RHIC and diffraction in \( pp \) Spin-flip.*

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Abstract

We argue that diffraction (Pomeron) contribution is present in the \( pp \) spin-flip amplitude. RHIC polarization data will be able to prove (or disprove) this conjecture.

1 Introduction

Following the hints given in very old pioneer papers [1, 2], in a recent work [3] we discussed the possible presence of a diffractive-like (Pomeron) contribution in the "reduced" spin-flip amplitude, that is, the spin-flip amplitude when the kinematical zero is removed. The procedure used was to remove this zero factorizing a \( \sin \theta \) factor (eq. (6) of [3]) according to the suggestion of [1, 2].

The result obtained in [3] predicted a very small polarization \( P \) at the Relativistic Heavy Ion Collider (RHIC) energies but, upon reconsideration of the problem, it is necessary to verify how much may this extrapolation is influenced by the use of the factor \( \sin \theta \), since this is not the only way to remove the kinematical zero. This could, alternatively, be done using a \( \sqrt{-t} \) factor [4, 5, 6] which entails a \( \sqrt{s} \) factor as compared with the \( \sin \theta \) option leading, possibly, to a significantly larger \( P \) at high energies.

Using \( \sqrt{-t} \) appears more in line with the Regge behavior prescription but only the data can, ultimately, clarify the issue. For this, we revise here our conclusions reached in [3] using \( \sqrt{-t} \) to show how RHIC should be able to settle the point.

To investigate \( pp \) scattering, it is necessary to specify five independent helicity amplitudes in terms of which the polarization \( P \) is given by

\begin{equation*}
P = \frac{1}{2} \left( |A_{+}\rangle |A_{-}\rangle \langle A_{+}| + |A_{-}\rangle |A_{+}\rangle \langle A_{-}| \right)
\end{equation*}

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1We are grateful to the referee and to Prof. O.V. Selyugin for stressing this point to us.
\[ P = 2 \frac{\text{Im}[(\phi_1 + \phi_2 + \phi_3 - \phi_4)\phi_5^*]}{|\phi_1|^2 + |\phi_2|^2 + |\phi_3|^2 + |\phi_4|^2 + 4|\phi_5|^2}, \]  

(1)

where \( \phi_1, \phi_3 \) are spin-non-flip amplitudes, \( \phi_2, \phi_4 \) are double spin-flip amplitudes and \( \phi_5 \) is the single spin-flip amplitude.

Following [3], the amplitudes \( \phi_2 \) and \( \phi_4 \) will be neglected and we write

\[ \phi_1 \sim g(s, t), \quad \phi_5 = h(s, t) \]  

(2)

assuming \( \phi_1 = \phi_3 \). \( g(s, t) \) and \( h(s, t) \) will be considered effective spin-non-flip and spin-flip amplitudes, respectively. These definitions are not exactly those used in [3] but this is immaterial since \( \phi_1, \phi_3 \) and \( \phi_5 \) were not explicitly given there.

Using eq. (2), \( P \) can now be rewritten as

\[ P = 2 \frac{\text{Im}[g(s, t)h^*(s, t)]}{|g(s, t)|^2 + 2|h(s, t)|^2}. \]  

(3)

We use the same set of data used in [3] for \( pp \) polarization data [7] and the parametrization for the \( pp \) spin-non-flip amplitude is again taken from reference [8]. Removing the kinematical zero with \( \sqrt{-t} \) (see below) we reach the following conclusions:

a) As in [3], we find that the presence of a diffractive-like behavior in the (reduced) spin-flip amplitude leads to good fit of the data but, differently from the sin \( \theta \) case, the reduced spin-flip amplitude is now only about 10% of the imaginary part of the spin-non-flip amplitude.

b) The data fitting with the same energy dependence in \( g(s, t) \) and \( h(s, t) \) seems the best choice; the zero of the polarization moves towards zero with energy as noticed earlier [3].

c) The magnitude of the polarization decreases as the energy increases but the extrapolation to 500 GeV predicts a non-negligible contribution if the same Pomeron trajectory for both spin-flip and spin-non-flip amplitudes is used;

d) if the intercepts of the trajectories of the Pomeron in \( g(s, t) \) and \( h(s, t) \) are not the same, the extrapolation to 500 GeV becomes quite unrealistic (and just as small as it was obtained with the factor sin \( \theta \)).

Our conclusion is that RHIC will really be able to give a clear cut answer to the long standing question: is diffraction (the Pomeron) contributing to spin-flip in \( pp \)?

