The $u - d$ quark mass difference and nuclear charge symmetry breaking

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The group theoretical analysis of the Coleman-Glashow tadpole picture of "meson-mixing" is quantitatively reproduced by the $u - d$ constituent quark mass difference in quantum loop calculations of the self-energies of mesons. This demonstration that the Coleman-Glashow scales can be directly calculated from the constituent $u - d$ quark mass difference finishes the link between charge symmetry breaking in low energy nuclear physics and the $u - d$ quark mass difference in particle physics.

1. INTRODUCTION

The very recent precision measurement of the $\Xi^0$ mass \[1\] has revived interest in the electromagnetic ($em$) mass splittings of the baryons, because of the resistance of the forty-year old Coleman-Glashow relation \[2\] to substantial symmetry-breaking effects in quark masses. This relation

$$M_{\Xi^-} - M_{\Xi^0} = M_{\Sigma^-} - M_{\Sigma^+} + M_p - M_n,$$

(1)

was derived assuming unbroken flavor $SU(3)$ and is now satisfied to within $4 \pm 3\%$ (scaled by $M_{\Sigma^+} - M_{\Sigma^-} \sim 8$ MeV), or broken at the one sigma level depending on your point of view \[3\]. While eq. (1) was derived from only the isospin breaking electromagnetic interaction, the individual $\Delta I = 1$ baryon pairs should, however, reflect $SU(2)$ breaking caused by $\Delta I = 1$ quark mass differences. Subsequent to Ref. \[2\] (and before the quark picture), Coleman and Glashow \[4\] suggested that symmetry-violating processes are dominated by symmetry-breaking tadpole diagrams with scalar mesons linking the tadpole to the $SU(3)$ invariant strong interactions. To describe electromagnetic splittings, they combined the tadpole Hamiltonian $H_{tad}^3$ (transforming like $\lambda_3$ in an $SU(3)$ context) together with the current-current operator $H_{JJ}$ (corresponding to the first order in $\alpha$ contribution due to photon exchange) to form an effective $\Delta I = 1$ electromagnetic ($em$) Hamiltonian density operator,

$$H_{em} = H_{JJ} + H_{tad}^3.$$

(2)

In a group-theoretical sense, eq. (2) gives a universal $\Delta I = 1$ picture \[4\] of

(a) hadron electromagnetic mass splittings of pseudoscalar (P) and vector (V) mesons, along with the splittings of octet baryons (B) and decuplet (D) baryons,

(b) off-diagonal $em$ transitions $\langle \Sigma^0 | H_{em} | \Lambda \rangle$, $\langle \pi^0 | H_{em} | \eta_{NS} \rangle$, and $\langle \rho^0 | H_{em} | \omega \rangle$. 

where \( \eta_{NS} \) is the non-strange \( \bar{q}q \) component of the \( \eta \) and the \( \omega \) is 97% nonstrange.

Quite soon after Ref. [4], Dalitz and von Hippel [6] applied the Coleman-Glashow (CG) Hamiltonian operator to the issue of charge symmetry for the \( \Lambda \) hyperon and, in particular the charge asymmetry of the \( \Lambda N \) interaction (i.e. \( \Lambda n \) versus \( \Lambda p \)). The CG operator, illustrated in Figure 2 for this case, suggests an appreciable \( \Delta I = 1 \) transition between the isospin-pure \( \Sigma^0 \) and \( \Lambda \) hyperons (often referred to as electromagnetic mixing). This transition allows an exchange of the isospin one pion between the \( \Lambda \) and nucleon, otherwise forbidden by isospin conservation, which contributes to hypernuclear charge symmetry breaking. Unfortunately our experimental knowledge of the \( \Lambda N \) interaction has not progressed much in the last thirty six years [7], so the success of the Coleman-Glashow off-diagonal \( em \) transitions \( \langle \Sigma^0 | H_{em} | \Lambda \rangle \), \( \langle \pi^0 | H_{em} | \eta_{NS} \rangle \), and \( \langle \rho^0 | H_{em} | \omega \rangle \) in describing hypernuclear charge symmetry breaking remains problematic at present [8].

