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Bearing capacity of shell strip footing on reinforced sand

W.R. Azzam *, A.M. Nasr

Department of Structural Engineering, Faculty of Engineering, Tanta University, Tanta, Egypt

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ABSTRACT

In this paper, the ultimate load capacities of shell foundations on unreinforced and reinforced sand were determined by laboratory model tests. A series of loading tests were carried out on model shell footing with and without single layer of reinforcement. The tests were done for shell foundation at different shell embedment depth and subgrade density. The results were compared with those for flat foundations without reinforcement. The model test results were verified using finite element analysis using program PLAXIS. The experimental studies indicated that, the ultimate load capacity of shell footing on reinforced subgrade is higher than those on unreinforced cases and the load settlement curves were significantly modified. The shell foundation over reinforced subgrade can be considered a good method to increase the effective depth of the foundation and decrease the resulting settlement. Also the rupture surface of shell reinforced system was significantly deeper than both normal footing and shell footing without reinforcement. The numerical analysis helps in understanding the deformation behavior of the studied systems and identifies the failure surface of reinforced shell footing.

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Introduction

Shell foundation has been considered the best shallow foundation for transferring heavy load to weak soils, where a conventional shallow foundation undergoes excessive settlement due to its economic advantage in area having high material-to-labor cost ratio. Kurian [1] and Fareed and Dawoud [2]. The conical shell raft foundation, which is a combined foundation, is suitable for water tanks and tower like structures. Concept of shell is not new in foundation design, considering construction in past with inverted brick arch foundation in this category. The use of inverted brick arches as foundation has been in practice in many parts of the world for a long time. Shells are essentially thin structures, thus structurally more efficient than flat structures. This is an advantage in situation involving heavy super structural loads to be transmitted to weaker soils. Shell footing is limited to a few geometries, such as conical, pyramidal, hyper and spherical footings. The structural performance of the shell foundation with respect to membrane stresses, bending moment, shear, deflection and ultimate strength of the shell itself was investigated in a wide range as stated by Paliwal and Rai [3], Paliwal and Sinha [4] and Melerski [5]. However, the geotechnical behavior of shell foundation to determine the soil response with respect to settlement, bearing capacity, contact pressure distribution...
and deformation within the soil mass has taken little attention. The experimental and numerical investigations that carried out to determine the geotechnical behavior of shell foundation were restricted. Abdel-Rahman [6], Hanna and Abdel-Rahman [7] reported experimental results on conical shell footings on sand for plain strain condition. Maharaj [8], Huat and Mohamed [9] and Kentaro et al. [10] conducted a finite element and experimental analysis for shell foundation to study the effects of increasing soil modulus in addition to investigating the geotechnical behavior of shell foundation. Most papers in literature were only studied the behavior of a variety of shell foundation on unreinforced sand without considering the existence of a reinforced element below this type. All works were done only on flat foundation placed on single or multi layers of reinforcement as discussed by many investigators as Latha and Somwanshi [11] and Patra et al. [12], except for Shaligram [13], who studied the behavior of shell triangular footing on reinforced layered sand. His investigation presents a surface study which explained only the effect of such technique on the bearing capacity without identification the stress and deformation of the adopted system. Consequently, in this research a new approach was adopted to study the geotechnical behavior of strip shell foundation resting on single layer of reinforcement to validate the reinforcement effect in the conjunction of adopting shell foundation. The present study was done using both experimental and numerical analysis to confirm the model test results and identify the deformation characteristics of the studied system.

**Experimental**

**Testing tank**

Fig. 1a shows a schematic view of the experimental model steel apparatus used in this research. The test box, having inside dimensions of 90 × 30 cm in plane and 120 cm in depth, the walls thickness of the tank is 6 mm. The tank box was built sufficiently rigid to maintain plane strain conditions by minimizing the out of plane displacement in all directions. The tank walls were braced from the outer surface using horizontal steel beam fitted at the mid depth of the tank. The inside walls of the tank are polished smooth to reduce friction with the soil. The inside walls were braced from the outer surface using horizontal steel beam fitted at the mid depth of the tank. The inside walls of the tank are polished smooth to reduce friction with the soil as much as possible by using galvanized coat in the inside wall.

