Type IIB string theory on $AdS_5 \times T^{nn'}$

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ABSTRACT

We study Kaluza-Klein spectrum of type IIB string theory compactified on $AdS_5 \times T^{nn'}$ in the context of $AdS/CFT$ correspondence. We examine some of the modes of the complexified 2 form potential as an example and show that for the states at the bottom of the Kaluza-Klein tower the corresponding $d = 4$ boundary field operators have rational conformal dimensions. The masses of some of the fermionic modes in the bottom of each tower as functions of the $R$ charge in the boundary conformal theory are also rational. Furthermore the modes in the bottom of the towers originating from $q$ forms on $T^{11}$ can be put in correspondence with the BRS cohomology classes of the $c = 1$ non critical string theory with ghost number $q$. However, a more detailed investigation is called for, to clarify further the relation of this supergravity background with the $c = 1$ strings.

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1. Introduction

It has recently been suggested by Klebanov and Witten\[1\] that the world volume super Yang Mills theory of parallel $D_3$-branes near a conical singularity of a Calabi-Yau manifold are related to string theory on $AdS_5 \times T^{11}$. They argue that string theory on $AdS_5 \times T^{11}$, with $T^{11} = (SU(2) \times SU(2))/U(1)$ can be described in terms of an $\mathcal{N} = 1$ superconformal $SU(N) \times SU(N)$ gauge theory. In the absence of a rigorous proof of the AdS/CFT correspondence \[2\][3] one needs to have the Kaluza-Klein spectrum of the supergravity in the above geometry. The Kaluza-Klein masses are used to identify the anomalous dimensions of the operators on the Yang Mills side\[4\][5].

The manifold $T^{11}$ also enters in the geometrical description of $c = 1$ matter compactified on a circle at the self dual radius coupled to the $d = 2$ gravity, the so called non critical $d=2$ string theory. It has been shown in [6] that the symmetry algebra of the volume preserving diffeomorphisms of the cone based on $T^{11}$ is in fact in a $1 - 1$ correspondence with the symmetry algebra of ($1,0$) and $(0,1)$ conformal primary fields of the $c = 1$ theory at the self dual radius. It is therefore tempting to ask if there is any relationship between the $c = 1$ theory and the type IIB string theory compactified on $AdS_5 \times T^{11}$. We will also touch upon the relation of Kaluza-Klein spectrum on $T^{nn}$ and $c = 1$ string theory at $n$-times the self dual radius in the last section.

The Kaluza-Klein modes are characterised by a set of quantum numbers $\{l, l', s\}$, where $l$ and $l'$ characterise the $SU(2) \times SU(2)$ multiplets and $s$, related to the $R$ charge in the Yang Mills side, is a $U(1)$ charge taking integer or $1/2$ integer values. The lowest values of $l$ and $l'$ will depend on $s$. For any given $s$ there is an infinite tower of Kaluza Klein modes. We shall show that for any given $s$ and for some ( but not all) of the modes with the smallest possible values of $l$ and $l'$ there corresponds well defined cohomology classes in the ghost numbers 0, 1 and 2 sectors of the $c = 1$ theory. More precisely we shall show that the Kaluza Klein modes in $AdS_5$ originating from $q$ forms in $T^{11}$ are in correspondence with the cohomology classes with ghost number $q$ on the $c = 1$ side. They have identical $SU(2) \times SU(2)$ quantum numbers. However the modes originating from the components of the metric in the internal space seem to be unmatched. Along the way we shall show that the derivation of the Kaluza-Klein spectrum can be reduced to the problem of the diagonalization of differential operators acting on tensors and spinors in the background of monopole fields on $S^2 \times S^2$. As an example we give the Kaluza-Klein masses of some of the bosonic and fermionic fields.
Possible correspondence between $c=1$ theory at self dual radius and the type IIB strings on conifold singularities have been noted in the past by Ghoshal and Vafa [7]. We shall comment on this paper at the end of the present note.

2. The manifolds $T^{nn'}$ and the Kaluza-Klein modes

Although manifolds $T^{nn}$ are related to the $c=1$ string theory, Kaluza-Klein spectrum can be studied on any $T^{nn'}$ for $n \neq n'$. Below we will study the Kaluza-Klein spectrum of Type IIB string theory compactified on $T^{nn'}$. In the end when we will relate our results to $c=1$ string theory we will take $n=n'$.

