Extra dimensions and color confinement

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Abstract

We consider an extension of the ordinary four dimensional Minkowski space by introducing additional dimensions which have their own Lorentz transformation. Particles can transform in a different way under each Lorentz group. We show that only quark interactions are slightly modified and that color confinement is automatic since these degrees of freedom run only in the extra dimensions. No compactification of the extra dimensions is needed.

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It is very well known that in relativistic field theories all massless particles have the same speed in vacuum: the speed of light \( c \). However, if the true space-time were direct product of several four dimensional Minkowski space-time manifolds the Lorentz symmetry would be
\[ L \otimes L' \otimes L'' \otimes \cdots, \]
with each factor \( L, L', L'' \ldots \) having different limit velocity, say \( c, c', c'' \ldots \) respectively.

In the following, we will consider only two of such factors, say \( L \otimes L' \). The space-time is then, in this case, an eight dimensional manifold. However, the four dimensions \( x^\mu \), which we will call the \( x \)-world, transform under \( L \otimes L' \) like \( (4, 1) \), while \( x'^\mu \) (the \( x' \)-world) transforms as \( (1, 4) \). Here, \( \mu, \mu' = 0, 1, 2, 3 \).

Now, let us introduce fields. Under \( L \otimes L' \) the photon is assumed to transform as \( (4, 1) \), while gluons transform as \( (1, 4) \). We can write the usual tensors \( F_{\mu\nu} \) for photons in the \( x \)-world and \( F^a_{\mu'\nu'} \) for gluons \( (a \) is the color index) in the \( x' \)-world.

Next, we will assume that two type of spinor fields do exist. One type transforms as a Dirac spinor under \( L \) but as a singlet under \( L' \) and the other one transforms as Dirac spinor under both factors. The former ones will be identified with leptons and the later ones with quarks. For leptons the Dirac equation is as usual in the \( x \)-world. We denote quark fields as \( q_{\alpha\alpha'}(x, x') \), where \( \alpha, \alpha' \) represent spinor indices in the \( x \)- and \( x' \)-worlds, respectively. Thus, the Dirac equation must be satisfied in each index \( \alpha \) and \( \alpha' \) separately
\[
\left( i\hbar c \gamma^\mu \partial_\mu - mc^2 \right)_{\alpha\beta} \left( i\hbar c' \gamma'^{\mu'} \partial_{\mu'} - mc'^2 \right)_{\alpha'\beta'} q_{\beta\beta'}(x, x') = 0.
\]
where \( \alpha, \beta, \alpha', \beta' \) denote, in an obvious notation, spinor indices. The free Lagrangian density reads then
\[
\mathcal{L}_q(x, x') = \bar{q}_{\alpha\alpha'}(x, x') \left( i\hbar c \gamma^\mu \partial_\mu - mc^2 \right)_{\alpha\beta} \left( i\hbar c' \gamma'^{\mu'} \partial_{\mu'} - mc'^2 \right)_{\alpha'\beta'} q_{\beta\beta'}(x, x')
\]
Notice that \( q_{\alpha\alpha'}(x, x') \) is a double-spinor since it transforms as a spinor under each factor of \( L \otimes L' \). It is different from the usual second rank spinor. The latter one has two spinor indices with respect to the same Lorentz transformation. Notice that in general \( q_{\alpha\alpha'}(x, x') \neq q_{\alpha}(x)q_{\alpha'}(x') \).
Since gluons run only over the $x'$ world, this implies that color confinement is automatic in this context. It seems at first sight that in our scheme there is not, besides the color confinement, any other observable effect. This is not the case but, in order to treat this issue it is necessary to consider interactions.

With all those fields defined above, we can build interactions by introducing covariant derivatives. For leptons we have the usual quantum electrodynamics (QED). In fact, we can assume that the intermediate vector bosons $W^\pm$ and $Z^0$ transform like the photon, and the Higgs boson is an scalar field under both kind of Lorentz transformations. Hence, the standard electroweak theory [1] remains the same for the lepton case. On the other hand, for quarks we obtain the gluon-photon-quark interactions which are different, in general, from those of QED and quantum chromodynamics (QCD) [2]. However, the deviations from pure QED and QCD are, in principle, calculable.

From Eq. (2) quark interactions arise in the usual way if we define the covariant derivatives as $\partial_\mu \rightarrow D_\mu = \partial_\mu - ieA_\mu$ and $\partial_{\mu'} \rightarrow D^a_{\mu'} = \partial_{\mu'} + igG^a_{\mu'}\lambda^a/2$, with $\lambda^a$, $a = 1, \ldots, 8$ the $SU(3)$ Gell-Mann matrices. We also could have started with a theory which is invariant under $SU(3)_c \otimes SU(2)_L \otimes U(1)_Y$, i.e., $\partial_\mu \rightarrow D^f_\mu$ and $\partial_{\mu'} \rightarrow D^a_{\mu'}$, where $D^f_\mu$ is the $SU(2) \otimes U(1)$ covariant derivative for the flavor $f$, but here we only will consider the first case. As we said before, $q_{\alpha\alpha'}$ is a general double-spinor under $L \otimes L'$. We have three possibilities for introducing interactions in the lagrangian of Eq. (2).

