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Chapter 11

XAO Analysis – AO’s and Their Populations in Crystal Fields

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1. Introduction

There are two ways to the analysis of electron density distribution (EDD) measured by X-ray diffraction. Multipole refinement (Stewart, 1969, 1973; Hansen & Coppens, 1978) expresses EDD as a linear combination of spherical harmonics. Using the determined functions, EDD is analyzed with Bader topological analysis (Bader, 1994). The concept of critical points of Bader theory defines the characters of chemical bonds. However, it does not give atomic orbital (AO) and its electron population, except for orbitals in high symmetry crystal fields where AO's are defined by theory. The other is based on quantum-mechanical orbital-models and gives the physical quantities just mentioned. The objective of this chapter is to introduce one of the second methods, the X-ray atomic orbital analysis (XAO) (Tanaka et al., 2008; hereinafter referred to as I), and its application to metal and rare-earth complexes, where advantages of XAO are revealed quite clear.

XAO is closely related to the electron population analysis (Stewart, 1969; Coppens et al., 1971). However, it was abandoned since the orthonormal relationships between orbital functions caused severe parameter interactions in conventional least-squares refinements. Later, the method incorporating the ortho-normal relationships between AO or MO (molecular orbital) was formulated by employing Lagrange’s unknown multiplier method (Tanaka, 1988; hereinafter referred to as II).

The AO-based EDD refinement has been started from 3d-transition metal complexes with so high crystal symmetry that the AO's are known by the crystal field theory. Spin states of the metals in perovskites, KCoF₃ and KMnF₃ (Kijima et al., 1981, 1983) and KFeF₃ (Miyata et al., 1983) were determined to be high spin. On the other hand, mixed orbitals of 3dₓ²₋ᵧ² and 3d₂ in Jahn-Teller distorted KCuF₃ were determined (Tanaka et al., 1979). The hybrid
orbital of $3d_2$ and $4s$ of Cu$^+$ which made O-Cu-O straight bond in CuAlO$_2$ was also determined (Ishiguro et al., 1983). The investigations on KNiF$_3$ and KCuF$_3$ are, to our knowledge, the first ones that determined spin states and AO's by X-ray diffraction, respectively. The Cu$^{2+}$ ion in [Cu(daco)$_2$(NO$_3$)$_2$] (daco: diazacyclooctane) is in the crystal field $C_i$. Five d-orbitals were determined with the least-squares method stated in II.

These investigations revealed that the anharmonic vibration (AHV) of atoms could not be ignored, since significant peaks still remained after removing the $3d$-peaks on difference density maps. They were attributed to the AHV of the metal atoms (Tanaka & Marumo, 1982; Ishiguro et al. 1983). For these investigations, a method proposed by Dawson et al., (1967), which had been applied only to high symmetry crystals, was made applicable to atoms in a general crystal field (Tanaka & Marumo, 1983).

Since the ratio of the number of bonding electrons to those in the unit cell becomes smaller as the atomic number increases, very accurate structure factors are necessary for X-ray EDD investigations of rare-earth compounds. Thus, the EDD analysis based on chemical-bond theories had not been done when we started the study on CeB$_6$. Actually, the ratio in CeB$_6$ is 1/88, which demands us to measure structure factors with the accuracy less than 1 %. Rare-earth crystals are usually very hard and good resulting in enhanced extinction and multiple diffraction (MD). Therefore, MD was investigated using the method by Tanaka & Saito (1975) in which an effective way to detect MD and correct for it were proposed. It introduced the time-lag between the relevant reflections, which usually do not occur at exactly the same time, to the method by Moon and Shull (1964). The effect of MD was demonstrated through the study of PtP$_2$ (Tanaka, et al., 1994) by measuring intensities avoiding MD and compared to those measured at the bisecting positions.

The frontier investigations aiming to measure 4f-EDD were done for CeB$_6$ (Sato, 1985) at 100 and 298 K and for nonaaqualanthanoids (Ln:La, Ce, Pr, Nd, Sm, Eu, Gd, Tb, Dy, Yb and Lu) (Chatterjee at al. 1988). Significant peaks were found around these rare-earth atoms on the residual density maps. However, the aspherical features of 4f-EDD were not analyzed quantitatively by the X-ray scattering factors calculated with AO's in a crystal field. The first 4f-EDD analysis of CeB$_6$ with the crystal field model but without spin-orbit interaction exhibited $T_{uu}$ state was occupied (Tanaka et al. 1997). The ground state of the 4f-state was found to be $4f^{5/2}I_g$ by the inelastic neutron scattering (Zirngiebel et al., 1984). When the spin-orbit interaction is taken into account, the total sum $J$ of the orbital and spin angular momentum $L$ and $S$, $J=L+S$, becomes a good quantum number. In the following discussion $J$ values of $p$, $d$, $f$ orbitals are attached as a subscript on the right-hand side of each orbital. The experiment does not contradict to our results. It was, however, investigated again at 100, 165, 230 and 298 K (Tanaka & Ōnuki, 2002) by introducing the spin-orbit interaction. Our experiments ascertained that 4f electron occupied $4f^{5/2}I_g$ orbitals. It revealed further that the electron population of them, $n(4f^{5/2}I_g)$, decreased on lowering the temperature. $4f^{5/2}I_g$ electrons are transferred to the $B_6$ moiety below room temperature. In addition to it just significant amount of $4f^{5/2}I_g$-electrons were found at 298 K, which was expected to be due to the thermal
excitation. Thus, the electron population above room temperature became very interesting. Since the energy gap between the two 4f-5/2 orbitals is about 500 K, the X-ray intensities were measured at 430 K expecting for 4f-5/2\(\Gamma_7\) to be more populated (Makita et al., 2007). The results were surprising. The 4f-5/2\(\Gamma_7\) is more populated than 4f-5/2\(\Gamma_8\) and 5d-5/2\(\Gamma_8\) is fully occupied at 430 K, which is expected to be at least a few ten thousands degrees higher than the 4f orbitals. The electron configurations of CeB\(_6\) at 340 and 535 K were further investigated to confirm the results (Makita et al., 2008). The EDD of SmB\(_6\) (4f\(^5\)) was studied (Funahashi, et al., 2010) below room temperature. 4f-5/2\(\Gamma_7\) and 4f-5/2\(\Gamma_8\) orbitals are fully or partially occupied. It is noted that 5d-5/2\(\Gamma_8\) occupation was also found. The physical properties such as electric resistivity are correlated to the electron configuration.

