Tolman-Oppenheimer-Volkoff equations and their implications for the structures of relativistic stars in $f(T)$ gravity

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Abstract We investigate in this paper the structures of neutron and quark stars in $f(T)$ theory of gravity where $T$ denotes the torsion scalar. Attention is attached to the TOV type equations of this theory and numerical integrations of these equations are performed with suitable EoS. We search for the deviation of the mass-radius diagrams for power-law and exponential type correction from the $TT$ gravity. Our results show that for some values of the input parameters appearing in the considered models, $f(T)$ theory promotes more the structures of the relativistic stars, in consistency with the observational data.

Keywords $f(T)$ gravity

1 Introduction

The current acceleration of the universe is widely accepted through various independent observational data, as supernovae Ia (Weyl 1919; Eddington 1923; Riess et al. 1998; Starobinsky 1980; Perlmutter et al. 1999), cosmic microwave background radiation (Sotiriou and Faraoni 2010; Khoury and Weltman 2004a,b; De Felice and Tsujikawa 2010), the large scale structure of the universe (Brax et al. 2008; Psaltis 2008), cosmic shear through gravitational weak lensing surveys (Schmidt et al. 2007) and the Lyman alpha forest absorption lines (Spergel et al. 2003). The well known standard equivalent theories, the General Relativity (GR) and the Tele-parallel Theory, are the first theories used for explaining the acceleration of the universe, including the existence of the dark energy as a new component of the universe (Capozziello 2002; Capozziello et al. 2003; Nojiri and Odintsov 2003a,b, 2006, 2011; Carroll et al. 2004; Olmo 2011; Nojiri and Odintsov 2007, 2013; Capozziello and Faraoni 2010; Capozziello and De Laurentis 2011; de la Cruz-Dombriz and Saez-Gomez 2012).

Another way of explaining this acceleration of the universe is modifying a standard theory, GR or $TT$. In this theory we are interested to the modification of $TT$, getting the so-called $f(T)$ theory of gravity where $T$ denotes the torsion scalar. Instead of the Levi-Civita connection in the GR, the $TT$ and $f(T)$ are based on the Weitzenbock connection. Note that in $f(T)$ theory of gravity, new scalar degrees of freedom appear, constraining the consideration of the torsion scalar into dynamics way as effective new scalar field, instead of the torsion scalar in $TT$ defined by the pressure and the density inside the stars. In the theories based on the curvature scalar, the semi-classical approach to quantum gravity is often used where the higher and logarithmic terms are included in the action because their relevance for the strong field regime in the interior of the relativistic stars (Nojiri and Odintsov 2004; Guo and Frolov 2013; Baibosunov et al. 1990; Vilenkin 1985; Shore 1980). Neutron stars probe high baryon densities where baryon density in the stellar interior can be of the order of magnitude beyond the nuclear saturation density ($\rho_c = 2.7 \times 10^{17}$ kg m$^{-3}$ (Hamzeh and Weller 2013). It is also important to mention that, more precisely in the framework of $f(R)$ this kind of work has been developed with interesting results. Artyom et al considered...
neutron stars with strong magnetic fields in the framework of \( f(R) \) where the effect of strong magnetic field is investigated within models involving quadratic and cubic corrections in the Ricci scalar to the Einstein-Hilbert action. Their results show that for cubic terms, a significant increasing of the maximal mass is possible (Astashenok et al. 2015). Moreover the authors extend the work about neutron star models but in the framework of perturbative \( f(R) \) gravity with realistic equations of state. They show that interesting result can be achieved in modified gravity with only a cubic correction, and more, for some EoS, the upper limit of neutron star mass increases and the realistic star configurations can be described (Astashenok et al. 2013). Still in order to investigate others aspect of neutron stars, the authors took into account models derived from \( f(R) \) and \( f(G) \) where \( G \) denotes the Gauss-Bonnet invariant and show that in general, other branches of massive neutron stars are possible considering the extra pressure contributions coming from gravity extensions (Astashenok et al. 2014a). The authors also undertake the case of \( f(R) = R + \alpha R^2 \) mimetic gravity where it is shown that the contribution of scalar field in mimetic gravity can lead to possible existence of extreme neutron stars with large masses (Astashenok and Odintsov 2015). In higher-derivative models with power-law terms as \( f(R) = R + \alpha R^2 + \beta R^3 \), where \( \alpha \) and \( \beta \) denote two real constants, realistic Mass-Radius relation and account for hyperons in equation of state (Astashenok et al. 2014b).

