PHOTON FEEDBACK: SCREENING AND THE EDDINGTON LIMIT

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ABSTRACT

Bright star-forming galaxies radiate well below their Eddington limits. The value of the flux-mean opacity that mediates the radiation force onto matter is orders of magnitude smaller than the UV or optical dust opacity. On empirical grounds, it is shown that high-redshift ULIRGs radiate at two orders of magnitude below their Eddington limits, while the local starbursters M82 and Arp 220 radiate at a few percent of their Eddington limits. A model for the radiative transfer of UV and optical light in dust-rich environments is considered. Radiation pressure on dust does not greatly affect the large-scale gas dynamics of star-forming galaxies.

Key words: dust, extinction – galaxies: general – galaxies: starburst – ISM: jets and outflows – radiative transfer

Online-only material: color figure

1. INTRODUCTION

Star formation is inefficient. The timescale at which gas is converted into stars is long in comparison to the gravitational collapse time. In order to explain this, some form of self-regulation or “feedback” is often invoked.

There are many different mechanisms: core-collapse supernovae, thermonuclear supernovae, radiation pressure on dust, cosmic ray pressure, stellar winds, heating due to ionizing radiation, jets from proto-stellar disks, turbulent stresses, ram-pressure stripping, pulsars winds, cosmic ray heating, dark matter annihilation as well as the various forms of energy release from supermassive black hole growth (see the Appendix for a brief literature review).

The UV and optical radiation force that directly results from star formation seems to be a promising candidate and has recently gained a significant amount of attention. Second only to prompt neutrinos from core-collapse supernovae, starlight is extremely well coupled to the interstellar medium even the most massive galaxies. In addition, UV and optical starlight is extremely well coupled to the interstellar medium via scattering and absorption onto dust grains. An assessment of this mechanism is the subject of this Letter.

2. THE EDDINGTON LIMIT

The radiation force density is given by (cf. Rybicki & Lightman 1979; Blaes & Socrates 2003; Andrews & Thompson 2011)

\[ \frac{\rho}{c} \int d\nu \kappa_\nu \mathbf{F}_\nu, \]

where \( \kappa_\nu \) is the sum of scattering and absorption opacities at photon frequency \( \nu \), \( \mathbf{F}_\nu \) is the radiative flux, and \( \rho \) is the fluid mass density. By equating this to the gravitational force and integrating over the galactic surface, we arrive at the Eddington limit

\[ L_{\text{Edd}} = \frac{4\pi GM_{\text{enc}} c}{\kappa_f}, \]

where \( M_{\text{enc}} \) is the enclosed dynamical mass and \( \kappa_f \) is the flux-mean opacity, which can be defined as

\[ \kappa_f \equiv \frac{\int d\nu \kappa_\nu L_\nu}{\int d\nu L_\nu}, \]

if the spectral energy distribution (SED) and the opacity law are uniform on the surface of the galaxy, so that

\[ \int d\mathbf{A} \int d\nu \kappa_\nu \mathbf{F}_\nu = \int d\nu \int d\mathbf{A} \cdot \kappa_\nu \mathbf{F}_\nu = \kappa_f L. \]

Note that the Eddington limit is a statement of hydrostatic balance that is independent of geometry and regardless of whether the flow is optically thick or thin (Abramowicz et al. 1980; Socrates 2012).

The most luminous starburst galaxies in the Universe have luminosities of the order of \( L_{\text{max}} \lesssim 10^{47} \text{ erg s}^{-1} \). The starlight couples to gas by scattering and absorption onto dust grains. The UV opacity on dust is \( \kappa_{\text{UV}} \sim \kappa_{\text{esc}} \), where \( \kappa_{\text{esc}} \) is the electron scattering opacity. The most massive galaxies in the Universe possess an enclosed mass of the order of \( M_{\text{enc}} \sim 10^{12} \, M_\odot \).

With this combination of \( L_{\text{max}}, \kappa_{\text{UV}} \) and \( M_{\text{enc}} \), it seems as though \( L_{\text{max}} \approx L_{\text{Edd}} \), when choosing \( \kappa_f \sim \kappa_{\text{UV}} \). This serves as the central argument behind the Murray et al. (2005) photon feedback model.

Such an argument is appealing. It provides a simple and compelling theoretical framework for understanding the coupling of the various forms of energy release that accompany star formation with the gravitational content of the galaxy in question. Ultimately, many of the principles outlined in Murray et al. (2005) may contribute to our understanding of the Faber-Jackson (1976) relation, the fundamental plane of elliptical galaxies, and the maximum luminosity of star-forming galaxies.

