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**Title:** Squeeze Free Space with Nonlocal Flat Optics

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Squeeze free space with nonlocal flat optics: supplemental document

This document provides supplemental information to "Squeeze free space with nonlocal flat optics". It consists of three sections. In Sec. 1, we provide the effective Hamiltonian for the second design with hexagonal lattice. In Sec. 2, we provide more details of the second design. In Sec. 3, we provide the polarization response of the second design.

1. EFFECTIVE HAMILTONIAN FOR HEXAGONAL LATTICE

Here we derive the $k \cdot p$ effective Hamiltonian near $\Gamma$ point for a photonic crystal slab having $C_{6v}$ symmetry. We consider a $2 \times 2$ effective Hamiltonian starting from a doubly degenerate pair at $\Gamma$. We choose the $E_1$ representation at $\Gamma$ point, and choose $|x\rangle$ and $|y\rangle$ as basis for the state, $k = (k_x, k_y)$ is the in plane wavevector. The effective Hamiltonian takes the form [1]:

$$ \hat{H}(k) = \hat{A}(k) - i\hat{B}(k) \quad (S1) $$

where $\hat{A}$ and $\hat{B}$ are Hermitian matrices, and can be decomposed as:

$$ \hat{A}(k) = f(k)\sigma_+ + f^*(k)\sigma_- + g(k)\sigma_z + h(k)I + \omega_0 I \quad (S2) $$

$$ \hat{B}(k) = r(k)\sigma_+ + r^*(k)\sigma_- + s(k)\sigma_z + t(k)I + \gamma_0 I \quad (S3) $$

Here $f$ and $r$ are complex, $g, h, s, t$ are real since $\hat{A}$ and $\hat{B}$ are chosen to be Hermitian. By virtue of the two fold degeneracy, we can choose a $\omega_0$ and $\gamma_0$ such that $f, g, h, r, s, t$ are zero at $k = 0$.

We know that the Hamiltonian of the system is constrained by the symmetry operations $g$ from the symmetry group $G$ of the system, in the following way:

$$ D(g)\hat{H}(k)D^{-1}(g) = \hat{H}(D(g)k) \quad (S4) $$

Here $D(g)$ and $D'(g)$ are the representations of $g$ in the Hilbert space spanned by the two states of $\Gamma$ and in the Cartesian space, respectively. We consider a photonic crystal slab with a hexagonal lattice, the symmetry group is $C_{6v} = \{E, 2C_3, C_3, 3\sigma_v, 3\sigma_d\}$. Due to our basis choice, the operator $D$ and $D'$ has the same matrix form. We notice that the Hermitian part and anti-Hermitian part in Eq. (S4) is decoupled, which means $\hat{A}$ and $\hat{B}$ will be constrained in the same way. Therefore, we only discuss the constraint on $\hat{A}$ in the following.

A. Inversion symmetry $C_2$

$$ D(C_2) = D(C_2) = \begin{pmatrix} -1 & 0 \\ 0 & -1 \end{pmatrix} \quad (S5) $$

Apply this symmetry operation in Eq. (S4), we have:

$$ \begin{pmatrix} h(k) + g(k) & f(k) \\ f^*(k) & h(k) - g(k) \end{pmatrix} = \begin{pmatrix} h(-k) + g(-k) & f(-k) \\ f^*(-k) & h(-k) - g(-k) \end{pmatrix} \quad (S6) $$

For this equation to hold for any $k, f, g, h$ cannot have terms proportional to $k_x$ or $k_y$. The lowest order terms must be quadratic.

B. Mirror symmetry $\sigma_v$

$$ D(\sigma_v) = D(\sigma_v) \equiv M_y = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \quad (S7) $$
Apply this symmetry operation in Eq. (S4), we have:

$$
\begin{pmatrix}
  h(k_x, k_y) + g(k_x, k_y) - f(k_x, k_y) \\
  -f^*(k_x, k_y) h(k_x, k_y) - g(k_x, k_y)
\end{pmatrix}
= \begin{pmatrix}
  h(k_x, -k_y) + g(k_x, -k_y) & f(k_x, -k_y) \\
  f^*(k_x, -k_y) & h(k_x, -k_y) - g(k_x, -k_y)
\end{pmatrix}
$$  \hspace{1cm} (S8)

For a general quadratic form, $f(k) = f_x k_x^2 + f_y k_y^2 + f_{xy} k_x k_y$, $g(k) = g_x k_x^2 + g_y k_y^2 + g_{xy} k_x k_y$, $h(k) = h_x k_x^2 + h_y k_y^2 + h_{xy} k_x k_y$ the constraint implies $f_x, f_y, g_{xy}$, and $h_{xy}$ are zero.

