On the evolution of the velocity–mass–size relations of disc-dominated galaxies over the past 10 billion years

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ABSTRACT
We study the evolution of the scaling relations between the maximum circular velocity, stellar mass and optical half-light radius of star-forming disc-dominated galaxies in the context of Λ cold dark matter-based galaxy formation models. Using data from the literature combined with new data from the Deep Extragalactic Evolutionary Probe 2 (DEEP2) and All-wavelength Extended Groth Strip International Survey (AEGIS) surveys, we show that there is a consistent picture for the evolution of these scaling relations from $z \sim 2$ to $z = 0$, both observationally and theoretically. The evolution of the observed stellar scaling relations is weaker than that of the virial scaling relations of dark matter haloes, which can be reproduced, both qualitatively and quantitatively, with a simple, cosmologically motivated model for disc evolution inside growing Navarro–Frenk–White dark matter haloes. In this model optical half-light radii are smaller, both at fixed stellar mass and at maximum circular velocity, at higher redshifts. This model also predicts that the scaling relations between baryonic quantities (baryonic mass, baryonic half-mass radii and maximum circular velocity) evolve even more weakly than the corresponding stellar relations. We emphasize, though, that this weak evolution does not imply that individual galaxies evolve weakly. On the contrary, individual galaxies grow strongly in mass, size and velocity but in such a way that they move largely along the scaling relations. Finally, recent observations have claimed surprisingly large sizes for a number of star-forming disc galaxies at $z \sim 2$, which has caused some authors to suggest that high-redshift disc galaxies have abnormally high spin parameters. However, we argue that the disc scalelengths in question have been systematically overestimated by a factor of ~2 and that there is an offset of a factor of ~1.4 between Hα sizes and optical sizes. Taking these effects into account, there is no indication that star-forming galaxies at high redshifts ($z \sim 2$) have abnormally high spin parameters.

Key words: galaxies: evolution – galaxies: formation – galaxies: fundamental parameters – galaxies: haloes – galaxies: high-redshift – galaxies: spiral.

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1 INTRODUCTION

In the current paradigm of galaxy formation, galaxy discs are considered to form from the accretion of gas inside hierarchically growing cold dark matter (CDM) haloes (White & Rees 1978). The dark matter (DM) and gas acquire angular momentum via tidal torques in the early universe (Peebles 1969). When the gas accretes onto the central galaxy, this angular momentum may eventually halt the collapse and lead to the formation of a rotationally supported disc (Fall & Efstathiou 1980). Under the assumption that the total specific angular momentum of the pre-collapse gas is similar to that of the DM and is conserved during collapse, this picture leads to predictions of present-day disc sizes that are in reasonable agreement with observations (Blumenthal et al. 1984; Dalcanton, Spergel & Summers 1997; Mo, Mao & White 1998; de Jong & Lacey 2000; Firmani & Avila-Reese 2000; Pizagno et al. 2005; Dutton et al. 2007).

The observational relation between the sizes of galaxy discs and their characteristic rotation velocities (the RV relation) provides a measure of the specific angular momentum of galaxy discs (e.g. Navarro & Steinmetz 2000). In the simplest model for disc-galaxy evolution, the sizes and rotation velocities of galaxy discs are proportional to the sizes and circular velocities of their host DM haloes, and the evolution of disc sizes at fixed circular velocity scales inversely with the Hubble parameter (e.g. Mo et al. 1998). For a ΛCDM cosmology (with ΩM = 0.3, ΩΛ = 0.7) this simple model predicts that discs should be a factor of ≃1.8 smaller, at fixed circular velocity, at z = 1, compared to z = 0, and a factor of ≃3.0 smaller at z = 2. For a standard CDM cosmology (with ΩM = 1, ΩΛ = 0), the predicted evolution is even stronger: a factor of ≃2.8 to z = 1 and a factor of ≃5.2 to z = 2.

Early observations indicated evolution in the RV relation with a scaling of ≃(1 + z)−1 out to z ∼ 1, in agreement with theoretical expectations (Mao, Mo & White 1998). More recently, Bouché et al. (2007) found that the disc scale size–rotation velocity relation at z ≃ 2.2 has roughly the same zero-point as at z = 0, in disagreement with simple theoretical expectations. While there are concerns about sample selection, small statistics, errors at high-z, etc., it is none the less appropriate to start thinking about the implications of this result for galaxy formation. A proposed explanation for this non-evolution is that disc galaxies at z ≃ 2.2 have spin parameters1 a factor of 3 higher than those predicted by ΛCDM (Bouché et al. 2007; Burkert et al. 2009). If confirmed, this could imply a significant modification of the current paradigm of galaxy disc formation, either through a decoupling of the angular momentum of baryons and DM or more drastically by changing the mechanism by which galactic angular momentum is generated.

A decoupling between the angular momentum of baryons and DM haloes is actually seen in cosmological hydrodynamical simulations of disc-galaxy formation (e.g. Navarro & Steinmetz 2000; Piontek & Steinmetz 2009; Sales et al. 2009). However, the result is that baryons lose angular momentum to the halo, which only makes high disc spin parameters harder to explain. A possible mechanism for increasing the specific angular momentum of disc galaxies is the removal of material with low specific angular momentum through galactic outflows. This mechanism has been shown theoretically to be effective in dwarf galaxies (Maller & Dekel 2002; Dutton 2009; Governato et al. 2010). Galactic outflows appear ubiquitous in star-forming galaxies at redshifts z ∼ 3 (Shapley et al. 2003) and z = 1.4 (Weiner et al. 2009). But whether they are powerful enough to increase the specific angular momentum of massive (Vrot = 200 km s−1) galaxy discs by a whole factor of 3 remains to be seen.

The relation between galaxy size and stellar mass (the RM relation) of disc-dominated galaxies also shows only weak evolution to z = 1 (Barden et al. 2005). This weak evolution can be understood in the ΛCDM scenario as a consequence of the increase of dark halo concentrations since z = 1 (Somerville et al. 2008) and also because individual disc galaxies tend to evolve roughly along the RM relation (Firmani & Avila-Reese 2009). At higher redshifts, there is evidence of a factor of ≃2 decrease in half-light sizes of disc-dominated galaxies at fixed stellar mass between z = 0 and z ≃ 2.5 (Trujillo et al. 2006). Again, this evolution is weaker than the evolution in the halo RM relation, but it is in agreement with the models of Somerville et al. (2008) and Firmani & Avila-Reese (2009). We note that elliptical galaxies have been found to evolve even more strongly with redshift (e.g. Trujillo et al. 2006; van Dokkum et al. 2008), and thus there is clear evidence that galaxies of all types were smaller, at fixed stellar mass, at higher redshifts.

It is possible that the non-evolution of the RV relation and the evolution of the RM relation of disc galaxies are consistent. This would require evolution in the MV relation [i.e. the stellar mass Tully–Fisher (TF) relation; Tully & Fisher 1977], with higher M*, at fixed Vrot, at higher z. However, the opposite has been claimed, with ≃0.4 dex lower Mstar at fixed Vmax at z ∼ 2.2 compared to z = 0 (Cresci et al. 2009). Thus, the various data sets for the evolution of the VMR relations are inconsistent.

In this paper, we examine the VMR relations of disc galaxies using data from the literature as well as new results from the Deep Extragalactic Evolutionary Probe 2 (DEEP2) survey (Davis et al. 2003; Newman et al., in preparation). We discuss, and resolve, sources of discrepancies among the various data sets and compare the data to predictions of a simple ΛCDM-based disc-galaxy evolution model. We show that the observations can be reproduced by a model with a constant spin parameter in agreement with predictions from ΛCDM, and a constant galaxy mass fraction. Hence, there is no need to invoke abnormally high spin parameters in order to explain the scaling relations of disc galaxies at z ∼ 2. Throughout, we assume a flat ΛCDM cosmology with (ΩM, ΩΛ, h) = (0.3, 0.7, 0.7).

2 OBSERVATIONS

2.1 Evolution of the stellar mass Tully–Fisher relation

The relation between stellar mass2 (or luminosity) and rotation velocity (or linewidth) is also commonly known as the TF relation (Tully & Fisher 1977). We define the velocity as the maximum rotation velocity, Vmax. Using slit spectroscopy from the DEEP1 survey (Vogt et al. 2005; Weiner et al. 2005), together with optical and K-band imaging, Conselice et al. (2005) found that, at fixed Vmax, stellar masses are lower by 0.07 ± 0.12 dex at z ∼ 0.45 and lower by 0.11 ± 0.13 dex at z ∼ 0.85, compared to the z = 0 relation from Bell & de Jong (2001). This weak evolution from z = 1 to 0 was confirmed by Kassin et al. (2007), who studied the stellar mass TF relation for a larger sample of galaxies from the DEEP2 survey, but adopting a different velocity indicator, S0.5 = (0.5Vrot + σ)1/2.

1 The spin parameter is a dimensionless measure of the specific angular momentum of DM haloes.

2 Note that stellar mass throughout is total stellar mass, not disc stellar mass.
which combines ordered motions (i.e. rotation, $V_{rot}$) and disordered motions (i.e. dispersion, $\sigma$) (Weiner et al. 2006a). This new parameter allowed inclusion of mergers and incompletely settled galaxies, which is useful at high redshifts where galaxies may be dynamically ‘young’.

Here we use data from Kassin et al. (2007) and calculate the evolution of stellar mass at fixed $S_{0.5}$ for galaxies with $S_{0.5} > 90$ km s$^{-1}$ [i.e. $\log_{10} V_{\text{max}}/(\text{km s}^{-1}) > 2.1$] and inclinations between 45° and 70°. As a comparison relation at redshift $z = 0$, we use the TF relation from Bell & de Jong (2001), corrected to a Chabrier (2003) initial mass function (IMF) as given in Section 2.3 and with $V_{\text{rot}}$ converted into $S_{0.5}$ assuming $\sigma = 0$. We note that since the galaxies at $z = 0$ are rotation dominated, adopting a more realistic value of $\sigma \sim 10$ km s$^{-1}$ (for the cold atomic hydrogen disc) for these galaxies will not change the $z = 0$ relation by any significant amount. The median offsets in stellar mass of the DEEP2 data with respect to the $z = 0$ relation, in three redshift bins from $z = 0.2$ to $1.1$, are given in Table 1. These results are consistent with no evolution or at most a weak decrease in stellar masses at fixed circular velocity at higher redshifts.

