Singularities in Graviton-Dilaton System:
Their Implications on the PPN Parameters
and the Cosmological Constant

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ABSTRACT. Alternatives to Einstein’s theory of general relativity can be distinguished by measuring the parametrised post Newtonian parameters. Two such parameters $\beta$ and $\gamma$, equal to one in Einstein theory, can be obtained from static spherically symmetric solutions. For the graviton-dilaton system, as in Brans-Dicke or low energy string theory, we find that if $\gamma \neq 1$ for a charge neutral point star, then there exist naked singularities. Thus, if $\gamma$ is measured to be different from one, then it cannot be explained by these theories, without implying naked singularities. We also couple a cosmological constant $\Lambda$ to the graviton-dilaton system, a la string theory. We find that static spherically symmetric solutions in low energy string theory, which describe the gravitational field of a point star in the real universe at least up to a distance $r_\ast \simeq O(\text{pc})$, always lead to curvature singularities. These singularities are stable and much worse than the naked ones. Requiring their absence up to a distance $r_\ast$ implies a bound $|\Lambda| < 10^{-102}(r_\ast/\text{pc})^{-2}$ in natural units. If $r_\ast \simeq 1\text{Mpc}$ then $|\Lambda| < 10^{-114}$, and if $r_\ast$ extends all the way up to the edge of the universe ($10^{28}\text{cm}$) then $|\Lambda| < 10^{-122}$ in natural units.
1. Introduction

In Einstein’s theory of general relativity, the gravitational field of a point star is described by the static, spherically symmetric Schwarzschild solution. Its predictions have been verified to a very good accuracy. However, for various reasons, as described in detail in [1], it is worthwhile to consider alternative theories of gravity. Among the popular ones are the Brans-Dicke (BD) theory, which is parametrised by a constant $\omega > 0$, and the string theory. A common feature among these generalised theories is the presence of a scalar field $\phi$, called BD scalar or dilaton. There are other generalisations of BD theory, where $\omega$ is a function of $\phi$, or the matter couplings to gravity depend on another function of $\phi$, etc. For details see [1]. We will consider here only BD theory and the low energy limit of the string theory.

These alternative theories can be distinguished by measuring a set of parameters called parametrised post Newtonian (PPN) parameters. Two such parameters, $\beta$ and $\gamma$ can be obtained from static spherically symmetric solutions of the graviton-dilaton system. In Einstein’s theory $\beta = \gamma = 1$. Experimentally, their measured values are given by $\frac{1}{3}(2 + 2\gamma - \beta) = 1.003 \pm 0.005$ and $\gamma = 1 \pm 0.001$. The parameter $\beta$ is a measure of non linearity in the superposition law for gravity, and $\gamma$ is a measure of the space time curvature [1]. In this paper, we study the static spherically symmetric solutions for BD and string theory, including only the graviton and the dilaton field. They describe the gravitational field of a point star, in these theories. We find that the only acceptable solutions all lead to the same predictions for the values of $\beta$ and $\gamma$ as in Einstein’s theory, namely $\beta = \gamma = 1$. There are more general static spherically symmetric solutions [2]-[6], which predict $\beta = 1$, $\gamma = 1 + \epsilon$. However, these solutions always have naked curvature singularities proportional to $\epsilon^2$ and, hence, are unacceptable.

These general solutions can be better understood by coupling the electromagnetic field [3, 4]. They lead to non trivial PPN parameters for a point star of mass $M$ and charge $Q$. In these solutions there is an inner and an outer horizon. The curvature scalar is singular at the inner horizon, but this singularity is hidden behind the outer horizon. A charge neutral star can then be obtained in two ways: in one, corresponding to the Schwarzschild solution, the PPN parameters are trivial and there is no naked singularity, while in the other, the PPN parameters are non trivial but there is a naked singularity.
Therefore neither BD nor low energy string theory can predict non trivial values for PPN parameters $\beta$ and $\gamma$, for a charge neutral star, without introducing naked singularities. Thus if naked singularities are forbidden then, for a charge neutral star, both the BD and the low energy string theory lead to the same predictions for $\beta$ and $\gamma$ as in Einstein theory. In particular, if the parameter $\gamma$ for a charge neutral point star is known to be different from one, then it cannot be explained by either BD or low energy string theory, without implying the existence of a naked singularity. In that case an alternative theory is needed that can predict a non trivial value for $\gamma$ for a charge neutral star, without any naked singularity.

In Einstein’s theory, one can also add a cosmological constant $\Lambda$. The only modifications to the static spherically symmetric solutions are that, the space time is not flat asymptotically and, for $\Lambda > 0$, it develops a new cosmological horizon [7]. In the second half of this paper, we couple the cosmological constant to the dilaton $\phi$, in a way analogous to the coupling of a tree level cosmological constant in low energy string theory [8].

The time dependent, expanding universe type of solutions to such system have been extensively studied in various space time dimensions [8]. In the low energy limit of the string theory, the dilaton is expected to develop a potential and acquire a mass. Hence, $\Lambda$ can be considered as a function of $\phi$, leading to a dilaton potential. Taking it to be of the form $e^{-\phi}\Lambda(\phi) = m(\phi - \phi_0)^2 + \cdots$ near the minimum, the authors of [9] have thoroughly analysed the implications of such a massive dilaton for the static spherically symmetric case. Depending on the choice of $m$, the system is expected to develop one, two, or three horizons. Also the static solutions to $d + d_i + 2$ dimensional gravity with a higher dimensional cosmological constant have been studied in [10], where $d_i$ is the number of internal dimensions.

In this paper, we take $\Lambda$ to be a constant and analyse the static spherically symmetric solutions in $d = 4$ space time. They describe the gravitational field of point stars, and continue to do so to a very good approximation even when the stars have non relativistic velocities with respect to each other. For example, the Schwarzschild solution describes very well the gravitational effect of the sun on earth even though the earth is revolving around the sun with a speed of $O(10\text{km/sec})$. Also, in Einstein’s theory, when a cosmological constant $\Lambda$ is present, the static spherically symmetric solutions still describe the gravitational field of point stars and also the redshift of distant objects [11]. Therefore one expects that in our present case also, when a cosmological
constant $\Lambda$ is present, the static spherically symmetric solutions will describe the gravitational field of stars at least up to a distance $r_*$, even though the real universe is not static but expanding, characterised by the Hubble constant $H_0 = 100h_0\text{km/sec/Mpc}$, $0.5 \leq h_0 \leq 1$. Hence, $r_*$ can reasonably be taken to be of $\mathcal{O}(\text{pc})$. Therefore the study of static spherically symmetric solutions is important and physically relevant even when a cosmological constant $\Lambda$ is present.

