The PFDL-Model-Free Adaptive Predictive Control for a Class of Discrete-Time Nonlinear Systems
Feilong Zhang, Bi Zhang, Xingang Zhao

State Key Laboratory of Robotics, Chinese Academy of Science, P. R. China

Abstract—In this paper, a novel partial form dynamic linearization (PFDL) data-driven model-free adaptive predictive control (MFAC) method is proposed for a class of discrete-time single-input single-output nonlinear systems. The main contributions of this paper are that we combine the concept of MPC with MFAC together to propose a novel MFAC method. We prove the bounded-input bounded-output stability and tracking error monotonic convergence of the proposed method; Moreover, we discuss the possible relationship between the current PFDL-MFAC and the proposed PFDL-MFAC. The simulation and experiment are carried out to verify the effectiveness of the proposed MFAC.

Index Terms—model-free control, discrete-time nonlinear systems, stability

I. INTRODUCTION

Traditional feedback control methods and modern control theory methods have encountered many problems in practical applications. Most of them are typical model-based control methods and require the off-line model of the systems in controller design [1-4]. However, the accurate physical model of the nonlinear time-varying system is hard to be identified in most industrial settings. Consequently, the idea of self-tuning control was firstly proposed by Kalman [5] in optimal control system design in 1958. Afterwards, minimum variance self-tuning regulator was proposed by Astrom and Wittenmark, but it is not applicable in non-minimum phase system for involving zero-pole cancellation [6-7]. Then, a generalized minimum variance control method was proposed by Clarke to extend the application in non-minimum phase system [8]. In addition, the stability and the convergence of several kinds of adaptive and generalized predictive control methods were analyzed by [9-15], which promotes a variety of adaptive control methods proposed and applied in industrial settings [16-19].

Nowadays, the data-driven model-free adaptive control (MFAC) firstly proposed by Hou has drawn much attention. Similar to above adaptive methods, it is not necessary to build the off-line model of the system. The traditional ARMAX model is replaced by the equivalent dynamic linearization data models, which is shown as the increment form of the LTI DARMA model in [20][21]. The pseudo-gradient (PG) vector, whose components act as the coefficients of the equivalent dynamic linearization data models, is based on the deterministic estimation algorithms and merely estimated by the I/O measurement data of the controlled system [20][22]. Moreover, unmodeled dynamics do not exist in the data-driven model-free adaptive control method, which gives a simplified discrete control structure to MFAC [20]. These advantages make it suitable for many practical applications through computer. For example, MFAC has been successfully implemented in chemical industry, linear motor control and injection molding process, PH value control, and robotic welding process [20].

In order to further improve the stability and robustness of the current PFDL-MFAC method, we propose the PFDL-MFAC, which can make full use of I/O measurement data in the past time to predict the output of the system and use more future information of the reference trajectory to adjust the system input appropriately before the reference trajectory changes. The above advantages of the MFAC can be attributed to that the index function of the MFAC takes multiple prediction errors into consideration. While the index function of the MFAC is only optimal for the error at the current time. Besides, the MFAC can be regarded as a matrix extension of the MFAC. The future coefficients of MFAC need more iterations to predict, which can make further use of I/O measurement data in the past time. This may improve the robustness of the system against the disturbance.

The direct motivation is to design a predictive model-based adaptive control method. In control engineering community, the model predictive control (MPC) shows many superior properties and broad prospects in the robotic systems, such as MIT’s Cheetah 3 controlled by MPC can apply the right forces on the ground. However, MPC may not work well under model mismatches. To this end, we combine the concept of MPC and MFAC together to introduce the MFAC. More interestingly, this paper shows an important finding: the proposed PFDL-MFAC can be considered as an elegant extension of the current PFDL-MFAC, sharing its general structure, which hasn’t been discussed so far, to the author’s best knowledge. Along with this, PFDL-MFAC has all the characteristics of the PFDL-MFAC, whose characteristics are detailed in [20][21].

The main contributions of this work are summarized as follows.
1) This paper proposes a method of PFDL-MFAC with adjustable parameters and analyses the relationship between the proposed PFDL-MFAC and the PFDL-MFAC.
2) The bounded-input bounded-output stability and the monotonic convergence of the tracking error dynamics of the PFDL-MFAC method are analyzed.
3) The effectiveness and merits of the proposed method are verified by the simulation and experiment.

