Isospin splitting in heavy baryons and mesons

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Abstract

A recent general analysis of light-baryon isospin splittings is updated and extended to charmed baryons. The measured $\Sigma_c$ and $\Xi_c$ splittings stand out as being difficult to understand in terms of two-body forces alone. We also discuss heavy-light mesons; though the framework here is necessarily less general, we nevertheless obtain some predictions that are not strongly model-dependent.

13.40.Dk, 12.40.Aa, 14.20.-c, 12.70.+q
I. BARYONS

Mass splittings among baryons and mesons containing different valence quarks provide a good probe of hadronic structure and of the forces that determine this structure \[1\]. Recently, results were reported for a general class of models in which the baryon octet and decuplet splittings were assumed to arise solely from two-body interactions \[2\]. It was found that the central masses of isospin multiplets could be fitted with a model error of 8.9 MeV. A similar parametrization of isospin splittings, also with a small model error, showed that the two measured $\Delta$ masses were the hardest to fit.

In this paper, we shall extend the analysis to baryons containing heavy quarks (thereby severely testing the assumption that three-body forces are negligible), and also to mesons. Since reporting on the fit in Ref. \[2\] we have recognized that the two sets of input $\Delta$ values that had been averaged \[3\] were obtained using essentially the same set of input experimental data. Very similar analysis methods were also used, so the differences in the results should be treated as a systematic error. In addition, we have concluded, through discussions with colleagues \[4\], that there are additional systematic errors arising from differences in the treatment of the coupling to open channels. We estimate these theoretical uncertainties to be, at this time, about 0.5 MeV for the $\Delta$ states. Similar systematic uncertainties would also occur in $\Sigma^*$ and $\Xi^*$ states. Since their widths are smaller and the errors are larger, we expect that this would give a negligible additional uncertainty to the masses of these states.

With the enlarged error for the $\Delta$ masses, the fit to octet and decuplet masses has a $\chi^2$ of 3.2 for 4 degrees of freedom, so that no model error needs to be invoked. However, almost all of this $\chi^2$ still arises from the $\Delta$ masses. The results from this modified fit are shown in Fig. \[1\]. The parameters from this fit will be used in fitting the heavy quark sector.

Although a breakdown of the spectator approximation in the charmed baryon sector would not be at all surprising (since the heavy charm quark may significantly alter the environment in which the light quarks interact), we will in fact see that there appears to be little indication of such a breakdown in most of the data.

The central masses of the charmed baryons may be expressed in terms of four pair energies $S_{nc}$, $T_{nc}$, $S_{sc}$, and $T_{sc}$ (in addition to the non-charm pair energies introduced in Ref. \[2\]) or in terms of the spin-dependent and spin-independent combinations $D \equiv (T - S)/4$ and $A \equiv (3T + S)/4$, respectively. ($T$ indicates a spin-triplet quark pair, $S$ a spin-singlet quark pair, and the subscripts give the flavor content.) However, there are only four data, so we introduce an additional assumption. In Ref. \[2\] it was verified that $x = D_{ns}/D_{nn} \sim \frac{2}{3}$, as is often assumed in models. Here we also assume that $x_c = D_{sc}/D_{nc} \sim \frac{2}{3}$. To give this constraint some flexibility, we introduce the auxiliary quantity $y_c = \frac{3}{2}D_{sc} - D_{nc}$, with an input value of 0.0 ± 1.6 MeV. Tables \[1\] and \[1\] summarize the fit of these pair energies to the $\Lambda_c$, $\Sigma_c$, $\Xi_c$, and $\Omega_c$ masses \[6\]. Including the correlated parameter errors from the non-charm sector shown in Table \[1\] and the same model error of 8.9 MeV, we find an overall $\chi^2$ of 0.37 with one degree of freedom, thus the central masses are easily fitted with a sum of two-body energies.

We may also express the central masses of the remaining (unmeasured) singly-charmed baryons in terms of pair energies to obtain the predictions shown in the left side of Table \[1\]. (Note that there are two $\Xi_c$ states, since there is no symmetry restriction on the spin wavefunction of a $usc$ baryon. We denote the $\chi^\rho$ state by $\Xi_c$ and the $\chi^3$ state by $\Xi'_c$. These
states actually mix slightly — and this was taken into account in our calculations — but the effects are negligible. (See also Ref. [4].)

