Neutron radii and neutron skin of neutron-rich nuclei deduced from proton-nucleus total reaction cross sections

I. A. M. Abdul-Mageed, Eman Hamza, Badawy Abu-Ibrahim

Department of Physics, Cairo University, Giza 12613, Egypt

A new method is proposed to deduce the neutron radii of neutron-rich nuclei. This method requires measuring the reaction cross sections of both the neutron-rich nucleus and its stable isotope at the same energy on a proton target. Using this method and the available experimental data of $^{12,22}$C + $p$ and $^{9,11}$Be + $p$ at 40 A MeV, the neutron radii of $^{22}$C and $^{14}$Be have been deduced for the first time.

PACS numbers: 25.60.Dz, 21.10.Gv, 25.60.-t, 21.60.-n

Neutron halo structure has been observed among the neutron-rich nuclei, it is manifested by the neutron density extending to extraordinary large radial distance. The neutron halo structure is indicated first by an increase in density extending to extraordinary large radial distance. The mass radii ($\bar{r}_m$) play an important role in nuclear physics and as-target [2] by the Glauber theory. Neutron radii ($\bar{r}_n$) of unstable nuclei are usually calculated by measuring their reaction (interaction) cross sections on Carbon target [3] or proton target [2] by the Glauber theory.

The root-mean-square point neutron radii (neutron radii) play an important role in nuclear physics and astrophysics. Neutron radii ($\bar{r}_n = (\bar{r}_n^2)^{1/2}$) are essential to extract the neutron skin thickness ($S_n = \bar{r}_n - \bar{r}_p$). The neutron skin thickness is strongly correlated to the radius of the low-mass neutron stars and to the density dependence of the symmetry energy [3, 4], (often encoded in a quantity denoted by $L$) which is related to the pressure of the pure neutron matter at the saturation density $\rho_s$. The origin of the size of both the thickness of the neutron skin and the neutron star is the pressure of the neutron-rich matter, pushing either against surface tension in an atomic nucleus or against gravity in a neutron star. The stellar radius and the neutron skin are sensitive to the same equation of state $\rho_s$. Recently, neutron star physics has received renewed interest since the LIGO-Virgo collaboration made the first direct detection of gravitational waves from the coalescence of a neutron star binary system [5]. The analysis of the data placed constraints on the tidal effects of the coalescing bodies, which were then translated to constraints on neutron star radii [6]. Also, the knowledge of the nuclear symmetry energy is relevant for the Standard Model tests via atomic parity violation [3].

Electron-nucleus scattering and measurement of muonic x-rays are used to determine the charge radii of stable nuclei. These techniques cannot be used for neutron-rich isotopes due to the weak beam intensities. High precision measurements of the charge radii ($\bar{r}_e = (\bar{r}_e^2)^{1/2}$) of He [3, 10], Li [11], Be [12, 13], and Mg [14] isotopes have been achieved using the isotope-shift technique. Combining the isotope-shift data with the data deduced from the reaction (interaction) cross section, one can obtain the proton radii of the neutron-rich isotopes from the relation $\bar{r}_p^2 = (A/N)\bar{r}_m^2 - (Z/N)\bar{r}_n^2$ and hence their neutron skin thickness [15].

Due to the limitation of the isotope-shift technique, mainly due to the low luminosity of rare isotopes close to the neutron drip line, the measurement of the charge-changing cross section ($\sigma_{cc}$) has been used to determine the proton radii of neutron-rich Carbon-isotopes [16]. The authors of Ref. [16] report that they scaled the experimental cross section to reproduce the known charge radius of $^{12}$C. They also combined the charge-changing cross section data with the reaction cross section measurements to obtain the neutron radii of $^{15,16}$C, and hence their neutron skins. Refs. [17, 18] used change-changing cross section to deduce the proton radii of $^{12-17}$B and $^{12-19}$C without any scaling factor.

The knowledge of the neutron radii of stable nuclei is scarce compared to that of proton radii, its main sources are parity-violation in electron scattering [12] and hadron scattering experiments, see for example [20]. Currently, these methods cannot be used for neutron-rich nuclei. In this letter, a new method is proposed in order to deduce the neutron radii of neutron-rich nuclei. The method requires only measuring the difference between the reaction cross sections of the studied neutron-rich nucleus and its stable isotope on proton targets at the same energy. Also, the efficiency of this method is demonstrated and it is applied to determine the neutron radii of $^{22}$C and $^{14}$Be.

