Some considerations about NS5 and LST Hawking radiation

Oscar Lorente-Espín

Departament de Física i Enginyeria Nuclear, Universitat Politècnica de Catalunya, Comte Urgell, 187, E-08036 Barcelona, Spain.
E-mail: oscar.lorente-espin@upc.edu

Abstract

We have studied the Hawking radiation corresponding to the NS5 and Little String Theory (LST) black hole models using two semi-classical methods: the complex path method and a gravitational anomaly. After summarizing some known concepts about the thermodynamics of these theories, we have computed the emission rates for the two black hole models. The temperature calculated from, e.g. the well-known surface gravity expression, is shown to be identical to that obtained from both the computation of the gravitational anomaly and the complex path method. Moreover, the two semi-classical methods show that NS5 exhibits non-thermal behavior that contrasts with the thermal behavior of LST. We remark that energy conservation is the key factor leading to a non-thermal profile for NS5. In contrast, LST keeps a thermal profile even when energy conservation is considered because temperature in this model does not depend on energy. In addition, we address the question of the seemingly paradoxical validity of cluster decomposition for a non-local model such as LST. We propose a solution to the paradox that is based on using a Yukawa-like interaction between particles.
1. Introduction

Since the pioneering proposal of Hawking that black holes can radiate [1], much work has been done in order to obtain a complete theory of quantum gravity. When Hawking announced his amazing results, a new powerful paradox emerged. The information loss paradox with the apparent violation of unitarity principle has consequences on well established quantum mechanics. A recent effort in order to solve this paradox has been done studying different semi-classical approaches such as the tunneling method proposed by Parikh and Wilczek [2, 3], the complex path analysis [4, 5, 6] or the cancellation of gravitational anomalies [7, 8, 9].

We have studied the Hawking radiation for NS5 and LST stringy black holes using different semi-classical methods obtaining equal results for each method. For good reviews on LST and NS5, we address the readers to [10, 11, 12, 13, 14, 15]. We have verified that the NS5 model shows a non-thermal emission whereas LST shows a thermal emission. This last conclusion matches with the Hagedorn properties of LST, namely the temperature of LST corresponds to the Hagedorn temperature.

The paper is organized as follows: in section 2, we briefly summarize the LST theory with some properties and thermodynamics. In section 3, we reduce the ten dimensional metric of LST to two dimensional one. All the physics will be analyzed within the propagation of massless particles in the r-t sector of the metric. In section 4, we study two different semi-classical methods in order to compute the emission rate for NS5 and LST. Complex path method and anomalies yields the same results as the tunneling method, analyzed in [16],
for the temperature and the emission rate. It is worth mentioning that in the classical computation of the Bogoliubov coefficients all the results for emission rates show thermal profiles due to the lack of energy conservation. This fact had driven Hawking to state that all the information that falls into the black hole is lost for ever, establishing in this way the information loss paradox. Nevertheless, one hopes to overcome this weird conclusion using semi-classical methods.

2. A glance at LST thermodynamics

Little string theory is a non-gravitational six dimensional and non-local field theory [10, 11, 12, 17, 18], believed to be dual to a string theory background, defined as the decoupled theory on a stack of N NS5-branes. In the limit of a vanishing asymptotic value for the string coupling $g_s \to 0$, keeping the string length $l_s$ fixed while the energy above extremality is fixed, i.e. $E_{\text{max}} = \text{fixed}$, the processes in which the modes that live on the branes are emitted into the bulk as closed strings are suppressed. The theory becomes free in the bulk, but strongly interacting on the brane. In this limit, the theory reduces to Little String Theory or more precisely to (2,0) LST for type IIA NS5-branes and to (1,1) LST for type IIB NS5-branes [15].

The throat geometry corresponding to N coincident non-extremal NS5-branes in the string frame [19] is

$$ds^2 = -f(r)dt^2 + \frac{A(r)}{f(r)}dr^2 + A(r)r^2d\Omega_3^2 + \sum_{j=2}^{6} dx_j^2,$$

(2.1)

where $dx_j^2$ corresponds to flat spatial directions along the 5-branes, $d\Omega_3^2$ corresponds to 3-sphere of the transverse geometry and the dilaton field is defined as $e^{2\Phi} = g_s^2 A(r)$. The metric functions are defined as

$$f(r) = 1 - \frac{r_0^2}{r^2}, \quad A(r) = \chi + \frac{N}{m_s^2 r^2},$$

(2.2)

$r_0$ is the non-extremality parameter, so the extremal configuration is obtained by the limit $r_0 \to 0$ and the location of the event horizon corresponds to $r = r_0$. We define the parameter $\chi$ which takes the values 1 for NS5 model and 0 for LST, these are only the values for which exists a supergravity solution. In addition to the previous fields there is a $NS - NS H_{(3)}$ form along the $S^3$, $H_{(3)} = 2N \epsilon_3$. According to the holographic principle the high spectrum of this dual string theory should be approximated by certain black hole in the background (2.1).
The geometry transverse to the 5-branes is a long tube which opens up into the asymptotic flat space with the horizon at the other end, in the limit $r \to r_0$ appears the semi-infinite throat parametrized by $(t,r)$ coordinates, in this region the dilaton grows linearly pointing out that gravity becomes strongly coupled far down the throat. The string propagation in this geometry should correspond to an exact conformal field theory [20]. The boundary of the near horizon geometry is $R^{5,1} \times R \times S^3$. The geometry (2.1) is regular as long as $r_0 \neq 0$.

In order to construct the thermal states of the black holes we identify the period of the Euclidean time as in [21]

$$\beta = \frac{2\pi \sqrt{N + \chi m^2 r_0^2}}{m_s},$$

whereby we avoid a conical singularity. At this point we want to mention that the same result for the temperature is obtained whether we calculate the surface gravity of the black hole, $\beta^{-1} = T = \frac{\kappa}{2\pi}$. Regarding LST model, we notice that the temperature is independent of the black hole radius and therefore of the black hole mass. In this way we could identify this temperature with the Hagedorn temperature of the superstring theory.

One can compute the energy density for the LST background in 10 dimensions, $e \equiv \frac{E}{V_5}$ and the entropy density, $s \equiv \frac{S}{V_5}$, where $V_5$ is the volume of the flat 5-branes space and $S$ is the standard Bekenstein-Hawking entropy calculated from the area of the event horizon of the black hole, $S = \frac{\text{Area}}{4G_{10}}$. Either in Einstein frame metric [21, 22] or in string frame metric [14, 19] it is satisfied the usual thermodynamic relation $S = \beta E$. This relation implies that the free energy of the system $F = E - TS$ vanishes. At very high energies the equation of state is of the Hagedorn form which leads to an exponentially growing density of states [13]:

$$\rho(E) = e^{S(E)} \sim e^{\beta E}.$$

At first sight one could think that a phase transition is present when the system evolves from NS5 to the near horizon limit of NS5, i.e. LST, but we have checked that it is not the case. Computing the Bekenstein-Hawking entropy

$$S = \frac{\text{Area}}{4G_{10}} = \frac{V_5}{2G_{10}} \pi^2 (N + \chi m_s^2 r_0^2)^{3/2},$$

and plotting it versus the temperature we do not detect any critical point (Davies point) [23] that would signal a phase transition. Even working in thermodynamic geometry [24], writing the LST metric like a Ruppeiner metric $ds^2 = -3\sqrt{\frac{\pi G}{\hbar M}}dS^2$, we do not detect any divergence in the scalar curvature that would signal a possible phase transition. However calculating the specific heat as $C = T \frac{\partial S}{\partial T}$, we have found that it has a negative value, $-3S$, showing that the theory is unstable. In the work [13], the authors show that loop/string corrections to the Hagedorn density of states of LST would be of the form $\rho(E) \sim E^\alpha e^{\beta E}(1 + O(\frac{1}{E}))$.
and the temperature-energy relation becomes $\beta = \frac{\partial \log \rho}{\partial E} = \beta_0 + \frac{\alpha E}{E^2} + O\left(\frac{1}{E^2}\right)$. They found that since $\alpha$ is negative the high energy thermodynamics corresponding to near-extremal 5-branes is unstable as well as the temperature is above the Hagedorn temperature and the specific heat is negative. This instability would be associated to the presence of a negative mode (tachyon) in string theory, the high temperature phase of the theory yields the condensation of this mode. The authors are lead again to the conclusion that the Hagedorn temperature is reached at a finite energy being associated with a phase transition.

