The possible assignments of the scalar \(K_0^*(1950)\) and \(K_0^*(2130)\) within the \(3P_0\) model

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We have evaluated the strong decays of the \(K_0^*(1950)\) and \(K_0^*(2130)\) within the \(3P_0\) model, by employing the meson wave functions from the relativized quark model. By comparing with the experimental measurements, the \(K_0^*(2130)\) could be assigned as \(K_0^*(3^3P_0)\), while the \(K_0^*(1950)\) seems like an exotic state, because its width can not be reasonably reproduced within the \(3P_0\) model. We also predict that the \(K_0^*(2^3P_0)\) state has a mass of about 1811 MeV and a width of about 656 MeV, while the \(K_0^*(4^3P_0)\) state has a mass of about 2404 MeV and a width of about 180 MeV.

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I. INTRODUCTION

According to the theory of quantum chromodynamics (QCD), in addition to the conventional \(q\bar{q}\) mesons, the so-called exotic states are also permitted, such as tetraquarks, molecules, glueballs, and hybrids [1–3]. Although many exotic states have been observed experimentally, such as \(X(3872), Z_c(3900), Z_c(4025), P_c, T_{cc}\), it is still difficult to distinguish between the exotic states with conventional quantum numbers and the ordinary \(q\bar{q}\) mesons.

One puzzle in hadron spectra is the scalar mesons, since there are too many states to be accommodated within the quark model without difficulty [4]. For example, \(K_0^*(700)\) state (also known as \(\kappa\)), together with its multiple partners \(a_0(980), f_0(500)\), does not fit well into the predictions of the quark model, since the observed mass ordering of these lowest scalar states is \(m_{\kappa} < m_{a_0} < m_{f_0}\) [5].

Within the naive quark model, it is natural to assume that the \(a_0(1450), K_0^*(1430), f_0(1710)\), and \(f_0(1370)\) are the \(1^3P_0\) members of the \(SU(3)\) flavor nonet [5]. The isovector scalar mesons \(a_0(2020)/a_0(1950)\) are suggested to be the good candidates of the \(a_0(3^3P_0)\) in our previous work [10]. However, the assignments of the excited scalar \(K_0^*\) states are still unclear. Up to now, above the \(K_0^*(1430)\) mass, only two scalar states \(K_0^*(1950)\) and \(K_0^*(2130)\) are reported. Their masses and widths are listed in Table I.

| state          | mass       | width       | ref  |
|----------------|------------|-------------|------|
| \(K_0^*(1950)\) | 1945±10±20 | 201±34±79   | [5]  |
| \(K_0^*(2130)\) | 2128±31±9  | 95±42±76   | [11] |

Since \(K_0^*(1950)\) was first reported in the \(K\pi\) invariant mass distribution of the \(K^- p \rightarrow K^- \pi^- n\) reaction by LASS in 1988 [12], it is difficult to interpret its properties within the quark model. The \(K_0^*(1950)\) mass is close to the \(K_0^*(2^3P_0)\) mass of about 1890 MeV predicted by the Godfrey-Isgur (GI) quark model [13]. However, it is expected that the \(K_0^*(2^3P_0)\) with a mass of 1850 MeV has a width of about 450 MeV within the \(3P_0\) decay model [14], larger than the \(K_0^*(1950)\) width. In addition, Ref. [15] recently analyzed the kaon family within the modified GI model involving the color screening effect, and predicted the mass and width of the \(K_0^*(2^3P_0)\) to be \(M = 1829\) MeV and \(\Gamma = 1000\) MeV, respectively, both of which disfavor the assignment of \(K_0^*(1950)\) as the candidate the \(K_0^*(2^3P_0)\) state.

The \(K_0^*(2130)\) was recently observed in the \(\eta\) decays by the BABAR Collaboration [11], and its mass is close to the \(K_0^*(3^3P_0)\) mass of 2176 MeV predicted by the modified GI model [15]. In addition, we have estimated that the \(n\bar{n}(3^3P_0)\) mass is about 1.9 ~ 2.0 GeV [10], thus one can naturally expect the \(K_0^*(3^3P_0)\) mass should be about 100 ~ 200 MeV larger than the \(a_0(3^3P_0)\) mass. Based on its mass information, the \(K_0^*(2130)\) seems a good candidate of the \(K_0^*(3^3P_0)\).

