Dynamic loading of the beam on the Pasternak base initiated by the sudden settlement of part of the base

V I Travush¹, V A Gordon², V I Kolchunov³, and E V Leontiev⁴

¹ Russian academy of architecture and construction science, 24/1, Bolshaya Dmitrovka St., Moscow, Russia, 107031
² Orel State University named after I.S. Turgenev, 95, Komsomolskaya St., Orel, Russia, 302026
³ Southwest State University, 94, 50 let Oktyabrya, Kursk, Russia, 305040
⁴ Main State Expertise of Russia, 42/1-2, Bolshaya Yakimanka St., Moscow, Russia, 119049

E-mail: travush@mail.ru; asiorel@mail.ru

Abstract. The article presents a methodology for the analytical assessment of the dynamic loading of a beam based on a mathematical model of a transient dynamic process in a loaded structure resting on an elastic two-parameter Pasternak base, initiated by sudden settlement of a part of the base. Determine that the sudden formation of a defect leads to a decrease in the overall rigidity of the structure and a violation of the static balance of the beam-base system. The arising inertia forces cause a dynamic response, redistribution and growth of deformations and stresses. As a result, there may be a violation of the regular functioning of the structure, loss of bearing capacity and destruction. From the standpoint of structural mechanics, the problem boils down to an analysis of the manifestations of structural nonlinearity of a loaded elastic system. It is shown that the factor of sudden formation of a defect significantly increases internal forces compared with the quasistatic formation of the same defect. It is also shown that taking into account the Pasternak parameter reduces (compared with the one-parameter Winkler base) the level of dynamic bending moments during sudden partial destruction of the base.

1. Introduction

In domestic and foreign researches and requirements documents on the issues of accounting for special effects and protecting buildings and structures from progressive collapse [1-4]. In the construction «beam-base» systems, along with this scenario, a scenario of defect formation in the form of sudden settlement in the base part and, accordingly, structural restructuring of the entire structural system is possible. In this regard, a new task arises for the computational analysis of the static and dynamic deformation of the beam on a structurally inhomogeneous base.

The mathematical model of the system includes the differential equations of static and dynamic bending of the beam fully and partially supported on the basis of Pasternak. The equations are written in matrix form. The solution is constructed by the method of initial parameters using state vectors of sections and matrices of the influence of initial parameters on the state of arbitrary sections. The
influence of the generalized stiffness parameters of the “beam-base” system and the size of the damage on the stress-strain state of the beam is analyzed.

2. Methods

Equation of natural bending vibrations of the «beam-base» systems (Figure 1)

\[ \frac{\partial^4 w_i}{\partial \xi_i^4} - 4\beta^2 \frac{\partial^2 w_i}{\partial \xi_i^2} + 4\alpha^4 \left( w_i + \frac{\partial^2 w_i}{\partial \tau^2} \right) = 0, \]  
(1)

where \( \xi = x / L; w_i = v_i / L \) is deflection; \( \alpha^2 = \frac{K_i L_i}{4EI}; \beta^2 = \frac{K_2 L^2}{4EI}; \nu = \frac{L_i}{L}; K_1 = \bar{K}_1 B; K_2 = \bar{K}_2 B; \bar{K}_i, \bar{K}_2 \) - respectively is bending and shear stiffness of the base; \( B \) is beam cross section width.

Figure 1. Beam partially supported on elastic foundation

We transform equation (1) by introducing two parameters having a frequency dimension called “conditional” frequencies \( \omega_{01} = \frac{\bar{K}_1}{\rho A}, \omega_{02} = \frac{\bar{K}_2}{2L\sqrt{\rho A}} \), where \( \rho \) is material density, \( A \) is beam cross-sectional area, to the form

\[ \frac{\partial^4 w_i}{\partial \xi_i^4} - 4\omega_{02}^2 \frac{\partial^2 w_i}{\partial \xi_i^2} + \omega_{01}^2 \left( w_i + \frac{\partial^2 w_i}{\partial \tau^2} \right) = 0, \]  
(2)

where \( \omega_{0i} = \frac{\omega_{01}}{\omega_{02}}, (i=1,2) \) are relative "conditional" frequencies, \( \omega_{02} = \frac{1}{L^2} \sqrt{\frac{EI}{\rho A}} \) is calibrating frequency, \( \tau = \omega_{01} t \) is nondimensional time.

