Discrete R-symmetry anomalies in heterotic orbifold models

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Abstract

Anomalies of discrete R-symmetries appearing in heterotic orbifold models are studied. We find that the mixed anomalies for different gauge groups satisfy the universal Green-Schwarz (GS) condition, indicating that these anomalies are canceled by the GS mechanism. An exact relation between the anomaly coefficients of the discrete R-symmetries and one-loop beta-function coefficients is obtained. We also find that the discrete R-symmetries have a good chance to be unbroken down to the supersymmetry breaking scale. Even below this scale a $Z_2$ subgroup is unbroken, and it may be an origin of the R-parity of the minimal supersymmetric standard model. Relations between the R-symmetry anomalies and T-duality anomalies are also investigated.
1 Introduction

Discrete symmetries play an important role in model building of particle physics. For example, abelian and non-abelian discrete flavor symmetries are useful to derive realistic quark/lepton masses and their mixing [1]. Discrete non-abelian flavor symmetries can also be used to suppress flavor changing neutral current processes in supersymmetric models [2, 3]. Furthermore, discrete symmetries can be introduced to forbid unfavorable couplings such as those leading to fast proton decay [4, 5].

Superstring theory is a promising candidate for unified theory including gravity and may provide with an origin of such discrete symmetries [6]. It is widely assumed that superstring theory leads to anomaly-free effective theories. In fact the anomalous $U(1)$ symmetries are restored by the Green-Schwarz (GS) mechanism [7, 8, 9]. For this mechanism to work, the mixed anomalies between the anomalous $U(1)$ and other continuous gauge symmetries have to satisfy a certain set of conditions, the GS conditions, at the field theory level. In particular, in heterotic string theory the mixed anomalies between the anomalous $U(1)$ symmetries and other continuous gauge symmetries must be universal for different gauge groups up to their Kac-Moody levels [10, 11]. A well-known discrete symmetry in heterotic string theory is T-duality symmetry, and its effective theory has T-duality anomalies [12]. It has been shown that the mixed anomalies between T-duality symmetry and continuous gauge symmetries are universal except for the sector containing an $N = 2$ subsector and are exactly canceled by the GS mechanism [13]. That has phenomenologically interesting consequences which have been studied in early 90’s [13, 14, 15].

Heterotic orbifold construction is one of interesting 4D string models [17, 18]. (See also for resent works Ref. [19, 20] and for review [21].) Geometrical structures of their compact spaces are simple compared with other types of 4D string model constructions. Phenomenological aspects in effective theory are related with geometrical aspects of orbifolds. Discrete symmetries which may be used as non-abelian flavor symmetries and also certain discrete $R$-symmetries originate from the geometrical structure of orbifolds [6, 22, 19, 23]. In this paper we consider discrete $R$-symmetries. Stringy-originated discrete symmetries are strongly constrained due to stringy consistency, and it is phenomenologically and theoretically important to study anomalies of discrete symmetries, as it is pointed out in [16] and the example of T-duality shows. We shall investigate the mixed anomalies between the discrete $R$-symmetries and the continuous gauge symmetries in concrete orbifold models. We will also study relations between the discrete $R$-anomalies, one-loop beta-function coefficients (scale anomalies) and T-duality anomalies.

This paper is organized as follows. In section 2, we give a brief review on heterotic orbifold models to fix our notation. In section 3, we define discrete $R$-charges, which is one of our main interests. In section 4, we calculate the mixed anomalies between the discrete $R$-symmetries and the continuous gauge symmetries in concrete models. We also study the relations of $R$-anomalies with one-loop beta-function coefficients and T-duality
Table 1: \( H \)-momenta for \( \mathbb{Z}_3 \), \( \mathbb{Z}_4 \), \( \mathbb{Z}_6 \)-I, \( \mathbb{Z}_6 \)-II and \( \mathbb{Z}_7 \) orbifolds

| \( v_i \) | \( \mathbb{Z}_3 \) | \( \mathbb{Z}_4 \) | \( \mathbb{Z}_6 \)-I | \( \mathbb{Z}_6 \)-II | \( \mathbb{Z}_7 \) |
|---|---|---|---|---|---|
| \( T_1 \) | (1,1,1)/3 | (1,1,2)/4 | (1,1,4)/6 | (1,2,3)/6 | (1,2,4)/7 |
| \( T_2 \) | — | (2,2,0)/4 | (2,2,2)/6 | (2,4,0)/6 | (2,4,1)/7 |
| \( T_3 \) | — | — | (3,3,0)/6 | (3,0,3)/6 | — |
| \( T_4 \) | — | — | — | (4,2,0)/6 | (4,1,2)/7 |

anomalies. In section 5, we discuss phenomenological implications of our results. Section 6 is devoted to conclusion and discussion.

2 Heterotic orbifold models

Here we review briefly heterotic orbifold models. First we give a review on \( \mathbb{Z}_N \) orbifold models, and next explain \( \mathbb{Z}_N \times \mathbb{Z}_M \) orbifold models. Heterotic string theory consists of 10D right-moving superstrings and 26D left-moving bosonic strings. Their common 10 dimensions correspond to our 4D space-time and 6D compact space. The other 16D left-moving bosonic strings correspond to a gauge part. Here, we consider the \( E_8 \times E_8 \) heterotic string theory, where momenta of 16D left-moving bosonic strings span \( E_8 \times E_8 \) root lattice.

The following discussions are also applicable to \( SO(32) \) heterotic string theory.