2. Basic formalism of XAO

2.1. AO’s in crystal fields

The i-th AO, \(\psi_{\alpha,i}(r)\) (i=1,2,...,M) of the \(\alpha\)-th atom are expressed with N basisfunctions, \(\psi_k(r)\),

\[
\psi_{\alpha,i}(r) = \sum_k c_{ik} \psi_k(r), \quad i = 1,2,...,M,
\]

where \(c_{ik}\) is a constant to be determined in the XAO analysis with the least-squares method incorporating orthonormal relationship between AO’s (Tanaka, 1988). Matrices and vectors are written in upper-case letters and bold lower-case letters, and superscript ‘ means a transposed matrix or a row vector, respectively. The basis functions, \(\psi_k(r)\), are listed in Table 1 of I. They are expressed using polar coordinate \((r, \theta, \phi)\) as a linear combination of a product of a radial function \(R_{nl}(r)\) and a spherical harmonic \(Y_{lm}(\theta, \phi)\) as,

\[
\psi_k(r) = \sum_{m=-l}^l d_{k,m} R_{nl}(r) Y_{lm}(\theta, \phi),
\]

where \(n, l\) and \(m\) are principal, azimuthal and magnetic quantum numbers. The non-relativistic radial functions, \(R_{nl}(r)\) (Mann, 1968) and relativistic functions calculated by HEX (Liberman et al., 1971) were mainly used for the XAO analysis. \(d_{k,m}\)’s are listed in Table 1 of I. Since approximate \(c_{ik}\)’s were necessary at the start of the non-linear least-squares calculation, they were calculated by taking the crystal field, which was calculated by placing a proper point charge on each atom, as a perturbation to the system (Kamimura, et al., 1969). For details, see eqs. (52)-(56) of I. The adjustable variables \(c_{ik}\)’s in the least-squares calculation are listed for all point group symmetries in Tables 4 and 5 in I.

2.2. Least-squares method incorporating ortho-normal condition

In a non-linear least-squares refinement, mathematical and physical relationships between parameters should be taken into account to avoid parameter interaction. The conventional
one used in X-ray crystallography was improved to obtain AO/MO by taking into account
the orthonormal relationship between wave functions (Tanaka, 1988),

$$\sum_{m}^{N} \sum_{n}^{N} c_{nm}^{*} s_{mn} = \delta_{ij}$$  \hspace{1cm} (3)

where $s_{mn} = \int \psi_{m}^{*} \psi_{n} \, dx$ and $N$ is the number of basis functions. $\delta_{ij}$ is Kronecker’s $\delta$. Assuming $c_{nm}$’s are real, it is rewritten with $(M,N)$ matrix $C=[c_{nm}]$ and $(N,N)$ matrix $S=[s_{mn}]$,

$$CSC = I,$$  \hspace{1cm} (4)

where $I$ is the unit matrix of order $M$. Using Lagrange's unknown multiplier $\lambda_{ij}$, the value $Q$ to be minimized in the least-squares method becomes,

$$Q = v' M_{f}^{-1} v - \sum_{i}^{M} \sum_{j}^{M} \lambda_{ij} (\sum_{m}^{N} \sum_{n}^{N} c_{nm}^{*} s_{mn} - \delta_{ij})$$  \hspace{1cm} (5)

For definition of $v$ see eqs. (10a) to (10d) of II. Putting $c_{nm}$ as the sum of the true value $c_{nm}^{0}$ and its small shift $\Delta c_{nm}$, $Q$ is expressed by ignoring the terms higher than second order of $\Delta c_{nm}$,

$$Q = v' M_{f}^{-1} v - \sum_{i}^{M} \sum_{j}^{M} \lambda_{ij} (\sum_{m}^{N} \sum_{n}^{N} (\Delta c_{nm} s_{mn} + c_{nm}^{0} s_{mn}) - \delta_{ij}),$$  \hspace{1cm} (6)

Putting $\lambda_{ij}=(1+\delta_{ij})\lambda_{ij}/2$, $Q$ is differentiated to give,

$$\delta Q = 2\delta x' (A' M_{f}^{-1} A x - A' M_{f}^{-1} f) - 2 \sum_{i}^{M} \sum_{j}^{M} \lambda_{ij} \sum_{m}^{N} \sum_{n}^{N} \delta (\Delta c_{nm} s_{mn}) = 0,$$  \hspace{1cm} (7)

where $x$ represents least-squares estimates of parameters. The first term is obtained following the usual procedure. For $x$, $A$ and $M_{f}$ see Hamilton (1964). After aligning $\Delta c_{nm}$’s linearly and adding at the end of $x$ for simplicity, $\lambda_{ij}$ is expressed in terms of $c_{nm}$ by using the orthonormal relationship in eq. (3). Then the final secular equation is obtained,

$$(I - R) A' M_{f}^{-1} A x = (I - R) A' M_{f}^{-1} f,$$  \hspace{1cm} (8)

where $I$ is unit matrix of order $P$, total number of variables. The $(P,P)$ matrix $R$ is expressed in terms of $c_{nm}$’s. It makes the inverse matrix of $(I - R) A' M_{f}^{-1} A$ in (8) calculable. The explicit form of $R=[r_{ij}]$ under the limiting conditions (10) for $i, j, k$ and $l$ is,

$$r_{ij} = [\text{SC}' \text{C}]_{kl},$$  \hspace{1cm} (9)

$$P = MN + (m - 1)N + 1 \leq i, j \leq P - MN + mN,$$

$$k = i - \{P - MN + (m - 1)N\}, \hspace{0.5cm} l = j - \{P - MN + (m - 1)N\},$$  \hspace{1cm} (10)
where \( m \) runs from 1 to \( M \). The other elements \( r_{ij} \) not defined here are zero.

