Theoretical Analysis and Finite Element Simulation of the Flat Compressive Stiffness of Oblique Column Reinforced Foam Sandwich Composites

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Abstract. In this paper, the finite element analysis of Ansys software and theoretical analysis was used to study the variation of the flat compression stiffness of the oblique column reinforced sandwich structure with the inclination of the rod and the radius of the rod. The results showed that the inclination angle of the rod increased from 45°. At 90°, the equivalent flat modulus increases by 2 times. When the tilt angle increases from 45° to 70°, as the angle increases, the equivalent flat modulus increases and the rate of change also increases, when the angle of inclination increases. When the angle is greater than 70°, the angle continues to increase, and the rate of change of the equivalent flat compressive modulus gradually becomes slower. The increase in the slanting column radius increases the flat compressive modulus, but increases the upper limit, and at the same time, the increase in radius causes an increase in mass, resulting in a decrease in the specific rigidity. The functional equivalent relationship between the modulus of elasticity and the structural parameters was deduced using the theory of mechanical equivalence. The functional relationship was used to predict the modulus of elasticity. The predicted results were in good agreement with the results of the finite element simulation. The configuration parameter selection provides reference guidance, and the functional relationship can also easily predict the influence of other structural parameters on the Z-stiffness of the sandwich structure.

1. Introduction
Foam sandwich composite materials are widely used in aviation, aerospace, navigation and transportation, due to their excellent bending stiffness and multi-functionality advantages [1]. In spite of this, foam sandwich composites still have two shortcomings: First, the compressive performance of the structure is mainly determined by the core material, generally it has relatively poor performance. Generally, the modulus, strength and other mechanical properties of the foam are generally low; the interface between the panel and the core material is usually weak, and under the bending load, the panel and the core material are easily deboned.

In order to improve the mechanical properties of the sandwich structure without losing its advantages of light weight and multi-functionality, different Z-direction (thickness direction) reinforcement methods are commonly used. These methods mainly include stitch-reinforced foam sandwich structures [2-6], Z-pin reinforced foam sandwich structure [7], dot matrix reinforced foam sandwich structure [8],

three-dimensional integral braided sandwich structure \cite{9, 10} and hybrid reinforced foam sandwich structure \cite{11}, these methods often There is no obvious boundary, and it has the sameness in both the molding process and the mechanical bearing mode. The enhancement effect of these methods is mainly determined by the structural parameters and distribution of the Z-direction reinforcement structure and the molding preparation process. After selecting a specific reinforcement method, the structural parameters and distributions need to be designed to achieve the expected mechanical performance index. Therefore, the structural parameters are studied. The influence of the overall mechanical properties has a very important significance.

In this paper, the oblique column lattice reinforced foam sandwich structure as the research object, using the finite element analysis software Ansys to simulate, combined with the theoretical analysis method. This paper studies the effect of the structural parameters such as the radius of the reinforced composite column, the angle of inclination on the overall flat compressive modulus, provides reference for optimization of oblique column structure design.

2. Parametric modelling and equivalent stiffness acquisition of lattice configuration
Lattice-enhanced lattice structures in foam sandwich structures are commonly known as straight-pillar, oblique-pillar, pyramidal, and tetrahedral types. In a broad sense, the above-mentioned types of lattice structures can be classified as one. Class that is oblique column lattice structure. The repetitive element model of the pyramid lattice configuration and slanted lattice configuration is shown in Fig. 1. In the geometry, the slanted lattice configuration can be translated, rotated, and combined to obtain the pyramid lattice structure. In the meantime, if the assumption that the constraint of the panel on the reinforced fiber column is an ideal fixed constraint, the force characteristics of the pyramid structure are also equivalent to the oblique column structure. Therefore, this paper only applies the finite element construction to the most basic oblique column repeat unit. Modal analysis is reasonable both in geometry and mechanics, and the analysis results have wide representativeness and reference significance.

