Mathematical, detailed and parametric modelling for smart spinal orthoses

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Abstract. The paper presents a mathematical model and the design details to manufacture a prototype of a smart orthosis for posture correction while sitting on a chair at your office or at home. The global geometry for developing the mathematical model was obtained by direct measuring a human 3D model, developed by scanning one of the authors’ body and 3D printing it. Based on the mathematical model, the detailed design for the entire product was performed in accordance with the product architecture. The stages of detailed design for the critical systems within the final assembly are also presented in the paper. The posture correction orthosis must be customized for each user, thus, once the design for the critical components has been validated, a parametric design has been generated to facilitate the manufacturing process. The technical solutions are generated and analysed for a vertebral element of the orthosis, representing the critical component of the product. The vertebral element is subject to mechanical stresses, so that a finite element analysis was performed. The geometry of the component and its material were validated by using FEA on the CAD model of the vertebral element. The further work will include research regarding the changes needed for transforming the orthosis into a medical device for spinal column illnesses.

1. Introduction

The mathematical modelling and mathematical simulations play an important role in biological sciences, and they are essential for a better understanding of the functions of human body [1].

Physical correction of human posture must use a device able to follow the anatomical shape of the spinal column. Studies show that improper body posture during sitting has detrimental effect to the human spinal column region over an extended period [2]. The paper proposes a mathematical model to represent the spinal column of human body during sitting tasks, based on a two-link kinematic open chain in two dimensional spaces. Thus, all movement joints and their effect to every part of vertebral bodies of human body can be thoroughly analysed.

The global geometry for developing the mathematical model was obtained by direct measuring a human 3D model, developed by scanning one of the authors’ body and 3D printing it. A global Cartesian coordinate system was established wherein the X-Y plane is mid-sagittal, the X-Z plane is transverse and tangent to the inferior surface of the ischium, and Y-Z is a frontal plane [3].

The increasing demands have motivated progress in the analysis and simulation of complex mechanical systems. The development of mathematical models of mechanical systems, however, requires both knowledge of the best methods of mechanics as well as practical experience. For

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dynamic systems, such as vehicles, mechanisms, machinery, and robots, discrete models have long proved themselves useful. Such multibody models are called mechanical systems even if they also contain devices which are electric, magnetic, hydraulic, or pneumatic in nature [4].

On the other side, due to the advantages of low cost and small size, wearable sensors have been drawing attention. They can meet most needs of human motion monitoring and have been adopted in many applications, such as interactive gaming and learning, animation, health care, personal navigation, and security monitoring [5].

Following all these tendencies, the paper presents the elements that were the basis for the development of a mathematical model for the manufacture a prototype of orthosis for posture correction while sitting on a chair at your office or at home. The detailed design process of the solution that allows the materialization of the mathematical model into a mechatronic product is also described, together with the process of optimizing the design of critical components.

2. Mathematical modelling
To develop a mathematical model for detecting and correcting the user's posture, the anatomy of the spine, its curves and joints were studied. The spine acts as a pillar that supports the human body and protects the spinal cord. Viewed from the side (Figure 1), it normally has the shape of letter 'S' due to three natural curves, which is why the spine can be divided into three regions: cervical, thoracic, and lumbar. The spine consists of 24 vertebral bones and two bones in the axial section of the pelvic girdle, sacrum, and coccyx. There are 7 cervical vertebrae, 12 thoracic vertebrae and 5 vertebrae [6].

The mathematical model describes the shape of the column based on the angles read by the sensors on the spinal orthoses prototype (Figure 2).

The input data of the calculation algorithm are:
- coordinates of the initial positions of the sensors: \( S_1 (x_1, y_1), S_2 (x_2, y_2), S_3 (x_3, y_3) \) and \( S_i (x_i, y_i) \);
- coordinates of the final positions of the sensors: \( S'_1 (x'_1, y'_1), S'_2 (x'_2, y'_2), S'_3 (x'_3, y'_3) \) and \( S'_i (x'_i, y'_i) \);
- the values of the angles between the rigid segments: \( \alpha_1, \alpha_2, \alpha_3 \) and \( \alpha_i \);
- length of rigid segments: \( l_1, l_2, l_3 \) and \( l_i \).

