Modeling of loading and unloading processes

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Abstract. The article theoretically examines the choice of the most optimal of the possible trajectories of cargo in the process of moving goods from one place to another, placing them in the most optimal shape and size, taking into account the characteristics of the cargo. At the same time, using the method of mathematical modeling, the position of objects in space was determined using the functions of the Cartesian and polar coordinate systems. For optimal planning and management of the placement of goods, as well as transport operations along any spatial trajectory, the matrix method of mathematical modeling of these processes was used. Practically all possible cases of cargo movement were investigated and a mathematical expression was formed taking into account the geometry and kinematics of their placement in the given volumes. Based on the results of theoretical studies, three conclusions were drawn

1. Introduction
Improving the efficiency and cost-effectiveness of the process of moving goods from one place to another can be achieved through optimal planning, that is, selection of the most optimal of the possible transport trajectories of cargo and their placement in the most optimal shape and size, taking into account their characteristics. Such a problem can be solved using the method of mathematical modeling of these processes [1].

2. Materials and methods
It is known that the position of objects in space [1] is determined by the Cartesian coordinate system

\[(X, Y, Z) = 0\]  \hspace{1cm} (1)

and in the polar coordinate system by the functions

\[F (\rho, \alpha, \beta) = 0\]  \hspace{1cm} (2)

Moving objects in space in the form of different loads can be done with different trajectories, including in a straight line, in an arc whose radius of curvature is constant or variable (Figure.1).
The relationship between expressions (1) and (2) is defined for pic. 1 as follows:

\[
\begin{align*}
X_A &= \rho_A \cos \alpha \cos \beta \\
Y_A &= \rho_A \cos \alpha \sin \beta \\
Z_A &= \rho_A \sin \alpha
\end{align*}
\] (3)

The distance of movement of loads in space, for example, from \(A_i\) state to \(A_{i+1}\) state, can be determined by the following expression:

\[
d = \sqrt{(X_{A_{i+1}} - X_{A_i})^2 + (Y_{A_{i+1}} - Y_{A_i})^2 + (Z_{A_{i+1}} - Z_{A_i})^2}
\] (4)

If it is necessary to express this distance in polar coordinates, then the expressions (3) are changed as follows:

\[
\begin{align*}
X_{A_i} &= \rho_{A_i} \cos \alpha_{A_i} \cos \beta_{A_i} \\
Y_{A_i} &= \rho_{A_i} \cos \alpha_{A_i} \sin \beta_{A_i} \\
Z_{A_i} &= \rho_{A_i} \sin \alpha_{A_i}
\end{align*}
\] \(i = 1, 2, \ldots, m\)

\[
\begin{align*}
X_{A_{i+1}} &= \rho_{A_{i+1}} \cos \alpha_{A_{i+1}} \cos \beta_{A_{i+1}} \\
Y_{A_{i+1}} &= \rho_{A_{i+1}} \cos \alpha_{A_{i+1}} \sin \beta_{A_{i+1}} \\
Z_{A_{i+1}} &= \rho_{A_{i+1}} \sin \alpha_{A_{i+1}}
\end{align*}
\] \(i = 1, 2, \ldots, m\)

(5)

In this case,

\[
d = \sqrt{(\rho_{A_{i+1}} \cos \alpha_{A_{i+1}} \cos \beta_{A_{i+1}} - \rho_{A_i} \cos \alpha_{A_i} \cos \beta_{A_i})^2 + (\rho_{A_{i+1}} \cos \alpha_{A_{i+1}} \sin \beta_{A_{i+1}} - \rho_{A_i} \cos \alpha_{A_i} \sin \beta_{A_i})^2 +}
\]

\[
+ (\rho_{A_{i+1}} \sin \alpha_{A_{i+1}} - \rho_{A_i} \sin \alpha_{A_i})^2
\]

(6)

Mathematical modeling of these processes is necessary for optimal planning and management of load placement and transportation along any spatial trajectory. It is preferable to use the matrix method for this [2, 3, 4, 5].

