Pseudo-drag of a polariton superfluid

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The indications of a drag of a polariton superfluid by an electric current have been reported in the recent experimental work [6] demonstrating that the speed of a superfluid polariton condensate formed by a non-resonant optical pumping can be controlled by the electric current. The proposed mechanism is based on the stimulated relaxation of moving uncondensed excitons dragged by the electric current. The stimulated relaxation process favors the formation of a moving condensate in a quantum state that is characterised by the lowest condensation threshold. We also show that the electron-mediated inelastic scattering of the reservoir excitons to the condensate leads to the transfer of a non-zero mean momentum to the electron gas thus contributing to the electric current. We predict the generation of circular electric currents in a micropillar cavity in the presence of a nonresonant laser pumping at normal incidence.

Introduction.—A superfluid state of matter is characterized by zero viscosity, hence it is perfectly protected from being perturbed by a weak external force. This protection constitutes one of the main experimental signatures of a superfluid such as a liquid helium [1, 2]. However, this paradigm is challenged for the out-of-equilibrium systems. In particular, optically pumped bosonic condensates of exciton-polaritons (or simply polaritons) in semiconductor microcavities represent such nonequilibrium systems, where several phenomena consistent with superfluidity have been experimentally demonstrated. Polaritons are hybrid quasiparticles arising due to the strong coupling between excitons and photons. Polaritons obey the bosonic statistics and they may undergo a phase transition into the condensed phase. In this phase, the polariton fluid demonstrates a dissipationless propagation with a subsonic velocity through a weak defect, that manifests its superfluid behaviour [3, 4].

One would naturally expect that in the subsonic regime the polariton superfluid should not be dragged by an electric current flowing either in the same quantum well or in the neighbouring conducting layer. On the other hand, the coupling is possible for non-condensed polaritons which do not belong to the superfluid. Recently, Berman and co-authors [5] predicted the existence of the mutual drag between the normal fraction of a polariton gas and the electric current. Such drag effect is mediated by the long-range interaction between the excitonic component of polaritons and charge carriers, leading to the appearance of the flow of the normal polariton fraction induced by the electric current and vice versa.

The indications of a drag of a polariton superfluid by an electric current have been reported in the recent experimental work [6] demonstrating that the speed of a superfluid polariton flow is sensitive to the magnitude and the direction of the electric current flowing in the same quantum well. Although the particular mechanism governing the detected drag effect remains unclear, the observed effect indicates that the interaction between the polaritons and the electric current is not limited only to the conventional Coulomb drag of the normal fraction.

In this Letter, we show theoretically that an electric current can affect the propagation of a polariton superfluid. The mechanism that governs this effect is related to the usual drag of the normal excitonic component in the presence of an electric current. We demonstrate that this flow of non-condensed excitons (exciton-polaritons) leads to formation of the condensate with a non-zero momentum, and thus provides the pseudo-drag effect to the condensate.

It is well-known that a driven-dissipative polariton condensate, which is formed by the nonresonant pumping, does not necessary occupy the lowest energy stationary state. In fact, the state in which the condensate forms is determined by the balance between the gain and the losses in the polariton system that defines the threshold of polariton lasing [7]. In the case of distributed losses and/or gain, the condensate is formed in the state with the lowest threshold. If the exciton reservoir moves, the gain is changed with respect to the static case, and the condensate may be formed in a moving state as well.

In particular, if the exciton reservoir is dragged by the electric current, the wave-vector of the forming condensate will depend on the direction and strength of the current. This effect is phenomenologically equivalent to the drag of a superfluid. However, the predicted phenomenon is not a direct drag effect, strictly speaking. It is mediated by the excitonic reservoir. That is why we shall refer to it as a pseudo-drag effect.

The model system.—To be specific, we consider an optical microcavity containing both an undoped quantum well and a conducting layer which confines a free elec-
tron gas. The polariton condensate is created in the microcavity by non-resonant laser pumping. This kind of pumping implies excitation of high-momentum excitons which then relax in energy feeding the condensate at the bottom of the lower polariton branch [8].

