^{10}}.$$  

As shown in fig.3, reasonable parameters give a still very small heating rate (below $10^{-6}\text{s}^{-1}$ at 100 nm distance). Even on a timescale of hours, the spinless atom is thus decoupled from the thermal excitations of the solid. We note that in evanescent field traps, the interaction with light scattered off surface/bulk impurities may be a dominant heating mechanism.

Conclusion. – In the vicinity of a solid surface at room temperature, fluctuating electric and magnetic fields couple to trapped particles and induce a finite lifetime of the trap’s ground state. If one wants a lifetime longer than about one second, ions must not be closer to the surface than about some tens of micrometers. This means that miniaturized coherent ion traps are difficult to realize at room temperature. Magnetic proximity fields are weaker, and atoms with spin live for many seconds in the ground state even at distances of a micron. The best candidates for long-time storage are spinless atoms because they are nearly insensitive to stray

\begin{footnotesize}
(1) C. Henkel, R. Adams, H. Gauck, D. Schneble, T. Pfau, C. I. Westbrook, and J. Mlynek, in preparation.
\end{footnotesize}
fields. Their ground state lifetime is much longer than hours and is mainly limited by surface waves that distort the electrostatic image potential.

The preceding estimates have been obtained from simple models for the trap geometry, neglecting the finite height of the trap potential and assuming a homogeneous substrate. An obvious extension would be to allow for layered media. Another point is the inclusion of a cross-coupling between fluctuating and static trapping fields. This increases the heating due to the Stark potential, e.g., because the atom acquires a static electric dipole moment in the trap. Finally, in some traps (magnetic or near-resonant optical) the coupling to non-trapped internal states of the particle, e.g., hyperfine or magnetic states, may lead to a relevant loss rate.

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