Multi-angle Five-Brane Intersections

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ABSTRACT

We find new solutions of IIA supergravity which have the interpretation of intersecting NS-5-branes at $Sp(2)$-angles on a string preserving at least $3/32$ of supersymmetry. We show that the relative position of every pair of NS-5-branes involved in the superposition is determined by four angles. In addition we explore the related configurations in IIB strings and M-theory.
1. Introduction

Most of the recent progress in understanding the various dualities of superstring theories as well as their applications in black holes and supersymmetric Yang-Mills is due to the investigation of intersecting brane configurations. There are many ways to view such configurations. One way is as classical solutions of supergravity theories which are the effective theories of superstrings and M-theory. There are many such configurations. Here we shall be mainly concerned with the M-theory configuration which has the interpretation of M-5-branes intersecting on a string. This configuration is related via reduction and via IIA/IIB T-duality to a large number of ten-dimensional configurations. These include those of [1] that have been used to investigate four-dimensional N=2 supersymmetric Yang-Mills as well as configurations that have many novel properties like those of [2] (and refs within).

There are many ways to superpose two M-5-branes so as to intersect on a string preserving a proportion of spacetime supersymmetry. The simplest case is that of orthogonally intersecting M-5-branes, i.e.

\begin{equation}
(i) \text{M} - 5 : \quad 0, 1, 2, 3, 4, 5, *, *, *, *, *
(ii) \text{M} - 5 : \quad 0, 1, *, *, *, *, 6, 7, 8, 9, * .
\end{equation}

In this notation, the M-5-branes are in the directions 0, 1, 2, 3, 4, 5 and 0, 1, 6, 7, 8, 9, respectively, and the string is in the directions 0, 1. The associated supergravity solution was found in [3] and its interpretation within M-theory was given in [4]. This solution depends on two harmonic functions. However, unlike other intersecting M-brane configurations [5, 6, 7] the harmonic function associated with one of the M-5-branes depends on the worldvolume coordinates of the other brane which are transverse to the common intersection. The rest of the configurations involve superpositions of M-5-branes at angles [8]. For this, one of the M-5-branes is rotated relative to the other with an element of a subgroup $G$ of $SO(8)$. An example of a supergravity solution with the interpretation of M-5-branes intersecting at $Sp(2)$-angles on a string was given in [2]. This was achieved by T-dualizing twice the
product of an eight-dimensional toric hyper-Kähler metric with a two-dimensional Minkowski one (as solution of IIA supergravity) and then lifting the resulting solution to eleven dimensions.*. The M-theory configuration found in [2] has the properties that there is one independent angle between every pair of intersecting M-5-branes and it preserves 3/16 of supersymmetry.

The intersecting M-5-brane configuration in [2] is a special case of a more general solution for which there are four independent angles between every pair of M-5-branes. One indication that more general solutions exist from those of [2] was given in [14] by comparing Yang-Mills configurations with M-5-brane configurations in the context of matrix theory [15]. In matrix theory, longitudinal M-5-branes correspond to four-dimensional Yang-Mills instantons. Using this, it turns out that intersecting M-5-branes at $Sp(2)$-angles on a string correspond to Yang-Mills configurations for which the curvature two-form† is in the $sp(2)$ subalgebra of $so(8)$ [16, 14]. Interpreting a class of solutions of this Yang-Mills BPS [17] condition as a superposition of four-dimensional instantons, it was found in [14] that there are four independent angles between every pair of four-dimensional instantons. Some more indications for the existence of other supersymmetric multi-angle intersecting brane solutions were also given in [18]. This was done by investigating the supersymmetry projections associated with two M-5-branes.

In this paper we shall present a method to superpose IIA NS-5-brane solutions that yields new solutions of IIA supergravity with the interpretation of intersecting IIA NS-5-branes at $Sp(2)$-angles on a string. They generalize those of [2]. We shall find that there are four independent angles between every pair of NS-5-branes involved in the intersection and that our solutions preserve at least 3/32 of supersymmetry. We shall also show that for a given pair of NS-5-branes and generic asymptotic metric, the angles matrix has two independent eigenvalues. We shall then present the superpositions of these solutions with a fundamental string and a

* For other brane intersections at angles see [9-13].
† The gauge group is $U(N)$ for some $N$. 
We shall investigate the IIB duals of these configurations as well as their M-theory interpretation.