### 2.3. Electron density and structure factors

The electron density \( \rho_\alpha(r) \) of the \( \alpha \)th atom centered at \( r_{\alpha}^{\text{atom}} \) is divided into that of the core and valence orbitals, \( \rho_{\alpha,\text{valence}}(r) \). It is further expressed from eq. (1),

\[
\rho_{\alpha,\text{valence}}(r) = \sum_{i} n_{\alpha,i} \rho_{\alpha,i}(r - r_{\alpha}^{\text{atom}}) = \sum_{i} n_{\alpha,i} \Psi_{\alpha,i}^*(\kappa_i(r - r_{\alpha}^{\text{atom}})) \Psi_{\alpha,i}(\kappa_i(r - r_{\alpha}^{\text{atom}})),
\]

where \( n_{\alpha,i} \) is the number of electrons of \( \Psi_{\alpha,i} \). \( \kappa_i \) expresses the expansion \( (\kappa_i > 1) \) or contraction \( (\kappa_i > 1) \) of \( i \)-th orbital (Coppens et al., 1979). Atom \( \alpha \) in the asymmetric unit is translated to \( r_{\alpha}^{\text{atom}} \) by the \( \sigma \)-th crystal symmetry operation. Then the structure factor is,

\[
F(k) = \sum_{\alpha} \sum_{\sigma} f_{\alpha\sigma}(k) \exp[i \mathbf{k} \cdot (r_{\alpha}^{\text{atom}} + \mathbf{u})],
\]

where \( f_{\alpha\sigma}(k) \) is an atomic scattering factor and \( \mathbf{k} \) is a scattering vector. The \( \alpha \)th atom at \( r_{\alpha}^{\text{atom}} \) is displaced by atomic vibration to \( r_{\alpha}^{\text{atom}} + \mathbf{u} \). It perturbs the periodicity of the crystal and reduces diffracted intensity. Then the atomic displacement parameter (ADP),

\[
\tau_{\alpha\sigma}^{\text{atom}}(k) = \langle \exp(i \mathbf{k} \cdot \mathbf{u}) \rangle,
\]

is calculated as an average in space and time of \( \exp(i \mathbf{k} \cdot \mathbf{u}) \) by assuming each atom vibrates independently from each other. \( f_{\alpha\sigma} \) is the sum of the scattering factors of core and valence electrons, \( f_{\alpha\sigma}^{\text{core}}(k) \) and \( f_{\alpha\sigma}^{\text{valence}}(k) \). \( f_{\alpha\sigma}^{\text{valence}}(k) \) is expressed as,

\[
f_{\alpha\sigma}^{\text{valence}}(k) = \sum_{i} n_{i} f_{\alpha,i}^{\text{valence}}(k),
\]

where \( n_{i} \) is the number of electrons of the \( i \)-th valence orbitals. \( f_{\alpha,i}^{\text{valence}}(k) \) is a Fourier transform of \( \rho_{\alpha,i}^{\text{valence}}(r) \). Final expression of \( f_{\alpha,i}^{\text{valence}}(k) \) becomes,

\[
f_{\alpha,i}^{\text{valence}}(k) = \sum_{K=M-K}^{M} \sum_{m=-l}^{l} \sum_{m'=-l}^{l} c_{K} c_{m}^* d_{K,m}^{*} d_{K,m'} \langle lm_{i}, l' m_{i'} \rangle e^{i \hbar M \gamma_{\sigma}} \]

\[
\times \sum_{K+l+l'}^{K+l+l'} \sum_{m=-l}^{l} \sum_{m'=-l}^{l} c_{K+l+l'} \langle lm_{i}, l' m_{i'} \rangle e^{i \hbar M \gamma_{\sigma}}
\]

where \( K+l+l' \) is even. \( M = m_{j} - m_{i} \) and \( |l-l'| \leq K \leq l+l' \). \( \langle j_{K} \rangle \) is expressed as,

\[
\langle j_{K} \rangle = \int R_{n}^{*}(r) R_{n'}^{*}(r) j_{K}(k \mathbf{r}) \rho^{2} d\mathbf{r}
\]
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is Bessel function of order \( K \). For \( \beta_{\sigma r} \), see Appendix B of I.

2.4. Anharmonic vibration (AHV)

Gaussian probability density functions (p.d.f.) of atoms are Fourier transformed to give harmonic ADP. Gram-Charier (G-C) formalism is widely used to introduce AHV. However, it expresses aspherical EDDs both from orbital functions and AHV, as shown clearly by Mallinson et al. (1988). Thus, it may not be adequate for accurate EDD researches. The method based on Boltzmann statistics (Dawson et al. 1967, Willis, 1969) was employed,

\[
T_{\text{atom}}(k) = \int \exp(-V(u)/k_B T) \exp(i \mathbf{k} \cdot \mathbf{u}) d\mathbf{u} / \int \exp(-V(u)/k_B T) d\mathbf{u},
\]

where \( k_B \) is Boltzmann constant, and \( T \) is absolute temperature. Assuming each atom vibrates independently and defining \( u_1(u_2, u_3) \) on the Cartesian coordinates parallel to the principal axes of the thermal ellipsoid of harmonic ADP, potential \( V(u) \) is expressed as,

\[
V(u) = V_0 + \frac{1}{2} \sum_{i=1}^{3} b_i u_i^2 + \sum_{i,j,k} c_{ijk} u_i u_j u_k + \sum_{i,j,k,l} q_{ijkl} u_i u_j u_k u_l + \ldots
\]

where \( i, j, k, l \) run from 1 to 3 and \( ' \) means they are permutable. There are 10 cubic and 15 quartic terms. Potential expansion terminates at the quadratic terms in the harmonic ADP.

By assuming the sum of the third and the fourth terms in (18) is much smaller than \( k_B T, b_i, c_{ijk} \) and \( q_{ijkl} \) are introduced into structure factor formalism using (12), (17) and (18).

3. Experimental

3.1. Multiple diffraction (MD)

MD occurs when two or more reflections are on the Ewald sphere. When the incident beam with wave length \( \lambda \) enters parallel to the unit vector \( \mathbf{j} \), and a primary reflection, the intensity of which is measured, is at the reflecting position, the conditions for a secondary reflection, \( h1h2h3 \), to occur become as follows (Tanaka & Saito, 1975),

\[
|\mathbf{e}_3 \cdot (\mathbf{j}/\lambda + h_1 \mathbf{b}_1)| \leq 1/\lambda
\]

\[
|\mathbf{e}_2 \cdot (\mathbf{j}/\lambda + h_2 \mathbf{b}_2 + h_3 \mathbf{b}_3)| \leq \left[ 1/\lambda^2 - \left| \mathbf{e}_3 \cdot (\mathbf{j}/\lambda + h_3 \mathbf{b}_3) \right|^2 \right]^{1/2} = r_3
\]

\[
\mathbf{e}_1 \cdot (\mathbf{j}/\lambda + h_1 \mathbf{b}_1 + h_2 \mathbf{b}_2 + h_3 \mathbf{b}_3) = \pm \left[ r_2^2 - \left| \mathbf{e}_2 \cdot (\mathbf{j}/\lambda + h_2 \mathbf{b}_2 + h_3 \mathbf{b}_3) \right|^2 \right]^{1/2}
\]

\( \mathbf{b}_1, \mathbf{b}_2, \mathbf{b}_3 \) are reciprocal-lattice vectors when the primary reflection exactly fulfills the reflecting condition and \( \mathbf{e}_1, \mathbf{e}_2, \mathbf{e}_3 \) are the unit vectors defined as,
Secondary reflections only around the surface of the Ewald sphere are searched using eqs. (19) to (21). Integers $h_3$ are selected from (19) and integers $h_2$ are calculated from (20) for each $h_3$. Then the value of $h_1$ is evaluated from eq. (21) for each $(h_2, h_3)$. The point $(h_1, h_2, h_3)$ is judged to be a reciprocal lattice point if $h_1$ is an integer. The Ewald sphere has width due to the divergence of the incident beam, finite size and mosaicity of the crystal, and wavelength spread. It permits some margin from an integer to $h_1$.

