Research on spatial feature differentiation of basin river network geomorphology based on multifractal algorithm

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Abstract. Using multi-fractal algorithm to scientifically quantify watershed morphology is one of the hot spots of geomorphology research. In this paper, based on the DEM data of the Northwest China Basin (Daxiahe), the remote sensing images of the Daxia River Basin were processed. The multi-fractal model was used to extract and analyze the multi-fractal characteristics and quantified characteristic parameters of the Daxia River and 7 sub-basins of the river system, and discussed the geomorphic significance of the spatial characteristics of the watershed. The results showed that the multi-fractal spectrum of the watershed was in the shape of a right hook (\( \Delta f < 0 \)). The span of the singular index \( \Delta a \) was between 0.860 and 1.160, and the value was relatively small. This indicated that the Daxia River watershed was evenly distributed in most areas, and the water system was not fully developed. The average values of the multi-fractal spectral height difference (\( \Delta f \)) and the singular index distribution range (\( \Delta a \)) from downstream to upstream were increasing (1.184~1.570) and decreasing (1.160~0.933) in sequence, which indicated that the proportion of the geomorphic unit with the smallest slope in the watershed increased sequentially from downstream to upstream, and the change of geomorphic shape from upstream to downstream tended to be complicated. This study is of great significance to effectively solve the complex problems in geomorphology and to master the characteristics and laws of various geomorphological phenomena.

1. Introduction

Multi-fractal theory has been used for describing and distinguishing many complex figures, systems and processes in nature since it was first proposed by Mandelbrot in the 1970s [1]. The multifractal spectrum it provides can characterize the singular distribution of theoretical and experimental forms [2]. For example, multifractal spectrum can be used to reveal a nonlinear relationship between hydrological variables [3], the complexity of the soil and the heterogeneity of the environment can also be measured using its singularity index [4]. Multifractal spectrum analysis can also be used in time series. For example, the multiplicity of daily average temperature [5], the scale characteristics of daily rainfall time series and their spatial and temporal changes [6].

Since its formation, the river network has been continuously damaged [7]. It is necessary to further explore the changes of the spatial heterogeneity structure of the river network of the river basin. Cao et al. [8] used multi-fractal methods to study the relief features and changing trends of the Loess Plateau.
Song et al. [9] used multi-fractal spectra to explain the morphological characteristics of the river network under the multi-fractal lens to reveal the filling capacity and spatial heterogeneity of the main rivers in the basin. Sun et al. [10] used the multifractal theory to provide an accurate analysis method for the impact of meadow degradation on soil properties. To study the lithology control of the multifractal of the river network, Gaudio et al. [11] reconstructed two types of spectra with different lithologies. To analyze the spatial differentiation characteristics of the Dali River Basin, Wang et al. [12] calculated the multifractal parameters of its geomorphology.

Therefore, multi-fractal can comprehensively quantitatively describe the overall characteristics of landforms, and the obtained parameters can have a positive significance in studying the distribution and characteristics of water systems in various regions [13]. This paper uses GIS remote sensing technology to process and analyze the DEM data of the Daxia River Basin, combined with the multifractal algorithm to conduct a comprehensive and distributed analysis and discussion of the multifractal characteristics and quantitative parameters of the study area.

2. Materials and methods

2.1. Overview of the study area

Daxia River belongs to the first-class tributary of the upper Yellow River and is located in the transitional zone between the Qinghai-Tibet Plateau and the Loess Plateau (East longitude 102°02′~103°23′, North latitude 34°51′~35°48′). The Daxia River flows into the northeast of Xiahe County, unearths the gate and enters the Linxia Basin. It traverses Linxia City from the west to the east, turns to the bridge and turns north to the Xiehu Gorge of Dongxiang County, and is injected into the Liujiaxia Reservoir in the village of Lamachuan Tazhang in Kangjiawan [14]. The total length of the main river channel is 203 km, the total watershed area is 7152 km², the annual average flow is 34.3 m³/s, the annual sand content is 3.49 kg/m³, and the forest area is 3.27×10⁶ hm² [15].

2.2. Multifractal algorithm

2.2.1. Multifractal spectrum. The calculation of the multifractal spectrum should first be the distribution probability of the statistical feature on the corresponding fractal structure. In this paper, the length of the water system is used as the target quantity, and the box method [16] is used to calculate and calculate the distribution probability of the length of the water system.

Cover the entire watershed with a box of scale \( \varepsilon \times \varepsilon \). Define the distribution probability of the water system in each box as \( Q_i(\varepsilon) \):

\[
Q_i(\varepsilon) = \frac{M_i(\varepsilon)}{M}
\]

(1)

Where \( M_i(\varepsilon) \) is the total length of the water system in Box \( i \), and \( M \) is the total length of the entire watershed.