The experimental knowledge of charge asymmetry in the \( NN \) interaction, on the other hand, is rather good and the off-diagonal \( em \) transitions, \( \langle \pi^0 | H_{em} | \eta_{NS} \rangle \) and \( \langle \rho^0 | H_{em} | \omega \rangle \), from the Coleman-Glashow \( em \) Hamiltonian operator embedded in single meson exchange (tree) diagrams analogous to those of Fig. 2 give the dominant and satisfactory description of nuclear charge asymmetry [9]. But it is the group theory structure and single universal Coleman-Glashow scale which “does the heavy lifting” of this description (although the measured properties of their postulated \( I = 1 \) scalar meson, now called the \( a_0 \), do recover the tadpole scale [10]). Indeed, Coleman and Glashow emphasized that “our explanation of symmetry-breaking phenomena suggests, but does not require, the existence of scalar mesons” [4]. It is the purpose of this talk to show that the group theoretical analysis of the CG tadpole (reviewed in more detail in [1]) is quantitatively reproduced by the \( u - d \) constituent quark mass difference in quantum loop calculations of the self-energies of mesons. But first I review the tadpole scale and show how the scale of the baryons is the same as the tadpole scale established by electromagnetic meson mass splittings.
2. MASS SPLITTINGS AND THE TADPOLE SCALE

Returning to electromagnetic mass splitting of the baryons, the new measurement [1] is within two standard deviations of the the earlier CG tadpole prediction [5] of eq. (2) for the octet baryons, reproduced in Table I. In this Table, the self-energy shift of a hadronic state arising from single photon exchange \((H_{JJ})\) has been evaluated via the known dominance of the Born terms in the dispersive evaluation of the Cottingham formula. I show two sets of results for the octet baryons, and note that an early evaluation (including estimates of smaller resonance and other contributions) in 1969 [11] is strikingly confirmed by a recent (1997) lattice QCD calculation which summed the electric and magnetic Born contributions over discretized bosonic momenta of the finite lattice [12]. To estimate the universal value of the tadpole from the baryon octet, one should concentrate on line three where only the small magnetic Born terms contribute to \(H_{JJ}\) of \(m_{\Sigma^+} - m_{\Sigma^-}\) and \(H^3_{tad}\) is isolated. Then one fills out Table 1 with

\[
(H^3_{tad})_{p-n} = \frac{2}{3}(H^3_{tad})_{\Sigma^0-\Sigma^-} = \frac{1}{3}(H^3_{tad})_{\Sigma^+ - \Sigma^-} = \frac{1}{2}(H^3_{tad})_{\Xi^0-\Xi^-} \approx -2.5 \text{ MeV}
\]

assuming the same semi-strong and electromagnetic \(d/f\) ratio of \(-1/3\) (see eg. Ref. [5]), and obtains the CG tadpole prediction [5] of eq. (2) for the octet baryons. One could attempt to improve the description of \(m_p - m_n\) by quoting the standard QED current-current (ie. photon exchange) self-energy for charged protons

\[
(H_{JJ})_p \simeq \Sigma(m) = \frac{3\alpha m_p}{2\pi} \left[ \ln\left(\frac{\Lambda}{m_p}\right) + \frac{1}{4} \right] \simeq 1.2 \text{ MeV},
\]

