DENSITY CORRELATIONS OF MAGNETIC IMPURITIES AND DISORDER

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Abstract: We consider an electron coupled to a random distribution of point vortices in the plane (magnetic impurities). We analyze the effect of the magnetic impurities on the density of states of the test particle, when the magnetic impurities have a spatial probability distribution governed by Bose or Fermi statistic at a given temperature. Comparison is made with the Poisson distribution, showing that the zero temperature Fermi distribution corresponds to less disorder. A phase diagram describing isolated impurities versus Landau level oscillations is proposed.

IPNO/TH 96-03
June 1996

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1. Introduction

The problem of a 2 dimensional electron gas coupled to a static random magnetic field has been a subject of interest in the past few years [1]. Particular attention has been paid to localisation properties of such systems. In the case of Gaussian disorder with zero mean, all states seem to be localised [2]. On the contrary, they are delocalised in the case of an uniform magnetic field. Therefore, the question arises about the role played by a mean-field description of a random magnetic field [3,4].

The model we are interested in consists of a planar electron of electric charge $e$ coupled to a random magnetic field [4]. What we mean by random magnetic field is a distribution of infinitely thin vortices carrying a flux $\phi$, modelizing some sort of magnetic impurities, characterized by the dimensionless Aharonov-Bohm coupling $\alpha = e\phi/2\pi$ (i.e. $\phi$ in unit of the quantum of flux). This system is periodic in $\alpha$ with period 1 and since there is no privileged orientation of the plane, it is invariant by changing $\alpha$ into $-\alpha$, implying that $\alpha$ can be restricted to the interval $[0, 1/2]$.

In order to study the effect of statistics on the disordered magnetic impurity systems, one may evaluate perturbatively in $\alpha$ the average one electron partition function, i.e the Laplace transform of the average density of states.

In previous works [4], we focused on magnetic impurities obeying a Poisson distribution (which actually corresponds to the Bose case at zero temperature, or at infinite temperature). We observed in particular a transition for $\alpha_c \approx 0.35$ between an almost free density of states for an isolated impurities system ($\alpha > 0.35$) and an oscillating Landau like density of states ($\alpha < 0.35$).

In this letter, we will consider the random magnetic impurities system as a gas of particles of a given density $\rho$, with a distribution obeying Fermi or Bose statistics at a
temperature $T_v$. We will first properly define the perturbative expansion of the average partition function. Then, we will explicitly compute at order $\alpha^2$ contributions to the average partition function. We will argue that in the Fermi case, at zero temperature, the average density of states always displays Landau like oscillations, implying that there is no transition between an isolated impurity disordered phase and a mean magnetic field phase. This result is not unexpected, since Fermi statistics clearly tends to homogeneize the impurity configurations, leading to a less disordered situation.

2. The Model

2.1. General Formalism

Let us consider an electron coupled to a random magnetic field given by a distribution $\rho(\mathbf{r})$ of magnetic impurities. This means that $\rho(\mathbf{r})d\mathbf{r}$ is the number of impurities at position $\mathbf{r}$ in the infinitesimal volume $d^2\mathbf{r}$. The Hamiltonian is given by

$$H = \frac{1}{2m}\left(\mathbf{p} - \alpha \int d^2\mathbf{r}'\rho(\mathbf{r}')\frac{\mathbf{k} \times (\mathbf{r} - \mathbf{r}')}{|\mathbf{r} - \mathbf{r}'|^2}\right)^2 \pm \frac{\alpha m}{\mu} \rho(\mathbf{r})$$

where we have explicitly taken into account the coupling of the magnetic field to the spin-up (+) or down (−) degree of freedom of the electron endowed with a magnetic moment $\mu = -\frac{e}{2m}$ (thus an electron with a gyromagnetic factor $g=2$).

