Charmonium decay widths in magnetized nuclear matter
– effects of (inverse) magnetic catalysis and PV mixing

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Abstract

The in-medium partial decay width of charmonium state, $\psi(3770)$ to open charm ($D$ and $\bar{D}$) mesons in magnetized isospin asymmetric nuclear matter is studied using a field theoretic model of composite hadrons. The medium modification of the decay width is computed using the in-medium masses of the charmonium and the open charm mesons calculated within a chiral effective model. The mass modifications of the $D$ and $\bar{D}$ mesons arise due to their interactions with the nucleons and the scalar mesons ($\sigma(\sim \langle \bar{u}u \rangle + \langle \bar{d}d \rangle)$, $\zeta(\sim \langle \bar{s}s \rangle)$, $\delta(\sim \langle \bar{u}u \rangle - \langle dd \rangle)$), and the mass shift of the charmonium state is obtained from the medium modification of a scalar dilaton field, $\chi$, which mimics the gluon condensates of QCD within the chiral effective model. The contributions of the Dirac sea of the nucleons are taken into account, which are observed to lead to an enhancement (reduction) of the quark condensates with increase in magnetic field, an effect called the (inverse) magnetic catalysis. In the presence of the external magnetic field, there is mixing of spin 0 (pseudoscalar) and spin 1 (vector) states (PV mixing) which modifies their masses and hence affects the charmonium decay width. The mass modifications due to the $D - D^*$ and $\bar{D} - \bar{D}^*$ mixings are observed to be much more dominant as compared to the $\psi(3770) - \eta_c'$ mixing. The effect due to (inverse) magnetic catalysis arising from the Dirac sea contributions of the nucleons on the decay width of $\psi(3770) \to D\bar{D}$ is observed to be quite significant. This should have observable effects on the production of the charmonium states and open charm mesons arising from ultra-relativistic peripheral heavy ion collision experiments, at RHIC and LHC, where extremely large magnetic fields are estimated to be produced.

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I. INTRODUCTION

The study of the in-medium properties of the heavy flavour mesons [1], in particular in the presence of strong magnetic fields, has been a topic of intense research due to its relevance in relativistic heavy ion collision experiments. The magnetic fields produced in ultra-relativistic peripheral heavy ion collision experiments, e.g., at LHC, CERN and RHIC, BNL, are estimated to be huge [2], the time evolution of the magnetic field, however, is still an open question. There have been a lot of studies of the effects of the magnetic field on the heavy flavour mesons in the literature, since the heavy flavour mesons are created at the early stage when the magnetic field can still be extremely large. The heavy quarkonium states and the open heavy flavour mesons have been studied extensively in the literature using the potential models [3–13], the QCD sum rule approach [14–31], the coupled channel approach [32–38], the quark meson coupling (QMC) model [39–47], as well as using a chiral effective model [48–55]. Studies of heavy quarkonium states (\(\bar{Q}Q\) bound states, \(Q = c, b\)) in presence of a gluon field, assuming the distance between \(Q\) and \(\bar{Q}\) to be small compared to the scalar of the gluonic fluctuations, show that the mass modifications of these states arise from the medium modification of the scalar gluon condensate in the leading order [56–59]. In a chiral effective model, the in-medium masses of the heavy quarkonium (charmonium and bottomonium) have been computed from the medium changes of a scalar dilaton field [50, 51, 55], which simulates the gluon condensates of QCD within the effective hadronic model. The mass modifications of the open heavy flavour (charm and bottom) mesons within the chiral effective model arise due to their interactions with the baryons and scalar mesons in the hadronic medium [48–54].

The chiral effective model, in the original version with three flavours of quarks (SU(3) model) [60–63], has been used extensively in the literature, for the study of finite nuclei [61], strange hadronic matter [62], light vector mesons [63], strange pseudoscalar mesons, e.g. the kaons and antikaons [64–67] in isospin asymmetric hadronic matter, as well as for the study of bulk matter of neutron stars [68]. Within the QCD sum rule framework, the light vector mesons [69, 70], as well as, the heavy quarkonium states [16–18], in (magnetized) hadronic matter have been studied, using the medium changes of the light quark condensates and gluon condensates calculated within the chiral SU(3) model. The effects of the magnetic fields on the masses of the \(K\) and \(\bar{K}\) mesons have been studied within the chiral SU(3)
model in Ref. [71]. The model has been used to study the partial decay widths of the heavy quarkonium states to the open heavy flavour mesons, in the hadronic medium [51] using a light quark creation model [72], namely the $^3P_0$ model [73–76] as well as using a field theoretical model for composite hadrons [77, 78]. The effects of magnetic field on the charmonium partial decay widths to $D\bar{D}$ mesons have been studied using the $^3P_0$ model [79] and, charmonium (bottomonium) decay widths to $D\bar{D}$ ($B\bar{B}$) using the field theoretic model of composite hadrons [80, 81].