We separate the variables in equation (2) by representing that the oscillations are harmonic

\[ w_i(\xi, \tau) = W_i(\xi) \sin \omega \tau = 0, \]

where \( \omega = \frac{\omega}{\omega_{01}} \) is dimensionless desired frequency, we obtain the equation of the forms of natural vibrations

\[ W_i^{iv} - 4\omega_{02}\omega_i W_i'' + \left( \omega_{01}^2 - \omega^2 \right) W_i = 0, \]
(3)

where \( \tilde{\omega} = \frac{\omega}{\omega_{02}} \) is dimensionless desired frequency.

The structure of equation (3) suggests the possibility of the following solutions:
If the $\hat{\omega} = \overrightarrow{\omega}_{01}$, then the deflection function is as follows

$$W_i = A_i + A_2\xi_i + A_4\sin 2\beta \xi_i + A_4\sin 2\beta \xi_i,$$

(4)

where $A_j (j = 1+4)$ are constant integration.

In works [5-19], close to topic, the efficiency of applying the initial parameter method in combination with a vector-matrix representation of the state of an arbitrary beam section for analyzing displacements and stresses when it interacts with the base is shown. A similar approach is used in this article.

Replacing the integration constants $A_j$ with the initial parameters of the first section $w_{0i} = w_{0i}(0), w_{0i}^f = w_{0i}^f(0), w_{0i}^{ij} = w_{0i}^{ij}(0), w_{0i}^{iii} = w_{0i}^{iii}(0)$, we bring function (4) to the form:

$$W_i = w_{0i} + \xi_i w_{0i}^f + \frac{1}{4\beta^2}(ch2\beta \xi_i - 1)w_{0i}^{ij} + \frac{1}{8\beta^2}(sh2\beta \xi_i - 2\beta \xi_i)w_{0i}^{iii}.$$

Now the state of an arbitrary section $\xi_i$ can be represented by the matrix equation:

$$\overrightarrow{W_i(\xi_i)} = V_{1i}(\xi_i)\overrightarrow{W_{0i}},$$

(5)

where $\overrightarrow{W_{0i}} = \{W_{0i}, W_{0i}^f, W_{0i}^{ij}, W_{0i}^{iii}\}$ is vector of initial parameters;

$$V_{1i}(\xi_i) = \begin{pmatrix} 1 & \xi_i & \frac{1}{4\beta^2}(ch2\beta \xi_i - 1) & \frac{1}{8\beta^2}(sh2\beta \xi_i - 2\beta \xi_i) \\ 0 & 1 & \frac{1}{2\beta}sh2\beta \xi_i & \frac{1}{4\beta^2}(ch2\beta \xi_i - 1) \\ 0 & 0 & ch2\beta \xi_i & \frac{1}{2\beta}sh2\beta \xi_i \\ 0 & 0 & 2\beta sh2\beta \xi_i & ch2\beta \xi_i \end{pmatrix}$$

is functional matrix of the influence of the initial parameters on the state of the section $\xi_i$.

Equations similar to equation (5) are constructed for the cases $\hat{\omega}_{0i} > \hat{\omega}$

$$\overrightarrow{W_i(\xi_i)} = V_{12}(\xi_i)\overrightarrow{W_{0i}},$$

(6)

and $\hat{\omega} < \hat{\omega}_{0i}$

$$\overrightarrow{W_i(\xi_i)} = V_{13}(\xi_i)\overrightarrow{W_{0i}},$$

(7)

where matrix $V_{12}(\xi_i)$ and $V_{13}(\xi_i)$ and their elements have the form:

$$V_{12}(\xi_i) = \begin{pmatrix} B_1(\xi_i) & B_2(\xi_i) & B_3(\xi_i) & B_1(\xi_i) \\ (rp)^2B_1(\xi_i) & B_2(\xi_i) & B_3(\xi_i) & B_1(\xi_i) \\ (rp)^2B_2(\xi_i) & (rp)^2B_1(\xi_i) & B_2(\xi_i) & B_1(\xi_i) \\ (rp)^2B_3(\xi_i) & (rp)^2B_2(\xi_i) & B_2(\xi_i) & B_1(\xi_i) \end{pmatrix};$$