2. IIA NS-5-branes at angles

The reduction of intersecting M-5-branes on a string configuration of eleven-dimensional supergravity in a direction transverse to the branes leads to a IIA string configuration with the interpretation of intersecting NS-5-branes on a string. It turns out that it is more convenient to carry out our computations in the context of IIA supergravity. The M-theory configurations can then be found by simply lifting the ten-dimensional ones to eleven dimensions.

We are interested in a solution of IIA supergravity that involves only NS-5-branes. This allows us to set all Ramond$\otimes$Ramond fields to zero since this is a consistent truncation of IIA supergravity. The resulting field equations in the string frame are

\[
\begin{align*}
R_{MN} - H_{MPQ}H_N^{PQ} + 2\nabla_M \partial_N \phi &= 0 \\
\nabla_P (e^{-2\phi} H^{PMN}) &= 0,
\end{align*}
\]

where $\phi$ is the dilaton and $H$ is the 3-form field strength of the NS$\otimes$NS sector, $\nabla$ is the Levi-Civita connection of the metric $g$ and $M, N, P, Q = 0, \ldots, 9$. We raise and lower indices with the metric $g_{MN}$. There is also a third field equation associated with the dilaton. However it is well known that this field equation is implied from those in (2.1) up to a constant.

The IIA NS-5-brane solution [19] is

\[
\begin{align*}
ds^2 &= ds^2(\mathbb{E}^{(1,5)}) + F ds^2(\mathbb{E}^4) \\
H &= -\frac{1}{2} \star dF \\
e^{2\phi} &= F
\end{align*}
\]

where

\[
F = 1 + \sum_i \frac{\mu_i}{|y - y_i|^2}
\]
is a harmonic function on $E^4$ and the Hodge star is that of $E^4$.

We are seeking solutions of IIA supergravity with the interpretation of intersecting NS-5-branes at angles on a string. Motivated by (2.2), we write the ansatz for the metric and three-form field strength as

$$
\begin{align*}
(ds^2)^2 &= ds^2(E^{(1,1)}) + ds^2_{(8)} \\
H &= H_{(8)} \\
e^{8\phi} &= g_{(8)}
\end{align*}
$$

(2.4)

where $ds^2_{(8)}$ is an eight-dimensional metric, $H_{(8)}$ is a closed three-form that depends on the same coordinates as those of $ds^2_{(8)}$ and $g_{(8)}$ is the determinant of $ds^2_{(8)}$. The coordinates in $E^{(1,1)}$ are the worldvolume directions of the string lying at the intersection $\star$.

The main task is to determine $ds^2_{(8)}$ and $H_{(8)}$. For this we consider the linear maps $\dagger$, $\tau$, from $E^8$ into $E^4$ given by

$$
y^\mu = p_{i\lambda}x^{i\lambda}\delta^\mu_0 + \delta^\mu_a(J^a)^\lambda p_{i\lambda}x^{i\rho} - a^\mu
$$

(2.5)

where $\{y^\mu; \mu = 0, 1, 2, 3\}$ are the standard coordinates of $E^4$, $\{x^{i\mu}; i = 1, 2, \mu = 0, 1, 2, 3\}$ are the standard coordinates $\ddagger$ of $E^8$, and $\{J^a; a = 1, 2, 3\}$ are constant complex structures on $E^4$ associated with a basis of anti-self-dual two-forms. The real numbers $\{p_{i\mu}, a^\mu; \mu = 0, \ldots, 3, i = 1, 2\}$ are the parameters of the linear maps.

The parameters $\{p_{i\mu}\}$, eight in total, are rotational while the parameters $\{a^\mu; \mu = 0, \ldots, 3\}$, four in total, are translational. Next using the above linear map, we pull

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$\star$ We remark that there is no worldvolume soliton on the NS-5-brane associated with this intersection.

$\dagger$ These linear maps are chosen such that they preserve certain quaternionic structures on $E^4$ and $E^8$.

