Impact of phonons on dephasing of individual excitons in deterministic quantum dot microlenses

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Optimized light-matter coupling in semiconductor nanostructures is a key to understand their optical properties and can be enabled by advanced fabrication techniques. Using in-situ electron beam lithography combined with a low-temperature cathodoluminescence imaging, we deterministically fabricate microlenses above selected InAs quantum dots (QDs) achieving their efficient coupling to the external light field. This enables to perform four-wave mixing micro-spectroscopy of single QD excitons, revealing the exciton population and coherence dynamics. We infer the temperature dependence of the dephasing in order to address the impact of phonons on the decoherence of confined excitons. The loss of the coherence over the first picoseconds is associated with the emission of a phonon wave packet, also governing the phonon background in photoluminescence (PL) spectra. Using theory based on the independent boson model, we consistently explain the initial coherence decay, the zero-phonon line fraction, and the lineshape of the phonon-assisted PL using realistic quantum dot geometries.

Owing to the progress in the semiconductor growth, the self-assembled quantum dots (QDs) offer nowadays optimal quality of the residing exciton transitions [1] with enhanced emission efficiency [2–4] and close to ideal quantum optical properties [5]. Forthcoming applications emerging from combining QDs and nanophotonics – such as, quantum light sources on-chip photonic networks – call for scalability and deterministic QD positioning. In this regard, in-situ electron beam lithography [6] (EBL) has evolved into a suited technique for the deterministic fabrication of quantum light sources [5]. When combined with a low temperature cathodoluminescence imaging, EBL permits to sculpt microlenses above individual QDs, enhancing collection efficiency over a broad spectral range.

Here, using four-wave mixing (FWM) micro-spectroscopy we reveal coherences of single QDs, deterministically embedded in microlenses realized by EBL. The resulting optical signals in these nanophotonic structures exhibit an enhanced signal to noise ratio. This enables us to infer the impact of acoustic phonons on the coherence dynamics of individual QDs. In particular, we report on the exciton zero phonon line (ZPL) dephasing close to the radiative limit in a single QD at 5 K, distinguishing it from its spectral wandering. Phonons are known to play a crucial role in the optical control of QDs [2] [11]. With increasing temperature, we observe an increasing impact of phonon-induced dephasing [12] owing to the polaron formation and wave packet emission [13] [14] and a broadening of the homogenous width γ, attributed to a quadratic coupling between carriers and acoustic phonons [15] [17]. Single QD micro-spectroscopy permits to associate the measured dephasing during the polaron formation with the spectral shape of phonon-assisted transitions, here accessed via photoluminescence (PL). We thus go beyond the FWM experiments performed on QD ensembles [11] [12], and we consistently explain the initial FWM decay, the zero-phonon line (ZPL) fraction, and the lineshape of the phonon-assisted PL using a realistic QD geometry. Additionally, the optical parameters of QDs - i.e. dephasing, lifetime, dipole moments, phonon coupling - are to some extent averaged in QD ensemble measurement due to stochastic distribution of their shapes and alloy composition. This issue is naturally overcome in a single QD spectroscopy carried out here.

When performing FWM on single emitters on a simple planar structure one is confronted with a huge ratio of the resonant background (typically 10^6 – 10^8 in the field, and 10^5 when assisted with high quality antireflection coatings [18]) and the induced FWM. We have recently shown that, using suitable photonic nanostructures [15] [20], one can boost the experimental sensitivity by bringing a large amount of the field amplitude to the vicinity of a QD. This enhances its interaction with the excitonic dipole, and hence reduces the required external power constituting the background. Furthermore, using nanophotonic devices, the FWM is efficiently collected by the detection optics, avoiding the total internal reflection affecting planar structures: assuming the gain in the collection efficiency η and an n−time enhancement of the local excitation intensity, the FWM amplitude is in-
FIG. 1. (a) Spectrally-resolved FWM amplitude generated by a few QD excitons embedded in a lens structure. The targeted QD trion (GX±) is labeled with the green star. The horizontal bar indicates a neutral exciton-biexciton system (GXB) in a QD located at the lens periphery. Inset: Calculated distribution of the near-field intensity for the QD-lens structure. The semiconductor-air interface is shown by the solid black line and the DBR starts below the dashed line. (b) Spectrally-integrated FWM amplitude of a trion in the target QD as a function of the pulse area \( \theta_1 \propto \sqrt{P_1} \) of \( E_1 \). The blue line shows the fit to the expected \( \sin(\theta_1/2) \) amplitude dependence of the FWM.

