A new 4-D hyperchaotic hyperjerk system with a single equilibrium, its dynamic properties and circuit design

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Abstract. This paper announces a new four-dimensional hyperchaotic hyperjerk system with a single equilibrium and discusses its dynamic properties such as Lyapunov exponents, phase portraits, Kaplan-Yorke dimension and equilibrium points. Hyperjerk systems have a nice triangular structure in their dynamics and they have many engineering applications. Our new hyperjerk system has three nonlinearities in total. New synchronization results based on active backstepping control are also derived for the new hyperjerk system. In addition, an electronic circuit implementation of the new hyperjerk system is designed carefully and examined well in MultiSIM. A good qualitative agreement has been shown between the MATLAB simulations of the theoretical hyperchaotic hyperjerk model and the MultiSIM results.

1. Introduction

Chaotic dynamical systems have applications in several areas of science and engineering [1-4]. Some popular applications of chaotic systems can be cited as oscillators [5-10], weather systems [11-12], chemical reactors [13-15], robotics [16-17], communication systems [18-20], circuits [21-26], etc.

In 1996, Gottlieb [27] exhibited that 3-D chaotic systems can be expressed as single ordinary differential equations that can be termed as jerk differential equations. The jerk differential equations arise in many physical models of science such as jerk circuits [28-30], thermal arc plasma [31], biological reactions [32], mechanical oscillations [33-34], etc.

In physics, a jerk differential equation can be expressed as the third order dynamics

\[ D^3x = f(x, Dx, D^2x) \]  

where \( D = \frac{d}{dt} \). If \( x(t) \) represents the displacement of a body, then \( Dx = \dot{x} \) represents its velocity and \( D^2x = \ddot{x} \) its acceleration. The third derivative \( D^3x = \dddot{x} \), is called as the jerk of the body.
Generalizing the jerk system (1), we obtain the hyperjerk system given by the dynamics

\[ D^{(n)} x = f(x, Dx, \ldots, D^{(n-1)} x), \quad (n \geq 4) \]  

(2)

It is convenient to express the hyperjerk dynamics (3) in the form of a system of ordinary differential equations. For this purpose, we define phase variables as

\[ x_1 = x, \quad x_2 = Dx, \quad \ldots, \quad x_n = D^{(n-1)} x \]  

(3)

Using the phase variables (3), we can rewrite the hyperjerk dynamics (2) in system form as

\[
\begin{align*}
\dot{x}_1 &= x_2 \\
\dot{x}_2 &= x_3 \\
& \vdots \\
\dot{x}_{n-1} &= x_n \\
\dot{x}_n &= f(x_1, x_2, \ldots, x_n)
\end{align*}
\]

(4)

Hyperjerk systems have generated significant attention in the chaos literature in view of their simple structure and complex dynamic properties [35-40]. Hyperchaotic systems are chaotic systems with more than one positive Lyapunov exponent, which have high level of complexity [41-45].

In this paper, we modify the Daltzis hyperjerk system [46] to obtain a new hyperjerk system. It is interesting that our new hyperjerk system is hyperchaotic and has a unique equilibrium at the origin. We detail our new system with phase portraits, an analysis of its Lyapunov exponents, Kaplan-Yorke dimension, dissipativity, etc.

As a control application, we describe the synchronization of the new hyperjerk system with backstepping control [47-48]. As an engineering application, we design an electronic circuit of the new hyperjerk system using MultiSIM.

The structure of this paper is as follows. Section 2 announces the new hyperchaotic hyperjerk system with a unique equilibrium and describes its properties. Section 3 details the synchronization of the new hyperjerk system via backstepping control. Section 4 describes an electronic circuit design of the new hyperjerk system using MultiSIM. Section 5 summarizes the paper with conclusions.

