Scale dependence of twist-three contributions to single spin asymmetries

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(Dated: December 22, 2009)

We reexamine the scale dependence of twist-three correlation functions relevant for the single transverse spin asymmetry in the framework of collinear factorization. Evolution equations are derived for both the flavor–nonsinglet and flavor–singlet distributions and arbitrary parton momenta. Our results do not agree with the recent calculations of the evolution in the limit of vanishing gluon momentum. Possible sources for this discrepancy are identified.

PACS numbers: 12.38.Bx, 13.88.+e, 12.39.St
Keywords: single spin asymmetry; evolution equation; higher twist

I. INTRODUCTION

Large transverse single spin asymmetries (SSAs) have been observed in different hadronic reactions and these observations generated a lot of interest. Such experiments are conceptually rather simple, but their theoretical description proved to be challenging as the leading-twist contributions to such asymmetries vanish, see [1 2 3] for a review. Over the past few years there was a splash of theoretical activity in this field, which mainly followed two lines: the $k_T$ factorization in terms of the transverse-momentum dependent (TMD) distributions (e.g. $F_{1g}^{x \perp}$) in Ref. [30], or, alternatively, collinear factorization including twist-three contributions in terms of multiparton correlation functions [12 13 14 15 16 17 18 19 20 21 22]. These two techniques have their own domain of validity and were shown to be consistent with each other in the kinematic regime where they both apply [23 24 25 26].

However, practically all existing calculations have been so far at the leading order (LO) which corresponds, roughly speaking, to the (generalized) parton model. In order to test the QCD dynamics and eventually also reduce the dependence of theory predictions on both the factorization scale and the renormalization scale of the strong coupling, it is necessary to calculate the scale dependence (evolution) of the relevant nonperturbative functions. The corresponding calculation was done in Refs. [27 28] where the evolution equation was derived for the gluonic pole contributions to twist-three correlation functions that are relevant to SSA. Unfortunately, it turns out that the equations derived in [27 28] in contradiction to earlier results obtained in the framework of the operator product expansion (OPE). A resolution of this discrepancy presents the main motivation for this study.

Although we do reproduce the terms that appear in Ref. [27], we obtain two additional contributions to the evolution equation both for flavor-nonsinglet and flavor-singlet operators. We have found that part of the disagreement is due to omission of cut diagrams corresponding to the mixing of conventional twist-three correlation functions that are discussed in the context of SSA with the contributions involving a gluon and the quark-antiquark pair on the opposite sides of the cut. These are the same contributions that have been discussed for some time for semi-inclusive DIS [20 26 29], and the new observation in this work is that they necessarily contribute to the leading logarithmic accuracy through the (factorization) scale dependence. For another part we do not have an intuitive explanation. Our OPE-based calculation contains additional contributions (compared to [27 28]) for which sending the gluon momentum to zero has to be done with caution, as will be explained in the text.

Another question that we want to address is to clarify the relation of these results to earlier calculations of the scale dependence of twist-three correlation functions that contribute to inclusive reactions, pioneered by the study of the structure function $g_2(x, Q^2)$ in Ref. [30]. We would like to emphasize that the scale dependence of an arbitrary twist-three operator in QCD, hence arbitrary twist-three light-cone correlation function can be determined in terms of the two-particle evolution kernels introduced by Bukhvostov, Frolov, Lipatov and Kuraev (BFLK) [31]. The BFLK approach has become standard in calculations of the spectrum of anomalous dimensions (dilatation operator) in supersymmetric theories that are relevant to the AdS/CFT correspondence (see e.g. [32 33]), but, unfortunately, remains to be largely unknown to the broad QCD community. We will show that evolution equations for the particular parton distributions relevant for SSA [27] do not require an independent calculation but can be obtained from the BFLK kernels by simple algebra. Using an updated version of this technique [34 35] we derive the complete evolution equations for all relevant three-particle correlation functions for arbitrary gluon momenta. This generalization is necessary since the gluon-pole contributions considered in [27 28] do not have autonomous scale dependence.

The presentation is organized as follows. Section 2 is introductory. We review basic properties of three-particle correlation functions and introduce a very convenient decomposition of momenta (coordinates) and the field
operators in the spinor representation. This rewriting makes the symmetries explicit and drastically simplifies the forthcoming algebra. Section 3 contains a detailed derivation of the flavor-non-singlet evolution equation in the BFLK approach and the comparison with \textsuperscript{27,28}. Our result coincides identically with the corresponding equation for the quark-antiquark-gluon correlation function relevant for the structure function $g_2(x,Q^2)$. The flavor-singlet evolution is considered in Section 4. In this case there are two independent evolution equations for positive and negative C-parity that involve three-gluon correlation functions involving $SU(3)$ structure constants $f^{abc}$ and $d^{abc}$, respectively. The equation for positive C-parity coincides, again, with the corresponding equation for the structure function $g_2(x,Q^2)$, whereas the equation for negative C-parity is, to our knowledge, a new result (in this form). Finally, in Section 5 we summarize. The full list of the BFLK kernels in the momentum representation is collected in the Appendix.

II. GENERAL DISCUSSION

A. Definition and support properties of three-particle light-cone correlation functions

Following Ref. \textsuperscript{27} we will consider the correlation functions $\bar{T}_{q,F}, \bar{T}_{\Delta q,F}$, corresponding to the nucleon matrix elements of the quark-antiquark-gluon light-ray operators

$$T_\mu(z_1, z_2, z_3) = g\bar{q}(z_1)\gamma_+ F_{\mu+}(z_2)q(z_3),$$

$$\Delta T_\mu(z_1, z_2, z_3) = g\bar{q}(z_1)\gamma_+\gamma_5 iF_{\mu+}(z_2)q(z_3),$$

(1)

and also the correlation functions $\bar{T}^{(f,d)}_{G,F}$ and $\bar{T}^{(f,d)}_{\Delta G,F}$ corresponding to the three-gluon operators

$$G_{\mu\rho\lambda}^\pm(z_1, z_2, z_3) = gC_{\mu}^{abc} F^a_{+\rho}(z_1)F^b_{+\mu}(z_2)F^c_{+\lambda}(z_3).$$

(2)

Here $n_\mu$ is the light-like vector, $n^2 = 0$, the “plus” projection is defined as $a_+ = a^{\mu}n_\mu$. Note that there are two gluon operators with a different color structure; the factors $C_{\mu}^{abc}$ are written in terms of the $SU(3)$ structure constants

$$C_+^{abc} = i f^{abc},$$

(3)

$$C_-^{abc} = d^{abc}.$$

In all cases the path-ordered Wilson lines are implied that ensure gauge invariance. They are not shown for brevity.

The definitions of parton distributions used by Kang and Qiu are (cf. Eqs. (12),(16),(14),(23) in \textsuperscript{27}):

$$\bar{T}_{q,F}(x, x_2) = \frac{1}{2} \int \frac{dz_1 dz_2}{(2\pi)^2} e^{ip_+}(z_1, x z_2) \langle P, s_T | \bar{s}^\mu T_\mu(0, z_2, z_1) | P, s_T \rangle,$$

$$\bar{T}_{\Delta q,F}(x, x_2) = \frac{1}{2} \int \frac{dz_1 dz_2}{(2\pi)^2} e^{ip_+}(z_1, x z_2) \langle P, s_T | \bar{s}^\mu \Delta T_\mu(0, z_2, z_1) | P, s_T \rangle,$$

$$\bar{T}^{(f,d)}_{G,F}(x, x_2) = \frac{g_\lambda}{P^+} \int \frac{dz_1 dz_2}{(2\pi)^2} e^{ip_+}(z_1, x z_2) \langle P, s_T | \bar{s}^\mu G_{\mu\rho\lambda}^\pm(0, z_2, z_1) | P, s_T \rangle,$$

(4)

$$\bar{T}^{(f,d)}_{\Delta G,F}(x, x_2) = \frac{\epsilon_\lambda}{P^+} \int \frac{dz_1 dz_2}{(2\pi)^2} e^{ip_+}(z_1, x z_2) \langle P, s_T | \bar{s}^\mu \Delta G_{\mu\rho\lambda}^\pm(0, z_2, z_1) | P, s_T \rangle.$$

Here $s_\mu$ is the nucleon spin vector normalized by the condition $s^2 = -1$, $\bar{s}^\mu = -\epsilon^{\mu\nu\sigma\rho}s_\nu n_\rho n_\sigma$ with $n$ being the second light-like vector, $n^2 = 0$, $n_\perp = 1$ and $\epsilon^{\lambda} = -\epsilon^{\lambda\mu\nu}\epsilon_{0123} = 1$. The spin vector $s_\mu$ is assumed to be transverse, $ns = \bar{n}s = 0$. In the last two equations $\bar{T}^{(f)}$ and $\bar{T}^{(d)}$ correspond to matrix elements of $G_{\mu\rho\lambda}^\pm$ and $\Delta G_{\mu\rho\lambda}^\pm$, respectively, cf. Eq. (3).

