Marginal-quality Based Incentive Mechanisms for Long-term Crowd Sensing Applications

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SUMMARY

Crowd sensing is a new paradigm that leverages pervasive sensor-equipped mobile devices to provide sensing services like forensic analysis, documenting public spaces, and collaboratively constructing statistical models. Extensive user participation is indispensable for achieving good service quality. Nowadays, most of existing mechanisms focus on guaranteeing good service quality based on instantaneous extensive user participation for crowd sensing applications. Little attention has been dedicated to maximizing long-term service quality for crowd sensing applications due to their asymmetric interests, preferences, selfish behaviors, etc. To fill these gaps, in this paper, we derive the closed expression of the marginal sensing data quality based on the monopoly aggregation in economics. Furthermore, we design marginal-quality based incentive mechanisms for long-term crowd sensing applications, not only to enhance extensive user participation by maximizing the expected total profits of mobile users, but also to stimulate mobile users to produce high-quality contents by applying the marginal quality. Finally, simulation results show that our mechanisms outperform the existing solutions. Copyright © 2014 John Wiley & Sons, Ltd.

KEY WORDS: maximizing profits; long-term crowd sensing; monopoly platforms; marginal quality.

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1. INTRODUCTION

Recently, numerous urban sensing applications have been widely developed in various fields such as traffic and environmental monitoring. However, traditional sensing techniques require high installation and maintenance cost [1, 2]. By contrast, crowd sensing provides a novel and efficient paradigm to leverage pervasive mobile devices to collect and upload sensing data. Some typical crowd sensing applications include Ear-Phone [3] and NoiseTube [4] for making noise maps, Nericell [5], SignalGruru [6], and VTrack [7] for providing timely traffic flow information.

The service quality of the above crowd sensing applications is mainly determined by two key factors: adequate user participation and high quality sensing data. Although some incentive mechanisms are proposed to attract users to participate in crowd sensing applications [8, 9, 10, 11, 12], they do not account for the following two aspects. On one hand, if some users obtain low rewards temporarily due to competition, they may drop out of the applications, which will decrease the level of adequate participation, i.e., participation ratio (i.e., the ratio of selected users to total users) [11, 12], thereby reducing the total service quality. For example, in the incentive mechanisms based on the reverse auction, the participants with higher true valuation become starved frequently to win. Accordingly, they lose their interest in continuous crowd sensing actions due to low rewards and drop out of the reverse auction. On the other hand, existing mechanisms can not prevent users from submitting low quality sensing data such as junk and spam. Although the authors of [13] attempt to eliminate or hide low-quality sensing data by rewarding sensing data providers with incentives, they fail to account for the adequate user participation issue. Therefore, simultaneously addressing the adequate user participation and high quality sensing data submission issues becomes particularly challenging for crowd sensing applications.

In this paper, we mainly focus on the long-term crowd sensing applications. Thus, beyond the above-mentioned challenges, we also consider the evaluation of the sensing data quality based on the long-term utility maximization. To this end, we first analyze the three parties of two-sided markets—*sensing data providers, platforms, and sensing data viewers*—on the basis of a typical virtual currency system supported by crowd sensing applications (see Fig. 1). Providers expect to
participate in crowd sensing to maximize their profits for submitting high quality sensing data under the condition of satisfying the participation ratio. At the same time, platforms optimally allocate their sensing data to viewers to maximize their utilities in terms of viewers’ preferences. Thus, we design the mechanisms to enable platforms to offer providers advertising revenue as economic incentives (i.e. subsidizing) to produce high quality sensing data under the condition of satisfying the participation ratio, which correspondingly attract more viewers for viewing (see Fig. 2). Our main contributions are summarized as follows:

- We explore the long-term maximal profits based on viewers’ preferences for crowd sensing applications under the condition of satisfying the participation ratio. Moreover, we derive the closed expression of the marginal sensing data quality based on the monopoly aggregation in economics. Platforms can update the quality states of sensing data from providers according to the marginal sensing data quality based on viewers’ preferences, thereby updating the threshold parameter. According to the threshold parameter, providers are subsidized to satisfy the participation ratio.

- We introduce a sensing data quality evolution process based on the observation that the HIGH/LOW quality state of sensing data is similar to the ACK/NACK feedback state in a time-varying wireless channel. Further, we apply the restless multi-armed bandit processes (RMBP) and a Whittle’s indexability analysis to design an incentive mechanism \( CQI \) to maximize the long-term expected total profits of monopoly platforms for crowd sensing applications.

- We extend the mechanism \( CQI \) to the mechanism \( HCQI \) for the scenario with heterogenous sensing costs. Simulation results indicate that our mechanisms outperform the existing solutions.

The rest of the paper is organized as follows. In Section 3, we present our system model and related definitions. In Section 4, we introduce the sensing data quality model and formulate our problem. We design the mechanism \( CQI \) for maximizing the profits over the infinite horizon in
Section 5 and extends it to the mechanism $HCQI$ for the heterogenous scenario in Section 6. Finally, we present the performance evaluation in Section 7 and conclude the paper in Section 8.

2. BACKGROUND AND RELATED WORK

Incentives are a powerful and common mechanism for crowd sensing applications such as task-driven applications, citizen science applications, and urban sensing applications [14]. Recently, various incentive mechanisms have been proposed. For instance, the authors of [8] consider a user-generated content platform monetized through advertising to incentivize users to produce high quality content. The authors of [13] propose a game-theoretic model to incentivize high quality content submission. The authors of [15, 16] explore the issue of high-quality contents by applying the sequential sensing technology. The authors of [17, 18] apply an entirely different approach to design incentive mechanisms by identifying the social psychological motives of providers so as to better reward those motives to encourage participation. However, they do not consider the impact of the number of providers on the content quality and derive the closed expression of the marginal content quality. Their incentive mechanisms are designed only for web based online social networks. Thus, these mechanisms do not apply for realistic crowd sensing in the physical world. On the other hand, the authors in [19, 20, 21, 22, 23] found that the quality of the work performed by participants was affected by the number of the incentive, compensation rate and work length. But they do not consider the impact of viewer preferences on the content quality.

