THE POSSIBLE SIGNALS
FROM THE D=6 SPACE-TIME

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Abstract

Based on the exceptional properties of $U(1_{em})$- sterile neutrinos and their possible Majorana-Weyl space-time nature we continue to discuss our ideas about the existence in Standard Model some phenomena or hidden symmetries which are related to the extra global dimensions. We carry on discussing some open questions of the Standard Model from the $D = p_s + q_t = 6$ space-time geometry. The origin of the three families could be considered in analogue to the existing of two species- matter-antimatter- in $D = 3 + 1$ Minkowski space-time what can give the ternary relation for neutrinos masses. Apart from the possible existence of a “hidden neutrino boost” $c_{new} > v_{neut} > c_{light}$ discussed in [24], [25], [26],[27],[28],[29] and a possible indication on $v_{neut} > c_{light}$ first observed in [33], there could be some other signatures of a $D = 6$-vacuum, for example, the possible signals of the breaking $CPT$- invariance and $Q_{em}$-conservation [28], [29].
Duality between internal and external space-time symmetries.

After long searches of decisions so a great number of the Standard Model (Glashow–Salam–Weinberg Model) extensions realized within the limits of the $D = 3 + 1$-Special Theory of Relativity, and in numerous ways of extensions of the internal symmetries of the Standard Model, physicists began to introduce also the new concepts in geometry of the world surrounding us which assumes inclusion of new global additional dimensions.

This means that we stand at the beginning of expansion of some concepts of the Special Theory of the Relativity. First of all it concerns to multidimensional ($D > 4$) to generalization of Lorentz group and more careful analysis, that role which the velocity of light plays in this theory and space-time nature of neutrinos. What parameters will define applicability of the subsequent theory of a relativity there is the most important question in opening of new Special Theory of Relativity.

All known physical theories are described by internal and external space-time symmetries. To understand these theories much deeper one should find the correlations between them. Therefore we follow for the evolution of the main physical theories looking for the correlations between internal and external space-time sym-
We started from the classical mechanics and ended by the E(8) × E(8) heterotic superstring in $R^{10}_3$ and with $E(8) \times E(6)$ in $R^{3,1}_3 \times K_6$ space[13].

It becomes more and more reasonable to propose that the Standard Model and its extensions could have an intrinsic link with a more fundamental symmetry, than the Cartan-Lie symmetries. Of course, this fundamental symmetry should generalize the symmetries of the Standard Model, since a lot of experimental data confirm its.

Therefore a hypothetical symmetry should be a natural generalization of the Lie symmetries with stronger constraints leading to diminishing the number of free parameters of the Standard Model. In principle, in (super)string approach we already have the interesting example of the generalization of finite Lie algebras by an infinite-dimensional affine Kac-Moody algebra with a central charge. Superstring theory intrinsically contains a number of infinite-dimensional algebraic symmetries, such as the Virasoro algebra associated with conformal invariance and affine Kac-Moody algebras. On the first traditional way to get in superstring the Standard Model at low energy limit there should be solved the problem of compactification extra dimensions, to get the Minkowski $R^{4,1}$ space-time. On this way it was discovered for physics the very interesting geometrical objects-Calabi-Yau manifolds, more exactly, the heterotic $E(8) \times E(8)$ superstring have been compactified on the 6-dimensional Calabi-Yau space $CY_3$, having the $SU(2), SU(3)$ group of holonomy [13], [14], [17]. Lucky happened that the structure of $CY_2 \subset CY_3$ spaces are directly connected to the affine Cartan-Lie symmetries[14]. This link of $CY_2$ spaces with some algebras from $A_r^{(1)}, D_r^{(1)}, E_6^{(1)}, E_7^{(1)}, E_8^{(1)}$ algebras can be explained by the crepant resolution of ADE-type singular structures $C^2/G$, where $G$ is one the five finite subgroups of $SU(2)$. Thus the superstring approach generated by ideas of Kaluza-Klein of the link the internal symmetries and the singularities to the cycle structures of the compactified manifolds like $CY_2, CY_3$ [14],[17],[18], [40].

The main argument to look for new symmetry with some extra ordinary properties is that the symmetries linked to the simple Cartan-Lie groups are not suf-
icient for a description of many parameters and features of the Standard Model. Therefore a hypothetical symmetry could be a natural generalization of the Lie symmetries with stronger constraints which could lead to diminishing the number of the free parameters of the Standard Model. There could be exist some ways to find these algebras and symmetries. One very traditional way is going from the theory of numbers. For example to consider the new complex numbers $q^D = 1, -1$ ($D = 3, 4, 5, ...$) [34],[35],[36], [38], [37],[39],[19], [40] to find the geometrical figures of defined by equations

$$z \cdot \bar{z} \cdot \bar{\bar{z}} \cdot \ldots = 1, \quad z \in R^D,$$

(2)

where $R^D$ is the $D$-dimensional Euclidean space. First one can find the $D = 3, 4, ...$-dimensional Abelian extensions of $U(1)$-group and then to look for the non-Abelian extensions. Another way is the geometrical way to study the singular structure of CY$_3$-, CY$_4$-spaces. But also there exists another important way connected to the study of the external symmetries connected to the very big extra dimensions.

In quantum electrodynamics and in Standard Model the external symmetries are connected with the ambient geometry of our (3+1) space-time. These symmetries can be be described by Lorentz group $SO(3, 1)$, having 6-parameters, 6=3-rotations + 3-boosts, and plus four translations $x'_\mu = x_\mu + \xi_\mu$. Also the external symmetries include some discrete symmetries, like, $P$, $T$, and $C$ symmetries, which are unified into the principal discrete symmetry of the SM - $CPT$ [4],[5],[9]. It has been already considered a lot of the models extended the Standard Model at very big energy range and, correspondingly, at very small distances, where it was proposed that the Special Theory of Relativity is still correct?! As one can see from the table the Grand Unified Models [12],[10],[11],[13], [15],[16] with $SU(5)$, $SO(10)$, $E(6)$ or $E(8) \times E(6)$- groups with the scale of unification about $10^{16}$ GeV, used on this scale the external Lorentz space-time symmetry $SO(3, 1)$ and, correspondingly, the Special Theory of Relativity?! The transformation of the special theory of a relativity on these scales could cause the non observability those channels of the proton decays predicted by Grand Unified Theorys. Thus in some Grand Unified Theorys the questions about the external space-time symmetry and other vacuum features often remain opened which can lead to the wrong experimental predictions.