For our purposes it is convenient to use a more symmetric notation with quark, antiquark and gluon momentum fractions treated equally

$$\langle P, s_T | \bar{s}^\mu T_\mu(0, z_2, z_3) | P, s_T \rangle = 2P^2_+ \int dx e^{-ip_+(\Sigma x z_k)} T_{qFq}(x_1, x_2, x_3),$$

$$\langle P, s_T | \bar{s}^\mu \Delta T_\mu(0, z_2, z_3) | P, s_T \rangle = 2P^2_+ \int dx e^{-ip_+(\Sigma x z_k)} \Delta T_{qFq}(x_1, x_2, x_3),$$

$$g_\lambda \langle P, s_T | \bar{s}^\mu G_{\mu\rho\lambda}^+(0, z_2, z_3) | P, s_T \rangle = P^3_+ \int dx e^{-ip_+(\Sigma x z_k)} T_{3F}^+(x_1, x_2, x_3),$$

$$\epsilon_\lambda \langle P, s_T | \bar{s}^\mu G_{\mu\rho\lambda}^-(0, z_2, z_3) | P, s_T \rangle = P^3_+ \int dx e^{-ip_+(\Sigma x z_k)} \Delta T_{3F}^+(x_1, x_2, x_3),$$

(5)
where the integration measure is defined as

\[ \int D\mathbf{x} = \int_{-1}^{1} dx_1 dx_2 dx_3 \delta(x_1 + x_2 + x_3). \] (6)

We assume here the standard (relativistic) normalization of states; the factor \( P^2 \) is necessary to ensure the reparametrization invariance to the choice of the light-cone vector \( n \mu \rightarrow \alpha n \mu \). Obviously

\[ \tilde{T}_{\mathbf{q},F}(x,x+2x_2) \equiv T_{\mathbf{q},F}(x-x_2,x_2,x), \] (7)

and similarly for the other distributions. Writing the definition in such a form (Fourier-transformed) makes explicit the support properties of the correlation functions: they are defined on the surface \( x_1 + x_2 + x_3 = 0 \) and effectively are functions of two variables only.

The support of parton correlation functions in the notation by Kang and Qiu, i.e. in the \((x,x_2)\) plane, is shown in Fig. 1. It can be separated in six different regions where \( x, x_2 \) and \( x + x_2 \) are positive or negative, respectively. The parton-model interpretation of each region is different. As discussed in detail in Ref. [38], interpretation of light-cone correlation functions as describing sequential emission or absorption of quark-, antiquark- and gluon-partons by the target arises by choice of a particular representation in terms of the sum over intermediate states (cut diagrams) that does not involve semidisconnected contributions. In particular, the upper-right region in Fig. 1 corresponds to emission of a pair of a quark-parton and a gluon with momentum fractions \( x > 0, x_2 > 0 \), respectively, and subsequent absorption of the quark-parton with \( x + x_2 \).

The picture can be made more symmetric going over to the correlation functions [31] and using analogue to barycentric coordinates [37] as shown in Fig. 2

\[ \tilde{d} = x_1 \tilde{e}_1 + x_2 \tilde{e}_2 + x_3 \tilde{e}_3 = x_1 \tilde{E}_1 + x_2 \tilde{E}_2. \]

The six different regions, labeled \( (12)^+3^- \), \( 2^+(13)^- \), etc., correspond to different subprocesses at the parton level [36]: For each parton \( k = 1, 2, 3 \) “plus” stands for emission \( (x_k > 0) \) and “minus” for absorption \( (x_k < 0) \).

Alternatively, one may think of “plus” and “minus” labels as indicating whether the corresponding parton appears in the direct or the final amplitude in the cut diagram [38].

It is important that different regions do not have autonomous scale dependence; they “talk” to each other and get mixed under the evolution. The particular mixing pattern can be understood as follows.

It is easy to see that to one loop accuracy any contribution to the evolution equation only involves two of the three partons. For example, parton–1 and parton–2 can exchange a gluon whereas parton–3 stays a spectator. The full evolution kernel can, therefore, be split in three two-particle kernels that involve parton pairs \( (12), (23) \) and \( (31) \), respectively. Schematically

\[ \frac{d}{d\mu} \mu T_{\mathbf{q},F} = \frac{\alpha_s}{2\pi} [H_{12} + H_{21} + H_{31}] \otimes T_{\mathbf{q},F}. \] (8)

Each two-particle kernel is, obviously, a function of the contributing parton momentum fractions, e.g. \( H_{12} \equiv H_{12}(x_1, x_2; x_1', x_2') \) and due to energy conservation \( x_1' + x_2' = x_1 + x_2 \). In other words, the rate for the scale variation of a three-particle parton correlation function with given values of momentum fractions \( x_1, x_2 \) can only involve this function on the line of constant \( x_1 + x_2 = -x_3 \) (for the contribution of \( H_{12} \), as shown in Fig. 2 (thick blue line AB).

Since \( x_3 \neq 0 \) (in general), the total momentum fraction carried by the two participating partons is non-zero, \( x_1 + x_2 \neq 0 \). This situation is familiar from studies of the scale dependence of leading-twist generalized parton distributions (GPDs) (see e.g. [39, 40]) and the three regions \( 2^+(13)^-, (12)^+3^-, 1^+(23)^- \) traversed by the thick blue line AB in Fig. 2 are in one-to-one correspondence to the two DGLAP regions \( 2^+1^- \) and \( 1^+2^- \) and the ERBL (central) region \( 1^+2^+ \) in the corresponding evolution equations. As well known [39, 40], the scale dependence of GPDs in the DGLAP regions is autonomous, whereas in the ERBL mode there are also terms describing “leakage” from the DGLAP regions. In the present
context, this result implies that evolution equation for a
generic three-particle light-cone correlation function for
the momentum fractions $x_1, x_2$ in the $(12)^+3^−$ region, as
in Fig. 2 will receive nontrivial contributions from the
$2^+(13)^−, 1^+(23)^−, 2^−(13)^+$ and $1^−(23)^+$ regions as well,
which have a different partonic interpretation.

So far we have considered the contribution of $H_{12}$ only.
The contributions of $H_{23}$ and $H_{13}$ are, in turn, kinemati-
cally constrained to the lines of constant $x_2′ + x_3′$
and $x_1′ + x_3′$, respectively, and, for the particular choice
of $x_1, x_2$ in the $(12)^+3^−$ region, correspond to the DGLAP
modes of the corresponding GPD-like evolution equa-
tions. Recall that the DGLAP evolution is ordered in
the momentum fraction; hence only parts of the kine-
matically allowed regions contribute, as shown in Fig. 2
by the thick red CD ($H_{13}$) and thick green EF ($H_{23}$)
lines.

Note that evolution of the parton correlation function
for given values of $x_1, x_2$ only involves the regions of
momentum fractions outwards from the center. If one draws
a small(er) hexagon (see Fig. 2) inside the big one, then
evolution of the correlation function outside of this small
hexagon does not depend on the latter inside the small
hexagon. This “radial” ordering is a generalization of the
usual momentum fraction ordering in DGLAP equations.
In contrast, there is no ordering/restrictions in the “az-
imuthal” direction and, in principle, all regions “talk”
to each other. This property may result in the increase
of the evolution rate in the small $x$ region.

B. Spinor representation and symmetry properties

Analysis of symmetry properties and the scale depend-
ence of the light-cone correlation functions can be sim-
plified significantly by going over to the spinor represen-
tation. Each covariant four-vector $x_\mu$ is mapped to a
hermitian $2 \times 2$ matrix $x$:

$$x_{a\dot{\alpha}} = x_\mu (\sigma^\mu)_{a\dot{\alpha}} , \quad \bar{x}^{\dot{\alpha} \alpha} = x_\mu (\bar{\sigma}^\mu)^{\dot{\alpha} \alpha} ,$$

where $\sigma^\mu = (1, \bar{\sigma}), \bar{\sigma}^\mu = (1, -\bar{\sigma})$ and $\bar{\sigma}$ are the usual
Pauli matrices. In components

$$x = \begin{pmatrix} x_0 + x_3 & x_1 - i x_2 \\ x_1 + i x_2 & x_0 - x_3 \end{pmatrix}$$

(9)

so that instead of the usual components $x_\mu = \{x_0, -\bar{x}\}$
each four-vector is described by its light-cone coordinates
$x_\pm = x_0 \pm x_3$ and two complex coordinates in the trans-
verse plane $x_1 \pm i x_2$.

The Dirac (quark) spinor $q$ is written as

$$q = (\psi_\alpha \chi^{\dot{\beta}}) , \quad \bar{q} = (\bar{\chi}^{\dot{\beta}} , \bar{\psi}_{\dot{\alpha}}) ,$$

(10)

where $\psi_\alpha, \bar{\chi}^{\dot{\beta}}$ are two-component Weyl spinors, $\bar{\psi}_{\dot{\alpha}} =
(\psi_\alpha)^\dagger, \bar{\chi}^{\dot{\beta}} = (\chi^{\dot{\beta}})^\dagger$.

