Universal invariant renormalization for supersymmetric theories.

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Abstract

Manifestly invariant renormalization scheme for supersymmetric gauge theories is proposed. This scheme is applied to supersymmetric quantum electrodynamics.

1 Introduction.

This paper addresses the problem of a manifestly invariant renormalization procedure for supersymmetric (SUSY) gauge theories.

A peculiar feature of supersymmetric gauge theories is nonpolynomial structure of the action. The only possibility to reduce the action to a polynomial form is to use the Wess-Zumino gauge. However in this case the manifest supersymmetry is lost and the powerful superdiagram technique cannot be used.

In supersymmetric gauges an infinite number of primitively divergent diagrams is present, which makes the theory formally nonrenormalizable. Nevertheless it can be shown that supersymmetry and gauge invariance reduce the number of independent counterterms, and the usual charge and wave functions renormalizations are sufficient.

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The corresponding procedure was firstly constructed for SUSY QED by one of the present authors [1] and then applied to Abelian and non-Abelian models in papers [2, 3, 4]. However the proof given in these papers relies on the assumption of a possibility to use an intermediate regularization preserving the symmetry of the theory. Most popular gauge invariant regularizations like dimensional regularization [5], dimensional reduction [6] or lattice regularizations break supersymmetry [7]. Higher covariant derivative regularization [8, 9, 10] in principle may be used, but the calculations are quite involved (see however [11, 12]).

A possible alternative is presented by algebraic renormalization. In this method one uses an arbitrary regularization or subtraction scheme, breaking the symmetry of the theory (for example the momentum cut-off regularization or the differential renormalization [13, 14]). The symmetry is restored at the second step by tuning finite counterterms to provide relevant Generalized Ward Identities (GWI) for the renormalized Green functions. This method was applied successfully to SUSY gauge theories in the papers [15, 16, 17], where the invariant renormalizability of \( N = 1 \) and \( N = 2 \) non-Abelian SUSY gauge models was proven in the framework of algebraic renormalization. However a practical implementation of the algebraic renormalization is rather cumbersome as the procedure requires a tuning of a large number of (noninvariant) counterterms.

Recently a new method of invariant renormalization was proposed [18, 19], which provides automatically the renormalized Green functions possessing a relevant symmetry for arbitrary intermediate regularization. This method was formulated as a special subtraction procedure which incorporates GWI. Solving explicitly the corresponding GWI one reduces the problem of renormalization in an arbitrary regularization scheme to explicitly gauge invariant procedure. So instead of the two step algebraic renormalization we have the one step procedure which guarantees the symmetry of the renormalized theory.

In the present paper we discuss how this method may be generalized to supersymmetric gauge theories. Renormalization of SUSY QED is described in details. A corresponding procedure for non-Abelian SUSY gauge models is under consideration.

The paper is organized as follows:

In Section 2 we introduce the notations and remind some information about supersymmetric quantum electrodynamics. The universal invariant renomalization scheme for the model is constructed in the next Section 3 and illustrated by an example of the one-loop renormalization in Section 4. The results are discussed in the Conclusion.

## 2 Supersymmetric quantum electrodynamics.

In the superspace \( N = 1 \) supersymmetric electrodynamics is described by the following action:
\[ S_0 = \frac{1}{4e^2} \text{Re} \int d^4x d^2\theta W_aC^{ab}W_b + \frac{1}{4} \int d^4x d^4\theta \left( \phi^+ e^{2V} \phi + \bar{\phi}^+ e^{-2V} \bar{\phi} \right) + \]
\[ + \frac{1}{2} \int d^4x d^2\theta m \bar{\phi} \phi + \frac{1}{2} \int d^4x d^2\bar{\theta} m \bar{\phi} \phi^*, \]

where \( \phi \) and \( \bar{\phi} \) are chiral superfields, \( V \) is a real superfield. The superfield \( W_a \) in the abelian case is defined as

\[ W_a = \frac{1}{16} \bar{D} (1 - \gamma_5) D (1 + \gamma_5) D_a V, \]

where \( D \) is the usual supersymmetric covariant derivative

\[ D = \frac{\partial}{\partial \theta} - i \gamma^\mu \theta \partial_\mu. \]

