Higgs inflation: consistency and generalisations

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ABSTRACT: We analyse the self-consistency of inflation in the Standard Model, where the Higgs field has a large non-minimal coupling to gravity. We determine the domain of energies in which this model represents a valid effective field theory as a function of the background Higgs field. This domain is bounded above by the cutoff scale which is found to be higher than the relevant dynamical scales throughout the whole history of the Universe, including the inflationary epoch and reheating. We present a systematic scheme to take into account quantum loop corrections to the inflationary calculations within the framework of effective field theory. We discuss the additional assumptions that must be satisfied by the ultra-violet completion of the theory to allow connection between the parameters of the inflationary effective theory and those describing the low-energy physics relevant for the collider experiments. A class of generalisations of inflationary theories with similar properties is constructed.

KEYWORDS: Inflation, Higgs boson, Non-minimal coupling, Effective field theory.
1. Introduction

It was proposed recently [1] that the inflationary expansion of the early Universe can be incorporated within the Standard Model (SM). The SM already contains a particle—the Higgs boson—with appropriate quantum numbers to play the role of the inflaton. The key point of the Higgs-inflation is the non-minimal coupling of the Higgs field to gravity. Namely, the SM-gravity action is taken as

\[ S = \int d^4x \sqrt{-g} \left( -\frac{M_P^2}{2}R - \xi H^\dagger H R + \mathcal{L}_{SM} \right), \]  

(1.1)

where \( R \) is the scalar curvature, \( H \) is the Higgs doublet, \( M_P \) is the Planck mass, \( \mathcal{L}_{SM} \) represents the SM Lagrangian and \( \xi \) is a new coupling constant (see [2, 3, 4, 5, 6, 7, 8] for previous studies of inflation with non-minimally coupled scalar fields).

As found in [1], for the appropriate choice of \( \xi \) of the order \( 10^4 \), the resulting model leads to successful inflation, provides the graceful exit from it, and produces the spectrum of primordial fluctuations in good agreement with the observational data.\(^1\) Thus one arrives

\(^1\)For the minimal coupling to gravity corresponding to \( \xi = 0 \) an unacceptably large amplitude of primordial inhomogeneities is generated for a realistic quartic Higgs self-interaction [9].
at an economical scenario, where inflation does not require introduction of any new degrees of freedom, with all necessary ingredients being present in the SM. This scenario was further explored in [10, 11, 12, 13, 14, 15, 16].

However, the self consistency of Higgs inflation was questioned in [17, 18, 19]. It was pointed out there that the operator describing non-minimal coupling, when written in terms of canonically normalised fields, has dimension 5 and is suppressed by the scale

$$\Lambda_0 = \frac{M_P}{\xi}. \quad (1.2)$$

This was interpreted as the ultra-violet (UV) cutoff, above which the SM has to be replaced by a more fundamental theory. If true, this would make the Higgs inflation “unnatural”. Indeed, for large $\xi$, the scale $\Lambda_0$ is considerably lower than the Planck mass. At the same time, the value of the Hubble expansion rate is close to $\Lambda_0$ during inflation, making the contribution of unknown effects beyond the SM substantial [17]. Moreover, $\Lambda_0$ is much smaller than the value of the Higgs field during inflation. According to the standard lore, one would argue that the action of the theory must be supplemented by other higher-order operators suppressed by $\Lambda_0$, including additional terms in the Higgs potential of the form

$$\left(\frac{H^\dagger H}{\Lambda_0^2}\right)^n \quad (1.3)$$

with $n \geq 3$. These terms would spoil the slow-roll regime. Based on these observations it was concluded in [18] that the validity of the Higgs inflation is very sensitive to the UV completion of the theory.

Our aim in the present paper is to re-assess the self-consistency of the Higgs inflation in order to disentangle the UV-sensitive aspects of the model from those which can be analysed by means of an effective field theory (EFT) description. To make the analysis clear we concentrate on a toy model of a single non-minimally coupled scalar $\phi$, representing the radial mode of the Higgs field, $H^\dagger H = 2\phi^2$. This allows to get rid of the complications related to the gauge fields; effect of additional SM fields will be discussed at the end of the paper.

We start by revisiting the calculation of the cutoff scale $\Lambda$, to determine the region of validity of the theory with large non-minimal coupling. We find that the domain of energies $E < \Lambda$, were the model can be considered as a valid EFT depends on the background value of the scalar field. Its upper boundary coincides with (1.2) at $\phi = 0$ and becomes higher for large background values of $\phi$. We show that the background dependent cutoff is parametrically higher than the energy scales characterising the dynamics of the system throughout the whole history of the Universe. In particular, during inflation, it coincides with the Planck mass (in the Einstein frame\textsuperscript{2}), which is much higher than the Hubble rate at that moment. Thus the necessary requirements for the validity of the semiclassical treatment of the model are satisfied.

\textsuperscript{2}The Einstein frame is the frame where the non-minimal coupling between the inflaton and curvature is eliminated in favour of essentially non-linear inflaton self-interaction. It is related to the Jordan frame, where the action (1.1) is originally formulated, by a conformal transformation, see Sec. 2.2.
We next turn to the analysis of quantum corrections in the model. Of course, the theory (1.1) is non-renormalisable. In the Jordan frame, the non-renormalisability is due to the coupling to gravity, while in the Einstein frame it manifests itself in essentially non-linear interactions of the scalar field. To remove the divergences, one has to add an infinite number of counter-terms and the corresponding finite terms with arbitrary coefficients. We show that the counter-terms can be chosen in such a way that they respect the classical symmetries of the model. The most important for us is the asymptotic symmetry of the action under the shifts of the inflaton field in the Einstein frame, corresponding to the asymptotic scale invariance in the Jordan frame. This symmetry exists in the domain $\phi \to \infty$ relevant for inflation.

It is a well-known property of an approximate shift symmetry that it allows to preserve the flatness of the inflaton potential under radiative corrections \[20\]. In this paper we develop a systematic scheme which takes into account breaking of the symmetry order by order in perturbation theory. It leads to the classification of the operators generated by quantum corrections according to their order in the parameter that controls breaking of the shift invariance at finite values of the field. This leads to an EFT description, close in spirit to that proposed in \[21, 22\], where the expansion is effectively controlled by the inflationary slow-roll parameters. Importantly, the asymptotic symmetry precludes the generation of counter-terms of the type (1.3). So, the Higgs inflation is self-consistent and “natural”. Note that a similar conclusion was achieved recently in \[23\].

It is worth stressing that in this paper we take a ‘minimalistic’ attitude to the self-consistency issue. Namely, we consider only those quantum corrections which are forced by the inflationary theory itself. In particular, the asymptotic shift symmetry which is the property of the inflationary dynamics is assumed to be valid also at the level of the UV-complete theory. Alternatively, this can be considered as a restriction on the UV-completion which must be satisfied for our results to remain in force. We do not address the origin of the asymptotic shift symmetry in the UV theory. This question has been discussed recently in \[24\] (see also references therein).

The EFT approach to inflation that we develop in this paper is general. Besides the model (1.1), we show how it can be applied to a wide class of inflationary Lagrangians with asymptotic shift symmetry.

Finally, we discuss under which conditions the parameters of the inflationary EFT can be connected to those describing the low-energy physics relevant for collider experiments. We find that this connection is sensitive to the details of the UV completion. Thus, no relation between these parameters can be established in general, without specific assumptions about the physics beyond the cutoff. We determine explicitly the requirements to the UV completion, which lead to the relation between the low-energy and inflationary domains. We discuss, in particular, the sensitivity of the connection of the Higgs boson mass and the inflationary parameters \[10, 14, 13, 15, 16, 25\] to UV physics.

The paper is organised as follows. In Sec. 2 we perform the calculation of the background dependent cutoff. In Sec. 3 we address the structure of quantum corrections and develop the effective field theory for the inflationary epoch. Section 4 discusses the generalisations of the Higgs-inflation to a wide class of inflationary Lagrangians with asymptotic...
shift symmetry. Section 5 is devoted to conclusions. Appendices contain analysis of the model with addition of fermions and gauge bosons.

