Model of river channel for timber transportation

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Abstract. The model of river channel for timber transportation allows the most efficient delivery of timber to consumers; ensures the minimization of material loss and negative environmental impacts. The model is especially useful in case of rivers with short shipping season, since the delivery of goods is carried out in a very limited time frame when the levels change dynamically, when the shipping conditions are rarely favorable and when the information about the transport dimensions is absent. Information received from the pilot charts when the levels differ from the design ones is worth being verified, which might be time consuming. However, with the help of these charts and at rather low cost the model can be designed. The purpose of this work is to solve the aforementioned issue. The simulation model under study differs from most digital surface models. The riverbed is depicted as a combination of its cross sections. They are connected by a line passing through the water surface at a design level above the waterway. According to the size of the channel and radius of waterway curvature, the allowable raft sizes are calculated. The minimum sizes are referred to for calculating the route.

1. Introduction.
In most of the cases rivers are navigable only when the water level significantly rises during the spring flood. The duration of such period is short; usually it lasts for about one month. The rivers are used to ship timber. During the spring flood, a great number of rafts of the winter bundling are drawn along them. Besides, during this period, timber is actively shipped along these rivers.

Rivers with short navigation season are not usually served by basin authority of waterways and navigation. Hence, the navigable situation is not set for such rivers; information about the cargo size at certain levels is missing. This significantly complicates the transportation of goods, increases the likelihood of accidents and other abnormal situations associated with significant material losses and negative environmental consequences. Pilot charts of rivers can be useful in preventing the situations mentioned. A fragment of such a chart is presented in Figure 1.
Undoubtedly, being useful, pilot charts have significant drawbacks. A flat image of the charts allows you to set the width of the channel for timber transportation only at the depth corresponding to the isobaths, and at the design water level. Changes in the levels during the spring flood are very significant. The determination of mentioned width can be performed with great inaccuracy. When determining the size of channel for timber transportation, on which the allowable sizes of rafts and other transport units depend, it is necessary to consider the entire route. The length of the route is usually quite significant, which implies significant labor costs and the duration of the process of this definition. The latter is unacceptable for operational management. The digital model of the channel for timber transportation allows us to get rid of the aforementioned shortcomings.

The simulation of channel for timber transportation (in case of rivers) is similar to the modeling of river course. Creating a digital model of the river course similarly to the well-known development of the terrain simulation in this case is not appropriate. A unified horizontal and vertical basis of river courses under consideration, if such exists, is for a very small number of cross-sections, as a rule, for hydrometric cross-sections of water posts. The distances between these sections are quite significant. The available data is not enough to develop a conventional digital model. Obtaining of a large number of detailed course profiles with a unified horizontal and vertical basis built up over many points based on the materials of instrumental surveys is a very laborious task. Moreover, the solution of this problem does not contribute to high accuracy of determining the sizes of the channel for timber transportation, since the uniformity of water levels at different time in the cross-section of the reference water post does not guarantee corresponding uniformity in other cross-sections. As a result of re-formation of the course, the survey data becomes obsolete with time, which also does not induce the implementation of such labor-intensive works. We believe that approximately the same accuracy acceptable for timber transportation can be obtained using the aforementioned models created at relatively low cost using existing pilot charts. Information on the development of such simulation in literature was not found. Based on the above, it was decided to do this work.

The purpose of the paper is to obtain a digital model of timber transportation channel of the river basing on pilot charts with its implementation in a software application designed to improve the safety and efficiency of timber supply by water.

The research method is theoretical. The river mechanics and regularities of the structure of river course were taken into account in this work [2, 5, 6].

2. Materials and methods.
Defining the simulation concept, the following basic provisions were defined. The river course is a combination of a large number of its transverse profiles (Figure 2). Their relative position is established by means of a line passing along the surface of the water above the fairway at a design level. The distance between the sections of the profiles is set along this line. It is perpendicular to the planes of transverse profiles. The difference in mark-ups of the free water surface in the alignment (transverse slopes of the water surface) is neglected. The size and shape of each transverse profile are defined by several characteristic points. Each transverse profile is assigned a radius of curvature of the fairway $R_i$ in the adjacent section. The number of transverse profiles is determined by the degree of detail of simulation.
and the length of the river site under consideration. The transverse profiles are built for the cross-sections with the smallest values of the depth, width, radius of curvature of timber transportation channel in the adjacent section.

