Accelerating Universe from Extra Spatial Dimension

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Abstract
We present a simple higher dimensional FRW type of model where the acceleration is apparently caused by the presence of the extra dimensions. Assuming an ansatz in the form of the deceleration parameter we get a class of solutions some of which shows the desirable feature of dimensional reduction as well as reasonably good physical properties of matter. Interestingly we do not have to invoke an extraneous scalar field or a cosmological constant to account for this acceleration. One argues that the terms containing the higher dimensional metric coefficients produces an extra negative pressure that apparently drives the inflation of the 4D space with an accelerating phase. It is further found that in line with the physical requirements our model admits of a decelerating phase in the early era along with an accelerating phase at present. Further the models asymptotically mimic a steady state type of universe although it starts from a big type of singularity. Correspondence to Wesson's induced matter theory is also briefly discussed and in line with it it is argued that the terms containing the higher dimensional metric coefficients apparently creates a negative pressure which drives the inflation of the 3-space with an accelerating phase.

KEYWORDS: Higher dimensions; Accelerated expansion; Deceleration parameter
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1. INTRODUCTION

One of the greatest challenges of modern cosmology is understanding the nature of the observed late-time acceleration of the universe. Recent measurements of type Ia Supernovae (SNIa) [1] at redshifts $z \sim 1$ and also the observational results coming from the cosmic microwave background radiation along with the Maxima [2] and Boomerang data [3] indicate that the expansion of the present universe is accelerated. In fact the present day results show that the supernovae look fainter than that expected from the luminosity redshift relationship in a decelerating universe. One is tempted to believe that in the...
universe there exists an important matter component which, in its most simple description, has the characteristic of a cosmological constant i.e., a vacuum energy density which contributes to a large component of a negative pressure. However, the inability of the particle physicists to compute the energy of the quantum vacuum - contributions from well understood physics amount to $10^{55}$ times the critical density - casts a dark shadow on the feasibility or otherwise of the presence of the constant. A rather important issue is the coincidence problem: dark energy seems to start dominating the energy budget and accelerating the expansion of the universe, just around the present time. To circumvent this difficulty and a host others an evolving cosmological constant or a time dependent scalar field termed *quintessence* with an appropriate self interaction potential $V(\phi)$ are introduced in the theory. The apparent proliferation of quintessence dominated models is due to the fact that it is trivial[4] to chose the 'appropriate' potential $V(\phi)$ such that one can always explain the observations for any given pair of scale factor and matter density. A number of other candidates have also been proposed: Brans-Dicke type of scalar fields [6] and a network of frustrated topological defects, to name a few. As the large scale structure and CMBR observations referred to earlier suggest that the universe is spatially flat, with matter density equal to about one third of the critical density the idea of a dark energy component is gaining momentum. While these and other models have some motivation and also attractive features none of them are compelling because of the fact that the cosmological constant or an evolving quintessence energy require extremely small number to fit the data: the present value of the energy density, $\rho_c \sim 10^{-12} eV$, and in the case of quintessence, the tiny mass smaller than the current Hubble parameter $m_Q < H_0 \sim 10^{-33} eV$ and sub-gravitational couplings to visible matter to satisfy fifth force constraints [7]. In view of the fact that the observational data coming from the supernova studies probe length scales $l \sim H_0^{-1} \sim 10^3 Mpc$, which are so far inaccessible to any particle physics experiments, various alternative proposals [8] have cropped up time to time to account for the current accelerated expansion without assuming any form of dark energy. They include phantom energy[9], certain modification of GR [10], Chaplygin gas as also its variable form [11]. Very relevant to mention that Vishwakarma [12] in a series of works took a completely opposite view to argue that it is possible to explain the dimming of supernovae within the framework of of the conventional decelerating model itself. Another line of argument is put forward by to suggest [13] that the radiation coming from the SNe are partially absorbed due to encounter by an obstacle. But observationally there is no frequency dependence of the dimming of the supernovae and so the mechanism must be very achromatic which, in turn, seems to rule out a medium of material particles. Because they may absorb light in the optical spectrum but will re-emit it in IR, affecting the CMBR in unacceptable ways. The dimming of the supernova is also sought to be explained on the basis of the so called *flavor oscillation* (see reference 7) so that light travelling through inter galactic magnetic fields can partially be converted into axions, and evade detection on earth. So the source would appear fainter and hence more distant than it is actually is although the Universe is not accelerating. In view of what has been stated above we have thought it worthwhile to incorporate the phenomenon of accelerating universe in the framework of higher dimensional spacetime itself. Here we have examined a scenario in multidimensional spacetime where the accelerated expansion of the universe at the current epoch may be made possible without forcing ourselves to
invoke any time dependent extraneous scalar field or vacuum energy. We have here taken a five dimensional spatially flat, homogeneous spacetime with perfect fluid as matter field and assuming a specific form of the deceleration parameter we have been able to show that the universe decelerates at the early era (a good news for structure formation) and after a certain instant starts accelerating in conformity with the present day observations. Higher dimensional spacetime is now an active field of activity in both general relativity and particle physics in its attempts to unify gravity with all other forces of nature [14]. These theories include kaluza-klein, induced matter, super string, supergravity and string. In these (4+d) dimensional models the d-spacelike dimensions are generally spontaneously compactified and the symmetries of this space appear as gauge symmetries of the of the effective 4D theory. At present these extra dimensions are not observed presumably because with time they shrink to an unobservably small length, say plankian. However standard cosmology indicates that the scale factor for the extra dimensions at some epoch in the past could have been comparable with or even larger than,that of the usual three dimensional space. Renewed interests to these models also stem from their recent applications to brane- cosmology. Our paper is organised as follows: After Introduction in section 1 we discussed the mathematical formalism and its implications in section 2 where we have, for simplicity, discussed two solutions from the class of solutions obtained. Interesting to point out that depending on the choice of the arbitrary constants our solutions exhibit the desirable property of dimensional reduction. Further our cosmology assumes a steady state type behaviour with time although preserving the big bang type singularity. We argue that the late acceleration may be explained on the basis of Wesson’s induced matter theory according to which the extra dimension creates an effective 4D pressure, which in the present case is incidentally negative. The paper ends with a short discussion in section 3.

