Measurements of Charmless Two-Body Charged $B$ Decays with Neutral Pions and Kaons

The BABAR Collaboration

March 25, 2022

Abstract

We present preliminary results of the analyses of $B \to h\pi^0$ and $B \to hK^0$ decays (with $h = \pi^\pm, K^\pm$) from a sample of approximately 60 million $B\bar{B}$ pairs collected by the BABAR detector at the PEP-II asymmetric-energy $B$ Factory at SLAC. We find evidence for a signal in $B^+ \to \pi^+\pi^0$, and we measure the branching fraction

$$B(B^+ \to \pi^+\pi^0) = (4.1^{+1.1}_{-1.0} \pm 0.8) \times 10^{-6}.$$ 

We also measure the following branching ratios and charge asymmetries: $B(B^+ \to K^+\pi^0) = (11.1^{+1.3}_{-1.2} \pm 1.0) \times 10^{-6}$, $B(B^+ \to \pi^+K^0) = (17.5^{+1.8}_{-1.7} \pm 1.3) \times 10^{-6}$, $B(B^+ \to K^+\bar{K}^0) < 1.3 \times 10^{-6}$ (90% CL), $A_{\pi^+\pi^0} = -0.02^{+0.27}_{-0.26} \pm 0.10$, $A_{K^+\pi^0} = 0.00 \pm 0.11 \pm 0.02$, $A_{\pi^+K^0} = -0.17 \pm 0.10 \pm 0.02$, where the errors are statistical and systematic, respectively.
The BABAR Collaboration,

B. Aubert, D. Boutigny, J.-M. Gaillard, A. Hicheur, Y. Karyotakis, J. P. Lees, P. Robbe, V. Tisserand, A. Zghiche

Laboratoire de Physique des Particules, F-74941 Annecy-le-Vieux, France

A. Palano, A. Pompili

Università di Bari, Dipartimento di Fisica and INFN, I-70126 Bari, Italy

G. P. Chen, J. C. Chen, N. D. Qi, G. Rong, P. Wang, Y. S. Zhu

Institute of High Energy Physics, Beijing 100039, China

G. Eigen, I. Ofte, B. Stugu

University of Bergen, Inst. of Physics, N-5007 Bergen, Norway

G. S. Abrams, A. W. Borgland, A. B. Breon, D. N. Brown, J. Button-Shafer, R. N. Cahn, E. Charles, M. S. Gill, A. V. Gritsan, Y. Groysman, R. G. Jacobsen, R. W. Kadel, J. Kadyk, L. T. Kerth, Yu. G. Kolomensky, J. F. Kral, C. LeClerc, M. E. Levi, G. Lynch, L. M. Mir, P. J. Oddone, T. Orimoto, M. Pripstein, N. A. Roe, A. Romosan, M. T. Ronan, V. G. Shelkov, A. V. Telnov, W. A. Wenzel

Lawrence Berkeley National Laboratory and University of California, Berkeley, CA 94720, USA

T. J. Harrison, C. M. Hawkes, D. J. Knowles, S. W. O’Neale, R. C. Penny, A. T. Watson, N. K. Watson

University of Birmingham, Birmingham, B15 2TT, United Kingdom

T. Deppermann, K. Goetzen, H. Koch, B. Lewandowski, K. Peters, H. Schmuecker, M. Steinke

Ruhr Universität Bochum, Institut für Experimentalphysik 1, D-44780 Bochum, Germany

N. R. Barlow, W. Bhimji, J. T. Boyd, N. Chevalier, P. J. Clark, W. N. Cottingham, B. Foster, C. Mackay, F. F. Wilson

University of Bristol, Bristol BS8 1TL, United Kingdom

K. Abe, C. Hearty, T. S. Mattison, J. A. McKenna, D. Thiessen

University of British Columbia, Vancouver, BC, Canada V6T 1Z1

S. Jolly, A. K. McKemey

Brunel University, Uxbridge, Middlesex UB8 3PH, United Kingdom

V. E. Blinov, A. D. Bukin, A. R. Buzykaev, V. B. Golubev, V. N. Ivanchenko, A. A. Korol, E. A. Kravchenko, A. P. Onuchin, S. I. Serednyakov, Yu. I. Skovpen, A. N. Yushkov

