GENERATION AND EVOLUTION OF MAGNETIC FIELDS IN THE GRAVITOMAGNETIC FIELD OF A KERR BLACK HOLE

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I study the generation and evolution of magnetic fields in the plasma surrounding a rotating black hole. Attention is focused on effects of the gravitomagnetic potential. The gravitomagnetic force appears as battery term in the generalized Ohm's law. The generated magnetic field should be stronger than fields generated by the classical Biermann battery. The coupling of the gravitomagnetic potential with electric fields appears as gravitomagnetic current in Maxwell's equations. In the magnetohydrodynamic induction equation, this current re-appears as source term for the poloidal magnetic field, which can produce closed magnetic structures around an accreting black hole. In principle, even self-excited axisymmetric dynamo action is possible, which means that Cowling's anti dynamo theorem does not hold in the Kerr metric. Finally, the structure of a black hole driven current is studied.

1 Introduction

The influence of the gravitomagnetic field of a rotating compact object on electromagnetic fields has been studied for some 25 years (Wald 1974, Bicak & Dvork 1976). It was shown by Blandford & Znajek (1977) that the coupling of the gravitomagnetic potential with a magnetic field results in an electromotive force. Currents driven by this electromotive force may extract rotational energy from a black hole. Cast in the language of the 3+1 split of the Kerr metric, Maxwell's equations, together with the ingoing wave boundary condition for electromagnetic fields at the horizon, led to the Membrane Paradigm (Thorne et al. 1986), enforcing the analogy between a rotating black hole, immersed in an external magnetic field, with pulsars. There are, however, some difficulties with this pulsar analogy, namely that (i) it is not easy to transport Poynting flux from close to the black hole's horizon out to infinity (Punsly & Coroniti 1989, Punsly 1996), and (ii) a black hole does not carry its own magnetic field, not to mention kGauss fields. Magnetic fields have to be accreted onto the black hole either from the outer accretion disk, or must be generated in the plasma surrounding the black hole.

In this talk I address the question of the generation of magnetic fields by a battery operating in the plasma close to a rotating black hole and the evolution and the structure of magnetic fields that are brought in by an accretion disk.
I show that the gravitomagnetic force may play a crucial role in the battery and that the coupling between the gravitomagnetic potential and an electric field is a source for the poloidal magnetic field, which may produce closed magnetic loops in the accreting plasma around the hole. In principle, even an axisymmetric gravitomagnetic dynamo is possible, i.e. Cowling’s anti-dynamo theorem is not valid in the Kerr-metric.

2 The MHD description of an electron-ion plasma

Here I summarize the relativistic definition of a plasma as center-of-mass fluid of its components, and the derivation of the generalized Ohm’s law (Khanna 1998a). The plasma is assumed to be a perfect fluid and is defined by the sum of the ion and electron stress-energy tensors, which contain a collisional coupling term:

\[ (\rho'_m + p'_m)W^\alpha W^\beta + p'_e g^{\alpha\beta} \equiv T^{\alpha\beta} = \sum_{x=i,e} (\rho'_{mx} + p'_{sx})W^\alpha W^\beta + p'_{sx}g^{\alpha\beta} + T^{\alpha\beta}_{x\text{coll}}. \]  

Subscripts \( i, e \) refer to ion and electron quantities, respectively. Superscripts denote the rest-frame in which the quantity is defined, where ‘\\( \cdot \)’ refers to the plasma rest-frame. In the 3+1 split (into hypersurfaces of constant Boyer-Lindquist time \( t \), filled with stationary zero angular momentum fiducial observers) \( T^{\alpha\beta} \) splits into the total density of mass-energy \( \epsilon \) and momentum density \( \vec{S} \):

\[ \epsilon \equiv (\rho'_m + p'_m \gamma^2)\gamma^2 \approx \rho'_m \gamma^2, \quad \vec{S} \equiv (\rho'_m + p'_m)\gamma^2 \vec{v} \approx \rho'_m \gamma^2 \vec{v} \]  

and the stress-energy tensor of 3-space with metric \( \vec{h} \):

\[ \vec{T} \equiv (\rho'_m + p'_m)\gamma^2 \vec{v} \otimes \vec{v} + p'_m \vec{h} \approx \rho'_m \gamma^2 \vec{v} \otimes \vec{v} + p'_m \vec{h}. \]