The geometry of the manifolds $T^{nn'}$ is defined in terms of the metric

$$ds^4 = c^2(dy_5 - n \cos y_1 dy_2 - n' \cos y_3 dy_4)^2 + a^2(dy_1^2 + \sin^2 y_1 dy_2^2) + a'^2(dy_3^2 + \sin^2 y_3 dy_4^2).$$

All the coordinates $y^\alpha$ are angles, such that $y = (y^1, y^2)$ parametrise a $S^2$ of radius $a$ while $y' = (y^3, y^4)$ parametrise a $S^2$ of radius $a'$. The angle $y^5$ ranges from 0 to $4\pi$. The constant $c$ is the radius of the circle defined by $y^5$. The constants $n$ and $n'$ will be taken to be integers. We shall denote these manifolds by $T^{nn'}$. Locally they look like $S^2 \times S^2 \times S^1$. Their isometry group is $SU(2) \times SU(2) \times U(1)$, where the $U(1)$ factor is due to the translational invariance of the coordinate $y^5$. It has been argued by Klebanov and Witten that the $U(1)$ factor should be identified with the $R$ symmetry of the world volume $\mathcal{N} = 1, d = 4$ superconformal field theory which arises as a consequence of the AdS/CFT correspondence. We shall see that it can also be put in correspondence with the $U(1)$ group generated by the Liouville mode of the $c=1$ string theory. In this way the $R$ charges of the boundary $d = 4$ superconformal theory will be set in correspondence with the Liouville momenta of the $c=1$ theory.

In order for $AdS_5 \times T^{nn'}$ to be a supersymmetric solution of the type IIB field equations it is necessary that $a = a' = \frac{1}{\sqrt{6|e|^3}}$, $n = \pm n'$ and $ec = -\frac{1}{3n}$, where, $e^2$ is related to the $AdS_5$ cosmological constant through $R_{\mu\nu} = -4e^2 g_{\mu\nu}$. For the present discussion we can keep the background parameters arbitrary.

Our strategy for the Kaluza Klein expansion is to dispose of the $y^5$ coordinate by Fourier expansion. As a result of this we obtain infinite number of fields living on $AdS_5 \times S^2 \times S^2$. Writing every object in an orthonormal basis of the $S^2 \times S^2$ manifold we obtain infinite number of fields coupled to a magnetic monopole field of charge $ns$ on the first $S^2$ and $n'$s on the second $S^2$. The harmonic expansion on the magnetic monopole background
has been studied in detail in [9]. We will use the formalism of this reference to write down
the eigenvalues of the $S^2 \times S^2$ Laplacian acting on any field. In this way we will obtain
the Kaluza Klein modes which depend on the coordinates of $AdS_5$ only.

To implement this idea we need the components of the 5-bein and the spin connections
on $T^{mn'}$. They are given by

$$e^1_1 = \frac{1}{a}, \quad e^2_2 = \frac{1}{a \sin y_1}, \quad e^5_2 = \frac{n}{a \cot y_1},$$
$$e^3_3 = \frac{1}{a}, \quad e^4_4 = \frac{1}{a' \sin y_3}, \quad e^5_4 = \frac{n'}{a' \cot y_3}, \quad e^5_5 = 1. \tag{2}$$

From these one can calculate the components of the spin connections. They are given by

$$\omega_a[b] = \varepsilon_a[e_b, \varepsilon_{a'b}], \quad \omega_{a[b]} = \omega_a \varepsilon_{b}, \quad \omega_a[b] = -\omega_{a'b} \varepsilon_{ab},$$
$$\omega_m[n] = \omega_m \varepsilon_{n}, \quad \omega_{5m}[n] = \omega_{5} \varepsilon_{mn}, \quad \omega_m[n] = -\omega_{m} \varepsilon_{mn}, \tag{3}$$

where $\omega_a = (\omega_1 = 0, \omega_2 = -\frac{1}{a} \cot y_1)$ and $\omega_m = (\omega_3 = 0, \omega_3 = -\frac{1}{a'} \cot y_3)$. These are
just the components of a magnetic monopole potential on each $S^2$. For what follows it is
important to note that $e^5_5 = -n \omega_a$ and $e^5_5 = -n' \omega_m$.

