Repeated Mergers of Black Hole Binaries: Implications for GW190521

Oliver Anagnostou\textsuperscript{1}, Michele Trenti\textsuperscript{1,2,3} ©, and Andrew Melatos\textsuperscript{1}

\textsuperscript{1}School of Physics, The University of Melbourne, VIC 3010, Australia; oanagnostou@student.unimelb.edu.au
\textsuperscript{2}Australian Research Council Centre of Excellence for Gravitational Wave Discovery (OzGrav), Australia
\textsuperscript{3}Australian Research Council Centre of Excellence in All Sky Astrophysics in 3 Dimensions (ASTRO 3D), Australia

Received 2021 December 22; revised 2022 October 19; accepted 2022 October 20; published 2022 December 7

Abstract

The gravitational wave event GW190521 involves the merger of two black holes of $\sim 85 \, M_\odot$ and $\sim 66 \, M_\odot$, forming an intermediate-mass black hole (IMBH) of mass $\sim 142 \, M_\odot$. Both progenitors are challenging to explain within standard stellar evolution as they are within the upper black hole mass gap. We propose a dynamical formation pathway for this IMBH based on multiple mergers in the core of a globular cluster. We identify such scenarios from analysis of a set of 58 N-body simulations using NBODY6-gpu. In one of our simulations, we observe a black hole undergoing a chain of seven binary mergers within 6 Gyr, attaining a final mass of $97.8 \, M_\odot$. We discuss the dynamical interactions that lead to the final IMBH product, as well as the evolution of the black hole population in that simulation. We explore statistically the effects of gravitational recoil on the viability of such hierarchical mergers. From the analysis of all 58 simulations we observe additional smaller chains, tentatively inferring that an IMBH formation through hierarchical mergers is expected in the lifetime of a median-mass globular cluster with probability $0.01 \lesssim p \lesssim 0.1$ without gravitational merger recoil. Using this order-of-magnitude estimate we show that our results are broadly consistent with the rate implied by GW190521, assuming that gravitational recoil ejection of progenitors has a low probability. We discuss implications for future gravitational wave detections, emphasizing the importance of studying such formation pathways for black holes within the upper mass gap as a means to constrain such modeling.

Unified Astronomy Thesaurus concepts: Gravitational wave sources (677); N-body simulations (1083); Intermediate-mass black holes (816); Black hole physics (159); Stellar mergers (2157); Globular star clusters (656)

1. Introduction

On 2020 September 2, the Laser Interferometer Gravitational-Wave Observatory (LIGO) and Virgo collaboration announced the detection of a gravitational wave signal consistent with a black hole binary (BHB) merger producing a remnant BH of mass $142^{+16}_{-18} \, M_\odot$ (The LIGO Scientific Collaboration et al. 2009). The event, known as GW190521, involved the merger of component masses $m_1 = 85^{+21}_{-14} \, M_\odot$ and $m_2 = 66^{+14}_{-15} \, M_\odot$. These are the heaviest two merging BHs yet observed by LIGO/Virgo (Stoyan et al. 2008; Abbott et al. 2016a, 2016b, 2016c, 2017a, 2017b, 2020a, 2020b, 2020c; The LIGO Scientific Collaboration et al. 2018). A similarly heavy merger was announced in the second half of LIGO’s third observational run, known as GW200220_061928 (Collaboration et al. 2021). Interestingly, both events have component masses that potentially lie within the supposed upper black hole mass gap, a range of BH masses that is outside what is predicted through standard stellar evolution. The mass gap is thought to originate from pair-instability supernovae (Heger et al. 2003; Belczynski et al. 2016; Spera & Mapelli 2017; Woosley 2017, 2019; Marchant et al. 2019). Current theoretical modeling places the lower cutoff at $\sim 50^{+5}_{-3} \, M_\odot$ (Farmer et al. 2019, 2020; Leung et al. 2019; Marchant et al. 2019; Belczynski et al. 2020; Costa et al. 2020; Mapelli et al. 2020; Marchant & Moriya 2020; Renzo et al. 2020), although there are uncertainties pertaining to hydrodynamics, relativistic effects, and nuclear physics. The modeling therefore implies that at least the primary masses are well within the mass gap and cannot have originated directly from a stellar progenitor.

The existence of physics beyond the standard model could lead to modifications in the canonical pair-instability supernova process, enabling the production of heavier stellar BHs than current theories permit. Phenomena such as the existence of new light particles (Croon et al. 2021), a different magnetic dipole moment of the electron neutrino (Barger et al. 1999; Heger et al. 2009), and the addition of extra spatial dimensions (Padilla 2015) can all alter the supernova process to produce heavier BHs. Indeed Sakstein et al. (2020) propose such new physics as potential explanations for GW190521.

However, without invoking nonstandard physics, there are other possible mechanisms that can form higher-mass BHs, particularly in dense environments through dynamical interactions. If a globular cluster (GC) is sufficiently dense at formation, massive stars can undergo one or more collisions before stellar core collapse. The result may be an evolved star with an oversized hydrogen envelope, which collapses to form a BH within the upper mass gap (Spera et al. 2019). Kremer et al. (2020) explored the formation of BHs with masses within the upper mass gap through collisions of young massive stars in dense star clusters. They found that $\sim 20\%$ of BH progenitors undergo one or more collisions before collapse, leading to $\sim 1\%$ of all stellar BHs forming within the upper mass gap. Di Carlo et al. (2020) found a slightly higher fraction of mass-gap BHs through young stellar collisions, with up to $\sim 6\%$ of all simulated BHs $\gtrsim 60 \, M_\odot$ depending on the progenitor’s metallicity.

BHBs may also form and evolve via gravitational interactions with other compact bodies in a dense stellar environment such as a GC (O’Leary et al. 2006, 2007; Banerjee et al. 2010;
Tanikawa 2013; Choksi et al. 2018). In these dense environments, the BH product of a previous BHB merger can go on to form a new BHB and merge again, which we henceforth refer to as a second-generation BH. This process can continue, creating a chain of mergers (O’Leary et al. 2016; Fishbach et al. 2017; Gerosa & Berti 2017; Samsing & Ilan 2018; Antonini et al. 2019; Rodriguez et al. 2019a; Yang et al. 2019; Safarzadeh et al. 2019a; Baibhav et al. 2020; Doctor et al. 2020; Gayathri et al. 2020; Gerosa et al. 2020; Kimball et al. 2020). Such hierarchical mergers allow BHs to reach masses above what is achievable through stellar evolution. In particular, it has been shown that GCs are effective nurseries to host such hierarchical mergers, and that populating the mass gap is possible through this dynamical channel (Gerosa & Berti 2017; Samsing & Hotokezaka 2020).

While intermediate-mass black holes (IMBHs) with $M_{\text{IMBH}} \gtrsim 100 M_\odot$ have not been reported to originate in dynamical simulations of hierarchical BHB mergers, indicating such a scenario is uncommon, recent work suggests that repeated mergers between BHs and very massive stars in young star clusters are a promising channel to form an IMBH within the first 15 Myr after star cluster formation (Rizzuto et al. 2020). One of the main barriers to forming an IMBH through hierarchical mergers is the potentially large gravitational recoil velocities of merger products (Peres 1962; Bekenstein 1973; Fitchett & Detweiler 1984; Campanelli et al. 2007; Lousto et al. 2010; van Meter et al. 2010). Such recoil velocities can exceed a typical cluster’s escape velocity, ejecting the second-generation BH and preventing future mergers. Although full calculations for gravitational recoil require numerical relativity, researchers have derived analytic formulæ that can reasonably approximate the recoil behavior seen in numerical simulations (Lousto et al. 2010; van Meter et al. 2010).

