Stability of strange stars (SS) under radial oscillation.

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A realistic Equation of State (EOS) leads to strange stars (ReSS) which are compact in the mass radius plot, close to the Schwarzschild limiting line \( [1] \). We carry out a stability analysis under radial oscillations and compare with the EOS of other SS models. We find that the ReSS is stable and an M-R region can be identified to that effect.

I. INTRODUCTION

The radial mode oscillation, being the simplest mode of neutron star has been considered first to be investigated more than 35 years ago \([2]\). It can give information about stability of the stellar object under consideration.

The radial modes of neutron star have been studied thoroughly by many authors for cold nuclear matter EOS \([3, 4, 5, 6, 7]\). With this radial modes have been investigated for other type of star, namely Strange Star \([8]\) protoneutron star \([9]\), Hybrid Star \([10]\).

We here present our analysis of radial mode oscillation for Realistic Strange Star (ReSS) Equation of State (EOS).

II. RADIAL OSCILLATIONS OF A RELATIVISTIC STAR

Thirty five years ago Chandrasekhar \([2]\) investigated these radial modes. Following him we investigate the same for ReSS.

The spherically symmetric metric is given by the line element

\[
\mathrm{d}s^2 = -e^{2\nu} \mathrm{d}t^2 + e^{2\mu} \mathrm{d}r^2 + r^2 (\mathrm{d}\theta^2 + \sin^2 \theta \mathrm{d}\phi^2).
\]

Together with the energy-momentum tensor for a perfect fluid, Einstein’s field equations yield the Tolman-Oppenheimer-Volkoff (TOV) equations which can be solved if we have an EOS, \( p(n_B) \) and \( \epsilon(n_B) \). Given the central density \( \epsilon_c \), we can arrive at an \( M - R \) curve by solving the TOV. Without disturbing the spherical symmetry of the background we define \( \delta r(r, t) \), a time dependent radial displacement of a fluid element located at the position \( r \) in the unperturbed model which assumes a harmonic time dependence, as

\[
\delta r(r, t) = u_n(r)e^{i\omega_n t}.
\]

The dynamical equation governing the stellar pulsation in its \( n \)th normal mode (\( n = 0 \), is the fundamental mode) has the Sturm-Liouville’s form (for details, see \([11]\)).

\[
P(r) \frac{d^2 u_n(r)}{dr^2} + \frac{dP}{dr} \frac{du_n}{dr} + [Q(r) + \omega_n^2 W(r)] u_n(r) = 0,
\]

where \( u_n(r) \) and \( \omega_n \) are the amplitude and frequency of the \( n \)th normal mode, respectively. The functions \( P(r) \), \( Q(r) \) and \( W(r) \) are expressed in terms of the equilibrium configuration of the star and are given by

\[
P(r) = \frac{\Gamma p}{r^2} e^{\mu + 3\nu}
\]

\[
Q(r) = e^{\mu + 3\nu} \left[ \frac{(p')^2}{r^2(\epsilon + p)} - \frac{4p'}{r^3} - \frac{8\pi}{r^2} (\epsilon + p) p e^{2\mu} \right]
\]
\[ W = \frac{(\epsilon + p)}{r^2} e^{\lambda u + \nu}, \]  

(6)

where the varying adiabatic index \( \Gamma \) is given by

\[ \Gamma = \frac{(\epsilon + p)}{p} \frac{dp}{d\epsilon}, \]  

(7)

\( \epsilon \) and \( p \) being the energy density and pressure of the unperturbed model, respectively. Eigenfrequencies can be obtained with the boundary conditions,

1. at the centre \( r = 0, \delta r = 0 \) and

2. at the surface \( \delta p = 0 \) leading to \( \Gamma p u(r)' = 0 \).

Since \( \omega \) is real for \( \omega^2 > 0 \), the solution is oscillatory. However for \( \omega^2 < 0 \), the angular frequency \( \omega \) is imaginary, which corresponds to an exponentially growing solution. This means that for negative values of \( \omega^2 \) the radial oscillations are unstable. For a compact star the fundamental mode \( \omega_0 \) becomes imaginary at some central density \( \epsilon_c \) less than the critical density \( \epsilon_{\text{critical}} \) for which the total mass \( M \) is a maximum. At \( \epsilon_c = \epsilon_0^0 \), \( \omega_0 \) vanishes. All higher modes are zero at even higher central densities. Therefore, the star is unstable for central densities greater than \( \epsilon_c^0 \). To illustrate, we plot the eigen frequencies \( \omega_n \) against \( \epsilon_c \), the central density in Fig. 1. The fundamental frequency \( \omega_0 \) does vanish at some \( \epsilon_c^0 \) while the higher modes remain nonzero.