However, \( \kappa_{\text{UV}} \neq \kappa_f \). In fact, for starbursting galaxies \( \kappa_f \ll \kappa_{\text{UV}} \). In Figure 1, the frequency, or wavelength, dependent dust opacity (solid red line) is super-imposed upon the SED of the compact—presumably—starbursting galaxies Arp 220 and
Figure 1. SED (black solid line) and total luminosity (black dashed line), wavelength-dependent opacity \( \kappa \lambda \) (red solid line, including both scattering and absorption) and flux-mean opacity (red dashed line) and the Eddington limit (blue dotted line) for luminous star-forming galaxies. For M82 and Arp 220, the SED is taken from Silva et al. (1998). The enclosed dynamical mass is estimated as \( M_{\text{enc}} \sim 2 \times 10^{9} \, M_\odot \) for M82 (Greco et al. 2012) and \( M_{\text{enc}} \sim 4 \times 10^{10} \, M_\odot \) for Arp 220 (Silva et al. 1998). The average SED and median enclosed mass for ULIRGs are taken from the sample of Michałowski et al. (2010). The opacity law is assumed to follow that of the Milky Way as calculated by Weingartner & Draine (2001) for \( RV = 3.1 \) and a gas-to-dust mass ratio \( f_{GD} = 100 \). The opacity model by Semenov et al. (2003) yields similar results.

![Figure 1](image1.png)

Figure 1 indicates that the ratio of the UV to FIR luminosity in bright star-forming galaxies is low—of the order of a few percent. As a result, the flux-mean opacity \( \kappa_F \) is correspondingly diminished and the galaxy in question radiates well below its Eddington value.

An increase in the gas surface density leads to a corresponding increase in the rate of star formation. Therefore, dense gas serves both as a source of UV/optical starlight radiation pressure via star formation, while simultaneously screening itself via absorption and re-processing of the UV light ultimately into the FIR. A depiction of the UV/optical screening in a dense star-forming galaxy is shown in Figure 2.

![Figure 2](image2.png)

Figure 2. A simple model of a star-forming galaxy. The starlight is assumed to be injected uniformly within the radius \( R \). Due to the large optical depth at UV/optical wavelengths, the cold gas in the bulk of the galaxy screens the starlight except for an outer region whose thickness is of the order of the screening length \( l \). The bolometric output generated from within the galaxy ultimately escapes in the form of quasi-thermal FIR radiation. Similar arguments for explaining the ratio of UV/FIR power can be made in cylindrical geometry as well with the same result.

3. UV/OPTICAL SCREENING

The primary cause of this shortfall in the radiation force is that most of the light belongs to wavelengths where the opacity is small, or, conversely, there is relatively little light at frequencies where the opacity is large.

The situation is similar to the radiative transfer in a stellar envelope where the opacity is large for values of photon energy near atomic transitions. Yet, photons in stellar envelopes escape primarily by avoiding such regions of high opacity, which leads to deep absorption features. Consequently, the flux-mean opacity in stellar envelopes remains close to the low continuum value.

\[ \frac{L_{\text{UV}}}{L_{\text{FIR}}} \sim \frac{l_{\text{UV}}}{R} \]

The expression above may be a good approximation for cool dust-rich galaxies that are opaque to starlight, where the FIR luminosity \( L_{\text{FIR}} \) is responsible for most of the bolometric energy release.

The simple picture outlined above may explain the small ratio of UV to FIR photon power in starbursting galaxies. An attempt to quantify the properties of starlight screening by considering the equation of radiative transfer is given below.

3.1. Radiative Transfer

The spatial and directional dependence of the starlight is quantified by the specific intensity \( I_\nu \). Dust grains are the primary source of scattering and absorption of starlight, where
absorption acts to redistribute the input stellar photon energy
longward to IR and FIR frequencies. At these wavelengths, the
dust opacity is relatively small and the radiant energy can more
easily escape.