One can address the question whether an observer in a moving frame observes a temperature above the Hagedorn temperature, being this the maximum temperature that can be achieved by the system. We have evaluated the simplest case, a scalar particle-like observer which moves on a NS5-brane with constant velocity at fixed distance $r$ from the horizon of NS5 and LST black hole systems. We consider the orbit for which $y_1 = vt$. Relating the time coordinate $t$ with the proper time $\tau$ through $d\tau^2 = -g_{\mu\nu}dx^\mu dx^\nu$, one obtains

$$\frac{d\tau}{dt} = \sqrt{f(r) - v^2}.$$  

(2.5)

The velocity is bounded by the local velocity of light thus we have to impose the constraint $v^2 \leq f(r)$. This relation brings us to a new coordinate of the horizon position, where causality is lost, seen by the moving particle, $r = \frac{r_0}{\sqrt{1 - v^2}}$. Furthermore the Killing vector relevant for the process is $\zeta = -\partial_t + v\partial_{y_1}$. Therefore evaluating the surface gravity at this new coordinate $r$, we obtain the local temperature for the moving scalar particle

$$T' = \frac{(1 - v^2)m_s}{2\pi \sqrt{N + \chi \frac{r_0^2}{(1 - v^2)}}},$$  

(2.6)

where we have worked out in natural units, $c = 1$ and $v < 1$. We notice two important features. First of all, we see that in the $v \to 0$ limit we recover the result (2.3). Secondly, comparing the temperature for the particle-like observer (2.6) with the temperature defined by (2.3) for an asymptotic static observer, we see that the former is lower than the second one. We conclude that the Hawking temperature of LST is a maximum bound and corresponds to the Hagedorn temperature. Unfortunately, we are not able to perform the same analysis for an accelerating particle-like observer. The main problem is that the path which the particle follows is not generated by a Killing vector field, this fact prevent us from using the surface gravity method in order to calculate the temperature.
3. Effective two dimensional theory

We are going to reduce the metric (2.1) to the r-t sector, relevant for the forthcoming sections. At first step we take the scalar field action

\[ S = \frac{1}{2\kappa_{10}^2} \int_M d^{10}x \sqrt{-g} \left( R - \frac{1}{2} \partial_\mu \phi \partial^\mu \phi - \frac{1}{12} e^{-\Phi} H^2_{(3)} \right) . \]  (3.1)

Performing a change to tortoise coordinate, see (2.1) \( dr^* = \frac{\sqrt{A(r)}}{f(r)} dr \), we expand the 10-dimensional action as

\[ S = \frac{1}{2\kappa_{10}^2} \int dtdr^* d\theta d\phi d\psi \prod_{j=2}^6 dx_j r^3 A(r)^2 \sin^2 \theta \sin \varphi (g_{s}e^{-\Phi})^{5/2} \left[ \frac{f(r)}{\sqrt{A(r)}} \left( R - \frac{e^{-\Phi}}{12} H^2_{(3)} \right) + \left( \frac{1}{2\sqrt{A(r)}} (\partial_t^2 - \partial_r^2) - \frac{f(r)}{2r^2 A^{3/2}} \left( \partial_\theta^2 + \frac{1}{\sin^2 \theta} \partial_\varphi^2 + \frac{1}{\sin^2 \theta \sin^2 \varphi} \partial_\psi^2 \right) - \frac{f(r)}{2\sqrt{A(r)}} \sum_{j=2}^6 \partial_j^2 \right) \phi(t,r) \right] , \]  (3.2)

where we have decomposed the scalar field into \( r-t \), 3-angular and 5-brane parts. Our following approximations are based on three main steps:

1. We consider only the propagation mode of an s-wave.

2. We take into account only a subset of states of the Hilbert space such that the eigenstates of momentum parallel to the NS5-brane vanish.

3. We take the near horizon limit, \( r \to r_0 \).

Eventually we come back to the original \( r \) radial coordinate obtaining for the action

\[ S = \frac{V(S^3) V_5}{2\kappa_{10}^2} \int dtdr A(r)^2 e^{-2\Phi} \left( -\frac{1}{f(r)} \partial_t^2 + \frac{f(r)}{A(r)} \partial_r^2 \right) \phi(t,r) , \]  (3.3)

where \( V(S^3) \) is the volume of the 3-sphere and \( V_5 \) the volume of 5-branes. From (3.3) we find out that the scalar field can be seen as \((1+1)\)-dimensional scalar field \( \phi(t,r) \) propagating in the background

\[ ds^2_{eff} = -f(r)dt^2 + \frac{A(r)}{f(r)} dr^2 , \]  (3.4)
together with an effective dilaton field

\[ e^{2\Phi} = g_s^2 A(r) . \quad (3.5) \]

Henceforth we are going to work with this two dimensional effective metric.

4. **Semiclassical Black Hole emission**

4.1. **Complex path method**

In this section we are going to show a semi-classical method to obtain the Hawking radiation. Eventually we will obtain the same thermal behavior than in [1], where the particle production is computed using the Bogoliubov transformation. The complex path method has been developed in [4], in order to calculate particle production in Schwarzschild-like space-time and it was extended for a different coordinate representations of the Schwarzschild space-time [5, 6]. Nevertheless complex path analysis had already been discussed by Landau [25], where it was used to describe tunneling processes in non-relativistic semi-classical quantum mechanics.

We will follow the reference [4] where the authors avoid to work in Kruskal representation. They use the standard coordinates in the r-t sector. However the method presents a disadvantage because one would find a coordinate singularity at the horizon. Nevertheless using the techniques of complex integration one bypasses the singularity. We also want to mention that the method of complex path leads to the same results that in [26]. In both methods, for Schwarzschild space-time and also as we will see in this section for NS5 and LST space-time, it has been found that the relation between emission and absorption probabilities is of the form

\[ P_e = e^{-\beta \omega} P_a , \quad (4.1) \]

where \( \omega \) is the energy of the emitted particles. We are tempting to compare this relation with the standard thermal Boltzmann distribution for blackbody radiation where \( \beta^{-1} \) is identified with Hawking temperature. We have verified that this is the case, if we compare our results versus the temperature calculated using the definition of surface gravity for example. It is noteworthy to say that this method allows one to get temperatures for black holes only comparing probabilities of emission and absorption but is not able to calculate the spectrum of thermal radiation. In that sense the tunneling method proposed is so far incomplete. To amend this shorts the authors in [27] present a new mechanism.
In order to apply the complex path method to NS5 and LST we have constructed the semi-classical action obtained from Hamilton-Jacobi equations. Afterwards we have computed the semi-classical propagator $K(r_2, t_2; r_1, t_1)$. Eventually we have calculated the emission and absorption probabilities.

We consider the equation of motion of a massless scalar particle $\Box \phi = 0$, in the background (3.1)

$$
- A(r) \frac{\partial^2}{\partial t^2} \phi(t, r) + \frac{f(r)}{r^3} \frac{\partial}{\partial r} \left[ r^3 f(r) \frac{\partial}{\partial r} \phi(t, r) \right] = 0 .
$$

Using the standard ansatz solution

$$
\phi(t, r) \sim e^{i S(t,r) / \hbar} ,
$$

and substituting in (4.2) we get an expression in terms of the action $S(t, r)$

$$
\left[-A(r)(\frac{\partial S}{\partial t})^2 + f(r)^2(\frac{\partial S}{\partial t})^2\right] + \frac{\hbar}{i} \left[-A(r) \frac{\partial^2 S}{\partial t^2} + f(r)^2 \frac{\partial^2 S}{\partial r^2} + \frac{f(r) d(r^3 f(r))}{dr} \frac{\partial S}{\partial r} \right] = 0 ,
$$

where we have collected the terms with $\hbar$ dependence. The following step is to write the action as an expansion in a power series of $(\hbar$)

$$
S(t, r) = S_0(t, r) + \left( \frac{\hbar}{i} \right) S_1(t, r) + \left( \frac{\hbar}{i} \right)^2 S_2(t, r) + ... .
$$

Substituting the expansion in (4.3) and neglecting terms of the order $(\hbar / i)$ and higher, we obtain the equation of motion to the lowest order in $S$

$$
- \frac{A(r)}{f(r)} \left( \frac{\partial S_0(t, r)}{\partial t} \right)^2 + f(r) \left( \frac{\partial S_0(t, r)}{\partial r} \right)^2 = 0 .
$$

We are interested in the evaluation of the semi-classical propagator which inform us about the amplitude for a particle going from $r_1$ at time $t_1$ to $r_2$ at time $t_2$. In the saddle point approximation we get

$$
K(r_2, t_2; r_1, t_1) = N \exp \left[ \frac{i}{\hbar} S_0(r_2, t_2; r_1, t_1) \right] ,
$$

where $N$ is a normalization constant. From (4.6) we get

$$
S_0(r_2,t_2; r_1, t_1) = -\omega(t_2 - t_1) \pm \omega \int_{r_1}^{r_2} \sqrt{\frac{A(r)}{f(r)}} dr ,
$$

7
the plus/minus sign correspond to ingoing/outgoing particles respectively and $\omega$ is the energy of the emitted or absorbed particles.

The integral (4.8) is not well behaved if the horizon $r_0$ is within the region of integration. However this turns to be the case, we are interested in the emission of particles through the event horizon, so the region of integration runs from inside the horizon to outside.

First we consider the propagation of an outgoing particle in the inner region $r_1 < r_0$. Applying the usual complex analysis tools, we deform the contour of integration around the pole $r_0$ in the upper complex half-plane. Obtaining for the radial part of (4.8)

$$S_e^0 = \frac{i\pi\omega}{2}r_0\sqrt{A(r_0)}.$$  

(4.9)

We will call it emission action because all we have done is nothing more than the emission of an outgoing particle, propagating from inside the horizon to the outside.

By the same talk one proceeds with analogous analysis to evaluate the action at lowest order for absorbed particles. In that case we are considering the propagation of an ingoing particle in the outer region, $r_0 < r_2$. Deforming the contour of integration in the upper complex half-plane, eventually we obtain the same result as the emission process up to change of sign. Now we are obtaining the absorption action for a particle that propagates from outside the horizon to the inside

$$S_a^0 = \frac{-i\pi\omega}{2}r_0\sqrt{A(r_0)}.$$  

(4.10)

We are interested in the expressions (4.9) and (4.10) in order to evaluate the probabilities of the emission and absorption processes. Thereby using the definition of the probability $P = |K(r_2, t_2; r_1, t_1)|^2$, and substituting the expression for the corresponding actions, we finally obtain for the emission and absorption probabilities

$$P_e \sim \exp\left[-\frac{\pi}{\hbar}\omega r_0\sqrt{A(r_0)}\right],$$  

$$P_a \sim \exp\left[\frac{\pi}{\hbar}\omega r_0\sqrt{A(r_0)}\right],$$  

(4.11)

where we have omitted the normalization constants. Eventually we are interested in to write the relation between emission and absorption probabilities

$$P_e = \exp\left[-\frac{2\pi}{\hbar}\omega r_0\sqrt{A(r_0)}\right]P_a.$$  

(4.12)

At first sight we observe that absorption process dominate over the emission, it is more easy for the system to absorb than to radiate particles. Also we note some misleading form in the expression for the absorption probability (4.11) because we could think that one might get a probability absorption greater than 1. Nevertheless we only have considered the spatial
contribution of the action in order to calculate the probabilities of emission and absorption processes. Instead of this we should also have considered the time contribution as proposed in the work [28].

