Candidates for dark matter in the
SU(3)$_C$ $\otimes$ SU(3)$_L$ $\otimes$ U(1)$_N$ models

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It has recently been pointed out that the 511 keV emission line detected by INTEGRAL/SPI from the bulge of our galaxy could be explained by annihilations of light dark matter particles into $e^+ e^-$. We present the possibility that dark matter could be made of scalar candidates, namely, of the Higgs bosons in the models based on SU(3)$_C$ $\otimes$ SU(3)$_L$ $\otimes$ U(1)$_N$ (3-3-1) gauge group. These particles are singlet of the SU(2)$_L$ $\otimes$ U(1)$_Y$ group, so they do not interact with the ordinary particles, except the Higgs boson in the standard model. The Spergel-Steinhardt condition for self-interacting dark matter gives a bound on the mass of the candidates to be a few MeVs. Besides the scalar candidates, which exist in both non-SUSY and SUSY 3-3-1 models with right-handed neutrinos, the spin $\frac{1}{2}$ candidate exists in a variant 3-3-1 version with exotic neutral lepton. In contrast to the singlet models, where an extra symmetry must be imposed to account the stability of the dark matter, here the decay of the candidates is automatically forbidden in all orders of perturbative expansion. This is because of the following feature: these scalars are singlets, i.e., in bottom of the Higgs triplet. Therefore, the standard model fermions and the standard gauge bosons cannot couple with them.

1 Introduction

Observation of the cosmic microwave background, the primordial abundances of light elements and large scale structure have revealed that a great deal of the mass of our universe consists of dark matter (DM). It is an amazing fact that even as our understanding of cosmology progresses by leaps and bounds, we remain almost completely ignorant about the nature of most of the matter in the universe [1]. It has recently been pointed out that the 511 keV emission line detected by INTEGRAL/SPI from the bulge of our galaxy could be explained by annihilations of light dark matter particles into $e^+ e^-$. The nature of DM is still a challenging question in cosmology.
Cosmological models with a mixture of roughly 35% collisionless cold DM such as axions, WIMPs, or any other candidate interacting through the weak and gravitational forces only, and 65% vacuum energy or quintessence match observation of the cosmic microwave background and large scale structure on extra-galactic scales with remarkable accuracy [2, 3]. It is known that only a fraction of the dark matter can be made of ordinary baryons and its enormous amount has unknown, nonbaryonic origin [4]. Until a few years ago, the more satisfactory cosmological scenarios were those ones composed of ordinary matter, cold DM and a contribution associated with the cosmological constant. To be consistent with inflationary cosmology, the spectrum of density fluctuations would be nearly scale-invariant and adiabatic. However, in recent years it has been pointed out that the conventional models of collisionless cold DM lead to problems with regard to galactic structures. They were only able to fit the observations on large scales (≫ 1 Mpc). Also, $N$-body simulations in these models result in a central singularity of the galactic halos [5] with a large number of sub-halos [6], which are in conflict with astronomical observations. A number of other inconsistencies are discussed in Refs. [7, 8]. Thus, the cold DM model is not able to explain observations on scales smaller than a few Mpc. It has recently been shown that an elegant way to avoid these problems is to assume the so called self-interacting dark matter [9]. One should notice that, in spite of all, self-interacting models lead to spherical halo centers in clusters, which is not in agreement with ellipsoidal centers indicated by strong gravitational lens observations [10] and by Chandra observations [11]. However, self-interacting dark matter models are self-motivated as alternative models. It is a well-accepted fact that the plausible candidates for DM are elementary particles. The key property of these particles is that, they must have a large scattering cross-section and negligible annihilation or dissipation. The Spergel-Steinhard model has motivated many follow-up studies [4, 12, 13]. Several authors have proposed models in which a specific scalar singlet that satisfies the self-interacting dark matter properties is introduced in the standard model (SM) in an ad hoc way [4, 13]. The SM offers no options for DM. The first gauge model for SIDM is the extended SM based on the $SU(3)_C \otimes SU(3)_L \otimes U(1)_N$ (3-3-1) gauge group [14,15]. The 3-3-1 model founded by Fregolente and Tonasse [16] to be SIDM differs from another one founded by Long and Lan [17]. Models with $SU(3)_C \otimes SU(3)_L \otimes U(1)_N$ gauge symmetry (called 3-3-1 models for short) are interesting possibilities for the physics at the TeV scale [14,15]. At low energies they coincide with the standard model and some of them give at least partial explanation to some fundamental questions that are accommodated but not explained by the standard model. For instance, in order to cancel the triangle anomalies, together with asymptotic freedom in QCD, the model predicts that the number of generations must be three and only three; (ii) the model of Ref. [14] predicts that $(g'/g)^2 = \sin^2 \theta_W / (1 - 4 \sin^2 \theta_W)$, thus there is a Landau pole at the energy scale $\mu$ at which $\sin^2 \theta_W (\mu) = 1/4$. According to recent calculations $\mu \sim 4$ TeV [18, 19]; (iii) the quantization of the electric charge [20] and the vectorial
character of the electromagnetic interactions \cite{21} do not depend on the nature of the neutrinos i.e., if they are Dirac or Majorana particles; (iv) as a consequence of item ii) above, the model possesses perturbative $\mathcal{N} = 1$ supersymmetry naturally at the $\mu$ scale \cite{22,23}; (v) the Peccei-Quinn \cite{24} symmetry occurs naturally in these models \cite{25}; (vi) since one generation of quarks is treated differently from the others this may be lead to a natural explanation for the large mass of the top quarks \cite{26}. Moreover, if right-handed neutrinos are considered transforming non-trivially \cite{15}, 3-3-1 models \cite{14} can be embedded in a model with 3-4-1 gauge symmetry in which leptons transform as $(\nu_l, l, l^c)_L \sim (1, 4, 0)$ under each gauge factors \cite{27,28}. The $SU(3)_L$ symmetry is possibly the largest symmetry involving the known leptons (and $SU(4)_L$ if right-handed neutrinos do really exist). This make 3-3-1 or 3-4-1 models interesting by their own, and it has been the source of interest recently \cite{29} because it requires that the number of fermion families be a multiple of the quark color in order to cancel anomalies, which suggest a path to the solution of the flavor problem. The Peccei-Quinn \cite{24} symmetry occurs naturally in these models \cite{25}. Another important feature of these models is that the $SU(2)_L$ group is totally embedded in $SU(3)_L$. A subject that has not been given much attention by particle physicists in the past, could prove to be a remarkable powerful and precise probe of the properties of dark matter. This paper is organized as follows: In Sec. 2 we briefly introduce necessary elements of the non-SUSY 3-3-1 models, and the candidates for dark matter. Sec. 3 is devoted to supersymmetric 3-3-1 model with right-handed neutrinos. We summarize our result and make conclusions in the last section. The aim of this review is to show that the 3-3-1 models contains candidates for dark matter. The outline of this paper is as follows. In Sec. 3, the 3-3-1 models are briefly recalled. Properties of DM are introduced. The candidates for SIDM in four 3-3-1 models are explored. The last section is devoted for conclusions.

