Warm Forming Response of ZEK100 Sheet obtained under Biaxial Stretching with Full-Field Displacement Measurements

Bruce W. Williams1*, Jonathan McKinley1, Kevin P. Boyle1, Lucian Blaga1, Srihari Kurukuri2 and Michael J. Worswick2

1CanmetMATERIALS, Natural Resources Canada, 183 Longwood Road South, Hamilton, ON L8P 0A5, Canada
2University of Waterloo, 200 University Avenue West, Waterloo, ON N2L 3G1, Canada
*bruce.williams@canada.ca

Abstract. The warm forming response of ZEK100 sheet was studied between 150 °C and 250 °C using hydrostatic bulging coupled with full-field displacement mapping. Due to the strain-rate sensitivity of magnesium alloys at elevated temperature, it was important to ensure that the strain-rate remained reasonably constant during the bulge test. Various gas pressure versus time profiles were used to achieve strain-rates in the range of 0.01 s⁻¹ to 0.1 s⁻¹. The results from equi-biaxial bulge testing will be detailed in addition to bulge testing performed using elliptical dies with various aspect ratios to generate data under different biaxial stretching conditions. Supplementary characterization was provided using shear specimen data and tension test data from the rolling and transverse directions of the sheet. The biaxial, tensile, and shear data was used to calibrate a linear transformation-based anisotropic yield function at 200 °C. The yield function coefficients were optimized by matching the surface displacement data between experiments and finite element simulations of the bulge test. Agreement of the surface displacement data for a given applied pressure ensured that both the stress and strain in the bulged dome were accurately captured. Finite element simulations of the bulge tests were compared to experimental data to confirm accuracy of the calculated yield function.

1. Introduction
Magnesium sheet alloys are under consideration for application in lightweight automotive applications. It has been shown that warm forming can significantly improve the ductility of Mg sheet alloys by the activation of additional slip system at elevated temperatures. Anisotropic yield functions are required to adequately describe the plastic deformation response of hexagonal closed packed magnesium sheet alloys [1]. To generate data for yield surface calibration, several mechanical tests are required. Green et al. [2] used cruciform specimens to generate deformation data under biaxial tension of an aluminum sheet alloy to calibrate an anisotropic yield function under stretching conditions. However, complex cruciform specimen geometry and the potential for low strains to failure have limited adoption of this method. The current work considers the use of elliptical bulge tests to generate biaxial tension data under linear strain paths at the apex of the dome. Rees [3] details a theoretical framework for elliptical bulge testing with use of the Hill48 anisotropic yield function with experimental results for brass and steel sheet.
Traditionally, either stress or strain values (in the form of $r$-values) or a combination of both are used to calculate yield function coefficients through an optimization scheme [4]. The current work, considers elliptical bulge testing to generate biaxial tension data for yield function calibration. The stress at the peak of the dome in an equi-biaxial bulge test can be calculated from thin-membrane shell theory assuming a shell of revolution [5]. Elliptical bulge testing leads to the formation of an ellipsoid, which is not a shell of revolution and the stresses at the peak/pole of the dome are unknown [5]. Consequently, only the strain ($r$-values) could be used in any scheme to optimize anisotropic yield function coefficients.

Recent investigations have used full-field displacement measurements obtained from Digital Image Correlation (DIC) techniques to calibrate anisotropic yield functions. Rossi and Pierron [6] present a theoretical framework using full-field displacement predictions from finite element models of tension tests with the Hill48 yield function. Gross and Ravi-Chandar [7] utilized full-field displacement data from tension tests of a steel alloy to calibrate coefficients for the Hill48 yield function. In both cases, stress and strain data was not used in the yield function calibration. In the current work, the yield function coefficients for ZEK100 at 200 °C were calibrated by matching the surface displacements between experiment and finite element models of the elliptical bulge test. For a given surface displacement, the (gas) pressure applied in the simulations corresponded to the applied pressure in experiment.

2. Experimental Method
The sheet sample was clamped between heated tooling using a 500-ton hydraulic press. There were no draw-beads used in the setup, so only the clamp force would prevent draw-in on the sheet during bulging. Four dies of Elliptical Ratio (ER) equal to unity (equi-biaxial), 1.5, 2.0, and 2.5 with the dimensions given in Figure 1 were used in the bulge test; each die had a corner radius of 6.35 mm. The dimensions of the elliptical dies were selected to minimize the effects of sheet bending during bulging, particularly for the elliptical die with ER=2.5. The bulge test setup is depicted in Figure 2. For ER = 1.5, 2, and 2.5, specimens were tested with the rolling direction of the sheet oriented at either zero or 90 degrees relative to the major axis of the ellipse. For the equi-biaxial case, the sheet was oriented with the rolling direction of the sheet along the major axis. Therefore, there were seven configurations tested at a specified test temperature and strain-rate.

