A Compact Source of Terahertz Radiation Based on an Open Corrugated Waveguide

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Abstract

We show that it is possible to produce terahertz wave generation in an open waveguide, which includes a multilayer dielectric plate. The plate consists of two dielectric layers with a corrugated interface. Near the interface, there is a thin semiconductor layer (quantum well), which is an electron-conducting channel. The generation and amplification of terahertz waves occur due to the efficient energy exchange between electrons, drifting in the quantum well, and the electromagnetic wave of the waveguide. We calculate the inhomogeneous electric fields induced near the corrugated dielectric interface by electric field of fundamental mode in the open waveguide. We formulate hydrodynamic equations and obtain analytical solutions for density waves of electrons interacting with the inhomogeneous electric field of the corrugation. According to numerical estimates, for a structure with a plate of quartz and sapphire layers and silicon-conducting channel, it is possible to generate electromagnetic waves with an output power of 25 mW at a frequency of 1 THz.

Keywords: terahertz source, corrugated waveguide, open waveguide, drifting electrons, interaction of electrons with a wave

1. Introduction

For past few decades terahertz radiation, which occupies the bandwidth from approximately 0.3–10 THz, have received a great deal of attention. Devices exploiting this waveband are set to become increasingly important in a very diverse range of applications, including medicine and biology [1, 2]. Nevertheless, despite significant progress in the study of terahertz sources in recent years (see, for example, [3, 4] and references therein), this range is mastered much less than its neighboring frequency ranges: the optical range, in which optoelectronic devices are used, and the microwave range, in which electro-vacuum microwave devices are mainly used.

Currently, the key problem lies in the creation of a sufficiently intense, and at the same time compact, terahertz source that can be adapted for a variety of applications.

Here we consider a scheme of a compact terahertz generator, which uses some methods and ideas that have been successfully applied in vacuum microwave
generators. In the schemes of terahertz generator discussed here, as in microwave generators (such as backward wave and traveling wave tubes), electrons interact with the waveguide wave and transfer their energy to this wave. However, a straight-forward adaptation of microwave generator circuit for terahertz generators causes serious difficulties. It is known that the most efficient energy transfer between electrons and an electromagnetic wave occurs when the electron velocity is close to the phase velocity of the wave. However, since the electron velocity is much less than the speed of the electromagnetic wave in a vacuum (light speed), slow-wave structures are used to reduce the wave velocity. The characteristic dimensions of such slow-wave structures should be comparable with the length of the amplified electromagnetic wave. There is a strong absorption of electromagnetic radiation in slow-wave structures with small characteristic dimensions, which are necessary for slowing down terahertz waves. This significantly affects the generation conditions. It is extremely difficult both to slow the wave and to avoid losses in this frequency range.

Here, we offer another phase-matching method. In the proposed terahertz generator, a speed of the electromagnetic wave, which propagate along a waveguide, is close to the speed of light in vacuum, but electrons are still able to effectively transfer their energy to the wave. Such a situation may occur in corrugated waveguides, in which the corrugated dielectric surface is located in the region of a high electric field of an electromagnetic wave, and electrons move over this corrugated surface. The electromagnetic wave causes the polarization of the dielectric in the zone of the corrugation, and this polarization, in turn, induces an alternating electric field near the corrugated dielectric interface. The characteristic scale that determines the induced electric field is determined by the corrugation period. Since the corrugated interface has a periodic structure, the induced electric field near the interface can be regarded as the sum of an infinite set of harmonics. By selecting the size of the period, we can ensure that electrons, moving at a small distance above the corrugation, would effectively interact with the first harmonic of the induced electric field and would give its energy to the wave.

Here we consider a scheme of an open corrugated waveguide, which includes a thin multilayer dielectric plate (Figure 1). The plate consists of two different dielectric layers with a dielectric interface corrugated periodically in the transverse direction of the waveguide (in the direction perpendicular to the direction of wave propagation). The electric field of the transverse electromagnetic wave (TE wave) induces near the corrugation an electric field that is non-uniform in the transverse direction of the waveguide. There is a quasi-two-dimensional conducting channel (quantum well) in one of the dielectric layers. In the quantum well located near the corrugation, electrons drift in the transverse direction of the waveguide, and interact with the transverse electric field of the induced surface electromagnetic wave. Phase matching ($v_e \approx v_f$) can be achieved by selecting the applied voltage (which determines the electron drift velocity $v_e$) and the corrugation period $L$ (which defines wave number $k_e = 2\pi/L$ and the phase velocity of the surface wave $v_f = \omega/k_e$, where $\omega$ is the frequency of the waves). The phase velocity of electromagnetic waves with a frequency of $\sim 10^{12}$ s$^{-1}$ is a value of the order of $\sim 10^6 - 10^7$ cm/s for the corrugation period of $L \sim 0.1-1$ μm (structures with such parameters are achievable at the present technological level). The drift velocity of electrons in a conducting channel can have a value of the order of $\sim 10^6 - 10^7$ cm/s. Thus, the corrugation period and the parameters of the electronic system can be selected so as to satisfy the conditions of the most effective interaction of the carriers with the electromagnetic wave. As a result of the interaction, the amplitude of the electric field of the
transverse electromagnetic wave increases, while the amplified electromagnetic wave itself propagates along the waveguide at a speed close to the speed of light in vacuum ($c >> v_f$).

