Multiple item batch scheduling model on a batch processor to minimize total actual flowtime parts through the shop

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Abstract. This research is to address a problem of scheduling batches consisting of multiple item parts processed on a batch processor where each item requires a certain processing time which may differ from others. A number of parts are defined as a batch and processed simultaneously with sharing a set up time. A batch can be formed of the parts with different type of item and the processing time of batch equals the longest processing time of parts in the batch. It is assumed that an excessive duration of processing the parts will not lead to any defect, but increase the operating cost. The completed part must simultaneously be delivered at common due date, d. The objective is to minimise total actual flowtime of parts through the shop. A solution procedure to solve the problem is proposed and numerical examples are shown.

1. Introduction
Batch scheduling is completed through the batching stage, grouping parts into a number of batches and the scheduling stage, determining the sequence of the resulted batch to be processed on a batch processor. All parts in a batch are processed simultaneously for a certain time as a batch processing time. Therefore the rules in batching stage for grouping parts into a batch could affect the batch processing time. Behnamiana [1], Kashan [2] and Noroozi [3] stated the batch processing time is equal to the highest processing time of part in the batch. It is assumed that the excessive of processing time could not cause any defect in parts.

Actual flowtime developed by Halim and Ohta applies the backward scheduling [4], determining the sequence of job starting from due date and then moving backward to the time zero. Backward scheduling could obtain a schedule that will satisfy the due date. Halim and Ohta defined actual flowtime as the time that the parts spend in the shop from the starting time of processing until the delivery time of the completed parts [4]. It is assumed that the arriving of parts can be arranged so that the processing of part is started rightly after arriving and the processed parts must be delivered at their due date. Hidayat developed the actual flowtime for batch processors [5].

This research was carried out in a furniture factory on a drying machine. Some woods in a batch are processed simultaneously with a certain processing time. The thickness and dimensions of the wood determine the time needed for the drying process. Wood with different thickness and dimensions are categorized as parts with different items, and each type of item requires a different processing time. Less duration of processing wood could lead the low quality of the dried wood. While an excessive duration of processing wood will not lead to any defect but increase the operating cost.
The rule in batching stage is that woods from different items can be grouped into the same batch. The batch processing time is equal to the highest processing time of woods in the batch. However, the difference between the real processing time and the required processing time will cause the time of woods in the shop longer and the operating costs higher. The company guarantees for fulfilling the due date agreed with the customer. So that dealing with the backward scheduling approach and minimization of the time of woods in the shop is a must. This is the reason for adopting total actual flowtime through the shop ($F_a$) as an objective.

Hidayat developed $F_a$ model for the shop with m heterogeneous batch processors [6]. Hidayat also developed $F_a$ model for the condition that the completed parts are delivered at different due dates but the parts remained single item [7]. In Halim $F_a$ model is developed for the condition that the batch processor need an addition time for set up after processing several batches [4]. Then Hidayat developed $F_a$ model for condition that the demanded parts are from different items [8]. However, in Hidayat only parts from the same item can be grouped into a batch. This difference becomes an important reason for developing multiple items batch scheduling model on a batch processor with the rules that the part from different item can be grouped into the same batch [8].

2. Problem statement

There are n parts from k different types of item must be processed in a batch processor and the processing times of each item is different. The part processing time of item g is $t_g$ in units of time for $g = 1, 2, ..., k$. The parts from different types of items can be grouped into a batch and the batch processing time equals to the highest part of processing time in the batch. Before processing a batch the batch processor requires a set up time, $s$ in units of time. All parts that have been processed must be delivered simultaneously at the common due date, $d$. Figure 1 illustrates the problem of batch scheduling on a batch processor for multiple items with common due date.

![Figure 1. Batch scheduling for multiple items with common due date.](image)

The following the symbols and notations are used in this paper.