Comparing (4.12) with the same relation in a thermal bath of particles (4.1) we can identify the temperature of our system (taking $\hbar = 1$ and $m_s = 1$)

$$T = \frac{1}{2\pi r_0 \sqrt{A(r_0)}} = \frac{1}{2\pi \sqrt{\chi r_0^2 + N}},$$

that coincides with the value of temperature obtained in (2.3).

So far we have studied NS5/LST systems without backreaction. The next step is to consider the backreaction of the metric due to the emission process. For all the details we address the reader to [16] where we had studied the tunneling method. Generically the process consists in the emission of a particle with energy $\omega$ from a black hole to the background. Taking into account the energy conservation the metric backreacts. Hence the total ADM mass is conserved and consequently the black hole mass must decrease by the same amount of the energy that it has been released. Our starting point in the evaluation of the backreaction is the expression of the action for the emission process (4.9). In our NS5/LST model $r_0^2 \sim M$, where $M$ is the mass of the black hole and the factors missed are not relevant in our study. When the metric backreacts in the emission process the energy conservation implies that $r_0^2 \to r_0^2 - \omega$. The shrink of the event horizon rides the tunneling emission between turning points defined just inside and just outside of the event horizon. Once the emission has been carried out we can perform the previous change in (4.9)

$$S^e_0 = \frac{i\pi}{2} \sqrt{\chi(r_0^2 - \omega) + N},$$

expanding for low energies we get

$$S^e_0 = \frac{i\pi}{2} \left( \omega \sqrt{\chi r_0^2 + N} - \frac{\chi \omega^2}{2 \sqrt{\chi r_0^2 + N}} + O(\omega)^3 \right).$$

(4.15) already shows the seemingly different features of both models (4.7). If we calculate the emission probability for both models we obtain

$$P_e \sim \begin{cases} 
\exp \left[ -\frac{\pi}{\hbar} \left( \omega \sqrt{r_0^2 + N} - \frac{\omega^2}{2 \sqrt{r_0^2 + N}} + \ldots \right) \right] & \text{if } \chi = 1 \text{ (NS5)}; \\
\exp \left[ -\frac{\pi}{\hbar} \omega \sqrt{N} \right] & \text{if } \chi = 0 \text{ (LST)}. 
\end{cases}$$

(4.16)
We see higher order correction terms corresponding to the NS5 emission probability, which indicates that the emission is not purely thermal. On the other hand, the emission probability expression corresponding to the LST model is exact, which indicates that the emission is purely thermal.

As a final comment, we want to stress the crucial fact of energy conservation in order to get a non-thermal emission in the tunneling formalism. In this work, we have concluded that the results obtained from the tunneling formalism in [2] are nothing more than an extension of the Hamilton-Jacobi formalism taking into account the energy conservation, which induces the backreaction of the event horizon. In particular, we are facing with an anomalous model which does not accomplish the previous expectations about non-thermal emissions. The LST model emits thermal radiation in any case with or without energy conservation.

4.2. Anomalies

In this section, we want to present another successful semi-classical method to compute the Hawking radiation from an evaporating black hole. The method is based on the cancellation of gravitational anomalies in a two-dimensional chiral theory taken as effective theory near the event horizon. This method was first proposed in [7].

Gravitational anomalies are anomalies in general covariance, i.e., general coordinate transformations (diffeomorphism), and they manifest as the non-conservation of the energy-momentum tensor. The authors in [7, 8] managed the treatment of gauge and covariant anomalies deriving an effective two-dimensional theory close to the horizon. They built an effective action performing a partial wave decomposition in tortoise coordinate and dropping potential factors which vanish exponentially fast near the horizon. Thus, physics near the horizon can be described by an infinite collection of (1+1) fields with the metric reduced to the r-t sector. In these previous works, the authors derived the Hawking radiation flux by anomaly cancellation, splitting the space-time into the near horizon region where the anomaly works and outside region when the conservation law is preserved. They carried out the calculation using the consistent chiral anomaly form of the energy-momentum tensor, see [29, 30],

$$\nabla_\mu T^\mu_\nu = \frac{1}{96\pi\sqrt{-g}}\epsilon^{\beta\delta}\partial_\beta\partial_\delta\Gamma_\nu^\alpha$$

(4.17)

together with the covariant boundary condition at the horizon. It is the cancellation of the covariant anomaly which leads us to the appearance of the Hawking radiation flux.

On the other hand, it is known the existence of two types of anomalies. Covariant anomalies, which transform covariantly under gauge or general coordinate transformation but they
do not satisfy the Wess-Zumino consistency condition. And consistent anomalies, which satisfy the consistency condition but they do not transform covariantly under gauge or general coordinate transformation. In our study we adopt the procedure carried out in [9] where the authors use a more coherent frame, working with covariant forms both for the expression of the chiral anomaly and for the boundary conditions. Unlike the previous works it is not necessary split the space-time into different regions, near the horizon region and outside.

First of all we consider the physics near the horizon of the NS5 and LST models described by an infinite collection of (1+1) scalar field particles propagating in the background [33,4]. In this frame we can consider that only the outgoing modes are present. The ingoing modes are lost into the black hole and they do not affect at the classical level. Nevertheless the total effective action must be covariant. Thereby the quantum contribution of these irrelevant ingoing modes will supply the extra term, a Wess-Zumino term, in order to cancel the gravitational anomaly providing the Hawking flux, [8]. The loss of the ingoing modes behind the horizon of the black hole causes that the effective theory becomes chiral, obtaining consequently a gravitational anomaly [29,30]. Following [9], we adopt the expression for the covariant form of the gravitational anomaly

\[ \nabla_\mu T^{\mu \nu} = \frac{1}{96\pi}\sqrt{-g} \epsilon^{\nu \mu} \nabla_\mu R , \]  

(4.18)

where \( R \) is the Ricci scalar and \( \epsilon^{\nu \mu} \) is the Levi-Civita tensor that in our case takes the values \( \epsilon^{tr} = -\epsilon^{rt} = 1 \) and zero for other contributions. The covariant boundary condition at the event horizon is

\[ T^r_t (r = r_0) = 0 . \]  

(4.19)

Noticing that we are working with a static metric, we evaluate the equation (4.18) for the effective two dimensional theory in the r-t sector. Eventually we get

\[ \partial_r (\sqrt{-g} T^r_t) = \frac{1}{96\pi} g_{tt} \partial_r R . \]  

(4.20)

The Ricci scalar for NS5 and LST models is

\[ R = \frac{f' A'}{2A^2} - \frac{f''}{A} , \]  

(4.21)

where primes denotes derivative with respect to the coordinate \( r \). Defining the new function

\[ N^r_t \equiv \frac{1}{96\pi} \left( -\frac{f f' A'}{2A^2} + \frac{f'^2}{2A} + \frac{f f''}{A} \right) , \]  

(4.22)

we can write (4.20) as
\[ \partial_r(\sqrt{-g} T^\tau_r) = \partial_r N^r_t . \] (4.23)

Then integrating the equation (4.23) we obtain

\[ \sqrt{-g} T^\tau_r = b_0 + (N^r_t(r) - N^r_t(r_0)) , \] (4.24)

where \( b_0 \) is an integration constant that can be evaluated implementing the covariant boundary condition (4.19). Doing so it yields the value \( b_0 = 0 \). Hence (4.24) becomes

\[ T^\tau_t = \frac{1}{\sqrt{-g}} (N^r_t(r) - N^r_t(r_0)) . \] (4.25)

The Hawking radiation flux is measured at infinity where the covariant gravitational anomaly vanishes. Therefore we compute the energy flux by taking the asymptotic limit of (4.25)

\[ T^\tau_t(r \to \infty) = - \frac{1}{\sqrt{-g}} N^r_t(r_0) . \] (4.26)

Evaluating (4.22) at the event horizon \( r_0 \) and considering the value of the surface gravity \( \kappa = \frac{1}{\sqrt{N + \chi r_0^2}} \), we finally obtain for the energy flux at infinity

\[ T^\tau_t(r \to \infty) = \frac{1}{\sqrt{-g}} \frac{\kappa^2}{48\pi} , \] (4.27)

which it is of course the Hawking radiation flux for a black hole.

5. Conclusion-Discussion

In this work we have started reviewing briefly some aspects about LST thermodynamics. We have exposed the thermal emission of LST due to the non-energy dependence of the Hagedorn temperature. Also we have evaluated the temperature experienced by a scalar particle-like observer, thereby we have verified that the Hagedorn temperature of LST is a maximum bound. Furthermore we have studied the Hawking radiation of the NS5 and LST black hole models using two semi-classical emission methods: the complex path method and the cancellation of the gravitational anomaly. We want to mention that using both methods we have recovered the previous results in [16] where we worked using the tunneling formalism. The complex path method [4, 6] shows how to evaluate the emission rate in the framework of the Hamilton-Jacobi formalism. We have shown that imposing energy conservation, in order to take into account the backreaction of the metric during the emission
process, we reproduce exactly the same results that in the tunneling formalism proposed in [2, 3]. We would like to point out the advantage that represents the complex path method with respect to the tunneling formalism. First of all, we avoid heuristic explanations about the tunneling mechanism in the process of the emission. Secondly, we work with the well known Hamilton-Jacobi equations plus the imposition of the energy conservation. And finally, it is not necessary to change the standard coordinates of the metric into Painlevé coordinates. We conclude that the tunneling method is nothing more than the complex path method plus energy conservation. Nevertheless none of the above methods are able to clarify the information loss paradox since they do not calculate the spectrum.

