Smith-Purcell radiation from a chain of spheres

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Abstract. Smith-Purcell and diffraction radiation were investigated. These types of radiation appear when a charged particle moves close to a conducting target. Spectral and angular distribution of diffraction radiation from the non-periodic chain of spheres is obtained analytically; local field effects are discussed. Analytical expression for the distribution of Smith-Purcell radiation from the periodic chain of spheres is obtained as well. For the first time it has been shown, that Smith-Purcell radiation for such a system is distributed over the cone. The results are investigated for the particles of different sizes, dielectric and metal, and for both ultrarelativistic and nonrelativistic cases.

1. Introduction

Smith-Purcell radiation arises when charged particles fly above a grating at the distance less than \( \beta \gamma \lambda \), where \( \beta = v / c \), \( v \) is the velocity of charged particles, \( \gamma = 1 / \sqrt{1 - \beta^2} \) is the Lorentz factor and \( \lambda \) is a wavelength. This effect for the first time was predicted by I.M. Frank [1] and was experimentally observed by Smith and Purcell in optical wavelength range, using 300 keV electrons [2]. Of late years, close attention has been paid to investigation of Smith-Purcell radiation from one and two-dimensional systems, that consists of small spherical particles [3]-[11]. In [3] radiation emission probability and electron energy loss spectra for finite and infinite strings of Al and silica spheres were obtained by solving Maxwell’s equations in the frames of a multipole expansion approach. The results obtained in Ref. [3] are general enough, but they are expressed in terms of spherical harmonic expansion, which makes the formulas rather complicated. Besides, nonrelativistic case only was analysed in that work.

Another way to describe electromagnetic processes in matter consists in calculation of the local field that acts on a single particle in a physical system. This work is devoted to development of theoretical basis of interaction between a moving charge and a chain of spherical particles characterized by arbitrary dielectric function. Spherical particles are assumed to be of different sizes, dielectric or metal. As dipole approximation had been considered, wavelength of emitted radiation must be much greater than a size of particles: \( \lambda_{rad} >> R \), where \( R \) - is sphere diameter. Electron energy losses are assumed to be negligible to its kinetic energy: \( E_{los} << E_{kin} \).

The local field theory for the chain of spheres was developed. Local field effects relate to physical interaction between scattering particles and also proved to lead to a sharp increase of the radiation intensity at some frequencies [6], [12].
2. Local field effects in diffraction radiation from the non-periodic chain of spheres.

Charge travelling above a system of spherical particles generates the polarisation current density \( j_{\text{mic}}(r',\omega) \), defined at frequency \( \omega \) and at \( r' \). It defines the radiation distribution with respect to an angle and a frequency [14]:

\[
d^2E(n,\omega) = \frac{\alpha^2 d\omega d\Omega}{c^3} \left[ d^2r' \exp\left(-\frac{i\mathbf{n} \cdot \mathbf{r}'}{c}\right) \right] \left[ n, j_{\text{mic}}(r',\omega) \right]^2.
\] (1.1)

In dipole approximation:

\[
j_{\text{mic}}(r',\omega) = -i\omega \alpha(\omega) \sum_p E_{\text{mic}}(R_p,\omega) \delta(r' - R_p) = -i\omega \sum_p d(R_p,\omega) \delta(r' - R_p),
\] (1.2)

where \( E_{\text{mic}}(R_p,\omega) \) is exact microscopic field acting on each spherical particle in the chain positioned at \( R_p \), \( d(R_p,\omega) \) is the particle dipole moment, \( \alpha(\omega) \) is the polarizability of a spherical particle and summation is performed over all particles located at \( R_p \).

Using microscopic Maxwell’s equations exact microscopic field can be obtained as:

\[
E_{\text{mic}}(r,\omega) = E_0(r,\omega) - \frac{1}{2\pi^2} \int \frac{d^2l}{l^2 - \alpha^2} \sum_p \left[ I[l, d(R_p,\omega)] \exp(l(r - R_p)) \right],
\] (1.3)

where \( E_0(r,\omega) \) is the field of external origin or electron self-field, \( l \) is the Fourier transform variable.