Increased by \( \sqrt{n \mu^2} \), while maintaining the external power of exciting laser pulses \( E_{1,2,3} \).

In contrast to our previous works, the EBL overcomes the issue of a low yield of optimally functioning devices, when patterning the sample containing randomly distributed QDs. Microlenses processed with EBL can be defined deterministically, spatially matched to QDs with about 30 nm alignment accuracy \[21\], combined with frequency matching guaranteed by their broadband operation beyond 100 nm.

Microlenses with a height of 0.35 \( \mu \)m and 2 \( \mu \)m diameter have been etched, as described in detail in Ref. \[9\], so as to create the hot spot of the excitation and detection mode field exactly at the QD plane, located between the semiconductor-air interface and the microlens surface \[8\] - see inset in Figure 1a. We use InAs QDs grown by metalorganic chemical vapor deposition \[23\]. As a result of the high light-extraction efficiency \[22\] of around 30% in our devices, we routinely note a bright QD photoluminescence, with spectrally-integrated count rate of 200 kHz below the PL saturation (not shown). For comparison, up to 1 kHz PL count rates are typically observed in our setup for high quality QDs embedded in planar samples.

For the FWM spectroscopy we use radio-frequency acousto-optic deflectors providing frequency shifts of \( \Omega_{1,2,3} \) for \( E_{1,2,3} \), respectively. FWM signals, in the lowest order proportional to \( E_1^* E_2 E_3 \) (\( * \) denoting complex conjugate), are then detected by performing optical heterodyning. To select the required heterodyne beat component, we interfere the reflected field with a frequency shifted reference field \( E_R \). Spectrally-resolved interferograms are recorded by a CCD camera \[18\], \[20\] and analyzed by spectral interferometry to retrieve amplitude and phase of the signal. In Figure 1b we show an exemplary two-pulse FWM spectrum, (driving with \( E_1 \) and \( E_2 \) and detecting at the heterodyne frequency \( 2 \Omega_3 - \Omega_1 \), over a range of 25 meV displaying several excitonic transitions. Owing to a strongly improved excitation and collection compared to planar structures \[24\], we here achieve a gain in the signal-to-noise ratio of the measured FWM amplitude by two-orders of magnitude. Exciton-biexciton pairs can be identified (an example is denoted with a green bar above 1375 meV) by employing FWM polarization and delay selection rules \[23\] (not shown). A similar FWM signal was found on another investigated microlens, pointing toward the deterministic character and high-quality of the EBL nano-processing platform.

In the following, we study the dominating transition at 1360.4 meV labeled as \( * \) in Figure 1a. This is to minimize the time required to perform following FWM sequences and thus to avoid drifts. We observe no FWM at \( \tau_{23} < 0 \), showing that it stems from a charged exciton (trion) transition \( GX^\pm \) \[25\]. To illustrate the enhanced in-coupling of \( E_{1,2,3} \) offered by the microlenses, in Figure 1b we present the FWM amplitude as a function of the pulse area \( \theta_1 \propto \sqrt{P_1} \), where \( P_1 \) represents the intensity of \( E_1 \), while \( P_2 \) is fixed to 1.5 \( \mu \)W. The FWM signal displays a Rabi rotation following the expected \( \sin(\theta_1/2) \) dependence \[26\] with the first maximum at \( \sqrt{P_1} = 0.75 \mu \)W, corresponding to a pulse area of \( \theta_1 = \pi/2 \). For higher intensities the FWM signal deviates from this behavior.

To measure the exciton density lifetime \( T_1 \), we employed the three-pulse FWM, where the signal is detected at \( \Omega_1 + \Omega_2 - \Omega_3 \), as a function of the second delay \( \tau_{23} \), displayed in Figure 2. From its exponential decay we determine the lifetime \( \tau = (347 \pm 12) \) ps at \( T = 5 \) K. Such a rather short lifetime, compared to about 1 ns typically observed \[27\] in these structures, is attributed to the selectivity of the FWM technique favoring particularly bright QDs with high dipole moment and thus displaying fast population decay dynamics (less intense transitions in Figure 1a) are expected to exhibit longer \( T_1 \)). Additionally, the radiative lifetime is slightly shortened due to a Purcell effect of the microlenses \[19\].