In the case of a discrete distribution $\rho(\mathbf{r}) = \sum_i \delta(\mathbf{r} - \mathbf{r}_i)$, where the index $i$ indices the impurities, the spin-term is a sum of contact terms. It corresponds to a choice of a peculiar self adjoint extension [5]: in the (+) case, the wave functions vanish at the location of the impurities (hard-core boundary condition), whereas in the (−) case singular wave functions are considered at the location of the impurities (attractive-core conditions). In order to extract the short distance behaviour of the wave functions, a non-unitary wave
function redefinition has been used [6]:

$$\psi_{N}^{\pm}(r) = \prod_{i=1}^{N} |r - r_{i}|^{\pm \alpha} \tilde{\psi}_{N}^{\pm}(r)$$  \hspace{1cm} (2)

The generalisation of this transformation to a continuous distribution \( \rho(r) \) is

$$\psi^{\pm}(r) = e^{\pm \alpha \int d^{2}r' \rho(r') \ln |r - r'|} \tilde{\psi}^{\pm}(r)$$  \hspace{1cm} (3)

The Hamiltonian \( \tilde{H}^{\pm} \) acting on \( \tilde{\psi}^{\pm}(r) \) rewrites

$$\tilde{H}^{+} = -\frac{2}{m} \partial_{z} \partial_{\bar{z}} - \frac{2\alpha}{m} \int dz' d\bar{z}' \frac{\rho(z', \bar{z}')}{{z'} - {z'}} \partial_{z}$$  \hspace{1cm} (4)

$$\tilde{H}^{-} = -\frac{2}{m} \partial_{z} \partial_{\bar{z}} + \frac{2\alpha}{m} \int dz' d\bar{z}' \frac{\rho(z', \bar{z}')}{{z'} - {z'}} \partial_{\bar{z}}$$  \hspace{1cm} (5)

where the complex coordinates in the plane have been used \( z = x + iy \), \( \partial_{z} = \frac{1}{2}(\partial_{x} - i\partial_{y}) \) and \( dzd\bar{z} = d^{2}r \). \( H \) or \( \tilde{H} \) can be used indifferently to compute the partition function, since it is by definition the trace of a function of \( H \). In the sequel we will concentrate on the spin up coupling, keeping in mind that the spin down analysis could be easily done following the same lines.

Up to now \( \rho(r) \) has not been specified. If a Poissonian distribution is choosen, \( \rho(r) \) is defined by its cumulents

$$\rho(r_{1})...\rho(r_{k}) = \rho \delta(r_{1} - r_{2})\delta(r_{2} - r_{3})...\delta(r_{k-1} - r_{k})$$  \hspace{1cm} (6)

Here however, we deal with quantum statistics for the impurities themselves, so \( \rho(r) \) has to be defined as

$$\rho(r) = \psi^{+}(r)\psi(r)$$  \hspace{1cm} (7)

with

$$\psi(r) = \frac{1}{2\pi} \int d^{2}k a(k)e^{-ikr}$$  \hspace{1cm} (8)
\[ \psi^+(\mathbf{r}) = \frac{1}{2\pi} \int d^2 k a^+(\mathbf{k}) e^{i \mathbf{kr}} \]  

(9)

\( a^+(\mathbf{k}) \) and \( a(\mathbf{k}) \) are the creation and annihilation Fock space particle operators, with the commutation rules

\[ [a(\mathbf{k}), a^+(\mathbf{k}')] = \delta(\mathbf{k} - \mathbf{k}') \]  

(10)

for bosons, and

\[ \{a(\mathbf{k}), a^+(\mathbf{k}')\} = \delta(\mathbf{k} - \mathbf{k}') \]  

(11)

for fermions.

Thus, the impurities, considered as a quantum gas, have a temperature \( T_v \) and a chemical potential \( \mu \), which determines their mean density \( \rho \). The average over disorder of an operator \( Q \) consists in

\[ < Q > = \frac{T_v [e^{-\beta_v H_v} Q]}{T_v [e^{-\beta_v H_v}]} \]  

(12)

where the impurity second quantized Hamiltonian

\[ H_v = \int d^2 k \left( \frac{k^2}{2m} - \mu \right) a^+(\mathbf{k})a(\mathbf{k}) \]  

