Estimations of $\Omega^+/\Omega^-$ at RHIC from a QGP Model with Diquarks

Hong Miao$^1$, Chongshou Gao$^{1,2}$

$^1$School of Physics, Peking University, Beijing 100871, China
$^2$Institute of Theoretical Physics, Academia Sinica, Beijing 100080, China

Abstract

Assuming that axial-vector and scalar diquarks exist in the Quark-Gluon Plasma near the critical temperature $T_c$, baryons can be produced from quark-diquark interactions. In RHIC conditions ($\sqrt{s_{NN}} = 130\text{GeV}$ and $200\text{GeV}$), the ratio $\Omega^+/\Omega^-$ may be larger than 1, based on the concept that QGP with diquarks would exist. This unusual result might be a helpful evidence for QGP existing in RHIC.

PACS number(s): 12.38.Mh 25.75.-q
Key words: QGP, plasma, diquark, strangeness, baryon

1. Introduction

Diquarks$^1$ may exist as bound states$^2$ in the Quark-Gluon Plasma (QGP) as well as quarks and gluons. If axial-vector and scalar diquarks exist near the critical temperature $T_c$ and approximate thermal equilibrium could form, baryon production can be described as the process of quark and diquark forming ($\frac{1}{2}^+$) and ($\frac{3}{2}^+$) baryon states$^4$. Ratios of different baryons can be estimated through this method. Since strange baryon production is widely discussed$^5$$^7$$^8$$^9$$^{10}$ and has a upper limit in the Hadronic Gas Model$^9$, the ratio beyond that limit can support the idea of QGP productions in the relativistic heavy ion collisions.

QGP with diquarks has a much higher energy density than general Quark-Gluon Plasma$^{11}$. Another interesting phenomena there is about the ratio of $\Omega^+ / \Omega^-$. In the conditions of hadronic matter, the ratio will smaller than 1$^{12}$. General QGP model will also predict the ratio $\Omega^+ / \Omega^- = 1$ $^5$$^7$ or near it.

While, since there are diquarks together with quarks in the QGP, strange particles are not only $s$ and $\bar{s}$, but also $V(ss)$, $V(ss)$, $V(us)$, $V(us)$, $V(ds)$, $V(ds)$, $S(us)$, $S(ds)$ and $S(ds)$. So strangeness conservation will not simply requires $N(s) = N(\bar{s})$. When $\mu_B > 0$, $(us)$ and $(ds)$ diquarks’ amounts will be larger than those of $(\bar{u}\bar{s})$ and $(\bar{d}\bar{s})$ diquarks, but $s$ quark’s and $(ss)$ diquark’s amounts will be smaller than those of $\bar{s}$ quark and $(\bar{s}\bar{s})$ diquark. That means the amount of $\Omega^- (sss)$ would be a little smaller than $\Omega^+ (\bar{s}\bar{s}\bar{s})$. This is a general subsequence based on diquark’s appearance. Some models from strings$^{13}$ also predict $\Omega^+ / \Omega^- > 1$ in p-p collisions, when diquarks are imported.

When concerning that the baryon chemical potential and strangeness chemical potential may have changed during the process of freeze-out, the result will be very complicated. And as a simplification, such influence has been neglected in the calculations. In addition, hadron interactions after phase transition may lower the ratios of $\Omega^+ / \Omega^-$, and can not be easily estimated so far.
2. Diquark Model and Baryon Production in QGP

In the SU(6) quark-diquark model, baryon wave functions can be described as combinations of quarks and diquarks \[1\] [14] [15] [16], and some baryons can be rewritten as \[4\]

\[
| \Lambda \rangle = \frac{1}{\sqrt{3}} \left[ B_{\frac{1}{2}}(V_{ud}, s) + \sqrt{\frac{3}{4}} B_{\frac{1}{2}}(V_{us}, d) - \sqrt{\frac{3}{4}} B_{\frac{1}{2}}(V_{ds}, u) + \sqrt{\frac{1}{4}} B_{\frac{1}{2}}(S_{us}, d) - \sqrt{\frac{1}{4}} B_{\frac{1}{2}}(S_{ds}, u) \right],
\]

