The Theory of Gravitation in the Space - Time with Fractal Dimensions and Modified Lorents Transformations

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In the space and the time with a fractional dimensions the Lorents transformations fulfill only as a good approach and become exact only when dimensions are integer. So the principle of relativity (it is exact when dimensions are integer) may be treated also as a good approximation and may remain valid (but modified) in case of small fractional corrections to integer dimensions of time and space. In this paper presented the gravitation field theory in the fractal time and space (based on the fractal theory of time and space developed by author early). In the theory are taken into account the alteration of Lorents transformations for case including \( v \approx c \) and are described the real gravitational fields with spin equal 2 in the fractal time defined on the Riemann or Minkowski measure carrier. In the theory introduced the new "quasi-spin", given four equations for gravitational fields (with different "quasi spins" and real and imaginary energies). For integer dimensions the theory coincide with Einstein GR or Logunov- Mestvirichvili gravitation theory.

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I. INTRODUCTION

The general relativity theory (GR) \([1]\) is one of the most known and used the theories of gravitation field. It is elegant, beautiful from physical point of view and explains all experimental facts (truly, it must be point out that others gravitational theories explain them too, as example see \([2]\)). The one of the known lacks of the GR consists in impossibility to include other physical fields (except gravitation field) in the frame of GR, i.e. in impossibility to look on other fields as on the characteristics of metric of Riemann geometry. The equations of GR based on assumption that all systems of frame are equivalent, the absolute systems of references are absent thou the space and the time in the presence of gravitational fields are inhomogeneous. In the fractal theory of time and space (see \([3],[4]\)) all the physical fields are included in the fractional dimensions of the time and the space, the time and the space fields are real fields and any system of frames is an absolute system of frame, the Lorents transformations and known physical laws fulfilled as not rigorously laws and are only very good approach for describing characteristics of the time and the space. For the case when fractional corrections to integer dimensions are small all the equations of the fractal theory practically coincide with equations of known physical theories or may give only non-sufficient corrections. Here we present the theory for the fields with spin equal two in the fractal time and space on the base of the equations given in \([3],[12],[4]\) (in the paper \([12]\) used the modified Lorents transformations with corrections given by the fractal theory of time for moving with speed of light\((\frac{d}{dt})\) for constructing the theories of scalar, vector and spinor field). The equation of this theory differs from equations of the theories \([13],[8]\) because the modified Lorents transformations give four systems of gravitational equations for four different gravitational fields (two with real energies and two with imaginary energies) instead of only one system in theories \([1],[2]\).

II. GENERALIZED FRACTIONAL DERIVATIVES

Following \([3],[4]\), we will consider both time and space as the initial real material fields existing in the world and generating all other physical fields by means of their fractional dimensions. For the gravitational fields equations construction may be used the principle of minimum fractional dimensions functional as it was made in \([3],[4]\). The aim of the paper is to include (in addition to Lagrangians used in \([3],[4]\)) in Lagrangians of fields the additional members that give corrections to Lorents transformations \([3],[4],[12]\) in the domain of velocities \(v \approx c\). For describing the functions defined on multifractal sets it is necessary to introduce the generalized fractional derivatives (see \([5],[6],[9]\)) and replace by GFD the usual derivatives (see \([5],[6],[9]\)) which are suitable to describe the dynamics of functions defined on multifractal sets of time and space (generalized fractional derivatives (GFD), see \([5],[6],[9]\)) and replace by GFD the usual derivatives and integral respect to time and space coordinates in the fractional dimensions functional. These functionals GFD are simple and natural generalization of the Riemann-Liouville fractional derivatives and integrals:

\[
D^\alpha_{+}\delta f(t) = \left( \frac{d}{dt} \right)^n \int_0^t \frac{f(t')dt'}{\Gamma(n-d(t'))(t-t')^{d(t')-n+1}} \tag{1}
\]
\[ D^d_{-t} f(t) = (-1)^n \left( \frac{d}{dt} \right)^n \int_t^b \frac{f(t')dt'}{\Gamma(n-d(t'))(t-t')^{d(t')-n+1}} \]  

