Systematic Investigation of X-Ray Spectral Variability of TeV Blazars during Flares in the RXTE Era

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Abstract

Utilizing all the 16 yr RXTE observations, we analyze the X-ray spectra of 32 TeV blazars and perform a systematic investigation of X-ray spectral variability for the five brightest sources during their major flares that lasted several days. We obtain photon spectral index (\(\alpha\)), flux and synchrotron radiation peak energy (\(E_p\)) from empirical spectral fitting, and electron spectral index (\(p\)) from theoretical synchrotron radiation modeling. We find that both \(\alpha\) and \(p\) generally display a harder-when-brighter trend, confirming the results of many previous works. Furthermore, we confirm and strengthen the result that \(p\) must vary in order to explain the observed X-ray spectral variability during flares, which would have useful implications for interpreting the associated higher-energy spectral variability. We see apparent electron spectral hysteresis in many but not all \(p\)-flux plots that takes a form of “loop” or oblique “8.” We obtain a tight \(p\)-hardness ratio (HR) relation and a tighter \(p\)-\(\alpha\) relation using spectra of flaring periods, both of which are also applicable to stacked data of quiescent periods. We demonstrate that these two empirical relations can be used efficiently to estimate \(p\) from HR or \(\alpha\) that is readily achieved. Finally, we find that, when considering TeV blazars as a whole, \(\alpha\) and X-ray luminosity are positively correlated, \(E_p\) is negatively correlated with \(p\) and \(\alpha\), and \(E_p\) is positively correlated with HR; all these correlations are in line with the blazar sequence. However, after correcting for the Doppler boosting effect, \(\alpha\) and intrinsic X-ray luminosity follow an anticorrelation.

Key words: BL Lacertae objects: individual (Mrk 421, Mrk 501, PKS 2155–304, PKS 2005–489, 1ES 1959+650) – galaxies: active

1. Introduction

Blazars, including flat-spectrum radio quasars (FSRQs) and BL Lac objects, are a subclass of active galactic nuclei (AGNs), with one of their relativistic jets pointing to the observers at small angles (Urry & Padovani 1995). They are the most important contributor to the cosmic TeV background radiation among extragalactic sources (e.g., Holder 2014; Sushch & H.E. S.S. Collaboration 2015; Inoue & Tanaka 2016). Their jet emission presents double-hump broadband spectral energy distributions (SEDs) and intense variability in multiple wavelengths across different timescales. The low-energy peak is located between infrared and X-ray energies, and the high-energy peak is located at hard X-ray up to TeV \(\gamma\)-ray emission (e.g., Abdo et al. 2010a). The first hump is widely believed to be produced by the synchrotron radiation of relativistic electrons and/or positrons in the jet, while the origin of the second hump is still in dispute. In leptonic scenarios, the high-energy hump is dominated by the inverse Compton radiation derived from relativistic electrons scattering synchrotron photons (e.g., Maraschi et al. 1992; Kirk et al. 1998) and/or external photons, e.g., from the accretion disk, broad-line region, or cosmic microwave background (e.g., Dermer et al. 1992; Sikora et al. 1994). In hadronic scenarios, the high-energy radiation is due to the proton emission processes (e.g., Aharonian 2000; Mücke & Protheroe 2001; Atoyan & Dermer 2003; Mücke et al. 2003; Böttcher et al. 2013; Fraija & Marinelli 2015).

According to the peak frequency of the low-energy hump (\(\nu_p\)), BL Lac objects can be divided into high-energy peaked BL Lac objects (HBLs), intermediate-energy peaked BL Lac objects (IBLs), and low-energy peaked BL Lac objects (LBLs; e.g., Padovani & Giommi 1995; Fossati et al. 1998; Abdo et al. 2010a; Fan et al. 2016). For HBLs, the synchrotron peak is located in the UV to X-ray domain (\(\nu_p > 10^{15}\) Hz); for IBLs, the peak is between optical and UV regimes (\(10^{14} < \nu_p \leq 10^{15}\) Hz); for LBLs, the peak is in the infrared band (\(\nu_p \leq 10^{14}\) Hz; Abdo et al. 2010a). Another type of blazars, FSRQs, are the high-luminosity sources whose synchrotron peak is located in the broad regime from the far-infrared to optical and even to UV wavelengths, and whose X-ray emission is from the inverse Compton radiation process. As one type of sources detected at TeV energies, TeV blazars mainly belong to HBLs whose X-ray spectrum is usually dominated by synchrotron emission; therefore, we use the synchrotron emission model to fit X-ray spectra of TeV blazars in this work.

The intense variability of blazars has been illustrated by, e.g., their multiple discrete X-ray flares at timescales from several months to days to minutes (e.g., Cui 2004; Xue & Cui 2005), with some extremely rapid flares even having characteristic rising timescales down to half a minute (Zhu et al. 2018). The flaring activities are often thought to be associated with several physical processes, such as the internal shocks generated in the jet (Rees 1978; Spada et al. 2001), the magnetic reconnection processes in the jet (Lyutikov 2003; Giannios et al. 2009), or...
the ejection events of relativistic particles into the jet (Boettcher et al. 1997; Mastichiadis & Kirk 1997). Furthermore, many studies have revealed a harder-when-brighter trend in X-ray flares of blazars, which manifests itself in hardening of spectra with increasing fluxes (e.g., Giommi et al. 1990; Sambruna et al. 1994; Xue et al. 2006; Abdo et al. 2010b). Xue et al. (2006) used the synchrotron model to investigate the X-ray spectral variability of Mrk 421 and Mrk 501 during flares that lasted for several days. Among the four key parameters (particle spectral index, maximum Lorentz factor, total energy density, and magnetic field), they found that the electron spectral index ($\gamma$) must vary during the flaring period and tends to decrease with increasing flux. Therefore, studying the evolution of physical parameters during flares could help us understand the underlying physical mechanism in the flaring process.

In this paper, we make use of all the 16 yr archival data of TeV blazars from the Proportional Counter Array (PCA) on board the Rossi X-Ray Timing Explorer (RXTE), a synchrotron radiation model, and the Markov chain Monte Carlo (MCMC) method to carry out a systematic investigation of the $3-25$ keV X-ray spectral variability during flares of Mrk 421, Mrk 501, PKS 2155–304, PKS 2005–489, and 1ES 1959+650. This work builds on and extends further the work of Xue et al. (2006), and some significant improvements over Xue et al. (2006) are as follows: our target sources for detailed analysis increase from two (i.e., Mrk 421 and Mrk 501) to five, the search of target flares covers all the RXTE/PCA observations throughout its entire life span (∼16 yr), and we utilize a new method that greatly improves calculation efficiency. One primary goal of this paper is to test the universality of the conclusion in Xue et al. (2006) that multiple parameters (which characterize the electron distribution and magnetic field), in particular, the electron spectral index, must vary, in order to account for the observed X-ray spectral variability of TeV blazars during flares that lasted for days to weeks. One thing worth noting is that we only focus on the evolution of electron spectral index in this paper, as the other parameters are generally constrained poorly (see the detailed discussion in Xue et al. 2006).

2. Data and Data Reduction

RXTE, carrying the All Sky Monitor (ASM), PCA, and the High-Energy X-Ray Timing Experiment, started operation in 1996 January and completed its scientific mission in 2012 January. During its 16 yr life span, RXTE observed 52 blazars (Rivers et al. 2013), including 32 TeV blazars (see Table 1) verified in the catalog of TeV sources (i.e., TeVCat7). These TeV blazars are 2 FSRQs, 1 LBL, 5 IBLs, and 24 HBLs.

