WATER TAXES AND FINES IMPOSED ON LEGAL AND ILLEGAL FIRMS EXPLOITING GROUDWATER

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Abstract. This paper uses a differential game approach to investigate a model that represents the exploitation of groundwater, taking into account the strategic and dynamic interactions among users of the resource and public authority. Agents’ behaviour may influence their gains but also the overexploitation of the aquifer. The effects of legal and illegal firms’ actions and the contribution of taxes and penalties imposed by public authorities, are analysed by studying Feedback equilibria in order to capture the problem of non-compliance with resource management regimes and to discuss policy options in a non-cooperative and cooperative context. We show that illegal extractions can be a significant stumbling block on the path towards implementing of better management and environmental policies and we explain how, in order to fight this phenomenon, the public authority must increase controlled activity rather than taxation, but also encourage cooperation between legal firms under appropriate conditions.

1. Introduction. Overexploitation of aquifers is a serious problem in many regions of the world. In fact, the intensive use of groundwater leads to a wide array of social, economic and environmental consequences such as land subsidence, increased agricultural vulnerability and strained use of other necessary water applications due to water increased in pumping costs. Groundwater resources are often exploited under a common property regime, that access is restricted to land owners situated over the aquifer. [13] propose their seminal work about an aquifer management regime and they estimate that welfare gains from policy intervention are inefficient compared with competitive outcomes. Gisser and Sanchez’s theoretical prediction is that if the storage capacity of the aquifer is relatively large, then the two systems would be very close. These results have produced a bulk of literature about groundwater management (see [14]). Using the game theory approach, some authors assume that

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farmers are myopic and make decisions based on short time span, without considering the impact of the other users on the available water stock. Other authors propose the exploitation of the water resource over a long period. \[16\] determines open-loop and Feedback Nash solutions showing that the open-loop equilibrium captures only the pumping cost externality, however, in the Feedback approach, the strategic externality also emerges. Moreover, if the objective function of the problem is concave, \[20\] show that the Feedback solution increases in inefficiency in comparison with the socially optimal outcome. Starting from \[13\] model, \[21, 22\] propose a differential game and determine analytical solutions of the socially optimal, open-loop and Feedback scenarios over an infinite planning horizon. Their results are in line with those of \[16\] and \[13\]. In fact, they show that strategic behaviour increases the exploitation of the aquifer compared with the open-loop solution, but if the groundwater storage capacity is large, then the difference between the social optimum and private extraction is negligible. \[11\] introduce ecosystem damages showing that these environmental externalities can change results substantially. \[2, 3\] consider heterogeneous farmers in terms of behaviour, cooperators and outsiders, in the exploitation of the water resource. In \[4\] authors determine the efficient extraction of groundwater among overlapping generations. However, none of the previously mentioned literature addresses the topic of illegal exploitation of groundwater resources.

Illegal water extraction, particularly for use in agriculture, is widespread in many regions of the world. \[15\] cite official estimates which indicate that in the Western La Mancha, half of all firms pump illegally. \[8\] describe several means of unauthorised groundwater use, including new wells drilled in aquifers that enable overexploitation. \[9\] suggest that in several Southern European countries unauthorised extraction may account for 30-50% of total extractions for agricultural use. \[5\] shows that agricultural expansion and increasing water use in Chile have become contentious among farmers. In particular, the illegal abstraction of groundwater is estimated at approximately double that of the sum of its granted rights.

The study of groundwater depletion is a question of developing adequate countermeasures in order to preserve natural systems. In fact, the Water Framework Directive (2000/60/EC) supplements existing legislation by expanding the scope of water protection to all waters, surface waters and groundwater. It requires that Member States enforce appropriate measures to achieve a good ecological and chemical status of all water bodies. Without external regulation, farmers do not internalise environmental damage that their activity is generating. For these reasons, public authority considers different interventions to enable firms to internalise these consequences by reducing their extractions. Water taxes represent a highly efficient instrument for groundwater protection, supported by a system of fines and adequate monitoring by the public authority.

The motivation of our paper is based on the idea that, to the best of our knowledge, there are no studies in the literature that use hydro-economic modeling with a differential game approach to evaluate groundwater policies in a context with illegal behaviour. Illegal water use is, in fact, a key issue to understand many of the problems related with depleting and overexploited stocks. We propose a model to study the dynamics of legal and illegal firms in regard to groundwater pumping in a non-cooperative and cooperative context. To have access to the aquifer resource, firms must request a permit from the public authority. This kind of action by firms is considered legal. The legal extraction of water is subjected to a taxation instrument
by public authority, which guarantees control of the level of groundwater extraction (see [12] and [10]). The model assumes that the public authority commits to a constant taxation rate over time and the water tax is proportional to the amount of water pumped. However, firms could decide to bypass authorisation and therefore pump water illegally, facing the risk of penalisation by public authority. Some authors have analysed the problem of non-compliance with resource management regimes, based on social norms literature in combination with property resources using [17] pioneer paper. However, these applications are relative to the case of overexploitation in fisheries ([1] and [23]) and forests ([19]). The novelty of our work is the introduction of policy tools in the field of water exploitation, aimed to fight illegal withdrawal and preserving the water table. Within the context of differential games, the analysis of the Feedback equilibria and the numerical applications show how the public authority, in order to preserve the aquifer, can act on the fiscal policy (determining the taxes) and on the control and sanctions. Moreover, the public authority can favour cooperation among legal firms in order to preserve the resource, under appropriate conditions.

