Momentum transport away from jet in expanding medium

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We study the transport dynamics of momenta deposited from jets in ultra-relativistic heavy-ion collisions. Assumining that the high energy partons traverse expanding quark-gluon fluids and are subject to lose their energy and momentum, we simulate di-jet asymmetric events by solving relativistic hydrodynamic equations numerically without linearization in the fully (3+1)-dimensional coordinate. Mach cones are formed and strongly broadened by radial flow of the background medium. As a result, the yield of low-\(p_T\) particles increases at large angles from the jet axis and compensates the di-jet momentum imbalance inside the jet-cone. This provides a novel interpretation of di-jet asymmetric events observed by the CMS Collaboration.

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\textbf{Introduction.}— The quark gluon plasma (QGP), supposed to have filled the early universe a few microseconds after the Big Bang, is the deconfined state of quarks and gluons realized under an extremely hot and dense condition \cite{1}. In heavy ion collisions at Relativistic Heavy Ion Collider (RHIC) in BNL and Large Hadron Collider (LHC) in CERN, the QGP is experimentally created. By colliding relativistically accelerated heavy nuclei, extremely high-temperature is achieved in the experiments. From the analysis of experimental data of elliptic flow, it has turned out that the QGP behaves like an almost-perfect fluid because of the strong interaction among the constituent particles \cite{2–6}.

Jets, namely partons with large transverse momenta, are created in hadron/nuclear collisions at collider energies. In nuclear collisions, these partons are subject to traverse a hot and dense medium. During traversing the medium, the parton loses its energy through strong interaction between them \cite{7–13}. Through the amount of lost energies, one can extract one of the fundamental properties of the medium, namely stopping power of the QGP against high energy partons. Besides, the energy-momentum deposition from jets excites the medium and propagates at a specific angle from the direction of the energetic parton. Mach cones are commonly used in various theoretical studies such as hydrodynamics with \cite{16,18} or without \cite{19–21} linearization, AdS/CFT calculations \cite{22,23}, and a parton transport model \cite{24,26}. Here we emphasize that the background QGP medium is no longer static, but expands with relativistic flow velocity. The resultant Mach cones should be distorted by the expansion of the QGP \cite{19,21,26,27}.

In this Letter, the dynamical transport process of energy and momentum deposited from energetic partons traversing the expanding QGP fluid is studied. We give a novel interpretation of the low-\(p_T\) enhancement at the large angles from the jet axis as observed at LHC as a consequence of the medium response to the energy-momentum deposition of the high energy partons. Here we use relativistic hydrodynamic framework to describe the medium response. This is the first attempt to numerically solve relativistic hydrodynamic equations with source terms without linearization in fully (3+1)-dimensional Minkowski coordinates. In this way, we properly take account of the interplay dynamics between the hydrodynamical expansion of the QGP and the collective flow induced by the energy-momentum deposition of jets.

In the following, we first overview the current experimental situation of di-jet asymmetry in high energy nuclear collisions at the LHC energy. Motivated by these findings, we formulate relativistic hydrodynamic equations with source terms which correspond to deposition of jets’ energies and momenta. We next solve these fully non-linear hydrodynamic equations without resort to linearization of the equations. Finally we investigate energy and momentum balance in di-jet events.

\textbf{Di-jet asymmetry.}— At the leading order, back-to-back...
back partons are created with equal transverse momenta. Contrary to the conventional di-jet events in hadron-hadron collisions, jet energies are apparently imbalanced in nuclear collisions: One parton going toward the outside of the medium is observed as a leading jet and the other one going inside is observed as a sub-leading jet. Thus the amount of the lost energy is different between the pair due to the position of the pair creation. The asymmetry ratio to quantify the di-jet transverse momentum imbalance is defined as

$$A_J = \frac{p_{T1} - p_{T2}}{p_{T1} + p_{T2}},$$ \hspace{1cm} (1)