2. The cutoff scale revisited

Let us start by discussing the definition of the cutoff scale. The useful criterion for the validity of perturbation theory is the tree unitarity [26], which means that all $N$-particle tree amplitudes $M_N$ have at most a high-energy behaviour

$$M_N \propto E^{4-N},$$

(2.1)

where $E$ is the energy. This is the case in renormalisable theories which thus can be considered as fundamental theories valid at arbitrary momenta (we leave aside the issue of Landau poles which, if present, occur at exponentially high energies and are irrelevant for our discussion). If instead the tree amplitudes grow with energy or fall slower than (2.1), the perturbation theory fails at some energy $\Lambda$, which can be called an ultra-violet cutoff. Whether the theory gets inconsistent at energies higher than $\Lambda$, or just enters into a strongly interacting phase, can not be deduced a priori. In any event, the theory is only predictive with the use of traditional perturbative methods at energies $E < \Lambda$. Thus we arrive at the following definition of the cutoff scale $\Lambda$. Compute all tree amplitudes with $N$ particles and find the energy $\Lambda_N$ at which the unitarity bound in each of them is violated. Then define the cutoff as\(^3\) $\Lambda = \min N \Lambda_N$.

The cutoff scale is not just a number. It depends, in general, on background bosonic field(s). For example, the cutoff of the 4-fermion low energy Fermi theory of weak interactions is proportional to the expectation value of the Higgs field. So, to define the region of validity of the theory (1.1), one should fix the background and consider the asymptotic high energy behaviour of tree $N$ particle amplitudes. This is exactly what we have to do to understand the viability of the Higgs inflation, because during inflationary evolution of the Universe, the system is not described by its perturbations about the vacuum solution, but rather by excitations above some classical background. Thus the fields are naturally divided in the slowly varying classical part and excitations on it

$$\Phi(x, t) = \Phi(t) + \delta\Phi(x, t),$$

(2.2)

where $\Phi$ stands for the generic set of fields in the theory (inflationary scalars, gravitational metric, etc.). The perturbations relevant for the cutoff determination have high frequencies corresponding to short time scales. These are much shorter than the typical time scale of the background evolution. Thus the background can be approximated as static with a good accuracy.

Instead of actually calculating the $N$ particle amplitudes, we will estimate the cutoff by power counting of the operators, present in the expansion of the action in $\delta\Phi$. That is,

\(^3\)Some care is needed in applying this definition. One should check that it does not put too much weight into the multiparticle amplitudes with $N \gg 1$, for which conventional perturbation theory breaks down even in the case of renormalisable field theories, see e.g. [27].
if we have operators with dimension larger than four, divided by some scale $\Lambda^{(n)}$, 

$$\frac{\mathcal{O}_{(n)}(\delta \Phi)}{[\Lambda^{(n)}(\Phi)]^{n-4}}, \quad (2.3)$$

then we expect the tree level unitarity to be violated in some amplitudes at the scale $\Lambda^{(n)}$. Clearly, the cutoff scale determined in this way does depend on the background values of the fields, which is indicated explicitly in (2.3). Some non-trivial cancellations may alter the result raising the unitarity violation scale. Thus, strictly speaking, this approach provides a lower estimate of the cutoff. We will neglect possible cancellations in what follows: after all, having the lower bound on the cutoff is enough for our purposes. Besides, we are going to see that the simple power counting estimates for the cutoff agree in two different representations of the theory which favours the identification of these estimates as the true value of the cutoff.

2.1 Cutoff in the Jordan frame

We now turn to the calculation of the cutoff scale for the Higgs-inflation model. We start by performing the analysis in the Jordan frame where the model was originally formulated. Throughout the main part of the paper we work with a toy model of a single real scalar field with non-minimal coupling to gravity (the effects of matter fermions and gauge bosons will be discussed later). The action of the model is

$$S = \int d^4x \sqrt{-g} \left( \frac{M^2 + \xi \phi^2}{2} R + \frac{(\partial \mu \phi)^2}{2} - \frac{\lambda \phi^4}{4} \right). \quad (2.4)$$

We have neglected the mass term for $\phi$ since it is not important at large values of the field. This is the scalar part of the action for the SM Higgs inflation in the unitary gauge [1]. The way inflation proceeds in this model is described in detail in [1], while the reheating was studied in [11, 12]. For our purposes we need the following results. The non-minimal coupling with curvature modifies the kinetic term for the scalar field for large values of the field, leading to the slow-roll evolution even with relatively large quartic coupling constant $\lambda$. Normalisation of the primordial density fluctuations to the observed value fixes the relation between $\lambda$ and non-minimal coupling $\xi$

$$\xi \approx 47000\sqrt{\lambda}. \quad (2.5)$$

This means, that if $\lambda$ is not very small, as in the case of the SM Higgs boson, $\xi$ should be rather large. In this limit inflation happens for the values of the Jordan frame field $\phi > \phi_{\text{END}} \approx M/\sqrt{\xi}$.

To obtain the scale of tree-level unitarity violation we expand the metric and the scalar around their background values,

$$g_{\mu\nu} = \bar{g}_{\mu\nu} + h_{\mu\nu}, \quad (2.6)$$

$$\phi = \bar{\phi} + \delta \phi. \quad (2.7)$$
The quadratic Lagrangian for the excitations has the form,
\[ \mathcal{L}^{(2)} = -\frac{M_P^2 + \xi \bar{\phi}^2}{8} (h^{\mu\nu} \Box h_{\mu\nu} + 2 \partial_{\nu} h^{\mu\nu} \partial^\rho h_{\mu\rho} - 2 \partial_{\nu} h^{\mu\nu} \partial_{\mu} h - h \Box h) + \frac{1}{2} (\partial_{\mu} \delta \phi)^2 + \xi \bar{\phi} (\Box h - \partial_{\lambda} \partial^{\lambda} h^{\lambda\rho}) \delta \phi , \]
(2.8)
where \( h = h_{\mu\mu} \). We retained here only the terms with two derivatives of the excitations as they determine the UV behaviour of the scattering amplitudes and hence the unitarity violation scale. Notice, that in the nontrivial background there is a large kinetic mixing between the trace of the metric and the scalar perturbations \([10, 13, 16, 25]\). The change of variables
\[ \delta \phi = \sqrt{\frac{M_P^2 + \xi \bar{\phi}^2}{M_P^2 + \xi \phi^2 + 6 \xi ^2 \bar{\phi}^2}} \bar{\phi} , \]
(2.9)
\[ h_{\mu\nu} = \frac{1}{\sqrt{M_P^2 + \xi \phi^2}} \hat{h}_{\mu\nu} - \frac{2 \xi \bar{\phi}}{\sqrt{(M_P^2 + \xi \bar{\phi}^2)(M_P^2 + \xi \bar{\phi}^2 + 6 \xi ^2 \bar{\phi}^2)}} \delta \phi , \]
(2.10)
diagonalises the kinetic term. The unitarity violation scale is now read out of the operators with dimension higher than four. The leading operator is the cubic scalar–graviton interaction \( \xi (\delta \phi)^2 \Box h \), which in terms of the canonically normalised variables has the form,
\[ \frac{\xi \sqrt{M_P^2 + \xi \phi^2}}{M_P^2 + \xi \phi^2 + 6 \xi ^2 \phi^2} (\delta \phi)^2 \Box h . \]
(2.11)
The cutoff is identified as the inverse of the coefficient in this operator,
\[ \Lambda^J(\bar{\phi}) = \frac{M_P^2 + \xi \bar{\phi}^2 + 6 \xi ^2 \bar{\phi}^2}{\xi \sqrt{M_P^2 + \xi \bar{\phi}^2}} , \]
(2.12)
where the superscript \( J \) reminds that this cutoff is obtained in the Jordan frame. The expression for the cutoff simplifies in three regions of background fields:

- \( \bar{\phi} \ll M_P/\xi \), low field region. This region corresponds to the present-day Universe. The cutoff is
  \[ \Lambda^J \simeq \frac{M_P}{\xi} . \]
  (2.13)
  This coincides with the zero background result of \([17, 18, 19]\). It is smaller than the Planck mass, but for the observationally required value of \( \xi \sim 10^4 \) it is still way above the reach of collider experiments.

