Study on a New Measurement Method of Small Polarization Rotation Using the Common-Path Heterodyne Interferometry and a Half-Wave Plate

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Abstract. This thesis proposes a novel method for small polarization rotation measurement using a half-wave plate and a high-stability common-path heterodyne interferometry. The phase with azimuth angle 22.5° of a half-wave plate has a distinct change resulting from the small polarization rotation of the incident light. The optical rotation properties of a material, such as glucose solution, can be determined in this phase from the concentration of the solution. Finally, the phase can be measured using a lock-in amplifier, and the relevant formula between the glucose concentration and the polarization rotation can also be identified with a recording table.

1. Introduction
In the optical measurement domain, common-path heterodyne interferometry is a critical technology that features numerous advantages such as stability, precision, high linearity, and minimum environmental interferences. Many models that measure phase delay of birefringence materials are based on common-path heterodyne interferometry.

The objective of this study was to identify a new simple method that is easy to operate to measure slight phase delay in a sample and further derive the birefringence or concentration properties of the material. Therefore, this study combined common-path heterodyne interferometry and half-wave plates to create a measurement system.

Many methods for measuring wave plate phase delay have been proposed [1]–[8]. Feng et al. [9] developed an optical heterodyne interferometer by adding two acousto-optic modulators (AOMs) to a Mach–Zender interferometer to measure the concentration of glucose. The advantage of this method is that its phase measurement is related only to solution concentrations and irrelevant to the index of refraction of the sample. Moreover, the left-handed and right-handed polarized lights share one polarization path and are unlikely to be affected by the environment. Cameron and Coté [10] designed a closed loop control system that uses common-path heterodyne interferometry and two Faraday rotator polarization controllers to modulate polarization vectors. They also used a lock-in amplifier to measure phase shifts. Chiu [11] employed an electro-optic modulator and lasers with two wavelengths to measure the wave plates’ phase delay and the direction their principal axis. The advantage of such a system is the ability to measure wave plates as well as phase delay and axial position of ordinary birefringent materials. Márquez et al. [12] proposed a technique for measuring phase difference between positive and negative eigenvectors of liquid crystal spatial light modulator and added sawtooth wave voltage into an electro-optic modulator to provide time-varying phase shift and measure phase delay of two eigenvectors. The technique was defective for being unable to accurately measure actual phase delay of
any eigenvector alone. Lo and Hsu [13] proposed a method to measure birefringence by driving electro-optic modulators through a sinusoidal signal in the common-path heterodyne interferometer. This technique uses wider modulation frequencies than those of conventional techniques. Wang and Oakberg [14] developed a device to measure low-level linear birefringence of optical materials. This device uses the polarization of a modulated laser in a photoelastic modulator, and has been proved to be able to measure different low-level linear birefringence with high sensitivity and favorable repeatability. Ohkubo and Umeda [15] developed a near-field scanning optical microscope with birefringence contrast imaging. This system uses right and left circularly polarized lights and can simultaneously measure the angle of the principal axis and the phase delay of linear birefringence using a Zeeman laser.

The present study combined half-wave plates and common-path heterodyne interferometry to develop an experimental model. Because a half-wave plate at an azimuth of 22.5° exhibited considerable phase shifts, it was used to measure the phase delay of birefringent materials to determine a simple and easy-to-operate method.

2. Principle

2.1 Common-path heterodyne interferometry

The principles of a common-path heterodyne interferometer involve using a heterodyne source with difference frequency. When the source passes through a sample, s- and p-polarized lights generate different phase delay variations, which are related to the physical quantity of the sample. Fig. 1 illustrates the principles. An assumption is made to have the heterodyne source wave proceed along the z-axis. When the source passes through the sample, its incident ray enters the analyzer ANt (45°). Then, electric field strength along the x and y-axes can be obtained to determine interference. The interference signal can be converted into digital signals of periodic cosines Iout. Reference signals that have not passed through the sample and the test signals that have passed through the sample are simultaneously input into a lock-in amplifier or a phase meter to determine the phase difference values.