The paper is organised in the following way. Section 2 presents the exploitation model of the groundwater resource and Section 3 determines the Feedback equilibria and the evolution of groundwater level in a non-cooperative context. Section 4 proposes numerical simulations about the effects that policy instruments have on the height of water table, on the legal and illegal profits and on the level of pumping. Section 5 analyses the case of cooperation among legal firms and proposes some numerical applications that compare the non-cooperative and cooperative results. Finally, Section 6 concludes and summarises the policy implications about water management.

2. The exploitation model of the groundwater resource. Let \( N \) denote the number of firms present on the land. \( L \) are the number of firms that require a permit to pump water and pay the water tax if they withdraw water and \( I \) are the ones who don’t require permission to pump. They evade the water tax and face the risk of being sanctioned by the regulating authority if they are caught. The first are considered legal farmers while the latter are considered illegal ones.

Let \( w_l(t) (l = 1, \ldots, L) \) and \( w_i(t) (i = 1, \ldots, I) \) be the pumping rate of the legal and illegal farmers at time \( t \), respectively. So that the amount of groundwater pumped is

\[
\sum_{l=1}^{L} w_l(t) + \sum_{i=1}^{I} w_i(t) = W(t)
\]

The differential equation which describes the dynamics of the water table is defined as the difference between natural recharge and net extractions:

\[
\dot{H}(t) = \frac{1}{\Delta S} [R + (\gamma - 1)W(t)]; \quad H(0) = H_0
\]

where \( H \) is the height of the aquifer, \( R \) denotes the natural recharge, \( \Delta S \) is the area of the aquifer, \( 0 < \gamma < 1 \) is the constant return flow coefficient of irrigation water, \( W \) is the amount of the groundwater pumped and \( H_0 \) is the initial groundwater table level at \( t = 0 \). Following [10] and [18], the production function is \( y(w_h) \) with the following function form:

\[
y(w_h(t)) = \alpha' \omega_h(t) - \frac{\beta'}{2} \omega_h^2(t); \quad \text{for} \quad h = l, i
\]
where \( \alpha' \) and \( \beta' \) are non-negative parameters that describe the effects of irrigation on the crop and that satisfy the following properties:

\[
\frac{dy}{d\omega_h} = \alpha' - \beta' \omega_h(t) \geq 0 \iff \omega_h(t) \leq \frac{\alpha'}{\beta'}; \\
\frac{d^2y}{d\omega_h^2} = -\beta' \leq 0;
\]

for \( h = i, l \), i.e. the production function has decreasing marginal returns. We assume that firms are price-takers and the agricultural price is constant and equal to \( p \). Consequently, the revenue is determined by multiplying the price by the production function, i.e.

\[
py_h(t) = p(\alpha' \omega_h(t) - \frac{\beta'}{2} \omega_h^2(t)); \quad \text{for} \quad h = l, i
\]

where \( \alpha = p\alpha' \) and \( \beta = p\beta' \). Based on [13] and [21], irrigation costs are:

\[
C(\omega_h(t), H(t)) = [c_0 - c_1 H(t)] \omega_h(t); \quad \text{for} \quad h = l, i
\]

where \( c_0 > 0 \) represents the fixed cost with respect to the aquifer height linked with the hydrologic cone and \( c_1 > 0 \) is the marginal pumping cost of water pumped. Irrigation costs must satisfy the following properties:

\[
\frac{\partial C}{\partial \omega_h} = c_0 - c_1 H(t) \geq 0 \iff H(t) \leq \frac{c_0}{c_1}; \\
\frac{\partial^2 C}{\partial \omega_h^2} = 0; \quad \frac{\partial C}{\partial H} \leq 0; \quad \frac{\partial^2 C}{\partial H^2} = 0; \quad \frac{\partial^2 C}{\partial \omega_h \partial H} \leq 0
\]

The value \( \tilde{H} := \frac{c_0}{c_1} \) represents the maximum water table elevation, according to [21]. In regards to public intervention to safeguard the level of the aquifer, we assume that legal firms acquire permission to withdraw and, if they pump, then pay an uniform water tax rate \( \delta \) on individual withdrawals \( \omega_l \). Instead, illegal farmers do not require any permission and only are affected by penalties associated with illegal behaviour when pumping. In fact, if an illegal firm is caught, it is obliged to pay a sanction \( \sigma > 0 \) proportional to its pumping \( \omega_i \). To avoid a moral hazard, we assume that \( \sigma > \delta \), i.e. the sanction is higher than the water tax rate. The enforcement intensity \( \phi \) is the probability of catching illegal firms, i.e. represents the regulator authority’s monitoring effort. Following [23], we would endogenise the subjective probability of being monitored. We let \( \phi \) be a function of legal size \( x \), where \( x = L/(L + I) \). It is reasonable to assume that if \( x \) is high, then the monitoring effort will be expected to decrease. The opposite is expected when \( x \) is low, that is the low ratio of legality to induce an increase in monitoring effort. The form of \( \phi(x) \) is:

\[
\phi(x) = (1 - x^\theta)^\eta
\]

with \( \theta, \eta > 0 \). \( \phi(x) \in [0, 1] \) assumes the properties of a sigmoid function desirable in our model. This assumption ensures monitoring more than proportional when the legality rate is low, and vice versa when it is high.

