THE STATISTICS OF WIDE-SEPARATION LENSES

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The probability that high-redshift sources are gravitationally-lensed with large image separations (\textit{i.e.}, greater than can be produced by galactic deflectors) is determined by the cosmological population of group- and cluster-sized halos. Thus the observed frequency of wide-separation lensed quasars can be used to constrain not only the halo distribution, but also a number of cosmological parameters. A calculation of the optical depth due to collapsed, isothermal halos is a useful guide to the lens statistics, and illustrates that the number of wide-separation lenses is a sensitive probe of the mean density of the universe and the present day density variance whilst being nearly independent of the cosmological constant.

1 Introduction

Gravitational lensing is a direct probe of the geometry of the universe, and so observations of lensing place constraints on both the distribution of mass and the underlying cosmological model. A large variety of lensing experiments are possible: weak shear surveys; searches for microlensing by compact objects; observations of giant arcs in clusters; measurement of Shapiro delay in binary pulsars; surveys for multiply-imaged quasars; and so on. The focus here is on the statistics of wide-separation lensed quasars (which are produced by group- and cluster-mass objects). Using a simple model of the halo population (Section \textsuperscript{2}), the optical depth to multiple imaging and the expected distribution of image separations can be calculated (Section \textsuperscript{3}). The results obtained are independent of the source population, which then implies that a more sophisticated analysis is required to constrain model parameters from the observed frequency wide-separation lenses\textsuperscript{1,4}.
Deflector population

The population of cluster-sized halos is well approximated by the Press-Schechter mass function, which depends on the cosmological model and the matter power spectrum. An approximate cold dark matter (CDM) power spectrum with a spectral slope of $n = 1$ is assumed, and only the normalisation is allowed to vary. The scale of the density fluctuations are normalised to match $\Delta_8$, the present day variance in spheres of 8 Mpc radius, and so is linked to a (co-moving) scale, rather than a mass. The clusters themselves are modelled as singular isothermal spheres, which are characterised by their line-of-sight velocity dispersion, $\sigma$, rather than their mass, $M$. General arguments imply that $M \propto \sigma^3$, but this conversion is somewhat ambiguous, and is an important source of systematic error in the calculation of lensing cross-sections. The resultant halo population (Fig. 1) is most strongly dependent on $\Delta_8$, and can be made consistent with galaxy counts if the Press-Schechter form is assumed only for $\sigma > 200$ km s$^{-1}$.

Lensing optical depth

Given the population of deflectors and a lens model, it is reasonably straightforward to calculate the optical depth, $\tau(z_s)$, to multiple imaging. Whilst $\tau$ is too crude an estimate of the lensing probability to meaningfully constrain model parameters, it is useful in an illustrative sense, particularly if the source-dependent aspects of the full probability (e.g., the magnification bias) can be factored out of the integral over the deflector population. This is the case for the isothermal sphere, and, in addition, the image separation, $\Delta \theta$, is completely determined by $z_d$, $z_s$ and $\sigma$, and so image separation cut-offs (i.e., $\Delta \theta_{\text{min}}$ and $\Delta \theta_{\text{max}}$) can be included in $\tau$. Thus the optical depth can be thought of as the fraction of the sphere at redshift $z_s$ inside the Einstein radius ($\theta_E$) of any cluster for which $\Delta \theta_{\text{min}} \leq 2 \theta_E \leq \Delta \theta_{\text{max}}$. Choosing $\Delta \theta_{\text{min}} \simeq 3$ arcsec removes the galactic lenses from the calculation (their population being poorly approximated by the naive Press-Schechter form); the value $\Delta \theta_{\text{max}}$ is determined by the breadth of the companion search.

