Hydrodynamics of Circumbinary Accretion: Angular Momentum Transfer and Binary Orbital Evolution

Diego J. Muñoz1,2,3, Ryan Miranda3,4, and Dong Lai3

1 Center for Interdisciplinary Exploration and Research in Astrophysics, Physics and Astronomy, Northwestern University, Evanston, IL 60208, USA
diego.munoz@northwestern.edu
2 Steward Observatory, University of Arizona, Tucson, AZ 85721, USA
3 Cornell Center for Astrophysics and Planetary Science, Department of Astronomy, Cornell University, Ithaca, NY 14853, USA
4 Institute for Advanced Study, School of Natural Sciences, Einstein Drive, Princeton, NJ 08540, USA

Received 2018 October 16; revised 2018 December 10; accepted 2018 December 10; published 2019 January 24

Abstract

We carry out 2D viscous hydrodynamical simulations of circumbinary accretion using the moving-mesh code AREPO. We self-consistently compute the accretion flow over a wide range of spatial scales, from the circumbinary disk (CBD) far from the central binary, through accretion streamers, to the disks around individual binary components, resolving the flow down to 2% of the binary separation. We focus on equal-mass binaries with arbitrary eccentricities. We evolve the flow over long (viscous) timescales until a quasi-steady state is reached, in which the mass supply rate at large distances \( M_0 \) (assumed constant) equals the time-averaged mass transfer rate across the disk and the total mass accretion rate onto the binary components. This quasi-steady state allows us to compute the secular angular momentum transfer rate onto the binary, \( \langle J_0 \rangle \), and the resulting orbital evolution. Through direct computation of the gravitational and accretional torques on the binary, we find that \( \langle J_0 \rangle \) is consistently positive (i.e., the binary gains angular momentum), with \( I_0 \equiv \langle J_0 \rangle / M_0 \) in the range of \((0.4 - 0.8)\Omega_{h} a_{h}^{2}\), depending on the binary eccentricity (where \( a_{h} \) and \( \Omega_{h} \) are the binary semimajor axis and angular frequency); we also find that this \( \langle J_0 \rangle \) is equal to the net angular momentum current across the CBD, indicating that global angular momentum balance is achieved in our simulations. In addition, we compute the time-averaged rate of change of the binary orbital energy for eccentric binaries and thus obtain the secular rates \( \langle \dot{E}_b \rangle \) and \( \langle \dot{q}_b \rangle \). In all cases, \( \langle \dot{E}_b \rangle \) is positive; that is, the binary expands while accreting. We discuss the implications of our results for the merger of supermassive binary black holes and for the formation of close stellar binaries.

Key words: accretion, accretion disks – binaries: general – black hole physics – stars: pre-main sequence

1. Introduction

Circumbinary accretion disks are a natural byproduct of binary star formation via disk fragmentation (e.g., Boss 1986; Bonnell & Bate 1994a, 1994b; Kratter et al. 2008). A number of these disks have been observed around Class I/II young stellar binaries—well-known objects include GG Tau, DQ Tau, and UZ Tau E (e.g., Dutrey et al. 1994; Mathieu et al. 1996, 1997)—and recently even around much younger binary Class 0 objects like L1448 IRS3B (Tobin et al. 2016). Circumbinary disks (CBD) are also expected to form around massive binary black holes following galaxy mergers (e.g., Begelman et al. 1980; Milosavljevi\'ć & Merritt 2001; Escala et al. 2005; Milosavljevi\'ć & Phinne\ 2005; Dotti et al. 2007; Mayer et al. 2007; Cuadra et al. 2009; Chapon et al. 2013), and even from the fallback ejecta around post-common envelope binaries (e.g., Kashi & Soker 2011). The interaction between the two central bodies and the surrounding gas via accretion and gravitational torques dictates the long-term evolution of the binary orbit. Thus, understanding the details of binary–disk interaction is essential to explaining the end-state distribution of stellar binary properties, and to deciphering how, or if, a CBD can assist the orbital decay of black hole binaries, initiating the process toward merger.

It is commonly assumed that the binary inevitably loses specific angular momentum by interacting with the outer disk or ring (e.g., Pringle 1991), which results in the migration or coalescence of the binary toward smaller orbital separations. This reasoning follows from extending the classical theory of satellite–disk interaction for extreme mass ratios (e.g., Lin & Papaloizou 1979, 1986; Goldreich & Tremaine 1980; Ward 1977) into the regime of binaries (mass ratios of order unity): the binary and disk are coupled via Lindblad resonance torques, and because the CBD rotates at an angular frequency that is lower than the binary’s, the nonaxisymmetric perturbations exerted in the gas “lag” the binary, thus exerting a negative torque on it. Numerous studies (e.g., Gould & Rix 2000; Armitage & Natarajan 2002, 2005; Cuadra et al. 2009; Haiman et al. 2009; Chang et al. 2010; Farris et al. 2015; Rafikov 2016; Kelley et al. 2017) have modeled the extraction of angular momentum from the binary by the outer disk, suggesting that a “gas-assisted” orbital migration may be important in facilitating binary black hole mergers at separations that are too wide for gravitational wave radiation to remove orbital angular momentum efficiently (what is known as the “final parsec” problem; Milosavljević & Merritt 2003a, 2003b).

Two complications can alter this picture of binary migration. The first, and most obvious, is that binaries can accrete from the CBD, and this must be taken into account when computing the torques exerted on the binary by the disk. Accretion torques and gravitational torques can be of a similar order of magnitude: the change in angular momentum by accretion alone is proportional to the mass accretion rate \( M \), while the gravitational Lindblad torque is proportional to the inner disk surface density, which is itself proportional to \( M \) provided that the disk is in a steady state. Because of the time-dependent forcing from the binary, it is only possible to reach a “quasi-steady state,” in which the time-averaged mass accretion...
rate onto the binary matches the mass transfer rate in the CBD. To achieve this quasi-steady state, simulations require a steady mass supply at large distances from the binary and a long integration time. Only under these conditions can it be guaranteed that the (arbitrary) initial conditions have no effect on the torque balance of the system. Recently, Muñoz & Lai (2016) and Miranda et al. (2017) have shown that such quasi-steady state can indeed be achieved, at least when the disk viscosity is sufficiently large, in which case the CBD transfers, on average, the same amount of gas to the binary as it would to a single object. Thus, the deep cavity carved in the gas by the gravitational torques can only modulate (see Artyomowicz & Lubow 1996; Günther & Kley 2002; MacFadyen & Milosavljević 2008; Hanawa et al. 2010; de Val-Borro et al. 2011; Shi et al. 2012; D’Orazi et al. 2013; Farris et al. 2014; Shi & Krolik 2015), but not suppress, the net accretion onto the binary (although see Ragusa et al. 2016 for a claim to the contrary at low viscosities). Numerical studies have shown that accretion can be modulated at either $\sim 5P_b$ or $P_b$, the binary orbital period, and that the transition between these two regimes depends on the binary mass ratio ($D’Orazi$ et al. 2013; Farris et al. 2014) or binary eccentricity, as shown by Muñoz & Lai (2016) and Miranda et al. (2017; see also Artyomowicz & Lubow 1996; Hayasaki et al. 2007, for qualitatively similar results using smoothed particle hydrodynamics).

The second complication is the unexplored contribution of the “circumsingle” disks (CSDs) to the total gravitational torque. In the same way that the outer CBD is expected to contribute negatively to the torque on the binary, the CSDs are expected to contribute positively (Goldreich & Tremaine 1980), with the total torque consisting of the net balance of the inner and outer Lindblad torques (which is typically negative for mass ratios of $\lesssim 10^{-4}$; Ward 1986, 1997). Savonije et al. (1994) computed the nonlinear effect of the CSDs on the binary orbit in the large mass ratio ($\lesssim 1$) regime, concluding that the gas loses angular momentum to the binary, which results in wave-induced accretion onto the primary (see also Ostriker 1994; Muñoz et al. 2015; Winter et al. 2018). Whether these inner/CSD torques can be of significance or even compete with the outer CBD torque is an open, and largely unexplored, question.

In order to compute the full balance of torques, one needs to (1) resolve the gas dynamics well within the Roche lobes of the primary and secondary and also (2) guarantee that the numerical scheme is able to capture the nonlinear response of the CSDs to the tidal forcing by the binary companions. These requirements can be met by shock-capturing hydrodynamical simulations at sufficiently high resolution. Recently, novel Godunov-type numerical schemes with good shock-capturing properties—such as the DISCO (Duffell & MacFadyen 2011) and AREPO (Springel 2010; Pakmor et al. 2016) methods—have been used to compute the joint gas dynamical evolution of CBDs and CSDs down to small scales over a large number of binary orbits (Farris et al. 2014, 2015; Muñoz & Lai 2016; Tang et al. 2017). Our simulations using AREPO (Muñoz & Lai 2016) have uncovered several new features, of short-term ($\sim P_b$) and long-term ($\sim 200 P_b$) variability, in the accretion rate onto eccentric binaries.

In a systematic exploratory work, Miranda et al. (2017; hereafter MML17) simulated viscous disks around binaries with mass ratio $\sim 1$ and computed the net angular momentum transfer within a CBD as a function of radius in quasi-steady state. They found that the net angular momentum transfer rate at the disk’s edge is positive (i.e., angular momentum is transferred inward) for various values of the binary eccentricity. This implies that in quasi-steady state, the binary gains angular momentum from the gas, possibly leading to binary expansion instead of coalescence. However, the work of MML17 is subject to an important limitation: the gas within the binary’s orbit was not simulated (see also MacFadyen & Milosavljević 2008; D’Orazi et al. 2013). Although capturing a portion of the circumbinary cavity, the simulations of MML17 excluded the high-density region where the CSDs form, excising this region by imposing a diode-like boundary condition on an annulus of radius similar to the binary separation. As a consequence, the complex inflow–outflow nature of the gas inside the circumbinary cavity (Muñoz & Lai 2016; Tang et al. 2017) was not captured correctly (this type of boundary may even be prone to some numerical artifacts; Thun et al. 2017). The purpose of the present work is to remove this limitation by self-consistently computing the gas dynamics within the circumbinary cavity down to scales of 2% of the binary separation, while simultaneously evolving a CBD that extends out to a distance of almost two orders of magnitude larger than the binary separation.

As in Muñoz & Lai (2016; hereafter ML16), we use the moving-mesh code AREPO to simulate circumbinary accretion. We consider CBDs with a steady mass supply far from the binary. This supply is enforced in the form of a boundary condition (also implemented by MML17), which allows for the possibility of a global steady state, in which the time-averaged accretion rate onto the binary equals the mass supply rate. We demonstrate through explicit computations that, in quasi-steady state, the angular momentum transfer rate within the CBD equals the total torque (from gravitational forces and accretion) acting on the binary. This provides unambiguous evidence that the binary gains angular momentum from the CBD. Furthermore, by tracking the orbital energy and angular momentum of the accreting binary, we compute the combined orbit-eccentricity evolution of the binary for the first time on secular timescales.

This paper is organized as follows. In Section 2, we describe our use of moving-mesh hydrodynamics to simulate circumbinary accretion flows and discuss how one can compute the various torques acting on the accreting binary. In Section 3, we describe and analyze our simulation results, identifying the similarities and differences between circular and eccentric binaries. In Section 4, we describe our calculation of the orbital energy and angular momentum changes, and we translate our results into orbital element evolution. Finally, in Section 5, we summarize our results and discuss the implications of this work for the formation and evolution of stellar binaries and supermassive binary black holes.