The fits and predictions for $j = \frac{1}{2}$ and $j = \frac{3}{2}$ baryons are shown in Fig. 2. For the fits, the quantities plotted are the contributions of the spin-dependent terms $D$. The spin-independent terms $A$ have been subtracted from the experimental data. For the non-charm baryons, we assume that $D_{ss} = D_{ns}^2/D_{nn}$.

We have also considered an additional stronger assumption, that the $A$'s are linear in the quark masses. In the non-charm sector, the three $A$'s are then fitted with two parameters, with $\chi^2 = 1.86$, which is not excessive. In the charm sector, however, fitting the two new $A$'s with one new parameter gives $\chi^2 = 24.2$. This indicates that the presence of the charm quark does affect the baryon structure, even though the fit to central masses shows no clear evidence for the existence of three-body effects.

The fit to isospin splittings proceeds similarly. There is no advantage in discussing the individual $\Sigma_c$ and $\Xi_c$ charge states, because their experimental errors are correlated in a complicated way. We therefore give results for the isospin tensorial quantities

$$\Sigma_c^1 = \Sigma_{c^+} - \Sigma_{c^0},$$
$$\Sigma_c^2 = \Sigma_{c^+} + \Sigma_{c^0} - 2\Sigma_{c^+},$$
$$\Xi_c^1 = \Xi_{c^+} - \Xi_{c^0}.$$

The new pair terms, $D_c^1$ and $A_c^4$ are shown in Table IV. Note that $D_c^1$ is very small and has little influence on the splittings; rather than treat it as a free parameter, we constrained it in the fit to $(0.03 \pm 0.05)$ MeV, an estimate obtained from the sizes and uncertainties of various terms in the simple model described in Ref. [2]. The best-fit value was $D_c^1 = 0.027$ MeV. The total $\chi^2$ for the fit is 6.23. The final two rows of the table are clearly the most intriguing: We obtain a poor fit to $\Xi_c^1$ and a worse fit to $\Sigma_c^2$. If these measurements are correct, it appears that three-body effects are important for isospin splittings but not for the isospin-averaged central masses.

The errors in the non-charm pair terms are correlated; our fit uses the full correlation matrix, and the shifts in the values of these terms are shown in Table V. The fit to the individual $\Sigma_c$ and $\Xi_c$ masses gives slightly adjusted values for the central $\Sigma_c$ and $\Xi_c$ masses, which were used as the input in Table IV.

As before, the pair term analysis gives rise to some predictions. These are shown in the right side of Table III. It is interesting that the combination $2\Xi_c^1 - \Sigma_c^1 = (-1.88 \pm 0.51)$ MeV is given by $-T^1 + 2T_s^1$, i.e., it is independent of the charm pair terms. Note also that in our model, $\Sigma_c^* = \Sigma_c^2 = \Sigma^2 = \Sigma_{c^*}$.

**II. MESONS**

A description of mesonic splittings in terms of pair energies is not useful, since a new parameter must be introduced for each meson. While we are thus forced to commit ourselves to stronger assumptions here, we still strive for results which are as model-independent as possible. We focus on the differential vector-pseudoscalar isospin splittings, which we denote by $\Delta(M)$. In particular, we define
\[
\begin{align*}
\Delta(B) &= (B^- - \bar{B}^0) - (B^- - \bar{B}^0) \quad (2.1a) \\
\Delta(D) &= (D^{*0} - D^{*+}) - (D^0 - D^+) \quad (2.1b) \\
\Delta(K) &= (K^{*-} - \bar{K}^{*0}) - (K^- - \bar{K}^0). \quad (2.1c)
\end{align*}
\]

These combinations, which have been considered previously by a number of authors [1], are interesting for several reasons:

\(i\) \(\Delta(D)\) has recently been measured very precisely: \(\Delta(D) = 1.48 \pm 0.1\) MeV (Ref. [3]). This datum constitutes a severe test (or useful calibration point) for model calculations.