Since we aim at building a theory that describes the condensation of polaritons in different points of the reciprocal space, we feel it appropriate to start with the semiclassical kinetic equations [8] for the polariton occupation $N_k$ of the quantum state characterized by a wave vector $k$:

$$\frac{dN_k}{dt} = P_k - (\Gamma_k + W^{\text{out}}_k) N_k + W^{\text{in}}_k (N_k + 1), \quad (1)$$

where $P_k$ describes the generation of particles in the state $k$ by the incoherent pumping, $\Gamma_k$ is the decay rate to the exterior of the microcavity, $W^{\text{out}}_k = \sum_{k'} W_{k \to k'} (N_{k'} + 1)$ and $W^{\text{in}}_k = \sum_{k'} W_{k' \to k} N_{k'}$ are the outgoing and incoming rates for the $k$-state due to scattering to and from the $k'$-states, respectively. The single-polariton rates $W_{k \to k'}$ account for the cumulative effect of polariton-phonon, polariton-electron and polariton-polariton interactions.

Describing the condensation dynamics we assume that at the initial moment of time the pump $P_k$ is switched on, starting the competition between the states with different $k$ for the particles created by the pump. The mode with $k = k_\ast$, which wins this competition, accumulates a macroscopically large number of polaritons, thereby manifesting the formation of the condensate. If initially all the polariton states are uniformly populated with small initial seeds, it is natural that the condensate will be formed in the state with the maximum growth rate, which we are aimed to determine in what follows.

First, we neglect the transitions of particles from the lower energy states to the reservoir, since these transitions are unlikely at low temperatures. Thus the outgoing rate $W^{\text{out}}_k$ for the $k$-state corresponds to the downward transitions only and $W^{\text{out}}_k = \sum_{|\mathbf{k}'| < |\mathbf{k}|} W_{\mathbf{k}' \to \mathbf{k}} (N_{\mathbf{k}'} + 1)$. In the parabolic region of the polariton dispersion one can approximate $W^{\text{out}}_k \simeq \gamma k^2$, where the factor $\gamma$ is defined by the relaxation mechanism [9][10].

Next, we note that the pump $P_k$ feeds mainly the exciton-like large wave vector states. These excitons form an incoherent reservoir a particle concentration $N_r$, and they are responsible for the pumping of the lower energy polariton states with the rate $W^{\text{in}}_k$. With the same accuracy, the $k$-dependence of the incoming rate is assumed to be parabolic. For the static in average reservoir, it is natural to assume that the $k = 0$ state has the maximum gain and $W^{\text{in}}_k \propto (1 - sk^2)$. Then, if the reservoir is in motion characterized by the mean wave vector $\mathbf{k}_r$ (defined below), we can write

$$W^{\text{in}}_k = r(k)N_r, \quad r(k) \simeq r_0 \left[1 - s(\mathbf{k} - \mathbf{k}_r)^2\right]. \quad (2)$$

The parameter $s$ in this expression can be estimated by assuming that excitons in the reservoir obey the Boltzmann statistic. We shall also assume that the wave vector dependence of the scattering probability is governed by the characteristic scattering cross-section dependent on the thermal De Broglie wavelength of the scatterer $\lambda_\text{th} = \sqrt{\frac{2\pi\hbar^2}{m_\text{ex} k_\text{B} T}}$, where $k_B$ is the Boltzmann constant [11]. We estimate $s = \lambda^2_\text{th} \approx 0.058 \, \mu m^2$ for the exciton mass $m_\text{ex} \sim 0.1 m_0$ and at the temperature $T \sim 1 \, K$.

In order to account for the superfluid dynamics we supplement the rate equations (1) with the generalised Gross-Pitaevskii equation for the condensate wave function $\Psi$, similar to [12], which in our case takes into account the $k$-dependencies of the gain and the loss rates:

$$\partial_t \Psi = \hat{D} \Psi - i\hbar^{-1} \hat{H} \Psi, \quad \partial_t N_r = P - [\Gamma_r + R(\Psi)] N_r - \nabla \cdot J_r. \quad (3a)$$