$$V_{13}(\xi_i) = \begin{pmatrix} B_1(\xi_i) & B_2(\xi_i) & B_3(\xi_i) & B_1(\xi_i) \\ B_2(\xi_i) & B_3(\xi_i) & B_3(\xi_i) & B_2(\xi_i) \\ B_3(\xi_i) & B_3(\xi_i) & B_3(\xi_i) & B_3(\xi_i) \\ B_2(\xi_i) & B_3(\xi_i) & B_3(\xi_i) & B_3(\xi_i) \end{pmatrix};$$
\[ B_1 = \frac{\text{rshp} \xi_1 - p \sin r \xi_2}{rp(r^2 + p^2)}; B_2 = \frac{\text{chp} \xi_1 - \cos r \xi_1}{r^2 + p^2}; B_3 = \frac{r^3 \text{shp} \xi_1 - p^3 \sin r \xi_1}{rp(r^2 + p^2)};\]

\[ B_4 = \frac{\varepsilon^2 \text{chp} \xi_1 - p^2 \cos r \xi_1}{r^2 + p^2}; p = \sqrt{r(\rho + \gamma + \beta^2)}; r = \sqrt{2(\rho + \gamma^2 - \beta^2)};\]

\[ \gamma = \sqrt{\frac{d^2 - \omega^2}{4}} \]

\[ V_{ij} = \begin{pmatrix} F_1(\xi) & F_2(\xi) & F_3(\xi) & F_4(\xi) \\ F_1(\xi) & F_2(\xi) & B_2(\xi) & B_3(\xi) \\ F_3(\xi) & F_4(\xi) & F_5(\xi) & F_6(\xi) \\ F_4(\xi) & F_5(\xi) & F_6(\xi) & F_7(\xi) \end{pmatrix}; \]

\[ F_i = \frac{\text{cha} \xi_1 \sin b \xi_2 - \text{sha} \xi_1 \cos b \xi_2}{2b(a^2 + b^2)}; F_2 = \frac{\text{sha} \xi_1 \sin b \xi_2}{2ab}; \]

\[ F_3 = \frac{3a^2 - b^2}{2a(a^2 + b^2)} \text{sha} \xi_1 \cos b \xi_2 - \frac{a^2 - 3b^2}{2b(a^2 + b^2)} \text{cha} \xi_1 \sin b \xi_2; \]

\[ F_4 = \text{cha} \xi_1 \text{cjsb} \xi_1 - \frac{a^2 - b^2}{2ab} \text{sha} \xi_1 \sin b \xi_2; \]

\[ a = \sqrt{\delta^2 + \beta^2}; b = \sqrt{\delta^2 - \beta^2}; \delta = \sqrt{\frac{\omega_{01} - \omega^2}{4}} \]

The equation of natural bending vibrations of this section has the form [20-23]

\[ \frac{\partial^4 w_2}{\partial \xi^4} + 4a^4 \frac{\partial^2 w_2}{\partial \xi^2} = 0. \quad (8) \]

Moreover, the state of an arbitrary section of this section \( \xi_2 \) is determined by the matrix equation

\[ \overline{W}_2(\xi_2) = V_2(\xi_2)V_0^2(\nu)\overline{W}_{01}, \quad (j = 1 \pm 3) \quad (9) \]

where the matrix of the influence of the initial parameters of the first section on the state of an arbitrary section \( \xi_2 \) of the second section has the form

\[ V_2 = \begin{pmatrix} R_4(\xi) & R_3(\xi) & R_2(\xi) & R_1(\xi) \\ \beta_3^4 R_4(\xi) & \beta_3^4 R_3(\xi) & \beta_3^4 R_2(\xi) & \beta_3^4 R_1(\xi) \\ \beta_3^3 R_2(\xi) & \beta_3^3 R_1(\xi) & R_4(\xi) & R_3(\xi) \\ \beta_3^2 R_4(\xi) & \beta_3^2 R_3(\xi) & \beta_3^2 R_2(\xi) & R_1(\xi) \end{pmatrix}; \]

\[ R_1(\xi_2) = \frac{\text{sh} \beta_3 \xi_2 - \sin \beta_3 \xi_2}{2\beta_3}; R_2(\xi_2) = \frac{\text{ch} \beta_3 \xi_2 - \cos \beta_3 \xi_2}{2\beta_3}; \]

\[ R_3(\xi_2) = \frac{\text{sh} \beta_3 \xi_2 + \sin \beta_3 \xi_2}{2\beta_3}; R_4(\xi_2) = \frac{\text{ch} \beta_3 \xi_2 + \cos \beta_3 \xi_2}{2}; \]

When deriving equation (9), the conditions of conjugation of sections were used \( \overline{W}_2(0) = \overline{W}_1(\nu). \)
For further constructions, it is necessary to determine the boundary conditions: below we consider the free ends of the beam with conditions of the form

\[ W_{01}^H = W_{01}^{III} = 0 \]
\[ W_2^H (1 - \nu) = W_2^{III} (1 - \nu) = 0 \]  

(10)

Natural frequencies and forms of bending vibrations of a beam with free ends partially supported by Pasternak’s two-parameter base.