In the present work, the in-medium partial decay widths of $\psi(3770)$ to $D\bar{D}$ are studied in magnetized (nuclear) matter using the field theoretic model of composite hadrons, from the mass modifications of these mesons calculated within the chiral effective model. The Dirac sea (DS) contributions to the self-energy of the nucleons (through summing over the tadpole diagrams) are also taken into consideration in the chiral effective model. As the matter created in ultra-relativistic peripheral heavy ion collisions is dilute, we study the partial decay width of $\psi(3770)$ (which is the lowest charmonium state which can decay to $D\bar{D}$ in vacuum) for $\rho_B = 0$ as well as for $\rho_B = \rho_0$, the nuclear matter saturation density, for symmetric as well as asymmetric nuclear matter in the presence of an external magnetic field. Within the chiral effective model, the Dirac sea contributions are observed to lead to an enhancement (reduction) of the quark condensates (through the scalar $\sigma(\sim \langle \bar{u}u \rangle + \langle \bar{d}d \rangle)$ and $\zeta(\sim \langle \bar{s}s \rangle)$ fields), with increase in the magnetic field, an effect called ‘(inverse) magnetic catalysis’ [82–85]. The Dirac sea effects have been studied extensively in the literature in the quark matter sector using the Nambu-Jona-Lasinio model [86–88]. In Ref. [89], the effects of Dirac sea have been studied for the nuclear matter using the Walecka model and an extended linear sigma model. These are observed to lead to magnetic catalysis (MC) for zero temperature and zero density, which is observed as a rise in the effective nucleon mass with the increase in magnetic field. In Ref. [90], the contributions of Dirac sea of the nucleons to the self-energies of the nucleons have been studied in the Walecka model by summing over the scalar ($\sigma$) and vector ($\omega$) tadpole diagrams. These have been calculated in a weak field approximation of the fermion propagator. At zero density, the effects of the Dirac sea are seen to lead to magnetic catalysis (MC) effect at zero temperature [90]. When the anomalous magnetic moments (AMMs) of the nucleons are taken into account, at a finite density and zero temperature, there is observed to be a drop in the effective nucleon mass with increase in the magnetic field. This behaviour with the magnetic field is observed when
the temperature is raised from zero to non-zero values, up to the critical temperature, $T_c$, when the nucleon mass has a sudden drop, corresponding to the vacuum to nuclear matter phase transition. The decrease in $T_c$ with increase in value of $B$ is identified with the inverse magnetic catalysis (IMC) [90]. This effect is also observed in some lattice QCD calculations [91], where the $T_c$ is seen to decrease with increase in the magnetic field.

In the present work, we incorporate the effects of the Dirac sea to the nucleon propagator, through summation of scalar ($\sigma, \zeta$ and $\delta$) and vector ($\omega$ and $\rho$) tadpole diagrams within the chiral effective model. When the anomalous magnetic moments (AMMs) of the nucleons are not taken into account, for zero density as well as for $\rho_B = \rho_0$, magnetic catalysis (MC) is observed. However, when the AMMs of nucleons are considered, for $\rho_B = \rho_0$ (both for symmetric and asymmetric nuclear matter), inverse magnetic catalysis (IMC) is observed, i.e., the quark condensate is observed to be reduced with rise in the magnetic field. In the presence of the magnetic field, the effects of the pseudoscalar meson-vector meson (PV)mixings [80, 81, 92, 93] are taken into account in the present study. These modify the masses of the pseudoscalar and the longitudinal component of the vector meson, leading to a decrease (increase) in the mass of the pseudoscalar (longitudinal component of the vector) meson. The PV mixing effects are taken into account to study the masses of the charm mesons. Additionally, the charged open charm mesons have contributions from the Landau levels to their masses. Including the Dirac sea contributions for the nucleons, as well as including the PV mixing, the masses of the open charm mesons [94] and the heavy quarkonium states [95] have been studied using the chiral effective model. The effects on the masses due to the PV mixing of the open charm mesons ($D - D^*$ and $\bar{D} - \bar{D}^*$ mixings) are observed to be much more dominant as compared to the $\psi(3770) - \eta_c'$ mixing. In the present work, the decay width of $\psi(3770) \rightarrow D\bar{D}$ is calculated using the mass modifications of these mesons in magnetized nuclear matter, including the effects of Dirac sea of nucleons [94, 95]. There is observed to be substantial increase in the decay width of $\psi(3770) \rightarrow D\bar{D}$, for the neutral $D\bar{D}$ channel, when the mixing of open charm mesons are taken into account, as compared to when the contributions of PV mixing for the masses of $D(\bar{D})$ are not considered.

