Cyclic universe from a new chameleon scalar field

Changjun Gao† and Youjun Lu‡
The National Astronomical Observatories, Chinese Academy of Sciences, Beijing, 100012, China

You-Gen Shen‡
Shanghai Astronomical Observatory, Chinese Academy of Sciences, Shanghai 200030, China
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We explore a cyclic universe by introducing a new chameleon scalar field. In the original version of chameleon scalar field, the mass of the chameleon scalar depends on the environment, specifically on the ambient matter density. But in this new version, the ambient energy density determines not its mass but its kinetic energy which is achieved by the Lagrange multiplier field. We find the new chameleon scalar is dominant both in the very early universe and in the far future of the universe such that a cyclic universe is found. In this model of universe, there are infinite cycles of expansion and contraction. Different from the inflationary universe, the corresponding cosmic space-time is geometrically complete and quantum stable. But similar to the Cyclic Model, the flatness problem, the horizon problem and the large scale structure of the universe can be explained in this cyclic universe.

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I. MOTIVATION

The theory of inflationary cosmology has become the standard paradigm for the early universe, which was firmly established in the past several decades. Inflationary cosmology not only naturally solves the flatness problem, the horizon problem and the magnetic monopole problem, but also elegantly explains the large scale structure of the universe. Despite of its success, it may be not perfect yet and suffer from some conceptual problems as follows.

First, a few common questions may be almost always asked as follows: What is the inflaton field? How to fine-tune the scalar potential in order that it is sufficiently flat? How did the universe begin and why did it start to inflate? These questions are of great importance for the theory consummation. Second, inflationary cosmology suffers from the trans-Planckian problem [1]. In inflationary cosmology, the fluctuations observed in the cosmic microwave background had wavelengths at the beginning of inflation smaller than the Planck scale. It is generally believed that the quantum gravity would become important at sub-Planckian scales. So the quantum gravity may produce different distribution on sub-Planckian wavelengths. This different distribution would be inflated and generate an uncertain correction to the predictions for the cosmic microwave background anisotropy. Third, Borde et al. [2] showed that the inflationary space-time is geometrically incomplete in the past directions of the history of the universe as the expansion rate averaged along the geodesic is positive, i.e., $\langle H \rangle > 0$. This incomplete-
it shows the new chameleon scalar could be dominant both in the very early universe and in the far future of the universe such that a cyclic universe is found. In this cyclic universe, there are infinite cycles of expansion and contraction. Different from the inflationary universe, the corresponding cosmic space-time is geometrically complete and quantum stable. Similar to the Cyclic Model, the flatness problem, the horizon problem and the large scale structure of the universe can be explained in this cyclic universe.

This paper is organized as follows. In section II, we propose the new chameleon scalar field theory using the Lagrange multiplier method. The equations of motion are derived. Then we construct a cyclic model for the universe and investigate its cyclic evolution in section III. The total cosmic energy density is zero at the transition points, either from expansion to contraction or from contraction to expansion. Therefore, the Big-bang and Big-crunch singularity problem is automatically vanishing. In section V, we demonstrate that the cosmic space-time is geometrically complete for both time-like and null geodesics. The particle horizon and the event horizon are all infinite in this space-time. So the horizon problem is solved. In section VI using the Wheeler-DeWitt equation, we find the system is quantum stable although a negative scalar potential is present. In section VII, we argue that flatness problem and the primordial scale-invariant power spectrum problem could be solved in this cyclic model. Finally, conclusions and discussions are given in section VIII. Throughout this paper, we adopt the system of units in which $G = c = \hbar = 1$ and the metric signature $(-, +, +, +)$.

II. A NEW CHAMELEON SCALAR FIELD

In this section, we shall introduce a new chameleon scalar field $\phi$. To this end, we start from the action as follows

$$S = \int d^{4}x \sqrt{-g} \left[ \frac{R}{16\pi} + \lambda \left( \frac{1}{2} \nabla_{\mu} \phi \nabla^{\mu} \phi - \frac{1}{6} G_{\mu \nu} U^{\mu} U^{\nu} \right) V(\phi) + L_{m} \right].$$  \tag{1}

