Difficulties in Modeling Natural Ventilation Systems on a PC

S V Biryukov

Kaf. TGV, National research Moscow state university of civil engineering, Yaroslavskoe shosse, 26, Moscow, 129337, Russia.

E-mail: biryukovsv@mgsu.ru

Abstract The article describes the equations, methods and difficulties in calculating the local resistance coefficients of T-fittings during the calculation of natural ventilation systems of a building. The choice of formulas for determining resistances has a significant impact on the calculation speed and accuracy of modeling ventilation systems. A method for accounting for ejection during air movement in T-fittings is proposed.

1. Introduction
The main problem in developing the program for the aerodynamic calculation of the natural ventilation network of a building was the choice of formulas for the analytical calculation of the local resistance coefficients of the ventilation network elements. There is an extensive literature that provides coefficients of local resistances in tabular form and in the form of formulas for its approximate estimation values in bends, T-fittings and crosspieces of 90, 45 degrees with a round and rectangular cross section of the air duct. Based on the analysis of extensive literature, the formulas of p. N. Kamenev [3] and I. E. Idelchik were selected. [2], which meet the requirements of automated calculation and at the same time have sufficient accuracy.

2. Selection of formulas for analytical calculation of local resistance coefficients
Fig. 1 shows the design scheme of the tee with the designation of the main parameters of the T-fitting and the air flow in it, and Fig. 2 diagrams of the tees and crosses encountered in the calculation.

![Diagram of T-fitting](image)

**Figure 1.** Principal scheme of T-fitting

- α-angle between the axis of the barrel and the passage;
- β-angle between the axis of the trunk and the branch;
- 1, 2, 3-the numbers of the tee elements correspond to: 1-passage, 2-branch, 3-main duct; the same numbers are used in indexes for the corresponding values.
- \( v_1, v_2, v_3 \) – average flow rates, m/s;
- \( f_1, f_2, f_3 \) – cross-sectional area, m²;
- \( Q_1, Q_2, Q_3 \)– expenses kg/h (kg/s).
Figure 2. Schemes of calculated tees and crosses

1) T-fitting, crosspiece for merging (exhaust) according to the scheme in Fig. 2 (b, d, g, i) according to [3] passage:

\[ \zeta_1 = \left( \frac{V_1}{V_3} \right)^2 - \left( \frac{V_1}{V_3} \right)^2 ; \zeta_1 = \zeta_1^* + \zeta_1^* \]  

(1) branch:
\[
\zeta' = \left( \frac{v_2}{v_3} \right)^2 - \left( \frac{v_1}{v_3} \right)^2; \quad \zeta'' = \zeta' + \zeta^*;
\]

(2)

additional pressure loss coefficient:

\[
\zeta^* = \left( \frac{v_3}{v_3} \right)^2 - 1)^2
\]

(3)

Average flow rate after mixing:

\[
v_3 = \frac{Q_1}{Q_3} v_1 \cos \alpha + \frac{Q_2}{Q_3} v_2 \cos \beta
\]

(4)

2) T-fitting or crosspiece for division (supply):
diagram in Fig. 2 (c, e, h, j) by [3]

passage:

If \( v_1 = v_3 \cos \alpha \)
\[
\zeta_1 = \sin^2 \alpha
\]

(5)

If \( v_1 < v_3 \cos \alpha \)
\[
\zeta_1 = \sin^2 \alpha + (\cos \alpha - v_1/v_3)^2
\]

(6)

If \( v_1 > v_3 \cos \alpha \)
\[
\zeta_1 = \sin^2 \alpha + \zeta_\alpha(v_1/v_3)^2
\]

(7)

branch:

If \( v_2 = v_3 \cos \beta \)
\[
\zeta_2 = \sin^2 \beta
\]

(8)

If \( v_2 < v_3 \cos \beta \)
\[
\zeta_2 = \sin^2 \beta
\]

(9)

If \( v_2 > v_3 \cos \beta \)
\[
\zeta_2 = \sin^2 \beta + \zeta_\alpha(v_2/v_3)^2
\]

(10)

3) T-fitting 90 [2]
fig. 2 (a)
– for two-way merge

\[
\zeta_2' = 1 + \left( \frac{f_3}{f_2} \right)^2 + 3 \left( \frac{f_3}{f_2} \right)^2 \left[ \left( \frac{Q_2}{Q_3} \right)^2 - \left( \frac{Q_2}{Q_3} \right)^2 \right]
\]