III. DISCUSSIONS AND SUMMARY

ReSS are stable against radial oscillations close to the maximum attainable mass. For example, the EOS of SS1 sustains gravitationally, \( M_{\text{max}} \sim 1.4M_\odot \), \( R = 7 \) km with a central number density \( n_c \sim 16n_0 \). However, the fundamental frequency of radial oscillations becomes zero at around \( n_c \sim 9.5 \) \( n_0 \), destabilizing the star after \( M = 1.36M_\odot \) with \( R = 7.24 \) km (Table I). It is still on the \( \frac{dM}{dR} > 0 \) region. Thus the maximum mass star which is stable against radial oscillations has a number density \( \sim 9.5n_0 \) at the centre and \( \sim 4.7n_0 \) at the surface. Macroscopically, upto this density small vibrations may be sustained.

![Graph](image.png)

**FIG. 1:** Angular frequency of three different modes against central density for SS1.

Numerical values of masses, radii, central densities and the corresponding eigen frequencies \( \omega_0, \omega_1 \) and \( \omega_2 \) are given in Tables IV and V for EOS1 and EOS3 respectively (SS1 and SS2 of Dey et al. 1998). Tables IV and V are for the bag model EOS with different parameters.
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### TABLE I: Data for EOS1 (SS1)

| $\rho_1 \times 10^{14}$ g/cc | $n_1/n_0$ | $M/M_\odot$ | $\omega_0 \times 10^3$ km/sec | $\omega_1 \times 10^3$ km/sec | $\omega_2 \times 10^3$ km/sec |
|-----------------------------|-----------|--------------|-------------------------------|-------------------------------|-------------------------------|
| 14.85                       | 5.462     | 0.407        | 5.262                         | 90.661                        | 199.957                       |
| 15.85                       | 5.798     | 0.502        | 5.611                         | 79.884                        | 179.851                       |
| 16.85                       | 6.122     | 0.787        | 5.940                         | 69.870                        | 162.018                       |
| 17.85                       | 6.429     | 0.893        | 6.643                         | 47.528                        | 125.267                       |
| 18.85                       | 6.735     | 0.991        | 6.828                         | 40.784                        | 114.643                       |
| 19.85                       | 7.036     | 1.077        | 6.970                         | 34.751                        | 105.455                       |
| 20.85                       | 7.321     | 1.133        | 7.050                         | 30.694                        | 99.662                        |
| 21.85                       | 7.605     | 1.182        | 7.113                         | 26.848                        | 94.509                        |
| 22.85                       | 7.886     | 1.226        | 7.161                         | 23.077                        | 89.825                        |
| 23.85                       | 8.159     | 1.261        | 7.193                         | 19.749                        | 86.070                        |
| 24.85                       | 8.427     | 1.288        | 7.214                         | 16.693                        | 83.005                        |
| 25.85                       | 8.692     | 1.312        | 7.228                         | 13.435                        | 80.192                        |
| 26.85                       | 8.955     | 1.333        | 7.236                         | 9.648                         | 77.588                        |
| 27.85                       | 9.212     | 1.349        | 7.240                         | 4.943                         | 75.483                        |
| 28.85                       | 9.466     | 1.363        | 7.240                         | 5.899                         | 73.592                        |
| 30.85                       | 9.969     | 1.381        | 7.235                         | –                             | 70.168                        |
| 32.85                       | 10.233    | 1.433        | 7.130                         | –                             | 64.144                        |
| 35.85                       | 11.176    | 1.417        | 7.194                         | –                             | 105.945                       |
| 40.85                       | 12.333    | 1.433        | 7.130                         | –                             | 100.169                       |
| 46.85                       | 13.669    | 1.437        | 7.055                         | –                             | 95.361                        |