- \( M_P/\xi \ll \bar{\phi} \ll M_P/\sqrt{\xi} \), the intermediate region (relevant for reheating, see \([11, 12]\)). The cutoff scales as
  \[ \Lambda^J \simeq \frac{\xi \bar{\phi}^2}{M_P} . \]
  (2.14)
  It is still below the Planck mass but starts to grow.
Figure 1: The dependence of the Jordan frame cutoff on the background value of the inflaton field in log-log scale. The effective field theory description is applicable at energies below the thick curve.

- $\bar{\phi} \gg M_P/\sqrt{\xi}$, large fields (inflationary period). The cutoff becomes

$$\Lambda^J \simeq \sqrt{\xi \phi}.$$  \hspace{1cm} (2.15)

Note that this coincides with the cutoff in the gravitational sector. The latter is given by the effective Planck mass defined as the coefficient in front of the $R$ term in the Lagrangian, $M^2_{\text{eff}} = \sqrt{M^2_P + \xi \bar{\phi}^2}$.

The behaviour of the cutoff is illustrated in Fig. 1.

2.2 Cutoff in the Einstein frame

It is instructive to repeat the calculation of the cutoff scale in the Einstein frame. In this frame the gravitational part of the action coincides with that of the usual Einstein’s gravity, while all non-trivial interactions are moved exclusively into the scalar sector. This makes the analysis in the Einstein frame conceptually simpler and we will work in this frame from now on. The Jordan and Einstein frames are related by the conformal transformation

$$g_{\mu \nu} = \Omega^{-2} \tilde{g}_{\mu \nu},$$ \hspace{1cm} (2.16)

with the conformal factor

$$\Omega^2 = 1 + \frac{\xi \bar{\phi}^2}{M^2_P}.$$ \hspace{1cm} (2.17)

Substituting this into (2.4) we obtain,

$$S = \int d^4x \sqrt{-\tilde{g}} \left( -\frac{M^2_P}{2} \tilde{R} + \frac{M^2_P(M^2_P + (6\xi + 1)\xi \bar{\phi}^2)}{(M^2_P + \xi \bar{\phi}^2)^2} (\partial_{\mu} \phi)^2 - \frac{\lambda M^4_P \phi^4}{4(M^2_P + \xi \bar{\phi}^2)^2} \right),$$ \hspace{1cm} (2.18)
where tilde denotes the geometrical quantities calculated in the Einstein frame. Note that the scalar potential flattens out and tends to a constant at large $\phi \gg M_P/\sqrt{\xi}$. This is the origin of the slow-roll inflation in the Einstein frame picture [1].

To proceed we have two options. One is to work directly with the field $\phi$. Then, expanding it around the background, one reads out the cutoff as the scale suppressing higher-order interactions appearing from the kinetic term. We take another route and perform the field redefinition which casts the kinetic term into the canonical form. This is achieved by introducing field $\chi$ related to $\phi$ by

$$\frac{d\chi}{d\phi} = \frac{M_P \sqrt{M_P^2 + (6\xi + 1)\xi \phi^2}}{M_P^2 + \xi \phi^2}. \tag{2.19}$$

In terms of this new field all non-linearities are moved into the potential

$$U(\chi) = \frac{\lambda M_P^4 \phi(\chi)^4}{4(M_P^2 + \xi \phi(\chi)^2)^2}. \tag{2.20}$$

Expanding as usual above the background,

$$\chi = \bar{\chi} + \delta \chi, \tag{2.21}$$

one calculates the Taylor expansion of the potential

$$U(\bar{\chi} + \delta \chi) = U(\bar{\chi}) + \sum_{n=1}^{\infty} \frac{1}{n!} d^n U d\chi^n (\delta \chi)^n. \tag{2.22}$$

In this expansion the operators with $n > 4$ will contribute to the $n$-particle scattering amplitudes in a non-unitary way starting from the energy scale

$$\Lambda_{(n)} \sim \left(\frac{d^n U}{d\chi^n}\right)^{-\frac{1}{n-4}}. \tag{2.23}$$

Let us analyse this expansion in the same three regions of the background field as before.

- $\bar{\phi}, \bar{\chi} \ll M_P/\xi$. In this case inverting (2.19) one finds

$$\phi(\chi) = \chi \left(1 + \sum_{k=1}^{\infty} c_k \left(\frac{\xi^2 \chi^2}{M_P^2}\right)^k\right), \tag{2.24}$$

where $c_k$ are numerical coefficients. Substituting this into (2.20), (2.23) one obtains,

$$\Lambda_{(n)} \sim \frac{M_P}{\xi} \lambda^{-1/(n-4)} \tag{2.25}$$

for even $n$. This differs from the Jordan frame expression (2.13) by a power of $\lambda$. However, this factor tends to unity for operators of higher dimension. If $\lambda$ is not too small, the factor becomes close to one already for moderately large $n$. Thus we omit it in the determination of the cutoff and arrive at the expression (2.13).
• Intermediate region \( M_P/\xi \ll \ddot{\phi} \ll M_P/\sqrt{\xi} \). For the Einstein frame field this corresponds to the region \( M_P/\xi \ll \bar{\chi} \ll M_P \). The calculation of the cutoff using (2.23) requires calculation of higher derivatives of the composite function \( U(\phi(\chi)) \) using (2.20), (2.19). Direct calculation is rather tedious\(^4\) (though obviously possible), so we will sketch here a simplified way to retain the leading behaviour of the higher derivatives for this region of \( \bar{\chi} \). To be precise, the next-to-leading term is

\[
U(\chi) = \frac{\lambda M_P^4}{4\xi^2} \left( 1 - \exp \left( -\frac{2\chi}{\sqrt{6}M_P} \right) \right)^2. \tag{2.31}
\]

\( M_P/\xi \ll \ddot{\phi} \ll M_P/\sqrt{\xi} \) only for relatively large \( n \). For small \( n \) the estimate (2.29) is true only for the lower part of the intermediate region.

\( \Lambda \) requires calculation of higher derivatives of the composite function \( U(\phi(\chi)) \).

\[
\Lambda_{(n)} \sim \frac{\ddot{\phi}^2\xi}{M_P} \cdot \left[ \frac{\xi^6\ddot{\phi}^6}{\lambda M_P^6} \right]^{1/(n-4)}. \tag{2.29}
\]

which reproduces (2.14) at \( n \to \infty \).

It is worth mentioning that a technically simpler way to obtain the cutoff in this region is to analyse the theory with additional fermions. Being out of the main line of the article this analysis is given in Appendix A.

• \( \ddot{\phi} \gg M_P/\sqrt{\xi} \), inflationary region. For the Einstein frame field this corresponds to \( \bar{\chi} \gg M_P \). In this region one can neglect \( M_P^2 \) in the numerator of (2.19) which yields simple analytic solution

\[
1 + \frac{\xi\ddot{\phi}^2}{M_P^2} \simeq \exp \left( \frac{2\chi}{\sqrt{6}M_P} \right). \tag{2.30}
\]

Substituting this into (2.20) we obtain the expression for the potential

\[
U(\chi) = \frac{\lambda M_P^4}{4\xi^2} \left( 1 - \exp \left( -\frac{2\chi}{\sqrt{6}M_P} \right) \right)^2. \tag{2.31}
\]

\( \Lambda \) requires calculation of higher derivatives of the composite function \( U(\phi(\chi)) \).

\[4\] Note that in this calculation one cannot use the approximate expression for the potential (2.31) as it does not lead to the required precision for higher derivatives.

\[5\] To be precise, the next-to-leading term is

\[
U''(\chi)(f(\chi))^2(f'(\chi))^{n-2} \propto \lambda/(\xi^2 M_P^{-4}). \]

It can be neglected in the region \( M_P/\xi \ll \ddot{\phi} \ll M_P/\sqrt{\xi} \) only for relatively large \( n \). For small \( n \) the estimate (2.29) is true only for the lower part of the intermediate region.
Expanding it around a background we get a series of interactions which have the form

$$\frac{\lambda M_P^4}{\xi^2} \exp \left( -\frac{2\bar{\chi}}{\sqrt{6}M_P} \right) \frac{(\delta\chi)^n}{M_P^n}.$$  

(2.32)

Note that these interactions contain an overall exponential suppression. However, for any fixed $\bar{\chi}$ the energy cutoff scale goes to

$$\Lambda \sim M_P$$  

(2.33)

for large $n$. To compare this result with that in the Jordan frame (2.15) we should rescale it using the conformal factor (2.17). The latter is approximately equal to $\sqrt{\xi}\bar{\phi}/M_P$ for large field. Multiplied by (2.33) this reproduces (2.15). Let us note again, that in the inflationary region the cutoff in the scalar sector coincides with the gravitational cutoff (which is just $M_P$ in the Einstein frame or $\sqrt{\xi}\bar{\phi}$ in the Jordan frame).