2.1. All Data

In this paper, we utilized data from PCA, which consists of five nearly identical proportional counter units (PCUs). For the 16 yr observations of the 32 TeV blazars, we followed Rivers et al. (2011) to extract the first xenon layer data from PCU 0, PCU 1, and PCU 2 before 1998 December 23; PCU 0 and PCU 2 between 1998 December 23 and 2000 May 12; and PCU 2 after 2000 May 12, given that PCUs 1, 3, and 4 suffered from high-voltage breakdown issues during their on-source time, and PCU 0 had been operating without its propane layer since 2000 May 12. In this work, we made use of Standard2 data exclusively and binned each individual PCA observation into one data point when producing light curves (the median exposure time of all observations of each source is more than 1000 s). The numbers of PCA observations of these objects are summarized in Table 1 (Column (5)).

We followed Xue & Cui (2005) and Xue et al. (2006) to perform data reduction and analysis using FTOOLS version 6.19. First, we created the data filter file and good time interval (GTI) file for each observation following the standard procedure. Second, according to the suggested criterion,9 we used the latest faint background model (pca_bkgd_cmfbrightvle_eMv20051128.mld) for observations with count rates $<40$ counts $s^{-1}$ PCU$^{-1}$ and the bright background model (pca_bkgd_cmfbrightvle_eMv20051128.mld) for observations with count rates $\geq 40$ counts $s^{-1}$ PCU$^{-1}$ to simulate background events. Finally, we extracted spectra for both observational data and simulated background events using corresponding GTIs and grouped the spectra appropriately using GRPPHA in order to improve the signal-to-noise ratio (S/N) for subsequent spectral analysis.

2.2. Data of Flaring Periods

Since one of the major goals of this work is to study the $3-25$ keV X-ray spectral variability during flares, the subsequent analysis has been limited to objects with high X-ray fluxes and at least five observations during one flare.

As such, among 32 TeV blazars, we singled out five objects (i.e., Mrk 421, Mrk 501, PKS 2155–304, PKS 2005–489, and 1ES 1959+650, hereafter “the five sources”; see Table 2) for which RXTE/PCA data allow us to obtain high-quality spectra (detailed analysis of the other 27 TeV blazars will be presented in a future study). Furthermore, the flares were selected with the following criteria: (1) individual flares lasted for several days and were covered by at least two observations in both the rise and decay periods, and (2) the minimum total count rate (summed over available PCUs) in $3-25$ keV is above 30 counts s$^{-1}$. In addition, adjacent outbursts following or followed by those flares were also included. Finally, we picked out 20.5 flares for Mrk 421 (an outburst with observations only in the rise or decay period was considered as 0.5 flares), seven flares for Mrk 501, four flares for PKS 2155–304, and only one flare for both PKS 2005–489 and 1ES 1959+650 from the 16 yr data (see Figures 1 and 2 for the typical flares of each source; see also the observations annotated with “$\triangleright$” in Table 2).

2.3. Data of Quiescent Periods

As a comparison, we also extracted spectra for the above five objects when they stayed in the relatively quiescent periods, with variability amplitude being relatively small over several days. In view of the low S/N of each spectrum, we stacked multiple spectra within a certain time range. Finally, we produced two stacked spectra for both Mrk 421 and Mrk 501, one stacked spectrum for both PKS 2155–304 and 1ES 1959+650, and no stacked spectrum for PKS 2155–304, respectively (see the rows annotated with “$\triangleright$” in Table 2). For PKS 2155–304, its two stacked spectra of the quiescent period are concave and thus not included in Table 2, because its $3-25$ keV X-ray emission likely

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7. The TeVCat online catalog is provided by Scott Wakely & Deirdre Horan [http://tevcat.uchicago.edu/].

8. Details are at http://heasarc.gsfc.nasa.gov/docs/xte/recipes/cook_book.html.

9. Details can be found under “Important Downloads and Links” at http://heasarc.gsfc.nasa.gov/docs/xte/pca_news.html.
3. Spectral Fitting, Modeling, and Method

3.1. Photon Spectral Analysis

For all the spectra of 32 TeV blazars, we performed spectral analysis with the XSPEC software package (version 12.9.0; Arnaud 1996). For each spectrum, we experimented with four empirical models: power law, broken power law, power law with an exponential cutoff, and log-parabola. For each object, we fixed the Galactic hydrogen absorption parameter ($N_H$) that was from the survey by Dickey & Lockman (1990), as reported in Table 1. According to the distribution of reduced chi-square when fitting each source, we found that both the cutoff power-law and log-parabolic models provided better fits to the data than power-law and broken power-law models. And it is often difficult to decide which is the best-fit model between the cutoff power-law and log-parabolic models (see Table 1). Here, we fitted the spectra with the cutoff power-law model to obtain the photon spectral index ($\alpha$; see details in Section 5.1) and with log-parabolic model to obtain the peak energy ($E_p$) of the synchrotron radiation hump in SED (see details in Section 5.4). The intrinsic SEDs (i.e., corrected for Galactic absorption) derived with the best fits were subsequently used for synchrotron radiation modeling, where we adopted the following cosmological parameters: $H_0 = 70$ km s$^{-1}$ Mpc$^{-1}$, $\Omega_m = 0.28$, and $\Omega_\Lambda = 0.72$ (Hinshaw et al. 2013).
| Object  | Date       | MJD     | $\Gamma$ | $F_{\nu, 25-50 \text{ keV}}$ | $\chi^2_{\nu}(v)$ | $a$ | $b$ | $K$ | $F_{\nu, 25-50 \text{ keV}}$ | $\chi^2_{\nu}(v)$ | $p$ | $\chi^2_{\nu}(v)$ |
|---------|------------|---------|----------|-----------------------------|-------------------|-----|-----|-----|-----------------------------|-------------------|-----|-----------------|
| Mrk 421 | 2001 Mar 20 | 51,988.44 | 2.45 (0.04) | 4.52 (0.34) | 2.34 (0.26) | 2.22 (0.10) | 0.28 (0.06) | 0.36 (0.03) | 4.53 | 0.22 (0.26) | 4.08 | 0.42 (24) |
| (flaring) | 2001 Mar 20 | 51,988.73 | 2.23 (0.04) | 7.49 (0.53) | 2.34 (0.32) | 1.97 (0.08) | 0.29 (0.05) | 0.37 (0.03) | 7.50 | 0.33 (26) | 3.58 | 0.63 (30) |
| Mrk 501 | 1997 Apr 12 | 50,550.19 | 1.72 (0.05) | 5.25 (0.78) | 2.34 (0.28) | 1.61 (0.10) | 0.12 (0.06) | 0.09 (0.01) | 5.25 | 0.67 (28) | 2.43 | 0.83 (29) |
| (flaring) | 1997 Apr 12 | 50,550.45 | 1.71 (0.04) | 6.00 (0.73) | 2.34 (0.29) | 1.62 (0.10) | 0.09 (0.06) | 0.09 (0.01) | 6.00 | 0.64 (29) | 2.39 | 0.66 (30) |
| PKS 2155-304 | 1996 May 19 | 50,222.65 | 2.32 (0.07) | 1.10 (0.47) | 2.34 (0.25) | 2.16 (0.15) | 0.22 (0.10) | 0.07 (0.01) | 1.10 | 0.43 (25) | 3.78 | 0.45 (22) |
| (flaring) | 1996 May 19 | 50,222.80 | 2.28 (0.07) | 1.36 (0.36) | 2.34 (0.28) | 2.15 (0.14) | 0.17 (0.09) | 0.08 (0.01) | 1.37 | 0.35 (26) | 3.65 | 0.37 (23) |
| PKS 2005-489 | 1998 Oct 22 | 51,108.51 | 2.17 (0.08) | 1.42 (0.45) | 2.34 (0.23) | 1.97 (0.17) | 0.25 (0.11) | 0.07 (0.01) | 1.43 | 0.39 (23) | 3.45 | 0.45 (19) |
| (flaring) | 1998 Nov 04 | 51,121.71 | 2.03 (0.06) | 2.67 (0.60) | 2.34 (0.27) | 1.82 (0.12) | 0.25 (0.07) | 0.09 (0.01) | 2.68 | 0.55 (26) | 3.13 | 0.47 (23) |
| IES 1959+650 | 2002 May 19 | 52,413.00 | 1.92 (0.13) | 1.46 (0.92) | 2.34 (0.21) | 1.83 (0.26) | 0.13 (0.16) | 0.09 (0.02) | 1.46 | 0.94 (21) | 2.83 | 1.03 (17) |
| (flaring) | 2002 May 19 | 52,413.78 | 1.56 (0.05) | 1.96 (0.86) | 2.34 (0.26) | 1.26 (0.11) | 0.33 (0.07) | 0.08 (0.01) | 1.96 | 0.78 (26) | 1.99 | 0.93 (24) |