## 2. Numerical Methods

### 2.1. Moving-mesh Hydrodynamics

We carry out hydrodynamical simulations using the Godunov-type moving-mesh code AREPO (Springel 2010; Pakmor et al. 2016) in its Navier–Stokes version (Muñoz 2013; Muñoz et al. 2013). AREPO has been implemented for viscous accretion disk simulations in 2D (Muñoz et al. 2014; ML16) and 3D (Muñoz et al. 2015). In the present work, we carry out circumbinary accretion simulations closely following the methodology of ML16. In brief, we simulate 2D Newtonian viscous flow,
with a locally isothermal equation of state \( P = \sum c_v^2 \). The gravitational acceleration due to the binary’s potential \( \phi_b(r) \) is evaluated at each cell center, and the motion of the moving cells by action of this external potential is integrated in a leapfrog-like fashion; that is, gravity and pure hydrodynamics are “operator split” (see Springel 2010 and Pakmor et al. 2016). The positions of the primary and secondary, \( n_1(t) \) and \( n_2(t) \), respectively, as well as their velocities \( v_1(t) \) and \( v_2(t) \), are prescribed functions of time, determined at each \( t \) by solving the Kepler equation given the binary eccentricity \( e_b \) and the mass ratio \( q_b = M_2/M_1 \) (e.g., Murray & Dermott 2000). Both \( e_b \) and the binary semimajor axis \( a_0 \) are fixed constants throughout the duration of the simulation.

Such a “prescribed” binary is chosen over a “live” binary to minimize noise or high-frequency “jittering” of the binary’s orbital elements. The sound speed \( c(r) \) is a prescribed function of \( r \), and is fed into an iterative isothermal Riemann solver at each Voronoi cell interface (e.g., Toro 2009; Springel 2010). The temperature of the disk is set by the global parameter \( h_0 \), the disk’s vertical aspect ratio. Like the sound speed, the kinematic viscosity coefficient \( \nu(r) \) is also a function of \( r \) and proportional to a Shakura-Sunyaev coefficient \( \alpha \). The functions \( c(r) \) and \( \nu(r) \) are constructed such that \[ c_s^2 \approx h_0^2 (M_1 + M_2) / |r| \] and \[ \nu \approx \alpha h_0^2 \sqrt{(M_1 + M_2) / |r|} \] when \( |r| \gg a_0 \), and that \[ c_s^2 \approx h_0^2 (M_1 + M_2) / |r - r_i| \] and \[ \nu \approx \alpha h_0^2 \sqrt{(M_1 + M_2) / |r - r_i|} \] \((i=1,2)\) when \( |r - r_i| \ll a_0 \); that is, both the CBD and the two CSDs behave like traditional \( \alpha \) disks when the flow is approximately Keplerian (for more details, see ML16).

We allow the gas to be accreted by the individual members of the binary (from here on, we will refer to the binary components as “stars,” although the results presented here are equally applicable to black hole binaries). Accretion is thus carried out by a sink particle algorithm: as gas cells get closer than a critical distance \( r_{acc} \) to one of the stars, they are flagged for their contents to be drained (cells are not eliminated) at the end of the time step. In order to keep the transition at \( |r - r_i| = r_{acc} \) smooth and avoid severe depletion of cells (which can affect the convergence speed of the Voronoi tessellation algorithm), we accrete cells with \( |r - r_i| \approx 0 \) more aggressively than those with \( |r - r_i| \approx r_{acc} \). This is accomplished by using a smooth spline function (Springel et al. 2001) that returns the fraction \( f \) of mass and linear momentum that is to be drained from each cell (the spline function is 0 at \( |r - r_i| = r_{acc} \) and 1 at \( |r - r_i| = 0 \). In practice, we cap \( f \) at a maximum value of 0.5 per time step to avoid excessive cell depletion (e.g., see Krumholz et al. 2004). As cells drift toward the “star,” subsequent time steps will continue to drain their mass and momentum, eventually fully emptying them from their original mass content or merging them into other cells.

The distribution of cells in the CBD is initially “quasi polar” (Muñoz et al. 2014), with a logarithmic spacing in radius; this type of cell allocation is roughly preserved throughout the simulation, during which cells are not allowed to become too distorted, nor to depart too significantly from their initial size. Within the circumbinary cavity, this polar-like resolution criterion smoothly transitions to one that is mass-based; that is, a constant mass per cell is sought. The quasi-Lagrangian nature of AREPO tends to preserve the cell mass along flow trajectories (e.g., Vogelsberger et al. 2012). However, whenever the finite-volume mass fluxes change a cell’s mass, a procedure of cell refinements and derefinements will enforce the desired mass resolution (Springel 2010). The target mass is set to be \( m_{target} = 1.2 \times 10^{-6} \Sigma_0 a_0^2 \), where we have introduced a natural scaling for the surface density,

\[
\Sigma_0 \equiv \frac{M_0}{3 \pi \alpha h_0^2 \Omega_b a_0^2},
\]

with \( M_0 \) the prescribed (constant) mass supply rate at the outer boundary (the cylindrical radius measured from the binary center of mass \( R = R_{out} = 70 a_0 \)). We enforce the outer boundary conditions at \( R = R_{out} \) by fixing the primitive variables to their standard viscous disk values, that is, the surface density \( \Sigma \approx M_0 / (3 \pi v_r) \), radial velocity \( v_r \approx -(3/2) v / R \), and the azimuthal velocity \( v_\phi \approx R \Omega \) (see Equations (20), (23), (25), and (26) below) with the viscosity given by \( \nu = \alpha h_0^2 R^2 \Omega K \). We also add a narrow damping zone (between \( 60 a_0 \) and \( R_{out} \)) to minimize oscillations and nonaxisymmetric features (de Val-Borro et al. 2006; Muñoz et al. 2014). We note that the constant-mass resolution is not strictly respected because the complex gas dynamics and density contrast of the circumbinary cavity can lead to extremely large cells in the dilute regions as well as extremely small cells within the dense regions. In order to circumvent these complications, we impose a maximum cell area of \( \delta a_{max} = 1.5 \times 10^{-3} a_0^2 \) and a minimum of \( \delta a_{min} = 10^{-5} a_0^2 \).

As a rule of thumb, we require \( \delta a_{min} \) to be smaller than \( \sim \pi a_{acc}^2 \) by a factor of at least 100.

The binary properties are fully determined by \( e_b \) and the mass ratio \( q_b = M_2 / M_1 \). In this paper, we only vary \( e_b \), fixing \( q_b = 1 \). We work in distance units relative to the binary semimajor axis, so \( a_0 = 1 \). Similarly, we set \( G = 1 \) and \( M_0 = M_1 = M_2 = 1 \). As noted above, the disk properties are set by the viscosity parameter \( \alpha \) and the dimensionless aspect ratio \( h_0 \). Throughout this work, we fix \( \alpha = h_0 = 0.1 \). The fiducial accretion radius is set at \( r_{acc} = 0.02 a_0 \), which is the same as the softening length of the Keplerian potential of each “star.”

2.2. Torque Acting on an Accreting Binary

In the following, we describe how we compute the angular momentum change of the binary from simulation output in postprocessing. In particular, we detail how the direct torque on the binary can be decomposed into all of its relevant contributions (Section 2.2.1). We also explain how the torque on the binary can be related to the angular momentum transfer as a function of radius in the CBD (Section 2.2.2).

2.2.1. Direct Computation of Torque Acting on a Binary

The orbital angular momentum of the binary is subject to torques due to accretion and gravitational interactions:

\[
\frac{dL_b}{dt} = \left. \frac{dL_b}{dt} \right|_{acc} + \left. \frac{dL_b}{dt} \right|_{grav} = \mu_b \dot{v}_b + \mu_b \frac{d\dot{v}_b}{dt} + \mu_b \frac{d\dot{v}_b}{dt}.
\]

where \( \dot{L}_b = a_0^2 \Omega_b (1 - e_b^2)^{1/2} \) is the binary’s specific angular momentum and is defined by \( L_b = \mu_b \dot{v}_b \), where \( \mu_b = M_1 M_2 / M_0 = q_b M_0 / (1 + q_b)^2 \) is the binary’s reduced mass.

For the low-mass disks of interest in this paper, the quantities
$M_b$, $q_b$, and $l_b$ in Equation (2) change a negligible amount over the timescales of the simulations (i.e., these quantities evolve over a long, secular timescale, $\sim M_b/M_b$), and we determine their time derivatives from simulation output in postprocessing. Hence, the binary evolution is not “live” (see Section 2.1 above).

The rate of change of total angular momentum in the binary $\dot{J}_b$ should also include the angular momentum transfer to the spins of the “stars.” Thus

$$\dot{J}_b = \dot{L}_b + \dot{S}_1 + \dot{S}_2. \quad (3)$$

Below, we describe in detail how the different contributions to Equations (2) and (3) are computed from numerical simulations.

**Mass change:** Even in the absence of specific torques in Equation (2), the angular momentum of the binary changes via changes in the reduced mass $\mu_b$. These changes are simply related to the changes in mass ratio and total mass:

$$\frac{\dot{\mu}_b}{\mu_b} = \frac{\dot{M}_b}{M_b} + \frac{1 - q_b}{1 + q_b} \frac{\dot{q}_b}{q_b}, \quad (4)$$

with

$$M_b = M_1 + M_2 \quad \text{and} \quad \dot{q}_b = \frac{1 + q_b}{M_b} (M_2 - q_b M_1). \quad (5)$$

As in ML16, we compute the accretion rate by keeping track of $\Delta m_{acc,i}(t)$, the cumulatively accreted mass at time $t$, over the course of the simulation. We compute $M_i$ in postprocessing by taking the time derivative using a centered-differences approximation:

$$M_i(t) \equiv \frac{d}{dt} \Delta m_{acc,i}(t). \quad (6)$$

**Specific gravitational torque:** The specific gravitational torque acting on the binary has the general form

$$\frac{dL_b}{dt}_{grav} = \left. \frac{M_1}{\mu_b} r_1 \times f_{grav,1} + \frac{M_2}{\mu_b} r_2 \times f_{grav,2} \right|_{t}$$

$$= r_b \times (f_{grav,1} - f_{grav,2}), \quad (7)$$

where $r_i = r_i - r_2$, and $f_{grav,i}$ is the force per unit mass acting on the primary/secondary:

$$f_{grav,i} = -G \sum_k m_k \frac{r_i - r_k}{|r_i - r_k|^3}, \quad (8)$$

where $m_k$ and $r_k$ are the mass and position vector of the $k$th computational cell, and the sum extends over all cells in the simulation domain. A proper account of the acceleration history of the “stars” requires a densely sampled force evaluation in time. In practice, we follow the gravitational “kicks” that act on the primary and secondary, evaluating them twice per time step, mimicking the kick-drift-kick operation that one would implement for a truly “live” binary orbit integration. As with the accreted mass, at each time $t$, we store the cumulative velocity change $\Delta v_{grav,i}(t)$. In postprocessing, the gravitational force per unit mass is computed as

$$f_{grav,i} = \frac{d}{dt} \Delta v_{grav,i}(t). \quad (9)$$

This approach is based on momentum conservation; as such, it takes into account all time steps in the simulation and does not rely on the instantaneous forces computed from sparse simulation snapshots.