and choosing the value of the ultraviolet cutoff \(\Lambda = 1.05 \text{ GeV}\) so that (2) is compatible with the observed proton-neutron mass difference of \(-1.29 \text{ MeV}\) [14]. From an effective field theory viewpoint, the discrepancy between the \(g\)-factor of the relativistic point spin \(1/2\) proton and the measured \(g\) of the proton suggests that the cutoff in nucleon QED must be of order of the nucleon’s mass \(m_p\) [15], consistent with (4). This cutoff exercise (which neglects magnetic moment contributions to the fermion’s self-energy) yields an \(H_{JJ}\) somewhat larger than the Born contributions of Table I and suggests that one should not limit oneself to estimates of \(H_{JJ}\) and \(m_d - m_u\) made only from the non-strange sector [16], ie. \(m_p - m_n\) and electromagnetic pion mass differences, the latter to which we now turn.
The Coleman-Glashow postulate (1) combined with the Dashen PCAC observation [17] that, in the soft limit, neutral pseudoscalar meson $H_{JJ}$ matrix elements vanish ($H_{JJ})_{\pi^0} = (H_{JJ})_{\bar{K}^0} = (H_{JJ})_{\pi^+\eta} = 0$ but charged matrix elements $(H_{JJ})_{\pi^+} = (H_{JJ})_{K^+}$ do not vanish, can be related by group theory to the measured electromagnetic mass splittings

$$\Delta m_K^2 \equiv m_{K^+}^2 - m_{K^0}^2 \approx -3960 \text{ MeV}^2, \quad \Delta m_{\pi}^2 \approx 1260 \text{ MeV}^2.$$  \hspace{1cm} (5)

This predicts

$$(H_{em})_{\Delta\pi} = (H_{JJ})_{\Delta\pi} + (H_{tad}^3)_{\Delta\pi} = (H_{JJ})_{\Delta K} + 0 = \Delta m_{\pi}^2,$$  \hspace{1cm} (6)

$$(H_{em})_{\Delta K} = (H_{JJ})_{\Delta K} + (H_{tad}^3)_{\Delta K} = \Delta m_K^2,$$  \hspace{1cm} (7)

and subtracting (6) from (7) fixes the kaon tadpole scale [5]

$$(H_{tad}^3)_{\Delta K} = \Delta m_K^2 - \Delta m_{\pi}^2 \approx -5220 \text{ MeV}^2.$$  \hspace{1cm} (8)

A number of model and lattice calculations suggest that for physical pseudoscalar mesons $(H_{JJ})_{\Delta\pi} \approx \Delta m_{\pi}^2 \approx 1260 \text{ MeV}^2$, but that $(H_{JJ})_{\Delta K} \approx 1900 - 2600 \text{ MeV}^2$ [18]. If so, then the kaon tadpole scale increases slightly from (8) to $-5800 - -6500 \text{ MeV}^2$. In section 3, we will show how to recapture the soft kaon tadpole scale of (8) and Ref. [5]. Also SU(3) symmetry predicts the off-diagonal $\Delta I = 1$ transitions as [2,19]

$$\langle \pi^0|H_{tad}^3|\eta_{NS}\rangle = \Delta m_K^2 - \Delta m_{\pi}^2 \approx -5220 \text{ MeV}^2,$$  \hspace{1cm} (9)

$$\langle \rho^0|H_{tad}^3|\omega\rangle = \Delta m_K^2 - \Delta m_{\rho}^2 \approx -5130 \text{ MeV}^2.$$  \hspace{1cm} (10)

The meson scale of about $-5200 \text{ MeV}^2$ in (8), (9), and (10), extendable to vector mesons via $SU(6)$ [3] or by the measured properties of the $a_0$ [10], can be related to the fitted baryon scale [3] by multiplying the latter by the normalization of the baryon spinors: $\bar{u}u = 2M_{baryon} \approx 2300 \text{ MeV}$. Then the mass$^2$ version of (8) is $(H_{tad}^3)_B \approx -5700 \text{ MeV}^2$ indicating a universal Coleman-Glashow scale and a picture of electromagnetic mass splittings which almost certainly rests upon the up-down quark mass difference [20].

