Exploring the birefringence of gravitational waves using ground- and space-based gravitational-wave interferometers

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Abstract

In this work we explore the CPT symmetry breaking and the birefringence in the gravitational sector. The CPT symmetry breaking forces two polarization modes of gravitational waves to travel at slightly different velocities, leading to the birefringence of gravitational waves. Different phase evolution of the two modes induced by the birefringence can be constrained by gravitational wave detectors, and thus an absence of such modifications in a given gravitational wave detection can place an upper limit on the CPT violation. Using the Fisher information matrix, we estimate the projected constraints on the CPT violation and hence on the birefrigence of gravitational waves from advanced ground- and space-based gravitational wave interferometers, including the Advanced Laser Interferometer Gravitational-Wave Observatory, the Einstein Telescope, and the Laser Interferometer Space Antenna.

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I. INTRODUCTION

In the special relativity, the Lorentz symmetry is a fundamental symmetry of Minkowski spacetime. The CPT symmetry is exact in any Lorentz invariant local quantum field theory with a Hermitian Hamiltonian. Both Lorentz and CPT symmetries have been well tested by experiments of high energy physics (see review in Ref. [1]). In the gravitational sector, the local Lorentz symmetry is one of the fundamental pillars of the general relativity (GR). At extremely high energy scales, however, the gravitational Lorentz symmetry is generically expected to be broken in theories of quantum gravity, such as deformed special relativity [2–5], Horava-Lifshitz gravity [6], loop quantum gravity [7, 8], non-commutative geometry [9, 10], and superstring theory [11]. If the local Lorentz symmetry was absent, the CPT symmetry may not hold any more, while the CPT violation necessarily violates the Lorentz symmetry in an interacting theory [12]. The tests of the CPT symmetry are as essential as the tests of the Lorentz symmetry.

An effective field theory (EFT) of gravitational local Lorentz violation has been constructed in the framework of Standard Model Extension (SME) [13]. The local Lorentz-violating operators of arbitrary dimensions have been introduced into the gravitational action. In the limit of flat spacetime, this EFT has been linearized around the Minkowski metric, and a covariant dispersion relation of gravitational waves (GWs) has been deduced out correspondingly [14]. The birefringence of GWs is implied by the separation of two polarization modes due to the two branches of the modified dispersion relation (MDR), which are induced by the CPT-odd Lorentz-violating operators of dimensions higher than four. The group speeds of GWs with the two modes should be slightly different from each other. This fact leads to an arrival-time difference between the two modes in the time domain, or equivalently different dephasing of the two modes in the frequency domain. This phenomena behaves like a correspondence to the birefringence in photon sector [15]. Therefore, one can test the CPT symmetry in the gravitational sector via performing the polarization observations of GWs from compact binary coalescences.

The recent discovery of GWs from compact binary coalescences, reported by the Advanced Laser Interferometer Gravitational-Wave Observatory (Adv.LIGO) and Virgo Collaborations [16–21], opens an observational window to explore the CPT symmetry breaking in the gravitational sector. Due to the arrival-time difference between the two polarization modes of GWs in the presence of birefringence, there would be a slight splitting of the peak at the maximum amplitude of the observed GW signal. Nevertheless, there were no obvious evidences for this splitting reported by the Adv.LIGO detectors [16], and hence an upper limit on the birefringence of GWs can be
obtained using the width of the peak as an upper bound on the splitting. Based on the GW150914 signal in the time domain, an upper limit of \( \sim 2 \times 10^{-14} \) m has been put on the dimension-five CPT-odd Lorentz-violating operators in the gravitational sector [14].

In this work, we aim to study the projected constraints on the CPT symmetry breaking in the gravitational sector and hence on the birefringence of GWs from three GW experiments, which include the second-generation ground-based Adv.LIGO [22], the third-generation ground-based Einstein Telescope (ET) [23], and the space-based Laser Interferometer Space Antenna (LISA) [24]. Introducing the CPT-violating dispersion relation phenomenologically, we explore the impact of the birefringence on the propagation of GWs from distant sources, and deduce the relating modification to the gravitational waveform following Ref. [25]. We try to find relations between this modification and the parameters in the parameterized post-Einstein framework (ppE) [26, 27]. In addition, we employ the Fisher information matrix to estimate the projected constraints on the CPT-violating parameters from these GW interferometers.

