Integrated scheduling methods of maintenance support resource

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Abstract. Maintenance support resource scheduling (MSRS) problem has attracted increasing attention in modern battle. Its objective is minimizing execution time of allocating multiple resources from multiple supply centers to tasks. As a NP-hard problem, there exist various constraints in the MSRS problem, including resource demands of tasks, resource reserves of supply centers and transport paths constraint. In this paper, we first build two models taking route planning and resource scheduling into consideration simultaneously. Then, with cutting plane approach and dual algorithm, we can obtain the optimal scheduling schemes corresponding to two models. Besides, the performance of the two approaches are verified with different scale instances.

1. Introduction
Scheduling method of maintenance support resource aims to minimize the time of allocating multiple maintenance resources to tasks on the premise of satisfying various constraints, such as resource requirements of tasks, resource reserves of supply centers and transport paths constraint. Considering the characteristics of high technology, high speed and high consumption in the modern battle, an efficient scheduling method which can minimize time completed and satisfy constraints is necessary.

To the best of our knowledge, the MSRS problem is always abstracted into three types of common problems, including vehicle routing problem[1], project scheduling problem[2] and shop scheduling problem[3], which focus on a single aspect. However, in the MSRS problem, both the resource allocation and route planning need considering. In addition, there are various resources in a real scenario, which is easy to be ignored. In order to solve the problems above, the MSRS problem is divided into two sub-problems: route planning problem and resource scheduling problem. In this way, the shortest path planning among tasks and supply centers is completed first. Then, two integrated approaches are designed considering the shortest routes and resource situations simultaneously. Besides, the various resources are transformed into single resource in various supply centers. Finally, we verify the feasibility and reliability of the designed approaches with various scale instances.

The rest of the paper is organized as follows. In Section 2, we build the mathematics model of the problem and its NP-hard proving is described in Section 3. The integrated approaches are introduced in Section 4. Experimental results and analyses are presented in Section 5. Section 6 concludes this paper.

2. Mathematical model of MSRS problem
The MSRS problem is to minimize the time consuming of allocating $n$ tasks to $m$ supply centers. The time consuming $c_{ij}$ is approximated using path length between supply center $i$ and task $j$. $x_{ij}$ means
allocating resource requirement of task \( j \) to supply center \( i \) or not. When binary variable \( x_{ij} \) is equal to 1, task \( j \) will be allocated to supply center \( i \), otherwise it will not be allocated. \( a_{ij} \) represents resource amount allocated from supply center \( i \) to task \( j \) and \( a_{ij} = 1 \). \( b_i \) stands for resource reserve of supply center \( i \). The mathematical model can be described as follows:

\[
\min \sum_{i=1}^{m} \sum_{j=1}^{n} c_{ij} x_{ij}
\]

\[
s.t. \sum_{j=1}^{n} a_{ij} x_{ij} \leq b_i, \sum_{i=1}^{m} x_{ij} = 1, x_{ij} \in \{0,1\} \quad i = 1,\ldots,m; j = 1,\ldots,n
\]

The objective is to minimize the time consuming of allocating all tasks to supply centers. Besides, the first constraint stands for resource reserve of each supply center. The second means that resource requirement of each task can be only assigned to one supply center.

### 3. NP-hard proving

Suppose that there are 10 supply centers and 20 tasks. Then the solution is a 10*20 matrix and the number of solutions is \( 2^{10*20} \). If the computer needs 1 ns to acquire one solution, then all the solutions need \( 5*10^{43} \) years. Obviously, the solving time is too long to imagine, and it will grow longer as the scale of the problem expands.

Except for the time analysis, we use generalized assignment problem (GAP) [4] to further prove the NP-hard characteristic of the MSRS problem. In a GAP problem, there are \( n \) tasks for \( m \) agents and the objective of GAP is finding the minimum cost of allocating tasks to agents. The model of the problem is also as Equation 1 shows. Here, \( c_{ij} \) stands for the expense of assigning task \( j \) to agent \( i \). \( a_{ij} \) represents the capacity requirement of assigning task \( j \) to agent \( i \). \( b_i \) means capacity reserve of agent \( i \). The binary variable \( x_{ij} \) means assigning task \( j \) to agent \( i \) or not. When \( x_{ij} \) is 1, the assignment will be executed, otherwise there is no assignment. In addition, the objective is to minimize the cost of assigning all tasks to agents. The first constraint is the capacity constraint of agent \( i \) and the second ensures that one task can be only assigned once. Besides, an agent can be assigned to more than one task ensuring one task is completed exactly once and the GAP is NP-hard [5].