We propose a generator circuit of an open waveguide, which includes a thin multilayer dielectric plate. In such waveguide, the field of an electromagnetic wave is focused in a region that includes both the plate itself and some region near the plate. Ohmic losses of electromagnetic waves are only into the dielectric plate, while wave energy is concentrated mainly outside the plate. So, in open waveguides, it is possible to reduce the energy loss of an electromagnetic wave (and, thus, to improve the generation conditions).

In our works [5, 6], a similar synchronization scheme was proposed for a open corrugated waveguide, in which electrons move ballistically in vacuum above a corrugated plate surface and interact with the non-uniform electric field induced near the corrugation. Such electro-vacuum terahertz generators can have an output power of the order of watts, and efficiency up to 80% [5].

However, in such schemes it is necessary to stabilize the electron beam position above the plate, avoiding the bombardment of the plate by electrons. In [5], we proposed a method for stabilizing the electron beam, but this greatly complicated the scheme of the terahertz generator.

In the scheme considered here, the position of the electrons is already stabilized, since the electrons drift in the quantum well, and the conducting channel can be located close to the corrugated dielectric interface. Such a terahertz generator is compact and, moreover, quite simple to produce. However, the parameters of such a generator are low compared with the parameters of the generator described in [5]. In the terahertz generator scheme considered here, electrons drift in a semiconductor quantum well and experience a large number of collisions. As a result, there are significant losses in Joule heat in such a waveguide. According to our estimates, in such a scheme, it is possible to generate terahertz waves with an output power of tens and hundreds of milliwatts and efficiency of the order of 1%.
A brief description of the terahertz generator, based on the interaction of electrons in a quantum well with an electromagnetic wave of a corrugated waveguide, was previously presented in our works [7, 8].

In the second section of this chapter, we present calculations of the induced inhomogeneous electric field of the wave near the corrugated dielectric interface. In the third section, we use a hydrodynamic approach to describe a system of charged carriers, which drift in a quantum well and interact with the inhomogeneous electric field of the wave in the zone of the corrugation. We define the parameters of the system under which the amplification of the electromagnetic field is the most effective. In the fourth section, we give a brief conclusion.

2. Wave electric fields in an open waveguide with a plate consisting of two dielectric layers with a corrugated interface

The generator circuit considered here is an open waveguide, which includes a thin dielectric plate consisting of two dielectric layers with a dielectric interface, periodically corrugated in the transverse direction of the waveguide. An electron conducting channel is included in one of the dielectric layers of the plate close to the corrugated dielectric interface (Figure 2), and the electrons drift in the conducting channel in the transverse direction of the waveguide (in the direction perpendicular to the direction of wave propagation).

We assume that the conducting channel is sufficiently thin (its thickness is much smaller than the corrugation amplitude). In this case, the problem of calculating of electric fields in the waveguide, and the problem of calculating of interactions of electrons with electromagnetic waves can be solved separately. In this section, we solve the problem of calculating of electric fields without electrons. In the next section, we introduce electrons, which interact with wave electric fields, into the picture.

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**Figure 2.** Plate profile of an open waveguide. The plate consists of two dielectric layers with permittivities $\varepsilon_1$ (light gray) and $\varepsilon_2$ (dark gray) having a periodically corrugated dielectric interface. Electrons drift with a mean velocity $v_0$ in a semiconductor layer 1. $E_0$ is an electric field of a principal wave of the waveguide (TE-wave).
Electromagnetic waves, including waves in the terahertz frequency range, can propagate in the proposed open waveguide. The waveguide contains a dielectric plate of thickness $2a$ consisting of two dielectric layers with different permittivities $\varepsilon_1$ and $\varepsilon_2$ ($\varepsilon_1 < \varepsilon_2$). For sufficiently thin plate, $a < c/\omega$, and for not too large values of $\varepsilon_1$ and $\varepsilon_2$ ($\varepsilon_i < 10$) the electromagnetic wave energy is mostly concentrated in the region outside the plate. In the waveguide, an electromagnetic wave propagates with the velocity close to the speed of light.