2 The non-SUSY 3-3-1 models

To frame the context, it is appropriate to recall briefly some relevant features of the 3 - 3 - 1 models \cite{14,15}. We first introduce the minimal version proposed by Pisano, Pleitez and Frampton \cite{14}

2.1 The 3-3-1 model with exotic lepton

The model treats the leptons as $SU(3)_L$ triplets \cite{30}

$$ f^a_L = \left( \begin{array}{c} \nu^a_L \\ e^a_L \\ P^a \end{array} \right) \sim (1, 3, 0), \quad e^a_R \sim (1, 1, -1), \quad P^a_R \sim (1, 1, +1), $$

where $a = 1, 2, 3$ is the generation index. Each charged left-handed fermion field has its right-handed counterpart transforming as a singlet of the $SU(3)_L$ group. Two
of the three quark generations transform as antitriplets and the third generation is
treated differently - it belongs to a triplet:

\[ Q_{iL} = \begin{pmatrix} d_{iL} \\ -u_{iL} \\ D_{iL} \end{pmatrix} \sim (3, \bar{3}, -1/3), \tag{2} \]

\[ u_{iR} \sim (3, 1, 2/3), \quad d_{iR} \sim (3, 1, -1/3), \quad D_{iR} \sim (3, 1, -4/3), \quad i = 1, 2, \]

\[ Q_{3L} = \begin{pmatrix} u_{3L} \\ d_{3L} \\ T_L \end{pmatrix} \sim (3, 3, 2/3), \tag{3} \]

\[ u_{3R} \sim (3, 1, 2/3), \quad d_{3R} \sim (3, 1, -1/3), \quad T_R \sim (3, 1, 5/3). \]

Of the nine gauge bosons \( W^a (a = 1, 2, \ldots, 8) \) and \( B \) of \( SU(3)_L \) and \( U(1)_N \), four are light: the photon \( A \), \( Z \) and \( W^\pm \). The remaining five correspond to new heavy gauge bosons \( Z_2 \), \( Y^\pm \) and the doubly charged bileptons \( X^{\pm\pm} \). Symmetry breaking and fermion mass generation can be achieved by three scalar \( SU(3)_L \) triplets

\[ SU(3)_C \otimes SU(3)_L \otimes U(1)_N \]

\[ \downarrow \langle \chi \rangle \]

\[ SU(3)_C \otimes SU(2)_L \otimes U(1)_Y \]

\[ \downarrow \langle \eta \rangle, \langle \rho \rangle \]

\[ SU(3)_C \otimes U(1)_Q, \]

where the minimally required scalar multiplets are summarized as

\[ \chi = \begin{pmatrix} \chi^- \\ \chi^- \\ \chi^0 \end{pmatrix} \sim (1, 3, -1), \]

\[ \eta = \begin{pmatrix} \eta^0 \\ \eta_1^- \\ \eta_2^+ \end{pmatrix} \sim (1, 3, 0), \]

\[ \rho = \begin{pmatrix} \rho^+ \\ \rho^0 \\ \rho^{++} \end{pmatrix} \sim (1, 3, +1). \tag{4} \]

The vacuum expectation value (VEV) \( \langle \chi^T \rangle = (0, 0, w/\sqrt{2}) \) yields masses for the exotic quarks, the heavy neutral gauge boson \( (Z_2) \) and two charged gauge bosons \( (X^{++}, Y^+) \). The masses of the standard gauge bosons and the ordinary fermions are related to the VEVs of the other scalar fields, \( \langle \eta^0 \rangle = v/\sqrt{2}, \langle \rho^0 \rangle = u/\sqrt{2} \). In order to be consistent with the low energy phenomenology we have to assume that
$u \gg v, \omega$. By matching the gauge coupling constants we get a relation between $g$ and $g_N$ – the couplings associated with SU(3)$_L$ and U(1)$_N$, respectively:

$$\frac{g^2_N}{g^2} = \frac{6 s_W^2 (M_{Z_2})}{1 - 4 s_W^2 (M_{Z_2})},$$

(5)

where $e = g s_W$ is the same as in the SM. The most economical, gauge invariant and renormalizable Higgs potential is

$$V (\eta, \rho, \chi) = \mu^2_1 \eta \eta + \mu^2_2 \rho \rho + \mu^2_3 \chi \chi + a_1 (\eta^\dagger \eta)^2 + a_2 (\rho^\dagger \rho)^2 + a_3 (\chi^\dagger \chi)^2$$

$$+ (\eta^\dagger \eta) (a_4 \rho^\dagger \rho + a_5 \chi^\dagger \chi) + a_6 (\rho^\dagger \rho) (\chi^\dagger \chi) + a_7 (\rho^\dagger \eta) (\eta^\dagger \rho)$$

$$+ a_8 (\chi^\dagger \eta) (\eta^\dagger \chi) + a_9 (\rho^\dagger \chi) (\chi^\dagger \rho) + \frac{1}{2} (f \epsilon^{ijk} \eta_i \rho_j \chi_k + H. c.),$$

(6)

Here the $\mu$'s and $f$ are coupling constants with dimension of mass with $a_3 < 0$ and $f < 0$ from the positivity of the scalar masses [31].

Symmetry breaking is initiated when the scalar neutral fields are shifted as $\phi = v \phi + \xi \phi + i \zeta \phi$, with $\phi = \eta^0, \rho^0, \chi^0$. The details of the physical spectrum of the neutral scalar sector are crucial for our results. It is given in Refs. [31] and we summarize them here. Firstly we notice that real part of the shifted fields leads to the three massive physical scalar fields $H^0_1, H^0_2, H^0_3$ defined by

$$\begin{pmatrix} \xi_\eta \\ \xi_\rho \end{pmatrix} \approx \frac{1}{v_W} \begin{pmatrix} v & u \\ u & -v \end{pmatrix} \begin{pmatrix} H^0_1 \\ H^0_2 \end{pmatrix}, \quad \xi_\chi \approx H^0_3,$$

(7)

where we are using $w \gg v, u$. The scalar $H^0_1$ is the one that we can identify with the standard model Higgs, since its squared mass,

$$m^2_1 \approx 4 \frac{a_2 v^4 - a_1 v^4}{v^2 - u^2},$$

(8)

carries no any feature from the 3-3-1 breakdown to the standard model. On the other hand $H^0_3$, with squared mass

$$m^2_3 \approx -4 a_3 w^2,$$

(9)

is a typical 3-3-1 scalar. So, there is no any massless Goldstone boson rising from the real part of the neutral sector. On the other hand, from the imaginary part we have two Goldstone and one massive physical state $h^0$ with eigenstate

$$\zeta_\chi \approx h^0,$$

(10)

and squared mass

$$m^2_h = -f^2 v_W^2 w^2 + v^2 u^2.$$

(11)
It is important to notice that $\zeta_\eta$ and $\zeta_\rho$ are pure massless Goldstone states. The approximation in Eqs. (7), (8), (9) and (10), is valid for $w \gg v, u$. This condition leads to relations among the parameters of the scalar potential (6). One of them, which enters in the $H_0^h h_0^h$ interaction, is

$$a_5v^2 + 2a_6u^2 \approx -\frac{vu}{2}$$

We must consider also the matter coupling through the scalar fields. In the model of Ref. [30], the full Yukawa Lagrangians that must be considered are

$$\mathcal{L}_\ell = -\sum_{ab} \left( \frac{1}{2} \epsilon^{ijk} G^{(v)}_{ab} \psi_{aL}^c \bar{\psi}_{bL} \eta_k + G^{(e)}_{ab} \psi_{aL}^c \bar{\psi}_{bL} \rho - G^{(P)}_{ab} \psi_{aL}^c \bar{\psi}_{bL} \chi \right) + \text{H. c.}$$

$$\mathcal{L}_Q = \sum_{b} \left( G^{(U)}_{1b} \bar{U}_{bR} \eta + G^{(D)}_{1b} \bar{D}_{bR} \rho + G^{(J)} J_{1R} \chi \right) +$$

