Deformability of Steel-Fiber Beams with External Tape Reinforcement

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Abstract. In 2013, the Lviv National Agrarian University patented the construction of a steel-fiber concrete beam, additionally reinforced with an external steel ribbon armature with end anchors. This technical solution allows reducing the cost of the structure by reducing the complexity of its manufacture, due to the absence of a reinforcing frame in the structure. The purpose of this work is to develop proposals for a refined calculation method of deflections of steel-fiber concrete beams with tape reinforcement on the basis of our experimental studies. 3 beams with dimensions 1500x150x60 mm were manufactured. Beam B-1, reinforced with steel tape 1500x60x3 mm with end stops, did not contain fiber. Beams BF-2 and BF-3 in addition to the specified reinforcement contained fiber, corresponding the coefficient of fiber reinforcement by volume, \( \rho_{fv} = 1.5\% \) i 2\% respectively. Compressed prisms of 400x100x100 mm and stretched samples of 700x100x60 mm were also tested to determine the mechanical characteristics of concrete and steel-fiber concrete. Loose beams with a working run of 1400 mm were tested with concentrated force applied along the middle of the length. Mass production fiber produced both in Ukraine and abroad with bent ends HE 1050 1 mm in diameter and 50 mm in length was used for reinforcement. The percentage of fiber reinforcement by volume of concrete beams was taken \( \rho_{fv} = 1.5\% \) and 2\% to provide enough bearing capacity of the inclined sections. The cement of grade 400 (activity 42.3 MPa) of the Ivano-Frankivsk plant was used to obtain C20 /25 concrete. The test samples were made of fine-grained concrete containing sand from the Yasinets quarry with a fineness modulus of not more than 2.5. Mixtures for beams were made in a forced mixer. The composition of the mixture was chosen so that the settling of the cone did not exceed 4-6 cm and that the fiber did not settle to the bottom of the form. The composition of the mixture per 1 m\(^3\) of the mixture was as follows: cement - 549 kg, sand - 1647 kg, water - 285.5 l. The deflections of the steel-concrete beams were smaller than the steel-concrete beams at equal moments. For example, at the moment of 210 kN cm the experimental deflection values of beams B-1, BF-2 and BF-3 were equal to 0.116; 0.081 and 0.064 cm, and at the moment 420 kN cm - 0.380; 0.213 and 0.140 cm, respectively. In this paper, it is proposed to determine deflections using the Mora integral and taking into account the results of the calculation of beams by deformation method. The ratio of theoretical and experimental values of deflections was equal to 0.93 ... 1.14.

1. Introduction
The construction of reinforced concrete beam, containing fibrous reinforcement and a steel strip with external anchor stops located outside the fibrebetical cross-section, allows, in certain cases, to reject...
the transverse reinforcement, which is a part of the frames [1]. This solution allows to reduce the complexity of manufacturing the structure, as there is no need for the manufacture of frames, and the role of the transverse reinforcement is performed by the fiber falling into the front of the inclined crack. In addition, the fiber reinforcement in normal sections perceives some of the tensile forces from the action of bending moments, which can reduce the thickness of the ribbon reinforcement.

In this work [1], the results of experimental studies of the bearing capacity of three beams with the specified tape reinforcement, two of which contained different amounts of fiber reinforcement, are presented. Positive influence of fiber reinforcement on bearing capacity of beams was revealed and it was suggested to determine it by deformation method using idealized deformation diagrams of steel fiber.

The deformation method of calculating both the load-bearing capacity of the fibrous concrete bending elements and the bends, as well as the width of the crack opening, has not yet become widespread, which can be explained by the peculiarities of the work of fiber concrete with different types of tensile fibers and difficulties in the analytical description of deformation diagrams [2, 3, 4, 5, 6]. Such circumstances, however, cannot be the reason for rejection of the deformation calculation methodology, which is stipulated by design rules [7, 8].

The purpose of this work is to experimentally determine the deflections of steel-fiber beams with ribbon reinforcement, deformation of concrete and ribbon reinforcement and to develop proposals for the method of its calculation.