3. QUARK LOOPS AND UP-DOWN CONSTITUENT QUARK MASS DIFFERENCE

A constituent quark mass difference $m_d - m_u \approx 4 \text{ MeV}$ follows from the observed $(s\bar{d})K^0 - (s\bar{u})K^+$ mass difference: $m_{K^0} - (497.67) - m_{K^+} - (493.68) \approx m_d - m_u = 3.99 \text{ MeV}$, with the common $\bar{s}$ spectator quark subtracting out of the kaon mass difference. Also the $\Sigma$ baryon mass difference in quark language for $(sdd)\Sigma^- - (suu)\Sigma^+$ is $m_{\Sigma^-} - (1197.45) - m_{\Sigma^+} - (1189.37) \approx 2(m_d - m_u) = 8.08 \text{ MeV}$, with common $s$ spectator mass subtracting out and the photon interaction $H_{JJ}$ at a minimum (Table I). These estimates are consistent and hint that a consistent Coleman-Glashow picture of hadronic mass splitting and mixings could be obtained from the differences of constituent quark loop diagrams [14,21].

Returning to the kaon tadpole scale of $-5220 \text{ MeV}^2$ in eq. (8), in quark language this is due to the quark line graphs of Figure 3. They are the $u - d$ quark bubble graphs plus the difference of those $u - d$ quark loops which look like a $\Delta I = 1$ $a_0$ tadpole with $a_0KK$ coupling [14,21]. Evaluation of these quark loop graphs results in the soft
momentum limit to

\[
(\Delta m_K^2)_{qk\,\text{loops}} = (m_u - m_d)[2(2\hat{m} - m_s) + 6\hat{m}^2(m_s - \hat{m})/(2\hat{m}^2 + m_s^2)]
+ 8g_{a_0KK}(m_u - m_d)\hat{m}^2/gm_0^2,
\]

(11)

where \( \hat{m} = (m_d + m_u)/2 \approx 337\) MeV, and \( m_s/\hat{m} \approx 1.44 \). Each of the two pairs of \( u - d \) tadpole-like graphs of Figure 3, summed in the second line of (11), includes the integral

\[
I = \int d^4p/2(2\pi)^4 \left[ \frac{m_u}{p^2 - m_u^2} - \frac{m_d}{p^2 - m_d^2} \right] = i(m_u - m_d)\hat{m}^2/N_c g^2,
\]

(12)

with \( N_c g^2 = 4\pi^2 \) [22]. The value of this \( u - d \) difference loop integral \( I \), which looks like the head of a tadpole and appears repeatedly in the following, is independent of the regularization scheme used to obtain it [23]. To evaluate the four tadpole graphs summed in the second line of (11), we use the linear \( \sigma \) model Lagrangian quark-quark-meson coupling \( g = 2\pi/\sqrt{3} \approx 3.63 \) [22] but deviate somewhat from the linear \( \sigma \) model Lagrangian tri-meson coupling \( g_{a_0KK} = (m_{a_0}^2 - m_K^2)/(2f_K) \approx 3.15\) GeV, and use instead \( g_{a_0KK} \approx 2.7\) GeV, the latter value an average of this chiral symmetry estimate and the \( U(3) \) symmetry estimate of \( g_{a_0KK} = g_{\sigma\pi\pi}/2 = m_\sigma^2/2f_\pi \approx 2.55 \) GeV [14]. Given the \( d - u \) quark mass difference of about 4 MeV, eq. (11) leads to [21]

\[
(\Delta m_K^2)_{qk\,\text{loops}} \approx -(2384 + 2800)\text{ MeV}^2 \approx -5184\text{ MeV}^2.
\]

(13)

Note that eq. (13) is in agreement with the soft Coleman-Glashow \( \lambda^3 \) kaon tadpole in (8) as found in Ref. [3].

