GRAVITATIONAL MICROLENSING BY THE ELLIS WORMHOLE

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ABSTRACT
A method to calculate light curves of the gravitational microlensing of the Ellis wormhole is derived in the weak-field limit. In this limit, lensing by the wormhole produces one image outside the Einstein ring and another image inside. The weak-field hypothesis is a good approximation in Galactic lensing if the throat radius is less than 10^{11} km. The light curves calculated have gutters of approximately 4% immediately outside the Einstein ring crossing times. The magnification of the Ellis wormhole lensing is generally less than that of Schwarzschild lensing. The optical depths and event rates are calculated for the Galactic bulge and Large Magellanic Cloud fields according to bound and unbound hypotheses. If the wormholes have throat radii between 100 and 10^7 km, are bound to the galaxy, and have a number density that is approximately that of ordinary stars, detection can be achieved by reanalyzing past data. If the wormholes are unbound, detection using past data is impossible.

Key words: gravitational lensing: micro

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1. INTRODUCTION

A solution of the Einstein equation that connects distant points of space–time was introduced by Einstein & Rosen (1935). This “Einstein–Rosen bridge” was the first solution to later be referred to as a wormhole. Initially, this type of solution was just a trivial or teaching example of mathematical physics. However, Morris & Thorne (1988) proved that some wormholes are “traversable,” i.e., space and time travel can be achieved by passing through the wormholes. They also showed that the existence of a wormhole requires exotic matter that violates the null energy condition. Although they are very exotic, the existence of wormholes has not been ruled out in theory. Inspired by the Morris–Thorne paper, there have been a number of theoretical works (see Visser 1995; Lobo 2008, and references therein) on wormholes. The curious properties of wormholes, such as time travel, energy conditions, space–time foams, and growth of a wormhole in an accelerating universe, have been studied. Although there have been enthusiastic theoretical studies, studies searching for real evidence of the existence of wormholes are scarce. Only a few attempts have been made to show the existence or nonexistence of wormholes.

A possible observational method that has been proposed to detect or exclude the existence of wormholes is the application of optical gravitational lensing. The gravitational lensing of wormholes was pioneered by Cramer et al. (1995), who inferred that some wormholes show “negative-mass” lensing. They showed that the light curve of the negative-mass lensing event of a distant star has singular double peaks. Several authors subsequently conducted theoretical studies on detectability (Safonova et al. 2002; Bogdanov & Cherepashchuk 2008). Another gravitational lensing method employing gamma rays was proposed by Torres et al. (1998), who postulated that the singular negative-mass lensing of distant active galactic nuclei causes a sharp spike of gamma rays and may be observed as double-peaked gamma-ray bursts. They analyzed BASTE data and set a limit for the density of the negative-mass objects.

There have been several recent works (Shatskii 2004; Perlick 2004; Nandi et al. 2006; Rahaman et al. 2007; Dey & Sen 2008) on the gravitational lensing of wormholes as structures of space–time. Such studies are expected to unveil lensing properties directly from the space–time structure. One study (Dey & Sen 2008) calculated the deflection angle of light due to the Ellis wormhole, whose asymptotic mass at infinity is zero. The massless wormhole is particularly interesting because it is expected to have unique gravitational lensing effects. The Ellis wormhole is expressed by the line element

\[ ds^2 = dt^2 - (r^2 + a^2)(d\theta^2 + \sin^2(\theta)d\phi^2), \]

where \( a \) is the throat radius of the wormhole. This type of wormhole was first introduced by Ellis (1973) as a massless scalar field. Later, Morris & Thorne (1988) studied this wormhole and proved it to be traversable. The dynamical features were studied by Shinkai & Hayward (2002), who showed that Gaussian perturbation causes either explosion to an inflationary universe or collapse to a black hole. Das & Kar (2005) showed that the tachyon condensate can be a source for the Ellis geometry.

