Abstract

The motion of a multi-electronic atom in an external electro-magnetic field is reconsidered. We prove that according to classical mechanics and electrodynamics, the assumption that the interaction with the magnetic field is described by means of a potential energy is no valid, and the trajectory of the center of mass can be deflected by a magnetic field, even if the internal angular momentum is zero. The characteristic equation of the corresponding Hamiltonian is not separable in three degrees of freedom for the hydrogen atom.

PACS 03.53.-w Quantum Mechanics, atom in crossed fields

1 Introduction

In a previous paper [1] we showed that the correspondence principle implies that the operator of angular momentum for a particle in an electromagnetic field is:

\[ \hat{L} = \vec{r} \times (-i\hbar \nabla - \frac{q}{c} \vec{A}) \],

where \( \vec{A} \) is the vector potential. As we mentioned there, it is required to guarantee that the expected values of angular momentum are gauge-invariant.

Also, from the general relation

\[ \frac{d\hat{f}}{dt} = \frac{i}{\hbar} [\hat{H}, \hat{f}] + \frac{\partial \hat{f}}{\partial t}, \]

we realized that the term \(-\frac{2}{\hbar} \vec{r} \times \vec{A}\) has to be included as part of the angular momentum, if the correct contribution of the electric field to the torque is going to be obtained—because

\[ \vec{E} = -\nabla V - \frac{1}{c} \frac{\partial \vec{A}}{\partial t}, \]

where \( V \) is the scalar potential.

From this we concluded that the theory of angular momentum and the theory of spin, in particular, had to be revised.
For example, in view of (1), the characteristic equation for the $z$ component of the angular momentum in presence of a magnetic field $\vec{H} = H_0 \hat{k}$, that can be obtained from the vector potential

$$A = \frac{1}{2} \vec{H} \times \vec{r}$$

is

$$-i\hbar \frac{\partial \psi}{\partial \phi} - \frac{q}{c} H \nu^2 \sin^2(\theta) \psi = m\hbar \psi,$$

that has not mono-valuated eigenfunctions, showing how the interpretation of the magnetic and azimuthal quantum numbers—that label the eigenfunctions of the operator $-i\hbar \vec{r} \times \nabla$—is disrupted by the presence of the magnetic field.

According to the axioms of quantum mechanics, the only values that an observable can assume are the eigenvalues of its operator. If this is true, the component of the angular momentum along the magnetic field has not allowed values, neither the corresponding component of the magnetic moment.

In another paper [3], we reconsidered the hydrogen atom in presence of an external magnetic field. The first thing we noticed was that, following a classical lagrangian approach, it can be proved that the motion of the center of mass and the internal motion are not physically independent, and the classical trajectory of the center of mass can be deflected if the field is inhomogeneous, no matter if the internal angular momentum is zero; an effect that is also predicted by Schrödinger theory, in view of Ehrenfest’s theorem—challenging the common belief about the function of the Stern-Gerlach apparatus, as resolving the eigenstates of an intrinsic angular momentum.

Also, we saw that the main evidence we have of the failure of Schrödinger’s theory to explain the properties of atoms in presence of magnetic fields is not completely reliable, because the usual formulation of the problem [4] pp. 71 & 541 [5] pp. 359-60 is not accurate.

We concluded that there is a basis for an explanation of the phenomena associated to spin as consequences of the Laws of Electrodynamics as applied to systems of electrical charges as wholes, but not as manifestations of intrinsic properties of punctual particles, as was also sustained in a different way by Bohr, who believed that the spin was only an abstraction, useful to compute the angular momentum [6].

In this paper we obtain similar and other results for multi-electronic atoms. We show that the assumption that the interaction of a multi-electronic atom with an external magnetic field can be fully described by means of a potential energy

$$\Phi = \frac{e}{2m_e c} \vec{L} \cdot \vec{H},$$

where $\vec{L}$ is the internal angular momentum, is incompatible with classical mechanics and electrodynamics.