In orbifold models, the 6D compact space is chosen to be 6D orbifold. A 6D orbifold is a division of 6D torus \( T^6 \) by a twist \( \theta \), while the torus \( T^6 \) is obtained as \( R^6/\Lambda^6 \), where \( \Lambda^6 \) is 6D lattice. Eigenvalues of the twist \( \theta \) are denoted as \( e^{2\pi iv_1}, e^{2\pi iv_2} \) and \( e^{2\pi iv_3} \) in the complex basis \( \mathbb{Z}_i \) \((i = 1, 2, 3)\). To preserve 4D N=1 supersymmetry (SUSY), they must satisfy the following condition,

\[
v_1 + v_2 + v_3 = \text{integer}. \tag{1}\]

When one of \( v_i \) is integer, N=2 SUSY is preserved. In the case with \( v_i \neq \text{integer} \), only N=1 SUSY is preserved. Such \( \mathbb{Z}_N \) orbifolds are classified into \( \mathbb{Z}_3, \mathbb{Z}_4, \mathbb{Z}_6 \)-I, \( \mathbb{Z}_6 \)-II, \( \mathbb{Z}_7 \), \( \mathbb{Z}_8 \)-I, \( \mathbb{Z}_8 \)-II, \( \mathbb{Z}_{12} \)-I and \( \mathbb{Z}_{12} \)-II, and their twists are explicitly shown in Table 1 and Table 2.

On the orbifold, closed string satisfies the following boundary condition,

\[
X(\sigma = \pi) = \theta^k X(\sigma = 0) + V, \tag{2}\]

where \( V \) is a shift vector on the 6D lattice \( \Lambda^6 \). The complex basis of \( X \) corresponds to \( \mathbb{Z}_i \). The \( \theta^k \)-twisted sector is denoted by \( T_k \), while the sector with \( k = 0 \) is the so-called untwisted sector.

It is convenient to bosonize right-moving fermionic strings. Here we write such bosonized fields by \( H^t \) \((t = 1, \cdots, 5)\). Their momenta \( p_t \) are quantized and span the SO(10)
weight lattice. Space-time bosons correspond to SO(10) vector momenta, and space-time fermions correspond to SO(10) spinor momenta. The 6D compact part, i.e. the SO(6) part, $p_i$ ($i = 1, 2, 3$) is relevant to our study. All of $\mathbb{Z}_N$ orbifold models have three untwisted sectors, $U_1$, $U_2$ and $U_3$, and their massless bosonic modes have the following SO(6) momenta,

$$U_1 : (1, 0, 0), \quad U_2 : (0, 1, 0), \quad U_3 : (0, 0, 1).$$  \tag{3}$$

On the other hand, the twisted sector $T_k$ has shifted SO(6) momenta, $r_i = p_i + kv_i$. Table 1 and Table 2 show explicitly $H$-momenta $r_i$ of massless bosonic states. That implies their $SO(6)$ $H$-momenta are obtained as

$$r_i = |kv_i| - \text{Int}(|kv_i|),$$  \tag{4}$$

where Int[$a$] denotes an integer part of fractional number $a$. This relation is not available for the untwisted sectors, and $r_i$ is obtained as Eq. (3).

The gauge sector can also be broken and gauge groups smaller than $E_8 \times E_8$ are obtained. Matter fields have some representations under such unbroken gauge symmetries.

Massless modes for 4D space-time bosons correspond to the following vertex operator \cite{24, 25},

$$V_{-1} = e^{-\phi} \prod_{i=1}^{3} (\partial Z_i)^{N_i} (\partial \bar{Z}_i)^{\bar{N}_i} e^{i\tau H^I} e^{iP^I X^I} e^{ikX} \sigma_k,$$  \tag{5}$$

in the $(-1)$-picture, where $\phi$ is the bosonized ghost, $kX$ corresponds to the 4D part and $P^I X^I$ corresponds to the gauge part. Oscillators of the left-mover are denoted by $\partial Z_i$ and $\partial \bar{Z}_i$, and $N_i$ and $\bar{N}_i$ are oscillator numbers, which are included in these massless modes.

| $v_i$  | $\mathbb{Z}_8$-I | $\mathbb{Z}_8$-II | $\mathbb{Z}_{12}$-I | $\mathbb{Z}_{12}$-II |
|-------|-----------------|-----------------|-----------------|-----------------|
| $T_1$ | $(1, 2, -3)/8$  | $(1, 3, 4)/8$   | $(1, 4, 7)/12$  | $(1, 5, 6)/12$  |
| $T_2$ | $(2, 4, 2)/8$   | $(2, 6, 0)/8$   | $(2, 8, 2)/12$  | $(2, 10, 0)/12$ |
| $T_3$ | $(4, 0, 4)/8$   | $(3, 1, 4)/8$   | $(3, 0, 9)/12$  | $(3, 3, 6)/12$  |
| $T_4$ | $(5, 2, 1)/8$   | $(4, 4, 0)/8$   | $(4, 4, 12)$    | $(4, 8, 0)/12$  |
| $T_5$ | $-$             | $-$             | $-$             | $(5, 1, 6)/12$  |
| $T_6$ | $-$             | $-$             | $-$             | $(6, 1, 6)/12$  |
| $T_7$ | $-$             | $-$             | $-$             | $(7, 4, 1)/12$  |
| $T_8$ | $-$             | $-$             | $-$             | $-$             |
| $T_9$ | $-$             | $-$             | $(9, 0, 3)/12$  | $-$             |
| $T_{10}$ | $-$           | $-$             | $-$             | $(10, 2, 0)/12$ |

