Altitude Control Design of LSU-05 Aircraft Using Abstraction Method

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Abstract. LAPAN Surveillance-05 (LSU-05) is a fixed wing type aircraft developed by LAPAN. This aircraft can be used for monitoring and mapping a very wide range of areas. In carrying out operational missions, the aircraft requires reliable control of autopilot to control flying altitudes, thus facilitating and able to support the mission and for aircraft safety. In the design of the control, we apply the abstraction method using software Pessoa. The design of the control with abstraction aims to design the control such that the closed loop system behavior satisfies the specifications in the form of simple temporal logic. In the experiments, we conduct two scenarios. For each scenario, we use different specification and different abstraction parameter.

1. Introduction
Unmanned aircraft or more commonly called Unmanned Aerial Vehicle (UAV) is an aircraft that can be controlled by remote control. Unmanned aircraft can be equipped with cameras, sensors, radar, and other equipment with weights that depend on the weight of the aircraft [1]. One of the unmanned aircraft developed in Indonesia is LAPAN Surveillance-05 (LSU-05). LSU-05 was developed by Lembaga Penerbangan dan Antariksa Nasional (LAPAN). LSU-05’s vision and mission is to test scientific equipment and observe areas under its flight path in cruise conditions [2]. This aircraft can be used for monitoring and mapping a very wide range of areas. To support the mission and for the safety of LSU-05, a reliable autopilot control system for LSU-05 is absolutely needed. The autopilot control system is a control system that is often used in aircraft control without human intervention. In carrying out missions such as mapping and monitoring, stable flight conditions are needed, one of which is stable at certain altitude. Thus, a reliable control is needed to control the altitude of aircraft.

The research about control systems in LSU-05 aircraft has been carried out using various methods. Some of these methods are Linear Quadratic Regulator (LQR) and Linear Quadratic Gaussian (LQG). LQG method was used to control the pitch angle [3] and LQG method was used to control the pitch angle and roll angle [4]. In this paper, abstraction method will be applied for altitude control design of LSU-05 aircraft. Control using abstraction is a method for designing control of a system, so that the behavior of closed loop systems will satisfy the desired specifications. In designing the control using abstraction method, Pessoa software will be used. Pessoa is a software for designing control systems that use abstraction methods and support simple temporal logic specifications [5]. In this paper, we simulate the control design in two
scenarios with different abstraction parameter and different specification. Then the simulation results of the two scenarios will be compared to obtain the best abstraction parameter.

2. Modelling of LSU-05 Aircraft
LSU-05 aircraft is a product of the Aeronautic Technology Center LAPAN with a vision and mission to test scientific equipment and observe areas under its flight path in cruise conditions [2]. The characteristic properties of LSU-05 are shown in Table 1 and Table 2:

| Symbol | Name              | Quantity                   |
|--------|------------------|----------------------------|
| $m$    | mass             | $11.278 \text{ kg}$        |
| $\rho$ | air density      | $0.6601 \text{ kg/m}^3$    |
| $s$    | wing area        | $3.32 \text{ m}^2$         |
| $I_y$  | moment of inertia| $11.278 \text{ kg.m}^2$    |
| $c$    | chord length     | $0.612 \text{ m}$          |
| $U_0$  | cruise speed     | $27.78 \text{ m/s}$        |

Table 2. Aerodynamic stability and control derivatives of LSU-05 [2].

| Symbol | Quantity       | Symbol | Quantity       |
|--------|----------------|--------|----------------|
| $C_{zq}$ | $-2$           | $C_{mq}$ | $-11.229$     |
| $C_{z\alpha}$ | $-5.8177$   | $C_{m\alpha}$ | $-2.1755$ |
| $C_{z\delta_e}$ | $-0.246$    | $C_{m\delta_e}$ | $-0.71$        |

