Noether gauge symmetry for the Bianchi type I model in $f(T)$ gravity

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Abstract
In this paper, we present the Noether symmetries of a class of the Bianchi type I anisotropic model in the context of $f(T)$ gravity. By solving the system of equations obtained from the Noether symmetry condition, we obtain the form of $f(T)$ as a teleparallel form. This analysis shows that teleparallel gravity has the maximum number of Noether symmetries. We derive the symmetry generators and show that there are five kinds of symmetries, including time and scale invariance under metric coefficients. We classify the symmetries and we obtain the corresponding invariants.

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1. Introduction

In the last decade, one of the big challenges for research in the physicist community is an explanation of the essence and mechanism of the acceleration of our universe [1] in this era of the universe, which has been confirmed by some observation data such as supernova type Ia [2], baryon acoustic oscillations [3], weak lensing [4] and the large scale structure [5]. Finding the phenomenological explanation of cosmic acceleration has been one of the main problems of cosmology and high energy theoretical physics [6]. Reviews of some recent and old attempts to resolve the issue of dark energy and its related problems can be found in [7].

In order to explain the current accelerated expansion without introducing dark energy, one may use a simple generalized version of the so-called teleparallel gravity [8], namely $f(T)$ theory. Although teleparallel gravity is not an alternative to general relativity (they are dynamically equivalent), its different formulation allows one to say: gravity is not due to curvature, but to torsion. In other words, using the curvature-less Weitzenböck connection instead of the torsion-less Levi-Civita connection in standard general relativity leads to, subsequently, replacing curvature by torsion. We should note that one of the main requirements of $f(T)$ gravity is that a class of spin-less connection frames exist where its torsion does not vanish [9]. Considering the above, $f(T)$ theory leads to interesting cosmological behavior whose various aspects have been examined in the literature [10].

On the other hand, we could not ignore the important role of continuous symmetry in mathematical physics. In particular, the well-known Noether symmetry theorem is a practical tool in theoretical physics, which states that any differentiable symmetry of an action of a physical system leads to a corresponding conserved quantity, the so-called Noether charge [11]. In the literature, applications of Noether’s symmetry in generalized theories of gravity have been studied (see [12] and references therein). In particular, Wei et al [12] calculated Noether symmetries of $f(T)$ cosmology containing matter and found a power-law solution $f(T) \sim \mu T^n$. Further, they showed that if $n > 3/2$, the expansion of our universe can be accelerated without invoking dark energy. We reconsider their model in the frame of anisotropic Bianchi type I models and find the corresponding symmetries. Noether gauge symmetry is also applied to $f(T)$ gravity minimally coupled with a canonical scalar field to determine the unknown functions of the theory: $f(T)$, $V(\phi)$, $W(\phi)$; it is shown that the behavior of the hubble parameter in the model closely matches the $\Lambda$CDM model [13]. A detail discussion on Noether symmetries and the conserved quantities is given in [14]. An investigation of the Noether symmetries of $f(T)$ cosmology involving...
matter and dark energy, where dark energy is represented by a canonical scalar field with potential and it is shown that $f(T) \sim T^\frac{1}{2}$ and $V(\phi) \sim \phi^2$ [15].

To explain the accelerated expansion of the universe, $f(T)$ gravity is considered with cold dark matter and Noether symmetries are utilized to study the cosmological consequences [16]. Recently we applied the Noether gauge symmetry in Bianchi type I models in some generalized scalar field models [17]. Further, there are some exact solutions for Bianchi type I models in $f(T)$ gravity [18]. In this paper we want to find the symmetries of a Bianchi type I spacetime filled with perfect fluid with barotropic EoS using the Noether gauge symmetry approach.

The plan of our letter is as follows. In section 2, we review the basics of $f(T)$ gravity. In section 3, we write down the Lagrangian of our model and the related dynamical equations. In section 4, we present the Noether symmetries and invariants of our model. Finally, we conclude the results in section 5.

