Electron Spin Resonance in Triple Quantum Dots

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Abstract. Electron spin resonance in triple quantum dots in series is analyzed by means of the density matrix formalism. We show that an ac magnetic field is able both to remove and to induce spin blockade.

1. Introduction
In solid state physics on the nanoscale one of the main protagonists is the electron spin. The two-level system formed by electron spin-up and down is a good candidate for a qubit and can easily be realized in quantum dots[1]. How to manipulate electron spins in quantum dots is one of the aims of recent experiments: Crossed DC and AC magnetic fields and also AC electric fields in combination with spin-orbit or hyperfine interaction are used to excite coherent spin rotations of one single electron in single and double quantum dots (DQD)[2, 3]. Electron spin resonance (ESR) has also been investigated quite recently theoretically for a DQD[4, 5].

Lately, a next step towards quantum dot arrays has been achieved: triple quantum dots (TQD), both in series[6] and triangular configurations[7]. Theoretical works in these systems concerned the analysis of eigenstates and stability diagrams, the effect of a magnetic field piercing the TQD area[8] and also strongly correlated electrons in the Kondo regime[9]. TQDs have also been proposed as spin entanglers[10] and as three-level systems where coherent electron trapping occurs[11, 12]. In the present paper, we analyze transport through a TQD driven by a time-dependent magnetic field $B_{AC}$ in a serial configuration, coupled to two electric baths, cf. Fig. 1. We discuss the electron spin dynamics and the electronic current for the case of two extra electrons in the TQD. We show that in the spin blockade regime, an AC magnetic field is able to break spin blockade if its frequency matches the Zeeman splitting in the sample. However, modifying the frequency, we found an antiresonance in the current, in which the two electrons form spin-blockaded entangled states.

2. Model
We consider a system of three weakly coupled quantum dots, where the dots are coupled through tunnel barriers, and dots 1 and 3 are connected to source and drain contacts respectively, shown schematically in Fig. 1. The Hamiltonian of the system is given by $\hat{H}(t) = \hat{H}_{sys} + \hat{H}_{QD-leads} + \hat{H}_{leads}$. The first term, $\hat{H}_{sys} = \hat{H}_{QD} + \hat{H}_{tun} + \hat{H}_B$, includes the on-site energies of the uncoupled TQD, $\hat{H}_{QD} = \sum_{i\sigma} \epsilon_i \hat{c}^\dagger_i \hat{c}_i \sigma$, the coherent tunneling between the dots, $\hat{H}_{tun} = -\sum_{ij\sigma} (t_{ij} \hat{c}^\dagger_i \hat{c}_j + h.c.)$ and the magnetic field term (see below). The second term
Figure 1. Schematic diagram of a TQD in the presence of crossed $B_{\text{DC}}$ and $B_{\text{AC}}$. The dots are coupled coherently to each other by tunneling amplitudes $t_{12,23}$ and incoherently to leads by rates $\Gamma_{L,R}$.

describes the coupling between the TQD and the leads, $\hat{H}_{\text{QD-leads}} = \sum_{l \in L,R} \gamma_l \hat{d}_{lk}\sigma \hat{c}_{lk}\sigma + \text{h.c.}$, and the last one describes the leads, $\hat{H}_{\text{leads}} = \sum_{lk\sigma} \epsilon_{lk} \hat{d}_{lk\sigma} \hat{c}_{lk\sigma}$.

We introduce now a magnetic field with two components: a DC one along the z-axis that induces a Zeeman-splitting $\Delta_i = g_i B_{zi}$, and a circularly polarized AC component in the $xy$-plane that rotates the $S_z$-component when its frequency fulfills the ESR condition $\hbar \omega_i = \Delta_i$. The time-dependent Hamiltonian of the magnetic field is then

$$\hat{H}_B(t) = \sum_{i=1}^{3} [\Delta_i S_{zi} + B_{\text{AC}}(\cos(\omega t) S_{xi} + \sin(\omega t) S_{yi})],$$

with the spin operators $S_i = \frac{1}{2} \sum_{\sigma\sigma'} c_{i\sigma}^\dagger \sigma \sigma' c_{i\sigma'}$.

