Optimization Design and Virtual Prototype Modeling and Simulation Analysis of FSAE Car Steering Trapezium

Bin Chen¹ *, Yunxiang Liu¹ and Wei Shi²
¹School of Computer Science and Information Engineering, Shanghai University of Applied Sciences, Shanghai, China
²Jiangsu Automobile Technician College, School of Automotive Engineering, Yangzhou, China

*Corresponding author e-mail:965564679@qq.com

Abstract. Based on the overall basic parameters and design requirements of FSAE racing car, a method for optimizing the design of FSAE racing steering system was proposed. Firstly, the mathematical model of the steering trapezium of the steering system was established, and the plane steering trapezoidal parameters were optimized by MATLAB programming. Then the virtual prototype model of steering, forking arm and front suspension was established by using ADMAS dynamic simulation analysis software. The relation schema of the inner wheel and outer wheel rotation angle, the parallel wheel travel and the varying parameters of the up-and-down travel were analyzed to ensure the stability of the racing car in the steering process and reduce the wear of the tire. The results showed that the optimized steering trapezoidal mechanism attained the expected goal and offered reference for the steering system design of FSAE racing car.

1. Introduction

The Chinese Formula Student Car Competition was established in 2010 and has been successfully held for eight times. The main purpose of the Formula Student Competition is to focus on developing students' ability in car design, processing, cost control and team members' collaboration. The selection of applicable talents provides a good platform; in addition, through competition, it can create a good academic competition atmosphere, provide a broad communication platform between major participating institutions, and promote academic exchanges between institutions. [1]

The steering system of the FSAE racing car determines the steering flexibility and steering stability of the car. In this paper, the relevant parameters of steering trapezoidal are analyzed and optimized for FSAE steering trapezoidal software, and the steering trapezoidal is reasonably designed. It is ensured that no slip occurs when the internal and external wheels are fully rolled, reducing the eccentric wear of the tire.

At home and abroad, most formula cars use rack and pinion steering gears and disconnected steering trapezoidal mechanisms. Many domestic universities use the mathematical model of steering trapezoids when designing the steering, and then use MATLAB software to optimize the steering trapezoidal parameters. However, the MATLAB software is optimized for the plane turning trapezoid. The steering trapezoid is a three-dimensional body. At this time, there is a certain error in the steering trapezoidal parameters. This paper mainly analyzed and optimized the steering system of the formula car that our school participated. [2] According to the overall layout of the school’s racing car, the plane
mathematical model of the front steering trapezoid was first established, and the MATLAB software was used to optimize the trapezoidal parameters. Then the ADAMS dynamic simulation software was used to build the virtual prototype model, and the internal and external angle relationship curves with the percentage of Ackermann target were analyzed. The interference of the suspension member and the steering member motion was analyzed to see the main changes. System design provided a technical reference. [3]

2. Theoretical Ackermann steering geometry

When the car is driving at low speed, if the influence of the slip and the side of the wheel is neglected, the wheels are guaranteed to be purely rolling, and the inner and outer wheels satisfy a certain geometric relationship, that is, the theoretical Ackermann geometry [4], as shown in Fig.1:

\[
\cot \theta_0 - \cot \theta = \frac{K}{L}
\]

In the formula (1), \( \theta_0 \) is the outer wheel angle, \( \theta \) is the inner wheel angle, \( K \) is the distance between the pins on both sides, and \( L \) is the wheelbase of the car.

According to the track and other colleges and universities participating in the competition, most of the equations are high-speed. At this time, the whole vehicle will generate centrifugal force and roll. The four-wheel load will be redistributed, and the outer steering wheel load will be larger than the inner wheel load. When the car is turning at a higher speed, the tires will have a side angle due to the elastic tire side yaw angle. At this time, the actual turning angle of the outer steering wheel is larger than the theoretical outer wheel angle, so the theoretical Ackermann steering relationship cannot satisfy the design of the equation steering system, and a new target angle relationship needs to be defined. [3,4]

3. Three-hearted theorem determines the steering disconnection point

Since the front suspension of the car used a double wishbone suspension, the tie rods were segmented. Whether the one-side wheel bounce affects the travel route of the other side wheel depends on whether the position of the steering gear break point is reasonable. The position of the turning point of the steering gear of the racing car can be determined by the three-heart theorem diagram method, as shown in Fig.2, which is an illustrative method of the ideal position of the steering gear breaking point.