Finally, the gluon strength tensor $F_{\mu\nu}$ and its dual $\tilde{F}_{\mu\nu}$
can be decomposed as

$$F_{a\beta,\dot{\alpha}\dot{\beta}} = \sigma^\mu_{\alpha\dot{\alpha}} \sigma^\nu_{\beta\dot{\beta}} F_{\mu\nu} = 2 \left( \epsilon_{\dot{a}\dot{\alpha}} f_{a\beta} - \epsilon_{\alpha\dot{\alpha}} \bar{f}_{\dot{a}\dot{\beta}} \right) ,$$

$$i \tilde{F}_{a\beta,\dot{a}\dot{\alpha}} = 2 (\epsilon_{\dot{a}\dot{\alpha}} f_{a\beta} + \epsilon_{\alpha\dot{\alpha}} \bar{f}_{\dot{a}\dot{\beta}}) .$$

(11)

Here $f_{a\beta}$ and $\bar{f}_{\dot{a}\dot{\beta}}$ are chiral and antichiral symmetric
tensors, $f^* = \bar{f}$, which belong to $(1,0)$ and $(0,1)$
representations of the Lorenz group, respectively. The
antisymmetric tensors, $\epsilon_{a\beta} = \epsilon^{a\beta}$ and $\epsilon_{\dot{a}\dot{\beta}} = \epsilon^{\dot{a}\dot{\beta}}$ are
normalized by $\epsilon_{12} = -\epsilon_{12} = 1$ and used for rising and lowering
indices

$$u^\alpha = \epsilon^{\alpha\beta} u_\beta , \quad u_\alpha = u^\beta \epsilon_{\beta\alpha} ,$$

$$\bar{u}^{\dot{\beta}} = \bar{u}_{\dot{\beta}} \epsilon_{\dot{\alpha}\dot{\beta}} , \quad \bar{u}_{\dot{\alpha}} = \epsilon_{\dot{\alpha}\dot{\beta}} \bar{u}^{\dot{\beta}} .$$

(12)

The contractions $(ab)$ and $(\bar{a} \bar{b})$ are defined as

$$(ab) = a^\alpha b_\alpha = -a_\alpha b^\alpha , \quad (\bar{a} \bar{b}) = \bar{a}_{\dot{\alpha}} \bar{b}^{\dot{\beta}} = -\bar{a}^{\dot{\beta}} b_{\dot{\alpha}} .$$

(13)

The scalar product of two four-vectors $a$ and $b$ takes the
form $a_\mu b^\mu = \frac{1}{2} g_{\alpha\beta} \tilde{b}^{\alpha\beta} .$

For convenience, we present the expressions for Dirac matrices in the spinor basis:

$$\gamma^\mu = \begin{pmatrix} 0 & \sigma^\mu_{a\dot{a}} \\ \bar{\sigma}^\mu_{\dot{a}a} & 0 \end{pmatrix} , \quad \gamma_5 = \begin{pmatrix} 0 & \sigma_5^{\alpha\dot{\alpha}} \\ \bar{\sigma}_5^{\dot{\alpha}\alpha} & 0 \end{pmatrix} ,$$

(14)

where $\gamma_5 = i \gamma^0 \gamma^1 \gamma^2 \gamma^3$. More expressions and various
useful identities can be found in [34, 55].

A covariant generalization of the decomposition in
terms of light-cone coordinates and a transverse plane
(14) can be found by observing that any light-like vector
that can be represented as a product of two spinors. In
particular one can parameterize the light-like vectors $n$ and
$\bar{n}$ as follows

$$n_{a\dot{\alpha}} = \lambda_{\alpha} \bar{\lambda}_{\dot{\alpha}} , \quad \bar{n}_{a\dot{\alpha}} = \mu_{\alpha} \bar{\mu}_{\dot{\alpha}} ,$$

(15)

where $\lambda = \lambda^\dagger, \bar{\mu} = \mu^\dagger$. The standard convention $(n\bar{n}) = 1$
corresponds to the normalization of auxiliary spinors

$$(\mu \lambda) = (\bar{\lambda} \bar{\mu}) = \sqrt{2} .$$

(16)

The basis vectors in the plane transverse to $n, \bar{n}$ can be
chosen as $\mu_{\alpha} \bar{\lambda}_{\dot{\alpha}}$ and $\lambda_{\alpha} \bar{\mu}_{\dot{\alpha}}$ and an arbitrary four-vector $x$
represented as

$$x_{a\dot{\alpha}} = z \lambda_{\alpha} \bar{\lambda}_{\dot{\alpha}} + \bar{z} \mu_{\alpha} \bar{\mu}_{\dot{\alpha}} + w \lambda_{\alpha} \bar{\mu}_{\dot{\alpha}} + \bar{w} \mu_{\alpha} \bar{\lambda}_{\dot{\alpha}} ,$$

(17)

where $z$ and $\bar{z}$ are real and $w, \bar{w} = w^\dagger$ complex coordinates
in the two light-like directions and the transverse plane,
respectively, cf. Eq. (14). In particular the spin vectors $s_\mu$ and $\bar{s}_\mu$ take the form

$$s_{a\dot{\alpha}} = \frac{1}{2} \begin{pmatrix} \lambda_{\alpha} \bar{\mu}_{\dot{\alpha}} s_{\mu\lambda} + \mu_{\alpha} \bar{\lambda}_{\dot{\alpha}} s_{\lambda\bar{\mu}} \\ \lambda_{\alpha} \bar{\mu}_{\dot{\alpha}} s_{\mu\lambda} - \mu_{\alpha} \bar{\lambda}_{\dot{\alpha}} s_{\lambda\bar{\mu}} \end{pmatrix} ,$$

$$\bar{s}_{a\dot{\alpha}} = -i \frac{1}{2} \begin{pmatrix} \lambda_{\alpha} \bar{\mu}_{\dot{\alpha}} s_{\mu\lambda} + \mu_{\alpha} \bar{\lambda}_{\dot{\alpha}} s_{\lambda\bar{\mu}} \\ \lambda_{\alpha} \bar{\mu}_{\dot{\alpha}} s_{\mu\lambda} - \mu_{\alpha} \bar{\lambda}_{\dot{\alpha}} s_{\lambda\bar{\mu}} \end{pmatrix} .$$

(18)
where $s_{\mu \lambda} = \mu^\alpha s_{\alpha \lambda} \tilde{\chi}^\alpha \text{def} (\mu s \lambda)$.

The “+−” fields (“good components”) are defined as the projections onto $\lambda$:

$$\psi_+ = \lambda^\alpha \psi_\alpha, \quad \chi_+ = \lambda^\alpha \chi_\alpha, \quad f_{++} = \lambda^\alpha \lambda_\beta f_{\alpha \beta},$$
$$\bar{\psi}_+ = \bar{\lambda}^\alpha \bar{\psi}_\alpha, \quad \bar{\chi}_+ = \bar{\lambda}^\alpha \bar{\chi}_\alpha, \quad \bar{f}_{++} = \bar{\lambda}^\alpha \bar{\lambda}^\beta \bar{f}_{\alpha \beta}.$$

The scale dependence of the correlation functions [4], [5] is determined by the renormalization properties of the corresponding light-ray operators [1], [2]. In order to make a connection to the existing results, it is convenient to go over, for quark-antiquark-gluon operators, to another basis [41].

$$S^\pm_{\rho}(z) = g\bar{q}(z_1) [F_{\rho +}(z_2) \pm i\gamma_5 \bar{F}_{\rho +}(z_2)] q(z_3). \quad (19)$$

Here and below we use a shorthand notation $q(z_3) \equiv q(z_3 n)$ etc., and also $S^\pm_{\rho}(z) \equiv S^\pm_{\rho}(z_1, z_2, z_3)$. The operators $S^\pm_{\rho}$ contribute to the structure function $g_{2}(x, Q^2)$ for polarized deep-inelastic scattering and their renormalization properties are studied in much detail [11], [12], [13], [14], [15], [16]. It is easy to check that

$$\hat{s}^p T_\rho(z) = \frac{1}{2} \hat{s}^p \left( S^+_{\rho}(z) + S^-_{\rho}(z) \right),$$
$$\hat{s}^p \Delta T_\rho(z) = -\frac{1}{2} \hat{s}^p \left( S^+_{\rho}(z) - S^-_{\rho}(z) \right). \quad (20)$$

The second identity is a consequence of the known representation for the product of two antisymmetric $\epsilon$-tensors as the determinant of the $4 \times 4$ matrix in terms of the metric tensors $g^{\mu \nu}$.

In the spinor basis one finds for the operators $S^\pm_{\alpha \dot{\alpha}} = S^\pm_{\rho} \sigma^\rho_{\alpha \dot{\alpha}}$ the following expressions:

$$S^+_{\alpha \dot{\alpha}}(z) = 2g \left[ \tilde{\lambda}_\alpha \bar{\psi}_+(z_1) f_{+ \alpha}(z_2) \psi_+(z_3) + \lambda_\alpha \chi_+(z_1) \bar{f}_{+ \alpha}(z_2) \bar{\chi}_+(z_3) \right],$$
$$S^-_{\alpha \dot{\alpha}}(z) = 2g \left[ \tilde{\lambda}_\alpha \chi_+(z_1) f_{+ \alpha}(z_2) \bar{\chi}_+(z_3) + \lambda_\alpha \bar{\psi}_+(z_1) \bar{f}_{+ \alpha}(z_2) \psi_+(z_3) \right]. \quad (21)$$

Taking into account transformation properties of the quark and gluon fields under charge conjugation,

$$C \psi C^{-1} = \chi, \quad \bar{C} \bar{\psi} C^{-1} = \bar{\chi},$$
$$C f C^{-1} = -f^T, \quad \bar{C} f C^{-1} = -\bar{f}^T, \quad (22)$$

where $f = f^{\alpha \mu}$, one derives

$$C S^+_{\rho}(z_1, z_2, z_3) C^{-1} = S^-_{\rho}(z_3, z_2, z_1). \quad (23)$$

Since operators of different C-parity do not mix under renormalization, it is convenient to introduce the C-even and C-odd combinations

$$\mathcal{S}^\pm(z) = S^+(z_1, z_2, z_3) \pm S^-(z_3, z_2, z_1), \quad (24)$$

where $S^\pm = \hat{s}^p S^\pm_{\rho}$. Note that these are not the same combinations as the ones appearing in Eq. [20] since the quark and antiquark field coordinates are interchanged. The expressions for the operators $\mathcal{S}^\pm$ in terms of chiral fields read

$$\mathcal{S}^\pm(z) = -\frac{ig}{\sqrt{2}} \left\{ s_{\mu \lambda} Q^\pm(z) - s_{\lambda \mu} \bar{Q}^\pm(z) \right\}, \quad (25)$$

where

$$Q^\pm(z) = \bar{\psi}_+(z_1) f_{++}(z_2) \psi_+(z_3) \pm \chi_+(z_3) f_{++}(z_2) \bar{\chi}_+(z_1),$$
$$\bar{Q}^\pm(z) = \chi_+(z_1) f_{++}(z_2) \bar{\chi}_+(z_3) \mp \bar{\psi}_+(z_2) f_{++}(z_2) \psi_+(z_1).$$

It is easy to see that $\bar{Q}^\pm(z) = \pm[Q^\pm(z)]^\dagger$, so that the C-even (“plus”) and C-odd (“minus”) $\mathcal{S}$-operators are hermitian and antihermitian, respectively, i.e. $(\mathcal{S}^\pm(z))^\dagger = \pm \mathcal{S}^\pm(z)$.