Here the Hamiltonian dynamics is described by $\hat{H} = -\frac{\hbar^2}{2m_\text{pol}} \nabla^2 + \alpha_c |\Psi|^2 + \alpha_r N_r$, with $m_\text{pol}$ being the polariton mass and the $\alpha_c$ and $\alpha_r$ terms describing the energy shifts due to interaction of polaritons between themselves and with reservoir excitons, respectively. The gain-dissipation term with $\hat{D} = N_r \hat{\Gamma} - \hat{\Gamma}$ is given by the gain operator $\hat{\Gamma} \equiv r(-i\nabla)$ and the dissipation operator $\hat{\Gamma} = \Gamma_0 - \gamma \nabla^2$, where $\Gamma_0$ is the radiative loss rate, which is taken identical for all the states at the bottom of the lower polariton branch. In Eq. (3b), the term $P$ accounts for the effective pumping of the reservoir, $\Gamma_r$ describes the decay rate of the reservoir population. We impose the local conservation of the number of polaritons in the process of their relaxation to the condensate, so that the corresponding reservoir depletion rate is given by $R(\Psi) = 1/2 \langle \Psi^* \hat{\Gamma} \Psi + \Psi \hat{\Gamma} \Psi^* \rangle$.

The last term in Eq. (3b) provides the continuity of the mass flow in the presence of a spatially inhomogeneous current $J_r$ of reservoir excitons. This current is related to the wave vector $\mathbf{k}_r$ entering Eq. (2) by $J_r = \hbar k_r N_r / m_\text{ex}$. The reservoir flux can be created either by the exciton density gradient or by the external force, in particular, by the Coulomb drag. If the electric current flows in the adjacent conducting layer, the exciton current may be induced by the exciton interaction with the moving carriers. The value of the exciton current is dependent on the exciton and carrier densities as well as on their relative velocity. Namely,

$$J_r = -D \nabla N_r + \lambda \left( \mathbf{v} - \frac{\hbar k_r}{m_\text{ex}} \right) n N_r, \quad (4)$$

where $D$ is the exciton diffusion coefficient, $\lambda$ is the drag coefficient, $\mathbf{v}$ and $n$ are the drift velocity and the surface density of the carriers. Note that the term responsible for the drag vanishes where the excitons and the charge carriers move with equal velocities.
The drag coefficient $\lambda$ can be estimated from Ref. [3], to be $\lambda \approx 1.1 \times 10^{-11}$ cm$^2$ for the case of the conducting layer represented by the n-doped GaAs quantum well with $n = 10^{11}$ cm$^{-2}$ and the electron mobility $\mu = 10^3$ cm$^2$V$^{-1}$s$^{-1}$ [13].

Polariton pseudo-drag effect in a microcavity stripe.— A remarkable manifestation of the pseudo-drag effect could be found in the double layer system embedded in a microcavity stripe, see Fig. 1a. We shall consider a one-dimensional exciton polariton condensate excited by a wide spot of the nonresonant continuous wave laser pumping, that for simplicity we assume to be homogeneous in space.

The voltage applied at the metal contacts [3] induces the electric current in the conducting layer, which drags the reservoir excitons in the second quantum well with the mean wave vector $k_r = 1.35 \mu$m$^{-1}$. The shaded domain shows the net gain $G(k)$ at the pump power corresponding to the red curve. (c) The polariton dispersion. The red point indicates the condensate wave vector at the threshold pump power $P = P_{th}$. The green point corresponds to the state with the maximum net gain at $P = 1.5P_{th}$. (d)-(f) Distribution of the condensate phase Arg($\Psi$) (lower bar) for different values of the electric current flowing in the adjacent quantum well (upper bar) at different pump powers $P$. The white arrows on the upper bar indicate the magnitude and the direction of the condensate wave vector $k_c$. Panels (g) and (h) show respectively the dependencies of the condensate wave vector $k_c$ on the pump power $P$ for different values of the electric current and on the electric current $I$ at different pump powers.

The polariton dispersion is given by the equation $\epsilon(k, \omega) = \epsilon_0 e^{-i\omega t + ik_{c}x}$, where $\epsilon_0$ is the dielectric constant of the medium, $\omega$ is the angular frequency, and $k_{c}$ is the wave vector of the condensate.

The drag coefficient $\lambda$ can be estimated from Eq. (4) with $\mu = m_e \lambda/(\hbar (1 + \lambda n))$ for the considered parameters equals to $5.4 \times 10^{-7}$ A$^{-1}$. It implies that for $d = 4 \mu$m stripe width the reservoir flow with $k_r = 1 \mu$m$^{-1}$ can be induced by an electric current of $I = jd \approx -7.5 \mu$A. This value agrees with the recent experimental studies of the drag effect in a polariton wire structure [6].