The change of intensity by MD was first formulated by Moon & Shull (1964). Primary and secondary beams usually come on the Ewald sphere with an angle-lag $\Delta \varepsilon$. Using $\Delta \varepsilon$ the formalism was modified for a spherical crystal with a radius $r$ (Tanaka & Saito, 1975),

$$\Delta I_1/I_1 = \frac{\sqrt{\pi r^2}}{8} Q_0 \sum_i \left[ -g_{01;01} \left( \frac{Q_{0i}}{Q_{01}} \right) - g_{01;1i} \left( \frac{Q_{0i}}{Q_{01}} \right) + g_{01;11} \left( \frac{Q_{0i}}{Q_{01}} \right) \left( \frac{Q_{0i}}{Q_{01}} \right) \right].$$  \hspace{1cm} (23)

Subscripts '0', '1' and 'i' stand for the incident, primary and secondary beams. $Q_{mn}$ is the integrated reflectivity per unit volume of a crystal for the $m$-th incident beam and $n$-th diffracted beam ($m-n$ process). It is proportional to the squares of the structure factor. In $i-1$ process, for example, a secondary beam $i$ acts as an incident beam and is diffracted along the direction of the primary beam $1$, which enhances the intensity of the primary beam. $g_{ij;mn}$ is a function of $\Delta \varepsilon$, Lorentz and polarization factors of the simultaneous $i-j$ and $m-n$ processes. The polarization factor for a general multi-diffracted process was recently formulated (Tanaka et al., 2010). For $g_{ij;mn}$ see Tanaka & Saito (1975). Since $\Delta I_1/I_1$ depends on the ratio of $Q_{mn}$’s and $\Delta \varepsilon$ of sharp primary and secondary reflections, $\Delta I_1/I_1$ fluctuates sharply. Eq. (23) was applied to diformohydrazide (Tanaka, 1978) and two peaks on the N-N bond in the difference density map became one peak at the middle of the bond.

The EDD analyses of compounds with metals and rare-earth elements need accurate measurements of structure factors, however MD affects the measured intensity seriously since $Q_{mn}$ is large. However correction for it does not seem fruitful, since each primary reflection has many secondary reflections and MD depends sharply on crystal orientation and half-width of each Bragg peak. Thus it is better to avoid MD by rotating a crystal around the reciprocal vector of the primary reflection ($\psi$-rotation) so that secondary reflections move apart from the Ewald sphere and measure at $\psi$ where $\Delta I_1/I_1$ is minimum. For actinoid compounds, MD cannot be avoided and the correction for it becomes necessary.

4. XAO analysis for crystals in the $O_h$ crystal field

4.1. 3d, 4f and 5d orbitals in the $O_h$ crystal field

In the present chapter EDD investigations on the first-transition metal complexes and rare earth compounds in $O_h$ crystal field are mainly stated. Thus it is worth while to represent the
energy level splitting of d- and f-states in Figs.1(a) and (b) (Funahashi, 2010). The spin-orbit splitting is neglected in a strong field model. For (3d)$^n$ systems strong field model is often employed. In (4f)$^n$ and (5d)$^n$ systems the two splittings are of the same order of magnitude. Weak field models are employed for CeB$_6$ and SmB$_6$. Their quantization axes point to the face-centre of the cubic unit cell, where no B atom exists. Note that the order of the levels in Fig. 1(a) is inverted, that is, $t_{2g}$ orbitals lies lower than $e_{g}$, when ligands are on the quantization axes like KCoF$_3$ (section 4). In weak field 5d- and 4f-states split into $j=3/2$ and $j=5/2$ states, and into $j=5/2$ and $j=7/2$ states by the spin-orbit interaction. The $O_h$ crystal field splits them further. Lobes of 4f$_{5/2} F_8$ extend along <100> directions while those of 4f$_{5/2} F_7$ orbitals along <110> and <111> as illustrated in Fig. 7 of Makita et al. (2007).

Figure 1. Energy-level splitting of (a) d- and (b) f-states in the $O_h$ crystal field with the degeneracy of each state in parentheses. Each orbital of a strong field can have two electrons.

4.2. 3d-EDD and spin states in KCoF$_3$, KMnF$_3$ and KFeF$_3$

The peak due to 3d electrons was first reported for [Co(NH$_3$)$_6$][Co(CN)$_6$] (Iwata & Saito, 1975) around the Co atom in $3\bar{1}$ crystal field. Later electron distribution at 80 K was reported (Iwata, 1977). 3d-EDD in $\gamma$-CoSiO$_4$ (Marumo et al., 1977) and CoAl$_2$O$_4$ (Toriumi, et al., 1978) were also observed. The electron density around Co in KCoF$_3$ was first analysed quantitatively based on the 3d-orbitals in $O_h$ crystal field (Kijima et al., 1981). A crystal was shaped into a sphere of diameter 0.13 mm, and intensities were measured up to $2\theta=130^\circ$ with a four-circle diffractometer using MoK$\alpha$ radiation without avoiding MD. $c_{i}$'s in (1) are fixed constants because of the high symmetry as listed in Tables 1(b) and 5(b) of $I$. $\kappa$ in eq. (11) was not introduced to the investigations in sections 4.1.1 and 5.2. Scattering factors of $d_{x^2-y^2}$, $d_{z^2}$, $d_{xy}$ and $d_{yz}$ orbitals are calculated with eq. (15). When electrons occupy the five d-orbitals equally, that is $(t_{2g})^2(eg)^{2.8}$ for Co$^{2+}$, atoms have spherical EDD. Hereinafter the refinement with the spherical scattering factors and without assuming AHV is called as spherical-atom refinement, in which type I or II anisotropic extinction (Becker &
Coppens, 1974a, b, 1975) was assumed. The difference density map assuming the spherical-atom, high and low spins, $t_{2g}^5(e_g)^2$ and $(t_{2g})^6(e_g)^1$, are shown in Figs. 2(a) to (c). Peaks remained after spherical-atom refinement were reduced and enhanced after high- and low-spin refinements, respectively. Large and almost no peaks in Fig. 2(c) and Fig. 2(b) demonstrate that Co$^{2+}$ is in the high-spin state. Spin-state was first determined by X-ray diffraction in this study. In the similar way, the spin-states of 3d-transition metals in KMnF$_3$ (Kijima et al., 1983) and KFeF$_3$ (Miyata et al., 1983) were determined to be high-spin.