where \( \Gamma(x) \) is Euler’s gamma function, and \( a \) and \( b \) are some constants from \([0, \infty)\). In these definitions, as usually, \( n = \{d\} + 1 \) where \( \{d\} \) is the integer part of \( d \) if \( d \geq 0 \) (i.e. \( n-1 \leq d < n \)) and \( n = 0 \) for \( d < 0 \). If \( d = \text{const.} \), the generalized fractional derivatives (GFD) \( [9] \) coincide with the Riemann - Liouville fractional derivatives \((d \geq 0)\) or fractional integrals \((d < 0)\). When \( d = n + \varepsilon(t), \varepsilon(t) \to 0 \), GFD can be represented by means of integer derivatives and integrals. There are relations between GFD and ordinary derivatives for integer values. If \( d_{n} \to n \) where \( n \) is an integer, \( ( \text{for} \ \text{example} \ d_{\alpha} = 1 + \varepsilon(r(t), t), \alpha = r, t) \), in that case it is possible represent GFD by approximate relations (see [3], [9]).

\[ D^{1+\varepsilon}_{r,x_{\alpha}} f(r(t), t) = \frac{\partial}{\partial x_{\alpha}} f(r(t), t) + \frac{\partial}{\partial t} [\varepsilon(r(t), t)f(r(t), t)] \]  

For \( n = 1, \text{i.e.} \ d = 1 + \varepsilon, |\varepsilon| << 1 \) it is possible to obtain:

\[ D^{1+\varepsilon}_{r,x_{\alpha}} f(t) = \frac{\partial}{\partial t} f(t) + a \frac{\partial}{\partial t} [\varepsilon(r(t), t)f(t)] \]  

where \( a \) is constant and defined by the choice of the rules of regularization of integrals ([3]-[2]) (for more detail see [3], [4], [5]). The selection of the rule of regularization that gives a real additives for usual derivative in \([2]\) yield \( a = 0.5 \) for \( d < 1 \) and \( a = 1.077 \) for \( d > 1 \) [3]. The functions under integral sign in \([3]-[5]\) we consider as the generalized functions defined on the set of the finite functions [3]. The notions of GFD, similar to \([1]-[2]\), also defined and for the space variables \( r \).

In the definitions of GFD \([1]-[2]\) the connections between fractal dimensions of time \( d_i(r(t), t) \) and characteristics of physical fields (say, potentials \( \Phi_i(r(t), t), \alpha = 1, 2, .. \)) or densities of Lagrangians \( L_i \) are determined, following [3], by the relation

\[ d_i(r(t), t) = 1 + \sum \beta_i L_i(\Phi_i(r(t), t)) \]

where \( \beta_i \) are densities of energy of physical fields, \( \beta_i \) are dimensional constants with physical dimension of \( [L_i]^{-1} \) (it is worth to choose \( \beta_i \) in the form \( \beta_i = a^{-1} \beta_i \) for the sake of independence from regularization constant). The definition of time as the system of subsets and definition the FD \( d \) (see \([2]\)) connects the value of fractional (fractal) dimension \( d_i(r(t), t) \) with each time instant \( t \). Thus \( d_i \) depends both on time \( t \) and coordinates \( r \). If \( d_i = 1 \) the absence of physical fields) the set of time has topological dimension equal to unity. The multifractal model of time allows, as will be shown early ([3], [8]) , to consider the divergence of energy of masses moving with speed of light in the special relativity theory as the result of the requirement of rigorous validity of the conservation laws in the presence of physical fields that is valid only for closed systems. In our theory there are an approximate fulfillment of conservation laws as in the fractal time of theory and space the Universe is treated as an open system defined on the measure carrier (the closed system is the Universe together with the measure carrier).

