Luttinger liquid coupled to Bose-Einstein condensation reservoirs

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We investigate the transport properties for a Luttinger liquid coupled to two identical Bose-Einstein condensation reservoirs. Using the approach of equation of motion for the Green function of the system, we find that the distance between the two resonant transmission probability peaks of the system is determined by the bosonic interaction strengths, and the sharpness of these resonant peaks is mainly determined by the Rabi frequency and phase of the Bose-Einstein condensation reservoir. These results for the proposed system involving a Luttinger liquid may build a bridge between the controling transport properties of cold atom in atom physics and the interacting boson transport in low-dimensional condensed matter physics.

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I. INTRODUCTION

The physics of ultracold one-dimensional (1D) Bose system is very different from that of ordinary three-dimensional cold gases [1, 2]. The possibility of Bose-Einstein condensation (BEC) in one dimension has been discussed for the noninteracting Bose gas [3]. The interaction between bosons plays an essential role due to the strong constraint in phase space in 1D case. Monien et al. [4] have shown that a trapped quasi-one-dimensional system of interacting bosons under the experimental conditions can be described by a Luttinger liquid (LL) Hamiltonian. As is known that the low-energy physics of 1D single channel conductors can not be described by the Fermi liquid theory if the particle-particle interactions are taken into account [5]. Such system falls into the so-called LL regime. However, unlike the Fermi liquids, the LL liquids may also include 1D interacting bosonic systems. Bosonic systems can display fermion-like properties and vice versa [6, 7, 8]. One well-known example in the field of cold atoms is the behavior of the Tonks-Girardeau gas [9], where the bosons interact so strongly that they effectively behave as free fermions.

In LL model theory, the main assumption is the linearization of the free-particle dispersion relation near the eigenenergy points of the system. Fermionic systems which are believed to be described by the LL model include quasi-one-dimensional organic metals [10], quantum wires [11, 12], and edge states in the quantum Hall system [13]. The actual system considered is finite length and are attached to two identical reservoirs at its end points. This is to say, these systems are always embedded in a three-dimensional matrix. So they will show a crossover to three-dimensional behavior at low temperature, while the trapped 1D Bose gas would provide a clean testing ground for the concept of LL model.

In nanoelectronics the control of electron quantum wires or quantum dots is performed by the biased conducting leads attached to them. But in nanobosonics the role of the “leads” is replaced by the finite superfluid reservoirs (given particle numbers) which can be coupled to a particular atom by optical transitions. With regard to this field, the dynamics of an atomic quantum dot coupled to a BEC reservoir via laser transitions has also been studied recently [14, 15].

In this paper, we consider a system consisting of a LL coupled to two identical BEC reservoirs. The bosons in the BEC reservoirs are confined in a shallow trap, while the atom in the LL is confined in a very tight potential. Atoms in both the LL and BEC reservoirs correspond to the different internal atomic states connected by Raman transition with Rabi frequency \( \omega \) and detuning \( \delta \). Using the approach of standard equation of motion for Green function (GF), we investigate the frequency-dependent transport properties for this system. Our results show that the distance between the two resonant transmission probability peaks is determined by the interaction strengths, and the sharpness of the resonant peak is mainly determined by the Rabi frequency and the phase of the BEC reservoir. The results for the proposed system involving a LL may build a bridge between the atomic transport in atomic physics and the interacting electron transport in low-dimensional condensed matter physics.

II. MODEL

The total Hamiltonian of the system consists of three parts, i.e.,

\[
H = \sum_{a=L,R} H_a + H_{cen} + H_T,
\]

where \( H_a \) is the Hamiltonian for the isolated left or right BEC reservoir, \( H_{cen} \) is the Hamiltonian of the isolated LL, and \( H_T \) is the Hamiltonian describing the transfer of a particle from the BEC to the LL.

