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Modelling conditional skewness: Heterogeneous beliefs, short sale restrictions and market declines

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ABSTRACT

This paper tests the relationship among heterogeneous beliefs, short sale restrictions and time-varying conditional skewness under different market conditions. The results show that heterogeneous beliefs and short sale restrictions have negative impacts on conditional skewness during periods of market decline but have negative, positive or no impacts during periods of market growth. This evidence reconciles conflicting evidence in recent empirical studies on the relationship among heterogeneous beliefs, short sale restrictions and conditional skewness.

The skewness of conditional return distributions has been widely recognized as a common phenomenon in financial markets. Some important aspects of skewness in returns have also been studied. First, a substantial body of literature documents that negative unconditional skewness is caused by asymmetric volatility (Engle & Ng, 1993; Glosten, Jagannathan, & Runkle, 1993; Nelson, 1990). Second, many authors have noted that financial time series behave differently during market declines and periods of market growth. Officer (1973), Schwert (1989a,b), and Campbell and Lettau (1999) indicated that volatility is higher when markets decline than it is when they expand, while Perez-Quiros and Timmermann (2001) identified a more pronounced negative conditional skewness during late expansions and early recessions. Third, starting from Hansen (1994), several empirical studies have documented that conditional skewness in market returns is time varying and predictable. Harvey and Siddique (1999), Jondeau and Rockinger (2003), Brooks, Burke, Heravi, and Persand (2005), Leon, Rubio, and Serna (2005), and Lanne and Saikkonen (2007) found a significant presence of time-varying conditional skewness in market returns. These studies viewed conditional skewness as analogous to heteroskedasticity and modeled conditional skewness as a function of lagged skewness.

However, more recent studies show evidence of the substantial instability of skewness over time, and thus, scholars have argued against the use of lagged skewness as the sole determinant of time-varying conditional skewness (Boyer, Mitton, & Vorkink, 2010). Motivated by the theory proposed by Hong and Stein (2003), which predicts negative skewness to be more pronounced when investors disagree more and short selling is restricted, a number of empirical studies have addressed this question by analyzing the relationship among skewness, short sale constraints, and heterogeneous beliefs. Tests on the validity of the theory, however, have returned conflicting findings. Daouk and Charoenrook (2005), Chang, Cheng, and Yu (2007), and Hueng and McDonald (2005) found either an insignificant or positive relationship among short sale restriction, heterogeneous beliefs, and skewness, whereas Chen, Hong, and Stein (2001) and Boyer et al. (2010) found a negative relationship among these factors. The details about these empirical findings and implications for financial market analysis are elaborated in Shum (2019).
The empirical analyses of skewness mentioned above pays no attention to the behavior of skewness under different market conditions. This paper argues that the relationship among short sale restrictions, heterogeneous beliefs, and conditional skewness behaves differently during periods of market decline and periods of market growth. The time series models that ignore this difference are highly likely to be mis-specified if the sample skewness measured during market declines behaves differently from that in expansion periods. Several theoretical and empirical papers motivate our empirical investigation on the effect of heterogeneous beliefs, short sale restrictions, and market direction on conditional skewness. Section 1 reviews the theoretical and empirical works that motivate our study. Section 2 discusses the theoretical background of our empirical work. Section 3 presents the conditional skewness model, which helps us test our idea. Section 4 discusses our empirical results, and Section 5 concludes the study.

1. Literature review

A number of theories have been proposed to explain the existence of skewness. One of the most influential theories of skewness, first documented by Black (1976), attributes the negative relationship between current stock prices and future volatility to leverage. An increase in financial leverage followed by a period of price decline increases future volatility and thus introduces negative skewness when the debt level is fixed. Although the empirical effect of leverage on volatility has been proven to be statistically significant (Christie, 1982), the effect is not sufficiently large to account for all the asymmetries in stock prices (Figlewski & Wang, 2000; Schwert, 1989b).

The second theory is the rational bubble theory (Blanchard & Watson, 1983; Diba & Grossman, 1988). A sharp fall in price followed by a period of sustained stock price increase contributes to the overall negative unconditional skewness in the market. The rational bubble theory, however, does not help us model skewness. Just as the bubble theory itself cannot predict when the bubble will burst, the model tells us nothing about when the distribution of returns will become more negatively skewed.

The third theory is the volatility feedback model (French, Schwert, & Stambaugh, 1987; Pindyck, 1983), which assumes that both good news and bad news generate uncertainty and, hence, volatility shocks with respect to future prices. Risk-averse investors will therefore require a higher rate of return and consequently a lower current price to compensate for the higher risk level, regardless of the nature of the news. This volatility feedback effect strengthens the effect of the negative impact of bad news but moderates the effect of the positive impact of good news. As a result, on average, the magnitude of the effect of negative events is larger than that of positive events, contributing to negative skewness in equity returns. The volatility feedback assumption is consistent with the empirical findings in Nelson (1990), Engle and Ng (1993), Glosten et al. (1993), and other empirical asymmetric GARCH studies.

The above theories are representative-agent-based and assume rationality. The fourth theory, the heterogeneous-agent-based theory proposed by Hong and Stein (2003), however, suggests the use of a mild assumption of investor irrationality together with some institutional frictions that offer us some insights into the abnormal behavior of skewness. They assume that some investors are overconfident and believe in their own private signals, which in turn generates differences of opinion; when differences of opinion are large and short selling is not allowed, the market prices reflect only the valuation of the optimist investors since short sale constraints prevent negative information from being revealed in the market. Aligned with earlier studies of the effects on asset pricing caused by heterogeneous beliefs and short sale constraints (Harrison & Kreps, 1978; Miller, 1977), Hong and Stein (2003) theory suggests that short sale constraints with heterogeneous beliefs lead to overpricing. Furthermore, Hong and Stein (2003) theory extended earlier studies by suggesting that if other previously optimistic investors bail out of the market, then the originally pessimistic group may become the marginal buyers and more will be learned about their negative signals. Thus, the accumulated hidden information is revealed during the market decline, and these returns will be more negatively skewed with high trading volume, which is a proxy of the degree of heterogeneous beliefs.

The positive relationship between negative asymmetries and the degree of heterogeneous beliefs is an implication of Hong and Stein (2003) paper. Chen et al. (2001) developed a series of cross-sectional analyses in an attempt to test this idea. In their analysis, they found that a higher detrended turnover, a proxy of the degree of heterogeneous beliefs, can predict more negatively skewed daily returns. Thus, they found evidence to confirm the theory proposed by Hong and Stein (2003). Through the use of a similar methodology, Boyer et al. (2010) found similar results by showing that firms that have high turnover usually have more negatively skewed returns.

