Heat transfer performance of nano-suspension of $Al_2O_3$ in ([C4mim][NTf2]) ionic liquid around a circular cylinder

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Abstract. IoNanofluids are a new category of heat transfer fluids synthesized by suspending fine nanoparticles in ionic liquids. These IoNanofluids show superior heat transfer characteristics than conventional nanofluids and are more suitable for medium to high temperature applications. In this study, a numerical analysis of heat transfer performance of nano-suspension of $Al_2O_3$ in ([C4mim][NTf2]) ionic fluid around a circular cylinder has been presented. A 2-D, laminar, steady and forced convective flow around a hot circular cylinder at a constant temperature has been taken into account at $10 \leq Re \leq 40$ and $0 \% \leq \phi \leq 2.5 \%$. Governing equations of flow and heat transfer are solved using SIMPLE algorithm based Finite Volume Method (FVM). An unique aspect of this study is the consideration of the influence of temperature on the thermo-physical properties of the IoNanofluids. Heat transfer characteristics are quantified in terms of mean Nusselt numbers and the thermal field around the circular cylinder has been visualized using isotherms. Influence of flow Reynolds number, particle volume fraction and inlet temperature over the local and mean Nusselt numbers has been discussed in detail. Evidently, increase in flow velocity and addition of nanoparticles resulted in heat transfer augmentation. Additionally, heat transfer performance of $Al_2O_3$--([C4mim][NTf2]) IoNanofluid is compared with the conventional $Al_2O_3$--$H_2O$ nanofluid. Comparatively, IoNanofluids outperformed conventional water based nanofluids with 59 % higher heat transfer enhancement ratios. Also, the heat transfer enhancement ratios were noted to be higher at high temperatures. Thus, the new class of cooling liquids (IoNanofluids) are promising working fluids for advanced real time high temperature engineering applications.

Keywords — Ionic Nanofluids, Thermal Stability, Bluff Body, Heat Transfer.

1. Introduction
Today’s advanced technology requires efficient cooling for miniaturized systems with high intensity heat fluxes. The demand for such ultra-high performance cooling systems led to the discovery of a new class of heat transfer fluids called nanofluids. S.U.S Choi conceived the idea of nanofluids [1], and proved that the thermal conductivity of liquids can be improved by adding nanoparticles. Nanofluids contain fine nanoparticles stably suspended in base fluids [2]. In the past two decades, several researchers have carried numerous research attempts to analyze the flow and thermal behavior of nanofluids [3]. Water, oil, and ethylene glycol are the traditionally used base fluids in the synthesis of nanofluids [3]. Though these nanofluids showed promising heat transfer performance, their applications were limited due to poor thermal...
stability. At higher temperatures the base fluids (water, oil, ethylene glycol) would vaporise, leaving behind the nanoparticles. At the wake of problems regarding the limitations of use traditional base fluids at high temperatures, a new class of nanofluids called IoNanofluids were discovered [4]. They were similar to the conventional nanofluids, but had Ionic fluids as their base fluids. These ionic fluids are molten salts or salts that stay in liquid phase above room temperature (293 K) to extremely high temperatures (> 1000 K) [5]. Being relatively new in its conception, the field of IoNanofluids is growing with alacrity, and has seen the emergence of several literatures on performed experimental and numerical studies to determine the thermophysical properties[4, 5, 6, 7]. Based on these studies, as a testament to their superior properties over the conventional nanofluids, they possess higher thermal stability and better thermophysical properties.