2. MATHEMATICAL FORMALISM

We here discuss a spatially flat 5D homogeneous cosmological model with the topology $M^1 \times R^3 \times S^1$ where $S^1$ is taken in the form of a circle such that

$$ds^2 = dt^2 - R^2(t) \left( dr^2 + r^2 d\theta^2 + r^2 \sin^2 \theta d\phi^2 \right) - \Phi^2(t) dy^2$$

(1)

where $R(t)$ is the scale factor for the 3D space and $\Phi(t)$, that for the extra dimension and $y$ is the fifth dimensional coordinate. The metric (1) admits a number of isometries. For the case $R^1 X S^3 X S^1$ the symmetry group of the group of spatial section is $O(4) X O(2)$. The stress tensor whose form will be dictated by Einstein’s equations must have the same invariance. Therefore the energy momentum tensor may be written as follows [15]

$$T_{00} = \rho(t), T_{ij} = p(t) g_{ij}, T_{55} = p_5 g_{55}$$

where the rest of the components vanish. The conservation of the energy-momentum tensor $T^{ab}, b = 0$ yields

$$\dot{\rho} + (p + \rho) \frac{\dot{R}}{R} + (p_5 + \rho) \frac{\dot{\Phi}}{\Phi} = 0$$

(2)
where $p_5$ is the pressure in the fifth dimension while that for the 3D space is isotropic and is given by $p$

The independent field equations for the metric (1) and matter field (2) are given by

$$\frac{3\ddot{R}^2}{R^2} + \frac{3\dot{R}}{R} \frac{\dot{\Phi}}{\Phi} = \rho$$  \hspace{1cm} (3)$$

$$2\frac{\ddot{R}}{R} + \frac{\dot{R}^2}{R^2} + 2\frac{\dot{R}}{R} \frac{\dot{\Phi}}{\Phi} + \frac{\ddot{\Phi}}{\Phi} = -p$$  \hspace{1cm} (4)$$

$$3\frac{\ddot{R}}{R} + 3\frac{\dot{R}^2}{R^2} = -p_5$$  \hspace{1cm} (5)$$

As we have five unknowns ($R$, $\Phi$, $\rho$, $p$ and $p_5$) with three independent equations we are at liberty to choose two connecting relations among them. We assume that $p = p_5$ so the pressure is isotropic including the extra space. From this condition we get from the field equations

$$\ddot{\Phi} + 2\frac{\dot{\Phi}}{\dot{R}} \frac{\dot{R}}{R} - \Phi\left(\frac{\ddot{R}}{R} + 2\frac{\dot{R}^2}{R^2}\right) = 0$$  \hspace{1cm} (6)$$

