Rapidity Gaps and Color Evolution in QCD Hard Scattering

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Abstract

We discuss rapidity-gap events between two jets produced at high momentum transfer in $p\bar{p}$ scattering, from the point of view of the soft energy flow into the interjet region. We define a gap cross section and, in perturbative QCD (pQCD), resum all the leading logarithms in the soft intermediate energy. We show that the numerical result from our cross section reproduces the shape of the D0 and CDF [1, 2, 3] experimental data.

1Talk given at the International Euroconference on Quantum Chromodynamics (QCD ’98), Montpellier, France, July 2-8, 1998.
1 Introduction

Dijet rapidity-gap events, identified by very low hadron multiplicity in the rapidity region between two jets produced at high momentum transfer, have been observed both at Fermilab [1, 2] and DESY [4, 5]. We refer in particular to the experimental papers of the CDF and D0 collaborations [1, 2, 3], where an excess of opposite-side dijet events with respect to a background of same-side events is reported in the inclusive process \( p(p_A) + \bar{p}(p_B) \rightarrow J_1(p_1) + J_2(p_2) + X \), for low hadron multiplicity in the central region of the calorimeter detector.

This phenomenon has been originally predicted from the exchange of two or more hard gluons in a color-singlet configuration, so that color recombination between nearly opposite moving particles, and consequent interjet hadronization, is avoided [6].

In our analysis of the problem [7, 8], we consider energy flow instead of particle multiplicity. In these terms, we first identify a dijet rapidity-gap cross section and, using pQCD, write it in a factorized form that enables us to compute the evolution towards long distances of the possible color components exchanged in the partonic hard scattering. Such evolution is due to soft, but still perturbative, interactions between the active partons. In our analysis, gaps turn out to have a more complicated structure than just color singlet exchange [9, 10]. The numerical results we obtain from our cross section closely resemble the qualitative behavior of the experimental data of Ref. [3].

2 Dijet rapidity-gap cross section

In terms of the (pseudo)rapidity variable \( y = (1/2) \ln \cot(\theta/2) \), with \( \theta \) the polar angle, we identify by the condition \(|y| > y_0\) the forward and backward regions of the calorimeter, where the two jets are to be found, with transverse energies above the experimental threshold \( E_T \) (see Fig. 1 of Ref. [7]). We define \( Q_c \) the amount of energy flowing into the symmetric central region, of width \( \Delta y = 2y_0 \).

The inclusive dijet cross section for all events with energy in the central region equal to \( Q_c \) can be written in the factorized form:

\[
\frac{d\sigma}{dQ_c}(S, E_T, \Delta y) = \sum_{f_A, f_B = u, d} \int d\cos \hat{\theta} \\
\times \int_0^1 dx_A \int_0^1 dx_B \phi_{f_A/p}(x_A, -\hat{t}) \phi_{f_B/\bar{p}}(x_B, -\hat{t}) \\
\times \sum_{f_1, f_2 = u, d} \frac{d\hat{s}(\hat{t})}{dQ_c} d\cos \hat{\theta} \left( \hat{s}, \hat{t}, y_{JJ}, \Delta y, \alpha_s(\hat{t}) \right) ,
\]

where, for simplicity, we only consider the contribution of valence quarks and of the partonic process \( q(k_A) + \bar{q}(k_B) \rightarrow q(k_1) + \bar{q}(k_2) + X \). We identify by \( \phi_{f_A/p} \) and \( \phi_{f_B/\bar{p}} \) the valence parton distributions, evaluated at scale \(-\hat{t}\), the dijet momentum.
transfer. \(d\hat{\sigma}^{(t)}/dQ_c\,d\cos\hat{\theta}\) is a hard scattering function, starting with the Born cross section at lowest order. The index \(f\) denotes \(f_A + \bar{f}_B \rightarrow f_1 + \bar{f}_2\). The detector geometry determines the phase space for the dijet total rapidity, \(y_{JJ}\), the partonic center-of-mass (c.m.) energy squared, \(\hat{s}\), and the partonic c.m. scattering angle \(\hat{\theta}\), with 

\[-\hat{s}^2\left(1 - \cos\hat{\theta}\right) = \hat{t}.