Define the partition function \( X_q(\varepsilon) \):

\[
X_q(\varepsilon) = \sum_{i=1}^{N(\varepsilon)} Q_i^q(\varepsilon)
\]

(2)

Where \( N(\varepsilon) \) is the number of non-empty boxes at the \( \varepsilon \) scale of the whole watershed, and \( q \in (-\infty, +\infty) \) is the weighting factor.
If $Q_i(\varepsilon)$ follows the multifractal model, the partition function has a simple power-law relationship with the box size $\varepsilon$:

$$X_q(\varepsilon) \propto \varepsilon^{\tau(q)}$$  \(3\)

Where $\tau(q)$ is the power index (quality index) of the $q$ order distance. If the measurement under study satisfies the multifractal, then a series of $\tau(q)$ values can be obtained by the slope of the $\ln(X_q(\varepsilon)) \sim \ln(\varepsilon)$ fitted straight line.

The multi-fractal generalized fractal dimension $D_q$ can be obtained with a quality index $\tau(q)$:

$$D_q = \begin{cases} \frac{\tau(q)}{q-1} & (q \neq 1) \\ \tau'(1), q = 1 \end{cases}$$  \(4\)

When $q = 1$, $\tau(q)$ can be differentiated. According to the different values of $q$, the fractal dimension has different meanings.

The Legendre transform of the $f(a) \sim a$ function:

$$a(q) = \frac{dq}{d\tau(q)} \quad f(a) = q a(q) - \tau(q)$$  \(5\)

According to formula (5), the multi-fractal spectral function $f(a) \sim a$ can be obtained.

2.2.2. Parameter meaning of multifractal spectrum. The parameters of the multifractal spectrum are of great significance to the complex system of watershed geomorphology [12, 17-18]. Generally, the multifractal spectrum $f(a)$ is a single-peak convex function with respect to the singular exponent $a$.

When $q > 0$, the curve increases monotonously. When $q < 0$, the curve decreases monotonously. When $q = 0$, $f(a)$ takes the maximum value. As shown in Figure 1.

**Figure 1.** Typical multifractal spectrum.
In the multi-fractal spectrum $f(a) \sim a$, the singular index $a$ represents the morphological differences of the sub-regions in the watershed with uneven, irregular and complex geomorphology. Fractal spectrum $f(a)$ reveals the status and trend of landform types with the same singular index $a$ in the watershed.

The multi-fractal spectrum index distribution range $\Delta a$ quantitatively characterizes the contrast relationship between the maximum probability subset and the minimum probability subset within the fractal. The larger the value of $\Delta a$, the greater the internal difference in fractal, and the more obvious the polarization trend of each subset probability. Conversely, the internal differences in fractals are reduced, and the distribution of each subset tends to be concentrated and uniform.

The magnitude of the height difference $\Delta f$ of the multifractal reflects the difference between the number of the smallest subset of probability measures and the number of the largest subset of probability measures. When the high probability subset is dominant, $\Delta f > 0$, the $f(a) \sim a$ curve is a hook to the left. When the small probability subset is dominant, $\Delta f < 0$, the $f(a) \sim a$ curve is a hook to the right. When the two are equal, $\Delta f = 0$, the $f(a) \sim a$ curve is symmetrical bell-shaped.

3. Results and analysis

3.1. Watershed classification and division

According to the latitude and longitude of the boundary of the Daxia River Basin, the DEM image data containing the study area were obtained from the geospatial data cloud (https://www.gscloud.cn/). According to the two watershed classification methods of Strahler and Shreve [19], combined with ArcGIS software, the Daxia River Basin is classified. Divide the watershed into 7 sub-watersheds (Figure 2).

![Figure 2. Division of the Daxiahezi Basin.](image)

3.2. Distributed calculation and analysis of multifractal spectrum

This paper takes the threshold of 10000 for the vector diagrams of the water systems of the Daxia River and 7 sub-basins in ArcGIS. Perform element-to-grid and grid analysis on this vector image. In the cell
size (A), fill in 10 sets of values at different scales to get the raster map corresponding to different side lengths. Perform grid-to-polyline transformation on this and count the length of the water system in the grid corresponding to different sizes $\varepsilon$ in the polyline diagram. The specifications of A are: 10×10, 20×20, 50×50, 60×60, 80×80, 90×90, 100×100, 150×150, 200×200, 500×500. According to the above algorithm, the multi-fractal relationship curve of the geomorphological features of the Daxia River and 7 sub-basins is finally obtained (Figure 3-6).

