Muon anomalous magnetic moment in technicolor models

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Contributions to the muon anomalous magnetic moment are evaluated in the technicolor model with scalars and topcolor assisted technicolor model. In the technicolor model with scalars, the additional contributions come from the loops of scalars, which were found sizable only for a very large $f/f'$ disfavored by the experiment of $b \to s \gamma$. The topcolor effect is also found to be large only for an unnaturally large $\tan\beta$, and thus the previously evaluated loop effects of extended technicolor bosons, suppressed by $m_{\nu}/M_{ETC}$, must be resorted to account for the E821 experiment. So, if the E821 experiment result persists, it would be a challenge to technicolor models.

Although both frameworks are well motivated, SUSY has been more favored by precision electroweak experiment than technicolor. The SUSY contributions to the muon anomalous magnetic moment were computed by many authors\textsuperscript{1}\textsuperscript{2}\textsuperscript{3}\textsuperscript{4}, which is likely to give significant contributions since the couplings of scalars to muon could be significantly enhanced by the parameter $f/f'$. Then we give an analysis for the topcolor-assisted technicolor model\textsuperscript{11}\textsuperscript{12}, which also seemingly can give large contributions since this model predicts a new gauge boson $Z'$. Finally, for other technicolor models, we will give a comment.

**Technicolor with scalars**

Technicolor with scalars\textsuperscript{8}\textsuperscript{9} has a minimal $SU(N)$ technicolor sector, consisting of two techniflavors $p$ and $m$. The technifermions transform as singlet under color and as fundamentals under the $SU(N)$ technicolor group. In addition to the above particle spectrum, there exists a scalar doublet $\phi$ to which both the ordinary fermions and technifermions are coupled. Unlike the SM Higgs doublet, $\phi$ does not cause electroweak symmetry breaking but obtains a non-zero effective vacuum expectation value (VEV) when technicolor breaks the symmetry.

If we write the matrix form of the scalar doublet as

$$\Phi = \begin{pmatrix} \phi^0 & \phi^+ \\ -\phi^- & \phi^0 \end{pmatrix} \equiv \frac{(\sigma + f')}{\sqrt{2}} \Sigma',$$

and adopt the conventional non-linear representation $\Sigma = \exp\left(\frac{2im}{f'}\right)$ and $\Sigma' = \exp\left(\frac{2im'}{f'}\right)$ for technipions, with fields in $\Pi$ and $\Pi'$ representing the pseudoscalar bound states of the technifermions $p$ and $m$, then the kinetic terms for the scalar fields are given by

$$L_{K.E.} = \frac{1}{2} \partial_\mu \sigma \partial^\mu \sigma + \frac{1}{4} f'^2 Tr(D_\mu \Sigma \Sigma^\dagger D^{\mu \Sigma}) + \frac{1}{4} (\sigma + f')^2 Tr(D_\mu \Sigma \Sigma^\dagger D^{\mu \Sigma'}).$$

Here $D'^\mu$ denotes the $SU(2)_L \times SU(2)_R$ covariant derivative, $\sigma$ is an isosinglet scalar field, $f$ and $f'$ are the technipion decay constant and the effective VEV, respectively.

The mixing between $\Pi$ and $\Pi'$ gives

$$\pi_a = \frac{f \Pi + f' \Pi'}{\sqrt{f^2 + f'^2}},$$

$$\pi_p = \frac{-f \Pi + f \Pi'}{\sqrt{f^2 + f'^2}},$$

with $\pi_a$ becoming the longitudinal component of the W and Z, and $\pi_p$ remaining in the low-energy theory as an

\textsuperscript{1}There are also some attempts to explain the reported deviation in other approaches\textsuperscript{11}. 

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isotriplet of physical scalars. From Eq. (2) one can obtain the correct gauge boson masses providing that $f^2 + f'^2 = v^2$ with the electroweak scale $v = 246$ GeV.

Additionally, the contributions to scalar potential generated by the technicolor interactions should be included in this model. The simplest term one can construct is

$$\mathcal{L}_T = c_1 4\pi f^3 T \left[ \Phi \left( \begin{array}{c} h^+_1 \\ 0 \\ h^-_1 \end{array} \right) + h.c. \right],$$

where $c_1$ is a coefficient of order unity, $h^+_1$ and $h^-_1$ are the Yukawa couplings of scalars to $p$ and $m$. From Eq. (3) the mass of the charged scalar at lowest order is obtained as

$$m_{\pi}^2 = 2\sqrt{2}(4\pi f/f')v^2 h$$

with $h = (h^+_1 + h^-_1)/2$. When the largest Coleman-Weinberg corrections for the $\sigma$ field are included in the effective chiral Lagrangian [4], one obtains the constraint