The outline of the paper is as follows. In section II, we describe briefly the chiral effective model used to calculate the masses of the charmonium and open charm mesons, accounting for the effects of the Dirac sea for the nucleons. The anomalous magnetic moments (AMMs) of the nucleons are also taken into account in the present work. The PV mixing effects
are also taken into account which modifies the masses of the charmonium as well as open charm mesons. As has already been mentioned, the $D$ and $\bar{D}$ masses are calculated from their interaction with the nucleons and the scalar fields, whereas the charmonium masses are calculated within the model from the modification of the scalar dilaton, which mimics the scale symmetry breaking of QCD. The magnetic field effects considered on the masses of the charmed mesons are the pseudoscalar-vector meson (PV) mixing and the (inverse) magnetic catalysis effects (in addition to contributions of Landau levels for the charged mesons). The former corresponds to the mixing of the pseudoscalar (spin 0) and the vector (spin 1) mesons in the presence of a magnetic field. The latter, the (inverse) magnetic catalysis effect, arises from the Dirac sea contributions of nucleons to their self-energies. In section III, the computation of the decay width of $\psi(3770) \rightarrow D\bar{D}$ using the field theoretic model of composite hadrons is briefly described. The results of these in-medium decay widths in magnetized isospin asymmetric nuclear matter are discussed in section IV and the summary of the present work are given in section V.

II. MASS MODIFICATIONS OF OPEN CHARM AND CHARMONIUM STATES

We describe the chiral effective model used to study the open charm ($D$ and $\bar{D}$) mesons and the charmonium states in magnetized nuclear matter. The model is based on a nonlinear realization of chiral symmetry, and, the breaking of scale invariance of QCD is incorporated into the model through the introduction of a scalar dilaton field, $\chi$ [60–63]. The Lagrangian of the model, in the presence of a magnetic field, has the form [96–98]

$$\mathcal{L} = \mathcal{L}_{\text{kin}} + \sum_{W} \mathcal{L}_{BW} + \mathcal{L}_{\text{vec}} + \mathcal{L}_{0} + \mathcal{L}_{\text{scalebreak}} + \mathcal{L}_{SB} + \mathcal{L}_{\text{mag}}^{B\gamma},$$  

where, $\mathcal{L}_{\text{kin}}$ corresponds to the kinetic energy terms of the baryons and the mesons, $\mathcal{L}_{BW}$ contains the interactions of the baryons with the meson, $W$ (scalar, pseudoscalar, vector, axialvector meson), $\mathcal{L}_{\text{vec}}$ describes the dynamical mass generation of the vector mesons via couplings to the scalar fields and contains additionally quartic self-interactions of the vector fields, $\mathcal{L}_{0}$ contains the meson-meson interaction terms, $\mathcal{L}_{\text{scalebreak}}$ is a scale invariance breaking logarithmic potential given in terms of a scalar dilaton field, $\chi$ and $\mathcal{L}_{SB}$ describes the explicit chiral symmetry breaking. The masses of the baryons are generated by the interaction with the scalar mesons. The term $\mathcal{L}_{\text{mag}}^{B\gamma}$ describes the interaction of the baryons
with the electromagnetic field, which includes a tensorial interaction $\sim \bar{\psi}_i \sigma^{\mu\nu} F_{\mu\nu} \psi_i$, whose coefficients account for the anomalous magnetic moments of the baryons \[96\, 98\].

In the present study of magnetized (nuclear) matter, the meson fields are treated as classical and nucleons as quantum fields and the self energy of the nucleons include the contributions from the Dirac sea. The mass of $i$-th baryon ($i = p, n$ in the present study), generated from the baryon-scalar field interaction term, is given as

$$M_i = -g_{\sigma i} \sigma - g_{\zeta i} \zeta - g_{\delta i} \delta,$$ \hspace{1cm} (2)

where, $\sigma$, $\zeta$ and $\delta$ are the non-strange iso-scalar, strange iso-scalar, and nonstrange isovector scalar fields. The effects of the Dirac sea of the nucleons are incorporated through summing the tadpole diagrams for the nucleon propagator, arising from their interactions with the scalar and vector fields. The masses of the open charm ($D$ and $\bar{D}$) mesons in magnetized (nuclear) matter, are obtained by solving their dispersion relations \[96\, 97\]

$$-\omega^2 + \vec{k}^2 + m_{D(\bar{D})}^2 - \Pi_{D(\bar{D})}(\omega, |\vec{k}|) = 0,$$ \hspace{1cm} (3)

where $\Pi_{D(\bar{D})}$ denotes the self energy of the $D$ ($\bar{D}$) meson in the medium. The masses are given as $m_{D(\bar{D})}^* = \omega(|\vec{k}| = 0)$, which depend (through the self energies) on the values of the scalar fields ($\sigma$, $\zeta$ and $\delta$) as well as the number and scalar densities of the nucleons. The scalar densities of the proton and neutron, $\rho_p^s$ and $\rho_n^s$, include the Dirac sea contributions in the weak magnetic field approximation for the nucleon propagator \[90\]. The anomalous magnetic moments (AMMs) of the nucleons are taken into account in the present study. For $\rho_B = \rho_0$, these are observed to lead to the effect of inverse magnetic catalysis (IMC), whereas, there is seen to be magnetic catalysis (MC), when the AMMs are ignored. However, for zero density, there is observed to be magnetic catalysis arising from the Dirac sea contributions, even when the AMMs of the nucleons are taken into account. The Dirac sea (DS) contributions are observed to lead to significant modifications to the open charm as well as charmonium states, which in turn, modify appreciably the decay width of $\psi(3770) \rightarrow D\bar{D}$. In the presence of a magnetic field, the lowest Landau level (LLL) contributions are taken into account for the charged $D^\pm$ mesons. The effective masses of the $D$ and $\bar{D}$ mesons are thus given as

