Gauge Mediation with Sequestered Supersymmetry Breaking

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Abstract

Gauge mediation models have two drawbacks, that is, the so-called $\mu$-problem and a lack of predictability of the gravitino dark matter abundance. We show that conformal sequestering in the supersymmetry breaking sector offers attractive solutions to both problems. The correct mass scale of the $\mu$ and $B_\mu$ terms is generated by taking the gravitino mass of $O(100)$ GeV without causing the flavor-changing neutral-current problem. Moreover, a large anomalous dimension of the supersymmetry breaking field naturally realizes the small stau and neutralino mass difference required for the coannihilation to work yielding the right dark matter abundance.
1 Introduction

Gauge-mediated supersymmetry (SUSY) breaking (GMSB) models [1] are very attractive, since those models can naturally solve the flavor-changing neutral-current (FCNC) problem in the SUSY standard model (SSM). This is because non-renormalizable operators at the Planck scale $M_{\text{Pl}}$ are irrelevant for generating the soft masses in gauge mediation. However, the GMSB models have two drawbacks. First, the origin of the so-called $\mu$ term is not clear at all. If it is induced by the Planck suppressed operators, the $\mu$ parameter becomes of the order of the gravitino mass $m_{3/2}$, which, however, is too small for the successful electroweak symmetry breaking. The reason for this is that the gravitino mass is required to satisfy $m_{3/2} < 1 \text{ GeV}$ in order to suppress FCNC in the GMSB scenario, provided that the non-renormalizable operators at the Planck scale induce generic squark and slepton masses of order of the gravitino mass. Second, for such a light gravitino mass, it is the gravitino that is a candidate for dark matter (DM) in the universe, since the lightest SUSY particle in the SSM is not stable, and decays into the gravitino. The density of the gravitino depends crucially on the reheating temperature after inflation, and hence we lose a predictability of the DM density in the universe without knowledge of the inflation dynamics.

We see that the above two problems originate from the small gravitino mass. Thus, if we increase the gravitino mass up to $\mathcal{O}(100) \text{ GeV}$, both problems can be simultaneously solved. In general, for the gravitino mass of $\mathcal{O}(100) \text{ GeV}$, the soft masses for squarks and sleptons given at the Planck scale induce too large FCNC. However, this is not always the case. In this paper we show that, if the conformal sequestering occurs in the SUSY breaking sector [2, 4], the above two problems are naturally solved in the GMSB models without causing the FCNC problem.

In the present model we consider a parameter region where the lightest neutralino is lighter than the gravitino and hence the stable lightest SUSY particle (LSP). Surprisingly enough, the present model naturally predicts the small mass difference between the lightest neutralino and the stau required for the coannihilation to work yielding the correct DM density in the present universe [3]. We would like to stress that a large anomalous dimension of the SUSY breaking field $S$ is crucial to realize the coannihilation region.
naturally.

This paper is organized as follows. In Sec. 2 we review a model for the conformal sequestering, and see how the $\mu$-problem is solved. In Sec. 3 we study the neutralino DM density in detail. We discuss the cosmological implications of our scenario in Sec. 4. The last section is devoted for conclusions.

2 A model for conformal sequestering of SUSY breaking

Let us consider the conformal sequestering of SUSY breaking, which offers a natural solution to the $\mu$-problem as we will see at the end of this section. We first review a model of conformal sequestering which was proposed in Ref. [4]. While we focus on the model for concreteness, any models of conformal sequestering containing a singlet SUSY breaking field $S$ may work as well.

2.1 A hidden sector model

We consider the IYIT SUSY breaking model [5], which is based on an $SP(N)$ gauge theory with $2N + 2$ chiral superfields $Q^i$ transforming in the fundamental representation of the gauge group. Here, $i = 1, \cdots, 2N + 2$ is the flavor index and we suppress the gauge indices for simplicity. We also introduce $\frac{1}{2}(2N + 2)(2N + 1)$ gauge singlet chiral fields, $S_{ij} = -S_{ji}$. The tree level superpotential of this theory is given by

$$W = hS_{ij}Q^iQ^j.$$  \hfill (1)

Here we have assumed an $SU(2N + 2)$ global symmetry which acts on the indices $i, j$ in the hidden sector, for simplicity. This global symmetry is also imposed on the Kähler potential as an exact symmetry for conformal sequestering to work properly, because such operators that correspond to conserved currents are not sequestered [2]. However, we can relax this exact symmetry to an $SP(N + 1)$, a subgroup of the $SU(2N + 2)$. We will come back to this point later.

[1] In this paper we neglect subtlety regarding quantum gravitational effects on global symmetry.
This theory exhibits a quantum deformation of the moduli space \[6\], and the low energy effective superpotential is given by

\[ W_{\text{eff}} = X (\text{Pf}(Q^i Q^j) - (\Lambda_{\text{SUSY}})^{2N+2}) + h S_{ij} Q^i Q^j, \]  

(2)

where \( X \) is a Lagrange multiplier and \( \Lambda_{\text{SUSY}} \) is a dynamical scale of the gauge theory, around which SUSY is broken. The equation of motion of \( X \) requires \( \text{Pf} \langle Q^i Q^j \rangle = (\Lambda_{\text{SUSY}})^{2N+2} \). Then singlet fields \( S_{ij} \) have \( F \)-term of order \( F_S \sim h \langle QQ \rangle \sim h (\Lambda_{\text{SUSY}})^2 \), and SUSY is broken.