$$\sum_{\alpha} \bar{Q}_{aL} \left( F^{(U)}_{ab} \bar{U}_{bR} \eta^* + F^{(D)}_{ab} \bar{D}_{bR} \rho^* + \sum_{\beta} F^{(J)}_{ab} J_{\beta R} \chi^* \right) + \text{H. c.},$$

where $G^{(v)}_{ab}, G^{(e)}_{ab}, G^{(P)}_{ab}, G^{(U)}_{1b}, F^{(U)}_{ab}, G^{(D)}_{1b}, F^{(D)}_{ab}, G^{(J)}$ and $F^{(J)}_{ab}$ are the Yukawa coupling constants. $\eta^*, \rho^*$ and $\chi^*$ denote the $\eta, \rho$ and $\chi$ antiparticle fields, respectively. The main properties that a good dark matter candidate must satisfy are stability and neutrality. Therefore, we go to the scalar sector of the model, more specifically to the neutral scalars, and we examine whether any of them can be stable and in addition whether they can satisfy the self-interacting dark matter criterions [9]. In addition, one should notice that such dark matter particle must not overpopulate the Universe. On the other hand, since our dark matter particle is not imposed arbitrarily to solve this specific problem, we must check that the necessary values of the parameters do not spoil the other bounds of the model.

We can check through a direct calculation by employing the Lagrangians (6), (13) and (14) and by using the eigenstates (7) and (10) that the Higgs scalar $h^0$ and $H_3^0$ can, in principle, satisfy the criterions above. Remarkably they do not interact directly with any standard model field except for the standard Higgs $H_1^0$. However, $h^0$ must be favored, since we have checked that it is easier to obtain a large scattering cross section for it, relative to $H_3^0$, by a convenient choice of the parameters. In contrast to the singlet models of the Refs. [4, 13], where an extra symmetry must be imposed to account the stability of the dark matter, here the decay of the $h^0$ scalar is automatically forbidden in all orders of perturbative expansion. This is because of the following features: i) this scalar comes from the triplet $\chi$, the one that induces the spontaneous symmetry breaking of the 3-3-1 model to the standard model. Therefore, the standard model fermions and the standard gauge bosons cannot couple with $h^0$. ii) the $h^0$ scalar comes from the imaginary part of the Higgs triplet $\chi$. As we mentioned above, the imaginary parts of $\eta$ and $\rho$ are pure massless Goldstone bosons. Therefore, there is not physical scalar fields which can mix with
$h^0$. So, the only interactions of $h^0$ come from the scalar potential and they are $H_0^3h^0h^0$ and $H_0^1h^0h^0$. This latter has strength $2i(a_5v^2 + a_6u^2)/v_W \equiv 2i \Theta$. We can check also that $h^0$ does not interact with other exotic particle.

Hence, if $v \sim u \sim (100 - 200)$ GeV and $-1 \leq a_5 \sim a_6 \leq 1$, the $h^0$ can interact only weakly with ordinary matter through the Higgs boson of the standard model $H_0^1$. The relevant quartic interaction for scattering is $h^0h^0h^0h^0$, whose strength is $-i a_3$. Other quartic interactions evolving $h^0$ and other neutral scalars are proportional to $1/\sqrt{w}$ and so we neglect them. The cross section of the process $h^0h^0 \rightarrow h^0h^0$ via the quartic interaction is

$$\sigma = \frac{a_2^3}{64\pi m_h^2}.$$  

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The contribution of the trilinear interactions via $H_0^1$ and $H_0^3$ exchange are negligible. There is no other contribution to the process involving the exchange of vector or scalar bosons. A self-interacting dark matter candidate must have mean free path $\Lambda = 1/n\sigma$ in the range $1 \text{ kpc} < \Lambda < 1 \text{ Mpc}$, where $n = \rho/m_h$ is the number density of the $h^0$ scalar and $\rho$ is its density at the solar radius [6]. Therefore, with $a_3 = -1$, $-0.208 \times 10^{-7}$ GeV $\leq f \leq -0.112 \times 10^{-6}$ GeV, $w = 1000$ GeV, $u = 195$ GeV and $\rho = 0.4$ GeV/cm$^3$, we obtain the required Spergel-Steinhardt bound, i.e., $2 \times 10^3$ GeV$^{-3} \leq \sigma/m_h \leq 3 \times 10^4$ GeV$^{-3}$ [9].

With this set of parameter values, we see from Eq. (11) that $5.5 \text{ MeV} \leq m_h \leq 29 \text{ MeV}$. This means that our dark matter particle is non-relativistic in the decoupling era (decoupling temperature $\sim 1$ eV) and, for a standard model Higgs boson mass $\sim 100$ GeV [32], it is produced by a thermal equilibrium density of the standard Higgs scalar to $h^0h^0$ pairs [13]. The density of the $h^0$ scalar from the $H_0^1$ decay can be obtained following the standard procedure, i.e., we must solve the Boltzmann equation

$$\frac{dn_h}{dt} + 3Hn_h = \langle \Gamma_H \rangle n_H^{(\text{eq})},$$  

where $n_h$ is the number density of the $h^0$ scalar at the time $t$, $H$ is the Hubble expansion rate,

$$\Gamma_H = \frac{\Theta^2}{4\pi E}$$  

is the decay rate for the $H_0^1$ with energy $E$ and

$$n_H^{(\text{eq})} = \frac{1}{2\pi^2} \int_{m_1}^{\infty} \frac{E \sqrt{E^2 - m_1^2}}{e^{E/T} - 1}$$  

is the thermal equilibrium density of the standard $H_0^1$ at temperature $T$ [33]. We are using the condition that the temperature is less than the electroweak phase transition $T_{\text{EW}} \geq 1.5m_1$ [13]. The thermal average of the decay rate is given by

$$\langle \Gamma_H \rangle = \frac{\alpha \Theta^2}{8\pi^3 n_H^{(\text{eq})} e^{m_1/T}}.$$  

where $\alpha$ is an integration parameter that can be taken to be 1.87 [13]. We define $\beta \equiv m_h/T^3$ and in the radiation-dominated era we write the evolution equation (15) as

$$\frac{d\beta}{dT} = -\frac{(\Gamma_H)_{\beta}^{(\text{eq})}}{K T^3} = -\frac{\alpha}{8\pi^2 K e m_1/T} \left( \frac{\Theta}{T^2} \right)^2,$$

(19)

where $K^2 = \frac{4\pi^3 g(T)}{45 m_{\text{Pl}}^2}$, $\beta^{(\text{eq})} = m_h^{(\text{eq})}/T^3$ is the $\beta$ parameter in the thermal equilibrium, $m_{\text{Pl}} = 1.2 \times 10^{19}$ GeV is the Planck mass and $g(T) = g_B + 7g_F/8 = 136.25$ for the model of the Ref. [30]. $g_B$ and $g_F$ are the relativistic bosonic and fermionic degrees of freedom, respectively. Here we are taking $T = m_1$ since this regime gives the larger contribution to $\beta$ [13]. Hence,

$$\beta = \frac{\alpha \Theta^2}{4\pi^3 K m_1^3}.$$

(20)

Now, the cosmic density of the $h^0$ scalar is

$$\Omega_h = 2g(T) T_\gamma^3 \frac{m_h \beta}{\rho_c g(T)},$$

(21)

where $T_\gamma = 2.4 \times 10^{-4}$ eV is the present photon temperature, $g(T_\gamma) = 2$ is the photon degree of freedom and $\rho_c = 7.5 \times 10^{-47} h^2$, with $h = 0.71$, being the critical density of the Universe. Let us take $m_h = 7.75$ MeV, $v = 174$ GeV, $a_5 = 0.65$, $-a_6 = 0.38$ (actually in our calculations, we have used a better precision for $a_5$ and $a_6$) and $m_1 = 150$ GeV. Thus, from Eqs. (20) and (21) we obtain $\Omega_h = 0.3$. Therefore, without imposing any new fields or symmetries, the 3-3-1 model possesses a scalar field that can satisfy all the properties required for the self-interacting dark matter and that does not overpopulate the Universe.