In this paper, we derive the light curve of lensing by the Ellis wormhole and discuss its detectability. In Section 2, we discuss gravitational lensing by the Ellis wormhole in the weak-field limit. The light curves of wormhole events are discussed in Section 3. The validity of the weak-field limit is discussed in Section 4. The optical depth and event rate are discussed in Section 5. The results are summarized in Section 6.
The deflection angle \( \alpha(r) \) of the Ellis wormhole was derived by Dey & Sen (2008) to be

\[
\alpha(r) = \pi \left\{ \sqrt{\frac{2(r^2 + a^2)}{2r^2 + a^2}} - 1 \right\},
\]

where \( r \) is the closest approach of the light. In the weak-field limit (\( r \to \infty \)), the deflection angle becomes

\[
\alpha(r) \to \pi \frac{a^2}{4r^2} - 5\pi \frac{a^4}{32r^4} + o\left(\frac{a^6}{r^6}\right).
\]

The angle between the lens (wormhole) and the source \( \beta \) can then be written as

\[
\beta = \frac{1}{D_L}b - \frac{D_{LS}}{D_S} \alpha(r),
\]

where \( D_L, D_S, D_{LS}, \) and \( b \) are the distances from the observer to the lens, from the observer to the source, and from the lens to the source, and the impact parameter of the light, respectively. In the asymptotic limit, Schwarzschild lensing and massive Janis–Newman–Winnicour (JNW) wormhole lensing (Dey & Sen 2008) have the same leading term of \( o(1/r) \). Therefore, the lensing property of the JNW wormhole is approximately the same as that of Schwarzschild lensing and is difficult to distinguish. As shown in Equation (3), the deflection angle of the Ellis wormhole does not have the term of \( o(1/r) \) and starts from \( o(1/r^2) \). This is due to the massless nature of the Ellis wormhole and indicates the possibility of observational discrimination from the ordinary gravitational lensing effect. In the weak-field limit, \( b \) is approximately equal to the closest approach \( r \). For the Ellis wormhole, \( b = \sqrt{r^2 + a^2} \to r (r \to \infty) \). We thus obtain

\[
\beta = \frac{r}{D_L} - \frac{\pi}{4} \frac{D_{LS} a^2}{D_S r^2} \quad (r > 0).
\]

The light passing through the other side of the lens may also form images. However, Equation (5) represents deflection in the wrong direction at \( r < 0 \). Thus, we must change the sign of the deflection angle:

\[
\beta = \frac{r}{D_L} + \frac{\pi}{4} \frac{D_{LS} a^2}{D_S r^2} \quad (r < 0).
\]

It is useful to note that a single equation is suitable both for \( r > 0 \) and \( r < 0 \) images in the Schwarzschild lensing. However, such treatment is applicable only when the deflection angle is an odd function of \( r \).

If the source and lens are completely aligned along the line of sight, the image is expected to be circular (an Einstein ring). The Einstein radius \( R_E \), which is defined as the radius of the circular image on the lens plane, is obtained from Equation (5) with \( \beta = 0 \) as

\[
R_E = \sqrt{\frac{\pi}{4} \frac{D_L D_{LS} a^2}{D_S}}.
\]

The image positions can then be calculated from

\[
\beta = \theta - \frac{\theta_E^3}{\theta^2} \quad (\theta > 0)
\]

and

\[
\beta = \theta + \frac{\theta_E^3}{\theta^2} \quad (\theta < 0),
\]

where \( \theta = b/D_L \approx r/D_L \) is the angle between the image and lens and \( \theta_E = R_E/D_L \) is the angular Einstein radius. Using reduced parameters \( \hat{\beta} = \beta/\theta_E \) and \( \hat{\theta} = \theta/\theta_E \), Equations (8) and (9) become simple cubic formulas:

\[
\hat{\theta}^3 - \hat{\beta} \hat{\theta}^2 - 1 = 0 \quad (\hat{\theta} > 0)
\]

and

\[
\hat{\theta}^3 - \hat{\beta} \hat{\theta}^2 + 1 = 0 \quad (\hat{\theta} < 0).
\]

