Neutron star composition in strong magnetic fields

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Abstract

We study the neutron star composition in the presence of a strong magnetic field. The effects of the anomalous magnetic moments of both nucleons and electrons are investigated in relativistic mean field calculations for a β-equilibrium system. Since neutrons are fully spin polarized in a large field, generally speaking, the proton fraction can never exceed the field free case. An extremely strong magnetic field may lead to a pure neutron matter instead of a proton-rich matter.

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The discovery of pulsars [1] substantially stimulated the study of neutron stars, a class of extreme objects in astrophysics. Indeed, shortly after their discovery pulsars were identified as rotating magnetized neutron stars [2] with a surface magnetic field of $10^{12} - 10^{14}$ G [3]. Recent observations of soft gamma repeaters (SGRs) have confirmed that they are newly born neutron stars with very large surface magnetic fields (up to $10^{15}$ G) [4]. Such stars are named as magnetars [5]. It is expected that appreciably higher fields could exist in the interior of neutron stars although the total strength of internal magnetic fields remains unknown. The allowed internal field strength of a star is constrained by the scalar virial theorem [6, 7]. According to it, the maximum interior field strength could reach $\sim 10^{18}$ G. Naturally, one may expect considerable influences of such high fields on the properties of neutron stars.

Theoretical investigation of a free electron gas in intense fields related to the neutron star crusts [7] and white dwarfs [8] as well as ideal noninteracting neutron-proton-electron (n-p-e) gas for the internal properties of neutron stars [9] has been carried out by several authors. The equation of state (EOS) of pure neutron matter under large magnetic fields was investigated in Ref. [10] where the strong interaction was taken into account. Theoretical calculations of spin polarized isospin asymmetric nuclear matter based on the Brückner-Hartree-Fock formalism has been performed in Ref. [11]. Analogous calculations employing various mean-field models can be seen in Ref. [12]. The effects of strong magnetic fields on a dense n-p-e system under the beta equilibrium and the charge neutrality conditions were studied in Ref. [13]. Extremely large fields up to the proton critical field (here the critical field is defined by the condition that the particle’s cyclotron energy is comparable to its rest mass [14]) $B_p^c \sim 10^{20}$ G and beyond were considered in their calculations. It was found that the nuclear matter in beta equilibrium practically converts into stable proton-rich matter. However, in these investigations the effects of the anomalous magnetic moments (AMM) were neglected, which may play an important role in the presence of such high fields. Recent studies demonstrated [15] that the contributions from the AMM of nucleons become significant when the magnetic field $B > 10^5 B_e^c$. Here $B_e^c = 4.414 \times 10^{13}$ G is the electron critical field. Consequently, the softening of
the EOS caused by Landau quantization [13] is overwhelmed by the stiffening due to the incorporation of the nucleon anomalous magnetic moments. But a dramatic increase of the proton fraction with the increase of the magnetic field remains. In the study of Ref. [15] the AMM of electrons was not taken into account.

The anomalous magnetic moment of an electron [16] may not be negligible due to its relatively large Bohr magneton. Because of the charge neutrality of the system, a significant effect on electrons will in turn influence protons. Considering that the proton fraction is crucial in determining the direct URCA process which leads to the cooling of neutron stars [17, 18], in this work we examine the problem with the inclusion of the AMM not only for nucleons but also for electrons. One may argue that the electron self-energy may not change substantially in magnetic fields when high-order terms are taken into account. However, systematic incorporation of high-order contributions beyond the AMM is not yet clear, which will be the topic of our forthcoming works. Here the effects of magnetic fields on different particles within the same system are taken into account at the equal footing. We consider the magnetic fields up to $10^6 B^e_c$. Though we do not yet know any object generating such fields, some possibilities have been suggested [19].

For the n-p-e system in a uniform magnetic field $B$ along the $z$ axis, the interacting Lagrangian can be written as [20]

$$\mathcal{L} = \bar{\psi} [i\gamma_\mu \partial^\mu - e\frac{1}{2}\gamma_\mu A^\mu - \frac{1}{4}\kappa_p \mu_N \sigma_{\mu\nu} F^{\mu\nu} - M_N + g_\sigma \sigma - g_\omega \gamma_\mu \omega^\mu - \frac{1}{2} g_\rho \gamma_\mu \mathbf{\tau} \cdot \mathbf{R}^\mu] \psi$$

$$+ \bar{\psi}_e [i\gamma_\mu \partial^\mu - e\gamma_\mu A^\mu - \frac{1}{4}\kappa_e \mu_B \sigma_{\mu\nu} F^{\mu\nu} - m_e] \psi + \frac{1}{2} \partial_\mu \sigma \partial^\mu \sigma - U(\sigma)$$

