Granular material pressure to reinforced concrete walls of cylindrical slender silos: Analysis and Experimental studies according to Eurocodes

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Abstract. Silos act as storage structures used for containing granular materials. The solids-induced loads acting on the walls and internals of silos are not easily determined, and inconsistent between commonly used design codes. This paper presents principles and a brief description of Eurocode standard to determine the solids-induced loads acting on reinforced concrete (RC) walls of cylindrical slender silos. That code is being considered as the most complete and modern standard for silo design up to now. A calculation example is provided to clarify the procedure. An experimental program was conducted to determine the friction coefficients between some granular materials and RC walls. The experimental results show that the values introduced in Eurocode can be used reliably.

1. Introduction
Silos are the storage structures used for containing granular materials such as coal, cement, cement clinker, wheat, etc. They can be in rectangular or circular forms in plan and the height is normally greater than 1.5 times of characteristic inside dimension of silo cross-section [1]. To design silo structure, all loads likely applied to the silo shall be determined, including wind, seismic, external and loads induced by stored bulk solids (solids-induced loads). There are numerous standards specifying the ways to calculate materials-induced loads. The four commonly used in the world are Eurocode standard EN 1991-4:2006 (EC1-4) [2], American standard ACI 313-16 [3], American society of agricultural engineers ANSI/ASAE EP433 [4], Australian standard AS 3774-1996 [5]. However, these codes give inconsistent guidance to engineers, and many common silo design conditions are not covered yet.

The solids-induced loads acting on the walls and internals of the structures are however not easily determined, and inconsistent between the commonly used design codes. If the loadings are not accurately determined, a silo failure may occur, causing significant losses. Cause of damage and failures in silo structures around the World has been reported and discussed extensively [6-7].

This paper presents principles and a brief description of Eurocode standard to determine the solids-induced loads acting on reinforced concrete (RC) walls of cylindrical slender silos with a detailed calculation example. An experimental program to determine the friction coefficients between some granular materials and RC walls was conducted and discussed in this paper.
2. Granular material pressure to reinforced concrete walls of slender silos to the Eurocodes

The Eurocode is considered as the most complete and modern standard for silo design until now within the above-mentioned design codes, since it exploits extensive European research works on silos over the past 30 years. On the other hand, it is quite complicated because the stipulations can be found in four different codes [2, 8-10]. The code stipulates that different silo geometrical arrangements require different design situations, depending on silo slenderness and hopper geometries, stored granular materials, filling and discharge arrangements.

The code divides silos into three assessment classes, ranging from simple/small silos with the capacity smaller than 100 ton (Action assessment class 1) to large/complex structures with the capacity over 10,000 ton or over 1,000 ton but with a large eccentric discharge (Action assessment class 3). The silos in Action assessment class 2 are classified in between the two extremes. This classification allows to apply an appropriate design procedure, avoiding any over-simplifications and disputations.

Silos are then classified by aspect ratio $h_c/d_c$, where $h_c$ is height of vertical-walled segment of silo from the transition to the equivalent surface and $d_c$ is characteristic dimension of inside of silo cross-section. Silos are classified slender ($h_c/d_c \geq 2$), intermediate slenderness ($1 < h_c/d_c < 2$), squat ($0.4 < h_c/d_c \leq 1$) and retaining ($h_c/d_c \leq 0.4$). Each geometry shall be treated with different loading features and design situations. Since slender silos are the most critical, this paper focuses only on those.

2.1. Design situations and properties of particulate solids

Silo aspect slenderness, the stored bulk solid, and conditions of filling and discharge are the factors affecting design situations. EC1-4 is the first silo design code to define many different conditions and regulate stipulations for each case. Considering fill and discharge conditions, three flow types are specified, namely, mass flow, pipe flow and mixed flow. On the other hand, even in the simple case of symmetric fill and discharge, accidental asymmetries also can occur resulting in non-uniform pressures around the circumference of a silo. EC1-4 deals with this situation by stipulating patch loads, which the other codes do not.

Three key material properties (coefficient of wall friction between the stored bulk solid and the wall surface $\mu$, bulk unit weight $\gamma$, and lateral pressure ratio $k$) are recommended to be determined by tests with test instruction given in Annex C [2]. Otherwise, the values given in Annex E can be used. EC1-4 is the first code to fully consider the fact that characteristic actions for different parts of a silo should be treated differently. Therefore, the code introduces upper and lower characteristic values for particulate solids properties used for different design situations and design purposes. For examples, when calculating maximum normal pressure on vertical wall, the lower values of wall friction coefficient $\mu$ and of angle of internal friction $\phi$ shall be used while the upper value of the lateral pressure ratio $K$ shall be used.