As depicted in Fig. 1, if the source proceeds along the z-axis, the electric field of difference frequency light generated through heterodyne sources along the x-axis (parallel to the optical table) can be represented as $E_x e^{i(\omega_1 t - \phi_1)}$; the electric field along the y-axis (vertical to the optical table) can be represented as $E_y e^{i(\omega_2 t - \phi_2)}$. The Jone’s matrix of the incident light into the sample can be written as

$$J = \begin{bmatrix} A_x e^{i\phi_1} & 0 \\ 1 & A_y e^{i\phi_2} \end{bmatrix}$$

(1)

The x-axis and y-axis polarized light from the sample induces phase delays as $\phi_1$ and $\phi_2$. After the light passes through the analyzer at 45° (ANt), the photodetector (Dt) detects an electric field ($E_{out}$) as follows:

$$E_{out} = \begin{bmatrix} A_x e^{-i\phi_1} \\ 1 \end{bmatrix} \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} E_x e^{i(\omega_1 t - \phi_1)} \\ E_y e^{i(\omega_2 t - \phi_2)} \end{bmatrix}$$

(2)

where $A_x$ and $A_y$ are, respectively, reflected coefficients and transmission coefficients of amplitudes of p-polarized light and s-polarized light passing through the sample. The final interference signal strength $I_{out}$ is as follows:

$$I_{out} = (A_x E_x)^2 + (A_y E_y)^2 + 2A_x A_y E_x E_y \cos[2\pi(f_1 - f_2)t - (\phi_x - \phi_y) - (\phi_1 - \phi_2)]$$

(3)
As revealed in Eq. (3), the interference signal is a cosine signal where \((f_1 - f_2)\) represents the difference frequency of two orthogonal linear polarization lights, \((\phi_1 - \phi_2)\) represents the phase difference of the sample, and \((\phi_x - \phi_y)\) represents the initial phase. Because the physical quantity of the sample is related to the phase variation of the sample, this study compared the reference signal that had not passed through the sample with the interference signal at the same frequency in terms of phase. The phase difference was \((\phi_1 - \phi_2)\), which was further used to determine the physical quantity of the sample. Compared with conventional interferometry, the proposed method was more convenient and accurate, making it suitable for applications related to precision measurements.

2.2 Acousto-optic modulator [16]–[18]

An AOM uses acousto-optical effects. When a piezo-electric transducer produces periodically varying vibrations under a radio frequency, grating is formed inside a crystal because of different indices of refraction; following the movement of grating, the incident ray is deflected. As shown in Fig. 2, when the incident ray passes through the AOM, it generates rays of +1, 0, and −1 orders according to the Doppler effect, yielding frequencies \(f - f_s\), \(f\), and \(f + f_s\). The frequencies range between dozens of MHz and hundreds of MHz. In this study, the AOM was used as the heterodyne source for the measurement experiment.

![Diagram of the common-path heterodyne interferometer.](image)

Fig. 2 Diagram of the common-path heterodyne interferometer.

In this experiment, the AOM was used to generate the necessary beat frequency for the experiment. The principle is to affect light transmission by forming grating, which can be achieved by using different indices of refraction of a crystal during acoustic wave transmission.

![Internal structure of an AOM.](image)

Fig. 3 Internal structure of an AOM.

Fig. 3. shows the movement of an incident ray in an AOM, where \(\vec{k}_t\) is the wave phasor of the incident ray, \(\vec{k}_r\) is the wave phasor of the emerging light, \(\vec{k}_s\) is the wave phasor of the acoustic wave, and \(\lambda\) is the wavelength of the acoustic wave.

According to the vector space, the relationship between \(\vec{k}_r\), \(\vec{k}_t\), and \(\vec{k}_s\) is written as follows:

\[
\vec{k}_s + \vec{k}_t = \vec{k}_r
\]  

(4)
The vector strength of $\vec{k}_s$ can be expressed as Eq. (5):

$$k_s = k_i \sin \theta_i + k_r \sin \theta_r$$

Because $k_i \equiv k_r = \frac{2\pi}{\lambda}$, $\theta_i \equiv \theta_r = \theta$, and $k_s = \frac{2\pi}{\lambda}$, Eq. (5) is revised as

$$\frac{1}{\lambda} = \frac{2}{\lambda} \sin \theta$$

When the incident ray is of a +1 order, then $\lambda = \frac{V_m}{f_m}$ and Eq. (6) is revised as

$$\sin \theta = \frac{\lambda f_m}{2V_m}$$

where $V_m$ is the velocity of the acoustic wave and $f_m$ is the modulating frequency of the acoustic wave.

To determine the frequency of the ray of +1 order, the relationship between $V_i$, $V_r$, and $V_m$ must be identified first.