2.1. Longitudinal Equation of Motion
Longitudinal dynamics is a mathematical model that describes the dynamics of the motion of an aircraft for movement in a vertical direction such as climbing or dipping. The motion is described by the axial force $X$, the normal force $Z$ and the pitching moment $M$. Inputs for longitudinal motion consist of elevator deflection and engine thrust [2]. For simplicity, it is assumed that only elevator deflection is involved in the control of the aircraft’s longitudinal motion. So, the longitudinal equation of motion can be written as in [2]:

\[
\begin{align*}
\frac{mU_0}{s} \ddot{u}' - C_{zu} u' - C_{x\alpha} \dot{\alpha} - \frac{c}{2U_0} C_{x\alpha} \dot{\alpha} - (C_w \cos \Theta_0) \dot{\theta} - \frac{c}{2U_0} C_{zq} \dot{\theta} &= C_{z\delta_e} \delta_e \\
- C_{zu} u' + \left( \frac{mU_0}{s} - \frac{c}{2U_0} C_{z\alpha} \right) \dot{\alpha} - C_{z\alpha} \dot{\alpha} + \left( \frac{mU_0}{s} - \frac{c}{2U_0} C_{zq} \right) - (C_w \sin \Theta_0) \dot{\theta} &= C_{z\delta_e} \delta_e \\
- C_{m\alpha} u' - C_{m\alpha} \dot{\alpha} - \frac{c}{2U_0} C_{m\alpha} \dot{\alpha} + \frac{I_y}{S_q c} \ddot{q} - \frac{c}{2U_0} C_{mq} q &= C_{m\delta_e} \delta_e \\
q &= \dot{\theta}
\end{align*}
\]

where $q' = \frac{1}{2} \rho U_0^2$ and state variables of longitudinal equation consist of linear velocity in the x axis direction ($u'$), angle of attack ($'\alpha$), pitch rate ($q$), and pitch angle ($\theta$). Input variable system is elevator deflection ($\delta_e$).
2.2. Short Period Approximation

Short period approximation is an approach of longitudinal motion to obtain a simpler motion equation by observing the short period mode. In short period mode, the dominant variables are $q$ and $\alpha$. The short period model can be written as in [6]:

\[
\dot{\alpha} = \frac{s q' C_{z\alpha}}{m U_0} \alpha + \left(1 + \frac{s q' C_{zq}}{2m U_0^2}\right) q + \frac{s q' C_{z\delta_e}}{m U_0} \delta_e
\]

\[
\dot{\alpha} = \frac{s q' C_{m\alpha}}{I_y} \alpha + \frac{s q' C_{mq}}{2U_0 I_y} q + \frac{s q' C_{m\delta_e}}{I_y} \delta_e
\]

\[
\dot{\alpha} = \frac{s q' C_{z\alpha}}{m U_0} \alpha + \left(1 + \frac{s q' C_{zq}}{2m U_0^2}\right) q + \frac{s q' C_{z\delta_e}}{m U_0} \delta_e
\]

\[
\dot{\alpha} = \frac{s q' C_{m\alpha}}{I_y} \alpha + \frac{s q' C_{mq}}{2U_0 I_y} q + \frac{s q' C_{m\delta_e}}{I_y} \delta_e
\]

2.3. Model For Altitude Control

In linearized form, the change of altitude equation can be written as in [7]:

\[
\dot{h} = -U_0 \alpha + U_0 \theta
\]

and from (1), the change of pitch angle is equal to the pitch rate. So, model for altitude control is obtained by augmenting the change of altitude equation and the change of pitch angle equation to the short period model. Model for altitude control can be written as:

\[
\begin{bmatrix}
\dot{\alpha} \\
\dot{q} \\
\dot{\theta} \\
\dot{h}
\end{bmatrix} =
\begin{bmatrix}
-22.8962 & 0.9133 & 0 & 0 \\
-99.8299 & -5.6759 & 0 & 0 \\
0 & 1 & 0 & 0 \\
-27.78 & 0 & 27.78 & 0
\end{bmatrix}
\begin{bmatrix}
\alpha \\
q \\
\theta \\
h
\end{bmatrix}
+ \begin{bmatrix}
-0.9682 \\
-32.5807 \\
0 \\
0
\end{bmatrix}
\delta_e
\]

Based on parameter value of LSU-05 in Table 1 and Table 2, model for altitude control of LSU-05 aircraft can be written as:

\[
\begin{bmatrix}
\dot{\alpha} \\
\dot{q} \\
\dot{\theta} \\
\dot{h}
\end{bmatrix} =
\begin{bmatrix}
-0.5292 & 0.9133 & 0 & 0 \\
-2.6759 & -5.6759 & 0 & 0 \\
0 & 1 & 0 & 0 \\
-27.78 & 0 & 27.78 & 0
\end{bmatrix}
\begin{bmatrix}
\alpha \\
q \\
\theta \\
h
\end{bmatrix}
+ \begin{bmatrix}
-0.9682 \\
-32.5807 \\
0 \\
0
\end{bmatrix}
\delta_e
\]