(13)

describes the equilibrium of the impurity gas in the grand-canonical ensemble. Note that we consider here quenched impurities, which are in thermodynamical equilibrium. Note also that the Poissonian distribution \( dP(\mathbf{r}_i) = d\mathbf{r}_i/V \) can be seen as the particular case of Bose distribution at \( T_v = 0 \). The impurities indeed condensate in the zero energy \( N \)-body wave function \( \psi(\mathbf{r}_1, \mathbf{r}_2, \ldots, \mathbf{r}_N) = \left( \frac{1}{\sqrt{V}} \right)^N \), leading to the \( N \)-impurity Poissonian distribution \( dP = \psi^*(\mathbf{r}_1, \mathbf{r}_2, \ldots, \mathbf{r}_N)\psi(\mathbf{r}_1, \mathbf{r}_2, \ldots, \mathbf{r}_N) d\mathbf{r}_1 d\mathbf{r}_2 \ldots d\mathbf{r}_N \).

We wish to evaluate perturbatively the average one electron partition function at inverse temperature \( \beta \)

\[ < \mathbf{r}|e^{-\beta \hat{H}}|\mathbf{r} > = \sum_{p=0}^{\infty} \left( \frac{2\alpha}{m} \right)^p \int_0^\beta d\beta_1 \ldots \int_0^{\beta_{p-1}} d\beta_p G_{\beta - \beta_1}(\mathbf{r}, \mathbf{r}_1) \psi^+(\mathbf{r}_1') \psi(\mathbf{r}_1') \]
where integrations over the position variables $r_i$ and $r'_i$ are implicit. $G_{\beta}(r, r')$ is the free electron propagator

$$G_{\beta}(r_1, r_2) = \frac{m}{2\pi^2} e^{-\frac{m}{2\beta} |r_1 - r_2|^2}$$

Averaging over disorder yields expressions like

$$<\rho(r'_1)\ldots \rho(r'_p)> = \frac{Tr[e^{-\beta_v H_v} \psi^+(r'_1)\psi(r'_1)\ldots \psi^+(r'_p)\psi(r'_p)]}{Tr[e^{-\beta_v H_v}]}$$

which can be evaluated using the contractions

$$g_{\pm}^f(r, r') = \langle \psi(r)\psi^+(r') \rangle = \int \frac{d^2k}{4\pi^2} (1 \pm n_k) e^{i k (r - r')}$$

$$g_{\pm}^b(r, r') = \langle \psi^+(r)\psi(r') \rangle = \int \frac{d^2k}{4\pi^2} n_k e^{i k (r - r')}$$

$n_k$ stands for the Bose-Einstein (upper sign) or Fermi-Dirac (lower sign) distributions.

$$n_k = \frac{1}{e^{\beta_v (k^2/2m - \mu)} \mp 1}$$

One has the relation

$$g_{f}^\pm(r, r') = g_{b}^\pm(r, r') \pm \delta(r - r')$$

To summarize, the perturbative expansion of the one electron average partition function can be represented in terms of Feynmann diagrams given by rules quite analogous to those of finite temperature second quantised formalism:

electron line: $G_{\beta}(r_i, r_j)$
forward impurity line: $g_{f}^\pm(r_i', r_{\sigma(i)'})$
backward impurity line: $g_{b}^\pm(r_i', r_{\sigma(i)'})$
impurity loop: $g_{b}^\pm(r_i', r_i') = \rho$
electron-impurity vertex: $\frac{2\alpha}{m} \frac{1}{z_i - z'_i} \partial z_i$


For a given diagram of order $p$, the electron propagates from its initial to its final position $r$ via $r_p, r_{p-1}, ..., r_1$, the location of electron-impurity interaction, with temperatures $0, \beta_p, ..., \beta_1, \beta$. The $p$ vortices located at position $r'_1, ..., r'_p$, undergo a permutation $\sigma$. At each $\sigma(i)$, it corresponds a vortex line:

- backward line if $\sigma(i) > i$
- forward line if $\sigma(i) < i$
- and a loop if $\sigma(i) = i$.

In the Fermi case, each diagram is affected by the signature of the permutation $\sigma$.

The dimensionless parameters at work are the rescaled average density $\lambda^2 \rho$ in unit of the electron thermal wavelength $\lambda^2 = 2\pi\beta/m$, the rescaled average density $\lambda^2 v \rho$ in unit of the impurity thermal wavelength $\lambda^2 v = 2\pi\beta v/m$, and the Aharonov-Bohm coupling constant $\alpha$.

