Spatial and Temporal Analysis of Direct Communications From Static Devices to Mobile Vehicles

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Abstract—This paper proposes a framework to analyze a wireless architecture where vehicles collect data from devices. Roads and vehicles are modeled by a Poisson line process and a Cox point process, respectively. At any given time, each vehicle is assumed to communicate with a roadside device in a disk of radius $r$ centered at the vehicle, which is referred to as the coverage disk. We study these direct communications from roadside devices to vehicles by investigating the network performance in both space and time domains. For the space domain analysis, we explicitly derive the signal-to-interference ratio distribution of the typical vehicle and the area spectral efficiency of the proposed network. For the time domain analysis, we characterize the evolution of the area fraction of the coverage disks over time and then evaluate the minimum association delay for an arbitrarily located roadside device to be covered by a disk. Leveraging the derived network performance, we investigate the optimization of network utility functions given by linear combinations of the performance metrics.

Index Terms—Device-to-device, vehicular networking, stochastic geometry, Cox point process, vehicle-to-all.

I. INTRODUCTION

A. Motivation and Background

This paper studies an emerging wireless architecture where devices collect data and passing-by vehicles harvest their data. The idea of using vehicles as key network components—like base stations or access points—is widely investigated in both industry [2]–[4] and academia [5]–[8]. Concrete examples of such networks range from ad hoc networking (where public transit vehicles provide large-scale Internet connectivity for pedestrians [9]), to vehicular-to-all—or equivalently device-to-device—networks (where pedestrians’ mobile devices send safety information to nearby vehicles [10], [11]), and to Internet-of-Things (IoT) networks (in which roadside sensors opportunistically forward their data toward nearby vehicles [6], [8]). More recently, device-to-device communications have been proposed as an effective way of offloading cellular data traffic [12]. These examples share the same basic structure where the data devices are distributed in space and vehicles directly communicate with nearby devices.

To this end, this paper proposes a network model based on such direct communications and analyzes its performance. Specifically, this paper develops a stochastic geometry model for this architecture based on various mathematical tools of this theory [13], [14]. The analysis presented in this paper not only sheds light on the performance of the proposed network but also provides a framework to quantify the potential of general network architectures leveraging vehicles.

B. Related Work

The proposed network architecture is an example of random mobile ad hoc network or device-to-device network in the sense that it aims at expanding the limited coverage of infrastructure or enabling high-speed and low-distance communication between devices, without infrastructure [15]–[22]. The performance of these networks has been studied extensively, with some studies using stochastic geometry to model the random locations of network components [23]–[26]. For instance, the homogeneous planar Poisson point process has been widely used for its analytical tractability [25], [27]. Specifically, under the Palm distribution of the Poisson point process [28], the distribution of the signal-to-interference-plus-noise ratio (SINR) of a typical user and the network area spectral efficiency were derived in [24], [29], [30].

However, modeling the locations of vehicles as a planar Poisson point process is inaccurate since in the planar Poisson point process [31], almost surely, no more than two points can be found on a line, and yet the locations of vehicles exhibit linear patterns, e.g., when they are on the same straight road. In order to address the location dependencies, a stochastic geometry Cox model was proposed in [32], where roads and vehicles are conditionally generated in the Euclidean plane.

More recently, this model was further studied in [33]–[35] to derive the signal-to-interference ratio (SIR) distribution of various links between vehicles and mobiles on the plane. These papers analyzed the typical network performance by considering an instantaneous snapshot of the network geometry, under

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the Palm distribution of the vehicle point process. This paper
uses the same approach to characterize short-term performance
properties such as the distribution of the SIR and the area
spectral efficiency.

On the other hand, since vehicles are assumed to cover a
wide area as they move on roads, it is essential to analyze
the network behavior over time. This paper uses the theory of
random closed sets [14], [36] to analyze the area fractions
of the coverage disks and the progress of coverage over
time, respectively. In addition, as in the literature on delay-
tolerant networks [37]–[41] or on random networks with data
mules [42]–[44], the proposed network incurs delay for link
association. This association delay may be substantial when
the density of vehicles is small or the speed of vehicle is
low. To quantify this association delay, the paper derives the
distribution of the shortest time required for a typical roadside
device to be covered by any vehicle, or equivalently any disk.

C. Contributions

1) Modeling of the Proposed Network: The paper considers
a generic network architecture where data devices commu-
nicate with vehicles on roads. A Poisson line process and
conditional linear Poisson point processes on each line model
the road network and the vehicles on the roads, respectively.
Then, the roadside data devices are assumed to be located in
coverage disks of radius \( \nu \) centered at each vehicle. Vehicles
are assumed to move along the lines of the Poisson line process
at a constant speed and to collect data from various devices
as they move.

2) Space Domain Network Analysis: The space domain
network performance is analyzed by considering a snapshot
of the proposed stochastic geometry model. To be specific,
using the Palm distribution of the vehicle point process and
assuming rich scattering and a general power-law path loss,
we obtain integral formulas for the interference distribution
at a typical vehicle and the SIR coverage probability of the
typical vehicle. We derive the ergodic throughput of the typical
link and obtain an integral expression for the area spectral
efficiency.

3) Time Domain Network Analysis: The time domain
network performance is investigated by quantifying the evolution
of the coverage disks with respect to (w.r.t.) time. The evolution
of the coverage disks is characterized as the Minkowski
sum [36] of the trajectories of vehicles on lines and the
coverage disk. Using the stationarity of the associated random
closed sets and the capacity functional formula, we explicitly
derive the area fraction of the coverage disks and also charac-
terize how it varies w.r.t. time. The behavior of the coverage
disks illustrates how the data from roadside devices can be
harvested by vehicles over time. In addition, conditionally on
the fact that a typical roadside device is eventually covered
by a vehicle, the network association delay is derived as the
link association time, i.e., the amount of time for this device
to be covered by a vehicle for the first time. Its distribution is
derived.