A relatively strong evolution in the stellar mass TF relation since $z \sim 0.6$ has been reported by Puech et al. (2010). These authors find an increase of 0.34 dex in stellar mass at fixed velocity since $z \sim 0.6$. However, the same authors find no evidence for evolution in baryonic mass at fixed velocity (the baryonic TF relation) over the same redshift range. The large difference between the evolution of the stellar and baryonic TF relations implies that gas fractions evolve significantly. However, the inferred gas fractions of their galaxies at $z \sim 0.6$ are on average 30 per cent, which is not much higher than that of local galaxies of the same mass. This implies that the evolution of the stellar mass TF relation should be only of the order of 0.1 dex different than that of the baryonic TF relation. The difference in the evolution of 0.4 dex found by Puech et al. (2010) can thus be traced to the use of inconsistent local baryonic and stellar mass TF relations.

At even higher redshifts, evidence has been reported of significant evolution compared to redshift $z = 0$. Using data from Spectroscopic Imaging survey in the NIR with SINFONI (SINS; Förster-Schreiber et al. 2009), Cresci et al. (2009) found that at fixed $V_{\text{max}}$, stellar masses at redshift $z \sim 2.2$ are lower by 0.41 $\pm 0.11$ dex compared to $z = 0$. Taken together, the observations indicate that there is a modest evolution in the zero-point of the TF relation out to $z \sim 1$ and a stronger evolution from $z \sim 1$ to 2.

| Redshift range | Median $z$ | $\Delta \log_{10} M_{\text{star}}|S_{0.5}$ | $N$ |
|----------------|------------|------------------------------------------|-----|
| 0.2–0.5        | 0.35       | $-0.06 \pm 0.09$                         | 19  |
| 0.5–0.8        | 0.73       | $-0.03 \pm 0.11$                         | 25  |
| 0.8–1.1        | 0.95       | $-0.15 \pm 0.09$                         | 29  |

Table 1. Evolution of the stellar mass–velocity relation of star-forming galaxies relative to $z = 0.0$, using DEEP2 data from Kassin et al. (2007).

2.2 Evolution in the size–stellar mass relation

Barden et al. (2005) found that there was little or no evolution in the circularized$^3$ optical half-light size–stellar mass relation of disc-dominated galaxies (defined as having Sérsic index $n < 2.5$) from redshift $z \sim 1$ to 0.1. Trujillo et al. (2006) measured the evolution of the circularized rest-frame $V$-band half-light radius–stellar mass relation since $z \sim 2.5$, finding that disc galaxies (Sérsic index $n < 2.5$) are a factor of 2–3 smaller, at fixed $M_{\text{star}}$, at $z \sim 2.5$ than at $z = 0.1$. Williams et al. (2010) measured the evolution of the circularized rest-frame $I$-band half-light size of galaxies with stellar masses greater than $6.3 \times 10^{10} M_\odot$ from $z = 0.5$ to 2, finding strong evolution for both star-forming and non-star-forming galaxies. In order to compare to $z = 0.1$, we use a mean $I$-band half-light size of 5.0 kpc. This has been determined using our Sloan Digital Sky Survey (SDSS) measurements (see Section 2.2.3) and applying a correction of $-0.06$ dex to go to the rest-frame $I$ band (see Section 3.4).

Here we present new results for the evolution of the disc size–stellar mass relation for blue-cloud disc-dominated galaxies from redshifts $z = 1.2$ to 0.1 using high-redshift data from DEEP2 (Davis et al. 2003; Newman et al., in preparation) and the All-wavelength Extended Groth Strip International Survey (AEGIS; Davis et al. 2007), and a low-redshift comparison sample from the SDSS (York et al. 2000). The main difference of our study with respect to previous studies (e.g. Barden et al. 2005; Trujillo et al. 2006) is our use of disc sizes, rather than total sizes. We use disc sizes because our main interest in this paper is the evolution of galaxy discs. Total sizes, even for disc-dominated galaxies, depend on the bulge fraction and bulge size and thus give a measurement that is more difficult to interpret.

2.2.1 DEEP2 data

Disc sizes have been measured using 2D bulge-disc fits using gim2d (Simard et al. 2002). The bulge component was assumed to have a de Vaucouleurs profile (Sérsic $n = 4$), while the disc was assumed to be exponential (i.e. Sérsic $n = 1$). The fits were performed simultaneously on $F606W$ ($V$) and $F814W$ ($I$) single-orbit Hubble Space Telescope (HST) Advanced Camera for Surveys (ACS) images. The bulge and disc sizes are constrained to be the same in each filter, but the bulge and disc fluxes are free to vary. This choice has been made to maximize the signal-to-noise ratio, at the expense of using bluer rest-frame wavelengths at higher redshifts.

The central rest-frame wavelength of the images (i.e. the average of the $F606W$ and $F814W$ images) varies from $\sim 500$ nm at $z = 0.4$ to $\sim 320$ nm at $z = 1.2$. This may cause a small ($\gtrsim 10$ per cent) systematic overestimation of the sizes at higher redshifts, because discs tend to have colour gradients (see Section 3.3). The sizes we present in this paper are major-axis (i.e. elliptical aperture) disc half-light sizes, i.e. 1.678 times the disc exponential scalelength.

We use both spectroscopic and photometric redshifts between $0.2 < z < 1.4$. We use high quality (‘z-quality’ $\geq 3$) spectroscopic redshifts from DEEP2 (Davis et al. 2003; Newman et al., in preparation) and photometric redshifts based on optical to Spitzer/IRAC photometry (Huang et al., in preparation). For the spectro-z sample, stellar masses were obtained from optical to near-infrared (NIR) spectral energy distribution (SED) fits from Bundy et al. (2006) which assumed a Chabrier (2003) IMF. For the photo-z sample we use stellar masses calculated using a relation between the $B$-band stellar mass-to-light ratio and rest-frame $(B - V)$ colour, which we calibrate against the Bundy et al. (2006) masses. Our calibration is similar to those reported in Lin et al. (2007) and Weiner et al. (2009), but makes use of only rest-frame $(U - B)$ colours, rather than $(U - V)$ and $(B - V)$. We compute rest-frame $(U - B)$ colours

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$^3$ For sizes measured in elliptical apertures, the quoted size is conventionally the major axis size, $R$. The circularized size is then often defined as $\sqrt{B/\pi R}$, where $b/a$ is the minor-to-major axial ratio.
and B-band luminosities by applying kCORRECT v4.1.4 (Blanton & Roweis 2007) to DEEP2 B, R, I magnitudes.

We select blue-cloud galaxies for this study by using the ‘green valley’ in the (U − B) colour–stellar mass plane as the division:

\[(U − B) < 1.05 − 0.1\log(M_{\text{star}}/10)\]  

(1)

In addition, we select only disc-dominated galaxies with disc light fractions greater than 50 per cent. This additional cut only removes a small fraction of galaxies because the vast majority of blue galaxies already have disc fractions greater than 50 per cent. In order to minimize the effects of extinction on disc sizes, colours and stellar masses, while at the same time keeping a significant sample size, we limit our sample to galaxies with disc minor-to-major axial ratios greater than 0.5 (corresponding to disc inclinations less than 60°, for a zero thickness disc).

2.2.2 Magnitude and surface brightness selection effects

The DEEP2 spectroscopic survey is limited by an apparent R-band magnitude of 24.1 AB. This results in a bias against redder galaxies at low stellar masses (Willmer et al. 2006). However, the stellar masses we use from Bundy et al. (2006) are limited by an apparent K-band magnitude of about 22.5 AB. This additional K-band selection is close to a selection on stellar mass and thus removes much of the bias by removing lower mass blue galaxies from the sample. In order to remove any remaining colour biases, we apply an additional redshift-dependent lower stellar mass limit of \[\log(M_{\text{min}}/10) = 8.5 + z\]. Our final DEEP2 sample consists of \(\sim 800\) galaxies with spectroscopic redshifts and \(\sim 2000\) galaxies with photometric redshifts.

A potentially important selection bias for galaxy size evolution studies is incompleteness at low-surface-brightness levels, especially at higher redshifts due to the cosmological surface brightness dimming, which goes as \((1 + z)^{4}\). To determine the impact of surface brightness selection effects it is customary to create model galaxies, insert them into real data and then run the same source detection algorithms that were applied to the real data. For the single-orbit HST/ACS F850LP observations from GEMS used by Barden et al. (2005) and Trujillo et al. (2006), these predict that surface brightness limits are significant for galaxies brighter than the magnitude limit of the COMBO-17 photometric redshift survey. However, as shown by Melbourne et al. (2007), typical large spiral galaxies at \(z \sim 1\) have significant amounts of high-surface-brightness substructure. This makes them much easier to detect and measure spectroscopic redshifts than smooth discs. In this paper, we use galaxies detected in an effective exposure of two orbits of HST/ACS imaging (single orbit each of F606W and F814W). Based on the results of Melbourne et al. (2007) we conclude that, at the depth of our images, we are not missing significant numbers of large star-forming galaxies with stellar masses greater than \(10^{10} M_\odot\) at \(z \sim 1\).

At the other extreme, small galaxies can be confused with stars, especially with ground-based seeing-limited imaging. However, stars can be reliably separated from galaxies based on the fact that stars tend to be brighter than compact high-redshift galaxies and also occupy a different locus from galaxies in \((B−R, R−I)\) space (Newman et al., in preparation). Furthermore, there is nothing to prevent small galaxies from being in our photo-z sample. The good agreement between the size–mass relations from our spectro-z and photo-z samples vindicates the DEEP2 star–galaxy separation method. We thus conclude that our sample of star-forming galaxies from DEEP2 is unlikely to be affected by biases against very small or very large galaxies above the (redshift-dependent) stellar mass limits of our sample.

