An input/output feedback linearization control strategy is developed for control of three-level three-phase neutral-point-clamped rectifier using its dual Lagrangian modeling. The load current of the rectifier can be given in two forms: (i) the load current involving the current of capacitor \( C_1 \) (ii) the load current involving the current of capacitor \( C_2 \). Applying the obtained load current to the Euler-Lagrange parameters of the rectifier, two nonlinear models of the system are derived. The models are based on the superposition law of the load current and the Euler-Lagrange description of the rectifier. Also, the two power-balance equations between the input and output sides are obtained by considering the effects of two load currents separately. Then, the two nonlinear models and power-balance equations are linearized using input–output feedback linearization, and the state feedback law controlled inputs are completed by pole placement. With the definition of outputs of the feedback linearization law, the zero dynamics of the system are avoided and a fast output voltages control scheme is designed. Finally, some integrators are added to robust control against parasitic elements. The proposed nonlinear controller is compared to the PI controller and the MATLAB/SIMULINK results verify the proposed control strategy.

**Key words:** Euler–Lagrange (EL) model, 3 level/phase NPC rectifier, Input/output feedback linearization, Power-balance equations, State feedback law controlled inputs

**1 INTRODUCTION**

Multilevel converters permit reaching voltage levels up to medium-voltage drives in the industrial environment. The converters are significantly used for medium-voltage and high-power applications [1-4]. Among these topologies, multilevel neutral-point-clamped (NPC) converters are the most widely used because of their advantages such as series connection of power switches and bi-directional power transfer capability.

However, one of the considerable problems of the NPC converters is how to control the oscillation in the neutral point potential (NPP). If the NPP is not controlled effec-
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tively, the output voltages of the converter wouldn’t reach its reference values and the devices and equipment might be damaged. There are some modulation techniques that are able to completely eliminate the oscillations in NPP [5-7]. In [8] a dynamic limiter and a control loop are presented that provide the system with a fast balancing dynamic and guarantee stability under all operating conditions. Also there are several nonlinear control methods [9-10] to maintain the output voltages balance. Reference [11] describes a new control method based on a simple feedback linearization procedure to obtain a linear time-invariant model for the NP voltage. The feedback linearization process takes almost no time, is direct, and yields a linear, time-invariant model. With this linear model, well-known and easy-to-implement classic control techniques for obtaining the desired system response and disturbance rejection can be applied with greater accuracy than in previous approaches. [12] A nonlinear control strategy based on feedback linearization technique is applied to the NPC converter. This control law robustness is validated for the various severe load and the system parameter variations. The input/output feedback linearization [13-15] control strategy that is proposed in this paper can enable the three phase/level NPC rectifiers to keep the output voltages and NPP at its desired values. Also, this control strategy is based on the dual lagrangian model [16] of the rectifier, which is obtained with the superposition law, the load current and the Euler-Lagrange description of the rectifier. Then, during the performance of the control strategy, the dynamics zero of the rectifier are avoided and the state feedback law controlled inputs are completed by pole placement.

The MATLAB/SIMULINK results comparing to the PI controller shown that the proposed control method gives good results in all the operating conditions such as: desired output voltages, very low THD, unity power factor and Neutral Point Potential (NPP) 0 V.

2 EULER-LAGRANGE MODELING OF THE 3 LEVEL/PHASE NPC RECTIFIER

The power circuit of the 3-level 3-phase NPC rectifier with balanced source is shown in Fig. 1. The output capacitors, line inductors and resistors are considered identical, i.e., $C_1 = C_2 = C, L_1 = L_2 = L_3 = L$ and $R_1 = R_2 = R_3 = R$ respectively. $E_{in}$ is the amplitude of the input voltage source. $S_{1a}, S_{1b}, S_{1c}$ and $S_{2a}$ ($j = a, b, c$) are switching functions describing the switching status of each phase as given by the following definition:

$$S_{ij} = \begin{cases} 0 & \text{if } T_{ij} = \text{off} \\ 1 & \text{if } T_{ij} = \text{on} \end{cases} \quad i = a, b, c \text{ and } j = 1, 2$$

Considering electric charges vector $\mathbf{q} = [q_{La} \ q_{Lb} \ q_{Lc} \ q_{C_1} \ q_{C_2}]$ corresponding to three boosting inductors and output capacitors respectively, the lagrangian of the system can be defined as:

$$\mathbf{L}(\mathbf{q}, \dot{\mathbf{q}}) = T(\mathbf{q}, \dot{\mathbf{q}}) - V(\mathbf{q}), \quad (1)$$

where $T(\mathbf{q}, \dot{\mathbf{q}})$ and $V(\mathbf{q})$ are magnetic co-energy and electric field energy of the circuit, respectively. In Fig. 1, the load current can be written both as:

$$i_L = (\dot{q}_{La} S_{a1} + \dot{q}_{Lb} S_{b1} + \dot{q}_{Lc} S_{c1}) - \dot{q}_{C_1}, \quad (2)$$