The condensate steady state reads $\Psi = \Psi_0 e^{-i\omega t + ik_{c}x}$, where the wave vector $k_{c}$ is selected during the condensate formation and corresponds to the quantum state characterised by the fastest growth rate (see Eq. (3a))

$$G(k) = \Psi_0^{-1} \hat{D} \Psi = N_c r(k) - \Gamma(k),$$

where $\Gamma(k) = \Gamma_0 + \gamma k^2$ and $r(k)$ is defined by Eq. (2). Note that the growth rate is time-dependent since it is governed by the reservoir density $N_c(t)$. As the pump power switches instantly, the reservoir, which grows with the rate $\Gamma_r$, approaches its trivial steady state $N_0 = N_c(|\Psi|^2 = 0) = P/\Gamma_r$ faster than the condensate forms, provided that $\Gamma_r \gg G(k_{c})$. The latter condition always holds in the vicinity of the threshold, where $G(k_{c})$ is small.

Above the threshold this condition is guaranteed in the limit $\Gamma_r \gg \Gamma_0$, which is frequently assumed for polariton systems [14,15]. As a result, the increase of the polariton population, which governs the choice of the condensate wave vector, occurs with the rate $G(k_{c}) \equiv G(k, N_0)$, characterized by the unsaturated gain $N_0 r(k)$.

If the reservoir is at rest, $k_r = 0$, in the absence of the electric current, the growth rate (6) is maximized at $k_c = 0$. Thus, above the threshold $P > P_{th} = \Gamma_r \Gamma_0/r_0$
FIG. 2. Generation of the circular electric current supported by the electron-mediated exciton scattering. (a) A pillar microcavity with the embedded two-layer system. The condensate density \( N_c \) distribution is sketched at the top surface of the micropillar. (b) The schematic illustration of the electric current generation mechanism. (c) The dependence of the electric current \( I \) and the corresponding magnetic field \( H \) at the center of the quantum well plane on the condensate density \( N_c \).

the condensate occupies the ground state. However as the reservoir is dragged by the electric current, the gain \( N_{opt}(k) \) maximum shifts towards \( k = k_r \), see Eq. (2) and Fig. [1]. In this case the maximum of the growth rate, which is indicated by the shaded region in Fig. [1], corresponds to the state

\[
k_c = \frac{sr_0N_0k_r}{sr_0N_0 + \gamma},
\]

(7)

which is the state where the condensate is formed in. The flow of the polariton condensate can be experimentally detected by the deviation of the condensate emission angle from the normal to the microcavity surface.

Note that the condensate forms with the non-zero wave vector even at the threshold pump power (green solid curve in Fig. [1]). The condensation threshold can be defined from the condition of the uniqueness of the solution of the equation \( N_r r(k) = \Gamma(k) \), see the red points in Figs. [1], [2], [3].

The flow of a polariton superfluid is indirectly controlled by the external electric current, as it is schematically illustrated in panels (d), (e) and (f) of Fig. [1]. The condensate moves along the direction of the motion of charge carriers having the wave vector \( k_c \) which is dependent on both the electric current and the incoherent pump power, according to Eq. (7). In particular, the condensate momentum plotted in Fig. [1] grows with the increase of the pump power approaching \( k_r \) in the limit of high pump power, see Figs. [1] and [2]. At the same time the momentum linearly increases with the electric current increase, see Fig. [1].

**Polariton whirl producing a circular electric current.**—So far, we neglected the back action of the exciton-polariton subsystem on the electron gas. This action is twofold. First, due to the inverse Coulomb drag effect \[5, 16\] the exciton flow induces an electric current in the conducting layer. This effect can be accounted for in a similar way to Eq. (4)

\[
j_{\text{drag}} = -\nu \left( \frac{\hbar k_r}{m_{\text{ex}}} - \mathbf{v} \right) n_N, \tag{8}
\]

where \( \nu \) is the drag coefficient and we consider again the electron-doped case, which is reflected by the minus sign before \( \nu \).

