STATUS OF DELAYED-NEUTRON PRECURSOR DATA:
HALF-LIVES AND NEUTRON EMISSION
PROBABILITIES

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ABSTRACT: — We present in this paper a compilation of the present status of experimental delayed-neutron precursor data; i.e. $\beta$-decay half-lives ($T_{1/2}$) and neutron emission probabilities ($P_n$) in the fission-product region ($27 \leq Z \leq 57$). These data are compared to two model predictions of substantially different sophistication: (i) an update of the empirical Kratz–Herrmann formula (KHF), and (ii) a unified macroscopic-microscopic model within the quasi-particle random-phase approximation (QRPA). Both models are also used to calculate so far unknown $T_{1/2}$ and $P_n$ values up to $Z = 63$. A number of possible refinements in the microscopic calculations are suggested to further improve the nuclear-physics foundation of these data for reactor and astrophysical applications.

INTRODUCTION

Half-lives ($T_{1/2}$) and delayed-neutron emission probabilities ($P_n$) are among the easiest measurable gross $\beta$-decay properties of neutron-rich nuclei far from stability. They are not only of importance for reactor applications, but also in the context of studying nuclear-structure features and astrophysical scenarios. Therefore, most of our recent experiments performed at international facilities such as CERN-ISOLDE, GANIL-LISE and GSI-FRS were primarily motivated by our current work on r-process nucleosynthesis. However, it is a pleasure for us to recognize that these data still today may be of interest for applications in reactor physics, a field which we practically left shortly after the "Specialists' Meeting on Delayed Neutrons" held at Birmingham in 1986.

Our motivation to put together this new compilation of $\beta$-decay half-lives and $\beta$-delayed neutron-emission came from recent discussions with T.R. England and W.B. Wilson from LANL about our activities in compiling and steadily updating experimental delayed-neutron data as well as various theoretical model predictions (Pfeiffer et al., 2000). They pointed out to us, that their recent summation calculations of aggregate fission-product delayed-neutron production using basic nuclear data from the early 1990’s (Brady, 1989; Brady and England, 1989, Rudstam 1993) show, in general, that a greater fraction of delayed neutrons is emitted at earlier times following fission than measured. As a consequence, the reactor response to a reference reactivity change is enhanced compared to that calculated with pulse functions derived from measurements (Wilson and England, 2000). Therefore, the use of updated $P_n$ and $T_{1/2}$ values is expected to improve the physics foundation of the basic input data used and to increase the accuracy of aggregate results obtained in summation calculations.

Since the tabulation of Brady (1989) and Rudstam (1993), about 40 new $P_n$ values have been measured in the fission-product region ($27 \leq Z \leq 57$), a number of delayed-neutron...
Figure 1: Chart illustrating the data available in the fission-product region. The new data evaluation represents a significant extension of measured $P_n$ values. Some data in the old data set are not present in the new data set.

branching ratios have also been determined with higher precision, and a similar number of ground-state and isomer decay half-lives of new delayed-neutron precursors have been obtained. These data are contained in our compilation (Table I), and are compared with two of our model predictions: (i) an update of the empirical Kratz-Herrmann formula (KHF) for $\beta$-delayed neutron emission probabilities $P_n$ and $\beta$-decay half-lives $T_{1/2}$ (Kratz and Herrmann, 1973; Pfeiffer, 2000), and an improved version of the macroscopic-microscopic QRPA model (Möller and Randrup, 1990) which can be used to calculate a large number of nuclear properties consistently (Möller et al., 1997). These two models, with quite different nuclear-structure basis, are also used to predict so far unknown $T_{1/2}$ and $P_n$ values in the fission-product region (see Table I).

**EXPERIMENTAL DATA**

Most of the new $\beta$-decay half-lives of the very neutron-rich delayed-neutron precursor isotopes included in Table I have been determined from growth-and-decay curves of neutrons detected with standard neutron-longcounter set-ups. As an example, the presently
Table 1: Experimental $\beta$-decay half-lives $T_{1/2}$ and $\beta$-delayed neutron-emission probabilities $P_n$ compared to three calculations.

| Isotope   | Exp. | $T_{1/2}$ (ms) | $P_n$ (%) | Exp. | KHF  | QRPA-1 | QRPA-2 |
|-----------|------|----------------|-----------|------|------|--------|--------|
| $^{66}_{27}$Co | 180  | 10  | 432  | 260 | 260 | 0.013  | 0.000  | 0.000  |
| $^{67}_{27}$Co | 425  | 20  | 400  | 76  | 120 | 1.132  | 0.156  | 0.061  |
| $^{68}_{27}$Co | 230  | 30  | 168  | 72  | 82  | 2.395  | 0.812  | 0.768  |
| $^{68m}_{27}$Co | 1600 | 300 |       |     |     |        |        |        |
| $^{69}_{27}$Co | 216  | 9   | 238  | 64  | 99  | 6.585  | 1.250  | 0.905  |
| $^{70}_{27}$Co | 120  | 30  | 101  | 43  | 60  | 7.469  | 3.213  | 2.154  |
| $^{72}_{27}$Co | 270  | 50  | 109  | 42  | 60  | 16.650 | 3.468  | 2.666  |
| $^{73}_{27}$Co | 100  | 50  | 58   | 29  | 52  | 13.350 | 5.810  | 3.721  |
| $^{74}_{27}$Co | 63   | 27  | 27   | 33  |     | 28.230 | 6.153  | 3.441  |
| $^{75}_{27}$Co | 35   | 17  | 25   |     |     | 20.700 | 9.858  | 5.962  |
| $^{76}_{27}$Co | 34   | 15  | 20   |     |     | 37.920 | 8.027  | 7.407  |
| $^{77}_{27}$Co | 21   | 11  | 17   |     |     | 29.690 | 11.034 | 24.677 |
| $^{78}_{27}$Co | 20   | 10  | 15   |     |     | 52.810 | 39.341 | 78.079 |
| $^{73}_{28}$Ni | 1570 | 50  | 859  | 814 | 814 | 0.000  | 0.000  | 0.000  |
| $^{74}_{28}$Ni | 840  | 30  | 251  | 2358| 2025| 0.097  | 0.193  | 0.226  |
| $^{75}_{28}$Ni | 500  | 200 | 114  | 890 | 696 | 0.632  | 3.098  | 2.853  |
| $^{76}_{28}$Ni | 641  | 6   | 153  | 428 |     | 2.880  | 6.706  | 3.252  |
| $^{77}_{28}$Ni | 440  | 126 | 920  | 593 |     | 4.108  | 26.207 | 23.952 |
| $^{78}_{28}$Ni | 61   | 372 | 323  |     |     | 4.859  | 36.401 | 38.724 |
| $^{79}_{28}$Ni | 66   | 332 | 326  |     |     | 10.810 | 40.662 | 55.747 |
| $^{73}_{29}$Cu | 3900 | 300 | 1936 | 4092| 2726| 0.029  | 0.045  | 0.018  |
| $^{74}_{29}$Cu | 1594 | 10  | 393  | 1308| 957 | 0.075  | 0.117  | 0.306  |
| $^{75}_{29}$Cu | 1224 | 3   | 458  | 1345| 844 | 2.6    | 5      | 3.982  |
| $^{76}_{29}$Cu | 641  | 6   | 153  | 657 | 428 | 2.4    | 5      | 2.880  |
| $^{76m}_{29}$Cu | 1270 | 300 |     |     |     | 14.330 | 92.135 | 81.490 |
| $^{73}_{29}$Cu | 29   | 40  | 40   |     |     | 61.680 | 98.931 | 98.931 |
| $^{74}_{29}$Cu | 28   | 90  | 170  |     |     | 72.270 | 99.985 | 100.000|
| $^{77}_{29}$Cu | 18   | 40  | 40   |     |     | 61.680 | 98.931 | 98.931 |
| $^{77}_{30}$Zn | 2080 | 50  | 689  | 8888| 8888| 0.000  | 0.000  | 0.000  |
| $^{77m}_{30}$Zn | 1050 | 100 |     |     |     |        |        |        |
| $^{78}_{30}$Zn | 1470 | 150 | 419  | 5022| 15694| 0.001  | 0.027  | 0.000  |
| $^{79}_{30}$Zn | 995  | 19  | 225  | 3925| 3098| 1.3    | 4      | 0.238  | 0.958  | 0.364 |
Table 1: Continued