The integration over the superspace in the equation (1) is defined as

\[ \int d^2\theta = \frac{1}{4} \frac{\partial}{\partial \theta} (1 + \gamma_5) \frac{\partial}{\partial \theta}; \quad \int d^2\bar{\theta} = \frac{1}{4} \frac{\partial}{\partial \bar{\theta}} (1 - \gamma_5) \frac{\partial}{\partial \bar{\theta}}; \]
\[ \int d^4\theta = \int d^2\theta d^2\bar{\theta} = \frac{1}{8} \left( \frac{\partial}{\partial \theta} \frac{\partial}{\partial \bar{\theta}} \right)^2. \]

Up to surface terms these expressions can be written in the explicitly supersymmetric form:

\[ \int d^4x d^2\theta = -\frac{1}{4} \int d^4x \bar{D} (1 + \gamma_5) D = -\frac{1}{2} \int d^4x D^2; \]
\[ \int d^4x d^2\bar{\theta} = -\frac{1}{4} \int d^4x \bar{D} (1 - \gamma_5) D = -\frac{1}{2} \int d^4x \bar{D}^2; \]
\[ \int d^4x d^4\theta = \frac{1}{4} \int d^4x D^2 \bar{D}^2 = \frac{1}{4} \int d^4x D^2 \bar{D}^2. \]

Here we use the following notations:

\[ D^2 \equiv \frac{1}{2} D (1 + \gamma_5) D; \quad \bar{D}^2 \equiv \frac{1}{2} \bar{D} (1 - \gamma_5) D. \]

Action (1) is invariant under the gauge transformations

\[ V \rightarrow V - \frac{1}{2} \left( \Lambda + \Lambda^+ \right); \]
\[ \phi \rightarrow e^{\Lambda} \phi; \quad \phi^* \rightarrow \phi^* e^{\Lambda^*}; \]
\[ \bar{\phi} \rightarrow e^{-\Lambda} \bar{\phi}; \quad \bar{\phi}^* \rightarrow \bar{\phi}^* e^{-\Lambda^*}, \]
where $\Lambda$ is an arbitrary chiral superfield. This invariance allows to gauge away some components of $V(x, \theta)$, resulting in the following equation:

$$V(x, \theta) = \frac{1}{2} \bar{\theta} \gamma^\mu \gamma_5 \theta A_\mu(x) + i \sqrt{2} (\bar{\theta} \theta) (\bar{\theta} \gamma_5 \chi(x)) + \frac{1}{4} (\bar{\theta} \theta)^2 D(x),$$

(8)

where $A_\mu$ is a gauge field, $\chi$ is a Majorana spinor and $D$ is a real scalar auxiliary field. In this gauge, which is called Wess-Zumino gauge, residual gauge transformations depend only on a single parameter, while the action is polynomial. However, this gauge breaks explicitly supersymmetry of the model.

Quantization of model (1) is described in details in book [21] and is not considered here. We only note, that the gauge fixing is made by adding the term

$$S_{gf} = \frac{1}{32 e^2 \xi} \int d^4 x d^4 \theta D^2 V \bar{D}^2 V,$$

(9)

$\xi$ being a constant.

### 3 Universal invariant renormalization.

In this section we consider a renormalization procedure, which may be used with arbitrary not necessary gauge invariant regularization providing renormalized correlators satisfying automatically SUSY GWI. In the language of counterterms it means, that no noninvariant counterterms are needed and renormalization freedom is restricted to the choice of gauge invariant local terms, which are not fixed by GWI. We assume, that a regularization used for calculations is manifestly supersymmetric allowing to use supergraph technique. The construction presented below may be used for arbitrary SUSY gauge models formulated in terms of corresponding superfields. However in this paper we concentrate on renormalization of SUSY QED and explicit equations will refer to this model. We follow the approach, developed in [18, 19], where the subtraction procedure incorporating automatically GWI was proposed.

To avoid the appearance of spurious infrared divergences we shall work in the "diagonal" gauge, corresponding to $\xi = -1$ in equation (9). Consideration of a general gauge requires additional infrared regularization, which in the Abelian case may be achieved by simply adding the mass term for gauge field and putting $m_A = 0$ after calculation of integrals (see [20]).