$$m_{D^\pm}^{\text{eff}} = \sqrt{m_{D^\pm}^*^2 + eB}, \quad m_{D^0(\bar{D}^0)}^{\text{eff}} = m_{D^0(\bar{D}^0)}^*,$$ \hspace{1cm} (4)

where $m_{D(\bar{D})}^*$ is the mass obtained as solution of the dispersion relation given by equation \[3\].
FIG. 1: Decay widths of (I) $\psi(1D) \rightarrow D^+D^-$, (II) $\psi(1D) \rightarrow D^0\bar{D}^0$, and (III) the sum of these two channels (I) and (II), as functions of $eB/m^2_\pi$, for $\rho_B = 0$ with the AMMs of nucleons taken into account. The effects due to the Dirac sea (DS) contributions are included. The effects of the $D - D^*$ ($\bar{D} - \bar{D}^*$) mixing on these decay widths are shown in (b) and (d), without and with the additional effect from $\psi(1D) - \eta'_c$ mixing, respectively. The results are compared with the cases when the AMMs of nucleons are not considered (shown as dotted lines).

The dilaton field $\chi$ of the scale breaking term $\mathcal{L}_{\text{scalebreak}}$ in the chiral effective model is related to the scalar gluon condensate of QCD and this relation is obtained by equating the trace of the energy momentum tensor in the chiral effective model and in QCD [50, 51, 55]. The mass shift of the charmonium state in the magnetized nuclear matter is hence computed
FIG. 2: Decay widths of (I) $\psi(1D) \to D^+ D^-$, (II) $\psi(1D) \to D^0 \bar{D}^0$, and (III) the sum of these two channels (I) and (II), as functions of $eB/m^2_\pi$, for $\rho_B = \rho_0$ and $\eta = 0$ with the AMMs of nucleons taken into account. The effects due to the Dirac sea (DS) contributions are shown in (b) and (d), without and with the $\psi(1D) - \eta_c'$ mixing, respectively. The results are compared with the cases when the AMMs of nucleons are not considered (shown as dotted lines).

from the medium change of the dilaton field from vacuum value, calculated within the chiral effective model, and is given as [50, 51]

$$\Delta m = \frac{4}{81} (1 - d) \int d|\mathbf{k}|^2 \langle |\frac{\partial \psi(\mathbf{k})}{\partial \mathbf{k}}|^2 \rangle \frac{|\mathbf{k}|}{|\mathbf{k}|^2/m_c + \epsilon} \left( \chi^4 - \chi_0^4 \right), \quad (5)$$

where

$$\langle |\frac{\partial \psi(\mathbf{k})}{\partial \mathbf{k}}|^2 \rangle = \frac{1}{4\pi} \int \left| \frac{\partial \psi(\mathbf{k})}{\partial \mathbf{k}} \right|^2 d\Omega. \quad (6)$$
FIG. 3: Same as Fig. 2, with additional mass modifications of the open charm mesons from $D - D^*$ and $\bar{D} - \bar{D}^*$ mixing effects.

In equation (5), $d$ is a parameter introduced in the scale breaking term in the Lagrangian, $\chi$ and $\chi_0$ are the values of the dilaton field in the magnetized medium and in vacuum respectively. The wave functions of the charmonium states, $\psi(k)$ are assumed to be harmonic oscillator wave functions, $m_c$ is the mass of the charm (bottom) quark, $\epsilon = 2m_c - m_\psi$ is the binding energy of the charmonium state of mass, $m_\psi$. The mass shifts of the heavy quarkonium states are thus obtained from the values of the dilaton field, $\chi$ (using equation (5)).

The Dirac sea contributions are included in the scalar densities of the nucleons, which occur in the equations of motion of the scalar fields, $\sigma$, $\zeta$ and $\chi$. For given values of
the baryon density, $\rho_B$, the isospin asymmetry parameter, $\eta = (\rho_n - \rho_p)/(2\rho_B)$ (with $\rho_n$ and $\rho_p$ as the neutron and proton number densities), the magnetic field, $B$ (chosen to be along z-direction), the fields ($\sigma$, $\zeta$, $\delta$ and $\chi$) are solved from their coupled equations of motion. Within the chiral effective model, the masses of the open charm mesons are given by equations (4). These are obtained from solutions of the dispersion relations given by equation (3) for $|\vec{k}| = 0$, with additional Landau level contributions for $D^\pm$ mesons.