$\ddagger$ The Euclidean metric on $E^8$ is $ds^2 = \delta_{ij}dx^{i\mu}dx^{j\nu}$.
back the metric

\[ ds^2 = \frac{1}{|y|^2} \delta_{\mu\nu}dy^\mu dy^\nu \]  \hspace{1cm} (2.6)

and the closed three-form

\[ \langle f_3 \rangle_{\mu\nu\rho} = -\frac{1}{2} \epsilon_{\mu\nu\rho}^\lambda \partial_\lambda \frac{1}{|y|^2} \]  \hspace{1cm} (2.7)

from \( \mathbb{E}^4 - \{0\} \) to \( \mathbb{E}^8 \), where \( \epsilon \) is the volume form of \( \mathbb{E}^4 \) with respect to the flat metric and \( \epsilon_{\mu\nu\rho}^\lambda = \epsilon_{\mu\nu\rho}^\sigma \delta^\sigma_\lambda \) and the norm \( I \cdot | \) is with respect to the Euclidean metric on \( \mathbb{E}^4 \). We remark that (2.6) and (2.7) is the NS-5-brane geometry above after removing the identity in the harmonic function \( F \). The pulled back metric is

\[ ds^2 = \frac{A_{ij}\delta_{\mu\nu} + B_{ijs}I_{\mu\nu}}{|p_i x^i \delta^\sigma_0 + \delta^\sigma_{\alpha}J^{\alpha\lambda}_\rho p_i x^i \rho - a^\sigma|^2} dx^i \mu dx^j \nu , \]  \hspace{1cm} (2.8)

where \( \{I_s; s = 1, 2, 3\} \) are the constant complex structures on \( \mathbb{E}^4 \) associated with a basis of self-dual two-forms and

\[ A_{ij} = p_i \cdot p_j = \delta^{\mu\nu}_i p_{i\mu} p_{j\nu} \]

\[ B_{ijs}I_{\mu\nu} = p_{i\mu} p_{j\nu} - p_{i\nu} p_{j\mu} + \epsilon_{\mu\nu\rho\sigma} p_{i\rho} p_{j\sigma} . \]  \hspace{1cm} (2.9)

To derive the above expressions, we have used the identity

\[ J^\rho_{\mu\nu}J_{\tau\alpha\beta} = \delta_{\mu\alpha} \delta_{\nu\beta} - \delta_{\nu\alpha} \delta_{\mu\beta} - \epsilon_{\mu\nu\alpha\beta} . \]  \hspace{1cm} (2.10)

The metric and closed three-form in the ansatz (2.4) are constructed by sum-

\[ ^\delta \text{We remark that } I_{r\mu\nu} = \delta_{\mu\lambda} I_{r}^\lambda \nu . \]
ming the pull-backs of $ds^2$ and $f_3$ over different choices of linear maps $\tau$, i.e.

$$ds^2_{(8)} = ds^2_\infty + \sum_\tau \mu(\tau)\tau^* ds^2$$

$$H_{(8)} = \sum_\tau \mu(\tau)\tau^* f_3,$$

(2.11)

where

$$ds^2_\infty = (U^{\infty}_{ij}\delta_{\mu\nu} + (V^{\infty})_{ij}I_{\mu\nu})dx^{i\mu}dx^{j\nu}$$

(2.12)

is a constant metric on $\mathbb{E}^8$ and $\mu(\tau)$ are real positive numbers. Observe that $ds^2_{(8)} \to ds^2_\infty$ as $|x^i| \to \infty$, i.e. $ds^2_\infty$ is the asymptotic metric of $ds^2_{(8)}$. Explicitly, the metric $ds^2_{(8)}$ is

$$ds^2_{(8)} \equiv g_{i\mu,j\nu}dx^{i\mu}dx^{j\nu} \equiv (U_{ij}\delta_{\mu\nu} + V_{ij}^sI_{\mu\nu})dx^{i\mu}dx^{j\nu}$$

$$= ds^2_\infty + \sum_{\{p,a\}} \mu(\{p,a\}) \frac{A(p)_{ij}\delta_{\mu\nu} + B(p)_{ij}^sI_{\mu\nu}}{|p_{i\lambda}x^{i\lambda}\delta_{\sigma0} + \delta_{\sigma0}aJ_{\rho}^i p_{i\lambda}x^{i\rho} - a^{2}\|d^{i\mu\lambda}dx^{j\nu}\.}{(2.13)}$$