\(ii\) \(\Delta(K)\) is also known rather precisely: \(\Delta(K) = -0.49 \pm 0.37\) MeV (Ref. [4]). Theorists have generally found it difficult to accommodate this value; even those models which successfully account for related splittings typically predict \(\Delta(K) \approx +3\) MeV (see Ref. [14]). It has been suggested that relativistic and/or unitarity effects (from \(K^* \to K\pi \to K^*\), for example) may be a source of the discrepancy.

\(iii\) \(B^- - \bar{B}^0\) is known to be \(-0.12 \pm 0.58\) MeV (Ref. [9]), but the \(B^*\) isospin splitting has not yet been measured, thus there is room for a prediction of \(\Delta(B)\).

\(iv\) As we shall presently see, it is possible to obtain predictions for the ratios \(\Delta(B) : \Delta(D) : \Delta(K)\) which are quite model-independent.

We write the quark-mass and electromagnetic contributions to \(\Delta(M)\) as \(\Delta^m(M)\) and \(\Delta^\gamma(M)\), respectively. To obtain \(\Delta^m(M)\), we employ an interpolation technique first suggested by Chan [11]. Consider, for the sake of definiteness, \(\Delta^m(D)\). From equation (2.11), we see that \(\Delta^m(D) \approx (m_u - m_d)(dE_{hyp}/dm_q)|_{m_q = m_u}\). This latter quantity may be accurately estimated with very little reliance on theory, since the function \(E_{hyp}(m_q)\) is rather well-determined by measurements of \(D^* - D\), \(D^*_s - D_s\) and \(J/\Psi - \eta_c\); an interpolating curve drawn through these points allows the extraction of \((dE_{hyp}/dm_q)|_{m_q = m_u}\) [12]. The same procedure can be applied to \(\Delta^m(B)\) and \(\Delta^m(K)\), and the relevant curves for all three cases are shown in Fig. 3.

In the case of D and B mesons, the hyperfine splittings turn out to be surprisingly independent of the lighter quark’s mass:

\[
\begin{align*}
D^* - D &= 141.4 \pm 0.1\ \text{MeV}, \\
D^*_s - D_s &= 141.5 \pm 1.9\ \text{MeV}, \\
B^* - B &= 46.0 \pm 0.6\ \text{MeV}, \\
B^*_s - B_s &= 47.0 \pm 2.6\ \text{MeV}.
\end{align*}
\]

(This is qualitatively understood in the quark model, where hyperfine splittings vary with the quark masses as \(|\psi(0)|^2/m_1m_2\); in a linear potential, \(|\psi(0)|^2\) is proportional to the reduced mass \(\mu\), so that \(E_{hyp} \propto (m_1 + m_2)^{-1}\), which becomes independent of \(m_1\) in the limit \(m_2 \gg m_1\). Thus \(\Delta^m(D)\) and \(\Delta^m(B)\) are both approximately zero. The second column of Table 7 shows the actual results of the interpolation procedure; we performed a quadratic fit to the three data points for D mesons, and a linear fit to the \(B^* - B\) and \(B^*_s - B_s\) data points for B mesons.

The strange meson sector differs in that \(E_{hyp}\) has significant \(m_q\)-dependence: the available data here are \(K^* - K = 398\ \text{MeV}\), \(D^*_s - D_s = 142\ \text{MeV}\), and \(B^*_s - B_s = 47\ \text{MeV}\). A cubic spline interpolation gives \((dE_{hyp}/dm_q)|_{m_q = m_u} = -0.36\), however, without a data point for the \(s\bar{s}\) mesons, the distance over which we must interpolate in this case is unsettlingly large.
The $s\bar{s}$ data point is not directly available, of course, because annihilation for ces rotate the pseudoscalar $s\bar{s}$ meson ($\eta_s$) and its nonstrange partner ($\eta_n$) into the physical $\eta$ and $\eta'$ states. Nevertheless, we may estimate the mass that $\eta_s$ would have in the absence of such forces by making use of the empirical rule [13] for the difference of like-flavor vector and pseudoscalar squared masses: $m_{\eta_s}^2 - m_{\eta_n}^2 \approx 0.56 \text{ GeV}^2$. This implies $m_{\eta_s} = \sqrt{m_{\phi}^2 - 0.56} = 690 \text{ MeV}$. Independently, an equal-spacing rule should hold for the squared masses of ideally-mixed pseudoscalars so that $m_{\eta_s}^2 = 2m_K^2 + m_{\pi}^2$, which gives $m_{\eta_s} = 687 \text{ MeV}$. Using this additional data point gives $d\left(E_{\text{hyp}}/dm_q\right)|_{m_q=m_u} = -0.32 \pm 0.06$, where the error estimate reflects the dependence of the fitted slope on our choice of constituent quark masses [14], and on the value of $m_{\eta_s}$.