Full-field surface displacements were measured using a DIC system. The DIC cameras were located outside of the clamping area and a mirror was used to reflect the surface image of the bulged samples. The surface displacement data was then used to determine a surface equation for the deformed dome according to,

$$z(x, y) = a_0x^2 + a_1y^2 + a_2xy + a_3x + a_4y + a_5$$  

which is provided in the equi-biaxial bulge test standard ISO16808 [8], but is also applied to the elliptical dome in the current work. For the equi-biaxial case, the radius of curvature, $\rho$, of the dome can be calculated from the above equation which is required to calculate the biaxial stress, $\sigma_b$, according to,

$$\sigma_b = \frac{P\rho}{2t}$$  

where $P$ is the applied pressure and $t$ is the current thickness of the sheet, which can be calculated from the thickness strain, $\varepsilon_{33} = \ln(t/t_0)$, obtained from DIC data.
At room temperature water was used as the fluid to apply pressure. Argon gas was used to test between 150 °C and 250 °C. At these temperatures, the sheet was clamped between heated tooling and allowed to increase to the required test temperature before proceeding with the bulge test. Due to the compressible nature of gas, the system was pre-pressurized to about 1.0 MPa. It was possible that this initial pre-pressure would cause slight deformation of the sheet. However, the pre-pressurization step was required to ensure that there was adequate stroke of the gas cylinder to fully pressurize the sheet to failure, with failure pressures reaching as high as 5.0 MPa.

Figure 1. Dimension of elliptical dies used in bulge testing.

Figure 2. Depiction of bulge test setup [9].

3. Experiment Results

3.1. Uniaxial, Shear, and Equi-Biaxial Tension at 200 °C

The effective stress versus plastic strain response is shown in Figure 3 for uniaxial tensile tests with the axis of major strain oriented in the Rolling Direction (RD) or Transverse Direction (TD) of the sheet. Also shown are the responses obtained from a shear test and the equi-biaxial bulge test where the effective stress and strain values were calculated based on isotropic (von Mises) deformation. In all cases, the thickness of the ZEK100 sheet was 1.63 mm and the tests were performed at 200 °C. The tensile tests were carried out at a displacement rate of 2.4 mm/min and the shear test at a rate of 0.3 mm/min. There is only a slight difference in the flow stress between the RD and TD direction of the sheet and only the equi-biaxial response shows significant strain hardening. The sheet demonstrates plastic anisotropy as the effective uniaxial and equi-biaxial tension responses do not match when assuming isotropic (von Mises) deformation. The shear data will be used to calibrate the anisotropic shear coefficient, as detailed below. A more detailed description of the tensile behaviour of ZEK100 at elevated temperatures is given in [10]. The local shear zone is shown in Figure 4 and the specimen geometry is detailed in [11].

3.2. Equi-Biaxial Response at Various Temperatures and Strain-Rates

To maintain a reasonably constant strain-rate during gas bulge testing, the displacement rate of the gas cylinder was varied with time (achieved mainly through a trial and error process). The resulting effective (von Mises) strain versus time response is shown in Figure 5a. The figure shows that the effective strain-rate in the slow and fast tests was about 0.01 s⁻¹ and 0.1 s⁻¹, respectively. Particularly for the slow tests, the strain-rate increases at higher effective strains but it was not plausible with the current test setup to decrease the strain-rate any further. It is speculated that this was due to the gas being highly compressed at these levels of strain such that even a minute amount of stroke on the gas cylinder led to large displacements in the bulge dome (hence, higher strain-rate).
Figure 3. Effective (von Mises) stress vs. plastic strain response for ZEK100 at 200 °C for tension, shear, and equi-biaxial tension.

Figure 4. Shear test with digital image correlation (contours of shear angle).

Figure 5b shows the effective stress (von Mises) versus effective (von Mises) strain response obtained from equi-biaxial bulge tests performed at 150, 200, and 250 °C at the slow and fast strain-rates. The response measured from a room temperature bulge test on ZEK100 sheet is also shown in the figure. In all cases, the effective stress was plotted at a level above an effective strain of about 5%. It was found that at lower effective strains, the stress calculated from Equation 2 was erroneous which was due to a large radius of curvature for the sheet at the start of the bulge test. Several fluctuations were seen in the elevated temperature response that were not observed in the room temperature response. This was attributed to heat rising from the die affecting the DIC measurements. For the slow tests, high effective (von Mises) strains, almost 80%, were obtained before failure of the sheet. For all cases, there is only a small, linear strain hardening response of the sheet metal. The increase in strength with strain-rate was consistent for the three temperatures with an average increase of about 15 MPa for a strain-rate increase of factor ten.