To clarify the scheme let us look at the derivatives of a $d = 10$ scalar $\Phi(x, y^5, y, y')$.
First we write

$$\Phi(x, y^5, y, y') = \sum_{s \in \frac{1}{2} \mathbb{Z}} e^{isy^5} \Phi^s(x, y, y') \tag{4}$$

Since the angle $y^5$ ranges over a period of $4\pi$, $s$ takes integer as well as $1/2$ integer values.
The derivative operator on $T^{mn'}$ then acts on the Fourier modes $\Phi^s$ according to

$$D_5 \Phi(x, y^5, y, y') = \frac{is}{c} \Phi(x, y, y')$$
$$D_2 \Phi^s(x, y, y') = (\partial_{-} - ins \omega_{2}) \Phi^s(x, y, y') = \nabla_a \Phi^s(x, y, y') \tag{5}$$
$$D_m \Phi^s(x, y, y') = (\partial_{m} - in' \omega_{m}) \Phi^s(x, y, y') = \nabla_m \Phi^s(x, y, y').$$

We thus see that $\nabla_a \Phi^s(x, y, y')$ and $\nabla_m \Phi^s(x, y, y')$ are precisely the orthonormal frame
components of the covariant derivatives of a charged scalar field coupled to $U(1)$ monopole
fields on each sphere. Furthermore the $U(1)$ charge of $\Phi^s$ on the first $S^2$ is $ns$ and on the
second $S^2$ it is $n's$. 3
The above covariant derivatives dictate the form of harmonic expansions for each \( \Phi^s \). Following the notation of [9] we can write

\[
\Phi^s(x, y, y') = \sum_{l \geq |n_1 s|, |\lambda| \leq l} (2l + 1) \frac{i}{2} \sum_{l' \geq |n' s|, |\lambda'| \leq l'} (2l' + 1) \frac{i}{2} \Phi_{\lambda, \lambda'}^{s, l, l'}(x) D_{n, \lambda}(L^{-1}y) D_{n, \lambda'}(L^{-1}y')
\]

(6)

In these formulae the spheres are regarded as the coset manifolds \( SU(2)/U(1) \). The \( SU(2) \) group element \( L_y \) represents the point \( y \in S^2 \) and \( D_{n, \lambda}(L^{-1}y) \) are the matrix elements of the unitary irreducible representations of \( SU(2) \) carrying spin \( l \). Their most important property for us is that

\[
\nabla_{\pm} D_{q, \lambda}(L^{-1}y) = \frac{i}{\sqrt{2a_1}} \sqrt{(l \mp q)(l \pm q + 1)} D_{q, \lambda}(L^{-1}y)
\]

(7)

The \( U(1) \) basis \( \pm \) on the first \( S^2 \) are defined by \( \nabla_{\pm} = \frac{1}{\sqrt{2}}(\nabla_1 \mp i \nabla_2) \). On the second \( S^2 \) we shall denote the \( U(1) \) basis by a prime, viz, \( \nabla'_{\pm} = \frac{1}{\sqrt{2}}(\nabla_3 \mp i \nabla_4) \).

From these relations we can easily read the eigenvalues of the Laplacian action on the charge \( q \) objects on \( S^2 \), viz,

\[
\nabla_{\pm} \nabla_{\pm} D_{q, \lambda}(L^{-1}y) = (\nabla_+ \nabla_+ + \nabla_- \nabla_-) D_{q, \lambda}(L^{-1}y) = -\frac{1}{a^2}(l(l+1) - q^2) D_{q, \lambda}(L^{-1}y)
\]

(8)

The eigenvalues of the 5 dimensional Laplacian on \( \Phi^s \) can now be read immediately. As an example let us consider the complex dilaton. The linearised equation for this field is given by

\[
(D_{\mu} D^\mu + D_{\alpha} D^\alpha) \Phi(x, y^5, y, y') = 0
\]

(9)

Noting that

\[
D_{\alpha} D^\alpha = -\left(\frac{s}{c}\right)^2 + \nabla_\alpha \nabla_\alpha + \nabla_m \nabla_m
\]

(10)

we can simply read the \( AdS_5 \) mass of the mode \( \Phi_{\lambda, \lambda'}^{s, l, l'}(x) \) as

\[
m^2 = \frac{s^2}{c^2} + \frac{1}{a^2}(l(l+1) - (ns)^2) + \frac{1}{a'^2}(l'(l'+1) - (n's)^2)
\]

(11)

where \( l \geq |ns| \) and \( l' \geq |n's| \).