Table 2: $H$-momenta for $\mathbb{Z}_8$-I, $\mathbb{Z}_8$-II, $\mathbb{Z}_{12}$-I and $\mathbb{Z}_{12}$-II orbifolds
In addition, $\sigma_k$ denotes the twist field for the $T_k$ sector. Similarly, we can write the vertex operator for 4D space-time massless fermions as

$$V_{-\frac{1}{2}} = e^{-\frac{1}{2}\phi} \prod_{i=1}^{3} (\partial Z_i)^{N_i} (\partial \bar{Z}_i)^{\bar{N}_i} e^{i r_i^{(f)} H_i} e^{i P_i X^I} e^{i k X} \sigma_k,$$

in the $(-1/2)$-picture. The $H$-momenta for space-time fermion and boson, $r_i^{(f)}$ and $r_i$ in the same supersymmetric multiplet are related each other as

$$r_i = r_i^{(f)} + (1, 1, 1)/2.$$ 

We need vertex operators $V_0$ with the 0-picture when we compute generic n-point couplings. We can obtain such vertex operators $V_0$ by operating the picture changing operator, $Q$, on $V_{-1}$, 

$$Q = e^{\phi} (e^{-2\pi i v_i^1 H_i} \partial Z_i + e^{2\pi i v_i^2 H_i} \bar{\partial} \bar{Z}_i),$$

where $r_1^v = (1, 0, 0)$, $r_2^v = (0, 1, 0)$ and $r_3^v = (0, 0, 1)$.

Next we briefly review on $\mathbb{Z}_N \times \mathbb{Z}_M$ orbifold models. In $\mathbb{Z}_N \times \mathbb{Z}_M$ orbifold models, we introduce two independent twists $\theta$ and $\omega$, whose twists are represented by $e^{2\pi i v_i^1}$ and $e^{2\pi i v_i^2}$, respectively in the complex basis. Two twists are chosen such that each of them breaks 4D $\mathcal{N}=4$ SUSY to 4D $\mathcal{N}=2$ SUSY and their combination preserves only $\mathcal{N}=1$ SUSY. Thus, eigenvalues $v_i^1$ and $v_i^2$ are chosen as

$$v_i^1 = (v^1, -v^1, 0), \quad v_i^2 = (0, v^2, -v^2),$$

where $v^1, v^2 \neq$ integer. In general, $\mathbb{Z}_N \times \mathbb{Z}_M$ orbifold models have three untwisted sectors, $U_1, U_2$ and $U_3$, and their massless bosonic modes have the same $SO(6)$ $H$-momenta $r_i$ as Eq. (3). In addition, there are $\theta^k \omega^\ell$-twisted sectors, and their $SO(6)$ $H$-momenta are obtained as

$$r_i = |k v_i^1| + |\ell v_i^2| - \text{Int}(|k v_i^1| + |\ell v_i^2|).$$

Vertex operators are also constructed in a similar way. Recently, non-factorizable $\mathbb{Z}_N \times \mathbb{Z}_M$ orbifold models have been studied. The above aspects are the same for such non-factorizable models.

3 Discrete R-symmetries

Here we define R-charges. We consider n-point couplings including two fermions. Such couplings are computed by the following n-point correlation function of vertex operators,

$$\langle V_{-1/2} V_{-1/2} V_0 \cdots V_0 \rangle.$$  

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They must have the total ghost charge $-2$, because the background has the ghost number 2. When this n-point correlation function does not vanish, its corresponding n-point coupling in effective theory is allowed. That is, selection rules for allowed n-point correlation functions in string theory correspond to symmetries in effective theory.

The vertex operator consists of several parts, the 4D part $e^{i k X}$, the gauge part $e^{i P X}$, the 6D twist field $\sigma_k$, the 6D left-moving oscillators $\partial Z_i$ and the bosonized fermion $e^{i r H}$. Each part has its own selection rule for allowed couplings. For the 4D part and the gauge part, the total 4D momentum $\sum k$ and the total momentum of the gauge part $\sum P$ should be conserved. The latter is nothing but the requirement of gauge invariance. The selection rule for 6D twist fields $\sigma_k$ is controlled by the space group selection rule $[25, 28]$.

Similarly, the total $H$-momenta can be conserved

$$\sum r_i = 1. \quad (12)$$

Here we take a summation over the $H$-momenta for scalar components, using the fact that the $H$-momentum of fermion component differs by $-1/2$. Another important symmetry is the twist symmetry of oscillators. We consider the following twist of oscillators,

$$\partial Z_i \to e^{2\pi i v_i} \partial Z_i, \quad \partial Z_i \to e^{-2\pi i v_i} \partial Z_i, \quad \partial \bar{Z}_i \to e^{2\pi i v_i} \partial \bar{Z}_i, \quad \partial \bar{Z}_i \to e^{-2\pi i v_i} \partial \bar{Z}_i. \quad (13)$$

Allowed couplings may be invariant under the above $Z_N$ twist.

Indeed, for 3-point couplings corresponding to $\langle V_{-1} V_{-1/2} V_{-1/2} \rangle$, we can require $H$-momentum conservation and $Z_N$ twist invariance of oscillators independently. However, we have to compute generic n-point couplings through picture changing, and the picture changing operator $Q$ includes non-vanishing $H$-momenta and right-moving oscillators $\bar{\partial} Z_i$ and $\bar{\partial} \bar{Z}_i$. Consequently, the definition of the $H$-momentum of each vertex operator depends on the choice of the picture and so its physical meaning remains somewhat obscure. We therefore use a picture independent quantity as follows,

$$R_i \equiv r_i + N_i - \bar{N}_i, \quad (14)$$

which can be interpreted as an R-charge $[19]$. This R-symmetry is a discrete surviving symmetry of the continuous $SU(3) \subset SU(4)$ R-symmetry under orbifolding. Here we do not distinguish oscillator numbers for the left-movers and right-movers, because they have the same phase under $Z_N$ twist. Indeed, physical states with $-1$ picture have vanishing oscillator number for the right-movers, while the oscillator number for the left-movers can be non-vanishing. Thus, hereafter $N_i$ and $\bar{N}_i$ denote the oscillator number for the left-movers, because we study the physical states with $-1$ picture from now. For simplicity, we use the notation $\Delta N_i = N_i - \bar{N}_i$. Now, we can write the selection rule due to $R$-symmetry as

$$\sum R_i = 1 \mod N_i, \quad (15)$$
| \( R_i \) | \( (1/2, 1/2, 1/2) \) |
| \( U_1 \) | \( (1/2, -1/2, -1/2) \) |
| \( U_2 \) | \( (-1/2, 1/2, -1/2) \) |
| \( U_3 \) | \( (-1/2, -1/2, 1/2) \) |
| \( T_k \) | \( kv - \text{Int}\{kv\} - 1/2 + \Delta N_i \) |