Recently, various research groups investigated repeated BH mergers within star clusters with the inclusion of gravitational merger recoil, either through Monte Carlo (Rodriguez et al. 2019b) or direct $N$-body methods (Banerjee 2020, 2021). In our work, gravitational recoil is not calculated while the simulations run, but it is instead implemented via postprocessing. This choice offers a hybrid solution that affords benefits of a Monte Carlo approach while having a direct $N$-body code for highly accurate stellar dynamics. In particular, we are able to turn recoil kicks on and off to explicitly investigate the effects of recoil on BHB merger chains. We are also able to investigate the distribution of possible recoil velocities by sampling over different BH spin orientations for a given merger, letting us explore the probability of finding merger chains of different lengths within GCs. The latter would be particularly challenging to investigate otherwise because of the limited number of BH mergers in a single simulation and the computational cost of running direct $N$-body simulations.

In this paper, we use a set of direct $N$-body simulations of dense star clusters that include both gravity and stellar evolution (Anagnostou et al. 2020) to investigate hierarchical BHB mergers as a potential formation channel for IMBHs within GCs over timescales comparable to their typical ages (i.e., $\sim 10$ Gyr). In Section 2, we briefly describe our simulation framework, including initial conditions and our treatment of gravitational recoil. In Section 3, we explore in detail one simulation that shows the evolution of a specific BH—henceforth referred to as “Snowball”—undergoing seven BHB mergers in 6.5 Gyr to reach a final mass $m_{\text{BH}} = 97.8 M_\odot$. In Section 4, we discuss the effects of gravitational recoil on the viability of such a merger chain, highlighting limitations in the calculations. From the length distribution of merger chains within the full set of simulated clusters, in Section 5 we estimate the comoving rate of IMBH formation in a typical GC. Finally, in Section 6 we discuss implications of this formation channel for the GW190521 detection, as well as for future gravitational wave detections.

2. Method

2.1. Simulations

This work is based on a set of 58 direct $N$-body simulations of mid-sized star clusters previously presented in Anagnostou et al. (2020), run using NBODY6 with graphics processing unit (GPU) support (Nitadori & Aarseth 2012). The set of simulations contains $5 \times 10^4 \leq N \leq 2 \times 10^5$ initial stars, and utilizes a range of initial conditions (see Anagnostou et al. 2020 for more detail).

Single stars and binaries evolve according to the “SSE” and “BSE” algorithms, respectively (Hurley et al. 2000, 2013), which are designed to handle complex evolutionary processes, including common envelope evolution and mass transfer, collisions and supernova kicks. The version of SSE available in our simulation code utilizes the Hurley et al. (2000) supernova scheme, which is based on work done by Eggleton et al. (1989). Under these prescriptions SSE calculates BH birth masses based on the formulation given by Belczynski et al. (2008), whereby BH mass is solely a function of the core mass of the progenitor star. Note that the code version we use does not include any prescriptions for oversized black holes, such as those presented in Spera et al. (2019). As stars evolve they lose mass and angular momentum due to stellar winds. These winds are calculated using the semiempirical prescriptions of Belczynski et al. (2010) whereby mass loss rate is a function of metallicity, stellar surface temperature, and luminosity. One of the key features of this model is a “bistability jump” that occurs at surface temperatures of $\sim 25,000$ K, which leads to a significant increase in mass loss due to an increase in the ratio of Fe III to Fe IV, which is a more effective wind driver. Binary orbital changes due to gravitational waves and subsequent merger times are calculated by integrating the orbit-averaged Peters equations (Peters 1964). NBODY6 does not include post-Newtonian corrections to the equations of motion.

In this paper we focus on a specific simulation, with the following specifications: initial number of particles $N_0 = 100,000$, primordial binary fraction $f = 0$, metallicity $Z = 0.001$, initial half-mass radius $r_{h,0} = 2.5$ pc and dimensionless King concentration parameter $W_0 = 7$. Stellar masses are drawn from a Kroupa (2001) initial mass function, between $0.08 M_\odot$ and $100 M_\odot$. The initial core radius is 0.56 pc with a tidal radius of 71.5 pc. The cluster initially underfills its tidal radius by a factor of three. The tidal radius is calculated by placing the cluster in a circular orbit around a point-mass galaxy with a galactocentric distance of 23.3 kpc. We also initialize the cluster in dynamical equilibrium and with no mass segregation. The cluster model is representative of an average mid-sized GC (see Heggie & Hut 2003 Table 1.1). The stellar metallicity of $Z = 0.001 ([Fe/H] \approx -1.33)$ is approximately the median metallicity for Milky Way GCs (Harris 1996). The cluster is evolved for a total of 12.5 Gyr.

Natal kicks are applied to neutron stars and black holes (no natal kicks are given to white dwarfs), drawn from a
Maxwellian distribution with velocity dispersion $\sigma_v \approx 1000$ m s$^{-1}$, the initial velocity dispersion of the cluster (see de Vita et al. 2019 for further explanation). Our choice of Maxwellian velocity dispersion is significantly smaller than the default \texttt{NBODY6} value of $\sigma_v \approx 265$ km s$^{-1}$, which is based on observations of pulsar proper motion (Hobbs et al. 2005). We choose to impart small natal kicks to create conditions such that a large fraction of BHs are retained in the model clusters after formation. This is justified as it somewhat mimics the effects of fallback-modulated BH natal kicks on BH retention statistics (Janka 2013; Repetto & Nelemans 2015; Banerjee et al. 2020). This treatment of natal kicks is the same for neutron stars and BHs, because the public version of \texttt{NBODY6} does not contain prescriptions for mass fallback or momentum-conserving BH natal kicks. One caveat of these small natal kicks is that an unusually large number of neutron stars are retained inside the host cluster after formation compared to similar studies (Chatterjee et al. 2017; Kremer et al. 2020, 2022; Ye et al. 2020). However, for most of a cluster’s lifetime neutron stars are too light to be part of the set of most massive particles that migrate to the cluster core. Hence the presence of neutron stars is unlikely to significantly affect the dynamics of the cluster core until very late in the cluster’s evolution. In addition, in the current study we are only interested in BHB interactions and mergers, and so the effects of the high retention fraction of neutron stars on our results are expected to be minimal. For information on the complete set of simulations, further detail on the code used, and full discussion of initial conditions see Anagnostou et al. (2020) and de Vita et al. (2019) (see also MacLeod et al. 2016).

2.2. Gravitational Recoil

Recent investigations implemented gravitational recoil calculations on-the-fly (Morawski et al. 2018; Rodriguez et al. 2018; Banerjee 2020). The public version of \texttt{NBODY6} does not include gravitational recoil nor the effects of BH spin, thus we added these additional physical ingredients in postprocessing. We calculate kick velocities using a numerical relativity-based fitting formula (detailed below) in a similar manner to Morawski et al. (2018), Rodriguez et al. (2018), and Banerjee (2020).

