A New Formula for Predicting Solar Cycles

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Abstract

A new formula for predicting solar cycles based on the current theoretical understanding of the solar cycle from the flux transport dynamo is presented. Two important processes—fluctuations in the Babcock–Leighton (BL) mechanism and variations in the meridional circulation (MC), which are believed to be responsible for irregularities of the solar cycle—are constrained using observational data. We take the polar field near minima of the cycle as a measure of the randomness in the BL process, and the decay rate near the minima as a consequence of the change in MC. We couple these two observationally derived quantities into a single formula to predict the amplitude of the future solar cycle. Our new formula suggests that cycle 25 would be a moderate cycle. Whether this formula for predicting the future solar cycle can be justified theoretically is also discussed using simulations from the flux transport dynamo model.

Key words: Sun: activity – Sun: interior – Sun: magnetic fields – sunspots

1. Introduction

The sunspot cycle with the approximate period of 11 yr is one of the most intriguing natural cycles known to mankind. Solar disturbances, which become more frequent during the peak of this cycle, control the space environment of the Earth and affect our lives in various ways. Developing a method for predicting the strength of a solar cycle in advance is of utmost societal importance (Pesnell 2008; Petrovay 2010; Choudhuri 2018). The aim of this paper is to propose a formula for predicting solar cycles. To apply this formula, we need values of certain quantities that become available toward the end of the previous cycle. Once these values are known, it will be possible to use this formula to predict the forthcoming cycle.

We discuss how we arrive at this formula by analyzing the data of the last few solar cycles. We also look at the question of whether this formula can be justified on the basis of the flux transport dynamo model—a theoretical model that has been successful in explaining many aspects of the solar cycle.

It has been known that there is a correlation between the polar field of the Sun during the solar minimum and the strength of the next cycle, allowing us to use this polar field as a predictor (Schatten 2005; Svalgaard et al. 2005). The theoretical explanation of this correlation on the basis of the flux transport dynamo model was provided by Jiang et al. (2007). The poloidal field is generated in the flux transport dynamo model by the Babcock–Leighton (BL) mechanism from the decay of tilted bipolar sunspots. As there is a scatter in the tilt angles of bipolar sunspots around the average given by Joy’s law (Longcope & Choudhuri 2002; Wang et al. 2015), the BL mechanism has an inherent randomness, leading to the unequal production of the poloidal field in different cycles (Karak & Miesch 2017). Because this poloidal field is brought to the polar region by the meridional circulation (MC) to produce the polar field at the end of the cycle and also diffuses to the bottom of the convection zone to act as the seed of the next cycle, we have this correlation between the polar field at the end of a cycle and the strength of the next cycle. Choudhuri et al. (2007) developed a methodology of incorporating the randomness of the BL mechanism in the theoretical flux transport dynamo model and predicted cycle 24 before its onset. Their prediction turned out to be the first successful prediction of a solar cycle from a theoretical dynamo model.

When the works mentioned in the last paragraph were being done, it was not yet realized that there could be another important source of irregularities in the solar cycle—fluctuations in the MC. It is the timescale of MC that sets the period of the flux transport dynamo. A slower MC makes cycles longer. Although we have actual measurements of MC only during the last few years, durations of past cycles give an indirect indication of how the strength of MC varied with time (Karak & Choudhuri 2011). If the diffusion timescale is shorter than the MC timescale—which is the case if we assume a value of diffusion based on simple mixing length arguments—then longer cycles become weaker due to a more prolonged action of diffusion. In other words, there would be an anticorrelation between the duration of the cycle and the cycle strength. This anticorrelation helps in explaining some features of observational data such as the Waldmeier effect (Karak & Choudhuri 2011). It seems that there is a time delay in the effect of MC on the cycle strength. As a result, the peak of a cycle depends not on the value of MC at that time, but on the value a few years earlier (Hazra et al. 2015). If the MC was weaker a few years earlier, then the decay rate of the previous cycle would be smaller. We actually find a correlation between the strength of a cycle and the decay rate of the previous cycle (Hazra et al. 2015).

