Analysis of point cloud data of tunnel cross-section using cubic B-spline curve

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Abstract. In order to overcome the problem of extraction and deformation information analysis of tunnel cross-section by 3D laser scanning technology in tunnel engineering application, a method of extraction and fitting of tunnel cross-section based on cubic B-spline curve was proposed. First, obtain the initial cross-sectional point cloud by using the cross-sectional intercept method based on the spatial point cloud subset; Then, the cross-section was fitted with a cubic B-spline curve, and the missing part of the point cloud was repaired automatically. In the end, specific points are extracted from the fitted cross-section point cloud to realize cross-section deformation analysis. And cross-section analysis of point cloud data of a section of subway tunnel shows that this method can quickly and accurately analyze the deformation information of the tunnel cross-section.

1. Introduction

The cross-sectional survey is an indispensable key technology in the construction of subway tunnels or other underground projects. After the completion of the tunnel, in addition to the completion survey, it is necessary to regularly survey the cross-section of the tunnel, monitor and analyze the deformation of the tunnel, so as to secure the safety of the tunnel. There are many kinds of tunnels in our country, and the engineering geology is complicated, the construction environment is harsh, and the requirement of tunnel survey is also very high. At present, the traditional method of tunnel section measurement in China is mainly total station instrument, section instrument and follow-up data processing software, which requires more surveyors, long working hours, heavy workload and low efficiency, and insufficient data density. Especially in the face of large elongated tunnel, there are great limitations. How to obtain a large number of point cloud data with high quality and high efficiency is a major problem to be solved by tunnel construction units at present. The extraction, fitting and deformation analysis of tunnel point cloud section have become urgent problems to be resolved at present.

Research scholars at home and abroad of 3D laser scanning technology [1-2] in tunnel deformation analysis to do a mass of work and many achievements, Rinskevan Gosliga et al. performed cylindrical fitting analysis on the tunnel[3], and Walton et al. also used the least square algorithm to fit the tunnel to the ellipse[4], the tunnel profile fitting method is normally assumed round curve or elliptic curve, but these methods cannot fitting non-circular and elliptic tunnel, even in the circle (ellipse) tunnel, tunnel deformation, also can produce irregular is not applicable. Therefore, in this paper, the cubic B spline
curve is adopted to extract the horseshoe tunnel cross-section of fitting, and the introduction of point cloud missing fitting method for adaptive threshold, eventually to extract the feature points to analyze the deformation of cross-section.

2. Materials and Methods

2.1. Point cloud data pre-processing
In order to obtain the point cloud data of the entire tunnel, multiple stations need to be set up for scanning. In the meantime, there will inevitably be some noise when point cloud data are acquired, which will seriously affect the accuracy and processing efficiency of subsequent data analysis. In addition, the original point cloud data should be compressed for specific applications to improve the efficiency of subsequent data processing. The pre-processing of tunnel point cloud data mainly includes point cloud data registration, denoising and compression.

2.2. Tunnel cross-section extraction
In three-dimensional space, the place where the cross-section point cloud of the tunnel is located is a three-dimensional plane perpendicular to the axis of the tunnel at the intersection point. Therefore, when the principal axis of the tunnel is determined, the attitude of the tunnel can be expressed effectively, and then the cross-section of the tunnel can be extracted directly. Considering that the method of intercepting the subset of cross-sectional point cloud of tunnel point cloud has high precision and is easy to be realized, this method is adopted to intercept the initial cross-sectional area at the target mileage.

Firstly, integrating the center line with the specified mileage is a so important step to find the position of the center line, and then the program can easily extract the section point the position according to the attitude information of the center line for the position. As showed in figure signal, to extract the tunnel axis \( P \) (approximate \( y \)-axis direction) cross-sectional point cloud data, the tangent \( v \) at the point \( P \) is expressed as \( \begin{pmatrix} \frac{dx}{dy} & 1 & \frac{dz}{dy} \end{pmatrix} \), then calculate the tangent plane \( N \) at point \( P \), the tangent plane equation is:
In the above formula, \( k_p x + y + k_p' z - (y_p + k_p x_p + k_p' z_p) = 0 \) \( \ldots \) (1)

In the above formula, \( k_p = \frac{dx}{dy} \) \( y = y_p \) \( k_p' = \frac{dz}{dy} \) \( y = y_p \).