Not all evidence points to the same conclusion. There is empirical evidence against Hong and Stein (2003) theory. Charoenrook and Daouk (2004) found that a higher detrended turnover predicts more negative unconditional skewness in countries where short selling is allowed than in countries where short selling is not allowed. Chang et al. (2007) found that the skewness of unconditional returns increases when stocks are not allowed to be sold short and decreases when stocks are allowed to be sold short. Blau and Pinegar (2009), who approximate short sale constraints by using relatively short interest, show that there are positive relationships among turnover, relatively short interest, and unconditional skewness. Huang and McDonald (2005) tested the behavior of time-varying conditional skewness by assuming a skewed t distribution on conditional returns. They show that a larger variance today is positively related to contemporaneous skewness at the market level.

Finally, the price convexity theory (Xu, 2007) provides the fifth potential explanation for market skewness. Unlike the above-mentioned theories that explain negative asymmetries, the price convexity theory explains positive market asymmetries. The model assumes that people may react to the same signal to a different extent. Since high precision traders react more than low precision traders to the same signal, price reactions are stronger to positive than to negative signals with the presence of short sale constraints. When the distribution of the signal is symmetric and no other factors affect the skewness of market return, the overall level of market skewness is positive. The price convexity theory, however, does not rule out the possibility of overall negative skewness. The model
allows the other skewness factors such as the volatility feedback effect, the stochastic bubble effect and the Hong and Stein’s “hidden bad news” effect to bring negativity to the overall level of market skewness. However, the theory does predict that the relative skewness conditional on the degree of price convexity remains positive even with the presence of the other skewness factors. Similar to the empirical studies in Charoenrook and Daouk (2004), Chang et al. (2007), Blau and Pinegar (2009), and Huang and McDonald (2005) mentioned above, empirical tests conducted by Xu (2007) find evidence against the Hong and Stein (2003) theory. In sum, they find that stock prices react more to good news than to bad news, short sale constrains increase skewness and heterogeneous beliefs increase skewness.

2. Theoretical background

None of the above empirical analyses of skewness include the behavior of skewness in different market conditions. We argue that the relationship among short sale restrictions, heterogeneous beliefs, and conditional skewness behaves differently during periods of market decline and periods of market growth. We believe that when the market is on the rise and a sustained increase in the market price is expected, a sudden price fall will not trigger a bearish investor who believes in his/her own negative signal to enter the market since it is profitable for other rational buyers to buy now and sell later, not only because shares would gain in dividends but also because rational buyers expect that prices will rise more quickly than discounts (Diba & Grossman, 1988).

To elaborate on our point, we take a closer look at the original Hong and Stein (2003) theory which assumes stock price depends on the terminal value of the dividend paid and then we modified their theory by allowing stock gains in dividends as well as its expected price when sold.

2.1. The original model

The original Hong and Stein (2003) theory assumes that a market with one stock to trade involves trader A, trader B, and a group of risk-neutral rational arbitrageurs. Traders A and B are subject to short sale constraints, but the arbitrageurs are not. The traders and the arbitrageurs will trade for four periods: time 0, 1, 2, and 3. Time 0 is the starting time of the trade, where no one receives any information. At time 1, trader B observes $S_{B1}$, and at time 2, trader A observes $S_{A1}$. The signals are equally informative for the arbitrageurs, and the stock will pay a terminal dividend of $D$ at time 3 where $D = (S_{A3} + S_{B3})/2 + \varepsilon$ and $\varepsilon$ is a normally distributed shock with zero mean and unit variance. The market clearing price of the stock at time 2 is the terminal value of the dividend paid, $P_2 = E(D)$. To obtain the equilibrium price, let us assume that trader A, trader B, the arbitrageurs, and the auctioneer are sitting in a room to “trade”. The auctioneer starts by announcing a “high” trial price $p$, which is known to be higher than anybody’s highest possible valuation. The initial “high” trial price creates negative excess demand. The auctioneer continues to lower the price when the excess demand remains negative. Conversely, once he/she reaches a point where the excess demand is positive, he/she raises the price. The process continues until the auctioneer reaches a price where the excess demand is zero and the market is cleared.

During the entire trading process, the arbitrageurs know that the average of signals $S_{A1}$ and $S_{B1}$ is the best estimate of the price. However, the arbitrageurs may not always see both signals. To further examine this, we look at the trading process at different time periods. First, at time 1, trader B obtains a bad signal $S_{B1}$, so their valuation of the stock is well below that of A. Because of short sale constraints, trader B will not enter the market, and the only trade will be between trader A and the arbitrageurs. The arbitrageurs can deduce that B’s signal is well below A’s but cannot determine the exact amount. Therefore, the market price at time 1 impounds A’s prior information but cannot reflect B’s time 1 signal.

Next, at time 2, two possible scenarios arise. If A receives a good signal at time 2, then B’s signal remains hidden. If, however, A gets a bad signal at time 2, then it is possible that B’s previously hidden time 1 signal may be revealed at time 2. For example, if B steps in and is willing to buy after the price drops by 5%, the arbitrageurs learn that B’s time 1 signal was not very negative. If, however, B does not step in even after the price drops by 20%, the arbitrageurs must conclude that B’s time 1 signal was more negative than their prior expectation. In other words, trader B’s sideline behavior in the face of A’s selling is additional bad news for the arbitrageurs above and beyond the bad news implicit in A’s selling.

The original model described above captures two elements of a market crash. First, the price movement may be totally out of proportion to the news arriving in the market (i.e., the signal received by A). Second, at time 2, when A gets a positive signal, it is revealed in the price, but the same is not true for the bad signal $S_{B1}$ received at time 1. In contrast, when A gets a negative signal, not only is the bad signal $S_{A1}$ received at time 2 revealed but the bad signal $S_{B1}$ received at time 1 may also be revealed. This result implies that the largest price movements will be declines. Apart from capturing market crashes, the original model also captures the skewness of return distributions: hidden bad news at time 1 translates into overpricing and thus positive asymmetry, while a piece of related bad news at time 2 generates negative asymmetry with a larger magnitude.