where $p_{T1}$ and $p_{T2}$ are the transverse momentum of the leading jet and of the sub-leading jet, respectively. In central Pb-Pb collisions at LHC, a mean shift of $A_J$ to higher values is observed [14 28]. The increase of highly-asymmetric di-jet events compared with p+p collisions indicates a substantial amount of jet energy loss in the medium at LHC. To see the balance of the whole transverse momentum in an event, one can define transverse momentum along the jet axis as

$$p_T^{||} = \sum_i p_i^T \cos(\phi_i - \phi_1),$$ \hspace{1cm} (2)

where the sum is taken over all tracks in a di-jet event and its transverse momentum is projected onto the sub-leading jet axis $\phi_1 = \phi_1 + \pi$ in the azimuthal direction. $p_T^{||}$ is measured in di-jet events in Pb-Pb collisions at LHC by the CMS Collaboration [14]. The transverse momentum averaged over events, $\langle p_T^{||} \rangle$, turned out to vanish within uncertainties even in large di-jet asymmetric events. The leading jet dominantly contributes to negative $\langle p_T^{||} \rangle$, which is balanced by the lower momentum particles with $0.5 < p_T < 8$ GeV/$c$ in the direction of the sub-leading jet outside the cone $\Delta R = \sqrt{\Delta \phi^2 + (\Delta \eta)^2} > 0.8$. Thus an apparent imbalance of the di-jet momenta only inside the cone is compensated by the low-$p_T$ particles at large angles from the jet axis. Since the low-$p_T$ particles play an important role in momentum balance of di-jet asymmetric events, it has been suggested that the energy deposition from the traveling partons wakes the QGP medium and induces collective flow to enhance low momentum particles at large angles from the axis of the quenched jet.

**Hydrodynamic Model with source terms.**—Motivated by these observations, we investigate the mechanism of energy and momentum transported away from jet in di-jet events. Assuming local thermal equilibrium, we perform relativistic hydrodynamic simulations to describe the space-time evolution of the QGP medium. We introduce source terms in the hydrodynamic equations which exhibit the energy and momentum deposition from these partons:

$$\partial_\mu T^{\mu\nu}(x) = J^{\nu}(x).$$ \hspace{1cm} (3)

Here $T^{\mu\nu}$ is the energy-momentum tensor of the QGP fluid and $J^{\nu}$ is the 4-momentum density deposited from the traversing jet partons. We solve the non-linear hydrodynamic equations numerically without linearization with a new high-precision scheme in fully (3+1)-dimensional coordinates [29].

For perfect fluids, the energy momentum tensor can be decomposed as

$$T^{\mu\nu} = (e + P)u^\mu u^\nu - Pg^{\mu\nu},$$ \hspace{1cm} (4)

where $e$, $P$, $w$, and $g^{\mu\nu} = \text{diag}(1, -1, -1, -1)$ are energy density, pressure, 4-flow velocity, and the Minkowski metric, respectively. Assuming that the deposited energy and momentum are instantaneously thermalized inside a fluid cell, the source terms for a pair of massless particles traveling through the medium are given by

$$J^\mu(x) = \sum_{a=1,2} J^\mu_a(x),$$ \hspace{1cm} (5)

$$J^\mu_a(x) = -\frac{dp^\mu_a}{dt} \delta^{(3)}(x - x_a(t)),$$ \hspace{1cm} (6)

$$J_a(x) = \frac{P_a}{p^\mu_a} J^\mu_a(x),$$ \hspace{1cm} (7)

where the index $a$ denotes the each energetic parton which is to be observed as the leading ($a = 1$) or the sub-leading jet ($a = 2$). For a given equation of state, we solve Eq. (3) in the (3+1)-dimensional Milne coordinates $(\tau, x, y, \eta_s)$ numerically without linearization. $\tau = \sqrt{t^2 + z^2}$ is the proper time and $\eta_s = (1/2) \ln [(t + z) / (t - z)]$ is the space-time rapidity. As an equation of state, we employ that of the ideal gas of massless quarks and gluons, $P(e) = e/3$, for simplicity. In this framework, we can handle an expanding background QGP fluid created in heavy ion collisions together with its response to propagation of di-jets. It should be noted that very small deposited energy and momentum relative to the total energy and momentum of the medium are treated here, so it is necessary to keep the energy-momentum conservaton in the whole system at very high precision. A new and robust scheme, which we developed and employed here, plays a crucial role to conserve the energy and momentum of the fluid accurately in full (3+1)-dimensional Milne coordinates and is essential for this calculation [29].

