Machine Learning at Wireless Edge with OFDM and Low Resolution ADC and DAC

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Abstract

We study collaborative machine learning (ML) systems where a massive dataset is distributed across independent workers which compute their local gradient estimates based on their own datasets. Workers send their estimates through a multipath fading multiple access channel (MAC) with orthogonal frequency division multiplexing (OFDM) to mitigate the frequency selectivity of the channel. We assume that the parameter server (PS) employs multiple antennas to align the received signals with no channel state information (CSI) at the workers. To reduce the power consumption and the hardware costs, we employ complex-valued low resolution digital to analog converters (DACs) and analog to digital converters (ADCs), respectively, at the transmitter and the receiver sides to study the effects of practical low cost DACs and ADCs on the learning performance of the system. Our theoretical analysis shows that the impairments caused by low-resolution DACs and ADCs, including the extreme case of one-bit DACs and ADCs, do not prevent the convergence of the learning algorithm, and the multipath channel effects vanish when a sufficient number of antennas are used at the PS. We also validate our theoretical results via simulations, and demonstrate that using low-resolution, even one-bit, DACs and ADCs causes only a slight decrease in the learning accuracy.

Index Terms

Distributed machine learning, federated learning, stochastic gradient descent, wireless channels, OFDM, low-resolution DAC and ADC, one-bit DAC and ADC.

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I. INTRODUCTION

The rapid growth of data sensing and collection capabilities of computation devices facilitates the use of massive datasets enabling machine learning (ML) systems to make more intelligent decisions than ever. However, this growth makes processing of all the data in a central processor troublesome due to energy inefficiency and privacy concerns. Recently, as an alternative to using a central processor, performing the ML task in a distributed manner, called federated learning, where each device connected to the central processor performs the task on its local dataset has drawn significant attention [1], [2].

Different aspects of federated learning is studied in recent literature. In [3], digital and analog distributed stochastic gradient descent (D-DSGD and A-DSGD) algorithms over a Gaussian multiple-access channel (MAC) are proposed where the authors use the superposition property of the MAC to recover the mean of the local gradients computed at remote workers. In D-DSGD, workers digitally compress their locally computed gradients into a finite number bits while in A-DSGD workers use an analog compression similar to what is done in compressed sensing to obey the bandwidth limitations over wireless channels. In [4] and [5], the channel between the PS and the workers is modeled as a fading MAC. Ref. [4] performs power allocation among the gradients to schedule workers according to their channel state information (CSI), and shows that the latency reduction of the proposed method scales linearly with the device population. Ref. [5] proposes a gradient sparsification method which is followed by a compressive sensing (CS) to reduce the dimension of the large parameter vector. By reducing the dimensionality of the gradients and designing a power allocation scheme, the authors obtain significant improvements in the performance compared to the existing schemes.

In addition to the studies that decrease the communication load, [6] considers transmission energy due to the limited energy budget of workers for local computations. Hence, the authors formulate the joint learning/computation and communication process as an optimization problem to minimize the total sum of the completion time of the federated learning by taking into consideration the trade-off between the energy consumption and latency. In [7], the authors focus on the minimization of the convergence time of a federated learning system by jointly considering user selection and resource allocation. The aim of the PS is to include as many workers as possible into the learning process for convergence to the global model with limited resources. Hence, the authors design a probabilistic user selection algorithm via an optimization
problem to minimize the convergence time by jointly considering user selection and resource allocation. Furthermore, there are several studies on data exchange rate reduction by quantization on federated learning systems [8]–[11]. Specifically, in [11], the authors introduce lossy federated learning (LFL) system which directly quantizes both the global model and the local model parameters to reduce the communication loss without employing any modulation scheme. They show that the convergence of the learning algorithm is guaranteed despite the quantization of global and local models. When the training data is randomly split to the workers, FLF with small quantization levels performs as good as a system with infinite resolution.

In this paper, our main objective is to study distributed learning algorithms over wireless channels in more realistic settings considering practical implementation issues including the wireless channel effects. We model the communication link as a frequency selective fading channel, and transmit the local gradients using orthogonal frequency division multiplexing (OFDM). Furthermore, in an effort to reduce the hardware complexity and power consumption, we employ low-resolution, digital to analog converters (DAC) at the transmitter side (at each worker), and analog to digital converters (ADCs) at the receiver side which employs multiple (even a massive number of) receive antennas. In fact, this is nothing but the over-the-air machine learning, except that here we consider the effects of the wireless medium as well as the use of low resolution DACs and ADCs. While OFDM transmission with low resolution ADCs and DACs has been studied from a communication theory perspective in the literature (e.g., [12]–[19]), this is the first work on federated learning where the parameters are transmitted through OFDM taking into account the practical limitations and realistic wireless channel effects.

The main contributions of our work on distributed machine learning over wireless channels can be summarized as follows:

- Different from previous works regarding federated learning ([3]–[5], [8]–[11], [20]), we consider a more realistic channel scenario where the channel between the workers and PS is modeled as multipath fading MAC channel.
- To cope with the realistic channel impairments, we transmit the local gradients using OFDM. Further, use cyclic prefix (CP) addition to mitigate the ISI caused by the multipath channel. Thus, different from [11], we consider the transmission and reception of actual OFDM signals, not gradients directly, as would be necessitated in a practical implementation.
- Since one of our main concerns is practical implementation of distributed learning systems, we also employ low-resolution DACs and ADCs separately at the workers and the PS
side, respectively. Also, we extend our studies to the case of a system which utilizes both low-resolution DACs and ADCs.

- Via both theoretical analysis and extensive simulations, we confirm that the effect of imperfections due to finite resolution DAC and/or ADC can be alleviated using sufficient number of receive antennas at the PS, and the convergence of the distributed learning algorithm is guaranteed even if we employ low cost DACs and/or ADCs.

The paper is organized as follows. Section II introduces the system model and preliminaries. DSGD with low-resolution DACs is analyzed in Section III, and the effect of low-resolution ADCs at the receiver side is studied in Section IV, respectively. The joint utilization of low-resolution DACs and ADCs are considered in Section V whose results are also valid for extreme one-bit case. Performance of distributed machine learning systems over fading multipath channels with OFDM and finite resolution DACs and ADCs is studied via simulations in Section VI, and the paper is concluded in Section VII.

