Conformal Supersymmetry Breaking
in Vector-like Gauge Theories

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Abstract

A new class of models of dynamical supersymmetry breaking is proposed. The models are based on $SU(N_C)$ gauge theories with $N_F(< N_C)$ flavors of quarks and singlets. Dynamically generated superpotential exhibits runaway behavior. By embedding the models into conformal field theories at high energies, the runaway potential is stabilized by strong quantum corrections to the Kähler potential. The quantum corrections are large but nevertheless can be controlled due to superconformal symmetry of the theories.
1 Introduction

In a supersymmetric (SUSY) $SU(N_C)$ gauge theory with $N_F$ flavors of massless quarks $Q$ and antiquarks $\bar{Q}$, the quantum superpotential implies runaway behavior of vacua for $N_F < N_C$. It has been, recently, shown that the runaway potential is stabilized by quantum corrections to the Kähler potential in a simple extension of the original SUSY gauge theory [1]. The stabilization of the potential leads to a dynamical SUSY breaking.

The model is based on an introduction of a singlet field $S$ with a tree level superpotential $W = \lambda SQ\bar{Q}$. We see again a runaway potential for $S$ in the large $|S|$ region, provided the minimal Kahler potential for the $S$. However, we should take into account quantum corrections to the Kahler potential for the $S$ to examine the dynamics of the $S$ field, since the full potential for $S$ is given by both contributions from the superpotential and the Kahler potential. The Kahler potential for the singlet $S$ is given by the anomalous dimension $\gamma_S$ of the $S$, which is determined by the Yukawa coupling $\lambda$ at the one-loop level. We find that the potential is an increasing function of $|S|$ if $\bar{\gamma}_S(|S|)/2 > 1 - N_F/N_C$ is satisfied ($\bar{\gamma}_S$ will be defined later).

However, because $\bar{\gamma}_S$ is loop suppressed, it is necessary to tune $N_F$ and $N_C$ such that $1 - N_F/N_C \ll 1$ for perturbative computation to be reliable. Furthermore, in Ref. [1], asymptotic non-free nature of the Yukawa coupling $\lambda$ is utilized to stabilize the potential, which implies that $\lambda$ eventually hits a so-called Landau pole at some high energy scale. Then, the theory becomes ill defined in the high energy regime.

In this paper, we embed the above SUSY theory into a conformal one. Because of (approximate) superconformal symmetry, the Kähler potential can be well controlled even in a strong coupling regime. Furthermore, there is no Landau pole problem since all couplings can be assumed to take the fixed point values.

Our discussion in this paper is mainly concerned with the behavior of the potential at the large $|S|$ regime. We will see that the potential is an increasing function of $|S|$, so that the potential minima are at the small $|S|$ regime. Because couplings are strong there, a detailed investigation of the minima is difficult. Nevertheless we will argue in the last section that SUSY is certainly broken, and our theories are indeed dynamical SUSY breaking models.
2 Embedding to conformal field theory

We consider a SUSY $SU(N_C)$ gauge theory with $N_F$ flavors of quarks $Q^i$ and anti-quarks $\tilde{Q}_i$ ($i = 1, \cdots, N_F$) which belong to the fundamental and anti-fundamental representations of the $SU(N_C)$, respectively [2]. We also introduce a gauge singlet chiral superfield $S$. To promote the model to a superconformal theory, we introduce additional $N'_F$ flavors of quarks $P^a$ and $\tilde{P}_a$ ($a = 1, \cdots, N'_F$) which also belong to the fundamental and anti-fundamental representations of the $SU(N_C)$, respectively. We adopt a tree level superpotential

$$W = \lambda S Q^i \tilde{Q}_i + m P^a \tilde{P}_a. \quad (1)$$

When the mass $m$ vanishes, this theory has an infrared conformal fixed point if $N_C$, $N_F$, and $N'_F$ satisfies $3N_C/2 < N_F + N'_F < 3N_C$, as shown in Ref. [2]. (This is an extension of the result of Ref [4]. We show the existence of the infrared fixed point perturbatively in Appendix A and prove the relation $3N_C/2 < N_F + N'_F$ by using general properties of conformal field theories later.)

In a region where the vacuum expectation value (vev) of $S$ is large, the $Q$ and $\tilde{Q}$ are massive, which we can integrate out. Furthermore, in a low-energy regime, the $P$ and $\tilde{P}$ can also be integrated out, and the low-energy dynamical scale of the $SU(N_C)$ gauge theory is given by

$$ \Lambda^{3N_C} = m^{N'_F} \Lambda^{3N_C - N_F - N'_F}, \quad (2)$$

where $\Lambda$ is the dynamical scale of the high-energy theory. Then the low-energy effective superpotential of $S$ is given by

$$W_{\text{eff}} = N_C \Lambda^{3N_C} = N_C [m^{N'_F} \Lambda^{3N_C - N_F - N'_F} (\lambda S)^{N_F}]^{\frac{1}{N_C}}. \quad (3)$$

We take the number of flavors of $Q$, $\tilde{Q}$ to be $N_F < N_C$, and in this case the superpotential (3) is of the runaway type. As discussed in Ref. [1], we must know the effective Kähler potential to determine whether the total potential (as opposed to superpotential) is a runaway one. In Ref. [1], a weak coupling analysis is done to compute quantum corrections to the effective Kähler potential. We do not assume the weak couplings here, and instead, we use a superconformal symmetry to determine the Kähler potential even in a strongly coupling theory in the following.
3 Low-energy effective Kähler potential

Now let us consider the low-energy effective Kähler potential of the singlet $S$ when massive quarks $Q$, $\tilde{Q}$ and $P$, $\tilde{P}$ are integrated out. We consider in the region where the vev of $S$ is such that the mass of $Q$, $\tilde{Q}$ is much larger than the mass of $P$, $\tilde{P}$.

