Research on Equivalent Fault Injection Method Based on Dependency Matrix

Wenhao Qiu\textsuperscript{1, 2,*}, Guangyao Lian\textsuperscript{2, a}, Haitao Man\textsuperscript{2, b} and Huijie Li\textsuperscript{2, c}

\textsuperscript{1}Shijiazhuang Campus, Army Engineering University, Shijiazhuang 050003, China; \textsuperscript{2}Unit 32181 of PLA, Shijiazhuang 050000, China \textsuperscript{3}Unit 72373 of PLA, Luoyang 471000, China

*Corresponding author: 997750944@qq.com, aIgy63112@163.com, b315639739@qq.com, c2217247715@qq.com

Abstract. An equivalent fault injection method based on dependency matrix is proposed to deal with the problem of incapability of fault injection or permanent damage to the equipment in the fault injection-based testability verification test, and the problem that is caused by complex structure and high integration of new equipment is ubiquitous. Firstly, the definition of the testability Petri net and its matrix representation method are given, based on this, the algorithm for generating fault-state dependency matrix based on Petri net is proposed. Then, the equivalence of non-injectable faults is analyzed by defining fault behavior vectors and behavioral equivalent faults, and the equivalent injection process for adding samples that can be injected in the testability verification test is proposed. The results of application in an equipment control system show that this method is capable of equivalent injection for non-injectable faults, which increasing the sample size of fault injection effectively.

1. Introduction

Testability verification test based on fault injection is an effective means to evaluate the testability level of equipment [1-3]. Compared with the testability verification test based on simulation and using data, the fault injection-based testability verification test for the actual equipment does not require the complex simulation model, which can simulate the equipment failure mode quickly and comprehensively, and can save the test cost and improve the test accuracy effectively.

Compared with discrete components-based systems, integrated circuit system is tightly packaged, highly integrated, and complex in structure, meanwhile, the access depth of the existing fault injector is limited [4], which lead to some failure modes that can detect detection capabilities not being effectively injected. Although combined-level fault injection before system integration can be performed during the development phase, but for complex equipment, the propagation and coupling relationship of the fault between the functional units are very complicated, and the fault that can be detected at the combined level are not necessarily detectable at the system level, so testability verification test results at the combined level are difficult to reflect the testability level at the system level.

Equivalent fault injection is an effective means to solve the problem of non-injection faults and optimize fault injection strategies. At present, there are few studies on equivalent fault injection. In [5],
a detailed analysis on the major factors affecting the effectiveness of failure injection in light of failure propagation characteristics is presented, which effectively deal with the issue of injection incapability of failure. However, this method requires that the behavior state vector set is consistent, while some failure modes are difficult to be completely equivalent. The equivalent fault injection method based on multi-signal flow graph is proposed in [6], but method for calculating correlation matrix is not presented.

In this, an equivalent fault injection method is proposed to deal with the issue of injection incapability of failures in testability verification test. Firstly, the Petri net is used to calculate the dependency matrix. Then, based on the dependency matrix, the fault is equivalent, and the non-injectable fault is replaced with equivalent fault. Finally, the effectiveness of the method is analyzed by an example.

2. Dependency matrix generation algorithm based on Petri net

2.1. Definition of testability Petri net

The so-called dependency matrix is a full-order dependency matrix, which includes not only first-order correlations but also high-order correlations. High-order correlation refers to fault-state indirect causality. For simple testability directed graph, the dependency matrix can be obtained by direct analysis, column vector method, and row vector method. For complex directed graphs, it is difficult to obtain dependency matrices, but Petri nets can effectively describe system state changes, processes, sequences, concurrency, conflicts, and synchronization relationships [7].