3 The partonic cross section: factorization and evolution in color space

The partonic cross section \(d\hat{\sigma}^{(t)}/dQ_c\,d\cos\hat{\theta}\) is an IR safe quantity. It can be further factorized in color space into a hard function, \(H\), describing quanta of high virtuality, and a soft function, \(S\), accounting for soft gluon emission into the central region, as follows [11, 12, 13]:

\[
Q_c \frac{d\hat{\sigma}^{(t)}}{dQ_c} \left(\hat{s}, \hat{t}, y_{JJ}, \Delta y, \alpha_s(-\hat{t})\right) = H_{IL} \left(\frac{\sqrt{-\hat{t}}}{\mu}, \sqrt{\hat{s}}, \alpha_s(\mu^2)\right) \times S_{LI} \left(\frac{Q_c}{\mu}, y_{JJ}, \Delta y\right).
\]

(2)

Here we can identify a hard scale, \(\sqrt{-\hat{t}}\), a soft scale, \(Q_c\), and a new factorization scale, \(\mu\). We sum over the indices \(I\) and \(L\), which label the possible color structures of the hard interaction. For the scattering of valence quarks and antiquarks these are just singlet and octet and, in a basis of \(t\)-channel projectors, are given respectively by [13]

\[
c_1 = \delta_{r_A,r_1} \delta_{r_B,r_2}
\]

\[
c_2 = -\frac{1}{2N_c} \delta_{r_A,r_1} \delta_{r_B,r_2} + \frac{1}{2} \delta_{r_A,r_B} \delta_{r_1,r_2}.
\]

(3)

The soft matrix \(S_{LI}\) in Eq. (2) coincides with the effective “eikonal” cross section, in which the hard scattering is replaced by a product of recoilless Wilson lines [11, 12, 13] in the directions of the incoming partons and the outgoing jets. It starts to receive contributions at zeroth order in \(\alpha_s\), where it is given by just a set of color traces,

\[
S^{(0)}_{LI} = \begin{pmatrix}
N_c^2 & 0 \\
0 & \frac{1}{4} (N_c^2 - 1)
\end{pmatrix},
\]

(4)

with \(N_c\) the number of colors.

\(H_{IL}\), on the other hand, starts at the level of Born amplitudes. The contribution of \(t\)-channel gluon exchange, which is pure color octet, is dominant, and we have \(H_{IL}^{(1)} = \delta_{l_2} \delta_{L_2} \hat{\sigma}_t\), where \(\hat{\sigma}_t\) is the \(t\)-channel partonic cross section, including the coupling \(\alpha_s(-\hat{t})\).
From the independence on $\mu$ of the left-hand side of Eq. (2), we immediately deduce the evolution equation satisfied by the soft matrix $S_{LI}$,

\[
\left( \mu \frac{\partial}{\partial \mu} + \beta(g) \frac{\partial}{\partial g} \right) S_{LI} = -(\Gamma_S^\dagger)_{LB} S_{BL} - S_{LA}(\Gamma_S)_{AI},
\]

\[\] (5)

where $\Gamma_S(\alpha_s)$ is a soft anomalous dimension matrix. The solution of this equation will enable us to resum all the leading logarithms of the soft scale $Q_c$. In the singlet-octet basis of Eq. (3) the anomalous dimension matrix is given by [7]

\[
\Gamma_S(y_{JJ}, \Delta y, \hat{\theta}) = \frac{\alpha_s}{4\pi} \left( \begin{array}{cc}
\rho + \xi & -4C_F i\pi \\
-8i\pi & \rho - \xi
\end{array} \right),
\]

\[\]

