General model and the mechanics of concrete elements and structures deformation

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Abstract. The article presents a critical analysis of the model development of reinforced concrete elements and structures resistance by force influences. Their relationship with the defining provisions of the mechanics of reinforced concrete elements deformation was evaluated. The main disadvantages of the models considered are identified. A universal deformation and force model of deformation of reinforced concrete elements and structures is proposed, which is free of most of the disadvantageous characteristic of other models. The main advantage of the deformation-force model of resistance is indicated, due to the expansion of the known system of static, geometric and physical relationships to the level of statically definable. It becomes noticeable when calculating reinforced concrete elements and structures for deflection and crack resistance. This is one of the main features of the deformation-force model, which makes it possible to quite simply “embed” the technical theory of reinforcement to concrete adhesion into the general theory of reinforced concrete deformation. Due to the averaged design cross-section and the averaged materials deformations in the block between cracks, all parameters of cracks formation and propagation in the deformation-force model are directly related to the parameters of reinforcement to concrete adhesion.

1. Introduction

Reinforced concrete theory is not just a set of provisions and statements about the regularities of concrete and reinforcement deformation and their joint interaction. As a true scientific theory, it must simultaneously describe the properties of these materials, explain the holistic picture of the reinforced concrete elements and structures deformation laws and provide opportunities to determine their true stress-strain state at any stage of deformation. Therefore, its development has always been accompanied by the improvement of the following structural elements:

- prerequisites – mainly in the form of fundamental concepts and principles, laws and axioms, assumptions and hypotheses specific to reinforced concrete;
- an idealized object, mainly in the form of abstract models of the definitions, properties and interconnections of reinforced concrete as a solid body;
- logic – in the form of a set of specific relationships (rules, methods and means) aimed at clarifying the structure of knowledge about concrete and aimed at their development
- conclusions – as a set of laws and statements, methods and techniques capable of predicting the condition of reinforced concrete elements and structures at any stage of their deformation.

The degree of mutual coherence of these structural elements continues to affect the completeness and effectiveness of the developed methods of calculating reinforced concrete elements.
2. Analysis of recent research and publications

In the early stages of its development, concrete was modeled as a completely solid (elastic) body. His condition was described by a system of static, geometric and physical relations, called the corresponding equations of solid-state mechanics. These relationships were based on the following assumptions, hypotheses and preconditions:

- the reinforcement and concrete of the compressed zone are deformed elastically by Hooke's law;
- the concrete does not accept the extended area of participation in the work of the reinforced concrete element;
- in sections normal to the longitudinal axis of the element, the flat section hypothesis holds.

All this together allowed us to form a methodology for calculating reinforced concrete elements and structures by “allowable stresses”. However, it was later found that the actual stress distribution in concrete over the height of the cross-section of the element does not correspond to the triangular stress plot, and therefore concrete should not be considered as elastic material.

At the second stage of the reinforced concrete theory development, since the early 1930's, it began to be modeled as a deformed solid. Since taking into account the elastic-plastic properties of concrete required correction of the corresponding ratios (equations) of the mechanics of deformed solid (MDS), it was implemented due to the following assumptions:

- the hypothesis of boundary equilibrium has been advanced, according to which the structure material is in the limiting state before destruction and the cross-section of the reinforced concrete element is in equilibrium [1];
- the performance of the compressed zone concrete was described by nonlinear deformation diagrams;
- the performance of the stretched concrete in deforming the reinforced concrete element was not taken into account.

It is on their basis in the late 1930's, a new method of calculating cross-sections of reinforced concrete structures by “destructive efforts” was formed and introduced into the normative documents of many countries of the world. Since then, the development of general theory of reinforced concrete has been divided into two separate directions.

In most countries of the world, it continued to develop on a conditionally “deformation” model, which involved the use of additional prerequisites:

- the operation of the compressed zone concrete in the cross section of the element is described by idealized but nonlinear deformation diagrams;
- in cross sections normal to the longitudinal axis of the reinforced concrete element, the flat section hypothesis remains valid throughout the deformation process.

In the USSR and some other countries, a “power” model of reinforced concrete deformation was adopted. According to her, it was assumed that the destruction of structures in the limiting stage is due to the formation of so-called “hinges of plasticity”. It was suggested to use in the calculations a rectangular plot of stresses in the concrete of the compressed zone [2] without involving the hypothesis of flat sections. This simplification made it possible to significantly simplify the calculations and achieve a significant economic effect for many types of reinforced concrete slabs, non-cut beams and frames compared to the allowable stress method. However, simplified calculations of reinforced concrete structural elements sections by “destructive forces” could not cover the issue of their rigidity and fracture toughness. In addition, possible deviations of actual loads and strength characteristics of materials from their calculated values were taken into account by one synthesizing factor of safety margin. And this made it very difficult to adjust it. But the biggest methodological disadvantage of simplified calculations for “destructive efforts” was that the use of the “plastic hinge” model removed the theory of reinforced concrete from the methods of construction mechanics.