The mass information alone is insufficient to identify the \(K_0^*(2130)\) as the \(K_0^*(3^3P_0)\) state. We shall discuss the possibility of the \(K_0^*(2130)\) as the \(K_0^*(3^3P_0)\) state by studying its strong decay properties.

In this work, we will investigate the possible assignment of \(K_0^*(2130)\) by analyzing the strong decay behaviors within the \(3P_0\) decay model. For completeness, we also check the possibility of the \(K_0^*(1950)\) as the ordinary scalar mesons, since it is natural and necessary to exhaust the possible \(q\bar{q}\)
II. MODEL AND PARAMETERS

The $^3P_0$ model has been widely used to study the Okubo-Zweig-Iizuka (OZI)-allowed open flavor two-body strong decays, it was originally introduced by Micu [16] and further developed by Le Yauanc et al. [17–19]. In the $^3P_0$ model, the meson strong decay takes place by producing a quark-antiquark pair with vacuum quantum number $J^{PC} = 0^{++}$. The newly produced quark-antiquark pair, together with the $q\bar{q}$ within the initial meson, regroups into two outgoing mesons in two possible quark rearrangement ways, as shown in Eq. 1. The $^3P_0$ model has been widely applied to study strong decays of hadrons with considerable success [10, 14, 20–33].

![Diagram of Two-body-decay diagrams of $A \rightarrow BC$ according to the $^3P_0$ model.](image)

**FIG. 1:** Two-body-decay diagrams of $A \rightarrow BC$ according to the $^3P_0$ model, where a pair of quark-antiquark are created to form the final two mesons. In the left diagram, Meson $B$ is formed by quark in Meson $A$ combined with the created antiquark, and Meson $C$ is formed by antiquark in Meson $A$ combined with the created quark. In the right diagram, Meson $B$ is formed by antiquark in Meson $A$ combined with the created quark, and Meson $C$ is formed by quark in Meson $A$ combined with the created antiquark.

Following the conventions in Refs. [20, 21], the transition operator $T$ of the decay $A \rightarrow BC$ in the $^3P_0$ model is given by

$$T = -3\gamma \sum_m \langle 1m1 - m|00\rangle \int d^3p_3d^3p_4\delta^3(p_3 + p_4)\mathcal{Y}_m^0(p_3 - p_4)\chi_{1-m}^{34}\phi_0^{34}\omega_0^{34}\bar{b}_3^i(p_3)d_4^j(p_4),$$

where the $\gamma$ is a dimensionless parameter corresponding to the production strength of the quark-antiquark pair $q_3\bar{q}_4$ with quantum number $J^{PC} = 0^{++}$, $p_3$ and $p_4$ are the momenta of the created quark $q_3$ and antiquark $\bar{q}_4$, respectively. $\chi_{1-m}^{34}$, $\phi_0^{34}$, and $\omega_0^{34}$ are the spin, flavor, and color wave functions of $q_3\bar{q}_4$ system, respectively. The solid harmonic polynomial $\mathcal{Y}_m^0(p) = |p|^2\mathcal{Y}_m^0(\theta_p, \phi_p)$ reflects the momentum-space distribution of the $q_3\bar{q}_4$.

The $S$ matrix of the process $A \rightarrow BC$ is defined by

$$\langle BC|S|A\rangle = I - 2\pi\delta(E_A - E_B - E_C)\langle BC|T|A\rangle,$$

where $|A\rangle$ ($|B\rangle$, $|C\rangle$) are the wave functions of the mock mesons defined by Ref. [34].

The transition matrix element $\langle BC|T|A\rangle$ can be written as

$$\langle BC|T|A\rangle = \delta^3(p_A - p_B - p_C)\mathcal{M}^{M_A M_B M_C}(p),$$

where $\mathcal{M}^{M_A M_B M_C}(p)$ is the helicity amplitude.

