Conceal an entrance by means of superscatterer

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Abstract

By using the novel property of the rectangular superscatterer, we propose a design which can conceal an entrance from electromagnetic wave detection. Such a superscatterer is realized by coating a negative index material shell on a perfect electrical conductor rectangle cylinder. The results are numerically confirmed by full-wave simulations both in the far-field and near-field.

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Electromagnetic (EM) metamaterials \[1, 2\] are artificially engineered materials with sub-wavelength composites, their effective parameters of permittivity and permeability are desirable and can attain more wider ranges than those in natural materials. The progress in metamaterial fabrication provides more freedom for designing striking devices, such as perfect lens \[3, 4\], cloaks \[5, 6, 7, 8, 9, 10\], hyperlens \[11, 12, 13, 14\] and other invisibility devices \[16, 17, 18\]. These devices seem unphysical or unattainable before, because it needs very special permittivity and permeability tensors, for example, the negative index material (NIM) \[3, 21\].

More recently, a new device called superscatterer \[24\] was proposed by means of the concept of complementary media \[19\]. In EM wave detection, it looks like a scatterer bigger than the physical size of the device. Since the outer region in the pair of complementary media is just the vacuum or host medium, it might be regarded as a kind of building blocks used to construct more complex objects, for example, a penetrable barrier or a closed two dimensional (2D) box, which are opaque by EM waves but penetrable by small particles. However, the building blocks need to be nicely fitted together to obtain the effect so that the circular shaped blocks may be inapplicable. In this letter, we design a 2D rectangular cylindrical superscatterer and demonstrate its interesting applications in concealing entrances.

For simplicity, let’s consider a perfect electrical conductor (PEC) wall \(|y| \leq b_3\) in vacuum, any incident EM waves will be reflected from it. If we remove the material in \(|x| \leq a_3 + \delta\) from the wall, an entrance appears, and it is inescapably detectable by analyzing the scattering EM waves. However, superscatterer provides a possibility that the entrance is kept but can not be detected by EM waves. Fig. 1 is the schematic demonstration and the total procedure is displayed as follows. Firstly, if we only remove the PEC materials in \(a_3 \leq |x| \leq a_3 + \delta\), it still looks like a wall when the non-negative parameter \(\delta\) tends to zero. Secondly, we compress the big rectangular cylinder, \(|x| \leq a_3\) and \(|y| \leq b_3\), into a small rectangular cylinder, \(|x| \leq a_1\) and \(|y| \leq b_1\). Now, the surfaces of these rectangular cylinders are denoted as \(\Gamma_3\) and \(\Gamma_1\), respectively. The material parameters of the compressed cylinder are obtained by the corresponding coordinate transformation. But in this case, it is a PEC cylinder that we compressed, so the PEC boundary condition is also kept in the surface \(\Gamma_1\). Finally, in order to conceal the entrance in EM detection, the gap between \(\Gamma_1\) and \(\Gamma_3\) should be filled by a pair of complementary media: One is the vacuum between \(\Gamma_3\) and \(\Gamma_2\), the other is the NIM shell between \(\Gamma_2\) and \(\Gamma_1\). Here \(\Gamma_2\) is the outer rectangular surface of the
FIG. 1: The schematic demonstration of how to design an electromagnetic wave concealed entrance by rectangular superscatterer with a pair of complementary media. The dashed rectangular is the domain to be concealed from electromagnetic detection, the smaller rectangular within solid line is the physical size of the superscatterer.

NIM shell with dimensions $2a_2$ and $2b_2$.

This device is a superscatterer and scatters the same fields as the uncompressed rectangular cylinder, so that the entrance between $\Gamma_2$ and $\Gamma_3$ is concealed in EM detection. The effective rectangular cylinder with surface $\Gamma_3$ is also called a virtual rectangular cylinder. Although the surface of $\Gamma_2$ can be any shape bounded between $\Gamma_1$ and $\Gamma_3$, here we only consider a simple shape: In the $x$-$y$ plane, $\Gamma_1$, $\Gamma_2$ and $\Gamma_3$ are rectangles with the same center and diagonals. In addition, the magnification factor $\eta$ is defined as $a_3/a_2$ or $b_3/b_2$, which is the ratio of the size of the virtual cylinder to the real size of this device.

