Evaluating the Force Concept Inventory for different student groups at the Norwegian University of Science and Technology

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Abstract
The Force Concept Inventory (FCI) was developed by Hestenes, Wells and Swackhamer, in order to assess student understanding of the concept of force. FCI has been used for over 20 years and in different countries. When applying the inventory in a new context it is important to evaluate the reliability and discrimination power of this assessment tool. In this study the reliability and discrimination power are evaluated in the context of Engineering education at a Norwegian university, using statistical tests, focusing on both item analysis and on the entire test. The results indicate that FCI is a reliable and discriminating tool in most cases. As there are exceptions, statistical tests should always be done when FCI is administered in a new context.

1 Introduction
Standardised multiple-choice tests can be used as a tool in physics education to assess student learning. A number of such tests have been developed covering a range of different domains in physics. One of the most commonly used test, the Force Concept Inventory (FCI), was introduced by Hestenes, Wells and Swackhamer [1]. FCI has since then been used as a tool in assessing the efficiency of a number of developed teaching methods (see for example [2]). FCI is limited to the understanding of the concept of force, but an increased understanding of force should work as a more general indication of learning in mechanics as a whole. Considering the extended use of FCI, it should also work as a tool in assessing the learning of Norwegian students in their introduction physics courses. In order to investigate the reliability and discrimination power of FCI at a Norwegian university, a number of statistical tests focusing both on individual
items and on the test as a whole, has been performed. There exist two aspects of test reliability; consistency and discriminatory power. A test is said to be reliable if it is consistent within itself and over time. If a test is shown to be reliable, one can assume that the same students would get the same score if they would take the test again after a period of time. A large variance in the test score of a reliable test will then depend on a systematic variation in the student population, where different levels of understanding or mastery will give different scores on the test. Both these aspects of test reliability can be assessed statistically. In order to evaluate the reliability of the FCI in a Norwegian context, the test was administrated to different student groups at the Norwegian University of Science and Technology (NTNU) in Trondheim. Even if the test is intended to be of general use, the level of the students understanding or mastery will affect the usefulness of the test, especially when the group has a higher degree of understanding or mastery.

2 Background

A concept inventory is a criterion-referenced test designed to evaluate if students have an accurate knowledge of a specific set of concepts within a defined area. Concept inventories are typically organized as multiple-choice tests in order to ensure that they are objectively scored in a reproducible manner and possible to administrate in large classes. Unlike a teacher-made multiple-choice test, questions and response choices in concept inventories are a subject of extensive research and development. The aims of the research may include ascertaining (a) the range of what individuals think a particular question is asking and (b) the most common responses to the questions. In the concept inventory, each question includes one correct answer and several distractors. The distractors are incorrect answers that are usually (but not always) based on students’ commonly held misconceptions. Ideally, the scores should reflect the amount of content knowledge students has mastered. The purpose of a criterion-referenced test is to ascertain whether students master a predetermined amount of content knowledge. The distractors are often based on ideas commonly held by students, as determined by years of research on misconceptions.

The Force Concept Inventory (FCI) \cite{1} is a multiple-choice test, designed to assess student understanding of the most basic concepts in Newtonian physics, particular forces. The test has 30 questions covering six areas of understanding: kinematics, Newton’s First, Second, and Third Laws, the superposition principle, and types of forces (such as gravitation, friction). Each question has only one correct Newtonian answer, with distractors based on student’s common misconceptions. A low score indicates that the student has an Aristotelian view while a high score (typically around 60% correct or higher) indicates a Newtonian understanding. The Norwegian version of FCI used, was translated and developed by Angell and collaborators at University of Oslo \cite{3}. 

\begin{thebibliography}{1}
\bibitem{1} Force Concept Inventory (FCI).
\bibitem{3} Angell, University of Oslo.
\end{thebibliography}
| Course   | Pre-test | Post-test |
|----------|----------|-----------|
| TFY4104  | 182      | 105       |
| TFY4115  | 91       | 58        |
| TFY4145  | 140      | 91        |

Table 1: Number of students taking the FCI.

