Study of Low Energy Spin Rotons in the Fractional Quantum Hall Effect

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Motivated by the discovery of extremely low energy collective modes in the fractional quantum Hall effect (Kang, Pinczuk et al.), with energies below the Zeeman energy, we study theoretically the spin reversed excitations for fractional quantum Hall states at $\nu = 2/5$ and $3/7$ and find qualitatively different behavior than for $\nu = 1/3$. We find that a low-energy, charge-neutral “spin roton,” associated with spin reversed excitations that involve a change in the composite-fermion Landau level index, has energy in reasonable agreement with experiment.

Inelastic light scattering has proved to be an extremely useful tool for investigating the neutral excitations of the fractional quantum Hall effect (FQHE). The initial focus was at $\nu = 1/3$, where three modes have been observed: the long wavelength collective mode, the roton mode, and the spin wave mode. The last has an energy, in the small wave vector limit, equal to the Zeeman splitting, denoted below by $E_z$. Recently, substantial progress has been made in extending the experiments to other fractions, e.g., $\nu = 2/5$ and $3/7$, where a much richer structure has emerged. In particular, Kang et al have reported observation of spin-reversed modes other than the spin wave at these fractions. At $\nu = 2/5$, they find a mode with energy approximately equal to $2E_z$; the near absence of $\sqrt{B_\perp}$ dependence of the energy indicates that it is not modified by the Coulomb interaction. Another striking observation was of a mode at $\nu = 3/7$ which has an energy smaller than $E_z$, roughly $0.4E_z$. These findings have prompted us to investigate the spin-reversed neutral excitations at $\nu = 2/5$ and $3/7$. We find that the nature of the low-energy spin reversed excitations is in general qualitatively different than that at $\nu = 1/3$, and involves composite-fermion Landau level transitions in conjunction with spin reversal.

The low energy excitations that conserve spin find a good description in terms of composite fermions. The incompressible ground state at $\nu = n/(2n+1)$ is interpreted as a state in which composite fermions completely fill $n$ composite-fermion (CF) Landau levels. The neutral excitation is a particle-hole pair of composite fermions, i.e. an excition of composite fermions. For fully polarized excitations, the lowest energy exciton corresponds to exciting a composite fermion from the topmost occupied CF-Landau level (LL) into the lowest unoccupied CF-LL; for such excitations, the CF theory is in good agreement with experiment, both for the roton minima at $1/3$, $2/5$, and $3/7$, and for the long wavelength neutral mode at $1/3$. The focus in this work is on spin reversed excitations. At $\nu = 1/3$, which maps into $\nu^* = 1$ of composite fermions, the lowest energy spin reversed mode clearly is the one in which the composite fermion flips its spin while remaining within the lowest CF-LL. For other fractions, the situation is more complicated. Consider, for example, $\nu = 2/5$, where two CF-LLs, labeled 0 and 1, are fully occupied. In the simplest approximation, neglecting excitations involving the third CF-LL, there are three possible low-energy excitations: (i) $\uparrow \rightarrow 0\downarrow$, (ii) $\uparrow \rightarrow 1\downarrow$, and (iii) $0\uparrow \rightarrow 0\downarrow$. The last two conserve the CF-LL index, whereas the composite fermion lowers its LL index in the first. One combination of these must produce the spin wave excitation. It is not a priori obvious which of these is the lowest energy mode. Our aim is to see if theory finds low energy excitations at general fractions that can be identified with the experimentally observed modes.

The relation between the spin reversed excitations of the FQHE and the corresponding integral quantum Hall states is implicit in the composite fermion description of these modes, and has been investigated in the past within the Chern Simons and the Hamiltonian formulations. While our approach below does not produce exactly the same quantitative dispersions as these studies, there are qualitative similarities. The behavior of the three modes at $\nu = 2/5$ is analogous to that for the $\nu = 2$ state, and the spin-wave mode of $1/3$ is qualitatively similar to that of $\nu = 1/5$. In particular, many of the qualitative features of the results below (the nature of the lowest mode and the presence of a spin roton) were obtained in an extensive theoretical study of magnetoexcitations by Murthy.

