Removing the Outlier from the Production Data for the Decline Curve Analysis of Shale Gas Reservoirs: A Comparative Study Using Machine Learning

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ABSTRACT: Decline curve analysis (DCA) is one of the most common tools to estimate hydrocarbon reserves. Recently, many decline curve models have been developed for unconventional reservoirs because of the complex driving mechanisms and production systems of such resources. DCA is subjected to some uncertainties. These uncertainties are mainly related to the data size available for regression, the quality of the data, and the selected decline curve model/s to be used. In this research, first, 20 decline curve models were summarized. For each model, the four basic equations were completed analytically. Second, 16 decline curve models were used with different data sizes and then a machine learning (ML) algorithm was used to detect the outlier from shale gas production data with different thresholds of 10, 15, and 20%. After that, the 16 models were compared based on different data sizes and the three levels of data quality. The results showed differences among all models’ performances in the goodness of fitting and prediction reliability based on the data size. Also, some models are more sensitive to removing the outlier than others. For example, Duong and Wang’s models seemed to be less affected by removing the outlier compared to Weng, Hesieh, stretched exponential production decline (SEPD), logistic growth (LGM), and fractional decline curve (FDC) models. Further, the extended exponential decline curve analysis (EEDCA) and the hyperbolic–exponential hybrid decline (HEHD) models tended to underestimate the reserves, and by removing the outlier, they tended to be more underestimators. This work presented a comparative analysis among 16 different DCA models based on removing the outlier using ML. This may motivate researchers for further investigations to conclude which combination of the outlier removers and DCA models could be used to improve production forecasting and reserve estimation.

1. INTRODUCTION

Any piece of data always carries information. The kind of data itself being analyzed is critical. The kind of analysis differs from one study to another. The effort, time, and cost of any study are essential factors to be considered in choosing the type of analysis. Engineers always seek to get as much information as they could using methods that are fast, effective, and with minimal analyzing effort and cost.

Analyzing different kinds of data can result in a valuable understanding of the whole field and therefore lead to better planning of the field, well spacing, characterizing the reservoir, determining the reserves, sizing the production facilities, deciding to invest profile, and more. The major reservoir engineering data analysis methods are (1) data-driven empirical models, (2) analytical or physics-based models: pressure transient analysis (PTA) and rate transient analysis (RTA), (3) numerical simulation models, and recently, (4) machine learning (ML) models.

Generally, each method has its limitations. Using one of them is specified based on many factors such as cost of the study, computational power available, availability of the data, and of course the questions to be answered. The data itself used for any of the aforementioned methods could suffer from anomalies or outliers, which increased the uncertainties around the final results.

In this paper, the focus was on empirical decline curve analysis (DCA) in shale gas wells and how to reduce the uncertainties related to the estimated ultimate recovery (EUR). Many studies compared different models based on different data sizes, but the sensitivity of each decline curve...
model to the data quality has not been studied yet. In this work, an ML algorithm called the angle-based outlier detection (ABOD) was used to enhance the production data quality. Overall, 16 decline curve models were used with different data sizes and different data quality. A comparative analysis was conducted to identify the sensitivity of models to data quality.

1.1. Shale Gas Characteristics. Shale gas reservoirs are characterized by very tight pores within the shale matrix. The sizes of these pores vary between micro- to nanosize. The porosity of the shale matrix is less than 10%, and the permeability is ultralow (less than 10−8 Darcy). The shale permeability is affected by the bedding trend as well as the in situ stresses due to the compaction of shale. Permeability could be as low as 10−18 Darcy. Due to stresses matrix shrinkage, shale gas reservoirs could have natural fractures. Shale gas reservoirs have huge volumes of trapped gas that contain more than 94% methane. This gas could be free in the matrix and the fractures, adsorbed by organic matter and clay minerals, or dissolved in asphaltenes and shale oil. The main production from the shale gas reservoirs comes from the free gas. The dissolved gas volume is ignored compared to the adsorbed gas, which could range between 20 and 85% of the total gas volume. Because of such rock and fluid properties, the driving mechanisms producing this gas are complex. The main mechanisms that exist are slippage, diffusion, and desorption.

1.2. Challenges Related to Analyzing the Production of Shale Gas Wells. Under natural conditions, producing from shale gas reservoirs with economic value is difficult. The optimum method used to produce shale gas reservoirs is to drill horizontal wells with multistages of hydraulic fracturing. This creates traverse or longitudinal fractures and blocks of the stimulated reservoir volume (SRV). The longer the horizontal length of the well and the higher the number of stages, the bigger the SRV that could be produced. However, this production mode creates a complex system that has a combination of interconnected natural fractures, hydraulic fractures, and an ultralow permeability matrix. The result is complex flow regime sequences.