2.2.3 SDSS comparison sample

As a low-redshift comparison sample, we use data from the SDSS DR7 (Abazajian et al. 2009). We use a similar methodology to the DEEP2 sample in order to minimize systematic biases in the comparison between high- and low-redshift data. Thus, we measure disc half-light radii with 2D bulge+disc fits using gimi2o, performed simultaneously on g and r images with the bulge and disc sizes constrained to be the same in each filter (Simard et al., in preparation). We use galaxies with spectroscopic redshifts from 0.02 < \(z < 0.2\) and stellar masses calculated using the relation between the r-band mass-to-light ratio and \((g−r)\) colour from Bell et al. (2003) with an offset of −0.1 dex to correspond to a Chabrier (2003) IMF. We select blue-cloud galaxies for this study using the valley in the \((g−r)−\text{stellar mass plane and requiring}\n
\[g−r < 0.71 − 0.067 \log(M_{\text{star}}/10)\]

(2)

As with the DEEP2 sample, we select galaxies with disc light fractions greater than 50 per cent and disc minor-to-major axial ratios greater than 0.5 (corresponding to disc inclinations less than 60° for a zero thickness disc). We use gimi2o model g and r magnitudes, with k-corrections to \(z = 0\) based on SDSS Petrosian ugriz magnitudes.

The SDSS spectroscopic survey is limited by the apparent r-band magnitude. In the colour–stellar mass plane, this results in a bias against redder galaxies at low stellar masses. In order to remove this effect, we apply a minimum stellar mass of \[\log(M_{\text{min}}/10) = 10.4 + \log(z/0.1)\]. Our final SDSS sample consists of \(\sim 130000\) galaxies.

Fig. 1 shows the disc half-light radius–total stellar mass relation from our SDSS sample. We fit the median SDSS disc size–mass relation with the following double power law:

\[R = R_0 \left(\frac{M}{M_0}\right)^{α} \left[\frac{1}{2} + \left(\frac{M}{M_0}\right)^{β}\right]^{γ} = \alpha/\gamma\]

(3)

Here \(α\) is the slope at low masses \((M \ll M_0)\), \(β\) is the slope at high masses \((M \gg M_0)\), \(γ\) controls the sharpness of the transition between the two slopes, \(M_0\) is the transition mass and \(R_0\) is the value of \(R\) at \(M_0\). We find that \[\log(M_0/(M_\odot)) = 10.44, \log(R_0/(kpc)) = 0.72, \alpha = 0.18, β = 0.52, γ = 1.8\] provide a good fit to the data.

We assume that the scatter in the SDSS disc size–mass relation is lognormal with \(s = \sigma_{\text{ln}M}/M\), with the following relation:

\[s = s_2 + (s_1 − s_2)/\left[1 + (M/M_0)^β\right]\]

(4)

Here \(s_1\) is the scatter at low masses, \(s_2\) is the scatter at high masses, \(M_0\) is the transition mass and \(γ\) controls the sharpness of the transition. We find that \(s_1 = 0.47, s_2 = 0.27, \log(M_0/(M_\odot)) = 10.3\) and \(γ = 2.2\) provide a good fit to the data.

For comparison, the total half-light radius–stellar mass relation for disc galaxies (Sersic \(n < 2.5\)) from the SDSS study of Shen et al. (2003) is given by the long-dashed line and the disc half-light size \((1.678 \times \text{scalelength})–\text{stellar mass relation from Dutton et al. (2007), using the spiral galaxy sample of Courteau et al. (2007), is given by the short-dashed line. The relation from Dutton et al. (2007) is in reasonable agreement with our result considering a single power law was used by Dutton et al. (2007). The relation from Shen et al. (2003) has a shallower slope at high masses and is offset to smaller sizes at all masses. These differences can be understood as a consequence of two factors: circularization and bulges. The half-light radii used by Shen et al. (2003) are measured using circular...
Figure 1. Disc half-light radius–stellar mass ($R_M$) relation of blue-cloud disc-dominated galaxies (bulge fraction less than 0.5) from the SDSS. The colour corresponds to the observed number density of galaxies on a logarithmic scale. The error bars show the median and error on the median in stellar mass bins of a width of 0.2 dex. The dotted lines show the 84th and 16th percentiles of the size distribution. The solid line shows our fit to the median relation using equation (3). For comparison, we also show the total half-light radius–stellar mass relation of disc-dominated ($S´r$ is $< 2.5$) galaxies from Shen et al. (2003, long-dashed line) and the disc half-light radius–stellar mass relation from Dutton et al. (2007, short-dashed line). The scatter in the size–mass relation is given in the lower panel. The points show our measurements from SDSS, the solid line shows our fit to these data using equation (4), while the long-dashed line shows the result from Shen et al. (2003).

apertures. Since the median major-to-minor axial ratio of galaxy discs is $\approx 2$, a circular size measurement will underestimate the true (face-on) half-light radius by a factor of $\approx 1.4$ (0.15 dex). This effect explains most of the discrepancy at low stellar masses. At high stellar masses there is an additional difference, which can plausibly be explained by the increased bulge fractions in higher mass galaxies (e.g. Dutton 2009). Since bulges tend to be smaller than discs, total half-light radii will be smaller than disc half-light radii.

We find that the scatter in disc sizes is mass dependent, with larger scatter at lower masses, in qualitative agreement with Shen et al. (2003). At low masses we find that the scatter of 0.47 in $\ln R$ is in agreement with Shen et al. (2003), but at high masses we find a scatter of 0.27 in $\ln R$, which is smaller than the scatter of 0.34 in $\ln R$ reported by Shen et al. (2003). As with the difference in the slopes at high masses, this difference is plausibly due to use of total half-light radii by Shen et al. (2003). We note that in the simplest $\Lambda$CDM-based disc formation models (e.g. Mo et al. 1998), the scatter in disc sizes at fixed mass is equal to the scatter in the halo spin parameter, $\lambda$, which is $\sigma_{\ln \lambda} \approx 0.5$ (e.g. Bullock et al. 2001b; Macciò et al. 2007). This is in rough agreement with the observed scatter at low stellar masses ($M_{\text{star}} < 10^{10} \, M_{\odot}$), but at high stellar masses ($M_{\text{star}} > 10^{11} \, M_{\odot}$) the observed scatter is a factor of $\approx 2$ lower than predicted by the simple model. This smaller than expected scatter has been noted by previous authors (e.g. de Jong & Lacey 2000; Shen et al. 2003; Pizagno et al. 2005; Dutton et al. 2007). This discrepancy may indicate that massive disc-dominated galaxies form in a subset of haloes with a biased distribution of halo spin parameters or that the distribution of disc sizes has been modified by secular evolution (e.g. Shen et al. 2003).

2.2.4 Evolution

The evolution of the disc half-light radius–stellar mass relation in six redshift bins from $z = 0.1$ (SDSS) up to $z \approx 1.2$ (DEEP2) is shown in Figs 2 and 3. The upper left panel shows fits to the SDSS disc size–stellar mass relation from Fig. 1. For both SDSS and DEEP2, the stellar mass is the total (i.e. disc plus bulge) stellar mass. The long-dashed line shows the median relation and the dotted lines show the 1$\sigma$ scatter.

Galaxies at redshifts $z > 0.2$ from DEEP2 are shown as green circles. The solid red lines show fits to the DEEP2 data obtained by calculating the median offset in size with respect to the $z = 0.1$ relation. The offsets are given in the top right corner of each
Evolution of velocity–mass–size relations

Figure 2. Evolution of the disc half-light radius–stellar mass (RM) relation of disc-dominated blue-cloud galaxies from DEEP2 using spectroscopic redshifts. The upper left panel shows the RM relation we derive for disc-dominated blue-cloud galaxies from SDSS. The long-dashed line shows the median relation and the dotted lines show 1σ scatter. In the other panels, the green points show observations from DEEP2 using HST imaging in five redshift bins. The median redshift in each bin is given in the upper left corner of each panel, together with the central rest-frame wavelength of the imaging used to measure galaxy sizes and the number of galaxies. The solid red line shows a fit to the DEEP2 data by offsetting the z = 0.1 relation (shown by the dashed line) in size. The red circle shows the median size for a stellar mass of 3 × 10^{10} M⊙. The offset is given in the upper right corner in each panel and is summarized in Table 2. The red short-dashed lines show a power-law fit to the data in each panel. The vertical dotted lines show the lower stellar mass limit at the upper redshift of each redshift bin.

The evolution of the zero-point (size evolution at fixed M_{star}) is summarized in Fig. 4. The data from DEEP2 are shown with blue pentagons (open for photo-z and filled for spectro-z), and the data from Trujillo et al. (2006) are shown with magenta squares. Note that the latter are for total half-light radii, not disc half-light radii, but nevertheless the two data sets are consistent within the error bars. The DEEP2 data, however, show stronger evidence for evolution both internally and in comparison to SDSS. For DEEP2, the evolution of the spectro-z sample is well fitted by

\[ \Delta \log_{10} R_{50} = 0.018 \pm 0.002 - 0.44 \pm 0.04 \log_{10}(1 + z), \]

where \( \Delta \log_{10} R_{50} \) is the size evolution relative to \( z = 0.1 \).

2.3 Evolution of the size–rotation velocity relation

The evolution of the size–velocity (RV) relation has been studied by Bouché et al. (2007) using data from the SINS survey (Förster-Schreiber et al. 2009) at \( z \approx 2 \) and from Courteau (1997) at \( z = 0 \). Using half-width half-maximum (HWHM) sizes interpreted as exponential disc scalelengths, Bouché et al. (2007) found that the RV relation exists at \( z = 2 \) with the same zero-point as that at \( z = 0 \).