The profit of a legal firm is

\[
\Pi_l = \int_0^{+\infty} e^{-rt} \pi_l(t) \, dt
\]
while the profit of an illegal one is

$$\Pi_i = \int_0^{+\infty} e^{-rt}\pi_i(t)\,dt$$

where \(r > 0\) is the constant discounted rate and \(\pi_l(t)\) and \(\pi_i(t)\) are:

$$\pi_l(t) = \alpha_\omega(t) - \frac{\beta}{2}\omega_\omega^2(t) - [c_0 - c_1H(t)]\omega_l(t) - \delta\omega_l(t)$$  \hspace{1cm} (3)

$$\pi_i(t) = \alpha_\omega(t) - \frac{\beta}{2}\omega_\omega^2(t) - [c_0 - c_1H(t)]\omega_i(t) - \phi_\sigma\omega_i(t)$$  \hspace{1cm} (4)

In order to guarantee positive profits, the following proposition holds.

**Proposition 1.** Let \(\bar{H} = \frac{\delta + c_0 - \alpha}{c_1}\) and \(\tilde{H} = \frac{\phi_\sigma + c_0 - \alpha}{c_1}\). When water tax \(\delta\) is smaller than the expected value of sanction \(\phi_\sigma\), i.e. \(\delta < \phi_\sigma\), then \(\bar{H} < \tilde{H}\) and it results:

- If \(H(t) < \bar{H} < \bar{H}\) and \(H(t) < \tilde{H}\) then both profits are negative;
- If \(\bar{H} < H(t) < \bar{H}\) and \(H(t) < \tilde{H}\) then only \(\pi_l(t) \geq 0\) within \(\omega_l(t) \leq \frac{\alpha}{\beta}\);
- If \(\bar{H} < \bar{H} < H(t)\) and \(H(t) < \tilde{H}\) then both profits are positive within \(\omega_h(t) \leq \frac{\alpha}{\beta}\), \(h = l, i\).

Otherwise, when water tax \(\delta\) is greater than the expected value of sanction \(\phi_\sigma\), i.e. \(\phi_\sigma \leq \delta < \alpha\), then \(\bar{H} < \tilde{H}\) and it results:

- If \(H(t) < \tilde{H} < \bar{H}\) and \(H(t) < \tilde{H}\) then both profits are negative;
- If \(\bar{H} < H(t) < \tilde{H}\) and \(H(t) < \tilde{H}\) then only \(\pi_i(t) \geq 0\) within \(\omega_i(t) < \frac{\alpha}{\beta}\);
- If \(\bar{H} < \tilde{H} < H(t)\) and \(H(t) < \tilde{H}\) then both profits are positive within \(\omega_h(t) \leq \frac{\alpha}{\beta}\), \(h = l, i\).

**Proof of Proposition 1.** The legal profit \(\pi_l\) is positive if:

$$0 \leq \omega_l(t) \leq \frac{2[\alpha - (c_0 - c_1H)] - \delta}{\beta}$$

and

$$\frac{2[\alpha - (c_0 - c_1H) - \delta]}{\beta} > 0 \iff H(t) > \bar{H} = \frac{\delta + c_0 - \alpha}{c_1} \quad \text{with } \bar{H} \text{ positive if } (\delta + c_0 - \alpha) > 0$$

The illegal profits \(\pi_i\) is positive if:

$$0 \leq \omega_i(t) \leq \frac{2[\alpha - (c_0 - c_1H) - \phi_\sigma]}{\beta}$$

and

$$\frac{2[\alpha - (c_0 - c_1H) - \phi_\sigma]}{\beta} > 0 \iff H(t) > \tilde{H} = \frac{\phi_\sigma + c_0 - \alpha}{c_1} \quad \text{with } \tilde{H} \text{ positive if } (\phi_\sigma + c_0 - \alpha) > 0$$

Comparing \(\bar{H}\) and \(\tilde{H}\), Proposition 1 holds. \hfill \Box

3. Feedback equilibria in the non-cooperative case. As it is well known, in Feedback equilibria, farmers’ behaviour in regards to accessing the water table is dependent on the decisions of the others players. We solve a differential game computing Feedback equilibria that allows obtaining the optimal level of water pumped and the water table trajectory. The objective of each firm is to maximise
its profit. Legal firms bind to a level of extraction that maximises the discounted value of their payoffs:

$$\max_{\omega_l} \Pi_l$$

(5)

Illegal firms try to avoid taxes on water pumping and they do not take into account the effects of their water extractions over time on the aquifer. For these reasons, a myopic behaviour captures the illegal firms’ actions (see [10], [7] and [6]). Illegal firms play a static game maximising their profits in a myopic way:

$$\max_{\omega_i} \pi_i$$

(6)

The Hamilton-Jacobi-Bellman equation for a legal firm is:

$$rV_l(t, H(t)) = \max_{\omega_l} \left\{ \alpha \omega_l(t) - \frac{\beta}{2} \omega_l^2(t) - [c_0 - c_1 H(t)] \omega_l(t) - \delta \omega_l(t) + \frac{V'_i(t, H(t))}{\Delta S} \left( R + (\gamma - 1) \sum_{i=1}^L w_i(t) + (\gamma - 1) \sum_{i=1}^I w_i(t) \right) \right\}$$

(7)

where $V_l(t, H(t))$ and $V_i(t, H(t))$ are the optimal control value functions and $V'_i(t, H(t))$ are the first derivatives with respect to the state variable $H$.