We prove also that the separation of the motion of the center of mass and the internal motion—neglecting quadratic terms in the magnetic field—leads
to what is known as the hamiltonian in orthogonal crossed fields that is not separable \[7\] and has been proved to have monodromy: “A dynamical property that makes a global definition of angle-action variables and of quantum numbers impossible” \[8\] \[9\], at least in the case of the hydrogen atom.

2 On the Motion of a Neutral System in an External Electromagnetic Field

We consider a system made of \(N\) charged particles, with masses \(m_i\) and charges \(q_i\) \((i = 1, \ldots, N)\), under the action of an external time independent electromagnetic field, derivable from the potentials \(V_0(\vec{r})\) and \(\vec{A}(\vec{r})\):

\[
\vec{E}_0 = -\nabla V_0, \quad \vec{H}_0 = \nabla \times \vec{A}
\]

We’ll suppose that the system as a whole is electrically neutral, in such way that:

\[
\sum_{i=1}^{N} q_i = 0 \quad (4)
\]

We will also neglect internal magnetic interactions, and write the Lagrange’s function of the system as:

\[
L = K - V(\vec{r}_1, \ldots, \vec{r}_N) + \sum_{i=1}^{N} q_i \left( -V_0(\vec{r}_i) + \frac{\vec{v}_i}{c} \cdot \vec{A}(\vec{r}_i) \right). \quad (5)
\]

Here

\[
V(\vec{r}_1, \ldots, \vec{r}_N) = \sum_{i<j} \frac{q_i q_j}{\|\vec{r}_i - \vec{r}_j\|}, \quad (6)
\]

Our last assumption is that the external electromagnetic field remains almost constant across the system and, therefore, that, inside the system, the corresponding potentials can be reasonably approximated by linear functions.

Let \(\vec{R}\) be the position vector of the center of mass and:

\[
\vec{r}_i = \vec{R} + \vec{\rho}_i \quad (7)
\]

The kinetic energy of the system is

\[
K = \frac{1}{2} M \dot{\vec{R}}^2 + \frac{1}{2} \sum_{i=1}^{N} m_i \dot{\vec{\rho}}_i^2, \quad (8)
\]

and the internal electrical energy is easily written as a function of the vectors \(\vec{\rho}_i\):

\[
V(\vec{\rho}_1, \ldots, \vec{\rho}_N) = \sum_{i<j} \frac{q_i q_j}{\|\vec{\rho}_i - \vec{\rho}_j\|},
\]

whilst the other terms are replaced by their first-order approximations.
Our last assumption implies that
\[ V_0(\vec{r}_i) \approx V_0(\vec{R}) + \nabla_\vec{R} V_0 \cdot \vec{p}_i = V_0(\vec{R}) - \vec{E}_0(\vec{R}) \cdot \vec{p}_i. \]

From this and (9) we get
\[
\sum_{i=1}^{N} q_i V_0(\vec{r}_i) \approx -\vec{P} \cdot \vec{E}_0(\vec{R}),
\]
where
\[
\vec{P} = \sum_{i=1}^{N} q_i \vec{p}_i,
\]
is the electrical dipole. As to the other terms, we have:
\[
\vec{v}_i \cdot \vec{A}(\vec{r}_i) = \vec{v}_i \cdot \vec{A}(\vec{R} + \vec{p}_i) \approx \vec{A}(\vec{R}) \cdot \vec{v}_i + ((\vec{v}_i \cdot \nabla_\vec{R}) \vec{A}(\vec{R}) + \vec{v}_i \times \vec{H}_0(\vec{R})) \cdot \vec{p}_i.
\]