The partial wave amplitude $\mathcal{M}^{LS}(p)$ can be given by [35].

$$\mathcal{M}^{LS}(p) = \sum_{M_A M_B M_C} \langle LM_LS M_S|J_A M_J\rangle \langle J_B M_J M_C|S M_S\rangle \times \int d\Omega Y_{LM_S}^{M_A M_B M_C}(p).$$

Various $^3P_0$ models exist in literature and typically differ in the choices of the pair-production vertex, the phase space conventions, and the meson wave functions employed. In this work, we restrict to the simplest vertex as introduced originally by Micu [16] which assumes a spatially constant pair-production strength $\gamma$, adopt the relativistic phase space, and employ the relativized quark model (RQM) wave functions [13].

With the relativistic phase space, the decay width $\Gamma(A \rightarrow BC)$ can be expressed in terms of the partial wave amplitude

$$\Gamma(A \rightarrow BC) = \frac{|\mathcal{C}|^2}{4M_A^2} \sum_{LS} |\mathcal{M}^{LS}(p)|^2,$$

where $|\mathcal{C}| = \sqrt{[M_A^2 - (M_B + M_C)^2][M_B^2 - (M_B - M_C)^2]/(2M_A)}$, and $M_A$, $M_B$, and $M_C$ are the masses of the mesons $A$, $B$, and $C$, respectively.

III. RESULTS

In our calculations, the parameters involve the $q\bar{q}$ pair production strength $\gamma$, and the ones in relativized quark model, as used in the work of Godfrey and Isgur [13]. The flavor wave functions for the mesons are adopted by following the conventions of Refs. [13, 14] except for (1) $f_1(1285) = -0.28n\bar{n} + 0.96s\bar{s}$ and $f_1(1420) = -0.96n\bar{n} - 0.28s\bar{s}$ as Ref. [36], (2) $f_2(1295) = (n\bar{n} - s\bar{s})/\sqrt{2}$ and $f_2(1475) = (n\bar{n} + s\bar{s})/\sqrt{2}$ as Ref. [37], where $n\bar{n} = (u\bar{u} + d\bar{d})/\sqrt{2}$.

The total widths of the final mesons are taken from the Review of Particle Physics [5].

We take $\gamma = 0.52$ by fitting to the total width of $K_0^*(1430)$ as the $^3P_0$ state. The decay widths of $K_0^*(1430)$ as the $K_0^*(1^2P_0)$ state are listed in Table II. According to our results, the dominant decay mode of $K_0^*(1430)$ is $K\pi$, which is consistent with the experimental data [5].

With the parameter $\gamma = 0.52$, we have calculated the decay widths of $K_0^*(1950)$ as the $K_0^*(2^1P_0)$ and $K_0^*(3^3P_0)$ states, respectively, which are listed in Table III. The total widths of the $K_0^*(2^1P_0)$ and $K_0^*(3^3P_0)$ states with a mass of 1945 MeV are expected to be 1308 MeV and 74 MeV, respectively. By comparing with the $K_0^*(1950)$ width $\Gamma = 201 \pm 34 \pm 79$.
TABLE II: Decay widths of $K_0^*(1430)$ as $1^3P_0$ state (in MeV).

| Channel | Mode | $\Gamma_0(1^3P_0)$ |
|---------|------|-------------------|
| $0^\pi \to 0^\pi + 0^\pi$ | $K\pi$ | 262.53 |
| $0^\pi \to 0^\pi + 1^\pi$ | $\pi K_{18}$ | < 0.1 |
| Total width | $271.95$ |
| Experiment | $270 \pm 80$ |

TABLE III: Decay widths of $K_0^*(1950)$ as $2^3P_0$ and $3^3P_0$ states (in MeV). The initial state mass is $1945$ MeV.