C. Six fold rotation symmetry $C_6$

$$
\mathcal{D}(C_6) = \mathcal{D}(C_6) = \begin{pmatrix}
  \frac{1}{2} & -\frac{\sqrt{3}}{2} \\
  \frac{\sqrt{3}}{2} & \frac{1}{2}
\end{pmatrix}
$$  \hspace{1cm} (S9)

Apply this symmetry operation in Eq. (S4), consider $f, g$, and $h$ only to nonzero quadratic orders. For the constraint to hold for any $k$, we arrive at the following:

$$
\begin{align*}
  h_x &= h_y \\
  g_x &= -g_y \\
  \text{Re}(f_{xy}) &= 2g_x \\
  \text{Im}(f_{xy}) &= 0
\end{align*}
$$  \hspace{1cm} (S10) to (S13)

Written in a more concise form, the coefficients in the effective Hamiltonian are:

$$
\begin{align*}
  f(k) &= 2\beta k_x k_y \\
  g(k) &= B(k_x^2 - k_y^2) \\
  h(k) &= A(k_x^2 + k_y^2)
\end{align*}
$$  \hspace{1cm} (S14) to (S16)

where $\beta$ and $B$ are real coefficients. The same form applies to the anti-Hermitian part of the Hamiltonian, with the coefficients:

$$
\begin{align*}
  r(k) &= 2B' k_x k_y \\
  s(k) &= B'(k_x^2 - k_y^2) \\
  t(k) &= A'(k_x^2 + k_y^2)
\end{align*}
$$  \hspace{1cm} (S17) to (S19)

where $A'$ and $B'$ are real coefficients. Therefore, the effective Hamiltonian can be written as:

$$
\mathcal{H}(k) = \omega_0 I - i\gamma_0 + a|k|^2 I + b(k_x^2 - k_y^2)\sigma_z + 2\beta k_x k_y \sigma_x
$$  \hspace{1cm} (S20)

Here $a = A - iA'$, $b = B - iB'$ are complex coefficients. This give rise to bands with isotropic dispersion:

$$
\omega_{\pm}(k) = \omega_0 - i\gamma_0 + (a \pm b)|k|^2
$$  \hspace{1cm} (S21)

2. DETAILS OF THE SECOND DESIGN

In this section, we provide more details of the second design. The structure is shown in Fig. 5a in the main text, which is a single layer photonic crystal slab with a hexagonal array of circular air holes. The slab supports a pair of guided resonances that are doubly degenerate at the $\Gamma$ point. As derived in the previous section, the effective Hamiltonian is isotropic near $\Gamma$. Such an isotropic Hamiltonian leads to the effect of single-band excitation: $s/p$-polarized light only couples to the upper/lower band, respectively, for every direction of incidence. This can be proved analogously as the case of square lattice [1]. Fig. S1a and S1b depict the magnitude $|t_{pp}|$ and phase $\text{arg}(t_{pp})$, respectively, at a general azimuthal angle $\phi = \arctan(k_y/k_x) = 14^\circ$. Due to the isotropic band structure, the results are essentially the same for any other $\phi$. The plots clearly show that $p$-polarized light only excites the lower band. The transmission exhibits sharp dip in magnitude and rapid variation in phase near the band dispersion of the guided resonances.
Fig. S1. The transmission properties of the hexagonal photonic crystal slab device shown in Fig. 5a. (a) Transmission magnitude $|t_{pp}|$ of the incident $p$ polarized light at different frequencies and in-plane wavevectors. (b) Transmission phase $\arg(t_{pp})$ of the incident $p$-polarized light at different frequencies and in-plane wavevectors. (c) $|t_{pp}|$ as a function of frequency, for $p$-polarized light with an incident angle $0^\circ$ and $6^\circ$. (d) $\arg(t_{pp})$ as a function of frequency, for $p$ polarized light with an incident angle $0^\circ$ and $6^\circ$. In all subfigures, the orange and blue lines correspond to the $0^\circ$ and $6^\circ$ incidence, respectively, and the green line indicates the operating frequency $\omega_{op} = 0.471 \times 2\pi c / a$.

