The behaviour of Couette–Taylor flow in an azimuthal plane

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Abstract. We investigated the behaviour of Couette-Taylor flow in an azimuthal cross section by flow visualization using Kalliroscope flakes, and by image analysis of the obtained movie. Kalliroscope flakes are platelet and can make shear flow visible. Wavy vortex flow mode (WVF) in which an azimuthal wave mode superposes Taylor vortex flow mode (TVF) was investigated. Wave number and phase velocity of the azimuthal wave appearing in WVF at higher Reynolds number were determined by image analysis. Azimuthal flow structure of modulated wavy vortex flow mode (MWVF) was clearly visualized and the modulation component of MWVF was investigated by Fourier analysis of a temporal variation of brightness distribution extracted from a movie of visualized flow. Continuous wavelet analysis shows that spatial-temporal behaviour of MWVF on an azimuthal plane; the modulation first appears near the outer cylinder and spreads globally.

1. Introduction
Appearing a modulation wave in Couette-Taylor flow (CFT), flow between coaxial rotating cylinders, has been received great interests relating to chaos and also to a route to turbulence [1]. The modulation wave has two different propagation modes; in Gorman & Swinny (GS) mode, an additional thin vortex tube appears and affects the wavy motion of Taylor vortices [2,3]; in Zhang & Swinny (ZS) mode, an additional fluctuation component which has faster traveling speed appears and creates a small structure on the Taylor vortices [4]. Behavior of the modulation components with respect to Reynolds number was investigated quantitatively in detail [5,6]. Recently Thoroddsen and Bauer [7] attempted to visualize typical vortex pattern due to the modulation using reflective flakes and colored light sheets [8]. However, behavior of the modulation wave in an azimuthal plane, especially appearance and propagation of the wave in the plane, is still unclear.

Couette-Taylor flow has been investigated in great detail (collected in [8]). However, the almost all studies dealt with the axial-radial cross section on CTF experimentally or numerically. In this study, we investigated the azimuthal cross section of CTF by flow visualization using Kalliroscope flakes, which are platelets and can make shear flow visible as a contrast of brightness. It is aimed to clarify how the modulation of the flow appears at the transition from wavy vortex flow mode (WVF) to modulated wavy vortex flow mode (MWVF). The critical Reynolds number for the transition from WVF to MWVF is estimated using temporal variations of the brightness information obtained from the visualized image of the azimuthal cross section. We perform time-frequency analysis using continuous wavelet transform to investigate how the modulation affects wavy motion of the Taylor vortices.
2. Experimental setup

Fig. 1 shows experimental setup and main symbols. The coaxial rotating system of the cylinders has radius ratio $\eta = r_i/(r_i + d) = 0.905$ and aspect ratio $\Gamma = L/d = 20$, where $L$ is the height of fluid layer, 200 mm. The outer cylinder and the end walls are made of acrylic resin to allow the penetration of light. Angular velocity of the inner cylinder $\Omega$ is controlled by stepping motor. Reynolds number in this system is defined as $R = r_i d \Omega / \nu$, where $r_i$ is the radius of the inner cylinder, 95 mm, $d$ is the gap width, 10 mm. Flow transition including critical Reynolds numbers is consistent with early works [9]. Further details of the apparatus are specified in references [5,6]. Working fluid is water with Kalliroscope flakes, AQ-1000, are suspended in water with 1% concentration of volume. (refer to internet web site [10] and papers [11,12] for information about Kalliroscope for detail). The visualized flow in the azimuthal cross section was recorded by a high-speed video camera. Recording conditions are: 60 fps in flame rate, 1/1000 sec in shutter speed, 68 sec in sampling time and distance from the end wall of the cylinder to the lens of the camera is 1 m. The light sheet has a thickness of 3 mm thickness and is emitted to the cylinder perpendicular to the rotating axis. A reduced Reynolds number, $R^* = R/R_c$, is used instead of $R$, where $R_c$ is the critical Reynolds number from Couette flow mode (CF) to Taylor vortex flow mode (TVF), 134.8 in this configuration. An incident light sheet illuminated an azimuthal cross section between two vortices where the outflow occurs. Azimuthal flow motion was measured at each Reynolds number from $R^* = 7.5$ to 8.5 with a step of 0.1.