The possible combinations of the flow regimes that exist when producing the shale gas wells and impact the decline trends of the production are:

- Linear flow: It could be the dominant flow during the well lifetime. It is perpendicular from the matrix to the hydraulic fractures. That transient linear flow can be identified by plotting the flow rate versus time on a log–log plot. The plot shows a −1/2 slope if the reservoir has no natural fractures.
- Linear–boundary-dominated flow (BDF): The transient linear appears and lasts for a period of time followed by the BDF. It is usually related to reaching the boundaries of the SRV. This could be identified on the log–log plot when deviation from the −1/2 slope starts to appear.
- Bilinear–linear: The initial flow regime assumed in this case is bilinear flow (linear followed by another linear). It is usually related to natural fractures when it exists and lasts for a very short period in the early time. In this case, the flow moves linearly from the fractures toward the well and from the matrix linearly to the fractures simultaneously. So on the log–log plot, it shows a −1/4 slope that identifies the bilinear flow followed by a −1/2 slope that identifies the linear flow.
- Bilinear–linear–BDF: In this case, the difference from the upper case is when the flow reached the SRV boundaries, and the log–log plot shows deviation from the −1/2 slope once again.

Figure 1 shows how these flow regimes are defined from the log–log plot using actual data. It should be pointed out that the appearance of one of these cases is related to whether the reservoir is naturally fractured or not. Also, this is related to fracture conductivity.

During production, controlling the bottom hole pressure (BHP) is essential to increase EUR. This allows control of the proppant backflow, prevents associated water to accumulate in the wellbore, and decreases the stress effect. Although creating SRV and controlling BHP are the key factors to developing the shale gas reservoirs and properly achieving them helps in improving the EUR, calculating the EUR itself is challenging due to these practices. During the early producing time, the flow rate is high as the flow mainly comes from the SRV. After reaching the boundaries of the SRV, the flow changes from transient flow into the boundary-dominated flow (BDF), causing a rapid drop in the flow rate profile and continuing with the long tail production profile. On the other hand, controlling the BHP leads to dramatic changes in the production data itself and the noise level may be too high.

Figure 1. Identifying the different flow regimes based on the slope value on the log–log plots.
Table 1. Arps Formulae of the Three Types of DCA

| Type                  | Exponential Decline | Hyperbolic Decline | Harmonic Decline |
|-----------------------|---------------------|--------------------|------------------|
| Parameter b           | 0                   | 0 < b < 1          | b = 1            |
| Equation              | Equation 1          | Equation 2         | Equation 4       |

Equation 1: \( D(t) = D_i \)

Equation 2: \( q_t = q_i \exp(-D_it) \)

Equation 3: \( G_p(t) = \left( \frac{q_i - q_f}{D_i} \right) t \)

Equation 4: \( G_p(t) = \left( \frac{q_i}{b} \right) \ln \left( \frac{b}{b - 1} \right) \)

So, the results of these two practices make it challenging to fit the production data accurately with decline curve models. An alternative approach is to use the ordinary least squares (OLS) versus weighted least squares (WLS) regression with a DCA model.

2. DECLINE CURVE ANALYSIS

Arnold and Anderson introduced the first mathematical model of DCA. This form was developed assuming a constant decline in the production rate. Many researchers till now had a big interest in that research area. Johnson and Bollens introduced the definition of loss ratio. Pirson noticed that the loss ratio could be constant or has constant differences at equal time intervals for some decline curves. For referencing, eqs 1–3 define the decline parameter (D), the loss ratio, and its second derivative, the decline exponent (b), respectively. Equation 4 gives the cumulative production as a function of time.

\[
D(t) = \frac{1}{q(t)} \frac{dq(t)}{dt} \quad (1)
\]

\[
\frac{1}{D(t)} = \frac{q(t)}{D(t)} \frac{dq(t)}{dt} \quad (2)
\]

\[
\frac{d}{dt} \left[ \frac{1}{D(t)} \right] = \frac{d}{dt} \left[ \frac{q(t)}{dq(t)/dt} \right] \quad (3)
\]

\[
G_p = \int_{t_1}^{t_2} q(t) dt \quad (4)
\]

where \( q_t \) = gas flow rate at time \( t_i \) (Mscf/day), \( t \) = time (day), \( D(t) \) = decline rate (day\(^{-1}\)), \( b \) = decline curve exponent, and \( G_p(t) \) = cumulative production (Mscf).

These parameters are simple to be obtained directly from the definition or by regression. Analyzing these parameters properly carries a lot of valuable information that could help in characterizing the reservoir. This is because they represent the changes in the production rate with time, which results in all that happened from the reservoir boundaries, driving mechanisms, wellbore, and production conduit till the wellhead operation conditions.

2.1. Arps DCA. Arps’ DCA models are classified into three types based on the b-value (Table 1). The main assumptions of Arps’ DCA are stabilized reservoir and production conditions and boundary-dominated flow. For conventional reservoirs, these assumptions could be achieved. But unconventional reservoirs have a rapid decline and transient flow in the early time of the production before reaching BDF. After that, the production mode shows a long tail of a slow decline. For these reasons and more, Arps’ models became less effective for estimating the EUR in unconventional reservoirs and therefore, many DCA models have been developed after Arps’ models. Appendix 1 summarizes 20 decline curve models. The four equations of \( q(t), G_p(t), b(t), \) and \( D(t) \) were completed analytically based on eqs 1–4 for each model. Graphing the four relationships could help in diagnosing the well and characterizing the reservoir.

The 16 models selected to be compared. Because Arps’s exponential and harmonic models are special cases of Arps’s hyperbolic model, only Arps’s hyperbolic model was
used. The modified Arps approach and multistage approach were excluded too. They are modifications of Arps’s approach with many more steps rather than just regression.

