Expansion of Bubbles in Inflationary Universe

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Abstract

We show that particle production during the expansion of bubbles of true vacuum in the sea of false vacuum is possible and calculate the resulting rate. As a result the nucleated bubbles cannot expand due to the transfer of false vacuum energy to the created particles inside the bubbles. Therefore all the inflationary models dealing with the nucleation and expansion of the bubbles (including extended inflation) may not be viable.

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1 Introduction

The idea of inflation ([1], [2]) solves many problems of old cosmology such as horizon problem. The main idea in inflationary models is the rapid expansion of the early universe ($e^{50}$ times the initial value) in a very short time. (In fact inflationary models are not the only candidate that can solve the horizon problem. There is a recent claim that this problem can be solved as a by-product of attributing a variable dimension to the universe [3].) The original inflationary scenario [1] was that the universe started from a de Sitter space-time and bubbles formed randomly in the sea of the de Sitter space-time through tunnelling of the inflaton field. The bubbles expand with the speed of light after their nucleation. The bubbles expand because the energy of the false vacuum is transferred to the kinetic energy of their wall [4]. The bubbles, however, never percolate in the old inflationary model (graceful exit problem).

Among other surviving models of inflation is Extended Inflation [5]. In the extended inflationary model the problem of percolation of bubbles is solved by substituting scalar-tensor gravity theories (such as extended Brans-Dicke actions) instead of Einstein Hilbert action. The bubbles nucleate with a variable rate so that their number will be enough to fill the false vacuum and the graceful exit problem of the original inflationary model is solved.

So far it has been assumed that the bubbles of true vacuum can expand in the false vacuum up to the time when they meet one another and mix into one. This is the way the inflation is ended and the universe ends up in true vacuum.

Figuring out the exact space time in the interior of the bubble is a formidable task. For the case when the space-time is homogenous but anisotropic it has been shown that the anisotropy suppresses bubble nucleation rate [6] but, so far, there is no discussion about the interior of the bubbles. For the case of isotropic models, it is usually assumed that the space-time inside the bubble’s wall is Minkowski while that of the outside is de Sitter or de Sitter
like (as is the case for extended inflation) [5].

An expanding bubble causes an observer which was located in the space-time of false vacuum finds himself in the space-time of true one after the bubble’s wall expands and passes him. The very change of space-time should cause particle production and therefore some part of the energy of the false vacuum transfers to them. As a result the energy of the false vacuum will be divided between two parts one part is the kinetic energy of the bubble’s wall and the other the produced particles. If the transfer of energy to the created particles is high enough (more than the kinetic energy of the bubble’s wall) the bubble won’t expand and therefore it may stop growing. As a result the universe will be full of bubbles some of them stationary and some other expanding slowly in de Sitter space-time. The problem to guess the fate of these bubble is not known yet. The production of particles during vacuum tunnelling in Minkowski space-time has been widely studied [7], [8] and [9], however the quantum situation when bubbles expand is not known and is the main subject of the present work.

Here we investigate the quantum fluctuations of vacuum during the expansion of bubble in an homogenous and isotropic universe. In what follows we will bring, after stating the idea of this work, the calculations of the amount of particles created during the bubble expansion. Here we take the space-time inside and outside the bubbles as Minkowski and de Sitter respectively. This rate is caculated in the approximation when the bubble is large enough to attribute zero temperature to the inside and also for large conformal time to be able to define the vacuum state. Making these assumptions is important in order to make our quantum field theory method valid. The exact evaluation of the matrix elements needs the knowledge of the metric all over the space-time. The metod we use for matching the metrics inside and outside the bubble follows [10] and will be published later [11].

An improved method of calculating the rate at the early times of bubble nucleation will be given in [11]. Our calculation may also be generalized to domain wall and any other thin walls when they expand [12].
2 Particle production due to the expansion of bubbles

Here we consider the evolution of bubbles with false or true vacuum inside. The general view of the problem is that due to the expansion of the bubbles the space-time changes with time. As a result we expect to have particle production during bubble expansion. This has crucial effects on the inflationary universe models involving bubble formation and expansion e.g. old and extended inflation. In what follows we calculate the rate of energy loss by bubble expansion.

Assume two different space-times, denoted by the metrics $g^{(1)}_{\mu\nu}$ and $g^{(2)}_{\mu\nu}$, separated by a thin wall. For a massive scalar fields in the background of each of these space-times, the general Lagrangians are

$$L^{(i)} = \frac{1}{2} \left[ -g^{(i)}_{\mu\nu} \phi^{(i)}_{,\mu} \phi^{(i)}_{,\nu} - \left[ m^{(i)}^2 + \xi R^{(i)} \right] \phi^{(i)2} \right]$$

where $i$ is 1 or 2, $m^{(i)}$ is the mass of the field $\phi^{(i)}$, $R^{(i)}$ is the scalar curvature of the space-time $(i)$ and $\xi$ is a numerical factor.

The field equation for the above lagrangian is

$$(\Box^{(i)} + m^{(i)2} + \xi R^{(i)})\phi^{(i)} = 0$$

$\xi = 0$ is the choice for the minimal coupling and $\xi = \frac{1}{4} \frac{(n-2)}{(n-1)} \equiv \xi(n)$, where $n$ is the space-time dimension, is the conformal coupling when the action for massless field is invariant under conformal transformation.

