A Weakly Supervised Model for Solving Math word Problems

Oishik Chatterjee  
Department of CSE  
IIT Bombay

Aashish Waikar  
Department of CSE  
IIT Bombay

Vishwajeet Kumar  
IBM Research  
Bangalore India

Ganesh Ramakrishnan  
Department of CSE  
IIT Bombay

Kavi Arya  
Department of CSE  
IIT Bombay

Abstract

Solving math word problems (MWPs) is an important and challenging problem in natural language processing. Existing approaches to solve MWPs require full supervision in the form of intermediate equations. However, labeling every math word problem with its corresponding equations is a time-consuming and expensive task. In order to address this challenge of equation annotation, we propose a weakly supervised model for solving math word problems by requiring only the final answer as supervision. We approach this problem by first learning to generate the equation using the problem description and the final answer, which we then use to train a supervised MWP solver. We propose and compare various weakly supervised techniques to learn to generate equations directly from the problem description and answer. Through extensive experiment, we demonstrate that even without using equations for supervision, our approach achieves an accuracy of 56.0 on the standard Math23K dataset (Wang et al., 2017). We also curate and release a new dataset for MWPs in English consisting of 10227 instances suitable for training weakly supervised models.

1 Introduction

A Math Word Problem (MWP) (e.g. is a numerical problem expressed in natural language (problem description)), that can be transformed into an equation (solution expression), which can be solved to obtain the final answer. In Table 1, we present an example of a Math Word Problem (MWP). Automatically solving MWPs has recently gained lot of research interest in natural language processing (NLP) and Artificial Intelligence (AI). The task of automatically solving MWPs is challenging owing to two primary reasons: i) the unavailability of large training datasets with problem descriptions, equations as well as corresponding answers; as depicted in Table 1, the equations provide full supervision, since equations can be solved to obtain the answer (which amounts to weak supervision only) ii) parsing problem description and representing it suitably for effective decoding of the equations is challenging. Paucity of completely supervised training data can post a severe challenge in training MWP solvers. Most existing approaches assume the availability of full supervision in the form of both intermediate equations and answers for training. However, annotating MWPs with equations is an expensive and time consuming task. There exists only one sufficiently large dataset (Wang et al., 2017) in Chinese consisting of MWPs with annotated intermediate equations for supervised training.

We propose a novel two-step weakly supervised technique to solve MWPs by making use only of the weak supervision (in the form of answers). In the first step, using only the answer as supervision, we learn to generate equations for questions in the training set. In the second step, we use the generated equations along with answers to train any state-of-the-art supervised model. We illustrate the effectiveness of our weakly supervised approach on our newly curated reasonably large dataset in English.

Our main contributions are as follows:

1) An approach, WARM, (c.f., Section 4) for generating equations from math word problems,
given (weak) supervision only in the form of the final answer.

2) An extended semi-supervised training method to leverage a small amount of annotated equations as strong/complete supervision.

3) A new and relatively large dataset in English (with more than 10k instances), for training weakly supervised models for solving Math word problems (c.f., Section 3).

## 2 Related Work

Automatic math word problem solving has recently drawn significant interests in the natural language processing (NLP) community. Existing MWP solving methods can be broadly classified into four categories: (a) rule-based methods, (b) statistics-based methods, (c) tree-based methods, and (d) neural-network-based methods.