The optimisation problem of illegal firms is:

$$\max_{\omega_i} \pi_i(t) = \max_{\omega_i} \left\{ \alpha \omega_i(t) - \frac{\beta}{2} \omega_i^2(t) - [c_0 - c_1 H(t)] \omega_i(t) - \phi \sigma w_i(t) \right\}$$

(8)

Applying the first order condition to Eqs. (7) and (8), we have:

$$\alpha - \beta w_l - c_0 + c_1 H - \delta + \frac{V'_l(t, H(t))}{\Delta S} (\gamma - 1) = 0$$

$$\alpha - \beta w_i - c_0 + c_1 H - \phi \sigma = 0$$

The solutions of the above equations give us:

$$w_l = \frac{(\alpha - c_0 + c_1 H - \delta)}{\beta} + \frac{(\gamma - 1)V'_l}{\Delta S \beta}$$

(9)

$$w_i = \frac{(\alpha - c_0 + c_1 H - \phi \sigma)}{\beta}$$

(10)

Substituting $w_l$ and $w_i$ in Eq. (7), it follows:

$$rV_l = \frac{(\gamma - 1)^2(2L - 1)(V'_l)^2}{2\beta \Delta S^2} + \frac{\left\{ \frac{(\gamma - 1)}{\beta} \left( (\alpha - c_0 + c_1 H - \delta) L + (\alpha - c_0 + c_1 H - \phi \sigma) I \right) \right\}}{\Delta S} \frac{R}{\Delta S} V'_l + \frac{(\alpha - c_0 + c_1 H - \delta)^2}{2 \beta}$$

(11)

In order to compute the solutions of Eq. (11), given the linear quadratic structure of the model, we guess that the optimal value function is quadratic and consequently the equilibrium strategy is linear with respect to the state variable. Precisely, we postulate a quadratic function of the form:

$$V_l(t, H(t)) = \frac{1}{2} AH^2(t) + BH(t) + C$$

(12)

with the first derivative:

$$V'_l(t, H(t)) = AH(t) + B$$

(13)
where $A, B$ and $C$ are constant parameters of the unknown value function, which are to be determined. Substituting Eqs. (12) and (13) in Eq. (11), we obtain a system of three Riccati equations for the coefficients of the value function:

\[
\begin{align*}
    rA &= \frac{(2L-1)(\gamma-1)^2}{\beta \Delta S^2} A^2 + 2\frac{(\gamma-1)(L+I)c_1 A + c_1^2}{\beta \Delta S} A + c_1^2 \\
    rB &= \frac{(2L-1)(\gamma-1)^2}{\beta \Delta S^2} AB + \frac{c_1(L+I)(\gamma-1)}{\beta \Delta S} B + \frac{(\gamma-1)(L+I)(\alpha - c_0) - \phi \sigma I - \delta L + R\beta}{\beta \Delta S} A + \frac{(\alpha - c_0 - \delta)c_1}{\beta} \\
    rC &= \frac{(2L-1)(\gamma-1)^2}{2\beta \Delta S^2} B^2 + \frac{(\gamma-1)(L+I)(\alpha - c_0) - \phi \sigma I - \delta L + R\beta}{\beta \Delta S} B + \frac{(\alpha - c_0 - \delta)^2}{2\beta} 
\end{align*}
\]

Eq. (14) admits two real and distinct solutions $A_1$ and $A_2$ with $A_2 > A_1 > 0$:

\[
A_{1,2} = -\frac{\Delta S}{2(\gamma-1)(2L-1)} \left[ 2(\gamma-1)(L+I)c_1 - \Delta S \alpha - \delta \pm \sqrt{D} \right]
\]

where

\[
D = 4c_1^2(\gamma-1)^2[L^2 + I^2 + 2L(I-1)] - 4\Delta S \beta (L+I)r(\gamma-1)c_1 + \Delta S^2 \beta^2 r^2
\]

is always positive. The stability condition $\frac{dH}{dH} < 0$ selects the solution. We substitute the linear strategy in the dynamic constraint of the water table obtaining the following differential equation:

\[
\dot{H} = \frac{R}{\Delta S} + \frac{(\gamma-1)L}{\Delta S} \left[ \frac{\alpha + c_1 H - \delta}{\beta} + \frac{(\gamma-1)}{\Delta S} V_1 \right] + \frac{(\gamma-1)I}{\Delta S} \left[ \frac{(\alpha + c_1 H - \phi \sigma)}{\beta} \right]
\]

Since $V_1' = AH + B$, the stability condition becomes:

\[
\frac{dH}{dH} < 0 \iff \frac{(\gamma-1)}{\beta \Delta S^2} [(L+I)\Delta Sc_1 + L(\gamma-1)A] < 0
\]