$$i_L = - (\dot{q}_{La} S_{a2} + \dot{q}_{Lb} S_{b2} + \dot{q}_{Lc} S_{c2} + \dot{q}_{C_2}), \quad (3)$$

with $S_{ai} = 1 - S_{ai}$. Of course, equation (2) is due to the capacitor $C_1$, and equation (3) is due to the capacitor $C_2$. Considering the load current presented above, the EL parameters describing the EL dynamics of the 3-level 3-phase NPC VSR can be written as:

$$T = \frac{1}{2} L (\dot{q}_{La}^2 + \dot{q}_{Lb}^2 + \dot{q}_{Lc}^2), \quad (4)$$

$$V = \frac{1}{2} C \dot{q}_{C_1}^2 + \frac{1}{2} C \dot{q}_{C_2}^2, \quad (5)$$

$$D = \frac{1}{2} R (\dot{q}_{La}^2 + \dot{q}_{Lb}^2 + \dot{q}_{Lc}^2) + \frac{1}{2} L \left\{ \dot{q}_{C_1} - (\dot{q}_{La} S_{a1} + \dot{q}_{Lb} S_{b1} + \dot{q}_{Lc} S_{c1}) \right\}^2, \quad (6)$$

or

$$D = \frac{1}{2} R (\dot{q}_{La}^2 + \dot{q}_{Lb}^2 + \dot{q}_{Lc}^2) + \frac{1}{2} L \left\{ \dot{q}_{C_2} + (\dot{q}_{La} S_{a2} + \dot{q}_{Lb} S_{b2} + \dot{q}_{Lc} S_{c2}) \right\}^2, \quad (7)$$

$$F = \begin{bmatrix} e_{a} & e_{b} & e_{c} & 0 & 0 \end{bmatrix}^T. \quad (8)$$

In these equations, the functions $D$ and $F$ are Rayleigh dissipation co-faction and the generalized forcing faction. Considering to the general from of the Euler–Lagrange (EL) equation:

$$\frac{d}{dt} \left( \frac{\partial \mathbf{L}}{\partial \dot{\mathbf{q}}} \right) - \frac{\partial \mathbf{L}}{\partial \mathbf{q}} + \frac{\partial \mathbf{D}}{\partial \dot{\mathbf{q}}} = \mathbf{F}, \quad (9)$$
and by substituting the elements described above, the following equations can be achieved:

$$\frac{d}{dt} \left( \frac{\partial T}{\partial q_{La}} \right) + \frac{\partial T}{\partial q_{La}} + \frac{\partial V}{\partial q_{La}} + \frac{\partial D}{\partial q_{La}} = e_a, \quad (10)$$

$$\frac{d}{dt} \left( \frac{\partial T}{\partial q_{Lb}} \right) + \frac{\partial T}{\partial q_{Lb}} + \frac{\partial V}{\partial q_{Lb}} + \frac{\partial D}{\partial q_{Lb}} = e_b, \quad (11)$$

$$\frac{d}{dt} \left( \frac{\partial T}{\partial q_{Lc}} \right) + \frac{\partial T}{\partial q_{Lc}} + \frac{\partial V}{\partial q_{Lc}} + \frac{\partial D}{\partial q_{Lc}} = e_c, \quad (12)$$

$$\frac{d}{dt} \left( \frac{\partial T}{\partial q_{C1}} \right) + \frac{\partial T}{\partial q_{C1}} + \frac{\partial V}{\partial q_{C1}} + \frac{\partial D}{\partial q_{C1}} = 0, \quad (13)$$

$$\frac{d}{dt} \left( \frac{\partial T}{\partial q_{C2}} \right) + \frac{\partial T}{\partial q_{C2}} + \frac{\partial V}{\partial q_{C2}} + \frac{\partial D}{\partial q_{C2}} = 0. \quad (14)$$

Since $D$ is presented in equations (6) and (7), two direct calculation of (10)-(14) with the EL parameters are obtained.

The first calculation: considering the EL parameters (4)-(6), the following is obtained:

$$L \frac{d}{dt} \left( \dot{q}_{La} \right) + R \dot{q}_{La} - R_{La} S_{a1} \left[ \dot{q}_{C1} - (\dot{q}_{La} S_{a1} + \dot{q}_{Lb} S_{b1} + \dot{q}_{Lc} S_{c1}) \right] = e_a, \quad (15)$$

$$L \frac{d}{dt} \left( \dot{q}_{Lb} \right) + R \dot{q}_{Lb} - R_{Lb} S_{b1} \left[ \dot{q}_{C1} - (\dot{q}_{La} S_{a1} + \dot{q}_{Lb} S_{b1} + \dot{q}_{Lc} S_{c1}) \right] = e_b, \quad (16)$$

$$L \frac{d}{dt} \left( \dot{q}_{Lc} \right) + R \dot{q}_{Lc} - R_{Lc} S_{c1} \left[ \dot{q}_{C1} - (\dot{q}_{La} S_{a1} + \dot{q}_{Lb} S_{b1} + \dot{q}_{Lc} S_{c1}) \right] = e_c, \quad (17)$$

$$\frac{q_{c1}}{C} + R \dot{q}_{c1} - (\dot{q}_{La} S_{a1} + \dot{q}_{Lb} S_{b1} + \dot{q}_{Lc} S_{c1}) = 0, \quad (18)$$

$$\frac{q_{c2}}{C} + R \dot{q}_{c2} + (\dot{q}_{La} S_{a1} + \dot{q}_{Lb} S_{b1} + \dot{q}_{Lc} S_{c1}) = 0. \quad (19)$$