Each transverse profile (Figure 2) has its own two-dimensional coordinate system. Its starting point is located at the lowest point of the profile (on the fairway). The horizontal axis is \( y \), the vertical axis is \( z \). The \( x \) coordinate characterizes the distance of this profile from the course of the river. Measurement of the \( x \) coordinate is performed along the fairway line.

![Figure 2. Cross-section of the river course: 1 - lower floodplain; 2 – steep bank; 3 - upper floodplain; 4 - flooded edge; 5 – low-water bed](image)

The mark-ups of the water surface at the considered level are determined at each transverse profile through the level increase in the corresponding cross-section above the design one. The specified increase is set by the corresponding level increase in the cross-sections of the reference water gauges and distances to them.

The height of the points of the transverse course profiles is actually determined relative to the water surface at a design level. In this regard, it is preferable that the design level is as low as possible. In this case, the dispersion of water surface mark-ups in one or another cross-section is minimized at a level equal to the design one at the reference gauge post. Thus, the calculation error decreases.

Pilot charts of the river (Figure 1), as a rule, depict the fairway with the depths indicated on it at a certain interval, isobaths, shore horizons, flooded and non-flooded bank edges. On the maps of rivers of the type under consideration, one isobate is usually depicted on each bank. Behind the shore horizons, the heights of individual points above the design level are indicated partially. On the fairway line the distances from the course of the river are given in kilometers.

To the previously mentioned characteristic points that determine both size and shape of the transverse profiles, the points corresponding to the fairway, isobaths, shore horizons, the foot of steep bank and bank edges are shown (Figure 2). The development of the algorithm for determining the coordinates of these points was carried out taking into account the regularities of the course structure of lowland rivers set forth below [3, 11]. The lower floodplains are usually adjacent to the low-water bed from both sides (Figure 1, Figure 2). As a rule, they are falling a bit towards this course. At the concave banks, the lower floodplain from the side farthest from the fairway usually turns into an almost vertical steep bank. For rectilinear and, specifically, for convex banks the presence of steep bank is not typical. If they have one, then its steepness is significantly less than that of the concave banks. If the edge of the steep bank is flooded, then the upper floodplain is located behind it. Flooded edges have mark-ups similar in size to
those on the water surface with average long-term maximum expenditures of spring flood \[11\]. In some places on the pilot charts, the height of the edges above the water at the design level can be indicated. Information on the presence of a connection between mark-ups of flood-free edges and flow characteristics has not been revealed. According to the accepted classification \[3\], the transverse profiles can be simpler in configuration than shown in Figure 2. They can be considered as special cases of the depicted option.

In the process of simulation working with the i-th profile (Figure 2) and using the pilot chart (Figure 1), the \( x_i \)-coordinate corresponding to the chosen cross-section is specified. Next, we define the distance in horizontal direction from the line of the fairway to the edge, shore horizon, the isobaths of the left bank, i.e. the coordinates \( y_{1i}, y_{2i}, y_{3i} \), then the coordinates \( y_{4i}, y_{5i}, y_{6i} \), i.e. the horizontal distance from the fairway to the isobaths, shore horizon, edge of the right bank. If there are two or more eyebrows on one or another bank, the distance is measured to the nearest edge of the fairway. Also, using the map, the depth in the cross-section on the fairway is specified, as well as the depth corresponding to the isobaths, the height of the edges above the water surface (given in some cases) and the radius of curvature of the fairway line \( Ri \). When turning right, the radius of curvature is considered positive, while at the left it is considered negative. The established profile characteristics correspond to the design water level. Practically, only the distance from the fairway to the edges \( y_{1i}, y_{6i} \) and the distance of the cross-section from the estuary \( x_i \) are not related to it.