In the view of the results obtained in the framework of \( f(R) \) and \( f(G) \), it is evident to observe that the choice of the action models to be used in the investigation of neutron stars features only depends on the cosmological observational data. However, in \( f(T) \) theory of gravity the choice of the action models, not only depends on the observational data but also on the choice of the tetrads. Note that for diagonal tetrad, the action is constrained to the Teleparallel theory (added to the cosmological constant) one and the results to be obtained in this case are the same as that of the General Relativity ones. On the other hand when the tetrad is not diagonal, algebraic functions can be obtained for the action and being constrained by the observational data later in order to obtain realistic cosmological models. The main aspect of the \( f(T) \) theory to be pointed out here is that, contrary to the equivalence between Teleparallel Theory and the General Relativity, there is any evident equivalence between the \( f(T) \) and \( f(R) \) theories. For example one can observe from the interesting work of Bamba et al. (2011) that \( R^a \) term addition to the Einstein-Hilbert one could cure the curvature singularity and viable \( f(R) \) gravity models could become free of such a singularity. Just the case of FRW universe can be used to illustrate the non-equivalence between the \( f(T) \) and \( f(R) \) theories. Observe that the torsion scalar is proportional to the square of the Hubble parameter, that is \( T = -6H^2 \), while the Ricci scalar reads \( R = -6(2H^2 + \dot{H}) \).

It is then obvious that the exponent \( \beta \) to be used in the additive term \( T^\beta \) to the TT has to be different from \( \alpha \) for a particular cosmological feature, explaining a kind of difference between the \( f(T) \) and \( f(R) \) theories.

In this paper, inspired by the fact that in dense medium the strong nuclear force play a crucial role, we consider the effect of corrections to the \( TT \) action involving terms of power-law and exponential in \( T \) on the observational features of the neutron stars and quark stars with suitable equations of state (EoS). For the neutron stars, we consider the polytropic EoS and the SLy EoS, while for the quark stars, we assume a simplest EoS in the so-called bag model. In this way, Tolman-Oppenheimer-Volkoff Equations are established from the generalized field equation in \( f(T) \). First we consider diagonal tetrad and fall into the constraint where the algebraic action function is reduced to the \( TT \) one. Since the goal is to introduce correction terms to the \( TT \) theory, we consider in the second step a non-diagonal tetrad through which the previous constraint fall down. According to our results it comes that for some values of the input parameters \( b_1, b_2, n \) and \( q \), structures of relativistic stars, both neutron and quark stars, are able to be found within \( f(T) \) theory of gravity.

The paper is organized as follows. Section 2 is devoted to the generality on \( f(T) \) gravity and the generalization of the TOV equations within diagonal tetrad fashion. The TOV equations are still developed in Sect. 3 but within the non-diagonal fashion; then the structures of the relativistic stars, both neutron and quark stars, are analyzed through numerical integrations of the field TOV equations. Our conclusion and perspectives are presented in Sect. 4.

2 Generality on \( f(T) \) gravity and generalised TOV equations

The action \( S \) of \( f(T) \) gravity, is given by

\[
S = \int d^4x \left[ \frac{f(T)}{2\kappa^2} + \mathcal{L}_m \right],
\]

where \( \mathcal{L}_m \) is the matter Lagrangian density assumed to depend only on the tetrad, and not on its covariant derivatives; \( e \) is the determinant of the tetrad and \( f \) a generic function depending on the scalar torsion.

The variation of the action (1) with respect to the tetrad leads to (Daouda et al. 2011, 2012)

\[
S_{\mu}^{\nu} \partial_{\rho} Tf_{\rho T T} + \left[ e^{-1} e^\lambda_\mu \partial_\rho (ee^\lambda_\nu S^{\lambda \nu}) \right] r
+ T^\lambda_\mu S^{\nu \lambda} \right] f_T + \frac{1}{4} \eta^{\nu \mu} = \frac{k^2}{2} T^{\nu}_{\mu},
\]

where \( T^{\nu}_{\mu} \) is the energy-momentum tensor. We consider that the matter content is an isotropic fluid, such that the corre-
sponding energy-momentum tensor reads
\[(\rho + p_t)u_\mu u^\nu - p_t \delta_\mu^\nu + (p_r - p_t)u_\mu v^\nu,\] (3)
with \(u^\mu, v^\mu\) the four-velocity and the unit space-like vector in the radial direction. The parameters \(\rho, p_r,\) and \(p_t\) denote the energy density, the pressure in the direction of \(v^\mu\) (normal pressure) and \(p_t\) the pressure orthogonal to \(u^\mu\) (transversal pressure), respectively. In an isotropic case, one has the equality \(p_r = p_t\).

