Models for the magnitude-distribution of brightest cluster galaxies

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ABSTRACT

The brightest, or first-ranked, galaxies (BCGs) in rich clusters show a very small dispersion in luminosity, making them excellent standard candles. This small dispersion raises questions about the nature of BCGs. Are they simply the extremes of normal galaxies formed via a stochastic process, or do they belong to a special class of atypical objects? If they do, are all BCGs special, or do normal galaxies compete for the first rank? To answer these questions, we undertake a statistical study of BCG magnitudes using results from extreme value theory. Two-population models do better than one-population models. A simple scenario where a random boost in the magnitude of a fraction of bright normal galaxies forms a class of atypical galaxies best describes the observed distribution of BCG magnitudes.

Key words: methods: statistical – galaxies: clusters: general – galaxies: elliptical and lenticular, cD – galaxies: evolution.

1 INTRODUCTION

Among the most luminous bodies in the Universe are the brightest, or first-ranked, galaxies in rich clusters. These galaxies have absolute magnitudes between $-21.5$ and $-23.3$ and are among the farthest observable objects. In addition, the magnitudes of these brightest cluster galaxies (BCGs) are highly uniform, with a dispersion of 0.32 mag (Hoessel & Schneider 1985). Their uniformity and large luminosity make BCGs excellent standard candles. The uniformity of BCG magnitudes raises a particularly important question regarding their nature (Peebles 1968; Sandage 1972). Are BCGs simply the brightest of a statistical set of galaxies or do they belong to a special class of objects? If a special class of galaxies exists, do all clusters have special galaxies and are they always first-ranked (Bhavsar 1989)? We investigate these questions using extreme value theory (Fisher & Tippett 1928).

2 EXTREME VALUE THEORY

The motivation for studying extreme phenomena is practical. Many of the memorable experiences in our lives can be classified as statistical extremes. Examples of maximum extremes are floods, the hottest summer temperatures and the lengths of the longest caterpillars. Examples of minimum extremes are draughts, floods, the hottest summer temperatures and the lengths of the longest hummingbirds. Some extremes do not effect our lives and others turn them upside down. The desire to understand these types of phenomena prompts the study of extreme value theory.

Fisher & Tippett (1928) show that the distribution of statistically largest or smallest extremes tends asymptotically to a well-determined and analytic form for a general class of parent distributions. Extremes drawn from sufficiently large and steeply falling parent distributions have this form. One may find the original argument in Fisher & Tippett (1928). Their derivation is reconstructed in greater detail by Bhavsar & Barrow (1985), who apply extreme value theory in an analysis of BCG magnitudes. Fisher & Tippett’s result states that the cumulative distribution of maximum extremes is given by

$$F(x) = e^{-e^{-a(x-x_0)}}.$$  \hfill (1)

This distribution is known as the Gumbel distribution. (For smallest extremes, one substitutes $x \rightarrow -x$.) From $F$ we may calculate the differential distribution (or probability density)

$$f(x) = ae^{-a(x-x_0)-e^{-a(x-x_0)}},$$  \hfill (2)

where $f(x) = F'(x)$, $x_0$ is the mode of the extremes and $a > 0$ is a measure of the steepness of fall of the parent distribution. The probability density is normalized to unity. The mean, median and standard deviation of the distribution given in Bhavsar & Barrow (1985) correspond to

$$\langle x \rangle = x_0 + \frac{0.577}{a}; \quad \text{med}(x) = x_0 + \frac{0.367}{a}; \quad \sigma^2 = \frac{\pi^2}{6a^2},$$  \hfill (3)

where $0.577 \approx -\Gamma'(1)$ is Euler’s constant, $0.367 \approx \ln[\ln(2)]$ and $\sigma$ is the standard deviation of the extremes. The standard form for the Gumbel, $F(x)$ and $f(x)$, is shown in Fig. 1. Note that for BCGs,
3 BRIGHTEST CLUSTER GALAXIES

3.1 Past results

Researchers have described BCGs as special, statistical extremes of a normal population, and as a mixture of the two (Peebles 1968; Peach 1969; Sandage 1972, 1976; Bhavsar & Barrow 1985; Bhavsar 1989; Postman & Lauer 1995).