![Diagram of Experimental setup and main symbols](image)

3. Result and discussions

3.1. Frequency analysis of brightness variation

The azimuthal cross section is illuminated at an extent of 90 degree, and brightness distribution expresses wavy motion of Taylor vortices (Fig. 2 (a)). Azimuthal distribution of the brightness in this figure gives the azimuthal wave number of the wavy vortex flow as $m = 6$. Fig. 2 (b) is a temporal variation of the brightness information extracted from the movie at angle of 15 degree, where the radial position $r$ is normalized by $r^* = (r-r_i)/d$. There is a periodic variation of the brightness and its period agrees with the period of the wavy motion. Fig. 2 (c) shows a spatially averaged power spectrum of the brightness variation from $r^* = 0.1$ to 0.9. There are large peaks in the spectrum. These peaks are corresponding to the frequency component of WVF ($f_0$) and its harmonics ($2f_0$, $3f_0$ and $4f_0$). Fig. 3 shows a measurement results at $R^* = 8.5$, at which the flow state is MWVF. Brightness variation shown in Fig. 3 (b) has also a certain period but it is slightly modulated. Spatially averaged power spectrum of the brightness variation has peaks corresponding with $f_0$ and its harmonics. It also has a
peak at a lower frequency region, which corresponds to the modulation frequency \(f_m\). Additionally, there are sidebands as \(f_w - f_m\) and \(f_w + f_m\) around WVF component \(f_w\).

Fig. 4 shows the variation of the power spectrum with respect to Reynolds number from \(R^* = 7.5\) to 8.5 with a step of 0.1, where the power of the spectrum is normalized by a peak value \(f_w\) at each Reynolds number. With the increment of \(R^*\), \(f_w\) increases as well. The additional wave component \((f_m)\) appears around \(R^* = 7.8\). At the same \(R^*\), there are two peaks around \(f_w\) indicating an interaction of two components of \(f_w\) and \(f_m\). The critical Reynolds number for the transition from WVF to MWVF, \(R_m\), can be determined from this variation to be \(R_m = 1051\) \((R_m / R_c = 7.8)\). Meanwhile another frequency appears at \(f = 0.2\) Hz, which corresponds to the rotating frequency of the inner cylinder. Subsequently, we might not exclude a doubt that the appearance of the component may be caused by the defects of the apparatus.

Fig. 2 (a) visualized image of an azimuthal cross section, (b) temporal variations of the brightness information, and (c) power spectrum of the brightness variation \((R^* = 7.5)\)

Fig. 3 (a) visualized image of an azimuthal cross section, (b) temporal variations of the brightness information, and (c) power spectrum of the brightness variation \((R^* = 8.5)\)
Fig. 4 Variation of the power spectrum with respect to Reynolds number, which is constructed by 11 power spectrums from $R^* = 7.5$ to $8.5$ with a step of $0.1$.

3.2. Time-frequency analysis using continuous wavelet transform

Continuous wavelet transform of time signal $s(t)$ ($t$ is time) was computed as

$$ W(a,b) = \frac{1}{\sqrt{a}} \int s(t) \phi\left(\frac{t-b}{a}\right) dt, \quad (1) $$

where $W$ is wavelet coefficients and $\phi(t)$ is a mother wavelet which is a basis function. $a$ is a time scale of $\phi(t)$, namely it expresses a length on the time axis. $b$ is a central location of $\phi(t)$ on the time axis. We can obtain time-frequency behavior of the signal by changing $a$ and $b$. We accommodated the mother wavelet with Mexican Hat wavelet,

$$ \phi(T) = \left(1 - T^2\right)\exp\left(-\frac{|T|^2}{2}\right), \quad T = \frac{t-b}{a}. \quad (2) $$