3. UNCERTAINTIES RELATED TO DECLINE CURVE STUDIES

DCA models have many uncertainties, especially for shale gas wells, which led to the development of many different empirical correlations to fit the decline behavior. Sources of uncertainties can be summarized as follows:

- Uncertainties related to determining the parameters themselves graphically or from nonlinear regression. For example, three fitting techniques could be used: ordinary least square regression (OLS), weighted least-squares regression (WLS), and maximum likelihood estimation (MLE).
- Joshi et al. concluded that the best fit could be obtained by OLS or WLS in different cases. However, WLS usually is better than OLS. In DCA, OLS is usually optimistic, while WLS is conservative, as shown in Figure 2. Hong et al. indicated that the least-squares estimation is a special case of MLE but a more-accurate data point will have more weight than a less-accurate data point using MLE, which improves the goodness of fitting.
- Size of the historical data: the less the production data size, the higher the uncertainty.
- Production data quality: due to changes in the operating conditions with time to control BHP and multiple shut-ins, noise, outliers, and fluctuations of the historical production data appear and affect the fitting and therefore the reliability of the forecasting.
- Flow regime variations through the production life: different transient flow regimes appear and could last for a very long time.
- There are many steps to decrease the degree of uncertainty related to DCA. These steps are:
  - Improving the data quality by processing the production data, smoothing the production curve, and removing the outlier.
  - Choosing the proper DCA method is based on the suggestions and limitations related to each method. For example, SEPD requires a production history of more than 35 months, Wang is used with a slow decline rate, while VDMA is used with a fast decline rate.
  - Performing DCA with more than one empirical DCA model is highly recommended.

4. REMOVING THE OUTLIER USING THE ML ALGORITHM

Hawkins defines an outlier “as an observation that deviates so much from other observations as to arouse suspicion that it was generated by a different mechanism.” Removing such anomalous observations from the data manually may cause bias or remove meaning observations.

There is no one standard and unified method to quantify the deviation of these anomalous observations from the core data that can be used uniformly across all cases. However, studies of outlier detection techniques started early in the 19th century. Many studies were performed and published for that particular. Although many of them were specifically developed for certain applications and they differ in the technique by

Table 2. Major Algorithms to Detect the Outlier

| Algorithm Type            | Algorithm                  | Reference                                  |
|---------------------------|----------------------------|--------------------------------------------|
| 1. Linear based           | Deviation-based outlier detection (LMDD) | (Schillkopf et al.)^36                     |
|                           | Minimum covariance determinant (MCD)   | (Shyu et al.)^37                           |
|                           | One-class support vector machines (OCSVM) | (Hardin and Rocke)^28                      |
|                           | Principal component analysis (PCA)      | (Shyu et al.)^37                           |
| 2. Probabilistic based    | Median absolute deviation (MAD)         | (Iglewicz and Hoaglin)^39                   |
|                           | Angle-based outlier detection (ABOD)    | (Kriegel et al.)^38                         |
|                           | Fast angle-based outlier detection (FABOD) | (Kriegel et al.)^38                      |
|                           | Stochastic outlier selection (SOS)      | (Angiulli and Pizzuti)^31                   |
|                           | Empirical cumulative outlier detection (ECOD) | (Li et al.)^32                           |
| 3. Proximity based        | Local outlier factor (LOF)              | (Breunig et al.)^34                         |
|                           | k-nearest neighbor (kNN)                | (Ramawan et al.)^35                         |
|                           | Average kNN                            |                                            |
|                           | Median kNN                             |                                            |
|                           | Connectivity-based outlier factor (CBOF) | (Breunig et al.)^36                         |
|                           | Memory efficient connectivity-based outlier factor (MEC-BOF) | (Chen, Yu, and Liu)^37                      |
|                           | Local outlier correlation integral (LOCI) | (Papadimitriou et al.)^39                  |
|                           | Subspace outlier detection (SOD)        | (Kriegel et al.)^39                         |
|                           | Histogram-based outlier score (HBOS)    | (Goldstein and Dengel)^39                   |
|                           | Rotation-based outlier detection (RBOD) | (Almarsen et al.)^32                        |
| 4. Ensembles based        | Feature bagging (FB)                   | (Lazaric and Kumar)^42                       |
|                           | Isolation forest (IF)                   | (Liu et al.)^45                             |
|                           | Lightweight one-line detector of anomalies (LODA) | (Pevny)^46                      |
|                           | Extreme boosting-based outlier detection (XGBOD) | (Zhao and Hryniewicki)^47                  |
|                           | Locally selective combination of parallel (LSCP) | (Zhao, Nasrullah, and Li)^45              |
|                           | Accelerating large-scale unsupervised heterogeneous outlier detection (ALSUHOD) | (Zhao et al.)^46                          |
| 5. Neural network based   | Variational autoencoder (VAE)           | (Kingma and Welling)^47                      |
|                           | Fully connected autoencoder (FCAE)      | (Aggarwal)^46                               |
|                           | Deep one-class classification (DOCC)    | (Ruff et al.)^59                            |
|                           | β variational autoencoder (BVAE)        | (Burgess et al.)^50                         |
|                           | Single-objective generative adversarial active learning (SO-GAAL) | (Liu et al.)^51                           |
|                           | Multiple-objective generative adversarial active learning (MO-GAAL) | (Liu et al.)^52                           |
Table 2 introduces the major five categories and some of the popular algorithms developed under each category. In our study, not all of these algorithms were investigated but a recommended method from the literature was used. Jha et al. investigated the five popular outlier detection techniques after adding artificial noise to simulated data. The investigated methods were one-class-supported vector machine (SVM), distance-based outlier detection, density-based outlier detection, angle-based outlier detection, and isolation forest. After investigation, it was concluded that the best method is the ABOD.