The output data of the calculation algorithm are:
- the angular values of rotation for each of the rigid segments: \( u_1, u_2, u_3 \) and \( u_i \);

![Figure 1. Side view of anatomical Spinal Column [7]](image1)

![Figure 2. Mathematical model of Spinal Column](image2)
2.1. Initial assumptions
Known data for the mathematical model are orientation angles and distances. The chosen approach refers to a mathematical model based on straight segments: the equations needed to calculate the coordinates of the target points are only first-degree relations. Thus, a linear system of equations is reached that is easier and faster to solve.

The following hypotheses are considered:
- The carrier monitoring system will offer the possibility of orientation on the directions of the 3 axes, but only one axis will be studied;
- The proposed mathematical model is based on the approximation of real curves with line segments and geometric functions;
- The model inputs will be the measured distances between the sensors on the flexible frame. Orientation angles will be provided by inertial sensors, as follows:
- The segments formed between two sensors are approximated with segments of length \( l_1, l_2, l_3 \) and \( l \), and the angle between two consecutive segments \( \alpha_1, \alpha_2, \alpha_3 \) and \( \alpha \);
- The right segments will coincide with each other, and a smooth curve will be obtained.

The model can precisely calculate the coordinates \((x, y)\) of the position of the sensors and can reconstruct the natural curves of the spine.

2.2. Methodology stages
The mathematical model development has the following stages:
- Initialize the coordinates \((x_0, y_0)\);
- Define the centre rotation \( S_0 \);
- Calculate the angles of rotation \( u_i \);
- Reset the coordinates \((x_i, y_i)\);
- Repeat the last two steps for the required number of iterations.

2.3. Coordinates initialization
The coordinates for the established points on the spine are initialized. The initialization of the coordinates is done iteratively for each joint as follows:

\[ S_0: (x_0, y_0) - \text{remains fixed.} \]

\[ S_1: \text{A centre rotation } S_1 \text{ is defined:} \]
\[
\begin{pmatrix} x_1 \\ y_1 \\ 0 \\ 1 \\ 0 \end{pmatrix} = l_1 \begin{pmatrix} \cos \alpha_1 & -\sin \alpha_1 \\ \sin \alpha_1 & \cos \alpha_1 \end{pmatrix} \begin{pmatrix} 0 \\ 1 \end{pmatrix} + \begin{pmatrix} x_0 \\ y_0 \end{pmatrix}
\]
\[ (1) \]

\[ S_2: \text{A centre rotation } S_2 \text{ is defined:} \]
\[
\begin{pmatrix} x_2 \\ y_2 \\ 0 \\ 1 \\ 0 \end{pmatrix} = \begin{pmatrix} l_2 \\ l_2 \end{pmatrix} \begin{pmatrix} \cos \alpha_2 & -\sin \alpha_2 \\ \sin \alpha_2 & \cos \alpha_2 \end{pmatrix} \begin{pmatrix} x_1 \\ y_1 \\ 0 \\ 1 \\ 0 \end{pmatrix} + \begin{pmatrix} x_2 \\ y_2 \end{pmatrix}
\]
\[ (2) \]

\[ S_{i+1}: \text{A centre rotation } S_i \text{ is defined:} \]
\[
\begin{pmatrix} x_{i+1} \\ y_{i+1} \\ 0 \\ 1 \\ 0 \end{pmatrix} = \begin{pmatrix} l_i \\ l_i \end{pmatrix} \begin{pmatrix} \cos \alpha_i & -\sin \alpha_i \\ \sin \alpha_i & \cos \alpha_i \end{pmatrix} \begin{pmatrix} x_{i-1} \\ y_{i-1} \\ 0 \\ 1 \\ 0 \end{pmatrix} + \begin{pmatrix} x_i \\ y_i \end{pmatrix}
\]
\[ (3) \]

2.4. Determination of rotation angle \( u_1 \)
The values of the rotation angles \( u_1 \) are determined. With the help of the obtained values the new coordinates are calculated \((x_2, y_2)\).