### 2.3 Cutoff and energy scales in the early Universe

Let us compare the background-dependent cutoff derived above with the characteristic energy scales during the evolution of the Universe. We perform the discussion in the Einstein frame. At the inflationary stage the typical momentum of the relevant excitations is equal to the Hubble parameter which for the potential (2.31) is of the order $H \sim \sqrt{\lambda}M_P/\xi$. This is much smaller than the cutoff (2.33) during inflation. Similarly, the energy density of the Universe during inflation, $\rho_I = \lambda M_P^4/\xi^2$, is much smaller than $\Lambda^4$.

During reheating the scalar field $\phi$ oscillates with the amplitude that decreases from $M_P/\sqrt{\xi}$ down to $M_P/\xi$. Detailed analysis in [11] shows that typical momenta of relativistic particles produced in this period are $\sqrt{\lambda}\xi\bar{\phi}/M_P$ for the Higgs boson and $g\bar{\phi}$ for other light particles, where $g$ is the gauge coupling. These are again parametrically smaller than the corresponding cutoff value (2.14) with the suppression provided by the coupling constants.\(^6\)

Finally, after the Universe reheats below the temperature $T \sim M_P/\xi$, all relevant particle energies drop below the small-field cutoff (2.13).

We see that during the whole evolution of the Universe the relevant energy scales are parametrically below the background dependent cutoff $\Lambda(\bar{\phi})$. Clearly, this is a necessary requirement for the validity of the semiclassical treatment of the model, which we thus find fulfilled. However, this requirement is not sufficient. To establish the sufficient conditions one has to analyse the loop corrections to the tree-level picture. We presently turn to this task.

### 3. Size of quantum corrections, counterterms and all that

#### 3.1 Effective field theory for inflation

Of course, the theory (2.4), or equivalently (2.18) is non-renormalisable. This implies that the loop corrections will generate infinite number of divergent counterterms. Naively this

\(^6\)This is the only place where the presence of the gauge bosons make things more subtle. As discussed in Appendix B, the cutoff may be comparable to the momentum of generated Higgs excitations during reheating. This may affect the details of the description of reheating, but does not alter it qualitatively.
seems to imply the loss of predictive power. We now argue that at large fields which are relevant for inflation, $\chi > M_P (\phi > M_P/\sqrt{\xi})$, it is possible to consistently account for quantum corrections in a way similar to that of effective field theory. The crucial property which enables to do this is the approximate symmetry of the action in the inflationary region. In the Einstein frame it manifests itself as the symmetry under the shifts of the scalar field, while in the Jordan frame it corresponds to the scale invariance.

Let us analyse the structure of possible counterterms. As usual, we work in the background field formalism. We stick to the Einstein frame language where all non-trivial interactions are concentrated in the scalar sector. After canonically normalising the scalar field we obtain the Lagrangian

$$\mathcal{L} = \frac{(\partial_\mu \chi)^2}{2} - U(\chi),$$  \hspace{1cm} (3.1)

where $U(\chi)$ has at large fields the generic form

$$U(\chi) = U_0 \left( 1 + \sum_{n=1}^{\infty} u_n e^{-n\chi/M} \right).$$  \hspace{1cm} (3.2)

For the concrete choice of the potential (2.31) we have

$$U_0 = \frac{\lambda M_P^4}{4\xi^2}, \quad M = \sqrt{\frac{6}{2}} M_P$$  \hspace{1cm} (3.3)

and $u_n$ coincide with the Taylor coefficients of the function $(1 - x)^2$. At $\chi \gg M$ the potential becomes constant giving rise to the shift symmetry

$$\chi \mapsto \chi + \text{const}. \hspace{1cm} (3.4)$$

If all coefficients $u_n$ were zero the shift symmetry would be exact and the flatness of the potential would be preserved by radiative corrections. However, for viable inflation it is important that the potential is not exactly flat due to the presence of exponential terms and the shift symmetry is only approximate. We will see nevertheless that the fact that the symmetry is only weakly broken allows to consistently take into account the quantum corrections during the inflationary stage (cf. [21]).

Let us now split the field into smooth background $\bar{\chi}$ and fluctuations $\delta \chi$ as in (2.21). The potential takes the form,

$$U = U_0 \left( 1 + \sum_{n=1}^{\infty} u_n e^{-n\bar{\chi}/M} \sum_{k=0}^{\infty} \frac{1}{k!} \left[ \frac{n \delta \chi}{M} \right]^k \right) = U_0 \left( 1 + \sum_{k=0}^{\infty} \frac{1}{k!} \left[ \frac{\delta \chi}{M} \right]^k \sum_{n=1}^{\infty} n^k u_n e^{-n\bar{\chi}/M} \right).$$  \hspace{1cm} (3.5)

The Wilsonian effective action is given by integrating out the perturbations $\delta \chi$. Technically, this amounts to computing all the loop diagrams generated by the interaction (3.5) without external legs of $\delta \chi$. The background field $\bar{\chi}$ in this procedure must be treated as classical. Let us see what kind of divergences this may produce.
Consider first the contributions to the effective potential. Clearly, all divergences are proportional to the positive powers of the exponent $e^{-\bar{\chi}/M}$. In other words, all counterterms can be organised into a series of the form (3.2). Thus they may be absorbed by renormalisation of $U_0$ and the coefficients $u_n$.

Let us now discuss the loop divergences that require counterterms with derivatives. For example, at two and three loops one obtains divergences of the following schematic form,

$$U_0 u_n M^4 e^{-n\bar{\chi}/M} \propto \frac{1}{\epsilon} \cdot \frac{U_n^2}{M^8} u_n u_m (\partial_{\mu} \bar{\chi})^2 e^{-(n+m)\bar{\chi}/M} ,$$  

(3.6)

$$U_0 u_n M^4 e^{-n\bar{\chi}/M} \propto \frac{1}{\epsilon} \cdot \frac{U_n^2}{M^8} u_n u_m \left( \frac{(\partial^2 \bar{\chi})^2}{M^2} + \frac{(\partial \bar{\chi})^4}{M^4} \right) e^{-(n+m)\bar{\chi}/M} ,$$  

(3.7)

where we assumed dimensional regularisation ($4 - 2\epsilon$ is the space-time dimension) and omitted numerical coefficients of order one and combinatorial factors. One makes two observations. First, the derivatives of $\bar{\chi}$ in these expressions are suppressed by powers of the cutoff scale $M$ appearing from the differentiation of the exponents. Second, the $\bar{\chi}$-dependent coefficients in front of the derivative terms are again proportional to positive powers of the exponent $e^{-\bar{\chi}/M}$. Thus we conclude that to absorb all loop divergencies the Lagrangian (3.1) must be promoted to

$$\mathcal{L} = \sum_{i=1}^{\infty} f^{(i)}(\chi) \left(\frac{\partial_{\mu} \chi}{2} - U(\chi) + \frac{f^{(2)}(\chi)}{M^2} (\partial^2 \chi)^2 + \frac{f^{(3)}(\chi)}{M^4} (\partial \chi)^4 \right) + \cdots ,$$  

(3.8)

where dots stand for terms with more derivatives. Here the coefficient functions are (formal) series in the exponent,

$$f^{(i)}(\chi) = \sum_{n=0}^{\infty} f^{(i)}_n e^{-n\chi/M} ,$$  

(3.9)

where $f^{(i)}_n$ are numerical coupling constants. It is straightforward to convince oneself that this form of the Lagrangian is preserved by quantum corrections, i.e. no new counterterms are generated by any loops including those coming from the higher order terms.\(^7\) In this sense we can speak about renormalisation of the non-renormalisable Lagrangian (3.8) with the coefficient functions (3.9).\(^8\) Clearly, the coefficient function $f^{(1)}(\chi)$ can be absorbed into redefinition of the field leaving the kinetic term canonical without spoiling the form (3.9) of the other terms. Thus we will omit this function in what follows.