For the next example we consider the complex, 2-form potential \( B_{MN} \). Firstly, The components \( B_{\mu \nu} \) are \( T^{mn'} \) scalars. Therefore they must be treated exactly in the same way
as $\Phi$. The remaining components can be decomposed into various tensors on $S^2 \times S^2$. Here we list the result,

$$D_\alpha D_\alpha B_{\mu \bar{\nu}} = \left[ -\left( \frac{s}{c} \right)^2 + \nabla_a \nabla_a + \nabla_m \nabla_m - 4e^2 \right] B_{\mu \bar{\nu}} - 2e\varepsilon_{ab} \nabla_a B_{\mu \bar{\nu}} - 2e\varepsilon_{mn} \nabla_m B_{\mu \bar{\nu}}$$  

(12)

$$D_\alpha D_\alpha B_{\mu \pm} = \left[ -\left( \frac{s}{c} \pm e \right)^2 + \nabla_a \nabla_a + \nabla_m \nabla_m - e^2 \right] B_{\mu \pm} \mp 2ie\nabla_\pm B_{\mu \bar{\nu}}$$  

(13)

$$D_\alpha D_\alpha B'_{\mu \pm} = \left[ -\left( \frac{s}{c} \mp e \right)^2 + \nabla_a \nabla_a + \nabla_m \nabla_m - e^2 \right] B'_{\mu \pm} \mp 2ie\nabla'_\pm B_{\mu \bar{\nu}}$$  

(14)

In these relations the covariant derivatives are defined by the $U(1)$ charge of each object. For an object $B_{pq}$ of charge $p$ on the first $S^2$ and charge $q$ in the second $S^2$ we have,

$$\nabla_a B_{pq} = (\partial_a - ip\omega_a)B_{pq} \quad \text{and} \quad \nabla_m B_{pq} = (\partial_m - iq\omega_m)B_{pq}$$

where $\partial_a$ and $\partial_m$ denote the partial derivatives in the orthonormal basis. The $(p, q)$ charges of $B_{\mu \bar{\nu}}$, $B_{\mu \pm}$, $B'_{\mu \pm}$, and $B_{\pm(\pm)}$ are, respectively, $(ns, n's)$, $(ns \pm 1, n's)$, $(ns, n's \pm 1)$ and $(ns \pm 1, n's \pm 1)$. These charges determine the lower bounds of $l$ and $l'$ in the harmonic expansion of $B_{pq}$ on $S^2 \times S^2$. The expansion of $B_{pq}$ is identical to the one of $\Phi$ given above except that we should put the lower bounds $l \geq |p|$ and $l' \geq |q|$.

Other bosonic fields can be treated in a similar way. We tabulate the $(p, q)$ charges of all of the bosonic fields of the type IIB supergravity in table I.

The technique outlined above can be used to obtain the Kaluza Klein masses of small perturbations around the background solution $AdS_5 \times T^{nn}$. The full analysis, although straightforward, is quite lengthy and will not be given here. Here we shall give the result of calculation of the masses of those modes which decouple without too much labour from the rest of the spectrum. The easiest one is the complex scalar for which we gave the spectrum of the masses in the previous section. We shall next consider the complex 2-form potential $B_{MN}$.

