Standard Model expectations on $\sin 2\beta(\phi_1)$ from $b \to s$ penguins

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Recent results of the standard model expectations on $\sin 2\beta_{\text{eff}}$ from penguin-dominated $b \to s$ decays are briefly reviewed.

I. INTRODUCTION

Although the Standard Model is very successful, New Physics is called for in various places, such as neutrino-oscillation, dark matter (energy) and baryon-asymmetry. Possible New Physics beyond the Standard Model is being intensively searched via the measurements of time-dependent CP asymmetries in neutral $B$ meson decays into final CP eigenstates defined by

$$\Gamma'(B(t) \to f) - \Gamma(B(t) \to \bar{f}) = S_f \sin(\Delta m t) + A_f \cos(\Delta m t),$$

where $\Delta m$ is the mass difference of the two neutral $B$ eigenstates, $S_f$ monitors mixing-induced CP asymmetry and $A_f$ measures direct CP violation. The CP-violating parameters $A_f$ and $S_f$ can be expressed as

$$A_f = -\frac{1 - |\lambda_f|^2}{1 + |\lambda_f|^2}, \quad S_f = \frac{2 \text{Im}\lambda_f}{1 + |\lambda_f|^2},$$

where

$$\lambda_f = \frac{q_B}{p_B} \frac{A(B^0 \to f)}{A(B^0 \to \bar{f})}. \quad (3)$$

In the standard model $\lambda_f \approx \eta_f e^{-2i\beta}$ for $b \to s$ penguin-dominated or pure penguin modes with $\eta_f = 1 (-1)$ for final $CP$-even (odd) states. Therefore, it is expected in the Standard Model that $-\eta_f S_f \approx \sin 2\beta$ and $A_f \approx 0$ with $\beta$ being one of the angles of the unitarity triangle.

The mixing-induced CP violation in $B$ decays has been already observed in the golden mode $B^0 \to J/\psi K_S$ for several years. The current world average the mixing-induced asymmetry from tree $b \to c\bar{s}s$ transition is

$$\sin 2\beta = 0.687 \pm 0.032.$$ \quad (4)

However, the time-dependent CP-asymmetries in the $b \to sq\bar{q}$ induced two-body decays such as $B^0 \to (\phi, \omega, \pi^0, \eta', f_0)K_S$ are found to show some indications of deviations from the expectation of the Standard Model (SM) \cite{1} (see Fig. 1). In the SM, CP asymmetry in all above-mentioned modes should be equal to $S_{f_{J/\psi K}}$ with a small deviation at most $O(0.1)$ \cite{2}. As discussed in \cite{2}, this may originate from the $O(\lambda^2)$ truncation and from the subdominant (color-suppressed) tree contribution to these processes. Since the penguin loop contributions are sensitive to high virtuality, New Physics beyond the SM may contribute to $S_f$ through the heavy particles in the loops. In order to detect the signal of New Physics unambiguously in the penguin $b \to s$ modes, it is of great importance to examine how much of the deviation of $S_f$ from $S_{f_{J/\psi K}}$,

$$\Delta S_f \equiv -\eta_f S_f - S_{f_{J/\psi K}},$$

is allowed in the SM \cite{2} \cite{3} \cite{4} \cite{5} \cite{6} \cite{7} \cite{8} \cite{9} \cite{10} \cite{11} \cite{12}.

The decay amplitude for the pure penguin or penguin-dominated charmless $B$ decay in general has the form

$$M(B^0 \to f) = V_{ub} V_{us}^{*} F^{u} + V_{cb} V_{cs}^{*} F^{c} + V_{tb} V_{ts}^{*} F^{t}.$$ \quad (6)

![FIG. 1: Experimental results for $\sin 2\beta_{\text{eff}}$ from $b \to s$ penguin decays \cite{2}.](attachment:image.png)
Unitarity of the CKM matrix elements leads to

\[ M(B^0 \to f) = V_{ub} V_{us}^* A_f^u + V_{cb} V_{cd}^* A_f^c \]

\[ \approx \lambda A_0^f R_0 e^{-i \gamma} A_f^u + \lambda A_0^f A_f^c, \]