(6)

where the functions $\xi$ and $\rho$ are

\[
\xi(\Delta y) = -2N_c \Delta y + 2i\pi \frac{N_c^2 - 2}{N_c},
\]

\[\]

(7)

\[
\rho(y_{JJ}, \Delta y, \hat{\theta}) = \frac{N_c^2 - 1}{N_c} \times \left[ \ln \left( \frac{\cos(\hat{\theta}) + \tanh \left( \frac{\Delta y}{2} - y_{JJ} \right)}{\cos(\hat{\theta}) - \tanh \left( \frac{\Delta y}{2} - y_{JJ} \right)} \right) \right.
\]

\[
+ \ln \left( \frac{\cos(\hat{\theta}) - \tanh \left( -\frac{\Delta y}{2} - y_{JJ} \right)}{\cos(\hat{\theta}) + \tanh \left( -\frac{\Delta y}{2} - y_{JJ} \right)} \right) \left. \right] + \frac{2}{N_c} \Delta y - 2i\pi \frac{N_c^2 - 2}{N_c}.
\]

\[\]

(8)

### 4 Color Evolution

It is natural to solve the evolution equation (5) in the basis of the eigenvectors of $\Gamma_S$,

\[
e_1 = \left( \frac{\xi}{\xi - \frac{1}{N_c}} \right)^{-1} \left( \begin{array}{c}
1 \\
\frac{\xi}{\xi - \frac{1}{N_c}} \end{array} \right),
\]

\[
e_2 = \left( \frac{i}{8\pi} \left( \xi + \frac{1}{N_c} \right) \right),
\]

\[\]

(9)

where we define

\[
\eta(\Delta y) \equiv \sqrt{N_c \left[ \xi(\Delta y) \right]^2 - 32C_F \pi^2}.
\]

\[\]

(10)

The eigenvectors only depend on the geometry, through $\Delta y$. In the limit of a wide central region $\Delta y \gg 1$ they become pure color states, $e_1$ a singlet and $e_2$ an octet. However, for a typical D0 geometry [1, 3], $\Delta y = 4$ at $\sqrt{S} = 1800$ GeV, we have in general color mixed states. In the following we will refer to $e_1$ and $e_2$ as “quasi-singlet”
and “quasi-octet” respectively, and we shall use Greek indices to identify the basis in which $\Gamma_S$ is diagonal [13].

The eigenvalues of $\Gamma_S$ corresponding to the eigenvectors of Eq. (8) are given by $\lambda_{\beta} = \alpha_s \hat{\lambda}_{\beta} + \cdots$, where we define

$$
\begin{align*}
\hat{\lambda}_1(y_{JJ}, \Delta y, \hat{\theta}) &= \frac{1}{2\pi} \left[ \frac{1}{2} \rho - \frac{1}{2\sqrt{N_c}} \eta \right], \\
\hat{\lambda}_2(y_{JJ}, \Delta y, \hat{\theta}) &= \frac{1}{2\pi} \left[ \frac{1}{2} \rho + \frac{1}{2\sqrt{N_c}} \eta \right].
\end{align*}
$$

(11)

The $\hat{\lambda}_\beta$‘s are in general complex. Over most of the $\hat{\theta}, y_{JJ}$ kinematical region $\text{Re} \hat{\lambda}_2 > \text{Re} \hat{\lambda}_1$ as well as $|\hat{\lambda}_2| > |\hat{\lambda}_1|$. The evolution equation (5) is easily solved when $\Gamma_S$ is diagonal, to give the partonic cross section [4]

$$
\frac{d\hat{\sigma}^{(f)}}{dQ_c d\cos \theta} (\hat{s}, \hat{t}, y_{JJ}, \Delta y, \alpha_s(-\hat{t})) = H_{\beta\gamma}^{(1)} (\Delta y, \sqrt{\hat{s}}, \sqrt{-\hat{t}}, \alpha_s(-\hat{t})) S_{\gamma\beta}^{(0)} (\Delta y) \\
\times \frac{E_{\gamma\beta}}{Q_c} \left[ \ln \left( \frac{Q_c}{\Lambda} \right) \right]^{E_{\gamma\beta}-1} \left[ \ln \left( \frac{\sqrt{-\hat{t}}}{\Lambda} \right) \right]^{-E_{\gamma\beta}}.
$$