**Specific accretion torque:** Accretion itself can induce a specific torque on the binary (e.g., see Roedig et al., 2012), separate from the torque inherent to the change in binary mass. Similar to Equation (7), we have

$$\frac{dL_b}{dt}_{acc} = r_b \times (f_{acc,1} - f_{acc,2}). \quad (10)$$

We compute the accretion forces by keeping track of the velocity kicks as mass is incorporated into the “stars.” As one would do in “live” accretion integrations, conservation of linear momentum dictates that the velocity of an accreting body of mass $M_i$ is updated after each time step according to

$$\dot{v}_{acc,i} = \frac{v_i(t) M_i + \delta p_{acc,i}}{M_i + \delta m_{acc,i}} - v_i(t) \quad \text{(11a)}$$

$$\approx \frac{\delta p_{acc,i}}{M_i} - \frac{\delta m_{acc,i}}{M_i} v_i(t), \quad (11b)$$

where $\delta m_{acc,i}$ and $\delta p_{acc,i}$ are the changes in mass and momentum during a single time step. The accumulation of $\delta v_{acc,i}$ over time then defines $\Delta v_{acc,i}(t)$. As in Equation (9), the accretion force is computed in postprocessing as

$$f_{acc,i} = \frac{d}{dt} \Delta v_{acc,i}(t), \quad (12)$$

from which we compute the associated torques (Equation (10)). This specific torque vanishes if accretion onto the “star” is isotropic, in which case, only the mass—but not the velocity—changes. Therefore, we typically refer to the torque resulting from Equation (12) as the anisotropic accretion torque.

Note that, had we stored the cumulative evolution of accreted linear momentum $\Delta p_{acc,i}(t)$ instead of $\Delta v_{acc,i}(t)$, the total accretion torque could be written as

$$\frac{dL_b}{dt}_{acc} = r_1 \times \frac{d}{dt} \Delta p_{acc,1}(t) + r_2 \times \frac{d}{dt} \Delta p_{acc,2}(t)$$

$$\approx \mu_b r_b \times (f_{acc,1} - f_{acc,2}) + (r_b \times v_b) \frac{d\mu_b}{dt}, \quad (13)$$

where $v_b = \dot{r}_b = v_1 - v_2$. Note that the second term in Equation (13) is already accounted for by the first term in Equation (2), which describes the accretion-induced torque at constant specific angular momentum.

**Spin torque:** Gas cells typically orbit around the “stars” before being drained, therefore carrying a small amount of angular momentum relative to a moving center. This residual angular momentum is irrevocably lost from the system due to accretion (e.g., Bate et al., 1995). The amount of angular momentum—or “spin”—$\Delta S_{acc,i}$ that is transferred to the “star” can be tracked as follows: each time some mass and momentum are drained from the $k$th gas cell, the spin angular
mum momentum \( \mathbf{r}_k - \mathbf{r}_l \) \times f_k \mathbf{p}_k \) (where \( f_k \) is the fraction of mass and linear momentum drained from the \( k \)th cell) is added to \( \Delta S_{\text{acc},i} \). As before, the spin torque is computed in postprocessing as

\[
S_{\text{acc},i} = \frac{d}{dt} \Delta S_{\text{acc},i}(t). \tag{14}
\]

In summary, in order to compute \( L_b \) and \( \dot{J}_b \) (Equations (2) and (3)) from simulation output, we need \( M_b \) and \( \dot{q}_b \) from Equations (5) and using Equation (6), \( (dL_b/dt)_{\text{grav}} \) from Equation (7), \( (d\dot{J}_b/dt)_{\text{acc}} \) from Equation (10), and \( S_{1.2} \) from Equation (14).

To determine the secular orbital evolution of an eccentric binary, we also need to compute the rate of change of the orbital energy, which is fully determined from \( M_b, f_{\text{grav}},i, \) and \( f_{\text{acc},i} \). This is discussed in Section 4.

### 2.2.2. Angular Momentum Transfer in the CBD

In quasi-steady-state, the time-averaged angular momentum transfer rate across the CBD, \( \dot{J}_b(R, t) \), is constant in \( R \) and equal to the angular momentum transfer rate onto the binary \( \dot{J}_b \). The balance of angular momentum currents throughout the CBD is (see Appendix A of MML17)

\[
\dot{J}_b = \dot{J}_{\text{adv}} - \dot{J}_{\text{visc}} - \dot{J}_{\text{grav}}, \tag{15}
\]

where \( \dot{J}_{\text{adv}}, \dot{J}_{\text{visc}}, \) and \( \dot{J}_{\text{grav}} \) are the advective, viscous, and gravitational contributions, respectively (in the notation of MML17, \( \dot{J}_{\text{adv}} \) is \( T^{\text{adv}}_{\text{rad}} \)). The computation of the angular momentum current in the disk \( \dot{J}_b \) from the AREPO output is more involved than in the PLUTO case, since there is no polar grid. In order to obtain \( \dot{J}_{\text{adv}}, \dot{J}_{\text{visc}}, \) and \( \dot{J}_{\text{grav}} \), we first compute 2D maps of angular momentum fluxes \( F_j(R, \phi) \) by remapping or interpolating the primitive variables of AREPO output onto a structured polar grid. The currents \( J(R) \) are then calculated from

\[
J(R) = \int_0^{2\pi} \dot{J}_b(R, \phi) R d\phi = 2\pi \left( F_j(R, \phi) R \right)_{\phi}. \tag{16}
\]

The advective, viscous, and gravitational fluxes are, respectively,

\[
F_{j, \text{adv}} = -R \sum \nu_{kR} v_{\phi k} = -\Sigma \frac{xy(y^2 - v_{\phi k}^2) + v_{\phi l} v_x(x^2 - y^2)}{\sqrt{x^2 + y^2}}, \tag{17}
\]

\[
F_{j, \text{visc}} = -R^2 \nu \Sigma \left( \frac{\partial}{\partial R} \left( \frac{v_{\phi}}{R} \right) + \frac{1}{R^2} \frac{\partial v_{\phi}}{\partial \phi} \right) = \frac{1}{\sqrt{x^2 + y^2}} \left[ 2xy \left( \frac{\partial v_{\phi}}{\partial y} - \frac{\partial v_x}{\partial x} \right) + (x^2 - y^2) \left( \frac{\partial v_x}{\partial y} + \frac{\partial v_y}{\partial x} \right) \right], \tag{18}
\]

and

\[
F_{j, \text{grav}} = -\int_R^{R_{\text{out}}} \sum \frac{\partial P_{b}}{\partial \phi} R dR' = -\int_R^{R_{\text{out}}} \Sigma (-x a_y + y a_x) dR', \tag{19}
\]

where we have made the dependence on Cartesian coordinates (the ones used by AREPO) explicit. In Equation (19), \( a = (a_x, a_y) \) is the cell-centered acceleration of the gas due to the binary’s potential \( \Phi_b \). These maps are computed from simulation snapshots, stacked to produce a time average and then integrated over azimuth (Equation (16)) to produce the time-averaged angular momentum currents of Equation (15).

### 2.3. Initial Setup

As in ML16, we prescribe an initially axisymmetric disk model (their Equation (2)) with fixed parameters of Shakura–Sunyaev viscosity \( \alpha \) and disk aspect ratio \( h_0 \). Motivated by the lessons learned from the numerical exploration of MML17, we use a slightly modified initial condition of the form

\[
\Sigma(R) = \frac{\left( \frac{R}{a_b} \right)^{3/2}}{1 + \frac{\Sigma(0)}{\Sigma_0} \left( \frac{R}{a_b} \right)^{3/2}} \times \mathcal{F}(R; R_{\text{cav}}), \tag{20}
\]

with \( \Omega_b = (GM_b/a_b^3)^{1/2} \) and \( \Sigma_0 \) given by Equation (1), where \( \Sigma(0) \) is an approximate initial guess for the angular momentum per unit accreted mass \( \Sigma(0) \equiv (J_b)/(M_b) \), carried by the accretion flow from the CBD onto the binary. We adopt the initial guess \( \Sigma(0) = 0.75 + 0.35e_b \). The factor \( \mathcal{F}(R; R_{\text{cav}}) \) in Equation (20) is a tapering function responsible for carving a cavity in the density profile (it goes to 0 as \( R \rightarrow 0 \) and to 1 for \( R \gtrsim 2R_{\text{cav}} \)). We find that a model function of the form

\[
\mathcal{F}(R; R_{\text{cav}}) = \exp \left[ -\left( \frac{R_{\text{cav}}}{R} \right)^4 + \left( \frac{R_{\text{cav}}}{R} \right)^2 \right] \tag{21}
\]

provides a good fit for \( \Sigma \) for all of the values of \( e_b \) explored by MML17 (\( h_0 = 1 \) fixed). The fitted values of \( R_{\text{cav}} \) increase nearly monotonically with binary eccentricity; we find

\[
R_{\text{cav}} \approx \begin{cases} 
2.45a_b & \text{for } e_b < 0.1 \\
0.6a_b(1.5e_b + 2.3) & \text{for } e_b \geq 0.1.
\end{cases} \tag{22}
\]

The profile (20) is discretized into 400 azimuthal zones and 600 radial zones logarithmically spaced between \( R_{\text{in}} = (1 + e_b)a_b \) and \( R_{\text{out}} = 70a_b \).

As usual, the azimuthal velocity profile is supplied to satisfy centrifugal equilibrium:

\[
v_{\phi}^2(R) = \frac{GM_b}{R} \left[ 1 + 3 \frac{Q}{R^2} \right] - c_s^2(R) \left[ 1 - \frac{R d\Sigma}{\Sigma dR} \right]. \tag{23}
\]
where the sound speed $c_s^2(R) \propto h_0^2 R^{-1}$ is a fixed function of radius alone for $R \gg a_b$, and where the quadrupole correction $Q$ to the binary’s (orbit-averaged) gravitational potential is given by

$$Q = \frac{a_b^2}{4} \frac{q_b}{1 + q_b} \left(1 + \frac{3}{2} q_b^2 \right). \quad (24)$$

Finally, we assume that radial transport is dominated by viscosity, and we use

$$v_R^{(\text{visc})}(R) = \frac{1}{R \Sigma} \frac{\partial}{\partial R} \left[ \nu \Sigma R \frac{d \Omega}{d R} \left( \frac{d}{d R} (R^2 \Omega) \right) \right]^{-1}$$

$$= -3 \alpha h_0^2 \Omega \dot{a}_b \left( \frac{R}{a_b} \right)^{-1/2} \left[ \frac{R}{\Sigma} \frac{d \Sigma}{d R} + 1 \right] \quad (25)$$
as an estimate of the initial radial velocity, where we have taken $\Omega \approx \sqrt{GM_b/R} = \Omega_b(R/a_b)^{-3/2}$. For our initial density profile (Equation (20)), Equation (25) roughly matches the radial velocity profile of a steady-supply disk,

$$v_R^{(\text{steady})} = -\frac{\dot{M}_0}{2\pi R \Sigma}, \quad (26)$$for $R \gtrsim 15a_b$. We have also experimented with setting $v_R = v_R^{(\text{steady})}$ as an initial condition and have found that quasi-steady state can be reached faster. We stress that any axisymmetric initial condition is an inadequate representation of the gas dynamics for $R \lesssim 8a_b$, and that the exact choice of the initial condition in this region is not critical to our results. It is of importance, however, that the simulation be evolved for a viscous time at a radius where the CBD can be considered axisymmetric and treated approximately as a conventional $\alpha$ disk. For an integration time of $t_{\text{int}} = 300P_b$, the disk is viscously relaxed out to a radius $R_{\text{rel}} = a_b \left(15/2\right)^{3/2} \approx 12a_b$; (Rafikov 2016; ML16). Therefore, outside this radius, the surface density will not have time to relax and will partially retain its initial configuration. It is then desirable that, already at $t = 0$, the accretion profile roughly satisfies $M(R) \approx M_0$ for $R \gtrsim 20a_b$; the initial condition (Equation (20)) meets this requirement.