![Figure 1. Theoretical Ackermann geometry](image1)

The theoretical Ackermann geometry mathematical expression is:

\[
\cot \theta_0 - \cot \theta = \frac{K}{L}
\]

![Figure 2. Ideal position of the steering gear break point](image2)
suspension were determined according to Fig. 2. Assuming a center point U of the tie rod ball joint, the position of the steering off point T can be obtained according to the method shown in Fig.2.\[4\]

4. Tire characteristics

Tires are used to connect the entire vehicle to the ground and to transmit the driving force. The car will run because of the friction of the tires. For the design of an equation steering system, the mechanical properties of the tire are an important part. It mainly includes the side angle, the side biasing force and the positive moment. Due to the elasticity of the tire, the direction of travel of the wheel when the vehicle is running will deviate from the plane of the wheel and form an angle with the plane, is the side angle. The corresponding wheel also produces lateral forces, as shown in Fig.3 for the tire coordinate system.\[2\]

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{tire_coordinate_system.png}
\caption{Tire coordinate system}
\end{figure}

In summary, FSAE car used the "Hoosier slick tire", the slick tire car can bend at a higher speed. In most cases, the equation is a high-speed cornering and the vehicle will generate centrifugal force. The vehicle has a roll and the tire will produce a side angle. At this time, the outer wheel angle is larger than the theoretical angle, and the steering at this time tends to be a parallel steering. A 30% Ackermann steering trapezoid was chosen.\[5,6,7\] The goal turn relationship is:

$$\theta_i = 0.3 \arctan \frac{L \tan \theta_0}{L - B \tan \theta_0} + 0.7 \theta_0$$  \hspace{1cm} (2)

In the formula (2), $\theta_i$ is the actual inner wheel angle; $\theta_0$ is the actual outer wheel angle; $L$ is the wheelbase, mm; $B$ is the distance between the two pins mm.

5. Mathematical model building

5.1. Determining the target steering relationship

Ordinary passenger cars use the theoretical Ackermann steering. For the Formula car, the car has a side angle and the body has a roll at high speed. The four-wheel load redistribution has an effect on the tire stiffness and is high speed. When the curve is bent, the outer wheel load is larger, and the outer wheel angle is gradually increased. At this time, the steering relationship tends to be parallel. Parallel steering is a steering geometry with the same inner and outer wheel angles. In order to ensure that the tires are purely rolling and reduce the eccentric wear of the tires, refer to the design of other college steering trapezoids, and define the Ackermann correction coefficient to be 30%.\[6,7\] The goal turn relationship is:

$$\theta = 0.3 \arctan \frac{L \tan \theta_0}{L - B \tan \theta_0} + 0.7 \theta_0$$  \hspace{1cm} (3)

In the formula (3), $\theta$ is the actual inner wheel angle and $\theta_0$ is the actual outer wheel angle.

5.2 Determine the optimization objective function

This year's Formula One car uses an independent suspension, which is matched with a split front steering trapezoid. When the wheel is turned to the left, the motion relationship of the wheels on both
sides is as shown in Fig.4. L1 is the length of the steering trapezoidal arm, L2 is the length of the tie rod, M is the length of the steering gear, h is the distance between the front axle and the steering gear, N is the center distance of the pins on both sides, $\lambda$ is the trapezoidal bottom angle, $\theta$ is inside Turning angle, $\theta_0$ is the outer wheel corner. \cite{7, 8, 9}

![Figure 4. Actual angle diagram](image)

According to Figure 4, the geometric relationship can be obtained:

$$w = \frac{N - M}{2}$$ (4)

$$l_2 = \sqrt{(w - l_1 \cos \lambda^2 - l_1 \sin \lambda - h)^2}$$ (5)

$$S = l_1 \cos(\lambda - \theta_0) + \sqrt{l_2^2 - \left[l_1 \sin(\lambda - \theta_0) - h\right]^2} - w$$ (6)

$$\theta = \arctan \frac{L \tan \theta_0}{L - K \tan \theta_0}$$ (7)

$$\theta_0 = \arctan \frac{h}{w - s} + \arccos \frac{l_1^2 + h^2 + (w - s)^2 - l_2^2}{2lh^2 + (w - s)^2} - \lambda$$ (8)

In the formula, S is the rack stroke, the operator calculated by W, and $\theta_0$ is the actual inner wheel angle.