Let us consider the case of a TE wave, in which there is an antinode of the electric field in the center of the cross section of the plate (where the dielectric plate is) [7] (Figure 1).

For a thin plate ($a < 2\pi c/\omega$) with a flat dielectric interface, the electric field of the TE wave is directed along the X-axis, and is given by

$$E_{TE}(r, t) = e_x E_x(y) \cdot \exp(ikz - i\omega t),$$

(1)

The field $E_x$ can be represented as:

$$E_x \approx E_{in} \cos(\chi y), \text{ if } |y| \leq a, \text{ and}$$
$$E_x \approx E_{out} \exp(-s|y|), \text{ if } |y| \geq a$$

(2)

Here, $e_x$ is a unit vector along the x-axis, $E_{out} \approx E_{in} = E_0$, $\chi$ and $s$ are transverse components of the wave vector in a dielectric and vacuum, respectively. The values $\chi$ and $s$ are related by $sa = \frac{1}{\varepsilon} \cdot \chi a \cdot \tan(\chi a)$ and $(\chi a)^2 + (sa)^2 = \frac{\omega^2}{c^2} (\varepsilon - 1)$, where $\varepsilon = \sqrt{\frac{\varepsilon_1}{\varepsilon_2}}$ is the average value of permittivity of the dielectric plate.

However, the electric field $E_x$ of the TE-wave is uniform for a homogeneous dielectric plate, as well as multi-layer plates with a flat dielectric interface. In this case, the effective interaction between electron and the wave should be absent.

We propose a scheme of a waveguide with a thin multilayered dielectric plate, and the interface between two dielectric layers is a corrugated surface (Figure 1). The waveguide with this plate can also serve as a waveguide for the TE wave, and the wave propagation speed along the Z-axis is still being close to the speed of light. But in this case, in the vicinity of corrugated dielectric interface, an additional surface wave is induced in a field of the TE wave. The induced inhomogeneous electric field of the surface wave is comparable with the electric field of the volume electromagnetic wave only in a very narrow region near the dielectric interface. And the inhomogeneous electric field decreases exponentially (in Y-direction) with distance from the interface. The induced electric field is inhomogeneous in the Z-direction of the waveguide, and electrons also drift in X-direction in quasi-two-dimensional conductance channel, located in the zone of the corrugation (Figure 2). Such electrons can interact with an inhomogeneous field of the wave.

Let the coordinates of the dielectric interface in the waveguide cross section vary according to a periodic law, for example, to the law $y(x) = R \cos(k_x x)$. Here $k_c = 2\pi / L$, and the corrugation amplitude $R$ is significantly smaller than the plate thickness, $R < < 2a$, and the corrugation period $L$ is much less than the plate width.

We accept the condition

$$k_c > > k,$$

(3)

and, as will be clear from the following, it is worth considering the case $k_c R \sim 1$, in which the interaction of an electromagnetic wave with electrons (and the amplification of this wave) will be most effective. We assume that the conducting
channel is thin enough \((h << R, \text{where } h \text{ is the channel width})\), so that the electrons in the channel do not deform the electromagnetic fields in the dielectric.

In such waveguide, the electric field of the wave can be represented as \(E (r, t) = E_{TE} (r, t) + F (r, t)\), where \(E_{TE} (r, t)\) is given by (1) and (2) \([7]\). The electric field \(F (r, t)\) of the wave, which is induced near the corrugation, is comparable with the TE wave field only in a very narrow region (with a width of about \(R\)) near the dielectric interface. And \(F (r, t)\) is significantly less than \(E_{TE} (r, t)\) outside this region.

Let us proceed to calculation of the electric field \(F (r, t)\). The field \(F (r, t)\) is caused by influence of the electric field \(E_{TE} (r, t)\) of the wave, which is induced near the corrugation, is comparable with the TE wave field only in a very narrow region (with a width of about \(R\)) near the dielectric interface. And \(F (r, t)\) is significantly less than \(E_{TE} (r, t)\) outside this region.

