Magnetic Foehn Effect in Nonadiabatic Transition

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The magneticization curves as a response of sweeping magnetic field in the thermal environment are investigated using the quantum master equation. In a slow velocity region where the system alm ost behaves adiabatically, the magnetic plateau appears which has been observed in the recent experiment of \( V_{15} \) [Phys. Rev. Lett. (2000)]. We investigate this mechanism and propose that this phenomenon is quite universal in the quasi-adiabatic transition with small and low of the heat, and we call it "Magnetic Foehn Effect". We observe the crossover between this mechanism and the Landau-Zener-Stueckelberg mechanism changing the velocity. Some experiment is proposed to clarify the inherent mechanism of this effect.

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Properties of quantum dynamics have been studied extensively for nanoscale magnets and also in microscopic systems with nanostructure. There the nonadiabatic transition plays in prominent roles. Landau [3], Zener [3], and Stueckelberg [3] (LZS) derived the well-known nonadiabatic transition probability for a two-level system with a sweeping field. It depends on the sweeping velocity and energy gap. Although the LZS formula is derived in two-level system, it can also describe nonadiabatic transitions at the avoided crossings in the uniaxial system. Therefore, it can be widely applied to analyze phenomena related to the nonadiabatic transition in various materials [3]. However, real experiments are always done in a thermal environment. Hence the studies on the nonadiabatic transition with an environment are quite crucial for further understanding of the ependent phenomena in magnetic systems [3]. There universal aspects of them all are independent of details of reservoirs which are very important. As an example of such universal aspects, we have studied the magnetization process in uniaxial molecule magnets such as \( Mn_2 \) and \( Fe_2 \) at very low temperatures in thermal environments. There the step-wise magnetization process is observed, which is due to the nonadiabatic transition and a fast damping process to the ground state. We call it apparent (or deceptive) nonadiabatic process. We also find that the transition probability of purely quantum mechanics is deducible from this process. This mechanism does not depend on the detailed structure of reservoir and fluctuation of noise at the resonant fields.

In this Letter, we propose another universal qualitative aspect of the thermal effect on the quasi-adiabatic transition where the LZS probability is almost one. We understand that the effect is quite general in systems which behave almost adiabatically in the dissipative environment. The present study is directly related to the recent experiment for the molecule \( V_{15} \) which is a defectively regarded as a two-level system [11]. This molecule has 15 atoms of \( V \) but they divided into subclusters of 6,3, and 6 spins. The subcluster of 6 spins forms a single state and contributes little to the magnetization and only the cluster of 3 spins mainly contributes to the magnetization. Therefore, the model can be regarded as a two-level system. This molecule is a very simple system and we may expect to see the LZS process clearly [12]. However, it was observed that the scattered population (i.e., that at the excited level) decreases when the sweeping velocity becomes faster. This is opposite to what we expect in view point of the LZS mechanism, where the probability of the adiabatic transition should decrease for faster change of the field and the population of the excited state should increase.

Chiorescu et al. [13] explained this behavior in the view of the phonon bottleneck effect which means a lack of phonon number which contributes to the excitation from the ground state at near resonant magnetic fields. In the experiment, the heat reservoir has a double-structure, i.e., the spin system is attached to the phonon system of the crystal, and the phonon system is attached to the external reservoir which is the liquid He. The contact between the phonon system and the external reservoir is so weak that in short time scale, the thermal field in the spin system is caused by only phonons. Due to this effect, the population of the excited state does not increase enough, and saturates at some value, which causes a magnetic plateau.

In this Letter, we show that plateau in the magnetization curve is also observed in the case that the spin system is connected with a single heat reservoir with slow relaxation rate, and propose that this qualitative property is universal with regard to the detailed structure of environment. We investigate a magnetization process for a sweeping field by making use of the quantum master equation which we have used in our studies of quantum dynamics in dissipative environments [14]. Thereby, we investigate the magnetization plateau for various sweeping velocities and temperatures.

The Hamiltonian we shall consider is given by

\[
H = H(t) S^z + \dot{S}^z;
\]  

where \( H(t) \) is the sweeping field, \( H(t) = vt \) and \( \dot{S}^z \) is the transverse field. Transverse field represents a term causing quantum fluctuation and does not commute with the magnetization \( S_z \). This simple system is realized in...
m any cases, e.g., the isotropic anti-ferromagnetic Heisenberg chain with odd number of spins has the doublet in the ground state. Actually $V_{15}$ is in this situation \[12\].