In the Standard Model one can also consider a link between external and internal global conservation laws. As the most important example one should study the correlation between the electric charge conservation and $CPT$ invariance[28]:

$$CPT \text{ – invariance} \leftrightarrow Q_{em} \text{ – conservation}. \quad (3)$$
The conservation of the electromagnetic charge is the consequence of the $U(1)$-global symmetry. There is remarkable link between the charges of proton and electron:

$$|Q_p + Q_e| \leq 10^{-21} q_e$$

The $CPT$ theorem can be proved in any Hermitian Lorentz symmetric quantum field theory (local)[5]. The $CPT$-invariance gives the very important equality for the masses (time life) of particle and antiparticle:

$$m(\Psi) = m(\bar{\Psi}) \rightarrow \text{binary } CPT - \text{ invariance}$$

We have got a lot of experimental and theoretical indications that $Q_{em}$-conservation law is valid in the $D = 3 + 1$ Minkowski space-time. One can thus in this approach the $CPT$-invariance and $Q_{em}$-conservation law are the prerogatives only for Minkowski $R_{3,1}^4$-space-time where are valid the $SO(3,1)$ Lorentz group (Poincaré) symmetry and $U(1)_{em}$ gauge symmetry[24],[27],[26], [29]. This validity of duality between global internal and external symmetries laws can be checked on some experiments [28].

There is the well-known the Poincaré duality in algebraic topology between the Homological group of manifold and cohomological group of the differential forms connected with this manifold. It means that if you knows something about the topological structure of the closed but non exact manifolds (Betti numbers) it is possible to connect them to the Hodge numbers characterize the Rham cohomological group of the closed differential forms but non exact differential forms determined on this manifold and opposite.

One can suggest the following relations between the topological geometry and physics:

$$\text{Homology Manifold} \Rightarrow \text{Space- Time; }$$
$$\text{Cohomology Forms} \Rightarrow \text{Fundamental Particles}$$

In physics the analogue of this duality means that the properties of the fundamental particles (like electron and positron in quantum electrodynamics) give very important information about the structure of the space-time, for example, about its dimension.

Thus, if such a duality exists, processes violating the $CPT$-invariance should accompanied by electromagnetic charge violation too. May be, this was one of the reasons why we did not see some rare decays, such as proton decay. In this case, the idea of grand unification is not enough to predict baryon or lepton-violating processes. Also a similar problem could be happened with searches for the rare flavour-changing processes in the channels, such as $\mu \rightarrow e + \gamma$, $\mu \rightarrow 3e$ etc (if the
observed broken family symmetry has an external space-time origin. Our idea of looking such processes is related to the large extra dimensions.

Thus one can say that to achieve in the superstring the progress it was shown one the ways how to solve the very important question i.e., the internal symmetries can be got by the geometrical way, from the study the singularities of the compactified CY_n spaces. This way begun from the old idea of Kaluza-Klein when the C/Z_2 singularity produces the U(1_{em}) = S^1 symmetry. The compactification on CY_3 spaces could expand the Kaluza-Klein case at very big class of simple Cartan-Killing-Lee algebras [14].

There is another very perspective way to consider non shrink global extra dimensions [20],[21], [22]. There is immediately appears very difficult problem - what is the Special Theory of Relativity in D > 4? What could be the Lorentz group extension? In superstring approach there was used the D- dimensional non compact group Lorentz with quadratic (p,q)-metrics (+... + −,...,−)?! But we don't know how to get the point limit and what there is no examples of the renormalizable quantum field theory in D > 4 (except φ^3 in D = 6?)[4].

The main experience what we have got from Kaluza-Klein, superstring/D-brane approaches and from the study of the structure of CY_n-manifolds is that we begun to understand the role of the compact small and non compact large dimensions in constructing the theories including the internal or external space-time symmetries. We can suppose that the non compact large dimensions are related to the extra dimensional space-time symmetries of our ambient world. The compact small dimensions are connected to the origin of the internal symmetries. For the Standard Model this circumstance was very important since we thought that the problem of three neutrino specie could be solved by including in our D = 3 + 1 space-time the some global non compact dimensions[24],[28]. So we begun to think that the appearance three families is related to the big extra dimensions like it was happened with two "families", particle-antiparticle, what was explained by Dirac in his relativistic equations in the D = 3+1- Minkowski space-time. Earlier a lot of publications has been devoted to the possibility to solve the three family problems through the introduction the internal gauge symmetries [15],[16]. But inside the three family problem there is the neutrino problems: the the Dirac-Weyl-Majorana nature [1],[2],[3], their masses and etc. We plan to consider the 3-neutrino problems by introducing the the two extra "non-compact" large dimensions and considering the corresponding geometry using the procedure of ternary complexification of Euclidean R^3 space and its extensions into the higher dimensions D ≥ 3. Note that all the previous geometrical ideas have been linked to the two dimensional Euclidean, Lobachevski, Riemann spaces and its the D = 2, 3, 4, ... hyper surfaces extensions with the symmetry-invariant quadratic metrics [6].
In $D = 6$ we can consider some cubic and quartic geometrical surfaces, the tetrahedral Pythagoras theorem, the $3 \times 3$ matrices an analogue of the Pauli matrices, n-ary complex analysis in $R^D$ [34],[35],[36],[38], [37],[19], [39],[40]. which give us some new interesting geometrical spaces having some applications Standard Modeland in Standard Cosmology Model.

2 The $D = (3 + 1)$ STR and the space-time structure of neutrinos

It can be distinguished the main principles of Special Theory of Relativity as the principle of relativity, principle of maximal velocity and the symmetry of the electromagnetic vacuum structure which has been determined by the 4-dimensional space-time Lorentz $SO(3, 1)$- group. The principle of maximality velocity can be formulated like the maximal velocity of the material objects cannot be more than the light velocity in the vacuum. The vacuum structure will be very important for the next discussions since the notion of the vacuum should be defined correctly and it can depend on some extra conditions of our ambient space-time. One can think that, if our visible world is determined only by the electromagnetic vacuum, the Special Theory of Relativity should be correct. But if the observed until now the $D=1+3$ dimensional world is only a subspace of the higher dimensional space there emerge the requirements to check the principles of STR on the experiments. It the case when one transforms the geometrical properties of the vacuum structure by embedding the additional dimensions, it is the most plausible that the principle of the maximal velocity should be crucially changed i.e. there could exist some specific particles propagated faster than light (neutrinos, dark matter "fermions") [24],[26],[25],[28], [33], [32], [31].

To realize the program to embed the "visible" $SO(3, 1)$- ambient world into higher dimensional space-time with large non compact extra dimensions one should take in mind all achievements of $D = 3 + 1$ approach in a new multidimensional Special Theory of Relativity. Our interests are connected to the checking the origin of some fundamental constants characterized the Special Theory of Relativity understanding of which could give the progress in solving some problems like as 3-neutrino species, three generations, the fermion and $W, Z$ bosons masses. Now we do accent on some peculiarities of Special Theory of Relativity Lorentz group and its spinor representations[4],[5],[9],[6].