**Centre rotation**

To determine \( u_1 \), a centre rotation \( S_1 < u_1 \) is defined:
\[
\begin{pmatrix} x'_2 \\ y'_2 \\ 0 \\ 1 \\ 0 \end{pmatrix} = \begin{pmatrix} \cos u_1 & -\sin u_1 \\ \sin u_1 & \cos u_1 \end{pmatrix} \begin{pmatrix} x_2 - x_1 \\ y_2 - y_1 \end{pmatrix} + \begin{pmatrix} x_1 \\ y_1 \end{pmatrix}
\]
\[ (4) \]

\[
\begin{pmatrix} x'_2 \\ y'_2 \\ 0 \\ 1 \\ 0 \end{pmatrix} = \begin{pmatrix} \cos u_1 & -\sin u_1 \\ \sin u_1 & \cos u_1 \end{pmatrix} \begin{pmatrix} x_2 - x_1 \\ y_2 - y_1 \end{pmatrix} + \begin{pmatrix} x_1 \\ y_1 \end{pmatrix}
\]

\[
\begin{pmatrix} x'_2 \\ y'_2 \\ 0 \\ 1 \\ 0 \end{pmatrix} = \begin{pmatrix} x'_2 - x_1 \\ y'_2 - y_1 \\ 0 \\ 1 \\ 0 \end{pmatrix} = \begin{pmatrix} x_2 - x_1 \\ y_2 - y_1 \end{pmatrix}
\]

\[
\begin{pmatrix} x'_2 \\ y'_2 \\ 0 \\ 1 \\ 0 \end{pmatrix} = \begin{pmatrix} x'_2 - x_1 \\ y'_2 - y_1 \\ 0 \\ 1 \\ 0 \end{pmatrix} = \begin{pmatrix} x_2 - x_1 \\ y_2 - y_1 \end{pmatrix}
\]

\[
\begin{pmatrix} x'_2 \\ y'_2 \\ 0 \\ 1 \\ 0 \end{pmatrix} = \begin{pmatrix} x'_2 - x_1 \\ y'_2 - y_1 \\ 0 \\ 1 \\ 0 \end{pmatrix} = \begin{pmatrix} x_2 - x_1 \\ y_2 - y_1 \end{pmatrix}
\]
Reset coordinates
Considering the performed centre rotation, the coordinates are reset, thus determining the new coordinates of the points of interest are as follows:
\[
\begin{pmatrix}
  x_i' \\
  y_i'
\end{pmatrix} = \begin{pmatrix}
  \cos u_i & -\sin u_i \\
  \sin u_i & \cos u_i
\end{pmatrix} \begin{pmatrix}
  x_i - x_1 \\
  y_i - y_1
\end{pmatrix} + \begin{pmatrix}
  x_1 \\
  y_1
\end{pmatrix}
\]
(5)

2.5. Determination of rotation angle \( u_i \)
The values of the rotation angles \( u_i \) are determined. With the help of the obtained values the new coordinates are calculated \((x_i, y_i)\).

Centre rotation
To determine \( u_i \), a centre rotation \( S_i < u_i \) is developed:
\[
\begin{pmatrix}
  x_{i+1} \\
  y_{i+1}
\end{pmatrix} = \begin{pmatrix}
  \cos u_i & -\sin u_i \\
  \sin u_i & \cos u_i
\end{pmatrix} \begin{pmatrix}
  x_i - x_1 \\
  y_i - y_1
\end{pmatrix} + \begin{pmatrix}
  x_1 \\
  y_1
\end{pmatrix} 
\Rightarrow \cos u_i = \frac{\begin{vmatrix}
  x_{i+1} - x_i - (y_{i+1} - y_i) \\
  y_{i+1} - y_i - (x_{i+1} - x_i)
\end{vmatrix}}{\begin{vmatrix}
  x_{i+1} - x_i \\
  y_{i+1} - y_i
\end{vmatrix}^2} 
\]
(6)

Reset coordinates
Considering the new centre of rotation, the coordinates are reset, thus determining the new coordinates of the points of interest are:
\[
\begin{pmatrix}
  x_i' \\
  y_i'
\end{pmatrix} = \begin{pmatrix}
  \cos u_i & -\sin u_i \\
  \sin u_i & \cos u_i
\end{pmatrix} \begin{pmatrix}
  x_i - x_1 \\
  y_i - y_1
\end{pmatrix} + \begin{pmatrix}
  x_1 \\
  y_1
\end{pmatrix}
\]
(7)