We conclude that the irregularities of solar cycle are caused by the combined effect of two factors: (i) randomness in the BL mechanism for poloidal field generation, and (ii) fluctuations in MC. Choudhuri & Karak (2012) developed a theoretical model of the grand minima of solar cycles by including both of these in their dynamo model. Now our aim is to develop a method for predicting a future cycle by taking both of these factors into consideration. The polar field $P$ at the sunspot minimum captures the effect of randomness in the BL mechanism in the previous cycle. On the other hand, the decay rate $R$ of the previous cycle provides information about MC during the phase that is important for determining the strength of the next cycle. So we expect the peak strength $A$ of the cycle to be given...
by a formula of the type

\[ A \propto P^\alpha R^\beta. \]  

Our job now is to check whether the strengths of the past cycles can be matched by some suitable choices of the indices \( \alpha \) and \( \beta \). We also look at the question of whether we find any support for such a formula in the simulations of the flux transport dynamo.

Section 2 is devoted to a discussion of the possibility of a formula of the type given in Equation (1) based on observational data. Section 3 discusses the support for this possibility in theoretical dynamo simulations. Whether or not such a formula can help us to predict the upcoming cycle is discussed in Section 4. Our conclusions are summarized in Section 5.

2. Observational Study

We have reasonably trustworthy data regarding sunspot number from at least the beginning of the twentieth century. We can easily obtain reliable values of the peak sunspot number \( A \) and the decay rate \( R \) for several previous solar cycles from these data. To check whether a formula like in Equation (1) worked for past cycles, we need the values of the polar field \( P \) during several sunspot minima. We have actual regular measurements of the polar field only from the mid-1970s. However, there are several proxies that indicate how the polar field might have evolved at earlier times. The important proxies we consider in this paper are (i) polar flux obtained from polar faculae number as presented by Muñoz-Jaramillo et al. (2012), (ii) the parameter \( A(t) \) obtained by Makarov et al. (2001) from the position of neutral lines (indicated by filaments) on the solar surface, and (iii) the polar network index (PNI) developed by Priyal et al. (2014) from chromospheric networks seen in Kodaikanal Ca K spectroheliograms. The bottom panel of Figure 1 plots polar fields inferred from these three different proxies, below the sunspot number shown in the upper panel. For all of these three proxies for the polar field, we plot normalized values by putting their maximum values in the range equal to 1. We see that the three proxies give very similar values of the polar magnetic field during much of the time. However, during a few solar minima (such as the minimum before cycle 16), we find that some of the proxies diverge widely. We do not know the reason behind this. The polar faculae number happens to be the most widely studied proxy for the polar field (Sheeley 1991; Muñoz-Jaramillo et al. 2012), so we use the polar field \( P \) obtained from this proxy in our analysis. We should keep in mind that the value of \( P \) obtained from this proxy may not be very reliable during the times when the different proxies give very different results. Another possible proxy for the polar field, which we do not include in this paper, is the geomagnetic \( aa \) index. Wang & Sheeley (2009) have studied the correlation of this index at the solar minima with the strength of the next cycle.

For the sunspot number, we have used two data sets. One provides the group sunspot number from the Greenwich Observatory\(^8\) and the other provides the calibrated monthly total sunspot number (Clette et al. 2014) from WDC-SILSO, Royal Observatory of Belgium, Brussels.\(^4\) Both of the data sets are smoothed by using Gaussian filter with FWHM = 2 yr and are shown in the upper panel of Figure 1. Several authors (Cameron & Schüssler 2007; Podladchikova et al. 2008, 2017; Petrovay 2010) have suggested possibilities of using some features of sunspot number variations for predicting future cycles. As we mentioned earlier, the decay rate during the late

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\(^{8}\) https://solarscience.msfc.nasa.gov/greenwch/spot_num.txt

\(^{4}\) http://www.sidc.be/silso/datafiles
phase of the cycle is a good precursor to predict the next cycle. We now calculate the decay rate during the late phase of the cycle following the procedure used by Hazra et al. (2015) in which the decay rate is taken as the slope between two points with a separation of 1 yr with the second point 1 yr before the minimum at the end of the cycle. The decay rate defined in this way is found to have a good correlation with the next cycle, which is not the case if the decay rate is defined in other ways (Hazra et al. 2015). Presumably, the decay rate calculated in one particular way captures the information about the strength of MC at a relevant phase of the cycle that affects the next cycle.

