Modelling the reusable space transport system for small payloads delivery

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Abstract. The problem of delivering a payload from orbit without the consumption of jet fuel is considered. To solve this problem, it is proposed to use a reusable space transport system that consists of a satellite in a circular orbit, a weightless tether, a capsule with a payload, and a lower module. The purpose of the paper is to develop a mathematical model of the transport system, and to determine the system’s parameters ensuring safe payload descent without entangling the tether. Equations of motion were obtained by Lagrange formalism. The capsule and the lower module were considered as rigid bodies, and the friction force between the capsule and the tether was taken into account. Numerical simulation showed that when the capsule approaches the lower end of the tether, the tether segment under the capsule can transfer into rotation. In turn, this will cause the capsule to stop and entangle the tether. The influence of the mass of the lower module on the dynamics of the system was investigated. The dependence of the entanglement point position on the mass of the module was identified. For safe descent from orbit, the separation of the capsule must occur before it reaches this dangerous point.

1. Introduction
The delivering a payload from orbit is one of the most topical issues of modern cosmonautics. Today, various re-entry capsules translated into the descent orbit with the help of jet engines are used. Alternative approach based on the use of space tethered systems are widely discussed in scientific literature. A detailed review of the different projects and methods of applying tethered systems in space is given in [1-3]. Tethered systems allow to deorbit a payload due to the momentum exchange between the connected by the tether bodies [4], or using the Earth's electromagnetic field [5, 6]. The projects of space tethered systems for delivering payloads based on using variable-length tethers are the most elaborated and close to practical implementation [7-10]. At the moment, three orbital transport experiments have been conducted: SEDS-1 in 1993, SEDS-2 in 1994 and YES2 in 2007 [3, 10]. In these projects variable-length tethers with the capsules at their ends were used [11]. In this paper, the reusable transport system based on the space tethered system of constant length is proposed. The payload is delivered to Earth by a capsule that slides along the tether. Similar mechanical systems with one or several capsules moving along the tether were investigated in [12-14]. In [15] the motion of a system consisting of two material points connected by weightless tether, and the load which can move freely along the tether was described. In paper [16] the problem of the motion of a mass point along a space elevator was considered. The space elevator was regarded as a weightless inextensible absolutely flexible tether. Conditions of the tether slackness were found, stationary motions were obtained and their stability was investigated. In [17] a simple analysis of a material point moving along an orbital tower was conducted. In the article [18] the dynamics of a flexible tethered system with a climber was analyzed. In [19], the dynamics of a tethered system with
a movable climber is studied for the case of circular orbit. The tether tension control, which avoids the tether deflection on the final segment of the capsule descent along tether, was proposed. In contradistinction to previous works our study takes into consideration the motion of the capsule around its center of mass, and the friction force between the capsule and the tether.

2. Formulation of the problem

The paper considers a reusable transport space system, which consists of a satellite in a circular orbit, a weightless tether, a capsule with a payload, and a lower module attached to the tether end. The system is designed to deliver small payloads to the Earth without the cost of jet fuel. The payload is placed into the capsule, which slides along tether. At some distance from the lower module, the separation of the capsule from the tether occurs by dividing the capsule into two parts. Then these parts move into the descent orbit. An alternative approach involving braking and docking of the capsule with the lower module, and its further separation from the module by its automation, is also possible. The capsule delivery scheme is shown in Figure 1. The capsule deviates from the local vertical of the satellite in the direction of its orbital motion as the result of the Coriolis force action. This can cause the swing-up and rotation of the tether segment bellow the capsule, which can lead to entanglement of the tether and make it impossible to continue using the transport system. The aim of the work is to develop a mathematical model of the reusable transport space system, and to define the system parameters ensuring safe descent of the payload without the tether entanglement.

3. Mathematical model

Let's consider the tethered system, consisting of a satellite, a tether, a lower module and a sliding capsule (Figure 2). The satellite moves in a circular orbit of the radius $R$ with constant angular velocity $\Omega$ and it is considered as a material point of mass $m_s$. The inextensible and weightless tether, which length is $l$, is attached to the satellite. It is assumed that the tether does not change its length during the transport operation. The descending capsule is a rigid body of a spherical shape. Its mass is $m_c$. The lower module is attached to the end of the tether. It is considered as a rigid body with mass $m_l$. The state of the mechanical system is described by five generalized coordinates $q = (\varphi_1, \varphi_2, \varphi_3, \varphi_4, s_1)$, where $\varphi_1, \varphi_2, \varphi_3$ are the angles of tether deviation the from the local vertical, $\varphi_4$ - is the angle of deviation of the lower module axis from the local vertical, $s_1$ is the length of the tether segment from the satellite to the capsule. The length of the tether segment inside the capsule is denoted as $s_2$. The distance from the capsule to the lower module is $s_3$, the distance from the tether attachment point to the center of mass of the lower module is $s_4$.