So the Galileo’s transformations should be changed by Lorentz’s transformations($\mu = \{0, i = 1, 2, 3\}$):

$$x'^\mu = \Lambda^\mu_\nu x^\nu$$

(7)
what conserve in Minkowski space $R^4_1$ the following space-time interval:

$$ds^2 = g_{\mu\nu} dx^\mu dx^\nu = c^2 (dt)^2 - (dx^1)^2 - (dx^2)^2 - (dx^3)^2,$$

(8)

where it was suggested $x_0 = ct$ and the Lorentz metrics is:

$$g_{\mu\nu} = g^{\mu\nu} = \text{diag}(1, -1, -1, -1)$$

(9)

A Lorentz transformation of $R^4_1 \rightarrow R^4_1$ is a linear map $\Lambda$ with

$$(x' \cdot x') = (\Lambda x \cdot \Lambda x) = (x \cdot x) = g_{\mu\nu} \cdot x^\mu \cdot x^\nu$$

$$\Lambda^\mu_\rho \cdot g_{\mu\nu} \cdot \Lambda^\nu_\tau = g^\rho_\tau.$$  

(10)

In STR the speed of light is identical in all inertial systems i.e. for two coordinate systems $\{t, x^1, x^2, x^3\}$ and $\{t', (x')^1, (x')^2, (x')^3\}$ the corresponding intervals must be equal:

$$c^2 t'^2 - (x')^2 = c^2 t^2 - (x)^2 - (x')^2 - (x')^2.$$

(11)

For the world line of the material particle in Minkowski space-time $R^4_1$

$$x^0 = ct, \quad x^1 = x^1(t), \quad x^2 = x^2(t), \quad x^3 = x^3(t),$$

(12)

one can consider the space vector velocity

$$v = \{\dot{x}^1, \dot{x}^2, \dot{x}^3\}.$$  

(13)

Then the principle of maximality velocity can be formulated like the maximal velocity of the material objects cannot be more than the light velocity in the vacuum, i.e.

$$|v| \leq c.$$  

(14)

The Lorentz group $L = O(3, 1)$ consists of 4 connected components depending on the sign of the determinant $det L = \pm 1$ and the magnitudes of the $L^0_0$-components, $L^0_0 \geq 1$ or $L^0_0 \leq 1$. Any $\Lambda \in L^4_1$ can be got from boost or rotation.

For each $x^\mu$ one can construct a complex Hermitian $2 \times 2$ matrices

$$\sigma(x) = x^\mu \cdot \sigma_\mu \rightarrow det \sigma(x) = (x^\mu \cdot x_\mu),$$

(15)

where $\sigma_i$ and $\sigma_0$ are three Pauli matrices and $2 \times 2$ unit matrix, respectively. The transformation of $\sigma(x)$ with a complex matrix $\hat{A} \in SL(2C)$, $det \hat{A} = 1$:

$$\sigma(x) \Rightarrow \sigma(y) = \hat{A} \cdot \sigma(x) \cdot \hat{A}^\dagger,$$

(16)
conserves the norm of $\sigma(x)$, i.e. $\det \sigma(x)$. The mapping $\hat{A} \to L_A$ is a group homomorphism from $SL(2C)$ onto $L^\uparrow_+$. 

$SL(2C)$ is a simply connected Lie group and is the universal covering of $L^\uparrow_+$. Any $\hat{A} \in SL(2C)$ can be written as 

$$\hat{A} = \hat{B} \cdot \hat{J},$$

(17)

where the Hermitian matrix $\hat{B}$ and the unitary matrix $\hat{J}$ determine the boost and the 3-space rotation, respectively:

$$\hat{B} = \exp \left[ \frac{1}{2} \omega \cdot (\vec{n} \cdot \vec{\sigma}) \right],$$

$$\hat{J} = \exp \left[ -i \frac{1}{2} \varphi (\vec{n} \cdot \vec{\sigma}) \right].$$

(18)

The matrix $L_j$ is a pure rotation around the axis $\vec{n}$ through an angle $\varphi$. The operator $L_{\hat{B}}$ is a pure $\vec{n}$-boost with a velocity

$$\vec{v} = \vec{\beta} = \tanh (\omega) \cdot \vec{n}. $$

(19)

If the system $\{x'\}$ is moving with respect to the system $\{x\}$ along the axis with the speed $v$ the boost transformation takes the next form:

$$ (ct) = (ct') \cosh \omega + x'_1 \sinh \omega, $$

(20)

$$ x_1 = (ct') \sinh \omega + x'_1 \cosh \omega, $$

(21)

$$ x_2 = x'_2, $$

(22)

$$ x_3 = x'_3. $$

(23)

where

$$ \tanh \omega = \frac{v}{c} = \beta, \ \gamma = \frac{1}{\sqrt{1-\beta^2}} $$

(25)

$$ \cosh \omega = \gamma, \ \sinh \omega = \beta \cdot \gamma $$

(26)

For the energy-momentum 4-Lorentz vector one can get the following Lorentz invariant expression:

$$ E^2 - \vec{p}^2 \cdot c^2 = m_0^2 \cdot c^4, $$

(27)

where $E$ and $\vec{p}$ are the energy and the 3-dimensional momentum vector, respectively. $m_0$ is the mass parameter in the rest.
Thus the Lorentz group transformations are determined by the fundamental constants: light speed in vacuum $c$, the rest mass $m_0$ and six free parameters: three angles and three-vector $\vec{\beta}$ with the change region:

$$|\vec{\beta}| \leq 1.$$  \hspace{1cm} (28)

Time by time there were appeared some ideas how to extend this region and to find signals expending faster than light, for example, searching for tachyons. The other ideas have been initiated by cosmology problems to solve some of them by proposal that the fundamental constants like $C_{\text{light}}$ depends on the time? And the last comment is concerning the our space-time. Our representations about the world change with getting more and more information from astrophysics and our progress in particle physics. So the extension space-time due to including some additional dimensions immediately causes the questions about the validity of Special Theory of Relativity.

The questions about mass origin of the particles and their properties are very important. These questions are very closely linked to the Lorentz representations [5], [9],[7].

There are well-known six Lorentz spin generators, $S^{\mu\nu}$, and the following combinations of them:

$$J^i = (J^1, J^2, J^3) = (-iS^{23}, -iS^{13}, -iS^{23})$$

$$B^i = (B^1, B^2, B^3) = (-iS^{01}, -iS^{02}, -iS^{03})$$  \hspace{1cm} (29)

obey to the commutation relations

$$[J^i, J^j] = i\epsilon_{ijk}J^k,$$

$$[B^i, B^j] = -i\epsilon_{ijk}B^k,$$

$$[J^i, B^j] = i\epsilon_{ijk}B^k.$$  \hspace{1cm} (30)

To get the isomorphism of $so(3, 1) \equiv sl(2, C)$ algebra to the algebra of the semi-simple group $SU(2)_L \times SU(2)_R$ it is easy to see from the commutation relations of the 6-operators $I^\pm = \frac{1}{2} \cdot (J^i \pm B^i)$:

$$[I^{-i}, I^{-j}] = \epsilon_{ijk}I^{-k}$$
\[
[I^{+i}, I^{+j}] = \epsilon_{ijk} I^{+k}
\]
\[
[I^{+i}, I^{-j}] = 0
\]