The underlying distribution of spin magnitude for stellar BHs is currently unknown and not well constrained. Two previous studies have simply assumed a uniform distribution of spin magnitude in which spins are randomly assigned at birth (Morawski et al. 2018; Rodriguez et al. 2018). Unlike those studies, we assign spin magnitudes using a fitting formula based on the MESA stellar evolution code, leading to a nonuniform distribution of spin magnitude (see Equation (4) in Belczynski et al. 2020 for the fitting formula). MESA calculates spins by modeling the loss of angular momentum from the progenitor core via stellar winds ( Paxton et al. 2010). The recoil velocities are then calculated using the analytic approximation (van Meter et al. 2010)

$$ V = V_{\perp m} \hat{\mathbf{e}}_1 + V_{\perp s} (\xi) \hat{\mathbf{e}}_1 + \sin(\xi) \hat{\mathbf{e}}_2 + V_{\parallel} \hat{\mathbf{e}}_3, $$

where the components perpendicular to the orbital axis are defined as

$$ V_{\perp m} = A \eta^2 \sqrt{1 - 4 \eta (1 + B \eta)}, $$

and

$$ V_{\perp s} = \frac{H - \eta^2}{1 + q} \left[ |\alpha_3^s| - q |\alpha^s_2| \right], $$

and the component parallel to the orbital axis is

$$ V_{\parallel} = \frac{K_2 \eta^2}{q + 1} (q |\alpha^s_1| \cos(\phi_1 - \Phi_1) - |\alpha^s_2| \cos(\phi_2 - \Phi_2)) $$

$$ + \frac{K_3 (q - 1) \eta^2}{(q + 1)^3} (q^2 |\alpha^s_1| \cos(\phi_1 - \Phi_1) - |\alpha^s_2| \cos(\phi_2 - \Phi_2)). $$

Here $A, K_2, K_3, \text{ and } K_4$ are constants (units: km s$^{-1}$), $B$ is a dimensionless constant, and $\xi$ measures the angle between the unequal mass and spin contributions to the recoil velocity in the orbital plane, and is taken to be 145$^\circ$ (Lousto et al. 2010). The mass ratio is defined as $q = m_2/m_1 \ll 1$, and the symmetric mass ratio as $\eta = q/(1 + q)^2$, where $m_1$ and $m_2$ are the primary and secondary black hole masses, respectively. Additionally, $\hat{\mathbf{e}}_1$ is the unit vector in the direction of the separation between the two BHs at the end of the last orbit prior to the final plunge, $\hat{\mathbf{e}}_2$ is the unit vector in the direction of the orbital axis, and $\hat{\mathbf{e}}_3 \equiv \hat{\mathbf{e}}_1 \times \hat{\mathbf{e}}_2$. The symbols $\parallel$ and $\perp$ refer to the components of the dimensionless spin vectors perpendicular and parallel to the orbital angular momentum, respectively. For example, $\alpha^s_i$ represents the projection of the $i$th black hole’s dimensionless spin onto the orbital plane, where $i = 1$ or 2. The symbol $\phi_i$ represents the angle between $\alpha^s_i$ and the displacement vector between the black holes, as measured at the end of the last orbit before the final plunge. The symbol $\Phi_i$ represents the amount by which this angle precesses before the merger. The $\Phi_i$ parameters are not constants because they depend on the specifics of a BHB inspiral, such as the mass ratio, initial separation, and spins. For the purposes of the current work, we set $\Phi_1 = \Phi_2 = 0$, as suggested by Lousto et al. (2010) for statistical simulations of BHB mergers. For more details on these analytic formulae, including parameter fitting, see van Meter et al. (2010).

Spin directions are expected to be randomly oriented for dynamically formed BHBs. The relative orientation of two BH spin vectors can have a significant impact on the recoil velocity. To deal with this large range of potential kick velocities based on spin orientation, we adopt a Monte Carlo method. For all mergers found by \texttt{NBODY6}, we randomly assign a spin direction to the merging BHs, drawing from an isotropic distribution. All other properties such as mass, initial separation, and spin magnitudes are fixed, as taken from \texttt{NBODY6} and MESA. We calculate the corresponding merger recoil velocity for the second-generation BH. If the resulting BH velocity (vector sum of the binary’s center-of-mass velocity prior to merger and the recoil velocity) exceeds the cluster escape velocity at the time of merger, then we exclude it from any future mergers, treating the BH as if it escaped the cluster. For any post-merger BH that is retained after recoil, we assign a new spin magnitude via numerical relativity-based fitting formulae presented in Rezzolla et al. (2008, Equations (8) and (10)). We repeat this postprocessing procedure $10^4$ times for the full merger history generated by each of the 58 simulations, in order to compute the probability of various escape and retention outcomes attached to every merger chain in every
simulation by assuming randomized spin. Because of the variable merger recoil velocities between calculations, the number of mergers is not always the same, as some hierarchical mergers can be cut short due to the ejection of one of the post-merger BHs.

This postprocessing implementation of recoil does come with some inherent disadvantages. Since we are not applying recoil velocities directly in \texttt{NBODY6}, our method only approximates the full effects of recoil kicks. Any post-merger BH that would be ejected due to recoil still remains in the cluster (as merger recoils are only calculated in postprocessing), and we simply ignore any future mergers involving it. Keeping these massive BHs in the cluster when they should be ejected does impact the dynamics of the cluster, particularly in the dense core. It may lower the probability of other BHB mergers, as BHs tend to primarily interact with the most massive BH in the core. However, this method does account for the most important impact of kicks when regarding hierarchical BHB mergers—the prevention of long merger chains due to the ejection of merger products. However, there are distinct advantages to calculating recoil outside of the N-body code. With this approach, we can enable and disable recoil kicks for our set of simulations—meaning we can directly assess the impact of merger recoils on the distribution of merger chain lengths. Furthermore, our Monte Carlo method for assigning spin magnitudes inherently requires postprocessing. In fact, when recoils are calculated within the simulation code, a single spin orientation must be selected, leading to a specific recoil velocity and specific outcome for the post-merger BH. Given the rarity of BH mergers in a single simulation and the computational resources required to run a single realization of a direct N-body simulation, it would be practically impossible to build sufficient information on the expected distribution of BH recoil events from run-time recoil implementation.

3. The Merger Chain without Recoils

We start by studying the merger chain without merger recoils caused by gravitational radiation reaction. This study serves two goals. (i) It provides a control experiment, or baseline, against which the strength and impact of the recoil effect can be assessed and quantified. Having such a benchmark allows us to directly investigate the impact of recoils in isolation by turning them on and off. This is important at a time when general relativistic fitting formulas for recoil outcomes are still under development and not yet settled. (ii) It yields an estimate of the maximum plausible mass of an IMBH like Snowball within our cluster models. This upper bound is important, because the physics of merger recoils remains a topic of active research in numerical relativity, and the recoil prescriptions predicted by theory are likely to be refined in the future. Note that Snowball’s final mass does not represent the true upper bound for the mass of a BH through repeated merger—it is merely an estimate for the upper bound in our cluster models.

In addition, the details of how a long merger chain occurs without recoil can still provide insight into chains with recoil, because long merger chains, although less likely, are still possible with gravitational recoil and should evolve via the same dynamical processes, as demonstrated explicitly in this paper. Understanding these dynamical processes is vital to progressing our understanding of repeated mergers.

