Heavy meson three body decay:  
Three decades of Dalitz plot amplitude analysis\[1].

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Abstract  
Three body heavy meson decays allows one to extract important quantities for  
flavour physic, however the phenomenological analysis has some open problems to  
be solved in order to make possible to extract physical parameters with precision.  
After many years and some experience accumulated, we have learned new features  
and we expect to learn much more from data coming mainly from B three body  
decays. In this paper we address these aspects and we present a proposal to modify  
the Dalitz fit amplitude method in order to allow further studies.

1 Introduction  
Three body heavy meson decay analysis have been treated as a coherent sum of indi-
vidual two body resonances amplitudes plus a non-resonant component, each amplitude  
with a strong final state interaction phase (FSI). This simple configuration based in the  
dominance of the quasi two body component, was proposed in the early eighties by Mark  
collaboration \[1\]. Important changes were made to improve this approach during these  
already three decades, however remains the basic structure of the original model. This  
analysis method have been working well to represent low statistics charm three body  
decay but have been presenting some problems to represent the equivalent three body  
beauty decays.

The basic concept used by Mark collaboration in its initial three body analysis was  
inspired in the amplitude analysis used for π - nucleon and K - nucleon interaction giving  
three or four final state \[2, 3, 4, 5, 6, 7\].\[2\]. Nevertheless, there is an important difference  
between hadron hadron interaction and heavy meson decay: while the last one use a  
full coherence sum, in hadronic collisions this quantum effect had been used in a more  
restrictively form, assuming partially the coherence between different two body resonances  
amplitudes present in the same phase space region.

Recently, for some charm meson three body decay analysis \[9, 10\], a significative change  
was partially introduced by FOCUS collaboration. To take in account interferences among

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\[2\] most of these experimental studies were to search for axial-vector particle properties of a \( J^P = 1^+ \)  
mesons decaying in three pseudo-scalar mesons \[8\]
resonances with the same quantum number, FOCUS used the two body unitary K-matrix approach to represent the s-wave component.

The K-matrix in Dalits analysis and the usual approach of coherent sum of resonances, reflected two different treatment for particle interferences well characterized by Azimov in a recent paper[11]. According to this author, interference between particles with the same quantum number would be classified as “direct interference”, otherwise it would said to be by “re-scattering interference”. Among other differences between they two, the locality is one of more important characteristic pointed out in this paper. Direct interferences have a large phase space range, while re-scattering interference between two resonances is limited at the region where resonances have intercepting phase space.

However, if for one side the K-matrix approach tried to treat correctly interferences between resonances belonging to the same partial wave, on the other side neglecting possible re-scattering interferences between particles with different quantum numbers, that is a singularity triangle type diagram [12]. This approximation seems to work for $D_s^+ \rightarrow \pi^- \pi^+ \pi^+$ decay [13], but not for $D^+ \rightarrow K^- \pi^+ \pi^+$. In fact the $K\pi$ phase motion of the last decay, extracted with Partial Wave Amplitude (PWA) analysis first provided by E791 collaboration and then by CLEO and FOCUS [14, 15, 16], shown a distribution substantially different from the one observed in scalar partial wave $K\pi$ elastic scattering by LASS collaboration [17] and also different from the recently PWA analysis performed for the semi-leptonic $D^+ \rightarrow K^- \pi^+ e^+ \nu_e$ decays [18]. However the same analysis for the $D_s^+ \rightarrow \pi^- \pi^+ \pi^+$ decays seems to have similar phase motion behaviour than the one obtained by $\pi^- \pi^+$ elastic scattering [13].

Since both kinds of interferences, with different behaviour, can be present in three body heavy meson decay, a complete amplitude analysis has to take in account the existence of both effects and also provide from the fit parameters, the relative contribution of each effect for each intermediary state.

