The external circuit effect on the steady states of a vacuum diode with a decelerating electron beam

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Abstract. It was found out that a reactive load presence in the external circuit of a diode with an electron beam can crucially change its operation modes. Using a non-relativistic Bursian diode with a decelerating electron beam as the example, it was demonstrated that the presence of inductance in the external circuit leads to the development of instability in the diode–external circuit system. Values of the parameters for which the diode can generate oscillations are determined. The instability growth rates and frequencies are calculated.

1. Introduction
Microwave oscillations in electron tubes were discovered about 100 years ago (i.e., the Barkhausen-Kurtz generator). Study of non-linear oscillations in diodes with an electron beam remains to be up today. This is due to various applications of such diodes [1], [2]. As a rule, large amplitude oscillations occur when the particles move collisionless in the diode interelectrode gap. The existence of such oscillations is due to the Bursian-Pierce instability [3] – [5].

The study of the dispersion properties of plasma diodes with an electron beam has a long history. In [6], for example, authors made an attempt to explain the results of their experiments in the Bursian diode (i.e., a diode in which a monoenergetic electron flow moves through the inter-electrode gap) with an electric field that slows down the electrons. They analysed the solutions of the dispersion equation with a purely active external circuit. In this paper, we’ll show that instability can not develop in such a system. We’ll show that in the diode studied in [6], oscillations can develop due to the presence of an parasitic inductance in the external circuit.

When studying non-linear oscillations in vacuum diodes, as a rule, numerical calculations of the processes in the inter-electrode gap of the diode are carried out taking into account the change in the voltage at the collector caused by the presence of an reactance in the external circuit. There is no analyse at what parameters of the system such oscillations can arise. In the current paper we analyse the solutions of the dispersion equations for the Bursian diode, taking into account the presence of inductance in the external circuit. As a result, we could opportunity construct the diode generation regions and determine the growth rates and frequencies of increasing perturbation.

2. Problem statement
We consider the planar electrodes placed at a distance $d$ from each other and infinitely extent in the transverse direction. Across them, a potential difference $U$ is applied. The $z$-axis is directed
perpendicular to the emitter surface \((z = 0)\). A non-relativistic monoenergetic electron flow is supplied by the emitter with density \(n_0\) and injection velocity \(v_0\) perpendicular to the emitter surface with no proper magnetic field (Fig. 1). The electrons move in a self-consistent electric

![Electron beam](image)

**Figure 1.** Diode scheme.

field with no collisions and are absorbed at the electrode if they reach it. Such a device was called the Bursian diode in the literature. We use \(\lambda_D\) and \(W_0\) as units of length and energy, respectively:

\[
\lambda_D = \left[ \frac{2e_0W_0}{e^2n_0} \right]^{1/2} \approx 0.3240 \cdot 10^{-2} \frac{V_0^{3/4}}{J_0^{1/2}} [cm], \quad W_0 = \frac{1}{2} m_e v_0^2 = eV_0.
\]

Here \(J_0 = en_0v_0\) is the current density of electrons entering in the inter-electrode gap, \(V_0\) is the accelerating voltage, \(e\) and \(m\) are the charge and mass of the electron, and \(\epsilon_0 \approx 8.854 \cdot 10^{-12} C^2/Nm^2\). The dimensionless coordinate \(\zeta = z/\lambda_D\), the time \(\tau = t\omega_0\), the velocity \(u = v/v_0\), the potential \(\eta = e\Phi/(2W_0)\), the electric field strength \(\varepsilon = eE\lambda_D/(2W_0)\) and the current density \(j = J/J_0\). The characteristic frequency \(\omega_0\) is determined by the formula

\[
\omega_0 = \frac{v_0}{\lambda_D} \approx 0.3754 \cdot 10^8 \frac{J_0^{1/2}}{V_0^{1/4}} [c^{-1}].
\]

We denote the inductance of the external circuit in the dimensionless form by \(L = \tilde{L}/L_0\)

\[
L_0 = 5.3277 \cdot 10^{-8} \frac{V_0^{5/4}}{S J_0^{3/2}} [H].
\]

Here \(S\) is the surface area of the electrode. Examples of typical values of the parameters are given in the table 1.