From an analysis of such static spherically symmetric solutions, we find [14] that for BD theory, they are likely to be regular outside the Schwarzschild horizon with no curvature singularities. However, for low energy string theory, the presence of a non-zero cosmological constant leads to a curvature singularity, which is much worse than a naked one as explained in the text. This singularity is argued to persist when generic perturbations and higher order string effects are included. However such naked singularities have not been observed in our universe. Hence, one should require that they be absent, at least up to a distance $r_*$, up to which the static spherically symmetric solutions analysed here are expected to describe the gravitational field of point stars. This will then impose a bound $|\Lambda| < 10^{-102}(r_*/\text{pc})^{-2}$ in natural units. Thus if $r_* \simeq 1\text{Mpc}$ then $|\Lambda| < 10^{-114}$, and if $r_*$ extends all the way up to the edge of the universe ($10^{28}\text{cm}$) then $|\Lambda| < 10^{-122}$ in natural units.

This paper is organised as follows. In section 2, the action and the equations of motion for the graviton and the dilaton are given, for the static spherically symmetric case. In section 3, we consider the solutions when $\Lambda = 0$, and analyse the PPN parameters and the singularities. In section 4, $\Lambda$ is taken to be non-zero. We show that for low energy string theory, non-zero $\Lambda$ leads to a naked curvature singularity, and give arguments for its persistence when generic perturbations and higher order string effects are included. In section 5, we conclude with a summary.

### 2. Equations of motion for graviton and dilaton

Consider the following action for graviton ($\tilde{g}_{\mu\nu}$) and dilaton ($\phi$) fields,

$$S = -\frac{1}{16\pi\kappa} \int d^4x \sqrt{\tilde{g}} e^\phi (\tilde{R} - \tilde{a}(\nabla^2\phi)^2 + \Lambda(\phi))$$  \hspace{1cm} (1)

in the target space with coordinates $x^\mu$, $\mu = 0, 1, 2, 3$, where $\kappa (= 1$ in the following) is Newton’s constant. In our notation, $R_{\mu\nu\lambda\tau} = \frac{\partial^2 g_{\mu\nu}}{\partial x^\lambda \partial x^\tau} + \cdots$. 

When $\tilde{a} = 1$, the action $S$ in equation (1) corresponds to the target space effective action for low energy string theory, whose equations of motion give the $\beta$-function equations for $\tilde{g}_{\mu\nu}$ and $\phi$ in the sigma model approach to the string theory. $\Lambda(\phi)$ is the dilaton potential which, if constant, would act as a tree level cosmological constant in low energy string theory [8], given by $\Lambda = \frac{1}{2}(d + d_{\text{int}} - 10)$, which is zero for a critical string and non zero for a non critical string. The field $e^{-\frac{2}{\tilde{a}}}$ acts as a string coupling. When $\tilde{a} = -\omega$, the above action corresponds to Brans-Dicke (BD) theory, where $\omega > 0$ is the BD parameter.

In the effective action (1), which is written in a frame (called physical frame in the following) with metric $\tilde{g}_{\mu\nu}$, the curvature term is not in the standard Einstein form. However, the standard form, where the equations of motion are often easier to analyse, can be obtained by a dilaton dependent conformal transformation

$$\tilde{g}_{\mu\nu} = e^{-\phi}g_{\mu\nu}$$

to the Einstein frame with metric $g_{\mu\nu}$. The curvature scalars in these two frames are related by

$$\tilde{R} = e^{\phi}(R - 3\nabla^2\phi + \frac{3}{2}(\nabla\phi)^2)$$

where $^\prime$ refers to the physical frame. The effective action now becomes

$$S = -\frac{1}{16\pi} \int d^4x \sqrt{\tilde{g}} (R + \frac{a}{2}(\nabla\phi)^2 + e^{-\phi}\Lambda(\phi)),$$

where $a \equiv 3 - 2\tilde{a} = 1$ for string theory and $= 2\omega + 3$ for BD theory. The equations of motion for $g_{\mu\nu}$ and $\phi$ that follow from this action, with $\Lambda_{\phi} \equiv \frac{\partial\Lambda}{\partial\phi}$, are

$$2R_{\mu\nu} + a\nabla_\mu\phi\nabla_\nu\phi + g_{\mu\nu}\Lambda e^{-\phi} = 0$$

$$a\nabla^2\phi + (\Lambda - \Lambda_{\phi})e^{-\phi} = 0.$$ (4)

There is no specified form for the function $\Lambda(\phi)$, either in Brans-Dicke theory or in string theory. However, in the low energy limit of the string theory, the dilaton is expected to acquire a mass, and consequently develop a potential of the form $e^{-\phi}\Lambda(\phi) = m(\phi - \phi_0)^2 + \cdots$ around the minimum of the potential. In two excellent papers [9], the implications of such a massive dilaton have been thoroughly analysed for static spherically symmetric
solutions. Hence, in the following we analyse only the case where $\Lambda(\phi)$ is a constant, which corresponds to a tree level cosmological constant in low energy string theory. Furthermore, since $a \geq 1$ in string and BD theory, we also consider only $a \geq 1$.

We will look for static, spherically symmetric solutions to equations (4). In the Schwarzschild gauge where $ds^2 = -f dt^2 + f^{-1} d\rho^2 + r^2 d\Omega^2$, $d\Omega^2$ being the line element on an unit sphere, and where the fields $f$, $r$, and $\phi$ depend only on $\rho$, the equations (4) become

\[
\frac{(fr^2)''}{2} - 1 = (fr^2)'
\]

\[
= a(\phi' fr^2)' - \Lambda \phi r^2 e^{-\phi} = -\Lambda r^2 e^{-\phi}
\]

\[
4r'' + ar \phi'^2 = 0
\]

(5)

where $'$ denotes $\rho$-derivatives.