The loading system consists of a hand-operated hydraulic jack and pre-calibrated load ring to apply the load manually to the footing soil system and the settlement was measured by dial gauges fixed at footing surface.

**Foundation models**

The strip shell footing models were made of steel plates with constant width \( B = 150 \text{ mm} \) in horizontal projection, with different embedment depth, \( a \) (60, 75 and 112.50 mm) and 20 mm thickness. The transverse footing length is 29 cm to satisfy the plain strain condition. Sketches of the foundation models are illustrated in Fig. 1b. A rough base condition was achieved by fixing thin layer of sand onto the base of the model footing with Epoxy glue. The load is transferred to the footing through a steel loading arm which was fixed rigidly by welding at the mid of foundation models as shown in the relevant Fig. 1b.

**Testing materials**

The sand used in this study is medium to coarse silica sand. A homogenous bed of dry silica sand was formed. The mean grain size \( D_{50} = 0.33 \text{ mm} \) and the uniformity coefficient is 3.5. The physical properties of tested sand are: Specific gravity was determined using gas jar method and it was found to be 2.65; the maximum and minimum dry density were obtained using Japanese method and were found to be 17.96 and 15.6 kN/m², respectively.

To prepare the compacted sand bed, the Japanese method [14] was adopted, using manual compactor. The sand depths were kept constant during the tests. Three test series were carried out on loose, medium and very dense sand. The unit weight of sand and thus the required relative density was controlled by pouring a pre-determined weight of sand into the testing tank, to fill each layer, and then the sand surface was leveled and compacted. A loose sand deposit was achieved by a placement soil layers 50 mm thickness in zero fall height. In order to obtain a compacted sand structure, the sand is placed in layers, each layer has 50 mm thickness and compacted using manual compactor 35 N. The numbers of compaction passes are pre-evaluated for each layer at the beginning of the program to achieve the required sand density. For medium and dense case, the falling height is 40 cm and 90 cm respectively. The relative density achieved during the tests was monitored and evaluated by collecting samples in small cans of known volume placed at different arbitrary locations in the test tank. The relative densities during the testing program were found to be 50%, 72% and 83%. The corresponding angles of shear resistance are 31°, 36° and 41°, respectively, which were obtained by applying a series of direct shear box tests at the corresponding relative density under different normal stresses.

In order to prepare the soil core under the shell model, the space under the shell was filled with sand according to the required unit weight as stated by Hanna and Abdel-Rahman [7]. The sand filling process of a shell model was done by placing a thin steel plate on the bottom of the shell model before placing it on its location. The steel plate was then slowly pulled out horizontally underneath the shell from the side.

The reinforcement adopted in the present research was a heat bonded nonwoven geotextile (Typar-3857) manufactured from polypropylene multifilament fibers. According to the manufacturer’s data, it has a nominal thickness of 2 mm and mass per unit area of 290 g/m². The wide-width tensile strength from the strip test method is 20.1 kN/m and the elongation at maximum load is 10%.

**Experimental testing program**

A total number of 34 tests were conducted on prearranged foundation models using three different sand densities and under different embedment depth \( a/B \). A series of loading tests were done for the foundation on both unreinforced and reinforced sand subgrade using Geotextile that was placed at fixed distance equal to 0.5B below the foundation tip with constant length equal to 4B as stated by Androwes [15], Abdel-Baki and Raymond [16] and Abu-Farsakh et al. [17]. In all testing program both plate sides of the shell foundations were embedded in the sand.
The increase in the ultimate load of a shell footing as compared to its flat counterpart is recognized in the present study as the shell efficiency factor ($\eta$). It is defined as given in Eq. (1), as the ratio between the differences in ultimate loads of shell footings over the ultimate load of flat footing.

$$\eta = \frac{Q_u - Q_{uf}}{Q_{uf}} \quad (1)$$

where $\eta$: shell efficiency; $Q_u$: ultimate load of shell footing; $Q_{uf}$: ultimate load of flat footing.