The candidate for self-interacting dark matter that we propose here differs from the singlet models of Refs. [4, 13] in an important point. As we have discussed above it comes from a gauge model proposed with another motivation that has an independent phenomenology. Therefore, the values of the parameters that we impose here must not spoil the preexisting bounds. We can obtain $m_1 \approx 150$ GeV from Eq. (8) with $a_1 = 1.2$ and $a_2 = 0.36$. From Eq. (9) we have $m_3 = 1$ TeV. On the other hand, one should notice that $m_h$ has a small value since $-f \sim (10^{-7} - 10^{-6})$ GeV and $u \sim 195$ GeV. However, $h^0$ does not couple to the particles of the standard model for the Higgs boson. Thus, it evades the present accelerator limits. The constants $a_5$ and $a_6$ do not enter in the masses of the particles of the model and so, it is free in this work [31].

2.2 The 3-3-1 model with right-handed neutrinos

In this model the leptons are in triplets, and the third member is a RH neutrino:

$$f^a_L = (\nu^a_L, e^a_L, (\nu^a_L)^T) \sim (1, 3, -1/3), e^a_R \sim (1, 1, -1).$$

(22)
Each charged left-handed fermion field has its right-handed counterpart transforming as a singlet of the SU(3)$_L$ group. The first two generations of quarks are in antitriplets while the third one is in a triplet:

$$Q_{iL} = (d_{iL}, -u_{iL}, D_{iL})^T \sim (3, \bar{3}, 0),$$  
$$u_{iR} \sim (3, 1, 2/3), d_{iR} \sim (3, 1, -1/3), D_{iR} \sim (3, 1, -1/3), \; i = 1, 2,$$

$$Q_{3L} = (u_{3L}, d_{3L}, T_L)^T \sim (3, 3, 1/3),$$

$$u_{3R} \sim (3, 1, 2/3), d_{3R} \sim (3, 1, -1/3), T_R \sim (3, 1, 2/3).$$

The charged gauge bosons are defined as

$$\sqrt{2} W^+_\mu = W^1_\mu - iW^2_\mu, \sqrt{2} Y^-_\mu = W^6_\mu - iW^7_\mu,$$

$$\sqrt{2} X^0_\mu = W^4_\mu - iW^5_\mu.$$  

The physical neutral gauge bosons are again related to $Z, Z'$ through the mixing angle $\phi$. The symmetry breaking can be achieved with just three SU(3)$_L$ triplets

$$\chi = (\chi^0, \chi^-, \chi^0)^T \sim (1, 3, -1/3),$$

$$\rho = (\rho^+, \rho^0, \rho^+)^T \sim (1, 3, 2/3),$$

$$\eta = (\eta^0, \eta^-, \eta^0)^T \sim (1, 3, -1/3).$$

The necessary VEVs are

$$\langle \chi \rangle = (0, 0, \omega/\sqrt{2})^T, \; \langle \rho \rangle = (0, u/\sqrt{2}, 0)^T, \; \langle \eta \rangle = (v/\sqrt{2}, 0, 0)^T.$$  

After symmetry breaking the gauge bosons gain masses

$$m^2_W = \frac{1}{4} g^2 (u^2 + v^2), \; M^2_X = \frac{1}{4} g^2 (v^2 + \omega^2), \; M^2_Y = \frac{1}{4} g^2 (u^2 + \omega^2).$$

Eqn.(28) gives us a relation

$$v^2_W = u^2 + v^2 = 246^2 \; \text{GeV}^2.$$  

In order to be consistent with the low energy phenomenology we have to assume that $\langle \chi \rangle \gg \langle \rho \rangle, \langle \eta \rangle$ such that $m_W \ll M_X, M_Y$. The symmetry-breaking hierarchy gives us splitting on the bilepton masses [34]

$$|M^2_X - M^2_Y| \leq m^2_W.$$  

Our aim in this paper is to show that the 3-3-1 model with RH neutrinos furnishes a good candidate for (self-interacting) dark matter. The main properties that a good dark matter candidate must satisfy are stability and neutrality. Therefore, we
go to the scalar sector of the model, more specifically to the neutral scalars, and we examine whether any of them can be stable and in addition whether they can satisfy the self-interacting dark matter criterions [9]. In addition, one should notice that such dark matter particle must not overpopulate the Universe. On the other hand, since our dark matter particle is not imposed arbitrarily to solve this specific problem, we must check that the necessary values of the parameters do not spoil the other bounds of the model. Under assumption of the discrete symmetry $\chi \rightarrow -\chi$, the most general potential can then be written in the following form $[35]$

\[
V(\eta, \rho, \chi) = \mu_1^2 \eta^+ \eta + \mu_2^2 \rho^+ \rho + \mu_3^2 \chi^+ \chi + \lambda_1 (\eta^+ \eta)^2 + \lambda_2 (\rho^+ \rho)^2 + \lambda_3 (\chi^+ \chi)^2 \\
+ (\eta^+ \eta) [\lambda_4 (\rho^+ \rho) + \lambda_5 (\chi^+ \chi)] + \lambda_6 (\rho^+ \rho) (\chi^+ \chi) + \lambda_7 (\rho^+ \eta) (\eta^+ \rho) \\
+ \lambda_8 (\chi^+ \eta) (\eta^+ \chi) + \lambda_9 (\rho^+ \chi) (\chi^+ \rho) + \lambda_{10} (\chi^+ \eta + \eta^+ \chi)^2.
\] (31)

We rewrite the expansion of the scalar fields which acquire a VEV:

\[
\eta_o = \frac{1}{\sqrt{2}} (v + \xi \eta + i\zeta \eta) ; \rho_o = \frac{1}{\sqrt{2}} (u + \xi \rho + i\zeta \rho) ; \chi_o = \frac{1}{\sqrt{2}} (w + \xi \chi + i\zeta \chi).
\] (32)

For the prime neutral fields which do not have VEV, we get analogously:

\[
\eta'_o = \frac{1}{\sqrt{2}} (\xi' \eta + i\zeta' \eta) ; \chi'_o = \frac{1}{\sqrt{2}} (\xi' \chi + i\zeta' \chi).
\] (33)

Requiring that in the shifted potential $V$, the linear terms in fields must be absent, we get in the tree level approximation, the following constraint equations:

\[
\mu_1^2 + \lambda_1 v^2 + \frac{1}{2} \lambda_4 u^2 + \frac{1}{2} \lambda_5 w^2 = 0, \%
\mu_2^2 + \lambda_2 u^2 + \frac{1}{2} \lambda_4 v^2 + \frac{1}{2} \lambda_6 w^2 = 0, \%
\mu_3^2 + \lambda_3 w^2 + \frac{1}{2} \lambda_7 v^2 + \frac{1}{2} \lambda_6 u^2 = 0.
\] (34)