(31)

This circumstance allows to classify the irreducible representations of \( SL(2, C) \) by two integer or semi-integer numbers \((m, n)\) of the finite-dimensional representations of the \( SU(2)_L \times SU(2)_R \) group. These representations one can use for spinor representations of \( S^\mu \nu \). The minimal representations of \( S^\mu \nu \) (except scalar \((0, 0)\) representation) are the Weyl spinors left-handed \((\frac{1}{2}, 0)\) and right-handed \((0, \frac{1}{2})\) - representations:

\[
\vartheta_a(x) \in (\frac{1}{2}, 0), \quad a = 1, 2
\]

(32)

\[
\bar{\vartheta}^\dot{a}(x) \in (0, \frac{1}{2}), \quad \dot{a} = 1, 2
\]

(33)

Two \( SU(2)_L \) - and \( SU(2)_R \) - are related by \( P \)-parity operation:

\[
P : \quad x_0 \to x_0, \quad \vec{x} \to -\vec{x}
\]

(34)

In the terms of generators \( \vec{J} \) and \( \vec{B} \) the \( SL(2, C) \) transformations take the forms:

\[
S = \exp i[\vec{\varphi} \cdot \vec{J} + \vec{\omega} \cdot \vec{B}]
\]

\[
S^P = \exp i[\vec{\varphi} \cdot \vec{J} - \vec{\omega} \cdot \vec{B}]
\]

(35)

The Weyl spinors cannot have \( m \neq 0 \). The Dirac fermions which are in the representation: \((\frac{1}{2}, 0) + (0, \frac{1}{2})\),

where

\[
\Psi = \begin{pmatrix}
\vartheta_a \\
\bar{\eta}^{\dot{a}}
\end{pmatrix}
\]

(36)

In Majorana basis the Dirac matrices are pure imaginary \((\gamma^\mu)^* = -\gamma^\mu\):

\[
\gamma^0 = \begin{pmatrix}
0 & \sigma^2 \\
\sigma^2 & 0
\end{pmatrix}, \quad \gamma^1 = \begin{pmatrix}
i\sigma^3 & 0 \\
0 & -i\sigma^3
\end{pmatrix}, \quad \gamma^2 = \begin{pmatrix}
0 & -\sigma^2 \\
\sigma^2 & 0
\end{pmatrix}, \quad \gamma^3 = \begin{pmatrix}
-i\sigma^1 & 0 \\
0 & -\sigma^1
\end{pmatrix}.
\]

(37)

In this basis the generators of Lorentz transformations \( S^\mu \nu \) are real and the fermion can be also real

\[
\Psi = \Psi^*
\]

(38)
This condition is preserved under Lorentz transformation. The operation of the complex conjugation can be extended for a general Dirac basis taking only

\[(\gamma^0)^\dagger = \gamma^0, \quad \gamma^i = -\gamma^i.\]  

(39)

Then it was introduced the charge conjugation operator \(C = 4 \times 4\)-matrix with the following conditions:

\[C^\dagger C = 1, \quad C^\dagger \gamma^\mu C = -(\gamma^\mu)^*.\]

(40)

Under action of the charge conjugation Dirac fermion transforms into

\[\Psi^C = \hat{C}\Psi^*\]

(41)

The charge conjugated fermion \(\Psi^C\) transforms by Lorentz covariant way and also satisfies to Dirac equation if the \(\Psi\) satisfies. Thus the condition of the Majorana fermion reality is

\[\Psi^{(C)} = \Psi\]

(42)

which is Lorentz invariant. In Majorana basis this condition reduces to \(C = 1\) and \(\Psi^* = \Psi\). In chiral basis

\[
\gamma^0 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \quad \gamma^i = \begin{pmatrix} 0 & \sigma^i \\ -\sigma^i & 0 \end{pmatrix}, \quad \gamma^5 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}, \quad C = i\gamma^2 = \begin{pmatrix} 0 & \epsilon_{\alpha\beta} \\ -\epsilon^{\alpha\beta} & 0 \end{pmatrix}.
\]

(43)

Note that the Majorana spinor can be expressed in terms of Weyl spinors as

\[
\Psi_M = \begin{pmatrix} \vartheta_a \\ -\epsilon^{\alpha\beta} \bar{\vartheta}^\beta \end{pmatrix}
\]

(44)

Theory of a massless neutrino in \(D = 3 + 1\) can be described either a theory of a massless Weyl fermion or as a theory of a massless four-component Majorana fermion. In Majorana case the neutrinos have the following mass term:

\[m_M(\nu)\bar{\Psi}_M \Psi_M = m_M(\nu)(\partial^\alpha \partial_a + \partial^\alpha \partial_a)\]

(45)

It is very important to establish a link between masses of three families. To get some additional relations between the masses of three Majorana neutrino species one can try to study the possible \(D = 6\) space-time global symmetries how it was.
done in $D = 3 + 1$ Minkowski space-time. This symmetry could give us a new relation which could be a signal from the extra dimension space-time. Our goal is to check the logic chain of the discovery the space-time nature, of spin, anti-particles and the three families (?). After detail study the space-time properties of neutrinos in $D = 3 + 1$ it is tempting to consider all three neutrinos in one $D = 6$ "ternary six-dimensional field"

\[(\psi_{\nu_e}, \psi_{\nu_\mu}, \psi_{\nu_\tau}) \rightarrow \Psi_{3\nu}.\]  

(46)

Taking into account already known six space-time degrees of freedom of the three Lorentz-Majorana neutrinos one can consider them in the $D = 6$ space-time. In this field each "internal" 2-component spinor bears the information about its $D = (3,1)$ space-time structure. In this $D = 6$-space apart from ordinary complex conjugation it is possible to embed the procedure of ternary conjugation using the $C_3$-ternary complex numbers with the generator $q$ satisfying to the following conditions:

\[q^3 = 1.\]  

(47)

We remind that if for the cyclic group $C_2$ there are two conjugation classes, 1 and $i$ and two one-dimensional irreducible representations:

| $C_2$   | 1   | $i$ |
|---------|-----|-----|
| $R^{(1)}$ | 1   | $i$ |
| $R^{(2)}$ | 1   | $-1$ |

(48)

the cyclic group $C_3$ already has three conjugation classes, $q_0$, $q$ and $q^2$, and, respectively, three one dimensional irreducible representations, $R^{(i)}$, $i = 1, 2, 3$. We write down the table of their characters, $\xi_l^{(i)}$:

\[
\begin{pmatrix}
-1 & q & q^2 \\
\xi^{(1)} & 1 & 1 & 1 \\
\xi^{(2)} & 1 & j & j^2 \\
\xi^{(3)} & 1 & j^2 & j \\
\end{pmatrix}
\]  

(49)

for $C_3$ ( $j_3 = \equiv j = \exp\{2\pi/3\})$.