The expression of $H = H_{(8)}$ and dilaton $\phi$ in terms of $U, V_r$ is

$$H_{i\mu,j\nu,k\rho} = \frac{1}{3!} \left[ -3\epsilon_{\mu\rho\tau} \partial_{(i\tau}U_{j)k} + 2\{\delta_{\mu\nu}(\partial_{(i\rho}U_{j)k} + I_{\sigma}^s\partial_{(i\tau}V_{j)k}) - I_{\sigma}^s\partial_{(i\rho}V_{j)k} + \text{cyclic (}i\mu,j\nu,k\rho)\} \right]$$

(2.14)

and

$$e^{8\phi} = \left( \det(U_{ij}) - \sum_{r=1}^3 \det(V_{ij}^r) \right)^4,$$

(2.15)

respectively. We have verified using

$$g^{i\mu,j\nu}\partial_{i\mu}g_{j\nu,k\lambda} = \frac{1}{4}g^{i\mu,j\nu}\partial_{k\lambda}g_{i\mu,j\nu}$$

(2.16)

and some of the results in [24] that the above configuration is a solution of the IIA supergravity field equations.
Our solutions preserve at least $3/32$ of supersymmetry. To see this, we introduce the complex structures

$$J_{r}^{i\mu} \gamma_{j\nu} = J_{r}^{\mu} \gamma_{\nu} \delta^{i} j$$

(2.17)

on $\mathbb{E}^8$ and $r = 1, 2, 3$. Then after some computation one can show that these complex structures are covariantly constant with respect to the connection

$$\nabla^{(+)} = \nabla + H,$$

where $\nabla$ is the Levi-Civita connection of $ds^2_{(8)}$. This implies that for generic choices of linear maps $\tau$ the holonomy of $\nabla^{(+)}$ is exactly $^* Sp(2)$. Using this, one can show that the above solution admits at least three Killing spinors and therefore it preserves at least $3/32$ of the supersymmetry $^\dagger$.

Our solution includes that of [2]. To find the latter, we repeat the construction as before but in this case we sum over linear maps $\tau$ with rotational parameters

$$\{p_{i\mu}\} = \{p_{i}, 0, 0, 0\}.$$  

(2.18)

It is then straightforward to show that the solution (2.4) reduces to that of [2].

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* Therefore, the eight-dimensional geometry $(ds^2_{(8)}, H_{(8)})$ admits an hyper-Kähler structure with torsion [20-23] with respect to the pair $(\nabla^{(+)}, J_{r})$.

$^\dagger$ More details will be given elsewhere [25].
3. Angles

The new solution (2.4) of IIA supergravity has been constructed as a superposition of NS-5-branes intersecting on a string. To find the angles amongst any two NS-5-branes involved in the superposition, we take the location of the branes involved in the configuration to be determined by the poles of the metric (2.11), i.e. the kernels of the linear maps \(\tau\). Given two such maps depending on the parameters \(\{p_i, a\}\) and \(\{q_i, b\}\), respectively, the kernels are given by the equations

\[
\begin{align*}
p_{i\lambda} x^{i\lambda} \delta^\mu_0 + \delta^\mu_\rho J^{a\lambda}_\rho p_{i\lambda} x^{i\rho} - a^\mu &= 0 \\
q_{i\lambda} x^{i\lambda} \delta^\mu_0 + \delta^\mu_\rho J^{a\lambda}_\rho q_{i\lambda} x^{i\rho} - b^\mu &= 0 .
\end{align*}
\]

(3.1)

It is straightforward to observe that each of the four-dimensional planes associated with the above equation depend on at most four rotational parameters up to a redefinition of the translational ones \(\dagger\). The normal vectors of the four-dimensional planes, associated with the kernels of the linear maps, at infinity are

\[
\begin{align*}
n^{(\mu)} &= g^{i\lambda,j\rho}_\infty \partial_{i\lambda} \tau^\mu(p, a) \partial_{j\rho} \\
m^{(\nu)} &= g^{i\lambda,j\rho}_\infty \partial_{i\lambda} \tau^\nu(q, b) \partial_{j\rho} .
\end{align*}
\]