The starting point for the calculations of \( H_a(\alpha = L,R) \) is the Hamiltonian

\[
H_a = \int dr [-\psi_{a}^*(r) \frac{1}{2m} \nabla^2 \psi_{a}(r) + V(r) \psi_{a}^*(r) \psi_{a}(r)]
\]
where $\psi_a(r)$ and $\psi^\dagger_a(r)$ are annihilation and creation operators for bosons in the BEC reservoir respectively, $m$ is the atomic mass, $V(r)$ is the potential confining bosons system in a trap, and $U(r-r')$ is the interaction potential between two particles in the BEC reservoir. (We have adopted the unit of $\hbar=1$ throughout this paper.) To take into account the quantum fluctuations of the state in which all the atoms are condensed in a single quantum state, the operator $\psi_a(r)$ can be represented in the form $\psi_a(r)=\sqrt{N_0}\delta\varphi_0+\delta\varphi_a(r)$, where $N_0$ is the particle number in the zero-momentum state, $\varphi_0$ is the wave function of the condensed state, and $\delta\varphi_a(r)$ denotes the fluctuation operator of momentum $k\neq 0$, i.e. the excitation above the ground state. Within the Bogoliubov approach one assumes that $\delta\varphi_a(r)$ is small and retains in the interaction all terms which have two powers of $\psi_a(r)$ or $\psi^\dagger_a(r)$. This is equivalent to including terms which are no more than quadratic in $\delta\varphi_a(r)$ or $\delta\varphi^\dagger_a(r)$.

Performing the Fourier transformation
\[
\delta\varphi_a(r) = \sum_k a_{a,k} e^{ikr},
\]
replacing $\delta\varphi_a(r)$ and $\delta\varphi^\dagger_a(r)$ by $a_{a,k}$ and $a^\dagger_{a,k}$ for $k \neq 0$, we obtain
\[
\begin{align*}
H_a &= \sum_{k(k\neq 0)} \left[ (\epsilon_{a,k}^0 + U_1(k))(a_{a,k}^\dagger a_{a,k} + a_{a,-k}^\dagger a_{a,-k}) \\
&+ U_2(k)(a_{a,k}^\dagger a_{a,-k} + a_{a,k} a_{a,-k}) \right],
\end{align*}
\]
where $\epsilon_{a,k}^0$ is the single particle energy, and $U_1(k)$ ($U_2(k)$) is the Fourier transformation of $U(r-r')$. Here the operators $a_{a,k}$ and $a_{a,k}^\dagger$ are destroy and create bosons in the state with momentum $k$ satisfy the usual Bose commutation relations.

We take the effect when two atoms are close to each other into account by using the effective interaction, and the Hartree-Fock terms are both equal to $n_0U_0$ in which $n_0$ is the number density of the BEC, $U_0$ the contact interaction in Hartree-Fock approximation, so the Hamiltonian for the isolated left or right BEC reservoir reads [16]
\[
H_a = \sum_{k(k\neq 0)} \left[ (\epsilon_{a,k}^0 + n_0U_0)(a_{a,k}^\dagger a_{a,k} + a_{a,-k}^\dagger a_{a,-k}) \\
+ n_0U_0(a_{a,k}^\dagger a_{a,-k}^\dagger + a_{a,k} a_{a,-k}) \right],
\]
where the single particle energy $\epsilon_{a,k}^0 = k^2/(2m)$, and $a_{a,k}^\dagger (a_{a,k})$ is the creation (annihilation) operator of the bosons in the left or right reservoir. Note that the prime on the sum indicates that it is to be taken only over one half of momentum space.

Considering the Raman detuning between LL and the reservoir and the phase of the reservoir, the Hamiltonian of the isolated left or right BEC reservoir, i.e., the first term $H_a$ ($\alpha = L, R$) of Eq. (1), is given by
\[
H_a = \sum_{k(k\neq 0)} \left[ (\epsilon_{a,k} + |\Delta|)(a_{a,k}^\dagger a_{a,k} + a_{a,-k}^\dagger a_{a,-k}) \\
+ (|\Delta| a_{a,k}^\dagger a_{a,-k} + |\Delta|^* a_{a,k} a_{a,-k}) \right],
\]
where $\epsilon_{a,k} = \epsilon_{a,k}^0 + \delta\alpha$ in which $\delta\alpha$ is the Raman detuning between LL and the reservoir $\alpha$; $\Delta = n_0U_0e^{i\phi_0}$ in which $\phi_0$ is the phase of the reservoir $\alpha$. In the absence of the drive ($\delta\alpha = 0$), the particles can also pass through the system because of the phase difference between two BEC reservoirs.