- $k$: the number type of item
- $g$: index for the type of item part, $g = 1, 2, ..., k$
- $n_g$: the number of demanded parts item $g$
- $n$: total demanded part, $n = \sum_{g=1}^{k} n_g$
- $i$: index for the position of batch on a production schedule which is counted from the end position on a time scale.
- $b_i$: set of parts in a batch sequenced at position $i$ ($i = 1, 2, ..., N$)
- $B_i$: the starting time of processing batch $b_i$
- $d$: the due date
- $s$: the set up time of all of item
- $s_i$: the set up time of batch $b_i$, $s_i = s$
- $t_i$: the processing time of batch $b_i$
3. Problem formulation

Hidayat et al. developed a model of total actual flowtime through the shop ($F^a$) for batch processors under the conditions of single item part and simultaneous delivery [5]. If on the shop there are $N$ batches, then $F^a$ is formulated with Equation (1).

$$F^a = \sum_{i=1}^{N} \left( \sum_{j=1}^{N} (t_j + s_j) - s_i \right) Q_i$$  \hspace{1cm} (1)$$

Hidayat states that the item type of part does not determine $F^a$ [8], therefore $F^a$ model for this research is the same as $F^a$ model in Hidayat [5] that is equation (1). Under the condition of multiple items, the processing time of each batch depends on the type of item part in the batch. Therefore a binary variable i.e $x_{ig}$ is needed to indicate the item type of parts in batch $b_i$. Where if batch $b_i$ consist of items $g$ then $x_{ig} = 1$ otherwise $x_{ig} = 0$.

Subject to:

$$\sum_{i=1}^{N} (t_i + s_i) - s_N \leq d$$  \hspace{1cm} (2)$$

$$\sum_{i=1}^{m} q_{ig} = n_g \quad \text{for} \quad g = 1, 2, \ldots, k$$  \hspace{1cm} (3)$$

$$\sum_{g=1}^{N} q_{ig} = Q_i \quad \text{for} \quad i = 1, 2, \ldots, N$$  \hspace{1cm} (4)$$

$$N_g = \sum_{i=1}^{N} x_{ig} \quad \forall \ g = 1, 2, \ldots, k$$  \hspace{1cm} (5)$$

$$t_i = \max\{t_{ig}, t_g\} \quad \text{for} \quad i = 1, 2, \ldots, N \text{ and } g = 1, 2, \ldots, k.$$  \hspace{1cm} (6)$$

$$B_1 = d - t_1$$  \hspace{1cm} (7)$$

$$B_i = B_{(i-1)} - s - t_i \quad \forall \ i = 2, 3, \ldots, N$$  \hspace{1cm} (8)$$

$$x_{ig} = \begin{cases} 1 & \forall \ i = 1, \ldots, N \ ; \ \forall \ g = 1, \ldots, k \\ 0 & \end{cases}$$  \hspace{1cm} (9)$$

$$q_{ig}, Q_i, B_i \geq 0. \quad \forall \ i = 1, 2, \ldots, N$$  \hspace{1cm} (10)$$

Equation (1) is the objective function of minimizing the total actual flowtime of parts through the shop. Equation (2) to ensure that the processing of all batches does not exceed due dates. Equation (3) guarantees that the number of processed parts is equal to the number of demanded parts. Equation (4) states that the size of each batch is the sum of the volume of all items in the batch. Constraint (5) to ensure that the number of batches which consist of items $g$ equals the sum of the binary variables for each type of item. Equation (6) states that the processing time of batch $b_i$ is equal to the highest
4. Method

Scheduling on a batch processor is done through batching and scheduling stages. In this study parts of different items can be grouped into the same batch and the batch processing time equals the highest part processing time in the batch. It is necessary to select the types of items to be combined in one batch such that $F^a$ is minimal. These conditions distinguish this study from Hidayat et al [8], therefore we need to develop the batching algorithm in Hidayat [8].

4.1. Proposition 1

If there are two batches contain parts from different type of items with different processing times for each item but the set up time is the same then the two items can be combined if and only if it meets the following condition.

$$t_{\text{join}} \leq t_1 + \frac{q_2}{q_1 + q_2} (t_2 + s)$$

Proof:

Suppose there are batch $b_1$ containing part item 1, batch $b_2$ containing part item 2 and the batch processing time are $t_1$ and $t_2$ respectively. The set up time is $s$.