Further:
\[
\sum_{i=1}^{N} \frac{q_i}{c} \vec{A}(\vec{R}) \cdot \vec{v}_i = \frac{1}{c} \left( \sum_{i=1}^{N} q_i \right) \vec{A}(\vec{R}) \cdot \vec{v}_i + \frac{1}{c} \vec{A}(\vec{R}) \cdot \sum_{i=1}^{N} q_i \vec{p}_i = \frac{1}{c} \vec{A}(\vec{R}) \cdot \vec{P};
\]
\[
\sum_{i=1}^{N} \frac{q_i}{c} ((\vec{v}_i \cdot \nabla_\vec{R}) \vec{A}(\vec{R})) \cdot \vec{p}_i = \frac{1}{c} \left( \sum_{i=1}^{N} q_i \right) ((\vec{v}_i \cdot \nabla_\vec{R}) \vec{A}(\vec{R})) \cdot \vec{p}_i;
\]
\[
\sum_{i=1}^{N} \frac{q_i}{c} (\vec{v}_i \times \vec{H}_0(\vec{R})) \cdot \vec{p}_i = \sum_{i=1}^{N} \frac{q_i}{c} (\vec{p}_i \times \vec{v}_i) \cdot \vec{H}_0(\vec{R})
\]
\[
= \vec{H}_0(\vec{R}) \cdot \sum_{i=1}^{N} \frac{q_i}{c} \vec{p}_i \times \vec{v}_i + \vec{H}_0(\vec{R}) \cdot \sum_{i=1}^{N} \frac{q_i}{c} \vec{p}_i \times \vec{p}_i.
\]

Now we can write our first approximation to the Lagrange’s Function:
\[
L(\vec{R}, \vec{p}_1, \ldots, \vec{p}_N, \vec{\dot{r}}_1, \ldots, \vec{\dot{r}}_N) = \frac{1}{2} M \vec{R}^2 + \frac{1}{2} \sum_{i=1}^{N} m_i \vec{\dot{r}}_i^2 - V + \vec{P} \cdot \vec{E}_0(\vec{R}) + \frac{1}{c} \vec{\dot{A}}(\vec{R}) \cdot \vec{P} - \frac{1}{c} \vec{A}(\vec{R}) \cdot \vec{P};
\]
where the terms
\[
\frac{1}{c} \vec{\dot{A}}(\vec{R}) \cdot \vec{P} + \frac{1}{c} \vec{A}(\vec{R}) \cdot \vec{P}
\]
have been omitted, because they add up to a total derivative.
Let’s consider a typical summand in \( \sum_{i=1}^{N} q_i ((\dot{\rho}_i \cdot \nabla_{\vec{R}}) \vec{A}(\vec{R})) \cdot \vec{p}_i: \)

\[
((\dot{\vec{p}} \cdot \nabla_{\vec{R}}) \vec{A}(\vec{R})) \cdot \vec{p} = \rho_i \dot{\rho}_j - \rho_j \dot{\rho}_i \frac{\partial}{\partial j} A_i + \rho_j \dot{\rho}_i = -\frac{1}{2} \vec{H}(\vec{R}) \cdot (\vec{p} \times \vec{p}) + \frac{1}{2} (\vec{p} \cdot ((\dot{\vec{p}} \cdot \nabla_{\vec{R}}) \vec{A}) + \dot{\vec{p}} \cdot ((\dot{\vec{p}} \cdot \nabla_{\vec{R}}) \vec{A}))
\]

The last term can be shown to be equal to a total-derivative plus a term that includes second derivatives of the external vector potential, that can be neglected since we have supposed that, inside the system, the external potentials can be reasonably approximated by linear functions. On this basis, we substitute the Lagrange’s Function by

\[
L = \frac{1}{2} M \dot{\vec{R}}^2 + \frac{1}{2} \sum_{i=1}^{N} m_i \dot{\vec{p}}_i^2 - V + \vec{P} \cdot \left( \vec{E}_0(\vec{R}) + \frac{\dot{\vec{R}}}{c} \times \vec{H}_0(\vec{R}) \right) + \frac{1}{2} \vec{H}_0(\vec{R}) \cdot \sum_{i=1}^{N} q_i \vec{p}_i \times \vec{p}_i
\]

(16)

Given that the vectors \( \vec{p}_i \) are subject to the condition

\[
\sum_{i=1}^{N} m_i \vec{p}_i = 0
\]