We now turn to the measurement of coherence dynamics as a function of temperature, to determine the impact of the phonon-interaction on the exciton dephasing. The FWM transient, generated in our time-averaged and multi-repetition heterodyne experiment, is emitted after the arrival of \( E_2 \). It is expected to exhibit a Gaussian echo \[19\], \[24\], owing to a Gaussian spectral wandering of standard deviation \( \sigma \), with the maximum at \( t = \tau_{12} \) and temporal full width at half maximum (FWHM) of
To retrieve $\sigma$, we apply the pulse sequence depicted in Figure 3, namely we keep $\tau_{12} = 110\,\text{ps}$ fixed, while scanning the delay $\tau_{2R}$ between $\mathcal{E}_2$ and $\mathcal{E}_R$. As such, the temporal sensitivity of the experiment $S(t)$ (green curve centered around $\mathcal{E}_R$), originating from the finite spectral resolution of the spectrometer, is scanned through a broad FWM transient. A measurement of $S(t)$ is given in the Supplementary Figure S1. The FWM integrated overlap between $S(t)$ and the echo plotted against $\tau_{2R}$ in Figure 3 indeed reproduces a Gaussian form, with the expected maximum at $\tau_{12} = 110\,\text{ps}$ and $\sigma = 8.2\,\text{meV}$. The measured inhomogeneous broadening (FWHM) $\sqrt{8\ln(2)\sigma}$ is plotted versus temperature in the inset, where we find that it varies only marginally within the investigated temperature range.

To extract the ZPL dephasing rate $\gamma = 1/T_2$ we measure the time-integrated FWM amplitude as a function of $\tau_{12}$. For a fixed $\tau_{2R}$, the echo moves through $S(t)$ when varying $\tau_{12}$. This has previously been compensated by correcting the signal in the time domain by $S(t)$ \cite{19, 20, 24}. However, for sufficiently large $\tau_{12}$, the echo is generated at times not accessible via $S(t)$, such that the signal cannot be retrieved via spectral interference. Here, this issue is overcome by simultaneously increasing $\tau_{2R}$ towards positive times when increasing $\tau_{12}$, such that $S(t)$ probes the same time portion of the echo for every $\tau_{12}$, as depicted in Figure 3. We initially set $\tau_{2R} = 70\,\text{ps}$ to assure the time ordering between $\mathcal{E}_R$ and FWM, as required to perform spectral interferometry. The resulting FWM amplitude for several temperatures is presented in Figure 3.

After the echo has developed for $\tau_{12} > 150\,\text{ps}$, the decay of the signal is given by a single exponential, yielding the dephasing time $T_2$. At low temperature the latter reaches $T_2 \approx 1.3T_1$, close to the radiative limit ($T_2 = 2T_1$), in spite of the significant inhomogeneous broadening. As shown in the inset, with increasing temperature, $T_2$ shortens rapidly consistently with previous measurements on ensembles \cite{12} and more recent complementary approaches employing photon-correlation techniques \cite{27}. The dominant term in the electron-phonon coupling in semiconductors is linear in the lattice displacement, i.e., it is linear in the phonon creation and annihilation operators. For the present case of a QD excited at the lowest exciton transition, which represents a two-level system, this reduces to the independent boson model \cite{15}. This model, for the 3D acoustic phonon density of states, provides a band of phonon assisted transitions and an unbroadened ZPL. The finite width of the ZPL and its temperature dependence are explained by phonon processes, which are of second order in the
Once the wavepacket has left the QD, the phonon-assisted transitions have dephased and are not further contributing to the FWM. In the $\tau_{12}$-dependence of the FWM signal this manifests itself as a fast decay on a timescale of about 2 ps [29] clearly revealed in Figure 4.

The final value after the initial drop $FWM_{\text{drop}}$ is plotted as a function of temperature in Figure 4. With increasing temperature the phonon coupling becomes more effective and, accordingly, already at $T = 30$ K the coherence decays by a factor of 5 during the first 5 ps. This explains why the initial value of the signal seen in Figure 3b, where this initial drop is not resolved, rapidly decays with increasing temperatures.

In the spectral domain, the initial decay is associated with a broad phonon background around a ZPL [29, 30]. This is seen in the PL spectra taken from the same QD, shown in Figure 4. For low temperatures the background is asymmetric reflecting the dominance of phonon emission processes over absorption processes, while at higher temperatures, when the thermal occupation of the involved phonons becomes much larger than one, the phonon background becomes symmetric.