**Notation:** Throughout this paper, the real and imaginary parts of $x \in \mathbb{C}$ are represented by $x^R$ and $x^I$, respectively. We use the notation $[a \ b]$ to indicate the integer set \{a, \ldots, b\} where $a \leq b$, $a$ and $b$ are positive integers, and $[b] = [1 \ b]$. We denote $l_2$ norm of a vector $x$ by $||x||_2$. The entry in the $i$-th row and $j$-th column of a matrix $A$ is denoted by $A[i,j]$. $N$-point Discrete Fourier Transform (DFT) of vector $x \in \mathbb{R}^N$ is defined as

$$X[u] = \sum_{n=1}^{N} x[n] e^{-j2\pi nu/N}. \quad (1)$$

while the $N$-point inverse discrete Fourier Transform (IDFT) of vector $X \in \mathbb{R}^N$ is given by

$$x[n] = \frac{1}{N} \sum_{u=1}^{N} X[u] e^{j2\pi nu/N}. \quad (2)$$

**II. System Model**

We consider a distributed ML system where each worker calculates its gradient estimate and sends the signal to a central PS through a multipath fading MAC with OFDM as illustrated in Fig.1. At the receiver side, OFDM demodulation, signal combining and global model parameter update are performed, and the global parameter is broadcast to the workers over an error-free link. We assume that there is no transmit side CSI, and that the PS employs multiple antennas to recover the average of the workers’ gradients. With the use of higher number of workers and
many antennas, a significant amount of power at the transmitter and receiver is consumed by the DACs and ADCs [21]. As the power consumption of DACs and ADCs increases linearly, and their hardware cost increases exponentially with the number of quantization bits [22], we consider a distributed learning system where the transmitters and receivers are equipped with low-resolution, even one-bit, DACs and ADCs, respectively, to keep the implementation cost and power consumption small.

In the distributed learning at the wireless edge setup, we jointly train a learning model by using iterative stochastic gradient descent (SGD) to minimize a loss function $f(\cdot)$. During the $t$-th iteration, worker $m \in [M]$ calculates the gradient estimate $g^t_m \in \mathbb{R}^d$ by processing its local dataset $B_m$ according to $\frac{1}{|B_m|} \sum_{u \in B_m} \nabla f(\theta_t, u)$ where $\theta_t \in \mathbb{R}^d$ is the vector of model parameters, $d$ is the number of model parameters, and $g^t_m[n]$ represents the $n$-th entry of the gradient estimate vector.

Since the local gradient vector has real components, we obtain the frequency domain representation of the gradients as

$$
\hat{g}^t_m = \left[ g^t_m[1] + jg^t_m[e + 1], g^t_m[2] + jg^t_m[e + 2], \cdots, g^t_m[e] + jg^t_m[2e] \right],
$$

where $e = \lfloor d/2 \rfloor$, $\hat{g}^t_m \in \mathbb{R}^e$, and $g^t_m[2e]$ is assigned as zero if $d \equiv 1 \pmod{2}$. Then, the first step is to form the OFDM signal by taking an $N$-point inverse discrete Fourier Transform (IDFT) of
the gradient vector as

\[ G'_m[u] = \frac{1}{N} \sum_{n=1}^{N} \hat{g}'_m[n] e^{j2\pi nu/N}, \]

(4)

for \( u \in [N] \). If \( e < N \), \( \hat{g}'_m[n] = 0 \) for \( n > e \), i.e., \( \hat{g}'_m \) is zero padded.

The channel between the worker \( m \) and the \( k \)-th antenna of the PS is modeled as a (wireless) multipath MAC. We assume that the channel does not change during the transmission of one OFDM word, while it may be different for different OFDM words. The impulse response of the channel is

\[ h'_{mk}[n] = \sum_{l=1}^{L} h'_{mk\ell} \delta[n - \tau_{mk\ell}], \]

(5)

where \( n \in [N + N_{cp}] \), \( L \) is the number of channel taps, \( \tau_{mk\ell} \) is the time delay and \( h'_{mk\ell} \in \mathbb{C} \) is the gain of the \( l \)-th channel tap from the \( m \)-th worker to the \( k \)-th antenna of the PS. Note that this is nothing but the machine learning over-the-air framework. We assume that \( h'_{mk\ell} \) are zero-mean complex Gaussian with \( \mathbb{E}[(h'_{mk\ell}) \cdot (h'_{mk'\ell'})^*] = 0 \) for \( (m, k, l) \neq (m', k', l') \), and \( \mathbb{E}[(h'_{mk\ell})^2] = \sigma^2_{h,k} \), i.e., all the channel taps experience Rayleigh fading.

To mitigate the ISI caused by the multipath channel, CP addition is performed by

\[ \bar{G}'_m = [G'_m[N - N_{cp} + 1] \ldots G'_m[N] \ G'_m[1] \ldots G'_m[N]], \]

(6)

where \( \bar{G}'_m \in \mathbb{C}^{N+N_{cp}} \) is the OFDM word to be transmitted by the \( m \)-th worker. The CP length \( N_{cp} \) is chosen to be greater than the delay spread of all the channels. The resulting (depending on the setup – quantized or full resolution) OFDM words are transmitted to the PS which are equipped with \( K \) receive antennas. The PS uses the received signal to update the model and sends it back to all the receivers over an error-free link.

At the \( k \)-th receive chain, after removing the CP, the \( n \)-th entry of the received vector at the input of the \( k \)-th receive antenna during iteration \( t \) is written as

\[ Y'_k[n] = \sum_{m=1}^{M} \sum_{l=1}^{L} h'_{mk\ell} G'_m[n - \tau_{mk\ell}] + z'_k[n], \]

(7)

where we model the communication link as a frequency selective fading channel whose impulse response is given in (5), and \( G'_m[n] \) is the transmitted signal by the \( m \)-th worker. The additive noise terms \( z'_k[n] \) are independent and identically distributed (i.i.d.) circularly symmetric zero mean complex Gaussian random variables, i.e., \( z'_k[n] \sim \mathcal{CN}(0, \sigma^2_z) \) for \( k \in [K] \).
Ideally, the PS updates the model parameter according to $\theta_{t+1} = \theta_t - \mu_t \frac{1}{M} \sum_{m=1}^{M} g_t^m$, and it is shared with the workers. However, in our setup, the local gradients are not available at the PS, instead the PS uses noisy and corrupted version (by low resolution DAC and/or ADCs) of the local gradients to recover the estimate of the gradient vector as will become apparent in the subsequent sections. In the subsequent sections, we drop the subscripts referring to iteration count $t$ for ease of exposition.

III. DSGD WITH LOW RESOLUTION DACS AT THE WORKERS

In this section, we study the effects of employing low-resolution DACs at the workers on the distributed learning process in an effort to reduce the hardware complexity and power consumption.