It is well known [21], that the degree of divergency of a diagram with $E_\phi$ external lines of chiral and antichiral superfields and $E_V$ external lines of the gauge superfield is equal to

$$\omega = 2 - P - E_\phi,$$

(10)
where \( P \) is a number of \( \phi \phi, \tilde{\phi} \tilde{\phi}, \phi^+ \phi^+ \) or \( \tilde{\phi}^+ \tilde{\phi}^+ \) propagators. Therefore, divergencies are present in the following Green functions:

\[
(2\pi)^4 \delta^4 \left( k_\mu + q_\mu + (p_1)_\mu + \ldots + (p_n)_\mu \right) \times \\
\times \Gamma \left[ (\theta_{x_1}, p_1), \ldots, (\theta_{x_n}, p_n); (\theta_y, q), (\theta_z, -q - p_1 - \ldots - p_n) \right] = \\
\equiv \int d^4 x_1 \ldots d^4 x_n \frac{\delta^{n+2} \Gamma}{\delta V_{x_1} \ldots \delta V_{x_n} \delta \phi_y} \delta \phi^+_x \bigg|_{\phi, \bar{\phi} = 0} \times \\
\times \exp \left( i(p_1)_\mu x^\mu_1 + \ldots + i(p_n)_\mu x^\mu_n + iq_\mu y^\mu + ik_\mu z^\mu \right). 
\]

\[
(2\pi)^4 \delta^4 \left( (p_1)_\mu + \ldots + (p_n)_\mu \right) \Pi \left[ (\theta_{x_1}, p_1), \ldots, (\theta_{x_n}, -p_1 - \ldots - p_{n-1}) \right] = \\
\equiv \int d^4 x_1 \ldots d^4 x_n \frac{\delta^n \Gamma}{\delta V_{x_1} \ldots \delta V_{x_n}} \bigg|_{\phi, \bar{\phi} = 0} \exp \left( i(p_1)_\mu x^\mu_1 + \ldots + i(p_n)_\mu x^\mu_n \right) 
\]

and in the functions \( \tilde{\Gamma} \), which are constructed similar to functions \( \Gamma \), but differentiation is performed over \( \tilde{\phi} \)-fields. Note, that functions \( \Pi \) for odd \( n \) are equal to 0, because contributions of diagrams with loops of \( \phi \) are cancelled with contributions of corresponding diagrams with \( \tilde{\phi} \)-loops.

The functions \( \Gamma, \tilde{\Gamma} \) and \( \Pi \) satisfy Ward identities [4], which in our notations can be written in the following form:

\[
(D_{x_1}^2 + \bar{D}_{x_1}^2) \Gamma \left[ (\theta_{x_1}, p_1), (\theta_{x_2}, p_2), \ldots, (\theta_{x_n}, p_n); (\theta_y, q), (\theta_z, -q - p_1 - \ldots - p_n) \right] = \\
= 2D_{x_1}^2 \delta^4 (\theta_y - \theta_{x_1}) \times \\
\Gamma \left[ (\theta_{x_2}, p_2), \ldots, (\theta_{x_n}, p_n); (\theta_{x_1}, q + p_1), (\theta_z, -q - p_1 - \ldots - p_n) \right] + \\
+ 2D_{x_1}^2 \delta^4 (\theta_{x_1} - \theta_z) \times \\
\times \Gamma \left[ (\theta_{x_2}, p_2), \ldots, (\theta_{x_n}, p_n); (\theta_y, q), (\theta_{x_1}, -q - p_2 - \ldots - p_n) \right]; 
\]

\[
(D_x^2 + \bar{D}_x^2) \left( \Pi \left[ (\theta_x, p), (\theta_y, -p) \right] - \frac{1}{2e^2 \xi} p^2 \delta^4 (\theta_x - \theta_y) \right) = 0; 
\]

\[
(D_{x_1}^2 + \bar{D}_{x_1}^2) \Pi \left[ (\theta_{x_1}, p_1), \ldots, (\theta_{x_{n-1}}, p_{n-1}); (\theta_{x_n}, -p_1 - \ldots - p_{n-1}) \right] = 0, \quad n > 2. 
\]

The supersymmetric covariant derivative in momentum representation is written as

\footnote{Note, that for an invariant regularization divergencies in functions \( \Pi \) will be absent for \( n > 2 \). However, in the general case it is impossible to ignore their existence.}
\[ D_x = \frac{\partial}{\partial \theta} - \gamma^\mu p_\mu, \]

where \( p \) is the momentum, corresponding to \( x \)-coordinate. Ward identities for functions \( \tilde{\Gamma} \) has the same structure.

As the chiral projector \( E_c \) can be presented as

\[ E_c \equiv \frac{1}{16\partial^2}(\bar{D}^2D^2 + D^2\bar{D}^2) = \frac{1}{16\partial^2}(\bar{D}^2 + D^2)^2, \]

these identities express the Green functions with at least one “chiral” gauge field component in terms of correlators with less number of chiral gauge components.