We first present the results obtained by using the Greenwich sunspot number. Then we present results based on SIDC data. Figure 2 presents the results we get by using the Greenwich sunspot number. Figure 2(a) shows the correlation between the decay rate calculated during the late phase of the cycle and the peak amplitude of the next cycle. The correlation coefficient \( r = 0.69 \) is less than \( r = 0.83 \), which Hazra et al. (2015) obtained by considering 23 cycles with same Gaussian smoothing with FWHM = 2 yr (see Table 1 of Hazra et al. (2015)). In all of our calculations, we have considered the data from cycle 15 up to the present time, as the polar faculae data are not available before cycle 15.

The different campaigns of the polar faculae data are calibrated with the direct measurement of the solar fields from Wilcox solar observatory from the mid-1970s. We use the calibrated long-term polar flux data set for the solar fields presented by Muñoz-Jaramillo et al. (2012). We calculate the polar field near the solar minima by averaging over a span of 1 yr before the minimum and 1 yr after the minimum. The correlation between polar field near minima and the next cycle amplitude is shown in Figure 2(b). For the polar field \( P \), we have used the polar flux (in Mx), which is obtained by multiplying the normalized polar field shown in Figure 1 by a factor of \( 6.25 \times 10^{22} \) following the calibration given by Muñoz-Jaramillo et al. (2012). We see that the correlation \( r = 0.46 \) is rather poor—mainly due to the fact that cycles 16 and 21 show large scatters. Note that Figure 2(a) of Muñoz-Jaramillo et al. (2013) presented essentially the same correlation \( r = 0.69 \) that we present in our Figure 2(b), except for some differences (they use sunspot area data, whereas we use sunspot number data). We point out that we get a lower correlation than Muñoz-Jaramillo et al. (2013). We discuss below that the correlation improves considerably when using SIDC sunspot number data (Figure 3(b)) rather than the Greenwich sunspot number data.

**Figure 2.** The correlation of various precursors with the next cycle amplitude is plotted. The precursors are (a) the decay rate at the late phase of the cycle, (b) the polar field near minima of the cycle \( (P \) is polar flux in Mx divided by \( 10^{22} \)), (c) the new precursor formula \( A = \sqrt{P \times R} \), and (d) \( A = P \times R \).
Finally, to check whether \( A = \sqrt{P \times R} \) and \( A = P \times R \) can be used as good precursors for predicting the next cycle, Figures 2(c) and (d) show the correlations between them and the peak of the next cycle. We find that among all possible values of \( \alpha \) and \( \beta \) appearing in (1), the combination of \( \alpha = \beta = 0.5 \) (case (i)) and \( \alpha = \beta = 1 \) (case (ii)) gives the highest correlation with the amplitude of the next cycle. The correlation coefficient for case (i) is 0.70 (null hypothesis rejected with probability 96.3%) and for case (ii) is 0.73 (null hypothesis rejected with probability 97.3%). These correlation coefficients are significantly higher than the correlation coefficient in the case of polar fields alone and slightly higher than the correlation coefficient in the case of decay rates alone.