Almost all model calculations are consistent with $m_d - m_u = 5 \pm 1 \text{ MeV}$ [15]. We adopt this value and show $\Delta m(M)$ in the third column of Table VI.

The electromagnetic contribution to $\Delta(M)$ is given by,

$$\Delta^\gamma(M) = \alpha_{\text{em}}q_h \left\{ \frac{1}{r} \right\}_0 - \left\{ \frac{1}{r} \right\}_1 + \frac{8\pi}{3} \frac{|\psi(0)|^2}{m_h m_n},$$

(2.3)

where the subscript $h$ refers to the heavy-flavor quark ($h = s, c, b$), and the subscript $n$ refers to the light anti-quark. The quantity $\alpha_{\text{em}}q_f \left\{ \frac{1}{r} \right\}_0$ is the Coulomb energy in the pseudoscalar (vector). The difference $\left\{ \frac{1}{r} \right\}_0 - \left\{ \frac{1}{r} \right\}_1$ is induced by hyperfine distortion of the wavefunctions; though it is often assumed to be negligible, explicit potential-model calculations indicate that this Coulomb term is only about 30% smaller than the magnetic term in (2.3) [16]. (We find it most convenient to discuss $\Delta^\gamma(M)$ in a potential-model framework, however, our results will turn out to be nearly independent of the parameters — and even the functional form — of the potential, hence we suspect that our conclusions are more reliable than our method.)

Consider calculating $\Delta^\gamma(M)$ in a Hamiltonian whose spin-independent part is

$$H_{\text{si}} = \frac{p^2}{2\mu} + br - \frac{4\alpha_0^0}{3} \left( \frac{b}{\mu^2} \right)^{1/3} \left( \frac{b}{\mu^2} \right)^{1/3}.$$ (2.4)

This is a standard Coulomb-plus-linear Hamiltonian, with a particular ansatz for the way in which the effective strong coupling constant varies with the reduced mass of the system: $\alpha_s(\mu) = \alpha_s^0 \cdot (b/\mu^2)^{1/3}$. We shall presently show that this form for $\alpha_s(\mu)$ gives excellent phenomenology, but for the moment we just capitalize on the fact that it yields simple scaling laws for the wavefunctions and energies of $H_{\text{si}}$. Since

$$\mu H_{\text{si}} = \frac{p^2}{2} + (\mu b)r - \frac{4\alpha_0^0}{3} (\mu b)^{1/3},$$ (2.5)

it follows that the energies are proportional to $(b^2/\mu)^{1/3}$, and wavefunction properties with dimension (length)$^n$ are proportional to $(\mu b)^{-n/3}$. Upon adding the spin-dependent term,

$$H_{\text{hyp}} = \frac{32\pi}{9} \alpha_s(\mu) \frac{S_h \cdot S_n}{m_h m_n} \delta^3(r),$$ (2.6)
we find the following scaling laws for the hyperfine, magnetic, and Coulomb energies:

$$E_{hyp} \equiv \frac{32\pi}{9} \alpha_s(\mu) \frac{\langle \psi(0)^2 \rangle}{m_h m_n} \propto \alpha_s(\mu) \frac{\mu b}{m_h m_n}$$  \hspace{1cm} (2.7)$$

$$E_{mag} \equiv \alpha_{em} q_h \frac{8\pi}{3} \frac{\langle \psi(0)^2 \rangle}{m_h m_n} \propto \frac{\mu b}{m_h m_n}$$  \hspace{1cm} (2.8)$$

$$E_{coul} \equiv \alpha_{em} q_h \left\{ \left\langle \frac{1}{r} \right\rangle_0 - \left\langle \frac{1}{r} \right\rangle_1 \right\} = \frac{64\pi}{9} \frac{\alpha_s(\mu)}{m_h m_n} \sum_n \frac{\langle 0 | \delta^2(r) | n \rangle \langle n | \frac{1}{r} | 0 \rangle}{E_0 - E_n} \propto \frac{\mu b}{m_h m_n} \right.  \hspace{1cm} (2.9)$$