Equations (13), (14) and (15) are written for SUSY QED. However they have essentially the same structure in non-abelian SUSY models, differing by the RHS which includes in this case also correlators with Faddeev-Popov ghost lines.

Our strategy may be formulated as follows: We firstly consider one-loop diagrams and renormalize in arbitrary (infrared safe) way the diagrams at the RHS of equations (13) – (15). Then having in mind that in an anomaly free theory GWI for correlators calculated with arbitrary subtraction scheme may be violated only by local terms, we define the renormalized Green functions at the LHS of equations (13) – (15) in such a way, that these identities are satisfied automatically. This renormalization still may be incomplete, as GWI obviously allow to add to the vertex function at the LHS an arbitrary gauge invariant counterterm. These counterterms as usual are free parameters which may be fixed by normalization conditions.

In the case of SUSY QED this procedure looks as follows:

First of all it is necessary to renormalize one-loop two-point Green function of matter superfields. The terms, proportional to \( \phi^*\phi \) and not containing \( V \) in the effective action, have the following structure

\[ 4 \int d^4x d^4\theta \phi^*(x)\Sigma(\sqrt{-\partial^2})\phi(x). \]

Hence, the two-point function of the matter field can be written as

\[ \Gamma[(x, q), (y, -q)] = \bar{D}_x^2 D_y^2 \delta^4(\theta_x - \theta_y) \Sigma(q). \]

Renormalized two-point function is defined by subtraction

\[ \Sigma'(q) = \Sigma(q) - \Sigma(\mu_\sigma), \]

where \( \mu_\sigma \) is a normalization point.

After renormalization of two-point function it is necessary to construct renormalized vertex functions \( \Gamma \) and \( \tilde{\Gamma} \). Due to supersymmetry the function \( \Gamma \) can be presented in the form

\[ \Gamma[\theta, p] = \sum_i B_i(\theta, p) F_i(p), \]

where \( p \) is the momentum, corresponding to \( x \)-coordinate. Ward identities for functions \( \tilde{\Gamma} \) has the same structure.
where \( p \) and \( \theta \) denote all set of arguments, \( B_i(\theta, p) \) are polynomials in \( p \) and \( \theta \), which are some linear independent combinations of covariant derivatives, acting on the product of \( \delta^4(\theta_k - \theta_l) \), and \( F_i(p) \) are scalar functions of external momenta. Renormalization is performed by subtracting polynomials \( P_i(p) \) from the functions \( F_i(p) \). We choose these polynomials in such a way, that the resulting function satisfies GWI (13) – (15) where the RHS includes renormalized functions with less number of chiral external lines.

Let us define "partially renormalized" function

\[
\gamma^r[\theta, p] = \sum_i B_i(\theta, p)\left(F_i(p) - P_i(p)\right),
\]

where \( P_i(p) \) are some polynomials. Then the LHS of equation (13) may be written in the form

\[
(D^2_{x_1} + \bar{D}^2_{x_1})\gamma^r[\theta, p] = \sum_i (D^2_{x_1} + \bar{D}^2_{x_1})B_i(\theta, p)\left(F_i(p) - P_i(p)\right).
\]

The combinations \( (D^2_{x_1} + \bar{D}^2_{x_1})B_i(\theta, p) \) in the general case are not independent. It is convenient to introduce linear independent polynomials \( Q_j \), proportional to the chiral parts of \( B_i \):

\[
(D^2_{x_1} + \bar{D}^2_{x_1})B_i(\theta, p) = \sum_j c_{ij}Q_j(\theta, p).
\]

Expanding the RHS of equation (13) over \( Q_j \) it is possible to rewrite the Ward identity as

\[
\sum_{ij} c_{ij}Q_j(\theta, p)\left(F_i(p) - P_i(p)\right) = \sum_j Q_j(\theta, p)R_j(p).
\]

Taking into account, that \( Q_j \) are linear independent, this equation is equivalent to the following system of linear equations:

\[
\sum_i c_{ij}\left(F_i(p) - P_i(p)\right) = R_j(p),
\]

which represents Ward identity (13) expanded in terms of linear independent polynomials \( Q_i(p, \theta) \). If the polynomials \( P_i(p) \) satisfy system (26), the function \( \gamma^r \) defined by equation (22) satisfies SUSY GWI. Indeed,

\[
(D^2_{x_1} + \bar{D}^2_{x_1})\gamma^r[\theta, p] = \sum_i c_{ij}Q_j(\theta, p)\left(F_i(p) - P_i(p)\right) = \sum_j Q_j(\theta, p)R_j(p).
\]