The authors of [24, 9] identified well-suited participants for sensing services. But, they focused only on the user selection, instead of the incentive mechanism design. In [25, 9], participants are provided much stronger incentive mechanisms. One of their proposed motivations is to better exploit the available channels. But their network-wide throughput is not optimal. Even if the mechanism in principle are proper to attain social optimum by some setting, it is often assumed that pricing schemes are impractical since they need an accounting infrastructure [26].

Compared with the above mentioned pricing schemes, virtual currency based incentive schemes have been extensively popular since users provides the content collected without the actual currency.
payments. For instance, in a reputation-based mechanism, where profits and punishments are identified based on differentiated services policy, the authors of [27, 28, 29] explore incentives to address the social dilemma between the ex-post payment and ex-ante payment. However, they do not take into account the impact of the content quality on the platform profits and the maximum profits over the infinite horizon [30] [31]. Although the authors in [13, 8] explore the content quality issues about maximizing the profits, they neglect the long-term maximum profits for crowd sensing application. Thus our work complements the the above reputation based incentive mechanism literature.

3. SYSTEM MODEL AND RELATED DEFINITIONS

In this section, we introduce the model with sensing data viewers, sensing data providers, and monopoly platforms. For the ease of notations, we occasionally use viewers, providers and platforms to refer to sensing data viewers, sensing data providers, and monopoly platforms respectively, and use “sensing data” and “content” interchangeably.

Fig. 1 and Fig. 2 illustrate a crowd sensing system. The system consists of monopoly platforms, sensing data providers, sensing data viewers. The platforms publishes a crowd sensing task. Receiving the task, sensing data providers produce sensing data and upload these sensing data

Figure 1. A crowd sensing system framework for maximizing long-term service quality.
to the platforms by leveraging mobile devices such as sensor-equipped smartphones, which are connected to the cloud by cellular networks (e.g., GSM/3G/4G) or WiFi connections. Sensing data viewers view these sensing data on the platforms. Well-known examples include YouTube, Facebook, Twitter, and Yahoo! Answers (an online community where people share knowledge).

A key characteristic of these platforms is that the crowd sensing data can be viewed for free by the users and the sensing data providers are not obliged to produce sensing data for the platforms. Thus, to achieving good sensing data quality, incentive mechanisms are necessary. To highlight our concern, next, we start to specify the modeling details in the order of sensing data viewers, sensing data providers, and monopoly platforms.

3.1. Sensing Data Viewers

In our scenario, to decrease the number of variables, in terms of the numeraire denoted as a representative sensing data, there are two viewer types. One is a representative sensing data viewer viewing the sensing data quality denoted as $q_0$, and the other is an aggregate sensing data viewer viewing the aggregate sensing data quality denoted as $y$. Although the sensing data viewers are diverse in terms of preferences to the contents from providers, the decision of a representative sensing data viewer may be convenient to characterizing the aggregate sensing data viewing decisions towards all the sensing data viewers. It is intuitively true that a higher quality sensing data from providers will attract more sensing data views from viewers (and bring a higher utility for its sensing data original provider, too) than the a provider with a lower quality. Thus, the sensing decision of the sensing data providers has a threshold structure. In particular, there exist marginal...
sensing data providers whose sensing data has a quality denoted by $q^*$. More formally, we refer to $q^*$ as the marginal sensing data quality. The potential range of related contents is labeled $1, 2, 3, \cdots$.

We denote the amounts of the various contents as $x(q_0)$ and $x(y)$. To characterize the sensing data quality, we assume that the sensing data quality $q$ follows a cumulative probability density distribution function (CDF), called as $F(q)$, across the unit mass of sensing data providers. In other words, $F(q)$ denotes the fraction of sensing data providers whose sensing data has a quality less than or equal to $(q \in [q_l, q_h])$, where $q_l$ and $q_h$ are the lowest and highest sensing data quality on all sensing data providers, respectively.

According to the well-known Dixit-Stiglitz function, we have the following Lemma 1.

**Lemma 1 (General Marginal Sensing Data Quality)**

Assume that there are the $S > 0$ viewers assigned at random to receive views of providers, a general marginal sensing data quality $\bar{q}^*$ given by $\bar{q}^* = \frac{1}{2}\left[y^2 + 4(y^o + \frac{1}{\rho+1})^{\frac{1}{2}} - y\right]$, where $y = \left(\sum_{i=1}^{N} q_i^o\right)^{\frac{1}{\rho}}$ is dual quantity, and $0 < \rho < 1$.

The detailed proof of the Lemma 1 is provided in the Appendix Appendix A.

The marginal sensing data quality as a threshold summarized in Fig. 2 will be used to calculate the observable variable HIGH/LOW in the following section.

