Double Asymptotic Scaling in Drell-Yan Processes

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Abstract

Double scaling may be observed in the Drell-Yan process at the Tevatron and in the ALICE detector at the Large Hadron Collider in a window of masses. In the double scaling limit of the cross section, the higher order QCD corrections are quite large, and are driven by the rise in gluon densities in the double scaling regime. The naive parton model cannot be valid even at asymptotic energies in the region of phase space where double scaling holds.
The HERA experiments have observed double asymptotic scaling of the deep inelastic structure function \( F_2(x, Q^2) \) by probing a region of kinematics with large momentum transfer, \( Q^2 \), and small Bjorken variable, \( x \). In this paper, we discuss the possibility of future hadron colliders observing similar double scaling phenomena. We examine the simplest such process here—that of Drell-Yan production of a lepton pair. The appropriate kinematic range will be accessible to the ALICE experiment at the CERN Large Hadron Collider and to experiments at the Tevatron.

In this process a lepton pair is produced in a hadron collider at a centre of mass energy \( \sqrt{S} \). The kinematics of the pair is specified by its invariant mass, \( Q \), and the longitudinal momentum fraction, \( x_F = 2 q_F / \sqrt{S} \). If \( x_F = 0 \), then at large \( \sqrt{S} \) and \( Q \), the ratio \( \tau = Q^2 / S \) can be small. The process will probe parton densities in the double scaling region \( x^0_1 = x^0_2 = \sqrt{\tau} \ll 1 \) and scale \( Q^2 \) large. The pair mass must satisfy the conditions \( Q_0 \leq Q \leq x_0 \sqrt{S} \) with appropriate choices of \( Q_0 \) and \( x_0 \). In this region the higher order QCD corrections are computable in the double asymptotic scaling (DAS) approximation and are not negligible. This is what we show in this communication.

The Drell-Yan cross section is the sum of the \( \bar{q}q \) annihilation process and the QCD Compton process \([4]\). The cross section at leading order (LO) in QCD \((i.e. \text{ up to } O(\alpha_s) \text{ in the strong coupling constant})\) comes from the diagrams shown in Figure [1]. We treat the two sets separately. Take the double differential cross section for the annihilation contribution to the Drell-Yan cross section—

\[
\frac{d\sigma_a}{d\log \tau dx_F} = \frac{4\pi\alpha^2}{9S/\sqrt{x_F^2} + 4\tau} \left\{ H(x^0_1, x^0_2, Q^2) \left[ 1 + \frac{2\alpha_s}{3\pi} \left( \frac{4}{3} \tau^2 + 1 \right) \right] + \frac{2\alpha_s}{3\pi} \left[ \int x^0_1 \int x^0_2 H(x^0_1, x^0_2, Q^2) f_q \left( \frac{x^0_1}{x^0_1} \right) + \int x^0_2 \int x^0_1 H(x^0_1, x^0_2, Q^2) f_q \left( \frac{x^0_2}{x^0_2} \right) \right. \\
+ \left. \int x^0_1 \int x^0_2 H(x^0_1, x^0_2, Q^2) \tilde{f}_q(z, y^*) \right\}.
\]

The DAS approximation must be consistently applied to the whole expression, \(i.e.\), to both the “finite parts”,

\[
f_q(z) = (1 + z^2) \left( \frac{\log(1 - z)}{1 - z} \right) + \frac{3}{2} \left( \frac{1}{1 - z} \right) - 2 - 3z,
\]

\[
\tilde{f}_q(z, y^*) = -2(1 - z) + \frac{1 + z^2}{(1 - z)_+} \left[ \frac{1}{(1 - y^*_+)} + \frac{1}{y^*_+} \right],
\]

and the parton density combination,

\[
H(x_1, x_2, Q^2) = \sum_f e^2_f [q_f(x_1, Q^2)\bar{q}_f(x_2, Q^2) + (x_1 \leftrightarrow x_2)].
\]

Before moving on to technicalities, we would like to set out the justification for this claim. Recall that in defining the parton densities through the DIS process, we absorbed collinear divergent parts of the higher order correction into the definition of these densities through renormalisation group equations which are the DGLAP equations. Of the collinear singular
part, the terms proportional to \( \log(Q^2/\Lambda^2) \) give the splitting functions (Mellin transforms of the anomalous dimensions of the leading twist operators) and the remainder are called “finite parts”. These latter are process dependent. In the DAS approximation for deep-inelastic lepton scattering, the DGLAP equations are solved analytically by expanding the anomalous dimensions about their right-most singularity at \( n = 1 \) in Mellin space i.e. they are expanded in Mellin space in the quantity \( \Delta = n - 1 \). The leading terms are of the form \( 1/\Delta \) (and come entirely from the splitting functions), and the first sub-leading terms are independent of \( \Delta \).