The rest of the paper is organized as follows. In Section II, the equivalent PFDL data predictive model is presented for a class of discrete time nonlinear systems. In Section III, we present the PFDL-MFAC method design and its stability analysis results. In Section IV, the effectiveness of the proposed PFDL-MFAC method are validated by the simulation and experiment. Section V gives the conclusions. At last, Appendix presents the detailed stability analysis of the proposed method.
II. DYNAMIC LINEARIZATION DATA PREDICTIVE MODELS FOR DISCRETE-TIME NONLINEAR SYSTEMS

A. System Model

In this section, an equivalent dynamic linearization data predictive model is given for general nonlinear discrete-time systems. Then, it is used in Sections III and IV to design and analyze the PF-DL-MFPC.

The discrete-time SISO nonlinear system is given as follows:

\[ y(k+1) = f(y(k), \ldots, y(k-n_u), u(k), \ldots, u(k-n_u)) \]  

where \( f(\cdot) \in \mathbb{R} \) is an unknown nonlinear function, \( n_u, n_f \in \mathbb{Z} \) represent the unknown orders of system input \( u(k) \) and the system output \( y(k) \) at time of \( k \), respectively.

The PF-DL of the nonlinear system (1) satisfies the following assumptions:

**Assumption 1:** The partial derivatives of \( f(\cdot) \) with respect to control input \( u(k), \ldots, u(k-L) \) are continuous.

**Assumption 2:** System (1) satisfies the following generalized Lipschitz condition

\[ |y(k_i + 1) - y(k_{i+1})| \leq b \|U(k_i) - U(k_{i+1})\| \]  

where \( U(k) = [u(k), \ldots, u(k-L+1)]^T \) is a vector that contains control input within a time window \([k-L+1, k]\). \( L \leq L \leq n_u \) is called pseudo orders of the system. For more detailed explanations about Assumption 1 and Assumption 2 please refer to [20][21].

**Theorem 1:** For the non-linear system (1) satisfying Assumptions 1 and 2, there must exist a time-varying vector \( \phi_i(k) \) called PG vector, if \( \Delta U(k) \neq 0, 1 \leq L \leq n_u \), system (1) can be transformed into the PF-DL data model shown as follows

\[ \Delta y(k+1) = \phi_i(k) \Delta U(k) \]  

(3)

For any time \( k \), we have \[ \|\phi_i(k)\| \leq b \], where \[ \phi_i^T(k) = [\phi_1(k), \ldots, \phi_{n_u}(k)] \}, \Delta U(k) = [\Delta u(k), \ldots, \Delta u(k-L+1)] \}

**Proof:** For details, please refer to [20][21].

**Remark 1:** For detailed meaning and significances about the dynamic linearization data modeling method, please refer to [20][21].

The relationship between LTI DARMA model and the dynamic linearization data model is also presented in [20][21], which give the suggestions of how to choose the pseudo-orders \( L \) of the model.

B. Predictive System Model

Rewrite Equation (3) into the \( N \) step forward prediction equation:

\[ y(k+1) = y(k) + \phi_i(k) \Delta U(k) \]  

(4)

Here, we define

\[ A = \begin{bmatrix} 0 & 1 & 0 & \cdots & 0 \\ 1 & 0 & \cdots & \cdots & \cdots \\ \vdots & \ddots & \ddots & \cdots & \cdots \\ 0 & \cdots & 1 & 0 \end{bmatrix}_{L \times L} \]

\[ B = \begin{bmatrix} 1 & 0 & \cdots & 0 \\ \end{bmatrix}_{1 \times L} \]

Based on (4), we have

\[ y(k+1) = y(k) + \phi_i(k) \Delta U(k) \]

\[ y(k+2) = y(k+1) + \phi_i(k+1) \Delta U(k+1) \]

\[ y(k+3) = y(k+2) + \phi_i(k+2) \Delta U(k+2) \]

\[ \vdots \]

\[ y(k+N) = y(k+N-1) + \phi_i(k+N-1) \Delta U(k+N-1) \]

\[ y(k+N) = y(k) + \sum_{i=0}^{N-1} \phi_i(k+i) \Delta U(k+i) \]  

(5)

where, \( N \) is the predictive step length, \( \Delta y(k+i) \) and \( \Delta u(k+i) \) represent the increment values of the predictive output and the predictive input of the system in the future time \( k+i \) \((i=1,2,\ldots,N)\), respectively. Here, we define \( Y_n(k), \Delta Y_n(k+1), \Delta U_n(k), \Delta U_{n_u}(k), \bar{\Psi}(k) \) and \( \Psi(k) \) as follows:

\[ \Delta Y_n(k+1) = Y_n(k+1) - Y_n(k) \]

\[ y(k+1) = \begin{bmatrix} y(k+1) \\ \vdots \\ y(k+N) \end{bmatrix}_{N \times 1} \]

\[ E = [1, \ldots, 1]_{N \times 1} \]