4.1. Angle-Based Outlier Detector Algorithm (ABOD). A data point is classified as an outlier or inlier using the ABOD algorithm based on the value of the angles between that point and every arbitrary pair of points in the data set. To make the algorithm faster and less computationally intensive, the k-nearest neighbors (kNN) are utilized. In other words, instead of measuring the angle between each point in the data set and every arbitrary pair of points in the data set, we only measure the angle between the point and a predetermined number (k) of the nearest points, as shown in Figure 3 (i.e., k = 3).

The cosine of the angles between all possible pairs of vectors weighted by the distance between the points is then obtained by computing the scalar product of all possible pairs of different vectors and dividing it by the squares of the relevant vector magnitudes, as shown in eq 5. The angle-based outlier factor (ABOF) is then calculated (A). ABOF(A) calculates the variance of the distance-weighted cosines of the angles between all arbitrary pairs of vectors that originate from A. (B and C), as shown in eq 6. The dot product could be written in terms of the cosine of the angle between the vectors

\[
\cos \theta = \frac{\langle AB \cdot AC \rangle}{\|AB\| \|AC\|}
\]

(5)

\[
ABOF(A) = \text{VAR}_{B,C \in N_k(A)} \left( \frac{\langle AB \cdot AC \rangle}{\|AB\|^2 \|AC\|^2} \right)
\]

(6)

where \(\langle AB \cdot AC \rangle\) is the dot product of vectors \(AB\) and \(AC\) and \(\|AB\|\) and \(\|AC\|\) are the lengths of the respective vectors.

The potential of the associated data point to be an outlier increases as ABOF decreases. As a result, ranking data points can be classified as outliers by applying a subjective definition to an arbitrary ABOF threshold value or ABOF threshold. As shown in Figure 4, the data point representing the head of the angle \(\Theta_1\) was considered an outlier compared to the other points. This means If a point’s ABOF falls below the ABOF threshold, it is categorized as an outlier. Depending on the degree of trust in the accuracy of the data, the ABOF threshold can be described as a percentage quantile of ABOF for all data points.

The ABOD technique was used in this study to filtrate inlier/outlier data considering three levels of assumptions that 10, 15, and 20% of the data are contaminated. The role of the ABOD technique in each level is to determine which are the highest 10, 15, and 20% of the data to be considered outliers.

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5. METHODOLOGY AND DATA

DCA is based on fitting a portion of the historic production data with a selected model. Regression is used to determine the fitting parameters within the model. After the fitting parameters are determined, predicting future production could be extended.

First, 16 different DCA models were compared based on the data size and data quality. For data size, regression was made for each model based on different sizes of the production data (i.e., 20, 40, 60, and 80% of the data) to determine the value of the fitting parameters for each regression step. After that, the models were extended to the end of the production data to compare the performance of each model by increasing the fitted data size, as shown in Figures 5 and 6.

The data used in this study are production data of a shale gas well (well_12). These data were released on SPE’s official website and dedicated to research interests. Table 3 summarizes the characteristics of the well_12 and the reservoir.

Table 3. Characteristics of Well_12

| Specification               | Value        |
|----------------------------|--------------|
| State                      | LA           |
| TVD (ft)                   | 11,258.854   |
| Formation/Reservoir        | HAYNESVILLE SHALE |
| Initial pressure estimate (psi) | 9939        |
| Reservoir temperature (deg F) | 285.21375   |
| Net pay (ft)               | 268.39703    |
| Wellbore diameter (ft)     | 0.7          |
| Porosity                   | 0.088000059  |
| Water saturation           | 0.183792612  |
| Oil saturation             | 0            |
| Gas saturation             | 0.816207388  |
| Gas specific gravity (API) | 30           |
| Condensate gravity (API)   | 30           |
| Dew point pressure (psi)   | 9893.27      |
| Sep. temperature (deg F)   | 100          |
| Sep. pressure (psi)        | 100          |
| Data points                | 4032         |

Figure 6. Methodology of testing the model’s performance with different data qualities.

Figure 7. Actual production data before removing the outliers.

Figure 8. (a) Production data after removing 10% outliers and (b) removed data.
Figure 7 shows the typical production profile of the selected well, while Figures 8–10 show the production profiles after removing 10, 15, and 20% as outliers using the ABOD algorithm, respectively, and the removed data in each step successfully.

From the first look, it could be inferred that Figures 9 and 10 are the same as Figure 7. But with a deep look, we could see how the density of the removed point increased, while the profile of the original data got smoother. This is one of the advantages of the ABOD algorithm. In the beginning, it removes the isolated points. After that, it removes closer by closer points without masking any trends within the production profile.