In the Drell-Yan cross section in eq. (4) some of the collinear divergent parts of the process have been absorbed into the scale dependent parton density combination in eq. (4). If this is treated in the DAS limit, then certainly the remaining part of the collinear divergent terms, in eq. (4), must be given the same treatment, to wit the \( 1/\Delta \) and constant terms must be retained in the Mellin transform (with respect to \( \tau \)) of eq. (4).

In order to be able to use the DAS forms of the quark and gluon distribution functions in the double probability distributions given in eq. (4), we need to first express the quark distributions in terms of the singlet (S) and non-singlet (NS) components. We define these through the equations

\[
q^S_f = n_F \sum_f (q_f + \bar{q}_f) \quad \text{and} \quad q_{NS,f} = 2n_F q_f - q^S_f.
\]  

(5)

Here \( n_F \) is the number of active flavours. This allows us to express the quark density for flavour \( f \), \( q_f \), in terms of \( q^S \) and \( q_{NS,f} \). In the DAS limit where the non-singlet can be neglected (up to subleading order) compared to the singlet— \( q_f \) is just given by the singlet quark densities.

Thus in the DAS limit the DGLAP equations can be solved exactly to give

\[
G(x, Q^2) = xg(x, Q^2) = \left(\frac{N^2}{4\pi\gamma\sigma}\right)^{1/2} \exp \left[ 2\gamma\sigma - \delta + \frac{\sigma}{\rho} \right] \left\{ 1 + O(1/\sigma) \right\},
\]

(6)

\[
Q(x, Q^2) = xq^S(x, Q^2) = N'G(x, Q^2)\gamma/\rho.
\]

(7)

where \( \rho \sigma = \log(x_0/x) \) and \( \sigma/\rho = \log(t/t_0) \) with \( t = \ln(Q^2/\Lambda^2) \) (\( t_0 \) which is \( t \) evaluated at \( Q_0^2 \), and \( x_0 \) are chosen so that the approximation holds). The usual gluon density function is denoted by \( g(x, Q^2) \). The constants appearing in these equations are given by

\[
\gamma^2 = \frac{12}{\beta_0}, \quad \delta = \frac{1}{\beta_0} \left[ 11 + \frac{2}{27}n_F \right], \quad \beta_0 = 11 - \frac{2}{3}n_F,
\]

(8)

and the running coupling \( \alpha_s(t) = 4\pi/(\beta_0 t) \). \( N \) and \( N' \) are overall normalisation constants which will be fitted with data or parametrisations. These are expressions valid to lowest order in the strong coupling constant \( g_s \).

There are no \( 1/\Delta \) terms in the Mellin transform of \( f_q \) and \( \tilde{f}_q \). The constant terms are identifiable by taking the first moment of these quantities—

\[
M[f_q] = -\frac{7}{2} + O(\Delta), \quad \text{and} \quad M[\tilde{f}_q] = -1 + O(\Delta).
\]

(9)
As a result, the leading part of the finite terms can be modelled by the $z$-space functional forms—

$$f_g(z) \simeq -\frac{7}{2}\delta(1-z) \quad \text{and} \quad \tilde{f}_g(z, y^*) \simeq -\delta(1-z). \quad (10)$$

The delta-functions tell us that this subleading correction is due entirely to the soft part of the phase space in the QCD corrections.

Substituting these into eq. (1) then gives us—

$$\left. \frac{d\sigma_s}{d\log \tau dx_F} \right|_{x_F=0} = \frac{2\pi\alpha_s^2 H(\sqrt{\tau}, \sqrt{\tau}, Q^2)}{9S\sqrt{\tau}} \left[ 1 + \frac{2\alpha_s}{3\pi} \left( \frac{4}{3} \pi^2 - 6 - \frac{1}{2\sqrt{\tau}} \right) \right]. \quad (11)$$

The appropriate expression for the parton density combination is

$$H \left( x_1^0, x_2^0, Q^2 \right) = H_0 \left( \frac{1}{4\pi\sqrt{\sigma_1 \sigma_2}} \right) \exp \left[ 2\gamma(\sigma_1 + \sigma_2) - 2\delta \log(t/t_0) \right] \frac{\gamma}{\rho^2 x_1^0 x_2^0}, \quad (12)$$

where $x_1^0 = x_2^0 = \sqrt{\tau}$, and hence $\sigma_1 = \sigma_2 = \log(x_0/\sqrt{\tau})$. The overall normalisation constant $H_0$ can be fixed from data on parton densities.

