Tomographic analyses of the CMB lensing and galaxy clustering to probe the linear structure growth

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Abstract. In a tomographic approach, we measure the cross-correlation between the CMB lensing reconstructed from the Planck satellite and the galaxies of the photometric redshift catalogue based on the combination of the South Galactic Cap u-band Sky Survey (SCUSS), Sloan Digital Sky Survey (SDSS), and Wide-field Infrared Survey Explorer (WISE) data. We perform the analyses considering six redshift bins spanning the range of $0.1 < z < 0.7$. From the estimates of the galaxy auto-spectrum and the cross-spectrum, we derive with high significance the galaxy bias and the amplitude of the cross-correlation at each redshift bin. We have finally applied these tomographic measurements to estimate the linear structure growth using the bias-independent $\hat{D}_G$ estimator introduced by [1]. We find that the amplitude of the structure growth with respect to the fiducial cosmology is $A_D = 1.02 \pm 0.14$, in agreement with the predictions of $\Lambda$CDM model. We perform tests for consistency of our results, finding no significant evidence for systematic effects.
1 Introduction

Progress in the sensitivity of astronomical photometric surveys dedicated to the study the large-scale structure (LSS) has been providing valuable information about the features of the Universe at several scales and redshifts [2–6]. The prospects of using LSS data to constrain cosmology are very promising. Several upcoming astronomical surveys will produce extensive photometric data covering a wide area of the sky such as the Large Synoptic Survey Telescope (LSST) [7] and the Wide-Field Infrared Survey Telescope (WFIRST). On the other hand, the cosmic microwave background radiation (CMB) allow us to test the primordial characteristics of the Universe. However, before reaching us, the CMB photons are affected by inhomogeneities along their path producing a range of secondary effects, beyond the primary CMB temperature fluctuations at the last scattering surface [8–10]. The gravitational deflection of the CMB photons by the mass distribution along their path, namely weak gravitational lensing, is one of these secondary effects.

The CMB lensing has been investigated by several methods and experiments in the past [11–15]. Recently, through observations of the Planck satellite, it was possible not only to detect the lensing effect with high statistical significance but also to robustly reconstruct the lensing potential map in almost full-sky [16, 17]. Such a reconstructed map contains unique information of the LSS since it is related to the integral of all photons deflections between the last scattering surface and us.

Although the CMB lensing signal covers a broad redshift range, from local to high redshifts, it is not possible to obtain the evolution of the LSS along the line of sight using only the CMB lensing data. The cross-correlation between the CMB lensing map with another tracer of matter provides additional astrophysical and cosmological information. Several galaxy catalogs, such as those from the Wide Field Survey Infrared Explorer (WISE) [18],
NRAO VLA Sky Survey (NVSS) [19], Canada-France-Hawaii Telescope (CFHT) [20], Sloan Digital Sky Survey (SDSS) [21–23], 2MASS [24, 25] and WISExSuperCOSMOS [25, 26] have already been cross-correlated with the CMB lensing potential. In addition, analysis in a deeper Universe has been extended through the cross-correlation of CMB lensing with density tracers at high redshifts, e.g. quasars [27–29] and sub-mm galaxies from Herschel H-ATLAS survey [30]. Particularly, the galaxy auto-correlation and cross-correlation with the CMB lensing provides the opportunity to constrain the linear growth of the density fluctuations. By probing the evolution of perturbations over time it is possible to understand the mechanism that sources the late-time accelerated expansion of the Universe and shed light to distinguish a variety of gravity models [31–33].

Analysis of the Redshift-Space Distortions [34] from spectroscopic surveys is a traditional way to measure the linear growth rate since it is commonly parameterized on large-scales by $f\sigma_8$, where $f \equiv d\ln D / d\ln a$ is the logarithmic derivative of the growth factor with respect to the scale factor $a$, and $\sigma_8$ is the linear matter variance in a spherical shell of radius 8 Mpc$h^{-1}$. However, this quantity can not be accurately measured for photometric surveys. Alternatively, the $\hat{D}_G$ statistic introduced by [1] establishes the linear growth for photometric redshift surveys. This estimator combines properly the auto and cross-correlation of the galaxy clustering and CMB lensing in such way that is a bias-independent on linear scales.

The main aim of the present work is to constrain the linear growth factor $D$ in a tomographic approach and, therefore, to measure the evolution of the linear growth function. For that, we consider the $\hat{D}_G$ statistic using the CMB lensing map reconstructed by the Planck team [16] with the galaxy overdensity from a multi-band photometric data [35] based on imaging from South Galactic Cap u-band Sky Survey (SCUSS), SDSS and WISE. The analyses are performed in six redshift bins spanning $0.1 < z < 0.7$, being complementary to the linear growth measures previously found for others photometric catalogs [1, 24, 25]. Although the $\hat{D}_G$ estimator is linear galaxy bias independent, additionally, we use the measured galaxy-CMB lensing cross-correlations and galaxy auto-correlation to infer the correlation amplitude and the linear bias over the redshift bins.