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This is specified by the present day normalised matter density, $\Omega_{\text{m0}}$, the similarly normalised cosmological constant, $\Omega_{\Lambda 0}$, and Hubble’s constant (although its value is unimportant in this calculation).
The optical depth is shown as a function of source redshift in Fig. 2. The standard increase of $\tau$ with $z_s$ is apparent, and comparing Fig. 2 (a) and (b) shows the expected dependence on $\Delta_8$. More interesting is the variation with cosmological model and $\Delta_\theta_{\text{max}}$. The optical depth to lensing by galaxies is primarily dependent on the cosmological constant as the differential volume element is so much larger in high-$\Omega_\Lambda$ models. This effect is present here, but it is not dominant, for two reasons. Firstly, clusters form earlier in low-density cosmologies – most large-scale structure formed before $z \simeq \Omega_\Lambda^{-1} - 1$ (if $\Omega_\Lambda = 0$) or, more recently, before $z \simeq \Omega_\Lambda^{-1/3} - 1$ (in flat models). Thus the lensing probability is greater if $\Omega_m$ is small, there being more high-redshift collapsed deflectors along a given line-of-sight. For a fixed $\Omega_m$, however, increasing the cosmological constant reduces the number of halos with $\Omega_m^{-1/3} - 1 < z_d < \Omega_m^{-1} - 1$, somewhat offsetting the usual increase of $\tau$ with $\Omega_\Lambda$. Secondly, the mass of a cluster that has collapsed from a given co-moving scale (e.g., 8 Mpc) is proportional to $\Omega_m$. For the isothermal sphere model the lensing cross-section scales as $\sigma^4 \propto M^{4/3} \propto \Omega_m^{4/3}$. It is because of this strong dependence that standard CDM models with $\Omega_m = 1$ are so inconsistent with the low number of wide separation lenses.

The expected distribution of image separations can be estimated by computing $d\tau/d\Delta_\theta$, which is illustrated in Fig. 3. As shown, these distributions are normalised to unity, but the vertical scaling can be estimated from the known wide-separation pairs. The immediate implication of this is that the fall-off with increasing $\Delta_\theta$ is far too shallow to be consistent with the complete absence of any (confirmed) lenses with $\Delta_\theta > 10$ arcsec. One inference that could be drawn is that the mass profiles of clusters are shallower than isothermal, but it is difficult to reconcile such models (including the Navarro, Frenk & White profile) with observations of arcs and arclets.

4 Conclusions

The statistics of wide-separation lenses are a useful cosmological probe. The lensing probability is sensitive to the geometry of the universe, and the population of collapsed halos at intermediate redshifts whilst being independent of the complexities of galaxy formation. As large-scale structure is dependent on the linear growth of perturbations (and also because the mass in a co-
Figure 3: The normalised distribution of image separations produced by a cosmological population of isothermal halos, for a source at $z_q = 3$. Results are shown for several cosmological models: $\Omega_m = 1$ and $\Omega_\Lambda = 0$ (solid lines); $\Omega_m = 0.3$ and $\Omega_\Lambda = 0$ (dashed lines); and $\Omega_m = 0.3$ and $\Omega_\Lambda = 0.7$ (dot-dashed lines), with $\Delta_8 = 0.5$ in (a) and $\Delta_8 = 1.0$ in (b).

Moving sphere of a given size is proportional to the density parameter) the likelihood of lensing is most sensitive to $\Omega_m$ and $\Delta_8$. The Large Bright Quasar Survey is devoid of wide-separation lenses which implies that $\Omega_m < 0.3$ (assuming $\Delta_8 > 0.4$) or that $\Delta_8 < 0.6$ (assuming $\Omega_m < 0.1$) with 99 per cent confidence. In the future such analyses could be extended to the 2 degree Field quasar survey (with $\sim 3 \times 10^4$ quasars) and the Sloan Digital Sky Survey (which will contain $\sim 10^5$ sources). These samples will be large enough to constrain $\Omega_m$ and $\Delta_8$ to within several per cent from the number of wide-separation lenses alone.

Acknowledgments

Much of this research was done whilst supported by an Australian Postgraduate Award.

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