For the Bose gas in a cylindrical symmetric trap confined to the $z$ axis by a tight trapping potential in $x$-$y$ plane, if the extension $L$ of the trap in the $z$ direction is much larger than its radius $R$ and the temperature is much lower than the energy of the lowest radial excitation, the ground state is described by a LL [4]. The starting point for the calculations of $H_{cen}$ is also the Hamiltonian
\[
H_{cen} = \int dr [-\psi_1^\dagger(r) \nabla^2 \psi_1(r) + V(r)\psi_1^\dagger(r)\psi_1(r)] \\
+ \frac{1}{2} \int dr \int dr' \psi_1^\dagger(r)\psi_1^\dagger(r')U(r-r')\psi_1(r)\psi_1(r'),
\]
where $\psi_1(r)$ and $\psi_1^\dagger(r)$ are annihilation and creation operators for bosons in the LL respectively, $V(r)$ is the potential confining bosons system in a trap, and $U(r-r')$ is the interaction potential between two particles in the LL. Through the same procedures as above, the operator $\psi_1(r)$ can be represented in the form $\psi_1(r)=\sqrt{N_0}\varphi_0+\delta\varphi_1(r)$. And the fluctuation operator
\[
\delta\psi_1(r) = \sum_q b_q e^{iqr},
\]
where the operators $b_q$ and $b_{-q}^\dagger$ that destroy and create bosons in the state with momentum $q$ satisfy the usual Bose commutation relations
\[
[b_q, b_{-q}^\dagger] = \delta_{q,q'}, \quad [b_q, b_{-q}^\dagger] = 0, \quad [b_q^\dagger, b_{-q}^\dagger] = 0.
\]
Using the Bogoliubov approach and replacing $\delta\varphi_1(r)$ and $\delta\varphi^\dagger_1(r)$ by $b_q$ and $b_{-q}^\dagger$ for $q \neq 0$, we obtain
\[
H_{cen} = \sum_{q(q\neq 0)} \left[ (\epsilon_q^0 + U_1(q))(b_q^\dagger b_q^0 + b_{-q}^\dagger b_{-q}^0) \\
+ U_2(q)(b_q^\dagger b_{-q}^\dagger + b_q b_{-q}) \right],
\]
where $\epsilon_q^0$ is the single particle energy, and $U_1(q)$ ($U_2(q)$) is the Fourier transformation of $U(r-r')$.

It is known that in the LL model, the main assumptions are: (1) the linearization of the dispersion relation; (2) only small momenta exchanges included. In terms of the two assumptions, the Hamiltonian of the isolated LL, i.e., the second term $H_{cen}$ of Eq. (1), is given by [17]
\[
H_{cen} = \sum_{q\neq 0} \left[ V_1(q + k_L) + \frac{V_2}{2\pi}(b_q^\dagger b_q^0 + b_{-q}^\dagger b_{-q}^0) \\
+ \frac{V_2}{2\pi}(b_q^\dagger b_{-q}^\dagger + b_q b_{-q}) \right].
\]
Here the single particle energy $\epsilon_q^0 = V_2(q + k_L)$ because of the linearization of the dispersion relation. In Hamiltonian (11), $b_q^\dagger (b_q)$ is the creation (annihilation) operator of the bosons in
the LL, $v_L$ is the eigen-velocity in the channel, $k_L$ is the eigen-wavevector, and $V_j$ ($j = 2, 4$) is the interaction potential when $q = 0$, different values of $V_j$ may represent the interaction potential between the left- and right-moving boson branches, while $V_4$ represents the interaction potential within a momentum branch.