From (18), it follows:

$$\dot{q}_{C1} - (\dot{q}_{La} S_{a1} + \dot{q}_{Lb} S_{b1} + \dot{q}_{Lc} S_{c1}) = -q_{c2} / CR_L. \quad (20)$$

By substituting (20) in equations (15)-(17), and by noting that $\dot{q}_{Lk} = i_k$, $k = a, b, c$ and that voltage of the output capacitor $C_1$ can be written as: $V_{C_1} = q_{c1}/C$, the EL model corresponding to the PWM converter can be described as:

$$L \frac{d}{dt} i_a + R i_a + S_{a1} V_{C_1} = e_a, \quad (21)$$

$$L \frac{d}{dt} i_b + R i_b + S_{b1} V_{C_1} = e_b, \quad (22)$$

$$L \frac{d}{dt} i_c + R i_c + S_{c1} V_{C_1} = e_c. \quad (23)$$

$$C \frac{dV_{C_1}}{dt} = (S_{a1} i_a + S_{b1} i_b + S_{c1} i_c) + \frac{V_{C_1}}{R_L} = 0. \quad (24)$$

$$C \frac{dV_{C_2}}{dt} = (\overline{S_{a2}} i_a + \overline{S_{b2}} i_b + \overline{S_{c2}} i_c) + \frac{V_{C_2}}{R_L} = 0. \quad (25)$$

Considering equations (21)-(25), it is possible to present them in the matrix form:

$$M \ddot{\omega} + \xi_1 (\omega) \omega + R \omega = F, \quad (26)$$

with

$$M = \begin{bmatrix} L & 0 & 0 & 0 & 0 \\ 0 & L & 0 & 0 & 0 \\ 0 & 0 & C & 0 & 0 \\ 0 & 0 & 0 & C & 0 \end{bmatrix},$$

$$\xi_1 (\omega) = \begin{bmatrix} 0 & 0 & 0 & S_{a1} & 0 \\ 0 & 0 & 0 & S_{b1} & 0 \\ -S_{a1} & -S_{b1} & -S_{c1} & 0 & 0 \\ \frac{S_{a2}}{S_{b2}} & \frac{S_{a2}}{S_{b2}} & \frac{S_{c2}}{S_{c2}} & 0 & 0 \end{bmatrix},$$

$$R = \begin{bmatrix} R & 0 & 0 & 0 & 0 \\ 0 & R & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{R_L} & 0 \\ 0 & 0 & 0 & 0 & \frac{1}{R_L} \end{bmatrix},$$

$$F = \begin{bmatrix} e_a \\ e_b \\ e_c \\ 0 \\ 0 \end{bmatrix}^T,$$

where $\omega = [i_a \ i_b \ i_c \ V_{C_1} \ V_{C_2}]$ and $F$ is the vector consisting of voltage sources.

The second calculation: by considering the EL parameters (4), (5) and (7) similar to first calculation and by applying (10)-(14) to (4), (5) and (7), and by noting that $\dot{q}_{Lk} = i_k$, $k = a, b, c$ and that voltage of the output capacitor $C_2$ is $V_{C_2} = q_{c2}/C$, and by considering that:

$$\dot{q}_{C2} + (\dot{q}_{La} S_{a1} + \dot{q}_{Lb} S_{b1} + \dot{q}_{Lc} S_{c1}) = -q_{c2} / CR_L, \quad (27)$$

the EL model corresponding to the converter can be written as:

$$L \frac{d}{dt} i_a + R i_a - \overline{S_{a1}} V_{C_2} = e_a, \quad (28)$$

$$L \frac{d}{dt} i_b + R i_b - \overline{S_{b1}} V_{C_2} = e_b, \quad (29)$$

$$L \frac{d}{dt} i_c + R i_c - \overline{S_{c1}} V_{C_2} = e_c, \quad (30)$$

$$C \frac{dV_{C_1}}{dt} - (S_{a1} i_a + S_{b1} i_b + S_{c1} i_c) + \frac{V_{C_1}}{R_L} = 0, \quad (31)$$

$$C \frac{dV_{C_2}}{dt} - (\overline{S_{a2}} i_a + \overline{S_{b2}} i_b + \overline{S_{c2}} i_c) + \frac{V_{C_2}}{R_L} = 0. \quad (32)$$
The above equations will result in a matrix as follows:

\[
M \dot{\omega} + \xi_2(u)\omega + R\omega = F, \tag{33}
\]

with

\[
\xi_2(u) = \begin{bmatrix}
0 & 0 & 0 & 0 & -S_{a1} \\
0 & 0 & 0 & 0 & -S_{b2} \\
S_{a2} & S_{b1} & -S_{a1} & 0 & 0 \\
S_{b2} & S_{c1} & -S_{b1} & 0 & 0
\end{bmatrix},
\]

where \(F, \omega, M\) and \(R\) are identical for both calculations. \(M\) is a positive-definite diagonal matrix, \(R\) is the dissipation matrix and \(\xi_i (u), (i = 1, 2)\) is the interconnection matrix with switches. The energy balance equation is:

\[
H(T) - H(0) + \int_0^T \omega^T R\omega dt = \int_0^T \omega^T F dt, \tag{34}
\]

where \(H = T + V = (1/2)\omega^T M\) represents the total energy of the circuit. Based on the energy balance equation:

\[
\begin{align*}
\text{stored energy } & (H(T) - H(0)) + \text{dissipated energy } \int_0^T \omega^T R\omega dt = \text{supplied energy} \int_0^T \omega^T F dt.
\end{align*}
\]

Absence of skew-symmetry property in the matrix \(F_i, (i = 1, 2)\) has no effect on the energy balance equation, the derivative of energy balance equation (34) may be easily found by taking the derivative of \(H\) along the trajectory of (26) or (33). It is clear that the EL models (21)-(25) and (28)-(32) of 3-level 3-phase NPC rectifier are according to Kirchhoff’s circuit theory and superposition law.

3 PROPOSED NONLINEAR CONTROL STRATEGY

3.1 Input/output feedback linearization based on the first equations (21-25)

In this section, it is considered that the first equations to regulate output voltage \(V_{C1}\) and PF (power factor), using input/output feedback linearization technique. By applying \(dq\) transformation \cite{16} to (21-25), the first equations of the rectifier in a rotating reference frame d-q may be written as below:

\[
L\dot{x}_1 = -Rx_1 + \omega Lx_2 - S_{d1}x_3 + e_d, \tag{35}
\]
\[
L\dot{x}_2 = -Rx_2 - \omega Lx_1 - S_{q1}x_3, \tag{36}
\]
\[
C\dot{x}_3 = \frac{3}{2} \left[ (S_{d1}x_1 + S_{q1}x_2) - \frac{2}{3R_L} x_3 \right], \tag{37}
\]
\[
C\dot{x}_4 = \frac{3}{2} \left[ (S_{d2}x_1 + S_{q2}x_2) - \frac{2}{3R_L} x_4 \right], \tag{38}
\]

with: \(\begin{bmatrix} x_1 & x_2 & x_3 & x_4 \end{bmatrix} = \begin{bmatrix} i_d & i_q & V_{C1} & V_{C2} \end{bmatrix} \).

On the other hand, according to (2), we will only consider the effect of capacitor \(C_1\) to obtain the power balance between dc and ac sides, i.e.,

\[
\frac{3}{2} e_d x_1 = x_3 \left( i_L + C_1 \frac{d x_3}{dt} \right). \tag{39}
\]

For applying input/output feedback linearization to a non-linear system, the number of control inputs \(u_i\) should be equal to the number of controlled outputs \(y_i\). The control inputs of the system based on the first equations are:

\[
u_{11} = S_1, \quad u_{21} = S_{q1}. \tag{40}\]

Thus, the system controlled outputs will be reduced as below:

\[
y_{11} = x_2, \quad y_{21} = x_3. \tag{41}\]

In order to obtain an effective linearized system, we should repeatedly differentiate each output \(y_{1i} (i = 1, 2)\) until at least one input \(u_{1i} (i = 1, 2)\) appears explicitly in its expression. After a straightforward calculation:

\[
\dot{y}_{11} = \dot{x}_2 = \dot{i}_q = \frac{1}{L} (-R_i - \omega L_i q - S_{d1} V_{C1}), \tag{42}
\]
\[
\dot{y}_{21} = \dot{x}_3 = \ddot{V}_{C1} = -\frac{3 R e_d d}{2 C V^2_{C1}} + \frac{3 \omega e_d d}{2 C V^2_{C1}} - \frac{3 e_d S_{d1}}{2 L C} + \frac{3 e_d E_m}{2 L C V_{C1}} - \frac{9 e_d^2 q^2}{4 C V^2_{C1}} + \frac{3 e_d i_L}{2 C V^2_{C1}} - \frac{i_L}{C}. \tag{43}
\]

The new inputs \(u_{1i} (i = 1, 2)\) based on the first equations will be obtained as:

\[
u_{11} = \dot{y}_{11}, \quad v_{21} = \dot{y}_{21}. \tag{44}\]