Fig. 5 shows the obtained time-frequency maps of the brightness variation near the outer cylinder, $r^* = 0.8$, at each Reynolds number ((a) $R^* = 7.5$ and (b) $R^* = 8.5$), where the gray scale represents absolute value of the wavelet coefficient. Vertical axis and horizontal axis are normalized by inner cylinder rotating frequency $\Omega$. Figures beneath the time-frequency maps are original brightness variation for the wavelet transform. There is a short-periodic variation with small value of $a$ around $a\Omega = 0.12$. This corresponds to a component of WVF ($f_\nu$). The number of peaks of the variation is twice compared to the original data because absolute value is used in the time-frequency maps. The WVF component in Fig. 5 (b) is fluctuated slightly toward an axis of the time scale $a$. This implies that the WVF component is affected by the modulation. Weak but broad peaks appear at a larger region of the time scale in Fig. 5 (b). This having a broad range of the scale (frequency) appears with longer period which is roughly 4 times longer than that of WVF and is considered to reflect the nature of modulating wave. Assuming that each band corresponds to the wave period of WVF, it might indicate that modulation induces different characteristics for each wave appearing in the azimuthal direction.

Fig. 6 shows the results of the wavelet transform for each radial position; (a) $r^* = 0.3$, (b) $r^* = 0.5$, and (c) $r^* = 0.7$, where the Reynolds number is fixed at $R^* = 8.5$ (MWVF). In comparison of them, there is phase difference of the wave components on the radial position: similar pattern of fundamental wave components first appears near the inner cylinder later it appears at the outer one in turn (see parts enclosed by broken line in each figure). In contrast, the modulating wave components first appear at the outer position and propagate to the inner position. Moreover, the delay of the modulation components is smaller between Fig. 6 (a) and Fig. 6 (b) than between Fig. 6 (b) and Fig. 6 (c). The modulating wave first appears near the outer cylinder and spreads globally.
Fig. 5 (top) Time-frequency maps of the brightness variation obtained near the outer cylinder and (bottom) corresponding brightness variations extracted at $r^* = 0.8$, (a) $R^* = 7.5$ (WVF) and (b) $R^* = 8.5$ (MWVF).

Fig. 6 Time-frequency map of the brightness variation obtained at each radial point (a) $r^* = 0.3$, (b) $r^* = 0.5$, and (c) $r^* = 0.7$, where $R^* = 8.5$ (MWVF); broken and solid lines represent phase delay of wave components.
4. Conclusion
We investigated the transition from WVF to MWVF by flow visualization of azimuthal cross section using Kalliroscope flakes, and obtained the following results: (1) The critical Reynolds number from WVF to MWVF is determined as $R = 1051$. (2) The modulation induces different characteristics for each wave appearing in the azimuthal direction. (3) The modulating wave first appears near the outer cylinder and spreads globally.

References
[1] Gollub J B and Swinney H L 1975 Onset of turbulence in a rotating fluid Phys. Rev. Lett. 35 927
[2] Gorman M and Swinney H L 1982 Spatial and temporal characteristics of modulated waves in the circular Couette system J. Fluid Mech. 117 123
[3] Coughlin K T, et al. 1991 Distinct quasiperiodic modes with like symmetry in a rotating fluid Phys. Rev. Lett. 66 1161
[4] Zhang L H and Swinney H L 1985 Nonpropagating oscillatory modes in Couette-Taylor flow Phys. Rev. A 31 1006
[5] Takeda Y, et al. 1993 Experimental observation of the quasiperiodic modes in a rotating Couette system Phys. Rev. E 47 4130
[6] Takeda Y 1999 Quasi-periodic state and transition to turbulence in a rotating Couette system J. Fluid Mech. 389 81
[7] Thoroddsen S T and Bauer J M 1999 Qualitative flow visualization using colored lights and reflective flakes Phys. Fluids 11 1702
[8] Koschmieder E L 1993 Benard Cells and Taylor Vorteces (Cambridge: Cambridge University Press)
[9] DiPrima C and Swinney H L 1985 Instabilities and transition in flow between concentric rotating cylinder (Swinnt H L and Gollub J P ed. 1985 Hydrodynamic instabilities and the transition to turbulence (Springer)) 139
[10] http://www.kalliroscope.com/
[11] Park K, et al. 1981 Determination of transition in Couette flow in finite geometries Phys. Rev. Lett. 47 1448
[12] Tasaka Y, et al. 2008 Visualization of a Rotating Flow under Large-deformed Free Surface using Anisotropic Flakes J. Visualization 11 163