$$\tilde{M}_f^2 f' + \frac{\sqrt{8}}{2} = 8\sqrt{2}\pi c_1 h f^3$$

and the isoscalar mass as

$$m_\sigma^2 = \tilde{M}_f^2 + \frac{2}{3\pi^2} (\pi m_{\pi}^4 + Nh^4) f'^2$$

in limit (i) where the shifted $\phi^4$ coupling $\tilde{\lambda}$ is small and can be neglected [8], and

$$m_\sigma^2 = \frac{3}{2} \tilde{\lambda} f'^2 - \frac{1}{4\pi^2} (\pi m_{\pi}^4 + Nh^4) f'^2$$

in limit (ii) where the shifted mass of the scalar doublet $\phi$, $\tilde{M}_f$ is small and can be neglected [8].

The advantage of this model is that it can successfully account for fermion masses without generating large flavor-changing neutral current effects, and without exceeding the experimental bounds on oblique electroweak radiative corrections. Furthermore, we stress at the lowest order, only two independent parameters $(f/f', m_{\pi})$ in the limits (i) and (ii) mentioned above are needed to describe the phenomenology.

The contributions to the muon anomalous magnetic moment stem from the diagrams shown in Fig. 1. The relevant interactions can be extracted from Eq. (2) and Eq. (3):[3]

$$\mathcal{L} = \left( \begin{array}{c} v \end{array} \right) \frac{g_{\mu\nu}}{2m_{\mu\nu}} \mu \mu' - \left( \begin{array}{c} f \\ f' \end{array} \right) \frac{g_{\mu\nu}}{2m_{\mu\nu}} \mu \mu' \gamma_5 \mu' + \frac{f}{f'} \left[ \pi^+_p \bar{\nu}_\mu (1 + \gamma_5) \mu - \pi^-_p \bar{\mu} (1 - \gamma_5) \nu_\mu \right]$$

$$-ieA_{\mu} \pi^+ p \downarrow \bar{\mu} \pi^- p.$$
According to this limit, $f/f'$, the mass of $m_{\sigma_p}$ is varied from 107.7 GeV to 1 TeV. The dashed lines denote the current lower and upper experimental bounds on new physics contributions at 90% C. L.

positive or negative and not large enough except for very large $f/f'$; whereas the charged scalar can only provide the contribution to the imaginary part which is much smaller than the real one.

2. Unlike the case in limit (i), the additional contributions are negligible in limit (ii). No very large $f/f'$ is allowed by the model.

3. As expected, for any fixed value of $f/f'$, the total contribution decreases rapidly as the mass of the charged scalar increases.

The limit on $f/f'$ has been investigated by several authors, and obtained from the studies of $b \to X_\tau \tau \nu$ $[16]$, $b \to X_\tau \gamma$ $[16,17]$, $Z \to b\bar{b}$ $[17]$, $B \to X_\mu \mu^+ \mu^-$ $[18]$ and B-$\bar{B}$ mixing $[8,11,12]$, which is given by $[16]$

$$f/f \leq 0.03 \left( \frac{m_{\pi_p}}{1 \text{ GeV}} \right) \quad (95\% \text{ C. L.}). \quad (14)$$

According to this limit, $f/f' > 200$, which is needed to give large contributions to the muon anomalous moment, seems unlikely.

**Topcolor-assisted technicolor model**

The other competitive candidate, which might provide large additional contributions to the muon anomalous moment, is the topcolor-assisted technicolor model $[10,12]$. The model assume: (i) electroweak interactions are broken by technicolor; (ii) the top quark mass is large because it is the combination of a dynamical condensate component, generated by a new strong dynamics, together with a small fundamental component, generated by an extended technicolor (ETC) $[19]$; (iii) the new strong dynamics is assumed to be chiral critically strong but spontaneously broken by technicolor at the scale $\sim 1 \text{ TeV}$, and it generally couples preferentially to the third generation. This needs a new class of technicolor models incorporating “top-color”. The dynamics at $\sim 1 \text{ TeV}$ scale involves the gauge structure:

$$SU(3)_1 \times SU(3)_2 \times U(1)_{Y_1} \times U(1)_{Y_2} \rightarrow SU(3)_{QCD} \times U(1)_{EM}$$

where $SU(3)_1 \times U(1)_{Y_1}$ [$SU(3)_2 \times U(1)_{Y_2}$] generally couples preferentially to the third (first and second) generation, and is assumed to be strong enough to form chiral $< \bar{t}t >$ but not $< \bar{b}b >$ condensation by the $U(1)_{Y_1}$ coupling. A residual global symmetry $SU(3)' \times U(1)'$ implies the existence of a massive color-singlet heavy $Z'$ and an octet $B_{\mu}^A$. A symmetry-breaking pattern outlined above will generically give rise to three top-pions, $\tilde{\pi}$, near the top mass scale.

