MULTI-CRITERIA SYNTHESIS OF FREQUENCY-MODULATED DISCRETE CONTROL OF SEMI-ACTIVE VEHICLE SUSPENSION - PART 1 ANALYSIS AND CONTROL STRATEGIES

J Genov
Technical University of Sofia, 8 Kl. Ohridski Blvd, Sofia 1000, Bulgaria
E-mail: j_genov@mail.bg

Abstract. The paper discusses the contradiction influence of system damping coefficient in the different excitation’s frequency diapasons on the raid comfort and the vehicles stability on road. It is analyzing the discrete “on-off” control laws for the semi-active suspension included in a model reduced to the vehicle wheel and accounting the vertical dynamics of the so-called “Quarter car model”. Based on analysis of the frequency characteristics it is propose a frequency modulation of the control realized by a signals filtration.

1. Introduction

The damping characteristics of the suspension of vehicles do not affect unambiguously on the quality of the vibroisolation. Due to the phase shifting, during certain time parts of the oscillation period; the damper directs energy into the object of vibroisolation instead of dissipating it. The amount of this induced energy depends on the damping coefficient and the frequency spectrum of the external excitation defining the phase shifting. Generally the vehicle’s comfort is associated with low values of the damping coefficient, while the stability of the vehicle with high values [1-7]. Results close to the optimal could be achieved by usage of semi-active damper which rheology changes through a control electromagnetic field.

Figure 1. Characteristic of a semi-active damper [8]
On Figure 1 is shown a sample characteristic of the damping force as a function of the deformation velocity, for different values of the control current.

The basic idea for the control of the semi-active damper is that when the energy is directed into the isolation object, to be switched to the lower value of damping coefficient, respectively, when the damper dissipates energy, to realize the maximum value of the damping. This must be realized under the condition for a compromise between ride comfort and road holding because when the control is targeted to the comfort criterion the stability makes worse and vice versa. In this sense and also in a case of presence of a poly-harmonic excitation, this “on-off” approach does not give the best possible results. Additionally, the sudden stiffening and softening of the suspension, leads to effects of internal impacts in the system.

2. Damping force analysis

On figure 2 is shown a one-dimensional linear suspension model for kinematic excitation. The shown dimensions are:

![One degree of freedom model](image)

**Figure 2.** One degree of freedom model

Parameters of the model are:

- $m$ - sprung mass;
- $k$ - coefficient of elasticity;
- $c$ - damping coefficient;
- $z(t), m$ - vertical displacement of the sprung mass considered from the static equilibrium position;
- $\xi(t)$ kinematic excitation from road roughness.

The Phase - frequency characteristic between vibration velocity $\dot{z}$ and damping force $F^d = c (\ddot{z} - \dot{z})$ is:

$$\Phi_{\dot{z}, F^d}(\omega) = \arctan\left(\frac{2n\omega}{\omega_n^2}\right) - \pi,$$  

(1)

where: $\omega_n$ – natural frequency, $\omega$ – kinematic excitation frequency, $n = c/(2m)$ – damping.

![Sprung mass velocity and deformation velocity of damper](image)

**Figure 3.** Sprung mass velocity and deformation velocity of damper.
On figure 3 is shown the result of the simulation $\dot{z}$ vs. $(\ddot{z} - \dot{z}) - F^d$, for linearly increasing frequency over time.

The time corresponding to the phase shift as function of the frequency is:

$$t_0(\omega) = \Phi_{\phi_F}(\omega) / \omega,$$

and percentage part of the oscillation period during which the damper force and the mass velocity are in the same direction is calculated by the dependence:

$$100\left(\frac{2t_0(\omega) - \tau}{\tau}\right) = 100\left(\frac{\Phi_{\phi_F}(\omega)}{\pi} - 1\right)\%,$$

where $\tau = 2\pi / \omega$ is the period of oscillation.

Graphics of the time intervals when the damper directs the energy to the mass, in percentage relation to the period of oscillation as functions of the frequency, and for different values of the damping, are shown in figure 4.