**Table 3:** Discrete \( R \)-charges of fermions in \( \mathbb{Z}_N \) orbifold models

where \( N_i \) is the minimum integer satisfying \( N_i = 1/\hat{v}_i \), where \( \hat{v}_i = v_i + m \) with any integer \( m \). For example, for \( Z_6 \)-II orbifold, we have \( v_i = (1, 2, -3)/6 \), and \( N_i = (6, 3, 2) \). Thus, these are discrete symmetries. Note that the above summation is taken over scalar components.

Discrete \( R \) symmetry itself is defined as the following transformation,

\[
| R_i \rangle \rightarrow e^{2\pi i v_i R_i} | R_i \rangle,
\]

for states with discrete \( R \)-charges, which are defined mod \( N_i \). For later convenience, we show discrete \( R \)-charges for fermions in Table 3. As shown there, gaugino fields always have \( R \)-charge \((1/2, 1/2, 1/2)\).

## 4 Anomalies of \( R \)-symmetry

### 4.1 Discrete \( R \) anomalies

Let us study anomalies of discrete \( R \)-symmetry. Under the \( R \)-transformation like Eq. (16), the path integral measure of fermion fields is not invariant, but changes as

\[
\mathcal{D}\psi\mathcal{D}\psi^\dagger \rightarrow \mathcal{D}\psi\mathcal{D}\psi^\dagger \exp \left[ -2\pi i v_i \sum_{G_a} A_{G_a}^{R_i} \int d^4x \frac{1}{16\pi^2} F^{(G_a)}_{\mu\nu} \tilde{F}^{(G_a)}_{\mu\nu} \right],
\]

where \( \tilde{F}^{(G_a)}_{\mu\nu} = \frac{1}{2} \varepsilon^{\mu\nu\rho\sigma} F^{(G_a)}_{\rho\sigma} \). The anomaly coefficients \( A_{G_a}^{R_i} \) are obtained as

\[
A_{G_a}^{R_i} = \sum_{G_a} R_i T(\mathbf{R}_{G_a}),
\]

where \( T(\mathbf{R}_{G_a}) \) is the Dynkin index for \( \mathbf{R}_{G_a} \) representation under \( G_a \). The winding number of the gauge field configuration, i.e., the Pontryagin index,

\[
\nu \equiv \frac{T(\mathbf{R}_{G_a}^{(f)})}{16\pi^2} \int d^4x F^{(G_a)}_{\mu\nu} \tilde{F}^{(G_a)}_{\mu\nu},
\]

for states with discrete \( R \)-charges, which are defined mod \( N_i \). For later convenience, we show discrete \( R \)-charges for fermions in Table 3. As shown there, gaugino fields always have \( R \)-charge \((1/2, 1/2, 1/2)\).
is integer, where \( T(R_{Ga}) \) denotes the Dynkin index of a fundamental representation of \( G_a \). Thus, the anomaly coefficients \( A_{Ga}^{Ri} \) are defined modulo \( N_i T(R_{Ga}) \).

By use of our discrete \( R \) charge, the anomaly coefficients are written as

\[
A_{Ga}^{Ri} = \frac{1}{2} C_2(G_a) + \sum_{\text{matter}} \left( r_i - \frac{1}{2} + \Delta N_i \right) T(R_{Ga}),
\]

(20)

where \( C_2(G_a) \) is quadratic Casimir. Note that \( r_i \) denotes the SO(6) shifted momentum for bosonic states. The first term in the right hand side is a contribution from gaugino fields and the other is the contribution from matter fields.

If these anomalies are canceled by the Green-Schwarz mechanism, these mixed anomalies must satisfy the following condition,

\[
\frac{A_{Ga}^{Ri}}{k_a} = \frac{A_{Gb}^{Ri}}{k_b},
\]

(21)

for different gauge groups, \( G_a \) and \( G_b \), where \( k_a \) and \( k_b \) are Kac-Moody levels. In the simple orbifold construction, we have the Kac-Moody level \( k_a = 1 \) for non-abelian gauge groups. Note again that anomalies are defined modulo \( N_i T(R_{Ga}) \). The above GS condition has its meaning mod \( N_i T(R_{Ga})/k_a \).

As illustrating examples, let us study explicitly one \( Z_3 \) model and one \( Z_4 \) model. Their gauge groups and massless spectra are shown in Table 4 and Table 5. First, we study \( R \)-anomalies in the \( Z_3 \) orbifold model. Since \( v_i = (1, 1, -2)/3 \), we have \( N_i = 3 \). For both \( E_6 \), mixed \( R \)-anomalies are computed as

\[
A_{E_6}^{Ri} = \frac{3}{2} + 9n_{E_6}^i,
\]

(22)

where \( n_{E_6}^i \) is integer. The second term in the right hand side appears because anomalies are defined modulo \( N_i T(27) \) with \( N_i = 3 \) and \( T(27) = 3 \) for \( E_6 \). Similarly, mixed \( R \)-anomalies for \( SU(3) \) are computed as

\[
A_{SU(3)}^{Ri} = -12 + \frac{3}{2} n_{SU(3)}^i,
\]