2. Basics of $f(T)$ gravity

General relativity is a gauge theory of the gravitational field; it is based on the equivalence principle. However it is not necessary to work with Riemannian manifolds. There are some extended theories such as Riemann–Cartan; in them the geometrical structure of the theory is not the metric. In these extensions, there is more than one dynamical quantity (metric). For example, this theory may be constructed from the metric, non-metricity and torsion [20]. Ignoring the non-metricity of the theory, we can leave the Riemannian manifold and go to the Weitzenbock spacetime, with its torsion and zero local Riemann tensor. One sample of such a theory is called teleparallel gravity in which we are working in a non-Riemannian manifold. The dynamics of the metric is determined using the scalar torsion $T$. The fundamental quantities in teleparallel theory is the vierbein (tetrade) basis $e_i^\mu$. This basis is an orthogonal, coordinate free basis, defined by the following equation:

$$g_{\mu \nu} = e_i^\mu e_i^\nu.$$  

This tetrade basis must be orthognormal; $e_i^\mu e_i^\nu = \delta_i^\nu$. There is a simple extension of the teleparallel gravity, called $f(T)$ gravity, where $f$ is an arbitrary function of the torsion $T$. One suitable form of action for $f(T)$ gravity in the Weitzenbock manifold is [21]

$$S = \frac{1}{2\kappa^2} \int d^4x \sqrt{-e}(T + f(T) + \mathcal{L}_m).$$

Here $e = \det(e_\mu^i)$, $\kappa^2 = 8\pi G$. The dynamical quantity of the model is the scalar torsion $T$ and $\mathcal{L}_m$ is the matter Lagrangian. The field equation can be derived from the action by varying the action with respect to $e_i^\mu$, which reads

$$e_i^\mu \delta_{\mu} (\mathcal{L}_m) (1 + f_T) - e_i^{\mu} T^\mu_\rho S_{\rho \mu} f_T + S_{\mu \nu} (T) f_T - \frac{1}{4} e_i^{(1 + f(T))} = 4\pi G e_i^\rho T^\rho_\mu,$$

where $T^\rho_\mu$ is the energy–momentum tensor for matter sector of the Lagrangian $\mathcal{L}_m$, defined by

$$T_{\mu \nu} = - \frac{2}{\sqrt{-g}} \frac{\delta \int (\sqrt{-g} \mathcal{L}_m d^4x)}{\delta g^{\mu \nu}}.$$  

Here $T$ is defined by

$$T = S_{\rho}^{\mu \nu} T^\rho_\mu,$$

where

$$T^\mu_\nu = e_i^\nu (\partial_\mu e_i^\nu - \partial_\nu e_i^\mu),$$

$$S_{\rho}^{\mu \nu} = \frac{1}{2}(K^{\mu \nu} + \delta^\mu_\rho T^\theta_\nu - \delta^\nu_\theta T^\theta_\mu)$$

and the contorsion tensor reads $K^{\mu \nu}$ as

$$K^{\mu \nu} = -\frac{1}{2}(T^\nu_\rho - T^\rho_\nu - T^\mu_\nu).$$

It is straightforward to show that this equation of motion reduces to the Einstein gravity when $f(T) = 0$; indeed, it is the equivalency between the teleparallel theory and Einstein gravity (Hayashi and Shirafuji [8]). The theory has been found to address the issue of cosmic acceleration in the early and late evolution of the universe [22] but this crucially depends on the choice of suitable $f(T)$, for instance an exponential form containing $T$ cannot lead to phantom crossing (Wu and Yu [10]). The reconstruction of $f(T)$ models has been reported in [23] while the thermodynamics of $f(T)$ cosmology, including a generalized second law of thermodynamics, has recently been investigated [24].

3. The model

We start from the standard gravitational action (chosen units are $c = 8\pi G = 1$) [18]

$$S = \int d^4x \sqrt{-g} \left[ T + f(T) - (1 + f_T(T)) \right] \left[ T + 2 \left( \frac{\dot{A}B}{AB} + \frac{\dot{A}C}{AC} + \frac{\dot{C}B}{CB} \right) + \mathcal{L}_m \right].$$

We consider the Bianchi-I metric

$$ds^2 = -dt^2 + A^2 dx_2 + B^2 dy_2 + C^2 dz^2.$$  

Here the metric potentials $A$, $B$ and $C$ are functions of $t$ alone.