This section defines the eigenfrequencies \( \tilde{\omega}_n \) and corresponding vibration modes of a beam with free ends, interacting with the Pasternak elastic base for various possible combinations of the desired \( \tilde{\omega}_n \) and known "conditional" \( \tilde{\omega}_{01} \) frequencies.

At the beginning, we accept the condition according to which the desired frequency \( \tilde{\omega}_n \), partially supported by the beam base, is equal to the "conditional" frequency \( \tilde{\omega}_{01} \).

This condition means the formulation of the following problem: for a given beam oscillating together with a base with an unknown frequency \( \tilde{\omega}_n \) to match a certain "conditional" free (without base) beam, the natural frequency of which will be the same as that of this beam, then \( \tilde{\omega}_n = \tilde{\omega}_{01} \).

When the problem is solved, that is, the \( \tilde{\omega}_n \) (or \( \tilde{\omega}_{01} \)) is found from the condition

\[ \frac{\omega_{01}}{\omega_n} = \frac{K_1}{K_2} \frac{1}{L} \frac{EI}{\rho A} = \sqrt{\frac{K_1 L}{EI}} \]

the combination of the parameters of the beam-base system \((K_1, L, E, I)\) can be obtained with a known length of the bearing section and a given characteristic of the base \( K_2 \), providing the frequency and the corresponding form of vibrations.

In this case \(( \tilde{\omega} = \tilde{\omega}_{01} \) ), the state of the sections is described by equations (5) and (9). Satisfying the boundary conditions (10), we obtain the frequency equation

\[ \text{ch}(\beta_1(1 - \nu)) \cos(\beta_1(1 - \nu)) = 1, \]  
\[ \text{ch}(\beta_2(1 - \nu)) \cos(\beta_2(1 - \nu)) = 1, \]  
\[ \text{ch}(\beta_3(1 - \nu)) \cos(\beta_3(1 - \nu)) = 1, \]  
\[ \text{ch}(\beta_4(1 - \nu)) \cos(\beta_4(1 - \nu)) = 1, \]  

(11)

whose roots \( \beta_{z1} = 0; \beta_{z2}(1 - \nu) = 4.73; \beta_{z3}(1 - \nu) = 7.853; \beta_{z4}(1 - \nu) = \frac{Z + 1}{2} \pi \) at \( n > 3 \) give physically parallel results when, at \( \nu = 1 \), with full support \( \lim_{\nu \to 1} \beta_{z1} = \lim_{\nu \to 1} \sqrt{\beta_0} = \lim_{\nu \to 1} \frac{4.73}{1 - \nu} = \infty \) at \( \nu = 1 \), which means the unrealizability of the accepted condition \( \tilde{\omega} = \tilde{\omega}_{01} \).

Within the framework of the adopted model of the «beam-base» system, the presence of an arbitrarily small length \( \nu \neq 0 \) of the beam portion interacting with the base excludes the possibility of beam movement with free ends as an absolutely rigid body. In this case, the calculation of the main, first, natural frequency should be performed according to option (7) and (9), that is, provided \( \tilde{\omega}_n < \tilde{\omega}_{01} \) beginning with \( \tilde{\omega}_1 = 0 \) at \( \nu \to 0 \) and \( \tilde{\omega}_{01} \neq 0 \).