The target rotation angle $\theta$ and the actual rotation angle $\theta_0$ should be guaranteed in the design. The two should be as close as possible. The weighting factor $\omega$ is introduced to form the objective function as follows:

$$f(x) = \sum_{i=0}^{n_m} \left[\frac{(\theta - \theta_0)}{\theta_0}\right] \times 100\%$$ (9)

Determine the weighting factors as follows:

$$\omega = 1.5 \quad (0 < \theta_0 < 10^\circ)$$ (10)

$$\omega = 1 \quad (10^\circ < \theta_0 < 20^\circ)$$ (11)

$$\omega = 0.5 \quad (20^\circ < \theta_0 < \theta_{\text{max}})$$ (12)

5.3 Optimization variables and constraints

In the steering trapezoidal geometry shown in Figure 4, the center distance of the king pin and the distance M of the steering gear break point are fixed values, so the design variable is the trapezoidal
bottom angle \( \lambda \) the trapezoidal arm length \( l_1 \), the tie rod length \( l_2 \) and the steering gear. The distance \( h \) to the front axle.

When assembling the steering system and the suspension system, the constraints of the steering system design are complicated. For example, in the interference with the suspension components, the pressure angle of each member should not be too large. Determine the scope of each variable constraint as shown in Table 1.\(^\text{[9,10]}\)

| Optimization variable                  | Initial value | Constraint range |
|----------------------------------------|---------------|------------------|
| Trapezoidal corner (°)                 | 95            | 90-120           |
| Trapezoidal arm (mm)                   | 80            | 60-100           |
| Distance from the front axle (mm)      | 75            | 60-80            |
| Tie rod length (mm)                    | 330           | 300-350          |

5.4 steering trapezoidal optimization results

Using MATLAB software to input the optimal variable constraint range in the steering ladder program, the trapezoidal parameter optimization results were obtained, as shown in Table 2. The curve of the inner and outer rotation angles of the steering wheel was generated by parameters, and the curve is as shown in Fig.5.\(^\text{[11,12]}\)

| Optimization variable                  | result |
|----------------------------------------|--------|
| Trapezoidal corner (°)                 | 105    |
| Trapezoidal arm (mm)                   | 80     |
| Distance from the front axle (mm)      | 80     |
| Tie rod length (mm)                    | 336.72 |

![Figure 5. Inner and outer wheel angle relationship curve](image)

6. Model and Analysis of ADMAS Virtual Prototype

This design established a virtual prototype model of the formula car to analyze all aspects of the performance of the car. The CATIA sketch used the three-heart theorem and the vehicle assembly
drawing to determine the hard point coordinates of the steering system. As shown in Table 3, the hard point coordinates were in the steering system.

**Table 3. Steering system hard point coordinates**

| Name                                             | X axis | Y axis | Z axis |
|--------------------------------------------------|--------|--------|--------|
| Steering gear housing mounting point (mm)        | -1680  | -235   | 180    |
| Front beam rod inner point (mm)                  | -1680  | -268   | 180    |
| Intermediate shaft front point (mm)              | -1375  | 0      | 431    |
| Intermediate shaft rear point (mm)               | -1300  | 0      | 486    |
| Upper swing arm front point (mm)                 | -1680  | 0      | 180    |
| Steering wheel center (mm)                       | -1062  | 0      | 601    |

A steering virtual model is built based on known hard point coordinates. The model steering gear was a rack and pinion. The rack and the steering gear housing were reciprocating, and the gear and the steering gear housing were in a rotary motion. The steering model is shown in Fig. 6. [9,10,11]

**Figure 6. steering model**

### 6.1 Equation front fork system modeling

The front fork arm model was established according to the known front fork arm hard point coordinates, as shown in Table 4 for the front fork arm hard point coordinates.