2 Isobaric model and experimental results.

With the increase of statistics, more resonances with different spins started to contribute significantly in three body D decays, Argus collaboration introduced a generalization of the isobar model [20]: each possible resonance amplitude is represented by a Breit-Wigner multiplied by angular distribution function associated with the spin of the resonance. The Non-Resonant is parametrized as a constant. The various contributions are combined in a coherent sum with complex coefficients that are extracted from maximum likelihood fits to the data. The absolute value of the coefficients are related to the relative fraction of each contribution and the phases takes into account final state interaction (FSI) between the resonance and the bachelor pion. This phase is considered constant because it depends only on the total energy of the system, i.e. the heavy meson mass.

With this simple model plus the E791 method [21] to determine the mass and the

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3 Both, the semi-leptonic and the $K\pi$ elastic scattering scalar phase motion agree each other in a clear verification of the Watson theorem [19].
width of some not yet well defined resonances, most of three body charm meson decay were successfully described\(^4\). On the other hand, B charmless three body decay scenario is not successfully describe by the use of the Isobar model. The most important problem is associated with the non-resonant (NR) contribution. Differently from charm decays, the NR contribution has playing a significant role in \(B\) meson studies. At a very early stage of the \(B\) factory analysis programme the ‘constant amplitude’ description of the NR term was abandoned and replaced by alternative approaches. When studying the decay \(B^+ \to K^+\pi^+\pi^-\) the Belle collaboration proposed an empirical exponential function to represent the NR contribution \(^{24}\). BABAR instead, used for the same decay mode \(^{25}\) a phase and magnitude parametrization obtained by the LASS experiment \(^{17}\) for S-wave \(K\pi\) elastic scattering.

The empirical function proposed by Belle \(^{24}\) has two terms, one for each Dalitz plot variable \((s_{13} \equiv M^2(K^+\pi^-)\) and \(s_{23} \equiv M^2(\pi^+\pi^-))\), and is given by:

\[
A_{nr}(K^+\pi^+\pi^-) = a_{nr1}e^{-\alpha s_{13}}e^{i\delta_{nr1}} + a_{nr2}e^{-\alpha s_{23}}e^{i\delta_{nr2}},
\]  

\((1)\)

with five free parameters: one magnitude and one phase for each of these terms and the slope of the exponential function \(\alpha\). The fit results exhibit three noteworthy and correlated characteristics: i) a large NR fraction of around 34\%, ii) a phase difference between \(\delta_{nr1}\) and \(\delta_{nr2}\) around 180\(^\circ\), iii) a sum of fractions from all components, resonant plus NR of around 150\%. In an amplitude analysis the sum of fractions can of course be bigger or less than 100\% and this is not inconsistent with the unitary concept \(^5\). However, such an outcome indicates the existence of destructive or constructive interference among two or more amplitudes. In practice, a sum of fit fractions \(\gg 100\%\), that is a large destructive interference, is often indicative of underlying problems in the fit model. In the study under consideration it may be that the apparent large sum of fractions is in fact an artifice, and that in the coherent sum the significant NR contribution is largely “eliminated” through the destructive interference arising from the \(\sim 180^\circ\) phase difference between the two NR amplitudes.

Subsequently, several amplitude analyses, including Belle’s study of the mode \(B^0 \to K_0^0\pi^+\pi^-\) \(^{26}\) and the BABAR analysis of \(B^0 \to K_S^0K^+K^-\) \(^{27}\), have used this exponential amplitude. The fit results for share the same three characteristics discussed above for \(B^+ \to K^+\pi^+\pi^-\).

This approach has been also used by both BABAR and Belle for three-body decays involving two identical particles, namely \(B^+ \to K^+K^+K^-\) \(^{24,28}\) and \(B^+ \to \pi^+\pi^+\pi^-\) \(^{29}\). Again, in both analyses a large contribution is found for the NR component, and the overall sums of fractions are significantly above 100\%.

\(^4\) There is at least one important exception, the \(D^0 \to K_0^0\pi^-\pi^+\). Because CP violation studies this decay have been analysed by the B-factories with a large samples and also many problems \(^{22,23}\).