This paper is structured as follows. We first introduce the formalism in sec. 2, and describe our data in sec. 3. The methodology is described in section 4. We then present our results in sec. 5, and discuss our conclusions in sec. 6.

## 2 Background

The gravitational lensing effect remaps the CMB temperature anisotropies by a 2D angular gradient of the lensing potential, $\alpha(\mathbf{n}) = \nabla \psi(\mathbf{n})$, where $\nabla$ is the 2D gradient operator on the sphere and $\psi(\mathbf{n})$ is the lensing potential. The 2D Laplacian of the lensing potential is related to the convergence $\kappa(\mathbf{n})$, which can be written as a function of the three-dimensional matter density contrast $\delta$ (see e.g. [36])

$$
\kappa(\mathbf{n}) = \int_{0}^{\infty} dz W^\kappa(z) \delta(\chi(z)\hat{\mathbf{n}},z),
$$

where the lensing kernel $W^\kappa$ is

$$
W^\kappa(z) = \frac{3\Omega_m}{2c} \frac{H_0^2}{H(z)} (1 + z) \chi(z) \frac{\chi_* - \chi(z)}{\chi_*},
$$
where we are considering a flat universe, $c$ is the speed of light, $H(z)$ is the Hubble parameter at redshift $z$, $H_0$ and $\Omega_m$ are the present-day parameters of Hubble and the matter density, respectively. The comoving distances $\chi(z)$ and $\chi_*$ are set to redshift $z$ and to the last scattering surface at $z_* \approx 1090$, respectively.

On the other hand, the galaxy overdensity $\delta_g$ from a galaxy catalogue with normalized redshift distribution $dn/dz$ also provides an estimate of the projected matter density contrast, given by

$$\delta_g(\hat{n}) = \int_0^\infty dz W^g(z) \delta(\chi(z)\hat{n}, z),$$

(2.3)

where the galaxy kernel $W^g$ for a linear, deterministic and scale-independent galaxy bias $b(z)$ is

$$W^g(z) = b(z) \frac{dn}{dz}.$$  

(2.4)

Under the Limber approximation [38], the two-point statistics in the harmonic space of the galaxy-galaxy and galaxy-CMB lensing correlations become

$$C^{gg}_\ell = \int_0^\infty \frac{dz}{c} \frac{H(z)}{\chi^2(z)} \left[ W^g(z) \right]^2 P\left(k = \frac{\ell}{\chi(z)}, z\right),$$

$$C^{kg}_\ell = \int_0^\infty \frac{dz}{c} \frac{H(z)}{\chi^2(z)} W^\kappa(z) W^g(z) P\left(k = \frac{\ell}{\chi(z)}, z\right),$$

(2.5)

where $P(k, z)$ is the matter power spectrum. The Limber approximation is quite accurate when $\ell$ is not too small ($\ell > 10$) [38], which is the regime considered in this work. Moreover, is possible to rewrite the equations 2.5 in terms of the linear growth function $D(z)$, since $P(k, z) = P(k, 0)D^2(z)$. Therefore,

$$C^{gg}_\ell \propto b^2(z) D^2(z),$$

$$C^{kg}_\ell \propto b(z) D^2(z).$$

(2.6)

Thus, by properly combining the two quantities of the equation 2.6, it is possible to eliminate the bias dependence and break the degeneracy between the galaxy bias and the linear growth through the estimator introduced by [1]:

$$\hat{D}_G \equiv \left\langle \frac{(C^{gg}_\ell)_{\text{obs}}}{(C^{gg}_\ell)_{\text{th}}} \right\rangle_{\ell} \sqrt{\left(\frac{(C^{gg}_\ell)_{\text{th}}}{(C^{gg}_\ell)_{\text{obs}}}\right)},$$

(2.7)

In the above equation, the $\hat{D}_G$ depends on the observed and theoretical slashed correlation functions, being the theoretical quantities $\Phi^{gg}_\ell$ and $\Phi^{kg}_\ell$ evaluated at $z = 0$ and therefore, they have the growth function removed.

In order to obtain the theoretical predictions for the matter power spectrum $P(k, z)$, we use the public Boltzmann code CAMB\textsuperscript{1} [39] with the Halofit [40] extension to non-linear evolution. Throughout the paper, we use the Planck 2015 cosmology [41] described by the TT,TE,EE+lowP+lensing+ext with parameters $\{\Omega_b h^2, \Omega_c h^2, \Omega_m, \tau, n_s, A_s, h\} = \{0.0223, 0.118, 0.308, 0.066, 0.966, 2.1 \times 10^{-9}, 0.677\}$.  

\textsuperscript{1}https://camb.info/
3 Data

3.1 Galaxy catalogue

In this study, we use the photometric redshift catalogue [35] based on multi-band data from three independent surveys: the South Galactic Cap u-band Sky Survey (SCUSS; [42]), Sloan Digital Sky Survey (SDSS; [43]), and Wide-field Infrared Survey Explorer (WISE; [44]).