2.3.1. Steady-state Tests

In order to test how close our initial condition is to a true steady state, we carry out three test integrations with $\epsilon_b = 0$: (1) we run an AREPO simulation with an open, diode-like boundary condition at $R = a_b$ for $300P_b$ and then “release” the boundary to allow for direct accretion onto the binary; integrating for an additional 500 orbits (see ML16); (2) we allow for direct accretion onto the “stars” immediately at the start-up of the simulation and integrate for 500 orbits; (3) we use the results of the PLUTO simulation from MML17 at $t = 300P_b$, remapping it onto an AREPO grid and rescaling the units such that the measured accretion rate at $R = 15a_b$ equals unity; then we proceed to evolve this setup by integrating, as in cases (1) and (2), for another 500 orbits. The accretion rates corresponding to these three tests are depicted in Figure 1. In all cases, after gas crosses the $R = a_b$ boundary, only 5–20 orbits are needed for the CSDs to form. Another 20–30 orbits are needed for the binary to start accreting with an averaged rate equal to $M_0$ to within a few percent. Eventually, $M_0$ is matched to within 1% in all cases, although this can take from 100 orbits (top panel) to 300 orbits (bottom panel). Overall, these three simulations are indistinguishable from each other, and we conclude that once quasi-steady state is reached, the initial conditions become irrelevant.

From here on, we define quasi-steady state as the state of the simulation for which the averaged accretion rate onto the binary $\langle \dot{M}_b \rangle$ over an interval $T = 100P_b$ is equal to the mass supply rate $\dot{M}_0$ to within 2% and where the time-averaged mass accretion profile in the CBD $\langle \dot{M}_b \rangle$ (R) is also $\approx M_0$ (i.e., flat to within an rms tolerance of 5%). Unless stated otherwise, all simulations presented henceforth include a preliminary phase (300$P_b$) in which the disk is evolved using a diode boundary condition at $R_{\text{in}} = a_b(1 + \epsilon_b)$. If a flat $\langle \dot{M}_b \rangle$ profile is reached within that time period, the boundary is removed and the simulation is resumed in the self-consistent fashion described in the preceding paragraphs.

3. Simulation Results

3.1. Results for a Circular Binary

Here we explore the accretion behavior of a circular, equal-mass binary. We focus on the simulation output after 3500 binary orbits.

3.1.1. Angular Momentum Transfer

The simulation output is processed to produce time series for $M_b$ and $q_b$, the specific torques $d\dot{b}_b/dt_{\text{grav}}$, and $d\dot{b}_b/dt_{\text{acc}}$, and the spin change rate $\dot{S}_b = \dot{S}_1 + \dot{S}_2$ (see Section 2.2.1). These quantities and the total $\dot{J}_b$ are shown in Figure 2 for a time interval of 200$P_b$. We find that in quasi-steady state, the time-averaged $\dot{J}_b$ is given by

$$\langle \dot{J}_b \rangle \approx 0.676M_0\Omega_b a_b^2. \quad (27)$$

Of the five different contributions to $\dot{J}_b$, two quantities dominate over the rest: the mass change $\langle \dot{M}_b \rangle \approx M_0$ (first panel) and the specific gravitational torque $\langle \dot{J}_{b,\text{grav}} \rangle \approx 1.567(M_0/M_0)\Omega_b a_b^2$ (third panel).

The stationary nature of $M_b$ and $\dot{J}_b$ means that, in an averaged sense, a constant amount of angular momentum is transferred to the binary by the disk per unit accreted mass. This constant is the eigenvalue of the accretion flow (e.g., Paczynski 1991; Popham & Narayan 1991), that is,

$$\frac{\langle \dot{J}_b \rangle}{\langle \dot{M}_b \rangle} \equiv I_0. \quad (28)$$

Our result in Figure 2 implies

$$I_0(e_b = 0) \approx 0.68\Omega_b a_b^2. \quad (29)$$

The eigenvalue $I_0$ can also be obtained by computing $\langle \dot{J}_b \rangle(R)$, the angular momentum current as a function of radius in the CBD (Equation (15)). We compute $\dot{J}_b(R)$ from the combination of $\dot{J}_{\text{adv}}, \dot{J}_{\text{visc}}$, and $\dot{J}_{\text{grav}}$, in turn computed from the time-averaged angular momentum flux maps $F_J$ as described in Section 2.2.2. We compute these angular momentum flux maps from simulation snapshots produced at a high output rate (15 snapshots per orbit) followed by time-averaging. Figure 3 shows the result. Importantly, we see that the net angular
Figure 1. Total binary accretion rate $M_b = M_1 + M_2$ for three steady-state tests over a period of 75.8 orbits (chosen to cover approximately 16 cycles of the accretion burst with frequency $\sim 2\Omega_b$). The same quasi-steady state is reached regardless of the different initial conditions (see Section 2.3.1).

momentum current $\langle \dot{J}_b \rangle (R)$ (dark red) is nearly flat for $R$ between $a_b$ and 10$a_b$. Furthermore, $\langle J_b \rangle (R)$ is very close to $\langle J_b \rangle$. In other words, all of the angular momentum crossing the CBD is received by the binary. This provides convincing evidence that the exchange of mass and angular momentum between the CBD and the binary are in a true quasi-steady state, and that our computed eigenvalue (Equation (29)) is reliable.

Dependence on sink radius: In our simulations, the “stars” behave as sink particles. In most of our simulations, we adopt the “stellar radius” of $r_{\text{acc}} = 0.02a_b$, but it is natural to evaluate how sensitive our results are to the choice of $r_{\text{acc}}$. Recently, Tang et al. (2017) posited that the sign of the net angular momentum transfer rate $\langle J_b \rangle$ (note that Tang et al. 2017 only computed $\langle J_b \rangle$ and did not include spin torques) can depend critically on the way gas is drained from the computational domain by the stars (see Section 5.2.1). We test the influence of the accretion algorithm on our results by carrying out two additional simulations with $r_{\text{acc}} = 0.04a_b$ and $r_{\text{acc}} = 0.06a_b$ (the softening lengths are updated accordingly, but all other parameters remain unchanged). The results of this test are depicted in Figure 4, where we contrast the gas density field (to illustrate the effect of a larger sink radius) with the evolution of $M_b$, $S_b$, and $J_b$. The mean values $\langle \dot{J}_b \rangle$ and $\langle M_b \rangle$ are robust against $r_{\text{acc}}$, but $\langle S_b \rangle$ grows almost proportionally to $r_{\text{acc}}$. Since $\langle J_b \rangle$ appears unmodified, the increase in $\langle S_b \rangle$ must be accompanied by a decrease in the torque $\mu_b\langle \dot{J}_{b,\text{grav}} \rangle$ (not shown). Indeed, over the interval [3500, 3700]$P_b$, the specific gravitational torque $\langle \dot{J}_{b,\text{grav}} \rangle$ is 1.524($M_0/M_b$)$\Omega_ba_b^2$ when $r_{\text{acc}} = 0.04a_b$ and 1.482($M_0/M_b$)$\Omega_ba_b^2$ when $r_{\text{acc}} = 0.06a_b$, both smaller than the value of 1.567($M_0/M_b$)$\Omega_ba_b^2$ in our fiducial simulation (Figure 2). The anisotropic accretion torque $\mu_b\langle \dot{J}_{b,\text{acc}} \rangle$ remains negligible in all cases. For all the values of $r_{\text{acc}}$ explored, $\langle S_b \rangle$ is a small contribution to the total transfer of angular momentum to the binary $\langle J_b \rangle$. Presumably, a much larger value of $r_{\text{acc}}$ than explored here—one that forbids the formation of CSDs—could make $\langle S_b \rangle$ an important contributor to $\langle J_b \rangle$, one that competes with $\mu_b\langle \dot{J}_{b,\text{grav}} \rangle$, perhaps to the point of erasing the positive torque contribution of the CSDs, turning all of the positive gain in orbital angular momentum into a growth in spin.

3.1.2. Understanding the Origin of the Positive Torque on the Binary: Spatial Distribution of Gravitational Torques

A major result of this paper (see Figures 2–3) is that $\langle J_b \rangle \approx \langle \dot{J}_b \rangle > 0$; that is, an accreting binary gains angular momentum from a steady-supply disk. From Figure 2, we see that a major contribution to $\langle J_b \rangle$ comes from the total gravitational torque $\mu_b\langle \dot{J}_{b,\text{grav}} \rangle$ (third panel), which is positive, in apparent contradiction with what one might expect from theoretical work (Goldreich & Tremaine 1980; Artymowicz & Lubow 1994; Lubow & Artymowicz 1996), since the linear theory of Lindblad torques of Goldreich & Tremaine (1979) predicts that an outer disk exerts negative torques on the inner binary. Note that our result for the angular momentum current
$J_{d, grav}$ in the CBD (Figure 3, yellow curve) is indeed consistent with this theoretical expectation. However, a self-consistent calculation of $J_b$ needs to include the entire distribution of gas and not only that of the CBD.

Figure 5 depicts the dissection of the total gravitational torque $T_{b, grav}$ into three components (from top to bottom): (1) the outer disk torque $T_{b, grav}^{(out)}$ ($R > R_{cav}$; with $R_{cav}$ as defined in Equation (22)), (2) the “cavity+streams” torque $T_{b, grav}^{(cav)}$ ($a_b \leq R \leq R_{cav}$), and (3) the “inner” torque $T_{b, grav}^{(in)}$ ($R < a_b$). The short-term evolution of these torques is depicted on the left panels of the figure, and the long-term evolution is shown on the right. On average, the sum of these three different torques must equal $\langle T_{b, grav} \rangle = \mu_b \langle \dot{M}_b \rangle = 0.392 M_0 a_b^2 \Omega_b$.

From the torque dissection, we confirm that the gravitational torque acting on the binary due to the material outside $R = a_b$ is indeed negative, but that the material inside $R = a_b$ exerts a positive torque that nearly doubles in magnitude the negative outer torque. We also find that the oscillation amplitudes of these three torque contributions can be much larger than their mean values. In order to capture both the rapid sign changes in
The Astrophysical Journal, 871:84 (20pp), 2019 January 20

We carry out this average over intervals of \( \sim 30 \) orbits in duration, at different times in the simulation. In addition, we perform the average over 200 orbits (rightmost panels of Figure 6). All of the averaging sets produce essentially the same result, implying that most of the variability takes place on timescales shorter than 30 orbits. Figure 6 reveals that the streamers are the dominant nonaxisymmetric feature that persists in time. From the torque density maps (bottom panels), it can be seen that different portions of the gas can “torque up” (regions in red) or “torque down” (regions in blue) the binary, but only the persistent nonaxisymmetric features contribute to the nonzero net torque.