(7)

where \( A_f^u = F_u^0 - F_u^t, A_f^c = F_c^0 - F_c^t, R_0 \equiv |V_{ub} V_{us}^* / (V_{cb} V_{cd}^*)| = \sqrt{\rho^2 + \eta^2}. \) The first term is suppressed by a factor of \( \lambda^2 \) relative to the second term. For a pure penguin decay such as \( B^0 \to \phi K^0 \), it is naively expected that \( A_f^u \) is in general comparable to \( A_f^c \) in magnitude. Therefore, to a good approximation \(-\eta_f S_f \approx \sin 2\beta \approx S_f / \omega K_f \). For penguin-dominated modes such as \( \omega K_f, \rho^0 K_f, \pi^0 K_f \), \( A_f^u \) also receives tree contributions from the \( b \to u + \bar{u}s \) tree operators. Since the Wilson coefficient for the penguin operator is smaller than the one for the tree operator, \( A_f^u \) could be significantly larger than \( A_f^c \). As the first term carries a weak phase \( \gamma \), it is possible that \( S_f \) is subject to a significant “tree pollution”. To quantify the deviation, it is known that to the first order in \( r_f \equiv (\lambda \omega A_f^u) / (\lambda \omega A_f^c) \),

\[ \Delta S_f = 2|r_f| \cos \beta \sin \gamma \sin \delta_f, \]

\[ A_f = 2|r_f| \sin \gamma \sin \delta_f, \]

with \( \delta_f = \arg(A_f^u / A_f^c) \). Hence, the magnitude of the CP asymmetry difference \( \Delta S_f \) and direct CP violation are both governed by the size of \( A_f^u / A_f^c \). However, for the aforementioned penguin-dominated modes, the tree contribution is color suppressed and hence in practice the deviation of \( S_f \) is expected to be small [2]. It is useful to note that \( \Delta S_f \) is proportional to the real part of \( A_f^u / A_f^c \) as shown in the above equation.

Below I will review the results of the SM expectations on \( \Delta S_f \) from short-distance and long-distance calculations. Recent reviews of results obtained from the SU(3) approach can be found in [17].

II. \( \Delta S_f \) FROM SHORT-DISTANCE CALCULATIONS

There are several QCD-based approaches in calculating hadronic \( B \) decays [12, 19, 24]. \( \Delta S_f \) from calculations of QCDF [9, 10], pQCD [11], SCET [12] are shown in Table 1. The QCDF calculations on \( PP \) and \( VP \) modes are from [9, 25], while those in \( SP \) modes are from [10]. It is interesting to note that (i) \( \Delta S_f \) are small and positive in most cases, while experimental central values for \( \Delta S_f \) are all negative, except the one from \( J_0 K_S \); (ii) QCDF and pQCD results agree with each other, since the main difference of these two approaches is the (penguin) annihilation contribution, which hardly affects \( S_f \); (iii) The SCET results involve some non-perturbative contributions fitted from data. These contributions affect \( \Delta S_f \) and give results in the \( \eta' K_S \) mode different from the QCDF ones.

It is instructive to understand the size and sign of \( \Delta S_f \) in the QCDF approach [9], for example. Recall that \( \Delta S_f \) is proportional to the real part of \( A_f^u / A_f^c \). We follow [9] to denote a complex number \( x \) by \(|x| \) if \( \text{Re}(x) > 0 \). In QCDF the dominant contributions to \( A_f^u / A_f^c \) are basically given by [9, 21].

\[ A_{\phi K_S}^u \sim \left(-a_4^u + r_x a_6^u\right) \sim \left[-P^u\right], \]

\[ A_{\phi K_S}^c \sim \left(-a_4^c + r_x a_6^c\right) \sim \left[-P^c\right], \]

\[ A_{\rho^0 K_S}^u \sim \left(-a_4^u + r_x a_6^u\right) \sim \left[-P^u\right], \]

\[ A_{\rho^0 K_S}^c \sim \left(-a_4^c + r_x a_6^c\right) \sim \left[-P^c\right], \]

\[ A_{\pi^0 K_S}^u \sim \left(-a_4^u + r_x a_6^u\right) \sim \left[-P^u\right], \]

\[ A_{\pi^0 K_S}^c \sim \left(-a_4^c + r_x a_6^c\right) \sim \left[-P^c\right], \]

\[ A_{\eta' K_S}^u \sim \left(-a_4^u + r_x a_6^u\right) \sim \left[-P^u\right], \]

\[ A_{\eta' K_S}^c \sim \left(-a_4^c + r_x a_6^c\right) \sim \left[-P^c\right], \]

where \( a_x^u \) are effective Wilson coefficients [24], \( r_x = O(1) \) are the chiral factors and \( R^{(uu)} \) are (real and positive) ratios of form factors and decay constants.