In order to get solutions describing stellar objects, we assume a spherically symmetric metric with two independent functions \(\alpha\) and \(\beta\) both depending on the radial coordinate, as
\[ds^2 = e^{\alpha(r)}dt^2 - e^{\beta(r)}dr^2 - r^2(d\theta^2 + \sin^2\theta d\phi^2).\] (4)
According to this metric, the field equation can be decoupled as
\[
\frac{f}{4} - \left[ T - \frac{1}{r^2} - \frac{e^{-\beta}}{r}(\alpha' + \beta') \right] \frac{fr}{2} = 4\pi \rho, \tag{5}
\]
\[
\left( T - \frac{1}{r^2} \right) \frac{fr}{2} - \frac{f}{4} = 4\pi p_r, \tag{6}
\]
\[
\left[ T + \frac{e^{-\beta}}{2}\left( \frac{\alpha''}{2} + \left( \frac{\alpha'}{4} + \frac{1}{2r} \right)(\alpha' - \beta') \right) \right] \frac{fr}{2} - \frac{f}{4} = 4\pi p_t, \tag{7}
\]
\[
\cot \theta \frac{fr}{2r^2} T' \beta' = 0, \tag{8}
\]
where the prime denotes the derivative with respect to the radial coordinate \(r\). The torsion scalar reads
\[T = \frac{2e^{-\beta}}{r^2}(1 + r\alpha').\] (9)

From the conservation Law for the stress tensor, i.e., \(\nabla_\mu T^\mu_\nu = 0\), one gets
\[
\frac{dp_r}{dr} = -\frac{1}{2}(\rho + p_r) \frac{d\alpha}{dr} + \frac{2}{r}(p_t - p_r). \tag{10}
\]
In the usual cases, i.e., with an isotropic fluid, where \(p_t = p_r = p\), and setting \(\alpha \to 2\alpha\), one gets \(\frac{dp}{dr} = -\frac{1}{2}(\rho + p) \frac{d\alpha}{dr}\).

By taking into account the trace of the field equations, one gets
\[
\frac{e^{-\beta}}{4r^2} \left[ 4(e^{-\beta} - 3) + 4r(\beta' - 3\alpha') + r^2(\alpha'' \beta' - 2\alpha'' \alpha') \right] fr + f = 4\pi(\rho - p_t - 2p_t). \tag{11}
\]
In order to get the TOV equations, it is suitable to replace the metric function by the following expression
\[e^{-\beta} = 1 - \frac{2GM}{r}, \tag{12}\]
For the following considerations, it is convenient to adopt dimensional variables \(M \to mM_\odot\) and \(r \to r_\odot r\) with \(r_\odot = GM_\odot\). Then, from (12) one can extract
\[
e^{-\beta} = 1 - \frac{2m}{r} \implies \beta' = \frac{2m}{r^2} \times \frac{1}{\frac{2m}{r} - 1}. \tag{13}
\]
Then it is easy to get stellar structures. It is important to note here that some asymptotic flatness are required as the radial coordinate evolves, that is
\[
\lim_{r \to \infty} T(r) = 0, \lim_{r \to \infty} m(r) = cste. \tag{14}
\]
By making use of (9), (10) within the condition of isotropy \(p_t = p_r = p\) and (13), one gets in terms of the mass, the pressure and the energy density, the following TOV equations
\[
\left[ \frac{2}{r(\rho + p)} \left( 1 - \frac{2m}{r} \right) \frac{dp}{dr} + \frac{1}{2r^2} \frac{dm}{dr} - \frac{1}{2r^2} + \frac{m}{r^3} \right] f_T + \frac{1}{4} f - 4\pi p = 0,
\]
\[
\left[ -\frac{2}{\rho + p} \left( \frac{1}{r^2} - \frac{6}{r} + \frac{11m}{r^2} \right) \frac{dp}{dr} + \frac{4}{(\rho + p)^2} \left( \frac{2m}{r} - 1 \right) \left( \frac{dp}{dr} \right)^2 \right.
\]
\[
- 2 \left( \frac{2m}{r} - 1 \right) \frac{d}{dr} \left( \frac{1}{\rho + p} \frac{dp}{dr} \right) - \frac{2}{r^2} \frac{dm}{dr} - \frac{2}{r^2} + \frac{4m}{r^3} \right] f_T + f - 4\pi(\rho - 3p) = 0. \tag{17}
\]
In order to solve the system (15)–(17), we need the expression of the algebraic function \(f(T)\), an explicit form of the equation of state \(p = f(\rho)\), and some initial conditions on the mass \(m\) and the pressure. As initial conditions for the mass and the pressure one can assume \(m(0) = 0\) and \(p(0) = f(\rho_\odot)\), where \(\rho_\odot = \rho(0)\). From (8) two cases can be distinguished, i.e., \(T = cste\) or \(f(T) = T - 2\Lambda\), where \(\Lambda\) is viewed as the cosmological constant. Let us first work with the realistic case, that is \(f(T) = T - 2\Lambda\).