### 3.2. Results for an Eccentric Binary

We repeat the numerical simulations of Section 3.1, this time for binary eccentricity \( e_b = 0.6 \). Several differences between these two calculations are expected. First, the lump that modulates the accretion about every five binary orbits when \( e_b = 0 \) (e.g., MacFadyen & Milosavljević 2008; Shi et al. 2012; ML16; MML17) is no longer present (ML16; for these values of \( \alpha \) and \( \eta_0 \), MML17 find that the lump disappears at \( e_b \geq 0.05 \)). Second, accretion will be preferential and alternating (Dunhill et al. 2015; ML16); that is, the primary and secondary will alternate over precessional timescales which one receives most of the mass. Third, the gas distribution and kinematics in the region \( R \lesssim a_b \) differ significantly from the circular case (material enters and leaves this region in a periodic fashion; see Figure 2 of ML16), affecting the amount of torque that this region can provide to counterbalance the torque from the CBD.

#### 3.2.1. Angular Momentum Transfer

In Figure 7 we show the total angular momentum transfer rate \( \dot{J}_b \) and its various contributions for \( q_b = 1 \) and \( e_b = 0.6 \) binary. As expected, we find that \( \dot{M}_b \) is no longer modulated at a frequency of \( \sim \Omega_b \), but that the dominant frequency \( \sim \Omega_b \) (ML16). There are variations over much longer timescales: the mass ratio change \( \dot{q}_b \) flips signs every \( \sim 415 P_b \). The gravitational specific torque \( \dot{h}_{b,\text{grav}} \) is again positive and larger than in the circular case. The contribution of the anisotropic accretion torque \( \dot{\mu}_b(\dot{h}_{b,\text{acc}}) \approx -0.21 \dot{M}_b \Omega_b a_b^2 \) is no longer negligible. As in the circular case, the contribution of the spin torque \( \dot{S}_b \) to the total torque \( \dot{J}_b \) is small (\( \sim 2\% \)).

As in Section 3.1, we can compute the accretion eigenvalue \( l_0 \) directly from the result shown in Figure 7, giving

\[
   \left| \frac{\dot{J}_b}{\dot{M}_0} \right| \approx 0.81 \Omega_b a_b^2, \quad (31)
\]

We also compute the net angular momentum current in the CBD \( \dot{J}_b(R) \) (Section 2.2.2) averaged over 300 orbits, as shown in Figure 8. We see that \( \dot{J}_b(R) \) is approximately constant (independent of \( R \)) and equal to \( \dot{J}_b \), indicating that the global angular momentum balance is achieved. In this case, the match between \( \dot{J}_b / \dot{M}_0 \) and \( l_0 = 0.81 \Omega_b a_b^2 \) is remarkable for \( R > 4a_b \) and \( R < 2a_b \).

#### 3.2.2. Spatial Distribution of Gravitational Torques

As in the case of the circular binary, the eccentric binary gains angular momentum, which, as before, is largely a result of the positive net gravitational torque \( \dot{h}_{b,\text{grav}} = \mu_b(\dot{h}_{b,\text{grav}}) \approx 0.796 \dot{M}_b \Omega_b a_b^2 \). As before, we study the spatial distribution of
Figure 4. Mass and angular momentum transfer for a circular binary for different values of the accretion (sink) radius $r_{\text{acc}}$. The top panels depict the gas surface density rendered at $t = 3600 P_b$ for $r_{\text{acc}} = 0.02 a_b$ (left), $r_{\text{acc}} = 0.04 a_b$ (middle), and $r_{\text{acc}} = 0.06 a_b$ (right). The bottom panels show the joint evolution of $\dot{M}_b$, $\dot{S}_b$, and $\dot{J}_b$ over the interval $[3550 P_b, 3650 P_b]$ for $r_{\text{acc}} = 0.04 a_b$ (middle) and $r_{\text{acc}} = 0.06 a_b$ (right). Over the same time interval, all simulations exhibit highly consistent behavior in $\dot{M}_b$ and $\dot{J}_b$, with $(\dot{J}_b)/\dot{M}_b \approx 0.67 - 0.68$ (see Figure 2). The net spin torque $\dot{S}_b$ increases almost linearly with $r_{\text{acc}}$, while $\dot{J}_b$ is not affected in a significant way.

Figure 5. Gravitational torques dissected according to distance $R$ from the binary barycenter (Section 2.3) as a function of time, covering the time interval between 3500 and 3535 $P_b$ (left panels) and between 3500 and 3700 $P_b$ (right panels). The outer torque (yellow, top panels) corresponds to the torque exerted by gas cells located at $R > R_{\text{cav}}$ (see Equation (22)); the cavity torque (blue, middle panels) corresponds to the total torque exerted by cells with radii $a_{R, \text{cav}} < R < R_{\text{cav}}$; the inner torque (red, bottom panels) corresponds to cells with $R < a_{\text{cav}}$. The mean values of these torques are indicated in each panel. Both the cavity and outer torques are negative, collectively amounting to $\sim -0.42 \dot{M}_b \Omega_b a_b^2$, in agreement with $\dot{J}_{b, \text{grav}}$ at $R = a_b$ (Figure 3). A net negative gravitational torque due to gas outside $R = a_b$ is in agreement with theoretical expectations and past numerical work. The net positive sign of $(\dot{J}_{b, \text{grav}})$ is due to the large inner torque $(\dot{T}_{b, \text{grav}})$. Over 200 orbits (right panels), the net gravitational torque $(\dot{T}_{b, \text{grav}}) = 0.392 \dot{M}_b \Omega_b a_b^2$ is equal to the specific torque $(\dot{J}_{b, \text{grav}}) = 1.567 \dot{M}_b/\dot{M}_b \Omega_b a_b^2$ (third panel of Figure 2) multiplied by the reduced mass $\mu_b = \dot{M}_b/\dot{M}_b$. The inner torque exhibits rapid variability on timescales $\sim P_b/100$, and its mean value, when properly measured, can double in magnitude the negative torque due to the outer/cavity gas. Comparison between the left and right panels shows that a few tens of orbits are sufficient to capture the basic stationary behavior of these torque time series.
gravitational torques in the simulation by splitting $T_{b,\text{grav}}$ into three components: $T_{b,\text{grav}}^{\text{out}}$, $T_{b,\text{grav}}^{\text{cav}}$, and $T_{b,\text{grav}}^{\text{in}}$ (Figure 9). We see a behavior similar to that of a circular binary (Figure 5): the sum $T_{b,\text{grav}}^{\text{out}} + T_{b,\text{grav}}^{\text{cav}}$ is negative (dominated by $T_{b,\text{grav}}^{\text{cav}}$), while the positive sign of the total torque $T_{b,\text{grav}}^{\text{in}}$ is entirely due to the positive sign of $T_{b,\text{grav}}^{\text{in}}$. The torque time series shown in Figure 9 exhibit some degree of regularity, but they differ from the nearly exact periodicity seen in Figure 5. As noted above, the accretion flow onto eccentric binaries is modulated over long secular timescales, and it is not surprising that a few tens of orbits cannot capture the entire range of time variability.

The fact that tens of orbits is too short a time span to reveal the underlying stationary behavior of the accretion flow can be illustrated by repeating the map-stacking analysis for the $e_b = 0.6$ case (see Figure 10 and Figure 6). The stacking of these images is trickier than in the circular case: in order for the two “stars” to line up at all snapshots, we need to introduce a rotating–pulsating coordinate system, a transformation that is well known in the study of the elliptical restricted three-body problem (ER3BP; e.g., Nechvila 1926; Kopal & Lyttleton 1963; Szebehely & Giacaglia 1964; Szebehely 1967; Musielak & Quarles 2014). The scaled, rotated coordinates $\xi$ and $\eta$ are related to the barycentric coordinates $x$ and $y$ by $x = r_b(\xi \cos \theta_b - \eta \sin \theta_b)$ and $y = r_b(\xi \sin \theta_b + \eta \cos \theta_b)$, where $r_b = a_b(1 - e_b^2)/(1 + e_b \cos \theta_b)$ and $\theta_b$ is the true anomaly of the binary. This transformation is not area preserving, so scalars must be transformed by multiplying by the Jacobian of the transformation ($=r_b^2$). In Figure 10 (top row, left to right), we show $\Sigma$ stacked over the time intervals $[3470 \, P_b, \, 3500 \, P_b]$, $[3530 \, P_b, \, 3560 \, P_b]$, $[3590 \, P_b, \, 3620 \, P_b]$, and $[3670 \, P_b, \, 3799 \, P_b]$. These reveal a striking inherent property of accretion onto eccentric binaries: the “loading” of the CSDs alternates in time; that is, the gas surface density is higher in one CSD than in the other. This disk loading can be directly tied to the sign of $q_b$ in Figure 7. When we average over $370 \, P_b$—almost the entire precessional period identified from the time series of $q_b$—we see that most of the asymmetries disappear (the last column of Figure 10), and that the distribution of gas starts to resemble that of the circular case (Figure 6). Note how the shape of the cavity also changes as preferential accretion switches recipients, as is expected if alternation is caused by the precession of the cavity edge.
We use this rotating system to study the spatial distribution of torques, albeit only in a qualitative manner (Figure 10). Some similarities can be found with the analogous plot for the circular case (Figure 6, bottom row), although the spatial distribution of torques in the eccentric case is more diffuse and variable. A direct comparison to the spatial torque dissection of Figure 9 is not possible, since the barycentric radial coordinate \( R \) changes from snapshot to snapshot in the rotating–pulsating coordinate system.

### 3.3. Other Values of Binary Eccentricity

We have repeated the experiments of Sections 3.1 and 3.2 for other values of the binary eccentricity. Figure 11 shows the results for a low-eccentricity binary (\( e_b = 0.1 \), top) and an intermediate-eccentricity binary (\( e_b = 0.5 \), bottom). In both cases, the circumbinary cavity lacks the accretion lump (MML17), and accretion exhibits long-term alternation. As in...
The Astrophysical Journal, 871:84 (20pp), 2019 January 20

Muñoz, Miranda, & Lai

Hadjidemetriou (1963) derived the evolution equation for the binary orbital elements due to time-varying mass loss by introducing a fictitious force. Here, we give a much simpler derivation of \( \dot{a}_b \) and \( \dot{e}_b \) for a mass-losing/gaining binary using energy and angular momentum conservation. We employ two methods to evaluate \( \langle \dot{a}_b \rangle \) and \( \langle \dot{e}_b \rangle \) using our simulation output.

Consider the specific angular momentum \( \mathbf{l}_b = \mathbf{r}_b \times \mathbf{\dot{r}}_b \) and specific energy \( \mathcal{E}_b = \frac{1}{2} \dot{r}_b^2 - \frac{GM_b}{r_b} \) of the binary. The equation of motion is

\[
\frac{d\mathcal{E}_b}{dt} = -\frac{GM_b}{r_b} \dot{r}_b + \mathbf{r}_b \cdot \mathbf{f}_{\text{ext,1}} + f_{\text{ext,2}}.
\]

Figure 8. Similar to Figure 3, except for an eccentric binary (\( e_b = 0.6 \)). The thin red line indicates the eigenvalue \( \lambda_b = 0.81a_b^2 \Omega_b \) (Equation (31)) obtained from the evolution of the accreting eccentric binary (Figure 7). The gray region represents the portion of the computational domain left out by the diode-like momentum to the binary, which is positive and represents the portion of the computational domain left out by the diode-like momentum to the binary. There are multiple consequences of this period not matching the apsidal precession period of the disk, in addition to the intrinsic difficulties of estimating the advective flux of angular momentum from reconstructed primitive variables.