**Note.** Column (1): object name. Columns (2) and (3): date and Modified Julian Date (MJD) of the RXTE/PCA observation. Columns (4)–(6): photon index ($\Gamma$; $\Gamma = \alpha + 1$, and $\alpha$ is the photon spectral index), 3–25 keV flux (in units of $10^{-10}$ erg cm$^{-2}$ s$^{-1}$), and best-fit reduced chi-square ($\chi^2_{\nu}$; degree of freedom $v$) with cutoff power-law fitting, respectively. Columns (7)–(11): parameters $a$, $b$, and $K$ (in units of photons cm$^{-2}$ s$^{-1}$ keV$^{-1}$) in Equation (3), 3–25 keV flux (in units of $10^{-10}$ erg cm$^{-2}$ s$^{-1}$), and best-fit $\chi^2_{\nu}$ with log-parabolic fitting, respectively. Columns (12) and (13): electron spectral index ($p$) and best-fit $\chi^2_{\nu}$ with synchrotron model fitting (see Section 3), respectively. All errors represent 1σ errors. For each source, the rows marked with "\textbf{A}" represent the results of five observations occurring during an example flare as plotted in Figure 1 (i.e., the five colorfully encircled data points in the light curves), and the rows marked with "\textbf{V}" represent the results of stacked spectra during quiescent periods (the date and MJD indicate the time range of stacked quiescent-period observations).
3.2. Synchrotron Model

We used the homogeneous synchrotron radiation model presented in Xue et al. (2006) to fit the time-resolved flaring-period spectra (see Section 2.2) and the stacked quiescent-period spectra (see Section 2.3) of the aforementioned five sources. It was based on the assumption that a single flare is generated from a localized region of the jet (i.e., jet blob\(^{10}\)), where the spatial distribution of electrons and magnetic field is homogeneous.

3.2.1. Electron Spectral Distribution

Full details on the synchrotron radiation model were presented in Section 3 of Xue et al. (2006); here we only provide a brief introduction. We assume that the emitting electrons follow the power-law spectral distribution with power-law index \( p \) and low- and high-energy cutoffs, \( \gamma_{\text{min}} \) and \( \gamma_{\text{max}} \), and are homogeneously distributed in the emitting region. In addition, the emitting region is assumed to be a spherical zone with a radius \( r \) that is compatible with the duration of the flare. Then, we can evaluate the spectrum of the emission by integrating the differential power of synchrotron radiation over the entire Lorentz factor range (i.e., \( \gamma_{\text{min}} \leq \gamma \leq \gamma_{\text{max}} \)) within the jet blob.

We chose electron spectral index (\( p \)), magnetic field (\( B \)), maximum Lorentz factor of electrons (\( \gamma_{\text{max}} \)), and total energy density of electrons (\( E_{\text{tot}}/m_e c^2 \)) as free parameters when performing synchrotron radiation modeling. During the fitting process, the Doppler factor (\( \delta \)) and the minimum Lorentz factor of the electrons (\( \gamma_{\text{min}} \)) were frozen, i.e., \( \delta = 15 \) (a nominal value for TeV blazars) and \( \gamma_{\text{min}} = 10^4 \). We had verified that a reasonable change of \( \delta \) and \( \gamma_{\text{min}} \) values had little impact on the distribution of \( p \). We found that our homogeneous synchrotron radiation model with the above parameter settings can well reproduce the observed 3–25 keV spectra, as in Xue et al. (2006).

3.2.2. Fitting Method

In Xue et al. (2006), the statistically acceptable solutions were obtained through a grid search. In this paper, we used a new method to obtain the solutions. At first, we defined the sufficiently wide preliminary ranges of the four parameters, i.e., \( p \) in 1.00–5.00, \( B \) in \( 10^{-3}–10^2 \) G, \( \gamma_{\text{max}} \) in \( 10^2–10^{11} \) (note that the final solutions are selected in the realistic range of \( 10^2–10^6 \)), and \( E_{\text{tot}}/m_e c^2 \) in \( 10^{-10}–10^{10} \) ergs \( cm^{-3} \), respectively. We adopted linear steps for \( p \), \( \log (B) \), \( \log (\gamma_{\text{max}}) \), and \( \log (E_{\text{tot}}) \) in the grid search; the step was 0.01 for \( p \) and 0.02 for the other three parameters, respectively. Starting with the preliminary parameter ranges, we used MPFIT to obtain a set of best-fit parameters that were then selected as the initial values for subsequent MCMC fitting. We utilized the MCMC method for fitting in order to narrow down the corresponding range of each parameter for each spectrum. Subsequently, we carried out a grid search to find acceptable solutions (i.e., \( \chi^2 < 1 + \sqrt{2/\nu} \), where \( \chi^2 \) is reduced chi-square and \( \nu \) is degree of freedom) within the parameter ranges constrained by the MCMC method.

The acceptable solutions usually cover a small range of the entire preliminary parameter space mentioned above. The usual way of performing a grid search from end to end would cover the whole parameter space uniformly, which is, however, very time-consuming. Conversely, the MCMC method could reduce the computing time to one-eighth of the time needed by the grid search, but the solutions might sometimes be trapped within a local minimum so that a meaningful parameter distribution cannot be obtained. Therefore, we decided to first use the MCMC method to restrict the parameter range from the preliminary range, and then we used the grid search to obtain the final solutions (and thus the \( p \) distribution). We had verified that this fitting method would obtain the same parameter distributions as the grid search method adopted by Xue et al. (2006) and could greatly improve computation efficiency. In this paper, we focus only on the \( p \) distribution that is reasonably constrained, given that the distributions of \( B \), \( \gamma_{\text{max}} \), and \( E_{\text{tot}} \) are usually constrained poorly. As indicated by Xue et al. (2006), the SED modeling suffers from serious degeneracy among the other three parameters (\( B \), \( \gamma_{\text{max}} \), and \( E_{\text{tot}} \)); our result draws the same conclusion. The 1σ errors of \( p \) are obtained based on the range of \( p \) when the chi-square (\( \chi^2 \)) equals 1 plus the minimum of \( \chi^2 \) (i.e., the best-fit \( \chi^2 \)) in the plot of \( \chi^2 \) versus \( p \).