\(^5\) The concept of fraction in a Dalitz plot analysis is directly related to the branching ratio of a particular amplitude. It can be classically associated to that fraction of the total number of events in an incoherent sum of amplitudes.

\(^6\) In these cases the expression Eq. (1) contracts to a single amplitude because of the symmetry under the exchange of identical particles.
Another important problem for almost all the charmless three-body $B$ decays is that it has been necessary to include some scalar contributions to get a good fit to data. The mass and width values obtained in some analysis involving $\pi^+\pi^-$ in final state do not match among these two experiments [24, 25] and some times do not match between different channels of the same experiment [29]. The same situation occurs with high mass scalar contribution involving the final state $K^+K^-$ [24, 28].

The non-resonant component which, in general, spread within all the populated phase space, can mimic other dynamical components [30] through the interferences with the resonances present in the same phase space. High mass scalar resonances have also similar capability, due this angular distribution and the large width, these contribution populates homogeneously large portion of the di-hadrons phase space, without a clear signature like vector or tensor contributions. This shadowing phenomenon was observed in charm meson three body decays at the E791 experiment, where the overestimated contribution of the non-resonant amplitude use to be able to mimic the real contributions coming from the scalar mesons $\sigma$ and $\kappa$ [21, 31].

3 Interference among resonances with the same quantum numbers.

In the isobar model, regarding interferences, all contributions are treated in the same way, independent of its origin. On the other hand K-matrix approach proposed by FOCUS [9, 10] use a especial treatment only to represents the S-wave scalar amplitude contribution unitary amplitude extract from two body elastic scattering. This approach take into account the peculiarity of long range phase space interference between two body amplitudes with the same quantum numbers. The high spin resonances was treated as the isobar model.

There are alternative ways to write two or more resonances with same quantum numbers in a unitary fashion, Tornqvist [32], per example, proposed to use a couple Breit-Wigner. There is a more recent representation that can be very useful for three body amplitude analysis, it was proposed by Svec [33] based in a Hu method [34]. As we can see below, this approach can be useful, because it makes explicit the unitary constraint in the isobar model, while keeping its usual structure. The proposal consist to represent a multi-resonance contribution through a unitary product of isolated Breit-Wigner. Since one resonance is represented by an unitary S-matrix Breit-Wigner function the product of Breit-Wigner will be unitary also. This idea is suitable to Breit-Wigner or other amplitude to represent a resonance.

With this assumption one can arrive to a simples T equation for two resonances [33]:

$$T_{res} = BW_1(s) + BW_2(s) + 2iBW_1(s)BW_2(s)$$

where $BW_i$ is the usual Breit-Wigner function. This equation can be re-written as coherent sum of Breit-Wigners with specific complex coefficient, depending of the resonances
parameters as we can see below:

\[ T_{\text{res}} = C_1^{(2)}(s)BW_1(s) + C_2^{(1)}(s)BW_2(s) \]  

(3)

the coefficients are given by \( C_1^{(2)}(s) = 1 - 2i/(z_1 - z_2) \) and \( C_2^{(1)}(s) = 1 + 2i/(z_1 - z_2) \), where \( z_i = m_i^2 - im_i\Gamma_i(s) \) with a smooth dependence in the energy variable. So to guarantee unitarity, the phase differences between two resonances with the same quantum number should be write as:

\[ C_1^{(2)}(s) - C_2^{(1)}(s) = \frac{4(\Delta m\Gamma(s) - i\Delta m^2)}{\Delta^2m\Gamma(s) + \Delta^2m^2} \]  

(4)

where \( m \) is the mass of the resonance, \( \Gamma(s) \) the width, \( \Delta m^2 \) and \( \Delta m\Gamma(s) \) are respectively the mass and the \( m\Gamma(s) \) differences between the two resonance. Since the \( \Delta m^2 \) is in general bigger than the \( \Delta m\Gamma(s) \), and the only dependence in \( s \) variable comes from the width, one can expect a little changes in the phase difference. In some sense, Equation 2 justify the use of coherent sum of Breit Wigner with constant parameters, at least for charm three body decay taking in account the reduct phase space of this decay. With the parameters extracted from data, one can check whether or not these values are compatible with expected by a two body unitary behaviour of resonances in the same partial wave.