Sometimes, it is more convenient to work in the standard gauge where the line element is given by $ds^2 = -f dt^2 + G dr^2 + r^2 d\Omega^2$, and where the fields $f$, $G$, and $\phi$ depend only on $r$. Equations (4) then become

\[
\frac{(fr^2)''}{2} - (fr^2)'G' - G = (fr^2)' - \frac{G' fr^2}{2G}
\]

\[
= a(\phi' fr^2)' - \frac{a\phi' G' fr^2}{2G} - \Lambda \phi Gr^2 e^{-\phi} = -\Lambda Gr^2 e^{-\phi}
\]

\[
2G' - ar \phi'^2 = 0
\]

(6)

where $'$ denotes $r$-derivatives now in the standard gauge. The curvature scalar $\tilde{R}$ in the physical frame is given by

\[
\tilde{R} = \frac{(3 - a) f \phi'^2 e^\phi}{2G} + \frac{(3 - 2a)\Lambda}{a} - \frac{3\Lambda \phi}{a}.
\]

(7)

Using (6), it is easy to obtain the following equation for $R_1 \equiv f \phi'^2 e^\phi$:

\[
R_1' + \left(\frac{4}{r} + \frac{f'}{f} - \phi'\right) R_1 = -2(\Lambda - \Lambda \phi) \phi'.
\]

(8)

If $\Lambda$ is a constant then $\Lambda \phi = 0$ and the second equality in (8) can be integrated to obtain

\[
\frac{f'}{f} - a \phi' = \frac{r_0 \sqrt{G}}{fr^2}
\]

(9)
where \( r_0 \) is an integration constant proportional to the mass of the star.

The metric can also be written in isotropic gauge where the line element is given by \( ds^2 = -f dt^2 + F(dh^2 + h^2 dΩ^2) \) where \( f \) and \( F \) are functions of \( h \) only. In this gauge, the observable parameters of the metric \( ˜g_{\mu\nu} \) in the physical frame can be extracted as follows. The mass of the star \( M \) and the relevant PPN parameters \( \beta \) and \( \gamma \) are obtained \([1]\) by expanding the metric components \( ˜f \) and \( ˜F \) in the physical frame, as \( h \to \infty \). These observables are defined by

\[
\begin{align*}
\tilde{f} &= 1 - \frac{2M}{h} + \frac{2\beta M^2}{h^2} + \cdots \\
\tilde{F} &= 1 + \frac{2\gamma M}{h} + \cdots.
\end{align*}
\]

For Einstein’s theory \( \beta = \gamma = 1 \). The physical significance of the PPN parameters \( \beta \) and \( \gamma \) is that \( \beta \) measures the non linearity in the superposition law of gravity, while \( \gamma \) measures the space time curvature. Experimentally, \( \beta \) and \( \gamma \) are obtained by measuring the precession of the perihelia of the planets’ orbits and the time delay of radar echoes near the sun respectively; their measured values are given by \( \frac{1}{3}(2+2\gamma-\beta) = 1.003\pm.005 \) and \( \gamma = 1\pm.001 \).

From now on, we will take \( \Lambda(\phi) \equiv \Lambda = constant \) and \( a \geq 1 \).

### 3. Solutions when \( \Lambda \) is zero

Consider first the solutions when \( \Lambda = 0 \). One then has the standard Schwarzschild solution

\[
\tilde{f} = 1 - \frac{\rho_0}{\rho} , \quad \tilde{r} = \rho , \quad \phi = \phi_0 ,
\]

where \( \rho_0 \) and \( \phi_0 \) are constants, which describes the gravitational field of a point star of mass \( M = \frac{\rho_0}{2} \). There is a horizon at \( \rho = \rho_0 \) where \( \tilde{g}_{tt} = \tilde{f} = 0 \). The curvature scalar \( \tilde{R} \) in the physical frame is regular everywhere, except at \( \rho = 0 \). This is the well known black hole singularity and is hidden behind the horizon. In the isotropic gauge, the solution becomes

\[
\begin{align*}
\tilde{f} &= \left(1 - \frac{\rho_0}{1 + 4\rho_0 \rho} \right)^2 , \\
\tilde{F} &= \left(1 + \frac{\rho_0}{4\rho} \right)^4 .
\end{align*}
\]
where \( h \) and \( \rho \) are related by

\[ \rho = h \left( 1 + \frac{\rho_0}{4h} \right)^2 . \tag{10} \]

The PPN parameters are given by \( \beta = \gamma = 1 \) and are trivial.

However, there are also more general solutions \([2]-[6]\) where the dilaton field \( \phi \) and the PPN parameters are non trivial. They are given, in the Schwarzschild gauge in the Einstein frame, by \([2, 6]\)

\[
\begin{align*}
    f &= \left(1 - \frac{\rho_0}{\rho}\right)^{\frac{1-k^2}{1+k^2}}, \\
    r^2 &= \rho^2 \left(1 - \frac{\rho_0}{\rho}\right)^{\frac{2l^2}{1+k^2}}, \\
    e^{\phi - \phi_0} &= \left(1 - \frac{\rho_0}{\rho}\right)^{\frac{2l}{1+k^2}} \tag{11}
\end{align*}
\]

where \( k \) is a parameter and \( l \equiv \frac{k}{\sqrt{a}} \). In the physical isotropic gauge, the metric components become

\[
\begin{align*}
    \tilde{f} &= \left(1 - \frac{\rho_0}{\rho}\right)^{\frac{1-k^2-2l}{1+k^2}}, \\
    \tilde{F} &= \frac{\rho^2}{h^2} \left(1 - \frac{\rho_0}{\rho}\right)^{\frac{2(k^2-1)}{1+k^2}} \tag{12}
\end{align*}
\]

where \( h \) and \( \rho \) are related as in (10). Expanding the functions \( \tilde{f} \) and \( \tilde{F} \) in inverse powers of \( h \), as \( h \to \infty \), one gets the mass \( M \) and the PPN parameters \( \beta \) and \( \gamma \) as

\[
\begin{align*}
    2M &= \frac{1 - k^2 - 2l}{1 + k^2} \rho_0, \\
    \beta &= 1, \\
    \gamma &= 1 + \frac{2l \rho_0}{(1 + k^2)M} . \tag{13}
\end{align*}
\]

The parameter \( \beta \) is trivial while \( \gamma \) is non trivial if \( l \rho_0 \neq 0 \).
The curvature scalar $\tilde{R}$ in the physical frame is given by

$$\tilde{R} = \frac{\tilde{a} M^2 (\gamma - 1)^2 e^{\phi_0}}{\rho^4} \left( 1 - \frac{\rho_0}{\rho} \right)^{\frac{-1 + k^2 - 2l}{1 + k^2}}. \quad (14)$$

In the above equations $\rho_0$ is positive, so that one obtains the standard Schwarzschild solution when $k = 0$. Also the physical mass $M$, given by $(13)$, must be positive which then implies that $1 - k^2 - 2l > 0$. Hence, the metric component $\tilde{g}_{tt}$ in the physical frame vanishes at $\rho = \rho_0$. The above condition on $k$, discussed below in more detail, also implies that $1 + 3k^2 - 2l > 0$. Hence, the curvature scalar $\tilde{R}$ in $(14)$ becomes singular there, unless $\gamma = 1$, i.e. unless the PPN parameters are trivial. This singularity is naked, as will be shown presently.