Next we compute the nonstrange (NS) \( \Delta I = 1 \) cm transition amplitude \( \langle \pi^0|H_{\text{tad}}^3|\eta_{\text{NS}}\rangle \) for \( m_{\eta_{\text{NS}}} \approx 760\) MeV (the weighted average of the \( \eta(548) \) and \( \eta'(958) \) [19]) in terms of \( u - d \) quark bubble and \( u - d \) \( a_0 \) tadpole graphs analogous to those of Figure 4. These graphs give [21]

\[
(H_{\pi\eta_{\text{NS}}}^3)_{qk\,\text{loops}} = (m_u - m_d)(2\hat{m} + 16\hat{m}^3/m_{a_0}^2)
\]

\[
\approx -2696\text{ MeV}^2 - 2535\text{ MeV}^2 \approx -5231\text{ MeV}^2,
\]

(14)
where the \((m_u - m_d)2\tilde{m}\) factor in (14) derives from the difference of \(u - d\) bubble graphs:

\[
-4ig^2N_c \int d^4p \left[ \frac{1}{p^2 - m_u^2} - \frac{1}{p^2 - m_d^2} \right].
\]

The integrand of (13) can immediately be turned into \(\frac{(m_u^2 - m_d^2)}{(p^2 - m_u^2)^2}\) so that the \(u - d\) bubble graphs expression (13) is simply \((m_u^2 - m_d^2)\) times the logarithmic divergent gap equation

\[
1 = -4iN_c g^2 \int \frac{d^4p}{(p^2 - m_u^2)(p^2 - m_d^2)},
\]

which arises from the quark loop generation of the the decay constant \(f_\pi\) in the quark-level Goldberger-Treiman relation \(f_\pi g = \tilde{m}\) [22]. The 16\(\tilde{m}^3/m_u^2\) term in (14) stems from the \(u - d\) quark loop which looks like a \(\Delta I = 1\) tadpole with \(a_\pi\eta_{NS}\) coupling. This latter \(u - d\) quark loop difference integral \(I\) is again evaluated via equation (12) above. To obtain the number (and the cubic power of \(\tilde{m}\)) in (14) we have replaced the \(a_\pi\eta_{NS}\) coupling \((m_u^2 - m_d^2)/2f_\pi\) by its Lagrangian linear \(\sigma\) model analogue [22] \((m_u^2 - m_d^2)/2f_\pi = (4\tilde{m})^2/2f_\pi\). Then \(f_\pi \approx 93\) MeV and \(\tilde{m} \approx 337\) MeV, so that the Goldberger-Treiman relation \(f_\pi g = \tilde{m}\) remains valid (for \(g = 2\pi/\sqrt{3}\) [23]).

![Figure 4: Quark loop contributions to meson mixing via \(H_{\text{tad}}^a\). These diagrams could stand for either \(\langle \rho^a|H_{\text{tad}}^a|\omega_{NS}\rangle\) or \(\langle \pi^a|H_{\text{tad}}^a|\omega_{NS}\rangle\).](image)

Lastly we study the \(\Delta I = 1\) \(em\) transition amplitude \(\langle \rho^a|H_{\text{tad}}^a|\omega_{NS}\rangle\) in terms of the \(u - d\) quark diagrams of Figure 4. For this vector meson transition one must work with the QED-like nonstrange quark bubble polarization tensor [24], taking the \(SU(3)\) value \(g_\omega = 3g_\rho\),

\[
\Pi_{\mu\nu} = (-k^2 g_{\mu\nu} + k_\mu k_\nu)\Pi(k^2, m_q^2)g_\rho g_\omega/12,
\]

where \(g_\rho \approx 5.03\) and \(g_\omega \approx 17.05\) (for \(e = \sqrt{4\pi\alpha} \approx 0.30282\)) follow from electron-positron decay rates [11,13]. The \(\omega\) is 97% nonstrange and 3% strange so the \(\Delta I = 1\) transition between the physical particles is well described by the difference of the \(u - d\) polarization function

\[
\Pi(k^2, m_u^2) - \Pi(k^2, m_d^2) = \frac{N_c(m_u^2 - m_d^2)}{2\pi^2k^2}
\]