In this paper, we redetermine the zero-point evolution using data from SINS at \( z \approx 2 \) adding data from Cresci et al. (2009) to those of Förster-Schreiber et al. (2009). Cresci et al. (2009) determined maximum rotation velocities for 18 galaxies at \( z \approx 2 \) from the SINS survey using kinematic modelling of 2D Hα velocity fields. Stellar masses were obtained from optical to NIR SED fits assuming solar metallicity Bruzual & Charlot (2003) stellar population synthesis models with a Chabrier (2003) IMF. Galaxy sizes were measured from Hα emission maps using two methods: Cresci et al. (2009) gave HWHM sizes, \( R_{\text{HWHM}} \), while Förster-Schreiber et al. (2009) gave circular half-light sizes, \( R_{1/2} \). Both size measurements have been made on the 18 galaxies from Cresci et al. (2009).

The HWHM sizes are obtained from a linear Gaussian fit to the major axis of the Hα line intensity maps. According to Bouché et al. (2007), the derived HWHM, once corrected for the observed seeing, corresponds to the exponential scalelength, \( R_d \), of the disc. Using simulations of model discs, these authors claim that \( R_d \) measured this way is likely to be overestimated by no more than 15 per cent. The half-light sizes from Förster-Schreiber et al. (2009) are obtained from the Hα curves-of-growth measured in circular apertures and corrected for seeing.
Figure 3. Evolution of the disc half-light radius–stellar mass ($R_M$) relation of disc-dominated blue-cloud galaxies from DEEP2 using photometric redshifts. The upper left panel shows the $R_M$ relation we derive for disc-dominated blue-cloud galaxies from SDSS. The long-dashed line shows the median relation and the dotted lines show 1σ scatter. In the other panels, the green points show observations from DEEP2 using HST imaging in five redshift bins. The median redshift in each bin is given in the upper left corner of each panel, together with the central rest-frame wavelength of the imaging used to measure galaxy sizes and the number of galaxies. The solid red line shows a fit to the DEEP2 data by offsetting the $z=0.1$ relation (shown by the dashed line) in size. The red circle shows the median size for a stellar mass of $3\times10^{10}M_\odot$. The offset is given in the upper right corner in each panel and is summarized in Table 2. The red dashed lines show a power-law fit to the data in each panel. The vertical dotted lines show the lower stellar mass limit at the upper redshift of each redshift bin.

Table 2. Evolution of the disc half-light radius–stellar mass relation of blue-cloud disc-dominated galaxies relative to SDSS galaxies at $z=0.1$, using data from DEEP2 from Figs 2 and 3.

| Redshift range | Median $z$ | $\Delta \log_{10} R_{50}/M_{\text{star}}$ | $N$ |
|----------------|------------|-----------------------------|-----|
| Spectroscopic redshifts | | | |
| 0.2–0.5 | 0.38 | $-0.05 \pm 0.03$ | 95 |
| 0.5–0.7 | 0.61 | $-0.08 \pm 0.02$ | 145 |
| 0.7–0.9 | 0.77 | $-0.08 \pm 0.03$ | 267 |
| 0.9–1.1 | 0.99 | $-0.12 \pm 0.02$ | 192 |
| 1.1–1.4 | 1.22 | $-0.14 \pm 0.03$ | 118 |
| Photometric redshifts | | | |
| 0.2–0.5 | 0.37 | $-0.02 \pm 0.01$ | 360 |
| 0.5–0.7 | 0.62 | $-0.07 \pm 0.01$ | 321 |
| 0.7–0.9 | 0.77 | $-0.09 \pm 0.01$ | 613 |
| 0.9–1.1 | 1.00 | $-0.11 \pm 0.01$ | 414 |
| 1.1–1.4 | 1.21 | $-0.18 \pm 0.02$ | 253 |

For an exponential disc, $R_d = 0.60 R_{1/2}$, and thus if $R_{\text{WHHM}} = R_d$ then $R_{\text{WHHM}}$ should be smaller than $R_{1/2}$ by 0.22 dex, whereas Fig. 5 shows that the opposite is true: the HWHM radii are 0.12 dex larger. The fact that $R_{\text{WHHM}} \simeq 1.3 R_{1/2}$ was also noted by Förster-Schreiber et al. (2009). The discrepancy may be somewhat exaggerated, as the half-light radii are measured through circular apertures, not along the major axis. If the galaxies studied by Cresci et al. (2009) were a random sample of galaxies, the median disc inclination angle would be 60°, and thus the circular radii would be too small on average by a factor of $\simeq 1.4$. However, the distribution of disc inclinations of the galaxies in Cresci et al. (2009) appears skewed towards low inclinations: only 3/18 galaxies have inclination greater than 60°. The median inclination is 42°, which implies that the circularization effect is of the order of 0.07 dex. In what follows we use both size measurements, assuming that $R_{\text{WHHM}}$ is equivalent to an exponential scalelength and $1.16 \times R_{1/2}$ is equivalent to a major-axis half-light radius. We will let consistency between various data sets determine which, if either, size measurement is more likely to be correct.

Fig. 6 shows the observed evolution of the VMR relations from $z \simeq 2$ to 0. The SINS data at $z \simeq 2$ are shown with circles. Two versions of radii are shown: HWHM Hα radii by Cresci et al. (2009) (upper panels) and (de-circularized) half-light Hα radii by Förster-Schreiber et al. (2009) (lower panels). The redshift $z = 0$ relations are shown as dashed lines and are obtained as follows. We use the $M_{\text{star}} - V_{\text{max}}$ relation from Bell & de Jong (2001) derived from K-band luminosities using a mass-dependent extinction correction, subtracting 0.1 dex from the stellar masses to correspond to a Chabrier (2003) IMF, and using a bisector fit:

$$\log_{10}\frac{M_{\text{star}}}{(10^{10} M_\odot)} = -0.61 + 4.51 \log_{10}\frac{V_{\text{max}}}{(100 \text{ km s}^{-1})}.$$
We note that the velocity used by Bell & de Jong (2001) is not $V_{\text{max}}$, but actually $V_{\text{flat}}$, the velocity in the flat outer part of the rotation curve. However, as shown by Dutton et al. (2010b), the TF relation derived using $V_{\text{flat}}$ for these galaxies is identical to that derived using $V_{\text{max}}$.

We use the local $I$-band disc scalelength–stellar mass relation from Dutton et al. (2007) which used an orthogonal fit:

$$\log_{10}\frac{R_{d,I}}{(\text{kpc})} = 0.348 + 0.281 \log_{10}\frac{M_{\text{star}}}{(10^{10} M_\odot)},$$

where we have adopted stellar masses from Bell et al. (2003) of $-0.1$ dex. Taking these $MV$ and $RM$ relations, we infer the mean relation between $R_{d,I}$ and $V_{\text{max}}$:

$$\log_{10}\frac{R_{d,I}}{(\text{kpc})} = 0.177 + 1.267 \log_{10}\frac{V_{\text{max}}}{(100 \text{ km s}^{-1})}.$$

In order to compare with disc half-light radii, we use $R_{50,I} = 1.678 R_{d,I}$. For simplicity, we use the $I$-band sizes to compare with the $z \simeq 2$ data. But we note that the evolution would not change significantly; in fact it would be slightly stronger, if we used the $V$-band $RM$ relation that we derived from the SDSS.

Due to the small numbers of galaxies in the $z \simeq 2$ sample, we do not attempt to fit the slopes of the high-redshift relations. Instead, we fix the slopes to the $z = 0$ values and measure the median offset of the $z \simeq 2$ galaxies with respect to the $z = 0$ relations. The values of these offsets are given at the bottom of each panel.

The upper right panel shows the disc scalelength–maximum rotation velocity relation using the Cresci et al. (2009) SINS data. The $z \simeq 2$ data have a marginally higher zero-point ($0.09 \pm 0.05$ dex) compared to the redshift zero data, which is in agreement with the findings of Bouché et al. (2007). Given that the TF relation evolves to lower stellar masses ($-0.44 \pm 0.08$ dex)$^4$ at fixed velocity at higher redshifts (lower left panel), this implies that at fixed stellar mass, sizes should be larger at higher redshifts. As is shown in the upper middle panel, this is indeed the case, with sizes larger by $0.20 \pm 0.05$ dex at $z \simeq 2$ than at $z = 0$. The evolution of the $R_{d,I}$–$M_{\text{star}}$ relation from the SINS Cresci et al. (2009) data is thus of opposite sign to that of previous studies (see Section 2.2). In particular, it is inconsistent with the $z \sim 2$ results from Trujillo et al. (2006) and Williams et al. (2010), who found that disc/star-forming galaxies were a factor of $\simeq 2$ smaller at fixed stellar mass.

The lower middle and right panels show the $RM$ and $RV$ relations using the SINS curve-of-growth $H_\alpha$ half-light radii from Förster-Schreiber et al. (2009). These panels show that the half-light sizes are smaller at a given stellar or maximum rotation velocity, relative to $z = 0$ galaxies. The offset for the $RM$ relation is $-0.07 \pm 0.05$ dex, while for the $RV$ relation it is $-0.22 \pm 0.06$ dex. The evolution in the $RM$ relation is still, however, weaker than obtained by Trujillo et al. (2006), who found $-0.28$ dex. As we discuss in Section 3.3, this remaining difference can be accounted for by the difference between half-light radii measured in $H_\alpha$ and those measured in rest-frame $I$-band light.

$^4$ Note that the difference in evolution of 0.05 dex between our result and that of Cresci et al. (2009), who used the same data, is caused by our different normalization of the $z = 0$ relation from Bell & de Jong (2001). In order to convert the stellar masses derived assuming a "diet"-Salpeter IMF of Bell & de Jong (2001) into a Chabrier (2003) IMF, which is used at $z \sim 2$, we adopt $-0.1$ dex, whereas Cresci et al. (2009) adopted $-0.15$ dex.
3 THEORETICAL EXPECTATIONS

We now compare these observational results to the predictions of various theoretical models.

3.1 Dark matter haloes

The zeroth-order prediction for the evolution of the VMR scaling relations of disc galaxies is given by the evolution of the VMR relations of DM haloes. This is obtained by assuming the following.