**Remark 2:** The dynamic linearization data model (5) is the predictive output of the system in the future time \( k+i \) \((i=1,2,\ldots,N)\) and the \( \bar{\Psi}(k) \) and \( \Psi(k) \) are of \( \bar{\Psi}(k) = [\bar{\Psi}(k), \bar{\Psi}_1(k), \ldots, \bar{\Psi}_L(k)]_{N \times L} \)

where, \( \bar{\Psi}_j(k) \) is the \( j \)-th column of the \( \bar{\Psi}(k) \).
where

\[ Y_n(k+1) = E y(k) + \Psi(k) \Delta U_n(k) + \overline{\Psi}(k) \Delta U_L(k-1) \]  

(6)

\[ N_u \] is the control step length. If \( \Delta u(k+j-1) = 0 \), \( N_u < j \leq N \), we can rewrite equation (6) into

\[ Y_n(k+1) = E y(k) + \Psi(k) \Delta U_n(k) + \overline{\Psi}(k) \Delta U_L(k-1) \]  

(7)

Where

\[ \Psi(k)_{N_u} = \begin{bmatrix} \phi_0^{(k)} B \\ \phi_{0}^{(k+1)} B \\ \vdots \\ \phi_{N_u-1}^{(k+i)} A B \\ \phi_{N_u}^{(k+i)} A B \\ \vdots \\ \phi_{N_u}^{(k+N_u)} A B \\ \phi_{N_u}^{(k+N_u-i)} A^{i-N_u+1} B \end{bmatrix} \]

III. MODEL-FREE ADAPTIVE PREDICTIVE CONTROL DESIGN AND STABILITY ANALYSIS

In this section, the design of PFDL-MFAPC method will firstly be presented. In addition, the relationship between the PFDL-MFAPC and PFDL-MFAC is presented. After that, the stability analysis with some necessary Theorems and Lemma are presented.

A. Design of PFDL Model Free Adaptive Predictive Control

A weighted control input index function is given as

\[ J = \left[ Y_n(k+1) - Y_n(k+1) \right] \left[ Y_n(k+1) - Y_n(k+1) \right]^T + \lambda \Delta U_n^T(k) \Delta U_n(k) \]

(8)

where, \( \lambda \) is a positive weighted constant.

\[ Y_n(k+1) = \left[ y^*(k+1), \ldots, y^*(k+N_u) \right]^T \]

is the desired system output vector, where \( y^*(k+i) \) is the desired output of the system at the future time of \( k+i \) \( (i=1, \ldots, N_u) \).

Considering that the PG vector can be obtained, combining Equation (7) with Equation (8) and solving the optimization condition \( \frac{\partial J}{\partial \Delta U_n(k)} = 0 \), we have the optimal output vector:

\[ \Delta U_n(k) = \left[ \hat{\Psi}(k) \hat{\Psi}(k) + \lambda I \right]^{-1} \hat{\Psi}^T(k) \bullet \]

(9)

\[ \hat{\Psi}(k) = \left[ \hat{\phi}_0(k) \hat{\phi}_0(k), \ldots, \hat{\phi}_{N-1}(k) \hat{\phi}_{N-1}(k) \right] \]

where, \( \phi_i(k) \) \( (i=1, \ldots, L+1) \) about the above adjustable parameters \( \rho_1 \) and \( A = \text{diag}[\rho_2, \ldots, \rho_{L+1}] \) are introduced to make the controller algorithm more flexible and to analysis the stability of the system. Then, we have the optimal current input

\[ u^o(k) = u(k-1) + g^T \Delta U_n^o(k) \]

(10)

where \( g = [1, 0, \ldots, 0]^T \).

Remark 2: The unknown \( \hat{\phi}_i(k+i) \) \( (i=0, 1, 2, \ldots, N-1) \), which make up unknown \( \hat{\Psi}_n(k) \) and \( \hat{\Psi}(k) \) in Equation (9), need to be replaced by their estimated and predicted values \( \hat{\phi}_i(k+i) \).

