Structure Functions of the Nucleon in a Soliton Model

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We study nucleon structure functions in the soliton picture of the bosonized Nambu-Jona-Lasinio model. We focus on their vacuum contributions and examine whether they are outweighed by their valence quark counterparts.

KEYWORDS: Nambu-Jona-Lasinio model, Chiral soliton, Nucleon structure functions

1. Introduction

We study nucleon structure functions within the Nambu-Jona-Lasinio (NJL) model. In this model the nucleon emerges as a chiral soliton that polarizes the quark fields. Stability of the soliton is achieved by balancing the binding energies of the valence levels and the vacuum polarization energy [1]. The Compton tensor is a nucleon matrix element of a time ordered product and can be computed from the regularized action. Since the hadron tensor is the absorptive part of the Compton tensor this is the ideal point of departure to unambiguously extract the contributions of the polarized vacuum to the structure functions. These contributions have never before been computed directly from the fully regularized [2] action and we numerically examine whether or not they considerably modify their valence counterparts. We also investigate the sum rules entering these structure functions. In addition to their physical content they serve as consistency checks on the heavy numerical endeavor. Finally the total structure functions undergo a perturbative DGLAP evolution to enable the comparison with experimental data.

2. The NJL Chiral Soliton Model

The simplest $SU(2)$ NJL model only contains scalar and pseudoscalar fields. Its Lagrangian is given by [3]

\[ \mathcal{L}_{NJL}(q) = \bar{q} (i \not{\partial} - m^0) q + \frac{G}{2} \left[ (\bar{q}q)^2 + (\bar{q}i\gamma_5 \vec{\tau}q)^2 \right], \]  

where $G$ is the coupling constant of the chirally invariant four fermion interaction. Introducing scalar $(S)$ and pseudoscalar $(P)$ meson $2 \times 2$ matrix fields that couple to $\bar{q}q$ and $\bar{q}i\gamma_5 \vec{\tau}q$, respectively, as $M = S + iP$ allows to integrate out the quark fields. This yields the effective action

\[ \mathcal{A}[M, M^\dagger] = -\frac{1}{4G} \int d^4x \text{tr} \left[ MM^\dagger - m^0(M + M^\dagger) \right] - iN_C \text{Tr}_A \log \left\{ i \not{\partial} - (MP_R + M^\dagger P_L) \right\}. \]  

This action is quadratically divergent requiring regularization. For definiteness we adopt the Pauli-Villars subtraction scheme with a single cut-off $\Lambda$. In this model chiral symmetry is dynamically broken as reflected by the non-zero vacuum expectation value, $\langle S \rangle = m$. In total there are three model parameters, $\Lambda, G$ and the current quark mass $m^0$. We identify the fluctuations $P$ as the pion field and impose the empirical values $m_\pi = 138$ MeV and $f_\pi = 93$ MeV so that $m$ is the only tunable parameter.
To construct the static soliton configuration, we impose the hedgehog ansatz which defines the Dirac Hamiltonian $h = -i\bar{\psi} \cdot \partial + \beta m \exp (i\vec{r} \cdot \vec{\gamma} F(r))$. Its diagonalization produces the energy eigenvalues $\epsilon_i$ and the eigenspinors $\Psi_i(r)$ as functionals of the chiral angle $F(r)$. Computing the functional trace in Eq. (2) in that basis yields the classical energy functional

$$E[F(r)] = \frac{m_n^2 f_\pi^2}{4} \int d^3r \left[1 - \cos(F(r))\right] + N_c \sum_i \left[\epsilon_i e_i - \frac{1}{2} \sum_\alpha \left[\epsilon_\alpha - \sqrt{\epsilon_\alpha^2 + \Lambda^2} + \frac{\Lambda^2}{2\sqrt{\epsilon_\alpha^2 + \Lambda^2}}\right]\right],$$

(3)

the subscripts ‘v’ denotes the distinct valence level which is strongly bound in the soliton background and is added to guarantee unit baryon number via its occupation number $\eta_v = [1 + \text{sign}(\epsilon_v)]/2$. The chiral angle $F(r)$ is obtained by extremizing $E[F(r)]$ (3) \cite{1}. Finally, nucleon states are generated by canonically quantizing the zero modes of the soliton. Then the nucleon wave function depends on the $SU(2)$ rotation matrix $A$ that parameterizes the flavor rotational zero modes.