So far we have been discussing explicitly purely scalar theory. However, it is clear that the expansion formulated above holds when the theory is coupled to gravity. Indeed, this coupling respects the asymptotic shift symmetry\(^9\) (3.4), and the violation of the latter still

\(^7\)To avoid confusion we stress that this statement refers to the divergent terms generated by loops. Besides these there are of course finite quantum corrections which must be consistently taken into account.

\(^8\)Note that within purely scalar theory a somewhat more restricted choice with all $f^{(i)}(\chi)$, $i \geq 2$, vanishing at asymptotically large $\chi$ (this corresponds to taking $f^{(i)}_0 = 0$ for $i \geq 2$) is also stable under quantum corrections. However, we do not impose this restriction as non-zero values of $f^{(i)}_0$ are inevitably generated once the theory is coupled to gravity.

\(^9\)Recall that we are working in the Einstein frame where coupling of $\chi$ to gravity is minimal.
comes only in the form of the exponents $e^{-\chi/M}$. This guarantees that all counterterms that appear in the perturbation theory\textsuperscript{10} can be arranged into shift-invariant operators multiplied by the coefficient functions of the form (3.9).

The above statement remains true also upon inclusion of other fields in the theory so far as their coupling to $\chi$ obeys the asymptotic shift symmetry. To illustrate this point let us consider a fermion field with $\chi$-dependent mass term,

$$\mathcal{L}_Y = m(\chi) \bar{\psi} \psi.$$ \hspace{1cm} (3.10)

The asymptotic shift symmetry implies

$$m(\chi) = \sum_{n=0}^{\infty} m_n e^{-n\chi/M} \hspace{1cm} (3.11)$$

at large $\chi$. The divergent part of the one-loop contribution of this fermion into the scalar potential is

$$\Delta U_{\psi}^{\text{div}} \propto \frac{1}{\epsilon} [m(\chi)]^4.$$ \hspace{1cm} (3.12)

Clearly, this has the structure (3.2) and is absorbed by the redefinition of $U_0$ and $u_n$. Note that in the case when the fermion remains massive at large $\chi$ (i.e. $m_0 \neq 0$) the structure (3.2) is also shared by the finite part of the loop,

$$\Delta U_{\psi}^{\text{fin}} \propto [m(\chi)]^4 \ln[m(\chi)/\mu],$$ \hspace{1cm} (3.13)

where $\mu$ is the normalisation point, meaning that this correction need not be considered separately and may be included in the general expression (3.2) for the inflaton potential.

The asymptotic shift symmetry of the inflaton couplings to other fields naturally arises in the SM Higgs inflation model \cite{14}. It corresponds to the asymptotic invariance of the original Jordan frame action (1.1) under the scale transformations $\Phi(x) \rightarrow \lambda^D \Phi(\lambda x)$, where $D$ is the canonical dimension of the field $\Phi$.

At first sight it seems that the presence of infinite number of coupling constants $u_n$, $f_n^{(i)}$ implies the loss of predictive power. In fact, this is not the case. At large values of $\chi$ the exponent $e^{-\chi/M}$ is small. Thus requiring only finite accuracy, we can keep only finite number of terms in the exponential series. Also being interested in characteristic momenta lower than the cutoff $M$, one can neglect the higher-derivative terms.\textsuperscript{11} Then the theory is determined by a finite number of parameters and (to this accuracy) the predictive power is recovered. This is similar to the situation in the standard EFT. From the practical point of view the expansion in $e^{-\chi/M}$ translates into the expansion in the slow-roll parameters

$$\varepsilon \equiv \frac{M_P^2}{2} \left( \frac{1}{U} \frac{dU}{d\chi} \right)^2 \approx \frac{1}{3} u_1^2 e^{-2\chi/M},$$ \hspace{1cm} (3.14)$$

$$\eta \equiv \frac{M_P^2}{U} \frac{d^2U}{d\chi^2} \approx \frac{2}{3} u_1 e^{-\chi/M}.$$ \hspace{1cm} (3.15)

\textsuperscript{10}We do not discuss possible non-perturbative gravitational effects, which are model dependent and can be exponentially suppressed \cite{28}.

\textsuperscript{11}Or keep a finite number of them which gives rise to the expansion in $E/M$, where $E$ is the energy scale of interest. We are not going to discuss this expansion as it is completely standard.
This means that the inflationary predictions can be sensibly calculated in the model, provided sufficient amount of the coefficients $u_n$ is fixed form the observations (fixing just $u_1$ is enough for all modern applications). For example, the well-known property of inflation with the potential (3.2), which follows from (3.14), (3.15), is the suppression of the $\varepsilon$-parameter compared to $\eta$, $\varepsilon \approx 3\eta^2/4$. Thus fixing $\eta \simeq -0.015$ from the tilt of the spectrum of scalar perturbations yields the prediction for the tensor to scalar perturbation ratio $r = 16\varepsilon \simeq 0.003$ [1] (unfortunately, too small to be observed with modern experimental techniques). The results of this section show that this prediction is robust under radiative corrections.

Let us stress once more that the crucial property which has enabled us to develop the above EFT of inflation is the asymptotic shift symmetry of the inflaton action. This property naturally appears in the Einstein frame and is preserved by quantum corrections with the standard renormalisation prescriptions. In this approach the original Jordan frame action (1.1) appears merely as a convenient shorthand representation which ensures the asymptotic shift symmetry of all inflaton couplings at tree level. The suitable language for the analysis of the quantum aspects of the theory is provided by the Einstein frame.

It is worth comparing this approach to that of Refs. [10, 13, 16, 25] and prescription II of [14, 15]. There the loop corrections are evaluated in the Jordan frame. This calculation produces, among others, the following contribution into the inflaton potential,

$$\Delta V^J \propto \phi^4 \ln[\phi/\mu] ,$$

(3.16)

where $\phi$ stands, as usual, for the Jordan frame field. This breaks the scale invariance at large $\phi$, and as a consequence breaks the asymptotic shift symmetry in the Einstein frame. The reason is that choosing the standard renormalisation prescription with fixed normalisation point $\mu_J$ in the Jordan frame corresponds to a field-dependent normalisation point $\mu_E$ in the Einstein frame (see discussion in [14]),

$$\mu_E = \mu_J/\Omega(\chi) .$$

(3.17)

This relation is, of course, nothing but a manifestation of the conformal anomaly. Vice versa, fixing the Einstein frame normalisation point leads to a field-dependent $\mu_J$ which eliminates the logarithmic factor in (3.16) and preserves the asymptotic scale invariance of the Jordan frame action. This matches with the fact that fixed $\mu_E$ preserves the asymptotic shift symmetry in the Einstein frame. Thus the choice of fixed $\mu_E$ is favoured by the EFT framework developed in this section which contains the asymptotic shift symmetry as the key ingredient.