6. RESULTS AND DISCUSSION

By fitting all models on 20, 40, 60, and 80% of the production data and then extending the prediction to the end of the production profile, Figures 11–14 show the performance of the models based on increasing the data size. The results show that the performance of all models improved by increasing the
data size. It could be easily noticed how models converged closer to the actual flow rate and the cumulative production by increasing the data size. However, some models are more sensitive to the data size than others. For example, K-model, Duong, Wang, and HEHD are less sensitive to the data size compared to other models such as Weng, SEPD, Hesieh, and Pan models.

After removing the outlier from the production data with three different levels (i.e., 10, 15, and 20%) using the ABOD algorithm, we fitted 80% of the production data and extended the prediction to the end of the production profile. Figures 15—17 show the performance of all models after removing the outlier. It could be noticed how the performance of all models improved and converged closer and closer to the actual flow rate and the actual cumulative production. However, some models are more sensitive to the data quality than others. For example, Arps, MLM, K-model, Duong, Wang, and HEHD are less sensitive to the data size compared to other models such as Weng, VDMA, LGM, EEDCA, FDC, Pan, HEHD, Wang, and VDMA model.

Figures 18 and 19 show how the goodness of fitting of all models get improved by increasing the data size and by removing more outlier from the production data. The improvement due to increasing the data size is small to fair, while there is a significant improvement for all models when just removing 10% of the data as outliers. Table 4 summarizes the models’ sensitivities to increasing the data size and...
improving the data quality by removing more outliers, while Table 5 summarizes the prediction reliability of all models according to the data size and data quality.

7. RESEARCH LIMITATIONS

In this research, one well with moderate noise within the production data was tested. Also, one recommended ML algorithm was used to remove the outlier. Although of the clear results, it is recommended to test more than one algorithm from the mentioned ones in Table 3, a large number of wells with different levels of noise and different production modes to conclude which combination of outlier removers and DCA models could be used effectively to improve the production forecasting and reserve estimation.

8. CONCLUSIONS AND RECOMMENDATIONS

In this paper, 20 decline curve models were summarized. The four basic functions of each model were completed analytically. Also, 16 models were compared based on different data sizes.

| Table 4. Decline Curve Models’ Sensitivities to Data Size and Data Quality |
|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|
| sensitivity to data size | sensitivity to data quality |
| less sensitive | moderate sensitive | highly sensitive | less sensitive | moderate sensitive | highly sensitive |
| K-model | Arps | Weng | Arps | Weibull | Weng |
| Duong | MLM | Weibull | MLM | PLE | Hesieh |
| HEHD | T-model | Hesieh | K-model | SEPD | LGM |
| Wang | LGM | SEPD | T-model | EEDCA | Pan Model |
| FDC | EEDCA | Duong | FDC | VDMA |
| Pan model | HEHD | VDMA | Wang |

| Table 5. Decline Curve Models’ Prediction Performance According to Data Size and Data Quality $^{a,b,c}$ |
|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|
| model | after fitting 20% | after fitting 40% | after fitting 60% | after fitting 80% | fitting 80% after removing 10% as outliers | fitting 80% after removing 15% as outliers | fitting 80% after removing 20% as outliers |
| Arps | + | + | + | + | + | + | + |
| MLM | + | + | + | + | + | + | + |
| K-model | + | * | * | * | * | * | * |
| Weng model | - | - | - | - | - | - | - |
| Weibull model | - | - | - | - | - | - | - |
| Hesieh model | - | - | - | - | - | - | - |
| PLE model | - | - | - | * | * | * | * |
| T-model | + | + | + | + | + | + | + |
| SEPD model | - | - | - | - | - | - | - |
| Duong model | + | + | + | + | + | + | + |
| LGM | - | - | - | - | - | - | - |
| EEDCA model | - | - | * | * | - | - | - |
| FDC model | + | + | + | + | + | + | + |
| Pan model | - | - | - | - | - | - | - |
| HEHD model | - | - | * | * | - | - | - |
| Wang model | + | + | * | * | - | - | - |
| VDMA model | - | - | - | - | * | * | * |

$^a$ (+) This means that the model tends to overestimate the reserve. $^b$ (-) This means that the model tends to underestimate the reserve. $^c$ (*) This means that the model tends to be best-estimate the reserve.
The ABOD algorithm was used to remove the outlier from actual production data. The performance of the models was measured before and after removing the outlier. It could be concluded as follows:

1. Generally, some models tend to overestimate the reserve such as Arps, MLM, T-Model, Duong, and FDC, while some others tend to underestimate the reserve such as Weng, Weibull, Hesieh, SEPD, and LGM.

2. The goodness of fitting and the reliability of prediction for almost all models get improved by increasing the data size and improving the data quality.

