Stress-strain state of the "wheel-rail" system under different movement conditions

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Abstract. A mathematical model of wheel-rail interaction for unworn profiles in the final-element modeling environment under different movement conditions, i.e. with the addition of additional torque and frictional forces, has been developed and tested in the article. Thus, the movement of rolling stock in the mode of braking (related to all cars and locomotives) and gaining traction (related to locomotives) is simulated. A significant influence of friction forces in the contact zone and additional torque on the stress-strain state during the interaction of the wheel and the rail are shown. The data obtained are additional refinements of one of the important factors in testing the durability of the material from which the rails are made – the loading of the samples and load application frequency. The work also approximated the results to maximize compressing and stretching stresses on the rail surface not only in a few variants of the studied original data. With the help of approximation, it is possible to study the stress state of the "wheel – rail" system at any point in a given range of vertical forces and additional torque.

1. Introduction

The "Russian Railways Holding Development Strategy for the period up to 2030" shows one of the main tasks of the development of the infrastructure business unit of the Russian Railways Holding: the development of infrastructure in order to switch to heavy traffic with train weight standards of nine thousand tons or more. To solve the problems, it is necessary to further modernize the elements of the upper structure of the track and running gears of the rolling stock, to improve the scientific and research base associated with the processes of interaction in the "wheel-rail" system to specify the maximum compressing and stretching stresses arising from the contact of wheels of rolling stock with the railway track.

The increase in axial loads inevitably causes an increase in mechanical stresses in the contact area of the wheel and rail, as well as all elements of the subrail base. That is, one of the main factors limiting the increase in axial loads are the maximum mechanical stresses (equivalent and normal) of the wheel-rail system. The quantitative parameters of the interaction between the rail and the wheel largely depend on the traffic safety and the basic feasibility indicators of the track and rolling stock facilities, including: the intensity of the wear and tear of the rails, the development of contact-fatigue defects, guaranteed handled tonnage, etc. [1-4].

This paper presents a mathematical model of the contact interaction between the wheel and the rail in the environment of the finite-element modeling. Different modes of rolling stock movement were
simulated at different values of axial loads, including the addition of auxiliary friction and torque in the contact zone. Additionally, contact stresses for the new wheel and rail profiles were calculated, and a qualitative and quantitative characteristic of their area of impact on the rail steel matrix in the contact area with all sorts of combinations of raw data was given.

As a result of the work, a conclusion on the stress-strain state of the rail and wheel was prepared. In particular, the effect of additional frictional forces in the contact zone was taken into account, and the dependences of the change in normal stresses on the rail surface during interaction with the rolling stock wheels on the values of the axial load and movement mode were obtained for experimental studies of the rail steel durability.

2. Description of the finite-element model

The wheel comes into contact with the head of the rail on a certain platform. The magnitudes and law of stress distribution over the contact platform depend on the dynamic load of the wheel, the ratio of the normal and tangential components of the wheel load vector, the shape of the contacting surfaces, etc.

For the first time, the solution of the main problems about contact stresses and deformations was obtained on the basis of the methods of elasticity theory in 1881—1882 by G. Hertz.

Based on the normal Hertz’s problem, the maximum contact stress \( P_{\text{max}} \) can be calculated according to the formula:

\[
P_{\text{max}} = \frac{3 \cdot F \cdot E^2}{2 \cdot \pi^3 \cdot r_e^2 \cdot (1 - \nu^2)^2},
\]

where \( E \) is the elasticity module; \( F \) is the normal loading force of the wheel and rail; \( r_e \) is the equivalent radius, depending on the characteristic radii of the wheel and rail interaction at the point of contact; \( \nu \) is Poisson’s coefficient.

It should be kept in mind that Hertz’s contact theory is true with the following assumptions:

- the contact surfaces are homogeneous and isotropic;
- friction forces in the contact zone are not active;
- the size of the contact area is small compared to the size of the contact bodies and the characteristic curvature radii of non-deformed surfaces;
- the solution of linear elastic semispace is used for the contact problem.
- contact surfaces are smooth.