3.1. Snowball’s History

Snowball starts as a 58.1 $M_{\odot}$ main-sequence star. After $\sim$6.4 Myr the star goes supernova and forms a 26 $M_{\odot}$ remnant BH, the second most massive BH produced through stellar evolution in the simulation. As this specific simulation does not include primordial stellar binaries, any BH that emerges must form dynamically through strong interactions. Remarkably, throughout the simulation Snowball is involved in nine mergers: seven BHB mergers, and two involving a main-sequence star (when a star-BH merger occurs, \texttt{NBODY6} treats the merger as if the BH fully consumes the star, with no mass loss or merger recoil), as shown in Figure 1. All seven BH mergers occur within the cluster core. This is expected, as Snowball spends 95% of its lifetime at $r \lesssim 0.7 r_{\text{c}}$ due to mass segregation resulting from energy equipartition in star clusters over the two-body relaxation timescale (see Trenti & van der Marel 2013). While in the core, Snowball tends to wander around the cluster center, as has been shown previously of IMBHs within GCs (de Vita et al. 2018). Figure 2 shows the position of Snowball as a function of its mass. The BH’s typical displacement from the cluster center appears to be independent of its mass. Snowball’s mass only changes by a factor of $\sim$4, and the expected BH displacement depends on $m_{\text{out}}^{0.44}$ (see Equation (16) in de Vita et al. 2018). Snowball gains mass through each successive BH merger until it reaches its final mass of 97.8 $M_{\odot}$. All nine mergers occur within the first 6.5 Gyr. Note that all BHB mergers in this simulation occur within this single branch, i.e., there are no BH mergers outside of this merger chain. Hence all Snowball merger companions are first-generation BHs. This is partly due to heavier BHs being dynamically favourable. Once Snowball becomes the most massive body in the cluster after approximately 230 Myr, it dominates the dynamics of the core, suppressing the retention of other BHs.

Once Snowball migrates to the core by $t \approx 11.7$ Myr, it experiences multiple close interactions with other mass-segregated BHs before forming its first BH, with an eccentricity of 0.92 and a period of $P = 99$ days. The binary remains in the core for 64 Myr, undergoing various three-body interactions, hardening (with the period reducing to 63 days) and altering the eccentricity before being disrupted, exchanging the secondary component for a 26.14 $M_{\odot}$ BH, the most massive BH in the cluster at that time. The lower panel in Figure 3 displays the evolution of Snowball’s binary eccentricity. It is clear that the many scattering events significantly alter the BHB eccentricity, with extremes of $e = 0.0008$ (near circular) and $e = 0.999$ (maximally eccentric). After five more three-body interactions the binary is again disrupted at $t \approx 230$ Myr, exchanging the 26.14 $M_{\odot}$ BH for a 25.34 $M_{\odot}$ BH and ejecting the heavier former companion from the cluster, making Snowball the most massive body in the cluster. The newly formed binary is fairly eccentric, with $e = 0.97$.

The process of forming a high-eccentricity BHB, undergoing multiple strong interactions and an eventual exchange event, occurs twice more, until Snowball forms its first merging BHB with a companion of $m_{\text{s}} = 23.2 M_{\odot}$, corresponding to a mass ratio of 0.89. The binary forms at $t = 258.7$ Myr with $P = 1732$ days and $e = 0.41$. After undergoing nine hardening events the binary separation is lowered sufficiently to allow for significant gravitational radiation, leading to the inspiral and eventual merger at $t \approx 300$ Myr. This merger nearly doubles Snowball’s mass to 37.1 $M_{\odot}$.
Soon after this first merger, at $t = 322$ Myr, Snowball forms a new BHB with a 23.34 $M_\odot$ companion at $r = 0.2r_c$. The remaining six BHB mergers follow a relatively similar evolution. In the top panel of Figure 3 we display the evolution of Snowball’s binary period. The seven BHB mergers are indicated with vertical black lines. The binaries often form with long periods that rapidly decrease through successive dynamical interactions with other BHs and bodies in the core. We indicate the 26 interactions that cause an exchange by dotted vertical green lines. The rate of BHB hardening tends to decrease as a binary evolves, first hardening rapidly and then slowly reducing the separation until an inspiral occurs, wherein most of a binary’s life is spent with small separations. This is partly because the cross section for three-body interactions of hard binaries is proportional to the semimajor axis (Celoria et al. 2018). Hence the hardening rate decreases as the binary orbit tightens.

Although most scattering events follow Heggie’s law (Heggie 1975), some soften the BHB involved instead. Most of these softening events occur due to an exchange. The attendant interval of chaotic three-body dynamics can dramatically alter the orbital parameters of the newly formed binary, leading to significant softening. This is most evident in the string of exchange events that occur at 400 Myr $\lesssim t \lesssim 500$ Myr, leading up to the second BHB merger. The top panel of Figure 3 shows the effects of such interactions, whereby the period varies nonmonotonically between 320 and 592 days.

Scattering events harden binaries and can impart significant recoil velocities that send even IMBHs out of the cluster core (de Vita et al. 2018). Anagnostou et al. (2020) found that $\sim 80\%$ of all ejections follow a three-body encounter with another BH, with the rest being ejected via binary–binary scattering. Although the various BHBs that count Snowball as a member experience significant three-body recoil, the resulting velocities are not sufficient to eject the binary. As can be seen in Figure 2, at most, the BHB is flung outside the cluster core, where it migrates back toward the center to undergo further scattering events. One particular interaction leads to a Snowball binary being flung out to 8.45$r_c$ $\approx 1.3$ pc when Snowball is $\sim 82\ M_\odot$. Although Snowball is never ejected, over the 12.5 Gyr approximately 67 binary–single interactions lead to the ejection of a BH. In particular, Snowball plays a large part in the depletion of BHs in the core, being responsible for ejecting 43 BHs. As the number of BHs in the core is depleted over time (starting from an initial $N_{BH} = 74$ formed from stellar evolution and retained in the cluster after natal kicks) the number of three-body hardening events likewise decreases.

Why is it that no other BHs merge with anything apart from Snowball in this cluster? Throughout the simulation, 10 other BHBs not containing Snowball form in the core; however, none remains bound long enough to either merge or be ejected, with...
the longest binary only surviving $\sim$31 Myr. All 10 binaries are disrupted by encounters with Snowball, or with a binary containing Snowball. Seven out of these 10 BHBs undergo an exchange encounter with Snowball, wherein the preexisting binary exchanges a component for Snowball, with the remaining three BHBs simply destroyed with no exchange. As a result of the exchange events, six of the seven newly unbound BHs are ejected from the cluster. Snowball’s presence in the core is essentially preventing the survival of any other BHB. Exchange encounters are more frequent within the first few gigayears when there is still a sufficiently high number of relatively massive BHs in the core, with only four out of 28 exchanges occurring after $t = 2$ Gyr. This is because exchange events, in general, require BHs of comparable mass (Heggie 1975). On the other hand, scattering events that do not result in an exchange remain relatively common, occurring every few million years at least until $\sim$10 Gyr, after which the BH population has mostly evaporated (see also Morscher et al. 2013 for a Monte Carlo investigation of the BH population in star clusters).

The specifics of Snowball’s mass gain depend on the mass of the companion BHs involved in the merger. However, our results clearly show that any BHB within a cluster core is likely to undergo multiple exchange events. Previous scattering experiments also show that the lighter of the two binary components is preferentially ejected from the system in favor of the intruder (Sigurdsson & Hernquist 1993), meaning that BHBs with mass ratios closer to one are dynamically favored. This process systematically increases the companion mass before merger. Hence, heavier BHs are more likely to merge than lighter ones, which is indeed what we see here. All seven Snowball BHB mergers involve a secondary BH that is in the heaviest 5% of all BHs at the time of the merger. This is why most of the BHs that merge with Snowball have masses between 20 and 25 $M_\odot$, as this is approximately the maximum mass of BHs produced by our cluster models and supernovae prescriptions.