We have verified that another successful method to evaluate Hawking radiation in NS5 and LST models is based on the cancellation of the gravitational anomaly [7].

We have shown that all the above methods lead to a non-thermal emission for the NS5 model and thermal emission for the LST model, see (4.16).

In the line of the works [31, 32] where the authors linked the existence of correlations among tunneled particles and the entropy conservation of the full system (black hole plus Hawking radiation), we have calculated the successive emission probabilities for two particles of energies $\omega_1$ and $\omega_2$ using (4.16) for each model respectively. We have found that the NS5 model does not satisfy cluster decomposition

$$\ln | \Gamma(\omega_1 + \omega_2) | - \ln | \Gamma(\omega_1)\Gamma(\omega_2) | = \frac{\omega_1\omega_2}{2\sqrt{N + r_0^2}}. \quad (5.1)$$

On the other hand we have found that the LST model satisfies cluster decomposition as we expected

$$\ln | \Gamma(\omega_1 + \omega_2) | - \ln | \Gamma(\omega_1)\Gamma(\omega_2) | = 0. \quad (5.2)$$

With these results at hand we can conclude that in the NS5 black hole exists correlations between emitted particles. This fact is intimately related with the non-thermal emission rate (4.16). Regarding (5.1) one hopes that the successive Hawking emissions could preserve unitarity avoiding in such a way the information loss paradox. However it is not the case for the LST black hole where the thermal emission rate (4.16) lead us to cluster decomposition. Therefore the successive emissions of particles are independent one of each other, thus the information of the initial states remain hidden.

It has been demonstrated that LST is a non-local theory. LST exhibits T-duality therefore ensures that it is a non-local theory, moreover LST shows an impossibility to be Fourier transformed into position space [10, 11, 12, 17]. On the other hand LST exhibits cluster decomposition pointing out a local behavior. In order to match the non-locality of LST
with cluster decomposition property, one is lead to the presence of Yukawa-like interactions mediated by non-massless particles within the theory.

REFERENCES

[1] S. W. Hawking, “Particle Creation by Black Holes,” Commun. Math. Phys. 43 (1975) 199-220.

[2] M. K. Parikh, F. Wilczek, “Hawking radiation as tunneling,” Phys. Rev. Lett. 85 (2000) 5042-5045. [hep-th/9907001].

[3] M. K. Parikh, “A Secret tunnel through the horizon,” Int. J. Mod. Phys. D13 (2004) 2351-2354. [hep-th/0405160].

[4] K. Srinivasan, T. Padmanabhan, “Particle production and complex path analysis,” Phys. Rev. D60 (1999) 024007. [gr-qc/9812028].

[5] S. Shankaranarayanan, K. Srinivasan, T. Padmanabhan, “Method of complex paths and general covariance of Hawking radiation,” Mod. Phys. Lett. A16 (2001) 571-578. [gr-qc/0007022].

[6] S. Shankaranarayanan, T. Padmanabhan, K. Srinivasan, “Hawking radiation in different coordinate settings: Complex paths approach,” Class. Quant. Grav. 19 (2002) 2671-2688. [gr-qc/0010042].

[7] S. P. Robinson, F. Wilczek, “A Relationship between Hawking radiation and gravitational anomalies,” Phys. Rev. Lett. 95 (2005) 011303. [gr-qc/0502074].

[8] S. Iso, H. Umetsu, F. Wilczek, “Hawking radiation from charged black holes via gauge and gravitational anomalies,” Phys. Rev. Lett. 96 (2006) 151302. [hep-th/0602146].

[9] R. Banerjee, “Covariant Anomalies, Horizons and Hawking Radiation,” Int. J. Mod. Phys. D17 (2009) 2539-2542. [arXiv:0807.4637 [hep-th]].

[10] D. Kutasov, “Introduction to little string theory,” Lectures given at the Spring School on Superstrings and Related Matters. Trieste, 2-10 April 2001.

[11] O. Aharony, “A Brief review of 'little string theories','” Class. Quant. Grav. 17 (2000) 929-938. [hep-th/9911147].
[12] A. Kapustin, “On the universality class of little string theories,” Phys. Rev. D63 (2001) 086005. [hep-th/9912044].

[13] D. Kutasov, D. A. Sahakyan, “Comments on the thermodynamics of little string theory,” JHEP 0102 (2001) 021. [hep-th/0012258].

[14] M. Rangamani, “Little string thermodynamics,” JHEP 0106 (2001) 042. [hep-th/0104125].

[15] O. Aharony, M. Berkooz, D. Kutasov et al., “Linear dilatons, NS five-branes and holography,” JHEP 9810 (1998) 004. [hep-th/9808149].

[16] O. Lorente-Espin, P. Talavera, “A Silence black hole: Hawking radiation at the Hagedorn temperature,” JHEP 0804 (2008) 080. [arXiv:0710.3833 [hep-th]].

[17] A. W. Peet, J. Polchinski, “UV / IR relations in AdS dynamics,” Phys. Rev. D59 (1999) 065011. [hep-th/9809022].

[18] N. Seiberg, “New theories in six-dimensions and matrix description of M theory on T**5 and T**5 / Z(2),” Phys. Lett. B408 (1997) 98-104. [hep-th/9705221].

[19] J. M. Maldacena, A. Strominger, “Semiclassical decay of near extremal five-branes,” JHEP 9712 (1997) 008. [hep-th/9710014].

[20] D. Kutasov, N. Seiberg, “Noncritical superstrings,” Phys. Lett. B251 (1990) 67-72.

[21] A. L. Cotrone, J. M. Pons, P. Talavera, “Notes on a SQCD-like plasma dual and holographic renormalization,” JHEP 0711 (2007) 034. [arXiv:0706.2766 [hep-th]].

[22] T. Harmark, N. A. Obers, “Hagedorn behavior of little string theory from string corrections to NS5-branes,” Phys. Lett. B485 (2000) 285-292. [hep-th/0005021].

[23] H. La, “Davies Critical Point and Tunneling,” [arXiv:1010.3626 [hep-th]].

[24] R. Banerjee, S. K. Modak, S. Samanta, “A New Phase Transition and Thermodynamic Geometry of Kerr-AdS Black Hole,” [arXiv:1005.4832 [hep-th]].

[25] L. D. Landau, E. M. Lifshitz, “Textbook On Theoretical Physics. Vol. 2: Quantum Mechanics.” Pergamon Press, New York, (1975).
[26] J. B. Hartle, S. W. Hawking, “Path Integral Derivation of Black Hole Radiance,” Phys. Rev. D13 (1976) 2188-2203.

[27] R. Banerjee, B. R. Majhi, “Hawking black body spectrum from tunneling mechanism,” Phys. Lett. B675 (2009) 243-245. [arXiv:0903.0250 [hep-th]].

[28] A. de Gill, D. Singleton, V. Akhmedova et al., “A WKB-like approach to Unruh Radiation,” Am. J. Phys. 78 (2010) 685-691. [arXiv:1001.4833 [gr-qc]].

[29] L. Alvarez-Gaume, E. Witten, “Gravitational Anomalies,” Nucl. Phys. B234 (1984) 269.

[30] R. A. Bertlmann, E. Kohlprath, “Two-dimensional gravitational anomalies, Schwinger terms and dispersion relations,” Annals Phys. 288 (2001) 137-163. [hep-th/0011067].

[31] B. Zhang, Q. -y. Cai, L. You et al., “Hidden Messenger Revealed in Hawking Radiation: A Resolution to the Paradox of Black Hole Information Loss,” Phys. Lett. B675 (2009) 98-101. [arXiv:0903.0893 [hep-th]].

[32] B. Zhang, Q. -y. Cai, M. -s. Zhan et al., “Entropy is Conserved in Hawking Radiation as Tunneling: a Revisit of the Black Hole Information Loss Paradox,” Annals Phys. 326 (2011) 350-363. [arXiv:0906.5033 [hep-th]].
Some considerations about NS5 and LST Hawking radiation

Oscar Lorente-Espín

Departament de Física i Enginyeria Nuclear, Universitat Politècnica de Catalunya, Comte Urgell, 187, E-08036 Barcelona, Spain.

Abstract

We have studied the Hawking radiation corresponding to the NS5 and Little String Theory (LST) black hole models using two semi-classical methods: the complex path method and a gravitational anomaly. After summarizing some known concepts about the thermodynamics of these theories, we have computed the emission rates for the two black hole models. The temperature calculated from, e.g. the well-known surface gravity expression, is shown to be identical to that obtained from both the computation of the gravitational anomaly and the complex path method. Moreover, the two semi-classical methods show that NS5 exhibits non-thermal behavior that contrasts with the thermal behavior of LST. We remark that energy conservation is the key factor leading to a non-thermal profile for NS5. In contrast, LST keeps a thermal profile even when energy conservation is considered because temperature in this model does not depend on energy.

KEYWORDS: Black Holes, String Theory, Hawking radiation.

E-mail: oscar.lorente-espin@upc.edu
1. Introduction

Since the pioneering proposal of Hawking that black holes can radiate [1], much work has been done in order to obtain a complete theory of quantum gravity. When Hawking announced his amazing results, a new powerful paradox emerged. The information loss paradox with the apparent violation of unitarity principle has consequences on well-established quantum mechanics. A recent effort in order to solve this paradox has been done studying different semi-classical approaches such as the tunneling method proposed by Parikh and Wilczek [2, 3], the complex path analysis [4, 5, 6] or the cancellation of gravitational anomalies [7, 8, 9].