4) Trade-Off Relationship and Optimization: We identify
trade-off relationships between the network’s short-term
behavior—characterized by the space domain analysis—
and long-term behavior—characterized by the time domain
analysis. For instance, when the coverage disk radius
increases, the long-term performance metrics, (e.g., area
fraction and network association delay) improve while the
short-term performance metrics (e.g., coverage probability
and ergodic throughput) worsen. To shed light on the potential
of the proposed architecture, we consider utility functions that
incorporate such key metrics and we optimize them w.r.t.,
e.g., the coverage disk radius.

D. Organization

Section II contains the system model of the proposed
architecture. Section III provides the space domain analysis
and Section IV provides the time domain analysis. Section V
presents various trade-offs and the optimization of network
utility. Section VI concludes the paper.

II. SYSTEM MODEL

This section introduces the spatial model for the devices,
vehicles, and coverage disks. Then, the channel model and
performance metrics are discussed.

A. Spatial Model

We model the road network using a stationary and isotropic
Poisson line process \( \Phi_l \) with intensity \( \lambda_l/\pi \) [14]. The Poisson
line process is generated as follows: a Poisson point process
with intensity \( \lambda_l/\pi \) is generated on the cylinder \( C : \mathbb{R} \times (0, \pi) \).
Then, each point of the Poisson point process, say \((r, \theta)\),
produces an undirected line in \( \mathbb{R}^2 \) where \( r \) is the algebraic
displacement from the origin to the line and \( \theta \) is the angle of
the normal vector to the line.

As other stationary point processes, such as the homoge-
neous Poisson point process [28], the stationary structure of
the line process allows one to analyze the network perform-
ance seen by a typical point [1], [34], [35], [45], which
will also give the network performance spatially-averaged
over all points in a large ball [28]. Furthermore, the model
based on the Poisson line process [45] can reproduce various
road topologies by varying \( \lambda_l \) and the distribution of \( \theta \); for
instance, the Poisson line process can model Manhattan-like
road structures by restricting the value of \( \theta \) to be either
0 or \( \pi/2 \) [14]. In this paper, the analysis will be focused on
the isotropic Poisson line process, where \( \theta \) is uniformly
distributed on \((0, \pi)\).

Conditionally on the line process \( \Phi_l \), the locations of
vehicles on each road are modeled by an independent one-
dimensional Poisson point process with intensity \( \mu \) on each
line. The collection of vehicles on the line process hence
forms a Poisson line Cox point process \( \Phi \) [45]. The Cox point
process is stationary and its spatial density, or equivalently its
intensity, is equal to \( \lambda_l \mu / \pi \) [45].

In order to describe the motion of vehicles, we consider a
simple dynamic. Each vehicle is assumed to move at a constant
speed \( v \) along the line on which it is located. At time \( 0 \), the
direction of motion is randomly determined by an independent
coin tossing at each vehicle. Once it is determined, each vehicle maintains its initial direction and speed. Let \( \Phi(t) \) denote the locations of vehicles at time \( t \) and \( \Phi \) denote \( \Phi(0) \). This Poisson line structure gives a simple and tractable parametric way to represent the dynamics of vehicles.

### B. Roadside Devices and Coverage Disks

Roadside devices are assumed to be Poisson distributed with intensity \( \lambda_d \) on the plane. Let \( \nu \) denote the maximum transmission range of direct communications from roadside devices. Equivalently, the area where roadside devices can communicate with a vehicle can be characterized by a disk of radius \( \nu \) centered at the vehicle. Let us denote the collection of such coverage disks centered on vehicles by

\[
S(t) = \bigcup_{X \in \Phi(t)} B_X(\nu),
\]

where \( B_X(\nu) \) denotes the disk of radius \( \nu \) centered at \( X \). Since vehicles change their locations over time, so do the coverage disks. Note that these disks might overlap in space.

In this paper, we assume time is slotted and each time slot is sufficiently small so that the displacement of the vehicles during a time slot is negligible. At every time slot, each vehicle selects one of its roadside users at random for data harvesting. When the density of roadside devices is large, the mean number of roadside devices per disk is also large: \( \lambda_d \gg \lambda \mu \) and \( \lambda_d \pi \nu^2 \gg 1 \). Thus, the probability that a given roadside is selected by more than two vehicles due to disk overlap is small. In addition, the probability that a disk contains no device is small. As a result, when the density of roadside devices is high, the locations of roadside devices that vehicles select are approximately uniformly and independently distributed inside the disks of radius \( \nu \) centered at the vehicles. In the remainder of this paper, we refer to these selected roadside devices as devices. The point process of devices in a snapshot at time \( t \) can hence be represented as

\[
\Xi(t) = \sum_{X_i \in \Phi(t), U_i \in \text{Uniform}(B_0(\nu))} \delta_{X_i + U_i},
\]

where \( \delta_x \) denotes the Dirac measure at \( x \) and \( \{U_i\}_{i \in \mathbb{Z}} \) is a sequence of independent and identically distributed (i.i.d.) vectors uniformly distributed in the disk \( B_0(\nu) \).

Fig. 1 illustrates the vehicle point process, the coverage disks, and the roadside devices. We consider \( \lambda_l = 3/\text{km} \), \( \mu = 3/\text{km} \), and \( \nu = 100 \text{m} \).

### C. Propagation Model

To investigate the basic performance of the proposed network architecture, we assume a classical power law path loss model with Rayleigh fading. The received power at distance \( d \) is \( pHd^{-\alpha} \), with \( p \) the device transmit power, \( H \) the power of the Rayleigh fade (that is assumed to be following an exponential distribution with mean one), and \( \alpha \) the path loss exponent (\( \alpha > 2 \)). Note that the techniques provided in this paper can be extended to other fading or path loss models. This is left for future work.

### D. Performance Metrics

#### 1) Space Domain

We focus on a typical link, namely a link from a roadside device to the typical vehicle at the origin. The Palm distribution of the vehicle point process is used, with the typical vehicle at the origin. The network performance seen by the typical vehicle coincides with the performance averaged over all vehicle in a large ball in the proposed stationary ergodic framework [28]. We compute the coverage probability of the typical vehicle and then the area spectral efficiency.