Note that for each $s$ there is an infinite tower of modes in each field. We shall consider only some of these towers whose mass can easily be deduced. For each $s$ this happens for the modes with the smallest values of $l$ and $l'$. Consider first the modes with $ns \geq 1$. In this case the modes $B_{\mu ns^{-1},ns}^{s,ns^{-1},ns}$ decouple from the rest of the system. Using the background values of $\frac{1}{a^2} = 6e^2$, $\frac{1}{ec} = -3n$ we obtain

$$2D^\mu D_{[\mu} B_{\nu]}^{s,ns^{-1},ns} - e^2[(3ns + 1)^2 - 1]B_{\nu}^{s,ns^{-1},ns} = 0$$  

(15)
Identical equation will result for $B^{s,ns,ns-1}_{\nu-}$.

For the range of $ns \leq -1$ it is the leading modes of $B^{s,-ns-1,-ns}_{\nu+}$ and $B^{s,-ns,-ns-1}_{\nu+}$ which decouple. Their equation of motion becomes

$$2D^{\mu}D_{[\mu}B^{s,-ns-1,-ns}_{\nu+]} - e^2[(3ns-1)^2 - 1]B^{s,-ns-1,-ns}_{\nu+} = 0$$

(16)

and an identical equation for $B^{s,-ns,-ns-1}_{\nu+}$.

Following [10] we shall identify $e^2[(3ns+1)^2 - 1]$ as the AdS mass of $B^{s,-ns-1,ns}_{\nu+}$ and $B^{s,-ns,ns-1}_{\nu+}$. Likewise $e^2[(3ns-1)^2 - 1]$ will be identified with the AdS mass of $B^{s,-ns-1,-ns}_{\nu+}$ and $B^{s,-ns,-ns-1}_{\nu+}$.

According to the AdS/CFT correspondence every bulk field in $AdS_5$ should correspond to a well defined gauge invariant operator in the boundary $d=4$ theory. For an AdS mode of mass $m$ which originates from a $p$ form field in the internal space the conformal dimension $\Delta$ of the $d=4$ theory is given by $(\Delta - p)(\Delta + p - 4) = m^2$. Clearly only very small subset of modes will produce a rational solution for $\Delta$. This is what happens to the modes singled out above, namely, the conformal dimension of the fields dual to $B^{s,-ns-1,ns}_{\nu+}$ and $B^{s,-ns,ns-1}_{\nu+}$ turn out to be $3(1+ns)$ and those of the fields corresponding to $B^{s,-ns-1,-ns}_{\nu+}$ and $B^{s,-ns,-ns-1}_{\nu+}$ are equal to $3ns+1$.

The same phenomenon also happens for the complex dilaton modes. This was observed by Gubser [11]. From our equation (11) it follows that for any given $s$ the smallest masses are obtained for $l = l' = |ns|$, which is $m^2 = e^2[(3|ns|+2)^2 - 4]$. Plugging this in the formula for the dimension with $p = 0$ produces the positive root $\Delta = 4 + 3|ns|$. Gubser has conjectured that for a given $s$ this will happen to every mode in the bottom of the tower with that value of $s$.

We can also obtain the fermion masses. The easiest one is the dilatino. It satisfies the equation

$$(\not\! D_x + i\not\! D_y + e)\chi = 0$$

(17)

We categorise the spectrum of $i\not\! D_y$ using the quantum number $s$. For the sake of simplicity let us set $n = n' = 1$. Let us take $|s| \geq 1/2$. In this case we have four subsectors. We will use the notation $\epsilon(s)$ to denote sign of $s$ and define $\epsilon(0) = 1$ in the fourth sector below. We shall also introduce the function $\nu(l, s)$ through $\nu(l, s) = \frac{1}{a}\sqrt{(l + \frac{1}{2})^2 - s^2}$ and define $\nu'(l', s) = \nu(l', s)$.

In the first sector we have only one eigenvalue

$$\lambda(l, l') = \lambda(-\frac{1}{2} + |s|, -\frac{1}{2} + |s|) = -\epsilon(s)e(1 + 3|s|).$$

(18)
The second sector is defined by $l = |s| - 1/2, l' \geq |s| + 1/2$. Here we have two towers parametrised by $l'$

$$\lambda_{\pm}(-\frac{1}{2} + |s|, l') = -\epsilon(s)\frac{e}{2} \pm \sqrt{\nu'^2 + \frac{e^2}{4}[1 + 12 |s| (1 + 3 |s|)]}$$