We will utilize the superconformal symmetry of the theory which is realized in the limit $m \to 0$. For this purpose, let us adopt a mass-independent renormalization scheme, where counter terms of dimensionless couplings do not depend on mass parameters. This is possible, because any divergent part of amplitudes can be renormalized by counter terms which do not depend on mass parameters. Thus we can choose (finite as well as infinite part of) counter terms such that they do not depend on mass parameters.

In mass-independent renormalization scheme, $\beta$ functions of dimensionless couplings do not depend on mass parameters. Thus in our case, even if $m \neq 0$, we can tune the gauge and Yukawa couplings so that they are just on the fixed point values of the massless theory. So the coupling constants are really “constant” (i.e. renormalization group invariant).

3.1 Kähler potential in the massless limit

After integrating out $Q$, $\tilde{Q}$ but not $P$, $\tilde{P}$, the singlet field $S$ decouples from the gauge sector, and our theory becomes an $SU(N_C)$ gauge theory with $N'_F$ flavors of quarks $P$, $\tilde{P}$. For $N'_F < N_C$, the corresponding mesonic degrees of freedom diverge, that is, $|P\tilde{P}| \to \infty$ as $m \to 0$. On the contrary, for $N'_F > N_C$, they are vanishing as $|P\tilde{P}| \to 0$ for $m \to 0$, which implies that the effective Kähler potential of $S$ suffers from no divergent contribution in the massless limit. We assume the latter is the case, hereafter.

The conformal piece of the effective Kähler potential is given by

$$\ln \frac{\partial^2 K_{\text{eff}}^{\text{conf}}}{\partial S \partial \bar{S}} = - \int_{\mu=M}^{\ln|S|} \tilde{\gamma}_S(\mu) \, d(\ln \mu),$$

where the renormalization scheme of Ref. [5] has been used (see also Ref. [6] for a Wilsonian approach). Here, $M$ is the renormalization point, and the $\tilde{\gamma}_S(\mu)$ is given in terms of

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1Even in the presence of non zero $m$, we use the same counterterms as those in the massless case to maintain mass-independent renormalization. Then, the right hand side of Eq. (4) differs from the effective Kähler potential $K_{\text{eff}}$ by the mass dependent terms $K_{\text{eff}}^{\text{mass}}$, i.e. $K_{\text{eff}} = K_{\text{eff}}^{\text{conf}} + K_{\text{eff}}^{\text{mass}}$. 

the anomalous dimension $\gamma_S$ of $S$ by 
\[ \tilde{\gamma}_S(\mu)/2 = \frac{\gamma_S(\mu)/2}{1 + \gamma_S(\mu)/2}. \] (5)

In our theory, $\tilde{\gamma}_S(\mu) = \tilde{\gamma}_{S*}$ is constant, which is the fixed-point value of the massless theory. In this case, we obtain
\[ K_{\text{conf}}^{\text{eff}} = (1 - \tilde{\gamma}_{S*}/2)^{-2} M^{\tilde{\gamma}_{S*}} |S|^{2 - \tilde{\gamma}_{S*}}. \] (6)

Because the effective Kähler potential has the scaling dimension two, the scaling dimension $\Delta_S$ of $S$ is given by
\[ \Delta_S = \frac{2}{2 - \gamma_{S*}} = 1 + \frac{\gamma_{S*}}{2}. \] (7)

We give a more explicit perturbative computation for the effective Kähler potential by taking DR scheme in Appendix A. There, momentum dependences (i.e. higher derivative terms) are also investigated.

### 3.2 Mass-dependent corrections and higher derivative terms

The mass-dependent corrections may be written as
\[ K_{\text{eff}}^{\text{mass}} = |\hat{S}|^2 f \left( |\hat{m}/\hat{S}| \right), \] (8)
where $\hat{S}$ and $\hat{m}$ denote such variables that are independent of the renormalization point $M$, and are given by
\[ \hat{S} = M^{\tilde{\gamma}_{S*}/2} S^{1 - \tilde{\gamma}_{S*}/2} = (M^{\gamma_{S*}/2} S)^{1/(1 + \gamma_{S*}/2)}, \] (9)
\[ \hat{m} = (M^{-\gamma_{P*} m})^{1/(1 - \gamma_{P*})}. \] (10)

Note that we do not need the dynamical scale $\Lambda$ in this dimensional argument, since $\Lambda$ is defined as $\Lambda^3 N_C - N_P - N'_P = M^3 N_C - N_P - N'_P \exp(-8\pi^2/g_{*}^2)$ and the gauge coupling $g_{*}$ is constant.