| Isotope | \(T_{1/2}\) (ms) | \(P_n\) (%) |
|---------|-----------------|-------------|
|         | Exp.  | KHF | QRPA-1 | QRPA-2 | Exp.  | KHF | QRPA-1 | QRPA-2 |
| \(^{80}\)Zn | 545 16 | 255 | 3025 | 2033 | 1.0 5 | 0.668 | 10.889 | 9.980 |
| \(^{81}\)Zn | 290 50 | 64 | 646 | 2401 | 7.5 30 | 2.343 | 21.952 | 60.505 |
| \(^{82}\)Zn | 52 | 211 | 734 | 17.170 | 35.281 | 99.972 |
| \(^{83}\)Zn | 43 | 22 | 818 | 23.300 | 9.816 | 99.982 |
| \(^{84}\)Zn | 43 | 65 | 387 | 25.480 | 33.273 | 100.000 |
| \(^{79}\)Ga | 2847 3 | 999 | 3463 | 2062 | 0.080 14 | 0.119 | 0.110 | 0.055 |
| \(^{80}\)Ga | 1697 11 | 301 | 1575 | 2413 | 0.85 6 | 0.468 | 0.865 | 0.626 |
| \(^{81}\)Ga | 1217 5 | 404 | 1684 | 1852 | 12.1 4 | 3.776 | 6.666 | 6.956 |
| \(^{82}\)Ga | 599 2 | 96 | 496 | 1817 | 22.3 22 | 5.548 | 13.248 | 24.235 |
| \(^{83}\)Ga | 308 1 | 82 | 202 | 891 | 38.7 98 | 33.790 | 76.668 | 98.359 |
| \(^{84}\)Ga | 85 10 | 56 | 22 | 1644 | 70 15 | 28.150 | 15.387 | 99.982 |
| \(^{85}\)Ga | 48 | 71 | 686 | 60.390 | 99.953 | 100.000 |
| \(^{86}\)Ga | 29 | 22 | 409 | 42.400 | 67.188 | 99.994 |
| \(^{87}\)Ga | 29 | 24 | 118 | 73.540 | 99.930 | 100.000 |
| \(^{83}\)Ge | 1850 60 | 249 | 2115 | 70415 | 0.019 | 0.109 | 2.198 |
| \(^{84}\)Ge | 954 14 | 207 | 1046 | 16208 | 10.2 9 | 1.747 | 8.565 | 76.205 |
| \(^{85}\)Ge | 540 50 | 131 | 40 | 9900 | 14 3 | 4.297 | 1.615 | 99.088 |
| \(^{86}\)Ge | 95 | 184 | 2168 | 6.044 | 6.647 | 65.566 |
| \(^{87}\)Ge | 64 | 44 | 1356 | 11.430 | 5.104 | 93.931 |
| \(^{88}\)Ge | 66 | 24 | 118 | 17.480 | 5.595 | 65.740 |
| \(^{89}\)Ge | 39 | 17 | 20 | 19.090 | 15.824 | 9.194 |
| \(^{84}\)As | 4020 30 | 392 | 3548 | 16635 | 0.18 10 | 0.026 | 0.302 | 0.373 |
| \(^{84m}\)As | 650 150 | | | | | | | |
| \(^{85}\)As | 2022 9 | 280 | 2485 | 9431 | 55 14 | 7.935 | 17.599 | 48.990 |
| \(^{86}\)As | 945 8 | 191 | 187 | 5023 | 26 7 | 9.290 | 10.392 | 92.592 |
| \(^{87}\)As | 560 110 | 137 | 269 | 2458 | 17.5 25 | 17.890 | 32.629 | 100.000 |
| \(^{88}\)As | 112 | 61 | 2263 | 23.060 | 35.870 | 99.924 |
| \(^{89}\)As | 59 | 66 | 374 | 29.690 | 90.576 | 100.000 |
| \(^{90}\)As | 43 | 23 | 21 | 30.800 | 41.952 | 22.786 |
| \(^{91}\)As | 44 | 36 | 73 | 58.130 | 99.784 | 100.000 |
| \(^{92}\)As | 27 | 36 | 36 | 40.550 | 90.468 | 90.468 |
| \(^{84}\)Se | 15300 900 | 1063 | 12602 | 12602 | 0.000 | 0.000 | 0.000 |
| \(^{84}\)Se | 5500 140 | 657 | 677 | 1885875 | 0.36 8 | 0.020 | 0.012 | 3.109 |
| \(^{84}\)Se | 1520 30 | 327 | 403 | 12312 | 0.67 30 | 0.193 | 0.231 | 0.986 |
| \(^{85}\)Se | 410 40 | 232 | 114 | 9050 | 7.8 25 | 1.198 | 0.519 | 9.187 |
| \(^{86}\)Se | 161 134 | 1127 | 2.991 | 0.859 | 0.923 |
| \(^{87}\)Se | 270 50 | 104 | 34 | 40 | 21 10 | 8.353 | 1.524 | 3.045 |
| Isotope | \( T_{1/2} \) (ms) | \( P_n \) (%) |
|---------|-----------------|--------------|
| \( ^{79}\text{Br} \) | 55600 | 2.52 |
| \( ^{80}\text{Br} \) | 16360 | 6.55 |
| \( ^{81}\text{Br} \) | 4400 | 13.7 |
| \( ^{82}\text{Br} \) | 1910 | 24.9 |
| \( ^{83}\text{Br} \) | 541 | 31.3 |
| \( ^{84}\text{Br} \) | 343 | 33.7 |
| \( ^{85}\text{Br} \) | 102 | 65 |
| \( ^{86}\text{Br} \) | 70 | 68 |
| \( ^{87}\text{Br} \) | 4492 | 0.011 |
| \( ^{88}\text{Br} \) | 1286 | 1.95 |
| \( ^{89}\text{Br} \) | 200 | 5.7 |
| \( ^{90}\text{Br} \) | 780 | 4.14 |
| \( ^{91}\text{Br} \) | 1840 | 0.033 |
| \( ^{92}\text{Br} \) | 102 | 65 |
| \( ^{93}\text{Br} \) | 70 | 68 |
| \( ^{94}\text{Br} \) | 4400 | 13.7 |
| \( ^{95}\text{Br} \) | 68400 | 100.0 |
| \( ^{96}\text{Br} \) | 16360 | 6.55 |
| \( ^{97}\text{Br} \) | 4400 | 13.7 |
| \( ^{98}\text{Br} \) | 68400 | 100.0 |
| \( ^{99}\text{Br} \) | 16360 | 6.55 |
| \( ^{100}\text{Br} \) | 4400 | 13.7 |
| \( ^{101}\text{Br} \) | 68400 | 100.0 |
| \( ^{102}\text{Br} \) | 16360 | 6.55 |
| \( ^{103}\text{Br} \) | 4400 | 13.7 |
| \( ^{104}\text{Br} \) | 68400 | 100.0 |
| \( ^{105}\text{Br} \) | 16360 | 6.55 |
| \( ^{106}\text{Br} \) | 4400 | 13.7 |
| \( ^{107}\text{Br} \) | 68400 | 100.0 |
| \( ^{108}\text{Br} \) | 16360 | 6.55 |
| \( ^{109}\text{Br} \) | 4400 | 13.7 |
| \( ^{110}\text{Br} \) | 68400 | 100.0 |
| \( ^{111}\text{Br} \) | 16360 | 6.55 |
| \( ^{112}\text{Br} \) | 4400 | 13.7 |
| \( ^{113}\text{Br} \) | 68400 | 100.0 |
| \( ^{114}\text{Br} \) | 16360 | 6.55 |
| \( ^{115}\text{Br} \) | 4400 | 13.7 |
| \( ^{116}\text{Br} \) | 68400 | 100.0 |
| \( ^{117}\text{Br} \) | 16360 | 6.55 |
| \( ^{118}\text{Br} \) | 4400 | 13.7 |
| \( ^{119}\text{Br} \) | 68400 | 100.0 |
| \( ^{120}\text{Br} \) | 16360 | 6.55 |

Table 1: Continued
Table 1: Continued

| Isotope | \( T_{1/2} \) (ms) | \( P_n \) (%) |
|---------|---------------------|---------------|
|         | Exp. | KHF | QRPA-1 | QRPA-2 | Exp. | KHF | QRPA-1 | QRPA-2 |
| \(^{102}\)Rb 37 | 37 | 5 | 36 | 13 | 13 | 18 | 8 | 25.270 | 20.312 | 20.312 |
| \(^{103}\)Rb 37 | 39 | 17 | 17 | 48.740 | 52.410 | 52.410 |
| \(^{104}\)Rb 37 | 27 | 13 | 13 | 37.980 | 58.332 | 58.332 |
| \(^{96}\)Sr 1070 | 10 | 854 | 1079 | 817 | 0.000 | 0.000 | 0.000 |
| \(^{97}\)Sr 429 | 5 | 556 | 1179 | 119 | 0.02 | 1 | 0.109 | 0.232 | 0.232 |
| \(^{98}\)Sr 653 | 2 | 626 | 724 | 724 | 0.40 | 17 | 0.161 | 0.380 | 0.380 |
| \(^{99}\)Sr 269 | 1 | 373 | 359 | 359 | 0.25 | 10 | 0.504 | 0.227 | 0.227 |
| \(^{100}\)Sr 202 | 3 | 289 | 495 | 495 | 1.11 | 34 | 0.168 | 0.437 | 0.437 |
| \(^{96m}\)Y 9600 | 200 | \( \leq 0.08 \) |
| \(^{97m1}\)Y 3750 | 30 | 1148 | 288 | 5030 | 0.045 | 20 | 0.066 | 0.014 | 0.015 |
| \(^{97m2}\)Y 1170 | 30 | \( \leq 0.08 \) |
| \(^{98}\)Y 1470 | 7 | 602 | 167 | 167 | 2.2 | 5 | 3.385 | 0.492 | 0.492 |
| \(^{100}\)Y 735 | 7 | 496 | 318 | 318 | 1.16 | 32 | 0.951 | 0.309 | 0.309 |
| \(^{100m}\)Y 940 | 30 | 235 | 149 | 149 | 2.3 | 8 | 3.936 | 1.212 | 1.212 |
| \(^{101}\)Y 426 | 20 | 325 | 189 | 189 | 5.0 | 12 | 3.689 | 1.191 | 1.191 |
| \(^{102}\)Y 360 | 40 | 352 | 189 | 189 | 27.540 | 36.014 | 36.014 |
| \(^{103}\)Y 224 | 19 | 181 | 89 | 89 | 8.3 | 30 | 8.487 | 3.519 | 3.519 |
| \(^{104}\)Y 180 | 60 | 127 | 30 | 30 | 11.560 | 3.241 | 3.241 |
| \(^{105}\)Y 88 | 48 | 48 | 48 | 20.420 | 14.012 | 14.012 |
| \(^{106}\)Y 66 | 35 | 35 | 35 | 24.010 | 16.345 | 16.345 |
| \(^{107}\)Y 74 | 31 | 31 | 31 | 31.730 | 32.062 | 32.062 |
| \(^{108}\)Y 48 | 23 | 23 | 23 | 25.540 | 36.014 | 36.014 |
| \(^{109}\)Zr 1300 | 100 | 779 | 1866 | 1866 | 0.000 | 0.000 | 0.000 |
| \(^{110}\)Zr 1200 | 300 | 598 | 1839 | 1839 | 0.012 | 0.023 | 0.023 |
| \(^{111}\)Zr 600 | 100 | 289 | 100 | 100 | 0.127 | 0.029 | 0.029 |
| \(^{112}\)Zr 270 | 367 | 367 | 1.476 | 0.614 | 0.614 |
Table 1: Continued