From Eq. (8), it is clear that \( \Delta S_f > 0 \) for \( \phi K_S, \omega K_S, \pi^0 K_S \), since their \( \text{Re}(A_f^u / A_f^c) \) can only be positive. Furthermore, due to the cancellation between \( a_4 \) and \( r_x a_6 \) in the \( \omega K_S \) amplitude, the corresponding penguin contribution is suppressed. This leads to a large and positive \( \Delta S_{\omega K_S} \) as shown in Table 1. For the cases of \( \rho^0 K_S \) and \( \eta' K_S \), there are chances for \( \Delta S_f \) to be positive or negative. The different signs in front of \( [P] \) in \( \rho^0 K_S \) and \( \eta' K_S \), are originated from the second term of the wave functions \( (u \bar{u} + d \bar{d})/\sqrt{2} \) of \( \omega \) and \( \rho^0 \) in the \( B^0 \to \omega \) and \( B^0 \to \rho^0 \) transitions, respectively. The \( [P] \) in \( \rho^0 K_S \) is also suppressed as the one in \( \omega K_S \), resulting a negative \( \Delta S_{\rho^0 K_S} \). On the other hand, \( [\bar{P}] \) in \( \eta' K_S \) is not only unsuppressed (no cancellation in the \( a_6 \) and \( a_6 \) terms), but, in fact, is further enhanced due to the constructive interference.
TABLE II: Direct CP asymmetry parameter $\mathcal{A}_f$ and the mixing-induced CP parameter $\Delta S_f^{SD+LD}$ for various modes. The first and second theoretical errors correspond to the SD and LD ones, respectively.

| Final State | $\Delta S_f^{SD+LD}$ | Expt | $\mathcal{A}_f(\%)$ | SD | SD+LD | Expt |
|-------------|----------------------|------|---------------------|----|--------|------|
| $\phi K_S$  | 0.09$^{+0.01}_{-0.02}$  | 0.04$^{+0.01+0.01}_{-0.02-0.02}$ | -0.22$^{+0.19}_{-0.20}$ | 0.8$^{+0.5}_{-0.2}$ | 2.3$^{+0.5+0.2}_{-1.0-1.5}$ | 9$^{+1.4}_{-1.0}$ |
| $\omega K_S$ | 0.13$^{+0.06}_{-0.05}$ | 0.09$^{+0.01+0.01}_{-0.04-0.04}$ | -0.06$^{+0.30}_{-0.40}$ | -6.8$^{+2.4}_{-4.0}$ | -13.5$^{+5.3+5.3}_{-5.7-5.7}$ | 44$^{+22}_{-23}$ |
| $\rho^0 K_S$ | -0.08$^{+0.03}_{-0.10}$ | -0.04$^{+0.01+0.01}_{-0.12-0.12}$ | -0.52$^{+0.58}_{-0.60}$ | 7.5$^{+4.5}_{-2.0}$ | 89.4$^{+15.8+5.8}_{-13.7-12.5}$ | -64$^{+48}_{-52}$ |
| $\eta' K_S$  | 0.04$^{+0.01}_{-0.02}$ | 0.06$^{+0.01+0.00}_{-0.02-0.02}$ | -0.19$^{+0.09}_{-0.20}$ | 1.4$^{+0.4}_{-0.2}$ | 2.1$^{+0.4}_{-0.2}$ | 1.0$^{+0.4}_{-0.2}$ |
| $\eta K_S$   | 0.04$^{+0.03}_{-0.03}$ | 0.07$^{+0.01+0.01}_{-0.03-0.03}$ | -0.03 | -5.7$^{+2.0}_{-3.9}$ | 1.8$^{+2.5}_{-1.8}$ | -0.5$^{+1.0}_{-0.5}$ |
| $\pi^0 K_S$  | -0.12$^{+0.03}_{-0.3}$ | -0.14$^{+0.01+0.01}_{-0.2-0.2}$ | -0.38$^{+0.26}_{-0.44}$ | 3.2$^{+1.1}_{-2.3}$ | 3.7$^{+1.9+1.7}_{-1.8-1.7}$ | 2$^{+1.3}_{-1.3}$ |