3.2. Final State

By the time Snowball has completed the seventh BHB merger, there are only a total of 19 BHs left in the cluster. Snowball forms another binary at $t = 6.65$ Gyr, shortly after the last merger. Snowball spends the last 6 Gyr bound to its companion BH at the center of the cluster, only briefly leaving the core four times due to recoil from scattering events (spending a total of 0.032 Gyr out of 6 Gyr at $r > r_\odot$).

At $t \approx 7.9$ Gyr, this final binary forms a hierarchical triple system. The system undergoes Kozai–Lidov oscillations (Kozai 1962; Lidov 1962), driving the inner binary to as high as $e = 0.99$ (see the red highlighted section in Figure 3). During this time the inner binary still experiences 10 three-body scattering events. Throughout the next 2 Gyr, the outer component of the triple system is exchanged nine times, ejecting the previously bound outer component from the cluster in all but one of these cases. Eventually, the BHB triple system is disrupted at $t \approx 9.94$ Gyr.

After 10.8 Gyr Snowball and its companion are the only remaining BHs in the cluster. During the last $\sim$200 Myr, the BHB once again forms a triple system with a white dwarf. This hierarchical triple remains until the end of the simulation.

4. The Merger Chain with Recoils

The results in Section 3 do not include merger recoils caused by gravitational radiation reaction. Such recoils are known to impart kicks comparable to or greater than the escape velocity of the GC (Campanelli et al. 2007; Lousto et al. 2010; Rodriguez et al. 2018; Morawski et al. 2018; Banerjee 2020). Our Monte Carlo method for assigning spin orientations allows us to investigate all possible recoil velocities for each merger so that we can determine how likely it is for the Snowball chain to reach a given length, a probability that is prohibitively expensive to sample adequately with in-run spin assignments. In Section 4.1, we explore statistically how merger recoils impede the formation of a heavy BH like Snowball. In Section 4.2 we discuss the impact of BH spin orientation on the retention or ejection outcome of a merger due to the resulting range of recoil velocities for each of the seven Snowball mergers.

4.1. Preventing the Formation of Snowball

Including merger recoil velocities prevents the formation of the 97.8 $M_\odot$ BH. The primary issue hindering such a merger chain is that all seven mergers involve a first-generation secondary BH. Hence the mass ratio becomes relatively biased.
toward low values after the initial merger, leading to significant recoil velocities for subsequent mergers in the chain. Only once the primary BH has grown massive enough to form a BHB with $q \lesssim 0.4$ can recoils once again be low enough for retention, as recoil velocity peaks at $q = 0.4$ (based on Equation (2)).

BH spins are assigned retroactively in postprocessing and are used to calculate BH merger recoils (see Section 2.2). The two first-generation BHs involved in the initial merger in the Snowball chain are assigned spin magnitudes of $\alpha_1 = 0.08$ and $\alpha_2 = 0.18$, based on the MESA code. From the merger of these two BHs, the resulting second-generation BH is ejected from the cluster in approximately 85.2% of the $10^4$ merger recoil calculations. In approximately 14.5% of the $10^4$ recoil calculations the second-generation BH merges again, and the resulting third-generation BH is ejected. In approximately 0.430% of the $10^4$ recoil calculations the third-generation BH merges and the resulting fourth-generation BH is ejected. The longest Snowball chain, comprising four mergers, occurs twice in $10^4$ recoil calculations.

4.2. Spin

The above results highlight the strong dependence of spin direction on merger recoil velocity. As discussed in Section 2.2, we calculate $10^4$ different recoils for each $\text{NBODY6}$ merger, fixing the mass ratio and spin magnitude but randomly assigning spin directions. As a pedagogical device, Figure 4 shows the histogram of possible recoil velocities for each of the seven mergers in the original, zero-recoil Snowball chain, ignoring temporarily that recoils truncate the chain after a maximum of four mergers. As the mass ratio decreases with subsequent mergers, larger recoils become more likely, until $q$ drops below 0.4, as discussed above. The BHs produced through mergers also tend to have significantly higher spin magnitudes than those formed through core collapse, because the majority of the BHB orbital angular momentum is transferred to the spin of the merger product. Higher spin magnitudes increase the impact of spin directions on the final velocity. From Figure 4 we see why the Snowball chain never extends past four mergers, as the minimum recoil velocity of the fourth merger is $\sim 129 \text{ km s}^{-1}$ (green line in Figure 4), far above the host cluster’s central escape velocity. The recoil distribution for the seventh merger (brown line in Figure 4) displays a uniquely narrow range of possible velocities due to the extreme mass ratio of $q = 0.089$. The histogram peaks at $<10 \text{ km s}^{-1}$, with a median of 6.14 km s$^{-1}$ and a standard deviation of 4.83 km s$^{-1}$. However, due to the larger recoils from earlier mergers in the chain, Snowball would be ejected long before it could undergo this seventh merger.

Although gravitational recoils eject Snowball after at most only four mergers, longer merger chains can occur rarely. One could foresee a scenario whereby an initial merger forms a BH that is not only retained but is also substantially larger than the remaining BH population. This massive second-generation BH could then merge multiple times with relatively light BHs without being ejected, as the low mass ratios would lead to low recoil velocities.

5. Merger Chain Statistics

Although the Snowball chain only reaches seven mergers if we ignore the effects of gravitational recoil, it is still instructive to discuss statistics of such a chain under idealized conditions, as a baseline benchmark against which simulations with recoil can be compared as the understanding of recoil physics develops in the future. Generally, the no-spin case represents the optimal path to form a massive BH via repeated mergers for a system with a shallow potential such as a GC, hence this scenario gives an upper limit to the probability of finding an IMBH through repeated mergers. Current models for gravitational recoils are only approximate because they rely on fitting and/or extrapolation to provide a general analytical solution to an intrinsically complex numerical relativity problem (Campanelli et al. 2007; Lousto et al. 2010; van Meter et al. 2010; Zlochower & Lousto 2015). By comparing results from this section with future GW detections of massive BHs, it will be possible not only to constrain dynamical formation pathways, but also to falsify theoretical parameterizations of BH recoil in a specific astrophysical context.

To estimate the likelihood of encountering Snowball-like chains in typical compact star clusters, we consider our full set of $N = 58$ simulations and construct a probability density function of merger chain length. We define the merger chain length $L$ as the total number of BH mergers involved in producing the final multi-generation BH. Based on this definition, Snowball has a chain length of seven. We also denote the generation of a BH as $\text{BH}_{\text{gen}}$. Across all 58 simulations, we observe 18 chains with $L = 1$, 28 chains with $2 \leq L \leq 4$, and Snowball with $L = 7$.