| Channel | Mode | $\Gamma_0(2^3P_0)$ | $\Gamma_0(3^3P_0)$ |
|---------|------|-------------------|-------------------|
| $0^\pi \to 0^\pi + 0^\pi$ | $\pi K$ | 78.26 | 1.64 |
| $\eta K$ | 1.74 | < 0.01 |
| $\eta K'$ | 11.39 | 1.37 |
| $\pi(1300)K$ | 170.82 | 6.79 |
| $\eta K(1295)$ | 147.80 | 4.15 |
| $\pi K(1460)$ | 91.15 | 0.34 |
| $0^\pi \to 0^\pi + 1^\pi$ | $K_{18}(1260)$ | 57.37 | 0.03 |
| $K_{b1}(1235)$ | 106.81 | 10.46 |
| $h_{1}(1415)K$ | 11.70 | 2.77 |
| $h_{1}(1170)K$ | 5.87 | < 0.01 |
| $K_f(1420)$ | 2.05 | 0.02 |
| $K_f(1285)$ | 4.59 | < 0.01 |
| $\pi K_{1A}$ | 132.10 | 1.13 |
| $\pi K_{1B}$ | 25.27 | 2.08 |
| $\eta K_{1A}$ | 54.80 | 6.70 |
| $0^\pi \to 1^\pi + 1^\pi$ | $K'(892)\eta$ | 204.87 | 18.60 |
| $K'(892)\eta(1020)$ | 9.17 | 0.04 |
| $K'(892)\eta(1475)$ | 192.59 | 17.84 |
| Total width | $1308.38$ | 74.01 |
| Experiment | $201 \pm 34 \pm 9$ |

TABLE IV: Decay widths of $K_0^*(2130)$ as $3^3P_0$ and $4^3P_0$ states (in MeV). The initial state mass is $2128$ MeV.

| Channel | Mode | $\Gamma_0(3^3P_0)$ | $\Gamma_0(4^3P_0)$ |
|---------|------|-------------------|-------------------|
| $0^\pi \to 0^\pi + 0^\pi$ | $\pi K$ | 8.58 | 0.08 |
| $\eta K$ | 0.07 | < 0.01 |
| $\eta K'$ | 0.21 | 0.29 |
| $\pi(1300)K$ | 4.35 | 4.09 |
| $\eta K(1295)$ | 7.38 | 3.55 |
| $\pi K(1460)$ | 8.83 | 2.83 |
| $\eta K(1460)$ | < 0.01 | 0.05 |
| Total width | $271.95$ |
| Experiment | $270 \pm 80$ |

MeV [5], it is hard to assign the $K_0^*(1950)$ as the ordinary scalar $q\bar{q}$ meson. Then we will discuss the possible assignment of the $K_0^*(2130)$ state. We have shown the decay widths of $K_0^*(2130)$ as the $K_0^*(3^3P_0)$ and $K_0^*(4^3P_0)$ states in Table IV, and total width is expected to be about 105 MeV and 20 MeV, respectively for the cases of the $K_0^*(3^3P_0)$ and $K_0^*(4^3P_0)$. The dependences of the total widths of $K_0^*(3^3P_0)$ and $K_0^*(4^3P_0)$ on the initial state mass are shown in Fig. 2 and Fig. 3, respectively. Within the uncertainties of the $K_0^*(2130)$ mass, the total width of $K_0^*(3^3P_0)$ is in agreement with the experimental data $\Gamma = 95 \pm 42 \pm 76$ MeV [5], which implies that the $K_0^*(2130)$ should be the good candidate of the $K_0^*(3^3P_0)$ state.

Phenomenologically, it is suggested that the light mesons could be grouped into the following Regge trajectories [29, 33, 38],

$$M_n^2 = M_0^2 + (n-1)\mu^2,$$

where $M_0$ is the lowest-lying meson mass, $n$ is the radial quantum number, and $\mu^2$ is the slope parameter of the corresponding trajectory. In the presence of $K_0^*(1430)$ and $K_0^*(2130)$ being the $K_0^*(1^3P_0)$ and $K_0^*(3^3P_0)$ states, we can roughly estimate the $K_0^*(2^3P_0)$ mass to be about 1811 MeV and $K_0^*(4^3P_0)$ mass to be about 2404 MeV as shown in Fig. 4.

It should be pointed out that, in our previous work [10], the mass scale for the $n\bar{q}q\bar{q}$ nonets is expected to be $1700 \sim 1800$ MeV. In addition, the mass of $K_0^*(2^3P_0)$ state is predicted.