\[ ^2 \text{Our definition of } \gamma \text{ is } -2 \text{ times that used in Ref. [5].} \]
\[ ^3 \text{Although the conformal symmetry is spontaneously broken by the vev of } S, \text{ the relation between } \Delta_S \text{ and } \gamma_{S*} \text{ is the same as in the unbroken conformal field theories.} \]
In fact, the effective Kähler potential (11) and the superpotential (3) can also be rewritten by using $\hat{S}$ and $\hat{m}$. It is easy to see that

$$K_{\text{eff}}^{\text{conf}} = (1 - \hat{\gamma}_{S^*}/2)^{-2} |\hat{S}|^2.$$  \hspace{1cm} (11)$$

The superpotential (3) can be rewritten as follows. Using the relation between $\Lambda$ and $M$,

$$\Lambda^{3N_C - N_F - N'_F} = M^{3N_C - N_F - N'_F} \exp(-8\pi^2/g_s^2),$$

we obtain

$$W_{\text{eff}} = N_C \left[ M^{3N_C - (1 + \gamma_{S^*}/2)N_F - (1 - \gamma_{P^*})N'_F} \exp(-8\pi^2/g_s^2)(M^{-\gamma_{P^*}m})^{N'_F}(\lambda_s M^{\gamma_{S^*}/2}S)^{N_F} \right]^{1/N_C}.$$  \hspace{1cm} (12)$$

Then using the relations $\gamma_{S^*} + 2\gamma_{Q^*} = 0$ and $3N_C - (1 - \gamma_{Q^*})N_F - (1 - \gamma_{P^*})N'_F = 0$, which are the conditions for $\beta$ functions of $\lambda$ and $g$ to vanish, we obtain

$$W_{\text{eff}} = N_C \left[ \exp(-8\pi^2/g_s^2)\hat{m}^{(1 - \gamma_{P^*})N'_F}(\lambda_s \hat{S}^{(1 + \gamma_{S^*}/2)})^{N_F} \right]^{1/N_C}.$$  \hspace{1cm} (13)$$

In this form it is manifest that the dynamical superpotential is indeed renormalization group invariant (6). Because a mass-independent renormalization is used, effects of the non-zero $m$ in the Kähler potential are all contained in the function $f\left(|\hat{m}/\hat{S}|\right)$, which is non-singular in the limit $m \to 0$ in the case we have assumed above. Then, the behavior of the effective Kähler potential in the massless limit $m \to 0$ implies that the $K_{\text{eff}}^{\text{mass}}$ increases at most as $|\hat{S}|^2$ for $|S| \to \infty$. Hence it does not alter the leading behavior of the effective Kähler potential from the conformal term (11) in the regime of large $|S|$. We neglect the term (8) from now on.

Next let us consider the higher derivative terms in the Kähler potential. Because $S$ interacts only through massive quarks $Q$, $\tilde{Q}$, the effective Kähler potential may be expanded in powers of derivatives. Then, for example, terms like

$$\int d^4\theta \frac{|D^2\hat{S}|^2}{|S|^2} \sim \frac{\hat{F}_S^4}{|S|^4}$$  \hspace{1cm} (14)$$

contribute to the potential of $S$, where $\hat{F}_S$ is the $F$ component of the superfield $\hat{S}$ and $D$ is the superderivative. In fact, these terms are effectively suppressed by positive powers of $\hat{m}/\hat{S}$. To see this, let us first assume that these terms are negligible compared to the
Table 1: R-charges of the fields. $x$ is a parameter which represents an ambiguity of the definition of $U(1)_R$. $x$ is completely fixed if we require the $U(1)_R$ to be the one appearing in the superconformal algebra of the theory. The charge of $m$ seen as a spurion field is also listed.

leading term coming from Eq (11). Then, by Eq (13), equation of motion of $\hat{F}_S$ gives

$$\hat{F}_S^* \sim \hat{m}^{(1-\gamma_{P*})\frac{\mathcal{N}_F}{\mathcal{N}_C}} \hat{S}^{(1+\gamma_{S*}/2)\frac{\mathcal{N}_F}{\mathcal{N}_C}-1} = \hat{S}^{2} \left( \frac{\hat{m}}{\hat{S}} \right)^{(1-\gamma_{P*})\frac{\mathcal{N}_F}{\mathcal{N}_C}},$$

(15)

where the relations $\gamma_{S*} + 2\gamma_{Q*} = 0$ and $3\mathcal{N}_C - (1 - \gamma_{Q*})\mathcal{N}_F - (1 - \gamma_{P*})\mathcal{N}_F = 0$ have been used. Thus $|\hat{F}_S/\hat{S}^2|$ is suppressed by positive power of $|\hat{m}/\hat{S}|$, and our initial assumption of neglecting the higher order terms in $\hat{F}_S$ is justified.

### 3.3 Value of the scaling dimension

If all couplings are small, we can use perturbation to compute the anomalous dimension (or equivalently the scaling dimension) of $S$ as in Appendix A. In fact, we do not assume weak couplings, because we allow large anomalous dimension of $S$ at the fixed point. Then the perturbative computations of Appendix A is not reliable.

Even then, we can determine $\Delta_S$ by using a general property of superconformal field theory. In $\mathcal{N} = 1$ superconformal field theory, $U(1)_R$ charges $R$ and scaling dimensions $\Delta$ of chiral (primary) operators are related by $\Delta = \frac{3}{2}R$. So we can obtain the scaling dimensions $\Delta$ from the $R$ charges of the chiral operators.