Budker Institute of Nuclear Physics, Novosibirsk 630090, Russia

D. Best, M. Chao, D. Kirkby, A. J. Lankford, M. Mandelkern, S. McMahon, D. P. Stoker

University of California at Irvine, Irvine, CA 92697, USA

K. Arisaka, C. Buchanan, S. Chun

University of California at Los Angeles, Los Angeles, CA 90024, USA

D. B. MacFarlane, S. Prell, Sh. Rahatlou, G. Raven, V. Sharma

University of California at San Diego, La Jolla, CA 92093, USA

2
F. Palombo

Università di Milano, Dipartimento di Fisica and INFN, I-20133 Milano, Italy

J. M. Bauer, L. Cremaldi, V. Eschenburg, R. Kroeger, J. Reidy, D. A. Sanders, D. J. Summers

University of Mississippi, University, MS 38677, USA

C. Hast, J. Y. Nief, P. Taras

Université de Montréal, Laboratoire René J. A. Lévesque, Montréal, QC, Canada H3C 3J7

H. Nicholson

Mount Holyoke College, South Hadley, MA 01075, USA

C. Cartaro, N. Cavallo, G. De Nardo, F. Fabozzi, C. Gatto, L. Lista, P. Paolucci, D. Piccolo, C. Sciacca

Università di Napoli Federico II, Dipartimento di Scienze Fisiche and INFN, I-80126, Napoli, Italy

J. M. LoSecco

University of Notre Dame, Notre Dame, IN 46556, USA

J. R. G. Alsmiller, T. A. Gabriel

Oak Ridge National Laboratory, Oak Ridge, TN 37831, USA

J. Brau, R. Frey, E. Grauges, M. Iwasaki, C. T. Potter, N. B. Sinev, D. Strom

University of Oregon, Eugene, OR 97403, USA

F. Colecchia, F. Dal Corso, A. Dorigo, F. Galeazzi, M. Margoni, M. Morandin, M. Posocco, M. Rotondo, F. Simonetto, R. Strolli, E. Torassa, C. Voci

Università di Padova, Dipartimento di Fisica and INFN, I-35131 Padova, Italy

M. Benayoun, H. Briand, J. Chauveau, P. David, Ch. de la Vaissière, L. Del Buono, O. Hamon, Ph. Leruste, J. Ocariz, M. Pivk, L. Roos, J. Stark

Universités Paris VI et VII, Lab de Physique Nucléaire H. E., F-75252 Paris, France

P. F. Manfredi, V. Re, V. Speziali

Università di Pavia, Dipartimento di Elettronica and INFN, I-27100 Pavia, Italy

E. D. Frank, L. Gladney, Q. H. Guo, J. Panetta

University of Pennsylvania, Philadelphia, PA 19104, USA

C. Angelini, G. Batignani, S. Bettarini, M. Bondioli, F. Bucci, G. Calderini, E. Campagna, M. Carpinelli, F. Forti, M. A. Giorgi, A. Lusiani, G. Marchiori, F. Martinez-Vidal, M. Morganti, N. Neri, E. Paoloni, M. Rama, G. Rizzo, F. Sandrelli, G. Triggiani, J. Walsh