(19)

and in particular for $l' = |s| + 1/2$ the eigenvalue is rational

$$\lambda_{\pm}(-\frac{1}{2} + |s|, \frac{1}{2} + |s|) = -\epsilon(s)\frac{e}{2} \pm \frac{|e|}{2}(6 |e| + 5).$$

(20)

In the third sector, which is defined by $l \geq |s| + 1/2, l' = |s| - 1/2$, for each $s$ again we have two towers parametrised by $l$

$$\lambda_{\pm}(l, -\frac{1}{2} + |s|) = -\epsilon(s)\frac{e}{2} \pm \sqrt{\nu'^2 + \frac{e^2}{4}[1 + 12 |s| (1 + 3 |s|)]}$$

(21)

and as in the case of sector two, for $l = |s| + 1/2$ the eigenvalue is rational. i.e.,

$$\lambda_{\pm}(\frac{1}{2} + |s|, -\frac{1}{2} + |s|) = -\epsilon(s)\frac{e}{2} \pm \frac{|e|}{2}(6 |e| + 5).$$

(22)

In the fourth sector, we will relax the condition to $|s| \geq 0$. Thus the $s = 0$ modes are included in this sector. This sector is defined by $l \geq |s| + 1/2, l' \geq |s| + 1/2$. Here we have four towers and they are given by

$$\lambda_{1}^\pm(l, l', s) = -\epsilon(s)\frac{e}{2} \pm \sqrt{\nu'^2 + \nu'^2 + e^2(3 |s| + \frac{1}{2})^2}$$

$$\lambda_{2}^\pm(l, l', s) = \epsilon(s)\frac{e}{2} \pm \sqrt{\nu'^2 + \nu'^2 + e^2(3 |s| - \frac{1}{2})^2}.$$ 

(23)

In this case, however, we find that for $l = l' = |s| + 1/2$, only two eigenvalues, $\lambda_2$ become rational, except for the case $s = 0$ when $\lambda_1$ also becomes rational.

$$\lambda_{1}^\pm(\frac{1}{2} + |s|, \frac{1}{2} + |s|) = -\epsilon(s)\frac{e}{2} \pm \frac{|e|}{2} \sqrt{36s^2 + 108 |s| + 49}$$

$$\lambda_{2}^\pm(\frac{1}{2} + |s|, \frac{1}{2} + |s|) = \epsilon(s)\frac{e}{2} \pm \frac{|e|}{2}(6 |s| + 7).$$

(24)

From the above list of the eigenvalues of $iD_\mu$ one can also obtain the masses of the $AdS$ gravitinos as well as the gamma trace of the gravitino, $\gamma^\mu \psi_\mu$. This latter field satisfies an
equation similar to (17) in which $e$ is replaced by $3e$, whereas the $D_\mu$ traceless and $gamma$ traceless gravitino $\phi_\mu$ satisfies

$$D_x \phi_\mu^j - (\lambda^j - e) \phi_\mu^j = 0.$$  

where $\lambda^j$ indicates the eigenvalues of $iD_y$ and $j$ runs over various sectors.

The fermionic modes in the direction of the Killing spinor in $T^{nn}$ satisfy equations similar to those in refs[10] .

3. Relation to c=1 string theory

The fundamental fields of the $c = 1$ theory are two scalars $X$ and $\phi$ and the $b,c$ ghost fields. $X$ is targeted on a $S^1$ while $\phi$ is coupled to a background charge. At the self dual radius $R = 1/\sqrt{2}$ of the circle there is a $SU(2) \times SU(2)$ symmetry. The BRS cohomology classes are organised according to the representations of this group. These classes are also labeled by their ghost numbers. Of interest to us are the ghost number zero, one and two operators given respectively by $O_{s,p}(z)\overline{O}_{s,p'}(\bar{z})$, $Y_{s+1,p}(z)\overline{O}_{s,p'}(\bar{z})$ as well as $a(z,\bar{z})O_{s,p}(z)\overline{O}_{s,p'}(\bar{z})$ and $Y_{s+1,p}(z)\overline{Y}_{s,p'}(\bar{z})$. The complex conjugates of these operators should also be added to the list. In each case the subscript $s$ characterises the $SU(2) \times SU(2)$ content of each object. For a given integer or 1/2 integer $s$ the indices $p$ and $p'$ range from $-s$ to $+s$[6].