As the discriminant of Equation (10) is \( -4\hat{\beta}^3 - 27 < 0 \), Equation (10) has two conjugate complex solutions and a real solution:

\[
\hat{\theta} = \frac{\hat{\beta}}{3} + U_{1+} + U_{1-},
\]

with

\[
U_{1\pm} = \frac{\sqrt{27}}{27} \left( 1 \mp \frac{1}{3} \sqrt{1 + \frac{27}{4} \left( \frac{2\hat{\beta}}{27} \right)^2 - \frac{4}{27}} \right).
\]
The real positive solution corresponds to the physical image. The discriminant of Equation (11) is \(4 \beta^3 - 27\). Thus, it has a real solution if \(\beta < \sqrt{27/4}\):

\[
\hat{\theta} = \frac{\beta}{3} + U_{2+} + U_{2-},
\]

where

\[
U_{2\pm} = \omega^3 \left[ \frac{\beta^3}{27} - \frac{1}{2} \pm \left( \frac{1}{4} \left( 1 - \frac{2 \beta^3}{27} \right)^2 \right) - \frac{\beta^6}{27^2} \right],
\]

with \(\omega = e^{(2\pi i)/3}\). This solution corresponds to a physical image inside the Einstein ring. For \(\beta > \sqrt{27/4}\), Equation (11) has three real solutions. However, two of them are not physical because they do not satisfy \(\hat{\theta} < 0\). Only the solution

\[
\hat{\theta} = \frac{\beta}{3} + \omega U_{2+} + U_{2-}
\]

Corresponds to a physical image inside the Einstein ring.

Figure 2 shows the calculated images for source stars at various positions on a straight line (source trajectory). The motion of the images is similar to that of Schwarzschild lensing. Table 1 shows the Einstein radii and angular Einstein radii for a bulge star (\(D_S = 8\) kpc and \(D_L = 4\) kpc are assumed) and a star in the Large Magellanic Cloud (LMC; \(D_S = 50\) kpc and \(D_L = 25\) kpc are assumed) for various throat radii. The detection of a lens for which the Einstein radius is smaller than the star radius (<10 km) is very difficult because most of the features of the gravitational lensing are smeared out by the finite-source effect. Thus, detecting a wormhole with a throat radius less than 1 km from the Galactic gravitational lensing of a star is very difficult.

### 3. LIGHT CURVES

The light curve of Schwarzschild lensing was derived by Paczyński (1986). The same method of derivation can be used for wormholes. The magnification of the brightness \(A\) is

\[
A = A_1 + A_2 = \left| \frac{\dot{\theta}_1 d\theta_1}{\beta d\beta} \right| + \left| \frac{\dot{\theta}_2 d\theta_2}{\beta d\beta} \right|,
\]

where \(A_1\) and \(A_2\) are the magnifications of the outer and inner images and \(\dot{\theta}_1\) and \(\dot{\theta}_2\) correspond to the outer and inner images, respectively. The relation between the lens and source trajectory in the sky is shown in Figure 3. The time dependence of \(\hat{\beta}\) is

\[
\hat{\beta}(t) = \sqrt{\hat{\beta}_0^2 + (t - t_0)^2 / t_E^2},
\]

where \(\hat{\beta}_0\) is the impact parameter of the source trajectory and \(t_0\) is the time of closest approach. \(t_E\) is the Einstein radius crossing time given by

\[
t_E = R_E / v_T,
\]

where \(v_T\) is the transverse velocity of the lens relative to the source and observer. The light curves obtained from Equations (18) and (19) are shown as thick red lines in Figure 4. The light curves corresponding to Schwarzschild lensing are shown as thin green lines for comparison. The magnifications by the Ellis wormhole are generally less than those of Schwarzschild lensing. The light curve of the Ellis wormhole for \(\beta_0 < 1.0\) shows characteristic gutters on both sides of the peak immediately outside the Einstein ring crossing times \((t = t_0 \pm t_E)\). The depth of the gutters is about 4% from the baseline. Amazingly, the star becomes fainter than normal in terms of apparent brightness in the gutters. This means that the Ellis wormhole lensing has off-center divergence.