5.1. Chains with and without Recoil

Our postprocessing implementation of gravitational recoil allows us to directly compare the merger chain statistics for recoil enabled and disabled. Table 1 lists the number of merger chains both with and without gravitational merger recoil taken into account in postprocessing. Without recoil, there are a total of 97 mergers. We highlight the presence of four chains of length $L = 4$, the absence of chains with $5 \leq L \leq 6$, and the existence of Snowball with $L = 7$. Moreover, only three chains involve two BHs with $\text{BH}_{\text{gen}} > 1$, because BHs with $\text{BH}_{\text{gen}} \geq 2$
correspond to 97, 70, and 58 BHB mergers for no-recoil, recoil, and spin recoil, respectively. This leads to two extra chains.

We report the number of chains both with and without gravitational recoil taken into account. Note that the numbers listed for the Recoil (spin) case are when we simply select the most likely recoil velocity for each merger. The total number of merger chains increases to 49 when including spin. The increase is a result of the two \( L = 4 \) chains, which involve the merger of \( \text{BH}_{\text{gen}} = 2 \) and \( \text{BH}_{\text{gen}} = 3 \) BHs (which are present in the no-recoil and recoil (no spin) cases) being cut short, as the \( \text{BH}_{\text{gen}} = 3 \) object is ejected due to the high merger recoil. This leads to two extra \( L = 2 \) chains. The merger chains correspond to 97, 70, and 58 BHB mergers for no-recoil, recoil, and spin recoil, respectively.

constitute a small fraction of the BH population. The number of \( L > 1 \) chains is greatly reduced when including recoil, although without spin there are still three \( L = 3 \) and \( L = 4 \) chains. There are only 70 BH mergers with recoil (no spin) included. To account for the effects of spin orientation on merger recoil, we simulate each of the 47 merger chains 10,000 times (see Section 2.2). Figure 5 displays \( P(L) \), the percentage of chains with length \( L \) from our Monte Carlo method. We find that approximately 99.3% of the chains reach \( L = 1 \), approximately 0.719% reach \( L = 2 \), approximately 0.0112% reach \( L = 3 \), and approximately \( 3.06 \times 10^{-4} \) reach \( L = 4 \). We find no \( L \geq 5 \) chains. The third column in Table 1 displays the number of chains if the most probable recoil velocity for each merger is chosen.

This is expected, as longer chains are less likely regardless of gravitational recoil, because longer chains are multiplicative in nature. At the most basic level, we expect the probability of a certain chain length to follow

\[
P(L) \propto P_0 \cdot L^p,
\]

where \( P_0 \) is the probability of getting any \( L > 1 \) chain. Notably, the functional form of Equation (5) is consistent with Figure 5. This explains the rarity of longer merger chains. Fitting to the zero-recoil case, we estimate a probability of a Snowball merger to be \( P(L = 7) \approx 0.022 \), consistent with observing its occurrence once out of 47 chains. If we instead fit for the recoil case without spin, we can loosely extrapolate out to \( L = 7 \), finding \( P(L = 7) \approx 1.4 \times 10^{-4} \). For the case with spin, we can extrapolate from our results in Figure 5, finding \( P(L = 7) \approx 2.5 \times 10^{-12} \).

Without recoil, approximately half of the 97 in-cluster mergers only involve BHs produced directly through core collapse. 18 of the resulting second-generation BHs never merge again, increasing to 33 when merger recoils are accounted for (without BH spin). 47 mergers involve a BH with \( \text{BH}_{\text{gen}} \geq 2 \). In 36 of these mergers \( m_2 \) is a first-generation BH, with \( m_1 \) being the multi-generational object. This is expected, because hierarchically formed BHs are on average heavier than those formed through core collapse, and thus are more likely to be the heavier element in a BHB. Mergers involving an object with \( \text{BH}_{\text{gen}} \geq 2 \) involve a first-generation BH merging with the higher-generation BH, with only three exceptions. One exception is a merger between two second-generation BHs, and the other two are between a second- and a third-generation BH. These three mergers represent the only such mergers between two previous merger remnants. When we take into account BH spins, the only merger between two multi-generational BHs is the merger between the two second-generation BHs. Hence we conclude two or more chains are unlikely to exist simultaneously within a mid-sized GC (\( \sim 10^5 \) stars). This is expected, because once a BHB merges, its product is likely to be the most massive body in the cluster, and thus to sink to the cluster center due to dynamical friction, even if merger recoil is imparted (as long as the resulting BH remains bound). The massive BH dominates dynamically within the dense core, likely undergoing many dynamical interactions with other single and binary BHs. Hence, any BHB that forms is likely to interact with the second-generation BH, in which case an exchange event will preferentially unbind \( m_2 \), forming a new BHB with the second-generation BH (Sigurdsson & Hernquist 1993). It is important to note that recoil velocity is approximately independent of the total binary mass, but does depend on \( q \). Hence light and heavy mergers have the same chance of reaching a given escape velocity.

Including recoil significantly increases the number of post-merger BH ejections, resulting in most longer merger chains no longer being viable. The \( L = 7 \) chain no longer occurs, now only reaching a chain length of four at most before the BH is ejected. Even without spin, the recoil is often large enough to eject the majority of the 50 second-generation BHs, with only 23 being retained, 17 of which undergo further mergers. Only five of the 13 third-generation BHs are retained, all of which merge again. This suggests that any retained BHs with \( \text{BH}_{\text{gen}} \geq 1 \) have a high chance of merging again. Including spins in the recoil calculations tends to increase the size of the kicks, with now only approximately 0.731% of the second-generation BHs retained after formation, \( \approx 73.9\% \) of which

---

**Table 1**

| \( L \) | No recoil | Recoil (no spin) | Recoil (spin) |
|--------|-----------|-----------------|--------------|
| 1      | 18        | 33              | 41           |
| 2      | 16        | 8               | 7            |
| 3      | 8         | 3               | 1            |
| 4      | 4         | 3               | 0            |
| 5      | 0         | 0               | 0            |
| 6      | 0         | 0               | 0            |
| 7      | 1         | 0               | 0            |
| Total  | 47        | 47              | 49           |

**Note.** We report the number of chains both with and without gravitational recoil taken into account. Note that the numbers listed for the Recoil (spin) case are when we simply select the most likely recoil velocity for each merger. The total number of merger chains increases to 49 when including spin. The increase is a result of the two \( L = 4 \) chains, which involve the merger of \( \text{BH}_{\text{gen}} = 2 \) and \( \text{BH}_{\text{gen}} = 3 \) BHs (which are present in the no-recoil and recoil (no spin) cases) being cut short, as the \( \text{BH}_{\text{gen}} = 3 \) object is ejected due to the high merger recoil. This leads to two extra \( L = 2 \) chains. The merger chains correspond to 97, 70, and 58 BHB mergers for no-recoil, recoil, and spin recoil, respectively.

**Figure 5.** The percentage of chains with length \( L \) from our Monte Carlo method used to calculate recoil velocities with randomized spin in postprocessing, plotted on a vertical log scale. Approximately 99.3% of the chains reach \( L = 1 \), approximately 0.719% reach \( L = 2 \), approximately 0.0112% reach \( L = 3 \), and approximately \( 3.06 \times 10^{-4} \) reach \( L = 4 \). There are no chains with length \( L \geq 5 \).
merge again. Approximately 85% of the resulting $L = 3$ chains correspond to a merger between two BH$_{\text{gen}} = 2$ objects, forming a 58 $M_\odot$ BH. This is likely the main way an $L = 3$ chain can occur in GCs of these sizes, as the other pathway involves a BH$_{\text{gen}} = 1$ object merging with a BH$_{\text{gen}} = 3$ object (as in the Snowball merger chain). This pathway is less likely because it requires the retention of a BH$_{\text{gen}} = 3$ object, which only occurs approximately 0.450% of the time in the case of Snowball’s merger chain. If we take the most likely recoil for each merger based on all spin orientations, we find only 58 BHB mergers, meaning that only 3.3% of all the BHs that remain bound after natal kicks end up merging.