In the presence of a magnetic field, there is mixing between the spin 0 (pseudoscalar) meson and spin 1 (vector) mesons, which modifies the masses of these mesons [80, 92, 93, 99–103]. The PV mixing leads to a drop (rise) in the mass of the pseudoscalar (longitudinal component of the vector) meson. The mass modifications have been studied using an effective

FIG. 4: same as Fig. 2 with $\eta=0.5$. 

![Graphs showing mass versus magnetic field strength for different cases with and without DS and c']
FIG. 5: same as Fig. 4, with additional mixing effects from $D - D^*$ and $\bar{D} - \bar{D}^*$.

Lagrangian density of the form $[100, 103]$

$$
\mathcal{L}_{PV} = \frac{g_{PV}}{m_{av}} e \tilde{F}_{\mu \nu} (\partial^\mu P^\nu),
$$

(7)

for the heavy quarkonia $[80, 100]$, the open charm mesons $[92]$ and strange ($K$ and $\bar{K}$) mesons $[93]$. In equation (7), $m_{av} = (m_V + m_P)/2$, $m_P$ and $m_V$ are the masses for the pseudoscalar and vector charmonium states, $\tilde{F}_{\mu \nu}$ is the dual electromagnetic field. In equation (7), the coupling parameter $g_{PV}$ is fitted from the observed value of the radiative decay width, $\Gamma(V \to P + \gamma)$.

The masses of the open charm pseudoscalar and vector charmonium mesons as calculated in the chiral effective model, are modified due to the PV mixing effects, taken into account
through the Lagrangian given by (7). There is observed to be appreciable mass modifications
due to the PV mixings ($J/\psi - \eta_c$, $\psi(2S) - \eta_c(2S)$ and $\psi(1D) - \eta_c(2S)$). in Ref. [80], and, due
to $D(\bar{D}) - D^*(\bar{D}^*)$ mixing effects [92]. These are observed to lead to substantial modification
of the partial decay width of $\psi(1D) \rightarrow D\bar{D}$ [80] due to $\psi(1D) - \eta_c(2S)$ mixing [80], as well as,
due to $D(\bar{D}) - D^*(\bar{D}^*)$ mixing effects [92]. In the present work, the Dirac sea contributions
are taken into account to compute the masses of the open charm and charmonium states
in the chiral effective model. These masses (for $D$ and $\bar{D}$ given by equation [4] and for
$\psi(3770)$ and $\eta_\prime_c$ obtained from [5])) undergo further modifications due to PV mixing effects,
calculated using the phenomenological Lagrangian given by equation (7). The AMMs of the
nucleons are considered in the present study [96–98], which are observed to lead to drastic
modifications to the masses of the open charm mesons, when the Dirac sea contributions
are taken into account.

III. DECAY WIDTHS OF CHARMONIUM STATES TO $D\bar{D}$

Using a field theoretic model of composite hadrons [104–106], the in-medium decay widths
of the vector charmonium state $\psi(3770)$ to $D\bar{D}$ in magnetized (nuclear) matter are computed
in the present work. The model describes the hadrons comprising of quark (and antiquark)
constituents. The constituent quark field operators of the hadron in motion are constructed
from the constituent quark field operators of the hadron at rest, by a Lorentz boosting.
Similar to the MIT bag model [107], where the quarks (antiquarks) occupy specific energy
levels inside the hadron, it is assumed in the present model for the composite hadrons that
the quark (antiquark) constituents carry fractions of the mass (energy) of the hadron at rest
(in motion) [104, 105].

With explicit constructions of the charmonium state and the open charm mesons, the
decay width is calculated using the light quark antiquark pair creation term of the free
Dirac Hamiltonian for constituent quark field [77]. The relevant part of the quark pair
creation term is through the $d\bar{d}(u\bar{u})$ creation for decay of the charmonium state, $\Psi$, to the
final state $D^+D^- (D^0\bar{D}^0)$. For $\Psi \rightarrow D(p) + \bar{D}(p')$, this pair creation term is given as

$$\mathcal{H}_{q\bar{q}}(x, t = 0) = Q_q^{(p)}(x)^\dagger(-i\alpha \cdot \nabla + \beta M_q)\bar{Q}_{q'}(x')$$

(8)

where, $M_q$ is the constituent mass of the light quark, $q = (u, d)$. The subscript $q$ of the field
operators in equation [8] refers to the fact that the light antiquark, \( \bar{q} \) and light quark, \( q \) are the constituents of the \( D \) and \( \bar{D} \) mesons with momenta \( p \) and \( p' \) respectively in the final state of the decay of the charmonium state, \( \Psi \).