(12)

Here $S_{\gamma\beta}^{(0)}$ and $H_{\beta\gamma}^{(1)}$ are obtained by transforming the corresponding quantities, defined in the basis (3), into the color eigenspace [4]. The exponents $E_{\gamma\beta}$ are given by

$$
E_{\gamma\beta} (y_{JJ}, \hat{\theta}, \Delta y) = \frac{2\pi}{\beta_1} \left( \hat{\lambda}_\gamma^* + \hat{\lambda}_\beta \right),
$$

(13)

where $\beta_1$ is the first coefficient in the expansion of the QCD $\beta$-function, $\beta_1 = \frac{11}{3} N_c - \frac{2}{3} n_f$. The magnitude of the quasi-singlet exponent $E_{11}$ is less than one for most kinematical configurations, while the magnitude of the quasi-octet exponent $E_{22}$ is always greater than one. As a consequence, Eq. (13) shows that the quasi-singlet component of the hard scattering is a decreasing function of $Q_c$, dominant for $Q_c < 1$ GeV and formally divergent at $Q_c = \Lambda$, whereas the quasi-octet component increases up to a maximum, until the inverse power behavior $1/Q_c$ takes over, causing a fast decrease.

5 Numerical results

By using in Eq. (1) the partonic cross section of Eq. (12) and the set CTEQ4L of parton distribution functions [13], and by performing the numerical integrations with the routine VEGAS, we have obtained the results shown in Fig. 1. These are to be compared with the experimental plot in Fig. 1 of Ref. [3], showing the measured
Figure 1: The cross section (solid line) and the contributions from quasi-octet (dotted line) and quasi-singlet (dashed line), for $\sqrt{S} = 630$ GeV, $\Delta y = 3.2$, and $\sqrt{S} = 1800$ GeV, $\Delta y = 4$, respectively. Compare Fig. 1 of Ref. [3]. Units are arbitrary.

number of events as a function of the number of towers counted in the central region of the calorimeter, clearly related to our $Q_c$. The shape of the experimental data is reproduced by our result. We emphasize that this result is perturbative. For quantities independent of overall normalizations we have found similarity with the data even at the quantitative level. For example, the minimum-maximum ratio of the cross section, is about 30% at $\sqrt{S} = 630$ GeV, and about 15% at $\sqrt{S} = 1800$ GeV, close to estimates we can make from the experimental data. Also an analog of the “hard singlet fraction” [1, 2], defined as the ratio of the area under the quasi-singlet curve to the area under the overall curve, is found to be about 5% at $\sqrt{S} = 630$ GeV and about 3% at $\sqrt{S} = 1800$ GeV, of the same order of magnitude, although somewhat higher, than the roughly 1% found at the Tevatron using track or tower multiplicities. The precise origin of this discrepancy might be related to the non-perturbative part of the survival probability [14], and remains to be explored. The decrease of the singlet fraction as a function of the center of mass energy is also in agreement with the experiment, although the trend we find is slower than the measured one.

6 Summary

We have computed by means of pQCD a dijet rapidity-gap cross section, defined in terms of energy flow. We have shown that the numerical results from this cross section qualitatively reproduce the behavior of the experimental data of Ref. [3]. Our analysis still needs to be refined by including the contributions of gluons and sea-quarks [8].
Acknowledgments

I would like to thank George Sterman, with whom the work I presented was done, for his suggestions and constant encouragement. I am also grateful to Jack Smith for his valuable advice in the implementation of the numerical simulation. This work was supported in part by the National Science Foundation, grant PHY9722101.

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