The motivation for proposing that BCGs are special is a result of the small dispersion observed in BCG magnitudes (Peach 1969; Sandage 1972, 1976). These authors argue that such a small dispersion is not sufficiently explained by the steepness of the luminosity function. In addition, astronomers observe a class of BCGs that are morphologically different, called cD galaxies. These galaxies are giant ellipticals and often have features, such as multiple nuclei and large envelopes, that distinguish them from normal galaxies.

On the other hand, Peebles (1968) argues that BCGs are just the extreme tail-end of normal galaxies that form in clusters via some stochastic process. In this case, the brightest galaxy in a given cluster is simply the brightest normal galaxy and, therefore, the distribution of BCG magnitudes is a Gumbel. (It is interesting to note that Peebles, independently of Fisher & Tippett, derived the Gumbel distribution for BCGs for the special case of an exponential luminosity function.)

Bhavsar (1989) contends neither of these scenarios adequately describes the observed distribution of BCG magnitudes and argues for a mixed population. Suppose that a special class of Galaxies exists but that not all clusters have a special galaxy. In clusters with no special galaxy, the BCG is simply the brightest normal galaxy. In a cluster containing at least one special galaxy, either all the normal galaxies are fainter, or the brightest normal galaxy (or galaxies) outshines the special one(s) and attains the first rank.

For BCGs, we consider three different two-population models. The first is the case discussed above with the brightest normal galaxies comprising one population and a special class of galaxies comprising the other. We call this case model C and write the total distribution as \( f_{G} \) (where ‘G’ stands for ‘Gumbel + gaussian’). To obtain the final form of \( f_{G} \) we note that

\[
I_{G} = \int_{M}^{\infty} f_{G}(M') \, dM' = F(M),
\]

where \( F(M) \) is given by equation (1) with \( x = -M \) and \( x_0 = M_{G} + (0.577/\alpha) \). Secondly, we note that

\[
I_{G} = \int_{M}^{\infty} f_{G}(M') \, dM' = (1 \pm erf[M - M_{G}])/2,
\]

where \( erf \) is the error function. The upper sign is for \( M < M_{G} \) and the lower sign is for \( M > M_{G} \). Thus, we may rewrite \( f_{G} \) by substituting in \( I_{G} \) and \( I_{G} \)

\[
f_{G}(M) = d[f_{G}I_{G} + f_{G}I_{G}] + (1 - d) f_{G}.
\]

Other possible combinations of assigning \( f_{G} \) and \( f_{G} \) to the two populations result in models D and E. In the case of model D, both distributed (Peach 1969; Sandage 1972, 1976; Postman & Lauer 1995). In this case, referred to henceforth as model A, the probability distribution of special galaxies, \( f_{sp} \), is a Gaussian, \( f_{sp} \), with mean \( M_{sp} \), standard deviation \( \sigma \) and normalization such that the integral over all magnitudes, \( M \), is unity. The distribution function is as follows:

\[
f_{sp}(M) = f_{sp} = \frac{1}{\sigma \sqrt{2\pi}} e^{-\frac{(M - M_{sp})^{2}}{2\sigma^{2}}}. \tag{4}
\]

If BCGs are simply the brightest of a normal set of galaxies (Peebles 1968); henceforth referred to as model B, the probability distribution of their magnitudes, \( f_{nor} \), is a Gumbel, \( f_{G} \), given by equation (2), with \( x = -M \) and \( x_0 = M_{G} + (0.577/\alpha) \) (Bhavsar & Barrow 1985)

\[
f_{nor}(M) = f_{nor} = e^{-(M - M_{G})}, \tag{5}
\]

where \( M_{G} \) is the mean of the extremes and \( \alpha \) is a measure of the steepness of fall of the parent distribution.