Figure 3.a Geometrical model for \( S_2 \) coordinates
Figure 3.b Geometrical model for \( S_3 \) coordinates

3. Detail design
In engineering design, the product architecture is related to the functional elements and physical components of products and is used to define the basic physical building blocks of the product in terms of what they do and what their interfaces are to the rest of the device [8]. One of the most widely accepted definitions of product architecture was given by Ulrich [9]. The spinal orthoses architecture shown in Figure 4 identifies the product’s functions, materialised in specific systems and assemblies.

Any engineering design process is based on two factors: (1) the state of technical and technological knowledge and (2) the project theme that includes the initial computer data about the future product [10]. Starting from the established objective specifications and the selected concept, the detailed design has been elaborated (Figure 4). The posture correction orthosis has four main systems, and each system has few assemblies. Further in this paper, the detailed design of the Reading assembly, the most critical component in the final product, is discussed, being marked in green in Figure 4. The other systems are normalized regardless of the user, while the Reading assembly is critical because the user can perform the movements that the system cannot, thus resulting in mechanical stress.
3.1. Reading assembly detailed design

The reading assembly has five main components, and it represents a mechanical copy of the spine. The Reading system is designed to accurately mimic the movements of the user's spine and read the position in which he is in real time. Thoracic and lumbar joints are represented by a single piece in the final assembly, while the number of the vertebral bodies, sensors and axes are added depending on the desired reading accuracy. The reading accuracy depends on the number of segments added to the system component [11]. Thus, for a 100% accurate reading, 24 segments are needed, one segment for each vertebra. The current design version is constructed with four vertebral bodies (Figure 5.a and 5.b), meaning that the reading accuracy is 1/6 from real user’s posture. The final Posture correction orthosis prototype will be manufactured with eight segments (reading accuracy will be 1/3 from real user’s posture).

![Spinal Column Sketch](image1)

**Figure 5.a** Spinal column sketch

![Spinal Column Detailed Design](image2)

**Figure 5.b** Spinal column detailed design

The vertebral element - VE (Figure 6.a) materializes the mathematical model defined in section 2. At each end, the VE has a sensor slot and a joint so that it can be assembled with another VE or with one of the two joints, thoracic or lumbar (Figure 6.b). The body is rigid and has a fixed length, depending on the desired accuracy, the number of segments is kept constant and the length of the VE varies.

During the design process, the assembly tolerance design is an important issue. Assembly tolerances reflect functional requirements of fitting, which can be used to control assembling qualities and production costs. Assembly constraints are essentially constraints between assembly feature surfaces of parts. Assembly tolerances are crucial for many activities in the product’s life cycle, which
not only affects assembly qualities but also determines manufacturing costs [12].

![Figure 6.a Vertebral element - VE](image1)

![Figure 6.b Components of for one VE](image2)

A challenge in the design of the VE was to establish the tolerances of different dimensions, because the VE assembles with most components of the orthosis. Because the VE is the most important component of the posture correction orthosis, it has gone through several design stages. Thus, the final shape of the VE was obtained after several iterations (V1-V5). Some versions of the concept are shown in the figure 7.a-e.

![Figure 7.a VE - v1](image3)

![Figure 7.b VE – v2](image4)

![Figure 7.c VE – v3](image5)

![Figure 7.d VE – v4](image6)

![Figure 7.e VE - 5](image7)

3.2. Finite Element Analysis

Because the vertebral bodies articulate both with each other and with the thoracic and lumbar joints, they have not been subjected to a finite element analysis on the frontal direction. The working stress scenario was considered when the user tilts to one side. In the lateral plane the components are rigid, thus a finite element analysis (FEA) was performed in this plane. The FEA analysis was performed in same conditions for two plastic materials, namely ABS and PET. The properties of the selected materials are presented in Table 1.

The first simulated material was ABS, and the result of the analysis is presented in Figure 8.a. The safety factor in this analysis is less than 1, ~ 0.41, which indicates that the part will not withstand the action of the applied force $F = 100$ N.

