Geometric modelling of the contact point between the bushing and sprocket in chain drives

R Saulescu, R Velicu and M Lates
Product Design, Mechatronics and Environment Department, Transilvania University of Brasov, 29 Eroilor Blvd, 500036, Brasov, Romania
E-mail: rsaulescu@unitbv.ro

Abstract. An important problem of the bush chains dynamics is represented by the calculus of the normal and transversal forces on all the contacts; these forces are producing vibrations in the chain and due to this, the chain is affected by the wear. One aspect of that dynamics is referring directly on the sprockets geometry and on the bushing and sprocket contact. The paper presents a calculus method for the contact angle between the bushing and the sprocket; this angle is a variable one depending on the bushing’s number being in contact (i) and on the specific elongation of the chain (x) due to the functioning of it. Based on the presented calculus model, a comparative analysis is proposed for these factors by using sprockets with different teeth numbers and different specific elongations of the chain. The results of the numerical simulations allow the dissemination of recommendations regarding the contact angle’s evolution, from the beginning to the end of the contact and regarding the influence of the chain’s specific elongations on the out of use of it.

1. Introduction
The bush chains are mainly used when the transmission of the high torques with a constant transmission ratio is needed, in the case that due to the big distances between the axes, the use of the gears is not suitable; the main disadvantage in this case is represented by the specific elongation of the chain.

In [1] is concluded that the chain’s elongation is limiting the physical functioning capacity of the chain. In this way, there are needed improvement criteria for the chain drives physical and mechanical characteristics, in particular for the geometry of the friction surfaces by considering the rotations of the bushings in the entering moment of the contact with the sprocket.

The reference [2] is studying the bushings friction in chains and is concluding that the measuring of the wear is hard due to the complex geometry of the surfaces; this wear is influencing the geometry and the configuration of the component elements of the chain.

The paper [3] proposes a new geometry design of a sprocket with the teeth number z=17 instead of the usual case of z=25, by considering some hypotheses as: the calculus are not taking into account the tolerances for the sprockets dimensions; the calculus are no taking into account the limiting misalignments of the chain links lengths; the chain elongation is made by constant pitch. These hypotheses are making possible to set up a new solution for the sprockets profile; this new profile increases the life time and the acoustics due to the vibrations of the transmission. The new profile allows the parametric optimization of the sprocket by investigating the geometry of it.
In [4] are proposed two solutions regarding the modelling of the contact between the bushing and the sprocket. These two solutions are hard to be developed and are referring on a real tooth profile and a circular one. The case study is describing a chain drive transmission used in a diesel engine from a boat. The paper’s conclusion is that the action of the polygonal effect of the transmission in concordance with the impact between the bushing and the sprocket make impossible to avoid the longitudinal and transversal vibrations, so the chain’s flexibility has an important influence. These effects may be decreased by studying the influence of the chain’s elongation on the transmission functioning and on the bushing and sprocket contact.

A graphic analysis of the bushing – sprocket contact is presented in [5]; there are studied two profiles for the sprockets – one from the American standard and one from the European standard.

The sprockets geometry and the elongation of the chains are representing a starting point in order to establish the contact between the bushing and the sprocket which means a right calculus for the contact static forces.

2. Problem formulation

A realistic model of the contact forces is achieved by assuming a right geometric model of the contact angles between the sprocket and the bushing.

The paper presents the evolution of the contact angle ($\alpha$) which is influencing the contact forces; this evolution is correlated to the chains pitch deviation (a maximum deviation of 0.2 % from the standard value). This deviation is produced mainly by the chains elongation due to the wear which appears during its running.

The evolution of the contact angle for the gearing between the chain and the bushing is analyzed by considering the sprockets with different teeth numbers ($z=16, 24, 96$), with the transmission ratio equal with 1; it is recommended that the first position of the chain on the sprocket should be considered the vertical contact between the bushing and the chain ($i=0$) due to the reason that the contact angle is $\alpha = 0$ and in this point the tangential forces are equal with zero. This evolution is useful in order to identify the maximum contact angle which appears at the last bushing which is coming out from the contact with the chain (approximatively one quarter of sprocket).

In the study presented in the paper, it has been chosen a standardized chain with bushings and small links; for this chain there are known the sprockets teeth numbers ($z$), the pitch of the chain ($p$) and the diameter of the bushing ($d_b$).

3. Modeling the contact angle

The aim of this modelling is to find out the contact angle between the sprocket and the bushing, according to the sprocket’s geometry. In order to achieve this modelling there are considered as known the following parameters: the chain’s pitch ($p$), the sprocket’s teeth number ($z$) and the diameter of the bushing ($d_b$); it is recommended to use an even number of teeth and links for the chain.

According to these parameters the sprocket’s geometry can be determined [6]; in order to determine the contact angle it is necessary to know the following geometrical parameters: the radius of the sprocket’s pitch circle ($R_A$), the radius of the roller’s hole of the sprocket ($r_A$), the radius of the bushing ($r_B$) and the sprocket’s angular pitch ($\tau$) [5].