The further development of the theory of reinforced concrete structural elements deformation by the model of “plastic hinge” was the calculation method of structures cross sections by boundary states, proposed in the late 1950's [3, 4]. Its implementation was associated with a system of coefficients, which ensured that such conditions would not follow either in the most unfavorable
combination of all loads or in the lowest values of the strength materials characteristics. The load bearing strength of reinforced concrete elements remained virtually unchanged. The introduction of the system of the above calculated reliability coefficients allowed to develop “empirical” methods of calculating reinforced concrete elements for rigidity and fracture toughness, but, unfortunately, did not ensure the methodological unity of structures calculations by boundary states.

By the beginning of the 21st century, it became clear to everyone that the “power” model had completely exhausted itself. Since then, the theories of reinforced concrete, in general, and the method of calculating cross-sections of structures by boundary states, in particular, have been oriented to the "deformation" model. With the development of computer technology, this model is not only finally established in the Western countries, but also gained recognition in the countries of Eastern Europe and the former USSR. However, now it seems that the “deformation” model already needs significant refinement, since often “idealized objects” (abstract models of reinforced concrete) are rather poorly linked to the "logic" of his research, and therefore:

- technical theory of reinforcement with concrete coupling is extremely poorly integrated into the general theory of reinforced concrete;
- the question of methodological (dialectical) unity of structures calculations by boundary states is almost never solved;
- the internal static uncertainty of reinforced concrete elements, even in the load bearing strength calculations, is in most cases revealed by numerous iterations;
- in the calculations of fracture toughness and deflection, there is a need not only for numerous iterative operations, but also for the use of various empirical parameters, dependencies and coefficients;
- the lack of universal calculating methods of reinforced concrete elements and structures that could be implemented both programatically and by engineering express methods, leads not only to the loss of engineering complexity of the calculation results, but also to the physical nature of the deformation processes.

3. The purpose and objectives of research

These studies are aimed at forming a universal theory of deformation of reinforced concrete, which is based on:

- defining hypotheses and preconditions not only of deformation, but also of power models;
- an idealized object of study in the form of an abstract model of a deformed rigid body with elastic-plastic properties;
- sufficiently substantiated logic of research in the form of the generally recognized “extended” system of MDS equations;
- ability to calculate or predict the condition of reinforced concrete elements and structures at any stage of their deformation.

The construction of a universal theory of reinforced concrete on these principles will help to solve all the above problems related to the “logic” of research.

4. Results and discussion

Modern analysis of the research state in the field of reinforced concrete suggests that one of the most successful idealized objects of the general (universal) theory of its deformation can be considered a model of deformed solid. This model allows not only to reproduce the elastic-plastic properties of the concrete itself, but also to offer their variant solutions in the form of force, deformation and universal deformation-force models [5].

The variants themselves or the ways in which the general theory of concrete is developed depends to a great extent on the basic initial hypotheses and assumptions. And in this the decisive role should be given to the hypotheses of boundary equilibrium, flat sections and “nonlinearity” of rigidity [6].

In the universal deformation-force model, whose development is initiated by the author, the limit equilibrium hypothesis \( M_{Ed,max} = M_u \) is one of the most important. It is not only the basis for static
relationships characterizing the reinforced concrete deformation, but also serves as a general criterion for the exhaustion of the load-bearing strength of a reinforced concrete element or structure [7].

Equally important is the hypothesis of flat sections, by which geometric relationships of reinforced concrete elements and structures deformation are formed. In general, there are a number of studies where the validity of this hypothesis is generally denied [8-10]. Indeed, in some sections of a reinforced concrete element with cracks, the flat section hypothesis does not work. But V. Ya. Bachinsky [11, 12] substantiated why the general stress-strain state (SSS) of a reinforced concrete element cannot be estimated by deformations and stresses of materials measured only in a separate section. It is not necessary to consider even the average, but the “focal” section of the element in the block between the cracks (Figure 1). And for him, in such circumstances, the conjecture of flat sections generally remains valid throughout the deformation process of an element. Perhaps this is why the vast majority of alternative solutions [8, 10, 13] also boil down to the “hidden” use of the flat cross-section hypothesis, rather than being constructed by the “deplanation” of normal cross sections.