Let us introduce additional gauge symmetries and matter chiral superfields so that the theory flows into a conformal fixed point above the SUSY breaking scale. We take an \( SP(N) \times SP(N')^2 = SP(N) \times SP(N')_1 \times SP(N')_2 \) model of Ref. \[4\] as a specific example. In this model, there are matter chiral fields \( Q^i \) and \( S_{ij} \) as above, and additional chiral fields \( Q'_1 \) and \( Q'_2 \). The \( Q'_1(2) \) transforms as a bi-fundamental representation under \( SP(N) \times SP(N')_{1(2)} \) and as a singlet under \( SP(N')_{2(1)} \). See Table \[1\]. We take the superpotential of this model to be

\[ W = h S_{ij} Q^i Q^j + m (Q'_1 Q'_1 + Q'_2 Q'_2), \]  

(3)

where \( m \) is a mass parameter of \( Q' \) at the Planck scale \( M_{PL} \sim 2.4 \times 10^{18} \text{ GeV} \). (The mass parameter \( m \) is not equal to the physical mass of \( Q' \), \( m_{\text{phys}} \), because of a large anomalous dimension of \( Q' \).) If \( N \) and \( N' \) are appropriately chosen, we can expect (or can explicitly show in the cases that we can use perturbation) that this theory flows into a nontrivial fixed point \[4\].

Basic picture of this model is as follows. As we lower a renormalization scale \( \mu_R \) from the Planck scale \( M_{PL} \), the theory enters conformal regime at some scale \( M_\ast \), which we
Table 2: Values of $\gamma$ and $\beta'$. This table is taken from the Table 4 of Ref. [4]. $\beta'$ is the lowest eigenvalue of the matrix $M$ of Ref. [4].

| $\text{SP}(3) \times \text{SP}(1)^2$ | $\gamma_Q \cdot \gamma_{Q'} \cdot \gamma_S$ | $\beta'$ |
|----------------------------------|---------------------------------|---------|
| $\text{SP}(5) \times \text{SP}(3)^2$ | -1, -1, 2 | non-perturbative |
| $\text{SP}(7) \times \text{SP}(5)^2$ | -0.8, -0.8, 1.6 | non-perturbative |
| $\text{SP}(13) \times \text{SP}(7)$ | -0.2, -0.8, 0.4 | 0.06 |
| $\text{SP}(20) \times \text{SP}(11)$ | -0.1, -0.8, 0.2 | 0.04 |

assume to be much larger than $m_{\text{phys}}$, but slightly smaller than the Planck scale. For $m_{\text{phys}} \lesssim \mu_R \lesssim M_*$, the coupling constants of the theory are almost fixed at a conformal fixed point, and the conformal sequestering occurs. For the energy scale below the mass of $Q'$, i.e., $\mu_R \lesssim m_{\text{phys}}$, we can integrate out the massive fields $Q'$, and the theory becomes identical to the IYIT model, and SUSY is broken at $\mu_R \simeq \Lambda_{\text{SUSY}}$ close to $m_{\text{phys}}$.

Next let us discuss the suppression of higher dimensional operators in a Kähler potential. From a point of view of low energy effective field theory, it is expected that there are higher dimensional terms in a Kähler potential, which couple the hidden sector fields $A_i (= Q, Q'$ and $S)$ and the visible sector fields $q_a$,

$$\Delta K = \frac{C_{ijab}}{M_{PL}^2} q_a^\dagger q_b A_i^\dagger A_j$$ \hspace{1cm} (4)$$

with $C_{ijab}$ expected to be $\mathcal{O}(1)$. If $C_{ijab}$ is generic, that is, if $C_{ijab}$ is not diagonal in the visible sector flavor indices $a, b$, then these terms lead to the severe FCNC problem. Conformal sequestering can solve this problem by suppressing the terms in $\Delta K$ by renormalization group flow from the scale $M_*$ to the physical mass scale of $Q'$, $m_{\text{phys}}$. The suppression factor is roughly given by $(\mu_R/M_*)^{\beta'}$, where $\beta' = \partial \beta(\alpha)/\partial \alpha$ is a derivative of a beta function $\beta(\alpha) = \mu_R(d\alpha/d\mu_R)$ with respect to a coupling constant $\alpha = g^2/4\pi$ of the theory. \footnote{Actually, the suppression factor is determined by the smallest eigenvalue of the matrix $(\partial \beta_k/\partial \alpha_l)$ if there are more than one coupling constant. In that case, $\beta'$ of this section should be regarded as the smallest eigenvalue. See Ref. [4] for details.} So, if $\beta'$ is sufficiently large, we expect a large suppression when we take the energy scale $\mu_R$ equal to the physical mass scale of $Q'$, $m_{\text{phys}}$. 


The soft scalar masses of the visible fields receive contribution from Eq. (4),
\[ \Delta m_{\text{vis}}^2 \sim C \left( \frac{m_{\text{phys}}}{M_*} \right)^{\beta'} m_{3/2}^2, \]  
(5)
where \( m_{3/2} \) is the gravitino mass and \( C \) collectively represents \( C_{ijab} \). Since we will take \( m_{3/2} = \mathcal{O}(100) \) GeV in our scenario, \( m_{\text{phys}} \) is \( \mathcal{O}(10^{10}) \) GeV. We also assume \( M_* \lesssim M_{PL} \simeq 2.4 \times 10^{18} \) GeV. The ratio of \( \Delta m_{\text{vis}} \) to \( m_{3/2} \) is then given by
\[ \left( \frac{\Delta m_{\text{vis}}}{m_{3/2}} \right)^2 \sim C \left( 10^{-8} \cdot \frac{m_{\text{phys}}}{10^{10} \text{ GeV}} \cdot \frac{10^{18} \text{ GeV}}{M_*} \right)^{\beta'}. \]  
(6)
For \( C = \mathcal{O}(1) \) and a relatively large value of \( \beta' \), the ratio is small enough to satisfy the constraints from FCNC. Note that phenomenological constraints from FCNC are rather mild compared to the case of anomaly mediation (\( m_{3/2} = \mathcal{O}(100) \) TeV) due to the smaller gravitino mass.