3 Stars structures through non-diagonal tetrad

In this section we consider a non-diagonal tetrad, in order to escape the constraint leading to the linear form of the alge-
Fig. 1 These figures show the evolution of the mass as the radius evolves for the case \( n < 0 \), for instance \( n = -0.5 \) (left diagram); for the case \( 0 < n < 1 \), for instance \( n = 0.5 \) (middle diagram) and for the case \( n > 1 \), for instance \( n = 2 \) (right diagram). For the three diagrams, the representative of the \( TT \) are the black ones \((b_1 = 0)\). The red, magenta, blue and green curves are plotted for \( b_1 = -2; -1; 1; 2 \), respectively. The graphs here are related to neutron stars within polytropic EoS for power-law correction to \( TT \) algebraic function \( f(T) \). We chose the non-diagonal tetrad as

\[
[e^\mu_\nu] = \begin{pmatrix}
e^\alpha & 0 & 0 & 0 \\
e^\beta \sin \theta \cos \phi & r \cos \theta \cos \phi & -r \sin \theta \sin \phi & 0 \\
e^\beta \sin \theta \sin \phi & r \cos \theta \sin \phi & r \sin \theta \cos \phi & 0 \\
e^\beta \cos \theta & -r \sin \theta & 0 & 0 \\
\end{pmatrix}
\]

(18)

The TOV equations in this case read

\[
-\frac{1}{2} \frac{\sqrt{r}}{r^2} \left[ \sqrt{r-2m} \left( \sqrt{\frac{r}{r-2m}} - 1 \right) T^f_{TT} + \frac{1}{2} f \right] + \frac{1}{r^2} \sqrt{r(r-2m)} \left[ (\alpha' r^2 - 2m r - 2a' m r + 2r - 2m) \left( \frac{r}{r-2m} - \alpha' r^2 + 2a' m r - 2r + 4m \right) \right] f_T + \frac{1}{4} f = 4\pi \rho, \tag{19}
\]

\[
-\frac{1}{4} f = 4\pi p_r, \tag{20}
\]

\[
\frac{1}{4r^2} \sqrt{r(r-2m)} \left[ \left( \frac{\alpha'}{r} + 2 \right) \frac{\sqrt{r-2m}}{r-2m} \right] T^f_{TT} - \frac{1}{8r^3} \left[ 4r (\alpha' r + 2) \left( \frac{r}{r-2m} - 2\alpha'' r^3 - \alpha' r^3 \right) + 2a' m r^2 + 4a'' m r^2 + 2a'^2 m r^2 - 6a' r^2 + 4m' r + 10a' m r - 8r + 4m \right] f_T - \frac{1}{4} f = 4\pi p_t, \tag{21}
\]

where \( p_r \) and \( p_t \) mean the radial and tangential pressures respectively. In this case, the trace of the equation of motion leads to

\[
\left[ \frac{1}{r} \left( \sqrt{1 - \frac{2m}{r}} + \alpha' m - 2 \right) + \frac{4m}{r^2} - \frac{\alpha'}{2} \right] T^f_{TT} + \left( \frac{\alpha' m}{2r} + \frac{\alpha'' m}{r} + \frac{\alpha'^2}{2r} - \frac{3\alpha' 2m'}{r^2} + \frac{11\alpha' m'}{2r^2} - \frac{4m'}{r^2} \right) f_T + f = 4\pi (\rho - p_r - 2p_t). \tag{22}
\]
In this rubric we will still work in the frame of MIT bag model where the simple equation of state for quark matter is (27). Here, we will distinguish two models, the power law and exponential models.