4. Results

4.1. X-Ray Spectra during Flares

Figure 1 presents one typical flare and its corresponding X-ray spectra during the flare for each of the aforementioned five sources. It shows that the synchrotron radiation model can describe the spectra very well (see the fitting results in Table 2). It is apparent that the X-ray spectrum varies significantly during flares and is harder when flux becomes higher (i.e., harder when brighter), which has been widely studied before for the following sources: Mrk 421 (e.g., Fossati et al. 2000, 2008; Brinkmann et al. 2003; Ravasio et al. 2004; Acciari et al. 2011; Baloković et al. 2013; Pian et al. 2014; Aleksic et al. 2015; Baloković et al. 2016; Kapanadze et al. 2017a), Mrk 501 (e.g., Pian et al. 1998; Krawczynski et al. 2000; Xue & Cui 2005; Gliozzi et al. 2006; Anderhub et al. 2009; Kapanadze et al. 2017b), 1ES 1959+650 (e.g., Giebels et al. 2002; Kapanadze et al. 2016b, 2018), PKS 2155–304 (e.g., Zhang et al. 2006a, 2006b; Kapanadze et al. 2014; Bhagwan et al. 2016), and PKS 2005–489 (e.g., Perlman et al. 1999).

In the observational energy band (i.e., 3–25 keV shown in Figure 1), the spectral shape is different for the five sources, which indicates that the synchrotron radiation peak is located at different energies. Combining the 3–25 keV spectral shape information and the synchrotron peak energy obtained by fitting all PCA spectra with the log-parabolic model detailed in Section 5.4, we found the following: for Mrk 421, the peak energy of all the spectra is below 6 keV, which is consistent with the results in Massaro et al. (2004), Tanihata et al. (2004), and Tramacere et al. (2007) (but Tramacere et al. 2009 show that its peak energy could be up to 30 keV); for Mrk 501, the peak energy of most spectra is above 3 keV, and Massaro et al. (2008) show that its peak energy could be up to 100 keV; for PKS 2005–489 and PKS 2155–304, the peak energy of spectra is below 3 keV; and for 1ES 1959+650, the peak energy of most spectra is below 30 keV. In fact, it was sometimes difficult to evaluate the exact location of the SED peak, which could fall beyond our limited spectral band coverage. Therefore, we could only provide a rough range of peak energy here.
Figure 1. The 3–25 keV PCA light curves, X-ray spectra, photon spectral index variations, and normalized distributions of electron spectral index during a typical flare of Mrk 421, Mrk 501, PKS 2155–304, PKS 2005–489, and 1ES 1959+650, respectively (one column for each source). Top row: 3–25 keV X-ray light curves (typical errors on count rates in units of counts s$^{-1}$ PCU$^{-1}$ are 0.16 for Mrk 421, 0.11 for Mrk 501, 0.09 for PKS 2155–489, 0.08 for PKS 2005–489, and 0.14 for 1ES 1959+650, respectively, which are very small and therefore not plotted). Second row: X-ray spectra, with solid curves representing best-fit synchrotron models that were obtained using χ$^2$ statistics (see Table 1). Third row: variations of photon spectral index ($\alpha$) over time during flares. Bottom row: normalized distributions of electron spectral index ($p$) derived with the synchrotron model fitting. For each source (column), the same color represents the same observation.
4.2. Electron Spectral Evolution

As we mentioned before, a general trend, in which the spectrum hardens with increasing flux, has been observed in blazars in X-ray observations. There are several conjectures for leading to such a trend. One of them is hardening or softening in the electron spectral distribution. Xue et al. (2006) demonstrated that variation of electron spectral index ($p$) is indispensable during a flare. They found that the quality of RXTE/PCA spectra enables utilizing the synchrotron model to place reasonable constraints on $p$ evolution during major flares of two TeV blazars, Mrk 421 and Mrk 501, i.e., $p$ variation is required and the electron spectrum tends to be harder/softer with the increase/decrease of flux, in addition to accompanying changes of some other key parameters. We confirm and strengthen the results of Xue et al. (2006) by finding that such a trend of $p$ evolution widely exists in multiple flares of five TeV blazars (see Figures 1 and 2) and that variation of $p$ over time is synchronous with variation of flux over time. In addition, the trend of $p$ evolution is consistent with that of $\alpha$ evolution (see Figure 9 in Appendix A). Noting the fact that the above five TeV blazars are all HBLs, we further examined the behaviors of BL Lacertae (the brightest IBL in Table 1) and 3C 279 (the brightest FSRQ in Table 1) in the $\alpha$-flux plot and found that both of them also show a harder-when-brighter trend.

However, there are a few exceptions that show a complex or even opposite evolution of $p$ rather than the simple harder-when-brighter trend during flares. For example, in panels (6), (9), (13), and (16) of Figure 2, the count-rate light curve and $p$ “light curve” somehow lose track of each other and thus do not
follow the general trend seen in the other panels, where $p$ evolution and count-rate evolution generally track each other in a synchronous way. We show the spectra and spectral variations of the case that exhibits the most apparent exception (i.e., panel (13)) in Figure 11. These exceptions might be due to the complexity of physical conditions in the emission region and/or the interaction between multiple populations of emitting electrons in two adjacent and comparative flares. For the latter case, the one-zone synchrotron radiation scenario could not be valid, and the introduction of multiple populations of emitting electrons might be essential.

4.3. Electron Spectral Hysteresis

In a conventional hardness–flux plot, spectral hardness can be different in the rising and falling periods of flares, which is known as "spectral hysteresis" and related to both acceleration and cooling timescales. In fact, spectral hysteresis could reveal itself as a "loop" shape in the hardness–flux plot (where the spectrum becomes harder along the positive $y$-axis direction and the flux becomes higher along the positive $x$-axis direction). Generally, a "hard lag" should result in a counterclockwise loop, while a "soft lag" would lead to a clockwise

**Figure 2.** Light curves (black segmented lines; left $y$-axis) and evolution of electron spectral index (red segmented lines; right $y$-axis, which is in descending order) over time during some typical flares for the five sources. The horizontal segments mark Flares A–L, which are further examined in Figure 3.
loop (e.g., Abeysekara et al. 2017). X-ray spectral hysteresis has been found in many blazars (e.g., Kataoka et al. 2000; Böttcher & Chiang 2002; Sembay et al. 2002; Zhang et al. 2002; Böttcher & Reimer 2004; Cui 2004; Ravasio et al. 2004; Brinkmann et al. 2005; Xue & Cui 2005; Gliozzi et al. 2006; Acciari et al. 2009; Tramacere et al. 2009; Kapanadze et al. 2016a, 2017a, 2017b, 2017c; Abeysekara et al. 2017; Kapanadze et al. 2018), and UV–optical spectral hysteresis has also been seen in nonblazar jet knots (e.g., Perlman et al. 2011).