Now consider a Kaluza-Klein tower originating from a $q$-form field in $T^{11}$. For a given $U(1)$ charge $s$ we consider the modes at the bottom of each tower ( those which presumably have rational conformal dimensions in the boundary gauge theory). Our observation is that these Kaluza-Klein modes are in correspondence with the ghost number $q$ cohomology classes in the $c = 1$ theory. In Table 1 we have listed all the Kaluza-Klein modes and the corresponding objects in the $c = 1$ model. Note that the modes originating from the components of the metric in $T^{11}$ ( which are not $q$-forms in the internal space!) do not seem to have a counterpart in the $c = 1$ side. Furthermore the modes corresponding to an operator containing $a(z,\bar{z})$ in the $c = 1$ side can actually be gauged away in the Kaluza-Klein side.

The $c = 1$ theory has an infinite dimensional algebra given in terms of the volume preserving diffeomorphisms of the quadric cone $a_1 a_2 - a_3 a_4 = 0$. It is known that the base of this cone is isomorphic to $T^{11}$. In the context of present discussion the 3 complex dimensional Ricci flat cone is in fact identical to the subspace transverse to the $D_3$-brane solution.
of the type IIB supergravity. Our Kaluza-Klein background is a near horizon approximation to this $D_3$ brane geometry. Thus the cone seems to be the common geometrical entity in the two very different looking theories\(^1\).

Let us consider the quadric cone $a_1a_2 - a_3a_4 = 0$. This cone is singular at its apex $a_1 = a_2 = a_3 = a_4 = 0$. One can resolve the singularity by deforming the defining equation into $a_1a_2 - a_3a_4 = \mu$. From the point of $c = 1$ theory $\mu$ corresponds to the 2-dimensional cosmological constant. One can also consider a topological $\sigma$ model targeted on a CY three fold near a conical singularity, for which the local equation is the same as our quadratic expression. For both of these theories the free energies can be evaluated as a function of $\mu$ and can be expressed as a genus expansion. In [7] Ghoshal and Vafa argued that in fact the two theories must be the same. They observed that, at the self dual radius, the $g = 0, 1$ and 2 contributions to the free energy of the $c = 1$ theory agree with the corresponding terms of the free energy of the topological sigma model near the conifold singularity. Subsequently, assuming the type II-Heterotic duality, the results of[13] gave further support to the Ghoshal Vafa conjecture. These authors calculated the coefficient of the term $R^2F^{2g-2}$ in the effective action of the Heterotic theory compactified on $K_3 \times T^2$ and realised that for any $g$ the coefficient is also given by the genus $g$ term of the partition function of the $c = 1$ theory at the self dual radius. More recently Gopakumar and Vafa [14] have calculated the $\sigma$ model partition function near a conifold singularity and have proven the conjecture made in [7].

A better understanding of the correspondences noted above may require the unraveling of the relevance of the volume preserving diffeomorphisms of the cone in the $D_3$ brane context. On the basis of the observations made in this note we would like to think that the $c = 1$ theory at the self dual radius has a role to play in organising the chiral primaries of the boundary $SU(N) \times SU(N)$ superconformal gauge theory.

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\(^1\) Similar remarks can be made about the manifolds $T^{mn}$. In this case we should consider the $c = 1$ string theory at $n$ times the self-dual radius [12].
Table 1: The quantum numbers of the KK spectrum of type IIB string theory on \( AdS_5 \times T^{n'n'} \). Correspondence with the \( c = 1 \) model at the self dual radius holds for \( n=n'=1 \) and for each KK tower only for the modes at the bottom of the tower. The rank of the differential form on \( T^{11} \) is mapped to the ghost number on the \( c = 1 \) side. The \( c = 1 \) spectrum given in the table is at self dual radius and for \( s \geq 0 \) only. Similar analysis can be done for negative \( s \)