After constructing the OFDM word corresponding to the gradient vectors, a complex-valued low-resolution DAC is employed to generate the transmitted signal at each worker. A $b$-bit complex-valued DAC consists of two parallel real-valued DACs with quantization function $Q_b(\cdot)$ that independently quantizes the real and imaginary parts into $\beta = 2^b$ reconstruction levels. The reconstruction levels are denoted by $\hat{a} = [\hat{a}_1 \hat{a}_2 \cdots \hat{a}_\beta] \in \mathbb{R}^\beta$ while the boundaries of the quantization regions are denoted by $\hat{x} = [\hat{x}_1 \hat{x}_2 \cdots \hat{x}_{\beta+1}] \in \mathbb{R}^{\beta+1}$ where $\hat{x}_1 = -\infty$ and $\hat{x}_{\beta+1} = +\infty$ for convenience. Also, we have, $\hat{a}_i < \hat{a}_j$, if $1 \leq i < j \leq \beta$, $\hat{x}_i < \hat{x}_j$ if $1 \leq i < j \leq \beta + 1$, and $\hat{x}_i \leq \hat{a}_j < \hat{x}_k$ if $1 \leq i \leq j < k \leq \beta + 1$. The corresponding real valued quantizer is $Q_b(z) = \hat{a}_i$ for $\hat{x}_i \leq z < \hat{x}_{i+1}$, $i \in [\beta]$, $z \in \mathbb{R}$. The complex-valued DAC operation can be expressed as $Q_b(x) = Q_b(x^R) + jQ_b(x^I)$. We assume that the quantizer output is chosen such that $Q_b(x) = \mathbb{E}[x|Q_b(x)]$, i.e., for each quantization region the reconstruction level is selected to minimize the mean squared error. The corresponding signal to quantization noise ratio (SQNR) of the input vector $x$ is calculated as

$$\text{SQNR} = \frac{\mathbb{E}[|x|^2]}{\mathbb{E}[|Q_b(x) - x|^2]} = 1 - \frac{\mathbb{E}[Q_b(x)x^*]}{\mathbb{E}[|Q_b(x) - x|^2]}.$$  

We model the OFDM words as wide-sense stationary (WSS) Gaussian processes based on an argument similar to the one made in [23]. That is, if the input data which forms the OFDM word is i.i.d. and bounded, the convex envelope of the OFDM word weakly converges to a Gaussian random process as the number of subcarriers goes to infinity through an application of central limit theorem (CLT). Similarly, if we assume that the elements of the gradient vector in the learning process are i.i.d. and bounded, then the real and imaginary parts of the baseband
Fig. 2: Histogram of the real and imaginary parts of the OFDM word.

Table I: Distortion factors with different quantization levels \[26\], \[27\].

| Number of bits | Distortion factor ($\eta$) |
|----------------|--------------------------|
| 1              | 0.3634                   |
| 2              | 0.1175                   |
| 3              | 0.03454                  |
| 4              | 0.009497                 |
| 5              | 0.002499                 |

OFDM word obtained from the gradient vector can be modeled as independent zero-mean stationary Gaussian processes. As a verification, we examine histograms of several OFDM word samples obtained by a certain learning task with our setup, demonstrating the OFDM samples are approximately Gaussian. An instance of an exemplary histogram of the OFDM word samples obtained through the 100-th iteration is given in Fig. 2, which is consistent with our assumption. Our extensive experiments further confirm that the corresponding OFDM word samples at different time indexes have almost same variance. Note that, even if the OFDM words are not Gaussian processes, the Bussgang theorem that will be used to model the nonlinear input-output relationship for DACs and ADCs is still a good approximation as illustrated extensively in the literature, see, e.g., \[24\]-\[25\].
Denoting the autocorrelation matrix of the OFDM words by $C_{\bar{G}_m\bar{G}_m}$ with equal diagonal elements denoted by $\sigma_{\bar{G}_m}^2$, and using the Bussgang decomposition [28]-[29], we can decompose the quantized signal into two parts: the desired signal component and the quantization distortion which is independent of the desired signal. Thus, we can write the quantized signal as

$$\bar{G}_m^Q[n] = Q(G_m[n]) = (1 - \eta)G_m[n] + q_m[n],$$

(9)

where $\eta = 1/$SQNR is the distortion factor which is the inverse of SQNR, and the variance of the distortion noise is $\sigma_{q_m}^2 = \eta(1 - \eta)\sigma_{\bar{G}_m}^2$. When a unit variance Gaussian input is processed by a non-uniform scalar minimum mean-square-error quantizer, the values of corresponding distortion factors are listed in Table I [26]-[27].

At the $k$-th receive chain, after removing the CP, the $n$-th entry of the received vector is written as

$$Y_k[n] = \sum_{m=1}^{M} \sum_{l=1}^{L} h_{mk} G_m[n - \tau_{mk}],$$

(10)

$$= \sum_{m=1}^{M} \sum_{l=1}^{L} h_{mk} ((1 - \eta) \cdot G_m[n - \tau_{mk}] + q_m[n - \tau_{mk}]) + z_k[n]$$

(11)

$$= (1 - \eta) \sum_{m=1}^{M} \sum_{l=1}^{L} h_{mk} G_m[n - \tau_{mk}] + w_k[n],$$

(12)

where the total non-Gaussian noise term $w_k[n]$ has variance $\sigma_z^2 + \eta(1 - \eta)\sigma_{\bar{G}_m}^2 \sum_{m=1}^{M} \sum_{l=1}^{L} |h_{mk}|^2$.

To perform the demodulation, we take the DFT of (10) which gives

$$r_k[i] = (1 - \eta) \sum_{m=1}^{M} H_{mk}[i]g_m[i] + \sum_{m=1}^{M} H_{mk}[i]Q_m[i] + Z_k[i],$$

(13)

where $Q_m[i]$ is the DFT of the quantization distortion noise and $H_{mk}[i]$'s are the channel gains from the $m$-th worker to the $k$-th receive chain for the $i$-th subcarrier, given by

$$H_{mk}[i] = \sum_{n=0}^{N-1} h_{mk}[n]e^{-j2\pi in/N}$$

$$= \sum_{n=0}^{N-1} \left( \sum_{l=1}^{L} h_{mk} \delta[n - \tau_{mk}] \right) e^{-j2\pi in/N}$$

$$= \sum_{l=1}^{L} h_{mk} e^{-j2\pi i\tau_{mk}/N}. $$

(14)
Since the channel taps are zero mean circularly symmetric complex Gaussian (i.e., Rayleigh fading), \( H_{mk}[i] \)'s are zero-mean complex Gaussian random variables with variance \( \sigma_H^2 = \sum_{l=1}^{L} \sigma_{h,l}^2 \).

Taking DFT of the channel noise vector, \( Z_k[i] \) is evaluated as

\[
Z_k[i] = \sum_{n=0}^{N-1} z_k[n] e^{-j2\pi in/N}.
\] (15)

The noise terms are i.i.d. circularly symmetric complex Gaussian, i.e., \( Z_k[n] \sim \mathcal{CN}(0, \sigma_{Z_k}^2) \) where \( \sigma_{Z_k}^2 = N\sigma_z^2 \).