![Figure 2. Alternative batch merged with different item.](image)

There are two alternatives, as shown in Figure 2. The first is to combine the two different items into one batch so that $Q = q_1 + q_2$. The batch processing time equals the highest processing time of the two items and it is expressed as $t_{\text{join}} = \max\{t_1, t_2\}$. The second alternative is not to be combined, but still divided into two different batches, batch $b_1$ and $b_2$. Suppose $F^{a1}$ and $F^{a2}$ respectively are the total actual flowtime of parts through the shop for the first and second alternatives. Figure 3 shows the gantt chart for the first alternative and is obtained equation (12).

![Figure 3. The gantt chart for the first alternative.](image)

$$F^{a1} = (q_1 + q_2)t_{\text{join}}$$

(12)
To get a minimum $F^a$ in the second alternative, according to Hidayat the two batches must be arranged by the following rules [8].

Figure 4 shows the gantt chart for the second alternative and obtained equation (13).

$$F^{a2} = q_1 t_1 + q_2 (t_1 + s + t_2)$$

(13)

The total actual flowtime part through the shop of the first alternative is better than the second alternative if $F^{a1} \leq F^{a2}$. Using equations (12) and (13), it is obtained:

$$t_{\text{join}} \leq t_1 + \frac{q_2}{(q_1 + q_2)} (s + t_2)$$

(14)

If $t_{\text{join}} = \max \{t_1, t_2\}$ satisfies condition (14) then 2 batches consisting of different items can be combined. If there are $k$ items with different processing times for each item, then combining items in one batch is done in stages, starting from two items to $k$ items.

5. Numerical example and result

Examples of calculations is performed on the 5 items type of part with the number of demanded part and the processing time of each item are listed in Table 1. The set-up time is 1 day and completed parts must be delivered on due date, $d = 70$ days.

| $g$ | 1 | 2 | 3 | 4 | 5 |
|-----|---|---|---|---|---|
| $n_g$ | 4 | 3 | 1 | 1 | 1 |
| $t_g$ | 7 | 9 | 15 | 17 | 20 |

The search for a solution is done using the proposed solution procedure.

First iteration
- Step 0 Set parameters $d = 70$, $k = 5$, $s = 1$, $n_g$, $t_g$
- Step 1 Each item of demanded part, $n_g$ grouped into one batch, obtained $N = 5$ batches.
- Step 2 Arrange 5 resulted batches in order with following rule: $\frac{t_1 + s}{q_1} \leq \frac{t_2 + s}{q_2} \leq \cdots \leq \frac{t_5 + s}{q_5}$
- Step 3 For combining batch $b_1$ and batch $b_2$.
  - $Q_1 = 4m^3$; $t_1 = 7$ days; $Q_2 = 3m^3$; $t_2 = 9$ days; $t_{\text{join}} = \max \{t_1, t_2\} = 9$ days.
  - Calculate $t_1 + \frac{q_2}{(q_1 + q_2)} (s + t_2) = 11.28$ days $\rightarrow 9$ days $\leq 11.28$ days
  - $t_{\text{join}} \leq t_1 + \frac{q_2}{(q_1 + q_2)} (s + t_2)$ is satisfied. Therefore batch $b_1$ and $b_2$ can be combined. For the next iteration, the number of batch become 4. The result is shown in table 2.
| Batch \(b_i\) | 1   | 2   | 3   | 4   |
|------------|-----|-----|-----|-----|
| \(g\)     | 1 and 2 | 3   | 4   | 5   |
| \(Q_t\)   | \(4+3=7\) | 1   | 1   | 1   |
| \(t_i\)   | 9   | 15  | 17  | 20  |

We did the second and the third iteration. Table 3 is shown the result of numerical example.

| Batch \(b_i\) | 1   | 2   |
|------------|-----|-----|
| Item \(g\) | 1 and 2 | 3, 4 and 5 |
| \(Q_t\)   | \(4+3=7\) | 1+1+1=3 |
| \(t_i\)   | 9   | 20  |

6. Conclusion
Based on this research, it can be seen the developed model is able to solve the multiple items batch scheduling problem on a batch processor with the batch processing time depending on the type of items in a batch. The model can be used to find the number of batches, the size of each batch and the sequence of the resulted batches so that the total actual flowtime parts through the shop \(F^a\) is minimized.

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