We show that in a Randall-Sundrum II type braneworld, the vacuum exterior of a spherical star is not in general a Schwarzschild spacetime, but has radiative-type stresses induced by 5-dimensional graviton effects. Standard matching conditions do not lead to a unique exterior on the brane because of these 5-dimensional graviton effects. We find an exact uniform-density stellar solution on the brane, and show that the general relativity upper bound $GM/R < \frac{4}{\alpha}$ is reduced by 5-dimensional high-energy effects. The existence of neutron stars leads to a constraint on the brane tension that is stronger than the big bang nucleosynthesis constraint, but weaker than the Newton-law experimental constraint. We present two different non-Schwarzschild exteriors that match the uniform-density star on the brane, and we give a uniqueness conjecture for the full 5-dimensional problem.

I. INTRODUCTION

String theory and M-theory describe gravity as a truly higher-dimensional interaction, which becomes effectively 4-dimensional at low enough energies. In braneworld models inspired by these theories, the observable universe is a 3-brane (domain wall) to which standard-model fields are confined, while gravity can access the extra spatial dimensions. Randall and Sundrum [1] showed how gravity could be localized near the brane at low energies even with a noncompact extra dimension. The warped spacetime metric satisfies the 5-dimensional Einstein equations with negative cosmological constant. Their models have been generalized to allow for arbitrary energy-momentum tensor on the brane [2].

The cosmological implications of these braneworld models have been extensively investigated (see e.g. the review [3] for further references). Significant deviations from Einstein's theory occur at very high energies, as in the very early universe. Gravitational collapse can also produce very high energies where 5-dimensional corrections would become significant. If an horizon forms, then the high-energy effects eventually become disconnected from the outside region on the brane. However, they could leave a signature on the brane. In addition to local high-energy effects, there are also nonlocal corrections arising from the imprint on the brane of Weyl curvature in the bulk, i.e. from 5-dimensional graviton stresses. These nonlocal Weyl stresses arise on the brane whenever there is inhomogeneity in the density; the inhomogeneity on the brane generates Weyl curvature in the bulk which ‘backreacts’ on the brane. Anyway we can have these nonlocal Weyl stresses even if the density is homogeneous, as we show in the case of static stars.

The high-energy (local) and bulk graviton stress (nonlocal) effects combine to significantly alter the matching problem on the brane, compared with the general relativistic case. For spherical compact objects (uncharged and non-radiating), matching in general relativ-
form star case is even more complicated. It is in principle possible to integrate numerically into the bulk (assuming appropriate boundary conditions) to find the 5-dimensional metric for which these stellar solutions are brane boundaries. However, even in the much simpler case of black hole solutions, further investigation is needed into the 5-dimensional aspects of stellar solutions and their exteriors. Perturbative studies of the static weak-field regime show that the leading order correction to the Newtonian potential on the brane is given by

$$\Phi = \frac{GM}{r} \left(1 + \frac{2r^2}{3\ell^2}\right),$$  

(1)

where $\ell$ is the curvature scale of 5-dimensional anti de Sitter spacetime (AdS$_5$). This result assumes that the bulk perturbations are bounded in conformally Minkowski coordinates, and that the bulk is nearly AdS$_5$. It is not clear whether there is a covariant way of uniquely characterizing these perturbative results, and therefore it remains unclear what the implications of the perturbative results are for very dense stars on the brane. However, it seems reasonable to conjecture that the bulk should be asymptotically AdS$_5$, and that its Cauchy horizon should be regular. Then perturbative results suggest that on the brane, the weak-field potential should behave as in Eq. (1). In fact, perturbative analysis also constrains the weak-field behaviour of other metric components on the brane, as well as of the nonlocal stresses on the brane induced by the bulk Weyl tensor.

II. FIELD EQUATIONS AND MATCHING CONDITIONS

The local and nonlocal extra-dimensional modifications to Einstein’s equations on the brane may be consolidated into an effective total energy-momentum tensor:

$$G_{\mu\nu} = \kappa^2 T_{\mu\nu}^{\text{eff}},$$  

(2)

where $\kappa^2 = 8\pi G$ and the bulk cosmological constant is chosen so that the brane cosmological constant vanishes. The effective total energy density, pressure, anisotropic stress and energy flux for a perfect fluid are

$$\rho^{\text{eff}} = \rho \left(1 + \frac{\rho}{2\lambda}\right) + \frac{6}{\kappa^4\lambda} U,$$  

(3)

$$p^{\text{eff}} = p + \frac{\rho}{2\lambda} (\rho + 2p) + \frac{2}{\kappa^4\lambda} U,$$  

(4)

$$\sigma^{\text{eff}}_{\mu\nu} = \frac{6}{\kappa^4\lambda} P_{\mu\nu},$$  

(5)

$$q_\mu^{\text{eff}} = \frac{6}{\kappa^4\lambda} Q_\mu,$$  

(6)

where $\lambda$ is the brane tension, and general relativity is regained in the limit $\lambda^{-1} \to 0$.

