Archimedean Choice Functions
An axiomatic foundation for imprecise decision making

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IPMU 2020
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An axiomatic foundation for imprecise decision making

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home…
FLip
Foundations Lab
for imprecise probabilities
imprecise stochastic processes
(such as imprecise Markov chains)
FLip Foundations Lab for imprecise probabilities
FLip

Foundations Lab
for imprecise probabilities
choice functions

\[ C(\text{sun}, \text{rain}, \text{car}) = ? \]
$C(\text{sun} , 10 , 6 , -2 , 10 , -2 , 0 ) = ?$
\( C(\text{sun}, \text{rain}, \text{umbrella}, \text{car}) = ? \)
a set of states $\mathcal{X}$

a finite set $A$ of options, which are bounded (utility) functions on $\mathcal{X}$

$$C(A) = ?$$

$$C(\text{walk}, \text{run}, \text{drive}) = ?$$
a set of states $\mathcal{X}$

a finite set $A$ of options, which are bounded (utility) functions on $\mathcal{X}$

\[
C(A) = B
\]

\[
\begin{bmatrix}
10 & 6 & -2 \\
-10 & -2 & 0
\end{bmatrix}
\]

\[
C\left(\begin{bmatrix}
\text{walk} & \text{umbrella} & \text{car}
\end{bmatrix}\right) = \begin{bmatrix}
\text{walk}
\end{bmatrix}
\]
a set of states $\mathcal{X}$

a finite set $A$ of options, which are bounded (utility) functions on $\mathcal{X}$

$C(A) = B$

$C($

\[
\begin{array}{ccc}
u_1 & u_2 & u_3 \\
10 & 6 & -2 \\
-10 & -2 & 0 \\
\end{array}
\]
Maximising expected utility

uncertainty model:

A finitely additive probability measure $P$ on subsets of $\mathcal{X}$ so it is actually a probability charge...

$P$ also denotes the corresponding expectation operator called linear prevision in the paper

choice function:

$$C_P(A) := \arg\max_{u \in A} P(u)$$

An option is chosen if it has the highest expected utility
Imprecise decision making

uncertainty model: a set $\mathcal{P}$ of finitely additive probability measures $P$
E-admissibility

uncertainty model:
a set $\mathcal{P}$ of finitely additive probability measures $P$

choice function:

$$C^E_\mathcal{P}(A) := \bigcup_{P \in \mathcal{P}} C_P(A)$$

an option is chosen if it has the highest expected utility according to at least one $P$ in $\mathcal{P}$
Maximality

**uncertainty model:**

a set $\mathcal{P}$ of finitely additive probability measures $P$

**choice function:**

$$C^M_{\mathcal{P}}(A) := \{ u \in A : (\nexists v \in A)(\forall P \in \mathcal{P})P(v) > P(u) \}$$

an option $u$ is chosen if there is no option $v$ in $A$ that is better, in the sense that it has higher expected value according to every $P$ in $\mathcal{P}$
uncertainty model

(set of) probability measures
interval probability
(set of) lower previsions
belief function
...

decision rule

maximising expected utility
E-admissibility
maximality
interval dominance
...

direct approaches
Why would you use any of these particular choice functions to make decisions under uncertainty?

Why use (sets of) probability measures?

Why use these particular decision rules?
axioms on a general choice function

uncertainty model + decision rule
Archimedean choice functions

decision making with sets of lower previsions

maximality
applied to a set of probability measures

E-admissibility
applied to a set of probability measures

maximising expected utility
applied to a single probability measure
Archimedean choice functions

decision making with sets of lower previsions

abstract definition in terms of sets of lower previsions

! !

direct definition that imposes axioms on the choices made

translation invariance + coherence + strict desirability
Archimedean choice functions

- Maximality: applied to a set of probability measures.
- Maximising expected utility: applied to a single probability measure.
- Decision making with sets of lower previsions.
- E-admissibility: applied to a set of probability measures.
decision making with sets of lower previsions

E-admissibility applied to a set of probability measures
Archimedean choice functions

decision making with sets of lower previsions

binarity

maximality

applied to a set of probability measures
Archimedean choice functions

decision making with sets of lower previsions

binarity

maximality
applied to a set of probability measures

E-admissibility
applied to a set of probability measures

mixingness
Archimedean choice functions

- Maximality
  - Applied to a set of probability measures

- Binarity
  - Decision making with sets of lower previsions

- Maximising
  - Expected utility
    - Applied to a set of probability measures

- Mixingness
  - E-admissibility
    - Applied to a set of probability measures
  - Maximising expected utility
    - Applied to a single probability measure