FIG. 2: The dependence of the total width of $K_0^*(3^3P_0)$ on the initial state mass. The experimental total width of the $K_0^*(2130)$ is denoted by the dashed line with a green band.
The dominant decay modes of \( K_0^*(2^3P_0) \) are presented in Table V. The initial state mass is 1811 MeV.

| Channel      | Mode       | \( \Gamma_i(2^3P_0) \) |
|--------------|------------|-------------------------|
| \( 0^+ \rightarrow 0^- + 0^- \) | \( \pi K \) | 57.13                   |
| \( \pi(1300)K \) | 96.06      |
| \( K\eta \) | 1.01       |
| \( K\eta' \) | 0.77       |
| \( K\eta(1295) \) | 100.95     |
| \( \pi K(1460) \) | 99.91      |
| \( 0^+ \rightarrow 0^- + 1^- \) | \( h_0(1170)K \) | 4.42                    |
| \( K\eta_1(1260) \) | 16.35      |
| \( Kf_0(1285) \) | 0.42       |
| \( Kb_1(1235) \) | 33.29      |
| \( \pi K_{1A} \) | 64.31      |
| \( \pi K_{1B} \) | 24.37      |
| \( 0^+ \rightarrow 1^- + 1^- \) | \( K^*(892)\rho \) | 81.42                  |
| \( K^*(892)\omega \) | 75.93      |
| Total width | 656.34     |

FIG. 3: The dependence of the total width of \( K_0^*(4^2P_0) \) on the initial state mass.

FIG. 4: The Regge trajectory of the \( K_0^*(n^3P_0) \) \( (n = 1, 2, 3, 4) \) states, where \( M_n \) is its mass, and the red line is the fitting result.

FIG. 5: The dependence of the total width of \( K_0^*(2^3P_0) \) on the initial state mass.

88 \( \sim \) 246 MeV.

to be 1890 MeV by the GI model [13] and to be 1829 MeV by the modified GI model [15]. The prediction of the \( K_0^*(2^3P_0) \) mass from the Regge trajectories is consistent with these previous predictions. The strong decays of \( K_0^*(2^3P_0) \) with a mass of 1811 MeV are presented in Table V. The dependence of the total width of \( K_0^*(2^3P_0) \) on the initial state mass is shown in Fig. 5. When the initial state mass varies from 1700 MeV to 1900 MeV, the total width of the \( K_0^*(2^3P_0) \) varies from about 215 MeV to 1111 MeV. The decay width varies greatly, since some decay modes are open gradually. The total width of \( K_0^*(2^3P_0) \) is expected to be about 656 MeV. The dominant decay modes of \( K_0^*(2^3P_0) \) are \( \pi(1300)K, K\eta(1295), \) and \( \pi K(1460) \). The \( K_0^*(2^3P_0) \) is predicted to be a broad state, which could be the reason that the \( K_0^*(2^3P_0) \) candidate is not yet observed experimentally.

We show the partial decay widths and the total decay width of the \( K_0^*(4^3P_0) \) state with a mass of 2404 MeV in Table VI. The total width of \( K_0^*(4^3P_0) \) is expected to be about 180 MeV. The dominant decay modes of \( K_0^*(4^3P_0) \) include \( K^*(1410)^+\omega, K^*(1410)^+\rho, K^*(892)^-\omega(1420) \). The dependence of the total width of the \( K_0^*(4^3P_0) \) state on the initial state mass is shown in Fig. 3. When the initial state mass varies from 2300 MeV to 2500 MeV, the total width of the \( K_0^*(4^3P_0) \) varies from about