| KK excitations of IIB | Constraints on \( l \) and \( l' \) | \( c = 1 \) Analog |
|-----------------------|----------------------------------|--------------------|
| \( h_{\mu\nu}, A_{\mu\nu\rho\sigma}, B_{\mu\nu}, \Phi \) | \( l \geq | s_n |, l' \geq | s'_n | \) | \( O_{s,p}(z)\overline{O}_{s,p'}(\overline{z}) \) |
| \( h_{\mu+}, B_{\mu+}, A_{\mu\nu\rho+} \) | \( l \geq | s_n + 1 |, l' \geq | s'_n | \) | \( Y^+_{s+1,p}(z)\overline{O}_{s,p'}(\overline{z}) \) |
| \( A_{\mu\nu\rho-}, h_{\mu-}, B_{\mu-} \) | \( l \geq | s_n |, l' \geq | s'_n - 1 | \) | \( Y^+_{s,p}(z)\overline{O}_{s-1,p'}(\overline{z}) \) |
| \( h_{\mu-}, B_{\mu-}, A_{\mu\nu\rho-} \) | \( l \geq | s_n - 1 |, l' \geq | s'_n | \) | \( O_{s-1,p}(z)\overline{Y}^+_{s,p'}(\overline{z}) \) |
| \( A_{\mu\nu\rho+}, h_{\mu+}, B_{\mu+} \) | \( l \geq | s_n |, l' \geq | s'_n + 1 | \) | \( O_{s,p}(z)\overline{Y}^+_{s+1,p'}(\overline{z}) \) |
| \( h_{55}, A_{\mu\nu55}, B_{55} \) | \( l \geq | s_n |, l' \geq | s'_n | \) | \( a(z, \overline{z})O_{s,p}(z)\overline{O}_{s,p'}(\overline{z}) \) |
| \( h_{\pm \pm} \) | \( l \geq | s_n + 2,0 |, l' \geq | s'_n | \) | \( 0 \) |
| \( h_{\pm(\pm)'} \) | \( l \geq | s_n + 1,0 |, l' \geq | s'_n | \) | \( 0 \) |
| \( h_{(\pm)'(\pm)'} \) | \( l \geq | s_n |, l' \geq | s'_n + 2,0 | \) | \( 0 \) |
| \( h_{\pm 5} \) | \( l \geq | s_n + 1 |, l' \geq | s'_n | \) | \( 0 \) |
| \( h_{(\pm)'5} \) | \( l \geq | s_n |, l' \geq | s'_n + 1 | \) | \( 0 \) |
| \( h_{55} \) | \( l \geq | s_n |, l' \geq | s'_n | \) | \( 0 \) |
| \( A_{\mu\nu\rho-}, A_{\mu\nu\rho-}, B_{\mu-}, B_{\mu-} \) | \( l \geq | s_n |, l' \geq | s'_n | \) | \( Y^+_{s,p}(z)\overline{Y}^+_{s,p'}(\overline{z}) \) |
| \( A_{\mu\nu\rho+}, B_{\pm(\pm)'} \) | \( l \geq | s_n + 1 |, l' \geq | s'_n + 1 | \) | \( Y^+_{s+1,p}(z)\overline{Y}^+_{s+1,p'}(\overline{z}) \) |
| \( A_{\mu\nu55}, B_{55} \) | \( l \geq | s_n + 1 |, l' \geq | s'_n | \) | \( a(z, \overline{z})Y^+_{s+1,p}(z)\overline{O}_{s,p'}(\overline{z}) \) |
| \( A_{\mu\nu-5}, B_{-5} \) | \( l \geq | s_n |, l' \geq | s'_n - 1 | \) | \( Y^+_{s,p}(z)\overline{Y}^+_{s-1,p'}(\overline{z}) \) |
| \( A_{\mu\nu-5}, B_{-5} \) | \( l \geq | s_n - 1 |, l' \geq | s'_n | \) | \( a(z, \overline{z})O_{s-1,p}(z)\overline{Y}^+_{s,p'}(\overline{z}) \) |
| \( A_{\mu\nu+5}, B_{55} \) | \( l \geq | s_n |, l' \geq | s'_n + 1 | \) | \( a(z, \overline{z})O_{s,p}(z)\overline{Y}^+_{s+1,p'}(\overline{z}) \) |
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