Figure 2. Difference density after refinements of (a) spherical-atom, (b) high-spin, (c) low-spin models. Contours at 0.2 eÅ$^{-3}$. Negative and zero in broken and dashed-dotted lines.

5. Experimental AO determination

When symmetries of crystal fields are low, $c_{ik}$'s in (1) become unknown variables, and it is necessary to determine them by the least-squares method keeping the orthonormal relationship between AO's. Independent variables, $c_{ik}$'s, and the relation between them are listed for 32 point group symmetries in Tables 4 and 5 of I.

5.1. Jahn-Teller distortion in KCuF$_3$ and mixed $d_{x^2-y^2}$ and $d_{z^2}$ orbital

In KCuF$_3$ each F ion between Cu$^{2+}$ ions (3d$^9$) shifts from the centre by Jahn-Teller effect. It makes short Cu-F$_{s}$, medium Cu-F$_{m}$ parallel to c-axis, and long Cu-F$_{l}$ bonds, resulting in $mmm$ point group symmetry for Cu$^{2+}$. Difference density after spherical-atom refinement exhibits non-equivalent four holes in Fig. 3 (b) (Tanaka et al. 1979). Table 5(b) of I indicates a mixing of $3d_{z^2}$ and $3d_{x^2-y^2}$ in the crystal field $mmm$,

$$\psi_G = \cos(\phi / 2)d_{z^2} + \sin(\phi / 2)d_{x^2-y^2},$$
$$\psi_E = \sin(\phi / 2)d_{z^2} - \cos(\phi / 2)d_{x^2-y^2}. \quad (24)$$

The peaks on Cu-F$_{l}$ in Fig. 3(a) correspond to the lone pair of $\psi_G$. Putting two and one electrons to $\psi_G$ and $\psi_E$, cos(φ/2) became 0.964(18). 3d-peaks in Fig. 3 (a) and (b) were
deleted in (c) and (d). It confirms orbitals in eq. (24) are quite reasonable. The orbital functions except the phase factor were determined for the first time from X-ray diffraction experiment. However, significant peaks still remained. The positive and negative peaks along <100> and <110> in Fig. 3(c) indicate preferential and inhibitive vibration of Cu$^{2+}$, respectively. Therefore, AHV of Cu$^{2+}$ was analysed (Tanaka & Marumo, 1982). The obtained potential in (18) corresponds to the peaks and removes them as seen in the residual density in Fig. 4. Our study on AHV started from this investigation. Since the AHV peaks appear in Fig. 3(c) and (d) after the $d$-electron peaks were removed, the aspherical peaks due to electron configuration and vibration of Cu$^{2+}$ seem to be separated well.

Figure 3. Difference density after spherical-atom refinement on (a) (001) and (b) (220) planes passing Cu$^{2+}$ at (1/2,0,0). That after $d$-orbital analysis on (c) (001) and (d) (220). Contours are the same as those in Fig. 2.

Figure 4. Difference density after AHV analysis. Contours are the same as those in Fig. 2.

5.2. 3d orbitals in $\overline{1}$ crystal field of Cu(daco)$_2$(NO$_3$)$_2$

The Cu$^{2+}$ ion ($3d^9$) of Cu(daco)$_2$(NO$_3$)$_2$(daco: 1,5-diazacyclooctane) is on the centre of symmetry and forms a coordination plane with the four N atoms in daco (Hoshino, et al., 1989). Difference densities on and perpendicular to the Cu-N$_4$ plane after the spherical-atom refinement are shown in Fig. 5(a) and (b). Negative four peaks in Fig. 5 (a) near Cu$^{2+}$ correspond to a 3d-hole. However, the peaks do not locate on the Cu-N bonds. Since the crystal field of the Cu atom is $C_s$, $d$-orbitals of Cu$^{2+}$ are expressed as a linear combination of $d_{x^2-y^2}, d_{z^2}, d_{yz}, d_{zx}$, and $d_{xy}$ orbitals and all the 25 coefficients $c_{ik}$ were refined with the
Since approximate $c_k$'s are necessary for the non-linear least-squares refinement, they are calculated putting a point charge on each atom (Kamimura et al., 1969). $\langle j_k \rangle_{3d}$ of Cu$^{2+}$ were taken from International Tables for X-ray crystallography (1974, vol. IV).

After each cycle of refinement, new set of coefficients were orthonormalized by the Löwdin’s method (Löwdin, 1950). Refinement was converged and the coefficients $c_k$'s, population $n_i$ and $\kappa_i$ are listed in Table 1. The number of significant $c_k$'s is seven among 25 coefficients. Orbital 1 in Table 1 is the hole orbital. It is not a pure $d_{x^2-y^2}$ but a mixed orbital of $d_{x^2-y^2}$ and $d_{z^2}$ orbitals, $c_{14}$=0.57(59) is not significant. The hole and large negative and positive peaks in Fig. 5 were removed in Fig. 6. Further AHV refinement removed the peaks in Fig.6, but that prior to the d-orbital analysis did not improve Fig. 5.

**Figure 5.** Difference density after spherical-atom refinement on (a) Cu-Ni plane and (b) the plane perpendicular to it. Contours are the same as those in Fig. 2.

