On the Localization Properties of Weyl Particles

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In this work, it is shown that Weyl particles can exist at different states in zero electromagnetic field, either as free particles or at localized states. In addition, it is shown that the localization, as well as the energy, of the particles can be fully controlled using simple electric fields, which can easily be realized in practice. These results are particularly important regarding possible practical applications of Weyl particles, both considering solid-state physics in materials supporting these particles and laser physics using ions trapped by laser beams, which can simulate the behavior of Weyl particles.

1. Introduction

Let us consider the Weyl equations in the form
\[ i\sigma^\mu \partial_\mu \psi + a_\mu \sigma^\mu \psi = 0 \]
where
\[ \sigma^0 = \sigma^0, \sigma^1 = -\sigma^3, \sigma^2 = -\sigma^1, \text{ and } \sigma^3 = -\sigma^2. \]

In a recent work,\(^2\) we have shown that all spinors of the form
\[ \psi = \begin{pmatrix} \cos \left( \frac{\mu t}{2} \right) \\ e^{i\varphi(t)} \sin \left( \frac{\mu t}{2} \right) \end{pmatrix} \exp \left[ i\hbar \left( r, t \right) \right] \]
(4)
satisfy the Weyl equation for particles with positive helicity (Equation 1) for the 4-potentials given by

\[ \left( a_0, a_1, a_2, a_3 \right) = \left( \frac{\partial h}{\partial t} + \frac{1}{2} \frac{\partial \varphi}{\partial t}, \frac{\partial h}{\partial x}, \frac{1}{2} \sin \varphi \frac{\partial \varphi}{\partial t}, \frac{\partial h}{\partial y}, \frac{1}{2} \cos \varphi \frac{\partial \varphi}{\partial t}, \frac{\partial h}{\partial z}, \frac{1}{2} \frac{\partial \varphi}{\partial t} \right) \]
(5)

In the same work it was also shown that all spinors of the form
\[ \psi' = \begin{pmatrix} -\sin \left( \frac{\mu t}{2} \right) \\ e^{i\varphi(t)} \cos \left( \frac{\mu t}{2} \right) \end{pmatrix} \exp \left[ i\hbar \left( r, t \right) \right] \]
(6)
are solutions to the Weyl equation for particles with negative helicity (Equation 2 or, equivalently, Equation 3) for the 4-potentials
\[ \left( a_0', a_1', a_2', a_3' \right) = \left( \frac{\partial h}{\partial t} + \frac{1}{2} \frac{\partial \varphi}{\partial t}, \frac{\partial h}{\partial x}, \frac{1}{2} \sin \varphi \frac{\partial \varphi}{\partial t}, \frac{\partial h}{\partial y}, \frac{1}{2} \cos \varphi \frac{\partial \varphi}{\partial t}, \frac{\partial h}{\partial z}, \frac{1}{2} \frac{\partial \varphi}{\partial t} \right) \]
(7)

where \( \theta(t) \) and \( \varphi(t) \) are arbitrary real functions of time and \( h(r, t) = h(x, y, z, t) \) is an arbitrary real function of the spatial coordinates and time.

Furthermore, according to Theorem 3.1 in ref. [3], the spinors given in Equation (4) will also be solutions of the Weyl equation for particles with positive helicity (Equation 1) for an infinite number of 4-potentials
\[ b_\mu = a_\mu + \kappa_\mu s \left( r, t \right), \mu = 0, 1, 2, \text{ and } 3 \]
(8)
where
\[ \left( \kappa_0, \kappa_1, \kappa_2, \kappa_3 \right) = \left( 1, - \frac{\psi' \sigma^1 \psi}{\psi^1}, - \frac{\psi' \sigma^2 \psi}{\psi^2}, - \frac{\psi' \sigma^3 \psi}{\psi^3} \right) = \left( 1, - \sin \theta \cos \varphi, - \sin \theta \sin \varphi, - \cos \theta \right) \]
(9)
and $s(r, t)$ is an arbitrary real function of the spatial coordinates and time.