(17)

to obtain the equations of motion, we have to introduce a time dependent vectorial multiplier, which leads to the modified Lagrange’s Function:

\[
L' = \frac{1}{2} M \dot{\vec{R}}^2 + \frac{1}{2} \sum_{i=1}^{N} m_i \dot{\vec{p}}_i^2 - V + \vec{P} \cdot \left( \vec{E}_0(\vec{R}) + \frac{\dot{\vec{R}}}{c} \times \vec{H}_0(\vec{R}) \right) + \frac{1}{2} \vec{H}_0(\vec{R}) \cdot \sum_{i=1}^{N} q_i \vec{p}_i \times \vec{p}_i
\]

(18)

The equations of motion are:

\[
\frac{d}{dt} \left( M \dot{\vec{R}} + \frac{1}{c} \vec{H}_0 \times \vec{P} \right) = (\vec{P} \cdot \nabla_{\vec{R}}) \vec{E}_0(\vec{R}) + \left( \left( \frac{1}{c} \vec{P} \times \dot{\vec{R}} + \frac{1}{2c} \sum_{i=1}^{N} q_i \vec{p}_i \times \dot{\vec{p}}_i \right) \cdot \nabla_{\vec{R}} \right) \vec{H}_0(\vec{R})
\]

(19)

and

\[
\frac{d}{dt} \left( m_i \dot{\vec{p}}_i + \frac{q_i}{2c} \vec{H}_0(\vec{R}) \times \vec{p}_i \right) = -\frac{\partial V}{\partial \vec{p}_i} + q_i \left( \vec{E}_0(\vec{R}) + \frac{\dot{\vec{R}}}{c} \times \vec{H}_0(\vec{R}) \right) + \frac{q_i}{2c} \vec{p}_i \times \vec{H}_0(\vec{R}) + m_i \ddot{\vec{p}}_i
\]

(20)
Considering that
\[
\frac{d}{dt}(\vec{H}_0 \times \vec{P}) = ((\dot{\vec{R}} \cdot \nabla)\vec{H}_0) \times \vec{P} + \vec{H}_0 \times \dot{\vec{P}} = \nabla_{\vec{R}}(\dot{\vec{R}} \cdot \vec{H}_0) \times \vec{P} + \vec{H}_0 \times \dot{\vec{P}}
\]
and
\[
\frac{d}{dt}(\vec{H}_o \times \vec{\rho}_i) = \nabla_{\vec{R}}(\dot{\vec{R}} \cdot \vec{H}_0) \times \vec{\rho}_i + \vec{H}_0 \times \dot{\vec{\rho}}_i
\]
(Where we have used the identity
\[
\nabla (\vec{a} \cdot \vec{b}) = (\vec{a} \cdot \nabla)\vec{b} + (\vec{b} \cdot \nabla)\vec{a} + \vec{a} \times (\nabla \times \vec{b}) + \vec{b} \times (\nabla \times \vec{a})
\]
and the fact that
\[
\nabla_{\vec{R}} \times \vec{H}_0(\vec{R}) = \vec{0},
\]
since \(\vec{H}_0\) is an external field.), eqs. \((19\text{ & }20)\) can be rewritten as:
\[
M \ddot{\vec{R}} = (\vec{P} \cdot \nabla)\vec{E}_0(\vec{R}) + \frac{1}{c} \dot{\vec{P}} \times \vec{H}_0(\vec{R}) + \frac{1}{c} \vec{P} \times \nabla(\dot{\vec{R}} \cdot \vec{H}_0(\vec{R})) + (21)
\]
and
\[
m_i \ddot{\vec{\rho}}_i = -\frac{\partial V}{\partial \vec{\rho}_i} + q_i \left( \vec{E}_0(\vec{R}) + \frac{\dot{\vec{R}}}{c} \times \vec{H}_0(\vec{R}) \right) + (22)
\]
\[
\frac{q_i}{2c} \dot{\vec{\rho}}_i \times \nabla_{\vec{R}}(\dot{\vec{R}} \cdot \vec{H}_0(\vec{R})) + \frac{q_i}{c} \dot{\vec{\rho}}_i \times \vec{H}_0(\vec{R}) + m_i \ddot{\vec{\lambda}}
\]
From \((14, 22)\) and the relations
\[
\sum_{i=1}^{n} m_i \vec{\rho}_i = \vec{0}, \quad \sum_{i=1}^{n} \frac{\partial V}{\partial \vec{\rho}_i} = \vec{0},
\]
we get:
\[
\ddot{\vec{\lambda}} = -\frac{\vec{P}}{2Mc} \times \nabla_{\vec{R}}(\dot{\vec{R}} \cdot \vec{H}_0(\vec{R})) - \frac{\dot{\vec{P}}}{Mc} \times \vec{H}_0(\vec{R}).
\]
Therefore
\[
m_i \ddot{\vec{\rho}}_i = -\frac{\partial V}{\partial \vec{\rho}_i} + q_i \left( \vec{E}_0(\vec{R}) + \frac{\dot{\vec{R}}}{c} \times \vec{H}_0(\vec{R}) \right) + (24)
\]
\[
\frac{1}{2c} (q_i \vec{\rho}_i - \frac{m_i}{M} \dot{\vec{P}}) \times \nabla_{\vec{R}}(\dot{\vec{R}} \cdot \vec{H}_0(\vec{R})) + \frac{1}{c} (q_i \vec{\rho}_i - \frac{m_i}{M} \dot{\vec{P}}) \times \vec{H}_0(\vec{R})
\]
3 Multi-Electronic Atom in a Magnetic Field