We will first discuss the constraints on $k$. The positivity of the physical mass $M$ in $(13)$ implies that $1 - k^2 - 2l > 0$, which restricts the parameter $k$ to be in the range

$$-\frac{1}{\sqrt{a}} - \sqrt{1 + \frac{1}{a}} < k < -\frac{1}{\sqrt{a}} + \sqrt{1 + \frac{1}{a}}. \quad (15)$$

However, the above equation turns out to be only a weak constraint on $k$. A stronger one follows requiring the PPN parameter $\gamma$ to lie within the experimentally observed range $\gamma = 1 \pm .002$. In fact, from equations $(13)$, requiring $\gamma = 1 + \epsilon$ gives

$$k = -\frac{1}{\sqrt{a}} \left( 1 + \frac{1}{\epsilon} \right) \pm \sqrt{1 + \frac{1}{a} \left( 1 + \frac{1}{\epsilon} \right)^2}.$$

Taking into the account the constraint on $k$ given by equation $(15)$, which implies that one should take the $+$ sign for the square root above, we get

$$k = -\frac{1}{\sqrt{a}} \left( 1 + \frac{1}{\epsilon} \right) + \sqrt{1 + \frac{1}{a} \left( 1 + \frac{1}{\epsilon} \right)^2} \simeq -\frac{\epsilon \sqrt{a}}{2(1 + \epsilon)}.$$

Hence, if $\gamma$ is required to be such that $|\gamma - 1| \leq |\epsilon|$, then one gets the following stronger constraint on $k$:

$$|k| < \frac{|\epsilon| \sqrt{a}}{2(1 + \epsilon)}, \quad (16)$$
where $|\epsilon| < .002$.

Now we will discuss the nature of the singularity at $\rho = \rho_0$.

1. As can be seen from equation (14), the curvature scalar is singular at $\rho = \rho_0$; hence, this singularity is not a coordinate artifact and cannot be removed by any coordinate transformation.

2. The metric on the surface $\rho = \rho_0$ has the signature $0 + ++$, and hence, this surface is null and the singularity is a null one.

3. Consider an outgoing radial null geodesic, which describes an outgoing photon. Since $ds^2 = 0$ for such a geodesic, its equation is given by

$$\frac{dt}{d\rho} = \left(1 - \frac{\rho_0}{\rho}\right)^{\frac{k^2 - 1}{k^2 + 1}},$$

where $t$ is the external time. This gives

$$t = \rho_* + \text{const} \tag{17}$$

where $\rho_*$, the analog of the ‘tortoise coordinate’, is defined by

$$\rho_* = \int d\rho \left(1 - \frac{\rho_0}{\rho}\right)^{\frac{k^2 - 1}{k^2 + 1}}.$$

For $k = 0$, $\rho_* = \rho + \rho_0 \ln(\rho - \rho_0)$ is the standard tortoise coordinate for Schwarzschild geometry, and it tends to $-\infty$ as $\rho \to \rho_0$. For $k \neq 0$, $\rho_*$ given above cannot be explicitly evaluated for arbitrary $k$. However, it can be shown that $\rho_*$ does not diverge as $\rho \to \rho_0$. Near $\rho_0$, let $y = \rho - \rho_0 \to 0$. Then, if $k \neq 0$ and obeys the bound given by equation (16), then

$$\rho_* = \rho_0 \int dy \, y^{\frac{k^2 - 1}{k^2 + 1}} + \cdots = \frac{\rho_0(k^2 + 1)}{2k^2} \frac{2k^2}{y^{\frac{k^2 + 1}{2}}} + \cdots$$

where $\cdots$ denote higher order terms in $y$. The right hand side of the above equation is finite as $y \to 0$, and thus $\rho_*$ does not diverge as $\rho \to \rho_0$.

The outgoing radial null geodesic equation (17) then implies that a radially outgoing photon starting from $\rho_i \geq \rho_0$ at external time $t_i$ will reach an outside observer at $\rho_f \ (\rho_i < \rho_f < \infty)$ at a finite external time $t_f$ given by

$$t_f - t_i = \rho_*(\rho_f) - \rho_*(\rho_i).$$
Since, as shown above, $\rho^* (\rho)$ has no divergence even when $\rho = \rho_0$, it follows that a photon can travel from arbitrarily close to the singularity to an outside observer within a finite external time interval. Hence, the singularity at $\rho = \rho_0$ is naked.

4 a. Similarly a material particle can also travel from arbitrarily close to the singularity to an outside observer in a finite external time interval. This can be shown as follows. Let the line element be given by

$$d\tilde{s}^2 = -g_0 dt^2 + g_1 d\rho^2 + g_2 d\Omega^2,$$

where, for our case,

$$g_0 = f e^{-\phi}, \quad g_1 = \frac{e^{-\phi}}{f}, \quad g_2 = r^2 e^{-\phi},$$

with $f$, $e^\phi$, and $r^2$ given by equation (11). The corresponding geodesic equation for a material particle travelling radially outward, which can be derived in a standard way as in [13], is given by

$$\frac{dt}{d\rho} = \sqrt{\frac{g_1}{g_0 (1 + E g_0)}}, \quad \frac{d\rho}{d\tau} = \text{const} \sqrt{\frac{1 + E g_0}{g_0 g_1}},$$

where $\tau$ is the proper time (or equivalently the proper distance), $E$ is the energy of the particle which is negative in our notation, and $1 + E g_0 > 0$. For our case, these equations give

$$t = \int d\rho \left(1 - \frac{\rho_0}{\rho}\right)^{\frac{k^2 - 1}{k^2 + 1}} \left(1 + E \left(1 - \frac{\rho_0}{\rho}\right)^{\frac{1 - k^2 - 2l}{k^2 + 1}}\right)^{-\frac{1}{2}} + \text{const},$$

$$\tau = (\text{const}) \int d\rho \left(1 - \frac{\rho_0}{\rho}\right)^{-\frac{2l}{k^2 + 1}} \left(1 + E \left(1 - \frac{\rho_0}{\rho}\right)^{\frac{1 - k^2 - 2l}{k^2 + 1}}\right)^{-\frac{1}{2}} + \text{const}.$$

Since $(1 + E g_0) > 0$, the only potential divergences in the above integrals are when $\rho \to \rho_0$. However, analysing these integrals near $\rho \to \rho_0$ as before, it can be seen that they do not diverge as $\rho \to \rho_0$. Hence, just as in the case of a photon above, it follows that a material particle can travel from arbitrarily
close to the singularity to an outside observer within a finite external time interval. This again implies that the singularity at \( \rho = \rho_0 \) is naked.

