Deep Echo State Network (DeepESN):
A Brief Survey

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Abstract

The study of deep recurrent neural networks (RNNs) and, in particular, of deep Reservoir Computing (RC) is gaining an increasing research attention in the neural networks community. The recently introduced Deep Echo State Network (DeepESN) model opened the way to an extremely efficient approach for designing deep neural networks for temporal data. At the same time, the study of DeepESNs allowed to shed light on the intrinsic properties of state dynamics developed by hierarchical compositions of recurrent layers, i.e. on the bias of depth in RNNs architectural design. In this paper, we summarize the advancements in the development, analysis and applications of DeepESNs.

Keywords: Deep Echo State Network, DeepESN, Reservoir Computing, Echo State Networks, Recurrent Neural Networks, Deep Learning, Deep Neural Networks

1 Introduction

In the last decade, the Reservoir Computing (RC) paradigm \cite{1,2} has attested as a state-of-the-art approach for the design of efficiently trained Recurrent Neural Networks (RNNs). Though different instances of the RC methodology exist in literature (see e.g. \cite{3,4}), the Echo State Network (ESN) \cite{5,6} certainly represents the most widely known model, with a strong theoretical ground (e.g. \cite{7,8,9,10,11,12}) and a plethora of successful applications reported in literature (see e.g. \cite{13,14} and references therein, as well as more recent works e.g. in \cite{15,16,17}). Essentially, ESNs are recurrent randomized neural networks \cite{18,19} in which the state dynamics are implemented by an untrained recurrent hidden layer, whose activation is used to feed a static output module that is the only trained part of the network. In this paper we deal with the extension of the ESN approach to the deep learning framework.

The study of deep neural network architectures for temporal data processing is an attractive area of research in the neural networks community \cite{20,21}. Investigations in the field of hierarchically organized Recurrent Neural Networks
(RNNs) showed that deep RNNs are able to develop in their internal states a multiple time-scales representation of the temporal information, a much desired feature e.g. when approaching complex tasks in the area of speech or text processing [22, 23].

Recently, the introduction of the Deep Echo State Network (DeepESN) model [24, 25] allowed to study the properties of layered RNN architectures separately from the learning aspects. Remarkably, such studies pointed out that the structured state space organization with multiple time-scales dynamics in deep RNNs is intrinsic to the nature of compositionality of recurrent neural modules. The interest in the study of the DeepESN model is hence twofold. On the one hand, it allows to shed light on the intrinsic properties of state dynamics of layered RNN architectures [41]. On the other hand it enables the design of extremely efficiently trained deep neural networks for temporal data.

Previous to the explicit introduction of the DeepESN model in [24], works on hierarchical RC models targeted ad-hoc constructed architectures, where different modules were trained for discovery of temporal features at different scales on synthetic data [26]. Ad-hoc constructed modular networks made up of multiple ESN modules have also been investigated in the speech processing area [27, 28]. More recently, the advantages of multi-layered RC networks have been experimentally studied on time-series benchmarks in the RC area [29]. Differently from the above mentioned works, the studies on DeepESN considered in the following aim to address some fundamental questions pertaining to the true nature of layering as a factor of architectural RNN design [41]. Such basic questions can be essentially summarized as follows: (i) Why stacking layers of recurrent units? (ii) What is the inherent architectural effect of layering in RNNs (independently from learning)? (iii) Can we extend the advantages of depth in RNN design using efficiently trained RC approaches? (iv) Can we exploit the insights from such analysis to address the automatic design of deep recurrent models (including fundamental parameters such as the architectural form, the number of layers, the number of units in each layer, etc.)?

This paper is intended both to draw a line of recent developments in response to the above mentioned key research questions and to provide an up-to-date overview on the progress and on the perspectives in the studies of DeepESNs, which are presented in Section 3. Before that, in Section 2 we recall the major characteristics of the DeepESN model.

2 The Deep Echo State Network Model

As for the standard shallow ESN model, a DeepESN is composed by a dynamical reservoir component, which embeds the input history into a rich state representation, and by a feed-forward readout part, which exploits the state encoding provided by the reservoir to compute the output. Crucially, the reservoir of a DeepESN is organized into a hierarchy of stacked recurrent layers, where the output of each layer acts as input for the next one, as illustrated in Figure 1.