The coverage probability is the probability that the SIR of the typical vehicle be greater than some threshold \( \tau \), i.e., the complementary cumulative distribution function (c.c.d.f.) of the SIR random variable. Using the notation in Eq. (2), the coverage probability, \( p_c(\tau) \), is defined by

\[
P^0_\Phi(\text{SIR} \geq \tau) = P^0_\Phi \left( \frac{pH||U_0||^{-\alpha}}{\sum_{i \in \mathbb{Z} : 0 < pH||X_i + U_i||^{-\alpha} \geq \tau} \right),
\]

where \( P^0_\Phi \) denotes the Palm distribution w.r.t. the vehicle point process \( \Phi \). The above expression can be interpreted as the fraction of links with SIRs greater than threshold \( \tau \).

Using the Shannon formula, the area spectral efficiency is defined by

\[
\text{ASE} = \frac{\lambda_l \mu}{\pi} \mathbb{E}_\Phi \left[ \log_2(1 + \text{SIR}) \right],
\]

where the expectation is w.r.t. the Palm distribution \( \Phi \) and SIR is the same as in Eq. (2).

#### 2) Time Domain

We consider the mean area fraction of the coverage disks and the minimum association delay. Both metrics are related to the evolution of the coverage disks. The shaded region in Fig. 1 illustrates the union of the coverage disks. The area fraction of the coverage disks is defined by

\[
\text{AF}(S(t)) = \lim_{r \to \infty} \frac{E[\ell_2(S(t) \cap B_0(r))]}{\ell_2(B_0(r))},
\]

where \( \ell_2(A) \) denotes the area of set \( A \). The mean area fraction of the stationary random set \( A \) is also the probability that the
origin lies in $A$ [46]. Moreover, the union of all coverage disks between time 0 and time $t$ is given by

$$S(t) = \bigcup_{0 \leq \xi \leq t} S(\xi) = \bigcup_{0 \leq \xi \leq t} \left( \bigcup_{x \in \Phi(\xi)} B_x(\nu) \right).$$  \hfill (5)

Similarly, the area fraction of the set $\hat{S}(t)$ is defined by

$$AF(\hat{S}(t)) = \lim_{r \to \infty} \frac{\mathbb{E}[\ell_2(\hat{S}(t) \cap B_0(r))]}{\ell_2(B_0(r))}. \hfill (6)$$

We define the network association delay as the minimum time required for an arbitrarily located roadside device to be located within a coverage disk, namely to be able to communicate with a vehicle. The minimum association delay $W$ is defined conditionally on the fact that a typical roadside device is in $S(\infty)$ as follows:

$$W = \inf_{\tau > 0} \{ \tau \text{ such that } \hat{S}(\tau) \cap 0 \neq \emptyset \mid 0 \in S(\infty) \}. \hfill (7)$$

This paper analyzes on the distribution of the minimum association time.

**Remark 1**: We use the terminology “short-term” and “long-term” to refer to the performance metrics obtained from the space domain and time domain analysis, respectively. The short-term behavior is characterized by the performance metrics that are obtained by the analysis of a snapshot of the network geometry under the stationary framework proposed in this paper. In contrast, the long-term behavior concerns the performance metrics that are obtained by analyzing the evolution of the network elements—in particular, the coverage disks—over time.

### III. Performance Analysis in the Space Domain

This section focuses on the space domain using the instantaneous layout of the network geometry. The space domain metrics capture the short-term behavior of the proposed architecture.

#### A. Interference

Roadside devices directly communicate with vehicles and they are uniformly located in the coverage disks centered at vehicles. Hence, we focus on the distribution of the interference power measured at vehicles. Specifically, we derive the interference under the Palm distribution, seen by the typical vehicle at the origin.

**Lemma 1**: The Laplace transform of the interference at the typical vehicle, $\mathcal{L}_I(s)$, is given by Eq. (8).

**Proof**: We consider a typical vehicle located at the origin. Consequently, there exists a line containing the origin [45]; we will refer to it as the typical line. We denote by $\phi(0)$ the linear Poisson point process with intensity $\mu$ on the typical line. Then, under the Palm distribution of the vehicle point process, the interference can be decomposed as follows:

$$I = \sum_{X_i \in \Phi} pH\|X_i + U_i\|^{-\alpha} + \sum_{X_i \in \phi(0)} pH\|X_i + U_i\|^{-\alpha},$$

where $I_2$ denotes the interference from all devices activated by vehicles on the typical line and $I_1$ accounts for the interference from all devices activated by vehicles on the rest of the lines. Due to Slivnyak’s theorem applied to the Poisson point process on the cylinder set, the points on the typical line—considered under the Palm distribution of the vehicle point process—and the other points on the rest of the lines are independent [45]. Consequently, the random variables $I_1$ and $I_2$ are independent and the Laplace transform of the interference is given by

$$\mathcal{L}_I(s) = \mathbb{E}_\Phi [\exp(-sI)] = \mathbb{E}_{\ell_1} [\exp(-sI_1)] \mathbb{E}_{\ell_2} [\exp(-sI_2)].$$

To begin with, $\mathcal{L}_{I_1}(s)$ is given by

$$\mathbb{E}_\Phi \left[ \prod_{X_i \in \Phi} \mathbb{E}_U \left[ e^{-spH\|X_i + U\|^{-\alpha}} \Phi, U \right] \right] \overset{(a)}{=} \mathbb{E}_\Phi \left[ \prod_{X_i \in \Phi} \mathbb{E}_U \left[ \frac{1}{1 + sp\|X_i + U\|^{-\alpha}} \Phi \right] \right] \overset{(b)}{=} \mathbb{E}_\Phi \left[ \prod_{X_i \in \Phi} \frac{1}{\ell_2(B_0(\nu))} \int_{B_0(\nu)} \frac{du}{1 + sp\|X_i + u\|^{-\alpha}} \right]. \hfill (9)$$

In order to obtain (a), we use the fact that the locations of devices are given by $Y_i = X_i + U_i$, where $\{U_i\}$ denote i.i.d. uniform vectors in the disk $B_0(r)$.