3. Control Design of LSU-05 Aircraft Model

Control design with abstraction is a method for designing control of a system, so that the behavior of closed loop systems will satisfy the desired specifications. The starting point of this method is designing controller $S_{cont}$ such that the composition $S_{cont} \times S_{\tau}(\Sigma)$ satisfies the desired specification. $S_{\tau}(\Sigma)$ is a transition system with time quantization parameter $\tau \in \mathbb{R}^+$. If the specification is given as another system $S_{spec}$, then we seek to synthesize a controller $S_{cont}$ so that:

$S_{cont} \times S_{\tau}(\Sigma) \cong_{AS} S_{spec}$

with $\cong_{AS}$ is $\varepsilon$-approximate alternating bisimulation relation notation [5, 8]. In general, this problem is not solvable algorithmically since $S_{\tau}(\Sigma)$ is an infinite system. We overcome this difficulty by replacing $S_{\tau}(\Sigma)$ by $S_{abs}$ (abstract system) for which we have the guarantee that if a controller satisfying:

$S_{cont} \times S_{abs} \cong_{AS} S_{spec}$

exist then a controller $S'_{cont}$ satisfying:

$S'_{cont} \times S_{\tau}(\Sigma) \cong_{AS} S_{spec}$

exist then a controller $S'_{cont}$ satisfying:
also exist. $S'_{cont}$ is the refinement of $S_{cont}$ [5, 8].

$S_{abs}$ is a system with finite input, finite set of states, and discrete time. The aircraft model has four state variables, they are $\alpha$, $q$, $\theta$, and $h$. The input variable is deflection elevator ($\delta_e$). So, the set of states $S_{abs}$ is:

$$[D]_{\eta} = \{ x \in D \mid x_i = k_i \eta, \text{ for some } k_i \in \mathbb{Z} \text{ and } i = 1, 2, 3, 4 \}$$

with $D \subseteq \mathbb{R}^4$, $D$ is a compact set, $\eta$ is space quantization parameter, and $D = D_{\alpha} \times D_q \times D_\theta \times D_h$ with $D_i$ is state set of variable $i$. The input space of $S_{abs}$ is:

$$[\cup]_{\mu} = \{ x \in \cup \mid x = k \mu, \text{ for some } k \in \mathbb{Z} \}$$

with $\cup \subseteq \mathbb{R}$ and $\mu$ is input quantization parameter.

Control design of aircraft model is simulated using Pessoa software in a computer with 4 GB of RAM and processor Intel core i5. We simulate the closed-loop system in two scenarios with different abstraction parameter and different specification. The simulation using Pessoa will produce a controller for the system (2) and will be known as system response.

3.1. Scenario 1

In scenario 1, the set of states $D$ for the abstract system is:

$$D = [-0.01, 0.01] \times [-0.1, 0.1] \times [-0.01, 0.01] \times [3000, 3001]$$

Input space is $\cup = [-0.04, 0.04]$, abstraction parameters are $\eta = 0.004$, $\mu = 0.015$, $\tau = 0.1$, $\varepsilon = 1$, the relation is $\varepsilon$-approximate alternating bisimulation relation, and the specification is reach and stay with initial condition $(\alpha_0, q_0, \theta_0, h_0) = (0, 0, 0, 3000.25)$ and target set $Z = [-0.01, 0.01] \times [-0.1, 0.1] \times [-0.01, 0.01] \times [3000.75, 3000.8]$.

The simulation results of scenario 1 are shown in Figure 1 - 3.

Based on Figure 1, the simulation results show the aircraft’s initial altitude is 3000.25 m. After that the altitude of aircraft reaches target 3000.75 m until 3000.8 m and then always in target. This result shows that the closed loop system behavior satisfies the specifications. Aircraft altitude response is more clearly shown in Figure 2. The altitude of aircraft reaches

Figure 1. Aircraft altitude response for scenario 1.

