A proposal of a spin separator based on the spin Zeeman effect in Y-shaped nanostructure with a quantum point contact is presented. Our calculations show that the appropriate tuning of the quantum point contact potential and the external magnetic field leads to the spin separation of the current: electrons with opposite spins flow through the different output branches. We demonstrate that this effect is robust against the scattering on impurities. The proposed device can also operate as a spin detector, in which – depending on the electron spin – the current flows through one of the output branches.

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used both for the separation and detection of the electron spin. Moreover, if the nanodevice operates in the regime, in which the current is carried through the edge states, then – in analogy to the quantum Hall effect – the spin separation is robust against the scattering.

We consider the Y-shaped two-dimensional nanostructure with the QPC located near the contact 2 [Fig. 1(c)]. Experimentally, similar construction but in the quantum ring geometry has been fabricated by the use of the lithography technique. In the presence of the external magnetic field \( B = (0, 0, B) \), the Hamiltonian of the system takes on the form

\[
\hat{H} = \left[ -i \hbar \nabla + e \mathbf{A} \right]^2 + U(r) 1 + \frac{1}{2} g \mu_B B \sigma_z ,
\]

where \( \mathbf{A} = (yB, 0, 0) \) is the vector potential, \( m_e \) is the conduction-band mass, \( \mathbf{1} \) is the \( 2 \times 2 \) unit matrix, and \( \sigma_z \) is the \( z \)-spin Pauli matrix. Potential energy \( U(r) = U_\ell(x, y) + U_{QPC}(x, y) \) is the sum of the Y-shaped confinement potential energy \( U_\ell(x, y) \) (we assume the hard-wall confinement in the \( y \) direction) and the electron potential energy in the QPC

\[
U_{QPC}(x, y) = \frac{1}{2} m_e \omega^2 y^2 \exp\left[-\frac{(x - x_0)^2}{2d^2}\right] ,
\]

where \( d \) determines the \( x \)-extension of the QPC, \( x_0 \) defines its position, and \( \hbar \omega \) is the energy of the transverse parabolic confinement in the QPC region. We assume that the confinement in the \( z \) direction is so strong that electrons occupy the ground-state resulting from the size quantization along this axis. In the calculations, we adopt the following geometrical parameters: width of the channel \( W = 40 \text{ nm} \), length of the nanostructure \( L = 1000 \text{ nm} \), \( d = 40 \text{ nm} \), and \( x_0 = 200 \text{ nm} \). We use the material parameters corresponding to In_{0.5}Ga_{0.5}As, i.e. \( m_e = 0.0465 m_0 \) and \( g = -8.97 \), however the spin separation effect will be observed for any semiconductor material with sufficiently large spin Zeeman splitting. The numerical calculations have been performed using the tight-binding method on the square lattice with \( \Delta x = \Delta y = 2 \text{ nm} \) and the hopping energy \( t = \hbar^2 / (2m_e \Delta x^2) \). We have used the Kwant package to determine the spin-dependent conductance \( G_{i,j}^{u(d)} \) between the contacts \( i \) and \( j \) \( (i, j = 1, 2, 3) \), and \( u \) and \( d \) correspond to spin-up and spin-down, respectively. In the proposed nanostructure electrons are injected from the lead 1 acting as the input and flow out from the device via the leads 2 and 3 (see Fig. 1). Figure 2 (upper panels) presents the spin-dependent conductance \( G_{i,j}^{u(d)} \) as a function of the QPC confinement energy \( \hbar\omega \) for magnetic field (a) \( B = 1 \text{ T} \) and (b) \( B = 3 \text{ T} \). We see that the rapid decrease of the conductance between the contacts 1 and 2 for the spin-up electrons occurs for the higher energy than for the spin-down electrons. Simultaneously, for both the spin polarizations the decrease of the electron transmission into the channel 2 is accompanied by the increase of the transmission into the channel 3. This means that in the confinement energy regime, for which the conductance of the spin-up electrons through the channel 2 is still high, the spin-down electrons are reflected from the QPC and flow through the channel 3. The current splits into the two spin-polarized beams. The splitting effect is quantitatively presented in Figs. 2 (c,d) which show the differences of the conductances \( P_{12} = G_{12}^{u} - G_{12}^{d} \) and \( P_{13} = G_{13}^{u} - G_{13}^{d} \). In panel (d), the vertical arrows mark the values of \( \hbar\omega \) chosen to present the results in Fig. 3.