A large anomalous dimension \( \gamma_S \) of \( S \), \( \gamma_S \gtrsim 1 \), will play a crucial role to account for the right DM abundance as we see in the next section. From Table 2 we see that we have \( \gamma_S = 2, \gamma_S = 1.6 \) and \( \gamma_S = 1.4 \) for the cases of \( SP(3) \times SP(1)^2 \), \( SP(5) \times SP(2)^2 \) and \( SP(7) \times SP(3)^2 \), respectively. In those cases, we cannot calculate the precise values of \( \beta' \) because the gauge and Yukawa couplings are very large and we cannot use perturbation. Thus in this paper we simply assume that the suppression is large enough to be consistent with FCNC constraints.

There are other higher dimensional operators in the Kähler potential, which must be suppressed as well. First, there are terms linear in \( S \), such as \( Sq^\dagger q/M_{PL} \). These terms are actually suppressed by a factor of \( (m_{\text{phys}}/M_*)^{\frac{\gamma_S}{2}} \) and therefore negligible. Second, there are terms which are cubic or quartic in the hidden sector fields, e.g.,
\[ \frac{1}{M_{PL}^2} (Q'Q')(Q'^\dagger Q'^\dagger)q^\dagger q. \]  
(7)
This term is suppressed by \( 1/M_{PL}^2 \), and so, it may seem that this is also negligible at a first glance. But in fact, terms like Eq. (7) are dangerous. To see this, consider the case of \( SP(3) \times SP(1)^2 \), in which the anomalous dimensions of \( Q, Q' \) and \( S \) are
\[ \gamma_Q = -1, \quad \gamma_{Q'} = -1, \quad \gamma_S = 2, \]  
(8)
and those operators $QQ$ and $Q'Q'$ saturate the unitarity bound of conformal field theory \cite{8}. If there is no vertex renormalization, the anomalous dimension of $(Q'Q')(Q'^{\dagger}Q'^{\dagger})$ is $\gamma_{Q'Q'^{\dagger}Q'^{\dagger}}/2 = -2$, and the operator $(\Pi)$ is enhanced by a factor of $(M_*/m_{\text{phys}})^2$. If $M_* \sim M_{PL}$, the operator is effectively suppressed only by $1/M_{PL}^2$, and is not negligible. Indeed, if $Q'Q'$ has non-vanishing $F$-term $F(Q'Q') \neq 0$ which is comparable with $F_S$, the operator $(\Pi)$ leads to too large flavor dependent soft masses of the visible sector, causing a FCNC problem. Actually, however, it is suppressed by a factor $(M_*/M_{PL})^2$ if $M_* \lesssim M_{PL}$, and so, the FCNC problem can be avoided if we take $M_* \ll M_{PL}$ \footnote{T. T. Y. thanks Y. Nakayama and M. Ibe for serious discussions on this problem.}. In the numerical analysis of the next section, we take $SP(3) \times SP(1)^2$ model with $M_* \sim 10^{16}$ GeV as an example. In that case $(M_*/M_{PL})^2 \sim 10^{-4}$, and there is no FCNC problem. We will neglect the soft masses induced from Eq. (7) in the following analysis.

2.2 Coupling the hidden sector to messenger fields

In our GMSB model, we introduce a Yukawa interaction between a singlet field and messenger fields in the superpotential,

$$W = \lambda S \Psi \Psi,$$

(9)

where $\Psi$ and $\Psi$ are the messenger superfields charged under standard model gauge groups. We would like to make two comments concerning the introduction of this term.

First, we need to single out one singlet field $S$ from the singlets $S_{ij}$. This can be done by reducing the global symmetry of the theory from $SU(2N + 2)$ to $SP(N + 1)$. Then, $S_{ij}$ can be decomposed as $S_{ij} = S'_{ij} + SR_{ij}$, where $R_{ij}$ is the $SP(N + 1)$ invariant tensor and $(R^{-1})^{ij}S'_{ij} = 0$. We then have to allow two different couplings $h_1$ and $h_2$ in the superpotential,

$$W = h_1 SR_{ij} Q'^i Q'^j + h_2 S'_{ij} Q'^i Q'^j.$$  

(10)

It is reasonable to assume that also in this case the theory flows into a conformal fixed point which is stable in the infrared. At the fixed point, vanishing of $\beta$ functions of $h_1$ and $h_2$ requires $\gamma_S + 2\gamma_Q = \gamma_{S'_{ij}} + 2\gamma_Q = 0$, and we have $\gamma_S = \gamma_{S'_{ij}}$. This suggests that the
fixed point of this theory is the same as in the case that we impose $SU(2N+2)$ symmetry. In other words, there is an enhanced $SU(2N+2)$ symmetry at the fixed point \footnote{This is an example of the “emergent symmetries” discussed in Ref. [9].}

Conserved currents $A^\dagger T^\alpha A$ ($T^\alpha$ are generators of $SU(2N+2)$ and $A = \{S_{ij}, Q^i\}$) have vanishing anomalous dimensions, and so, operators like $\Delta K = C_{ab\alpha}q^a_d q_b A^\dagger T^\alpha A$ are not sequestered. Then we have to worry about non-sequestering of such conserved currents [2, 9]. The $SU(2N+2)$ adjoint representation can be decomposed into symmetric and traceless-anti-symmetric representation of $SP(N+1)$ (trace is taken by contracting indices with $R_{ij}$), and there is no trivial representation. If we impose the $SP(N+1)$ symmetry on the Kähler potential, therefore, there is no conserved current which can appear in the Kähler potential. Thus non-sequestering of conserved currents does not occur in our case.