In order to examine the settlement characteristics of shell footings versus that of the conventional flat one, a non-dimensional settlement factor ($F_\delta$) was introduced. The settlement factor was calculated at the ultimate load ($Q_u$) to reflect the settlement characteristics of the footings throughout the loading process. The settlement factor is presented in Eq. (2). It should be noted that a lower value of settlement factor indicates better settlement characteristics.

$$F_\delta = \frac{\delta_u \gamma A_b}{Q_u} \quad (2)$$

where $\delta_u$: settlement at ultimate load; $\gamma$: soil unit weight; $A_b$: area of footing in horizontal projection; $Q_u$: ultimate load.

**Results and discussion**

*The load settlement curves of shell footing with and without reinforcement*

The load settlement data are summarized for given tests due to space limitation, and some of the results are presented in Fig. 2.
It presents the load settlement curves for flat and shell footing with and without reinforcement at different sand density. It has been found that the load settlement curves were significantly modified as the subgrade density increased. The existence of shell footing can improve and increase the ultimate load compared with flat footing. It can be seen that the ultimate load increases due to both shell and reinforcement effects as illustrated in the relevant figure, at shell embedment depth \((a/B = 0.5)\). It can also be observed from this figure that, the ultimate load increases with the increase in the angle of shearing resistance, also the shell footings have higher ultimate loads than flat one. The existence of the reinforcement below the shell footing can significantly improve and increase the ultimate bearing capacity of shell footing. The load carrying capacity of shell footing over reinforced subgrade is higher than the shell footing without reinforcement; this indicates that the reinforcement has a considerable effect in increasing the footing load capacity with the increase in shell embedment depth. Shell footing ensures better enclosibility of the shell inside the space of the footing by preventing the soil from flowing outward. Also, the soil wedge inside the shell footing is gradually compacted during the loading stages; thus, the subgrade soil is improved and the settlement is decreased. This can be very significant, particularly when the density of the soil is poor/low.

The bearing capacity of shell footing on loose sand was increased compared with flat footing on the same soil. On the other hand, the reinforcement can cause an additional improvement with the shell, where the soil wedge between the shell and the soil above reinforcement was effectively interlocked and the subgrade densification was achieved. This is backed to the reinforcement which controls and decreases the vertical deformation and a progressive densification is induced. It can be seen that a combined effect was induced which is represented in the shell effect and the reinforcement effect. Hence, both the soil inside the shell wedge and the soil over the reinforced layer became stiffer, one unit and effectively interlocked. As a result, the footing load capacity increased and the settlement decreased.

The degree of the improvement in the ultimate bearing capacity of the system depends on the ratio \((a/B)\) and soil density or shear angle. These results were in agreement with Hanna and Adel-Rahman [7].

The effect of shell embedment depth and reinforcement on the ultimate load capacity

In order to study the effect of both the shell embedment depth and the reinforcement on the ultimate foundation capacity, the relation between the angles of shearing resistance against ultimate load was plotted in Fig. 3, at different shell embedment depth for both shell footing with and without reinforcement. It is noticed that the increase in embedment depth increases the ultimate load capacity of the shell footing compared with flat footing. Because the increase in embedment depth leads to an effective increase in foundation depth and confined zone, thus the ultimate bearing capacity increases. As the shear angle of the subgrade increases, the footing load capacity also increases. The present reinforced layer below the toe of the shell reduces the pressure induced within the subgrade and increases the ultimate load capacity as shown in the relevant Fig. 3, for different reinforced cases. The combined effect of such reinforcement can substantially reduce the distortion rate in the sheared zone and limit the induced tensile strains which were produced at failure. As well, this Figure again justifying that the reinforcement can distinctly improve the subgrade capacity due to the combined effect that resulted (shell and reinforcement effect).