We assume that the spacetime is filled with a perfect fluid. The Lagrangian of Bianchi type-I model can be written as

$$\mathcal{L}(A, B, C, T, \dot{A}, \dot{B}, \dot{C}) = ABC \left[ T + f(T) - (1 + f_T) \right] \left[ T + 2 \left( \frac{\dot{A}B}{AB} + \frac{\dot{A}C}{AC} + \frac{\dot{C}B}{CB} \right) + \mathcal{L}_m \right].$$

Note that the Lagrangian does not depend on $\dot{T}$. We must emphasize here that when the global geometry of spacetime remains homogeneous, for both isotropic and anisotropic cases, there exists an isothermal process for
tending the whole system to the maximum entropy. Via such an isothermal process, it is possible to define a global (not local) average pressure. In an expanding universe, we define an average Hubble parameter by $H = \frac{1}{3} \Sigma_i H_i$, where $H_i = \frac{\dot{a}_i}{a_i}$. Such an average Hubble parameter leads to the effective global pressure of the anisotropic spacetime [25].

The equations of motion corresponding to Lagrangian (3) are

$$f_{TT} \left[ T + 2 \left( \frac{AB}{AB} + \frac{AC}{AC} + \frac{CB}{CB} \right) \right] = 0,$$  \hspace{1cm} (4)

$$2T f_{TT} \left( \frac{B}{B} + \frac{C}{C} \right) + 2(1 + f_T) \left( \frac{B}{B} \right) + 2 \frac{B}{B} \frac{C}{C} = 2(1 + f_T) \frac{B}{B} \frac{C}{C} - (f - T f_T) - p_1,$$  \hspace{1cm} (5)

$$2T f_{TT} \left( \frac{A}{A} + \frac{C}{C} \right) + 2(1 + f_T) \left( \frac{A}{A} + \frac{C}{C} \right) = 2(1 + f_T) \frac{A}{A} - (f - T f_T) - p_2.$$  \hspace{1cm} (6)

$$2T f_{TT} \left( \frac{B}{B} + \frac{A}{A} \right) + 2(1 + f_T) \left( \frac{B}{B} \right) + 2 \frac{B}{B} \frac{A}{A} = 2(1 + f_T) \frac{B}{B} \frac{A}{A} - (f - T f_T) - p_3.$$  \hspace{1cm} (7)

From equation (4), there are two possibilities: (i) $f_{TT} = 0$, which indicates teleparallel gravity (we will be back to this case later) and (ii) the other possibility is

$$T = -2 \left( \frac{AB}{AB} + \frac{AC}{AC} + \frac{CB}{CB} \right),$$

which is the definition of scalar torsion for the Bianch I model. Further, it is easy to show that by applying $e^\mu_\nu = \text{diag}(1, A, B, C)$, we can obtain the definition of torsion scalar. By substituting $f = 0$ and $A = B = C = a$, we obtain the usual second Friedmann equation. We can redefine the three anisotropic pressures as the components of the energy–momentum tensor $T_{\mu\nu} = \text{diag}(\rho, -p_1, -p_2, -p_3)$.

Adding equations (5)–(7) yields

$$12T H f_{TT} + 2(1 + f_T) \left[ 3 \left( H + \sum_i H_i^2 \right) - T (1 + f_T) \right] - 3(f - T f_T) - P,$$

$$- 8HT f_{TT} - (4H - T) (4H - 2T) f_T - f = \rho.$$

The first Friedmann equation in this model is obtained by the Hamiltonian constraint equation [18]

$$- T - 2T f_T + f = \rho.$$  \hspace{1cm} (8)

In this paper, we will be working only with $p_1 = p_2 = p_3$ which comes from the definition of pressure $P$.

### 4. Noether gauge symmetry of the model

In this section, we carry out the Noether gauge symmetry equations for the matter Lagrangian $L_m = P = P_0 (ABC)^{1/(1+w)}$.

A vector field

$$X = \xi(t, A, B, C, T) \frac{\partial}{\partial t} + \eta^1(t, T, A, B, C) \frac{\partial}{\partial T} + \eta^2(t, T, A, B, C) \frac{\partial}{\partial A} + \eta^3(t, T, A, B, C) \frac{\partial}{\partial B} + \eta^4(t, T, A, B, C, T) \frac{\partial}{\partial C}$$

is a Noether gauge symmetry of the Lagrangian [19], if there exists a vector valued gauge function $G \in \mathcal{U}$, where $\mathcal{U}$ is the space of differential functions, such that

$$X^{[1]} L + L (D_t \xi) = D_t G.$$  \hspace{1cm} (9)