The operators $\mathcal{S}^\pm$ have autonomous evolution for the flavor-nonsinglet sector, whereas in the singlet sector they mix with three-gluon operators $\mathcal{F}^\pm$ of the same C-parity:

$$\mathcal{F}^\pm(z) = -\frac{ig}{\sqrt{2}} \left\{ s_{\mu \lambda} f_{++}(z_1) f_{++}(z_2) f_{++}(z_3) - s_{\lambda \mu} f_{++}(z_1) f_{++}(z_2) f_{++}(z_3) \right\}. \quad (26)$$

The color factors $C^{abc}_{\pm}$ are given in Eq. [23]. In the vector notation this definition corresponds to

$$\mathcal{F}^\pm(z) = 2g C^{abc}_{\pm} g^{p q} (1 \mp P_{23} \mp P_{12}) \times F_{\pm \nu}(z_1) F^p_{+ \nu}(z_2) F^q_{+ \nu}(z_3), \quad (27)$$

where $P_{23}$ and $P_{12}$ are the permutation operators acting on the field coordinates, e.g.

$$P_{23} F_{\pm \nu}(z_1) F^p_{+ \nu}(z_2) F^q_{+ \nu}(z_3) = F_{\pm \nu}(z_1) F^q_{+ \nu}(z_2) F^p_{+ \nu}(z_3) \quad (28)$$

etc.

Taking the nucleon matrix elements of the operators $\mathcal{S}^\pm$ and $\mathcal{F}^\pm$ one obtains the corresponding parton correlation functions in momentum space

$$\langle P, s T | \mathcal{S}^\pm(x) | P, s T \rangle = 2P_+^\dagger \int Dx e^{-iP_+ \cdot \Sigma x_k z_k} \mathcal{S}^\pm(x),$$
$$\langle P, s T | \mathcal{F}^\pm(x) | P, s T \rangle = 2P_+^\dagger \int Dx e^{-iP_+ \cdot \Sigma x_k z_k} \mathcal{F}^\pm(x). \quad (29)$$

Here $x = (x_1, x_2, x_3)$, the integration measure $Dx$ is defined in Eq. [25]. We use the same notation for the correlation functions as for the corresponding operators which, hopefully, will not result in a confusion. Note that although definitions of the operators $\mathcal{S}^\pm$ and $\mathcal{F}^\pm$, Eqs. [24] and [20], involve the spin-vector $s^\mu$, the dependence on $s^\mu$ actually drops out in the matrix elements [20].

Hermiticity and C-invariance imply that (cf. [17])

$$(\mathcal{S}^\pm(x))^* = \mp \mathcal{S}^\mp(-x), \quad \mathcal{S}^\pm(x) = \pm \mathcal{S}^\mp(-x),$$
$$(\mathcal{F}^\pm(x))^* = \mp \mathcal{F}^\mp(-x), \quad \mathcal{F}^\pm(x) = \mp \mathcal{F}^\mp(-x). \quad (30)$$
This means, in particular, that the correlation functions \( \mathcal{S}^\pm(x) \) and \( \mathcal{F}^\pm(x) \) are real functions. Notice also that the function \( \mathcal{F}^- \) is symmetric and \( \mathcal{F}^+ \) antisymmetric under the interchange of the last two arguments:

\[
\mathcal{F}^\pm(x_1, x_2, x_3) = \mp \mathcal{F}^\pm(x_1, x_3, x_2).
\]

The six correlation functions defined in Eq. (5) (or, equivalently, Eq. (4)) can be written in terms of two independent quark-antiquark-gluon functions \( \mathcal{S}^\pm(x) \) and two three-gluon functions \( \mathcal{F}^\pm(x) \) as follows:

\[
\begin{align*}
T_{qF}(x) &= \frac{1}{4} \left[ (1 + P_{13}) \mathcal{S}^+(x) + (1 - P_{13}) \mathcal{S}^-(x) \right], \\
\Delta T_{qF}(x) &= -\frac{1}{4} \left[ (1 - P_{13}) \mathcal{S}^+(x) + (1 + P_{13}) \mathcal{S}^-(x) \right], \\
T_{3F}^\pm(x) &= \frac{1}{2} (\mp P_{13}) \mathcal{F}^\pm(x), \\
\Delta T_{3F}^\pm(x) &= -\frac{1}{2} (\mp P_{13}) \mathcal{F}^\pm(x).
\end{align*}
\]

Here \( T_{qF}(x) \equiv T_{qF}(x_1, x_2, x_3) \), etc., and \( P_{ik} \) are the permutation operators for the corresponding momentum fractions, e.g. \( P_{12} \mathcal{F}^\pm(x_1, x_2, x_3) \equiv \mathcal{F}^\pm(x_2, x_1, x_3) \). As follows from (31) the functions \( T_{3F}(x) \) and \( \Delta T_{3F}(x) \) are not independent,

\[
\Delta T_{3F}^\pm(x_1, x_2, x_3) = \pm \left[ T_{3F}^\pm(x_1, x_3, x_2) - T_{3F}^\pm(x_2, x_1, x_3) \right]
\]

(32)

It follows from (31) and (32) that the correlation functions satisfy the following symmetry relation

\[
\begin{align*}
T_{qF}(x_1, x_2, x_3) &= T_{qF}(-x_3, -x_2, -x_1), \\
\Delta T_{qF}(x_1, x_2, x_3) &= -\Delta T_{qF}(-x_3, -x_2, -x_1), \\
T_{3F}^\pm(x_1, x_2, x_3) &= T_{3F}^\pm(-x_3, -x_2, -x_1), \\
\Delta T_{3F}^\pm(x_1, x_2, x_3) &= -\Delta T_{3F}^\pm(-x_3, -x_2, -x_1).
\end{align*}
\]

(33)

Authors of (27) also introduce symmetrized quark-antiquark-gluon parton distributions

\[
\begin{align*}
\bar{T}_{q,F}(x, x') &= \frac{1}{2} \left( \tilde{T}_{q,F}(x, x') + \tilde{T}_{q,F}(x', x) \right), \\
\bar{T}_{\Delta q,F}(x, x') &= \frac{1}{2} \left( \tilde{T}_{\Delta q,F}(x, x') - \tilde{T}_{\Delta q,F}(x', x) \right).
\end{align*}
\]

(34)

As follows from Eq. (33) such a symmetrization is not necessary since \( \tilde{T}_{q,F}(x, x') = \tilde{T}_{q,F}(x', x) \) and \( \Delta \tilde{T}_{q,F}(x, x') = -\Delta \tilde{T}_{q,F}(x', x) \). In the expressions given below we drop the “tilde” notation for these functions.

III. FLAVOR-NONSINGLET EVOLUTION

The flavor-nonsinglet light-ray operators \( \mathcal{S}^+ \) and \( \mathcal{S}^- \) satisfy the same evolution equation,

\[
\left( \mu \frac{\partial}{\partial \mu} + \beta(g) \frac{\partial}{\partial g} + \frac{\alpha_s}{2\pi} \mathbb{H} \right) \mathcal{S}^\pm = 0,
\]

(35)

where \( \mathbb{H} \) is an integral operator. The explicit expression for \( \mathbb{H} \) (at one-loop) can be restored from the corresponding result for \( \mathcal{S}^\pm \) in (41) (see also (43) (46) (47)). Expanding this equation at short distances \( z_i \to 0 \) one reproduces the mixing matrix for the twist-three local operators (30) (41) (48) (49).

Alternatively, the answer for \( \mathbb{H} \) can be obtained by simple algebra from the known expressions for the one-loop two-particle kernels (31) (32). This technique is general and applicable to arbitrary twist-three (and twist-four (32)) evolution equations. In the next Section we will use this approach for a more complicated case of flavor-singlet operators so it makes sense to explain the details on the present (simpler) example.

The starting observation is that to one-loop accuracy any contribution to the evolution can only involve two partons; hence \( \mathbb{H} \) can be represented as a sum of two-particle kernels. Schematically

\[
\mathbb{H} \psi_i(t) \psi_j(t') = i\epsilon_{ab}[\phi^{1a}, \psi^{b}_{++} + \psi^{b}_{++} + \psi^{b}_{++} + \psi^{b}_{++} + \psi^{b}_{++} + \psi^{b}_{++}]
\]

(36)

where the contractions correspond to the sum of relevant Feynman diagrams (in light-cone gauge). The corresponding expressions were derived originally by Bukhvostov, Frolov, Lipatov and Kuraev (BFLK) (31). The complete list of the BFLK kernels for arbitrary chiral fields is given in Ref. (55) so that it only remains to contract the color indices. For the reader’s convenience we collect all the kernels in Table 1 in the Appendix.