Figure 7, the time interval in the x axis is chosen to cover an entire precessional period, estimated as the time it takes for \( q_b \) to change sign twice. This period is measured to be \( \sim 800P_b \) for \( e_b = 0.1 \) and \( \sim 400P_b \) for \( e_b = 0.5 \). Note that this alternation period does not exactly match the apsidal precession period of test particles at the location of the cavity edge (e.g., Thun et al. 2017), since the latter scales with cavity size and eccentricity as \( \left( \frac{R_{\text{cav}}}{a_0} \right)^{1/2} \left[ 1 + \frac{3}{2} e_b^2 \right]^{-1} \) (Equation (7) in MML16), which, if we take Equation (22) at face value, implies that the precession period scales as \( e_b^{7/2} \). However, the precession rate of the inner CBD can be coherent out to \( \sim 20a_0 \) (e.g., Figure 9 in MML17), and then it does not need to be set by the precession rate at \( R_{\text{cav}} \), but rather by some mean rate over that range of radii.

The main conclusion drawn from Figure 11 is that, provided the steady state is reached, the net transfer of angular momentum to the binary \( \langle \dot{J}_b \rangle \) is positive for various values of \( e_b \). As in the circular binary case, to obtain a reliable measurement of \( \langle \dot{J}_b \rangle \), it is important to capture accurately the gravitational torques from the CSDs, which are positive and can counterbalance the negative torques due to the CBD.

We summarize the results of our simulations in Table 1. In all cases explored—a circular binary with three values of \( n_{\text{acc}} \) and three eccentric binaries—the binary experiences a net gain in angular momentum. There are multiple consequences of having \( \langle \dot{J}_b \rangle > 0 \). In particular, we are interested in translating \( \langle \dot{M}_b \rangle \) and \( \langle \dot{J}_b \rangle \) into secular changes in the orbital elements of the binary. We explore this in the next section.

4. Orbital Evolution of Accreting Binaries

For a circular, equal-mass binary, the time-averaged rates \( \langle \dot{M}_b \rangle \) and \( \langle \dot{J}_b \rangle \) completely determine the secular semimajor axis evolution \( \langle \dot{a}_b \rangle \) of the binary (assuming that the binary remains circular). For an eccentric binary, more information is needed. The orbital evolution of variable-mass binaries has long been studied (e.g., Brown 1925; Jeans 1925; Hadjidemetriou 1963; Verbist 1972; Veras et al. 2011).

\[
\frac{d\mathcal{E}_b}{dt} = -\frac{GM_b}{r_b} \dot{r}_b + \mathbf{r}_b \cdot \mathbf{f}_{\text{ext,1}} + f_{\text{ext,2}}.
\]

Because \( e_b^2 = 1 + 2\dot{J}_b \mathcal{E}_b/(GM_b)^2 \) and \( \mathcal{E}_b = -GM_b/(2a_0) \), we have

\[
2e_b \dot{\mathcal{E}}_b = \frac{\dot{a}_b}{a_0} + \frac{\dot{\mathcal{E}}_b}{\dot{\mathcal{E}}_b} - \frac{2M_b}{MB}.
\]

Applying Equation (33) to our simulation output (and using the same procedure as described in Section 2.2.1), we can evaluate \( \dot{\mathcal{E}}_b/\dot{\mathcal{E}}_b \) for the accreting binary. This is shown, together with \( \dot{M}_b/M_b \) and \( \dot{a}_b/a_0 \), in Figure 12 for \( e_b = 0 \) (left panels) and \( e_b = 0.6 \) (right panels). The linear combination of these quantities gives \( \dot{a}_b/\dot{\mathcal{E}}_b \) (Equation (34)) and \( \dot{\mathcal{E}}_b/\dot{\mathcal{E}}_b \) (Equation (35)).

For the \( e_b = 0 \) binary, we find \( \langle \dot{a}_b \rangle/\dot{\mathcal{E}}_b \approx \langle \dot{M}_b \rangle/M_b \) (i.e., it is consistent with zero) and \( \langle \dot{a}_b \rangle/a_0 \approx 2.15\langle \dot{M}_b \rangle/M_b \). This implies that the binary expands while remaining circular. For the \( e_b = 0.6 \) binary, \( \langle \dot{a}_b \rangle/\dot{\mathcal{E}}_b \approx -2.84\langle \dot{M}_b \rangle/M_b \), or \( \langle \dot{a}_b \rangle/\dot{\mathcal{E}}_b \approx -2.34\langle \dot{M}_b \rangle/M_b \), and \( \langle \dot{a}_b \rangle/a_0 \approx 0.47\langle \dot{M}_b \rangle/M_b \). In this case, the binary still expands—although five times slower than its circular counterpart—but it tends to circularize while doing so. These results present a significant challenge to the standard view that binaries surrounded by CSDs must shrink (see Section 5).

We note that Equations (34) and (35) are equivalent to the more complex-looking expressions derived by previous authors. For example, substituting Equations (32) and (33) into Equation (34), and using \( \mathbf{r}_b \cdot \mathbf{f}_{\text{ext}} = \Omega_b a_0 (1 - e_b^2)^{-1/2} (e_b \sin \phi \mathbf{f}_{\text{ext,1}} + (1 + e_b \cos \phi) \mathbf{f}_{\text{ext,2}}) \) and \( |\mathbf{r}_b \times \mathbf{f}_{\text{ext}}| = a_0 (1 - e_b^2) (1 + e_b \cos \phi)^{-1} \mathbf{f}_{\text{ext,1}} \), one can write the following in terms of...
of the true anomaly $\phi$:

$$
\dot{e}_b = \frac{a_b(1-e_b^2)}{GM_b} \left[ \sin \phi \hat{f}_{ext,r} + \left( \frac{e_b + 2 \cos \phi + e_b \cos^2 \phi}{1 + e_b \cos \phi} \right) f_{ext,\phi} \right] - \frac{M_b}{M_b}(e_b + \cos \phi)
$$

and

$$
\dot{a}_b = \frac{\sqrt{\frac{a_b^3}{GM_b(1-e_b^2)}} [e_b \sin \phi \hat{f}_{ext,r} + (1 + e_b \cos \phi)f_{ext,\phi}]}{1 - e_b^2} - a_b \frac{M_b(1 + 2 e_b \cos \phi + e_b^2)}{M_b(1 - e_b^2)}.
$$

Figure 9. Similar to Figure 5, this time for an eccentric binary ($e_b = 0.6$). The total gravitational torque is dissected into an outer component (yellow, top panels) for $R > R_{grav}$, a cavity component (blue, middle panel) for $a_b(1 + e_b) < R <= R_{grav}$, and an inner component (red, bottom panels) for $R <= a_b(1 + e_b)$. The gravitational torque due to gas outside the boundary $R = a_b(1 + e_b)$ is negative, but smaller in magnitude than the positive torque due to gas inside that boundary. Over 200 orbits (right panels), the net gravitational torque $\langle \tau_{grav} \rangle = 0.807 M_b \Omega_b a_b^2$ is equal to the specific torque $\langle \tau_{grav} \rangle = 3.229 (M_b/M_0) \Omega_b a_b^2$ (extracting the interval $[3500, 3700] P_b$ from the third panel of Figure 7) and multiplied by the reduced mass $\mu_b = M_b/4$.

When setting $f_{ext} = 0$, we recover the known expressions for the effect of mass loss on $e_b$ and $a_b$ (Hadjidemetriou 1963). If $M_b = 0$, we simply recover the perturbation equations for osculating orbital elements (Burns 1976; Murray & Dermott 2000). Furthermore, these expression for $\dot{e}_b$ and $\dot{a}_b$ are also the coplanar equivalent to those derived by Veras et al. (2013, their Equations (29) and (30)), in which case the external forces are a result of anisotropic mass loss.

We can also use Equations (34) and (35) to directly compute the instantaneous rates of change $\dot{e}_b$ and $\dot{a}_b$. The results are shown in Figure 13. The long-term time average of these time series provides the secular evolution of the orbital elements. For the circular binary, we obtain that $\langle \dot{a}_b \rangle \approx 2.15 a_b M_0/M_b$ and $\langle \dot{e}_b \rangle \approx 0$, in agreement with Figure 12. For the $e_b = 0.6$ binary, again we find agreement with Figure 12: $\langle \dot{a}_b \rangle \approx 0.47 a_b M_0/M_b$ and $\langle \dot{e}_b \rangle \approx -2.34 a_b M_0/M_b$.

Table 1 includes the measured values of $\langle \dot{a}_b \rangle$ and $\langle \dot{e}_b \rangle$ for the six simulations presented in this work. In all configurations explored, $\langle \dot{a}_b \rangle > 0$, that is, binaries expand as they accrete. The behavior of $\langle \dot{e}_b \rangle$ is not monotonic in $e_b$: circular binaries remain circular and high-eccentricity binaries circularize; however, the moderate-eccentricity binary ($e_b = 0.1$) has $\langle \dot{e}_b \rangle > 0$. This behavior in $\langle \dot{e}_b \rangle$ is reminiscent of a result reported by Roedig et al. (2011, simulations of very massive CBD around live binaries), in which a “fixed point” for which $\langle \dot{e}_b \rangle = 0$ can be inferred for some intermediate value of $e_b$. This behavior is perhaps related to the bifurcation in CBD precession rates measured by Thun et al. (2017). This result is intriguing and warrants a more thorough exploration of different values of $e_b$.

5. Summary and Discussion

5.1. Summary of Key Results

We have carried out a suite of two-dimensional, viscous hydrodynamical simulations of CBD accretion using AREPO (Springel 2010), a finite-volume method on a freely moving Voronoi mesh. As in our previous work (Muñoz & Lai 2016; ML16), which also used the AREPO code, we can robustly simulate accretion onto eccentric binaries without the constraints imposed by structured-grid numerical schemes. Most importantly, using the AREPO code allows us to follow the mass accretion through a wide radial extent of the CBD (up to $70 a_{bin}$, where $a_{bin}$ is the binary semimajor axis), leading to accretion onto the individual members (“stars” or “black holes”) of the binary via circumsingle disks (down to spatial scales of $0.02 a_{bin}$). In ML16, we focused on accretion variability on different timescales. In this paper, we study the angular momentum transfer to the binary and, for the first time, determined the long-term evolution rates of orbital elements (semimajor axis and eccentricity) for binaries undergoing circumbinary accretion. This work also goes significantly beyond our other recent paper (Miranda et al. 2017; MML17), which used the polar-grid code PLUTO (excising the innermost “cavity” region surrounding the binary) to explore the dynamics and angular momentum transfer within CBD.
As in ML16, our simulations focus on the well-controlled numerical setup where mass is supplied to the outer disk ($R_{\text{out}} = 7a_b$) at a fixed rate. We consider equal-mass binaries, use a locally isothermal equation of state (corresponding to $H/R = 0.1$), and adopt the $\alpha$-viscosity prescription with $\alpha = 0.1$. The “stars” or “black holes” are treated as absorbing spheres of radii $r_{\text{acc}} \ll a_b$ (our canonical value is $r_{\text{acc}} = 0.02a_b$). In all our simulations, we evolve the system for a sufficiently long time that a quasi-steady state is reached, in which the time-averaged mass accretion rate onto the binary matches the mass transfer rate across the disk, and both are equal to the mass supply rate at $R_{\text{out}}$. Our key findings pertain to the angular momentum transfer and the secular evolution of the binary in such a quasi-steady state:

(i) For all cases studied in this paper (with various values of binary eccentricities; see Table 1), the time-averaged angular momentum transfer rate to the binary is positive ($\langle J_b \rangle > 0$); that is, the binary gains angular momentum. We determined this $\langle J_b \rangle$ by directly computing the gravitational torque and accretion torque on the binary. In addition, we computed the net angular momentum current (transfer rate) $\dot{J}_b$ (Equation (15)) across the CBD and find that its time average $\langle \dot{J}_b \rangle$ is constant (independent of $R$; see Figures 3 and 8) and equal to $\langle J_b \rangle$. This agreement shows that global balance is reached in our simulations in terms of both mass and angular momentum transfer.