We assume that the CSI is available at the PS, hence the received signals from the \( K \) antennas can be combined to align the gradient vectors using

\[
y[i] = \frac{1}{(1 - \eta)} \cdot K \sum_{k=1}^{K} \left( \sum_{m=1}^{M} (H_{mk}[i])^* r_k[i] \right) g_m[i],
\] (16)
as in [20]. By substituting (13) into (16), we obtain

\[
y[i] = \frac{1}{K} \sum_{k=1}^{K} \sum_{m=1}^{M} |H_{mk}[i]|^2 g_m[i] \tag{17a}
\]

signal term

\[
+ \frac{1}{K} \sum_{k=1}^{K} \sum_{m=1}^{M} \sum_{m' = 1}^{M} (H_{mk}[i])^* H_{m'k}[i] g_{m'}[i] \tag{17b}
\]

interference term

\[
+ \frac{1}{(1 - \eta)K} \sum_{k=1}^{K} \sum_{m=1}^{M} \sum_{m' = 1}^{M} (H_{mk}[i])^* H_{m'k}[i] Q_{m'}[i] \tag{17c}
\]

distortion noise term

\[
+ \frac{1}{(1 - \eta)K} \sum_{k=1}^{K} \sum_{m=1}^{M} |H_{mk}[i]|^2 Q_m[i] \tag{17d}
\]

second type of distortion noise term

\[
+ \frac{1}{(1 - \eta)K} \sum_{k=1}^{K} \left( \sum_{m=1}^{M} H_{mk}[i] \right)^* Z_k[i]. \tag{17e}
\]

channel noise term

There are five different terms in (17): the signal component, interference, distortion noise term, the second type of distortion noise term, and channel noise.
To analyze the interference term (17b), we write it as a summation of \( M \) terms

\[
\frac{1}{K} \left[ \left( \sum_{k=1}^{K} \sum_{m=2}^{M} (H_{mk}[i])^* H_{1k}[i] \right) g_1[i] + \cdots + \left( \sum_{k=1}^{K} \sum_{m=1}^{M} (H_{mk}[i])^* H_{jk}[i] \right) g_j[i] + \cdots + \left( \sum_{k=1}^{K} \sum_{m=1}^{M-1} (H_{mk}[i])^* H_{Mk}[i] \right) g_M[i] \right],
\]

and consider the coefficient of each interference term \( g_j[i] \) separately.

Let us define

\[
\kappa_j[i] = \frac{1}{K} \sum_{k=1}^{K} \sum_{m=1}^{M} (H_{mk}[i])^* H_{jk}[i],
\]

for the coefficient of \( j \)-th interfering gradient \( g_j[i] \) in (17b) where \( i \in [N] \), and \( j \in [M] \). Since \( H_{mk}[i] \) and \( H_{jk}[i] \) are independent for \( j \neq m \), the mean and variance of \( \kappa_j[i] \) are calculated as

\[
\mathbb{E}[\kappa_j[i]] = 0, \quad (20a)
\]
\[
\mathbb{E}[|\kappa_j[i]|^2] = \frac{(M - 1)\sigma_H^4}{K}. \quad (20b)
\]

We have such \( M \) different interference terms in (17b) each for different interfering workers with zero mean, and its variance scales with \( M/K \). Thus, all of \( M \) interference terms approach zero as \( K \to \infty \).

To analyze the distortion noise term (17c), we define the coefficient of each uncorrelated distortion term \( Q_j[i] \) separately as in the case of (17b) by

\[
\delta_{1j}[i] = \frac{1}{(1 - \eta)K} \sum_{k=1}^{K} \sum_{m=1}^{M} (H_{mk}[i])^* H_{jk}[i],
\]

where \( i \in [N] \), and \( j \in [M] \) for all uncorrelated \( M \) terms in the summation (17c).

Similar to the analysis of \( \kappa_j[i] \), the mean and variance of \( \delta_{1j}[i] \) are calculated as

\[
\mathbb{E}[\delta_{1j}[i]] = 0, \quad (22a)
\]
\[
\mathbb{E}[|\delta_{1j}[i]|^2] = \frac{(M - 1)\sigma_H^4}{(1 - \eta)^2 K}. \quad (22b)
\]
This implies that each of \( M \) interfering terms in (17c) goes to zero if \( K \) is large enough. Thus, the detrimental effect of the distortion noise term can also be diminished by employing large number of receive antennas.

To analyze the second type of distortion noise term (17d), we consider each distortion interference \( Q_j[i] \) separately for \( j \in [M] \), and define the coefficient of the interfering distortion term caused by the \( j \)-th one as

\[
\delta_{2j}[i] = \frac{1}{(1 - \eta)K} \sum_{k=1}^{K} |H_{jk}[i]|^2,
\]

(23)

where \( i \in [N] \), and \( j \in [M] \).

The mean of \( \delta_{2j}[i] \) is

\[
\mathbb{E}[\delta_{2j}[i]] = \frac{\sigma_H^2}{(1 - \eta)}.
\]

(24)

For the variance of \( \delta_{2j}[i] \), we have

\[
\mathbb{E}[|\delta_{2j}[i]|^2] = \frac{1}{(1 - \eta)^2K^2} \sum_{k_1=1}^{K} \sum_{k_2=1}^{K} \mathbb{E}[|H_{jk_1}[i]|^4] \mathbb{E}[|H_{jk_2}[i]|^2]
\]

(25)

- If \( k_1 = k_2 \) (case 2.1)

\[
\mathbb{E}[|\delta_{2j}[i]|^2] \mid_{\text{case 2.1}} = \frac{1}{(1 - \eta)^2K^2} \sum_{k=1}^{K} \mathbb{E}[|H_{jk}[i]|^4] = \frac{1}{(1 - \eta)^2K^2} \mathbb{E}[|H_{jk}[i]|^4].
\]

(26)

- If \( k_1 \neq k_2 \) (case 2.1)

\[
\mathbb{E}[|\delta_{2j}[i]|^2] \mid_{\text{case 2.2}} = \frac{1}{(1 - \eta)^2K^2} \sum_{k_1=1}^{K} \sum_{k_2=1, k_2 \neq k_1}^{K} \mathbb{E}[|H_{jk_1}[i]|^2] \mathbb{E}[|H_{jk_2}[i]|^2]
\]

\[
= \frac{(K^2 - K)\sigma_H^4}{(1 - \eta)^2K^2}
\]

(27)

\[
\approx \frac{\sigma_H^4}{(1 - \eta)^2},
\]

(28)

for \( K \gg 1 \). Thus, the mean and variance of the second distortion term of the \( j \)-th worker is
calculated as

\[
\mathbb{E} [\delta_{2j}[i]] = \frac{\sigma_H^2}{(1 - \eta)}, \quad (29a)
\]

\[
\text{Var}(\delta_{2j}[i]) \approx \frac{1}{(1 - \eta)^2 K} \text{E} [\|H_{jk}[i]\|^4], \quad (29b)
\]

for \( K \gg 1 \) where \( \text{Var}(\delta_{2j}[i]) \) approaches zero as \( K \to \infty \) while it has a finite mean. Further, we know that the mean of distortion term, \( Q_j[i] \) for all \( j \in [M] \), is zero. Accordingly, using the law of large numbers, the summation will converge to the mean of \( Q_j[i] \), which is zero, for large enough \( M \), resulting approximately zero second type of distortion noise.

