Algorithm for building a group incentive system in the implementation of engineering projects

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Abstract. Analysis of unsuccessful projects shows that the main reason is the violation of the time limits of the project. This implies the importance of project time management processes. It is known that measures aimed at reducing the time, as a rule, are associated with the occurrence of additional costs for performers. Naturally, the project manager must compensate for these costs, which occurs within the framework of the incentive system used. Given that engineering is a key area of the state economy, which largely determines its innovative development, it is very important to improve project management methods. It is shown how it is possible to carry out the development of a group incentive system aimed at ensuring a given value for reducing the duration of work. In this case, all the work, the duration of which must be reduced, is divided into groups and each such group has its own individual individual incentive system for all the works of this group. Naturally, such an incentive system is more expensive than an individual, but less than a universal one. The problem arises of designing group incentive systems that minimize the costs allocated to these goals.

1. Introduction

Modern engineering is one of the key sectors of the economy that determines the effectiveness of the development of the state and the degree of independence of its policy in the international arena. In the context of the dynamic development of the world economy, mechanical engineering makes it possible to implement an innovative scenario for the development of the country's economy. Unfortunately, Russian engineering is in a general systemic crisis when past, unresolved, problems have overlapped with modern ones. Evidence of the trouble in the industry is the issue of the “Crimean turbines”, which has already made a noise in the press, but is incomplete for the industry. As it turned out, Russian engineering is not able to produce power units of such power. This is due to the fact that engineering is a fairly high-tech industry, requiring the rapid development of scientific research. And here it is impossible to start from the middle, that is, from applied research, since it has long been known that the foundation for applied research is fundamental. But with the latter things are quite unimportant. This directly affects applied research, since they do not receive the proper material for work. That is why the ideas developed 30-40 years ago are still in use.

Against the background of the fact that the industry still exploits the basic ideas generated several decades ago, the management efficiency implemented in the industry is particularly acute. In this
regard, it is worth emphasizing that the engineering industry, especially focused on the production of unique products, such as high-power turbines, is an ideal model for applying modern management technology for project management. But, as you know, one of the key functions of project management is the development of incentive systems for performers for the results of the project [1-6].

The key characteristic of the project is the timing; that is, the project must be completed within the specified time frame, since very often outside of them the need for project results is greatly reduced. There is a task of managing the timing of the project.

2. Materials and methods

There is a project consisting of \( n \) works. During the preliminary design of the project implementation process, that is, the development of its schedule, it turned out that under the conditions considered, the project cannot be implemented within the time period set by the customer. There is a need to reduce the duration of the project to meet the deadlines.

Naturally, the reduction in the duration of the work is associated with additional costs for the contractor, which the project manager, in principle, must compensate. The value of costs, generally speaking, will be proportional to the value of the reduction in the timing of work, that is, determined by the value

\[
Z_i = k_j \cdot \Delta_j, \tag{1}
\]

where \( \Delta_j \) – the amount of reduction in the duration of the \( j \)-th job; \( k_j > 0 \) – proportionality coefficient.

In this case, expression (1) describes a linear incentive system [3] and the total cost for a particular group \( i \) will be

\[
S_i = \lambda_i \cdot T_i, \tag{2}
\]

where \( \lambda_i = \max k_j, T_i = \sum \Delta_j, Q_i \) – many group work \( i \).

If a jump-like incentive system is chosen for group \( i \) [3], then the minimum incentive fund for compensating the costs of performers for group \( i \) will be

\[
S_i = n_i \cdot \max (k_j \cdot \Delta_j), \tag{3}
\]

where \( n_i \) – number of jobs in the group \( i \).

There is a task of designing a system of incentives for performers, ensuring the implementation of the planned terms of the project.

In this case, it is necessary to determine the breakdown of the project work into groups in order to reduce the duration of the project and for each group to choose an incentive system so that the total cost of organizing the reduction of the deadlines is minimal. We will consider this problem in three versions. In the first, for all groups, only a linear stimulation system of type (1) is used, in the second - only systems of the jump-type stimulation class of type (3), and in the third case, systems of both types can be used.

Let’s consider another way of dividing into groups, namely, each group may include the number of jobs \( l \) within certain boundaries: \( l_1 \leq l \leq l_2 \). For example, if \( l_1=2, l_2=3 \), then each group may include either 2 or 3 jobs. For this case, it is also necessary to carry out an optimal breakdown of work into groups.

3. Results

Consider possible methods for solving the formulated problems. Suppose that for each group a linear stimulation system is applied. We arrange the work in ascending order (not descending order) \( k_i \), i.e. \( k_1 \leq k_2 \leq \cdots \leq k_n \). In this case, the following statement will be true:

Statement 1 [4]. If the group includes works with numbers \( i \) and \( j > i + 1 \), then the optimal solution will include all intermediate works in the same group.