### Table 1

| \(a (\text{km})\) | Bulge\(^a\) | LMC\(^b\) |
|-------------------|-------------|-------------|
| \(R_E (\text{km})\) | \(\theta_0 (\text{mas})\) | \(R_E (\text{km})\) | \(\theta_0 (\text{mas})\) |
| 1                 | 3.64 x 10\(^5\) | 0.001       | 6.71 x 10\(^5\) | <0.001 |
| 10                | 1.69 x 10\(^6\) | 0.003       | 3.12 x 10\(^6\) | 0.001  |
| \(10^2\)          | 7.85 x 10\(^6\) | 0.013       | 1.45 x 10\(^7\) | 0.004  |
| \(10^3\)          | 3.64 x 10\(^7\) | 0.061       | 6.71 x 10\(^7\) | 0.018  |
| \(10^4\)          | 1.69 x 10\(^8\) | 0.283       | 3.12 x 10\(^8\) | 0.083  |
| \(10^5\)          | 7.85 x 10\(^8\) | 1.31        | 1.45 x 10\(^9\) | 0.387  |
| \(10^6\)          | 3.64 x 10\(^9\) | 6.10        | 6.71 x 10\(^9\) | 1.80   |
| \(10^7\)          | 1.69 x 10\(^{10}\) | 28.3     | 3.12 x 10\(^{10}\) | 8.35   |
| \(10^8\)          | 7.85 x 10\(^{10}\) | 131       | 1.45 x 10\(^{11}\) | 38.7   |
| \(10^9\)          | 3.64 x 10\(^{11}\) | 610        | 6.71 x 10\(^{11}\) | 180    |
| \(10^{10}\)       | 1.69 x 10\(^{12}\) | 2832      | 3.12 x 10\(^{12}\) | 835    |
| \(10^{11}\)       | 7.85 x 10\(^{12}\) | 13,143     | 1.45 x 10\(^{13}\) | 3874   |

**Notes.** \(a\) is the throat radius of the wormhole, \(R_E\) is the Einstein radius, and \(\theta_0\) is the angular Einstein radius.

\(^a\) \(D_S = 8\) kpc and \(D_L = 4\) kpc are assumed.

\(^b\) \(D_S = 50\) kpc and \(D_L = 25\) kpc are assumed.
Figure 3. Sketch of the relation between the source trajectory and the lens (wormhole) in the sky. All quantities are normalized by the angular Einstein radius $\theta_E$.

In conventional gravitational lensing theory (Schneider et al. 1992), the convergence of light is expressed by a convolution of the surface mass density. Thus, we need to introduce negative mass to describe divergent lensing by the Ellis wormhole. However, negative mass is not a physical entity. As the lensing by the Ellis wormhole is convergent at the center, lensing at some other place must be divergent because the wormhole has zero asymptotic mass. For $\hat{\beta}_0 > 1.0$, the light curve of the wormhole has a basin at $t_0$ and no peak. Using these features, discrimination from Schwarzschild lensing can be achieved. Equations (7) and (20) indicate that the physical parameters ($D_L$, $a$, and $v_T$) are degenerate in $t_E$ and cannot be derived by fitting the light-curve data. This situation is the same as that for Schwarzschild lensing. To obtain or constrain these values, observations of the finite-source effect (Nemiroff & Wickramasinghe 1994) or parallax (Alcock et al. 1995) are necessary.

The detectability of the magnification of the star brightness depends on the timescale. The Einstein radius crossing time $t_E$ depends on the transverse velocity $v_T$. There is no reliable estimate of $v_T$ for wormholes. Here we assume that the velocity of the wormhole is approximately equal to the rotation velocity of stars ($v_T = 220 \text{ km s}^{-1}$) if it is bound to the Galaxy. If the wormhole is not bound to our Galaxy, the transverse velocity would be much higher. We assume $v_T = 5000 \text{ km s}^{-1}$ (Safonova et al. 2002) for the unbound wormhole. Table 2 shows the Einstein radius crossing times of the Ellis wormhole lendings for the Galactic bulge and LMC in both bound and unbound states.

Figure 4. Light curves for $\hat{\beta}_0 = 0.2$ (top left), $\hat{\beta}_0 = 0.5$ (top right), $\hat{\beta}_0 = 1.0$ (bottom left), and $\hat{\beta}_0 = 1.5$ (bottom right). Thick red lines are the light curves for wormholes. Thin green lines are corresponding light curves for Schwarzschild lenses. (A color version of this figure is available in the online journal.)
To find very long timescale events (which the timescale is less than one day) is difficult to detect. Observations are limited to once every few hours, an event for unbound scenarios. As the frequencies of current microlensing detections are small and decreases quickly with $\hat{\beta}$ ($\Lambda_2/A = 0.034$ for $\beta = 2$ and 0.013 for $\beta = 3$). On the other hand, the absolute value of the corresponding $\hat{\theta}$ does not decrease as quickly ($\hat{\theta} = -0.618$ for $\beta = 2$ and $-0.532$ for $\beta = 3$). Thus, the contribution of the higher-order effect of the second image to the total brightness is expected to be small.