### 3.2. Sensing Data Providers

Consider $N$ independent providers, each with sensing data quality $q_i (i = 1, 2, \cdots, N)$. Without loss of generality, we normalize the mass of sensing data providers to one. For the ease of analysis, we emphasize the endogenous choice by whether or not there are the maximal expected total discounted reward. Further we assume that providers decide whether or not to be activated based on the concept of subsidy for passivity. To this end, we assume that monopoly platforms provides economic incentives to the providers (i.e. the subsidy mentioned above) to produce more high-quality contents. As such, the high-quality sensing data attracts more views of viewers. The sensing data quality produced by providers is presented in this paper as a scalar quantity, actually may be obtained by a combination of the well known technical specification specifications, for instance,
video resolution and image processing, etc. We assume also that there are the constraint on the participation ratio to guarantee the extensive user participation sensing, i.e., \( M/N \) in the paper. Moreover, we define \( c(q_i) > 0 \) and \( 0 < q_i < 1 \) as the sensing cost and the sensing data quality produced by providers (a distribution in a normalized interval \([0, 1]\)). To characterize the sensing cost heterogeneity, we assume that the sensing cost \( c(q_i) > 0 \) is a constant when we consider the sensing costs are homologous.

### 3.3. Monopoly Platforms

To subsidize the providers in the passive state and reward the ones in the active state, monopoly platforms share with providers (part of) its advertising profits as an economic incentive like YouTube Partner, and so on. Moreover, monopoly platforms are responsible for evaluating the marginal sensing data quality and updating the current sensing data quality in terms of the number of time slots that viewers view the sensing data with advertisements. Monopoly platforms are also responsible for noticing the sensing providers about the states of their submitted sensing data, and subsidizing the providers in the passive state, illustrated in Fig. 2.

### 4. PROBLEM FORMULATION OF MAXIMIZING LONG-TERM PROFITS

In this section, we first define the sensing data quality model. Based on the model, furthermore, we explore the long-term maximal profits, followed by the marginal-quality incentive mechanism based on homologous production costs.

#### 4.1. Sensing Data Quality Model

Since the marginal sensing data qualities produced by providers are not directly observable before the production is made. Providers can, however, infer the sensing data quality according to its decision and observation history.

**Definition 1 (Sensing Data Quality Model)**

A sensing data quality is defined as a “HIGH” or “LOW” state if it is not less than the marginal
sensing data quality (to be elaborated in Section 7) or smaller than the marginal sensing data quality from the feedback of monopoly platforms, respectively, where a “HIGH” state means that it gets a qualification for a candidate, and vice versa.

Thus, a sensing data quality produced from providers evolves in the state space $S = \{LOW, HIGH\}$ or $\{0, 1\}$ across time slots in terms of transition probability matrix $P$ as follows:

$$P = \begin{bmatrix} 1 - r & r \\ 1 - p & p \end{bmatrix}$$

where $r$ and $p$ denote the transition probability from “LOW” to “HIGH” and “HIGH” to “HIGH” respectively, as shown in Fig. 3. For specific mobile user $i$, its corresponding transition probability is $r_i$ and $p_i$. Throughout the paper, we assume the transition probability matrix $P$ of every provider is known to the platform. In practice, the matrix $P$ for provider $i$ may be learned in an initial training period, in which the provider continuously sensing data and upload to the platform in every slot. In this period, we compute a sample average $\bar{X}$ of the durations $(X_1, X_2, X_3, \cdots)$ that the sensing data quality from the provider is continuously “HIGH”. It is obvious that $X_i$ is i.i.d over $k$ with $E[X_k] = 1/(1 - p)$. As a result, we may use $1 - 1/\bar{X}$ as an estimate of $p$. The transition probability $r$ can be estimated similarly.

Assume that $\omega_i(\tau)$ is the probability that the sensing data quality from provider $i$ is “HIGH”, which is larger than the marginal sensing data quality from the feedback of viewers, in time slot $t$ conditioning on the past observation history from the feedback of monopoly platforms. For simplicity of representation, we omit the subscript $i$, i.e. $\omega(\tau) = P\{s(\tau) = HIGH| \text{all past observation of some user}\}$, where $s(\tau)$ denotes the state of the sensing data quality $q(\tau)$. In an
initial training period $l$, we can estimate $\omega_i$ by the average of the durations that the sensing data quality is continuously in the “HIGH” state. According to Definition 1, if the $q_i(\tau)$ is larger than or equal to the marginal sensing data quality $q^*$, the value of the $s_i(\tau)$ is HIGH, otherwise the value of the $s_i(\tau)$ is LOW. Based on the current state $s_i(\tau)$, the platform determines whether the provider $i$ is selected or not. If the provider is selected, we have $a_i(\tau) = 1$, 0 otherwise. Given the selection action and the observation in time slot $\tau$, the sensing data quality $\omega_i(\tau)$ is updated in the following equation.

$$
\omega_i(\tau + 1) = \begin{cases} 
    r_i & a_i(\tau) = 1 \text{ and } s_i(\tau) = LOW \\
    p_i & a_i(\tau) = 1 \text{ and } s_i(\tau) = HIGH \\
    \omega_i(\tau)p_i + (1 - \omega_i(\tau))r_i & a_i(\tau) = 0
\end{cases}
$$

(2)

Generally speaking, for any fixed provider $i$, if we observe its sensing data quality in time slot $(\tau - n)$ for some $n \leq \tau$ through the feedback of monopoly platforms, $\omega_i(\tau)$ is equal to the n-step transition probability $P^n_{i,j}(j \in S)$, which take the value in the set $F_i = \{P^n_{i,01}, P^n_{i,11}\} \cup \{\omega_i(0)\}$, where $\omega_i(0)$, the stationary probability that the sensing data quality of user $i$ is larger than or equal to the marginal sensing data quality $q^*$, is given respectively as follows:

$$
\omega_i(0) = \frac{r_i}{1 + r_i - p_i}
$$

(3)

If $\omega_{s,l}(s \in S)$ denotes the probability that the most recent sensing data quality is always observed in the “HIGH” state during continuous $l$ time slots, it is obtained according to (2) as follows:

$$
\omega_{0,l} = \frac{r - (p - r)^l}{1 + r - p}, \omega_{1,l} = \frac{r + (p - r)^l(1 - p)}{1 + r - p}
$$

Thus we can know that $\omega(\tau)$ of every user evolves over sensing data quality evolution space $\mathbb{C} = \{\omega_{0,l}, \omega(1), \omega_{1,l}, l \in \mathbb{Z}^+\}$. The rest of the variables are defined in Table I.