Figure 1. Repeated element model of pyramid configuration and oblique column configuration

2.1. The establishment of a finite element model
The APDL parametric language of Ansys software is used to model the repetitive elements of slanted lattice lattice configuration. The structural parameters can be set as variables, which can be calculated after convenient assignment. In this paper, the finite element analysis is performed using the slanting column's radius and inclination as variables, and the influence of the parameters on the equivalent flat modulus is obtained.

The overall model is a regular quadrangular prism with a bottom width of \( W = 40 \text{mm} \) and a height of \( H = 50 \text{mm} \). The thickness of the upper and lower panels is \( D = 5 \text{mm} \). The angle of the oblique fiber column \( R \) is in the range of 1.5mm to 5mm, and the angle of inclination of the fiber column is in the range of 45° ~ 90°. Considering the actual use of puncture or suturing in preparation, the fiber column reinforcement method is used to model the upper and lower panels. The oblique sandwich reinforced sandwich model is shown in Figure 2. The material types and corresponding engineering constants for panels, fiber columns and foams are shown in Table 1.
Table 1. The Material Constants of Panels, Fiber Columns and Foam

|                      | Fiberglass twill/epoxy resin | Glass Fiber/Epoxy Resin | Polyurethane foam |
|----------------------|-----------------------------|-------------------------|-------------------|
| Engineering constant | Engineering constant value  | Engineering constant value | Engineering constant value |
| $E_1/E_2$/GPa        | 21                          | $E_1$/GPa               | 29                |
| $E_3$/GPa            | 6.42                        | $E_3$/GPa               | 3.75              |
| $G_{13}/G_{23}$/GPa  | 3.0                         | $G_{13}/G_{23}$/GPa     | 2.45              |
| $G_{12}$/GPa         | 3.5                         | $G_{12}$/GPa            | 2.0               |
| $\nu_{12}$          | 0.04                        | $\nu_{12}=$            | 0.26              |
| $\nu_{13}$          | 0.3                         | $\nu_{23}$              | 0.3               |
| $\rho$/kg·m$^{-3}$   | 2093                        | $\rho$/kg·m$^{-3}$      | 2093              |

Figure 2. The Repeat unit geometry model of oblique column

2.2. Calculation of equivalent stiffness

The equivalent stress and strain of the finite element model is calculated according to equations (1) and (2) for the geometrical average of the stress and strain of all elements. The equivalent modulus is then obtained from the ratio of equivalent stress and strain according to (3).

\[
\sigma_z = \frac{1}{V} \sum \sigma \cdot \Delta V \quad (1)
\]

\[
\varepsilon_z = \frac{1}{V} \sum \varepsilon \cdot \Delta V \quad (2)
\]

\[
E_z = \frac{\sigma_z}{\varepsilon_z} \quad (3)
\]

After establishing the finite element model, in order to obtain its equivalent flat compressive modulus, it is necessary to calculate the equivalent stress strain according to equations (1) and (2), but this average calculation will have errors, so the method of adopting displacement constraint is adopted. Loading, the bottom node adopts fixed boundary conditions, constrains all degrees of freedom, the upper floor is loaded with the freedom degree constraint, and the same Z-degree of freedom is applied. The final equivalent strain is the ratio of the applied Z-degree of freedom to $H$. The error introduced by calculating the average strain is avoided.
Using Ansys batch mode, command flow files with different parameters are run, and the results are saved in the output file to obtain equivalent modulus data corresponding to different parameters. The calculation of the equivalent modulus is also performed in the command flow. Using the element table operation and matrix calculation function of Ansys, the equivalent modulus can be calculated directly.

3. Theoretical analysis and FEM simulation of the flat compressive stiffness of composite materials with oblique columns and reinforced foam sandwich

The theoretical derivation of the equivalent flat compressive modulus of sloped-core foam-sandwich structural composites is based on the basic formulae of compressive and bending deformation in material mechanics, and reasonable mechanical equivalents and assumptions are made for complex sandwich structures, simplifying the mechanical model.

