Radiative decays of the \((0^+, 1^+)\) strange-bottom mesons

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Abstract

In this article, we assume that the \((0^+, 1^+)\) strange-bottom mesons are the conventional \(b\bar{s}\) mesons, and calculate the electromagnetic coupling constants \(d, g_1, g_2\) and \(g_3\) using the light-cone QCD sum rules. Then we study the radiative decays \(B_s^0 \rightarrow B_s^*\gamma\), \(B_{s1} \rightarrow B_{s1}\gamma\), \(B_{s1} \rightarrow B_{s1}^*\gamma\) and \(B_{s1} \rightarrow B_{s0}\gamma\), and observe that the widths are rather narrow. We can search for the \((0^+, 1^+)\) strange-bottom mesons in the invariant \(B_s\pi^0\) and \(B_s^*\pi^0\) mass distributions in the strong decays or in the invariant \(B_s^*\gamma\) and \(B_s\gamma\) mass distributions in the radiative decays.

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1 Introduction

In 2007, the CDF Collaboration reported the first observation of two narrow \(B_s\) mesons with the spin-parity \(J^P = (1^+, 2^+)\) using 1 fb\(^{-1}\) of \(p\bar{p}\) collisions at \(\sqrt{s} = 1.96\) TeV collected with the CDF II detector at the Fermilab Tevatron [1], the masses are \(M_{B_{s1}^*} = (5829.4 \pm 0.7)\) MeV and \(M_{B_{s2}^*} = (5839.7 \pm 0.7)\) MeV. The D0 Collaboration reported the direct observation of the \(B_{s2}^*\) in fully reconstructed decays to \(B^+K^-\), the mass is \((5839.6 \pm 1.1 \pm 0.7)\) MeV [2]. While the \((0^+, 1^+)\) strange-bottom mesons are still lack experimental evidence, they may be observed at the Tevatron or more probably at the LHCb. The LHCb will be the most copious source of all the \(B\) hadrons, where the \(b\bar{b}\) pairs will be copiously produced with the cross section about 500 \(\mu\)b [3].

The \((0^+, 1^+)\) doublet \(B_s\) mesons have been studied with the potential quark models, the heavy quark effective theory and the lattice QCD [4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16], the predicted masses are different from each other.

In the previous work [17], we have studied the \((0^+, 1^+)\) strange-bottom mesons using the QCD sum rules, and observed the central values of the masses are below the corresponding \(BK\) and \(B^*K\) thresholds, respectively. It is a special property, the strong decays \(B_{s0} \rightarrow BK\) and \(B_{s1} \rightarrow B^*K\) are kinematically forbidden. They can decay through the isospin violation precesses \(B_{s0} \rightarrow B_s\eta \rightarrow B_s\pi^0\) and \(B_{s1} \rightarrow B_{s1}^*\eta \rightarrow B_{s1}^*\pi^0\) respectively, and the widths are narrow [18]. They can also decay through the radiative processes.

Radiative decays are important processes in probing the structures of the hadrons and serve as valuable testing grounds to select the best phenomenological model.

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The radiative decays $D^* \to D\gamma$ and $B^* \to B\gamma$ have been studied extensively by various theoretical approaches, such as the constituent quark model \[19, 20, 21, 22\], the light-cone QCD sum rules \[23, 24, 25\], the heavy quark effective theory \[26, 27\], the chiral perturbation theory \[28\], the light-front quark model \[29\], etc (For more literatures, one can consult the excellent review "Phenomenology of heavy meson chiral Lagrangians" \[30\] or the book "Heavy quark physics" \[31\]). The works on the radiative decays of the $P$-wave heavy-light mesons are relatively fewer and focus on the charm mesons $D_{s0}$ and $D_{s1}$. The radiative decays of the $D_{s0}$ and $D_{s1}$ have been studied using the constituent quark model \[6, 32, 33, 34, 35\], the vector meson dominance ansatz in the heavy quark limit \[36, 37\], the heavy-hadron chiral perturbation theory \[38\], the light-cone QCD sum rules \[39, 40\], the effective SU(4) theory with dynamically generated scalar resonances \[41\], etc.

In Ref.\[39\], the radiative decays $D_{s0} \to D_{s}^*\gamma$, $D_{s1} \to D_{s}\gamma$, $D_{s1} \to D_{s0}\gamma$ and $D_{s1} \to D_{s0}\gamma$ are studied using the light-cone QCD sum rules. Experimentally, the branching fractions listed in the particle data group are $\text{Br}(D_{s1} \to D_{s}\gamma) = (18 \pm 4)\%$, $\text{Br}(D_{s1} \to D_{s0}\gamma) < 8\%$ and $\text{Br}(D_{s1} \to D_{s0}\gamma) = 3.7^{+5.1}_{-2.4}\%$ \[42\].

The $(0^+, 1^+)$ $D_s$ and $B_s$ mesons have similar properties \[17, 18, 43\], we extend our previous works to make systematic studies. The mesons $B_{s0}$ and $B_{s1}$ may be observed in the invariant $B_{s}^*\gamma$ and $B_{s}\gamma$ mass distributions, and the radiative decays are suitable to understand the nature of the strange-bottom mesons. In this article, we study the radiative decays $B_{s0} \to B_{s}^*\gamma$, $B_{s1} \to B_{s}\gamma$, $B_{s1} \to B_{s}^*\gamma$ and $B_{s1} \to B_{s0}\gamma$ using the light-cone QCD sum rules.