4.2. Problem Formulation

As the mentioned above, for the convenience of analysis, we assume the crowd sensing system with $N$ users as providers, $S$ users as viewers and one platform, which is similar to the model in [32] by exploiting interference to transmit concurrently to increase network capacity. Different from [32], it
Table I. Summary of Notations

| Variable | Description |
|----------|-------------|
| $N, S$   | number of providers, number of viewers |
| $M$      | number of providers selected in each time slot |
| $K$      | number of types with different cost |
| $q_i(\tau)$ | content quality of provider $i$ at time slot $\tau$ |
| $\vec{q}$ | content quality vector of all providers |
| $\vec{q}^*$ | a general marginal sensing data quality |
| $q^*, \vec{q}^*$ | marginal content quality, marginal content quality vector |
| $\tau$ | duration of a time slot |
| $t$ | type of a provider under the condition of heterogeneity |
| $a_i(\tau)$ | action of whether to select provider $i$ at time slot $\tau$ |
| $s_i(\tau)$ | state of the content quality of provider $i$ at time slot $\tau$ |
| $r, p$ | state transition probability from “0” to “1” and “1” to “1” |
| $S, C$ | state space and the quality evolution space |
| $Q, A$ | sensing data quality space and providers’ action space |
| $\phi$ | a function that maps from the space $Q$ to the space $A$ |
| $\omega_i$ | probability that the quality state of provider $i$ is “HIGH” |
| $\beta$ | a discount factor |
| $c$ | a data sensing cost |
| $\lambda, \delta$ | a subsidy and randomization factor of the RMBP |
| $l$ | number of time slots every period |
| $c_t$ | a data sensing cost of a provider with type $t$ |

is desired to satisfy the participation ratio $M/N$ for $N$ providers, i.e., to choose $M$ providers out of $N$ providers to guarantee the extensive user participation so as to maximize the long-term expected rate of reward. This is the classic restless bandit problem proposed by Whittle [33]. Each provider
in our sensing data quality model is considered as an arm and the state of arm \( i \) in time slot \( \tau \) is the state of the sensing data quality \( q_i(\tau) \) mentioned above. Thus, a policy \( \phi : Q(\tau) \rightarrow A(\tau) \) is a function that maps from the sensing data quality vector \( Q(\tau) \) to the selection action set \( A(\tau) \) in time slot \( \tau \).

Furthermore, in our model, to encourage more users to participate, we define the following sustaining cooperation incentive conditions. We assume that monopoly platforms provide providers with economic incentives (i.e. the subsidy mentioned above) to produce more high-quality contents. Our objective is to design the optimal policy \( \phi^* \in \Phi \) to maximize the expected long-term reward and keep users complying with the sustaining incentive condition. Generally speaking, our problem is given as follows:

\[
V(\vec{q}, M) = \max_{\phi} \lim_{T \to \infty} \frac{1}{T} \lim_{l \to \infty} \frac{1}{T_l} \mathbb{E}_{\phi}\left[ \sum_{\tau=0}^{T_l-1} \sum_{i=1}^{N} \beta^\tau q_i(\tau) \omega_i(\tau) a_i^\phi(\tau) \right]
\]

subject to

\[
\lim_{T \to \infty} \sup_{\phi} \frac{1}{T_l} \mathbb{E}_{\phi}\left[ \sum_{\tau=0}^{T_l-1} \sum_{i=1}^{N} a_{i}^{\phi}(\tau) \right] = M,
\]

where \( a_{i}^{\phi}(\tau) = 1 \) if user \( i \) is taken in time slot \( \tau \) and is zero otherwise.

5. OPTIMAL POLICY OF MAXIMIZING PROFITS OVER THE INFINITE HORIZON

We first introduce a Whittle Indexability concept to solve the problem optimally. Furthermore, to encourage more users to participate, we define a sustaining cooperation incentive conditions and propose a marginal-quality based incentive mechanism for crowd sensing applications.

5.1. Whittle Indexability

Definition 2 (Whittle Indexability)

A long-term crowd sensing task is called Whittle Indexability if \( D_i(\lambda) \) increases monotonically from \( \emptyset \) to the full state space for provider \( i \) as \( \lambda \) increases from \(-\infty\) to \(+\infty\), where \( D_i(\lambda) \) be the set of values of \( s_i \), for which provider \( i \) would be rested under a \( \lambda \)-subsidy policy [33].
According to [34], our policy is Whittle Indexable, thereby the whittle’s index value \( \lambda_i \in \inf \{ \lambda : \omega \in \mathbb{D}_i(\lambda) \} \) holds. Thus, the optimal solution of our policy takes the following expression: provider \( i \) with \( \lambda_i(\omega_i(\tau)) > \lambda \) is activated according to [34, 35], where \( \lambda \) is a lagrange multiplier viewed as a constant ‘subsidy for passivity’.

**Definition 3 (Active Provider)**

An indexable provider is called an active provider if under a \( \lambda \)-subsidy policy, \( \lambda_i(\omega_i(\tau)) > \lambda^* \) holds for the provider \( i \) and \( \lambda_i(\omega_i(\tau)) = \lambda^* \) holds with probability \( \delta^* \) for the provider \( i \), \( \lambda(\omega_i(\tau)) \) can be obtained according to [34].