It is satisfied by the solution $A = A_1$. In order to find the Feedback equilibrium water table trajectory, starting from Eqs. (9) and (10), we substitute $V_1' = A_1 H + B_1$ and we obtain:

\[
\begin{align*}
    w_l &= \frac{H}{\beta} \left( c_1 + \frac{(\gamma-1)A_1}{\Delta S} \right) + \frac{1}{\beta} \left( \alpha - c_0 - \delta + \frac{(\gamma-1)B_1}{\Delta S} \right) \\
    w_i &= \frac{c_1 H}{\beta} + \frac{(\alpha - c_0 - \phi \sigma)}{\beta}
\end{align*}
\]

where, from Eq. (15), we have that:

\[
\begin{align*}
    B_1 &= \frac{2\Delta S \{(\alpha - c_0 - \delta)[c_1 \Delta S + L(\gamma-1)A_1] + I(\gamma-1)(\alpha - c_0 - \phi \sigma)A_1 + R\beta A_1\}}{(\beta r \Delta S + \sqrt{D}) \Delta S}
\end{align*}
\]

The solution $B_1$ is positive as $\alpha - c_0 - \delta < 0$ and $\alpha - c_0 - \phi \sigma < 0$ from Prop. 1 and $c_1 \Delta S + L(\gamma-1)A_1$ is negative.

Sustituting Eqs. (19) and (20) in the dynamic (1), we get:

\[
\dot{H} = \dot{Y} + Y
\]
where:
\[
\dot{Y} = \frac{(\gamma - 1)[c_1(L + I)\Delta S + L(\gamma - 1)A_1]}{\beta \Delta S^2} 
\]
\[
Y = \frac{\{(\gamma - 1)[(\alpha - c_0 - \delta)L + (\alpha - c_0 - \phi \sigma)I] + R\beta\} \Delta S + L(\gamma - 1)^2B_1}{\Delta S^2 \beta \delta \delta} 
\]

The Feedback Nash equilibrium is:
\[
H^* = -\frac{Y}{\dot{Y}} = \frac{\{(\gamma - 1)[(\alpha - c_0 - \delta)L + (\alpha - c_0 - \phi \sigma)I] + R\beta\} \Delta S + L(\gamma - 1)^2B_1}{(\gamma - 1)[c_1(L + I)\Delta S + L(\gamma - 1)A_1]} 
\]

and the Feedback Nash equilibrium water table trajectory is given by:
\[
H(t) = H^* + (H_0 - H^*)e^{\gamma t} 
\] (24)

where the initial water table level is \(H(0) = H_0\). The equilibrium \(H^*\) is positive since \(\dot{Y}\), given by Eq. (22), is a negative quantity that coincides with the stability condition (18) and \(Y\) is positive. From this result, we affirm that the equilibrium water table path converges to the stationary equilibrium and so the Feedback Nash equilibrium is globally asymptotically stable.

4. Numerical applications. For our numerical simulation, we estimate the number of firms to be fixed at \(N = 50\), the return flow coefficient \(\gamma = 0.10\), the natural recharge \(R = 1750\) mm/year, the interest rate \(r = 0.10\), the area of aquifer \(\Delta S = 2000\) ha, the fixed cost, with respect to the aquifer height, linked with the hydrologic cone \(c_0 = 2.3\) euro/mm-ha, the marginal pumping cost of water pumped \(c_1 = 0.003\) mm-ha per m and the initial water table elevation \(H_0 = 600\) m above sea level. Finally, the selling price \(p = 0.50\) euro/Kg, \(\alpha' = 2.20\) and \(\beta' = 0.014\).

We estimate the production function parameters as \(\alpha = 1.10\), \(\beta = 0.007\) and the probability monitoring parameters given in [23] are \(\theta = 2\) and \(\eta = 2\).

Based on our parameters, we have that \(H = c_0/c_1 = 766.66\) m above sea level and \(\alpha/\beta = 157.14\).

Figs. (1) and (2) propose the analysis of pumping by legal and illegal firms \(\omega_l(t)\) and \(\omega_i(t)\) and the respective evolution of the water table trajectory \(H(t)\) given by Eq. (24), assuming the sanction is fixed at \(\sigma = 2\) while the tax imposed by the public authority is variable. In particular, we analyse different scenarios as the number of firms, \(L\), that request pumping authorisation changes. As already mentioned in Section 1, the phenomenon of illegality varies from 30% to 50%. For this reason we have assumed, as a minimum value of the legal firms \(L\), a number equal to half of the total number of firms present on the land. Obviously, the number of legal firms determines the intensity of the monitoring activity \(\phi\) committed by the public authority. In detail, as illustrated in Figs. 1(a)-1(b), when the number of legal firms coincides with almost all the firms present, the pumping of illegal firms exceeds that of the legal ones, which decreases as the water tax \(\delta\) increases. We remark how the pumping rates satisfy the constraint \(\omega_l(t) \leq 157.14\) with \(h = l, i\), for all \(t \geq 0\).

Relative to the evolution of the water height, if \(\delta\) increases, the aquifer is more preserved. If \(\delta = 0.20\) we have that \(\phi \sigma \leq \delta\) and \(\bar{H} < H < H(t)\); so both firms pump and the water table height decreases over time. If \(\delta = 0.40\), there is limited pumping by the legal firms, which results in a minimal increase in the height of the aquifer. When the water tax exceeds the value \(\delta = 0.50\), then from Proposition 1 it results that \(\bar{H} < H(t) < \bar{H}\) and so legal firms will avoid pumping anymore water.
For instance, when $\delta = 1.50$, the illegal firms are the only ones to withdraw, and so, considering that they are the minority, the water level goes up over time.

