Some considerations on skewness and kurtosis of vertical velocity in the convective boundary layer

Alberto Maurizi and Francesco Tampieri
Institute of Atmospheric Sciences and Climate

October 17, 2013

Abstract

Data of skewness $S$ and kurtosis $K$ of vertical velocity in the convective boundary layer from different datasets have been analysed. Vertical profiles of $S$ were found to be grouped into two classes that display different slopes with height: one is nearly constant and the other is increasing. This behaviour can be explained using a simple model for the PDF of vertical velocity and assuming two distinct vertical profiles of updraft area fraction from literature. The possibility of describing the explicit dependence of $K$ on $S$ was revised critically, also considering the neutral limit as well as the limit for very small non-dimensional height. It was found that the coefficients of the relationship depends on both the Obukhov length scale $L$ and inversion height $z_i$.

1 Introduction

Although the empirical knowledge of atmospheric turbulence is increasing steadily, a number of still unresolved questions remain, due in large part to the intrinsic nature of the atmospheric boundary layer (ABL). In general, a combination of field observations, laboratory and numerical experiments is necessary to improve our understanding.

For instance, third- and fourth-order moments of vertical velocity are difficult to measure because of the concurrent intrinsic unsteadiness of the ABL on the one hand, and the need for long, steady samples for reliable high-order moment estimations on the other. Nevertheless, third-order moment data have been considered at least since Chiba (1978) and, along with fourth-order moments, continue to receive attention. Very reliable data in neutral conditions have been collected for years in wind tunnel experiments (see Durst et al. (1987), for instance). In the convective boundary layer (CBL), a large amount of observations and large eddy simulations (LES) are available: a summary can be found in Lenschow et al. (2014, hereinafter LLMSC).

Closures for Reynolds-averaged Navier-Stokes equation models of ABL are of crucial importance in numerical weather prediction, as well as climate and atmospheric composition modelling. As far the CBL is concerned, the need for closures based on moments higher than 2 arises from the nonlocal nature of the vertical mixing mechanism. A rather common approach is to solve equations for the third-order moments, and use a closure for the fourth ones (see, for instance, Canuto et al. (1994)).
The earliest closure on fourth-order moments was the quasi normal (QN) approximation dating to Millionshchikov (1941). It assumes that fourth-order cumulants of velocity correlations are zero as in the Gaussian distribution, and expresses the principle of “maximum ignorance”[^1]. It was formulated for two point statistics (spatial correlation) but can be simply applied to one point statistics, i.e., for null separation. The QN approximation has been used for instance by Losch (2004) and Canuto et al. (2007).

On purely statistical ground, the relationship $K \geq S^2 + 1$ defines the region of existence of probability density functions (PDFs) in the ($S$, $K$) space (see, e.g., Pearson, 1916; Kendall and Stuart, 1977). Consequently, the QN approximation is unrealizable for $S^2 > 2$. Based on this consideration, Maurizi (2007) suggested that the least biased assumption on fourth-order statistics, a “revised” QN approximation, is $K = 3(S^2 + 1)$, which is always realizable and reduces to Gaussian for $S = 0$.

Early attempts to organize third- and fourth-order moment data in the ($S$, $K$) space are found in Durst et al. (1987) for the neutral boundary layer and in Maurizi and Tampieri (1999) for different stability conditions. A parabola-like relationship is often observed in different turbulence regimes (Durst et al., 1987; Shaw and Seguin, 1987; Maurizi and Tampieri, 1999) and is used to write closure relationships (Alberghini et al., 2002; Grvanik and Hartmann, 2002; Cheng et al., 2005; Gryanik et al., 2005).

It is noteworthy that empirical relationships and theoretical approaches (e.g., based on mass-flux) give basically the same results, consistent with the “revised” QN hypothesis.

Moreover, the same qualitative results are found for data of completely different nature (see, e.g., Sattin et al., 2009b; Cristelli et al., 2012), reflecting the purely statistical nature of the relationship (Sattin et al., 2009a), which, due to the curve that limits the existence of PDFs, produces a parabolic curvature of the ($S$, $K$) space. This also reflects on other properties, e.g., Lagrangian correlation time and information entropy (Maurizi and Lorenzani, 2001; Quan et al., 2012).

The purpose of this article is to extend the analysis by LLMSC of the closure problem in ($S$, $K$) space by reviewing the recent literature and bringing together information from already available data. The different data are compared and merged in Section 2, where an interpretation is given of the skewness behaviour for different convection classes. Moreover, the behaviour very near the ground is analysed. In Section 3, the skewness-kurtosis relationship is discussed in terms of changing stability and distance from the wall (ground surface). In the final Section, some conclusions are drawn.