The relationship between the ultimate load \((Q_u)\) and the shear angle of the subgrade \((\phi)\) for shell footing with and without reinforcement can be expressed in the following nonlinear relationship which is based on regression analysis:

\[
Q_u = C_1 e^{C_2 \tan \phi}
\]

where \(C_1\) and \(C_2\) are factors related to the ratio \((a/B)\) and the existence of the reinforcement layer. The values of the factors \(C_1\) and \(C_2\) at different cases were extracted from Fig. 3, and
plotted against ratio \((a/B)\) for shell footing with and without reinforcement layer as shown in Fig. 4. It has been found that increasing the shell embedment depth can increase the values of the factor \(C_1\) for both shell footing with and without reinforcement. However, the values of the factor \(C_1\) of the reinforced cases are higher than the shell footing without reinforcement (Fig. 4a). This can also confirm the effect of the reinforcement in increasing the ultimate load capacity of the shell footing on reinforced sand.

On the other hand, it has been found that a sharp decrease in the factor \(C_2\) was achieved for unreinforced shell footing when the embedment ratio \(a/B\) increased from 0.5 to 0.75 (Fig. 4b). The values of the factor \(C_2\) of the reinforced case is higher than that of the unreinforced shell footing but there is a trivial difference between reinforced and unreinforced case. It was also found that the factors \(C_1\) and \(C_2\) depend on the initial density of the subgrade especially angle of internal friction. This equation may be used as a rough guide to determine the ultimate capacity of shell footing under the studied conditions. It can be seen that, based on the above equation, the ultimate theoretical values are nearly equal to the ultimate laboratory values. Because the difference between the obtained values is insignificant, this equation fairly expressed the measured values of \(Q_u\) in the laboratory test once the factor \(C_1\), \(C_2\) and the angle of shear resistance are known.

**Effect of shell and reinforcement on the footing efficiency**

Fig. 5 presents the calculated shell efficiency factors \(\eta\) which were deduced in the present experimental investigation. In general, it can be concluded that shell efficiency increases with the increase in the shell embedment depth \((a/B)\). It can be seen that, the effect of shell configuration diminishes when the soil becomes denser. Moreover, the shell efficiency factor reduces remarkably when the soil is denser. This opinion is similar to the opinion stated by Hanna and Adel-Rahman [18]. The shell efficiency increases remarkably in the tests conducted on reinforced subgrade as compared to shell footing without reinforcement.

The shell efficiency factors also decrease with the increase in the angle of shearing resistance as confirmed in Fig. 6. From this Figure, the variation of shell efficiency \(\eta\) with shear angle \(\phi\) at different shell embedment depth is presented. It is noticed that there is a sharp decrease in the shell efficiency when the shear angle increases and the values of the shell efficiency increase with the increase in shell embedment depth. It has been found that, increasing the subgrade density reduces significantly the shell efficiency factor for both reinforced and unreinforced shell footing. It can be concluded that at higher subgrade density, the improvement range is small compared with loose and medium relative density. This is because of increasing of the degree of improvement in loose condition due to shell effect and better improvement due to existence of the reinforced layer.

**Effect of shell configuration and reinforcement on settlement characteristics**

In this part, an attempt was made to study the effect of shell foundation as well as the existence of the reinforced layer on...
the resulting settlement at failure. The calculated settlement factor ($F_s$) which was deduced from the present experimental investigation at different studied parameters is plotted in Fig. 7. In general, for any footing, the settlement factor decreases for denser sand. The comparison between shell and flat footings for any given sand state indicates that the shell footings possess a lower settlement factor which demonstrates better settlement characteristics for shell footings. The comparison between the shell footing without reinforcement and with reinforcement shows that the settlement factor decreases remarkably for the shell footing with reinforcement. Also the settlement factors are affected by the shell embedment depth. The increase in the shell embedment depth ($a/B$) obviously decreases the settlement of the shell footing soil system in both reinforced and unreinforced conditions. But the reduction in settlement for reinforced shell footing is higher than that of unreinforced cases. It has been found that for low relative density and at embedment depth ($a/B = 0.75$ reinforced condition), the improvement in the settlement factor reach 50% of its initial value of flat footing, while this value is 26% for shell footing without reinforcement. On the other hand, at a dense state, these values reaches 55% for reinforced shell footing at ($a/B = 0.75$) and 31% for unreinforced shell footing. This again confirmed the effectiveness of the reinforced layer in controlling the vertical settlement of the shell footing due to resulting combined effect.