In order to prove this statement, we arrange the groups of works in ascending (non-decreasing) parameter \( \lambda_j \), i.e. \( \lambda_1 \leq \lambda_2 \leq \cdots \leq \lambda_m \). Let \( j, k \in Q \) and there exists \( i < s < k \) such that \( s \in Q_j, p \neq j \) (figure 1).

\[...\]
Figure 1. Ranking of a group of jobs in increasing order of $\lambda_j$.

Let $Q_1=(1,3)$, $Q_2=(4,5)$. Swap jobs 3 and 2. If $k_3 \leq k_5$, then $\lambda'_1 = \max(k_1,k_2) < \lambda_1$, and $\lambda'_2 = \lambda_2$. Consequently

$$\lambda'_1 - k_3 + \lambda'_2 - k_2 > (\lambda'_1 - k_2) + (\lambda'_2 - k_3).$$

If $k_3 > \lambda'_2$, then $\lambda'_1 < \lambda_1$, and $\lambda'_2 = k_5$. Consequently

$$\lambda'_1 - k_3 + (\lambda'_2 - k_3) > (\lambda'_1 - k_2) + (\lambda'_2 - k_3).$$

The statement is proven.

Using the proved statement, the subsequent solution of the problem will be performed as follows. We form an oriented $(m+1)$-vertex graph without contours (figure 2) [7, 8]. The vertices of which (with the exception of the entrance) will correspond to the jobs, and $k_1 \leq k_2 \leq \ldots \leq k_n$. Draw arcs $(i, j)$ in the graph if the jobs from $(i+1)$ to $j$ form a group.

Figure 2. Auxiliary oriented (m+1) - vertex graph without contours.

We take the length of the arc $(i, j)$ equal to the stimulation fund of this group:

$$l_{ij} = \max_{x \in [i+1:j]} \sum_{y \in [i+1:j]} \Delta_y.$$

The main property of the graph shown in Figure 2 consists in the fact that any path in it connecting the input to the output and consisting of $m$ arcs determines the option of dividing the project $Q_j$, $j = 1, m$ work into groups, and the length of this path is equal to the size of the incentive fund. The problem boiled down to the following: determine the shortest path from $m$ arcs in a given graph [9, 10, 11].

To solve this problem, based on the original graph shown in Figure 2, we define the auxiliary network shown in figure 3. For the case of 4 works, the data are shown in table 1.

The constructed auxiliary graph is characterized in that it has four end vertices marked in Roman numerals in figure 3. The peak marked with the Roman numeral I corresponds to the system of unified stimulation, the peak IV - individual stimulation; the other two vertices describe different variants of
the group incentive system: vertex II corresponds to the case when the division is carried out by two jobs in a group, and vertices III to three trips. Thus, the original problem was reduced to the problem of determining the shortest path from the entrance to vertex II, if the condition was accepted to divide the project into groups of no more than two, that is $m=2$, and from the entrance to vertex III when the group is supposed to have three jobs, that is $m=3$ [3, 12].

Figure 3. Auxiliary network.

| $i$ | 1 | 2 | 3 | 4 |
|-----|---|---|---|---|
| $\Delta$ | 10 | 4 | 5 | 3 |
| $k_i$ | 1 | 2 | 3 | 4 |

Table 1. Initial data for building an auxiliary network.

Assume that the project manager accepted that a jump-like incentive system is used in each group. We arrange the work in order of increasing costs $M_j = k_i \Delta$, aimed at reducing the duration of the work, that is, a condition of the form $M_1 \leq M_2 \leq \cdots \leq M_n$.

In this case, statement 1 will still be true. The proof in this case will be completely analogous to the previous case [13]. The network of Figure 2 and the network of Figure 1. The lengths of arcs in this case are equal

$$l_{ij} = n_j \max_{s \in [i+1; j]} M_j.$$  

The optimal partition is determined by the shortest paths to the corresponding vertices.

Finally, we consider the option when for each group both a linear system and a jump-like stimulation system can be applied. In the general case, there is no numbering under which Statement 1 is true. Therefore, we have to consider all partitions of the set into $m$ subsets, the number of which grows exponentially with increasing $n$, следовательно, задача является $NP$-hard.

It is clear that the third, mixed option, when different incentive systems are applied in groups, will be most beneficial.

The main difficulty in using the obtained results is that the initial data should be the number of groups into which it is necessary to break down the whole set of project work and the number of work in groups, although there are options where the number of work in groups can be different. For such a case, a heuristic algorithm is proposed that gives results that are fairly close to optimal.

In order to determine the approximate number of groups into which the whole set of works can be divided, we prove the following two statements.

**Statement 2.** The larger the number of groups, the lower the total cost of organizing incentives.
The proof of the statement is based on the fact that the minimum amount of incentive costs in this case will correspond to the individual incentive system. By increasing the number of groups into which the entire set of work on the project is divided, we are thereby approaching the individual incentive system.