where \(N_c\) is the number of colors in the quark model. We emphasize that the difference between the \(u\) and \(d\) quark contributions to the polarization function is finite. In order for the inverse propagator \(\Delta_{\mu\nu}^{-1}(k) = (k_\mu k_\nu - k^2 g_{\mu\nu})[1 + \Pi(k^2)] = -g_{\mu\nu}(k^2 - m^2)\) terms in \(k_\mu k_\nu\) to actually simulate a reciprocal vector meson propagator, it is clear that the
polarization function $-k^2 \Pi(k^2)$ in (18) acts as a squared (quark) mass. Note that this discussion and eq. (18) allows us to interpret the (squared mass) bubble Hamiltonian density $\rho - \omega$ transition matrix element as

$$\langle \rho | (H_{\rho \omega})_{\text{bubble loops}} | \omega \rangle = -k^2 \left[ \Pi(k^2, m_u) - \Pi(k^2, m_d) \right] g_\rho g_\omega / 12$$

$$= g_\rho^2 N_c \left( m_u^2 - m_d^2 \right) / 8\pi^2. \quad (19)$$

Besides this $u - d$ quark bubble term we must add in the $\Delta I = 1 a_0$ tadpole term. It was shown in Ref. [10] that the measured decays of the $a_0$ meson with the aid of the vector dominance model lead to $g_{a_0 \omega} = g_{a_0 \pi \eta_{NS}}$. Then the tadpole term for the $\rho \omega$ transition is the same as the second term of (14), and the $\rho^0 - \omega$ effective Hamiltonian becomes

$$\langle \rho^0 | (H_{J J})_{\text{tadpoles}} | \omega \rangle = (g_\rho / e)(g_\omega / e) m_V^2 \approx 644 \text{ MeV}^2 \quad (21)$$

on the vector meson mass shell $k^2 = m_V^2$ in the spirit of vector meson dominance (VMD). In (21) we have used the average $\rho^0 - \omega$ mass $m_V = 776$ MeV along with the updated VMD ratios $g_\rho / e \approx 16.6$ and $g_\omega / e \approx 56.3$, with the latter $g_\rho$ and $g_\omega$ couplings found from electron-positron decay rates [11,12]. Combining (20) with the $H_{J J}$ term in (21) according to the Coleman-Glashow decomposition (2) requires

$$\langle H_{em} \rangle_{\rho \omega} \approx (644 - 5127) \text{ MeV}^2 \approx -4483 \text{ MeV}^2. \quad (22)$$

This latter scale derived from quark loops and photon exchange is quite near the empirical $\Delta I = 1 \text{ em}$ transition $-4520 \pm 50$ MeV$^2$ found from the measured $\omega \rightarrow \rho^0 \rightarrow 2\pi$ decay rate [26,10].

To conclude, the Coleman-Glashow group-theoretical decomposition (2) leads to the universal $H_{\text{tail}}^3$ $\Delta I = 1$ scale of $\approx -5200$ MeV$^2$ in eqs. (8,9,10) which is close to the fitted baryon tadpole scale $(H_{\text{tail}}^3)_{p-n} \approx -2.5$ MeV in (3). Both of these latter scales are reproduced in the alternative quark-loop scheme, and again result in the universal quark-loop $\Delta I = 1$ transitions in eqs. (13,14,20), which are based on $m_d - m_u \approx 4$ MeV.

REFERENCES

1. V. Fanti et al., “Precision measurement of the $\Xi^0$ mass and the branching ratios of the decays $\Xi^0 \rightarrow \Lambda \gamma$ and $\Xi^0 \rightarrow \Sigma^0 \gamma$”, Eur. Phys. J. C 12, 69 (2000).
2. S. Coleman and S. Glashow, “Electrodynamic properties of baryons in the unitary symmetry scheme”, Phys. Rev. Lett. 6, 423 (1961).
3. J. L. Rosner, “Improved test of relations for baryon isomultiplet splittings”, Phys. Rev. D 57, 4310 (1998); G. Dillon and G. Morpurgo, “On the miracle of the Coleman-Glashow and other baryon mass formulas”, Phys. Lett. B 481, 239 (2000); E. J. Jenkins and R. F. Lebed, “Naturalness of the Coleman-Glashow mass relation in the $1/N_c$ expansion: an update”, hep-ph/0005038.
4. S. Coleman and S. Glashow, “Departures from the eightfold way: theory of strong interaction symmetry breakdown”, Phys. Rev. 134, B671 (1964).