Figure 3. $\ln(X_q(\varepsilon)) \div \ln(\varepsilon)$ relationship curve of Daxia River and 7 sub-basins.

Figure 4. Quality index relationship.

Figure 5. $D_q \sim q$ relationship curve.
Figure 3 shows the double logarithmic curves of \( q = 3 \) to \( q = 3 \), and the interval of 1 for Daxia River and 7 sub-basins. Although Daxiahe and Sangke are floating at \( q = -3 \) and \( q = 2 \) in the figure, the whole shows a good linear relationship. Therefore, \( X_q(\varepsilon) \) and \( \varepsilon \) satisfy the power law relationship, indicating that the distribution of the river network water system within a certain scale change range is scale-free. Figure 4 shows the relationship between the quality index of the study area. \( \tau(q) \) is a convex function, namely there is a non-linear relationship between \( \tau(q) \) and \( q \), which indicates that the geomorphological characteristics of the Daxia River watershed have obvious multi-fractal characteristics. Figure 5 shows the generalized fractal dimension of the study area. \( D_q \) is not a constant, which means that the image is not evenly distributed, and \( D_q \) is a function that decreases with the increase of \( q \), indicating that \( D_q \) has multi-fractal characteristics. Figure 6 shows the mult fractal spectrum of the study area respectively. Because the multifractal spectrum is in the shape of a right hook, \( \Delta f < 0 \), namely the distribution of the water system depends to a large extent on the distribution of the water system with a small probability, which shows that the watershed of the Daxia River has a uniform distribution in most areas, the water system is less developed and simple, and the density of the river network is relatively small. From figure 6, we can see that there is an obvious curvature change at the end of the right hook of the multifractal spectrum of the Daxia River and 7 sub-basins, which shows that the geomorphology of the watershed has undergone sudden changes within the distribution range of this section. It can be seen from the distribution map of the water system (Figure 7) that the distribution of the water system in the direction of each area is small, and this difference is consistent with the analysis results of the multifractal spectrum calculated.

3.3. Quantification and analysis of distributed parameters of multifractal spectrum
Multifractal spectrum is an important quantitative index to study the characteristics of multifractal. Through the self-similar description of the heterogeneity and singularity of the whole watershed, it expresses the different hierarchical structure and characteristics caused by the evolution of different geomorphic local conditions.

According to the division of the Daxiahe sub-basin in Figure 2, the seven sub-basins in the study area can be divided into upper reaches (Sangke, Tongren), middle reaches (Ganja, Xiahe), and lower reaches (Hezuo, Shuangcheng, Zheqiao). From the above calculations, the multi-fractal quantization parameter values of Daxiahe and 7 sub-basins can be obtained, as shown in Table 1.
Table 1. Multifractal parameters of geomorphology in the upper, middle and lower reaches of the Daxia River.

| Position     | Watershed   | $a_{\text{min}}$ | $f(a_{\text{min}})$ | $a_{\text{max}}$ | $f(a_{\text{max}})$ | $\Delta a$ | $\Delta f$ |
|--------------|-------------|-------------------|----------------------|-------------------|----------------------|------------|------------|
| Total watershed | Daxiahe     | -0.546            | -1.337               | 0.574             | 0.234                | 1.120      | 1.571      |
| Upstream     | Sangke      | -0.040            | -0.591               | 0.820             | 0.462                | 0.860      | 1.053      |
|              | Tongren     | -0.213            | -0.887               | 0.792             | 0.428                | 1.005      | 1.315      |
|              | average value | -0.127          | -0.739               | 0.806             | 0.445                | 0.933      | 1.184      |
| Midstream    | Ganji       | -0.246            | -0.989               | 0.809             | 0.477                | 1.055      | 1.466      |
|              | Xiahe       | -0.202            | -0.831               | 0.783             | 0.369                | 0.985      | 1.199      |
|              | average value | -0.224          | -0.910               | 0.796             | 0.423                | 1.020      | 1.333      |
| Downstream   | Hezuo       | -0.232            | -0.926               | 0.788             | 0.455                | 1.020      | 1.381      |
|              | Shuangcheng | -0.256            | -0.957               | 0.814             | 0.441                | 1.070      | 1.398      |
|              | Zheqiao     | -0.584            | -1.537               | 0.806             | 0.395                | 1.390      | 1.931      |
|              | average value | -0.357          | -1.140               | 0.803             | 0.430                | 1.160      | 1.570      |