2.2. The modified model

The model can be modified if we assume the stock gains in dividends as well as its expected price when sold. The discounted future price remains positive if an agent can rationally plan to sell the overvalued asset to another agent at some finite future date (O’Connell & Zeldes, 1988; Tirole, 1982). Therefore, instead of $P_2 = E(D)$, we assume the following terminal payoff of the stock at time 2:
where $\beta$ is the terminal value of a rational bubble process that arises when the expected future price is growing faster than the discount factor. A few good examples of these rational bubbles are provided in Blanchard and Watson (1982) and Diba and Grossman (1988). We continue assuming that traders $A$ and $B$ only believe in their signals, but we also assume that the arbitrageurs expect that they can sell the stock at time 3 to another agent with a markup $b$ added onto the fundamental price of the stock. Therefore, the terminal value of the bubble process $\beta$ can be written as:

$$\beta = b + \varepsilon$$

where $\varepsilon$ is a random variable (or combination of random variables) generated by a stochastic process that satisfies $E(\varepsilon) = 0$. This process is similar to the process in Diba and Grossman (1988). Under the assumptions of perfect information, rational expectation, risk neutrality, constant marginal utility, and constant discount rates, Diba and Grossman (1988) show that a negative rational bubble ends at a negative price. Given free disposal, a negative bubble is not possible since investors cannot rationally expect the value of the stock to become negative in a finite time. Therefore, bubbles can only be positive, i.e., $\beta \geq 0$.

With the assumption of positive bubbles, trader $B$’s signal $S_B$ remains hidden as long as $b$ is high enough. When $b$ is high, before the trial price $p$ called out by the auctioneer ever reaches $S_B$, it reaches the arbitrageurs’ conditional estimate of the stock value, and the market is cleared. This is true for both time 1 and time 2. Therefore, when trader $A$ gets a negative signal at time 2, trader $B$ will not enter the market, since the arbitrageurs will clear the market with an overvalued price. Their negative signals will be revealed if and only if the bubble bursts, i.e., $\beta = 0$, or when $b$ is small enough.

In other words, the Hong and Stein (2003) theory of rational bubbles predicts negative asymmetries if and only if the bubble bursts and the market enters a downward adjustment period. On the other hand, when the bubble is forming, traders enter the market not only because shares would gain in dividends but also because rational buyers expect that prices will rise more quickly than discounts (Diba & Grossman, 1988). This implies that rational traders can plan to sell an overvalued asset to another agent at some finite future date (O’Connell & Zeldes, 1988; Tirole, 1982). It follows that a sudden price fall (or a technical downward adjustment) will not trigger traders with bad private signals about the terminal dividend to enter the market since it is profitable for other rational buyers to buy now and sell later to clear the market even at an overvalued price. As a result, the modified theory predicts less negative (i.e., more positive) asymmetries or no asymmetries during the bubble phase since the bubble process prevents traders with bad private signals to enter the market.

Therefore, the theory proposed by Hong and Stein (2003), together with the bubble process, predicts more pronounced negative skewness during market declines than expansions, and thus, negative asymmetries are more positively related to the degree of heterogeneous beliefs and short sale restrictions during periods of general market declines.

The aim of this paper is therefore to examine the effects of short sale restrictions, heterogeneous beliefs, and market direction on time-varying conditional skewness. In particular, we would like to see how the relationship among heterogeneous beliefs, short sale restrictions, and conditional skewness changes under different market conditions. In the next section, we present a skew normal generalized autoregressive conditional heteroskedasticity (SN-GARCH) model that helps us test this idea.

3. Modeling conditional skewness

The SN-GARCH model extends the GARCH model to allow for time-varying conditional skewness by assuming that conditional returns follow a skew normal distribution (Azzalini, 1985). The class of skew normal distributions extends the normal distribution by including an additional shape parameter to regulate skewness and allows a continuous transition from normality to non-normality. There are other alternatives such as the inverse hyperbolic sine function proposed by Johnson (1949), the stable Paretian distributions proposed by Mandelbrot (1963), the skew-$t$ distribution proposed by Hansen (1994), the skewed generalized error distribution proposed by Hansen, McDonald, and Newey (2010), and the centered parametrized skew-$t$ distribution proposed by Azzalini (2013). All these distributions allow us to model both skewness and kurtosis. Since our paper focuses on skewness but not kurtosis and the parametric tests can still perform well with data that are fat-tailed if the sample size is large enough, we have chosen the skew normal distribution since the skew normal distribution is mathematically tractable even under a multivariate setting, and no transformation due to parameter constraints imposed by the distribution is necessary (Arellano-Valle & Azzalini, 2008; Azzalini & Dalla Valle, 1996). Thus, the skew normal distribution is a natural choice for modeling skewness when the data under analysis exhibits unimodal empirical distribution with some skewness present. First, we look at the properties of the skew normal distribution, and then we present our SN-GARCH model.

3.1. Skew normal distribution

To define the skew normal random variables, let $X$ and $Y \sim N(0,1)$. For any $z, \alpha \in \mathbb{R}$, the random variable

$$Z = \frac{\alpha}{\sqrt{1 + \alpha^2}} |X| + \frac{1}{\sqrt{1 + \alpha^2}} Y$$

with the density function

$$f(z) = 2\phi(z)\Phi(\alpha z).$$
where \(-\infty < z < \infty\) is referred to as a standard skew normal variable and where \(\phi()\) and \(\Phi()\) are the standard normal density and distribution function, respectively. The class of standard skew normal variables is denoted by \(SN(\alpha)\), with \(\alpha\) representing the skewness coefficient of the distribution. When \(\alpha\) is zero, the skew normal distribution reduces to a simple standard normal distribution. The distribution is left-skewed if \(\alpha\) is negative and right-skewed if \(\alpha\) is positive. The moment generating function of the random variable \(Z\) above is given by

\[M(t) = 2\exp(t^2/2)\Phi(\delta),\]

where \(\delta = \alpha/(1 + \alpha^2)^{1/2}\). From the moment generating function, we can obtain the first two moments of the skew normal random variables

\[E(Z) = b\delta\] and \(E(Z^2) = 1,\]

where \(b = (2/\pi)^{1/2}\). Note that the second moment of the skew normal variate is the same as the second moment of the normal variate; in fact, both have the same \(\chi^2\) distribution when squared. Indeed, all even moments of a skew normal variable coincide with the corresponding even moments of the normal. It follows from the first two moments presented above that the variance of the skew normal distribution is given by

\[\text{Var}(Z) = 1 - b^2\delta^2.\]

In practice, one can apply a linear transformation to generate an entire family of skew normal distributions, as follows:

\[V = \xi + \omega Z,\quad SN(\alpha),\]

where the random variable \(V\) is a skew normal variate with the location parameter \(\xi\) and the scale parameter \(\omega\). We denote this distribution as \(SN(\xi, \omega^2, \alpha)\). When \(\alpha = 0\), we obtain the normal distribution, \(N(\xi, \omega^2)\). The expected mean, variance, and skewness of the skew normal variable \(V\) are

\[E(V) = \xi + \omega\sqrt{2/\pi}\delta\]
\[\text{Var}(V) = \omega^2(1 - 2\delta^2/\pi)\]

and

\[\text{Skew}(V) = \frac{4 - \pi}{2} \frac{E(Z)^3}{\text{Var}(Z)^{3/2}}\]

with

\[\delta = \alpha/\sqrt{1 + \alpha^2},\quad E(Z) = \sqrt{2/\pi}\delta,\quad \text{Var}(Z) = 1 - 2\delta^2/\pi\]

We can thus apply the usual transformation to have a more general skew normal density function
where \(-\infty < v < \infty\).