The neutral strange-bottom mesons and charged strange-charm mesons have different electromagnetic properties besides they have different masses. The present work is far from trivial as just a replacement $c \to b$, we can borrow some ideas from the magnetic moments of the nucleons.

In the isospin limit, the proton and neutron have degenerated mass, however, their electromagnetic properties are quite different. If we take them as point particles, their magnetic moments are $\mu_p = 1$ and $\mu_n = 0$ (in unit of nucleon magneton) from Dirac’s theory of relativistic fermions. In 1933, Otto Stern measured the magnetic moment of the proton, which deviates from one significantly and indicates the proton has under-structures. The neutron is neutral, its (anomalous) magnetic originates from the Pauli form-factor. The electromagnetic form-factors (Dirac and Pauli form-factor) are excellent subjects to under the under-structures of the nucleon, and have been extensively studied both experimentally and theoretically.

The radiative decays embody the nature of the hadron’s constituents and the dynamics that binds the constituents together, the present work is necessary.

The light-cone QCD sum rules approach carries out the operator product expansion near the light-cone $x^2 \approx 0$ instead of the short distance $x \approx 0$ while the non-perturbative matrix elements are parameterized by the light-cone distribution amplitudes instead of the vacuum condensates \[44, 45, 46, 47, 48, 49\]. The coefficients in the light-cone distribution amplitudes are calculated with the conventional QCD sum rules and the values are universal \[50, 51, 52\].
The article is arranged as: in Section 2, we derive the electromagnetic coupling constants \( d, g_1, g_2 \) and \( g_3 \) using the light-cone QCD sum rules; in Section 3, the numerical result and discussion; and Section 4 is reserved for conclusion.

## 2 Electromagnetic coupling constants \( d, g_1, g_2 \) and \( g_3 \) with light-cone QCD sum rules

In the following, we write down the definitions for the electromagnetic coupling constants \( d, g_1, g_2 \) and \( g_3 \) among the \((0^-,1^-), (0^+,1^+)\) strange-bottom mesons and the photon \([39]\).

\[
\langle \gamma(q)B^*_s(p)|B_{s0}(k)\rangle = \frac{ed}{\sqrt{2}} \{e^* \cdot \bar{\eta} p \cdot q - e^* \cdot \bar{\eta}^* p \cdot q \} ,
\]
\[
\langle \gamma(q)B_s(p)|B_{s1}(k)\rangle = \frac{eg_1}{\sqrt{2}} \{e^* \cdot \eta p \cdot q - e^* \cdot \eta^* p \cdot q \} ,
\]
\[
\langle \gamma(q)B^*_s(p)|B_{s1}(k)\rangle = i e g_2 \varepsilon_{\alpha \beta \sigma \tau} \bar{\eta}^* \bar{e}^\alpha \bar{e}^\beta p^\sigma q^\tau ,
\]
\[
\langle \gamma(q)B_{s0}(p)|B_{s1}(k)\rangle = i e g_3 \varepsilon_{\alpha \beta \sigma \tau} \bar{e}^\alpha \bar{e}^\beta p^\sigma q^\tau ,
\]

where the \( \varepsilon, \eta \) and \( \bar{\eta} \) are the polarization vectors of the photon, \( B_{s1} \) and \( B^*_s \), respectively, and \( \varepsilon \) is the electric charge. In Ref.\([39]\), the radiative decays of the \((0^+,1^+)\) strange-charm mesons are studied using the light-cone QCD sum rules, in this article, we follow the routine and study the radiative decays of the \((0^+,1^+)\) strange-bottom mesons.

We study the electromagnetic coupling constants \( d, g_1, g_2 \) and \( g_3 \) with the two-point correlation functions \( F_\mu(p,q), T_\mu(p,q), T_{\mu \nu}(p,q) \) and \( W_\mu(p,q) \), respectively,

\[
F_\mu(p,q) = i \int d^4x \, e^{ip \cdot x} \langle \gamma(q)|T \{ J^\dagger_\mu(x)J_0(0) \} |0\rangle ,
\]
\[
T_\mu(p,q) = i \int d^4x e^{ip \cdot x} \langle \gamma(q)|T \{ J^\dagger_5(x)J^A_\mu(0) \} |0\rangle ,
\]
\[
T_{\mu \nu}(p,q) = i \int d^4x e^{ip \cdot x} \langle \gamma(q)|T \{ J^\dagger_\mu(x)J^A_\nu(0) \} |0\rangle ,
\]
\[
W_\mu(p,q) = i \int d^4x e^{ip \cdot x} \langle \gamma(q)|T \{ J^A_0(x)J^\dagger_\mu(0) \} |0\rangle ,
\]

where

\[
J_0(x) = \bar{b}(x)s(x) , \\
J_\mu(x) = \bar{b}(x)\gamma_\mu s(x) , \\
J^A_\mu(x) = \bar{b}(x)\gamma_\mu \gamma_5 s(x) , \\
J_5(x) = \bar{b}(x)\gamma_5 s(x) .
\]