It should be clear from Figure 2 that the points corresponding to cycles 16 and 21 in all the plots make correlation coefficients lower than what they would otherwise be. These points are indicated in brown. Interestingly, we see in Figure 1 that the various proxies of the polar field preceding these cycles did not match each other well. We have no explanation for this. However, this raises the question of whether polar faculae counts at these times were good indicators for the polar field. In particular, the cycle 16 was the weakest cycle in the century. Because according to the flux transport dynamo model with reasonably high diffusivity (Jiang et al. 2007), the polar field strengths during the minima preceding the cycles are expected to be correlated with the strengths of the cycles, we expect the polar field before cycle 16 to be weak. But in the polar faculae count and PNI, the polar field is comparable to the polar field preceding the strongest cycle (cycle 19). However, the polar field from \( A(t) \) index of Makarov et al. (2001) before cycle 16 is weak as expected from theory. We also point out that, during the minima preceding cycles 20 and 21, there was a dearth of spectroheliogram plates and some error may be introduced in the PNI count during these times (Priyal et al. 2014). If we had used \( A(t) \) as the proxy of the polar field rather than the polar faculae count, then the points corresponding to cycles 16 and 21 in Figure 2 would have considerably less scatter. In Figure 3 of Wang & Sheeley (2009), in a plot of \( aa \) index at minima against the next cycles, we see that the points for cycles 16 and 21 are not so far from the best-fit straight line. Because we are unsure of the polar field during the minima preceding cycles 16 and 21, we quote the correlation coefficients for our newly proposed precursors without points 16 and 21. The correlation coefficient for case (i) with square root dependence on polar field and decay rate is 0.97 with null hypothesis rejected with probability 99.9%. For case (ii), the correlation coefficient without cycles 16 and 21 is 0.98 (99.9%). Having these high values of correlation coefficients, we argue that the newly

![Figure 3](image-url). Same as Figure 2 but for SIDC sunspot number data.
proposed precursor formulae hold a promise to predict the future solar cycle based on the polar field measurement $P$ preceding the cycle and the decay rate $R$ at a late phase of the previous cycle. These two quantities represent the two physical causes behind irregularities in the solar cycles, namely fluctuations in the BL mechanism and fluctuations in the MC respectively. We also point out that, without cycles 16 and 21, the correlation coefficient for polar field alone with the next cycle amplitude is around 0.90 (99.5%). This is significantly higher than what we get when these cycles are included, but less than the correlation coefficients obtained with the newly suggested precursors. Therefore, the precursors we are suggesting would be very helpful in predicting a future solar cycle with better accuracy.

Next, we repeat the same exercise with the newly calibrated international sunspot numbers from SIDC, Royal Observatory of Belgium. The results are presented in Figure 3. Each panel in Figure 3 is similar to Figure 2. It is clear from Figure 3 that the newly proposed precursors are more highly correlated with the peak amplitude of the cycle compared to either the decay rate of the previous cycle alone or the polar field during the preceding minima alone. We find that the correlation of decay rate with the next cycle amplitude is 0.62 (94.5%) and the correlation of polar fields near minima with the next cycle is also 0.62 (94.5%). On the other hand, the newly proposed precursors (Figures 3(c) and (d)) have correlation coefficients of 0.69 (97.4%) for case (i) and 0.70 (97.6%) for case (ii), which are higher than the correlation coefficients found from the polar field alone (Figure 3(a)) and the decay rate alone (Figure 3(b)). The correlation coefficients do get improved if we exclude points 16 and 21. However, when we use SIDC data, we find that the correlation coefficient obtained from the polar field alone becomes quite high and the correlation coefficients obtained with the new precursor formulae (for each case (i) and case (ii)) become slightly less. This shows the hazard of doing statistical analysis with very few data points.

We also checked whether the trends we found exist if we analyze the data of two hemispheres separately. Because the data for two hemispheres are available separately only from the Greenwich Observatory, we use these data to calculate strength of the cycle and the decay rate $R$—by using the same method we are using, but now doing the analysis for the two hemispheres separately. Figure 4, which is similar to Figures 2 and 3, shows results of this analysis by indicating data points corresponding to northern and southern hemispheres in blue and red respectively. Table 1 shows the

![Figure 4](image_url)
correlation coefficients we would get if we treat the data of the two hemispheres separately instead of combining them together, as we have done in Figure 4. Even in this analysis, we always find the trend that \( A = \sqrt{P \times R} \) and \( A = P \times R \) are correlated better with the next cycle than \( P \) or \( R \), giving an indication that this trend may not be an artifact of a small data set.

Although our analysis is severely restricted by the very limited number of data points and the values of the polar field are uncertain in some cases even among these few data points, we still find tantalizing hints that the new precursor formulae we are suggesting are better for predicting a future cycle than the polar field at the minimum alone or the decay rate of the previous cycle alone. We have to wait for a few more cycles (at least for half a century) before one can draw firmer conclusions.