(23)

where \( n_{SU(3)}^i \) is integer. The second term in the right hand side appears through \( N_i T(3) \) with \( N_i = 3 \) and \( T(3) = 1/2 \) for \( SU(3) \). Thus, in this model, mixed \( R \)-anomalies satisfy

\[
A_{E_6}^{Ri} = A_{SU(3)}^{Ri} \quad (\text{mod } 3/2).
\]

(24)

\footnote{See for explicit massless spectra Ref. [29], where a typographical error is included in the \( U_3 \) sector of the \( Z_4 \) orbifold model. It is corrected in Table 5.}
Next, we study R-anomalies in the $Z_4$ orbifold model with the gauge group $SO(10) \times SU(4) \times SO(12) \times SU(2) \times U(1)$. Since the $Z_4$ orbifold has $v_i = (1, 1, -2)/4$, we have $N_i = (4, 4, 2)$. Mixed anomalies between $R_1,2$ and $SO(10)$ are computed as

$$A_{SO(10)}^{R_{1,2}} = 1 + 4n_{SO(10)}^{1,2},$$

with integer $n_{SO(10)}^{1,2}$, where the second term appears through $N_i T(R_a)$ with $N_i = 4$ and $T(10) = 1$ for $SO(10)$. Similarly, mixed anomalies between $R_3$ and $SO(10)$ is computed as

$$A_{SO(10)}^{R_3} = -9 + 2n_{SO(10)}^3,$$

with integer $n_{SO(10)}^3$. Furthermore, mixed R-anomalies for other non-abelian groups are obtained as

$$A_{SU(4)}^{R_{1,2}} = -7 + 2n_{SU(4)}^{1,2}, \quad A_{SU(4)}^{R_3} = -9 + n_{SU(4)}^3,$$

$$A_{SO(12)}^{R_{1,2}} = 1 + 4n_{SO(12)}^{1,2}, \quad A_{SO(12)}^{R_3} = 3 + 2n_{SO(12)}^3,$$

$$A_{SU(2)}^{R_{1,2}} = -15 + 2n_{SU(2)}^{1,2}, \quad A_{SU(2)}^{R_3} = 3 + n_{SU(2)}^3,$$

with integer $n_{G_a}^i$, where the second terms appear through $N_i T(R_a)$ with $N_i = (4, 4, 2)$, and $T(12) = 1$ for $SO(12)$, $T(4) = 1/2$ for $SU(4)$ and $T(2) = 1/2$ for $SU(2)$. These
anomalies satisfy the GS condition,
\[ A^{R_{1,2}}_{SO(10)} = A^{R_{1,2}}_{SU(4)} = A^{R_{1,2}}_{SO(12)} = A^{R_{1,2}}_{SU(2)} \quad \text{(mod 2)}, \]
\[ A^{R_{3}}_{SO(10)} = A^{R_{3}}_{SU(4)} = A^{R_{3}}_{SO(12)} = A^{R_{3}}_{SU(2)} \quad \text{(mod 1)}. \]

### 4.2 Relation with beta-function

Here we study the relation between discrete R anomalies and one-loop beta-functions. We find
\[ \sum_{i=1,2,3} r_i = 1, \]  
from Eqs. (11) and (10) as well as Table 1 and Table 2. By using this, we can write the sum of R-anomalies as
\[ A^R_G = \sum_{i=1,2,3} A^R_i \]
\[ = \frac{3}{2} C_2(G_a) + \sum_{\text{matter}} T(R_G)(-\frac{1}{2} + \sum_i \Delta N_i). \]

Thus, when \( \sum_i \Delta N_i = 0 \), the total anomaly \( A^R_G \) is proportional to the one-loop beta-function coefficient, i.e. the scale anomaly, \( b_{G_a} \),
\[ b_{G_a} = 3C_2(G_a) - \sum_{\text{matter}} T(R_{G_a}). \]

When we use the definition of R charge \( \tilde{R}_i = 2R_i \), we would have \( A^R_{G_a} = b_{G_a} \). It is not accidental that \( A^R_{G_a} \) is proportional to \( b_{G_a} \) \[30, 31\]. The sum of the R-charges \( \sum_{i=1,2,3} R_i \) of a supermultiplet is nothing but the R-charge (up to an overall normalization) associated with the R-current which is a bosonic component of the supercurrent \[32\], when the R-charge is universal for all of matter fields, i.e. \( \sum_i \Delta N_i = 0 \). Using the supertrace identity \[33\] it is in fact possible to show \[31\] that \( A^R_{G_a} \) is proportional to \( b_{G_a} \) to all orders in perturbation theory.

In explicit models, non-abelian groups except \( SU(2) \) have few massless matter fields with non-vanishing oscillator numbers, while massless matter fields with oscillators can appear as singlets as well as \( SU(2) \) doublets. Thus, in explicit models the total R-anomaly \( A^R_{G_a} \) is related with the one-loop beta-function coefficient \( b_{G_a} \),
\[ 2A^R_{G_a} = b_{G_a}, \]
modulo \( N_i T(R_a) \) for most of non-abelian groups. Since the total R-anomalies satisfy the GS condition, \( A^R_{G_a} = A^R_{G_b} \), the above relation between \( A^R_{G_a} \) and \( b_{G_a} \) leads to
\[ b_{G_a} = b_{G_b}, \]
\[ (33) \]
modulo $2N_i T(R_a)$.