Since dark matter has to be neutral, then we consider only neutral Higgs sector. In the $\xi, \xi, \xi, \xi, \xi'$ basis the square mass matrix, after imposing of the constraints $[31]$, has a quasi-diagonal form as follows:

\[
M^2_H = \begin{pmatrix}
M^2_{3H} & 0 \\
0 & M^2_{2H}
\end{pmatrix},
\] (35)

where

\[
M^2_{3H} = \frac{1}{2} \begin{pmatrix}
2\lambda_1 v^2 & \lambda_4 vu & \lambda_5 vw \\
\lambda_4 vu & 2\lambda_2 u^2 & \lambda_6 uw \\
\lambda_5 vw & \lambda_6 uw & 2\lambda_3 w^2
\end{pmatrix},
\] (36)

and

\[
M^2_{2H} = \left( \frac{\lambda_8}{4} + \lambda_{10} \right) \begin{pmatrix}
w^2 & vw \\
vw & v^2
\end{pmatrix}.
\] (37)
The above mass matrix shows that the prime fields mix themselves but do not mix with others. In the limit
\[
\lambda_1 v, \lambda_2 u, \lambda_4 u \ll \lambda_5 w, \lambda_6 w,
\] (38)
we obtained physical eigenstates \(H_1(x)\) and \(\sigma(x)\)
\[
\begin{pmatrix}
H_1(x) \\
\sigma(x)
\end{pmatrix} = \frac{1}{(\lambda_5^2 v^2 + \lambda_6^2 u^2)^{1/2}} \begin{pmatrix}
\lambda_6 u & -\lambda_5 v \\
\lambda_5 v & \lambda_6 u
\end{pmatrix} \begin{pmatrix}
\xi_\eta \\
\xi_\rho
\end{pmatrix},
\] (39)
with masses [35]
\[
m_{H_1}^2 \approx \frac{v^2}{4\lambda_6} (2\lambda_1 \lambda_6 - \lambda_4 \lambda_5) \approx \frac{u^2}{4\lambda_5} (2\lambda_2 \lambda_5 - \lambda_4 \lambda_6),
\] (40)
\[
m_{\sigma}^2 \approx \frac{1}{2} \lambda_1 v^2 + \frac{\lambda_4 \lambda_6 u^2}{4\lambda_5} \approx \frac{1}{2} \lambda_2 u^2 + \frac{\lambda_4 \lambda_5 v^2}{4\lambda_6}.
\] (41)
Eqs. (40) and (41) also give us relations among coupling constants and VEVs. Another massive physical state \(H_3\) with mass:
\[
m_{H_3}^2 \approx -\lambda_3 w^2.
\] (42)
The scalar \(\sigma(x)\) is the one that we can identify with the SM Higgs boson [35]. In the approximation \(w \gg v\), mass matrix \(M_2^2\) gives us one Goldstone \(\xi_\eta\) and one physical massive field \(\xi_\eta'\) with mass
\[
m_{\xi_\eta'}^2 = \left(\frac{\lambda_8}{4} + \lambda_{10}\right) w^2.
\] (43)
In the pseudoscalar sector, we have three Goldstone bosons which can be identified as follows: \(G_2 \equiv \zeta_\eta\), \(G_3 \equiv \zeta_\rho\), \(G_4 \equiv \zeta_\chi\) and in the \(\zeta_\eta^{\prime 0}, \zeta_\chi^{\prime 0}\) basis
\[
M_{2A}^2 = \left(\frac{\lambda_8}{4} + \lambda_{10}\right) \begin{pmatrix}
w^2 & v w \\
v w & v^2
\end{pmatrix}.
\] (44)
We easily get one Goldstone \(G_3'\) and one massive pseudoscalar boson \(\zeta_\eta'\) with mass
\[
m_{\zeta_\eta'}^2 = \left(\frac{\lambda_8}{4} + \lambda_{10}\right) w^2.
\] (45)
It is to be emphasized that, both \(\xi_\eta'\) and \(\zeta_\eta'\) are in an singlet of the SU(2). Therefore they do not interact with the SM gauge bosons \(W^\pm, Z^0\) and \(\gamma\). Unlike the 3-3-1 model considered in [16], here we have two fields which can be considered as dark
matter. To get the interaction of dark matter to the SM Higgs boson, we consider the following relevant parts

\[
L_{int}(\sigma, \zeta_\eta) = \frac{1}{4} \lambda_1 \left[ v^2 + 2v \xi_\eta + \zeta_\eta^2 + \zeta_\eta' + \zeta_\eta'' + 2\eta^+ \eta^- \right]^2 + \frac{1}{4} \lambda_4 \left[ v^2 + 2v \xi_\eta + \zeta_\eta^2 + \zeta_\eta' + \zeta_\eta'' + 2\eta^+ \eta^- \right] \times \left[ u^2 + 2u \xi_\rho + \zeta_\rho^2 + \zeta_\rho' + 2\rho^+ \rho^- + 2\rho^+ - \rho^- + \right]
\]

Substituting (39) we get couplings of SIDM with the SM Higgs boson \( \sigma \)

\[
L(\sigma, \zeta_\eta) = \left[ \frac{\sigma(x)}{\lambda_2 v^2 + \lambda_3 u^2} \right] \left( \lambda_1 \lambda_5 v^2 + \frac{\lambda_4 \lambda_6}{2} u^2 \right) + \frac{H_1(x) \sigma(x)}{\lambda_5^2 v^2 + \lambda_6^2 u^2} \left( \lambda_1 - \frac{\lambda_4}{2} \right) \lambda_5 \lambda_6 u v
\]

\[
+ \frac{\sigma^2(x)}{2(\lambda_3^2 v^2 + \lambda_4^2 u^2)} \left( \lambda_5^2 v^2 + \frac{\lambda_6^2}{2} u^2 \right) \left( \zeta_\eta^2 + \zeta_\eta' \right) \tag{47}
\]

From Yukawa couplings, we see that our candidates do not interact with ordinary leptons and quarks [36].

\[
\mathcal{L}^{uk}_{Yuk} = \lambda_{3a} \bar{Q}_3 L u a R \eta + \lambda_{3ia} \bar{Q}_i L d a R \eta + h.c.
\]

\[
= \lambda_{3a} (\bar{u}_3 L \eta^0 + \bar{d}_3 L \eta^- + \bar{T}_L \eta^0) u a R + \lambda_{3ia} (\bar{d}_i L \eta^0 - \bar{u}_i L \eta^+ + \bar{D}_i L \eta^{+0}) d a R + h.c.
\]

We see that the candidates for dark matter in this model have not couplings with all the SM particles except for the Higgs boson. For stability of DM, we have to put mass of the SM Higgs boson is twice bigger mass of the candidate

\[
\frac{1}{2} \lambda_1 v^2 + \frac{\lambda_4 \lambda_6 u^2}{4 \lambda_5} \approx \frac{1}{2} \lambda_2 v^2 + \frac{\lambda_4 \lambda_5 v^2}{4 \lambda_6} \geq - \left( \frac{\lambda_8}{4} + \lambda_{10} \right) w^2 \tag{48}
\]

To avoid the interaction of DM with Goldstone boson, we have

\[
\lambda_1 = \frac{\lambda_4}{2} \tag{49}
\]