After a simple algebra one gets

\[
\mathbb{H} = N_c \mathbb{H}_0 - \frac{1}{N_c} \mathbb{H}_1 - 3C_F,
\]

(37)

where, in notation of Ref. (32),

\[
\begin{align*}
\mathbb{H}_0 &= \mathcal{H}_{12} + \mathcal{H}_{23} - 2\mathcal{H}_{12}, \\
\mathbb{H}_1 &= \mathcal{H}_{13} - \mathcal{H}_{13} - P_{23} \mathcal{H}_{23}^{(1)} + 2\mathcal{H}_{12}.
\end{align*}
\]

(38)

Here \( \mathcal{H}_{ik} \) are two-particle integral operators that act on the light-cone coordinates of the \( i \)-th and \( k \)-th partons:

\[
[\mathcal{H}_{ik} \phi](z_i, z_k) = \int dz'_i dz'_k H(z_i, z_k|z'_i, z'_k) \phi(z'_i, z'_k).
\]

(39)

These kernels are \( SL(2) \)-invariant and depend on the conformal spins of partons that they are acting on. The corresponding values are \( j = 1 \) for quarks and \( j = 3/2 \) for gluons (for the “plus” components) so one has to use \( j_1 = 1, j_2 = 3/2, j_3 = 1 \). One obtains, for example,

\[
[\mathcal{H}_{13} \mathcal{S}^+](z_1, z_2, z_3) = \int_0^1 d\bar{\alpha} \int_0^{\bar{\alpha}} d\beta \mathcal{S}^+(z_{13}^\alpha, z_2, z_{31}^\beta),
\]

where

\[
z_{ik}^\alpha = z_i \bar{\alpha} + z_k \alpha, \quad \bar{\alpha} = 1 - \alpha.
\]
Taking the nucleon matrix element of Eq. (35) one obtains the evolution equation for the corresponding parton distribution function. Technically, this corresponds to going over from coordinate to the momentum fraction space \( \{ z_1, z_2, z_3 \} \rightarrow \{ x_1, x_2, x_3 \} \). Thanks to energy conservation, two-particle kernels in momentum fraction space can be written in the following generic form

\[
\mathcal{H}_{ik}(x, x_k) = \int_{-\infty}^{\infty} dx' \mathcal{H}(x, x_k | x', x'_k) \varphi(x', x'_k),
\]

where \( \mathcal{H}(x, x_k | x', x'_k) \) is our final result. The “Hamiltonian” \( \mathcal{H} \) is defined in Eqs. (37), (38). Explicit expressions for the two-particle kernels \( \mathcal{H}_{ik}, \mathcal{H}_{ik}^{+}, \mathcal{H}_{ik}^{-(1)}, \mathcal{H}_{ik}^{-} \) are given in the Appendix. Note that when applying two-particle kernels to a three-parton parton distribution one has to treat the latter as a function of three independent variables; the condition \( x_1 + x_2 + x_3 = 0 \) is applied afterwards.

The last step, using the first two relations in Eq. (31) we obtain

\[
\mu \frac{d}{d\mu} T_{qFq}(x) = -\frac{\alpha_s}{4\pi} \left( \mathbb{H} + P_{13} \bar{H}_{13} \right) T_{qFq}(x) + \frac{\alpha_s}{4\pi} \left( \mathbb{H} - P_{13} \bar{H}_{13} \right) \Delta T_{qFq}(x),
\]

\[
\mu \frac{d}{d\mu} \Delta T_{qFq}(x) = -\frac{\alpha_s}{4\pi} \left( \mathbb{H} + P_{13} \bar{H}_{13} \right) \Delta T_{qFq}(x) + \frac{\alpha_s}{4\pi} \left( \mathbb{H} - P_{13} \bar{H}_{13} \right) T_{qFq}(x),
\]

where \( \Delta T_{qFq}(x) \) comes from support properties of the kernels and the parton distributions; the corresponding expressions are collected in the Appendix.

The partonic interpretation of different contributions in Eq. (33) is illustrated in Fig. 3. Note that the condition

\[
\mu \frac{d}{d\mu} T_{qFq}(x) = dx' dx'_k \delta(x_1 + x_2 - x'_1 - x'_2).
\]

It is assumed that restrictions on integration regions over \( x'_1, x'_2 \) come from support properties of the kernels and the parton distributions; the corresponding expressions are collected in the Appendix.
tion of zero gluon momentum $x_2 = 0$ corresponds to the choice of $\vec{x}$ on the (positive) horizontal axis. The first term $\sim T_{q,F}(\xi,\xi)$ on the r.h.s. of Eq. (43) corresponds to the integration over the horizontal line segment $xD$ shown in red in Fig. 3. The terms in $T_{q,F}(x,\xi)$ are due to the integration along the blue, $AB$, and green, $EF$, lines in the regions $1^+ (23)^-$ and $(12)^+3^-$, and the ones in $T_{q,F}(x,\xi)$ correspond to the contributions along the same lines in the regions $2^+ (13)^-$ and $(13)^+2^-$, respectively. By construction, the function $T_{q,F}(x,x')$ is symmetric and $T_{\Delta q,F}(x,x')$ antisymmetric under the interchange of arguments, $x \leftrightarrow x'$. In Fig. 3 this corresponds to a reflection symmetry around the horizontal axis.

The evolution equation (43) differs from the corresponding result by Kang and Qiu (see Eq. (99) in [27]) by the two extra terms in the second line. The last term, proportional to $1/N_c$, originates from the kernels $P_{23}\mathcal{H}_{23}^{c,(1)}$ and $\mathcal{H}_{23}$ in Eq. (45). On the diagrammatic level this contribution corresponds to the “exchange” diagrams of the type shown in Fig. 4 which correspond to mixing of the regions that have different partonic interpretation: $2^+ (13)^- \leftrightarrow 1^+ (23)^-$, or $(13)^+2^- \leftrightarrow (12)^+3^-$. Similar contributions have been discussed in a somewhat different context in Ref. [28].

The other difference is the extra term $-N_c T_{q,F}(x,x)$ in the second line in Eq. (43) for which we do not see any obvious explanation. The origin of this term in our calculation can be traced to the kernels $\mathcal{H}_{12}$ and $\mathcal{H}_{23}$. E.g. the second term in the expression for $\mathcal{H}_{12}$, Eq. (A.13), gives

$$
\mu \frac{d}{d\mu} T_{q,F}(x, x + x_2) \xrightarrow{x_2 \to 0^+} = \sum \frac{\alpha_s N_c}{4\pi} \int_{x_2}^{\infty} \frac{x_2 dx_2}{x_2 (x_2 - x_2')} \left[ T_{q,F}(x, x + x_2) - \frac{x_2}{x_2'} T_{q,F}(x, x + x_2') \right].
$$

Because of an overall $x_2$ factor, it is tempting to put this contribution to zero in the $x_2 \to 0$ limit. However, it is easy to see that in the same limit the integral becomes linearly divergent so that at the end a finite contribution arises. This term can easily be missed if the gluon momentum is put to zero at the beginning of the calculation.

It is instructive to analyze Eq. (43) in the large-$N_c$ limit. Neglecting $1/N_c$ terms in (43) one is left with

$$
\mu \frac{d}{d\mu} T_{q,F}(x, x) = \frac{\alpha_s N_c}{2\pi} \left\{ -T_{q,F}(x, x) + \int_{x}^{1} \frac{d\xi}{\xi} \left[ (\hat{P}_{qq}(z) + z) T_{q,F}(x, \xi) - T_{\Delta q,F}(x, \xi) \right] \right\},
$$

(46)

where $\hat{P}_{qq}(z)$ is obtained from $P_{qq}(z)$, Eq. (44), omitting the $C_F$ factor. One sees that the scale dependence of $T_{q,F}(x, x)$ is determined by $T_{q,F}(x, \xi)$ in the region $x \leq \xi \leq 1$, which corresponds to the contribution of the blue and green line segments $xB$ and $xF$ in Fig. 3. It receives no contribution from the “diagonal” region $T_{q,F}(\xi, \xi)$ (the red line segment $xD$ in Fig. 3), the corresponding contributions cancel out between the first and the second terms in Eq. (43) (to the $O(1/N_c)$ accuracy). By this reason the conclusion in Ref. [27] that the evolution of the function $T_{q,F}(x, x)$ mainly follows a pattern determined by the quark splitting function can be misleading.

The evolution does simplify, however, in the large $x$ limit in which case the integration regions shrink to a point. One obtains

$$
\mu \frac{d}{d\mu} T_{q,F}(x, x) = \frac{\alpha_s N_c}{\pi} \int_{x}^{1} \frac{d\xi}{\xi} P_{q,F}^{NS,z-1}(z) T_{q,F}(\xi, \xi),
$$

(47)

where, retaining singular terms at $z \to 1$ only

$$
P_{q,F}^{NS,z-1}(z) = 2C_F \left[ \frac{1}{(1-z)_+} + \frac{3}{4} \delta(1-z) \right] - N_c \delta(1-z).$$

(48)

This result can be compared with the evolution of the usual $F_1(x, Q^2)$ structure function which involves, to the same accuracy

$$
P_{qq}^{NS,z-1}(z) = 2C_F \left[ \frac{1}{(1-z)_+} + \frac{3}{4} \delta(1-z) \right],
$$

(49)

and the twist-three contribution to the structure function $g_2(x, Q^2)$ (42, 46)

$$
P_{g_2}^{NS,z-1}(z) = 2C_F \left[ \frac{1}{(1-z)_+} + \frac{3}{4} \delta(1-z) \right] - \frac{N_c}{2} \delta(1-z).
$$

(50)

The last term in $\delta(1-z)$ is written in the large-$N_c$ limit. The contributions $\sim 1/(1-z)_+$ are the same in all three cases, which indicates that all three functions $T_{q,F}(x, x; Q^2)$, $F_1(x, Q^2)$ and $g_2^{u,w,q}(x, Q^2)$ may have the same functional dependence on the Bjorken variable $x$ in the $x \to 1$ limit. Different terms $\sim \delta(1-z)$ suggest, on the other hand, that twist-three functions are suppressed at large scales $Q^2$ compared to the twist-two distribution.
where\
\[ H \]
the evolution equation takes the matrix form
\[ F \]
the function
\[ T \]
exactly corresponds to the gap between the lowest anomalous dimension in the spectrum of twist-three operators and the usual twist-two anomalous dimension. For the function \( T_{q,F}(x, Q^2) \) we predict a stronger suppression which translates to scaling violation in SSA. This result can be phenomenologically relevant.