In fact, it may even be possible that we impose no symmetry on the Kähler potential at all. The Kähler potential of $A$ with dangerous conserved current operators is

$$K = A^\dagger A + \epsilon_\alpha A^\dagger T^\alpha A,$$

where $\epsilon_\alpha = C_{ab\alpha}q^a_d q_b$. For the purpose of calculating the soft masses, we can suppose that $q_a$ are constants. Then we can transform the hidden fields $A$ as

$$A \rightarrow \left(1 - \frac{1}{2}\epsilon_\alpha T^\alpha\right) A$$

so that the Kähler potential becomes

$$K \rightarrow A^\dagger A + O(\epsilon^2).$$

The point is that because this transformation corresponds to the symmetry transformation of the whole theory, which is respected even by the conformal symmetry breaking mass term of $Q'$, we can completely transform away the visible fields $\epsilon_\alpha = C_{ab\alpha}q^a_d q_b$ and no soft mass is generated. For more discussions on conserved currents, see Ref. [9]. The above argument suggests that even if we do not impose any symmetry at all on the Kähler potential, we may achieve conformal sequestering without the danger caused by conserved currents. The only requirement is that the theory should flow into the infrared stable fixed point for arbitrary Yukawa couplings $h_{kli}^i S_{ij} Q^k Q^l$. 

Second, there is a danger that introducing the coupling (9) may significantly deform the original theory. We argue that this interaction is in fact harmless for the hidden sector dynamics. Suppose that the value of $\lambda$ in Eq. (9) and the standard model gauge couplings are not so large at the scale $M_*$. Then, the anomalous dimension of $\Psi$ and $\bar{\Psi}$, $\gamma$, is small. In this case the renormalization group equation of $\lambda$ is given by

$$\mu \frac{d}{d\mu} |\lambda| = \left( \frac{\gamma_S}{2} + \gamma_\Psi \right) |\lambda| \simeq \frac{\gamma_S}{2} |\lambda| > 0.$$  \hspace{1cm} (14)

As we lower the energy scale $\mu$, $\lambda$ becomes smaller and smaller, and so does the contribution of Eq. (9) to $\gamma_\Psi$. The effect of the interaction (9) to the hidden sector dynamics therefore becomes totally negligible. In other words, the operator of Eq. (9) is an irrelevant operator of renormalization group flow. Even if $\lambda$ is somewhat large at the scale $M_*$, at least in the leading order of perturbation theory, the Yukawa coupling gives positive contribution to $\gamma_\Psi$, and so, the relation $\gamma_S + 2\gamma_\Psi > 0$ still holds. This fact makes the above discussion more robust.

While $\lambda$ at the scale $M_*$ is naturally expected to be $\mathcal{O}(1)$, it gets suppressed at the SUSY breaking scale (and therefore at the messenger mass scale) due to strong conformal dynamics. The value of $\lambda$ at the scale $m_{\text{phys}}$ is given by

$$\lambda|_{m_{\text{phys}}} \simeq \left( 10^{-8} \cdot \frac{m_{\text{phys}}}{10^{10} \text{ GeV}} \cdot \frac{10^{18} \text{ GeV}}{M_*} \right)^{\frac{\gamma_S}{2}} \lambda_0,$$  \hspace{1cm} (15)

where we have defined the value of $\lambda$ at the scale $M_*$ as $\lambda_0 \equiv \lambda|_{M_*}$. As we will see in the next section, the smallness of $\lambda|_{m_{\text{phys}}}$ is essential for the coannihilation to occur in a wide parameter region of $B_\mu/\mu$.

### 2.3 The origin of $\mu$ and $B_\mu$ terms

Before closing this section, let us explain the origin of $\mu$ and $B_\mu$ terms in our model. Although in our model the soft masses of the visible sector are generated by gauge mediation, the $\mu$ term and $B_\mu$ term are generated by supergravity effects \[11\]. We assume that there is some global $U(1)_R$ symmetry in the theory, under which Higgs doublets are neutral. Then, arbitrary $\mu$ term is forbidden by this symmetry, but the following terms
in the Kähler potential \(^5\) and the superpotential are allowed:

\[
K \supset c H_u H_d + \text{h.c.}, \quad (16)
\]

\[
W \supset c' (m_{3/2})^* H_u H_d. \quad (17)
\]

The interaction (17) is allowed because \((m_{3/2})^* \propto W_0\) has \(U(1)_R\) charge 2, where \(W_0\) is a constant term in the superpotential \(^4\). We expect that \(c\) and \(c'\) are \(O(1)\) parameters. In the compensator formalism of supergravity \(^10\), we have to put the compensator field, \(\Phi\), so the Kähler potential and the superpotential become

\[
K \supset c H_u H_d \frac{\Phi^*}{\Phi} + \text{h.c.}, \quad (18)
\]

\[
W \supset c' (m_{3/2})^* H_u H_d \Phi. \quad (19)
\]

Substituting a vacuum expectation value (vev) \(\langle \Phi \rangle = 1 + m_{3/2} \theta^2\) and integrating over \(d\theta^2\) and/or \(d\bar{\theta}^2\), we obtain the \(\mu\) and \(B_\mu\) terms

\[
\mu = (c + c') (m_{3/2})^*, \quad (20)
\]

\[
B_\mu = (-c + c') |m_{3/2}|^2. \quad (21)
\]

The correct mass scale of \(\mu\) and \(B_\mu\) can be generated for the gravitino mass of \(O(100)\) GeV with \(c, c' = O(1)\). Note that the FCNC problem is absent thanks to the conformal sequestering of the SUSY breaking. Other terms such as \(A\)-terms which are generated by the anomaly mediation are suppressed by one-loop factor. An alternative solution to the \(\mu/B_\mu\) problem was proposed in Refs. \(^7\).

\(^5\)The Kähler potential \(K\) in this section is that in the conformal frame of supergravity. The usual Kähler potential in the Einstein frame \(K_{\text{sugra}}\) is related to this Kähler potential by \(K_{\text{sugra}} = -3 M_{PL}^2 \log(1 - K/3 M_{PL}^2)\). Conformal sequestering occurs in the conformal frame of supergravity.