**Relative velocity of $V_i$ against $V_m$**

$$V_i = V_m \sin \theta$$

**Relative velocity of $V_r$ against $V_m$**

$$V_r = V_m \sin \theta$$

**Relative velocity of $V_i$ against $V_r$**

$$V_i = -2V_m \sin \theta$$

Eq. (7) is substituted into Eq. (10)

**Relative velocity of $V_r$ against $V_i$**

$$V_r = \lambda f_m$$

Because $V_r = \lambda f_r$, it is substituted into Eq. (11) as

$$f_r = f_i - f_m$$

2.3 Experiment framework

The experimental framework is depicted in Fig. 4. Common-path heterodyne interferometry was combined with a half-wave plate to measure a sample with an internal diameter of 1 cm that contained glucose solution.

The motivation of this thesis is to discover a new simple method to measure small polarization rotation. Due to a large phase change near the azimuth angle of 22.5° of a half-wave plate, the small polarization rotation from the test medium generates a large phase change between the two orthogonal polarizations of the test light. Thus, the optical rotation can be measured using the common-path heterodyne interferometry and a half-wave plate.

First, in order to generate the heterodyne light source, the laser light is divided into $p$-polarized light and $s$-polarized light using a polarization beam splitter (PBS1), with the optical setup as presented in Fig. 4. The frequency of the $p$-polarized light passing through an acousto-optic modulator (AOM1) is of $f_0 = 80.00\text{MHz}$ for the +1 order diffraction light. Likewise, the $s$-polarized light after reflection by the mirror (M2), and incident into another acousto-optic modulator (AOM2), the +1 order diffraction light has frequency $f_0 = 80.01\text{MHz}$, where $f_0$ represents the optical frequency of the laser. Finally, the two lights are combined through another polarization beam splitter (PBS2) to obtain a heterodyne source with a frequency difference of 10kHz.

A beam splitter (BS) can split a heterodyne light source into transmitted and reflected light. The reflected light passes through an analyzer (ANr) with an azimuth angle of 45°, and is received by a photodetector (Dr). The detected signal is the reference signal $I_r$. The transmitted light passing through a sample, a half-wave plate ($W_{1/2}$) and another analyzer (ANt) at an azimuth angle of 45°, and detected...
by a photodetector (Dt), is called the. Finally, the phase difference between the reference signal $I_r$ and the test signal $I_t$ can be captured using a lock-in amplifier (SR830).

![Experimental system architecture](image)

**Fig. 4 Experimental system architecture**

### 2.4 Formula development

This section mainly involves formula development as depicted in Fig. 5. During the formula development process, the optimal measurement conditions were identified to obtain an innovative measurement method.

![Experimental framework](image)

**Fig. 5 Experimental framework.**

In Fig. 5, the Jone’s matrix of the reference signal can be written as

$$E_r = AN_r(45^\circ)E_1 = \frac{1}{2} \left[ A_x e^{j(\omega_1 t)} + A_y e^{j(\omega_2 t)} \right]$$

(14)
\[ I_r = |E_r|^2 = \frac{1}{2} A_x^2 + \frac{1}{2} A_y^2 + A_x A_y \cos(\omega t) \]  
\[ (15) \]

where \( A_x \) and \( A_y \) are electric field amplitudes along the x and y axes, respectively; \( \omega = \omega_2 - \omega_1 \).

If \( \Gamma \neq \pi \), the Jones’s matrix of the test signal can be expressed as

\[ E_t = AE_x e^{j(\omega_1 t + \delta)} + NE_y e^{j(\omega_2 t + \phi_0 + \delta)} \]
\[ (16) \]

where \( \phi_0 \) is the phase delay difference between \( E_2 \) and \( E_1 \); \( \delta \) is the phase delay difference of the sample \( \delta_2 \) - \( \delta_1 \), and \( \theta \) is the azimuth of the wave plate.