Another possibility of deviation from the weak-field approximation is the contribution of relativistic images. Recently, gravitational lensing in the strong-field limit (Virbhadra & Ellis 2000) has been studied for lensing by black holes. In this limit, light rays are strongly bent and wound close to the photon sphere. As a result, a number of relativistic images appear around the photon sphere. However, it has been shown that there is no photon sphere (Dey & Sen 2008) in Ellis wormhole lensing. Therefore, there is no contribution of relativistic images to the magnification in Ellis wormhole lensing. We thus conclude that the weak-field hypothesis is a good approximation unless the throat radius is comparable to the galactic distance.

### 5. OPTICAL DEPTH AND EVENT RATE

The probability of a microlensing event to occur for a star is expressed by the optical depth $\tau$:

$$\tau = \pi \int_0^{D_L} n(D_L) R_E^2 dD_L,$$

where $n(D_L)$ is the number density of wormholes as a function of the line of sight. Here we simply assume that $n(D_L)$ is constant ($n(D_L) = n$):

$$\tau = \pi n \int_0^{D_L} \pi D_L^2 \frac{D_L (D_S - D_L)}{D_S} a^{2/3} \sqrt{x(1 - x)} dx$$

$$\approx 0.785 D_L^{5/3} n a^{4/3} D_S^{1/3}.$$

The event rate expected for a source star $\Gamma$ is calculated as

$$\Gamma = 2 \int_0^{D_L} n(D_L) R_E v_T dD_L,$$

$$\approx 0.978 n v_T D_L^{4/3} D_S^{1/3}.$$

There is no reliable prediction of the number density of wormholes. Several authors (Krasnikov 2000; Lobo 2008) have speculated that wormholes are very common in the universe, at least as abundant as stars. Even if we accept such speculation, there are still large uncertainties in the value of $n$ because the distribution of wormholes is not specified. Here, we introduce two possibilities. One is that wormholes are bound to the Galaxy and the number density is approximately equal to the local stellar density. The other possibility is that wormholes are not bound to the Galaxy and are approximately uniformly distributed throughout the universe. For the bound hypothesis, we use $n = \rho_{\text{loc}}/(M_{\text{star}})$, where $\rho_{\text{loc}}$ is the local stellar density in the solar neighborhood, $\rho_{\text{loc}} = 0.044 M_\odot pc^{-3}$, and $M_{\text{star}}$ is the average mass of stars. We use $M_{\text{star}} = 0.3 M_\odot$, a

### Table 2: Einstein Radius Crossing Times for Bulge and LMC Lensings

| $a$ (km) | Bulge$^a$ $t_E$ (day) | LMC$^b$ $t_E$ (day) |
|---------|----------------------|---------------------|
| Bound$^c$ Unbound$^d$ | Bound$^c$ Unbound$^d$ | Bound$^c$ Unbound$^d$ |
| 1       | 0.019 0.001          | 0.035 0.002          |
| 10      | 0.089 0.004          | 0.164 0.007          |
| $10^2$  | 0.413 0.018          | 0.761 0.033          |
| $10^3$  | 1.92 0.084           | 3.53 0.155           |
| $10^4$  | 8.90 0.392           | 16.4 0.721           |
| $10^5$  | 41.3 1.82            | 76.1 3.35            |
| $10^6$  | 192 8.44             | 353 15.5             |
| $10^7$  | 890 39.2             | 1639 72.1            |
| $10^8$  | 4130 182             | 7608 335             |
| $10^9$  | $>10^4$ 843          | $>10^4$ 1553         |
| $10^{10}$ | $>10^4$ 3915         | $>10^4$ 7212         |

Notes: $a$ is the throat radius of the wormhole and $t_E$ is the Einstein radius crossing time.

$^a$ $D_S = 8$ kpc and $D_L = 4$ kpc are assumed.

$^b$ $D_S = 50$ kpc and $D_L = 25$ kpc are assumed.

$^c$ $v_T = 220$ km s$^{-1}$ is assumed.