Similarly, the spinors given in Equation (6) will also be solutions to the Weyl equation for particles with negative helicity (Equation 2 or, equivalently, Equation 3) for an infinite number of 4-potentials

$$b'_{\mu} = a'_{\mu} + k's(r, t), \mu = 0, 1, 2, \text{ and } 3$$

(10)

where

$$\left(\kappa'_{0}, \kappa'_{1}, \kappa'_{2}, \kappa'_{3}\right) = \left(1, \frac{\psi'_{i}^{\dagger} \sigma^{i} \psi'_{j}^{\dagger} \sigma^{j}}{\psi'_{i}^{\dagger} \psi'_{j}^{\dagger}}, \frac{\psi'_{i}^{\dagger} \sigma^{i} \psi'_{j}^{\dagger} \sigma^{j}}{\psi'_{i}^{\dagger} \psi'_{j}^{\dagger}}\right) = \left(1, -\sin \theta \cos \varphi, -\sin \theta \sin \varphi, -\cos \theta\right) = \left(\kappa_{0}, \kappa_{1}, \kappa_{2}, \kappa_{3}\right)$$

(11)

Assuming that $\frac{d\theta}{dt} = \frac{d\varphi}{dt} = 0$ and setting

$$h(r, t) = E_{0} \left[\kappa \sin \theta \cos \varphi + \gamma \sin \theta \sin \varphi + z \cos \theta - t\right]$$

(12)

the above solutions correspond to free Weyl particles with energy $E_{0}$ moving along a straight line in space with parallel angle and azimuthal angle $\varphi$. As it will be shown in the following sections, the solutions of the Weyl equations given in Equations (4) and (6) are much more general and have very important and unexpected properties.

2. On the Property of Weyl Particles to Exist at Different Quantum States in Zero Electromagnetic Field

The electromagnetic fields corresponding to the 4-potentials given by Equations (5) and (7) can be easily calculated through the formulae:

$$E = -\nabla U - \frac{\partial A}{\partial t}, \quad B = \nabla \times A$$

(13)

where $U = b_{0}/q$ is the electric potential and $A = -(1/q)(b_{i}i + b_{j}j + b_{k}k)$ is the magnetic vector potential. The choice of the minus sign in the definition of the magnetic potential is related to the form of the Weyl equations used in this article. Thus, in the case of particles with positive helicity, the electromagnetic field corresponding to the 4-potential $a_{\mu}$ is

$$E = \frac{1}{2q} \left( \cos \varphi \frac{d\theta}{dt} \frac{d\varphi}{dt} + \sin \varphi \frac{d^{2}\varphi}{dt^{2}} \right) i + \frac{1}{2q} \left( \sin \varphi \frac{d\theta}{dt} \frac{d\varphi}{dt} - \cos \varphi \frac{d^{2}\varphi}{dt^{2}} \right) j - \frac{1}{2q} \frac{d^{2}\varphi}{dt^{2}} k$$

$$B = 0$$

(14)

Similarly, in the case of particles with negative helicity, the electromagnetic field corresponding to the 4-potential $a'_{\mu}$ takes the form $E' = -E$ and $B' = 0$.

It is easily confirmed that, if the following condition holds

$$\frac{d^{2}\varphi}{dt^{2}} = \frac{d^{2} \varphi}{dt^{2}} = \frac{d\theta}{dt} \frac{d\varphi}{dt} = 0$$

(15)

$$E = B = 0$$

(16)

implying that Weyl particles in zero electromagnetic field can either move on a straight line $(d\theta/dt = dq/dt = 0)$, or move with constant angular velocity regarding the polar angle $(d\theta/dt = \omega_{0}, dq/dt = 0)$, or move with constant angular velocity with respect to the azimuthal angle $(d\theta/dt = 0, dq/dt = \omega_{0})$.