Rule-based systems (Fletcher, 1985; Bakman, 2007; Yuhui et al., 2010) were amongst the earliest approaches to solve MWP. They heavily rely on hand-engineered rules that might cover a limited domain of problems, whereas math word problems have potentially diverse descriptions in real-world settings. On the other hand, statistics-based methods (Hosseini et al., 2014; Kushman et al., 2014; Sundaram and Khemani, 2015; Mitra and Baral, 2016; Liang et al., 2016a,b) use pre-defined logic templates and employ traditional machine learning models to identify entities, quantities, and operators from the problem text and employ simple logical inference to yield the numeric answer. Upadhyay et al. (2016) employ a semi-supervised approach by learning to predict templates and corresponding alignments using both explicit and implicit supervision. These approaches involve an additional overhead of annotating the problem text with the relevant template, which can be prohibitive when learning from large-scale datasets or when requiring an MWP solver to be trained on a language with little or no existing dataset. Moreover, the pre-defined templates used by these methods are not very robust to dataset diversity. Tree-based methods (Roy and Roth, 2015; Koncel-Kedziorski et al., 2015; Roy et al., 2016; Roy and Roth, 2017, 2018) replaced the process of deriving an equation by constructing an equivalent tree structure, step by step, in a bottom-up manner. More recently, neural network-based MWP solving methods (Wang et al., 2017, 2018a,b; Huang et al., 2018; Chiang and Chen, 2019; Wang et al., 2019; Liu et al., 2019; Xie and Sun, 2019) have been proposed. These employ an encoder-decoder architecture and train in an end-to-end manner without the need of hand-crafted rules or templates.

Wang et al. (2017) were the first to propose a sequence-to-sequence (Seq2Seq) model, viz., Deep Neural Solver, for solving MWP. They employ an RNN-based encoder-decoder architecture to directly translate the problem text into equation templates and also release a high-quality large-scale dataset, Math23K, consisting of 23,161 math word problems in Chinese. Wang et al. (2018a) extend the idea of the Deep Neural Solver by introducing equation normalization techniques. Huang et al. (2018) propose the incorporation of copy and alignment mechanisms in the standard Seq2Seq model.

Chiang and Chen (2019) propose a Seq2Seq model with an encoder designed to understand the semantics of the problem, and a decoder equipped with a stack to facilitate tracking semantic meanings of the operands. Wang et al. (2019) propose a two-step pipeline that employs i) a Seq2Seq model to convert the input text into a tree-structure template with quantities representing inferred numbers as leaf nodes and unknown operators as inner nodes ii) a recursive neural network to encode the quantities, and infer the unknown operator nodes in a bottom-up manner.

Unlike the earlier neural-network based methods, (Liu et al., 2019) and (Xie and Sun, 2019) propose tree-structured decoding that generates the syntax tree of the equation in a top-down manner. In addition to applying tree-structured decoding, (Zhang et al., 2020) propose a graph-based encoder to capture relationships and order information among the quantities. Wang et al. (2018b) make the first attempt of applying deep-reinforcement learning to solve MWP. They equivalently expressed the output equation as an expression tree and proposed MathDQN, a customised version of the deep Q-network (DQN) framework by tailoring the definitions of states, actions and rewards functions and use this for iterative construction of the tree.

For a more comprehensive review on automatic MWP solvers, readers can refer to a recent survey paper (Zhang et al., 2018). Unlike all the previous works that require equations for supervision, we propose a novel weakly supervised model, WARM, (c.f., Section 4) for solving MWP using only the
a decoder network with fully connected units (described in Section 4.3). We call this model WARM. Using this model, we create a noisy equation-annotated dataset. We use only those instances to create the dataset for which the equation generated by the model yields the correct answer. Note that the equations are noisy as there is no guarantee that the generated equation will be the shortest or even correct. In the second step we use this noisy data to train a supervised model.

Figure 2: WARM: Our Weakly Supervised Approach

4 Our Approach: WARM

We propose a weakly supervised model, WARM\(^2\), for solving the math word problem using only the answer as supervision. We employ a two-step cascaded approach for solving the given task. For the first step, we propose a model that predicts the equation given a problem text and answer. This model uses reinforcement learning to search the space of possible equations given the question and the correct answer only. It consists of a two layer bidirectional GRU (Cho et al., 2014) encoder and

\(^1\)https://in.ixl.com/

\(^2\)WARM stands for WeAkly supeRvised Math solver.

4.1 Equation Generation

Given a problem text \(P\) and answer \(A\) we first pass the encoded representation obtained from the encoder to the decoder. At each time step, the decoder generates an operator and its two operands from the operator and operand vocab list. The operation is then executed to obtain a new quantity. This quantity is checked against the ground truth and if it matches the ground truth, the decoding is terminated and a reward of +1 is assigned. Else a reward of -1 is given and the generated quantity is added to the operand vocab list. In the following subsection, we describe this process in more detail.