From big bang nucleosynthesis constraints, $\lambda \gtrsim 1$ MeV$^4$, but a much stronger bound arises from null results of sub-millimetre tests of Newton’s law: $\lambda \gtrsim 10^8$ GeV$^4$.

The local effects of the bulk, arising from the brane extrinsic curvature, are encoded in the quadratic terms, $\sim (T_{\mu\nu})^2/\lambda$, which are significant at high energies, $\rho \gtrsim \lambda$. The nonlocal bulk effects, arising from the bulk Weyl tensor, are carried by nonlocal energy density $U$, nonlocal energy flux $Q_\mu$ and nonlocal anisotropic stress $P_{\mu\nu}$. Five-dimensional graviton stresses are imprinted on the brane via these nonlocal Weyl terms.

Static spherical symmetry implies $Q_\mu = 0$ and

$$P_{\mu\nu} = P(r u_\mu - \frac{1}{\lambda} h_{\mu\nu}),$$  

(7)

where $\rho$ is a unit radial vector, and $h_{\mu\nu} = g_{\mu\nu} + u_\mu u_\nu$ projects into the rest space of static observers with 4-velocity $u^\mu$. The brane energy-momentum tensor separately satisfies the usual conservation equations, $\nabla^\nu T_{\mu\nu}^{\text{eff}} = 0$, and the 4-dimensional Bianchi identities on the brane imply that the effective energy-momentum tensor is also conserved: $\nabla^\nu T_{\mu\nu}^{\text{eff}} = 0$. For static spherical symmetry, these conservation equations reduce to

$$D_\mu p + (\rho + p) A_\mu = 0,$$  

(8)

$$\frac{4}{3} D_\mu U + \frac{4}{3} U A_\mu + D^\nu P_{\mu\nu} = -\frac{4}{3} \kappa^4 (\rho + p) D_\mu \rho,$$  

(9)

where $D_\mu$ is the covariant spatial derivative and $A_\mu$ is the 4-acceleration. In static coordinates the metric is

$$ds^2 = -A^2(r)dt^2 + B^2(r)dr^2 + r^2 d\Omega^2,$$  

(10)

and Eqs. (3)–(10) imply

$$\frac{1}{r^2} - \frac{1}{B^2} \left(1 - \frac{2 B'}{B} \right) = 8\pi G \rho^{\text{eff}},$$  

(11)

$$-\frac{1}{r^2} + \frac{2}{B^2} \left(\frac{A'}{r} + \frac{2 A'}{A}\right) = 8\pi G \left(p^{\text{eff}} + \frac{4}{\kappa^4\lambda} P\right),$$  

(12)

$$p' + \frac{A'}{A} (\rho + p) = 0,$$  

(13)

$$U' + 4 \frac{A'}{A} U + 2 P' + 2 \frac{A'}{A} P + \frac{6}{r} P = -2(4\pi G)^2 (\rho + p)'.$$  

(14)

The exterior is characterized by

$$\rho = 0 = p, \quad U = U^+, \quad P = P^+, \quad (\rho + p)',$$  

(15)

so that in general $\rho^{\text{eff}}$ and $p^{\text{eff}}$ are nonzero in the exterior: there are in general Weyl stresses in the exterior, induced by bulk graviton effects. These stresses are radiative, since their energy-momentum tensor is traceless ($p^{\text{eff}} = \frac{1}{3} \rho^{\text{eff}}$). The system of equations for the exterior is not closed until a further condition is given on $U^+$, $P^+$ (e.g., we could impose $P^+ = 0$ to close the system). In other words, from a brane observer’s perspective, there are many possible static spherical exterior metrics, including the simplest case of Schwarzschild ($U^+ = 0 = P^+$).
The interior has nonzero $\rho$ and $p$; in general, $\mathcal{U}^-$ and $\mathcal{P}^-$ are also nonzero, since by Eq. (14), density gradients are a source for Weyl stresses in the interior. For a uniform density star, we can have $\mathcal{U}^- = 0 = \mathcal{P}^-$, but nonzero $\mathcal{U}^-$ and/or $\mathcal{P}^-$ are possible, subject to Eq. (14) with zero right-hand side.

