Design of a scanning polarimetric scatterometer for rough surface scattering measurements

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Abstract. This paper presents the mechanical design of a goniometric scatterometer to be used to measure the light scattered in rough surfaces. Advances in the instrumentation of a polarization measurement system are also presented. It has been found that the linear part of the retardance vs. voltage curves for a liquid crystal retarder corresponds to retardance between $\lambda$ and $\lambda/2$ and that this range of retardance leads to variations in the frequency components of the time-varying detected signal for sinusoidal voltage. Preliminary results for a Stokes polarimeter are presented.

1. Introduction

Rough surface scattering has applications in many scientific and technological areas, for example, in the interpretation of remote sensing images or in the testing and imaging of printed circuits [1-6]. While many of the recent theoretical, numerical and experimental advances in this area have used 1D surface structure, for example lines on a flat substrate, many applications involve scattering from surfaces with 2D surface roughness. This type of surface has proved to be more difficult to analyze theoretically and more difficult to measure experimentally. However, recent advances in the theoretical and numerical aspects of scattering of vector-electromagnetic waves in rough surfaces have opened up the possibility of studying the 2D roughness problem. To verify the validity of the numerical models developed for these cases is important that experimental results be obtained to compare with the numerical results [7-11]. For a 2D rough surface this involves measuring the scattered light in the hemisphere above the rough surface. This compares to the measurement in a single plane for 1D rough surfaces. The measurement of the light in the full hemisphere can be achieved by using a mirror to collect the scattered light and direct it to a detector, but with this method the polarization of the scattered light is affected by the optical system and it is very difficult to compensate the effects of all the optical components to separate only the polarization of the scattering process. To measure the polarization of the scattered light we have chosen to build a scanning scatterometer which uses two rotational movements to scan a detector over the hemisphere of interest. The polarization will be measured by using a system of variable liquid crystal retarders. In section 2 we discuss the mechanical construction of the scatterometer and we present the advances in the polarization measurement system in section 3.
2. Mechanical Design

Figure 1 shows a diagram of the design of the scatterometer. The position of the rough surface is indicated in the figure and can be adjusted in height to account for different types and thicknesses of substrates. The illumination optics consist of a laser diode with a wavelength of 635nm, a collimating lens and polarization optics (see section 3). The arm of the illumination optics can be rotated to give the desired incidence angle of the light on the rough surface. The detection optics consist of a detector (PMT or photodiode) a collecting lens and polarization optics. This arm of the scatterometer can also be rotated to any inclination angle to position the detector. The third rotation motor rotates the sample/illumination arm assembly 360° to produce the scan of the detector over azimuthal angle to cover the entire hemisphere above the rough surface. The only part of the hemisphere which cannot be measured is the part blocked by the arm holding the illumination optics and this corresponds to approximately 1.5°. The resolution of all the rotations is ± 0.25°. The rotations and reading of the detector data will be performed via a LabView program.

![Figure 1](image)

Figure 1. the mechanical design of the scatterometer

3. Polarization Measurement

The measurement of the polarization (Stokes vector or Mueller matrix) of light is well established [12]. Recently, more use has been made of variable retarders, for example, liquid crystal retarders which have changes of the retardance depending on the voltage applied to the liquid crystal system. The use of this type of systems involves the application of sinusoidal variations of voltage to give sinusoidal variations of the retardance, and thus harmonic variations of the signal detected after transmission in a linear polarizer. We have acquired a liquid crystal system manufactured by Meadowlark Optics® and we have implemented a Stokes polarimeter as an intermediate step to producing a Mueller matrix polarimeter. However, because of linearity considerations for the
retardance variation, we have had to modify the equations relating the detected signal to the Stokes parameters measured.

**Figure 2.** Retardance vs voltage for a typical liquid crystal retarder for a wavelength of 633nm

Figure 2 shows the typical variation of the retardance with the voltage applied. It can be seen that the curve is not linear, especially for higher values of the voltage which give lower values of retardance. If the retardance does not depend linearly on the voltage then the voltage variations (which are usually sinusoidal) cannot be related directly to the retardance variations, and the analysis of the signals becomes very complicated. To avoid these problems we use the linear region of retardance between $\lambda$ and $\lambda/2$ for a wavelength of 633nm, corresponding to voltage values between approximately 1.5V to 2.5V. An analysis of the detected signal is given below.

**Figure 3.** the set-up for a Stokes polarimeter. The angles associated with each component refer to the relative angle of the axis of that component.