**Bearing capacity failure mechanism of the system**

The following analysis reports some useful comments about the failure of shell footing soil system with and without single reinforced layer. Fig. 8 shows experimentally and theoretically the failure modes for shell footing with and without reinforcement. Generally, in case of normal flat footing located at medium and dense state, it can be seen that the general shear failure is a well-defined pattern, which consists of a continuous failure surface that develops from one edge of the footing to the ground surface. The mechanism of soil collapse of normal flat footing on reinforced layer placed at a given depth below the footing was extensively investigated by Yahmamoto and Kusuda [19] and Michalowski and Shi [20]. Their study proved that the failure was induced and formed directly below the reinforcement. The reinforcement can contribute to increase the bearing capacity through significantly changing the geometry of the collapse pattern, preventing the mechanism from reaching deep into the soil. The reinforcement prevents the most adverse mechanisms from occurring, leading to an increase in the limit load. The main role of the inclusion is to reduce the distortion rate in the sheared zone and reduce the ultimate shear stress mobilized in the shear zone. The reinforcement provides an effective restraint and has a useful role in preventing the vertical soil spreading. As a result, the shear strength of subgrade is distinctly increased and the failure pattern is modified as stated by Michalowski and Shi [20].

Applying this terminology for tested shell footing on reinforced sand, it can be concluded that the presence of such reinforced layer beneath the shell footing causes a progressive densification to the confined subgrade and acted as an improved zone. The zone between the shell and reinforcement can be gradually densified during the loading stages and behaves as if embedded block or one unit (as mentioned by densified triangle or wedge as shown in Fig. 8a with imaginary footing width $B'$ according to load transfer mechanism). As a result, the soil shear failure takes place below the reinforced element due to higher deformation of reinforced layer at failure. The shell footing and the soil inside the shell which is located above the reinforcement can inhibit deep footing effect. This confirms that the shell footing and confined soil over reinforcement behaves like embedded foundation or stiff block and the soil failure is distributed directly below the reinforcement as confirmed by experimental results shown in Fig. 8b and c. This figure demonstrated that the shear failure planes are commenced and dissipated below the reinforced layer.

It is necessary to refer that, not only in the shape of foundations and soil density, but also in other governing factors above mentioned have an effect on the modification of the induced failure pattern. Such as, increasing the embedment depth can significantly increase the effective stress over the reinforcement as a result the bearing capacity is increased and the failure mechanism is modified. Also the imaginary shell footing width at the face of reinforced layer can play an important role in the modification of the failure plane ($B'$). Increasing the shell width has increased the imaginary width hence the bearing capacity is increased. The failure surfaces or shear planes took place at the bottom of the reinforced layer (Fig. 8c). This figure shows load transfer mechanism and the concentration of stress that is mostly located under the reinforcement.

The finite element analysis confirms and shows the modification of the failure pattern of tested shell footing. On the other hand, for the shell footing with and without reinforcement, the ruptured surface is modified as shown in Fig. 8a, b and c, and the bearing capacity failure takes place at the toe of the shell. The wedge of the failure surface of shell footing is deeper than that for flat footing due to the embedded effect. It can be concluded that using the shell foundation can be considered a good method to increase the effective foundation depth as clearly obtained in the conjunction figures. In the same manner, the reinforced layer below the toe of shell footing can also noticeably increase the effective foundation depth and the failure surface occurs directly below the reinforced layer. It is noticed that the wedge of rupture surface for the
shell footing with reinforcement is deeper than that for other systems. This is because the resulting soil wedge inside the shell and above the reinforcement is more than that obtained in shell footing without reinforcement. This also indicates that the shell footing with reinforcement has higher bearing capacity than other systems. While, at low relative density, the reinforced shell footing can significantly reduce the induced punching shear failure in the form of an elastic settlement compared with large settlement induced in case of flat footing.

