N* Spectrum using an $O(a)$-Improved Fermion Action

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The construction of operators and calculational methods for the determination of the $N^*$ spectrum are discussed. The masses of the parity partners of the nucleon and delta are computed from the $O(a)$-improved data of the UKQCD Collaboration, and a clear splitting observed between the mass of the nucleon and its parity partner.

1. INTRODUCTION

There has been burgeoning interest in the calculation of the excited spectrum of nucleons, spurred by the experimental programmes at Jefferson Laboratory and elsewhere. Coinciding with this experimental interest, there has been a flurry of activity in the lattice community. Two calculations have appeared; the first employed the highly-improved $D_{234}$ fermion action\[1,2\], whilst the second employed domain-wall fermions\[3\]. Both calculations exhibited a clear splitting between the masses of the nucleon and its parity partner.

The construction of operators and calculational methods for the determination of the $N^*$ spectrum are discussed. The masses of the parity partners of the nucleon and delta are computed from the $O(a)$-improved data of the UKQCD Collaboration, and a clear splitting observed between the mass of the nucleon and its parity partner.

2. CALCULATIONAL DETAILS

There are three local interpolating operators employed for a positive-parity nucleon at rest:

\[
N_1^{1/2+} = \epsilon_{ijk} (u_i^T C \gamma_5 d_j) u_k \tag{1}
\]

\[
N_2^{1/2+} = \epsilon_{ijk} (u_i^T C d_j) \gamma_5 u_k \tag{2}
\]

\[
N_3^{1/2+} = \epsilon_{ijk} (u_i^T C \gamma_4 \gamma_5 d_j) u_k. \tag{3}
\]

The “diquark” part of both $N_1$ and $N_3$ couples upper spinor components, whilst that in $N_2$ involves lower components and vanishes in the non-relativistic limit. In practice, $N_1$ and $N_3$ have a much greater overlap with the nucleon ground state than $N_2$, and are used in spectroscopy calculations to obtain the nucleon mass. The operators of eqns. (1)-(3) are appropriate for positive-parity states; interpolating operators for negative-parity states are constructed by simply multiplying by $\gamma_5$.

Correlators constructed from these operators receive contributions from states of both parities. The best delineation that can be achieved is that of forward-propagating positive-parity states and backward-propagating negative-parity states, or the converse, through the use of the parity projection operator $(1 \pm \gamma_4)$. On a lattice anti-periodic in time, the correlator may be written

\[
C_{N_1^{1/2+}}(t) = \sum_{\vec{x}} \left( (1 \pm \gamma_4)_{\alpha\beta} \langle N_i,\alpha(\vec{x},t) \overline{N}_i,\beta(0) \rangle + (1 \mp \gamma_4)_{\alpha\beta} \langle N_i,\alpha(\vec{x},N_t-t) \overline{N}_i,\beta(0) \rangle \right), \tag{4}
\]

where $N_t$ is the temporal extent of the lattice. At large distances, when $t \gg 1$ and $N_t - t \gg 1$, the correlators behave as

\[
C_{N_1^{{1/2}^+}}(t) \rightarrow A_1^+ e^{-M_1^+ t} + A_1^- e^{-M_1^- (N_t-t)} \tag{5}
\]

\[
C_{N_1^{{1/2}^-}}(t) \rightarrow A_1^- e^{-M_1^- t} + A_1^+ e^{-M_1^+ (N_t-t)} \tag{6}
\]

where $M_1^+$ and $M_1^-$ are the lightest positive- and negative-parity masses respectively. In the pres-
ence of unbroken chiral symmetry, the positive- and negative-parity baryons would form mass-degenerate doublets. The mass splitting between them is a manifestation of spontaneously broken chiral symmetry.

A local interpolating operator for the lowest-lying spin states of the $I = 3/2$ $\Delta$ baryon is

$$\Delta^{3/2,1/2} = \epsilon_{ijk} (u_i^T C \gamma_\mu u_j) u_k.$$  

(7)

This has an overlap onto both spin-$3/2$ and spin-$1/2$ states, but these can be distinguished using a suitable projection, and the positive- and negative-parity states delineated as described above.

3. CALCULATIONAL DETAILS

The calculation is performed using the gauge configurations and propagators generated by the UKQCD Collaboration. Two values of the coupling, $\beta = 6.0$ and $\beta = 6.2$, are employed, and propagators are computed using the clover fermion action, with the clover coefficient determined non-perturbatively thus removing all $O(a)$ discretisation errors. The quark propagators are computed from both local and fuzzed sources to local and fuzzed sinks. The parameters of the calculation are summarised in Table 1, and further details are contained in ref. [4].