3.3. Elliptical Bulge Test Response at 200 °C

The strains paths in the RD and TD directions obtained from the elliptical bulge tests of ZEK100 at 200 °C are shown in Figure 6. The effective (von Mises) strain-rate in the tests was about 0.01 s⁻¹. The figure shows there was a good range of strain paths obtained for the elliptical ratio tests and that the strain paths at the apex are linear. There was good formability for the equi-biaxial and ER=1.5, RD case. Less formability was obtained under different paths, particularly for the ER=2.5 cases where the sheet samples failed in the clamping region from contact with the elliptical die.
Figure 5. a) Effective strain vs. time and b) Effective stress vs. strain from equi-biaxial bulge tests performed between 150 °C and 250 °C and strain-rates between 0.01 s⁻¹ (slow) and 0.1 (fast) s⁻¹.

Figure 6. Transverse versus rolling direction strain measured for seven elliptical configurations for ZEK100 tested at 200 °C.

4. FEM Model and Optimization
The FEM models for each of the seven elliptical bulge test configurations comprised about 13,000 solid elements. Simulations were performed using the implicit solvers in the software WARP3D [12]. The effective stress (\(\bar{\sigma}\)) versus plastic strain (\(\dot{\varepsilon}^p\)) response was specified as \(\bar{\sigma} = 109 + 100\dot{\varepsilon}^p\) (MPa) which corresponded to the behaviour of ZEK100 at 200 °C in the rolling direction of the sheet (Figure 3). A non-evolving CPB06 asymmetric/anisotropic yield function was used to describe the deformation response of ZEK100, which is given by [1],

\[
\bar{\sigma} = B[(|\Sigma_1| - k\Sigma_1)^a + (|\Sigma_2| - k\Sigma_2)^a + (|\Sigma_3| - k\Sigma_3)^a]^{1/a}
\]  

(3)
where $k$ is a parameter describing the strength difference in tension and compression and $\Sigma_1$, $\Sigma_2$, $\Sigma_3$ and are the principal components of $\Sigma=Cs$. $C$ is the anisotropic transformation matrix and $s$ are the deviatoric stresses. In this equation $B$ is given by,

$$
B = \left[\frac{1}{(\phi_1 - k\phi_2)^a + (\phi_2 - k\phi_3)^a + (\phi_3 - k\phi_1)^a}\right]^{\frac{1}{a}}
$$

$$
\phi_1 = \left(\frac{2}{3}C_{11} - \frac{1}{3}C_{12} - \frac{1}{3}C_{13}\right)
$$

$$
\phi_2 = \left(\frac{2}{3}C_{12} - \frac{1}{3}C_{22} - \frac{1}{3}C_{23}\right)
$$

$$
\phi_3 = \left(\frac{2}{3}C_{13} - \frac{1}{3}C_{23} - \frac{1}{3}C_{13}\right)
$$

The parameters $C_{ij}$, $C_{ij}^s$, $C_{12}$, $C_{13}$, $C_{22}$, $C_{23}$, and $C_{33}$ were calculated using both the tensile data and the bulge test data, as detailed below. The shear parameter, $C_{44} (=C_{55} = C_{66})$, was determined from a shear test, after the in-plane coefficients were determined. In this work, $k=0$ as only the tension-tension strain space was considered. The yield function was implemented as a user-subroutine in WARP3D as detailed in [9].

For a given set of yield function coefficients, a set of seven simulations was performed corresponding to the seven elliptical bulge test configurations. The applied pressure in the bulge test corresponded to the measure pressured at a given point in the experiment. After the simulations were completed, at an effective strain corresponding to about 10% (the effective strain varies slightly depending on the yield function coefficients), the predicted bulge surface was fit to Equation 1. The corresponding data from the full-field surface measurement from DIC, also fit using Equation 1, was compared to the predicted surface and the $R^2$ value was calculated. The algorithms within Dakota [13] were used to find the optimal set of yield function coefficients by maximizing the agreement of $R^2$ between experiment and simulation based on an average of the seven configurations with equal weighting. It is important to note that the $r$-values and biaxial stresses were not used in the optimization, only the full-field displacement data.

5. Results and Discussion

The CPB06 yield surface determined to match the experiment bulge data is shown in Figure 7 for two values of the yield function exponents, $a=2$ and 6. The corresponding YF coefficients are given in Table 1. The YF exponent $a$ was not part of the optimization scheme and was required to have an integer value. The predicted elliptical bulge stresses based on the resultant yield function coefficients are shown for the seven configurations. Also shown in the figure are the predicted directions of plastic flow for the seven bulge test configurations.