Roughening of the silo walls affects significantly the wall friction coefficient. Only EC1-4 has taken it into consideration by specifying wall surface into four categories, namely, slippery (D1), smooth (D2), raspy (D3) and irregular (D4). This is an advantage of EC1-4 compared to the other silo design codes.

2.2. Filling loads on vertical walls of slender silos

2.2.1. Symmetrical filling load

The values of horizontal pressure $p_{ho}$, wall friction traction $p_{w}$ and vertical pressure $p_v$ at any depth after filling can be calculated by Equation (1). The distribution shape of the pressure is shown in Figure 1.

$$p_{ho}(z) = p_{ho}Y_f(z)$$
$$p_{w}(z) = \mu p_{ho}Y_f(z)$$
$$p_{v}(z) = \frac{p_{ho}}{K}Y_f(z)$$
where: $\gamma$ is the characteristic value of the unit weight; $\mu$ is the characteristic value of the wall friction coefficient; $K$ is the characteristic value of the lateral pressure ratio; $z$ is the depth below the equivalent surface of the solid; $A$ is the plan cross-sectional area of the silo; $U$ is the internal perimeter of the plan cross-section of the silo.

**Figure 1.** Symmetrical filling pressures in the vertical-walled segment [2].

### 2.2.2. Filling patch load

The reference magnitude of the filling patch pressure $p_{pf}$ can be calculated by Equation (3). It should be noted that only EC1-4 stipulates filling and discharge patch loads.

$$
\begin{align*}
  p_{pf} &= C_{pf} p_f \\
  C_{pf} &= 0.21 C_{op} \left[1 + 2E^{5}\left(1 - e^{-\frac{(k \cdot l^t - 1)}{5}}\right)\right] \\
  &\geq 0
\end{align*}
$$

For thick-walled circular silos ($d_c/t \leq 200$): $p_{pf} = p_f / 7$.

For thin-walled circular silos ($d_c/t > 200$): $p_{pf} = p_f \cos \theta$.

The form of the filling patch pressure depends on type of the silo wall as shown in **Figure 2**.
2.3. Discharge loads on vertical walls of slender silos

2.3.1. Symmetrical discharge load. The symmetrical discharge pressures $p_{he}$ and $p_{we}$ should be determined by Equation (4).

$$
\begin{align*}
    p_{he} &= C_h p_{hf} \\
    p_{we} &= C_w p_{wf}
\end{align*}
$$

where $C_h$ and $C_w$ are the discharge factors for horizontal pressure and for wall frictional traction, respectively.

2.3.2. Discharge patch load. The reference magnitude of the discharge outward patch pressure $p_{pe}$ should be determined by Equation (5).

$$
    p_{pe} = C_{pe} p_{he}
$$

For $h_c/d_c > 1.2$: $C_{pe} = 0.42C_{op} \left[1 + 2E^2 \left(1 - e^{-1.5(h_c/d_c)}\right)\right]$

For $h_c/d_c \leq 1.2$: $C_{pe} = 0.272C_{op} \left[h_c/d_c - 1 + E\right] \geq 0$

For thick-walled circular silos: $p_{pe} = p_{pe}/7$. For thin-walled circular silos: $p_{pe} = p_{pe} \cos \theta$. The form of the discharge patch pressure depends on the silo form as shown in Figure 3.

**Figure 2.** Circular silos: side elevation and plan view of the filling patch load [2].

The height of the zone on which the patch load is applied should be taken as: $s \approx 0.2d_c$. 
2.4. Alternative method to determine filling and discharge patch loads

An alternative method to determine filling and discharge patch loads is to multiply the symmetrical loads with a factor to take account of asymmetries in the filling and discharge processes.

For thick-walled circular silos, the total symmetrical horizontal pressures for filling \( (p_{hf,u}) \) and discharge \( (p_{he,u}) \) are calculated by Equation (6).

\[
p_{hf,u} = p_{hf} \left( 1 + \zeta C_{pf} \right)
\]
\[
p_{he,u} = p_{he} \left( 1 + \zeta C_{pe} \right)
\]
\[
\zeta = 0.5 + 0.01 (d_c / t) \geq 1.0
\]

For thin-walled circular silos, the total symmetrical horizontal pressures for filling \( (p_{hc,u}) \) and discharge \( (p_{hc,a}) \), and the total symmetrical frictional traction for filling \( p_{wf,u} \) and discharge \( p_{wf,a} \) are calculated by Equation (7).

\[
p_{hf,u} = p_{hf} \left( 1 + 0.5C_{pf} \right)
\]
\[
p_{wf,u} = p_{wf} \left( 1 + C_{pf} \right)
\]
\[
p_{he,a} = p_{he} \left( 1 + 0.5C_{pe} \right)
\]
\[
p_{we,a} = p_{we} \left( 1 + C_{pe} \right)
\]