We evaluate the energy dispersions of the spin-reversed excitations numerically using the wave functions of the composite fermion theory. The method has been described previously and will not be repeated here. The spherical geometry is used in the calculations, where the total orbital angular momentum $L$ is related to the wave vector of the planar geometry as $k = L/R$, where $R = \sqrt{Ql_0}$ is the radius of the sphere, $Q$ is the strength of the monopole at the center (corresponding to a total flux of $2Q\hbar/e$), and $l_0 = \sqrt{\hbar c/eB_\perp}$ is the magnetic length. The thermodynamic ($N^{-1} \rightarrow 0$) values of the Coulomb energies of the spin reversed excitations as well as of the fully polarized ground state are calculated by extrapolating the results for up to $N = 50$ particles, and up to $10^7$ Monte Carlo steps have been used to obtain each energy. The Coulomb energy of the exciton measured relative to the ground state is denoted by $\Delta_{\text{ex}}^\nu$; the Zeeman energy $E_z = |\mu_B B|$ must be added to it to obtain the full energy of the spin-reversed excitation.
FIG. 1. The upper panel gives the dispersions of three spin reversed non-orthogonal modes at \( \nu = 2/5 \) (\( 1^\uparrow \rightarrow 0^\downarrow \), \( 1^\uparrow \rightarrow 1^\downarrow \), and \( 0^\uparrow \rightarrow 0^\downarrow \)) for \( N = 30 \) particles for the Coulomb potential \( V(r) = e^2/\epsilon r \). The ground state is assumed to be fully polarized. The error bars indicate the estimated statistical error in Monte Carlo. The lower panel shows the modes obtained after diagonalization of the Hamiltonian in the properly orthonormalized basis, labeled (a), (b), and (c). The curve (d) in the lower panel shows the energy of the spin reversed excitation \( 0^\downarrow \rightarrow 1^\uparrow \) of the unpolarized \( 2/5 \) state for \( N = 38 \). The lines are a guide to the eye.

We begin with \( \nu = 2/5 \) and compute the energies of the excitations \( 1^\uparrow \rightarrow 0^\downarrow \), \( 1^\uparrow \rightarrow 1^\downarrow \), and \( 0^\uparrow \rightarrow 0^\downarrow \), shown in the upper panel of Fig. 1 for \( N = 30 \). However, the wave functions for three states are not necessarily mutually orthogonal for a given \( L \). This, for example, is the reason why the gapless spin-wave mode is absent. In order to obtain meaningful results, it is necessary to diagonalize the Hamiltonian in an orthonormal basis. The orthonormal basis can be obtained by the Gram-Schmidt procedure in the subspace of three states. The orthonormalization as well as the diagonalization of the Hamiltonian \( H = \frac{1}{2} \sum_{j\neq k} V(r_{jk}) \) requires a calculation of the off-diagonal matrix elements of the type \( \langle u_l|u_m \rangle \) and \( \langle u_l|H|u_m \rangle \), where \( u_l \) and \( u_m \) (\( l \neq m \)) are unorthonormalized wave functions. Since the Monte Carlo evaluation is most efficient when the integrand is positive definite, we determine these using the equation

\[
\langle u_l|u_m \rangle = \frac{\langle u_l + u_m|u_l + u_m \rangle - \langle u_l|u_l \rangle - \langle u_m|u_m \rangle}{2},
\]

and a similar equation for the Hamiltonian matrix elements.

The three Coulomb eigenvalues obtained in this way are shown in Fig. 1 labeled (a), (b), and (c). For \( \nu = 3/7 \), we diagonalize the Coulomb Hamiltonian in the subspace defined by six modes: \( 2^\uparrow \rightarrow 0^\downarrow ; \ 2^\uparrow \rightarrow 1^\downarrow ; \ 1^\uparrow \rightarrow 0^\downarrow ; \ 2^\uparrow \rightarrow 2^\downarrow ; \ 1^\uparrow \rightarrow 1^\downarrow ; \) and \( 0^\uparrow \rightarrow 0^\downarrow \). The resulting spectrum is shown in Fig. 2. (The discrete points correspond to the discrete \( L \) values. Because of certain properties of the single particle eigenstates in the spherical geometry, there is no state at \( L = 0 \) or \( L = 1 \) for the \( 2^\uparrow \rightarrow 0^\downarrow \) transition, and none at \( L = 0 \) for \( 1^\uparrow \rightarrow 0^\downarrow \) and \( 2^\uparrow \rightarrow 1^\downarrow \) transitions.) Of particular interest is the lowest energy excitation, which may be expected to be the sharpest due to the absence of lower energy modes into which it may decay. Given that the ground state wave function is extremely accurate, our results provide a variational upper bound on the energy of this mode. We also note that the energy of the lowest mode at \( \nu = 2/5 \) is in semi-quantitative agreement with that obtained in the Hamiltonian approach.

FIG. 2. The spin reversed excitations of fully polarized \( 3/7 \) state. The results are for the Coulomb potential \( V(r) = e^2/\epsilon r \) for \( N = 39 \) particles.
0↓ at ν = 2/5 and 2↑ → 0↓ at ν = 3/7. The energy of this excitation is surprisingly low, which can be understood physically by noting that the energy increase due to the self energies of the particle and hole is offset to a large degree by the decrease in the CF cyclotron energy. Another noteworthy aspect is that this excitation has a roton minimum, termed the “spin roton”. The presence of the roton is significant because, due to a divergent density of states associated with it, the roton is more readily observable in Raman scattering. (The roton wave vector is much larger than that of light, but it can be activated by disorder, which breaks the translational invariance and hence wave vector conservation. The roton has also been investigated by ballistic phonon scattering.\cite{B})

In the 2D limit, the energies of the spin rotons at ν = 2/5 (for the 1↑ → 0↓ mode) and ν = 3/7 (2↑ → 0↓) are estimated to be −0.0024(18)ε2/ε0 and −0.0091(36)ε2/ε0, respectively. Fig. 3 shows the dependence of the thermodynamic energies on the carrier density for the hetero junction geometry as well as for the quantum well geometry for two typical widths. Only the interaction energy is shown in these figures; the Zeeman energy must be added to obtain the total energy of the excitation.