The SCUSS is a u-band (354 nm) imaging survey using the 2.3m Bok telescope located on Kitt Peak, USA. The data products were released in 2015 containing calibrated single-epoch images, stacked images, photometric catalogs, and a catalogue of star proper motions. The released catalogue covers an area of approximately 4000 square degrees of the South Galactic Cap and overlaps roughly 75% of the area covered by the SDSS [45]. The detailed information about the SCUSS and the data reduction can be found in [45] and [46].

The SDSS is a multi-spectral imaging and spectroscopic redshift survey, encompassing an area of about 14000 square degrees. The SDSS uses a wide-field camera that is made up of 30 CCDs. The survey is carried out imaging in five broad bands $u,g,r,i,z$, with limit-magnitude with 95% completeness 22.0, 22.2, 22.2, 21.3 and 20.5 mag, respectively. The data have been released publicly in a series of roughly annual data releases. Specifically, the photometric data from the Data Releases 10 (DR10) [47] is considered to obtain the final catalogue used in this paper [35].

WISE is an infrared astronomical space telescope that scanned all-sky at 3.4, 4.6, 12 and 22 µm, known as W1, W2, W3, and W4, respectively. In September 2010, the frozen hydrogen cooling the telescope was depleted and the survey continued as NEOWISE, with the W1 and W2 bands. In order to match properly the official all-sky WISE catalogs with the SDSS data, is considered a technique to measure model magnitudes of the SDSS objects in new coadds of WISE images, called as forced photometry. This provides an extensive extragalactic catalogue resulting in a sample of more than 400 million sources.

The catalogue we use has been built by using the 7 photometric bands ranging from the near-ultraviolet to near-infrared. A local linear regression algorithm [6] is adopted using a spectroscopic training set composed mostly of galaxies from the SDSS DR13 spectroscopy, in addition to several other surveys. The model magnitudes utilize the shape parameters from SDSS r-band and also the SDSS star/galaxy separation to characterize the source type. The final catalogue contains $\sim 23.1$ million galaxies \(^2\) with $\sim 99\%$ of the sources spanning the redshift interval of $z \leq 0.9$ [35]. The multi-band information allows to estimate the photo-z’s for the sources more accurately and less biased than the SDSS photometric redshifts, with the average bias of $\Delta z_{\text{norm}} = 2.28 \times 10^{-4}$ and standard deviation of $\sigma_z = 0.019$.

In order to apply a tomographic approach, we split the full catalogue into six redshift bins of width $\Delta z = 0.1$ over $0.1 < z < 0.7$. We ignore the extreme redshift bins where the fractional photo-z errors become large and the galaxy density became small. We use the position of the sources to create a pixelized overdensity map, for each redshift bin, using $\delta_g(x) = \frac{n_g(x) - \bar{n}}{\bar{n}}$, where $n_g$ is the number of observed galaxies in a given pixel and $\bar{n}$ is the mean number of objects per pixel in the unmasked area, in the HEALPix scheme [48] with a resolution parameter $N_{\text{side}} = 512$. The figure 1 shows the overdensity map in these six redshift bins, where the grey area indicates the masked regions. However, we discard the stripes located in the galactic longitude range $180^\circ < l < 330^\circ$ due to the low density, remaining

\(^2\)Available for download from http://batc.bao.ac.cn/zouhu/doku.php?id=projects:photoz:start
about $f_{\text{sky}} = 0.08$ in each map for analyses. The specifics of each bin are summarized in the table 1.

\begin{align*}
0.1 < z < 0.2 & \quad 0.2 < z < 0.3 \\
0.3 < z < 0.4 & \quad 0.4 < z < 0.5 \\
0.5 < z < 0.6 & \quad 0.6 < z < 0.7
\end{align*}

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{figure1.png}
\caption{All-sky projections of the galaxy overdensity of the photometric redshift catalogue in the six photo-z bins adopted in the analysis. The maps are constructed in the \texttt{HEALPix} pixelization scheme, with the resolution parameter $N_{\text{side}} = 512$. The gray areas correspond to the masked regions.}
\end{figure}

As discussed in the section 2, we need the overall redshift distribution $dn/dz$ and the galaxy bias to connect the galaxy overdensity $\delta_g$ to the underlying matter overdensity $\delta$. However, we need take into account the effect of the photometric redshift errors [49, 50]. We can accurately reconstruct the true $dn/dz$ distribution by the convolution of the sample’s photometric redshift distribution $dn/dz(z_{\text{ph}})$ with the catalog’s photo-z error function $p(z|z_{\text{ph}})$:

\begin{equation}
\frac{dn}{dz} = \int_0^\infty dz_{\text{ph}} \frac{dn(z_{\text{ph}})}{dz} p(z|z_{\text{ph}}) W(z_{\text{ph}}),
\end{equation}

where $p(z|z_{\text{ph}})$ is parameterized as a Gaussian distribution with zero mean and dispersion $\sigma_z$. 