Accepting this condition, the state of the beam in the sections is described by

\[ \overline{W}_1(\xi_1) = V_{13}(\xi_1) \overline{W}_{01} \]
\[ \overline{W}_2(\xi_2) = V_2(\xi_2) V_{13}(\nu) \overline{W}_{01} \]

From the second matrix equation we obtain the frequency equation

\[ z_1 z_2 - z_3 z_4 = 0, \]  

(12)
where

\[
\begin{align*}
    z_1 &= \beta_1^i \left[ R_2(1-\nu)F_4(v) + R_1(1-\nu)F_4^1(v) + R_4(1-\nu)F_4^ii(v) + R_3(1-\nu)F_4^iii(v) \right]; \\
    z_2 &= \beta_2^i \left[ R_3(1-\nu)F_3(v) + R_1(1-\nu)F_3^1(v) + R_4(1-\nu)F_3^ii(v) + R_3(1-\nu)F_3^iii(v) \right]; \\
    z_3 &= \beta_3^i \left[ R_3(1-\nu)F_2(v) + R_2(1-\nu)F_2^1(v) + R_4(1-\nu)F_2^ii(v) + R_3(1-\nu)F_2^iii(v) \right]; \\
    z_4 &= \beta_4^i \left[ R_3(1-\nu)F(v) + 2R_2(1-\nu)F^1(v) + R_4(1-\nu)F^ii(v) + R_3(1-\nu)F^iii(v) \right].
\end{align*}
\]

The vibration forms of the sections after calculating frequencies from (12) are described by functions

\[
W_{1n}(\xi_1) = F_{4n}(\xi_1) - UF_{3n}(\xi_1)
\]

\[
W_{2n}(\xi_2) = R_{3n}(\xi_2)F_3(v) + R_{4n}(\xi_2)F_3^1(v) + R_{2n}(\xi_2)F_3^ii(v) + R_{1n}(\xi_2)F_3^iii(v) - U \left[ R_{4n}(\xi_2)F_3(v) + R_{3n}(\xi_2)F_3^1(v) + R_{2n}(\xi_2)F_3^ii(v) + R_{1n}(\xi_2)F_3^iii(v) \right],
\]

\[
U = \frac{z_1}{z_4}.
\]

The bending moments in the sections are determined by the functions

\[
M_{1n}(\xi_1) = F_{4n}(\xi_1) - UF_{3n}(\xi_1)
\]

\[
M_{2n}(\xi_2) = \beta_2^i \left[ R_{2n}(\xi_2)F_4(v) + R_{3n}(\xi_2)F_4^1(v) + R_{4n}(\xi_2)F_4^ii(v) + R_{3n}(\xi_2)F_4^iii(v) \right] - U \left[ \beta_2^i \left( R_{2n}(\xi_2)F_4(v) + R_{3n}(\xi_2)F_4^1(v) + R_{4n}(\xi_2)F_4^ii(v) + R_{3n}(\xi_2)F_4^iii(v) \right) \right].
\]

We accept the condition \( \hat{\omega}_n = \hat{\omega}_{01} \). The state of the plots in this case can be represented by the matrix equations (14) and (26)

\[
W_1(\xi_1) = V_{12}(\xi_1) \bar{W}_{01}
\]

\[
W_2(\xi_2) = V_2(\xi_2) V_{12}(\nu) \bar{W}_{01}.
\]

We obtain the frequency equation

\[
T_1 T_2 - T_3 T_4 = 0,
\]

where

\[
T_1 = \beta_1^i \left( R_2(1-\nu)B_4(v) + (rp)^2 \left( \beta_1^i R_1(1-\nu)B_1(v) + R_4(1-\nu)B_2(v) + R_3(1-\nu)B_3^2(v) \right) \right);
\]

\[
T_2 = \beta_2^i \left( R_3(1-\nu)B_4(v) + R_1(1-\nu)B_4(v) + (rp)^2 \left( R_4(1-\nu)B_1(v) + R_3(1-\nu)B_2(v) \right) \right);
\]

\[
T_3 = \beta_3^i \left( R_3(1-\nu)B_4(v) + (rp)^2 \left( R_1(1-\nu)B_1(v) + R_4(1-\nu)B_2(v) + (rp)^2 R_3(1-\nu)B_3^2(v) \right) \right);
\]

\[
T_4 = \beta_4^i \left( R_3(1-\nu)B_4(v) + R_2(1-\nu)B_4(v) + (rp)^2 R_4(1-\nu)B_1(v) + (rp)^2 R_3(1-\nu)B_2(v) \right).
\]