The wrong muon decay \((\mu^- \rightarrow e^- \nu_e \bar{\nu}_\mu)\) gives a lower limit for singly charged bilepton \(M_Y \sim 230 \text{ GeV}\). Combining Eqns. [58, 59] with [60] we obtain the following relation: \(u \sim v \approx 100 - 200 \text{ GeV} \) and \(w \approx (500 - 1000) \text{ GeV}\). The cross section for \(hh \rightarrow hh\) (where \(h\) stands for \(\xi_\eta^0\) and \(\zeta_\eta^0\)) with quartic interaction is \(\sigma = \frac{\lambda_4^2}{4 \pi m_h^2}\). The requirement on the quality \(\sigma_{el}/(m_h[GeV])\) denoting the ration of the DM elastic cross section to its mass (measured in GeV) is that [9, 13, 37]

\[
2.05 \times 10^3 \text{ GeV}^{-3} \leq \frac{\sigma}{m_h} \leq 2.57 \times 10^4 \text{ GeV}^{-3} \tag{50}
\]
Taking $\lambda_1 = 1$ we get $4.7 \text{ MeV} \leq m_h \leq 23 \text{ MeV}$. The SIDM candidates interact with the SM Higgs boson by strength 0.65 if $\lambda_5 = \lambda_6 = 1$ and $u = v = 175 \text{ GeV}$ are taken. Now consider the cosmic density of the $h$ scalar given by \[16\]:

$$
\Omega_h = 2g(T_\gamma) T_\gamma^3 \frac{m_h \beta}{\rho_c g(T)},
$$

(51)

where $T_\gamma = 2.4 \times 10^{-4} \text{ eV}$ is the present photon temperature, $g(T_\gamma) = 2$ is the photon degree of freedom and $\rho_c = 7.5 \times 10^{-47} h^2$ with $h = 0.71$, being the critical density of the Universe. Taking $m_h = 4.7 \text{ MeV}$, we obtain $\Omega_h = 0.18$. This means that the SIDM candidates do not overpopulate the Universe.

### 2.3 The 3-3-1 model with exotic neutral lepton

In this model the leptons are in triplets, and the third member is a neutral exotic lepton and left-handed fermion field has its right-handed counterpart transforming as a singlet of the SU(3)$_L$ group \[38\]:

$$
f_L^a = (\nu_L^a, e_L^a, N_L^a)^T \sim (1, 3, -1/3), e_R^a \sim (1, 1, -1), N_R^a \sim (1, 1, 0).
$$

(52)

The first two generations of quarks are in antitriplets while the third one is in a triplet:

$$
Q_{iL} = (d_{iL}, -u_{iL}, D_{iL})^T \sim (3, 3, 0),
$$

(53)

$$
u_{iR} \sim (3, 1, 2/3), d_{iR} \sim (3, 1, -1/3), D_{iR} \sim (3, 1, -1/3), \ i = 1, 2,
$$

(54)

$$
Q_3L = (u_{3L}, d_{3L}, T_L)^T \sim (3, 3, 1/3),
$$

$$
u_{3R} \sim (3, 1, 2/3), d_{3R} \sim (3, 1, -1/3), T_R \sim (3, 1, 2/3).
$$

(55)

The charged gauge bosons are defined as

$$
\sqrt{2} \ W^+_{\mu} = W^1_{\mu} - i W^2_{\mu}, \sqrt{2} \ Y^-_{\mu} = W^6_{\mu} - i W^7_{\mu},
$$

$$
\sqrt{2} \ X^0_{\mu} = W^3_{\mu} - i W^5_{\mu}.
$$

(55)

The physical neutral gauge bosons are again related to Z, Z' through the mixing angle $\phi$. The symmetry breaking can be achieved with three SU(3)$_L$ triplets and an sextet

$$
\chi = \left(\chi^0, \chi^-, \chi^0\right)^T \sim (1, 3, -1/3),
$$

$$
\rho = \left(\rho^+, \rho^0, \rho^+\right)^T \sim (1, 3, 2/3),
$$

(56)

$$
\eta = \begin{pmatrix}
\sigma^0_1 & \frac{h^-_1}{\sqrt{2}} & \frac{\sigma^0_2}{\sqrt{2}} \\
\frac{h^-_2}{\sqrt{2}} & \frac{\sigma^0_2}{\sqrt{2}} & \frac{h^-_2}{\sqrt{2}} \\
\frac{\sigma^0_2}{\sqrt{2}} & \frac{h^-_2}{\sqrt{2}} & \sigma^0_3
\end{pmatrix} \sim (1, 6, -2/3).
$$
The necessary VEVs are
\[ \langle \chi \rangle = (0,0,\omega/\sqrt{2})^T, \quad \langle \rho \rangle = (0,u/\sqrt{2},0)^T, \quad \langle \eta \rangle = (v/\sqrt{2},0,0)^T. \quad (57) \]

The VEV of sextet has the form
\[ \langle S \rangle = \begin{pmatrix} 0 & 0 & v_{\sigma_2} \\ 0 & 0 & 0 \\ v_{\sigma_2} & 0 & 0 \end{pmatrix} \]

After symmetry breaking the gauge bosons gain masses
\[ m_W^2 = \frac{1}{4} g^2(u^2 + v^2 + v_4), \quad M_Y^2 = \frac{1}{4} g^2(v^2 + \omega^2 + v_4^2), \quad M_X^2 = \frac{1}{4} g^2(u^2 + \omega^2 + v_4^2). \quad (58) \]

Eqn.\[58\] gives us a relation
\[ v_W^2 = u^2 + v^2 + v_4^2 = 246^2 \text{ GeV}^2. \quad (59) \]

In order to be consistent with the low energy phenomenology we have to assume that \( \langle \chi \rangle \gg \langle \rho \rangle, \langle \eta \rangle \) such that \( m_W \ll M_X, M_Y \). The symmetry-breaking hierarchy gives us splitting on the bilepton masses \[34\]
\[ |M_X^2 - M_Y^2| \leq m_W^2. \quad (60) \]

The Yukawa couplings are given
\[ \mathcal{L}_Y = f_1 \bar{L}_a \phi \nu_R + \frac{1}{2} f_1 \bar{L}_a S L_b \quad (61) \]

The mass eigenstates \((\nu_e, \tilde{N})\) are related to the weak eigenstates \((\nu_e, \tilde{N})\)
\[ \tilde{\nu} = \nu \cos \theta - N \sin \theta, \quad \tilde{N} = \nu \sin \theta + N \cos \theta \quad (62) \]

where \(\theta\) is a mixing angle. A mass distribution extended well beyond the visible galaxy \[39\]
\[ M(R) = \int_R^\infty \rho(r) dV = v^2 RG^{-1} \quad (63) \]

Fig.1: Two diagrams for the decay \(N \to \nu_e \gamma\)
Candidates for Dark Matter

The decay of $N$ particle into neutrino and photon contains three graphs depicted in Fig.1 which give [38]

$$\Gamma(N \rightarrow \nu_e\gamma) = \frac{9 m_N^5 G_F^2 \alpha}{4 \times 512 \pi^2} \sin^2(2\theta)$$  \hspace{1cm} (64)$$

which corresponds to lifetime

$$\tau \approx 4.67 \times 10^{14} \sin^{-2}(2\theta) (1\text{keV}/m_Nc^2)^5 \text{yr}$$ \hspace{1cm} (65)$$

It is obvious that the $N$- particle’s life can be longer than the age of the universe, if $m_N \sim 1\text{ kEV}$. It was shown that the Higgs contribution is much smaller than those of the two first, and the $N$- particle meets the constraints on dark matter from cosmology and galaxy formation.