Thus the Coulomb and magnetic terms scale in the same way, and we have

$$\Delta^\gamma(B) : \Delta^\gamma(D) : \Delta^\gamma(K) = q_h \frac{E_{hyp}(B)}{\alpha_s(\mu_B)} : q_c \frac{E_{hyp}(D)}{\alpha_s(\mu_D)} : q_s \frac{E_{hyp}(K)}{\alpha_s(\mu_K)}.  \hspace{1cm} (2.10)$$

In fact, it is easy to check that the result (2.10) holds not just for the specific potential in (2.4), but for any potential of the form

$$V_\beta(r) = C r^\beta - \frac{4 \alpha_s^0}{3} \frac{C}{r^{\beta+1}} \right.  \hspace{1cm} (2.11)$$

(And also for $V(r) = C \log(r/r_0) - (4/3)(\alpha_s^0/r)(C/\mu)^{1/2}$, which behaves in many ways like the $\beta \to 0$ limit of $V_\beta(r)$; we will refer to this logarithmic potential as $V_0(r)$.) In the general case, the ratios of electromagnetic splittings are

$$\Delta^\gamma(B) : \Delta^\gamma(D) : \Delta^\gamma(K) = q_h \mu_b^\epsilon \frac{E_{hyp}(B)}{\alpha_s(\mu_B)} : q_c \mu_d^\epsilon \frac{E_{hyp}(D)}{\alpha_s(\mu_D)} : q_s \mu_k^\epsilon \frac{E_{hyp}(K)}{\alpha_s(\mu_K)}.  \hspace{1cm} (2.12)$$

where $\epsilon \equiv (\beta + 1)/(\beta + 2)$.

With regard to the phenomenological soundness of equation (2.11), Fig. 4 shows the “running” of $\alpha_s(\mu)$ for three cases of interest, namely $\beta = 0, 1, 2$ (corresponding to logarithmic, linear, and harmonic confinement, respectively). In each case, the curve lies close to the empirical values of $\alpha_s(\mu_K)$, $\alpha_s(\mu_D)$, and $\alpha_s(\mu_B)$, which were determined by fitting to the measured $2P - 1S$ and $^3S_1 - ^1S_0$ meson splittings [17]. (Of course, the success of these fits is not mysterious; there is little difficulty in accurately parametrizing a monotonic, closely spaced set of data points ($\mu_B - \mu_K$ is only about 0.1 GeV) with an adjustable fitting function which is similarly monotonic.) Thus the potentials $V_\beta(r)$, originally constructed just to give simple scaling behaviour, actually coincide closely with purely phenomenological potentials tuned to fit the meson spectrum. This indicates that we may apply equation (2.12) with some confidence: fitting $\Delta^\gamma(D)$ to $\Delta(D)_{\text{expt}} - \Delta^m(D) = 1.50 \pm 0.12$ MeV, and taking $\beta = 1$ we obtain the results shown in columns 4 and 5 of Table V. The results are quite insensitive to $\beta$ and to the constituent quark masses; $\Delta^\gamma(B)$ and $\Delta^\gamma(K)$ change by less than 5% when beta is varied between 0 and 2, and by less than 10% when the quark masses are varied [14]. We thus assign a 10% “parameter uncertainty error” to each of $\Delta^\gamma(B)$ and $\Delta^\gamma(K)$, and add it in quadrature with the error arising from $\Delta^\gamma(D)$.
Both of the predictions in Table VI are interesting. First, we obtain a negative value for $\Delta(B)$, in disagreement with most other authors (see Table VII). The sign of our prediction follows just from the near-vanishing of $\Delta^m(B)$ (as implied by the near-equality of the $B$ and $B_s$ hyperfine splittings), and the fact that $\Delta^\gamma(B)$ is negative definite (since the Coulomb and magnetic terms in equation (2.3) have the same sign). Second, our result for $\Delta(K)$ is in agreement with the measured value, and even has the correct sign (though in this case there is a large cancellation between the quark-mass and electromagnetic terms), so that $\Delta(K)$ can apparently be explained without invoking large relativistic or unitarity effects.