Furthermore, using the law of large numbers, as the number of antennas at the PS \( K \to \infty \), the signal term can be approximated as

\[
y_{\text{sig}}[i] = \sigma_H^2 \sum_{m=1}^{M} g_m[i]. \quad (30)
\]

Thus, with low-resolution DAC at the transmit antennas, the PS can recover the \( i \)-th entry of the desired signal

\[
\frac{1}{M} \sum_{m=1}^{M} g_m[i] = \begin{cases} 
\frac{y_R[i]}{M\sigma_H^2}, & \text{if } 1 \leq i \leq e, \\
\frac{y_I[i-e]}{M\sigma_H^2}, & \text{if } e < i \leq 2e.
\end{cases} \quad (31)
\]

This result clearly shows that the destructive effect of low-resolution DACs can be effectively alleviated using sufficient number of PS antennas. Thus, the convergence of the learning process is guaranteed even if we employ low cost low-resolution DACs at the workers which significantly reduces the cost of designing distributed learning setup with higher number of workers.

**IV. DSGD with Low Resolution ADCs at the PS**

In this section, we consider a system where the workers transmit the OFDM words corresponding to the local gradients with full-resolution through a multipath fading channel while the PS employs low-resolution ADCs at each receive antenna. The aim of the PS is to obtain an estimate for the gradient vector using quantized OFDM words received from multiple antennas. We analyze the effect of multipath fading and low-resolution ADCs at the receiver on the convergence of the distributed ML solutions where the channel model is the same as the previous section.
At each receive chain, after removing the CP, the $n$-th entry of the received OFDM word $Y_k$ is
\[ Y_k[n] = \sum_{m=1}^{M} \sum_{l=1}^{L} h_{mkl} G_m[n - \tau_{mkl}] + z_k[n]. \] (32)

The $(k, k')$-th element of the auto-correlation matrix of $Y[n] = [Y_1[n] \cdots Y_K[n]]$ received by different antennas can be written as
\[ C_{YY}[k, k'] = \mathbb{E} \left[ \sum_{m=1}^{M} \sum_{m'=1}^{M} \sum_{l=1}^{L} \sum_{l'=1}^{L} h_{mkl} h_{m'k'l'}^* G_m[n - \tau_{mkl}] G_{m'}[n - \tau_{m'k'l'}] \right] + \sigma_z^2 \mathbb{1}_{\{k=k'\}} \] (33)
\[ = \sum_{m=1}^{M} \sum_{l=1}^{L} \sum_{l'=1}^{L} h_{mkl} h_{m'k'l'}^* \mathbb{E} \left[ G_m[n - \tau_{mkl}] G_{m'}[n - \tau_{m'k'l'}] \right] + \sigma_z^2 \mathbb{1}_{\{k=k'\}} \] (34)
which is only a function of $k$ and $k'$ since the OFDM words are modeled as WSS.

Variance of the received signal at the $k$-th antenna $Y_k[n]$ is given by
\[ \sigma_{Y_k}^2 = \mathbb{E} \left[ \sum_{m=1}^{M} \sum_{m'=1}^{M} \sum_{l=1}^{L} \sum_{l'=1}^{L} h_{mkl} h_{m'k'l'}^* G_m[n - \tau_{mkl}] G_{m'}^*[n - \tau_{m'k'l'}] \right] + \sigma_z^2 \] (35)
\[ = \sum_{m=1}^{M} \sum_{l=1}^{L} \sum_{l'=1}^{L} h_{mkl} h_{m'k'l'}^* \mathbb{E} \left[ G_m[n - \tau_{mkl}] G_{m'}^*[n - \tau_{m'k'l}] \right] + \sigma_z^2, \] (36)
which only depends on $k$.

A complex-valued low-resolution ADC employed at each receive antenna performs quantization with the quantizer output denoted by $Q_b(\cdot)$. As in the case with low-resolution DACs at the workers described in the previous section, we describe $b$-bit quantization with quantization function $Q_b(\cdot)$ that independently quantizes the real and imaginary parts into $\beta = 2^b$ reconstruction levels such that the quantizer output is chosen as $Q_b(x) = \mathbb{E}[x|Q_b(x)]$.

With element-wise quantization, we can decompose the quantized signal into two parts as the desired signal component and quantization distortion which is uncorrelated with the desired signal. Analytically, we can write the quantized signal as
\[ R_k[n] = (1 - \eta_k) \left( \sum_{m=1}^{M} \sum_{l=1}^{L} h_{mkl} G_m[n - \tau_{mkl}] + z_k[n] \right) + w_q^k[n], \] (37)
where $\eta_k$ is the distortion factor which is the inverse of the SQNR due to quantization of $Y_k$ which can be determined using Table [1] and $w_q^k[n]$ is a non-Gaussian distortion noise whose variance for the $k$-th antenna is $\sigma_{w_q^k}^2 = \eta_k(1 - \eta_k) \sigma_{Y_k}^2$. 

In our setup, each receive antenna at the PS is equipped with identical ADCs. As explained in [29], while it maybe tempting to think that the quantization noise at different ADCs is uncorrelated, this is generally not the case since each antenna receives different (delayed) linear combinations of the same set of OFDM words generated at the workers. On the other hand, as shown in [30], the distortion can be safely approximated as uncorrelated for massive MIMO systems with sufficient number of users. We have also validated this approximation for our system which reveals that the correlation across the antennas of the PS is near-zero, even for the one-bit ADC case. Therefore, the correlations can be ignored as in the additive quantization noise model (AQNM) leading to a tractable scheme [31]. We further note that there are different studies on low-resolution ADCs which also neglect the distortion correlation among antennas as in our approach [26], [32]-[33]. For zero-mean Gaussian processes, this approach is equivalent to the Bussgang decomposition, except that it ignores the correlation among the elements of the distortion term.

If we define the total effective noise caused by the channel and quantization as

\[ w_k[n] = (1 - \eta_k) z_k[n] + w_q[k] \]  

(38)

the output of the complex ADC can be written as

\[ R_k[n] = (1 - \eta_k) \sum_{m=1}^{M} \sum_{l=1}^{L} h_{mkl} G_m[n - \tau_{mkl}] + w_k[n], \]  

(39)

where \( w_k[n] \) is non-Gaussian total noise with variance \( \sigma^2_{w_k} = \sigma^2_{w_q} + (1 - \eta_k)^2 \sigma^2_z \), and assumed to be uncorrelated across the antennas.

To perform OFDM demodulation, we take the discrete Fourier Transform (DFT) of (39) which results in

\[ r_k[i] = (1 - \eta_k) \sum_{m=1}^{M} H_{mk}[i] g_m[i] + W_k[i], \]  

(40)

where \( H_{mk}[i] \)'s are the channel gains from the \( m \)-th worker to the \( k \)-th receive chain for the \( i \)-th subcarrier, given by (14) which are zero-mean Gaussian random variables with variance \( \sigma^2_H = \sum_{l=1}^{L} \sigma^2_{h,lt} \).

Taking the DFT of the effective noise, \( W_k[i] \) is evaluated as

\[ W_k[i] = \sum_{n=0}^{N-1} w_k[n] e^{-j2\pi in/N}. \]  

(41)
We know that the channel noise is i.i.d., and we assume that the distortion noise is \(m\)-dependent to decorrelate fast enough, i.e., \(m \ll N\). Hence, \(W_k[i]\) converges absolutely to a Gaussian random variable by an application of the central limit theorem (CLT) \([34]\), i.e., \(W_k[n] \sim \mathcal{CN}(0, \sigma^2_{W_k})\) where \(\sigma^2_{W_k} = N\sigma^2_w\).