5. S. A. Coon and M. D. Scadron, “Universality of $\Delta I = 1$ meson mixing and charge symmetry breaking”, Phys. Rev. C 51, 2923 (1995).

6. R. H. Dalitz and F. von Hippel, “Electromagnetic $\Lambda\Sigma^0$ mixing and charge symmetry for the $\Lambda$-hyperon”, Phys. Lett. 10, 153 (1964).

7. R. H. Dalitz and F. von Hippel, “Electromagnetic $\Lambda\Sigma^0$ mixing and charge symmetry for the $\Lambda$-hyperon”, Phys. Lett. 10, 153 (1964).

8. S. A. Coon and M. D. Scadron, “Vector meson dominance and $\rho - \omega$ mixing”, Phys. Rev. C 58, 2958 (1998).

9. U. van Kolck, J. A. Niskanen, and G. A. Miller, “Charge symmetry violation in $pn \to d\pi^0$ and chiral effective field theory”, Phys. Rev. D 59, 113006.

10. R. Delbourgo, D.-L. Liu, and M. D. Scadron, “Electromagnetic properties of hadrons via the $u - d$ mass difference and direct photon exchange”, Phys. Rev. D 59, 2089.

11. J. F. Donoghue, B. R. Holstein, and D. Wyler, “Electromagnetic self-energies of pseudoscalar mesons and Dashen’s theorem”, Phys. Rev. D 47, (1993) 2089.

12. J. F. Donoghue, B. R. Holstein, and D. Wyler, “Electromagnetic self-energies of pseudoscalar mesons and Dashen’s theorem”, Phys. Rev. D 47, (1993) 2089.

13. J. F. Donoghue, B. R. Holstein, and D. Wyler, “Electromagnetic self-energies of pseudoscalar mesons and Dashen’s theorem”, Phys. Rev. D 47, (1993) 2089.

14. J. F. Donoghue, B. R. Holstein, and D. Wyler, “Electromagnetic self-energies of pseudoscalar mesons and Dashen’s theorem”, Phys. Rev. D 47, (1993) 2089.

15. J. F. Donoghue, B. R. Holstein, and D. Wyler, “Electromagnetic self-energies of pseudoscalar mesons and Dashen’s theorem”, Phys. Rev. D 47, (1993) 2089.

16. J. F. Donoghue, B. R. Holstein, and D. Wyler, “Electromagnetic self-energies of pseudoscalar mesons and Dashen’s theorem”, Phys. Rev. D 47, (1993) 2089.

17. J. F. Donoghue, B. R. Holstein, and D. Wyler, “Electromagnetic self-energies of pseudoscalar mesons and Dashen’s theorem”, Phys. Rev. D 47, (1993) 2089.

18. J. F. Donoghue, B. R. Holstein, and D. Wyler, “Electromagnetic self-energies of pseudoscalar mesons and Dashen’s theorem”, Phys. Rev. D 47, (1993) 2089.

19. J. F. Donoghue, B. R. Holstein, and D. Wyler, “Electromagnetic self-energies of pseudoscalar mesons and Dashen’s theorem”, Phys. Rev. D 47, (1993) 2089.

20. J. F. Donoghue, B. R. Holstein, and D. Wyler, “Electromagnetic self-energies of pseudoscalar mesons and Dashen’s theorem”, Phys. Rev. D 47, (1993) 2089.