III. CONCLUSIONS

In summary, we have analyzed baryon and meson isospin splittings and isomultiplet central masses in a constituent quark picture, avoiding (as much as possible) strong reliance on particular models or parameter sets. Our results indicate that improved measurements of the $\Delta$, $\Sigma_c$, $\Xi_c$, and $B^*$ splittings would be most interesting. The $\Delta$ masses are currently the most discrepant in the non-charm sector, the $\Sigma_c$ and the $\Xi_c$ seem to indicate the existence of substantial three-body effects, and measurement of the sign of $\Delta(B) = (B^{*-} - B^{*0}) - (B^- - \bar{B}^0)$ will provide a significant test of model calculations.

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FIGURES

FIG. 1. Octet and decuplet isospin splittings (deviations from multiplet centers) in MeV, for multiplets with \( I > 0 \). The fitted masses of \( j = \frac{1}{2} \) states are indicated by filled circles, and the fitted masses of \( j = \frac{3}{2} \) states are indicated by open circles. The crosses give the experimental data and errors. For clarity, \( j = \frac{1}{2} \) and \( j = \frac{3}{2} \) points have been slightly offset.

FIG. 2. Multiplet hyperfine energies, in MeV. A spin-independent mass term that depends on the constituent quark flavors has been subtracted from each energy. The fitted masses of \( j = \frac{1}{2} \) states are indicated by filled circles, and the fitted masses of \( j = \frac{3}{2} \) states are indicated by open circles. The data and errors are shown by crosses, but only the \( \Xi_c \) and \( \Omega_c \) have errors of noticeable size.

FIG. 3. Hyperfine splittings in \( s\bar{q}, c\bar{q}, \) and \( b\bar{q} \) mesons, as a function of the antiquark mass. (See the text for an explanation of the \( s\bar{s} \) data point.) Typical interpolating curves are also shown.

FIG. 4. The data points show effective values of \( \alpha_s(\mu_K), \alpha_s(\mu_D), \alpha_s(\mu_B) \), obtained by fitting meson splittings to harmonic+Coulomb, linear+Coulomb, and logarithmic+Coulomb potentials. The curves show the corresponding behaviour of \( \alpha_s(\mu) \) predicted by equation (2.11).
### TABLE I. Contributions to the fit to charmed baryon central masses.

|        | $T_{nn}$ | $T_{ss}$ | $S_{nn}$ | $S_{ns}$ | $A_{nc}$ | $A_{sc}$ | $D_{nc}$ | $D_{sc}$ | total | data | ± $\chi^2$ |
|--------|----------|----------|----------|----------|----------|----------|----------|----------|-------|------|-------------|
| $\Lambda_c$ | 0        | 0        | 213.5    | 0        | 2068.7   | 0        | 0        | 0        | 2282.3 | 2284.9 | 0.6 0.053    |
| $\Sigma_c - \Lambda_c$ | 412.1 | 0        | -213.5   | 0        | 0        | -27.0    | 0        | 0        | 171.5 | 167.8 | 0.3 0.047     |
| $\Xi_c$ | 0        | 0        | 0        | 350.6    | 1034.4   | 1088.5   | 0        | 0        | 2473.3 | 2470.0 | 1.5 0.130   |
| $\Omega_c$ | 0      | 557.0    | 0        | 0        | 0        | 2176.9   | 0       | -17.6    | 2716.3 | 2719.0 | 7.4 0.054    |
| $\eta_c$ | 0        | 0        | 0        | 0        | 0        | 0        | -6.8    | 6.6      | -0.15  | 0.00  | 1.60 0.008  |

### TABLE II. Comparison of the non-charm pair terms obtained from fits to charmed and non-charmed baryon central masses.