For this system \[1\], the LZS transition probability is given by,

$$P_{LZS}(v) = \frac{1}{2} \exp \left( \frac{-2}{v} \right);$$

(2)

in the case of $S = 1=2$ spin system \[3\]. Thus the normalized magnetization at $t = 1$ is given by,

$$M_{\text{out}} = 1 - 2P_{LZS}(v);$$

(3)

We should note that this expression of $M_{\text{out}}$ is also exact for any values for $S$ although \[3\] is derived for $S = \frac{1}{2}$.

We introduce a thermal environment taking the phonon system as the bath, $H_B = -\sum_i \omega_i b_i^\dagger b_i$, where $b_i$ and $b_i^\dagger$ are the annihilation and creation boson operators of the frequency $\omega_i$. We adopt the spectral density of the boson bath $I(\omega)$ in the form $I(\omega) = I_0(\omega)$ for $\omega > 0$; $0$ for $\omega < 0$, with $I_0 = 2$. In the experimental situation in the magnetic molecules such as $S = 10$ in $Mn_{12}$ and $Fe_{13}$, the hyper fine interaction and the dipole interaction are not negligible at very low temperatures \[13\]. In the case of $V_{15}$, the phonon gives a dominant contribution \[4\].

In the case of phonon bath, we can derive an equation of motion of the reduced density matrix tracing out the degree of freedom of the bath in the following form (the quantum master equation \[4\]):

$$\frac{\partial \rho(t)}{\partial t} = -i[H; \rho(t)] + \frac{1}{2} [R; \rho(t)] + [R; \rho(t)]^\dagger;$$

(4)

where $X$ is a system operator through which the system and the bath couple with the constant $\hbar$. The last term on the right-hand side describes the pure quantum dynamics of the system while the second term represents effects of environment at a temperature $T = 1$. There $R$ is defined as follows:

$$\rho_{jk} \rho_{in} = (E_k E_m) \rho_{jk} \rho_{in} + i \hbar [\rho_{jk}, \rho_{in}] ;
\rho(\omega) = I(\omega)^\dagger I(\omega) \text{ and } n(\omega) = (\omega + 1)^{\frac{1}{2}} ;$$

where $\rho_{jk}$ and $\rho_{in}$ represent the eigenstates of $H$ with the eigenenergies $E_j$ and $E_n$, respectively.

We simulate the evolution given by Eq. (4) for various sweeping velocities in $S=2$ spin system \[4\]. Throughout this Letter, we set $\hbar$ to be unity. From now on, we set parameters as $\beta = 0.5; T = 150$, and $v = 0.001$. In Fig. 1(a), we present the magnetization curves for fast sweeping rates, $v = 0.1; 0.2$, and $0.4$. Here we clearly see that the magnetic plateau decreases with $v$ increases, which is consistent with \[3\]. On the other hand, in the case of slow sweeping rates we also find the magnetic plateau as shown in Fig. 1(b) although in these sweeping rates the LZS transition probabilities \[3\] are almost one. Here we should note that the magnetic plateau increases when $v$ increases, which is an opposite property to the fast sweeping case. This is the same phenomenon as the experiment \[3\].

![Image](image-url)

**Fig. 1.** Magnetization process for (a) $v = 0.1; 0.2$, and $0.4$, and (b) $v = 0.001; 0.006$, and $0.01$.

In Fig. 2 we show the non monotonic dependence of the plateau height $M_{\text{out}}$ on velocities. Trivially when $v$ is very large, $M_{\text{out}}$ shows linear dependence on $1/v$ from Eq. \[3\]. In the slow sweeping rate region with $P_{LZS} = 1$, $M_{\text{out}}$ goes down.

![Image](image-url)

**Fig. 2.** Plateau height $M_{\text{out}}$ as a function of $1/v$. 

### References

1. [Refer to relevant literature](#).
2. [Refer to relevant literature](#).
3. [Refer to relevant literature](#).
4. [Refer to relevant literature](#).

