Coding Technique With Two and Three Stars, \( SSML(N_4, L_2) \) and GMJ CODE

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Abstract. In this paper, we introduce a coding technique for converting a text message using sub super mean labeling on two and three star graphs to a picture coding message. The first part of this paper there are some definitions and discussions for the existence graph and in the second part there are some illustrations to explain how the text message is converted in to picture code, by using graph labeling, and it maintain secrecy, possess intricacy and provide a sense of sufficiency. Secrecy is the practice of hiding information from certain individuals or groups who do not have the need to know, perhaps while sharing it with other individuals that which is kept hidden is known as secret.

Keywords: GMJ, Star, \( SSML(N_4, L_2) \)

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1. Introduction

In the sense of cryptography, graph problems are typically trivial, but often through suitable generalization, they may suggest ideas that are not only non-trivial, but may also be of some interest, and a few examples are given by R. C. Read [6]. Many family secrets are protected by using a mutually agreed upon construct (official family story) when communicating to any outside members. Graph theory may be used to modelling a problem that can be easier to see and find a solution to the problem. One of the graph subject is graph labeling topic. A Graph Labeling is an assignment of integers to the nodes or links, or both, subject to certain conditions. In the present investigations \( SSML(N_4, L_2) \) denotes Sub Super Mean Labeling assigning numbers to the nodes which differ by four and so the numbers assigned to the links differ by two which lead to the concepts of Omissions and Repetitions of some numbers denoted by RO (Repetitions and Omissions). We introduce \( t \) RO for two star and \( g \) RO for three star graphs. Proceeding further using these concepts and GMJ code (Graph Message Jumbled code), we arrived at two coding techniques which are dealt in this paper.

2. Literature Review:

Perhaps of all the creations of man, language is the most astonishing. The first business of language is simply to be transparent. With the dynamic living of the humans and the increase of civilization, the need to update transparency through secrecy increases. In any area, while...
messages and data want to be communicated only to a group of people or to an individual, one isn’t capable of using a common language. The communication becomes very a lot restricted among the sender and the receiver and now no longer to be understood through others. Cryptography is the technology containing strategies to convert an intelligible message into one which it’s far unintelligible and reworking the message return to its original form. The oldest kinds of ciphers (algorithm) became advanced through Julius Caesar ([4], [5]). The idea of coding using labeled trees was introduced by Saverio Caminiti et al., [8]. R.C.Read [6] presented a paper using graph theory and cryptography. Authenticated key distribution using given set of primes for secret sharing was given by Chandramowliswaran et al., [1]. The security of communication is a important trouble on global wide web. It is about confidentiality, integrity, authentication during access or editing of confidential internal documents and are given by a research paper [7]. Motivated by these work we have introduced a few techniques of coding by using super mean labeling on a two star graph [9]. The concept of Sub Super Mean labeling on two and three star graphs was introduced by Uma Mahswari et all [12], [10], [13], [14], [15]. In this paper, Coding is done on a two and three star graphs after applying SSML \((N_4, L_2)\) on them.

**Notations:**

Let \(G = (N(G), L(G))\) be a graph with \(a\) nodes and \(b\) links.

3. Definitions

**Definition 3.1 Sub Super Mean Labeling:**

Let \(G\) be a \((a,b)\) graph and \(\psi : N(G) \rightarrow \{1, 2, 3, \cdots, a+b\}\) be an injection. For each link \(\ell = uv\), let \(\psi^*(\ell) = \frac{\psi(u) + \psi(v)}{2}\) if \(\psi(u) + \psi(v)\) is even and \(\psi^*(\ell) = \frac{\psi(u) + \psi(v) + 1}{2}\) if \(\psi(u) + \psi(v)\) is odd. Then \(f\) is called sub super mean labeling if \(\psi(N) \cup \{\psi^*(\ell) : \ell \in L(G)\} \subset \{1, 2, 3, \cdots, a+b\}\). A graph that admits a sub super mean labeling is called a sub super mean graph.