### 3. Theoretical Interpretation

As explained in Section 2, our observational study motivated us to introduce new precursors for predicting the future solar cycle. In this section, we discuss whether we can provide any justification for these new precursors from a theoretical flux transport dynamo model. Presently, the flux transport dynamo model is the most promising model to explain the various features of the solar cycle and its irregularities.

The cyclic oscillation between the poloidal field and the toroidal field produces the solar cycle in this model (Choudhuri et al. 1995; Durney 1995; Choudhuri & Dikpati 1999; Chatterjee et al. 2004). The differential rotation stretches the poloidal field to generate the toroidal field. Then the toroidal field rises up to the photosphere due to magnetic buoyancy and creates bipolar magnetic regions that appear with tilts produced by the Coriolis force (D’Silva & Choudhuri 1993). The decay of tilted bipolar magnetic regions due to turbulent diffusion produces the poloidal field via the BL mechanism (Babcock 1961; Leighton 1969). For a detailed explanation of the model, please see the reviews by Choudhuri (2011), Charbonneau (2014), and Karak et al. (2014). The irregularities in the solar cycle mainly arise because of the inherent randomness in the mechanism for generating the poloidal field, as first pointed out by Choudhuri (1992) and then analyzed by many authors (Charbonneau & Dikpati 2000; Karak & Choudhuri 2011; Hazra et al. 2015) who successfully reproduced many observed irregularities in the solar cycle. But some of the irregular properties (e.g., the Waldmeier Effect, the correlation between decay rate, and the amplitude of the next cycle) are not reproduced using only the fluctuations in the BL mechanism. Karak (2010) pointed out that the fluctuations in the MC can be another source of irregularities in the solar cycle. By including fluctuations in the MC, Karak & Choudhuri (2011) successfully reproduced the Waldmeier effect and Hazra et al. (2015) reproduced the correlation of the decay rate with the next cycle amplitude.

We now present some theoretical results obtained by introducing fluctuations into both the BL process and the MC in our dynamo model. We solve the time evolution equations of the poloidal and toroidal fields as given below:

\[
\begin{align*}
\frac{\partial A}{\partial t} + \frac{1}{s} (\nu \nabla)(sA) &= \eta_p \left( \nabla^2 - \frac{1}{s^2} \right) A + S(r, \theta, t), \\
\frac{\partial B}{\partial t} + \frac{1}{r} \left( \frac{\partial}{\partial r} (\nu r B) + \frac{\partial}{\partial \theta} (\nu \theta B) \right) &= \eta_t \left( \nabla^2 - \frac{1}{s^2} \right) B \\
&+ \frac{1}{r} \left( \frac{d \eta}{dr} \partial(r B) \right) + \frac{1}{r} \frac{d \eta_t}{dr} (r B),
\end{align*}
\]

where \( A_{\phi} \) is the magnetic vector potential corresponding to the poloidal field, \( B \) is the toroidal component, and \( s = r \sin \theta \). The diffusion coefficients \( \eta_p \) and \( \eta_t \) correspond to poloidal and toroidal components of magnetic field respectively. The source term \( S(r, \theta) \) takes care of magnetic buoyancy and the BL mechanism. We have considered a local \( \alpha \)-parameterization of magnetic buoyancy (Choudhuri & Hazra 2016). The parameters for the flux transport models are chosen with the same approach as Section 4.3 of Hazra et al. (2015). If the toroidal field near the bottom of the convection zone is stronger than a critical value \( B_{c} \), some amount of toroidal field is removed from there and applied to the surface layers, which produces the poloidal field via the BL mechanism. This introduces a nonlinearity in the problem that limits the dynamo growth.