[20][21][22] give the projection algorithm to estimate \( \hat{\phi}_i(k) \) and reset the \( \hat{\phi}_i(k) \) by the initial vector according to the following algorithm.

\[ \hat{\phi}_i(k) = \hat{\phi}_i(1) \]

(11)

\[ \left[ \left( \left( Y_n(k+1) - E y(k) \right) - \hat{\Psi}(k) \Delta U_L(k-1) \right) \right] \]

The actual control law at the current instant is

\[ u(k) = u(k-1) + g^T \Delta U_n(k) \]

(13)

Remark 3: The methods of how to choose \( N_u \) and \( N_v \) are detailed in [21].

Remark 4: The special cases of the proposed PFDL-MFAPC method are shown below.

When \( N_u = 1 \), we have the following simplified control output, which does not have the inverse calculation of matrix

\[ \Delta U_n(k) = \frac{1}{\left[ \hat{\Psi}^T(k) \right]_{N_u, N}} \left[ \hat{\Psi}(k) \right]_{N_u, N+1, N} + \lambda \]

(14)

\[ \left[ \begin{array}{c} \rho_2 \\ \vdots \\ \rho_{L+1} \end{array} \right] \left[ \begin{array}{c} \Delta u(k-1) \\ \vdots \\ \Delta u(k-L+1) \end{array} \right] \]

When \( N = 1 \) and the corresponding \( N_u = 1 \), the PFDL-MFAPC degrades into the PFDL-MFAC.

\[ \Delta u(k) = \frac{\hat{\phi}_1(k)}{\lambda + | \hat{\phi}_1(k) |^2} \left[ \rho_2 (y^* - y(k)) \right] \]

(15)

\[ \left[ \begin{array}{c} \hat{\phi}_1(k) \\ \hat{\phi}_2(k) \\ \vdots \\ \hat{\phi}_{L+1}(k) \end{array} \right] \]

From (15), we can conclude that the proposed PFDL-MFAPC can be considered as an elegant extension of the current PFDL-MFAC, whose meaning and analysis are shown in [20][21][22].
B. Stability Analysis of MFAPC

This section gives some Lemmas, assumptions, and the proof of stability of PFDL-MFAPC.

Lemma 1 ([27]): Let 
\[ A = \begin{bmatrix} a_1 & \cdots & a_{L-1} & a_L \\ 1 & 0 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & 1 \end{bmatrix} \]
\[ \sum_{i=1}^{L} |a_i| < 1 \], then \( \sigma(A) < 1 \), where \( \sigma(A) \) is the spectral radius of \( A \).

Lemma 2: ([28]) When \( A \in \mathbb{R}^{n \times n} \), for any given \( \varepsilon > 0 \), there exists an induced consistent matrix norm such that
\[ \|A\|_\varepsilon \leq \sigma(A) + \varepsilon \]
where \( \sigma(A) \) is the spectral radius of \( A \).

Theorem 2: If the system is described by (1) and controlled by the MFAPC method (9)-(10) with the desired trajectory \( y_d(k) = y_d = \text{constant} \), there exists a \( \hat{\lambda} \) such that, when \( \lambda > \hat{\lambda} \), it guarantees: 1) \( \lim_{k \to \infty} |y(k+1) - y^\prime| = 0 \); 2) the control system is BIBO stable.

Proof: Appendix presents the proof of Theorem 2, which is inspired by [20][21].

IV. SIMULATIONS

[21][29] give a number of examples to compare MFAC with other typical DDC methods, like data-driven PID (DD-PID), iterative feedback tuning (IFT), and virtual reference feedback tuning (VRFT). The conclusion is that the tracking performance of MFAC is better than the above methods in its simulations. Therefore, we only need to show the effectiveness and the advantages of MFAPC methods by comparing with MFAC.

Example 1: We choose an example from [21] to make comparisons between MFAPC and MFAC, and the following discrete-time SISO nonlinear structure-varying system is considered.

\[
y(k) = \begin{cases} 
2.5y(k-1)y(k-2) + 1.2u(k-1) + 1.4u(k-2) \\
1 + y^\prime(k-1) + y^\prime(k-2) + 0.7\sin(0.5(y(k-1) + y(k-2))) \\
-0.1y(k-1) - 0.2y(k-2) - 0.3y(k-3) + 0.1u(k-1) \\
+ 0.02u(k-1) + 0.03u(k-1) & 0 < k \leq 200 \\
0.1y(k-1) + 0.2y(k-2) + 0.3y(k-3) - 0.1u(k-1) \\
+ 0.02u(k-1) + 0.03u(k-1) & 200 < k \leq 400 
\end{cases}
\]

(16)

The system is structure-varying and discontinuous, and we suppose that the system is unknown to the controller design process. The desired output trajectory is \( y^\prime(k+1) = 5x(-1)^{\mu_{(k-1)}}\), \( 1 \leq k \leq 400 \).