III. FSI CONTRIBUTIONS TO $\Delta S_f$

Evidence of direct CP violation in the decay $B^0 \to K^- \pi^+$ is now established, while the combined BaBar and Belle measurements of $B^- \to \rho^+ \pi^-$ imply a sizable direct CP asymmetry in the $\rho^+ \pi^-$ mode. In fact, direct CP asymmetries in these channels are much bigger than expectations (of many people) and may be indicative of appreciable LD rescattering effects, in general, in $B$ decays. The possibility of final-state interactions in bringing in the possible tree pollution sources to $S_f$ are considered. Both $A_f^T$ and $A_f^f$ will receive long-distance tree and penguin contributions from rescattering of some intermediate states. In particular there may be some dynamical enhancement of light $u$-quark loop. If tree contributions to $A_f^f$ are sizable, then final-state rescattering will have the potential of pushing $S_f$ away from the naive expectation. Take the penguin-dominated decay $\bar{B}^0 \to \omega K^0$ as an illustration. It can proceed through the weak decay $\bar{B}^0 \to K^* \pi^+$ followed by the rescattering $K^* \pi^+ \to \omega K^0$. The tree contribution to $\bar{B}^0 \to K^* \pi^+$, which is color allowed, turns out to be comparable to the penguin one because of the absence of the chiral enhancement characterized by the $a_6$ penguin term. Consequently, even within the framework of the SM, final-state rescattering may provide a mechanism of tree pollution to $S_f$. By the same token, we note that although $\bar{B}^0 \to \phi K^0$ is a pure penguin process at short distances, it does receive tree contributions via long-distance rescattering. Note that in addition to these charmless final states contributions, there are also contributions from charmful $D_s^{(*)} D^{(*)}$ final states, see Fig. 2. These final-state rescatterings provide the long-distance $u$- and $c$-penguin contributions.

An updated version of results in are shown in Table II. Several comments are in order. (i) $\phi K_S$ and $\eta' K_S$ are the theoretical and experimental cleanest modes for measuring $\sin 2\beta_{\text{eff}}$ in these penguin modes. The constructive interference behavior of penguins in the $\eta' K_S$ mode is still hold in the LD case, resulting a tiny $\Delta S_f^{\eta' K_S}$. (ii) Tree pollutions in $\omega K_S$ and $\rho^0 K_S$ are diluted due to the LD $c$-penguin contributions.

It is found that LD tree contributions are in general not large enough in producing sizable $\Delta S_f$, since their contributions are overwhelmed by LD $c$-penguin contributions from $D_s^{(*)} D^{(*)}$ rescatterings. On the other hand, while it may be possible to have a large $\Delta S_f$ from rescattering models that enhance the contributions from charmless states, a sizable direct CP violation will also be generated. Since direct CP violations are sensitive to strong phases generated from FSI, these approaches will also give a sizable direct CP violation at the same time when a large $\Delta S_f$ is produced. The present data on the $\phi K_S$ and $\eta' K_S$ modes do not support large direct CP violations in these modes. Consequently, it is unlikely that FSI will enlarge their $\Delta S_f$. In order to constrain or to
refine these calculations, it will be very useful to have more and better data on direct CP violations.

IV. $\Delta S_f$ IN $KKK$ MODES

$B^0 \to K^+K^0$ and $B^0 \to K^0K^-K^+$ are penguin-dominated and pure penguin decays, respectively. They are also used to extract sin$2\beta_{\text{eff}}$ with results shown in Fig. 1. Three-body modes are in general more complicated than two-body modes. For example, while the $K_SK_SK_S$ mode remains as a CP-even mode, the $K^+K^-0$ mode is not a CP-eigen state \cite{27}. Furthermore, the mass spectra of these modes are in general complicated and non-trivial.

A factorization approach is used to study these $KKK$ modes \cite{14}. In the factorization approach, the $B^0 \to K^+K^-K_S^+$ amplitude, for example, basically consists of two factorized terms: $(B^0 \to K^0_S\times (0 \to K^+K^-)$ and $(B^0 \to K^+K^-\times (0 \to K^-)$, where $A(B)$ denotes a $A \to B$ transition matrix element. The dominant contribution is from the $(B^0 \to K^0_S\times (0 \to K^+$ term, which is a penguin induced term, while the sub-leading $(B^0 \to K^+K^-\times (0 \to K^-)$ term contains both tree and penguin contributions. In fact, $B^0 \to K^+K^0$ transition is a $b \to u$ transition, which has a color allowed tree contribution.

Results of CP asymmetries for these modes are given in Table III. The first uncertainty is from hadronic parameter in $B^0 \to K^+K^-K_S^+$ transition in $K^+K^-0$ mode (and a similar term in $K_SK_SK_S$ mode), the second uncertainty is from other hadronic parameters, while the last uncertainty is from the uncertainty in $\gamma$.