(3.2)

The angles amongst the normal vectors \(n^{(\mu)}\) and \(m^{(\nu)}\) are given by the “angles matrix”

\[
\cos(\theta^{\mu\nu}) = \frac{g^{i\lambda,j\rho}_\infty n^{(\mu)} n^{(\nu)}_{i\lambda} m^{(\nu)}_{j\rho}}{|n^{(\mu)}||m^{(\nu)}|}
\]

(3.3)

where

\[
|n^{(\mu)}|^2 = g^{i\lambda,j\rho}_\infty n^{(\mu)}_{i\lambda} n^{(\mu)}_{j\rho} = \sqrt{g^{i\lambda,j\rho}_\infty p_{i\lambda} p_{j\rho}}
\]

(3.4)

is the length of the vector \(n^{(\mu)}\) as measured by the asymptotic metric and similarly for \(|m^{(\nu)}|\). To find the angles in terms of the parameters \(\{p_i, \{q_j\}\}\) of the solution,

\(\dagger\) This is also true for the metric and three-form field strength of the solution. It turns out that they depend on at most four rotational parameters for every five-brane involved in the intersection.
we simply substitute the expressions of the normal vectors (3.2) into (3.3). Thus we find
\[
\cos(\theta^{\mu\nu}) = \frac{g^{i\lambda,j\rho} p_{i\lambda} q_{j\rho}\delta^{\mu\nu} + (g^{\infty})^{i\lambda,j\sigma}(J^c)^{\rho\lambda} p_{i\rho} q_{j\sigma} I_c^{\mu\nu}}{\sqrt{g^{\infty} P_{i\alpha} P_{j\beta}} \sqrt{g^{\infty} q_{k\alpha} q_{\ell\beta}}}
\]  
(3.5)

From this, we conclude that, for generic choices of the rotational parameters \{p, q\}, there are four independent angles between every pair of intersecting NS-5-branes in the natural coordinates that we have chosen to express the brane solution. It turns out that these angles are $Sp(2)$ angles. The most convenient way to establish this is to use an alternative construction for the solutions (2.4) which will be presented in [25] and so we shall not pursue this point further here.

We proceed to find the number of independent angles between a given pair of five branes. We shall take them to be the minimal number of independent eigenvalues of the angles matrix $A = \{\cos(\theta^{\mu\nu})\}$ over the different parameterizations of the four-dimensional planes (3.1). To diagonalize the angles matrix in our case, we write its elements as
\[
\cos(\theta^{\mu\nu}) = a\delta^{\mu\nu} + b^c I_c^{\mu\nu}
\]  
(3.6)

where $\{a, b^c; c = 1, 2, 3\}$ can be easily computed from (3.5). Then we choose a complex basis with respect to the complex structure
\[
K = \frac{b^c}{|b|} I_c ,
\]  
(3.7)

where $|b|^2 = \delta_{ac} b^a b^c$. Since $\delta_{\mu\nu}$ is hermitian with respect to $K$, we have
\[
\cos(\theta^{\alpha\bar{\beta}}) = (a + i|b|)\delta^{\alpha\bar{\beta}} ,
\]  
(3.8)

where $a, |b|$ depend on the form of the asymptotic metric and the parameterization of the four-dimensional planes (3.1). For generic asymptotic metric it seems that there is no relation between $a$ and $|b|$ and hence there are two independent angles. For Euclidean asymptotic metric, if we place the first four-dimensional plane along the
first four coordinates of $E^8$ and parameterize the second one using four rotational parameters, then the angles matrix has one independent eigenvalue.

We remark that the above complex basis that diagonalizes the angles matrix depends on the pair of branes involved in the intersection. There is no choice of basis that diagonalizes simultaneously the angles matrix of all pairs. Hence there are in general four angles parameterizing the relative position of the branes involved in the configuration.