From Eq. (11) we obtain
\begin{equation}
B^2(r) = \left[1 - \frac{2Gm(r)}{r}\right]^{-1},
\end{equation}
where the mass function is
\begin{equation}
m(r) = 4\pi \int_a^r \rho^{\text{eff}}(r')r'^2dr',
\end{equation}
and $a = 0$ for the interior solution, while $a = R$ for the exterior solution. Equation (13) integrates in the interior for $\rho = \text{const}$ to give
\begin{equation}
A^-(r) = \frac{\alpha}{\rho + p(r)},
\end{equation}
where $\alpha$ is a constant.

The Israel-Darmois matching conditions at the stellar surface $\Sigma$ give [1]
\begin{equation}
\left[G_{\mu\nu}r^\nu\right]_{\Sigma} = 0,
\end{equation}
where $[f]_{\Sigma} \equiv f(R^+) - f(R^-)$. By the brane field equation (2), this implies $[\mathcal{T}^{\text{eff}}_{\mu\nu}r^\nu]_{\Sigma} = 0$, which leads to
\begin{equation}
\left[p^{\text{eff}} + \frac{4}{\kappa^2 \lambda} \mathcal{P}\right]_{\Sigma} = 0.
\end{equation}
Even if the physical pressure vanishes at the surface, the effective pressure is nonzero there, so that in general a radial stress is needed in the exterior to balance this effective pressure.

Assuming that the physical pressure vanishes on the surface, i.e. $p(R) = 0$, this becomes
\begin{equation}
(4\pi G)^2 \rho^2(R) + \mathcal{U}^-(R) + 2\mathcal{P}^-(R) = \mathcal{U}^+(R) + 2\mathcal{P}^+(R).
\end{equation}
Note that we have multiplied through by $\lambda$ to obtain this form, so that there is no general relativity limit of the equation.

In general relativity, Eq. (20) implies
\begin{equation}
p(R) = 0,
\end{equation}
whereas for the braneworld model, we take this as a (physically reasonable) assumption.

Equation (21) gives the matching condition for any static spherical star with vanishing pressure at the surface. If there are no Weyl stresses in the interior, i.e. $\mathcal{U}^- = 0 = \mathcal{P}^-$, and if the energy density is non-vanishing at the surface, $\rho(R) \neq 0$, then there must be Weyl stresses in the exterior, i.e. the exterior cannot be Schwarzschild.

Equivalently, if the exterior is Schwarzschild and the energy density is nonzero at the surface, then the interior must have nonlocal Weyl stresses.

We will further assume that $\mathcal{P}^- = 0$, which is consistent with the isotropy of the physical pressure in the star, so that
\begin{equation}
\mathcal{U}^-(r) = \frac{\beta}{[A^-(r)]^4},
\end{equation}
where $\beta$ is a constant. The matching condition in Eq. (21) then reduces for a uniform star to
\begin{equation}
(4\pi G)^2 \rho^2 + \frac{\beta}{\alpha \pi \rho^4} = \mathcal{U}^+(R) + 2\mathcal{P}^+(R).
\end{equation}
It follows that the exterior of a uniform star cannot be Schwarzschild if there are no Weyl stresses in the interior.

The Weyl stresses arise from the projection of the bulk Weyl tensor, which responds nonlocally to the gravitational field on the brane, and ‘backreacts’ on the brane. Thus in general, we expect that Weyl stresses will occur in both the interior and exterior. However, it is possible to find consistent solutions on the brane with Weyl stresses only in the exterior. The general case of an interior with Weyl stresses is much more complicated.