This table defines the operation of ternary complex conjugations:

\[\tilde{q} = jq, \quad \bar{q} = j^2q, \quad \bar{\tilde{q}} = q,\]
\[ \tilde{q}^2 = j^2 q^2, \quad \tilde{\tilde{q}}^2 = j q^2 \quad (50) \]
\[ j = \exp \left( \frac{2\pi i}{3} \right), \quad 1 + j + j^2 = 0. \quad (51) \]

So, the main idea is to use the cyclic groups \( C^n \) (and new algebras/symmetries) to find the new geometrical "irreducible" substructures in \( R^n \) spaces, which are not the consequences of the simple direct n-multiplication of the known structures of Euclidean \( R^2 \) space which could help to build the \( D = 6 \) extension of the Lorentz group.

Than one can introduce the "family" ternary quantum charge numbers which should be conserved by a new ternary Abelian group. If this symmetry in \( D = 6 \) exists it means that in this space-time can exist a new "light" expanding with a velocity faster than electromagnetic light?! The structure of this ternary Abelian symmetry we discuss in the next section.

Then 3-neutrino field can be represented as

\[ \Psi = \begin{pmatrix} \vartheta \\ \tilde{\eta} \\ \tilde{\tilde{\xi}} \end{pmatrix} \quad (53) \]

The action of the new charge operator on the field can be constructed with the operator expressed in \( 6 \times 6 \)-matrix form:

\[ \Gamma^1 = \begin{pmatrix} 0 & \sigma^0 & 0 \\ 0 & 0 & \sigma^0 \\ \sigma^0 & 0 & 0 \end{pmatrix}, \quad (\Gamma^1)^2 = \begin{pmatrix} 0 & 0 & \sigma^0 \\ \sigma^0 & 0 & 0 \\ 0 & \sigma^0 & 0 \end{pmatrix}, \quad (\Gamma^1)^3 = \begin{pmatrix} \sigma^0 & 0 & 0 \\ 0 & \sigma^0 & 0 \\ 0 & 0 & \sigma^0 \end{pmatrix}. \quad (54) \]

The consequent actions of this operator are

\[ C^g \Psi = \Gamma^1 \tilde{\Psi} = \Psi^g, \]
\[ C^g C^g \Psi = C^{g9} \Psi = \Psi^{g9} \]
\[ C^g C^g C^g \Psi = C^{g99} \Psi = \Psi^{g99} = \Psi, \quad (55) \]

where

\[ \Psi^g = \begin{pmatrix} \vartheta \\ \tilde{\eta} \\ \tilde{\tilde{\xi}} \end{pmatrix} \quad (56) \]
and

\[ \Psi^{gg} = \begin{pmatrix} \tilde{\vartheta} \\ \eta \\ \tilde{\xi} \end{pmatrix} \]  

(57)

So we presented the 3-neutrino specie with respect to a global symmetry in \( D = 6 \) as the ternary Dirac complex fermion each component we considered as Majorana with respect to the Lorentz symmetry. The action in \( D(p + q = 6) \) space-time of the discrete possible \([CgP^pT^q]^6\)-invariance for three neutrino species could lead to the mass relation as it was in \( D = (1 + 3) \) space-time with application \( CPT \) theorem:

\[ m_M(\nu_e) = m_M(\nu_\mu) = m_M(\nu_\tau) \quad \text{Ternary mass relation} \]  

(58)

The mechanism of oscillation could be done due to a hypothetical interaction existing in \( D = 6 \) closely reminding the mechanism of transitions of the \( K^0 - \bar{K}^0 \) mesons. As \( CPT \) theorem is acting only in \( D = 4 \) these oscillations can go with \( CPT \)-violation. All these consequences could be checked on experiments. Thus this symmetry could send a signal from the extra dimension space-time.

Moreover in this case one can consider the chiral ternary symmetry, which is acting in all \( D = 6 \) space-time for \( U(1_{em}) \) sterile particles. In this case the ternary chiral symmetry forbids all bilinear Majorana mass forms. Thus we could come to the consequence that all Majorana-Lorentz masses are equal zero?! It seems the same result could be also valid for Dirac-Lorentz mass bilinear terms. In general the negligibility of neutrino masses with respect to the Electroweak scale breaking\( (m_\nu/M_{EW} \sim 10^{-9}) \) can confirm that the extra-dimensional dynamics could be very important to understand this very intriguing question. For example, one can represent the existence the hot dark matter what emits a new "light" expending much faster than light? The massless of the particles could lead to a new symmetry.... There is the question with charged fermions, quarks and charged leptons. The mechanism of production of the masses is connected to the Electroweak scale breakingand as one can see the mass spectrum is confined by this scale. The charged particles are living in the \( D = 3 + 1 \) space-time and the ternary symmetry is breaking in this part of \( D = 6 \).

3 Towards a ternary theory of neutrino.

Among unresolved problems of Standard Model in a special way there are questions on an origin and a role of 3 types neutrinos in the Universe and, at last, their special
characteristics. On the one hand, neutrino participates only in weak interaction and consequently possesses deeply getting property at substance passage. On the other hand, neutrinos, unlike others fermion components of the Standard Model, have very small mass. This fact in itself is the extremely mysterious problem, but it leads to that neutrinos are long-living.

The third possible important property which can have neutrinos, is connected with their possible space-time nature in $D = 4$ or in $D = 6$. Neutrinos while are unique representatives of a matter known to us from the Standard Model which could be connected with the physical phenomena, originating from more extensive multidimensional geometry of our Universe filled with more fundamental dark matter? According to the Standard Cosmological Model and last astrophysical data, the dark matter makes about 22% from all structure of substance of the Universe. The contribution of dark energy makes 74%, and the matter observed in the form of planets, stars, galaxies, intergalactic gas makes all about 4%. Undoubtedly that these estimations are dependent from modelling are represent our understanding of world around at present. As a result we come to a conclusion that already observable quark-lepton the matter spectrum, a number of other researches of already spent experiments within the limits of Standard Model together with the resulted data received from astrophysical researches, specify in existence of absolutely new properties of structure of the vacuum connected with new space-time geometry of the Universe.

Electroweak symmetry in Standard Model is broken to $U(1_{em})$-symmetry using Higgs mechanism. But there was represented the very interesting direction, in which violation of electroweak symmetry is connected with additional global dimensions. So the problem of the neutrino masses could be decided at the assumption that ours $D = 3 + 1$ world is a part of 6-dimensional space-time in which some symmetries forbid the "usual" mass terms ( "threshold effect from $D = 6 \rightarrow D = 4$")?!