### Table 6. Expressions for the Gas Flow Rate $q(t)$

| Model Description                        | Expression                                                                                     | No. of Fitting Parameters | Model Structure |
|------------------------------------------|-----------------------------------------------------------------------------------------------|---------------------------|-----------------|
| exponential Arps (1945)                  | $q_i = q_i \exp(-D_i t)$                                                                     | 2                         | exponential     |
| hyperbolic Arps (1945)                   | $q_i = \frac{q_i}{(1 + bD_i t)^{\alpha}}$                                                    | 3                         | power function  |
| harmonic Arps (1945)                     | $q_i = \frac{q_i}{(1 + D_i t)^{\alpha}}$                                                    | 2                         | power function  |
| Matthews–Leikovits model (MLM) (1956)   | $q_i = \frac{1}{(a_{MLM} t / n_{MLM} + 1)^{n_{MLM}}}$                                       | 3                         | power function  |
| K (1970)                                 | $q_i = \frac{b_K}{b_K + t}$                                                                  | 2                         | power function  |
| Weibull (1995)                           | $q_i = q_i \exp\left(-\frac{\rho_{WB}}{\rho_{WB}}\right)$                                   | 3                         | exponential and power functions |
| boundary ‘b’ approach (2000)             | based on Arps models                                                                         | 2                         | exponential function |
| Hesieh (2001)                            | $q_i = q_i e^{-(n_i + m_i) t}$                                                                | 3                         | modified power function |
| multistage approach (2007)               | $q_i = q_{switch} \exp[-D_{switch}(t - t_{switch})]$                                         | 2                         | exponential function |
| modified Arps approach (2008)            | $q_i = \frac{(q_i D_{i=0})}{[1 + b(D_i q_i(t - t_{switch}))^{\alpha}]}$                    | 3                         | power function  |
| power low exponential (PLE) (2008)       | $D_m \neq 0$                                                                                 | 4                         | exponential and power functions |
| $D_m = 0$                                | $q_i(t) = q_i e^{-D_i(t-B_i)}$                                                                | 3                         | power function  |
| T-model (2009)                           | $q_i = G_i a_i t^{\chi+1} \exp\left[\frac{a_i}{b_i + 1} t^{\chi+1}\right]$                  | 3                         | power function  |
| stretched exponential production decline (SEPD) (2010) | $q_i = q_i \exp[-(t/t)^{\alpha_{SEPD}}]$                                                     | 3                         | exponential and power functions |
| Duong (2010, 2011)                       | $q_i = \frac{a_0}{1 - m_0} e^{-(t-\tau_0)\alpha}$                                           | 3                         | power function  |
| logistic growth model (LGM) (2011)       | $q_i = \frac{q_i}{(a_{LGM} \exp^{\alpha_{LGM}} e^{-t})}$                                    | 3                         | power function  |
| extended exponential DCA (EEDCA) (2015)  | $q_i = q_i \exp[-(\beta_i + \rho e^{-\epsilon}) t]$                                         | 4                         | exponential and power functions |
| fractional decline curve (FDC) (2016)    | $q_i = q_i \exp\left[-\frac{q_i}{t^\alpha}\right]$                                          | 3                         | power function  |
| Wang (2017)                              | $q_i = q_i \exp[-(\alpha_i \ln t)^\beta]$                                                   | 2                         | exponential function modified by the logarithmic function |
| variable decline modified Arps (VDMA) (2018) | $q_i = q_i \exp[-D_i (t^{1-m_{VDMA}})]$                                                      | 3                         | exponential function modified by the power function |
6. The advantage of using the ABOD algorithm is that it smooths the production profile without removing clear events through the profile. However, it is recommended to compare other algorithms with different data characteristics to determine the most effective algorithm for DCA.

### APPENDIX 1. DECLINE CURVE MODELS: APPROACHES, CHARACTERISTICS, AND REFERENCES

Expressions for the gas flow rate \( q \) as a function of time \( t \) and the numbers of fitting parameters for the various models are given in Table 6. Expressions for the decline rate \( D \) and the decline exponent \( b \) as functions of \( t \) are given in Table 7.