\[
| \Sigma^0 \rangle = \frac{1}{\sqrt{3}} \left[ B_{\frac{1}{2}}(V_{ud}, s) - \sqrt{\frac{3}{4}} B_{\frac{1}{2}}(V_{us}, d) - \sqrt{\frac{3}{4}} B_{\frac{1}{2}}(V_{ds}, u) + \sqrt{\frac{1}{4}} B_{\frac{1}{2}}(S_{us}, d) + \sqrt{\frac{1}{4}} B_{\frac{1}{2}}(S_{ds}, u) \right],
\]

\[
| \Omega^- \rangle = B_{\frac{3}{2}}(V_{ss}, s),
\]

where B represent a baryon state.

So the productions can be described as

\[
\frac{d\Lambda}{dt} = \frac{1}{3} \cdot \left[ \frac{3}{4} \Gamma(V_{us}, d, \Lambda) + \frac{3}{4} \Gamma(V_{ds}, u, \Lambda) + \frac{1}{4} \Gamma(S_{us}, d, \Lambda) + \frac{1}{4} \Gamma(S_{ds}, u, \Lambda) \right],
\]

\[
\frac{d\Sigma^0}{dt} = \frac{1}{3} \cdot \left[ \Gamma(V_{us}, s, \Sigma^0) + \frac{1}{4} \Gamma(V_{us}, d, \Sigma^0) + \frac{1}{4} \Gamma(V_{ds}, u, \Sigma^0) + \frac{3}{4} \Gamma(S_{us}, d, \Sigma^0) + \frac{3}{4} \Gamma(S_{ds}, u, \Sigma^0) \right],
\]

\[
\frac{d\Omega^-}{dt} = \Gamma(V_{ss}, s, \Omega^-),
\]

(1)

\( p, n, \Sigma^0, \Xi^- \) and other baryons can be calculated through similar methods.

As a simplification, baryon production can be described as a combination of different processes of quarks and diquarks forming \( \binom{1}{2}^+ \) or \( \binom{3}{2}^+ \) baryon states, as

\[
\frac{dB}{dt} = \sum C_{cq}^2(D_{q_1q_2}, q_3, B)\Gamma(D_{q_1q_2}, q_3, B),
\]

(2)

where \( C_{cq}^2(D_{q_1q_2}, q_3, B) \) is the Clebsch-Gordan coefficient to represent the state of quark-diquark coupling shown in equations (1), and one could get the result \[3|17\] after the integration of the producing cross-sections of the baryon states above under the conditions of quarks and diquarks are assumed to be under ideal Fermi and Bose distributions.

\[
\frac{dB}{dt} = \sum C_{cq}^2(D_{q_1q_2}, q_3, B) 3\omega_D\omega_q |M|^2 \frac{2\pi}{3} T^2 F_{\text{FB}}(q_3, D_{q_1q_2}, B, T),
\]

(3)

where \( \omega_D \) and \( \omega_q \) are the spin and color degeneracy of quarks and diquarks, while

\[
T^2 F_{\text{FB}}(q, D, B, T) = \int \int \frac{dE_q dE_D}{(e^{E_q/T} + 1)(e^{E_D/T} + 1)},
\]

The integrating ranges are

\[
m_q \leq E_q \leq \infty, m_D \leq E_D \leq \infty
\]

and

\[
E_q E_D \geq \frac{1}{4m_B^2} \left[ 4(E_q + E_D)(m_q^2 E_D + m_D^2 E_q) + [m_B^2 - (m_q + m_D)^2][m_B^2 - (m_q - m_D)^2] \right]
\]

For \( \binom{1}{2}^+ \) baryons from axial-vector diquarks, one has the effective lagrangian (with some corrections of the expressions in \[4\], which make few differences on the final results.)

\[
L_{\text{int}V_{1/2}} = igB\gamma_\mu\gamma_5 qV_\mu,
\]