![Figure 4. Percentage relation of time interval of importing energy to the period of oscillation](image)

The analysis of the results shows that in the under-resonance and resonance frequencies the phase shifting between the force in the damper and the direction of the velocity of the mass is close to 180°, i.e. the damper predominantly dissipates the vibrational energy. In the after-resonance region the
relative duration of the intervals, in which the damper introduces energy, increases. In that as much
greater the damping ratio is, these intervals are more prolonged while at the same time a higher
amount of energy is introduced through them - figure 5.
In the “Quarter car” model figure 6 the influence of the damping coefficient is even more
complicated.

![Figure 6. Quarter car model](image)

**Figure 7.** Magnitude responses of a) displacement of the unsprung mass b) dynamic force between
the tire and road pavement c) displacement of the sprung mass d) acceleration of the sprung mass.
On figure 7a) and on 7b) are given families of magnitude response for the vibro-displacement of the unsprung mass – $z_u$ and the normal dynamic force between the tire and road pavement, $F_{u,dyn} = k_u (-z_u + \xi) + c_u (-\ddot{z}_u + \dddot{\xi})$, related with the vehicle stability, and on figure 7c) and 7d) the vibro-displacement - $z_s$ and acceleration $\ddot{z}_s$ of the sprung mass both related with the ride comfort.

It is obvious from upper graphics that in each frequency range, determined by the damping invariant frequencies, the influence of the damping is changed. Furthermore, the invariant frequencies are different for each of the magnitude characteristics.

3. Essence and laws of semi-active control system

In the semi-active dampers, the base silicon fluid contains ferrite particles. Under the influence of an electromagnetic field, they form colonial structures, which lead to considerable rise of the viscosity and respectively of the damping. In this way the damping may to varying in a wide range. A linearized model of this variation may be presented in the following form:

$$c = c^l + \left(c^u - c^l\right)U$$

where $c^l$ and $c^u$ are respectively the lower and upper limits of the variation of the damping coefficient obtained for values 0 and 1 of the control signal $U$.

![Figure 8. Magnitude responses of a) displacement of the unsprung mass b) dynamic force between the tire and road pavement c) displacement of the sprung mass d) acceleration of the sprung mass](image-url)
The classical approach for vibroisolation of the sprung mass is a control that sets maximum damping when the damping force is in the opposite direction to the velocity and minimal value otherwise (so called “sky hook” type control):

\[ c_s = c_s' + (c_s'' - c_s')u_s, \quad U_s = \begin{cases} 
1, & (\ddot{z}_s - \ddot{z}_a) \ddot{z}_s \leq 0 \\
0, & (\ddot{z}_s - \ddot{z}_a) \ddot{z}_s > 0
\end{cases} \tag{5} \]

The results of this control compared with those from the passive suspension are shown of figure 8. It is seen that this control law gives good results between the two invariant frequencies.

For another type sky hook control:

\[ c_s = c_s' + (c_s'' - c_s')u_s, \quad U_s = \begin{cases} 
1, & (\ddot{z}_s - \ddot{z}_a) \ddot{z}_s \leq 0 \\
0, & (\ddot{z}_s - \ddot{z}_a) \ddot{z}_s > 0
\end{cases} \tag{6} \]

the obtained results are given on figure 9.

**Figure 9.** Magnitude responses for control of the type (6)

Here, the effect is exactly the opposite of the control type (5).
From the result analysis follows that the combination of the two control systems leads to maximum results of vibroisolation of the sprung mass. This can be accomplished by using combination filters – low-pass filter for control system (5) and band-pass filter for (6):

\[ c_s = c_s' + (c_s'' - c_s')U_s' \]

\[ U_s = \begin{cases} 
0 & , [q > 0] \cap [p > 0], \quad q = F_s^d \ddot{z}_s, \quad p = F_s^d \dot{z}_s \\
\left[ \frac{z_s^{bp}}{z_s^{bp} + k_s^{bp}} \right] & , [q \leq 0] \cap [p > 0], \quad U_s^b = \left[ \frac{z_s^{bp}}{z_s^{bp} + k_s^{bp}} \right], \quad [q > 0] \cap [p \leq 0] \\
U_s^v + U_s^a & , [q \leq 0] \cap [p \leq 0] 
\end{cases} \]

(7)

The frequency characteristics of the low-pass and band-pass filter are shown on figure 10.