We turn next to the contribution of the Compton process to the cross section. The full expression is—

$$\frac{d\sigma_c}{d\log \tau dx_F} = \frac{\alpha_s^2}{9S\sqrt{x_F^2 + 4\tau}} \left\{ \int_{x_1^0}^1 \frac{dx_1}{x_1} G(x_1, x_2, Q^2) f_g \left( \frac{x_1^0}{x_1} \right) + \int_{x_2^0}^1 \frac{dx_2}{x_2} G(x_2, x_1, Q^2) f_g \left( \frac{x_2^0}{x_2} \right) \right. \right.$$

$$+ \left. \int_{x_1^0}^1 \frac{dx_1}{x_1} \int_{x_2^0}^1 \frac{dx_2}{x_2} (1-z)(x_1 + x_2) \tilde{f}_g(z, 1-y^*) \right.$$

$$+ \left. \int_{x_1^0}^1 \frac{dx_1}{x_1} \int_{x_2^0}^1 \frac{dx_2}{x_2} (1-z)(x_1 + x_2) \tilde{f}_g(z, y^*) \right\} \quad (13)$$

where the finite parts,

$$f_g(z) = \left[ z^2 + (1-z)^2 \right] \log(1-z) + 1 - 6z(1-z), \quad (14)$$

$$\tilde{f}_g(z, y^*) = 2z(1-z) + (1-z)^2 y^* + \left[ z^2 + (1-z)^2 \right] \frac{1}{y^+_1}, \quad (15)$$

and the parton density combination,

$$G(x_1, x_2, Q^2) = g(x_2, Q^2) \sum_f e_f^2 [q_f(x_1, Q^2) + \bar{q}_f(x_1, Q^2)] \quad (16)$$

are both to be evaluated consistently in the DAS limit.

The leading parts of the finite terms are identifiable from the first moment—

$$M[f_g] = -\frac{13}{18} + \mathcal{O}(\Delta), \quad \text{and} \quad M[\tilde{f}_g] = \frac{1}{3} \left( 1 + y^* + \frac{2}{y^+_1} \right) + \mathcal{O}(\Delta). \quad (17)$$

In $z$-space, they can be modelled up to subleading order in $\Delta$ by the functions

$$f_g(z) = -\frac{13}{18} \delta(1-z) \quad \text{and} \quad \tilde{f}_g(z, y^*) = \left[ \frac{1}{3} \left( 1 + y^* + \frac{2}{y^+_1} \right) \right] \delta(1-z). \quad (18)$$
Substituting into eq. (13) gives the DAS result—
\[
\frac{d\sigma_c}{d\log\tau dx_F} \bigg|_{x_F=0} = \frac{2\pi\alpha_s^2}{9S\sqrt{\tau}} \left[ \frac{13}{9} + \frac{1}{2\sqrt{\tau}} \right].
\] (19)

The appropriate expression for the parton density combination is
\[
G\left(\sqrt{\tau}, \sqrt{\tau}, Q^2\right) = G_0 \left(\frac{1}{4\pi\alpha_s}\right) \exp\left[-2\delta\log\frac{t}{t_0}\right] \left\{ \frac{\gamma}{\rho\sqrt{\tau}} \exp[4\gamma] \right\}.
\] (20)

The constant $G_0$ can again be fixed from the numerical values of the parton densities.

This concludes the computation, since the Drell-Yan cross section
\[
S \frac{d\sigma_a}{d\log\tau dx_F} \bigg|_{x_F=0} = S \frac{d\sigma_a}{d\log\tau dx_F} \bigg|_{x_F=0} + S \frac{d\sigma_c}{d\log\tau dx_F} \bigg|_{x_F=0}
\] (21)

and the two terms on the right are given in eqs. (11,19). We make three remarks—

- Since $x_F = 0$, we have $x_1^0 = x_2^0 = \sqrt{\tau}$, and hence
  \[
  \sigma_1 = \sigma_2 = \sqrt{\log(t/t_0)} \log(x_0/\sqrt{\tau}).
  \] (22)

  Also, $t = \log(S\tau/\Lambda^2)$. Hence, the only free variable in eqs. (11,19) is $\tau$.