Figure 8 and Figure 9 compare the predicted and measured shape of the bulge for the equi-biaxial case and for the case with ER=2.5 with the RD being along the major axis. The results in Figure 8 show that the equi-biaxial bulge test was better predicted with $a=2$. With both $a=2$ and $a=6$, the elliptical bulge is well predicted as seen in Figure 9. The main reason for using $a=6$ was to obtain better agreement of the strain ($r$-values) in the uniaxial RD and TD direction, but this came at the expense of a poorer prediction for the equi-biaxial tension. With $a=2$, the predicted $r$-values in the (uniaxial tension) RD and TD directions were about 4.8 and 2.0, respectively with the measured values ranging from 0.6 to 1.0. With $a=6$, the predicted $r$-values were improved with values of 1.1 and 1.0 in the RD and TD, respectively. More accurate agreement with the measured response could be obtained by modifying the optimization procedure (e.g. inclusion of $r$-values), using an evolving-CPB06 yield function, or employing a yield function with more coefficients. The shear coefficients, $C_{ij} (=C_{55} = C_{66})$, were determined by performing an optimization to match the predicted and measured shear response, where the experimental shear response was shown in Figure 3. The optimization was carried out after the in-plane yield function coefficients were calculated for both $a=2$ and $a=6$. 



\begin{table}[h]
\centering
\begin{tabular}{|c|c|c|c|c|c|}
\hline
\textbf{Configuration} & \textbf{RD} & \textbf{TD} & \textbf{Biaxial} & \textbf{Equi-biaxial} & \textbf{Experiment} \\
\hline
1 & 4.8 & 2.0 & 0.6 & 1.0 & 1.0 \\
\hline
2 & 4.5 & 1.9 & 0.7 & 1.2 & 1.2 \\
\hline
3 & 4.3 & 1.8 & 0.8 & 1.3 & 1.3 \\
\hline
4 & 4.2 & 1.7 & 0.9 & 1.4 & 1.4 \\
\hline
5 & 4.1 & 1.6 & 1.0 & 1.5 & 1.5 \\
\hline
6 & 4.0 & 1.5 & 1.1 & 1.6 & 1.6 \\
\hline
7 & 3.9 & 1.4 & 1.2 & 1.7 & 1.7 \\
\hline
8 & 3.8 & 1.3 & 1.3 & 1.8 & 1.8 \\
\hline
9 & 3.7 & 1.2 & 1.4 & 1.9 & 1.9 \\
\hline
10 & 3.6 & 1.1 & 1.5 & 2.0 & 2.0 \\
\hline
\end{tabular}
\caption{Predicted and Measured Shear Response for Various Configurations.}
\end{table}
Table 1. CPB06 Yield Function Coefficients determined for ZEK100 at 200 °C.

| Parameter | Set | $a$ | $k$ | $C_{11}$ | $C_{12}$ | $C_{13}$ | $C_{22}$ | $C_{23}$ | $C_{33}$ | $C_{44}$ |
|-----------|-----|-----|-----|---------|---------|---------|---------|---------|---------|---------|
|           | PS1 | 2   | 0   | 1.158   | 0.015   | 0.200   | 1.010   | 0.700   | 1.000   | 2.194   |
|           | PS2 | 6   | 0   | 1.043   | 0.000   | 0.000   | 1.157   | 0.000   | 0.700   | 2.492   |

Figure 7. Resultant yield surface with yield function exponent a) $a=2$ and b) $a=6$.

Figure 8. Predicted vs. measured (magenta from DIC) deformation (1/4 symmetry) for the equi-biaxial bulge test on ZEK100 at 200 °C for a) $a=2$ and b) $a=6$ (predicted contours of effective plastic strain).

Figure 9. Predicted vs. measured (magenta from DIC) deformation (1/4 symmetry) for the bulge test with ER=2.5 (RD) on ZEK100 at 200 °C for a) $a=2$ and b) $a=6$ (predicted contours of effective plastic strain).
6. Conclusions
The warm forming response of ZEK100 sheet was studied between 150 and 250 °C using hydrostatic bulging coupled with full-field displacement mapping. Bulge tests were performed with strain-rates ranging from 0.01 s\(^{-1}\) to 0.1 s\(^{-1}\). It was shown that the strain-rate should remain relatively constant during the bulge test at these temperatures to isolate the influence of strain-rate on the flow stress response of the material. Bulge testing using elliptical dies was performed to provide experimental data for ZEK100 at 200 °C in the tension-tension quadrant of strain space. Anisotropic yield function coefficients were determined by matching the measured and predicted surface data from FE simulation through an optimization scheme. R-values and biaxial stress values were not used in the calibration. It was found that an accurate yield function of the sheet alloy could be calibrated by considering only full-field displacement measurements obtain from the elliptical bulge test.

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