The charmonium state, \( \psi(3770) \) with spin projection \( m \), at rest is given as

\[
|\psi^m(3770)(0)\rangle = \int dk c_r^i(k)^t u_r a^m_{\psi(1D)}(k) \bar{c}_s^i(-k)v_s|vac\rangle,
\]

where, \( i \) is the color index of the charm quark (antiquark) operator, \( u_r \) and \( v_s \) are the two component spinors for the quark and antiquark. The expression for \( a^m_{\psi(1D)}(k) \) is given in terms of the wave functions (assumed to be harmonic oscillator type) for the charmonium state, \( \psi(3770) \), which corresponds to 1D state, is given as [77][108]

\[
a^m_{\psi(1D)}(k) = \frac{1}{4\sqrt{3\pi}} u_{\psi(1D)}(k) \left( \sigma_m - 3(\sigma \cdot \hat{k})\hat{k}^m \right),
\]

where,

\[
u_{\psi(1D)}(k) = \left( \frac{16}{15} \right)^{1/2} \pi^{-1/4}(R^2_{\psi(1D)})^{3/4} \exp\left( -\frac{1}{2}R_{\psi(1D)}^2k^2 \right).
\]

The \( D(D^+,D^0) \) and \( \bar{D}(\bar{D}^-,\bar{D}^0) \) states, with finite momenta are contructed in terms of the constituent quark field operators, obtained from the quark field operators of these mesons at rest through a Lorentz boosting [106]. These states, assuming harmonic oscillator wave functions, are explicitly given as

\[
|D(p)\rangle = \frac{1}{\sqrt{6}} \left( \frac{R_D^2}{\pi} \right)^{3/4} \int dk \exp\left( -\frac{R_D^2k^2}{2} \right) c_r^i(k + \lambda_2 p)^t u_r \bar{q}^i(-k + \lambda_1 p)v_s dk,
\]

\[
|\bar{D}(p')\rangle = \frac{1}{\sqrt{6}} \left( \frac{R_{D'}^2}{\pi} \right)^{3/4} \int dk \exp\left( -\frac{R_{D'}^2k^2}{2} \right) q_r^i(k + \lambda_1 p')^t u_r \bar{c}^i(-k + \lambda_2 p')v_s dk.
\]

where, \( q = (d,u) \) for \( (D^+,D^-) \) and \( (D^0,\bar{D}^0) \) respectively. In equations [12] and [13], \( \lambda_1 \) and \( \lambda_2 \) are the fractions of the mass (energy) of the \( D(\bar{D}) \) meson at rest (in motion), carried by the constituent light \( (d,u) \) antiquark (quark) and the constituent heavy charm quark (antiquark), with \( \lambda_1 + \lambda_2 = 1 \). The values of \( \lambda_1 \) and \( \lambda_2 \) are calculated by assuming the binding energy of the hadron as shared by the quark (antiquark) to be inversely proportional to the quark (antiquark) mass [105].

The decay width of the charmonium state, \( \psi(3770)(\equiv \psi(1D)) \) to \( DD \) is calculated from the matrix element of the light quark-antiquark pair creation part of the Hamiltonian, between the initial charmonium state and the final state for the reaction \( \psi(3770) \to D(p) + \bar{D}(p') \) as given by

\[
\langle D(p)|\langle \bar{D}(p')|\mathcal{H}_{dd}(x,t = 0)dx|\psi^m(3770)(0)\rangle = \delta(p + p')A_{\psi(3770)}(|p|)p_m.
\]
where,
\[
A_{\psi(3770)}(|\mathbf{p}|) = 6c_\psi \exp[(a_\psi b_\psi^2 - R_D^2 \lambda_0^2)\mathbf{p}^2] \cdot \left(\frac{\pi}{a_\psi}\right)^{3/2} \left[F_0^\psi + F_1^\psi \frac{3}{2a_\psi} + F_2^\psi \frac{15}{4a_\psi^2}\right]. \tag{15}
\]

In the above, the parameters \(a_\psi, b_\psi\) and \(c_\psi\) are given in terms of \(R_D\) and \(R_{\psi(3770)}\), which are the strengths of the harmonic oscillator wave functions for the \(D(\bar{D})\) and the charmonium state, \(\psi(3770)\), and \(F_i^\psi (i = 0, 1, 2)\) are polynomials in \(|\mathbf{p}|\), the magnitude of the momentum of the outgoing \(D(\bar{D})\) meson \cite{77}. The expression for the decay width of the charmonium state, \(\psi(3770)\) to \(D\bar{D}\), without accounting for the PV mixing effects, is given by
\[
\Gamma(\psi(3770) \to D(\mathbf{p})\bar{D}(-\mathbf{p})) = \gamma_\psi^2 \frac{8\pi^2}{3} |\mathbf{p}|^2 \frac{p_D^0(|\mathbf{p}|) p_D^0(|\mathbf{p}|)}{m_{\psi(3770)}} A_{\psi(3770)}(|\mathbf{p}|)^2 \tag{16}
\]