According to the facts above, the MSRS problem is a special case of GAP, in which the task and supply center correspond to the task and agent of GAP respectively. As a consequence, the MSRS problem is also NP-hard.

### 4. Algorithm framework

Procedures of two integrated approaches are shown in Figure 1. Part (a) is the routing planning and part (b) represents model buildings and corresponding solving algorithms.
4.1 Route planning
In the route planning part, actual route conditions need abstracting into grid map like part (a) in Figure 1. Then, A* algorithm\([6]\) is applied to find the shortest routes among multiple points. Compared with the classical algorithm which needs searching the entire search space, the A* algorithm instructs the search direction to the end point with a heuristic factor, which needs shorter time to find a short path.

\[
f(n) = g(n) + h(n)
\] (2)

As shown in Equation 2, \(f(n)\) stands for the value of the specified point \(n\). Besides, the movement cost from start point to the specified point \(n\) is defined with \(g(n)\). \(h(n)\) is the heuristic value function, which represents the estimated cost from the specific point to the end.

4.2 Integrated approaches
After getting the shortest routes, two integrated approaches are designed, including cutting plane approach (CPA) based on an integer linear programming (ILP) model and dual algorithm (DA) based on a minimum cost maximum flow (MCMF) model.

4.2.1 Cutting plane approach based on the integer linear programming model. In the MSRA problem, a unit resource reserve of the same supply center is equivalent. Similarity, a unit resource demand of the same task is also equal in value. Therefore, the model can be changed into an integer linear programming model as follows.

\[
\min \sum_{i=1}^{m} \sum_{j=1}^{n} c_{ij} x_{ij}
\]
\[
\text{s.t. } \sum_{i=1}^{m} x_{ij} \geq d_{j}, \sum_{j=1}^{n} x_{ij} \leq S_{i}, x_{ij} \geq 0 \text{ and } x_{ij} \text{ is an integer } \quad i = 1, \ldots, m; j = 1, \ldots, n
\] (3)

Now, \(x_{ij}\) means the resource amount allocated from supply center \(i\) to task \(j\). \(S_{i}\) and \(d_{j}\) stand for resource reserve of supply center \(i\) and resource requirement of task \(j\), respectively.

With this approach, we will find the shortest routes among multiple tasks and supply centers with A* algorithm first. Afterwards, the initial model will be transformed into an integer linear programming model. Then, we will solve the new model with CPA for the optimal scheduling scheme.

4.2.2 Dual approach based on the minimum cost maximum flow model. Using the problem characteristics, our model is transformed into a minimum cost maximum flow (MCMF) model to reduce the problem complexity.

In Figure 2, we build the MCML model with a capacity network. Here, each edge has two parameters, which consist of the capacity \(c_{ij}\) of each edge \((v_{i}, v_{j})\) and the cost of unit flow \(d_{ij}(d_{ij} \geq 0)\). In practice, \(c_{ij}\) means the resource demand or reserve of the previous node and \(d_{ij}\) means the route length of two nodes. Besides, there exist a virtual start point and a virtual end point. \(E\) represents the edges. The objective is obtaining a feasible flow \(f = \{f_{ij}\}\), which stands for flow of edge \((v_{i}, v_{j})\), to get
the minimum cost when the flow is maximum as Equation 4. In addition, dual algorithm is applied to getting solutions.

\[ d(f) = \sum_{(x,y) \in E} d_{xy}f_{xy} \] (4)

5. Experiments and analyses

In this part, there are 24 kinds of instances and their parameter settings are listed in Table 1.

| Parameter                          | Notation | Value                        |
|------------------------------------|----------|------------------------------|
| The number of supply centers       | M        | \{5,10,15,20\}              |
| The number of tasks                | N        | \{10,30,50,70,90,110\}      |
| The average resource demand of tasks | RT      | 10                          |
| The average resource storage of supply centers | RS | N*RT/M+10                   |
| The average length of routes       | L        | 8                           |