Note that the LL arisen in our system describes Bose system, so the operators $b_q$ and $b_q^\dagger$ correspond to the destruction or creation of an individual particle (i.e., boson). However, when the LL describes the interacting electrons in one dimension, an individual particle is a fermion. By means of the bosonization technique, we can also write the Hamiltonian in a pair of conjugate operators $q$, where $a$, $b$, and $c$ are boson operators. But the operators $b_q$ and $b_q^\dagger$ are linear combination of the density fluctuations $\rho_q = \sum_k a_k^\dagger e^{i q k} a_k$, so they conserve the number of fermion particle and do not correspond to the destruction or creation of an individual particle.

The Bose field can also be describe by its density-phase representation: $\psi_{g}(r) = \sqrt{\rho(r)}e^{i \delta \varphi(r)}$. Expanding in small fluctuation of the phase $\delta \varphi$ and the density $\rho$ around the saddle point solution, $\psi_{g}(r) = \sqrt{\rho_0} + \delta \varphi(r) e^{i (\delta \varphi(r) + \delta \varphi_0)}$. The density fluctuation operator $\delta \rho$ and the phase fluctuation operator $\delta \varphi$ form a pair of conjugate operators $[\delta \rho(q), \delta \varphi(z)] = i \delta (z - q)$. With the same approximation as the equations of motion in Ref [4], we can also express the Hamiltonian of the isolated LL as (Eq. (10) in Ref. [4])

$$ H_{cen} = \int dz \left[ \frac{\rho}{2m} (\partial_z \delta \varphi)^2 + \frac{\kappa}{2 \rho^2} (\varphi^2 - \varphi_0^2) \right], $$

(12)

where $\rho$ is the number of particles per unit length, $m$ is the atomic mass, and $\kappa$ is the compressibility.

The starting point for the calculations of $H_T$ which describes the transfer of a particle from the BEC to the LL, is the Hamiltonian

$$ H_T = \Omega_L \psi_1^\dagger(r_1) \psi_1(r_1) + \Omega_R \psi_1^\dagger(r_2) \psi_2(r_2) + \Omega_L \psi_1^\dagger(r_1) \psi_1(r_1) + \Omega_R \psi_1^\dagger(r_2) \psi_1(r_2), $$

(13)

where $\Omega_\alpha (\alpha = L, R)$ is Rabi frequency. Here we have assumed that the atom in the LL is coupled to atoms in the reservoir $\alpha$ via Raman transition with Rabi frequency. Using the Bogoliubov approach and replacing $\delta \psi_\alpha (r)$ and $\delta \phi_\alpha (r)$ by $a_{\alpha,k}^\dagger$ and $a_{\alpha,k}$ for $\alpha \neq 0$, we obtain

$$ H_T = \Omega_L \sum_k e^{-i k r_1} a_{L,k}^\dagger \delta \psi_1(r_1) + \Omega_R \sum_k e^{i k r_2} a_{R,k} \delta \psi_1(r_2), $$

(14)

where $\delta \phi_\alpha (r) = \delta \phi_0^\alpha + \delta \phi_\alpha (r) = \sqrt{\rho_0} + \delta \phi_\alpha (r) e^{i (\delta \varphi_0 + \delta \varphi_\alpha (r) + \delta \varphi(r))}$, if replacing $\delta \psi_\alpha (r)$ by $e^{i \delta \phi_\alpha (r)}$, the Hamiltonian (14) becomes

$$ H_T = \Omega_L \sum_k e^{-i k r_1} a_{L,k}^\dagger \delta \phi_1(r_1) + \Omega_R \sum_k e^{i k r_2} a_{R,k} \delta \phi_1(r_2), $$

(15)

And replacing $\delta \psi_1 (r)$ and $\delta \phi_1 (r)$ by $b_q$ and $b_q^\dagger$ for $q \neq 0$, the Hamiltonian (14) becomes

$$ H_T = \sum_{k,q} (\Omega_q a_{\alpha,k}^\dagger b_q + \Omega_q^* b_q^\dagger a_{\alpha,k}). $$

(16)

Because the operator $a_{\alpha,k}^\dagger$ correspond to a creation of a particle in the BEC reservoir and the operator $b_q$ correspond to a destruction of a particle in the LL, terms such as $b_q a_{\alpha,k}$ would thus correspond to a destruction of a particle in the BEC and a creation of particles in the LL.

