Deformed dispersion relations and the Hanbury–Brown–Twiss Effect

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In the present work we analyze the possibility of detecting some deformed dispersion relations, emerging in some quantum–gravity models, resorting to the so–called Hanbury–Brown–Twiss effect. It will be proved that in some scenarios the possibilities are not pessimistic. Forssooth, for some values of the corresponding parameters the aforementioned effect could render interesting outcomes.

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I. INTRODUCTION

The possibility that Lorentz symmetry could depict only an approximate symmetry of quantum space has already been taken seriously, and lot of work has been devoted to this analysis. Within this realm several cases have been studied, for instance, quantum–gravity approaches based upon non–commutative geometry, or loop–quantum gravity models, etc. Though the question of the detection of these effects has always been considered an impossible task, recently, this issue has been addressed with a more optimistic spirit.

One of the predicted traits, emerging in these approaches, embodies a modified dispersion relation, the one renders a small energy–dependent speed for the photon. The feasibility around the detection of these corrections has already been analyzed, though it must be underlined that in the aforementioned cases, it seems that the corresponding experiments have always been considered in the realm of the so–called first–order coherence properties of light. Though this sort of experiments have already played a fundamental role in the context of gravitational physics, it must be also pointed out that they are very sensitive to vibrations and fluctuations in the relative phase of the two involved waves, for instance, due to the propagation through the atmosphere, and hence, they could not be very useful in the present situation, since we expect very tiny modifications.

In this spirit, in the present work the possibilities that the so–called Hanbury–Brown–Twiss (HBT) effect could open up in this realm are analyzed. The intention in our work is twofold. Forssooth, firstly, as it is already known, HBT requires the use of two photodetectors located at different points, and concurrent with this last factor, it is not sensitive to vibrations or atmospheric distortions, and in consequence it seems to fit better with our goals. Secondly, this approach allows us to introduce an additional parameter (the separation between the two detectors), which could help us with our attempt of confronting against the experiment some of these modified dispersion relations. Indeed, as it will be shown below, the measurement of this type of traits will depend (in a second–order coherence process) not only upon the order of magnitude of the corrections, but also upon the distance between the two involved photodetectors. It will be proved that for some of the proposed modifications the current technology could make possible the detection of the corresponding new extra terms. In some other more stringent scenarios, it will shown that a distance of $10^2$km., between the photodetectors, could render an interesting situation in the experimental realm for the case of gamma–ray bursts.

II. SECOND–ORDER COHERENCE AND SOME QUANTUM–GRAVITY EFFECTS

As already mentioned above several quantum–gravity models predict a modified dispersion relation, which can be characterized from a phenomenological point of view through corrections hinging upon Planck’s length, i.e., $l_p$

$$E^2 = p^2 \left[ 1 - \alpha \left( E l_p \right)^n \right]. \quad (1)$$

Here $\alpha$ is a coefficient, usually of order 1 and whose precise value depends upon the considered quantum–gravity model, and $n$, the lowest power in Planck’s length leading to a non–vanishing contribution, is also model dependent. Casting (1) in ordinary units

$$E^2 = p^2 c^2 \left[ 1 - \alpha \left( E \sqrt{G/c^5 \hbar} \right)^n \right]. \quad (2)$$

Recalling that

$$p = \hbar k. \quad (3)$$
It is readily seen that

\[ k = \frac{E/(\hbar c)}{\left[1 - \alpha \left(\sqrt{G/(c^3 \hbar)}\right)^n\right]^{1/2}} \tag{4} \]

Since we expect very tiny corrections, then the following expansion is justified

\[ k = \frac{E}{\hbar c} \left[1 + \frac{\alpha}{2} \left(\sqrt{G/(c^3 \hbar)}\right)^n + \frac{3}{8} \alpha^2 \left(\sqrt{G/(c^3 \hbar)}\right)^{2n} + \ldots\right] \tag{5} \]

Let us now consider two photons propagating along the axis defined by the unit vector \( \hat{e} \), but with different energy. At this point it must be mentioned that in order to avoid a more complicated experimental situation (namely, the consequences of a light source having a continuous frequency distribution, in connection with the presence of a deformed dispersion relation, have not yet been addressed) we introduce only two frequencies.

\[ \vec{k} = k\hat{e}, \tag{6} \]

\[ \vec{k}' = k'\hat{e}. \tag{7} \]

Let us now consider the detection of these photons resorting to HBT \( ^8 \). In other words, we have two photodetectors located at points \( A_1 \) and \( A_2 \), with position vectors, \( \vec{r}_1 \) and \( \vec{r}_2 \), respectively.