Assuming that the CSI is available at the PS as in the previous section, the received signals from the \(K\) antennas can be combined to align the gradient vectors by

\[
y[i] = \frac{1}{K} \sum_{k=1}^{K} \frac{1}{1 - \eta_k} \left( \sum_{m=1}^{M} (H_{mk}[i])^* \right) r_k[i].
\]

(42)

By substituting (40) into (42), we obtain

\[
y[i] = \frac{1}{K} \sum_{k=1}^{K} \sum_{m=1}^{M} |H_{mk}[i]|^2 g_m[i]
\]

signal term

\[+ \frac{1}{K} \sum_{k=1}^{K} \sum_{m=1}^{M} \sum_{m' \neq m} (H_{mk}[i])^* H_{m'k}[i] g_{m'}[i] \]

interference term

\[+ \frac{1}{K} \sum_{k=1}^{K} \frac{1}{1 - \eta_k} \left( \sum_{m=1}^{M} (H_{mk}[i])^* \right) W_k[i]. \]

(43a)

(43b)

(43c)

There are three different terms in (43): the signal component, interference and noise. Using the law of large numbers, as the number of antennas at the PS \(K \to \infty\), the signal term approaches

\[
y_{\text{sig}}[i] = \sigma^2_H \sum_{m=1}^{M} g_m[i].
\]

(44)

Thus, the PS can recover the \(i\)-th entry of the desired signal

\[
\frac{1}{M} \sum_{m=1}^{M} g_m[i] = \frac{y_{\text{sig}}[i]}{M\sigma^2_H}.
\]

(45)

To analyze the interference term (43b), we follow the same approach as in the previous section where each \(M\) interfering terms are analyzed separately. We define the coefficient of \(j\)-th interfering worker as

\[
\kappa_j[i] = \frac{1}{K} \sum_{k=1}^{K} \sum_{m=1}^{M} (H_{mk}[i])^* H_{jk}[i],
\]

(46)
where \( i \in [N] \), and \( j \in [M] \). Since \( H_{mk}[i] \) and \( H_{jk}[i] \) are independent for \( j \neq m \), the mean and variance of \( \kappa_j[i] \) are calculated as

\[
\mathbb{E} [\kappa_j[i]] = 0, \quad (47a)
\]
\[
\mathbb{E} \left[ |\kappa_j[i]|^2 \right] = \frac{(M - 1)\sigma_H^4}{K}. \quad (47b)
\]

Accordingly, for fixed gradient values, each of \( M \) interference terms in (43b) has zero mean and their variances scale with \( M/K \). Thus, similar to the ideal case (where the receive chains have infinite resolution as considered in [20]), the interference term approaches zero as \( K \to \infty \). Therefore, using a sufficiently large number of antennas at the PS diminishes the destructive effects of the interference on the learning process, and the estimate for the gradient vector can be determined by

\[
\frac{1}{M} \sum_{m=1}^{M} g_m[i] = \begin{cases} 
\frac{y^R[i]}{M\sigma_H^2}, & \text{if } 1 \leq i \leq e, \\
\frac{y^I[i-e]}{M\sigma_H^2}, & \text{if } e < i \leq 2e,
\end{cases} \quad (48)
\]

for \( i \in [d] \) from the noisy version of the received local gradients. This result clearly shows that the convergence of the learning process is guaranteed even if we employ low cost low-resolution ADCs at the receiver.

V. DSGD with Finite Resolution DACs and ADCs

We now consider a system where the workers and the PS employ low-resolution DACs and ADCs, respectively. Each worker uses a finite resolution DAC to quantize the OFDM words, and transmits them through a multipath fading channel. The PS receives the signal from multiple antennas where finite resolution ADCs are employed at each receive chain. The aim is to obtain an estimate of the gradients using the received signals which are distorted by ADCs and DACs as well as the multipath fading channel impairments. As in Sections III and IV, the joint effect of quantization on both the transmitter and receiver side is considered, and their impact on the convergence of the learning process is analyzed by utilizing the Bussgang decomposition and AQNM model for the quantization operation at the workers and PS, respectively.

Each worker calculates their local gradients and their corresponding OFDM words \( \bar{G}_m \in \mathbb{C}^{N+N_{cp}} \). As in Section III, each worker uses a finite resolution DAC, and quantize the OFDM
words corresponding to the local gradients where the $n$-th element of the transmitted signal by the $m$-th worker is

$$
\bar{G}_m^Q[n] = Q(\bar{G}_m[n]) = (1 - \eta)\bar{G}_m[n] + q_m[n].
$$

Using the Bussgang decomposition, $\eta = 1/$SQNR due to the quantization of $\bar{G}_m[n]$, and the variance of the distortion noise is $\sigma^2_{q_m} = \eta(1 - \eta)\sigma^2_{\bar{G}_m}$.

The quantized signals pass through a multipath fading channel whose impulse response is given by (5). After removing the CP, the received signal at the input of the finite resolution ADC of the $k$-th antenna of the PS is

$$
U_k[n] = \sum_{m=1}^{M} \sum_{l=1}^{L} \bar{h}_{mkl} \left( (1 - \eta)G_m[n - \tau_{mkl}] + q_m[n - \tau_{mkl}] \right) + z_k[n].
$$

The mean of $U_k[n]$ is zero, and its variance is given by

$$
\sigma^2_{U_k} = \sum_{m=1}^{M} \sum_{l=1}^{L} |\bar{h}_{mkl}|^2 \left( (1 - \eta)^2 + \eta(1 - \eta) \right) \sigma^2_{G_m}
$$

$$
+ (1 - \eta)^2 \sum_{m=1}^{M} \sum_{l=1}^{L} \sum_{l' = 1, l' \neq l}^{L} \bar{h}_{mkl} \bar{h}_{mlk'}^* \mathbb{E} \left[ G_m[n - \tau_{mkl}]G_m[n - \tau_{mlk'}] \right] + \sigma^2_z,
$$

which only depends on the receive antenna index $k$.

The PS employs finite resolution ADCs at each receive antenna. Similar to quantization with DAC, the quantization operation of the ADC can be modeled as a linear operation using AQNM model where the correlation of distortion noise across the antennas is ignored. The corresponding quantized signal at the $k$-th antenna is written as

$$
R_k[n] = (1 - \eta_k) \left( \sum_{m=1}^{M} \sum_{l=1}^{L} \bar{h}_{mkl}(1 - \eta)G_m[n - \tau_{mkl}] + \sum_{m=1}^{M} \sum_{l=1}^{L} \bar{h}_{mkl}q_m[n - \tau_{mkl}] + z_k[n] \right) + v_q[n],
$$

where $\eta_k$ is the distortion factor due to quantization of the received signal at $k$-th antenna ($U_k$), and calculated through the SQNR of the corresponding quantization operation as $\eta_k = 1/$SQNR. $v_q[n]$ is a non-Gaussian distortion noise whose variance is $\sigma^2_{v_q} = \eta_k(1 - \eta_k)\sigma^2_{U_k}$.