The main idea of the local field theory consists in replacement of exact microscopic field \( E_{\text{mic}}(R_p,\omega) \) by some effective field \( E_{\text{loc}}(R_p,\omega) \) called local field. Hence:

\[
E_{\text{loc}}(r,\omega) = E_0(r,\omega) - \frac{1}{2\pi^2} \int \frac{d^2l}{l^2 - \alpha^2} \sum_p \left[ I[l, d(R_p,\omega)] \exp(l(r - R_p)) \right].
\] (1.4)

The value of the local field can be obtained as a result of averaging over the distribution of all the rest particles relative to the certain \( a \)-th particle. It is assumed, that \( f(R_a) \) or \( g(R_a) = Nf(R_a) \) probability density of such a distribution, where \( R_a = R_p - R_a \).

From (1.4) we can derive the expression for the dipole moment of the spherical particle located at \( R_a \):

\[
d(R_a,\omega) = \alpha(\omega) E_0(R_a,\omega) - \frac{\alpha(\omega)}{2\pi^2} \sum_p \int \frac{d^2l}{l^2 - \alpha^2} \left[ I[l, d(R_p,\omega)] \exp(l(R_p - R_a)) \right].
\] (1.5)

Averaging of (1.5) over \( R_a \) and applying Fourier transform give:

\[
d(q,\omega) = \alpha(\omega) E_0(q,\omega) - 4\pi \alpha(\omega) \int \frac{d^2l dR_a}{l^2 - \alpha^2} g(q-1) \left[ I[l, d(q,\omega)] \right],
\] (1.6)

where \( q \) is the Fourier transform variable.

Applying inverse Fourier transform to (1.6) and rearranging lead us to the expression:

\[
A_i d_i(R,\omega) = C_i E_0^\omega(R,\omega),
\] (1.7)

where

\[
A_i(R,\omega) = 2\pi \delta(R_a) \delta_q - 4\pi \alpha(\omega) \int \frac{d^2l d\tilde{q}}{l^2 - \alpha^2} (\tilde{l} \delta_q - l l) g(q - 1) \exp(i \mathbf{q} \mathbf{R}_a).
\] (1.8)

Multiplying (1.8) by \( A_i^\dagger \) and rearranging give:

\[
d_i = \alpha(\omega) A_i^\dagger E_0^\omega(R,\omega).
\] (1.9)
Tensor $A_{ij}^{-}$ defines relation between the dipole moment of the sphere in the chain and the external field acting on this sphere.

Substituting (1.9) into (1.2) leads to the expression for the current density, generated by all the spherical particles at some certain point in the space, characterized by $r'$:

$$j(r', \omega) = -i\omega \sum_{k} \alpha(\omega) \hat{A}^{-} \cdot E(\mathbf{R}_k, \omega) \delta(r' - \mathbf{R}_k)$$  \hspace{1cm} (1.10)

Now let us define analytical expression for $A_{ij}$. Applying Fourier transform to (1.8) for $A_{ij}(\mathbf{q}, \omega)$ we obtain:

$$A_{ij}(\mathbf{q}, \omega) = \delta_{ij} - 4\pi \alpha(\omega) \int \frac{d^1 l}{l^2 - \alpha^2/c^2} g(q-1)(l^2 \delta_y - l_x)$$  \hspace{1cm} (1.11)

where $g(q-1)$ is the Fourier transform of the probability density $g(\mathbf{R}_w)$. Probability density at Oyz plane is defined by two-dimensional Gaussian distribution:

$$\eta(y, z) = \frac{1}{2\pi\Delta^2} \exp\left(-\frac{R^2}{2\Delta^2}\right),$$

with parameter $\Delta$, representing particle coordinate deviation along $y$ and $z$ axes. Since there is no correlation between distant spheres, therefore $f(x_w) = 0$, where $x_w >> n^{-1} (n = \frac{N}{L}$ is linear probability density along the x axis, $L$ is the chain length).