When the capsule moves along the tether, it experiences the influence of forces and torques (figure 3): gravity force $G$, centrifugal force of inertia $\Phi$, Coriolis force $\Phi'$, friction force from tether $F_f$, normal reaction from tether $N$ and moment of reaction $M_N$. If the reaction force is greater than the Coriolis force, the tangential component of the capsule orbital velocity gradually decreases. As a result, the angular velocity of the capsule around the Earth and the centrifugal force also decreases.
The centrifugal force ceases to compensate for the gravitational force, and the capsule descends down the tether.

**Figure 2.** Space tethered system.

**Figure 3.** Forces acting on the capsule.

The equations of motion can be obtained using the Lagrange equations of the second kind. In order to calculate the Lagrangian of the mechanical system \( L = T - \Pi \), the kinetic \( T \) and potential \( \Pi \) energy should be found. The kinetic energy is given by

\[
T = \sum_{i=1}^{3} \frac{m_i V_i^2}{2} + \frac{J_{12} (\phi_2 + \Omega)^2}{2} + \frac{J_{13} (\phi_3 + \Omega)^2}{2},
\]

where \( m_i \) is the mass of the \( i \)-th body, \( V_i \) is the velocity of the center of mass of the \( i \)-th body, \( J_{ij} \) is the moments of inertia of the \( i \)-th body; \( i = 1 \) denotes the satellite, \( i = 2 \) - the capsule, \( i = 3 \) - the lower module. The potential energy can be found as

\[
\Pi = -\sum_{i=1}^{3} \frac{m_i}{r_i} - \frac{\mu}{2} \left( \sum_{i=1}^{3} \frac{J_{ij}}{r_i^3} \right) - \frac{3\mu}{2} \left( \frac{J_{12} \cos^2 (\phi_2 + \beta_1) + J_{13} \sin^2 (\phi_3 + \beta_2)}{r_1^3} \right) - \frac{3\mu}{2} \left( \frac{J_{13} \cos^2 (\phi_4 + \beta_1) + J_{13} \sin^2 (\phi_4 + \beta_1)}{r_3^3} \right),
\]

where \( r_i = \sqrt{x_i^2 + y_i^2} \) is the distance from the center of the Earth \( E \) to the center of mass of the \( i \)-th body, angles \( \beta_1 \) and \( \beta_2 \) are shown on Figure 2 and can be calculated as

\[
\cos \beta_i = \frac{x_i x_i + y_i y_i}{r_i^2},
\]

\( x_i \) and \( y_i \) are the coordinates of the \( i \)-th points in the fixed coordinate reference frame \( Ex_y \). It is follows from the geometry (Figure 2) that

\[
x_2 = x_1 - x_1 \cos (\phi_1 + \nu) - \frac{s_2}{2} \cos (\phi_2 + \nu), \quad y_2 = y_1 - x_1 \sin (\phi_1 + \nu) - \frac{s_2}{2} \sin (\phi_2 + \nu);
\]

\[
x_3 = x_1 - \sum_{i=1}^{4} x_1 \cos (\phi_1 + \nu), \quad y_3 = y_1 - \sum_{i=1}^{4} x_1 \sin (\phi_1 + \nu);
\]

where \( \nu = \Omega t \) is the true anomaly angle of the satellite.

Let us write the general form of the Lagrange equations of the second kind

\[
d \frac{\partial L}{\partial \dot{q}_j} - \frac{\partial L}{\partial q_j} = Q_j,
\]

where \( Q_j \) is nonpotential generalized forces. To calculate the generalized forces arising from the friction, let us write

\[
Q_j = F^j \frac{\partial \vec{r}}{\partial q_j},
\]
where $F_f = \mu_j N \sin(\dot{s}_i)$ is the friction force, and $\mu_j$ is the friction coefficient. From (5) with regard to (3) we obtain:

$$Q_{\varphi_0} = 0, Q_{\varphi_1} = 0, Q_{\dot{\varphi}_1} = 0, Q_{\ddot{\varphi}_1} = -\mu_j N \sin(\dot{s}_i).$$

(6)

The tether reaction force $N$ can be found by Newton’s second law for the capsule in the projections on the axis of the inertial coordinate system (Figure 2):

$$m_2 \ddot{x}_2 = -\frac{\mu_m x_2}{r_2^2} + N \cos(\varphi_2 - \pi/2 + \nu) - F_f \sin(\varphi_2 - \pi/2 + \nu),$$

$$m_2 \ddot{y}_2 = -\frac{\mu_m y_2}{r_2^2} + N \sin(\varphi_2 - \pi/2 + \nu) + F_f \cos(\varphi_2 - \pi/2 + \nu).$$

(7)

Substitution of the coordinates (3) in (7) and expression of $N$ gives

$$N = m_2 \left[ \dot{\varphi}_1 s_1 \cos(\varphi_1 - \varphi_2) + \frac{s_1}{2} \ddot{\varphi}_1 + \ddot{x}_2 \sin(\varphi_1 - \varphi_2) - s_1 (\Omega + \dot{\varphi}_1)^2 \sin(\varphi_1 - \varphi_2) +$$

$$+ 2 \dot{s}_1 (\Omega + \dot{\varphi}_1) \cos(\varphi_1 - \varphi_2) - RQ^2 \sin \varphi_2 + \frac{\mu}{r_2^2} (R \sin \varphi_2 - s_1 \sin(\varphi_1 - \varphi_2)) \right].$$

(8)

Substituting (8) in (6) and then in (4), we obtain a system of five second-order differential equations describing the motion of the system. These equations are not presented here because of their bulkiness.

