Appendix A

We provide in Table S1 below the first three occurrences of deviations in the values of the initially estimated seed, $s_0$, from the square root of $x$, with $x$ being a square unsigned integer number. The value of a deviation is computed as $|s_0 - \sqrt{x}|$. 
| Square number $x$ | Exact square root ($\sqrt{x}$) | Initial estimation of seed value ($s_0$) | Deviation of $s_0$ from $\sqrt{x}$ |
|------------------|-----------------------------|--------------------------------------|---------------------------------|
| 1                | 1                           | 1                                    | 0                               |
| 4                | 2                           | 2                                    | 0                               |
| 9                | 3                           | 3                                    | 0                               |
| ...              | ...                         | ...                                  | ...                             |
| 400              | 20                          | 20                                   | 0                               |
| 441              | 21                          | 21                                   | 0                               |
| 484              | 22                          | 23                                   | 1                               |
| 529              | 23                          | 24                                   | 1                               |
| 576              | 24                          | 25                                   | 1                               |
| 625              | 25                          | 25                                   | 0                               |
| 676              | 26                          | 26                                   | 0                               |
| ...              | ...                         | ...                                  | ...                             |
| 1444             | 38                          | 38                                   | 0                               |
| 1521             | 39                          | 39                                   | 0                               |
| 1600             | 40                          | 41                                   | 1                               |
| 1681             | 41                          | 42                                   | 1                               |
| 1764             | 42                          | 43                                   | 1                               |
| 1849             | 43                          | 44                                   | 1                               |
| 1936             | 44                          | 46                                   | 2                               |
| 2025             | 45                          | 47                                   | 2                               |
| 2116             | 46                          | 48                                   | 2                               |
| 2209             | 47                          | 49                                   | 2                               |
| 2301             | 48                          | 50                                   | 2                               |
| 2500             | 50                          | 51                                   | 1                               |
| 2601             | 51                          | 52                                   | 1                               |
| 2704             | 52                          | 53                                   | 0                               |
| 2809             | 53                          | 53                                   | 0                               |
| 2916             | 54                          | 54                                   | 0                               |
| 3025             | 55                          | 55                                   | 0                               |
| ...              | ...                         | ...                                  | ...                             |
| 5476             | 74                          | 74                                   | 0                               |
| 5625             | 75                          | 75                                   | 0                               |
| 5776             | 76                          | 77                                   | 1                               |
| 5929             | 77                          | 78                                   | 1                               |
| 6084             | 78                          | 79                                   | 1                               |
| 6241             | 79                          | 80                                   | 1                               |
| 6400             | 80                          | 82                                   | 2                               |
| 6561             | 81                          | 83                                   | 2                               |
| 6724             | 82                          | 84                                   | 2                               |
| 6889             | 83                          | 85                                   | 2                               |
| 7056             | 84                          | 87                                   | 3                               |
| 7225             | 85                          | 88                                   | 3                               |
| 7396             | 86                          | 89                                   | 3                               |
| 7569             | 87                          | 91                                   | 4                               |
| 7744             | 88                          | 92                                   | 4                               |
| 7921             | 89                          | 93                                   | 4                               |
| 8100             | 90                          | 95                                   | 5                               |
| 8281             | 91                          | 96                                   | 5                               |
| 8464             | 92                          | 97                                   | 5                               |
| 8649             | 93                          | 97                                   | 4                               |
| 8836             | 94                          | 98                                   | 4                               |
| 9025             | 95                          | 99                                   | 4                               |
| 9216             | 96                          | 100                                  | 4                               |
| 9409             | 97                          | 100                                  | 3                               |
| 9604             | 98                          | 101                                  | 3                               |
| 9801             | 99                          | 102                                  | 3                               |
| 10000            | 100                         | 103                                  | 3                               |
| 10201            | 101                         | 103                                  | 2                               |
| 10404            | 102                         | 104                                  | 2                               |
| 10609            | 103                         | 105                                  | 2                               |
| 10816            | 104                         | 106                                  | 2                               |
| 11025            | 105                         | 107                                  | 2                               |
| 11236            | 106                         | 107                                  | 1                               |
| 11449            | 107                         | 108                                  | 1                               |
| 11664            | 108                         | 109                                  | 1                               |
| 11881            | 109                         | 110                                  | 1                               |
| 12100            | 110                         | 111                                  | 1                               |
| 12321            | 111                         | 112                                  | 1                               |
| 12544            | 112                         | 113                                  | 1                               |
| 12769            | 113                         | 113                                  | 0                               |
| 12996            | 114                         | 114                                  | 0                               |
| 13225            | 115                         | 115                                  | 0                               |
| ...              | ...                         | ...                                  | ...                             |

**Table S1.** First three occurrences of deviations of the initially estimated seed values ($s_0$) from the exact square root of a square unsigned number $x$. 