4. Other IIA solutions

The solution found in section two can be superposed with a fundamental string and a ten-dimensional pp-wave. The resulting solution is

$$ds^2 = F^{-1}(du dv + (K - 1)du^2) + ds_{(8)}^2$$

$$e^{2\phi} = F^{-1}(g_{(8)})^{1/4}$$

$$H = du \wedge dv \wedge dF^{-1} + H_{(8)}$$

where $F, K$ are harmonic-like functions satisfying,

$$\partial_{\mu}(g_{(8)})^{1/4} g^{\mu\nu} \partial_{\nu} F = 0 \quad (4.2)$$

and similarly for $K$, $u, v$ are light-cone coordinates. Using (2.16), we can rewrite (4.2) as

$$g^{\mu\nu} \partial_{\mu} \partial_{\nu} F = 0 \quad (4.3)$$

A class of solutions of this equation is given by

$$F = 1 + \sum_{\tau} \frac{\mu(\tau)}{|\tau(x)|^2} \quad (4.4)$$

We note that the parameters $\{p, a\}$ of the linear maps $\tau$ in the above sum are not necessarily those that have appeared in the sum for the metric (2.11).
Another possibility is to superpose D-4-branes at $Sp(2)$-angles intersecting on a 0-brane. In addition, we can place a D-0-brane on this configuration. The resulting solution is

$$
\begin{align*}
    ds^2 &= F^{\frac{3}{2}}(g_{(8)}^{\frac{1}{2}}[-F^{-1} dt^2 + ds^2_{(8)}] + g_{(8)}^\frac{1}{2} dz^2) \\
    e^{2\phi} &= F^{\frac{3}{2}}(g_{(8)})^{-\frac{1}{8}} \\
    G_2 &= dt \wedge dF^{-1} \\
    G_4 &= H_{(8)} \wedge dz,
\end{align*}
$$

where $F$ is a harmonic-like function, as in (4.3), associated with the D-0-brane, and $G_2$ and $G_4$ are the IIA Ramond⊗Ramond two-and four-form field strengths, respectively. All the above solutions preserve at least 3/32 of spacetime supersymmetry.

5. IIB 5-branes at Angles

The IIA supergravity solution described in section two is also a solution of IIB supergravity. Alternatively one can use T-duality along the string direction to transform the IIA solution to a IIB one. The field equations of IIB supergravity are invariant under $SL(2, \mathbb{R})$. Under the action of $SL(2, \mathbb{R})$, the metric in the Einstein frame remains invariant, the two three-form field strengths $H^1, H^2$ of IIB supergravity transform as doublets, and

$$
\tau = \ell + i e^{-\phi_B}
$$

transforms by fractional linear transformations, where $\ell$ is the axion and $\phi_B$ is the IIB dilaton (see e.g. [2]). To proceed, we choose $H^1 = H$ and $H^2 = H'$ the three-form field strengths associated with NS-5-brane and the D-5-brane, respectively. This symmetry can be used to construct new solutions from the one in (2.4). In particular, it is known that under S-duality (which is an element of $SL(2, \mathbb{R})$), the NS-5-branes transform to D-5-branes. Therefore it is expected that the solution
(2.4) as a solution of IIB supergravity transformed under S-duality will lead to a new solution of IIB with the interpretation of D-5-branes intersecting at $Sp(2)$-angles on a string. To perform the S-duality transformation, we first write the metric (2.4) in the Einstein frame using

$$ds_E^2 = e^{-\frac{1}{2}\phi} ds_B^2 .$$

(5.2)

Then we apply S-duality to find

$$ds_E^2 = g_{(8)}^{\frac{1}{2}} (ds^{(1,1)} + ds_{(8)})$$

$$e^{8\phi} = g_{(8)}^{-1}$$

$$H' = H_{(8)} .$$

(5.3)

More solutions in IIB can be found by T-dualizing the solutions (4.1) and (4.5) of IIA theory. In the former case, T-duality along the string direction will lead to a solution in IIB with the same interpretation as that in IIA. In the latter case, T-duality along $z$ will lead to a configuration with the interpretation of intersecting D-5-branes at $Sp(2)$ angles on a string superposed with a D-string at the intersection.