**Definition 3.2 Sub Super Mean Labeling SSML \((N(4), L(2))\)**

A specific SSML defined on a three star graph where the numbers assigned to the pendant nodes differ by four and the corresponding link values differ by two, almost everywhere is named as SSML \((N(4), L(2))\) aptly.

**Definition 3.3 \(t\) and \((t-1)\) RO Sub Super Mean Graphs**

If the number of repetitions = number of omissions = \(t\) or \((t-1)\) with respect to SSML\((N_4, L_2)\), then the two star graph is called \(t\) RO or \((t-1)\) RO sub super mean graph where \(\mu = \lambda + 2t\) or \(\mu = \lambda + (2t + 1)\) with \(|\lambda - \mu| > 3\).

**Definition 3.4**
Definition 3.5 **Coding method**

GMJ coding method, procedure for encoding a message referred by [12]. GMJ code stands for:

- Graph Message Jumble code. A coding technique to communicate a message through graphs, jumbling letters is named as GMJ code.

- It also refers to the name of one of the researchers (Gabriel Margaret Joan) who has conceived this method of coding.

4. Discussion and Findings on a Two Star Graph

The discussion of SSML \((N_4, L_2)\) for \(G = K_{1, \lambda} \cup K_{1, \mu}\) for all values of \(\lambda\) and \(\mu\), for \(\lambda \leq \mu\) is provided below, which is required for the following theorem.

Let \(G = K_{1, \lambda} \cup K_{1, \mu}\)

\[
a = 1 + \lambda + 1 + \mu = 2 + \lambda + \mu \\
b = \lambda + \mu \\
a + b = 2\lambda + 2\mu + 2
\]

In the first copy of \(G = K_{1, \lambda} \cup K_{1, \mu}\) the labeling is done as follows:

\[
\psi(u) = 1 \\
\psi(u_1) = 3 \\
\psi(u_i) = 3 + 4(i - 1), \ 2 \leq i \leq \lambda \\
\psi(u_\lambda) = 3 + 4(\lambda - 1) = 4\lambda - 1 \\
\psi^*(uv_1) = 2 \\
\psi^*(uv_i) = 2 + 2(i - 1), \ 2 \leq i \leq \lambda \\
\psi^*(uv_\lambda) = 2\lambda.
\]

In the second copy of \(G = K_{1, \lambda} \cup K_{1, \mu}\), the labeling is done as follows:

Define \(\psi(v) = \psi(u_\lambda) + 4 = 4\lambda + 3\)

\[
\psi(v_1) = 5 \\
\psi(v_j) = 5 + 4(j - 1), \ 2 \leq j \leq J, \\
\text{where } J = \left\lceil \frac{2\lambda + 2\mu - 3}{4} \right\rceil
\]

\[
\psi(v_{J+1}) = 2\lambda + 2\mu + 2 \\
\psi(v_{J+2}), \psi(v_{J+3}) \text{ etc are assigned the remaining odd integers.}
\]

From the theorem [10], we have, if \(G = K_{1, \lambda} \cup K_{1, \mu}\) admits a SSML\((N_4, L_2)\) with \(|\lambda - \mu| > 3\), then we have the following:

(i) If \(\lambda\) is odd and \(\mu = \lambda + 2t\) or \(\mu = \lambda + 2t + 1\), then the graph is a \(t\) RO graph, \(t \geq 2\).

(ii) If \(\lambda\) is even and \(\mu = \lambda + 2t\) or \(\mu = \lambda + 2t + 1\), the graph is \((t - 1)\) RO graph, \(t \geq 2\).

Figure 1 is an example for 2RO graph,

Consider the graph \(G = K_{1, 5} \cup K_{1, 9}\), \(a + b = 30\).
5. Illustrations

Illustration 1:

(i) **Message:** Meet me at zero dark night.