The gravitational equations may be received by using the principle of minimum to functional of fractal dimensions with dependencies of GFD \([3]-[5]\) or by replacing the ordinary derivatives in proper physical equations by GFD \([3]\) (the results will be the same). In this paper we generalize the theory of gravitational fields \([3]-[5]\) by including in the equations received in these papers the results of paper \([14]\) that took into account the modified Lorentz transformations.

### III. THE BASED EQUATIONS FOR PHYSICAL FIELDS WITH SPIN EQUAL TWO

For generalization of the gravitational field theory presented in \([3]-[5]\) by means of construction the equations in the fractal time and space with modified Lorentz transformations we write at first the field equations for scalar function \( \Phi \) of paper \([14]\)

\[ (\Box^2 - 4a_0^2 \frac{\partial^2}{\partial t^2}) \Phi(r,t) = E_0^2 \Phi(r,t) \]  

where \( \Box \) is D’Alamber operator \((\Box = \Delta - \frac{\partial^2}{\partial t^2}, \Delta \) is Laplasian), \( \Phi \) are functions describing particles or fields. For scalar \( \Phi \) equation \([3]\) describes the scalar field in the space with fractal dimensions of time that originate the all physical fields \([a_0 \neq 0]\) . The corrections in \([3]\) to the usual D’Alamber equation are the result of modifying the Lorentz transformations. The last is consequences of fractal nature of time. Now we may write the equations with taken into account both phenomenon: the influences of multifractal structure of time ( use in the equations the generalized Riemann-Liouville fractional derivatives (GFD) instead of ordinary derivatives) and corrections to equations from modified Lorentz transformations received in \([14]\). In that case the equation \([3]\) take the form

\[ (D^2_{-t}D^2_{+t} - \Delta)^2 \Phi(r,t) = [E_0^2 + 4a_0^2(D_{-t}^2D_{+t}^2)^2] \Phi(r,t) \]  

where functionals \( D^2_{-t}D^2_{+t} \) defined by \([1]-[2]\) and \([3]\). It is useful to receive from these equations of the fourth order \([2]\) the four equations of the second order. It is possible if use the Dirac type four-component matrices \( \alpha_i \) where \((i = 1, 2, 3, 4) : \alpha_i \alpha_j + \alpha_j \alpha_i = \delta_{ij} \). Than we have four equations of second order for the fields both
with real energies (two equations) and with imaginary energies (two equations):

\[
(D^\mu_{\nu t}D^{\nu}_{\mu t} - \Delta)I\Phi_{\nu}(r, t) = [\alpha_1 E^\mu_0 + \\
+ 2a_0\alpha_2(D^\mu_{\nu t}D^{\nu}_{\mu t})]\Phi_{\nu}(r, t)
\]

where \(\alpha_1\) and \(\alpha_2\) may be chosen as in [13].

\[
\alpha_1 = \begin{pmatrix}
1 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 \\
0 & 0 & 1 & 0 \\
0 & 0 & 0 & -1
\end{pmatrix}, \quad \alpha_2 = \begin{pmatrix}
0 & 0 & 0 & 1 \\
0 & 0 & 1 & 0 \\
0 & 1 & 0 & 0 \\
1 & 0 & 0 & 0
\end{pmatrix}
\]

and \(I\) is the 4-component unit matrix. This situation is the same as for vector, spinor or Proca fields described in the [14].

**IV. THE SELECTION OF THE MODELS OF GRAVITY**

The generalization of gravitational theory in fractal space on the base of equations (8) is possible by using the two models of describing the gravity fields:

a) The first model based on Einstein representations the space-time as the continuous set of points described by Riemannian geometry. In that case in the fractal theory of time and space the measure carrier must be defined as the Riemannian sets with integer dimensions. On this sets we construct the fractal sets of time and space with dimensions defined by Lagrangian densities of energy for all physical fields (see (3)). On the fractal sets the laws of physics will broken because of the sets of space and time become the open systems (see statistical theory of open system in [?] ) connected with the measure carrier. An arbitrary theories for any physical fields will include (in the fractal space) the influences of Riemannian geometry because of Riemannian carrier of measure. So the Riemannian geometry of fields will be not consequences of characteristics of fields (for example, gravitational field with spin equal two), but characteristics of measure carrier. In that case we may describe the gravitational field (if take into account the influences of all physical fields on behaviour of gravitational field) by using the principle of minimum of fractal dimensions functional and Euler equations with generalized fractional derivatives (GFD) as we introduced in the equations of paper [3], [4], [12]. In this model it is necessary to use the covariant derivatives in the fractal Riemannian time-space as it was made in [3], [12] and improve them by introducing the four equations for the gravitational field tensor \(\Phi^{\mu\nu}\) and introducing the corrections from modified Lorentz transformations (the introducing of the last corrections was demonstrated above). In that case the new gravitational equations will describe four fields: the two gravitational fields with real energies (if use the analogy with Dirac theory these fields may have different sign of the gravitational charges) and the two gravitational fields with imaginary energies. All these fields exist in the Riemann time-space with fractal dimensions. In the case of integer dimensions of time and space the received equations (and the theory) coincide with equations of ordinary GR. In this paper we consider also the another gravity model with more analogy to Logunov-Mestvirichvili model of gravity [2].

b) The second model for describing the gravitation fields in the fractal time and space (by GFD using ) consists in the selection of other the measure carrier. The selection the measure carrier is: the measure carrier selected as the flat four-dimensions pseudo Euclidean Minkowski time-space. Our fractal Universe in that case defined as the multifractal sets on the pseudo Euclidean Minkowski time-space (the model of a measure carrier selection in the case a) was the model of Riemannian time-space). Let us select (as a base) the system of reference which coincide for FD equal to unit with Cartesian system of reference (we remind that in the fractal theory of time and space there are only an absolute systems of reference but if FD of time and space near integer the principle of equivalence of all reference systems is valid with grate exactness). The equations of the gravitation fields in that case will be similar to the equations of the theory [2] in which all derivatives replaced on GFD and metric tensor \(\gamma^{\mu\nu}\) are contained the functions (functionals) originated by fractional dimensions (i.e. it must be the function of \(L\) where \(L\) is Lagrangians energy densities of gravitation fields). Beside these corrections must be taken into account the corrections (the main corrections) from modifying of Lorentz transformations and the presence as result of it of four sorts of the gravitational fields (two with real and two with imaginary energies). The differences the theory of gravitation based on above statements from the theory in that case are: 1) the real gravitational fields in the theory originated by fractional dimensions of time, but not postulated as in [2]; 2) the ordinary derivatives replaced by GFD for taking into account the FD of time; 3) we took into account also the modification of Lorentz transformations for time with fractional dimensions; 4) the time-space is fractal and only the measure carrier is Minkowski time-space with integer dimensions..

**V. THE GRAavitATIONAL EQUATIONS DEFINED ON MINKOWSKI TIME-SPACE MEASURE CARRIER**

It is convenient to use the designations of the theory for the equations construction of gravitation fractal theory in the multifractal Universe defined on the pseudo Euclidean Minkowski time-space (this space may have any dimensions but integer). The equations for gravitation field tensor \(\tilde{\Phi}^{\mu\nu} = \sqrt{-\gamma} \cdot \Phi^{\mu\nu} (\gamma = det(\gamma_{\mu\nu}), \tilde{\Phi}^{\mu\nu} =
\[ \sqrt{-\gamma} \cdot t^{\mu\nu}, \quad L - \text{is a Lagrangians density of physical fields (see in details \cite{2,3}) have form} \]
\[
\gamma^{\alpha\beta} D_{i,\alpha}^{d_i} D_{j,\beta}^{d_j} \Phi^{\mu\nu} = \alpha_1 b^2 \Phi^{\mu\nu} + \lambda \tilde{\Phi}^{\mu\nu}(\gamma^{\mu\nu}, \Phi_A) + 2\alpha_0 \gamma^{44} D_{i,\alpha}^{d_i} D_{j,\beta}^{d_j} \Phi^{\mu\nu} + \frac{\delta L}{\delta \gamma^{\mu\nu}} \]
\[ \tilde{\Phi}^{\mu\nu} = - \frac{2}{\lambda} \Phi^{\mu\nu} \]
\[ D_{ij,\mu}^{d_i} \Phi^{\mu\nu} = 0 \quad (10) \]