Fig. S2. Transfer function amplitude $|t_{pp}|$ of the hexagonal photonic crystal slab device as a function of in plane wavevector magnitude $|k|$. Fig. S1c and S1d plot the spectra of the transmission magnitude and phase at incident angles $\theta = 0^\circ$ and $\theta = 6^\circ$, respectively. For both incident angles, both the amplitude and phase follow the Fano lineshape formula as described by Eq. (12) and Eq. (13) in the main text, and the resonance shifts to higher frequencies as $\theta$ increases. At the operating frequency $\omega_{op} = 0.471 \times 2\pi c / a$ as indicated by the green dashed line, the magnitude of the transmission coefficient stays close to unity. The phase varies substantially for varying incident angles, as described by Eq. (19) in the main text. Also, there is no diffraction order at this frequency since the lattice constant is subwavelength.

Fig. S2 shows the magnitude of the transmission along the $\Gamma X$ direction at the operating frequency $\omega_{op}$. The transmission has a magnitude of unity over the entire wavevector range considered. Hence the structure behaves as an all-pass filter in this wavevector range. As shown in Fig. 5e and 5f in the main text, the phase is isotropic and shows a quadratic dependency of $|k|$. The magnitude and phase response therefore agrees with Eq. (19) in the main text.

In Fig. S3, we provide a numerical demonstration of the performance of our device. We consider a converging radially ($p$) polarized cylindrical vector beam at the operating frequency $\omega_{op}$. Its intensity distribution at $z = 0$ plane is shown in Fig. S3a. For demonstration, we directly simulate a cascade of $N = 90$ devices with an air gap of thickness $0.5a$ between every two neighboring devices. The total thickness of the 90 cascaded devices excluding the air gaps are $30a$. We
Fig. S3. Demonstration of the device with beam propagation. (a) The intensity distribution of an input converging radially (p) polarized cylindrical vector beam. (b) The radial intensity profile of the three beams in (a), (c), and (d). (c) The intensity distribution of the input beam immediately after passing through 90 devices with a total thickness 30a excluding the air gaps between the devices. (d) The intensity distribution of the input beam after a free space propagation of 378a.

We numerically calculate the intensity distribution of the transmitted beam after the cascaded devices as shown in Fig. S3c. The beam size significantly reduces after passing through the devices. We also numerically verify that the result for 90 devices is indeed the same as that for a single device repeated 90 times. As comparison, in Fig. S3d we plot the intensity distribution of the original beam after the propagation in free space by $90 \times (d_{\text{eff}} + 0.5a) = 378a$. Fig. S3c and Fig. S3d agree very well. In Fig. S3b we plot the radial profile for the input beam and the aforementioned two output beams. We see that our device can substitute free space propagation and reproduce the beam width and beam shape very well. The reduced peak intensity is caused by the deviation of the transfer function at large wavevectors.

3. POLARIZATION RESPONSE

In this section, we provide the polarization response of the second device consisting of a hexagonal array of holes. We consider four polarizations: P polarization, S polarization, right circular polarization (RCP) and left circular polarization (LCP). For all the four polarizations, our device serves as an all-pass quadratic phase filter, thus can be used to substitute free space.

For the four polarizations, Fig. S4 shows the phases of the transfer function $\arg(t)$ as functions of in-plane wavevectors $(k_x, k_y)$, while Fig. S5 shows $\arg(t)$ as a function of in-plane wavevector magnitude $|k|$, along with a fit to a quadratic function, for each polarization. These plots show that, for all the four polarizations, the phase is isotropic and exhibits a quadratic dependency of $|k|$. Here we note that the phase response of RCP and LCP is the same, and is approximately the average of that of P and S polarizations. From the fitting, we obtain the equivalent thickness $d_{\text{eff}} = 3.70a, 12.53a, 7.83a, 7.83a$ for P, S, RCP and LCP, respectively, corresponding to a compression ratio $d_{\text{eff}}/d = 11.2, 38.0, 23.7, 23.7$, respectively.

REFERENCES

1. C. Guo, M. Xiao, M. Minkov, Y. Shi, and S. Fan, “Photonic crystal slab Laplace operator for image differentiation,” Optica 5, 251 (2018).
Fig. S4. arg(t) as a function of in-plane wavevectors \((k_x, k_y)\) for (a) p polarization, (b) s polarization, (c) right circular polarization, and (d) left circular polarization.

Fig. S5. arg(t) as a function of in-plane wavevector magnitude \(|k|\) for (a) p polarization, (b) s polarization, (c) right circular polarization, and (d) left circular polarization, along with a fit to a quadratic function.