### III. Braneworld Generalization of Exact Uniform-Density Solution

With uniform density and $\mathcal{U}^- = 0 = \mathcal{P}^-$, we have the case of purely local (high-energy) modifications to the general relativity uniform-density solution, i.e. to the Schwarzschild interior solution [12]. The interior mass function is
\begin{equation}
m^-(r) = M \left[1 + \frac{3M}{8\pi \lambda R^3} \left(\frac{r}{R}\right)^3\right],
\end{equation}
where $M = 4\pi R^3 \rho/3$. Thus
\begin{equation}
B^-(r) = \frac{1}{\Delta(r)},
\end{equation}
and the pressure is given by
\begin{equation}
p(r) = \frac{\rho}{\Delta(r)} \left[\Delta(r) - \Delta(R)(1 + \rho/\lambda)\right]/[3\Delta(R) - \Delta(r)] \rho/\lambda,
\end{equation}
where
\begin{equation}
\Delta(r) = \left[1 - \frac{2GM}{r} \left(\frac{r}{R}\right)^3 \left\{1 + \frac{\rho}{2\lambda}\right\}\right]^{1/2}.
\end{equation}
In the general relativity limit, $\lambda^{-1} \to 0$, we regain the known exact solution [13]. The high-energy corrections considerably complicate the exact solution.

Since $\Delta(R)$ must be real, we find an astrophysical lower limit on $\lambda$, independent of the Newton-law and cosmological limits:
In particular, this implies $R > 2GM$, so that the Schwarzschild radius is still a limiting radius, as in general relativity. Taking a typical neutron star (assuming uniform density) with $\rho \sim 10^9$ MeV$^4$ and $M \sim 4 \times 10^{57}$ GeV, we find

$$\lambda > 5 \times 10^8 \text{ MeV}^4.$$  \hspace{1cm} (30)

This is the astrophysical limit, below which stable neutron stars could not exist on the brane. It is much stronger than the cosmological nucleosynthesis constraint, but much weaker than the Newton-law lower bound. Thus stable neutron stars are easily compatible with braneworld high-energy corrections, and the deviations from general relativity are very small. If we used the lower bound in Eq. (29) allowed by the stellar limit, then the corrections to general relativistic stellar models would be significant, as illustrated in Fig. 1.

We can also obtain an upper limit on compactness from the requirement that $p(r)$ must be finite. Since $p(r)$ is a decreasing function, this is equivalent to the condition that $p(0)$ is finite and positive, which gives the condition

$$\frac{GM}{R} \leq \frac{4}{9} \left[ 1 + \frac{7\rho/4\lambda + 5\rho^2/8\lambda^2}{(1 + \rho/\lambda)^2(1 + \rho/2\lambda)} \right].$$  \hspace{1cm} (31)

It follows that high-energy braneworld corrections reduce the compactness limit of the star. For the stellar bound on $\lambda$ given by Eq. (31), the reduction would be significant, but for the Newton-law bound, the correction to the general relativity limit of $\frac{4}{9}$ is very small. The lowest order correction is given by

$$\frac{GM}{R} \leq \frac{4}{9} \left[ 1 - \frac{3\rho}{4\lambda} + O\left(\frac{\rho^2}{\lambda^2}\right) \right].$$  \hspace{1cm} (32)

For $\lambda \sim 10^8$ GeV$^4$, the minimum allowed by submillimetre experiments, and $\rho \sim 10^9$ MeV$^4$, the fractional correction is $\sim 10^{-11}$.

As argued above, any exterior solution that matches this uniform-density solution cannot be a Schwarzschild exterior. We will now present two possible exterior solutions.

### IV. TWO POSSIBLE NON-SCHWARZSCHILD EXTERIOR SOLUTIONS

The system of equations satisfied by the exterior spacetime on the brane is not closed. Essentially, we have two independent unknowns $U^+$ and $P^+$ satisfying one equation, i.e. Eq. (14) with zero right-hand side. Even requiring that the exterior must be asymptotically Schwarzschild does not lead to a unique solution. Further investigation of the 5-dimensional solution is needed in order to determine what the further constraints are. We are able to find two exterior solutions (with $U^+ = 0 = \mathcal{P}^+$) that are consistent with all equations and matching conditions on the brane, and that are asymptotically Schwarzschild.

The first is the Reissner-Nördstrom-like solution given in (1), in which a tidal Weyl charge plays a role similar to that of electric charge in the general relativity Reissner-Nördstrom solution. We stress that there is no electric charge in this model: nonlocal Weyl effects from the 5th dimension lead to an energy-momentum tensor on the brane that has the same form as that for an electric field, but without any electric field being present. The formal similarity is not complete, since the tidal Weyl charge gives a positive contribution to the gravitational potential, unlike the negative contribution of an electric charge in the general relativistic Reissner-Nördstrom solution.