In equations \( \Phi^{\mu\nu} \) the tensors of gravitational fields \( \Phi^{\mu\nu} \) included in the form of column

\[ \Phi^{\mu\nu} = \begin{pmatrix} \Phi_{1,\mu}^{\nu} \\ \Phi_{2,\mu}^{\nu} \\ \Phi_{3,\mu}^{\nu} \\ \Phi_{4,\mu}^{\nu} \end{pmatrix} \]

The metric tensor \( \gamma^{\alpha\beta} \) is a function (or functional) of the tensors of gravitational fields \( \Psi^{\mu\nu} \) and defined on the fractal pseudo Euclidean Minkowski time-space. These dependencies the \( \gamma^{\alpha\beta} \) from \( \Phi^{\mu\nu} \) originated by dependencies the interval \( dS^2 \) from \( \Phi^{\mu\nu} \) because the last in the fractal Minkowski space has the complicated form (see \( \Phi \)) and may be expand in powers of \( \Phi^{\mu\nu} \). For expansion \( \gamma^{\mu\nu} \) in powers of \( \Phi^{\mu\nu} \) obtain

\[ \gamma^{\mu\nu}(\Phi^{\alpha\beta}) = \gamma^{\mu\nu} + \sum A_{\alpha\beta}^{\mu\nu} \Phi^{\alpha\beta} + ... \]

where \( A_{\alpha\beta}^{\mu\nu} \) are coefficients of expansion and depend at coordinate and time. So if it is possible when for large distances from center of gravity \( r_0 \) \( (r_0 << r) \) to limit oneself by two first members of \( \Phi^{\alpha\beta} \) we may write

\[ \gamma^{\mu\nu}(\Phi^{\alpha\beta}) \approx \gamma^{\mu\nu} + \Phi^{\mu\nu} \]

The equation \( \gamma^{\mu\nu} \) describes the boundary conditions for \( \Phi^{\mu\nu} \) on the Universe surface and \( b \) is a mass of graviton and play role of parameter expanding the domain of existence of GFD.

**VI. GRAVITATIONAL FIELDS DEFINED ON THE RIEMANN SPACE MEASURE CARRIER**

As the carrier of a measure is the Riemann space with an integer dimensions we obtain the determination for covariant derivatives in Riemann space with fractional dimensions

\[ D_{i,\alpha}^{d_i} t^{\mu\nu} = D_{i,\alpha}^{d_i} t^{\mu\nu} + \gamma^{\alpha\beta} t_{i,\beta}^{\mu\nu} \quad i = t, r \quad (12) \]

where \( t^{\mu\nu} \) is the energy-momentum tensor and \( \gamma^{\mu\nu} \) is the metric tensor of the Riemann "four-dimension space with fractional dimensions", \( D_{i,\alpha}^{d_i} \) are GFD, \( \gamma^{\alpha\beta} \) are Christoffel symbols

\[ \gamma^{\alpha\beta} = \frac{1}{2} \gamma^{\gamma\delta}(D_{i,\alpha}^{d_i} \gamma_{\beta}^{\gamma\delta} + D_{i,\beta}^{d_i} \gamma_{\alpha}^{\gamma\delta} + D_{i,\gamma}^{d_i} \gamma_{\alpha\beta}^{\gamma\delta} + D_{i,\delta}^{d_i} \gamma_{\alpha\beta}^{\gamma\delta}) \quad (13) \]