**Numerical modeling**

The following part introduces the verification of the numerical analysis by the model test results. The results obtained from the model tests were verified by carrying out numerical studies by using the finite element method. The plain strain elasto-plastic finite element analysis was carried out using the commercial program PLAXIS [21]. This analysis aims to identify the failure pattern and stress behavior of reinforced shell system. It is also considered a good method for verifying the parameters that cannot be measured in the laboratory like the scale effect for using large scale shell footing.

The soil in this analysis was modeled by the Mohr–Coulomb failure criteria. Which is a simple and rather compatible and agrees with experimental testing results compared with other models. The plain strain condition and 6-node triangle elements were used for this analysis. Modulus of elasticity of soil at different sand density was obtained from triaxial tests. The shell footing element used in this investigation is a beam element which is considered being very stiff and rough (the interface strength $R_{int}$ was taken 0.67, sand steel interfaces). The material properties of the beam are an elastic normal stiffness $EA$ and bending stiffness $EI$. Whereas, $E$: modulus of elasticity of beam material used, $A$: cross section area and $I$: moment of inertia of the shell footing model. The reinforced layer of the adopted model was modeled as geotextile element which is defined by the axial horizontal stiffness $EA$ (kN/m) for the Geotextile material. The virtual interface element with Geotextile element was simulated before mesh generation. Positive and negative interface elements with virtual thickness are simulated in the program.

In all calculations described in this research, force control technique is considered, where point forces are concentrated, forces that act on a geometry point at the center of shell footings. Point forces are actually line loads in the out-of-plane direction. The input values of point forces are given in force per unit of length (for example kN/m). The value of applied point (load system A) is taken according to the obtained value from the model test divided by the footing width in plane.

The properties of the adopted sand which were simulated and defined in the program are ($\gamma = 18$ kN/m$^3$, $\eta = 0.3$, $E = 7500$ kPa, friction angle $\phi = 41^\circ$ and angle of dilatancy $= 11^\circ$). The shell foundation is simulated as an elastic beam element and defined at embedment ratio ($a/B = 0.75$). The main footing properties are (axial stiffness, $EA = 20.1$ kN/m and bending stiffness, $EI = 151,200$ kN/m$^2$).

**Verification of finite element analysis**

A comparison between the load displacement responses was calculated using the finite element analysis and the results obtained from the relevant model tests for shell footing with reinforcement and the flat one is shown in Fig. 9. The finite element calculations are moderately correct for computed values of the ultimate loads. The finite element results are close to those of laboratory test models and agree with the same trends.

The results of the finite element analysis confirm the experimental value. However, there is a little difference between the results from the finite element analysis and those obtained from the model test. This difference is due to plain strain condition and scale effect in addition to environmental conditions in the laboratory.

![Fig. 8](image-url) The modified failure pattern for shell footing without and with reinforced single reinforcement layer, $a/B = 0.50$. 

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Numerical results

The results of the finite element analysis and its output are shown in Fig. 10a–g for different foundation cases which are flat, shell without and with reinforcement. The total displacement vector obtained from the analysis is shown in Fig. 10(a–c), at corresponding ultimate footing capacity. It may be seen that the shell and reinforcement can significantly modify the deformation direction compared with flat cases (Fig. 10a), whereas the deformation and flow of the soil particles for flat footing occur mostly under the footing and there is a trivial upward deformation/heave along side of flat footing as clearly shown, and the existence of shell makes the soil heave significantly along each side of the shell (Fig. 10b). In addition, the reinforcement can confine and decrease the soil deformation as shown in Fig. 10c. Generally, the comparison between flat and shell foundations indicates that the rupture surface for the shell footing is deeper than that for strip flat type. This also confirms the failure pattern of the system as shown in Fig. 10 and agrees with Abd-Al-Rhman [6].