3.2 Connection with the low energy physics

We have seen that the inflationary physics in the model at hand up to a given order in the slow-roll parameters is determined by a finite number of coupling constants. It is important to understand if these constants can be related to the observables measured at small energies (i.e. at small values of the field) in the modern collider experiments. It turns out that the answer to this question cannot be given within the EFT picture and depends on the UV-completion.
One way to see the problem is to notice that establishing the desired relation implies, in particular, the knowledge of the scalar potential \( U(\chi) \) in the whole range from \( \chi = 0 \) to \( \chi = \infty \). Let us assume that this potential is known at tree level and estimate quantum corrections to it. We concentrate on the divergent parts which require introduction of counterterms in the bare action. The expansion of the bare potential in the background field method has the form,

\[
\lambda U(\bar{\chi} + \delta \chi) = \lambda \left( U(\bar{\chi}) + \frac{1}{2} U''(\bar{\chi})(\delta \chi)^2 + \frac{1}{3!} U'''(\bar{\chi})(\delta \chi)^3 + \frac{1}{4!} U^{(IV)}(\bar{\chi})(\delta \chi)^4 + \cdots \right),
\]

where for convenience we have singled out explicitly the overall coupling \( \lambda \). Computing loop corrections in, say, cutoff regularisation scheme one generates the divergences of the form:

- in one loop: \( \lambda U''(\bar{\chi}) \bar{A}^2, \lambda^2 (U''(\bar{\chi}))^2 \log \bar{A}, \) (3.19)

- in two loops: \( \lambda U^{(IV)}(\bar{\chi}) \bar{A}^4, \lambda^2 (U''(\bar{\chi}))^2 \bar{A}^2, \lambda^3 U^{(IV)}(U'')^2 (\log \bar{A})^2, \) (3.20)

where \( \bar{A} \) is the loop cutoff. Similar results would be obtained in the Pauli–Fierz regularisation. According to the standard rules we have to add corresponding counterterms to the bare Lagrangian in order to absorb the divergences. But the important point is that these counterterms have a different functional dependence on the background than the original potential. So we cannot really absorb them in the redefinition of the coupling constants. Moreover, there is no natural hierarchy between the lower and higher loop contributions. In this situation it is impossible to keep the radiative corrections under control for all values of the fields.

The important role in this reasoning is played by power-law divergences. This type of divergences is particularly affected by the UV-completion. Thus one concludes that the form of the potential at the intermediate values of \( \chi \) is sensitive to the high-energy physics. To make this statement more precise let us consider a UV-completion which involves a heavy particle with the mass \( m(\chi) \) depending on the scalar field value. At large values of the field the \( \chi \)-dependence of the mass has to obey the asymptotic shift symmetry (3.11), but no further restrictions can be imposed on it without the detailed knowledge of the UV theory. Then, even if the mass is made much higher than the scale of interesting processes and at tree level the particle can be integrated out, on the one loop level it will contribute the Coleman–Weinberg potential of the form (3.13). In general this leads to large modification of the potential, which is uncontrollable by the EFT. Thus we conclude that the inflationary and the present day physics cannot be connected without specific assumptions about the UV completion. (Still, both can be separately described within the EFT framework.)

A possible choice of the additional assumption, which was effectively adopted in [14, 15], is that the UV theory is such that the power-law divergences must be discarded al-

\footnote{It is worth stressing that \( \bar{A} \) is just a technical parameter of the regularisation procedure and thus need not coincide with the tree-level unitarity violation scale found in Sec. 2. In particular, one is free to choose \( \bar{A} \) to depend or not on the background.}
It implies that the UV theory does not contain any heavy elementary particles beyond the SM (or alternatively, the effect of heavy degrees of freedom on the low-energy physics completely cancels out). Technically, this assumption is implemented by the use of the scale-free minimal subtraction scheme based on dimensional regularisation in all loop computations. Then we see from (3.19), (3.20) that the remaining (logarithmic) divergences are suppressed by an extra power of the coupling constant $\lambda$ for each additional loop. So one can arrange the perturbative expansion in such a way that the divergences are absorbed order by order in $\lambda$. Namely one writes the potential as a formal series in $\lambda$,

$$U(\chi) = \lambda U_1(\chi) + \lambda^2 U_2(\chi) + \lambda^3 U_3(\chi) + \cdots. \tag{3.21}$$

Then the divergences originating from $U_i$ feed in only into terms of order $i+1$ and larger in $\lambda$. The functions $U_j, j \geq i+1$ can be chosen in such a way as to absorb these divergences.

Note that there are two physically distinct types of ambiguities associated with this procedure. First, the functions $U_i, i \geq 2$, are determined up to the finite part of the counterterms generated by the loops with $U_j, j < i$, insertions. This uncertainty is unavoidable in any quantum treatment of the model. The second source of uncertainty is the freedom in the choice of the tree-level action. Indeed, all functions $U_i$ can, in principle, be modified by an addition of an arbitrary function. This ambiguity eventually amounts to defining the model and must be fixed by some physical considerations.

Let us illustrate this point by considering the function $U_2$. Representing it in the form

$$U_2(\chi) = c_{21} \cdot \left(U''_1(\chi)\right)^2 + \tilde{U}_2(\chi), \tag{3.22}$$

we see that the one-loop divergence coming from $U_1$ is absorbed into the renormalisation of the coefficient $c_{21}$. This coefficient is thus promoted to an independent coupling constant that must be fixed from measurements. Its a priori unknown value represents an uncertainty of the first type. On the other hand, the unrenormalised contribution $\tilde{U}_2(\chi)$ is related to the second type of uncertainties. Its choice makes part of the definition of the model. For example, it can be consistently put to zero.

It appears that the same rearrangement can be done with the divergences appearing in the kinetic term and higher derivative terms. When $\chi$ is coupled to other fields, say, fermions and gauge fields, one has to promote all functions of $\chi$ in the action to a formal series in all coupling constants including the Yukawa and gauge couplings. Then, at least formally, the physics to any finite order in the coupling constants is determined by finite number of parameters. Note though that to make predictions in this approach one still needs an additional principle to fix the ambiguities in the coefficient functions of the formal series, such as, e.g., the choice of $\tilde{U}_2(\chi)$ in (3.22).

In [14, 15] this ambiguity was fixed by choosing the finite parts of all the coefficient functions beyond $U_1$ to be zero (i.e. the $\overline{\text{MS}}$ subtraction scheme was used). Let us estimate what kind of uncertainty is introduced in the bounds on the Higgs mass obtained in [14, 15].

\footnote{The physics behind this hypothesis is discussed in [29, 30] and is associated with exact, but spontaneously broken quantum scale invariance.}
by variations of the subtraction scheme.\textsuperscript{14} This uncertainty is represented by the finite part of the counterterms which we thus need to estimate. For this end we consider the bare potential

\[ U_B = \lambda U_1 + \left( \frac{1}{\epsilon + c} \right) \frac{\lambda^2 m_H^2}{64\pi^2}, \tag{3.23} \]

where we wrote explicitly the loop suppression factor and \( c \) is an arbitrary constant. The variation of \( c \) does not change the predictions for inflation, since the contribution of the finite part of the counter-term is negligible in the inflationary region. However, this term modifies the relation between the low energy Higgs mass \( m_H \) and the high energy scalar self-coupling, leading to the change in the lower and upper Higgs mass bounds by an amount

\[ \delta m_H \simeq \frac{9c}{64\pi^2} \frac{m_H^3}{v^2}, \tag{3.24} \]

where \( v = 246.22 \) GeV is the SM Higgs field vacuum expectation value. Numerically, for \( c \sim 1 \), this change is \( \sim 0.5 \) GeV for the lower bound on the Higgs mass (\( \simeq 126 \) GeV) and is about 2 GeV for the upper bound (\( \simeq 194 \) GeV). This is smaller than other uncertainties, related, e.g. to the experimental error in the mass of the top quark \([15]\). However, if \( c \) is large (say, \(|c| > 10\)), the uncertainties introduced by the finite part of counter-terms will dominate the error bars.