We list the $R$ charges of the fields in Table 1. We have imposed that the $R$ charges of $Q$ ($P$) and $\tilde{Q}$ ($\tilde{P}$) are the same. Even then, because of the existence of an axial $U(1)_A$ symmetry, the definition of $U(1)_R$ is ambiguous. We represent this ambiguity by parametrizing the $U(1)_R$ by a parameter $x$ in Table 1. But at the conformal fixed point, there is a unique $U(1)_R$ symmetry which appears in the superconformal algebra. This $U(1)_R$ is the one whose $R$ charges are related to conformal dimensions.

This $U(1)_R$ symmetry can be determined by $a$-maximization technique [7]. According to Intriligator and Wecht, we can obtain the value of $x$ for the superconformal $U(1)_R$
Table 2: Scaling dimensions of $S$ for some values of $N_C$, $N_F$, and $N'_F$. In the text we have assumed that $N_C < N'_F$, but we also list the scaling dimensions in the case of $N_C \geq N'_F$ here.

| $(N_C, N_F, N'_F)$ | $(3, 2, 3)$ | $(3, 2, 4)$ | $(4, 3, 3)$ | $(4, 3, 4)$ | $(4, 3, 5)$ | $(5, 3, 5)$ |
|-------------------|------------|------------|------------|------------|------------|------------|
| $\Delta_S$       | 1.78       | 1.46       | 2          | 1.70       | 1.48       | 1.86       |

Charges by (locally) maximizing the following combination of 't Hooft anomalies:

$$a_{\text{trial}} \equiv \frac{3}{32} (3 \text{tr } R^3 - \text{tr } R)$$

$$= \frac{3}{32} [\left(N_C^2 - 1\right)(3 - 1) + \{3(-2x - 1)^3 - (-2x - 1)\} + 2N_CN_F\{3x^3 - x\}$$

$$+ 2N_CN'_F\{3(-N_C + N_Fx/N'_F)^3 - (-N_C + N_Fx/N'_F)\}]$$.

(16)

After straightforward calculations, we obtain the result

$$x = \frac{2 + N_C^2N_F/N'_F^2 - \sqrt{(2 + N_C^2N_F^2/N'_F^2)^2 + (-4 + N_CN_F - N_CN_F^3/N'_F^2)(8/9 + N_C^3N_F/N'_F^2)}}{-4 + N_CN_F - N_CN_F^3/N'_F^2}$$.

(17)

From Table 1, we can see that the $U(1)_R$ charge of $S$ is $-2x$, so the scaling dimension of $S$ is $\Delta_S = -3x$. One can check that in the limit $N_C, N_F, N'_F \gg 1$ and $n \equiv 3N_C - N_F - N'_F = \mathcal{O}(1)$, this exact expression for $\Delta_S = 1 + \gamma_S/2$ coincides with Eq. (A.5). We list some numerical results in Table 2.

In conformal field theories, there are unitarity bounds on scaling dimensions of operators [8] (see also Ref [9]). Scaling dimensions of gauge invariant operators must be equal to or greater than 1. The relation $\Delta = \frac{3}{2}R$ determines the scaling dimensions of operators $P\tilde{P}$ and $P\tilde{Q}$ to be [8]

$$\Delta_{P\tilde{P}} = 3\left[1 - (N_C + N_Fx)/N'_F\right], \quad \Delta_{P\tilde{Q}} = \frac{3}{2}\left[(1 + x) + 1 - (N_C + N_Fx)/N'_F\right].$$

(18)

By requiring $\Delta_{P\tilde{P}} \geq 1$ and $\Delta_{P\tilde{Q}} \geq 1$, we can obtain (assuming $N_F < N'_F$)

$$2 - \frac{2(N_F + N'_F) - 3N_C}{N'_F - N_F} \leq -3x \leq 2 + \frac{2(N_F + N'_F) - 3N_C}{N'_F - N_F}.$$

(19)

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4 Later we will introduce singlets $S_i^j$ and a tree level superpotential $W = \lambda S_i^j Q^i \tilde{Q}_j$. Then the operators $Q^i \tilde{Q}_j$ are not chiral primary because of the equations of motion of $S_i^j$, $\frac{1}{4} D^2(S_i^j)^* = \lambda Q^i \tilde{Q}_j$. Then $\Delta = \frac{3}{2}R$ cannot be used for this operator.
This gives a bound on $\Delta_S = -3x$. This bound suggests that $N_C$, $N_F$ and $N'_F$ must satisfy the relation
\[
\frac{3}{2}N_C \leq N_F + N'_F, \tag{20}
\]
in order that there is a conformal fixed point in our theory, as in Ref. [4].

3.4 Potential of $S$

From the effective superpotential (3) and the effective Kähler potential (6), the potential of $S$ is given by
\[
V(S) = \left( \frac{\partial^2 K_{\text{eff}}}{\partial S \partial S^*} \right)^{-1} | \frac{\partial W_{\text{eff}}}{\partial S} |^2 = M^{-\tilde{\gamma}_S} | N_F m^{N'_F} \lambda^{3N_C - N_F - N'_F} | N_C | S |^{\tilde{\gamma}_S + 2(1 - N_F / N_C)}. \tag{21}
\]
Since $\tilde{\gamma}_S$ is related to $\Delta_S$ by $\tilde{\gamma}_S / 2 = (\Delta_S - 1) / \Delta_S$, the potential is an increasing function of $S$ if the condition
\[
\frac{\Delta_S - 1}{\Delta_S} > 1 - \frac{N_F}{N_C} \tag{22}
\]
is satisfied. Using Eq. (17) (or Table 2), one can check that there exist many sets of values $N_C$, $N_F$, $N'_F$ which satisfy this condition.