TABLE V: Decay widths of \( K_0^*(2^3P_0) \) (in MeV). The initial state mass is 1811 MeV.
| Channel | Mode | $\Gamma_i(4^3P_0)$ |
|---------|------|-----------------|
| $0^+ \rightarrow 0^- + 0^-$ | $\pi K$ | 0.60 |
|  | $K \eta$ | < 0.01 |
|  | $K \eta^*$ | 0.28 |
|  | $\pi(1300)K$ | 0.16 |
|  | $K \eta(1295)$ | 0.07 |
|  | $\pi K(1460)$ | 0.12 |
|  | $K \eta(1475)$ | < 0.01 |
|  | $K(1460)\eta$ | 0.31 |
| $0^+ \rightarrow 0^- + 1^+$ | $K_{10}(1260)$ | 0.70 |
|  | $K b_1(1325)$ | 4.50 |
|  | $h_1(1415)K$ | 4.60 |
|  | $h_1(1170)K$ | 0.06 |
|  | $K f_1(1420)$ | 0.21 |
|  | $K f_1(1285)$ | 0.03 |
|  | $\pi K_{1A}$ | 0.02 |
|  | $\pi K_{1B}$ | 1.69 |
|  | $\eta K_{1A}$ | 2.74 |
|  | $\eta K_{1B}$ | 0.98 |
|  | $\eta K_{1A}$ | 0.08 |
|  | $\eta K_{1B}$ | 0.01 |
|  | $K_{10}(1640)$ | 0.23 |
| $0^+ \rightarrow 0^+ + 1^-$ | $K^*(1430)\rho$ | 0.09 |
| $0^+ \rightarrow 1^- + 1^-$ | $K^*(892)\rho$ | 8.76 |
|  | $K^*(892)\rho(1450)$ | 6.34 |
|  | $K^*(892)\omega(1420)$ | 19.83 |
|  | $K^*(892)\phi(1020)$ | 0.02 |
|  | $K^*(892)\omega$ | 8.15 |
|  | $K^*(1410)\rho$ | 32.46 |
|  | $K^*(1410)\omega$ | 36.79 |
| $0^+ \rightarrow 1^+ + 1^-$ | $K^*(892)_{10}(1260)$ | 2.50 |
|  | $K^*(892)_{b_1}(1235)$ | 0.03 |
|  | $K^*(892)_{b_1}(1170)$ | 0.13 |
|  | $K^*(892)_{b_1}(1415)$ | < 0.01 |
|  | $K^*(892)_{f_1}(1420)$ | 0.69 |
|  | $K^*(892)_{f_1}(1285)$ | 0.34 |
|  | $\rho K_{1A}$ | < 0.01 |
|  | $\omega K_{1A}$ | < 0.01 |
|  | $\phi K_{1B}$ | 0.25 |
|  | $\rho K_{1B}$ | 1.29 |
|  | $\omega K_{1B}$ | 0.22 |
| $0^+ \rightarrow 0^- + 2^-$ | $\pi K_{2}(1820)$ | 0.17 |
|  | $\pi K_{2}(1770)$ | 2.03 |
|  | $K \pi_{2}(1670)$ | 7.15 |
|  | $\eta_{2}(1645)K$ | 4.02 |
|  | $\eta_{2}(1870)K$ | 0.07 |
|  | $\eta K_{2}(1820)$ | 0.05 |
|  | $\eta K_{2}(1770)$ | < 0.01 |
| $0^+ \rightarrow 1^- + 2^-$ | $K^*(892)_{2}(1270)$ | 0.93 |
|  | $K^*(892)_{a_2}(1320)$ | 12.56 |
|  | $K_{2}(1430)\rho$ | 17.92 |
| Total width | | 180.23 |
IV. SUMMARY

In this work, we have discussed the possible assignments of $K_0'_{1s}(1950)$ and $K_0^*(2130)$ by calculating the strong decay widths within the $1^3P_0$ strong decay model. We suggest that the $K_0^*(2130)$ could be assigned as $K_0^*(3^3P_0)$ based on its mass and width. However, the $K_0^*$ (1950) seems like an exotic state, and its width cannot be reasonably reproduced within the $3^3P_0$ model.

With the assignment of the $K_0^*(2130)$ as the $K_0^*(3^3P_0)$ state, we have roughly estimated the masses of $K_0^*(2^3P_0)$ and $K_0^*(4^3P_0)$ to be about 1811 MeV and 2404 MeV, respectively, within the Regge trajectories. The total width of $K_0^*(2^3P_0)$ is predicted to be about 656 MeV, which implies that this state is not easy to be observed experimentally. The total width of $K_0^*(4^3P_0)$ is predicted to be about 180 MeV, which could be helpful to search for the $K_0^*(4^3P_0)$ state in future.

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