To produce irregularities in the cycles, 100% fluctuation is introduced in the BL \( \alpha \) with coherence time of one month, whereas a 30% fluctuation with 30 yr of coherence time is introduced in the meridional flow. This gives us reasonably irregular cycles comparable to observed solar cycles. The decay rate during the late phase of the cycle is calculated from the output of the theoretical simulation and its correlation with the amplitude of the next cycle is shown in Figure 5(a). This is the same as the results presented in Hazra et al. (2015). We have calculated the peak polar field near minima and the next cycle amplitude is shown in Figure 5(b). The two bottom panels of Figure 5 show how well the new precursors introduced by us on the basis of the observational data are correlated with the amplitude of the next cycle. It may be noted that the theoretical correlation between the polar field at the minima and the peak of the next cycle, as shown in Figure 5(b), is already very high—considerably higher than what is seen in the observational data (see Figures 2(b) and 3(b)). In such a situation, it is somewhat difficult to ascertain whether the new precursors give even better correlations. We made several independent runs of our code and found that the correlations computed in different runs are often slightly different, although they have the same statistical nature. In Figure 5 we have presented results from one run in which both the precursors \( \sqrt{(P \times R)} \) (case (i)) and \( (P \times R) \) (case (ii)) give better correlations with the next cycle amplitude than \( R \), but about the same as \( P \). In fact, the correlation coefficients for the precursors are marginally lower than that for \( P \). Because \( P \) alone gives such a good correlation in our theoretical model, precursors that combine it with the less strongly correlated quantity \( R \) tend to have slightly fewer correlations.
To verify theoretically whether our proposed precursors are really better for predicting a future cycle than either $P$ or $R$, we need a theoretical dynamo model that faithfully reproduces all the different features of the solar cycle. The model presented in Section 4.3 of Hazra et al. (2015) reproduced most of the features of the solar cycle. However, we now realize that it produces a tighter correlation between the polar field $P$ at the minima and the next cycle amplitude compared to what is observed. As pointed out by Jiang et al. (2007), this correlation becomes better as the value of turbulent diffusion increases. Thus we need to do calculations with a model that has a lower value of diffusion in order to increase the scatter in the correlation between $P$ and the next cycle. On the other hand, Chatterjee et al. (2004) showed that the dynamo solution tends to become quadrupolar (contrary to what is observed) when decreasing the value of diffusion. It is, therefore, necessary to construct a theoretical model that prefers dipolar parity but gives more scatter in the correlation between $P$ and the next cycle. On the basis of several trial runs using different combinations of parameters, so far we have not been able to come up with a theoretical model that has this property. So we are currently unable to give very strong arguments, on the basis of a theoretical dynamo model, that the precursors we have suggested are better at predicting a future cycle than $P$ and $R$.

Based on the theoretical calculations we have presented, we can certainly say that our precursors are very good at predicting the next cycle, although the correlation of $P$ with the next cycle is already so strong in the theoretical model that it is not clear whether the precursors correspond to significant improvements.

Another issue we should mention is that the effect of the variations of meridional flow on the polar field can be different in models that handle the BL mechanism differently (Muñoz-Jaramillo et al. 2010). Although treating the BL mechanism through an $\alpha$-parameterization led to reasonably realistic theoretical models matching observations, we should keep in mind that this is a gross oversimplification and the results should be interpreted with caution when there are variations in the MC.

In summary, we emphasize that, given the many uncertainties in both the theoretical model and the observational data, the comparison between the two should be taken to be of illustrative nature only and should not be taken too seriously as a suggested real physical interpretation.

Figure 5. Same as Figure 2 but for theoretically produced irregular solar cycles.
4. Future Cycle Prediction

As we believe that the two precursors we have suggested (case (i) and case (ii)) are particularly well suited to predict future cycles, we now write down appropriate formulae based on these precursors that can be readily used for predicting future cycles. The sunspot database from SIDC is the best calibrated and most trustworthy sunspot number database, so we consider the best straight line fits of the data points for the two cases of the SIDC database only (Figures 3(c) and (d)). We perform the least square fitting for both cases, i.e., case (i) with $\alpha = \beta = 0.5$ (Figure 3(c)) and case (ii) with $\alpha = \beta = 1.0$ (Figure 3(d)), to arrive at formulae from which the amplitude of the next solar cycle can be calculated, after knowing the values of the precursors $P$ and $R$ around the minimum before the cycle. The formula representing the case (i) ($\alpha = \beta = 0.5$) is