A little inspection shows that $\Phi = R$ is a solution of this equation, which makes the spacetime isotropic. For a more general solution we substitute $\Phi = Ru(t)$ such that

$$R\ddot{u} + 4\dot{R}\dot{u} = 0$$  \hspace{1cm} (7)$$

or $\dot{u} = \frac{\beta}{R^2}$ where $\beta$ is an arbitrary constant. As we have another choice we make the ansatz that the deceleration parameter,

$$q = -\frac{\dot{R}^2}{R^2}$$  \hspace{1cm} (8)$$

is given by

$$q = \frac{a - R^m}{b + R^m}$$  \hspace{1cm} (9)$$

where $m$ is any arbitrary constant. In taking this particular form of the expression we are guided by the consideration that the deceleration parameter should be amenable to both decelerated expansion at the early era and accelerated expansion at the current era to account for both structure formation as well as for interpretation of the current data from the supernova. Naturally both the constants $a$ and $b$ should be positive. The expression

$$\frac{\ddot{R}}{R} = \frac{a - R^m}{b + R^m} \left(\frac{\dot{R}}{R}\right)$$  \hspace{1cm} (10)$$

offers a first integral as

$$\dot{R} = (R^m + b)^2 R^{2 - \frac{2m}{n}}$$  \hspace{1cm} (11)$$

A little inspection shows that the expression $R = \sinh^{2/m} \omega t$, subject to the restrictions that $b = 1$ and $a = \frac{m^2}{2}$ will be a solution of this equation. For convenience we take $2/m = n$, which finally reduces $q$ to

$$q = \frac{(1 - n) - n \sinh^2 \omega t}{n \cosh^2 \omega t}$$  \hspace{1cm} (12)$$
The nature of variation of $q$ depends critically on the value of the exponent $n$, which in turn, determines the scale factor $R$. For $n > 1$ we get only accelerating model. However for $n < 1$ the desirable phenomenon of flip occurs. An initially decelerating model starts accelerating after a certain instant. But it is not obvious from our analysis at what value of redshift this flip occurs. We here consider only two cases corresponding to $n = \frac{1}{2}$ and $\frac{1}{4}$. As we are considering an expanding model any negative value of $n$ is ruled out. Skipping intermediate mathematical steps for economy of space we give the final expressions as follows:

**CASE I** ($n = \frac{1}{2}$)

In this case we get

$$\Phi = \sinh^{\frac{1}{2}}\omega t(\gamma - \beta \coth \omega t) \quad (13)$$

$$R = R_0 \sinh^{\frac{1}{2}}\omega t \quad (14)$$

With the above form of the metric coefficients the matter field reduce to

$$p = p_5 = -\frac{3\omega^2}{2} \quad (15)$$

$$\rho = \frac{3 \cosh^2 \omega t}{2\sinh^2 \omega t} + \frac{3\beta \cosh \omega t}{2\sinh^3 \omega t(\gamma - \beta \coth t)} \quad (16)$$