(4)
Then,
\[ |M|_{V_{1/2}}^2 = \frac{g^2}{3} \left( \frac{(m_B^2 - m_q^2)^2}{m_V^2} + m_B^2 + m_q^2 - 2m_V^2 + 6m_Bm_q \right), \]
(5)

For \((\frac{1}{2})^+\) baryons from scalar diquarks,
\[ L_{intS_{1/2}} = ig\bar{B}qS, \]
(6)
\[ |M|_{S_{1/2}}^2 = g^2 [(m_B + m_q)^2 - m_S^2], \]
(7)

For \((\frac{3}{2})^+\) baryons, the \(|M|^2\) are rather complicated and only ratios of anti-baryon/baryon are calculated, as the same matrices.

The diquark mass is preliminarily assumed as \(m_D(q_1q_2) = m_{d0} + m_{q_1} + m_{q_2}\) and the difference of axial-vector diquark mass and scalar diquark mass is neglected as a simple assumption. \(m_{d0}\) here is about 400-800 MeV and should not be smaller than the masses of constituent quarks. Additionally, it is assumed that \(g\) is same in these reactions.

3. Analysis

In the calculations, we set current quark mass,
\[ m_u = 3MeV, m_d = 6MeV, m_s = 122.5MeV, \]
as the mean masses. Different quark masses may cause systematic errors about 5% ~ 10%. The critical temperature is estimated at \(T_c(\mu=0) = 166.1MeV\) from a recent calculation based on [9] and [13]. The critical temperatures estimated from different methods (such as [19][20],) are similar and may cause systematic errors about 1% ~ 5%.

\(\Omega^+ / \Omega^-\) ratios calculated in different conditions are shown in Figure 1. It is clearly that the ratio is always greater than 1 when \(m_{d0}\) and \(\mu_B\) varies, even if ideal Fermi or Bose distributions are not formed. (*P.S. RHIC data [21] of \(\Omega^+ / \Omega^-\) at 130 GeV is before corrections such as annihilation of the daughter anti-protons with physical material in the detectors. After that correction the ratio may larger than 1, see in [22] Fig 6.)

Anti-baryon/baryon ratios are listed in Table 1, \(\mu_B \approx 47\) MeV and \(\mu_B \approx 30\) MeV are used to meet data from RHIC Au-Au at \(\sqrt{s_{NN}} = 130 GeV\) and 200 GeV [21][22][23][24][25][26][27][28][29][30][31][32], as \(m_{d0}\) is estimated at 600 MeV. Many of the calculations work well except \(\Xi\) productions, which also promote the inclusive \(\Lambda\) productions. Some strange baryon over proton ratios are listed in Table 2 and 3, compared with the upper limits from the Hadronic Gas Model. Some theoretical values are larger than the PHENIX preliminary results [31](, for which HG model works well). This may be caused by the condition that ideal QGP fluid is not completely formed at the temperature of \(T_c\).
Figure 1: Ratios of $\Omega^+ / \Omega^-$ from QGP with diquarks (lines larger than 1.0), general QGP (line at 1.0) and the upper limits of Hadronic Gas Model (lines smaller than 1.0) at different conditions, compared with preliminary results from RHIC\cite{21}. Systematic errors $\sim 5\%$.

| Ratios                  | $\sqrt{s_{NN}} = 130$ GeV | $\mu_B = 47$ MeV | $\sqrt{s_{NN}} = 200$ GeV | $\mu_B = 30$ MeV |
|-------------------------|-----------------------------|------------------|-----------------------------|------------------|
| $\bar{p}/p$             | $0.61 \pm 0.03 \pm 0.06$\cite{25} | 0.616            | $0.73 \pm 0.02 \pm 0.03$\cite{26} | 0.734           |
| $\bar{p}/p$ ($p, \Sigma^+$) | 0.72 $\pm 0.05$\cite{22}          | 0.698            | 0.78 $\pm 0.05$\cite{22}          | 0.795           |
| $\bar{p}/p$ (inclusive) | 0.65 $\pm 0.01 \pm 0.07$\cite{29}   | 0.747 $\pm 0.007 \pm 0.046$\cite{28} |                      |
| $\Lambda^{+}/\Lambda$   | 0.73 $\pm 0.03$\cite{25}          | 0.726            |                            | 0.815           |
| $\Lambda^{+}/\Lambda$ (inclusive) | 0.74 $\pm 0.04 \pm 0.03$\cite{30}       | 0.798            | $\approx > 0.8$\cite{32}       | 0.866           |
| $\Xi^{+}/\Xi^-$         | 0.75 $\pm 0.09 \pm 0.17$\cite{31}    | 0.921            | -                           | 0.949           |
| $\Omega^+ / \Omega^-$   | 0.95 $\pm 0.15 \pm 0.05^*$\cite{21} | 1.166            | 1.026 $\pm 0.075 \pm 0.12$\cite{21} | 1.103           |