The total effective non-Gaussian noise caused by the channel and quantization with ADC at the PS is

$$
p_k[n] = (1 - \eta_k)z_k[n] + v_q[n],
$$
with variance $\sigma_{p_k}^2 = (1 - \eta_k)^2 \sigma_z^2 + \sigma_{vq}^2$, and the output of the complex ADC can be rewritten as

$$R_k[n] = (1 - \eta_k)(1 - \eta) \sum_{m=1}^{M} \sum_{l=1}^{L} h_{mkl} G_m[n - \tau_{mkl}]$$

$$+ (1 - \eta_k) \sum_{m=1}^{M} \sum_{l=1}^{L} h_{mkl} q_m[n - \tau_{mkl}] + p_k[n].$$  \hspace{1cm} (54)

To perform OFDM demodulation, we take the DFT of (54) which results in

$$r_k[i] = (1 - \eta_k)(1 - \eta) \sum_{m=1}^{M} H_{mk}[i] g_m[i] + (1 - \eta_k) \sum_{m=1}^{M} H_{mk}[i] Q_m[i] + P_k[i],$$  \hspace{1cm} (55)

where $H_{mk}[i]$’s are defined by (14), and $Q_m[i]$ is the DFT of the quantization distortion noise.

Taking DFT of the effective noise, $P_k[i]$ is evaluated as

$$P_k[i] = \sum_{n=0}^{N-1} p_k[n] e^{-j2\pi in/N}.$$  \hspace{1cm} (57)

With a similar approach to the one used in Section IV, $P_k[i]$ converges absolutely to a Gaussian random variable by an application of CLT [34].

Since the CSI is not available at the PS as in [20], the received signals can be combined to align the gradient vectors as

$$y[i] = \frac{1}{K} \sum_{k=1}^{K} \frac{1}{(1 - \eta)(1 - \eta_k)} \left( \sum_{m=1}^{M} (H_{mk}[i])^* \right) r_k[i].$$  \hspace{1cm} (56)

This quantity can be written as the sum of five different terms as in Section III:

$$y[i] = \frac{1}{K} \sum_{k=1}^{K} \sum_{m=1}^{M} |H_{mk}[i]|^2 g_m[i]$$  \hspace{1cm} (57a)

signal term

$$+ \frac{1}{K} \sum_{k=1}^{K} \sum_{m=1}^{M} \sum_{m' = 1}^{M} (H_{mk}[i])^* H_{m'k}[i] g_{m'}[i]$$  \hspace{1cm} (57b)

interference term

$$+ \frac{1}{(1 - \eta)K} \sum_{k=1}^{K} \sum_{m=1}^{M} \sum_{m' = 1}^{M} (H_{mk}[i])^* H_{m'k}[i] Q_{m'}[i]$$  \hspace{1cm} (57c)

distortion noise term

$$+ \frac{1}{(1 - \eta)K} \sum_{k=1}^{K} \sum_{m=1}^{M} |H_{mk}[i]|^2 Q_{m'}[i]$$  \hspace{1cm} (57d)

second type of distortion noise term
\[
\frac{1}{(1-\eta)K} \sum_{k=1}^{K} \frac{1}{(1-\eta_k)} \left( \sum_{m=1}^{M} (H_{mk}[i])^* \right) P_k[i],
\]

which are the same as the terms given in (17) except for the last noise term. As in Section IV, the noise term, \( P_k[i] \), includes both the channel noise and the quantization noise due to ADCs which has zero mean and finite variance. The analyses of the interference term (57b), distortion noise term (57c), and the second type of distortion noise term (57d) are the same as those of (17b), (17c), and (17d), respectively. Hence, similar arguments on the convergence of the learning algorithm with finite resolution DAC are also valid for the combined effects of DACs and ADCs. In other words, using sufficiently large number of antennas at the PS, the gradients can be recovered via (31). The analysis shows that we can design a federated learning system with a large number of workers and receive antennas, and still have extremely low hardware cost and energy consumption. This is remarkable since it shows the practicality of the federated learning system with very low-cost hardware even for the case of realistic wireless channels.

VI. NUMERICAL EXAMPLES

In this section, we evaluate the performance of the distributed learning algorithms at the wireless edge with realistic channel effects and hardware limitations via simulations. Our main objective is to verify that the theoretical expectations on the low-cost federated learning systems over wireless channels are also valid in practice. We use the MNIST dataset \([35]\) with 60000 training and 10000 test samples to train a single layer neural network using the Adam optimizer \([36]\). At the beginning of the training process, each worker caches \( B = 1000 \) training samples randomly. The number of parameters is \( d = 7850 \).

Our system consists of \( M = 20 \) workers connected to a PS through a multipath fading channel with \( L = 3 \) taps and \( \sigma_{h,l}^2 = 1/L \), hence we have a normalized uniform multipath delay profile where each tap experiences Rayleigh fading. The number of subcarriers is taken as \( N = 4096 \).

We take the sampling period as \( T_s = T_w/N \) where \( T_w \) is the OFDM word duration without the CP. We assume that first tap has no delay and coherence time corresponds to \( 1000T_s \). Also, time delays are uniformly spaced, i.e., \( \tau_{mk1} = 0, \tau_{mk2} = 500T_s, \tau_{mk3} = 1000T_s \) for \( \forall m,k \).

The cyclic prefix length is set to \( N_{cp} = 1024 \), which is enough to remove the ISI effects caused by

\(^1\)We select this multipath delay profile for the ease of illustration and reproduction. More realistic multipath delay profiles, e.g., uniformly distributed time delays, can be selected, but it will not change the performance of the system.
Fig. 3: Test accuracy of the system with low-resolution DAC and channel noise variance $\sigma_z^2 = 8 \times 10^{-4}$.

The average transmit power of the OFDM word transmitted by the $m$-th worker is calculated as $P_T = \frac{1}{T} \sum_{t=1}^{T} \left\| \bar{G}_m^t \right\|_2^2$, which gives $P_T = 1.3267 \times 10^{-4}$ for this setup, where $T$ is the total iteration count. In our theoretical analysis, we model the OFDM words with the autocorrelation matrix $C_{G_mG_m}$ with equal nonzero diagonal elements denoted by $\sigma_G^2$, and zero off-diagonal elements. In our simulations, we do not make any assumption on the statistics of the gradients; we simply use the information which we obtain from our simulations to model the statistics of the gradients.
In Figs. 3a and 3b, the test accuracy for a system where each worker is equipped with low-resolution DAC and different number of antennas $K \in \{1, 5, M, 2M^2\}$ at the receiver side is illustrated for a system with $\sigma_z^2 = 8 \times 10^{-4}$. As the number of receive antennas increases, the test accuracy approaches that of the infinite resolution case since the variance of the distortion noise and interference decreases. At iteration $T = 1600$, the accuracy loss with one-bit DAC compared to infinite resolution case is 17.62%, 6.62%, 4.07%, and 0.37% for $K = 1$, $K = 5$, $K = 2M$, and $K = 2M^2$, respectively. Furthermore, the low complexity system achieves almost the same
accuracy with infinite resolution case when two-bit DACs are employed (except for $K = 1$ which has an accuracy loss of 2.64%). In Fig. 4a and 4b, we increase the channel noise variance to $\sigma_z^2 = 4 \times 10^{-3}$, i.e., there is 14 dB SNR reduction. As expected, the performance of the learning algorithm deteriorates, since the effect of noise term is increased. However, as shown in Figs. 4a and 4b, the convergence of the learning algorithm is still achieved, and the accuracy loss of one-bit DAC case compared to infinite resolution case is 27.54%, 13.95%, 4.71%, and 0.8% for $K = 1$, $K = 5$, $K = 2M$, and $K = 2M^2$, respectively. With two-bit DACs, the accuracy loss
(a) Number of receive antennas $K = 1, 5$.