The problem can be described by the scalar potential \(\phi^i(x, y)\), which satisfies to the Laplace equation. We denote the potential as \(\phi^i(x, y)\) inside the dielectric layer with the permittivity \(\varepsilon_1\), and as \(\phi^{(2)}(x, y)\) inside the layer with the permittivity \(\varepsilon_2\) \([7]\). The functions \(\phi^i(x, y)\) and \(\phi^{(2)}(x, y)\) satisfy to the Laplace equation \(\Delta \phi^i(x, y) = 0\) and \(\Delta \phi^{(2)}(x, y) = 0\) within its domain. Then the boundary conditions take the form

\[
\phi^i(x, y) \mid_c = \phi^{(2)}(x, y) \mid_c \quad \text{and} \quad \varepsilon_1 \frac{\partial \phi^i(x, y)}{\partial n} \mid_c = \varepsilon_2 \frac{\partial \phi^{(2)}(x, y)}{\partial n} \mid_c .
\]

The conditions (4)–(5) represent the continuity of the tangential electric field and normal component of an electric induction vector in a point with coordinates \((x, y)\) on the corrugation dielectric interface \([7]\).

Since the corrugated dielectric interface is described by a periodic function of \(x\), the electrostatic potential functions \(\phi^i(x, y)\) can be written as the sum of spatial harmonics:

\[
\phi^i(x, y) = \sum_{m=0}^{\infty} B_m^i \cos (mkx \chi) \exp (-mkc |y|) + \sum_{m=0}^{\infty} A_m^i \sin (mkx \chi) \exp (-mkc |y|) + E_0x
\]

In our problem, \(y(x) = R \cos (kx)\) is a continuously differentiable function. In this case, expressions (6) represent a complete set of basis functions \([9]\) for potentials \(\phi^i\) and for electric fields \(E^i = -\nabla \phi^i\). The expansion coefficients \(A_m^i\) and \(B_m^i\) of the sum (6) are found from the boundary conditions (4)–(5) at the point \((x, y) = (x, \cos kx)\) \([7]\). For the boundary described by \(y(x) = R \cos (kx)\), this system is consistent, if the coefficients \(B_m^i = 0\), and \(A_m^i \neq 0\). Then for the calculation of coefficients \(A_m^i\) we have limited the harmonic expansion to the seventh term. According to our calculations, when \(\varepsilon_2/\varepsilon_1 \geq 3\), taking into account the seven expansion terms is sufficient for calculating the coefficients, and the error will not exceed a few percent.
The resulting expression for the x-component $E^{(1)}_x$ of the electric field $E^{(1)} = -\nabla \phi^{(1)}$ induced in the upper layer near the corrugation (at $y > R \cos (k_c x)$) can be represented as

$$E^{(1)}_x(x, y) = E_0 \sum_{m=1}^{\infty} C^{(1)}_m e^{imk_c x} e^{-mk_c |y|}$$  \hspace{1cm} (7)$$

The coefficients $C^{(1)}_m$, which characterize the contribution of $m$th harmonic to the electric field induced above the corrugation, depend on the shape and size of corrugation and, moreover, on the parameters $\varepsilon_1$ and $\varepsilon_2$. We carried out numerical analysis on an example of a structure with a plate of quartz ($\varepsilon_1 = 4.5$) and sapphire ($\varepsilon_2 = 9.3$). In the case, when the dielectric boundary is described by $y(x) = R \cos (k_c x)$, each of the coefficients $C^{(1)}_m$ can be represented as a function of $k_c R$. \textbf{Figure 3} shows the calculated dependences $C^{(1)}_1 (k_c R)$, $C^{(1)}_2 (k_c R)$ and $C^{(1)}_3 (k_c R)$. For $k_c R \geq 1$, the value of $C^{(1)}_1$ is much greater than $C^{(1)}_2$ and $C^{(1)}_3$. The coefficients $C^{(1)}_m$ for $m > 3$ are much less than $C^{(1)}_1$, $C^{(1)}_2$ and $C^{(1)}_3$.

\textbf{Figure 4} shows the first harmonic of electric field $E^{(1)}_x / E_0$ at $y = R$ as a function of $k_c x$ for different values of the corrugation amplitude ($k_c R = 0.3, 1.2, 2$). At $y = R$, the amplitude of $E^{(1)}_x / E_0$ takes the maximum value for $k_c R \approx 1.2$. At the maximum we have $E^{(1)}_x / E_0 = 0.15$ and $C^{(1)}_1 \approx 0.5$. At the corrugation height $R \approx 1.2 \cdot (k_c)^{-1}$ the amplitude of the first harmonic $E^{(1)}_x$ of the electric field reaches its maximum value. The first harmonic electric field of the corrugation is smaller for both the higher and lower corrugation amplitude. Indeed, for lower corrugation heights ($k_c R << 1$), the induced electric field is small because of weak polarization of charges on the dielectric interface with weak corrugation. In the case of higher corrugation heights ($k_c R >> 1$), the charge-induced field largely decays at $y \approx R$.

