The manifestation of fractality in the architecture of buildings and structures

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Abstract. The article shows the presence of proportional correspondence of the parts and the whole using the principles of fractal geometry in the natural objects and phenomena beauty concept formation. The objects of the artificial environment are studied for the formation of images using the mathematical models’ example for calculating natural objects with fractal properties.

Introduction
The understanding of beauty coming from the proportional comparability of the parts and the whole has long been proved. The serious studies of this comparability have been known from the history up to this day.

It is believed that Pythagoras, who studied the secret sciences of the Egyptian priests in the pyramid country, for the first time called such harmonious proportional comparability of the parts and the whole the “golden proportion” and introduced this concept into the aesthetic canons of the ancient Greek culture.

In his studies, written in the III century BC and survived up to this day, Euclid built correct pentagons with the help of the golden ratio, and therefore such a pentagon was called the “golden one”.

The ancient Roman researcher Vitruvius, in his architectural work “10 books on architecture”, analysed and substantiated the harmonious proportionality of the most beautiful antique buildings.

At the time of the medieval Renaissance, the brilliant Italian mathematician Luca Pacioli wrote the first book about the golden proportion, called the “Divine Proportion”. In his opinion, even God used this principle to create the universe. This idea was later used by Kepler, the last book of whom was called “The Universe Harmony”.

Later, the proportional harmony was obtained in the process of sectioning a stereometric body formed by regular pentagons and then called the “golden section” by another brilliant scientist Leonardo da Vinci. The numerical expression of the golden section is named after the ancient Greek architect Phidias “phi” (denoted by the Greek letter φ) and is equal to 1.6180339887498948482 ..., the value is rounded off: φ = 1.618. In percentage terms, the golden section looks like 62% and 38% of the whole.

Thus, the principle of the golden section is the highest manifestation of the structural and functional perfection of the whole and its parts in nature, art, science, technology and architecture.

Since then, people have been fascinated by the mysterious connection between beauty and mathematics, the completely material harmony of forms and the perfect abstraction of numbers and
relationships. This connection is ephemeral, but tangible, it’s not for nothing that artists have been using the laws of geometry for many years and have been inspired by mathematical laws.

**Main part**

The still preserved architectural monuments causing admiration for their beauty, have already been investigated for the proportional correspondence of the parts and the whole and this harmony is not everywhere correlated with the presence of the “golden proportion”. To the question what is the beauty of certain architectural masterpieces of the past, an opportunity to answer only at the end of the twentieth century, when Benoit Mandelbrot introduced the term “fractal” in his book “Fractal geometry of nature”, published in 1977.

![Figure 1. Classic Fractal, Dynamic Fractal, Random Fractal](image)

This term is mainly from advanced maths. The term means a geometric figure that has the property of self-similarity, that is, composed of several parts, each of which is similar to the entire figure as a whole.

![Figure 2. The Sierpinski triangle, Peano curve, view of the Levi curve after 12 iterations](image)

The fractals naturally arise in physics in the simulation of nonlinear processes, such as turbulent fluid flow, complex diffusion-adsorption processes, flames, clouds, and others.
Figure 3. Graph of the Weierstrass function

Are they used in biology and botany? for example, to model plants, populations or to describe the systems of internal organs.

Figure 4. Tree, system of blood vessels and bronchial tree

After creating the Koch curve, it was proposed to use it when calculating the coastline length.
Figure 5. Koch curve and the coastline

Fractals are used in petrochemistry, for example, when modeling porous materials that have a very complex geometric structure.

Figure 6. Various porous materials samples micrographs

Fractals are widely used in computer graphics to build images of natural objects such as mountain landscapes, sea surfaces, and so on, achieving very believable realism and small amounts of data. One of the most famous fractals is the basis for computer construction of nature objects images (the Mandelbrot set) due to its color visualizations.
The architects of the past centuries intuitively used the fractal principles in their buildings because the sensation of beauty was given to them at the genetic level and they perceived spiritually the natural beauty surrounding them. The fractal principles were more often embodied in the fractal surfaces of buildings forming facades and their decorative elements, but fractality can also be observed in the spatial (internal and external) organization of buildings and structures. Since then, we have enjoyed the very interesting both in the image of the facades and in the construction of the spatial form architectural monuments [8], [9].
Figure 8. Examples of using the principles of fractality in architectural monuments

Hence, an understanding of the buildings and structures beauty law can only come after studying another one, causing interest in the scientific world, the proportional dependence of the parts and the whole - the principle of fractal geometry.