\(^6\)The phase of the gravitino mass \(m_{3/2}\) is determined as follows. In the compensator formalism of supergravity, the Lagrangian of the compensator field \(\Phi = 1 + F_\Phi \theta^2\) is \(\mathcal{L} = \int d\theta^2 d\bar{\theta}^2 [-3 M_{PL}^2 \Phi^* \Phi \exp(-K_{\text{sugra}}/3 M_{PL}^2)] + \int d\theta^2 W \Phi^3 + \text{h.c.} = -3 M_{PL}^2 |F_\Phi|^2 + 3 W_0 F_\Phi + \text{h.c.} + \cdots\) where dots denote terms irrelevant for the vev of \(F_\Phi\) and the lowest component of \(\Phi\) is gauge fixed to be 1. By solving the equation of motion of \(\Phi\), we have \(\langle F_\Phi \rangle = W_0^*/M_{PL}^2\). We define the phase of \(m_{3/2}\) such that \(m_{3/2} \equiv \langle F_\Phi \rangle = W_0^*/M_{PL}^2\).
3 A sequestered GMSB model and the neutralino relic density

We consider a simple GMSB model, where a SUSY breaking field \( S \) couples to \( N_5 \) pairs of messenger chiral superfields, \( \Psi \) and \( \bar{\Psi} \), which transform as 5 and 5* under the \( SU(5)_{GUT} \):

\[
W = \lambda S \Psi \bar{\Psi} + M \Psi \bar{\Psi},
\]

where \( M \) is the messenger mass and \( \lambda \) is set to be the value at \( m_{\text{phys}} \) throughout this section, i.e., \( \lambda = \lambda |_{m_{\text{phys}}} \). A priori \( \lambda \) is a free parameter, however, in our scenario, \( \lambda \) is naturally very small: \( \lambda \simeq 10^{-6} - 10^{-7} \) (see Eq. (15)). The SUSY breaking field \( S \) develop a vev \( \langle S \rangle = \theta \sqrt{3} F_S \), which is related to the gravitino mass as \(|F_S| = \sqrt{3} m_{3/2} M_{PL} \), assuming that the SUSY breaking is dominated by \( F_S \).

In the GMSB models, the SSM gaugino masses are generated from loop diagrams of the messengers. At the one-loop level, gaugino masses are given by

\[
M_a = \frac{N_5 \alpha_a}{4\pi} \Lambda_{eff} g(x),
\]

where we have defined \( \Lambda_{eff} = \lambda F_S/M, x = \lambda F_S/M^2 \), and

\[
g(x) = \frac{1}{x^2} [(1 + x) \log(1 + x) + (1 - x) \log(1 - x)].
\]

Here \( a = 1, 2, 3 \) labels \( U(1), SU(2) \) and \( SU(3) \) in the SSM, respectively, and we use the normalization \( \alpha_1 = 5 \alpha_{EM}/(3 \cos^2 \theta_W) \). The soft scalar masses arise at the two loop level, and given by

\[
m_{\phi_i}^2 = 2N_5 \Lambda_{eff}^2 \sum_a \left( \frac{\alpha_a}{4\pi} \right)^2 C_a(i) f(x),
\]

where \( C_a(i) \) are Casimir invariants for the visible particles \( \phi_i \) \( (C_1(i) = 3Y_i^2/5) \) and

\[
f(x) = \frac{1 + x}{x^2} \left[ \log(1 + x) - 2Li_2(x/[1 + x]) + \frac{1}{2} Li_2(2x/[1 + x]) \right] + (x \rightarrow -x).
\]

For \( x < 1 \), both \( f(x) \) and \( g(x) \) are \( O(1) \). We see that \( m_{\phi_i} \simeq M_a = O(1) \) TeV is realized for \( \Lambda_{eff} = O(10^5) \) GeV.

Since the above expressions for the soft masses are given at the messenger scale, one should solve the visible sector renormalization group (RG) equation to get the on-shell
masses and mixing matrices. To this end, we have used the program SOFTSUSY 2.0.18 \cite{12}, setting $\text{sgn}(\mu) = +1$. In our analysis, we choose $B_\mu/\mu$ at the messenger scale as a free parameter and $\tan \beta$ (ratio of two higgs expectation values) is determined by given parameters. This is because we can naturally expect $B_\mu/\mu = \mathcal{O}(m_3/2)$ at the SUSY breaking scale from Eqs. (20) and (21). Notice that $(B_\mu/\mu)_{\text{SUSY breaking}} = (B_\mu/\mu)_{\text{messenger}}$ at least at the one-loop level of RG equations, assuming that the other SSM soft parameters vanish above the messenger scale.

In the present GMSB model, the lighter stau ($\tilde{\tau}_1$), the lightest neutralino ($\tilde{\chi}^0_1$) or the gravitino becomes LSP. We mainly consider a parameter region where the neutralino, $\tilde{\chi}^0_1$, is the LSP and hence a candidate of the DM. Which of the two particles, $\tilde{\tau}_1$ or $\tilde{\chi}^0_1$, becomes the LSP mainly depends on the number of the messenger ($N_5$), the mass of the messenger ($M$), and $B_\mu/\mu$. As can be seen in Eqs. (23) and (25), the gauginos become heavier as increasing $N_5$. The stau becomes heavier for a larger mass of the messenger, while the gaugino masses are almost independent of the messenger mass. Hence, in the case of the heavy messenger, the lighter stau mass $m_{\tilde{\tau}_1}$ tends to become heavier than the mass of the lightest neutralino, $m_{\tilde{\chi}^0_1}$ (see. Fig. 1).

Larger $\tan \beta$ implies stronger tau’s Yukawa coupling. Therefore, the stau becomes
lighter through left-right mixing and RG effects for the larger \( \tan \beta \). The value of \( (B_{\mu}/\mu)_{\text{messenger}} \) is connected to the value of \( \tan \beta \). In general, a smaller \( B_{\mu}/\mu \) leads to a larger \( \tan \beta \).