Thus, we have the expression for the x-component of the first harmonic electric field of a wave at $y = R$ (in the region where the conducting channel is)

$$F^{(1)}_x = E_0 C^{(1)}_1 \exp (-k_c R) \exp (ik_c x + ikz - i\omega t).$$  \hspace{1cm} (8)$$

\textbf{Figure 3.}

Coefficients, characterizing a contribution of the first $C^{(1)}_1$ (black line), second $C^{(1)}_2$ (blue line) and third $C^{(1)}_3$ (green line) harmonics to the induced electric near the corrugation, as a function $k_c R$; $R$ is a corrugation amplitude, $k_c = 2\pi / \Lambda$, $\Lambda$ is a corrugation period.
The optimal corrugation amplitude \( R \) and period \( L \), in which the first harmonic amplitude of the non-uniform electric field takes its maximum, are determined by \( k_c R \approx \frac{1}{2} \) (where \( k_c = \frac{2\pi}{L} \)).

3. Electron interaction with electromagnetic wave

Let us now consider the problem of interaction of an electromagnetic wave with the electrons. In our study frame electrons drift in a quasi-two-dimensional conducting channel in the transverse direction (X direction) of the waveguide in a quasi-two-dimensional conducting channel (Figure 2). The conductive channel is located close to the corrugated dielectric interface, that is, in large inhomogeneous electric field region.

The most effective interaction between the first harmonic of a wave and electrons happens under the condition of synchronism, when phase velocity \( v_f = \omega/k_c \) of the wave is close to the electron drift velocity \( v_0 \). The corrugation period \( L \) and applied voltage are selected in such a way as to satisfy the condition \( v_0 \approx v_f \). (More precisely, the electron drift velocity must slightly exceed the phase velocity of the electromagnetic wave \cite{7}. ) Due to the change of the electron velocity in the electric field, the electron flow becomes velocity modulated. Electron velocity modulation process is accompanied an electron bunching, and electron density wave is formed. Parts of the electrons, which decelerated in the electric field of the corrugation, release their energy to the electromagnetic wave. The slightly amplified electromagnetic wave causes an increase in the amplitude of the electron density wave which, in turn, amplifies the electromagnetic wave even more \cite{5}. Thus, an oscillatory mode appears.

To describe the electrons interacting resonantly with the wave, a self-consistent system of equations is required. The system should take into account the change as electromagnetic wave fields, and electron velocity \( v(x, t) \) and density \( n(x, t) \). We assume that the electron density is quite low and the conducting channel is very thin, so that the screening of the corrugation electric field by electrons is negligible. TE-wave of the waveguide includes \( H_y \) and \( H_z \) components of the magnetic field.

Since the electron collision frequency is much greater than the cyclotron frequency,
the influence of magnetic field on the electron motion is negligible. The form of a solution describing the electric field of the first harmonic wave is described by (8).

Since the corrugated dielectric interface is homogeneous along the Z-axis we can take $z = 0$ without loss of generality. At the initial generation stage, while the linear approximation is correct, the amplitude of the electromagnetic increases with time by the exponential law with an increment $\gamma$. Then, taking into account (8), the contribution of the first harmonic to the wave’s electric field at $y = R$ can be presented as

$$F^{(1)}_{x}(x, t) = E_0 \cdot C_1^{(1)}(k, R) \exp(-k, R) \cdot \exp(ik, x - i\omega t) \cdot \exp(\gamma t)$$

We will consider only the effect of the electric field (directed along the X-axis) on the drifting electron in the quantum well located at $y \approx R$.

In the hydrodynamic approximation, functions $v(x, t)$ and $n(x, t)$ can be described by the motion equation

$$\frac{\partial v}{\partial t} + (v \cdot \nabla)v = \frac{e}{m} \left( E_{app} + F^{(1)} \right) - \frac{v}{\tau} - \frac{\nabla p}{m \cdot n}$$

and the equation of continuity

$$\frac{\partial n}{\partial t} + \text{div}(n \cdot v) = 0.$$

Here, $e$ and $m$ are the electron charge and the electron effective mass, respectively; $E_{app}$ is the applied dc electric field, $F^{(1)}$ is electric field of the wave in a waveguide, $\tau^{-1}$ is the electron collision frequency. The last term on the right-hand side of Eq. (7) describes the effect of the pressure gradient ($\nabla p$) in a non-uniform gas of carriers.