**Figure 6.** Difference density after 3d-orbital analysis. Contours are the same as in Fig. 2.
When G-C formalism was applied to an iron complex, it removed the peaks equally well as the multipole refinement did (Mallinson, et al., 1988). Our AHV analysis is based on the classical thermodynamics and does not have such a problem, though the p.d.f. function \( \exp\{-V(u)/k_B T\} \) is not integrable (Scheringer, 1977) at a place far from the nucleus. However, we can apply it safely as long as it is applied within the region where the sum of the third and fourth terms in (18) is much smaller than \( k_B T \). The condition is usually fulfilled in the investigations of EDD.

\[
\begin{array}{ccccccc}
i & n_l & \kappa_i & d_{x^2-y^2} & d_{z^2} & d_{\phi^2} & d_{\theta^2} \\
1 & 1.0 & 0.97(11) & 0.75(34) & 0.21(59) & 0.23(216) & 0.57(59) & 0.10(79) \\
2 & 2.0 & 1.37(7) & 0.08(28) & 0.12(17) & -0.19(19) & 0.10(26) & -0.97(6) \\
3 & 2.0 & 0.81(5) & 0.63(22) & -0.15(52) & -0.44(44) & -0.61(34) & 0.06(22) \\
4 & 2.0 & 0.71(4) & 0.04(32) & -0.93(13) & -0.10(29) & 0.34(41) & -0.06(10) \\
5 & 2.0 & 0.90(5) & 0.15(76) & -0.22(45) & 0.84(43) & -0.42(85) & -0.22(15) \\
\end{array}
\]

Table 1. 3d-orbital parameters \( c_i \)'s of Cu\(^{2+}\). Significant ones in thick lines

### 6. 4f-EDD in rare-earth hexa-borides

4f-EDD analysis has become more and more important since many rare-earth compounds with interesting physical properties, such as high-temperature superconductors, have been found. The EDD of rare-earth hexa-borides were investigated since they are famous for their properties related to the Kondo effect.

#### 6.1. XAO analysis of CeB\(_6\)

Ce and B atoms are at the body-centre and on the edges of the cubic unit cell with the B\(_6\) regular octahedra at the corners as shown in Fig. 7. Ce is in an \( O_h \) crystal field and B atoms are on 4-fold axes. CeB\(_6\) is a typical dense Kondo material with Kondo-temperature \( T_K = 2 \) K. The magnetic moment decreases with that of temperature and vanishes below \( T_K \). However, Kondo effect starts above room temperature (Onuki et al. 1984). Thus, change of EDD below room temperature is interesting although our diffractometer did not permit to measure below 2K. Intensities were measured at 100, 165, 230 and 298 K aiming to separate the two kinds of asphericity of EDD stated in 5.1 and 5.2 (Tanaka & Onuki, 2002). MD was avoided by \( \phi \)-scan with the program \( IUANGL \) (Tanaka et al., 1994). Crystal was shaped into a sphere with a radius 36 \( \mu \)m to reduce absorption, extinction and MD.

![Figure 7. Crystal structure of CeB\(_6\)](image-url)
Spin-orbit interaction was introduced into the XAO analysis. Four-fold degenerate $4f_{5/2}$ orbitals were taken from Table 4 (b) of I, as,

$$
\psi_{1}^{4f} = \frac{5}{6}\psi_{1} + \frac{1}{6}\psi_{5}, \psi_{2}^{4f} = \frac{1}{6}\psi_{2} + \frac{5}{6}\psi_{6}, \psi_{3}^{4f} = \psi_{3}, \psi_{4}^{4f} = \psi_{4},
$$

where $\psi_{i}$ (i=1 to 6) are basis functions in Table 1(c) of I. In the similar way $5p_{3/2}, 5p_{1/2}$ and $4f_{7/2}, 4f_{5/2}$ orbitals were introduced into the XAO analysis.

![Figure 8](image)

**Figure 8.** Difference density on (100) plane around Ce at (a) 100 K, (b) 165 K, (c) 230 K, (d) 298 K. Contours are the same as those in Fig. 2.

| Temp. | Ce | $\eta(\Gamma_{1})$ | $\kappa(\Gamma_{1})$ | $\eta(5p_{3/2})$ | $\eta(5p_{1/2})$ | $\kappa(2p_{x})$ | $\kappa(2p_{y})$ | Ce-B |
|-------|----|-------------------|-------------------|------------------|------------------|------------------|------------------|------|
| 100K  | 13(1) | 179(8) | 95(2) | 47(19) | -131(48) | 27(6) | 103(8) | 93(31) | 92(12) | 3.0416(3) |
| 165K  | 15(2) | 158(12) | 98(2) | -14(6) | 39(15) | 52(4) | 52(8) | 86(16) | 108(11) | 3.0440(4) |
| 230K  | 19(2) | 157(11) | 100(2) | 0(4) | -1(11) | 69(4) | 16(8) | 51(19) | 130(18) | 3.0459(7) |
| 298K  | 21(2) | 160(10) | 102(2) | 6(2) | -13(5) | 67(4) | 19(8) | 54(17) | 130(13) | 3.0439(4) |

*Orbital and AHV parameters. $n(X)$ and $\kappa(X)$ are a number of electrons and $\kappa$ of the orbital X. $n(x10^{2})$, $\kappa(x10^{2})$, $\eta_{II}$ (x10^{-19} J A^{2}), $5p_{3/2,1/2}$ and $2s$ are fully occupied. $n(2p_{x})=n(2p_{y})$, $\kappa(2p_{x})=\kappa(2p_{y})$. Unit of Ce-B is Å. $n(\Gamma_{7})$ is 4.2(3.6)x10^{-2} only at 298 K.

6.1.1. XAO analysis of CeB$_{6}$ below room temperature

After spherical-atom refinement for ions, Ce$^{3+}$ and B$^{0.5}$, the populations of the orbitals were refined keeping the sum of them equal to that of nuclear charges. When populations of them exceeded one/two or became negative, they were fixed to one/two or zero. In our program
QNTAO (Tanaka, 2000) each sub-shell is treated as a pseudo-atom sharing atomic parameters with the other sub-shells. It enables us to analyze the EDD of non-stoichiometric atoms, since valence electrons can be treated independently from the core electrons. Basis functions in Table 1 of I are automatically assigned to each AO labelled with $2l+1$ and the number of basis functions. Since high symmetry fixes $c^e$s in (1), the other orbital parameters and atomic parameters, including AHV were refined. Electron density around Ce on (001) after spherical-atom refinement is shown in Fig. 8. The peak heights around Ce increase from 0.6, 1.2 to 2.0 eÅ$^{-3}$ on lowering the temperature to 165 K and reduce to 0.6 eÅ$^{-3}$ at 100 K. After XAO analysis they were removed, exhibiting they were due to the 4f-electrons. Parameters of AO's and AHV are listed in Table 2. No electrons were found in 4f$_{7/2}$ orbitals but B-2s and Ce-5p are always fully occupied. $n(4f_{5/2}\Gamma_7)$ has value 0.042(36) only at 298 K but 4f$_{5/2}\Gamma_8$ is occupied at the four temperatures. It agrees with the previous experiments (Zirngiebl et al., 1984;Sato et al., 1984). Shorter Ce-B at 298 K than that at 230 K seems to be correlated to 4f$_{5/2}\Gamma_7$. It extends along $\langle 111 \rangle$ or to the centre of B$_6$ octahedron making them extend to reduce electrostatic repulsion. Expanded 2p$_x$ still has electrons. Total number of 4f-electrons mainly composed of 4f$_{5/2}\Gamma_8$ are 0.98(11), 0.77(8), 0.61(7) and 0.52(6) at 298, 230, 165 and 100 K. They decrease with temperature. Table 2 tells 2p$_{x}$ and 4f electrons are transferred to 2p$_{z}$, main contributor to the shortest covalent B-B bonds between B$_6$ octahedra (called as B-B$_{out}$).