In more detail, the velocity of the particles, can be defined as:

$$v = (\psi'^{\dagger} \sigma^{0} \psi') i + (\psi'^{\dagger} \sigma^{0} \psi') j + (\psi'^{\dagger} \sigma^{0} \psi') k$$

(17)

$$v' = (\psi'^{\dagger} \sigma^{0} \psi') i + (\psi'^{\dagger} \sigma^{0} \psi') j + (\psi'^{\dagger} \sigma^{0} \psi') k$$

(18)

in the cases of positive and negative helicity, respectively. Therefore, the speed vector of a Weyl particle in zero electromagnetic field will become constant in the case of $dq/dt = 0$, or it will take the form

$$v_{\varphi} = \sin (\theta_{0} + \varphi_{0} t) \cos \varphi_{0} i + \sin (\theta_{0} + \varphi_{0} t) \sin \varphi_{0} j + \cos (\theta_{0} + \varphi_{0} t) k$$

(19)

in the case that $d\theta/dt = \omega_{0}, dq/dt = 0$. In the above expressions $\theta_{0}$ and $\varphi_{0}$ are the initial values of the polar and azimuthal angles of the particles, respectively. It should also be mentioned that, in all cases, the modulus of the velocity of the particles $|v| = 1$ in natural units, as required by the special theory of relativity.

To gain more insight into the behavior of Weyl particles in zero electromagnetic field, we consider the motion of a classical particle with velocity equal to that of the Weyl particle. It is easy to verify that a classical particle with velocity $v_{\varphi}$ performs circular motion with radius

$$r_{\varphi} = \frac{1}{\omega_{1}}$$

(20)

On the other hand, a classical particle with velocity $v_{\varphi}$ performs helical motion with radius

$$r_{\varphi} = \frac{\sin \theta_{0}}{\omega_{2}}$$

(21)

and pitch

$$d_{\varphi} = \frac{2\pi \cos \theta_{0}}{\omega_{2}}$$

(22)

It is easy to confirm that, in the case that $\theta_{0} = \pi/2$ the particle still performs circular motion with radius $r_{\varphi} = 1/\omega_{2}$. Conse-
quently, Weyl particles in zero electromagnetic field exhibit one of the following behaviors:

- Move as free particles, assuming that $d\theta/dt = d\phi/dt = 0$, or
- Exist in a localized bounded state in the case that $d\theta/dt = \omega_1$, $d\phi/dt = 0$, or
- Exist in an intermediate state, bound on the $x$–$y$ plane, and free along the $z$-axis, corresponding to $d\theta/dt = 0$, $d\phi/dt = \omega_2$.

Thus, Weyl particles have the remarkable property to exist in different states in zero electromagnetic field.

Furthermore, let us consider the following electromagnetic fields

$$E_\theta (\mathbf{r}, t) = -\frac{1}{q} \left( [\sin \theta \cos \varphi \frac{\partial s}{\partial t} + \frac{\partial s}{\partial x} \sin \theta \sin \varphi \frac{\partial \varphi}{\partial t}] i + s \left( \cos \theta \cos \varphi \frac{\partial \varphi}{\partial t} - \sin \varphi \frac{\partial \varphi}{\partial t} \right) j - \frac{1}{q} \left( \cos \theta \tan \frac{\partial s}{\partial t} + \frac{\partial s}{\partial z} \sin \theta \sin \varphi \frac{\partial \varphi}{\partial t} \right) k \right)$$

and

$$B_\theta (\mathbf{r}, t) = \frac{1}{q} \left( -\sin \theta \sin \varphi \frac{\partial s}{\partial z} + \cos \theta \frac{\partial s}{\partial y} \right) i + \frac{1}{q} \left( \sin \theta \cos \varphi \frac{\partial \varphi}{\partial z} - \cos \varphi \frac{\partial \varphi}{\partial x} \right) j + \frac{1}{q} \sin \theta \left( -\cos \varphi \frac{\partial \varphi}{\partial y} + \sin \varphi \frac{\partial \varphi}{\partial x} \right) k \tag{23}$$

that correspond to the 4-potentials

$$(U, A) = (1, \sin \theta \cos \varphi, \sin \theta \sin \varphi, \cos \theta \sin \varphi) \frac{s(\mathbf{r}, t)}{q} \tag{24}$$

According to Theorem 3.1 in ref. [3], if $\mathbf{E}_\theta$ and $\mathbf{B}_\theta$ are added to $\mathbf{E}$, $\mathbf{B}$ or $\mathbf{E}', \mathbf{B}'$, then the state of the Weyl particles in the new electromagnetic field will not be affected.