3.2. The SN-GARCH model

In this section, we introduce the two variations on the SN-GARCH model we used in the empirical section: the Glosten-Jagannathan-Runkle GARCH (GJR-GARCH) model (Glosten et al., 1993) and the quadratic GARCH (Q-GARCH) model (Sentana, 1995). To model conditional skewness, we consider the specification of returns in which

\[ r_t = \mu + u_t, \quad u_t = \alpha \epsilon_t \]

where \( r_t \) is the daily return at day \( t \), \( \mu \) is the conditional mean, and \( u_t \) is the unexpected part of returns, which is generally referred to as “news” in the markets.

According to the GJR-GARCH model, the conditional variance follows an asymmetric GARCH process as follows:

\[ \sigma_t^2 = a_0 + a_1 \sigma_{t-1}^2 + a_2 u_{t-1}^2 + a_3 u_{t-1}^2 I^+ \]

where \( I^+ = 1 \) if \( u_{t-1} > 0 \) and \( I^- = 0 \) if \( u_{t-1} \leq 0 \). We expect the coefficient on the asymmetric term to be negative so that bad news has a larger impact than good news on the conditional variance of the return.

In the Q-GARCH model, the variance equation is specified as follows:

\[ \sigma_t^2 = a_0 + a_1 \sigma_{t-1}^2 + a_2 u_{t-1}^2 + a_3 u_{t-1}. \]

Since arbitrage forces unexpected returns to have zero mean, we assume the innovation term \( \epsilon_t \) to be a sequence of independent, identically distributed random variates from the general skew normal distribution, \( SN(\kappa, \tau^2, \alpha) \). Similar to Liseo and Loperfido (2006), the location parameter is

\[ \kappa = -\frac{2}{\pi} \frac{r^2 \alpha}{\sqrt{1 + \alpha^2}} \]

and the scale parameter is

\[ \tau^2 = \left(1 - \frac{2\alpha^2}{\pi(1 + \alpha^2)}\right)^{-1} \]

such that the innovation term \( \epsilon_t \) has zero mean, as required by the arbitrage-free condition, and the scale parameter \( \sigma_t^2 \) is retained to
be the conditional variance of the model. Unlike Liseo and Loperfido (2006), we allow the shape parameter $\alpha$, and thus, the skewness in the Q-GARCH model to be time varying.
Fig. 5. Yearly price differences.

Fig. 6. Quarterly price differences.
Note that when $b_3 > 0$, good news has a positive impact and bad news has a negative impact on the skewness of the returns. The use of $\varepsilon_{t-1}$ instead of $u_{t-1}$ in the skewness equation is to prevent conditional variance from having an effect on skewness. $S_i$ represents all the relevant skewness factors and can be defined as follows:

$$S_i = c_1DTO_t + c_2DSI_t + c_3DTO_tI_t + c_4DSI_tI_t + c_5DTO_tDSI_t + c_6DTO_tDSI_tI_t + c_7I_t$$

(20)

The model has three main skewness factors. $DTO_t$, a detrended turnover, which is a proxy for heterogeneous beliefs; $DSI_t$, a detrended short interest, which is a proxy for short sale restrictions. (For examples of these uses of proxies, see Miller (1977), Epps and Epps (1976), Figlewski (1981), Jones and Lamont (2002), and others.) The third skewness factor is the market direction indicator, $I_t$, which is equal to one during periods of general market declines and zero otherwise. We expect the signs of the coefficients on “$DTO_tI_t$” and “$DSI_tI_t$” to be negative since the Hong and Stein (2003) theory, together with the bubble process, predicts that negative asymmetries are more positively related to the degree of heterogeneous beliefs and short sale restriction during periods of general market decline. The other variables control any interaction effects among the three variables. We refer to the model that uses the GJR-GARCH conditional variance as the SN-GJR-GARCH model, and we refer to the model that uses the Q-GARCH conditional variance as the SN-Q-GARCH model.

An estimation has been performed using the “optimize” routine in Mata; the code is available from the author upon request.

4. Empirical tests

The stocks analyzed in this paper consist of all the components of the Hang Seng Index (HSI). Our data, including short interest, trading volume, and total shares outstanding of individual stocks that constitute the HSI over the period Jan 4, 1999 to May 31, 2011, were purchased from the Hong Kong Stock Exchange website (www.hkex.com.hk). The daily return series for individual stocks is calculated as $r_{i,t} = \ln(p_{i,t}) - \ln(p_{i,t-1})$, where the daily closing prices for individual stocks, $p_{i,t}$, are obtained from the Reuters EcoWin Pro database. Market return, turnover, and short interest are capitalization weighted and consist of all HSI components. Historical changes to the list of HSI constituents can be downloaded from the HSI website (www.hsi.com.hk). The turnover and short interest series are measured in the number of shares traded per day.

Following the methodology used by Chen et al. (2001), the normal level of heterogeneous beliefs and short sale restrictions in this paper are approximated by a centered moving average of market turnover and short interest over a 120-day window where the centered moving average with the $i$-day window is defined as $\tilde{x}_{i,t} = \frac{1}{2i+1} \sum_{j=-i}^{i} x_{t+j}$. Both the turnover and short interest series are
detrended by first taking their natural log and then subtracting the moving average trends from the logged series. The degree of heterogeneous beliefs and short sale restrictions are the highest (lowest) when the level of turnover or short interest has the highest positive (negative) deviation from the normal levels. We use a centered instead of a backward moving average since we are interested in detrending and not forecasting. Fig. 1 plots the market return, whereas Figs. 2 and 3 plot the detrended turnover and detrended short interest series, respectively.