In this case, at each time step $t$, the state computation proceeds by following
the pipeline of recurrent layers, from the first one, which is directly fed by the external input, up to the highest one in the reservoir architecture. In our notation we use $N_U$ to denote the external input dimension, $N_L$ to indicate the number of reservoir layers, and we assume, for the sake of simplicity, that each reservoir layer has $N_R$ recurrent units. Moreover, we use $u(t) \in \mathbb{R}^{N_U}$ to denote the external input at time step $t$, while $x^{(i)}(t) \in \mathbb{R}^{N_R}$ is the state of the reservoir layer $i$ at time step $t$. In general, we use the superscript $(i)$ to indicate that an item is related to the $i$-th reservoir in the stack. At each time step $t$, the composition of the states in all the reservoir layers, i.e. $x(t) = (x^{(1)}(t), \ldots, x^{(N_L)}(t)) \in \mathbb{R}^{N_R N_L}$, gives the global state of the network.

The computation carried out by the stacked reservoir of a DeepESN can be understood under a dynamical system viewpoint as an input-driven discrete-time non-linear dynamical system, where the evolution of the global state $x(t)$ is governed by a state transition function $F = (F^{(1)}, \ldots, F^{(N_L)})$, with each $F^{(i)}$ ruling the state dynamics at layer $i$. Assuming leaky integrator reservoir
In each layer and omitting the bias terms for the ease of notation, the reservoir dynamics of a DeepESN are mathematically described as follows. For the first layer we have that:

\[
x^{(1)}(t) = F(u(t), x^{(1)}(t-1)) = (1 - a^{(1)})x^{(1)}(t-1) + f(W^{(1)}u(t) + \hat{W}^{(1)}x^{(1)}(t-1)),
\]

while for successive layers \(i > 1\) the state update is given by:

\[
x^{(i)}(t) = F(x^{(i-1)}(t), x^{(i)}(t-1)) = (1 - a^{(i)})x^{(i)}(t-1) + f(W^{(i)}x^{(i-1)}(t) + \hat{W}^{(i)}x^{(i)}(t-1)).
\]

In the above equations 1 and 2, \(W^{(1)} \in \mathbb{R}^{N_R \times N_U}\) is the input weight matrix, \(W^{(i)} \in \mathbb{R}^{N_R \times N_R}\) for \(i > 1\) is the weight matrix for inter-layer connections from layer \((i-1)\) to layer \(i\), \(\hat{W}^{(i)} \in \mathbb{R}^{N_R \times N_R}\) is the recurrent weight matrix for layer \(i\), \(a^{(i)} \in [0, 1]\) is the leaking rate for layer \(i\) and \(f\) denotes the element-wise applied activation function for the recurrent reservoir units (typically, the tanh non-linearity is used).

Interestingly, as graphically illustrated in Figure 2, we can observe that the reservoir architecture of a DeepESN can be characterized, with respect to the shallow counterpart, by interpreting it as a constrained version of standard shallow ESN/RNN with the same total number of recurrent units. In particular, the following constraints are applied in order to obtain a layered architecture:

- all the connections from the input layer to reservoir layers at a level higher than 1 are removed (influencing the way in which the external input information is seen by recurrent units progressively more distant from the input layer);
- all the connections from higher layers to lower ones are removed (which affects the flow of information and the dynamics of sub-parts of the network’s state);
- all the connections from each layer to higher layers different from the immediately successive one in the pipeline are removed (which affects the flow of information and the dynamics of sub-parts of the network’s state).