Equations (21 & 24) clearly show that the internal motion and the motion of the center of mass are not independent in presence of an external electromagnetic field, as was supposed to formulate the quantum description of multi-electronic atoms.

In case of a purely magnetic external field, the equation of motion of the center of mass is simplified to:

$$M \ddot{\vec{R}} = \frac{1}{c} \dot{\vec{P}} \times \vec{H}_0(\vec{R}) + \frac{1}{2c} \sum_{i=1}^{N} q_i \dot{\vec{p}}_i \times \dot{\vec{p}}_i \cdot \nabla \vec{H}_0(\vec{R})$$

The term

$$\left( \frac{1}{2c} \sum_{i=1}^{N} q_i \dot{\vec{p}}_i \times \dot{\vec{p}}_i \right) \cdot \nabla \vec{H}_0(\vec{R})$$

is what is usually substituted by

$$- \frac{e}{2m_e c} (\vec{L} \cdot \nabla \vec{H}_0(\vec{R}))$$

which is valid only in case that all the particles have the same charge-mass relation, which is not true for common atoms.

It’s evident that, given that the mass of the nucleus is much bigger than the electronic mass, we can neglect the corresponding contribution in (26) to justify its substitution by (27). But this is just an approximation, and not a fundamental relation, as its usually presented, to attribute to classical physics the naive view that an atom in a magnetic field can be considered as a tiny circuit. Even more, if this approximation is introduced, (25) is transformed into:

$$M \ddot{\vec{R}} = \frac{1}{c} \dot{\vec{P}} \times \vec{H}_0(\vec{R}) + \frac{1}{c} \vec{P} \times \nabla \vec{H}_0(\vec{R}) - \frac{e}{m_e c} \vec{S} - \frac{e}{2m_e c} \vec{L} \cdot \nabla \vec{H}_0(\vec{R})$$

where

$$\frac{e}{m_e c} \vec{S} = \frac{1}{c} \vec{P} \times \dot{\vec{R}}$$

Equation (28) looks like the equation of motion of a particle with an intrinsic angular momentum and by no means implies that “The trajectory of a multi-electronic atom cannot be deflected by an inhomogeneous magnetic field if the internal angular momentum is equal to zero.” as it’s claimed to demonstrate that the result of the Stern-Gerlach experiment is not predicted by classical mechanics and cannot be explained but by introducing spin variables. In fact, equation (26) includes the force

$$\vec{f}_\parallel = \frac{1}{c} \vec{P} \times \nabla \vec{H}_0(\vec{R})$$
that appears when the component of the velocity along the magnetic field is not negligible, which is not predicted by the usual theory.