| Isotope | $T_{1/2}$ (ms) | $P_n$ (%) |
|---------|----------------|------------|
| $^{107}$Zr | 144 197 197 | 1.727 1.457 1.457 |
| $^{108}$Zr | 130 181 181 | 5.820 1.796 1.796 |
| $^{109}$Zr | 117 122 122 | 3.968 4.007 4.007 |
| $^{110}$Zr | 98 86 86 | 7.114 5.979 5.979 |
| $^{103}$Nb | 1500 200 3192 9535 9535 | 0.000 0.000 0.000 |
| $^{104}$Nb | 4900 300 1145 2790 2790 | 0.06 3 0.002 0.003 0.003 |
| $^{105}$Nb | 2950 40 | 0.05 3 |
| $^{106}$Nb | 193 17 218 365 365 | 1.7 9 0.241 0.273 0.273 |
| $^{107}$Nb | 98 86 86 | 4.5 3 0.823 0.178 0.178 |
| $^{108}$Nb | 300 9 440 657 657 | 1.7 9 0.241 0.273 0.273 |
| $^{109}$Nb | 190 30 229 377 377 | 31 5 12.180 15.499 15.499 |
| $^{110}$Nb | 170 20 109 253 253 | 40 8 9.959 17.144 17.144 |
| $^{111}$Nb | 113 | 22.060 59.599 59.599 |
| $^{112}$Nb | 69 85 85 | 21.090 64.316 64.316 |
| $^{113}$Nb | 56 56 56 | 55.760 90.942 90.942 |
| $^{109}$Mo | 530 60 484 1802 1802 | 0.002 0.000 0.000 |
| $^{110}$Mo | 300 40 594 1832 1832 | 0.074 0.000 0.000 |
| $^{111}$Mo | 237 978 978 | 0.313 0.025 0.025 |
| $^{112}$Mo | 287 672 672 | 1.233 0.308 0.308 |
| $^{113}$Mo | 133 133 133 | 1.806 3.030 3.030 |
| $^{114}$Mo | 144 113 113 | 4.255 3.881 3.881 |
| $^{115}$Mo | 92 52 52 | 5.330 4.984 4.984 |
| $^{108}$Tc | 5170 70 1515 702 702 | 0.000 0.000 0.000 |
| $^{109}$Tc | 870 40 2010 702 | 0.002 0.000 0.000 |
| $^{110}$Tc | 920 30 663 274 274 | 0.04 2 0.110 0.067 0.067 |
| $^{111}$Tc | 290 20 886 195 195 | 0.85 20 1.367 0.327 0.327 |
| $^{112}$Tc | 290 20 312 142 142 | 1.5 2 1.135 0.797 0.797 |
| $^{113}$Tc | 170 20 392 115 115 | 2.1 3 6.418 4.536 4.536 |
| $^{114}$Tc | 150 30 172 82 82 | 1.3 4 4.423 7.233 7.233 |
| $^{115}$Tc | 210 74 74 | 13.330 19.044 19.044 |
| $^{116}$Tc | 96 46 46 | 11.740 16.381 16.381 |
| $^{117}$Tc | 94 42 42 | 22.990 24.361 24.361 |
| $^{118}$Tc | 66 36 36 | 17.170 25.068 25.068 |
| $^{113}$Ru | 800 50 950 2200 2200 | 0.000 0.000 0.000 |
| $^{114}$Ru | 530 60 1354 491 491 | 0.000 0.009 0.009 |
| $^{115}$Ru | 740 80 47 753 753 | 0.003 1.021 1.021 |
Table 1: Continued

| Isotope | $T_{1/2}$ (ms) | $P_n$ (%) |
|---------|----------------|-----------|
|         | Exp. | KHF | QRPA-1 | QRPA-2 | Exp. | KHF | QRPA-1 | QRPA-2 |
| $^{116}$Ru | 556 | 612 | 612 | 0.053 | 0.002 | 0.002 |
| $^{117}$Ru | 237 | 175 | 175 | 0.287 | 0.369 | 0.369 |
| $^{118}$Ru | 287 | 233 | 233 | 1.029 | 1.120 | 1.120 |
| $^{119}$Ru | 162 | 185 | 185 | 1.712 | 2.616 | 2.616 |
| $^{120}$Ru | 149 | 118 | 118 | 2.599 | 2.945 | 2.945 |
| $^{114}$Rh | 1850 | 50 | 1244 | 2730 | 0.000 | 0.000 | 0.000 |
| $^{114m}$Rh | 1850 | 50 | 1244 | 2730 | 0.000 | 0.000 | 0.000 |
| $^{115}$Rh | 990 | 50 | 476 | 682 | 0.083 | 0.016 | 0.016 |
| $^{116}$Rh | 680 | 60 | 589 | 686 | 0.057 | 0.000 | 0.000 |
| $^{116m}$Rh | 900 | 400 | 857 | 245 | 9.614 | 0.940 | 0.940 |
| $^{117}$Rh | 440 | 40 | 857 | 245 | 0.000 | 0.000 | 0.000 |
| $^{118}$Rh | 346 | 125 | 125 | 1.102 | 0.924 | 0.924 |
| $^{119}$Rh | 411 | 111 | 111 | 4.196 | 3.203 | 3.203 |
| $^{120}$Rh | 177 | 87 | 87 | 5.856 | 3.547 | 3.547 |
| $^{121}$Rh | 215 | 65 | 65 | 11.300 | 7.620 | 7.620 |
| $^{122}$Rh | 108 | 56 | 56 | 9.057 | 8.540 | 8.540 |
| $^{120}$Pd | 500 | 100 | 1267 | 2686 | 0.000 | 0.000 | 0.000 |
| $^{121}$Pd | 428 | 1632 | 1632 | 0.002 | 0.002 | 0.002 |
| $^{122}$Pd | 541 | 1123 | 1123 | 0.039 | 0.044 | 0.044 |
| $^{123}$Pd | 244 | 476 | 476 | 0.224 | 0.313 | 0.313 |
| $^{124}$Pd | 257 | 328 | 328 | 0.552 | 0.656 | 0.656 |
| $^{119}$Ag | 2100 | 100 | 3567 | 985 | 0.000 | 0.000 | 0.000 |
| $^{119m}$Ag | 6000 | 500 | 1230 | 30 | 865 | 490 | 490 | <0.003 | 0.000 | 0.000 | 0.000 |
| $^{120}$Ag | 370 | 40 | 780 | 10 | 1337 | 412 | 412 | 0.076 | 5 | 0.135 | 0.040 | 0.040 |
| $^{120m}$Ag | 780 | 10 | 1337 | 412 | 0.076 | 5 | 0.135 | 0.040 | 0.040 |
| $^{122}$Ag | 550 | 50 | 488 | 190 | 190 | 0.186 | 10 | 0.120 | 0.175 | 0.175 |
| $^{122m}$Ag | 550 | 50 | 488 | 190 | 0.186 | 10 | 0.120 | 0.175 | 0.175 |
| $^{123}$Ag | 296 | 6 | 652 | 219 | 219 | 0.55 | 5 | 1.683 | 0.642 | 0.642 |
| $^{124}$Ag | 172 | 5 | 267 | 117 | 117 | 0.55 | 5 | 1.683 | 0.642 | 0.642 |
| $^{124m}$Ag | 166 | 7 | 288 | 116 | 116 | <0.003 | 0.000 | 0.000 | 0.000 |
| $^{125}$Ag | 107 | 12 | 145 | 118 | 118 | 0.058 | 3.389 | 3.389 |
| $^{126}$Ag | 107 | 12 | 145 | 118 | 118 | 0.058 | 3.389 | 3.389 |
| $^{126m}$Ag | 79 | 3 | 164 | 84 | 84 | 12.210 | 5.785 | 5.785 |
| $^{127}$Ag | 58 | 5 | 107 | 86 | 86 | 5.079 | 6.417 | 6.417 |
| $^{129}$Ag | 46$^\dagger$ | 46$^\dagger$ | 84 | 33 | 33 | 11.760 | 8.990 | 8.990 |

$^\dagger$This experimental data point was added after the manuscript was completed and is therefore not taken into account elsewhere in figures and tables.
Table 1: Continued