The total reaction cross section of a nucleus ($A = p + n$) incident on a proton is expressed in the Optical Limit Approximation (OLA) of Glauber theory [21] as

$$\sigma^R_A = \int dB \left(1 - e^{-2\text{Im}[\chi_N(b)] + \chi_N([b])}\right),$$  \hspace{1cm} (1)  

where $b$ is the impact parameter vector perpendicular to the beam (z) direction, and $\chi(b)$ is the phase-shift function defined as $i\chi_N(b) = -\int d\rho_N(s)\Gamma_{PN}(s + b)$, where $\chi_N$ implies the phase shift due to the protons ($N = p$) or neutrons ($N = n$) inside the nucleus. The function
\( \rho_N(s) \) is the z-integrated density distribution of the protons (neutrons). The finite-range profile function, \( \Gamma_{pN} \), for \( pp \) and \( pn \) scatterings, is usually parameterized in the form \( \Gamma_{pN}(b) = \frac{1 - i \alpha_{pN}}{4 \beta_{pN}} \sigma_{pN}^{\text{tot}} e^{-b^2/(2 \beta_{pN})} \), where \( \alpha_{pN} \) is the ratio of the real to the imaginary part of the \( pp \) (\( pn \)) scattering amplitude in the forward direction, \( \sigma_{pN}^{\text{tot}} \) is the \( pp \) (\( pn \)) total cross section, and \( \beta_{pN} \) is the slope parameter of the \( pp \) (\( pn \)) elastic scattering differential cross section. For the zero-range approximation, it is parameterized in the form \( \Gamma_{pN}(b) = \frac{1 - i \alpha_{pN}}{2 \beta_{pN}} \sigma_{pN}^{\text{tot}} \delta(b) \). The validity of the OLA has already been tested with stable and unstable nuclei incident on a proton target \( ^{22}_{12}\text{Be} \) \cite{22,23}. The reaction cross section shift between a neutron-rich nucleus and its stable isotope is defined as \( \delta \sigma_{A_2,A_1}^R = \sigma_{A_2}^R - \sigma_{A_1}^R \). Both \( \sigma_{A_2}^R \) and \( \sigma_{A_1}^R \) are measured on a proton target and at the same energy. From Eq. (1), the reaction cross section shift is given by

\[
\delta \sigma_{A_2,A_1}^R = 2\pi \int b e^{-2i\Delta\chi_N(b)} \times \left( 1 - e^{-2i\Delta\chi_p(b) + \Delta\chi_N(b)} \right) \db, (2)
\]

where, \( \Delta\chi_N(b) = \chi_{N_2}(b) - \chi_{N_1}(b) \) and

\[
-2i\Delta\chi_N(b) = -\frac{\sigma_{pN}^{\text{tot}}}{2\pi\beta_{pN}} \times \int ds \left( \rho_{N_2}(s) - \rho_{N_1}(s) \right) e^{-(b+s)^2/(2\beta_{pN})}. (3)
\]

For the zero range approximation, it reduces to

\[
-2i\Delta\chi_N(b) = -\sigma_{pN}^{\text{tot}} \left( \rho_{N_2}(b) - \rho_{N_1}(b) \right). (4)
\]

It has been shown, in Ref. \cite{24}, that various nuclear density distributions having the same matter radii provide almost the same values of the \( p \)-nucleus reaction cross section in the energy region 100A-800A MeV. Furthermore, charge radii obtained from the isotope-shift technique show that the maximum difference between the measured charge radii of the isotopes of the same element for \(^4\text{He}, ^6\text{He}, ^8\text{He}, ^9\text{Li}, ^{11}\text{Li}, ^{12}\text{Be}\), and \(^{24}\text{Mg}\) is 0.392±0.019 fm (23% from that of \(^4\text{He}\)) \cite{3}, 0.3±0.065 fm (12% from that of \(^6\text{Li}\)) \cite{11}, 0.162±0.03 fm (6% from that of \(^9\text{Be}\)) \cite{12}, and 0.158±0.008 fm (5% from that of \(^{24}\text{Mg}\)) \cite{14}, respectively. Also, the proton radii obtained from the charge-changing cross section indicate that the maximum difference between the measured proton radii of the isotopes of the same element for \(^{12}\text{Be}, ^{12}\text{C}, ^{12}\text{B}, ^{12}\text{O}, ^{16}\text{O}, ^{16}\text{C}, ^{16}\text{Ne}, ^{24}\text{Ne}, ^{32}\text{Ne}, ^{32}\text{Mg}\) and \(^{32}\text{Mg}\) is 0.35±0.04 fm (15% from that of \(^{10}\text{B}\)) \cite{17}, 0.09±0.012 fm (4% from that of \(^{12}\text{C}\)) \cite{22}, and 0.12±0.08 fm (5% from that of \(^{12}\text{C}\)) \cite{18}, respectively. As one sees, the maximum difference between the proton radii for the isotopes of the same element is less than 10% except for \(^{16}\text{O}, ^{16}\text{C}\), and \(^{16}\text{Ne}\). It has been shown in Ref. \cite{18} that the effect of the center-of-mass motion of the halo on the proton radius becomes smaller with increasing mass number. The values of the charge radii, given in \cite{26}, show that the differences between the charge radii of the isotopes of the same element are small. Based on the above discussion, we assume that the contribution of \( \Delta\chi_p(b) \) to the value of the reaction cross section shift is negligible. This means that \( \Delta\chi_p(b) \simeq 0 \). Under this assumption, Eq. (2) reduces to