where $L$ is the Lagrangian (3), $D_t \xi$ represents the total derivative of $\xi$ and the coefficients $\xi, \eta^i, (i = 1, 2, 3, 4)$ are determined from the Noether symmetry conditions. Now, if one wants to apply the generator $X$ to the Lagrangian or a differential equation consisting of independent variables, the dependent variables and the derivatives of the dependent variables with respect to the independent variable up to the order $n$, then one has to prolong (or to extend) the generator $X$ up to $n$th derivative, called the $n$th order prolongation, so that one can have the action of the generator on all the derivatives. The space, whose coordinates represent the independent variables, the dependent variables and the derivatives of the dependent variables up to order $n$, is called the $n$th order jet space of the underlying space consisting of only the independent and dependent variables. Here the Lagrangian contains only the first order derivative of the dependent variables along with the independent variables and the dependent variables. So we need the first order prolongation of the above symmetry in the first-order jet space comprising of all derivatives, which is given by

$$X^{[1]} = X + \eta^1 \frac{\partial}{\partial T} + \eta^2 \frac{\partial}{\partial A} + \eta^3 \frac{\partial}{\partial B} + \eta^4 \frac{\partial}{\partial C},$$

where

$$\eta^1 = D_t \eta^1 - \dot{T} D_t \xi, \quad \eta^2 = D_t \eta^2 - \dot{A} D_t \xi, \quad \eta^3 = D_t \eta^3 - \dot{B} D_t \xi, \quad \eta^4 = D_t \eta^4 - \dot{C} D_t \xi,$$

and $D_t$ is the total derivative operator

$$D_t = \frac{\partial}{\partial t} + \dot{T} \frac{\partial}{\partial T} + \dot{A} \frac{\partial}{\partial A} + \dot{B} \frac{\partial}{\partial B} + \dot{C} \frac{\partial}{\partial C}.$$  \hspace{1cm} (10)

If $X$ is the Noether symmetry corresponding to the Lagrangian $L(t, T, A, B, C, T, A, B, C)$, then

$$I = \xi L + \left( (\eta^1 - \xi T) \frac{\partial L}{\partial T} + (\eta^2 - \xi A) \frac{\partial L}{\partial A} + (\eta^3 - \xi B) \frac{\partial L}{\partial B} + (\eta^4 - \xi C) \frac{\partial L}{\partial C} \right)$$

is a first integral, an invariant or a conserved quantity associated with $X$. The Noether symmetry condition (10)
yields the following system of linear partial differential equations (PDEs):

\[ G_T = 0, \quad \xi_T = 0, \quad \xi_x = 0, \quad \xi_y = 0, \quad \xi_z = 0, \quad f(T) \text{ becomes linear and the above system of linear PDEs gives extra symmetries, along with the above mentioned four symmetries, which are} \]

\[ X_5 = \frac{\partial}{\partial T} + C \frac{\partial}{\partial C}, \quad (28) \]

\[ X = g(t, T, A, B, C) \frac{\partial}{\partial T}, \quad (29) \]

where \( g(t, T, A, B, C) \) is an arbitrary function of its arguments. This shows that we have the maximum number of Noether symmetries for the linear case.

The invariants or conserved quantities corresponding to the symmetry generators are

\[ I_1 = 2(1+m)(\dot{A}BC + \dot{A}BC + \dot{A}BC) + P_0(ABC)^{-w}, \quad (30) \]

\[ I_2 = 2(1+m)(\dot{A}BC - \dot{A}BC), \quad (31) \]

\[ I_3 = 2(1+m)(ABC - \dot{ABC}), \quad (32) \]

\[ I_4 = 2(1+m) \left[ \dot{ABC} \ln \left( \frac{B}{C} \right) + \dot{ABC} \ln \left( \frac{C}{A} \right) + \dot{ABC} \ln \left( \frac{A}{B} \right) \right], \quad (33) \]

\[ I_5 = 2(1+m)(\dot{ABC} + \dot{ABC} + \dot{ABC}) + t P_0(ABC)^{-w} - 2(1+m)(\dot{ABC} + ABC). \quad (34) \]

From the previous remarks on the symmetry properties of the model, as \( X_1 = \frac{\partial}{\partial t} \) so \( I_1 \) is the total energy or Hamiltonian of the system, \( I_2 \) and \( I_3 \) characterize some conserved rotational momenta of the system and \( I_4 \) and \( I_5 \) are the rescaling momentum invariants.