(i) The total mass profile of the galaxy and halo is isothermal.
(ii) The galaxy mass fraction, \( m_{\text{gal}} = M_{\text{gal}}/M_{\text{vir}} \), is a constant and independent of redshift (for a given halo). Here \( M_{\text{vir}} \) is the galaxy mass (i.e. the sum of stellar mass and cold gas) and \( M_{\text{vir}} \) is the total mass within the halo virial radius.
(iii) The halo concentration.
(iv) The galaxy spin parameter, \( \lambda_{\text{gal}} = (J_{\text{gal}}/m_{\text{gal}})\lambda \), is a constant and independent of redshift (for a given halo). Here \( J_{\text{gal}} = J_{\text{gal}}/J_{\text{vir}} \) is the angular momentum fraction, where \( J_{\text{gal}} \) is the total angular momentum of the galaxy and \( J_{\text{vir}} \) is the total angular momentum within the virial radius, and the halo spin parameter, \( \lambda \), is given by

\[
\lambda = \frac{J_{\text{vir}}/E_{\text{vir}}^{1/2}}{\sqrt{2} M_{\text{vir}} V_{\text{vir}}^{1/2}} = \frac{J_{\text{vir}}/M_{\text{vir}}^{1/2}}{\sqrt{2} R_{\text{vir}} V_{\text{vir}} f_{\text{s}}^{1/2}}.
\]

Here \( R_{\text{vir}} \) is the virial radius, \( V_{\text{vir}} \) is the circular velocity at the virial radius, \( E_{\text{vir}} \) is the halo’s energy and \( f_{\text{s}} = 1 \). For more general haloes, \( f_{\text{s}} \) measures the deviation of \( E \) from that of a singular isothermal sphere (SIS). For a Navarro–Frenk–White (NFW) halo \( f_{\text{s}} \approx 1/(c/21.5)^{0.7} \) see Mo et al. (1998), where \( c \) is the halo concentration.
(iv) The galaxy is 100 per cent stars, i.e. there is no cold gas.

We refer to this model as the SIS model. Under these assumptions, the disc size scales as the size and circular velocity of the halo (e.g. Mo et al. 1998): \( R_{\text{SIS}} = \lambda R_{\text{vir}} \sqrt{\frac{2}{3}} \propto V_{\text{vir}} \), and the stellar mass scales with the halo mass: \( M_{\text{star}} \propto M_{\text{vir}} \).

Assuming that the halo is defined by spherical top hat collapse: \( M_{\text{vir}} = \frac{4}{3}\pi R_{\text{vir}}^3 \Delta_{\text{vir}} \rho_{\text{crit}} \), the evolution of the \( MV \), \( RM \) and \( RV \) relations is given by

\[
\frac{M_{\text{vir}}(z)}{M_{\text{vir,0}}} = \left[ \frac{V_{\text{vir}}(z)}{V_{\text{vir,0}}} \right]^{3/2} \left[ \frac{\Delta_{\text{vir}}(z)}{\Delta_{\text{vir,0}}} \right]^{-1/2} \frac{H(z)}{H_0} \]
\]

\[
\frac{R_{\text{vir}}(z)}{R_{\text{vir,0}}} = \left[ \frac{M_{\text{vir}}(z)}{M_{\text{vir,0}}} \right]^{1/3} \left[ \frac{\Delta_{\text{vir}}(z)}{\Delta_{\text{vir,0}}} \right]^{-1/3} \frac{H(z)}{H_0}^{2/3}.
\]

The zero-point evolutions are obtained by setting the leading terms on the RHS = 1. Thus, the evolution is governed by the evolution of the Hubble parameter \( H(z) \) and the halo overdensity within the virial radius, \( \Delta_{\text{vir}}(z) \). The evolution of the Hubble parameter (in a flat cosmology) is given by

\[
H(z) = H_0(\Omega_{\Lambda} + \Omega_0(1+z)^3)^{1/2}.
\]

For the evolution of the halo overdensity, \( \Delta_{\text{vir}}(z) \), we use the fitting formula from Bryan & Norman (1998): \( \Delta_{\text{vir}} = 18\pi^2 + 82z - 39z^2 \), where \( z = \Omega(z) - 1 \) and \( \Omega(z) \) is defined as \( \Omega_0(1+z)^3[H(z)/H_0]^2 \).

Thus, to first order the evolutions are determined by the evolution of the Hubble parameter and to second order by the evolution of the...
halo overdensity. These predicted evolutions, for a ΛCDM cosmology with ΩM = 0.3 and ΩΛ = 0.7, are shown by the dashed lines in Fig. 7. For the Mvir–Vvir and Rvir–Vvir relations the evolution scales roughly as (1 + z)^−1.3, while for the Rvir–Mvir relation the evolution scales roughly as (1 + z)^−0.8. The observed evolution of the maximum circular velocity–stellar mass–half-light radius relations is given by the coloured symbols. For the stellar mass–velocity relation, we use data from Conselice et al. (2005, green triangles), DEEP2 (Kassin et al. 2007, blue pentagons) and SINS (Cresci et al. 2009, red open circle). For the size–stellar mass relation, we use data from Trujillo et al. (2006, magenta squares) and SINS (Cresci et al. 2009, red open square), Williams et al. (2010, green triangles) and DEEP2 (this paper, blue pentagons). For the size–velocity relation, we use data from SINS (Cresci et al. 2009, red open circle) and Förster-Schreiber et al. (2009, red open square). These observations are described in more detail in Section 2. As has been noted by previous authors, the observed evolution of all three VMR relations is weaker than this simple prediction from DM haloes.

The SIS model makes a number of simplifying assumptions that are likely to be incorrect. First, total mass density profiles are not expected to be globally isothermal. Dark matter haloes in cosmological simulations have Vmax > Vvir (e.g. Bullock et al. 2001a). In addition, the contribution of the baryons to the inner potential can increase the observed Vmax even further.

Using the models of Mo et al. (1998), Somerville et al. (2008) showed that including the expected evolution in dark halo concentrations, as well as the contribution of baryons to the potential, results in weaker evolution than predicted by the SIS model. However, Somerville et al. (2008) make the assumption that discs have no gas and thus that the stellar mass is equal to the baryonic mass and the V-band half-light radius is equal to the baryonic half-mass radius. In order to determine whether the stellar relations evolve differently from the baryonic relations, one needs to follow the evolution of stellar and gas discs in a self-consistent way. This requires realistic cosmological simulations with gas or semi-analytic models (SAMs). Here we use the latter.

3.2 Disc-galaxy semi-analytic model

In order to calculate the evolution of stellar discs as opposed to baryonic discs, we use the disc-galaxy SAM of Dutton & van den Bosch (2009). This model consists of discs that grow inside evolving NFW haloes (Navarro, Frenk & White 1997), with structure determined from cosmological N-body simulations (Bullock et al. 2001a; Macciò, Dutton & van den Bosch 2008), a median spin parameter λ, which is independent of redshift, and halo specific angular momentum distributions from Sharma & Steinmetz (2005), which are specified by a parameter α. The cumulative distribution of specific angular momentum P(<s), where s is the specific angular momentum in units of the total specific angular momentum, is given by

P(<s) = γ(α, αs),

where γ is the incomplete gamma function. In this model, discs are not formally exponential, but the stellar discs can often be well described by an exponential profile over several scalelengths (Dutton 2009).

To build a model as close as possible to the SIS model, we maintain assumptions (ii) and (iii) from above. As in Somerville et al. (2008), the baryonic disc is in dynamical equilibrium inside an NFW halo which evolves with redshift according to cosmological simulations (Bullock et al. 2001a). The key difference is that we do not make assumption (iv), i.e. that the disc is 100 per cent stars. In our model, the evolution of the stellar and gas discs (and hence stellar and gas mass) is governed by the radial variation of star formation, gas recycling and accretion. The stellar mass is thus always less than the baryonic mass and the stellar disc is usually smaller in radius than the baryonic disc. Because our model follows stellar populations as a function of radius, we calculate the sizes of model galaxies in optical to NIR light as well as stellar and baryonic mass. This allows us to test whether the sizes measured in different passbands and masses are equivalent.

An additional difference between our models and those of Somerville et al. (2008) is halo contraction. Somerville et al. (2008) assumed that haloes contract according to the Blumenthal et al.
(1986) adiabatic contraction model. In our model, we leave the haloes uncontracted, as models with dark halo contraction (and standard IMFs) are unable to reproduce the zero-points of the VMR relations (Dutton et al. 2007). There are a variety of astrophysical processes that could reverse the expected effect of halo contraction. These include dynamical friction from massive clumps of baryons (e.g. El-Zant, Shlosman & Hoffman 2001; Mo & Mao 2004; Elmegreen, Bournaud & Elmegreen 2008; Jardel & Sellwood 2009), dynamical friction due to bars (e.g. Weinberg & Katz 2002; Sellwood 2008) and rapid mass outflows due to supernova feedback (e.g. Mo & Mao 2004; Read & Gilmore 2005; Governato et al. 2010), so our choice of not including adiabatic contraction is at least physically plausible as well as empirically motivated. As we will see from the success of our models in matching the evolution of the VMR relations, halo contraction is also not required in order to reproduce the observed evolution of the VMR relations.

In the model used here, we adopt a cosmology with \((\Omega_{\text{M}}, \Omega_{\Lambda}, h, \sigma_{8}, n) = (0.3, 0.7, 0.7, 0.8, 1.0)\), which is close to that of the WMAP fifth-year results (Dunkley et al. 2009). We adopt a galaxy mass fraction of \(m_{\text{gal}} = (M_{\text{star}} + M_{\text{gal}})/M_{\text{vir}} = 0.04\), a median spin parameter of \(\lambda_{\text{gal}} = 0.035\) and a median angular momentum shape parameter of \(\bar{\lambda} = 0.9\). These parameters are motivated by observations of \(\lambda_{\text{gal}}\) (Hoekstra et al. 2005; Dutton et al. 2010b) and theoretical predictions for \(\lambda\) (Bullock et al. 2001b; Macciò et al. 2007) and \(\alpha\) (Sharma & Steinmetz 2005). They also result in models that roughly reproduce the observed VMR relations at \(z = 0\) (Fig. 8). However, this model does not exactly reproduce the slopes of the local VMR relations. Doing so requires either \(m_{\text{gal}}\) or \(\lambda_{\text{gal}}\) to vary with halo mass (e.g. Shen et al. 2003; Dutton et al. 2007).