In this case, the vibration modes and bending moments of the sections after calculating the frequencies from (13) are determined respectively by functions

\[
W_{1n}(\xi_1) = B_{4n}(\xi_1) - UF_{3n}(\xi_1);
\]

\[
W_{2n}(\xi_2) = R_{3n}(\xi_2)B_4(v) + (rp)^2 \left( R_{3n}(\xi_2)B_1(v) + R_{2n}(\xi_2)B_2(v) + R_{1n}(\xi_2)B_3^2(v) \right) - U \left[ R_{4n}(\xi_2)B_4(v) + R_{3n}(\xi_2)B_2(v) + (rp)^2 R_{2n}(\xi_2)B_1(v) + R_{1n}(\xi_2)B_3(v) \right],
\]

\[
M_{1n}(\xi_1) = (rp)^2 \left( B_{2n}(\xi_1) - UF_{3n}(\xi_1) \right);
\]

\[
M_{2n}(\xi_2) = \beta_2^i \left( R_{2n}(\xi_2)B_4(v) + (rp)^2 R_{4n}(\xi_2)B_1(v) + (rp)^2 R_{3n}(\xi_2)B_2(v) + R_{1n}(\xi_2)B_3^2(v) \right) - U \left[ \beta_2^i \left( R_{2n}(\xi_2)B_4(v) + R_{4n}(\xi_2)B_1(v) + (rp)^2 R_{3n}(\xi_2)B_2(v) + R_{1n}(\xi_2)B_3(v) \right) \right].
\]

\[
U_1 = \frac{T_3}{T_4}.
\]
3. Results and Discussion

We calculate the deflections and bending moments in the "beam-base" system with the following characteristics: beam length $L = 6.7$ m; cross section – a rectangle with sides: width $B = 0.25$ m, height $h = 0.18$ m; cross-sectional area $A = 0.045$ m$^2$; axial moment of inertia $I = 1.215 \cdot 10^{-4}$ m$^4$; Young's modulus of the beam material $E = 3.05 \cdot 10^{10}$ N/m$^2$; base material varies. The base material is gravel with a hardness parameter $K_1 = 7.5$ MPa m$^3$, $m = 7.5$ N/m$^3$.

The bedding value for this material, taking into account the width of the base of the beam $B = 0.25$ m, $K_1 B = 1.875 \cdot 10^6$ N/m$^3$.

One of the generalized parameters of the beam-base system $\alpha$ becomes $\alpha = \sqrt{\frac{K_1 L^2}{4EI}} = 3.976$.

The experimental data related to the second parameter $K_2$ of the «beam-base» system are clearly not enough, but according to the recommendations given in [11-17], we accept $K_2 = 0.35K_1 m^2 = 0.65 \cdot 10^6$ N, then $\beta = \sqrt{\frac{K_2 L^2}{4EI}} = 1.41$.

Given the known relations $4\alpha^4 = \bar{\omega}_{01}, \beta = \bar{\omega}_{02}$

We accept as the base values of relative "conditional" frequencies $\bar{\omega}_{01} = 32, \bar{\omega}_{02} = 1.41$

Table 1 shows the values of the first three dimensionless frequencies $\bar{\omega}_j$ obtained from the solution of equations (12) and (13) for two combinations of "conditional" frequencies $\bar{\omega}_{01}$ and $\bar{\omega}_{02}$ that characterize the generalized stiffness of the «beam-base» system and the length $\nu$ of the preserved section of the base after its partial destruction.

**Table 1. Natural frequencies of the beam, partially supported on the base of Pasternak**

| $\bar{\omega}_j/\bar{\omega}_{02}$ | $\bar{\omega}_1$ | $\bar{\omega}_2$ | $\bar{\omega}_3$ |
|-------------------------------|-----------------|-----------------|-----------------|
| $\nu = 0.25$                  | 11/0.23         | 0.812           | 2.746           |
|                               |                 | 5.857           | 23.24           |
|                               |                 | 23.73           | 24.35           |
|                               |                 | 61.9            | 62.2            |
|                               |                 | 62.4            |                 |
| $\nu = 0.75$                  | 32/1.41         | 2.15            | 5.64            |
|                               |                 | 12.46           | 33.48           |
|                               |                 | 34.06           | 38.4            |
|                               |                 | 63.9            | 67.5            |
|                               |                 |                 | 70.1            |

The modes of natural vibrations corresponding to these frequencies and the distribution of bending moments along the length of the beam are shown in Figure 2 for the values of $\nu = 0.5$. Forced vibrations of the beam on the elastic base of Pasternak.
Figure 2. The first three forms of vibration of the deflections and bending moments of the beam

Forced beam vibrations initiated by partial destruction of the Pasternak base are described by the differential equation

\[
\frac{\partial^4 w_d}{\partial \xi^4} - 4 \beta^2 \frac{\partial^2 w_d}{\partial \xi^2} + 4 \alpha^4 \left( w_d + \frac{\partial^2 w_d}{\partial \tau^2} \right) = q
\]  

(14)
where $w_d = w_d(\xi, \tau)$ - dimensionless intensity of evenly distributed load.