In the presence of the external magnetic field \( B = (0, 0, B) \), the Hamiltonian of the system takes on the form

\[
\hat{H} = \left[ -i \hbar \nabla + e \mathbf{A} \right]^2 + U(r) 1 + \frac{1}{2} g \mu_B B \sigma_z ,
\]

where \( \mathbf{A} = (yB, 0, 0) \) is the vector potential, \( m_e \) is the conduction-band mass, \( \mathbf{1} \) is the \( 2 \times 2 \) unit matrix, and \( \sigma_z \) is the \( z \)-spin Pauli matrix. Potential energy \( U(r) = U_\ell(x, y) + U_{QPC}(x, y) \) is the sum of the Y-shaped confinement potential energy \( U_\ell(x, y) \) (we assume the hard-wall confinement in the \( y \) direction) and the electron potential energy in the QPC

\[
U_{QPC}(x, y) = \frac{1}{2} m_e \omega^2 y^2 \exp\left[-\frac{(x - x_0)^2}{2d^2}\right] ,
\]

where \( d \) determines the \( x \)-extension of the QPC, \( x_0 \) defines its position, and \( \hbar \omega \) is the energy of the transverse parabolic confinement in the QPC region. We assume that the confinement in the \( z \) direction is so strong that electrons occupy the ground-state resulting from the size quantization along this axis. In the calculations, we adopt the following geometrical parameters: width of the channel \( W = 40 \text{ nm} \), length of the nanostructure \( L = 1000 \text{ nm} \), \( d = 40 \text{ nm} \), and \( x_0 = 200 \text{ nm} \). We use the material parameters corresponding to In_{0.5}Ga_{0.5}As, i.e. \( m_e = 0.0465 m_0 \) and \( g = -8.97 \), however the spin separation effect will be observed for any semiconductor material with sufficiently large spin Zeeman splitting. The numerical calculations have been performed using the tight-binding method on the square lattice with \( \Delta x = \Delta y = 2 \text{ nm} \) and the hopping energy \( t = \hbar^2 / (2m_e \Delta x^2) \). We have used the Kwant package to determine the spin-dependent conductance \( G_{i,j}^{u(d)} \) between the contacts \( i \) and \( j \) \( (i, j = 1, 2, 3) \), and \( u \) and \( d \) correspond to spin-up and spin-down, respectively. In the proposed nanostructure electrons are injected from the lead 1 acting as the input and flow out from the device via the leads 2 and 3 (see Fig. 1). Figure 2 (upper panels) presents the spin-dependent conductance \( G_{i,j}^{u(d)} \) as a function of the QPC confinement energy \( \hbar\omega \) for magnetic field (a) \( B = 1 \text{ T} \) and (b) \( B = 3 \text{ T} \). We see that the rapid decrease of the conductance between the contacts 1 and 2 for the spin-up electrons occurs for the higher energy than for the spin-down electrons. Simultaneously, for both the spin polarizations the decrease of the electron transmission into the channel 2 is accompanied by the increase of the transmission into the channel 3. This means that in the confinement energy regime, for which the conductance of the spin-up electrons through the channel 2 is still high, the spin-down electrons are reflected from the QPC and flow through the channel 3. The current splits into the two spin-polarized beams. The splitting effect is quantitatively presented in Figs. 2 (c,d) which show the differences of the conductances \( P_{12} = G_{12}^{u} - G_{12}^{d} \) and \( P_{13} = G_{13}^{u} - G_{13}^{d} \). In Figs. 2 (c,d) the spin separation of the current is revealed as the peak (dip), which corresponds to the spin-up (spin-down) polarization of the current flowing through the corresponding channel.

The spin separation mechanism proposed in this paper results from the joint effect of the spin Zeeman splitting and the formation of edge states. If the magnetic field is applied, the spin degeneration of transverse electron states is lifted by the Zeeman effect [cf. the dispersion relations in Fig. 3(a,b)]. In the calculations we have adjusted the Fermi level in the leads so that only the two lowest transverse states (one corresponding to spin-up and one corresponding to spin-down) are occupied [see Fig. 3(b)]. The spin-up and spin-down electrons with the chosen energy are injected into the system from the lead 1 and flow towards the QPC located in the channel 2. Due to the spin Zeeman splitting the reflection probabilities in the QPC region for spin-up and spin-down electrons are different. The increase of the confinement energy \( \hbar\omega \) leads to the increase of the transverse state energies in the QPC. In particular, we can tune \( \hbar\omega \) so that only the spin-up energy level is located below the Fermi energy [see Fig. 3(a)]. The absence of the available spin-down electron states in the QPC region results in a backscattering of spin-down electrons. On the other hand, the
FIG. 3. Electron density (a,c,e) and spin density (b,d,f) in the nanostructure for (a,b) $\hbar \omega = 2$ meV, (c,d) $\hbar \omega = 14$ meV, and (e,f) $\hbar \omega = 25$ meV. The gray rectangle represents the position of the QPC.