4.2 Encoder

The encoder takes as input a math word problem represented as a sequence of tokens \(P = x_1 x_2 x_3 \ldots x_n\). We replace each number in the question with a special token \(<\text{num}>\) to obtain this sequence. Each word token \(x_i\) is first transformed into the corresponding word embedding \(\hat{x}_i\) by looking up an embedding matrix \(\mathbf{M}_w\). Next, a binary feature is appended to the embedding to indicate whether the token is a word or a number. This appended embedding vector is then passed through a 2 layer bidirectional GRU (Cho et al., 2014) and the output of both the directions of the final layer are summed to get the encoded representation of the text. This is then passed on to the decoder.
4.3 Decoder

We have implemented a fully connected decoder network for generating equations in a bottom-up manner. Our decoder takes as input the previous decoded operand and the last decoder hidden state and outputs the operator, left operand, right operand and hidden state for the current time step. We initialize the decoder hidden state with the last state of the encoder.

\[
\begin{align*}
    \alpha^p_t, \alpha^l_t, \alpha^r_t, h^d_t &= \text{DecoderFCN}(\alpha^p_{t-1}, h^d_{t-1}) \\
\end{align*}
\]

Here, \( h^d_t \) is the decoder hidden state at \( t \)th time step. \( \alpha^p_t, \alpha^l_t \) and \( \alpha^r_t \) are probability distributions over operators, left operand and right operand respectively.
4.3.1 Operator generation

Inside our decoder, we learn an operator embedding matrix $E_{\text{op}}(op_{t-1})$ where $op_{t-1}$ is the operator sampled in the last time step. We use a gating mechanism to generate the operator hidden state $h_t^{\text{op}}$.

$$g_t^{\text{op}} = \sigma(W_{1}^{\text{op}}[E_{\text{op}}(op_{t-1}); h_{t-1}^{L}] + b_{1}^{\text{op}})$$

$$h_t^{\text{op}} = g_t^{\text{op}} \ast \tanh(W_{2}^{\text{op}}[E_{\text{op}}(op_{t-1}); h_{t-1}^{L}] + b_{2}^{\text{op}})$$

$$o_t^{\text{op}} = \text{softmax}(W_{3}^{\text{op}} h_t^{\text{op}} + b_{3}^{\text{op}})$$

Here $\sigma()$ denotes the sigmoid function and $\ast$ denotes elementwise multiplication. We sample operator $op_t$ from the probability distribution $o_t^{\text{op}}$.

4.3.2 Left Operand Generation

We use the embedding of the current operator $E_{\text{op}}(op_t)$ and the operator hidden state $h_t^{\text{op}}$ to get a probability distribution over the operands. We use a similar gating mechanism as used for generating operator.

$$g_t^{\text{ol}} = \sigma(W_{1}^{\text{ol}}[E_{\text{op}}(op_{t}); h_t^{\text{op}}] + b_{1}^{\text{ol}})$$

$$h_t^{\text{ol}} = g_t^{\text{ol}} \ast \tanh(W_{2}^{\text{ol}}[E_{\text{op}}(op_{t}); h_t^{\text{op}}] + b_{2}^{\text{ol}})$$

$$o_t^{\text{ol}} = \text{softmax}(W_{3}^{\text{ol}} h_t^{\text{ol}} + b_{3}^{\text{ol}})$$

We sample the left operand $ol_t$ from the probability distribution $o_t^{\text{ol}}$.