Consequently $g(\mathbf{R}_w)$ can be written as:

$$g(\mathbf{R}_w) = \frac{1}{2\pi\Delta^2} \exp\left(-\frac{R^2}{2\Delta^2}\right)(1 - f(x))$$  \hspace{1cm} (1.12)

where $\mathbf{R}_w$ is located at Oyz plane.

Applying the Fourier transform to (1.12) gives:

$$g(q-1) = \frac{1}{(2\pi)^4 L\Delta^2} \int d^1 R \exp\left(-\frac{R^2}{2\Delta^2}\right)(1 - f(x)) \exp(-i(q-1)\mathbf{R})$$  \hspace{1cm} (1.13)

Substituting (1.13) into (1.11) leads to:

$$A_{ij}(\mathbf{q}, \omega) = \delta_{ij} - \frac{4\pi \alpha(\omega)}{(2\pi)^4 L\Delta^2} \int \frac{d^1 l d^1 R}{l^2 - \alpha^2/c^2} (l^2 \delta_y - l_x)(1 - f(x)) \exp(-i(q-1)\mathbf{R} - \frac{R^2}{2\Delta^2})$$  \hspace{1cm} (1.14)

Integration over $\mathbf{R}$ and $l_x$ gives:

$$A_{ij}(\omega) = \left[1 + \frac{\Theta(\omega)}{L} \right][\delta_{ij} - e_i e_j] + \left[1 - \Theta(\omega)\right] e_i e_j;$$  \hspace{1cm} (1.15)

where

$$\Theta(\omega) = \frac{\alpha(\omega)}{L} \int dl_x [1 - \Phi \left(\frac{l_x^2 + l_y^2}{2} + \frac{l_x^2}{2(L - \Delta)}\right)] \exp\left(\frac{l_x^2}{2(L - \Delta)}\right)$$

where $\Phi$ is Errors function.

Taking into consideration, that:

$$\left[\left(\delta_{ij} - e_i e_j\right)B + e_i e_j C\right]\left(\delta_{ij} - e_i e_j\right) = \delta_{ij},$$

helps us to obtain $A_{ij}^{-}(\omega)$:
Substituting (1.16) into (1.9) gives the expression for the dipole moment of the spherical particle:

\[ d_i (\mathbf{k}, \omega) = \alpha (\omega) \left[ \frac{\delta_{ik} - \epsilon_i \epsilon_k}{1 + \Theta (\omega)} + \frac{\epsilon_i \epsilon_k}{1 - \Theta (\omega)} \right] E_0^\omega (\mathbf{k}, \omega) \]

\[ = \alpha (\omega) \left[ \frac{\delta_{ik} - \epsilon_i \epsilon_k}{1 + \Theta (\omega)} + \frac{3 \epsilon_i \epsilon_k \Theta (\omega)}{1 + \Theta (\omega)} \right] E_0^\omega (\mathbf{k}, \omega) \]

where polarizability of the particle is defined by:

\[ \alpha (\omega) = R^3 \frac{\epsilon (\omega) - 1}{\epsilon (\omega) + 2}, \]

where \( R \) is the diameter of the spherical particle and \( \epsilon (\omega) \) is dielectric conductivity function.

Distribution of the emitted radiation with respect to an angle and a frequency can be written as:

\[ d^2 E (\mathbf{k}, \omega) = (2\pi)^3 \omega^2 \frac{d\Omega}{c^3} \left[ n \cdot j (\mathbf{k}, \omega) \right]^2, \]

where the expression for the current density, generated in the chain, can be derived from (1.17):

\[ j_i (\mathbf{k}, \omega) = -\frac{i\omega}{2\pi} \alpha (\omega) \left[ \sum_{\omega = 1}^{N} \frac{1}{\Theta (\omega)} \delta_i^E (\mathbf{k}, \omega) \right] \left[ \sum_{\omega = 1}^{N} \frac{\Theta (\omega)}{1 - \Theta (\omega)} \right] + \frac{3 \epsilon_i \epsilon_k \Theta (\omega)}{1 + \Theta (\omega)} \right] E_0^\omega (\mathbf{k}, \omega) \]

\[ \exp (-i\mathbf{kR}) \]

where \( N \) is number of the spherical particles in the chain and

\[ \Theta (\omega) = \frac{\epsilon (\omega)}{L} \int d\mathbf{l}_1 d\mathbf{l}_2 d\mathbf{l}_3 f (\mathbf{l}_i) \left[ 1 - \Phi \left( \frac{l_i^2 + l_j^2}{2} \right) \right] \frac{l_i^2}{\sqrt{l_i^2 + l_i^2}} \exp \left( \frac{l_i^2 A^2}{2} \right). \]