Here $R$, $\phi$, $G_{\mu \nu}$, $U^{\mu}$, $V(\phi)$, $L_{m}$ are the Ricci scalar, the Lagrange multiplier field, the chameleon scalar field, the Einstein tensor, the four-velocity of isotropic observer, the chameleon scalar potential and the ordinary matter Lagrangian, respectively. We note that, different from the conventional scalar field, the Lagrange multiplier field $\lambda$ is present in the action which makes the evolution of the scalar very different from the quintessence. The Lagrange multiplier field is recently introduced into the investigation of cosmology by Eugene Lim et al. \cite{16} and one of the authors \cite{17} like this:

$$S = \int d^{4}x \sqrt{-g} \left[ \frac{R}{16\pi} + \lambda \left( \frac{1}{2} \nabla_{\mu} \phi \nabla^{\mu} \phi + V(\phi) \right) + L_{m} \right].$$  \tag{2}

Afterwards, this Lagrange multiplier field cosmology is studied by many authors, for example, see Ref. \cite{18}. Comparing Eq. (1) and Eq. (2), we find Eq. (1) is a new theory.

Variation of the action Eq. (1) with respect to the metric $g^{\mu \nu}$ gives the Einstein equations

$$G_{\mu \nu} = 8\pi \left[ \sum_{i} T^{(m)}_{\mu \nu} + T^{(\phi)}_{\mu \nu} \right],$$

$$T^{(m)}_{\mu \nu} = (\rho_{i} + p_{i}) U_{\mu} U_{\nu} + p_{i} g_{\mu \nu},$$

$$T^{(\phi)}_{\mu \nu} = -\lambda \nabla_{\mu} \phi \nabla^{\mu} \phi + \frac{1}{2} \lambda \nabla_{\sigma} \phi \nabla^{\sigma} \phi g_{\mu \nu} - \frac{1}{6} \lambda \nabla_{\alpha} (\nabla_{\beta} U^{\alpha}) - \frac{1}{6} \left[ 2 \nabla_{\alpha} \nabla_{\beta} (\lambda U^{\alpha}) U^{\beta} \right] g_{\mu \nu} - 4 \lambda U^{\alpha} R_{\alpha \beta \mu \nu} U^{\beta}.$$  \tag{3}

where $\rho_{i}$ and $p_{i}$ are the energy density and pressure of the $i$-th perfect fluid.

Secondly, variation of the action Eq. (1) with respect to $\lambda$ and $\phi$ give the equation of motion for $\lambda$

$$\frac{1}{2} \nabla_{\mu} \phi \nabla^{\mu} \phi - \frac{1}{6} G_{\mu \nu} U^{\mu} U^{\nu} = 0,$$  \tag{4}

and for $\phi$,

$$\nabla_{\mu} (\lambda \nabla^{\mu} \phi) - \frac{\partial V}{\partial \phi} = 0,$$  \tag{5}

respectively. Finally, the equation of motion of $i$-th perfect fluid is given by

$$\nabla^{\mu} T^{(m)}_{\mu \nu} = 0.$$  \tag{6}

Eq. (4) reveals that the kinetic energy of the scalar field is proportional to the background energy density of spacetime because of

$$G_{\mu \nu} U^{\mu} U^{\nu} \sim T_{\mu \nu} U^{\mu} U^{\nu} \sim \rho_{BG}.$$  \tag{7}

In the conventional chameleon scalar theory, the mass of the scalar is proportional to the background energy density \cite{12, 14}. So this is a new chameleon scalar field. In the following, we shall study the cosmic evolution of the chameleon scalar in the background of Friedmann-Robertson-Walker spacetime.

The line element of Friedmann-Robertson-Walker spacetime is given by

$$ds^{2} = -dt^{2} + a(t)^{2} \left( dx^{2} + x^{2} d\Omega^{2} \right),$$  \tag{8}

with $a(t)$ the cosmic scale factor. The four-velocity takes the form

$$U^{\mu} = (-1, 0, 0, 0).$$  \tag{9}
At first glance, the energy momentum tensor $T^\mu_\nu$ in Eq. (3) is rather complicated. But it is actually very simple in the background of Friedmann-Robertson-Walker spacetime. We find equations (3-6) are simply

$$3H^2 = 8\pi \left( V + \sum_i \rho_i \right),$$  \hspace{1cm} (10)

$$2\dot{H} + 3H^2 = -8\pi \left[ -V + \frac{1}{3}\lambda H^2 ight. + \frac{1}{3a^2} (\lambda a^2)^2 + \sum_i \rho_i \right],$$  \hspace{1cm} (11)

$$\dot{\phi} = H,$$  \hspace{1cm} (12)

$$\frac{1}{a^2} \frac{d}{dt} (\lambda a^3 \dot{\phi}) + \frac{\partial V}{\partial \phi} = 0,$$  \hspace{1cm} (13)

$$\dot{\rho}_i + 3H (\rho_i + p_i) = 0.$$  \hspace{1cm} (14)