We generate a Monte Carlo sample of 2000 galaxies with halo masses between \(M_{\text{vir}} = 10^{10.3}\) and \(10^{13.5} h^{-1} M_{\odot}\), lognormal scatter in spin parameter of \(\sigma_{\text{ln} \lambda} = 0.5\) (Bullock et al. 2001b), lognormal scatter in halo angular momentum profile of \(\sigma_{\text{ln} \alpha} = 0.25\) (Sharma & Steinmetz 2005) and lognormal scatter in halo concentration of \(\sigma_{\text{ln} c} = 0.25\) (Macciò et al. 2008). We determine the evolution of the zero-points of the model VMR relations by fitting the MV and RV relations for \(2.1 \leq \log_{10} [V_{\text{max}}/(\text{km s}^{-1})] \leq 2.5\) and the RM relations for \(9.0 \leq \log_{10} [M_{\text{star}}/(M_{\odot})] \leq 11.0\). We then calculate the evolution at \(\log_{10} [V_{\text{max}}/(\text{km s}^{-1})] = 2.3\) and \(\log_{10} [M_{\text{star}}/(M_{\odot})] = 10.5\).

The solid lines in Fig. 7 show evolution in the zero-points of the \(V_{\text{max}} - M_{\text{star}} - R_{50}\) (i.e. maximum circular velocity, stellar mass, V-band half-light radius) relations of this model from redshifts \(z = 4\) to 0. These relations show weaker evolution than the virial relations (dashed lines) but stronger evolution than the baryonic relations (dotted lines). The evolution of the model viral, baryonic, and stellar relations is given in Table 3.

One interpretation of the weak evolution of the galaxy scaling relations since \(z \sim 1\) is that individual galaxies also evolve weakly
The differences between the evolution of the baryonic and virial \(MV\) relations (dotted versus dashed lines in Fig. 7) are due solely to evolution in the ratio of \(V_{\text{max}}\) to \(V_{\text{vir}}\) and not due to an evolution in the ratio of the baryonic mass to virial mass, \(m_{\text{gal}}\), which is fixed to a constant in this model. The ratio of \(V_{\text{max}}\) to \(V_{\text{vir}}\) increases towards lower redshift due to higher concentrations in lower redshift haloes. This results in less evolution in \(V_{\text{max}}\) than \(V_{\text{vir}}\) at fixed \(m_{\text{gal}}\) or \(M_{\text{vir}}\) and hence less evolution in \(M_{\text{gal}}\) or \(M_{\text{vir}}\) at fixed \(V_{\text{max}}\) than at fixed \(V_{\text{vir}}\). The evolution of \(V_{\text{max}}\) and \(V_{\text{vir}}\) for an individual galaxy with redshift \(z = 0\) virial mass of \(M_{\text{vir}} = 10^{12} h^{-1} M_{\odot}\) and median halo parameters is shown in the left-hand panels of Fig. 8. This shows that for high redshifts \((z \gtrsim 2)\) \(V_{\text{max}} \simeq V_{\text{vir}}\), but for low redshifts \((z \lesssim 1)\) \(V_{\text{vir}}\) remains constant while \(V_{\text{max}}\) continues to increase. At redshift \(z = 0\), \(V_{\text{max}} \simeq 1.3 V_{\text{vir}}\), which is consistent with recent measurements Dutton et al. (2010b). The lower middle panel of Fig. 8 shows that \(R_{\text{gal}}\) and \(M_{\text{vir}}\) both continue to increase at low redshifts. The reason \(V_{\text{vir}}\) remains roughly constant while \(R_{\text{gal}}\) and \(M_{\text{vir}}\) increase is due to a trade-off between the addition of new mass (which increases \(V_{\text{vir}}\)) and the increase of the virial radius (which decreases \(V_{\text{vir}}\) because the circular velocity of the NFW profile declines at large radius).

since \(z \sim 1\). However, disc galaxies were forming stars at much higher rates in the past (e.g. Noeske et al. 2007a), and thus stellar masses are expected to grow significantly since \(z \sim 1\). Simple models for the evolution of the star formation rate–stellar mass relation suggest that galaxies with present-day stellar masses of \(3 \times 10^{10} M_{\odot}\) had stellar masses a factor of \(\simeq 2.5\) lower at \(z = 1\) (Noeske et al. 2007b). This amount of evolution in stellar masses is consistent with that predicted by our model. Fig. 8 shows the evolution of a galaxy with a present-day stellar mass of \(3 \times 10^{10} M_{\odot}\). Since \(z = 1\) its stellar mass has increased by a factor of \(\simeq 2.5\), its half-light size has increased by a factor of \(\simeq 2\) and its maximum circular velocity has increased by a factor of \(\simeq 1.15\). In terms of samples of galaxies, the evolution since \(z = 1\) is just a factor of \(\simeq 1.5\) increase in stellar masses, at fixed \(V_{\text{max}}\), and a factor of \(\simeq 1.3\) increase in disc size, at fixed stellar mass. Thus, the scaling relations between properties of galaxies \((V_{\text{max}} - M_{\text{star}} - R_{\text{gal}})\) evolve only weakly since \(z \sim 1\) because individual galaxies evolve roughly along the scaling relations. Similar theoretically based conclusions have been made previously for the stellar mass–velocity relation (Portinari & Sommer-Larsen 2007) and the size–stellar mass relation (Firmani & Avila-Reese 2009).

### Table 3. Evolution of the stellar, baryonic and virial velocity–mass–size relations of our disc-galaxy evolution model, with \(m_{\text{gal}} = 0.04\) and \(\lambda = 0.035\), relative to redshift \(z = 0\) as shown in Fig. 7.

| \(z\) | \(\log_{10}(1 + z)\) | \(\Delta \log_{10} M_{\text{star}} | V_{\text{max}}\) | \(\Delta \log_{10} M_{\text{gal}} | V_{\text{max}}\) | \(\Delta \log_{10} M_{\text{vir}} | V_{\text{vir}}\) |
|---|---|---|---|---|
| 0.00 | 0.000 | 0.000 | 0.000 | 0.000 |
| 0.10 | 0.041 | -0.001 | 0.006 | -0.035 |
| 0.30 | 0.114 | -0.027 | -0.014 | -0.106 |
| 0.51 | 0.179 | -0.065 | -0.039 | -0.177 |
| 0.70 | 0.230 | -0.105 | -0.066 | -0.238 |
| 1.01 | 0.303 | -0.171 | -0.115 | -0.327 |
| 1.41 | 0.382 | -0.264 | -0.190 | -0.435 |
| 2.00 | 0.477 | -0.418 | -0.307 | -0.568 |
| 2.50 | 0.544 | -0.543 | -0.394 | -0.665 |
| 3.00 | 0.602 | -0.661 | -0.472 | -0.749 |
| 4.00 | 0.699 | -0.872 | -0.610 | -0.892 |

| \(z\) | \(\log_{10}(1 + z)\) | \(\Delta \log_{10} R_{\text{gal}} | M_{\text{star}}\) | \(\Delta \log_{10} R_{\text{gal}} | M_{\text{gal}}\) | \(\Delta \log_{10} R_{\text{gal}} | M_{\text{vir}}\) |
|---|---|---|---|---|
| 0.00 | 0.000 | 0.000 | 0.000 | 0.000 |
| 0.10 | 0.041 | 0.003 | 0.012 | -0.024 |
| 0.30 | 0.114 | -0.019 | -0.001 | -0.071 |
| 0.51 | 0.179 | -0.046 | -0.022 | -0.118 |
| 0.70 | 0.230 | -0.071 | -0.042 | -0.159 |
| 1.01 | 0.303 | -0.116 | -0.071 | -0.218 |
| 1.41 | 0.382 | -0.175 | -0.112 | -0.290 |
| 2.00 | 0.477 | -0.270 | -0.170 | -0.379 |
| 2.50 | 0.544 | -0.339 | -0.224 | -0.443 |
| 3.00 | 0.602 | -0.402 | -0.276 | -0.500 |
| 4.00 | 0.699 | -0.519 | -0.368 | -0.595 |
Since the baryonic mass fraction we adopt is only $\simeq 25$ per cent of the universal baryon fraction, it is certainly plausible that the mechanisms responsible for making galaxy formation inefficient result in $m_{\text{gal}}$ varying with redshift. However, to first order, variation in $m_{\text{gal}}$ moves galaxies along the VM relation (e.g. Navarro & Steinmetz 2000; Dutton et al. 2007), and thus we expect that large changes in $m_{\text{gal}}$ with redshift would be needed in order to significantly change the evolution from that predicted by our constant $m_{\text{gal}}$ model.

The differences between the evolution of the baryonic and virial $RM$ relations in Fig. 7 are caused by evolution in the ratio of baryonic to virial sizes (because the baryon mass fraction is a constant), which are roughly given by $R_{\text{90gal}}/R_{\text{vir}} \propto \lambda_{\text{gal}}(V_{\text{vir}}/V_{\text{max}}) f_{\text{d}}^{-1/2}$. Thus as with the $MV$ relations, the differences between the baryonic and virial $RM$ relations are driven by the evolution in $V_{\text{max}}/V_{\text{vir}}$. Unlike the VM relation, the baryonic $RM$ relation is sensitive to the adopted baryon mass fraction (Dutton et al. 2007). Thus if $m_{\text{gal}}$ decreases (or increases) with increasing redshift, this will result in weaker (or stronger) evolution in the baryonic $RM$ relation.

The differences between the evolution of the baryonic and virial $RV$ relations are determined by evolution in both $R_{\text{90gal}}/R_{\text{vir}}$ and $V_{\text{max}}/V_{\text{vir}}$. Evolution in the size ratio accounts for $\simeq 2.3$ of the evolution difference, while evolution in the velocity ratio accounts for the remaining $\simeq 1/3$. As with the $RM$ relation, the $RV$ relation is sensitive to the galaxy mass fraction, with a similar sign dependence.