Clearly, one expects that in the limit $\beta_v \to 0$, i.e. Boltzmann statistics, with uncorrelated randomly dropped impurities, one recovers Poisson distribution. Also, as already advocated above, one expects that in the limit $\beta_v \to \infty$, in the Bose case, one has again the Poisson distribution, whereas the Fermi distribution leads to a less disordered situation. In the sequel, one will concentrate on the relative interplay between the dimensionless parameters $\lambda^2 \rho, \lambda^2_v \rho, \alpha$ to study the phase diagram of the magnetic impurity system.

2.2. Mean-field expansion

Consider first diagrams that are entirely built by impurity loops (Fig. 1a). These diagrams do not involve the many-body statistical correlations of the impurity distribution, and are thus independant of the statistic. Therefore, they yield the same contribution as in the Poisson case [4].
The order $\alpha^p (p > 0)$ term writes

$$-\frac{1}{\lambda^2} \frac{\zeta(1-p)}{(p-1)!} (-\lambda^2 \rho \alpha)^p$$

Summation over $p$ yields as it should the partition function per unit volume of the mean magnetic field $e < B > = 2\pi \rho \alpha$ (i.e. in the mean-field limit $\alpha \to 0$, $\rho \to \infty$, $\rho \alpha$ finite)

$$Z_{\langle B \rangle} = \frac{e < B >}{2\pi} \frac{1}{2 \sinh \beta e < B >} \exp(-\beta \frac{e < B >}{2m})$$ (20)

The global positive shift in the Landau spectrum is a direct manifestation of the hard-core boundary conditions on the wavefunctions [4].

2.3. Second order expansion

Non trivial diagrams (Fig.1b), i.e. not mean-field diagrams, appear at second order in $\alpha$. Let us denote the fugacity by $z = \pm e^{\beta \mu}$. In the interval $z \in [-1, 1]$ one has the expansion

$$n(k) = \pm \sum_{n=1}^{\infty} (-z)^n e^{-n^2 \beta \mu} \frac{k^2}{m}$$ (21)

As a result

$$g_f(r, r') = \delta(r - r') + \sum_{n=1}^{\infty} (-z)^n G_{n\beta \mu}(r, r')$$ (22)

$$g_b(r, r') = \pm \sum_{n=1}^{\infty} (-z)^n G_{n\beta \mu}(r, r')$$ (23)

The density $\rho$ is related to $z$ by

$$\rho = \pm \frac{1}{\lambda^2} \ln(1 + z)$$ (24)

Using (22, 23), the diagram of Fig. 1b has the contribution

$$D(z) = \frac{\rho \alpha^2}{2} - \frac{m \alpha^2}{4 \beta \mu} \sum_{n=1}^{\infty} \frac{(-z)^{n+p}}{n + p} \left( 1 - \int_0^1 dx \frac{x_{np}}{x_{np} + x(1-x)} \right)$$ (25)
where \( x_{np} = \frac{n p \beta}{n + p p} \).

At high temperature \( (T_v > T) \), in the Boltzmann limit, i.e. \( z \to 0 \) both in the Bose and Fermi cases, one gets \( D(z) \to \rho \alpha^2 /2 \), which is precisely the Poisson distribution result. The correction reads

\[
D(z) = \frac{\rho \alpha^2}{2} (1 - \frac{\pi}{2} \lambda^2 \rho) + \ldots
\]  

(26)

At low temperature \( (T_v < T) \), \( D(z) \) rewrites

\[
D(z) = \frac{\rho \alpha^2}{2} \left[ 1 + \sum_{q=1}^{\infty} \sum_{m=0}^{q-1} (-\lambda^2 \rho)^q \frac{C_m^q}{(2q+1)!} C_{q-1}^{m-1} \frac{h_{q-m}(z)h_{m+1}(z)}{\ln(1+z)^q+1} \right]
\]  