The rest of the paper is arranged as follows. In section II, we introduce the CPT-violating dispersion relation of GWs, and study the corresponding modification to the gravitational waveform. In section III, we introduce the Fisher information matrix, and the sensitivity curves of future GW detectors. In section IV, we show the projected constraints on the CPT violation and hence on the birefringence of GWs. The conclusions and discussions are summarized in section V.

II. GRAVITATIONAL WAVEFORM MODIFIED DUE TO THE BIREFRINGENCE

The birefringence of GWs can be induced by the CPT-odd local Lorentz-violating operators with dimensions higher than four in the SME [14, 28], and the modified dispersion of GWs can impact the propagation of GWs [25, 29]. Two polarization modes of GWs have slightly different propagating velocities due to the birefringence, and thus there is an arrival-time difference between the two modes. In other words, the birefringence induces different dephasing of the two polarization modes in the frequency domain.

Regardless of theoretical details, in this work, we propose a phenomenological framework to study the birefringence in the gravitational sector. In the gravitational sector, the CPT-violating dispersion relation of GWs is assumed to take the following form

\[
E^2 = p^2 \left(1 \pm \zeta p^\alpha \right) ,
\]

(1)

where the symbol \( \pm \) reflect the existence of birefringence, and \( \alpha \) denotes a dimensionless parameter.
Throughout this paper we appoint $G = c = 1$ in which $G$ and $c$ denote the Newton’s constant and the speed of light, respectively. We can define an effective energy scale to be $E_\alpha \equiv \zeta^{-1/\alpha}$ when $\alpha \neq 0$, while deduce a speed difference between the two polarization modes as $E_0 \equiv \delta v/c \simeq \zeta$ when $\alpha = 0$. In particular, the above dispersion with $\alpha = 1, 3, 5, \ldots$ corresponds to the rotation-invariant limit of the leading-order dispersion relation in the SME [14].

For any dispersion relation of the form $E^2 = p^2 + Ap^\alpha$, the dephasing of GW signal has been extensively studied in the frequency domain [25]. Therefore, we adopt the similar formula in this work, except introducing different dispersion relations to the two polarization modes, respectively. Without loss of generality, we assume that the plus mode of GWs takes “+” while the cross one takes “−” in Eq. (1), and appoint that both $\zeta$ and $\alpha$ are positive semi-definite, namely $\zeta \geq 0$ and $\alpha \geq 0$. In addition, $\alpha = 0$ implies that the speeds of GWs for the two polarization modes are constant, but different from each other.

Following the deduction in Ref. [25], the dephasing of the two polarization modes in the frequency domain, due to the MDR in Eq. (1), can be written as follows

$$\Psi(f; M_z, \eta, \phi_c, t_c; \alpha) = \Psi_{\text{GR}}(f; M_z, \eta, \phi_c, t_c) \pm \xi(\pi M_z f)^{\alpha+1}, \quad (2)$$

where $M_z = (m_1 + m_2)(1 + z)$ is the total mass of a compact binary coalescence in the observer frame, $\eta = m_1 m_2 (m_1 + m_2)^{-2}$ is the symmetric mass ratio, $M_z = M_z \eta^{3/5}$ is the chirp mass, $\phi_c$ and $t_c$ are coalescence phase and time, $f$ is the observed GW frequency, and $\xi$ is given by

$$\xi = \frac{h^\alpha \zeta}{\pi^\alpha (\alpha + 1) M_z^{\alpha+1}} \int_0^z \frac{(1 + z')^\alpha dz'}{H_0 \sqrt{\Omega_m (1 + z')^3 + \Omega_\Lambda}}, \quad (3)$$

where $z$ denotes a cosmological redshift to the GW source, and $h$ is the Planck constant. Here $m_i$ ($i = 1, 2$) denotes the mass of $i$-th component black hole in the binary system in the source frame, and $\Psi_{\text{GR}} \equiv \Psi_{\text{GR}}(f; M_z, \eta, \phi_c, t_c)$ is the phase of the GR waveform. We adopt the spatially-flat cosmological constant plus cold dark matter ($\Lambda$CDM) model, whose parameters are fixed, namely, the Hubble constant $H_0 = 67.74$ km s$^{-1}$ Mpc$^{-1}$, the matter density today $\Omega_m = 0.3089$, and the dark energy density today $\Omega_\Lambda = 1 - \Omega_m$ [30].