**Table 4. Front fork arm hard point coordinates**

| Name                                             | X axis | Y axis | Z axis |
|--------------------------------------------------|--------|--------|--------|
| Hinge arm front point (mm)                       | -1759  | -218   | 127    |
| Hinge arm back point (mm)                        | -1448.813 | -238 | 127    |
| Outer arm outer point (mm)                       | -1604  | -571   | 146    |
| Upper swing arm front point (mm)                 | -1718  | -288   | 335    |
| Upper arm rear point (mm)                        | -1473  | -288   | 335    |
| Upper swing arm (mm)                             | -1595  | -550   | 386    |
| Front beam rod inner point (mm)                  | -1680  | -267   | 180    |
| Outer tie rod outer point (mm)                   | -1677  | -584   | 180    |
| Tire center (mm)                                 | -1600  | -600   | 266    |
| Push arm external contact (mm)                   | -1604  | -551   | 146    |
| Push arm inner contact (mm)                      | -1604  | -320   | 600    |
| Rocker center (mm)                               | -1604  | -270   | 580    |
| Shock absorber external contact (mm)             | -1604  | -250   | 630    |
| Shock absorber internal contact (mm)             | -1604  | -40    | 630    |
This racing suspension used a double wishbone. The two hard ends of the double wishbone were connected to the frame. The shock absorber and the spring were connected to the frame through the rocker arm. The front fork arm model is shown in Fig. 7.

![Front fork arm model](image)

**Figure 7.** Front fork arm model

6.2 *Equation front suspension test bench modeling*

The front fork arm and the steering virtual prototype model were built, and the hard point coordinates of the model were modified according to the known front fork arm and steering hard point coordinates. The front suspension test rig is shown in Fig. 8.

![Front suspension test bench](image)

**Figure 8.** Front suspension test bench

6.3 *Steering wheel inner and outer wheel angle curve*

According to the 30% Ackermann angle relationship, the maximum rotation angle of the inner wheel can be calculated as 24°, and the maximum rotation angle of the outer wheel is 27°. After the front suspension test bench is built, the wheel rotation angle is verified. It can be seen from Fig. 9 that the inner and outer wheel angles can better follow the 30% Ackermann angle relationship.

![Inside and outside corner relationship curve](image)

**Figure 9.** inside and outside corner relationship curve
6.4 Analysis of wheel parallel bounce

The domestic formula competition rules stipulate that the suspension jumps up and down at least 25.4mm, so the suspension stroke of the suspension is set to 30mm. Since the car is symmetrical, it is only necessary to analyze the beating of one side of the wheel.

When driving on bumpy or uneven roads, if the suspension guide member and the steering moving member interfere, the front wheel toe angle will change accordingly. If the toe change is too large, the steering stability of the car will be deteriorated. As shown in Fig. 10, the toe angle of the wheel before and after the jump is changed by $0.05^\circ$.

![Figure 10. Variation of the toe angle before parallel bounce](image)

The toe angle changes little during the wheel bounce, which reduces the eccentric wear of the tire and improves the handling stability of the car.

6.5 Analysis of parameters related to the up and down of the wheel

The design of the toe and camber determines the portability of the car and the stability of the car. If the wheel toe and camber are not properly matched, the tire will accelerate wear and affect the performance of the car. The formula car is generally at a high speed in the dynamic game with more corners. In order to ensure the stability of the car and reduce the wear of the tires, this year's wheel camber and the toe are designed to be negative.

The rules of the competition stipulate that the up and down bounce of the suspension is $\pm 25.4\text{mm}$, simulating the curve of the toe and camber angle when the suspension is bouncing up and down. The up and down jitter parameter settings are shown in Fig. 11.\[13,15,16\]

![Figure 11. up and down jitter parameter settings](image)
(1) Front beam curve
When the suspension jumps up and down, the toe change curve is shown in Fig. 12.

![Figure 12. Front beam curve](image1)

(2) Curve of camber angle
When the suspension jumps up and down, the camber angle curve is shown in Fig.13.

![Figure 13. Camber angle curve](image2)

The car's toe was designed to be -0.537 degrees and the camber angle was -1 degree. According to Fig.12 and Fig.13, when the suspension jumps up and down mm, the toe angle varies from -0.7 degrees to -0.3 degrees, and the camber angle ranges from -0.1 degrees to -2.1 degrees. The two parameters vary greatly with the suspension up and down, which ensures the stability of the car and increases the contact area between the tire and the ground during cornering and reduces the wear of the tire.

7. Conclusion
A method for optimizing the design of the steering system of FSAE racing car was proposed, and the steering trapezoidal mathematical model and the space virtual prototype model were constructed. The plane steering trapezoidal parameters were optimized by MATLAB programming. Then the virtual prototype model of steering, forking arm and front suspension was established by using ADMAS dynamic simulation analysis software. The results showed that the optimized analysis method was real and reliable, providing practical reference for the design of steering system of formula racing car in the future.

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