A Poincaré-covariant current operator for interacting systems and deuteron electromagnetic form factors

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In front-form dynamics a current operator for systems of interacting particles, which fulfills Poincaré, parity and time reversal covariance, together with hermiticity, can be defined. The electromagnetic form factors can be extracted without any ambiguity and in the elastic case the continuity equation is automatically satisfied. Applications to the calculation of deuteron form factors are presented, and the effects of different nucleon-nucleon interactions, as well as of different nucleon form factors are investigated.

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1. INTRODUCTION

The electromagnetic (e.m.) current operator and the states of a system must have correct transformation properties with respect to the same representation of the Poincaré group. For systems of interacting particles some of the Poincaré generators are interaction dependent and, therefore, it is not a simple task to construct a current operator, able to fulfill the proper Poincaré, parity and time reversal covariance, as well as current conservation, hermiticity and charge normalization. Instead of including perturbatively the relativistic properties through $p/E$ series, or considering field-theoretic approaches, we adopt the front-form Hamiltonian dynamics with a fixed number of particles, which includes relativity in a coherent, non-perturbative way and allows one to retain a large amount of the successfull phenomenology developed within the “non-relativistic” domain. In this dynamics seven, out of ten, Poincaré generators are interaction free. In particular, the Lorentz boosts are interaction free and the states can be written as products of total momentum eigenstates times intrinsic eigenstates. In the two-body case, if the mass operator, $\tilde{M}$, for the intrinsic functions is defined according to $\tilde{M}^2 = M_0^2 + V$ (with $M_0$ the free mass operator and $V$ the interaction operator), then the mass equation has the same form as the nonrelativistic Schroedinger equation in momentum representation.

In Ref. (a) it was shown that all the requirements of Poincaré covariance can be satisfied, if, in the Breit frame where the momentum transfer, $\vec{q}$, is directed along the spin-quantization axis, $z$, the current operator is covariant with respect to rotations around $z$. Since in the front form the rotations around the $z$ axis are kinematical, the extended Poincaré covariance (i.e., Poincaré covariance plus parity and time reversal covariance) is satisfied by a current operator which in our Breit frame is given by the sum of free, one-body currents (i.e., $J_{\mu}(0) = \sum_{i=1}^{N} j_{\mu,i}^{\text{free}}$, with $N$ the number of constituents in the system). The hermiticity can be easily implemented and, in the elastic case, the extended Poincaré covariance plus hermiticity imply current conservation.

Our Poincaré covariant current operator has been already successfully tested in the case of deep inelastic scattering. In this paper we calculate deuteron e.m. properties; more details for the magnetic moment and the quadrupole moment and other results for the deuteron elastic form factors (f.f.) can be found in Refs. (e) and (c,d), respectively.

2. DEUTERON ELECTROMAGNETIC FORM FACTORS

In the elastic case, for a system of spin $S$ one has only $2S + 1$ non-zero independent matrix elements for the current defined in Ref. (a), corresponding to the $2S + 1$ elastic form factors. Then the extraction of elastic e.m. form factors is no more plagued by the ambiguities which are present when, as usual, the free current is considered in the reference frame where $q^+ = q_0 + q_z = 0$ (indeed, if the current is taken free in the $q^+ = 0$ frame, one has four independent matrix elements to calculate the three deuteron f.f.).

Our results for the deuteron magnetic and quadrupole moments, corresponding to different $N - N$ interactions, are reported in Fig. 1, together with the non relativistic
Figure 1. (a) Deuteron magnetic moment, \( \mu_d \), against the asymptotic normalization ratio \( \eta \), for the Av14 \( \text{[5]} \), RSC \( \text{[6]} \), Paris \( \text{[7]} \), CD-Bonn \( \text{[8]} \), Nijmegen1, Nijmegen93, RSC93 \( \text{[9]} \), and Av18 \( \text{[10]} \) \( N - N \) interactions (mentioned in decreasing order for \( \eta \)). Full dot represents the experimental values for \( \mu_d \) and \( \eta \); empty triangles and diamonds correspond to the non-relativistic and relativistic results, respectively, while dashed and solid lines are linear fits for these results. Full triangle and diamond are the results of the CD-Bonn interaction. (b) The same as in (a), but for the deuteron quadrupole moment, \( Q_d \). (After Ref. \( \text{[3]}\text{(e)} \))