In steady state, the equation of radiative transfer for the
specific intensity $I_\nu(x, \hat{s})$ along a direction $\hat{s}$ reads (Rybicki & Lightman 1979)
\[
\nabla \cdot \left[ I_\nu(\hat{s}) \hat{s} \right] = - (\alpha_\nu + \sigma_\nu) I_\nu(\hat{s}) + \sigma_\nu \int_{4\pi} I_\nu(\hat{s}') f_\nu(\hat{s} \cdot \hat{s}') d\Omega' + j_\nu(\hat{s}) 
\] (6)
where the spatial dependence on $x$ has been suppressed. Here, $\alpha_\nu(x)$ is the absorption coefficient and $\sigma_\nu(x)$ is the scattering coefficient. They are, respectively, equal to the inverse of the mean free paths for absorption and scattering of photons of frequency $\nu$. The emission coefficient, or source, is given by $j_\nu(x, \hat{s})$ and the normalized differential scattering cross section, $f_\nu(\hat{s} \cdot \hat{s}')$, satisfies
\[
\int_{4\pi} f_\nu(\hat{s} \cdot \hat{s}') d\Omega' = 1. 
\] (7)

By defining the mean intensity $J_\nu(x) = \int I_\nu(x, \hat{s}) d\Omega$, the radiative flux $F_\nu(x) = \int I_\nu(x, \hat{s}) \hat{s} d\Omega$ and the mean emissivity $q_\nu(x) = \int j_\nu(x, \hat{s}) d\Omega$, the radiative transfer equation averaged over the propagation angle reduces to
\[
\nabla \cdot F_\nu = -\alpha_\nu J_\nu + q_\nu. 
\] (8)

In the diffusion approximation, the specific intensity $I_\nu$ can be expressed as an isotropic part (i.e., the mean intensity $J_\nu$) plus a small directional flux $F_\nu$. That is,
\[
I_\nu(\hat{s}) = \frac{1}{4\pi} J_\nu + \frac{3}{4\pi} F_\nu \cdot \hat{s} 
\] (9)
where the factor of three on the right-hand side results from enforcing the Eddington closure approximation. Substitution of this expression into Equation (6), multiplying by $\hat{s}$, and integrating over all solid angles leads to a relation between the mean intensity and the flux of the following form
\[
F_\nu = -D_\nu \nabla J_\nu 
\] (10)
where
\[
D_\nu(x) \equiv \frac{1}{3[(1 - g)\alpha_\nu + \alpha_\nu]} = \frac{1}{3 \rho \kappa_\nu} 
\] (11)
is the photon diffusion coefficient, and $g$ is the average cosine of the scattering angle. The two relations in Equations (8) and (10) between the mean intensity and the flux can be combined to give a diffusion equation for the mean intensity alone
\[
\nabla \cdot (D_\nu \nabla J_\nu) - \alpha_\nu J_\nu = -q_\nu. 
\] (12)

In a uniform medium, the diffusion coefficient $D_\nu$ is independent of position, and under these conditions
\[
D_\nu \nabla^2 J_\nu - \alpha_\nu J_\nu = -q_\nu, 
\] (13)
which is in the form of a screened Poisson’s equation with screening length $l_\nu \equiv \sqrt{D_\nu/\alpha_\nu}$. The Green’s function $G_\nu(x, x')$ for the screened Poisson’s equation satisfies $\nabla^2 G_\nu(x, x') = -\delta(x - x')$ and is given by
\[
G_\nu(x, x') = \frac{\exp(-|x - x'|/l_\nu)}{4\pi |x - x'|} 
\] (14)
which is also referred to as the Yukawa potential. Under the assumptions that the radiation field is quasi-isotropic and photon diffusion proceeds through a uniform medium, the analogy of the radiative transfer problem with electrostatics is clear. The average intensity $J_\nu$, which is equivalent to the photon energy density, can be thought of as an electrostatic potential that is produced by some source of radiation $q_\nu$, which itself can be thought of as an electrostatic charge density. Furthermore, it follows that the radiative flux $F_\nu$, responsible for the radiation force, is analogous to an electric field. Absorption attenuates the mean intensity in starlight, which is therefore evanescent in the bulk of the galaxy, and its effect may be thought of as a screening.

The solution to the radiative transfer of starlight under these circumstances further motivates the arguments behind Figure 2 and Equation (5).