$$A = 11.35 \times \sqrt{P \times R} + 82.03, \quad (4)$$

whereas for case (ii) (with $\alpha = \beta = 1$) the formula is

$$A = 0.64 \times (P \times R) + 127.07. \quad (5)$$

Note that the best-fit straight lines in Figures 3(c) and (d) do not pass through the origin. In other words, a future cycle is never predicted to have zero strength for any combination of positive values of $P$ and $R$. It is thus clear that the formulae we have arrived at cannot handle the situation of a grand minimum. Presumably, these formulae would give good results when the various parameters lie within a reasonable range of values. When we know the appropriate values of the polar field $P$ and decay rate $R$ (of the previous cycle) at the time of a solar minimum, we can use these formulae for predicting the next cycle.

Finally, we calculate the peak sunspot number of cycle 25 based on our newly obtained precursor formulae given in Equations (4) and (5). As these formulae need $P$ and $R$ values, we calculate them individually. Ideally, to conform with what we are doing for the other cycles, we should calculate $P$ by averaging over 2 yr around the minimum and should calculate $R$ from the slope of sunspot number over a year ending 1 yr before the minimum. We are not sure how close we are to reaching the minimum. However, we believe that we are sufficiently close to the minimum to allow us to calculate $P$ and $R$ with a reasonable degree of reliability. Figure 6 shows a plot of SIDC sunspot number and the actual polar field data during the last few years, indicating how we are obtaining $P$ and $R$.

We calculate the polar field $P$ near the minimum preceding cycle 25 by using polar field data from the Wilcox Solar Observatory. We have taken a 1 yr average of the polar field before the present epoch, which is expected to be close to the minimum (see the red marked line in the polar field curve of Figure 6), to use this polar field as a precursor for predicting cycle 25. Although throughout the paper we have used the calibrated long-term polar flux data set from the Mount Wilson Observatory (Muñoz-Jaramillo et al. 2013), we use Wilcox Solar Observatory polar field data here because it is the most reliable directly measured available polar field data set. The long-term data of polar flux, which we used to obtain the new precursor formulae (Equations (4) and (5)), have three cycles overlapping with the WSO direct polar field data. Therefore, we have calibrated the MWO polar flux data with respect to the WSO polar field data. After implementing the calibration factor, the polar flux near the minimum preceding solar cycle 25 corresponding to the WSO data turns out to be $(1.648 \pm 0.147) \times 10^{22}$ Mx. We use this value of $P$ to predict solar cycle 25. Next, we need to calculate the decay rate $R$ just before the minimum preceding cycle 25. We have calculated it as indicated by the slope in Figure 6 with the red line. This is found to be $14.84$. We plug these values of $P$ and $R$ into the new precursor formulae (Equations (4) and (5)) to get an estimate of the peak amplitude of cycle 25. Please note that we have tentatively assumed the present epoch (April, 2019) to be the minimum (black dashed line in Figure 6) for the purpose of calculating $P$ and $R$. Because the actual minimum may occur 1–2 yr later, the recalculated values of $P$ and $R$ at that time may be slightly different, leading to a slightly modified prediction.

According the formula in Equation (4) (with $\alpha = \beta = 0.5$), the peak amplitude of cycle 25 would be 138 and with the formula in Equation (5) ($\alpha = \beta = 1$) the peak amplitude would be 143 of SIDC data. These calculations suggest that cycle 25 would be a moderate cycle.

5. Conclusion

Predicting a solar cycle before its onset is a challenge that interests both solar physicists and the general public. Even a decade ago, it was a rather uncertain art. Pesnell (2008) combined all the predictions that were made for cycle 24 in Figure 1 of his paper. It was clear that the various predictions covered virtually the entire range of all possible values of the peak sunspot number. During the intervening years, our understanding of the physical basis for the solar cycle prediction has deepened considerably. The aim of the present paper has been to come up with a simple formula for predicting the forthcoming cycle on the basis on this new understanding.