Figs. 1(c)-1(d) depict the case with 80% of legal firms. It is highlighted how, for a low taxation, the legal firms pump more than the illegal ones, but as $\delta$ increases, legal pumping decreases down to zero when the $\delta = 0.25$. The effect on the groundwater level is a greater exploitation when $\delta$ is low. However, when $\delta$ increases and legal pumping decreases, this results in greater conservation of the resource even with the increase of illegal pumping.

In Figs. 2(a)-2(b), the number of legal firms is 70%. We observe that, with a low water tax, the pumping level of legal firms is higher than that of the illegal ones, which do not withdraw after a certain period of time. If $\delta$ exceeds 0.60, the pumping of legal firms is equal to zero. In regard to the corresponding level of $H(t)$, as $\delta$ increases, the aquifer is preserved, but different scenarios occur. In correspondence with a low water tax, the absence of withdrawal by illegal firms causes the aquifer height to drop, but with less intensive use. Correspondingly, if $\delta = 0.50$, reduced pumping by both legal and illegal firms leads to an increase in the height of the water table. Finally, a higher tax gives a situation where only the illegal firms pump and the height of the aquifer is better preserved. Figs. 2(c)-2(d) show the case in which 50% of firms acting legally withdraw. In this context, considering the monitoring activity of the public authority, illegal firms are discouraged from extracting from the water table and therefore, when taxation is low, only the legal firms pump, while, at higher taxation, the legal firms will also not pump. Following that, the evolution of $H(t)$ shows a decrease in the height of the aquifer when the legal firms withdraw and linear growth in the absence of any pumping.

Figs. (3) and (4) study the pumping levels and the water table evolution when $\delta = 0.10$ and the sanction $\sigma$ changes. In particular, Fig. 3(a) shows the case in which almost all firms legally withdraw. It highlights how an increase in penalty leads to a reduction in illegal pumping until it less than legal pumping. Respectively, the levels of $H(t)$ improve slightly but remain very close to each other, as $\sigma$ grows (see Fig. 3(b)). This result is due to the high number of legal firms, which consequently results in limited monitoring activity. Figs. 3(c) and 4(a) show that, if the number of legal firms decreases, the monitoring activity increases and the withdrawal of illegal firms goes to zero when the sanction is large. In addition, the height of water is more protected when the penalty increases, as illustrated in Figs. 3(d) and 4(b). Finally, assuming a legality rate of 50% and consequently a growing monitoring activity, we observe a reduction of the maximum threshold of the sanction for which it is convenient to be illegal. The absence of illegal pumping results in an increase in height of the water table (see Figs. 4(c)-4(d)).

In Fig. (5), we compare the profits of legal and illegal firms. The red line corresponds to the profits of the illegal firms while the green line represents the legal ones. In this comparison, we assume as variables fines $\sigma$ and we propose different scenarios in which the water tax $\delta$ assumes a low, medium and high value, while the number of legal firms $L$ is larger than or equal to 25. First of all, it can be seen that the legal profits are not very sensitive to the variation of fines $\sigma$ or to the number of legal firms $L$, which entails a change in monitoring. Vice versa, an increase in the water tax leads to a reduction in legal profits while illegal profits decrease as penalties increase. Furthermore, as the number of legal firms changes, there is a greater reduction in illegal profit due to more intense monitoring by the public authority. The break even point between illegal and legal profits determines
the values of sanctions from which it is better to withdraw legally. Moreover, an increase in water tax reduces the legal profits and therefore increases the values of sanctions from which the legal profits exceed the illegal ones. This means that the public authority must tighten penalties to ensure that the legal profits are more appealing than the illegal ones.

In Fig. (6) we compare the profits of legal and illegal firms when the water tax $\delta$ changes and the fines are fixed. We observe how an increase in the water tax leads to a reduction in legal profits and an increase in illegal ones. However, the latter decrease considerably when the number of legal firms decreases, nonetheless, the profits of legal firms do not change significantly. Finally, a decrease in the number
of legal firms, which determines a growth in monitoring activity, moves to right the break even point. This means that public authority has a higher water tax range whereby the legal profits exceed the illegal ones.

The aim of the public authority is to preserve the aquifer. To do this, it must optimally combine taxation, sanction and monitoring. An increase in the water tax rate and in the sanction implements greater safety measures to protect the water resource, as long as this does not discourage the withdrawals of legal firms. The number of firms applying for authorisation has an effect on the authority’s policy as it determines the monitoring level. The growth of monitoring and the enforcement of penalties lead to higher profits for legal firms. The increase in sanctions $\sigma$ by
the authority can determine the degree of protection of the aquifer and favour legal profits over illegal ones. Consequently, if the public authority intends to encourage legal activity by increasing their profits respective to illegal profits, it must act more based on control and sanction rather than by increasing taxation.