4. Results of numerical simulation

Let us consider the motion of a mechanical system with the following parameters: the length of the tether is $l = 30000$ m, the radius of the satellite orbit is $R = 6645000$ m, the friction coefficient is $\mu_f = 0.1$, the diameter of the capsule is $s_2 = 0.4$ m, the initial velocity of the capsule is $\dot{s}_{i0} = 2.8$ m/s. The lower module is a cylinder, whose radius is $r_2 = 0.5$ m and length is $d = 2 \cdot s_1 = 2$ m. Mass of the satellite is $m_1 = 6300$ kg, mass of the capsule is $m_2 = 50$ kg. At the initial time, the tether and the capsule are oriented along the satellite’s local vertical, and the lower module is deflected by the angle $\varphi_1 = 0.1$ rad.

Integrating the equations of motion obtained in the previous section, we obtain the dependence of the deviation angles $\varphi_i$ on the distance $s_i$ for the case when $m_i = 100$ kg (Figure 4). During descent the capsule deviates from the local vertical in the direction of the orbital flight under the influence of the Coriolis force. This effect is well known and described, for example, in [13,18]. Calculations show that at the distance of $s_i = 29575$ m, the capsule stops. In this moment the tether segment below the capsule rotated to a large angle, and the lower module is above the capsule. This situation can be interpreted as entanglement of the tether. A sharp drop in the capsule speed relative the tether $\dot{s}_i$ is observed with entanglement (Figure 5). This is caused by the fact that in this position, the braking of the capsule is provided not only by the friction force, but also by the tension reaction force of the lower tether segment.

Let’s perform a series of numerical calculations, and investigate the influence of the mass of the lower module on the position of the entanglement point. It is seen from Figure 6 that when the mass of the lower module is increased, the tether entanglement occurs later. For example, in the case of $m_i = 1000$ kg, it occurs when the capsule is at a distance of $s_i = 24$ cm from the lower module (Figure 7), and in the case of $m_i = 2000$ kg, "entanglement" occurs at a distance of $s_i = 2$ mm, in fact, the capsule will strike the upper end of the lower module. Note, that as the mass of the lower module increases, the amplitude of oscillations of the angular $\varphi_1, \varphi_2, \varphi_3$ decreases. The tether is stronger, and a larger Coriolis force, and hence a larger relative velocity, is required to deflect the capsule.
5. The discussion of the results

The studies show that the radially oriented space tethered system, which is attached to the satellite in circular orbit, allows the descent of the capsule with a small payload without jet fuel costs. However, two problems can be noted. Firstly, the capsule develops a high velocity relative to the tether during its uncontrolled sliding (Figure 5). If the descent scheme does not involve the separation of the capsule before it reaches the lower module, then the docking with the lower module is assumed, and the capsule braking system is required. The second problem is entanglement of the tether at the final stage of the capsule motion in the case of a small relative mass of the lower module. This problem can be solved by installing jet engines in the lower module, or by separating the capsule at a great distance from the lower module. A sharp increase in the angles $\phi_2$, $\phi_3$ is observed at the final stage of the capsule motion even in the case of a large mass of the lower module. Therefore, the lower module must be equipped with capturing devices that allow to stop and fix the capsule before it reaches a dangerous point. After docking, the automation of the lower module should provide the separation of the capsule from the tether.

6. Conclusions
The article considers a reusable space transportation system aimed for the delivery of small payloads to the Earth and consisting of a satellite moving in a circular orbit, a weightless three-segment tether, a capsule with a payload and a lower module. The equations of motion of the system were obtained with help the Lagrange equations of the second kind. Within the framework of the model the capsule and the lower module were considered as a rigid bodies, and the friction force between the capsule and the tether was taken into account. Simulation of the uncontrolled sliding of the capsule along the tether was performed. It was shown that the motion of the capsule caused a deflection of the tether in the direction of the orbital flight. When the capsule approaches to the lower module, the segment of the tether bellow the capsule can transfer into rotation. In turn, this will cause the capsule to stop and entangle the tether. The influence of the mass of the lower module on entanglement of the tether was investigated. It was found that with the mass of the module increasing, the cable entanglement occurs later. Based on a series of numerical calculations, the graph that demonstrates the dependence of the distance to the point, at which the tether entanglement occurs, on the module mass was constructed. When the lower module has a relatively small mass, the separation of the capsule from the tethered system must occur before reaching the dangerous point of entanglement. In the case of a large mass of the lower module, a sharp increase in the angle of deflection of the cable segment between the capsule and the lower module is also observed, when the capsule approaches the lower end of the tether. Itcan lead to collision between the capsule and the lower module. Therefore, the automation of the module must ensure the capture and fixation of the capsule until it reaches the attachment point of the tether to the module.

The results obtained in the work can be used to create new transport space systems, including long tether and capsules moving along them.

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