We now believe that the irregularities of the solar cycle are produced primarily by two factors: fluctuations in the BL mechanism and fluctuations in MC. So, in order to predict a cycle, we need to include contributions from both of these factors. As the polar field $P$ at the beginning of the cycle provides the relevant information for fluctuations in the BL mechanism and the decay rate $R$ at the end of the previous cycle provides the relevant information for fluctuations in MC, we have looked for formulae combining $P$ and $R$, which have
good correlations with the peak of the next cycle. The formulæ have to be calibrated by using the data of the past cycles. The first bottleneck in this process is the lack of polar field data before the 1970s. We have pointed out three proxies for the polar field. During much of the time, these three proxies give very similar values for the polar field. However, there have been intervals during which some of the proxies diverged and we do not have reliable information about the polar field in those intervals. We saw that some of the data points in our correlation plots with the largest scatters corresponded to these intervals.

Because the polar field alone has often been used as a predictor for the next cycle (Schatten 2005; Svalgaard et al. 2005), we now come to the question of whether it is really necessary to include \( R \) in our formulæ. From the durations of past cycles, as shown in Figure 2 of Karak & Choudhuri (2011), it appears that there have been significant fluctuations in MC during the nineteenth century, but there have not been very large fluctuations during cycles 15–22 covering much of the twentieth century. This means that the polar field of the Sun alone would have been a reasonably good predictor for solar cycles during the twentieth century. Although we have no idea at the present time as to what causes these fluctuations in MC, there is no reason to expect that the nineteenth century was a very atypical era and we are quite likely to enter similar eras of large fluctuations in MC in the future. In such an era, using the polar field alone as a predictor for cycles would probably be inadequate and we have to include the effect of varying MC, as we have tried to do in this paper. We are forced to use data only for cycles 15–22 when some information about the polar field is available from various proxies. Comparing Figure 2(a) of this paper with Figure 2(b) of Hazra et al. (2015), we find that most of the data points we are considering now (except the points for cycles 19 and 22) have a rather narrow spread in the values of decay rate (horizontal axis) compared to what we see in Figure 2(b) of Hazra et al. (2015). This certainly makes it difficult to calibrate our formulæ by incorporating the dependence of \( R \) properly. So, our formulæ should be taken as provisional at the present time. Our formulæ will probably be properly calibrated only after we have an era of a few decades during which the MC has large fluctuations (like the nineteenth century).

We may point out that there is a periodic variation of MC with the solar cycle (Komm et al. 1993; Chou & Dai 2001; Basu & Antia 2010; González Hernández et al. 2010; Hathaway & Rightmire 2011), presumably due to the Lorentz force of the dynamo-generated magnetic field (Hazra & Choudhuri 2017). There is also some evidence of a migratory pattern in MC variations with migration of the activity belt, indicating flow toward this belt—at least at the surface (Snodgrass & Dailey 1996; Chou & Dai 2001; Cameron & Schüssler 2010; Howe et al. 2018; Komm et al. 2018; Lin & Chou 2018). Presumably, the amplitude of these inter-cycle variations would depend on the strength of the cycle, but we have not considered these so far poorly understood variations in this work, limiting the scope of our model by the inclusion of only a simple kind of nonlinearity in magnetic buoyancy (allowing the toroidal field to rise only when it is stronger than \( B_t \)) to restrain the dynamo growth. So far there have not been many studies of the nonlinear interaction between the MC and the dynamo (Karak & Choudhuri 2012). Future studies should include the tilt-angle quenching (Karak & Miesch 2017) and the nonlinear modulation due to active region inflows (Martin-Belda & Cameron 2017).

In summary, we have to say that the formulæ proposed in this paper are of somewhat provisional nature at the present time, as we have a very limited amount of past data to calibrate them. However, we believe that these formulæ show the right way for predicting future solar cycles. Based on the formulæ we have arrived at, we have presented our prediction for the upcoming cycle 25. We expect that the formulæ for predicting future cycles will be improved as solar astronomers get more data to calibrate them in the future and will eventually prove to be very powerful tools for predicting solar cycles before their onset.

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