Università di Pisa, Scuola Normale Superiore and INFN, I-56100 Pisa, Italy

M. Haire, D. Judd, K. Paick, L. Turnbull, D. E. Wagoner

Prairie View A&M University, Prairie View, TX 77446, USA

J. Albert, P. Elmer, C. Lu, V. Miftakov, S. F. Schaffner, A. J. S. Smith, A. Tumanov, E. W. Varnes

Princeton University, Princeton, NJ 08544, USA
1 Introduction

The study of $B$ meson decays into charmless hadronic final states plays an important role in the understanding of $CP$ violation in the $B$ system. Measurements of the $CP$-violating asymmetry in the $\pi^+\pi^-$ decay mode can provide information on the angle $\alpha$ of the Unitarity Triangle. However, in contrast to the theoretically clean determination of the angle $\beta$ in $B$ decays to charmonium final states \[1, 2\] the extraction of $\alpha$ in $\pi^+\pi^-$ decay is complicated by the interference of $b \to uW^-$ tree and $b \to dg$ penguin amplitudes. Since these amplitudes have similar magnitude but carry different weak phases, additional measurements of the isospin-related decays \[1\], $B^+ \to \pi^+\pi^0$ and $B^0 \to \pi^0\pi^0$, are required to provide a means of measuring $\alpha$ \[3\]. The measurement of the branching ratio of the $B^+ \to \pi^+\pi^0$ decay is, in fact, a crucial ingredient, since it is a pure tree amplitude in a very good approximation. Therefore, in this channel direct $CP$ violation, detected as a charge asymmetry ($A$), is expected to be zero. Moreover, measurements of $B \to K\pi$ decays are interesting since phenomenological models have been proposed for extracting the weak phase $\gamma$ with a global fit to the observables \[4, 5, 6\]. We also present here an analysis of the $B^+ \to \pi^+K^0$ and $B^+ \to K^+\bar{K}^0$ decays. The $BABAR$ collaboration has previously published \[7\] measurements of the branching fractions for $B$ mesons decaying into $K^+\pi^0$ and $B^+ \to \pi^+K^0$, but no significant signals were seen for $B^+ \to \pi^+\pi^0$ and $B^+ \to K^+\bar{K}^0$ decays. The results reported here are an update of these published analyses.

2 Data Sample

The data used in these analyses were collected with the $BABAR$ detector at the PEP-II $e^+e^-$ storage ring during the years 2000 and 2001. The sample corresponds to an integrated luminosity of about $54 \, \text{fb}^{-1}$ accumulated near the $\Upsilon(4S)$ resonance (“on-resonance”) and about $5 \, \text{fb}^{-1}$ accumulated at a center-of-mass (CM) energy about 40 MeV below the $\Upsilon(4S)$ resonance (“off-resonance”), which are used for continuum background studies. The on-resonance sample corresponds to $(60.2 \pm 0.7) \times 10^6$ $BB$ pairs. The collider is operated with asymmetric beam energies, producing a boost ($\beta\gamma = 0.55$) of the $\Upsilon(4S)$ along the collision axis. The boost increases the momentum range of two-body $B$ decay products from a narrow distribution centered near $2.6 \, \text{GeV}/c$ in the CM to a broad distribution extending from 1.7 to 4.3 GeV/c.

$BABAR$ is a solenoidal detector optimized for the asymmetric beam configuration at PEP-II and is described in detail in Ref. \[8\]. Charged particle (track) momenta are measured in a tracking system consisting of a 5-layer, double-sided, silicon vertex tracker and a 40-layer drift chamber filled with a gas mixture of helium and isobutane, both operating within a 1.5 T superconducting solenoidal magnet. Photon candidates are selected as local maxima of deposited energy in an electromagnetic calorimeter (EMC) consisting of 6580 CsI(Tl) crystals arranged in barrel and forward endcap subdetectors. In this analysis, tracks are identified as pions or kaons by the Čerenkov angle $\theta_c$ measured by a detector of internally reflected Čerenkov light (DIRC). The DIRC system is a unique type of Čerenkov detector that relies on total internal reflection within the radiating volumes (quartz bars) to deliver the Čerenkov light outside the tracking and magnetic volumes, where the Čerenkov ring is imaged by an array of $\sim 11000$ photomultiplier tubes.

\[\text{1Charge conjugate modes are assumed throughout this paper.}\]
3 Event Selection, $\pi^0$ and $K^0$ Reconstruction

Hadronic events are selected based on track multiplicity and event topology. Backgrounds from non-hadronic events are reduced by requiring the ratio of Fox-Wolfram moments, $H_2/H_0$, to be less than 0.95 and the sphericity [10] of the event to be greater than 0.01.

Candidate $\pi^0$ mesons are reconstructed as pairs of photons with an invariant mass within $3\sigma$ of the nominal $\pi^0$ mass [11], where the resolution $\sigma$ is about 8 MeV/$c^2$. Photon candidates are selected as showers in the EMC that have the expected lateral shape, are not matched to a track, and have a minimum energy of 30 MeV. The $\pi^0$ candidates are then kinematically fitted with their mass constrained to the $\pi^0$ nominal mass.