However, for a fermionic LL, if there is transfer of a particle from the BEC reservoirs to the fermionic LL, it must have a different form

$$ H_T = \Omega_L \psi_1^\dagger(r_1) \psi_1(r_1) + \Omega_R \psi_1^\dagger(r_2) \psi_2(r_2) + \Omega_L \psi_1^\dagger(r_1) \psi_1(r_1) + \Omega_R \psi_1^\dagger(r_2) \psi_1(r_2), $$

(17)

where $\psi_\alpha = \sum c_q e^{i q r}$ is the Fermi annihilation operator. Using the bosonization technique, the Fermi annihilation operator can be written as $[18]

$$ \Psi_F (r) \sim \sum_{p=1} \sum q \exp(i \theta (r) + \phi (r)) \sim \exp(i \sum q (e^{i q r} b_q + h.c.)), $$

(18)

where $\theta (r) \text{ and } \phi (r)$ obey the commutation relations $[\psi (r), \phi (r') = \theta (r) \delta (r - r') \phi (r') - \phi (r') \theta (r') \phi (r')] = i \theta (r - r') $. Note that the total Hamiltonian $H$ in our system is equal to the sum of Eq. (6), Eq. (11) and Eq. (16).

### III. FORMULATION

In terms of the Heisenberg equation of motion, the current of the reservoir $\alpha$ can be written as

$$ I_{\alpha,k} (t) = \frac{d N_{\alpha,k}}{dt} = -i \langle [N_{\alpha,k}, H] \rangle = -i \langle \sum_{kq} (\Omega_q a_{\alpha,k}^\dagger b_q + h.c.) \rangle, $$

(19)

where $N_{\alpha,k} = \sum a_{\alpha,k}^\dagger a_{\alpha,k}$ is the total number operator for the boson in the reservoir $\alpha$. Defining a $2 \times 2$ GF $G_{q,k \alpha}^\leq (t)$, then the current of the reservoir $\alpha$ becomes

$$ I_{\alpha,k} (t) = -i \sum_{kq} (G_{q,k \alpha}^\leq (t))_{11} + h.c., $$

(20)

where $(G_{q,k \alpha}^\leq (t))_{11} = \langle a_{\alpha,k}^\dagger b_q \rangle$ is the element in the first row and the first column of GF $G_{q,k \alpha}^\leq (t)$. Similarly,

$$ I_{\alpha,k} (t) = -i \sum_{kq} (G_{q,k \alpha}^\geq (t))_{22} + h.c., $$

(21)

where $(G_{q,k \alpha}^\geq (t))_{22}$ is the element in the second row and the second column of GF $G_{q,k \alpha}^\geq (t)$. Since the current is conserved,
the currents of the bosons with momentum $k$ and $-k$ are equal, i.e., $I_{\alpha,k} = I_{\alpha,-k}$.

Using the theorem of analytic continuation, we have

$$G^r_{\alpha,k} (t,i) = G^r (t,t_i) \Omega^\alpha_{\alpha,k} g^r_{\alpha,k} (t_i,i) + G^r (t,t_i) \Omega^\alpha_{\alpha,k} g^r_{\alpha,k} (t_i,i), \quad (22)$$

where $G^c/r$ is 2x2 lessor/retarded GF of LL with coupling between the LL and the reservoir, while $g^r_{\alpha,k}$ is 2x2 lessor/advanced GF of the isolated BEC reservoir, respectively. The GF $g^r_{\alpha,k} (E)$ in Fourier space is given by

$$g^r_{\alpha,k} (E) = \left[ g^{a}_{\alpha,k} (E) - g^{a}_{\alpha,k} (E) \right] f_a (E), \quad (23)$$

where $f_a (E) \ (\alpha = L, R)$ is the Bose distribution function. And based on the Landauer-Buttiker formula [19, 20, 21], the current in the Fourier space can be written as