Electron accumulation at B-B$_{out}$ enhances $q_{1111}$ and $q_{1122}$ of Ce at 165 K. They change signs and become more larger at 100 K. Negative and positive $q_{1111}$ at 165 K and 100K indicates that vibration of Ce at body-centre to the face-centre is favorable at 165 K, and becomes unfavorable at 100 K. This is the reason for the enhanced peak in Fig. 8(b) and reduced one in (a). Negative $q_{1122}$ at 100 K indicates the attractive force to the edge centre increased by the accumulated 2p$_{z}$ electrons. Since sum of the mean radii of B-2p and Ce-5p in free space, 2.2048+1.7947=3.9995 Å (Mann, 1968) is longer than Ce-B distance in Table 2, a slight expansion/contraction of them affects the potential seriously. Since the 5p orbitals are fully occupied, EDD's of them are spherical. B-2p$_{z}$ is closer than 2p$_{z}$ to the spherically distributed Ce-5p$_{3/2}$ electrons. $d(5p_{3/2})$ exhibits slight expansion on lowering the temperature. Expansion and contraction of 5p$_{3/2}$ and 4f$_{5/2}\Gamma_7$ in Table 2 reduces the potential of 4f orbitals lying closer to the nucleus. However, it contradicts to the decrease of 4f population. It indicates that the system resists against losing 4f electrons from Ce by making the potential of Ce more stable. It reminds us of the Le Chatelier’s law: the system resists against the change. Then why are 4f electrons allowed to flow out of Ce in spite of the reduced potential energy of the $\Gamma_8$ state? The enhanced AHV at 165 and 100 K produced new ways of vibration along $\langle 100 \rangle$ and $\langle 110 \rangle$ directions increasing the ways to attain the energy of the system. Therefore, the enhanced AHV at lower temperatures means an increase in entropy. Since the electron transfer itself from Ce to (B-B)$_{out}$ increases entropy, it cannot be stopped.

The XAO analysis of CeB$_6$ was applied to the weak-field model. The crystal structure, EDD, electron populations, expansion/contraction parameters and AHV parameters were quite consistent with each other at different temperatures. Therefore, the 4f-EDD is concluded to be measured and analyzed successfully.
6.1.2. 4f population inversion and fully occupied 5d states at 430 K

The electron population at 298 K exhibited slightly occupied \(4f_{5/2}^{7}\). Since the energy gap between \(4f_{5/2}^{7}\) and \(4f_{5/2}^{8}\) was reported to be 530-560 K (Loewenhaupt et al., 1985; Zirngiebl et al., 1984), XAO analysis at 430 K was performed to observe more electrons in \(4f_{5/2}^{8}\) (Makita et al., 2007). For details of the high-temperature diffraction equipment, see PhD thesis of Makita (Makita, 2008). Scattering factors of Ce were evaluated from relativistic AO’s calculated by the program GRASP (Dyall et al., 1989). Difference density around Ce on (001) after the spherical-atom refinement is shown in Fig. 9(a). The 4f-peaks closest to Ce elongate along <110> in contrast to those below room temperature in Fig. 8. They are surrounded by the four peaks of 0.65 eÅ\(^{-3}\), which extend along <100>, and disappeared after the analysis of the 4f peaks. However, the population of \(4f_{5/2}^{8}\) and \(4f_{5/2}^{7}\) were 0.06(3) and 0.36(11). Higher temperature reverses the populations of \(4f_{5/2}^{8}\) and \(4f_{5/2}^{7}\). The peaks outside the 4f-peaks, called 5d peaks, remained almost unchanged. Accordingly \(5d_{5/2}^{8}\) and \(5d_{5/2}^{7}\) orbitals were further refined. The population of \(5d_{5/2}^{8}\), \(n(5d_{5/2}^{8})\), exceeded 1.0 and was fixed. \(n(4f_{5/2}^{8}\) and \(5d_{5/2}^{8})\) were 0.06(2), 0.37(1) and 1.0, while \(5d_{5/2}^{7}\) was vacant. R factor was reduced from 1.25 to 1.16 %. The peaks outside the 4f peaks reduced slightly from 0.65 to 0.59 eÅ\(^{-3}\) in Fig. 10(b). Since 5d orbitals extend in a large area, in contrast to 4f orbitals, slight change of EDD is significant. Thus electron densities around 5d–peaks in Figs. 9 (a) and (b) were numerically integrated to give 3.41 and 1.57 electrons. XAO analysis explained 54 % of the 5d-peaks. Therefore \(5d_{5/2}^{8}\) is concluded to be occupied, though peaks still remained. It may be ascribed to the inaccurate 5d-AO’s (Claiser et al., 2004).

Why are 5d orbitals occupied? The energy level of the Ce-5d and B-2p calculated by Liberman et al., (1971) and Mann (1967) are -0.63 and -0.62 a.u. They are very close compared to the Ce-4f level at -0.75 a.u. Thus B-2p electrons are transferred to 5d levels predominantly according to the first-order perturbation theory. The radial distribution functions of the relevant AO’s are illustrated in Fig. 10. Putting the origin of B at \(r=3.045\) Å, 2p was drawn to the reverse side. 5d and 2p overlap well, and it is expected that 2p electrons which distribute all over the crystal through the network of B-B covalent bonds can be transferred most probably to 5d orbitals. Electrons traveling in the crystal are expected to stay at 5d-AO’s orbitals when they come close to Ce.
Figure 10. Radial distribution functions of Ca and B.

Figure 11. $4f\Gamma_8$ (red) and $5d\Gamma_8$ orbitals.

Figure 12. $5d$- and $4f$-energy levels expected from the electron populations.
Why does $4f_{5/2}^8 - 4f_{7/2}^8$ inversion occur? $5d_{5/2}^3$ orbitals are located outside of $4f_{5/2}^8$ having exactly the same symmetry (Fig. 11). Since $5d_{5/2}^3$ is fully occupied, the potential of the $4f_{5/2}^8$ orbital is enhanced. It is the reason for the population inversion of $4f_{5/2}^8$ and $f^7$. The energy level diagram expected from the electron populations obtained by the XAO analysis is shown in Fig.12. Since the quantization axes are defined parallel to $<100>$, $T_{2g}$ states locate higher than $E_g$, $5d_{5/2}^3$ orbital closely related to $T_{2g}$ is located highest among the 5d orbitals as shown in Fig. 12. From the populations obtained, 2p electrons seem to be transferred to $5d_{5/2}^3$ orbitals, directly or first to $5d_{3/2}^3$ and then to $5d_{5/2}^3$.