For example, the explicit $Z_3$ orbifold model and $Z_4$ orbifold model in Table $4$ and Table $5$ have only non-oscillated massless modes except singlets. The $Z_3$ orbifold model has the following total R-anomalies and one-loop beta-function coefficient,

\[ A^R_{E_6} = \frac{9}{2} + 9n_{E_6}, \quad b_{E_6} = 9, \]
\[ A^R_{SU(3)} = -36 + \frac{3}{2}n_{SU(3)}, \quad b_{SU(3)} = -72. \]  

(34)

Hence, this model satisfy $2A^R_{G_a} = b_{G_a}$ and its one-loop beta-function coefficients satisfy

\[ b_{E_6} = b_{SU(3)} \pmod{3}. \]  

(35)

Similarly, the $Z_4$ orbifold model in Table $5$ has the total R-anomalies and one-loop beta-function coefficients as,

\[ A^R_{SO(10)} = -7 + 2n_{SO(10)}, \quad b_{SO(10)} = -14 \]
\[ A^R_{SU(4)} = -23 + n_{SU(4)}, \quad b_{SU(4)} = -46 \]
\[ A^R_{SO(12)} = 5 + 2n_{SO(10)}, \quad b_{SO(12)} = 10 \]
\[ A^R_{SU(2)} = -27 + n_{SU(2)}, \quad b_{SU(2)} = -54. \]  

(36)

Thus, this model also satisfies $2A^R_{G_a} = b_{G_a}$ and its one-loop beta-function coefficients satisfy

\[ b_{SO(10)} = b_{SU(4)} = b_{SO(12)} = b_{SU(2)} \pmod{2}. \]  

(37)

4.3 Relation with T-duality anomaly

Here we study the relation between R-anomalies and T-duality anomalies. The relation between R-symmetries and T-duality has also been studied in Ref. $[22]$. The T-duality anomalies are obtained as $[12] [13]$

\[ A^T_{G_a} = -C_2(G_a) + \sum_{\text{matter}} T(R_{G_a})(1 + 2n_i), \]  

(38)

where $n_i$ is the modular weight of matter fields for the $i$-th torus. The modular weight is related with $r_i$ as

\[ n_i = -1 \quad \text{for} \quad r_i = 1, \]
\[ = 0 \quad \text{for} \quad r_i = 0, \]
\[ = r_i - 1 - \Delta N_i \quad \text{for} \quad r_i \neq 0, 1. \]  

(39)
Note that \( n_i = -r_i \) for \( r_i = 0, 1/2, 1 \). Thus, in the model, which includes only matter fields with \( r_i = 0, 1/2, 1 \), the T-duality anomalies and R-anomalies are proportional to each other,

\[
A_{G_a}^{T_i} = -2A_{G_a}^{R_i}. \tag{40}
\]

In generic model, such relation is violated, but T-duality anomalies and R-anomalies are still related with each other as

\[
A_{G_a}^{T_i} = -2A_{G_a}^{R_i} - 2 \sum_{r_i \neq 0, 1/2, 1} (2r_i - 1). \tag{41}
\]

T-duality should also satisfy the GS condition,

\[
\frac{A_{G_a}^{T_i}}{k_a} = \frac{A_{G_b}^{T_i}}{k_b}, \tag{42}
\]

for the \( i \)-th torus, which does not include the N=2 subsector. Thus, the requirement that T-duality anomalies and R-anomalies should satisfy the GS condition, leads to a similar condition for

\[
\Delta^i_a = 2 \sum_{r_i^b \neq 0, 1/2, 1} (2r_i^b - 1). \tag{43}
\]

For the \( i \)-th torus, which includes N=2 subsector, T-duality anomalies can be canceled by the GS mechanism and T-dependent threshold correction \[34\]. Thus, for such torus, the T-duality anomalies has no constrain from the GS condition. However, even for such torus, R-anomaly should satisfy the GS condition.

For example, the \( Z_4 \) orbifold model in Table 5 has the following T-duality anomalies,

\[
\begin{align*}
A_{SO(10)}^{T_{1,2}} &= -2, \quad A_{SO(10)}^{T_3} = 18, \\
A_{SU(4)}^{T_{1,2}} &= -2, \quad A_{SU(4)}^{T_3} = 18, \\
A_{SO(12)}^{T_{1,2}} &= -2, \quad A_{SO(12)}^{T_3} = -6, \\
A_{SU(2)}^{T_{1,2}} &= -2, \quad A_{SU(2)}^{T_3} = -6. \tag{44}
\end{align*}
\]

They satisfy the GS condition,

\[
A_{SO(10)}^{T_{1,2}} = A_{SU(4)}^{T_{1,2}} = A_{SO(12)}^{T_{1,2}} = A_{SU(2)}^{T_{1,2}}. \tag{45}
\]

On the other hand, for the third torus, T-duality anomalies \( A_{G_a}^{T_3} \) do not satisfy the GS condition, that is, anomalies \( A_{G_a}^{T_3} \) are not universal, because there is the N=2 subsector and one-loop gauge kinetic functions depend on the \( T_3 \) moduli with non-universal coefficients \[34\]. However, they satisfy

\[
\begin{align*}
A_{SO(10)}^{T_3} &= -2A_{SO(10)}^{R_3}, \quad A_{SU(4)}^{T_3} = -2A_{SU(4)}^{R_3}, \\
A_{SO(12)}^{T_3} &= -2A_{SO(12)}^{R_3}, \quad A_{SU(2)}^{T_3} = -2A_{SU(2)}^{R_3}. \tag{46}
\end{align*}
\]

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because this model has only massless modes with $r_3 = 0, 1/2, 1$. Indeed, all of $Z_4$ orbifold models include only massless modes with $r_3 = 0, 1/2, 1$. Furthermore, all of $Z_N$ orbifold models with $v_i = 1/2$ have only massless modes with $r_i = 0, 1/2, 1$. Thus, the above relation (40) holds true in such $Z_N$ orbifold models. That is also true for $R_1$-anomalies in $Z_2 \times Z_M$ orbifold models with $v_1 = (1/2, -1/2, 0)$ and $v_2 = (0, v_2, -v_2)$.