When the vehicle moves, the position of the wheel set in relation to the rails changes significantly, resulting in different combinations of the contact zones of the wheel and the rail. Even if the axial load is constant, normal stresses will change due to differences in curvature radii of the contact surfaces of these zones. If there is one curvature radius of the surface in the contact area, one can use Hertz’s solution. If there are two or more curvature radii in the contact area, "non-Hertz’s" solutions should be used to determine the contact area. This is fundamentally important when solving the problem of the stress-strain state of contact of worn wheels and rails, as well as in the presence of side contact. Classical methods of calculation, despite the increasing accuracy of methods, are able to solve problems only with their considerable idealization, replacing the real structure with its computational scheme.

Modern numerical modeling techniques allow such tasks to be solved with sufficient precision in the finite-element modeling environment, taking into account, among other things, the additional friction forces between the contact surfaces [5-10].

To analyze the distribution of stresses in the contact area of the wheel and the rail, a three-dimensional model of interaction between the new wheel and the new rail with full geometric similarity of full-scale structures – GOST 10791-2011 and GOST R 51685-2013 (Figure 1).
On the basis of three-dimensional geometric models, the finite-element models (FEM) of the interaction between the new wheel and the new rail are built. The power of the estimated models is more than 300,000 nodes and reaches 1 million depending on the area of the contact patch. In the contact area, the element mesh is noticeably thickened (the size of the element is 0.2 - 0.5 mm) for the most accurate display of the analysis results.

It has been experimentally determined that for a particular task, further reduction in the mesh size does not increase the accuracy of output, but only increases the time it takes to solve the system of differential equations.

The following boundary conditions and loads were used in the solution:
- A rigid restraint is modeled along the rail foot area.
- The contact surface of the wheel and axis moves at a single-point contact only in a vertical direction.
- The rest of the elements are not fixed;
- The force is applied with the help of a "remote point", which makes it possible to simulate the impact of the axial load closest to the actual physical process that occurs during contact.

The original data parameters:
- The car wheel is 950 mm in diameter;
- The axial load is 25, 27 and 30 t/axis;
- The wheel axis is in vertical position;
- The rail axis is with a slope of 1/20;
- Contact surfaces are according to specified profiles.

The stress-strain state of the wheel and the rail was also assessed at different modes of rolling stock movement: frictional forces between contacting surfaces and torque attached to the wheel rotation axis were added. The maximum torque is limited by the greatest friction between contacting surfaces, since the wheel slippage occurs further. The wheel and rail cohesion ratio varies from 0.05 to 0.3 under different contact conditions, so it is advisable to calculate the stress-strain state of the wheel-rail system with a smooth increase of tangential forces in the contact area. Torque values range from 20% to 80% of the maximum allowed one depending on the axial load (Figure 2).
Figure 2. The diagram of the application of additional torque, where $F_{ax}$ is the axial load, $F_{fr}$ is the friction force when the wheels and rail are in contact; $M$ is the torque.

The maximum value of friction forces depends on the axial load and the friction (cohesion) ratio. The maximum value of torque is limited to the maximum forces of friction, overcoming which, the wheel slippage occurs. Thus:

$$M_{max} = F_{fr} \cdot l,$$

where $l$ is the force lever and the distance from the wheel’s rotation axis to the point of contact. For the vertical contact of the new rail and the new wheel, $l$ is numerically equal to the wheel radius (475 mm for the car wheels). In other types of contact, the force lever is determined depending on the rail and wheel wear profile. In assessing the stress-strain state of side contact, the value of $l$ is defined as the distance from the wheel’s rotation axis to the point of ridge contact.

The torque values in modeling linearly increase from 0 (no additional friction force) to 80% of the maximum for further analysis of the results and the ability to approximate the resulting functions.