2. A new 4-D hyperchaotic hyperjerk system

In 2018, Daltzis et al. [46] proposed a new 4-D hyperjerk system given by the dynamics

\[
\begin{align*}
\dot{x}_1 &= x_2 \\
\dot{x}_2 &= x_3 \\
\dot{x}_3 &= x_4 \\
\dot{x}_4 &= -x_1 - x_2 - ax_3 - b |x_2| - cx_1^4 x_4
\end{align*}
\]

(5)

In Eq. (5), \( x_1, x_2, x_3, x_4 \) are the states and \( a, b, c \) are positive parameters. In [49], it was shown that the hyperjerk system (5) is hyperchaotic for the choice of parameters \((a, b, c) = (3.8, 0.1, 1.5)\).

Using Wolf’s algorithm [49], the Lyapunov exponents of the Daltzis hyperjerk system (5) are calculated for \((a, b, c) = (3.8, 0.1, 1.5)\), and \( X(0) = (0.1, 0.1, 0.1, 0.1) \) for \( T = 1E4 \) seconds as

\[ L_1 = 0.1201, \quad L_2 = 0.0210, \quad L_3 = 0, \quad L_4 = -1.2854 \]

(6)

Since the Lyapunov exponents \( L_1 \) and \( L_2 \) are positive in (6), we conclude that the Daltzis hyperjerk system (5) is hyperchaotic. Also, we can calculate the Kaplan-Yorke dimension of the Daltzis hyperjerk system (5) as follows:

\[ D_{KY} = 3 + \frac{L_4 + L_2 + L_3}{|L_4|} = 3.1098 \]

(7)
In this paper, we propose a new 4-D hyperchaotic hyperjerk system by modifying the Daltzis hyperjerk system (5) and obtaining a new system given as follows:

\[
\begin{align*}
\dot{x}_1 &= x_2 \\
\dot{x}_2 &= x_3 \\
\dot{x}_3 &= x_4 \\
\dot{x}_4 &= -x_1 - x_2 - ax_3 + b |x_3| - cx_4^4 x_4 - dx_2^2
\end{align*}
\]  

(8)

In Eq. (8), \(x_1, x_2, x_3, x_4\) are the states and \(a, b, c, d\) are positive parameters.

We show that the system (8) is hyperchaotic for the parameter values \((a, b, c, d) = (3.6, 0.02, 3, 0.05)\)

Using Wolf’s algorithm [49], the Lyapunov exponents of the new hyperjerk system (8) are obtained for the parameter values as in (9) and \(X(0) = (0.1, 0.1, 0.1, 0.1)\) for \(T = 1E4\) seconds as

\[
L_1 = 0.12479, \quad L_2 = 0.03995, \quad L_3 = 0, \quad L_4 = -1.22715
\]  

(10)

Since \(L_1\) and \(L_2\) are positive Lyapunov exponents, the new hyperjerk system (8) is hyperchaotic.

By adding all the Lyapunov exponents in (10), we get their sum as

\[
L_1 + L_2 + L_3 + L_4 = -1.06241 < 0
\]  

(11)

This shows that the new hyperchaotic hyperjerk system (8) is dissipative.

Comparing the equations (6) and (10), we notice that the positive Lyapunov exponents \(L_1, L_2\) of the new hyperjerk system (8) are greater than the positive Lyapunov exponents \(L_1, L_2\) of the Daltzis hyperjerk system (5).

The Kaplan-Yorke fractal dimension of the new hyperjerk system (8) is calculated as

\[
D_{KY} = 3 + \frac{L_1 + L_2 + L_3}{|L_4|} = 3.1342,
\]  

(12)

which is greater than the Kaplan-Yorke dimension \(D_{KY}\) of the Daltzis hyperjerk system (5).

Thus, we have shown that the new hyperjerk system (8) has more complex dynamic properties than the Daltzis hyperjerk system (5).

It is easy to verify that \(E_0 = (0, 0, 0, 0)\) is the unique equilibrium point of the new hyperjerk system (8). The Jacobian matrix of the new hyperjerk system (8) at \(E_0\) is calculated for the hyperchaotic case (9) as

\[
J_0 = \begin{bmatrix}
0 & 1 & 0 & 0 \\
0 & 0 & 1 & 0 \\
0 & 0 & 0 & 1 \\
-1 & -1 & -3.6 & 0
\end{bmatrix}
\]  

(13)

The eigenvalues of \(J_0\) are computed as

\[
\lambda_{1,2} = 0.1604 \pm 1.8395i, \quad \lambda_{3,4} = -0.1604 \pm 0.5172i
\]  

(14)

This shows that the equilibrium point \(E_0 = (0, 0, 0, 0)\) is in the critical case.