We define four different market direction indicators, namely, the crisis indicator $\mathcal{I}_{\text{crisis},t}$, the yearly market direction indicator $\mathcal{I}_{\text{year},t}$, the quarterly market direction indicator $\mathcal{I}_{\text{qtr},t}$, and the weekly market direction indicator $\mathcal{I}_{\text{week},t}$. We first define the crisis indicator. There were two major financial crises in Hong Kong over the period 1999–2012. First, shortly after the recovery from the financial crisis of 1997–1998, Hong Kong’s economy was hit by the global economic downturn in 2001 followed by the outbreak of severe acute respiratory syndrome (SARS) in 2003. The second major crisis was the global financial crisis that took place in 2008. We define the first crisis period as March 28, 2000 to April 25, 2003 and the second crisis period as October 30, 2007 to March 9, 2009. Fig. 4 shows the HSI series along with the start and end dates of the crises. Since the purpose of the crisis indicator is to indicate general market declines, we pick the starting and ending dates by using ex post data.

The crisis indicator is somewhat arbitrary; therefore, we also look at other market direction indicators. The yearly market direction indicator $\mathcal{I}_{\text{year},t}$ is equal to 1 at day $t$ for the year $y - 1$ if the yearly price difference calculated as $P_y - P_{y-1}$ is negative, where the variable $y$ records the number of years from the start of January 4, 1999 to May 31, 2011 and $P_y$ is the last observation of the daily price series in year $y$. The quarterly market direction indicator $\mathcal{I}_{\text{qtr},t}$ is equal to 1 at day $t$ for the quarter $q - 1$ if the quarterly price difference calculated as $P_q - P_{q-1}$ is negative, where the variable $q$ records the number of quarters from the start of January 4, 1999 to May 31, 2011, and $P_q$ is the last observation of the daily price series at quarter $q$. The weekly market direction indicator $\mathcal{I}_{\text{week},t}$ is equal to 1 at day $t$ for the week $w$ if the weekly price difference calculated as $P_w - P_{w-1}$ is negative, where the variable $w$ records the number of weeks from the start of January 4, 1999 to May 31, 2011 and $P_w$ is the last observation of the daily price series at week $w$.

Figs. 5–7 show the yearly, quarterly, and weekly price difference indicators, respectively.

### 4.1. HSI empirical skewness

Most financial data are characterized by time-varying and predictable skewness. For example, Hansen (1994), Harvey and
Siddique (1999), Jondeau and Rockinger (2003), Brooks et al. (2005), Leon et al. (2005), and Lanne and Saikkonen (2007) found a significant presence of time-varying conditional skewness in market returns.

To see whether skewness of our daily HSI return data exhibits any kind of time-varying property, we display the rolling monthly estimates of different unconditional skewness measures in Fig. 8. The first unconditional skewness measure is the classic fisher skewness coefficient defined as

$$\text{Fisher Coefficient } 1(\tau_t) = \frac{1}{n} \sum_{i=1}^{n} \frac{(r_t - \bar{r})}{s^3}$$

where $r_t$ the daily log return at time $t$, $\bar{r}$ is the sample mean and $s$ is the sample standard deviation. The second measure is the quantile-based measure developed by Hinkley (1975), which is defined as

$$RA_\theta(r_{i,n}) = \frac{[q_{0.75}(r_{i,n}) - q_{0.5}(r_{i,n})] - [q_{0.5}(r_{i,n}) - q_{1-\theta}(r_{i,n})]}{q_{1-\theta}(r_{i,n}) - q_{0.75}(r_{i,n})}$$

where $q_\theta(r_{i,n}) = F_{n}^{-1}(r_{i,n})$ for $\theta \in (0, 1]$ is the unconditional quantiles of the log continuously compounded $n$-period return $r_{i,n} = \sum_{j=0}^{n-1} r_{i,j}$. We set $\theta = 0.75$ as Yule and Kendall (1968) showed that when $\theta = 0.75$, $RA_\theta(r_{i,n})$ is very robust to outliers.

From Fig. 8, we can see that all the rolling unconditional skewness estimates show the variation in skewness month by month for the HSI index. From the unconditional estimates alone, however, we have no concrete evidence for instability of unconditional skewness over time. However, it has been shown that failure to detect unconditional skewness in the sample would not rule out the possibility of time-varying conditional skewness over the estimation period (Ghysels, Plazzi, & Valkanov, 2011). The same can also be applied to the stability of conditional and unconditional skewness; even if unconditional skewness proves to be stable, we cannot rule out the possibility of the presence of unstable conditional skewness. To see whether there is any instability in conditional skewness, we apply the robust conditional quantile-based skewness estimate proposed by Ghysels et al. (2011, 2016). The conditional quantile-based skewness measure is defined as

$$RA_{\theta,1-1}(r_{i,n}) = \frac{[q_{0.75,1-1}(r_{i,n}) - q_{0.5,1-1}(r_{i,n})] - [q_{0.5,1-1}(r_{i,n}) - q_{1-\theta,1-1}(r_{i,n})]}{q_{1-\theta,1-1}(r_{i,n}) - q_{0.75,1-1}(r_{i,n})}$$

where

$$q_{\alpha,1-1}(r_{i,n}) = \alpha_{0,n} + \beta_{0,n} Z_{1-1}(\kappa_{0,n})$$
is the conditional quantile of the log continuously compounded \( n \)-period return \( r_{t,n} = \sum_{j=0}^{n-1} r_{t+j} \), with a linear filter

\[
Z_{t-1}(x_{d,t}) = \sum_{d=0}^{d} a_d(x_{d,t}) x_{t-1-d}
\]

where \( x_{t-1-d} \) is a conditional variable representing daily conditioning information with lag of \( d \) days and the weight is

\[
a_d(x_{d,t}) = \frac{f \left( \frac{d}{\theta}, x_{d,t-1} ; \ldots, x_{d,t} \right)}{\sum_{d=0}^{D} f \left( \frac{d}{\theta}, x_{d,t-1} ; \ldots, x_{d,t} \right)}
\]

Following Ghysels et al. (2011), we set \( d = 250, n = 22, x_{d,t} = 1 \) and \( x_{t-1-d} = |r_{t-1-d}| \). We have also followed the convention to set \( \theta = 0.75 \) (Yule & Kendall, 1968) and denote the robust conditional quantile-based skewness estimate as SK75. Fig. 9 displays the robust conditional skewness measure and compares it with our crisis periods. We note that the robust conditional skewness measure is unstable over time and is more negative during the two crisis periods. This gives us the first piece of evidence for our modified theory, which predicts less negative (i.e., more positive) asymmetries during the bubble phase.
4.2. SN-GARCH estimation results

Table 1 shows the estimation results and various specification tests for the SN-GJR-GARCH and SN-Q-GARCH models presented in the previous section.