21. J. F. Donoghue, B. R. Holstein, and D. Wyler, “Electromagnetic self-energies of pseudoscalar mesons and Dashen’s theorem”, Phys. Rev. D 47, (1993) 2089.

22. J. F. Donoghue, B. R. Holstein, and D. Wyler, “Electromagnetic self-energies of pseudoscalar mesons and Dashen’s theorem”, Phys. Rev. D 47, (1993) 2089.

23. J. F. Donoghue, B. R. Holstein, and D. Wyler, “Electromagnetic self-energies of pseudoscalar mesons and Dashen’s theorem”, Phys. Rev. D 47, (1993) 2089.

24. J. F. Donoghue, B. R. Holstein, and D. Wyler, “Electromagnetic self-energies of pseudoscalar mesons and Dashen’s theorem”, Phys. Rev. D 47, (1993) 2089.

25. J. F. Donoghue, B. R. Holstein, and D. Wyler, “Electromagnetic self-energies of pseudoscalar mesons and Dashen’s theorem”, Phys. Rev. D 47, (1993) 2089.

26. J. F. Donoghue, B. R. Holstein, and D. Wyler, “Electromagnetic self-energies of pseudoscalar mesons and Dashen’s theorem”, Phys. Rev. D 47, (1993) 2089.

27. J. F. Donoghue, B. R. Holstein, and D. Wyler, “Electromagnetic self-energies of pseudoscalar mesons and Dashen’s theorem”, Phys. Rev. D 47, (1993) 2089.

28. J. F. Donoghue, B. R. Holstein, and D. Wyler, “Electromagnetic self-energies of pseudoscalar mesons and Dashen’s theorem”, Phys. Rev. D 47, (1993) 2089.

29. J. F. Donoghue, B. R. Holstein, and D. Wyler, “Electromagnetic self-energies of pseudoscalar mesons and Dashen’s theorem”, Phys. Rev. D 47, (1993) 2089.
scheme, see the appendix of D. Kekež, D. Klabučar, and M. D. Scadron, “Revisiting the $U_A(1)$ problems”, J. Phys. G 26(2000) 1335, hep-ph0003234.

20. G. A. Miller and W. T. H. van Oers, “Charge independence and charge symmetry”, in Symmetries and Fundamental Interactions in Nuclei, W. Haxton and E. M. Henley, eds. (World Scientific, Singapore, 1995).

21. S. A. Coon and M. D. Scadron, “Charge symmetry breaking via $\Delta I = 1$ group theory or by the $u − d$ quark mass difference and direct photon exchange”, Riv. Mex. Fis. 46 S1 (2000) 5.

22. For the $SU(2)$ linear $\sigma$ model with quarks, pions and the sigma, see R. Delbourgo and M.D. Scadron, “Dynamical Generation of the gauged $SU(2)$ linear sigma model”, Mod. Phys. Lett. A10, 251 (1995), hep-ph/9910242. For the $SU(3)$ linear $\sigma$ model, see R. Delbourgo and M.D. Scadron, “Dynamical Generation of linear $\sigma$ model $SU(3)$ Lagrangian and meson nonet mixing”, Int. J. Mod. Phys. A13, 657 (1998), hep-ph/9807504, and references therein.

23. R. Delbourgo, M.D. Scadron, and A.A. Rawlinson, “Regularizing the quark-level linear $\sigma$-model”, Mod. Phys. Lett. A13, 1893 (1998) hep-ph/9807505.

24. See for instance the review: R. Delbourgo, “How to deal with infinite integrals in quantum field theory”, Rep. Prog. Phys. 39, 345 (1976).

25. R. Gatto, “Effects of virtual vector mesons in $e^+e^−$ collisions”, Nuovo Cimento 28, 658 (1963).

26. S. A. Coon and R. C. Barrett, “$\rho − \omega$ mixing in nuclear charge asymmetry”. Phys. Rev. C36, 2189 (1987).