NLO QCD Corrections for $\chi_{cJ}$ Inclusive Production at $B$ Factories

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Abstract

The next-to-leading order (NLO) quantum chromodynamics (QCD) corrections for $\chi_{cJ}(^{3}P_{J}^{[1]}, ^{3}S_{1}^{[8]})$, the P-wave charmoniums inclusive production at $B$ factories are calculated utilizing the non-relativistic QCD (NRQCD) factorization formalism. Large NLO corrections are found, especially for $^{3}P_{0}^{[1]}$ and $^{3}S_{1}^{[8]}$ configurations. Numerical evaluation indicates that the total cross sections of $\chi_{cJ} + c + \bar{c}$ processes are about $1 \sim 100$fb, which are accessible in the super-B experiment.

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The advent of NRQCD factorization formalism placed the heavy quarkonium physics on a more solid ground \cite{1}. In the framework of NRQCD, nevertheless there are still many open questions about the quarkonium production and decay. A number of investigations indicate that the leading order QCD calculations are inadequate to explain experimental data. It seems so far that most of the discrepancies between leading order calculation and experimental observation can be rectified by including higher order corrections, which has encouraged more NLO QCD calculations on quarkonium production and decays. On this point, one typical example is the double charmonium production at $B$ factories \cite{2–8}.

The charmonium production at $B$ factories is one of the most interesting and challenging problems in quarkonium physics. The observed cross sections of charmonium production processes $e^+e^- \rightarrow J/\psi + \eta_c$ and $e^+e^- \rightarrow J/\psi + c + \bar{c}$ are much large than the leading order QCD theoretical results \cite{2, 3, 9–11}. Through tedious investigations on the next-to-leading order QCD corrections for these processes \cite{4–6}, people found that the large gaps between theory and experiment can be greatly narrowed almost to non-existence. Though at the moment there have not been much data collected, the exclusive processes $e^+e^- \rightarrow J/\psi + \chi_{cJ}(J = 0, 1, 2)$ at $B$ factories were investigated up to NLO accuracy \cite{12, 13}. The large NLO QCD corrections to S-wave charmonium productions, exclusively and inclusively, enlighten us that higher order corrections to P-wave charmonium inclusive production must be very, if not more, important, not to mention here the color-octet contribution will be non-negligible. In the literature \cite{14}, the leading order theoretical estimation for $e^+e^- \rightarrow \chi_{cJ} + c + \bar{c}(J = 0, 1, 2)$ processes was made. For the aims of phenomenological study and the deeper understanding of NRQCD framework, in this work we calculate the NLO QCD corrections for the P-wave charmonium $\chi_{cJ}(3P^3_1, 3S^1_1)$ inclusive production processes $e^+e^- \rightarrow \gamma^* \rightarrow \chi_{cJ} + c + \bar{c}(J = 0, 1, 2)$ at the $B$-factory energy. Note that $e^+e^- \rightarrow \gamma^* \rightarrow \chi_{cJ} + g + g(J = 0, 1, 2)$ processes do not exist.

In the calculation, Mathematica package of \textbf{FeynArts} \cite{15} was used to generate the LO and NLO Feynman diagrams, as schematically shown in Fig.1. The standard form of quarkonium spin projection operator was adopted \cite{16–18}. For color-singlet and
spin-triplet case of our concern, it reads:

$$v(p)\bar{u}(p) = \frac{1}{4\sqrt{2}E(E + m)}(\not{p} - m_c) \otimes \epsilon^*_S \otimes \epsilon^*_S (P + 2E)(\not{p} + m_c) \otimes \left( 1_c \sqrt{N_c} \right).$$ (1)

Here $p = \frac{P}{2} + q$, $\bar{p} = \frac{P}{2} - q$ respectively are the momenta of quark and antiquark, $\epsilon^*_S$ is a spin polarization vector, $E^2 = P^2/4 = m^2 - q^2$, $N_c = 3$, and $1_c$ represents the unit color matrix. For spin-singlet and color-singlet state, the projection operator may be obtained by replacing the $\epsilon^*_S$ in (1) by a $\gamma_5$, while for color-octet state the color matrix $1_c$ should be substituted by $\sqrt{2}T_c$, the Gell-Mann matrices.