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so that \( p(z|z_{ph}) \propto \exp\left(-0.5(z/\sigma_z(1+z))^2\right) \), where \( \sigma_z = 0.019 \) [35] and the \( W(z_{ph}) \) is the window function, such that \( W = 1 \) for \( z_{ph} \) in the selected interval and \( W = 0 \) otherwise. The redshift distribution for the total catalogue is shown as the solid black line in figure 2, while the distribution to each tomographic bin is shown as the dashed lines.

\[
\begin{array}{ccc}
\text{Redshift range} & N_{\text{tot}} & \bar{n} \text{ [gal sr}^{-1}] \\
0.1 - 0.2 & 2,208869 & 2.19 \times 10^6 \\
0.2 - 0.3 & 3,178981 & 3.14 \times 10^6 \\
0.3 - 0.4 & 3,686820 & 3.64 \times 10^6 \\
0.4 - 0.5 & 5,155408 & 5.09 \times 10^6 \\
0.5 - 0.6 & 4,348898 & 4.29 \times 10^6 \\
0.6 - 0.7 & 2,101281 & 2.08 \times 10^6 \\
\text{Total} & 20,680257 & 2.04 \times 10^7 \\
\end{array}
\]

Table 1: The number of sources and the galaxy number density of each tomographic bins.

Figure 2: Normalized redshift distributions for the total galaxy catalogue (black solid line) and for the six tomographic bins used in the analysis (dashed lines). These are obtained by convolving the photo-z distribution in each redshift interval with a photometric error distribution.

3.2 Planck CMB lensing

We consider the CMB lensing products of the Planck 2015 data release [3]. The lensing convergence map has been constructed based on the quadratic estimators that exploit the statistical information introduced by weak lensing in the CMB data [51]. The Planck team [16] has provided as an estimate of the CMB lensing, the convergence field \( \kappa \) reconstructed using the minimum-variance (MV) combination of the estimators applied to temperature (T)

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3http://pla.esac.esa.int/pla
and polarization (P) of the SMICA foreground-cleaned map. The total lensing signal measured from $\kappa$ is detected at about $40\sigma$.

The $\kappa$ map released is band-limited to the multipole range $8 \leq \ell \leq 2048$. The reconstructed map covers about $f_{\text{sky}} \sim 67.3\%$ of the sky, masking regions contaminated by Galaxy emissions and point sources. Jointly to the released lensing products, it is available the corresponding confidence mask and a set of 100 realistic simulations, which accurately incorporate the Planck noise levels and the $\kappa$ statistical properties [52]. The maps as well as the mask are provided in the HEALPix resolution parameter $N_{\text{side}} = 2048$. We use the HEALPix $\text{ud-}gr\text{ade}$ routine to convert in the resolution $N_{\text{side}} = 512$.

4 Method

In this work, we use the angular power spectrum (APS) of the galaxy overdensity and the angular cross-power spectrum (CAPS) between the galaxy overdensity and the CMB convergence map to estimate the cosmic growth information at several redshifts. In this section, we describe the procedure followed in the analysis of these two datasets.

4.1 Estimator

The APS and CAPS estimates for incomplete sky coverage are affected by the mask, which introduces coupling between different modes [53]. Therefore, we use a pseudo-$C_\ell$ estimator based on the MASTER approach [54], that provides a very good approximation to this issue, mainly on larger scales which is the regime we are considering, as detailed below.

Let us denote the two fields $X$ and $Y$ with the auto-power spectrum when $X = Y$ and the true cross-(auto-)spectrum denoted as $C_{XY}^X (C_{XX}^X)$ to the full sky. The pseudo-cross spectrum $\tilde{C}_{XY}$ measured in a fraction of the sky is

$$\tilde{C}_{XY}^\ell = \frac{1}{2\ell + 1} \sum_{m=-\ell}^{\ell} \tilde{X}_{\ell m} \tilde{Y}_{\ell m}^*, \quad (4.1)$$

where $\tilde{X}_{\ell m}$ and $\tilde{Y}_{\ell m}^*$ are the spherical harmonic coefficients of the maps. The mask acts as a weight modifying the underlying harmonic coefficients so that the pseudo-$C_\ell$ measured from the data can be related to the true spectrum by the mode-mode coupling matrix $M_{\ell\ell'}$ as

$$\tilde{C}_{XY}^\ell = \sum_{\ell'} M_{\ell\ell'} C_{XY}^{X^\ell'}, \quad (4.2)$$

where $M_{\ell\ell'}$ is inferred by the geometry of the mask [55], given by

$$M_{\ell\ell'} = \frac{2\ell' + 1}{4\pi} \sum_{\ell''}(2\ell'' + 1)W_{\ell''} \left( \begin{array}{ccc} \ell & \ell' & \ell'' \\ 0 & 0 & 0 \end{array} \right)^2. \quad (4.3)$$

Here $W_{\ell'}$ is the APS of the mask when $X = Y$, while in the cross-correlation corresponds to the two joint masks. In the cross-correlation analysis, we use the mask resulting from multiplying the $\kappa$ mask with the $\delta_g$ mask for each redshift bin.