The current operators \( J_0(x), J_5(x), J_\mu(x) \) and \( J^A_\mu(x) \) interpolate the mesons \( B_{s0}, B_s, B^*_s \) and \( B_{s1} \) respectively. The correlation functions \( F_\mu(p,q), T_\mu(p,q), T_{\mu \nu}(p,q) \)
and $W_{\mu}(p, q)$ can be decomposed as

$$
F_{\mu}(p, q) = F_A (p \cdot \varepsilon^* q_{\mu} - p \cdot q \varepsilon_{\mu}^*) + \cdots,
$$

$$
T_{\mu}(p, q) = T_B (p \cdot \varepsilon^* q_{\mu} - p \cdot q \varepsilon_{\mu}^*) + \cdots,
$$

$$
T_{\mu\nu}(p, q) = T_C \varepsilon_{\mu\nu\sigma\tau} \varepsilon^* \sigma q_{\tau} + T_1 p_{\mu} \varepsilon_{\nu\beta\sigma\tau} p^\beta \varepsilon^* \sigma q_{\tau} + \cdots
$$

$$
W_{\mu}(p, q) = W_D i \varepsilon_{\mu\alpha\sigma\tau} \varepsilon^* \alpha p_{\sigma} q_{\tau},
$$

(4)

due to Lorentz invariance. We choose the tensor structures $p \cdot \varepsilon^* q_{\mu} - p \cdot q \varepsilon_{\mu}^*$ and $\varepsilon_{\mu\sigma\tau} \varepsilon^* \rho q_{\tau}$ for analysis.

In this article, we consult the analytical expressions of Ref.[39], and make a simple replacement for the corresponding parameters of the strange-charm and strange-bottom mesons to obtain the following four sum rules. We would like not to follow the standard procedure of the light-cone QCD sum rules and repeat the straightforward but tedious calculations, one can consult Ref.[39] for the technical details. We perform detailed numerical calculations, analyze the effects originate from the electric charge difference between the $c$ and $b$ quarks in addition to the heavy quark symmetry. Taking into account our previous works [17, 18, 43], we make systematic studies about the properties of the $|0^+, 1^+\rangle$ $B_s$ mesons.

$$
d = \exp\left(\frac{M_{B_{s0}}^2 + M_{B_{s0}^*}^2}{2M_{B_{s0}}^2}\right) \left\{ \int_{\Delta}^{s_0} ds e^{-\frac{s}{2M^2}} \rho_A(s) - 2e_s f_{3\gamma} \frac{m_b e^{-m_b^2/2M^2}}{M^2} \Psi^v(u_0) \right.
$$

$$
+ e_b e^{-\frac{m_b^2}{2M^2}} \langle \bar{s}s \rangle \left[ 1 + \frac{m_s^2}{4M^2} + \frac{m_s^2 m_b^2}{2M^4} \right] + e_s \langle \bar{s}s \rangle \left( e^{-\frac{m_s^2}{2M^2}} - e^{-\frac{s_0}{2M^2}} \right) \frac{M^2 \chi_\phi(u_0)}{2M^2} \left[ 1 + \frac{m_b^2}{4M^2} \right] \left[ 1 + \frac{m_b^2}{4M^2} \right] \right.
$$

$$
+ \int_0^{1-u_0} dv \int_0^{1-v} du \alpha_g \frac{dA}{du} \left( u_0 - (1-v)u_0 + v u_0 - v \alpha_g, \alpha_g \right) \left[ 1 - u_0 + v \right]
$$

$$
+ \int_{1-u_0}^{1} dv \int_0^{1-v} du \alpha_g \frac{dA}{du} \left( u_0 - (1-v)u_0 + v u_0 - v \alpha_g, \alpha_g \right) \left[ 1 - u_0 + v \right],
$$

(5)
\[ g_1 = \exp \left( \frac{M_{B_1}^2 + M_{B_2}^2}{2M^2} \right) \left( m_b + m_s \right) \left\{ \int_{\Delta}^{s_0} d\pi \exp \left( -\frac{s_0}{2M^2} \right) \rho_B(s) + 2e_s f_{3\gamma} m_b e^{-\frac{m_b^2}{2M^2}} \Psi^v(u_0) \right\} \]

\[ = e_s \langle \bar{s}s \rangle (\exp \left( -\frac{m_b^2}{2M^2} \right) - e^{-\frac{\rho_0}{2M^2}}) M^2 \chi \phi_v(u_0) \]

\[ - e_s \langle \bar{s}s \rangle e^{-\frac{m_b^2}{2M^2}} \left[ - \int_0^{1-u_0} du \int_0^{u_0} d\alpha \Phi^v_B(u_0 - (1-v)\alpha, 1 - u_0 - v\alpha, \alpha) \right] \left\{ 1 - \frac{m_b^2}{M^2} \right\}, \] (6)

\[ g_2 = \exp \left( \frac{M_{B_1}^2 + M_{B_2}^2}{2M^2} \right) \left\{ \int_{\Delta}^{s_0} d\pi \exp \left( -\frac{s_0}{2M^2} \right) \rho_C(s) + e_s m_b e^{-\frac{m_b^2}{2M^2}} \langle \bar{s}s \rangle \left[ \frac{1}{M^2} \right] \right\} \left[ \frac{m_b^2}{4M^2} \bar{A}(u_0) - \bar{H}_\gamma(u_0)(1 - u_0) - \bar{H}_\gamma(u_0) \left( 1 - \frac{2m_b^2}{M^2} \right) \right] \]

\[ + e_s f_{3\gamma} M^2 (e^{-\frac{m_b^2}{2M^2}} - e^{-\frac{\rho_0}{2M^2}}) \left[ \frac{1}{4} (1 - u_0) \psi^v(u_0) - \frac{1}{4} \psi^v(u_0) \right] \]

\[ - \Psi^v(u_0) \left( 1 + \frac{2m_b^2}{M^2} \right) + (1 - u_0)\Phi^v(u_0) \]