Let us assume that $n(x, t) = n_0 + \delta n(x, t)$ and $v(x, t) = v_0 + \delta v(x, t)$, where $n_0$ and $v_0$ are the unperturbed concentration and velocity of the charge carriers. At $\tau^{-1} > > \omega$, the left side of Eq. (9) can be neglected, then

$$\delta v = \mu F_1 - D \frac{1}{n_0} \frac{\partial \delta n}{\partial x},$$

where $\mu$ is mobility and $D$ is diffusion coefficient of the charge carrier system in the quantum well.

We solve the system of Eqs. (9) and (11) for the boundary conditions $\delta n(0, t) = 0$ (that is, at the point of entry into the interaction space, the electron flow is uniform). In the linear approximation over $\delta n(x, t)$ and $\delta v(x, t)$, solutions take the form (in the region outside the boundary $x = 0$).

$$\delta n \approx n_0 k_e E_0 C_1^{(1)} \frac{\exp(-k, R)}{\delta^2 + (\gamma + D \cdot k_e^2)^2} \left\{ -\delta \cos(k, x - \omega t) - (Dk_e^2 + \gamma) \sin(k, x - \omega t) \right\} \exp(\gamma t),$$

$$\delta v \approx \frac{\mu E_0 C_1^{(1)} \exp(-k, R)}{\delta^2 + (\gamma + Dk_e^2)} \left\{ (Dk_e^2 \gamma + \delta^2) \cos(k, x - \omega t) + Dk_e^2 \delta \sin(k, x - \omega t) \right\} \exp(\gamma t).$$

Here, $\delta = k_e v_0 - \omega$, $\delta < < \omega$. The expressions (12)–(13) were obtained for condition $\gamma < < Dk_e^2$, which is satisfied for typical semiconductors with diffusion coefficient $D > 1$ cm$^2$/s.
Amplification of electromagnetic waves is achieved due to the fact that drifting charge carriers transfer their energy to the radiation field. Represent the current of carriers, which interact resonantly with the wave, as the sum.

\[ J(x, t) = J_n(x, t) + J_v(x, t), \]

where \( J_n(x, t) = e \cdot \delta n(x, t) \cdot v_0 \cdot L_z \) is the current of the electron density wave, and \( J_v(x, t) = e \cdot \delta v(x, t) \cdot n_0 \cdot L_z \) is the current due to velocity modulation of the electron stream, \( L_z \) is the length of the plate. (Usually, \( J_v \) is much smaller than \( J_n \).) The electron current does work on the electromagnetic wave, and gives its energy to the electromagnetic wave. Let \( P \) denote the energy transferred by electrons to an electromagnetic wave per unit time and per unit area of the plate, and in addition per unit path of the electron flow averaged over the distance traveled. Using (12) and (13), we obtain:

\[ P \approx - (1/2) \cdot e \cdot n_0 \cdot \left( C_1^2 E_0 e^{-kR} \right)^2 \mu \cdot \theta, \]  

(14)

where the function

\[ \theta = \frac{\delta \cdot (k \cdot v_0 + \delta)}{\delta^2 + (Dk_\parallel^2)^2} \]

characterizes the efficiency of energy transfer from the electron density waves to the electromagnetic wave. From \( \theta \) it follows that if the electron velocity \( v_0 \) is strictly equal to \( \omega/k \), (this corresponds to \( \delta = 0 \)), the amplification of an electromagnetic wave is absent. Indeed, in the case of \( \delta = 0 \), electromagnetic wave energy, expended for electron acceleration, is equal to the energy that is returned to the wave by decelerating electrons. Thus, there is no amplification of an electromagnetic wave. In order for the wave amplification mode to occur, the electron velocity must be slightly higher than \( \omega/k \) (\( \delta > 0 \)).