Electronics cooling, heat exchangers, solar collectors, chimney stacks, cooling towers, power generators and nuclear fuel element cooling, etc are some engineering applications, that mimic the flow past bluff bodies. Furthermore, flow and heat transfer around bluff bodies is a classical topic of academic interest. Numerous detailed works on this flow phenomenon is available in the literature [8, 9, 10]. Recently, with the advent of nanofluids, several researchers are working on numerical analysis of nanofluid flow and heat transfer around bluff bodies. Valipour and Ghadi [11] performed a numerical analysis of forced convective heat transfer of nanofluids around a circular bluff body. In this study, the thermo-physical properties of nanofluids were estimated using classical models of Hamilton-Crosser [12] and Brinkman [13]. A numerical study on flow and heat transfer of nanofluids over a square cylinder also produced synonymous results [14]. Vegad et al. [15] numerically analyzed the heat transfer behavior of nanofluids around a circular cylinder and reported an enhancement in heat transfer with addition of nanoparticles. It was noted that the heat transfer enhancement produced by the nanofluids is a function of properties of basefluids, nanoparticles and particle volume fraction [16]. During a numerical analysis of upward laminar mixed convection past a circular cylinder, Bing and Mohammed [17] reported that smaller nanoparticles produced higher enhancement in thermal conductivity. Furthermore, a numerical study by Farooji et al. [18] revealed that the maximum enhancement in heat transfer occurs at an optimum volume fraction for any given particle diameter. Adding to that, it was observed that particle shape is an important parameter that influences the heat transfer. Chamkha et al. [19] numerically analyzed the transient natural convection of nanofluids over a vertical cylinder and showed that nanoparticles in spherical shape resulted in better heat transfer enhancement that cylindrical nanoparticles. A detailed study on forced and mixed convective nanofluid flow past a circular cylinder at high Prandtl numbers by Sarkar et al. [20] showed a stabilizing effect in flow and enhancement in heat transfer by increasing the volume fraction. During a study on mixed convective flow of nanofluids around a circular cylinder at high Prandtl numbers, similar results were observed by Sarkar et al. [21]. A strong relation between nanoparticle volume fraction and mean Nusselt number was observed during a numerical study on mixed convective nanofluid flow around a square cylinder. Addition of nanoparticles to the basefluid resulted in more number of low frequency higher energy modes in a mixed convective flow around a square cylinder [22]. Furthermore, addition of nanoparticles led to a decrease in total entropy generation [23]. A recent study by Deepak Selvakumar et al. [24] highlighted the effects of of uncertainties in prediction of effective properties of nanofluids on flow and heat transfer performance around a circular bluff body. Choice of numerical approach is very important in numerically modeling of nanofluids [25, 26].

Detailed review of the archived literature reveals that all of the available studies on nanofluids flow and heat transfer around bluff bodies consider only conventional nanofluids. With respect to IoNanofluids, there are works reported in literature, that deal with experimental measurement of thermo-physical properties of IoNanofluids. Only, very few studies are recently reported in numerical analysis of flow and heat transfer performance of IoNanofluids [27, 4, 28, 29]. In
order to bridge this gap, present study deals with numerical analysis of flow and heat transfer behavior of $Al_2O_3 - ([C4mim][NTf2])$ IoNanofluids, around an heated circular cylinder, with temperature dependent thermo-physical properties. Additionally, the heat transfer enhancement of IoNanofluids is compared with conventional water based nanofluids.

2. Mathematical formulation and problem statement
A hot circular cylinder (at $T_w$) is exposed to a $2-D$, laminar, steady flow of $Al_2O_3 - ([C4mim][NTf2])$ IoNanofluid at ambient temperature $T_{\infty}$. The cylinder has a diameter $D$ and is considered to be infinitely long in the $z-$ direction. The IoNanofluid has a uniform velocity ($U_\infty$) in the positive $x-$ direction. The outer boundary is placed at a distance of $100D$ away from the cylinder as graphically shown in Fig. 1. Present study considers $Al_2O_3 - ([C4mim][NTf2])$ IoNanofluid to be a homogeneous fluid with effective thermo-physical properties adopted from experimental data available in literature [4, 27].

![Figure 1: Graphical view of the computation domain for flow around a circular cylinder.](image)

A single phase approach has been adopted and the continuity, momentum and energy equations for Newtonian fluid flow are solved in the dimensional form as follows, assuming that the viscous dissipation is negligible.