From Figure 2, we further selected a number of flares (i.e., Flares A–L) to examine a different version of spectral hysteresis that is in the form of electron spectral hysteresis (shown as Figure 3), which is consistent with the photon spectral hysteresis (see Figure 10 in Appendix A). In many of these p-flux plots, electron spectral hysteresis is apparent, rendering itself in a “loop” (e.g., panel (6) of Figure 3, which corresponds to Flare F of Mrk 421) or oblique “8” (e.g., panels (1), (2), (3), (5), (11), and (12), which correspond to Flares A, B, C, E of Mrk 421, Flare K of PKS 2005–489, and Flare L of 1ES 1959+650, respectively) shape, whereas some cases show no apparent hysteresis (e.g., panels (4), (7), (8), (9), and (10), which correspond to Flare D of Mrk 421, Flares G, H, I of Mrk 501, and Flare J of PKS 2155–304, respectively), given the relatively large errors of p. As in Sections 4.1 and 4.2, most panels of Figure 3 also show an overall trend that electron spectrum typically hardens with increasing flux and softens during decreasing phase, which might reflect a process of
electron acceleration, injection, or cooling. Interestingly, there are a few cases that seem to behave in a perplexing way. For instance, for Flare A of Mrk 421 (panel 1), the spectrum starts with almost no spectral variability but a flux increase, and then it suddenly hardens when flux remains invariable (the flare peak might happen to be between the third and fourth observations of this flare, which was not observed and led to such a case) and softens with a flux decrease; Flare K of PKS 2005–489 (panel 11) shows that the value of \( p \) is nearly invariable during the rising period of the flare, given the uncertainties on \( p \). Perlman et al. (1999) had analyzed the prominent flare of PKS 2005–489 (as shown in panel 10 of Figure 3) and found that the 2–10 keV X-ray spectral variability follows a counterclockwise “loop” in the spectral index–flux plane. The evolution of \( p \)-flux in this work is consistent with their result.

Electron spectral index \((p)\) represents the fraction of electrons in different energies. The fraction of high-energy electrons increases with decreasing values of \( p \). For most flares in Figure 3, it appears that the value of \( p \) in the rise phase of the flare is larger than (panels (2), (3), (6), (7), (8), and (12)) or approximately equal to (panels (4) and (9)) the value in the decay phase. In other words, at the beginning of the flare, the fraction of high-energy electrons is low and subsequently increases gradually, which leads to “hard lag.” However, for the flares in panels (1), (10), and (11), the trend is opposite; this means that the fraction of high-energy electrons is high in the beginning and then decreases, which leads to “soft lag.” We then used a cross-correlation function to search for likely time lags between soft-band (3–8 keV) and hard-band (8–25 keV) light curves, but we found no evidence for existence of time lags. These nondetections of time lags might be due to the fact that the actual time lag is likely intraday, which is difficult to resolve using our light curves of several-day time resolution. In Garson et al. (2010), two flares of Mrk 421 lasting for 0.5 days showed several-hour time lags between the 0.5–2 keV and 2–10 keV light curves and displayed different movements in the hardness–flux plot. Krawczynski et al. (2000) indicated that the time lag between 3 and 25 keV is smaller than 15 hr in Mrk 501. Therefore, more intraday observational data would be required to detect likely time lags of our sources that might be responsible for the observed electron spectral hysteresis.

### 4.4. Photon Spectral Index versus Luminosity

As Figure 4 shows, for a single object, photon spectral index \((\alpha)\) decreases with increasing flux (panel (4)) or luminosity (panels (5) and (6)), which is the so-called harder-when-brighter trend (see Section 4.1), while for the entirety of the five sources, photon spectral index seems to increase with increasing observed luminosity (panel (5); see median values denoted by pentagrams) but decreases with increasing intrinsic luminosity that has been corrected for the Doppler boosting effect (panel (6)). To control for spectral data quality in Figure 4, we only chose observations with fluxes above 0.25 \( \times (F_{\text{max}} - F_{\text{min}}) \) for each source, where \( F_{\text{max}} \) and \( F_{\text{min}} \) are the maximum and minimum fluxes of the source among the 16 yr data, respectively.

The positive correlation between photon spectral index and observed luminosity (panel (5) of Figure 4) is correlated with the “blazar sequence” (Fossati et al. 1998; Ghisellini et al. 1998). According to the blazar sequence, there is a negative correlation between the synchrotron peak energy and the observed bolometric luminosity \((L_{\text{bol}})\). The observed bolometric flux can be estimated roughly through the relation of \( L_{\text{bol}} \approx 5\nu_L F(\nu_L) \), where \( F(\nu_L) \) is the peak flux of the synchrotron hump (Massaro et al. 2004), such that \( L_{\text{bol}} \approx 5\nu_L L(\nu_L) \propto L_{25} \). In addition, there is an anticorrelation between X-ray photon spectral index and synchrotron peak energy (e.g., Lin et al. 1999; Giommi et al. 2005; Perlman et al. 2005). Therefore, when the peak moves to the lower-energy band, the X-ray luminosity will increase and the X-ray spectrum will tend to steepen. As a result, there would be a positive correlation between the X-ray luminosity and X-ray photon spectral index, as demonstrated in panel (5) of Figure 4 (the Spearman’s ranking correlation for the overall \( \alpha \)–\( L_{25} \) relation of the five sources is 0.31 with a significance of \( 3.42 \times 10^{-3} \)). However, the anticorrelation between synchrotron peak energy and luminosity (the blazar sequence) disappears after applying the Doppler boosting correction to the observed luminosity (e.g., Nieppola et al. 2008; Wu et al. 2009; Huang et al. 2014; Fan et al. 2017).

In this work, we performed approximate Doppler corrections using the equation \( L_{\text{intrinsic}} = L_{\text{observed}}/\delta^2 \), where \( L_{\text{intrinsic}} \) is the intrinsic luminosity, \( L_{\text{observed}} \) is the observed luminosity, and \( \delta \) is the lower limit of the \( \gamma \)-ray Doppler factor from Fan et al. (2013), which is estimated according to the pair-production optical depth (Mattox et al. 1993): 2.77 for Mrk 421, 2.83 for Mrk 501, 4.15 for PKS 2155-304, 3.30 for PKS 2005–489, and 2.32 for IES 1959+650. Our results are consistent with the previous studies (panel (6) of Figure 4): the Spearman’s ranking correlation for the overall \( \alpha \)–\( L_{25} \) intrinsic relation of the five sources is \(-0.46 \) with a significance of \( 2.33 \times 10^{-28} \).

## 5. Discussion

### 5.1. Electron Spectral Index versus Photon Spectral Index

We obtained the photon spectral index \((\alpha)\) by fitting spectra with the cutoff power-law model rather than the log-parabolic model. These two models both provided great spectral fitting results, but the photon spectral index in the log-parabolic model is energy dependent, so we did not adopt \( \alpha \) from this model. The cutoff power-law model follows the relation of \( F(E) \propto E^{-\alpha} \cdot \exp(-E/\beta) \), where \( \beta \) is the \( e \)-folding energy of exponential roll-off. In this work, we have assumed that the electron spectral distribution follows a power-law shape, which usually produces the optically thin synchrotron radiation spectrum with a power-law photon spectral index of \( \alpha = (p - 1)/2 \) (see the dashed line in Figure 5). However, in this work, the cutoff power-law model could provide better fits to all the spectra than the power-law model; therefore, we used the cutoff power-law model to obtain photon spectral index \((\alpha)\).

According to Figure 5, there is a significant linear relation between \( p \) and \( \alpha \) during flares of the five sources, which follows the formula of

\[
p = (2.27 \pm 0.03) \times \alpha + (0.81 \pm 0.03).
\]

(1)

This relation shows a slight deviation from the theoretical relation of \( \alpha = (p - 1)/2 \) for the power-law spectral distribution. This deviation might be due to the energy loss of electrons and the acceleration process of relativistic electrons, which could produce the spectrum not exactly following the
power-law distribution. Therefore, for the X-ray spectrum not following the power-law shape, it might not be suitable to use $\alpha = (p - 1)/2$ to calculate $p$ using $\alpha$ and vice versa.