The approximate expressions hold for a ‘cold’ plasma. Adding the current density 4-vectors of each plasma component and splitting into charge density and current density yields

\[ \rho_c \equiv \rho_{ci} + \rho_{ce} = Z\epsilon n\gamma_i - e\epsilon n_e \gamma_e, \quad \vec{j} \equiv \vec{j}_i + \vec{j}_e = Z\epsilon n\gamma_i \vec{v}_i - e\epsilon n_e \gamma_e \vec{v}_e. \]

All quantities resulting from the split are measured locally by FIDOs.

2.1 The generalized Ohm’s law in the 3+1 split of the Kerr metric

In the ‘cold’ plasma limit, the local laws of momentum conservation for each species can be re-written as equations of motion, which can then be combined
to yield the generalized Ohm’s law for an electron-ion plasma

\[
\frac{j}{\sigma\gamma_e} \approx \vec{E} + \frac{Zn_i\gamma_i}{n_e\gamma_e} \vec{v} \times \vec{B} - \frac{j}{en_e\gamma_e} + \frac{\nabla (\alpha_g B_e^e)}{en_e\gamma_e \alpha_g} + \frac{4\pi\gamma_e}{\omega_{pe}^2} \rho_e \vec{g} + \frac{\rho'_e \gamma \vec{v}}{\sigma\gamma_e}
\]

\[
- \frac{4\pi e (Zn_i \gamma_i^2 - n_e \gamma_e^2)}{\omega_{pe}^2 \gamma_e \gamma_i^2} \left( \frac{d(\gamma \vec{v})}{dt_p} - \nabla \cdot (\gamma \vec{v}) \right),
\]

Equation (5)

with the conductivity \( \sigma = e^2 n_e / m_e \nu_c \equiv \omega_{pe}^2 / 4\pi \nu_c \), the electron plasma frequency \( \omega_{pe} \), the factor of gravitational redshift \( \alpha_g \) (with \( \vec{g} = -\nabla \ln \alpha_g \)) and the gravitomagnetic tensor field \( \vec{H} \equiv \alpha_g^{-1} \nabla \vec{\beta} \), \( \vec{\beta} = \beta \phi \vec{e} \phi \equiv -\omega \vec{e} \phi \) is the gravitomagnetic potential, which drags space into differential rotation with angular velocity \( \omega \). Note that, in the single fluid description, the gravitomagnetic force drives currents only, if the plasma is charged in its rest frame. \( \tau_p \) is the proper time in the plasma rest frame. The derivation was made with the assumption that the species are coupled sufficiently strong that their bulk accelerations

\[
\frac{d(\gamma x \vec{v}_x)}{d\tau_p} \equiv \left[ \frac{\gamma_x}{\alpha_g} \frac{\partial}{\partial t} + \gamma_x \left( \vec{v}_x - \frac{\vec{\beta}}{\alpha_g} \right) \cdot \nabla \right] (\gamma_x \vec{v}_x)
\]

are synchronized. The same is required for the gravitomagnetic accelerations, i.e. \( | \vec{H} \cdot (\gamma_i^2 \vec{v}_i) - \vec{H} \cdot (\gamma_e^2 \vec{v}_e)| \ll | \vec{H} \cdot (\gamma_i^2 \vec{v}_i)| \). If the MHD-assumption of “synchronized accelerations” is not made, Ohm’s law contains further current acceleration terms, inertial terms and gravitomagnetic terms (Khanne 1998a), which may be important for collisionless reconnection and particle acceleration along magnetic fields. This topic will be discussed elsewhere.