Depending on the size of the sky cut, the relation 4.2 cannot be inverted to obtain $C_{XY}^\ell$ because in general, the coupling matrix is singular. To mitigate the coupling effect and also to reduce the errors on the resulting CAPS, it is appropriate to bin the power spectrum in $\ell$. We bin the power spectrum in $\ell$ in a linearly spaced band powers of width $\Delta \ell = 10$ in
the range $10 < \ell < 512$. We test different bin width values, however, we find no significant influence on the results. While we set the lowest value of $\ell$ based on the $l_{\text{min}}$ of the Planck map and the accuracy of the Limber approximation, we impose a conservative cut in scales $\ell > 70$ to avoid several effects significant at small scales that could affect our analysis, such as the non-linear galaxy bias and the thermal Sunyaev-Zel’dovich (tSZ) contamination in the $\kappa$ map. The impact of this $\ell_{\text{max}} = 70$ choice on our results is explored below.

An unbiased estimator of the true-bandpowers $\hat{C}^{XY}_{\ell}$ is then, given in terms of the binned coupling matrix $K_{LL'}$

$$\hat{C}^{XY}_{\ell} = \sum_{L' \ell} K^{-1}_{LL'} P_{L\ell} \hat{C}^{XY}_{\ell}, \quad (4.4)$$

where

$$K_{LL'} = \sum_{\ell \ell'} P_{L \ell} M_{\ell \ell'} B_{\ell}^X B_{\ell'}^Y p^2_{\ell} F_{\ell} Q_{L \ell'} . \quad (4.5)$$

Here $L$ denotes the bandpower index, $P_{L \ell}$ is the binning operator and $Q_{L \ell'}$ is its reciprocal corresponding to a piece-wise interpolation. The $B_{\ell}$ is a beam function for each $X$ and $Y$ observed field, $p_{\ell}$ is the pixel window function and $F_{\ell}$ is the effective filtering function.

We do not need to debias the noise in the CAPS estimator since the CMB lensing and the galaxy data are completely independent measurements and therefore have, in principle, uncorrelated noise signals. However, we correct the estimated APS, $\hat{C}^{g g}_{\ell}$, by subtracting the shot noise term:

$$N^{g g}_{\ell} = \frac{1}{\bar{n}} , \text{ where } \bar{n} \text{ is the average number density of galaxies per steradian.}$$

The errors on the estimated auto-(cross-)spectrum are determined by

$$\Delta \hat{C}^{XY}_{\ell} = \sqrt{\left(\frac{1}{2L + 1} f_{\text{sky}} \Delta \ell \left[ (\hat{C}^{XY}_{\ell})^2 + \hat{C}^{XX}_{\ell} \hat{C}^{YY}_{\ell} \right] \right)^{1/2}} , \quad (4.6)$$

where we assume in this equation that both fields behave as Gaussian random fields and the APS incorporates the associated noise which is $N^{g g}_{\ell}$ and $N^{\kappa}_{\ell}$ for the galaxy and CMB lensing, respectively. We need to take into account the errors associated with the APS and CAPS measurements of the equation 4.6 to obtain the $\hat{D}_G$ estimator properly. Thus, we use the weighted average in the $\hat{D}_G$ calculation, as described in detail in Appendix A in [24].

5 Results

5.1 Galaxy bias and lensing amplitude

We show the measurements of the $\ell$-binned APS (left panel) and the CAPS (right panel) in Figure 3. The six panels represent, from top to bottom, the estimates to each redshift bin. The error bars of the extracted APS and CAPS are calculated using the expression 4.6.

Although the $\hat{D}_G$ estimator is bias-independent for a narrow redshift bin, we can use the observed APS and CAPS to respectively estimate the best-fit bias $b$ and the amplitude of the cross-correlation $A = b A_{\text{lens}}$, where the later is introduced motivated by phenomenological reasons and $A_{\text{lens}}$ is the CMB lensing amplitude. Therefore, on the average, $A_{\text{lens}}$ is expected to be 1 if the underlying cosmology conforms to the fiducial model and then, the amplitude $A$ should be the same value as the galaxy bias $b$ determined from the auto-correlation.

We assume that the bias does not evolve within each redshift bin so that $A$ and $b$ are free parameters obtained by means of Bayesian analysis assuming uninformative flat priors and a Gaussian likelihood

$$L(x | \theta) \propto \exp \left[ -\frac{1}{2} (x - \mu(\theta))^T C^{-1} (x - \mu(\theta)) \right] , \quad (5.1)$$

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where $x$ is the extracted $\hat{C}^{gg}_{\ell}$ or $\hat{C}^{\kappa g}_{\ell}$, $\mu$ is the correspondent binned theoretical prediction for the parameters $\theta$, and $C$ is the covariance matrix. The covariance matrix is assumed diagonal, with its elements computed by the equation 4.6. In order to efficiently sample the parameter space, we use the Markov chain Monte Carlo (MCMC) method, employing emcee package [57]. We perform this analysis for each redshift bin and as a comparison, also for the full sample spanning $0.1 < z < 0.7$. Our results are stable against the length of the chain as well as the initial walker positions.

