Low wavelength dependency design for MMI (multi-mode interference) mode converter

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Abstract: Mode-division multiplexing (MDM) is widely researched, and we are researching mode converter by using multi-mode interference (MMI) waveguide. Wavelength division multiplexing (WDM) must be used in addition to the MDM, however, wavelength dependency exits for MMI in general as it works based on the interference of light. In this paper, we analyze the wavelength dependency of the MMI 0\textsuperscript{th} to 1\textsuperscript{st} mode-converter theoretically. As a result, we clarify the design scheme to suppress the wavelength dependency. Moreover, we show the sufficient performance of less than 1.0 dB in entire C-band for highly confined waveguide (Si/SiO\textsubscript{2} waveguide) as a design example.

Keywords: MMI, MDM, LP mode, multi-mode

Classification: Optoelectronics, Lasers and quantum electronics, Ultrafast optics, Silicon photonics, Planar lightwave circuits

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1 Introduction
Spatial multi-mode multiplexing (MDM) is expected to become one of the next generation transmission technologies [1, 2]. Nowadays, several mode converters for the MDM have been proposed and demonstrated [3, 4, 5, 6, 7, 8, 9, 10]. One important issue is to realize such that mode converter on photonic integrated circuit. Among them, MMI (Multi-mode Interference) mode converter is attractive of its high fabrication tolerance, and others [11, 12]. However, Generally speaking, MMI waveguide has wavelength dependency as it works based on the interference of propagating light inside of the device. Therefore, in this paper, we analyze the wavelength dependency on the excess loss of the MMI mode-converter theoretically. As a result, we clarify the scheme to suppress the excess loss. Moreover, according to the scheme explained above, we show the possibility of the excess loss less than 1.0 dB in entire C-band in case of Si/SiO\(_2\) waveguide.

2 Mode conversion principles
To realize 0\(^{th}\) to 1\(^{st}\) mode-converter, we utilize the following special 2 × 3-MMI configuration [13]. Fig. 1(a) shows the schematics of the waveguide. Here, \(W_{\text{MMI}}\) and \(W_a\) are the MMI width and access waveguide width, respectively, and \(L_e\) is the beat length which is calculated by Eq. (1) using effective width \(W_e\) [14]. As is shown in the figure, for the both case of light injection from upper port as well as from lower port, the resulting output goes as follows: 1) From the both side output ports, regular 0\(^{th}\) mode outputs come out, while 2) in the center port, 1\(^{st}\) order mode comes out. Please see the case that the phase of the injected 0\(^{th}\) mode is opposite for injecting lower port compared to the case of upper port injection. The phase relation is illustrated in Fig. 1(b) and Fig. 1(c). As is shown here, the 0\(^{th}\) modes at the both output of case Fig. 1(b) and Fig. 1(c) are opposite, while the 1\(^{st}\) modes at the center are the same each other. Therefore if we inject one of the input 0\(^{th}\) modes with \(\pi\) phase shift toward 2 × 3-MMI as is shown in Fig. 1(d), only 1\(^{st}\) mode comes out while 0\(^{th}\) mode is canceled [13]. Since no light is coming from two side output...
access waveguides, we can remove these waveguides and make the MMI coupler structure as is shown in Fig. 1(d).

\[ L_\pi = \frac{4n_r W_0^2}{3\lambda} \]  

To prepare the above two 0th mode injection, we use MMI 3 dB splitter, and phase shifter for making \( \pi \) phase shift. The full structure of the mode converter is shown in Fig. 2. The first MMI region is a MMI splitter to divide injected 0th mode into two 0th modes with same power and same phase. The second part of the structure is the phase shifter to make phase difference of \( \pi \), and the last MMI region is a MMI coupler to combine two 0th modes with \( \pi \) phase difference into 1st mode as in Fig. 1(d).

Fig. 1. Schematics of 2 \times 1 \text{ MMI mode-converter}.
Here, as is shown in Fig. 2, we use bend waveguide to achieve $\pi$ phase shift.

