Optically excited plasmaelastic waves in semiconductor plate-coupled plasma and elastic phenomena

D. M. Todorović ¹, S. Galović ², M. Popović ²

¹ Institute for Multidisciplinary Research, University of Belgrade, P.O.Box 33, 11030 Belgrade, Serbia,
² The "Vinča" Institute of Nuclear Sciences, P. O. Box 522, 11001 Belgrade, Serbia

E-mail: dmtodor@afrodita.rcub.bg.ac.rs

Abstract. A system of coupled plasma and elastic waves (the plasmaelastic waves) equations are analyzed. The treatment considers a semiconductor elastic plate with isotropic and homogeneous plasma and elastic properties. The solution of the coupled system of plasma and elastic equations are given for a typical photothermal configuration including the carrier surface and volume recombination processes. The analysis of the plasma density and elastic fields shows that the coupling plasmaelastic effects show the attenuation and disperse phenomena.

1. Introduction
Photoacoustic (PA) and photothermal (PT) science and technology have extensively developed new methods in investigation of semiconductors and microelectronic structures [1,2]. The PA and PT techniques were recently established as diagnostic methods with good sensitivity to the dynamics of photoexcited carriers [3]. Semiconducing materials and micro-electro-mechanical structures show a mechanical strain when electron-hole plasma is generated. The photoexcited carriers produce periodic elastic deformation in the sample - electronic deformation (ED) [4]. The ED mechanism is based on the fact that photogenerated plasma causes a deformation of the crystal lattice, i.e. it causes the deformation of the potential of the conduction and valence bands in the semiconductor. Thus, the photoexcited carriers may cause a local strain in the sample. This strain in turn may change the propagation of the plasma waves in the semiconductor, in a manner analogous to thermal wave generation by local periodic elastic deformation.

Photoexcitation and transport of carriers in semiconductor is conventional modeled as the carrier-diffusion wave, i.e. with the parabolic partial differential equations. In this work the diffusion – type carrier wave equations (parabolic type) were transformed in hyperbolic type. Then the governing field equations in this dynamic coupled theory are wave-type (hyperbolic) equations of carrier distribution and elastic displacements. The concept was introduced in analogy of generalized thermoelasticity proposed by Lord and Shulman in [5], in which, in comparison to the classical theory, a single relaxation time into consideration was introduced. This theory shows that the plasmaelastic disturbance propagates as a wave with a finite speed equal to or less than the speed $c$ defined by the relation time $t_o$. 

© 2010 IOP Publishing Ltd
Many authors analyzed the noncoupled system of plasma and elastic equations. The ED effect in semiconductors, i.e. partially coupled plasma and elastic waves, had been studied previously by Stearns and Kino [4] and Avanesyan, Gusev and Zheludov [6]. The thermoelastic and plasmaelastic effects in semiconductors were theoretically analyzed in recently published papers by Todorović [7,8]. In this work the system of coupled plasma and elastic waves, so-called the plasmaelastic (PE) wave [8] in an elastic semiconductor plate are analyzed.

2. Coupled plasma and elastic fields
The coupled plasma, \(n(r,t)\), and elastic displacement, \(u(r,t)\), transport equations in a vector form, for an elastic medium with isotropic and homogeneous electronic, thermal and elastic properties, are [7]:

\[
\frac{\partial}{\partial t} + \nabla \cdot \mathbf{u} = D_e \nabla^2 n(r,t) - \frac{n(r,t) - n_o}{\tau} - \gamma_r \left( \frac{\partial}{\partial t} + \frac{\partial^2}{\partial t^2} \right) \nabla u(r,t) + G(r,t) \tag{1}
\]

\[
\frac{\partial^2 \mathbf{u}(r,t)}{\partial t^2} = c_1^2 \nabla^2 \mathbf{u}(r,t) + (c_1^2 - c_2^2) \nabla \left( \nabla \cdot \mathbf{u}(r,t) \right) - \gamma_r \nabla n(r,t) \tag{2}
\]

where \(D_e\) is the carrier diffusivity, \(\tau\) is the photogenerated carrier lifetime, \(n_o\) is the equilibrium carrier concentration, \(\gamma_r\) is the elastic coupling factor, \(r\) is the position-vector, \(t\) is the time, \(\rho\) is the density, \(G(r,t)\) is the carrier photoexcitation 'source' term, \(c_1, c_2\) are the longitudinal and shear wave velocities in the medium, respectively, \(\gamma_N\), is plasma coupling factor (the difference of deformation potential of conduction and valence band). The phenomena described by the system of equations (1)-(2) can be indicated as a coupled dynamic generalized plasmaelastic (PE) problem. The first equation represents the hyperbolic plasma equation where it is assuming for the speed of carrier transport, \(c_E\), that identical to the longitudinal wave velocity/pulse speed \(c_1\). The plasma relaxation time \(t_o = D_E / c_1^2\), where \(D_E\) is the carrier diffusivity. The third term in the elastic equation (2) represents the 'source' term and describes the influence of the plasma waves on the elastic wave.