In the above, \(p_D^0((\mathbf{p}) = (m_{D(\bar{D})}^2 + |\mathbf{p}|^2)^{1/2}\), with
\[
|\mathbf{p}| = \left(\frac{m_\psi^2}{4} - \frac{m_D^2 + m_\bar{D}^2}{2} + \frac{(m_D^2 - m_\bar{D}^2)^2}{4m_{\psi(3770)}^2}\right)^{1/2}. \tag{17}
\]

In the above, the masses of the \(D(\bar{D})\) and charmonium state \(\psi(3770)\) are the in-medium masses in the magnetized nuclear matter calculated in the chiral effective model, as given by equations \(11\) and \(13\). The expression for \(A_{\psi(3770)}(|\mathbf{p}|)\) is given by equation \(15\). The parameter, \(\gamma_\psi\), in the expression for the charmonium decay width, is a measure of the coupling strength for the creation of the light quark antiquark pair, to produce the \(D\bar{D}\) final state. This parameter is adjusted to reproduce the vacuum decay widths of \(\psi(3770)\) to \(D^+D^-\) and \(D^0\bar{D}^0\) \cite{77}. The decay width of the charmonium state is observed to have the dependence on the magnitude of the 3-momentum of the produced \(D(\bar{D})\) meson, \(|\mathbf{p}|\), given by equation \(17\), as a polynomial part multiplied by an exponential term.

When we include the PV mixing effect, the expression for the decay width is modified to
\[
\Gamma^{PV}(\psi(3770) \to D(\mathbf{p})\bar{D}(-\mathbf{p})) = \gamma_\psi^2 \frac{8\pi^2}{3} \left[\left(\frac{2}{3} |\mathbf{p}|^2 \frac{p_D^0(|\mathbf{p}|) p_D^0(|\mathbf{p}|)}{m_{\psi(3770)}} A_{\psi(3770)}(|\mathbf{p}|)^2\right) + \left(\frac{1}{3} |\mathbf{p}|^2 \frac{p_D^0(|\mathbf{p}|) p_D^0(|\mathbf{p}|)}{m_{\psi(3770)}^{PV}} A^\psi(|\mathbf{p}|)^2\left(|\mathbf{p}| \to |\mathbf{p}||(m_{\psi(3770)} = m_{\psi(3770)}^{PV})\right)\right]\tag{18}
\]

In the above, the first term corresponds to the transverse polarizations for the charmonium state, \(\psi(3770)\), whose masses remain unaffected by the mixing of the pseudoscalar and vector charmonium states. The second term in \(18\) corresponds to the longitudinal component of the charmonium state, \(\psi(3770)\), whose mass is modified due to the mixing with the pseudoscalar meson, \(\eta'_c(\equiv \eta_c(2S))\) in the presence of the magnetic field.
IV. RESULTS AND DISCUSSIONS

We discuss the results obtained due to the effects of Dirac sea contributions for the nucleons, as well as, PV mixing on the decay width of $\psi(3770) \rightarrow D\bar{D}$ in magnetized isospin asymmetric nuclear matter. The scalar fields and the dilaton field are solved from their coupled equations of motion within the chiral effective model, for given values of the baryon density, $\rho_B$, isospin asymmetry parameter, $\eta$ and the magnetic field, $B$. In the present study, the AMMs of the nucleons are considered and the results for the charmonium decay width is compared to the case when AMMs are not taken into account.

There is enhancement of the quark condensates (through scalar fields $\sigma$ and $\zeta$) with increase in the magnetic field, due to Dirac sea contributions for zero density as well as for $\rho_B = \rho_0$ (when AMMs of nucleons are not considered) both for symmetric ($\eta=0$) and asymmetric (with $\eta=0.5$) nuclear matter, an effect called magnetic catalysis (MC). However, for $\rho_B = \rho_0$, there is observed to be inverse magnetic catalysis (IMC) in magnetized nuclear matter, when the AMMs of nucleons are taken into account. The Dirac sea contributions have appreciable effect on the meson masses, and hence on the decay width of $\psi(3770) \rightarrow D\bar{D}$.