Here $H \equiv \dot{a}/a$ is the Hubble parameter and dot represents the derivative with respect to the cosmic time $t$. Eq. (10) and Eq. (11) are the Friedmann equation and acceleration equation, respectively. Eq. (12) is exactly the equation of motion for $\lambda$ although $\lambda$ is in absence. Eq. (13) is the equation of motion for $\phi$ and Eq. (14) is the energy conservation equation for $i$-th perfect fluid.

From the Friedmann equation, we see a significant difference from the quintessence that the energy density of the chameleon scalar field is uniquely contributed by the potential energy. This makes the investigation of the cosmic evolution of the scalar field remarkably simple. In fact, from Eq. (12) we obtain

$$\dot{\phi} = \ln a.$$  \hspace{1cm} (15)

So the scalar potential $V(\phi)$ is nothing but $V(\ln a)$. For our purpose, we shall consider a negative scalar potential

$$V(\phi) = -\left( \rho_{\phi_0} e^{-m\phi} + \rho_\psi e^{n\phi} \right),$$  \hspace{1cm} (16)

with $\rho_{\phi_0}$, $\rho_\psi$, $m$, $n$ four positive constants. The reason for this negative potential will be given in the next section and we shall show that, with this negative scalar potential, a cyclic universe is found.

### III. CYCLIC UNIVERSE

Now the Friedmann equation could be written as follows

$$3H^2 = 8\pi \left( \frac{\rho_{\phi_0}}{a^m} + \frac{\rho_{\psi}}{a^3} + \frac{\rho_m}{a^3} + \frac{\rho_k}{a^2} + \rho_\Lambda - \rho_{\psi_0} a^3 \right).$$  \hspace{1cm} (17)

On the right hand side of Eq. (17), the energy densities are from the chameleon scalar field (the first and the last term), relativistic matter, dust matter (which includes dark matter and baryonic matter), cosmic spatial curvature and cosmological constant, respectively. The parameters $\rho_{\phi_0}$, $\rho_{\psi}$, $\rho_m$, $\rho_k$, $\rho_\Lambda$, and $\rho_{\psi_0}$ represent the present-day energy density of each component, respectively. The parameter $\rho_k$ can be positive, negative or 0, and the other five parameters are all positive.

Now we could understand why we introduce the negative scalar potential. According to the Friedmann equation for the standard $\Lambda$CDM cosmology

$$3H^2 = 8\pi \left( \frac{\rho_m}{a^3} + \frac{\rho_m}{a^3} + \frac{\rho_k}{a^2} + \rho_\Lambda \right),$$  \hspace{1cm} (18)

the evolution of the scale factor $a$ is controlled by the relativistic matter if $a$ is sufficiently small, and $a \propto \sqrt{t}$. At the Big-bang singularity, $t = 0$, the scale factor vanishes and the energy density is divergent. In order to erase the Big-bang singularity and turn the universe from contraction to expansion, namely, making the velocity of the universe $\dot{a}$ varying from negative (contracting universe) to positive (expanding universe), it is necessary to introduce the negative energy density. At the turning point, we should have $\dot{a} = 0$.

For simplicity, the negative energy density may be assumed to scale as $a^{-n}$ with $n > 4$ such that the velocity of the universe $\dot{a}$ could cross zero at the minimum of the scale factor. We take $n = 6$ in this paper. Similarly, with the increasing of the scale factor, dark energy dominates finally. In order to let the universe contract at sometime in the future, it is also necessary to introduce negative energy density that dominates at large $a$, and this density may be assumed to scale as $a^{-m}$ with $m > 0$ such that the velocity of the universe $\dot{a}$ could cross zero at the maximum of the scale factor. In this study, we simply take $m = 2$.