\[
\delta \sigma_{A_2,A_1}^R \simeq 2\pi \int b e^{-2i\Delta\chi_p(b) + \Delta\chi_N(b)} \times \left( 1 - e^{-2i\Delta\chi_N(b)} \right) \db. (5)
\]

Note that Eq. (5) does not depend on the proton density of the neutron-rich isotope. In Eq. (5), the neutron and proton density distributions of the stable isotope can be obtained from literature eg. \cite{27,28}, while the neutron density distribution of the neutron-rich isotope is the key quantity. To check the validity of the approximation in Eq. (5), the values of \( \delta \sigma_{A_2,A_1}^R \) calculated using Eq. (5) is compared to the exact values obtained from Eq. (2).

Thus, the factor FS is defined as

\[
FS = \frac{\delta \sigma_{A_2,A_1}^R [\text{Eq. (2)}]}{\delta \sigma_{A_2,A_1}^R [\text{Eq. (5)}]} (6)
\]

and its values are calculated for Be, B, C, O, Ne and Mg isotopes at different projectile energies. In the present calculations, the harmonic oscillator (HO) density \cite{20} has been used for Be and B with radii taken from \cite{14} and \cite{30}(SKM*), respectively. The densities of Carbon isotopes are taken from Ref. \cite{22}. For O, Ne and Mg isotopes, the Hartree-Fock densities have been used \cite{31}. The nucleon-nucleon interaction parameters are taken from Ref. \cite{22}.

A sample of the results is given in Fig. II showing the FS factor as a function of energy for \(^{11,12}\text{Be}, ^{15,17,19}\text{B}, ^{16,19,20}\text{C}, ^{21,23,24}\text{O}, ^{30,31,32}\text{Ne}\), and \(^{32,35,37}\text{Mg}\) where the reaction cross section shifts are calculated with respect to the stable nuclei \(^9\text{Be}, ^{10}\text{B}, ^{12}\text{C}, ^{16}\text{O}, ^{20}\text{Ne}\), and \(^{24}\text{Mg}\) respectively. As can be seen from the figure, the deviation between the values calculated using Eq. (5) and the exact ones calculated using Eq. (2) does not exceed 7% for all the considered isotopes in the considered energy range.

Figure II shows the FS factor as a function of the neutron number of all the considered isotopes at the energies 40A MeV and 800A MeV. Confirming that the deviation between the values calculated using Eq. (5) and the exact ones calculated using Eq. (2) does not exceed 7% irrespective of the neutron number.

In order to examine the effect of the increase of the proton radii of the neutron-rich nuclei on the validity of the approximation used in Eq. (5), a model analysis with HO density distribution for \(^{12,19}\text{C}\) has been performed. The configuration of the \(^{12}\text{C}\) wave function is assumed to be \((0s_1/2)^2 (0p_3/2)^4\) for both protons and neutrons. The HO length parameter is fixed in such a way to reproduce the proton radius, 2.33 fm, extracted
from the charge radius. Also, the neutron radius of $^{19}$C is fixed to be 3.37 fm\cite{22}, with the configuration $(0s_{1/2})^2(0p_{3/2})^4(0d_{5/2})^2(1s_{1/2})^1$. The HO length parameter is increased in order to increase the proton radius of $^{13}$C from 2.33 fm by 0.05 fm in each step. Figure 3 shows the relation between the FS factor and the difference in proton radii of $^{19}$C and $^{12}$C at different energy values. If the proton radius of $^{13}$C increases by almost 10%, 20%, 30% from that of $^{12}$C, the FS factor increases by almost 1% (6%), 3% (13%), 5% (20%), respectively at incident energy 40 MeV (800 MeV). From this figure, one notices that the approximation given in Eq. 3 is very effective at energies less than 300 MeV.