3. Results

During the simulation, the equivalent stresses in rails and wheels were obtained (Figure 3). But these stresses, which are based on the Mises-Hencky theory, also known as the theory of maximum distortion energy, have no sign, that is, they are always positive and do not show the transition of the material from a compressed state to a stretched and vice versa. Therefore, the drop in normal stresses along the axis (Figure 4), coinciding with the wheel trajectory, is of interest in terms of rail endurance tests, thus creating additional tangential forces before and after the contact patch depending on the mode of movement (traction or braking). Also it is important that due to the railway track elasticity, the wheel always performs extra work when moving upwards, as well as the rolling of the wheel through the elastic reverse half-wave.
Figure 3. Equivalent stresses (MPa) with an axial load of 30 tons in a vertical section

Figure 4. Normal stresses on the rail surface with an axial load of 27 tons, occurring directly on the trajectory of the wheel under an additional effect of a torque of 15.098 kN·m

The general results obtained are graphs of the difference in normal stresses on the rail surface (MPa) at various axial loads under the additional effect of a torque (Figure 5). The X-axis represents the length of the line of influence of contact forces in mm depending on the applied additional torque. The Y-axis represents the magnitude of the normal stresses occurring on the rail surface.
Studies of the graphs show that the maximum and minimum values do not qualitatively change with the addition of torque, that is, the extremum points practically do not depend on the value of the applied torque. The quantitative impact of the addition of the torque is significant, which indicates a substantial influence on the stress-strain state of the "wheel-rail" system of the rolling stock movement mode.

It should be noted that the influence of the braking mode has a more significant effect in connection with the quantitative overbalance of cars in relation to locomotives.

After the performed approximation, a compressive stress curve was obtained (Figure 6).

![Figure 5](image-url)  
**Figure 5.** Normal stress difference in the rail surface (MPa) with an axial load of 27 tons

![Figure 6](image-url)  
**Figure 6.** Change in compressive stresses on the rail surface with an axial load of 27 tons depending on the applied torque
Thus, using the approximation curves, it becomes possible to determine the local maximum and minimum stresses, as well as the required frequency of load application to conduct an experimental study of rail steel durability when using the method described earlier. The results are presented below:

Approximation equations with an axle load of 25 tons:

**Compression**

\[ \sigma = -0.2843 \cdot M^2 + 0.3697 \cdot M - 821.59 \]

**Elongation**

\[ \sigma = 16.6 \cdot M + 68.703 \]

Approximation equations with an axle load of 27 tons:

**Compression**

\[ \sigma = -0.3172 \cdot M^2 + 1.1418 \cdot M - 850.85 \]

**Elongation**

\[ \sigma = 17.689 \cdot M + 51.609 \]

Approximation equations with an axle load of 30 tons:

**Compression**

\[ \sigma = -0.2533 \cdot M^2 + 0.1609 \cdot M - 885.27 \]

**Elongation**

\[ \sigma = 16.428 \cdot M + 43.629 \]

Comparative analysis of the largest stress values without additional torque and with its addition is presented in tabular form in Table 1.

| Maximum value (the sign is ignored) at the maximum possible torque before slipping | Minimum value (without torque) |
|-----------------------------------------------------------------------|-----------------------------|
| -901.94                                                               | -821.59                     |
| 358.77                                                                | 68.7                        |
| -942.27                                                               | -850.85                     |
| 385.44                                                                | 51.61                       |
| -993.27                                                               | -885.27                     |
| 388.11                                                                | 43.63                       |

4. Results

During the simulation, equivalent stresses in rails and wheels were obtained, taking into account the influence of friction forces in the contact zone, as well as additional torque, on the stress-strain state during the interaction of the wheel and the rail. The results clearly indicate a significant increase in compressive normal stresses – from 5 to 9 times, compared with the Mises-Hencky theory. Thus, the addition of torque creates on the rail surface areas of sharp differences in normal stresses from tension to compression and vice versa. Such differences must be taken into account when testing the material for durability in order to more accurately determine the guaranteed service life of rails in the areas of action of additional tangential forces: prolonged ascents, descents, braking sections.

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