The relative position of characteristic points of transverse profile, corresponding to the lines of the fairway, isobates, and shore horizon is determined for almost all cross-sections by one algorithm. The algorithm for determining the location of characteristic points of the transverse profile that are above the water surface at a design level is set depending on the type of profile.

When constructing a part of the profile located between the shore horizon and the steep slope of concave bank, it is possible to make assumptions about the equality of the averaged slopes of this part and the profile fragment adjacent to it from the fairway. Besides, the value of the averaged slope of the considered area can be set. In the absence of information about this value, it can be taken equal to 200. This value obviously exceeds the actually encountered values \[9\]. The application of the second option is advisable, for example, in cases where the adjacent fragment of the transverse profile between the isobath and the shore horizon has an averaged slope of over 200. When assigning the slope of lower floodplain in this range, local conditions should be noted. They can be estimated using the same pilot chart, description of the river, materials of a reconnaissance expedition or remote sensing \[7, 8, 14, 15\]. If, for example, it is established that the lower floodplain has a large width, then its slope has to be less than 20\(^\circ\).

It is assumed that the part of the profile adjacent to the steep bank of concave bank extends to the vertical passing through the edge. The point of intersection of these lines was called the foot point of steep bank. Its mark-up should be less compared to the edge mark-up, which can often be determined. If specified condition is not fulfilled, the designated slope of the lower floodplain should be reduced. It is known that with an average annual consumption, the water surface is slightly higher than the top of the side \[11\] or, according to other data, approximately in the middle between the flooded edge and the side lines \[2\]. Thus, it is possible to focus on the mark-up of the foot of the steep bank, slightly exceeding the mark-up of the water surface with an average annual consumption if the mark-up of this surface can be identified.

The magnitude of an average long-term and average long-term maximum flow of spring flood is known for water gauging station. This also applies to the levels corresponding to these flows. The mark-ups of the water surface at these levels in the other cross-sections and, accordingly, the approximate values of mark-ups of flooded edges and feet of steep bank are determined in the same way as they are done at any other levels. The algorithm for this definition is discussed below.

An approximate determination of edges and feet of the steep bank at large values of the ratio of the course width to the depth typical for lowland rivers does not lead to significant errors when designing the transverse profiles.
Since a steep bank for the convex and straight-line banks is less typical, the foot point is not considered. After the point of shore horizon, the best and the most cautious option will be the edge point (Figure 2). It is acceptable to use similar method in the presence of doubts and in concave banks with flooded edges. Less cautious option is to adopt a slope of a part of profile adjacent to the shore horizon outside the convex and straight-line banks, in the same way as that of the concave bank. However, the considered fragment of profile stretches up to the intersection with the water surface with an average annual consumption or up to the outer boundary of coastal sands. When using the first method, it is necessary to check that the mentioned intersection is not located behind the flooded edge. If it happened, a fragment slope is required. The next profile section stretches up to the point corresponding to the mentioned edge.

In case of flooded edge absence regarding convex or straight-line banks, it is recommended to assign it with exaggeration, providing some reinsurance. It is advisable to use data reconnaissance, description of river or its specific section. Sometimes, large-scale topographic maps of relevant terrain may be useful. The rest of the profile near the flood-free edge is built without significant differences.

In case of designing a transverse profile of branched section of the river, one shipping (floating) sleeve is considered, the rest are ignored. In this case, the near center, the sleeves and other centers located behind it are considered as the lower floodplain. The near non-flooded island is considered shore. Let us note that the transverse profiles designed in this way can only be used when determining the sizes of timber transportation channel. The assessment of hydraulic characteristics assumed in the subsequent development of simulation on such profiles is not foreseen.