In the limit of quasi-neutral plasma \( (Zn_i \approx n_e) \) and \( \gamma_e \approx \gamma_i \approx \gamma \) Eq. (5) reduces to

\[
\vec{j} \approx \sigma \gamma (\vec{E} + \vec{v} \times \vec{B}) - \frac{\sigma}{en_e}(\vec{j} \times \vec{B}) + \frac{\sigma}{en_e \alpha_g} \nabla (\alpha_g B_e^e),
\]

which contains all the terms, familiar from the non-relativistic generalized Ohm’s law, but no gravitomagnetic terms.

2.2 The gravitomagnetic battery

The generation of magnetic fields by a plasma battery was originally devised by Biermann (1950) for stars. He showed that, if the centrifugal force acting on a rotating plasma does not possess a potential, the charge separation owing to the electron partial pressure cannot be balanced by an electrostatic field, and thus currents must flow and a magnetic field is generated.
In Khanna (1998b) I have re-formulated Biermann’s theory in the Kerr metric. The base of this battery theory is Ohm’s law of eq. (7). Assuming that electrons and ions have non-relativistic bulk velocities in the plasma rest frame, superscripts i, e, ′ can be dropped. With \( p = p_i + p_e = (n_i + n_e)kT \), the impressed electric field (IEF), \( \vec{E}^{(i)} = \vec{\nabla}(\alpha_g p_e)/en\gamma\alpha_g \), can be re-expressed with the aid of the equation of motion for a ‘cold’ quasi-neutral plasma to yield

\[
\vec{E}^{(i)} = \frac{m_i}{(Z + 1)e} \left( \gamma \vec{g} + \vec{H} \cdot (\gamma \vec{v}) \right) - d(\gamma \vec{v})/d\tau + \frac{Z \left( j \times \vec{B} + (\vec{v} \cdot \vec{j})\vec{E} \right)}{(Z + 1)e n\gamma}.
\]

\( \tau \) is the proper time in a FIDO frame; i.e. \( d/d\tau = \gamma d/d\tau \). The criterium for magnetic field generation is that \( \vec{\nabla} \times \alpha_g \vec{E}^{(i)} \neq 0 \). Here I restrict the discussion to the gravitomagnetic IEF \( \vec{E}^{(i)}_{\text{gm}} \). The function part of \( \alpha_g \vec{E}^{(i)}_{\text{gm}} \) is

\[
\left( \beta \cdot \vec{\nabla} + \vec{\nabla} \beta \right) (\gamma \vec{v}) = \left( \beta_i (\gamma v^i)_i + \gamma \beta^{ij} v_i \right) \vec{e}_j
\]

\[
= -\gamma v^\phi \omega^2 \vec{e}_\phi - \omega \left( (\gamma v^r)_r \vec{e}_r + (\gamma v^\phi)_\phi \vec{e}_\phi \right),
\]

where \( \omega = (h_{\phi\phi})^{1/2} \) and | denotes the covariant derivative in 3-space. In axisymmetry \( \alpha_g \vec{E}^{(i)}_{\text{gm}} \) is clearly rotational, unless some freak \( \gamma \) should manage to make \( \gamma v^\phi \omega^2 \) a function of \( \omega \) alone. Thus the gravitomagnetic force drives a poloidal current and generates a toroidal magnetic field. Only if \( v^\phi \) is non-axisymmetric, the gravitomagnetic IEF drives toroidal currents. The total IEF \( (\alpha_g \vec{E}^{(i)}_{\text{gm}} + \alpha_g \vec{E}^{(i)}_{\text{class}}) \) is likely to rotational in general. This will be clarified for specific velocity fields elsewhere.