The equations for gravitation field tensor \( \Phi^{\mu\nu} \) than read

\[ \gamma^{\alpha\beta} D_{i,\alpha}^{d_i} D_{j,\beta}^{d_j} \Phi^{\mu\nu} = b^2 \alpha_1 \Phi^{\mu\nu} + \lambda \tilde{\Phi}^{\mu\nu}(\gamma^{\mu\nu}, \Phi_A) + 2\alpha_0 \gamma^{44} D_{i,\alpha}^{d_i} D_{j,\beta}^{d_j} \Phi^{\mu\nu} \]

where \( b \) is a constant value that necessary to introduce for using more broad sets of functions with GFD and it after calculations must be put zero. The \( \Phi^{\mu\nu} \) is a four column matrix. So we have again four equations for gravitational fields with real and imaginary energies. The equation for curvature tensor (with GFD ) have an usual form, but it will be four equations for different the curvature tensors \( R(i), i = 1, 2, 3, 4 \) and necessary take into account corrections in the covariant derivatives from fractal nature of space and of modifying Lorents transformations

\[ R^{\mu\nu}_{i} = - \frac{1}{2} \gamma^{\mu\nu} R_{i} = \frac{8\pi}{\sqrt{-g}} T^{\mu\nu}_{i} \quad (15) \]

\[ D_{i,\alpha}^{d_i} \tilde{\Phi}^{\mu\nu} = 0 \quad (16) \]

The equation \( \gamma^{\mu\nu} \) describes the boundary conditions for \( g^{\mu\nu} \) on the Universe surface. For the case of weak fields the generalized covariant derivatives may be represented as (see \( \Phi \))

\[ D_{i,\alpha}^{d_i} t^{\mu\nu} \approx t D_{i,\alpha}^{d_i} t^{\mu\nu} + t D_{i,\alpha}^{d_i} t^{\mu\nu} \quad (17) \]

The \( t D_{i,\alpha}^{d_i} \) in \( \Phi \) describes the contribution from FD of time and space, the member \( t D_{i,\alpha}^{d_i} \) describes the contribution from Riemann space with integer dimensions.

Let us see what differences between very similar equations \( \Phi \) and \( \Phi \). The equation \( \Phi \) differs from equation \( \Phi \) based on the Riemannian measure by three aspects:

a) the metric tensor \( \gamma^{\mu\nu} \) in \( \Phi \) determined on the Minkowski space with fractional dimensions;

b) equations differ by dependencies of metrics tensor \( \gamma^{\mu\nu} \) from \( L \) (because in the \( \Phi \) there are no dependencies in the \( \gamma^{\mu\nu} \) from \( L \) originated by the Riemann metric tensor), there are only dependencies originating by FD;

c) the reason of appearance in equation \( \Phi \) of the dependencies the \( \gamma^{\mu\nu} \) at \( L \) lay in the originate it by the only fractal dimensions of time and space. If FD are integer the \( \Phi \) coincide with equation of the theory \( \Phi \). If FD integer in \( \Phi \) these equations coincide with equations of GR. For weak fields GFD may be represented only by FD covariant derivatives (only two members in the right part of \( \Phi \)) and in that case \( \Phi \) may be represented by metric tensor \( g^{\mu\nu} \) of an "effective" Riemann space with integer dimensions as in \( \Phi \) (see also \( \Phi \)). So \( \Phi \) gives
the equations GR too. We pay attention that the corresponding results of the theory \[1\] for connections between metric tensor $\gamma^{\mu\nu}$ of Minkowski space with "effective" metric tensor $g^{\mu\nu}$ of Riemann space and gravitation tensor (though they are valid) are the special case of our theory. In general case the metric tensor of Minkowski space are complicated function of gravitation field tensor. We took into account the alterations in equations originated by modified Lorentz transformations in models with both measures (Riemann and Minkowski).