\[ + m_b e_s \langle \bar{s}s \rangle e^{-\frac{m_b^2}{2M^2}} \left[ \int_0^{1-u_0} du \int_0^{u_0} d\alpha \Phi^v_B(u_0 - (1-v)\alpha, 1 - u_0 - v\alpha, \alpha) \right] \]

\[ + \int_0^{1-u_0} du \int_0^{u_0} d\alpha \Phi^v_B(u_0 - (1-v)\alpha, 1 - u_0 - v\alpha, \alpha) \]

\[ - e_s f_{3\gamma} M^2 (e^{-\frac{m_b^2}{2M^2}} - e^{-\frac{\rho_0}{2M^2}}) \left[ \int_0^{u_0} d\alpha \int_{u_0 - \alpha}^{1 - \alpha} d\alpha \Phi^v_C(1 - \alpha, \alpha, \alpha) \right] \]

\[ - \int_0^{u_0} d\alpha \frac{1}{u_0 - \alpha} \Phi^v_C(1 - u_0, \alpha, u_0 - \alpha) \right\}, \] (7)
\[ g_3 = \exp \left( \frac{M_B^2 + \frac{m_B^2}{2M^2}}{f_B \bar{f}_{B_0} M_B \bar{M}} \right) \left\{ \int_\Delta \frac{ds e^{-m_B^2 s}}{s} \rho_D(s) + e_b e^{-\frac{m_B^2}{M^2} s} \left( 1 + \frac{m_s m_b}{2M^2} + \frac{m_s^2 m_b^2}{8M^4} \right) \right\} \]

\[ + e_s \langle \bar{s}s \rangle \left( e^{-\frac{m_s^2}{s^2}} - e^{-\frac{m_B^2}{s^2}} \right) M^2 \phi_\gamma(u_0) \]

\[ + e^{-\frac{m_s^2}{s^2}} e_s \langle \bar{s}s \rangle \left[ -\frac{1}{4} \phi(u_0)(1 + \frac{m_B^2}{M^2}) \right] - \frac{m_b}{2} e_s \phi \psi^a(u_0) e^{-\frac{m_s^2}{s^2}} \]

\[ + e^{-\frac{m_s^2}{s^2}} e_s \langle \bar{s}s \rangle \left( 1 - u_0 - v \alpha_g \right) \right\}, \quad (8) \]

where

\[ \rho_A(s) = \frac{3e_s}{4\pi^2} \left\{ m_s \ln \left( \frac{s - m_B^2 + m_s^2 - \lambda \bar{s}(s, m_b^2, m_s^2)}{s - m_B^2 + m_s^2 + \lambda \bar{s}(s, m_b^2, m_s^2)} \right) - \frac{m_b - m_s}{s} \lambda \bar{s}(s, m_b^2, m_s^2) \right\} \]

\[ + \frac{3e_s m_b + m_s}{4\pi^2} \frac{\lambda \bar{s}(s, m_b^2, m_s^2)}{s} \left( 1 - \frac{m_s^2 - m_b^2}{s} \right) \]

\[ \rho_B(s) = -\frac{3e_s}{8\pi^2} \left\{ \frac{2m_s}{s} \ln \left( \frac{s - m_b^2 + m_s^2 - \lambda \bar{s}(s, m_b^2, m_s^2)}{s - m_b^2 + m_s^2 + \lambda \bar{s}(s, m_b^2, m_s^2)} \right) \right\} \]

\[ + \frac{(m_b - m_s)}{s^2} \left( \frac{m_b^2 - m_s^2 - s}{s} \right) \lambda \bar{s}(s, m_b^2, m_s^2) \}

\[ \rho_C(s) = \frac{3e_s}{4\pi^2} m_s m_b \ln \left( \frac{s - m_b^2 + m_s^2 - \lambda \bar{s}(s, m_b^2, m_s^2)}{s - m_b^2 + m_s^2 + \lambda \bar{s}(s, m_b^2, m_s^2)} \right) \]

\[ \rho_D(s) = -\frac{3e_s}{4\pi^2} \left\{ \frac{m_b + m_s}{s} \lambda \bar{s}(s, m_b^2, m_s^2) + m_s \ln \left( \frac{s - m_b^2 + m_s^2 - \lambda \bar{s}(s, m_b^2, m_s^2)}{s - m_b^2 + m_s^2 + \lambda \bar{s}(s, m_b^2, m_s^2)} \right) \right\} \]

\[ + (s \leftrightarrow b), \]

\[ \mathcal{F}_A = \mathcal{S} - \bar{\mathcal{S}} - T_1 + T_4 - T_3 + T_2 + 2v(-\mathcal{S} + T_3 - T_2), \]

\[ \mathcal{F}_B = \mathcal{S} + \bar{\mathcal{S}} - T_1 - T_2 + T_3 + T_4 + 2v(-\mathcal{S} - T_3 + T_2), \]

\[ \mathcal{F}_{C1} = \mathcal{S} + \bar{\mathcal{S}} + T_1 - T_2 - T_3 + T_4, \]

\[ \mathcal{F}_{C2} = \mathcal{A} + \mathcal{V}, \]

\[ \mathcal{F}_D = \mathcal{S} + \bar{\mathcal{S}} + T_1 + T_4 - T_2 - T_3 + 2v(-\mathcal{S} + T_3 - T_4), \]