4 b. By a similar analysis, it follows that the proper distance \( \tau \) between \( \rho_f \) and \( \rho_i \) (\( \rightarrow \rho_0 \)) is also finite. The integral for \( \tau \) given above does not diverge since \( 1 + k^2 - 2l > 0 \), which follows from the constraint \( 1 - k^2 - 2l > 0 \) discussed before equation (15).

5. The curvature scalar diverges as \( \rho \rightarrow \rho_0 \). The ensuing tidal forces will rip away any physical apparatus as it nears \( \rho_0 \). However, the information about this event can be communicated to the outside observer in a finite external time since, as shown above, a photon or a material particle can travel from arbitrarily close to the singularity to an outside observer within a finite external time interval.

For these reasons, the singularity at \( \rho = \rho_0 \) is naked and physically unacceptable. For recent detailed discussions on naked singularities and their various general aspects, such as their definition, physical unacceptability, various scenario for their formation in Einstein’s theory, etc., see ([15]).

We would like to make one further remark. The situation described here is different from those corresponding to other solutions in string theory where singular null horizons appear. This is because, if and when the singularities do appear for a charge neutral point star in the later case, they are always hidden behind a horizon. For a point star with extremal charge, singular null horizons can appear, but this situation again differs from the present one in which only point stars with no charge are considered. The naked, singular ‘horizon’ occurs in our case mainly because of the requirement that the PPN parameter \( \gamma \) for a charge neutral point star be non trivial, i.e. \( \gamma \neq 1 \). The motivation for this requirement has been discussed in the introduction.

One can gain more insight into the solution (11) by comparing it to that of [3, 4]. Consider, as in [3, 4], a \( U(1) \) gauge field \( A_\mu \), coupled to (3) through the action

\[
S_{em} = -\frac{1}{16\pi} \int d^4x \sqrt{g} e^{\frac{k}{\sqrt{4}}} F_{\mu\nu} F^{\mu\nu}
\]

where \( F_{\mu\nu} \equiv \partial_\mu A_\nu - \partial_\nu A_\mu \). The general solution for the above system with the graviton, dilaton, and a gauge field is given in the Schwarzschild gauge.
in the Einstein frame, by \[3, 4\]

\[
\begin{align*}
\tilde{f} &= \left(1 - \frac{\rho_1}{\rho}\right) \left(1 - \frac{\rho_0}{\rho}\right) \frac{1}{1 + k^2} \\
\tilde{r}^2 &= \rho^2 \left(1 - \frac{\rho_0}{\rho}\right) \frac{2l^2}{1 + k^2} \\
\tilde{e}^{\phi - \phi_0} &= \left(1 - \frac{\rho_0}{\rho}\right) \frac{2l^2}{1 + k^2} \\
\tilde{F}_{t\rho} &= \frac{Q}{\rho^2} \tag{19}
\end{align*}
\]

where \(l = \frac{k}{\sqrt{a}}\) and the remaining components of \(F_{\mu\nu}\) are zero. In the physical isotropic gauge, the metric components become

\[
\begin{align*}
\tilde{f} &= \left(1 - \frac{\rho_1}{\rho}\right) \left(1 - \frac{\rho_0}{\rho}\right) \frac{1 - k^2 + 2l}{1 + k^2} \\
\tilde{F} &= \frac{\rho^2}{h^2} \left(1 - \frac{\rho_0}{\rho}\right) \frac{2l^2}{1 + k^2} , \tag{20}
\end{align*}
\]

and, \(h\) and \(\rho\) are now related by

\[
\rho - \rho_0 = h(1 + \frac{\rho_1 - \rho_0}{4h})^2 .
\]

Expanding the functions \(\tilde{f}\) and \(\tilde{F}\) in inverse powers of \(h\) as \(h \to \infty\), one gets the mass \(M\), the charge \(Q\), and the PPN parameters \(\beta\) and \(\gamma\) as

\[
\begin{align*}
2M &= \rho_1 + \frac{1 - k^2 - 2l}{1 + k^2} \rho_0 \\
Q^2 &= \frac{\rho_1 \rho_0}{1 + k^2} \\
\beta &= 1 + \frac{(1 - l)Q^2}{2M^2} \\
\gamma &= 1 + \frac{2l\rho_0}{(1 + k^2)M} .
\end{align*}
\]
The parameter $\beta$ is non trivial if the charge $Q \neq 0$ while $\gamma$ is non trivial if $\rho_0 \neq 0$.

The curvature scalar $\tilde{R}$ in the physical frame is given by

\[ \tilde{R} = \frac{\tilde{a} M^2 (\gamma - 1)^2 e^{\phi_0}}{\rho^4} \left( 1 - \frac{\rho_1}{\rho} \right) \left( 1 - \frac{\rho_0}{\rho} \right)^{-\frac{1 + 3k^2 - 2l}{1 + k^2}}. \]  

(21)

The metric component $\tilde{g}_{\mu\nu}$ in the physical frame vanishes at $\rho = \rho_1$ and $\rho = \rho_0$. The curvature scalar $\tilde{R}$ is regular at $\rho = \rho_1$ but, since $1 + 3k^2 - 2l > 0$ for $a \geq 1$, it is singular at $\rho = \rho_0$ unless $\gamma = 1$. This singularity is hidden behind the horizon at $\rho_1$ if $\rho_1 > \rho_0$, and naked otherwise for the same reasons as given following equation (14).

Now, consider the charge neutral solution, i.e. $Q = 0$. This can be obtained by setting either $\rho_0 = 0$ or $\rho_1 = 0$. In the former case, one gets the usual Schwarzschild solution with trivial values for $\beta$ and $\gamma$. In the later case one gets the solution described in (11) where the parameter $\gamma$ is non trivial.