Figure 3 shows the setup used for the Stokes polarimeter. The light to be analyzed passes through two liquid crystal variable retarders with their axes at 45° to each other and finally through a linear polarizer with its transmission axis parallel to the axis of the first retarder. The system affects the Stokes vector of the light following the relation

$$S = MS_i$$

(1)
where $S_i$ is the Stokes vector to be measured, and $S$ is the Stokes vector of the light at the detector. In particular, the detected intensity is the first term of the Stokes vector $S$. The term $M$ is the Mueller matrix of the system and can be analyzed in terms of the Mueller matrices of the components:

$$M = M_p(0^\circ)M_R(\delta_2,45^\circ)M_R(\delta_1,0^\circ)$$

where $M_p(0^\circ)$ is the Mueller matrix of a linear polarizer with its transmission axis at $0^\circ$, and $M_R(\delta,\theta)$ is the Mueller matrix of a retarder of retardance $\delta$ with its fast axis at $\theta$. These matrices are given by

$$M_p(0^\circ) = \frac{1}{2} \begin{pmatrix} 1 & 1 & 0 & 0 \\ 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

$$M_R(\delta,\theta) = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos^2 \theta + \cos \delta \sin^2 \theta & (1 - \cos \delta) \sin 2\theta \cos 2\theta & -\sin \delta \sin 2\theta \\ 0 & (1 - \cos \delta) \sin 2\theta \cos 2\theta & \sin^2 \theta + \cos \delta \cos^2 \theta & \sin \delta \cos 2\theta \\ 0 & \sin \delta \sin 2\theta & -\sin \delta \cos 2\theta & \cos \delta \end{pmatrix}$$

We also have

$$\delta = \frac{3\pi}{2} + \frac{\pi}{2} \sin \omega t$$

for both retarders, with $\omega$ being the frequency of oscillation of the applied voltage. This gives

$$\sin \delta = -\cos \left( \frac{\pi}{2} \sin \omega t \right)$$

$$\cos \delta = \sin \left( \frac{\pi}{2} \sin \omega t \right)$$

Substituting equations (3), (4) and (6) in equation (2) the Mueller matrix for the system becomes

$$M_R(\delta,\theta) = \frac{1}{2} \begin{pmatrix} 1 & \sin \left( \frac{\pi}{2} \sin \omega_2 t \right) & \cos \left( \frac{\pi}{2} \sin \omega_1 t \right) & \cos \left( \frac{\pi}{2} \sin \omega_2 t \right) \\ \sin \left( \frac{\pi}{2} \sin \omega_2 t \right) & 1 & \cos \left( \frac{\pi}{2} \sin \omega_1 t \right) & \cos \left( \frac{\pi}{2} \sin \omega_2 t \right) \\ 0 & \cos \left( \frac{\pi}{2} \sin \omega_1 t \right) & 1 & \cos \left( \frac{\pi}{2} \sin \omega_2 t \right) \\ 0 & 0 & \cos \left( \frac{\pi}{2} \sin \omega_1 t \right) & 1 \end{pmatrix}$$

The detected intensity is the first element of the detected Stokes vector.
\[ S_0 = \frac{1}{2} \left\{ S_{i0} + \sin \left( \frac{\pi}{2} \sin \omega_2 t \right) S_{i1} + \cos \left( \frac{\pi}{2} \sin \omega_2 t \right) \right. \]
\[ \left. \cos \left( \frac{\pi}{2} \sin \omega_2 t \right) S_{i2} \right\} \]
\[ + \sin \left( \frac{\pi}{2} \sin \omega_2 t \right) \cos \left( \frac{\pi}{2} \sin \omega_2 t \right) S_{i3} \right\} \]  

(8)

Now, using the expansions

\[ \sin(\phi \sin \omega t) = 2 \sum_{n=0}^{\infty} J_{2n+1}(\phi) \sin[(2n+1)\omega t] = 2J_1(\phi) \sin(\omega t) \]
\[ \cos(\phi \sin \omega t) = J_0(\phi) + 2 \sum_{n=0}^{\infty} J_{2n}(\phi) \cos[2n \omega t] \]

(9)

where \( J_i(\phi) \) is the Bessel function of order \( i \) and frequencies up to the double of the original frequency have been taken. Substituting equation (9) in equation (8) gives

\[ S_0 = \frac{1}{2} \left\{ S_{i0} + 2J_1(\frac{\pi}{2}) \sin(\omega_2 t) S_{i1} \right. \]
\[ + \left( J_0(\frac{\pi}{2}) + 2J_2(\frac{\pi}{2}) \cos[2\omega_2 t] \right) \right. \]
\[ \left. \left. \left( J_0(\frac{\pi}{2}) + 2J_2(\frac{\pi}{2}) \cos[2\omega_2 t] \right) S_{i2} \right\} \]
\[ + 2J_1(\frac{\pi}{2}) \sin(\omega_2 t) \left( J_0(\frac{\pi}{2}) + 2J_2(\frac{\pi}{2}) \cos[2\omega_2 t] \right) S_{i3} \right\} \]  