$^d$ $v_T = 5000$ km s$^{-1}$ is assumed.

unbound scenarios. As the frequencies of current microlensing observations are limited to once every few hours, an event for which the timescale is less than one day is difficult to detect. To find very long timescale events ($t_E \geq 1000$ days), long-term monitoring of events is necessary. The realistic period of observation is $\leq 10$ years. Thus, the realistic size of the throat radius that we can search for is limited to $10^3 \leq a \leq 10^7$ km both for the Galactic bulge and LMC if wormholes are bound to our Galaxy. If wormholes are unbound, the detection is limited to $10^9 \leq a \leq 10^{10}$ km.

### 4. VALIDITY OF THE WEAK-FIELD HYPOTHESIS

First, we consider the outer image. In the previous section, we applied the weak-field approximation to the impact parameter $b$ and the deflection angle $a(r)$. As previously mentioned, the impact parameter $b$ is written as

$$b = \sqrt{r^2 + b^2} \approx r \left(1 + \frac{1}{2} \frac{a^2}{r^2}\right).$$

The condition to neglect the second term is $a \ll \sqrt{2} r$. As the image is always outside the Einstein ring,

$$a \ll \sqrt{2} R_E.$$  (22)

From the deflection angle, we obtain a similar relation from Equation (3):

$$a \ll \sqrt{\frac{8}{3}} R_E.$$  (23)

The values of $a$ and $R_E$ in Table 1 show that the weak-field approximation is suitable for $a \ll 10^{13}$ km in Galactic microlensing. More generally, $R_E \approx D_S^{1/3} a^{2/3}$ is derived from Equation (7) for $D_L \approx D_S/2$. This means that $R_E$ is much greater than $a$ if $a \ll D_S$. Thus, the weak-field approximation is suitable if the throat radius is negligibly small compared with the source distance. For the inner image, the higher-order effect is expected to be greater than that for the outer image. However, the contribution of the inner image to the total brightness is

$$\hat{\beta} (\Lambda_2/A = 0.034)$$

for $\beta = 2$ and 0.013 for $\beta = 3$. On the other hand, the absolute value of the corresponding $\hat{\theta}$ does not decrease as quickly ($\hat{\theta} = -0.618$ for $\beta = 2$ and $-0.532$ for $\beta = 3$). Thus, the contribution of the higher-order effect of the second image to the total brightness is expected to be small.

Another possibility of deviation from the weak-field approximation is the contribution of relativistic images. Recently, gravitational lensing in the strong-field limit (Virbhadra & Ellis 2000) has been studied for lensing by black holes. In this limit, light rays are strongly bent and wound close to the photon sphere. As a result, a number of relativistic images appear around the photon sphere. However, it has been shown that there is no photon sphere (Dey & Sen 2008) in Ellis wormhole lensing. Therefore, there is no contribution of relativistic images to the magnification in Ellis wormhole lensing. We thus conclude that the weak-field hypothesis is a good approximation unless the throat radius is comparable to the galactic distance.

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Table 3
Optical Depths and Event Rates for Bulge Lensing

| a (km) | Bounda | Unboundb | | Bounda | Unboundb |
|-------|--------|----------| |-------|--------|
| 10    | 8.24 × 10^{-12} | 2.45 × 10^{-8} | 2.78 × 10^{-19} | 1.88 × 10^{-14} |
| 10^2  | 1.77 × 10^{-10} | 1.14 × 10^{-7} | 6.00 × 10^{-18} | 8.73 × 10^{-14} |
| 10^3  | 3.82 × 10^{-9} | 5.27 × 10^{-7} | 1.29 × 10^{-16} | 4.05 × 10^{-13} |
| 10^4  | 8.24 × 10^{-8} | 2.45 × 10^{-6} | 2.78 × 10^{-15} | 1.88 × 10^{-12} |
| 10^5  | 1.77 × 10^{-6} | 1.14 × 10^{-5} | 6.00 × 10^{-14} | 8.73 × 10^{-12} |
| 10^6  | 3.82 × 10^{-5} | 5.27 × 10^{-5} | 1.29 × 10^{-12} | 4.05 × 10^{-11} |
| 10^7  | 8.24 × 10^{-4} | 2.45 × 10^{-4} | 2.78 × 10^{-11} | 1.88 × 10^{-10} |
| 10^8  | 1.77 × 10^{-3} | 1.14 × 10^{-3} | 6.00 × 10^{-10} | 8.73 × 10^{-10} |
| 10^9  | 3.82 × 10^{-2} | 5.27 × 10^{-2} | 1.29 × 10^{-8} | 4.05 × 10^{-9} |
| 10^{10}| 8.24 | 2.45 × 10^{-1} | 2.78 × 10^{-7} | 1.88 × 10^{-8} |