4 Adding more singlets

In the model of Ref. [1] and also in the present model, there are mesonic runaway directions $| \det Q_i \tilde{Q}_j | \to \infty$ with $S = 0$ (and $\text{tr} Q \tilde{Q} = Q_i \tilde{Q}_i = 0$ by the equation of motion of $S$). In Ref. [1], there are two parameters, the gauge coupling $g$ and Yukawa coupling $\lambda$. By appropriately choosing these parameters, $S$ can be stabilized at local minima where the vev of $S$ is large enough so that the local minima can be parametrically stable. In the present case, there are essentially one parameter, the mass parameter $m$, because couplings are uniquely fixed by the conformal dynamics. The potential of $S$ is a monotonically increasing function of $S$ in the regime where $m$ dependence in the Kähler potential can be neglected. Thus there is no local minimum in the regime where the vev of $S$ is large, and we have to worry about the mesonic runaway.
To avoid the above difficulty, we can introduce $N_F \times N_F$ singlets $S_{ij}^\tilde{\alpha} \tilde{\beta}$ instead of one singlet $S$, and adopt a tree level superpotential

$$W = \lambda S_{ij}^\tilde{\alpha} \tilde{\beta} Q_i \tilde{Q}_j + m P^a \tilde{P}_a.$$  

Then, as in Ref. [10], the vevs of $Q_i \tilde{Q}_j$ are blocked by the equations of motion of $S_{ij}^\tilde{\alpha} \tilde{\beta}$, and there is no danger of mesonic runaway.

Let us consider the low-energy effective superpotential and Kähler potential of this theory. The superpotential is exactly determined to be

$$W_{\text{eff}} = N_C \left( m^{N_F} \Lambda^{3N_F - N_F - N^\tilde{\alpha}} \det(\lambda S) \right)^{\frac{1}{N_C}},$$

where the $S$ denotes the matrix ($S_{ij}^\tilde{\alpha} \tilde{\beta}$). This form yields a definite scaling behavior $W_{\text{eff}}(\rho^{N_C} S) = \rho^{N_F} W_{\text{eff}}(S)$ for $\rho > 0$. On the other hand, the effective Kähler potential may be much more complicated than that in the case of one singlet. Nevertheless, from the superconformal symmetry, we see that the effective Kähler potential has a scaling behavior $K_{\text{eff}}(\rho^{\Delta_S} S) = \rho^2 K_{\text{eff}}(S)$, where we have neglected the corrections due to the mass $m$ for our purposes, as is the case for one singlet. Hence the Kähler metric is given by

$$g_{IJ} = \frac{\partial^2 K_{\text{eff}}}{\partial S^I \partial S^J},$$

where the index $I$ collectively denotes the indices $i$ and $\tilde{j}$.

Let us investigate the potential of $S$:

$$V = \sum_{I,J} g^{I*J} \left( \frac{\partial W_{\text{eff}}}{\partial S^I} \right)^* \frac{\partial W_{\text{eff}}}{\partial S^J},$$

where $g^{I*J}$ is the inverse of $g_{IJ}$. It is straightforward to see that the ‘vector’ $\partial W_{\text{eff}} / \partial S^I$ is nonzero for any direction $S^I$. Furthermore, the superpotential forces the singlets $S_{ij}^\tilde{\alpha} \tilde{\beta}$ to be a diagonal form $S_i^\tilde{\alpha} \propto \delta_i^\tilde{\alpha}$, up to $SU(N_F) \times SU(N_F)$ transformation, as in the model of Ref. [10]. We assume that the Kähler potential is not so singular as to change this behavior. For example, if the effective Kähler potential is of the form

$${\it K_{\text{eff}}} \propto \text{tr} \left\{ (S^I S)^{1/\Delta_S} \right\},$$

5It is helpful that we can diagonalize the fixed $S$ by means of the $SU(N_F) \times SU(N_F)$ symmetry.  
61-loop computation as in Appendix A suggests that the effective Kähler potential is of the form (27). However, the computation is not so reliable because there are many scales (vevs of $S_i^\tilde{\alpha}$) unlike the case of one singlet.
one can check that the above assumption is indeed satisfied. Therefore, in order to
determine whether the runaway of singlets is stabilized, it is enough to consider the
overall scaling behavior of the potential,

\[ V(\rho S) = \rho^2 \left( \frac{\Delta S - 1}{\Delta S} - 1 + \frac{N_F}{N_C} \right) V(S), \]

and see whether the \( S \) runs away or not, namely, the situation is almost the same as in
the case of one singlet.

There is one important difference from the case of one singlet: the value of \( \Delta_S \). One-
parameter family of \( U(1)_R \) symmetries is the same as in Table 1 but \( a_{\text{trial}} \) and thus the
superconformal \( U(1)_R \) are different. Using \( a\)-maximization, the value of \( x \) is obtained as

\[
2N_F^2 + N_C^2 N_F^2 / N_F'^2 - \sqrt{(2N_F^2 + N_C^2 N_F^2 / N_F'^2)^2 + (-4N_F^2 + N_C N_F - N_C N_F^3 / N_F'^2)(\frac{8}{9} N_F^2 + N_C^3 N_F / N_F'^2)} \\
-4N_F^2 + N_C N_F - N_C N_F^3 / N_F'^2
\]

Several numerical results are listed in Table 3. Using these values of \( \Delta_S \), we can check
that there exist many sets of values of \( N_C, N_F, N_F' \) which satisfy the condition (22).