4. Generalisation

Here we discuss the application of the EFT ideas developed in Sec. 3.1 to a general class of inflationary models having the property of asymptotic shift symmetry. Let us start from a scalar theory with the potential

\[ U(\chi) = U_0(1 + u_1 F(\chi/M) + \cdots), \tag{4.1} \]

where \( F(x) \) is a given function with the property

\[ F(x) \to 0 \quad \text{at} \quad x \to +\infty \tag{4.2} \]

together with its derivatives, and \( u_1 \) is a coefficient of order one. Potentials of this class appear in many inflationary models, see e.g. [31] and references therein. Inflation happens at \( \chi > M \). Dots in (4.1) stand for the terms which are yet to be determined and which, as we will see, are required for consistency. The case considered in Sec. 3.1 corresponds to \( F(x) = e^{-x} \). Another example to have in mind is

\[ F(x) = \frac{1}{x^\alpha}, \quad \alpha > 0. \tag{4.3} \]

The expansion of the potential in the background field formalism is

\[ U(\bar{\chi} + \delta \chi) = U_0 \left( 1 + u_1 \sum_{k=0}^{\infty} \frac{F^{(k)}(\bar{\chi}/M)}{k!} \left( \frac{\delta \chi}{M} \right)^k + \cdots \right), \tag{4.4} \]

\textsuperscript{14}In the terminology of Refs. [14, 15] we are considering the renormalisation prescription I.
where $F^{(k)}$ is the $k$-th derivative of the function $F(x)$. One observes that the loop integrals calculated using (4.4) generate divergences proportional to the products of derivatives of $F$, \[ F^{(k_1)} \left( \frac{\chi}{M} \right) F^{(k_2)} \left( \frac{\chi}{M} \right) \cdots F^{(k_n)} \left( \frac{\chi}{M} \right). \] (4.5)

In general, all possible combinations of this type appear with the only restriction that the sum
\[ k_1 + k_2 + \cdots + k_n \]
is even, following from the fact that all the lines of the perturbation $\delta \chi$ must be closed into loops. This requires inclusion of the corresponding counterterms in the bare action. Thus we conclude that (4.1) must be extended to the following formal series,
\[ U(\chi) = U_0 \left( 1 + \sum_{k_1,k_2,\ldots,k_n} u_{k_1,k_2,\ldots,k_n} F^{(k_1)} \left( \frac{\chi}{M} \right) F^{(k_2)} \left( \frac{\chi}{M} \right) \cdots F^{(k_n)} \left( \frac{\chi}{M} \right) \right), \] (4.6)

where $u_{k_1,k_2,\ldots,k_n}$ are arbitrary coefficients (coupling constants). Similar extension must be performed with all coefficient functions appearing in the Lagrangian, i.e. those multiplying the terms with derivatives of $\chi$ and the interactions of $\chi$ with other fields. It is straightforward to convince oneself that the resulting form of the Lagrangian is stable under (perturbative) quantum corrections. Note that for particular choices of the function $F(x)$ some subsets of the terms in the series (4.6) may collapse to a single term. For example, this happens for the choice $F(x) = e^{-x}$ when (4.6) becomes a simple series in the powers of the exponent considered in Sec. 3.1.

A comment is on order. It appears natural to identify the scale $M$ with the cutoff in the loop integrals. In this case the size of the loop corrections to the coefficients $u_{k_1,k_2,\ldots,k_n}$ is of the order $O[(U_0/M^4)^{n-1}]$, which can be obtained by iterative use of (4.4) in the perturbative diagrams. This gives the bound on the natural value of these coefficients:
\[ u_{k_1,k_2,\ldots,k_n} \gtrsim \left( \frac{U_0}{M^4} \right)^{n-1}. \] (4.7)

Note that the ratio on the r.h.s. of this bound is smaller that one. This follows from the requirement that the energy density during inflation must be smaller than the cutoff to the fourth power. Thus the choice of the coefficients $u_{k_1,k_2,\ldots,k_n}$ saturating the bound (4.7) corresponds to hierarchically small values of the coefficients with higher $n$.

One may wonder if the expression (4.6) with its infinite number of terms is of any use. The answer is yes, provided the subsequent terms in the series vanish at $\chi \to +\infty$ faster than the previous ones. Then at the inflationary epoch corresponding to large $\chi$ one can account for them in the framework of an EFT expansion analogous to that developed in Sec. 3.1. Loosely speaking this requires that the function $F(x)$ is such that
\[ F^{(k+1)}(x) < F^{(k)}(x) \quad \text{at} \quad x \to +\infty. \] (4.8)
This restriction is rather mild and is satisfied by reasonable choices\footnote{In this sense the choice $F(x) = e^{-x}$ is a limiting case, $F^{(k+1)} = F^{(k)}$. Additional property that enables to develop the EFT description in this case is the simplification of the series (4.6) pointed above.} of $F(x)$. Additionally, the higher terms in the series (4.6) may be suppressed if the coefficients $u_{k_1,k_2,...,k_n}$ are chosen to obey the hierarchy corresponding to the saturation of the bound (4.7).

Let us see explicitly how the EFT picture works for the power-law choice (4.3). Substituting the derivatives into (4.6) and combining the terms of the same form together we arrive at the following double series,

$$U(\chi) = U_0 \left( 1 + \sum_{n=1, m=0}^{\infty} u_{n,m} \left( \frac{M}{\chi} \right)^{n\alpha + 2m} \right).$$

(4.9)

Keeping only the leading term in the series one obtains certain expressions for the inflationary observables\footnote{[31].}. In particular, for the slow-roll parameters one has

$$\varepsilon = \frac{\alpha^2 M_P^2}{2 M^2} u_{1,0}^2 \left( \frac{M}{\chi} \right)^{2\alpha + 2},$$

(4.10)

$$\eta = \frac{\alpha(\alpha + 1) M_P^2}{M^2} u_{1,0} \left( \frac{M}{\chi} \right)^{\alpha + 2}.$$  

(4.11)

To be concrete, we assume that $M$ is of order $M_P$ and $u_{1,0} = O(1)$. Then we obtain the relation

$$\varepsilon \simeq \eta \frac{2\alpha + 2}{\alpha + 2}.$$  

(4.12)

Let us use (4.9) to estimate the size of quantum corrections to the inflationary observables. The first subleading terms in the series (4.9) are

$$u_{1,1} \left( \frac{M}{\chi} \right)^{\alpha + 2} \quad \text{and} \quad u_{2,0} \left( \frac{M}{\chi} \right)^{2\alpha}.$$  

(4.13)

According to the above discussion the natural size of the coefficients $u_{1,1}$, $u_{2,0}$ generated by loop effects is

$$u_{1,1} \sim 1, \quad u_{2,0} \sim \frac{U_0}{M^4}.$$  

(4.14)

We now show that for a realistic inflationary model the second term in (4.13) is negligible compared to the first. From the COBE normalisation we have $U_0/M^4 \ll \varepsilon$ and thus

$$u_{2,0} \left( \frac{M}{\chi} \right)^{2\alpha} \ll \varepsilon \eta \frac{2\alpha + 2}{\alpha + 2} \simeq \eta \frac{2\alpha + 2}{\alpha + 2},$$  

(4.15)

where we have used the relations (4.11), (4.12). On the other hand,

$$u_{1,1} \left( \frac{M}{\chi} \right)^{\alpha + 2} \sim \eta,$$  

(4.16)

which is indeed larger than (4.15) for any positive $\alpha$ and $\eta \ll 1$. Thus the main correction comes from the first term in (4.13). Its relative size compared to the leading term is
\((M/\chi)^2\). Expressed via the slow-roll parameter this translates into \(\eta^2/(\alpha+2)\). Note that the correction behaves as a fractional power of \(\eta\).

It is worth mentioning that the modification of the Higgs inflation proposed recently in [32] belongs to the class of models considered in this section. It corresponds to the power-law choice (4.3) for the function \(F(x)\) and \(\alpha = 2\). Note though that in the case of [32] the scale \(M\) is much lower than\(^{16}M_P\), so the formulas of the previous paragraph should be appropriately modified when applied to this case.

5. Conclusions

In this paper we addressed the sensitivity of the Higgs inflation scenario proposed in [1] to the details of the UV completion of the theory. We determined the cutoff of the theory which we identified with the scale of violation of the tree-level unitarity. With this definition the cutoff depends on the background value of the inflaton (Higgs) field and we found that it is larger than the characteristic energy scale involved in the physical processes during the whole history of the Universe.

For clarity we concentrated on a simplified toy model obtained from the model of [1] by suppressing fermion and gauge fields. This captures the main features, namely the background dependence of the cutoff and the properties of the quantum corrections to the potential. As discussed in Appendix A, inclusion of the SM fermions does not affect our results. The situation is slightly more complicated when the gauge bosons are taken into account. The cutoff in the gauge sector is lower than in the pure inflaton theory, see Appendix B. Still, it is parametrically higher than the relevant energy scales at inflation and during the subsequent evolution (modulo subtleties at the beginning of the reheating epoch, see Appendix B).