Such relation between T-duality anomalies and R-anomalies (40) would be important, because the GS condition on R-anomalies leads to a certain condition on the T-duality anomalies even including the N=2 subsector. For example, in the above $Z_4$ orbifold model, the following condition is required

$$A_{SO(10)}^{T_3} = A_{SU(4)}^{T_3} = A_{SO(12)}^{T_3} \equiv A_{SU(2)}^{T_3} \pmod{2}.$$ (47)

5 Phenomenological implications

5.1 Symmetry breaking of the discrete R-symmetries

5.1.1 Nonperturbative breaking

If the discrete R-symmetries are anomalous, they are broken by nonperturbative effects at low energy. This is because, for the GS mechanism to take place, the axionic part of the dilaton $S$ should transform non-linearly under the anomalous symmetry. This means that a term like $e^{-aS}$ with a constant $a$ has a definite charge $R_i^S$ under the anomalous symmetry. Nonperturbative effects can therefore induce terms like $e^{-aS}\Phi^1 \cdots \Phi^n$ with matter fields $\Phi^a$, where the total charge satisfies the condition for allowed couplings, i.e. $R_i^S + \sum_a R_i^a = 1 \pmod{N_i}$. This implies that below the scale of the vacuum expectation value (VEV) of $S$, such non-invariant terms can appear in a low-energy effective Lagrangian. The canonical dimension of the non-invariant operator $e^{-aS}\Phi^1 \cdots \Phi^n$ that can be generated by the nonperturbative effects depends of course on the R charge $R_i^S$. If the smallest dimension is larger than four, they will be suppressed by certain powers of the string scale. However, the operator can produce non-invariant mass terms like $m\Phi\Phi'$, because some of the chiral superfields may acquire VEVs. One should worry about such cases. Needless to say that small higher dimensional terms would be useful in phenomenological applications such as explaining fermion masses.

In the case that the smallest dimension is smaller than three, the anomalous discrete R symmetry has less power to constrain the low-energy theory.

5.1.2 Spontaneous breaking

In the discussion above, we have considered R-symmetry breaking by nonperturbative effects when R-symmetries are anomalous. Here we comment on another type of symmetry breaking: they can be broken spontaneously by the VEVs of scalar fields in the form $U(1) \times R \rightarrow R'$. That is, we consider a spontaneous symmetry breaking, where some
scalar fields with non-vanishing \( U(1) \) and \( R \) charges develop their VEVs and they break \( U(1) \) and \( R \) symmetries in such a way that an unbroken \( R' \) symmetry remains intact. (Its order is denoted by \( N' \) below.) Even in such symmetry breaking, we can obtain the GS condition for the unbroken \( R' \) from the GS condition for the \( U(1) \) and \( R \)-anomalies. Suppose that we have the GS condition for the \( U(1) \) symmetry as

\[
Tr QT(\mathbf{R}_{G_a})/k_a = Tr QT(\mathbf{R}_{G_b})/k_b, \tag{48}
\]

where \( Q \) is the \( U(1) \) charge. Since the unbroken \( R' \) charge is a linear combination of \( R \) and \( Q \), the mixed anomalies for \( R' \) should also satisfy the GS condition,

\[
Tr R'T(\mathbf{R}_{G_a})/k_a = Tr R'T(\mathbf{R}_{G_b})/k_b. \tag{49}
\]

Here the anomaly coefficients \( Tr R'T(\mathbf{R}_{G_a}) \) are defined modulo \( N'T(\mathbf{R}_{G_a}^{(f)}) \).

Through the symmetry breaking \( U(1) \times R \rightarrow R' \), some matter fields may gain mass terms like

\[
W \sim m \Phi \bar{\Phi}. \tag{50}
\]

Such a pair of the matter fields \( \Phi \) and \( \bar{\Phi} \) should form a vector-like representation of \( G_a \) and have opposite \( R' \) charges of the unbroken \( R' \) symmetry. The heavy modes of this type have therefore no contribution to the mixed anomalies between the gauge symmetry \( G_a \) and the unbroken \( R' \) symmetry. This implies that the above GS condition for the unbroken \( R' \) remains unchanged even after the spontaneous symmetry breaking. The symmetry breaking \( U(1) \times R \rightarrow R' \) also allows Majorana mass terms like

\[
W \sim m \Phi \Phi. \tag{51}
\]

This type of Majorana mass terms can appear for an even order \( N' \) of the \( R' \) symmetry if the \( R' \) charge of \( \Phi \) is \( N'/2 \) and \( \Phi \) is in a real representation \( \mathbf{R}_{G_a} \) of the unbroken gauge group \( G_a \). The field \( \Phi \) contributes to the anomaly coefficient as \( \frac{N'}{2} T(\mathbf{R}_{G_a}) \). That however may change only the modulo-structure of the anomaly coefficients. For \( SU(N) \) gauge group, this contribution is obtained as \( \frac{N'}{2} \times \) (integer). Thus, the modulo-structure does not change, that is, the anomaly coefficients \( Tr R'T(\mathbf{R}_{G_a}) \) are defined modulo \( N'/2 \). However, for other gauge groups, the modulo-structure of the anomaly coefficients may change.