Relation (44) shows that there is a linear relation between the outputs and the new inputs. By Substituting (44) into (42) and (43) and also after some calculations, the switching functions of \(S_{d1}, S_{q1}\) are given as:

\[
S_{q1} = \frac{1}{V_{C1}} \left( -R_i - \omega L d i - L u_{11} \right), \tag{45}
\]
\[
S_{d1} = \frac{-R_i d}{V_{C1}} + \frac{L \omega q}{V_{C1}} + \frac{e_m}{V_{C1}} \frac{3 L e_d d^2}{2 C V^2_{C1}} + \frac{L i_L d}{C V^2_{C1}} - \frac{2 L i_L}{3 e_d} - \frac{2 L C}{3 e_d} u_{21}. \tag{46}
\]

To guarantee unity power factor and regulate the voltage of \(C_1\) capacitor, the new inputs are defined as:

\[
i_q = v_{11} = \dot{i}_q = \lambda_{11} (i_q - \dot{i}_q^*) , \quad i_q^* = 0, \tag{47}
\]
\[
\ddot{V}_{C1} = v_{21} = \ddot{V}_{C1} = \gamma_{11} \left( \dot{V}_{C1} - \dot{V}_{C1}^* \right) - \gamma_{21} \left( V_{C1} - V_{C1}^* \right), \tag{48}
\]

where \(i_q^*\) and \(V_{C1}^*\) are the reference value of \(i_q\) and \(V_{C1}\), respectively. The gains \(\lambda_{11}\) and \(\gamma_{11}\), \((i = 1, 2)\) can be designed by using linear techniques (poles assignment, LQR, etc.).
3.2 Input/output feedback linearization based on the second equations (28-32)

The second equations (28-32) in synchronous d-q frame are obtained as follows:

\[ L\ddot{x}_1 = -Rx_1 + \omega Lx_2 - S_{d2}x_4 + E_m, \]  
\[ L\ddot{x}_2 = -Rx_2 - \omega Lx_1 - S_{q2}x_4, \]  
\[ C\ddot{x}_3 = \frac{3}{2} \left( S_{d1}x_1 + S_{q1}x_2 - \frac{2}{3R_L}x_3 \right), \]  
\[ C\ddot{x}_4 = \frac{3}{2} \left( S_{d2}x_1 + S_{q2}x_2 - \frac{2}{3R_L}x_4 \right), \]

with: \[ [x_1 \ x_2 \ x_3 \ x_4] = [i_d \ i_q \ V_{C1} \ V_{C2}]. \]

As it can be seen from equation (3), dc power is due to the effect of \( C_2 \) capacitor and the power balance between dc and ac sides will be as:

\[ \frac{3}{2} i_d x_1 = x_4 \left( i_L + C \frac{dx_4}{dt} \right). \]  

The control inputs of the system based on the second equations are:

\[ u_{12} = S_{d2}, \quad u_{22} = S_{q2}. \]  

It is known from (54) that the nonlinear control based on the second equations is completed by reducing the system outputs. Considering the reduced outputs, unity power factor and desired \( C_1 \) output voltage must be gained. Thus:

\[ y_{12} = x_2, \quad y_{22} = x_4. \]  

Applying input/output feedback linearization technique to (55):

\[ \dot{y}_{12} = \ddot{x}_2 = \hat{i}_q = 1 \left( -Ri_q - \omega L\dot{i}_q - S_{d2}V_{C2} \right), \]  
\[ \dot{y}_{22} = \ddot{x}_4 = \hat{V}_{C2} = \frac{3R_e d_4 i_d}{2LCV_{C2}} + \frac{3\omega e i_d q}{2CV_{C2}} + \frac{3e_d S_{d2}}{2LC} + \frac{9e_d^2 e_m}{4C^2 V_{C2}^2} \frac{2CV_{C2} - i_L}{C}. \]  

The new inputs \( v_{i2}, (i = 1, 2) \) are:

\[ v_{12} = \dot{y}_{12}, \quad v_{22} = \dot{y}_{22}. \]  

By Substituting (58) into (56) and (57), the \( S_{d2} \) and \( S_{q2} \) are obtained as:

\[ S_{q2} = \frac{1}{V_{C2}} \left( -Ri_q - \omega L\dot{i}_q - L\dot{v}_{12} \right), \]  
\[ S_{d2} = \frac{R i_d}{V_{C2}} + \frac{L\omega i_q}{V_{C2}} + \frac{E_m}{V_{C2}} - \frac{3Le_d^2}{2CV_{C2}^2} + \frac{L i_d q}{CV_{C2}^2} - \frac{2L i_L}{3e_d} - \frac{2LC}{3e_d} \dot{v}_{22}. \]  

The new inputs are considered as follows:

\[ i_q = v_{12} = \hat{i}_q - \lambda_1 i_q + \lambda_1 e_q, \quad i_q = 0, \]  
\[ \dot{V}_{C2} = \dot{v}_{22} = \hat{V}_{C2} - \gamma_1 \left( \dot{V}_{C2} - \hat{V}_{C2} \right) - \gamma_2 \left( V_{C2} - V_{C2}^* \right), \quad V_{C2}^* = 250 V, \]

where \( \hat{i}_q \) and \( \hat{V}_{C2} \) are the reference values of \( i_q \) and \( V_{C2} \), respectively.