Further, according to the Bellman equation and the expressions (2), we have the following expression.

\[
V(\omega) = \max \{ \omega_i(\tau) + \beta(\omega_i V(p) + (1 - \omega_i)V(r)) \}, \lambda + \beta V(\omega_i(\tau)p + (1 - \omega_i(\tau)r) \}
\]

(5)

From the preceding discussion and the above expression, after every time slot ends, if user \( i \) is activated to transfer the sensing data quality \( q_i \), the user will receive \( \omega_i(\tau) + \beta(\omega_i V(p) + (1 - \omega_i)V(r)) \) profits, Otherwise, a subsidy will be provided \( \lambda \) payments from platforms. Now, we discuss the participating incentive condition of the users.

5.2. Participating Incentive Condition

**Lemma 2 (Participating Incentive Condition)**

In our system, the participating incentive condition is individually reasonable only if the expression \( \lambda - c \leq 0 \) holds, where \( c \) is a data sensing cost.

The detailed proof of the Lemma 2 is provided in the Appendix Appendix B.

Obviously, Lemma 2 holds under the hypothesis that viewers can access to the sensing data for free. If the above expression does not hold, i.e. the sensing cost is too high, in principle, platforms may charge viewers due to viewing the sensing data, which we will explore in our future work.
Thus, it is true that under our policy $\phi$, all providers are willing to participate for the sensing data producing, no matter whether the sensing data quality is high or low.

**Algorithm 1** $T$-Period marginal Content Quality Incentive mechanism $CQI$.

**Input:** $\rho, q_l, q_t, n, \omega_0$, the previous period marginal sensing data quality $q^*$, the stationary sensing data quality $q_0$, and the capacity limit $M$.

**Output:** The total profit $V$.

1: The deadline are divided into $T$ periods, and each period consists of $l$ time slots;
2: platforms estimate $\omega_0$ and calculate $\lambda^*$ and $\delta^*$ according to [36] and the incentive constraint conditions in Lemma 2, and then announce the values of $\lambda^*$ and $\delta^*$ to providers by its feedback at the end of the initial period;
3: repeat
4: for (int $j = 1th$ time slot, $j < l, j + +$) do
5: **Production Decision:** providers update the value of the current sensing data quality based on the expression (2) and calculate their own index value $\lambda$, and then decide to whether to participate according to Definition 3;
6: **Viewing Decision:** Optimally the sensing data view of viewers are disseminated to maximize the utility of viewers;
7: **Threshold Parameter Decision:** According to view results from viewers, platforms calculate the values of $\lambda^*$ and $\delta^*$ according to [36] by inputting $q^*$ and $M$, and announce the values of $\lambda^*$ and $\delta^*$ to producers by its feedback at the end of the time slot $j$;
8: end for
9: Calculate the marginal sensing data quality $q^*$ for the next period;
10: until The number of total periods=$T$
11: Calculate the total profit $V$ according to equation (4);
12: return $V$. 
5.3. Optimal Algorithm over The Infinite Horizon

As mentioned above, at the beginning of the following algorithm, every period consists of \( l \) time slot durations. At the end of each period, the platform calculates and updates the marginal sensing data quality \( q^* \) for the next period. The process is repeated until all periods are exhausted. Finally, the platform outputs the total profit \( V \). The details of our marginal-quality based incentive mechanism for maximizing long-term profits is given in the following Algorithm 1.

**Lemma 3**

The per-period computational complexity in Algorithm 1 is \( O(lN) \).

**Proof:** The computational complexity of Algorithm 1 is dominated by sorting the index values in the line 7, which has complexity \( O((2\tau + 1)Nl \cdot \log((2\tau + 1)N)) \). After each period, Algorithm 1 takes a very simple form: in each time slot, if the index value a user as a provider is above the threshold, the user will participate in data sensing and transmit the sensing data. Therefore, the per-period computational complexity is \( O(lN) \). Thus, Lemma 3 holds.

6. GENERALIZATION TO HETEROGENOUS TYPE CONTENTS

In this section, to meet the different type sensing data demands of viewers, we generalize the above analysis to heterogenous type contents.

6.1. Heterogenous Sensing Data Quality Model

We define provider \( i \) with sensing data type \( q_{ik} \) as type \( k \) sensing data providers. We normalize the mass so that \( \sum_{i=1}^{K} n_i = 1 \). Under the continuum model, we can obtain the marginal sensing data quality illustrated in the following Lemma 4.

**Lemma 4 (Marginal Sensing Data Quality)**

Assume that there are the \( S > 0 \) viewers assigned at random to receive views of providers, a
marginal sensing data quality vector \( \vec{q}^* = < q^*_1, q^*_2, \cdots, q^*_K > \) is solved by \( q^*_t = \exp(-\frac{(\vec{q}^*_t)^2}{2\sigma^2}) \), where \( \vec{q}^* = < \vec{q}^*_1, \vec{q}^*_2, \cdots, \vec{q}^*_K > \) is solved, where \( \vec{q}^*_t = \frac{1}{2}[\bar{y}^2 + 4(y\rho + \sum_{t=1}^{K} \int_{q_t} q_t^t dF_t(q))]^{\frac{1}{2}} - y]. \)

The detailed proof of the Lemma 4 is provided in the Appendix Appendix C.