$K^0$ mesons are detected in the mode $K^0 \rightarrow K^0_S \rightarrow \pi^+\pi^-$ and are reconstructed from pairs of oppositely charged tracks that form a well-measured vertex and have an invariant mass within 11.2 MeV/$c^2$ (which corresponds to 3.5$\sigma$) of the nominal $K^0_S$ mass [11]. The measured proper decay time of the $K^0_S$ candidate is required to exceed five times its uncertainty.

4 $B$ Reconstruction

$B$ meson candidates are reconstructed by combining a $\pi^0$ or a $K^0_S$ candidate with a track $h$. The kinematic constraints provided by the $\Upsilon(4S)$ initial state and knowledge of the beam energies are exploited to efficiently identify $B$ candidates. We define a beam-energy substituted mass $m_{ES} = \sqrt{E_b^2 - p_B^2}$, where $E_b = (s/2 + p_i \cdot p_B)/E_i$, $s$ is the total energy of the $e^+e^-$ system in the CM and lab frames, respectively, and $p_i$ and $p_B$ are the momentum vectors in the lab frame of the $e^+e^-$ system and the $B$ candidate, respectively. An additional kinematic parameter $\Delta E$ is defined as the difference between the energy of the $B$ candidate and half the energy of the $e^+e^-$ system, computed in the CM system. The $m_{ES}$ resolution is dominated by the beam energy spread, while for $\Delta E$ the main contribution comes from the measurement of particle energies in the detector. These two variables are therefore substantially uncorrelated.

However, in the $h\pi^0$ (with $h = \pi^\pm, K^{\pm}$) final states both the $\Delta E$ and $m_{ES}$ distributions have a tail due to imperfect containment of the electromagnetic showers initiated by the $\pi^0$. In this case only, in order to reduce this source of correlation and to slightly improve the resolution, we fit the $B$ candidate with the energy constrained to the CM beam energy in the two cases of kaon and pion mass hypothesis for the track $h$. For the $h\pi^0$ decay the energy-constrained mass resolution is then found to be about 3 MeV/$c^2$ from the core Gaussian width of a Crystal Ball fit to Monte Carlo simulated signal events. For the $hK^0_S$ decay the $m_{ES}$ resolution is found to be 2.5 MeV/$c^2$ from a Gaussian fit. For both decay topologies the signal Monte Carlo resolutions are validated by comparing data and Monte Carlo resolutions for decays into open charm final states with large branching fractions, such as $B^- \rightarrow D^0\rho^-$, $(\rho^+ \rightarrow \pi^+\pi^0$ and $D^0 \rightarrow K^-\pi^0$) for the $h\pi^0$ analysis, and $B^- \rightarrow D^0\pi^- (D^0 \rightarrow K^-\pi^+)$ for the $hK^0_S$ analysis.

The $\Delta E$ variable is evaluated assuming the pion mass hypothesis for the track $h$. Its distribution for the signal $\pi^+\pi^0$ events is described by a Crystal Ball function centered near zero. Since the $\Delta E$ distribution has a mean that depends on the track $h$ momentum in the lab frame in the case of signal $K^+\pi^0$ events, we also calculate $\Delta E$ with the kaon mass hypothesis ($\Delta E(K)$) for those events. We empirically find that its distribution is described better by a sum of two Gaussians with different mean values. For $hK^0_S$ signal events the $\Delta E$ distribution is parametrized as a sum

\footnote{A core Gaussian with a power law to describe a tail at negative values is called the Crystal Ball function [12].}
of two Gaussians centered near zero with the core Gaussian accounting for 95% of the events, taking into account the momentum dependence for signal $B^+ \rightarrow K^+ K_s^0$. Based on Monte Carlo simulated $B^+ \rightarrow \pi^+ \pi^0$ and $B^+ \rightarrow \pi^+ K_s^0$ events, we estimate the resolution on $\Delta E$ for the core Gaussian width to be about 40 MeV and 26 MeV, respectively. Candidates are selected in the range $5.2 < m_{ES} < 5.3$ GeV/c$^2$. Different requirements on $\Delta E$ specific to each analysis are then applied.