Then, the locations of all points of $\Phi$ can be denoted as

$$X_i = \tau_j \rho_j^i + \kappa_j \bar{k}_j,$$

where $\rho_j^i$ is the unit vector normal to the line of $X_i$ and $\kappa_j$ is the unit vector of the line of $X_i$. Here,
In order to derive (b), we use the fact that \( r_j \) and \( t_k \) denote the coordinates of \( X_i \) with respect to the orthonormal basis \( (\hat{\rho}_j, \hat{\kappa}_j) \).

The random vector \( u \) of Eq. (9) can be written as \( u \hat{\rho}_j + u \hat{\kappa}_j \).

As a result, the Laplace transform is given by

\[
E[\Phi_0] = \prod_{j \in \Phi} \int_{B(\nu)} \frac{1}{\pi \nu^2} \frac{1}{1 + sp(\rho_j + u)^2} \, du \, dv \tag{10}
\]

where we use the probability generating function on the Poisson point process with intensity \( \mu \).

Finally, from the independence of random variables \( I_1 \) and \( I_2 \), we multiply Eqs. (10) and (11) to obtain the complete formula for the Laplace transform of interference.

\[
E[\Phi_0] = \prod_{j \in \Phi} \int_{B(\nu)1+sp(\rho_j + u)^2} \frac{1}{\pi \nu^2} \frac{1}{1 + sp(\rho_j + u)^2} \, du \, dv \tag{11}
\]

Remark 2: In the proposed vehicular architecture, the interference power of the typical vehicle is given by Eq. (8). In contrast, the interference seen by a typical location in the plane does not follow the same distribution. Specifically, from the perspective of an arbitrarily located point, it is almost surely not on any road. Therefore, its Laplace transform is given by Eq. (10). Consequently, the typical vehicle experiences an additional interference from the devices on the typical road, compared to a randomly located point in space. A similar phenomenon was also discussed in [34].

Fig. 2 illustrates the Laplace transforms of the interference, evaluated by Monte Carlo simulations and derived by Theorem 1, respectively. By comparing marks and lines, we find that the integral expression in Theorem 1 exactly matches the simulation results. For the computation, we use \( p = 0.01 \), \( \alpha = 3 \), \( \lambda_1 = \mu = 5 \) and \( \nu = 0.1 \). The x-axis is the Laplace transform argument.
B. Coverage Probability

Theorem 1: The SIR coverage probability of the typical vehicle is given by Eq. (12), shown at the top of the next page.

Proof: Denote by $U$, the location of the device associated with the typical vehicle at the origin. Then, the SIR coverage probability of the typical vehicle is given by

$$p_c(\tau) = P_0^0 \left( \frac{\sum_{i \in \Phi} pHi \|X_i + U_i\|^{-\alpha}}{\sum_{i \in \Phi + b_0} pHi \|X_i\|^{-\alpha}} > \tau \right).$$

We obtain (a) from Slivnyak’s theorem and (b) by using the density of $\|U_0\|$ as $2z/\nu^2$ for $0 \leq z \leq \nu$. The proof is completed by using Eq. (8).

Fig. 3 illustrates the coverage probability of the typical vehicle. For the considered parameters, the figure exhibits two trends w.r.t. parameters $\alpha$ and $\nu$: (1) a higher path loss exponent provides better coverage due to better spatial separation of the interference for the same topology of vehicles and devices; (2) a smaller disk yields a better coverage performance mainly because it implies that the devices are closer to their corresponding vehicles. These observations show that the proposed architecture provides a better SIR performance in a dense urban scenario where the path loss is greater and the distances from the devices to roads are shorter.

C. Area Spectral Efficiency

The area spectral efficiency is defined as the product of the achievable rate of the typical link and the density of vehicles. Consequently, the area spectral efficiency of the network is interpreted as the spatial average of the achievable throughput per unit area.

Theorem 2: The area spectral efficiency of the proposed architecture is given by

$$\text{ASE} = \frac{\lambda \mu}{\pi} \int_0^\infty \int_0^{\nu} \frac{2\rho^{1-\alpha}}{\nu^2(1 + z\rho^{-\alpha})} \mathcal{L}_I(z) \, dz, \quad (14)$$

where $\mathcal{L}_I(z)$ is the Laplace transform of the interference given in Theorem 1.

Proof: To obtain the ergodic throughput of the typical link we use the following expression in [47]. For two independent random variables $X > 0$ and $Y > 0$, we have

$$E \left[ \log_2 \left( 1 + \frac{X}{Y} \right) \right] = \int_0^\infty z^{-1} \left[ 1 - E \left[ e^{-zX} \right] \right] E \left[ e^{-ZY} \right] \, dz.$$  

As a result, the area spectral efficiency is given by

$$\frac{\lambda \mu}{\pi} E_0^0 \left[ \log_2 \left( 1 + \frac{\sum_{i \in \Phi} pHi \|X_i + U_i\|^{-\alpha}}{\sum_{i \in \Phi + b_0} pHi \|X_i\|^{-\alpha}} \right) \right]$$

(a) $$= \frac{\lambda \mu}{\pi} \int_0^\infty z^{-1} \left[ 1 - E_{H,U} \left[ e^{-z\rho H\|U\|^{-\alpha}} \right] \right] E_0^0 \left[ e^{-zI} \right] \, dz$$

(b) $$= \frac{\lambda \mu}{\pi} \int_0^\infty z^{-1} \left[ 1 - \int_0^{\nu} \frac{2\rho}{\nu^2(1 + z\rho^{-\alpha})} \, dr \right] E_0^0 \left[ e^{-zI} \right] \, dz$$

$$= \frac{\lambda \mu}{\pi} \int_0^\infty z^{-1} \left[ \int_0^{\nu} \frac{2\rho z^{-\alpha}}{\nu^2(1 + z\rho^{-\alpha})} \, dr \right] E_0^0 \left[ e^{-zI} \right] \, dz$$

$$= \frac{\lambda \mu}{\pi} \int_0^\infty \frac{2\rho z^{-\alpha}}{\nu^2(1 + z\rho^{-\alpha})} \mathcal{L}_I(z) \, dz. \quad (15)$$

To derive (a), we use the interference seen by the typical vehicle in Theorem 1. To get (b), we use the fact $U$ is randomly distributed in disk $B_0(\nu)$. Finally, applying Eq. (8) to (15) completes the proof.