In the first one, the covariant derivative (Abelian like QED or non-Abelian like the electroweak interactions) is introduced only in the $x$-world part of the lagrangian being, the $x'$-world part of quark fields an spectator. For an Abelian QED-like gauge symmetry, the interaction lagrangian density is

$$L_{q}^{QED} = e \int d^3x d^3x' \bar{q}_{\alpha\alpha'}(x, x')\gamma_\mu^\alpha(x, x')q_{\beta\alpha'}(x, x').$$

We see that concerning the indices related to the $x$-world, Eq. (3) is in fact the usual QED interaction lagrangian density.

In the second case, it is the $x$ part of the quark fields which acts as an spectator, while
the QCD-like covariant derivative is introduced in the $x'$-world part. Thus, the interaction reads

$$L_{QCD}^q = g \int d^3x' \bar{q}_{\alpha'}(x, x') \gamma_{\alpha'\beta'}^\mu A_{\mu}^a(x, x') \frac{\lambda^a}{2} q_{\alpha\beta}(x, x'),$$

(4)

where we have omitted the color index in the quark fields. In this case, concerning the indices running over the $x'$-world we obtain the QCD interaction lagrangian.

Finally, we have both possibilities at the same time. Then, interactions are

$$L_{new}^q = e g \int d^3x' \bar{q}_{\alpha'}(x, x') \gamma_{\alpha'\beta'}^\mu A_{\mu}^a(x, x') \frac{\lambda^a}{2} q_{\alpha\beta}(x, x'),$$

(5)

Notice that, since the gluon fields transform as singlet under $L$, from the point of view of an observer in the $x$-world the interaction in Eq. (5) is of the form

$$L_{new}^q = e \int d^3x \bar{q}(x) \gamma^\mu q(x)A_{\mu}(x) \phi(x),$$

(6)

where $\phi(x)$ is the gluon fields viewed from the $x$-world. So, the interaction in Eq. (5) is of the non-renormalizable type in the $x$-world.

Next, for simplicity, let us assume that $q_{\alpha\beta'}(x, x') = q_{\alpha}(x)q_{\alpha'}(x')$ in Eqs. (3), (4) and (5). Hence, in this simplified situation both $x$- and $x'$-world decouple from each other even in the quark sector. The interaction in Eq. (3) becomes

$$L_{QED}^q = e \hbar c \int d^3x \bar{q}(x) \gamma^\mu q(x)A_{\mu}(x) \cdot W_G,$$

(7)

where we have defined

$$W_G = \int d^3x' \bar{q}(x')(i\hbar c \gamma^\mu \partial_\mu - mc^2)q(x').$$

(8)

On the other hand, the interactions Eq. (4) is now

$$L_{QCD}^q = g \hbar c' \int d^3x' \bar{q}_i(x') \gamma^\mu \frac{\lambda^a_i}{2} q_j(x')A_{\mu}^a_i \cdot W_A,$$

(9)

with

$$W_A = \int d^3x \bar{q}(x)(i\hbar c \gamma^\mu \partial_\mu - mc^2)q(x).$$

(10)
Finally, the interactions in Eq. (5) is written as

\[ L_{q}^{\text{new}} = \left[ e\hbar c \int d^{3}x \bar{q}(x)\gamma^{\mu}q(x)A_{\mu}(x) \right]_{\text{em}} \cdot \left[ g\hbar c' \int d^{3}x' \bar{q}_{i}(x')\gamma^{\mu} \lambda_{ij}^{a}\frac{\lambda_{j}^{a}}{2}q_{j}(x')A_{\mu}^{a} \right]_{\text{color}}. \]  

(11)

Defining \( \alpha'_{s} \equiv g^{2}/\hbar c' \) and \( \alpha_{s} \equiv g^{2}/\hbar c \) we have

\[ \alpha'_{s} = \alpha_{s} \cdot \frac{c}{c'}. \]  

(12)

Thus, i) if \( c' < c \), it means that \( \alpha'_{s} > \alpha_{s} \) or, ii) if \( c' > c \), that \( \alpha'_{s} < \alpha_{s} \). Of course the possibility \( c' = c \) is not excluded but we will argue below that an interesting possibility is the first one.