(27)

where the functions \( h_q(z) \)'s are defined in the interval \( ] -1, 1 [ \) by

\[
h_q(z) = \sum_{n=1}^{\infty} \frac{(-z)^n}{n^q}
\]  

(28)

The low temperature limit corresponds to \( z \to +\infty \) in the Fermi case and \( z \to -1 \) in the Bose case. It happens that (28) can be analytically continued in \( ] -1, +\infty [ \) by noticing that

\[
h_q(z) = -\frac{z}{1 + z}
\]  

(29)

and by using the recursive relation

\[
h_{q+1}(z) = \int_0^z \frac{h_q(x)}{x} dx
\]  

(30)

As a result, \( D(z) \) in (27) is valid in the interval \( z \in ] -1, +\infty [ \). It follows that in the Fermi case and in the low temperature limit \( T_v \to 0 \), one can use the expansion

\[
h_q(z) = -\frac{(\ln z)^q}{q!} + 2 \sum_{n=1}^{E(q/2)} h_{2n}(1) \frac{(\ln z)^{q-2n}}{(q-2n)!} - (-1)^q \sum_{n=1}^{\infty} \frac{(-1)^n}{n^q} \frac{1}{z^n}
\]  

(31)

which is valid for \( z > 1 \).
Which contributions does $D(z)$ yield in the low $T_v$ limit? In the Bose case, some care is required, since in (27) the limit $z \to 0$ cannot be interchanged with summations. We checked numerically that (27) indeed yields $\rho \alpha^2 / 2$, i.e. the Poissonian result as expected. One obtains for the average partition function $Z$ at order $\alpha^2$

$$Z = \frac{1}{\lambda^2}(1 - \frac{1}{2} \lambda^2 \rho \alpha + \frac{1}{2} (\lambda^2 \rho)^2 \alpha^2 (\frac{1}{6} + \frac{1}{\lambda^2 \rho}) + \ldots) \quad (32)$$

In the Fermi case, using the identity $\sum_{q=1}^{q-1} C_q^{q-1} \frac{1}{(q - m)! (m + 1)!} = \frac{2q!}{(q+1)!(q)!}$, (27) yields

$$\lim_{z \to +\infty} D(z) = \rho \alpha^2 / 2 \left[ 1 - \frac{1}{\lambda^2 \rho} (1 + \lambda^2 \rho - e^{-\lambda^2 \rho} - 2 \sqrt{\lambda^2 \rho} \int_0^{\sqrt{\lambda^2 \rho}} dy e^{-y^2}) \right] + \ldots \quad (33)$$

At this point one can consider either a low impurity density limit $\lambda^2 \rho \ll 1$, or a high impurity density limit $\lambda^2 \rho \gg 1$. At low density $\lambda^2 \rho \ll 1$, one finds that the average partition function per unit volume rewrites as

$$Z = \frac{1}{\lambda^2}(1 - \frac{1}{2} \lambda^2 \rho \alpha + \frac{1}{2} (\lambda^2 \rho)^2 \alpha^2 (\frac{1}{6} + \frac{1}{\lambda^2 \rho} + \frac{1}{30} \lambda^2 \rho) + \ldots) \quad (34)$$

At order $\alpha^2$, the $\rho^2 \alpha^2$ term is missing, a situation quite different from the Poissonian case (32), where this mean-field term precisely dominates in the mean-field limit. Therefore, in the Fermi case at low temperature, the low density expansion is not adapted to describe the mean-field limit. On the other hand, at high density $\lambda^2 \rho \gg 1$, (33) leads to

$$Z = \frac{1}{\lambda^2}(1 - \frac{1}{2} \lambda^2 \rho \alpha + \frac{1}{2} (\lambda^2 \rho)^2 \alpha^2 [\frac{1}{6} + \frac{1}{\lambda^2 \rho} (\sqrt{\frac{\pi}{\lambda^2 \rho}} - \frac{1}{\lambda^2 \rho} + \frac{1}{\lambda^2 \rho} e^{-\lambda^2 \rho} O(\frac{1}{\lambda^2 \rho})] + \ldots) \quad (35)$$

where the leading mean magnetic field term is indeed present.