The suitability of the this approach to represent data involving amplitudes with the same quantum numbers, could be observed in two occasions: the scalar amplitude and the phase motion obtained with the partial analysis analysis (PWA), performed for the \( D^+ \rightarrow K^-\pi^+\pi^+ \) [14, 15, 16] and for \( D_s^+ \rightarrow \pi^-\pi^+\pi^+ \) [13] decays. Both decays were well represented with a coherent sum of two scalar resonances contributions with a phase difference, as it was plotted in references [13, 14, 15, 16]. However the K-matrix was able to represent well only the \( D_s^+ \rightarrow \pi^-\pi^+\pi^+ \). The scalar \( K^-\pi^+ \) amplitude and the phase behaviour of the \( D^+ \rightarrow K^-\pi^+\pi^+ \) decay, obtained with PWA differed substantially from the \( K^-\pi^+ \) elastic scattering experimental results [17]. This observed difference should be due a strong re-scattering contribution in this three body final state [35], that can not be represented in the K-matrix formalism [12].

4 Interference among resonances with different quantum numbers.

We can divide the interference between resonances with different quantum numbers in Dalitz plot in two types: parallel, i.e. defined at the same Dalitz variable \( s_{ij} \) and that can interfere in a large portion of the phase space, like the vector resonance \( \rho(770) \) and the scalar one \( \sigma(500) \) or with the \( f_0(980) \) in a decay involving two charged mesons \( \pi \) in final state. The \emph{parallel interference} have been described in several models through local interferences for all phase space, but due to the angular distribution, the phase space integral should close to zero to parallel resonances with different spin.
The second type we can call crossing, i.e. interference between resonances that cross each other in the phase space with different $s_{ij}$ Dalitz plot variables, like the interference between the $\rho(770)$ resonance with the $K^*(890)$ in $B^+\rightarrow K^+\pi^+\pi^-$ decay. The crossing resonances has the interference in a limited phase space region and the integral in general is different from zero. With a coherent sum, this term make a real contribution to the total amplitude square. Due to the formal complexity to work with this contribution, some Isobar approach to describe hadron hadron interaction, preferred to make the approximation that interferences between crossing resonances zero. This approximation was used in SLAC manual for three body partial analysis in the eighties [7]. To performed the analysis for the final state $K^-\pi^+\pi^-p$, they assume the integral of the crossing term between the $\rho$ and $K^*(890)$ is zero, beside they can observe a significant amount of interference.

Since the crossing interference can not be estimated by first principles, a correct inclusion of the crossing contribution in amplitude analysis is a big challenger. The best known formal approach to use for study re-scattering effects is the Faddeev equation [11, 36]. Very preliminary studies using this method, have presented nice results with a natural explanation for the observed phenomena to a global phase difference between the P and S wave from $K\pi$ partial wave analysis (PWA) of the $D^+\rightarrow K^-\pi^+\pi^+$ decay to the $K\pi$ elastic scattering [37]. In fact other than the difference pointed out before between these two experimental results, there is a global phase difference bigger than $\pi/2$ degrees [14, 15, 16]. This difference between three body decay and elastic scattering was also observed in $D^+_s\rightarrow \pi^-\pi^+\pi^+$ partial wave analysis [13].

Experimental studies in the sixties and seventies presented a practical and empirical approach for the re-scattering problem. They treated this effect by a coherent sum of Breit Wigner also to the crossing resonances. In $K^+p\rightarrow K^0\pi^+p$ studies at CERN experiment [3], they introduced a sum of individual amplitudes with a constant phase difference, between the interfering resonances due a re-scattering phenomena. This determine the degree of constructive or destructive interference among them. Each amplitude is parametrized by a Breit-Wigner multiply by a real density term independent of the energies, that can be extracted from data. However there is an important difference between the original amplitude analysis and the one we use nowadays, the coherence sum was not generalized and fixed for all resonances, like we have been doing in the three body heavy mesons decays.