The value of \( \theta (\omega) \) can have an imaginary part, so the denominators in (1.19) and (1.20) are not zero. However, they can be very small, that implies the peaks of radiation intensity, defined by the condition:

\[ \begin{cases} 1 + \frac{\theta (\omega)}{2} = 0; \\ 1 - \frac{\theta (\omega)}{2} = 0. \end{cases} \]

In (1.19) and (1.20), according to the local field approach, particles of the chain are considered to interact with each other. We would like to emphasize that taking into account the interaction between the grating elements (spheres, in our case) means, inter alia, taking into consideration the so-called shadowing effect (see the report of Prof. X. Artru at RREPS-2009 and [13]).

### 3. Local field effects in Smith-Purcell radiation from the periodic chain of spheres.

Since (1.19) and (1.20) are obtained we shall consider the case when the chain of spheres is periodic and, as a first approximation, electron self-field only acts on each particle in the chain. Due to this fact...
shadowing effect is not taken into consideration. Other parameters remaining the same: we consider
dielectric or metal spherical particles of different sizes, characterized by arbitrary dielectric function;
electron energy losses are assumed to be negligible to its kinetic energy: \( E_{\text{loss}} \ll E_{\text{kin}} \); wavelength of
emitted radiation must be much greater than the size of the particles: \( \lambda_{\text{rad}} \gg R \).

Assuming that an electron travels with constant velocity \( v \) at a height \( z = h \) above the chain of
spheres, aligned along \( x \) axis, electron charge density can be written as:
\[
\rho_e(r,t) = e \delta(x-vt) \delta(y) \delta(z-h)
\]
(2.1)

Current density generated in the material of the spheres at the moment \( t \) and at the certain point in
the space, characterized by \( r \), can be calculated as:
\[
j_e(r,t) = ev \delta(x-vt) \delta(y) \delta(z-h)
\]
(2.2)

Applying the Fourier transform to (2.2) gives:
\[
\hat{j}_e(q,\omega) = \frac{ev}{2\pi} \int e^{i\omega \cdot q} \delta(x-vt) \delta(y) \delta(z-h) dt dr
\]
(2.3)

As it was proposed, to a first approximation the electron self-field acts only, therefore:
\[
E_{\text{self}}(r,\omega) \approx E_e(r,\omega)
\]
(2.4)

Using microscopic Maxwell’s equations electron self-field can be obtained as:
\[
E_e(q,\omega) = -\frac{4\pi i}{\omega} \mathbf{q} \cdot \left[ \frac{\mathbf{q} \cdot \mathbf{q}}{q^2 - \omega^2/c^2} \right] + \frac{1}{B} \hat{j}_e(q,\omega)
\]
(2.5)

Substituting (2.3) into (2.5) and applying Fourier transform to modified (2.5) lead us to the
expression for the electron self-field acting at the spherical particle located at \( b \):
\[
D^2 E(n,\omega) = \frac{\omega^2}{c^2} \frac{d\omega d\Omega}{d\omega d\Omega} \left[ \mathbf{n} \cdot \mathbf{j}(r',\omega) \right] \sin^2 \theta
\]
(2.7)

where
\[
\mathbf{j}(r',\omega) = -i\omega \sum_n \mathbf{d}(r',\omega) \delta(r' - R_n) = -i\omega \sum_n \alpha(\omega) E_n(r,\omega) \delta(r' - R_n)
\]
(2.8)

Substituting (2.8) into (2.7) gives:
\[
\frac{d^2 E(n,\omega)}{d\omega d\Omega} = \frac{\omega^2}{c^2} \left[ n \sum_n E_n(r,\omega) \exp(-ikR_n) \right]^2
\]
(2.9)