Employing the method used in Refs. [16,18] and by virtue of the symbolic computation package FeynCalc [19], the Born level results are readily obtained. For NLO QCD corrections, more complicated procedures have to be taken. The NLO corrections to the inclusive processes $e^+e^- \to \gamma^* \to \chi_{cJ} + c + \bar{c}$ ($J = 0, 1, 2$) include two parts, the virtual and real corrections to the leading order result. With virtual corrections, the
cross section can be formulated as:

\[ d\sigma_{\text{virtual}} = \frac{1}{4} \frac{1}{2s} \sum 2 \text{Re}(M^*_{\text{Born}}M_{NLO})d\text{PS}_3, \quad (2) \]

while with real corrections the cross section reads as:

\[ d\sigma_{\text{real}} = \frac{1}{4} \frac{1}{2s} \sum |M_{\text{real}}|^2d\text{PS}_4. \quad (3) \]

Here \( d\text{PS}_3 \) and \( d\text{PS}_4 \) stand for three- and four-body phase spaces respectively.

We use the phase space slicing method with two cutoffs in the calculation of real corrections \[20\]. The outgoing gluon with energy \( p_g^0 < \delta \) is considered to be soft, while \( p_g^0 > \delta \) the gluon will be taken as a hard one. The \( \delta \) here is a small quantity with energy-momentum unit. Under the soft condition of \( p_g^0 < \delta \) and in the Eikonal approximation,

\[ d\text{PS}_4|_{\text{soft}} = d\text{PS}_3 \frac{d^3p_g}{(2\pi)^3} p_g^{|p_g^0 < \delta}. \quad (4) \]

With the help of Eq.(4), the divergent and finite parts in real correction (3) will be properly separated through the method given in Ref. \[20\].

Throughout our calculation, the self-developed codes based on \texttt{FeynCalc} \[19\] are used to trace the matrices of spin and color, and to perform derivative on the heavy quark relative momentum \( q \) within quarkonium. The Mathematica package \texttt{Apart} \[21\] reduces the propagators of individual one-loop diagrams. After these procedures, the NLO virtual cross-section \( M^*_{\text{Born}}M_{NLO} \) is then expressed as linear combinations of one-loop integrals, such as:

\[ I_0^N(D, \{n_i\}) \equiv \int \frac{d^Dq_i}{(2\pi)^D} \frac{1}{D_1^{n_1}D_2^{n_2} \cdots D_N^{n_N} |_{N \leq 4}. \quad (5) \]

The linearly independent propagators have the form \( D_i \equiv (q_i + r_i)^2 - m_i^2 \) and the index \( n_i \) can be any integers except 0. The package \texttt{Fire} \[22\] is then employed to reduce all one-loop integrals \( I_0^N(D, \{n_i\}) \) to typical master-integrals \( (A_0, B_0, C_0, D_0) \), and the finite part of the master-integrals is computed by \texttt{LoopTools} \[23\].

We performed the calculation in the Feynman gauge, and the conventional dimensional regularization with \( D = 4 - 2\epsilon \) was adopted in regularizing the ultraviolet and
infrared divergences. In the end, all ultraviolet divergences are completely canceled by counter terms, while the Coulomb singularities are factorized out and attributed to the NRQCD long-distance matrix elements. Infrared divergences arising from loop integration, phase space and counter terms are partially canceled with each other. After combining various IR divergences, one obtains divergent terms proportional to the Born level amplitude of the color-octet $3S^3_1$ process, and the remaining divergent terms are then attributed to the NLO color-octet NRQCD matrix elements $\langle 0|O^{\chi_{cJ}}(3S^3_1)|0 \rangle$. By this procedure, the IR divergences appearing in the NLO correction for $3S^3_1$ state production cancel out completely in the end.

The ultraviolet and infrared divergences exist also in the renormalization constants $Z_2, Z_3, Z_m, Z_g$, corresponding respectively to the quark field, gluon field, quark mass, and strong coupling constant $\alpha_s$. Among them, in our calculation the $Z_g$ and $Z_3$ are defined in the modified-minimal-subtraction ($\overline{MS}$) scheme, while the other two are in the on-shell (OS) scheme. Thereafter, the counter terms read:

$$
\delta Z_2^{\text{OS}} = -C_F \frac{\alpha_s}{4\pi} \left[ \frac{1}{\epsilon_{\text{UV}}} + 2 \frac{\epsilon_{\text{IR}}}{\epsilon_{\text{IR}}} - 3\gamma_E + 3 \ln \frac{4\pi \mu^2}{m^2} + 4 \right],
$$