## 5 Conclusions and discussion

The runaway behavior of dynamical superpotential in certain gauge theories can be sta-
bilized by quantum corrections to the Kähler potential, leading to dynamical supersym-
metry breaking [1]. In this paper, we have extended such theories to the ones which
have superconformal symmetry at high-energy regime. By doing this, the effective Kähler
potential is well controlled even in the strong coupling theories, and we can obtain the
condition, Eq. (22) (with \( \Delta_S \) determined exactly), under which the stabilization by the
Kähler potential occurs. Furthermore, there is no Landau pole problem of the Yukawa
coupling, which the original model suffers from.
In the present model, the singlet $S$ has minima near the origin, $|\hat{S}| \lesssim \hat{m}$. There the superconformal symmetry is explicitly broken and the theory is very strongly coupled. Thus the investigation of the theory near the minima is a rather difficult problem, although not completely impossible. However, even without knowing anything about the minima, we can see that SUSY is broken, by the following Witten index \cite{11} argument.

Suppose that we add a term $\text{tr}(\kappa S) = \kappa^i_\bar{j} \hat{S}^i_\bar{j}$ to the superpotential of the theory. Here $\kappa = (\kappa^i_\bar{j})$ is some matrix with $\det \kappa \neq 0$. By taking $m$ and $\kappa$ to be very large, we can calculate the Witten index of this theory. First, $m$ makes $P$ and $\tilde{P}$ to be very massive, so we can forget about them. Second, $\kappa$ gives vevs to $Q \tilde{Q}$, and if $N_F < N_C$, $Q$, $\tilde{Q}$ and $S$ all become massive. The vevs of $Q \tilde{Q}$ break gauge symmetry $SU(N_C)$ to $SU(N_C - N_F)$. Finally, at low energy we obtain a pure $SU(N_C - N_F)$ gauge theory. Thus the Witten index is $N_C - N_F$. We can explicitly see $N_C - N_F$ vacua by adding to Eq. (24) the term $\text{tr}(\kappa S)$,

$$W_{\text{eff}} = N_C(m^{N_F} \Lambda^{3N_C - N_F - N_F} \det \lambda S)^{1/N_C} + \text{tr}(\kappa S). \quad (30)$$

By the equation of motion of $S$, we obtain

$$S = -\kappa^{-1} \left( \frac{\Lambda^{3N_C - N_F - N_F} m^{N_F}}{\det(-\lambda^{-1}\kappa)} \right)^{N_C - N_F}. \quad (31)$$

Here the $N_C - N_F$ vacua are represented by the $(N_C - N_F)$th root.

Let us take a limit $\kappa \to 0$. Then, all the vacua of Eq. (31) go to infinity. This does not necessarily mean that there is no SUSY vacuum at finite vevs of the fields. But because the Witten index is $N_C - N_F$, and $N_C - N_F$ vacua go to infinity, if SUSY vacua exist at finite vevs, there have to be the same number of “bosonic vacua” and “fermionic vacua”, and these vacua may not be protected by any invariant of the theory, such as Witten index. So, it seems unlikely that such vacua exist, and we can reasonably believe that SUSY is broken in our theory.

\footnote{An easy way to see this is as follows. When $N_F < N_C$, we can describe the theory by using gauge invariant “mesons” $M = (M^i_\bar{j}) = (Q^i \tilde{Q}_\bar{j})$, as long as $\det M \neq 0$. Then, the superpotential $W_{\text{tree}} = \lambda \text{tr}(SM) + \text{tr}(\kappa S)$ is nothing but the mass terms of $S$ and $M$, with $\langle S \rangle = 0$ and $\langle M \rangle = -\lambda^{-1}\kappa$.}

\footnote{Of course this argument is not rigorous, because Witten index may not be well defined at $\kappa = 0$.}
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Appendix A Explicit perturbative computation

In this appendix, we explicitly compute the 1PI effective action of $S$ at 1-loop level using $\text{DR}$ scheme $[12]$. $\text{DR}$ is conventional in perturbative computations in SUSY field theories, and this scheme (like $\text{MS}$) is also known to be a simple example of mass-independent renormalization $[13]$. It is assumed that the gauge and Yukawa coupling constants at the fixed point are small and perturbative calculation is reliable. The renormalization group (RG) argument done in this appendix is valid to all orders in perturbation theory.

First, consider the RG equations of the Yukawa coupling $\lambda$ and the gauge coupling $g$. We will establish that there is indeed an infrared conformal fixed point. By the perturbative non-renormalization theorem, the RG equation of $\lambda$ is

$$
\beta_\lambda \equiv M \frac{d}{dM} |\lambda|^2 = (\gamma_S + 2\gamma_Q) |\lambda|^2, \quad (A.1)
$$

where $\gamma_S$ and $\gamma_Q$ are the anomalous dimensions of $S$ and $Q(\tilde{Q})$ respectively, and $M$ is a renormalization scale. The RG equation of $g$ is given by the NSVZ $\beta$ function $[14]$ up to 2-loop order

$$
\beta_g \equiv M \frac{d}{dM} g^2 = -\frac{g^4}{8\pi^2} \frac{3N_C - (1 - \gamma_Q)N_F - (1 - \gamma_P)N_F'}{1 - N_Cg^2/8\pi^2}, \quad (A.2)
$$

where $\gamma_P$ is the anomalous dimension of $P(\tilde{P})$. 