(ii) **Clue:** Twinkling twins but not identical.
(Here \( \lambda \neq \mu \), \( |\lambda - \mu| > 3 \), not identical \( \Rightarrow \lambda \neq \mu \))

(iii) **Graph:** Two star graph.

(iv) **Numbering of alphabets:** VIBGYOR

**Clue:** If white is the universal set then black is the null set. (white contains the seven colours while black does not contain any of them). The 26 letters of English are divided into sets according to the letters in VIBGYOR and the sets are numbered in order.

\[
\begin{align*}
V & \quad \text{III} & \quad \text{IV} & \quad \text{II} \\
(A) & \quad (B \ C \ D \ E \ F) & \quad (G \ II) & \quad (I \ J \ K \ L \ M \ N) \\
VI & \quad \text{VII} & \quad \text{I} & \quad \text{V} \\
(O \ P \ Q) & \quad (R \ S \ T \ U) & \quad (V \ W \ X) & \quad (Y \ Z)
\end{align*}
\]
(v) **Coding a letter:**

Based on the theorem and a rule which are stated below the coding is developed.

Description is provided below:

The following results are required which are presented in Chapter 2.

(i) The two star graph $K_{1, \lambda} \cup K_{1, \mu}$, $\lambda \leq \mu$ is a 1 RO graph if $|\lambda - \mu| \leq 3$ and a $t$ RO graph if $\lambda$ is odd, $(t-1)$ RO graph if $|\lambda - \mu| > 3$ where $t = \mu - \lambda + 2$ or $\mu = \lambda + 2t + 1$.

(ii) The FEIO $= 2\lambda + 2$.

It is possible to write

$$FEIO = \begin{cases} 
\text{odd multiple of 7} + s, & \text{where } s \text{ is odd and } s \leq 6 \\
\text{even multiple of 7} + s, & \text{where } s \text{ is even and } s \leq 6 
\end{cases}$$

For example, if $\lambda = 12$,

$$2\lambda + 2 = 26 = (3 \times 7) + 5, \text{ FEIO } \equiv s \pmod{7}.$$ 

That is,

$M$ is denoted by $K_{12}^{18}$ and it can also be denoted by $K_{12}^{19}$.

For one more letter the explanation is provided.

$E$ belongs to the third set and it is the fourth number of the set.

If $\lambda = 15$, $2\lambda + 2 = 32 = (4 \times 7) + 4, \text{ FEIO } \equiv s \pmod{7}$.

That is, $M$ is denoted by $K_{15}^{22}$ and it can also be denoted by $K_{15}^{21}$.

$E$ is given by $K_{15}^{22}$, the next $E$ can be denoted by $K_{21}^{15}$.

(vi) **Coding:** (wordwise)

- **MEET** $- K_{12}^{11}K_{29}^{15}K_{21}^{15}K_{26}^{15}$
- **ME** $- K_{12}^{11}K_{29}^{15}$
- **AT** $- K_{11}^{12}K_{25}^{15}$
- **ZERO** $- K_{13}^{14}K_{21}^{15}K_{30}^{15}$
- **DARK** $- K_{25}^{17}K_{30}^{18}K_{27}^{18}K_{32}^{18}$
- **NIGHT** $- K_{25}^{17}K_{21}^{16}K_{39}^{25}K_{16}^{11}$

(vii) **Presenting the letter codes:**

The coding is written along a horizontal string in the reverse order. The last word is written first.
(viii) **Horizontal string:**

\[ K_{16}^{14} K_{17}^{17} K_{23}^{11} K_{18}^{18} K_{31}^{10} K_{29}^{10} K_{26}^{25} K_{30}^{25} K_{32}^{14} K_{22}^{15} K_{26}^{15} K_{11}^{17} \]

(ix) **About the picture coding:**

Figure 2 represents the picture coding, the white colour contains all the seven colours and the black colour contains none of them. So, the letter U representing the universal set and the letter \( \phi \) representing the null set are used here.

