A Bethe-Salpeter treatment of Cooper pairs (CPs) based on an ideal Fermi gas (IFG) “sea” yields the familiar negative-energy, two-particle bound-state if two-hole CPs are ignored, but is meaningless otherwise as it gives purely-imaginary energies. However, when based on the BCS ground state, legitimate two-particle “moving” CPs emerge but as positive-energy, finite-lifetime resonances for nonzero center-of-mass momentum, with a linear dispersion leading term. Bose-Einstein condensation of such pairs may thus occur in exactly two dimensions as it cannot with quadratic dispersion.

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Shortly after the publication of the BCS theory of superconductivity charged Cooper pairs (CPs) observed in magnetic fluid quantization experiments with 3D conventional superconductors, and much later with quasi-2D cuprate superconductors, suggested CPs as an indispensable ingredient. Although BCS theory admits the presence of Cooper “correlations,” several boson-fermion (BF) models with real, bosonic CPs have been introduced after the pioneering work of Refs. [9]-[11]. However, with one exception [12], all such models neglect the effect of two-hole (2h) CPs treated on an equal footing with two-particle (2p) CPs—as Green’s functions [20] can naturally guarantee.

The BCS condensate consists of equal numbers of 2p and 2h Cooper correlations; this is evident from the perfect symmetry about \( \mu \), the electron chemical potential, of the well-known Bogoliubov coefficients \( \beta^2(\epsilon) \) and \( u^2(\epsilon) \) coefficients [see just below (4) later on], where \( \epsilon \) is the electron energy. Some motivation for this Letter comes from the unique but unexplained role played by hole charge carriers in the normal state of superconductors in general, as well as from the ability of the “complete” (in that both 2h- and 2p-CPs are allowed in varying proportions) BF model” of Refs. [9]-[11] to “unify” both BCS and Bose-Einstein condensation (BEC) theories as special cases. Substantially higher \( T_c \)’s than BCS theory are then predicted without abandoning electron-phonon dynamics. Compelling evidence for a significant presence of this dynamics in high-\( T_c \) cuprate superconductors from angle-resolved photoemission spectroscopy data has recently been reported [12].

In this Letter the Bethe-Salpeter (BS) many-body equation (in the ladder approximation) treating both 2p and 2h pairs on an equal footing is used to show that, while the ordinary CP problem [based on an ideal Fermi gas (IFG) ground state (the usual “Fermi sea”)] does not possess stable energy solutions: i) CPs based not on the IFG-sea but on the BCS ground state survive as positive energy resonances; ii) their dispersion relation in leading order in the total (or center-of-mass) momentum (CMM) \( \hbar \mathbf{K} = \hbar (\mathbf{k}_1 + \mathbf{k}_2) \) is linear rather than the quadratic \( \hbar^2 K^2/4m \) of a composite boson (e.g., a deuteron) of mass \( 2m \) moving not in the Fermi sea but in vacuum; and iii) this latter “moving CP” solution, though often confused with it, is physically distinct from another more common solution sometimes called the Anderson-Bogoliubov-Higgs (ABH) [23] (4 p. 44), collective excitation. The ABH mode is also linear in leading order and goes over into the IFG ordinary sound mode in zero coupling. A new feature emerging from our present 2D results, compared with a prior 3D study outlined in Ref. [20], is the imaginary energy term leading to finite-lifetime CPs. We focus here on 2D because of its interest [24] for quasi-2D cuprate superconductors. In general, our results will be crucial for Bose-Einstein condensation (BEC) scenarios employing BF models of superconductivity, not only in exactly 2D as with the Berezinskii-Kosterlitz-Thouless transition, but also down to \( (1 + \epsilon)D \) which characterize the quasi-1D organo-metallic (Bechgaard salt) superconductors [22]-[24]. Striking experimental confirmation of how superconductivity is “extinguished” as dimensionality \( d \) is diminished towards unity has been reported by Tinkham and co-workers [25]-[28]. They measured resistance vs. temperature curves in superconducting nanowires consisting of carbon nanotubes sputtered with amorphous \( Mo_75Ge_{21} \) and of widths from 22 to 10 nm, showing \( T_c \) vanishes for the thinnest widths. Our results also apply, albeit with a different interaction, to neutral-atom superfluidity as in liquid \( ^3He \) as well as to ultracold trapped alkali Fermi gases such as \( ^6Li \) and \( ^40K \) since pairing is believed to occur there also.
For bosons with excitation energy $\varepsilon_K = C_s K^s + o(K^s)$ (for small CMM $K$) BEC occurs in a box of length $L$ if and only if $d > s$, since $T_c \equiv 0$ for all $d \leq s$. The commonest example is $s = 2$ as in the textbook case of ordinary bosons with $\varepsilon_K = h^2 K^2/2m$ exactly, giving the familiar result that BEC is not allowed for $d \leq 2$. The general result for any $s$ is seen as follows. The total boson number is