We set up the initial QGP fluid at $\tau_0 = 0.6$ fm/$c$. Around the mid-rapidity region, the initial energy density is flat in the $\eta_s$ direction like the Bjorken scaling solution [30]. The flat region is smoothly connected to vacuum at the both ends by using a half Gaussian [31]:

$$H(\eta_s) = \exp \left[ -\frac{(\eta_s - \eta_{\text{flat}}/2)^2}{2\sigma^2_{\eta}} \right] \theta \left( \eta_s - \eta_{\text{flat}}/2 \right),$$ \hspace{1cm} (8)

where $\eta_{\text{flat}}$ and $\sigma_{\eta}$ are the rapidity length of the flat region and the width of the Gaussians, respectively. Then full
initial energy density distribution is factorized as
\[ e(τ = τ_0, x, y, η_h) = e_T(x, y) H(η_h). \] (9)

Here \( e_T \) is the smooth transverse profile of the initial energy density for central (0-5%) Pb-Pb collisions. We calculate the number density of participants and binary collisions using Monte-Carlo Glauber model. We assume the entropy density distribution is proportional to the linear combination of these two densities. The distribution is normalized by comparison of final multiplicity at mid-rapidity with the LHC data [32]. Then, by using the equation of state, \( e_T \) is obtained. We choose \( η_{\text{flat}} = 10 \) and \( σ_η = 0.5 \) for Pb-Pb collision at LHC [32].

For the energy loss of the partons, we employ the collisional energy loss [34]
\[-\frac{dp_0^0}{dt} = A \times \frac{0.5}{3} πα_s^2 T^2 \left( 1 + \frac{1}{6} n_f \right) \log \frac{\sqrt{4T p_0^0}}{m_D}. \] (10)

Here \( α_s = g^2/4π \) is the strong coupling constant, \( n_f \) is the number of active flavors in the QGP medium, \( m_D = \sqrt{1 + \frac{1}{3} n_f g T} \) is the Debye mass, and \( A \) is a parameter which allows us to control the strength of the energy loss. We set \( n_f = 3 \) (u, d, s), \( α_s = 0.3 \), and \( A = 15 \).

The Cooper-Frye formula [32] is used to obtain the momentum distribution of particle species \( i \) from hydrodynamic outputs,
\[ p_0^0 dN_i = g_i \left( \frac{2π}{(2π)^3} \right)^3 \int \frac{p^μ dσ_μ(x)}{\exp[p^μ u_μ(x)/T(x)] \pm 1}. \] (11)

where \( g_i \) is the degeneracy and ± corresponds to Fermi/Bose distribution for particle species \( i \). The freeze-out is supposed to occur at fixed proper time \( τ_f = 9.6 \text{ fm}/c \), which is a typical value for central Pb-Pb collisions and not crucial for results presented here. Thus \( p^μ dσ_μ = p_T \cos(η_p - η_a) τ_f dx dy dη_p \), where \( η_p \) is the momentum rapidity. Here we set as a rapidity cut \( |η_p| < 2.4 \). Suppose \( (p_T^a) = (-p_T \cos(φ_p - φ_1)) \) contains contribution from particles originated from fluids, it is calculated from Eq. (11) as
\[ \langle p_T^a \rangle_{\text{fluid}} = -\int dφ_p dϕ_p p_T cos(ϕ_p - φ_1) \frac{dN_i}{dp_T dϕ_p}. \] (12)

Adding the transverse momentum of the traversing parton pair to this, we obtain \( (p_T^a) \) to be compared with the data.

**Results.**— A pair of back-to-back partons is supposed to be created at \( (τ = 0, x = x_0, y = 0, η_p = 0) \) with the common energy, \( p_0^0(τ = 0) = 200 \text{ GeV} \). Until \( τ_0 \), these partons travel without interacting medium. Then they start to interact the expanding QGP fluid at \( τ_0 \) and travel in the opposite direction along the x-axis. We can control jet asymmetric parameter \( A_J \) by changing the initial position of pair creation: When the position of the pair creation is off-central, namely \( x_0 \neq 0 \), the amount of energy loss is different between these two partons.