(10)

Combining the trigonometric terms the measured signal can be written in terms of the frequencies of the variations of the retardances:

\[ S_0 = \frac{1}{2} \left\{ S_{i0} + J_0(\frac{\pi}{2}) J_0(\frac{\pi}{2}) S_{i1} + J_1(\frac{\pi}{2}) S_{i1} \sin(\omega_2 t) + J_0(\frac{\pi}{2}) J_1(\frac{\pi}{2}) S_{i2} \sin(\omega_2 t) \right. \]
\[ + J_0(\frac{\pi}{2}) J_2(\frac{\pi}{2}) S_{i2} \left( \cos[2\omega_2 t] + \cos[2\omega_2 t] \right) \]
\[ + J_1(\frac{\pi}{2}) J_2(\frac{\pi}{2}) S_{i3} \left( \cos[(\omega_1 - 2\omega_2) t] + \cos[(\omega_1 + 2\omega_2) t] \right) \]
\[ + J_2(\frac{\pi}{2}) J_2(\frac{\pi}{2}) S_{i3} \left( \cos[(2\omega_1 - 2\omega_2) t] + \cos[(2\omega_1 + 2\omega_2) t] \right) \]  

(11)

And finally, resolving this equation for the components of the unknown Stokes vector, in terms of the frequency components of the detected signal
4. Results

We have measured the Stokes vector of light passing through a linear polarizer and a half-wave retarder, as the retarder is rotated, and also for a linear polarizer and a quarter-wave retarder as the retarder is rotated. In the first case the polarization measured should shift between $S_1$ (horizontal/vertical linear polarization) and $S_2$ (±45° linear polarization), and in the second case the polarization should shift between linear and circular $S_3$ polarization. The experimental results are shown in figures 4 and 5 for these two cases. It can be seen that the behaviour of the results is as expected.

\[
\begin{align*}
S_{10} &= 2S_0(0) - J_0\left(\frac{\pi}{2}\right)J_0\left(\frac{\pi}{2}\right)S_{12} \\
S_{11} &= \frac{S_0(\omega_2)}{J_1\left(\frac{\pi}{2}\right)} \\
S_{12} &= \frac{S_0(2\omega_2)}{J_0\left(\frac{\pi}{2}\right)J_2\left(\frac{\pi}{2}\right)} = \frac{S_0(2\omega_1 \pm 2\omega_2)}{J_2\left(\frac{\pi}{2}\right)J_2\left(\frac{\pi}{2}\right)} \\
S_{13} &= \frac{S_0(\omega_1 \pm 2\omega_2)}{J_1\left(\frac{\pi}{2}\right)J_2\left(\frac{\pi}{2}\right)} = \frac{S_0(\omega_1)}{J_0\left(\frac{\pi}{2}\right)J_1\left(\frac{\pi}{2}\right)}
\end{align*}
\]
5. Conclusions
The mechanical design and polarization optics of a goniometric scatterometer have been presented. The scatterometer has an angular resolution of 0.25° and a small range of angles which cannot be measured because of blocking by the illumination arm. Advances in the construction of the polarization measurement system have been presented and preliminary results have shown the expected behaviour.

References
[1] Jakeman E and Hoenders BJ 1982 Optica Act, 29 1587.
[2] Depine RA and Skigin DC 1994 J. Opt. Soc. Am. A 11 2844.
[3] Mendoza-Suárez A and Méndez ER 1997 Applied Optics 36 3521.
[4] Bruce NC 2003 Applied Optics 42 2398.
[5] Bruce NC 2005 Applied Optics 44 784.
[6] Ogilvy JA 1991 Theory of Wave Scattering from Random Rough Surfaces (Bristol, IOP Publishing)
[7] Rodriguez-Herrera OG, Rosete-Aguilar M and Bruce NC Rev. Sci. Inst. 75 4820
[8] Rinder T and Rothe H 2002 SPIE Proceedings on Surface Scattering and Diffraction for Advanced Metrology II 115.
[9] Mattison P, Dombrowski M 1998 and Lorenz J SPIE Proceedings on Scattering and Surface Roughness II 240.
[10] Mainguy S, Olivier M, Josse M and Guidon M 1991, SPIE Proceedings on Optical Scatter: Applications, Measurement and Theory 269.
[11] Ford JN, Tang K, and Buckius RO 1995 J. Heat Transfer 117 955.
[12] Goldstein D 2003 Polarized Light, Second Edition (New York, Marcel Dekker) 2003