We have studied the Hawking radiation for NS5 and LST stringy black holes using different semi-classical methods obtaining equal results for each method. For good reviews on LST and NS5, we address the readers to [10, 11, 12, 13, 14, 15]. We have verified that the NS5 model shows a non-thermal emission whereas LST shows a thermal emission. This last conclusion matches with the Hagedorn properties of LST, namely the temperature of LST corresponds to the Hagedorn temperature.

The Letter is organized as follows: in section 2, we briefly summarize the LST theory with some properties and thermodynamics. In section 3, we reduce the ten-dimensional metric of LST to two-dimensional one. All the physics will be analyzed within the propagation of massless particles in the $r - t$ sector of the metric. In section 4, we study two different semi-classical methods in order to compute the emission rate for NS5 and LST. Complex path method and anomalies yields the same results as the tunneling method, analyzed in [16].
for the temperature and the emission rate. It is worth to mentioning that in the classical computation of the Bogoliubov coefficients all the results for emission rates shows thermal profiles due to the lack of energy conservation. This fact had driven Hawking to state that all the information that falls into the black hole is lost for ever, establishing in this way the information loss paradox. Nevertheless, one hopes to overcome this weird conclusion using semi-classical methods.

2. A glance at LST thermodynamics

Little string theory is a non-gravitational six-dimensional and non-local field theory [10, 11, 12, 17, 18], believed to be dual to a string theory background, defined as the decoupled theory on a stack of $N$ NS5-branes. In the limit of a vanishing asymptotic value for the string coupling $g_s \to 0$, keeping the string length $l_s$ fixed while the energy above extremality is fixed, i.e. $\frac{E}{m_s} = \text{fixed}$, the processes in which the modes that live on the branes are emitted into the bulk as closed strings are suppressed. The theory becomes free in the bulk, but strongly interacting on the brane. In this limit, the theory reduces to Little String Theory or more precisely to (2,0) LST for type IIA NS5-branes and to (1,1) LST for type IIB NS5-branes [15].

The throat geometry corresponding to $N$ coincident non-extremal NS5-branes in the string frame [19] is

$$ds^2 = -f(r)dt^2 + \frac{A(r)}{f(r)}dr^2 + A(r)r^2d\Omega_3^2 + \sum_{j=1}^{5} dx_j^2,$$

where $dx_j^2$ corresponds to flat spatial directions along the 5-branes, $d\Omega_3^2$ corresponds to 3-sphere of the transverse geometry and the dilaton field is defined as $e^{2\phi} = g_s^2 A(r)$. The metric functions are defined as

$$f(r) = 1 - \frac{r_0^2}{r^2}, \quad A(r) = \chi + \frac{N}{m_s^2 r^2},$$

$r_0$ is the non-extremality parameter, so the extremal configuration is obtained by the limit $r_0 \to 0$ and the location of the event horizon corresponds to $r = r_0$. We define the parameter $\chi$ which takes the values 1 for NS5 model and 0 for LST, these are only the values for which exist a supergravity solution. In addition to the previous fields there is an $NS-NS H_{(3)}$ form along the $S^3$, $H_{(3)} = 2N \epsilon_3$. According to the holographic principle the high spectrum of this dual string theory should be approximated by certain black hole in the background [2.1].
The geometry transverse to the 5-branes is a long tube which opens up into the asymptotic flat space with the horizon at the other end, in the limit $r \to r_0$ appears the semi-infinite throat parametrized by $(t, r)$ coordinates, in this region the dilaton grows linearly pointing out that gravity becomes strongly coupled far down the throat. The string propagation in this geometry should correspond to an exact conformal field theory [20]. The boundary of the near horizon geometry is $R^5 \times R \times S^3$. The geometry (2.1) is regular as long as $r_0 \neq 0$.

In order to construct the thermal states of the black holes we write the corresponding metric in Rindler coordinates as

$$ds^2 = -\kappa^2 \rho^2 dt^2 + d\rho^2 + A(r) r^2 d\Omega_3^2 + \sum_{j=1}^5 dx_j^2,$$

(2.3)

where we have introduced the radial Rindler coordinate $\rho$ (proper length), also we see that the quantity $\kappa = \frac{f(r_0)}{2\sqrt{A(r_0)}}$ coincides with the surface gravity of the NS5 and LST black holes. Then performing a Wick rotation $t_E = it$, the Euclidean time coordinate has to be periodic with period $2\pi$ in order to avoid a conical singularity. Thus we identify the period of the Euclidean time as in [21]

$$\beta = \frac{2\pi}{\kappa} = \frac{2\pi \sqrt{N + \chi r_0^2}}{m_s}.$$

(2.4)

The temperature obtained, $T = \beta^{-1}$, does not depend on which frame we are using, the string frame and Einstein frame are related by a local rescaling that does not affect the result.

Regarding LST model, we notice that the temperature is independent of the black hole radius and therefore of the black hole mass. In this way we could identify this temperature with the Hagedorn temperature of the superstring theory. One can compute the energy density for the LST background in ten dimensions, $e \equiv \frac{E}{V_5}$ and the entropy density, $s \equiv \frac{S}{V_5}$, where $V_5$ is the volume of the flat 5-branes space and $S$ is the standard Bekenstein-Hawking entropy calculated from the area of the event horizon of the black hole, $S = \frac{\text{Area}}{4G_{10}}$. Either in Einstein frame metric [21, 22] or in string frame metric [14, 19] it is satisfied the usual thermodynamic relation $S = \beta E$. This relation implies that the free energy of the system $F = E - TS$ vanishes. At very high energies the equation of state is of the Hagedorn form which leads to an exponentially growing density of states [13]: $\rho(E) = e^{S(E)} \sim e^{\beta E}$.

At first sight one could think that a phase transition is present when the system evolves from NS5 to the near horizon limit of NS5, i.e. LST, but we have checked that it is not the case. Computing the Bekenstein-Hawking entropy

$$S = \frac{\text{Area}}{4G_{10}} = \frac{V_5}{2G_{10}} \pi^2 (N + \chi m_s^2 r_0^2)^{3/2},$$

(2.5)
and plotting it versus the temperature we do not detect any critical point (Davies point) \[23\] that would signal a phase transition. Even working in thermodynamic geometry \[24\], writing

\[
ds^2 = -3\sqrt{\frac{\pi G}{M}} dS^2
\]

we do not detect any divergence in the scalar curvature that would signal a possible phase transition. However calculating the specific heat as \(C = T \frac{\partial S}{\partial T}\), we have found that it has a negative value, \(-3S\), showing that the theory is unstable. In the work \[13\], the authors show that loop/string corrections to the Hagedorn density of states of LST would be of the form \(\rho(E) \sim E^\alpha e^{\beta E}(1 + O(\frac{1}{E}))\) and the temperature-energy relation becomes \(\beta = \frac{\partial \log \rho}{\partial E} = \beta_0 + \frac{\alpha}{E} + O(\frac{1}{E^2})\). They found that since \(\alpha\) is negative the high energy thermodynamics corresponding to near-extremal 5-branes is unstable as well as the temperature is above the Hagedorn temperature and the specific heat is negative. This instability would be associated to the presence of a negative mode (tachyon) in string theory, the high temperature phase of the theory yields the condensation of this mode. The authors are lead again to the conclusion that the Hagedorn temperature is reached at a finite energy being associated with a phase transition.

Next we would like to address the question whether an observer in a moving frame observes a temperature above the Hagedorn temperature. We know that in the near horizon limit of NS5, i.e. LST, the system reaches the maximum temperature, namely the Hagedorn temperature. One could think that a boosted observer may observes a temperature bigger than the Hagedorn one, for this reason we want to verify the validity of this statement. We have evaluated the simplest case, a scalar particle-like observer which moves on an NS5-brane with constant velocity at fixed distance \(r\) from the horizon of the LST black hole. We consider the orbit for which \(y_1 = vt\). Relating the time coordinate \(t\) with the proper time \(\tau\) through

\[
d\tau^2 = -g_{\mu\nu} dx^\mu dx^\nu,
\]

one obtains

\[
\frac{d\tau}{dt} = \sqrt{f(r) - v^2}.
\]

The velocity is bounded by the local velocity of light thus we have to impose the constraint \(v^2 \leq f(r)\). This relation brings us to a new coordinate of the horizon position, where causality is lost, seen by the moving particle, \(r = \frac{r_0}{\sqrt{1-v^2}}\). Furthermore the Killing vector relevant for the process is \(\zeta = -\partial_t + v \partial_{y_1}\). Therefore evaluating the surface gravity at this new coordinate \(r\), we obtain the local temperature for the moving scalar particle

\[
T' = \frac{(1 - v^2) m_s}{2\pi \sqrt{N + \chi \frac{r_0^2}{(1-v^2)}}},
\]

where we have worked out in natural units, \(c = 1\) and \(v < 1\). We notice two important features. First of all, we see that in the \(v \to 0\) limit we recover the result \(2.3\). Secondly,
comparing the temperature for the particle-like observer (2.7) with the temperature defined by (2.4) for an asymptotic static observer, we see that the former is lower than the second one. We conclude that the Hawking temperature of LST is a maximum bound and corresponds to the Hagedorn temperature. Unfortunately, we are not able to perform the same analysis for an accelerating particle-like observer. The main problem is that the path which the particle follows is not generated by a Killing vector field, this fact prevent us from using the surface gravity method in order to calculate the temperature.