First, we look at the variance equation in Table 1. In the variance equations of both models, all the signs are consistent with our expectations. Coefficients on the asymmetric terms in the variance equations are negative and strongly significant at the 1% level for various specifications, implying that negative shocks tend to cause higher volatility than positive shocks. This result is consistent with the studies of Nelson (1990), Engle and Ng (1993), Glosten et al. (1993), and other empirical asymmetric GARCH studies.

Second, in the skewness equations of both models in Table 1, the coefficients are generally significant at the 5% level, indicating that conditional skewness is time-varying, which is consistent with the findings of Harvey and Siddique (1999), Jondeau and Rockinger (2003), Brooks et al. (2005), Leon et al. (2005), and Lanne and Saikkonen (2007). The coefficients on $t_{1}$ in the skewness equations of both models are positive, implying that good news has a positive impact and bad news has a negative impact on the skewness of the returns. This shows that our asymmetric terms in the variance equations and skewness equations are consistent with each other. The asymmetric terms in the variance equations tell us that the distribution of the returns is negatively skewed for the estimation period, whereas the asymmetric terms in the skewness equations tell us that today’s returns are more negatively skewed when the market receives bad news on the previous day.

Third, in the skewness factors equation of Table 1, the coefficients on $DTO_{Crisis t}, I$ and $DSI_{Crisis t}, I$ are negative and strongly significant for various specifications in both models, indicating that both turnover and market short interest have a statistically significant power in predicting conditional skewness during the two crisis periods. Specifically, during the two crisis periods, negative conditional skewness is more pronounced when people disagree about the market more or the short sale constraints are increasingly binding. However, heterogeneous beliefs and short sale restrictions do not seem important to the determination of market conditional skewness during market growth since the coefficients on $DTO_{t}$ and $DSI_{t}$ in our baseline specifications, specification (1) for the SN-GJR-GARCH model and specification (4) for the SN-Q-GARCH model, respectively, are not statistically significant. In other words, our results indicate that the short sale constraints and differences in opinions contributed to negative asymmetries during the two crisis periods but did not do so during the periods of market growth. These results confirmed our prediction, since the Hong and Stein (2003) theory of rational bubbles predicts negative asymmetries if and only if the bubble bursts and the market enters a downward adjustment period. Note that in specifications (3) and (6), when we omit the crisis indicator and its related terms, heterogeneous 

![Fig. 10. Conditional skewness decomposition.](image-url)
beliefs and short sale restrictions, i.e., $DTO_t$ and $DSI_t$ have either positive, negative, or no impacts on conditional skewness. This result may shed light on why studies on the effect of heterogeneous beliefs and short sale restrictions on skewness have come to conflicting conclusions.

Fourth, we decompose the skewness equation in our baseline specifications, i.e., specification (1) for the SN-GJR-GARCH model and specification (4) for the SN-Q-GARCH model in Table 1, into two parts:

$$s_t = s_0 + S_t$$  \hspace{1cm} (27)

where part I is defined as

$$s_t = b_0 + b_1 s_{t-1} + b_2 \varepsilon_{t-1}^2 + b_3 \varepsilon_{t-1}$$  \hspace{1cm} (28)

and part II is defined as

$$S_t = c_1DTO_t + c_2DSI_t + c_3DTO_t \tilde{z}_t + c_4DSI_t \tilde{z}_t + c_5DTO_t DSI_t \tilde{z}_t + c_6DTO_t DSI_t \tilde{z}_t + c_7 \tilde{z}_t$$  \hspace{1cm} (29)

The first part of the skewness equation $s_t$ viewed conditional skewness as analogous to heteroskedasticity and modeled conditional skewness as a function of lagged skewness and shocks. Under the specification $s_t$ is stable over time and thus, the instability of the conditional skewness is brought to us by $S_t$. Fig. 10 plot the conditional skewness decomposition for the SN-GJR-GARCH and SN-Q-GARCH model. It shows us how the skewness factors bring instability to conditional skewness. First, the skewness factors contribute more negative skewness during the crisis periods. Second, conditional skewness clearly does not mean reverting during the crisis periods.

Last, we compare the robust conditional skewness measure proposed by Ghysels et al. (2011, 2016) in Fig. 9 with the conditional skewness estimated by the SN-GJR-GARCH and SN-Q-GARCH model (i.e., specification (1) and (4) in Table 1 respectively). The comparison is shown in Fig. 11. From the figure, we can see that both measures predict less negative (i.e., more positive) asymmetries during the bubble phase and, thus, give us additional evidence to support our modified theory.

Table 2 shows the estimation results of the SN-GJR-GARCH and SN-Q-GARCH models with different market direction indicators. As mentioned in the previous section, the crisis indicator is somewhat arbitrary; therefore, as a robustness check, we also test the yearly, quarterly, and weekly market direction indicators. As shown in Figs. 4 and 5, the yearly market indicator spots similar turning points in the market, whereas the quarterly market indicator spots more frequent changes in the market direction than the crisis.
The crisis and yearly indicators help us test how conditional skewness responds to the general market decline for a relatively long period of time. The quarterly market indicator, which can be seen in Fig. 6, helps us test how conditional skewness responds to more frequent changes in the market direction, and the weekly market direction indicator, which can be seen in Fig. 7, helps us test how conditional skewness responds to short-term changes in the market direction. The estimation results are presented in Table 2. All coefficients on \( \text{DTO}_{t-1} \) and \( \text{DSI}_{t-1} \) are negative and strongly significant when the yearly and quarterly market indicators are used. However, we find that market turnover has only mild statistical power in forecasting conditional skewness, while market short interest is not important in determining the skewness of market return even during periods of general market declines when the weekly market direction indicator is used. This result may imply that the degree of heterogeneous beliefs and short sale restrictions in the market has an effect on conditional skewness when the downward trend in the market persists for more than a week. It is also possible that the weekly market direction indicator that we have used to obtain the above results may be a noisy proxy for indicating real changes in the market direction.