Interestingly as $t \sim 0$, $\rho = \rho_0 = \frac{3\omega^2}{2}$ and as $t \sim \infty$, $\rho_\infty = \frac{3\omega^2}{2}$. So asymptotically the mass density tends to assume a constant value and the cosmology mimics a steady state type of behaviour though not exactly following the type advocated by Bondi and Gold or Hoyle and Narlikar. With time the density assumes a constant value as $\rho_\infty = \frac{3\omega^2}{2}$. Further as $t \sim 0$, $R \sim t^{1/2}$ and $A \sim t^{-1/2}$ So at the early era the spacetime resembles the well known form given by Chodos and Detweiler [16]. The temporal behaviour of the model depends critically on the initial conditions. If the arbitrary constant $\gamma$ is made zero the extra dimension starts from an infinite extension, shrinks to a minimum and then expands again indefinitely so that there is no dimensional reduction in this case. However the large extra dimensions in the theory are not that much out of favour these days as it attempts to address the well known hierarchical problem in quantum field theory. Further with time the model isotropises in this case and the 4D volume expands. On the other hand with non vanishing $\gamma$ the extra dimension contracts and ultimately vanishes exhibiting the desirable feature of dimensional reduction. In this case the model again becomes singular. Theorists try to save the situation via assuming varied stabilising mechanisms [17] like quantum gravity, Casimir effect etc. so that they produce a sort of repulsive potential to halt the shrinkage at a very small constant value, say Planckian length. At this stage the extra dimensions essentially decouple as all its derivatives vanish in the field equations. The cosmology now enters the standard 4D phase following the FRW model without having any reference to the extra dimensions.

A serious shortcoming of this analysis is that there is no dynamical evolution in the expression for pressure. We shall shortly see that this defect is overcome in our next section.

**CASE II** ($n = \frac{1}{4}$)
Skipping the intermediate mathematical steps like case I we write the final results as

\[ R = R_0 \sinh \frac{1}{4} \omega t \] (17)

\[ \Phi = \sinh \frac{1}{4} \omega t (\beta \ln \tanh \frac{\omega t}{2} + \gamma) \] (18)

With the above values of the R and Φ we get the following expressions for the pressure matter density as

\[ p = p_5 = -\frac{3\omega^2}{8\sinh^2 \omega t} (\sinh^2 \omega t - 1) \] (19)

\[ \rho = \frac{3\omega^2 \cosh \omega t}{8 \sinh^2 \omega t} (\cosh \omega t + \frac{2\beta}{\beta \ln \tanh \omega t + \gamma}) \] (20)

As commented earlier this model does not suffer from the disqualification of a constant pressure. Here both pressure and mass density are evolving and start from an infinite value as a big bang singularity. But it has not escaped our notice that both the physical quantities assume steady values asymptotically at \( \frac{3\omega^2}{8} \). So unlike the big bang type it resembles more a steady state type cosmology. But the pressure changes signature from positive to negative at \( \sinh \omega t = 1 \). However it is not difficult to explain the asymptotically steady value of the matter field because a little algebra shows that with time the 4D volume,

\[ V = R^3 \Phi = \sinh \omega t (\beta \ln \tanh \omega t + \gamma) \] (21)

stabilises at some finite value. Another striking difference from the earlier case is that here both the scales start from zero and depending on the signature of the arbitrary constant \( \gamma \) the fifth dimension either expands indefinitely or collapses at a finite time.

**ACCELERATED UNIVERSE**

As commented in our introduction this model admits of both deceleration at the early phase and acceleration at present. The early deceleration is physically relevant in the sense that it allows structure formation to take place while the present day acceleration is in conformity with the current observations. For the general case \( R = \sinh^n \omega t \), we get for deceleration parameter \( q = -\frac{\ddot{R}R}{\dot{R}^2} = -\frac{\cosh^2 \omega t - 1}{\cosh^2 \omega t} \). Thus for \( n \geq 1 \), \( q < 0 \). Hence always accelerating.

For case I, \( n = \frac{1}{2} \) and \( q = \frac{1 - \sinh^2 \omega t}{1 + \sinh^2 \omega t} \). Let \( \sinh \omega t_c = 1 \). Hence for \( t < t_c \), \( q > 0 \) (deceleration) and for \( t > t_c \), \( q < 0 \) (acceleration).

For case II, \( n = \frac{1}{4} \) and \( q = \frac{3 - \sinh^2 \omega t}{\cosh^2 \omega t} \). Let \( \sinh \omega t_c = \sqrt{3} \). Hence for \( t < t_c \), \( q > 0 \) (deceleration) and for \( t > t_c \), \( q < 0 \) (acceleration).

Hence acceleration starts later in the second case.