and \( \Delta = (m_b + m_s)^2 \), \( \bar{H}_\gamma(u) = \int_0^u du' H_\gamma(u'), H_\gamma(u) = \int_0^u du' h_\gamma(u'), \Psi^\gamma(u) = \int_0^u du' \psi^\gamma(u'). \) The explicit expressions of the light-cone distribution amplitudes \( \mathcal{S}, \mathcal{S}, \mathcal{T}_1, \mathcal{T}_2, \mathcal{T}_3, \mathcal{T}_4, \mathcal{A}, \mathcal{V}, \mathcal{A}, \phi_\gamma, \psi^\alpha, \psi^\gamma \) and \( h_\gamma \) are given in the appendix [33]. In
Eqs. (5-8), the standard definitions for the decay constants have been used,

\begin{align*}
\langle 0 | J_{\mu}^i (0) | B_s^* (p) \rangle &= f_{B_s^*} M_{B_s^*} \bar{\eta}_\mu, \\
\langle 0 | J_{\mu}^{A1} (0) | B_{s1} (p) \rangle &= f_{B_{s1}} M_{B_{s1}} \eta_\mu, \\
\langle 0 | J_{\mu}^i (0) | B_s (p) \rangle &= \frac{f_{B_s} M_{B_s}^2}{m_b + m_s}, \\
\langle 0 | J_{\mu}^i (0) | B_{s0} (p) \rangle &= f_{B_{s0}} M_{B_{s0}}.
\end{align*}

The parameters in the light-cone distribution amplitudes are scale dependent and calculated using the QCD sum rules [53]. In the heavy quark limit, the bound energy of the \((0^+, 1^+)\) strange-bottom mesons is about \(\Lambda = \frac{3 M_{B_{s1}} + M_{B_{s0}}}{4} - m_b \approx 1\) GeV, which can serve as a typical energy scale and validate our choice \(\mu = 1\) GeV, one can choose another typical energy scale \(\mu = \sqrt{M_B^2 - m_b^2} \approx 2.4\) GeV. The physical quantities would not depend on the special energy scale we choose, we expect that scale dependence of the input parameters is canceled out approximately with each other, the values of the electromagnetic coupling constants which calculated at the energy scale \(\mu = 1\) GeV can make robust predictions.

The masses of the strange-bottom mesons are \(M_{B_{s1}} = 5.72\) GeV, \(M_{B_{s0}} = 5.70\) GeV, \(M_{B_{s1}} = 5.412\) GeV and \(M_{B_{s0}} = 5.366\) GeV,

\[
\frac{M_{B_{s0}}^2}{M_{B_{s0}}^2 + M_{B_{s1}}^2} \approx \frac{M_{B_{s1}}^2}{M_{B_{s1}}^2 + M_{B_{s0}}^2} \approx \frac{M_{B_{s1}}^2}{M_{B_{s1}}^2 + M_{B_{s0}}^2} \approx \frac{M_{B_{s1}}^2}{M_{B_{s1}}^2 + M_{B_{s0}}^2} \approx 0.50 - 0.53.
\]

There exists an overlapping working window for the two Borel parameters \(M_1^2\) and \(M_2^2\). It is convenient to take the value \(M_1^2 = M_2^2 = 2 M^2\) and \(u_0 = \frac{1}{2}\).

In the four sum rules, the terms originate from the nonperturbative interactions between the photon and quarks can be classified as \(O(M^2), O(1), O(1/M_B), \ldots\), the terms of order \(O(M^2)\) are greatly enhanced by the large Borel parameter \(M^2\), their contributions are large and continuum subtraction is necessary. We introduce the threshold parameter \(s_0\) (denotes \(s_0^A, s_0^B, s_0^C\) and \(s_0^D\)) and make the simple replacement \(e^{-\frac{m_b^2}{M^2}} \rightarrow e^{-\frac{m_b^2}{M^2}} - e^{-\frac{s_0^A}{M^2}}\) for the terms of order \(O(M^2)\) to subtract the contaminations from the high resonances and continuum states. For technical details, one can consult Ref. [54].

The \((0^+, 1^+)\) \(D_s\) and \(B_s\) mesons may have \(c\bar{s}\) and \(b\bar{s}\) kernels of the typical \(c\bar{s}\) and \(b\bar{s}\) mesons size respectively, strong couplings to the virtual intermediate hadronic states (or virtual mesons loops) may result in smaller masses than the conventional \(c\bar{s}\) and \(b\bar{s}\) mesons in the potential models [43, 55, 56, 57]. In Ref. [58], Guo et al. take the masses from the potential models as bare masses, and calculate the mass shifts for the scalar heavy mesons due to the hadronic loops, the numerical results indicate the masses from the quark models can be reduced significantly. In the previous works, we have calculated the strong coupling constants \(g_{D_0 D K}, g_{D_s D^* K}, g_{B_{s0} B K}\) and \(g_{B_{s1} B^* K}\) using the light-cone QCD sum rules [43, 55, 56, 57], the large
strong coupling constants support the hadronic dressing mechanism \cite{62, 63, 64}. In this article, we assume that the hadronic loops reduce ”bare” masses from the potential models, not the (renormalized) physical masses from the QCD sum rules, and neglect possible contaminations from the BK and $B^*K$ thresholds.