Given the Friedmann equation, Eq. (17), and the equation of conservation for energy Eq. (14), we derive the acceleration equation

$$\frac{6}{H_0^2} \frac{\ddot{a}}{a} = -8\pi \left( -4\rho_{\phi_0} a^6 + 4\rho_\psi a^4 - \rho_m a^3 - 2\rho_\Lambda + 4\rho_{\psi_0} a^2 \right).$$  \hspace{1cm} (19)

For the present-day universe, we have the Friedmann equation

$$3H_0^2 = 8\pi \rho_0,$$  \hspace{1cm} (20)

where $H_0$ and $\rho_0$ are the present-day Hubble parameter and total cosmic energy density.

Dividing the acceleration equation Eq. (19) by Eq. (20), we obtain

$$\frac{2}{H_0^2} \frac{\ddot{a}}{a} = \frac{4\Omega_{\phi_0}}{a^6} - \frac{2\Omega_\psi}{a^4} - \frac{\Omega_m}{a^3} + 2\Omega_\Lambda - 4\Omega_{\psi_0} a^2,$$  \hspace{1cm} (21)

where the dimensionless constants (the ratio of each component in the present-day cosmic energy density) are defined as

$$\Omega_{\phi_0} = \frac{\rho_{\phi_0}}{\rho_0}, \quad \Omega_\psi = \frac{\rho_\psi}{\rho_0}, \quad \Omega_m = \frac{\rho_m}{\rho_0},$$

$$\Omega_\Lambda = \frac{\rho_\Lambda}{\rho_0}, \quad \Omega_{\psi_0} = \frac{\rho_{\psi_0}}{\rho_0}, \quad \Omega_k = \frac{\rho_k}{\rho_0}.\quad \text{(22)}$$
We can let the cosmic time $t$ absorb $H_0$ in Eq. (21). Then
the unit of cosmic time is the Hubble age
\[ \tau = \frac{1}{H_0}. \]
(23)

Eq. (21) is reduced to
\[ \frac{2\ddot{a}}{a} = \frac{4\Omega_{\phi_0}}{a^3} - \frac{2\Omega_{r_0}}{a^4} - \frac{\Omega_{m_0}}{a^3} + 2\Omega_\Lambda - 4\Omega_{\psi_0}a^2. \]
(24)

As an example, we consider the cosmology model with the following values of parameters, $\Omega_{r_0} = 8.1 \cdot 10^{-5}$, $\Omega_{m_0} = 0.27$, $\Omega_\Lambda = 0.73$, and $\Omega_k = 0$, which are consistent with current observations [19]. As for the values of $\Omega_{\phi_0}$ and $\Omega_{\psi_0}$, they must be sufficiently small in order to be not contradict with the well constrained standard $\Lambda$CDM model. However, with decreasing scale factor, the universe is finally dominated by both the $\Omega_{\phi_0}$ term and the radiation. A maximum energy density appears in this epoch. It is generally believed that the Planck energy density,
\[ \Omega_{\text{Planck}} \simeq 10^{123}, \]
(25)
is the highest one. So $\Omega_{\phi_0}$ is constrained to be in the order of $\Omega_{\phi_0} \simeq 10^{-68}$. For $\Omega_{\psi_0}$, we simply set $\Omega_{\psi_0} \simeq 10^{-10}$, which is not inconsistent with astronomical observations.

Figures 1 and 2 show the evolution of the scale factor $\ln a$ and the velocity $v \equiv 10^{-4}\dot{a}$ of the universe with respect to the cosmic time $t$. Apparently there are infinite cycles, expansion and contraction, in the evolution of the universe. The present-day universe locates at $\ln a = 0$ as we have set $a = 1$ for the present-day universe. The minimum and the maximum of the scale factor are $a_{\text{min}} \simeq (\Omega_{\phi_0}/\Omega_{r_0})^{1/2} \sim e^{-73}$ and $a_{\text{max}} \simeq (\Omega_{\psi_0}/\Omega_\Lambda)^{1/2} \sim e^{11}$, respectively (see Fig. 1). Note here that what characters the physical size of the universe is not the scale factor, but the apparent horizon, event horizon and particle horizon. We shall come back to this point in the next section.