Spatial matrix in the Cartesian coordinate system is defined by the expression \(A = \| a_{ik} \|\), here \(i\) stands for rows, \(j\) – columns, \(k\) – height. In this case 1 \(\leq i \leq m\), 1 \(\leq j \leq n\), 1 \(\leq k \leq v\) (Fig. 2), hence the dimensions of the matrix are represented by \(m \times n \times v\), and the cells – by \(a_{ijk}\).
For example, the load needs to move from the cell $a_{i,j,k}$ to the cell $a_{i_2,j_2,k_2}$. Naturally, the most rational way to do this is with a straight line defined by the following expression:

$$d = \sqrt{(i_2 - i_1)^2 + (j_2 - j_1)^2 + (k_2 - k_1)^2}$$

In this expression, $d$, $i_1$, $i_2$, $j_1$, $j_2$, $k_1$, $k_2$ are taken as dimensionless quantities, but can be easily converted to dimensional quantities when needed.

Depending on the skill of the operator operating the load-carrying device and the characteristics of the obstacles in the load path, the transport trajectory may be straight-line, curvilinear with a constant and variable radius of curvature, and broken-line. In these cases, expression (7) is written differently accordingly.

If the load is moved in the $YOX$ plane, $k = const$; if the transport plane is $ZOX$, $j = const$; if the load is moved in the $ZOY$ plane, then $i = const$.

The advantage of using a matrix system is that operations are performed with dimensionless indices $i$, $j$ and $k$. In addition, this system allows mathematical planning and programmatic control of the transfer of loads from any place to the desired location with an optimal trajectory.

Let’s consider the problem of modeling the properties and law of motion of loads.

For example, if the load $A$ is transferred from the starting point $A_1$ to the last point $A_2$ by a linear law, then we can write [6, 7, 8]:

$$X_A = \frac{X_1 + \lambda X_2}{1 + \lambda}, \quad Y_A = \frac{Y_1 + \lambda Y_2}{1 + \lambda}, \quad Z_A = \frac{Z_1 + \lambda Z_2}{1 + \lambda}$$

Here:

$$\lambda = \frac{A_1A}{AA_2} = \frac{A_1A}{A_1A_2 - A_1A}$$

It can be seen that $\lambda = \dot{\lambda}(t)$ – the law of motion of a load is expressed in terms of time $t$. If it is necessary to switch to a polar number system, then
\[ \rho_A = \sqrt{X_A^2 + Y_A^2 + Z_A^2} = \sqrt{(X_1 + \lambda X_2)^2 + (Y_1 + \lambda Y_2)^2 + (Z_1 + \lambda Z_2)^2} = \]
\[= \sqrt{\rho_1^2 + \rho_2^2 + 2\lambda(X_1 X_2 + Y_1 Y_2 + Z_1 Z_2)} \]
\[= \frac{\rho_1^2 + \rho_2^2 + 2\lambda(X_1 X_2 + Y_1 Y_2 + Z_1 Z_2)}{1 + \lambda} \]  \hfill (9)

here: \( \rho_1^2 = X_1^2 + Y_1^2 + Z_1^2 \); \( \rho_2^2 = X_2^2 + Y_2^2 + Z_2^2 \)

The law (5) of change of angles can be found from expression:
\[ \frac{Y_{A_1}}{X_{A_1}} = \tan \beta_{A_1}; \quad \frac{Y_{A_2}}{X_{A_2}} = \tan \beta_{A_2}; \quad \frac{Y_{A}}{X_{A}} = \tan \beta_{A} \]

then
\[ X_{A_1} + Y_{A_1}^2 = \rho_{A_1}^2 \cos^2 \alpha_{A_1} \]
\[ X_{A_2} + Y_{A_2}^2 = \rho_{A_2}^2 \cos^2 \alpha_{A_2} \]
\[ X_{A}^2 + Y_{A}^2 = \rho_{A}^2 \cos^2 \alpha_{A} \]

3. Results and discussion

Thus, modeling the position and movement of loads by the matrix method allows moving it along any spatial trajectory, depending on the nature of the obstacles in the load path and the design of the load-carrying vehicle. It will also be necessary to take into account the characteristics of the loads and the obstacles and constraints they may encounter in the planning and management of the placement of loads in different positions in space and their optimal movement along the desired trajectory. Objects (loads) transported in space can have different parameters in terms of mass, configuration and shape. Therefore, it is necessary to address the issue of taking into account their dimensions when moving loads to a certain capacity. In this regard, we try to mathematically describe the configuration of different bodies of volumetric nature [9, 10, 11].