Finally, in Table 3, we test the effect of past returns on conditional skewness by including cumulative returns in our model. Both Hong and Stein (2003) and Hueng and McDonald (2005) found that past returns as far back as 36 months are negatively related to conditional skewness. Therefore, following Hueng and McDonald (2005) and Hong and Stein (2003), we define cumulative returns as $R_{t} = \prod_{i=1}^{750} (1 + R_{t-i}) - 1$, where $R_{t-i}$ is the simple return between times $t - i$ and $t$. For $t < 750$, we set $r$ as the number

### Table 2

| Indicator (I) | SN-GJR-GARCH | SN-Q-GARCH |
|---------------|--------------|------------|
| Variance Equation $h_t$ | $\eta_{t-1}$ | $\eta_{t-1}$ |
| $h_{t-1}$ | 0.921061*** | 0.924689*** |
| ($0.0075$) | ($0.0076$) |
| $u_{t-1}$ | 0.068102*** | 0.094890*** |
| ($0.0070$) | ($0.0097$) |
| $u_{t-1}I^+$ | $-0.000647***$ | $0.000002***$ |
| ($0.0001$) | ($0.0000$) |
| $u_{t-1}$ | $-0.0000588***$ | $0.000002***$ |
| ($0.0001$) | ($0.0000$) |
| _cons | 0.000002*** | 0.000002*** |
| ($0.0000$) | ($0.0000$) |

**Skewness Equation** $\alpha_t = \alpha_1 + \alpha_2$

**Skewness Factors I-$\alpha$**

| $s_{t-1}$ | $-0.395437***$ | $-0.393245***$ |
| ($0.1067$) | ($0.1094$) |
| $s_{t-1}^2$ | $0.190641***$ | $0.185546***$ |
| ($0.0651$) | ($0.0689$) |
| $s_{t-1}$ | $0.449871***$ | $0.435419***$ |
| ($0.1296$) | ($0.1279$) |
| _cons | 0.203194 | 0.182494 |
| ($0.4893$) | ($0.4891$) |

**Skewness Factors II-$\alpha$**

| DTO | 0.914384 | 0.938103 |
| ($0.5817$) | ($0.5739$) |
| DSI | $-0.167603$ | $-0.169346$ |
| ($0.3310$) | ($0.3174$) |
| DTO $\bar{\alpha}_t$ | $-2.075701***$ | $-2.056578***$ |
| ($0.7184$) | ($0.7188$) |
| DSI $\bar{\alpha}_t$ | $-1.822093***$ | $-1.794744***$ |
| ($0.4740$) | ($0.4642$) |
| DTO DSI | $-0.125198$ | $-0.127797$ |
| ($0.6540$) | ($0.6251$) |
| DTO DSI $\bar{\alpha}_t$ | 1.441900 | 1.437034 |
| ($1.0968$) | ($1.0962$) |
| $\bar{\alpha}_t$ | $-0.951439*$ | $-0.992285*$ |
| ($0.5381$) | ($0.5457$) |

**Mean Equation**

| _cons | 0.000359 | 0.000355 |
| ($0.0003$) | ($0.0003$) |
| N | 3061 | 3061 |
| LL | 8841.2225 | 8841.9794 |

Standard errors in parentheses.

* $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$. 

indicator. The crisis and yearly indicators help us test how conditional skewness responds to the general market decline for a relatively long period of time. The quarterly market indicator, which can be seen in Fig. 6, helps us test how conditional skewness responds to more frequent changes in the market direction, and the weekly market direction indicator, which can be seen in Fig. 7, helps us test how conditional skewness responds to short-term changes in the market direction. The estimation results are presented in Table 2. All coefficients on \( \text{DTO}_{t-1} \) and \( \text{DSI}_{t-1} \) are negative and strongly significant when the yearly and quarterly market indicators are used. However, we find that market turnover has only mild statistical power in forecasting conditional skewness, while market short interest is not important in determining the skewness of market return even during periods of general market declines when the weekly market direction indicator is used. This result may imply that the degree of heterogeneous beliefs and short sale restrictions in the market has an effect on conditional skewness when the downward trend in the market persists for more than a week. It is also possible that the weekly market direction indicator that we have used to obtain the above results may be a noisy proxy for indicating real changes in the market direction.

Finally, in Table 3, we test the effect of past returns on conditional skewness by including cumulative returns in our model. Both Hong and Stein (2003) and Hueng and McDonald (2005) found that past returns as far back as 36 months are negatively related to conditional skewness. Therefore, following Hueng and McDonald (2005) and Hong and Stein (2003), we define cumulative returns as $R_{t} = \prod_{i=1}^{750} (1 + R_{t-i}) - 1$, where $R_{t-i}$ is the simple return between times $t - i$ and $t$. For $t < 750$, we set $r$ as the number
of maximum possible days that we can use. We drop the first five observations such that our shortest cumulation period is five days. Fig. 12 plots cumulative returns ($RET_t$) against time. For each specification, we estimate the SN-GJR-GARCH and SN-Q-GARCH models with $RET_t$ as additional skewness factors measured in percentage. Table 3 presents the estimation results. We find that past returns have no predictive power on conditional skewness when skewness factors are included. However, when we omit the skewness factors, the coefficients on the cumulative return are negative and strongly significant. This result is similar to the results shown in Hong and Stein (2003) and Hueng and McDonald (2005).

5. Conclusion

In this paper, we have analyzed the relationship among short sale restrictions, heterogeneous beliefs, and conditional skewness by using a skew normal generalized autoregressive conditional heteroskedasticity model. Unlike previous studies, our paper considers the relationship under different market conditions. We show that negative conditional skewness is more pronounced when people disagree more about the market or the short sale constraints are increasingly binding during the general market declines, but the

Table 3
Estimation results with cumulative returns in the skewness equation.