spin-up electrons still have a high transmission probability through the QPC. Therefore, the current splits into the two spin-polarized electron beams. In order to obtain the full separation of the electrons with opposite spins we exploit the orbital effect, which causes that the transport of electrons is carried by the edge states that are formed in a sufficiently high magnetic field. Due to the orbital effect, the electrons with negative and positive velocities (along the $x$-axis) are spatially separated. The electrons flowing in a certain direction are always shifted, by the Lorentz force, to the right boundary of the conductive channel with respect to the direction of the current flow (cf. Fig. 3). This causes that the spin-down electrons backscattered from the QPC are injected into the channel 3. In other words, the orbital effect prevents the spin-down polarized electrons reflected from the QPC to flow back into the channel 1. The resulting spin separation effect is clearly demonstrated in Fig. 3. For $\hbar \omega = 5$ meV both the spin-up and spin-down electrons are transmitted through the QPC [Fig. 3(a)] and reach the contact 2. We see that the current is partially spin polarized [Fig. 3(a)], which is a consequence of unequal electron densities of states at the Fermi level which results from the spin Zeeman effect. For $\hbar \omega = 14$ meV [Fig. 3(b)] the Y-shaped nanostructure acts as the (almost) perfect spin splitter. Depending on their spins the electrons injected from the contact 1 are either transmitted through the QPC into the channel 2 or reflected into the channel 3. As described above, the backscattering of the spin-down electrons is very strong [cf. Fig. 3(d)]. We have found that the nanodevice with the parameters of Fig. 3(d) is the optimal realization of the spin separator, in which the electrons with the opposite spins are spatially separated and leave the nanostructure via the different conduction channels. The further increase of the confinement energy $\hbar \omega$ [Fig. 3(e,f)] causes that in the QPC region there are no available quantum states for both the spin-up and spin-down electrons. The electrons with either spin are fully reflected from the QPC and due to the orbital effect flow through the channel 3.

The results presented so far have been obtained with the neglect of the scattering. Nevertheless, form the experimental point of view it is desirable to construct the spin selector which acts in the non-ballistic regime. Now, we will show that the nanodevice proposed in our paper is robust against the scattering. Due to the orbital effect, the electrons flowing in a certain channel are moved towards the edge according to the Lorentz force direction. For the sufficiently strong magnetic field, the currents flowing in the opposite directions are transported through the edge states localized at two opposite sides of the channel. If we increase the magnetic field, the separation between edge states carrying the current in the opposite directions increases. This effect leads to the suppression of the backscattering since the electron can change its momentum only if it is scattered from the edge state localized on one side of the channel to that on the other side. The scattering is suppressed due the vanishingly small overlap between the wave functions localized on the opposite sides of the channel. This mechanism is well known and is the origin of the 'zero' resistance in the quantum Hall effect. We have quantitatively described this effect introducing the model according to which the spin-independent scattering is included by assuming that the transfer energy $t$ is uniformly distributed within the range $W/2 < t < W/2$. The relation between the strength of scatterers and the mean free path $\ell$ is given by:

$$\frac{W}{E_F} = \left( \frac{6\lambda_F^3}{\pi^3 \Delta x^2 \ell} \right)^{1/2},$$

where $\lambda_F$ is the Fermi wavelength. In our calculations, we have applied the realistic values of the mean free path, which was experimentally measured to be greater than 1 $\mu$m for the 2DEG in InGaAs. Fig. 4 shows $P_{12}$ and $P_{13}$ calculated in the non-ballistic regime as a function of $\hbar \omega$ for several values of the mean free path. The results presented in Fig. 4 have been obtained by averaging over $10^4$ computational runs for each value of the energy $\hbar \omega$. We see that – in the considered non-ballistic regime – the scattering does not affect the spin-splitting effect. The pronounced peak and dip, which demonstrate spin separation are still clearly visible. We should emphasize that the insensitivity of the scattering becomes greater, if
QPCs. We have shown that by the appropriate tuning of spin separator based on the Y-shaped 2DEG with the neglect the spin-orbit interaction in our calculations.

the effect of the external magnetic field is dominating, we neglect the spin-orbit interaction in our calculations.

In conclusion, we have proposed the non-ballistic spin separator based on the Y-shaped 2DEG with the QPC. We have shown that by the appropriate tuning of confinement energy in the QPC, the input unpolarized current can be splitted into two fully spin-polarized beams, whereas the electrons with opposite spins flow through the different branches of the nanostructure. The separation mechanism has been explained as the joint effect of the spin Zeeman splitting and transport via the edge states generated in the external magnetic field. We show that the proposed spin separation mechanism is robust against the scattering. Although the results have been presented as a function of the QPC confinement energy $\hbar \omega$, the input unpolarized current can be splitted into two fully spin-polarized beams, whereas the electrons with opposite spins flow through the different branches of the nanostructure. The separation mechanism has been explained as the joint effect of the spin Zeeman splitting and transport via the edge states generated in the external magnetic field. We show that the proposed spin separation mechanism is robust against the scattering. Although the results have been presented as a function of the QPC confinement energy $\hbar \omega$, in the experimental realization this energy can be tuned by changing the voltage applied to the QPC contacts. In this structure the spin separation effect can be easily switched on/off by the change of the voltage applied to the nearby gate. It is also worth noting that if the fully spin polarized current is injected from the input electrode, the current will flow through only one of the output branches. In this case, when measuring the current in both the output, we obtain the information about the spin polarization of the current injected into the system. This means that the proposed nanodevice can also acts as a detector of the spin polarized current.

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FIG. 4. Spin polarization $P_{12}$ and $P_{13}$ as a function of the QPC confinement energy $\hbar \omega$. Results calculated in the non-ballistic regime for different values of mean free path $\ell$.

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