## 2 Bringing data together

### 2.1 Vertical profiles

LLMSC produced a very accurate vertical profile dataset of vertical velocity in CBL measured by a LIDAR Doppler, also providing estimations of stability parameters. Another dataset considered here is the set of measurements taken in the Rome area (Mastrantonio et al., 1994) by a...
SODAR Doppler, which constitute the basis of the [Alberghi et al. (2002)] and [Tampieri and Maurizi (2003)] articles. The two datasets differ, in particular, in the range of height covered. Figure 1 displays the number distribution of measurements with respect to the normalised measurement height $z/z_i$ for the two datasets. The AMT dataset has a 90% quantile of 0.39, while the LLMSC one has a 10% quantile of 0.34. This highlights the complementarity of the two datasets.

Vertical profiles of $S$ and $K$ of both datasets are shown in Figures 2 and 3. TM and LLMSC provide independent classifications into two groups. The TM classification is heuristically based on the trend of $S$ with $z/z_i$, for $z/z_i < 0.5$. The two classes are “constant $S$” and “increasing $S$”. The LLMSC classification is based on the convection strength (“weak” and “strong”), measured as $z_i/L$. Figure 2 shows that the two classifications are consistent. The “constant $S$” TM class corresponds to the LLMSC “weak convection”, while the “increasing $S$” class corresponds to LLMSC “strong convection”. Independent linear fits of the two data sets were performed for $z/z_i < 0.8$. A similar slope was found between LLMSC and TM for each class (see Figure 2). The Figure clearly reveals a systematic difference ($\Delta S \simeq 0.15$) between the two datasets, which could originate from the different measuring systems: LIDAR and SODAR sample different volumes, leading to different cuts in the turbulence spectrum. It can also be observed that, according to TM, the “strong convection” class displays a smaller $S$ for small $z/z_i$ than the “weak convection” class.

As far as $K$ is concerned, the two classes cannot be distinguished and the two datasets are fully compatible.

A simple kinematic model can justify the two classes on the basis of the vertical profile of the area fraction of updrafts $A$. Assuming the [Baerentsen and Berkowicz (1984)] model for vertical velocity PDF, $S$ and $K$ can be expressed as a function of $A$ as

$$S = \frac{2}{A(1-A)(1-2A)}$$

(1)

and

$$K = \frac{51 - 3A + 3A^2}{2A(1-A)}$$

(2)

respectively. The observed vertical profile of $A$ shows a certain degree of variability. For instance, LLMSC suggest a model for the vertical velocity that implies a constant $A$, while data from [Young (1988)] and the
Figure 2: $S(z/L)$ (left) and $K(z/L)$ (right) from LLMSC and TM. Fit are performed on data with $z/z_i$ below 0.8. The second fit on TM data is performed adding a bias of 0.15 to $S$.

Large Eddy Simulations of Schumann and Moeng (1991) support a weak variation with height, with a minimum around $z/z_i = 0.5$. According to Equation (1), in the lower part of the CBL, decreasing $A$ corresponds to strongly increasing $S$, while $K$ displays smaller variations with height. This behaviour is quite general and not related to the specific PDF: it can be obtained even for the simplest case of two $\delta$ distributions. The kinematic model suggests that the vertical distribution of the updraft area fraction is a function of $L/z_i$.

2.2 Neutral limit

In order to investigate the neutral limit for skewness and kurtosis, new data (Maurizi and Robins, 2000) from a wind tunnel experiment performed at EnFle$^4$ are considered. Figure 4 shows that within the surface layer, skewness of the vertical component of velocity in neutral conditions is nearly constant at $S \simeq 0.05$ and $K \simeq 3.5$.

Data from the SGS2000 experiment using the ISFF facility$^3$ analysed by Barberis (2007) add further complexity to the picture. Measurements were performed over a relatively uniform surface, using two arrays of sonic anemometers with height above the ground that was varied during the experiment from 3.45 to 8.66 meters. A stationarity analysis was performed to select periods long enough to make fourth-order statistics significant.

$^2$http://www.surrey.ac.uk

$^3$ http://www.eol.ucar.edu/isf/facilities/isff/index.html
Figure 3: As in Figure 2 but for $K$ and without fitting lines.

Figure 4: Vertical profiles of $S$ and $K$ as a function of nondimensional height $z^+ = z/z_0$ in neutral conditions [Maurizi and Robins, 2000].
These data cover a relatively large range of stability, and a very small range of $z/z_i$ near the ground that cannot be covered by remote sensing instruments such as LIDAR and SODAR.

