Discussion of the deflagration index expression for cylindrical vessel

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Abstract. The cube-root law and the expression of the maximum overpressure rise rate available in the most literatures are accurate when they are used to calculate or assess the gas explosion intensity in a spherical vessel, however, large deviation will be occurred when they are extended to the vessels of other shapes and sizes. The deflagration index KG expression for the cylindrical vessel case is put forward and discussed, and the difference between the deflagration index formula of the spherical and cylindrical vessel are also simply discussed. The conclusions drawn by this paper may be useful to the gas explosion intensity calculation or assessment in cylindrical vessel explosions.

1. Introduction

Overpressure is the direct cause of all kinds of damage during a flammable gas explosion. In all overpressure characteristic parameters, the maximum overpressure rise rate and deflagration index are two important parameters for measuring gas explosion damage power, and widely used in engineering design and risk assessment of pressure vessel involving gas explosion [1,2]. In predicting and evaluating consequence of combustible gas explosions, the deflagration index KG is used in a greater extent than the maximum overpressure rise rate for examining the violence of explosions in closed vessels of various dimensions and shapes [2], and defined as:

$$KG=V^{1/3} \cdot (dp/dt)_{\text{max}}$$  \hspace{1cm} (1)

Where $(dp/dt)_{\text{max}}$ can be determined by Eq. (2) according to the study performed in a sphere vessel by Dahoe et al. (1996):

$$\left(\frac{dp}{dt}\right)_{\text{max}} = \frac{3(P_{\text{max}}-P_0)}{R_{\text{vessel}}} \left(\frac{P_{\text{max}}}{P_0}\right)^{2/3} S_L$$  \hspace{1cm} (2)

Where $R_{\text{vessel}}$ is the radius of the sphere vessel, $S_L$ is the laminar flame velocity.
From the Eq. (1) and (2), it can be seen that the deflagration index \( K_G \) is related to the volume of the vessel and the maximum overpressure rise rate. Razus et al. [3], Cammarota et al. [4], Nishimura et al. [5], Salzano et al. [6], Minu & Brandes [7] etc. all pointed out that the deflagration index \( K_G \) was closely related to not only the maximum overpressure rise rate and the volume of the vessel, but also the shape and the size of the vessel. Eq. (1) and (2) are accurate when they are used to calculate or assess the gas explosion intensity in a spherical vessel [1, 5], however, large deviation will be occurred when they are extended to the vessels of other shapes and sizes [2-6]. The main reason for this is that the flame shape, structure and speed will be influenced by the vessel shape and size, which will result in the increase of heat release rate and overpressure rise rate, eventually affects deflagration index \( K_G \) [5, 7, 8-10]. Therefore, in the calculation or estimation of the gas explosion intensity in a non-spherical vessel, the influence of the shape and size of the vessel must be taken into account.

In this paper, an expression for the deflagration index \( K_G \) when the influence of the shape and size of the vessel are taken into account will be deduced and discussed. The deflagration index \( K_G \) expression for cylindrical vessel is put forward and discussed. This may be useful to the gas explosion intensity calculation or assessment in cylindrical vessel explosions.

2. Derivation and discussion of the deflagration index expression for a cylindrical vessel

For spherical vessel, Cammarota et al. [4] conducted a formula to describe the relationship between the maximum pressure rise rate and the maximum pressure by analyzing results of the experiments in a cylindrical vessel with central ignition:

\[
\left( \frac{dp}{dt} \right)_{\text{max}} = \frac{A_v}{V} S_L \left( \frac{p_{\text{max}}}{p_0} - 1 \right) \left( \frac{p_{\text{max}}}{p_0} \right)^{1/3} \tag{3}
\]

Where \( A_v \) is the surface area of a spherical vessel with radius equivalent to the radius of the cylindrical experimental vessel, \( V \) is the volume of the cylindrical vessel, \( S_L \) is the laminar burning velocity, \( p_0 \) and \( p_{\text{max}} \) are, respectively, the initial pressure and the maximum pressure.