3. **Effective Two-Dimensional Theory**

We are going to reduce the metric (2.1) to the \( r-t \) sector, relevant for the forthcoming sections. At first step we take the scalar field action

\[
S = \frac{1}{2\kappa^2_{10}} \int_M d^{10}x \sqrt{-g} \left( R - \frac{1}{2} \partial_{\mu} \phi \partial^{\mu} \phi - \frac{1}{12} e^{-\Phi} H^2_{(3)} \right) .
\] (3.1)

Performing a change to tortoise coordinate, see (2.1) \( dr^* = \sqrt{A(r)} f(r) dr \), we expand the ten-dimensional action as

\[
S = \frac{1}{2\kappa^2_{10}} \int dt d\theta d\phi d\psi \prod_{j=1}^{5} dx_j \ r^3 A(r)^2 \sin^2 \theta \ \sin \varphi \ (g_s e^{-\Phi})^{5/2} \left[ \frac{f(r)}{\sqrt{A(r)}} \left( R - \frac{e^{-\Phi}}{12} H^2_{(3)} \right) + \right.
\]

\[
\left. \left( \frac{1}{2\sqrt{A(r)}} (\partial_t^2 - \partial_r^2) - \frac{f(r)}{2r^2 A^{3/2}} \left( \partial_\theta^2 + \frac{1}{\sin^2 \theta} \partial_\varphi^2 + \frac{1}{\sin^2 \theta \sin^2 \varphi} \partial_{\psi}^2 \right) - \frac{f(r)}{2\sqrt{A(r)}} \sum_{j=2}^{6} \partial_j^2 \right) \phi(t,r) S(\Omega_3) \ e^{i \sum k_j x_j} \right] ,
\] (3.2)

where we have decomposed the scalar field into \( r-t \), 3-angular and 5-brane parts. Our following approximations are based on three main steps:

1. We only consider the propagation mode of an s-wave.
2. We only take into account a subset of states of the Hilbert space such that the eigenstates of momentum parallel to the NS5-brane vanish.
3. We take the near horizon limit, \( r \to r_0 \).
Eventually we come back to the original $r$ radial coordinate obtaining for the action

$$S = \frac{V(S^3)V_5}{2\kappa_{10}^2} \int dt dr A(r)^2 e^{-2\Phi} \left(-\frac{1}{f(r)} \partial_t^2 + \frac{f(r)}{A(r)} \partial_r^2\right) \phi(t,r),$$

(3.3)

where $V(S^3)$ is the volume of the 3-sphere and $V_5$ the volume of 5-branes. From (3.3) we find out that the scalar field can be seen as $(1+1)$-dimensional scalar field $\phi(t,r)$ propagating in the background

$$ds_{\text{eff}}^2 = -f(r) dt^2 + \frac{A(r)}{f(r)} dr^2,$$

(3.4)

together with an effective dilaton field

$$e^{2\Phi} = g_s^2 A(r).$$

(3.5)

Henceforth we are going to work with this two-dimensional effective metric.

4. Semi-classical Black Hole emission

4.1. Complex path method

In this section we are going to show a semi-classical method to obtain the Hawking radiation. Eventually we will obtain the same thermal behavior than in [1], where the particle production is computed using the Bogoliubov transformation. The complex path method has been developed in [4], in order to calculate particle production in Schwarzschild-like space-time and it was extended for a different coordinate representations of the Schwarzschild space-time [5, 6]. Nevertheless complex path analysis had already been discussed by Landau [25], where it was used to describe tunneling processes in non-relativistic semi-classical quantum mechanics.

We will follow the reference [4] where the authors avoid to work in Kruskal representation. They use the standard coordinates in the $r-t$ sector. However the method presents a disadvantage because one would find a coordinate singularity at the horizon. Nevertheless using the techniques of complex integration one bypasses the singularity. We also want to mention that the method of complex path leads to the same results that in [26]. In both methods, for the Schwarzschild space-time and also as we will see in this section for NS5 and LST space-time, it has been found that the relation between emission and absorption probabilities is of the form.
\[
P_e = e^{-\beta \omega} P_a, \tag{4.1}
\]
where \( \omega \) is the energy of the emitted particles. We are tempting to compare this relation with the standard thermal Boltzmann distribution for blackbody radiation where \( \beta^{-1} \) is identified with Hawking temperature. We have verified that this is the case, if we compare our results versus the temperature calculated using the definition of surface gravity for example. It is noteworthy to say that this method allows one to get temperatures for black holes only comparing probabilities of emission and absorption but is not able to calculate the spectrum of thermal radiation. In that sense the tunneling method proposed is so far incomplete. To amend this shorts the authors in \cite{27} present a new mechanism.

In order to apply the complex path method to NS5 and LST we have constructed the semi-classical action obtained from Hamilton-Jacobi equations. Afterwards we have computed the semi-classical propagator \( K(r_2, t_2; r_1, t_1) \). Eventually we have calculated the emission and absorption probabilities.

We consider the equation of motion of a massless scalar particle \( \Box \phi = 0 \), in the background \( (3.4) \)

\[
-A(r) \frac{\partial^2 \phi(t, r)}{\partial t^2} + \frac{f(r)}{r^3} \frac{\partial}{\partial r} \left[ r^3 f(r) \frac{\partial \phi(t, r)}{\partial r} \right] = 0 . \tag{4.2}
\]
Using the standard ansatz solution
\[
\phi(t, r) \sim e^{\frac{i}{\hbar} S(t, r)} , \tag{4.3}
\]
and substituting in \( (4.2) \) we get an expression in terms of the action \( S(t, r) \)

\[
-A(r) \left( \frac{\partial S}{\partial t} \right)^2 + f(r)^2 \left( \frac{\partial S}{\partial r} \right)^2 + \frac{\hbar}{i} \left[ -A(r) \frac{\partial^2 S}{\partial t^2} + f(r)^2 \frac{\partial^2 S}{\partial r^2} + \frac{f(r) d(r^3 f(r))}{dr} \frac{\partial S}{\partial r} \right] = 0 , \tag{4.4}
\]
where we have collected the terms with \( \hbar \) dependence. The following step is to write the action as an expansion in a power series of \( \left( \frac{\hbar}{i} \right) \)

\[
S(t, r) = S_0(t, r) + \left( \frac{\hbar}{i} \right) S_1(t, r) + \left( \frac{\hbar}{i} \right)^2 S_2(t, r) + \ldots . \tag{4.5}
\]
Substituting the above expansion in \( (4.4) \) and neglecting terms of the order \( \left( \frac{\hbar}{i} \right)^2 \) and higher, we obtain a non-linear first order partial differential equation which corresponds to the Hamilton-Jacobi equation of motion to the leading order in the action \( S \).
\[-A(r) \left( \frac{\partial S_0(t, r)}{\partial t} \right)^2 + f(r)^2 \left( \frac{\partial S_0(t, r)}{\partial r} \right)^2 = 0. \] (4.6)

We are interested in the evaluation of the semi-classical propagator which inform us about the amplitude for a particle going from \( r_1 \) at time \( t_1 \) to \( r_2 \) at time \( t_2 \). In the saddle point approximation we get

\[ K(r_2, t_2; r_1, t_1) = N \exp \left[ \frac{i}{\hbar} S_0(r_2, t_2; r_1, t_1) \right], \] (4.7)

where \( N \) is a normalization constant. From (4.6) we get

\[ S_0(r_2, t_2; r_1, t_1) = -\omega(t_2 - t_1) \pm \omega \int_{r_1}^{r_2} \sqrt{A(r) \cdot f(r)} \, dr, \] (4.8)

the plus/minus sign correspond to ingoing/outgoing particles respectively and \( \omega \) is the energy of the emitted or absorbed particles.

The integral (4.8) is not well behaved if the horizon \( r_0 \) is within the region of integration. However this turns to be the case, we are interested in the emission of particles through the event horizon, so the region of integration runs from inside the horizon to outside.

First we consider the propagation of an outgoing particle in the inner region \( r_1 < r_0 \). Applying the usual complex analysis tools, we deform the contour of integration around the pole \( r_0 \) in the upper complex half-plane. Obtaining for the radial part of (4.8)

\[ S^e_0 = \frac{i \pi \omega}{r_0 \sqrt{A(r_0)}}, \] (4.9)

We will call it emission action because we simply consider the emission of an outgoing particle propagating from inside the horizon to the outside.

By the same talk one proceeds with analogous analysis to evaluate the action at lowest order for absorbed particles. In that case we are considering the propagation of an ingoing particle in the outer region, \( r_0 < r_2 \). Deforming the contour of integration in the upper complex half-plane, eventually we obtain the same result as the emission process up to change of sign. Now we are obtaining the absorption action for a particle that propagates from the region outside of the horizon to the inside

\[ S^a_0 = -\frac{i \pi \omega}{r_0 \sqrt{A(r_0)}}, \] (4.10)

We are interested in the expressions (4.9) and (4.10) in order to evaluate the probabilities of the emission and absorption processes. Thereby using the definition of the probability: \( P = |K(r_2, t_2; r_1, t_1)|^2 \), and substituting the expression for the corresponding actions, we finally obtain for the emission and absorption probabilities
\[ P_e \sim \exp \left[ -\frac{\pi}{\hbar} \omega r_0 \sqrt{A(r_0)} \right], \quad P_a \sim \exp \left[ \frac{\pi}{\hbar} \omega r_0 \sqrt{A(r_0)} \right], \quad (4.11) \]

where we have omitted the normalization constants. Eventually we are interested in to write the relation between emission and absorption probabilities

\[ P_e = \exp \left[ -\frac{2\pi}{\hbar} \omega r_0 \sqrt{A(r_0)} \right] P_a. \quad (4.12) \]

At first sight we observe that absorption process dominate over the emission, it is more easy for the system to absorb than to radiate particles. Also we note some misleading form in the expression for the absorption probability (4.11) because we could think that one might get a probability absorption greater than 1. Nevertheless we only have considered the spatial contribution of the action in order to calculate the probabilities of emission and absorption processes. Instead of this we should also have considered the time contribution as proposed in the work [28].