5 Background Rejection

The dominant background to these channels is from random combinations of a true $\pi^0$ ($K_s^0$) with a track, produced in $e^+e^- \rightarrow q\bar{q}$ continuum events (where $q = u, d, s,$ or $c$). Another source of background originates from $B$ decays into three (or more) light mesons. Detailed Monte Carlo simulation, off-resonance, and on-resonance data are used to study backgrounds. For this study we select on-resonance data in $\Delta E$ sideband regions defined by the ranges $0.20 < |\Delta E| < 0.45$ GeV for $h\pi^0$, and $-0.305 < \Delta E < -0.115$ GeV plus $0.075 < \Delta E < 0.265$ GeV for $hK_s^0$.

In the CM frame the continuum background typically exhibits a two-jet structure, in contrast to the isotropic decay of $B\bar{B}$ pairs produced in $\Upsilon(4S)$ decays. We exploit the topology difference between signal and background by making use of two event-shape quantities.

The first variable is the angle $\theta_s$ between the sphericity axes of the $B$ candidate and of the remaining tracks and photons in the event. The distribution of $|\cos \theta_s|$ in the CM frame is strongly peaked near 1 for continuum events and is approximately uniform for $B\bar{B}$ events. We require $|\cos \theta_s| < 0.8$ in the $h\pi^0$ analysis, but, given the lower level of background, only $|\cos \theta_s| < 0.9$ in the $hK_s^0$ analysis.

The second quantity is a Fisher discriminant $F$ \cite{1} constructed from the scalar sum of the CM momenta of all tracks and photons (excluding the $B$ candidate decay products) flowing into nine concentric cones centered on the thrust axis of the $B$ candidate. Each cone subtends an angle of $10^\circ$ and is folded to combine the forward and backward intervals. Monte Carlo samples are used to obtain the values of the Fisher coefficients, which are determined by maximizing the statistical separation between signal and background events. No requirement is applied on $F$; instead the distributions for signal and background events are included in a maximum likelihood fit as described in the next section.

On the other hand, $B$ background events tend to peak in $m_{ES}$, as do signal events, but have more negative $\Delta E$ values. They are particularly harmful for the $h\pi^0$ analysis given the poorer $\Delta E$ resolution. We use data in the negative $\Delta E$ sideband region to estimate the magnitude of this background and Monte Carlo techniques to choose a $\Delta E$ requirement that reduces this background to a negligible level. We finally require $-0.11 < \Delta E < 0.15$ GeV for $h\pi^0$ and $-0.115 < \Delta E < 0.075$ GeV for $hK_s^0$.

A total of 13661 candidates in the on-resonance data satisfy our $h\pi^0$ selection criteria and with the $hK_s^0$ analysis requirements we select 10668 candidates. These two samples enter into two separate maximum likelihood fits.

The final selection efficiency $\epsilon$ is $(25.6 \pm 1.7)\%$ $[(22.5 \pm 1.5)\%]$ for $B^+ \rightarrow \pi^+ \pi^0$ $[B^+ \rightarrow K^+ \pi^0]$ events, while it is $(47.5 \pm 2.2)\%$ $[(47.1 \pm 2.2)\%]$ for $B^+ \rightarrow \pi^+ K_s^0$ $[B^+ \rightarrow K^+ K_s^0]$ events. The errors on the efficiencies are statistical and systematic, combined in quadrature. The dominant component is due to the imperfect knowledge of $\pi^0$ and $K_s^0$ reconstruction efficiencies (5% and 3% relative errors, respectively).
6 Signal Extraction