Figure 1 shows the energy density distribution of the expanding QGP fluid at \( τ = 9.6 \text{ fm}/c \) in (a) transverse plane at \( η = 0 \) and in (b) reaction plane at \( y = 0 \). A pair of energetic partons is created at \( (τ = 0, x = 1.5 \text{ fm}, y = 0, η = 0) \) and travels in the opposite direction along the x-axis at the speed of light.

**FIG. 1:** Color Online) Energy density distribution of the expanding QGP fluid at \( τ = 9.6 \text{ fm}/c \) in (a) transverse plane at \( η = 0 \) and in (b) reaction plane at \( y = 0 \). A pair of energetic partons is created at \( (τ = 0, x = 1.5 \text{ fm}, y = 0, η = 0) \) and travels in the opposite direction along the x-axis at the speed of light.
they are included in the region of $p_T > 8 \text{ GeV}/c$. The contribution from the $p_T > 2 \text{ GeV}/c$ range is negative, i.e., on the leading jet side. This negative contribution is balanced by the positive contribution of the particles with $p_T < 2 \text{ GeV}/c$. We next analyze $\langle p_T^\parallel \rangle$ inside the jet cone and out of the jet cone separately, where the two cones with $\Delta R = 0.8$ around the leading and sub-leading jet axes are considered. The contribution of the particles inside and outside the cones are shown in Figs. 2 (b) and (c), respectively. The in-cone contribution to $\langle p_T^\parallel \rangle$ is negative and high-$p_T$ particles are dominant. In the out-of-cone region, $\langle p_T^\parallel \rangle$ is positive and only particles with $p_T < 2 \text{ GeV}/c$ contribute. These out-of-cone low-$p_T$ particles are originated from deposited energy and momentum transported by the collective flow in the QGP fluid.

**Discussion.**— The equation of state of massless ideal gas employed in the present study might have been too simplified. However, we find that the basic feature of momentum transport away from the jet axis due to radial expansion does not change when we employ the equation of state from recent lattice QCD calculations \cite{36,37}. Interestingly, momentum is transported at larger angle in this realistic equation of state than in the hardest equation of state employed here since the softer equation of state makes the resultant Mach angle sharper. This means that we estimate the minimum effect of momentum transport away from the jet axis. One would have had to employ the lattice equation of state in a more quantitative analysis. However, a drawback is that all the resonances should be considered at freezeout to see the subtle interplay of momentum balance. In this Letter, we respect the simple but strict momentum conservation of freezeout processes as well as that of hydrodynamic evolution without employing the lattice equation of state.

It should be noted that the purpose of the present study in this Letter is to demonstrate (and to claim its importance of) non-linear responses of the QGP fluid to the jet propagation and that we can in principle employ any kinds of energy loss model, which we postpone as future comprehensive studies.

**Summary.**— In this Letter, motivated by the current experimental situation, we studied the collective flow in the QGP induced by jet particles and the redistribution dynamics of the deposited energy and momentum. We formulated relativistic hydrodynamic equations with source terms introduced to account deposition of jets’ energies and momenta. By solving the hydrodynamic equations numerically without linearization in fully $(3 + 1)$-dimensional Milne coordinates, we performed the simulations of di-jet asymmetric events in heavy ion collisions. In the calculations, a new scheme was employed to solve the equations at very high precision. We found that jet particles induce Mach cones in the medium and these Mach cones are strongly distorted by radial flow in the transverse plane, but due to the expanding coordinates, not so much apparently in the longitudinal direction in the reaction plane. We also showed that low-$p_T$ particles are enhanced at large angles from the quenched jet axis and compensate a large fraction of the di-jet momentum imbalance. The enhancement of low-$p_T$ particles arises from deposited energy and momentum transported by the collective flow in the QGP, which gives a novel interpretation of the di-jet asymmetric events observed by the CMS Collaboration.

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