3.2. Boundary Value Problem in Spherical Symmetry

In a uniform sphere, the source function $q_\nu = q_0$ is a constant. The inhomogeneous solution for $J_\nu$ to the screened diffusion equation must be regular at the center of the galaxy and satisfy an appropriate boundary condition at the surface $r = R$. For the latter, we employ the two-stream approximation (Rybicki & Lightman 1979), which relates the flux $F_\nu$ to the mean intensity $J_\nu$ in such a way as to maintain the Eddington approximation for the starlight. With this, the mean intensity inside the spherical galaxy is given by
\[
J_\nu = \frac{q_0 l_\nu^2}{D_\nu} \left[ 1 - \frac{R \sinh(r/l_\nu)}{r/C} \right], 
\] (15)
where the constant $C = C(R)$ is determined by the boundary condition at $r = R$ and is written as
\[
C \equiv \sqrt{3D_\nu \alpha_\nu} \cosh \left( \frac{R}{l_\nu} \right) + \left( 1 - \sqrt{3D_\nu} \right) \sinh \left( \frac{R}{l_\nu} \right). 
\] (16)

For $l_\nu \ll R$, the mean intensity inside the sphere is almost constant and equal to $J_\nu \sim q_0 l_\nu^2/D_\nu \sim q_0/\alpha_\nu$, as expected for a homogeneous source with emissivity $q_0$ that is absorbed locally with absorption coefficient $\alpha_\nu$.

The radial component of the radiative flux $F_\nu = \hat{r} \cdot F_\nu$ is given by
\[
F_\nu = \frac{q_0 R l_\nu}{r^2} \left[ r \cosh(r/l_\nu) - l_\nu \sinh(r/l_\nu) \right]/C, 
\] (17)
which vanishes in the limit $r \to 0$, as expected. Near $r = 0$, $F_\nu$ rises linearly with $r$ up to $r \approx l_\nu$, at which point $F_\nu$ increases approximately exponentially with scale length $l_\nu$ until $r = R$, where it reaches its maximum value. Since the UV radiation force is proportional to $F_\nu$, it follows that, in the limit $l_{\nu \text{UV}}/R \approx L_{\nu \text{UV}}/L_{\text{FR}} \ll 1$, throughout the bulk of the flow, the UV radiation force effectively vanishes. Consequently, the so-called “single-scattering” approximation for UV photons
and absorption (i.e., Hopkins et al. 2012a, 2012b, 2012c) significantly—or exponentially—overestimates the UV radiation force throughout the bulk of a given star forming flow.

In the absence of absorption (set $g = 0$ in Equation (13)), while still enforcing the two-stream approximation at the surface, the unscreened intensity $J_v^{(0)}$ is given by

$$J_v^{(0)} = \frac{q_0}{6D_v^{(0)}} (R^2 - r^2) + \frac{q_0 R}{\sqrt{3}}$$ (18)

where $D_v^{(0)} = [3(1 - g)\sigma_v^{-1} - 2D_v]$ for isotropic diffusion (i.e., $g = 0$) and assuming equal mean free paths for scattering and absorption (i.e., $\sigma_v = \alpha_v$). The corresponding unscreened radiative flux is then $F_v^{(0)} = q_0 R / 3$.

For $R \gg \alpha_v^{-1}$ and $R \gg D_v$ (which also implies $R \gg l_v$), the ratio between screened and unscreened mean intensities at the center (i.e., $r = 0$) is

$$\left. \frac{J_v}{J_v^{(0)}} \right|_{r=0} = \frac{6D_v^{(0)} l_v^2}{D_v} \frac{12l_v^2}{R^2} \sim \frac{l_v^2}{R^2}$$ (19)

It follows that, for galaxies such as Arp 220 and ULIRGs whose starlight is highly screened, the UV/optical radiation pressure is strongly suppressed at depth.

The ratio of outward radial fluxes at the outer boundary of the galaxy (i.e., $r = R$) reads

$$\left. \frac{F_v}{F_v^{(0)}} \right|_{r=R} = \frac{3l_v}{R} \frac{1}{\sqrt{3}D_v\alpha_v + 1} \sim \frac{l_v}{R}$$ (20)

which corroborates the intuition that led to Figure 2.

4. DISCUSSION AND SUMMARY

In Section 2 an empirical method—the calculation of the Eddington limit—for determining the relative strength of the radiation force on the surface of galaxies is described. It is demonstrated that light resulting from star formation is too dim to affect the large-scale gas dynamics of actively star-forming galaxies.

The dust opacity is large at small wavelengths, where the emergent spectrum is relatively dark, while the dust opacity is small at the long photon wavelengths, where the emergent spectrum is bright. By combining these two properties, it is apparent that the radiation force is, in general, unexpectedly weak in gas-rich star-forming sources.

Galaxies, such as the Milky Way, that are not as rich in cool dense molecular gas do not suffer from the same level of self-shielding that renders radiation pressure ineffective in starbursting sources. However, the lack of molecular fuel also implies a relatively low level of star formation. Although the flux-mean opacity may be higher, due to a harder spectrum, the total luminosity is low. Thus, it is unlikely that unscreened galaxies radiate above their Eddington limit.