2. Test program, designs of prototypes, materials and their mechanical characteristics
The research program provides for the production of three beams measuring 1500 x 150 x 60 mm (Figure 1, Table 1) [1].

A detailed description of the composition of concrete beams is given in the paper [1]. The fine-grained concrete had the following composition based on 1 m³: cement of grade 400 with the activity of 42.3 MPa - 549 kg, sand with the modulus of size no more than 2.5 - 1647 kg, water - 285.5 l.

Fragments of tape reinforcement made of C245 steel with a cross-section of 40x3 mm were tested on the MUP-20 tensile machine, which allowed to establish a yield strength σy = 245.32 MPa, a tensile strength σt = 372.53 MPa and a modulus of elasticity Es = 2.04 · 10^5 MPa.

Simultaneously with the beams, compressed prisms with dimensions 400x100x100 mm, stretched specimens measuring 700x100x60 mm, and cubes of concrete with dimensions 100x100x100 mm, were produced. Based on the results of the tests of these elements, the deformative and strength properties of concrete and steel-fibroconcrete were researched.
Table 1. Characteristics of the test beams

| Brand of the sample | Percentage of fiber reinforcement by volume $\rho_{fv, \%}$ | Average cube strength of concrete $f_{cm, \text{cube}}, \text{MPa}$ | Average prism strength of concrete (steel fiber concrete) $f_{cm, \text{prism}}, (f_{fcm, \text{prism}})$, MPa | Average tensile strength of concrete (steel fiber concrete) $f_{ctm}, (f_{fctm})$, MPa | The average value of the modulus of concrete (steel fiber concrete) $E_{cm} \cdot 10^{-3}, (E_{fcm} \cdot 10^{-3})$, MPa |
|---------------------|------------------------|-----------------|-----------------|-----------------|-----------------|
| B1                  | 0                      | 23,25           | 2,03            | 27,11           |
| BF1                 | 1,5                    | 29,01           | 2,49            | 30,19           |
| BF2                 | 2                      | 29,34           | 3,33            | 32,21           |

For disperse reinforcement, anchor type fibers with bent ends HE 1050 of the Ukrainian production of OJSC «Silur» with 1 mm diameter and 50 mm length were used (Figure 2). Beams BF-2 and BF-3 in addition to reinforcement with strip reinforcement contained fiber in the amount of 1.59 kg and 2.12 kg, which corresponds to $\rho_{fv}=1.5\%, 2\%$, respectively.

Figure 2. Fiber reinforcement with bent ends HE 1050

3. Results and discussions

The test beams were loaded by means of a hydraulic jack with concentrated force applied in the middle of the run in steps (Figure 3). Before the expected formation of cracks and the destruction, the degree of loading was reduced to 2-3% of the destructive load.

Impressions were recorded for 10-15 minutes between loading stages. The values of the loads at which cracks appeared were recorded, at each stage the width of their opening was measured with the help of the microscope MPB – 2 and their peaks were marked, the increments of deflections and the subsidence of the supports were fixed (Figures 4, 5). The behavior of load beams is described in detail in the paper [1].
With increasing load, there was an increase in the deformation of both the tape reinforcement and the compressed fibers of concrete. At the same values of bending moments, the deformation of reinforcement and concrete of fibro-concrete beams were smaller than at beams B-1. The increase in fiber content also contributed to the reduction of deformation (Table 2, Figure 6).