As it can be observed in the above equation, the magnetic vector potential is the product of the electric scalar potential with the velocity of the particles. This holds also in the case of the Liénard–Wiechert potentials[7] that describe the magnetic vector potential and the electric scalar potential in the Lorenz gauge of a moving electric point charge. In natural units, the magnetic potential is related to the electric potential through the formula $A(\mathbf{r}, t) = v_\theta (t) U(\mathbf{r}, t)$, where $v_\theta (t)$ is the velocity of the source charge, evaluated at the retarded time $t_\theta = t - |r - r_\theta (t)|$, with $\mathbf{r}$ being the observation position vector and $\mathbf{r}_\theta$ the position vector of the point charge at the time of the signal’s emission from that point.

Further, let us assume two localized Weyl particles moving in closed circular orbits of radius $r$. The distance between the two particles is defined as the distance between the centers of the two circular orbits. From the above analysis, it follows that the distance between two successive particles must be equal to integer multiples of $2\pi r$ for the 4-potential of Equation (24) to be equal to the Liénard–Wiechert potential. It should be noted that the term “orbit” refers to the trajectory of a classical particle with the same velocity as that of the Weyl particle. Thus, if Weyl particles are arranged on a lattice of step equal to integer multiples of $2\pi r$, the interaction between the particles will not have any influence on their quantum state. This is another interesting implication of the fact that Weyl spinors are degenerate.

In the rest of the paper, we shall safely assume that the interaction between Weyl particles is negligible, since Weyl particles and consequently their charge are expected to be of the order of the electron charge. Therefore, the potentials and fields generated by these particles are expected to be negligible compared to the external fields. We shall also assume that function $s$ depends only on time to simplify the calculations and make the physical interpretation of the results more transparent.

In conclusion, in this section, we have shown that Weyl particles can exist in localized states, even in zero electromagnetic field. Thus, Weyl particles have the remarkable property to be able to exist at different states in zero electromagnetic field. At the same time, as shown in Theorem 3.1 in ref. [3], they can exist in the same state in a wide variety of electromagnetic fields. In the following section, we shall discuss in more detail the localization properties of Weyl particles, also providing some specific examples.

3. On the Localization of Weyl Particles and Its Control Through Simple Electric Fields

To gain more insight into the behavior of Weyl particles in localized states, we assume that the angles $\theta$ and $\phi$ change at a constant rate, namely $d\theta/dt = \omega_1$, $d\phi/dt = \omega_2$. The velocity $v$ of the particle, as given by Equation (16), in the time interval $t \in [0, 50\pi]$, for $\omega_1 = \sqrt{3}$, $\omega_2 = \sqrt{5}$, and $\varphi_0 = \theta_0 = \pi/4$, is shown in Figure 1.

The trajectory of a classical particle moving with the velocity shown in Figure 1 is graphically depicted in Figure 2. It is evident that the motion is bounded and, consequently, the Weyl particle is localized.

The electromagnetic field corresponding to these localized states is obtained from Equation (14)

$$\mathbf{E} = \frac{\omega_1 \omega_2}{2q} \left[ \cos (\varphi_0 + \omega_1 t) i + \sin (\varphi_0 + \omega_1 t) j \right]$$

$$\mathbf{B} = 0 \tag{25}$$

Thus, the localization of the particles is fully determined by the electromagnetic field in their region. This becomes more evident considering that one of the functions $\theta(t)$ and $\phi(t)$ is constant, while the other one is arbitrary. For example, supposing that $\theta(t) = \theta_0$ is constant, the electromagnetic field in Equation (14) becomes

$$\mathbf{E} = -\frac{1}{2q} \frac{d^2\varphi}{dt^2} k$$

$$\mathbf{B} = 0 \tag{26}$$
Figure 1. Plot of the velocity $v$ of a Weyl particle in natural units in the time interval $t \in [0, 50\pi]$, for $\omega_1 = \sqrt{3}$, $\omega_2 = \sqrt{5}$ and $\varphi_0 = \theta_0 = \pi/4$.