To obtain an approximated quantum representation of the motion, we'll suppose that the trajectory of the nucleus coincides with the trajectory of the center of mass, in such way that the Lagrange’s function can be approximated by:

$$L = \frac{1}{2} M \dot{\vec{R}}^2 + \frac{m_e}{2} \sum_{i=1}^{Z} \dot{\vec{p}}_i^2 - V + \frac{\vec{P}}{c} \cdot (\dot{\vec{R}} \times \vec{H}(\vec{R})) - \frac{e}{2m_e c} \vec{L} \cdot \vec{H}_0(\vec{R}) \tag{29}$$

where $Z$ is the atomic number,

$$V = -\sum_{i=1}^{Z} \frac{e^2}{\| \vec{R} - \vec{\rho}_i \|} + \sum_{i<j} \frac{e^2}{\| \vec{\rho}_i - \vec{\rho}_j \|},$$

$$\vec{P} = -e \sum_{i=1}^{Z} \vec{\rho}_i,$$

and

$$\vec{L} = m_e \sum_{i=1}^{Z} \vec{\rho}_i \times \dot{\vec{\rho}}_i$$

The corresponding momenta are:

$$\vec{p}_{\vec{R}} = M \dot{\vec{R}} + \frac{1}{c} \vec{H}(\vec{R}) \times \vec{P} \tag{30}$$

$$\vec{p}_{\vec{\rho}_i} = m_e \dot{\vec{\rho}}_i - \frac{e}{2c} \vec{H}_0(\vec{R}) \times \vec{\rho}_i, \tag{31}$$

from which we can get the energy, which is a constant of motion:

$$E = \frac{1}{2} M \dot{\vec{R}}^2 + \frac{m_e}{2} \sum_{i=1}^{Z} \dot{\vec{p}}_i^2 + V \tag{32}$$

Equation (32) clearly shows the unsoundness of the assumption that in conformity with the theoretical framework of classical mechanics and electrodynamics the interaction of an atom with a magnetic field is completely described by means of the potential energy

$$\Phi = \frac{e}{2m_e c} \vec{L} \cdot \vec{H}. \tag{33}$$

According to classical mechanics the change of potential energy is equal to the work done by the corresponding forces and, according to electrodynamics, the magnetic force is perpendicular to the velocity and, therefore, does not work. That’s why the energy (32) does not include magnetic terms, just the kinetic
energy plus the electrical potential energy, as predicted by electrodynamics, but
the Hamilton’s Function does:

\[ H = \left( \frac{\mathbf{p} \cdot \mathbf{R}}{2M} - \frac{1}{c} \mathbf{H}_0(\mathbf{R}) \times \mathbf{P} \right)^2 + \sum_{i=1}^{Z} \left( \frac{\mathbf{p} \cdot \mathbf{\rho}_i + \frac{e}{2c} \mathbf{H}_0(\mathbf{R}) \times \mathbf{\rho}_i}{2m_e} \right)^2 + V. \]  

(34)

The Hamiltonian is:

\[ \hat{H} = \left( -i\hbar \nabla \mathbf{R} - \frac{\mathbf{p} \cdot \mathbf{R}}{2M} \right)^2 + \sum_{i=1}^{Z} \left( -i\hbar \nabla \mathbf{\rho}_i + \frac{e}{2c} \mathbf{H}_0(\mathbf{R}) \times \mathbf{\rho}_i \right)^2 + V. \]  

(35)

After some algebra, considering that

\[ \nabla \mathbf{R} \times \mathbf{H}_0(\mathbf{R}) = 0, \]

the Hamiltonian is transformed into:

\[ \hat{H} = -\frac{\hbar^2}{2M} \nabla^2 - \frac{\hbar^2}{2m_e} \sum_{i=1}^{Z} \nabla^2 \mathbf{\rho}_i + V + \frac{i\hbar}{Mc} (\mathbf{H}_0(\mathbf{R}) \times \mathbf{P}) \cdot \nabla \mathbf{R} + \left( \frac{e}{2mc^2} \right) \sum_{i=1}^{Z} (\mathbf{H}_0(R) \times \mathbf{\rho}_i)^2, \]

(36)

where

\[ \hat{i} = \sum_{i=1}^{Z} -i\hbar \mathbf{\rho}_i \times \nabla \mathbf{\rho}_i. \]