### 3.1 Stars structures within power-law and exponential gravities

In this subsection we assume the algebraic power-law action function as \( f(T) = T + b_1 T^n \) where \( b_1 \) is real constant and \( n \) a real number with \( n \neq 0 \) and \( n \neq 1 \), such that \( f_T(T) = 1 + nb_1 T^{n-1} \) and \( f_{TT}(T) = n(n-1)b_1 T^{n-2} \). The exponential action function is assumed as \( f(T) = b_2 e^{\xi T} \), with \( b_2 \neq 0 \) and \( q \neq 0 \) such that \( f_T(T) = b_2 q^2 e^{\xi T} \) and \( f_{TT}(T) = b_2 q^2 e^{\xi T} \).

#### 3.1.1 Neutron stars

The structure of neutron stars has been developed in other kind of modified gravity, namely the \( f(R) \) gravity (Babichev and Langlois 2010; Kantiab and Afshordi 2011; Cisterna et al. 2015; Cooney et al. 2010; Orellana et al. 2013; Deliduman et al. 2012; Santos 2012). In our case we will explore the observable features such as the mass-radius relation of the neutron stars. In order to solve the generalized field equations it is needed the relation between the energy density and the pressures. We will assume for simplicity an homogeneous matter content such that the tangential and radial pressures are equal, and equations of state will be assumed for the neutron stars driving the information of the matter inside the star. Two types of equation of state are considered: the simpler polytropic EoS and a more realistic SLy EoS (Haensel and Potekhin 2004).

**Polytropic EoS** As simpler polytropic equation of state, we assume the following one

\[
\zeta = 2\xi + 5
\]  

(23)

with

\[
\xi = \log\left(\frac{p}{\text{g/cm}^3}\right), \quad \zeta = \log\left(\frac{p}{\text{dyn cm}^{-2}}\right)
\]  

(24)

By making use of Runge-Kutta method (Farinelli et al. 2013), we numerically solve the system formed by the equations (19), (20) and (24) through an integration from the center to the surface of the star. We assume that the density at the center of the star is increased from a non-null value \( \rho_c \), fixed to \( \rho_c = 2.7 \times 10^{17} \text{ kg m}^{-3} \), called nuclear saturation density. The numerical results are presented and the deviation from the \( TT \) theory of gravity can be seen for large values, both positive and negative, of the parameters \( b_1 \) and \( b_2 \) for the power-law gravity and exponential gravity, respectively. The evolution the mass-radius diagram for neutron stars within polytropic EoS are represented in Fig. 1 for power-law correction and Fig. 2 for exponential correction to the \( TT \) of gravity.

**SLy EoS** This type of equation of state characterizes the behaviour of nuclear matter at high densities and is expressed as

\[
\xi = \frac{a_1 + 2a_2 \xi + a_3 \xi^3}{1 + a_4 \xi} f_0 a_5(\xi - a_6) + f_0(a_7 + a_8 \xi) \\
\times \left[ a_9(a_{10} - \xi) + f_0(a_{11} + a_{12} \xi)[a_{13}(a_{14} - \xi)] \right] \\
+ f_0(a_{15} + a_{16} \xi)[a_{17}(a_{18} - \xi)].
\]  

(25)

Here the parameters \( \xi \) and \( \zeta \) are defined as in (24) and the function \( f_0 \) is defined by

\[
f_0(x) = \frac{1}{e^x + 1}
\]  

(26)

The values of coefficients \( a_i \) can be viewed in Table 1 (Haensel and Potekhin 2004).

As in the previous case, we assume that the density at the center of star evolves from a critical \( \rho_c \) to the total density at it surface. The deviation from the \( TT \) is presented for the both power-law gravity and the exponential one for different values of the parameters \( b_1 \) and \( b_2 \). The evolution of the mass-radius diagram for neutron stars within SLy EoS are represented in Fig. 3 for power-law correction and Fig. 4 for exponential correction to the \( TT \) of gravity.

#### 3.1.2 Quark stars

A quark star is a self-gravitating system consisting of deconfined \( u, d \) and \( s \) quarks and electrons (Khoury and Weltman 2004a,b). These deconfined quarks are the fundamental elements of the color superconductor system. In the comparision with the standard hadron matter, they lead to a softer equation of state. In the frame of the so-called bag model,
Fig. 2 The figures show the evolution of the mass as the radius evolves for the case \( q < 0 \), for instance \( q = -1 \) (left diagram) and for the case \( q > 0 \), for instance \( q = 1 \) (right diagram). For both diagrams, the representative of the TT are the black ones \((b_2 = 0)\). The red, magenta, blue and green curves are plotted for \( b_2 = -1; -0.5; 0.5; 1 \), respectively. The graphs here are related to neutron stars within polytropic EoS for exponential correction to TT.