3. Fermion case at zero Temperature: the Landau regime

3.1. The ordered phase
Before the system reaches the mean-field limit (20), we expect an intermediate regime characterised by smooth Landau oscillations in the spectrum. This intermediate regime is identified as the ordered phase by opposition to the phase with no oscillation (disordered phase). The order $\alpha^2$ is quite instructive to give information on the way the system reaches the mean-field limit. Consider indeed the case of Poissonian impurities (32).

We observe $1/\rho$ corrections to the mean-field at any order of the perturbative expansion in $(\alpha \rho)^n$. On the other hand, (35) shows corrections to the mean-field of order $1/(\rho \sqrt{\rho})$. This implies that the system approaches more rapidly its mean-field limit when the impurities are fermions at zero temperature, rather than Poissonian. In other words, the system is less disordered, since a Fermi distribution of impurities is more homogeneous than a Poissonian one.

Let us generalise these considerations at any order $\alpha^n$ of perturbative theory. One has to evaluate $\langle \rho(r_1) \rho(r_2) ... \rho(r_n) \rangle$, which can be rewritten as

$$\langle \rho(r_1) \rho(r_2) ... \rho(r_n) \rangle = \sum_{p=1}^{n} \sum_{f \in S^n_p} \frac{1}{p!} \int d\mathbf{r}_1'...d\mathbf{r}_p' \rho(\mathbf{r}_1',...\mathbf{r}_p') \prod_{q=1}^{n} \delta(\mathbf{r}_q - \mathbf{r}'_{f(q)})$$  

(36)

$S^n_p$ is the set of all possible surjections from $(1,..,n)$ to $(1,..,p)$ and $\rho(\mathbf{r}_1,..,\mathbf{r}_p)$ is the $p$-body correlation function

$$\rho(\mathbf{r}_1,..,\mathbf{r}_p) = \sum_{\sigma \in S^p} \epsilon(\sigma) g_b(\mathbf{r}_1 - \mathbf{r}_{\sigma(1)}) ... g_b(\mathbf{r}_p - \mathbf{r}_{\sigma(p)})$$  

(37)

In the case of Fermions at zero temperature, one has the correlator

$$g_b(\mathbf{r}) = \frac{1}{r} \sqrt{\frac{\mu}{\pi}} J_1(\sqrt{4\pi \rho r})$$  

(38)

For example, using

$$\langle \rho(\mathbf{r}_1) \rho(\mathbf{r}_2) \rangle = \rho^2 - \frac{1}{|\mathbf{r}_1 - \mathbf{r}_2|^2} \frac{\rho}{\pi} [J_1(\sqrt{4\pi \rho |\mathbf{r}_1 - \mathbf{r}_2|})]^2 + \rho \delta(\mathbf{r}_1 - \mathbf{r}_2)$$  

(39)
altogether with (14), yields the contribution (33), in addition to the mean-field term.

At high density ($\lambda^2 \rho \gg 1$), because of Fermi exclusion, the $p$-body correlation function becomes

$$\rho(r_1, \ldots, r_p) = \rho^p \left[ 1 - \frac{1}{\rho} \sum_{i<j} \delta(r_i - r_j) + O\left( \frac{1}{\rho^{\sqrt{\rho}}} \right) \right]$$  \hspace{1cm} (40)

When (40) is used for evaluating $\langle \rho(r_1)\rho(r_2)\ldots\rho(r_n) \rangle$, one indeed finds that corrections to the mean-field term $(\rho \alpha)^n$ are of order $1/(\rho^{\sqrt{\rho}})$.

### 3.2. Absence of a pure disordered phase

We have just seen that corrections to the average magnetic field limit are less important in the Fermi case. Could it be that the statistics of the impurities alter the occurrence of the transition itself [4]? First let us remark that the average partition function can be expressed as

$$Z = \frac{1}{\lambda^2} F(\lambda^2 \rho, \lambda^2 \rho, \alpha)$$  \hspace{1cm} (41)