Substituting (2.6) into (2.9) and integrating over the \( r, q, t \); bearing in mind that \( y_s = z_s = 0 \), lead to
Smith-Purcell radiation distribution:
\[
\frac{d^2 E(n,\omega)}{d\omega d\Omega} = \frac{\varepsilon^3 - k^2 R^2}{c^2 \pi^2 B^2 \varepsilon^2(\omega) + c^2 k^2 R^2} \sum_{n=0}^{\infty} \exp \left( i \left( \frac{n}{\beta} - n_s \right) kdnb \right) \left[ \frac{n}{\gamma} K_0 \left( \frac{k}{\beta} \right) + \frac{n}{\gamma} K_1 \left( \frac{k}{\beta} \right) \right]^2
\]
(2.10)

Taking into account, that:
\[
\sum_{j=0}^{N-1} \exp \left( i \left( \beta^{-1} - n_j \right) kdN \right) = \frac{\sin^2 \left( \frac{\left( \beta^{-1} - n_j \right) kd}{2} \right)}{\sin^2 \left( \frac{\left( \beta^{-1} - n_j \right) kd}{2} \right)}, \quad (2.11)
\]

helps to transform (2.10) to:

\[
d^2 E(n, \omega) = \frac{e^2}{c} \frac{k^4 R^4}{\pi^2 \beta^2} \left( \frac{\varepsilon(\omega) - 1}{\varepsilon(\omega) + 2} \right)^2 \sin^2 \left( \frac{\left( \beta^{-1} - n_j \right) kdN}{2} \right) \sin^2 \left( \frac{\left( \beta^{-1} - n_j \right) kd}{2} \right),
\]

\[
\times \left[ n_x K_1 \left( \frac{kh}{\beta \gamma} \right) \frac{1}{\gamma} + n_y K_1 \left( \frac{kh}{\beta \gamma} \right) \frac{1}{\gamma} + n_z K_1 \left( \frac{kh}{\beta \gamma} \right) \right], \quad (2.12)
\]

where \( R \) is the spherical particle diameter, \( \varepsilon(\omega) \) is the dielectric conductivity function of the sphere’s material, \( h \) is the impact parameter or the distance between the chain and the charged particle, \( d \) is the spacing between the spherical particles, \( K_1, K_2 \) are Bessel’s functions of first and second order accordingly.

Expressions for ultrarelativistic and nonrelativistic cases can be performed as well. In nonrelativistic case, when \( kh \ll \beta \ll 1 \), (2.12) can be transformed as:

\[
d^2 E(n, \omega) = \frac{e^2}{c} \frac{k^4 R^4}{\pi^2 \beta^2} \left( \frac{\varepsilon(\omega) - 1}{\varepsilon(\omega) + 2} \right)^2 \sin^2 \left( \frac{\left( \beta^{-1} - n_j \right) kdN}{2} \right) \sin^2 \left( \frac{\left( \beta^{-1} - n_j \right) kd}{2} \right) (1 - n_i^2), \quad (2.13)
\]

where \( n_j, n_i \) are x and z components of the unit vector between \( \mathbf{v} \) and \( \mathbf{k} \).

In ultrarelativistic case, when \( \gamma \gg 1 \), (2.12) can be written as:

\[
d^2 E(n, \omega) = \frac{e^2}{c} \frac{k^4 R^4}{\pi^2 \beta^2} \left( \frac{\varepsilon(\omega) - 1}{\varepsilon(\omega) + 2} \right)^2 \sin^2 \left( \frac{\left( \beta^{-1} - n_j \right) kdN}{2} \right) \sin^2 \left( \frac{\left( \beta^{-1} - n_j \right) kd}{2} \right) \left[ \gamma K_1 \left( \frac{kh}{\beta \gamma} \right) \right], \quad (2.14)
\]

where \( K_1 \) is Bessel’s function of the first order.

Detailed analysis of the results will be performed in the Discussion section.