In figure 1, we plot the decay widths of $\psi(3770) \rightarrow D\bar{D}$ for $\rho_B = 0$ including the Dirac sea contributions for the nucleons as well as effects from the PV mixing in the presence of a magnetic field. These decay widths in magnetized (nuclear) matter were studied using a field theoretical model of composite hadrons [80], including the effects of the mixing the charmonium states $\psi(3770)$ and $\eta_c(2S)$, as well as the PV mixing of the open charm mesons ($D - D^*$ and $\bar{D} - \bar{D}^*$ mixings) [92], in addition to the Landau level contributions for the charged $D^\pm$ mesons. However, the Dirac sea contributions to the self energy of the nucleons were not considered in Refs. [80] and [92]. In figure 1 panel (a) shows the decay widths of (I) $\psi(3770) \rightarrow D^+D^-$, (II) $\psi(3770) \rightarrow D^0\bar{D}^0$, and sum of these sub-channels, in the absence of the PV mixing of the charmonium states as well as open charm mesons. The masses of the charged $D^\pm$ mesons increase due to the lowest Landau level (LLL) contributions, which lead to a drop in the decay width to the charged open charm meson pair, whereas, the decay width to the neutral mesons $D^0\bar{D}^0$ is observed to change marginally with increase in the magnetic field. The mixings of the $D - D^*$ and $\bar{D} - \bar{D}^*$ mesons lead to drop of the masses of the open charm pseudoscalar mesons, and this is observed as a significant enhancement.
of the decay width in the neutral $D\bar{D}$ channel. In the presence of the $\psi(1D) - \eta_c$ mixing (which leads to increase in the mass of the longitudinal component of $\psi(1D)$), but without accounting for the mixing in the open charm meson sectors, there is observed to be a rise in the decay widths for both the sub channels for high values of magnetic field, as can be seen from panel (c). When both the mixings (for the charmonium as well as open charm mesons) are considered, there is observed to be significant rise in the decay width to the neutral $D\bar{D}$.

The decay widths of $\psi(1D) \to D\bar{D}$, along with for the subchannels (I) $\psi(1D) \to D^+D^-$ and (II) $\psi(1D) \to D^0\bar{D}^0$, are shown for $\rho_B = \rho_0$, accounting for the Dirac sea contributions to the scalar densities of the nucleons as well as with the $\psi(1D) - \eta'_c$ mixing. These are shown without and with the PV effects for the open charm $(D(\bar{D}) - D^*(\bar{D}^*)$ mixing) mesons, for symmetric ($\eta=0$) nuclear matter, in figures 2 and 3 respectively, and for asymmetric (with $\eta=0.5$) nuclear matter, in figures 4 and 5 respectively. When the AMMs of the nucleons are considered, the Dirac sea contributions are observed to modify the decay width to the neutral $D\bar{D}$ appreciably at high magnetic fields, which is further enhanced due to the $\psi(1D) - \eta'_c$ mixing, for $\eta = 0$, in the absence of PV mixing of open charm mesons as can be seen from panels (b) and (d) of figure 2. There is observed to be significant rise in the decay widths when the PV mixing in open charm sectors, is taken into account for the open charm mesons, as can be seen from figure 3. The effects of the isospin asymmetry is observed to be much less dominant as compared to the effects due to the Dirac sea contributions and the PV mixing effects. The AMMs of the nucleons however do play an important role, as the Dirac contribution effects lead to inverse magnetic catalysis (IMC) when the AMMs are considered, whereas, there is observed to be magnetic catalysis (MC) when the AMMs are neglected. Due to the opposite behaviour of the scalar fields (proportional to the light quark condensates), the behaviours of the open charm mesons are quite different without and with the inclusion of the AMMs of the nucleons.

The magnetic field effects considered on the decay width of the charmonium state $\psi(1D) \to D\bar{D}$ in the present work, are due to the Dirac sea effects of the nucleons, the PV $(\psi(1D) - \eta'_c, D - D^*$ and $\bar{D} - \bar{D}^*)$ mixing effects and Landau level contributions for the open charm mesons. The Dirac contributions are observed to lead to significant modifications to these decay widths, with the decay to the neutral $D\bar{D}$ to be much larger compared to the charged $D\bar{D}$. The strong magnetic field created at the early stage should have ob-
servable consequences on the production of the hidden and open charm mesons arising from ultra-relativistic heavy ion collision experiments.

V. SUMMARY

To summarize, we have studied the decay width of the charmonium states $\psi(1D)$ to $D\bar{D}$ in magnetized (nuclear) matter, accounting for the Dirac sea contributions for the self energies of the nucleons within a chiral effective model. Th open charm mesons ($D$ and $\bar{D}$) are calculated from their interactions to the nucleons and the scalar mesons, whereas, the charmonium masses are calculated within a chiral effective model from the medium change of a scalar dilaton field, which mimics the gluon condensates of QCD. There is observed to be magnetic catalysis effect, i.e., enhancement of the quark condensates (given in terms of the scalar fields) with rise in magnetic field, for $\rho_B = 0$, for both the cases of accounting and ignoring the AMMs of the nucleons. However, for $\rho_B = \rho_0$, there is observed to be inverse magnetic catalysis (IMC) when the AMMs of the nucleons are taken into account. The effects from PV ($\psi(1D) - \eta_c$, $D - D^*$ and $\bar{D} - \bar{D}^*$) mixings in the presence of the magnetic field are also taken into account, in addition to the Landau contributions for the charged open charm mesons. The effects of the Dirac sea as well as PV mixings are observed to be quite significant on the charmonium decay widths. These should have observable consequences on the production of charmonium states and open charm ($D$ and $\bar{D}$) mesons, as these are created at the early stage of the non-central ultra-relativistic heavy ion collision experiments, when the magnetic field can be large.

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