3 Numerical result and discussion

The input parameters are taken as $f_{3\gamma} = -(0.0039 \pm 0.0020)$ GeV$^2$, $\omega^V = 3.8 \pm 1.8$, $\omega^A = -2.1 \pm 1.0$, $\chi = -(3.15 \pm 0.3)$ GeV$^{-2}$ \cite{53}, $k = 0.2$, $\zeta_1 = 0.4$, $\zeta_2 = 0.3$, $\varphi_2 = k^+ = \zeta_1^+ = \zeta_2^+ = 0$ \cite{44}, $\langle \bar{s}s \rangle = 0.8\langle \bar{q}q \rangle$, $\langle \bar{q}q \rangle = (-0.24 \text{ GeV})^3$ \cite{50, 51, 52}, $m_s = (0.14 \pm 0.01)$ GeV, $m_b = (4.7 \pm 0.1)$ GeV, $M_{B_s} = 5.366$ GeV, $M_{B_s^*} = 5.412$ GeV \cite{12}, $M_{B_{s0}} = 5.70$ GeV, $M_{B_{s1}} = 5.72$ GeV, $f_{B_{s0}} = f_{B_{s1}} = 0.24$ GeV \cite{17}, and $f_{B_s^*} = f_{B_s} = 0.19$ GeV \cite{19, 65, 66}.

The threshold parameters are taken as $s_0^S = (37 \pm 1)$ GeV$^2$ and $s_A^0 = (38 \pm 1)$ GeV$^2$, which are chosen to below the corresponding masses of the first radially excited states, $M_{S_0} = 6.264$ GeV for the $B_{s0}$ and $M_{A_0} = 6.296$ GeV for the $B_{s1}$ in the potential models \cite{9}.

In 2006, the BaBar Collaboration observed a new $c\bar{s}$ state $D_s(2860)$ with the mass $M = (2856.6 \pm 1.5 \pm 5.0)$ MeV, width $\Gamma = (48 \pm 7 \pm 10)$ MeV and possible spin-parity $0^+, 1^-, 2^+, \cdots$ \cite{59}. It has been interpreted as the first radial excitation of the $0^+$ state $D_{s0}(2317)$ in Refs. \cite{60, 61}, although other identifications are not excluded. The energy gap between the $2P$ and $1P$ scalar $c\bar{s}$ states is about $\delta M_S = 0.539$ GeV.

If the masses of the $P$-wave strange-bottom mesons are of the same order (about 5.8 GeV \cite{11, 12}) and the energy gap between the ground state and the first radially excited state is about 0.5 GeV (just like the $c\bar{s}$ mesons), we can make a rough estimation for the masses of the first radially excited ($0^+, 1^+$) strange-bottom states, $M_r \approx (5.8 + 0.5)$ GeV. The threshold parameters should be chosen as $s_0 < M_r^2 \approx 40$ GeV$^2$, which are consistent with the predictions of the potential models \cite{9}.

The Borel parameters are chosen as $M^2 = (5 - 7)$ GeV$^2$, which are determined from the two-point QCD sum rules \cite{17}. In those regions, the contributions from the pole terms are larger than 50%, furthermore, the dominating contributions come from the perturbative terms.

The masses $M_{B_{s0}}$ and $M_{B_{s1}}$ obtained from the QCD sum rules have uncertainties, $M_{B_{s0}} = (5.70 \pm 0.11)$ GeV and $M_{B_{s1}} = (5.72 \pm 0.09)$ GeV \cite{17}, we can take the central values to avoid the possibility $M_{B_{s0}} > M_{B_{s1}}$, in that case the radiative decay $B_{s1} \rightarrow B_{s0}\gamma$ is kinematically forbidden. Furthermore, we neglect the uncertainties of the decay constants for consistence. The masses and decay constants of the ($0^+$, $1^+$) mesons are calculated using the QCD sum rules, some uncertainties originate from the Borel parameters and threshold parameters, so our approximation is not crude.

Taking into account the uncertainties of the input parameters, finally we obtain the numerical values of the electromagnetic coupling constants $d$, $g_1$, $g_2$, and $g_3$.
Figure 1: The electromagnetic coupling constant $d$ with the Borel parameter $M^2$. 

(which are shown in Figs. (1-4) respectively)

$$|d| = (0.09 - 0.29) \text{ GeV}^{-1},$$

$$|g_1| = (0.18 - 0.40) \text{ GeV}^{-1},$$

$$|g_2| = 0.25 - 1.20,$$

$$|g_3| = (0.31 - 0.64) \text{ GeV}^{-1},$$

and the radiative decay widths,

$$\Gamma_{B_{s0} \rightarrow B_{s}^* \gamma} = \frac{\alpha d^2 p^3}{3} = (1.3 - 13.6) \text{ KeV},$$

$$\Gamma_{B_{s1} \rightarrow B_{s}^* \gamma} = \frac{\alpha g_1^2 p^3}{3} = (3.2 - 15.8) \text{ KeV},$$

$$\Gamma_{B_{s1} \rightarrow B_{s}^* \gamma} = \frac{\alpha g_2^2 p^3 (M_{B_{s1}}^2 + M_{B_{s}^*}^2)}{3M_{B_{s1}}^2 M_{B_{s}^*}^2} = (0.3 - 6.1) \text{ KeV},$$

$$\Gamma_{B_{s1} \rightarrow B_{s0} \gamma} = \frac{\alpha g_3^2 p^3}{3} = (0.002 - 0.008) \text{ KeV},$$

where $\alpha$ is the fine structure constant, and $p$ is the momentum of the final particles in the central-of-mass frame of the initial meson. The values of the $p$ are about 0.28 GeV, 0.34 GeV, 0.30 GeV and 0.02 GeV for the radiative decays $B_{s0} \rightarrow B_{s}^* \gamma$, $B_{s1} \rightarrow B_{s} \gamma$, $B_{s1} \rightarrow B_{s}^* \gamma$ and $B_{s1} \rightarrow B_{s0} \gamma$ respectively. The decay widths are proportional to $p^3$, the decay $B_{s1} \rightarrow B_{s0} \gamma$ is kinematically suppressed and the width $\Gamma_{B_{s1} \rightarrow B_{s0} \gamma}$ is rather small.