For each topology \((h\pi^0\text{ and } hK_S^0)\), an unbinned maximum likelihood fit determines the signal and background yields \(n_i\) \((i = 1\text{ to } M)\), where \(M\) is the total number of signal and background species) and charge asymmetries \(A_i = (n_i^- - n_i^+)/n_i^- + n_i^+\), where \(n_i^- (n_i^+)\) is the fitted number of \(i^{th}\) type \(h^-\pi^0\) \((h^+\pi^0)\) \([h^-K_S^0\text{ and } h^+K_S^0]\) events. The input variables to the fit are \(m_{ES}\), \(\Delta E\), \(F\) and the Cherenkov angle \(\theta_c\) of the track from the candidate \(B\) decay. The extended likelihood function \(\mathcal{L}\) is defined as

\[
\mathcal{L} = \exp \left( -\sum_{i=1}^{M} n_i \right) \prod_{j=1}^{N} \left[ \sum_{i=1}^{M} \frac{1}{2} (1 - q_j A_i) n_i \mathcal{P}_i (\vec{x}_j; \vec{\alpha}_i) \right],
\]

where \(q_j\) is the charge of the track \(h\) in the \(j^{th}\) event. The \(M\) probabilities \(\mathcal{P}_i(\vec{x}_j; \vec{\alpha}_i)\) are evaluated as the product of probability density functions (PDFs) for each of the independent variables \(\vec{x}_j\), given the set of parameters \(\vec{\alpha}_i\). Monte Carlo simulation is used to validate the assumption that the fit variables are uncorrelated. The exponential factor in the likelihood accounts for Poisson fluctuations in the total number of observed events \(N\).

The parameters for the background \(m_{ES}\) and \(\Delta E\) PDFs are determined from events in the off-resonance data and in the \(m_{ES}\) sideband region, respectively. The \(m_{ES}\) shape is parameterized by a threshold function \(f(m_{ES}) \propto m_{ES}\sqrt{1 - x^2}\exp[-\xi(1 - x^2)]\), where \(x = m_{ES}/m_0\) and \(m_0\) is the average CM beam energy. The background shape in \(\Delta E\) is parameterized as a second-order polynomial. The signal distributions have been already described in Sect. [3].

Events from Monte Carlo simulated signal decays and from on-resonance \(m_{ES}\) sideband regions are used to parameterize the Fisher discriminant PDF for signal and background events as a Gaussian and a sum of two Gaussians, respectively. Alternative parameterizations for \(F\), obtained from off-resonance data (for background) and \(B^- \rightarrow D^0\pi^-\) fully reconstructed decays (for signal), are used to estimate systematic uncertainties. The \(\theta_c\) PDFs are derived from kaon and pion tracks in the momentum range of interest from a sample of \(D^{++} \rightarrow D^0\pi^+ (D^0 \rightarrow K^-\pi^+)\) decays. This control sample is used to parameterize the \(\theta_c\) resolution as a function of track polar angle.

The results of the fit are summarized in the first column of Table [3], where the statistical error for each mode corresponds to a 68\% confidence level interval and is given by the change in signal yield \(n_i\) that corresponds to a \(-2\ln \mathcal{L}\) increase of one unit. We define a signal statistical significance as the square root of the change in \(-2\ln \mathcal{L}\) when the signal yield is fixed to zero. For the \(\pi^+\pi^0\) mode, we find a 5.2\(\sigma\) statistical significance for the signal.

In order to increase the relative fraction of signal events of a given type for display purpose we choose events passing requirements on likelihood ratios. These likelihood ratios are defined as \(R_{\text{sig}} = \sum_s \mathcal{P}_s / \sum_i \mathcal{P}_i\) and \(R_k = \mathcal{P}_k / \sum_s \mathcal{P}_s\), where \(\sum_s\) denotes the sum over the probabilities for signal hypotheses only, \(\sum_i\) denotes the sum over all the probabilities (signal and background), and \(\mathcal{P}_k\) denotes the probability for signal hypothesis \(k\). These probabilities are constructed from all the PDFs except that describing the plotted variable. Figures [4] and [5] show the distributions in \(m_{ES}\) and \(\Delta E\) for events passing all such selection criteria. The likelihood fit projections, scaled by the relative efficiencies for the likelihood ratio requirements, are overlaid on each distribution. Since the sample projections in \(m_{ES}\) and \(\Delta E\) are obtained with requirements on different likelihood ratios, the number of signal events appearing in the two projections are not the same.
Figure 1: Distributions of $m_{ES}$ and $\Delta E$ for $\pi^+\pi^0$ events (left) and $K^+\pi^0$ events (right) after additional requirements on likelihood ratios, based on all variables except the one being plotted. Solid curves represent projections of the complete maximum likelihood fit result; dotted curves represent the background contribution.
Figure 2: Distributions of $m_{ES}$ (left) and $\Delta E$ (right) for $\pi^+ K^0_S$ events after additional requirements on likelihood ratios, based on all variables except the one being plotted. Solid curves represent projections of the complete maximum likelihood fit result; dotted curves represent the background contribution.