$$- \frac{1}{4} \omega_{\mu\nu} \omega^{\mu\nu} + \frac{1}{2} m_\omega^2 \omega_\mu \omega^\mu - \frac{1}{4} \mathbf{R}^{\mu\nu} \cdot \mathbf{R}^{\mu\nu} + \frac{1}{2} m_\rho^2 \mathbf{R}_\mu \cdot \mathbf{R}^\mu,$$

where the conventional notation has been adopted. $U(\sigma)$ is the self-interaction part of the scalar field [21]

$$U(\sigma) = \frac{1}{2} m_\sigma^2 \sigma^2 + \frac{1}{3} b(g_\sigma \sigma)^3 + \frac{1}{4} c(g_\sigma \sigma)^4.$$  

(1)

$A^\mu \equiv (0, 0, Bx, 0)$ refers to a constant external magnetic field and $\sigma^{\mu\nu} = \frac{i}{2} [\gamma^\mu, \gamma^\nu]$. $\mu_N$ and $\mu_B$ are the nuclear magneton of nucleons and Bohr magneton of electrons; $\kappa_p = 3.5856$, $\kappa_n = -3.8263$ and $\kappa_e = \alpha/\pi$ are the coefficients of the AMM for protons, neutrons and
electrons, respectively. The third set of parameters in Table II of Ref. [22] is used as the nucleon coupling strengths. It turns out $g_\sigma = 8.7818$, $g_\omega = 8.7116$, $g_\rho = 8.4635$, $b g_\sigma^3 = 27.9060$, $c g_\sigma^4 = -14.3989$. This yields a binding energy $B/A = -16.3$ MeV, saturation density $\rho_0 = 0.153$ fm$^{-3}$ and bulk symmetry energy $a_{sym} = 32.5$ MeV.

The general solutions for the Dirac equation with the inclusion of the AMM are in the form of $\psi(X) \propto e^{-iEt+i\nu y+i\rho_z x}\phi_p(p_y,p_z,x)$ for protons and electrons and $\psi(X) \propto e^{-iEt+i\rho_x x}\phi_n(p)$ for neutrons. We have derived the concrete expressions of the Dirac spinors $\phi_p(p_y,p_z,x)$ and $\phi_n(p)$ in the chiral representation [23]. The chemical potentials of protons and neutrons are defined as

$$\mu_p = \epsilon_f^p + g_\omega \omega_0 + \frac{1}{2} g_\rho R_{0,0},$$
$$\mu_n = \epsilon_f^n + g_\omega \omega_0 - \frac{1}{2} g_\rho R_{0,0}. \tag{3}$$

They are related to the respective Fermi momenta via following equations:

$$\left(k_{f,\nu,S}^p\right)^2 = \left(\epsilon_f^p\right)^2 - \left(\sqrt{m^* + 2eB\nu} + S\Delta\right)^2, \tag{5}$$
$$\left(k_{f,S}^n\right)^2 = \left(\epsilon_f^n\right)^2 - (m^* + S\Delta)^2. \tag{6}$$

Here $\Delta = -\frac{1}{2} \kappa_b \mu_N B$; $S = \pm 1$ for spin-up and spin-down particles. $\nu$ is the quantum number of Landau levels for charged particles [14]. The effective nucleon mass $m^* = M_N - g_\sigma \sigma$. Here $\sigma$, $\omega_0$ and $R_{0,0}$ are the mean values of the scalar field, the time-like component of the vector field and the time-like isospin 3-component of the vector-isovector field in neutron star matter, respectively. They are obtained by solving the non-linear equations of the meson fields. The total scalar density and baryon density are $\rho_S = \rho_S^p + \rho_S^n$ and $\rho = \rho_S^p + \rho_S^n$ respectively, where