3 Wavelength dependency — Theory, and scheme to control —

3.1 Theory of wavelength dependency on MMI mode converter

We have discussed about the mode conversion principle of the MMI mode converter in previous chapter. In this section, we analyze the wavelength dependency on the MMI mode converter theoretically. First of all, we discuss the theoretical length of the MMI because it contains several wavelength dependent factors. The MMI length of MMI splitter $L_{\text{MMI-splitter}}$ satisfies Eq. (2) while and the one of MMI coupler satisfies Eq. (3);

$$L_{\text{MMI-splitter}} = \frac{3L_e(\lambda)}{8}$$

$$L_{\text{MMI-coupler}} = \frac{3L_e(\lambda)}{4}$$

The both MMI lengths are proportional to the so called beat length $L_e(\lambda)$. And this $L_e(\lambda)$ contains several wavelength dependent factors. Equation (4) shows the $L_e(\lambda)$ itself;

$$L_e(\lambda) = \frac{4n_r(\lambda)W_e(\lambda)^2}{3\lambda}$$

As is indicated in the equation, it contains the following three wavelength dependent factors;

1) Effective width $W_e(\lambda)$
2) Refractive index of guiding region $n_r(\lambda)$, and cladding region $n_c(\lambda)$
3) Inverse of Wavelength $1/\lambda$.

Factor 1) Effective width $W_e(\lambda)$

The effective width $W_e(\lambda)$ is approximated in Eq. (5);

$$W_e = W_{MMI} + \left(\frac{\lambda}{\pi}\right) \left(\frac{n_c}{n_r}\right)^{2\sigma} (n_r^2 - n_c^2)^{-(1/2)}$$

where $\sigma$ is 0 for TE polarization and 1 for TM polarization [14]. As is shown here clearly, the second term of this equation, which corresponds to Goos-Hänchen shift, and it increases slightly when $\lambda$ goes longer. The behavior of $W_e(\lambda)$ as a function of $\lambda$ is shown in Fig. 4(a). In this figure, the case of $W_{MMI} = 20 \mu m$ is shown as an example.
Factor 2) Refractive index of guiding region \( n_r \), and cladding region \( n_c \)

Refractive index is dependent on wavelength \( \lambda \) in general. For instance, the ones of Si and SiO\(_2\) are approximated in Eq. (6) and (7), respectively [15]. Please note that this approximation is linear approximation which is available for only within C-band.

\[
\begin{align*}
n_{Si}(\lambda) &\approx 3.61 - 8.0 \times 10^{-5} \lambda \\
n_{SiO_2}(\lambda) &\approx 1.47 - 2.0 \times 10^{-5} \lambda
\end{align*}
\]

Therefore by using these relation of (6) and (7), \( n_r \) and \( n_c \) are approximated as follows in case of Si/SiO\(_2\) waveguide (here, the thickness of Si is set to be 260 nm). Fig. 3 shows the layer structure used for the calculations.

\[
\begin{align*}
n_r(\lambda) &\approx 3.76 - 5.0 \times 10^{-4} \lambda \\
n_c(\lambda) &= n_{Air}(\lambda) \approx 1.0
\end{align*}
\]

And as is shown clearly here, they decrease when \( \lambda \) goes longer. The behavior of \( n_r(\lambda) \) as a function of \( \lambda \) is shown in Fig. 4(b).

Factor 3) Inverse of Wavelength \( 1/\lambda \)

It is obvious that \( 1/\lambda \) decrease when \( \lambda \) goes longer. The behavior of \( W_e(\lambda) \) as a function of \( \lambda \) is shown in Fig. 4(c).

As a result of these three factors, \( L_x(\lambda) \) has clear dependency on \( \lambda \) and it decreases as \( \lambda \) goes longer (see Fig. 4(d)).