(11)

\[
\zeta_2' = \frac{\zeta_2}{Q_2 f_3} \left( \frac{Q_3}{Q_3 f_2} \right)^2, \text{ similarly for the second branch}
\]

(12)

– for two-way division
fig. 2 (f)

\[
\zeta_2' = 1 + 0.3 \left( \frac{v_2}{v_3} \right)^2
\]

(13)

4) tap (knee) [2]

\[
\zeta = 0.95 \sin^2 (\delta/2) + 2.05 \sin^4 (\delta/2),
\]

(14)

where: \( v_3' \) – average flow rate after mixing, m / s; indexes 1, 2, 3 - correspond to: 1 - passage, 2 - branch, 3 - main duct of a T or crosspiece fitting; \( \delta \) – angle of rotation (90 or 45 degrees).

When determining the pressure loss coefficient in T- and cross fittings, there are cases when the resulting coefficient is less than zero. This means that there is an ejection in the element, which creates additional air suction. This phenomenon is most often observed in the connection of branches on the
upper floor of the building to the main channel. Since negative values of the coefficient lead to negative values of the resistance characteristic $S$, which are later used under a power dependence, and the use of absolute values of $S$ is unacceptable, since the physical picture is distorted, special methods must be used in calculations that exclude the appearance of negative values in the subradical expression. Features of calculating natural ventilation systems in high-rise buildings were described in [4].

It is important to quickly and accurately calculate the coefficients of local resistances when modeling the air regime of the entire building. In this case, the natural ventilation system will only be part of the system of equations. Modeling of the air mod of a building taking into account temperature changes in different parts was described in [5].

3. **Iterative solution of a system of equations. The calculation of the resistance characteristics of the network elements**

There are several schemes for organizing channels in the natural vent network. Consider the most complex scheme with a common main channel and floor-by-floor branches (satellites), which is shown in Fig. 3.

At the outlet of the ventilation shaft at point n, there is a pressure $P_n$, which is equal to the atmospheric or discharge pressure developed by the fan installed at the outlet of the main channel.

At points 1,2,3... there is a pressure $P_i$ – the pressure at the nodal points;

At points 1A, 2A, 3A... there is a pressure $P_r$ – the pressure inside the room with the number i.

![Figure 3. Scheme of natural vent network](image)
For each row node shown in figure 3, you can write a system of equations:

\[
\begin{cases}
G_i = \left( \frac{|P_i - P_{i+1}|}{S_i} \right)^2 \cdot \text{sign}(P_i - P_{i+1}) \\
G_{i+1} = \left( \frac{|P_i - P_{i-1}|}{S_{i-1}} \right)^2 \cdot \text{sign}(P_i - P_{i-1}) \\
G_n = \left( \frac{|P_i - P_n|}{S_n} \right)^2 \cdot \text{sign}(P_i - P_n) \\
G_i + G_{i-1} + G_n = 0
\end{cases}
\]  

(15)

The system of equations for the top node:

\[
\begin{cases}
G_i = \left( \frac{|P_i - P_n|}{S_i} \right)^2 \cdot \text{sign}(P_i - P_n), \text{ если } P_n = 0 \Rightarrow G_i = \left( \frac{|P_i|}{S_i} \right)^2 \cdot \text{sign}(P_i) \\
G_{i+1} = \left( \frac{|P_i - P_{i-1}|}{S_{i-1}} \right)^2 \cdot \text{sign}(P_i - P_{i-1}) \\
G_n = \left( \frac{|P_i - P_n|}{S_n} \right)^2 \cdot \text{sign}(P_i - P_n) \\
G_i + G_{i-1} + G_n = 0
\end{cases}
\]  

(16)

The system of equations for the second node from the bottom:

\[
\begin{cases}
G_2 = \left( \frac{|P_2 - P_3|}{S_2} \right)^2 \cdot \text{sign}(P_2 - P_3) \\
G_{r2} = \left( \frac{|P_2 - P_{r2}|}{S_{r2}} \right)^2 \cdot \text{sign}(P_2 - P_{r2}) \\
G_{r1} = \left( \frac{|P_2 - P_{r1}|}{S_{r1}} \right)^2 \cdot \text{sign}(P_2 - P_{r1}) \\
G_2 + G_{r2} + G_{r1} = 0
\end{cases}
\]  

(17)

For the lower (first) node, \( P_1 \) is determined from the system for the second node.
where: $G_i$ - flow rate in the main channel from point $i$ to point $i+1$ (n), kg/h; $G_{ri}$ - flow rate through connection (branch), kg/h; $S_i$ - characteristic of the resistance of the network section from point $i$ to point $i+1$ (n), Pa/(kg / h)$^2$; $S_{ri}$ - resistance characteristic of the i-th branch, Pa/(kg / h)$^2$.