The $\gamma_S$, $\gamma_Q$, and $\gamma_P$ are given at 1-loop order by

$$
\gamma_S = N_C N_F \frac{|\lambda|^2}{8\pi^2}, \quad \gamma_Q = \frac{|\lambda|^2}{8\pi^2} - \frac{N_C^2 - 1}{N_C} \frac{g^2}{8\pi^2}, \quad \gamma_P = -\frac{N_C^2 - 1}{N_C} \frac{g^2}{8\pi^2}. \quad (A.3)
$$

Using these values in Eqs. (A.1,A.2) and requiring that $\beta_\lambda = \beta_g = 0$, we can obtain

$$
\frac{|\lambda_*|^2}{8\pi^2} \simeq \frac{2n}{N_C N_F N_{\text{tot}}}, \quad \frac{g_*^2}{8\pi^2} \simeq \frac{n}{N_C N_{\text{tot}}}. \quad (A.4)
$$

Here $n \equiv 3N_C - N_F - N'_F$, $N_{\text{tot}} \equiv N_F + N'_F$, and $\lambda_*$ and $g_*$ are the values of $\lambda$ and $g$ at the fixed point. We have assumed that $N_C, N_F, N'_F \gg 1$ and $n = O(1)$. One can easily check using Eqs. (A.1,A.2) that this fixed point is indeed infrared stable, that is, the couplings flow into it, not away from it. The value of $\gamma_S$ at the fixed point is given by

$$
\gamma_{S*} \simeq \frac{2n}{N_{\text{tot}}}. \quad (A.5)
$$

Next let us go to the calculation of the effective action of $S$. At 1-loop level, only a $Q, \tilde{Q}$ loop contributes to the two point function of $S$. One can easily compute the 2-point part of the effective Kähler potential by first computing the 2-point 1PI diagram of $\langle F_S F_S^* \rangle$, the $F$ component of the chiral field $S$, and then infer the entire effective Kähler potential by using supersymmetry. For completeness, we take the external momentum to be non zero. Using DR, the 1PI effective action $\Gamma_{1\text{-loop}}$ is obtained as

$$
\int d^4\theta \int \frac{d^4p}{(2\pi)^4} \left[ 1 - \frac{N_C N_F |\lambda|^2}{16\pi^2} \int_0^1 dz \ln \left( \frac{m_Q^2 + z(1-z)p^2}{M^2} \right) \right] |\tilde{S}(p,\theta)|^2 + \cdots \quad (A.6)
$$

where the background field $S(x,\theta) = S_0 + \tilde{S}(x,\theta)$ with $S_0$ as the zero mode of $S(x,\theta)$, $m_Q = |\lambda S_0|$, and

$$
\tilde{S}(p,\theta) = \int d^4x \tilde{S}(x,\theta)e^{-ipx}. \quad (A.7)
$$

The elipsis in Eq. (A.6) represents terms containing more than two $\tilde{S}$.

We now consider the RG improvement of Eq. (A.6). Suppose that the RG improved form of Eq. (A.6) is given by

$$
\Gamma = \int d^4\theta \int \frac{d^4p}{(2\pi)^4} D(p, m_Q, M, \lambda, g)|\tilde{S}(p,\theta)|^2 + \cdots. \quad (A.8)
$$

---

9 We follow the convention of Ref. [13].

10 We separate the background field $S$ to $S_0$ and $\tilde{S}$ to make computation fairly explicit.
Then, taking into account the fact that \( \Gamma \) is RG invariant and \( \tilde{S} \) is RG variant, with the anomalous dimension given by \( \gamma_S \), the Callan-Symanzik (CS) equation for \( D(p, m_Q, M, \lambda, g) \) is given by

\[
\left( M \frac{\partial}{\partial M} + \gamma_Q m_Q \frac{\partial}{\partial m_Q} + \beta_{\lambda} \frac{\partial}{\partial |\lambda|^2} + \beta_g \frac{\partial}{\partial g^2} - \gamma_S \right) D(p, m_Q, M, \lambda, g) = 0, \tag{A.9}
\]

where we have used the RG equation of \( m_Q \), namely,

\[
M \frac{\partial}{\partial M} m_Q = \gamma_Q m_Q.
\]

In the above analysis, we have shown that the theory flows into a conformal fixed point, which is infrared stable. Then the \( \beta \) functions \( \beta_{\lambda} \) and \( \beta_g \) vanish, and \( \gamma_S \) and \( \gamma_Q \) are constant.\footnote{The couplings might seem to run below the scale \( m_Q \), because conformal symmetry is spontaneously broken by the vev of \( S \). In fact, \( \text{DR} \) is a renormalization scheme in which RG runnings of dimensionless couplings are not affected by mass terms, so that \( \lambda \) and \( g \) can really be constant.}

In this case, Eq. (A.9) can be easily solved. If we rewrite \( D(p, m_Q, M) \) as

\[
D(p, m_Q, M) = (M \hat{D}(p, \hat{m}_Q, M))^\gamma_S, \tag{A.10}
\]

where \( \hat{m}_Q = (M^{-\gamma_Q} m_Q)^{1/(1-\gamma_Q)} \), and \( \hat{D}(p, \hat{m}_Q, M) \) has mass dimension \(-1\), then Eq. (A.9) becomes

\[
M \frac{\partial}{\partial M} \hat{D}(p, \hat{m}_Q, M) = 0. \tag{A.11}
\]

Thus the solution of the CS equation is simply \( \hat{D} = \hat{D}(p, \hat{m}_Q) \). This expression is valid to all orders in perturbation theory.