For decoding we reverse the process.

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6. **Discussion**

The discussion of SSML(\( N_4, L_2 \)) on \( G = K_1, \delta \cup K_1, \lambda \cup K_1, \mu \), \( \delta \leq \lambda \leq \mu \) for all values of \( \delta, \lambda \) and \( \mu \).

By the theorem referred by [13] we have, if \( G = K_1, \delta \cup K_1, \lambda \cup K_1, \mu \) admits SSML(\( N_4, L_2 \)) with \( 2 \leq \delta \), or \( 2 \leq (\mu - \lambda) + \delta \) and \( g = \left[ \frac{\mu + \delta - \lambda}{2} \right] \), then

(A) If \( a + b = 2\delta + 2\lambda + 2\mu + 3 = 4J + 3 \), \( J = \delta + K, K = \left[ \frac{\mu + \lambda - \delta}{2} \right] \), the graph is a \((g - 1)\) RO, \( g \) RO and \((g + 1)\) RO graph according as both FEIO, SEIO accommodated, any one is accommodated and both not accommodated respectively.

(B) If \( (a + b) = 2\delta + 2\lambda + 2\mu + 3 > 4J + 3 \), then the graph is \( g \) RO, \((g + 1)\) RO and \((g + 2)\) RO respectively as stated in (A).

The results are, for proof refer [13].

**Case (i):** When anyone FEIO or SEIO is accommodated, the number of even integers omitted is \((g + 1) - 1 = g\), so the graph is a \( g \) RO graph.

**Case (ii):** When both FEIO and SEIO are accommodated, the number of even integers omitted is \((g + 1) - 2 = (g - 1)\), so the graph is a \((g - 1)\) RO graph.

**Case (iii):** When both FEIO and SEIO are not accommodated, the number of even integers omitted is \((g + 1)\), so the graph is a \((g + 1)\) RO graph.

Figure 3 is the Super Mean Labeling and the Figure 4 is the Sub Super Mean labeling SSML(\( N_4, L_2 \)) are done on the graph \( G = K_1, 4 \cup K_1, 4 \cup K_1, 4 \), \( a + b = 27 \).
Note: In the discussion of the above theorem we required two conditions on \( \delta, \lambda \) and \( \mu \).

(i) \( 2 \leq \delta \) claim (iii)
(ii) \( 2 \leq (\mu - \lambda) + \delta \) claim (v)

We now prove that \( 2 \leq \delta \Rightarrow 2 \leq (\mu - \lambda) + \delta \)

If \( \mu = \lambda \), \( 2 \leq (\mu - \lambda) + \delta \) is true as \( 2 \leq \delta \).

If \( \mu \neq \lambda \), \( (\mu - \lambda) \geq 1 \), \( (\mu - \lambda) + \delta > 2 \)

Therefore \( 2 \leq \delta \Rightarrow 2 \leq (\mu - \lambda) + \delta \), for all values of \( \lambda \) and \( \mu \), but \( 2 \leq (\mu - \lambda) + \delta \) need not imply \( 2 \leq \delta \).

For if \( \mu \neq \lambda \) and \( \delta = 1, 2 \leq (\mu - \lambda) + \delta \) holds.

But as \( \delta = 1, 2 \leq \delta \) fails. When \( \delta = 1, \lambda = 2 \) and \( \mu = 3 \) the condition \( 2 \leq (\mu - \lambda) + \delta \) is true but \( 2 \leq \delta \) fails, the theorem holds good for the three star \( K_{1,1} \cup K_{1,2} \cup K_{1,3} \).

When \( \delta = \lambda = \mu = 1 \) the condition \( 2 \leq (\mu - \lambda) + \delta \) and \( 2 \leq \delta \) both fail. It is found that the labeling itself is not available for \( 4\delta + 7 \), the first pendant node of the third copy exceeds \( (a + b) \).