$$
\delta Z_m^{\text{OS}} = -3C_F \frac{\alpha_s}{4\pi} \left[ \frac{1}{\epsilon_{\text{UV}}} - \gamma_E + \ln \frac{4\pi \mu^2}{m^2} + \frac{4}{3} \right],
$$

$$
\delta Z_3^{\text{MS}} = \frac{\alpha_s}{4\pi} \left( \beta_0 - 2C_A \right) \left[ \frac{1}{\epsilon_{\text{UV}}} - \gamma_E + \ln(4\pi) \right],
$$

$$
\delta Z_g^{\text{MS}} = -\frac{\beta_0}{2} \frac{\alpha_s}{4\pi} \left[ \frac{1}{\epsilon_{\text{UV}}} - \gamma_E + \ln(4\pi) \right].
$$

We had taken several measures to check our calculation. We compared our leading-order result with Ref. [9] and got an agreement while having the same inputs. We also made use of Helac-Onia [25] to calculations of leading-order processes and the hard parts of real corrections. We took several different values of the soft cut $\delta$ and found the results are insensitive to the change. We apply our code to the NLO calculations of $J/\psi \rightarrow e^+e^-$ and $\eta_c \rightarrow \gamma\gamma$ processes, and found our analytic results agree with those in the literature. Moreover, we found the NLO results for $e^+e^- \rightarrow \gamma^* \rightarrow \chi_{cJ} + c + \bar{c}(J = 0, 1, 2)$ processes can also be expressed in the form

$$
\sigma_{\text{NLO}} = \sigma_{\text{LO}} \left( 1 + \frac{\alpha_s}{\pi} \frac{\beta_0}{2} \ln \left( \frac{\mu^2}{s} \right) + C \right),
$$

(7)
Table I, where the uncertainty comes mainly from \( m \) obtained. The cross sections for \( e \) inclusive production and integrate over the phase space, the numerical results can be two photon decay process, i.e. \( \Gamma(\chi \rightarrow \gamma \gamma) = \frac{128\pi\alpha^2 \langle \mathcal{O}_1 \rangle_{\chi cJ}}{405m_c^2}(1 - \frac{16\alpha_s}{3\pi}) \) \( 12 \), and it therefore is \( \langle \mathcal{O}_1 \rangle_{\chi cJ} \equiv \langle \chi cJ | O^{\chi c}(3P_2^{[1]}) | \chi cJ \rangle = 0.0166m_c^4 \) GeV; the magnitude of color-octet matrix element \( \langle 0| O^{\chi c}(3S_1^{[8]}) | 0 \rangle = 0.000748m_c^2 \) GeV \( 20 \), which was obtained by fitting the \( \chi_{cJ} \) hadroproduction theoretical result to the Tevatron data. The two-loop expression for the running coupling constant \( \alpha_s(\mu) \) reads

\[
\frac{\alpha_s(\mu)}{4\pi} = \frac{1}{\beta_0 L} - \frac{\beta_1 \ln L}{\beta_0^2 L^2}.
\] (8)

Here, \( L = \ln(\mu^2/\Lambda_{QCD}^2) \), \( \beta_0 = (11/3)C_A - (4/3)T_F n_f \), and \( \beta_1 = (34/3)C_A^2 - 4C_F T_F n_f - (20/3)C_A T_F n_f \), with \( \Lambda_{QCD} \) to be 296 MeV \( 28 \) and \( n_f = 4 \), the number of active flavors.

After substituting the above input parameters into the analytical expressions for \( \chi_{cJ} \) inclusive production and integrate over the phase space, the numerical results can be obtained. The cross sections for \( e^+e^- \rightarrow \gamma^* \rightarrow \chi_{cJ}(3P_0^{[1]}) + c + \bar{c} + X \) are presented in Table I, where the uncertainty comes mainly from \( m_c \). We chose the renormalization scale \( \mu \) running from \( 2m_c \) to \( \sqrt{s}/2 \). For color-octet, since the matrix elements have the relation: \( \langle 0| O^{\chi c}(3S_1^{[8]}) | 0 \rangle : \langle 0| O^{\chi c}(3S_1^{[8]}) | 0 \rangle : \langle 0| O^{\chi c}(3S_1^{[8]}) | 0 \rangle = 1 : 3 : 5 \), we merely need to calculate one of the three processes. The cross sections for \( e^+e^- \rightarrow \gamma^* \rightarrow \chi_{cJ}(3S_1^{[8]}) + c + \bar{c} + X \) are presented in Table II.