Table 1: Anti-baryon/baryon ratios at $m_{d0} = 600$ MeV. Where $p$ ($p, \Sigma^+$) includes the decay contributions of $\Sigma^+$, $p$ (inclusive) includes the decay contributions of $\Sigma^+$ and $\Lambda$ baryons, $\Lambda$ (inclusive) includes the decay contributions of $\Xi^0$ and $\Xi^-$, contributions of $\Omega$ are neglected. Systematic errors $\sim 5\%$. 
Table 2  Relative yields of baryons at $\sqrt{s_{NN}} = 130\text{GeV}$ and $\mu_B = 47\text{MeV}$ from QGP with diquarks compared with Ideal Hadronic Gas limit at $T = 170\text{ MeV}$. Systematic errors $\sim 15\%$.

| Ratios                  | Exp. | QGP$q$ | HG limit |
|-------------------------|------|--------|----------|
| $\Lambda + \Sigma$     | -    | 1.256  | 0.710    |
| $\Lambda + \Sigma$     | -    | 1.714  | 0.767    |
| $\bar{\rho}$           | -    | 1.347  | 0.880    |
| $\bar{\rho}(\text{inclusive})$ | -    | 1.588  | 0.964    |
| $\Sigma$                | -    | 1.305  | 0.692    |
| $\Lambda - \Sigma$     | -    | 1.294  | 0.692    |

Table 3  Relative yields of baryons at $\sqrt{s_{NN}} = 200\text{GeV}$ and $\mu_B = 30\text{MeV}$ from QGP with diquarks compared with Ideal Hadronic Gas limit at $T = 170\text{ MeV}$. Systematic errors $\sim 15\%$.

4. Discussion

Baryon production ratios could be researched in the model of Quark-Gluon Plasma with diquarks and strange baryon over proton ratios from these calculations are larger than those of Hadronic Gas[4]. Another result is that $\Sigma^+$ production is larger than $\Lambda$, which in Hadronic Gas Model is smaller. But it is hard to be observed, due to the short decay length. $\Omega^+ / \Omega^- > 1$ is the most unusual results, which has been slightly supported by $\Omega^- / h^-$ and $\Omega^+ / h^-$ measured in RHIC[21], although the statistical and systematic errors are too large to confirm it.

It is reported that local thermal equilibrium has been formed in RHIC with high energy density based on recent hydrodynamic analysis[33]. As well as other evidence such as jet quenching[34], QGP existing in RHIC is nearly proved. $\Omega^+ / \Omega^- > 1$ will be another strong evidence if future results with smaller errors could confirm it.

Acknowledgement

We would like to thank Doctor Gene Van Buren and Doctor Christophe Suire for helpful discussions about experimental data. This work was supported in part by the National Natural Science Foundation of China (90103019), and the Doctoral Programme Foundation of Institution of Higher Education, the State Education Commission of China (2000000147).

References

[1] Pavković M I, Phys. Rev. D. 13 (1976) 2128.
[2] Shuryak E, Zahed I, hep-ph/0403127.
[3] Mustafa M, Thoma M, Chakraborty P, hep-ph/0403279.
[4] Miao H, Ma Z B and Gao C S, J. Phys. G29 (2003) 2187-2192.
[5] Rafelski J, Letessier J, Phys. Lett B469 (1999) 12. nucl-th/9908024.
[6] Rafelski J, Letessier J, Tounsi A, Acta Phys. Polon. B27 (1996) 1037.
[7] Rafelski J, Letessier J, Acta Phys. Polon. B30 (1999) 3559, hep-ph/9910300.