We have defined the dimensionless quantities,

\[
\gamma_r = \frac{n_o}{c_1^2}, \quad \gamma_r = \frac{\omega_r}{c_1}, \quad \omega_r = \frac{c_1^2}{D_E},
\]

\[
r_r = \frac{\omega_r}{c_1}, \quad t_r = \frac{\omega_r}{c_1}; \quad n_r = n/o_n, \quad n_r = n/o_n, \quad \epsilon_r = \epsilon_r / \epsilon_n, \quad \omega_r = \epsilon_r / \omega_r,
\]

where \(q\) is the wave number, \(\sigma_{ij}\) the component of stress and \(\omega_r\) is the characteristic frequency of the medium (a quantity characteristic of the plasmaelastic medium).

For temporally periodic and space uniform optical excitation at the surface \(z = 0\) of elastic plate, it is possible to take in consideration the one–dimension (1-D) case. Using \(r = (0,0,z)\), \(u = (0,0,w)\) and \(w = \partial \phi / \partial z\), then the system of equations (1)–(2) becomes

\[
\frac{\partial^2}{\partial z^2} - \frac{1}{\tau} N - \frac{\partial^2 \phi}{\partial z^2} = -G \tag{2}
\]

\[
- \gamma_r N + \left( \frac{\partial^2}{\partial z^2} + \xi^2 \right) \phi = 0 \tag{3}
\]

where \(N(z,t) = n(z,t) - n_o\) is the photogenerated carrier density, \(G(z,t) = G_o \exp(-\alpha z + i \xi t)\) is the volume plasma source, \(\alpha\) is a real parameter and \(\chi = i \xi - \xi^2\) is the complex parameter for hyperbolic type of carrier transport equation.

3. Plasmaelastic waves
The solution of the system of equations (3) – (4) can be given as the sum of homogeneous part and particular integral

\[
N(z) = N^o \exp(\xi z) + N'^o \exp(-\alpha z), \quad \phi(z) = \Phi^o \exp(\xi z) + \Phi'^o \exp(-\alpha z),
\]

(4)
where $N^o$, $\Phi^o$, $N^p$ and $\Phi^p$ are dimensionless carrier density and potential wave amplitudes. Generally, in relations (5) the values $q$ and $\zeta$ can be complex constants. If the value $\zeta$ is the real constant then we have the PE wave of fixed frequency. On the other hand, if the value $q$ is the real constant we have the PE wave of fixed wavelength. For our analysis the waves of fixed frequencies are taken in consideration.

Substituting the solutions (5) into equations (3) and (4) for the homogeneous system ($G = 0$) and eliminating the ratio $N^o / \Phi^o$, it can be obtained the characteristic equation. The roots of the characteristic equation are four complex numbers $q_m(\zeta)$:

$$q_m(\zeta) = \pm \left[ \sigma_m(\zeta) + i \frac{\zeta}{v_m(\zeta)} \right] , \quad m = 1,2;$$

where $\sigma_m(\zeta) = \text{Re}[q_m(\zeta)]$ is the attenuation coefficient and $v_m(\zeta) = \frac{\xi}{\text{Im}[q_m(\zeta)]}$ is the phase velocity of the PE wave.

The general solution for coupled dynamic PE problem for an elastic plate has the form:

$$N(z,t) = N_w^o \exp \left[ \sigma_1 z + i \xi \left( t + \frac{z}{v_1} \right) \right] + N_z^o \exp \left[ - \sigma_1 z + i \xi \left( t - \frac{z}{v_1} \right) \right] + N_w^p \exp \left[ \sigma_2 z + i \xi \left( t + \frac{z}{v_2} \right) \right] + N_z^p \exp \left[ - \sigma_2 z + i \xi \left( t - \frac{z}{v_2} \right) \right]$$

$$w(z,t) = w_w^o \exp \left[ \sigma_1 z + i \xi \left( t + \frac{z}{v_1} \right) \right] + w_z^o \exp \left[ - \sigma_1 z + i \xi \left( t - \frac{z}{v_1} \right) \right] + w_w^p \exp \left[ \sigma_2 z + i \xi \left( t + \frac{z}{v_2} \right) \right] + w_z^p \exp \left[ - \sigma_2 z + i \xi \left( t - \frac{z}{v_2} \right) \right]$$

