Fokker-Planck equation with memory: the cross over from ballistic to diffusive processes in many particle systems and incompressible media

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The unified description of diffusion processes that crosses over from a ballistic behavior at short times to a fractional diffusion (sub- or superdiffusion), as well as to the ordinary diffusion at longer times, is proposed on the basis of a non-Markovian generalization of the Fokker-Planck equation. The relations between the non-Markovian kinetic coefficients and observable quantities (mean- and mean square displacements) are established. The problem of calculations of the kinetic coefficients using the Langevin equations is discussed. Solutions of the non-Markovian equation describing diffusive processes in the real (coordinate) space are obtained. For long times, such a solution agrees with results obtained within the continuous random walk theory but is much superior to this solution at shorter times, where the effect of the ballistic region is crucial.