The best-fit bias and amplitude with their $1\sigma$ errors are reported in the captions of the Figure 3 as well as the best-fit theoretical model with its $1\sigma$ uncertainties are shown as the solid lines and the gray shaded region, respectively. The significance of the detection is calculated as $S/N = \sqrt{\chi^2_{null} - \chi^2_{min}(\theta)}$, where the $\chi^2_{null}$ is the $\chi^2(\theta = 0)$ and $\chi^2_{min}(\theta)$ is the value for the best-fit. The parameter values, the $S/N$, and the $\chi^2_{min}$ for each redshift bin are summarized in Table 2.

In the tomography analysis, we have found the best-fit bias in agreement up to $1\sigma$ with the values of the cross-correlation amplitude, indicating the lensing amplitude consistent with unity. For all the redshift bins, the best-fit bias has $S/N \sim 13$. Although the constraints using CAPS is clearly weaker than in the APS case, we do find $S/N \sim 1.40 - 2.55$ in each bin. We also show that the reduced $\chi^2_{min}$ values reveal that our estimate of the covariance is realistic and the model provides a good fit to the data. The only exception is the bias from the auto-correlation in the last two redshift bins, $0.5 < z < 0.6$ and $0.6 < z < 0.7$, with reduced $\chi^2_{min}$ slightly greater than 1.

In the null tests, we perform a null hypothesis test of no correlation between the CMB lensing and the galaxy density maps. We do this by considering the cross-correlation of these two fields, being one of them the real map and the second one from simulations. As these maps do not contain a common cosmological signal, the mean correlation is expected to be consistent with zero.

We cross-correlate the real galaxy maps of each redshift bin with the 100 convergence simulations from the Planck 2015 release [16]. In addition, we cross-correlate the Planck CMB convergence map with 100 galaxy simulations constructed considering the corresponding best-fit bias, masks, shot noise and the same properties of the galaxy number density of each redshift tomographic bin. The Figure 5 shows the cross-power spectrum estimated in both bins.

5.2 Null tests

In order to check the validity of the cross-correlation estimate against the possibility of residual systematics or spurious signals, we perform a null hypothesis test of no correlation between the CMB lensing and the galaxy density maps. We do this by considering the cross-correlation of these two fields, being one of them the real map and the second one from simulations. As these maps do not contain a common cosmological signal, the mean correlation is expected to be consistent with zero.

We cross-correlate the real galaxy maps of each redshift bin with the 100 convergence simulations from the Planck 2015 release [16]. In addition, we cross-correlate the Planck CMB convergence map with 100 galaxy simulations constructed considering the corresponding best-fit bias, masks, shot noise and the same properties of the galaxy number density of each redshift tomographic bin. The Figure 5 shows the cross-power spectrum estimated in both bins.

\[4\http://dfm.io/emcee/current/\]
Figure 3: The galaxy auto-power spectrum (left panel) and the cross power spectrum (right panel) of the various tomographic slices. The panels refer to the photo-z bins, from low to high redshift (top to bottom). The points are the direct estimates while the solid line is the fiducial cosmology rescaled by the best-fit galaxy bias (for the auto-spectra) and by the cross-correlation amplitudes $A = b A_{\text{lens}}$ (for the cross-spectra). Both, the amplitudes and biases are reported in the captions with their 1$\sigma$. The best-fit theory was inferred using up to multipole $\ell < 70$. The shaded grey region indicates the 1$\sigma$ around the best-fit theory.
Figure 4: The galaxy auto-power spectrum (left panel) and the cross power spectrum (right panel) for the full galaxy sample in the redshift range $0.1 < z < 0.7$. As the Figure 3, the amplitudes and biases are reported in the captions with their $1\sigma$ and the shaded grey region indicates the $1\sigma$ around the best-fit theory in the solid line.

| Correlation    | Photo-z bin | $b \pm \sigma_b$ | S/N   | $\chi^2$/d.o.f |
|----------------|-------------|-------------------|-------|----------------|
| Gal-Gal        | $0.1 < z < 0.2$ | 0.76 ± 0.03       | 13.48 | 3.79/5         |
|                | $0.2 < z < 0.3$ | 0.78 ± 0.03       | 13.34 | 8.77/5         |
|                | $0.3 < z < 0.4$ | 0.85 ± 0.03       | 13.39 | 7.94/5         |
|                | $0.4 < z < 0.5$ | 0.90 ± 0.04       | 13.40 | 8.69/5         |
|                | $0.5 < z < 0.6$ | 0.88 ± 0.04       | 13.17 | 13.48/5        |
|                | $0.6 < z < 0.7$ | 0.85 ± 0.04       | 12.76 | 18.52/5        |
| full           | $0.1 < z < 0.7$ | 1.44 ± 0.04       | 13.80 | 8.60/5         |

| Correlation    | Photo-z bin | $A \pm \sigma_A$ | S/N   | $\chi^2$/d.o.f |
|----------------|-------------|-------------------|-------|----------------|
| Gal- CMB lensing | $0.1 < z < 0.2$ | 0.88 ± 0.49       | 1.54  | 1.48/5         |
|                | $0.2 < z < 0.3$ | 0.72 ± 0.36       | 1.40  | 4.16/5         |
|                | $0.3 < z < 0.4$ | 0.69 ± 0.34       | 1.47  | 2.75/5         |
|                | $0.4 < z < 0.5$ | 0.69 ± 0.32       | 1.52  | 2.50/5         |
|                | $0.5 < z < 0.6$ | 0.69 ± 0.28       | 1.94  | 1.31/5         |
|                | $0.6 < z < 0.7$ | 0.81 ± 0.30       | 2.55  | 1.49/5         |
| full           | $0.1 < z < 0.7$ | 0.67 ± 0.24       | 2.59  | 2.60/5         |