3 Supersymmetric 3-3-1 model with right-handed neutrinos

Supersymmetric version on the 3-3-1 model with right-handed neutrinos has been proposed in [22, 40]. Here we will follow the usual notation writing for a given fermion $f$, the respective sfermions by $\tilde{f}$ i.e., $\tilde{l}$ and $\tilde{q}$ denote sleptons and squarks respectively. Then, we have the following additional representations

$$\tilde{Q}_{3L} = \begin{pmatrix} \tilde{d}_3 \\ \tilde{u}_3 \\ \tilde{d}'_3 \end{pmatrix}_L \sim (3, 3^*, 0), \quad \tilde{Q}_{3L} = \begin{pmatrix} \tilde{d}_3 \\ \tilde{u}_3 \end{pmatrix}_L \sim (3, 3, 1/3),$$

$$\tilde{L}_{3L} = \begin{pmatrix} \tilde{\nu}_3 \\ \tilde{l}_3 \\ \tilde{\nu}'_3 \end{pmatrix}_L \sim (1, 3, -1/3),$$ \hspace{1cm} (66)$$

\begin{figure}
\centering
\begin{tikzpicture}
\node (N) at (0,0) {$N$};
\node (nu_e) at (-1,0) {$\nu_e$};
\node (e) at (1,0) {$e$};
\node (nu_e) at (2,0) {$\nu_e$};
\node (H) at (0,1) {$H$};
\node (H) at (0,-1) {$H$};
\draw[->] (N) -- (nu_e);
\draw[->] (N) -- (e);
\draw[<->] (nu_e) -- (nu_e);
\draw[->] (H) .. controls (-0.5,0) and (0.5,0) .. (H);
\draw[->] (H) -- (nu_e);
\draw[->] (H) -- (nu_e);
\node at (1,-2) {$\gamma$};
\end{tikzpicture}
\caption{Diagram for Higgs contribution to the decay $N \rightarrow \nu_e\gamma$}
\end{figure}
\[ \tilde{\nu}^c_{aL} \sim (1, 1, 1), \]
\[ \tilde{u}^c_{iL}, \tilde{d}^c_{iL} \sim (3^*, 1, -2/3), \quad \tilde{c}^c_{aL}, \tilde{d}^c_{aL} \sim (3^*, 1, 1/3), \]

with \( a = e, \mu, \tau; \; i = 1, 2, 3; \) and \( \alpha = 1, 2. \) However, when considering quark (or squark) singlets of a given charge we will use the notation \( u^c_{iL}, d^c_{iL} (\tilde{u}^c_{iL}, \tilde{d}^c_{iL}) \) with \( i(j) = 1, 2, 3. \) The supersymmetric partner of the scalar Higgs fields, the higgsinos, are

\[ \tilde{\eta} = \begin{pmatrix} \tilde{\eta}_0^0 \\ \tilde{\eta}_1^- \\ \tilde{\eta}_2^- \end{pmatrix}, \quad \tilde{\chi} = \begin{pmatrix} \tilde{\chi}_1^0 \\ \tilde{\chi}_2^- \\ \tilde{\chi}_2^0 \end{pmatrix} \sim (1, 3, -1/3), \]

\[ \tilde{\rho} = \begin{pmatrix} \tilde{\rho}_1^+ \\ \tilde{\rho}_0^0 \\ \tilde{\rho}_2^+ \end{pmatrix} \sim (1, 3, 2/3), \]

and the respective extra higgsinos, needed to cancel the chiral anomaly of the higgsinos in Eq. (68), are

\[ \tilde{\eta}' = \begin{pmatrix} \tilde{\eta}_0^0 \\ \tilde{\eta}_1^+ \\ \tilde{\eta}_2^- \end{pmatrix}, \quad \tilde{\chi}' = \begin{pmatrix} \tilde{\chi}_1^0 \\ \tilde{\chi}_2^+ \\ \tilde{\chi}_2^0 \end{pmatrix} \sim (1, 3^*, 1/3), \]

\[ \tilde{\rho}' = \begin{pmatrix} \tilde{\rho}_1^- \\ \tilde{\rho}_0^0 \\ \tilde{\rho}_2^- \end{pmatrix} \sim (1, 3^*, -2/3), \]

and the corresponding scalar partners denoted by \( \eta', \chi', \rho', \) with the same charge assignment as in Eq. (68), and with the following VEVs: \( v' = \langle \eta_1^0 \rangle / \sqrt{2}, \quad w' = \langle \chi_2^0 \rangle / \sqrt{2} \) and \( u' = \langle \rho^0 \rangle / \sqrt{2}. \) This complete the representation content of this supersymmetric model. Concerning the gauge bosons and their superpartners, if we denote the gluons by \( g^b \) the respective superparticles, the gluinos, are denoted by \( \lambda^b_\lambda, \) with \( b = 1, \ldots, 8; \) and in the electroweak sector we have \( V^b, \) the gauge boson of \( SU(3)_L, \) and their gauginos partners \( \lambda^b_A; \) finally we have the gauge boson of \( U(1)_N, \) denoted by \( V', \) and its supersymmetric partner \( \lambda_B. \) This is the total number of fields in the minimal supersymmetric extension of the 3-3-1 model of Refs. [15,30].

### 3.1 Superfields

The superfields formalism is useful in writing the Lagrangian which is manifestly invariant under the supersymmetric transformations [41] with fermions and scalars put in chiral superfields while the gauge bosons in vector superfields. As usual the superfield of a field \( \phi \) will be denoted by \( \hat{\phi} \) [42]. The chiral superfield of a multiplet \( \phi \) is denoted by

\[ \hat{\phi} \equiv \hat{\phi}(x, \theta, \bar{\theta}) = \hat{\phi}(x) + i \theta \sigma^m \bar{\theta} \partial_m \hat{\phi}(x) + \frac{1}{4} \theta \theta \bar{\theta} \bar{\theta} \Box \hat{\phi}(x) \]
The supersymmetric part of the Lagrangian is decomposed in the lepton, quark, and gauge sectors as follows:

\[ \mathcal{L}_{\text{SUSY}} = \mathcal{L}_{\text{Lepton}} + \mathcal{L}_{\text{Quark}} + \mathcal{L}_{\text{Gauge}} + \mathcal{L}_{\text{Scalar}}, \]  

where we have defined \( \hat{L}\), \( \hat{e}\), \( \hat{\nu}\), \( \hat{\nu}'\), \( \hat{\nu}''\), and similar expressions for \( \hat{c}\), \( \hat{\chi}', \hat{\chi}''\) and we must change (field) by (field). The vector superfield for the gauge bosons of each factor \( SU(3)_C, SU(3)_L \) and \( U(1)_N \) are denoted by \( \hat{V}_C, \hat{\nu}_C, \hat{V}_L; \) and \( \hat{V}' \), respectively, where we have defined \( \hat{V}_C = T^b \hat{V}^b_C, \) \( \hat{V} = T^b \hat{V}^b; \) \( \hat{V}_C = T^b \hat{V}^b_C, \) \( \hat{V} = T^b \hat{V}^b; \) \( T^b = \lambda^b/2, T^b = -\lambda^b/2 \) are the generators of triplet and antitriplet representations, respectively, and \( \lambda^b \) are the Gell-Mann matrices. The Lagrangian of the model has the following form

\[ \mathcal{L}_{331s} = \mathcal{L}_{\text{SUSY}} + \mathcal{L}_{\text{soft}}, \]

where \( \mathcal{L}_{\text{SUSY}} \) is the supersymmetric part and \( \mathcal{L}_{\text{soft}} \) the soft terms breaking explicitly the supersymmetry.