5.1 Feasibility of integrated approaches

Here, the parameters of the first instance are M = 2, N = 8, RS = 50, RT = 10, L = 8 and the first instance is generated in Table 2 to demonstrate the reliability of the two integrated approaches.

| Resource of supply centers | Resource of tasks | Route matrix |
|----------------------------|-------------------|--------------|
| [93; 23]                  | [10; 4; 20; 5;]   | [1 3 10 2 8 15 3 5;] |
|                           |                   | [9 9 3 3 14 14 7 10] |

Table 3. Scheduling schemes of two integrated approaches.

| Approach                  | Scheduling scheme | Cost | Time/s |
|---------------------------|-------------------|------|--------|
| CPA based on ILP          |                   | 411  | 0.031727 |
| ILP                       |                   |      |        |
| D A based on MCMF         |                   | 411  | 0.022368 |
| MCMF                      |                   |      |        |

In Table 3, the number in the scheduling scheme is the resource amount allocated from the particular supply center to the task. Obviously, the two exact approaches have the same scheduling scheme and approximate executed time. Besides, the scheduling schemes are optimal.

Then, the second instance is generated in Table 4. Here, M=5, N=10 and other parameters are invariable. Apparently, the bold scheduling schemes of the two approaches in Table 5 are not the same. The reason is that the bold routes from supply center 3 and 4 to task 5 are the same. However, they have the same optimal cost even with different specific schemes.

| Resource of supply | Resource of tasks | Route matrix |
|--------------------|-------------------|--------------|
| [5 4 39 26 55]     | [9; 4; 11; 19; 14; 8; 6; 17; 19; 8] | [13; 4 1 3 3 3 15 5 16 11; 14 6 4 3 9 5 5 1 10 16; 14 1 11 6 2 9 7 9 15 3; 7 1 7 12 2 11 5 10 3 7; 1 4 5 9 4 6 12 7 1 7] |

Table 4. The second instance generated.

| Approach                  | Scheduling scheme | Cost | Time/s |
|---------------------------|-------------------|------|--------|
| CPA based on ILP          |                   | 406  | 0.005337 |
| ILP                       |                   |      |        |
| D A based on MCMF         |                   |      |        |
| MCMF                      |                   |      |        |

Table 5. Scheduling schemes of two integrated approaches.
5.2 Reliability of integrated approaches

Then, we use 24 various scale instances to illustrate the reliability of two approaches. The parameter settings are listed in Table 1. In Table 6. We can find that CPA and DA obtain the same optimal solutions all the time even with big scale instances. Besides, as the two approaches are exact solution methods, the scheduling schemes are optimal.

Table 6. The cost comparison of two integrated approaches with 24 various scale instances.

| Instance | CPA  | DA  | CPA  | DA  | CPA  | DA  | CPA  | DA  |
|----------|------|-----|------|-----|------|-----|------|-----|
| 5*10     | 406  | 406 | 10*10| 406 | 406  | 15*10| 324  | 324 |
| 5*30     | 977  | 977 | 10*30| 717 | 717  | 15*30| 498  | 498 |
| 5*50     | 1773 | 1773| 10*50| 1522| 1522 | 15*50| 943  | 943 |
| 5*70     | 2439 | 2439| 10*70| 1589| 1589 | 15*70| 1363 | 1363|
| 5*90     | 3280 | 3280| 10*90| 2040| 2040 | 15*90| 1540 | 1540|
| 5*110    | 4292 | 4292| 10*110|2280| 2280 | 15*110|1830 | 1830|

6. Conclusion

In this paper, we build two integrated models to combine the route planning and resource scheduling of the MSRS problem organically. Then, the optimal scheduling schemes are generated with corresponding algorithms. Afterwards, we systematically analyze the feasibility and reliability of the designed approaches. The experiments results show that the two approaches can obtain the optimal costs even with different scheduling schemes. Moreover, we illustrate the reliability of our approaches with various scale problems. In the future, we would like to consider the other constraints, such as task timing constraint of the MSRS problem, which is more in line with the actual scene.

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