Figures 1-4 show the 2-D phase portraits of the new hyperjerk system (8).
Figure 1. Two-dimensional phase plot of the new hyperjerk system (8) in the \((x_1, x_2)\) plane with \(X_0 = (0.1, 0.1, 0.1, 0.1)\) and \((a, b, c, d) = (3.6, 0.02, 3, 0.05)\)

Figure 2. Two-dimensional phase plot of the new hyperjerk system (8) in the \((x_2, x_3)\) plane with \(X_0 = (0.1, 0.1, 0.1, 0.1)\) and \((a, b, c, d) = (3.6, 0.02, 3, 0.05)\)
Figure 3. Two-dimensional phase plot of the new hyperjerk system (8) in the \((x_3, x_4)\) – plane with \(X_0 = (0.1, 0.1, 0.1, 0.1)\) and \((a, b, c, d) = (3.6, 0.02, 3, 0.05)\)

Figure 4. Two-dimensional phase plot of the new hyperjerk system (8) in the \((x_1, x_4)\) – plane with \(X_0 = (0.1, 0.1, 0.1, 0.1)\) and \((a, b, c, d) = (3.6, 0.02, 3, 0.05)\)
3. Global hyperchaos synchronization of the new hyperjerk systems via backstepping control

In this section, we design an engineering application of the new hyperjerk system, viz. global hyperchaos synchronization of the new hyperjerk systems taken as master and slave systems via active backstepping control method.

As the master system, we take the new hyperjerk system given by

\[
\begin{align*}
\dot{x}_1 &= x_2 \\
\dot{x}_2 &= x_3 \\
\dot{x}_3 &= x_4 \\
\dot{x}_4 &= -x_1 - x_2 - ax_3 + b|x_3| - cx_1^4x_4 - d x_2^2
\end{align*}
\]

where \( X = (x_1, x_2, x_3, x_4) \) is the state and \( a, b, c, d \) are system parameters.

As the slave system, we consider the new hyperjerk system with control given by

\[
\begin{align*}
\dot{y}_1 &= y_2 \\
\dot{y}_2 &= y_3 \\
\dot{y}_3 &= y_4 \\
\dot{y}_4 &= -y_1 - y_2 - ay_3 + b |y_3| - cy_1^4y_4 - dy_2^2 + u
\end{align*}
\]

where \( Y = (y_1, y_2, y_3, y_4) \) is the state and \( u \) is the active backstepping control to be designed.

The synchronization error between new hyperjerk systems (15) and (16) is defined via the equation

\[
\begin{align*}
e_1 &= y_1 - x_1 \\
e_2 &= y_2 - x_2 \\
e_3 &= y_3 - x_3 \\
e_4 &= y_4 - x_4
\end{align*}
\]

The dynamics of the synchronization error is easily obtained as follows:

\[
\begin{align*}
\dot{e}_1 &= e_2 \\
\dot{e}_2 &= e_3 \\
\dot{e}_3 &= e_4 \\
\dot{e}_4 &= -e_1 - e_2 - ae_3 + b |y_3| - c(y_1^4y_4 - x_1^4x_4) - d(y_2^2 - x_2^2) + u
\end{align*}
\]

Using active backstepping control method, we establish the key result of this section.

**Theorem 1.** The master and slave hyperchaotic systems represented by the new hyperjerk systems (15) and (16) are globally and exponentially synchronized by means of the active backstepping feedback control law given by

\[
u = -4e_1 - 9e_2 - (9 - a)e_3 - 4e_4 - b |y_3| - c(y_1^4y_4 - x_1^4x_4) + d(y_2^2 - x_2^2) - k \xi_4
\]

where \( k > 0 \) is a gain constant and

\[
\xi_4 = 3e_1 + 5e_2 + 3e_3 + e_4
\]