In presence of a weak poloidal magnetic field the Biermann battery is limited due to modifications of the rotation law by the Lorentz force, rather than by ohmic dissipation. Then the contribution of the centrifugal force to the IEF becomes irrotational already at weak toroidal fields (Mestel & Roxburgh 1962). The gravitomagnetic battery term, on the other hand, is only linearly dependent on \( \vec{v} \). The equilibrium field strength should therefore be higher than for the Biermann battery.

3 The MHD induction equation in the 3+1 split of the Kerr metric

In this section I review the axisymmetric dynamo equations in the 3+1 split of the Kerr metric (Khanna & Camenzind 1996a). Ohm’s law is assumed to be of the standard form for a quasi-neutral plasma; Hall-term and IEF are neglected. Combining Maxwell’s equations (Thorne et al. 1986) with Ohm’s
law yields the MHD induction equation

\[
\frac{\partial \vec{B}}{\partial t} = \vec{\nabla} \times \left( \alpha \vec{v} \times \vec{B} - \frac{\eta}{\gamma} \left( \vec{\nabla} \times (\alpha \vec{B}) + (\vec{E}_p \cdot \vec{\nabla}) \dot{\omega} \hat{e}_\phi \right) \right) + (\vec{B}_p \cdot \vec{\nabla}) \dot{\omega} \hat{e}_\phi .
\]

(10)

The term standing with the magnetic diffusivity \(\eta\) is the current density, which, via Ampère’s law, contains the coupling of the gravitomagnetic field with the electric field. In axisymmetry this is simply the shear of the poloidal electric field in the differential rotation of space, \(\omega\). Another induction term is the shear of the poloidal magnetic field by \(\omega\). This generates toroidal magnetic field out of poloidal magnetic field even in a zero-angular-momentum flow.

### 3.1 The gravitomagnetic dynamo

Introducing the flux \(\Psi\) of the poloidal magnetic field and the poloidal current \(T\)

\[
\Psi = \frac{1}{2\pi} \int \vec{B}_p \cdot d\vec{A} = \dot{\omega} A^\phi \quad T = 2 \int \alpha \vec{j}_p \cdot d\vec{A} = \alpha \dot{\omega} B^\phi ,
\]

where \(A^\phi\) is the toroidal component of the vector potential, eq. (10) splits into

\[
\frac{\partial \Psi}{\partial t} + \alpha (\vec{v}_p \cdot \vec{\nabla}) \Psi - \frac{\eta \dot{\omega}}{\gamma} \omega^\phi (\vec{\nabla} \omega \cdot \vec{\nabla} \Psi) + \frac{\eta \omega^2}{\gamma} \vec{\nabla} \cdot \left( \frac{\alpha \vec{e}_\phi}{\omega^2} \vec{\nabla} \Psi \right)
= \frac{\eta \dot{\omega}}{\gamma \alpha} \left( \vec{v}_p - \frac{\eta}{\gamma} \vec{\nabla} T \right) \vec{\nabla} \cdot \vec{\nabla} \omega \quad \text{(12)}
\]

\[
\frac{\partial T}{\partial t} + \alpha (\vec{v}_p \cdot \vec{\nabla}) T + \alpha \omega^2 T \left( \vec{\nabla} \cdot \vec{v}_p \right) - \alpha \omega^2 \vec{\nabla} \cdot \left( \frac{\eta}{\gamma \omega^2} \vec{\nabla} T \right)
= \alpha \dot{\omega} (\vec{\nabla} \Psi \times \vec{e}_\phi) \cdot \vec{\nabla} \Omega .
\]

(13)

These equations are the relativistic equivalent of the classical axisymmetric dynamo equations. It is important to note, however, that no mean-field approach was made, but \(\Psi\) has source terms anyway. They result from \(\vec{E}_p \cdot \vec{\nabla} \omega\). Obviously, Cowling’s anti-dynamo theorem does not hold close to a rotating black hole. Growing modes of this gravitomagnetic dynamo were shown to exist for steep gradients of the plasma angular velocity \(\Omega\) (Núñez 1997). For simple accretion scenarios, growing modes could not be found in kinematic numerical simulations (Khanna & Camenzind 1996b). If, however, magnetic field is replenished by an outer boundary condition, the gravitomagnetic source terms generate closed loops around the black hole.
3.2 The magnetic field structure in the accretion disk close to the hole