$$\rho_S^p = \frac{eB m^*}{2\pi^2} \sum_S \sum_\nu \sqrt{m^* + 2eB\nu} + S\Delta \ln \left| \frac{k_{f,\nu,S}^p + \epsilon_f^p}{\sqrt{m^* + 2eB\nu} + S\Delta} \right|, \tag{7}$$
$$\rho_S^n = \frac{m^*}{4\pi^2} \sum_S \left[ \epsilon_f^n k_{f,S}^n - (m^* + S\Delta)^2 \ln \frac{k_{f,S}^n + \epsilon_f^n}{m^* + S\Delta} \right], \tag{8}$$
$$\rho^p = \frac{eB}{2\pi^2} \sum_S \sum_\nu k_{f,\nu,S}^p, \tag{9}$$
$$\rho^n_0 = \frac{1}{2\pi^2} \sum_S \left[ \frac{1}{3} \left(k_{f,S}^n\right)^3 + \frac{S\Delta}{2} \left(\frac{m^* + S\Delta}{k_{f,S}^n + \epsilon_f^n} \arcsin \frac{m^* + S\Delta}{\epsilon_f^n} - \frac{\pi}{2} \right) \right]. \tag{10}$$
The summation of $\nu$ runs up to the largest integer for which $(k_{T,\nu,S}^p)^2$ is positive. For spin-up protons $\nu$ starts from 1 while for spin-down protons 0. It should be pointed out that here the so-called spin up and spin down are just relative notes since the wave functions are no longer eigenfunctions of 3-component spin operator [23], mainly attributed to the coupling of the spin to the magnetic field. The electrons are assumed to move freely in the strong magnetic field. The chemical potential of electrons $\mu_e = \epsilon_f^e$. Its relation to the electron Fermi momentum as well as the definition of the electron density $\rho_0^e$ are the same as given in Eqs. (5) and (9) except that the corresponding quantities are replaced by the electron ones. Numerical calculations are performed under the constraints of the charge neutrality $\rho_0^p = \rho_0^e$ and the $\beta$-equilibrium $\mu_n = \mu_p + \mu_e$ [24]. These two constraint equations together with three meson equations are solved self-consistently in an iterative procedure.

Figure 1 displays the effective nucleon mass and proton fraction as functions of the baryon density and magnetic field strength. Two situations with and without taking into account the anomalous magnetic moments are distinguished. In agreement with the finding of Refs. [13, 15], the effective mass remains unaltered from the field free case when $B \leq 10^5 B^e_c$. If the field is increased to $10^6 B_c^e$, the effects of magnetic fields cause the $m^*/M_N$ to increase when the AMM is included ($\kappa \neq 0$) and decrease when the AMM is neglected ($\kappa = 0$). The proton fraction is shown to increase with the increasing of the magnetic field strength in the case of $\kappa = 0$. A considerable enhancement of $Y_p$ is observed at $B = 10^6 B^e_c$. For even stronger magnetic fields, one may speak about proton-rich matter [13]. The situation, however, becomes completely different if the anomalous magnetic moments are taken into account. At $B = 10^5 B^e_c$, $Y_p$ is a little enhanced at lower density and then quickly approaches the field free case as the density increases. In the case of $B = 10^6 B^e_c$, no protons survive at $\rho < 4\rho_0$. Evident suppression of the proton fraction is exhibited for the typical density range of neutron stars.

In order to understand the above results, in the upper panel of Fig. 2 we depict the proton fraction as a function of the magnetic field strength, where the effects of the AMM are neglected. It can be found that for a fixed density the proton fraction begins
to increase rapidly at a certain value of the magnetic field. This is the critical point where both protons and electrons are completely spin polarized (i.e., only one Landau level is occupied.) due to Landau quantization while no direct effects from the magnetic field act on neutrons. The Fermi energies of electrons and protons fall drastically. As a consequence, a large amount of neutrons converts to protons. The system approaches to a well proton-rich matter. The variation of the critical field with the baryon density is displayed in Fig. 3 as the dashed line.

In the lower panel of Fig. 2 we show the results including the effects of the AMM. Different curves are related to the different baryon densities. For each curve there exist two turning points. The first one corresponds to the critical field of electron polarization, its variation with density is depicted as the dotted line in Fig. 3. As soon as the electron is fully polarized, its Fermi energy will be considerably reduced. This causes the conversion of neutrons to protons in a beta-equilibrium system. The proton fraction is therefore enhanced substantially. Because of the different coefficients for the AMM of electrons and protons, the complete spin polarization of protons is reached later than electrons as shown in Fig. 3 with the dash-dotted line. There are no evident signals for the proton spin polarization in the lower panel of Fig. 2 since the rapid increase of the proton fraction has been induced by the electron polarization. When the field is further increased, the neutron can be fully spin polarized because of the coupling of the neutron spin to the magnetic field, an effect totally caused by the anomalous magnetic moment. The inverse process, $p \rightarrow n$, starts at the second turning points appeared on the curves plotted in the lower panel of Fig. 2. This causes the $Y_p$ to drop rapidly with the increase of $B$. A pure neutron matter becomes possible at very large field strength. The critical field for neutron polarization as a function of the baryon density is given in Fig. 3 as the solid line. One can see that the neutron spin polarization happens at $B > 10^5 B_e^c$. We have also considered the situation where the electron AMM is switched off but the nucleon AMM is taken into account. No enhancement of the proton fraction is observed compared to the case that both the electron and nucleon AMM are neglected. While comparing with the condition that the AMM of both electrons and nucleons are considered, the proton
fraction is less suppressed. For instance, protons start to appear at $\rho \approx 2\rho_0$ rather than $\rho \approx 4\rho_0$ for $B = 10^6B_c^e$. In order to check our numerical process we have revealed the results of Ref. [15], which is indicated in the lower panel of Fig. 1 as the long-dashed line. The above calculations have been repeated with other mean-field parameter sets used in Refs. [13, 15], the qualitative trend remains unchanged.