In the case of two populations (Bhavsar 1989), we derive the distribution that \( M \) should have from the contributions of the two individual populations. Consider \( N \) clusters of galaxies and suppose that \( n < N \) have at least one special galaxy. Let the independent magnitude-distribution of normal and special galaxies, respectively, be \( f_{nor} \) and \( f_{sp} \). The total magnitude-distribution function, \( f_{tot} \), is then given by

\[
f_{tot}(M) = d \left[ \int_{M}^{\infty} f_{nor}(M') \, dM' + \int_{M}^{\infty} f_{sp}(M') \, dM' \right] + (1 - d)f_{nor}, \tag{6}
\]

where \( d = n/N \). The first (second) term is the probability of picking a special (normal) galaxy, with absolute magnitude \( M \), from a cluster containing both populations with the condition that all the normal (special) galaxies are fainter. The third term gives the probability of picking a galaxy, with absolute magnitude \( M \), in clusters containing only normal galaxies. Equation (6) is true for all well-behaved functions \( f_{nor} \) and \( f_{sp} \). If \( f_{nor} \) and \( f_{sp} \) are normalized to unity, then so is the resulting total distribution function \( f_{tot} \). (Note that equation 6 works, in general, whenever there are two independent populations competing for first rank.)

For BCGs, we consider three different two-population models. The first is the case discussed above with the brightest normal galaxies comprising one population and a special class of galaxies comprising the other. We call this case model C and write the total distribution as \( f_{G} \) (where ‘G’ stands for ‘Gumbel + gaussian’). To obtain the final form of \( f_{G} \) we note that

\[
I_{G} = \int_{M}^{\infty} f_{G}(M') \, dM' = F(M), \tag{7}
\]

where \( F(M) \) is given by equation (1) with \( x = -M \) and \( x_0 = M_{G} + (0.577/\alpha) \). Secondly, we note that

\[
I_{G} = \int_{M}^{\infty} f_{G}(M') \, dM' = (1 \pm erf[M - M_{G}])/2, \tag{8}
\]

where \( erf \) is the error function. The upper sign is for \( M < M_{G} \) and the lower sign is for \( M > M_{G} \). Thus, we may rewrite \( f_{G} \) by substituting in \( I_{G} \) and \( I_{G} \)

\[
f_{G}(M) = d[I_{G} + f_{G}(I_{G})] + (1 - d)f_{G}. \tag{9}
\]
distributions are Gaussian and the total distribution function, \( f_\text{G} \), is given by

\[
f_\text{G}(M) = d[f_\text{G2}f_\text{G1} + f_\text{G1}f_\text{G2}] + (1 - d)f_\text{G1},
\]

where the notation is self-evident and the two Gaussians are characterized, respectively, by \( M_\text{G1}, a_1 \), and \( M_\text{G2}, a_2 \). In the case of model E, both distributions are Gumbels (\( f_\text{G} \) is also a Gumbel) and the total distribution function, \( f_\text{GG} \), is given by

\[
f_\text{GG}(M) = d[f_\text{G2}f_\text{G1} + f_\text{G1}f_\text{G2}] + (1 - d)f_\text{G1},
\]

where the two Gumbels are characterized, respectively, by \( M_\text{G1}, a_1 \), and \( M_\text{G2}, a_2 \). Table 1 summarizes the forms of the five models.