3 Student groups

The test was given in three different courses with different student groups, both as a pre-(instruction) and post-(instruction) test in the Full semester 2012. The Courses were traditional calculus-based introductory physics courses. As all engineering students at NTNU have to take at least one course in physics, it was possible to administer FCI to both physics masters and non-physics masters. However, different physics courses are given to different masters programs, but all courses contain about the same amount of content relevant for the FCI survey during lectures and are using the same textbook as the main source. The three groups consisted of students in different physics courses; Mechanical Physics (TFY4145/FY1001) for Physics Masters; Physics (TFY4104) for Master students in Marine Technology, Industrial Economics and Technology Management and Mechanical Engineering; and Physics (TFY4115) for Master students in Electronics, Engineering Cybernetics and Nanotechnology. It should also be noted that TFY4104 and TFY4115 include electromagnetics and thermodynamics, respectively, in addition to mechanics.

The test was voluntary with no extra credit given. The numbers of students taking the tests are given in Table 1. The result of the tests with respect to understanding will be presented elsewhere as we focus on the reliability of the test in this paper. The students in the different groups have a similar background, but one can assume that the Physics Masters has a more explicit interest and knowledge in physics and will subsequently score higher on the FCI. The Physics masters and Nanotechnology students are generally believed to be high-achieving students as admission grades are higher compared with the other Master programs. The Physics masters and Nanotechnology students are first year students while the others are second year students. By examining the results in the different groups it is possible to establish the reliability within each group. Using the data from the individual groups we performed five statistical tests: three focusing on individual items (item difficulty index, item discrimination index, item point biserial coefficient) and two on the test as a whole (Kuder-Richardson test reliability and test Ferguson’s $\delta$).

4 Item difficulty index

The item difficulty index ($P$) is a measure of the difficulty of each test item and of the test as a whole. It is defined as the ratio of the total number $N_1$ of correct answers to the total number $N$ of students who answered the specific item:
Table 2: Average difficulty index

|                | Pre-test (%) | Post-test (%) |
|----------------|--------------|---------------|
| TFY4104        | 70           | 72            |
| TFY4115        | 72           | 81            |
| TFY4145        | 78           | 86            |

The difficulty index is, however, somewhat misnamed, since it is simply the proportion of correct answers to a particular item, where the name “easiness index” would be more appropriate. The greater \( P \) value, the higher percentage of correct answers and consequently the easier the item is for the population. The difficulty index will thus depend on the population, something which is the case in this study. There are a number of different criteria for acceptable values of the difficulty index for a test [3]. The optimum value for an item should be \( P = 0.5 \), while it is useful to have a sensible range. A widely adopted criterion requires the difficulty index to be between 0.3 and 0.9 for each question. For a test with a large number \((M)\) of items it is more sensible to consider the test difficulty as the average difficulty index \((\bar{P})\) of all the items \((P_i)\):

\[
\bar{P} = \frac{1}{M} \sum P_i
\]  

Figures 1 and 2 plots the difficulty index \(P\) values for each question in FCI, for the three different student groups and the pre-test and post-test respectively. The difficulty index in the pre-tests, fall, in most cases, within the desired range of 0.3-0.9. There are maximum 4 items with difficulty index above 0.9 in the pre-test, something that is acceptable. In the post-test the number of questions with a difficulty above 0.9 rises to 14 and 7, for the TFY4145 and TFY4115 groups, respectively. The average difficulty indexes for the tests are given in table 2. The average difficulty indexes range from 0.70 to 0.78 in the pre-test to 0.72 to 0.86 in the post-test. Even if the results fall within the acceptable range, the values are very close to the limit of the acceptable range. Taking into account that the number of items with a difficulty index over 0.9 is large, the use of FCI in its original form as a post-test for a student group such as the TFY4145 group, is very questionable. However, if one want to study the weaker part of the student population, the test can still be used.

5 Item discrimination index

The item discrimination index \((D)\) is a measure of the discriminatory power for the individual items in a test. That is, the extent to which an individual test
Figure 1: Difficulty index pre-test

Figure 2: Difficulty index post-test
item distinguishes a student who know the material well from those who do not. A high discrimination index will indicate a higher probability for students with a higher level of knowledge to answer the item correctly, while those with less knowledge will get the wrong answer. The item discrimination index \( D \) is calculated by dividing the sample into two groups of equal size, a high (H) score group and a low (L) score group based on their individual total scores on the test. For each specific item, one counts the number of correct answers in both the high and low groups: \( N_H \) and \( N_L \). Using the total number of students taking the test (\( N \)), the discrimination index for a specific item can be calculated as

\[
D = \frac{N_H - N_L}{N/K}
\]

where K is a numerical factor based on how the division into the high and low group is made. If we split the sample in two, using the median, the high and low groups consist each of 50\% of the total sample, giving K=2. However, it is possible to use other groupings, for example taking the top 25\% as the high group and the bottom 25\% as the low group. The 50\%-50\% grouping may underestimate the discrimination power, since it takes all students into account even those where the difference is small. To reduce the probability of underestimating the discrimination power we use a 25\%-25\% grouping. The discrimination index is then expressed as:

\[
D = \frac{N_H - N_L}{N/4}
\]