(ii) Our computed angular momentum transfer rate per unit accreted mass, $\langle J_b \rangle / \langle M_b \rangle$, lies in the range $(0.4 - 0.8)\dot{M}_b \Omega_b$ (where $\Omega_b$ is the orbital angular velocity of the binary) and depends on the binary eccentricity (see Table 1). A small fraction of $\langle J_b \rangle$ contributes to the spin-up torque on the binary components (“stars”).

(iii) Using our simulations, we computed the time-averaged rate of change of the binary orbital energy $\langle \dot{E}_b \rangle$, taking account of gravitational forces and accretion. Combining $\langle \dot{E}_b \rangle$ and $\langle J_b \rangle$, we obtain, for the first time, the secular evolution rates for the binary semimajor axis and...
eccentricity (Section 4). In all the cases considered, we found that the binary expands while accreting (at a rate of order \(\dot{a}_b/\dot{e}_b \sim (M_b/M_0)^\alpha\), see Table 1). Circular binaries remain circular and high-eccentricity binaries circularize; however, moderate-eccentricity binaries \((e_b = 0.1)\) may become more eccentric.

(iv) We carried out a detailed analysis on the different contributors to the total binary torque \(\dot{J}_b\) [see (i)]. This analysis showed that while the gravitational torque from the CBD is negative (as expected based on theoretical grounds), the torque from the inner “circumstellar” disks \((R < a_b)\) is overwhelmingly positive and is responsible for the overall positive value of \(\langle \dot{J}_b \rangle\). A proper calculation of \(\dot{J}_b\) requires a careful treatment of gas flow within the binary cavity.

5.2. Discussion

5.2.1. Angular Momentum Transfer: Comparison to Previous Work

The most important result of our study is that in quasi-steady state the binary gains angular momentum. This result was also strongly suggested in our recent study (MML17) using the polar-grid code PLUTO, in which the innermost region of the circumbinary cavity is excised from the simulation domain. MML17 showed that a quasi-steady state can be reached in the CBD, with global balance of both \(\langle \dot{M}_b \rangle (R)\) and \(\langle \dot{L}_b \rangle (R)\). In fact, for \(e_b = 0\), the accretion eigenvalue obtained by MML17...
and adopted a polar grid in the domain. The Astrophysical Journal, 871:84 (20pp), 2019 January 20
Muñoz, Miranda, & Lai

Figure 12. Evolution of the rate of change in mass $M_\ast/M_0$ (top), specific angular momentum $h_\ast/h_0$ (middle), and specific energy $\dot{e}_\ast/e_\ast$ (bottom) for the $e_0 = 0$ binary (left panels) and the $e_0 = 0.6$ binary (right panels). These quantities determine the rate of change of the orbital elements $\dot{e}_b$ and $\dot{a}_b$ via Equations (34) and (35).

Figure 13. Binary semimajor axis change rate (top, red) and the binary eccentricity change rate (bottom, blue) for circular and eccentric binaries according to Equations (36) and (37) and the simulation results of Sections 3.1 and 3.2. Left panels ($e_0 = 0$): the mean values $\langle \dot{a}_b \rangle \approx 2.15a_b M_0/M_\ast$ and $\langle \dot{e}_b \rangle \approx 0$ indicate that a circular binary tends to remain circular, but that its semimajor axis grows in time. Right panels ($e_0 = 0.6$): the binary expands and circularizes ($\dot{a}_b \approx 0.38a_b M_0/M_\ast$ and $\dot{e}_b \approx -2.34M_0/M_\ast$). Both circular and eccentric binaries exhibit similar rms variability in $\dot{a}_b$ and $\dot{e}_b$, except during the irregular bursts that take place each time $\dot{q}_b$ switches signs in the eccentric case (at $t/P_b \sim 3520$ and $\sim 3740$).

($l_0 \approx 0.8a_b^2 \Omega_b$) is close to the value obtained in this paper ($l_0 \approx 0.68a_b^2 \Omega_b$; Equation (29)). At high eccentricities, however, the mismatch between our new results and those of MML17 grows (e.g., they found $l_0 \gtrsim a_b^2 \Omega_b$ for $e_0 = 0.6$). This discrepancy may be understood from the fact that, for finite eccentricities, the mass flow $M_b(R)$ at $R = a_0(1 + e_b)$ (the inner boundary of the computation domain adopted by MML17) can be both inward and outward (see Figure 2 of ML16), a behavior that a diode-like open boundary (adopted by MML17) cannot capture. The mismatch also underscores the importance of resolving the flows inside the cavity. Because the simulations of MML17 did not fully include the inner cavity, one should take those results as suggestive. In this paper, calculating $\langle \dot{J}_b \rangle$ in two different ways and obtaining agreement (direct torque and angular momentum current across the CBD), we have obtained robust evidence that the binary can gain angular momentum while accreting ($\langle \dot{J}_b \rangle > 0$). This result is insensitive to the accretion radius (the size of the “stars”) as long as $\varepsilon_{\text{acc}} \lesssim 0.1a_b$ (see Table 1).

Our result ($\langle \dot{J}_b \rangle > 0$) contradicts the widely held view that a binary loses angular momentum to its surrounding disk and experiences orbital decay. In fact, while there have been many numerical simulations of circumbinary accretion (see references in Section 1), only a few address the issue of angular momentum transfer in a quantitative way. Three such studies use simulations that excise the inner cavity (MacFadyen & Milosavljević 2008; Shi et al. 2012; MML17). The work of MacFadyen & Milosavljević (2008) was the most similar to MML17: these authors considered $H/R = 0.1$ and a disk viscosity with $\alpha = 0.01$ and adopted a polar grid in the domain between $R_{\text{in}} = a_b$ and $R_{\text{out}} = 100a_b$. They found a reduction of mass accretion onto the binary and the dominance of the gravitational torque relative to advective torque (therefore a negative net torque on the binary). However, with their small viscosity parameter (by contrast, MML17 used $\alpha = 0.1$ and 0.05), the “viscous relaxation” radius $t = 4000P_b$ (the typical duration of their runs) is only about $3a_b$, and their surface density profile is far from steady state even for $R \gg R_{\text{in}}$. We suggest that the result of MacFadyen & Milosavljević (2008) reflects a “transient” phase of their simulations. Shi et al. (2012) obtained a positive value of $\dot{J}_b$ in their 3D MHD simulations of CBD (truncated at $R_{\text{in}} = 0.8a_b$). However, the duration of their simulations is only $\sim 100P_b$, and it is unlikely that a quasi-steady state is reached. Their value of $l_0$, which is too small to cause orbital expansion, may not properly characterize the long-term evolution of the binary. As noted above, our previous study (MML17) using the polar-grid code...
PLUTO (with $R_m = a_0(1 + e_b)$ for eccentric binaries) did achieve quasi-steady state in terms of mass and angular momentum transport in the CBD. For $e_b = 0$, the value of $l_0$ measured by MML17 was close to the value presented in this paper (Table 1), while the $l_0$ values for $e_b > 0$ were unreliable for understandable reasons (see above).

In a recent paper, Tang et al. (2017) TMH17 carried out hydrodynamical simulations of accretion onto circular binaries using the DISCO code (Duffell & MacFadyen 2012; Duffell 2016). Their simulations are similar to ours in many respects (i.e., locally isothermal equation of state with $H/R = 0.1$, $\alpha = 0.1$). Their accretion prescription is different from ours: inside a “sink” radius (measured from each “star”), they assumed that the gas is depleted at the rate $d\Sigma/dt = -\Sigma/t_{\text{sink}}$, with $t_{\text{sink}} = 0.1a_b$ and $t_{\text{sink}}$ a free parameter. By contrast, the AREPO code in its Navier–Stokes formulation (Munoz et al. 2013) self-consistently incorporates viscous transport in all of our computation domain in a unified manner, and gas accretion onto the “stars” is treated using an absorbing sink algorithm with a single parameter $r_{\text{acc}} = 0.02a_0$ (see Section 2.1). Using a direct force computation, TMH17 found that the net torque on the binary is negative unless $t_{\text{sink}}$ is much smaller than $P_b$. The discrepancy between our results and those of TMH17 is important and deserves further study and comparison. For now, we offer the following comments: (1) In our simulations, we have achieved global mass transfer balance ($\langle M_i \rangle + \langle M_2 \rangle \approx \langle M_0 \rangle$; see Figures 1 and 2) and global angular momentum transfer balance ($\langle J_i \rangle \approx \langle J_0 \rangle$; see Figures 3 and 8); TMH17 did not demonstrate that they have achieved these. (2) The mass depletion prescription adopted by TMH17 may be problematic in terms of mass conservation. TMH17 found that the more “filled” the CSDs are (corresponding to larger $t_{\text{sink}}$), the more negative the net torque on the binary is (see their Figure 2); this is counterintuitive, as on physical grounds one expects that CSDs contribute a positive gravitational torque on the binary (Savonije et al. 1994). (3) It is unclear whether the definition and calculation of the accretion torque in TMH17 are consistent with angular momentum conservation (see their Equation (8)).

Turning to eccentric binaries, our paper represents the first work to compute the secular evolution rates of $a_0$ and $e_b$, consistently taking into account accretion via direct, long-term simulations. Most previous (analytic) works (such as Artymowicz & Lubow 1996; Lubow & Artymowicz 2000) assumed that accretion was negligible ($M_0 = 0$) and that $a_0$ and $e_b$ were solely due to gravitational Lindblad torques. Hayasaka (2009) carried out an analytic calculation including the effect of accretion coupled to the Lindblad torques from the CBD; in contrast to our new findings, he concluded that, for $a = h_0 = 0.1$, binaries migrate inward if they accrete at the same rate as the CBD.

5.2.2. Caveats and Limitations of Our Work

In this paper, we have “solved” the problem of circumbinary accretion in the “cleanest” scenario and in quasi-steady state, taking advantage of AREPO’s capability to resolve a wide dynamical range in space (from $7a_0$ down to $0.02a_0$). We should, however, bear in mind the limitations of our work when confronting real astrophysical applications.

Our simulations are two-dimensional and adopt a simple equation of state as well as the $\alpha$-viscosity prescription. Three-dimensional MHD simulations (Noble et al. 2012; Shi et al. 2012; Shi & Kroluk 2015) would represent the transport of angular momentum more realistically. In addition, general relativistic effects are bound to be important at close separations in the case of supermassive binary black holes (SMBBHs; Farris et al. 2012).