where $N_m(\zeta)$ and $W_m(\zeta)$ are corresponding complex parameters and $N_w^o, w_w^o, N_z^o, w_z^o$ are the wave amplitudes. The wave amplitudes can be obtained from the appropriate boundary conditions at the sample interfaces $z = 0, L$ (the plasma boundary conditions including the surface recombination and mechanical boundary conditions are for stress-free plate surfaces). The solutions (7) and (8) define the PE waves with terms $\exp[ \sigma_i z + i \xi (t + z / v_i) ]$ which correspond to modified plasma wave $(j=1)$ and modified elastic waves $(j=2)$.

4. Analysis of the plasmaelastic waves

The plasma (the carrier-density) and elastic (the displacement) fields for uncoupled and coupled wave equations in a elastic plate were calculated and analyzed. Fig.1 and 2 show amplitude and phase of carrier-density field $N(z,f)$ vs frequency at the surface $z = 0$ for a thin elastic Si plate (200 µm thick), excited with uniform laser beam for the uncoupled case – the plasma wave and coupled plasma and elastic waves - the plasmaelastic waves. Fig.3 shows phase of elastic bending $w(z,f)$ vs frequency at the surface $z = 0$ for the uncoupled case – the elastic waves and coupled plasma and elastic waves - the plasmaelastic wave.

Comparison solution for coupled plasmaelastic problem with solution for uncoupled problem, it is possible to see that the plasma waves are much damped waves. Amplitude of these waves decrease with distance, while the phase velocity is the function of the frequency, i.e. the dispersion phenomena exists.

Conclusion

The solution of the coupled hyperbolic system of plasma and elastic equations were given for a semiconductor elastic plate including the carrier surface and volume recombination processes.

A quantitative analysis of the coupling plasmaelastic effects was given. The analysis showed, for the wave of fixed frequency, that the plasmaelastic phenomena can be modeled with the superposition of the modified plasma and elastic waves. The plasmaelastic waves show the attenuation and disperse phenomena. These investigations are important for many practical experimental situation (PA and PT...
experiments, atomic force microscopy, elastic microscopy, etc) and sensors and actuators based on the MEMS (NEMS), OEMS.

Figure 1. Amplitude of carrier-density $N(z, f)$ vs. frequency at the surface $z = 0$ for a thin Si plate excited with uniform laser beam: ( - - ) uncoupled case – the plasma waves; ( - ) coupled plasma and elastic waves - the plasmaelastic waves.

Figure 2. Phase of carrier-density $N(z, f)$ vs. frequency at the surface $z = 0$ for a thin Si plate excited with uniform laser beam: ( - - ) uncoupled case – the plasma waves; ( - ) coupled plasma and elastic waves - the plasmaelastic waves.
Figure 3. Phase of elastic vibration $w(z, f)$ vs frequency at the surface $z=0$ of a thin Si plate excited with uniform excitation: (- - -) uncoupled case – the plasma waves; (- -) coupled plasma and elastic waves - the plasmaselatic waves.

Acknowledgement
This work is supported by Ministry of Science and Technological Development of the Republic of Serbia (Projects TR11027, 141013 and I.T.1.04.0062.B).

References
[1] D.Almond, P.Patel, *Photothermal Science and Techniques*, Chapman&Hall, London, 1996.
[2] A.Mandelis and K.H.Michaelian, Eds, *Photoacoustic and Photothermal Science and Engineering*, Special Section of *Opt.Eng.*, 36(2) 1997.
[3] D.M.Todorovic, P.M.Nikolic, Ch. 9 in *Semiconductors and Electronic Materials* (A.Mandelis and P.Hess, Eds., SPIE Opt.Eng. Press, Bellingham, Washington, 2000), p. 273-318.
[4] R.G.Stearns, G.S.Kino, *Appl. Phys. Lett.*, 47, 1048-1050 (1985).
[5] H.W.Lord, H.W.Lord, Y. Shulman, *J. Mech.Phys.Solids*, 1967, 15, 229-300.
[6] S.M.Avanesyan, V.E.Gusev, N.I.Zheludov, *Appl. Phys. A*, 40, 163-166 (1986).
[7] D.M.Todorović, *Rev.Sci.Instrum.*, 2003, 74(1), 582-585.
[8] D.M.Todorovic, *J. Phys. IV France* 125 (2005) 551-555.