According to figure 1, the eccentricity ($A_0B_0$) between the center of the sprocket’s foot circle ($A_0$) and the bushing’s center ($B_0$) it may be written as:

$$A_0B_0 = r_A - r_B,$$

where $A_0B_0 = A_1B_1 = A_2B_2 = \ldots = A_kB_k$

The longed pitch $B_iB_{i+1}$ is determined by considering that during its running the chain has an elongation with a constant length ($x$) through its entire length, relative to the chain’s pitch ($B_0B_{01}$):

$$B_iB_{i+1} = B_0B_1 = B_0B_{01} + x = p + x$$
where \( B_0B_1 = B_1B_2 = B_2B_3 = \ldots = B_kB_{k+1} = B_{k+1}B_1 \)

In order to establish the longed pitch it is used the figure 1, which is a deformed one according to the real case due to the reason that it should be seen the difference between the standard pitch and the longed one, for two types of deformations: \( x_1 \) and \( x_2 \), where \( x_2 > x_1 \).

**Figure 1.** The sketch used to find out the main geometrical parameters of the chain drive and specific elongation of the chain.

The geometrical parameters are determined by considering the initial mounting position where the contact angle \( (\alpha_0 = 0) \) (see figure 1) and the first contact between the bushing and the sprocket \((i=1)\) is noted with the index 1.

According to figure 1 the next mathematical model used to determine the contact angle can be proposed. This model is based on the geometrical parameters which have been obtained in the mounting position (the initial position) and by considering, step by step, each contact. The values of the current contact angle are depending on the previous contacts where \( i \) is considered as the number of the contact between the bushing and the chain; for \( i=0 \), in the mounting position, \( \alpha_0 = 0, \lambda_0 = 0 \) and \( OB_0 = R_A - r_B + r_A \):

\[
B_{i-1}A_i = \sqrt{OB_{i-1}^2 + OA_i^2 - 2OB_{i-1}OA_i \cos(\tau - \lambda_{i-1})} \tag{3}
\]

\[
\delta_i = \arccos \frac{B_{i-1}A_i^2 + OA_i^2 - OB_{i-1}^2}{2B_{i-1}A_i \cdot OA_i} \tag{4}
\]

\[
\psi_i = \arccos \frac{B_{i-1}A_i^2 + A_iB_i^2 - B_{i-1}B_i^2}{2B_{i-1}A_i \cdot A_iB_i} \tag{5}
\]

\[
\alpha_i = \psi_i - \delta_i \tag{6}
\]

\[
OB_i = \sqrt{OA_i^2 + A_iB_i^2 - 2OA_i \cdot A_iB_i \cos \alpha_i} \tag{7}
\]

\[
\lambda_i = \arcsin \frac{A_iB_i \sin \alpha_i}{OB_i} \tag{8}
\]

where \( OA_i = R_A \),
By using the algorithm from above, in the next step, the influence of the specific chain’s elongation is determined – figure 2a; this influence is determined by considering a variation of the chain’s specific elongation starting from 0.2% from the chain’s pitch and ending with a value four times bigger. If the value of the contact angle ($\alpha$) is bigger than the value of the sprocket’s roller’s hole angle ($\phi$) (figure 2b) the bushing is coming out from the contact with the radius of the roller’s hole of the sprocket ($r_A$) and due to that the previous proposed mathematical model is no longer useful.

Figure 2. Bushing chain transmission: the influence of the chain’s specific elongation on the contact angle (a) and a detail of a bushing-sprocket contact by considering the rolling hole’s angle (b).

4. Case study

The aims of this paper are referring on determining the influence of the specific elongation ($x$) on the contact angle ($\alpha$); this influence is established by varying the sprocket’s teeth number in the hypothesis of the transmission ratio equal with one. In the case study it is proposed a standard chain from STAS 3006-80 which has the pitch $p = 9.525$ mm and the bushing diameter which is in contact with the chain $d_B = 5.08$ mm. By considering $z=16$ there are established the sprocket’s profile geometrical parameters used in the analytical modelling of the contact angle [7, 8]:

- The angular pitch $\tau = \frac{360}{z} = 22.5$ grd
- The radius of the rolling’s hole $r_A = 0.505 \cdot d_B + 0.069 \frac{d_B}{3} = 2.68$ mm
- The radius of the pitch circle $R_A = \frac{1}{2} p \left( \sin \frac{\tau}{2} \right)^{-1} - d_B + r_A = 24.55$ mm
- The radius of the bushing $r_B = \frac{d_B}{2} = 2.54$ mm

The three radiuses from above are replaced in the proposed algorithm from before and, according to this, the diagrams with the variation of the contact angle ($\alpha_i$) are obtained; the diagrams show the variation of the contact angle depending on the variation of the specific elongation ($x$), starting with the allowable value (2 % from the chain’s pitch) and ending with the value of 8 % from the pitch. In order to establish the elongation’s influence ($x$) on the chain drive transmission, in the algorithm is modified the value of the elongation:
Figure 3. The influence of the specific elongation on the contact angle for $z=16$.

