Approach to Computer Implementation of Mathematical Model of 3-Phase Induction Motor

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Abstract. This article discusses the development of the computer model of an induction motor based on the mathematical model in a three-phase stator reference frame. It uses an approach that allows combining during preparation of the computer model dual methods: means of visual programming circuitry (in the form of electrical schematics) and logical one (in the form of block diagrams). The approach enables easy integration of the model of an induction motor as part of more complex models of electrical complexes and systems. The developed computer model gives the user access to the beginning and the end of a winding of each of the three phases of the stator and rotor. This property is particularly important when considering the asymmetric modes of operation or when powered by the special circuitry of semiconductor converters.

1. Introduction

Nowadays, mathematical modeling is one of the most important tools for the study of processes in electromechanics and the electric drive. In addition to writing adequate equations of the object of research, are important convenient and efficient implementation of mathematical models as a tool for computer calculations and simulations, as a tool for further work of engineers and scientists in the field of analysis and synthesis of the operation modes of electrical equipment. There are many proposed mathematical models and even more of their computer implementations are developed. There are examples of published detailed descriptions of computer models of induction motors (IM): [1, 2].

The author recommends reading this article in conjunction with [3]. The mathematical model of three-phase IM suggested in [3] is based on stator reference frame coordinate system axis \( \alpha, \beta, \gamma \), which are aligned with stator phase axes A, B, C. This mathematical model contains iron losses resistances \( r_\mu \) included in magnetization circuits in each phase of IM in parallel with magnetizing inductance \( L_\mu \) [4] (analogy with the T-shaped equivalent circuit of an asynchronous electric machine). This was adopted by the usual generalized electric machine assumptions: each of the phase stator windings creates in a smooth air gap the sinusoidal-allocated magnetomotive force; magnetic saturation coefficient is constant. In the expressions of the mathematical model, the following notation is adopted: \( v \) - voltage; \( i \) - current; \( t \) - time; \( r \) - resistance; \( \Psi \) - magnetic flux linkage; \( \omega_R \) - mechanical angular rotor speed, \( p \) - the number of pole pairs. Lower indexes \( \alpha, \beta, \gamma \) indicate the affiliation to the appropriate phase. Subscript \( s \) indicates the affiliation to the stator, index \( r \) belongs to the rotor, and index \( \mu \) belongs to the magnetization branch. \( L_{\alpha\sigma} \) - leakage inductance of the stator phase; \( L_{\sigma\alpha} \) - leakage
2. Requirements for computer model
In the process of the computer model of the IM according to equations (1) – (12) [3], development using OrCAD – computer-aided design system, intended primarily for the design and simulation of electronic and electrical devices [5], there was the task of ensuring the embedded model of IM in an electrical power supply circuit, including converters, by simple connection of virtual terminals. That is, relative to the model of the electrical circuit means that is attached to this circuit, the IM model should also have the properties of electric circuits: one can apply a potential difference to the terminals, providing a bidirectional electric current in the connecting circuits, including a pass-through current between them. Another task is the formation of such structure of the computer model which will be used perhaps as a universal template, which records the values of variables (parameters of IM), asked simultaneously for all equations. The resulting computer model is suitable for the description of both the squirrel-cage and wound rotor IM in any of four quadrants. The winding phases can be connected in schemes triangle, wye, independently joined with each of its voltage to have any other wiring of each other or of power source. The developed computer model gives the user access to the beginning and the end of a winding of each of the three phases of the stator and the rotor. This property is particularly important when considering the asymmetric modes of operation or when powered by the special circuitry of semiconductor converters [6].

3. The implementation of a computer model
The tasks solved by combining two approaches in the IM computer model development: circuit for electrical parts and the method of block diagrams for the magnetic and mechanical parts. The computer model of IM is packed into a hierarchical block with a specified list of variables (parameters of IM). Within a single OrCAD project, using the copy operation, one can get the required number of hierarchical blocks (models of IM), for each of which it is possible to set unique parameter values. Sensors and signal inputs in the IM model used controlled sources of currents and voltages. Such voltage source is controlled by current (VSCC), voltage source is controlled by voltage (VSCV) and current source is controlled by voltage (CSCV). A graphical image of the computer model of the phase A windings of the stator and the rotor of IM, composed of (1) to (7) [3] is shown in Figure 1.