So, on this graph SSML\( (N_4, L_2) \) cannot be assigned and \( g \) cannot be used in this case. When \( \delta = 1 \), the theorem may or may not hold. In the second illustration a technique is developed using the conditions for FEIO, SEIO or both to be accommodated as pendant nodes and the value of \( g = \left[ \frac{\mu + \delta - \lambda}{2} \right] \) which is connected with RO.
The discussion given below explains the conditions required for the same and $g = \left\lfloor \frac{\mu + \delta - \lambda}{2} \right\rfloor$.

$$2g \leq \mu + \delta - \lambda \leq (2g + 1)$$

$$(2g - \delta) \leq (\mu - \lambda) \leq (2g - \delta) + 1.$$ (1)

Take $(2g - \delta) = k$, choose a $\delta \geq 2$, choose a $\lambda$ such that $\delta + 2\lambda + 4$ is odd if FEIO to be accommodated; $\lambda$ can be odd or even but $\delta$ has to be odd.

Then $\mu$ gets fixed, $\mu = \lambda + k$ or $\mu = \lambda + k + 1$ for FEIO.

$$\delta + 3\lambda + 5 \text{ is odd for SEIO to be accommodated.}$$

If $\delta$ is odd then $\lambda$ is odd, if $\delta$ is even then $\lambda$ is even.

Take both $\delta$ and $\lambda$ as even numbers for SEIO.

$$\delta + 2\lambda + 4 \text{ and } \delta + 3\lambda + 5 \text{ both must be odd for FEIO, SEIO or both to be accommodated.}$$ (4)

Table 1 shows the above observations are tabulated:

| $\delta$ | $\lambda$ | FEIO $= \delta + 2\lambda + 4$ | SEIO $= \delta + 3\lambda + 5$ | Both |
| --- | --- | --- | --- | --- |
| odd | odd | ✓ | ✓ | ✓ |
| odd | even | ✓ | × | × |
| even | odd | × | × | × |
| even | even | × | ✓ | × |

We also need to make the following observations on the value of $\delta, \lambda$ and $\mu$.

$$g = \left\lfloor \frac{\delta + \mu - \lambda}{2} \right\rfloor$$

$$2g \leq (\delta + \mu) - \lambda \leq (2g + 1)$$

$$2g \leq \delta + (\mu - \lambda) \leq (2g + 1).$$

For a value of $g$, when $\delta$ and $\lambda$ are odd, from the condition $\delta = 2g + 1$, as the least value of $(\mu - \lambda)$ is zero, $\mu \geq \lambda$. For $F$ to be accommodated, $\delta$ is odd, $\lambda$ is odd ($\lambda$ can also be even) together imply $\mu$ is odd and $\mu = \lambda$.

$\delta$ odd, $\lambda$ even for $F$ to be accommodated.

If $\delta$ is even, the condition implies $\delta = 2g$ as $2g \leq \delta \leq (2g + 1)$, so $0 \leq (\mu - \lambda) \leq 1$, $\mu \leq (\lambda + 1)$.

Take $\lambda$ even and so $\mu$ is odd that is $\delta$ is even, $\lambda$ even and $\mu = (\lambda + 1)$ odd if $S$ has to be accommodated.

As $1 \leq g \leq 13$ and $2 \leq \delta$ with the conditions [5] and [6] decoding is made possible.

Now for the coding of a letter we need to describe the method of numbering alphabets.
Illustration 2:

(i) **Message:** Precious stones near BR (Blowing Rock).

(ii) **Clue:** Look for Orion in the diamond embedded beautiful sky. (Orion is a constellation of three stars. A three star graph is implied).

(iii) **Graph:** Three star graph.

(iv) **Numbering of alphabets:**

   **Clue:** Omission of one’s duty is condemnable. Repetition of good deeds is commendable. Divide the 26 alphabets into 2 sets of 13 letters. For the first set of letters from A to M, the condition for FEIO to be accommodated as a pendant node is taken. That is condition [5].