Notes. a is the throat radius of the wormhole, τ is the optical depth, and Γ is the event rate. $D_b = 8 \text{ kpc}$ is assumed.

b $\nu_T = 220 \text{ km s}^{-1}$ and $n = 0.147 \text{ pc}^{-3}$ are assumed.

c $\nu_T = 5000 \text{ km s}^{-1}$ and $n = 4.97 \times 10^{10} \text{ pc}^{-3}$ are assumed.

d $\nu_T = 220 \text{ km s}^{-1}$ and $n = 0.147 \text{ pc}^{-3}$ are assumed.

e $\nu_T = 5000 \text{ km s}^{-1}$ and $n = 4.97 \times 10^{10} \text{ pc}^{-3}$ are assumed.

Table 4
Optical Depths and Event Rates for LMC Lensing

| a (km) | Bounda | Unboundb | | Bounda | Unboundb |
|-------|--------|----------| |-------|--------|
| 10    | 1.75 × 10^{-10} | 2.82 × 10^{-7} | 5.90 × 10^{-18} | 2.17 × 10^{-13} |
| 10^2  | 3.76 × 10^{-9} | 1.31 × 10^{-6} | 1.27 × 10^{-16} | 1.01 × 10^{-12} |
| 10^3  | 8.11 × 10^{-7} | 6.37 × 10^{-6} | 2.74 × 10^{-15} | 4.67 × 10^{-12} |
| 10^4  | 1.75 × 10^{-5} | 2.82 × 10^{-5} | 5.90 × 10^{-14} | 2.17 × 10^{-11} |
| 10^5  | 3.76 × 10^{-4} | 1.31 × 10^{-4} | 1.27 × 10^{-12} | 1.01 × 10^{-10} |
| 10^6  | 8.11 × 10^{-3} | 6.37 × 10^{-3} | 2.74 × 10^{-11} | 4.67 × 10^{-10} |
| 10^7  | 1.75 | 2.82 × 10^{-2} | 5.90 × 10^{-10} | 2.17 × 10^{-9} |
| 10^8  | 3.76 | 1.31 × 10^{-1} | 1.27 × 10^{-8} | 1.01 × 10^{-8} |
| 10^9  | 8.11 | 6.37 × 10^{-2} | 2.74 × 10^{-7} | 4.67 × 10^{-8} |
| 10^{10}| 175 | 2.82 × 10^{-1} | 5.90 × 10^{-6} | 2.17 × 10^{-7} |

Notes. a is the throat radius of the wormhole, τ is the optical depth, and Γ is the event rate. $D_b = 8 \text{ kpc}$ is assumed.

b $\nu_T = 220 \text{ km s}^{-1}$ and $n = 0.147 \text{ pc}^{-3}$ are assumed.

c $\nu_T = 5000 \text{ km s}^{-1}$ and $n = 4.97 \times 10^{10} \text{ pc}^{-3}$ are assumed.

d $\nu_T = 220 \text{ km s}^{-1}$ and $n = 0.147 \text{ pc}^{-3}$ are assumed.

e $\nu_T = 5000 \text{ km s}^{-1}$ and $n = 4.97 \times 10^{10} \text{ pc}^{-3}$ are assumed.