The difference between HBT and the first–order correlation function lies in the fact that the former measures the square of the modulus of the complex degree of coherence, whereas a first–order correlation function approach measures also the phase \( ^{10} \).

The second–order correlation function reads \( ^3 \)

\[ G^{(2)}(\vec{r}_1, \vec{r}_2; t, t) = \mathcal{E} \left(1 + \cos \left[\left(\vec{k} - \vec{k}'\right) \cdot (\vec{r}_2 - \vec{r}_1)\right]\right). \tag{8} \]

Here \( \mathcal{E} \) is a constant factor with dimensions of electric field. With our previous expressions, and denoting by \( \Delta \theta^{(n)} \) the phase difference for \( n \), we have that the interference pattern reads (to second–order in \( \Delta E \))

\[ \Delta \theta^{(n)} = \frac{l \Delta E}{\alpha E^2 \sqrt{c^3 \hbar} G} \left(1 + \frac{n + 1}{2} \alpha [E \sqrt{G/(c^3 \hbar)}]^n + \frac{3}{8} \alpha^2 (2n + 1) [E \sqrt{G/(c^3 \hbar)}]^{2n}\right) \tag{9} \]

Here we have assumed that \( E = E' + \Delta E \), and in addition, \( l = \hat{e} \cdot (\vec{r}_2 - \vec{r}_1) \). The analysis of the feasibility of the detection of this kind of corrections depends upon the value of \( n \), at least in the context of a first–order correlation function, and in consequence, we will divide our situation in the same manner, namely, to first order in \( \Delta E \) we have (approximately) that

\[ \Delta \theta^{(1)} = \frac{l \Delta E}{\alpha E^2 \sqrt{c^3 \hbar} G} \left[1 + \alpha [E \sqrt{G/(c^3 \hbar)}]\right] \tag{10} \]

\[ \Delta \theta^{(2)} = \frac{l \Delta E}{\alpha E^2 \sqrt{c^3 \hbar} G} \left[1 + \frac{3}{2} \alpha [E \sqrt{G/(c^3 \hbar)}]^2\right] \tag{11} \]

### III. CONCLUSIONS

Let us now address the issue of the feasibility of this kind of experiments. In order to do this let us assume that \( \Delta E = \frac{E}{\gamma} \), with \( \gamma > 1 \). The possibility of measuring the involved corrections hinges upon the fact that if, \( \Delta \theta^{(0)} \) and \( \Delta \theta^{(exp)} \) denote the phase difference in the case in which \( \alpha = 0 \), and the experimental resolution, respectively, then \( \Delta \theta^{(n)} - \Delta \theta^{(0)} > \Delta \theta^{(exp)} \).

Let us now contemplate this issue from a different perspective, namely, we seek the value of \( l^{(n)} \), that renders the detection of the corrections. Hence (to first order in \( E \sqrt{G/(c^3 \hbar)} \))

\[ l^{(1)} \geq \frac{\sqrt{c^3 \hbar} G \Delta \theta^{(exp)}}{\alpha E^2}, \tag{12} \]

\[ l^{(2)} \geq \frac{2 c^3 \hbar^2 \gamma}{3 \alpha GE^3} \Delta \theta^{(exp)}. \tag{13} \]

If we assume the following values for our parameters, \( \Delta \theta^{(exp)} \sim 10^{-4} \), \( \alpha \sim 1 \), \( \gamma \sim 10^2 \), \( E \sim 10^{12} \text{e–V} \), then

\[ l^{(1)} \geq 10^{-3} \text{cm}, \tag{14} \]

\[ l^{(2)} \geq 10^{13} \text{cm}. \tag{15} \]

The energy that has been considered has the order of magnitude of the highest energy that nowadays can be produced in a laboratory \( ^3 \). These two last expressions mean that if the corrections to the dispersion relation entail \( n = 1 \), then a HBT type–like experiment with a distance between the photodetectors greater than \( 10^{-3} \text{cm} \) could detect the extra term. In the remaining case, \( n = 2 \),
the required distance implies the impossibility of detecting (with this energy) the correction.