5. Feedback equilibria in the cooperative case. In this section, we assume that all legal firms decide to cooperate in order to maximise their total benefit:

\[
\max_{\omega_l} \sum_{l=1}^{L} \Pi_l = \max_{\omega_l} \sum_{l=1}^{L} \int_{0}^{+\infty} e^{-rt} \pi_l(t)
\]
while the illegal firms act singularly and maximise their own profits:

$$\max_{\omega_i} \pi_i$$

The Hamilton-Jacobi-Bellman equation for the coalition composed of legal firms is:

$$r\bar{V}_l(t, H(t)) = \max_{\omega_l} \left\{ \sum_{i=1}^{L} \left[ \alpha \omega_i(t) - \frac{\beta}{2} \omega_i^2(t) - [c_0 - c_1 H(t)] \omega_i(t) - \delta \omega_i(t) \right] + \frac{\bar{V}_i'(t, H(t))}{\Delta S} \left( R + (\gamma - 1) \sum_{i=1}^{L} \omega_i(t) + (\gamma - 1) \sum_{i=1}^{L} \omega_i(t) \right) \right\}$$

(25)
Figure 5. Profits of legal and illegal firms when sanction $\sigma$ changes.
The first order conditions give us:

\[ \omega_l = \frac{(\alpha - c_0 + c_1 H - \delta)}{\beta} + \frac{(\gamma - 1)\bar{V}'_l}{\Delta S} \]  

(26)

\[ \omega_i = \frac{(\alpha - c_0 + c_1 H - \phi \sigma)}{\beta} \]  

(27)

Substituting \( \omega_l \) and \( \omega_i \) in Eq. (25), it follows:

\[ r\bar{V}_l = \frac{L(\gamma - 1)^2(\bar{V}'_l)^2}{2\beta \Delta S^2} + \left\{ \frac{(\gamma - 1)((\alpha - c_0 + c_1 H - \delta)L + (\alpha - c_0 + c_1 H - \phi \sigma)I)}{\beta \Delta S} + \frac{R}{\Delta S} \right\} \bar{V}'_l + \frac{L(\alpha - c_0 + c_1 H - \delta)^2}{2\beta} \]  

(28)

The optimal value function in the cooperative case assumes the following form:

\[ \bar{V}_l(t, H(t)) = \frac{1}{2} \bar{A} H^2(t) + \bar{B} H(t) + \bar{C} \]  

(29)

where \( \bar{A}, \bar{B} \) and \( \bar{C} \) are the parameters that need to be determined. Substituting the value function and its derivative, we obtain a system of equations for the coefficients.
of the value function:
\[ r\bar{A} = \frac{L(\gamma - 1)^2}{\beta \Delta S^2} \bar{A}^2 + \frac{2(\gamma - 1)(L + I)c_1}{\beta \Delta S} \bar{A} + \frac{Lc_1^2}{\beta} \]  
(30)
\[ r\bar{B} = \frac{L(\gamma - 1)^2}{\beta \Delta S^2} \bar{A}\bar{B} + \frac{c_1(L + I)(\gamma - 1)}{\beta \Delta S} \bar{B} + \frac{(\gamma - 1)(L + I)(\alpha - c_0 - \phi \sigma I - \delta L) + R\beta}{\beta \Delta S} \bar{A} + \frac{L(\alpha - c_0 - \delta)c_1}{\beta}, \]  
(31)
\[ r\bar{C} = \frac{L(\gamma - 1)^2}{2\beta \Delta S^2} \bar{B}^2 + \frac{(\gamma - 1)(L + I)(\alpha - c_0 - \phi \sigma I - \delta L) + R\beta}{\beta \Delta S} \bar{B} + \frac{L(\alpha - c_0 - \delta)^2}{2\beta}, \]  
(32)
Eq. (30) admits two real and distinct solutions \( \bar{A}_1 \) and \( \bar{A}_2 \) with \( \bar{A}_2 > \bar{A}_1 > 0 \):
\[ \bar{A}_{1,2} = -\frac{\Delta S}{2L(\gamma - 1)^2} \left[ 2(\gamma - 1)(L + I)c_1 - \Delta S \beta \pm \sqrt{\Delta} \right] \]
where
\[ \Delta = [2(\gamma - 1)(L + 2Lc_1 - \beta \Delta S)] \cdot [2(\gamma - 1)c_1 - \beta \Delta S] \]
is always positive. Only the solution \( \bar{A}_1 \) satisfies the stability condition \( \frac{d\bar{H}}{dt} < 0 \).