Because in the CBL $z/L \to 0$ as $z/z_i \to 0$, while approaching the surface the flow tends to neutrality. Consequently, vertical velocity statistics should approach the neutral $S$ and $K$ values. However, ISFF data (Figure 5) show that even in near neutral conditions a far more pronounced asymmetry can appear for flows having small negative values of $z/L$. The asymmetry introduced by a forcing mechanism like convection far more marked than that caused by the presence of the wall.

The rather abrupt sharp transition from the convective value of $S$ to the neutral one calls for a phenomenological analysis. Monin-Obukhov similarity theory (MOST) suggests that vertical velocity variance $w^2$ scale on $u^2_*$ for small $|z/L|$ and on $w^2_* \propto |z/L|^{2/3}$ for free convection conditions (Kader and Yaglom 1990). Analogous dependences can be derived for $w^3$. However, many observations and theoretical considerations suggest that the thickness of the boundary layer must enter into the scaling relationship, because of the influence of the largest eddies even on the near surface values. As far as the horizontal components of velocity are concerned, this point has been extensively discussed by Yaglom (1994).

As a working hypothesis, it can be suggested that a term depending on $z_i/L$ should also be included in the expression for the vertical velocity variance.

An empirical expression for skewness based on the previous considerations is

$$S = \frac{c_{00} + c_0 |z_i/L| + c |z/L|}{(b_{00} + b_0 |z_i/L|^{2/3} + b |z/L|^{2/3})^{3/2}}$$

which reduces to standard MOST form setting $c_0$ and $b_0$ equal to 0.

From the literature, a possible choice of constants is: $b = 3$, $b_{00} = 1$, $b_0 = 0.1$. Then, the third-order moment coefficients can be chosen in order to match the asymptotic values of skewness for neutral conditions (0.05) and for free convection (0.33). In Figure 5 Equation (3) is reported for the case $c_0 = 0$ $b_0 = 0$ and for the complete expression with $z_i/L = 500$. It can be seen that, in the second case, the drop is much sharper than in the first one. Thus, the skewness behaviour can be qualitatively explained using proper scaling relationships that account also for parameters not included in MOST. In Figure 5 the expression of Chiba (1978) is also reported: it shows a smoother transition, similar to our first case, consistent with the absence of the CBL height in the formulation.

3 Considerations on the S-K relationship

In $(S, K)$ space a lower bound to $K$ is known to exist: $K \geq S^2 + 1$ for any PDF. Less known is the fact that, due to the Gauss-Winckler inequality, $K \geq 1.8$ for a unimodal symmetric PDF [Kendall and Stuart, 1977, Vol I, p. 95]. This result can be extended also to non-symmetric PDFs ($S \neq 0$). Using the Gauss-Winckler inequality and substituting the expression of non-centered moments in terms of centered ones, a relationship between $K$ and a measure of the asymmetry can be established:

$$K \geq 1.8 + f(S, S_P),$$

where $S_P$ is a measure of skewness, defined by Pearson as:

$$S_P = \frac{\mu_1 - m}{\sigma}$$
in which $\mu_1$ is the mean of the distribution, $m$ is the (single) mode and $\sigma$ is the standard deviation. Even if the explicit form of $f$ is not known in general, it can be easily estimated for Pearson’s family of (unimodal) PDFs as, in this case, $S_P$ has a simple representation in terms of $S$ and $K$. This particular form of $f$ is reported in Figure 6. Although this is not the most general one, it can be argued that different PDFs would give a similar parabola-like relationship. In addition, numerical experiments (Maurizi and Lorenzani, 2001) confirm that PDFs based on simple models become bi-modal while approaching the lower bound. Since two-value velocity implies infinite accelerations, by extension, a highly bi-modal PDF cannot be considered as a reliable representation of turbulence statistics. Therefore, data are expected to lie well above the unimodal limit and models should account for it.

The $S$ and $K$ data used so far are reported in Figure 6. AMT and LLMSC data cover different regions of $S$. This makes the fit proposed by AMT, where $K \to 2.4$ as $S \to 0$, more suitable for small $S$, while the Gryvanik and Hartmann (2002) fit, cited by LLSMC, is more suitable for large $S$. Merging the two, a new fit can be proposed in the form of $K = a_2 S^2 + a_0$ with $a_2 = 1.8$ and $a_0 = 2.4$. The proposed relationship uses two parameters as in Gryvanik and Hartmann (2002), but provides a better description of the small $S$ limit, according to AMT. It is worth noting that this parameterisation is appropriate only for data below $z/z_i < 0.65$. At higher levels it can be found (mainly for the weak convective cases) that $K$ increases for decreasing (or constant) $S$, preventing the possibility of describing $K$ as an explicit function of $S$ for the whole CBL. Similar behaviour is observed also in LES (see LLMSC, their Figure 11).
Figure 6: Summary of $S$ and $K$ for different experiments. The LLMSC data above $z/z_i = 0.8$ are excluded. Curves are as described in the figure legend. Data connected by lines are LLMSC weak convective cases for $0.65 < z/z_i < 0.8$.