Figure 5 shows \( \theta \) as a function of \( \delta \), where \( \delta = k \cdot v_0 - \omega \). The region of values of \( \delta \), in which the function \( \theta \) takes positive values (and \( P < 0 \), corresponds to the amplification of the electromagnetic wave. The electromagnetic wave attenuation occurs for negative values of \( \theta \) (\( P > 0 \)). The maximum energy transfer from

![Figure 5](image)

\( \theta \) as a function of \( \delta = k \cdot v_0 - \omega \). The function \( \theta \) characterizes the efficiency of energy transfer from electrons to the electromagnetic wave.
electrons to the wave (the maximum of the electromagnetic wave amplification) corresponds to the maximum of the function $\theta$ which is attained at

$$\delta_g \approx Dk_c^2 \left(1 - \frac{Dk_c}{v_0}\right).$$

We carried out numerical estimates for the structure with p-Si conducting channel. The semiconductor p-Si has quite high values of the electrical breakdown field strength ($3 \cdot 10^7$ V/cm), and high values of the saturation electron velocity ($1.5 \cdot 10^7$ cm/s) in high electric fields and relatively low electron mobility at room temperature [10]. The crystal heterostructures Si O$_2$/Si /Si O$_2$ are mastered well enough by now [11].

For our estimates, we used the values of $\mu = 10^2$ cm$^2$V$^{-1}$s$^{-1}$ for room temperature mobility and $v_0 = 5 \cdot 10^6$ cm/s for the drift velocity in the applied field $E_{app} \approx 5 \cdot 10^4$ V/cm. Then, at the frequency $\omega = 10^{12}$ s$^{-1}$ and the wave number $k_c = \omega/v_0 = 2 \cdot 10^5$ cm$^{-1}$ (which corresponds to the corrugation period $L_c = 0.3 \mu$), we have $D \cdot k_c^2 \approx 1.2 \cdot 10^{11}$ s$^{-1}$. The hole collision frequency is $\tau = 10^{14}$ s$^{-1}$, so condition $\tau^{-1} > \omega$ is satisfied.

For amplification coefficient $\gamma = 10^8$ s$^{-1}$ ($\gamma < < D \cdot k_c^2$), we have found the optimum mismatch $\delta_g \approx 10^{11}$ s$^{-1}$, at which for the value of the energy transmitted from the electrons to the electromagnetic wave have its maximal in linear amplification regime.

The plate width $L_x = 2\pi/\chi \approx 0.87$ mm is chosen to be equal to half the natural wavelength with the frequency $\omega = 10^{12}$ s$^{-1}$. With the above parameters, a maximum value of the field $E_{max}$ reaches value of $10^4$ V/cm at which the exponential growth of electromagnetic field comes to set end, and the process goes to a quasistationary state. For our estimates, we used $E_0 \approx 170$ V/cm, for which a linear regime of generation is certainly satisfied.

Negative energy flow of the electron density wave (represented by the expression (14)) is converted into a positive energy flow of the electromagnetic wave.

The change per unit time of the electromagnetic field energy of the open waveguide is determined by the sum $W_v + W_d$ of the contributions of the field energy outside the plate (in vacuum)

$$W_v \approx \frac{1}{4\pi} \cdot S \cdot \gamma \cdot e^{2\tau t} \cdot E_0^2 \frac{c^2}{\omega^2 a} \frac{e}{(e - 1)},$$

and the field energy concentrated inside the plate

$$W_d \approx \frac{1}{4\pi} \cdot S \cdot \gamma \cdot e^{2\tau t} \cdot E_0^2 \left[(e + 1) \cdot \frac{\cos \alpha \sin \alpha \chi + a \chi}{\chi}\right],$$

where $S = L_x L_z$ is the area of the plate.

For a structure with a plate of quartz and sapphire with the thickness $2a = 400 \mu$, the wave vector takes the value of $s \approx a \omega^2 (e - 1)/(ec^2) = 1.8 \cdot \text{cm}^{-1}$ in vacuum and the value of $\chi \approx (e - 1)^{1/2} \omega/e = 72.3 \text{ cm}^{-1}$ in dielectric. In this case, the ratio of the wave energy in the dielectric to the wave energy in vacuum is $W_d/W_v \approx 10^{-3}$.

To start the electromagnetic oscillations, the gain $\gamma$ must be larger than the loss factor.

A Compact Source of Terahertz Radiation Based on an Open Corrugated Waveguide
DOI: http://dx.doi.org/10.5772/intechopen.89692
\[ \gamma_{\text{loss}} = \omega \cdot \tan \beta \cdot \frac{W_d}{W_v + W_d}, \]

which characterizes the attenuation of waves due to dielectric losses. Here, \( \tan \beta \) is dielectric loss tangent, which depends on dielectric materials. For the structure of quartz and sapphire \((\tan \beta \approx 10^{-4})\) with thickness \(2 \cdot a = 400 \ \mu \text{m}\), and at frequency \(\omega = 10^{12} \ \text{s}^{-1}\), the value of the dielectric loss coefficient is \(\gamma_t \approx 3 \cdot 10^6 \ \text{s}^{-1}\). Thus, the start-up condition \(\gamma >> \gamma_{\text{loss}}\) is fulfilled with a sufficient margin.