Using these values, we calculated the optical depths and event rates for bulge and LMC lensings. Table 3 presents the results for the bulge lensings. In an ordinary Schwarzschild microlensing survey, observations are made of more than 10 million stars. Thus, we can expect approximately $10^7 \Gamma$ events in a year. However, the situation is different in a wormhole search. As mentioned previously, the magnification of wormhole lensing is less than that of Schwarzschild lensing, and a remarkable feature of wormhole lensing is the decreasing brightness around the Einstein radius crossing times. Past microlensing surveys have mainly searched for stars that increase in brightness. The stars monitored are those with magnitudes down to the limiting magnitude or less. However, we need to find stars that decrease in brightness in the wormhole search. To do so, we need to watch brighter stars. Therefore, far fewer stars can be monitored than in an ordinary microlensing survey. Furthermore, the detection efficiency of the wormhole is thought to be less than that for Schwarzschild lensing because of the low magnification. Here we assume that the effective number of stars monitored to find a wormhole is $10^6$. To expect more than one event in a survey of several years, $\Gamma$ must be greater than $\sim 10^{-6}$. The values in Table 3 indicate that the detection of wormholes with $a > 10^4 \text{ km}$ is expected in the microlensing survey of the Galactic bulge in the case of the bound model. The results for the optical depths and the event rates for LMC lensing are presented in Table 4. On the basis of the same discussion as for bulge lensing, we expect $\Gamma > 10^{-6}$ to find a wormhole. The event rates expected for LMC lensing are greater than those for bulge lensing. We expect the detection of a wormhole event if $a > 10^2 \text{ km}$ for the bound model. If no candidate is found, we can set upper limits of $\Gamma$ and/or $\tau$ as functions of $\nu_T$. To convert these values to physical parameters ($n$ and $a$) requires the distribution of $\nu_T$. Right now, there is no reliable model of the distribution except for using the bound or unbound hypothesis. On the other hand, the event rates for the unbound model are too small for the events to be detected.

In past microlensing surveys (Alcock et al. 2000; Tisserand et al. 2007; Wyzykowski et al. 2009; Sumi et al. 2003), large amounts of data have already been collected for both the bulge and LMC fields. Monitoring more than $10^6$ stars for about 10 years can be achieved by simply reanalyzing the past data. Thus, discovery of wormholes can be expected if their population density is as high as the local stellar density and $10^5 \text{ km} < a < 10^7 \text{ km}$. Such wormholes of astronomical size are large enough for humans to pass through. Thus, they would be of interest to people discussing the possibility of space–time travel. If no candidate is found, the possibility of a rich population of large-throat wormholes bound to the Galaxy can be ruled out. Such a limit, however, may not affect existing wormhole theories because there is no prediction of the abundance. However, theoretical studies on wormholes are still in progress. The limit imposed by observation is expected to affect future wormhole theories. On the other hand, the discovery of unbound wormholes is very difficult even if their population density is comparable to that of ordinary stars. To discover such wormholes, the monitoring of a much larger number of stars in distant galaxies would be necessary. For example, $\Gamma \approx 1.7 \times 10^{-6}$ and $\nu_T \approx 380$ days for the M101 microlensing survey ($D_S = 7.4 \text{ Mpc}$) if the throat radius is $10^7 \text{ km}$. To carry out such a microlensing survey, observation from space is necessary because the resolving of a large number of stars in a distant galaxy is impossible through ground observations.

Only the Ellis wormhole has been discussed in this paper. There are several other types of wormholes (Shatskii 2004; Nandi et al. 2006; Rahaman et al. 2007) for which deflection angles have been derived. These wormholes are expected to have different light curves. To detect those wormholes, calculations of their light curves are necessary. The method used in this paper can be employed only when we know the analytic solutions of the image positions. If no analytic solution is found, the calculation must be made numerically.

6. SUMMARY

The gravitational lensing of the Ellis wormhole is solved in the weak-field limit. The image positions are calculated as
real solutions of simple cubic formulas. One image appears on the source star side and outside the Einstein ring. The other image appears on the other side and inside the Einstein ring. A simple estimation shows that the weak-field hypothesis is a good approximation for Galactic microlensing if the throat radius is less than $10^{11}$ km. The derived light curve has characteristic gutters immediately outside the Einstein ring crossing times. Optical depths and event rates for bulge and LMC lensings are calculated for simple bound and unbound hypotheses. The results show that the bound wormholes can be detected by reanalyzing past data if the throat radius is between $10^2$ and $10^7$ km and the number density is approximately equal to the local stellar density. If the wormholes are unbound and approximately uniformly distributed in the universe with average stellar density, detection of the wormholes is impossible using past microlensing data. To detect unbound wormholes, a microlensing survey of distant galaxies from space is necessary.

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