Another group of data (EnFlo and ISFF) clusters around a parabolic curve with a different parameter: $a_0 = 3.3$. These data encompass both purely neutral and convective cases with $z/z_i \ll 0.1$. It may be argued that the coefficients of the parameterisations depend on both $z/L$ and $z/z_i$, with a dependence on $z/z_i$, strong for the 0 to 0.1 range and very weak for $z/z_i > 0.1$.

4 Conclusions

The observations regarding skewness and kurtosis in the convective boundary layer analysed in this paper show that there is an intrinsic variability in turbulence behaviour which cannot be simply explained in terms of differences in surface fluxes.

The dependence of skewness on height is interpreted in terms of the area distribution of updrafts and downdrafts, which is an important aspect of convection physics (an extensive discussion was given by Hunt et al., 1988). The correct representation of the vertical velocity PDF is considered a key factor when dealing with dispersion problems, as well as in parameterisations for numerical weather prediction models. Thus, the present results suggest a careful consideration of these issues.

The transition from unstable to near neutral conditions in the surface layer is sharper than expected from currently used similarity functions for $\bar{w}^2$ and $\bar{w}^3$: the available data show values of skewness around 3.3 in the surface layer for $z/L < -0.1$ and 0.05 for $z/L = 0$, with a sub-
stantial variability. The Chiba [1978] formula did suggest a smoother transition. Also in this case, effects not represented by the standard similarity approach, e.g., the limitation to eddy size due to finite CBL height (Kader and Yaglom [1990]), may become important in shaping such transitions.

Finally, the discussion about the $S-K$ relationship highlights that it cannot be considered a universal one, but is likely to depend on the structure and dynamics of the boundary layer under consideration.

The conclusions are that some tiles into the mosaic are added but also questions for future research are raised.

Acknowledgements

The software used for the production of this article (data analysis, plotting, typesetting) is Free Software. The authors would like to thank the whole free software community, the Free Software Foundation (http://www.fsf.org) and the Debian Project (http://www.debian.org).

References

Alberghi, S., A. Maurizi, and F. Tampieri, 2002: Relationship between the vertical velocity skewness and kurtosis observed during sea-breeze convection. J. Appl. Meteorol., 41, 885–889.

Baerentsen, J. H. and R. Berkowicz, 1984: Monte-carlo simulation of plume diffusion in the convective boundary layer. Atmos. Environ., 18, 701–712.

Barberis, E., 2007: Analisi statistiche nello strato limite turbolento, thesis, Univ. Torino, Dip. Fisica.

Canuto, V. M., Y. Cheng, and A. Howard, 2007: Non-local ocean mixing model and a new plume model for deep convection. Ocean Modelling, 16, 28–46.

Canuto, V. M., F. Minotti, C. Ronchi, and R. Ypma, 1994: Second-order closure pbl model with new third-order moments: comparison with les data. Journal of the Atmospheric Sciences, 51, 1605–1618.

Cheng, Y., V. M. Canuto, and A. Howard, 2005: Nonlocal convective pbl model based on new third- and fourth-order moments. Journal of the Atmospheric Sciences, 62, 2189–2204.

Chiba, O., 1978: Stability dependence of the vertical velocity skewness in the atmospheric surface layer. Journal of the meteorological society in Japan, 56, 140–142.

Cristelli, M., A. Zaccaria, and L. Pietronero, 2012: Universal relation between skewness and kurtosis in complex dynamics. Physical Review E, 85.

Durst, F., J. Jovanovic, and L. J. Kanevce, 1987: Probability density distribution in turbulent wall boundary-layer flows, Turbulent Shear Flows 5, F. Durst, B. E. Launder, J. L. Lumley, F. W. Schmidt, and J. H. Whitelaw, eds., Springer Verlag, Berlin.
Gryanik, V. and J. Hartmann, 2002: A turbulence closure for the convective boundary layer based on a two-scale mass-flux approach. *Journal of the Atmospheric Sciences*, 59, 2729–2744.

