Research of High Resolution ISAR Algorithm Based on Coherent Registration and Fusion for Distributed Radar

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Abstract. According to the Inverse Synthetic Aperture Radar (ISAR), this thesis analyses the attributed to the physical restriction that the ISAR image resolution cannot be improved, therefore, a high resolution ISAR algorithm for distributed radar system is proposed. Depended on existing hardware development level, it combines multi-radar echo signals for one target and then utilize data fusion theory for multi-visual angle and multi-frequency band to obtain ultra-wide band and larger coherent accumulation angle, on this account, the much higher resolution 2D-image than the single radar is able to acquired. The main content illustrates the method for high resolution ISAR on target electromagnetic scattering two-exponential sum model, and the simulation is made to demonstrate the analysis is effective and validity.

1. Introduction
Since 2006, the Distributed Radar ISAR technology attracts dramatically attention, it is represented by More Input and More Output (MIMO) radar and Synthetic Impulse and Aperture Radar (SIAR)[1][2]. The principle of Distributed ISAR indicates that one target is emitted by a set of radar, where the observed information using width and angle relative to radar is utilized to realize associated imaging. In this way, the signal width and shifting angle is extended extremely to improving more two-dimension high resolution than single one[3][4]. In this paper, a two-dimension signal fusion ISAR on distributed radar method under the low SNR is proposed. What needs to be pointed out is that the method will organically combine distributed structure and data fusion theory in some way.

2. Geometric and Signal Model
Generally, there are three kinds of radar echo model, and we usually used including ideal scattering point model, index sum model and GTD model. Ideal scattering point model consider scattering intensity is fixed. But the GTD model take into account that the kind of scattering centre and position information[5]. In sum, the index sum model is applied to the followings, because the advantages about this method contain not only the simply and quickly mathematical operating, but also great matching when the observed width and angle is little needed.

2.1. One-Dimensional Model
On radar observed frequency and filed, target can be decomposed into several discrete scattering centre group, so we regard radar back scattering as some kinds of these group’s coherent composition. Thus, the total target frequency response consists of every scattering centre frequency[6][7].

**Figure 1. Geometry of far field scattering centre.**

As shown in Figure 1, the proposed incident wave spreads along the z axis in positive direction, and the incident electrical field in $r$ can be presented:

$$E(f) = \sum_{m} \phi_{m,f} e^{-j2\pi f_{m}r_{m}}$$  \hspace{1cm} (1)

The amplitude $\phi_{m,f}$ of scattering centre is affected by the change of frequency $(\beta\gamma) \leq \alpha$ from the GTD theory, and there $\alpha$ is model geometry type, hence, $\phi_{m,f}$ can be represent as:

$$\phi_{m,f} = A_{m}(j\frac{f}{f_{c}})^{y_{m}}$$  \hspace{1cm} (2)

From equation (1) and equation (2), the following result is obtained:

$$E(f) = \sum_{m} A_{m}(j\frac{f}{f_{c}})^{y_{m}} e^{-j\frac{2\pi f_{m}r_{m}}{f_{c}}}$$  \hspace{1cm} (3)

Where $r_{m} = \hat{z} \cdot r_{m}$, which is the target electromagnetic scattering model based on GTD theory. Model parameter $\{A_{m}, r_{m}, \alpha\}_{m=1}^{M}$ corresponds amplitude $A_{m}$, and the number of scattering center M.

The GTD frequency response model is established by the property of target electromagnetic scattering process in optics region, and it is difficult to solve the model parameters[8]. However, exponent sign $\beta_{m}$ can be substituted for exponential function $j^{\gamma_{m}}$ when the observed relative bandwidth is not wide enough. It is obtained that:

$$E(f) = \sum_{m=1}^{M} A_{m}(j\frac{f}{f_{c}})^{y_{m}} e^{-\frac{2\pi f_{m}r_{m}}{f_{c}}}$$  \hspace{1cm} (4)

Where $A_{m} = A_{m}(j\frac{f}{f_{c}})^{y_{m}}$.