| Isotope  | $T_{1/2}$ (ms) | $P_n$ (%) | $T_{1/2}$ (ms) | $P_n$ (%) |
|----------|----------------|-----------|----------------|-----------|
|          | Exp. KHF QRPA-1 QRPA-2 |          | Exp. KHF QRPA-1 QRPA-2 |          |
| $^{129\text{m}}\text{Ag}$ | 30 36 36 | 19.240 67.219 67.219 | 28 40 40 | 68.150 100.000 100.000 |
| $^{130}\text{Ag}$ | 20 34 34 | 61.090 100.000 100.000 |          |          |
| $^{131}\text{Ag}$ | 506 15 798 5146 5146 | 0.000 0.000 0.000 | 370 70 280 2329 2329 | 0.019 0.223 0.223 |
| $^{132}\text{Ag}$ | 340 30 289 924 924 | 0.079 0.752 0.752 | 270 40 135 2284 2284 | 0.766 0.944 0.944 |
| $^{133}\text{Ag}$ | 162 7 138 655 655 | 3.6 10 1.083 2.883 2.883 | 68 3 65 545 545 | 3.5 10 3.855 61.210 61.210 |
| $^{134}\text{Ag}$ | 97 10 56 563 563 | 60 15 20.210 99.976 99.976 |          |          |
| $^{126}\text{Cd}$ | 38 446 446 | 26.530 99.025 99.025 | 32 10 9 509 509 | 60 15 20.210 99.976 99.976 |
| $^{127}\text{Cd}$ | 12200 200 | 0.000 0.000 0.000 | 1600 100 909 552 552 | 0.000 0.000 0.000 |
| $^{128}\text{Cd}$ | 1640 50 |          |          |          |
| $^{129}\text{Cd}$ | 1090 10 1192 567 567 | 0.69 4 |          |          |
| $^{130}\text{Cd}$ | 3670 40 | 0.69 4 |          |          |
| $^{131}\text{Cd}$ | 776 24 527 480 480 | 0.038 3 0.023 0.027 0.027 | 776 24 | 0.038 3 0.023 0.027 0.027 |
| $^{132}\text{Cd}$ | 611 4 525 312 312 | 0.23 7 0.792 0.670 0.670 | 1230 30 | 0.23 7 0.792 0.670 0.670 |
| $^{133}\text{Cd}$ | 1230 30 | 3.6 4 |          |          |
| $^{134}\text{Cd}$ | 278 3 246 216 216 | 1.01 22 0.551 0.985 0.985 | 538 5 | 1.01 22 0.551 0.985 0.985 |
| $^{135}\text{Cd}$ | 280 30 216 146 146 | 2.2 3 3.685 3.817 3.817 | 550 10 | 1.65 18 |
| $^{136}\text{Cd}$ | 350 50 |          |          |          |
| $^{137}\text{Cd}$ | 320 60 |          |          |          |
| $^{138}\text{Cd}$ | 206 4 45 95 95 | 5.2 12 7.627 9.237 9.237 | 180 15 35 139 139 | 87 9 56.760 100.000 100.000 |
| $^{139}\text{Cd}$ | 138 8 32 97 97 | >17 56.760 100.000 100.000 |          |          |
| $^{140}\text{Cd}$ | 41 90 90 | 70.890 100.000 100.000 |          |          |
| Isotope | $T_{1/2}$ (ms) | $P_n$ (%) |
|---------|----------------|-----------|
| $^{136}_{49}$In | Exp. | 30 | 69 | 69 | Exp. | 56.880 | 100.000 | 100.000 |
| $^{137}_{49}$In | 31 | 48 | 48 | 79.100 | 100.000 | 100.000 |
| $^{133}_{50}$Sn | 1450 | 30 | 362 | 9479 | 9479 | 0.0294 | 24 | 0.002 | 0.040 | 0.040 |
| $^{134}_{50}$Sn | 1120 | 80 | 245 | 2196 | 2196 | 17 | 14 | 6.000 | 93.128 | 93.128 |
| $^{135}_{50}$Sn | 450 | 50 | 215 | 2789 | 2789 | 22 | 7 | 12.650 | 98.591 | 98.591 |
| $^{136}_{50}$Sn | 169 | 904 | 904 | 25 | 7 | 9.478 | 88.334 | 88.334 |
| $^{137}_{50}$Sn | 140 | 733 | 733 | 16.100 | 99.360 | 99.360 |
| $^{138}_{50}$Sn | 143 | 460 | 460 | 32.410 | 100.000 | 100.000 |
| $^{139}_{50}$Sn | 81 | 338 | 338 | 18.710 | 99.303 | 99.303 |
| $^{140}_{50}$Sn | 86 | 119 | 119 | 27.800 | 99.997 | 99.997 |
| $^{134}_{51}$Sb | 780 | 60 | 765 | 88293 | 973552 | 0.014 | 2.032 | 14.190 |
| $^{134m}_{51}$Sb | 10220 | 90 | 0.088 | 17 |
| $^{135}_{51}$Sb | 923 | 14 | 302 | 25810 | 27082 | 23.2 | 68 | 19.440 | 99.999 |
| $^{136}_{51}$Sb | 199 | 970 | 4727 | 25.710 | 96.361 | 99.999 |
| $^{137}_{51}$Sb | 168 | 41 | 1599 | 28.350 | 99.997 |
| $^{138}_{51}$Sb | 127 | 176 | 876 | 35.210 | 99.766 | 100.000 |
| $^{139}_{51}$Sb | 80 | 38 | 645 | 36.270 | 99.999 |
| $^{140}_{51}$Sb | 86 | 45 | 84 | 65.230 | 99.378 | 99.378 |
| $^{141}_{51}$Sb | 46 | 45 | 45 | 40.790 | 95.947 | 95.947 |
| $^{142}_{51}$Sb | 50 | 29 | 29 | 62.860 | 99.999 |
| $^{136}_{52}$Te | 17630 | 80 | 1079 | 49938 | 49938 | 1.26 | 20 | 0.128 | 10.451 |
| $^{137}_{52}$Te | 2490 | 50 | 711 | 119887 | 151542 | 2.86 | 24 | 0.440 | 54.228 | 69.222 |
| $^{138}_{52}$Te | 1400 | 400 | 438 | 25690 | 25690 | 6.32 | 21 | 0.978 | 3.683 | 3.683 |
| $^{139}_{52}$Te | 347 | 269 | 5329 | 3.304 | 2.295 | 52.042 |
| $^{140}_{52}$Te | 304 | 282 | 1138 | 3.880 | 2.947 | 7.825 |
| $^{141}_{52}$Te | 213 | 122 | 632 | 4.876 | 8.355 | 44.992 |
| $^{142}_{52}$Te | 200 | 108 | 108 | 7.381 | 10.457 | 10.457 |
| $^{143}_{52}$Te | 105 | 67 | 67 | 10.320 | 16.262 |
| $^{144}_{52}$Te | 117 | 63 | 63 | 14.790 | 22.622 |
| $^{145}_{52}$Te | 77 | 30 | 30 | 14.670 | 21.510 |
| $^{146}_{52}$Te | 75 | 38 | 38 | 18.540 | 35.513 |
| $^{137}_{53}$I | 24130 | 120 | 1995 | 1894022 | 3365424 | 7.02 | 54 | 1.426 | 72.004 |
| $^{138}_{53}$I | 6490 | 70 | 1152 | 2254 | 9020949 | 5.17 | 36 | 1.092 | 85.858 |
| $^{139}_{53}$I | 2282 | 10 | 920 | 2338 | 58145 | 10.8 | 12 | 7.645 | 59.749 |
| $^{140}_{53}$I | 860 | 40 | 518 | 302 | 17216 | 14.4 | 63 | 5.825 | 10.850 |
| $^{141}_{53}$I | 430 | 20 | 521 | 351 | 2347 | 30 | 9 | 14.190 |

**Table 1:** Continued
### Table 1: Continued

| Isotope | $T_{1/2}$ (ms) | $P_n$ (%) |
|---------|----------------|-----------|
| Isotope | Exp. | KHF | QRPA-1 | QRPA-2 | Exp. | KHF | QRPA-1 | QRPA-2 |
| $^{142}$I | 308 | 182 | 1400 | | 10.750 | 46.997 | 99.952 |
| $^{143}$I | 296 | 150 | 150 | | 21.460 | 77.120 | 77.120 |
| $^{144}$I | 194 | 58 | 58 | | 17.370 | 29.379 | 29.379 |
| $^{145}$I | 127 | 57 | 57 | | 38.580 | 46.359 | 46.359 |
| $^{146}$I | 80 | 29 | 29 | | 24.200 | 27.584 | 27.584 |
| $^{147}$I | 75 | 33 | 33 | | 41.940 | 59.854 | 59.854 |
| $^{148}$I | 55 | 30 | 30 | | 35.650 | 81.555 | 81.555 |
| $^{149}$I | 55 | 39 | 39 | | 53.140 | 97.687 | 97.687 |
| $^{141}$Xe | 1730 | 10 | 1290 | 725 | 83645 | 0.046 | 4 | 0.006 | 0.004 | 0.168 |
| $^{142}$Xe | 1220 | 20 | 1113 | 841 | 6634 | 0.42 | 3 | 0.027 | 0.020 | 0.113 |
| $^{143}$Xe | 300 | 30 | 654 | 464 | 4155 | 0.334 | 0.450 | 0.743 |
| $^{144}$Xe | 1150 | 200 | 647 | 291 | 291 | 0.651 | 0.693 | 0.693 |
| $^{145}$Xe | 900 | 300 | 417 | 233 | 233 | 1.510 | 3.805 | 3.805 |
| $^{146}$Xe | 369 | 93 | 93 | | 4.300 | 5.438 | 5.438 |
| $^{147}$Xe | 176 | 126 | 126 | | 6.168 | 9.592 | 9.592 |
| $^{148}$Xe | 119 | 94 | 94 | | 9.234 | 22.899 | 22.899 |
| $^{149}$Xe | 112 | 71 | 71 | | 11.440 | 25.968 | 25.968 |
| $^{150}$Xe | 83 | 33 | 33 | | 14.180 | 24.373 | 24.373 |
| $^{151}$Xe | 0 | 27 | 27 | | 0.000 | 52.705 | 52.705 |
| $^{141}$Cs | 24940 | 60 | 3636 | 9279 | 807516 | 0.038 | 8 | 0.035 | 0.026 | 4.653 |
| $^{142}$Cs | 1689 | 11 | 1731 | 1261 | 882522 | 0.091 | 8 | 0.121 | 0.031 | 39.801 |
| $^{143}$Cs | 1791 | 8 | 1411 | 1750 | 21384 | 1.59 | 15 | 1.588 | 1.067 | 30.965 |
| $^{144}$Cs | 993 | 13 | 692 | 1243 | 18073 | 3.41 | 40 | 1.871 | 3.754 | 66.359 |
| $^{145}$Cs | 582 | 6 | 436 | 412 | 932 | 13.1 | 7 | 4.884 | 8.146 | 17.980 |
| $^{146}$Cs | 323 | 6 | 381 | 784 | 784 | 13.4 | 10 | 10.240 | 36.444 | 36.444 |
| $^{147}$Cs | 225 | 5 | 206 | 234 | 234 | 27.5 | 21 | 8.264 | 16.383 | 16.383 |
| $^{148}$Cs | 158 | 7 | 207 | 165 | 165 | 25.0 | 43 | 21.700 | 34.467 | 34.467 |
| $^{149}$Cs | 112 | 3 | 172 | 219 | 219 | 19.130 | 64.646 | 64.646 |
| $^{150}$Cs | 82 | 7 | 123 | 158 | 158 | 20 | 10 | 18.110 | 71.808 | 71.808 |
| $^{151}$Cs | 109 | 101 | 101 | | 27.690 | 82.526 | 82.526 |
| $^{152}$Cs | 82 | 30 | 30 | | 27.290 | 47.453 | 47.453 |
| $^{153}$Cs | 77 | 56 | 56 | | 38.830 | 89.391 | 89.391 |
| $^{154}$Cs | 58 | 43 | 43 | | 33.580 | 83.477 | 83.477 |
| $^{146}$Ba | 2220 | 70 | 2538 | 2457 | 2457 | 0.000 | 0.000 | 0.000 |
| $^{147}$Ba | 893 | 1 | 1785 | 3559 | 3559 | 0.000 | 0.000 | 0.000 |
| $^{148}$Ba | 602 | 25 | 1054 | 603 | 603 | 0.12 | 6 | 0.062 | 0.076 | 0.076 |
Table 1: Continued