Note, that a choice of \( P_i \) is not unique, because any solution of Ward identity is defined up to a term \( \Pi_{1/2}f_i \), which can not be determined from the Ward identity. However, this freedom is irrelevant for our procedure.
To eliminate the remaining ultraviolet divergences it is sufficient to subtract from $\gamma^r$ gauge invariant local counterterms $P_{gi}$

$$\Gamma^r[p, \theta] = \gamma^r[p, \theta] - P_{gi}. \tag{28}$$

These counterterms obviously cannot be fixed by GWI. So we succeeded to reduce the subtraction procedure in an arbitrary regularization scheme to subtraction of gauge invariant counterterms.

Having obtained the function $\Gamma^r_3$, it is necessary to substitute it into the RHS of renormalized Ward identity (13) for the function $\Gamma^r_4$. Then the process is repeated. So, we constructed a recurrent procedure, which defines all functions $\Gamma^r_n$. This procedure is illustrated in the next Section by an example of one-loop renormalization of the function $\Gamma_3$ in the momentum cut-off regularization.

The functions $\tilde{\Gamma}^r$ are constructed in a similar manner.

At the next step it is necessary to renormalize Green functions $\Pi$, corresponding to diagrams without external lines of matter superfields. Due to supersymmetry they can be written in the form, similar to equation (21):

$$\Pi[\theta, p] = \sum_i B_i(\theta, p) F_i(p). \tag{29}$$

For example, the function $\Pi$ for $n = 2$ can be presented as

$$\Pi[(\theta_x, \theta_y), (\theta_y, -p)] = F_1(p) p^2 \Pi_{1/2} \delta^4(\theta_x - \theta_y) + F_2(p) \delta^4(\theta_x - \theta_y), \tag{30}$$

where

$$\Pi_{1/2} = -\frac{1}{16 e^2} D^a \bar{D}^2 C_{ab} D^b. \tag{31}$$

In this case

$$B_1(\theta, p) = p^2 \Pi_{1/2} \delta^4(\theta_x - \theta_y); \quad B_2(\theta, p) = \delta^4(\theta_x - \theta_y) \tag{32}$$

As before, the renormalized Green functions are obtained by subtracting from $F_i(p)$ some polynomials chosen to provide GWI for the function $\Pi^r$

$$\Pi^r[\theta, p] = \sum_i B_i(\theta, p) \left( F_i(p) - P_i(p) \right), \tag{33}$$

For the two-point Green function substitution of (30) into equation (14) gives the following equation:

$$\left( F_2(p) - P_2(p) - \frac{1}{2e^2 \xi} p^2 \right) \left( D_x^2 + \bar{D}_x^2 \right) \delta^4(\theta_x - \theta_y) = 0. \tag{34}$$

Therefore,
\[ P_2(p) = F_2(p) - \frac{1}{2e^2 \xi} p^2, \]  
(35)

while the function \( P_1 \) can not be defined from GWI and corresponds to a gauge invariant counterterm. It is convenient to choose

\[ P_1(p) = F_1(\mu_\pi) - \frac{1}{2e^2 \xi} p^2, \]  
(36)

where \( \mu_\pi \) is a normalization point, which can be different from \( \mu_\sigma \). Then the renormalized two-point Green function can be written as

\[ \Pi^r[(\theta_x, p), (\theta_y, -p)] = \frac{1}{32e^2 \xi} \left(D^2 \bar{D}^2 + \bar{D}^2 D^2\right) \delta^4(\theta_x - \theta_y) + \\
\left( F_1(p) - F_1(\mu_\pi) \right) p^2 \Pi^r_{1/2} \delta^4(\theta_x - \theta_y), \]  
(37)

This Green function satisfies the equation

\[ \left( D_x^2 + \bar{D}_x^2 \right) \left( \Pi^r[(\theta_x, p), (\theta_y, -p)] - \frac{1}{2e^2 \xi} p^2 \delta^4(\theta_x - \theta_y) \right) = 0, \]  
(38)

which is a supersymmetric generalization of transversality condition in the usual quantum electrodynamics.

Green functions, containing only external \( V \)-lines with \( E_V > 2 \), can be renormalized similarly. It is sufficient to put in equations (25) and (26) \( R_j = 0 \). Then equation (22) may be used to define the renormalized function \( \Pi^r \).