It is well known that providers will typically experience in real scenes. Compared with perfectly sensing data quality symmetry, such sensing data quality asymmetry faces a significant challenge about the design and analysis of maximizing long-term profits. Specifically, for all the sensing data qualities in type \( t \), their sensing data quality evolve based on the same Markov model. However, these characteristics are different between types. The state transition of some sensing data quality type \( t \) is illustrated in Fig. 4. Its transition probability matrix \( P_t \) is given as follows:

\[
P_t = \begin{bmatrix}
1 - r_t & r_t \\
1 - p_t & p_t
\end{bmatrix}
\]

where \( r_t \) and \( p_t \) denote the transition probability for sensing data quality in type \( t \) from “LOW” to “HIGH” and “HIGH” to “HIGH” respectively. Thus, sensing data qualities of the users in type \( t \) are updated according to the following expressions.

\[
\omega_t(\tau + 1) = \begin{cases}
  r_t & a_t(\tau) = 1 \text{ and } s_t(\tau) = LOW \\
  p_t & a_t(\tau) = 1 \text{ and } s_t(\tau) = HIGH \\
  \omega_t(\tau)p_t + (1 - \omega_t(\tau))r_t & a_t(\tau) = 0
\end{cases}
\]

If \( \omega_{s,l}(s \in S) \) denotes the probability that the most recent sensing data quality in type \( t \) is always observed in the “HIGH” state during continuous \( l \) time slots, according to (7), it is obtained by

\[
\omega_{0,l}^t = \frac{r_t - (p_t - r_t)r_t}{1 + r_t - p_t}, \quad \omega_{1,l}^t = \frac{r_t + (p_t - r_t)(1 - p_t)}{1 + r_t - p_t}.
\]

If a user sensing data quality is never updated, the sensing data quality monotonically converges to the stationary probability \( \omega_t(1) = \frac{r_t}{1 + r_t - p_t} \). Thus...
we can know that $\omega_t(\tau)$ of every provider with type $t$ evolves over sensing data quality evolution space $\mathbb{C} = \{\omega_{0,t}, \omega_t(1), \omega_{1,t}, t \in \mathbb{Z}^+\}$.

6.2. Heterogenous Incentive Condition

In our system, due to the heterogenous sensing data characteristic, the productions of different sensing data quality types need different costs. As such, the heterogenous sensing data quality demands heterogenous participating incentive condition to sustain stable crowd sensing system.

**Lemma 5 (Heterogenous Incentive Condition)**

Assume that users as providers need a production cost of $c_t \geq 0$ to provide type $k$ sensing data quality. the heterogenous participating incentive condition is given by the expression $\lambda(t) \geq c_t$.

Since the proof of the Lemma 5 is similar to the proof of the Lemma 2, the proof of the Lemma 5 need not be provided. It is true that from Lemma 5, all providers in type $t (t = 1, \cdots, K)$ are willing to participate for the sensing data producing no matter whether the sensing data quality is high or low under our policy $\phi$.

6.3. Heterogenous Threshold Parameter

In this subsection, we will determine the active condition for heterogenous sensing data quality.

**Definition 4 (Heterogenous Active Provider)**

An indexable provider $i$ with type $t$ is called a heterogenous active provider if under a $\lambda$-subsidy policy, $\lambda_t(\omega_i(\tau)) > \lambda^*$ holds for the provider $i$ type $t$ and $\lambda_t(\omega_i(\tau)) = \lambda^*$ holds with probability $\delta^*$ for the provider $i$, $\lambda_t(\omega_i(\tau))$ can be obtained according to [34, 35].

Thus, our heterogenous cost incentive mechanism $HCQI$ for long-term crowd sensing is illustrated in Algorithm 2.

7. EXPERIMENT SIMULATIONS AND PERFORMANCE ANALYSIS

In order to assess the performance of crowd sensing under our policy, we implement our mechanism $HCQI$ successfully with the NS-2 simulator. For the ease of comparisons, a marginal content
Algorithm 2 $T$-Period Heterogenous marginal Content Quality Incentive mechanism $HCQI$.

**Input:** $\rho$, $qh$, $ql$, $n$, $\omega_0$, and the previous period marginal sensing data quality vector $\vec{q}^*$. 

**Output:** The total profit $V$.

1. The deadline are divided into $T$ periods, and each period consists of $l$ time slots;
2. In an initial training period, platforms estimate the value of $\omega_0$ and calculate the values of $\lambda^*$ and $\delta^*$ according to [36] and the expected transmission time slots, and then announce the values of $\lambda^*$ and $\delta^*$ to providers by its feedback at the end of the initial period;
3. repeat
4. for (int $j = 1$th time slot, $j <= l$, $j + +$) do
5. **Production Decision:** providers update the value of the current sensing data quality vector based on the expression (7) and calculate their own index value $\lambda$, and then decide to whether to participate according to Lemma 4;
6. **Viewing Decision:** Optimally the sensing data view of viewers are disseminated to maximize the utility of viewers;
7. **Threshold Parameter Decision:** According to view results from viewers, platforms calculate the values of $\lambda^*$ and $\delta^*$ according to [36] by inputting $\vec{q}^*$ and $M$, and announce the values of $\lambda^*$ and $\delta^*$ to producers by its feedback at the end of the time slot $j$.
8. end for
9. Calculate the marginal sensing data quality vector $\vec{q}^*$ for the next period;
10. until The number of total periods=$T$
11. Calculate the total profit $V$;
12. return $V$.