   For the second set of letters from N to Z, the condition for SEIO to be accommodated as a pendant node is taken. That is condition [6].

   In both the cases, the value of \( g \) specifies the position of the letter in the set. Here \( g \) varies from 1 to 13.

   If the letter belongs to the first set containing A to M, then \( \delta \) is odd, \( \lambda = \mu \), even. The number representing the letter will be factored as \( \delta \times \lambda \times \lambda = \delta \times \lambda^2 \). (7)

   If the letter belongs to the second set containing N to Z, then \( \delta \) is even, \( \lambda \) is even and \( \mu = \lambda + 1 \). The number representing the letter will be factored as \( \delta \times \lambda \times (\lambda + 1) \), (\( \delta \) even, \( \lambda \) even, \( \mu = \lambda + 1 \)) (8)

   To summarize:

\[
\begin{align*}
&\text{FEIO (} \delta \text{ odd, } \lambda \text{ even and } \mu = \lambda) \\
&\text{A B C D E F G H I J K L M} \\
&(\text{SEIO (} \delta, \lambda \text{ even and } \mu = \lambda + 1) \\
&\text{N O P Q R S T U V W X Y Z}
\end{align*}
\]

(v) **Coding a letter:**

   After finding the suitable values for \( \delta, \lambda \) and \( \mu \) the letter is represented by a number which is the product of \( \delta, \lambda \) and \( \mu \) and which takes up the form \( \delta \times \lambda \times \lambda = \delta \times \lambda^2 \) or \( \delta \times \lambda \times (\lambda + 1) \) as given in (7) and (8).

   (i) \( P : (S, g = 3)(\delta \text{ even, } \lambda \text{ even, } \mu = \lambda + 1), \ 6 \leq \delta + (\mu - \lambda) \leq 7, \ \delta \text{ is even, } \delta = 6, \ (\mu - \lambda) \leq 1, \ (\lambda \text{ can take any even integer } \geq \delta), \ \lambda = 14, \ \mu = 15, \ P \text{ is denoted by } 6 \times 14 \times 15 = 1260. \)

   (ii) Consider the letter \( E \).

   \( E : (F, g = 5), \ (\delta \text{ odd, } \lambda \text{ even, } \lambda = \mu \text{ even}), \ 10 \leq \delta + (\mu - \lambda) \leq 11, \ \delta = 11, \ (\mu - \lambda) = 0, \ \lambda \text{ even}. \) Note that \( \lambda \) can assume any even number \( \geq \delta. \)

   Take \( \lambda = 18 \), then \( \mu = 18, \ E : 11 \times 18 \times 18 \) and it is denoted by 3564 and so on. The other letters in the message are allotted numbers proceeding in the same way.
(vi) **Coding:** (wordwise)

**PRECIOUS**  
\[ P : [(S,g) = 3], 6 \times 14 \times 15 = 1260], \quad R : [(S,g) = 5], 10 \times 16 \times 17 = 2720]  

\[ E : [(F,g) = 5], 11 \times 18 \times 18 = 3564], \quad C : [(S,g) = 3], 7 \times 8 \times 8 = 448]  

\[ I : [(F,g) = 9], 19 \times 22 \times 22 = 9196], \quad O : [(S,g) = 2], 4 \times 6 \times 7 = 168]  

\[ U : [(S,g) = 8], 16 \times 32 \times 33 = 16896], \quad S : [(S,g) = 6], 12 \times 16 \times 17 = 3264]  

**STONES**  
\[ S : [(S,g) = 6], 12 \times 16 \times 17 = 3264], \quad T : [(S,g) = 7], 14 \times 30 \times 31 = 13020]  

\[ O : [(S,g) = 2], 4 \times 8 \times 9 = 288], \quad N : [(S,g) = 1], 2 \times 34 \times 35 = 2380]  

\[ E : [(F,g) = 5], 11 \times 20 \times 20 = 4400], \quad S : [(S,g) = 6], 12 \times 34 \times 35 = 14280]  