4.3.3 Right Operand Generation

For generating the right operand, we use the additional context information that is already available from the generated left operand. So in addition to the operator embedding $E_{\text{op}}(op_t)$ and operator hidden state $h_t^{\text{op}}$ we also use the left operand hidden state to get the right operand hidden state $h_t^{\text{or}}$.

$$g_t^{\text{or}} = \sigma(W_{1}^{\text{or}}[E_{\text{op}}(op_{t}); h_t^{\text{op}}; h_t^{\text{ol}}] + b_{1}^{\text{or}})$$

$$h_t^{\text{or}} = g_t^{\text{or}} \ast \tanh(W_{2}^{\text{or}}[E_{\text{op}}(op_{t}); h_t^{\text{op}}; h_t^{\text{ol}}] + b_{2}^{\text{or}})$$

$$o_t^{\text{or}} = \text{softmax}(W_{3}^{\text{or}} h_t^{\text{or}} + b_{3}^{\text{or}})$$

We sample right operand $or_t$ from the probability distribution $o_t^{\text{or}}$. The hidden state $h_t^{\text{or}}$ is returned as the current decoder state $h_t^{d}$.

4.3.4 Bottom-up Equation Construction

For each training instance, we maintain a dictionary of possible operands $\text{OpDict}$. Initially, this dictionary contains the numeric values that occurred in the instance, i.e., the tokens we have replaced with $<\text{num}>$ during encoding. At $t^{\text{th}}$ decoding step we sample an operator $op_t$, left operand $ol_t$ and right operand $or_t$. We get an intermediate result by using the operator corresponding to $op_t$ on the operands $ol_t$ and $or_t$. This intermediate result is added to $\text{OpDict}$ which enables us to reuse the results of previous computations in future decoding steps. So $\text{OpDict}$ acts as a dynamic dictionary of operands and we use it to reach towards the final answer in a bottom-up manner.

4.4 Rewards, Loss

We use REINFORCE (Williams, 1992) algorithm for training the model using just the final answer as the ground truth. We model the reward as $+1$ if the predicted answer matches the ground truth and $-1$ if the predicted answer doesn’t equal the ground truth.

Let $R_t$ define the reward obtained after generating $y_t = (op_t, ol_t, or_t)$. The probability $P_t$ of generating the tuple $y_t$ is specified in the following equation

$$p_0(y_t) = \prod_{i=1}^{t} o_t^{\text{ol}} \times o_t^{\text{or}}$$

The loss is specified by $L = -\sum R_{p_{y_t}}(R_t)$ and the corresponding gradient is $\nabla_{\theta}L = \sum_{y_t} p_{\theta}(y_t)R_t\nabla_{\theta} \log p_0(y_t)$

Since the space of $y_t$ makes it infeasible to compute the exact gradient, we use the standardized technique of sampling $y_t$ from $p_0(y_t)$ to get an estimate of the gradient.

4.5 Beam Exploration in Training

Since the rewards space for our problem is very sparse, we have observed that the gradients while training our model goes to zero. Our model converges too quickly to some local optima and consequently training accuracy saturates to some fixed value despite training for large number of epochs. In order to counter this problem, we have employed beam exploration in the training procedure. Instead of sampling operator $op_t$, left operand $ol_t$ and right operand $or_t$ only once in each decoding step, we
sample \( w \) number of triplets \((o_{p_t}, o_{l_t}, o_{r_t})\) without replacement from the joint probability space in each decoding step, where \( w \) is the beam width. For selecting beams out of all possible candidates, we have tried multiple heuristics by looking at probability and reward values. We have observed empirically that selecting the beam which gives a positive reward at the earliest decoding step gives the best performance. This enables our model to explore more and mitigates the above problem significantly.

4.6 **WARM-S: Adding Semi-supervision**

We consider a model that benefits from a relatively small amount of strong supervision in the form of equation annotated data: \( D_1 \), in addition to potentially larger sized math problem datasets with no equation annotation \( D_2 \). For a data instance \( d: d.p, d.e, \) and \( d.a \) represent its problem statement, equation, and answer respectively. \( D_1 \) contains instances of the form \((d.p, d.e, d.a)\) while \( D_2 \) contains instances of the form \((d.p, d.a)\).