Thus, it can be seen from (14) and (21) that, in BD or low energy string theory, a non trivial value for the parameter $\gamma$ for a charge neutral point star implies the existence of a naked singularity. Conversely, in these theories, the absence of naked singularities necessarily implies that the PPN parameters $\beta$ and $\gamma$ for a charge neutral point star are trivial. Thus if naked singularities are forbidden then, for such a star, both the BD and the low energy string theory lead to the same predictions for $\beta$ and $\gamma$ as in Einstein theory. In particular, if the parameter $\gamma$ for such a star is found to be different from one, then it cannot be explained by either BD or low energy string theory, without implying the existence of a naked singularity. In that case an alternative theory is needed that can predict a non trivial value for $\gamma$ for such a charge neutral point star, without any naked singularity.

4. Solutions when $\Lambda$ is non zero

Consider now the case when $\Lambda \neq 0$. The equation involving $(\phi' f r^2)'$ in (3) is the equation of motion for $\phi$ that follows from (3). However, this equation will be absent if the dilaton $\phi$ is absent. Hence, in that case, this equation is to be ignored and $\phi$ is to be set to zero in the remaining equations. The solution to (3) is then given by

\[ f = 1 - \frac{r_0}{r} - \frac{\Lambda}{6} r^2, \quad G = 1. \]
The curvature scalar $\tilde{R} = \Lambda$. This solution describes the static, spherically symmetric gravitational field of a point star of mass $M = \frac{r_0^2}{2}$ in Einstein theory, in the presence of a cosmological constant $\Lambda$ \cite{7}.

In the presence of both the dilaton $\phi$, and the cosmological constant $\Lambda$, the solution to equations (6) is not known in an explicit form. Here we study this solution and its implications. The solution, required to reduce to the Schwarzschild one when $\Lambda = 0$, would describe the static, spherically symmetric gravitational field of a point star in the graviton-dilaton system \cite{1}, in the presence of a cosmological constant $\Lambda$.

For nonzero $\Lambda$, the following general features are valid for any solution to equations (6):

(i) The dilaton field $\phi$ cannot be a constant. In fact, the only case where $\phi$ can be a constant for a non zero $\Lambda$ is when $\Lambda = \Lambda_\phi$, i.e. $\Lambda = \lambda e^\phi$. But, as can be seen from (3), this corresponds to pure Einstein theory with a cosmological constant $\lambda$ and a free scalar field $\phi$.

(ii) $\ln G$, and hence $G$, strictly increases since $a \geq 1$ and consequently $(\ln G)' > 0$.

(iii) Consider the following polynomial ansatz for the fields as $r \to \infty$.

\[
\begin{align*}
  f &= A r^k + \cdots \\
  G &= B r^l + \cdots \\
  e^{-\phi} &= e^{-\phi_0} r^m + \cdots
\end{align*}
\]  

where $\cdots$ denote subleading terms in the limit $r \to \infty$ (it can be easily shown that if one of the fields has an asymptotic polynomial behaviour, then the others also have similar behaviour). Substituting these expressions into equations (6) gives, to the leading order, $2l = am^2$ and

\[
\frac{(k+2)}{2}(k+1-l/2)Ar^k - Br^l = k(k+1-l/2)Ar^k = -am(k+1-l/2)Ar^k = -B\Lambda e^{-\phi_0} Br^{l+m+2}.
\]  

The last two equalities above imply $k = -am = l + m + 2$ which, together with $2l = am^2$, lead to $(m+2)(m+2a) = 0$. This gives the solution $(k, l, m) = (2a, 2a, -2)$ or $(2, \frac{2}{a}, -\frac{2}{a})$. Using these relations and equations (23) it follows that

\[
k(k+1-l/2)A = -\Lambda e^{-\phi_0} B
\]
\[
(m + \frac{2}{a})\Lambda e^{-\phi_0} + \frac{4}{r^{m+2}}) B = 0.
\]

If \(a > 1\), as in BD theory, then there is always a non trivial asymptotic solution with non zero \(A\) and \(B\). For example, \((k, l, m) = (2, \frac{2}{a}, -\frac{2}{a})\) and \(B\) arbitrary. Note that in the second relation above, the term involving \(r^{m+2}\) can be ignored to the leading order, since \(m + 2 = 2(1 - \frac{1}{a}) > 0\). Also, as can be easily checked for this solution, the curvature scalar \(\tilde{R}\) in the physical frame is finite as \(r \to \infty\).

However, if \(a = 1\) as in low energy string theory, then the above equations are consistent only if \(A = B = 0\). Hence, in this case, equations (6) do not admit a non trivial solution where the fields are polynomials in \(r\) as \(r \to \infty\). A similar analysis will rule out the solutions where the fields have polynomial-logarithmic behaviour asymptotically, \(i.e.\) where the fields behave as \(r^m (\ln^n r)(\ln^p \ln r)\ldots\) to the leading order in \(r\) as \(r \to \infty\).

Thus, when \(a > 1\), which includes BD theory, but not the low energy string theory, a non trivial asymptotic solution for graviton and dilaton exists asymptotically, as \(r \to \infty\). Therefore it is very plausible, although not proved here, that a full solution can be constructed, perhaps numerically, starting from a Schwarzschild solution near the horizon and approaching the above asymptotic form as \(r \to \infty\). The curvature scalar \(\tilde{R}\) in the physical frame is also likely to remain finite everywhere outside the Schwarzschild horizon.

However, for low energy string theory where \(a = 1\), the situation is totally different. To start with, no non trivial solution exists for graviton and dilaton asymptotically as \(r \to \infty\). To further understand the solutions to (6), we start with the Schwarzschild solution and study how it gets modified when \(\Lambda \neq 0\) (from now on we set \(a = 1\)). Then the expression involving \(\Lambda\) in (6) acts as a source for the fields \(f\), \(G\), and \(\phi\), which can be solved iteratively to any order in \(\Lambda\). By construction, this would reduce to the Schwarzschild solution in the limit \(\Lambda \to 0\). One thus gets

\[
f = 1 - \frac{r_0}{r} - \frac{\Lambda r^2}{6} - \left(\frac{\Lambda^2 r^4}{120} u_2 - \frac{4 \Lambda^3 r^6}{2835} u_3 + \cdots\right)\]

\[
G = 1 + \left(\frac{\Lambda^2 r^4}{72} v_2 + \frac{2 \Lambda^3 r^6}{405} v_3 + \cdots\right)\]

\[
\phi = \phi_0 - \frac{\Lambda r^2}{6} (1 + \frac{2 r_0}{r} + \frac{2 r^2}{r^2} \ln(r - r_0))\]

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where $\phi_0$ is a constant which can be set to zero without any physical consequence, and $u_i$, $v_i$, $w_i$ are functions of $\frac{2}{r}$ and $\ln r$ which tend to 1 in the limit $\frac{2}{r} \ll 1$. Evaluating $u_i$, $v_i$, $w_i$ and/or further higher order terms will not illuminate the general features of the solution. Also, the series will typically have a finite radius of convergence beyond which it is meaningless. Although it is possible to construct convergent series in different intervals of $r$, it is difficult to extract general features. Hence we follow a different approach.