Define a five-tuple Petri net system \( \Sigma = (S, C, F, W, M_0) \) to represent the testability Petri net, where

1. \( S = \{s_1, s_2, \ldots, s_m\} \) is the set of all places in the net. The place \( s_i \) represents the fault, the number of places is \( m \).
2. \( C = \{c_1, c_2, \ldots, c_n\} \) is the set of all transitions in the net. The transition \( c_i \) represents the state, the number of transitions is \( n \).
3. \( F \) is the set of directed arcs connecting places and transitions. \( F \subseteq (S \times C) \cup (C \times S) \), that is, the directed arc exists only between the place and the transition, and there is no directed arc between any of the places or between the transitions. \( W \) is the weight function of the directed arc \( F, \ W : F \rightarrow \{1,2,3,\ldots\} \), which is generally 1.
4. The operating state of the network can be represented by the state vector \( M \) of the Petri net:
   \[ M = [M(s_1), M(s_2), \ldots, M(s_m)]^T \], where \( M_0 \) is the initial state of the Petri net:
   \[ M_0 = [M_0(s_1), M_0(s_2), \ldots, M_0(s_m)]^T \].
5. Rule of transition

   ① For transition \( c_j \in C \ (j = 1,2,\ldots,n) \), \( \forall s_i \in S \ (1,2,\ldots,m) \), if
   \[ s_i \in \overset{\cdot}{c}_j \& M(s_i) \geq W(s_i, c_j) \] (1)

   Then, the place \( c_j \) has the right of occurrence under the identifier \( M \), recorded as \( M[c_j > \ ] \), where \( \overset{\cdot}{c}_j \) is input set of the place \( c_j \), \( W(s, c_j) \) is weight of the directed arc \( s \rightarrow c_j \).

   ② If \( M[c_j > \ ] \), the place \( c_j \) will occur under the identifier \( M \), and the resulting transition will get a new identifier \( M' \), recorded as \( M[c_j > M'] \). Then \( \forall s_i \in S \),

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(6) Association matrix and state equation

Just as an identifier of a Petri net can be represented as an m-dimensional non-negative vector, the structure of the Petri net can also be represented by a matrix. Next, the linear algebra method is proposed to analyze the properties of Petri nets.

**Definition 1**: Assume $\Sigma = (S, C, F, M)$ is a Petri net, so the structure of the Petri net $(S, C, F)$ can be represented by a matrix of $n$ rows and $m$ columns $A = [a_{ij}]_{n \times m}$, and $A = A^+ - A^-$, where

$$A = \begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1m} \\ a_{21} & a_{22} & \cdots & a_{2m} \\ \vdots & \vdots & \ddots & \vdots \\ a_{n1} & a_{n2} & \cdots & a_{nm} \end{bmatrix}, \quad A^+ = \begin{bmatrix} a^+_{11} & a^+_{12} & \cdots & a^+_{1m} \\ a^+_{21} & a^+_{22} & \cdots & a^+_{2m} \\ \vdots & \vdots & \ddots & \vdots \\ a^+_{n1} & a^+_{n2} & \cdots & a^+_{nm} \end{bmatrix}, \quad A^- = \begin{bmatrix} a^-_{11} & a^-_{12} & \cdots & a^-_{1m} \\ a^-_{21} & a^-_{22} & \cdots & a^-_{2m} \\ \vdots & \vdots & \ddots & \vdots \\ a^-_{n1} & a^-_{n2} & \cdots & a^-_{nm} \end{bmatrix}.$$

Where $a_{ij} = a^+_i - a^-_j, \quad i = 1, 2, \ldots, n, \quad j = 1, 2, \ldots, m$.

$$a^+_i = \begin{cases} W(c_i, s_j), & (c_i, s_j) \in F \\ 0, & \text{others} \end{cases} (3)$$

$$a^-_j = \begin{cases} W(s_j, c_i), & (s_j, c_i) \in F \\ 0, & \text{others} \end{cases} (4)$$

$A$ is the association matrix of $\Sigma$, $A^+ = [a^+_i]_{n \times m}$ is the input matrix for $\Sigma$, $A^- = [a^-_j]_{n \times m}$ is the output matrix for $\Sigma$. $A_i^+, A_j^+$ are the $i$-th row and the $j$-th column of the matrix $A$. $A_i^+, A_j^+$ and $A_i^-, A_j^-$ are the $i$-th row and the $j$-th column of the matrix $A^+$ and $A^-$. In the pure network (excluding the self-loop), there is a one-to-one correspondence between the structure of the association matrix and the network, that is, there is at most one directly connected arc between any transition and any library, and $a^+_i$ and $a^-_j$ will not offset each other. Dependency models and multi-signal flow models commonly used in testability modeling are pure nets.