In problems of this type, the main difficulty is the need to first determine the number of groups into which it is necessary to break down all the project work. In many cases, this parameter is determined from the conditions of production of work and technological relations between them. But in some cases, this problem must be solved by the designer. In this case, the following statement will be true.

**Statement 3.** The maximum possible number of groups into which all project work can be divided will be determined from the expression \( m = \lfloor n/2 + 0.5 \rfloor \).

The proof follows from the assumptions that the minimum possible number of works in a group is two and the prohibition of placing one work in a group.

Consider the task of distributing project work into groups when designing a group incentive system under the most general assumption, that is, the number of groups is not specified, the number of works in groups, too. In this case, the following heuristic approach is possible, based on the idea that a series ranked by the size of the cost of organizing incentives will give an approximate distribution of work.

We arrange the project in descending order of costs. In principle, it can be intuitively assumed that the fewer jobs in each group, the lower the total cost of incentives. The ban on placement in groups of one job will look logical, since this practically means the use of an individual incentive system, which contradicts the original statement of the problem. Thus, the minimum number of jobs in the group should be two. Therefore, it is necessary to determine the feasibility of including third work in each of the groups. That is, to find out the question of whether this will increase costs compared with the case when in each group there will be two jobs or not.

In this case, we assume that the number of work in the project that needs to be divided into groups will be even. This is easily achieved in the process of organizational design of the process of performing work on a project. In this case, it is possible to split an individual project work into two operations, which should be considered as separate works or vice versa, to combine two work similar in technology into one. In any case, to achieve that the project has an even number of works is quite simple. For this case, it is advisable to propose a “matrix algorithm” of cost calculation. The procedure in this case is as follows:

1. **Step.** Arrange all project work in descending order of incentive costs to reduce the work duration.

2. **Step.** Generate a cost matrix of \( m \times n \), where \( m \) – is the number of groups into which the entire set of works is necessary to be divided, and \( n \) – is the number of works in the project. At the intersection of the \( i \)-th row and the \( j \)-th column, there will be the value of the total cost of organizing incentives in the case when the \( j \)-th job is included in the \( i \)-th group. Moreover, the matrix rows are filled under the assumption that some of the work has already entered into some groups. This means that the \( i \)-th row does not begin to be filled from the first column, but from the column that takes into account the presence of work that has not yet been included in the groups, that is, for the initial filling of the matrix, the condition \( j = i + 2 \) must be fulfilled. At the same time, we believe that the first group will contain two works, that is, the first and second, since, according to the initial condition, the group should not have one work. Subsequent groups will also initially contain two works and it is necessary to check the appropriateness of including a third work in each of the groups.

3. **Step.** Consider the process of forming an arbitrary \((i+1)\)-th group. In this case, the condition \( i < j \) must be satisfied. We find the element of the \((i+1)\)-th line different from zero and fixed its number \( j \). It is this element that needs to be checked for the appropriateness of inclusion in an existing group, that is, which will be more profitable: the inclusion of the work under consideration in the already created group \( i \) (при этом должно соблюдаться условие \( i \neq 0 \) must be met) or the work under consideration should serve as the basis for creating a new \((i+1)\)-th group. To do this, compare the costs that arise in both cases. To this end, we compare the elements of the cost matrix \( z_{i+1,j} \) and \( z_{j} \). In the event that the condition \( z_{i+1,j} < z_{j} \), is satisfied, this means that the inclusion of the work in question in the previous group \( i \) is impractical, since it leads to an increase in total costs and, therefore, it is necessary
to form a new \((i+1)\)-th group, which initially includes the works \(j\) and \(j+1\). Now, having fixed the number of the created group \((i+1)\) and the numbers of the works \(j\) and \(j+1\), included in this group, go to the beginning of step 3.

In the same case, when the condition \(z_{i+1,j} > z_{j,i}\) is satisfied, it follows that the \(j\)-th job must be included in the existing group with number \(i\). In this case, it is necessary to zero out the element \(z_{i+1,j}\) i.e. put \(z_{i+1,j} = 0\) and since now the elements \(j+1\) and \(j+2\) will be included in the proposed group. Accordingly, all elements of \(j+1\) and \(j+2\) columns in rows whose numbers are greater than the current one are also reset to zero. The number of the group \(i\) under consideration does not change, since it is supplemented by another work and the number of this work \(j\) is fixed. After which, you must go to the beginning of step 3. Since the number of rows and columns in the cost matrix is finite, the algorithm will complete in a finite number of steps.

4. Conclusion
Thus, the problems of synthesis of optimal group stimulation systems were considered. Effective solution algorithms have been proposed for some of them. Many synthesis problems are complex (in some cases – \(NP\)-hard). Heuristic algorithms are proposed for some of them.

A heuristic algorithm for the synthesis of a group incentive system is proposed when a preliminary specification of the number of groups and the number of jobs in groups is not required. Computational experiments have shown that in this case the optimal solution is often obtained, or the solution is close enough to the optimal.

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