To study $\Delta S_f$ and $A_f$, it is crucial to know the size of the $b \to u$ transition term ($A_f^s$). For the pure-penguin $K_SK_SK_S$ mode, the smallness of $\Delta S_f$ and $A_f$ can be easily understood. For the $K^+K^-0$ mode, there is a $b \to u$ transition in the $(B^0 \to K^-K_S^+)\otimes (0 \to K^+)$ term. It has the potential of giving large tree pollution in $\Delta S_f$. It requires more efforts to study the size and the impact of this term.

It is important to note that the $b \to u$ transition term in the $K^+K^-0$ mode is not a CP self-conjugated term, since under a CP conjugation, this term will be turned into a $(B^0 \to K^-K_S^+)\times (0 \to K^+)$ term, which is, however, missing in the original amplitude. Hence, this term contributes to both CP-even and CP-odd configurations with similar strength. Therefore, information in the CP-odd part can be used to constrain its size and its impact on $\Delta S_f$ and $A_f$. Indeed, it is found recently \cite{24} that the CP-odd part is highly dominated by $\phi K_S$, where other contributions $(m_{K^+K^-} \neq m_\phi)$ are highly suppressed. Since the $(B^0 \to K^+K^-)\times (0 \to K^+)$ term favors a large $m_{K^+K^-}$ region, which is clearly separated from the $\phi$-resonance region, the result of the CP-odd configuration strongly constrains the contribution from this $b \to u$ transition term. Consequently, the tree pollution is constrained and the $\Delta S_f$ should not be large. Note that results shown in Table III were obtained without fully incorporating these information. The first uncertainty in Table III will be reduced, if the CP-odd result is taken into account. To further refine the results it will be very useful to perform a detail Dalitz-plot analysis.

TABLE III: Mixing-induced and direct CP asymmetries $\Delta S_f$ (top) and $A_f$ (in $\%$, bottom), respectively, in $B^0 \to K^+K^-0$ and $K_SK_SK_S$ decays. Results for $(K^+K^-L)_{CP \pm}$ are identical to those for $(K^+K^-K)_{CP \pm}$.

| Final State | $\Delta S_f$ | | $A_f$(%) | Expt. |
|-------------|--------------|-----------------------------|-------------|
| $(K^+K^-K)_S$ | $0.03 \pm 0.08 \pm 0.02 \pm 0.00$ | $-0.12 \pm 0.08 \pm 0.17$ | $-0.12 \pm 0.08 \pm 0.17$ | |
| $(K^+K^-K)_S$ | $0.05 \pm 0.01 \pm 0.04 \pm 0.00$ | $0.03 \pm 0.01 \pm 0.02 \pm 0.00$ | $0.03 \pm 0.01 \pm 0.02 \pm 0.00$ | |
| $(K^+K^-K)_L$ | $0.03 \pm 0.01 \pm 0.02 \pm 0.00$ | $-0.60 \pm 0.34$ | $-0.60 \pm 0.34$ | |
| $(K^+K^-L)_S$ | $0.02 \pm 0.00 \pm 0.00 \pm 0.00$ | $0.19 \pm 0.23$ | $0.19 \pm 0.23$ | |
| $(K^+K^-K)_L$ | $0.02 \pm 0.00 \pm 0.00 \pm 0.00$ | $-0.19 \pm 0.17$ | $-0.19 \pm 0.17$ | |
| $(K^+K^-L)_S$ | $0.2 \pm 0.1 \pm 0.1 \pm 0.0$ | $-0.1 \pm 0.1 \pm 0.1 \pm 0.0$ | $-0.1 \pm 0.1 \pm 0.1 \pm 0.0$ | $-8 \pm 10$ |
| $(K^+K^-L)_S$ | $-0.1 \pm 0.1 \pm 0.1 \pm 0.0$ | $0.1 \pm 0.1 \pm 0.1 \pm 0.0$ | $0.1 \pm 0.1 \pm 0.1 \pm 0.0$ | $0.1 \pm 0.1 \pm 0.1 \pm 0.0$ | $31 \pm 17$ |

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[25] Results obtained agree with those in [13].
[26] In general, we have $\text{Re}(a_2) > 0$, $\text{Re}(a_6) < \text{Re}(a_4) < 0$.
[27] However, it is found that $K^+ K^- K^0$ is dominated by the CP-even part and hence it is still useful in extracting $\sin 2\beta_{ct}$. 