|        | $T_{nn}$ | $T_{ns}$ | $T_{ss}$ | $S_{nn}$ | $S_{ns}$ |
|--------|----------|----------|----------|----------|----------|
| From non-charm data | 412.2 ± 2.6 | 486.8 ± 3.0 | 556.8 ± 2.8 | 212.5 ± 5.4 | 351.4 ± 4.4 |
| Including charm data | 412.1 | 486.8 | 557.0 | 213.5 | 350.6 |
| $\chi^2$ | 0.002 | 0.000 | 0.005 | 0.034 | 0.033 |

### TABLE III. Predictions for charmed baryon masses and isospin splittings.

| Baryon | Predicted mass (MeV) | Splitting | Prediction (MeV) |
|--------|-----------------------|-----------|-----------------|
| $\Sigma_c^*$ | 2494 ± 16  | $\Sigma_c^{*1}$ | 1.00 ± 0.52 |
| $\Xi_c'$ | 2587 ± 12  | $\Sigma_c^{*2}$ | 1.64 ± 0.21 |
| $\Xi_c$ | 2621 ± 15  | $\Xi_c'^1$ | -0.52 ± 0.33 |
| $\Omega_c^*$ | 2743 ± 17  | $\Xi_c'^1$ | -0.44 ± 0.37 |

### TABLE IV. Contributions to the fit to charmed baryon isospin splittings.

|        | $T^1$ | $T^2$ | $S^1_s$ | $A^1_c$ | $D^1_c$ | total | data | ± $\chi^2$ |
|--------|-------|-------|--------|--------|--------|-------|------|-------------|
| $\Sigma_c^{*1}$ | -1.29 | 0     | 0      | 2.24   | -0.11  | 0.84  | 0.90 | 0.42 0.018 |
| $\Sigma_c^{*2}$ | 0     | 1.64  | 0      | 0      | 0      | 1.64  | -1.30 | 1.48 3.95  |
| $\Xi_c'^1$ | 0     | 0     | -4.03  | 1.12   | 0      | -2.91 | -6.30 | 2.30 2.17  |
TABLE V. Comparison of the non-charm pair terms obtained from fits to charm and non-charm isospin splittings.

|               | $T_1^1$ | $T_2^1$ | $T_3^1$ | $S_3^1$ |
|---------------|--------|--------|--------|--------|
| From non-charm data | $-1.29 \pm 0.0$ | $1.70 \pm 0.21$ | $-1.60 \pm 0.26$ | $-4.02 \pm 0.10$ |
| Including charm data | $-1.29$ | $1.64$ | $-1.59$ | $-4.03$ |
| $\chi^2$     | 0.000  | 0.082  | 0.000  | 0.006  |

TABLE VI. Quark mass and electromagnetic contributions to $\Delta(D)$, $\Delta(B)$, and $\Delta(K)$.

| Meson | $(dE_{hyp}/dm_q)|_{m_q=m_n}$ | $\Delta^m(M)$ | $\Delta^\gamma(M)$ | $\Delta(M)$ |
|-------|-------------------------------|----------------|---------------------|--------------|
| $D$   | $0.004 \pm 0.01$              | $-0.02 \pm 0.06$ | $1.50 \pm 0.12$    | $1.48 \pm 0.10$ $^a$ |
| $B$   | $0.005 \pm 0.01$              | $-0.02 \pm 0.06$ | $-0.26 \pm 0.03$   | $-0.29 \pm 0.07$ |
| $K$   | $-0.32 \pm 0.06$              | $1.60 \pm 0.44$  | $-1.75 \pm 0.22$   | $-0.15 \pm 0.49$ |

$^a$Used in fit.

TABLE VII. Various predictions for $\Delta(B)$. (The fourth entry is actually a constraint obtained by fitting the measured decay constants $f_D$ and $f_B$ to a nonrelativistic quark model.)

| Source | $\Delta(B)$ |
|--------|--------------|
| Ref. [11] | $0.3 \pm 0.03$ |
| Ref. [18] | 0.1 |
| Ref. [19] | 0.3 |
| Ref. [20] | $-0.05$ to $0.49$ |
| This work | $-0.29 \pm 0.07$ |
The text seems to be discussing the relationship between $\mu$ (GeV) and $\alpha_s(\mu)$, with three different curves labeled as harmonic, linear, and logarithmic. The diagram shows the variation of $\alpha_s(\mu)$ with $\mu$ for these different types of behavior.