### 4 MODELLING THE DATA

#### 4.1 Data sets

We utilize two data sets from the literature. First, we re-analyse the data used by Bhavsar (1989). This is a 93 member subset of 116 metric BCG visual-intrinsic (VI) magnitudes compiled by Hoesel, Gunn & Thuan (1980), henceforth referred to as ‘HGT’. These 93 are the data from clusters of richness 0 and 1 only; Bhavsar ignores the rest of the BCGs in order to keep the data set consistent with Sandage’s (1976) result that BCG magnitude is independent of cluster-richness. The internal consistency of the set is 0.014 mag, as published in HGT. Secondly, we analyse the 119 metric BCG magnitudes, taken in the Kron–Cousins \( R_0 \) band, as published in HGT. Secondly, we analyse the 119 metric BCG magnitudes, taken in the Kron–Cousins \( R_0 \) band, compiled by Lauer & Postman (1994), henceforth referred to as ‘LP’. The data were corrected for local and possible large scale motions. The 119 LP data are comprised of BCGs from 107 clusters of richness 0 and 1, and 9, 2 and 1 of richness 2, 3 and 4, respectively (Abell, Corwin & Olowin 1989). We find that removing the 12 BCGs from clusters of richness class \( 2 \) does not significantly change the distribution of the LP data. This is consistent with Sandage’s (1976) result that BCG magnitude is independent of cluster-richness. The two data sets have 34 galaxies in common. Comparing the subset of 34, we find that the HGT values are, on average, 0.06 ± 0.19 mag brighter than the LP values. A two-sample Kolmogorov–Smirnov (K–S) Test addresses the consistency of the two data sets in describing the same population of objects. The null hypothesis is that the same distribution describes both data sets. We find that the two data sets fail the null hypothesis at the 82 per cent confidence level. Therefore, we do not expect the same parameters or distribution to describe both sets. These discrepancies may need further investigation, but such an analysis is outside the scope of this work. We investigate each data set separately and present our results.

#### 4.2 Fitting method

We consider models A–E discussed above. The two-population distributions have five parameters each: two means, two standard deviations and the fraction, \( d \), of clusters that contain a special population of galaxies. If there is no population of special galaxies, then \( d = 0 \). We use maximum-likelihood fitting. The theory behind this method is discussed in Press et al. (1992). The maximum-likelihood fit to a data set of size \( N \) for a function, \( f \), are the parameters, \( a \), that maximize the likelihood function:

\[
L = \prod_{i=1}^{N} f(x_i; a).
\]

### Table 1. Distribution components for the five models.

| MODEL | \( f_{\text{G1}} \) | \( f_{\text{G2}} \) |
|-------|-----------------|-----------------|
| A     | –               | \( f_\text{G} \) |
| B     | \( f_\text{G} \) | –               |
| C     | \( f_\text{G} \) | \( f_\text{G} \) |
| D     | \( f_\text{G1} \) | \( f_\text{G2} \) |
| E     | \( f_\text{G1} \) | \( f_\text{G2} \) |

### Table 2. Fit-parameters for the HGT data for models A–E.

| MODEL A | MODEL B | MODEL C | MODEL D | MODEL E |
|---------|---------|---------|---------|---------|
| \( M_\text{G} = -22.63 \) | \( M_\text{G} = -22.66 \) | \( M_\text{G} = -22.30 \) | \( M_\text{G} = -22.29 \) | \( M_\text{G} = -22.40 \) |
| \( \sigma = 0.34 \) | \( a = 2.82 \) | \( M_\text{G} = -22.79 \) | \( M_\text{G} = -22.83 \) | \( M_\text{G} = -22.86 \) |
| \( D = 0.0876 \) | \( D = 0.1174 \) | \( d = 0.64 \) | \( d = 0.62 \) | \( d = 0.48 \) |
| \( P = 0.531 \) | \( P = 0.848 \) | \( \sigma_1 = 0.2 \) | \( \sigma_1 = 0.19 \) | \( a_1 = 3.7 \) |
| \( \sigma = 0.2 \) | \( \sigma_1 = 0.19 \) | \( a_1 = 3.7 \) | \( a_1 = 8.3 \) |
| \( D = 0.0562 \) | \( D = 0.0519 \) | \( D = 0.0525 \) | \( P = 0.063 \) | \( P = 0.032 \) |
| \( P = 0.063 \) | \( P = 0.032 \) | \( P = 0.036 \) | \( P = 0.063 \) | \( P = 0.032 \) |