Fig. 4 illustrates the ergodic throughput, i.e., the achievable rate of the typical vehicle. For the considered parameters, a higher path loss exponent and/or a lower road intensity provide a better link throughput. Moreover, as the size of coverage disk increases, the link throughput monotonically decreases because the mean distances from vehicles to their associated devices also increase.

Remark 3: Rayleigh fading is considered in this paper to derive the space domain metrics. Therefore, the Laplace
transform of the interference, the distribution of the SIR coverage probability, and the area spectral efficiency formula are applicable to scenarios where a rich scattering is justified, e.g., sub 6 GHz carrier frequency. Nevertheless, the technique presented in this paper could be used to evaluate the space domain metrics under a generalized fading model in vehicular networks [48]. Specifically, when fading is characterized as Nakagami-m distribution, the signal over a distance $d$ is given by $pHd^{-\alpha}$ with $H \sim \text{Gamma}(m, \Omega/m)$. Under Nakagami-m fading, the Laplace transform of the interference in Eq. (9) is given by

$$\mathcal{L}_{\text{Nakagami}(m, \Omega/m)}(s) = \mathcal{E}_\Phi \left[ \prod_{X_i \in \Phi} E_U \left[ E_H \left[ e^{-spH\|X_i+U\|^{-\alpha}} \right] \Phi, U \right] \right]$$

where $m, \Omega$ are the empirically estimated parameters for the Nakagami-m distribution [49]. Then, by following the steps provided in the proof of Lemma 1, the Laplace transform of the interference at the typical vehicle is given by Eq. (16), shown at the top of the next page.

The above Laplace transform of the interference with Nakagami-m fading can be used to compute the SIR coverage probability and area spectral efficiency accordingly. See Eqs. (13) and (14). The exact SIR coverage probability and area spectral efficiency are not provided in this paper.

IV. PERFORMANCE ANALYSIS IN THE TIME DOMAIN

We focus on the evolution of the coverage disks w.r.t. time by deriving the dynamics of area fraction and network association delay. Unlike in Section III, the Palm distribution of the vehicle point process will not be used in this section. Therefore, there is no typical vehicle at the origin.

A. Area Fraction

Recall that for a time $t$, the union of the coverage disks is given by $S(t) = \bigcup_{X \in \Phi(t)} B_X(\nu)$. Theorem 3: For any time $t$, $S(t)$ is time and motion invariant. Its area fraction is

$$AF(S(t)) = 1 - e^{-2\lambda t} \int_{\mathbb{R}} 1 - e^{-2\nu s\sqrt{\tau + \nu s - u}} du \, dt.$$  \hspace{1cm} (17)

Moreover, the area fraction of the cumulative coverage disks up to time $t$ is given by

$$AF(S(t)) = 1 - e^{-2\lambda t} \int_{\mathbb{R}} 1 - \exp(-2\nu(s + \sqrt{\tau + \nu s - r})) dr.$$  \hspace{1cm} (18)

In addition, its limit is given by

$$AF(S(\infty)) = 1 - \exp(-2\lambda \nu).$$  \hspace{1cm} (19)

Proof: We first show that $S(t)$ is time invariant. By a slight abuse of notation, let $\phi(r_i, \theta_i, t = 0)$ denote the Poisson point process on line $(r_i, \theta_i, t = 0)$ at time zero. Then, we have

$$S(t = 0) = \bigcup_{X \in \Phi(t = 0)} B_X(\nu) = \bigcup_{(r_i, \theta_i) \in \Phi(t = 0)} \left( \bigcup_{X \in \phi(r_i, \theta_i, t = 0)} B_X(\nu) \right),$$

where one can interpret the set $\bigcup_{X \in \phi(r_i, \theta_i, t = 0)} B_X(\nu)$ as the Boolean model [13, Chap.3] of finite radius balls centered on the Poisson point process $\phi(r_i, \theta_i, t = 0)$ at time zero. Notice that every vehicle is assumed to choose its direction of motion, according to an independent and identical coin toss at time zero. Therefore, at any $t$, due to the thinning and superposition of the Poisson point process on each line, the point process at time $t$ is still a Poisson point process with the same intensity. Furthermore, since the distributions of the centroids of the Boolean models are the same for time 0 and $t$, we can write

$$\bigcup_{X \in \phi(r_i, \theta_i, t = 0)} B_X(\nu) \overset{d}{=} \bigcup_{X \in \phi(r_i, \theta_i, t)} B_X(\nu),$$

where $\overset{d}{=}$ denotes equality in distribution. Because the Poisson lines are time-invariant, we have

$$S(t) \overset{d}{=} S,$$  \hspace{1cm} (21)

where $S$ denotes the set for the coverage disks at time zero, $S(0)$.

In order to show the planar motion invariance of $S$, we use [36, Prop. 4.3]; the random closed set $S$ is motion invariant if and only if its capacity functional, $T_S(K) := \mathbb{P}(S \cap K \neq \emptyset)$ for all compact set $K$, is motion invariant [14]. The capacity functional of $S$ is given by

$$T_S(K) = 1 - \mathbb{P}(\text{no point of } \Phi \text{ in } B_0(\nu)).$$

Furthermore, we also have

$$T_S(K + x) = 1 - \mathbb{P}(\text{no point of } S_x \Phi \text{ in } B_x(\nu)) = 1 - \mathbb{P}(\text{no point of } \Phi \text{ in } B_0(\nu)),$$

where $S_x \Phi$ is the translation of $\Phi$ by a shift $x \in \mathbb{R}^2$. We obtain the last expression because the vehicle point process $\Phi$ (the grains of the Boolean model) is a motion invariant point process [45]. Since $T_S(K) = T_S(K + x)$, the random closed set $S$ is motion invariant.
L_{Nikogosian \,(m, \Omega/m)}(s) = \exp \left( -\lambda t \int_0^\infty \exp \left( \frac{1}{1 + sp(\Omega/m)((r + u)^2 + (t + v)^2)^{\frac{k}{2}}} \right) \, du \, dv \, dt \right) \times \exp \left( -\mu t \int_0^\infty \exp \left( \frac{1}{1 + sp(\Omega/m)((r + u)^2 + (t + v)^2)^{\frac{k}{2}}} \right) \, du \, dv \, dt \right).