**Coannihilation**

Let us first calculate a naively expected range for the messenger scale in our scenario. The value of \( \lambda \) is related to the value of \( \lambda_0 \equiv \lambda|_{M_*} \) by

\[
\lambda \sim \left( \frac{\sqrt{F_S}}{M_*} \right)^{\gamma_S/2} \lambda_0,
\]

where we have substituted \( \sqrt{F_S} \sim m_{\text{phys}} \), assuming the Yukawa coupling \( h \) is of order unity. Using the relations \( \Lambda_{\text{eff}} = \lambda F_S/M \) and \( m_{3/2} = F_S/\sqrt{3}M_{PL} \), we obtain

\[
M \sim 10^{15-3\gamma_S} \text{ GeV} \times \lambda_0 \left( \frac{\Lambda_{\text{eff}}}{10^5 \text{ GeV}} \right)^{-1} \left( \frac{m_{3/2}}{10^2 \text{ GeV}} \right)^{1+\gamma_S/4} \left( \frac{10^{16} \text{ GeV}}{M_*} \right)^{\gamma_S/2}.
\]

If we adopt the hidden sector model of \( SP(3) \times SP(1)^2 \) and \( M_* \sim 10^{16} \text{ GeV} \), Eq. (28) leads to

\[
M \sim 10^9 \text{ GeV} \times \lambda_0 \left( \frac{\Lambda_{\text{eff}}}{10^5 \text{ GeV}} \right)^{-1} \left( \frac{m_{3/2}}{10^2 \text{ GeV}} \right)^{3/2}.
\]

Hence, the messenger mass is expected to be \( \mathcal{O}(10^9) \) GeV in the model, unless the value of \( \lambda_0 \) is fine-tuned to be much smaller than unity.

Next, let us discuss the parameter region in which the coannihilation takes place. Roughly speaking, the coannihilation occurs when the lighter stau mass becomes very close to the lightest neutralino mass [3]. In Fig. 2 we show the relation between \( M \) and \( B_{\mu}/\mu \) when \( m_{\tilde{\tau}_1} = m_{\chi_1^0} \) is met. From the figure, we can see that required coannihilation occurs for a wide region of \( B_{\mu}/\mu \) if the messenger mass \( M \) is approximately \( 10^8 \) GeV for \( N_5 = 3 \). Such value of \( M \) is realized naturally in our model for \( \lambda_0 = \mathcal{O}(10^{-1}) \) (see Eq. (29)). Note also that we can obtain the value \( N_5 = 3 \) not only by introducing 3 pairs

\[\text{[Footnote]}\]

\[\text{[Footnote]}\]

From the SSM soft parameters, we can rotate away all but one complex phases in our model. This remaining complex phase has a potential danger for the SUSY CP problem. An accurate bound for this phase from the CP constraint depends on details of the spectrum of the SUSY parameters. In fact, we see the CP problem becomes milder in the region where \( B_{\mu}/\mu \) is smaller than the wino mass. Here we neglect the remaining phase and take all parameters to be real in this paper, for simplicity.
We set $\Lambda_{\text{eff}} = 10^5, 5 \times 10^4, 3 \times 10^4$ GeV for $N_5 = 1, 2, 3$, respectively.

of messengers which transform as $5$ and $5^*$, but also by introducing one pair of messengers transforming as $10$ and $10^*$. In the case that $N_5 = 2$, $M = O(10^5)$ GeV is required for the coannihilation to occur with a wide parameter region of $B_{\mu}/\mu$. This is achieved by a rather small value of $\lambda_0 = O(10^{-4})$.

**Relic Density**

From the viewpoint of naturalness, the case that $N_5 = 3$ seems to be most interesting, since it naturally predicts a suitable value of messenger mass (Eq. (29)) for $B_{\mu}/\mu = O(m_{3/2})$ (see Fig. 2). Now we show that this model actually predicts the correct abundance of the neutralino DM. In Fig. 3, a contour plot of $\Omega_{\tilde{\chi}_1^0}h^2$ on the $(M, \Lambda_{\text{eff}})$ plane is shown. Here we set $(B_{\mu}/\mu)_{\text{messenger}} = 50$ GeV, 100 GeV for Fig. 3-(a) and (b), respectively. We have used the program MicroOmegas 2.2 [13] to estimate the cold dark matter density. Here, we set $m_{3/2} = 500$ GeV, and the red and blue lines represent $m_{\tilde{\chi}_1^0} = m_{\tilde{\tau}_1}$ and $m_{h^0} = 110$ GeV, respectively. We can see that $\Omega_{\tilde{\chi}_1^0}h^2 \simeq 0.1$ is realized for $M = 10^9 - 10^{10}$ GeV. This value of the messenger mass is nothing but the expected one from Eq. (29). In Fig. 3 we have taken into account the anomaly-mediation (AMSB) effects [14] to the SUSY breaking soft masses for the SSM particles.
Figure 3: Contour plot of $\Omega_{\tilde{\chi}}h^2$ on the $(M, \Lambda_{\text{eff}})$ plane for $N_5 = 3$ and (a) $B_\mu/\mu = 50$ GeV, (b) $B_\mu/\mu = 100$ GeV at the messenger scale. The black line represents $\Omega_{\tilde{\chi}}h^2 = 0.1$, and the shaded region shows the ambiguity from the AMSB effect. Here, we set $m_{3/2} = 500$ GeV. The red line represents $m_{\tilde{\chi}} = m_{\tilde{\tau}}$. On the left side of this red line, the stau becomes the LSP. The blue line represents $m_{h_0} = 110$ GeV. Above this blue line, $m_{h_0}$ becomes larger than 110 GeV.

4 Cosmology

Let us discuss the cosmological implications of our scenario. In the previous section, we have seen that the neutralino LSP, instead of the gravitino, can naturally account for the observed DM abundance. It does not necessarily mean, however, that the cosmological abundance of the gravitino is totally negligible. In fact, gravitinos can be produced thermally directly from the hot plasma, and non-thermally from the inflaton decay. It is known that the gravitinos can induce a severe cosmological problem \[15, 16, 17\].