For stepped frequency radar:

$$f = f_{0} + k\Delta f, \quad k = 0, \cdots, N-1$$  \hspace{1cm} (5)

Here $f_{c}$ the origins of stepped frequency, $\Delta f$ is stepped-frequency interval, N is step number, from equation (5) we can get:

$$E(f_{0} + k\Delta f) = \sum_{m} A_{m}\beta_{m}^{k+M} e^{-\frac{2\pi f_{0}r_{m}}{f_{c}}} e^{-\frac{2\pi k\Delta f r_{m}}{f_{c}}},$$  \hspace{1cm} (6)

If the target response $y(k)$ in frequency $f_{0} + k\Delta f$ is expressed by $E(f_{0} + k\Delta f)$, the equation (6) can be shown as:

$$y(k) = \sum_{m=1}^{M} d_{m}p_{m}^{k}, \quad k = 0, \cdots, N-1$$  \hspace{1cm} (7)
equation (7) is the one-dimension index sum model of target response. Model parameters \( \{d_m, p_m\}_{m=1}^M \) correspond amplitude \( d_m \), pole \( p_m \), and model order \( M \). Index sum model is applied to frequency response means make use of \( N \) frequency data to estimate parameter \( \{d_m, p_m\}_{m=1}^M \).

2.2. Two-Dimension Index Sum Model

The target’s back scattering field can be seen as scattering of several individual centres. A single centre back scattering field can be described as two-dimensional function containing frequency \( f \) and angle \( \theta \), thus the field will be presented by scattering synthesis of these points.

\[
E(f, \phi) = \sum_{m=1}^{M} \sigma_m \exp[-j \frac{4\pi f}{c}(x_m \cos \phi + y_m \sin \phi)]
\]

(8)

Here, \( c \) is light velocity, \( x_m \) and \( y_m \) is point two-dimensional coordinates. \( \sigma_m \) is complex amplitude, the number of scattering centre is \( M \). Only considering the frequency signal in a certain observed angle, the equation (8) is represented as:

\[
E(f) = \sum_{m=1}^{M} \sigma_m \exp[-j \frac{4\pi f}{c} R_m]
\]

(9)

Here, \( R_m = x_m \cos \phi + y_m \sin \phi \), \( c \) is light velocity.

From equation (8) it can indicate that under the certain observed angle it is correctly to get distance image with FFT transformation. But it is not correctly to get azimuth image, because phase is not a linear function for angle leading to image defocused with FFT transformation. Therefore, the phase decoupling is needed, and the polar coordinate transformation is:

\[
u = f \cos \phi \\
v = f \sin \phi
\]

(10)

So equation (8) can be represented as:

\[
E(u, v) = \sum_{m=1}^{M} \sigma_m \exp[-j \frac{4\pi}{c}(x_m u + y_m v)]
\]

(11)

\[
E(u, v) = \sum_{m=1}^{M} \sigma_m P_{um} P_{vm}^\ast
\]

In that, the estimation about \( \sigma_m \), \( P_{um} \), \( P_{vm} \) is applied to get echo data of any point in \( (u, v) \) domain. Here \( P_{um} = \exp(-j \frac{4\pi x_m}{c}) \), \( P_{vm} = \exp(-j \frac{4\pi y_m}{c}) \).

For sake of the radar data sampled uniform, the result transformed into \( f \) and \( \phi \) domain is non-uniform sampling, so the bilinear interpolation method is adopted to imaging. Searching a nearby four points in \( (f, \phi) \) domain structure function:

\[
\begin{align*}
& a_1 f_1 + a_2 \phi_1 + a_3 f_1 \phi_1 + a_4 = E(f_1, \phi_1) \\
& a_1 f_2 + a_2 \phi_2 + a_3 f_2 \phi_2 + a_4 = E(f_2, \phi_2) \\
& a_1 f_3 + a_2 \phi_3 + a_3 f_3 \phi_3 + a_4 = E(f_3, \phi_3) \\
& a_1 f_4 + a_2 \phi_4 + a_3 f_4 \phi_4 + a_4 = E(f_4, \phi_4)
\end{align*}
\]

(13)

And the vector is:

\[
\begin{bmatrix}
f_1 & \phi_1 & f_1 \phi_1 & 1 & a_1 \\
f_2 & \phi_2 & f_2 \phi_2 & 1 & a_2 \\
f_3 & \phi_3 & f_3 \phi_3 & 1 & a_3 \\
f_4 & \phi_4 & f_4 \phi_4 & 1 & a_4
\end{bmatrix}
= \begin{bmatrix}
E(f_1, \phi_1) \\
E(f_2, \phi_2) \\
E(f_3, \phi_3) \\
E(f_4, \phi_4)
\end{bmatrix}
\]

(14)

It is easy to obtain \( [a_1, a_2, a_3, a_4] \) from equation (14) and estimate the interpolation point value in order to extract echo data.