**NEAR**  
\[ N : [(S,g) = 1], 2 \times 26 \times 27 = 1404], \quad E : [(F,g) = 5], 11 \times 42 \times 42 = 19404]  

\[ A : [(F,g) = 1], 3 \times 60 \times 60 = 10800], \quad R : [(S,g) = 5], 10 \times 12 \times 13 = 1560]  

**BR**  
\[ B : [(F,g) = 2], 2 \times 42 \times 42 = 3528], \quad R : [(S,g) = 5], 10 \times 28 \times 29 = 8120]  

(vii) **Presenting the letter codes:**

The numbers for the letters are presented along a horizontal string jumbling them. Here odd positioned letters (written alternately) and then even positioned letters written alternately. There are 20 letters in the message, the letters with the position 1, 19, 3, 17 and so on, then 2, 20, 4, 18 and so on are written along the horizontal string making decoding a bit difficult and thereby increasing the measure of secrecy.

\[
\begin{array}{cccccccccc}
(1) & (19) & (3) & (17) & (5) & (15) & (7) & (13) & (9) & (11) \\
1260 & 3528 & 3564 & 10800 & 9196 & 1404 & 16896 & 4400 & 3264 & 288 \\
(2) & (20) & (4) & (18) & (6) & (16) & (8) & (14) & (10) & (12) \\
2720 & 8120 & 448 & 1560 & 168 & 19404 & 1872 & 14280 & 13020 & 2380 \\
\end{array}
\]

(viii) **Horizontal string:**

\[
\begin{array}{cccccccccccc}
1260 & 3528 & 3564 & 10800 & 9196 & 1404 & 16896 & 4400 & 3264 & 288 & 2720 & 8120 \\
448 & 1560 & 168 & 19404 & 1872 & 14280 & 13020 & 2380 \\
\end{array}
\]

(ix) **A brief explanation about decoding.**

Given a number, factorize it into three factors (not necessarily into prime factors.)

(i) Identify the least odd factor and two equal even factors, greater than the odd factors. The position of the letter represented by the number is \((\text{odd divisor} - 1) \div 2\) in the first set given by FEIO condition.

(ii) Identify the least even factor and two factors which are two consecutive numbers one of them greater than or equal to the even factor. The \((\text{least even factor}) \div 2\) specifies the position of the letter given by SEIO condition.

For example, 16896 is factored.

\[
16896 = 4 \times 4224 = 4 \times 12 \times 352 = 4 \times 12 \times 16 \times 22 \\
= 16 \times 2 \times 2 \times 3 \times 4 \times 2 \times 11 = 16 \times 32 \times 33 \\
\delta = 16, \lambda = 32, \mu = 33, g = 8.
\]

It represents the 8th letter in the set \(N\) to \(Z\) that is the letter \(U\).
About the picture coding:

The numbers are given in the form of squares, making the finding of the letter still more difficult. Figure 5 shows the picture coding for FEIO and SEIO.

![Figure 5. Coded message (FEIO AND SEIO)](image)

The sender should send the following to the receiver:

1. **Clue to guess the graph**
2. **Clue for the method used for numbering the alphabets without explanation**
3. **Letter codes along a pattern (horizontal string or picture coding)**
7. Conclusion and future work:
In this paper the researchers have formulated two coding techniques using GMJ coding and illustrated how the text message is converted into a picture message using graph labeling. The results we have obtained by introducing the concepts of omissions and repetitions with a labeling SSML ($N_4$, $L_2$) on a two and three star graphs and using the results we provided discussions for drawing the star graphs. This method is very useful in the field of coding theory, cryptography to mention a few to share the secret messages using sub super mean labeling and the clues, mathematical or non-mathematical are very creative to maintain secrecy and depend on the sender’s imagination and creativity and hence an computer based algorithm cannot be made available for the clues.

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