The braneworld solution is given by

$$(A^+)^2 = (B^+)^2 = 1 - \frac{2GM}{r} + \frac{q}{r^2},$$  \hspace{1cm} (33)

$$\mathcal{U}^+ = -\frac{\mathcal{P}^+}{2} = \frac{4}{3} \pi G q \lambda \frac{1}{r^4},$$  \hspace{1cm} (34)

where the matching conditions imply

$$q = -3GR \frac{\rho}{\lambda},$$  \hspace{1cm} (35)

$$\mathcal{M} = M \left[ 1 - \frac{\rho}{\lambda} \right],$$  \hspace{1cm} (36)

$$\alpha = \rho \Delta(R).$$  \hspace{1cm} (37)

Note that the Weyl energy density in the exterior is negative, so that 5-dimensional graviton effects lead to a strengthening of the gravitational potential (this is discussed further in [4]). Since $\mathcal{M} > 0$ is required for asymptotic Schwarzschild behaviour, we have a slightly stronger condition on the brane tension:

$$\lambda > \rho.$$  \hspace{1cm} (38)
However, this still gives a weak lower limit, \( \lambda > 10^9 \) MeV\(^4\). In this solution the horizon is at

\[
r_{h} = GM \left[ 1 + \left( 1 + \frac{3R}{2GM} - 2 \frac{\rho}{\lambda} + \frac{\rho^2}{\lambda^2} \right)^{1/2} \right].
\]  
(39)

Expanding this exact expression shows that the horizon is slightly beyond the general relativistic Schwarzschild horizon:

\[
r_{h} = 2GM \left[ 1 + \frac{3(R - 2GM)}{4GM} \frac{\rho}{\lambda} \right] + O \left( \frac{\rho^2}{\lambda^2} \right) > 2GM.
\]  
(40)

The exterior curvature invariant \( R^2 = R_{\mu\nu}R^{\mu\nu} \) is given by

\[
R = 8\pi G \left( \frac{\rho}{\lambda} \right)^2 \left( \frac{R}{r} \right)^4.
\]  
(41)

Note that for the Schwarzschild exterior, \( R = 0 \).

The second exterior is a new solution. Like the above solution, it satisfies the braneworld field equations in the exterior, and the matching conditions at the surface of the uniform-density star. It is given by

\[
(A^+)^2 = 1 - \frac{2GN}{r},
\]  
(42)

\[
(B^+)^2 = (A^+)^2 \left[ 1 + \frac{C}{\lambda(r - \frac{3}{2}GN)} \right],
\]  
(43)

\[
\mathcal{U}^+ = \frac{2\pi G^2 NC}{(1 - 3GN/2r)^2} \frac{1}{r^4},
\]  
(44)

\[
\mathcal{P}^+ = \left( \frac{2}{3} - \frac{r}{GN} \right) \mathcal{U}^+.
\]  
(45)

From the matching conditions:

\[
\mathcal{N} = M \left[ 1 + \frac{2\rho/\lambda}{1 + 3GM\rho/R\lambda} \right],
\]  
(46)

\[
C = 3GM\rho \left[ \frac{1 - 3GM/2R}{1 + 3GM\rho/R\lambda} \right],
\]  
(47)

\[
\alpha = \frac{\rho \Delta(R)}{(1 + 3GM\rho/R\lambda)^{1/2}}.
\]  
(48)

The horizon in this new solution is at

\[
r_{h} = 2GN,
\]  
(49)

which leads to

\[
r_{h} = 2GM \left[ 1 + \left( \frac{2R - 3GM}{2R} \right) \frac{\rho}{\lambda} \right] + O \left( \frac{\rho^2}{\lambda^2} \right) > 2GM.
\]  
(50)

The curvature invariant is

![FIG. 2. Qualitative behavior of the curvature invariant \( R^2 \): the upper curve is the Reissner-Nördstrom-like solution given by Eqs. (39) and (34); the lower curve is the new solution given by Eqs. (13) and (34) (\( \lambda = 5 \times 10^9 \) MeV\(^4\)).](image)

\[
\mathcal{R} = \sqrt{\frac{2}{3}} RC \left( \frac{4\pi R}{3M} \right)^2 \left( 1 - \frac{8GN/3r + 2G^2N^2/r^2}{1 - 3GN/2r} \right)^{1/2} \times \left( \frac{\rho}{\lambda} \right)^2 \left( \frac{R}{r} \right)^3.
\]  
(51)

Comparing with Eq. (41), it is clear that these two solutions are different. The difference in their curvature invariants is illustrated in Fig. 2.