From the value function obtained, it follows that:
\[ \hat{\omega}_1(t) = \frac{H(t)}{\beta} \left( c_1 + \frac{(\gamma - 1)\bar{A}_1}{\Delta S} \right) + \frac{1}{\beta} \left( \alpha - c_0 - \delta + \frac{(\gamma - 1)\bar{B}_1}{\Delta S} \right) \]  
(33)
\[ \hat{\omega}_1(t) = \frac{c_1H(t)}{\beta} + \frac{(\alpha - c_0 - \phi \sigma)}{\beta} \]  
(34)
indicate the level of pumping of legal and illegal firms in the case of cooperation, respectively and
\[ \hat{B}_1 = -\frac{\Delta S \{ (\gamma - 1)(L(\alpha - c_0 - \delta) + I(\alpha - c_0 - \phi \sigma)) + R\beta \bar{A}_1 + L(\alpha - c_0 - \delta)c_1 \Delta S \}}{L(\gamma - 1)^2 \bar{A}_1 + \Delta S c_1 (L + I)(\gamma - 1)} \]
is obtained by solving Eq. (31). Sustituting Eqs. (33) and (34) in the dynamic (1), we find that the cooperative Feedback Nash equilibrium water table trajectory is:
\[ \hat{H}_c(t) = H^*_c + (H_0 - H^*_c)e^{Y_c t} \]  
(35)
where the cooperative Feedback Nash equilibrium is:
\[ H^*_c = -\frac{Y_c}{Y_c} = -\frac{\left\{ (\gamma - 1)[(\alpha - c_0 - \delta)L + (\alpha - c_0 - \phi \sigma)I] + R\beta \right\} \Delta S + L(\gamma - 1)^2 \bar{B}_1}{(\gamma - 1)[c_1(L + I)\Delta S + L(\gamma - 1)\bar{A}_1]} \]
and
\[ \hat{Y}_c = \frac{(\gamma - 1)\left[ c_1(L + I)\Delta S + L(\gamma - 1)\bar{A}_1 \right]}{\beta \Delta S^2} \]  
(36)
\[ Y_c = \frac{(\gamma - 1)[(\alpha - c_0 - \delta)L + (\alpha - c_0 - \phi \sigma)I] + R\beta \Delta S + L(\gamma - 1)^2 \bar{B}_1}{\Delta S^2 \beta \Delta S} \]  
(37)
In order to support our analysis, we propose a numerical simulation that shows that a cooperation can preserve the groundwater resource. We assume that the number of firms is \( N = 50 \), the return flow coefficient \( \gamma = 0.10 \), the natural recharge \( R = 1750 \text{ mm/year} \), the interest rate \( r = 0.10 \), the area of aquifer \( \Delta S = 2000 \text{ ha} \), the marginal pumping cost of water pumped \( c_1 = 0.003 \text{ mm-ha per m} \) and the initial water table elevation \( H_0 = 150 \text{ m} \) above sea level. The selling price \( p = 0.50 \)
euro/Kg, $\alpha = 1.10$, $\beta = 0.007$ and the probability monitoring parameters are $\theta = 2$ and $\eta = 2$. We assume that $\sigma = 4$ and $\delta = 0.50$. Finally, the fixed cost, with respect to the aquifer height, assumes different values, i.e. $c_0 = 0.60$ and $c_0 = 1$ euro/mm-ha.

Fig. 7 shows the water table elevation when the size of legal cooperators changes. In particular, in Fig. 7(a) it is assumed a low value of the fixed cost linked to the idrologic cone, $c_0 = 0.60$ euro/mm-ha; instead in Fig. 7(b) the value of $c_0$ is increased to $c_0 = 1$ euro/mm-ha.

We observe that, when $L$ increases and the value of $c_0$ is low, then the water table level rises. However, when $L$ increases and the value of $c_0$ is high, then the groundwater level decreases. This means that greater legality makes it possible to achieve a social result that brings about greater protection of the water resource in the case in which the fixed cost $c_0$ is low. So, the cooperative analysis demonstrates the opportunities that can occur by sharing water resources and brings to light possibilities to protect the ecosystem. On the other hand, an increase in the fixed cost would result in a greater economic advantage for a larger coalition size, but could cause a negative effect on the height of the water. In particular, the curves for $L = 35$ and $L = 25$ in Fig. 7(b) are increasing as, in such case, the illegal firms decide to not withdraw due to the high costs, monitoring and penalties.

Figs. 8 and 9 compare the effects of the cooperative versus the non-cooperative case on water height when $c_0$ is low and high. First of all, we observe that the resource is better protected by cooperation when $c_0 = 0.60$ and, moreover, the increase in the size of the coalition favours the conservation of the water table. Instead, when the value of the fixed cost is $c_0 = 1$, then cooperation and an increase in the size of the coalition will cause a greater deterioration of the resource. In conclusion, the benefits of sharing water resources within the coalition are greater when the fixed costs are higher. These advantages involve more pumping to maximise profits.

6. Conclusions and Policy implications. In this paper we proposed a model detailing the exploitation of a common groundwater resource with a differential game approach, taking into account the strategic and dynamic interactions among
the users and public authority. Several factors may characterise the profits of firms as well as the overexploitation of the aquifer, which can be influenced by different behaviours of the agents and regulator. We considered that firms may act in a legal or illegal way within a context of cooperation and non-cooperation. The legal firms require authorisation to withdraw by agreeing to pay a water tax imposed by the public authority in order to preserve the resource. The illegal firms do not request authorisation for pumping and therefore face the risk of being sanctioned, if they are discovered. In the non-cooperative case, both legal and illegal firms maximise their own profit; on the other hand, in the cooperative case, the legal firms maximise the aggregate profit. The effects of the presence of legal and illegal firms and the contribution of policy intervention were analysed in the Feedback equilibria study in, both, the non-cooperative and cooperative case.

In the non-cooperative case, the analysis demonstrated the usefulness of the water tax, monitoring control based on the number of firms requesting authorisation and fines in order to preserve the height of the water table and its evolution over time. The analytical, numerical and graphical results have shown that a policy in which the authority increases the water tax with respect to the expected value of fines, protects the height of the water table until it does not discourage the pumping of
In the cooperative case, the analysis demonstrated how a cooperation between legal firms can result in greater conservation or, alternatively, an overexploitation of the water resource in relation to fixed costs. Our results indicate that cooperative management improves the economic benefits of water users, but it may have different effects on groundwater protection also with regulatory policies. Public authority can promote cooperative management by creating different incentives for cooperation, such as low fixed costs, but also water tax, monitoring mechanisms and sanctions. However, cooperation in water management may be challenging in practice because of the strategic behaviour of stakeholders.

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