3.1. Theoretical analysis of flat compressive modulus of sandwich structure

Sandwich structure before and after deformation shown in Figure 3, the Figure 1, 2, 3 respectively represents the panel, the oblique column and the foam, distinguish different material types. When the compression displacement of the sandwich structure under the load is $\delta$, the upper and lower panels and the foam are analyzed as a whole, and the oblique column is individually analyzed. $F_{1,3}$ is the force applied to the panel and foam, and $F_2$ is the oblique column. Under the influence of the force, the deformation of $\delta$ occurs entirely, and the equivalent elastic modulus can be deduced from the overall force and deformation.

![Figure 3. Schematic view of the foam sandwich structure with oblique columns reinforced before and after deformation](image)

Firstly, the contribution of the diagonal column to the stiffness is analyzed separately. As shown in Fig. 4, when the deformation of $\delta$ occurs, the force carried by the oblique column is $F_2$. The action on the oblique column can be decomposed into the axial force $F_A$ and the shear force $F_S$, respectively, resulting in axial deformation and bending deformation.
The relationship between $F_2$ and $\delta$ can be obtained from equation (4) by the compression and bending equation and the geometric relationship of the cylindrical rod.

$$
\begin{align*}
F_A &= \frac{E_s\pi R^2 \delta \sin \theta}{H / \sin \theta} \\
F_S &= \frac{3E_s\pi R^2 \delta \cos \theta}{4(H / \sin \theta)^3} \\
F_2 &= F_A \sin \theta + F_S \cos \theta
\end{align*}
$$

The theoretical analysis of the sandwich structure panel and foam as a whole is based on the following two reasonable assumptions: ① The Poisson effect of the upper and lower panels and the foam of the sandwich structure is ignored; ② The contribution of the panel and foam to the stiffness is equivalent to the solid body stiffness contribution without the slanted hole minus the stiffness contribution of the slanted pillar shaped foam pillar (as illustrated in Figure 5).

![Figure 4. Analysis of oblique column](image)

![Figure 5. Equivalent sketch of panel and foam stiffness](image)
For the panel and foam overall analysis, hypothesis ② can be used as shown in Fig. 6. Analysis of the panel and foam without oblique holes can obtain the following relation between the strength of the panel and the foam $F_{1+3}$:

$$
\begin{align*}
\delta &= \varepsilon_1 \cdot 2D + \varepsilon_3 \cdot (H - 2D) \\
F_{1+3}' &= (\delta / H) \cdot E'W^2
\end{align*}
$$

(5)

$E'$ is the equivalent modulus of the panel and foam; $F_{1+3}'$ is the force of the solid body, minus the force of the foam slanting column is the actual force $F_{1+3}$ of the panel and the foam. The above obtained force is superimposed, and an equivalent modulus can be obtained based on the distortion $\delta$.

$$
E_Z = \frac{E_1E_2H}{HE_1 + 2D(E_3 - E_1)} + \frac{\pi(E_2 - E_3)R^2 \sin^2 \theta}{W^2} + \frac{3\pi(E_2 - E_3)R^4 \cos^2 \theta \sin^3 \theta}{4W^2H^2}
$$

(6)

From (6), we can see that $H$, $D$, $R$, $W$, and $\theta$ are all variables that can affect $E_Z$.

3.2. Influence of Oblique Angle on Flat Compressive stiffness

The total thickness of the entire sandwich structure was fixed, and different angles were calculated using Ansys. The equivalent modulus $E_Z$ with a fixed oblique fiber column radius $R=2$mm and $\theta$ from 45° to 90° was obtained, and $\theta$ and $E_Z$ were made. The relationship is shown in Fig.7. The scatter plots are the results of the finite element simulation. The continuous lines are the results of theoretical analysis.
Figure 7. Influence of slant angle of oblique column on flat modulus