Figure 2. The trajectory of a classical particle moving with the velocity shown in Figure 1 for $t \in [0, 100\pi]$.

As mentioned in the previous section, in the absence of an electromagnetic field ($E = B = 0$), a classical particle moving with the same velocity as the Weyl one performs helical motion with radius
\[
r_\varphi = \sin \theta_0 \left( \frac{d\varphi}{dt} \right)^{-1}
\]

(27)

According to Equations (26) and (27), in the presence of an electric field $E$ along the $z$-axis, the particle follows a helical trajectory with a time-dependent radius $r_\varphi(t)$, related to the magnitude $|E|$ through the formula
\[
\frac{d}{dt} \left( \frac{1}{r_\varphi(t)} \right) = \pm \frac{2q |E|}{\sin \theta_0}
\]

(28)

with the plus and minus signs corresponding to $E$ being antiparallel and parallel to the direction of the angular velocity of the particle, respectively. Solving Equation (28) in the case of a constant electric field yields the following expression
\[
\frac{1}{r_\varphi(t)} = \frac{1}{r_{\varphi 0}} \pm \frac{2q |E|}{\sin \theta_0} t
\]

(29)

or equivalently
\[
r_\varphi(t) = \frac{r_{\varphi 0} \sin \theta_0}{\sin \theta_0 \pm 2qr_{\varphi 0} |E|} t
\]

(30)

where $r_{\varphi 0}$ is the initial value of the radius, prior to the application of $E$. According to Equation (30), in the case of $E$ parallel to the direction of the angular velocity of the particle, the radius $r_\varphi(t)$ becomes infinite, after a time interval
\[
 t_{\varphi 0} = \frac{\sin \theta_0}{2qr_{\varphi 0} |E|}
\]

(31)

Subsequently, the angular velocity changes sign and points to the opposite direction. Thus, in the presence of an electric field antiparallel to the vector of the angular velocity, the localization of the particle constantly increases. On the other hand, in the case of an electric field parallel to the vector of the angular velocity, the localization of the particle decreases initially becoming zero after a time interval given by Equation (31), and subsequently the particle becomes again localized with the vector of the angular velocity pointing to the opposite direction. The latter case is depicted in Figure 3. In Figure 4, we provide the projection of the motion of the particle on the $x$-$y$ plane, where the delocalization and re-localization are evident.

On the other hand, assuming that $\varphi(t) = \varphi_0$ is constant, the electromagnetic field of Equation (14) takes the form
\[
E = \frac{1}{2q} \frac{d^2 \theta}{dt^2} (\sin \varphi - \cos \varphi j)
\]

B = 0

(32)

In this case, assuming that $E = B = 0$, a classical particle with the same velocity as the Weyl one moves on a circular trajectory with radius
\[
r_\theta = \left( \frac{d\theta}{dt} \right)^{-1}
\]

(33)

According to Equations (32) and (33), in the presence of an electric field $E$ parallel to the vector $\sin \varphi i - \cos \varphi j$, the particle performs instantaneously circular motion with time-dependent radius $r_\varphi(t)$ given by the formula
\[
\frac{d}{dt} \left( \frac{1}{r_\varphi(t)} \right) = \pm 2q |E|
\]

(34)
Figure 3. The trajectory of a classical particle with the same velocity as the Weyl particle for \( \theta(t) = \pi/4 \) and \( \varphi(t) = 20t - 2t^2 \), corresponding to a constant electric field \( E = (2/q)k \), applied for \( t \in [0, 10] \).

As in the previous case, the plus and minus signs correspond to an electric field antiparallel and parallel to the direction of the angular velocity of the particle, respectively. Solving Equation (34) for \( r_\theta(t) \), assuming constant \( E \), yields

\[
\frac{1}{r_\theta(t)} = \frac{1}{r_{\theta 0}} \pm 2q |E| t \tag{35}
\]

or equivalently

\[
r_\theta(t) = \frac{r_{\theta 0}}{1 \pm 2qr_{\theta 0} |E| t} \tag{36}
\]

where \( r_{\theta 0} \) is the initial value of the radius, prior to the application of the electric field.