The controller parameters and initial setting for both the PFDL-MFAPC and PFDL-MFAC are listed in Table I, and all of them should be the same with [21].

We make comparisons among PFDL-MFAPC, PFDL-MFAC and the PID. [21] gives an appropriate group of PID parameters: \( k_p=0.15 \), \( T_i=0.5 \), \( T_d=0 \). The comparisons of tracking performance are shown in Fig. 1. The control inputs of these methods are shown in Fig. 2. The components of the PG estimation of both methods are shown in Fig. 3. The performance indexes for MFAPC and MFAC are shown in TABLE II.
From Fig. 1 and TABLE II, we can see that the respond speed and the precision of the systems controlled by MFAPC is better than that controlled by MFAC, and the systems controlled by PID cannot converge well after the time 200. The above advantages can be attributed to that PFDL-MFAPC can make full use of I/O measurement data in the past time and use more future information of the reference trajectory.

V. CONCLUSION
A novel model-free adaptive predictive control (MFAPC) method with adjustable parameters is proposed for a class of discrete-time single-input and single-output nonlinear systems. Then, we show the relationship between the PFDL-MFAC and the proposed PFDL-MFAPC. The bounded-input bounded-output (BIBO) stability analysis and the tracking error monotonic convergence of the MFAPC method are analyzed by the contraction mapping technique. The effectiveness of the proposed method has been illustrated by simulation and experiment.

APPENDIX: PROOF OF THEOREM 2
This section proves the convergence of the tracking error and the BIBO stability of the system controlled by the proposed PFDL-MFAPC.

We first define \( P = g \left[ \hat{\Psi}^T(k) \hat{\Psi}(k) + \lambda I \right]^{-1} \hat{\Psi}^T(k) \).

According to Section II, we can express \( \hat{\Psi}(k) \) as
\[
\hat{\Psi}(k) = [\hat{\Psi}_1(k), \hat{\Psi}_2(k), \ldots, \hat{\Psi}_{L_e}(k), 0]_{L_e+L},
\]
From (12) and (13), we have
\[
\Delta U_k = [\Delta u(k), \ldots, \Delta u(k-L+1)]^T
\]
\[
= A(k)[u(k-1), \ldots, u(k-L)]^T
+ \rho, g^T \hat{\Psi}^T(k) \hat{\Psi}(k) + \lambda I \right]^{-1} \hat{\Psi}^T(k) ECE(k)
\]
\[
= A(k) \Delta U_k + \rho, PEC(k)
\]
Then, \( A(k) \) may be rewritten as
\[
A(k) = \begin{bmatrix}
-g^T \hat{\Psi}^T(k) \hat{\Psi}(k) + \lambda I \right]^{-1} \hat{\Psi}^T(k) P \hat{\Psi}(k) A \\
I_{(L-e)(L-L)} (L-L)_{L_e+L}
\end{bmatrix}_{L_e+L}
\]
\[
= \begin{bmatrix}
-\rho_2 P \hat{\Psi}_1(k) - \rho_2 P \hat{\Psi}_2(k) \ldots - \rho_2 P \hat{\Psi}_{L_e}(k) \\
1 & 0 & \ldots & 0 & 0 \\
0 & 1 & \ldots & 0 & 0 \\
\vdots & \vdots & \ddots & \vdots & \vdots \\
0 & 0 & \ldots & 1 & 0
\end{bmatrix}_{L_e+L}
\]
\[
C = [1, 0, \ldots, 0]_{L_e+L}^T
\]
According to the sum of the first row of \( A(k) \) and the matrix norm inequalities between \( \| \cdot \|_\infty \) and \( \| \cdot \|_1 \), we have
\[
\sum_{l=2}^{L_e} \rho_l P \hat{\Psi}(k)_l \leq (\max_{l=2, \ldots, L_e} \rho_l) \sum_{l=2}^{L_e} \| P \hat{\Psi}(k) \|_1
\leq (\max_{l=2, \ldots, L_e} \rho_l) \left\| \left[ \hat{\Psi}(k)^T \hat{\Psi}(k) + \lambda I \right]^{-1} \hat{\Psi}^T(k) \hat{\Psi}(k) \right\|_1
\leq (\max_{l=2, \ldots, L_e} \rho_l) \left\| \left[ \hat{\Psi}(k)^T \hat{\Psi}(k) + \lambda I \right]^{-1} \hat{\Psi}^T(k) \hat{\Psi}(k) \right\|_1
\leq (\max_{l=2, \ldots, L_e} \rho_l) \left\| \left( \hat{\Psi}(k)^T \hat{\Psi}(k) + \lambda I \right)^{-1} \right\|_1 \| \hat{\Psi}(k) \|_1
\leq \rho_2 \left\| \left( \hat{\Psi}(k)^T \hat{\Psi}(k) + \lambda I \right)^{-1} \right\|_1 \| \hat{\Psi}(k) \|_1
\]
\[
\left[ \sum_{i=1}^{L_i} \rho_i |P_{\hat{\Psi}_i}(k)| \right]^{1/2} \leq \sqrt{\frac{1}{N_{\nu}} \min_{i=1,\ldots,N_{\nu}} [\lambda + b_j]} \left\| \hat{\Psi}^T(k) \right\| \left\| \hat{\Psi}(k) \right\|^{1/2} \\
\leq M_4 < 1
\]