ones, against the deuteron asymptotic normalization ratio \( \eta = A_D/A_S \). A remarkable linear behaviour appears for both quantities and in our Poincaré covariant calculation the relativistic effects bring both \( \mu_d \) and \( Q_d \) closer to the experimental values, except for the charge-dependent Bonn interaction \( \text{[8]} \).

In Fig. 2 we report our results for the deuteron f.f. \( A(Q^2) \) and \( B(Q^2) \). For \( A(Q^2) \) the dependence on the nucleon f.f. is very high, stronger than the effect of different \( N - N \) interactions (see also Refs. \( \text{[3]}\text{(c,d)} \)). If the poorly known neutron electromagnetic structure is properly fitted, the overall behaviour of the experimental data can be reproduced, in particular the recent \( A(Q^2) \) data of Refs. \( \text{[13]} \) and \( \text{[14]} \) (open dots and upward triangles, respectively, in Fig. 2(a)) are well described. With respect to the nucleon f.f. model of Ref. \( \text{[11]} \), small changes are obtained for \( G_{M}^{n} \) and higher values for \( G_{E}^{n} \) in the range \( 0.5 - 1(GeV/c)^2 \). The tensor polarization \( T_{20}(Q^2) \) has a considerable dependence on the interaction \( \text{[3]}\text{(c,d)} \) and only a weak dependence on the nucleon f.f., as well known, so that it cannot be described, together with \( A(Q^2) \) and \( B(Q^2) \), by a simple fit of the neutron form factors. In order to obtain a more precise description of the data one should explicitly introduce two-body currents, which will have to fulfill separately the constraints of extended Poincaré covariance and hermiticity. A more stringent comparison with new TJNAF data for \( B(Q^2) \) will be possible in the near future.
Figure 2. (a) The deuteron form factor $A(Q^2)/(G_D^2 * F)$ (with $G_D = (1 + Q^2/0.71)^{-2}$ and $F = (1 + Q^2/0.1)^{-2.5}$) obtained by the $N - N$ Reid soft core interaction \[1\] and the nucleon f.f. of Ref. \[11\] (solid line), and Ref. \[12\] (dot-dashed line). The dotted line is obtained using the proton f.f. of Ref. \[11\] and a fit for the neutron form factors. For the experimental data see Refs. \[3\](c,d). (b) The same as in (a), but for deuteron form factor $B(Q^2)/(G_D^2 * F_1)$ (with $F_1 = (1 + Q^2/0.1)^{-3}$).

3. CONCLUSION

In the Breit frame where the three-momentum transfer is directed along the spin quantization axis, the current operator has to be covariant for rotations around the $z$ axis. All the necessary requirements for extended Poincaré covariance, hermiticity, current conservation and charge normalization can be satisfied by a current obtained by free, one-body terms in that frame \[3\](a,e).

We have applied our results to the calculation of the deuteron elastic f.f., without any ambiguity. We were able to obtain, for the first time in a covariant relativistic approach without ad hoc assumptions on the values of specific matrix elements of the current, unambiguous results for both $\mu_d$ and $Q_d$, close to the experimental data \[3\](e). The f.f. $A(Q^2)$ and $B(Q^2)$ show remarkable effects from different models for the nucleon f.f., while $T_{20}(Q^2)$ has an higher dependence on different $N - N$ interactions.

Our approach, based on the reduction of the whole complexity of the Poincaré covariance to the SU(2) symmetry \[3\](a), can represent a simple framework where to investigate the many-body terms to be added to the free current.

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