3.2 Scheme to control wavelength dependency

To suppress the wavelength dependency on \( L_x(\lambda) \), here we consider to control the derivative of \( L_x(\lambda) \) by using MMI waveguide parameter (waveguide width) \( W_{MMI} \) at first. Fig. 5 shows the derivative \( dL_x(\lambda)/d\lambda \) as a function of \( W_{MMI} \) (at \( \lambda = 1550 \) nm). As is shown here, it decreases as \( W_{MMI} \) decreases, and it is possible to set it to be zero. The MMI width \( W_{MMI, dL_x/d\lambda=0} \), which make \( dL_x(\lambda)/d\lambda \) to be zero, is approximately derived as follow;

\[
W_{MMI, dL_x/d\lambda=0} = \frac{\lambda}{\pi \sqrt{n_r^2 - n_c^2}}
\]

Here, it is assumed that the refractive index \( n_r \) and \( n_c \) satisfy \( dn_r/d\lambda \ll 1 \), and \( dn_c/d\lambda \ll 1 \) in C-band. It becomes 0 when \( W_{MMI} = 200 \) nm in case of Si/SiO\(_2\).
Therefore, theoretically, the wavelength dependency is completely canceled when $W_{\text{MMI}} = 200\,\text{nm}$, however, such that narrow waveguide width is unrealistic for the presently considered device as higher order mode is not excited in such a narrow width in case of utilizing Si/SiO$_2$ waveguide for C-band at least. Much wider width may be necessary for actual device, therefore, we propose to compromise with choosing a certain value of the $W_{\text{MMI}}$. One possible criterion is to consider the defocusing length of approximately 10\,\mu m. In case of choosing $-dL_x/d\lambda < 0.9$, it corresponds to within approximately 10\,\mu m defocusing at the band-edge of C-band (1565\,nm). To make sure, it is derived as
Please note that there is no definite reason to suppress the defocusing length below 10 µm, however, in the following we verify the possibility to suppress the excess loss below 1.0 dB. From Fig. 5, the required condition for \( W_{\text{MMI}} \) goes below to satisfy \( \frac{dL_{\text{ex}}(\lambda)}{d\lambda} < 0.9 \, \mu\text{m}/\mu\text{m} \);

\[
W_{\text{MMI}} \leq 20 \, \mu\text{m} \tag{12}
\]

The excess loss itself, when the operation wavelength shifts from the center wavelength, occurs due to the defocusing that results in leak-out of the optical field and it spreads toward outside of the access waveguide. This leak-out is decreased when the width of the access waveguide \( W_a \) is set to be wider. For this reason, we consider to use \( W_a \) in addition to the MMI width \( W_{\text{MMI}} \) to achieve relatively low excess loss at the band-edge of C-band (1565 nm and 1530 nm). For instance, here we consider the case that \( W_{\text{MMI}} \) is set to be 20 µm, and \( W_a \) to be 2 µm. The excess loss was estimated by using beam-propagation-method (BPM), as it is difficult to derive theoretical excess loss value. The excess losses were estimated to be −1.1 dB (1565 nm) and −1.7 dB (1530 nm) for MMI splitter, and −1.8 dB (1565 nm) and −2.8 dB (1530 nm) for MMI coupler, respectively. However, when \( W_a \) is changed to be 3 µm, the estimated excess loss are improved to be −0.4 dB (1565 nm) and −0.6 dB (1530 nm) for MMI splitter, and −0.7 dB (1565 nm) and −0.9 dB (1530 nm) for MMI coupler, respectively. This is due to the total amount of field-matching between the optical fields of MMI waveguide edge and the one of access waveguide explained in the above. This effect is achieved in entire C-band as is shown in Fig. 6, Fig. 7, and Fig. 8. For this reason, to suppress the wavelength dependent excess loss, the excess loss is compensated by choosing relatively wider \( W_a \) after choosing proper \( W_{\text{MMI}} \). And thus, we could confirm that 10 µm defocusing length itself is a kind of realistic criteria when we design.