For \( A = B = C \) the invariants \( I_1 \) to \( I_5 \) becomes

\[ I_1 = 6(1+m)(\dot{A} \dot{A}) + P_0(A)^{-3w}, \quad (35) \]

\[ I_2 = 0, \quad (36) \]

\[ I_3 = 0, \quad (37) \]

\[ I_4 = 0, \quad (38) \]

\[ I_5 = 6(1+m) t (\dot{A} \dot{A}) + t P_0(A)^{-3w} - 4(1+m)(A^2 \dot{A}). \quad (39) \]

The equations of motion for the Lagrange (3) corresponding to the case \( f(T) = mT \) are

\[ T + 2 \left( \frac{\dot{A} \dot{B}}{\dot{A} \dot{C}} + \frac{\dot{A} \dot{C}}{\dot{A} \dot{B}} + \frac{\dot{B} \dot{C}}{\dot{B} \dot{C}} \right) = 0. \quad (40) \]
2(1 + m) \left( \frac{A}{B} + \frac{C}{B} \right) - \frac{w P_0}{w P_0} (A B C)^{(w + 1)} = 0. \quad (41)

2(1 + m) \left( \frac{\dot{A}}{A} + \frac{\dot{C}}{C} \right) - \frac{w P_0}{w P_0} (A B C)^{(w + 1)} = 0. \quad (42)

2(1 + m) \left( \frac{B}{B} + \frac{2 A}{B} + \frac{A}{A} \right) - \frac{w P_0}{w P_0} (A B C)^{(w + 1)} = 0. \quad (43)

Now the total energy \( E_L \) corresponding to the \((0, 0)\)-Einstein equation is

\[
E_L = T \frac{\partial L}{\partial T} + A \frac{\partial L}{\partial A} + \xi B \frac{\partial L}{\partial B} + C \frac{\partial L}{\partial C} - L
\]

which is equal to \(-I_1\), so \(I_1 = 0\). \(I_2\) and \(I_3\) are conserved quantities so these must be constants, say \(I_1 = a\) and \(I_2 = b\), where \(a\) and \(b\) are constants.

These are the constraints which the equations of motion (40)–(43) must satisfy. These are sufficient to find the solution of the equations of motion. Under these constraints, we have the solution of the equations of motion (40)–(43) for \(E_m = 0\) as follows:

\[
A(t) = \frac{1}{-b + a} \left[ (c_1 + c_2)(b a - b^2 - a^2)ight. \\
\left. + (-b + 2a) \sqrt{a^2 - b a} + b^2 \right] \tag{44}
\]

\[
B(t) = c_1 \exp \int A(\dot{A}^2 b + 2 A \dot{A} b - 3 A \dot{A} a) / A a (A \dot{A} + 2 \dot{A}^2) \ dt,
\]

\[
C(t) = \frac{(9 A \dot{A} \dot{A} a^2 b - 5 A \dot{A} a^3 - 4 A \dot{A} b a^2 + 2 a^3 \dot{A}^2 - 2 b^2 a \dot{A}^2)}{Q}, \tag{46}
\]

\[
T(t) = 0, \tag{47}
\]

where

\[
Q = 40 B A A b a + 2 B A^3 b a + 2 B A^3 b a m - 40 B A A b a m - 16 B A A b a m - 8 B A^3 b^2 m - 8 B A^3 b^2 - 16 B A A b a^2 - 28 B A A a^2 + 4 B A^3 a^2 m + 4 B A^3 a^2
\]

and \(c_1\), \(c_2\) and \(c_3\) are arbitrary constants.

### 5. Conclusion

In this paper, we investigated the Noether gauge symmetries of \(f(T)\) in a homogeneous but anisotropic Bianchi type I background. We solved the gauge equations and classified the model according to its generators. We showed that there are five symmetries, in which \(X_1\) corresponds to the time invariance of the model, \(X_{2,3}\) define the rescaling translational invariance and results in the conserve momentum, \(X_4\) is a combination of rescaled translational symmetry and finally, \(X_5\) shows the contraction in time of the rescaled invariance of the whole model. The related invariance \(I_1\) is the invariant under time translation, i.e. the energy or Hamiltonian of the system, \(I_{2,3}\) characterize some conserve rotational momentums of the system, \(I_4\) is the rescaling momentum invariance and \(I_5\) is the generalized conserved but non-holonomic momentum of the system under symmetry \(X_5\).

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