In fact in their studies, the coherence is include in the fit function only among resonances that they observe a priory in data, a signature of interference between them. For the $K^+p\rightarrow K^0\pi^+p$ study, they use a coherent sum only for the $K^*(1400)p$ and a baryonic resonance amplitude $K^0N^*(1236)$, but not between the dominant contributions $K^*(890)p$ and $K^0N^*(1236)$. This approach was generalized by the Berkeley group that studied the interaction $\pi^-p\rightarrow \pi^0\pi^+p$ [4]. In fact they introduced a coherence factor to take in account the observed re-scattering for all resonances present in the spectrum. The observed results between the $\rho^-$’s amplitudes and the different $N^*$ resonances variety from 0.6 to 0.8.

Other physics process involving particle interference among different quantum numbers used also a free parameter to manage with the non obvious source of coherence. In
order to observe the $\rho$ and $\omega$ isospin violation interaction proposed by Glashow [38], experimental studies were performed with low transverse momentum events from $\pi^{-}p \rightarrow \pi^{-}\pi^{+}n$ interaction [39]. The amplitude square used to fit the $\pi^{+}\pi^{-}$ spectrum, included one Breit-Wigner square for the $\rho$ decays and another one for the G-parity violating channel $\omega \rightarrow \pi^{+}\pi^{-}$, plus an interference term. The last one was wrote to represent the clear interference observed in the $\rho$ mass region. Besides the usual product of two Breit-Wigners and a relative strong phase, was also included a real multiplicative coherent factor that could change from 0 to 1. This factor was also include in other studies $\rho$ and $\omega$ interactions studies with different reactions [40, 41, 42].

More recently was proposed also a coherence parameter to take in account possible contamination in the dominant pseudoscalar exchange process, from the axial meson component in the $\pi^{-}p \rightarrow \pi^{0}\pi^{0}n$ interaction [43]. Also Cristal Barrel collaboration [44, 45] used a coherence parameter to take in account the many different partial waves in the initial and final $p\bar{p}$ interaction at rest, that part interfere and part does not. For the $pp \rightarrow \pi^{0}\eta\eta$ [45] they fit seven intermediary states, with seven complex number for each one resonance contribution plus twelve coherence factor. Some of them were set to zero because there was no overlapping region or apriori tests did not present any significant contribution.

Heavy meson three body decays has a simpler configuration since that it involves a spin 0 initial state and three spin zero final state. So the use of a coherence factor is less crucial than used in the above examples, were some times it is associated with and average of the initial or final helicity state for interaction involving baryons. However some problems remain in heavy meson decays, like the re-scattering, the existence of different isospin amplitudes in a same final state.

Also a technical problem can coming out in amplitude analysis. In fact, experimental resolution have not been considered fully in their analysis. In general it is not important to describe the mass spectrum of the usual resonances, as it was pointed out for the narrow $f_{0}(980)$ resonance analysed in the $D_{s}^{+}\rightarrow \pi^{+}\pi^{-}\pi^{+}$ [46]. However it might be different for describe the crossing region between two narrow resonances, the interference can created a strong variation in a short little phase space region and the experimental resolution can spread out this variations in this particular region. One could take this effect in account in an amplitude fit function using Monte Carlo studies. But the inclusion of phenomenological coherence factor could serve to mimic this effect and others possible associated at the non-parametrized simple re-scattering contributions and the average on the isospin amplitudes.