Fig. 3 shows the evolution of total cosmic energy density $\rho$ with respect to the cosmic time $t$. The maximum and minimum of the energy density are the Planck density and zero, respectively. As seen from Figure 3, the universe contracts from $B$ to $C$ and expands from $C$ to $D$ with the increasing of time. The contracting and expanding phase constitute a complete cycle of evolution. In order to understand the evolution in detail, let’s start from point $A$, which represents the time one cycle period before the present day (point Now in Fig. 3). In the first several Hubble ages (a plateau), the universe...
is dominated by the term $\Omega_\Lambda$. Then the term $-\Omega_{\psi_0} a^2$ grows and the total cosmic energy density evolves as $\rho \simeq \Omega_\Lambda - \Omega_{\psi_0} a^2$. The energy density decreases with the expansion of the universe. When the density vanishes, we find from the Friedmann equation Eq. (17) that the velocity $\dot{a}$ of the universe also vanishes but the acceleration of the universe $\ddot{a}$ becomes negative (from Eq. (24)). So after crossing point $B$, the universe evolves into contracting phase. $B$ (and $D$) corresponds to the maximum of the scale factor as clearly shown in Fig. 4.

In the contracting phase (from $B$ to $C$), the universe is first dominated by $\Omega_\Lambda - \Omega_{\psi_0} a^2$, and then by $\Omega_\Lambda$ (the plateau), by the matter, by the radiation, and by radiation and term $\Omega_{\psi_0}$ in a sequence. The maximum of energy density appears at $a \simeq \sqrt{\frac{\rho}{\rho_0}} a_{min}$. With the increasing of term $\Omega_{\psi_0}$, the cosmic density decreases sharply to 0 at $a = a_{min}$. At the minimum of the scale factor, both the cosmic density and the cosmic velocity vanishes (point $C$). After point $C$, the universe goes into the expanding phase. At the turning point $C$, we have $\dot{a} = 0$ and $\ddot{a} > 0$ which follows from Eq. (17) and Eq. (24).

In the expanding phase (from $C$ to $D$), the universe is firstly dominated by both the chameleon scalar and the radiation, $\rho \simeq \Omega_{\psi_0} a^4 - \Omega_{\psi_0} a^6$. Since the density of chameleon scalar decreases quickly with the expansion of the universe, the cosmic energy density grows significantly. Once the cosmic energy density crosses the maximum, the universe evolves as the standard $\Lambda\text{CDM}$ model (before arriving at point $D$). The present-day universe locates at $\rho = 1$ (one unit of present-day cosmic energy density, $\rho_0$). It is apparent the age of the universe (from the maximum energy density to Now) is within one Hubble age which is consistent with observations.

**IV. GEOMETRICALLY COMPLETE SPACE-TIME AND HORIZONS**

**A. geometrically complete space-time**

In this subsection, we show that the space-time of the cyclic universe constructed in section [11] is complete for both null and time-like geodesics. Therefore, there is no singularity in this space-time. To this end, let’s first derive the equation of motion for a photon and a massive particle. The metric of spatially-flat Friedmann-Robertson-Walker Universe is given by

$$ds^2 = dt^2 - a(t)^2 (dx^2 + dy^2 + dz^2).$$

Then the Lagrangian for a photon and a massive particle propagating in this space-time is given by [20]

$$\mathcal{L} = \frac{1}{2} \left[ \dot{t}^2 - a^2 (\dot{x}^2 + \dot{y}^2 + \dot{z}^2) \right] \equiv \epsilon,$$

where dot denotes the derivative with respect to the affine parameter $\tau$, and $\tau$ is exactly the proper time of a massive particle. We have $\epsilon = 0$ and $\epsilon = 1/2$ for the photon and massive particle (with mass of unit one), respectively [20]. Using the Euler-Lagrange equation, we obtain the equation of geodesics

$$i^2 - \frac{c^2}{a^2} - 2\epsilon = 0,$$

where $\epsilon = 0$ for the null geodesics and $\epsilon = 1/2$ for the time-like geodesics, and $\zeta$ is an integration constant that represents the momentum of the particle. Therefore, $\tau$ is integrated as

$$\tau = \int_0^{+\infty} \frac{1}{i \sqrt{2\epsilon + \zeta^2}} dt = \infty,$$

for the future-pointing geodesics, and

$$\tau = \int_0^{-\infty} \frac{1}{i \sqrt{2\epsilon + \zeta^2}} dt = \infty,$$

for the past-pointing geodesics, respectively. Equations (29) and (30) show that the space-time is complete for both null and time-like geodesics.