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We associate the mechanism of magnetic plateau with the well-known Föhn phenomenon in meteorology. The air with the vapor climbs up the mountain getting colder adiabatically and next the rain ensues. At this moment the vapor gives the heat to the air as the latent heat. Then, the air alone goes down the mountain and the temperature of the air increases higher than the original due to the influence of the latent heat at the mountain. In the present magnetic system, ‘climbing up the mountain’ corresponds to sweeping of a magnetic field to \( H = 0 \), ‘the latent heat’ to ‘the heat from the phonon’, and ‘the increase of temperature of the air’ to ‘the increase of temperature of the spin’. After the plateau the magnetic relaxation to the equilibrium value due to cooling through the albatraz, which corresponds to the cooling of the air by the land after hot air reaches to the ground. Thus these similarities lead us to call this magnetic phenomenon ‘Magnetic Föhn effect’. We conclude that the magnetic Föhn effect is also observed in the Hamiltonian \( \mathcal{H} \) with larger \( S \). The essential mechanism of this phenomenon is in the wave of heat during adiabatic process. Because the mechanism is quite simple, we can say that the ‘Magnetic Föhn effect’ is a universal phenomenon in the magnetic systems which behave adiabatically.

Because the LJS transition occurs only in vicinity of \( H = 0 \) and at other points only relaxations due to the dissipation term occur, the time evolution process may be divided into three regions: (i) \( \mathcal{H}_0 (t:0) \) and \( \mathcal{H}_L (t:0) \), and (ii) \( \mathcal{H}_L (t:s) \). In order to realize a visible plateau, the relaxation rate must be small compared with \( v \). Actually, because of \( \mathcal{H}_L (t:0) \) and \( \mathcal{H}_L (t:s) \), the relaxation is slow at around the resonant point due to small. We also studied the system with \( H = 0 \), where we found that the plateau sustain for large values of \( v \) (i) and the shape of the magnetic relaxation process seems very different.

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\[ H = J \sum_i S_i^x S_{i+1}^x + S_i^y S_{i+1}^y + A S_i^z S_{i+1}^z \]  

(7)

where \( S_i \) is an \( S = 1/2 \) spin and the open boundary condition is adopted. The gap becomes small with \( A \), and decreases with \( N \) exponentially. Thus we expect that the crossover would be observed in some system and there we can obtain many informations of effects due to combination of the quantum process and the thermal excitation.

In \( V_{15} \), Chiorescu et al. attributes the magnetic Foehn phenomenon to the lack of phonon as a mechanism of the insufficient supply of heat. In such a situation, we would propose an experiment to realize the adiabatic transition of an isolated spin system. That is, we first sweep the field from the large negative value to the eddy (\( H_p \)) where the magnetic plateau (\( M_p \)) is observed. At this point the number of phonon is very small and supply of phonons from outside is expected to be small. In this circumstance, if the field is swept in the opposite direction from \( H_p \) to \( H_p \), the spins at the lower spin cannot be excited by the phonon because no phonon is available, and behaves pure quantum mechanically. In the experiment the LLS probability is almost one. The spins at the higher level may emit the phonon and relax to the lower level. But the excited phonon will be used to excite the spin again because the population of the upper level is much smaller than that of equilibrium. Thus we expect that the magnetization simply changes the sign when \( H_p \) \( H_p \). In the iteration of this process \( H_p \) \( H_p \) \( H_p \) \( H_p \), the magnetization would maintain the same amplitude for a while, 1 \( M_p \) \( M_p \) \( M_p \) \( M_p \), before the heat flows in from the external environment and equilibrates the system. From this slow relaxation of magnetization we could know the relaxation rate between the phonon and the external bath. On the other hand, in the magnetic Foehn phenomenon due to slow relaxation but not short of phonon number, \( M_p \) relaxes with the thermalization rate. From Eq. (6), we can derive the relation of 22 for the iteration: \( (n) = p_1 + p_2 \) \( 22 \) \( 22 \) \( 22 \) \( 22 \) with \( (0) = 0 \) and \( p_2 = (1) \). The sequence \( M_p \) \( (n) \) is given by \( M \) \( (n) = 1 \). \( 22 \) \( 22 \) \( 22 \) \( 22 \) \( 22 \) \( 22 \) where \( (n) \) \( (n) \) \( (n) \) \( (n) \) \( (n) \). Here \( p_2 \) is given by

\[ p_2 = \exp \left( \frac{Z_{H_p} = v}{d} \right) \coth \left( \frac{H_p}{v} \right) \] 

(8)

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**Fig. 4.** Dependence of the gap on the size of the system \( N \) and the anisotropy \( A \).