### Table 3. Fit-parameters for the LP data for models A–E.

| MODEL A | MODEL B | MODEL C | MODEL D | MODEL E |
|---------|---------|---------|---------|---------|
| \( M_\text{G} = -22.43 \) | \( M_\text{G} = -22.45 \) | \( M_\text{G} = -21.84 \) | \( M_\text{G} = -22.11 \) | \( M_\text{G} = -22.18 \) |
| \( \sigma = 0.33 \) | \( a = 2.99 \) | \( M_\text{G} = -22.44 \) | \( M_\text{G} = -22.52 \) | \( M_\text{G} = -22.52 \) |
| \( D = 0.0565 \) | \( D = 0.1173 \) | \( d = 0.95 \) | \( d = 0.72 \) | \( d = 0.64 \) |
| \( P = 0.162 \) | \( P = 0.926 \) | \( \sigma_1 = 0.2 \) | \( \sigma_1 = 0.3 \) | \( a_1 = 3.65 \) |
| \( \sigma = 0.32 \) | \( \sigma_1 = 0.3 \) | \( a_1 = 3.65 \) | \( a_1 = 5.33 \) |
| \( D = 0.0570 \) | \( D = 0.0527 \) | \( D = 0.0421 \) | \( P = 0.158 \) | \( P = 0.098 \) |
| \( P = 0.158 \) | \( P = 0.098 \) | \( P = 0.014 \) | \( P = 0.158 \) | \( P = 0.098 \) |

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where the $f(x_i; a)$ are the values of the probability density, $f$, evaluated at each of the $N$ data points, $x_i$. For a certain $f$, one finds the set of parameters that maximizes the product, $L$.

5 RESULTS

5.1 Parameters and fits

After obtaining parameters from the maximum-likelihood method for models A–E for both data sets, we compute the K–S statistics. We list the results in Tables 2 and 3, respectively. Lower values of the K–S $D$-statistic correspond to lower values of rejection probability, $P$, and thus denote a better fit. Figs 2 and 3 illustrate the performance of each of the five models. Note that the distributions use the parameters obtained by the maximum-likelihood method, using every data point, and are not a fit to the particular histograms.

5.2 Comparison with previous work

We compare our results with Bhavsar (1989) and Postman & Lauer (1995). Bhavsar’s (1989) two-population model is our model C. He uses maximum-likelihood fitting and his best-fitting parameters are $M_G = -22.31$, $M_g = -22.79$, $d = 0.63$, $a = 4.01$ and $\sigma = 0.21$. Our parameters are in excellent agreement. Minor variation is expected due to differences in fitting techniques. Postman & Lauer (1995) argue against Bhavsar’s two-population model and claim that BCG magnitudes are Gaussian, based on a 26 per cent confidence level.

In agreement with both Bhavsar (1989) and Postman & Lauer (1995), it is clear from Tables 2 and 3 and Figs 2 and 3 that for both data sets no Gumbel distribution describes the BCG data. This rejects the Gumbel hypothesis (model B) with 85 per cent and 93 per cent confidence levels, respectively, for the HGT and LP sets. For the HGT data, the Gaussian fails at the 53 per cent confidence level, while for the LP data, the rejection confidence is 16 per cent. The difference between our value of 16 per cent and Postman & Lauer’s value arises because our result is for the maximum-likelihood fit Gaussian, while Postman & Lauer’s is for a Gaussian with the same mean and standard deviation as the LP data.

The relatively high rejection-confidence of the one-population models has motivated us to investigate two-population models. The presence of cD galaxies strongly suggests the possibility of another population. Overall, the two-population models fit the data much better than do the Gumbels and as well or better than do the respective Gaussians. The larger number of parameters is taken into account by the statistical estimators when calculating...
the confidence of rejecting the null hypothesis. Moreover, the parameters are physical quantities that are observationally verifiable (Bhavsar 1989).

Our result that no one model or set of parameters describes both data sets is consistent with the fact that a two-sample K–S Test indicates that the sets are not consistent with one another. Postman & Lauer (1995) have raised questions regarding HGT’s BCG classification and sky subtraction.

