Online Supplement for “An Approach for Analyzing and Managing Flexibility in Engineering Systems Design Based on Decision Rules and Multistage Stochastic Programming”

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Table I: Summary of notation

| Variable | Definition | Variable | Definition |
|----------|------------|----------|------------|
| $T$      | Number of time periods in the planning horizon | $\xi_t$ | Realization of uncertainty / Amount of food waste collected in year $t$ |
| $\eta_t$ | Amount of other organic waste collected in year $t$ | $x_t$ | Option chosen for in time $t$ |
| $\pi$    | Discount rate | $\xi$ | A scenario of uncertainty |
| $\Omega$ | Set of all possible uncertainty scenarios | $\xi^k$ | $k^{th}$ scenario of uncertainty |
| $p^k$    | Probability of scenario $k$ | $X_t$ | Set of feasible options in period $t$ |
| $x$      | Option sequence | $X$ | Set of all feasible option sequences |
| $\delta$ | Decision rule | $\theta$ | Set of parameters of the decision rule |
| $\delta_{\theta}$ | Decision rule with parameter $\theta$ | $D$ | Set of all mappings from $\Omega$ to $X$ |
| $\xi_{[t]}$ | The history of the uncertainty realization up to period $t$ | $r_t(\delta_{\theta}(\xi^k_{[t]}), \xi^k)$ | Profit function in period $t$ in scenario $k$ |
| $\delta_{\theta}(\xi)$ | The sequence of options chosen following the decision rule | $\delta_{\theta}(\xi^k_{[t]}), \xi^k$ | The option chosen at time $t$ in scenario $k$ |
| $r(\delta_{\theta}(\xi), \xi)$ | Total profit function | $\Delta$ | A subset of $D$ |
| $\delta^*$ | The optimal decision rule | $\theta^k$ | A replication of the decision rule parameters in scenario $k$ |
| $\bar{\theta}$ | The average of $\theta^k$ over its $K$ replications | $\lambda$ | Lagrange multipliers |
| $\lambda^k$ | Lagrange multipliers for scenarios $k$ | $L(\delta_{\theta}, \lambda)$ | Lagrangian of the original problem |
| $D(\lambda)$ | Optimal value of the Lagrangian relaxation problem with the Lagrange multiplier $\lambda$ | $Z^{LD}$ | Optimal value of the Lagrangian dual problem |
| $t_i$ | Step size in $i^{th}$ iteration in the gradient | $r_t$ | Profit function |
| Symbol | Description |
|--------|-------------|
| $R_t$  | Revenue function |
| $P_t$  | The penalty function due to the capacity shortage |
| $f_t'$ | Capacity shortage of gasifier |
| $\xi_{t+1,j}$ | Food waste amount in node order $j$ of the following $n$ nodes at time interval $t + 1$ |
| $\alpha$ | The severity level of the current capacity |
| $o_u$  | Capacity of the unit module |
| $h_t^k$ | The amount of capacity to be added |
| $\epsilon'$ | The capacity of gasifier initially installed |
| $\gamma_M$ | The maximum capacity allowed to be installed in gasifier |
| $\gamma_t$ | Capacity of gasifier in year $t$ |
| $T_t$  | Tipping fee function of hybrid WTE system |
| $R_{G_t}$ | Revenue function of gasifier in hybrid WTE system |
| $C_{G_t}$ | Cost function of gasifier in hybrid WTE system |
| $R_t(S_{t-1})$ | Function of total reward starting from period $t$ until the last period |
| $\rho_t$ | Vector of weights of in the linear value function approximation in ADP |
| $I$     | Number of sample paths generated in each iteration in LSPI algorithm |
| $C_t$  | Cost function |
| $f_t$  | Capacity shortage of AD |
| $n$    | Number of jumps from each node |
| $p_j$  | Probability of the state in node order $j$ |
| $\beta$ | The number of modules expanded each time |
| $e_t^k$ | Binary variables indicating whether to expand in year $t$ in scenario $k$ |
| $\epsilon$ | The number of modules initially installed in AD |
| $x_M$  | The maximum number of modules allowed to be installed in AD |
| $x_f$  | The capacity of the baseline inflexible design |
| $\gamma_{1t}, \gamma_{2t}, \gamma_{3t}$ | Coefficients of the linear decision rule for gasifier |
| $R_{A_t}$ | Revenue function of AD in hybrid WTE system |
| $C_{A_t}$ | Cost function of AD in hybrid WTE system |
| $S_t$  | Set of state variables in period $t$ |
| $\phi_t(\cdot)$ | Basis function in the linear value function approximation in ADP |
| $H$    | Number of iterations in LSPI algorithm |

* The variables with superscript $k$ correspond to the variables in scenario $k$. 

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Table II: List of assumptions for WTE system

| Parameters | Value       | Definition                                                                 |
|------------|-------------|-----------------------------------------------------------------------------|
| T          | 9           | Time span of the system considered                                          |
| p          | 70%         | Purity ratio of the food waste feedstock                                    |
| τ          | 10%         | Residue ratio of the AD process                                             |
| $z_1$      | $22,469/tpd$| Revenue from electricity generation per tpd food waste processed in AD unit |
| $z_2$      | $1,336/tpd$ | Revenue from compost per tpd waste processed in AD unit                     |
| $z_3$      | $28,105/tpd$| Revenue from tipping fee per tpd waste collected                            |
| $z_4$      | $700/tpd$   | Transportation cost per tpd food waste collected                            |
| $z_5$      | $75,000/tpd$| Capacity installation cost per unit AD capacity                             |
| $z_6$      | $816/tpd$   | Land rental cost per unit capacity installed                                |
| $z_7$      | $204/tpd$   | Land rental cost per unit capacity reserved                                 |
| $z_8$      | $675/tpd$   | Labor cost per unit capacity of AD unit                                     |
| $z_9$      | $225/tpd$   | Maintenance cost per unit capacity of AD unit                               |
| $z_{10}$   | $28,105/tpd$| Disposal cost per tpd waste disposed in landfill                            |
| π          | 8%          | Discount rate                                                               |
| $o_u$      | 30 tpd      | Capacity of an unit module                                                  |
| $p'$       | 70%         | Purity ratio of the other organic waste feedstock                           |
| $τ'$       | 20%         | Residue ratio of the gasifier                                               |
| $z_{11}$   | $62,678/tpd$| Revenue from electricity generation per tpd waste processed in the gasifier|
| $z_{12}$   | $5,840/tpd$ | Total of labor, admin, maintenance cost per tpd waste treated in the gasifier|
| $z_{13}$   | $2,920/tpd$ | Cost of the RDF process per tpd of waste treated in the gasifier            |
$z_{14}$  S$96,970/tpd  Capital cost of per tpd capacity of the gasifier

*tons per day (tpd) is used to as the unit of waste amount and the capacity of the AD plant.

Table III: Parameters of the main uncertainty drivers in the case study

| Parameters | Definition               | Food waste | Other organic waste |
|------------|--------------------------|------------|---------------------|
| $\mu$      | Annual growth rate       | 14.1%      | 6.0%                |
| $\sigma$   | Volatility               | 16.4%      | 4.1%                |
| $\xi_0/\eta_0$ | Waste amount in year 0 | 191 tpd    | 2,823 tpd           |