We extend the WARM model to include a Cross-Entropy loss component for instances belonging to \( D_1 \). The net loss is the sum of the REINFORCE and Cross-Entropy losses shown below:-

**Loss 1:**
\[
\sum_{d \in D_2} RL_{\text{WARM}}(d.p, d.a)
\]

**Loss 2:**
\[
\sum_{d \in D_1} \text{CrossEntropy}(d.e, \text{WARM}(d.p, d.a))
\]

5 **Experimental Setup**

In this section, we conduct experiments on three datasets to examine the performance of the proposed weakly supervised models compared to various baselines and fully supervised models.

5.1 **Datasets**

We performed all our experiments on the publicly available AllArith \((\text{Roy and Roth, 2017})\) and Math23K \((\text{Wang et al., 2017})\) datasets and also on our EMWP10K dataset. For each dataset, we have used a 80 : 20 train-test split.

AllArith contains 831 math word problems, annotated with equations and answer, populated by collecting problems from smaller datasets AL2 \((\text{Hosseini et al., 2014})\), IL \((\text{Roy and Roth, 2015})\), CC \((\text{Roy and Roth, 2015})\) and SingleEQ \((\text{Koncel-Kedzioriski et al., 2015})\) and normalizing all mentions of quantities to digits and filtering out near-duplicate problems (with over 80% match of unigrams and bigrams).

Math23K \((\text{Wang et al., 2017})\) contains 23,161 math word problems in Chinese with 2187 templates, annotated with equations and answers, for elementary school students and is crawled from multiple online education websites. It is the largest publicly available dataset for the task of automatic math word problem solving.

EW10K contain 10,227 math word problems in English for classes VI to X. Described in Section 3. We have used a 80 : 20 train-test split.

5.2 **Dataset Preprocessing**

We replace every number token in the problem text with a special word token \(<\text{num}>\) before providing it as input to the encoder. We also define a set of numerical constants \( V_{\text{const}} \) to solve those problems which might require special numeric values that may not be present in the problem text. For example, consider the problem “The radius of a circle is 2.5, what is its area?”, the solution is \(\pi \times 2.5 \times 2.5\)” where the constant quantity \(\pi\) cannot be found in the text. As our model does not use equations as supervision, we cannot know precisely what extra numeric values might be required for a problem, so we fix \( V_{\text{const}} = \{1, \pi\} \). Finally, the operand dictionary for every problem is initialised as \( \text{OpDict} = n_p \cup V_{\text{const}} \) where \( n_p \) is the set of numeric values present in the problem text.

5.3 **Implementation Details**

We implement all our models in PyTorch.\(^4\) We set the dimension of the word embedding layer to 128, and the dimension of all the hidden states for other layers to 512. We use REINFORCE \((\text{Williams, 1992})\) algorithm and Adam \((\text{Kingma and Ba, 2014})\) to optimize the parameters. The initial value of the learning rate is set to 0.001, and the learning rate is multiplied by 0.7 every 75 epochs. We also set the dropout probability to 0.5 and weight decay to 1e-5 to avoid over-fitting. Finally, we set the beam width to 5 in beam exploration for more exploration. We train our model for 100 epochs on a GTX 1080Ti machine with the batch size set to 256.

5.4 **Models**

We compare the math word problem solving accuracy of our weakly supervised models with beam exploration on various baselines and fully supervised models. We describe the models below:

\(^3\)Source code and pre-trained models will be released upon publication

\(^4\)https://pytorch.org/
**WARM (Weakly Supervised Math Solver)** is a weakly supervised model with a two-step approach with equation generation as the first step and then doing supervised training on the equations that yield the correct answer as the next step. It uses beam exploration (discussed in Section 4.5).

**WARM w/o Beam Exploration** is the same as WARM but does not use beam exploration while decoding.

**WARM-S** is a semi-supervised model which assumes the availability of a small amount of data with annotated equations. It employs semi-supervision for both equation generation and supervised training. It also uses beam exploration (discussed in Section 4.5).

**WARM-S w/o Beam Exploration** is the same as WARM-S but does not use beam exploration while decoding.

**Random Equation Sampling** consists of a random search over $k$ parallel paths of length $d$. For each path, an operator and its two operands are uniformly sampled from the given vocabulary and the result is added to the operand vocab (similar to WARM). The equation is terminated once the correct answer is reached. We set $k = 5$ and $d = 40$ for a fair comparison with our model in terms of the number of search operations.