5.2 Gravity-induced supersymmetry breaking and Gauge symmetry breaking

The most important difference of the discrete R-symmetries compared with T-duality in phenomenological applications comes from the fact that (for the heterotic orbifold string models) the moduli and dilaton superfields have vanishing R-charges. The VEVs of their bosonic components do not therefore violate the discrete R-symmetries in the perturbation
theory. (We have discussed above the nonperturbative effects due to the VEV of the dilaton, which may be small in a wide class of models.) However, the F-components of the moduli and dilaton superfields have non-zero R-charges. Therefore, since the VEVs of these F-components generate soft-supersymmetry breaking (SSB) terms at low energy, the SSB terms do not have to respect the discrete R-symmetries. Fortunately, in the visible sector, the scale of the R-symmetry breaking must be of the same order as that of supersymmetry breaking. If the order of the discrete R-symmetry is even, the VEVs of these F-components break the discrete R-symmetry down to its subgroup \( Z_2 \), an R-parity. That is an interesting observation because it may be an origin of the R-parity of the minimal supersymmetric standard model (MSSM).

Gauge symmetry breaking can be achieved by VEVs of chiral supermultiplets in a non-trivial representation of the gauge group or by non-trivial Wilson lines. Clearly, if the chiral supermultiplets have vanishing R-charges and only their scalar components acquire VEVs, the discrete R-symmetries remain unbroken. Similarly, the Wilson lines do not break the discrete R-symmetries because gauge fields have no R charge. As a consequence, the discrete R-symmetries have a good chance to be intact at low energy if the nonperturbative effects are small.

### 5.3 Constraints on low-energy beta-functions

Only anomaly-free discrete R-symmetries remain as intact symmetries in a low-energy effective theory. Obviously, the model with anomaly-free discrete R-symmetries corresponds to \( A_{G_a}^R = 0 \) (mod \( N_i T(R_{G_a}^f) \)). Consider for instance \( SU(N) \) gauge groups for which \( T(R_{G_a}^f) = 1/2 \) is usually satisfied. Then in models, which have no oscillator mode in a non-trivial representations of \( SU(N) \), the relation between R-anomalies and beta-function coefficients lead to

\[
b_a = 2A_{G_a}^R = 0,
\]

mod \( N_i \) for any gauge group \( G_a \). For example, the \( Z_3 \) orbifold model with anomaly-free R-symmetries leads to \( b_a = 3n_a \) with integer \( n_a \), while the \( Z_4 \) orbifold model with anomaly-free R-symmetries leads to \( b_a = 2n_a \). Similarly, \( b_a = 1 \) would be possible in \( Z_6 \)-II orbifold models because \( N_i = (6,3,2) \) as one can see from Table 1.

Even for anomalous discrete R-symmetries, the GS condition for R-anomalies and the relation between beta-function coefficients (21), (32), (33) would have phenomenological implications. As discussed at the beginning in this section, the non-perturbative effects can generate operators like \( e^{-aS}\Phi^1 \cdots \Phi^a \). If its canonical dimension is larger than four, its contribution to low-energy beta-functions may be assumed to be small.  

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2 Whether the nonperturbative effects due to the VEV of the dilaton do play an important roll in the SSB sector depends on the R charge of the dilaton, and one has to check it explicitly for a given model.

3 If the operator produces non-invariant mass terms like \( M\Phi\Phi' \) with \( M \) larger than the low-energy scale, the low-energy spectrum may change. Then the power of the discrete R-symmetries decreases.
As for the MSSM we find $b_3 = -3$ and $b_2 = 1$ for $SU(3)$ and $SU(2)$, respectively. That is, we have $b_2 - b_3 = 4$, implying the MSSM can not be realized, e.g. in $Z_3$ orbifold models, because $Z_3$ orbifold models require $b_a - b_b = 0 \mod 3$ if the effects of the symmetry breaking of the discrete R-symmetries can be neglected. Similarly, the model with $b_2 - b_3 = 4$ can not be obtained in the $Z_6$-$I$, $Z_7$ or $Z_{12}$-$I$ orbifold models.

6 Conclusion

We have studied anomalies of the discrete R-symmetries in heterotic orbifold models. They are remnants of $SU(4)_R$ symmetry which, along with extended $N = 4$ supersymmetry, is explicitly broken by orbifolding. We have found that the mixed anomalies for different gauge groups satisfy the universal GS condition. Therefore, these anomalies can be canceled by the GS mechanism, which remains to be proven at the string theory level. As a byproduct, we have found a relation between the anomaly coefficients of the discrete R-symmetries and one-loop beta-function coefficients. In particular, in the case that the contribution coming from the oscillator modes for the chiral matter fields in non-trivial representations of a gauge group vanishes, the anomaly coefficient corresponding to the sum of the discrete R-symmetry anomaly is exactly proportional to the one-loop beta-function coefficient of the corresponding gauge coupling.

In a wide class of models, the discrete R-symmetries may be unbroken at low energy. The main reason for this is that the moduli superfields have vanishing R-charges. This should be contrasted to the case of T-duality, where the moduli fields transform non-trivially under the T-duality transformation. We have studied the relation between anomalies of the discrete R-symmetries and T-duality. We have argued that the discrete R-symmetries have a good chance to be unbroken down to the supersymmetry breaking scale. Even below this scale a $Z_2$ subgroup is unbroken, which may be an origin of the R-parity of the MSSM. In fact, the R-parity of the MSSM is completely anomaly-free, indicating that it has a stringy origin.

Our investigation on the discrete R-symmetries in heterotic orbifold models could be extended to other types of heterotic models, e.g. free fermionic construction [35] and Gepner models [36] as well as Calabi-Yau models. Furthermore, our studies can be extended to type IIA and IIB string theories with D-branes, e.g. intersecting/magnetized D-brane models. This however would be beyond the scope of the present paper, and we will leave it to our future study. At last we emphasize that string models have other discrete symmetries. For example, heterotic orbifold models have non-abelian discrete flavor symmetries [23]. They may be identified with the non-abelian discrete flavor symmetries which have been recently introduced in constructing low-energy flavor models [1]. Further investigations in this direction are certainly necessary to link the non-abelian discrete flavor symmetries from the top and the bottom with each other.
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