The decisive role in the formation of general (universal) theory of reinforced concrete is given to the hypothesis of rigidity “nonlinearity”. It underlies the "deformation-force" model of reinforced concrete elements resistance to force influences [5]

\[ M \cdot r = A - B \cdot r^{-1} \cdot r_u - C \cdot M \cdot M_u^{-1}, \]  

where \( A, B, C \) are the parameters that reflect the strength and deformation characteristics of the element section, which are directly related to its initial stiffness (\( D_o \)), curvature (\( r_u^{-1} \)) and bearing strength (\( M_u \)).

According to this hypothesis, the generalized diagram of the reinforced concrete element condition (Figure 2) takes on a universal appearance

\[ M = \left[ D_o \cdot r^{-1} - M_u \cdot (r_u \cdot r^{-1})^2 \right] \left[ 1 + (D_o \cdot M_u^{-1} - 2 \cdot r_u \cdot r^{-1}) \cdot r^{-1} \right]^{-1} \]  

and is transformed, under certain initial conditions, into a generally recognized concrete deformation diagram

\[ \sigma_c = f_{ck} \cdot \left[ E_{co} \cdot \varepsilon_c \cdot (E_{cu} \cdot \varepsilon_{cu})^{-1} - (\varepsilon_c \cdot \varepsilon_{cu}^{-1})^2 \right] \left[ 1 + (E_{co} \cdot E_{cu}^{-1} - 2 \cdot \varepsilon_c \cdot \varepsilon_{cu}^{-1}) \right]^{-1}, \]  

where \( f_{ck} \) - the characteristic value of the concrete compressive strength; \( E_{co} \) - initial value of modulus (deformation) of concrete; \( E_{cu} \) - limit value of the concrete deformation module; \( \varepsilon_{cu} \) - value of compressed concrete boundary deformation.
Figure 2. Forms of the state diagrams of the bending reinforced concrete element depending on the percentage of its reinforcement (\( \rho_1 = 0.63 - 2.52 \)).

Thanks to the above transformation of diagrams \( M - r^{-1} \) and \( \sigma_c - \varepsilon_c \) methodological unity of reinforced concrete elements and structures calculation is provided for bearing strength, crack resistance and deflections. Moreover, the universal state diagram of the reinforced concrete element (2) acts as a link that not only links but also complements the MDS ratio system [5, 6]:

- static \( M = f(\varepsilon_c, \varepsilon_{ct}, \varepsilon_s) \), \( N = f(\varepsilon_c, \varepsilon_{ct}, \varepsilon_s) \);
- geometric \( r^{-1} = f(\varepsilon_c, \varepsilon_{ct}, \varepsilon_s) \);
- physical \( \sigma_c = f(\varepsilon_c) \), \( \sigma_{ct} = f(\varepsilon_{ct}) \), \( \sigma_s = f(\varepsilon_s) \).

\[
\begin{aligned}
\text{Equation system (4)}
\end{aligned}
\]

In general, the system of MDS equations (4) is complemented not only by the state diagram (2) but also by the function of compressed concrete boundary deformations [5]. It was obtained from the same diagram (2) by the application of the extreme criterion of bearing strength (the Fermi criterion). As a result, the “logic” of reinforced concrete elements and structures studies by deformation-force model allows to extend the mentioned system of MDS equations (4) to the level of statically significant. In such circumstances, it is possible to significantly reduce the number of iterative operations in the calculations and even completely eliminate them. And this, in turn, contributes to the development of engineering express methods of calculating reinforced concrete elements and structures. In the power model, similar methods have already been used. When using the “plastic hinge” system (4) became statically significant, and the need for the use of geometric and physical relations in general was eliminated [3].

The advantages of the deformation-force resistance model [5-7] with the extended relations system (4) become especially noticeable in the calculations of reinforced concrete elements and structures in the deflections and fracture toughness. According to the vast majority of existing methods [8, 14-16], the calculation of bends and cracks is based on the fact that the concrete element or structure SSS is first determined separately in the sections with cracks and in the middle sections between them. Subsequently, on the basis of already averaged parameters or characteristics of an element SSS, its
averaged settlement cross-section is formed. The characteristic difference between the deformation-force model is that it allows to determine directly the curved curvature of the reinforced concrete element at any deformation stage

\[
r^{-1} = \frac{2M_u \cdot r_u}{(1 - M \cdot M_u^{-1}) \cdot D_o \cdot r_u^{-1} + 2M - \sqrt{(1 - M \cdot M_u^{-1}) \cdot D_o \cdot r_u^{-1} + 2M}^2 - 4M \cdot M_u}
\]

and to relate it, in the flat section hypothesis, directly to the relative materials deformations in the central section of the block between cracks

\[
r^{-1} = (\varepsilon_{c2} + \varepsilon_{s1}) \cdot d^{-1},
\]

where \( \varepsilon_{c2} \) - current values of the most compressed face concrete relative deformations; \( \varepsilon_{s1} \) - current values of elongation of reinforcement relative deformations; \( d \) - the working height of the section of the rectangular profile element, respectively.