**B. horizons**

In general, there are three horizons in the Friedmann-Robertson-Walker space-time, i.e., the particle horizon, event horizon and apparent horizon. In a universe with a finite age, a photon would only propagate a finite distance in that finite time and the volume of space from which we can receive information at a given time is also limited. The boundary of this volume is called the particle horizon which is given by

$$d_p = a \int_{t_i}^t \frac{dt}{a},$$

where $t_i$ is the beginning of the universe. For the cyclic universe presented above, we have $t_i = -\infty$ and $a_{min} \leq a \leq a_{max}$. So the corresponding particle horizon is infinite.

The event horizon is defined as the complement of the particle horizon. It encloses the set of points from which photons sent at a given time $t$ will never be received by an observer in the future. For our cyclic universe, we find the physical size of the event horizon is also infinite,

$$d_e = a \int_{t}^{+\infty} \frac{dt}{a} = \infty.$$

The apparent horizon is a marginally trapped surface with vanishing expansion and has been argued to be a causal horizon for a dynamical space-time. The apparent horizon is associated with the Hawking temperature, gravitational entropy and other thermodynamical aspects [21][23]. The first law of thermodynamics for the
apparent horizon has been derived not only in general relativity but also in several alternative theories of gravity, including the Lovelock, nonlinear, scalar-tensor, and braneworld theories \[24–30\]. Therefore, it is of great importance for us to consider the apparent horizon in the cyclic model of the universe. For the spatially flat universe, the physical size of the apparent horizon is given by

\[ d_a = \frac{1}{H}. \]  

(33)

We recognize that it is exactly the Hubble scale or Hubble horizon. For our cyclic universe, we find there is a minimum of the physical size of the apparent horizon, namely, the Planck length. At the turning points from expansion to contraction (and from contraction to expansion), the Hubble radius is infinite. Since the particle horizon and event horizon are infinite, the cosmic horizon problem is solved.

V. QUANTUM STABILITY

In this section, we shall consider the quantum stability of this cyclic universe model. In the approximation of minisuperspace, the wave function of the universe depends only on one freedom, the scale factor \(a\). The Hamiltonian of the universe is given by

\[ \mathcal{H} = -\frac{1}{3\pi a} \left[ p_a^2 + U(a) \right], \]  

(34)

where

\[ p_a = \frac{3\pi}{2} a\dot{a}, \]  

(35)

is the momentum conjugate to \(a\) and the potential \(U(a)\) is given by

\[ U(a) = \frac{9\pi^2}{4} a^2 \left(1 - \frac{8\pi}{3} a^2 \rho \right), \]  

(36)

where \(\rho\) is total energy density of the universe. Then the Hamiltonian constraint \(\mathcal{H} \equiv 0\) gives exactly the Friedmann equation, Eq. (17).

In the theory of quantum cosmology, the universe is described by a wave function \(\Psi(a)\). The conjugate momentum \(p_a\) becomes a differential operator \(-i\frac{d}{da}\) and the Hamiltonian constraint is replaced by the Wheeler-DeWitt (WDW) equation \[31\]

\[ \mathcal{H}\Psi = 0, \]  

(37)

i.e.,

\[ \left(-\frac{d^2}{da^2} + U\right)\Psi = 0. \]  

(38)

Fig. 4 shows the potential \(U(a)\) with respect to the scale factor \(a\). The point C and D correspond to the minimum \(a_{min}\) and maximum \(a_{max}\) of the scale factor, respectively. The region between C and D is classically allowed. As the potential is divergent when \(a \to 0\) and \(a \to \infty\), the universe can not tunnel to the classically forbidden regions, \(a < a_{min}\) and \(a > a_{max}\). In other words, this model of cyclic universe is quantum stable.

VI. FLATNESS PROBLEM AND THE SCALE-INVARIANT PERTURBATIONS

In the Cyclic Model \[3\] of the universe, because the equation of state \(w \gg 1\) during the contraction phase, the universe is homogeneous, isotropic and flat \[34\] with a scale-invariant spectrum of density perturbations \[32, 33\]. The \(w \gg 1\) condition also ensures that anisotropy is small and first order perturbation theory remains valid until just before the bounce \[34\].