6. M-theory

The IIA supergravity solution described in section three can be easily lifted to M-theory. Let $z$ be the eleventh coordinate. The relevant Kaluza-Klein ansatz for the reduction from eleven dimensions to ten is

$$ds_{(11)}^2 = e^{4\phi} dz^2 + e^{-2\phi} ds_{(10)}^2$$

$$G_4 = H \wedge dz ,$$

(6.1)

where $\phi$ is the ten-dimensional dilaton, the ten-dimensional metric $ds_{(10)}^2$ is in the string frame, $G_4$ is the 4-form field strength of eleven-dimensional supergravity and

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* We have not included in this ansatz the Kaluza-Klein vector and the IIA four-form field strength because they vanish for our ten-dimensional solutions.
$H$ is the ten-dimensional NS$\otimes$NS three-form field strength. Lifting the solution (2.4) of IIA theory to eleven-dimensions, we find the M-theory solution

$$\begin{align*}
ds^2 &= (g^{(8)})^{-\frac{1}{8}} (ds^2(E^{(1,1)}) + ds^2_{(8)}) \\
G_4 &= H^{(8)} \wedge dz
\end{align*}$$

(6.2)

This solution has the interpretation of intersecting M-5-branes at $Sp(2)$-angles on a string separated along the direction $z$. We remark though that the solution is not localized in the $z$ direction.

The above solution can be superposed with a membrane without breaking any more supersymmetry. The membrane directions are those of the string intersection and that of $z$. The resulting solution is

$$\begin{align*}
ds^2 &= F^{-\frac{2}{3}} (g^{(8)})^\frac{1}{8} dz^2 + (g^{(8)})^{-\frac{1}{12}} \left[ F^{-\frac{2}{3}} ds^2(E^{(1,1)}) + F^\frac{1}{3} ds^2_{(8)} \right] \\
G_4 &= H^{(8)} \wedge dz + \text{Vol}(E^{(1,1)}) \wedge dF^{-1} \wedge dz
\end{align*}$$

(6.3)

where $F$ is a harmonic-like function as in (4.3) associated with the membrane.

Apart from these M-brane configurations we can allow a pp-wave to propagate along the string direction. The resulting M-theory configuration is

$$\begin{align*}
ds^2 &= F^{-\frac{2}{3}} (g^{(8)})^\frac{1}{8} dz^2 + (g^{(8)})^{-\frac{1}{12}} \left[ F^{-\frac{2}{3}} (du dv + (K - 1)du^2) + F^\frac{1}{3} ds^2_{(8)} \right] \\
G_4 &= H^{(8)} \wedge dz + \text{Vol}(E^{(1,1)}) \wedge dF^{-1} \wedge dz
\end{align*}$$

(6.4)

where $F, K$ are harmonic-like functions as in (4.3). This solution includes all previous ones and preserves $3/32$ of the supersymmetry. For example if $F$ and $K$ are one, then we recover the solution (6.2). Setting $K = 1$, we recover the solution (6.3). We can also set $F = 1$ in which case we find a new M-theory solution which has the interpretation of M-5-branes intersecting on a string at $Sp(2)$-angles and a wave propagating along the string. Reduction of this solution along the pp-wave direction gives the solution (4.5) of the IIA theory. We can also reduce (6.4) along
the same direction yielding a new IIA solution. We expect that our solutions will receive corrections due to the anomaly terms induced by the M-5-brane to the D=11 supergravity action as those in [26].

7. Conclusions

We have constructed new solutions with the interpretation of intersecting IIA NS-5-branes at $Sp(2)$-angles on a string preserving at least $3/32$ of supersymmetry. We have shown that there are four independent angles between every pair of intersecting NS-5-branes, respectively. We have described the superposition of the intersecting NS-5-brane solutions with a fundamental string and a pp-wave. We have also investigated the T-duals of these solutions as well as their M-theory interpretation.

The intersecting IIA NS-5-brane solutions that we have found are also solutions of the heterotic and type I strings. It would be of interest to investigate further our brane solutions in the context of the heterotic string using as Yang-Mills fields the instantons of [17] (see [27]). It is expected that consideration of the cancellation of chiral anomalies of the heterotic string will modify our solutions. A related problem is the investigation of heterotic sigma models with bosonic couplings given by the geometries found in section two and with Yang-Mills couplings provided by the instantons of [17]. These sigma models admit a $(4,0)$-supersymmetric extension. Therefore they are expected to be ultraviolet finite [21, 22]. However due to the presence of sigma model anomalies, their couplings may receive $\alpha'$ corrections.

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