3. Design example

An example is given to illustrate the form and magnitude of granular material pressure to RC walls of a cylindrical slender silo. The silo has a total height \( h_t \) of 50m, internal diameter \( d_c \) of 15m. The chamber height \( h_c \) is 40m, the effective storage height \( h_e \) is 39m. The cross-sectional of silo \( A \) is 176.71m², the perimeter \( U \) is 47.12m. The silo capacity is 4824 ton. The assumed thickness \( t \) is 300mm. The silo has no internal structure inside. Consider the silo has the wall surface category D3.
The silo contains limestone power with the upper characteristic value of unit weight of 13.0 kN/m³. The modification coefficient for lateral pressure ratio \( a_K \) is 1.2. The modification coefficient for wall friction coefficient \( a_\mu \) is 1.07. The mean value of lateral pressure ratio \( K_m \) is 0.54. The mean value of wall friction coefficient \( \mu_m \) is 0.51. The modification coefficient for internal angle friction \( a_\phi \) is 1.22. The mean value of internal angle friction \( \phi_m \) is 30°.

EC1-4 is applied because the following dimensional limitations are satisfied: \( h_b/d_c < 10 \), \( h_b < 100 \text{m}, \) \( d_c < 60 \text{m}. \) The silo is slender silo with thick wall since \( h_c/d_c \geq 2.0 \) and \( d_c/t < 200 \).

EC1-4 stipulates that for different design situations and purposes, the lower and upper characteristic values should be adopted correspondingly. Table 1 lists out all the lower and upper characteristic values to be adopted.

### Table 1. Design purposes and characteristic values adopted.

| Purpose:                      | Characteristic value adopted |
|-------------------------------|-----------------------------|
|                               | Wall friction coefficient \( \mu \) | Lateral pressure ratio \( K \) | Angle of internal friction \( \phi \) |
| For the silo vertical wall    | 0.48                         | 0.65                         | 25°                        |
| 1) Maximum normal pressure on |                               |                              |                            |
| vertical wall used to design  |                               |                              |                            |
| silo wall in circumferential  |                               |                              |                            |
| direction                     |                               |                              |                            |
| 2) Maximum frictional traction |                               |                              |                            |
| on vertical wall used to design|                               |                              |                            |
| silo wall in vertical direction|                               |                              |                            |
| 3) Maximum vertical load on hop|                               |                              |                            |
| per or silo bottom used to design connection between wall and hopper | 0.48 | 0.45 | 37° |
| For the hopper wall           | 0.48                         | 0.45                         | 25°                        |
| 4) Maximum hopper pressures on |                               |                              |                            |
| filling                       | 0.48                         | 0.65                         | 37°                        |
| 5) Maximum hopper pressures on |                               |                              |                            |
| discharge                     | 0.48                         | 0.45                         | 25°                        |

The filling and discharge patch load factors \( C_{pf} \) and \( C_{pe} \) are calculated as 0.0964 and 0.1928, respectively.

![Figure 4. Horizontal pressure and wall friction traction on silo wall due to filling and discharge.](image-url)
The calculation results are shown in Figure 4. The maximum values of horizontal pressure $p_{hf}$ and the wall friction traction $p_{of}$ due to symmetrical filling are 97.9 kN/m$^2$ and 47.0 kN/m$^2$, respectively. The symmetrical discharge pressures $p_{he}$ and $p_{we}$ reach the values of 112.6 kN/m$^2$ and 51.7 kN/m$^2$, respectively. The pressures during discharge are about 1.1 to 1.15 times of the pressures during filling. Using the alternative method to take into account of filling and discharge patch loads $p_{hf,u}$ and $p_{he,u}$, the maximum values of horizontal pressure during filling and discharge increase about 10% to 20%.

4. Experimental study on friction coefficient between granular materials and RC silo walls

To determine accurately filling and discharge loads on vertical walls, the key parameter is the friction coefficient between the stored material and the silo wall. In Vietnam, previously when design a silo the factors related to material properties are taken from Russian standard [11-13]. It may lead to inaccuracy in design since local conditions may be different. Therefore, an experimental study according to Eurocode principles is required.

4.1. Principle of the test

EC1-4 recommends that properties of stored particulate solids shall be obtained either from test results or from other relevant data. Therefore, Appendix C of EC1-4 introduces very detailed the principles to measure of properties of solids for silo load evaluation. Figure 5 shows the test method to determine the wall friction coefficient.

As shown in Figure 5, the stored material is contained in a box with dimension of $D \times H$. The total vertical force is $N$. Applying a horizontal force $F$ with a loading rate of 0.04 mm/s and measuring lateral displacement $\Delta$, the wall friction coefficient can be calculated as $\mu = F_r/N$. $F_r$ is the value at which the lateral force is stable.