Table 1: Summary of fitted signal yields, measured branching fraction $B$ and charge asymmetries $A$. The first error is statistical and the second is systematic. For the $K^+ K^0_S$ mode we quote the 90% confidence level (CL) upper limits for the signal yield and branching ratio, and give the central values in parentheses.

| Mode          | Signal Yield | $B \times 10^{-6}$ | $A$         |
|---------------|--------------|--------------------|-------------|
| $\pi^+ \pi^0$ | $62^{+14}_{-16} \pm 11$ | $4.1^{+1.4}_{-1.6} \pm 0.8$ | $-0.02^{+0.27}_{-0.26} \pm 0.10$ |
| $K^+ \pi^0$  | $149 \pm 17 \pm 8$ | $11.1^{+1.3}_{-1.2} \pm 1.0$ | $0.00 \pm 0.11 \pm 0.02$ |
| $\pi^+ K^0$  | $172 \pm 17 \pm 9$ | $17.5^{+1.8}_{-1.7} \pm 1.3$ | $-0.17 \pm 0.10 \pm 0.02$ |
| $K^+ K^0_S$  | $< 10 \left( -5.6^{+2.8}_{-5.5} \pm 2.5 \right)$ | $< 1.3 \left( -0.6^{+0.6}_{-0.7} \pm 0.3 \right)$ | $-$ |

7 Branching Fraction Results

The branching fractions are defined as

$$B(h \pi^0) = \frac{1}{B(\pi^0 \rightarrow \gamma \gamma)} \frac{n_h \pi^0}{\epsilon_h \pi^0 \cdot N_{BB}},$$

$$B(h K^0) = \frac{1}{B(K^0 \rightarrow K^0_S)} \frac{n_h K^0}{B(K^0 \rightarrow \pi^+ \pi^-) \epsilon_h K^0 \cdot N_{BB}},$$

where $n_h \pi^0$ ($n_h K^0_S$) is the signal yield from the fit and $\epsilon_h \pi^0$ ($\epsilon_h K^0_S$) is the reconstruction efficiency for the mode $h \pi^0$ ($h K^0_S$) in the detected $\pi^0$ ($K^0_S$) decay chain. $N_{BB} = (60.2 \pm 0.7) \times 10^6$ is the total number of $B \overline{B}$ pairs in our dataset. $B(\pi^0 \rightarrow \gamma \gamma)$, $B(K^0 \rightarrow K^0_S)$, and $B(K^0 \rightarrow \pi^+ \pi^-)$ are taken to be equal to 0.98798, 0.5 and 0.6861, respectively [11]. Implicit in the above equations is the assumption of equal branching fractions for $\Upsilon(4S) \rightarrow B \overline{B}$ and $\Upsilon(4S) \rightarrow B^+ B^-$. Systematic uncertainties on the branching fractions arise primarily from uncertainty on the final
selection efficiency and uncertainty on $n_{i}$ due to imperfect knowledge of the PDF shapes. The latter is estimated either by varying the PDF parameters within $1\sigma$ of their measured uncertainties or by substituting alternative PDFs from independent control samples. In the $h\pi^{0}$ analysis the most relevant systematic uncertainties on the signal yields are due to the background $m_{ES}$ parametrization and Fisher background shape (about 10% each), while for the $hK^{0}$ analysis the $\Delta E$ offset and resolution and Fisher signal shape contribute the largest errors (about 4% each). We estimate the systematic uncertainty on the signal yields due to the residual presence of $B$ decay backgrounds with Monte Carlo techniques and we find that it is negligible compared with the other effects.

In the case of the $\pi^{+}\pi^{0}$ final state, we evaluate how the imperfect knowledge of the PDF shapes can affect the significance of the signal. We recalculate the square root of the change in $-2\ln L$ with $n_{\pi^{+}\pi^{0}}$ fixed to zero for the worst case PDF variations and we find a 4.0$\sigma$ statistical significance for the signal.