The energy gap between the $P$-wave and $S$-wave strange-charm mesons is larger than the one between two $P$-wave (or two $S$-wave) strange-charm mesons, $M_{D_{s1}} -$
Figure 2: The electromagnetic coupling constant $g_1$ with the Borel parameter $M^2$.

Figure 3: The electromagnetic coupling constant $g_2$ with the Borel parameter $M^2$. 
Figure 4: The electromagnetic coupling constant $g_3$ with the Borel parameter $M^2$.

$M_{D_{s0}} = M_{D_s^*} - M_{D_s} = 0.14$ GeV and $\frac{3M_{D_{s1}}+M_{D_{s0}}}{4} - \frac{3M_{D_s^*}+M_{D_s}}{4} = 0.35$ GeV, the same relation holds for their bottom cousins. The radiative decays between the $P$-wave and $S$-wave strange-bottom mesons are kinematically favorable comparing with the internal transitions among the $P$-wave (or $S$-wave) mesons. There exists a possibility that the radiative decay $B_{s0} \rightarrow B_{s1}\gamma$ can take place, its width is about the same order of the width $\Gamma_{B_{s1} \rightarrow B_{s0}\gamma}$, much smaller than the width $\Gamma_{B_{s0} \rightarrow B_s^*\gamma}$. It is not an ideal channel to search for the $B_{s1}$ or $B_{s0}$ meson, we can search for the $B_{s1}$ and $B_{s0}$ mesons in the invariant $B_s\gamma$ and $B_{s^*}\gamma$ mass distributions in the radiative decays at the LHCb.

In Ref.[67], Faessler et al take the $(0^+,1^+)$ doublet $B_{s0}$ and $B_{s1}$ as the $B K$ and $B^*K$ molecules respectively, study the radiative decays, and obtain the narrow widths $\Gamma_{B_{s0} \rightarrow B_{s1}\gamma} = 3.07$ KeV and $\Gamma_{B_{s1} \rightarrow B_{s0}\gamma} = 2.01$ KeV, which are much smaller than the central values of the corresponding ones in the present work, see Eq.(12). We can probe the quark configurations of the mesons $B_{s0}$ and $B_{s1}$ using their radiative decays. If the $(0^+,1^+)$ $D_s$ and $B_s$ mesons are $Q\bar{s}$ cousins, the heavy quark symmetry warrants that they have analogous decay hierarchy $\Gamma_{B_{s1} \rightarrow B_{s}\gamma} \geq \Gamma_{B_{s1} \rightarrow B_{s^*}\gamma} \geq \Gamma_{B_{s1} \rightarrow B_{s0}\gamma}$ and $\Gamma_{D_{s1} \rightarrow D_s\gamma} \geq \Gamma_{D_{s1} \rightarrow D_s^*\gamma} \geq \Gamma_{D_{s1} \rightarrow D_{s0}\gamma}$ [39]. It is indeed the case from the present analysis. On the other hand, the magnitudes are quite different ($\Gamma_{D_{s1} \rightarrow D_s\gamma} = (19 - 29)$ KeV, $\Gamma_{D_{s1} \rightarrow D_s^*\gamma} = (0.6 - 1.1)$ KeV, $\Gamma_{D_{s1} \rightarrow D_{s0}\gamma} = (0.5 - 0.8$ KeV) due to the fact that the heavy quarks $b$ and $c$ have different electric charge, the strange-bottom mesons are neutral while the strange-charm mesons are charged. From Eqs.(5-9), we can see that the electromagnetic coupling constants are proportional to $C_s e_s + C_Q e_Q$, where the $C_s$ and $C_Q$ are formal notations, the heavy quark electric charge $e_Q$ have significant effects.
The strong couplings of the \((0^+, 1^+)\) \(B_s\) mesons to the nearby thresholds may result in some tetraquark components, whether the nucleon-like bound states or deuteron-like bound states \([43]\), the tetraquark components may lead to smaller radiative decay widths than the corresponding pure \(b\bar{s}\) states.

The central values of the masses of the \((0^+, 1^+)\) strange-bottom mesons from the QCD sum rules are below the corresponding \(BK\) and \(B^*K\) thresholds respectively \([17]\), the decays \(B_{s0} \rightarrow BK\) and \(B_{s1} \rightarrow B^*K\) are kinematically forbidden. In the previous works \([18]\), we have calculated the strong coupling constants \(g_{B_{s0}B_s\eta}\) and \(g_{B_{s1}B^*_s\eta}\) using the light-cone QCD sum rules, studied the strong isospin violation decays \(B_{s0} \rightarrow B_s\eta \rightarrow B_s\pi^0\) and \(B_{s1} \rightarrow B^*_s\eta \rightarrow B^*_s\pi^0\), and observed that the decay widths are about several KeV due to the small \(\eta - \pi^0\) transition matrix \([68]\), \(\Gamma_{B_{s1} \rightarrow B^*_s\pi^0} = (5.3 - 20.7)\) KeV and \(\Gamma_{B_{s0} \rightarrow B_s\pi^0} = (6.8 - 30.7)\) KeV.