6.1.3. Electron configuration at 340 and 535 K

In order to confirm the 5d-occupation in CeB$_6$ at 430 K, XAO analyses were done at 340 and 535 K. The difference densities, as well as electron configurations, are different from each other as shown in Fig. 13. Since the electrons are continually transferred between 2p and 5d orbitals, the crystal field seems to change with temperature resulting in different directions of the 4f and 5d peaks in Fig. 13. However, it is confirmed that 5d orbitals in CeB$_6$ are occupied above room temperature (Makita et al., 2008).

Figure 13. Bird-eye view of difference densities around Ce at (a) 340, (b) 430 and (c) 535 K. Contours as in Fig. 9 except the interval of 0.1 eÅ$^{-3}$. Frames are parallel to $<100>$ axes.

6.2. XAO analysis of SmB$_6$

SmB$_6$ formally has five 4f electrons. In order to extend XAO analysis to multi 4f-electron system, EDD of SmB$_6$ was measured at 100, 165, 230 and 298 K (Funahashi, et al., 2010). It is interesting to see how physical properties as a Kondo insulator are explained by the XAO analysis. The shadow in the difference density at 230 K in Fig. 14 specifies roughly the area of 5d peaks. $5p_{3/2}$, $4f_{5/2}^3$, $f^7$ and AHV were refined reducing the 4f peaks in Fig. 14(a). However, since 4f-peaks still remained along $<100>$, the populations of $4f_{5/2}^3$, $f^7$, $f^6$ were refined but only $f^6$ orbital which extends along $<100>$ more sharply than $4f_{5/2}^3$ survived. Since peaks (peak A) remained in the 5d-area, $5d_{5/2}^3$, $f^7$ and $5d_{5/2}^3$ were added in the refinement and only $5d_{5/2}^3$ orbital, which stems from $e_g$ orbital of the storing field model had population. Fig. 14(b) exhibits that peak A is reduced from 0.43 to 0.17 eÅ$^{-3}$ and 4f-peaks are almost deleted. Final parameters are listed in Table 3.
The electron configuration in Table 3 is correlated to the physical properties of SmB$_6$ as follows: (a) SmB$_6$ is a Kondo insulator. Its electric resistivity increases gradually like semiconductors below room temperature and begins to increase steeply below 30 K with a decrease in temperature. It also begins to increase like metals above room temperature (Ueda & Onuki, 1998). 2$p$ and 5$d_{5/2}$ orbitals consist of the conduction band (Kimura et al., 1990). Since B-2$p$ extends along the edge, it does not overlap with 5$d_{5/2}$ effectively and does not seem to contribute to the band. The population of 2$p$ in Table 3 as well as the resistivity steadily increases on lowering temperature. $n(5d_{5/2})$ also increases from 230 K. The increase of populations indicates that of localized electrons. It may be correlated to the increase of the resistivity. (b) 4$f_{7/2}$ are vacant only at 100 K, although they are occupied at the other three temperatures. It may be correlated to the band gap between the 4$f$ states, which is reported to start developing between 150 and 100 K (Souma, et al., 2002). (c) Among the 5$d$ orbitals only the 5$d_{5/2}$ are occupied. Since 5$d_{5/2}$ orbitals correspond to e$_g$ in the strong field model as illustrated in Fig. 1, it agrees with the band calculation of LaB$_6$ by Harima (1988), reporting that 5$d$-e$_g$ and 2$p$ of B consist of the conduction band.

| Temperature (K) | $n(4f_{5/2} \Gamma_8)$ | $n(4f_{7/2} \Gamma_6)$ | $n(5d_{5/2} \Gamma_8)$ | $n(2p_x)$ | $n(2p_z)$ |
|----------------|------------------------|------------------------|------------------------|----------|----------|
| 100K          | 0.96(9)                | 0.48(12)               | 0.0                    | 1.0      | 0.21(8)  | 0.45(17) |
| 165K          | 1.0                    | 0.67(13)               | 0.44(12)               | 0.75(24) | 0.16(13) | 0.55(39) |
| 230K          | 1.0                    | 0.61(13)               | 0.59(13)               | 0.56(16) | 0.13(7)  | 0.66(28) |
| 298K          | 0.77(3)                | 0.59(7)                | 0.47(6)                | 0.78(6)  | 0.07(3)  | 0.81(12) |

Table 3. Temperature dependence of electron populations of Sm 4$f$, 5$d$ and B 2$p$ orbitals.

7. Bright future for X-ray crystallography

EDD investigation was limited up to 3$d$-transition-metal complexes. However, XAO analysis made EDD investigations of rare-earth compounds as well as non-stoichiometric ones possible. Its application to organic compounds can be attained when it is developed to X-ray crystallography.
molecular orbital analysis (XMO). Since the least-squares method stated in 2.2 was formulated for MO models, XMO analysis will be accomplished in a near future. When actinoid compounds become our targets, however, MD-effect is expected to be so much that the avoidance of it by $\varphi$ – rotation is impossible and the correction for it will be inevitable.

CeB$_6$ is a possible quantum-material to emit UV light when electrons in $5d_{5/2}$ could be transferred to $4f_{5/2}$, as the investigation at 430 K revealed. The d-f transition is a permitted one by quantum mechanics. Since the 5d-occupation is found in the ground state of CeB$_6$, some external force is necessary to make the transition occur. A electron populations in CeB$_6$ and SmB$_6$ found by the XAO analysis demonstrate the importance of the EDD analysis based on quantum-mechanical orbitals.

As discussed in 5.3, the aspherical properties of EDD and AHV are separated better by the method based on classical Boltzmann statistics than the G-C method. However, recent development of neutron diffraction will make it possible to get intrinsic ADP's and use them as known parameters in X-ray EDD analyses. It will improve XAO analysis of rare-earth complexes and makes the XMO analysis surer and easier.

The accuracy of X-ray structure-factor measurements has been improved so much that every crystallographer will investigate EDD easily as a part of their X-ray structure analysis. The top-up operation with constant incident beam intensity at SR facilities has improved the accuracy of the structure factor measurements from 1 % to 0.1 %. Future of X-ray diffraction is bright.

8. References

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