$$I_{\alpha} = - \int_{\delta}^{\infty} \frac{dE}{2\pi} \left[ f_L (E) - f_R (E) \right] T (E), \quad (24)$$

where $E$ is the energy of the incident boson from the reservoir $\alpha$. And with the help of Keldysh equation: $G^r = G^r \Sigma^\alpha G^a$, the transmission probability $T (E)$ is solved as

$$T (E) = 4 \text{Tr} \left[ \Sigma^r (E-\delta_L) \sum_q G^r (E) \Sigma^\alpha_q (E-\delta_R) \sum_q G^a (E) \right]. \quad (25)$$

Here $\Sigma^r/\Sigma^c$ is lessor/retarded self energy, respectively. The retarded self energy is given by $\Sigma^r = \sum_{\alpha} \Sigma^r_\alpha$, where $\Sigma^r_\alpha$ is retarded self energy of reservoir $\alpha$ and defined as

$$\Sigma^r_\alpha = \sum_{k} t^*_r_{k,\alpha} g^r_{k,\alpha} t^*_r_{k,\alpha}, \quad (26)$$

where

$$t^*_r_{k,\alpha} = \begin{pmatrix} \Omega & 0 \\ 0 & -\Omega^* \end{pmatrix}, \quad (27)$$

and $g^r_{k,\alpha}$ is the retarded GF of the isolated left or right BEC reservoir and defined as

$$g^r_{k,\alpha} (t,i) = -i\theta (t-i) \times \left\{ \begin{pmatrix} \langle \{a_{\alpha,k}(t), a^\dagger_{\alpha,k}(i)\} \rangle \\ \langle \{b_{\alpha,k}(t), b^\dagger_{\alpha,k}(i)\} \rangle \end{pmatrix} \right\} \quad (28)$$

In terms of the equation of motion for the GF, $g^r_{\alpha,k}$ in the Fourier space can be written as

$$g^r_{\alpha,k} = \frac{1}{E^2 - \epsilon_{\alpha,k}^2 - 2|\Delta|\epsilon_{\alpha,k}} \begin{pmatrix} E + \epsilon_{\alpha,k} + |\Delta| & -\Delta \\ -\Delta^* & -E + \epsilon_{\alpha,k} + |\Delta| \end{pmatrix}, \quad (29)$$

Defining $g^r_{\alpha,k} = \sum_{k} g^r_{\alpha,k}$, by straightforward calculation, we obtain

$$g^r_{\alpha} = \rho_{\alpha} \begin{pmatrix} E & -\Delta \\ -\Delta^* & -E \end{pmatrix} \int \frac{d(\epsilon_{\alpha,k} + |\Delta|)}{(E + i0^+)^2 - (\epsilon_{\alpha,k} + |\Delta|)^2 + |\Delta|^2} + \rho_{\alpha} \begin{pmatrix} \epsilon_{\alpha,k} + |\Delta| & 0 \\ 0 & \epsilon_{\alpha,k} + |\Delta| \end{pmatrix} \int \frac{d(\epsilon_{\alpha,k} + |\Delta|)}{(E + i0^+)^2 - (\epsilon_{\alpha,k} + |\Delta|)^2 + |\Delta|^2} \quad (30)$$

Here we have changed $\sum_{k}$ into an integral $\int d\epsilon_{\alpha,k} \rho_{\alpha}$ with the help of the density of the states in the BEC reservoir $\rho_{\alpha}$. The second term of Eq. (30) vanishes, because in the second term both $g^r_{\alpha,11}$ and $g^r_{\alpha,22}$ are odd functions of $\epsilon_{\alpha,k} + |\Delta|$. So we finally obtain

$$g^r_{\alpha} = \rho_{\alpha} \begin{pmatrix} E & -\Delta \\ -\Delta^* & -E \end{pmatrix} \int \frac{d(\epsilon_{\alpha,k} + |\Delta|)}{(E + i0^+)^2 - (\epsilon_{\alpha,k} + |\Delta|)^2 + |\Delta|^2} \quad (31)$$