### 3.2 The supersymmetric Lagrangian

The supersymmetric part of the Lagrangian is decomposed in the lepton, quark, gauge, and the scalar sectors as follow:

\[ \mathcal{L}_{\text{SUSY}} = \mathcal{L}_{\text{Lepton}} + \mathcal{L}_{\text{Quark}} + \mathcal{L}_{\text{Gauge}} + \mathcal{L}_{\text{Scalar}}, \]

where

\[ \mathcal{L}_{\text{Leptons}} = \int d^4\theta \left[ \tilde{l}_{aL} (2g_\nu \hat{V}_C + g \hat{V}) \tilde{\nu}_L + \tilde{\nu}_c \tilde{\nu}_c \right], \]

in the lepton sector, we have omitted the sum over the three lepton family for simplicity, and

\[ \mathcal{L}_{\text{Quarks}} = \int d^4\theta \left[ \tilde{c}_i L \tilde{c}_i \tilde{c}_i \tilde{c}_i + \tilde{e}_i c_i \tilde{e}_i c_i + \tilde{\nu}_L \tilde{\nu}_L \tilde{\nu}_L + \tilde{d}_i \tilde{d}_i \tilde{d}_i \tilde{d}_i \right]. \]
in the quark sector, and we have denoted \( g_s, g, g' \) the gauge coupling constants for the \( SU(3)_C, SU(3)_L, U(1)_N \) factors, respectively. In the gauge sector we have

\[
\mathcal{L}_{\text{Gauge}} = \frac{1}{4} \int d^2 \theta \left[ \mathcal{W}_C \mathcal{W}_C + \mathcal{W} \mathcal{W} + \mathcal{W}' \mathcal{W}' \right] \\
+ \frac{1}{4} \int d^2 \bar{\theta} \left[ \bar{\mathcal{W}_C} \bar{\mathcal{W}_C} + \bar{\mathcal{W}} \bar{\mathcal{W}} + \bar{\mathcal{W}'} \bar{\mathcal{W}'} \right],
\]

where \( \mathcal{W}_C, \mathcal{W} \) and \( \mathcal{W}' \) are fields that can be written as follows:

\[
\mathcal{W}_C = -\frac{1}{8g_s} \bar{D} \bar{D} e^{-2g_s \bar{V}_C} D \zeta e^{2g_s \bar{V}_C}, \\
\mathcal{W}_\zeta = -\frac{1}{8g} \bar{D} \bar{D} e^{-2g \bar{V}} D \zeta e^{2g \bar{V}}, \\
\mathcal{W}'_\zeta = -\frac{1}{4} \bar{D} \bar{D} \zeta \bar{V}', \quad \zeta = 1, 2.
\]

Finally, in the scalar sector we have

\[
\mathcal{L}_{\text{Scalar}} = \int d^4 \theta \left[ \bar{\eta} e^{[2g \bar{V} - \frac{g'}{2} \bar{V}']} \eta + \bar{\chi} e^{[2g \bar{V} - \frac{g'}{2} \bar{V}']} \chi + \bar{\rho} e^{[2g \bar{V} + \frac{g'}{2} \bar{V}']} \rho \right] \\
+ \int d^2 \theta W + \int d^2 \bar{\theta} \bar{W},
\]

where \( W \) is the superpotential.

### 3.3 The Scalar Potential

In the present model the scalar potential is written as

\[
V_{331} = V_F + V_D + V_{\text{soft}},
\]

where

\[
V_F = -\mathcal{L}_F = \sum_m F^\dagger_m F_m
\]

and

\[
V_D = -\mathcal{L}_D = \frac{1}{2}(D^a D^a + D D) = \frac{g^2}{18} (-\eta^i \eta^i + \eta^i \eta^i - \chi^i \chi - \chi^i \chi' + 2 \rho^i \rho - 2 \rho^i \rho')^2 \\
+ \frac{g^2}{8} (\eta_i^i \lambda_{ij}^j \eta_j - \eta_i^i \lambda_{ij}^j \eta_j + \chi_i^i \lambda_{ij}^j \chi_j - \chi_i^i \lambda_{ij}^j \chi_j + \rho_i^i \lambda_{ij}^j \rho_j + \rho_i^i \lambda_{ij}^j \rho_j)^2.
\]
finally,
\[
V_{\text{soft}} = -\mathcal{L}_{\text{soft}} = m_\eta^2 \eta^\dagger \eta + m_\rho^2 \rho^\dagger \rho + m_\chi^2 \chi^\dagger \chi + m_{\eta'}^2 \eta'^\dagger \eta' \\
+ m_{\rho'}^2 \rho'^\dagger \rho' + m_{\chi'}^2 \chi'^\dagger \chi' - \epsilon_{ijk}(k_3 \rho_i \chi_j \eta_k + k_3' \rho'_i \chi'_j \eta'_k) \\
+ H.c. ,
\]
(83)
where we have used the scalar multiplets in Eqs. (57) and (69). With Eqs. (81)-(83) we can work out the mass spectra of the scalar and pseudoscalar fields by making the usual shift \(X^0 \rightarrow \frac{1}{\sqrt{2}}(v_X + H_X + iF_X)\). The analysis is similar to that of Ref. [22] and we will not write the constraints equation, etc. By using as input the following values for the parameters: \(\sin^2 \theta_W = 0.2314, \ g = 0.6532, \ g' = 1.1466; \ f_2 = 2, \ f_2' = 10^{-3}; \ k_1 = k_1' = 10 \text{ GeV}; \ \mu_\eta = \mu_\rho = \mu_\chi = -10^3 \text{ GeV}; \ m_\eta = 15 \text{ GeV}, \ m_\rho = 10 \text{ GeV}. \ m_\rho = 244.99 \text{ GeV}; \ m_{\chi_2} = m_{\chi'_2} = 10^3 \text{ GeV} \) and \(m_{\rho'} = 13 \text{ GeV}, \) we obtain the masses
\[
m_1 \approx 1702, \ m_2 \approx 1449, \ m_3 \approx 387, \\
m_4 \approx 380, \ m_5 \approx 361, \ m_6 \approx 130, \ (84)
\]
for the scalar sector (all masses are in GeV). Note that the lightest neutral scalar is heavier than the lower limit of the Higgs scalar of the standard model, i.e., \(m_H > \approx 114 \text{ GeV}. \) For the pseudoscalar sector we obtain
\[
M_1 \approx 1702, \ M_2 \approx 1449, \ M_3 \approx 363, \\
M_4 \approx 5, \ M_5 = 0, \ M_6 = 0, \ (85)
\]
only the two massless pseudoscalars are exact values, i.e., there are two Goldstone bosons as it should be. Notice that there is a light pseudoscalar. A carefully study shows that there is a Higgs boson satisfies the conditions for SIDM [43].

4 Conclusions

In conclusion, it is a remarkable fact that the 3-3-1 model has an option for self-interacting dark matter without the need of imposing any new symmetry to stabilize it. We have shown that the Spergel-Steinhardt bound for self-interacting dark matter [30] can be realized in the 3-3-1 model with a reasonable choice of the values of the parameters. The 3-3-1 model with RH neutrinos provides two Higgs bosons: one is scalar or \(CP\)-even and another is pseudoscalar or \(CP\)-odd particle having properties of candidates for dark matter. In difference with the previous candidate which introduced by hand, our self-interacting dark matter arises without impose new properties to satisfy all the criteria. From the conditions for SIDM one get the bounds for scalar Higgs bosons: \(m_h = 7.75 \text{ MeV} \) in the 3-3-1 model with exotic leptons [16] and \(4.7 \text{ MeV} \leq m_h \leq 23 \text{ MeV} \) in the 3-3-1 version with RH neutrinos [17].
Scalar dark matter candidates have been recently investigated in [44]. It means that the candidate for SIDM has not to be introduced ad hoc as in other models [45]. This feature remains valid in the supersymmetric 3-3-1 model with right-handed neutrinos. The spin $\frac{1}{2}$ exists in the version with exotic neutral lepton. This would turn out to be particularly interesting if observations reveal to be in favor of light dark matter. This work was supported in part by National Council for Natural Sciences of Vietnam contract No: KT - 04.1.2.

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