### 2 Definition of the amplitudes

The effective \( pp \) spin-non-flip amplitude will be written as

\[ g(s, t) = a^{nf}(s, t) = a_+(s, t) - a_-(s, t) \]  

(4)
with
\[
\begin{align*}
a_+ (s, t) &= a_F(s, t) + a_f(s, t), \\
a_- (s, t) &= a_O(s, t) + a_ω(s, t)
\end{align*}
\] (5)

where \(a_F(s, t)\) and \(a_O(s, t)\) are the Pomeron and Odderon amplitudes respectively and \(a_f(s, t)\) [\(a_ω(s, t)\)] are the even [odd] secondary Reggeons. These different amplitudes are taken directly from Ref. [8] and their explicit forms are given in Appendix A together with the values of their parameters.

In the effective spin-flip amplitude, \(h(s, t)\), we neglect the contribution of secondary Reggeons and, mutatis mutandis, we follow [3] to write
\[
\begin{align*}
h(s, t) &= a_{sf} (s, t) = (iγ_1 + β_1) \sqrt{-t} \tilde{s}^{α_{sf}(t)} e^{β_1 t} Θ(0.5 - |t|) \\
&+ (iγ_2 + β_2) \sqrt{-t} \tilde{s}^{α_{sf}(t)} e^{β_2 t} Θ(|t| - 0.5),
\end{align*}
\] (6)

where the mass of the proton, \(m_p\), is used to make the parameters dimensionless. In (6) \(\tilde{s} = \frac{s}{s_0} e^{-iπ/2}\); \(Θ\) is the step function and we assume \(s_0 = 1\) GeV\(^2\) as in [8]; the superscript \(sf\) (for “spin-flip”) will allow us to check if the Pomeron trajectory can be different for spin-flip and spin-non-flip. In the parametrization [3] the Pomeron contribution to the spin-flip amplitude is allowed a complex phase and this, we believe, can be justified by the CP-even contribution induced by the exchange of three(or more)-gluon ladders.

To start with, we take the spin-flip Pomeron trajectory \(α_{sf}(t)\) to be exactly the same as derived for the spin-non-flip amplitude
\[
α_{sf}(t) = α_F(t) = α_F(0) + α'_F t
\] (7)

where \(α_F(0)\) and \(α'_F\) are found in Appendix A. The data at \(\sqrt{s} = 13.8, 16.8\) and 23.8 GeV (a total of 64 points) are used in the fit and the values of the parameters, together with the \(χ^2\) are listed in Table 1.

| \(γ_1\) | \(1.35 \times 10^{-1}\) | \(γ_2\) | \(2.55 \times 10^{-2}\) |
| \(δ_1\) | \(2.64 \times 10^{-1}\) | \(δ_2\) | \(5.38 \times 10^{-2}\) |
| \(β_1\) (GeV\(^{-2}\)) | 4.74 | \(β_2\) (GeV\(^{-2}\)) | 2.29 |
| \(χ^2/d.f. = 1.1\) |

Table 1: Values of the parameters obtained from fitting polarization data at \(\sqrt{s} = 13.8, 16.8\) and 23.8 GeV with eqs. (3) and (5).

In Fig. 1 we show the polarization data together with our reconstruction. As a check of the validity of our solution, Fig. 2 shows how it accounts for the data at \(\sqrt{s} = 19.4\) Gev (not used in the fit). In Fig. 3 we show \(dσ/dt\) at various energies.

\^2Actually, \(a_f\) embodies both \(f\) and \(ρ\) contributions (and \(a_ω\) both \(ω\) and \(a_2\)).
Some considerations are in order:

a) the $IP$ contribution to the spin-flip amplitude is considerably smaller than to $g(s, t)$ (about 10%, roughly) contrary to the sin $\theta$ case\[3\];

b) the (small $|t|$) slope of the spin-flip amplitude $\beta_1 = 4.74 \text{ GeV}^{-2}$ is somewhat smaller than the one determined previously\[2, 3\] but this is not unexpected due to the changes made in eq. (6);

c) the extrapolation of our solution to 50 and 500 GeV predicts the polarization shown in Fig. 4. The curve at 500 GeV is smaller than that at 50 GeV but
Figure 2: The prediction for the polarization at 19.4 GeV (not used in the fit) compared with the experimental data at that energy.

considerably larger than the prediction with $\sin \theta$ [3]; most important, this polarization could be measured at RHIC;

d) our result (i.e., $h(s,t)$) cannot be extended to $|t|$ values much higher than few GeV$^2$ because the spin-non-flip amplitude utilized is valid at the Born level [8] and its description in the region after the dip ($|t| > 1.5$ GeV$^2$) is not very good. To extend our considerations to higher $|t|$, it would be necessary to adopt the more sophisticated eikonalized version. Anyway, the $t$-region of interest for RHIC is up to 1.5 GeV$^2$ [9] so we can confine our analysis to the not too high $|t|$-region.