The value of $\alpha$ chosen (0.1) in our simulations is in part due to computational expediency. A larger viscosity results in a shorter viscous time—still much longer than all other relevant timescales—which allows us to reach the quasi-steady state in the inner region ($R \lesssim 10a_0$) of the CBD. Very small values of $\alpha$ would prevent us from attaining such a steady state. MML17 considered $\alpha = 0.1$ and 0.05 in their PLUTO simulations and found the same $l_0$ value for circular binaries. At very low viscosities, one can expect more pronounced spiral arms (in the CBD and CSDs), which would increase the tidal torques; at the same time, the tidal cavity surrounding the binary becomes larger and the CSDs are truncated at smaller radii (Artymowicz & Lubow 1994; Miranda & Lai 2015), thus removing the gas that would exert the strongest torques. These opposing effects may cancel each other out (at least for $\alpha = 0.05 - 0.1$ and $e_b = 0$), as suggested by the results of MML17. On the other hand, the smoothed particle hydrodynamics simulations by Ragusa et al. (2016) suggest that accretion may be suppressed when the viscosity is sufficiently small (small $\alpha$ or small $h_0$), although it is not clear if those simulations were run for a sufficiently long integration time. Exploring the dependence of our results on $\alpha$ and $h_0$ will be the subject of future work.

We have assumed a steady mass supply at a large distance, which may not exist under realistic astrophysical conditions. We expect that our general results should hold even if the external supply rate $\dot{M}_0$ varies in time, provided it does so smoothly and slowly, on a timescale longer than the accretion time at the outer edge of the CBD, $\tau_{\text{acc}} \approx R_{20}^2/\nu$. As several of the quantities measured in this work are scaled by $\dot{M}_0$, such as $\dot{J}_b$, $\dot{a}_b$, and $\dot{e}_b$, a slow change in the supply rate will translate into a proportional amplification or reduction of these quantities, without affecting their qualitative behavior. On the other hand, clear violations of the steady-supply condition—such as accretion via gas clumps (e.g., Goicovic et al. 2016) or from tori of small radial extent—might change the response of the binary.

We have focused on $q_b = 1$ in this paper, but the behavior of $q_b < 1$ binaries may be different. In particular, the phenomenon of alternating accretion described in ML16, attributed to apsidal precession in the CBD, has not been thoroughly explored for mass ratios different from unity. The value of $\langle \dot{q}_b \rangle$—and whether it averages to zero over secular timescales—is very important for properly determining $\langle J_b \rangle$ when $q_b \neq 1$ (Equations (2) and (4)). The sign of $\langle \dot{q}_b \rangle$ from CBD accretion is still an unresolved question, and recent work by Young & Clarke (2015) suggests that it depends on the aspect ratio of the disk (see also Clarke 2008). No grid-based simulation has explored the sign of $\langle \dot{q}_b \rangle$ while systematically varying both $q_b$ and $e_b$, and this will be the subject of future work.

Finally, we do not include self-gravity of the disk in our simulations, as we have implicitly assumed that the local disk mass, $R^2\dot{\Sigma}(R)$ (with $R \sim \dot{M}_0$), is much less than the binary mass. A massive, self-gravitating disk may qualitatively change the dynamics of circumbinary accretion studied in this paper (e.g., Cuadra et al. 2009; Roedig et al. 2012), although some disk fragmentation studies have already reported outward migration of protobinaries (Kratter et al. 2010a); see below.
5.2.3. Implications for Disk-assisted Binary Decay

Supermassive binary BHs: It is widely assumed that CBDs around SMBBHs extract angular momentum from such binaries, shrinking their orbital radii from ~1 pc to ~0.01 pc, before gravitational radiation can push the binary toward merger within a Hubble time (e.g., Begelman et al. 1980; Armitage & Natarajan 2002; MacFadyen & Milosavljević 2008; Rafikov 2016; Tang et al. 2017). Our findings contradict this assumption, thus putting into question the relevance of circumbinary gas as a solution to the “final parsec problem.” At least for equal-mass binary black holes and non-self-gravitating disks, our results suggest that circumbinary accretion may prevent rather than assist the merger of such objects.

Formation of stellar binaries: The finding that binaries embedded in accretion disks are able to gain angular momentum poses problems to binary formation theory. Of the proposed mechanisms for binary formation at intermediate separations (<10 au; e.g., Tohline 2002) the still-qualitative “fragmentation then migration” scenario has emerged as a strong contender over the past decade (e.g., Krumholz et al. 2009; Kratter et al. 2010a; Zhu et al. 2012). This mechanism is grounded in the difficulty for massive disks to fragment at distances smaller than 50–100 au from the central object (Matzner & Levin 2005; Rafikov 2005; Boley et al. 2006; Whitworth & Stamatellos 2006; Stamatellos & Whitworth 2008; Cossins et al. 2009; Kratter et al. 2010b) and thus requires binaries to reduce their separations by one or two orders of magnitude after they have formed (see Moe & Kratter 2018 for a recent analysis of binaries at small separations and their possible formation mechanisms). If binaries embedded in disks tend to expand, the viability of this mechanism would be questionable (e.g., Satsuka et al. 2017).

How can binaries still shrink?: Given the idealized nature of our simulations, we cannot rule out the possibility of binary shrinkage. We envision several possibilities where binary decay may occur:

(i) Unequal-mass binaries and locally massive disks. We have only explored binaries with $q_b = 1$ and small disk masses, but systems with $q_b < 1$ and disks with local mass $\sim \Sigma_0 a^2 > M_2$ could behave very differently (Cuadra et al. 2009; Roedig et al. 2012). In particular, when $q_b \ll 1$, the binary would evolve in a fashion similar to Type II migration (e.g., Lin & Papaloizou 1986; Nelson et al. 2000) or a modified version of it (e.g., Duffell et al. 2014; Dürmann & Kley 2015). Such massive disks may exist in the early stage immediately following galaxy merger (producing a massive gas torus surrounding an SMBH binary) or the early (class zero) stage of stellar binary formation.

(ii) Three-dimensional angular momentum loss effects. Our 2D calculations cannot capture effects that are intrinsically 3D, such as winds and outflows. MHD winds can take away angular momentum in accretion disks, and such a mechanism may operate in circumbinary accretion as well. The 3D ideal MHD simulations of Kuruwita et al. (2017) suggest that newly formed binaries could contract significantly within hundreds of orbits as they grow in mass by a factor of ~100, again implying that binary shrinkage may take place at a much earlier phase of binary formation than that explored in the present work.

D.J.M. thanks Volker Springel for making AREPO available in this work. We thank Kaitlin Kratter, Yoram Lithwick, Zsolt Regály, Pawel Artymowicz, and Zoltán Haiman for useful discussion and Luke Zoltan Kelley for comments on the manuscript. This work was supported in part by the NSF grant AST1715246 and NASA grant NNX14AP31G. This research was supported in part through the computational resources and staff contributions provided for the Quest high-performance computing facility at Northwestern University, which is jointly supported by the Office of the Provost, the Office for Research, and Northwestern University Information Technology.

ORCID iDs

Diego J. Muñoz © https://orcid.org/0000-0003-2186-234X
Ryan Miranda © https://orcid.org/0000-0002-2804-3657
Dong Lai © https://orcid.org/0000-0002-1934-6250

References

Armitage, P. J., & Natarajan, P. 2002, ApJL, 567, L9
Armitage, P. J., & Natarajan, P. 2005, ApJ, 634, 921
Artymowicz, P., & Lubow, S. H. 1996, ApJL, 461, L51
Bate, M. R., Bonnell, I. A., & Price, N. M. 1995, MNRAS, 277, 362
Begelman, M. C., Blandford, R. D., & Rees, M. J. 1980, Natur, 287, 307
Boley, A. C., Mejía, A. C., Durisen, R. H., et al. 2006, ApJ, 651, 517
Bonnell, I. A., & Bate, M. R. 1995, MNRAS, 279, L45
Bonell, I. A., & Bate, M. R. 2014b, MNRAS, 271, 999
Boss, A. P. 1986, ApJS, 62, 519
Brown, E. W. 1925, PNAS, 11, 274
Burns, J. A. 1976, AmJPh, 44, 944
Chang, P., Stubbe, L. E., Menou, K., & Quataert, E. 2010, MNRAS, 407, 2007
Chapon, D., Mayer, L., & Teysseier, R. 2013, MNRAS, 429, 3114
Clarke, C. J. 2008, in ASP Conf. Ser. 390, Pathways Through an Eclectic Universe, ed. J. H. Knapen, T. J. Mahoney, & A. Vazdekis (San Francisco, CA: ASP), 76
Cossins, P., Lodato, G., & Clarke, C. J. 2009, MNRAS, 393, 1157
Cuadra, J., Armitage, P. J., Alexander, R. D., & Begelman, M. C. 2009, MNRAS, 393, 1423
de Val-Borro, M., Edgar, R. G., Artymowicz, P., et al. 2006, MNRAS, 370, 529
de Val-Borro, M., Gamm, G. F., Stempels, H. C., & Pepliński, A. 2011, MNRAS, 413, 2679
D’Orazio, D. J., Haiman, Z., & MacFadyen, A. 2013, MNRAS, 436, 2997
Dotti, M., Colpi, M., Haardt, F., & Mayer, L. 2007, MNRAS, 379, 596
Duffell, P. C. 2016, ApJS, 226, 2
Duffell, P. C., Haiman, Z., MacFadyen, A. I., D’Orazio, D. J., & Farris, B. D. 2014, ApJL, 792, L10
Duffell, P. C., & MacFadyen, A. I. 2011, ApJS, 197, 15
Duffell, P. C., & MacFadyen, A. I. 2012, ApJ, 755, 7
Dunhill, A. C., Cuadra, J., & Dougados, C. 2015, MNRAS, 448, 3545
Dürmann, C., & Kley, W. 2015, A&A, 574, A52
Dutrey, A., Guilloteau, S., & Simon, M. 1994, A&A, 286, 149
Escala, A., Larson, R. B., Coppi, P. S., & Mardones, D. 2005, ApJ, 630, 152
Farris, B. D., Duffell, P., MacFadyen, A. I., & Haiman, Z. 2014, ApJ, 783, 134
Farris, B. D., Duffell, P., MacFadyen, A. I., & Haiman, Z. 2015, MNRAS, 447, L80
Farris, B. D., Gold, R., Paschalidis, V., Etienne, Z. B., & Shapiro, S. L. 2012, PPhTuL, 109, 221102
Goicoe, F. G., Cuadra, J., Sesana, A., et al. 2016, MNRAS, 455, 1989
Goldreich, P., & Tremaine, S. 1979, ApJ, 233, 857
Goldreich, P., & Tremaine, S. 1980, ApJ, 241, 425
Gould, A., & Rix, H.-W. 2000, ApJL, 532, L29
 Günther, R., & Kley, W. 2002, A&A, 387, 550
Hadjidemetriou, J. D. 1963, Icar, 2, 440
Haiman, Z., Kossis, B., & Menou, K. 2009, ApJ, 700, 1952
Hanawa, T., Ochi, Y., & Ando, K. 2010, ApJ, 708, 485
Hayasaki, K. 2009, PASJ, 61, 65
Hayasaki, K., Mineshige, S., & Sudou, H. 2007, PASJ, 59, 427
Jeans, J. H. 1925, MNRAS, 85, 912