The abundance of the gravitinos produced from thermal scatterings is given by [18, 19, 20]

$$Y_{3/2}^{(TH)} \simeq 1.9 \times 10^{-12} \left[ 1 + \left( \frac{m_{\tilde{g}_3}^2}{3m_{3/2}^2} \right) \right] \left( \frac{T_R}{10^{10} \text{ GeV}} \right)$$

$$\times \left[ 1 + 0.045 \ln \left( \frac{T_R}{10^{10} \text{ GeV}} \right) \right] \left[ 1 - 0.028 \ln \left( \frac{T_R}{10^{10} \text{ GeV}} \right) \right], \quad (30)$$

where $T_R$ is the reheating temperature and $m_{\tilde{g}_3}$ is the gluino running mass evaluated at the reheating. Moreover, the gravitinos are generically produced by the inflaton decay, if
the inflaton has a non-vanishing vev (more precisely, a non-vanishing linear term in the Kähler potential) at the potential minimum \[21, 22, 23, 24\]. For an inflaton mass lighter than the SUSY breaking scale, the gravitino pair production becomes efficient \[21\] (see also Refs. \[25, 26\]). On the other hand, for the inflaton mass heavier than the SUSY breaking scale, the gravitinos are produced from the inflaton decay into the hidden gauge sector \[24, 23\]. The abundance of the non-thermally produced gravitinos is given by

\[ Y_{3/2}^{NT} \simeq 7 \times 10^{-11} x \left( \frac{g_*}{200} \right)^{- \frac{1}{2}} \left( \frac{\langle \phi \rangle}{10^{12} \text{GeV}} \right)^2 \left( \frac{m_\phi}{10^{12} \text{GeV}} \right)^2 \left( \frac{T_R}{10^6 \text{GeV}} \right)^{-1}, \tag{31} \]

where \( g_* \) counts the relativistic degrees of freedom, \( \langle \phi \rangle \) is the inflaton vev, and \( m_\phi \) the inflaton mass. Here \( x \) is a numerical coefficient given by

\[ x = \begin{cases} 
10^{-3} & \text{for } m_\phi < \Lambda_{\text{SUSY}} \\
1 \sim 10^{-1} & \text{for } m_\phi > \Lambda_{\text{SUSY}} 
\end{cases}, \tag{32} \]

The precise value of \( x \) depends on the detailed structure of the SUSY breaking sector.

In our scenario, the gravitino mass is set to be of the order of 100 GeV to generate the \( \mu \)-term of a right magnitude, and the gravitino is not the LSP and therefore unstable. For such unstable gravitino, the total gravitino abundance must satisfy

\[ Y_{3/2} \equiv Y_{3/2}^{TH} + Y_{3/2}^{NT} \lesssim \mathcal{O}(10^{-16}), \tag{33} \]

in order not to spoil the success of the big bang nucleosynthesis (BBN) \[19, 27, 28, 29\]. Substituting Eq. (30) into Eq. (33), we obtain an upper bound on \( T_R \):

\[ T_R \lesssim \mathcal{O}(10^6) \text{ GeV}. \tag{34} \]

It is non-trivial for an inflation model to satisfy the bound on \( T_R \) \[22, 23\]. Indeed, it rules out the smooth hybrid inflation \[30\] as well as a part of the parameter space of the hybrid inflation \[31\]. In addition, the non-thermal gravitino production excludes most of the inflation models such as the new \[32, 33\] and hybrid inflation. Note that one cannot avoid the gravitino overproduction simply by reducing the reheating temperature due to the peculiar dependence of \( Y_{3/2}^{NT} \) on \( T_R \).

Among possible solutions to the (non-thermal) gravitino overproduction, the simplest one is to suppress the inflaton vev by imposing a symmetry on the inflaton. As a concrete
example, let us consider a chaotic inflation model with a $Z_2$ symmetry [34]. In this model, we assume that the Kähler potential $K(\phi, \phi^\dagger)$ is invariant under the shift of $\phi$,

$$\phi \rightarrow \phi + iA,$$

where $A$ is a dimension-one real parameter. We also impose a $Z_2$ symmetry: $\phi \rightarrow -\phi$. Then, the Kähler potential is given by

$$K(\phi + \phi^\dagger) = \frac{1}{2}(\phi + \phi^\dagger)^2 + \cdots,$$

where we have dropped a linear term of $(\phi + \phi^\dagger)$ which is forbidden by the $Z_2$ symmetry.

We introduce a small breaking term of the shift symmetry in the superpotential to generate a potential for the inflaton:

$$W(\phi, \psi) = m_{\inf} \phi \psi,$$

where we have introduced a new chiral multiplet $\psi$ charged under the $Z_2$ symmetry: $\psi \rightarrow -\psi$. The inflaton mass $m_{\inf} \approx 2 \times 10^{13}$ GeV represents the breaking scale of the shift symmetry, and reproduces the density fluctuations of the right magnitude. The imaginary part of $\phi$ is identified with the inflaton field $\varphi \equiv \sqrt{2}\text{Im}[\phi]$, and the scalar potential is given by

$$V(\varphi, \psi) \approx \frac{1}{2}m_{\inf}^2 \varphi^2 + m_{\inf}^2 |\psi|^2,$$

after the real part of $\phi$ settles down to the minimum. For $\varphi \gg M_{PL}$ and $|\psi| < M_{PL}$, the $\varphi$ field dominates the potential and the chaotic inflation takes place (for details see Ref. [34]). Since the linear term in the Käher potential is absent thanks to the $Z_2$ symmetry, the non-thermal gravitino production does not occur.