Comparing (4.12) with the same relation in a thermal bath of particles (4.1) we can identify the temperature of our system (taking \( \hbar = 1 \) and \( m_s = 1 \))

\[ T = \frac{1}{2\pi r_0 \sqrt{A(r_0)}} = \frac{1}{2\pi \sqrt{\chi r_0^2 + N}}, \quad (4.13) \]

that coincides with the value of temperature obtained in (2.4).

So far we have studied NS5/LST systems without backreaction. The next step is to consider the backreaction of the metric due to the emission process. For all the details we address the reader to [16] where we had studied the tunneling method. Generically the process consists in the emission of a particle with energy \( \omega \) from a black hole to the background. Taking into account the energy conservation the metric backreacts. Hence the total ADM mass is conserved and consequently the black hole mass must decrease by the same amount of the energy that it has been released. Our starting point in the evaluation of the backreaction is the expression of the action for the emission process (4.9). In our NS5/LST model we have the following relation between the event horizon and the mass of the black hole: \( r_0^2 \sim M \), where \( M \) is the mass of the black hole and the factors missed are not relevant in our study. When the metric backreacts in the emission process the energy conservation implies that \( r_0^2 \rightarrow r_0^2 - \omega \). The shrink of the event horizon rides the tunneling emission between turning points defined just inside and just outside of the event horizon. Once the emission has been carried out we can perform the previous change in (4.9)

\[ S_0^e = \frac{i\pi}{2} \omega \sqrt{\chi (r_0^2 - \omega)} + N, \quad (4.14) \]
expanding for low energies we get

\[ S_0^e = \frac{i\pi}{2} \left( \omega \sqrt{\chi r_0^2 + N} - \frac{\chi \omega^2}{2\sqrt{\chi r_0^2 + N}} + O(\omega^3) \right). \quad (4.15) \]

Calculating the emission probability for both models we obtain

\[
P_e \sim \begin{cases} 
\exp \left[ -\frac{\pi}{\hbar} \left( \omega \sqrt{r_0^2 + N} - \frac{\omega^2}{2\sqrt{r_0^2 + N}} + \ldots \right) \right] & \text{if } \chi = 1 \text{ (NS5)}; \\
\exp \left[ -\frac{\pi}{\hbar} \omega \sqrt{N} \right] & \text{if } \chi = 0 \text{ (LST)}. 
\end{cases} \quad (4.16)
\]

We see higher order correction terms corresponding to the NS5 emission probability, which indicates that the emission is not purely thermal. On the other hand the emission probability expression corresponding to the LST model is exact, which indicates that the emission is purely thermal.

As a final comment we want to stress the crucial fact of the energy conservation in order to get a non-thermal emission in the tunneling formalism. In this work, we have concluded that the results obtained from the tunneling formalism in [2] are nothing more than an extension of the Hamilton-Jacobi formalism taking into account the energy conservation, which induces the backreaction of the event horizon. In our particular case we are facing with an anomalous model which does not accomplish the previous expectations about non-thermal emission. The LST model emits thermal radiation in any case with or without energy conservation.

In order to analyze the deviations from the thermal behavior of the NS5 model it would be relevant to perform the computation of the greybody factors. Thus we must solve the radial part of the equation of motion [4,12]. As far we know this equation cannot be solved analytically, therefore it is necessary a numerical analysis which enlighten the non-thermal aspects of the NS5 model. Even so, we can elucidate that the non-thermal behavior of the NS5 model comes from the throat region. In this region the dilaton grows linearly pointing out that gravity becomes strongly coupled far down the throat, and states with large quantum numbers exist. On the other hand the near horizon limit of the NS5, i.e. LST, decouples the mode interactions between the bulk and the brane, the spectrum reduces to less excited states, leading to a thermal behavior with the Hagedorn temperature, see [29] for a complete discussion.
4.2. Anomalies

In this section we want to present another successful semi-classical method to compute the Hawking radiation from an evaporating black hole. The method is based on the cancellation of gravitational anomalies in a two-dimensional chiral theory taken as effective theory near the event horizon. This method was first proposed in [7]. Gravitational anomalies are anomalies in general covariance, i.e. general coordinate transformations (diffeomorphism), and they manifests as the non-conservation of the energy-momentum tensor.

The authors in [7, 8] managed the treatment of gauge and covariant anomalies deriving an effective two-dimensional theory close to the horizon. They built an effective action performing a partial wave decomposition in tortoise coordinate and dropping potential factors which vanish exponentially fast near the horizon. Thus physics near the horizon can be described by an infinite collection of (1+1) fields with the metric reduced to the $r - t$ sector. In these previous works the authors derived the Hawking radiation flux by anomaly cancellation, splitting the space-time into the near horizon region where the anomaly works and outside region when the conservation law is preserved. They carried out the calculation using the consistent chiral anomaly form of the energy-momentum tensor, see [30, 31],

$$\nabla_\mu T^\mu_\nu = \frac{1}{96\pi\sqrt{-g}}\epsilon^{\beta\delta}\partial_\beta\partial_\alpha\Gamma^\alpha_{\nu\beta}$$

(4.17)

together with the covariant boundary condition at the horizon. It is the cancellation of the covariant anomaly which lead us to the appearance of the Hawking radiation flux.

On the other hand it is known the existence of two types of anomalies. Covariant anomalies, which transform covariantly under gauge or general coordinate transformation but they do not satisfy the Wess-Zumino consistency condition. And consistent anomalies, which satisfy the consistency condition but they do not transform covariantly under gauge or general coordinate transformation. In our study we adopt the procedure carried out in [9] where the authors use a more coherent frame, working with covariant forms both for the expression of the chiral anomaly and for the boundary conditions. Unlike the previous works it is not necessary split the space-time into different regions, near the horizon region and outside.

First of all we consider the physics near the horizon of the NS5 and LST models described by an infinite collection of (1+1) scalar field particles propagating in the background (3.4). It is not necessary to work with the full metric because only the $r - t$ sector is relevant for the emission processes, obtaining so the same results for the full theory as for the effective two-dimensional theory. In this frame we can consider that only the outgoing modes are present. The ingoing modes are lost into the black hole and they do not affect at the classical level. Nevertheless the total effective action must be covariant. Thereby the quantum
contribution of these irrelevant ingoing modes will supply the extra term, a Wess-Zumino term, in order to cancel the gravitational anomaly providing the Hawking flux \cite{8}. The loss of the ingoing modes behind the horizon of the black hole causes that the effective theory becomes chiral, obtaining consequently a gravitational anomaly \cite{30,31}. Following \cite{9}, we adopt the expression for the covariant form of the gravitational anomaly

\[ \nabla_{\mu}T^{\mu\nu} = \frac{1}{96\pi}e^{\nu\mu}\nabla_{\mu}R, \]

where \( R \) is the Ricci scalar and \( e^{\nu\mu} \) is the Levi-Civitá tensor that in our case takes the values \( e^{tr} = -e^{rt} = 1 \) and zero for other contributions. The covariant boundary condition at the event horizon is

\[ T_t^r (r = r_0) = 0. \]

Noticing that we are working with a static metric, we evaluate the equation (4.18) for the effective two-dimensional theory in the \( r-t \) sector. Eventually we get

\[ \partial_r(\sqrt{-g}T^r_t) = \frac{1}{96\pi}g_{tt}\partial_r R. \]

The Ricci scalar for NS5 and LST models is

\[ R = \frac{f'A'}{2A^2} - \frac{f''}{A}, \]

where primes denotes derivative with respect to the coordinate \( r \). Defining the new function

\[ N_t^r \equiv \frac{1}{96\pi}\left(-\frac{ff'A'}{2A^2} - \frac{f'^2}{2A} + \frac{ff''}{A}\right), \]

we can write (4.20) as

\[ \partial_r(\sqrt{-g}T^r_t) = \partial_r N_t^r. \]

Then integrating the equation (4.23) we obtain

\[ \sqrt{-g}T^r_t = b_0 + (N_t^r (r) - N_t^r (r_0)), \]

where \( b_0 \) is an integration constant that can be evaluated implementing the covariant boundary condition (4.19). Doing so it yields the value \( b_0 = 0 \). Hence (4.24) becomes

\[ T_t^r = \frac{1}{\sqrt{-g}}(N_t^r (r) - N_t^r (r_0)). \]
The Hawking radiation flux is measured at infinity where the covariant gravitational anomaly vanishes. Therefore we compute the energy flux by taking the asymptotic limit of (4.25)

\[ T^r_t (r \to \infty) = - \frac{1}{\sqrt{-g}} N^r_t (r_0) . \]  

(4.26)

Evaluating (4.22) at the event horizon \( r_0 \) and considering the value of the surface gravity \( \kappa = \frac{1}{\sqrt{N + \chi r_0^2}} \), we finally obtain for the energy flux at infinity

\[ T^r_t (r \to \infty) = \frac{1}{\sqrt{-g}} \frac{\kappa^2}{48\pi} , \]  

(4.27)

which it is of course the Hawking radiation flux for a black hole.