**Seq2Seq Baseline** is a GRU (Cho et al., 2014) based seq2seq encoder decoder model. REINFORCE (Williams, 1992) is used to train the model. Beam exploration is also employed to mitigate issues mentioned in Section 4.5.

**Hybrid model w/ SNI** (Wang et al., 2017) is a combination of the retrieval model and the RNN-based Seq2Seq model with significant number identification (SNI).

**Ensemble model w/ EN** (Wang et al., 2018a) is an ensemble model that selects the result according to generation probability among Bi-LSTM, ConvS2S and Transformer Seq2Seq models with equation normalization (EN).

**Semantically-Aligned** (Chiang and Chen, 2019) is a Seq2Seq model with an encoder designed to understand the semantics of the problem text and a decoder equipped with a stack to facilitate tracking the semantic meanings of the operands.

**T-RNN + Retrieval** (Wang et al., 2019) is a combination of the retrieval model and the T-RNN model comprising a structure prediction module that predicts the template with unknown operators and an answer generation module that predicts the operators.

**Seq2Tree** (Liu et al., 2019) is a Seq2Tree model with a Bi-LSTM encoder and a top-down hierarchical tree-structured decoder consisting of an LSTM which makes use of the parent and sibling information fed as the input, and also an auxiliary stack to help in the decoding process.

**GTS** (Xie and Sun, 2019) is a tree-structured neural model that generates the expression tree in a goal-driven manner.

**Graph2Tree** (Zhang et al., 2020) consists of a graph-based encoder which captures the relationships and order information among the quantities and a tree-based decoder that generates expression tree in a goal-driven manner.

As described earlier in Section 4, we use our weakly supervised models to generate labelled data (i.e., equations) which we then use to train a supervised model. We have performed experiments using GTS (Xie and Sun, 2019) and Graph2Tree (Zhang et al., 2020) as the supervised models since they are the current state-of-the-art.

### 6 Results and Analysis

| Weakly Supervised Models       | AllArith | Math23K | EW10K |
|-------------------------------|----------|---------|-------|
| WARM w/o Beam Exploration     | 42.1     | 14.5    | 45.2  |
| WARM                           | 95.8     | 80.1    | 98.8  |
| Baselines                      |          |         |       |
| Random Equation Sampling       | 53.4     | 17.6    | 46.3  |
| Seq2Seq Baseline               | 67.0     | 7.1     | 77.6  |

Table 2: Equation generation accuracy of our weakly supervised models compared to baselines.

### 6.1 Analyzing WARM

In Table 2, we observe that our model WARM yields far higher accuracy than random baselines with the accuracy values close to 100% on AllArith and EW10K. Thus we are able to more accurately generate equations for a given problem and answer which can then be used to train supervised models.

As has been discussed earlier in Section 4.5, our model WARM w/o Beam Exploration suffers from the problem of converging to local optima because of the sparsity of reward signal. Training our weakly supervised models with beam exploration alleviates the issue to a large extent as we explore the solution space much more extensively and thus sparsity in reward signal is reduced. We see vast improvement in training accuracy by
Ariel have in her garden?\)

\[ X = (4.0 + (2.0 \times 3.0)) \] (Correct)\)

\[ X = (100.0 \times (18.0 / (18.0 + 18.0))) \] (Correct)\)

\[ X = (7.0 + 7.0) \] (Incorrect)\)

\[ X = (7.0 + 6.0) / 3.0, \] but the model predicted \( X = 7.0 + 7.0 \).

In Table 4, we present some of the predictions. As can be seen, the model is capable of producing long complex equations as well. Sometimes, it may reach the correct answer but the equation might be wrong (like in the last example the correct equation would have been \( X = 7.0 \times 6.0 / 3.0 \), but the model predicted \( X = 7.0 + 7.0 \)).