**Proof.** We establish this result via backstepping control method and Lyapunov stability theory [50]. We define the Lyapunov function

\[
V_4(\xi_4) = \frac{1}{2} \xi_4^2
\]

where

\[
\xi_4 = e_1
\]
Differentiating $V_1$ along the error dynamics (18), we get
\[ \dot{V}_1 = \xi_1 \dot{\xi}_1 = e_1 e_2 = -\xi_1^2 + \xi_1 (e_1 + e_2) \]  
(23)

We set
\[ \xi_2 = e_1 + e_2 \]  
(24)

Using (24), we can rewrite (23) as
\[ \dot{V}_1 = -\xi_2^2 + \xi_1 \xi_2 \]  
(24)

Next, we define the Lyapunov function
\[ V_2(\xi_1, \xi_2) = V_1(\xi_1) + \frac{1}{2} \xi_2^2 = \frac{1}{2} (\xi_1^2 + \xi_2^2) \]  
(26)

Differentiating $V_2$ along the error dynamics (18), we get
\[ \dot{V}_2 = -\xi_1^2 - \xi_2^2 + \xi_2 (2e_1 + 2e_2 + e_3) \]  
(27)

Now, we set
\[ \xi_3 = 2e_1 + 2e_2 + e_3 \]  
(28)

Using (28), we can simplify Eq. (27) as
\[ \dot{V}_2 = -\xi_3^2 + \xi_2 \xi_3 \]  
(29)

Next, we define the Lyapunov function
\[ V_3(\xi_1, \xi_2, \xi_3) = V_2(\xi_1, \xi_2) + \frac{1}{2} \xi_3^2 = \frac{1}{2} (\xi_1^2 + \xi_2^2 + \xi_3^2) \]  
(30)

Differentiating $V_3$ along the error dynamics (18), we get
\[ \dot{V}_3 = -\xi_1^2 - \xi_2^2 - \xi_3^2 + \xi_3 (3e_1 + 5e_2 + 3e_3 + e_4) \]  
(31)

Next, we set
\[ \xi_4 = 3e_1 + 5e_2 + 3e_3 + e_4 \]  
(32)

Using (32), we can simplify the equation (31) as
\[ \dot{V}_3 = -\xi_1^2 - \xi_2^2 - \xi_3^2 + \xi_3 \xi_4 \]  
(33)

To simplify the notation, we set $\xi = (\xi_1, \xi_2, \xi_3, \xi_4)$.

Finally, we define the quadratic Lyapunov function
\[ V(\xi) = V_3(\xi_1, \xi_2, \xi_3, \xi_4) + \frac{1}{2} \xi_4^2 = \frac{1}{2} (\xi_1^2 + \xi_2^2 + \xi_3^2 + \xi_4^2) \]  
(34)

Clearly, $V$ is a quadratic and positive definite function on $\mathbb{R}^4$.

Differentiating $V$ along the error dynamics (18), we get
\[ \dot{V} = -\xi_1^2 - \xi_2^2 - \xi_3^2 - \xi_4^2 + \xi_4 (\xi_4 + \xi_3 + \xi_4) = -\xi_1^2 - \xi_2^2 - \xi_3^2 - \xi_4^2 + \xi_4 (S) \]  
(35)

where
\[ S = \xi_4 + \xi_3 + \xi_4 = \xi_4 + \xi_3 + (3\dot{e}_1 + 5\dot{e}_2 + 3\dot{e}_3 + \dot{e}_4) \]  
(36)

It is easy to see that
\[ S = 4e_1 + 9e_2 + (9-\alpha)e_3 + 4e_4 + b(|y_3| - |x_3|) - c(y_1^4y_4 - x_1^4x_4) - d(y_2^2 - x_2^2) + u \]  
(37)

Substituting the backstepping control law (19) into (37), we get
\[ S = -k \xi_4 \]  
(38)

Using (38), we can simplify Eq. (35) as
\[ \dot{V} = -\xi_1^2 - \xi_2^2 - \xi_3^2 - (1+k) \xi_4^2 \]  
(39)

this is a quadratic and negative definite function on $\mathbb{R}^4$.  