**Theorem 1**: Assume $\Sigma = (S, C, F, M)$ is a Petri net, and $A$ is the association matrix of $\Sigma$, $c_j \in C$. Then the necessary and sufficient condition for $M \geq c_j$ is

$$M^T \geq A_i^+ (5)$$

**Theorem 2**: Assume $\Sigma = (S, C, F, M)$ is a Petri net. If $M \geq c_j > M'$, then

$$M' = M + (A_i)^T (6)$$

**Theorem 3**: Assume $\Sigma = (S, C, F, M)$ is a Petri net, and $M_0$ is the initial state. If $M \in R(M_0)$, then there is an $n$-dimensional vector $X$ of non-negative integers, such that
\[ M = M_o + A^T \cdot X \]  

(7)

1.2 Dependency matrix generation algorithm

**Definition 2**: Fault-state dependency matrix \( D = [d_{ij}]_{n \times m} \) is the correlation between fault and state. Where \( d_{ij} = 1 \) indicates that the occurrence of fault \( F_i \) cause an abnormality in the state parameter \( S_j \), and \( d_{ij} = 0 \) indicates that the occurrence of fault \( F_i \) does not cause an abnormality in the state parameter \( S_j \).

Convert the problem of calculating the fault-state dependency matrix into the problem of accessibility of the Petri net under single fault condition.

Known: In the single fault state \( s_i \), that is, \( M_o = [0,0,0,0,0,0,1,0,0,0,0,0]^T \), where \( M_o(s_i) = 1 \), \( M_o(s_j) = 1,(k \neq i,k \in [1,m]) \).

Solve: Determine whether there is a state \( M \in R(M_o) \) making \( c_j (j = 1,2,\cdots,n) \) has the right to take, that is \( M[c_j > 1] \). If it exists, \( d_{ij} = 1 \), otherwise, \( d_{ij} = 0 \).

In the single fault \( s_i \), the initial identifier of the Petri network is \( M_o = [1,0,0,0,0,0,0,0,0,0,0,0]^T \), Determine whether \( s_i \) is related to \( c_j \) includes two steps:

(1) Assume \( M \in R(M_o) \), judge whether \( c_j \) has the right to occur according to the theorem 1, and if the formula (5) is satisfied, then \( d_{ij} = 1 \).

(2) If the first step is true, according to Theorem 2, \( M[c_j > 1] \), then \( M' = M + (A_x)^T \). Replace \( M \) with \( M' \), and return to the first step.

3. Equivalent fault injection based on dependency matrix

In multi-signal flow graphs-based testability analysis, the fault-test dependency matrix is often used to analyze hide faults, impersonate faults, and fuzzy groups [8]. The information of the test point is a part of the state parameters of the unit under test (UUT), and the masquerading fault is the same behavior vector obtained after a certain operation between the faults in the tested state space. The equivalence of faults is analyzed according to this idea.

**Definition 3**: Fault behavior vector \( B_{Fi} \). The fault behavior vector of \( F_i \) is represented as a vector composed of all the elements of the \( i \)-th row which is 1 in the matrix, that is, the abnormal state caused by \( F_i \).

**Definition 4**: Behavioral equivalent fault. At the level \( a \), if the behavior vector in \( D \) satisfies the relationship shown in (8)

\[ F_{Bi} = F_{Bj} + L + F_{Bi}; i,j,k = 1,2,\cdots,m \]

(8)

Then, the fault \( F_i \) is equivalent to the behavior of \( F_j;...,F_k \), and the fault equivalent to \( F_i \) is called the behavioral equivalent fault set of \( F_i \).