3.3 The state feedback law

To control the new inputs, we use the state feedback controlled inputs as shown in (47), (48), (61) and (62). Then using a linear control technique, the gains \( \lambda_{ij} \) \((i = 1, j = 1, 2) \) and \( \gamma_{ij} \) \((i, j = 1, 2) \) will be obtained. First we define the output errors as below:

\[ e_1 = i_q - \hat{i}_q, \]  
\[ e_2 = V_{C1} - \hat{V}_{C1}^*, \quad e_3 = V_{C2} - \hat{V}_{C2}^*. \]  

By applying (63) and (64) to (47), (48), (61) and (62), two equation sets are obtained as:

\[ \{ \dot{e}_1 + \lambda_{11} e_1 = 0 \} \quad \{ \dot{e}_1 + \lambda_{12} e_1 = 0 \} \]  
\[ \{ \dot{e}_2 + \gamma_{11} e_2 + \gamma_{21} e_1 = 0 \} \quad \{ \dot{e}_3 + \gamma_{12} e_3 + \gamma_{22} e_2 = 0 \} \]

By means of the pole placement technique the gains \( \lambda_{ij} \) \((i = 1, j = 1, 2) \) and \( \gamma_{ij} \) \((i, j = 1, 2) \) are obtained.

To control inner dynamics of system and eliminate the tracking error due to the presence of parasitic elements, we add integral controls to (65) and the new state feedback controlled inputs will be obtained as follows:

\[ \{ \dot{e}_1 + \lambda_{11} e_1 + \lambda_{21} \int e_1 dt = 0 \} \quad \{ \dot{e}_2 + \gamma_{11} e_2 + \gamma_{21} e_1 + \gamma_{31} \int e_2 dt = 0 \} \]

and

\[ \{ \dot{e}_3 + \lambda_{12} e_2 + \lambda_{22} \int e_1 dt = 0 \} \quad \{ \dot{e}_3 + \gamma_{12} e_3 + \gamma_{22} e_2 + \gamma_{32} \int e_3 dt = 0 \} \]

The integral controls will cause robust control against parasitic elements. Now the pole placement technique is applied to (66) and (67) and then the gains \( \lambda_{ij} \) \((i, j = 1, 2) \) and \( \gamma_{ij} \) \((i = 1, 2, j = 1, 2, 3) \) are obtained.

3.4 Three-Level SPWM Modulation Method

We use 3-level SPWM for controlling the rectifier. Though the three-level SPWM modulation is nonlinear, it has the advantages of lower switching frequency and easier to be realized. The obtained switching functions from (45), (46), (59) and (60) are transformed into reference abc frame (switching functions \( S_{11} \) and \( S_{12} \) \((i = a, b, c) \)) and then we define the switching signals \( V_i \) \((i = a, b, c) \) as below:

\[ V_i = S_{11} V_{C1} + S_{12} V_{C2}, \quad i = a, b, c. \]  

Modulation of phase "a" is denoted in Fig (2).
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Fig. 2. Three-level SPWM

Fig. 3. Block diagram of the PI controller

4 PI CONTROLLER TECHNIQUE

A PI control technique for the NPC rectifier using MATLAB/SIMULINK is applied to compare with the proposed control method. In order to maintain the $V_{C1}$ and $V_{C2}$ output voltages and the neutral point potential at desired set point, the desired value of input d-component current is given as below:

$$x_1^* = \left( x_3 - \frac{V_r}{2} \right) + \left( x_4 - \frac{V_r}{2} \right) + (V_{dc} - V_r). \quad (69)$$

Also, to guarantee unity power factor operation, the desired value of input q-component current is defined as:

$$x_2^* = 0. \quad (70)$$

5 SIMULATION RESULTS

To verify the proposed nonlinear control method, the three level/phase NPC rectifier has been studied by simulation using MATLAB/ SIMULINK and compared to the performance of the same system with a PI controller. Figure 4 shows the block diagram of the proposed nonlinear control method.

Using two desired values and PI controllers, the control method based on PI controller can be obtained as shown in Fig. 3.

Fig. 4. Block diagram of the proposed control method

Fig. 5. The voltage of $C_1$ capacitor
control system. The parameters of the system are considered as: $R = 0.1\Omega$, $L = 0.003mH$, $R_L = 30\Omega$, $\frac{C}{C_1} = \frac{C_2}{} = 2200\mu F$, $f = 50Hz$, switching frequency $= 2KHz$, $E_m = 150V$, $V_{dc} = V_r = 500V$, $V_{C_1} = V_{C_2} = 250V$.

As it can be seen from figures 5-7, the proposed non-linear controller can keep the output voltages near their desired values and also the Neutral Point Potential (NPP) approaches to zero (the voltage $V_{C_1} - V_{C_2}$ of is near zero).