The GR waveform, which includes the inspiral, merger, and ringdown evolution of a compact binary system, can be given by the IMRPhenomB waveform [31], which is available up to an asymmetric mass ratio of 4. To obtain the modified waveform, we take into account different dephasing of the two polarization modes in Eq. (2) to deform the GR waveform. For a given value of $\alpha$, therefore, the adopted gravitational waveform is given by

$$\tilde{h}(f) = C(d_L, \iota) B(f; M_z, \eta) e^{i\Psi(f; M_z, \eta, \phi_c, t_c, \zeta, \alpha)}, \quad (4)$$
where $\mathcal{B}$ encodes the information of intrinsic parameters, and $\mathcal{C}$ encodes the location (luminosity distance $d_L$) and the orientation (inclination angle $\iota$) in the sky frame. The IMRPhenomB waveform in the non-spinning limit can be explicitly found in Appendix A. This waveform is used in this work since it contains the extra information about the post-inspiral stage which can improve the constraints on the phase correction [29].

A mapping relation can be explicitly established between the waveform in Eq. (4) and the ppE waveform of the form $	ilde{h}_{\text{ppE}} = CB (1 + \alpha_{\text{ppE}} u_{\text{ppE}}) e^{i \Psi_{\text{GR}} + i \beta_{\text{ppE}} u_{\text{ppE}}} [26, 27]$, in which one defines a dimensionless frequency $u = \pi M z f$. To be specific, the mapping relation is given by $\alpha_{\text{ppE}} = 0$, $\beta_{\text{ppE}} = \pm \xi$, and $b_{\text{ppE}} = \alpha + 1$. One can constrain these additional parameters using the GW events from the Adv.LIGO and Virgo. However, a detailed study is beyond the scope of this paper, and we thus leave it to future work. In this paper, we focus on the projected constraints on these parameters from three GW experiments.

For the $i$-th GW interferometer of a given experiment considered here, the “observed” gravitational waveform in the frequency domain is written as [32]

$$
\tilde{h}^{(i)}(f) = \frac{\sqrt{3}}{2} F^{(i)}_+ (\tilde{\theta}, \tilde{\phi}, \psi) \tilde{h}_+ (f) + \frac{\sqrt{3}}{2} F^{(i)}_\times (\tilde{\theta}, \tilde{\phi}, \psi) \tilde{h}_\times (f),
$$

(5)

where $F^{(i)}_+$ and $F^{(i)}_\times$ denote the antenna pattern functions for the two polarization modes of GWs, respectively. One can explicitly write $F^{(1)}_+$ and $F^{(1)}_\times$ as follows

$$
F^{(1)}_+ = \frac{1}{2} \left(1 + \cos^2 \tilde{\theta}\right) \cos 2\tilde{\phi} \cos 2\psi - \cos \tilde{\theta} \sin 2\tilde{\phi} \sin 2\psi,
$$

$$
F^{(1)}_\times = \frac{1}{2} \left(1 + \cos^2 \tilde{\theta}\right) \cos 2\tilde{\phi} \sin 2\psi - \cos \tilde{\theta} \sin 2\tilde{\phi} \cos 2\psi,
$$

which are expressed in terms of the source orientation (polar angle $\tilde{\theta}$, azimuthal angle $\tilde{\phi}$) and polarization angle ($\psi$) in the detector frame. For the Adv.LIGO, we just use the above antenna pattern function for a single detector. For the ET [33], two additional sets of antenna pattern functions are given by

$$
F^{(2)}_+ (\tilde{\theta}, \tilde{\phi}, \psi) = F^{(1)}_+ (\tilde{\theta}, \tilde{\phi} + 2\pi/3, \psi),
$$