It can also be observed that, in the case of \( E \) parallel to the direction of the angular velocity of the particle, \( r_\theta(t) \) becomes infinite after a time interval

\[
t_{\theta 0} = \frac{1}{2qr_{\theta 0} |E|} \tag{37}
\]

For \( t > t_{\theta 0} \), \( r_\theta(t) \) changes sign, indicating that the angular velocity vector points to the opposite direction. Therefore, in the presence of an electric field \( E \) antiparallel to the vector of the angular velocity, the localization of the particle increases constantly. On the other hand, in the case that the electric field is parallel to the vector of the angular velocity, the localization of the particle initially decreases, becomes zero at \( t = t_{\theta 0} \), and subsequently the particle becomes localized again with the vector of the angular velocity pointing to the opposite direction. The delocalization and re-localization of the particle is shown in Figure 5.

In all cases, parameters \( \theta_0 \) and \( \varphi_0 \) represent the polar and azimuthal angles of the particles at the time of the application of the fields. From the above expressions, it is evident that the localization of the particles can be fully controlled using simple electric fields, which can be easily realized in practice. Also, the localization of the particles can increase or decrease depending on the direction of the applied electric field. Specifically, if the field is parallel (antiparallel) to the vector of the angular velocity of the particles, the localization of the particles decreases (increases). The opposite is true for particles with negative helicity.

Thus, the application of an electric field \( E \) perpendicular to the propagation direction of the Weyl particles enables the full control of their localization, as discussed above. This is a particularly interesting result with important possible applications in materials supporting Weyl particles.\[8–18\] The dynamics of Weyl particles
Consequently, the energy of the Weyl particles can be fully controlled applying an electric field along their direction of propagation. In addition, assuming that $E$ is constant, Equation (42) implies that $\frac{dE}{dt}$ is also constant, equal to $qE$. Obviously, the energy increases if the field is parallel to the propagation direction of the particles and decreases otherwise. In S.I. units the rate of the energy change in $J \cdot s^{-1}$ is equal to $|E|c$, where $|E|$ is expressed in $V \cdot m^{-1}$, $q$ is the charge of the particle in $C$, and $c$ is the speed of light in $m \cdot s^{-1}$. If we further assume that the charge of the particles is equal to the electron charge, the above analysis implies that the energy of the particles changes at a rate of $eV \cdot m^{-1}$ of propagation inside the constant field.

Thus, applying a constant electric field for the appropriate amount of time, it is even possible to annihilate the particle by making its energy zero. If the field $E$ remains applied, the particle will reappear moving in the opposite direction, increasing its energy with time. Consequently, it is possible to fully control the state of Weyl particles, regarding both their energy and localization, using simple electric fields. We believe that this result significantly enhances the potential of Weyl particles for practical applications.

It should be noted that the above analysis is also valid for massless Dirac particles described by degenerate spinors. However, in this case, it is not possible to change the propagation direction of the particles without changing the form of the solutions.

Another interesting remark is that, according to Equations (16)–(19), the modulus of the velocity of Weyl particles is always equal to one, which in free space is equal to the speed of light in vacuum, as expected for massless particles. In certain materials, the magnitude of the velocity of Weyl particles is expected to take lower values. However, since $c = 1$ in the natural system of units, Weyl equations do not depend on the magnitude of the velocity of the particles and they are expected to hold equally well in all materials supporting Weyl particles.

### 4. Conclusions

In this work, we have shown that Weyl particles can exist at different states in zero electromagnetic field, either as free particles or at localized states. Furthermore, it is shown that it is possible to fully control the localization, as well as the energy of Weyl particles, using simple electric fields, which can be easily realized in practice. We believe that these results significantly enhance the potential of Weyl particles for practical applications, both in the framework of solid-state physics and in the framework of laser physics, where the behavior of Weyl particles can be simulated using ions trapped by laser beams.

### Conflict of Interest

The authors declare no conflict of interest.

### Data Availability Statement

The data that support the findings of this study are available from the corresponding author upon reasonable request.

### Keywords

- electromagnetic fields
- localization
- Weyl equations
- Weyl particles
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