By substituting Eq. (3) into definition equation of the deflagration index \( K_G \), which is so called the cube-root law, they obtained the following equation for \( K_G \):

\[
K_G = \left( \frac{dp}{dt} \right)_{\text{max}}^{3/4} V = \frac{A_v}{V^{2/3}} S_L \left( \frac{p_{\text{max}}}{p_0} - 1 \right) \left( \frac{p_{\text{max}}}{p_0} \right)^{1/3} \tag{4}
\]

From Eq. (4), it seems that \( K_G \) has no relationship with the geometric parameters of the cylindrical vessel. In fact, however, some mistake maybe exist in the deduce process from Eq. (3) to Eq. (4). Assuming the radius and height (or length) of the cylindrical vessel are \( R \) and \( L \), respectively, the \( K_G \) can be derived by inserting Eq. (3) into the cube-root law:

\[
K_G = \left( \frac{dp}{dt} \right)_{\text{max}}^{3/4} V = \frac{A_v}{V^{2/3}} S_L \left( \frac{p_{\text{max}}}{p_0} - 1 \right) \left( \frac{p_{\text{max}}}{p_0} \right)^{1/3} = \left( \frac{16\pi R^2}{L^2} \right)^{1/3} S_L \left( \frac{p_{\text{max}}}{p_0} - 1 \right) \left( \frac{p_{\text{max}}}{p_0} \right)^{1/3} \tag{5}
\]

Where \( D \) is the diameter of the vessel. In contrast Eq. (5) with Eq. (4), the Eq. (4) is missing a coefficient of \( \left( \frac{16\pi}{L^2} \right)^{1/3} \), which just reflects the shape and size impact of the cylindrical vessel. In the
experimental set-up section of Cammarota et al. (2010), the diameter and height of the cylindrical vessel were 0.135 m and 0.4 m, respectively. So \( \left( \frac{16\pi D^2}{L^2} \right)^{1/3} \) is approximately equal to 1.789, which means Cammarota et al. [4] may have underestimated the deflagration index \( K_G \).

When the test vessel is spherical, a similar equation with a coefficient of \( (36\pi)^{1/3} \) for \( K_G \) was deduced by Dahoe et al. [1],

\[
K_G = \left( 36\pi \right)^{1/3} S_L \left( \frac{p_{\text{max}} - p^0}{p^0} \right)^{1/\gamma}
\]

(6)

It can be seen from Eq. (6) that the value of \( K_G \) is independent of the geometric parameters if in a sphere case. However, in cylindrical case, the \( K_G \) is closely related to the geometric parameters of the vessel. Specifically, the \( K_G \) of a cylindrical vessel is greatly influenced by inverse of the length to diameter ratio \( L/D \). In general, the length of a cylindrical vessel is larger than its corresponding diameter, therefore \( \left( \frac{16\pi D^2}{L^2} \right)^{1/3} \) for the cylinder is smaller than \( (36\pi)^{2/2} \) for the sphere. Maybe that is why Razus et al. [2], Mitu et al., [7], Movileanu et al. [11], Joseph and Patricia [12] etc. reported that the values of \( K_G \) were lower in elongated vessels in comparison with short vessels and they also claimed that the \( K_G \) in the cylindrical vessel was lower than that in spherical vessels.

3. Conclusions
The maximum overpressure rise rate and deflagration index are two important parameters for measuring gas explosion damage power. In this paper, the deflagration index \( K_G \) expression for the cylindrical vessel case is put forward and discussed, and the main conclusions can be drawn as follows:

(1) The cube-root law and the expression of the maximum overpressure rise rate available in most literatures are accurate when they are used to calculate or assess the gas explosion intensity in a spherical vessel, however, large deviation will be occurred when they are extended to the vessels of other shapes and sizes.

(2) When the influence of the shape and size of the vessel are taken into account, the deflagration index expression for a cylindrical vessel is

\[
K_G = \left( \frac{16\pi D^2}{L^2} \right)^{1/3} S_L \left( \frac{p_{\text{max}} - p^0}{p^0} \right)^{1/\gamma}
\]

(3) When the diameters are same, the values of \( K_G \) are lower in elongated vessels in comparison with short vessels and the \( K_G \) in the cylindrical vessel is also lower than that in spherical vessels.

Acknowledgments
This work was financially supported by xxx fund. Financial supports for this work, provided by the National Natural Science Foundation of China (No. 51704301) and Youth Scientific Research Foundation of LEU (No. YQ16-420802), are gratefully acknowledged.

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