**IV. FLAVOR-SINGLET EVOLUTION**

In the flavor-singlet sector one has to take into account mixing between the quark-antiquark-gluon and three-gluon operators with the same C-parity. Namely, \( G^+ \) gets mixed with \( F^+ \) and \( G^- \) with \( F^- \). For each case, the evolution equation takes the matrix form
\[
\left( \mu \frac{\partial}{\partial \mu} + \beta(g) \left( \frac{\partial}{\partial g} + \frac{\alpha_s(Q)}{4\pi} \right) \right) \left( \mathbb{H}^\pm \right) = 0 ,
\]
where
\[
\mathbb{H}^\pm = \begin{pmatrix} \mathbb{H}^\pm_{QQ} & \mathbb{H}^\pm_{QF} \\ \mathbb{H}^\pm_{FQ} & \mathbb{H}^\pm_{FF} \end{pmatrix}.
\]

In what follows we assume that the quark-antiquark-gluon flavor-singlet operator is defined including the sum over \( n_f \) light flavors, \( qFq = ¯uFud + \ldots \).

For the diagonal entries we obtain
\[
\mathbb{H}^+_{QQ} = \mathbb{H} + 4n_f P_{qq}^d , \\
\mathbb{H}^-_{QQ} = \mathbb{H} ,
\]
where \( \mathbb{H} \) is given by Eq. (37) and
\[
\mathbb{H}^\pm_{FF} = N_c \left( \hat{H}_{12} + \hat{H}_{23} + \hat{H}_{31} - 4(\hat{H}_{12} + \hat{H}_{13}) \right) \\
- 2(\hat{H}_{12} + \hat{H}_{13}) \pm 6(\hat{H}_{12} + \hat{H}_{13}) - b_0 ,
\]
with \( b_0 = \frac{11}{3} N_c - \frac{2}{3} n_f \). The off-diagonal entries in coordinate space take the form
\[
\mathbb{H}^\pm_{QF} = -in_f z_{13} \left( \mathcal{H}^+_{11} + \hat{\mathcal{H}}^+_{13} + 2\mathcal{H}_{13} \right) ,
\]
\[
\mathbb{H}^{+\mp}_{FQ} = i N_c (1 - P_{23}) \frac{1}{z_{13}} \left[ 2\mathcal{H}_{13}P_{13} + 1 \right] P_0 ,
\]
\[
\mathbb{H}^{\mp+}_{FQ} = -i \frac{N_c^2 - 4}{N_c} (1 + P_{23}) \frac{1}{z_{13}} \left[ 2\mathcal{H}_{13}P_{13} - 1 \right] .
\]

The corresponding expressions in momentum space are given in the Appendix, Eqs. (A.25) - (A.30).

The evolution equations for conventional \( T_{qFq}, \Delta T_{qFq}, T\hat{3}_{F}(x) \Delta T\hat{3}_{F} \) can readily be obtained from Eq. (52) by symmetrization in the arguments, as specified in Eq. (51).

In the limit of zero gluon momentum our result for the evolution of \( T_{q,F}(x, x) \equiv T_{qFq}(x, 0, 0) \) differs from the corresponding expression in Eq. (107) of [27] by the same two terms as in the nonsinglet case; the terms in \( P_{qg} \) (in our calculation due to \( \mathbb{H}^{QF} \)) coincide.

To compare our results for three-gluon distributions we introduce the functions
\[
T^\pm_F(x, x') = \frac{1}{x} T^\pm_{3F}(x' - x, x') , \\
\Delta T^\pm_F(x, x') = \frac{1}{x} \Delta T^\pm_{3F}(x' - x, x') ,
\]
which coincide with \( T^{(f)}_{G,F}(x, x'), T^{(d)}_{G,F}(x, x'), T^{(f)}_{\Delta G,F}(x, x'), T^{(d)}_{\Delta G,F}(x, x') \) defined in [27], respectively. We remind that the \( \Delta T^\pm_F(x, x') \) distributions and hence \( T^{(f)}_{\Delta G,F}(x, x'), T^{(d)}_{\Delta G,F}(x, x') \) can be expressed in terms of \( T^\pm_F(x, x') \) alias \( T^{(f)}_{G,F}(x, x'), T^{(d)}_{G,F}(x, x') \), so that they do not need to be considered separately. After some algebra one obtains the following equations for \( T^\pm_F(x, x) \):

\[
\frac{d}{d\mu} T^\pm_F(x, x) = \frac{\alpha_s N_c}{\pi} \left( -T^\pm_F(x, x) + \int_x^1 \frac{d\xi}{\xi} \left[ 2P_{gg}(z)T^\pm_F(\xi, \xi) + \frac{z}{1-z} \left[ T^\pm_F(\xi, x) - T^\pm_F(\xi, \xi) \right] \right] \right. \\
- (1 - z) \left( z + \frac{1}{z} \right) T^\pm_F(\xi, \xi) + \left. \frac{1 + z}{2} \left[ T^\pm_F(x, \xi) - \Delta T^\pm_F(x, \xi) \right] \right) \\
+ \frac{1}{2} (1 - z) \left[ T^\pm_F(x, x - \xi) - \Delta T^\pm_F(x, x - \xi) \right] + \frac{1}{2} \Delta T^\pm F_{gg}(z) \left( T_{q,F}(\xi, \xi) \mp T_{q,F}(\xi, -\xi) \right) ,
\]
We note in passing that the particular ansatz proposed
leading as they are modified by the evolution themselves.

\[ A^+ = 1, \quad A^- = \frac{N_c^2 - 4}{N_c^2} \]  

(59)

and

\[ P_{gg}(z) = \frac{z}{(1-z)_+} + \frac{1-z}{z} + z(1-z) + \frac{b_0}{4N_c} \delta(1-z), \]

\[ P_{gg}(z) = \frac{1 + (1-z)^2}{z}. \]  

(60)

Our result does not agree with that of Kang and Qiu [27]. Eqs. (109), (110) in an overall sign in front of the
\[ T_{G,F}^{(f,d)}(x,x) \pm \int_x^1 \frac{d\xi}{\xi}(T_{G,F}^+(x,x) - \Delta T_{G,F}^+(x,x)) \]

(61)

which seem to have the same origin as the extra contributions that we also have for the flavor-nonsinglet distributions:
The first term in (61) originates from contributions of the type in Eq. (55) that involve a subtlety in taking the
\[ (x_2 \rightarrow 0 \text{ limit}, \text{ and the second one corresponds to the contribution of } \text{“exchange” \ type diagrams, the kernel } H_{12} \text{ in (53)}, \text{ that give rise to mixing of regions with different partonic interpretation.} \]

Closing this section, we want to stress that the scale dependence has to be studied using complete evolution equations for the three-particle parton distributions, Eq. (62). Using the gluon-pole projected equations [58] with a certain ansatz for the “off-diagonal” correlation functions \[ T_{G,F}^{(f,d)}(x,x') \equiv T_{A}^{(f,d)}(x,x') \] may be misleading as they are modified by the evolution themselves. We note in passing that the particular ansatz proposed in [27]

\[ T_{G,F}^{(f,d)}(x_1,x_2) = \frac{1}{2} \left[ T_{G,F}^{(f,d)}(x_1,x_1) \right. \]

\[ + T_{G,F}^{(f,d)}(x_2,x_2) \left. e^{-(x_1-x_2)^2/2\sigma^2} \right] \]

(62)

and \[ T_{\Delta G,F}^{(f,d)}(x_1,x_2) = \Delta T_{G,F}^+(x',x' - x,x) = 0, \] is inconsistent with the constraint [32].

V. CONCLUSIONS

We have given a complete reanalysis of the scale dependence of twist-three three-particle correlation functions that are relevant for calculations of single transverse spin asymmetries in the framework of collinear factorization. The calculation is done using the two-particle kernels for the renormalization of light-ray operators in the spinor basis, which are available from Ref. [35]. Evolution equations are derived for arbitrary parton momentum fractions, for the flavor-nonsinglet quark-antiquark-gluon distribution, Eqs. (63), (67), (68), and for the mixing matrix of the flavor-singlet quark-antiquark-gluon and three-gluon distributions with both positive and negative C-parity, Eqs. (69)–(72) and (A.25)–(A.30). Specializing to the case of zero gluon momentum we have compared our results with the recent calculation in Ref. [27]. There are two terms where we disagree, and their origin could be identified. As a byproduct of our calculation we predict logarithmic scaling violation in the SSA at large values of Bjorken x which may be phenomenologically relevant. Numerical studies of the evolution effects on realistic models of parton distributions will be considered elsewhere.

Acknowledgements

We are grateful to Yu. Koike, K. Tanaka and V. Vogelsang for correspondence which initiated this study and useful comments. This work was supported by the German Research Foundation (DFG), grant 9209282, RNP grant 2.1.1/1575 and the RFFI grants 07-02-92166, 09-01-93108.