VII. CONCLUSION

In this paper we considered the two models of gravity theories defined on the multifractal sets of time and space. From our point of view there are two main approach to theories of gravity: the first is the approach of Einstein’s theory in which the gravitation fields and forces are no exist and its role play the curvature of the Riemannian time-space originated by Riemannian geometry. The second approach is the approach of postulating the real gravitational fields and forces made in the Logunov-Mestvirichvili theory \[4\]. The last theory treats the gravity as an usual real field in the flat pseudo Euclidean time-space and it is the very attractive feature of this theory. The results of both theories coincide on the distances far from the gravitational radius of centre of gravity. On the distances of order the gravitational radius of (if our Universe is multifractal set of space and time points defined on the measure carrier) the both theories it seems are not correct. In the Universe with multifractal dimensions of time and space on these distances the main role will play the integral characteristics of GFD and all equations become not differential but integral equations without containing of any infinity. We presented in these paper the new theory of gravity: gravity theory in the time and space with fractional dimensions. This theory use the idea and results of works \[3\] - \[6\], \[12\] - \[14\] and take into account the corrections to SR given by the theory of almost inertial systems in the time with fractional dimensions \[13\]. In other words this gravitation theory expand the main results of \[14\] on the gravitational fields.

Let us enumerate now the main results the theory presented in this paper:

1) The theory gives four sorts of different gravitation fields: two fields with real energy (these fields differs by the sign of their energies (the field for gravitons and the field for anti-gravitons) and two fields with imaginary energies. The situation is the same as for vector (electromagnetic) and spinor (electron-positron) fields considered in \[14\];

2) The interactions for each of both fields with real energies with imaginary energies fields are different. This gives the possibility to introduce the assumption about existence of new characteristics of gravitational fields ("quasi-spin") for explaining these facts;

3) The consideration of two models of a measure carriers (the measure on Riemannian space and the measure on Minkowski space) are made.

4) The presented theory coincide with GR or the theory Logunov-Mestvirichvili for case when FD of time and space become integer.

5) In the fractal time and space the ordinary derivatives and integral must be replaced by GFD. So the main idea of this work may be used for generalization of all gravitational theories not considered here, including quantum theories of gravitation.

6) In this paper adopted the point of view: our Universe is multifractal sets of time and space "points" (see detials in the \[3\] - \[4\]). As any multifractal set it defined on a measure carrier. Thus the Universe is an open system (statistical theory of open systems see in the \[19\], \[20\]) and the all physical conservation laws (energy, mass and so on) fulfill as the very good approach. The exact the conservation laws fulfill only for closed system: the Universe plus the measure carrier. So the correct selection of measure is a very serious task and it is the task of near future. In the domains of Universe where the correction to integer time and space dimensions are very small (in such domain of Universe we live and such domains are in distances far away from gravitational radius) the exchange by energy, mass, momentum and so on between the Universe and the measure carrier is very small too and it may be neglected. It is necessary nevertheless remember about continuous exchange by energy between Universe and the measure carrier (absorption and emission of energy) in every place of our Universe in the frame of presented fractal theory. The Universe never lost its energy it seems in that case and the far energy future of Universe is not so sad.

7) In this paper we considered the characteristics of gravitational fields (characteristics electromagnetic and electron-positron fields were considered in \[14\]. Naturally the algorithm used in the paper may be applied to any fields (electro-weak, Lee-Yang, quarks and so on) in domains where the fractional correction to the dimensions of time are small. In that case will be true the main results of this paper: every physical fields must be replaced by four fields with the real and imaginary energies. So it seems very likely that all physical fields must have their imaginary twins (if our Universe is fractal).

8) Nobody knows what the time and the space dimensions has our Universe. If the dimensions of time and space are fractional the presented in this paper the the-
ories of gravitational fields will be true (if at least one of the selections the measures carrier are valid) and will describe the reality of our Universe. As was stressed in [13] the one of methods of verification the fractal theory of time and space is to accelerate the charge particle to speed of light that in the time with fractional dimensions is possible (because for spaces with FD of time the SR was modified in the narrow domain of velocities near velocity of light (see [3]-[6]).

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