| Isotope  | \( T_{1/2} \) (ms) | \( P_n \) (%) |
|----------|---------------------|-------------|
|          | Exp. | KHF | QRPA-1 | QRPA-2 | Exp. | KHF | QRPA-1 | QRPA-2 |
| \(^{149}\)Ba 56 | 344 7 | 467 | 300 | 300 | 0.79 | 39 | 0.092 | 0.123 | 0.123 |
| \(^{150}\)Ba 56 | 300 | 389 | 438 | 438 | 1.0 | 5 | 0.700 | 0.806 | 0.806 |
| \(^{151}\)Ba 56 | 259 | 310 | 310 | 1.757 | 4.174 | 4.174 |
| \(^{152}\)Ba 56 | 228 | 205 | 205 | 2.763 | 4.534 | 4.534 |
| \(^{153}\)Ba 56 | 158 | 69 | 69 | 4.732 | 4.634 | 4.634 |
| \(^{154}\)Ba 56 | 157 | 94 | 94 | 6.361 | 9.117 | 9.117 |
| \(^{146}\)La 57 | 6270 100 | 3572 | 1212 | 1212 | 0.000 | 0.000 | 0.000 |
| \(^{149m}\)La 57 | 10000 | 100 | | | | | |
| \(^{147}\)La 57 | 4015 | 8 | 5033 | 13458 | 13458 | 0.032 | 11 | 0.004 | 0.008 | 0.008 |
| \(^{148}\)La 57 | 1050 | 10 | 1731 | 15129 | 15129 | 0.153 | 43 | 0.052 | 0.003 | 0.003 |
| \(^{149}\)La 57 | 158 | 30 | 2342 | 2255 | 2255 | 1.46 | 29 | 0.249 | 1.229 | 1.229 |
| \(^{150}\)La 57 | 510 | 30 | 1130 | 570 | 570 | 2.69 | 34 | 0.277 | 0.796 | 0.796 |
| \(^{151}\)La 57 | 778 | 874 | 874 | 1.856 | 12.933 | 12.933 |
| \(^{152}\)La 57 | 451 | 612 | 612 | 3.104 | 28.109 | 28.109 |
| \(^{153}\)La 57 | 342 | 345 | 345 | 7.539 | 50.360 | 50.360 |
| \(^{154}\)La 57 | 228 | 96 | 96 | 9.276 | 20.237 | 20.237 |
| \(^{155}\)La 57 | 184 | 142 | 142 | 17.560 | 59.075 | 59.075 |
| \(^{156}\)La 57 | 112 | 103 | 103 | 18.900 | 60.043 | 60.043 |
| \(^{152}\)Ce 58 | 1100 300 | 1831 | 3169 | 3169 | 0.000 | 0.000 | 0.000 |
| \(^{153}\)Ce 58 | 979 | 1814 | 1814 | 0.000 | 0.018 | 0.018 |
| \(^{154}\)Ce 58 | 775 | 870 | 870 | 0.019 | 0.095 | 0.095 |
| \(^{155}\)Ce 58 | 471 | 174 | 174 | 0.257 | 0.180 | 0.180 |
| \(^{156}\)Ce 58 | 369 | 306 | 306 | 0.697 | 0.734 | 0.734 |
| \(^{152}\)Pr 59 | 3630 120 | 3746 | 965 | 965 | 0.000 | 0.000 | 0.000 |
| \(^{153}\)Pr 59 | 4300 200 | 2607 | 863 | 863 | 0.000 | 0.001 | 0.001 |
| \(^{154}\)Pr 59 | 2300 100 | 1539 | 542 | 542 | 0.048 | 0.169 | 0.169 |
| \(^{155}\)Pr 59 | 1561 493 | 493 | | | | 0.021 | 0.026 | 0.026 |
| \(^{156}\)Pr 59 | 598 | 165 | 165 | 3.776 | 7.694 | 7.694 |
| \(^{157}\)Nd 60 | 5470 110 | 3229 | 7086 | 7086 | 0.000 | 0.000 | 0.000 |
| \(^{157}\)Nd 60 | 1906 | 508 | 508 | 0.000 | 0.000 | 0.000 |
| \(^{158}\)Nd 60 | 1331 | 1313 | 1313 | 0.000 | 0.000 | 0.000 |
| \(^{159}\)Nd 60 | 773 | 772 | 772 | 0.021 | 0.026 | 0.026 |
| \(^{157}\)Pm 61 | 10560 100 | 8084 | 2101 | 2101 | 0.000 | 0.000 | 0.000 |
| \(^{159}\)Pm 61 | 4800 500 | 4496 | 488 | 488 | 0.000 | 0.000 | 0.000 |
| \(^{161}\)Pm 61 | 2623 | 642 | 642 | 0.002 | 0.006 | 0.006 |
| \(^{161}\)Pm 61 | 1561 | 493 | 493 | 0.073 | 0.049 | 0.049 |
Table 1: Continued

| Isotope  | $T_{1/2}$ (ms) | $P_n$ (%) |
|----------|----------------|-----------|
|          | Exp. KHF QRPA-1 QRPA-2 | Exp. KHF QRPA-1 QRPA-2 |
| $^{161}_{61}$Pm | 1065 331 331 | 0.803 0.361 0.361 |
| $^{160}_{62}$Sm | 9600 300 9440 26147 26147 | 0.000 0.000 0.000 |
| $^{161}_{62}$Sm | 4800 4442 11207 11207 | 0.000 0.000 0.000 |
| $^{162}_{62}$Sm | 3099 6821 6821 | 0.000 0.000 0.000 |
| $^{163}_{62}$Sm | 1748 3580 3580 | 0.000 0.000 0.000 |
| $^{164}_{62}$Sm | 1226 2527 2527 | 0.001 0.000 0.000 |
| $^{165}_{62}$Sm | 764 701 701 | 0.066 0.020 0.020 |
| $^{166}_{62}$Sm | 570 624 624 | 0.288 0.469 0.469 |
| $^{162}_{63}$Eu | 10600 1000 9218 40430 40430 | 0.000 0.000 0.000 |
| $^{163}_{63}$Eu | 5219 23562 23562 | 0.000 0.000 0.000 |
| $^{164}_{63}$Eu | 2844 12047 12047 | 0.001 0.000 0.000 |
| $^{165}_{63}$Eu | 1794 7521 7521 | 0.117 0.144 0.144 |

used Mainz 4π neutron detector consists of 64 $^{3}$He proportional counters arranged in three concentric rings in a large, well-shielded paraffin matrix (Böhmer, 1998) with a total efficiency of about 45 %. The majority of the new $P_n$ values were deduced from the ratios of simultaneously measured β- and delayed-neutron activities. It was only in a few cases that γ-spectroscopic data were used to determine the one or other decay property (e.g. independent $P_n$ determinations for $^{93}$Br, $^{100}$Rb and $^{135}$Sn). Most of the new data were obtained at the on-line mass-separator facility ISOLDE at CERN (see, e.g. Fedoseyev et al., 1995; Kratz et al., 2000; Hannawald et al., 2000; Köster, 2000; Shergur et al., 2000). Data in the Fe-group region were obtained at the fragment separators LISE at GANIL (Dörfler et al., 1996; Sorlin et al., 2000) and FRS at GSI (Ameil et al., 1998; Bernas et al., 1998), and at the LISOL separator at Louvain-la-Neuve (Franchoo et al., 1998; Weissman et al., 1999; Mueller et al., 2000). Data in the refractory-element region were measured at the ion-guide separator IGISOL at Jyväskylä (Mehren et al., 1996; Wang et al., 1999). Finally, some new data in the $^{132}$Sn region came from the OSIRIS mass-separator group at Studsvik (Korgul et al., 2000; Mach et al., 2000).

In a number of cases, “old” $P_n$ values from the 1970’s deduced from measured delayed-neutron yields and (questionable) fission yields not yet containing the later well established odd-even effects, were – as far as possible – corrected, as was also done by Rudstam in his 1993 compilation (Rudstam, 1993). In those cases, where later publications explicitly stated that the new data supersede earlier ones, the latter were no longer taken into account. Multiple determinations of the same isotopes performed with the same method at the same facility by the same authors (e.g. for Rb and Cs precursors) were treated differently from the common practice to calculate weighted averages of experimental values, when a later measurement was more reliable than earlier ones. Finally, a number of “questionable” $P_n$ values, in particular those where no modern mass model would predict the $(Q_\beta - S_n)$ window for neutron emission to be positive (e.g. $^{146,147}$Ba and $^{146}$La), are still cited in our Table,
but should in fact be neglected in any application, hence also in reactor calculations.

MODELS

Theoretically, both integral $\beta$-decay quantities, $T_{1/2}$ and $P_n$, are interrelated via their usual definition in terms of the so-called $\beta$-strength function ($S_\beta(E)$) (see, e.g. Duke et al. (1970)).

$$1/T_{1/2} = \sum_{E_i \geq 0} S_\beta(E_i) \times f(Z, Q_\beta - E_i);$$

where $Q_\beta$ is the maximum $\beta$-decay energy (or the isobaric mass difference) and $f(Z, Q_\beta - E_i)$ the Fermi function. With this definition, $T_{1/2}$ may yield information on the average $\beta$-feeding of a nucleus. However, since the low-energy part of its excitation spectrum is strongly weighted by the energy factor of $\beta$-decay, $f \sim (Q_\beta - E_i)^5$, $T_{1/2}$ is dominated by the lowest-energy resonances in $S_\beta(E_i)$; i.e. by the (near-) ground-state allowed (Gamow-Teller, GT) or first-forbidden (ff) transitions.

The $\beta$-delayed neutron emission probability ($P_n$) is schematically given by

$$P_n = \frac{\sum_{Q_\beta} S_\beta(E_i) f(Z, Q_\beta - E_i)}{\sum_{Q_\beta} S_\beta(E_i) f(Z, Q_\beta - E_i)};$$

thus defining $P_n$ as the ratio of the integral $\beta$-strength to states above the neutron separation energy $S_n$. As done in nearly all $P_n$ calculations, in the above equation, the ratio of the partial widths for l-wave neutron emission ($\Gamma^l_n(E_n)$) and the total width ($\Gamma_{tot} = \Gamma^0_n(E_n) + \Gamma_\gamma$) is set equal to 1; i.e. possible $\gamma$-decay from neutron-unbound levels is neglected. As we will discuss later, this simplification is justified in most but not all delayed-neutron decay (precursor – emitter – final nucleus) systems. In any case, again because of the $(Q_\beta - E)^5$ dependence of the Fermi function, the physical significance of the $P_n$ quantity is limited, too. It mainly reflects the $\beta$-feeding to the energy region just beyond $S_n$. Taken together, however, the two gross decay properties, $T_{1/2}$ and $P_n$, may well provide some first information about the nuclear structure determining $\beta$-decay. Generally speaking, for a given $Q_\beta$ value a short half-life usually correlates with a small $P_n$ value, and vice versa. This is actually more that a rule of thumb since it can be used to check the consistency of experimental numbers. Sometimes even global plots of double-ratios of experimental to theoretical $P_n$ to $T_{1/2}$ relations are used to show systematic trends (see, e.g. Tachibana et al. (1998)).

Concerning the identification of special nuclear-structure features only from $T_{1/2}$ and $P_n$, there are several impressive examples in literature. Among them are: (i) the development of single-particle (SP) structures and related ground-state shape changes in the $50 \leq N \leq 60$ region of the Sr isotopes (Kratz, 1984), (ii) the at that time totally unexpected prediction of collectivity of neutron-magic ($N=28$) $^{44}$S situated two proton-pairs below the doubly-magic $^{48}$Ca (Sorlin et al., 1993), and (iii) the very recent interpretation of the surprising decay properties of $^{131,132}$Cd just above $N = 82$ (Kratz et al., 2000; Hannawald et al., 2000).