So, the one-loop renormalization procedure is finished. Due to the locality of the subtraction terms in the limit, when regularization is removed, the present scheme is equivalent to adding the counterterms

\[ \Delta S = -\sum_{n,i} \frac{1}{n!} \int d^4x \, d^4\theta_1 \ldots d^4\theta_n \, B_i(\theta, \partial) P_i(\partial) \, V_i(\theta_1, x) \ldots V(\theta_n, x) + \\
+ \frac{1}{4} \sum_{n=0}^{\infty} (Z_{\Gamma_n} - 1) \frac{1}{n!} \int d^4x \, d^4\theta \, \phi^+(2V)^n \phi + \\
+ \frac{1}{4} \sum_{n=0}^{\infty} (\bar{Z}_{\Gamma_n} - 1) \frac{1}{n!} \int d^4x \, d^4\bar{\theta} \, \bar{\phi}^+(-2\bar{V})^n \bar{\phi}. \]  
(39)

It is important to note, that due to the presence of \( \delta \)-functions in \( B_i \) all these terms can be presented as integrals over a single \( \theta \). For the noninvariant regularization these counterterms certainly can be not gauge invariant.

Having constructed one-loop counterterms, one can calculate two-loop diagrams and perform similar renormalization at the two-loop level. All combinatorics is given by the standard R-operation. After the renormalization in the each loop renormalized Green functions will automatically satisfy renormalized Ward identities. It means, that the present scheme provides gauge invariant result for the effective action even if a regularization is not gauge invariant.
4 Application of universal invariant renormalization at the one-loop level.

To illustrate application of the scheme, described above, let us consider one-loop renormalization of $N = 1$ supersymmetric QED, regularized by cutting-off loop momenta in the gauge with $\xi = -1$.

The diagram, corresponding to the one-loop two-point Green function of the matter field is presented at Figure 1. After simple calculations, using Feynman rules, described in book [21], in the Euclidean space its contribution to the effective action can be written as

$$\Delta \Gamma^{(1)}_{\phi} = - \int \frac{d^4q}{(2\pi)^4} d^4\theta e^2 \left( \phi^*(q)\phi(-q) + \tilde{\phi}^*(q)\phi(-q) \right) \frac{1}{\left(2\pi\right)^4} \frac{1}{2(k + q)^2(k^2 + m^2)},$$

(40)

where integration over loop momentum $k$ is defined as

$$\int d^4k \equiv \int_0^M dk k^3 \int_0^\pi d\theta_1 \sin^2 \theta_1 \int_0^\pi d\theta_2 \sin \theta_2 \int_0^{2\pi} d\theta_3$$

(41)

and $M$ is an ultraviolet cut-off. Therefore, according to equation (18) the function $\Sigma(q)$ can be written as

$$\Sigma(q) = - \int \frac{d^4k}{(2\pi)^4} \frac{e^2}{8(k + q)^2(k^2 + m^2)} = - \frac{e^2}{128\pi^2} \left( \ln \frac{M^2 + m^2}{q^2 + m^2} + 1 - \frac{m^2}{q^2} \ln \frac{q^2 + m^2}{m^2} \right).$$

(42)

Two-point function for the matter superfields is renormalized by subtraction (20), which corresponds to

$$\Delta S = \frac{\alpha}{8\pi} \left( \ln \frac{M^2 + m^2}{\mu^2 + m^2} + 1 - \frac{m^2}{\mu^2} \ln \frac{\mu^2 + m^2}{m^2} \right) \int d^4x d^4\theta \left( \phi^* \phi + \tilde{\phi}^* \tilde{\phi} \right).$$

(43)

The next diagram to be considered is the one-loop three-point vertex function, which is described by the diagrams, presented at Figure 2. Having calculated them in the chosen regularization we obtained, that the corresponding three-point function can be written as

$$\Gamma[(\theta_x, p); (\theta_y, q), (\theta_z, -q - p)] =$$
The structure functions $B_i$ entering equation (21) are

\[
B_1(\theta, p, q) = \bar{D}_x^2 \delta^4(\theta_y - \theta_x) D_x^2 \delta^4(\theta_z - \theta_x);
\]

\[
B_2(\theta, p, q) = \frac{1}{4} \bar{D}_x \gamma^\mu \gamma_5 D_x \left( \bar{D}_x^2 \delta^4(\theta_y - \theta_x) D_x^2 \delta^4(\theta_z - \theta_x) \right);
\]

\[
B_3(\theta, p, q) = p^2 (\Pi_{1/2})_x \left( \bar{D}_x^2 \delta^4(\theta_y - \theta_x) D_x^2 \delta^4(\theta_z - \theta_x) \right). \tag{45}
\]