The differences between the stellar and baryonic $MV$ relations in Fig. 7 can be understood as a result of the higher cold gas fractions at higher redshifts in our model. We note that the evolution in the cold gas fractions in our model is relatively modest, with a factor of $\lesssim 2$ increase in gas masses at fixed stellar mass between redshifts $z = 0$ and 2. As shown by Dutton, van den Bosch & Dekel (2010a), more general models (including cooling and outflows) also predict weak evolution in cold gas fractions but strong evolution in molecular gas fractions. Both of these predictions are consistent with recent observations (e.g. Erb et al. 2006; Daddi et al. 2010; Puech et al. 2010; Tacconi et al. 2010).

Higher cold gas fractions also contribute to different evolution in the stellar and baryonic $RV$ and $RM$ relations. But there are also contributions from differences between sizes in stellar mass and sizes in optical light, which are a consequence of the inside-out nature of stellar disc growth in our models. These differences are discussed in more detail below.

3.3 Reconciling SINS sizes with other observations

Our model nicely reproduces the observed zero-point evolution of the stellar mass TF ($M_{\text{star}}-V_{\text{max}}$) and size–stellar mass relations from $z = 2.2$ to 0 (Fig. 7). The model also predicts a factor of $\simeq 3$ evolution in the zero-point of the optical half-light size–velocity relation ($R_{\text{HWHM}}-V_{\text{max}}$) over this redshift range (solid black line in the right-hand panel of Fig. 7). However, the $R_{\text{g}}-V_{\text{max}}$ data from SINGS (using disc scalelengths from Cresci et al. 2009) indicate weak evolution in the opposite direction from $z = 2.2$ to 0 (red open circle in the right-hand panel of Fig. 7). Recall that we have derived the evolution of the $R_{\text{g}}-V_{\text{max}}$ relation by comparing with the $I$-band $R_{\text{g}}-V_{\text{max}}$ relation at $z = 0$ (see Section 2.3). Since the Cresci et al. (2009) data set also implies that higher redshift galaxies are larger at fixed stellar mass (red open circle in the middle panel of Fig. 7), this obviously means that the evolution of the scaling relations between different data sets is inconsistent. The offset between the observed $RM$ evolution from Trujillo et al. (2006) and the Cresci et al. (2009) SINS data is also a factor of $\simeq 3$. Thus if the Cresci et al. (2009) disc scalelengths of the SINS galaxies could be reduced by a factor of $\simeq 3$, then all of the data sets would be consistent. Furthermore, the data would be consistent with our simple, but cosmologically motivated, model for disc-galaxy evolution.

As discussed in Section 2.3, there are two size measurements now available for SINS galaxies, both based on Hz imaging: major-axis HWHM sizes by Cresci et al. (2009) (which have been interpreted as being equivalent to disc scalelengths) and circular half-light radii by Förster-Schreiber et al. (2009). In Section 2.3, we calculated the evolution of the SINS data at $z \simeq 2.2$ relative to the $I$-band $RM$ and $RV$ relations at $z = 0$. We found that the two size measurements resulted in very different amounts of evolution, with the half-light sizes giving a factor of $\simeq 2$ smaller sizes at higher redshift. Thus, using the SINS half-light sizes from Förster-Schreiber et al. (2009) (red open squares in Fig. 7) removes most of the discrepancy that exists when using the Cresci et al. (2009) HWHM sizes interpreted as disc scalelengths.

However, there is still a small discrepancy at the 0.2-dex level between the evolution of the SINS half-light sizes with those from Trujillo et al. (2006), Williams et al. (2010) and our models. Below, we discuss whether this difference can be explained by the difference between sizes measured in Hz compared to rest-frame $I$-band light. Since the SINS survey is not a volume-limited sample of galaxies at high redshift, and at least some of the SINS sample was selected on the basis of discy morphology with spatially resolved velocity gradients, another possibility is that a selection bias against small galaxies exists in the SINS survey.

3.4 The relation between sizes in Hz versus rest-frame optical light

How well do galaxy sizes measured in Hz trace those measured in rest-frame optical stellar light? We now address this question using the disc SAM discussed in Section 3.2. To make our comparison, we assume that Hz is a reliable tracer of recent star formation, where recent star formation in our models is the star formation within the last time-step (i.e. $\simeq 30-100$ Myr). For simplicity, we also ignore the effects of dust, which may modify the relation between sizes measured in Hz and optical light. Fig. 9 shows the results where we compare the half-mass radii of recent star formation, $R_{\text{SP}}$, in our disc formation model, to the half-light radii in the near-UV to NIR passbands.

We find that the half-mass radius of the recent star formation is on average a factor of $\simeq 2$ larger than the half-mass radius of the total stellar mass. This is a signature of the ‘inside-out’ nature of stellar disc growth in our models. The inside-out growth is due to a combination of two factors: (1) the cosmological inside-out growth, which means that for an individual galaxy the baryonic disc grows with time due to an increase in the specific angular momentum, and (2) the density dependence of the star formation law (star formation is less efficient at lower gas densities), which is star formation induced inside-out growth.

Using cosmological hydrodynamical simulations of galaxies at redshift $z \simeq 2$, Sales et al. (2009) found a similar factor of $\simeq 1.8$ difference between the half-mass sizes of the dense (star-forming) gas and the half-mass sizes of the stellar mass. However, as we go from stellar mass sizes to NIR sizes to optical sizes, the differences with respect to the star formation sizes decrease. In terms of optical half-light radii, the star formation size is just 0.1 dex higher than $V$-band sizes and 0.15 dex higher than $I$-band sizes at $z = 2$. Thus, Fig. 9 suggests that the differences between Hz and optical sizes by themselves are unlikely to explain the full factor of 3 discrepancy between the SINS disc scalelengths and the other observations.
shown in Fig. 7. But, as shown by the solid red circles, these are of the right magnitude to fully remove the lingering small discrepancy between the Förster-Schreiber et al. (2009) half-light radii and the Trujillo et al. (2006) data and our models.

To summarize, we have shown that the radii of SINS galaxies can plausibly be reconciled with other observational data and with theoretical models provided proper radii are used (half-light radii, not HWHM) and provided a small correction is applied to convert Hα radii to optical radii. Ideally, this agreement should be verified directly with sizes measured in rest-frame optical light, and preferably using the same techniques and definitions as used at lower redshifts. It would also be desirable to test this concordance by measuring the evolution of the VMR relations at redshifts $z \gtrsim 1$ using larger samples of galaxies and over a wider range of masses than current studies. The measurement of rest-frame optical sizes of galaxies at $z \sim 2$ is currently possible with $H$-band imaging with HST, and $H$- and $K$-band imaging with adaptive optics from the ground.

The right-hand panel of Fig. 9 predicts that not much evolution is expected in the size ratios between different passbands. Thus, as a test of how realistic the size ratios of our models are at high redshift, we can compare them to observations of size ratios at low redshift. Fig. 9 shows that at redshift $z = 0$, the sizes in the $B$ band should be larger than those in the $V$, $R$ and $H$ bands by 0.037, 0.061 and 0.145 dex, respectively. This is consistent with the observations of nearby spiral galaxies by MacArthur, Courteau & Holtzman (2003), who found differences of 0.029, 0.061 and 0.127 dex. While this does not prove that our model predicts the correct size ratios for $z = 2$ galaxies, it does provide indirect support.

Another prediction of the model is that sizes in molecular gas should be only slightly larger than sizes in Hα or UV light. This is a consequence of the almost linear relation between star formation rate surface density and molecular gas surface density in our model (see fig. A1 of Dutton et al. 2010a). At higher molecular gas densities, the slope of the star formation law in our model is steeper than unity, with an asymptotic value of 1.4, in agreement with the standard Kennicutt–Schmidt relation (Kennicutt 1998). Since molecular gas discs are predicted to be of higher density at higher redshifts, this results in a slight evolution in the ratio of molecular gas sizes to star formation rate sizes in our model (right-hand panel of Fig. 9). The similarity between molecular gas sizes and UV sizes has already been observed for a handful of star-forming galaxies at redshift $z \sim 1$ (Daddi et al. 2010), which provides further support for our model.

3.5 The evolution of the Tully–Fisher relation in optical to NIR luminosities

The evolution of our model TF relations in optical to NIR luminosities is shown in Fig. 10 and in Table 4. Contrary to the stellar and baryonic TF relations, which show a decrease in mass at fixed circular velocity (solid and dotted lines), the luminosity TF relations show an increase in luminosity at fixed circular velocity between today and redshift $z \sim 1$. Luminosities in bluer passbands show stronger evolution than redder passbands. In the $B$ band, the evolution is 0.36 dex (or 0.9 mag) since $z = 1$, while in the $K$ band, the evolution is just 0.04 dex (or 0.1 mag) since $z = 1$. A similar theoretical prediction of 0.85 mag for the evolution of the $B$-band TF relation since $z = 1$ has been shown by Portinari & Sommer-Larsen (2007) using cosmological hydrodynamical simulations. Using an SAM, Firmani & Avila-Reese (2003) find differences between the evolution of the $B$-band and $H$-band TF relations which are qualitatively similar to what we find in our models, though in detail there are differences in the absolute evolution.