Figure 9. Examples of using the principles of fractality in modern architecture
In Wikipedia, the concept of “fractal” is interpreted as from the Latin “fractus” - crushed, broken - a set that has the property of self-similarity (an object that exactly or approximately coincides with a part of itself, that is, the whole has the same form as one or more parts) [10].

That is, “fractals” have the following properties: self-similarity; ability to develop; have fractional dimension, blurriness and fuzziness of contours; the ability to describe the chaotic processes [1], [2].

Many modern architects, inspired by the complex geometry of fractals, embodied mathematical images in simple, understandable to the eye and logic forms, thereby bringing into the world a bright and unforgettable diversity of fractal geometry in architecture.

At present, the principles of fractal geometry make it possible to simulate the surrounding reality in the most convenient way, which is manifested in the ability of virtual computer-generated fractals to simulate the real objects of both living and non-living world quite well. A Japanese designer Takeshi Miyakawa used the principle of fractality when creating furniture, namely, one of the bedside tables’ models. It consists of 23 drawers, and the drawers are located so that they almost completely use all the space in the shape of a cube allocated to the bedside table.

![Figure 10. A bedside table created by the principle of fractality](image)

It was difficult for Henry Segerman to abandon this source of ideas, since he is a mathematician by profession, then armed with the formulas and numbers, he arranged his virtual world in a mathematical way, filling it with the unseen fractal figures, spirals and even tesseracts, four-dimensional hypercubes. Hilbert’s curves, visualized by Segerman, represent fractal structures in the form of a continuous line that fills the space of a figure’s volume without ever interrupting or intersecting with itself, and if you increase the scale, you can see that parts of this curve repeating the shape of the whole. Segerman created a little more than a hundred sculptures, and each of them is a visual and, as far as possible, the exact physical embodiment of abstract mathematical concepts and laws.

![Figure 11. 3D-sculptures by Henry Segerman](image)

Summary
The principles of fractal geometry provide the ability to perform the computer experiments reproducing such phenomena and processes with which, for a number of reasons, it is impossible to conduct experiments in the real world. Fractal models make it possible to detect certain patterns and orderliness even in the systems where, at first glance, there is only disorder and chaos, since the state of chaos is not equivalent to random behavior, and only at the first glance they seem random, in fact, their values are completely predefined by the system input parameters.

Fractal geometry can and should be used in modeling the structure of the artificial environment created by mankind by means of architecture. The process of designing both a single building and its integration into the urban environment, which in the process of formation also seems rather chaotic, falls under the principles of fractal geometry. Modeling these processes using the principles of fractal geometry allows not only to create the most comfortable artificial urban environment, but to monitor the flow of all the processes within it.

As a result of the project activity, the basic principles of fractal geometry were formed - this is a hierarchical organization of the whole; its continuous shaping; measure singularity; uncertainty of boundaries and dynamism of chaos. Benoit Mandelbrot was the first to write about fractality in nature and architecture, based on the study of world architecture the buildings of the past with these principles [7].

The fractal principle of the structural elements arrangement in architecture is used both in the construction of the building itself, in the organization of the internal space, and in the organization of the entire complex as a whole. Architects, using the fractal principle, create unique designs. They try to combine the constructions of amazing shape and structure [3], [4]. The fractal principle in architectural structures has a fractional dimension, in which the self-similar elements are used with a fractal dimension.

XXI century completely moved the whole process of designing architectural objects to the virtual space of computer modeling. The landscape design today is carried out using the computer simulation with the most advanced programs.

An unlimited number of architectural images is obtained using mathematical models for calculating natural objects that have fractal properties. Special programs are created for calculating all the physical and mechanical characteristics of structures and their work to counteract external influences on the object as a whole. Modern architecture at the design stage concerns the creation of new materials, technologies and structures necessary for the construction of ultra-modern buildings.

The aesthetic characteristics of the concept of the artificial environment beauty are the result of the interaction of stability and chaos, linear non-linear activity and the principles of fractality are in the hands of scientists as a tool for analyzing the beauty of historical monuments of architecture and the possibility of modeling the ultramodern architecture of buildings and structures based on these principles.

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