### 5.2. Estimation of Snowball Event Rate

The results in Section 5.1 can be converted into a rough estimate of the IMBH formation rate in GCs. First we consider the no-recoil event rate as a baseline estimate. To estimate the comoving rate of Snowball-like merger chains to an order of magnitude, we can use the equation

$$R_{\text{Snowball}} \approx R_\ast f_{\text{GC}} f_{\text{Snowball}} / M_{\text{GC}}, \quad (6)$$

where $R_\ast$ is the comoving star formation rate (units: $M_\odot$ yr$^{-1}$ Mpc$^{-3}$) at the characteristic time of star cluster formation and within the volume observed by LIGO, $f_{\text{GC}} \approx 10^{-3}$ is an order-of-magnitude estimate of the fraction of star formation that occurs within a dense star cluster, $f_{\text{Snowball}}$ is the fraction of those clusters that are likely to produce Snowball-like merger chains, estimated from our simulations, and $M_{\text{GC}} \approx 6.5 \times 10^4 M_\odot$ is the characteristic mass of a dense star cluster in our runs.

A detailed estimation of $R_{\text{Snowball}}$ would require us to take into account both the redshift dependence of $R_\ast(z)$ and the distribution of time delays between the formation of the cluster and a chain merger event. However, uncertainties in $f_{\text{GC}}$, $f_{\text{Snowball}}$, and $M_{\text{GC}}$ dominate Equation (6), thus to remain in the spirit of an estimate to only an order of magnitude, we can neglect redshift dependence for $R_\ast$, adopting an appropriate average value for it, and assume the distribution of merger time delays is a delta function at the cluster’s age. Given that LIGO is sensitive to high-mass merger events out to $z \sim 1–1.5$ (Chen et al. 2017), and that most of the volume is thus at cosmological distances, we assume $R_\ast \approx 0.1 M_\odot$ yr$^{-1}$ Mpc$^{-3}$, corresponding to the comoving star formation rate at $z \approx 1.2$ from Madau & Dickinson (2014, Figure 9, left panel). We also neglect the effects of a term representing cluster birth mass function. For our basic estimate, we simply ignore clusters with stellar masses smaller than our simulations, and use characteristic values for the population of globular clusters in the Milky Way. To an order of magnitude, a more detailed calculation would give the same result, because although $f_{\text{Snowball}}$ would be smaller in lower-mass clusters, one would have to increase $f_{\text{GC}}$ to reflect the fact that a greater proportion of star formation is considered in the calculation if lower-mass clusters are included.

Based on the results presented in Table 1, we assume $f_{\text{Snowball}} \approx 0.02$ (one IMBH chain event out of over 50 simulations). A more precise characterization is challenging from a single chain instance discovered in our set of simulations. Given these assumptions, we estimate the Snowball event rate to be

$$R_{\text{Snowball}} \approx 0.03 \text{ Gpc}^{-3} \text{ yr}^{-1}. \quad (7)$$

This is within the 90% confidence interval of the inferred rate of mergers similar to GW190521, estimated by Abbott et al. (2020d) as $0.13^{+0.3}_{-0.11} \text{ Gpc}^{-3} \text{ yr}^{-1}$. We can take $f_{\text{Snowball}} \approx 0.02$ as an estimate for the upper limit for our cluster models. The fact that this upper limit does not lead to an event rate exceeding the observed LIGO rate is reassuring and qualitatively supports the plausibility of IMBH formation through chain mergers in dense stellar systems, but of course, a deeper quantitative investigation is needed to further assess the scenario we propose.

Of course, the above agreement is only for the special baseline case without merger recoil. Assuming the rough estimate for a Snowball chain rate of $P(L = 7) \approx 10^{-12}$ with spin recoil, presented in Section 5.1, we obtain $f_{\text{Snowball}} \approx 0.02/(8.8 \times 10^3) = 2.3 \times 10^{-12}$, and hence

$$R_{\text{Snowball}} \approx 3.5 \times 10^{-12} \text{ Gpc}^{-3} \text{ yr}^{-1}. \quad (8)$$

Although this event rate is orders of magnitudes below even the lower bound of the inferred LIGO rate, an underprediction is expected, as there are other potential GW190521 formation channels. Our estimates are also based on our cluster models, and one would expect $f_{\text{Snowball}}$ to be larger for more massive clusters with larger escape velocities. Note that Equation (8) is simply based on Equation (5), extrapolated out to $L = 7$.

### 6. Discussion

In this work we analyze the hierarchical mergers of BHBs formed within simulated GCs, investigating the dynamical formation of an IMBH with and without merger recoils. The analysis is based on a set of direct NBODY6 simulations of realistic median-sized cluster models previously presented in Anagnostou et al. (2020). We present a detailed investigation into the evolution of “Snowball,” a BH that undergoes seven BHB mergers in a baseline system excluding merger recoil, attaining a final mass of $\sim 98 M_\odot$. We also explore the effects of gravitational merger recoil on this Snowball merger chain, and on the prevalence of merger chains of different lengths across our 58 simulations. Through our Monte Carlo method of calculating merger recoils, we estimate the probabilities of merging chains reaching a given length.

BH spins can lead to recoil velocities above $10^3$ km s$^{-1}$ (Campanelli et al. 2007), well above the central escape velocity of any known GC. Hence merger chains with $L > 2$ are unlikely, as backed up by our results. Based on Table 1, we predict that median-mass GCs are more likely to host a single $L > 1$ merger chain than multiple chains. We therefore conclude that long chains of seven or more mergers are too rare to explain the full rate inferred by LIGO data, without substantial modifications to the currently accepted fitting formulas for BH recoil, or some other modification to the simulation physics unrelated to recoil. However, we do find a small percentage of $L = 3$ and $L = 4$ chains, indicating that even with the current analytical formulation of BH recoils, there is a region of the parameter space of initial conditions (BH masses, spins) where the resulting recoil is insufficient to eject the merger product. The majority of the $L = 3$ chains involve the merger between two second-generation BHs. If we use our estimated probability for an $L = 3$ chain of $P(L = 3) = 4.95 \times 10^{-5}$, we estimate $R_{L=3} \approx 1.5 \times 10^{-4}$ Gpc$^{-3}$ yr$^{-1}$. This is a potential
formation pathway for GW190521. Although due to our initial conditions and supernovae prescriptions the second-generation BHs in these $L = 3$ chains are lighter than the primary and secondary masses of GW190521, a similar chain with heavier initial first-generation BHs could potentially explain the heavy merging masses observed by LIGO/Virgo. In fact, through the analytic formulae for gravitational recoil presented in Section 2.2 (Equations (1)–(4)) and merger mass loss (Lousto et al. 2010), we find that GW190521 could have formed in a median-mass GC from an $L = 3$ merger chain. In this scenario, both the primary and secondary components are second-generation BHs that have been retained post-merger due to sufficiently low recoil velocities (like the $L = 3$ chain we observed in our simulations), while still achieving the required mass observed by LIGO.