Fig. 3 These figures show the evolution of the mass as the radius evolves for the case \( n < 0 \), for instance \( n = -0.5 \) (left diagram); for the case \( 0 < n < 1 \), for instance \( n = 0.5 \) (middle diagram) and for the case \( n > 1 \), for instance \( n = 2 \) (right diagram). For the three diagrams, the representatives of the TT are the black ones \((b_1 = 0)\). The red, magenta, blue and green curves are plotted for \( b_1 = -2; -1; 1; 2 \), respectively. The graphs here are related to neutron stars within SLy EoS for power-law correction to TT.

A simple equation of state is obtained for quark matter (Astashenok et al. 2014c)

\[
p = \omega(\rho - 4\gamma)
\]

(27)

where \( \gamma \) is the bag constant. The value of the parameter \( \omega \) depends on the mass \( m_s \) of the strange quark. For the radiation, one has \( m_s = 0 \) and the parameter is \( \omega = 0 \), and for more realistic model where \( m_s = 250 \text{ MeV} \), the parameter is \( \omega = 0.28 \). The parameter \( \gamma \) belongs to the interval \( 58.8 < B < 91.2 \) in the unit of \( \text{MeV}/\text{fm}^3 \) (Spergel et al. 2003). The evolution the mass-radius diagram for quark stars are represented in Fig. 5 for power-law correction and Fig. 6 for exponential correction to the TT of gravity.
Fig. 4  The figures show the evolution of the mass as the radius evolves for the case $q < 0$, for instance $q = -1$ (left diagram) and for the case $q > 0$, for instance $q = 1$ (right diagram). For both diagrams, the representatives of the $TT$ are the black ones ($b_2 = 0$). The red, magenta, blue and green curves are plotted for $b_2 = -1; -0.5; 0.5; 1$, respectively. The graphs here are related to neutron stars within SLy EoS for exponential correction to $TT$.

Fig. 5  These figures show the evolution of the mass as the radius evolves for the case $n < 0$, for instance $n = -0.5$ (left diagram); for the case $0 < n < 1$, for instance $n = 0.5$ (middle diagram) and for the case $n > 1$, for instance $n = 2$ (right diagram). For the three diagrams, the representatives of the $TT$ are the black ones ($b_1 = 0$). The red, magenta, blue and green curves are plotted for $b_1 = -2; -1; 1; 2$, respectively. The graphs are related to the quark stars with the polytropic correction to $TT$.

4 Conclusion

The structures of neutron and quark stars structures have been analyzed in this work in the framework of $f(T)$, $T$ being the torsion scalar. The main goal here is searching the deviation of mass-radius diagram of both neutron and quark stars of the modified gravity, view as correction terms to the $TT$, from corresponding diagram of the $TT$. The first has been establishing the TOV equations for $f(T)$ theory. The fundamental fashion here is assuming the non-diagonal tetrad from which any constraint can occur about the choice of the algebraic action function. Therefore, we assume two
The figures show the evolution of the mass as the radius evolves for the case $q < 0$, for instance $q = -1$ (left diagram) and for the case $q > 0$, for instance $q = 1$ (right diagram). For both diagrams, the representatives of the TT are the black ones ($b_2 = 0$). The red, magenta, blue and green curves are plotted for $b_2 = -1; -0.5; 0.5; 1$, respectively. The graphs here are related to neutron stars within SLy EoS for exponential correction to TT.

interesting cases as correction terms; the power-law and exponential terms, including the parameters ($b_1$ and $n$) and ($b_2$ and $q$), respectively.

Our analysis are based on the numerical integration of the system formed by TOV equations and the different EoS, the polytropic and SLy EoS for neutron stars and a suitable EoS for the quark stars. Our results show that for some values of the input parameters in the neutron stars case, for both polytropic and SLy EoS, the deviation of the $f(T)$ terms from the TT is obvious for the beginning and the intermediate values of the radius. However, in some cases and for large values of the radius, the correction terms do not have effect on the evolution of the mass, with respect to the $TT$ case. In the quark stars case, it appears that for any value of the radius, the deviation of the $f(T)$ theory terms from the $TT$ is obvious.

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