It turns out that one can understand the general features of the solutions using only (i) the equations (6), (ii) the behaviour of the fields for small $r$, and (iii) their non polynomial-logarithmic behaviour in the limit $r \to \infty$.

Note that $G = 1$ for Schwarzschild solution. Let $G$ has no pole at any finite $r$. Then the requirement that any solution to (8) reduce to the Schwarzschild one when $\Lambda = 0$, combined with the fact that $G$ is a non decreasing function, implies that $G(\infty)$, and hence, $B$ must be non zero. Then the above analysis, which excludes polynomial behaviour for the fields with non trivial coefficients, implies in particular, that the fields cannot be constant, including zero, as $r \to \infty$.

Consider first the case where $r_0 = 0$. This will describe the static, spherically symmetric gravitational field of a star of negligible mass in low energy string theory when $\Lambda \neq 0$. With $r_0 = 0$ and setting $\phi_0 = 0$, equation (9) gives $e^\phi = |f|$. It also follows from (24) that the function $f$ has a local maximum (minimum) at the origin if $\Lambda$ is positive (negative). Away from the origin, the function $f$ can

(A) have no pole at any finite $r$ and go to either $\infty$ or a constant as $r \to \infty$, or

(B) have a pole at a finite $r = r_p$ (its behaviour for $r > r_p$ will not be necessary for our purposes). We will also consider the case where

(C) $f$ has a zero at $r = r_H$.

Case A: The function $f$, and hence $G$, has no pole at finite $r$. From the analysis preceding equation (24), it is already clear that $f(\infty)$ cannot be a constant. This can also be seen as follows. A necessary condition for $f(\infty)$ to be a constant is that $f$ must have at least one more critical point at $0 < r_c \leq \infty$. Let $f'(r_1) = 0$, where $r_1 \leq \infty$ is the first critical point after the origin (note that $r_1 = \infty$ corresponds to the function $f$ decreasing (increasing) to a constant monotonically if $\Lambda$ is positive (negative)). Then
it follows, from the behaviour of $f$ near the origin, that $f$ must have a local minimum (maximum) at $r = r_1$, i.e. $f''(r_1)$ must be positive (negative). This requirement holds good even when $r_1 = \infty$. However, from equations (6) we get

$$f''(r_1) = -\Lambda e^{-\phi}G$$

which is negative (positive) if $\Lambda$ is positive (negative). This is in contradiction to the above condition. Therefore $f'(r) \neq 0$ for any $r > 0$, including $r = \infty$. Hence, the function $f$ obeying equations (6) and which behaves as in (24) near the origin, cannot be constant in the limit $r \to \infty$. From this, and the asymptotic non polynomial-logarithmic behaviour of $f$, it follows that $f(\infty) \to \infty$.

Whether these singularities are genuine or only coordinate artifacts can be decided by evaluating the curvature scalar, $\tilde{R}$, or equivalently $R_1 \equiv \frac{f\phi' + \phi}{G}$ which obeys the equation

$$R_1' + 4\frac{R_1}{r} = -2\Lambda \frac{f''}{f}.$$  \hspace{1cm} (25)

See equations (7), (8), and (9). It can be seen that $R_1(\infty)$ cannot be a constant. For, if it were, then one gets $f(\infty) \to r^{-2\frac{R_1(\infty)}{\Lambda}}$, a polynomial behaviour for $f$ as $r \to \infty$, which is ruled out. Equation (25) can be solved to give

$$R_1 = -\frac{2\Lambda}{r^4} \int dr \frac{r^4 f'}{f}.$$  

From this it follows, as $r \to \infty$, that $\frac{f'}{f} > \frac{k}{r}$ for any constant $k$ (otherwise $R_1(\infty) \to constant$). This implies that $f$ grows faster than any power of $r$ when $r \to \infty$. Evaluating the above integral in this limit, one then gets $R_1(\infty) \to \infty$.

Case B: The function $f$ has a pole at a finite $r = r_p < \infty$. Then, from equation (25) it follows, near $r = r_p$, that

$$R_1(r_p) = -2\Lambda \ln f(r_p) + O(r - r_p) \to \pm \infty.$$  

Case C: The function $f$ has a zero at $r = r_H$. Then, from equation (25) it follows, near $r = r_H$, that

$$R_1(r_H) = -2\Lambda \ln f(r_H) + O(r - r_H) \to \pm \infty.$$
Thus we see that $R_1$, and hence, the curvature scalar $\tilde{R}$ in the string frame, always diverges at one or more points $r \equiv r_s = r_p, r_H, \infty$, in low energy string theory when the cosmological constant $\Lambda \neq 0$. These singularities, which will persist even when $r_0 \neq 0$ as argued below, are naked. In fact, they are much worse, as they are created by any object, no matter how small its mass is. Thus at any point of the string target space, there will be a singularity produced by an object located at a distance $r_s$ from that point.

The above analysis also goes through when $r_0 \neq 0$ (the well known black hole singularity present now at $r = 0$, independent of $\Lambda$ and hidden behind the Schwarzschild horizon, will not concern us here). The easiest way to see it is as follows. Let the radius of convergence of the series in (24) be $\gamma$, i.e. the series converges for $r < r_{con} \equiv \sqrt{\frac{|\Lambda|}{\psi}}$ (the expansion parameter in the series is $\Lambda r^2$). Thus, for $r_0 \ll r_{con}$, its effect on the fields will be negligible by the time $r$ is near $r_{con}$, and even more so beyond $r_{con}$, as can be seen from (24), where the functions $u_i, v_i, w_i \to 1$ in the limit $\frac{r}{r_{con}} \ll 1$. Hence such a non zero $r_0$ will not affect the poles and zeroes of $f, G$, and $\phi$ (which lie beyond $r_{con}$), and therefore, the curvature singularities found before will persist.