The issue of injection incapability of failures resulting from inaccessible locations is ubiquitous in new equipment, so it is unrealistic to inject all the faults. In the fault injection process, due to the restrictions of the plug-in and probe type, the loading of the fault signal must be based on bus injection. However, bus injection is also limited. For injections that cannot be injected by the bus, probes and other means are required as much as possible, and the equivalent injection for inaccessible locations must be considered [9,10]. Suppose that there are \( k \) sample for fault injection in failure mode set \( F_i = \{F_{I1},F_{I2},...,F_{Im}\} \), and other \( m-k \) samples are inaccessible due to factors such as encapsulation. In this case, the equivalent injection method for adding injectable samples is as follows:
Step 1: Establish a multi-signal flow model of the UUT, and determine the level of detection/isolation test.
Step 2: According failure mode set $F_I$ and the model, the fault-state dependency matrix can be obtained by using the testability Petri net.
Step 3: Establish the fault model based on the basic characteristics of the signal.
Step 4: For the fault $F_i$ that cannot be directly injected, search the behavioral equivalent fault set according to equation (8).

4. Equivalent fault injection for typical systems
The equivalent fault injection method based on correlation matrix is applied to an equipment control system to verify the effectiveness of the method. The combination is composed of 8 modules such as CPU module, synchronous communication unit, asynchronous communication unit, and analog-to-digital conversion unit and so on. According to the final FMEA of the test parties, the system has 43 failure modes, and size of the fault injection sample is 67 according to GJB 2072. Due to space limitations, without loss of generality, only the AD conversion unit is used as an example to analyze the equivalent fault injection. The multi-signal flow model of the AD conversion unit is shown in Figure 1.

![Figure 1. Multi-signal flow model of the AD conversion unit](image)

According to the dependency matrix generation algorithm in Section 1.2, The fault-state dependency matrix is
The fault behavior vector of each failure mode can be obtained from $D_e$. According to Definition 4, $F_5$ has the behavioral equivalent fault set $\{F_1 \cup F_2\}$. Similarly, the fault-state dependency matrix of all other modules of the control system can be obtained. Then, the non-injectable faults can be equivalent injected based on dependency matrix, and the test results after equivalent fault injection are shown in Table 1.

![Matrix Image]

$$D_e = \begin{bmatrix}
0 & 1 & 0 & 1 \\
0 & 0 & 1 & 1 \\
1 & 1 & 1 & 1 \\
0 & 0 & 0 & 1 \\
0 & 1 & 1 & 1
\end{bmatrix}.$$ 

Table 1. Test results after equivalent injection

| Module name                  | Direct injection | After equivalent injection | Non-injectable fault |
|------------------------------|------------------|----------------------------|----------------------|
| CPU module                   | 8                | 11(3)                      | 2                    |
| Asynchronous communication unit | 7                | 8(1)                       | 1                    |
| Asynchronous communication unit | 8                | 9(1)                       | 0                    |
| DA conversion unit           | 7                | 8(1)                       | 0                    |
| AD conversion unit           | 5                | 6(1)                       | 0                    |
| I/O module 1                 | 6                | 6(0)                       | 0                    |
| I/O module 2                 | 6                | 6(0)                       | 0                    |
| Power supply unit            | 9                | 9(0)                       | 0                    |
| Total                        | 56               | 63(7)                      | 1                    |
| Fault injection rate         | 83.6%            | 94.0%                      | -                    |

From the test results in Table 1, we see that the fault injection rate of direct fault injection using bus and other means is 83.6% due to the complexity of fault propagation and highly integrated design in new equipment, which is relatively low. However, equivalent fault injection increase the number of injectable sample of the control system from 56 to 63, and increase the fault injection rate by 10.4%, which effectively improving the effective sample size of fault injection.

5. Conclusion

To deal with the problem of some faults cannot be injected in the testability verification test caused by the complicated structure, high integration of the new equipment and insufficient access depth of the existing fault injectors, we have presented the equivalent fault injection method based on dependency matrix. The method uses Petri net to describe the correlation model, information flow model and multi-signal flow model, and gives the relevant definition of testability Petri net. The state space of the equipment is described based on the Petri net dependency matrix generation algorithm. Then, by referring to the impersonate faults and fuzzy group research ideas, the equivalence of faults is analyzed by defining the fault behavior vector and the behavioral equivalent fault, and the equivalent injection method of adding the implantable sample in the testability verification test is proposed. The results show that the method can search the equivalent faults of non-injectable faults, and effectively increase the amount of fault samples that can be injected, which improve the fault sample coverage of the testability verification test.
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