Generally, in the chain drive transmissions the first four contact pairs are influencing the transmission’s functionality; due to this there are varied the sprocket’s teeth number for the first four contacts between the bushing and the sprocket (see figure 4), for different specific elongations:
Figure 4. The influence of the specific elongation on the contact angle, for different teeth numbers; elongation (% from the chain’s pitch) is: 0.2 (a), 0.3 (b), 0.4 (c), 0.5 (d), 0.6 (e) and 0.8 (f).

5. Results and discussions
The main criteria which represents the basics of the chain’s functioning safety is the wear behavior because the wear produces the chain’s elongation and, as consequence, a bad functioning with supplementary vibrations and noises. Due to this the specific elongation is one of the factors which is influencing the right functioning of the chain drive and is determine the out of use of it. This elongation is allowable up to 0.225 % for the weak resistance chains and up to 0.28 % \[9\] from the nominal pitch, for the other chains. Due to the specific elongation of the chain, the contact point between the bushing and the sprocket is changing its position so, the contact angle is modifying its value and that is influencing directly the transmission’s dynamics, due to the variation of the normal and transversal forces which are inducing vibrations and wear.

In the paper there are analyzed the influence of the chain’s specific elongation \(x\), according to a proposed mathematical model which allows the determining of the contact angle between the sprocket and the bushing \(\alpha\), depending on the chain’s elongation and the teeth number.

The geometry of the sprocket is based on a standardized bushing chain which has the pitch \(p= 9.525\) [mm] and the diameter of the bushing \(d_B=5.08\) [mm]. For these parameters the sprocket’s geometry is depending directly on the sprocket’s teeth number; these values are presented in the table 1:

| \(z\) | 16 | 24 | 96 |
|---|---|---|---|
| \(R_A\) mm | 24.55 | 36.63 | 145.7 |
| \(r_A\) mm | 2.68 | 2.68 | 2.68 |
| \(r_B\) mm | 2.54 | 2.54 | 2.54 |

According to figure 3 it can be observed that by increasing the number of the bushings in contact \((i)\) and by increasing the specific elongation of the chain \((x)\), the contact angle is increasing, starting with the initial mounting position (see figures 1 and 2). The contacts between the bushings and the sprocket are decreasing with the increasing of the chain’s elongation; in this case the contact is achieved between the top of the sprocket and the bushing and due to that, the vibrations and as consequence, the wear are appearing; as example: \(z=16\) for \(x=0.3\%\) from the nominal pitch there will be in contact 6 bushings \((i=6)\); for \(x=0.8\%\) from \(p\) it could be in contact one tooth \((i=1)\), as exception considering the contact between the bushing and the chain in the initial position \(i=0\).
According to figure 4, the transmission’s behavior is influenced mainly by the specific elongation of the chain. The number of bushings being in contact with the sprocket \((i)\), by considering the increasing of the teeth number \((z)\), is decreasing with the increasing of the specific elongation \((x)\); as example: for \(x=0.5\%\) from the nominal pitch and for \(z=24\) there are 3 bushings in contact with the chain; for \(z=96\) there are 2 in contact with the chain (figure 4d).

By concluding, it can be observed that a right functioning of the chain (the chain’s life time) is depending on the specific elongation of the chain which is influencing the increasing of the wear. By increasing the specific elongation of the chain \((x)\), the contact angle \((\alpha)\) is increasing and the bushing number being in contact \((i)\) is decreasing, so the bushings / the chain will coming out from the gearing with the chain. The obtained diagrams represent a starting point to determining the contact angle; in this way, the contact point between the bushing and the sprocket is established and this allows the dynamics study of the transmission and of the friction loses in the chain with small links and bushings.

6. Conclusions
As it is known, the bushing chain drives have a longer life time due to the rolling contact between the sprocket and the bushing than the other chain drives; the rolling motion is produced due to the radius of the bushing which is smaller than the radius of sprocket’s foot and due to the pitch of the sprocket which is smaller than the chain’s nominal pitch. By comparing the results obtained in the paper it can be concluded that with the increasing of the chain’s elongation, the contact angle is increasing which has as consequence frictions on bigger surfaces between the bushing and the sprocket; another consequence could be the out coming of the chain from the gearing which produces higher vibrations in the transmission.

The paper is analyzing the influence of the specific elongation of the chain on the number of the bushings being in contact based on a generalized mathematical model used to determine the contact angle.

The wear which appears in a chain drive transmission is produced due to the chain’s elongation which is influencing the contact point between the bushing and the chain, so the position of the transversal forces during the functioning of the chain. The determination of these parameters is useful in the optimizing process of the bushing chain drives by considering supplementary criteria with directly connections to minimizing the transversal vibrations and the friction.

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