In this model, in addition to the previously evaluated loop effects of ETC bosons which generates the muon mass $[2]$, the muon anomalous magnetic moment receives additional contributions only from a gauge boson $Z'_\mu$, top-pions $\tilde{\pi}^0$, $\tilde{\pi}^+ \tilde{\pi}^- \pi^0$ and technipions with Feynman diagrams similar to Fig. 3 and $\sigma$, $\pi_p$ replaced by $Z'_\mu$, top-pions (technipions), respectively. The interaction of top-pions, neutral gauge boson with muon are given by $[1]$

$$\mathcal{L}_{eff} = \frac{g_1}{2} \tan^2 \theta' Z'_\mu \left[ \bar{\mu}_L \gamma^\alpha \mu_L + 2 \bar{\mu}_R \gamma^\alpha \mu_R \right] + \frac{m_{\mu}}{f_{\pi}} \left[ \frac{i}{\sqrt{2}} \bar{\mu}_L \gamma^\alpha \gamma_5 \mu_R + \bar{\mu}_L \pi^+ \mu_R + h.c. \right]. \quad (15)$$

where $g_1$ is the $U(1)_{Y_1}$ coupling constant at the scale $\sim 1 \text{ TeV}$. The SM $U(1)_Y$- and the $U(1)'$ field $Z'_\mu$ is then defined by orthogonal rotation with mixing angle $\theta'$.

In this letter, we don’t take small technipions effects into account. Note when the Lagrangian in Eq. (15) is used to calculate the additional technicolor effects to the muon anomalous magnetic moment, the small contribution from the charged top-pions, as well as neutral one, can be neglected safely, as in the case of technicolor with scalars. Now we present the contributions from the neutral gauge boson $Z'$

$$a_{\mu} = \frac{g_1^2}{32 \pi^2} \tan^2 \theta' \left\{ -a_{Z'} - b_{Z'} - \frac{11}{2} \right. \right.$$

$$+ \left. \frac{1}{a_{Z'} - b_{Z'}} \left[ a_{Z'} \left( a_{Z'}^2 + 5a_{Z'} - 6 \right) \ln \frac{a_{Z'}}{a_{Z'} - 1} - b_{Z'} \left( b_{Z'}^2 + 5b_{Z'} - 6 \right) \ln \frac{b_{Z'}}{b_{Z'} - 1} \right] \right\}, \quad (16)$$

where $a_{Z'}$, $b_{Z'}$ are defined in Eq. (13).

The additional contribution due to the neutral gauge boson $Z'$ to the muon anomalous magnetic moment as
a function of $\tan \theta'$ is shown in Fig. 3, with the mass of $Z'$ varying from 100 GeV to 1 TeV for any fixed value of $\tan \theta'$. One can see the topcolor-assisted model can also give large contributions in case of a large $\tan \theta'$, which, however, is not favored by the model because a small $\tan \theta' \ll 1$ is ultimately demanded to select the top quark direction for condensation. To account for the E821 experiment, the ETC loop contributions must be resorted, which is suppressed by $m_{\text{ETC}}^2/M_{\text{ETC}}^2$, and requires $M_{\text{ETC}} \sim 1$ TeV, as evaluated in [2].

We also scanned other technicolor models and found that models without ETC to generate the fermions masses are more unlikely to give the large contributions to the muon anomalous magnetic moment since the new particle couplings to the muon do not have any enhancement factors like $f/f'$. In summary, we evaluated technicolor contributions to the muon anomalous magnetic moment in two frameworks of technicolor: technicolor model with scalars and the topcolor-assisted technicolor model. We found that the technicolor model with scalars can give the large contributions required by the E821 experiment only for a very large $f/f'$ which, however, is disfavored by the experiment of $b \to s\gamma$. The $Z'$ loop in the topcolor-assisted model can also give large contributions in case of a large tan $\theta'$, which is not in accord with the motivation for building this model. Thus to account for the E821 experiment, the ETC loop contributions suppressed by $m_{\text{ETC}}^2/M_{\text{ETC}}^2$ must be resorted.

Therefore, if the current deviation of the muon anomalous moment from its SM prediction persists as the experiment is further developed, it would be a challenge to the technicolor models.

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