Today, in studies of nuclear-structure features, even of gross properties such as the $T_{1/2}$ and $P_n$ values considered here, a substantial number of different theoretical approaches are used. The significance and sophistication of these models and their relation to each other
should, however, be clear before they are applied. Therefore, in the following we assign the nuclear models used to calculate the above two decay properties to different groups:

1. **Models where the physical quantity of interest is given by an expression such as a polynomial or an algebraic expression.**

   Normally, the parameters are determined by adjustments to experimental data and describe only a single nuclear property. No nuclear wave functions are obtained in these models. Examples of theories of this type are purely empirical approaches that assume a specific shape of $S_\beta(E)$ (either constant or proportional to level density), such as the Kratz-Hermann formula (Kratz and Herrmann, 1973) or the statistical ”gross theory” of $\beta$-decay (Takahashi, 1972; Takahashi et al., 1973). These models can be considered to be analogous to the liquid-drop model of nuclear masses, and are —again— appropriate for dealing with average properties of $\beta$-decay, however taking into account the Ikeda sum-rule to quantitatively define the total strength. In both types of approaches, model-inherently no insight into the underlying single-particle (SP) structure is possible.

2. **Models that use an effective nuclear interaction and usually solve the microscopic quantum-mechanical Schrödinger or Dirac equation.**

   The approaches that actually solve the Schrödinger equation provide nuclear wave functions which allow a variety of nuclear properties (e.g. ground-state shapes, level energies, spins and parities, transition rates, $T_{1/2}$, $P_{xn}$, etc.) to be modeled within a single framework. Most theories of this type that are currently used in large-scale calculations, such as e.g. the FRDM+QRPA model used here (Möller et al., 1997) or the ETFSI+cQRPA approach (Aboussir et al., 1995; Borzov et al., 1996), in principle fall into two subgroups, depending on the type of microscopic interaction used. Another aspect of these models is, whether they are restricted to spherical shapes, or to even-even isotopes, or whether they can describe all nuclear shapes and all types of nuclei:

   (a) SP approaches that use a simple central potential with additional residual interactions. The Schrödinger equation is solved in a SP approximation and additional two-body interactions are treated in the BCS, Lipkin-Nogami, or RPA approximations, for example. To obtain the nuclear potential energy as a function of shape, one combines the SP model with a macroscopic model, which then leads to the macroscopic-microscopic model. Within this approach, the nuclear ground-state energy is calculated as a sum of a microscopic correction obtained from the SP levels by use of the Strutinsky method and a macroscopic energy.

   (b) Hartree-Fock-type models, in which the postulated effective interaction is of a two-body type. If the microscopic Schrödinger equation is solved then the wave functions obtained are antisymmetrized Slater determinants. In such models, it is possible to obtain the nuclear ground-state energy as $E = \langle \Psi_0 | H | \Psi_0 \rangle$, otherwise the HF have many similarities to those in category 2a but have fewer parameters.

In principle, models in group 2b are expected to be more accurate, because the wave functions and effective interactions can in principle be more realistic. However, two problems
still remain today: what effective interaction is sufficiently realistic to yield more accurate results, and what are the optimized parameter values for such a two-body interaction?

Some models in category 2 have been overparameterized, which means that their microscopic origins have been lost and the results are just parameterizations of the experimental data. Examples of such models are the calculations of Hirsch et al. (1992, 1996) where the strength of the residual GT interaction has been fitted for each element (Z-number) in order to obtain optimum reproduction of known \( T_{1/2} \) and \( P_n \) values in each isotopic chain.

To conclude this section, let us emphasize that there is no “correct” model in nuclear physics. Any modeling of nuclear-structure properties involves approximations of the true forces and equations with the goal to obtain a formulation that can be solved in practice, but that “retains the essential features” of the true system under study, so that one can still learn something. What we mean by this, depends on the actual circumstances. It may well turn out that when proceeding from a simplistic, macroscopic approach to a more microscopic model, the first overall result may be “worse” just in terms of agreement between calculated and measured data. However, the disagreements may now be understood more easily, and further nuclear-structure-based, realistic improvements will become possible.

**Prediction of \( P_n \) and \( T_{1/2} \) Values from KHF**

As outlined above, Kratz and Herrmann in 1972 (Kratz and Herrmann, 1973) applied the concept of the \( \beta \)-strength function to the integral quantity of the delayed-neutron emission probability, and derived a simple phenomenological expression for \( P_n \) values, later commonly referred to as the "Kratz-Hermann Formula"

\[
P_n \approx a \left[ \frac{(Q_\beta - S_n)}{(Q_\beta - C)} \right]^b \quad \text{[\%]} \tag{3}
\]

where \( a \) and \( b \) are free parameters to be determined by a log-log fit, and \( C \) is the cut-off parameter (corresponding to the pairing-gap according to the even and odd character of the \( \beta \)-decay daughter, i.e. the neutron-emitter nucleus).

This KHF has been used in evaluations and in generation of data files (e.g. the ENDF/B versions) for nuclear applications up to present. The above free parameters \( a \) and \( b \) were from time to time redetermined (Mann et al., 1984; Mann, 1986; England et al., 1986) as more experimental data became available. These values are summarized in Table 2. Using the present data set presented in Table 1, we now again obtain new \( a \) and \( b \) parameters from (i) a linear regression, and (ii) a weighted non-linear least-squares fit to about 110 measured \( P_n \) values in the fission-product region. For the present fits, the mass excesses to calculate \( Q_\beta \) and \( S_n \) were taken from the compilation of Audi and Wapstra (1995), otherwise from the FRDM model predictions (Möller et al., 1995). The cut-off parameter \( C \) was calculated according to the expressions given by of Madland and Nix (1988). With the considerably larger database available today, apart from global fits of the whole \( 27 \leq Z \leq 57 \) fission-product region, also separate fits of the light and heavy mass regions may for the first time be of some significance. The corresponding fits to the experimental \( P_n \) values in the different mass regions are shown in Figs. 1–3, and the resulting values of the quantities \( a \) and \( b \) are given in Table 3. It is quite evident from both the Figures and the Tables, that the new fit parameters differ significantly from the earlier ones; however, no clear trend
with the increasing number of experimental data over the years is visible. With respect to the present fits, one can state that – within the given uncertainties – parameter $a$ does not change very much, neither as a function of mass region, nor between the linear regression and the non-linear least-squares fit. However, for the slope-parameter $b$ there is a difference. Here, the least-squares fit consistently results in a somewhat steeper slope (by about one unit) than does the linear regression.

Based on the new non-linear least-squares fit parameters, the KHF was used to predict so far unknown $P_n$ values between $^{27}$Co and $^{63}$Eu in the relevant mass ranges for each isotopic chain. These theoretical values are listed in Table I.

In analogy with the $P_n$ values, the $\beta$-decay half-lives $T_{1/2}$ are to be regarded as “gross”
Table 2: Parameters from fits to the Kratz–Herrmann–Formula from literature. The two sets from Kratz and Herrmann (1973) derive from different atomic mass evaluations.

| Reference                | Parameters |
|--------------------------|------------|
|                          | $a$ [%]    | $b$       |
| Kratz and Herrmann (1973)| 25.        | 2.1 ±0.2  |
| Kratz and Herrmann (1973)| 51.        | 3.6 ±0.3  |
| Mann (1984)              | 123.4      | 4.34      |
| Mann (1986)              | 54.0 +31/-20| 3.44 ±0.51|
| England (1986)           | 44.08      | 4.119     |

Therefore, one can assume that the statistical concepts underlying the Kratz–Herrmann-formula for $P_n$ values can be applied for the description of $T_{1/2}$.

Figure 3: Fits to the Kratz-Herrmann-Formula in the region of “heavy” fission products. For an explanation of symbols, see Fig. 2.
Table 3: Parameters from fits to the Kratz–Herrmann–Formula in different mass regions. The sequence corresponds to Figs. 1 to 3.

| Region          | Lin. regression | Least-squares fit |
|-----------------|-----------------|-------------------|
|                 | $a$ [%] $b$  | $r^2$  | $a$ [%] $b$  | red. $\chi^2$ |
| $29 \leq Z \leq 43$ | 88.23 4.11 | 0.81 | 105.76 5.51 | 80.97 |
|                 | $\pm 37.67$ | $\pm 0.61$ |
| $47 \leq Z \leq 57$ | 84.35 3.89 | 0.86 | 123.09 4.68 | 57.49 |
|                 | $\pm 41.17$ | $\pm 0.38$ |
| $29 \leq Z \leq 57$ | 85.16 3.99 | 0.83 | 80.58 4.72 | 78.23 |
|                 | $\pm 20.72$ | $\pm 0.34$ |

The half-lives are inversely proportional to the Fermi-function $f(Z, E)$, which, in first order, is proportional to the fifth power of the reaction $Q_\beta$-value:

$$T_{1/2} \sim \frac{1}{f(Z, E)} \sim Q_\beta^{-5}$$

Figure 4: Fits to the Kratz-Herrmann-Formula for all fission products. For an explanation of symbols, see Fig. 2.
Table 4: Parameters from fits to $T_{1/2}$ of neutron–rich nuclides.

| lin. regression | least-squares fit |
|-----------------|-------------------|
| $a$ [ms]        | $a$ [ms]          |
| $b$             | $b$ red. $\chi^2$ |
| 2.74E06         | 7.07E05           |
| 4.5             | 4.0               |
| 0.72            | 1.1E04            |
| ±5.33E05        | ±0.4              |

Therefore, in a log-log plot of $T_{1/2}$ versus $Q_\beta$ one expects the data points to be scattered around a line with a slope of about -(1/5).

Pfeiffer et al. (2000) suggested to fit the $T_{1/2}$ of neutron-rich nuclides according to the following expression:

$$T_{1/2} \simeq a \times (Q_\beta - C)^{-b}$$

where the cut-off parameter $C$ is calculated according to the fit of Madland and Nix (1988), and the parameters $a$ and $b$ are listed in Table 4.

The gross theory has, basically, the same functional dependence on the $Q_\beta$-value, but underestimates the $\beta$-strength to low-lying states, which results in too long half-lives. We here compensate for this deficiency by treating the coefficient $a$ as a free parameter to be determined by a fitting procedure. The values obtained are listed in Table 4.