3.3.1. Holistic analysis. The paragraph text follows on from the subsubsection heading but should not be in italic. Among the parameters of the multi-fractal spectrum of watershed landform morphology, $f(a_{\text{max}})$ is actually equal to the simple dimension $D$ of the watershed landform morphology, which is a comprehensive approximate representation of the watershed landform morphology. According to the calculation, the values of $f(a_{\text{max}})$ in the Daxia River and the 7 sub-basins are from 0.234 to 0.477, and the results are relatively small, indicating that the overall distribution of the river system in the basin is relatively simple and regular. $a_{\text{min}}$ and $a_{\text{max}}$ represent the singular index when the probability of maximum water system distribution and the probability of minimum water system distribution change with $\varepsilon$, respectively. It can be calculated that $a_{\text{min}}$ and $a_{\text{max}}$ in the study area are bounded between -0.546--0.040 and 0.574--0.820, respectively, so the singular exponent span $\Delta a = a_{\text{max}} - a_{\text{min}}$ is bounded on 0.860--1.160. $\Delta a$ can quantitatively describe the uneven degree of distribution probability of the water system. Because all $\Delta a$ in the study area is small, it means that the water system in the basin has a small difference in density, good uniformity, and regular distribution. The development of the distribution of water systems in the whole and the local areas is roughly similar.

3.3.2. Distributed analysis. Geomorphic multifractal parameters can quantify the spatial differentiation characteristics of the watershed landforms such as fragmentation and complexity. In this paper, the distributed quantification and analysis of the study area can more reasonably reveal the spatial differentiation characteristics of landforms in the watershed.

According to the multi-fractal calculation model, the multi-fractal parameters of the upper, middle and lower reaches of the Daxia River and seven sub-basins are calculated in a distributed manner (Table 1). It can be seen that the average value of the minimum singularity index $a_{\text{min}}$ is from -0.127 in the upstream and mid-stream -0.224, down to -0.357, showing a decreasing trend. The average value of the multifractal spectrum value $f(a_{\text{min}})$ decreased from -0.739 in the upstream to -0.910 in the midstream, and finally decreased to -1.140 in the downstream, showing a decreasing trend, which indicates that the proportion of the largest slope geomorphic unit is the uppermost (Sangke, Tongren). The average maximum singularity index $a_{\text{max}}$ increased from 0.796 in the middle reaches to 0.803 in the downstream, and finally to 0.806 in the upstream, showing an increasing trend. The maximum geomorphic singularity index $a_{\text{max}}$ corresponding to the average value of the multifractal spectrum value $f(a_{\text{max}})$ increased from 0.423 in the middle reaches to 0.430 in the downstream, and finally...
increased to 0.445 in the upstream, showing an increasing trend, indicating that the proportion of the smallest slope topography units is the middle reaches (Ganja and Xiahe).

The average value of the height difference $\Delta f$ of the multifractal spectrum is 1.184 in the downstream, 1.333 in the middle, and 1.570 in the upstream. This indicates that the proportion of geomorphic units with the smallest slope in the upstream watershed is significantly increased compared with the proportion of geomorphic units with the largest slope compared with the middle and downstream watershed. The average value of the singular index distribution range $\Delta a$ of the topography is 0.933 in the upstream, 1.020 in the middle, and 1.160 in the downstream. This indicates that the geomorphological changes in the Daxia River Basin tend to be complicated from upstream to downstream.

4. Conclusion
Using the multi-fractal theory, the characteristic parameters of the multi-fractal spectrum of watershed (Daxia River) geomorphology in the water system of northwest China were extracted in a holistic and distributed manner. The research area was analyzed with the extracted feature parameters as a whole and distributed, which confirmed that the watershed geomorphology of the Daxia River has certain multifractal characteristics. Moreover, most areas of the basin are evenly distributed. The development of the water system is in its infancy, which is relatively inadequate and simple. The distribution of the river network is regular and the density is relatively small. Meanwhile, the proportion of geomorphic units with the smallest slope in the upstream (Sangke, Tongren) watershed of this area is significantly increased compared with the proportion of geomorphic units with the largest slope compared with the middle (Ganja, Xiahe) and downstream (Hezuo, Shuangcheng, Zheqiao) watersheds. The geomorphological features of the watershed gradually became more complicated from Sanke and Tongren, to Ganja and Xiahe, and finally to Hezuo, Shuangcheng and Zheqiao. Through the discussion of the multi-fractal features of watershed landform morphology, it can reveal the features of landform morphology, and has important theoretical significance for the correct understanding, description and quantification of landform development process.

Acknowledgements
This work was supported by the National Natural Science Foundation of China [61563047].

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