$$N = N_0(T) + \sum_{K \neq 0} [\exp \beta(\varepsilon_K - \mu_B) - 1]^{-1}$$

with $\beta \equiv k_B T$. Since $N_0(T_c) \simeq 0$ while the boson chemical potential $\mu_B$ also vanishes at $T = T_c$, in the thermodynamic limit the boson number density becomes

$$N/L^d \simeq A_d \int_{0^+}^{\infty} dK K^{d-1}[\exp \beta(C_s K^s + \cdots) - 1]^{-1}$$

where $A_d$ is a finite coefficient. Thus

$$N/L^d \simeq A_d(k_BT_c/C_s) \int_{0^+}^{K_{\text{max}}} dK K^{d-s-1} + \int_{K_{\text{max}}}^{\infty} \cdots,$$

where $K_{\text{max}}$ is small and can be picked arbitrarily so long as the integral $\int_{K_{\text{max}}}^{\infty} \cdots$ is finite, as is $N/L^d$. However, if $d = s$ the first integral gives $\ln K |_{K_{\text{max}}} = -\infty$; and if $d < s$ it gives $1/(d-s)K^{s-d} |_{K_{\text{max}}} = -\infty$. Hence, $T_c$ must vanish if and only if $d \leq s$, but is otherwise finite. This conclusion hinges only on the leading term of the boson dispersion relation $\varepsilon_k$. The case $s = 1$ emerges in the CP problem to be discussed now.

In dealing with the many-electron system we assume a BCS-like electron-phonon model $s$-wave inter-electron interaction, whose double Fourier transform $\nu([k_1 - k'_1])$ is just

$$\nu(k_1, k'_1) = -(k_F/k'_1)V$$

(1)

if $k_F - k_D < k'_1 < k_F + k_D$, and $= 0$ otherwise. Here $V > 0$, $k_F \equiv mv_F$ the Fermi momentum, $m$ the effective electron mass, $v_F$ the Fermi velocity, and $k_D \equiv \omega_D/v_F$ with $\omega_D$ the Debye frequency. The usual condition $\hbar \omega_D \ll E_F$ then implies that $k'_1/k_F \equiv \hbar \omega_D/2E_F \ll 1$.

The BS wavefunction equation \cite{29} in the ladder approximation with both particles and holes for the original IFG-based CP problem using \cite{3} leads to an equation for the wavefunction $\psi_k$ in momentum space for CPs with zero CMM $K \equiv k_1 + k_2 = 0$ that is

$$(2\xi_k - \mathcal{E}_0)\psi_k = V \sum_{k'}^{''} \psi_{k'} - V \sum_{k'}^{} \psi_{k'}.$$  \hspace{1cm} (2)