**PREDICTION OF $T_{1/2}$ AND $P_n$ VALUES FROM FRDM-QRPA**

The formalism we use to calculate Gamow-Teller (GT) $\beta$-strength functions is fairly lengthy, since it involves adding pairing and Gamow-Teller residual interactions to the folded-Yukawa single-particle Hamiltonian and solving the resulting Schrödinger equation in the quasi-particle random-phase approximation (QRPA). Because this model has been completely described in two previous papers (Krumlinde et al., 1984; Möller et al., 1990), we refer to those two publications for a full model specification and for a definition of notation used. We restrict the discussion here to an overview of features that are particularly relevant to the results discussed in this paper.

It is well known that wave functions and transition matrix elements are more affected by small perturbations to the Hamiltonian than are the eigenvalues. When transition rates are calculated it is therefore necessary to add residual interactions to the folded-Yukawa single-particle Hamiltonian in addition to the pairing interaction that is included in the mass model. Fortunately, the residual interaction may be restricted to a term specific to the particular type of decay considered. To obtain reasonably accurate half-lives it is also very important to include ground-state deformations. Originally the QRPA formalism was developed for and applied only to spherical nuclei (Hamamoto, 1965; Halbleib et al., 1967). The extension to deformed nuclei, which is necessary in global calculations of $\beta$-decay properties, was first described in 1984 (Krumlinde et al., 1984).

To treat Gamow-Teller $\beta$ decay we therefore add the Gamow-Teller force

$$V_{GT} = 2\chi_{GT} : \beta^{-} \cdot \beta^{+} :$$

(6)
to the folded-Yukawa single-particle Hamiltonian, after pairing has already been incorporated, with the standard choice $\chi_{\text{GT}} = 23 \text{ MeV}/A$ (Hamamoto, 1965; Halbleib et al., 1967; Krumlinde et al., 1984; Möller et al., 1990). Here $\beta^\pm = \sum_i \sigma_i t^\pm_i$ are the Gamow-Teller $\beta^\pm$-transition operators.

The process of $\beta$ decay occurs from an initial ground state or excited state in a mother nucleus to a final state in the daughter nucleus. For $\beta^-$ decay, the final configuration is a nucleus in some excited state or its ground state, an electron (with energy $E_e$), and an anti-neutrino (with energy $E_\nu$). The decay rate $w_{fi}$ to one nuclear state $f$ is

$$w_{fi} = \frac{m_0 c^2}{\hbar} \frac{\Gamma^2}{2\pi^3} |M_{fi}|^2 f(Z, R, \epsilon_0)$$

where $R$ is the nuclear radius and $\epsilon_0 = E_0/m_0 c^2$, with $m_0$ the electron mass. Moreover, $|M_{fi}|^2$ is the nuclear matrix element, which is also the $\beta$-strength function. The dimensionless constant $\Gamma$ is defined by

$$\Gamma \equiv \frac{g}{m_0 c^2} \left( \frac{m_0 c}{\hbar} \right)^3$$

where $g$ is the Gamow-Teller coupling constant. The quantity $f(Z, R, \epsilon_0)$ has been extensively discussed and tabulated elsewhere (Preston, 1962; Gove and Martin, 1971; deShalit and Feshbach, 1974).

For the special case in which the two-neutron separation energy $S_{2n}$ in the daughter nucleus is greater than $Q_\beta$, the energy released in ground-state to ground-state $\beta$ decay, the probability for $\beta$-delayed one-neutron emission, in percent, is given by

$$P_{1n} = 100 \frac{\sum_{0<E_f<Q_\beta} w_{fi}}{\sum_{0<E_f<Q_\beta} w_{fi}}$$

where $E_f = Q_\beta - E_0$ is the excitation energy in the daughter nucleus and $S_{1n}$ is the one-neutron separation energy in the daughter nucleus. We assume that decays to energies above $S_{1n}$ always lead to delayed neutron emission.

To obtain the half-life with respect to $\beta$ decay one sums up the decay rates $w_{fi}$ to the individual nuclear states in the allowed energy window. The half-life is then related to the total decay rate by

$$T_\beta = \frac{\ln 2}{\sum_{0<E_f<Q_\beta} w_{fi}}$$

The above equation may be rewritten as

$$T_\beta = \frac{\hbar}{m_0 c^2} \frac{2\pi^3 \ln 2 \Gamma^2}{\Gamma^2} \sum_{0<E_f<Q_\beta} |M_{fi}|^2 f(Z, R, \epsilon_0) = \sum_{0<E_f<Q_\beta} \frac{B}{|M_{fi}|^2 f(Z, R, \epsilon_0)}$$

with

$$B = \frac{\hbar}{m_0 c^2} \frac{2\pi^3 \ln 2}{\Gamma^2}$$
Figure 5: Calculated $\beta$-strength function for $^{95}$Rb in our standard model (Möller et al., 1997). However, the deformation is not taken from the standard ground-state mass and deformation calculation (Möller et al., 1995). Instead the ground-state shape is assumed spherical, in accordance with experimental evidence. The figure shows the sensitivity of the calculated $P_n$ value to small details of the model. Since there is no strength below the neutron separation energy, the calculated $\beta$-delayed neutron-emission probability is 100%. However it is clear from the figure that just a small decrease in the energy of the large peak just above the neutron binding energy would drastically change the calculated value.

For the value of $B$ corresponding to Gamow-Teller decay we use

$$B = 4131 \text{ s}$$

(13)

The energy released in ground-state to ground-state electron decay is given in terms of the atomic mass excess $M(Z, N)$ or the total binding energy $E_{\text{bind}}(Z, N)$ by

$$Q_{\beta^-} = M(Z, N) - M(Z + 1, N - 1)$$

(14)

The above formulas apply to the $\beta^-$ decays that are of interest here. The decay $Q$ values and neutron separation energies $S_{\nu}$ are obtained from our FRDM mass model when experimental data are unavailable (Möller et al., 1995). The matrix elements $M_{fi}$ are obtained from our QRPA model. More details are provided elsewhere (Möller et al., 1990).

We present here two calculations, QRPA-1 and QRPA-2 of $T_{1/2}$ and $P_n$. They are based on our standard $QRPA$ model described above, but with the following enhancements:
Figure 6: This calculation corresponds to the QRPA-1 model specification. However, this nucleus is known to be spherical although a deformed shape was obtained in the ground-state mass-and-deformation calculation (Möller et al., 1995). Therefore, in our QRPA-2 calculation in Fig. 7, this nucleus is treated as spherical in accordance with experiment.

For QRPA-1:

1. To calculate $\beta$-decay $Q$-values and neutron separation energies $S_{\nu n}$ we use experimental ground-state masses where available, otherwise calculated masses (Möller et al., 1995). In our previous recent calculations we used the 1989 mass evaluation (Audi 1989); here we use the 1995 mass evaluation (Audi et al., 1995).

2. It is known that at higher excitation energies additional residual interactions result in a spreading of the transition strength. In our 1997 calculation each transition goes to a precise, well-specified energy in the daughter nucleus. This can result in very large changes in the calculated $P_n$ values for minute changes in, for example $S_{1n}$, depending on whether an intense, sharp transition is located just below or just above the neutron separation energy (Möller et al., 1990). To remove this unphysical feature we introduce an empirical spreading width that sets in above 2 MeV. Specifically, each transition strength “spike” above 2 MeV is transformed to a Gaussian of width

$$\Delta_{sw} = \frac{8.62}{A^{0.57}}$$

This choice is equal to the error in the mass model. Thus, it accounts approximately for the uncertainty in calculated neutron separation energies.
Figure 7: This calculation corresponds to the QRPA-2 model specification. The calculation is identical to the calculation in Fig. 6 except that the ground-state shape here is spherical.

and at the same time it roughly corresponds to the observed spreading of transition strengths in the energy range 2–10 MeV, which is the range of interest here.

For QRPA-2:

1. In this calculation we retain all of the features of the QRPA-1 calculation and in addition account more accurately for the ground-state deformations which affect the energy levels and wave-functions that are obtained in the single-particle model. The ground-state deformations calculated in the FRDM mass model (Möller et al., 1992), generally agree with experimental observations, but in transition regions between spherical and deformed nuclei discrepancies do occur. In the QRPA-2 calculation we therefore replace calculated deformations with spherical shape, when experimental data so indicate. This has been done for the following nuclei: $^{67-78}$Fe, $^{67-79}$Co, $^{73-80}$Ni, $^{73-81}$Cu, $^{78-84}$Zn, $^{79-87}$Ga, $^{83-90}$Ge, $^{84-91}$As, $^{87-94}$Se, $^{87-96}$Br, $^{92-98}$Kr, $^{91-96}$Rb, $^{96-97}$Sr, $^{96-98}$Y, $^{134-140}$Sb, $^{136-141}$Te, $^{137-142}$I, $^{141-145}$Xe, and $^{141-145}$Cs.

To illustrate some typical features of $\beta$-strength functions we present the strength function of $^{95}$Rb calculated in three different ways in Figs. 5–7.

It is not our aim here to make a detailed analysis of each individual nucleus, but instead to present an overview of the model performance in a calculation of
a large number of $\beta$-decay half-lives. In Figs. 8 and 9 we compare measured and calculated $\beta$-decay half-lives and $\beta$-delayed neutron emission probabilities for the nuclei considered here. To address the reliability in various regions of nuclei and versus distance from stability, we present the ratios $T_{\beta,\text{calc}}/T_{\beta,\text{exp}}$, $P_{n,\text{calc}}/P_{n,\text{exp}}$ versus the quantity $T_{\beta,\text{exp}}$. Because the relative error in the calculated half-lives is more sensitive to small shifts in the positions of the calculated single-particle levels for decays with small energy releases, where long half-lives are expected, one can anticipate that half-life calculations are more reliable far from stability than close to $\beta$-stable nuclei.

Before we make a quantitative analysis of the agreement between calculated and experimental half-lives we briefly discuss what conclusions can be drawn from a simple
visual inspection of Figs. 8 and 9. As functions of $T_{\beta,\text{exp}}$ one would expect the average error to increase as $T_{\beta,\text{exp}}$ increases. This is indeed the case. In addition one is left with the impression that the errors in our calculation are fairly large. However, this is partly a fallacy, since for small errors there are many more points than for large errors. This is not clearly seen in the figures, since for small errors many points are superimposed on one another. To obtain a more exact understanding of the error in the calculation we therefore perform a more detailed analysis.