(6)

$$
F^{(3)}_+ (\tilde{\theta}, \tilde{\phi}, \psi) = F^{(1)}_+ (\tilde{\theta}, \tilde{\phi} + 4\pi/3, \psi).
$$

(7)

For LISA [34, 35], the second set of antenna pattern functions is given by

$$
F^{(2)}_+ (\tilde{\theta}, \tilde{\phi}, \psi) = F^{(1)}_+ (\tilde{\theta}, \tilde{\phi} - \pi/4, \psi).
$$

(8)

If $\psi = \pi/8$ was set fiducially, we can get a relation of $F^{(i)}_+ = F^{(i)}_\times$. In this work, we mainly study the optimally-oriented binaries which are defined to be face-on and located directly above the detectors, namely $\iota = \tilde{\theta} = 0$. In addition, we set a vanishing fiducial value to $\tilde{\phi}$, namely $\tilde{\phi} = 0$. 


III. FISHER INFORMATION MATRIX

The Fisher information matrix [36–38] can be performed to obtain the projected constraints on the CPT symmetry breaking and the birefringence of GWs, given a GW detection which is consistent with the GR. The Fisher matrix is defined as follows

\[ F_{ab} = \sum_{i=1}^{n} \left( \frac{\partial h^{(i)}_{1}}{\partial \theta_a} \right) \left( \frac{\partial h^{(i)}_{1}}{\partial \theta_b} \right), \]  

(9)

where \( \theta_a \) denotes the \( a \)-th parameter, \( n \) denotes the number of GW detectors, and we define an inner product between two frequency-domain gravitational waveforms \( \tilde{h}_1 \) and \( \tilde{h}_2 \) as follows

\[ (h_1 | h_2) = 2 \int_{f_{\text{low}}}^{f_{\text{high}}} \frac{\tilde{h}_1^\ast(f) \tilde{h}_2(f) + \tilde{h}_1(f) \tilde{h}_2^\ast(f)}{S_h(f)} df, \]  

(10)

where the symbol \( \ast \) denotes the complex conjugate, and \( S_h(f) \) denotes the noise power spectral density (PSD) of a GW detector. In Appendix B, we list the noise PSDs of Adv.LIGO [39], ET [40], and LISA [34, 35]. Here \( f_{\text{low}} \) is a lower-cutoff frequency of the detector, while \( f_{\text{high}} \) is an upper-cutoff frequency, at which the GW detection terminates. The signal-to-noise ratio (SNR) for a given gravitational waveform \( \tilde{h} \) can thus be simply defined as \( \text{SNR} = \sum_{i=1}^{n} \sqrt{(h^{(i)}_1 | h^{(i)}_1)} \).

The root-mean-square (rms) uncertainty on the parameter \( \theta_a \) is given by the diagonal component of the covariance matrix \( C_{ab} \), namely \( \Delta \theta_a = \sqrt{C_{aa}} \). The Cramer-Rao bound says an inequality of the form \( C \geq F^{-1} \). Therefore, a lower bound on the rms uncertainty on \( \theta_a \) is given by

\[ \Delta \theta_a = \sqrt{(F^{-1})_{aa}}. \]  

(11)

In this work we use this formula to obtain the projected constraints on the birefringence of GWs from ground- and space-based GW interferometers.