We define the equation of state as \(w' = \sum_i \frac{p_i}{\sum_i \rho_i}\), where \(i\) represents the \(i\)-th component. Fig. 5 shows the evolution of \(w'\) with the natural logarithm of the scale factor (lna). As shown in Fig. 5, \(w' \gg 1\) in the contracting phase. Thus we conclude that the flatness of spacetime and the scale-invariant primordial power spectrum can also be generated in the present cyclic universe model.

VII. CONCLUSION AND DISCUSSION

In conclusion, we have explored a cyclic model for the universe by simply introducing a new chameleon scalar field using the Lagrange multiplier method. In the original version of chameleon scalar field, the mass of the
FIG. 5: Evolution of the equation of state $w'$ with the natural logarithm of scale factor $\ln a$. We have $w' \gg 1$ in the vicinity of the turning point from expansion to contraction.

The chameleon scalar depends on the environment, specifically on the ambient matter density. But in this new version, the ambient energy density determines not its mass but its kinetic energy. Then a significant difference from the conventional chameleon scalar appears: the energy density of the new chameleon scalar field is uniquely contributed by the scalar potential in the evolution of the universe. Given a specific potential, the chameleon scalar could be important around both the minimum and the maximum of the cosmic scale factor. So a cyclic universe is found.

There are infinite cycles of cosmic expansion and contraction in this model. At the turning points from expansion to contraction or from contraction to expansion, the cosmic energy density is zero. So the Hubble horizon is infinite at these turning points. We also find that the cosmic space-time is geometrically complete for both time-like and null geodesics. Therefore, there is no Big-bang or Big-crunch singularity in the space-time. We calculate the particle horizon and the event horizon, respectively, and find they are infinite. So the horizon problem is vanishing. Using the Wheeler-DeWitt equation, we show the system is quantum stable although a negative scalar potential is present. Finally, we argue that the flatness of spacetime and the primordial scale-invariant power spectrum could be generated in this cyclic model with the help of findings by Gratton et al. [32] and Boyel et al. [33].