The range of the item discrimination index \( D \) is \([-1,+1]\), where +1 is the best value and -1 the worst. In the case where all students in the high score group and none in the low score group get the correct answer the discrimination index would be +1. If none in the high score group and all in the low score group get the correct answer the discrimination index would be -1. These extremes are very unlikely, but shows that items with a negative discrimination index should be removed. A question is typical considered to provide a good discrimination if \( D > 0.3 \), lower values indicate that students resort to guessing on that item. In a test with a large number of items it is possible to allow a few items with a lower discrimination index, but the majority should have higher discrimination indices in order to ensure that the test can distinguish students with strong and weak mastery. It is useful to calculate the averaged discrimination index \( \bar{D} \) for all items in the test.

\[
\bar{D} = \frac{1}{M} \sum D_i
\]

Figures \( 3 \) and \( 4 \) plots the discrimination index \( D \) values for the items in FCI, for the three different student groups and the pre-test and post-test respectively. A majority of the items in the pre-test has a discrimination index \( D > 0.3 \), only a few has a lower value, with variations between different groups. The average discrimination indices are 0.49, 0.49 and 0.45, for the different student groups
(TFY4145, TFY4104, and TFY4115, respectively). This indicates that the FCI has a good discriminating power in the pre-test situation. In the post-test the number of questions with the discrimination index \( D < 0.3 \), rises to 15 and 11, out of 30, for the TFY4145 and TFY4115 groups, respectively, while the averaged discrimination index decreases to 0.36 and 0.42, respectively. This raises serious doubts as how applicable the post-test is for the TFY4145 group. The discrimination power for the TFY4115 group is lower than in the pre-test but still within the accepted range. In the TFY4104 group, the discrimination index remains almost the same (0.48). Questions 6, 16 and 29, and to some extent question 19 are especially doubtful as they combine a high difficulty index, that is being quite simple, with a low discrimination index, not distinguishing the high score and low score groups in these student groups.

![Figure 3: Item Discriminating index pre-test](image)

### 6 Point biserial coefficient

The point biserial coefficient is another measure of the individual item reliability. It reflects the correlation between the total score and the score on individual items in the test. A positive coefficient indicates that a student with a high total score is more likely to answer the item correctly than a student with a low total score. Thus giving a complementary measure to the item discrimination index. In order to calculate the point biserial coefficient for an item, one obtain the correlation between the score for a question and the total scores. If the
number of items in the test is sufficiently large, >20, the test can be viewed as continuous. The point biserial coefficient can then be defined as:

$$r_{pbc} = \frac{\bar{X}_1 - \bar{X}_0}{\sigma_x} \sqrt{\frac{P}{1 - P}}$$

Where $\bar{X}_1$ is the average total score for those who answered an item correctly, $\bar{X}_0$ is the average total score for all participants, $\sigma_x$ is the standard deviation of the total scores and $P$ is the difficulty index for this specific item. For an item to be considered as reliable it should be consistent with the whole test, a high correlation between individual item scores and the total score is desirable. A satisfactory point biserial coefficient is $r_{pbc} > 0.2[4]$. Items with lower values may be used, as long as the number of these items is small, but the test as a whole should have an average higher than 0.2. The average point biserial coefficients for the different student groups are given in table 3. All values are greater than 0.2 so the overall test has a fairly high correlation with the whole test. Figures 5 and 6 shows the point biserial coefficients for the individual items in the pre- and post-tests for different student groups, respectively. It should be noted that questions 16, 19 and 29 overall show a lower degree of correlation than the others. As these questions also show a lower degree of discrimination and these might be subject to revision, at least in the context of the student groups in this study. There is a course-dependent variation of the point biserial coefficient for
|        | Pre-test | Post-test |
|--------|----------|-----------|
| TFY4104 | 0.45     | 0.48      |
| TFY4115 | 0.42     | 0.49      |
| TFY4145 | 0.50     | 0.34      |

Table 3: Average point biserial coefficients

the post-test. These variations might be due to statistical variations or different course context.