Also, it is necessary to consider still observable properties of exotic global symmetry 3 quark lepton generations which because of considerable distinction of the masses should be strongly broken. Quarks and leptons enter into Standard Model in the form of 3 generations which except for their mass spectrum can be considered as identical repetitions of one generation. Distinction of the masses of quarks of 3 generations is shown in occurrence of their mixing which has been measured on experiment in electroweak quark decays. Despite intensive and long experimental searches, a question on existence gauge interactions between the fermions of the various generations till now remains opened and represents the next riddle of the nature. There have done a lot of efforts to solve this problems by searching for local gauge family symmetries, for example in superstring there were suggested two
variants with 3-families with gauge symmetry $U(1) \times U(1) \times U(1)$ in [15] and $3 + 1$ families with $SU(3) \times U(1)$ gauge symmetry in [16]. Many experimental searching the rare processes with lepton number violation did not give any success. So there appeared the question that the family problem could be connected to the new space-time symmetries having an origin to the extra dimensions. So our way to try to solve the family problem by geometrical way through the new space-time symmetries and at first we concentrated on three neutrino problem. The charged quarks and leptons have considerably different properties. To solve this problem we decided to use the ternary symmetries which are connected to the ternary complex numbers and, correspondingly, to the complexification of $R^{3n}$ -Euclidean spaces. The matrices Pauli $\sigma_{\mu}$ have been used in non-relativistic (Galilei) quantum mechanics of theory of electron for description of spin:

$$\sigma_{i}\sigma_{j} + \sigma_{j}\sigma_{i} = \delta_{ij}$$

$$\sigma_{i}^{2} = \sigma_{0}, \quad \sigma_{i}\sigma_{j}\sigma_{k} = i\epsilon_{ijk}\sigma_{k}$$

(59)

These matrices can be used to construct the $4 \times 4 \gamma^\mu$, Dirac matrices which satisfy to the following relation:

$$\gamma_{\mu}\gamma^{\nu} + \gamma_{\nu}\gamma^{\mu} = 2g_{\mu\nu},$$

(60)

where $g^{\mu\nu} = diag(1, -1, -1, -1)$. The matrices Pauli have been used in non-relativistic (Galilei) quantum mechanics of theory of electron what used the 2-dimensional spinors for description of spin. The Dirac matrices $2^{2} \times 2^{2}$ have been used in relativistic Dirac equation of electron what expends the notion of spin in into $D4$-space-time and gave the origin of a new anti-fermion matter. If one uses this binary Clifford algebra for $D = 6$ space-time he should introduce the $2^{3} \times 2^{3}$ Dirac matrices what can build from Pauli matrices by the next rules:

$$\gamma_{1}^{(3)} = \sigma_{1} \otimes \sigma_{0} \otimes \sigma_{0}$$

$$\gamma_{2}^{(3)} = \sigma_{2} \otimes \sigma_{0} \otimes \sigma_{0}$$

$$\gamma_{3}^{(3)} = \sigma_{3} \otimes \sigma_{1} \otimes \sigma_{0}$$

$$\gamma_{4}^{(3)} = \sigma_{3} \otimes \sigma_{2} \otimes \sigma_{0}$$

$$\gamma_{5}^{(3)} = \sigma_{3} \otimes \sigma_{3} \otimes \sigma_{1}$$

$$\gamma_{6}^{(3)} = \sigma_{3} \otimes \sigma_{3} \otimes \sigma_{2}$$

For the consideration the $D = 2^{(2k+1)}$ dimensions one should introduces the analogue of $\sigma_{3}, \gamma_{5}$ - matrices.

If the origin of Lorentz symmetry, Dirac equations, $\gamma^{(k)}$-matrices and etc started from geometry of the plane- Pythagoras theorem and symmetries of the circle, hyperbola, ordinary complex analysis. In addition one can study new symmetries, new manifolds, new Pythagoras theorem and etc in $R^3$. In $R^3$ there could
be some new structures what you cannot get from the simple composition of the $R^2$-plane and $R^1$-line. It reminds the problem of three body interactions where to get correct result one should take into account the irreducible triple interaction. The extension of the $D = 3 + 1$-space-time till $D = 6$ by Dirac procedure gives the doubling increase of the fermion spectrum. To get the tripling increase of the fermions (three families) one should another way, i.e., "to study the theory of the number 3": the complexification of $R^3$, the cubic Pythagoras theorem, the new structures of $R^3$ and new symmetries and etc [34],[35],[36], [38],[37],[39],[19], [40]:

Let start from the ternary complex numbers:

\[
\begin{align*}
    z &= x_0 q_0 + x_1 q + x_2 q^2, \\
    \tilde{z} &= x_0 q_0 + j x_1 q + j^2 x_2 q^2, \\
    \tilde{\tilde{z}} &= x_0 q_0 + x_1 j^2 q + x_2 j q^2.
\end{align*}
\]

(61)

Now one can build the following cubic form:

\[
\begin{align*}
    <z>^3 &= z \tilde{z} \tilde{\tilde{z}} \\
    <z_1 z_2>^3 &= <z_1>^3 <z_2>^3
\end{align*}
\]

(62)

The following exponent expression

\[
U_T(1) = \exp (q \phi_1 + q^2 \phi_2)
\]

(63)

with two conjugation operations

\[
\begin{align*}
    \tilde{U}_T(1) &= \exp (j q \phi_1 + j^2 q^2 \phi_2) \\
    \tilde{\tilde{U}}_T(1) &= \exp (j^2 \phi + j q^2 \phi_2).
\end{align*}
\]

(64)

produces the Abelian two-parameter ($\phi_i$) group. This exponent has very important property:

\[
U_T \cdot \tilde{U}_T \cdot \tilde{\tilde{U}}_T = 1
\]

(65)

what can be expanded for n-dimensional ternary unitary operators:

\[
U_T \cdot U_T^\dagger \cdot U_T^{\dagger\dagger} = 1.
\]

(66)
The Abelian unitary $U_T$-group can be expanded in any $n$ what is defined by $2n^2 - 1$-parameters ($n\geq 1$).

The unit form determines the $U_T(1)$ invariant two-dimensional surface:

$$zz = x_0^3 + x_1^3 + x_2^3 - 3x_0x_1x_3 = 1$$  \hspace{1cm} (67)

This cubic surface (spherical paraboloid) is a ternary analogue of the $S^1$- circle defined by units complex numbers $z \cdot \overline{z} = 1$.

Similarly one can build the triple $U_T(1)$-unitary invariant form in the space of the fields:

$$\psi \bar{\psi} \bar{\psi}$$  \hspace{1cm} (68)

Thus the $U_T(1)$- transformations could be linked to the space symmetry of the cubic surface (spherical paraboloid) and to the invariant production of the norm functions.