APPENDICES

APPENDIX A: BFLK KERNELS IN MOMENTUM REPRESENTATION

Gauge-invariant N-particle quasipartonic light-ray operators can be defined as a product of “plus” fields

\[ O(z_1, \ldots, z_N) = CX(z_1) \otimes \cdots \otimes X(z_N) \equiv \]

\[ \equiv C_{i_1 \ldots i_N} \left[ (0, z_1) X(z_1) \right]^{i_1} \cdots \left[ (0, z_N) X(z_N) \right]^{i_N} \]

where \( X(z_k) = \{ \psi, \bar{\psi}, \chi, \bar{\chi}, f_{++}, \bar{f}_{++} \}, [0, z_k] \) are Wilson lines in the appropriate representation of the gauge group, \( i_1, \ldots, i_N \) are color indices and \( C_{i_1 \ldots i_N} \) is an invariant color tensor such that

\[ [(t_1)_{k_1 i_1} + (t_2)_{k_2 i_2} + \cdots + (t_N)_{k_N i_N}] S_{i_1 \ldots i_N} = 0. \]

(2.2)

Here and below it is implied that the generators \( t^a \) are taken in the appropriate representation,

\[ t^a X = \left\{ \begin{array}{ll}
(t^a \psi)^i = T^a_{ii'} \psi^{i'} & \text{for quarks } \psi, \bar{\psi} \\
(t^a \bar{\psi}) = -T^a_{ii'} \bar{\psi}^{i'} & \text{for antiquarks } \bar{\psi}, \chi \\
(t^a f)^b = i t^{ab'} f^{b'} & \text{for gluons } f, \bar{f}
\end{array} \right. \]

(2.3)

where \( T^a \) are the generators in fundamental representation. The condition in Eq. (A.2) ensures that \( O(z_1, \ldots, z_N) \) is a color singlet.
For each \( N \), the set of quasipartonic operators with the same quantum numbers is closed under renormalization \[31\]. A renormalized quasipartonic operator is written as

\[
[O_i(X)]_R = Z_{ik} O_k(X_0) ,
\]

where \( X_0 = Z_X X \) is the bare field. Renormalized operators satisfy the RG equation

\[
\left( \mu \frac{\partial}{\partial \mu} + \beta(g) \frac{\partial}{\partial g} + \gamma_{ik} \right) [O_k(X)]_R = 0 .
\]

Here \( \beta(g) \) is the (QCD) beta function and

\[
\gamma = -\mu \frac{d}{d\mu} Z Z^{-1}
\]

is the matrix of anomalous dimensions. To the one-loop accuracy one obtains in dimensional regularization \( D = 4 - 2\epsilon \)

\[
Z = I + \frac{\alpha_s}{4\pi\epsilon} H \quad \text{and} \quad \gamma = \frac{\alpha_s}{2\pi} H .
\]

The operator \( H \) (Hamiltonian) is given by the sum of two-particles kernels, \( H_{ik}^{(2-2)} \)

\[
H^{(N-N)} = \sum_{i,k} H_{ik}^{(2-2)} .
\]

The general structure of the kernels is

\[
H_{12}^{(2-2)} [X^{i_1}(z_1) \otimes X^{i_2}(z_2)] =
\sum_q \sum_i [C_{i_1 i_2} \tilde{H}_{12}^{(q)} X^{i_1} \otimes X^{i_2}](z_1, z_2) .
\]

Here \( [C_{i_1 i_2}] \) is a color tensor, \( \tilde{H}_{12}^{(q)} \) is an \( SL(2, \mathbb{R}) \) invariant operator which acts on coordinates of the fields, and \( q \) enumerates different structures.

Explicit expressions for the two-particle (BFLK) kernels in the light-cone gauge in coordinate space are given in Table I \[32\]. We tacitly assume existence of \( n_f \) quark flavors; the kernels \( B^{NS} \) and \( B^S \) correspond to the flavor-nonsinglet and flavor-singlet quark pairs, respectively. In the kernel \( B^S \) the generators \( \tau^a \) should be taken \( \tau^a = T^a \) for the \( \psi \otimes \bar{\psi} \) pair, and \( \tau^a = (-T^a)^t \) for the \( \chi \otimes \bar{\chi} \) pair.

The kernels in Table I are written in terms of several "standard" \( SL(2) \)-invariant operators defined as

\[
[\hat{H}^\alpha \varphi](z_1, z_2) = \int_0^1 \frac{d\alpha}{\alpha} \left[ 2\varphi(z_1, z_2) - 2(2\alpha_j - 1)\varphi(z_{12}^{\alpha_j}, z_{12}^j) \right],
\]

\[
[\hat{H}^d \varphi](z_1, z_2) = \int_0^1 \frac{d\alpha}{\alpha} \left[ 2\varphi(z_1, z_2) - 2(2\alpha_j - 1)\varphi(z_{12}^{\alpha_j}, z_{12}^j) \right],
\]

\[
[\hat{H}^+ \varphi](z_1, z_2) = \int_0^1 \frac{d\alpha}{\alpha} \left[ 2\varphi(z_1, z_2) - 2(2\alpha_j - 1)\varphi(z_{12}^{\alpha_j}, z_{12}^j) \right],
\]

\[
[\hat{H}^{-} \varphi](z_1, z_2) = \int_0^1 \frac{d\alpha}{\alpha} \left[ 2\varphi(z_1, z_2) - 2(2\alpha_j - 1)\varphi(z_{12}^{\alpha_j}, z_{12}^j) \right],
\]

\[
[\hat{H}_{12}^{(c,k)} \varphi](z_1, z_2) = \int_0^1 \frac{d\alpha}{\alpha} \left[ 2\varphi(z_1, z_2) - 2(2\alpha_j - 1)\varphi(z_{12}^{\alpha_j}, z_{12}^j) \right],
\]

where in the last line it is assumed that \( 0 < k < 2j_1 \). Here and below we use a shorthand notation \( z_{ik}^\alpha = z_i^\alpha + z_k^\alpha \), \( \bar{\alpha} = 1 - \alpha \), \( z_{12} = z_1 - z_2 \).

The kernels depend on the conformal spins of the partons they are acting on, \( j = 1 \) for quarks and antiquarks and \( j = 3/2 \) for gluons. Another shorthand notation is

\[
J^a(z_1, z_2) =
\sum_A \left[ \tilde{\psi}^A_+(z_1) T^a \psi^A_+(z_2) + \chi^A_+(z_1) T^a \bar{\chi}^A_+(z_2) \right],
\]

\[
J^{ac}(z_1, z_2) =
\sum_A \left[ \tilde{\psi}^A_+(z_1) T^a T^c \psi^A_+(z_2) - \chi^A_+(z_1) T^c T^a \bar{\chi}^A_+(z_2) \right],
\]

where \( A \) is the flavor index; the sum runs over all possible flavors. The constants

\[
\sigma_q = \frac{3}{4}, \quad \sigma_g = b_0/4N_c,
\]

where \( b_0 = \frac{11}{4} N_c - \frac{7}{2} n_f \), correspond to the "plus" quark field and transverse gluon field renormalization in the axial gauge

\[
Z_q = 1 + \frac{\alpha_s}{2\pi\epsilon} \sigma_q C_F, \quad Z_g = 1 + \frac{\alpha_s}{2\pi\epsilon} \sigma_g C_A .
\]

Finally, \( P_{12} \) and \( P_{ac} \) stand for permutation operators in position and color space, respectively. For example

\[
P_{12} J^{ac}(z_1, z_2) = J^{ac}(z_2, z_1), \quad P_{ac} J^{ac}(z_1, z_2) = J^{ca}(z_1, z_2) .
\]

Going over from the light-ray operator renormalization to evolution equations for parton distributions corresponds to the Fourier transformation of the functions
TABLE I: Summary of the BFLK kernels \[35\].

\[\mathcal{H}^d, \mathcal{H}^e,(k), \tilde{\mathcal{H}}, \mathcal{H}^+, \tilde{\mathcal{H}}^+, \mathcal{H}^- \] from coordinate to the momentum fraction representation.

A generic two-particle kernel in momentum space has the form

\[ [\mathcal{H} \varphi](u_1, u_2) = \int_{-\infty}^{\infty} Dv \mathcal{H}(u_1, u_2|v_1, v_2) \varphi(v_1, v_2), \tag{A.14} \]

where \(Dv = dv_1 dv_2 \delta(u_1 + u_2 - v_1 - v_2)\). In this Appendix we use the letters \( u, v \) for momentum fractions. Note that the integration regions over \( v_1 \) and \( v_2 \) in Eq. \(\text{(A.14)}\) are (formally) infinite; in practice they are constrained by the support properties of parton distributions and also (Heaviside) step-functions that are present in the kernels. For later convenience we introduce a notation

\[ \Theta(a_1, \ldots, a_n) = \prod_{k=1}^{n} \Theta(a_k) - \prod_{k=1}^{n} \Theta(-a_k). \tag{A.15} \]

We remind that the kernels depend on the conformal spins of fields they act on, cf. Eq. \(\text{(A.9)}\), so that, in principle, each one has to carry a pair of indices \((j_1, j_2)\) which we omitted for brevity.