In QCD hadrons are considered as being bound states of permanently confined quarks. However, until now, there is no proof neither of the existence of bound states nor of the confinement hypothesis in realistic \( 3 + 1 \) theories.

The confinement of quarks and gluons is supposed to be explained, in principle, in the context of QCD. Confinement means that we cannot isolate or produce particles carrying color. On the other hand, no fractional electric charge has also been found until now. Hence, since quarks carry out this sort of charge we can ask ourselves if the mechanism for the color confinement and charge screening are the same or not. It could be that quarks screen their color by vacuum polarization effects, but the Coulomb fields which give us information about their fractional electric charges remain unshielded. There is no answer for this question based on “first principles” but it was pointed out some years ago that in the massive Schwinger model the screening of the electric charge is not necessarily an indication for the color charges confinement. In other words, it means that the absence of colored states cannot be interpreted necessarily as a result of confinement in the usual sense, and colorless quarks are possible.

In the context we have considered above, it is possible that, instead of being the color confinement a dynamical consequence of the theory, it should be an implication of extra space-time dimensions. Hence, we do not understand the neutralization of the color degrees of freedom as the formation of hadrons but, as an effect due to the fact that these degrees
of freedom run out in extra space-time dimensions in which the speed limit is \( c' \). We do not know the actual value of \( c' \) but only that it could be different from the usual speed of light in vacuum \( c \).

Usually, in the quark picture, decays or reactions involving hadrons show symbolically the flow of quantum numbers during the process under consideration. However, this is of limited use when dynamical quantities are treated since we do not know how to represent the quark confinement in the context of a theory of strong interactions, say QCD. Hence, in the diagram representing the decays or reactions involving hadrons our ignorance is contained within the usual “shaded circles” (form factors).

According to the uncertainty principle, a fluctuation \( \Delta E \) in the energy of a particle is not observable if it occurs in a sufficiently small time interval \( \Delta t \). The maximum distance that a particle can travel in this time is (we assume that the value of \( \hbar \) is the same in both \( x \) and \( x' \)-worlds)

\[
R \approx c \Delta t = \frac{\hbar c}{\Delta E}.
\]  

(13)

For massless particles, since they can have arbitrary small amount of energy \( \Delta E = \hbar c/\lambda \), being \( \lambda \) the wavelenght of the particle, the range of the respective force is infinite. This happens for photons and gluons in both, the usual QED and QCD theories and also in the present approach. However, from the point of view of an observer in our space-time (unprimed one) the range of a gluon compared with a photon of the same energy is shorter:

\[
R_{\text{gluon}} = R_{\text{photon}} \cdot \frac{c'}{c}.
\]  

(14)

As long as \( c' \ll c \) we see that the range of the gluon as seeing in our laboratories is rather smaller than the range of a photon with the same energy. Hence, gluons would appear to us as being very massive.

Let us summarise the picture we have built up. In our context, the confinement which can be understood as: i) there are no colored states in Nature or, ii) scattering of hadrons produces only hadrons, are equivalent by construction. This is so because the space-time
dimensions accessible to the ordinary electron-photon devices are only sensitive to the electric part of the quarks. All electroweak degrees of freedom as leptons, photons, intermediate vector bosons and Higgs bosons, run on the usual $x$-world, while gluons run only in the $x'$ world. Only quarks are, in some sense, the “bridge” of both type of space-time since they carry color and electric charge. Since “observation” means that the system has experienced some interaction, such as the scattering off it of light or electron, in the absence of any interaction, the system is totally isolated from the outside world. It is in this sense that the color confinement is understood in our approach. According to the special theory of relativity, length and time measurements are dependent on the observer. In our scheme those measurements depend also on the type of fields used in carrying out the measurements: photons or gluons.

In despite of its speculative nature, our proposition can, at least in principle, be verified experimentally. Firstly, by observing colorless fractional charges. Within our context, even if it is possible to produce colored states, it is easy to see that it is not possible to detect, directly, these degrees of freedom. This is because all the experimental apparatus used so far are electron-photon devices. Hence, according to our theory they are not sensitive to the primed world space-time. Then, it is only throughout the electric interactions of quarks that we have information about the hidden degrees of freedom. Secondly, by evaluating the quark form factors $W_G$ and $W_A$ in Eq (8) and (10), respectively, using them for calculating the nucleon (hadrons) form factors and verifying if they coincide or not with the observed ones. Thirdly, looking for effects of non-renormalizable interactions like that in Eq. (5) or its simplified form in Eq. (11).

It is still possible that the space-time coordinates transform under $L \otimes L'$ like $(4, 4')$.

Finally, we call attention to the fact that although we have extended the number of space-time dimensions, any process like “compactification” of the extra dimension is not necessary. Thus, this ideas could be interesting in other contexts not necessarily in the one we are concerned here.
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