\[ M = \cos \frac{\Gamma}{2} - j(\cos 2\theta + \sin 2\theta) \sin \frac{\Gamma}{2} \]
\[ (17) \]

\[ N = \cos \frac{\Gamma}{2} + j(\cos 2\theta - \sin 2\theta) \sin \frac{\Gamma}{2} \]
\[ (18) \]

\[ \Gamma = |E_t|^2 \]
\[ = \frac{1}{2} \left[ \cos \left( \frac{\Gamma}{2} \right)^2 (A_x^2 + A_y^2) + \sin \left( \frac{\Gamma}{2} \right)^2 \left[ (\cos 2\theta + \sin 2\theta)^2 A_x^2 + (\cos 2\theta + \sin 2\theta)^2 A_y^2 \right] + 2\gamma \cos(\alpha - \beta) \right] \]
\[ (19) \]

where

\[ \gamma = \sqrt{\left( \cos \left( \frac{\Gamma}{2} \right)^2 - \sin \left( \frac{\Gamma}{2} \right)^2 \cos 4\theta \right)^2 + (\sin \Gamma \cos 2\theta)^2 A_x A_y} \]
\[ (20) \]

\[ \alpha = \omega t - \phi_0 + \delta \]
\[ (21) \]

\[ \beta = \tan^{-1}\left( \frac{\sin \Gamma \cos 2\theta}{\cos \left( \frac{\Gamma}{2} \right)^2 - \sin \left( \frac{\Gamma}{2} \right)^2 \cos 4\theta} \right) \]
\[ (22) \]

3. Results and Discussions

This study used glucose solution as the analyte sample. Because glucose solution is birefringent, a change in the concentration of the glucose solution results in slight polarization rotation. These results were observed in p- and s-polarized lights in common-path heterodyne interferometry. Consequently, a lock-in amplifier can be used to easily measure the phase variation. A half-wave plate was used without the sample to measure the phase delay of the half-wave plate. The experimental results are shown in Fig. 6. The solid line represents the MATLAB simulation results when \( \Gamma = +177^\circ \), and the dotted line represents the measurement results. The results confirmed that the phase delay (\( \Gamma \)) of the half-wave plate was +177^\circ.
Fig. 6 Experimental result of the half-wave plate.

When the azimuth angle of the half wave plate is 22.5°, the average slope of phase in the azimuth angle segment in the range [22.3°, 22.7°] is −103.29 (°/°) for a phase retardation of 177° of the wave plate. The phase change because of the optical rotation is greater than the retardation of the specimen. The degree of polarization rotation is defined as the azimuth rotation. This method measures the polarization change accurately, because it has a very high phase slope property, and a normal material has very small polarization rotation.

Fig. 7 Γ=177° degrees in the graph of 22.3° to 22.7°

The measured result when the half-wave plate is at θ = 0° is the phase retardation. Fig. 8. depicts the measurement results. The phase difference fluctuates, indicating that the change occurs only in the phase retardation, not in the polarization. The phase slope due to the polarization rotation at θ = 0° is zero. The phase distribution fluctuates with no correlation between the concentration of the sugar and polarization rotation, indicating that the phase change is only related to the axis of the birefringence of the sugar solution. The phase appears to fluctuate wildly when the speed axis changes irregularly and randomly.
The azimuth angle of half wave plate for the small polarization rotation measurement is $\theta = 22.5^\circ$. Fig. 9. shows the measurement results, showing an increasing phase change as the glucose concentration increases. The polarization rotation of the glucose solution causes this phase change is caused by, and is proportional to the phase difference.

Fig. 10. displays the measurement results of the degrees of polarization rotation of different glucose concentrations. The initial phase difference for the pure water is 92.5°. The phase change from the initial phase is divided by the average phase slope of $-103.29$ (°/°) is the level of polarization rotation. Thus, the relationship between the glucose concentration and the polarization rotation is obtained. If the phase resolution is 0.01°, then the optimal azimuth resolution (polarity rotation) $\approx 9.68 \times 10^{-5}$° = 0.35°.
Fig. 10 Relationship between glucose concentration and polarization angle

The polarization rotation can be regarded as the azimuth rotation. From Fig. 10, the negative value of the azimuth rotation indicates that the wave plate is equivalent to clockwise rotation, so the direction of the polarization rotation is inverse. In other words, the polarization has right-hand rotation.

4. Conclusions

If an optical rotation system with improved sensitivity can be used in future studies, glucose solutions with lower concentrations will be able to be measured, thereby elevating the analysis resolution to identify minute degrees of optical rotation. The application of such a technology in medical science can help measure the blood glucose concentration in patients with diabetes. In addition, the system can be applied in high-precision measurement to determine the optical rotation of materials.

This study demonstrates the feasibility of the proposed small polarization rotation measurement method. This method successfully measures small polarization variations of 0–0.035° for the phase resolution of 0.01°. The resolution of polarization rotation can reach to 0.35°, so can be analyzed for some research purposes that require the micro-rotation of polarization.

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