We obtain

\[ \mathcal{H}^d_{12}(u|v) = \frac{\Theta(u_1, u_2)}{u_1 + u_2} \left( \frac{u_1}{u_1 + u_2} \right)^{2j_1 - 1} \left( \frac{u_2}{u_1 + u_2} \right)^{2j_2 - 1}, \tag{A.16} \]

\[ \mathcal{H}^e,(k)(u|v) = \frac{\Theta(u_1, u_1 - u_1)}{u_1} \left( \frac{u_1}{u_1 - u_1} \right)^{2j_1 - k} \left( 1 - \frac{u_1}{u_1} \right)^{k-1}. \tag{A.17} \]

\[ [\tilde{\mathcal{H}}_{12}]^e (u|v) = \int Dv \left\{ \frac{u_1}{v_1} \Theta(u_1, v_1 - u_1) \left( \varphi(u) - \left( \frac{u_1}{v_1} \right)^{2j_2 - 2} \varphi(v) \right) \right\}. \tag{A.18} \]

Note that \(\mathcal{H}^d_{12} \[\text{(A.16)}\] vanishes in the DGLAP region.

For the remaining kernels the expressions for arbitrary conformal spins become too lengthy, so we specialize to the particular cases of interest.

The operator \(\mathcal{H}^+\) enters the BFLK kernels in Table \(\text{I}\) with the values of conformal spins \((j_1, j_2) = \{(1, 1), (3/2, 1), (1, 3/2), (3/2, 3/2)\}\}. In momentum fraction space one obtains

\[ \mathcal{H}^+,(j_1, j_2)(u|v) = \Theta(-u_1, u_2, u_1 - u_1) A^{(j_1, j_2)}(u|v) + \Theta(u_1, u_2, v_2 - u_2) B^{(j_1, j_2)}(u|v) + \Theta(u_1, u_1, v_1 - u_1) C^{(j_1, j_2)}(u|v). \tag{A.19} \]

The first term contributes to the DGLAP region only, the other two — to the ERBL region.
One derives the following expressions:

\[
A^{(1,1)}(u,v) = \frac{v_1 - u_1}{v_1 v_2}, \\
B^{(1,1)}(u,v) = \frac{u_2}{v_2 (u_1 + u_2)}, \\
C^{(1,1)}(u,v) = \frac{u_1}{v_1 (u_1 + u_2)}, \\
A^{(2,1)}(u,v) = \frac{1}{2} \left( \frac{v_1^2 - u_1^2}{v_1^2 v_2} \right), \\
B^{(2,1)}(u,v) = \frac{u_2 + 2u_1}{2 v_2 (u_1 + u_2)}, \\
C^{(2,1)}(u,v) = \frac{1}{2} \left( \frac{v_1^2 + u_1 u_2}{v_1 (u_1 + u_2)} \right), \\
A^{(2,2)}(u,v) = \frac{1}{2} \left( \frac{v_1 - u_1}{v_1 v_2} \right) \left( v_1 u_2 + u_1 v_2 - \frac{1}{3} (v_1 - u_1)^2 \right), \\
B^{(2,2)}(u,v) = \frac{1}{2} \left( \frac{u_2}{v_2} \right)^2 \frac{1}{u_1 + u_2} \left( 1 + \frac{u_1 v_2 - u_2 v_1}{u_1 + u_2} \right), \\
C^{(2,2)}(u,v) = \frac{1}{2} \left( \frac{u_1}{v_1} \right)^2 \frac{1}{u_1 + u_2} \left( 1 + \frac{u_2 v_1 - u_1 v_2}{u_1 + u_2} \right) \left( u_1^2 + u_2^2 \right). \\
\] (A.20)

Obviously

\[
H_{+(1,3/2)}(u_1, u_2 | v_1, v_2) = H_{+(3/2,1)}(u_2, u_1 | v_1, v_2).
\]

The “modified” \( \tilde{H}^+ \) kernel is needed for the conformal spins \((j_1, j_2) = (3/2, 3/2)\) only. It is given by a similar expression

\[
\tilde{H}^{+(j_1, j_2)}(u,v) = \Theta(-u_1, u_2, v_2 - v_1) A^{(j_1, j_2)}(u,v) + \Theta(u_1, u_2, v_2 - u_1) B^{(j_1, j_2)}(u,v) + \Theta(u_1, u_2, v_1 - u_1) C^{(j_1, j_2)}(u,v)
\]

(A.21)

with

\[
A(u,v) = \frac{(u_1 - v_1) v_2}{6 v_1^2 v_2^2}, \\
B(u,v) = \frac{1}{2} \left( \frac{v_2}{v_1} \right)^2 \frac{u_1 v_2 - v_1 u_2}{u_1 + u_2}, \\
C(u,v) = \frac{1}{2} \left( \frac{u_1}{v_1} \right)^2 \frac{u_2 v_1 - v_2 u_1}{u_1 + u_2} \left( u_1^2 + u_2^2 \right).
\] (A.22)

Note that the factors \(A, B, C\) (and, similar, \(\tilde{A}, \tilde{B}, \tilde{C}\)) satisfy the relation

\[
A^{(j_1, j_2)} - B^{(j_1, j_2)} + C^{(j_1, j_2)} = 0.
\]

The kernels \(H^+(u,v)\) and \(\tilde{H}^+(u,v)\) are continuous functions in the whole domain although they are given by different expressions in the DGLAP and ERBL regions.

The kernel \(H^-\) has a different structure of regions, namely

\[
H^-(u,v) = \Theta(u_1, -u_2, v_2 - v_1) D(u,v) + \Theta(u_1, v_2, -u_1) E(u,v) + \Theta(u_1, u_2, v_1 - v_2) F(u,v).
\] (A.23)

For the spins of interest \((j_1, j_2) = \{(3/2, 1), (3/2, 3/2)\}\) one gets

\[
D^{(3/2,1)}(u,v) = \frac{1}{2} \frac{(u_2 - v_1)}{v_1}, \\
E^{(3/2,1)}(u,v) = \frac{1}{2} \frac{(u_1)}{v_1}, \\
F^{(3/2,1)}(u,v) = \frac{u_2}{2 v_1} \frac{v_1 - u_2 v_2}{2 v_1^2}, \\
D^{(3/2,3/2)}(u,v) = \frac{1}{6} \frac{(u_2 - v_1)}{(u_1 + u_2)^3}, \\
E^{(3/2,3/2)}(u,v) = \frac{1}{2} \frac{(u_1)}{v_1} \frac{u_2 v_2 - v_1 u_1/3}{(u_1 + u_2)^3}, \\
F^{(3/2,3/2)}(u,v) = \frac{1}{2} \frac{(u_2)}{v_1} \frac{u_1 v_1 - u_2 v_2/3}{(u_1 + u_2)^3}.
\] (A.24)

We also give here the explicit expressions for the off-diagonal kernels in momentum representation. The \(H^{\pm}_{Q\bar{Q}}\)–kernel can be written as

\[
H^{\pm}_{Q\bar{Q}}(u,v) = n_f \left( V^+_{13}(u,v) + V^-_{13}(u,v) \right),
\] (A.25)

where \(V^+_{13}(u,v)\) and \(V^-_{13}(u,v)\) have the decomposition over different regions of the form \((A.19)\) and \((A.22)\), respectively. For the corresponding functions \(A_V, B_V, C_V\) and \(D_V, E_V, F_V\) one obtains

\[
A_V(u,v) = \frac{u_1 u_3}{v_1 v_3^2}, \\
B_V(u,v) = \frac{u_1 u_3}{v_1 v_3} \frac{v_1 + 3v_3}{(u_1 + u_3)^3}, \\
C_V(u,v) = \frac{u_1 u_3 v_3}{v_1^2 (u_1 + u_3)^3}, \\
D_V(u,v) = \frac{(u_3 - v_1)^2}{v_1^2 v_3^2}, \\
E_V(u,v) = -\frac{u_1^2}{v_1^2 (u_1 + u_3)^2} \left[ \frac{2u_3 v_3}{u_1 (u_1 + u_3)} - 1 \right], \\
F_V(u,v) = -\frac{u_1^2}{v_1^2 (u_1 + u_3)^2} \left[ \frac{2u_1 v_1}{u_3 (u_1 + u_3)} - 1 \right].
\] (A.26)

The \(H^{\pm}_{Q\bar{Q}}\)–kernel can be written as

\[
H^{\pm}_{Q\bar{Q}} = N_c (1 - P_{23}) \left[ W^+ + W^- - 2\Delta W \right],
\]

\[
H^{\pm}_{Q\bar{Q}} = -\frac{N_c^2 - 4}{N_c} (1 + P_{23}) \left[ W^+ + W^- \right].
\] (A.27)

Here

\[
\Delta W(u,v) = \Theta(u_3, v_3 - u_3) - \Theta(u_1, u_3) \frac{u_1^2 (3u_2 + u_1)}{(u_1 + u_3)^3}.
\] (A.28)

The kernel \(W^+(u_1, u_3 | v_1, v_3)\) has the structure \((A.19)\) with

\[
A_W(u,v) = 1, \quad B_W(u,v) = -C_W(u,v) = \frac{1}{2}.
\] (A.29)
and the kernel $W^-(u_1, u_3|v_1, v_3)$ takes the form (A.23) with

$$D_W(u|v) = -\frac{(v_1 - u_3)^2}{v_1 v_3},$$

$$E_W(u|v) = \frac{1}{2} - \frac{u_1^2}{v_3(u_1 + u_2)},$$

$$F_W(u|v) = \frac{1}{2} + \frac{u_3^2}{v_1(u_1 + u_2)}.$$  \hfill (A.30)

The kernels $W^\pm, \Delta W$ are antisymmetric under $(u_1, v_1) \leftrightarrow (u_2, v_2)$. Note that the kernels in (A.27) have discontinuities on the DGLAP–ERBL boundaries.

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