In order to induce the reheating into the visible sector, we consider the following interactions:

$$W_{\text{int}} = \frac{k}{2} \phi NN + \frac{1}{2} M_N NN,$$

where $N$ is a right-handed neutrino chiral multiplet. The $Z_2$ symmetry is explicitly broken by those interactions, and we will later discuss how small the breaking should be. For $m_{\inf} \gg 2M_N$, the decay rate is given by

$$\Gamma_N \approx \frac{k^2}{32\pi} m_{\inf}.$$
Assuming that the reheating occurs mainly through the decay into the right-handed (s)neutrinos, the reheating temperature is given by

\[ T_R \simeq 2 \times 10^6 \text{GeV} \left( \frac{k}{10^{-8}} \right) \left( \frac{m_{\text{inf}}}{2 \times 10^{13} \text{GeV}} \right)^{1/2}, \]  

(41)

where we have defined the reheating temperature as

\[ T_R \equiv \left( \frac{\pi^2 g_*}{10} \right)^{-1/4} \sqrt{\frac{\Gamma_N M_{PL}}{\pi M_N}}, \]  

(42)

The non-thermal leptogenesis occurs in this case \cite{35, 36, 37}, and the resultant baryon asymmetry is given by

\[ \frac{n_B}{s} \simeq 1 \times 10^{-10} \left( \frac{k}{10^{-8}} \right) \left( \frac{M_N}{10^{13} \text{GeV}} \right) \left( \frac{m_{\text{inf}}}{2 \times 10^{13} \text{GeV}} \right)^{1/2} \left( \frac{m_{\nu_3}}{0.05 \text{eV}} \right) \delta_{\text{eff}}, \]  

(43)

where \( m_{\nu_3} \) is the heaviest neutrino mass and \( \delta_{\text{eff}} \leq 1 \) represents the effective \( CP \)-violating phase. Note that a right amount of the baryon asymmetry is generated for \( k \sim 10^{-8} \) and \( M_N \sim 10^{13} \text{GeV} \), corresponding to \( T_R \sim 10^6 \text{GeV} \) being marginally compatible with the constraint (see Eq. (34)).

Now let us discuss the \( Z_2 \) symmetry breaking. We may interpret the first term in Eq. (39) breaks both the shift and \( Z_2 \) symmetries, with an assumption that \( NN \) is even under the \( Z_2 \) symmetry. Then, we may attribute the smallness of \( k \sim 10^{-8} \) to the breaking of the \( Z_2 \) symmetry, while the inflaton mass \( m_{\text{inf}}/M_{PL} \sim 10^{-5} \) represents the typical magnitude of the shift symmetry breaking \cite{8}. Since the \( Z_2 \) symmetry is explicitly broken, a linear term in the Kähler potential is induced at one-loop level: \( \delta K \sim 1/(16\pi^2)kM_N^2\phi + \text{h.c.} \). Or, since the \( Z_2 \) symmetry is not a true symmetry of the theory, we may expect the presence of a linear term, \( K = \tilde{c}M_{PL}(\phi + \phi^\dagger) \) with \( \tilde{c} \sim 10^{-8} \), from the beginning. Our concern is if such a tiny \( Z_2 \) breaking leads to the gravitino overproduction again. To satisfy the BBN constraint \cite{33}, the coefficient \( c \) must be suppressed as

\[ \tilde{c} \lesssim \mathcal{O}(10^{-7}) \cdot x^{-1/2}, \]  

(44)

\footnote{Alternatively, we can interpret that the first term in Eq. (39) breaks the shift symmetry while the second term breaks the \( Z_2 \) symmetry, by assigning a \( Z_2 \) odd charge to \( NN \).}
where we have substituted $\langle \phi \rangle \simeq \tilde{c}M_{PL}/\sqrt{2}$ and $m_\phi = m_{\text{inf}}$ into Eq. (31). Therefore, for $\tilde{c} \sim k \sim 10^{-8}$, we can avoid the non-thermal gravitino overproduction problem. In addition, the non-thermal leptogenesis is also possible.

Lastly let us make a comment on the Polonyi problem [42]. If the SUSY breaking field $S$ has a non-vanishing linear term in the Kähler potential, the initial position of $S$ during inflation is generically deviated from the origin [43]. Such a linear term may not exist at tree level, but it is necessarily generated due to the coupling to the messenger fields at one-loop level. If the deviation were large, the SUSY breaking field might produce too many gravitinos. Fortunately, due to the large anomalous dimension of the SUSY breaking field, $S$, the messenger mass scale is suppressed, and so does the linear term. Therefore there is no Polonyi problem in our scenario [44].

5 Conclusions

In this paper we have pointed out that a conformal sequestering of the SUSY breaking can naturally solve the two problem inherent in the gauge mediation; the $\mu/B_\mu$ problem and the lack of predictability of the gravitino DM abundance. First, since the dangerous higher dimensional operators in the Kähler potential are suppressed due to the conformal sequestering, we can increase the gravitino mass up to $\mathcal{O}(100)$ GeV without causing the FCNC problem. The correct mass scale of the $\mu$ and $B_\mu$ terms can be generated for such gravitino mass. Second, a large anomalous dimension of the SUSY breaking field makes the messenger scale very small, which results in a small mass difference between the neutralino and the stau, making the coannihilation to naturally occur. We have also discussed the cosmological implications of our scenario. The unstable gravitino of a mass of 100 GeV suffers from a severe gravitino overproduction problem, but we can find an example in which the problem is avoided and the right amount of the baryon asymmetry is generated through the non-thermal leptogenesis.

\footnote{If the gravitino is the LSP, a right amount of the gravitino DM can be produced for $k \sim \tilde{c} \sim m_{\text{inf}} \sim 10^{-5}$ (in the Planck unit), and the thermal leptogenesis becomes possible. The BBN bound can be avoided by including tiny violation of the $R$-parity and the decay of the unstable gravitino may explain the anomalies observed by HEAT and EGRET [40][41].}


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