The adopted parameter space is spanned by six independent parameters, which are explicitly written as \( \theta = \{ \ln d_L, \ln M, \ln \eta, \phi_c, t_c, \xi \} \). Here we use the chirp mass in the source frame, namely \( M = M_z(1+z)^{-1} \). Using Eq. (3), we can deduce the constraints on \( \zeta \) and thus on \( E_\alpha \) for a given \( \alpha \), once the constraints on \( \xi \) are obtained. Fiducially, we set vanishing values to \( \phi_c, t_c \) and \( \xi \), and fix \( d_L \) to be 1Gpc for the Adv.LIGO and the ET while to be 3Gpc for the LISA. The Adv.LIGO, ET, and LISA are sensitive to frequency ranges \( 10^{-1} - 10^4 \) Hz, \( 1 - 10^4 \) Hz, and \( 10^{-4} - 10^{-1} \) Hz, respectively. We fix fiducial values \( \iota = \tilde{\theta} = \phi = 0 \) and \( \psi = \pi/8 \). Our results can thus be viewed as optimal constraints on the birefringence in the gravitational sector. However, we also study the influence of these angles on our projected constraints on the birefringence by changing the fiducial values of these angles and extending the parameter space.
In this section, we show the $1\sigma$ uncertainty on $\xi$ for a given $\alpha$, and then convert it to an upper bound on the birefringence of GWs, namely an upper limit on $\zeta$. Or equivalently, we obtain an upper limit on the speed splitting $\delta v_g/c \simeq \zeta$ for $\alpha = 0$, and an lower bound on the effective energy scale $E_\alpha = \zeta^{-1/\alpha}$ for $\alpha > 0$. Fig. 1 and Fig. 2 show the projected constraints on the speed splitting $\delta v_g/c$ and the effective energy scale $E_\alpha$ of the birefringence of GWs, sourced by the optimally-oriented non-spinning compact binary coalescences, from the Adv.LIGO, the ET, and the LISA.

For $\alpha = 0$, there is a slight speed splitting $\delta v_g/c$, which is a constant independent of the GW frequency, between the two polarization modes of GWs. For each a mode, in fact, the modification to the gravitational waveform can be identified with a rescaled coalescence time. Due to opposite modifications to the two modes, however, we can obtain the upper limits on this speed splitting, which are showed in Fig. 1. Using the designed sensitivity of Adv.LIGO, the $1\sigma$ upper bound on $\delta v_g/c$ is $10^{-21} - 10^{-20}$ for the binaries with chirp mass $10 - 10^2 M_\odot$. In the similar mass range, the third-generation ground-based ET can improve the Adv.LIGO upper limits by around two orders of magnitude, and thus reach $\sim O(10^{-23})$ at around $10^2 M_\odot$. However, the ET can probe wider mass range than the Adv.LIGO, and obtain the upper bounds as $10^{-23} - 10^{-21}$ in the mass range $1 - 10^3 M_\odot$. In the more massive mass range such as $10^4 - 10^7 M_\odot$, the LISA shows the upper constraints on $\delta v_g/c$ to be $10^{-20} - 10^{-18}$, which are poorer than those obtained from the Adv.LIGO and the ET. The reason is that the chirp mass appears in the denominator of Eq. (3).

For a given experiment, our constraints on the birefringence of GWs become looser in the most massive mass range, since less frequency modes of GWs remain within the sensitive range of the experiment. They becomes looser in the least massive mass range due to smaller SNRs.

For $\alpha > 0$, the speed splitting between the two polarization modes depends on the GW frequency. For each a mode, the modification to the gravitational waveform can be naively identified as a high-order phase deformation. Due to opposite modifications to the two modes, we can thus obtain the lower limits on the effective energy scales, which describe the appearance of the birefringence. Our constraints on $E_\alpha$ with $\alpha = 1, 2, 3$ are depicted in Fig. 2 for three GW experiments. For $\alpha = 1$, we notice that at $\sim 10 M_\odot$ the designed sensitivity of Adv.LIGO can provide a lower bound of $\sim 1 \text{GeV}$, which corresponds to a length $\sim 1.2 \times 10^{-15} \text{m}$. Compared with the current constraint $\sim 2 \times 10^{-14} \text{m}$ from the first observing run of Adv.LIGO [14], this projected constraint is improved by around six times for the designed sensitivity of Adv.LIGO. In the similar mass range,
FIG. 1: The projected upper bounds on the speed splitting \( E_0 = \delta v_g/c \) between two polarization modes of gravitational waves from the optimally-oriented non-spinning sources (with chirp mass \( M \) in the source frame). For the ground-based Adv.LIGO (green) and the ET (red), we adopt the gravitational-wave sources located at 1Gpc, while for the space-based LISA (blue) we adopt sources at 3Gpc. The solid, dashed, and dash-dotted curves denote the symmetric mass ratio of 1/4, 2/9, and 4/25, respectively. The SNRs are also depicted here.