Gryanik, V., J. Hartmann, S. Raasch, and M. Schroter, 2005: A refinement of the millionshchikov quasi-normality hypothesis for convective boundary layer turbulence. *Journal of the Atmospheric Sciences*, 62, 2632–2638.

Hunt, J. C. R., J. C. Kaimal, and J. E. Gaynor, 1988: Eddy structure in the convective boundary layer - new measurements and new concepts. *Quart. J. Roy. Meteor. Soc.*, 114, 827–858.

Kader, B. A. and A. M. Yaglom, 1990: Mean fields and fluctuation moments in unstably stratified turbulent boundary layers. *J. Fluid Mech.*, 212, 637–662.

Kendall, S. M. and A. Stuart, 1977: *The Advanced Theory of Statistics*, vol. 1, 4th ed., C. Griffin & Co., London.

Lenschow, D. H., M. Lothon, S. D. Mayor, P. P. Sullivan, and G. Canut, 2012: A comparison of higher-order vertical velocity moments in the convective boundary layer from lidar with in situ measurements and large-eddy simulation. *Boundary-Layer Meteorol.*, 143, 107123.

Losch, M., 2004: On the validity of the millionshchikov quasi-normality hypothesis for open-ocean deep convection. *Geophysical Research Letters*, 31.

Mastrantonio, G., A. Viola, S. Argentini, G. Fiocco, L. Giannini, L. Rossini, G. Abbate, R. Ocone, and M. Casonato, 1994: Observations of sea-breeze events in rome and the surrounding area by a network of doppler sodars. *Boundary Layer Meteorology*, 71, 67–80.

Maurizi, A., 2007: Quasi normal hypothesis revised, *Advances in Turbulence XI, Proceedings of the 11th EUROMECH European Turbulence Conference*, vol. 117 of *Springer Proceedings in Physics*, Springer, pp. 603–605.

Maurizi, A. and S. Lorenzani, 2001: Lagrangian time scales in inhomogeneous non-Gaussian turbulence. *Flow, Turbulence and Combustion*, 67, 205–216.

Maurizi, A. and A. Robins, 2000: Boundary-layer flow and dispersion over a two-dimensional hill; high-order statistics of the flow and concentration fields, experiment performed at EnFlo, UniSurrey, UK.

Maurizi, A. and F. Tampieri, 1999: Velocity probability density functions in Lagrangian dispersion models for inhomogeneous turbulence. *Atmos. Environ.*, 33, 281–289.

Millionshchikov, M. D., 1941: Theory of homogeneous isotropic turbulence. *Dokl. Akad. Nauk SSSR*, 32, 611–614.

Pearson, K., 1916: Mathematical contributions to the theory of evolution. xix: second supplement to a memoir on skew variation. *Phil. Trans. Roy. Soc. A*, 216, 432.
Quan, L., E. Ferrero, and F. Hu, 2012: Relating statistical moments and entropy in the stable boundary layer. *Physica A*, **391**, 231247.

Sattin, F., M. Agostini, R. Cavazzana, G. Serianni, P. Scarin, and N. Vianello, 2009a: About the parabolic relation existing between the skewness and the kurtosis in time series of experimental data. *Physica Scripta*, **79**, 045006.

Sattin, F., M. Agostini, P. Scarin, N. Vianello, R. Cavazzana, L. Marrelli, G. Serianni, S. J. Zweben, R. J. Maqueda, Y. Yagi, H. Sakakita, H. Koguchi, S. Kiyama, Y. Hirano, and J. L. Terry, 2009b: On the statistics of edge fluctuations: comparative study between various fusion devices. *Plasma Phys. Control. Fusion*, **51**, 055013.

Schumann, U. and C.-H. Moeng, 1991: Plume fluxes in clear and cloudy convective boundary layers. *Journal of Atmospheric Sciences*, **48**, 1746–1757.

Shaw, R. H. and I. Seginer, 1987: Calculation of velocity skewness in real and artificial plant canopies. *Boundary-Layer Meteorology*, **39**, 315–332.

Tampieri, F. and A. Maurizi, 2003: Investigations on convective boundary layer turbulence using sodar data. *Annals of Geophysics*, **46**, 451–457.

Yaglom, A. M., 1994: Fluctuation spectra and variances in convective turbulent boundary layers: a reevaluation of old models. *Phys. of Fluids*, **6**, 962–972.

Young, G. S., 1988: Turbulence structure of the convective boundary layer. Part II: Phoenix 78 aircraft observations of thermals and their environment. *Journal of Atmospheric Sciences*, **45**, 727–735.