There are two degenerate \(P\)-wave strange-bottom doublets: the \(j_q = \frac{1}{2}\) states \(B_{s0}\) and \(B_{s1}\), and the \(j_q = \frac{3}{2}\) states \(B^*_{s1}\) and \(B^*_{s2}\). If kinematically allowed, the states with \(j_q = \frac{1}{2}\) can decay via an \(S\)-wave transition, while the \(j_q = \frac{3}{2}\) states undergo a \(D\)-wave transition; the decay widths of the states with \(j_q = \frac{1}{2}\) are expected to be much broader than the corresponding \(j_q = \frac{3}{2}\) states. Our numerical results indicate the widths of the \(j_q = \frac{1}{2}\) states are also narrow \([17, 18]\).

We can search for the \((0^+, 1^+)\) strange-bottom mesons \(B_{s0}\) and \(B_{s1}\) in the invariant \(B_s\pi^0\) and \(B^*_s\pi^0\) mass distributions in the strong decays or in the invariant \(B^*_s\gamma\) and \(B_s\gamma\) mass distributions in the radiative decays. Those mesons can be observed at the LHCb, where the \(b\bar{b}\) pairs will be copiously produced with the cross section about 500 \(\mu b\) \([3]\).

### 4 Conclusion

In this article, we assume that the \((0^+, 1^+)\) strange-bottom mesons \(B_{s0}\) and \(B_{s1}\) are the conventional \(b\bar{s}\) mesons, and calculate the electromagnetic coupling constants \(d, g_1, g_2, g_3\) using the light-cone QCD sum rules. Then we study the radiative decays \(B_{s0} \rightarrow B^*_s\gamma\), \(B_{s1} \rightarrow B_s\gamma\), \(B_{s1} \rightarrow B^*_s\gamma\) and \(B_{s1} \rightarrow B_{s0}\gamma\), and observe that the decay widths are rather narrow. We can search for the mesons \(B_{s0}\) and \(B_{s1}\) in the invariant \(B_s\pi^0\) and \(B^*_s\pi^0\) mass distributions in the strong decays or in the invariant \(B^*_s\gamma\) and \(B_s\gamma\) mass distributions in the radiative decays at the LHCb.

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Appendix

The light-cone distribution amplitudes of the photon are parameterized as

$$\phi_\gamma(u) = 6u\bar{u} \left[ 1 + \varphi_2 C_2^\frac{3}{2} (2u - 1) \right],$$

$$A(u) = 40u^2\bar{u}^2(3k - k^+ + 1) + 8(\zeta_2^+ - 3\zeta_2)[u\bar{u}(2 + 13u\bar{u})$$

$$+ 2u^3(10 - 15u + 6u^2)\ln u + 2u^3(10 - 15\bar{u} + 6\bar{u}^2)\ln \bar{u}],$$

$$h_\gamma(u) = -10(1 + 2k^+)C_2^\frac{1}{2} (2u - 1),$$

$$\psi^q(u) = 5 \left[ 3(2u - 1)^2 - 1 \right] + \frac{3}{64} \left[ 15\omega^V_\gamma - 5\omega^A_\gamma \right] \left[ 3 - 30(2u - 1)^2 + 35(2u - 1)^4 \right],$$

$$\psi^g(u) = \left[ 1 - (2u - 1)^2 \right] \left[ 5(2u - 1)^2 - 1 \right] \frac{5}{2} \left[ 1 + \frac{9}{16}\omega^V_\gamma - \frac{3}{16}\omega^A_\gamma \right],$$

$$V(\alpha_q, \alpha_g, \alpha_g) = 540\omega^V_\gamma (\alpha_q - \alpha_g)\alpha_q\alpha_q\alpha_g^2,$$

$$A(\alpha_q, \alpha_g, \alpha_g) = 360\alpha_q\alpha_q\alpha_g^2 \left[ 1 + \omega^A_\gamma \frac{1}{2} (7\alpha_g - 3) \right],$$

$$S(\alpha_q, \alpha_q, \alpha_g) = 30\alpha_g^2 \left\{ (k + k^+)(1 - \alpha_g) + (\zeta_1 + \zeta_1^+)(1 - \alpha_g)(1 - 2\alpha_g)$$

$$+ \zeta_2[3(\alpha_q - \alpha_q)^2 - \alpha_g(1 - \alpha_g)] \right\},$$

$$\tilde{S}(\alpha_q, \alpha_q, \alpha_g) = -30\alpha_g^2 \left\{ (k + k^+)(1 - \alpha_g) + (\zeta_1 - \zeta_1^+)(1 - \alpha_g)(1 - 2\alpha_g)$$

$$+ \zeta_2[3(\alpha_q - \alpha_q)^2 - \alpha_g(1 - \alpha_g)] \right\},$$

$$T_1(\alpha_q, \alpha_g, \alpha_g) = -120(3\zeta_2 + \zeta_2^+)(\alpha_q - \alpha_g)\alpha_q\alpha_q\alpha_g,$$

$$T_2(\alpha_q, \alpha_g, \alpha_g) = 30\alpha_g^2(\alpha_q - \alpha_q) \left\{ (k - k^+) + (\zeta_1 - \zeta_1^+)(1 - 2\alpha_g) + \zeta_2(3 - 4\alpha_g) \right\},$$

$$T_3(\alpha_q, \alpha_q, \alpha_g) = -120(3\zeta_2 - \zeta_2^+)(\alpha_q - \alpha_q)\alpha_q\alpha_q\alpha_g,$$

$$T_4(\alpha_q, \alpha_q, \alpha_g) = 30\alpha_g^2(\alpha_q - \alpha_q) \left\{ (k + k^+) + (\zeta_1 + \zeta_1^+)(1 - 2\alpha_g) + \zeta_2(3 - 4\alpha_g) \right\}. \quad (13)$$

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