### 6.2 Analysing Semi-supervision through WARM-S

For analyzing semi-supervision, we shuffled and combined AllArith with EW10K. In Table 5 we observe that with less than 10% of fully annotated data, our equation exploration accuracy increases from 45.2% to 92.0% when Beam Exploration is not used. With Beam Exploration, the accuracy goes from 98.8% to 99.3%.

### 7 Conclusion

We have proposed a two step approach for solving math word problems, using only the final answer for supervision. Our weakly supervised approach,
WARM, achieves a reasonable accuracy of 58.5 on the standard Math23K dataset even without leveraging equations for supervision. We also curate and release a large scale MWP dataset, EW10K, in English. We observed that the results are encouraging for simpler MWPs.

8 Acknowledgement

We would like to acknowledge Saiteja Talluri and Raktim Chaki for their contributions in the initial stages of the work.

References

Yefim Bakman. 2007. Robust understanding of word problems with extraneous information. arXiv preprint math/0701393.

Ting-Rui Chiang and Yun-Nung Chen. 2019. Semantically-aligned equation generation for solving and reasoning math word problems. In Proceedings of the 2019 Conference of the North American Chapter of the Association for Computational Linguistics: Human Language Technologies. ACL.

Kyunghyun Cho, Bart van Merriënboer, Caglar Gulcehre, Dzmitry Bahdanau, Fethi Bougares, Holger Schwenk, and Yoshua Bengio. 2014. Learning phrase representations using RNN encoder–decoder for statistical machine translation. In Proceedings of the 2014 Conference on Empirical Methods in Natural Language Processing (EMNLP), pages 1724–1734, Doha, Qatar. Association for Computational Linguistics.

Charles R. Fletcher. 1985. Understanding and solving arithmetic word problems: A computer simulation. Behavior Research Methods, 17:565–571.

Mohammad Javad Hosseini, Hannaneh Hajishirzi, Oren Etzioni, and Nate Kushman. 2014. Learning to solve arithmetic word problems with verb categorization. In Proceedings of the 2014 Conference on Empirical Methods in Natural Language Processing (EMNLP), pages 523–533. ACL.

Danqing Huang, Jing Liu, Chin-Yew Lin, and Jian Yin. 2018. Neural math word problem solver with reinforcement learning. In Proceedings of the 27th International Conference on Computational Linguistics, pages 213–223. ACL.

Danqing Huang, Shuming Shi, Chin-Yew Lin, Jian Yin, and Wei-Ying Ma. 2016. How well do computers solve math word problems? large-scale dataset construction and evaluation. In Proceedings of the 54th Annual Meeting of the Association for Computational Linguistics (Volume 1: Long Papers), pages 887–896, Berlin, Germany. Association for Computational Linguistics.

Diederik Kingma and Jimmy Ba. 2014. Adam: A method for stochastic optimization. International Conference on Learning Representations.

Rik Koncel-Kedziorski, Hannaneh Hajishirzi, Ashish Sabharwal, Oren Etzioni, and Siena Dumas Ang. 2015. Parsing algebraic word problems into equations. Transactions of the Association for Computational Linguistics, 3:585–597.

Nate Kushman, Yoav Artzi, Luke Zettlemoyer, and Regina Barzilay. 2014. Learning to automatically solve algebra word problems. In Proceedings of the 52nd Annual Meeting of the Association for Computational Linguistics (Volume 1: Long Papers), pages 271–281. Association for Computational Linguistics.

Chao-Chun Liang, Kuang-Yi Hsu, Chien-Tsong Huang, Chung-Min Li, Shen-Yu Miao, and Keh-Yih Su. 2016a. A tag-based English math word problem solver with understanding, reasoning and explanation. In Proceedings of the Demonstrations Session, NAACL HLT 2016, pages 67–71. The Association for Computational Linguistics.

Chao-Chun Liang, Kuang-Yi Hsu, Chien-Tsong Huang, Chung-Min Li, Shen-Yu Miao, and Keh-Yih Su. 2016b. A tag-based statistical English math word problem solver with understanding, reasoning and explanation. In Proceedings of the Twenty-Fifth International Joint Conference on Artificial Intelligence, IJCAI'16, page 4254–4255. AAAI Press.