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[1] R. H. Brandenberger, [arXiv:hep-th/1204.6108].
[2] A. Borde, A. H. Guth and A. Vilenkin, Phys. Rev. Lett. 90, 151301 (2003) [gr-qc/0110012].
[3] P. J. Steinhardt and N. Turok, Science 296, 1436 (2002); P. J. Steinhardt and N. Turok, Phys. Rev. D 65, 126003 (2002).
[4] L. Baum, P. H. Frampton, Phys. Rev. Lett. 98, 071301 (2007).
[5] R. H. Brandenberger, Phys. Rev. D 80, 043516 (2009); Y. Cai, D. A. Easson, R. H. Brandenberger, JCAP, 08(2012)020; C. Lin, R. H. Brandenberger, L. P. Levasseur, JCAP 1104:019,2011; T. Biswas, E. Gerwick, T. Koivisto, A. Mazumdar, Phys. Rev. Lett. 108, 031101 (2012); T. Biswas, A. S. Koshelev, A. Mazumdar, S. Y. Vernov, JCAP 08 (2012) 024; T. Biswas, E. Gerwick, T. Koivisto, A. Mazumdar, JCAP 1011:008,2010; T. Biswas, A. Mazumdar, W. Siegel, JCAP 0603 (2006) 009.
[6] G.F.R. Ellis and R. Maartens, Class. Quantum Grav. 21, 223 (2004) [gr-qc/0211082v4];
[7] J. D. Barrow, G. F. R. Ellis, R. Maartens, and C.G. Tsagas, Class. Quant. Grav. 20, L155 (2003) [gr-qc/0302094];
[8] S. del Campo, E. Guendelman, A.B. Kaganovich, R. Herrera and P. Labrana, Phys. Lett. B 699, 211 (2011), [arXiv:1105.0651];
[9] P. Wu and H. Yu, Phys. Rev. D81, 103522 (2010) [arXiv:0909.2821];
[10] P.W. Graham, B. Horn, S. Kachru, S. Rajendran, and G. Torroba, [arXiv:1109.0282].
[11] M. P. Dubowski, Ann. Phys. 248, 199 (1996) [gr-qc/9503017].
[12] J. Khoury and A. Weltman, Phys. Rev. Lett. 93, 171104 (2004).
[13] David F. Mota, John D. Barrow, Phys. Lett. B 581, 141 (2004).
[14] J. Khoury and A. Weltman, Phys. Rev. D 69, 044026 (2004).
[15] P. Brax, C. van de Bruck, A.-C. Davis, J. Khoury, and A. Weltman Phys. Rev. D 70, 123518 (2004); H. Wei and R-G. Cai, Phys. Rev. D 71, 043504 (2005).
[16] E. A. Lim, I. Sawicki, A. Vikman, JCAP 05 (2010) 012 [astro-ph:1003.5751].
[17] C. Gao, Y. Gong, X. Wang and X. Chen, Phys. Lett. B, 702, 107 (2011) [astro-ph:1003.6056].
[18] S. Capozziello, M. De Laurentis and S. D. Odintsov, Eur. Phys. J. C 72, 2068 (2012); S. Capozziello, J. Matsumoto, S. Nojiri, S.D. Odintsov, Phys. Lett. B 693, 198 (2010); S. Nojiri and S. D. Odintsov, Phys. Rev. D 83 (2011) 023001; J. Kluson, S. Nojiri and S.D. Odintsov, Phys. Lett. B 701, 117 (2011); Yi-Fu Cai, E.N. Saridakis, Class.Quant.Grav. 28, 035010 (2011); D. Saez-Gomez, Phys.Rev. D 85, 023009 (2012); L.N. Granda, E. Losia, JCAP 1209, 011 (2012); A Cid, P. Labrana, Phys. Lett. B 717, 10 (2012).
[19] D. N. Spergel et al., Astrophys. J. Suppl. Ser. 170, 377 (2007); Morad Amarzguioui, O. Elgaroy, David F. Mota
and T. Multamaki, Astrophys. J. 454, 707 (2006).
[20] S. Chandrasekhar, The Mathematical Theory of Black Holes, (Oxford University Press, New York, 1983).
[21] S. A. Hayward, S. Mukohyama, and M.C. Ashworth, Phys. Lett. A 256, 347 (1999); S.A. Hayward, Class. Quantum Grav. 15, 3147 (1998).
[22] D. Bak and S.J. Rey, Class. Quantum Grav. 17, L83 (2000).
[23] R.G. Cai and S.P. Kim, J. High Energy Phys. 02, 050 (2005).
[24] Y. Gong and A. Wang, Phys. Rev. Lett. 99, 211301 (2007).
[25] M. Akbar and R.G. Cai, Phys. Rev. D 75, 084003 (2007).
[26] R.G. Cai and L.M. Cao, Phys. Rev. D 75, 064008 (2007).
[27] R.G. Cai and L.M. Cao, Nucl. Phys. B 785, 135 (2007).
[28] M. Akbar and R.G. Cai, Phys. Lett. B 648, 243 (2007).
[29] A. Sheykhi, B. Wang and R.G. Cai, Phys. Rev. D 76, 023515 (2007).
[30] A. Sheykhi, B. Wang and R.G. Cai, Nucl. Phys. B 779, 1 (2007).
[31] B.S. DeWitt, Phys. Rev. 160, 1113 (1967); A. Vilenkin, Phys. Rev. D50, 2581 (1994); C. Kiefer and B. Sandhofer, arXiv:0804.0672 [gr-qc]; J. J. Halliwell, in Proceedings of the 1990 Jerusalem Winter School on Quantum Cosmology and Baby Universes, ed. by S. Coleman, J.B. Hartle, T. Piran and S. Weinberg (World Scientific, Singapore, 1991).
[32] S. Gratton, J. Khoury, P. J. Steinhardt and N. Turok, Phys. Rev. D 69, 103505 (2004).
[33] L. A. Boyle, P. J. Steinhardt and N. Turok, Phys. Rev. Lett. 96, 111301 (2006).
[34] J. K. Erickson, D. H. Wesley, P. J. Steinhardt and N. Turok, Rev. D 69, 063514 (2004).
[35] P. J. Steinhardt and N. Turok, New. Astron. Rev. 49, 43 (2005).