the ET can improve the Adv.LIGO's projected constraint by more than one order of magnitude, and even reach \( \sim 10\text{GeV} \) at \( 1 - 10^2M_\odot \). At \( 10^4 - 10^6M_\odot \), however, the tightest constraint from the LISA becomes \( \sim 10^{-6}\text{GeV} \), which is much looser than the Adv.LIGO and the ET. When we increase \( \alpha \) from 1 to 3, the lower bound on \( E_\alpha \) becomes lower by around 14 and 15 orders of mag-
FIG. 2: The projected lower bounds on the effective energy scale \( E_\alpha \) with \( \alpha = 1, 2, 3 \) of the birefringence of gravitational waves from the optimally-oriented non-spinning sources (with chirp mass \( M \) in the source frame). For the ground-based Adv.LIGO (green) and the ET (red), we adopt the gravitational-wave sources located at 1Gpc, while for the space-based LISA (blue) we adopt sources at 3Gpc. The solid, dashed, and dash-dotted curves denote the symmetric mass ratio of \( 1/4, 2/9, \) and \( 4/25 \), respectively. The SNRs are also depicted here.

Magnitude at \( \sim 10^2 M_\odot \) for the Adv.LIGO and the ET, respectively, while by 13 orders of magnitude at \( \sim 10^6 M_\odot \) for the LISA. The gap between the constraints for \( \alpha = 1 \) and \( \alpha = 2 \) is nearly three times that for \( \alpha = 2 \) and \( \alpha = 3 \). For each a mode, based on Eq. (2), a higher \( \alpha \) can induce a larger phase correction at higher frequencies. However, the GW interferometers can constrain the
phase deformations of higher orders at a worse level [26, 41, 42]. Our results obviously come from a balance between the above two ingredients.

The symmetric mass ratio ($\eta$) can impact our constraints on the birefringence of GWs. Here we consider $\eta$ within a range $0.25 - 0.16$, which corresponds to an asymmetric mass ratio $q = 1 - 4$. For a given experiment, a smaller $\eta$ (i.e. larger $q$) mainly makes our constraints to be looser in the relatively massive tail of the constraint curve in Fig. 1 and Fig. 2, since less frequency modes remain within the sensitive range of the experiment for more massive binary coalescences. By contrast, the $\eta$ impact on our constraints is not obvious for relatively less massive binary coalescences. In addition, we extend the parameter space to include the inclination angle ($\iota$), the polarization angle ($\psi$), and the source orientation ($\theta$ and $\phi$), and vary the fiducial values of these angle parameters. Though these angle parameters indeed let the $\xi$ constraint become looser, we find that our projected constraints on the birefringence of GWs remain robust, and the modifications to them are less than or around one order of magnitude. The reason is that we utilize only the relative phase difference between the two polarization modes of GWs in our study. A network of GW interferometers can help to reduce the uncertainties on these angle parameters [43].

V. CONCLUSION AND DISCUSSION

In this work, we proposed a phenomenological framework to explore the impact on the propagation of GWs from the CPT symmetry breaking and the birefringence in the gravitational sector. We found opposite dephasing for the two polarization modes of GWs due to the birefringence, and calculated the corresponding gravitational waveform which is modified by the CPT-violating parameters. We proposed to test the CPT violation and the birefringence using GWs from distant compact binary coalescences, and also estimated the projected constraints on the CPT-violating parameters from the second-generation ground-based Adv.LIGO, the third-generation ground-based ET, and the space-based LISA. Our study involves only the relative phase difference between the two polarization modes of GWs, and is thus model independent at some sense. Future GW observations are expected to shed light on the CPT symmetry breaking and the birefringence in the gravitational sector.