For the theoretical modeling of the signals we employ the standard model of a QD coupled to acoustic phonons via the pure dephasing mechanism [29, 30, 31], which has been proven to successfully describe a variety of optical phenomena in single QDs and QD ensembles. For this model, exact analytical formulas for linear and nonlinear optical signals after excitation with an arbitrary series of short laser pulses can be obtained within a generating function formalism [32, 33]. To be specific, we model the QD trion transition as a two-level system, which is coupled via deformation potential coupling to longitudinal acoustic (LA) phonons with a linear dispersion relation. Assuming an approximately harmonic confinement potential, we take Gaussian-shaped wave functions for electrons and holes with electron localization lengths $a_r$ in the in-plane direction and $a_z$ in the out-of-plane direction, i.e., we model a lens-shaped QD and treat the exciton wave function as a product of electron and hole wave functions.

The results of our calculations are shown as solid lines in Figure 4 and as dashed lines in Figure 3. We use electron localization lengths $a_r = 8$ nm and $a_z = 1$ nm (the respective hole localization lengths are scaled by 0.87) and assume GaAs parameters. We find an excellent agreement between theory and experiment for both the FWM signals and the PL spectra over the whole range of temperatures, however for an increased phonon coupling strength. Specifically, to achieve agreement with both the FWM signal and phonon background in PL simultaneously, we had to increase the phonon coupling constant by a factor of 1.5. This we model by increased deformation potentials $D^e = 10.5$ eV and $D^h = -5.25$ eV, with respect to the standard parameters $D^e = 7.0$ eV and $D^h = -3.5$ eV, which have previously been used to
quantitatively describe various optical signals from QD structures [2,4]. A more detailed discussion of the role of the parameters for the FWM signals and PL spectra can be found in the Supplemental Information. While we do not have a definite explanation, we note that similarly higher values for the deformation potentials can also be found in the literature [36], and an increased deformation potential has also been used to explain mobilities in a 2DEG [37,39]. One explanation could be that in the present sample there are additional mechanisms like piezo-electric coupling, which are usually negligible but contribute here, e.g., because of an increased spatial separation of electron and hole wave functions [25], leading to an effective increase of the coupling described by larger deformation potentials.

Finally, it is instructive to compare the FWM initial decay with the ZPL weight in PL (Z), defined as fraction of ZPL in the total absorption spectrum [12]. Within the formalism describing our experiment we obtain FWM_{drop} \propto Z^2. We approximate Z as the PL ratio between the measured ZPL and the entire PL, including the phonon background. In spite of finite spectral resolution and significant \sigma, for all considered temperatures we obtain close agreement between FWM_{drop} and Z^2 independently estimated from PL, as shown in Figure 4d.

In this Article, we have employed EBL to deterministically embed QDs within microlenses, providing a convenient nanophotonic platform to perform coherent nonlinear spectroscopy of individual QDs. Microlens structures enabled to efficiently penetrate across the dielectric boundary and to tightly focus the light field at the QD location, which has been exploited to perform FWM microspectroscopy. We have measured and modeled the role of acoustic phonons on the coherence of single QD excitons, in particular corroborating signatures of single phonon wave packet emission in FWM and PL. Our fundamental studies, aiming to understand the complex interplay between charges and lattice vibrations, are at the heart of condensed matter optics. They are relevant for a large class of individual emitters in solids, like epitaxial and colloidal QDs or colour centres in diamond, or emerging class of individual emitters in solids, like epitaxial and Research (BMBF) through the VIP-project QSOURCE (Grant No. 03V0630), and by the project EMPIR 14IND05 MIQC2 within the European Union’s Horizon 2020 research and innovation programme.

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**Supporting Information**

**SPECTROMETER RESPONSE FUNCTION**

The measured spectrometer response function $S(t)$ is shown in Supplementary Figure S1 together with the fit function

$$S(t) = 8288 \cdot \exp \left( \frac{-|t - \tau_1|}{51 \text{ ps}} \right) + 7888 \cdot \exp \left[ -\left( \frac{t - \tau_1}{76 \text{ ps}} \right)^2 \right].$$

(1)

Supplementary Figure S1. Measured spectrometer response function with fit.