Systematic uncertainties on the charge asymmetries are evaluated from PDF variations added in quadrature with the limit on intrinsic charge bias in the detector (0.01). The small yield of $\pi^{+}\pi^{0}$ channel is the origin of the systematic error on the charge asymmetry (0.10), dominated by the PDF variations.

In conclusion, we find evidence for the decay $B^{+} \rightarrow \pi^{+}\pi^{0}$ and measure a branching fraction of $\mathcal{B}(B^{+} \rightarrow \pi^{+}\pi^{0}) = (4.1^{+1.1}_{-1.0}\pm0.8) \times 10^{-6}$. We also measure $\mathcal{B}(B^{+} \rightarrow K^{+}\pi^{0}) = (11.1^{+1.3}_{-1.2}\pm1.0) \times 10^{-6}$ and $\mathcal{B}(B^{+} \rightarrow \pi^{+}K^{0}) = (17.5^{+1.8}_{-1.7}\pm1.3) \times 10^{-6}$, with significant improvements on the errors with respect to our previously published results. We do not observe any evidence of direct $CP$ asymmetry in these channels, measuring $A_{\pi^{+}\pi^{0}} = -0.02^{+0.27}_{-0.26} \pm 0.10$, $A_{K^{+}\pi^{0}} = 0.00 \pm 0.11 \pm 0.02$, and $A_{\pi^{+}\pi^{0}} = -0.17 \pm 0.10 \pm 0.02$. No evidence of a signal is found for the $K^{+}\overline{K}^{0}$ final state for which we set a 90% CL upper limit on the branching ratio of $1.3 \times 10^{-6}$.

8 Acknowledgements

We are grateful for the extraordinary contributions of our PEP-II colleagues in achieving the excellent luminosity and machine conditions that have made this work possible. The success of this project also relies critically on the expertise and dedication of the computing organizations that support BABAR. The collaborating institutions wish to thank SLAC for its support and the kind hospitality extended to them. This work is supported by the US Department of Energy and National Science Foundation, the Natural Sciences and Engineering Research Council (Canada), Institute of High Energy Physics (China), the Commissariat à l’Energie Atomique and Institut National de Physique Nucléaire et de Physique des Particules (France), the Bundesministerium für Bildung und Forschung (Germany), the Istituto Nazionale di Fisica Nucleare (Italy), the Research Council of Norway, the Ministry of Science and Technology of the Russian Federation, and the Particle Physics and Astronomy Research Council (United Kingdom). Individuals have received support from the A. P. Sloan Foundation, the Research Corporation, and the Alexander von Humboldt Foundation.

References

[1] BABAR Collaboration, B. Aubert et al., Phys. Rev. Lett. 87, 091801 (2001).

[2] Belle Collaboration, K. Abe et al., Phys. Rev. Lett. 87, 091802 (2001).
[3] M. Gronau, Phys. Rev. Lett. 65, 3381 (1990).
[4] M. Beneke, G. Buchalla, M. Neubert, and C.T. Sachrajda, Nucl. Phys. B 606, 245 (2001).
[5] M. Ciuchini, E. Franco, G. Martinelli, M. Pierini, and L. Silvestrini, Phys. Lett. B 515, 33 (2001).
[6] Y. Y. Keum, H. N. Li and A. I. Sanda, Phys. Rev. D 63, 054008 (2001).
[7] BABAR Collaboration, B. Aubert et al., Phys. Rev. Lett. 87, 151802 (2001).
[8] BABAR Collaboration, B. Aubert et al., Nucl. Instrum. and Methods A479, 1 (2002).
[9] G.C. Fox and S. Wolfram, Phys. Rev. Lett. 41, 1581 (1978).
[10] S.L. Wu, Phys. Rep. 107, 59 (1984).
[11] Particle Data Group, D.E. Groom et al., Eur. Phys. J. C 15, 1 (2000).
[12] E. Bloom and C. Peck, Ann. Rev. Nucl. and Part. Sci. 33, 143 (1983).
[13] ARGUS Collaboration, H. Albrecht et al., Z. Phys. C 48, 543 (1990).