Furthermore, at failure, a progressive densification is induced. Hence, the soil wedge inside the shell that is located directly over the reinforced element behaves as one unit and settles at the same time as regarded in Fig. 10c. This shows that the displacement vectors are distributed directly below the reinforcement and extends to depth equal to 0.5B which verifies the occurrence of the embedded block.

On the other hand, the shear strains associated at failure are shown in Fig. 10(d–f), for different footing types. The distribution of the extreme shear strains is presented in shading area, where the red shading refers to maximum strains. It is noticed that for flat footing, the maximum strains or high sheared zones are found directly below the footing within the depth equal to B and distinctly reduced in both lower depth and horizontally at adjacent footing sides (Fig. 10d). While for the tested shell footing without reinforcement, the maximum strains (high sheared zones) occur at the edge of the shell footing and reduce in lower soil depth. It is also extended to distance equal to 2B as illustrated in Fig. 10e. This again confirms that the shell can significantly make the failure surface deeper than that flat footing, whereas the existence of reinforcement below the shell footing modifies the resulting extreme strains. The maximum shear strains are found only at the toe of the shell and are extended to a distance equal to 0.5B along shell sides as clearly shown in red shading Fig. 10f. This refers to the effectiveness of shell and reinforcement in modifying the distribution of strains. It also justifies the effect of reinforcement in modifying the failure plane. It is noticed that the soil shear failure takes place below the reinforcement directly under the shell footing block which acts as an embedded foundation. This foundation block settles simultaneously and it transfers the stress below the reinforcement as illustrated in Fig. 10f. It showed that the maximum shear strains are induced below of the reinforced soil block. In this manner, Fig. 10g, confirmed and justified the occurrence of soil shear failure at the bottom of reinforced element. Observing this figure, the plastic points and tension cut off are found mostly at the confined zone and extended to a depth below the reinforcement. This validates and confirms that the soil shear failure is modified and becomes different from shell footing without reinforcement. It also confirms the findings obtained and expected before as presented in Fig. 8.

In order to study the effect of shell foundation and existence of reinforcement, the values of the contact pressure under the shell foundation with and without reinforcement were extracted numerically from the program outputs at different subgrade density and embedment depth (a/B). These values were determined at depth equal to distance (a/2) below the center line of the shell and within the confined region by shell sides.

Generally, it can be observed that, the contact pressure at failure increases with the increase in the shell embedment depth as shown in Fig. 11. The increase in shell embedment depth provided more confinement for a denser sand state, as the angle of shear resistance increases the contact pressure at failure increases. The comparison between the shell footing with and without reinforcement indicates that the reinforcement possessed more confined pressure as shown in the relevant Figure, while the values of the contact pressure of the flat one at the same depth below the footing was smaller than that of shell cases.

Scale effect

As in all small-scale model tests, particularly in sand, scale effects need to be considered. There are several important factors that invalidate the use of small-scale models, which had been constructed in sand and tested at 1 g. The work described in this research was performed on small-scale 1 g physical models. For such small-scale models, the soil particle size, construction techniques, boundary conditions, soil–reinforcement interfacing features, reinforcement stiffness and the dilatancy at low stress are the important factors that are to be considered. Kusakabe [22], summarized test data and indicated that the impact of the particle size on the footing bearing capacity becomes less marked for a (Dso/B) ratio which is smaller than 1/100. Therefore, the effect of the particle size in this research should be smaller as the ratio Dso/B used in the model was 0.0092. According to Bransby and Smith [23], with smooth side walls and a relatively wide tank, side friction and boundary
Fig. 10  Responses of normal and shell foundation with and without reinforcement ($a/B = 0.75$ and $\phi = 41^\circ$).
conditions do not have any significant effect on the results of the reduced scale model. Hence, the inside walls of the container are polished smoothly to reduce any friction with the sand as much as possible. Furthermore, for neglecting the effect of the boundary conditions, the length of the tank was taken 6 times the footing width and the soil layer thickness 7 times the footing width [24,25]. Also, to provide proper rigidity to the model tank and prevent any lateral movement of the container walls, its sides and top were strengthened by fitting steel angles. The construction techniques used to build the model layout in the lab were similar to the field requirements.