Qianying Liu, Wenyv Guan, Sujian Li, and Daisuke Kawahara. 2019. Tree-structured decoding for solving math word problems. In Proceedings of the 2019 Conference on Empirical Methods in Natural Language Processing and the 9th International Joint Conference on Natural Language Processing (EMNLP-IJCNLP), pages 2370–2379.

Arindam Mitra and Chitta Baral. 2016. Learning to use formulas to solve simple arithmetic problems. In Proceedings of the 54th Annual Meeting of the Association for Computational Linguistics (Volume 1: Long Papers), pages 2144–2153. Association for Computational Linguistics.

Subhro Roy and Dan Roth. 2015. Solving general arithmetic word problems. In Proceedings of the 2015 Conference on Empirical Methods in Natural Language Processing, pages 1743–1752. ACL.

Subhro Roy and Dan Roth. 2017. Unit dependency graph and its application to arithmetic word problem solving. In Proceedings of the Thirty-First AAAI Conference on Artificial Intelligence, page 3082–3088. AAAI Press.

Subhro Roy and Dan Roth. 2018. Mapping to declarative knowledge for word problem solving. Trans. Assoc. Comput. Linguistics, 6:159–172.
Subhro Roy, Shyam Upadhyay, and Dan Roth. 2016. Equation parsing: Mapping sentences to grounded equations. In Proceedings of the 2016 Conference on Empirical Methods in Natural Language Processing, EMNLP 2016, pages 1088–1097. The Association for Computational Linguistics.

Sowmya S Sundaram and Deepak Khemani. 2015. Natural language processing for solving simple word problems. In Proceedings of the 12th International Conference on Natural Language Processing, pages 394–402. NLP Association of India.

Shyam Upadhyay, Ming-Wei Chang, Kai-Wei Chang, and Wen-tau Yih. 2016. Learning from explicit and implicit supervision jointly for algebra word problems. In Proceedings of the 2016 Conference on Empirical Methods in Natural Language Processing, pages 297–306.

Lei Wang, Yan Wang, Deng Cai, Dongxiang Zhang, and Xiaojiang Liu. 2018a. Translating math word problem to expression tree. In Proceedings of the 2018 Conference on Empirical Methods in Natural Language Processing, pages 1064–1069. ACL.

Lei Wang, Dongxiang Zhang, Lianli Gao, Jingkuan Song, Long Guo, and Heng Tao Shen. 2018b. Mathdqn: Solving arithmetic word problems via deep reinforcement learning. In Proceedings of the Thirty-Second AAAI Conference on Artificial Intelligence, AAAI, pages 5545–5552. AAAI Press.

Lei Wang, Dongxiang Zhang, Jipeng Zhang, Xing Xu, Lianli Gao, Bing Tian Dai, and Heng Tao Shen. 2019. Template-based math word problem solvers with recursive neural networks. In Proceedings of the Thirty-Third AAAI Conference on Artificial Intelligence, AAAI, pages 7144–7151. AAAI Press.

Yan Wang, Xiaojiang Liu, and Shuming Shi. 2017. Deep neural solver for math word problems. In Proceedings of the 2017 Conference on Empirical Methods in Natural Language Processing, pages 845–854. ACL.

Ronald J. Williams. 1992. Simple statistical gradient-following algorithms for connectionist reinforcement learning. Mach. Learn., 8(3–4):229–256.

Zhipeng Xie and Shichao Sun. 2019. A goal-driven tree-structured neural model for math word problems. In Proceedings of the 28th International Joint Conference on Artificial Intelligence, pages 5299–5305. AAAI Press.

M. Yuhui, Z. Ying, C. Guangzuo, R. Yun, and H. Ronghuai. 2010. Frame-based calculus of solving arithmetic multi-step addition and subtraction word problems. In 2010 Second International Workshop on Education Technology and Computer Science, volume 2, pages 476–479.