5 Closing Remarks

Charmless three body B decays has an important appointment soon with the LHCb data [47]. In fact for the LHC run 2010-2012 it is expect more than decades of thousand events for the decays $B^{+}$ to three light mesons and thousand of events to three body decays channels involving a proton anti-proton [48]. This amount of events corresponds about
one order of magnitude bigger than the produced by the electron positron factories Belle and BaBar until today. With this statistics many new things could be understood for CP violation, that is the main proposal of LHCb experiment. Model independent approach was developed to search new sources of directly CP violation in Dalitz plot [49]. However if one want to go further and extract more information from Dalitz plot, like the CKM phase $\gamma$ [50, 51] or the relative amount from the Penguins and Tree contributions or also to know the relative CP violation from each intermediary resonance state, or even to extract the the $y$ parameter from $B^0_s$ [52] one have to work with amplitude analysis. But as we could see in this paper, many studies have to be performed before to get trust-able parameters from this kind of fits.

We could see here that the first results from Belle and BaBar presented some problems to describe the non-resonant contribution and the high mass scalar particles. The inclusion of an empirical exponential function increased significantly the confidence level of the fit. However we noted that this result imply in a strong destructive interference between the $s_{13}$ and $s_{23}$ non-resonant components. This destructive interference take place just in the crossing region between the $\pi^+\pi^-$ and $K^+\pi^-$ resonances. We think one of the first studies to do with the new coming data is to look deeply at this effect. As it was pointed out in the previews section, the correct inclusion of the crossing region is the big challenger and seems that the charmless B three body decays analysis have to take care of this problem in order to have a good representation of the big amount of events expected for these final state B decays.

The start point of the amplitude analysis for these studies could keep the general form of the isobaric model, but given more freedom to the fit function allowing more flexibility to understand better the data. Although Eq. 4 shown a smooth variation of the phase difference with data, this effect can be important for the fit involving a big amount of data as expected in LHCb and them could be add at the amplitude. In order to understand the importance of the coherence in hadronic interaction, it would be interesting includes a phenomenological real parameter in some interference terms of the fit function. The coherence parameters values, obtained from fit can give some light for understand better how works interferences in the crossing region among two resonances placed in different invariant mass, like $\pi^+\pi^-$ and $K^+\pi^-$ resonances in $B^+ \rightarrow K^+\pi^-\pi^+$ decay.

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References

[1] R.H. Schindler et al. Mark Collaboration Phys.Rev. D24, 78 (1981).

[2] De Baere et al, Il Nuovo Cimento 51A, (1967).

[3] B. Jongejans, in Methods in Subnuclear Physics, edited by M. Nikolic (Gordon and Breach, New York, 1970), Vol. IV.
[4] G. Gidal et al Phys. Let. **76B**, 657 (1973).

[5] W. Michael Phys. Rev. D 7, 1985 (1973).

[6] J.N. Macnaughton *et al.*, Nucl. Phys. B95, 197 (1975).

[7] D. Astom, T.S. Lasinski and P.K. Sinervo, ”The SLAC three-Body Partial Wave Analysis System” - SLAC-Report-287.

[8] I. Bediaga, J.M. Pires and A.F.S. Santoro, Rev.Bras.Fis.**11**,997 (1981).

[9] J.M. Link et al. FOCUS Collaboration, Phys.Lett. **585B** 200 (2004).

[10] J.M. Link et al FOCUS Collaboration, Phys.Lett. **653**, 1 (2007).

[11] Ya. Azimov, J.Phys. *37G* 023001 (2010) and hep-ph/0904.1376.

[12] V.V. Anisovich, A.V. Sarantsev, arXiv:0806.1601.

[13] BABAR Collaboration, Phys.Rev. **79D**, 032003 (2009).

[14] E.M. Aitala et al. E791 Collaboration, Phys.Rev. **73D** 032004 (2006), Erratum-ibid.D74:059901,2006.

[15] J.M. Link et al, FOCUS Collaboration, Phys.Lett. **681B**, 14 (2009).

[16] G. Bonvicini et al, CLEO-c Collaboration, Phys. Rev. **78D**, 052001 (2008).

[17] Aston, et al, LASS collaboration, Nucl. Phys. **B 296**, 493 (1988).