As a first application of the proposed method, the neutron radius of $^{22}$C is determined. The $^{22}$C nucleus is known as a two-neutron halo type. It has been studied in the three-body model of $^{20}$C+n+n predicting a binding energy in the range 0.122-0.480 MeV and a matter radius (neutron radius) 3.61-4.11 fm (3.96-4.58 fm)\cite{32}. Accurate experimental data on $^{22}$C binding energy is not yet available, so $^{22}$C is still attracting attention\cite{28}. The interaction cross section of $^{22}$C on a proton target at 40 A MeV was measured first by Tanaka et al.\cite{2}, in 2010, to be $\sigma_{22}^{R}[\text{Exp}]=1338 \pm 274$ mb, resulting in the rather large matter radius 5.4 ± 0.9 fm. On the other hand, Togano et al.\cite{24}, in 2016, measured the interaction cross section of $^{22}$C on a $^{12}$C target at almost 240 A MeV obtaining a matter radius 3.44 ± 0.08 fm. Nevertheless, the results of Ref. 2 have been used in the present calculations since the method requires a proton target. The available data of the reaction cross section of p+$^{12}$C at 40 MeV is for natural Carbon and has the value of 371±11 mb\cite{33}. Thus, the reaction cross section shift for $^{22}$C and $^{12}$C incident on a proton target at 40 A MeV is $\delta\sigma_{22}^{R}[\text{Exp}]=967\pm285$ mb.

Figure 4 shows the reaction cross section shift $\delta\sigma_{22}^{R}$ as a function of the neutron radius of $^{22}$C. The harmonic oscillator wave function is assumed for $^{22}$C, with the neutron configuration for the ground states being
(0s_{1/2})^2(0p_{3/2})^4(0p_{1/2})^2(0d_{5/2})^6(1s_{1/2})^2. From the figure, the neutron radius of $^{22}$C is found to be 5.2±0.95 fm. Using this value and assuming that the proton radius of $^{22}$C is 2.33 fm (same as $^{12}$C), the matter radius of $^{22}$C is found to be 4.59 fm which is consistent with that of Ref. [2]. Consequently, the neutron skin thickness of $^{22}$C is evaluated to be 2.87±0.95 fm. The authors of [23] tried to simultaneously reproduce the two measured cross sections of $^{22}$C given in [2] and [34], they concluded that the simultaneous reproduction of the two measured cross sections of $^{22}$C is not feasible.

As a second application, the neutron radius of $^{14}$Be will be determined. The $^{14}$Be nucleus is also known as a two-neutron halo type with the two-neutron separation energy 1.27±0.13 MeV [30]. The interaction cross sections of $^{14}$Be on Carbon and proton targets have been measured and the matter radius was deduced as 3.22±0.07 fm [37], 3.25±0.11 fm [38], and 3.1±0.15 fm [39]. Also, the charge-changing cross section of $^{14}$Be has been measured at 900A MeV on a Carbon target and the proton radius has been deduced to be 2.41±0.06 fm [40].

The reaction cross section of $^{14}$Be on a proton target at 41A MeV has been measured to be 712±14 mb [37]. On the other hand, the value of the reaction cross section of $^{9}$Be on a proton target at 39.7A MeV has been measured to be 398±12 mb [41]. Thus, the reaction cross section shift for $^{14}$Be and $^{9}$Be incident on a proton target at ∼40A MeV is $\delta\sigma_{14,9}^{R}$ [Exp]=314±26 mb. Assuming a HO wave function for the ground state of $^{14}$Be with neutron configuration (0s_{1/2})^2(0p_{3/2})^4(0p_{1/2})^2(1s_{1/2})^2, the result of the calculations is shown in Fig. 5. From the figure, the neutron radius of $^{14}$Be is found to be 3.36±0.11 fm, consistent with the results of Refs. [42, 43]. Assuming that the proton radius of $^{14}$Be is equal to that of $^{9}$Be, i.e. 2.36 fm [44], one gets the matter radius of $^{14}$Be the value 3.11 fm. On the other hand, using the proton radius of $^{14}$Be obtained from the charge-changing cross section, 2.41±0.06 fm, one gets the matter radius of $^{14}$Be to be 3.12 fm. Both agree well with the radii measured in Refs. [37-39]. The neutron skin thickness of $^{14}$Be is found to be (3.36±0.11-2.41±0.06) 0.95 ± 0.17 fm.

In summary, a new method has been introduced to determine the neutron radii of neutron-rich nuclei. This method requires measuring the difference between the reaction cross sections of the neutron-rich nucleus and that of its stable isotope on proton targets at the same energy. By applying this method, the neutron radii of $^{22}$C and $^{14}$Be have been determined. The obtained radii are consistent with the previously published values.