Here $\xi_k \equiv h^2 k^2/2m - E_F$, $\mathcal{E}_0$ is the eigenvalue energy and $K \equiv 1/2(k_1 - k_2)$ is the relative wavevector of a pair. The single prime over the first (2p-CP) summation term denotes the restriction $0 < \xi_k < \hbar \omega_D$ while the double prime in the last (2h-CP) term means $-\hbar \omega_D < \xi_k < 0$. Without this latter term we have Cooper’s Schrödinger-like equation \cite{2} for 2p-CPs whose implicit solution is clearly $\psi_k = (2\xi_k - \mathcal{E}_0)^{-1}V \sum_{k'}^{} \psi_{k'}$. Since the summation term is constant, performing that summation on both sides allows canceling the $\psi_{k'}$-dependent terms, leaving the eigenvalue equation $\sum_{k}(2\xi_k - \mathcal{E}_0)^{-1} = 1/V$ with the familiar solution $\mathcal{E}_0 = -2\hbar \omega_D/(\epsilon^2/\lambda - 1)$ (exact in 2D, and to a very good approximation otherwise if $\hbar \omega_D \ll E_F$) where $\lambda \equiv VN(E_F)$ with $N(E_F)$ the electronic density of states (DOS) for one spin. This corresponds to a negative-energy, stationary-state bound pair. For $K \geq 0$ the CP eigenvalue equation becomes

$$\sum_{k}(2\xi_k + h^2 K^2/2m - \mathcal{E}_K)^{-1} = 1/V.$$  \hspace{1cm} (3)

Note that a CP state of energy $\mathcal{E}_K$ is characterized only by a definite $K$ but not definite $k$, in contrast to a “BCS pair” defined \cite{1} with fixed $K$ and $k$ (or equivalently definite $k_1$ and $k_2$). Without the first summation term in \cite{2} the same result in $\mathcal{E}_0$ for 2p-CPs follows for 2h-CPs (apart from a sign change). However, using similar techniques to solve the complete equation \cite{2}—which cannot be derived from an ordinary (non-BS) Schrödinger-like equation in spite of its simple appearance—gives the purely-imaginary $\mathcal{E}_0 = \pm i2\hbar \omega_D/\sqrt{\epsilon^2/\lambda - 1}$, thus implying an obvious instability. This was reported in Refs. \cite{24} p. 44 and \cite{40} who did not stress the pure 2p and 2h cases just discussed. Clearly then, the original CP picture is meaningless if particle- and hole-pairs are treated on an equal footing as consistency demands. This is perhaps the prime motivation for seeking a new unperturbed Hamiltonian about which to, e.g., do perturbation theory.

A BS treatment not about the IFG sea but about the BCS ground state vindicates the CP concept. This substitution might seem an artificial mathematical construct but its experimental support lies precisely in Refs. \cite{3-5} and its physical justification lies in recovering two expected results: the ABH sound mode as well as finite-lifetime effects in CPs. In either 3D \cite{2} or 2D the BS equation yields two distinct solutions: the usual ABH sound solution and a highly nontrivial “moving CP” solution. The BS formalism gives rise to a set of three coupled equations, one for each (2p, 2h and ph) channel wavefunction for any spin-independent interaction such as \cite{1}. However, the ph channel decouples, leaving only two coupled wavefunction equations for the ABH solution. The equations involved are too lengthy, and will be derived in detail elsewhere. The ABH collective excitation mode energy $\mathcal{E}_K$ is found to be determined by an equation that for $K = 0$ gives $\mathcal{E}_0 = 0$ (Ref. \cite{24} p. 39) and reduces to $\int_{0}^{2\hbar \omega_D} d\xi/\sqrt{\xi^2 + \Delta^2} = 1/\lambda$, the familiar BCS $T = 0$ gap equation for interaction \cite{1} whose solution is $\Delta = \hbar \omega_D/\sinh(1/\lambda)$. Taylor-expanding $\mathcal{E}_K$ about
\( K = 0 \) and small \( \lambda \) gives
\[
\mathcal{E}_K \simeq \frac{\hbar v_F}{\sqrt{2}} K + O(K^2).
\]