Given \( 0 < \rho_1 < 1, \ldots, 0 < \rho_{L-1} < 1 \), we have \( (\max_{i=2,\ldots,L-1} \rho_i) < 1 \). Hence, we have
\[
\sum_{i=1}^{L_i} \rho_i \left| P_{\hat{\Psi}_i}(k) \right| \leq (\max_{i=2,\ldots,L-1} \rho_i) \sum_{i=1}^{L_i} \left| P_{\hat{\Psi}_i}(k) \right| \leq (\max_{i=2,\ldots,L-1} \rho_i) M_4^{-1} < 1
\]

According to Lemma 1 and (24), we can see that the sum of the absolute values of each element in the first row of matrix \( A(k) \) is less than 1. Then, it is obvious that all the eigenvalues of \( A(k) \) satisfy \( |\lambda| < 1 \). The characteristic equation of \( A(k) \) is
\[
z^L + \rho_1 \hat{P}_{\hat{\Psi}_1}(k) z^{L-1} + \cdots + \rho_L \hat{P}_{\hat{\Psi}_L}(k) = 0
\]
Based on \( |\lambda| < 1 \) and (25), we have the following inequality:
\[
|\lambda|^{L-1} \leq \sum_{i=1}^{L_i} \rho_i \left| P_{\hat{\Psi}_i}(k) \right| |\lambda|^{L-1} \leq (\max_{i=2,\ldots,L-1} \rho_i) M_4^{-1} < 1
\]
which means \( |\lambda| \leq (\max_{i=2,\ldots,L-1} \rho_i) M_4^{-1} < 1 \). Hence, according to Lemma 2 and (26), there exists an arbitrarily small positive \( \varepsilon \) that makes the following inequality hold.
\[
|\hat{A}(k)| \leq \sigma(\hat{A}(k)) + \varepsilon \leq (\max_{i=2,\ldots,L-1} \rho_i) M_4 + \varepsilon < 1
\]
where \( |\hat{A}(k)| \) is the compatible norm of \( A(k) \). Let
\[
d_2 = (\max_{i=2,\ldots,L-1} \rho_i) M_4^{-1}.
\]

Here, \( PE = g^T \left[ \hat{\Psi}^T(k) \hat{\Psi}(k) + \lambda I \right]^{-1} \hat{\Psi}^T(k)E \) is a number that equals to the sum of the each elements in the first row of \( \left[ \hat{\Psi}^T(k) \hat{\Psi}(k) + \lambda I \right]^{-1} \hat{\Psi}^T(k) \). Then, we have
\[
g^T \left[ \hat{\Psi}^T(k) \hat{\Psi}(k) + \lambda I \right]^{-1} \hat{\Psi}^T(k)E
\leq \left\| \hat{\Psi}^T(k) \hat{\Psi}(k) + \lambda I \right\|^{-1} \left\| \hat{\Psi}^T(k) \right\| E
\leq \sqrt{\frac{1}{N_{\nu}} \min_{i=1,\ldots,N_{\nu}} [\lambda + b_j]} \left\| \hat{\Psi}^T(k) \right\| E
\]

Similar to the proof process of (23), there exists positive \( \lambda_{\min 2} \) and \( M_2 \), such that \( \lambda > \lambda_{\min 2} \), then we obtain the following two inequations
\[
0 < M_1 \leq g^T \left[ \hat{\Psi}^T(k) \hat{\Psi}(k) + \lambda I \right]^{-1} \hat{\Psi}^T(k)E \leq M_2 < 1
\]
\[ |e(k+1)| < g(k+1), \quad k=1, 2, \ldots \]

where, \( g(2) = d_4 |e(1)| \), \( d_4 = 1 - M_3 > 0.5 > d_3 \)