Separating the variables in equation (14) using the series

$$w_d = \sum_{n=1}^{\infty} Q_n(\tau)W_n(\xi)$$

(15)

where $W_n = W_n(\xi)$ are eigenfunctions obtained by conjugating the eigenfunctions of sections $W_n(\xi_1)u, W_n(\xi_2); Q_n = Q_n(\tau)$ are the time functions $\tau$ to be determined, we obtain the equation for determining $Q_n(\tau)$

$$\frac{d^2Q_n}{d\tau^2} + \omega_n^2 Q_n = R_n$$

(16)

$$R_n = \frac{1}{\omega_n^2} \int_{0}^{1} \left( \frac{\partial w_n(\xi)}{\partial \xi} \right) d\xi$$

where

The general solution of equation (14) has the form

$$w_d = \sum_{n=1}^{\infty} (D_{1n} \cos \omega_n \tau + D_{2n} \sin \omega_n \tau + \frac{R_n}{\omega_n^2})W_n(\xi)$$

(17)

The integration constants $D_{1n}$ u $D_{2n}$ are determined from the initial conditions

$$w_d(\xi, 0) = w_{st}(\xi)$$

$$\frac{\partial w_d}{\partial \tau} \bigg|_{\xi, 0} = 0$$

(18)

where $w_d(\xi)$ is static bending of the beam, fully supported on an elastic base, determined from the equation

$$\frac{d^4w_{st}}{d\xi^4} + 4\alpha^4 w_{st} = q$$

(19)

The beam, freely lying on the elastic base of Pasternak and loaded with a uniformly distributed load $\bar{q} = const$, as follows from the solution of equation (19), plunges into the base without bending

$$w_{st} = -\frac{\bar{q}}{4\alpha^4} = -\frac{\bar{q}}{\omega_{01}^2}$$

by the amount

From the second condition (18) it follows $D_{2n} = 0$, and from the first condition we get

$$\sum_{n=1}^{\infty} (D_{1n} + \frac{R_n}{\omega_n^2})W_n(\xi) = w_{st}$$

(20)

Multiplying both sides of (20) by $W_n(\xi)$ and integrating over $\xi$ from 0 to 1, we get
Substituting the constants $D_{in}W_{2n}$ in the series (17), we obtain

$$w_d(\xi, \tau) = \sum_{n=1}^{\infty} (B_n \cos \omega_n \tau + C_n \sin \frac{\omega_n \tau}{2})W_n(\xi)$$  

where

$$C_n = \frac{2\eta}{\omega n^2} \int_0^1 W_n d\xi$$

$$M_d = W_{2n} = \sum_{n=1}^{\infty} (B_n \cos \omega_n \tau + C_n \sin \frac{\omega_n \tau}{2})W_n(\xi)$$

In figure 3 and 4 show: a graph of the change in the bending moment in the section $\xi=0.43$ during the onset of the dynamic process resulting from the loss of half of the elastic base under the beam ($\nu=0.5$) with the value $\omega_{01} = 18(\alpha = 3)$ in Figure 3, and the graph of stationary (steady-state) oscillations after $\tau>14$ in Figure 4. The maximum value of the bending moment reaches 0.389.

**Figure 3.** Bending moment in cross section $\xi = 0.43$ at the beginning of a dynamic process

**Figure 4.** Steady moment fluctuations in the cross section $\xi = 0.43$
In Figure 5 shows the plot of the distribution of bending moment along the length of the beam with the quasistatic formation of the same defect (ν=0.5).

![Bending moment diagram](image)

**Figure 5.** Bending moment diagram for quasistatic destruction of the base

4. Conclusions
Investigation of the «beam-base» system with a possible scenario of defect formation in the form of a sudden precipitation of part of the base, it was found that the formation of a defect leads to a decrease in the overall rigidity of the structure and a violation of the static balance of the beam-base system. The arising inertia forces cause a dynamic response, redistribution and growth of deformations and stresses, the scenario of defect formation in the form of sudden settlement in the base part and, accordingly, structural restructuring of the entire structural system is possible.

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