Note that the leading term is just the ordinary sound mode in an IFG whose sound speed \( c = v_F/\sqrt{d} \) in \( d \) dimensions which also follows trivially from the zero-temperature IFG pressure \( P = n^2[d(E/N)/dn] = 2nE_F/(d + 2) \) on applying the familiar thermodynamic relation \( dP/dn = mc^2 \). Here \( E = dE_F/(d + 2) \) is the IFG ground-state energy while \( n = N/L^d = k_F^d/d^d \) is the fermion-number density.

The second solution in the BCS-ground-state-based BS treatment is the moving CP solution for the pair energy \( \mathcal{E}_K \) which in 2D is contained in the equation
\[
\frac{1}{2\pi} \hbar v_F \int_{k_F - k_D}^{k_F + k_D} dk \int_0^{2\pi} d\varphi \sin \left( \varphi \right) \frac{u_{K/2+k}^2 - u_{K/2-k}^2}{\mathcal{E}_K^{k/2+k} + \mathcal{E}_K^{k/2-k} - \mathcal{E}_K^2} = 1,
\]
where \( \varphi \) is the angle between \( \mathbf{K} \) and \( \mathbf{k} \); \( \lambda = VN(E_F) \) as before with \( N(E_F) \equiv m/2\pi\hbar^2 \) the constant 2D DOS and \( V \) the interaction strength defined in \( \ref{footnote:1} \); \( \mathcal{E}_K \equiv \sqrt{\xi_k^2 + \Delta^2} \) with \( \Delta \) the fermionic gap; while \( u_k^2 = \frac{1}{2}(1 + \xi_k/E_k) \) and \( v_k^2 = 1 - u_k^2 \) are the Bogoliubov functions. In addition to the pp and hh wavefunctions (depicted graphically in Ref. \( \ref{footnote:2} \), Fig. 2), diagrams associated with the ph channel give zero contribution at \( T = 0 \). A third equation for the ph wavefunction describes the ph bound state but turns out to depend only on the pp and hh wavefunctions. Taylor-expanding \( \mathcal{E}_K \) in powers of \( K \) around \( K = 0 \), and introducing a possible damping factor by adding an imaginary term \( -iK \) in the denominator, yields to order \( K^2 \) for small \( \lambda \)
\[
\pm \mathcal{E}_K \simeq 2\Delta + \frac{\lambda}{2\pi} \hbar v_F K + \frac{\hbar v_F}{9k_D} e^{1/\lambda} K^2
\]
\[
- i \frac{\lambda}{2\pi} \hbar v_F K + \frac{\hbar v_F}{12k_D} e^{1/\lambda} K^2 + O(K^3)
\]
where the upper and lower sign refers to 2p- and 2h-CPs, respectively. A linear dispersion in leading order again appears, but now associated with the bosonic moving CP. The positive-energy 2p-CP resonance has a lifetime \( \tau_K \equiv \hbar/2\Gamma_K = \hbar/2 \left[ (\lambda/\pi)\hbar v_F K + (\hbar v_F/12k_D)e^{1/\lambda} K^2 \right] \) diverging only at \( K = 0 \), and falling to zero as \( K \) increases. Thus, “faster” moving CPs are shorter-lived and eventually break up, while “non-moving” ones are stationary states. The linear term \( (\lambda/2\pi)\hbar v_F K \) contrasts sharply with the \emph{coupling-independent} leading-term in \( \mathcal{E}_K = \mathcal{E}_0 - (2/\pi)\hbar v_F K + O(K^2) \) (or 1/2 in 3D \( \ref{footnote:1} \) instead of 2/\pi) that follows from the \emph{original} CP problem \( \ref{footnote:1} \) neglecting holes—for either interaction \( \ref{footnote:1} \) \( \ref{footnote:2} \) or an attractive delta inter-fermion potential \( \ref{footnote:3} \) \( \ref{footnote:4} \) (imagined regularized \( \ref{footnote:4} \) to have a single bound state whose binding energy serves as the coupling parameter). In the latter simple example, moreover, it is manifestly clear in 2D \( \ref{footnote:3} \) that the quadratic \( h^2K^2/4m \) stands alone as the leading term for any coupling only when \( E_F \equiv \frac{1}{2}mv_F^2 \) is \emph{strictly} zero, i.e., in the absence of the Fermi sea. Fig. 1 graphs the exact moving CP (mCP) energy extracted from \( \ref{footnote:3} \), along with its leading linear-dispersion term and this plus the next (quadratic) term from \( \ref{footnote:3} \). The interaction parameter values used in \( \ref{footnote:3} \) were \( \hbar \omega_D/E_F = 0.05 \) (a typical value for cuprates) and the two values \( \lambda = \frac{1}{4} \) and \( \frac{1}{2} \), giving for \( \mathcal{E}_0/E_F \equiv 2\Delta/E_F = 2\hbar \omega_D/E_F \sinh(1/\lambda) \approx 0.004 \) and 0.028, respectively (marked as dots in the figure). Remarkably enough, the linear approximation (thin short-dashed lines in figure) is better over a wider range of \( K/k_F \) values for weaker coupling in spite of a larger and larger partial contribution from the quadratic term in \( \ref{footnote:3} \); this peculiarity also emerged from the ordinary CP treatment of Ref. \( \ref{footnote:2} \) and might suggest the expansion in powers of \( K \) to be an asymptotic series that should be truncated after the linear term. For reference we also plot the linear term \( \hbar v_F K/\sqrt{2} \) of the sound solution \( \ref{footnote:4} \).