Continuity equation:

$$\frac{1}{r^*} \frac{\partial (r^* u_r^*)}{\partial r^*} + \frac{1}{r} \frac{\partial u_\theta^*}{\partial \theta^*} = 0 \tag{1}$$

Momentum equations:

$$\left(\frac{u_\theta^*}{r^*} \frac{\partial u_r^*}{\partial \theta^*} + u_r^* \frac{\partial u_r^*}{\partial r^*} + \frac{u_r^* u_r^*}{r^*}\right) = \frac{1}{\rho_{eff} r^*} \frac{\partial p^*}{\partial \theta^*} + \nu_{eff} \left(\frac{1}{r^*} \frac{\partial}{\partial r^*} \left(r^* \frac{\partial u_\theta^*}{\partial r^*}\right) + \frac{1}{r^*} \frac{\partial^2 u_\theta^*}{\partial \theta^*^2} + \frac{2}{r^*} \frac{\partial u_r^*}{\partial \theta^*} - \frac{u_\theta^*}{r^*}\right) \tag{2}$$

$$\left(\frac{u_\theta^*}{r} \frac{\partial u_r^*}{\partial \theta^*} + u_r^* \frac{\partial u_r^*}{\partial r^*} - \frac{u_r^* u_r^*}{r^*}\right) = -\frac{1}{\rho_{eff} r^*} \frac{\partial p^*}{\partial r^*} + \nu_{eff} \left(\frac{1}{r^*} \frac{\partial}{\partial r^*} \left(r^* \frac{\partial u_\theta^*}{\partial r^*}\right) + \frac{1}{r^*} \frac{\partial^2 u_\theta^*}{\partial \theta^*^2} - \frac{2}{r^*} \frac{\partial u_r^*}{\partial \theta^*} - \frac{u_\theta^*}{r^*}\right) \tag{3}$$
Energy equation:

\[
\left( \frac{u_\theta}{r^*} \frac{\partial T^*}{\partial \theta^*} + u_r \frac{\partial T^*}{\partial r^*} \right) = \alpha_{eff} \left( \frac{1}{r^*} \frac{\partial}{\partial r^*} \left( \frac{\partial T^*}{\partial r^*} \right) + \frac{1}{r^*2} \frac{\partial^2 T^*}{\partial \theta^*} \right)
\]  

(4)

2.1. Boundary conditions
Following are the conditions applied at various boundaries (see Fig. 1) in order to solve the flow problem.

Inflow boundary (AB): At the inflow boundary,

\[ u_\theta = \sin \theta; \quad u_r = -\cos \theta; \quad T = T_\infty \]  

(5)

Outflow boundary (BC):

\[ \frac{\partial u_\theta}{\partial r^*} = 0; \quad \frac{\partial u_r}{\partial r^*} = 0; \quad \frac{\partial T^*}{\partial r^*} = 0 \]  

(6)

On the cylinder surface:

\[ u_\theta = 0; \quad u_r = 0, T = 1 \]  

(7)

2.2. Definitions of certain numerical parameters
Some of the parameters used in this study are defined as follows:

2.3. Local Nusselt number (\(N_u_s\))
Local Nusselt number distribution along the cylinder surface is determined as

\[ N_u_s = \frac{k_{eff}}{k_i} \frac{\partial T}{\partial n_s} \]  

(8)

2.4. Mean Nusselt number (\(N_u_M\))
The net heat transfer from the cylinder surface is represented in terms of mean Nusselt number, which is calculated by integrating the local Nusselt number along the cylinder surface as follows:

\[ N_u_M = \frac{1}{\pi} \int_0^\pi N u \sin \theta \, d\theta. \]  

(9)

2.5. Heat transfer Enhancement Ratio (\(HER\))
In order to compare the heat transfer rates of water based nanofluids and IoNanofluids, Heat transfer Enhancement Ratio (\(HER\)) is calculated as follows:

\[ HER = \frac{N_u_M}{N_{u_{bf}}} \]  

(10)

where, \(N_{u_{bf}}\) is the mean Nusselt number obtained using basefluid and \(N_u_M\) is the mean Nusselt number at a specific volume fraction, \(\phi\) of IoNanofluid.
2.6. Numerical methodology
The modeling of geometry and meshing was carried out using ANSYS ICEM CFD 15.0. The governing equations were solved along with suitable boundary conditions using the commercial FVM solver ANSYS FLUENT 15.0. A SIMPLE algorithm based pressure-velocity coupling has been employed and convective terms are discretized using third-order accurate QUICK scheme. The discretized equations are sequentially solved using an iterative approach with a convergence criteria for residuals equal to $10^{-7}$.