- In general, QCD corrections involve parton densities at all momentum fractions in the range $\sqrt{\tau} < x < 1$, whereas in the DAS only the densities at $x = \sqrt{\tau}$ are involved. This is due to the delta-functions in eqs. (10,18). The same physics reduces the DGLAP equations for structure functions to the wave equation [1].

- The coefficient of $\alpha_s$ in eq. (11) is negative and larger than unity. This is not a problem, since this cross section is not physical, being only one part of the total physical cross section. Adding eq. (19) makes the total physical cross section positive.

Since we examine DAS at leading loop order, the parton densities to be used in fixing the normalisations $H_0$ and $G_0$ must be obtained at LO. For this reason we work with the GRV 94 LO [3] set of parton density parametrisations. We find that DAS is valid if we choose $Q_0^2 = 1$ GeV$^2$ and $x_0 = 0.1$. It turns out that $G_0/H_0 \approx 40$. Such a large value of this ratio is generic, since the gluon densities are substantially larger than the quark densities. As a result, the Compton process dominates over the annihilation process in the DAS.

Recall that the increase of the Drell-Yan K-factor over unity is due to the parts of the cross section proportional to $\alpha_s$. From the DAS expressions, and the fact that $G_0 \gg H_0$, it is clear that the K-factor is much larger than unity for $\sqrt{S} \to \infty$. In fact, from the behaviour of $\alpha_s$, one can see that the K-factor vanishes as $1/t$. The rest of the factors in eqs. (11,19) are almost independent of $S$ at fixed $\tau$. Since the K-factor in the DAS remains non-negligible at the largest accessible energies, the naive parton model cannot be recovered wherever DAS is valid. In contrast, the K-factor at larger values of $\tau$ are rather close to unity at these energies [1], and the parton model may be taken as a reasonable approximation to reality.
In Figure 2 we compare the DAS forms of the cross section with the full one-loop expression. It is interesting to note the disagreement between the two sets of computations at low $\tau$. This is expected, since low $\tau$ corresponds to low $Q^2$, where DAS breaks down. Similarly, at large $\tau$, DAS breaks down because $x$ is not small enough. Thus, the applicability of DAS to Drell-Yan is bounded both above and below, in $\tau$. This is to be contrasted with the situation in DIS.

In Figure 3 we show the differential cross section $S d\sigma/d \log \tau dx_F$ as a function of $\sqrt{\tau}$ for three different values of $\sqrt{S}$. Note that the cross section for these three values of $\sqrt{S}$ lie almost on one single universal curve. This is the double scaling curve. Departures from DAS are visible at $\sqrt{\tau} \approx x_0$ and $\tau S \approx Q^2_0$.

In the DAS limit of hadronic cross sections, the leading correction is universal—this is the $1/\Delta$ piece which comes entirely from the splitting functions, i.e., the collinear region of phase space, and is absorbed into the parton densities. The next term in the expansion in $\Delta$ is not universal. However, since it arises in the soft part of phase space, it should be easier to evaluate and sum the DAS phase space to all loop orders than to evaluate the full correction. In fact, this resummation might even be necessary. The reason is in the large value of the K-factor, as mentioned above. Since the one loop correction is so large, it may actually be necessary to sum the $\log Q^2 \log(1/\tau)$ contributions to all orders, so as to describe the physical process. This is an extension to hadronic collision of the usual DLL procedure in lepton-proton DIS. We leave this for the future.

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Figure 1: The diagrams that contribute to the Drell-Yan process up to $O(\alpha_s)$ in the strong coupling constant — (a) the annihilation diagrams and (b) the Compton diagrams.

Figure 2: The DAS prediction for the Drell-Yan cross section as a function of $\sqrt{\tau}$ for $x_F = 0$ compared with the full cross section at three different values of $\sqrt{S}$. In each case, the DAS curve is the lower one. The curves for 16 TeV and 1.8 TeV have been scaled by a factor of 100 and .01 respectively for clarity in display.
Figure 3: The DAS prediction for the Drell-Yan cross section as a function of $\sqrt{\tau}$ for $x_F = 0$ at the values of $\sqrt{S}$ shown.