Another peculiarity of the deformation-force model is that it allows to quite simply "embed" the technical theory of reinforcement with concrete coupling into the general theory of reinforced concrete deformation [17-19]. Due to the averaged cross section and the averaged materials deformation in the block between the cracks, all the crack formation parameters are directly related to the parameters of the reinforcement with concrete coupling (Figure 3).

![Figure 3. Calculation scheme of bending reinforced concrete element stress-strain state in the block between two cracks](image)

In this case, the solutions obtained are much simpler than those offered in the framework of fracture mechanics by models of “double console elements” or folded rods. The stated advantages of the mentioned models [8, 13, 20] in the general reinforced concrete theory are generally debatable. The fact is that the fracture mechanics can fairly effectively describe the laws of origin and heterogeneities development and defects in the structure of relatively homogeneous materials under different loading types. The application of the destruction mechanics to reinforced concrete, in which even before the loading start there are already many heterogeneities and defects in the form of pores and sinks, micro- and macro-cracks and dislocations of various kinds, leads to solutions whose implementation is extremely difficult, even with the help of special software complexes. In these circumstances, it is premature to speak about the fracture mechanics introduction into the practice of designing reinforced concrete structures.

The effectiveness of the proposed deformation-strength model of the reinforced concrete to the forces resistance and the methods of reinforced concrete elements and structures calculation developed
on its basis were evaluated by comparing the theoretical calculations with the experimental data results (Table 1). At the same time, similar comparisons were made with the results of calculations performed according to the current norms methods [15, 16]. All of them showed that the priority in the accuracy of the bearing strength estimation, deflections and width of the reinforced concrete elements normal cracks opening belongs to the calculation methods, which underlie the deformation-strength model of reinforced concrete resistance.

Table 1. Comparison of theoretical and experimental values of the studied parameters.

| Authors and research parameters | Deviation of the experimental data from the calculated methods |
|---------------------------------|----------------------------------------------------------|
|                                 | [15] | [16] | [5] |
|                                 | Δ    | σ    | v.% | Δ   | σ   | v.% |
| Bearing strength of bending elements |        |        |     |        |        |     |
| Pam H. J. [21]                  | 1.020| 0.083| 7.83| 1.016| 0.080| 7.83 |
| Sarkar S. [22]                  | 1.052| 0.170| 16.00| 1.048| 0.174| 16.65|
| Burns N., Siess C. [23]         | 1.019| 0.1553| 15.24| 1.022| 0.1585| 15.51 |
| Ernst G. C. [24]                | 1.011| 0.1798| 17.78| 1.009| 0.1881| 18.64 |
| Pundinaitė M. [25]              | 1.289| 0.3002| 23.29| 1.334| 0.3209| 24.06 |
| Gilbert R., Nejadi S. [26]      | 1.095| 0.3321| 30.33| 1.103| 0.3766| 34.14 |

| Deflections                      |        |        |     |        |        |     |
|                                  |        |        |     |        |        |     |
|                                  |        |        |     |        |        |     |
|                                  |        |        |     |        |        |     |
|                                  |        |        |     |        |        |     |

5. Conclusions

Thus, suggested by the authors direction of reinforced concrete general theory development by deformation-force model of deformed solid body allows:

- to achieve the methodological unity of the reinforced concrete elements and structures calculation by bearing strength, fracture resistance and deflections due to the dialectical transformation of the elements state diagrams into material deformation diagrams;
- significantly reduce the static uncertainty level of the generally recognized system of equations (ratios) of MDS or in general transform this system into statically significant due to its expansion by the element state diagram analytical dependence and the function of the compressed concrete boundary deformations;
- effectively predict not only the overall rigidity, but also any other parameter of the stressed-stain state of reinforced concrete elements and structures at any stage of their deformation with minimal use of empirical parameters and coefficients, or even without their involvement;
- due to its relative simplicity, to create engineering express methods of calculating reinforced concrete elements and structures by boundary states without the use of special software;
- it is quite simple to “integrate” the technical theory of reinforcement with concrete coupling into the general theory of reinforced concrete deformation due to the use of parameters and characteristics of the reinforced concrete elements and structures cross-section in the areas between normal cracks.

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