The unit ternary complex numbers $||q|| = 1$ can be realized in the set of the $3 \times 3$ matrices-nonions [36],[38], [37],[39],[40] - the three dimensional extension of Pauli matrices having the some common properties:

$$Q_a^3 = 1Q, \quad a = 0, 1, \cdots, 8,$$

$$Q_{3+i} = Q_i^2 = Q_i^\dagger, \quad i = 1, 2, 3,$$

$$Q_i = Q_{3+i}^2 = Q_{3+i}^\dagger, \quad i = 1, 2, 3$$

$$Q_7 = Q_8, \quad Q_8^2 = Q_7$$  \hspace{1cm} (69)

See also the full table of multiplication producing the ternary analogue of quaternions in [19].

$$Q_1 = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{pmatrix}, \quad Q_2 = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & j \\ j^2 & 0 & 0 \end{pmatrix}, \quad Q_3 = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & j^2 \\ j & 0 & 0 \end{pmatrix} \hspace{1cm} (70)$$

$$Q_4 = \begin{pmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix}, \quad Q_5 = \begin{pmatrix} 0 & 0 & j \\ 1 & 0 & 0 \\ 0 & j^2 & 0 \end{pmatrix}, \quad Q_6 = \begin{pmatrix} 0 & 0 & j^2 \\ 1 & 0 & 0 \\ 0 & j & 0 \end{pmatrix} \hspace{1cm} (71)$$

(72)
The matrices of the I-st and II-nd sets satisfy to some remarkable relations:

\[ Q_i Q_j Q_k + Q_j Q_k Q_i + Q_k Q_i Q_j = 3j^2 \eta_{ijk} Q_0 \]
\[ Q_{3+i} Q_{3+j} Q_{3+k} + Q_{3+j} Q_{3+k} Q_{3+i} + Q_{3+k} Q_{3+i} Q_{3+j} = 3j \eta_{ijk} Q_0 \]

with

\[ \eta_{111} = \eta_{222} = \eta_{333} = 1 \]
\[ \eta_{123} = \eta_{231} = \eta_{312} = j \]
\[ \eta_{321} = \eta_{213} = \eta_{132} = j^2 \]

where \( j = \exp(2\pi/3) \).

The surface

\[ x_0^3 + x_1^3 + x_2^3 - 3 x_0 x_1 x_2 = 1 \]

after changing coordinate system can get the famous expression. Introduce \( u_0 = x_0 + x_1 + x_2 \) and parametrise a point on the circle of radius \( r^2 = u_1^2 + u_2^2 \) around the trisectrice by its polar coordinates \( (r, u_1 = r \cos \theta, u_2 = r \sin \theta) \). The surface in these coordinates can be reduced to the well known spheric paraboloid:

\[ u_0(u_1^2 + u_2^2) = 1 \]

From this cubic surface one can also define the tetrahedron Pythagoras theorem:

\[ S_A^3 + S_B^3 + S_C^3 - 3S_A S_B S_C = S_D^3, \]

what gives in the tetrahedron the link between the areas of the \( S_{A,B,C,D} \) four triangle faces. This theorem can be easily extend to all \( D = 4, 5, 6, \ldots \)-dimensional cases!
From the exponent expression one can find the space structure $s$ of this ternary unitary group:

$$U_T = \exp (q \phi_1 + q^2 \phi_2) = \exp (q - q^2) \alpha \cdot \exp (q + q^2) \beta) \rightarrow SOP(2, 1)$$

(80)

The $SO(2') \times tP(2, 1)$ group of transformations produced the one-parameter three-dimensional orthogonal group and ”projective” parabolic group. Remind that the $SO(2')$-symmetry is acting in $R^3$-spaces, where the general orthogonal group $SO(3)$ has three parameters. The ”projective” group $P(1, 2)$ defines the set of parabolas in $R^3$-spaces.

Considering the ”real” case

$$U_T \rightarrow SOP(2, 1)$$

(81)

The subgroup $SO(2')$ can be expressed through the $3 \times 3$ matrix [36],[38], [37],[19], [39],[40].

The subgroup $SO(2')$ can be expressed through the $3 \times 3$ matrix:

$$SO(2') = \exp \{ \alpha(q_1 - q_2^2) \} = \begin{pmatrix} c_0 & s_0 & t_0 \\ t_0 & c_0 & s_0 \\ s_0 & t_0 & c_0 \end{pmatrix},$$

(82)

where we have the particular choice for the functions, $c, s, t$ giving the determinant equal to one:

$$c_0 = \frac{1}{3}(1 + 2 \cos(\phi))$$

$$s_0 = \frac{1}{3}(1 + 2 \cos(\phi + \frac{2\pi}{3})),$$

$$t_0 = \frac{1}{3}(1 + 2 \cos(\phi - \frac{2\pi}{3})).$$

(83)

where $j - j^2 = \sqrt{3}i$, $\phi = \sqrt{3} \alpha$.

Apart from that the $SO(2')$ transformations preserve the cubic form they also satisfy to binary condition $OgO^t = O^t gO = 1$, preserving the quadratic metric $g_{ij} = diag(1, 1, 1)$ what is equivalent to the additional two restrictions for the ordinary $3 \times 3$ orthogonal transformations:

$$c_0^2 + s_0^2 + t_0^2 = 1$$

$$c_0 s_0 + s_0 t_0 + t_0 c_0 = 0,$$

(84)
what in our case can be easily checked. Thus in the case \( \alpha = [(\phi_1 + \phi_2)/2] = -\beta = -[(\phi_1 - \phi_2)/2] \) the ternary symmetry gives the orthogonal symmetry in \( R^3 \) with additional ternary restriction.

In the case \( \alpha = \beta \), the ternary symmetry coincides with the other binary symmetry preserving the quadratic metrics:

\[
g_{ij} = \text{diag}(2,-1,-1), \quad P^t \cdot \hat{g} \cdot P = \hat{g}. \quad (85)
\]

\[
P = \exp\{\alpha(q_1 + q_1^2)\} = \begin{pmatrix} c_+ & s_+ & s_+ \\ s_+ & c_+ & s_+ \\ s_+ & s_+ & c_+ \end{pmatrix}, \quad (86)
\]

where \( c_+ = \frac{1}{3}(e^{2\alpha} + 2e^{-\alpha}) \), \( s_+ = \frac{1}{3}(e^{2\alpha} - e^{-\alpha}) \) and the cubic equation reduces to the next form:

\[
c_+^3 + s_+^3 + t_+^3 - 3c_+s_+t_+ = (c_+ - s_+)^2(c_+ + 2s_) = 1. \quad (87)
\]

Note that apart from the remarkable group properties the matrices

\[
O_T = \begin{pmatrix} c & s & t \\ t & c & s \\ s & t & c \end{pmatrix}, \quad \text{det}(O_T) = c^3 + s^3 + t^3 - 3cst, \quad (88)
\]

satisfy to the next equality:

\[
O_T \cdot O_T^J \cdot O_T^{JJ} = c^3 + s^3 + t^3 - 3cst \cdot \hat{1}, \quad (89)
\]

where we introduce the following operations of conjugations:

\[
O_T^J = \begin{pmatrix} c & j_2s & j_2t \\ j_2t & c & js \\ js & j_2t & c \end{pmatrix}, \quad O_T^{JJ} = \begin{pmatrix} c & j^2s & j^2t \\ j^2t & c & js \\ js & j^2t & c \end{pmatrix} \quad (90)
\]

Thus, the two parametric ternary group \( SOP(1,2) \) preserving the cubic form reduces exactly to two known binary symmetries, \( \alpha = -\beta \) and \( \alpha = \beta \), but in which these two binary symmetry are unified by non-trivial way.