|                      | SN-GJR-GARCH | SN-Q-GARCH |
|----------------------|--------------|------------|
|                      | (1)          | (2)        | (3)          | (4)          |
| **Variance Equation $h_t$** |              |            |              |              |
| $h_{t-1}$           | 0.925946***  | 0.923723***| 0.922395***  | 0.921480***  |
|                      | (0.0075)     | (0.0076)   | (0.0075)     | (0.0073)     |
| $u_{t-1}^2$         | 0.090851***  | 0.098176***| 0.066697***  | 0.068671***  |
|                      | (0.0095)     | (0.0092)   | (0.0069)     | (0.0067)     |
| $u_{t-1}^2I^+_t$   | -0.052000*** | -0.060109***|             |              |
|                      | (0.0103)     | (0.0094)   |              |              |
| $u_t$               |              |            |              |              |
|                      | 0.000002***  | 0.000002***| 0.000002***  | 0.000002***  |
|                      | (0.0000)     | (0.0000)   | (0.0000)     | (0.0000)     |
| **Skewness Equation $s_t = \gamma_t + \delta_t$** |              |            |              |              |
| $s_{t-1}$           | -0.449809*** | 0.193720   | -0.455548*** | 0.241970     |
|                      | (0.1034)     | (0.1619)   | (0.1024)     | (0.1594)     |
| $e_{t-1}^2$         | 0.157330***  | 0.232349***| 0.164145***  | 0.291517***  |
|                      | (0.0584)     | (0.0752)   | (0.0579)     | (0.0579)     |
| $\epsilon_{t-1}$   | 0.407628***  | 0.545841***| 0.422381***  | 0.462237***  |
|                      | (0.1213)     | (0.1206)   | (0.1224)     | (0.1225)     |
| $\gamma_{t-1}$     |              |            |              |              |
|                      | 0.247934     | -0.075924  | 0.264295     | -0.036921    |
|                      | (0.4766)     | (0.1867)   | (0.4743)     | (0.1529)     |
| **Skewness Factors II-$\delta_t$** |              |            |              |              |
| DTO                  | 0.898929     | 0.891090   |              |              |
|                      | (0.5598)     | (0.5572)   |              |              |
| DSI                  | 0.491968     | 0.481711   |              |              |
|                      | (0.4946)     | (0.4885)   |              |              |
| DTO·$\delta_{Crisis, t}$ | -2.091444*** | -2.155092***|            |              |
|                      | (0.7377)     | (0.7311)   |              |              |
| DSI·$\delta_{Crisis, t}$ | -2.573345*** | -2.551889***|      |              |
|                      | (0.6011)     | (0.6007)   |              |              |
| DTO·DSI              | -0.987776    | -0.969622  |              |              |
|                      | (0.9728)     | (0.9595)   |              |              |
| DTO·DSI·$\delta_{Crisis, t}$ | 2.285931*   | 2.236892*  | (1.3503)     |              |
|                      | (1.3760)     | (1.3503)   |              |              |
| $\delta_{Crisis, t}$ | -0.925569*   | -0.876804* |              |              |
|                      | (0.5385)     | (0.5311)   |              |              |
| RET                  | -0.005536    | -0.008793***| -0.005625    | -0.009379*** |
|                      | (0.0040)     | (0.0024)   | (0.0041)     | (0.0024)     |
| **Mean Equation**    |              |            |              |              |
| $\gamma_{t-1}$      | 0.0000495*   | 0.000607** | 0.000416     | 0.000443     |
|                      | (0.0003)     | (0.0003)   | (0.0003)     | (0.0003)     |
| N                    | 3056         | 3056       | 3056         | 3056         |
| LL                   | 8833.8753    | 8813.5079  | 8833.8377    | 8813.3772    |

Standard errors in parentheses *$p < 0.10$, **$p < 0.05$, ***$p < 0.01$. 
effect is undetermined during periods of market growth. We demonstrated the importance of market conditions on conditional skewness and reconciled conflicting evidence in recent empirical studies on the relationship among heterogeneous beliefs, short sale restrictions, and conditional skewness by combining Hong and Stein (2003) theory with bubble processes.

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Appendix A. Supplementary data

Supplementary data to this article can be found online at https://doi.org/10.1016/j.najef.2019.101070.

References

Arellano-Valle, R. B., & Azzalini, A. (2008). The centred parametrization for the multivariate skew-normal distribution. Journal of Multivariate Analysis, 99, 1362–1382.
Azzalini, A. (1985). A class of distributions which includes the normal ones. Scandinavian Journal of Statistics, 12, 171–178.
Azzalini, A. (2013). The skew-normal & related families (Institute of Mathematical Statistics Monographs). Cambridge: Cambridge University Press.
Azzalini, A., & Dall’Alba, G. (1996). The multivariate skew-normal distribution. Biometrika, 83, 715–726.
Black, F. (1976). Studies in stock price volatility changes. Proceedings of the 1976 business meeting of the business and economics statistics section, American Statistical Association, (pp. 177–181).
Blanchard, O. J., & Watson, M. W. (1983). Bubbles, rational expectations and financial markets. NBER Working Paper 945 National Bureau of Economic Research.
Blau, B. M., & Pinegar, J. M. (2009). Shorting skewness. SSRN Working Paper 1468047 Social Science Research Network.
Brooks, C., Burke, S., Heravi, S., & Persand, G. (2005). Autoregressive conditional kurtosis. Journal of Financial Econometrics, 3, 399–421.
Campbell, J. Y., & Lettau, M. (1999). Dispersion and volatility in stock returns: An empirical investigation. NBER Working Paper 7144 National Bureau of Economic Research.
Chang, E. C., Cheng, J. W., & Yu, Y. (2007). Short-sales constraints and price discovery: Evidence from the Hong Kong market. Journal of Finance, 62, 2097–2121.
Charoenrook, A., & Daouk, H. (2004). Conditional skewness of aggregate market returns. SSRN Working Paper 562153 Social Science Research Network.
Chen, J., Hong, H., & Stein, J. (2001). Forecasting crashes: Trading volume, past returns, and volatility. Journal of Financial Economics, 61, 345–381.
Christie, A. (1982). The stochastic behavior of common stock variances: Value, leverage, and interest rate effects. Journal of Financial Economics, 10, 407–432.
Daouk, H., & Charoenrook, A. (2005). A study of market-wide short-selling restrictions. SSRN Working Paper 687562 Social Science Research Network.
Diba, B. T., & Groisman, H. I. (1988). The theory of rational bubbles in stock prices. Economic Journal, 98, 746–754.
Engle, R. F., & Ng, V. K. (1993). Measuring and testing the impact of news on volatility. The Journal of Finance, 48, 1749–1777.
Epps, T. W., & Epps, M. L. (1976). The stochastic dependence of security price changes and transaction volumes: Implications for the mixture-of-distributions hypothesis. Econometrica, 44, 305–321.
Figlewski, S. (1981). The informational effects of restrictions on short sales: Some empirical evidence. The Journal of Financial and Quantitative Analysis, 16, 463–1376.
Figlewski, S., & Wang, X. (2000). Is the leverage effect a leverage effect? SSRN Working Paper 256109 Social Science Research Network.
French, K. R., Schwert, G., & Stambaugh, R. F. (1987). Expected stock returns and volatility. Journal of Financial Economics, 19, 3–29.
Glyns, E., Plazzi, A., & Valkanov, R. (2011). Conditional skewness of stock market returns in developed and emerging markets and its economic fundamentals, Swiss Finance Institute Research Paper Series 11-06, Swiss Finance Institute.