Figure 3. Beam test scheme [1]:
1, 2, 3 - microindicators for measuring the subsidence of supports and deflections; 4 - microindicator for measuring the deformation of extremely compressed fibers; 5 - microindicator for measuring deformations at the level of the bottom of the tape reinforcement

Figure 4. Test of the steel fiber concrete beam

Figure 5. Beams B-1, BF-2 and BF-3 after testing
Figure 6. Experimental values of the gaps of beams B-1, BF-2, BF-3

Table 2. Results of beams testing

| Code of beams | Bending moment $M$, kN·cm | The relative deformation of extreme compressed concrete fibers $\varepsilon_c$ | The relative deformation of the tape reinforcement $\varepsilon_s$ | Experimental values of deflections $a_{exp}$, mm | Theoretical values of deflections $a_{theor}$, mm | $\frac{a_{theor}}{a_{exp}}$ |
|---------------|---------------------------|------------------------------------------------|--------------------------------|-----------------|-----------------|-----------------|
| B-1           | 70                        | 0,00011                                     | 0,00020                       | 0,283           | 0,289           | 1,02            |
|               | 140                       | 0,00021                                     | 0,00055                       | 0,670           | 0,697           | 1,04            |
|               | 210                       | 0,00035                                     | 0,00101                       | 1,161           | 1,226           | 1,06            |
|               | 280                       | 0,00048                                     | 0,00163                       | 1,894           | 1,875           | 0,99            |
|               | 350                       | 0,00063                                     | 0,00238                       | 2,590           | 2,644           | 1,02            |
|               | 420                       | 0,00081                                     | 0,00325                       | 3,802           | 3,533           | 0,93            |
|               | 490                       | 0,00102                                     | 0,00423                       | 4,360           | 4,542           | 1,04            |
| BF-2          | 70                        | 0,00011                                     | 0,00012                       | 0,245           | 0,247           | 1,01            |
|               | 140                       | 0,00023                                     | 0,00028                       | 0,518           | 0,529           | 1,02            |
|               | 210                       | 0,00032                                     | 0,00051                       | 0,811           | 0,848           | 1,05            |
|               | 280                       | 0,00044                                     | 0,00075                       | 1,191           | 1,203           | 1,01            |
|               | 350                       | 0,00056                                     | 0,00104                       | 1,514           | 1,594           | 1,05            |
|               | 420                       | 0,00073                                     | 0,00132                       | 2,131           | 2,020           | 0,95            |
|               | 490                       | 0,00098                                     | 0,00177                       | 2,458           | 2,483           | 1,01            |
|               | 560                       | 0,00122                                     | 0,00232                       | 2,982           | 2,982           | 1,00            |
| BF-3          | 70                        | 0,0001                                      | 0,00010                       | 0,231           | 0,233           | 1,01            |
|               | 140                       | 0,00021                                     | 0,00021                       | 0,467           | 0,477           | 1,02            |
|               | 210                       | 0,00031                                     | 0,00034                       | 0,641           | 0,729           | 1,14            |
|               | 280                       | 0,00048                                     | 0,00055                       | 0,952           | 0,991           | 1,04            |
|               | 350                       | 0,00058                                     | 0,00066                       | 1,263           | 1,263           | 1,00            |
|               | 420                       | 0,000699                                    | 0,00082                       | 1,402           | 1,545           | 1,10            |
|               | 490                       | 0,000792                                    | 0,00105                       | 1,781           | 1,836           | 1,03            |
|               | 560                       | 0,000891                                    | 0,00120                       | 2,116           | 2,137           | 1,01            |
|               | 630                       | 0,001203                                    | 0,00140                       | 2,325           | 2,447           | 1,05            |
According to the results of deformation measurements of extremely compressed fibers of concrete and tape reinforcement, a curvature of $\frac{1}{r}$ was determined at each stage of beam loading by the formula:

$$\frac{1}{r} = \frac{\varepsilon_c + \varepsilon_s}{h},$$

where $h = 15 \text{ cm}$ – the height of the beam.