(b) Number of receive antennas $K = 2M, 2M^2$.

Fig. 6: Test accuracy of the system with low-resolution ADC and channel noise variance $\sigma_z^2 = 4 \times 10^{-3}$.

decreases to $3.26\%$ and $2.40\%$ for $K = 1$ and $K = 5$, respectively, while it gives almost same performance when the number of PS antennas is $K = 2M$ and $K = 2M^2$. These results clearly illustrate that when moderate number of receive antennas are employed, low-resolution, even two-bit, DACs can achieve a learning performance comparable with the infinite resolution case.

In Figs. 5a and 5b, the test accuracy for different number of antennas $K \in \{1, 5, M, 2M^2\}$ each equipped with a low-resolution ADC is illustrated for a system with $\sigma_z^2 = 8 \times 10^{-4}$, and compared with the error-free shared link case. As expected, using higher number of receive
antennas results in an improved learning accuracy. Indeed the results are very close to those of the infinite resolution case, especially with two-bit ADCs, while there is a minor drop on accuracy with one-bit ADCs. For instance, after the 1600-th iteration, using one-bit ADCs causes only 2.64\%, 0.95\%, and 0.13\%, accuracy loss compared to infinite resolution case for $K = 1$, $K = 5$, and $K = 2M$ respectively, while it achieves the performance of the infinite resolution with $K = 2M^2$ PS antennas. These results are due to the fact that increasing the number of antennas reduces the interference dramatically which makes the combined signal a very good
estimate of the gradient vector, even with low-resolution ADCs.

Without changing any other parameters of the setup described above, we increase the noise variance to $\sigma_z^2 = 4 \times 10^{-3}$ in Figs. 6a and 6b. As in the previous case, for the two-bit ADC case, the performance of the proposed scheme is very close to the error-free case for large number of receive antennas. When the number of antennas is decreased, with the detrimental effects of the channel noise and interference caused by shared multipath fading channels, the accuracy decreases. However, even for this high level of channel noise, using one-bit ADCs causes only
Fig. 9: Test accuracy of the system with separate one-bit DACs at the workers, one-bit ADCs at the PS antennas, and joint DACs and ADCs where the channel noise variance is \( \sigma_z^2 = 8 \times 10^{-4} \), and \( K = 5 \).

4.09\%, 2.55\%, 0.37\%, and 0.32\% accuracy loss compared to infinite resolution case for \( K = 1 \), \( K = 5 \), \( K = 2M \), and \( K = 2M^2 \), respectively after the 1600-th iteration.

In Figs. 7a and 7b, we consider a system which employs both low-resolution DACs at the workers and one-bit ADCs at the PS antennas with channel noise variance \( \sigma_z^2 = 8 \times 10^{-4} \). As expected, using low-resolution DAC and ADC at the same time increases the amount of interference in the gradient estimate at the PS, which decreases the learning accuracy of the distributed system. However, the combined effect of the interference terms is still negligible, especially for sufficiently large number of receive antennas. After the 1600-th iteration, using one-bit DACs and ADCs simultaneously causes only 17.91\%, 7.76\%, 4.18\%, and 0.39\% accuracy loss compared to infinite resolution case for \( K = 1 \), \( K = 5 \), \( K = 2M \), and \( K = 2M^2 \), respectively. When \( K = 1 \), using two-bit DAC and ADC results in a 2.95\% accuracy loss while it is almost same as the infinite resolution case when the number of PS antennas is higher. In the same system, we increase the channel noise variance to \( \sigma_z^2 = 4 \times 10^{-3} \) in Figs. 8a and 8b which causes 28.56\%, 15.66\%, 6.87\%, and 1.91\% accuracy loss compared to infinite resolution case for \( K = 1 \), \( K = 5 \), \( K = 2M \), and \( K = 2M^2 \), respectively after the 1600-th iteration with one-bit DAC and ADC. With two-bit DAC and ADC, the accuracy loss decreases to 3.35\% and 2.57\% for \( K = 1 \) and \( K = 5 \), respectively.

Finally, in Fig. 9 we compare the effect of one-bit quantization on the transmitter and receiver...
side, both separately and jointly, on the distributed machine learning system with a fixed number of receive antennas $K = 5$. As expected, the test accuracy of the system with one-bit DAC workers and infinite resolution PS antennas is lower than the case of infinite resolution workers and PS antennas with one-bit ADC. This is because, using DACs at the workers results in higher interference (due to two types of distortion noise terms) than using ADCs, and the performance is deteriorated. Further, simultaneously employing DACs and ADCs combines the consequential interference terms that reduce the test accuracy while the convergence of the learning algorithm is preserved. Another important implication of our results is that even though our analysis is based on a certain assumption on the statistics of the gradients, the simulation results (which are obtained without using Gaussian assumption on the OFDM words) are compatible with our theoretical expectations. Hence, with a slight sacrifice on the accuracy rate of the learning algorithm, power and hardware efficient systems (at both transmitter and receiver sides) can be designed and implemented for distributed learning at the wireless edge (or, federated learning) over realistic wireless channels.

VII. Conclusions

In this paper, we have investigated a distributed learning system at the wireless edge with OFDM based transmission and low-resolution, even one-bit, DACs and ADCs, respectively at the transmitter and receiver sides for a practical and inexpensive system design, and reduced power consumption. Our analytical results illustrate that with low-resolution DACs at the transmitter and ADCs at the receiver, the convergence of the distributed learning algorithms based on stochastic gradient descent is guaranteed when the number of receive antennas is increased as in the ideal case of infinite resolution DACs and ADCs. Moreover, the convergence is still attained with the joint use of DACs and ADCs which reduces the implementation costs further. The results are also valid for the extreme case of one-bit DACs and ADCs. Through extensive numerical examples, it is also illustrated that using a moderate number of antennas with low-resolution DACs and ADCs, e.g., using 5 antennas at the PS, can significantly approach to the performance of the infinite resolution case. It is also observed that, in case of low channel noise, the learning performance is decreased only slightly even for the extreme case of one-bit ADC and DACs.

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