We cannot presently address such matters as the nature of the normal state, the pseudogaps observed in underdoped cuprates, etc., but efforts in these directions are in progress.

Like Cooper’s \( \ref{footnote:2} \) [see Eq. \( \ref{footnote:3} \)], our BS CPs are characterized by a definite \( \mathbf{K} \) and \emph{not} also by definite \( \mathbf{k} \) as the pairs discussed by BCS \( \ref{footnote:1} \). Hence, the objection does not apply that CPs are not bosons because BCS pairs with definite \( \mathbf{K} \) and \( \mathbf{k} \) (or equivalently definite \( k_1 \) and \( k_2 \)) have
creation/annihilation operators that do not obey Bose commutation relations [Ref. 1, Eqs. (2.11) to (2.13)]. In fact, either $k$ or $k^*$ shows that a given “ordinary” or BS CP state labeled by either $k$ or $E_k$ can accommodate (in the thermodynamic limit) an indefinitely many possible BCS pairs with different $K$’s. This implies BE statistics for either ordinary or BS CPs as each energy state has no occupation limit.

To conclude, hole pairs treated on a par with electron pairs play a vital role in determining the precise nature of CPs even at zero temperature, only when based not on the usual ideal-Fermi-gas (IFG) “sea” but on the BCS ground state. Treatment them with the Bethe-Salpeter equation gives purely-imaginary-energy CPs when based on the IFG, and positive-energy resonant-state CPs with a finite lifetime for nonzero CMM when based on the IFG, and positive-energy resonant-state CPs with a situation analogous to that of the IFG-based CP problem that neglects holes, as sketched just below [2]. The BS “moving-CP” dispersion relation is gapped by twice the BCS energy gap, followed by a linear leading term in the CMM expansion about $K = 0$. Thus, this linearity is distinct from the better-known one associated with the sound or ABH collective excitation mode whose energy vanishes at $K = 0$. This linearity for the boson component instead of the quadratic $\hbar^2 K^2/4m$ can give BEC for all $d > 1$, including exactly 2D, and thus in principle apply not only to quasi-2D cuprate but also to quasi-1D organo-metallic superconductors.

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