There are various options for designing transverse profiles on the considered characteristic points (Figure 2). One may get an approximating curve on their bases. In this case, it is advisable to use high-order polynomials. The river reach valleys usually have asymmetrical transverse profiles [11]. To design them, it is recommended that approximating dependencies be selected separately for the left and right half-profiles. You can use splines [10, 13], that is, to form a profile from separate fragments of curves that are described by various functions and pass through characteristic points. Modern software allows you to effectively and quickly solve problems of this type. As a result, it is possible to obtain profiles that are quite close to real ones (shown as a smooth curve in Figure 2). Characteristic points can be connected with straight-line segments. It is not a rare case when this option of profile design is acceptable, as shown in Figure 2.

Based on the information presented, we obtained the following formula for calculating the mark-ups of the considered characteristic points. Since the counting of mark-ups in each profile was agreed to be carried out from its lowest point, for any $i$-th cross-section on the fairway the bottom mark is $z_{i1} = 0$ m. At the same time the bottom marks at the points of isobath

$$Z_{Pi} = h_{Pi} - h_{Pi1},$$

where $h_{Pi}$, $h_{Pi1}$ are the depth in the cross-section on the fairway and isobaths lines at a design level, m.

The mark-ups of the coarse on the lines of the water edges and, accordingly, the water surface at the design level are equal to the corresponding depth in the fairway, i.e.

The mark-up of the steep bank foot point

$$z_{Pi1} = z_{Pi1} + \Delta z_{Pi},$$

where $\Delta z_{Pi}$ is the excess of steep bank foot point above the water surface at a design level, m.

In case of adopting the option of equality of average slope of profile sections adjacent to the water edge on the left bank

$$\Delta z_{Pi1} = \frac{z_{Pi1} - z_{Pi}}{y_{2i} - y_{i1}}(y_{1i} - y_{2i}),$$

on the right

$$\Delta z_{Pi1} = \frac{z_{Pi1} - z_{Pi}}{y_{6i} - y_{5i}}(y_{5i} - y_{6i}).$$

The presence of additional symbols “L” or “R” in the indices means that these values refer to the left or right bank, respectively.

If the slope of the lower floodplain is set, then on the left bank it may be presented as follows:
The change of level is set and adjusted according to the recommendations given above. According to them, the mark-up of the steep bank foot is finally identified. The mark-ups of steep banks are also set in accordance with the recommendations given above. If the height of the edge of the water surface is indicated on the pilot charts in the vicinity of this cross section, then the mark-up of this border is better determined by adding this height to the mark-up of the water surface.

In the model, it is possible to use more detailed profiles of stream gauge of water gauge stations obtained from instrumental measurements. It is advisable to use these profiles not only to determine the sizes of the channel of timber transportation, but also to calculate the hydraulic characteristics using well-known formulas [4]. Thus, it is possible to estimate the magnitudes of the flow velocities in the section at different levels used in the course of solving tasks related to timber transportation. A similar assessment using simplified profiles will be less accurate.

To determine the size and shape of the flow section in the i-th cross section, it is necessary to know the water surface mark-up under the conditions considered in addition to the coordinates of its characteristic points. In accordance with the above provisions, it is defined by the following formula:

$$z_{Ti} = z_{PRI} + \Delta h_i,$$

where $\Delta h_i$ is the water level increase in the i-th cross section above the design one, m.

The level increase over the design one may slightly change at one time or another within a fairly long stretch of the river. This change depends on many factors, most of which are random variables. These include, for example, factors associated with the intensity of snow melting in different parts of the catchment area, with the passage of showers in relatively small areas, etc. The identification of mathematical dependence of $\Delta h_i$ on $x$, taking into account these factors in our opinion is unproductive. Taking into account relatively small change in $\Delta h_i$ along the stream, we believe that in most cases it is possible to obtain the results of accuracy that are satisfactory for timber transportation tasks when making the assumption that this change is linear. Then, $\Delta h_i$ can be determined by the following formula:

$$\Delta h_i = \Delta h_T - \frac{x_{i-1} - x_i}{x_{i-1} - x_B} (\Delta h_T - \Delta h_B),$$

where $x_i, x_{i-1}, x_B$ are the distances along the fairway from the estuary to the i – the cross section and the cross section of the nearest water gauging station located downstream and upstream, respectively, km; $\Delta h_T, \Delta h_B$ is the level increase at specified water gauging station above the design one, m.