The dynamics of the system within the Born-Markov-approximation is given by the equation of motion of the reduced density matrix elements:

$$\dot{\rho}_{ln}(t) = -i \langle l| [\hat{H}_{\text{TQD}} + H_i + \hat{H}_B(t), \rho] |n \rangle$$

$$+ \sum_{k \neq n} (\Gamma_{nk}\rho_{kk} - \Gamma_{kn}\rho_{nn})\delta_{ln} - \Lambda_{ln}\rho_{ln}(1 - \delta_{ln}).$$

The commutator accounts for the coherent dynamics in the TQD, $\Gamma_{ln}$ are the transition rates from state $|n\rangle$ to state $|l\rangle$ induced by the coupling to the leads - being $\Gamma_i = 2\pi|\gamma_i|^2$ when they occur through lead $i \in L,R$. Decoherence is considered by the terms $\Lambda_{ln} = \frac{1}{2} \sum_{k}(\Gamma_{kl} + \Gamma_{lk})$.

We consider a configuration where the dot coupled to the drain is permanently occupied by one electron (see Fig. 1), and only up to two electrons can be in the system. Double occupancy is only allowed in the drain dot. This is the case when the chemical potentials in the leads satisfy $\epsilon_3 + V < \mu_R < \epsilon_3 + U_3$ and $\mu_L < \epsilon_1 + 2V$. For resonant tunneling, $\epsilon_1 = \epsilon_2$ and $\epsilon_{1,2} + V = \epsilon_3 + U_3$. The system is described by the full two-electron TQD basis, out of which the relevant states for the dynamics are $|0,\sigma,\sigma\rangle$, $|0,\sigma,\sigma'\rangle$, $|\sigma,0,\sigma\rangle$, $|\sigma,0,\sigma'\rangle$ and $|0,0,\uparrow\rangle$, where $\sigma = \{\uparrow,\downarrow\}$ and $\sigma' = \{\uparrow,\downarrow\}$. We consider a bias high enough to have unidirectional transport (from left to right), so the current is given by $I = \sum_{\sigma} \Gamma_R \rho_{\sigma|0,0,\uparrow\rangle}$. 

3. Results

3.1. $B_{\text{AC}} = 0$

The DC magnetic field produces a Zeeman splitting in the three dots, which we consider to be inhomogeneous, here especially we treat the case $\Delta_1 = \Delta_2 \neq \Delta_3$. In general, different Zeeman
splittings occur due to the presence of hyperfine interaction, but can also be achieved by g-factor engineering[13]. The difference leads to a mixing of the two-electron triplet and singlet subspace. Without AC field applied to the system however, there is no continuous mixing between different spin channels: If the entering electron is in the same spin state as the confined electron in the third dot, Pauli principle forbids tunneling to the third dot. The occupation of the current blocking states increases and no current will flow to the drain, see Fig. 2.

3.2. $B_{AC} \neq 0$

Let us now apply a $B_{AC}$. The time-dependent Hamiltonian (1) for a circularly polarized magnetic field can be transformed into the rotating frame by means of $\hat{U}(t) = \exp\{-i\omega t \sum_{i=1}^{3} \hat{S}_{zi}\}$ and one ends up with a time-independent Hamiltonian[5, 12]:

$$\hat{H}_0 = \sum_{i=1}^{3} [\Delta_i - \hbar \omega] \hat{S}_{zi} + B_{AC} \hat{S}_{x3}$$  \hspace{1cm} (3)

As a first case, we set the AC frequency to be resonant with the Zeeman splitting induced by $B_{DC}$, i.e. $\hbar \omega = \Delta_{1,2}$.