We know from practice that bodies can have cubic, parallelepiped, prismatic, conical, cylindrical and other shapes. To accommodate such loads, we choose an ellipsoid as the most appropriate geometric figure (Fig. 3). This is due to the fact that by selecting one or another constant parameter of the ellipsoid, for example, \( a, b, c \), we can fully cover any configuration loads on it, including structural elements such as concrete columns, slabs, tanks of various shapes. It is known that the ellipsoid equation has the following form
\[ \frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1 \]  \hfill (10)

If it is necessary to move piles and columns of different shapes (Figure. 4), the ellipsoid enclosing them (see Figure. 3) will have an elongated shape at the expense of parameter \( a \). If the movable object is in the form of a cube (Figure. 5&6), such a shape can be placed in a sphere of radius \( r \), where \( a = b = c = r \). Thus, any object can be inserted into an ellipsoid. The question may arise, why are such actions necessary? The answer to this question is as follows: the object being moved has both a geometric and a center of gravity, and they may or may not be mutually compatible. In addition, it will be necessary to address the dynamics and kinematics of the movement of bodies, as well as the permeability and capacity of bodies between obstacles and constraints [12, 13, 14].
An ellipsoid that encloses them if necessary to move.

Figure 3.

Piles and columns of different shapes.

Figure 4.

If the object to be moved is in the shape of a cube.

Figure 5.

If the object to be moved is in the shape of a cube.

Therefore, we rewrite the ellipsoid equation in the following form:

\[
\frac{(X - X_A)^2}{a^2} + \frac{(Y - Y_A)^2}{b^2} + \frac{(Z - Z_A)^2}{c^2} = 1
\]  

(11)

Here we know from the above that

\[ X_A = X_A(\lambda), \quad Y_A = Y_A(\lambda), \quad Z_A = Z_A(\lambda) \]

Thus, we are now able to determine the spatial position of the load to be moved at any given moment depending on the time of movement. Furthermore, if the moving load is represented by an ellipsoid on the surface, then the load-bearing area (e.g., a rack cell) must also have an ellipsoidal shape, but the cell must cover the ellipsoid rather than the ellipsoid cell. In that case, by comparing the dimensions of the internal and external ellipsoids, we can model the spatial problems of load transport, that is, the rational use of space, that is, solve problems of an economic nature.

It should also be noted that depending on the nature and material of the loads to be transported, the load-carrying devices may be rotated at an angle of \( \theta \) to one side or the other around the \( OZ \) axis during transport [2]. In this case, for non-stationary trucks (Picture 7), we use the following transition equations [15,16]:

\[
\begin{align*}
\bar{X} &= (X - X_0) \cos \theta + (Y - Y_0) \sin \theta \\
\bar{Y} &= -(X - X_0) \sin \theta + (Y - Y_0) \cos \theta \\
\bar{Z} &= Z
\end{align*}
\]

or

\[
\begin{align*}
X &= X_0 + \bar{X} \cos \theta - \bar{Y} \sin \theta \\
Y &= Y_0 + \bar{X} \sin \theta - \bar{Y} \cos \theta \\
Z &= Z
\end{align*}
\]

(12)
Thus, considering almost all possible cases of movable loads, mathematical expressions of geometry and kinematics were formed in terms of placing them in a given volume.

4. Conclusion
- Mathematical models representing the location and trajectory of moving loads in space are obtained in Cartesian and polar coordinate systems.
- A matrix method has been proposed for mathematical modeling of optimal planning and control of load placement and movement along any spatial trajectory.
- Capacity models have been proposed that take into account their configuration for overcoming obstacles and constraints that may be encountered in the path of moving loads in space and for optimal placement.

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