2.3. Adaptive fitting method of tunnel cross-section

The extracted tunnel cross-section is always a large number of discrete points. These points have accurate three-dimensional coordinates, but do not constitute a continuous line. There are missing or offset points in many feature points. Therefore, it is necessary to denoise. Curve fitting is performed on the point cloud of the section to construct a continuous section curve, which is closer to the real shape of the cross-section of the tunnel, and is also more convenient for the mathematical expression of the section to obtain the required feature points. Cubic B-spline curve can fit closed contours well \([5]\), and can fit sections effectively and truly.

Each segment of the segmented cubic B-spline curve is defined by four adjacent vertices. For section point fitting, it can be regarded as fitting a curve that is connected end to end by \( m \) points. The curve is composed of \( m \) segments of cubic B-spline curves are spliced together. So a tunnel section model based on cubic B-spline curve fitting is constructed.

B-spline function model:

\[
P_{i,n}(t) = \sum_{x=0}^{n} P_{i,x} \cdot F_{k,n}(t)
\] \( \ldots \) (2)

In the above formula, \( t \in [0,1], i = 0,1,2, \ldots, m, k = 0,1,2, \ldots, n, P_{i,x} \) is the control vertex, \( F_{k,n}(t) \) is the \( n \)-th B-spline basis function, When \( n=3 \), the expression of the first paragraph is:

\[
F_{0,3} = \frac{1}{6} (1-t)^3
\]
\[
F_{1,3} = \frac{1}{6} (3t^3 - 6t^2 + 4)
\]
\[
F_{2,3} = \frac{1}{6} (-3t^3 + 3t^2 + 3t + 1)
\]
\[
F_{3,3} = \frac{1}{6} t^3
\] \( \ldots \) (3)

The first cubic B-spline function is:

\[
P_{0,3}(t) = P_{0,0} F_{0,3} + P_{1,1} F_{1,3} + P_{2,2} F_{2,3} + P_{3,3} F_{3,3}, \quad t \in [0,1]
\] \( \ldots \) (4)

Taking into consideration the point cloud holes caused by the scanning angle of the station and the presence of obstructions, etc., the point cloud missing threshold \( r \) is set to identify the missing part of the point cloud. As showing in Figure 3. The main idea is to judge the missing side of the point cloud based on the distance of the point cloud adjacent to the azimuth angle, so as to optimize the fitting process. For the fitting of missing point cloud, the algorithm will adaptively increase the density of the fitting parameter \( r \), so that the fitting result has a higher balance, so as to realize the point cloud repair of the missing point cloud. This method of identifying missing point clouds is used for adaptive cross-section fitting, which effectively solves the problems of uneven cross-section and missing information.
caused by missing point clouds, and further realizes the optimization of the fitting results, and finally obtains adaptive fitting cross-section of the closed tunnel.

2.4. Extraction of designated points on tunnel cross-section

The designated point analysis of the tunnel cross-section is to obtain the deformation information of the cross-section data at a certain mileage in the tunnel. Different projects have different requirements for the deformation analysis points required by the tunnel cross-section. Generally, the points that need to be extracted are the tunnel apex, the lowest point of the tunnel, the intersection of the tunnel central axis section and the tunnel cross-section, etc., which have location characteristics and can reflect tunnel deformation information. The position of the specific point is adjusted according to the actual project.