Based on (37) and (38), we get
\[
g(k + 2) - g(k + 1) = (d_4 - 1)d_4^i |e(i)| + (d_4 - 1)d_4^i \sum_{j=1}^{i-1} d_4^{i-j} |e(j)| + d_4^i \sum_{j=1}^{k} d_4^{k-j} |e(j)|
\]

\[
< (d_4 - 1)g(k + 1) + d_4^i \sum_{j=1}^{i-1} d_4^{i-j} |e(j)| + d_4^i g(k)
\]

\[
< (d_4 - 1)g(k + 1) + d_4^i \sum_{j=1}^{i-1} d_4^{i-j} |e(j)| + d_4^i (d_4^{-1} |e(1)|)
\]

\[
- d_4^i \sum_{j=1}^{k} d_4^{k-j} |e(j)|
\]

\[
= (d_4 - 1)g(k + 1) + d_4^i |e(i)| - d_4^i \sum_{j=1}^{i-1} d_4^{i-j} |e(j)|
\]

\[
= (d_4 + d_4 - 1)g(k + 1)
\]

(38)

Substituting (39) into (38), we get
\[
g(k + 2) < (d_4 + d_4)g(k + 1)
\]

(39)

When \( 0 < \rho < 1 \), \( (i = 2, \ldots, L+1) \), we have
\[
0 < \max_{(i=2, \ldots, L+1)} \rho^{1/2 \lambda + L-1} M_1 < M_3 < 1 , \text{ then we further get}
\]

\[
d_4 + d_4 = 1 - M_3 + (\max_{(i=2, \ldots, L+1)} \rho^{1/2 \lambda + L-1} M_1) < 1
\]

(40)

Substituting (41) into (40), we have
\[
limit_{k \to \infty} g(k + 2) < limit_{k \to \infty} (d_4 + d_4) g(k + 1) < \cdots < limit_{k \to \infty} (d_4 + d_4)^i g(2) = 0
\]

(41)

Therefore, the conclusion 1) of Theorem 2 is the result of (42) and (38) when \( \lambda > \lambda_{min} = \max \{ \lambda_{min 1}, \lambda_{min 2}, \lambda_{min 3} \} \).

Based on (31), (37), (38) and (40), we have
\[
||U_i(k)|| \leq \sum_{j=1}^{i} \|\Delta U_i(j)\|
\]

\[
\leq \sum_{j=1}^{i} \|d_4^j \Delta U_i(j)\| + \rho M_2 \sum_{j=1}^{i-1} d_4^{i-j} |e(j)|
\]

\[
< \rho M_2 \sum_{j=1}^{i} d_4^{i-j} |e(j)|
\]

\[
= \rho M_2 (|e(1)| + d_4^i |e(1)| + \cdots + d_4^{i-1} |e(1)|)
\]

\[
+ (d_4^i |e(1)| + \cdots + |e(k)|)
\]

\[
< \rho M_2 \frac{1}{1 - d_4} (|e(1)| + |e(2)| + \cdots + |e(k)|)
\]

\[
< \rho M_2 \frac{1}{1 - d_4} (g(2) + \cdots + g(k))
\]

\[
< \rho M_2 \frac{1}{1 - d_4} (g(2) + \cdots + g(k))
\]

\[
= \rho M_2 \frac{1}{1 - d_4} (g(2) + \cdots + g(k))
\]

(42)

Hence, (43) proves the boundedness of \( ||U_i(k)|| \). The conclusion 2) of Theorem 2 in Section III is proved.

We finished the proof of Theorem 2.

REFERENCES

[1] Y. J. Liu, S. Tong, C. L. P. Chen, D. J. Li. Neural controller design-based adaptive control for nonlinear MIMO systems with unknown hysteresis inputs[J]. IEEE transactions on cybernetics, 2015, 46(1): 9-19.

[2] Liu Y J, Tong S. Barrier Lyapunov functions for Nussbaum gain adaptive control of full state constrained nonlinear systems[J]. Automatica, 2017, 76: 143-152.

[3] Pekal L, Matuš R. A suboptimal shifting based zero-pole placement method for systems with delays[J]. International Journal of Control, Automation and Systems, 2018, 16(2): 594-608.

[4] Kugelmann B, Pulch R. Robust Optimal Control of Fishing in a Three Competing Species Model[J]. IFAC-PapersOnLine, 2018, 51(2): 7-12.