![Fig. 6. MMI splitter part excess loss as a function of wavelength, for variable \( W_a \).](image-url)
The eigen 1st mode profile of TE polarization is calculated in Fig. 9(a). Here, the access waveguide width is set to \( W_a = 2 \) µm, and the output waveguide width is calculated to 4 µm, by using the mode converter structure shown in Fig. 2. The region between two black dot lines shown in Fig. 9 represents the region of the output waveguide. We have verified the mode conversion efficiency by calculating the coupling efficiency between the output mode profile of the MMI mode converter and the eigen 1st mode profile shown in Fig. 9(a). For wavelength \( \lambda = 1550 \) nm, the conversion efficiency from 0th mode to 1st mode is 98%. When wavelength changes from \( \lambda = 1550 \) nm, the length of \( L_a \) defocuses as it contains several wavelength dependent factors (see Eq. 4 and Fig. 4), which affects the output mode profile. For example at the edge of C-band \( \lambda = 1565 \) nm, the output mode profile is shown in Fig. 9(b). In this band-edge, the conversion efficiency of 0th mode to 1st mode is approximately 76%. However, wider access waveguide \( W_a \) can suppress the influence of the defocus caused by wavelength dependency. For example, when the access waveguide is set to \( W_a = 4 \) µm, the mode profiles of the eigen 1st mode and the output mode from MMI converter at wavelength \( \lambda = 1565 \) nm is shown in Fig. 10(a) and Fig. 10(b), respectively. The mode con-

Fig. 7. MMI coupler part excess loss as a function of wavelength, for variable \( W_a \).

Fig. 8. Entire MMI mode-converter excess loss as a function of wavelength, for variable \( W_a \).
version efficiency from 0th mode to 1st mode at $\lambda = 1565 \text{nm}$ is 95% for $W_a = 4 \mu m$, which have improved nearly 20% compared to $W_a = 2 \mu m$.

The optical path of the phase shift region may change when wavelength shifts. The optical path change results in phase shift. This phase shift is expressed in Eq. (13)

$$\Delta \theta = \frac{2\pi n_r}{\lambda} \Delta L$$

Here, $\Delta \theta$ represents the phase shift, and $\Delta L$ is the length difference between the straight and bending waveguides in the phase shifter region. When wavelength is $\lambda = 1550 \text{nm}$, $\Delta L = 87 \text{nm}$ is needed to achieve ideal phase shift $\Delta \theta_{\text{ideal}} = 180^\circ$.

For $\Delta L = 87 \text{nm}$, the phase shift at $\lambda = 1530 \text{nm}$ and 1565 nm is calculated as 177.4$^\circ$ and 183.6$^\circ$ respectively, by using Eq. (8) and Eq. (13). The maximum phase shift difference with ideal phase shift $\Delta \theta_{\text{ideal}}$ among C-band is about 3.6$^\circ$ and the excess loss caused by the phase shift difference can be calculated by beam propagation method, and it is less than 0.007 dB among C-band.
4 Polarization dependency — Theory, and scheme to control —

4.1 Theory of polarization dependency

The center wavelength of MMI waveguide (here, of course, mode-converter) is different between TE and TM modes in general. This is another item that must be analyzed theoretically. To discuss the general polarization dependency on MMI, we introduce the following “mode depending center wavelength difference” $\Delta\lambda_{TE-TM}$ as the amount of the polarization dependency between TE mode and TM mode.

The way to derive the $\Delta\lambda_{TE-TM}$ is as follows;

1) At first, calculate the beat length on TE mode $L_{\pi TE}$ for a certain wavelength (for C-band, it corresponds to 1550 nm) $\lambda_{center-TE}$ (see Eq. (14)).
2) Next, calculate the center wavelength for TM mode $\lambda_{center-TM}$ with using the same beat length of $L_{\pi TE}$ (see Eq. (15)).
3) Then, derive the $\Delta\lambda_{TE-TM}$ from the difference between $\lambda_{center-TE}$ and $\lambda_{center-TM}$ (see Eq. (16)).