**THEORETICAL FIT OF THE INITIAL DECAY IN FWM AND PHONON ASSISTED TRANSITIONS IN PL**

The theoretical curves have been calculated on the basis of a two-level system using a generating function formalism. The FWM signal is obtained from the third-order nonlinear polarization given by Eqs. (5) and (6) in Ref. [1]. The PL spectra are calculated from the linear order nonlinear polarization given by Eqs. (12) and (13) in Ref. [1]. The imaginary part of the Fourier transform of the latter gives the absorption spectrum; the PL spectrum is then obtained as the mirror image with respect to the ZPL.

The wave functions $\psi^i$ for electron (i=e) and hole (i=h) in a lens-shaped QD with harmonic confinement potential read

$$\psi^i = \frac{1}{\pi^{3/4} a_r^i \sqrt{a_z^i}} \exp \left\{ -\frac{1}{2} \left[ \left( \frac{r}{a_r^i} \right)^2 + \left( \frac{z}{a_z^i} \right)^2 \right] \right\},$$

(2)

where $a_r^i$ are the localization lengths in the in-plane direction with $r^2 = x^2 + y^2$ and $a_z^i$ the localization lengths in the out-of-plane direction. Assuming the same confinement potential for electrons and holes, the ratio between electron and hole localization lengths is determined by the effective masses via $a^h/a^e = (m^e/m^h)^{1/4} = 0.87$. The wave functions enter in the matrix element for deformation potential coupling in the following way:

$$g_q = \frac{q}{\sqrt{2 \rho V \omega_q}} \left( D^e \exp \left\{ -D^e \left[ (q_r a_r^e)^2 + (q_z a_z^e)^2 \right] \right\} - D^h \exp \left\{ -D^h \left[ (q_r a_r^h)^2 + (q_z a_z^h)^2 \right] \right\} \right).$$

(3)

Here, $q$ is the phonon wave vector, $\rho = 5.37$ g/cm$^3$ the density, $V$ the volume and $\omega_q = c_{LA}|q|$ is the LA phonon dispersion relation, with the sound velocity $c_{LA} = 5115$ m/s. The deformation potential coupling constants are $D^e$ and $D^h$ for electron and holes, respectively. As discussed in the paper, to achieve agreement between theory and experiment over the whole temperature range we have taken $D^e = 10.50$ eV and $D^h = -5.25$ eV for the curves shown in Figure 4b of the paper. In principle, one could also change the values of the density $\rho$ or the sound velocity $c_{LA}$ to increase the coupling constant $g_q$. However, such scaling involves a factor of $1/1.5^2$, which brings these parameter well beyond the above-mentioned values, which are well established in the literature. The results for additional temperatures, namely $T = 9$, 15, 25 and 40 K are shown in Supplementary Figure S2. Also for these curves we find an agreement between theory and experiment.

Next, we discuss the theoretical fits for the standard parameters used in several combined experimental and theoretical studies [2,4] with $D^e = 7.0$ eV and $D^h = -3.5$ eV when fitting both the initial FWM drop and the PL spectra simultaneously. In this case, the only free parameters are the localization lengths of the wave
functions. In Supplementary Figure S3 we have taken $a_r = 8 \text{ nm}$ and $a_z = 1 \text{ nm}$. For the phonon background in the PL spectra we still get the same excellent agreement between theory and experiment because the shape of the spectra is not influenced by the total strength of the coupling. The height of the ZPL however is over estimated by the simulation. This is confirmed, when looking at the FWM signal in Supplementary Figure S3a, where we find that the final values of the drop $FWM_{\text{drop}}$, which are reached at $\tau_{12} = 5 \text{ ps}$, are different from the experimentally observed results indicating that the coupling is obviously too weak.

On the other hand, we can fit the initial decay in the coherence using, e.g., a spherical dot with $a_r = a_z = 3 \text{ nm}$ as shown in Supplementary Figure S4. Due to the smaller dot size the coupling is increased and we see that the final values of the initial drop $FWM_{\text{drop}}$ are now well reproduced. However, the decay is much faster than in the case of the lens shaped dot in Supplementary Figures S2 and S3. This fast drop is associated with an increased broadening of the phonon background spectra. Accordingly, the fits of the phonon background shown in Supplementary Figure S4 exhibit no agreement with the experimental measurements, while the height of the ZPL is well reproduced. Also for other combinations of in-plane and out-of-plane localization lengths no simultaneous fit of the spectral and temporal data could be found.

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