The masses of the $N^{1/2+}$ and $N^{1/2-}$ states are obtained from a simultaneous, four-parameter fit to the correlators $C_{N^+} (t)$ and $C_{N^-} (t)$ of Equation (4), using fuzzed sources and local sinks. The quality of the data and of the fits for both the positive- and negative-parity states is illustrated in Figure 1, and the importance of including the backward-propagating positive-parity state in the determination of the negative-parity mass is clear.

The masses in the chiral limit are obtained by linear extrapolation in the quark mass, as shown in Figure 2 for the data at $\beta = 6.2$. The results show a clear splitting between the masses of the positive- and negative-parity states persisting to the chiral limit. Also shown as the burst is the experimental determination of the lowest lying $J = 1/2^-$ mass, $N(1535)$, scaled with the lattice spacing determined from the nucleon mass; the agreement between the lattice determination of the $N^{1/2-}$ mass and the experimental value is striking. The quality of the extracted masses is poorer at $\beta = 6.0$ than at $\beta = 6.2$, reflecting the larger value of the masses in lattice units. As observed in ref. [3], the "bad" baryon operator $N_2$ yields a much higher mass for the positive-parity ground state, comparable with that of the negative-parity baryon obtained using $N_1$.

With only two values of the lattice spacing, it is not possible to perform a continuum extrapolation. However some indication of the magnitude of discretisation effects can be gleaned from Figure 3, where the masses of the $N^{1/2+}$ and $N^{1/2-}$ states are shown in units of Sommer’s $r_0$ at each of the two lattice spacings. The higher spin states are expected to be larger, and thus more susceptible to finite-volume corrections. A analysis including data at three values of the lattice spacing and at different lattice volumes is in progress [5].

The mass of the $\Delta^{3/2-}$ is also accessible, though subject to greater statistical noise. Here
Table 1
The parameters of the lattices used in the calculation.

| $\beta$ | $c_{sw}$ | $L^3 \cdot T$ | $L$ [fm] | $\kappa$ | #conf. |
|---------|---------|--------------|---------|---------|--------|
| 6.0     | 1.769   | $16^3 \cdot 48$ | 1.5     | 0.13344, 0.13417, 0.13455 | 496    |
| 6.2     | 1.614   | $24^3 \cdot 48$ | 1.6     | 0.13460, 0.13510, 0.13530 | 216    |

Figure 2. The masses of the $N^{1/2^{+}}$ (circles) and $N^{1/2^{-}}$ (diamonds) at $\beta = 6.2$. The lines are linear extrapolations to the chiral limit. The burst is the physical $N(1535)$ mass, expressed in lattice units as discussed in the text.

Figure 3. The masses of the $N^{1/2^{+}}$ and $N^{1/2^{-}}$ states in units of $r_0$ against the lattice spacing $a^2$ in units of $r_0^2$.

4. DISCUSSION

This calculation using the non-perturbatively improved clover fermion action is capable of resolving the mass splitting between positive- and negative-parity baryon states, and yields a value consistent with experiment. Under the $SU(6) \otimes O(3)$ symmetry of spin-flavour and orbital angular momentum, the low-lying negative parity baryons are assigned to the 70-plet representation. Calculations of the masses of these states both in the quark model and in large $N_C$ suggest that the spin-orbit contribution is surprisingly small, whilst the hyperfine contribution is crucial. The introduction of the clover term to the Wilson fermion action improves the prediction for hyperfine splittings (see e.g. ref. [8]), and this may account for earlier failures to present measurements of the negative-parity masses using the Wilson fermion action. A comparison of
results using the two actions will be included in a later paper[5].

The determination of the masses of spin states above $3/2$ requires the use of non-local baryon operators. Indeed, baryon spins above $5/2$ are contaminated by lower spins in the same irreducible representations of the cubic group[9]. Nevertheless, the \textit{ab initio} determination of the masses of the first few spin/parities of the $N^*$ system seems feasible, with the promise of invaluable information about the nature of QCD and hadronic physics.

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\begin{figure}[h]
\centering
\includegraphics[width=0.4\textwidth]{figure4}
\caption{The effective mass in the $\Delta^{3/2-}$ channel at $\beta = 6.2$ and $\kappa = 0.1351$ for the local-local (crosses) and fuzzed-local (circles) correlators. The curves are from a simultaneous, single-mass fit to both correlators.}
\end{figure}

\begin{figure}[h]
\centering
\includegraphics[width=0.4\textwidth]{figure5}
\caption{The masses of the $\Delta^{3/2+}$ (circles) and $\Delta^{3/2-}$ (diamonds) at $\beta = 6.2$. The lines are linear extrapolations to the chiral limit.}
\end{figure}

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