$$L_{\pi TE} = \frac{4n_r}{3\lambda} \left( W_{MMI} + \frac{\lambda_{center-TE}}{\pi n_r^2 - n_c^2} \right)^2$$  \hspace{1cm} (14)

$$\lambda_{center-TM} = L_{\pi TE} - B - \sqrt{(L_{\pi TE} - B)^2 - 4AC}$$ \hspace{1cm} (15)

$$\Delta\lambda_{TE-TM} = \lambda_{center-TE} - \lambda_{center-TM}$$

$$= \frac{\lambda_{center-TE} - L_{\pi TE} - B - \sqrt{(L_{\pi TE} - B)^2 - 4AC}}{2A}$$ \hspace{1cm} (16)

Where A, B, and C are defined as;

$$A = \frac{4n_r^4}{3\pi^2 n_c^2 (n_r^2 - n_c^2)}$$ \hspace{1cm} (17)

$$B = \frac{8n_r^2 W_{MMI}}{3\pi n_c \sqrt{n_r^2 - n_c^2}}$$ \hspace{1cm} (18)

$$C = \frac{4n_r W_{MMI}^2}{3}$$ \hspace{1cm} (19)

4.2 Scheme to control polarization dependency in addition to wavelength

As can be seen from Eq. (14)–(19), the difference $\Delta\lambda_{TE-TM}$ is clearly depending on MMI width $W_{MMI}$ again. Fig. 11 shows $\Delta\lambda_{TE-TM}$ as a function of $W_{MMI}$. On the contrary of the wavelength dependency case, it decreases as $W_{MMI}$ “increases”.

However, from Fig. 11, a certain polarization dependency always exits, and not going to be zero except the case when $W_{MMI}$ becomes infinite. As infinite width is of course impossible for actual device, therefore, we propose to compromise again with choosing a certain value of $\Delta\lambda_{TE-TM}$. One realistic criteria is $\Delta\lambda_{TE-TM} < 35$ nm, that corresponds to the band width of C-band. To achieve this value, $W_{MMI}$ must satisfy $W_{MMI} \geq 13$ μm (see Fig. 11) for Si/SiO₂ waveguide. On the other hand, we have to consider the wavelength dependency requirement explained in the previous chapter. Therefore, the design criteria for $W_{MMI}$ must satisfy the following condition for Si/SiO₂ waveguide;
The remaining thing is to suppress the excess loss by choosing proper $W_a$ as is already discussed in the previous chapter. In the following, we show one typical design procedure as an example;

Procedure 1) Choosing $W_{MMI}$ to satisfy Eq. (20)
Here, we choose $W_{MMI}$ to be 16 $\mu$m to satisfy (20).

Procedure 2) Choosing possible $W_a$ to suppress the excess loss at the both band edge
Here, we consider the case that $W_{MMI}$ is set to be 16 $\mu$m, and $W_a$ to be 4 $\mu$m. The entire MMI mode converter excess losses are $-0.3$ dB (1565 nm) and $-0.35$ dB (1530 nm) for TE polarization, and $-1.0$ dB (1565 nm) and $-0.1$ dB (1530 nm) for TM polarization, respectively. This result shows that the mode conversion from 0th mode to 1st mode is less than 1.0 dB excess loss among C-band, and the conversion efficiency is over 89%. The result is shown in Fig. 12. This result includes the affection caused by wavelength dependency in phase shift region.

$$13 \mu m \leq W_{MMI} \leq 20 \mu m$$

(20)

Fig. 11. $\Delta \lambda_{TE-TM}$ as a function of $W_{MMI}$
5 Summary

As is discussed in the above chapters, narrower $W_{\text{MMI}}$ is needed in case of low wavelength dependency while wider one is required in case of low polarization dependency. As we have discussed already, theoretical dependencies for the both wavelength and polarization are impossible to extract completely by using waveguide parameters, therefore, one realistic solution is to choose a certain value and suppress the excess loss by choosing proper access waveguide width. In this paper, we discuss based on Si/SiO$_2$ material system, however, the scheme explained here is exploitable for another waveguide material system including silica waveguide. We hope and believe that this design procedure explained here will contribute to future mode-division multiplexing.

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