According to the discussion in [24], the parameters of complementary media can be obtained by the approach of the transformation optics[8]. Here, we consider a simple continuous map between the pair of complementary media: The region of vacuum is mapped to the region of NIM shell, especially, the surface $\Gamma_3$ is mapped to $\Gamma_1$ but the surface $\Gamma_2$ is mapped to itself. For the 2D rectangular cylindrical superscatterer, the NIM shell is separated to 4
regions as shown in Fig. 1, and the coordinate transformation equations for region I are
\[
\begin{align*}
x' &= -a_2 - a_1 \cdot x + a_3 - a_1 \cdot a_2, \\
y' &= -a_2 - a_1 \cdot y + a_3 - a_1 \cdot a_2 \cdot y \left/ a_3 - a_2 \cdot x \right., \\
z' &= z.
\end{align*}
\]
(1)

The relations of \(a_1 < a_2 < a_3\) provide a folded geometry \([22, 23]\), so the NIM appears. In addition, it becomes the usual rectangular concentrator if the relations are \(a_1 < a_3 < a_2\), or it describes a rectangular cloak if \(a_1 < a_2\) and \(a_3 = 0\). For comparison, the similar transformation equations for square cloak can be found in paper \([17]\).

In the new coordinate system, using the Jacobian transformation matrix
\[
\Lambda_i' = \frac{\partial x_i'}{\partial x_i},
\]
(2)
the components of relative permittivity tensor \(\varepsilon'\) and relative permeability tensor \(\mu'\) \([15]\) can be put in the following form,
\[
\begin{align*}
\varepsilon'^{ij} &= \left[\det \Lambda\right]^{-1} \Lambda_i' \Lambda_j' \delta^{ij}, \\
\mu'^{ij} &= \left[\det \Lambda\right]^{-1} \Lambda_i' \Lambda_j' \delta^{ij},
\end{align*}
\]
(3)
where \(\delta^{ij}\) is equal to 1 for \(i = j\) and equal to 0 otherwise. By straightforward calculations, the relative permittivity and permeability tensors in region I are obtained as follows,
\[
\varepsilon' = \mu' = \begin{pmatrix} \Delta_1 & \Delta_2 \\ \Delta_2 & 0 \end{pmatrix} \begin{pmatrix} \Delta_1 & 0 \\ 0 & \Delta_2 \end{pmatrix},
\]
(4)
where
\[
\begin{align*}
\Delta_1 &= \frac{(a_3 - a_2)^2 x' - (a_3 - a_2)(a_3 - a_1) a_2}{x'^2}, \\
\Delta_2 &= \frac{(a_3 - a_1) a_2 y'}{(a_3 - a_2) x'^2}.
\end{align*}
\]
(5)

Due to the symmetry of this device, the material parameters \(\varepsilon'^{ij}\) and \(\mu'^{ij}\) in the region III can be readily obtained by rotating the tensors in (4) by \(\pi\) around the \(z\)-axis. But for the regions II and IV, besides of rotating the tensors in (4) by angle \(\pi/2\) and \(3\pi/2\), we need replace all \(a_k\) with \(b_k\), \(k = 1, 2, 3\). Here, it should be remarked that the obtained material parameters are continuous at the interfaces among these regions. Subsequently, the materials interpretation \([15]\) means the tensors in (4) and its counterparts in other regions.
FIG. 2: Snapshot of the total $E_z$ field around the superscatterer concealed entrance. The electric field was totally reflected so that the electric field on the other side of the entrance is barely zero.

just describe the material parameters of rectangular superscatterer, if we substitute the primed indices by unprimed indices.

In order to demonstrate the performance of the designed device in Fig. 1, full-wave simulations are performed by finite element solver of the Comsol Multiphysics software package. Here the device parameters \{a_1, a_2, a_3\} and \{b_1, b_2, b_3\} are \{0.03m, 0.06m, 0.12m\} and \{0.015m, 0.03m, 0.06m\}, respectively, so that the magnification factor $\eta$ is equal to 2. In this case, we consider 2D transverse electric (TE) polarization incident waves (whose electric field is along $z$-axis) with frequency 7 GHz, it means the distances between device and walls, or the width of entrance, are about $1.4\lambda$ where $\lambda$ is the wavelength. The ranges of material parameters along their local principle axis are taken as follows: $\mu_x, \mu_y \in [-20.5, -0.05]$, and $\epsilon_z \in [-8, -2]$, which are finite and in a range of possible fabrication. A tiny absorptive imaginary part ($\sim 10^{-12}$) is added to $\mu$ and $\mu$ due to the inevitable losses of the NIM.