In this study we made several approximations which may worsen our projected constraints on the birefringence. For example, we focused on the impact of the birefringence on the kinematical propagation of GWs while disregarded that on the dynamical generation and evolution of GWs. The amplitudes of the two polarization modes may be corrected by the modified theories of gravity.
Theoretical effects towards this direction should be given to the gravitational sector of the SME. For the binary coalescences, we did not take into account the additional parameters such as the eccentricity, the spins, the precession, etc. Recently, the spins of the compact objects in the binary coalescences have been showed to worsen the projected constraints on the Lorentz symmetry breaking [47]. A detailed analysis towards this direction may be also necessary for the CPT violation and the birefringence in the future.

There is one possible way to improve our projected constraints on the birefringence of GWs. In the presence of Lorentz symmetry breaking, the gravitational lensing due to a foreground massive object is expected to be frequency dependent in the time delay for the electromagnetic signal [48]. The situation would be similar to the GW signal. Due to the birefringence, the modifications to the dispersion take an opposite sign for the two polarization modes of GWs. Therefore, the gravitational lensing may contribute an additional arrival-time difference between the two modes. This naive estimation may be helpful to constrain tighter the CPT symmetry breaking and the birefringence of GWs. However, a detailed study on such a topic is beyond the scope of this paper, and we thus leave it to future work.

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Appendix A: The IMRPhenomB waveform

In the non-spinning limit, the IMRPhenomB waveform [31] takes the following form

\[ \tilde{h}(f) = C(d_L, \iota)B(f; M_z, \eta)e^{i\Psi_{GR}(f; M_z, \eta, \phi_c, \iota_c)}. \]  \hspace{1cm} (A1)

For the plus mode one has \( C_+ = d_L^{-1}(1 + \cos^2 \iota)/2 \), while for the cross mode one has \( C_\times = d_L^{-1} \cos \iota \).

The luminosity distance to a redshift \( z \) is given by

\[ d_L = \frac{1 + z}{H_0} \int_0^z \frac{dz'}{\sqrt{\Omega_m(1 + z')^3 + \Omega_\Lambda}}. \]  \hspace{1cm} (A2)

\( B \) has been expressed as follows

\[ B = \left( \frac{5\eta}{24} \right)^{1/2} \frac{M_z^{5/6}}{\pi^{2/3} f_1^{7/6}} \begin{cases} f^{-7/6} \left( 1 + \sum_{i=2}^{3} \alpha_i v^i \right) & \text{for } f < f_1 \\ \frac{w_m f^{-2/3}}{2} \left( 1 + \sum_{i=1}^{2} \epsilon_i v^i \right) & \text{for } f_1 \leq f < f_2 \\ w_r \frac{\sigma/2\pi}{(f-f_2)^2 + \sigma^2/4} & \text{for } f_2 \leq f \leq f_3 \\ 0 & \text{for } f > f_3 \end{cases} \]  \hspace{1cm} (A3)

where one defines \( f' = f/f_1 \) and \( v = (\pi M_z f)^{1/3} \) for simplicity, and \( w_m \) and \( w_r \) are two normalization constants that make \( B \) to be continuous. The phase \( \Psi_{GR} \) has the following form

\[ \Psi_{GR}(f) = 2\pi ft_c + \phi_c + \frac{3}{128\eta v^5} \left( 1 + \sum_{k=2}^{7} v^k \psi_k \right). \]  \hspace{1cm} (A4)

Note that there is a phase difference \( \pi/2 \) between two polarization modes. The parameters \( \alpha_i, \epsilon_i, f_1, \sigma \) and \( \psi_k \) can be expressed in terms of \( M_z \) and \( \eta \) as follows