Quality $q^*$ is the solution of normalizing the general marginal content quality by applying Gaussian similarity function [37, 38], and then $0 \leq q^* \leq 1$ holds. Thus, the marginal content quality is obtained as follows:

$$q^* = \exp(-\frac{(\bar{q}^*)^2}{2\sigma^2}).$$  \hspace{1cm} (8)
At the same time, a marginal content quality vector $\vec{q}^* = < q_1^*, q_2^*, \cdots, q_K^* >$ is the solution of normalizing the general marginal content quality vector by applying Gaussian similarity function [37, 38], and then $0 \leq q_t^* \leq 1$ holds. Thus, the $t$-th component $q_t^*$ of the marginal content quality $\vec{q}^*$ is obtained as follows:

$$q_t^* = \exp\left(-\frac{(\bar{q}_t^*)^2}{2\sigma^2}\right).$$ (9)

We attempt to validate our mechanism $HCQI$ on a very small data example (some random 100 time slots) from RollerNet [39] for discounted total profits, the sensing data quality and costs respectively, where an Android phone as user’s mobile device equipped with GPS to continuously journal the location of the phone, a phone application to mark the locations of turns, and some WiFis are used as platforms. We compare our final algorithm $HCQI$ with three algorithms, the user-generated sensing data algorithm (UGC) [8], the Perseus algorithm [30], and the Witness [31]. We also randomly generate an observation model from viewers for each example and fix the discounted factor $\beta = 0.98$ in the infinite horizontal time slot direction (in our experiment, we set $10^5$ ms as the infinite horizontal direction).

Fig. 5(a) indicates that although the performance of UGC outperforms Perseus, and Witness after $q = 0.72$, the returned profit value for our algorithm $HCQI$ is always higher than the profit value returned by UGC, Perseus, and Witness. For each experiment, we calculate the profit value (the discounted sum of rewards) of the four algorithms ($HCQI$, UGC, Perseus, and Witness) returned with 100 samples. Each profit value in Fig. 5(a) displays the average profit value over all examples.

Fig. 5(b) indicates that our algorithms outperform the algorithm in [8]. The main reason is that maximizing profits of the algorithm in [8] also lead to a reduction of the sensing data quality diversity. Thus, the number of participating in providing the candidate decreases. However, our algorithm has a good recovery and diversity due to subsidy for passivity. Notice that Perseus and Witness is optimal when allowed an infinite tree depth. However, we must confine them to a tractable number of steps. As such, it is natural that the outcome limited at the thirtieth time slot is suboptimal. Obviously, $HCQI$ and UGC outperforms Perseus, and Witness at the thirtieth time slot.
Figure 5. (a) Discounted total profit versus the sensing data quality $q$ ($c = 1.2, M = 15$); (b) Discounted total profit versus the number of truncated slots $\tau$ ($c = 1.2, M = 15$); (c) Discounted total profit versus cost $c$ ($M = 15$).

From Fig. 5(c), it can be seen that the proposed HCQI can significantly increase the discounted total profit compared to the three algorithms (UGC, Perseus, and Witness) when the payment cost $c \geq 0.5$. We also observe that by optimally choosing the payment cost $c$, the discounted total profit increases with the decrease of the payment cost $c$. Numerical results indicate that our algorithms outperform the three algorithms (UGC, Perseus, and Witness).

8. CONCLUSIONS AND FUTURE WORK

In this paper, we first analyze the main stages of two-sided markets—sensing data providers, platforms, and sensing data viewers. Then we derive the closed expression of the marginal sensing data quality based on the monopoly aggregation in economics. According to the marginal sensing
data quality, we introduce a sensing data quality evolution process to form our goal—the long-term maximal profits for crowd sensing applications. Furthermore, to solve the utility equation, we design a marginal-quality based incentive mechanism $CQI$ for achieving good quality by applying the Whittle Indexability method and our participation incentive condition. Moreover, we generalize the $CQI$ mechanism to $HCQI$ mechanism for solving the heterogenous type sensing data quality. Finally, numerical results show that our marginal-quality based mechanisms outperform existing relative algorithms. In future works, we will explore a secure multi-platform incentive mechanism based on sensing data quality to avoid the active attacks of the malicious nodes like the colluding attacks for crowd sensing.

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APPENDIX A.

Proof of Lemma 1:

Proof

According to the previous definitions of variables, we formulate a separable utility function for the utility maximization problem for the representative content viewer as follows:

$$u = U(x(q_0), x(y)) \quad \text{(Appendix A.1)}$$

where

$$y = \left( \sum_{i=1}^{n} q_i^\rho \right)^{\frac{1}{\rho}} \quad \text{is dual quantity, and } 0 < \rho < 1 \quad \text{(for concavity, and } 0 < \rho \text{ for several } q_i = 0 \text{).}$$

According to the well-known Dixit-Stiglitz function, we have

$$y^\rho = \sum_{i=1}^{n} q_i^\rho \quad \text{(Appendix A.2)}$$

Integrating both sides of the equation above, we obtain

$$\int_0^y dy^\rho = \sum_{i=1}^{n} \int_{q_i}^{q_i} q_i^\rho dq_i$$

Thus,

$$y = \left[ \frac{n(q_i^\rho + 1 - q_i^\rho + 1)}{1 + \rho} \right]^{\frac{1}{\rho}} \quad \text{(Appendix A.3)}$$

Definition 5 (General Marginal Content Quality)

Assume that there are the $S > 0$ viewers assigned at random to receive views of providers, a general marginal content quality is the solution of the following utility maximizing problem:

$$U(x(q_0), x(y)) = \int_{q_0}^{1} q \cdot x(q)^{\frac{\rho - 1}{\rho}} dF(q) + y x(y)^{\frac{\rho - 1}{\rho}} \quad \text{(Appendix A.4)}$$

Subject to

$$\bar{q}^* = \arg \max_{x(q_0) \geq 0, x(y) \geq 0} \int_{q_0}^{1} q \cdot x(q) dF(q) + x(y) \leq S,$$

where $y$ is presented in the expression (Appendix A.3).