We analysed the quantum corrections to the tree-level Lagrangian of the theory and formulated the assumptions about the UV completion that allow to keep these corrections under control. At the proper inflationary stage the sufficient conditions are concisely formulated as the requirement of asymptotic symmetry of the theory at large values of the inflaton field. Depending on whether one works in Jordan or Einstein frame, the corresponding symmetry is either scale invariance or the symmetry under the shifts of the field. This asymptotic symmetry allows to arrange the quantum corrections to the inflaton potential in an infinite series where subsequent terms are suppressed by a small parameter which is physically identified with (a power of) the slow-roll parameter \(\eta\). Besides, the Lagrangian of the low-energy theory contains standard EFT contributions with higher derivatives of the inflaton field considered in [21, 22]. Thus our results extend the EFT approach to inflation to the case of the inflaton potential.

The approach developed in this paper is general and we showed how it can be applied to a wide class of inflationary models with asymptotic shift (or scale) symmetry. In particular, the modification of the Higgs inflation model proposed in [32] belongs to this class. On the other hand, our method does not apply to the “new Higgs inflation” of [33, 34] which does\(^ {16}\)Namely, \(M = M_P/\sqrt{\xi}\), where \(\xi \gg 1\) is the coefficient of the non-minimal coupling of the inflaton to curvature in the Jordan frame.
not seem to possess any asymptotic shift or scale invariance, and thus requires a separate study of stability against radiative corrections.

We also analysed in the context of the model of [1] under which conditions the parameters of the inflationary EFT can be related to the observables measured at low energies. We showed that this requires further assumptions about the UV completion. We considered the assumption that the UV completion is such that all power-law divergences appearing in the loop diagrams must be discarded. Physically, this amounts to the statement that the effects of possible heavy degrees of freedom present in the full theory on the low-energy physics completely cancel out. We demonstrated that this (admittedly, strong) assumption is sufficient to establish the link between inflation and the low-energy physics once the tree-level action of the model is fixed.

Let us conclude with the following remark. The approach we adopted in this paper is that of the effective field theory. Thus the asymptotic shift symmetry that was crucial for us to develop the consistent perturbation scheme for inflationary calculations was just assumed: we did not address the question how it appears at the level of UV-complete theory. One may speculate in this connection that the truly fundamental property is the scale invariance of the Jordan frame action at large values of the inflaton (which is, of course, equivalent to the Einstein frame shift symmetry). This may indicate that the UV-complete theory has an exact, but spontaneously broken quantum scale invariance, relevant also for gauge hierarchy, cosmological constant, and dark energy problems [29, 30]. This may also be related to the existence of a scale invariant UV fixed point. Recently similar ideas were expressed in [23]. It would be interesting to understand the connection between this work and the results of the present paper.

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A. Fermions

Let us analyse the effect of the fermion field with the Lagrangian in the Jordan frame,

$$\mathcal{L}_Y^J = i\bar{\psi}\partial\psi + y\phi\bar{\psi}\psi, \quad (A.1)$$

where $y$ is the coupling constant. After conformal transformation (2.16) with the appropriate rescaling of the fermion

$$\psi \rightarrow \Omega^{3/2}\psi, \quad (A.2)$$

the interaction in the Einstein frame becomes

$$\mathcal{L}_Y^E = y\frac{\phi(\chi)}{\sqrt{1 + \xi\phi^2(\chi)/M_P^2}}\bar{\psi}\psi. \quad (A.3)$$

With the help of the relation (2.19) it is easy to reproduce the results of the Sec. 2 in all three regions of the background field.
• \( \tilde{\phi}, \tilde{\chi} \ll M_P/\xi \). Using (2.24) we get

\[
\Lambda_{(n)} \sim \frac{M_P}{\xi} y^{-1/(n-4)},
\]  

which coincides (apart from the obvious change of the coupling constant) with (2.25).

• Intermediate region \( M_P/\xi \ll \tilde{\phi} \ll M_P/\sqrt{\xi} \). In this region the relation between the \( \phi \) and \( \chi \) fields is

\[
\phi \approx \left( \frac{2}{3} \right)^{1/4} \sqrt{\frac{M_P \chi}{\xi}},
\]

and thus (A.3) takes the form

\[
\mathcal{L}_Y^E \approx y \left( \frac{2}{3} \right)^{1/4} \sqrt{\frac{M_P \chi}{\xi}} \bar{\psi} \psi .
\]

Clearly, expansion of this expression in the perturbations \( \delta \chi \) produces an infinite series of higher-order operators of the form

\[
y \left( \frac{2}{3} \right)^{1/4} \sqrt{\frac{M_P \chi}{\xi}} \frac{(\delta \chi)^n \bar{\psi} \psi}{\chi^{n-1}}.
\]

This shows that for moderately small Yukawa couplings the scale suppressing higher interactions is essentially equal to \( \tilde{\chi} \) which gives the cutoff

\[
\Lambda \simeq \tilde{\chi}.
\]  

Given the relation (A.5) this coincides with the expression (2.14) obtained in the Jordan frame.

• \( \tilde{\phi} \gg M_P/\sqrt{\xi} \). Using the relation (2.30) we obtain the exponential interaction

\[
\frac{y M_P}{\sqrt{\xi}} \left( 1 - \frac{1}{2} e^{-\frac{3}{2} \bar{\phi}/M_P} + \ldots \right) \bar{\psi} \psi,
\]

which upon expanding in the excitations \( \delta \chi \) yields the Planck mass cutoff (2.33).

Thus we conclude that the addition of fermions does not change the value of the cutoff.

B. Gauge bosons

The addition of the gauge bosons is also most simple to analyse in the Einstein frame. As the vector fields do not change under the conformal transformation (2.16), the only change is in the gauge-Higgs interactions. For example, in the unitary gauge the only change is in the mass terms of the gauge bosons,

\[
y^2 \phi^2 A_\mu A_\mu \rightarrow g^2 \phi(\chi)^2 \frac{\Omega^2(\chi)}{\Omega^2(\chi)} A_\mu A_\mu,
\]  

(B.1)
where $A_{\mu}$ is a gauge boson, and $g$ is the weak coupling constant. Full action in arbitrary gauge can be obtained similarly to the chiral electroweak model and is described in detail in [15].

We see that at nonzero background the Higgs field excitations $\delta \chi$ interact with the gauge bosons weaker than in the normal Higgs model. Thus the Higgs mechanism fails at the inflationary and reheating epochs—the diagrams with the perturbations of the Higgs field do not cancel the growth of the amplitudes with non-abelian vector bosons. Thus the gauge fields will enter into strong coupling at the energy $m_A(\bar{\chi})/g$, where $m_A(\bar{\chi})$ is the mass of the vector bosons. Reading out the expressions for $m_A(\bar{\chi})$ from (B.1) [1, 11, 15] we obtain the (Einstein frame) cutoff in the gauge sector,

$$\Lambda_A(\bar{\chi}) \simeq \begin{cases} \bar{\phi} \simeq \sqrt{\frac{M_P \bar{\chi}}{\xi}} & \text{at } \frac{M_P}{\xi} \lesssim \bar{\chi} \lesssim M_P, \\ M_P \sqrt{\xi} & \text{at } M_P \lesssim \bar{\chi}. \end{cases} \quad (B.2)$$

This is lower than the values (2.29), (2.33) obtained in the pure inflaton model. Still, during inflation $\Lambda_A$ is parametrically higher than all the characteristic energy scales discussed at the end of Sec. 2. Also during reheating the momenta of the vector bosons, which are of order $m_A$ [11], are safely below $\Lambda_A$. Some tension arises for the momenta of the Higgs excitations, $p_{\text{Higgs}} \sim \sqrt{\xi} \bar{\phi}^2/M_P$ [11], which may be larger than $\Lambda_A$ at the beginning of the reheating period corresponding to $\bar{\phi} \simeq M_P/\sqrt{\xi}$. However, significant energy is transfered to the relativistic Higgs excitations only at later stages. Thus one does not expect this lower cutoff to alter qualitatively the analysis of reheating in [11, 12].

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