Renormalized vertex function is defined by subtracting the corresponding polynomials:

\[
\Gamma^r_3[(\theta_x, p); (\theta_y, q), (\theta_z, -q - p)] =
\]

\[
= \left\{ - \int\frac{d^4k}{(2\pi)^4} \left( \frac{e^2}{8k^2((k + q)^2 + m^2)} + \frac{e^2}{8k^2((k + q + p)^2 + m^2)} \right) - P(p) - \right.
\]

\[
- \int\frac{d^4k}{(2\pi)^4} \frac{e^2(k + q + p/2)_\mu}{4k^2((k + q)^2 + m^2)((k + q + p)^2 + m^2)} \frac{1}{4} \bar{D}_x \gamma^\mu \gamma_5 D_x +
\]

\[
+ \int\frac{d^4k}{(2\pi)^4} \frac{e^2}{8k^2((k + q)^2 + m^2)((k + q + p)^2 + m^2)} p^2 (\Pi_{1/2})_x \times
\]

\[
\times \left( \bar{D}_x^2 \delta^4(\theta_y - \theta_x) D_x^2 \delta^4(\theta_z - \theta_x) \right). \tag{46}
\]

Note, that we do not subtract anything from the last two terms, because the corresponding integrals are convergent. Taking into account, that

\[
D^2 \bar{D}_\gamma \gamma_5 D = 4i D^2 \partial_\mu;
\]

\[
\bar{D}_x^2 \gamma_5 D = -4i \bar{D}_x^2 \partial_\mu;
\]

\[
D^2 (\Pi_{1/2}) = D^2 (\Pi_{1/2}) = 0, \tag{47}
\]
the left hand side of Ward identity can be written as

\[ \left( D_x^2 + \bar{D}_x^2 \right) \Gamma^r_3[(\theta_x, p); (\theta_y, q), (\theta_z, -q - p)] = \]

\[ = \bar{D}_x^2 \delta^4(\theta_y - \theta_x) D_x^2 \delta^4(\theta_x - \theta_z) \times \]

\[ \times \left( - \int M \frac{d^4k}{(2\pi)^4} \frac{e^2}{8k^2((k + q + p)^2 + m^2)} - P(p) \right) + \]

\[ + D_x^2 \delta^4(\theta_x - \theta_z) \bar{D}_x^2 \delta^4(\theta_x - \theta_y) \times \]

\[ \times \left( - \int M \frac{d^4k}{(2\pi)^4} \frac{e^2}{8k^2((k + q)^2 + m^2)} - P(p) \right) \]

\[ (48) \]

while the right hand side of the Ward identity is written as

\[ 2 \bar{D}_x^2 \delta^4(\theta_y - \theta_x) \bar{D}_x^2 \delta^4(\theta_x - \theta_z) \Sigma^r(q + p) + 2 D_x^2 \delta^4(\theta_x - \theta_z) D_x^2 \delta^4(\theta_x - \theta_y) \Sigma^r(q). \]

Thus, in this case

\[ Q_1(\theta, p, q) = \bar{D}_x^2 \delta^4(\theta_y - \theta_x) \bar{D}_x^2 D_x^2 \delta^4(\theta_x - \theta_z); \]

\[ Q_2(\theta, p, q) = D_x^2 \delta^4(\theta_x - \theta_z) \bar{D}_x^2 \delta^4(\theta_x - \theta_y). \]

Taking into account, that

\[ \Sigma^r(q) = - \int M \frac{d^4k}{(2\pi)^4} \frac{e^2}{8k^2((k + q)^2 + m^2)} - \Sigma(\mu_\sigma) \]

\[ (51) \]

we obtain, that

\[ P(p) = \Sigma(\mu_\sigma). \]

\[ (52) \]