The results show that the transient state of FL is shorter than PI controller and maintains better output voltages at desired values in steady state. Figures 8-9 show that the system can reach unity power factor and the line-current is nearly a sine wave. In comparison, the proposed control method is superior to PI controller. Figure 10 shows the values of the line-current THD in the steady state. However, PI controller has a slight superiority, the obtained THD value is 2.3% for FL and therefore meets the standards IEC 61 000-3-2/4.
Fig. 11. The input line-line voltage $V_{ab}$ for FL.

Fig. 12. The input line-line voltage $V_{ab}$ for PI.

Fig. 13. Three-phase input current for FL.

Fig. 14. Three-phase input current for PI.

Fig. 15. The switching signals for FL.

Fig. 16. The switching signals for PI.
Fig. 17. (a-c): Response of output voltages with a step changes in the load current from zero to nominal value in $t = 0.6\,s$.

Fig. 18. Response of current and voltage of phase a with a step changes in the load current from zero to nominal value in $t = 0.6\,s$.

Fig. 19. Response of three phase input currents with a step changes in the load current from zero to nominal value in $t = 0.6\,s$ (a) FL, (b) PI.
The input line-to-line voltage \( V_{ab} \) of the rectifier is shown in figures 11-12. Figures 13-14 show three phase input currents in steady state operation for FL and PI controller, respectively. The switching signals of the two controllers are shown in Figs 15 and 16. In all of the mentioned figures, FL controller has an extremely short transient state and can reach their desired values quickly.

The response of the output voltages for a step change in the load current from zero to nominal value at \( t = 0.6 \) s is shown in Fig. 17 (a-c). The proposed controller reduces the steady-state error and indicates faster performance and less undershoot compared to the PI controller. Fig. 18 shows the response of current and voltage of phase “a” with a step changes in the load current from zero to nominal value at \( t = 0.6 \) s. It clearly shows that FL controller performs well by allowing input current to be tracked at the desired value with maintaining unit PF. Figs. 19(a-b) and 20(a-b) show the response of three phase input currents and switching signals with step changes in the load current from zero to nominal value at \( t = 0.6 \) s validate the proposed nonlinear control robustness against the sudden change of load.

Fig. 20. Response of the switching signals with a step changes in the load current from zero to nominal value in \( t = 0.6 \) s (a)FL, (b) PI

Fig. 21. (a-c). Response of dc-link output voltage with a step changes in the load current from nominal value to zero in \( t = 0.6 \) s
Figure 21 (a-c) shows the response of dc-link output voltages with a step changes in the load current from nominal value to zero in $t = 0.6$ s. Due to a change in system load, the output voltages are disturbed and are again sta-

**Fig. 22.** Response of current and voltage of phase a with a step changes in the load current from nominal value to zero in $t = 0.6$ s

**Fig. 23.** Response of three phase input currents with a step changes in the load current from nominal value to zero in $t = 0.6$ s (a) FL, (b) PI

**Fig. 24.** Response of the switching signals with a step changes in the load current from nominal value to zero in $t = 0 : 6$ s (a) FL, (b) PI

**Fig. 25.** Power Factor under different output power

Figure 21 (a-c) shows the response of dc-link output voltages with a step changes in the load current from nominal value to zero in $t = 0.6$ s. Due to a change in system load, the output voltages are disturbed and are again sta-
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Fig. 26. THD under different output power

bilized faster by using the proposed nonlinear controller in comparison to the PI controller. The output voltage overshoots are extremely small and the proposed nonlinear controller can keep the output voltages at their desired value. Also the response of current and voltage of phase “a” with a step changes in the load current from nominal value to zero in $t = 0.6$ s is shown in Fig. 22. Figure 22 shows the performance superiority of the proposed controller in reaching the current of phase “a” to zero during the load variation. It can be understood from Fig. 23 (a-b) and Fig. 24 (a-b) that the three phase input currents and the switching signals variations are extremely small and also in comparison with PI controller, the proposed nonlinear control law has better input currents and switching signals in keeping at their desired value. Figures 25 and 26 show the power factor and THD of phase “a” current, under the different load conditions for FL.

6 CONCLUSION

In this paper, a nonlinear controller based on input/output feedback linearization technique and a new model of the three level/phase NPC rectifier has been proposed. After obtaining dual lagrangian model of the rectifier using the Euler-Lagrange description of the rectifier, the superposition law and the two obtained forms of the load current, two the power-balance equations between the ac and dc sides are obtained by considering its corresponding load current. Then, the system is linearized and also the state feedback law controlled inputs are designed. To achieve a robust control against parasitic elements, some integrators are added to the state feedback law and then the new controlled inputs are completed by pole placement technique. Defining the appropriate outputs, the presented linearizing control law can avoid the zero dynamics of the system. The proposed control technique is also validated for dc-bus voltages tracking, unity power factor and very low THD reaching by MATLAB/SIMULINK. In comparison with PI controller, the proposed controller has a significant superiority. REFERENCES

[1] J. Li, S. Bhattacharya, and A. Q. Huang, “A New Nine-Level Active NPC (ANPC) Converter for Grid Connection of Large Wind Turbines for Distributed Generation,” IEEE Trans on Power Electronics, Vol. 26, No. 3, March 2011, pp. 961–972.