From Fig. 7, comparing the results of theoretical derivation and finite element simulation results, it can be seen that the influence of $\theta$ on $E_Z$ is approximately the same, and the contribution of angle increase to the flat modulus increases, in the angle range of 45° to 70°, the increase of $\theta$ caused a significant increase in $E_Z$. When the angle continued to increase, the increase of $E_Z$ gradually slowed to 90° and reached the maximum value. The change trend was consistent with the experimental results obtained by Qiu Yanhui et al. [2]. After comparison and analysis of the data, the difference between the theoretical derivation results and the finite element simulation results is within 25%, where the difference is large when $\theta$ is small. The reason for this difference is that the Poisson ratio of the panel and foam is not considered in the theoretical derivation. Different effects of transverse stress; second, when $\theta$ is less than 90°, there is an error in the equivalent method proposed when analysing the panel and the foam equivalent modulus. The smaller the $\theta$, the greater the error is, specifically because the inclination of the oblique column is larger when the angle is smaller. The load on the slanted column consists of two parts: compression and bending. The bending component increases with the degree of inclination. The bending deformation resistance against the external load is derived in the theoretical derivation according to the formula of pure bending of the material mechanics. The actual bending is shear deformation. The contribution of shear deformation to stiffness is proportional to the shear modulus, and the fiber column shear modulus is much smaller than the axial modulus, so the oblique fiber is overestimated. The rigidity of the column leads to a large theoretical derivation.

3.3. Influence of Oblique Column Radius on Flat Compressive stiffness

The influence of the radius $R$ of the oblique column on the flat compressive modulus of the whole sandwich structure under the same inclination angle $\theta$ of the oblique column is studied. In this paper, fixed $\theta = 90^\circ$, and radius $R$ varies from 1.5 to 6mm, a series of change data is obtained by finite element analysis, Figure 8 shows the trend of $E_Z$ with $R$, the scatter is the result of finite element simulation, the curve is the result of theoretical analysis.
Figure 8. Effect of slanting radius on flat modulus

From the figure, it can be seen that as $R$ increases, the modulus of flat compression increases significantly, and the slope of the curve also increases. The result indicates that the sensitivity of the flat modulus to $R$ will also increase, but the upper limit is not shown in the figure because $R$ cannot be infinite increased, there is an upper limit of the flat modulus, and the maximum is the axial modulus of the fiber column. At the same time, we can see from the figure that the theoretical derivation results are in good agreement with the results of the finite element simulation. The reason is that when simulating the effect of $R$ on $E_Z$, $\theta$ is fixed at 90°. In the previous section, the angle was analysed. When the scale is large, the theoretical derivation is in good agreement with the finite element simulation. At this time, the flexural loading of the fiber column is the compressive load, and there is no shearing effect due to the bending deformation. The theoretical derivation results are more accurate, so the $\theta$ is relatively large. Under the circumstances, the study of the effect of $R$ on $E_Z$ can be easily predicted by using functional relationships.

4. Conclusion

Using the theoretical analysis of finite element simulation and reasonable simplification, the laws of the effect of slanting column radius $R$ and inclination angle $\theta$ on the overall Z-direction stiffness are obtained, and the following main conclusions can be obtained:

(1) When $\theta$ increases from 45° to 90°, $E_Z$ increases to 2 times. At the same time, when $E_Z$ increases from 45° to 70°, $E_Z$ increases significantly; when it exceeds 70°, the angle continues to increase, the increase rate of $E_Z$ gradually slowed down, with 90° reaching the maximum, which can fully exert the Z-direction reinforcing effect of the fiber column;

(2) When $R$ increases, $E_Z$ increases significantly, but it cannot increase indefinitely, and $R$ is not as large as possible. When $R$ is larger, the quality will increase significantly, and the specific stiffness will decrease. Therefore, in the actual design of $R$, the mass of the column need to take into further consideration;

(3) The variation law of $E_Z$ with $\theta$ and $R$ is studied by finite element simulation and theoretical analysis. It showed that the theoretical derivation results are in good agreement with the finite element results when $\theta$ is close to 90°. The theoretical formula has certain rationality. In the design of structural parameters, it can be designed according to the formula. The thickness $D$ of the panel, the overall thickness $H$, and the width $W$ of the repeating unit (the spacing between the oblique columns) that can be designed in the formula can all be designed to meet the required stiffness. Claim.
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