The above mentioned constraints, that graphically correspond to layering, have been explicitly and extensively discussed in our previous work in [24]. Under this point of view, the DeepESN architecture can be seen as a simplification of the corresponding single-layer ESN, leading to a reduction in the absolute number of recurrent weights which, assuming full-connected reservoirs at each layer, is quadratic in both the number of recurrent units per layer and total number of layers [31]. As detailed in the above points, however, note that this peculiar architectural organization influences the way in which the temporal information is processed by the different sub-parts of the hierarchical reservoir, composed by recurrent units that are progressively more distant from the external input.
Figure 2: The layered reservoir architecture of DeepESN as a constrained version of a shallow reservoir. Compared to the shallow case with the same total number of recurrent units, in a stacked DeepESN architecture the following connections are removed: from the input to reservoir levels at height > 1 (blue dashed arrows), from higher to lower reservoir levels (green dash dotted arrows), from each reservoir at level $i$ to all reservoirs at levels $> i + 1$ (orange dotted arrows).

Furthermore, differently from the case of a standard ESN/RNN, the state information transmission between consecutive layers in a DeepESN presents no temporal delays. In this respect, we can make the following considerations:

- the aspect of sequentiality between layers operation is already present and discussed in previous works in literature on deep RNN (see e.g. [23, 22, 32, 33]), which actually stimulated the investigation on the intrinsic role of layering in such hierarchically organized recurrent network architectures;

- this choice allows the model to process the temporal information at each time step in a “deep” temporal fashion, i.e. through a hierarchical composition of multiple levels of recurrent units;

- in particular, notice that the use of (hyperbolic tangent) non-linearities applied individually to each layer during the state computation does not allow to describe the DeepESN dynamics by means of an equivalent shallow system.

Based on the above observations, a major research question naturally arises and drives the motivation to the studies reported in Section 3, i.e. how and to
what extent the described constraints that rule the layered construction and the hierarchical representation in deep recurrent models have an influence on their dynamics.

As in the standard RC framework, the reservoir parameters, i.e. the weights in matrices $W^{(i)}$ and $\hat{W}^{(i)}$, are left untrained after initialization under stability constraints, which are given through the analysis of the Echo State Property for deep reservoirs provided in [34].

As regards the output computation, although different choices are possible for the pattern of connectivity between the reservoir layers and the output module (see e.g. [23, 35]), a typical setting consists in feeding at each time step $t$ the state of all reservoir layers (i.e. the global state of the DeepESN) to the output layer, as illustrated in Figure 3. Note that this choice enables the readout component to give different weights to the dynamics developed at different layers, thereby allowing to exploit the potential variety of state representations in the stacked reservoir. Denoting by $N_Y$ the size of the output space, in the typical case of linear readout, the output at time step $t$ is computed as:

$$y(t) = W_{\text{out}} x(t) = W_{\text{out}} (x^{(1)}, \ldots, x^{(N_L)}),$$

where $W_{\text{out}} \in \mathbb{R}^{N_L \times N_Y}$ is the readout weight matrix that is adapted on a training set, typically in closed form through direct methods such as pseudo-inversion or ridge-regression.

## 3 Advances

Here we briefly survey the recent advances in the study of the DeepESN model. The works described in the following, by addressing the key questions summarized in the Introduction, provide a general support to the significance of the DeepESN, also critically discussing advantages and drawbacks of its construction.

- The DeepESN model has been introduced in [24], which extends the preliminary work in [25]. The analysis provided in these papers revealed, through empirical investigations, the hierarchical structure of temporal data representations developed by the layered reservoir architecture of a DeepESN. Specifically, the stacked composition of recurrent reservoir layers was shown to enable a multiple time-scales representation of the temporal information, naturally ordered along the network’s hierarchy. Besides, in [24] layering proved effective also as a way to enhance the effect of known RC factors of network design, including unsupervised reservoir adaptation by means of Intrinsic Plasticity [36]. The resulting effects have been analyzed also in terms of state entropy and memory.

- The hierarchically structured state representation in DeepESNs has been investigated by means of frequency analysis in [37], which specifically considered the case of recurrent units with linear activation functions. Results
Figure 3: Readout organization for DeepESN in which at each time step the reservoir states of all layers are used as input for the output layer.

pointed out the intrinsic multiple frequency representation in DeepESN states, where, even in the simplified linear setting, progressively higher layers focus on progressively lower frequencies. In [37] the potentiality of the deep RC approach has also been exploited in predictive experiments, showing that DeepESNs outperform state-of-the-art results on the class of Multiple Superimposed Oscillator (MSO) tasks by several orders of magnitude.