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Thus, the proof is complete by Lyapunov stability theory. ■

For numerical simulations, we take the parameters of the hyperjerk systems as in the hyperchaotic case, viz. \((a, b, c, d) = (3.6, 0.02, 3, 0.05)\). Also, we take \(k = 25\).

The initial conditions of the master system (15) are taken as \(X(0) = (3.6, 1.7, 0.1, 1.4)\).

The initial conditions of the slave system (16) are taken as \(Y(0) = (2.1, 0.4, 1.2, 2.5)\).

Figure 5 shows the complete synchronization of the new hyperjerk systems represented by the master system (15) and the slave system (16), while Figure 6 depicts the time-history of the synchronization error between the systems (15) and (16).

![Figure 5](image1.png)

**Figure 5.** Complete synchronization of the hyperjerk systems (15) and (16) with \(X(0) = (3.6, 1.7, 0.1, 1.4)\) and \(Y(0) = (2.1, 0.4, 1.2, 2.5)\)

![Figure 6](image2.png)

**Figure 6.** Time-history of the synchronization error between the new hyperjerk systems (15) and (16) with \(X(0) = (3.6, 1.7, 0.1, 1.4)\) and \(Y(0) = (2.1, 0.4, 1.2, 2.5)\)
4. Circuit realization of the new hyperjerk system

The new chaotic system described by (1) is easily implemented with an electronic circuit via op-amps and multipliers [51-54], as shown in Figure 7. As shown in this figure, the circuit includes twenty two resistors, four capacitors, ten operational amplifiers (TL082CD), four analog multipliers (AD633JN) and two diodes (1N4148).

Chaotic differential equations of the new circuit are given below.

\[
\begin{align*}
\dot{x}_1 &= \frac{1}{C_1 R_1} x_2 \\
\dot{x}_2 &= \frac{1}{C_2 R_2} x_3 \\
\dot{x}_3 &= \frac{1}{C_3 R_3} x_4 \\
\dot{x}_4 &= -\frac{1}{C_4 R_4} x_1 - \frac{1}{C_4 R_5} x_2 - \frac{1}{C_4 R_6} x_3 + \frac{1}{C_4 R_7} x_1 |x_1| - \frac{1}{100 C_4 R_8} x_1^4 x_4 - \frac{1}{C_4 R_9} x_2^2
\end{align*}
\]

(40)

In which the voltages of capacitors are denoted as \(x_1, x_2, x_3, x_4\). When the system parameters are \((a, b, c, d) = (3.6, 0.02, 3, 0.05)\), the circuit elements have the values \(R_6 = 27.77 \Omega, R_7 = 5 \text{ M}\Omega, R_8 = 333.33 \Omega, R_9 = 2 \text{ M}\Omega, R_1 = R_2 = R_3 = R_4 = R_5 = R_{10} = R_{11} = R_{12} = R_{13} = R_{14} = R_{15} = R_{16} = R_{17} = R_{18} = R_{19} = R_{20} = R_{21} = R_{22} = 100 \text{ k}\Omega, C_1 = C_2 = C_3 = C_4 = 3.2 \text{ nF}\). Phase portrait outputs of the electronic circuit simulation are shown in Figure 8. Circuit simulation outputs (see Figure 8) show good qualitative matching with the numerical simulations (see Figures 1-4).

5. Conclusions

A new hyperjerk system is constructed and analyzed. This system possesses one quadratic term and single equilibrium point. The dynamical behaviors are investigated by stability analysis, various phase portraits and Lyapunov exponent analysis. The obtained results confirm the chaotic behaviors. As an engineering application, we have derived new synchronization results for the new hyperchaotic hyperjerk system using active backstepping control. In addition, the new hyperjerk have been executed for its simulations using the designed electronical circuit in MultiSIM program. The results of the MultiSIM simulation agree with the numerical simulation.
continued
Figure 7. Schematic of the circuit which realizes new hyperjerk system
continued
Figure 9 MultiSIM chaotic attractors in the (a) $x_1 - x_2$ plane, (b) $x_2 - x_3$ plane (c) $x_3 - x_4$ plane and (d) $x_1 - x_4$ plane

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