The unknowns are pressures at nodal points and flow rates through branches, as well as individual sections of the trunk. Known are the pressures in rooms where exhaust grilles are installed for this ventilation network.

To determine pressures at nodal points and flow rates, it is necessary to know the resistance characteristics for each section, which depend on the geometric characteristics of the section, the type of local resistance, and the flow rate through this section. The dependence of the coefficient of pressure drop on the flow rate significantly complicates the solution of the system.

The iterative process of obtaining a solution to a general system of equations for the entire network consists of the following steps:

1) setting initial approximations
- expenses on branches are accepted according to the project norm. They are used to determine expenses on main sections.
- pressures at nodal points are assigned based on the accepted costs on branches and the average resistance characteristic $S=0.0001$ Pa/(kg / h)$^2$:

$$P_i = P_{ri} - 0.0001 \cdot G_{ri}^2,$$

(18)

- Coefficient of pressure drop for vent network elements that do not depend on the flow rate (speed) of the flow passing through them are set in the reference book. These include fittings, grilles, diffusers, confusers, umbrellas and other elements.

2) Determination of the density of the transported air in the vent network
In the branches and main sections that are adjacent to the branches, densities are accepted, determined by the temperatures of the internal air in the corresponding rooms. The dependence of air density on temperature is calculated using the formula. kg/m$^3$:

$$\rho = \frac{353}{273.16 + t},$$

(19)

where $t$ – is the temperature of the transported air, °C.

3) Determination of the section resistance characteristics
The resistance characteristics of each section are made up of the resistance characteristics of the individual elements of this section $S=\Sigma S_i$. The elements are a tee, a straight section, a branch, a grid, and so on.

When deriving the formulas for determining $S$ and $A$, the Darcy-Weisbach formula is used to calculate the friction pressure loss:

$$\Delta P = \frac{\lambda}{d} \frac{1}{2 g} v^2,$$

(20)

where $\lambda$ – coefficient of hydraulic friction; $l$ – length of the section, m; $v$ – flow rate in the element cross-section, m/s; $d$ – air duct diameter equivalent in speed, m; $g$ – acceleration of gravity, $g=9.81$ m/s$^2$.

The resistance characteristic of the element of the section $s$, Pa/(kg/h)$^2$, is determined by the formula:
S = A\left(\frac{2}{d}\ln(1 + \zeta)\right), \quad (21)

where A – specific hydrodynamic pressure in the element, Pa/(kg/h)^2, which occurs at a flow rate of 1 kg/h:

A = \frac{1}{2f^2\rho 3600^2}, \quad (for\ round\ ducts\ A = \frac{6.25}{10^8 \rho d^2}); \quad (22)

L – length of the element's conducting channel, m; d – speed-equivalent element diameter, m; \zeta – coefficient of local resistance; f – live cross-section of the element, m^2; \rho – density of the transported air, kg/m^3;

As mentioned, during the calculation process, situations arise when the found resistance characteristic of the section S becomes less than zero, due to the phenomenon of ejection. To exclude a negative value under the power dependence, an additional point (the point of fictitious pressure) is adopted in the work, where the pressure corresponding to the compensated S will be found. The fictitious pressure is determined by the formula:

P_{\text{fi}} = P_i + S_ri \* G_{ri}^2, \quad (23)

where S_ri – resistance characteristic (negative) of the tee branch element, Pa/(kg/h)^2;

P_i – pressure at the node point of the tee, Pa;

G_{ni} – flow through the section (branch), kg/h

4) Determination of air flow in the sections

After determining the characteristics of the resistances of the sections, new expenses are determined on the network sections from the expressions...

5) Calculation of the residual in the node

For each node, there is a discrepancy in expenses.

\varepsilon = G_i + G_{i+1} + G_{ni}, \quad \text{kg/h} \quad (24)

Finding the discrepancy is required to estimate the proximity of the solution of the system of node equations to the required accuracy and to further adjust the pressure at the node points.

6) Pressure adjustment at key points

Depending on the chosen numerical method for solving systems of equations, the pressures at the nodal points are corrected. When solving systems of equations, the Newton method is used with Seidel iteration substitution or using a matrix.

4. References

[1] Altshul A D, Zhivatovsky L S, Ivanov L P 1987 Hydraulics and aerodynamics (M: stroizdat) 414 p
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