Therefore, the renormalized three point function is equal to

\[ \Gamma^r[(\theta_x, p); (\theta_y, q), (\theta_z, -q - p)] = \left\{ \Sigma^r(q) + \Sigma^r(q + p) - \]

\[ - \int \frac{d^4k}{(2\pi)^4} \frac{e^2(k + q + p/2)_\mu}{4k^2((k + q)^2 + m^2)((k + q + p)^2 + m^2)} \frac{1}{4} \bar{D}_x \gamma_5 \gamma_5 D_x + \]

\[ + \int \frac{d^4k}{(2\pi)^4} \frac{e^2}{8k^2((k + q)^2 + m^2)((k + q + p)^2 + m^2)} p^2(\Pi_{1/2})_x \right\} \times \]

\[ \times \left( \bar{D}_x^2 \delta^4(\theta_y - \theta_x) D_x^2 \delta^4(\theta_x - \theta_z) \right). \]

\[ (53) \]
It is evident, that our subtraction (together with the corresponding subtraction for $\tilde{\Gamma}$) is equivalent to adding the following counterterms:

$$\Delta S = \frac{\alpha}{8\pi} \left( \ln \frac{M^2 + m^2}{\mu^2 + m^2} + 1 - \frac{m^2}{\mu^2} \ln \frac{\mu^2 + m^2}{m^2} \right) \int d^4x \, d^4\theta \left( \phi^+ 2V\phi - \tilde{\phi}^+ 2V\tilde{\phi} \right),$$

(54)

where we substituted the explicit expression for $\Sigma(\mu)$, constructed above. So, the renormalization of the three-point function is finished. At the next step it is necessary to consider four-point function with two external matter superfields and continue the recurrent process.

The one-loop contribution to the two-point Green function of the gauge field is defined by diagrams, presented at Figure 3 and is equal to

$$\int \frac{d^4p}{(2\pi)^4} \left\{ \int d^2\theta W_a(p)C_{ab}W_b(-p) \int_{-M}^M \frac{d^4k}{(2\pi)^4} \left[ \frac{1}{2(k^2 + m^2)((k+p)^2 + m^2)} \right] + \right.$$ $${\left. + \int d^4\theta V(p)V(-p) \int_{-M}^M \frac{d^4k}{(2\pi)^4} \left( \frac{1}{(k+p)^2 + m^2} - \frac{1}{k^2 + m^2} \right) \right\}.\quad (55)$$

Note, that due to using the regularization, which breaks gauge invariance in equation (55) noninvariant terms are present. Equation (55) corresponds to

$$F_1(p) = \int_{-M}^M \frac{d^4k}{(2\pi)^4} \left( \frac{1}{(k^2 + m^2)((k+p)^2 + m^2)} \right) ;$$

$$F_2(p) = \int_{-M}^M \frac{d^4k}{(2\pi)^4} \left( \frac{2}{(k+p)^2 + m^2} - \frac{2}{k^2 + m^2} \right) = -\frac{p^2}{16\pi^2} + O(1/M^2).\quad (56)$$

in equation (30). (It is not difficult to find explicit expression for $F_1(p)$, but it is rather lengthy and we do not present it here.) After subtraction (37) renormalized contribution of diagrams with two external $V$-lines to the effective action can be presented as

$$\frac{1}{2} \int \frac{d^4p}{(2\pi)^4} d^2\theta W_a(p)C_{ab}W_b(-p) \left( F_1(p) - F_1(\mu) \right),\quad (57)$$

so all noninvariant terms disappear and the result becomes finite. The $\beta$-function, corresponding to expression (57), is equal to

$$\beta = \frac{\alpha^2}{\pi} + O(\alpha^3)\quad (58)$$

and agrees with calculations, made by dimensional reduction [21].

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It is easy to verify, that the above subtraction is equivalent to adding the following counterterms:

\[
\Delta S = -\frac{1}{2} F_1(\mu_x) \Re \int d^4x \, d^2\theta \, W_a C_{ab} W_b - \frac{1}{32\pi^2} \int d^4x \, d^4\theta \, V \, \partial^2 V, \tag{59}
\]

which are not gauge invariant.

5 Conclusion.

In this paper we presented a renormalization procedure for SUSY QED, which guarantees gauge invariance of the renormalized theory for any intermediate regularization. SUSY Ward identities are incorporated into subtractions, which allows to avoid the appearance of noninvariant counterterms.

The same procedure obviously may be applied to non-abelian supersymmetric models provided they are formulated in terms of corresponding superfields. Some technical problems related to more complicated structure of non-abelian theories will be considered elsewhere.

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Figure 1: Feinman diagram, giving nontrivial contribution to the one-loop anomalous dimension.

Figure 2: Feinman diagrams, giving nontrivial contribution to the one-loop three-point vertex function in the $\mathcal{N} = 1$ supersymmetric electrodynamics.
Figure 3: Feinman diagrams, giving nontrivial contribution to the one-loop $\beta$-function of $N = 1$ supersymmetric electrodynamics.