In the following, calculating the integral by using the residual theorem, Eq. (31) can be reduced as

$$g^r_{\alpha} = \rho_{\alpha} \frac{iv\pi}{\sqrt{E^2 + |\Delta|^2}} \begin{pmatrix} E & -\Delta \\ -\Delta^* & -E \end{pmatrix}, \quad (32)$$

with $v = 1$ for $E > 0$ and $v = -1$ otherwise. Inserting Eqs. (27) and (32) into (26), we obtain

$$\Sigma^r_{\alpha} = - \frac{i v \Gamma_{\alpha}}{2 \sqrt{E^2 + |\Delta|^2}} \begin{pmatrix} E & \Delta \\ -\Delta & -E \end{pmatrix}, \quad (33)$$

where the linewidth function $\Gamma_{\alpha} = 2\pi |\Omega^2|^2 |\rho_{\alpha}|$. Under the so called wide-band approximation, the self energy of the lead is not sensitive to the energy and can be taken as a constant independent of the energy $E$. The non-diagonal term in the expression of the self energy $\Delta = |\Delta| e^{i\phi}$ is $|\Delta|$ when the phase of the BEC reservoir $\phi = 0$.

In Eq. (25), $G^r (E)$ and $G^a (E)$ denote the Fourier transforms of the GF $G^r (t)$ and $G^a (t)$ respectively. $G^r$ can be obtained by Dyson equation in matrix form

$$G^r = G^0_r + G^r \Sigma^r G^r_0, \quad (34)$$

where $G^0_r$ is the retarded GF of the isolated LL which is defined as

$$G^0_r (t,i) = -i\theta (t-i) \times \left\{ \begin{pmatrix} \langle \{b_{\alpha,q}(t), b^\dagger_{\alpha,q}(i)\} \rangle \\ \langle \{b_{\alpha,q}(t), b^\dagger_{\alpha,q}(i)\} \rangle \end{pmatrix} \right\} \quad (35)$$

In terms of the equation of motion for the GF, we can obtain
where the parameter $g$ is the strength of the interaction which is defined as $g = (1 + V/(\pi V_c))^{-1/2}$. Here we have assumed that $V_2 = V_4 = V$. This definition follows that of the fermions. The LL parameters $g$ also can be extracted from the Lieb-Liniger equation \cite{22,23}. For repulsive bosons, $g = 1$ corresponds to the hard-core limit, while $g > 1$ for repulsion, with $g \to \infty$ in the limit of weak interactions. In the case of fermion, non-interacting fermion corresponds to $g = 1$ and repulsive interaction corresponds to $g < 1$.

IV. QUANTUM TRANSPORT

In the following we show some numerical examples calculated according to Eq. (25) for the transport properties of this system with the experimental parameters: \cite{24} for $^{87}$Rb, $T = 1$ nK and the eigenenergy of the LL $E_L = 2.0$ kHz. By analyzing the form of $G'_0$ for the isolated LL, because $q/k_L \approx 0$ or $q/k_L \approx 2$, and the energy is equal the sum of the excitation energy and the eigenenergy of the LL, there should appear peaks near the eigenenergy and near three times of the eigenenergy in the transmission probability versus the energy of the incident boson. And peaks near $E/E_L = 1$ or near $E/E_L = 3$ will evidently differ from the resonant peak in the case of quantum dot coupled to BEC reservoirs. Because $q$ has a range of values, this will open some new channels for transmission. Here we will consider the symmetric case, i.e., $\Gamma_L \equiv \Gamma_R = \Gamma/2$.