3. Multi-Radar and Multi-view ISAR High Resolution Imaging
Pointing at bistatic radar, on the one hand if the observed position is different as shown in Figure 2, there will be two radar scanning curved surface in the space, on the other hand, if two radar observe under small angle, the curved surface will be mostly flat. Generally, the scanning surface is not co-plane, namely different imaging surface.

Given objective real rotation axis and radar sight line remains unchanged during the whole imaging, thus the two radar scanning will form two conical surface around the axis of rotation. Usually obtaining the same under the view point of coherent accumulation time is more than the single imaging time, therefore, in such a short period of time actual rotation make approximation to the rotation is unreasonable. If the actual rotation axis is equal with two radar line of sight angle, so the radar scanning plane on the same cone surface as shown in Figure 3. Particularly, if the actual rotation axis is perpendicular to radar line of sight, both two radar scanning surface will be in \( k_z - k_y \) surface which means coplanar imaging.

From the GTD scattering field theory, in the region of the high frequency radar, target backscatter can be treated as a scattering field of the coherent superposition of multiple independent scattering centre. When there is a relatively smaller bandwidth and observed angle, the scattering field can be expressed as:

\[
E(f, \phi) = \sum_{m=1}^{M} \sigma_m \exp[-j \frac{4\pi f}{c}(x_m \cos \phi + y_m \sin \phi)]
\]  

(15)

Here, \( c \) is light velocity, \( x_m \) and \( y_m \) is the Two-Dimensional coordinates of the scattering points in the target coordinates. \( \sigma_m \) is complex amplitude, and \( M \) is the number of scattering centre.

\[
u = f \sin \phi
\]

So, equation (15) can be written as:

\[
E(u,v) = \sum_{m=1}^{M} \sigma_m \exp[-j \frac{4\pi}{c}(x_m u + y_m v)]
\]

(17)

\[
u = v_0 + \delta v n, \quad n \in [0, \ldots, N-1]
\]

(18)

And then equation (18) is able to be simplified into Two-Dimensional all-pole model:

\[
E(n_1,n_2) = \sum_{m=1}^{M} \sigma_m \exp[-j \frac{4\pi}{c}(x_m \delta n_1 + y_m \delta n_2)]
\]

(19)
Here, $p_m$ and $q_n$ in equation (19) is correspondingly indicate $u$ and $v$ directional pole, and $a_m$ means model amplitude coefficient. Equation (20) and equation (21) represent the $p_m$ and $q_n$ specifically.

4. Simulations

In order to validate the effectiveness of the ISAR high resolution in the multiple radar and multiple angle with Two-Dimensional fusion technology, First of all, some point targets are used to deal with simulation, and among them five scattering points is (-0.8,-0.8), (-0.8,0.8), (0.8,0.8), (0.8,-0.8), (1.6,0) and all of amplitude is 1, where the scattering points are shown in Figure 4. Working in different perspectives and different frequency bands of two radar parameters are shown in the following Table 1.

Table 1. Simulation Parameters.

|      | Frequency Bandwidth | Frequency Interval | Angle Interval | SNR |
|------|---------------------|--------------------|----------------|-----|
| Radar1 | 5.9–6.0 GHz         | 5MHz               | -2°–0°         | 0.1° | 13dB |
| Radar2 | 6.0–6.1 GHz         | 5MHz               | 0°–2°          | 0.1° | 13dB |
| Radar3 | 5.9–6.1 GHz         | 5MHz               | -2°–2°         | 0.1° | 13dB |

Figure 5. Radar 1 ISAR Image

Figure 6. Radar 2 ISAR Image
Figure 5–Figure 8 present the result in different method, whereby, Figure 7 shows the imaging result for the full spectrum domain signal in FFT, and then Figure 8 shows the result for Two-Dimensional fusion of the multiple radar and multiple angles. As a result, five scattering points are clearly visible, thus, it proves that both the imaging result in two method is roughly same. Figure 5 and Figure 6 are the imaging result for radar 1 and radar 2. Caused by a lack of resolution, there are three scattering points shown in the figure cannot be separated.

5. Conclusion
This paper devotes to studying the radar signal fusion of multiple bands and multiple sights high ISAR imaging technology. Firstly, the paper elaborates the electromagnetic scattering model, as well as the linear frequency modulation signal pre-processing and construct Two-Dimension index sum model. And then a correction method is proposed to extract model on the basis of the index sum model leading to improving the efficiency and stability pole estimation dramatically. In the end, the coefficient of model is achieved with least squares solution. The method in this paper is so useful to extrapolate or interpolation spectrum data for obtaining effectiveness of super resolution and high quality of ISAR image. The simulation demonstrates the method is effective.

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