In addition, values of $p$ and $\alpha$ in relatively quiescent periods (i.e., pentagrams in Figure 5) seem to follow nicely the above $p-\alpha$ relation that was derived with flaring periods, which

Figure 3. Evolution of electron spectral index ($p$) with flux for the five sources during Flares A–L as annotated in Figure 2, with red arrows indicating time sequences.
5.2. Electron Spectral Index versus Hardness Ratio

We define hardness ratio (HR) as $HR = H/S$, where $H$ and $S$ are count rates in the 8–25 keV and 3–8 keV bands, respectively. As expected, Figure 6 also presents a linear relation between $p$ and HR during flares, which follows the formula of

$$p = (-8.28 \pm 0.16) \times HR + (5.54 \pm 0.05).$$

We had verified that a similar linear relation between $p$ and $HR_{(10-25 keV)/(3-10 keV)}$ would also be obtained, which is not presented here. The $p$–HR relation (i.e., Equation (2)) is not as tight as the $p$–$\alpha$ relation (i.e., Equation (1)), and the former has significantly larger scatters than the latter, which is also expected given the following two facts: the derivation of $\alpha$ makes use of full spectral information, while the calculation of HR only utilizes crude spectral information; and the influence of Galactic absorption was not taken into account for deriving HR, which should introduce additional small scatters. Furthermore, values of $p$ and HR in relatively quiescent periods (i.e., pentagrams in Figure 6) also seem to generally follow the $p$–HR relation that was derived with flaring periods (see Section 5.1 and Figure 5).

5.3. Application of $p$–$\alpha$ and $p$–HR Relations

The $p$–$\alpha$ and $p$–HR relations provide us with two quick and straightforward empirical approaches to roughly estimate the electron spectral index ($p$) simply based on values of $\alpha$ and/or HR, without resorting to detailed synchrotron radiation modeling. The distribution of $p$ can only be obtained through fitting spectra with the synchrotron radiation model, which not only relies on high-quality spectral data but also takes a long time. Therefore, if one is only interested in knowing the approximate range of $p$ for a particular spectrum, then it would be efficient to estimate $p$ using one of the $p$–$\alpha$ and $p$–HR relations or even both.

For the purpose of verifying the reliability of these two approaches, we compared values of $p$ estimated using $\alpha$ with that estimated using HR, utilizing all HBL spectral data among the 32 TeV blazars (note that the 3–25 keV spectra of FSRQs, LBLs, and IBLs might be dominated by both synchrotron and inverse Compton scattering components, which are not suitable for simple synchrotron radiation modeling). Figure 7 shows a reasonably good agreement between $p_{\alpha}$ and $p_{HR}$, which were derived using the $p$–$\alpha$ and $p$–HR relations, respectively. As the flux increases, the estimation of $p$ becomes more reliable, leading to an improved agreement between $p_{\alpha}$ and $p_{HR}$, thanks to the better quality of data. Therefore, although the $p$–HR relation shows a larger scatter compared with the $p$–$\alpha$ relation, it is still a reliable way to estimate $p$ quickly.

One thing worth noting is that there is a nearly horizontal low-density tail to the top of the distribution. We excluded the likely “pileup”-like effect for this feature because the fluxes of these observations are not very high. The possible reason is that $p$–$\alpha$ and $p$–HR relations do not completely follow the linear correlation, i.e., the $p$ derived from the data is smaller than the best-fit $p$–$\alpha$ relation at $\alpha > 1.5$ (see Figure 5) and is larger than the best-fit $p$–HR relation at $HR < 0.2$ (see Figure 6). Therefore, if we use the best-fit relations to estimate $p$ in the high $p$ range, the $p_{\alpha}$ tends to be larger and the $p_{HR}$ tends to be smaller, which will lead to a nearly horizontal tail. Even so, for the vast majority of observations, $p$–$\alpha$ and $p$–HR relations can provide consistent $p$ estimates.

As demonstrated above, both the $p$–$\alpha$ and $p$–HR relations are suitable for estimation of $p$ in the case that the radiative process is dominated by synchrotron radiation in jets. We note that the $p$–HR relation should be instrument dependent because HR is closely related to the instrument response, therefore being valid only for RXTE/PCA data, while the $p$–$\alpha$ relation should be instrument independent.
5.4. Peak Energy versus Spectral Parameters

For many cases in blazars, the log-parabolic model could greatly reproduce the spectra around the synchrotron peak in the SED, which provides a valid method to estimate the energy and flux of the peak (e.g., Massaro et al. 2004; Tanihata et al. 2004; Tramacere et al. 2007, 2009). Therefore, we used the log-parabolic model to estimate $E_p$, following Massaro et al. (2004), which is given by

$$F(E) = K(E/E_1)^{(-a+b \log(E/E_1))} \text{ (ph cm}^{-2} \text{s}^{-1} \text{keV}^{-1}).$$

$E_1$ is the reference energy that is generally fixed to 1 keV, $a$ is the photon spectral index at the energy of $E_1$, $b$ is the curvature parameter, and $K$ is the normalization factor. The values of these parameters can be derived from the spectral fitting process. The peak energy of the synchrotron radiation hump is given by $E_p = E_1(10^{2-a-b})^{1/2}$. The rest-frame peak energy is $E_p = (1 + z)E_p^{obs}$, where $z$ is the redshift. In some cases, parameter $b$ is below 0, which means that the fitted curve is concave, and the resulting peak energy is not the real peak energy of the synchrotron hump. There are many reasons for such a result, such as a concave spectrum and poor quality of data (especially in the high-energy band). For such cases, we could only obtain a rough range of $E_p$: if $b < 0$ and $E_{p, fit} < 3$ keV, then $E_p > 25$ keV or $E_p < 3$ keV; if $b < 0$ and $E_{p, fit} > 3$ keV, then $E_p < 3$ keV ($E_{p, fit}$ is the fitting result of the peak energy). Fortunately, there are only two observations with $b < 0$, and both of them belong to the second case.

For the flaring periods of the five sources, according to the location of the synchrotron radiation SED peak, we roughly divided a total of 276 spectra into three groups: $E_p < 5$ keV (202 spectra), 5 keV $< E_p < 15$ keV (32 spectra), and $E_p > 15$ keV (42 spectra). Figure 8 presents the distributions of three spectral parameters $p$, $\alpha$, and HR for these three groups of spectra. Although our sample is not complete, it still reveals a general trend that, with the synchrotron radiation SED peak energy increasing, both $p$ and $\alpha$ decrease while HR increases. This is consistent with the trend that the spectra are harder with higher peak frequencies seen in other works (e.g., Lin et al. 1999; Giommi et al. 2005; Perlman et al. 2005).

Additionally, some rough constraints on $E_p$ (see also Section 4.1) and the three spectral parameters can be obtained, according to Figure 8: if $E_p$ is lower than 5 keV, then $p$ is higher than $\sim 2.7$, $\alpha$ is higher than $\sim 0.8$, and HR is lower than $\sim 0.35$; if $E_p$ is higher than 5 keV, then $p$ is lower than $\sim 3.0$, $\alpha$ is lower than $\sim 1.0$, and HR is higher than $\sim 0.30$. The constrained range of $\alpha$ is in good agreement with the result in Perlman et al. (2005), where three hard ($\Gamma < 2$, i.e., $\alpha < 1$) spectra correspond to $E_p > 5$ keV.