Leveraging the invariance property of S(t), the area fraction of S(t) is
\[ \text{AF}(S(t)) = \lim_{r \to \infty} \frac{E[t_2(S(t) \cap B_0(r))]}{t_2(B_0(r))} \]
\[ = \lim_{r \to \infty} \frac{E[t_2(S \cap B_0(r))]}{t_2(B_0(r))} \]
\[ = \lim_{r \to \infty} \frac{\int_{B_0(r)} \mathbb{1}_{x \in S} \, dx}{\int_{B_0(r)} \mathbb{1}_x \, dx} \]
\[ = \lim_{r \to \infty} \frac{\int_{B_0(r)} \mathbb{1}_{x \in S} \, dx}{\int_{B_0(r)} \mathbb{1}_x \, dx} = E[\mathbb{1}_{0 \in S}], \]
where (a) is obtained by the time invariance of S and (b) follows from the fact that the area of a set is given by the Lebesgue integral of the indicator function of the set. We obtain (c) from Fubini's theorem and (d) from the stationarity of S, respectively. Therefore, the area fraction is
\[ E[\mathbb{1}_{0 \in S}] = P(0 \in S) = P(\min_{X \in \Phi} \|X_k\| \leq \nu) \]
\[ = 1 - E \left[ \prod_{X \in \Phi} \mathbb{1}_{\|X_k\| > \nu} \right], \quad (22) \]
where we obtain (e) because the probability of S containing the origin is equal to the probability that \(B_{\min_{X \in \Phi} \|X_k\|}(\nu)\), the disk centered at \(\min_{X \in \Phi} \|X\|\), contains the origin. As a result, we have
\[ E \left[ \prod_{X \in \Phi} \mathbb{1}_{\|X_k\| > \nu} \right] \]
\[ = E_{b_1}(=) \prod_{X \in \Phi} \mathbb{1}_{\|X_k\| > \nu} \]
\[ = E_{b_1} \prod_{X \in \Phi} \mathbb{1}_{\|X_k\| > \nu} \]
\[ = \exp \left( -2\lambda_1 \int_0^\nu 1 - e^{-\mu \sqrt{\nu^2 - \nu^2 - r_i^2}} \, dr \right), \quad (23) \]
where \(x \wedge y\) denotes the minimum of \(x\) and \(y\). We have (f) by conditioning on the Poisson line process \(\Phi_1\) and by representing the points of the Cox point process \(X\) as a Poisson point processes \(\phi(r_i, \theta_i)\). Equality (g) is obtained by denoting the two nearest point—w.r.t. the closest point of the line \(i\) to the origin—on each side by \(X_{i,0}\) and \(X_{i,1}\), respectively. We obtain (h) from the distribution function of the exponential random variable with parameter \(2\mu\). Applying the Laplace transform of the cylinder Poisson point process gives Eq. (23). Finally combining Eq. (23) into Eq. (22) completes the proof.

Now, let us focus on the cumulative coverage disks \(\hat{S}(t)\). The area fraction is given by
\[ \text{AF}(\hat{S}(t)) = \lim_{r \to \infty} \frac{E[t_2(\hat{S}(t) \cap B_0(r))]}{t_2(B_0(r))} \]
\[ = \lim_{r \to \infty} \frac{\int_{B_0(r)} \mathbb{1}_{x \in \hat{S}(t)} \, dx}{\int_{B_0(r)} \mathbb{1}_x \, dx} \]
\[ = \lim_{r \to \infty} \frac{\int_{B_0(r)} \mathbb{1}_{x \in \hat{S}(t)} \, dx}{\int_{B_0(r)} \mathbb{1}_x \, dx} = E[\mathbb{1}_{0 \in \hat{S}(t)}]. \]
Therefore, we have \(\text{AF}(\hat{S}(t)) = P(0 \in \hat{S}(t)) = 1 - P(0 \notin \hat{S}(t))\), where \(0 \notin \hat{S}(t)\) indicates that the origin is not an element of the set \(\hat{S}(t)\). In order to have the set \(\hat{S}(t)\) not containing the origin at time \(t\), the distances from the origin to the lines are greater than \(\nu\). For lines \(|r_i| < \nu\), let \(0\) denote the point on line \(i\) closest to the origin. The point process on line \((r_i, \theta_i)\) consists of two Poisson point processes based on their moving directions: \(\phi(r_i, \theta_i) = \phi_1(r_i, \theta_i) + \phi_2(r_i, \theta_i)\) with intensity \(\mu/2\) each and moving directions \((p, q) \in \{(+, -), (-, +)\})^1\). For any \((p, q)\), the points of the two Poisson point processes, \(\phi_1(r_i, \theta_i)\) and \(\phi_2(r_i, \theta_i)\), must satisfy \(\|X - 0\|_i > \sqrt{\nu^2 - r_i^2} + vt\).