Table 1: Grid sensitivity analysis for flow around a circular cylinder using 2.5 % nanofluid at $Re = 10$ and 40.

| S. No | Re | Grid       | $C_D$  | % Difference | $Nu_M$ | % Difference |
|-------|----|------------|--------|--------------|--------|--------------|
| 1     | 10 | 200 × 100  | 2.8079 | 0.014        | 24.3511| 0.0030       |
| 2     | 10 | 400 × 200  | 2.8075 | 0.003        | 24.3520| 0.0008       |
| 3     | 10 | 600 × 300  | 2.8074 | -            | 24.3522| -            |
| 4     | 40 | 200 × 100  | 1.5133 | 0.006        | 55.0214| 0.007        |
| 5     | 40 | 400 × 200  | 1.5132 | 0.006        | 55.0256| 0.002        |
| 6     | 40 | 600 × 300  | 1.5131 | -            | 55.0271| -            |

2.7. Grid sensitivity analysis
A detailed grid sensitivity analysis has been carried out to ensure the grid independency of the solution. A two-dimensional, non-uniform structured mesh, which is more fine near the cylinder surface to capture the minute variations in temperature and flow field. Grid sensitivity analysis was performed at the extreme values of Reynolds number (10 and 40) with 2.5% IoNanofluid. The results of grid independence study are presented in Table 1.

![Figure 2](image)

(a) Comparison of coefficient of drag ($C_D$)  
(b) Comparison of mean Nusselt number ($Nu_M$)

Figure 2: Comparison of present code with literature data for flow around a circular cylinder with air ($Pr = 0.71$) as working fluid.

2.8. Code validation
The reliability of the results presented in this study are ensured by comparing the coefficient of drag and mean Nusselt number obtained from the present code with the data of Lange et al. [9], Dennis and Chang [30] and Soares et al. [31]. The comparison is presented in Figs. 2a and 2b. Results of the present code match closely with the standard literature data.
3. Results and discussion
The heat transfer characteristics of IoNanofluids are discussed in detail in the following section:

![Graphs showing mean Nusselt number](image1)

(a) $T = 293$ K
(b) $T = 333$ K

Figure 3: Mean Nusselt number at $10 \leq \text{Re} \leq 40$ and $0 \% \leq \phi \leq 2.5 \%$.

3.1. Mean Nusselt number ($Nu_M$)
In Fig. 3, variation of mean Nusselt Number with Reynolds number, particle volume fraction and inlet temperature are presented. As expected, it is observed that $Nu_M$ gradually increases with flow Reynolds number. It is noted that the $Nu_M$ is minimum at $Re = 10$ and has a maximum value at $Re = 40$, for a given volume fraction and inlet temperature. As the volume fraction increases, it is observed that the $Nu_M$ values also raise. $Nu_M$ is also a function of inlet temperature. Low Mean Nusselt number values are observed at higher inlet temperatures. Reduction in temperature difference between the cylinder and inlet temperature reduces the thermal gradient and leads to lower Nusselt number values. Furthermore, IoNanofluids exhibit a decrease in thermal conductivity at elevated temperatures.

![Thermal field isotherms](image2)

Figure 4: Thermal field (isotherms) distribution around the circular cylinder at $Re = 40$ with $0 \% \leq \phi \leq 2.5 \%$ and $293 K \leq T \leq 333 K$.