In the context of first-order correlation functions, the possibility of detecting the case \( n = 2 \) is, currently, completely an impossible task. Forsooth, the time difference in the arrival between the two photons is given by \( 10^{-18} \)s. Nevertheless, our approach introduces an additional parameter, and therefore, if we consider the case of an energy of \( E \sim 10^{19} \text{e-V} \) (which is tantamount to the energy that could be involved in the observation of gamma-ray bursts), then

\[
\ell(2) \geq 10^5 m. \tag{16}
\]

Summing up, our input data has been the modified dispersion relation that emerges in some quantum-gravity scenarios and, resorting to the so-called second-order correlation function, it has been proved that the most favorable case \((n = 1)\) could be analyzed within the current technological sensitivity. Additionally, it has been shown that a more difficult situation \((n = 2)\) could be tested with gamma-ray bursts.

A fleeting glimpse at the literature shows us, that up to now, this last case \((n = 2)\) has been considered very difficult to attack, experimentally. In the present model the presence of our extra parameter \( \ell \) allows us to get closer to its possible detection.

Let us now address the topic of the feasibility of the present proposal. There is already some experimental evidence which purports that the case \((n = 1)\) should be discarded. In other words, experimentally we must consider \( n = 2 \) as, physically, more relevant than \( n = 1 \). Therefore, we will analyze the feasibility in the context of \( n = 2 \), which is a tougher situation, experimentally, to handle than \( n = 1 \).

The experimental parameter that should be measured is the normalized correlation coefficient of the fluctuations in the photoelectric current outputs, \( C(l) \). The connection with difference in time of arrival stems from HBT, namely, the squared modulus of the degree of coherence function, \( \gamma \), is proportional to the normalized correlation function of the photocurrent fluctuations, \( |\gamma| \), namely,

\[
C(l) = \delta |\gamma(l)|^2. \tag{17}
\]

The parameter \( \delta \) is the average number of photoelectric counts due to light of one polarization registered by the detector in the corresponding correlation time. Experimentally, for thermal sources of temperature below \( 10^5 \) K, \( \delta \) is always smaller than \( 1 \). In order to enhance the effect, i.e., to have a larger value of \( \delta \), we may consider the fact that the number of average photons, as a function of the involved frequency, \( \nu \), reads

\[
\delta = 2\xi(3)\frac{\nu^3}{c^2\pi^2}. \tag{18}
\]

Here \( \xi \) is the so-called Riemann zeta function. Clearly, higher energy implies larger mean number of photons. Hence, for an energy of \( E \sim 10^{19} \text{e-V} \) we expect a value of \( \delta \) not as small as in the case of \( 10^5 \) K. In other words, the higher the energy of the light beam, the larger the constant between \( C(l) \) and \( \gamma \) becomes. Of course, this last fact cannot be considered a shortcoming of the proposal.

In terms of the photocurrents fluctuations at the two photodetectors

\[
C(l) = \frac{<\Delta I_1(t)\Delta I_2(t)>}{(\langle |\Delta I_1(t)|^2 \rangle)^{1/2} \langle |\Delta I_2(t)|^2 \rangle^{1/2}}. \tag{19}
\]

The feasibility of detecting a deformed dispersion relation in this context depends upon the aforementioned fluctuations. We may find already in the extant literature some models that explain the pulse width of a Gamma Ray Burst (GRB), in terms of the involved energy, as a power law expression, at least in the case in which the sources are observed as fireballs. Though the aforementioned result is a model it implies that fluctuations in energy shall be present in GRB, and in consequence, as they impinge upon the corresponding photodetectors they entail current fluctuations. In other words, we may find non-vanishing sources for \( C(l) \), a fact that sounds promising, and that tells us that we shall resort to those GRB (considering that we perform this experiment within the range of \( E \sim 10^{19} \text{e-V} \)) which show the largest pulse width. In this case we expect to have a better experimental situation, and in consequence we may assert that this proposal is a feasible one.

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