The second simulated material was PET, and the result of the analysis is presented in Figure 8.b. The safety factor in the case of this analysis is more than 1, ~ 1.2, which indicates that the part will withstand the action of the applied force $F = 100$ N.
Table 1. Material properties [13]

| Properties                      | ABS            | PET            |
|---------------------------------|----------------|----------------|
| Thermal conductivity [W/(m*K)]   | 3.000E-01      | 1.600E-01      |
| Specific heat [J/(g*C)]         | 2.287          | 1.500          |
| Thermal expansion [µm/(m*C)]    | 0.702          | 87.500         |
| Young’s module [GPa]            | 2.758          | 2.240          |
| Poisson’s ratio                 | 0.420          | 0.380          |
| Shear modulus [MPa]             | 1240.00        | 805.000        |
| Density [g/cm³]                 | 1.541          | 1.060          |
| Yield strength [MPa]            | 54.400         | 20.000         |
| Tensile Strength [MPa]          | 55.100         | 29.600         |

Because the posture correction orthosis is used by humans, the most important specification is safety during use. For this reason, the safety factor obtained ~ 1.2 is enough for this purpose. In case of emergency, the user will not be blocked by the mechanical spine but will be able to easily damage the component and will be able to move freely. Thus, the safety factor obtained is good both, from a technical point of view and from the safety in use point of view.

Figure 8.a FEA with ABS

Figure 8.b FEA with PET

4. Parametric design
Design optimization is an engineering design methodology that uses a mathematical formulation of a design problem to support the selection of the optimal design from several generated alternatives. Parametric design is a method in which parameters are used to set the size values of a part or the whole model. In the case of the VE, parametric optimization was performed on the model optimized for 3D printing, as shown in Figure 9. Each element from the initial design has its own parameters and it is important for the components, as they are required to define the final parametric model. Figure 10 shows the elements of the model.

Because the dimensions of the VE bores depend on the dimensions of the components to be assembled, they were left as parameters of the model, thus they were not customized. Firstly, the dimensions that influence the geometry of the vertebral body were parameterized according to the
user's height (Figure 11.a,b). Dependencies were created between the dimensions of the model.

![3D model of VE](image1)

**Figure 9** 3D model of VE

![Model parameters of VE](image2)

**Figure 10** Model parameters of VE

The next step was to create parameters that define the dimensions of the VE depending on the height of the user. Two user parameters were defined (Figure 12.a). The first parameter was the user's height in mm, and this is the only parameter that is modified by the product manufacturing team. The second parameter is the height factor that proportionally converts the dimensions of the VE according to the height of the user (Figure 12.b).

![Initial model parameters](image3)

**Figure 11.a** Initial model parameters

![Customized model parameters](image4)

**Figure 11.b** Customized model parameters

![Initial model parameters](image5)

**Figure 12.a** Initial model parameters

![Customized model parameters](image6)

**Figure 12.b** Customized model parameters

In figures 13.a-c several dimensional variations of the VE, depending on the height of the user, are developed and presented. For a user with a height of 1.5 m, each of the eight vertebral bodies has a
length between the centres of the joint points of 50.3 mm (Figure 13.a). For a user with a height of 2.0 m, the distance between the joint points of the model is 67 mm (Figure 13.c).

Apart from the commercial components, all components went through several design iterations until the optimal solution was chosen. For all components that are subjected to mechanical stresses during operation, FEA was performed. With the help of this analysis the geometry and the right material for the given application were chosen (Figure 14.a,b).
5. Conclusions
For the development of the smart posture correction orthosis prototype a mathematical model was generated considering the principle of anatomical functioning of the human body and especially the particularities of the spine. Regardless of the user, the operating principle remains the same and the technical solution can be adapted for each individual user. Because the technical solution is customized for each user, along with the detailed design, a parametrical design was developed. This allows the use of the same technological production process for each device depending on the needs of the given user.

The current orthosis is designated for posture correction of the healthy subjects. As a future work we consider the process of transforming the product into a medical device for correcting the spinal column illnesses, including the cervical vertebrae, not only lumbar ant thoracic. In this case the subject’s axial rotation and frontal movement will also be considered.

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