This allowed to obtain the dependence of the curvature on the values of the bending moments $M$. Using the Excel spreadsheet, second-degree polynomials approximated the functions $\frac{1}{r} = f(M)$ for each beam (Figure 7). The bending moment in cross-section at a distance $x$ from the left support of the beam due to the action of the concentrated force $P$ applied in the middle of the run, changes for the left half of the length of the beam according to the equation:

$$M_x = \frac{P}{2} x.$$ 

Therefore, the change in curvature along the left half of the beam B-1 can be written as:

$$\left(\frac{1}{r}\right)_x = 1 \cdot 10^{-9} M_x^2 + 2 \cdot 10^{-7} M_x,$$

for a beam BF-2:

$$\left(\frac{1}{r}\right)_x = 5 \cdot 10^{-10} M_x^2 + 1 \cdot 10^{-7} M_x,$$

for a beam BF-3:

$$\left(\frac{1}{r}\right)_x = 8 \cdot 10^{-11} M_x^2 + 2 \cdot 10^{-7} M_x.$$
The theoretical deflection values for each of the beams were determined by the Mohr integral:

\[ a = 2 \cdot \int_{0}^{l/2} \left( \frac{1}{r(x)} \right) M_{1x} dx, \]  

Where \( M_{1x} = \frac{1}{2} x \) – the law of change of the bending moment from a single force applied in the middle of the beam run for its left half-span.

The solution of equation (6) with respect to equations (3), (4) and (5) has the following form:

\[ a = 2 \int_{0}^{l/2} \left[ k_1 \left( \frac{1}{2} x \right)^2 + k_2 \left( \frac{1}{2} x \right) \right] dx = \frac{k_1 p^2 l^4}{256} + \frac{k_2 p l^3}{48}, \]

where \( k_1 = 1 \cdot 10^{-9} \frac{1}{kN \cdot cm^2}; \)  
(7)  
\( k_1 = 5 \cdot 10^{-10} \frac{1}{kN \cdot cm^2}; \)  
\( k_1 = 8 \cdot 10^{-11} \frac{1}{kN \cdot cm^2}; \) for beams B-1, BF-2, BF-3 respectively;

\( k_2 = 2 \cdot 10^{-7} \frac{1}{kN \cdot cm^2}; \)  
\( k_2 = 1 \cdot 10^{-7} \frac{1}{kN \cdot cm^2}; \)  
\( k_2 = 2 \cdot 10^{-7} \frac{1}{kN \cdot cm^2}; \) for beams B-1, BF-2, BF-3 respectively;

\( l = 140 \text{ cm} \) - the span of the beam.

The theoretical values of the deflections calculated by the formula (6) are given in table 2. The ratio of theoretical deflections to the experimental ones is in the range of 0.93 ... 1.14. For beams reinforced with tape and fiber reinforcement with costs within 1.5 - 2% of the volume of concrete, the coefficients of polynomials (4) and (5) before the accumulation of experimental data can be taken by linear interpolation.

It should be noted that in the ribbon reinforcement with relative deformations 0.0012 the process of the flow begins. Therefore, according to the experimental data in beams B-1, BF-2, BF-3 at bending moments 280 kN·cm, 420 kN·cm and 560 kN·cm respectively, which is 50%, 60% and 80% of the bearing capacity of these beams, in the reinforcing strip of deformation reached fluidity. Therefore, it is advisable to provide welding to the tape of the reinforcement bar in addition to the end stops to ensure the joint work of the tape with concrete part of which may not be beyond the internal edges of the supports.

4. Conclusions
This paper presents the results of experimental and theoretical studies of the deformability of beams containing fibrous and external tape fittings with end stops. The deformations of the ribbon reinforcement, the outer compressed fibers of the concrete and the deflections of the beams with fiber were smaller in comparison with beams without fiber. For example, at the moment of 350 kN·cm, the deflection of the beam B-1 was 2,590 mm, and in respect of the beams with fibrous reinforcement BF-2 and BF-3 – were 1,514 mm and 1,263 mm, respectively.

It is proposed to present the curvature based on the results of experiments as a function which bending moment is the argument and to approximate this function by a polynomial using an Excel spreadsheet. This allows defining deflections by the Mora integral. The ratio of theoretical deflections to experimental ones is in the range 0.93 ... 1.14.

Additional studies require the calculation and design of end stops, the use of additional rod reinforcement welded to the tape to counteract the displacement of concrete relative to the tape.
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