![Figure 5](image)

(a) Principle to measure wall friction  
(b) Typical shear-displacement curve

**Figure 5.** Test method to determine wall friction coefficient [2].

4.2. Test setup and test specimens

An experimental study was conducted by the authors at the Laboratory of Construction Testing and Inspection, National University of Civil Engineering. The test setup is shown in Figure 6 showing the location of instrumentations.
Figure 6. Test setup.

A 1200 long × 1200 wide × 60 thick (in mm) reinforced concrete plate (1) was placed horizontally, its smooth surface is upwards to represent a silo wall. Stored granular material with a total weight of 900N was stored in a 600 long × 500 wide × 320 high (in mm) wood box (2). There were few cast-iron weights (4) above the box to achieve the predetermined testing weight. The box was connected to a jack (9) and a reaction frame (7). Loading was applied to the jack through a hydraulic pump (8). A load cell (6) was attached to measure accurately the force. A linear variable displacement transducer (LVDT) (5) was used to measure lateral displacement of the box. A data logger (10) and a computer (11) were used to record testing data.

Five granular materials were tested, including crushed stone (D), coarse sand (CV), cement (XM), maize (NH) and rice (G). Each material was tested three times, corresponding with a total vertical load $N$ equals to 1800, 2200 and 2600N, respectively.

4.3. Test results and discussion

Table 2 shows a summary of the test results, indicating that the results are quite consistent between there tests.

| Material   | Specimen 1 (N=1800N) | Specimen 2 (N=2200N) | Specimen 3 (N=2600N) | Mean value µTN |
|------------|-----------------------|-----------------------|-----------------------|----------------|
|            | $F_r$ (N) | $\mu^{TN}$ | $F_r$ (N) | $\mu^{TN}$ | $F_r$ (N) | $\mu^{TN}$ | $\mu^{TN}$ |
| Stone (D)   | 800      | 0.444       | 950      | 0.432       | 1175     | 0.452       | 0.443       |
| Sand (CV)   | 650      | 0.361       | 875      | 0.398       | 1100     | 0.423       | 0.394       |
| Cement (XM) | 800      | 0.444       | 950      | 0.432       | 1325     | 0.510       | 0.462       |
| Maize (NH)  | 650      | 0.361       | 725      | 0.330       | 825      | 0.317       | 0.336       |
| Rice (G)    | 750      | 0.417       | 925      | 0.420       | 1000     | 0.385       | 0.407       |

The test results are then compared to the values from EC1-4, ACI 313-16 and Russian Code [11-13] as shown in Table 3. The results of wall friction coefficient are quite close with the values recommended in EC1-4 for D2 wall (smooth), since the mean coefficient is 0.979 with COV is only 0.078, while compared to ACI 313 the mean coefficient is 0.912, and only 0.891 for Russian code.
It can be concluded that the values recommended in EC1-4 are comparable with the experimental values and can be used in Vietnam. On the other hand, EC1-4 provides different values of wall friction coefficient for different types of silo walls, that is very practical and advancing.

5. Conclusions
The new Eurocode design method for silos is time consuming and more complicated than previous design codes, but it is quite comprehensive and covers many common silo design situations. Therefore, the Eurocodes should be used to design RC silos.

The principles and the brief description of EC1-4 to determine the solids-induced loads acting on RC silo walls have been introduced, together with a calculation example. An independent experiment was conducted to investigate the friction coefficients of some granular materials with concrete surfaces, showing that the values introduced in EC1-4 can be used reliably in Vietnam conditions.

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Table 3. Comparison between test results with design codes.

| Material | Test value (µTN) | EC1-4 - µTN/µIC (µIC) | ACI 313 | Russian Code |
|----------|-----------------|------------------------|----------|--------------|
|          |                 | D1 wall | D2 wall | D3 wall | µTN/µIC (µIC) | µTN/µIC (µIC) |
| Stone (D)| (0.443)         | 1.136 | 0.904 | 0.751 | 0.984 | 0.886 |
| Sand (CV)| (0.394)         | 1.037 | 0.821 | 0.691 | 0.876 | 0.788 |
| Cement (XM)| (0.462) | 1.127 | 1.004 | 0.906 | 0.991 | 0.924 |
| Maize (NH)| (0.336)        | 1.527 | 0.933 | 0.634 | 0.794 | 0.840 |
| Rice (G)| (0.407)         | 1.696 | 1.233 | 0.848 | 0.917 | 1.018 |

Mean / COV 1.305 / 0.145 0.979 / 0.078 0.766 / 0.056 0.912 / 0.041 0.891 / 0.044
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