- The fundamental RC conditions related to the Echo State Property (ESP) have been generalized to the case of deep RC networks in [34]. Specifically, through the study of stability and contractivity of nested dynamical systems, the theoretical analysis in [34] gives a sufficient condition and a necessary condition for the Echo State Property to hold in case of deep RNN architectures. Remarkably, the work in [34] provides a relevant conceptual and practical tool for the definition, validity and usage of DeepESN in an “autonomous” way with respect to the standard ESN model.

- The study of DeepESN dynamics under a dynamical system perspective has been pursued in [31, 38], which provide a theoretical and practical framework for the study of stability of layered recurrent dynamics in terms
of local Lyapunov exponents. This study also provided interesting insights in terms of the quality of the developed system dynamics, showing that layering has the effect of naturally pushing the global dynamical regime of the recurrent network closer to the stable-unstable transition condition known as the edge of chaos.

- The study of the frequency spectrum of deep reservoirs enabled to address one of the fundamental open issues in deep learning, namely how to choose the number of layers in a deep RNN architecture. Starting from the analysis of the intrinsic differentiation of the filtering effects of successive levels in a stacked RNN architecture, the work in [39] proposed an automatic method for the design of DeepESNs. Noticeably, the proposed approach allows to tailor the DeepESN architecture to the characteristics of the input signals, consistently relieving the cost of the model selection process, and leading to new state-of-the-art results in speech and music processing tasks.

- A first extension of the deep RC framework for learning in structured domains has been presented in [40], which introduced the Deep Tree Echo State Network (DeepTESN) model. The new model points out that it is possible to combine the concepts of deep learning, learning for trees and RC training efficiency, taking advantages from the layered architectural organization and from the compositionality of the structured representations both in terms of efficiency and in terms of effectiveness. Overall, DeepTESN provides a first instance of an extremely efficient approach for the design of deep neural networks for learning in cases where the input data is represented in the form of tree structures.

- For what regards the experimental analysis in applications, DeepESNs were shown to bring several advantages in both cases of synthetic and real-world tasks. Specifically, DeepESNs outperformed shallow reservoir architectures (under fair conditions on the number of total recurrent units and, as such, on the number of trainable readout parameters) on the Mackey-Glass next-step prediction task [41], on the short-term Memory Capacity task [24, 42], on MSO tasks [37], as well as on a Frequency Based Classification task [39], purposely designed to assess multiple-frequency representation abilities. As pertains to real-world problems, the DeepESN approach recently proved effective in a variety of domains, including Ambient Assisted Living (AAL) [43], medical diagnosis [44], speech and polyphonic music processing [39].

4 Conclusions

In this survey we have provided a brief overview of the extension of the RC approach towards the deep learning framework, describing the salient features of the DeepESN model. Noticeably, DeepESNs enable the analysis of the intrinsic
properties of state dynamics in deep RNN architectures, i.e. the study of the bias due to layering in the design of RNNs. At the same time, DeepESNs allow to transfer the striking advantages of the ESN methodology to the case of deep recurrent architectures, leading to an efficient approach for designing deep neural networks for temporal data.

The analysis of the distinctive characteristics and dynamical properties of the DeepESN model has been carried out first empirically, in terms of entropy of state dynamics and system memory. Then, it has been conducted through more abstract theoretical investigations that allowed the derivation of the fundamental conditions for the ESP of deep networks, as well as the characterization of the developed dynamical regimes in terms of local Lyapunov exponents. Besides, studies on the frequency analysis of DeepESN dynamics allowed us to develop an algorithm for the automatic setup of (the number of layers of) a DeepESN. Current developments already include model variants and applications to both synthetic and real-world tasks. Finally, a pioneering extension of the deep RC approach to learning in structured domains has been introduced.

Overall, the final aim of this paper is to summarize the successive advances in the development, analysis and applications of the DeepESN model, providing a document that is intended to contain a constantly updated view over this research topic.

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