In summary, we have studied the properties of neutron star matter under strong magnetic fields, with an emphasis on the neutron star composition. After taking into account the effects of the AMM of nucleons and electrons, the proton fraction is found to never exceed the field free case. Our results demonstrate that extremely strong magnetic fields may lead to a pure neutron matter rather than a proton-rich matter observed in Refs. [13, 15], mainly attributed to the complete neutron spin polarization induced by the AMM effects.

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References

[1] A. Hewish, S.J. Bell, J.D.H. Pilkington, P.F. Scott, and R.A. Collins, Nature 217, 709 (1968).

[2] T. Gold, Nature 218, 731 (1968).

[3] F.C. Michel, Theory of Neutron Star: Magnetospheres, (The University of Chicago Press, Chicago, 1991).

[4] E.P. Mazets, S.V. Golenetskii, V.N. Il’inskii, R.L. Aptekar’, and Yu.A. Guryan, Nature 282, 587 (1979); R.E. Rothschild, S.R. Kulkarni, and R.E. Lingenfelter, Nature 368, 432 (1994); C. Kouveliotou et al., Nature 393, 235 (1998); P.M. Woods et al., Astrophys. J. 519, L139 (1999).
[5] R.C. Duncan and C. Thompson, Astrophys. J. **392**, L9 (1992).

[6] S.L. Shapiro and S.A. Teukolsky, *Black Holes, White Dwarfs, and Neutron Stars*, (Wiley-Interscience, New York, 1983).

[7] D. Lai and S.L. Shapiro, Astrophys. J. **383**, 745 (1991).

[8] In-Saeng Suh and G.J. Mathews, Astrophys. J. **530**, 949 (2000).

[9] In-Saeng Suh and G.J. Mathews, Astrophys. J. **546**, 1126 (2001).

[10] A.S. Vshivtsev and D.V. Serebryakova, JETP **79**, 17 (1994).

[11] I. Vidana and I. Bombaci, Phys. Rev. **C66**, 045801 (2002).

[12] Jia Huan-yu, Sun Bao-xi, Meng Jie, and Zhao En-guang, Chin. Phys. Lett. **18**, 1571 (2001); Zhang Feng-shou and Chen Lie-wen, Chin. Phys. Lett. **18**, 142 (2001).

[13] S. Chakrabarty, D. Bandyopadhyay, and S. Pal, Phys. Rev. Lett. **78**, 2898 (1997).

[14] L.D. Landau and E.M. Lifshitz, *Quantum Mechanics*, (Pergamon, Oxford, 1977).

[15] A. Broderick, M. Prakash, and J.M. Lattimer, Astrophys. J. **537**, 351 (2000).

[16] J. Schwinger, Phys. Rev. **73**, 416 (1948); Phys. Rev. **76**, 790 (1949).

[17] J. Boguta, Phys. Lett. **B106**, 255 (1981).

[18] J.M. Lattimer, C.J. Pethick, M. Prakash, and P. Haensel, Phys. Rev. Lett. **66**, 2701 (1991); C.J. Pethick, Rev. Mod. Phys. **64**, 1133 (1992).

[19] R.C. Duncan, astro-ph/0002442.

[20] B.D. Serot and J.D. Walecka, Adv. Nucl. Phys. **16**, 1 (1986).

[21] J. Boguta and A.R. Bodmer, Nucl. Phys. **A292**, 413 (1977).

[22] N.K. Glendenning and S.A. Moszkowski, Phys. Rev. Lett. **67**, 2414 (1991).
[23] G. Mao, A. Iwamoto, and Z. Li, astro-ph/0109221; Chin. J. Astron. & Astrophys. accepted.

[24] N.K. Glendenning, *Compact Stars*, (Springer, New York, 1997).
Figure 1: The effective nucleon mass $m^*/M_N$ (upper panel) and proton fraction $Y_p$ (lower panel) as functions of the baryon density. Different curves are related to the different cases of magnetic fields $B$ and with or without the inclusion of the anomalous magnetic moments $\kappa$ as indicated in the figure. In the lower panel, the long-dashed line reveals the case considered in Ref. [15].
Figure 2: The proton fraction as a function of the magnetic field strength. The upper panel shows the results without the AMM while in the lower panel the AMM is taken into account.
Figure 3: The critical field of particle spin polarization as a function of the baryon density. Different curves correspond to the different cases as indicated in the figure and discussed in the text.