Figure 2. Oscillation of aircraft altitude response for scenario 1.
target at 2.25 seconds, after that altitude response is always oscillating in range 3000.75 m until 3000.8 m with average oscillation frequency is 2.5 oscillations every second. It shows that the aircraft is always at an altitude 3000.75 m until 3000.8 m with up and down motion. The aircraft altitude controller can be seen in Figure 3 as follows:

![Figure 3. Aircraft altitude controller for scenario 1.](image)

Figure 3 shows the magnitude of the aircraft’s elevator angle against time. From 2.2 second and so on, the elevator angle is always change between 0.03 rad to -0.03 rad. The magnitude of elevator angle from 0 to 2.25 seconds causes the altitude of aircraft reaches target. While the magnitude of elevator angle in the next second is used to maintain the altitude of aircraft such that the altitude always in target.

3.2. Scenario 2
In scenario 2, the set of states $D$ for the abstract system is:

$$D = [-0.01, 0.01] \times [-0.2, 0.2] \times [-0.01, 0.01] \times [3000, 3001]$$

Input space is $\cup = [-0.1, 0.1]$, abstraction parameters are $\eta = 0.004$, $\mu = 0.05$, $\tau = 0.05$, $\varepsilon = 1$, the relation is $\varepsilon$-approximate alternating bisimulation relation, and the specification is reach and stay with initial condition $(\alpha_0, q_0, \theta_0, h_0) = (0, 0, 0, 3000.25)$ and target set $Z = [-0.01, 0.01] \times [-0.2, 0.2] \times [-0.01, 0.01] \times [3000.75, 3000.8]$. The simulation results of scenario 2 are shown in Figure 4 - 6.

![Figure 4. Aircraft altitude response for scenario 2.](image)

![Figure 5. Oscillation of aircraft altitude response for scenario 2.](image)
Based on Figure 4, the simulation results show the aircraft’s initial altitude is 3000.25 m. After that the altitude of aircraft reaches target 3000.75 m until 3000.8 m and then always in target. This result shows that the closed loop system behavior satisfies the specifications. Aircraft altitude response is more clearly shown in Figure 5. The altitude of aircraft reaches target at 2.59 seconds, after that altitude response is always oscillating in range 3000.75 m until 3000.8 m with average oscillation frequency is 5 oscillations every second. It shows that the aircraft is always at an altitude 3000.75 m until 3000.8 m with up and down motion.

The aircraft altitude controller can be seen in Figure 6 as follows:

![Figure 6. Aircraft altitude controller for scenario 2.](image)

Figure 6 shows the magnitude of the aircraft’s elevator angle against time. From 2.51 second and so on, the elevator angle changes between 0.1 rad to -0.1 rad. The magnitude of elevator angle from 0 to 2.59 seconds causes the altitude of aircraft reaches target. While the magnitude of elevator angle in the next second to maintain the altitude of aircraft to always in target.

4. Conclusions
Based on the result of simulation in the previous section, we can obtain the following conclusions:

(i) The design of specifications used in this paper are:
   (a) Scenario 1 uses reach and stay specification with initial conditions (0, 0, 0, 3000.25) and target set $Z = [-0.01, 0.01] \times [-0.1, 0.1] \times [-0.01, 0.01] \times [3000.75, 3000.8]$.
   (b) Scenario 2 uses reach and stay specifications with initial conditions (0, 0, 0, 3000.25) and target set $Z = [-0.01, 0.01] \times [-0.2, 0.2] \times [-0.01, 0.01] \times [3000.75, 3000.8]$

(ii) Scenario 1 uses the abstraction parameters $\eta = 0.004, \mu = 0.015, \tau = 0.1, \varepsilon = 0.002$, whereas scenario 2 uses abstraction parameters $\eta = 0.004, \mu = 0.05, \tau = 0.05, \varepsilon = 0.002$. From the result of simulation, the aircraft altitude response in scenario 1 is faster to reach target than the aircraft altitude response in scenario 2. The average oscillation of altitude response in scenario 1 is smaller than scenario 2. So abstraction parameter values recomended for the control design are $\eta = 0.004, \mu = 0.015, \tau = 0.1$, and $\varepsilon = 0.002$.

(iii) The simulation results are:
   (a) The closed loop system behavior in the scenario 1 and scenario 2 satisfies specifications. Aircraft altitude response starts from a certain initial condition then reaches the target and after that always on target.
   (b) In the scenario 1, the time needed for the aircraft to reach the target is 2.25 seconds. In the scenario 2, the time needed for the aircraft to reach the target is 2.59 seconds.
(c) In the scenario 1, the average oscillation frequency of aircraft altitude response is 2.5 oscillations every second. In the scenario 2, the average oscillation frequency of aircraft altitude response is 5 oscillations every second.

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