4. Discussion

Expressions (1.19) and (1.20) for the distribution of diffraction radiation and the current density in the chain of spheres were obtained. They are valid for the particle chains, consisting of different numbers of spherical particles. The possible sharp enhancement of the radiation intensity can be observed at the frequencies determined in (1.22).

Expression for the distribution of Smith-Purcell radiation (2.12) defined in rather general form, what allows us to obtain expressions for ultrarelativistic and nonrelativistic cases. The radiation distribution is proportional to the square of the sphere’s polarizability, where polarizability itself can be written as [14],[15]:

\[
\alpha(\omega) = R^2 \frac{\varepsilon(\omega) - 1}{\varepsilon(\omega) + 2}. \quad (3.1)
\]

Expression for the distribution of Smith-Purcell radiation is valid for the spherical particles of the diameter \( R \), made from materials, characterized by the dielectric conductivity function \( \varepsilon(\omega) \). The formula (3.1) is obtained in the frames of Reyleigh theory and describes polarizability of small particles very well.

Taking into account the condition for the local field effects:
\[
\frac{\sin^2 \left( \frac{p \phi N}{2} \right)}{\sin^2 \left( \frac{p \phi}{2} \right)} \xrightarrow{n \to 1} 2\pi N \sum_m \delta \left( p \phi - 2\pi m \right),
\]

(3.2)

where \( p = \beta^{-1} - n_x \) and \( \phi = kd \), leads us to the definition of the Smith-Purcell relation:

\[
\frac{d}{\lambda} \left( \beta^{-1} - n_x \right) = m, \quad m = 1, 2, \ldots
\]

(3.3)

where \( n_x \) is the x component of the unit vector between \( \mathbf{v} \) and \( \mathbf{k} \), and \( m \) is the order of emitted radiation.

Smith-Purcell radiation from the chain of spheres was analyzed graphically. We are considering two geometries. First can be described as:

\[
\sin \theta \cos \phi, n_x = \sin \theta \sin \phi, n_y = \cos \theta;
\]

the second one as:

\[
\cos \theta', \sin \cos \phi', n_x = \sin \theta' \sin \phi'.
\]

In Figure 1(a, b) the angular distribution of Smith-Purcell radiation from the chain of polyethylene spheres for two different wavelengths is performed. Two narrow peaks on the sides of the broad one correspond to Smith-Purcell radiation. The peak at \( \phi = 0 \) relates to the forward radiation.

Figure 1. Angular distribution of Smith-Purcell radiation for (a) \( \lambda = 700nm \) and (b) \( \lambda = 3mm \) in ultrarelativistic case. Distribution of radiation is presented in first geometry for three different values of the impact parameter and the rest parameters chosen as (a): \( \gamma = 100, d = 700nm, R = 100nm, N = 10, \epsilon_{\text{more}} = 2,37, m = 1, \theta = \frac{\pi}{2}, \phi \in [-\pi, \pi] \); (b): \( \gamma = 100, d = 4nm, R = 0,5nm, N = 10, \epsilon_{\text{more}} = 2,37, m = 1, \theta = \frac{\pi}{2}, \phi \in [-\pi, \pi] \).

Analysis of the radiation distribution lets us assume that Smith-Purcell radiation distributed over the cone and the cone’s axis coincides with the axis x of both geometries. Moreover second geometry provides us with information how exactly the Smith-Purcell radiation is distributed over the cone, what is shown in Figure 2(a, b).
Figure 2. Angular distribution of Smith-Purcell radiation over the cone in second geometry for (a) $\lambda = 700\text{nm}$ and (b) $\lambda = 3\text{mm}$. These distributions correspond to the peaks of Smith-Purcell radiation in the first geometry (a) $\phi = \pi/2$ and (b) $\phi = \arccos(0.25)$.

From (3.3) it is possible to derive the region of validity for (2.12). Smith-Purcell radiation for a nonrelativistic charged particle ($\beta \ll 1$) is possible when:

$$\lambda \sim \frac{d}{\beta m} \gg d,$$

and for an ultrarelativistic charge ($\gamma \gg 1$) when:

$$\frac{\gamma^2 d}{2 m} \ll \lambda \ll \frac{2 d}{m}.$$

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