6. Summary and Conclusions

During its entire life span ($\sim 16$ yr), RXTE had observed 32 TeV blazars, including 2 FSRQs, 1 LBL, 5 IBLs, and 24 HBLs. In this paper, we analyzed the 16 yr RXTE/PCA observational data of the 32 TeV blazars and further selected out five brightest sources to carry out a systematic investigation of X-ray spectral variability during their major flares in the RXTE era, using both empirical spectral fitting (to obtain values of $\alpha$, flux, and $E_p$) and theoretical synchrotron radiation modeling (to obtain $p$ distributions). Our work builds on Xue et al. (2006), which studied only two TeV blazars, confirms and strengthens their main results with a larger sample, and provides many further insights regarding X-ray spectral variability of TeV blazars. We summarize our main results as follows:

**Figure 5.** $p-\alpha$ (electron spectral index–photon spectral index) relation of the five sources. Colored filled squares represent flaring observations; pentagrams represent quiescent-period observations (see Section 2.3 and Table 2). Squares and pentagrams in the same color represent the same source. The black solid line and its shaded region represent the best fit and 1σ uncertainty to all data of flaring periods, i.e., $p = (2.27 \pm 0.03) \times \alpha + (0.81 \pm 0.03)$. The dashed line indicates the relation of $\alpha = (p - 1)/2$ as expected in the optically thin synchrotron radiation spectrum in power-law shape.

**Figure 6.** $p$–HR (electron spectral index–hardness ratio) relation of the five sources. All symbols have the same meaning as in Figure 5. The best fit to all data of flaring periods is $p = (-8.28 \pm 0.16) \times HR + (5.54 \pm 0.05)$. 


1. The cutoff power-law and log-parabolic models could provide evenly good fitting results to the X-ray spectra of all sources. The X-ray spectra, characterized by $\alpha$, display a harder-when-brighter trend during a number of flares of the five brightest sources (i.e., Mrk 421, Mrk 501, PKS 2155–304, PKS 2005–489, and 1ES 1959+650), which is consistent with previous studies.

2. The high quality of the PCA data of the five sources enables detailed synchrotron radiation modeling upon their spectra. It seems clear that the evolution of $p$ also generally follows a harder-when-brighter trend, and the variation of $p$, accompanied by changes of other key parameters, is required to explain the observed X-ray spectral variability of TeV blazars during flares, which would have useful implications for interpreting the associated higher-energy (i.e., gamma-ray) spectral variability that the same population of ultrarelativistic electrons is responsible for. These results confirm and strengthen those of Xue et al. (2006). However, there are some cases that do not follow the harder-when-brighter trend exactly, which might be related to the complex physical conditions in the emitting region or the “contamination” of electron populations from adjacent flares.

3. Electron spectral hysteresis is clearly seen in many but not all $p$-flux plots, rendering itself in a “loop” or oblique “8” shape. Although this phenomenon is often associated with time lags between the soft and hard bands, no apparent hard or soft lag is identified based on our several-day-timescale light curves. Intraday observations might help resolve likely intraday time lags.

4. A tight $p$–$HR$ relation and a tighter $p$–$\alpha$ relation are obtained using spectra of flaring periods, both of which are also applicable to stacked data of quiescent periods, indicating that both relations are independent of flux level. These two relations can be used to estimate $p$ quickly and straightforwardly, and the reliability of $p$ estimation improves with improved data quality.

5. Collectively (i.e., TeV blazars being treated as a whole), $\alpha$ and X-ray luminosity are positively correlated, $E_p$ is negatively correlated with $p$ and $\alpha$, and $E_p$ is positively correlated with HR. All these correlations are in line with the blazar sequence. However, after correcting for the Doppler boosting effect, $\alpha$ and intrinsic X-ray luminosity follow an anticorrelation.

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Appendix A
Photon Spectral Evolution and Photon Spectral Hysteresis

In Figures 2 and 3, we have shown the electron spectral evolution and electron spectral hysteresis, respectively. Given that most of the previous works studied spectral variability through photon spectral index, in this appendix we also show the photon spectral evolution in Figure 9 and photon spectral hysteresis in Figure 10. The photon spectral evolution shows a harder-when-brighter trend (see Figure 9), which is consistent with previous studies, and also shows a similar trend to that revealed by $p$ evolution (see Figure 2). In addition, the photon spectral hysteresis (see Figure 10) shows a similar trend to that seen with electron spectral hysteresis (see Figure 3). Therefore, consistent results on spectral evolution and hysteresis are obtained using either the $\alpha$ or $p$ representation, which is expected given the tight $p$–$\alpha$ relation (see Figure 5).

Appendix B
Opposite Evolution of Spectral Index with Flux

During the period between MJD 50,641 and 50,645 (i.e., panel (13) of Figure 2), Mrk 501 shows a softer-when-brighter trend in terms of $p$ variation, which is opposite to the harder-when-brighter trend existing in most of the studied cases (see Figure 2 and Section 4.2). The spectra and $\alpha$ variations of these observations are shown in Figure 11, which also show a softer-when-brighter trend. Several cases
also present a similar opposite trend, e.g., Mrk 421 in the period between MJD 54,509 and 54,511 (see panel (6)), and PKS 2155–304 in the period between MJD 50,229.2 and 50,230.2 (see panel (16)). This opposite trend of these three cases exists in the transition region between two individual flares, which indicates that it might be due to the interaction between multiple populations of emitting electrons in the two adjacent flares.

Figure 10. Same as Figure 3, but for the evolution of photon spectral index (α).
Figure 11. Same as Figure 1, but for Mrk 501 during the period between MJD 50,641 and 50,645 (see panel (13) of Figure 2).