The radiative transfer of starlight in luminous star-forming galaxies can be modeled in the diffusion approximation, including the effect of absorption by dust grains. Dust-screening greatly diminishes the value of the starlight intensity, or pressure, at depth. Only an outer shell with thickness equal to the screening length contributes to the outgoing flux of UV/optical starlight. An increase in the density of molecular gas increases the rate of starlight production and the Eddington limit may be approached. Yet, increasing the fuel supply necessarily leads to an increase in dust-screening, with corresponding downward departures from the Eddington limit, due to the corresponding decrease in the flux-mean opacity.

Since the sources under consideration radiate well below their Eddington limit, the large-scale radiation force due to FIR photons is also small in comparison to the galaxy-scale gravitational force at the surface. It is therefore unlikely that the FIR radiation pressure will be large enough to be dynamically important at depth. In the diffusion approximation, radiation pressure is proportional to optical depth, which is proportional to column density. Since gas pressure is also proportional to column density, the ratio between gas and FIR radiation pressure should remain approximately a constant with depth.

There may be some class of galaxies in the Universe or objects within luminous star-forming galaxies themselves (e.g., giant molecular clouds), where starlight pressure on interstellar dust leads to a radiation force that exceeds the local source of gravity (Murray et al. 2010). It may be useful to empirically determine the Eddington limit for these objects with the method described in Section 2.

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APPENDIX

REVIEW OF RADIATION FEEDBACK

In recent years, the idea that radiation pressure due to starlight interacting with dust grains may be comparable to gravity was considered by Scoville (2003), Martin (2005) and Murray et al. (2005, hereafter MQT) hypothesized that radiation pressure on dust grains may be responsible for driving large-scale galactic winds in analogy to line-driven stellar winds of massive stars. MQT utilize the large UV/optical opacity on dust to justify that, despite the low light-to-mass ratio of the galaxies, it is possible for them to radiate with super-Eddington luminosities. Thompson et al. (2005) focus on the effects of coupling the IR and FIR radiation to dust and thus, to the gaseous component, in an attempt to construct a starlight-powered galactic disk solution. More recently, Murray et al. (2010) changed focus from large scales to the scale of giant molecular clouds (GMCs), where they posit that fully populated star clusters born in GMCs unbind the site of their birth once the star to gas ratio becomes large. In a series of recent papers (Hopkins et al. 2011, 2012a, 2012b, 2012c), both the GMC-scale starlight-driven hypothesis as well as an approximate formulation of the large-scale radiation force are utilized in a number of massive numerical simulations meant to describe a variety of star-forming galaxies. They find that the radiation force is capable of significantly altering the gas dynamics of star-forming galaxies.

Recently, Andrews & Thompson (2011) and Zhang & Thompson (2012) note that, at face value, star-forming galaxies are sub-Eddington with respect to the Rosseland mean opacity for dust. By taking the Rosseland mean, they assume a functional form for the SED of the galaxy. As a result of this, they are led
to believe that there is some level of theoretical ambiguity when assessing the strength of the radiative forcing, particularly when discerning between optically thick and optically thin regimes. One of the most attractive features of the Eddington limit is that it is an empirical tool. In Section 2, the observed SED of galaxies is integrated against a typical dust opacity law in order to obtain the strength of the radiative forcing on the surface of a given galaxy, with relatively little ambiguity. Furthermore, the Eddington limit is independent of whether the flow is optically thick or thin.

In describing the feedback of cosmic ray protons during star formation, Socrates et al. (2008; see also Sironi & Socrates 2010 for the case of quasars) attempt to draw a distinction between cosmic ray feedback and the feedback from starlight. In their Appendix, they note that UV photons decouple and are reprocessed into the FIR close to the massive star of their origin. Once starlight is converted into the FIR band, they argue that it is unlikely to affect the dynamics of the interstellar medium. A critical reexamination of photon feedback is the subject of many recent numerical studies (Novak et al. 2012; Krumholz & Thompson 2012; Wise et al. 2012; Agertz et al. 2013; Jiang et al. 2013). They all show that, for various reasons, the starlight pressure may not be efficient in its coupling to the interstellar medium. However, to solve the full radiation hydrodynamical problem of photon feedback is incredibly challenging from a numerical perspective due to large ratio of the photon diffusion to gas sound speed (cf. Fernández & Socrates 2013). In Section 2, an attempt is made to circumvent this debate by empirically determining the strength of the radiative forcing on the surface of star-forming galaxies.

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