Or, one can repeat the above analysis. Now, one does not start at $r = 0$, where there is the well known black hole singularity if $r_0 \neq 0$, but at some point beyond the horizon, where the cosmological constant term, $\frac{\Lambda r^2}{6}$, in the expression for $f$ in (24) dominates the mass term, $\frac{m}{r}$; that is, near when $r^3 > \frac{6m}{|\Lambda|}$. This value of $r$ can be ensured to fall within the radius of convergence $r_{con}$ by choosing, for a given non zero $\Lambda$, a sufficiently small $r_0$, i.e. $6r_0 < \sqrt{\frac{|\Lambda|}{\psi}}$. Then, the analysis proceeds as before. If $\Lambda$ is positive (negative), then the function $f$ will be decreasing (increasing), as $r$ is increasing beyond the value $\frac{6m}{|\Lambda|}$, where the cosmological constant term in $f$ has started dominating the mass term. One can then consider the cases (A), (B), and (C) as before, and arrive at the same conclusion.

Thus, it is very likely that these singularities will also persist for any $r_0$, since the restriction on $r_0$ above is only due to the limitation of our analysis. The negligible effect of $r_0$, in the presence of a cosmological constant, is also physically reasonable since the cosmological constant can be thought of as vacuum energy density and, as $r$ increases, the vacuum energy will overwhelm any non zero mass of a star, which is proportional to $r_0$.

Similarly, one can consider a point star with charge $Q$. The fields then
will be modified by the presence of terms involving $\frac{Q^2}{r^2}$, which will become negligible when $Q \ll r$. Thus, again by an analysis similar to the above, the singularities can be shown to persist even when the star is charged. Physically, the curvature singularities arise because of the run away feed back effect of the cosmological constant $\Lambda$ on the fields, as can be seen from (6). Therefore, the effect worsens as $r$ increases. But, the effect of mass, charge, etc. of a point star decreases as $r$ increases, cannot compensate for the effects of $\Lambda$, and hence cannot remove the singularities arising due to a nonzero $\Lambda$. From this, it is also clear that any generic perturbation such as aspherical mass/charge distribution, non zero angular momentum, etc. will not remove the above singularities either, since the effects of these perturbations decrease with increasing $r$.

Thus we see that the static spherically symmetric gravitational field produced by a star in low energy string theory has a naked curvature singularity when the cosmological constant $\Lambda \neq 0$. The singularity is in fact much worse than a naked one, and is stable under generic perturbations such as the ones discussed above.

Now, as discussed in the introduction, the static spherically symmetric solution describes the gravitational field of a spherical star at least up to a distance $r_\ast \simeq \mathcal{O}(\text{pc})$, in our universe regardless of its non static nature. Therefore, the singularities described here must be absent at least up to a distance $r_\ast$. This will then translate into a constraint on the cosmological constant $\Lambda$, in the sigma model approach to low energy string theory. If we take, somewhat arbitrarily, that the curvature becomes unacceptably strong when $|\Lambda| r^2 \simeq 1$, then requiring the absence of singularity up to a distance $r_\ast$ would give

$$|\Lambda| r_\ast^2 < 1,$$

which gives the bound

$$|\Lambda| < 10^{-102} \left( \frac{r_\ast}{\text{pc}} \right)^{-2}$$

in natural units. Thus if $r_\ast \simeq 1\text{Mpc}$ then $|\Lambda| < 10^{-114}$, and if $r_\ast$ extends all the way up to the edge of the universe ($10^{28}\text{cm}$) then $|\Lambda| < 10^{-122}$ in natural units.

The existence of the naked singularity in low energy string theory when the cosmological constant, $\Lambda \neq 0$ also means the following. If $\Lambda$ was zero during some era in the evolution of the universe, then the mechanism (if
exists) that enforces cosmic censorship - no evolution of singularities from a generic, regular, initial configuration - would also enforce the vanishing of $\Lambda$ in the long run, when the universe would be evolving sufficiently slowly for the static solutions to be applicable. Otherwise, cosmic censorship would be violated by the singularities presented above.

We now remark on the validity of the low energy effective action in string theory. This action is only perturbative and will be modified by higher order corrections in the regions of strong curvature. Hence, when these corrections are included, the singularities seen here may not be present. However, these corrections will kick in only when the curvature is strong, and the low energy effective action, and thus our analysis, is likely to remain valid until then. Therefore while the fields and the curvature may never actually become infinite, even when $\Lambda \neq 0$, in the full string action with higher order corrections, the present analysis indicates that they will become sufficiently strong as to be physically unacceptable, thus justifying the above conclusions.

5. Conclusion

We have analysed the static, spherically symmetric solutions to the graviton-dilaton system, with or without electromagnetic couplings and the cosmological constant. These solutions describe the gravitational field of a point star. The main results of the present analysis can be summarised as follows.

1. For a charge neutral point star, neither BD nor low energy string theory predicts non trivial PPN parameters, $\beta$ and $\gamma$, without introducing naked singularities. Thus, if the naked singularities are forbidden, then these theories lead to the same predictions as in Einstein theory in the static spherically symmetric regime. In particular, if the parameter $\gamma$ for a charge neutral star is observed to be different from one, then it cannot be explained by either BD or low energy string theory, without implying the existence of a naked singularity.

2. Upon coupling the cosmological constant $\Lambda$ as in the action (\ref{eq:action}), in a way analogous to the coupling of a tree level cosmological constant in low energy string theory, we find the following for the static spherically symmetric solutions. For BD type theories, these solutions are likely to exist with no naked curvature singularities. However, for low energy string theory, the presence of a non zero cosmological constant leads to a curvature singularity.
in the universe, which is much worse than a naked singularity and is stable under generic perturbations. As discussed before, the static spherically symmetric solutions describe the gravitational field of a point star at least up to a distance $r_* \simeq O(\text{pc})$, in our universe regardless of its non static nature. Therefore, the singularities described here must be absent at least up to a distance $r_*$. This implies a bound $|\Lambda| < 10^{-102}(\frac{r_*}{\text{pc}})^{-2}$ in natural units. If $r_* \simeq 1\text{Mpc}$ then $|\Lambda| < 10^{-114}$, and if $r_*$ extends all the way up to the edge of the universe ($10^{28}\text{cm}$) then $|\Lambda| < 10^{-122}$ in natural units. We have also argued that this result, and the consequent bound on $\Lambda$, are unlikely to change even when the higher order string effects are included.

**Note Added:** After the completion of our work, we were informed by C. P. Burgess of reference [4], where spherically symmetric, six parameter family of four dimensional string solutions have been studied.

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