One often analyzes the error in a calculation by studying a root-mean-square deviation, which in this case would be

$$\sigma_{\text{rms}}^2 = \frac{1}{n} \sum_{i=1}^{n} (T_{\beta,\text{exp}} - T_{\beta,\text{calc}})^2$$

(16)
However, such an error analysis is unsuitable here, for two reasons. First, the quantities studied vary by many orders of magnitude. Second, the calculated and measured quantities may differ by orders of magnitude. We therefore study the quantity \( \log(T_{\beta,\text{calc}}/T_{\beta,\exp}) \), which is plotted in Fig. 8, instead of \((T_{\beta,\exp} - T_{\beta,\text{calc}})^2\). We present the formalism here for the half-life, but the formalism is also used to study the error of our calculated \( P_n \) values.

To facilitate the interpretation of the error plots we consider two hypothetical cases. As the first example, suppose that all the points were grouped on the line \( T_{\beta,\text{calc}}/T_{\beta,\exp} = 10 \). It is immediately clear that an error of this type could be entirely removed by introducing a renormalization factor, which is a common practice in the calculation of \( \beta^- \)-decay half-lives. We shall see below that in our model the half-lives corresponding to our calculated strength functions have about zero average deviation from the calculated half-lives, so no renormalization factor is necessary.

In another extreme, suppose half the points were located on the line \( T_{\beta,\text{calc}}/T_{\beta,\exp} = \)

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### Table 5: Analysis of the discrepancy between calculated and measured \( \beta^- \)-decay half-lives shown in Fig. 8.

| Model | \( n \) | \( M_{r_1} \) | \( M_{r_1}^{10} \) | \( \sigma_{r_1} \) | \( \sigma_{r_1}^{10} \) | \( T_{\beta,\exp}^{\max} \) |
|-------|---------|-------------|--------------|--------------|--------------|-----------------|
| KHF   | 115     | -0.11       | 0.77         | 0.33         | 2.15         | 1               |
| QRPA-1| 115     | -0.02       | 0.95         | 0.50         | 3.14         | 1               |
| QRPA-2| 115     | 0.13        | 1.37         | 0.61         | 4.04         | 1               |
| KHF   | 187     | -0.40       | 0.58         | 0.41         | 2.56         | all             |
| QRPA-1| 187     | -0.06       | 0.87         | 0.59         | 3.88         | all             |
| QRPA-2| 187     | 0.22        | 1.67         | 0.75         | 5.75         | all             |

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### Table 6: Analysis of the discrepancy between calculated and measured \( \beta^- \)-delayed neutron-emission probabilities \( P_n \) values shown in Fig. 9.

| Model | \( n \) | \( M_{r_1} \) | \( M_{r_1}^{10} \) | \( \sigma_{r_1} \) | \( \sigma_{r_1}^{10} \) | \( P_{n,\exp}^{\min} \) |
|-------|---------|-------------|--------------|--------------|--------------|-------------------|
| KHF   | 86      | -0.31       | 0.49         | 0.36         | 2.31         | 1                 |
| QRPA-1| 86      | -0.12       | 0.76         | 0.60         | 4.02         | 1                 |
| QRPA-2| 86      | 0.12        | 1.34         | 0.65         | 4.51         | 1                 |
| KHF   | 118     | -0.29       | 0.51         | 0.44         | 2.76         | all               |
| QRPA-1| 118     | -0.18       | 0.66         | 0.62         | 4.14         | all               |
| QRPA-2| 118     | 0.11        | 1.28         | 0.75         | 5.62         | all               |
10 and the other half on the line $T_{\beta,\text{calc}}/T_{\beta,\text{exp}} = 0.1$. In this case the average of $\log(T_{\beta,\text{calc}}/T_{\beta,\text{exp}})$ would be zero. We are therefore led to the conclusion that there are two types of errors that are of interest to study, namely the average position of the points in Fig. 8, which is just the average of the quantity $\log(T_{\beta,\text{calc}}/T_{\beta,\text{exp}})$, and the spread of the points around this average. To analyze the error along these ideas, we introduce the quantities

\begin{align*}
  r & = T_{\beta,\text{calc}}/T_{\beta,\text{exp}} \\
  r_1 & = \log_{10}(r) \\
  M_{r_1} & = \frac{1}{n} \sum_{i=1}^{n} r_i \\
  M_{r_1}^{10} & = 10^{M_{r_1}} \\
  \sigma_{r_1} & = \left[ \frac{1}{n} \sum_{i=1}^{n} (r_i - M_{r_1})^2 \right]^{1/2} \\
  \sigma_{r_1}^{10} & = 10^{\sigma_{r_1}} \tag{17}
\end{align*}

where $M_{r_1}$ is the average position of the points and $\sigma_{r_1}$ is the spread around this average. The spread $\sigma_{r_1}$ can be expected to be related to uncertainties in the positions of the levels in the underlying single-particle model. The use of a logarithm in the definition of $r_1$ implies that these two quantities correspond directly to distances as seen by the eye in Figs. 8–9, in units where one order of magnitude is 1. After the error analysis has been carried out we want to discuss its result in terms like “on the average the calculated half-lives are ‘a factor of two’ too long.” To be able to do this we must convert back from the logarithmic scale. Thus, we realize that the quantities $M_{r_1}^{10}$ and $\sigma_{r_1}^{10}$ are conversions back to “factor of” units of the quantities $M_{r_1}$ and $\sigma_{r_1}$, which are expressed in distance or logarithmic units.

**DISCUSSION AND SUMMARY**

In Tables 5 and 6 we show the results of an evaluation of the quantities in Eq. (17) for $T_{1/2}$ and $P_n$ corresponding to $\beta$ decay of the nuclei in table 4. In the QRPA calculations the ratio between calculated and measured decay half-lives is close to 1.0. This shows, as pointed out earlier (Möller and Randrup, 1990) that no renormalization of the calculated strength is necessary. The mean deviation between calculated and experimental half-lives is a factor of 2–5 depending on model and half-life cutoff. Also the calculated $P_n$ values agree on the average with the experimental data. Here the mean deviation between calculated and experimental data is a factor of 3–6, again depending on model and half-life cutoff. All half-life calculations agree better with data for shorter half-lives, cf. Fig. 8 and Table 6. Therefore one can expect the models to perform better far from stability than what is indicated by the table. The $\beta$-delayed neutron emission rates are also better calculated in the region of short half-lives and high $P_n$ values, cf. Fig 6 and Table 7. Again, this suggests calculated $P_n$ values are more reliable far from stability than indicated by Table 6.
The KHF results appear more reliable than the QRPA results. This may seem surprising at first, because the KHF has minimal microscopic content compared to the QRPA. However, an advantage of the QRPA is that it provides so much more detail about $\beta$-decay than does the KHF, namely the $ft$ values of the individual decays, and the transition energies associated with those decays. A very detailed discussion of the possible sources of discrepancies between our QRPA results and experimental data is presented in Ref. (Möller and Randrup, 1990). One difficulty the calculations face is that the calculated half-lives depend on the energy of the transitions as $(Q_\beta - E)^5$.

As an example we note that calculated half-lives for $^{95}$Rb, for which $Q_\beta = 9.28$ MeV, change by a factor 1.5 for a change in transition energies by only 0.4 MeV. It is very difficult to reproduce transition energies to this accuracy in a global nuclear-structure model.

For the QRPA-2 calculation we observe that the average of $T_{\beta,\text{calc}}/T_{\beta,\text{exp}}$ is considerably larger than 1, which corresponds to a correct average. One would have a priori assumed that this calculation would be in better agreement with experiment since we substitute calculated deformations for spherical deformations when so indicated by experimental data. However, since we do not include $\beta$-strength due to forbidden transitions in our model, one would indeed expect that calculated half-lives be too low on the average. The non-spherical deformations that occur, contrary to experimental observations, in the QRPA-1 calculations in some sense simulate the missing low-lying forbidden $\beta$-strength. However, a much more satisfying description would be to use correct ground-state deformations and develop some model to account for the strength related to forbidden transitions.

The $P_n$ values calculated in the QRPA-1 are on the average too low. At present we have no clear explanation for this result. An obvious correction to the model is to take competition with $\gamma$ emission into account, in particular for emission of $l_n \geq 3$ neutrons. However, such a correction would further lower the ratio $T_{n,\text{calc}}/T_{n,\text{exp}}$. One may speculate that an accounting for both this effect and forbidden transition strength in QRPA-2 would bring about satisfactory agreement. This possibility need to be investigated.

We feel strongly that in a global, unified nuclear-structure model a single set of constants must be used over the entire chart of the nuclides, otherwise the basic foundation of the model is violated. However, for the purpose of generating the best possible data bases of half-lives and $\beta$-delayed neutron-emission probabilities a complementary approach is reasonable. Just as we feel it is appropriate to use experimental ground-state deformations, experimental single-particle levels, when known, could also be used as the starting point for the QRPA calculations. In practice the situation would be that in some regions, such as near the doubly magic $^{132}$Sn, many half-lives and $P_n$ values would be unknown, but considerable information on single-particle level order and energies would be available. This experimental information could then be taken into account by locally adjusting the single-particle model proton and neutron spin-orbit strengths and the diffuseness of the single-particle well to obtain optimum agreement with the observed single-particle data such as the observed neutron single-particle sequence $f_7/2$, $p_3/2$, $p_1/2$, and $h_9/2$ near $^{132}$Sn. The hope would be that the local agreement would be retained in some limited extrapolation away from the known region. Such a fairly limited extrapolation would be all that is required to reach the
isotopes in the fission-product region where experimental data are not yet available, cf. Fig. [1]. Limited studies along these lines have been undertaken by, for example, Hannawald et al. (2000). Other highly desirable enhancements to the calculations would be to include first-forbidden strength, perhaps first in a gross-theory approach and later from a new microscopic model. The cut-off parameter $C$ in the KHF formula could be taken from the Lipkin-Nogami microscopic calculation instead of from the Madland-Nix macroscopic expression. The energy window $(Q_\beta - S_n)$ could be reduced by 150 to max 500 keV to account for the angular-momentum barrier for emission of $l \geq 3$ neutrons in for example $^{137}$I.

In conclusion we note that we now have available about 40 new experimental $T_{1/2}$ and $P_n$ values in the fission-product region. Data for additional nuclei in this region that are required as input in reactor criticality, astrophysical and other applications are provided from theoretical calculations. The substantial increase in available experimental data since the compilations by Brady (1989) and Rudstam (1993) is expected to have a significant impact on applied calculations.

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