We first simulate the case of plane wave incidence with unit amplitude. It is interesting to calculate the scattering properties of a penetrable barrier, which is a layer of 2D photonic crystal and the primitive cell is the superscatterer and a rectangular PEC cylinder. In the
FIG. 3: Snapshot of the distribution of the total electric field with a line source. (a) The total electric field distribution when a PEC wall is present. (b) The total electric field distribution when an entrance is concealed by the superscatterer. (c) - (d) The total electric fields with an entrance unblocked and blocked by a rectangular PEC cylinder with the same physical size of the superscatterer, respectively.

$x$-$y$ plane, the scale of the virtual cylinder is $0.24\text{m}$ $\times$ $0.12\text{m}$ and those of PEC cylinder is $0.06\text{m}$ $\times$ $0.12\text{m}$. The primitive cell is shown in Fig. 2 and the pattern of electric field is calculated by the finite element solver with periodic boundary condition at $x = \pm 0.15\text{m}$. One can find the superscatterers do work as building blocks and the layer of 2D photonic crystals looks like a mirror (a PEC wall) although it is penetrable for small particles. Here, the bounds of the amplitude of electric field in Fig. 2 set from $-2$ to $2$ for clarity. The white flecks show the regions where the value of fields exceed the bounds. It appears in the complementary media and the highest value of the field in the flecks is about $10^1$, which comes from the dominative high-$m$ modes as in the circular version [24].

We also investigated the near-field performance of the device by cylindrical wave inci-
dence. In this case, there is a concealed entrance in the wall, and the electric line source carries a current of 0.001A in z-axis and located at (−0.105m, 0.26m). The surrounding regions are perfect match layer (PML) regions, and an object with PEC boundary is placed under the wall. Fig. 3(a) is the snapshot of total electric field induced by a PEC wall, and Fig. 3(b) demonstrates how the entrance is concealed by a rectangular superscatterer in EM detection. For comparison, we have also simulated the results of removing the device (See in Fig. 3(c)), or replacing the device with a rectangular cylinder with PEC boundary, which has the same dimensions as the device (See in Fig. 3(d)). There have some white flecks, in which the value of fields exceeds the bounds [−5V/m, 5V/m], around electric line source, because the field trending to source will increase quickly. Other white flecks appears in the complementary media and the highest value of the field in the flecks is about $10^1 V/m$. Comparing the patterns of electric field in Fig. 3, one can find the entrance is indeed concealed in EM detection.

The concealed entrance is a successful case that rectangular superscatterer can be regarded as building blocks, in which the virtual rectangular cylinder is nicely fitted to the left and right walls. More wider concealed entrance can be obtained if a larger magnification factor $\eta$ is taken. Consequently, in the region of microwave frequency, it is obvious that such a striking device as “platform nine and three-quarters” at King’s Cross Station, where Harry Potter and his friends usually get on the train to Hogwarts, is realizable. However, when another object is placed in the virtual cylinder, the transformation optical approach can not tell us what happens. In Fig. 4, we have simulated numerically the results when an object with PEC boundary traverses the concealed entrance. Comparing the patterns of electric field in the virtual cylinder and under the walls, we find the concealment effect is destroyed more or less when object is overlapped with the virtual cylinder.

In conclusion, we proposed a rectangular cylindrical superscatterer and demonstrated its concealment effect on an entrance in EM detection. The device performance are numerically confirmed by full-wave simulations both in the far-field and near-field. This kind functional devices can be regarded as special building blocks, which are opaque by EM waves but penetrable by small particles, and may be useful in concealing important entrance.

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FIG. 4: The total electric field distribution when an object with PEC boundary go through the superscatterer concealed entrance, the property of total reflection is broken during the object pass through the entrance.

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