3. Hadronic Tensor in the NJL Model

Deep inelastic scattering measures the hadronic tensor which is obtained from the nucleon matrix element of the commutator of two electromagnetic currents \( [J_\mu(x), J_\nu(0)] \). In the Bjorken limit, when the four-momentum $q$ of the virtual photon approaches negative spacelike infinity, the form factors of this tensor turn into structure functions that only depend on the Bjorken variable $x = -q^2/2q \cdot p$, where $p$ is the nucleon momentum. In the NJL model the electromagnetic current is written as $J_\mu = \bar{\psi} Q \gamma_\mu \psi$, where $Q$ is the flavor quark charge matrix. We wish to compute expectation values of products of these currents by introducing an auxiliary source field in the bosonized action and taking appropriate derivatives. The bosonized action, Eq. (2) is obtained from a path integral formalism in which such derivatives yield (matrix elements of) time ordered products. In case of two current operators, this is the Compton tensor. We then take advantage of the fact that the hadronic tensor equals the absorptive part of the Compton tensor. Details of this analysis are reported in Ref. \cite{4}. For orientation we display the resulting leading term in the 1/$N_C$ counting for the vacuum contribution to the hadronic tensor

$$W_{\mu\nu}(q) = -i \frac{MN_C \pi}{8} \int \frac{d\omega}{2\pi} \sum_\alpha \int d^3 \xi \int \frac{d\lambda}{2\pi} \epsilon^{\mu\nu\alpha\lambda}$$

$$\times \langle N, \bar{s} | \left[\Psi_\alpha(\xi)Q_A^\dagger \gamma_\mu \gamma_\nu \Psi_\alpha(\xi + \lambda \hat{q}) e^{-i\lambda \omega} - \Psi_\alpha(\xi + \lambda \hat{q})e^{-i\lambda \omega} \gamma_\nu \gamma_\mu \Psi_\alpha(\xi)Q_A^\dagger \right] f^{\alpha}_{\nu}(\omega) \rangle_p$$

$$+ \left[\Psi_\alpha(\xi)Q_A^\dagger \gamma_\mu \Psi_\alpha(\xi + \lambda \hat{q}) e^{-i\lambda \omega} - \Psi_\alpha(\xi + \lambda \hat{q})e^{-i\lambda \omega} \gamma_\nu \gamma_\mu \Psi_\alpha(\xi)Q_A^\dagger \right] f^{\alpha}_{\nu}(\omega) \rangle_p |N, \bar{s}\rangle,$$

(4)

where $n^\mu = (1, \hat{q})^\mu$ is the light-cone vector defined by the direction of the virtual photon. In the above

$$f^{\alpha}_{\nu}(\omega) = \frac{\omega \pm \epsilon_\alpha}{\omega^2 - \epsilon_\alpha^2 + i\epsilon} - \frac{\omega \pm \epsilon_\alpha}{\omega^2 - \epsilon_\alpha^2 - \Lambda^2 + i\epsilon} + \frac{\Lambda^2}{\omega^2 - \epsilon_\alpha^2 + i\epsilon},$$

(5)

are Pauli-Villars regularized spectral functions with ‘p’ extracting their pole contributions. Furthermore $Q_A = AQA^\dagger$ is the flavor rotated quark charge matrix. The subscript ‘5’ refers to a particular treatment of the axial component of $\gamma_\mu \gamma_5$ for consistency of regularization \cite{4, 5}. It now remains to adopt particular components and kinematics to project Eq. (4) onto the relevant structure functions.

4. Numerical Results

In these proceedings we can only present a small sample of the vast numerical results for nucleon structure functions in the NJL model. More details will be presented elsewhere \cite{6}.