5.3 Physical motivation

Researchers have suggested various mechanisms whereby a second population with a brighter average metric magnitude could evolve from the bright normal galaxies. Cannibalism, the process by which large galaxies in the central regions of rich clusters grow at the expense of smaller galaxies (Ostriker & Hausman 1977; Hausman & Ostriker 1978), is one possibility. The existence of giant elliptical and cD galaxies near the centre of approximately half of all rich clusters supports this hypothesis. These galaxies always lie at the tail-end of their cluster-luminosity functions. The occurrence of cannibalism continues to be debated (Merritt 1984).

Motivated by the existence in the literature of strong arguments for such a process, we build a very simple schematic to study its statistical effects on the population of first-ranked galaxies. We make two assumptions: (i) at an early epoch the BCGs all belonged to one population, and (ii) galaxies from the bright end of this population evolve, resulting in a random boost to their luminosity. We construct a set of \( N \) galaxies with an exponential luminosity function between absolute magnitudes \(-22.0\) and \(-23.0\). This represents the galaxies at the bright end of cluster luminosity functions that are candidates for a boost. A random number, \( n_0 \), of these galaxies undergo a random boost between 0.1 and 0.9 mag. We label the boosted subset as \( n_b \). We choose this range for the following reasons. First, Hausman & Ostriker (1978) show via a simulation that one would expect a large galaxy to gain, on average, 0.5 mag during its first cannibalistic encounter. This is consistent with Aragon-Salamanca, Baugh & Kauffmann (1998), who state that BCGs were approximately 0.5 mag fainter at \( z = 1 \). Secondly, we limit ourselves to one encounter because Merritt (1984) argues that the time scale for galactic encounters is too long for cannibalism to be common in the Universe. We wish to investigate the magnitude-distribution of the resulting boosted population. These represent the special galaxies mentioned previously. Specifically, this distribution could give us insight into the form of \( f_{sp} \).

To our surprise, we find that the distribution, \( f_{sp} \), of \( n_b \) is a Gumbel! The K–S Test rejects the Gaussian hypothesis at the 98 per cent confidence level. Conversely, the Gumbel distribution, with the same mean and deviation as the data, fits well, with only

![Figure 3. Left-hand column shows cumulative distribution function for the LP data and the maximum-likelihood fits for each of the five models. Right-hand column shows LP histogram with a plot of the differential distribution for each of the five models.](https://academic.oup.com/mnras/article-abstract/322/3/625/953705)
a 7 per cent confidence level for rejection. We summarize these results in Table 4 and Fig. 4. Thus, the two-population model E (a combination of two Gumbels), which is best-fitting for the newer LP data, has a physical basis.

6 CONCLUSION

For more than 30 years, cosmologists have debated the nature of the magnitude-distribution of brightest cluster galaxies. Peebles (1968); Sandage (1972, 1976) and Peach (1969) reach markedly different conclusions. More recently, Bhavsar (1989) and Postman & Lauer (1995) differ regarding the population(s) that comprise the first-ranked galaxies. In light of this controversy, we have conducted a new examination of the distribution of BCG magnitudes. We consider the BCGs as a class of objects to which we may apply well established results from extreme value theory. We find that there are a number of models that perform well in describing the HGT and LP data sets. Though a Gaussian fits both data sets, the confidence limits warrant further investigation of two-population models.

Tables 2 and 3 clearly show that we should reject the Gumbel (model B) as a fit, i.e., the hypothesis that all BCGs are statistical extremes. The Gaussian (model A) is marginally acceptable but without physical basis. Two-population models, in particular, the three combinations of $f_G$ and $f_g$, describe the data very well. Tables 2 and 3 show their relative merits. Model E stands out as giving the best overall fit and is motivated by a physical basis. Therefore, it is most likely that there are two populations of BCGs: the extremes of a normal population and a class of atypical galaxies with a brighter average mean.

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