The scale effect and the validation of using such reinforcement with small scale model shell footing were ensured and compared by the results of the laboratory model footing as presented before.

This part of study aims at investigating the scale effect of the adopted shell foundation on reinforced soils using finite element analysis as stated by DeMerchant et al. [26] and Chen and Abu-Farsakh [27]. The finite element model was first verified by the results of laboratory model footing tests as presented in Fig. 11 and then was used to numerically investigate the load–settlement response of different large shell footing sizes and embedment depth (a/B) on reinforced soil foundations. In this study, the adopted shell footing width is 2 m and the embedment ratio is varied and taken as mentioned in this research. The results of large scale model shell footings were compared with model tests in dimensionless manner. The improving in the ultimate load capacity of shell footing for both small and large footing was obtained and compared with flat footing. The load ratio for shell footing on reinforced sand were determined at different embedment depth (a/B). The load ratio can be obtained from the following expression (Lr = QultR/QultF), where QultR is the ultimate shell footing capacity on reinforced sand and QultF is the ultimate load capacity of flat footing without reinforcement. Fig. 12 shows the variation of the load ratio against embedment ratio for both model and analytical large scale shell footing at dense state. It was noticed that the numerical results of full-scale shell footing on reinforced sand were agreement with the model laboratory test result and has the same trend. But there is a little discrepancy in the results around 7%. As can be seen in this figure, the values of numerical analysis (full-scale) are close to those of laboratory test models, validating the results obtained in both studies. Of course, the small differences between the experimental (small model) and the numerical values (full-scale) are related to errors and environmental conditions in the laboratory. In addition to the variation of stress level which applied on the reinforced element in both model test and program, it can be concluded that the current model test results can validate the full-scale foundation as introduced by DeMerchant et al. [26] and Chen and Abu-Farsakh [27].

Conclusions

In the present paper, the geotechnical behavior of shell foundation with and without single layer of reinforcement was investigated experimentally and compared with flat footing. The following major conclusions are put in a quantitative form to the extent possible. Even though the values so given apply to the specific data used in the analysis, they can be considered indicative of the general trend of these results.

1. The soil wedge between the shell and the soil above reinforcement is effectively interlocked and the subgrade densification is achieved and as a result the footing load capacity increases and the settlement decreases.
2. The load carrying capacity of the shell footing on reinforced dense subgrade was found to increase by around 2.5 times of flat footing when the embedment depth ratio a/B increased from 0.40 to 0.50, and increased by 2.9 times when the embedment depth ratio increased from 0.5 to 0.75.
3. The improvement in the load carrying capacity of shell footing on reinforced loose subgrade was reached to 2.80 times of flat footing at embedment depth ratio of 0.75.
4. The increase in the angle of shear resistance of subgrade from $31^\circ$ to $41^\circ$ for reinforced shell footing reduce the settlement factor of flatted type by as much as 200% to 230% of flat footing at $a/B = 0.75$.

5. The settlement factor of shell footing on reinforced loose subgrade was reduced by 200% of flatted footing at embedment depth ratio of $a/B = 0.75$ and reduced by 230% for dense condition.

6. There is a sharp decrease in the shell efficiency when the shear angle decreases and the values of the shell efficiency increase with the increase in shell embedment depth.

7. The shell efficiency increases remarkably for the tests conducted on the shell footing on reinforced subgrade as compared with shell footing without reinforcement.

8. The existence of reinforced layer below the shell toe significantly modifies the bearing capacity failure. The wedge of rupture surface for the shell footing with reinforcement layer is deeper than those of flat and shell footing without reinforcement.

9. The finite element analysis was validated by the model test results and identifies the failure patterns for the shell footing with and without reinforcement.

10. It is recommended for future work to ensure the results on the large scale footing size in the field to draw general and comprehensive conclusions based on this manuscript.

Conflict of interest

The author declares that there is no conflict of interest.

Compliance with Ethics Requirements

This article does not contain any studies with human or animal subjects.

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