[18] BABAR Collaboration, hep-ex/1012.1810.

[19] K.M. Watson, Phys.Rev. **95**, 228 (19540.

[20] H. Albrecht et al, Phys. Lett **308B** 435 (1993)

[21] E.M. Aitala et al. E791 Collaboration, Phys.Rev.Lett. **86** 770 (2001).

[22] Belle Collaboration, Phys.Rev.Lett. **89**, 121801 (2002). **Acknowledgments:** This work was supported by the NSF under the grant number PHY-0807959 and by CNPq. One of us (IB) thanks the CBPF for the kind hospitality extended to him, while this work was completed. We also thanks Jōao Torres for clarifying conversations about issues related Astrophysics.

[23] BaBar Collaboration, Phys. Rev. Lett. 95, 121802 (2005).

[24] Belle Collaboration, Phys. Rev. **D 71**, 092003 (2005) and Phys. Rev. Lett. **96**, 251803 (2006).
[25] BABAR Collaboration, Phys. Rev. D 72, 072003 (2005) and Phys. Rev. D 78, 012004 (2008).

[26] Belle Collaboration, Phys. Rev. D 75, 012006 (2007).

[27] BABAR Collaboration, arXiv:0808.0700 [hep-ex], submitted to ICHEP2008. and Phys. Rev. Lett. 99, 161802 (2007).

[28] BABAR Collaboration, Phys. Rev. D 74, 032003 (2006).

[29] BABAR Collaboration, Phys. Rev. D 79, 072006 (2009).

[30] I. Bediaga, D.R. Boito, G. Guerrer, F.S. Navarra, M. Nielsen, Phys.Lett. B665, 30-34 (2008).

[31] E.M. Aitala et al. E791 Collaboration, Phys.Rev.Lett. 89, 121801 (2002).

[32] N.A. Tornqvist, Z.Phys. 68C 647 (1995).

[33] M. Svec, Phys. Rev. 64D, 096003 (2001).

[34] N. Hu, Phys. Rev. 74, 131 (1948).

[35] M.R.Pennington, hep-ph/1003.2549.

[36] L. D. Faddeev, Sov. Phys. JETP 12 1014 (1961).

[37] K.S.S.F Guimar??es et al. Proceedings of the workshop Light Cone 2009. Published in Nucl. Phys. 199, 341 (2010).

[38] S.L. Glashow, Phys. Rev. Let. 7, 469 (1961).

[39] B. N. Ratcliff et al, Phys. Lett. B 38, 345 (1972)

[40] G. Goldhaber et al, Phys. Rev. Lett. 23, 1351 (1969)

[41] W.W.M. allison et al, Phys. Rev. Lett. 24, 618 (1970)

[42] S. Hagopian et al, Phys. Rev. Lett. 25, 1050 (1970)

[43] N. N. Achasov and G. N. Shestakov, Phys. Rev. 67 D, 114018-1 (2003).

[44] J. Adomeit et al, Z. fur Physik bf 71C, 227 (1996)

[45] A. Abele et al, Eur. Phys. J. 8C, 67 (1999)

[46] Aitala et al, Phys.Rev.Lett. 86, 765 (2001).

[47] LHCb Collaboration, JINST 3, S08005 (2008).

[48] I. Bediaga, behalf LHCb Collaboration in ICHEP2010.
[49] I. Bediaga, I.I. Bigi, A. Gomes, G. Guerrer, J. Miranda and A.C.dos Reis, Phys.Rev. 80D, 096006 (2009).

[50] I. Bediaga, R.E. Blanco, C. Gobel, R. Mendez-Galain, Phys.Rev.Lett. 81, 4067 (1998).

[51] I. Bediaga, G. Guerrer and J. Miranda, Phys.Rev. 76D, 073011 (2007).

[52] M. Gronau, D.Pirjol, A. Soni and J. Zupan, Phys.Rev. 75D, 014002 (2007).