**Figure 10.** Frequency characteristics of modulating filters a) low-pas b) band-pass

The obtained results are shown on figure 11.

**Figure 11.** Magnitude responses for control of type (7)
This control realizes the best vibroisolation of the sprung mass, but the unsprung mass dynamics is increased. In order to reduce the dynamics of the unsprung mass, respectively the stability of the vehicle on the road, the following control is considered (as called “Ground hook” control type):

\[
c_s = c_s' + (c_s'' - c_s')U_u, \quad U_u = \begin{cases} 1, & (\dot{z}_u - \ddot{z}_u)\dot{z}_u \leq 0 \\ 0, & (\dot{z}_u - \ddot{z}_u)\dot{z}_u > 0 \end{cases}
\]  

(8)

The obtained results are shown on figure 12. Inefficiency of the control system is established between the 1st and 2nd invariant frequency for \(z_s\) and \(F_u\).

The results are shown on figure 12. The control similar to (6) due to the phase shift is not effective and it is consider the another form:

\[
c_s = c_s' + (c_s'' - c_s')U_u, \quad U_u = \begin{cases} 1, & (\dot{z}_u - \ddot{z}_u)\dot{z}_u \leq 0 \\ 0, & (\dot{z}_u - \ddot{z}_u)\dot{z}_u > 0 \end{cases}
\]  

(9)

The results are shown of figure 13. Those results are good between the invariant frequencies, but outside of them they are worse than for the control of type (8) and both control laws may by combined as same way as in the control (7).
Figure 13. Magnitude responses for control of type (9)

Acknowledgements: This work was supported by OP "Science and education for smart growth" – centers of excellence TU Sofia „National center of Mechatronics and clean technologies”, BG05M2OP001-1.001-0008-C01 Sect. Design, synthesis and testing of noise and vibration protection

References
[1] Dahlberg T., 1978, Ride Comfort and Road Holding of a 2-DOF Vehicle Travelling on a Randomly Profiled Road, *Journal of Sound and Vibration* 58 (2), pp.179-187
[2] Nikolov V. 2003 Simulation Model of Forced Oscillations of a Mechanical System with Four Degrees of Freedom *Mechanics of Machines* 2003 5 pp.126-129
[3] Bala Raju A., Shruti R., Venkatachalam R., 2014 Effect of Damping on Comfort Level of a *Int. Jour. of Innovative Research in Science, Engineering and Automation Problems* 2014 pp.53-58
[4] Bachev V., Nikolov V., Angelov I. 2014 A Study of the Natural Frequencies of the Free Undamped Related Oscillations of a Car *Int. Jour. Engineering and Automation Problems* 2014 pp.53-58
[5] Guiggiani M., 2014 *The Science of Vehicle Dynamics. Handling, Braking and Ride of Road and Race Cars*, Springer Dordrecht Heidelberg New York London, p.356
[6] Kralov I., Nedelchev K., 2016 Mass and Elasticity Synthesis of the Support of a Generator for Vibration Energy Harvesting, *Journal of the Balkan Tribological Association*, 22, 2016 №3 A-I, pp.3213-3227
[7] Kralov I., 2017 New Solution for Transport and Industrial Noise Protection through Reflective Noise Barriers, BulTrans2017, Sozopol, MATEC Web of Conferences 133, 06001(2017) p.5
[8] Lord Corporation 2006 *MR Damper RD-1005-3 Product Bulletin, MR Solutions Customer Service*