Let us return to 1-loop computation. To obtain the 1-loop improved effective action, we have to obtain \( \hat{D}(p, \hat{m}_Q) \) to zero-th order in small couplings. Expanding the solution \( D(p, m_Q, M) = (M \hat{D}(p, \hat{m}_Q, M))^{\gamma_S} \) in powers of small couplings and comparing with Eq. (A.6), we obtain

\[
\gamma_S \ln(M \hat{D}(p, m_Q)) = - \frac{N_C N_F |\lambda|^2}{16 \pi^2} \int_0^1 dz \ln \left( \frac{m_Q^2 + z(1-z)p^2}{M^2} \right). \tag{A.12}
\]

From this equation, we can correctly obtain the 1-loop anomalous dimension \( \gamma_S = N_C N_F |\lambda|^2/8 \pi^2 \), and our final expression for the RG improved \( \Gamma \) is

\[
\Gamma = \int d^4 \theta \int \frac{d^4 p}{(2\pi)^4} \exp \left[ -\frac{\gamma_S}{2} \int_0^1 dz \ln \left( \frac{\hat{m}_Q^2 + z(1-z)p^2}{M^2} \right) \right] |\hat{S}(p, \theta)|^2 + \cdots. \tag{A.13}
\]

As a check, consider the limit \( m_Q \to 0 \). Then Eq. (A.13) becomes

\[
\Gamma \to \int d^4 \theta \int \frac{d^4 p}{(2\pi)^4} \exp(\gamma_S) \left( \frac{M^2}{p^2} \right)^{\gamma_S/2} |\hat{S}(p, \theta)|^2 + \cdots. \tag{A.14}
\]
From this effective action, we can compute two point correlation functions. For example, the two point correlation function of the lowest component $\tilde{S}(p)$ of $\hat{S}(p, \theta)$ is

$$\langle \tilde{S}(p)\tilde{S}^*(p') \rangle = (2\pi)^4 \delta^{(4)}(p-p') \exp(-\gamma_S) \frac{M^{-\gamma_S}}{p^{2-\gamma_S}},$$  \hspace{1cm} (A.15)

exactly as expected from conformal invariance of the theory at $S_0 = 0$.

Our real interest is the limit $S_0 \to \infty$. In this limit, Eq. (A.13) becomes

$$\Gamma = \int d^4\theta \int d^4p \left( \frac{M}{\hat{m}_Q} \right)^{\gamma_S} |\tilde{S}(p, \theta)|^2 + \cdots$$

$$= \int d^4\theta \int d^4x \left( \frac{M}{|\lambda S_0|} \right)^{\gamma_S/(1+\gamma_S/2)} |\tilde{S}(x, \theta)|^2 + \cdots, \hspace{1cm} (A.16)$$

where we have used the relation $\hat{m}_Q = (M^{-\gamma_Q} m_Q)^{1/(1-\gamma_Q)}$ and $\gamma_S + 2\gamma_Q = 0$ at the conformal fixed point. Eq. (A.16) is what we need to compute the potential of $S$ when the vev of $S$ is large.

In fact, if we assume that $\hat{D}(p, \hat{m}_Q)$ is non-singular in the limit $p \to 0$, which seems plausible because $S$ only interacts through massive quarks $Q$, $\hat{Q}$, then dimensional analysis tell us that $\hat{D}(p = 0, \hat{m}_Q) \propto \hat{m}_Q^{-1}$. Thus we can conclude that

$$\Gamma = C \int d^4\theta \int d^4x \left( \frac{M}{|\lambda S_0|} \right)^{\gamma_S/(1+\gamma_S/2)} |\tilde{S}(x, \theta)|^2 + \cdots, \hspace{1cm} (A.17)$$

to all orders in perturbation theory, where $C$ is some constant which may depend on coupling constants. From the 1-loop result, we know $C = 1 + \mathcal{O}(g^2, |\lambda|^2)$.

The effective action should not depend on $S_0$ and $\tilde{S}(x, \theta)$ separately, but depend only on the combination $S(x, \theta) = S_0 + \tilde{S}(x, \theta)$. Thus the first term and the dots in Eq. (A.17) should combine together to give

$$\Gamma = \tilde{C} \int d^4\theta \int d^4x M^{\gamma_S/(1+\gamma_S/2)} |S(x, \theta)|^{2/(1+\gamma_S/2)} + \text{higher-derivative terms}, \hspace{1cm} (A.18)$$

where the higher-derivative terms come from expansion of $\hat{D}(p, \hat{m}_Q)$ in powers of $p^2/\hat{m}_Q^2$, and $\tilde{C}$ is defined as $\tilde{C} = C(1+\gamma_S/2)^{-2} |\lambda|^{-\gamma_S/(1+\gamma_S/2)}$. This is the effective Kähler potential in $\overline{\text{DR}}$ scheme.
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