As a result, we have
\[ \text{AF}(\hat{S}(t)) = 1 - P(0 \notin \hat{S}(t)) \]
\[ = 1 - E \left[ \prod_{|r_i| < \nu} \mathbb{1}_{X \in \phi(r_i, \theta_i)} \mathbb{1}_{X - 0\|_i > \sqrt{\nu^2 - r_i^2} + vt} \right] \]
\[ = 1 - E \left[ \prod_{|r_i| < \nu} e^{-\mu \sqrt{\nu^2 - \nu^2 - r_i^2}} \right] \]
\[ = 1 - e^{-2\lambda_1 \int_0^\nu 1 - e^{-\mu \sqrt{\nu^2 - \nu^2 - r_i^2}} \, dr}. \]

The set of moving directions, \(\{(+, -), (-, +)\}\), is given to illustrate the fact that \(\phi_1\) and \(\phi_2\) have different moving directions.
To derive (i), we use the fact that $\phi_1^2(r, \theta)$ and $\phi_2^2(r, \theta)$ are independent and the minimum of two exponential with parameter $\lambda$ is exponential random variables with parameter $2\lambda$. To have the limit value of the area fraction, we use $t = \infty$.

Theorem 3 shows that the area fractions of $S(0)$ and $S(t)$ have the same distribution for all $t$; in particular, the areas covered at time 0 and at time $t$ are the same on average. In Fig. 1, the area fraction can be interpreted as the mean area of the shaded region, divided by the total area. Eq. (17) gives the area fraction as a function of the parameters. It shows that the area fraction is increasing with the radius of the coverage disk $\nu$, the intensity of vehicles $\mu$, and the density of lines $\lambda_l$. Fig. 5 illustrates the area fraction of the coverage disk w.r.t. the size of the coverage disk.

Now, let us discuss $\text{AF}(\tilde{S}(t))$: (1) it is increasing w.r.t. time $t$; (2) it is a function of the radius $\nu$ and the intensity $\lambda_l$. Fig. 7 depicts the behaviors of $\text{AF}(\tilde{S}(t))$ w.r.t. time, based on two sets of parameters. Note that, as time tends to infinity, $\text{AF}(\tilde{S}(t))$ tends to $\text{AF}(\tilde{S}(\infty))$ specified by Eq. (19). For moderate speeds $v = \{36, 72, 108\}$ km/h, their limiting values $\text{AF}(\tilde{S}(\infty))$ are achieved in less than 60 seconds. Notice that the limit is increasing w.r.t. the density of roads and the radius of the coverage disk. Eqs. (18) and (19) show how long-term network properties are affected by parameter changes.

Remark 4: Fig. 8 illustrates the area fraction of the covered area in Eq. (19) w.r.t. the radius of the coverage disk, for three different cases: (1) urban, (2) suburban, and (3) rural, which are devised according to the density of roads and vehicles. For the urban case, the road density is high $\lambda_l = 9$/km; for the suburban area, it is moderate, $\lambda_l = 6$/km; and for the rural area case, it is low, $\lambda_l = 3$/km. For all cases, the limits tend to one as the radius of the coverage disk tends to infinity. Furthermore, the urban area case dominates both suburban and rural cases. In the urban case, a smaller disk may suffice to cover a significant part of the Euclidean plane. For instance, when the radius of the disk is 100 meters—the typical transmission range of devices with limited power source [50]—about 80% of the entire plane is covered in the urban case whereas only 30% is covered in the rural case.

The following example sheds light on Theorem 3 for different mobility model.

Example 1: We have assumed that vehicles move at a constant speed $v$ and that their trajectories strictly follow
roads. An extension of the proposed model could feature a randomized-speed model where each vehicle initially determines its speed according to an independent and identical distribution, such as Uniform on $[0, v_{\text{max}}]$, and maintains that speed. By the displacement and superposition principles of the Poisson point process [14], the locations of the vehicles after time $t$ on each line are again given by a Poisson point process with intensity $\mu$. The area fraction is still given by Eqs. (17) and (18).

B. Network Association Delay

In this paper, the minimum association delay is defined as the time required for an arbitrarily located device to be contained in any coverage disk. Let $\Psi$ denote the planar Poisson point process for roadside devices. It is clear from Section IV-A that if a point of $\Psi$ is not in $\bar{S}(\infty)$, it will not be covered by any vehicle at any time. See Fig. 6 for an illustration of the set $\bar{S}(\infty)$. Therefore, the association delay is defined more precisely conditionally on the fact that the point is in $\bar{S}(\infty)$. Fig. 9 illustrates the association delay with coverage disks at $t = \{0, 200\}$, respectively.

Theorem 4: The c.d.f. of the association delay is given by

$$P(W \leq w|0 \in \bar{S}(\infty)) = \frac{1 - e^{-2\lambda t} \int_{0}^{\nu} 1 - e^{-2\mu(\sqrt{u^2 - w^2} + vw)} \, du}{1 - e^{-2\lambda \nu}},$$

(24)

where $\nu$ is the speed of vehicles.

Proof: We can write the distribution of the association delay as follows:

$$P_{\Psi}^{0}(W \leq w, 0 \in \bar{S}(\infty)) = 1 - E \left[ \prod_{|r_i| < \nu} \Phi_{\bar{r}_i} \left[ \prod_{X < \phi^+(\|X - 0_i\| > \nu w + \sqrt{\nu^2 - \bar{r}_i^2})} \Phi_l \right] \right],$$

$$= 1 - E \left[ \prod_{|r_i| < \nu} \exp \left( -2\mu \left( \sqrt{\nu^2 - r_i^2} + vw \right) \right) \right],$$

$$= 1 - \exp \left( -2\lambda t \int_{0}^{\nu} 1 - e^{-2\mu(\sqrt{u^2 - w^2} + vw)} \, du \right),$$

where we have used the same technique as in deriving Eq. (23). Note that the association delay is integrated w.r.t. the density functions of the Poisson line process and of the Poisson point process on each line. On the other hand, from Theorem 3 we have $P_{\Psi}^{0}(0 \in S(\infty)) = P(0 \in \bar{S}(\infty)) = 1 - e^{-2\lambda \nu}$. Therefore, we have

$$P_{\Psi}^{0}(W \leq w|0 \in \bar{S}(\infty)) = \frac{1 - \exp \left( -2\lambda t \int_{0}^{\nu} 1 - e^{-2\mu(\sqrt{u^2 - w^2} + vw)} \, du \right)}{1 - \exp(-2\lambda \nu)}.$$

We derive the distribution of the association delay. The formula can be directly used to derive network parameters satisfying pre-defined association delay requirements. For instance, assume that $\nu$ is given as 100 meters and $\lambda_t = 3/\text{km}$. Then, if one requires that at least 90% of roadside devices have less than 60 seconds of delay in the proposed framework, we can write this as $P(W < 60 \, \text{sec}) = 0.9$. Thus, using the above formula, one can explicitly find the minimum speed $\nu$ that meets this network requirement.