[5] Kalman R E. Design of self-optimizing control systems[J]. Trans. ASME, 1958, 80: 468-478.

[6] Åström K J, Wittenmark B. Adaptive control[M]. Courier Corporation, 2013.

[7] Åström K J, Wittenmark B. On self tuning regulators[J]. Automatica, 1973, 9(2): 185-199.

[8] Clarke, David W., and Peter J. Gawthrop. "Self-tuning controller." Proceedings of the institution of electrical engineers. Vol. 122. No. 9. IET, 1975.

[9] Goodwin G C, Sin K S. Adaptive filtering prediction and control[M]. Courier Corporation, 2014.

[10] Goodwin G C, Long R S, McNlins B C. Adaptive control of bilinear systems[M]. 1980.

[11] Goodwin G, Johnson C, Sin K. Global convergence for adaptive one-step-ahead optimal controllers based on input matching[J]. IEEE Transactions on Automatic Control, 1981, 26(6): 1269-1273.

[12] Narendra K S, Lin Y H, Valavani L S. Stable adaptive controller design, Part II: Proof of stability[J]. IEEE Transactions on Automatic Control, 1980, 25(3): 440-448.

[13] Morse A. Global stability of parameter-adaptive control systems[J]. IEEE Transactions on Automatic Control, 1980, 25(3): 433-439.

[14] Narendra K S, Peterson B B. Bounded error adaptive control[C]//Decision and Control including the Symposium on Adaptive Processes, 1980 19th IEEE Conference on. IEEE, 1980, 19: 605-610.

[15] Goodwin G, Sin K. Adaptive control of nonminimum phase systems[J]. IEEE Transactions on Automatic Control, 1981, 26(2): 478-483.

[16] Chai T Y. An indirect stochastic adaptive scheme with on-line choice of weighting polynomials[J]. IEEE Transactions on Automatic Control, 1990, 35(1): 82-85.

[17] Chai T Y, He S J. A simple adaptive control algorithm with fault-tolerance[C]//Decision and Control, 1991., Proceedings of the 30th IEEE Conference on. IEEE, 1991: 1794-1795.

[18] Chai T Y, Zheng L X. Inferential Self-tuning Controller And Global Convergence Analysis[C]//American Control Conference, 1992. IEEE, 1992: 2740-2744.

[19] Åström K J, Häggblad T, Hang C C, et al. Automatic tuning and adaptation for PID controllers—a survey[M]//Adaptive Systems in Control and Signal Processing 1992. 1993: 371-376.

[20] Z. S. Hou and S. T. Jin, “A novel data-driven control approach for a class of discrete-time nonlinear systems,” IEEE Transaction on Control Systems Technology, vol. 19, no. 6, pp. 1549-1558, 2011.

[21] Z. S. Hou and S. T. Jin, Model Free Adaptive Control: Theory and Applications, CRC Press, Taylor and Francis Group, 2013.

[22] Z. S. Hou and S. T. Jin, “Data-driven model-free adaptive control for a class of MIMO nonlinear discrete-time systems,” IEEE Transaction on Neural Networks, vol. 22, no.12, pp. 2173-2188, 2011.

[23] Yu Q, Hou Z, Bu X, et al. RBFNN-Based Data-Driven Predictive Iterative Learning Control for Nonaffine Nonlinear Systems[J]. IEEE transactions on neural networks and learning systems, 2019.

[24] Hou Z, Liu S, Tian T. Lazy-learning-based data-driven model-free adaptive predictive control for a class of discrete-time nonlinear systems[J]. IEEE transactions on neural networks and learning systems, 2016, 28(8): 1914-1928.

[25] Hou Z, Liu S, Yin C. Local learning-based model-free adaptive predictive control for adjustment of oxygen concentration in syngas manufacturing industry[J]. IET Control Theory & Applications, 2016.
10(12): 1384-1394.

[26] Dong N, Feng Y, Han X, et al. An Improved Model-free Adaptive Predictive Control Algorithm For Nonlinear Systems With Large Time Delay[C]//2018 IEEE 7th Data Driven Control and Learning Systems Conference (DDCLS). IEEE, 2018: 60-64.

[27] E. I. Jury, Theory and Application of the z-Transform Method. New York: Wiley, 1964.

[28] Abadir K M, Magnus J R. Matrix algebra[M]. Cambridge University

[29] Hou Z, Zhu Y. Controller-dynamic-linearization-based model free adaptive control for discrete-time nonlinear systems[J]. IEEE Transactions on Industrial Informatics, 2013, 9(4): 2301-2309.