[8] Letessier J, Rafelski J, Acta Phys. Polon. B30 (1999) 153.

[9] Gao C S, Wu T, J. Phys. G27 (2001) 459-463.

[10] Braun-Munzinger P, Redlich K, Stachel J, Invited review for Quark Gluon Plasma 3, eds. Hwa R C and Wang X N, World Scientific Publishing, nucl-th/0304013.

[11] Ma Z B, Miao H and Gao C S, Chin. Phys. Lett. 20 (2003) 1691-1693.

[12] Mitrovski M, (for the NA49 collaboration), J. Phys. G30 (2004) S357-S362.

[13] BLEICHER M, et al., Phys. Rev. Lett. 88 (2002) 202501.

[14] Ma B Q, Qing D, Schmidt I, Phys. Rev. C65 (2002) 035205, hep-ph/0202015.

[15] Carlitz R, Phys. Lett. B58 (1975) 345; Kaur J, Nucl. Phys. B128 (1977) 219; Schäfer A, Phys. Lett. B208 (1988) 175.

[16] Ma B Q, Schmidt I, Yang J J, Phys. Lett. B477 (2000) 107-113, hep-ph/9906424.

[17] Gao C S, in JingShin Theoretical Physics Symposium in Honor of Professor Ta-You Wu T Y, edited by Hsu J P and Hsu L, (World Scientific, 1998) 362.

[18] Gao C S, Commun. Theor. Phys. 40 (2003) 188.

[19] Karsch F, Nucl. Phys. (Proc Suppl) 83-84 (2000) 14.

[20] Fodor Z, Katz S, JHEP 0203 (2002) 014, hep-lat/0106002.

[21] Suire C, (for the STAR Collaboration), Nucl. Phys. A715 (2003) 470-4735, nucl-ex/0211017; Suire C, (for the STAR Collaboration), Quark Matter 2002.

[22] Van Buren G, (for the STAR Collaboration), Nucl. Phys. A715 (2003) 129-139, nucl-ex/0211021.

[23] Xu N, Kaneta M, Nucl. Phys. A698 (2002) 306-313, nucl-ex/0104021.

[24] Baran A, Broniowski W and Florkowski W, Acta Phys. Polon. B35 (2004) 779-798, nucl-th/0305075.

[25] Harris J, Overview of First Results from Star, (for STAR Collaboration), Quark Matter 2002.

[26] Wosiek B, (for the PHOBOS Collaboration), Nucl. Phys. A715 (2003) 510-513, nucl-ex/0210037.

[27] Chujo T, (for the PHENIX Collaboration), Nucl. Phys. A715 (2003) 151-160, nucl-ex/0209027.

[28] Adler S S, et al., PHENIX Collaboration, nucl-ex/0307022.

[29] Adler C, et al., the STAR Collaboration, Phys. Rev. Lett. 86 (2001) 4778-4782.

[30] Adler C, et al., the STAR Collaboration, Phys. Rev. Lett. 89 (2002) 092301.

[31] Adcox K, et al., the PHENIX Collaboration, Phys. Rev. Lett. 89 (2002) 092302.

[32] Van Buren G, (for the STAR Collaboration), J. Phys. G28 (2002) 2103-2108, nucl-ex/0201009.

[33] Hirano T, Quark Matter 2004.

[34] Back B B, et al., the PHOBOS Collaboration, Phys. Rev. Lett. 91 (2003) 072302; Adler S S, et al., the PHENIX Collaboration, Phys. Rev. Lett. 91 (2002) 072303; Adams J, et al., the STAR Collaboration, Phys. Rev. Lett. 91 (2002) 072304; Arsene I, et al., the BRAHMS Collaboration, Phys. Rev. Lett. 91 (2002) 072305.