In order to make contact with experiment, we obtain the thermodynamic limits for the energies of the rotons at 2/5 and 3/7. For this purpose, we will use the un

\[ \Delta_{c} \]  

\[ \text{[e}^{2}/\text{cm}^{-2}] \]

\[ \text{density [10}^{10} \text{cm}^{-2}] \]

\[ 0.015 \]  

\[ 0.005 \]  

\[ 0.000 \]  

\[ -0.005 \]  

\[ -0.010 \]  

\[ -0.015 \]  

\[ 0 \]  

\[ 10 \]  

\[ 20 \]  

\[ 30 \]  

\[ 40 \]  

\[ 50 \]  

\[ 60 \]  

FIG. 3. Thermodynamic energies of the spin rotons of the fully polarized ν = 2/5 and 3/7 states as a function of the carrier density. Both the heterojunction and the square well geometries are considered. The dash-dotted line represents the heterojunction geometry. The square wells width widths 150Å and 300Å are denoted by the solid and dashed lines respectively. The typical Monte Carlo uncertainty is shown on the left.

The thermodynamic energy of the spin roton at 3/7 gets contribution from both the Zeeman and the interaction energies. Such an instability may be expected, because no other such low energy mode is known, and because the calculated energy of the spin roton is in reasonable agreement with the observed energy. (We note that the energy of the spin roton at 3/7 gets contribution from both the Zeeman and the interaction energies. As a result, its dependence on B is expected to have both B and \( \sqrt{B_{\perp}} \) terms.) The observation of a spin roton at ν = 2/5, which may be complicated by its proximity to the much stronger spin wave excitation, will provide additional support to the above physics of the low energy mode at ν = 3/7.

One may also speculate that the 2\( E_{Z} \) mode observed by Kang et al.\cite{A} is a zero-wave-vector two-roton mode, formed from the combination of two spin rotons with equal and opposite momenta. This is analogous to the two-roton mode for the fully polarized long-wavelength excitation.\cite{C} The nearly vanishing interaction energy of this mode is consistent with the linear-B dependence of the energy found experimentally. Again, if this assignment is correct, one may anticipate a two-roton mode at ν = 3/7 as well. Kang et al.\cite{A} have also reported 2\( E_{Z} \) modes at other filling factors, which we do not address in the present work.

Because \( \Delta_{c} \epsilon^{2} \) is negative for the roton, both 2/5 and 3/7 are susceptible to a roton instability at sufficiently small Zeeman energies. Such an instability may be expected, because the \( E_{Z} = 0 \) ground state at these filling factors is not fully polarized. It is interesting to compare the lines of instability that follow from these results with the spin phase diagram of the FQHE states obtained in an earlier study.\cite{D} (The lines of instability do not necessarily coincide with phase boundaries.) In the zero thickness limit, the above results imply that the unpolarized 2/5 state is unstable for the densities greater than \( \sim 8.8(0.3) \times 10^{10} \text{ cm}^{-2} \) (assuming \( B = B_{\perp} \)) and the fully polarized state is...
unstable for the densities less than $\sim 1.6(0.9) \times 10^9$ cm$^{-2}$. There is thus a range of densities for which both the unpolarized and fully polarized states are stable against quantum fluctuations. This suggests that the transition is first order, occurring before the roton energy vanishes.

For completeness, we have also considered excitations of the spin-unpolarized ground state of $\nu = 2/5$, in which both spin states of lowest CF-LL are fully occupied. The unpolarized ground state is relevant at sufficiently small Zeeman energies. The lowest energy excitation, namely $0\downarrow \rightarrow 1\uparrow$, always involves a spin-reversal. (In this case, the Zeeman energy $E_z$ must be subtracted from the interaction energy to obtain the full energy.) The dispersion for this excitation is shown in Fig. 1 (curve d in the lower panel). It also has a roton minimum, with energy 0.0178(34) $c^2/\epsilon l_0$ in the thermodynamic limit.

To summarize, we have studied spin reversed excitations at $\nu = 2/5$ and 3/7 and found qualitatively different behavior compared to $\nu = 1/3$. Our results provide a possible explanation for the low energy modes observed in recent light scattering experiments in terms of spin rotons.

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