VSCC1 performs a function of the phase current of the stator \(i_{sa}\) sensor, similarly VSCC2 – sensor of rotor phase current \(i_{ra}\) given to the stator winding. CSCV1 is used for entering \(i_{ra}\) into the scheme. VSCV1 is introduced into the phase magnetizing circuit \(v_{qa}\) voltage. A similar function is performed by VSCV2 – entering into the circuit of the rotor winding, the difference is between the electromotive force of a branch of magnetization and rotation electromotive force, i.e., the voltage drop value. Active resistances \(r_{serv}\) (see Figure 1) have a large value, for example 10 \(M\Omega\). These resistors introduced for service purposes. Without affecting to numerical calculation results, they stabilize the solution (simulation) by maintaining the current circuit (the physical sense – the way for leakage current), which is especially important when a discrete change of the resistance of a IM circuit occurs, for example, when powered from the frequency converter or in case of the phase circuit breaking. A similar solution is described in [7].

The rest of the equations of three-phase IM mathematical model is implemented by the author in the computer model in the form of block diagrams [8]. Figure 2 shows part of the model, where one obtains instantaneous value of \(L_{\mu}\) - complete phase inductance of the stator winding from the main
magnetic flux in relative units. $\Psi_{\mu \Sigma m}$ is the rated value of $\Psi_{\mu \Sigma m}$ - the instantaneous value of the amplitude of the representing vector of the flux linkage of mutual induction.

![Graphical Image](image)

**Figure 1.** The graphical image of the computer model of the phase A windings of the stator and the rotor of IM

Signal $\Psi_{\mu \Sigma m}$ can be calculated as:

$$\Psi_{\mu \Sigma m} = \sqrt{\psi^2_{\mu x} + \psi^2_{\mu y}}, \quad (1)$$

where $\Psi_{\mu x}$ and $\Psi_{\mu y}$ are the projections of the representing vector of the flux linkage of mutual induction on orthogonal coordinate axes X and Y. In the case of three-phase stator reference frame of the coordinate system axis, these projections are derived from mutual induction flux linkages of each phase.

$$\Psi_{\mu x} = \frac{2}{3} \left( \Psi_{\mu a} + \Psi_{\mu b} \cos \left( -\frac{2\pi}{3} \right) + \Psi_{\mu c} \cos \left( \frac{2\pi}{3} \right) \right); \quad (2)$$

$$\Psi_{\mu y} = -\frac{2}{3} \left( \Psi_{\mu b} \sin \left( -\frac{2\pi}{3} \right) + \Psi_{\mu c} \sin \left( \frac{2\pi}{3} \right) \right). \quad (3)$$
Figure 2. The block diagram for obtaining instantaneous values of complete phase inductance of the stator winding from the main magnetic flux in relative units $L_{\mu}^*$.

In Figure 3 there is the model part corresponding to above mentioned (1) – (3) equations. Figure 4 shows a part of the IM model, designed to determine the rotation electromotive force in each phase. Model rotor speed $\omega_{r, \text{model}}$ in $p$ times is higher than the real mechanical speed of rotor $\omega_r$. Figure 5a shows a part of the IM model corresponding to equations (2) [3]. Figure 5b shows a part of the model, which forms the signals of electromagnetic torque $T_{em}$, angular speed $\omega_r$, and the angle of rotation $\Theta_r$ of the rotor. In Figure 5b, $n_r$ - mechanical angular rotor speed, rotation per minute.

Figure 3. The block diagram that implements the calculation of the instantaneous value of the amplitude of the representing vector of the flux linkage of mutual induction.

Figure 4. Part of the IM model, designed to determine the rotation electromotive force in each phase.
4. Conclusion
The proposed mathematical model of three-phase IM and the method of its computer implementation confirmed their adequacy and effectiveness to address some of the problems of analysis and design of electrotechnical complexes and systems, having IM in its composition [3].

Figure 5. Block diagrams: a – corresponding to equations (2) [3]; b – which forms the signals of electromagnetic torque $T_{em}$, angular speed $\omega_r$ and the angle of rotation of rotor $\Theta_r$. 
In conclusion, the author would like to identify ways of improving the suggested mathematical model of three-phase IM [3]. Priority for criticism might be the lack in the mathematical model accounting for the skin effect in the conductors of the rotor winding. In fact, this is a notable disadvantage since in some cases, the current displacement in the rotor is able to accelerate the transitional process of IM start-up in several times [9, 10].

There are two types of problems whose solution requires a mathematical model accounting for the skin effect in the conductors of the IM rotor winding: the correct description of transient processes at direct start-up and correct account of the losses in the rotor winding in case of feeding from the source of non-sinusoidal voltage or current. When deciding tasks of the first type, the author sometimes puts active resistance and leakage inductive resistance of the rotor, which change in dependence on the slip. The solution of tasks of the second type requires the coordination of the complex resistance components of the rotor values to current frequency in it. The second approach is more versatile as it can be used for solution of both types of abovementioned tasks. According to the author, to implement the second approach, the best way is to create the computer implementation of the mathematical model of three-phase IM in the original coordinates.

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