The extraction method of the designated location point, in the completed section of the fitting, the highest point (the maximum $z$ value) is searched as the vertex, the reason that part of the track has been laid during scanning. And some tunnels cannot get the lowest point of the tunnel section, however, select the center line the lowest point serves as the bottom point. After obtaining the center point of the tunnel, according to the design drawing, take $n$ predetermined vertical distances up or down respectively to obtain $n$ points on the left and right sides, and obtain the three-dimensional coordinates of these characteristic points $\left(x_i, y_i, z_i\right)$, as shown in Figure 4.

![Figure 3. Point cloud missing threshold](image1)

![Figure 4. Schematic diagram of designated points on the cross-section](image2)

The vertex of the tunnel section can be used to analyze the settlement and deformation information about the section [6-7]. The lateral displacement of the tunnel section can be analyzed through the peripheral points of the rock wall of the cross-section. The local position of the section deformation can also be located according to the geometric relationship between the characteristic point and the track center. As showed in the figure, marked points include the arch vertices and the designated points on the tunnel wall. The local deformation information of the cross-section can be calculated by calculating the three-dimensional coordinates of these points. The height of the crack plane is given by the design parameters.

3. Results & Discussion

3.1. Continuous extraction of tunnel cross-section

For the reprocessed point cloud, continuous extraction of the tunnel cross-section is performed. In order to get the cross-section information of a specific location, the cross-section point cloud at that location must be extracted for analysis. In actual engineering, the entire tunnel can be extracted in detection, a
curved tunnel is usually taken at 5 meters, and a straight tunnel at 6 meters is taken. Of course, it will be adapted according to the actual situation of the tunnel. In order to verify this algorithm, a smaller distance and an interval of 2 meters are used for continuous extraction of the tunnel cross-section.

3.2. Adaptive fitting of tunnel cross-section
After the cross-section is intercepted, it is still in point cloud form. Since the subway tunnel section is not a regular curve such as a circle, an ellipse or a rectangle, in order to provide a more realistic section curve, the constraint conditions of the section fitting curve are not specified, but the cubic B-spline curve method is used to fit the section. A large number of experiments have demonstrated that the higher the degree of the piecewise fitting polynomial, the smoother the curve will be. Therefore, the higher degree polynomial will cause the fitting result to deviate greatly from the control vertex, and cannot reflect the original characteristic information; If it is too low, an effective fitting cannot be achieved and the cubic B-spline curve interpolation method has an ideal fitting effect. By writing an algorithm, the cross-section is fitted by interpolation between two points of the cross-section. On this basis, the point cloud missing threshold $\tau$ is used to determine the point cloud missing. The algorithm for the location of the larger point cloud missing will be adaptive. The interpolation density has been improved to fit the best section curve, which is convenient for analysis of convergence deformation.

3.3. Information extraction of designated points in tunnel cross-section
Different projects have different requirements for the deformation analysis points required by the tunnel section. By extracting the characteristic points of the tunnel, the three-dimensional coordinates of the distinguishing points are obtained (see Table 2). According to the comparison between the measured coordinates of these points and the design coordinates, the tunnel can be measured. As-built measurement, the comparison between specific points of multi-phase scan data can be utilized to analyze the deformation of the tunnel section, and then obtain the overall tunnel deformation information. Generally, the points that need to be extracted are the apex of the tunnel, the lowest point of the tunnel,
the intersection of the central axis of the tunnel and the cross-section of the tunnel, and other points that have location characteristics and can reflect the deformation information of the tunnel. The location of specific points is adjusted according to the actual project.

![Figure 7. Designated point distribution](image)

**3.4. Tunnel cross-section analysis**

![Figure 8. The difference between the first and second period data](image)