**V. INTERIOR SOLUTION WITH SCHWARZSCHILD EXTERIOR**

If we assume that the exterior is the Schwarzschild exterior (\( \mathcal{U}^+ = 0 = \mathcal{P}^+ \)), then Eqs. (23) and (24) imply that the interior must have negative Weyl energy density:

\[
\mathcal{U}^- (r) = - \left( \frac{4\pi G}{\rho} \right)^2 \left[ \rho + \rho(r) \right]^4.
\]  
(52)

This means that the tidal effects on the brane from bulk gravitons reinforce the gravitational field in the star. (See [38] for further discussion of the meaning of \( \mathcal{U} < 0 \).)

It follows that the mass function in Eq. (23) becomes

\[
m^{-} (r) = M \left( 1 + \frac{\rho}{2\lambda} \right) \left( \frac{r}{R} \right)^3 - 6\pi \frac{\alpha}{\rho \lambda^2} \int_{0}^{r} \left[ \rho + \rho(r') \right]^4 r'^2 dr',
\]  
(53)

which is reduced by the negative Weyl energy density, relative to the solution in the previous section and to the general relativity solution. The effective pressure is given by

\[
p^{\text{eff}} = p - \frac{\rho}{2\lambda} (2 + 6w + 4w^2 + w^3),
\]  
(54)
where $w = p/\rho$. Thus $p^{\text{eff}} < p$, so that 5-dimensional high-energy effects reduce the pressure in comparison with general relativity.

VI. CONCLUSIONS

We have investigated how 5-dimensional gravity can affect static stellar solutions on the brane. We found the exact braneworld generalization of the uniform density stellar solution, and used this to estimate the local (high-energy) effects of bulk gravity. We derived an astrophysical lower limit on the brane tension $\lambda$, given by Eq. (23), which is much stronger than the big bang nucleosynthesis limit, but much weaker than the experimental Newton-law limit. We also found that the star is less compact than in general relativity, as shown by Eqs. (21) and (22). The smallness of high-energy corrections to stellar solutions flows from the fact that $\lambda$ is well above the energy density $\rho$ of stable stars. However nonlocal corrections from the bulk Weyl curvature (5-dimensional graviton effects) have qualitative implications that are very different from general relativity.

The Schwarzschild solution is no longer the unique asymptotically flat vacuum exterior; in general, the exterior carries an imprint of nonlocal bulk graviton stresses. The exterior is not uniquely determined by matching conditions on the brane, since the 5-dimensional metric is involved via the nonlocal Weyl stresses. We demonstrated this explicitly by giving two exact exterior solutions, both asymptotically Schwarzschild. Each exterior which satisfies the matching conditions leads to a bulk metric, which could in principle be determined locally by numerical integration. However, this is very complicated even in the simpler case of black holes on the brane. Without any exact or approximate 5-dimensional solutions to guide us, we do not know how the properties of the bulk metric, and in particular its global properties, will influence the exterior solution on the brane.

Guided by perturbative analysis of the static weak field limit [1][3][4][5][6][7][8][9][10][11][12], we make the following conjecture: if the bulk for a static stellar solution on the brane is asymptotically AdS$_5$ and has regular Cauchy horizon, then the exterior vacuum which satisfies the matching conditions on the brane is uniquely determined, and agrees with the perturbative weak-field results at lowest order. An immediate implication of this conjecture is that the exterior is not Schwarzschild, since perturbative analysis shows that there are nonzero Weyl stresses in the exterior (these stresses are the manifestation on the brane of the massive Kaluza-Klein bulk graviton modes). In addition, the two exterior solutions that we present would be ruled out by the conjecture, since both of them violate the perturbative result for the weak-field potential, Eq. (5).

The static problem is already complicated, so that analysis of dynamical collapse on the brane will be very difficult. However, the dynamical problem could give rise to more striking features. Energy densities well above the brane tension could be reached before horizon formation, so that high-energy corrections could be significant. We expect that these corrections, together with the nonlocal bulk graviton stress effects, will leave a non-static, but transient, signature in the exterior of collapsing matter. This is currently under investigation.

Acknowledgements: We thank Edward Anderson, Bruce Bassett, Marco Bruni, Malcolm MacCallum and Kei-ichi Maeda for useful discussions. CG is supported by PPARC.