The scenario above requires fine-tuning of a specific set of mass and spin combinations for the precursor first-generation BH. We can calculate the recoil and mass losses for all realistic combinations of BH mass ratio and spin, and find the proportion of combinations that lead to a retained post-merger BH matching the masses of the GW190521 primary and secondary BHs. Not all combinations are viable, as binaries with low mass ratio are dynamically unfavourable, due to being more likely to be disrupted via exchange events. To account for this we restrict our calculation to mass ratios above 0.4, as this is the lowest mass ratio of any first- and secondary mass combination leading to retention of a sufficiently massive primary ($\sim 85 M_\odot$), and 0.38% of combinations for a secondary ($\sim 66 M_\odot$). Although the chance of both mergers occurring in the same cluster and the products then merging again in an event such as GW190521 is small, this is still a viable formation scenario. The set of viable combinations is increased if the cluster has a higher escape velocity, which likewise increases the likelihood of such a merger chain. If we instead set the escape velocity at $50 M_\odot$, the proportion of viable mass and spin combinations increases to 1.5% and 1.7% for the primary and secondary masses, respectively. This may suggest that GW190521 originated in a star cluster more massive than those we simulated.

On average across our 58 simulations, the last merger in chains with $L > 1$ occurs at $t = 4735$ Myr. Thus it appears that hierarchical mergers in relatively young GCs ($t \lesssim 6$ Gyr) are preferred, when there is still a sufficient population of BHs in the core. If true, this means that hierarchical mergers are more likely to occur at cosmological distances, when the star formation rate is higher and there was a higher proportion of young GCs. In principle, such distant sources are more difficult to detect. However, the larger average chirp mass from hierarchical mergers leads to a larger gravitational strain.

The hierarchical merger pathway provides a natural method for forming BHs within the BH mass gap, without the need for new physics, but that does not mean that such new physics is absent. Although gravitational recoil lowers the likelihood of longer merger chains occurring, the formation of multi-generation BHs, including those similar to the final BH of GW190521 is still possible within mid-sized GCs. Regardless of the origin of GW190521, this remarkable detection represents the first confirmed detection of an IMBH and will likely spark increased interest in the formation and evolution of these stellar behemoths.

We would like to thank Morgan Macleod for useful discussion. Parts of this research are supported by the Australian Research Council Centre of Excellence for Gravitational Wave Discovery (OzGrav) (project number CE170100004). Numerical simulations have been performed on HPC clusters at the University of Melbourne (Spa) and Swinburne University (OzSTAR).

**ORCID iDs**

Michele Trenti @ https://orcid.org/0000-0001-9391-305X

**References**

Abbott, B. P., Abbott, R., Abbott, T. D., et al. 2016a, PhRvL, 116, 061102
Abbott, B. P., Abbott, R., Abbott, T. D., et al. 2016b, PhRvX, 6, 041015
Abbott, B. P., Abbott, R., Abbott, T. D., et al. 2016c, PhRvL, 116, 241103
Abbott, B. P., Abbott, R., Abbott, T. D., et al. 2017a, PhRvL, 118, 221101
Abbott, B. P., Abbott, R., Abbott, T. D., et al. 2017b, ApJ, 851, L35
Abbott, B. P., Abbott, R., Abrahams, S., et al. 2020a, ApJL, 896, L44
Abbott, R., Abbott, T. D., Abrahams, S., et al. 2020b, PhRvD, 102, 043015
Abbott, R., Abbott, T. D., Abrahams, S., et al. 2020d, ApJ, 900, L13
Anagnostou, O., Trenti, M., & Melatos, A. 2020, PASA, 37, 44
Antonini, F., Gieles, M., & Gualandris, A. 2019, MNRAS, 486, 5008
Baibhav, V., Gerosa, D., Berti, E., et al. 2020, PhRvD, 102, 043002
Banerjee, S. 2020, MNRAS, 500, 3002
Banerjee, S. 2022, A&A, 665, 20
Banerjee, S., Baumgardt, H., & Kroupa, P. 2010, MNRAS, 402, 371
Banerjee, S., Belczynski, K., Fryer, C. L., et al. 2020, A&A, 639, A41
Bargery, V., Han, T., Kao, C., & Zhang, R.-J. 1999, PhLB, 461, 54
Bekenstein, J. D. 1973, ApJ, 183, 657
Belczynski, K., Bulik, T., Fryer, C. L., et al. 2010, ApJ, 714, 1217
Belczynski, K., Heger, A., Gladysz, W., et al. 2016, A&A, 594, A97
Belczynski, K., Kalogera, V., Rasio, F. A., et al. 2008, ApJS, 174, 223
Belczynski, K., Klencki, J., Fields, C. E., et al. 2020, A&A, 636, A104
Campbell, M., Lousto, C., Zlochower, Y., & Merritt, D. 2007, ApJ, 659, L5
Cerolino, M., Oliveri, R., Sesana, A., & Mapelli, M. 2018, arXiv:1807.11489
Chatterjee, S., Rodriguez, C. L., & Rasio, F. A. 2017, ApJ, 834, 68
Chen, H.-Y., Holz, D. E., Miller, J., et al. 2021, CQGra, 38, 055010
Choksi, N., Gnedin, O. Y., & Li, H. 2018, MNRAS, 480, 2343
Collaboration, T. L. S., et al. 2020, the Virgo Collaboration, the KAGRA Collaboration, et al. 2021, arXiv:2111.03606
Costa, G., Bressan, A., Mapelli, M., et al. 2020, MNRAS, 501, 4514
Croon, D., McDermott, S. D., & Sakstein, J. 2021, PDU, 32, 100801
de Vla, R., Trenti, M., & MacLeod, M. 2018, MNRAS, 475, 1574
de Vla, R., Trenti, M., & MacLeod, M. 2019, MNRAS, 485, 5752
Di Carlo, U., Mapelli, M., Bouffanais, Y., & Bouffanais, Y., et al. 2020, MNRAS, 497, 1043
Doctor, Z., Wysocki, D., O’Shaughnessy, R., Holz, D. E., & Farr, B. 2020, ApJ, 893, 35
Egginton, P. P., Fitchett, M. J., & Tout, C. A. 1989, ApJ, 347, 998
Farmer, R., Renzo, M., de Mink, S., Fishbach, M., & Justham, S. 2020, ApJ, 902, 36
Farmer, R., Renzo, M., de Mink, S. E., Marchant, P., & Justham, S. 2019, ApJ, 887, 53
Fishbach, M., Holz, D. E., & Farr, B. 2017, ApJ, 840, L24
Fitchett, M. J., & Detweiler, S. 1984, MNRAS, 211, 933
Gayathri, V., Bartos, I., Haiman, Z., et al. 2020, ApJ, 890, L20
Gerosa, D., & Berti, E. 2017, PhRvD, 95, 124046
Gerosa, D., Vitale, S., & Berti, E. 2020, PhRvL, 125, 101103
Hafès, W. E. 1996, ApJ, 462, 446
Heeger, A., Friedlund, A., Giannotti, M., & Cirigliano, V. 2009, ApJ, 696, 608
Heeger, A., Fryer, C. L., Weisz, S. E., Langer, N., & Hartmann, D. H. 2003, ApJ, 591, 288
Heggie, D., & Hut, P. 2003, The Gravitational Million-Body Problem: A Multidisciplinary Approach to Star Cluster Dynamics (Cambridge: Cambridge Univ. Press)
Heggie, D. C. 1975, MNRAS, 173, 729