#### Table 7. Expressions for the Decline Rate \( D(t) \) and Decline Exponent \( b \)

| Model                        | Expression for \( D(t) \) | Expression for \( b \) |
|------------------------------|---------------------------|------------------------|
| exponential Arps (1945)     | \( D = D_t \)             | \( b = 0 \)            |
| hyperbolic Arps (1945)      | \( D = \frac{D_t}{1 + D_t b t} \) | \( 0 < b < 1 \)        |
| harmonic Arps (1945)        | \( D = \frac{a_n}{an + at + n} \) | \( b = 1 \)            |
| MLM (1956)                  | \( D = \frac{1}{b_X + t} \) | \( b = 1 \)            |
| K (1970)                    | \( D = \frac{1}{b_X + t} \) | \( b = 1 \)            |
| Weng (1984)                 | \( D = \frac{1}{b_{CW}} \cdot \frac{a_{CW}}{t} \) | \( b = \frac{b_{CW} \cdot a_{CW}}{(t - b_{CW} \cdot a_{CW})^2} \) |
| Weibull (1995)              | \( D = \frac{n_{WB}^{1-n_{WB}}}{a_{WB}} \) | \( b = n_{WB} \)        |
| boundary “b” approach (2000) | \( D = \frac{D_t}{1 + D_t b t} \) | \( b = 1 - \frac{(\beta \cdot \gamma)}{2} \cdot \frac{m(p) - m(p_m)}{\left( \frac{b}{\gamma} - z_n \right)} \) |
| Hsieh (2001)                | \( D = m_i \ln(t) - \frac{m_{1i} - m_{0i} t}{t} \) | \( b = \frac{m_{1i} t - m_{0i}}{[m_{0i} t \ln(t) + m_{1i} t + m_{0i}]} \) |
| multistage approach (2007)  | \( D = \frac{\Delta q_i}{q_i (\Delta t)} \) | \( b = \frac{1}{\Delta t} \left( -\frac{\Delta q_i}{\Delta t} \right) \) |
| modified Arps approach (2008) | \( D = \frac{\Delta q_i}{q_i (\Delta t)} \) | \( b = \frac{1}{\Delta t} \left( -\frac{\Delta q_i}{\Delta t} \right) \) |
| PLE (2008)                  | \( D = D_m t^{-\alpha} \) | \( b = D_m + D_t^{(1-\alpha)} \) |
| T-model (2009)              | \( D = \frac{a_t t^{b_t + 1} + b_t}{t} \) | \( b = D_t^{(1-\alpha)} \) |
| SEPD (2010)                 | \( D = \frac{n_{SEPD}^{1-n_{SEPD}}}{e^{n_{SEPD}}} \) | \( b = e^{n_{SEPD} - n_{SEPD}} \) |
| Duong (2010, 2011)          | \( D = \frac{m t}{t - \frac{a}{t}} \) | \( b = m \left[ t^{(1 - a\tau)} - a\tau \right] \) |
| LGM (2011)                  | \( D = \frac{a(1 - n) (1 + n) t^n}{(a + t^n)} \) | \( b = a\tau \left[ n \tau - n (n - 1) t^n + (1 + n) \tau t^n \right] \) |
| EEDCA (2015)                | \( D = \beta_1 + \beta_2 (-n_1 \tau^{1 - n_1} + c^{1 - n_1}) \) | \( b = \beta_1 \left[ (1 - n_1 \tau^{1 - n_1} - n_1 (n_1 \tau^{1 - n_1}-1 + n_1 \tau^{1 - n_1 n_1})) \right] \) |
| FDC (2016)                  | \( D = \frac{\alpha}{t} \) | \( b = \alpha \) |
| Wang (2017)                 | \( D = \frac{2 \lambda_w}{t} \) | \( b = \frac{1}{\lambda_w} \) |
| VDMA (2018)                 | \( D = D_t e^{-n_{VDMA} (1 + n_{VDMA})} \) | \( b = \frac{n_{VDMA} e^{n_{VDMA} - 1}}{D_t (n_{VDMA} + 1)} \) |
Table 8. Strengths and Weaknesses of the Models