In doubtful and most critical cases, the level increase can be taken throughout the considered section to be equal to the smallest of the corresponding increase at the nearest water gauging station. If necessary, the accuracy of $\Delta h_i$ can be improved if observation is organized at the simplest intermediate water gauging station during the period of timber transportation.

The formulae (7), (8) are also used to determine the water surface mark-up in the design cross sections with the long-term average annual and the long-term average annual maximum rate. As it was noted, the mark-ups of the steep bank foot and flooded ridges can be established in this way.

You can enter an unlimited number of rectangular contours of the channel for timber transportation with any ratios of its width $b_i$ and depth $h_i$ at the edges. It is advisable to set the $h_i$ value, guided by certain precipitations of transport units T and bottom stock $\Delta a$, as well as to determine the value $b_i$ corresponding to a given depth $h$. The following formula shows the depth $h$ relation to the allowable draft $T$:

$$h = T + \Delta z,$$

The width of the channel for timber transportation is represented as a sum of the two components:

$$b_i = b_{1i} + b_{2i},$$

where $b_{1i}, b_{2i}$ located to the left and to the right of the fairway, respectively, constituting the width of the channel of timber transportation, m.

Before determining the values of $b_{1i}$ and $b_{2i}$, the bottom mark-up is calculated at the edges of the channel of timber transportation using the following formula:

$$z_{Cri} = Z_{Ti} - h.$$
Then, using the dependencies describing this profile, the magnitudes of \( z_{Kr} \) are used to determine the ordinates of two bottom points at the edges of the channel for timber transportation. In magnitude, the value of one of them is equal to \( b_1 \), and of the other value is equal to \( b_2 \). If the transverse profile of the course is based on the parts described by various functions, the calculation algorithm provides for the selection of corresponding function according to interval \( z \) where it is used. This relates to the option with the use of splines, as well as to the option of the connection of characteristic points of the profile considered below by straight line segments, where \( b_{ij} \) equal to - \( y_{ij} \) is calculated using the following formulae.

\[
\begin{align*}
\text{if } z_{Cr} & \leq z_{Pi}, \quad b_{1i} = \frac{|y_{2i}|}{z_{Pi}} z_{Cr}; \quad (12) \\
\text{when } z_{Pi} & < z_{Cr} \leq z_{Ri}, \quad b_{1i} = |y_{3i}| + \frac{|y_{2i}|}{z_{Ri}-z_{Pi}} (z_{Cr} - z_{Pi}); \quad (13) \\
\text{if } z_{Ri} & < z_{Cr} \leq z_{Pi}, \quad b_{1i} = |y_{2i}| + \frac{|y_{3i}|}{z_{Pi}-z_{Ri}} (z_{Cr} - z_{Pi}); \quad (14) \\
\text{when } z_{Cr} & > z_{Pi}, \quad b_{1i} = |y_{2i}|. \quad (14)
\end{align*}
\]

If the profile is completed by a segment connecting the points of the shore horizon and the edge of the bank when \( z_{Cr} > z_{Pi} \), instead of (13), (14), the following formula is used:

\[
b_{1i} = |y_{2i}| + \frac{|y_{3i}|}{z_{Pi}-z_{Ri}} (z_{Cr} - z_{Pi}). \quad (15)
\]

The calculation of \( b_2 \) values is carried out using similar formulae with the only difference in ordinate indices where instead of numbers 1, 2, 3, numbers 6, 5, 4 are written, respectively. The module signs in this case can be ignored.

For specific profiles, the algorithm can be partially adjusted without changing the general concept of the solution.

The determination of size of the channel for timber transportation is assumed for all transverse profiles on specified route, the minimum sizes are taken as final ones.