Before ending the section a final remark may be in order. For example if we take the case II it follows that the acceleration starts when \( \sinh \omega t > \sqrt{3} \). On the other hand the
pressure starts becoming negative at \( \sinh \omega t > 1 \). Evidently acceleration of the universe starts later than the instant when the pressure becomes negative. One can argue that the fact that the pressure becomes negative does not guarantee the present acceleration of the universe. For this to happen it has to dominate long enough to overcome the gravitational attraction produced by ordinary matter.

**WESSON’S FORMALISM**

At this stage one may digress a little and call attention to the *Induced Matter Theory* recently developed and formulated by Wesson [18] according to which it is possible to interpret most properties of matter as a result of 5D Riemannian geometry. Accordingly the 5D field equations for the apparent vacuum for Einstein tensor are given by

\[
G_{AB} = 0 \tag{22}
\]

By contrast the 4D equations for Einstein’s equations are given by

\[
G_{ij} = T_{ij} \tag{23}
\]

where \( A, B \) run from 0 to 4 where as \( i, j \) from 0 to 3. The central idea of the induced matter theory (see Wesson, p.42 for more details) is that the equations (23) are a subset of (22) with an effective or induced energy momentum tensor \( T_{ij} \) which contain the classical properties of matter. It follows from the theorem of Cambell that any analytic \( N \) dimensional Riemannian manifold can be locally embedded in an \((N+1)D\) Ricci-flat Riemannian manifold [19]. Though not exactly similar (we are here not dealing with a 5D vacuum) we can collect the terms containing the fifth dimension from the expression of pressure to call them \( p_\phi \) such that from the equation (4) it follows that

\[
p_\phi = -\frac{\ddot{\Phi}}{\Phi} - 2 \frac{\dot{R}}{R} \frac{\dot{\Phi}}{\Phi} \tag{24}
\]

With our solution for \( n = 1/4 \) the above equation yields

\[
p_\phi = \frac{1 - 3 \sinh^2 \omega t}{16 \sinh^2 \omega t} \tag{25}
\]

So long as \( 1 > 3 \sinh^2 \omega t \) the fifth dimension induced pressure is positive which, in turn, makes the deceleration parameter, \( q \) is positive but soon after \( p_\phi < 0 \) and this apparently drives the acceleration of the universe making the 3D deceleration parameter finally negative.

**3.DISCUSSION**

While vast literature exists to address the observational fact of the current accelerated expansion of the universe we are not aware of models of similar kind in the framework of higher dimensional spacetime. We have here discussed a scenario in homogeneous 5D spacetime which admits a decelerating expansion in the early epoch along with an
accelerated phase at present in line with the current observational results. The most important finding, in our opinion is the result that it is possible to achieve this acceleration without introducing any external quintessence-like scalar field or vacuum energy into the theory—the presence of the extra dimension, so to say, seems to cause the expansion to accelerate. One can naively attempt to interpret the result as follows: It is well known that a higher dimensional spacetime with a Ricci-flat extra dimension is equivalent to an effective 4D theory with an extra massless scalar field, which may be instrumental in driving the acceleration. In this context one may also call attention to Wesson’s induced matter theory to argue that negative induced pressure is responsible for the accelerated expansion of the universe. Another desirable feature is the phenomenon of dimensional reduction so that the model finally reduces to an effective 4D one. This takes place in both the cases discussed here. Although in both the cases the cosmology starts from an initial big bang type of singularity the matter field asymptotically becomes steady pointing to a steady state like model which is at variance with the standard FRW type of models. This may be of some interests because we do not have to hypothesise concepts like the so-called ‘matter creation from nothing’ or any type of ‘creation field’. To sum up a last remark may be in order. The first case suffers from the disqualification that the pressure is always a constant—it has no dynamical evolution. This defect is, however, not present in the second case. But the most serious shortcoming is the absence of any mechanism to achieve the stabilisation of the extra dimension at a very short length. It is not apparent from our analysis how that stabilisation work in our model. However in an earlier work Guendelman and Kaganovich [20] studied the Wheeler-Dewitt equation in presence of a negative cosmological constant and dust. They got the interesting result that the quantum effects do stabilise the volume of the universe, thus providing a mechanism of quantum avoidance of the singularity. It is also shown (see reference 10) that if one one starts with more than one extra dimension that the extra space may generate a repulsive potential to halt the indefinite shrinkage.

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