[1] I. Tanihata et al., Phys. Rev. Lett. 55, 2676 (1985).
[2] K. Tanaka et al., Phys. Rev. Lett. 104, 062701 (2010).
[3] F.J. Fattoyev, J. Piekarewicz, and C.J. Horowitz, Phys. Rev. Lett. 120, 172702 (2018).
[4] X. Roca-Maza, G. Colo, and H. Sagawa, Phys. Rev. Lett. 120, 202501 (2018).
[5] K. Iida, K. Oyamatsu, and B. Abu-Ibrahim, Phys. Lett. B 576, 273 (2003).
[6] B. P. Abbott et al. (LIGO Scientific Collaboration and Virgo Collaboration), Phys. Rev. Lett. 119, 161101 (2017).
[7] B. P. Abbott et al. (LIGO Scientific Collaboration and Virgo Collaboration), Phys. Rev. Lett. 121, 161101 (2018).
[8] T. Sil, et al., Phys. Rev. C 71, 045502 (2005).
[9] P. Mueller et al., Phys. Rev. Lett. 99, 252501 (2007).
[10] M. Brodeur et al., Phys. Rev. Lett. 108, 052504 (2012).
[11] R. Sanchez et al., Phys. Rev. Lett. 96, 033002 (2006).
[12] W. Nortershauser et al., Phys. Rev. Lett. 102, 062503 (2009).
[13] A. Krieger et al., Phys. Rev. Lett. 108, 142501 (2012).
[14] D. T. Yordanov et al., Phys. Rev. Lett. 108, 042504
(2012).
[15] T. Suzuki et al., Phys. Rev. Lett. 75, 3241 (1995).
[16] T. Yamaguchi et al., Phys. Rev. Lett. 107, 032502 (2011).
[17] A. Estrade et al., Phys. Rev. Lett. 113, 132501 (2014).
[18] R. Kanungo et al., Phys. Rev. Lett. 117, 102501 (2016).
[19] S. Abrahamyan et al. (PREX Collaboration), Phys. Rev. Lett. 108, 112502 (2012).
[20] B. C. Clark, L.I. Kerr and S. Hama, Phys. Rev.C 67, 054605 (2003).
[21] R.J. Glauber, in Lectures in Theoretical Physics (Interscience, New York, 1959), Vol. 1, p.315.
[22] B. Abu-Ibrahim, Phys. Rev.C, 77, 034607 (2008).
[23] T. Nagahisa, and W. Horiuchi, Phys. Rev. C,97, 054614 (2018).
[24] K. Kaki, Prog. Theor. Exp. Phys. 093D01 (2017).
[25] D. T. Tran et al, Phys. Rev. C 94, 064604 (2016).
[26] I. Angeli et al., Atomic data and nuclear data table 99, 69 (2013).
[27] H. De Vries, C. W. De Jager, and C. De Vries, Atomic data and nuclear data table 36, 495 (1987).
[28] J.D. Patterson and R.J. Peterson, Nucl. Phys. A717, 235 (2003).
[29] B. Abu-Ibrahim et al., J. Phys. Soc. Jpn. 78, 044201 (2009).
[30] E. Tel et al., Commun. Theor. Phys. 49, pp. 696 (2008).
[31] RIPL-3, Reference Input Parameter Library of IAEA, Nuclear Masses Segment available online http://www-nds.iaea.org
[32] W. Horiuchi et al., Phys. Rev. C,75, 044607 (2007).
[33] N. B. Shulgina et al., Phys. Rev. C,97, 064307 (2018).
[34] Toyano et al., Phys. Lett. B 761, 412 (2016).
[35] J.J. H. Menet et al., Phys. Rev. C 4, 1114 (1971).
[36] M. Wang et al., Chin. Phys. C 36, 1603 (2012).
[37] T. Moriguchi et al., Nucl. Phys. A 929, 83 (2014).
[38] S. Iliev et al., Nucl. Phys. A 875, 8 (2012).
[39] T. Suzuki et al., Nucl. Phys. A 658, 313 (1999).
[40] S. Terashima et al. Prog. Theor. Exp. Phys. 101D02 (2014).
[41] W. F. McGill et al., Phys. Rev. C 10, 2237 (1974).
[42] Y. Kanada-En’yo, Phys. Rev. C,91, 014315 (2015).
[43] G.A. Lalazissis, D. Vretenar, and P. Ring, Eur. Phys. J. A 22, 37 (2004).
[44] S. Ahmed et al., Phys. Rev. C, 96, 064602 (2017).