\[ \alpha_2 = -323/224 + 451\eta/168, \quad \alpha_3 = 0 \]  \hspace{1cm} (A5)
\[ \epsilon_1 = -1.8897, \quad \epsilon_2 = 1.6557 \]  \hspace{1cm} (A6)
\[ \pi M_z f_1 = (1 - 4.455 + 3.521) + 0.6437\eta - 0.05822\eta^2 - 7.092\eta^3 \]  \hspace{1cm} (A7)
\[ \pi M_z f_2 = (1 - 0.63)/2 + 0.1469\eta - 0.0249\eta^2 + 2.325\eta^3 \]  \hspace{1cm} (A8)
\[ \pi M_z \sigma = (1 - 0.63)/4 - 0.4098\eta + 1.829\eta^2 - 2.87\eta^3 \]  \hspace{1cm} (A9)
\[ \pi M_z f_3 = 0.3236 - 0.1331\eta - 0.2714\eta^2 + 4.922\eta^3 \]  \hspace{1cm} (A10)
\[ \psi_2 = 3715/756 - 920.9\eta + 6742\eta^2 - 1.34 \times 10^4\eta^3 \]  \hspace{1cm} (A11)
\[ \psi_3 = -16\pi + 1.702 \times 10^2\eta - 1.214 \times 10^3\eta^2 + 2.386 \times 10^5\eta^3 \]  \hspace{1cm} (A12)
\[ \psi_4 = 15293365/508032 - 1.254 \times 10^5\eta + 8.735 \times 10^5\eta^2 - 1.694 \times 10^6\eta^3 \]  \hspace{1cm} (A13)
\[ \psi_5 = 0 - 8.898 \times 10^5\eta + 5.981 \times 10^6\eta^2 - 1.128 \times 10^7\eta^3 \]  \hspace{1cm} (A14)
\[ \psi_7 = 0 + 8.696 \times 10^5\eta - 5.838 \times 10^6\eta^2 + 1.089 \times 10^7\eta^3 \]  \hspace{1cm} (A15)
Appendix B: The noise power spectral densities of ground- and space-based gravitational-wave detectors

For the designed sensitivity of the second-generation ground-based Adv.LIGO [49], we use an analytic fitting formula to the noise PSD as follows [39]

\[ S_h(f) = 10^{-48} \left( 0.0152x^{-4} + 0.2935x^{9/4} + 2.7951x^{3/2} - 6.5080x^{3/4} + 17.7622 \right) \text{Hz}^{-1}, \quad (B1) \]

where \( x = f/245.4\text{Hz} \). For the third-generation ground-based ET, we employ a fitting formula for the noise PSD as follows [40]

\[ S_h(f) = 10^{-50} \left( 2.39 \times 10^{-27}x^{-15.64} + 0.349x^{-2.145} + 1.76x^{-0.12} + 0.409x^{1.1} \right)^2 \text{Hz}^{-1}, \quad (B2) \]

where \( x = f/100\text{Hz} \). For the space-based LISA, we employ an analytic form for the noise PSD as follows [34, 35]

\[ S_h(f) = \frac{20}{3} \left( \frac{4S_n^{\text{acc}} + 2S_n^{\text{loc}} + S_n^{\text{sn}} + S_n^{\text{omn}}}{L^2} \right) \left[ 1 + \left( \frac{2Lf}{0.41} \right)^2 \right] \text{Hz}^{-1}, \quad (B3) \]

where \( L = 2.5 \times 10^9\text{m} \) is the arm length, and one has the following expressions

\[ S_n^{\text{acc}} = \left( \frac{1\text{Hz}}{2\pi f} \right)^4 \left[ 9 \times 10^{-30} + 3.24 \times 10^{-28} \left( \frac{3 \times 10^{-5}\text{Hz}}{f} \right)^{10} + \left( \frac{10^{-4}\text{Hz}}{f} \right)^2 \right] \text{mHz}^{-1}, \quad (B4) \]

\[ S_n^{\text{loc}} = 2.89 \times 10^{-24} \text{m}^2\text{Hz}^{-1}, \quad (B5) \]

\[ S_n^{\text{sn}} = 7.92 \times 10^{-23} \text{m}^2\text{Hz}^{-1}, \quad (B6) \]

\[ S_n^{\text{omn}} = 4.00 \times 10^{-24} \text{m}^2\text{Hz}^{-1}. \quad (B7) \]

Here \( S_n^{\text{acc}} \), \( S_n^{\text{loc}} \), \( S_n^{\text{sn}} \), and \( S_n^{\text{omn}} \) are noises due to the low-frequency acceleration, the local interferometer noise, the shot noise, and other measurement noise, respectively.
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