3.2. Isotherms
Thermal field around a circular cylinder is shown in Figs. 4 and 5. Heat transfer is noted to increase at higher concentrations. This is substantiated with the observation from the isotherms that show a decrease in thermal boundary layer thickness, at higher volume fractions. This can
Figure 5: Comparison of distribution of isotherms around the circular cylinder using 
[C4mim][NTf2] based alumina IoNanofluids and water based nanofluids at Re = 40 and 
T = 333 K.

also be identified by short and sharp thumb like projections being formed with the addition 
of nanoparticles. Further, the length of the isotherms are observed to increase with inlet 
temperature. A decrease in the net temperature difference between the cylinder and the fluid 
lead to longer isotherms, indicating lower heat transfer. Also, at higher Re, the thermal plumes 
are seen to move closer to the cylinder in the rear side, indicating enhanced heat transfer. This 
is due to more pronounced recirculation at higher Reynolds number. Additionally, it is noted 
in all the cases the isotherms are crowded at the upstream side of the cylinder. Fig. 5 present 
a comparison between IoNanofluids and water based nanofluids. It can be noted that at higher 
volume fractions boundary layer thickness reduces and length of the plume is seen to decrease 
for both the base fluids. This indicates higher heat transfer rates. Uniquely for IoNanofluids, the 
overall thermal field is seen to be crowded and the thermal plumes are noted to be sharper and 
shorter. This shows enhanced heat transfer characteristics for IoNanofluids than water based 
nanofluids.

Figure 6: Comparison of enhancement in heat transfer around circular cylinder at \( \phi = 2.5 \% \) 
for water based nanofluid and [C4mim][NTf2] IoNanofluid at 10 \( \leq Re \leq 40 \).
3.3. Heat transfer Enhancement Ratio (HER)

Heat transfer enhancement ratio for IoNanofluids are compared with water based nanofluids at 293 $K$ and 333 $K$ in Fig. 6. It is observed that, irrespective of temperature, HER values are always greater than 1 for IoNanofluids. Moreover, results indicate that heat transfer enhancement of IoNanofluids is also a function of temperature. Though, the Nusselt number values are low at 293$K$, IoNanofluids show higher heat transfer rates at higher temperatures. This shows that IoNanofluids are better working fluids for high temperature heat transfer applications. It is also noted that HER values for IoNanofluids are always higher than that of water based nanofluids, at a given volume fraction, for all temperatures. This shows that IoNanofluids are highly promising and act as better base fluids, as they are thermally stable and have better heat transfer characteristics than traditional base fluids.

4. Conclusion

Forced convective heat transfer around infinitely long circular cylinder during a steady, laminar flow of $Al_2O_3-([C_4mim][NTf_2])$ IoNanofluid has been numerically studied. Effects of Reynolds number, particle volume fraction and temperature on the heat transfer characteristics have been investigated. Findings of the numerical investigation can be summarized as follows:-

- Evidently, the heat transfer increased with increase in flow Reynolds number.
- The mean Nusselt number is noted to increase with the addition of nanoparticles to the base fluid. For instance, there was a 10.2 % increase in the $Nu_M$ for the addition of 0.5 % $Al_2O_3$ nanoparticles to the basefluid (293 $K$, $Re = 10$).
- The local Nusselt number values are observed to be high at the front stagnation points and then show a gradual decrease in the downstream direction and a local increase is seen near the rear stagnation point due to recirculation at the wake region.
- The local Nusselt number follows a trend of decrease with an increase in temperature and is noted to be directly proportional to volume fraction of the nanoparticles.
- Thermal boundary layer thickness is observed to decrease with increasing Reynolds numbers and volume fractions, implying higher heat transfer rates. This trend is reversed for increase in temperature, where thermal boundary layer increases at higher temperatures.
- Results show that IoNanofluids are up to 51 % more effective than water based nanofluids at $Re = 10$ for $\phi = 2.5$ %
- It is observed that IoNanofluids show higher enhancement in heat transfer compared to pure ionic basefluids, at elevated temperatures.

It can be concluded that IoNanofluids are better working fluids in real time high temperature engineering applications due to their high thermal stability and higher thermal transport capability at elevated temperatures.

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