| Model                     | Model strengths                                                                 | Model weaknesses                                                                 |
|---------------------------|---------------------------------------------------------------------------------|----------------------------------------------------------------------------------|
| Arps (1945)               | simple to apply.                                                                 | overestimate the EUR.                                                            |
| MLM (1956)                | applied to depletion reservoirs with low pressure where gravity is the only driving mechanism. | not consider the flow behavior of shale gas.                                    |
| K (1970)                  | simple to apply.                                                                 | not consider the flow behavior.                                                  |
| Weng (1984)               | simple with only two fitting parameters.                                        | underestimate the EUR.                                                           |
| Weibull (1995)            | simple with only two fitting parameters.                                        | not consider the flow behavior.                                                  |
| boundary “b” approach (2000) | consider the flow behavior of shale gas.                                      | not consider the flow behavior.                                                  |
| Hsieh (2001)              | consider the flow behavior of shale gas and the reservoir’s unique properties. | underestimate the EUR with small production data size even with large data size. |
| multistage approach (2007) | consider the flow behavior with the same structure of Arps.                    | switch point ($D_{\text{switch}}$ at $t_{\text{switch}}$) is still hard to be determined. |
| modified Arps approach (2008) | consider the flow behavior and applicable for shale gas.                   | multiple steps are required to be performed.                                    |
| PLE (2008)                | specially developed to match shale gas behavior.                               | four fitting parameters are being regressed on.                                 |
| $D_m \neq 0$              |                                                                                  |                                                                                  |
| $D_m = 0$                 |                                                                                  |                                                                                  |
| T-model (2009)            | applicable on shale gas and could consider the flow behavior                    | overestimate the EUR.                                                            |
| SEPD (2010)               | applicable to shale gas and consider the flow behavior of shale gas.           | underestimated EUR with small production data size.                             |
| Duong (2010, 2011)        | applicable to shale gas, consider the flow behavior, and fit data from various shale plays very well. | in the case of water breakthrough, the model parameters are overestimated.        |
| LGM (2011)                | simple and consider the flow behavior of shale gas.                            | based on growth and this requires a certain data size to be applicable.          |
| EEDCA (2015)              | applicable with both early and late time production.                           | $J_1$ is a fixed value as it has no effect on fitting the curve.                  |
| FDC (2016)                | applicable to shale gas production behavior, especially late long tail behavior. | EUR is underestimated.                                                           |
| Wang (2017)               | based on the same assumptions of Duong but more general.                       | compared to other models that consider the flow behavior of the shale gas, EUR is slightly overestimated. |
| VDMA (2018)               | applicable to shale gas production behavior and simple as it is a modification of Arps exponential model. | compared to other models that consider the flow behavior of the shale gas, EUR is slightly underestimated. |
| Model                          | Expression                                                                 | Reference(s) |
|-------------------------------|---------------------------------------------------------------------------|---------------|
| exponential Arps (1945)       | \[ G_p(t) = \frac{(q_i - q_f)}{D} \]                                      | 53            |
| hyperbolic Arps (1945)        | \[ G_p(t) = \frac{q_i}{D (1 - b)} \left( \frac{q_i}{q_f} \right) \left( 1 - \frac{q_i}{q_f} \right) \] | 53            |
| harmonic Arps (1945)          | \[ G_p(t) = \frac{q_i}{D} \ln \left( \frac{q_f}{q_i} \right) \]            | 53            |
| MLM (1956)                    | \[ G_p = \frac{q_i}{\alpha} (\alpha t + n)^{n+1} \left( \frac{\alpha t + n}{\alpha t + n + 1} \right) \] | 22, 54        |
| K (1970)                      | \[ G_p = q_i \cdot b_c^2 \ln(b_k + t) \]                                   | 22            |
| Weng (1984)                   | \[ G_p = b_{GW}^{-\omega+1} q_t \Gamma \left( d_{GW} + 1, \frac{t}{b_{GW}} \right) \] | 55            |
| Weibull (1995)                | \[ G_p = q_i \cdot \frac{1}{\nu_{WB}} \Gamma \left( 1 - \frac{1}{\nu_{WB}}, \frac{\nu_{WB}}{\nu_{WB}} \right) \] | 56            |
| boundary "b" approach (2000)  | \[ G_p(t) = \frac{q_i}{D (1 - b)} \left( \frac{q_i}{q_f} \right) \left( 1 - \frac{q_i}{q_f} \right) \] | 7, 57         |
| Hsieh (2001)                  | \[ G_p = \sum q_i \]                                                       | 22, 58        |
| multistage approach (2007)    | \[ G_p(t) = \frac{q_i}{D (1 - b)} \left( \frac{q_i}{q_f} \right) \left( 1 - \frac{q_i}{q_f} \right) \] | 7, 59, 60     |
| modified Arps approach (2008) | \[ G_p = \left( \frac{q_i (q_{G_{inv}})}{D (q_{G_{inv}} - 1)} \right) \left( \frac{q_i}{q_{G_{inv}}} \right)^{(1-b)} \] | 7, 61         |
| PLE (2008)                    | \[ G_p = \sum q_i \]                                                       | 62, 63        |
| T-model (2009)                | \[ G_p = G_k \exp \left( \frac{a}{b+1} \right)^{(b+1)} \]                | 7, 64         |
| SEPD (2010)                   | \[ G_p = q \cdot \left( \Gamma \left( \frac{1}{m} \right) - \Gamma \left( \frac{1}{m} \right) - \left( \frac{1}{m} \right)^{\frac{a}{m}} \right) \] | 65            |
| Duong (2010, 2011)            | \[ G_p = \frac{a}{a-m} \exp \left( \frac{a}{1-m} \right) \left( t^{1-n} - 1 \right) \] | 66            |
| LGM (2011)                    | \[ G_p = (EUR)^{a^n} \]                                                     | 67, 68        |
| EEDCA (2015)                  | \[ G_p = \sum q_i \]                                                       | 69            |
| FDC (2016)                    | \[ G_p(t) = \sum_{k=1}^{k} q_i = \sum_{i=1}^{k} m E_i(-\lambda_t^n) \]   | 11            |
| Wang (2017)                   | \[ G_p = \frac{q_t^{2+\lambda_t+1}}{2_{\lambda_t} + 1} \]                | 70            |
| VDMA (2018)                   | \[ G_p = \frac{D_{1/\nu_{VDMA}}^{1-\phi} q_i \Gamma \left( \frac{1}{\nu_{VDMA} - 1}, D_{1/\nu_{VDMA}}^{1-\phi} \right) \cdot \Gamma \left( \frac{1}{\nu_{VDMA} - 1}, D_{1/\nu_{VDMA}}^{1-\phi} \right)\} \cdot \nu_{VDMA} - 1 \] | 71            |
Strengths and weaknesses of the models are presented in Table 8. Expressions for the cumulative production $G_\text{rp}$ as a function of $t$ and references are given in Table 9. Symbols with the subscript of the model’s initials are the fitting parameters of the model.

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**Notes**

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### ACRONYMS

- ABOD = angle-based outlier detection
- ABOF = angle-based outlier factor
- BDF = boundary-dominated flow
- BHP = bottom hole pressure
- DCA = decline curve analysis
- EEDCA = extended exponential decline curve
- EUR = estimated ultimate recovery
- FDC = fractional decline curve
- HEHD = hyperbolic–exponential hybrid decline
- $kNN = k$-nearest neighbors
- LGM = logistic growth model
- MLE = maximum likelihood estimation
- MLM = Matthews–Lefkovits model
- OLS = ordinary least-squares
- PLE = power-law equation
- SEPD = stretched exponential decline model
- SVM = supported vector machine
- VDMA = variable decline modified Arps
- WLS = weighted least-squares

### Nomenclature

- $b =$ decline curve exponent
- $D =$ decline rate (day$^{-1}$)
- $D_i =$ initial decline rate (day$^{-1}$)
- $G_\text{rp} =$ gas cumulative production (Mscf)
- $q =$ gas flow rate (Mscf/day)
- $t =$ time (day)

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