This induces the rotation of the electron spin. Due to the mixing of singlet and triplet states, the AC field is able to break the spin blockade by inducing transitions like $|0, \uparrow, \uparrow\rangle \rightarrow |0, \downarrow, \uparrow\rangle$ and $|0, \downarrow, \uparrow\rangle \rightarrow |0, \uparrow, \uparrow\rangle$, such that a finite current flows. For resonant transport, the current is shown as a function of $B_{AC}$ and for different values of the Zeeman splitting difference $\delta = \Delta_{1,2} - \Delta_3$, see Fig. 3. As it is expected, the smaller $\delta$, the smaller the current, since then the electrons stay mainly in the triplet subspace. The maximum current is shifted to higher intensities $B_{AC}$, the bigger $\delta$.

If the resonance conditions $\omega = \Delta_{1,2}$ or $\omega = \Delta_3$ are not fulfilled though, the behavior of the current changes. In Fig. 4 we plot the current versus the AC frequency: For $\omega = \Delta_{1,2}$ and $\omega = \Delta_2$, the current reaches a maximum, whereas at $\omega_0 = \frac{\Delta_{1,2} + \Delta_2}{2}$, current is completely blocked. This can be explained by having a look at the eigenstates of (3). The Hamiltonian (3) can be diagonalized at $\omega_0$, and two of its eigenstates are:

$$|\Psi_1\rangle = \frac{1}{2} (|\uparrow,0,\uparrow\rangle - |\downarrow,0,\downarrow\rangle - |0,\uparrow,\uparrow\rangle + |0,\downarrow,\downarrow\rangle)$$  \hspace{1cm} (4)

$$|\Psi_2\rangle = \frac{1}{2} (|\uparrow,0,\uparrow\rangle + |\downarrow,0,\downarrow\rangle - |0,\uparrow,\downarrow\rangle + |0,\downarrow,\downarrow\rangle)$$  \hspace{1cm} (5)
Figure 3. $I$-$B_{AC}$ in a serial TQD exposed to crossed DC and AC magnetic fields, for $\omega = \Delta_{1,2} \neq \Delta_3$. For large $B_{AC}$ current eventually vanishes due to a competition between tunneling oscillations and spin rotations: the larger $B_{AC}$, the longer the electrons remain performing spin rotations and thus do not tunnel to the subsequent dot. The smaller the difference $\Delta_{1,2} - \Delta_3$, the smaller the current, due to less mixing of singlet and triplet subspace. Parameters: $\Gamma = 0.001$, $t = 0.01$, $\Delta_{1,2} = 0.013$, units in meV.

Figure 4. $I$-$\omega$ in a serial TQD exposed to crossed DC and AC magnetic fields. The current shows an antiresonant behavior once the AC frequency approaches $\hbar \omega = \Delta_1 + \Delta_3$, independently of the system parameters $t_{ij}$ and $B_{AC}$. Left: $I$-$\omega$ for $t_{12} = t_{23} = t$ and different intensities $B_{AC}$ of the AC magnetic field. Right: $I$-$\omega$ for different tunneling amplitudes $t_{12}, t_{23}$. Parameters: $\Gamma = t = 0.01$, units in meV.

These states only consist of parallel spin states, i.e., they are not coupled to the transport state $|0,0,\uparrow\rangle$. Thus, at $\omega_0$, the magnetic field forces the system back into spin blockade. This antiresonant behavior is a general property of the AC magnetic field, as it occurs also in triangular TQDs and DQDs[12]. The width of the antiresonance depends mainly $B_{AC}$, as one can appreciate in Fig. 4.

4. Conclusion
In summary, we present results for single electron transport through a specific configuration of a triangular TQD driven by crossed DC and AC magnetic fields. In the undriven case we calculated analytically the current as a function of both detuning and magnetic flux, showing the formation of a dark state as well as AB oscillations. The special configuration considered here leads to asymmetric current characteristics. ESR conditions allow to consider the TQD as a qubit where Rabi oscillations between two dark states can be contolled. Both in the undriven
and driven case, finite spin scattering destroys the coherent electron trapping and allow for a finite current to flow through the TQD.

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