Table 2: Summary of the results estimated from the APS and CAPS for the 5 redshift bins and for sample between $0.1 < z < 0.7$: the top half table shows the best-fit linear bias $b$ to the galaxy auto-correlations, while the lower half shows the best-fit to the cross-correlations amplitudes $A = bA_{lens}$. The signal-to-noise ($S/N$) and the best-fit $\chi^2$ are also shown.

Considering the covariance matrices obtained from these simulations, we calculate the $\chi^2$ and the probability-to-exceed (PTE) for the scales $10 \leq \ell \leq 70$ with $dof \nu = 6$. The results are displayed in the Table 3. Although we found different values of the PTE for each test and redshift bin, no significant signal is detected in either case and then, they are consistent with zero.
Figure 5: Null tests for the cross-power spectrum to the six redshift bins. In the right panel, is the mean correlation between the Planck CMB convergence map and 100 galaxy overdensity simulations obtained considering the respective features of each redshift bin. In the left panel, is the mean correlation between the galaxy overdensity and the 100 simulated Planck CMB lensing maps. The errors bars are given by the standard deviation of the simulated cross-power spectra divided by $\sqrt{100}$.

Table 3: Null hypothesis cross-correlation tests.

| Correlation                         | Photo-z bin | $\chi^2$ | PTE (%) |
|------------------------------------|-------------|----------|---------|
| $\kappa$ Planck Sims × Gal         | 0.1 < $z$ < 0.2 | 6.64     | 35.45   |
|                                    | 0.2 < $z$ < 0.3 | 8.73     | 18.89   |
|                                    | 0.3 < $z$ < 0.4 | 6.92     | 32.81   |
|                                    | 0.4 < $z$ < 0.5 | 8.89     | 17.96   |
|                                    | 0.5 < $z$ < 0.6 | 10.15    | 11.81   |
|                                    | 0.6 < $z$ < 0.7 | 8.83     | 18.30   |
| $\kappa$ Planck × Gal Sims        | 0.1 < $z$ < 0.2 | 4.50     | 60.92   |
|                                    | 0.2 < $z$ < 0.3 | 9.38     | 15.28   |
|                                    | 0.3 < $z$ < 0.4 | 4.21     | 64.79   |
|                                    | 0.4 < $z$ < 0.5 | 5.08     | 53.24   |
|                                    | 0.5 < $z$ < 0.6 | 4.64     | 59.01   |
|                                    | 0.6 < $z$ < 0.7 | 1.62     | 95.09   |

5.3 Constraints of $\hat{D}_G$

We calculate the $\hat{D}_G$ estimator, using the extracted APS and CAPS from the datasets. The figure 6 shows the result for each redshift bin with the corresponding $1\sigma$ error bars. The error bars for each redshift bin are estimated from the dispersion of the $\hat{D}_G^{sim}$, establish from auto- and cross- spectra of 500 correlated Gaussian realizations [59, 60] of $\kappa$ and $\delta_g$ considering their statistical properties consistent with the data. The solid black line is the expected in the fiducial Planck ΛCDM model $D_G^{fid}(z)$. As the expected function $D_G^{fid}(z)$ is directly related to the cosmological parameters $\Omega_m\sigma_8H_0^2$, we consider the Planck chains to randomly draw 3000 points and calculate the linear growth function for each cosmology. The gray shaded region
around the $D_G^{fid}(z)$ is the $2\sigma$ scatter for the 3000 cosmologies. It is worth mention that for each model $i$, we normalize the curve by multiplying by the factor $(\Omega_m \sigma_8 H_0^2)^i/(\Omega_m \sigma_8 H_0^2)^{fid}$ as [1, 24].

We can assess the amplitude of the linear growth function $A_D$, with respect to the fiducial prediction, assuming a template shape of the $D_G$ to be fixed by the $D_G^{fid}(z)$ [1, 24], such that

$$D_G(z) = A_D D_G^{fid}(z),$$

where to each tomographic bin we use the median of the redshift distribution as input of $z$. We use the MCMC method with a flat prior to fit the amplitude. We find $A_D = 1.02 \pm 0.14$, in excellent agreement with the fiducial value $A_D = 1$.

Similar analyses using other galaxy samples, for nearer [24, 25] and for deeper [1, 61] redshifts than the considered in this work, also indicate agreement with the fiducial cosmology established by Planck. Although, for the DES Science Verification galaxies, it revealed a mild $\sim 1.7\sigma$ discrepancy away from the fiducial cosmology. In this sense, our analysis is complimentary, as we consider another survey that covers a different region of the sky and therefore, extends to probing other possible systematics effects and redshifts intervals.