When towing rafts along a route, their sizes allowable are determined by the sizes and radius of curvature of the latter. The maximum raft width in the case of one-way traffic is calculated using the following formula [12]:

\[
B_{1nsi} = \frac{b_1}{1.5}; \quad (16)
\]

with two-way traffic

\[
B_{2nsi} = \frac{b_1 - 1.3R_{k}}{1.3}; \quad (17)
\]

where \( B_s \) is the width of the caravan of oncoming ships, m

During the model trial it was established that when \( B_s \) is less than 0.102b, (possibly with a large width of the channel for timber transportation), formula (17) is more effective compared to (16). Therefore, under the above condition, formula (16) should also be used in two-way traffic.

The maximum section length is calculated using the following formula [12]:

\[
L_{ci} = \frac{R_l}{3.0 ... 3.5}; \quad (18)
\]

maximum raft length

\[
L_{rai} = \sqrt{0.0025R_l^2 + 4.4R_l b_1 + 7.78b_1^2 + 3.42 b_1^3/R_l}. \quad (19)
\]

From the sizes of transport units obtained in this way for all calculated points of the route, the minimum ones are selected. They are accepted as the maximum allowable on the route.
Using the described algorithm, a model of the channel for timber transportation of the river Vaga was developed. On the basis of this model, a software application was created that ensures its practical use. The interface of the created application is clear (Figure 3). In the upper part of the I/O window, there are fields for entering input data. These are the levels at water gauge stations, the estimated draft of rafts, the distance of the upper and lower boundaries of the route from the estuary. After entering data, click on “Calculate” button. In the fields in the lower part of the window, there will appear information about the appropriate width of the channel for timber transportation, the maximum allowable section and raft lengths, as well as its width in cases of one-way and two-way traffic.

![Software I/O Window](image)

**Figure 3. Software I/O Window**

In the course of operational management, it is assumed to introduce current values of levels at reference water gauge stations, and in the course of preliminary planning of actions, i.e. the values of corresponding interest ratio, established through statistical processing of observational data, performed according to a known method [1, 12].

An application created for a specific river is intended for very limited number of users. The use of special expensive software products and non-cheap services of IT companies in this case is inexpedient, especially considering that due to changes in the channel and newly discovered data, it is assumed that the model should be updated once in while. In this regard, for model implementation, one of the most accessible software products MS Office Excel was used. Having read the instructions, even not very advanced user will be able to make necessary changes to the model according to the characteristic points of profiles. The spreadsheets allow data recording on profiles in separate lines, which makes it easier to change data. In this case, the calculation algorithm for each profile can be adjusted in accordance with its specific features. In the process of working with the application, opening of tables is not required. To use it, it is enough to have minimum computer skills. It is only data input/output window that the user can see (Figure 3). It was created in the integrated software environment MS Visual Studio using C# programming language. Data exchange between the aforementioned window and spreadsheets is provided using an open source project NPOI.

A working version of this application was also created for the upper section of the river Pinega. It was successfully used to justify the sizes of rafts of several modifications. The subsequent towing of these rafts made it possible to verify the justification performed.

### 3. Conclusions

Application of digital model of the river channel for timber transportation improves the efficiency of operational management in the field of transportation of timber by water and planning this process, reducing the likelihood of significant material losses and negative environmental impacts. The simulation is especially useful in case of rivers with short shipping season, since the delivery of goods is carried out in a very limited time frame when the levels change dynamically, when the shipping conditions are rarely favorable and when the information about the transport sizes is absent.

A digital model of timber transportation channel of the Vaga river was designed. It was developed using pilot charts taking into account the regularities of the structure of the channels of lowland...
watercourses. The model was implemented in a software application. A working version of this model was created for the upper section of the river Pinega. By analogy, the appropriate models can be designed for other lowland rivers. The completed designs and their application were not costly. They make it possible to determine not only the sizes of timber transportation channel, but also the sizes of rafts permissible under the terms of the route at given water levels at reference water gauge stations. The trial of the application made it possible to verify its normal functioning. The results obtained with the help of models were considered acceptable. The results obtained for the Pinega River were tested under production conditions.

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