[2] M. Chaves, E. Margato, J.F. Silva, S.F. Pinto, J. Santana, “Fast optimum-predictive control and capacitor voltage balancing strategy for bipolar back-to-back NPC converters in high-voltage direct current transmission systems,” IET Gener. Transm. Distrib., 2011, Vol. 5, Iss. 3, pp. 368–375.

[3] H. Akagi, K. Isozaki, “A Hybrid Active Filter for a Three-Phase 12-Pulse Diode Rectifier Used as the Front End of a Medium-Voltage Motor Drive,” IEEE Trans on Power Electronics, Vol. 27, NO. 1. January 2012 pp. 69–77.

[4] G. Abad, M. Á. Rodríguez, J. Poza, “Three-Level NPC Converter-Based Predictive Direct Power Control of the Doubly Fed Induction Machine at Low Constant Switching Frequency,” IEEE Transactions on Industrial Electronics, VOL. 55, NO. 12, DECEMBER 2008, pp. 4417-4429.

[5] J. Pou, J. Zaragoza, P. Rodríguez, S. Ceballos, V. Sala, R. Burgos, and D. Boroyevich, “Fast-processing modulation strategy for the neutral-point-clamped converter with total elimination of the low-frequency voltage oscillations in the neutral point,” IEEE Trans. Ind. Electron., vol. 54, no. 4, pp. 2288–2299, Aug. 2007.

[6] S. Busquets-Monge, J. D. Ortega, J. Bordonau, J. A. Beristain, and J. Rocabet, “Closed-loop control design for a three-level three-phase neutral-point-clamped inverter using the optimized nearest-three virtual space-vector modulation,” in Proc. IEEE PESC, 2006, pp. 1–7.

[7] S. Busquets-Monge, J. Bordonau, D. Boroyevich, and S. Somavilla, “The nearest three virtual space vector PWM—A modulation for the comprehensive neutral-point balancing in the three-level NPC inverter,” IEEE Power Electron. Lett., vol. 2, no. 1, pp. 11–15, Mar. 2004.

[8] J. Zaragoza, J. Pou, S. Ceballos, E. Robles, C. Jaen, and M. Corbalán “Voltage -Balance Compensator for a Carrier-Based Modulation in the Neutral-Point-Clamped Converter,” IEEE Trans on Industrial Elec., vol. 56, no. 2, pp. 305–314, Feb 2009.

[9] A. Isidori, Nonlinear Control Systems, 2nd ed. Berlin, Germany: Springer-Verlag, 1995.

[10] J. E. Slotine and W. Li, Applied Nonlinear Control. Englewood Cliffs, NJ: Prentice–Hall, 1991.

[11] S. Cobreces, J. Bordonau, J. Salaet, E. J. Bueno, F. J. Rodriguez, “Exact Linearization Nonlinear Neutral-Point Voltage Control for Single-Phase Three-Level NPC Converters,” IEEE Trans on Power Elec., vol. 24, no. 10., pp. 2357–2362, Oct. 2009.

[12] L. Yacoubi, K.-I. Al-Haddad, F. Fnaiech and L.A. Dessaint, “A DSP-Based Implementation of a New Nonlinear Control for a Three-Phase Neutral Point Clamped Boost Rectifier Prototype,” IEEE Transactions on Industrial Electronics vol. 52, No.1. Feb. 2005, pp. 197-205.
[13] T.-S. Lee, “Input-output linearization and zero-dynamics control of three-phase ac/dc voltage-source converters,” IEEE Transactions on Power Electronics, vol. 18, pp. 11–22, Jan. 2003.

[14] D.-C. Lee, G.-M. Lee, and K.-D. Lee, “Dc-bus voltage control of three-phase ac/dc pwm converters using feedback linearization,” IEEE Transactions on Industry Applications, vol. 36, pp. 826–833, May-June 2000.

[15] A. E. Leon, J. A. Solsona, C. Busada, H. Chiacchiarini, M. I. Valla “A Novel Feedback/Feed forward Control Strategy for Three-Phase Voltage-Source Converters,” 1-4244-0755-9/07 ‘2007 IEEE, pp. 3391–3396.

[16] M. Mehrasa, S. Lesan, S. N. Hoseini Emeni, A. Sheikholeslami, “Passivity-Based Control With Dual Lagrangian Model Of Four-Wire Three-Level Three-Phase NPC Voltage-Source Rectifier”, Special Power Electronics Systems and Applications, CPE, IEEE, pp.411-418., 2009.

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Received: 2012-04-01
Accepted: 2012-12-21