The extension of group \( SOP(2,1) \) from \( R^3 \) space to the \( R^n \) spaces \( (n > 3) \) should have the \( 2 \times C_3^2 \)-parameters and construction the transformation in matrix forms is following to the known extensions of usual orthogonal groups which have the \( C_2^n \)-parameters.
4 Preliminary conclusions from \( D = (3 + 1 = 4) \subset D = (p + q = 6) \).

The three species of neutrinos could have a ternary complex structure which could directly related to the origin of three quark-lepton families. In this approach the family symmetry can have the space-time origin as we already have met with similar question of the origin matter-antimatter in Dirac relativistic theory of electron. Since there are three quark-lepton families the procedure of Dirac cannot use any more. In his approach we could hope only to get in every stage the doubling of fermion states: \( 2^4 \). To get tripling or to get three families we should include in consideration new properties of the space-time. Before all our progress was connected to theory the well known properties of the two-dimensional Euclidean, Lobachevski and Riemann surfaces and its extensions to the n-dimensional cases. In this case all symmetries and algebra are based on the binary structures, for example, the possible invariant quadratic metrics \( g^{\mu\nu} \) has the signature \( \{ (+...+)_p; (-...-)_q \} \).

So if in the old approach there were used the Pythagoras theorem for triangle, the complex analysis for the plane but in our case we need also to include the ”irreducible” structures going only from \( R^3 \) spaces, where exists the tetrahedron extension of Pythagoras theorem, the ternary complex numbers and the ternary complex analysis with the cubic differential Laplace equations [36],[37],[38],[19], [39],[40]. There is very big difference between the charge matter fermions and neutrinos what can be see from the structure of Lorentz group representations: Dirac, Weyl chiral and Majorana fermions. Suggesting the two dimensional structure of neutrino field ( Majorana or Majorana-Weyl we came to the consideration the \( (D = 2 \text{(spin)} \times 3 \text{(fam)} = 6 \)-space-time. It gives us a chance to include in our ambient space-time just two additional extra dimensions. To realize this project we studied the \( 3 \times 3 \) matrices- nonions- the three dimensional extension of the 4-Pauli matrices. The algebraic structure of nonions is much complicate than the algebraic structure of Pauli matrices. The ternary complex analysis allowed us to discover the Abelian groups preserving the cubic surfaces and the n-dimensional extensions of them to non Abelian case. To observe the new extra dimensions it was suggested to make the neutrino experiments to measure the neutrino velocity which as was predicted in [24] is more than light. Such scenario with some fundamental boost-velocities could give a push to go beyond the standard Big Bang model in the time before the SU(2)U(1) phase and could explain the horizon, atness problems. This scenario is different with respect to the other scenario of the varying speed of light [30] in spite of the similar problems to construct such mechanisms. The restriction of sterility for neutrino in our conjecture means that
with the electromagnetic charged particles to observe new boost (new topological circuit) at the now available energies is very difficult or may be impossible now. For U(1)_{em} charged particles getting to the new SU(2)U(1) vacuum structure could be a threshold effect like as Vavilov-Cherenkov effect with emitting of lot of energy [24]. May be, this threshold could be linked to the scale of electroweak symmetry violation.

The other signatures which could confirm the result [?] are the searching for the processes going with violation CPT-invariance and $Q_{em}$ conservation. For example, it is important to search for the CPT invariance violation in neutrino oscillations [32].

The new ternary structure structure of Majorana neutrino can lead to discovery the new interactions linked the electromagnetic charge matter to the dark matter. There is very important the progress in making the new experiments to check is neutrino of Majorana-Weyl nature or not. The other possibility to check the Majorana-Weyl nature of neutrino can be related with the proton decay problem where the proton non-stability problem could be solved also in the electron, $\mu^-$, $\tau^-$ lepton and $b^-$ meson rare decay experiments [28] which are going with $Q_{em}$-conservation law violation.

We started our discussion from the exceptional properties of the Lorenz group structure - Dirac-Weyl-Majorana neutrinos- which may be have non-Majorana structure with respect to a new D=6 space-time symmetry- and step by step want to come to an idea of an existence of completely new physics at energy scale $O(1 - 10)$TeV which is related to the D=6 space-time geometry [28] and may be with origin of the charge fermions in Standard Model.

The new ternary structure could be linked to the some global dark symmetries in Standard Model, for example:

$$N(colors) = N(families) = N(space \, dim.) = 3.$$  

(91)

Expected new experimental and theoretical consequences from carrying out further neutrino experiments cause so huge interest in scientific and public circles of which wasn’t since opening of the Special theory of a relativity in the beginning of 20th century. Multidimensional generalization of the special theory of a relativity will result and in respective generalization of the General Theory of the Relativity and will bring very important attention to the question on speed of distribution of gravitational waves.

The expected results of the neutrino measurements of velocity could be also depend on the possible signature $(p + q) = 6, p \geq 3, q \geq 1$ of two extra dimensions
in D=6- space-time:

\[
D = p + q = 6 \quad \text{spacedim. timedim.}
\]

\[
\begin{align*}
R_{5,1}^6 & \quad p = 5 & q = 1 \\
R_{4,2}^6 & \quad p = 4 & q = 2 \\
R_{3,3}^6 & \quad p = 3 & q = 3 \\
\end{align*}
\]  \quad (92)

At [24] we suggested to check the possible effect \( V_{\text{neut}} > c \) in three types neutrino experiments:

- 1. The experiments with a long base: MINOS, OPERA, K2K
- 2. The experiments with the short base
- 3. Beam-dump experiments

The main question for us was not only to fix the effect but to understand the dynamics of the possible effect, for example, dependence on the neutrino energy or on the mass of the neutrino sources, dependence on the neutrino species and Majorana-Weyl structure, how this possible effect could depend energy-distance-direction-time?

At the end of this article we would like to stress that to get correct results there should be done a lot of experimental and theoretical investigations in future.

1. The Majorana-Weyl-Dirac structure of neutrinos in \( D+(3+1) \rightarrow D = (p+q) \) space-time

2. Complexification of \( R^3, R^4, R^5, R^6 \) and the corresponding Abelian symmetries

3. Classification of ternary non-Abelian symmetries

4. The new space-time structure of neutrinos in cosmology with extra dimensions \( D > 4 \)
5. A link neutrino with dark matter

6. Dark ternar external and internal symmetries symmetries in the SM

7. Poincaré duality: $CPT$-invariance and $Q_{em}$-conservation

8. The link between masses of quarks/charged leptons and electro-weak symmetry breaking

9. New interactions and proton/electron non-stability

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