5. Conclusion-Discussion

In this work we have started reviewing briefly some aspects about LST thermodynamics. We have exposed the thermal emission of LST due to the non-energy dependence of the Hagedorn temperature. Also we have evaluated the temperature experienced by a scalar particle-like observer, thereby we have verified that the Hagedorn temperature of LST is a maximum bound. Furthermore we have studied the Hawking radiation of the NS5 and LST black hole models using two semi-classical emission methods: the complex path method and the cancellation of the gravitational anomaly. We want to mention that using both methods we have recovered the previous results in [16] where we worked using the tunneling formalism. The complex path method [4, 6] shows how to evaluate the emission rate in the framework of the Hamilton-Jacobi formalism. We have shown that imposing energy conservation, in order to take into account the backreaction of the metric during the emission process, we reproduce exactly the same results that in the tunneling formalism proposed in [2, 3]. We would like to point out the advantage that represents the complex path method with respect to the tunneling formalism. First of all, we avoid heuristic explanations about the tunneling mechanism in the process of the emission. Secondly, we work with the well-known Hamilton-Jacobi equations plus the imposition of the energy conservation. And finally, it is not necessary to change the standard coordinates of the metric into Painlevé coordinates. We conclude that the tunneling method is nothing more than the complex path method plus energy conservation. Nevertheless none of the above methods are able to clarify the information loss paradox since they do not calculate the spectrum.

We have verified that another successful method to evaluate Hawking radiation in NS5 and LST models is based on the cancellation of the gravitational anomaly [7].
We have shown that all the above methods lead to a non-thermal emission for the NS5 model and thermal emission for the LST model, see (4.16).

Finally, cluster decomposition principle is a crucial physical requirement which states that very distant experiments produced uncorrelated results, thus establishing the local behavior of the field theory. Cluster decomposition principle states that if multi-particle processes are performed in $N$ very distant laboratories, then the S-matrix element for the overall process factorizes. This factorization ensures a factorization of the corresponding transition probabilities, corresponding to uncorrelated experimental results. In the line of the works [32, 33] where the authors linked the existence of correlations among tunneled particles and the entropy conservation of the full system (black hole plus Hawking radiation), we have calculated the successive emission probabilities for two particles of energies $\omega_1$ and $\omega_2$ using (4.16) for each model respectively. We have found that the NS5 model does not satisfy cluster decomposition

$$\ln | \Gamma(\omega_1 + \omega_2) | - \ln | \Gamma(\omega_1)\Gamma(\omega_2) | = \frac{\omega_1\omega_2}{2\sqrt{N + r_0^2}}. \quad (5.1)$$

On the other hand we have found that the LST model satisfies cluster decomposition as we expected

$$\ln | \Gamma(\omega_1 + \omega_2) | - \ln | \Gamma(\omega_1)\Gamma(\omega_2) | = 0, \quad (5.2)$$

where $\Gamma(\omega_1)$ and $\Gamma(\omega_2)$ are the emission probabilities corresponding to a particle of energy $\omega_1$ and $\omega_2$ respectively; $\Gamma(\omega_1 + \omega_2)$ is the emission probability of a particle with energy $\omega_1 + \omega_2$. We have found that $\Gamma(\omega_1,\omega_2) = \Gamma(\omega_1 + \omega_2)$ is accomplished at low energies, namely, the emission probability of a particle $\omega_2$ conditioned by the previous emission of a particle $\omega_1$ is the same as the emission of a single particle of energy $\omega_1 + \omega_2$. With these results at hand we can conclude that in the NS5 black hole exists correlations between emitted particles. This fact is intimately related with the non-thermal emission rate (4.16). Regarding (5.1) one hopes that the successive Hawking emissions could preserve unitarity avoiding in such a way the information loss paradox. However it is not the case for the LST black hole where the thermal emission rate (4.16) lead us to cluster decomposition. Therefore the successive emissions of particles are independent one of each other, thus the information of the initial states remain hidden.

Acknowledgments:
I thank Pere Talavera for fruitful discussions and insightful comments on this work. Also I
appreciate a lot the instructive comments formulated by F. J. Perez-Reche.

REFERENCES

[1] S. W. Hawking, “Particle Creation by Black Holes,” Commun. Math. Phys. 43 (1975) 199-220.

[2] M. K. Parikh, F. Wilczek, “Hawking radiation as tunneling,” Phys. Rev. Lett. 85 (2000) 5042-5045. [hep-th/9907001].

[3] M. K. Parikh, “A Secret tunnel through the horizon,” Int. J. Mod. Phys. D13 (2004) 2351-2354. [hep-th/0405160].

[4] K. Srinivasan, T. Padmanabhan, “Particle production and complex path analysis,” Phys. Rev. D60 (1999) 024007. [gr-qc/9812028].

[5] S. Shankaranarayanan, K. Srinivasan, T. Padmanabhan, “Method of complex paths and general covariance of Hawking radiation,” Mod. Phys. Lett. A16 (2001) 571-578. [gr-qc/0007022].

[6] S. Shankaranarayanan, T. Padmanabhan, K. Srinivasan, “Hawking radiation in different coordinate settings: Complex paths approach,” Class. Quant. Grav. 19 (2002) 2671-2688. [gr-qc/0010042].

[7] S. P. Robinson, F. Wilczek, “A Relationship between Hawking radiation and gravitational anomalies,” Phys. Rev. Lett. 95 (2005) 011303. [gr-qc/0502074].

[8] S. Iso, H. Umetsu, F. Wilczek, “Hawking radiation from charged black holes via gauge and gravitational anomalies,” Phys. Rev. Lett. 96 (2006) 151302. [hep-th/0602146].

[9] R. Banerjee, “Covariant Anomalies, Horizons and Hawking Radiation,” Int. J. Mod. Phys. D17 (2009) 2539-2542. [arXiv:0807.4637 [hep-th]].

[10] D. Kutasov, “Introduction to little string theory,” Lectures given at the Spring School on Superstrings and Related Matters. Trieste, 2-10 April 2001.

[11] O. Aharony, “A Brief review of ‘little string theories’,” Class. Quant. Grav. 17 (2000) 929-938. [hep-th/9911147].
[12] A. Kapustin, “On the universality class of little string theories,” Phys. Rev. D63 (2001) 086005. [hep-th/9912044].

[13] D. Kutasov, D. A. Sahakyan, “Comments on the thermodynamics of little string theory,” JHEP 0102 (2001) 021. [hep-th/0012258].

[14] M. Rangamani, “Little string thermodynamics,” JHEP 0106 (2001) 042. [hep-th/0104125].

[15] O. Aharony, M. Berkooz, D. Kutasov et al., “Linear dilatons, NS five-branes and holography,” JHEP 9810 (1998) 004. [hep-th/9808149].

[16] O. Lorente-Espin, P. Talavera, “A Silence black hole: Hawking radiation at the Hagedorn temperature,” JHEP 0804 (2008) 080. [arXiv:0710.3833 [hep-th]].

[17] A. W. Peet, J. Polchinski, “UV / IR relations in AdS dynamics,” Phys. Rev. D59 (1999) 065011. [hep-th/9809022].

[18] N. Seiberg, “New theories in six-dimensions and matrix description of M theory on T**5 and T**5 / Z(2),” Phys. Lett. B408 (1997) 98-104. [hep-th/9705221].

[19] J. M. Maldacena, A. Strominger, “Semiclassical decay of near extremal five-branes,” JHEP 9712 (1997) 008. [hep-th/9710014].

[20] D. Kutasov, N. Seiberg, “Noncritical superstrings,” Phys. Lett. B251 (1990) 67-72.

[21] A. L. Cotrone, J. M. Pons, P. Talavera, “Notes on a SQCD-like plasma dual and holographic renormalization,” JHEP 0711 (2007) 034. [arXiv:0706.2766 [hep-th]].

[22] T. Harmark, N. A. Obers, “Hagedorn behavior of little string theory from string corrections to NS5-branes,” Phys. Lett. B485 (2000) 285-292. [hep-th/0005021].

[23] H. La, “Davies Critical Point and Tunneling,”

[arXiv:1010.3626 [hep-th]].

[24] R. Banerjee, S. K. Modak, S. Samanta, “A New Phase Transition and Thermodynamic Geometry of Kerr-AdS Black Hole,”

[arXiv:1005.4832 [hep-th]].

[25] L. D. Landau, E. M. Lifshitz, “Textbook On Theoretical Physics. Vol. 2: Quantum Mechanics.” Pergamon Press, New York, (1975).
[26] J. B. Hartle, S. W. Hawking, “Path Integral Derivation of Black Hole Radiance,” Phys. Rev. D13 (1976) 2188-2203.

[27] R. Banerjee, B. R. Majhi, “Hawking black body spectrum from tunneling mechanism,” Phys. Lett. B675 (2009) 243-245. [arXiv:0903.0250 [hep-th]].

[28] A. de Gill, D. Singleton, V. Akhmedova et al., “A WKB-like approach to Unruh Radiation,” Am. J. Phys. 78 (2010) 685-691. [arXiv:1001.4833 [gr-qc]].

[29] J. D. Madrigal, P. Talavera, “A Note on the string spectrum at the Hagedorn temperature,” [arXiv:0905.3578 [hep-th]].

[30] L. Alvarez-Gaume, E. Witten, “Gravitational Anomalies,” Nucl. Phys. B234 (1984) 269.

[31] R. A. Bertlmann, E. Kohlprath, “Two-dimensional gravitational anomalies, Schwinger terms and dispersion relations,” Annals Phys. 288 (2001) 137-163. [hep-th/0011067].

[32] B. Zhang, Q. -y. Cai, L. You et al., “Hidden Messenger Revealed in Hawking Radiation: A Resolution to the Paradox of Black Hole Information Loss,” Phys. Lett. B675 (2009) 98-101. [arXiv:0903.0893 [hep-th]].

[33] B. Zhang, Q. -y. Cai, M. -s. Zhan et al., “Entropy is Conserved in Hawking Radiation as Tunneling: a Revisit of the Black Hole Information Loss Paradox,” Annals Phys. 326 (2011) 350-363. [arXiv:0906.5033 [hep-th]].