If the last two terms can be neglected, the Hamiltonian is simplified to

\[ \hat{H} = -\frac{\hbar^2}{2M} \nabla^2 - \frac{\hbar^2}{2m_e} \sum_{i=1}^{Z} \nabla^2 \mathbf{\rho}_i + V + \frac{i\hbar}{Mc} (\mathbf{H}_0(\mathbf{R}) \times \mathbf{P}) \cdot \nabla \mathbf{R} + \frac{e}{2mc^2} \mathbf{H}_0(\mathbf{R}) \cdot \hat{i}. \]  

(37)

Compare this operator to the operator in [4, pp. 71 & 541] to see that the conclusion that the properties of atoms in magnetic fields cannot be explained but through the introduction of spin variables is, at least, precipitated, and probably wrong. To avoid come to the conclusion that, after all, (33) represents a sort of magnetic potential energy, notice that, in view of (30), the operators

\[ -i\hbar \nabla \]  

in (37) do not correspond to linear momentum and the operator

\[ \frac{e}{2mc^2} \hat{i} \]

does not correspond to the magnetic moment. If the missing term is introduced in the Hamiltonian, as well as the spinorial terms—that have the form (33) that we have already discarded—the agreement of the predicted spectra with the experimental results will be disrupted.

Let’s suppose that the magnetic field is uniform and make the substitution

\[ \Psi(\mathbf{\tilde{R}}, \mathbf{\tilde{\rho}_1}, \cdots, \mathbf{\tilde{\rho}_Z}) = e^{\frac{i}{\hbar} \mathbf{\tilde{R}} \cdot \mathbf{\tilde{P}}} \psi(\mathbf{\rho}_1, \cdots, \mathbf{\rho}_Z) \]  

(38)
The characteristic equation for (37) is transformed into:

\[
\left( -\frac{\hbar^2}{2m_e} \sum_{i=1}^{2} \nabla^2_{\rho_i} + V - \frac{\vec{p} \cdot \vec{H}_0}{M_c} + \frac{e}{2m_e} \vec{H}_0 \cdot \vec{L} \right) \psi = \epsilon \psi, \tag{39}
\]

where

\[
\epsilon = E - \frac{\vec{p}^2}{2M}
\]

This is the characteristic equation of what is known as the Hamiltonian in orthogonal crossed fields—\(\vec{E}_0 = \vec{p} \times \vec{H}_0 \) and \(\vec{H}_0\)—that is not separable in three degrees of freedom, for the hydrogen atom, because the presence of the electric field breaks the azimuthal symmetry \([7]\). Furthermore, recently, it has been proved that the hydrogen atom in orthogonal crossed fields has monodromy: “A dynamical property that makes a global definition of angle-action variables and of quantum numbers impossible” \([8][9]\) which might explain the need to introduce changes in Schrödinger’s formalism.

References

[1] O. Chavoya-Aceves; Remarks on the Theory of Angular Momentum.; arXiv:quant-ph/0305049 (2003).
[2] H. Weyl; The Theory of Groups and Quantum Mechanics; Dover (1950).
[3] O. Chavoya-Aceves; An Explanation of Spin Based on Classical Mechanics and Electrodynamics.; arXiv:quant-ph/0305137 (2003).
[4] A. Messiah; Quantum Mechanics; Dover (1999).
[5] D. Bohm; Quantum Theory; Dover (1989).
[6] B. M. Garraway, S. Stenholm; Does the Flying Electron Spin?; Contemp. Phys. 43, 147 (2002).
[7] J. Main, M. Schwacke, and G. Wunner; Hydrogen Atom in Combined Electric and Magnetic Fields with Arbitrary Mutual Orientations; Physical Review A 57 No. 2 pp. 1149-1157 (1998).
[8] R. H. Gushman, D. A. Sadovski; Monodromy in Perturbed Kepler Systems: Hydrogen Atom in Crossed Fields; Europhysics Letters 47(1) (1999).
[9] R. H. Gushman, D. A. Sadovski; Monodromy in the Hydrogen Atom in Crossed Fields; Physica 14D 166-96 (2000).