3. Results

3.1. Stationary states

The steady states of the Bursian diode are corresponded to monotonically decreasing distributions of the potential or one minimum ones. The corresponding solutions are determined by only two dimensionless parameters: the inter-electrode distance \(\delta = d/\lambda_D\) and the external voltage \(V = eU/(2W_0)\). The solutions are written in terms of the Lagrangian coordinates as

\[
\zeta(\tau) = \frac{1}{6} \tau^3 - \frac{\epsilon_0}{2} \tau^2 + \tau, \quad \eta(\tau) = \frac{1}{2} (u^2 - 1), \quad u(\tau) = \frac{1}{2} \tau^2 - \epsilon_0 \tau + 1.
\]
Table 1. Typical values of the parameters.

| $V_0$, V | $J_0$, A/cm$^2$ | $\lambda_D$, cm | $v_0$, cm/c | $\omega_0$, c$^{-1}$ | $L_0$, $\mu$H |
|----------|----------------|--------------|------------|----------------|-------------|
| 10       | 1              | 0.0182       | 0.384 x 10$^6$ | 21.11 x 10$^6$ | 0.95        |
| 100      | 1              | 0.1025       | 1.22 x 10$^6$  | 11.87 x 10$^6$ | 16.85       |

The relation between the time-of-flight of an electron between electrodes $T$ and the values $\delta$ and $V$ takes the following form [7]:

$$\delta = -\frac{1}{12} T^3 + \frac{1}{2} \left( 1 + \sqrt{1 + 2V} \right) T. \quad (5)$$

$$T = 2\sqrt{P} \cos \left[ \frac{1}{3} \left( \pi + \arccos \frac{6\delta}{P^{3/2}} \right) \right], \quad P = 2 \left( 1 + \sqrt{1 + 2V} \right). \quad (6)$$

Steady state solutions is convenient to display by continuous curves in the plane $\{\varepsilon, \delta\}$, corresponding to fixed value $V$ (Fig. 2) [5]. All these solutions are stable ones.

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Figure 2. Branches of solutions for various values of an external voltage $V$: (1) $V = 0$, (2) $V = -0.2$, (3) $V = -0.4$. Closed circles mark position of the SCL point, closed triangles – the minimum $\varepsilon_0$ point, circles – the boundaries of solutions with no electron reflection.

3.2. The dispersion equation

The condition for matching the diode with the external circuit: $Z + Z_{ext} = 0$ with diode impedance $Z = -\tilde{U}/\tilde{I}$ and the external circuit impedance $Z_{ext}$, takes the form

$$-\frac{1}{\Omega^2} \left[ (2 - i\Omega T) \exp(i\Omega T) - i\Omega^3 \delta - i\Omega T - 2 \right] = i\Omega L = 0, \quad (7)$$

Here $T$ is related to $\delta$ and $V$ by the formula (6).

We have studied the solutions of the dispersion equation (6). It is known that there are a countable number of dispersion branches, i.e. dependences of growth rate $\Gamma$ and frequency $\Omega$ of the eigenmode on $\delta$, for each fixed value of the $V$. Three of the branches, that have the largest growth rates, for the diode with a purely active external circuit ($L = 0$) are shown in Fig. 3.
Figure 3. Dispersion curves of the Bursian diode with a purely active external circuit ($L = 0$) corresponding to the branch 3 ($V = -0.4$) in Fig. 2. $A$ and $O$ are aperiodic and oscillation branches, respectively; (a) relates to the growth rates, (b) – to the frequencies.

Figure 4. Dispersion curves of the Bursian diode with an inductive external circuit ($L = 0.01$) corresponding to branch 3 ($V = -0.4$) in Fig. 2. The lower digit of the symbol $O$ indicates an oscillation branch corresponding to a similar branch in the diode with $L = 0$; the lower symbol $L$ indicates a new branch appearing in a diode with $L > 0$.

Occurrence of the inductance $L$ in the external circuit leads to the appearance of a new branch (it is lettered by the symbol $L$ in Fig. 4.) Its growth rate runs into the region of positive values, i.e. the solutions turn out to be unstable at relevant values of $\delta$. In this case, nonlinear oscillations have to develop in the diode, which, apparently, were observed authors [6]. We have also revealed that for each value of the external voltage $V$, the instability develops only for inductance values being fallen in the interval $(0, 0.0311 \cdot (1 + \sqrt{1 + 2V})^{5/2})$. Dependencies of new dispersion branch on the inductance value are shown in Fig. 5.

4. Conclusion
We studied the problem of an external circuit effect on the stability of steady state solutions of the Bursian diode. It is shown that an inductive load in the external circuit can initiate the
oscillation instability in the diode regime with a decelerating electron flow. The value of the inductance at which the instability develops is estimated. The regions in which the diode can generate are found. The growth rates and frequencies being specific for the instability in the generation region are calculated. In order to see to what state the system diode – the external circuit will fall as the perturbation develops, the non-linear stage of the process must be studied. For this purpose, it is necessary to carry out calculations of the process at the non-linear stage.

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Figure 5. Dispersion curves $O_L$ of the Bursian diode with an inductive external circuit corresponding to branch 3 ($V = -0.4$) in Fig. 2 for various values of an inductance $L$: (1) – $L = 0.001$, (2) – 0.01, (3) – 0.03, (4) – 0.07 and (5) – 0.08.