Through the acquired three-dimensional coordinates of the designated point, the left and right lateral distance of each tunnel cross-section can be obtained for monitoring the lateral displacement of the tunnel, and the elevation of the point can ulteriorly analyze the tunnel settlement information. In the experimental data, the denoised point cloud data is used as the first phase data, and the noisy point cloud data is used as the second phase data, and a total of 11 sections of the two phases are obtained. Each section extracts one vertex and eight designated points on the tunnel wall. The specific coordinates are shown in Table 1 below. Due to the large amount of data, only nine designated point coordinates of the
first section are listed in Table 1. Through the designated point coordinate information, the tunnel cross-section and the tunnel height at the mileage can be obtained and the analysis of the tunnel cross-section can be completed well. The specific values are shown in Table 2 below (The chart unit is meters). Table 2 lists 4 transverse distances S and a tunnel height H in the first cross section. Through the deviation graph 8, 9, we can see the absolute value of the small deformation value caused by the noise in the two periods of data. It can be seen that the horseshoe-shaped tunnel also retain the curve information well under the cubic B-spline curve fitting. Designated points have high accuracy whether they are used for as-built measurement or deformation analysis.

![Figure 9. Tunnel height difference](image)

| Coordinate | Designated point of the first phase cross-section | Designated point of the second phase cross-section |
|------------|--------------------------------------------------|-------------------------------------------------|
| Ve         | X 3.9963  Y -0.8552  Z 3.9841 | X 3.9963  Y -0.8552  Z 3.9841 |
| L1         | X 3.0453  Y -2.1001  Z 3.4998 | X 3.0453  Y -2.1001  Z 3.4998 |
| R1         | X 4.8792  Y 0.37308  Z 3.5001 | X 4.8792  Y 0.37308  Z 3.5001 |
| L2         | X 2.4029  Y -2.9509  Z 2.0004 | X 2.4017  Y -2.9501  Z 2.0002 |
| R2         | X 5.5394  Y 1.2186  Z 2.0002 | X 5.5394  Y 1.2186  Z 2.0002 |
| L3         | X 2.3663  Y -3.0391  Z 1.1997 | X 2.3125  Y -2.9993  Z 1.2001 |
| R3         | X 5.5982  Y 1.2936  Z 1.1997 | X 5.5982  Y 1.2936  Z 1.1997 |
| L4         | X 3.3742  Y -3.8292  Z 1.2002 | X 3.3741  Y -3.7892  Z 1.1998 |
| R4         | X 6.6232  Y 0.5221  Z 1.1997 | X 6.6232  Y 0.5221  Z 1.1997 |
| L3         | X 3.4461  Y -3.7970  Z 0.7000 | X 3.4031  Y -3.7655  Z 0.6981 |
| R4         | X 6.6023  Y 0.49240  Z 0.7007 | X 6.6023  Y 0.49240  Z 0.7007 |
Table 2. Cross-section 1 cross distance and tunnel height.

| Designated point of the first phase cross-section | Designated point of the second phase cross-section |
|-----------------------------------------------|-----------------------------------------------|
| S          | H          | S          | H          |
| L1-R1      | 3.07894    | 3.07894    |
| L2-R2      | 5.21757    | 5.21763    |
| L3-R3      | 5.40538    | 5.40607    |
| L4-R4      | 5.33884    | 5.33884    |

4. Conclusions
In view of the tunnel situation in this project, the use of 3D laser scanning technology for processing has obvious advantages, which are mainly reflected in the large measurement range, high scanning efficiency, simple operation and accuracy that can meet the actual needs of the project. Additionally, the simulation of the tunnel cross-section in this paper, the point cloud missing recognition threshold is added on the basis of cubic B-spline curve fitting, so that the fitted section has a certain degree of smoothness and can effectively reflect the actual deformation characteristics of the section. Experiments on non-circular tunnels it is proved that this method can truly reflect the uneven deformation information of the tunnel section. And it makes up for the inadequacy of the circular curve (elliptic curve) which cannot be fitted to the non-circular (elliptical) tunnel cross-section; the corresponding feature point coordinates are extracted to realize the completion of the tunnel measurement and deformation monitoring. All in all, in addition to subway tunnels, this example can also be an excellent instance for a variety of traditional surveying and mapping fields, all of which can obtain higher accuracy and efficiency.

Acknowledgments
This research was supported by the major scientific and technological innovation projects of Shandong Province under Grant No. 2019JZZY020103.

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