### TABLE II: Data for EOS3 (SS2)

| $\rho_1 \times 10^{14}$ g/cc | $n_1/n_0$ | $M/M_\odot$ | $\omega_0 \times 10^3$ km/sec | $\omega_1 \times 10^3$ km/sec | $\omega_2 \times 10^3$ km/sec |
|-----------------------------|-----------|--------------|-------------------------------|-------------------------------|-------------------------------|
| 17.17                       | 6.067     | 0.423        | 5.070                         | 86.879                        | 195.416                       |
| 18.17                       | 6.382     | 0.539        | 5.460                         | 74.016                        | 172.966                       |
| 19.17                       | 6.695     | 0.659        | 5.794                         | 63.013                        | 153.699                       |
| 20.17                       | 7.006     | 0.781        | 6.078                         | 53.079                        | 136.745                       |
| 21.17                       | 7.298     | 0.855        | 6.227                         | 47.332                        | 127.643                       |
| 22.17                       | 7.588     | 0.923        | 6.351                         | 42.074                        | 119.663                       |
| 23.17                       | 7.876     | 0.986        | 6.453                         | 37.131                        | 112.402                       |
| 24.17                       | 8.156     | 1.036        | 6.524                         | 33.069                        | 106.694                       |
| 25.17                       | 8.428     | 1.075        | 6.575                         | 29.646                        | 102.152                       |
| 26.17                       | 8.699     | 1.110        | 6.615                         | 26.321                        | 98.030                        |
| 27.17                       | 8.967     | 1.142        | 6.647                         | 22.992                        | 94.193                        |
| 28.17                       | 9.227     | 1.167        | 6.667                         | 20.163                        | 91.206                        |
| 29.17                       | 9.485     | 1.188        | 6.682                         | 17.335                        | 88.518                        |
| 30.17                       | 9.741     | 1.207        | 6.693                         | 14.307                        | 86.066                        |
| 31.17                       | 9.994     | 1.224        | 6.691                         | 10.827                        | 83.642                        |
| 32.17                       | 10.242    | 1.237        | 6.702                         | 6.982                         | 81.741                        |
| 33.17                       | 10.488    | 1.249        | 6.703                         | 3.351                         | 79.959                        |
| 35.17                       | 10.974    | 1.270        | 6.698                         | –                             | 76.694                        |
| 40.17                       | 12.148    | 1.301        | 6.650                         | –                             | 70.680                        |
| 45.17                       | 13.278    | 1.316        | 6.622                         | –                             | 66.380                        |
| 50.17                       | 14.371    | 1.323        | 6.573                         | –                             | 63.141                        |
| 55.17                       | 15.537    | 1.352        | 6.518                         | –                             | 60.616                        |
### TABLE III: Data for bag model with $B=60$ & $m_s=150$

| $\rho_c \times 10^{14}$ g/c.c. | $n_c/n_0$ | $M/M_\odot$ | R km | $\omega_0 \times 10^3$ sec$^{-1}$ | $\omega_1 \times 10^3$ sec$^{-1}$ | $\omega_2 \times 10^3$ sec$^{-1}$ |
|-----------------|-----------|-------------|-------|-----------------|-----------------|-----------------|
| 6.20            | 2.421     | 0.691       | 8.549 | 38.964          | 92.549          | 142.426         |
| 7.20            | 2.778     | 1.019       | 9.544 | 27.409          | 73.557          | 114.644         |
| 8.20            | 3.122     | 1.240       | 10.021| 20.733          | 63.870          | 100.636         |
| 9.20            | 3.454     | 1.393       | 10.263| 15.915          | 57.854          | 92.024          |
| 10.20           | 3.776     | 1.501       | 10.393| 11.910          | 53.693          | 86.161          |
| 11.20           | 4.089     | 1.581       | 10.452| 7.900           | 50.628          | 81.883          |
| 12.20           | 4.396     | 1.639       | 10.460| 2.159           | 48.272          | 78.626          |
| 13.20           | 4.695     | 1.683       | 10.462| 6.156           | 46.395          | 76.048          |
| 15.20           | 5.277     | 1.741       | 10.405| -               | 43.526          | 72.203          |
| 17.20           | 5.839     | 1.775       | 10.321| -               | 41.429          | 69.514          |
| 23.70           | 7.560     | 1.805       | 10.012| -               | 37.316          | 64.404          |

### TABLE IV: Data for bag model with $B=75$ & $m_s=150$

| $\rho_c \times 10^{14}$ g/c.c. | $n_c/n_0$ | $M/M_\odot$ | R km | $\omega_0 \times 10^3$ sec$^{-1}$ | $\omega_1 \times 10^3$ sec$^{-1}$ | $\omega_2 \times 10^3$ sec$^{-1}$ |
|-----------------|-----------|-------------|-------|-----------------|-----------------|-----------------|
| 9.83            | 3.573     | 1.072       | 8.923 | 24.809          | 73.494          | 115.482         |
| 10.83           | 3.892     | 1.198       | 9.148 | 20.030          | 67.175          | 106.403         |
| 11.83           | 4.203     | 1.293       | 9.281 | 16.097          | 62.623          | 99.944          |
| 12.83           | 4.506     | 1.366       | 9.356 | 12.565          | 59.194          | 95.089          |
| 13.83           | 4.804     | 1.422       | 9.396 | 9.063           | 56.480          | 91.314          |
| 14.83           | 5.095     | 1.467       | 9.412 | 4.502           | 54.280          | 88.272          |
| 15.83           | 5.318     | 1.502       | 9.411 | 4.887           | 52.470          | 85.769          |
| 20.83           | 6.748     | 1.594       | 9.294 | -               | 46.505          | 77.834          |
| 25.83           | 8.031     | 1.622       | 9.123 | -               | 43.109          | 73.610          |
| 28.83           | 8.769     | 1.626       | 9.020 | -               | 41.638          | 71.868          |