References

Abdo, A. A., Ackermann, M., Agudo, I., et al. 2010a, ApJ, 716, 30
Abdo, A. A., Ackermann, M., Ajello, M., et al. 2010b, ApJ, 710, 1271
Abeysekara, A. U., Archambault, S., Archer, A., et al. 2017, ApJ, 834, 2
Acciari, V. A., Aliu, E., Arlen, T., et al. 2011, ApJ, 738, 25
Acciari, V. A., Aliu, E., Aune, T., et al. 2009, ApJ, 703, 169
Aharonian, F. A. 2000, NewA, 5, 377
Aleksić, J., Ansoldi, S., Antonelli, L. A., et al. 2015, A&A, 576, A126
Anderhub, H., Antonelli, L. A., Antoranz, P., et al. 2009, ApJ, 705, 1624
Arnaud, K. A. 1996, in ASP Conf. Ser. 101, Astronomical Data Analysis Software and Systems V, ed. G. H. Jacoby, & J. Barnes (San Francisco, CA: ASP), 17
Atoyan, A. M., & Dermer, C. D. 2003, ApJ, 586, 79
Baloković, M., Ajello, M., Blandford, R. D., et al. 2013, EPJWC, 61, 04013
Baloković, M., Puech, D., Madejski, G., et al. 2016, ApJ, 819, 156
Bhargava, J., Gupta, A. C., Papadakis, I. E., & Wiita, P. J. 2016, NewA, 44, 21
Boettcher, M., Mase, H., & Schlickeiser, R. 1997, A&A, 324, 395
Böttcher, M., & Chiang. J. 2002, ApJ, 581, 127
Böttcher, M., & Reimer, A. 2004, ApJ, 609, 576
Böttcher, M., Reimer, A., Sweeney, K., & Prakash, A. 2013, ApJ, 768, 54
Brinkmann, W., Papadakis, I. E., den Herder, J. W. A., & Haberl, F. 2003, A&A, 402, 929
Brinkmann, W., Papadakis, I. E., Raether, C., Mimica, P., & Haberl, F. 2005, A&A, 443, 397
Cui, W. 2004, ApJ, 605, 662
Dermer, C. D., Schlickeiser, R., & Mastichiadis, A. 1992, A&A, 256, L27
Dickey, J. M., & Lockman, F. J. 1990, A&A, 28, 215
Fan, J. H., Yang, J. H., Liu, Y., et al. 2016, ApJS, 226, 20
Fan, J.-H., Yang, J.-H., Liu, Y., & Zhang, J.-Y. 2013, RAAS, 13, 259
Fan, J. H., Yang, J. H., Xiao, H. B., et al. 2017, ApJL, 835, L38
Fossati, G., Buckley, J. H., Bond, I. H., et al. 2008, ApJ, 677, 906
Fossati, G., Celotti, A., Chiaberge, M., et al. 2000, ApJ, 541, 166
Fossati, G., Maraschi, L., Celotti, A., Comastri, A., & Ghisellini, G. 1998, MNRAS, 299, 433
Fraija, N., & Marinelli, A. 2015, ApJ, 70, 54
Garson, A. B., III, Baring, M. G., & Krawczynski, H. 2010, ApJ, 722, 358
Ghisellini, G., Celotti, A., Fossati, G., Maraschi, L., & Comastri, A. 1998, MNRAS, 301, 451
Giannios, D., Uzdensky, D. A., & Begelman, M. C. 2009, MNRAS, 395, L29
Giebels, B., Bloom, E. D., Focke, W., et al. 2002, ApJ, 571, 763
Giannios, D., Uzdensky, D. A., & Begelman, M. C. 2009, MNRAS, 395, L29
Giommi, P., Barr, P., Garilli, B., Maccagni, D., & Pollock, A. M. T. 1990, ApJ, 356, 432
Giommi, P., Piranomonte, S., Perri, M., & Padovani, P. 2005, A&A, 434, 385
Gliozzi, M., Sambruna, R. M., Jung, I., et al. 2006, ApJ, 646, 61
Harris, D. E., Biretta, J. A., Junor, W., et al. 2003, ApJL, 586, L41
Harris, D. E., Cheung, C. C., Biretta, J. A., et al. 2006, ApJ, 640, 211
Hinshaw, G., Larson, D., Komatsu, E., et al. 2013, ApJS, 208, 19
Holder, J. 2014, BrPth, 44, 450
Huang, B., Zhang, X., Xiong, D., & Zhang, H. 2014, JApA, 35, 381
Inoue, Y., & Tanaka, Y. T. 2016, ApJ, 818, 187
Kaplanadze, B., Dorner, D., Romano, P., et al. 2017a, ApJ, 848, 103
Kaplanadze, B., Dorner, D., Romano, P., et al. 2017b, MNRAS, 469, 1655
Kaplanadze, B., Dorner, D., Vercellone, S., et al. 2016a, ApJ, 831, 102
Kaplanadze, B., Dorner, D., Vercellone, S., et al. 2018, MNRAS, 473, 2542
Kaplanadze, B., Romano, P., Vercellone, S., et al. 2016b, MNRAS, 457, 704
Kaplanadze, B., Romano, P., Vercellone, S., & Kaplanadze, S. 2014, MNRAS, 444, 1077
Kaplanadze, S., Kaplanadze, B., Romano, P., Vercellone, S., & Tabagari, L. 2017c, Ap&SS, 362, 196
Kataoka, J., Takahashi, T., Makino, F., et al. 2000, ApJ, 528, 243
Kirk, J. G., Rieger, F. M., & Mastichiadis, A. 1997, A&A, 320, 19
Krawczynski, H., Coppi, P. S., Maccarone, T., & Aharonian, F. A. 2000, A&A, 353, 97
Lin, Y. C., Bertsch, D. L., Bloom, S. D., et al. 1999, ApJ, 525, 191
Lytutkov, M. 2003, NewAR, 47, 513
Maraschi, L., Ghisellini, G., & Celotti, A. 1992, ApJL, 397, L5
Marshall, H. L., Miller, B. P., Davis, D. S., et al. 2002, ApJ, 564, 683
Massaro, E., Perri, M., Giommi, P., & Nesci, R. 2004, A&A, 413, 489
Massaro, F., Tramacere, A., Cavaliere, A., Perri, M., & Giommi, P. 2008, A&A, 478, 395
Mastichiadis, A., & Kirk, J. G. 1997, A&A, 320, 19
Mattox, J. R., Bertsch, D. L., Chiang, J., et al. 1993, ApJ, 410, 609
Mücke, A., & Protheroe, R. J. 2001, ApJ, 561, 121
Mücke, A., Protheroe, R. J., Engel, R., Rachen, J. P., & Stanev, T. 2003, ApJ, 18, 593
Nieppola, E., Valtaoja, E., Tornikoski, M., Hovatta, T., & Kotiranta, M. 2008, Ap&SS, 362, 196
Perlman, E. S., Adams, S. C., Cara, M., et al. 2011, ApJ, 743, 119
Perlman, E. S., Madejski, G., & Georganopoulos, M., et al. 2005, ApJ, 625, 727
Perlman, E. S.,Madejski, G., & Stocke, J. T., & Rector, T. A. 1999, ApJL, 523, L11
Pian, E., Turler, M., Fiocchi, M., et al. 2014, A&A, 570, A77
Pian, E., Vacanti, G., Tagliaferri, G., et al. 1998, ApJL, 492, L17

Ravasio, M., Tagliaferri, G., Ghisellini, G., & Tavecchio, F. 2004, A&A, 424, 841
Rees, M. J. 1978, MNRAS, 184, 61P
Rivers, E., Markowitz, A., & Rothschild, R. 2011, ApJS, 193, 3
Rivers, E., Markowitz, A., & Rothschild, R. 2013, ApJ, 772, 114
Sambruna, R. M., Barr, P., Giommi, P., et al. 1994, ApJ, 434, 468
Sembay, S., Edelson, R., Markowitz, A., Griffiths, R. G., & Turner, M. J. L. 2002, ApJ, 574, 634
Sikora, M., Begelman, M. C., & Rees, M. J. 1994, ApJ, 421, 153
Spada, M., Ghisellini, G., Lazzati, D., & Celotti, A. 2001, MNRAS, 325, 1559
Sushch, I. & H.E.S.S. Collaboration 2015, AASP, 5, 59
Tanihata, C., Kataoka, J., Takahashi, T., & Madejski, G. M. 2004, ApJ, 601, 759
Tramacere, A., Giommi, P., Perri, M., Verrecchia, F., & Tosti, G. 2009, A&A, 501, 879
Tramacere, A., Massaro, F., & Cavaliere, A. 2007, A&A, 466, 521
Urry, C. M., & Padovani, P. 1995, PASP, 107, 803
Wu, Z.-Z., Gu, M.-F., & Jiang, D.-R. 2009, RAA, 9, 168
Xue, Y., & Cui, W. 2005, ApJ, 622, 160
Xue, Y., Yuan, F., & Cui, W. 2006, ApJ, 647, 194
Zhang, Y. H., Bai, J. M., Zhang, S. N., et al. 2006a, ApJ, 651, 782
Zhang, Y. H., Treves, A., Celotti, A., et al. 2002, ApJ, 572, 762
Zhang, Y. H., Treves, A., Maraschi, L., Bai, J. M., & Liu, F. K. 2006b, ApJ, 637, 699
Zhu, S. F., Xue, Y. Q., Brandt, W. N., Cui, W., & Wang, Y. J. 2018, ApJ, 853, 34