From Tables I and II we notice that after NLO QCD corrections, cross sections for \( 3P_0^{[1]} \) and \( 3S_1^{[8]} \) production are significantly enhanced, while cross sections for \( 3P_1^{[1]} \) and \( 3P_2^{[1]} \) production are depressed. Relatively, the NLO corrections for \( 3P_0^{[1]} \), \( 3P_2^{[1]} \) and \( 3S_1^{[8]} \) states are large, similar to cases of other charmonium production processes \( 4, 5, 27 \). In case we express the NLO cross sections as \( \sigma_{NLO} = \sigma_{LO}(1 + \frac{16\alpha_s}{3\pi} \ln(\frac{\mu^2}{\Lambda^2}) + C)) \), large \( C \)s yield from \( 3P_0^{[1]} \) and \( 3S_1^{[8]} \) production processes. Note that the large \( C \) not only produces notable NLO correction, but also induces evident renormalization scale dependence. From Tables I and II one can also read that for \( \chi_{c0} + c + \bar{c} \) production, the dominant contribution comes from the color-singlet configuration, while for \( \chi_{c2} + c + \bar{c} \) production
The next-to-leading order cross sections of $\chi$ but much less $\mu$

In Figure 2 we show the renormalization scale dependence, which in principle becomes weaker with higher order corrections.

For fixed order calculation, one of the main uncertainties in final result comes from the scale dependence, which in principle becomes weaker with higher order corrections.

In Table I we present the improved upper limits for the production of these states, i.e. 77 fb for $\chi_{c1}$ and 79 fb for $\chi_{c2}$, which include our results. It is worth noting that the radiative processes $e^+e^- \rightarrow \chi_{cJ} + \gamma$ also contribute remarkably to the $\chi_{cJ}$ inclusive production at B factories. It produces similar amount of $\chi_{c1}$ and $\chi_{c2}$, but much less $\chi_{c0}$, in comparison with the processes of our concern.

For fixed order calculation, one of the main uncertainties in final result comes from the scale dependence, which in principle becomes weaker with higher order corrections. In Figure 2 we show the renormalization scale $\mu$ dependence of leading order and the next-to-leading order cross sections of $e^+e^- \rightarrow \gamma^* \rightarrow \chi_{cJ}(^3P_J^1, ^3S_J^1) + c + \bar{c} + X$ processes.
The Figure exhibits that only the $\mu$ dependence of $^3P_1^{[1]}$ production process is slightly depressed by NLO QCD corrections, while no evident improvement for others. This happens, we think, because of the large positive NLO correction for $^3P_0^{[1]}$, $^3S_1^{[8]}$ and large negative correction for $^3P_2^{[1]}$ production processes, that is the large $C$ in (7). For these processes, the scale dependence would be reduced when even higher order corrections are taken into account.

In summary, we have calculated the NLO QCD corrections to $e^+e^- \rightarrow \gamma^* \rightarrow \chi_{\pm}$.
\( \chi_{cJ}(^3P_0^{[1]}, ^3S_1^{[8]}), c + \bar{c} + X \) processes at B factories. The NRQCD factorization works well, that is all divergences can be properly handled, when both color-singlet and color-octet mechanisms are taken into account. Large positive NLO corrections have been found for \( ^3P_0^{[1]} \) and \( ^3S_1^{[8]} \) production processes, while for \( ^3P_2^{[1]} \), the cross section is substantially depressed by NLO QCD correction. For \( \chi_{c0} + c + \bar{c} \) production, the dominant contribution comes from the color-singlet mechanism, while for \( \chi_{c1,c2} + c + \bar{c} \) production neither color-singlet or color-octet mechanism yields overwhelmingly more than the other. From the results in this work, one can understand why previous BaBar measurement gave only the upper limits for \( \chi_{c1} \) and \( \chi_{c2} \) production, and one may also expect that the \( \chi_{cJ} \) will be measurable in BELLE II(super-B) experiment.

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