Besides this, there is the stimulated electron-assisted scattering of excitons from the reservoir to the condensate. Each act of electron-assisted scattering transfers the momentum \( \hbar \delta k = \hbar (k_r - k_c) \) to the electron gas. This momentum transfer can be described in terms of a force acting on a single electron:

\[
F = \beta \frac{\hbar N_r R(\Psi)}{n} \delta \mathbf{k}, \tag{9}
\]

where \( N_r R(\Psi) \) is the scattering rate defined in Eq. (3b), and the factor \( \beta < 1 \) accounts for the part of the total number of electron-mediated scattering events. Then, the current \( j_{\text{scat}} \) induced by the electron-mediated scattering can be calculated using the classical Drude theory of conductivity:

\[
j_{\text{scat}} = \sigma \delta \mathbf{k} N_r R(\Psi), \tag{10}
\]

where \( \sigma = \hbar \beta \mu \), which for the considered parameters and \( \beta = 0.05 \) is \( \sigma = 5.26 \times 10^{-9} \text{ C cm}^2 \).

Note, that both discussed mechanisms are capable of inducing an electric current at zero voltage. However, in contrast to the Coulomb drag effect, the electron-mediated scattering induces the current which is dependent on the state of polariton superfluid. It allows for nontrivial manifestations of the effect. For instance, a spectacular property of the exciton-polariton condensate is its ability to form a persistent circular current or vorticity. Once such a circular current is created in a non-resonantly pumped polariton condensate, its non-zero
angular momentum is partially transferred to the electron gas producing a circular current as Fig. 2b schematically shows.

Let us study these phenomenon in detail. Although polariton condensates carrying non-zero angular momenta were obtained in many different configurations [13,17], to be specific we focus on the system shown in Fig. 2a. We shall assume that the double layer system is embedded in a cylindrical microcavity pillar, where the formation of a ring-shaped condensate carrying the persistent polariton current was recently demonstrated [18].

The distribution of the condensate density \( N_c = |\Psi|^2 \) is characterized by the mean radius \( R_0 \) and the width \( 2a \), see Fig. 2a. Then, assuming that \( N_c \) takes the constant value over the width of the ring [19], we estimate the rate of scattering as \( N_c R(\Psi) = r_0 \left( 1 - s(\mathbf{k} - \mathbf{k}_c)^2 \right) N_c N_c^{\text{(st)}} \), where the condensate wave vector is defined by the winding number \( m \) as \( \mathbf{k} = m/R_0 \). The reservoir steady-state density \( N_c^{\text{(st)}} \) can be obtained from Eq. (3b).

\[
N_c^{\text{(st)}} = \frac{\Gamma_0 + \gamma k^2}{r_0 \left( 1 - s(\mathbf{k} - \mathbf{k}_c)^2 \right)}. \tag{11}
\]

In order to estimate the value of the induced current \( I_{\text{scat}} = 2a j_{\text{scat}} \), we assume that the reservoir is at rest, \( \mathbf{k}_r = 0 \), and take \( m = 1 \), \( a = 1 \mu m \) and \( R_0 = 10 \mu m \). So, for the typical value of the condensate density \( N_c = 10^{10} \text{cm}^{-2} \) we obtain that the current is on a picoampere scale. According to Eq. (5) this current induces a negligibly small flow of reservoir excitons (see Fig. 2b), that is consistent with our assumption of the stationary reservoir.

The existence of the predicted circular electric current can be experimentally detected measuring the current-induced magnetic field. Assuming that the circular current density \( j_{\text{scat}} \) follows the condensate density \( N_c \) distribution, we estimate the magnetic field in the center of the ring \( H = I_{\text{scat}}/2R_0 \), which can be detected with the state of the art SQUID magnetometers [20]. The dependence of the induced electric current given by Eqs. (10), (11) and the corresponding magnetic field on the condensate density is shown in Fig. 2b.

In conclusion, we demonstrate that propagation of out-of-equilibrium polariton condensate can be controlled by an electric current, as the latter influences the gain rate of different single-polariton quantum states. The effect appears due to drift of the excitonic reservoir. The existence of this pseudo-drag effect paves the way for the engineering of the integrated optical circuits operating with polariton condensates. It may be promising for the creation of superfluid gyrosopes and quantum interferometers. The reciprocal effect of acceleration of the charge carriers by the moving polariton superfluid is also described. Being sustained by the stimulated exciton-electron scattering, the carrier’s flow is evidenced by the circular electric current in a cylindrical micropillar excited by a nonresonant laser pump.

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