In the accretion disk, magnetic field may be advected into the near-horizon area, where gravitomagnetic effects may become important. This can be simulated by advection/diffusion boundary conditions for $F = \Psi$, $T$

$$\frac{\partial F}{\partial n} + \frac{\gamma |v^f|}{\eta} F = \frac{\partial F_{\text{out}}}{\partial n} + \frac{\gamma |v^f|}{\eta} F_{\text{out}}, \quad (14)$$

where $\partial/\partial n$ is the derivative along the outer boundary normal. Figure 1 shows the stationary final state of a time-dependent simulation, in which $|B_{p,\text{out}}|/|B_{t,\text{out}}| = 1/50$. For such a dominantly toroidal magnetic field the gravitomagnetic source terms are strong enough to change the topology of $\Psi$. This may influence the efficiency of the electromagnetic extraction of rotational energy from the hole.

3.3 Black Hole driven currents in accretion flows

It was mentioned above that the shear of space does also induce a toroidal magnetic field (cf. eq. [10]). In eq. (13) this shear term is obscure, but still there, hidden in $(\nabla \Psi \times \epsilon_0) \cdot \nabla \Omega \times \hat{B}_p \cdot \nabla \Omega = \hat{B}_p \cdot \nabla (\alpha_p v^\phi + \omega)$. In a zero-angular-momentum flow $v^\phi = 0$ (or, equivalently $\Omega = \omega$) and thus the current $T$ is
Figure 2: Poloidal magnetic field (left) and corresponding black hole generated current (right) in a zero-angular-momentum accretion flow. The specific angular momentum of the black hole is $a = 0.998M$. Dotted contours have negative values. Dashed lines indicate the disk scale height $H$.

solely generated by the shear of space. Such a scenario can be used to assess, which fraction of the total current in a general accretion-ejection flow is driven by the hole. Figure 2 shows the steady state of an initially homogenous vertical magnetic field that has been dragged onto the hole by radial accretion. The major part of the black hole driven current is confined within the ergosphere. The toroidal field is of the same order as the poloidal field. The current’s energy will be dissipated in the accretion flow and in the corona. In a realistic accretion-ejection flow, the plasma shear in the ergosphere of a rapidly rotating black hole will be similar to the shear of space. Thus the current system should have a structure similar to the current of Fig. 2. The energy extracted from the hole will therefore likely be deposited in the plasma close to the hole.

4 Conclusions

It was shown that a rotating black hole can generate magnetic fields in an initially un-magnetized plasma. In axisymmetry a plasma battery can only generate a toroidal magnetic field, but then the coupling of the gravitomagnetic potential with toroidal magnetic fields generates poloidal magnetic fields. Even
an axisymmetric self-excited dynamo is theoretically possible, i.e. Cowling’s theorem does not hold close to a Kerr black hole. Due to the joint action of gravitomagnetic battery and gravitomagnetic dynamo source term, a rotating black hole will always be surrounded by poloidal and toroidal magnetic fields (probably of low field strength, though). The gravitomagnetic dynamo source may generate closed poloidal magnetic field structures around the hole, which will influence the efficiency of the Blandford-Znajek mechanism.

The “shear-of-space” driven fraction of a global current can be assessed by a kinematic simulation of a zero-angular-momentum flow. The major part of the resulting current system is generated and closed in the corona near the hole. In a realistic accretion-ejection flow, the plasma shear in the ergosphere of a rapidly rotating black hole will be similar to the shear of space. The current system should therefore have a similar structure as in the example shown here, which means that the energy extracted from the hole is likely to be deposited in the disk corona.

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