Remark 5: Note $W = 0$ if the typical point is contained in a disk at $t = 0$. In Eq. (24), the probability mass at zero, $P_{\Psi}^{0}(W = 0|0 \in \bar{S}(\infty))$, is given by

$$AF(S(0)) = \frac{1 - \exp \left( -2\lambda t \int_{0}^{\nu} 1 - e^{-2\mu(\sqrt{u^2 - w^2} + vw)} \, du \right)}{1 - \exp(-2\lambda \nu)}.$$

V. DISCUSSIONS

In this section, we discuss a few trade-off relationships in the proposed network, in particular, trade-off relations represented as a linear combination of short-term and long-term metrics, w.r.t. the network parameters.

A. Trade-Offs

Notice that a trade-off relationship exists between the short-term and the long-term performance results—specifically, a trade-off w.r.t. the radius of the coverage disk exists between the SIR coverage and the association delay. For instance, if the radius increases, a shorter waiting time is achieved according to Theorem 4; i.e., the association delay decreases. However, the SIR coverage probability of the typical vehicle diminishes according to Theorem 1. This contrasting behavior occurs because the size of the disk is closely related to both the distance to the desired signal device and the total area occupied by the coverage disks.

Another trade-off relationship exists between coverage (or equivalently rate) and association delay w.r.t. the density of vehicles $\mu$. For instance, if the density $\mu$ is high, more vehicles are present on each road, creating a higher interference at the typical vehicle. Consequently, the SIR coverage probability decreases. However, due to the increased number of vehicles, a typical device is more likely to be geometrically covered within a shorter period of time.

This is not very surprising in the context of delay-tolerant networks where the coverage or capacity are known to be improved at the expense of excessive delays. For instance,
[37] showed that by allowing an infinite delay, one can significantly increase the network throughput. Similar trade-off relationships were investigated under different network topologies in [39], [41]. The core mechanism of classical delay-tolerant networks is very simple: nodes transmit only when they are proximal; this increases the signal power and improves the network performance. In this context, the proposed network is very similar to traditional delay-tolerant networks. However, a noticeable difference is present in the proposed model: In this paper, the trade-off phenomena can be controlled or even exploited by changing parameters such as the density $\mu$ or the radius $\nu$. For instance, by decreasing the density of vehicles, the coverage or rate of the typical vehicle can be significantly improved. Similarly, by increasing the radius of the coverage disk, the network’s association delay can be substantially improved. Therefore, one can leverage the trade-off and design the proposed network while meeting some delay or SIR coverage constraints.

### B. Example: Network Optimization

To illustrate the use of our results for network design, we consider a simple linear combination of the coverage probability and the area fraction. This is captured by the following aggregate utility function $J(\nu, \mu) = w_1 p_c(\tau) + w_2 AF(S(\infty))$, where $p_c(\tau)$ is the coverage probability of the vehicle with SIR threshold $\tau$, $AF(S(\infty))$ is the limit of the area fraction of the cumulative coverage disks, and $w_1 > 0$, and $w_2 > 0$ are fixed weights. The coverage probability and the area fraction are both functions of $\nu$ and $\mu$. The above utility function is given by a linear combination of time and space domain performance metrics that behave differently w.r.t. $\nu$ and $\mu$, respectively. See Table II.

A classical way of interpreting $J(\nu, \mu)$ is to view it as a network utility and to maximize it w.r.t. network parameters. For convenience, this paper assumes that the utility would scale linearly with the values of the SIR coverage probability and the area fraction over time. The weights $w_1, w_2$ are positive numbers characterizing the utility. For instance, if reliable communication is desirable at the expense of high association delay, one uses $w_1 > w_2$. Similarly, if one wishes that vehicles sweep a wider area, one considers $w_2 > w_1$. In general, depending on high-level design principles, different weights can be used. Given $J$, one can maximize the revenue or utility by jointly finding the best combination of coverage disks or inter-vehicle distance. Note that such an unconstrained formulation is maybe infeasible due to various practicalities such as hardware constraints or radio regulations.

Fig. 10 illustrates the utility when the weights are equal, $w_1 = w_2 = 0.5$. The utility function is concave w.r.t. $\nu$ for a given value of $\mu$. Consequently, the optimum pair $\nu^*, \mu^*$ can be found numerically.

**Example 2:** Any objective function—consisting of metrics with contrasting behaviors—can be used to provide a way to balance the various aspects of the proposed vehicular network architecture. For instance, the optimization problem can be also written as

$$\arg \max_{\nu, \mu > 0} \left\{ w_1 p_c(\tau) + w_2 AF(S(\infty)) \right\}$$

subject to $P(\Phi(W < w) | \overline{S}(\infty)) = c$.

The set can be numerically obtained by using the methods for integral equation e.g., Taylor expansion [51]. A detailed analysis of the network optimization including the delay aspect is left for future work.

### VI. Conclusion

In this paper, we propose a new network architecture where vehicles harvest data from devices randomly distributed in space. The proposed system leverages the idea that moving vehicles can communicate, even if briefly, with a significant number of nearby roadside devices and thus can service a wide area over time. We analyze the proposed network in both the space and time domain. First, we analyze the SIR coverage probability and the area spectral efficiency based on a snapshot of the network geometry. Then, we investigate the evolution of the network geometry to compute the area fraction of the coverage disks and to derive the distribution of the network association delay. Using the derived metrics, we discuss various trade-off relationships and show how to optimize network utility based on those trade-offs.

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