One open question remains the possibility that $\alpha^{sf}(0)$ be not the same as in the spin-non-flip amplitude. If $\alpha^{sf}(0)$ is left free to vary below unity (i.e., no Pomeron contribution), one can still fit the data (albeit with a negative $\alpha^{sf}(0)$) and the description of the polarization remains essentially the same. The quality of the fit (the $\chi^2/d.f.$) is practically the same as can be seen in Fig. 5 and 6. Table 2 shows the values obtained for the parameters of eq. (6) when $\alpha^{sf}(0) \neq \alpha_P(0)$. On the other hand, in this case all parameters change drastically. In particular, the couplings $\gamma_i$ and $\delta_i$ become totally absurd so that we incline to discard entirely this solution. However, this can be left to the experiment. In this case, in fact, when the fit to the polarization is extrapolated from $\sqrt{s} = 23.8$ GeV to RHIC energies (Fig. 7), the polarization predictions are negligibly small (and very similar to those obtained with $\sin \theta$, see Fig. 7 on [3]).
Figure 3: The differential cross section obtained in this work taking into account the spin-flip amplitude. The highest set of data correspond to 23.5 and 27.4 GeV grouped together. The other sets (multiplied by powers of $10^{-2}$) are 30.5, 44.6, 52.8 and 62 GeV.

Figure 4: The polarization predictions for 50 and 500 GeV with the parameters of Table 1.
\[
\alpha_{P}^{sI}(0) = -0.129
\]

| \(\gamma_1\) | -1031 | \(\gamma_2\) | -46.8 |
| \(\delta_1\) | 187 | \(\delta_2\) | 3.67 |
| \(\beta_1\) (GeV\(^{-2}\)) | 7.84 | \(\beta_2\) (GeV\(^{-2}\)) | 2.29 |

\(\chi^2/d.f. = 1.1\)

Table 2: Values of the parameters in the absence of diffraction (\(\alpha^{sI}(0) < \alpha_{P}(0)\)).

![Graphs showing polarization vs. |t| for different energies](image)

Figure 5: Results from fitting polarization data at various energies (13.8, 16.8 and 23.8 GeV) without diffraction (parameters of Table 2).

## 3 Conclusions

We performed the fit of polarization data for \(pp\) scattering with a Pomeron spin-flip amplitude where the kinematical zero is removed by the factor \(\sqrt{-t}\), instead
Figure 6: The prediction for polarization at 19.4 GeV compared with the experimental data using the parameters of Table 2 (dashed line).

Figure 7: The polarization predictions for 50 and 500 GeV using the parameters of Table 2 (i.e., without diffraction). A detailed view of the 500 GeV is shown in the inset.

of $\sin \theta$ as done earlier. The motivation for this reanalysis is that, hidden inside
the sinθ option, there is a $1/\sqrt{s}$ dependence which affects the extrapolation to RHIC energies.

Although $\sqrt{-t}$ is more in line with a Regge parametrization, we cannot find a clear cut theoretical argument against using sinθ in the Pomeron spin-flip amplitude but the differences predicted in the high energy extrapolations obtained here and in Ref. [3] marks the importance of RHIC to solve this question.

If the kinematical zero is removed using $\sqrt{-t}$ and diffraction is present in the spin-flip amplitude, the Pomeron contribution to the spin-flip amplitude is smaller than to the spin-non-flip amplitude (about 10%). At the same time, the extrapolation to RHIC energies is predicted to be non-negligible and considerably larger than using sinθ. The slope $\beta_1$ is somewhat smaller ($\beta_1 = 4.74$ GeV$^{-2}$ instead of 6.25 GeV$^{-2}$ in [3]) but $\beta_2$ has approximately the same value ($\beta_2 = 2.29$ GeV$^{-2}$ instead of 2.30 GeV$^{-2}$ in [3]). The $P$ values at $\sqrt{s} \sim 500$ GeV may be accessible to measurement in this case since they can reach 10 percent near the region of the dip in $d\sigma/dt$.