Furthermore, we can obtain the optimal number of viewing the general marginal content quality $\bar{q}^*$

$$x^*(\bar{q}^*) = \frac{S(\rho + 1)\bar{q}^*\rho}{(\rho + 1) \cdot y^\rho + (1 - q_i^\rho + 1)}$$
Additionally, \( x^*(q^*) = S \cdot \frac{q^*}{q + y} \). Thus, we can obtain the closed-form solution of a general marginal content quality based on the above expression mentioned as follows:

\[
q^* = \frac{1}{2}[y^2 + 4(y^p + \frac{1 - q_{0}^p+1}{\rho + 1})^\frac{1}{\rho} - y] .
\]

(Appendix A.5)

Thus, the Lemma holds.

\[\square\]

**APPENDIX B.**

**Proof of Lemma 2:**

**Proof**

We consider two cases according to the above Bellman equation. The first case is that when \( \omega_i(\tau) + \beta(\omega_i(p) + (1 - \omega_i)r) > \lambda + \beta V(\omega_i(\tau)p + (1 - \omega_i)\tau) \), according the above Bellman equation, provider \( i \) is activated by the platform. For the simplicity of notation, we drop the subscript \( i \) and assume that “F” and “D” denote “carry out” and “defuse to carry out” respectively. Assume that a content sensing needs cost \( c \). If the provider complies with the platform’s decision, i.e., it carries out the sensing action, the expected payoff of the provider is given by \( V_{\omega}(F_u; F_p) = -c + \omega + \beta V(\omega V(p) + (1 - \omega)V(\tau)) \), where the first \( F_u \) means that the user as a provider carries out the sensing action and the second \( F_p \) means that the platform selects the user for achieving the participatory ratio requirement. Similarly, if the provider does not comply with the platform’s decision, i.e., it defuses to carry out the sensing action, the profit of the provider is given in the following expression, \( V_{\omega}(D_u; F_p) = \beta V(\omega(\tau)p + (1 - \omega(\tau))\tau) \). Further, we have \( V_{\omega}(F_u; F_p) = -c + \omega + \beta (\omega V(p) + (1 - \omega)V(\tau)) > -c + \lambda + \beta V(\omega_i(\tau)p + (1 - \omega_i)\tau) = -c + \lambda + V_{\omega}(D_u; F_p) \). If \( \lambda - c \leq 0 \), then the provider will comply the platform’s decision for itself interest since \( V_{\omega}(F_u; F_p) \geq V_{\omega}(D_u; F_p) \).

The other case is that when \( \lambda + \beta V(\omega_i(\tau)p + (1 - \omega_i(\tau))\tau) \geq \omega_i(\tau) + \beta(\omega_i(p) + (1 - \omega_i)\tau) \), according to the above Bellman equation, the provider \( i \) is not selected. If the provider complies with the platform’s decision, i.e., it does not carry out sensing action, \( V_{\omega}(D_u; D_p) = \lambda + \beta V(\omega_i(\tau)p + (1 - \omega_i(\tau))\tau) \). If it does not comply with the platform’s decision, i.e., it carries out sensing action, \( V_{\omega}(F_u; D_p) = \omega_i(\tau) + \beta(\omega_i(p) + (1 - \omega_i)\tau) \). The participating incentive condition is derived in the following Lemma 2. Since \( V_{\omega}(D_u; D_p) \geq -c + \lambda + \beta V(\omega_i(\tau)p + (1 - \omega_i(\tau))\tau) \geq -c + \omega_i(\tau) + \beta(\omega_i(p) + (1 - \omega_i)\tau) \), the provider must comply with the platform’s decision for itself interest.

Putting these together, the Lemma 2 holds. \[\square\]
APPENDIX C.

Proof of Lemma 4:

Proof

Under the continuum model, the corresponding CDF and the general marginal content quality is $F_t(q)$ and $\bar{q}_t^*$ respectively.

**Definition 6 (General Marginal Content Quality Vector)**

Assume that there are the $S > 0$ viewers assigned at random to receive views of providers, a general marginal content quality vector $\vec{\bar{q}}^* = < \bar{q}_1^*, \bar{q}_2^*, \ldots, \bar{q}_K^* >$ is the solution of the following utility maximizing problem:

$$U(x(q_0), x(y)) = \left[ \sum_{t=1}^{K} \int_{q_{0t}}^{1} n_t q^x(q) \frac{dF}{\rho q} dF_t(q) + y x(y) \right]^{\frac{1}{\rho - 1}}$$  \hspace{1cm} (Appendix C.1)

Subject to

$$\bar{q}_t^* = \arg \max_{x(q_0) \geq 0, x(y) \geq 0} \sum_{t=1}^{K} \int_{q_{0t}}^{1} n_t x(q) dF_t(q) + x(y) \leq S.$$  \hspace{1cm} (Appendix C.1)

where $y$ is presented in the expression (Appendix A.3).

Furthermore, we can obtain the optimal number of viewing the general marginal content quality $\bar{q}_s$ as follows:

$$x^*(\bar{q}_t^*) = \frac{S(\rho + 1)(\bar{q}_t^*)^\rho}{(\rho + 1) \cdot y^\rho + (\rho + 1) \sum_{t=1}^{K} \int_{q_{0t}}^{1} n_t q^\rho dF_t(q)}$$

Additionally, $x^*(\bar{q}_t^*) = S \frac{\bar{q}_t^*}{q_t + y}$. Thus, we can obtain the closed-form solution of a general marginal content quality based on the above expression mentioned as follows:

$$\bar{q}_t^* = \frac{1}{2} [y^2 + 4(y^\rho + \sum_{t=1}^{K} \int_{q_{0t}}^{1} n_t q^\rho dF_t(q))]^{\frac{1}{2}} - y].$$  \hspace{1cm} (Appendix C.2)

Thus, the Lemma holds. \hfill \square