The metric around the bubble’s wall can be written as

$$ds^2 = C^2(\bar{\eta})(-d\bar{\eta}^2 + d\bar{r}^2 + \bar{r}^2(d\Omega^2))$$

$$ds^2 = -d\eta^2 + dr^2 + r^2 d\Omega^2$$

where $\bar{s}$ refers to the metric outside the bubble and for the de Sitter space-time we have $C(\eta) = \frac{1}{H\eta}$ with $H$ a constant. For small values of the conformal
time $\bar{\eta}$ it is nontrivial to have a vacuum state outside the bubble. However assuming $\bar{\eta}$ to be sufficiently large we can have an adiabatic vacuum

$$\lim_{\bar{\eta} \to \infty} \frac{\dot{C}(\bar{\eta})}{C(\bar{\eta})} \to 0$$

(2.5)

Therefore a vacuum state can be defined asymptotically for outside region.

The state inside is not a real vacuum state, but rather a thermal state \[9\], however it is Lorentz invariant \[9\]. Here we assume a large bubble so that the temperature inside the bubble be low enough to be able to assume a vacuum state inside.

Now we can decompose a field in each of these space times in terms of creation and annihilation operators $a_k$ and $a_k^+$

$$\phi^{(i)}(x) = \sum_k [a_k^{(i)} u_k^{(i)} + a_k^{(i)+} u_k^{(i)*}]$$

(2.6)

The solution for $u_k^{(i)}$ is given by \[13\]

$$u_k^{(1)} = \frac{1}{\sqrt{2\pi}} e^{ikx} \sqrt{\pi} H_{\nu}^{(2)}(k\bar{\eta})$$

(2.7)

where $H$ is a Hankel function with $\nu^2 = \frac{9}{4} - 12(m^2 R^{-1} + \xi)$ and

$$u_k^{(2)} = \frac{1}{\sqrt{2\pi}} e^{ikx} e^{i\omega_k \eta}$$

(2.8)

where (1) and (2) refer to outside and inside the bubble, respectively.

Now we write the solution over all space time as follows

$$\phi = \phi^{(2)} \theta(R_b - r) + \phi^{(1)} \theta(r - R_b)$$

(2.9)

where $\phi^{(i)}$ is the quantum field inside (for $i = 2$) or outside (for $i = 1$) of the bubble.

The coefficient $\beta$ in the Bogolubov transformation is then the inner product of the mode solution
\[
\beta_{k'k} = -(u^{(1)}_{k'}, u^{(2)}_{k}) = i \int_{\eta_0}^{\eta} u^{(1)}_{k'}(\bar{x}(x)) \bar{\eta} u^{(2)*}_{k}(x) d^3x \tag{2.10}
\]

Note that \(\bar{x}\) refers to the coordinate outside the bubble.

In fact the arguments in the inner product are in different coordinates and with the methods (e.g. \([10]\)) of matching of the metrics the relation between the two coordinates can be found \([11]\).

The total number of particles per mode will be

\[
N_k = \Sigma_{k'} |\beta_{k'k}|^2 \tag{2.11}
\]

The local observer previously in de Sitter space-time, now finds itself in Minkowski, where he sees particle production by the rate predicted by (2.11).

From (2.10) it is clear that when \(u^{(1)}_{k} = u^{(2)}_{k}\), the coefficient is zero. Therefore in our approximation i.e. \(\bar{\eta} \to \infty\), the coefficient, \(\beta\), is very small. However as we go back in time the discrepancy between \(u^{(1)}_{k}\) and \(u^{(2)}_{k}\) gets larger and therefore the rate of particle production inside the bubble gets amplified. As a result the bubble may not have expanded to what we have considered as an approximation.

3 Conclusion

The particle production rate due to the sudden change of vacuum in time caused by the expansion of bubbles that separate two different space times has been calculated. Some part of the energy of the false vacuum then transfers to create particles, rather than accelerating the wall and therefore the bubble will slow down expanding. Therefore the inflationary models concerning the production and expansion of bubbles may have serious graceful exit problem, that the de Sitter universe will be filled with static, or slowly growing, bubbles of true vacuum inside. In fact the state inside these static bubble is a hot thermal one partly induced by the particle production.
Here we made the assumption of big bubble and large conformal time in order to be able to define vacuum states inside and outside of the bubble. With these assumptions we observed that the expansion of the wall of the bubble causes particle creation although very small. However, this result suggests that if we go back in time the quantum effect will be amplified and presumably the bubble could not have expanded to what we have considered as an approximation, that is the bubbles would have stopped growing at the early times after their nucleation. In fact in the recent work [14], it has been shown that the particle production is enhanced in a thermal state, which corresponds to small bubbles or at small conformal time in our model, and justifies our suggestion.

So this work shows that the inflationary models concerning the creation and expansion of bubbles have the difficulty explaining the graceful exit problem.

The method explained in this work may also be applied to many other situations, when domain wall or bubbles are concerned (such as [12]).

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