We conclude that, only in the case of a diffractive (Pomeron) contribution to the spin-flip $pp$ amplitude, will RHIC be able to obtain a measurable polarization. Such a contribution was suggested to be present more than 30 years ago [1] and is not ruled out by present data. On the other hand, the fit without diffractive contribution in the spin-flip amplitude leads to a solution whose set of parameters is quite absurd.

Acknowledgements. One of us (AFM) would like to thank the Department of Theoretical Physics of the University of Torino for its hospitality and the FAPESP of Brazil for its financial support. Several discussions with Prof. E. Martynov and Prof. M. Giffon and the comments of Prof. O.V. Selyugin are gratefully acknowledged.

APPENDIX

A The spin-non-flip amplitude

The spin-non-flip amplitude utilized in this work is

$$a^{nf}(s, t) \equiv a_+(s, t) - a_-(s, t),$$  \hspace{1cm} (8)

where

$$a_+(s, t) = a_P(s, t) + a_f(s, t)$$  \hspace{1cm} (9)

and

$$a_-(s, t) = a_O(s, t) + a_\omega(s, t).$$  \hspace{1cm} (10)

The expressions for the two Reggeons used in [3] are

$$a_R(s, t) = a_R S^\alpha_R(t) e^{b_R t},$$  \hspace{1cm} (11)

and

$$\alpha_R(t) = \alpha_R(0) + \alpha'_R t, \ (R = f \text{ and } \omega)$$  \hspace{1cm} (12)
with $a_f(a_\omega)$ real (imaginary).

For the Pomeron, the non spin-flip amplitude is

$$a_P^{(D)}(s,t) = a_P s^{\alpha_P(t)} [e^{b_P(\alpha_P(t)-1)}(b_P + \ln \tilde{s}) + d_P \ln \tilde{s}]$$  \hspace{1cm} (13)

while for the Odderon, we choose

$$a_O(s,t) = (1 - \exp(\gamma t))a_O s^{\alpha_O(t)}$$

$$\times [e^{b_O(\alpha_O(t)-1)}(b_O + \ln \tilde{s}) + d_O \ln \tilde{s}],$$  \hspace{1cm} (14)

and again $a_P(a_O)$ real (imaginary). We use $\alpha_i(t) = \alpha_i(0) + \alpha_i' t$ where $i = P, O$.

Our definition for the amplitude follows [8] so that

$$\sigma_t = \frac{4\pi}{s}\text{Im}\{a_{nf}(s,t=0)\},$$  \hspace{1cm} (15)

$$\frac{d\sigma}{dt} = \frac{\pi}{s^2}(|a_{nf}(s,t)|^2 + 2|a_{sf}(s,t)|^2).$$  \hspace{1cm} (16)

In this work we retain the same parameters for the spin-non-flip amplitude as in [8] and we keep them fixed while fitting the parameters of the spin-flip amplitude. We utilize the dipole model at the Born level since most of the polarization data is contained in the $t$-domain corresponding to the region before the dip in $d\sigma/dt$ (well described without eikonalization). The values of the parameters of the spin-non-flip amplitude [8] are shown in Table 3.

|            | Pomeron | Odderon | $f$-Reggeon | $\omega$-Reggeon |
|------------|---------|---------|-------------|-----------------|
| $\alpha_i(0)$ | 1.071   | 1.0     | 0.72        | 0.46            |
| $\alpha_i'$ | 0.28 GeV$^{-2}$ | 0.12 GeV$^{-2}$ | 0.50 GeV$^{-2}$ | 0.50 GeV$^{-2}$ |
| $a_i$      | 0.066   | 0.100   | -14.0       | 9.0             |
| $b_i$      | 14.56   | 28.10   | 1.64 GeV$^{-2}$ | 0.38 GeV$^{-2}$ |
| $d_i$      | 0.07    | -0.06   | -           | -               |
| $\gamma$   | -       | 1.56 GeV$^{-2}$ | -           | -               |

Table 3: Parameters of the dipole model at the Born level with $i = P, O, f, \omega$ (from Ref. [8]).

To calculate the polarization we utilized the form

$$P = 2\frac{\text{Im}(a_{nf}(s,t)(a_{sf}(s,t))^*)}{|a_{nf}(s,t)|^2 + 2|a_{sf}(s,t)|^2};$$  \hspace{1cm} (17)

where the star on the numerator means complex conjugate.

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