Most observations find no or weak evolution in the $J$- and $K$-band TF relations since $z \sim 1$ (Conselice et al. 2005; Flores et al. 2006; Weiner et al. 2006b; Fernández Lorenzo et al. 2010), which is in good agreement with our model (but see Puech et al. 2008) who
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Figure 10. Evolution of the TF relation using luminosities measured in rest-frame optical to NIR pass bands in our disc-galaxy SAM. At fixed circular velocity, luminosities increase out to redshift $z \lesssim 1.5$, with stronger evolution in bluer passbands than the NIR. For comparison, the evolution of the stellar and baryonic TF relations is shown with solid and dotted lines, respectively, which are the same as those in the left-hand panel of Fig. 7.

find 0.6 mag of brightening between $z \sim 0.6$ and $z = 0$. For the $B$-band TF relation, a wide range of evolution has been reported, but almost all studies find a brightening in $B$-band luminosities at higher redshifts (e.g. Vogt et al. 1996, 1997; Simard & Pritchet 1998; Ziegler et al. 2002; Böhm et al. 2004; Bamford et al. 2005; Bamford, Aragón-Salamanca & Milvang-Jensen 2006; Weiner et al. 2006b; Chiu, Bamford & Bunker 2007; Fernández Lorenzo et al. 2010). Most studies are consistent with a brightening of $1 \pm 0.5$ mag from $z = 0$ to $z \sim 1$, and thus in agreement with our model.

Finally, we note that no evolution in the $J$- or $K$-band TF relation does not imply no evolution in the stellar mass TF relation. In our model the stellar masses at fixed velocity decrease towards higher redshifts, but the light-to-mass ratios increase monotonically towards higher redshifts (because the stellar populations are progressively younger). The net effect is very weak evolution in $J$- and $K$-band TF relations between $z \sim 2$ and $z = 0$.

4 SUMMARY

We study the evolution in the zero-points of the relations between maximum rotation velocity, $V$, stellar mass, $M$, and rest-frame optical disc size, $R$, of disc galaxies in the context of ΛCDM-based galaxy formation models.

Using data from the DEEP2 survey to measure the evolution of the $RM$ relation since $z = 1.2$, together with published results from Conselice et al. (2005), Trujillo et al. (2006), Kassin et al. (2007) and the SINS survey (Cresci et al. 2009; Förster-Schreiber et al. 2009), we show that there is a consistent observational picture, with one exception, for the evolution of the $VMR$ relations from redshifts $z \lesssim 2$ to 0. The exception is that the $H\alpha$ exponential disc scalelengths of galaxies from the SINS survey measured by Cresci et al. (2009) appear to be a factor of $\pm 3$ higher at fixed $M$ than found by other observations. This apparent discrepancy can be traced to two factors.

First, Bouché et al. (2007) and Cresci et al. (2009) measure HWHM sizes and interpret these as exponential disc scalelengths. However, the $H\alpha$ half-light radii from Förster-Schreiber et al. (2009) of the same galaxies do not support this interpretation. Instead, they suggest that the HWHM overestimates the disc scalelength by a factor of $\pm 2$.

Secondly, using a ΛCDM-based disc-galaxy formation model (Dutton & van den Bosch 2009) we show that disc galaxies at redshifts $z = 0–3$ are expected to have half-mass radii of recently formed stars, $R_{\rm gh}$, a factor of $\pm 2$ higher than the half-mass radius of the total stellar mass, $R_{\rm stars}$. This is a direct consequence of the inside-out disc growth. In terms of optical half-light radii, our models predict that $R_{\rm gh} \simeq 1.4R_{\star} \simeq 1.25R_{\ell}$ with little dependence on redshift.

Additionally, since the SINS survey is not a volume-limited sample of galaxies at high redshifts, and at least some of the SINS sample was selected on the basis of discy morphology with spatially resolved velocity gradients, another possibility (which we do not invoke here) is that a selection bias against small galaxies exists in the SINS survey. In order to rule out this possibility, the size-mass relation of SINS galaxies needs to be compared to that of a volume-limited control sample, with the same methods used to derive sizes and masses.

We further show that the observed evolution of the $VMR$ relations is consistent with a simple ΛCDM-based model of discs growing inside evolving NFW DM haloes. This model adopts a constant disc-to-halo mass fraction of $m_d = 0.04$ and a median spin parameter of $\lambda = 0.035$, independent of redshift. The galaxy mass fraction is consistent with observations at low redshifts (Hoekstra et al. 2005; Dutton et al. 2010b), and the spin parameter is consistent with expectations from cosmological simulations (e.g. Bullock et al. 2001b; Macciò et al. 2007). While our model is certainly an over-simplification of disc-galaxy evolution, it demonstrates that there is no need to invoke abnormally high spin parameters to explain

| $z$ | $\log_{10}(1+z)$ | $\Delta \log_{10} L_\beta/V_{\text{max}}$ | $\Delta \log_{10} L_\gamma/V_{\text{max}}$ | $\Delta \log_{10} L_I/V_{\text{max}}$ | $\Delta \log_{10} L_J/V_{\text{max}}$ | $\Delta \log_{10} L_K/V_{\text{max}}$ |
|-----|----------------|---------------------------------|---------------------------------|---------------------------------|---------------------------------|---------------------------------|
| 0.00 | 0.000          | 0.000                           | 0.000                           | 0.000                           | 0.000                           | 0.000                           |
| 0.10 | 0.041          | 0.123                           | 0.068                           | 0.022                           | 0.001                           | −0.005                          |
| 0.30 | 0.114          | 0.236                           | 0.151                           | 0.076                           | 0.034                           | 0.021                           |
| 0.51 | 0.179          | 0.296                           | 0.200                           | 0.107                           | 0.051                           | 0.034                           |
| 0.70 | 0.230          | 0.329                           | 0.225                           | 0.123                           | 0.060                           | 0.038                           |
| 1.01 | 0.303          | 0.357                           | 0.250                           | 0.137                           | 0.060                           | 0.036                           |
| 1.41 | 0.382          | 0.374                           | 0.263                           | 0.139                           | 0.048                           | 0.018                           |
| 2.00 | 0.477          | 0.353                           | 0.234                           | 0.094                           | −0.010                          | −0.051                          |
| 2.50 | 0.544          | 0.319                           | 0.191                           | 0.043                           | −0.070                          | −0.118                          |
| 3.00 | 0.602          | 0.279                           | 0.143                           | −0.016                          | −0.132                          | −0.186                          |
| 4.00 | 0.699          | 0.192                           | 0.047                           | −0.123                          | −0.245                          | −0.311                          |
the scaling relations of star-forming disc galaxies at high redshifts ($z \simeq 2$) as has been claimed by Bouché et al. (2007) and Burkert et al. (2009).

The weak evolution of the galaxy scaling relations since $z \sim 1$ in our model is due to the fact that individual galaxies grow roughly along the scaling relations, and not due to weak evolution in the properties of individual galaxies themselves. Similar conclusions for the stellar mass–velocity relation have been reached by Portinari & Sommer-Larsen (2007) using cosmological simulations and for the size–stellar mass relation by Firmani & Avila-Reese (2009) using an SAM similar to that used here. For example, for a galaxy with a present-day stellar mass of $3 \times 10^{10} M_{\odot}$, since $z = 1$, its stellar mass has increased by a factor of $\simeq 2.5$, its half-light size has increased by a factor of $\simeq 2.0$ and its maximum circular velocity has increased by a factor of $\simeq 1.15$. In terms of samples of galaxies, the evolution is just a factor of $\simeq 1.5$ increase in stellar mass, at fixed $V_{\text{max}}$, and a factor of $\simeq 1.3$ increase in disc size at fixed stellar mass, since $z = 1$. Evolution in stellar mass by a factor of $\simeq 2.5$ for galaxies with present-day stellar masses of $3 \times 10^{10} M_{\odot}$ is also consistent with the evolution of the star formation rate–stellar mass relation since $z = 1$ (Noeske et al. 2007b).

In our models, the evolution of the stellar scaling relations is \textit{stronger} than that of the baryonic scaling relations (maximum circular velocity, baryonic mass and baryonic half-mass size). This is due to a combination of the inside-out nature of stellar disc growth in $\Lambda$CDM cosmologies, coupled to a decrease in cold gas fractions with cosmic time.

In our models, the evolution of the baryonic scaling relations is \textit{weaker} than that of the virial scaling relations of DM haloes, assuming a constant galaxy mass fraction. For example, the baryonic TF and baryonic size–velocity relations evolve by just $\simeq 0.11 \text{dex}$ from $z = 1$ to 0, whereas the corresponding relations between halo virial quantities evolve by $\simeq 0.33 \text{dex}$ from $z = 1$ to 0. This difference can be understood as a consequence of the ratio of maximum circular velocity to virial circular velocity, $V_{\text{max}}/V_{\text{vir}}$, increasing towards lower redshifts. This in turn is largely a consequence of the increase in halo concentrations with time (e.g. Bullock et al. 2001a).

While we have shown that there is a consistent observational and theoretical picture for the evolution of the $V/M$ relations out to redshift $z \sim 2$, there is much room for progress. Our theoretical model makes a number of simplifying assumptions which are unlikely to be correct in detail. We assume that the galaxy mass fraction and galaxy spin parameters do not evolve with time. While this is a reasonable assumption to start with, we note that this is not a natural outcome of our model when we include cooling and outflows. Our model also assumes that galaxy discs are smooth and 100 per cent supported by rotation. This assumption is valid in the local universe, and perhaps up to redshift $z \sim 1$, but may break down at higher ($z \gtrsim 2$) redshifts, especially for lower mass galaxies.

Current observational samples at $z \gtrsim 1$ are small and/or subject to measurement uncertainties and selection biases. For the $RM$ relation, it will be possible, in the near future, to measure robust rest-frame optical sizes at $z \lesssim 2.5$ using large NIR surveys with $HST$/Wide Field Camera 3. Measuring maximum circular velocities is currently a challenge at high redshifts, as it is limited to ground-based NIR spectroscopy. These observations are typically seeing limited, which complicates the measurement of maximum rotation velocities. However, coming generations of adaptive-optics-assisted ground-based telescopes will open up the field in concert with the Atacama Large Millimeter/submillimeter Array (ALMA) and the Square Kilometer Array (SKA). With this suite of frontier instrumentation, it will be possible to measure resolved rotation curves and gas density profiles in molecular and atomic gas out to high redshifts. Coupled to observations of stellar masses and sizes, these observations will enable the evolution of the baryonic $VMR$ scaling relations over a large fraction of cosmic time to be measured. These scaling relations will provide a complete set of observational constraints with which to test models of disc-galaxy formation.

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