Fig. 1 illustrates the transmission probability $T$ as a function of the incident boson energy $E$ (in units of $E_L$) where the parameters are $\delta_L = \delta_R = 0$ and $\phi = 0$. For the system with fixed $|\Delta| = 2\pi \times 0.41$ kHz and $g = 10$, Fig. 1(a) shows the dependence of two different Rabi frequency $\Omega = 0.02$ kHz (red solid line) and 0.03 kHz (blue dashed line) on the transmission probability, respectively. From this figure we can find that the resonant peak becomes wider as the Rabi frequency increases. It is because $|\Omega|^2 \propto \Gamma$, and $\Gamma$ describes how well the reservoir is in contact with the LL. The larger linewidth function corresponds to the stronger coupled case, and the stronger coupling corresponds to the wider the resonant peak. Fig. 1(b) shows the result of the transmission probability $T$ versus $E$ with fixed Rabi frequency $\Omega = 0.02$ kHz and $|\Delta| = 2\pi \times 0.41$ kHz for two different interaction strengths, where the red solid line for $g = 2$ and the blue dashed line for $g = 10$, respectively. The distance between the two resonant peaks become smaller when the particle-particle interaction parameter $g$ is larger. From this figure, we can conclude that the interaction parameter plays an important role on the relative position of the two resonant peaks. Fig. 1(c) illustrates $T$ as a function of $E$ with fixed Rabi frequency $\Omega = 0.02$ kHz and fixed interaction strength $g = 10$ for two different $|\Delta| = 0.2\pi$ kHz (red solid line) and $0.82\pi$ kHz (blue dashed line), respectively. From this figure, we can not find more visible difference between the two cases, which show that the width and height of the peaks are not sensitive to $|\Delta|$.

![Figure 1](image1.png)

FIG. 1: (Color Online) The transmission probability $T$ as a function of incident boson energy $E$ (in units of $E_L$, $E_L = 2.0$ kHz) where the parameters are $\delta_L = \delta_R = 0$ and $\phi = 0$. (a) for two different Rabi frequencies $\Omega = 0.02$ kHz (red solid line) and 0.03 kHz (blue dashed line) with $|\Delta| = 2\pi \times 0.41$ kHz and $g = 10$, (b) for two different interaction strengths $g = 2$ (red solid line) and 10 (blue dashed line) with $\Omega = 0.02$ kHz and $|\Delta| = 2\pi \times 0.41$ kHz, and (c) for two different $|\Delta| = 0.2\pi$ kHz (red solid line) and $0.82\pi$ kHz (blue dashed line) with $\Omega = 0.02$ kHz and $g = 10$, respectively.
or $\phi=3\pi/2$ and the green dashed line for $\phi=\pi$, respectively. From this figure we can see that in the range of 0 to $\pi$ the resonant peak becomes sharper as the phase increases, while in the range of $\pi$ to $2\pi$ the resonant peak becomes wider as the phase increases. Fig. 2 is very similar to Fig. 1(a), which makes clear that, through varying the off-diagonal term of the self energy, the phase of the BEC reservoir play the similar role as the Rabi frequency on the transmission probability $T$.

There may be possible experimental realizations for our system. Firstly, two BEC reservoirs can be realized in current experiments with atomic gases. Secondly, Luttinger liquid arises in our systems can be realized in current experiments with atomic gases. With the current technology there appears no difficulty in making transverse frequency $\omega_\perp>100\omega_z$, where $\omega_z$ is the longitudinal frequency. In such limit, one can produce atomic gases with all the atoms lying in the lowest harmonic oscillator state in the $x$-$y$ plane, leaving the motion along $z$ (the only degree of freedom). The system then behaves like a 1D Bose gas. For steeper magnetic traps, $\omega_\perp \sim 50$ kHz, particle densities of $\rho \sim 10^4$ particle/cm, and assuming a scattering length of 110 $a_B$ for Rb, it should be possible to observe the LL behavior [4].

V. CONCLUSION

In conclusion, using the equation of motion for Green function, we have investigated the transport properties for a Luttinger liquid coupled to two identical Bose-Einstein condensation reservoirs. It is demonstrated how the transmission probability is determined by Rabi frequency, interaction strength, $|\Delta|$, and phase of the BEC reservoir, respectively. We have found that the distance between the two resonant transmission probability peaks is determined by the interaction strengths, while the sharpness of the resonant peak is mainly determined by the Rabi frequency and phase of the reservoir. The further theoretical investigation on taking into account impurity, spin or other interactions are worthy to be carry out. These results for the proposed system involving a LL may be useful to control transport properties of cold atom.

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