The carrier density \(n_0\), which can ensure the energy flow of the charge density wave for generation of terahertz radiation, is determined by the law of conservation: \(P = W_v + W_d\). For the structures with the above parameters, we have \(n_0 \approx 2 \cdot 10^{12} \ \text{cm}^{-2}\). Then the generation power is 25 mW for plate area \(S = 1 \ \text{cm}^2\).

The formation and amplification of the charge density wave are taking place in the system of the scattering carriers. Let us denote by \(Q\) the energy that drifting carriers lose in collisions per unit time:

\[ Q = e n_0 \mu \cdot E_{\text{app}}^2 \cdot L_x \cdot L_z. \]

The efficiency of the generator can be estimated from the ratio of the useful energy \(P\) to the total energy \(P + Q\) expended by the electrons both for amplification of electromagnetic waves and for collisions: \(\text{eff} = |P|/(|P| + Q)\).

For the scheme with the above parameters, the generation efficiency is 1%, and the generation power is 25 mW at frequency of \(\omega = 10^{12} \ \text{s}^{-1}\).

Note that one of the possible ways to improve the output power is to increase the area of the plate. Moreover, the output power can be increased (according to expression (14)) by increasing the average electron concentration in the channel, as well as increasing the electron mobility. However, the mobility cannot be very high, since in the framework of the model described above, condition \(\tau^{-1} >> \omega\) must be fulfilled (see conditions for Eq. (11) above). But the generation efficiency for the considered circuit does not depend on the plate area, electron concentration, and mobility.

### 4. Conclusion

Currently, there are many approaches to the problem of terahertz generator creation, and there are working devices. However, the problem is still not solved completely. Now efforts are focused not on the development of a single device with record-breaking parameters, but on devices that are suitable for wide application.

Here we consider the concept of a compact terahertz generator, for which neither low temperatures nor strong magnetic fields are required. So, it may have some advantages compared other terahertz generator.

We propose a terahertz generator scheme, which is an open waveguide with a thin dielectric plate, since in such a waveguide the ohmic losses of electromagnetic waves can be small. Indeed, in such waveguides, ohmic losses are present only in the dielectric plate, while the wave energy is concentrated mainly outside the plate.

The device plate consists of two dielectric layers with different values of permittivity and dielectric interface, periodically corrugated in the transverse direction of the waveguide. The transverse TE wave propagating in the waveguide causes polarization of the dielectric in the zone of the corrugation, and this polarization, in turn, induces an alternating electric field near the corrugated dielectric interface. An electron conducting channel is included in one of the dielectric layers of the
plate close to the corrugated dielectric interface. Electrons drift in the conducting channel in the transverse direction of the waveguide (in the direction perpendicular to the direction of wave propagation). The drifting electrons interact with the inhomogeneous electric field which is induced near the dielectric interface by the TE wave of the waveguide. The corrugation period and the parameters of the electronic system are selected in such a way as to ensure the regime of the most effective interaction of drifting electrons with an electromagnetic wave. The generation and amplification of terahertz waves occurs due to the efficient energy exchange between electrons and the electromagnetic wave of the waveguide [7]. It is important that in the considered scheme, the interaction of the electron flow with the electromagnetic wave is quite effective without slowing down the wave propagating in the waveguide.

One of the advantages of the proposed scheme is its low sensitivity to the spread of heterostructure parameters. Before fabrication the corrugated structure, it is possible to define corrugation parameters, under which the synchronism condition will be satisfied.

We have calculated the induced inhomogeneous electric field of the wave near the corrugated dielectric interface, and we have obtained the optimal values of the corrugation amplitude and period at which the amplitude of the non-uniform electric field takes its maximum.

In the hydrodynamic approximation, we have obtained analytical solutions of the equations taking into account the diffusion and drift of the charge carriers in the quantum well. The solutions describe density wave of charge carriers that interact with the first spatial harmonic wave of an inhomogeneous electric field near the corrugation. We have presented numerical estimates for a structure with a plate of quartz and sapphire and the silicon conductance channel. We have found that it is possible to generate electromagnetic waves at the frequency $\omega = 10^{12}$ s$^{-1}$ with the output power of 25 mW and efficiency 1% in waveguides with plate area 1 cm$^2$ and the average electron concentration in the channel $2 \cdot 10^{12}$ cm$^{-2}$.

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