Once the structure functions are computed from Eqs. (4) and (5) the valence quark counterparts need to be added. They were computed earlier and assumed to be dominant \cite{7–9}. The numerical
simulation is quite costly because reliable Fourier transforms of all eigenfunctions $\Psi_\alpha$ must be obtained. Whenever applicable we have verified sum rules that relate integrated structure functions to coordinate space matrix elements of the $\Psi_\alpha$.

4.1 Unpolarized Structure Functions

In figure 1 we show the numerical results of the unpolarized structure function that enters the Gottfried sum rule using $m = 400$ MeV. Indeed we see that the valence part dominates. Since the soliton breaks translational invariance, the model structure functions are not guaranteed to be localized in $x \in [0, 1]$. This is particularly reflected by the vacuum part having a small negative contribution slightly above $x = 1$. Translational invariance is restored by transformation to the infinite momentum frame (IMF) \cite{10}. Then the structure functions vanish for $x > 1$ and are subjected to perturbative QCD evolution (DGLAP formalism). The resulting structure function is compared to data \cite{11} in the right panel of figure 1. Though the gross structure is reproduced, in the vicinity of $x \lesssim 1$ the negative part of the vacuum contribution has an inauspicious impact. The integral $S_G = \int_0^\infty \frac{dx}{x} (F_p^2 - F_n^2)$ is the Gottfried sum rule and we list our model prediction in table I. As there are obvious cancellations when integrating the vacuum part, the total sum rule essentially equals its valence contribution. In total we obtain reasonable agreement with the experimental value $0.235 \pm 0.026$ \cite{11}; in particular when confronted with the naive parton model prediction of $1/3$.

4.2 Polarized Structure Functions

The polarized spin structure functions $g_1(x)$ and $g_2(x)$ are obtained from the antisymmetric contribution $W_{\mu\nu}(q) - W_{\nu\mu}(q)$. In figure 2 we show typical results for the axial structure functions of the proton. The data are well produced. When combined with the neutron, the corresponding (so-called Bjorken) sum rule gives the axial charge $g_A$ whose empirical value is $1.2601 \pm 0.0025$ \cite{12}. As typical in soliton models, this value is underestimated by about 30%-40%, cf. table II. Yet the computed axial singlet charge, which is subleading in $1/N_C$, agrees with the empirical value $\Delta \Sigma \sim 0.31 \pm 0.07$ \cite{15}.

Table I. The Gottfried sum rule for various values of $m$. The subscripts ’v’ and ‘s’ denote the valence and vacuum contributions, respectively. The last column contains their sums.

| $m$ [MeV] | $|S_G|_v$ | $|S_G|_s$ | $S_G$ |
|-----------|-----------|-----------|-------|
| 400       | 0.214     | 0.000156  | 0.214 |
| 450       | 0.225     | 0.000248  | 0.225 |
| 500       | 0.236     | 0.000356  | 0.237 |
Fig. 2. Predicted polarized structure functions computed for $m = 400\text{MeV}$. The entry 'RF' refers to the actual model calculation while IMF and DGLAP denote projection and evolution, respectively. Data are from Refs. [13, 14].

Table II. The axial-vector and -singlet charges for various values of $m$. Subscripts are as in table I.

| $m$ [MeV] | $g^A_{1\nu}$ | $g^A_{1s} | g^A | g^A_{0\nu}$ | $g^A_{0s} | g^A |
|----------|---------------|---------------|----------|---------------|---------------|----------|
| 400      | 0.734         | 0.0648        | 0.799    | 400           | 0.344         | 0.00157  |
| 450      | 0.715         | 0.0509        | 0.766    | 450           | 0.327         | 0.00214  |
| 500      | 0.704         | 0.0289        | 0.733    | 500           | 0.316         | 0.00282  |

5. Conclusion

We have computed nucleon structure functions from the chiral soliton of the bosonized NJL model. This approach has the very important feature that the regularization of the vacuum contribution is self-contained when computed from the Compton tensor. Our numerical simulations confirm that indeed the valence level contribution to the structure functions dominates over the vacuum counterpart. In comparison with data we find some shortcomings for the unpolarized structure functions while the polarized structure functions of the proton are nicely reproduced.

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