![Chart](chart.png)

Figure 5: Point biserial coefficient pre-test

7 Test analysis

The reliability of single items in the test is measured by the point biserial coefficient. In order to examine the reliability of the test as a whole, other methods have to be used. In this work we use two measures of the reliability for the test as a whole: Kuder-Richardson reliability index and Ferguson’s delta ($\delta$).

7.1 Kuder-Richardson reliability index

A not very practical way to evaluate the reliability of a test, is to administer it twice to the same sample. In such a case we would expect a significant correlation between the two test scores, provided the students’ performance is
The correlation coefficient between the two sets of scores will be defining the reliability index of the test. It is obvious that this method is not practical to use. In the case of a test that has been designed specifically for a certain knowledge domain and with parallel questions, the Spearman-Brown formula \([5]\) can be used to calculate the reliability index. This equation connects the reliability index with the correlation between any parallel equally sized subsets in the test. Kuder and Richardson \([6]\) developed this idea further by dividing the test into the smallest possible subsets, that is individual items. This means that each item is considered as a single parallel test, and assuming that the means, variance and standard deviation is the same for all items in the whole test. The result derived gives the reliability index as:

\[
r_{\text{test}} = \frac{M}{M-1} \left( 1 - \frac{\sum \sigma^2_i}{\sigma^2_x} \right)
\]

where \(M\) is the number of items in the whole test, \(\sigma^2_i\) is the standard deviation for the \(i\)th item score and \(\sigma^2_x\) is the standard deviation of the total test score. This expression takes the different variances of the individual items into account, relaxing the assumption that all items must have the same means, variance and standard deviation. For multiple-choice tests the formula can be rewritten as:

Figure 6: Point biserial index post-test
Table 4: Kuder-Richardson reliability index

|        | Pre-test | Post-test |
|--------|----------|-----------|
| TFY4104 | 0.87     | 0.88      |
| TFY4115 | 0.84     | 0.86      |
| TFY4145 | 0.90     | 0.87      |

7.2 Ferguson’s delta

Ferguson’s delta is another widely used whole test statistic. It measures the discriminatory power of the whole test by investigating how the students’ individual scores are distributed. In a test one aims at a broad distribution of total scores, as this is supposed to show a better discrimination. The expression of Ferguson’s delta can be written as [7, p 150]:

\[ \delta = \frac{N^2 - \sum f_i^2}{N^2 - \left( \frac{N^2}{M+1} \right)} \]

where \( N \) is the number of students taking the test, \( M \) is the number of items in the test and \( f_i \) is the frequency of cases with the same score. One should be aware that Ferguson’s delta is more a measure of the population than the test itself, since a change in population will change the result of the Ferguson’s delta formula, while not testing the test itself. If a test and population combined has a Ferguson’s delta greater than 0.90, it is considered to provide a good discrimination for this population [7, p 144]. In our study the Ferguson’s delta is greater than 0.90, in all cases as is shown in table 5.

8 Discussion

The reliability and discriminatory power of the Force Concept Inventory has been evaluated using five statistical tests in three different student groups, both
Table 5: Ferguson’s delta in pre-instructional and post-instructional tests, at the Norwegian University of Science and Technology (NTNU). The aim of this study was to test the applicability of the FCI in different contexts, as made possible with Physics majors and engineering students required to take at least one physics course at NTNU. We have found that the FCI is reliable and discriminating enough for pre-tests in all student groups. The post-test for Physics majors (TFY4145) can not be considered as applicable in the present form for the full group, the average difficulty index (86%) has reached a level where ceiling effects will cause problems. The average discrimination index, though still over the 0.3 level, is not a good indicator as half of the questions have a discrimination index below that level. A similar but not as serious problem can also be seen in the TFY4115 group. However, it is still possible to use the test for specific subgroups, that is low achieving students, in order to investigate their understanding. The Force Concept Inventory is a widely used instrument, but as has been shown here, it can not be used without taking the context and student groups into account. Used on a high-achieving group, there is a substantial risk of encountering ceiling effects, with a decrease in discriminatory power. It will still be useful for the students within this group that has not obtained an understanding of the fundamental concepts. Questions 6, 16, 19 and 29 in the FCI are somewhat problematic and might be replaced with other questions in a high-achieving group, such as TFY4145. However, one might also consider constructing a special high-achieving FCI suitable for Physics majors.

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