**Figure 6**: The linear growth factor estimated from the $\hat{D}_G$ estimator to the six redshift bins. The solid black line represents the theoretical growth function for the Planck fiducial cosmology. The $2\sigma$ scatter for 3000 cosmologies randomly drawn from the Planck chains is shown in the gray shaded region.

### 5.4 Consistency test

In order to avoid contamination of nonlinearities, we limited our analyses at scales up to $\ell \leq 70$ in all redshift bins. However, it is necessary to investigate the impact of our choice, since the scales subtended by the modes that are entering the nonlinear regime, that is, $\Delta^2(k_{NL}) = k_{NL}^3 P_{linear}(k_{NL})/(2\pi^2) \approx 1$, vary for each of the redshift bins considered. In this way, we explore the variation of the $D_G$ value, considering different choices of $\ell_{max}$ for each
redshift bin. Effectively, the question is to understand whether by extending the scales would lead to a significant change in the observed value of $\hat{D}_G$.

The figure 7 shows the $\hat{D}_G$ as a function of the maximum multipole $\ell_{\text{max}}$ considered, for each redshift bin.
for each redshift bin. The gray shaded region represents a 1σ error bar, estimated through the 500 correlated realizations described in the previous section. The dotted horizontal red line indicates the value found when considering scales up to ℓ ≤ 70. For better visualization, the x-axis range changes for each redshift bin, due to the scales where the nonlinearities are relevant.

We can observe that for the three first redshift bins and the last one there are no significant deviations of the ˆDG with ℓ max. Thus, it being unlikely that the linear growth factor inferred is affected by the inadequate inclusion of non-linear scales. However, for 0.4 < z < 0.5 and 0.5 < z < 0.6, we can see a slight increase in the ˆDG value with the ℓ max, although this behavior begins to occur at scales where the nonlinearities are not expected to be relevant. Therefore, a better investigation of possible systematics and effects in these redshift ranges is needed, in order to better determine the cause of this effect. For the purpose of this work, this aspect does not significantly impact the overall result, since the shift in the ˆDG value is in agreement in up to 2σ of the estimated value (represented by the dotted red line).

6 Conclusions

Recent reports witness the increasing importance of measuring the growth of the cosmic structures using the large deep surveys catalogues and CMB lensing data [1, 24, 25, 61], now available. In fact, the linear structure growth factor as a function of redshift, DG(z), have the potential to discriminate between alternative models of cosmic acceleration. In this work, we present a tomographic estimate of the linear growth factor by combining the auto- and cross-correlation of the CMB Planck convergence map, κ, and a galaxy density fluctuations map, δg, where the δg map was constructed from the photometric catalogue based on multi-band data from SCUSS, SDSS, and WISE [35]. We perform detailed analyses in six redshift bins of width δz = 0.1, in the redshift interval 0.1 < z < 0.7.

We have studied the evolution of the linear galaxy bias, b, and the amplitude of the cross-correlation, A, using the auto- and cross-angular power spectra, respectively. We found a significant detection of the best-fit parameters, although the galaxy clustering auto-correlation fit with higher S/N than the galaxy-CMB lensing correlations and the results are summarized in Table 2. We have found that b and A are consistent with each other in all redshift bins. However, when using the full galaxy sample at 0.1 < z < 0.7, we do find A < b, with a tension of ∼ 3σ (including only statistical errors). This result may indicate effects such as stochasticity or evolution of bias in this redshift range [62].

In addition, we perform null tests to check if our measured signal is contaminated by artifacts from the survey’s systematics or other undesirable effects. To this end, we have cross-correlated the real κ map (δg) with a set of δg (κ) simulations that include noise, where no significant signal is detected in the cases analyzed, which indicates that the cross-correlation is unlikely to be affected by such effects. Our results are displayed in Figure 5.

By combining the auto and the cross-correlation estimates, we measure the linear growth factor at different epochs of the Universe by using the bias-independent estimator ˆDG introduced by [1]. Our main result displayed in Figure 6, shows the measured linear structure growth factor in comparison with the expected in the fiducial ΛCDM scenario. Our result is consistent with the fiducial model, with the amplitude of the linear growth function AD = 1.02±0.14, being AD = 1 the fiducial value (see equation 5.2). Moreover, we have tested the stability of the ˆDG value against different choices of ranges of angular scales used in the analysis. We found no significant shift in the ˆDG value with the different scales, except
in the bins $0.4 < z < 0.5$ and $0.5 < z < 0.6$, although this does not significantly affect our overall result. The details of these analyses are displayed in Figure 7.

The CMB lensing tomography is an efficient method to test the linear growth of cosmic structures and, by extension, to test dark energy scenarios and/or alternative gravity models. In the near future, the CMB and galaxy surveys such as the Simons Observatory, CMB-S4, LSST, and WFIRST will produce comprehensive data and will enable to reach a deep mapping of the galaxies and a high sensitivity in reconstructing the CMB lensing potential. Thus, we may expect that the CMB lensing tomography, through analysis as the one used here, will be fundamental to find shrunken bounds in the scenario that better explains the history of the cosmic structure growth.

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