which means that the average density of states is a function of $E/\rho$, $\lambda^2 \rho$ and $\alpha$. This is due to the fact that $g_b(r)$ is $\rho$ times a function of $\sqrt{pr}$ and $\lambda^2 \rho$. In (14) together with (36), rescaling $\beta_i$ into $\beta_i/\beta$, $r_i$ into $r_i/\lambda$ and $r'_i$ into $\sqrt{\rho} r'_i$, immediately leads to (41). In particular at $T_v = 0$, $\lambda^2 Z$ is a function of $\lambda^2 \rho$ and $\alpha$. Considering the expansion

$$\lambda^2 Z = 1 + \frac{1}{2} \alpha(\alpha - 1) \lambda^2 \rho + c_2(\alpha)(\lambda^2 \rho)^2 + c_3(\alpha)(\lambda^2 \rho)^3 + \ldots$$  \hspace{1cm} (42)

we can show that $c_2 = 0$. Use simply

$$g_b(r) = \rho \sum_{k=0}^{\infty} \frac{(-\pi \rho r^2)^k}{k!(k+1)!}$$  \hspace{1cm} (43)

and conclude in general that $\rho(r_1, \ldots, r_p)$ is at least of order $\rho^p$. In particular

$$\rho(r_1, r_2) = \rho^2 - \left[ g_b(r_1 - r_2) \right]^2$$  \hspace{1cm} (44)
starts at order $\rho^3$, whereas $\rho(r) = \rho$. Therefore, no contribution of order $\rho^2$ is to be found in $\langle \rho(r_1)\rho(r_2)\ldots\rho(r_n) \rangle$, which means that the average partition function does not contain terms of order $\rho^2$ at any order $\alpha^n$.

The specific heat \[4\]
\[C = k\beta^2 \frac{d^2}{d\beta^2} \ln Z\] (45)
gives a lower bound for the critical value $\alpha_c$ at which the system does not exhibit any Landau oscillations. More precisely, if the correction to $C - C_0$ ($C_0 = k$) is negative when $\beta \rightarrow 0$, then oscillations are already present since, at small $\beta$,
\[C - C_0 = 2\pi^2 k\beta^2 \int_0^\infty \int_0^\infty dE dE' \frac{d}{dE} \frac{<\rho(E)/V>}{dE} \frac{d}{dE'} \frac{<\rho(E')/V>}{dE'} (E - E')^2 + \ldots\] (46)
Here, the small $\beta$ expansion \[34\] yields
\[C - C_0 = -\frac{1}{4} k(\frac{2\pi^2 \rho m}{\alpha})^2 \alpha^2 (1 - \alpha)^2 + \ldots\] (47)
which is always negative. Therefore the average density of states always displays Landau oscillations.

4. Conclusion

To complete this analysis, let us emphasize again that for intermediate magnetic impurity temperature, the average partition function has been shown in \[11\] to scale as $1/\lambda^2$ times a function of $\lambda^2 \rho$, $\lambda^4 \rho$ and $\alpha$. This implies that the average density of states is a function $<\rho(E/\rho, \lambda^2 \rho, \alpha>)$. Since Poisson distribution is recovered in the Boltzman limit $T_v \rightarrow \infty$ (since then $\rho(r_1, \ldots, r_p) \rightarrow \rho^p$), one interpolates between the Poisson and the zero temperature Fermi cases simply by varying $\lambda^2 \rho$ from 0 to $\infty$. When $\lambda^2 \rho = 0$, there is a transition at $\alpha_c \simeq 0.35$, whereas when $\lambda^2 \rho = \infty$, we have just shown that no transition
occurs at all. Therefore, we expect that, for $\lambda_{\rho}^2\rho$ sufficiently small, a transition still occurs at a critical value $\alpha_c' > 0.35$, and for $\lambda_{\rho}^2\rho$ sufficiently big, no transition occurs anymore, meaning that the system is always Landau like, in the whole interval $\alpha \in [0, 1/2]$ (Fig 2).

It is not clear if the transition observed in the Poissonian case can be interpreted as a phase transition, possibly related to a localisation-delocalisation transition. But magnetic impurity distributions do influence this transition by actually reordering the system when the density correlations are increased.

Acknowledgements: We dedicate this work to the memory of Claude Itzykson, who suggested to one of us (S.O.) to look at the effect of statistics on the impurity distribution.

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