ConArg: a Tool to Solve (Weighted) Abstract Argumentation Frameworks with (Soft) Constraints

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Abstract

ConArg is a Constraint Programming-based tool that can be used to model and solve different problems related to Abstract Argumentation Frameworks (AFs). To implement this tool we have used JaCoP, a Java library that provides the user with a Finite Domain Constraint Programming paradigm. ConArg is able to randomly generate networks with small-world properties in order to find conflict-free, admissible, complete, stable grounded, preferred, semi-stable, stage and ideal extensions on such interaction graphs. We present the main features of ConArg and we report the performance in time, showing also a comparison with ASPARTIX \cite{vioni}, a similar tool using Answer Set Programming. The use of techniques for constraint solving can tackle the complexity of the problems presented in \cite{bistarelli}. Moreover we suggest semiring-based soft constraints as a mean to parametrically represent and solve Weighted Argumentation Frameworks: different kinds of preference levels related to attacks, e.g., a score representing a “fuzziness”, a “cost” or a probability, can be represented by choosing different instantiation of the semiring algebraic structure. The basic idea is to provide a common computational and quantitative framework.

Keywords: Abstract Argumentation Frameworks, , Constraint Satisfaction Problems, Weighted Attacks, Tool for Argumentation.

1. Introduction

Argumentation \cite{dung} is based on the exchange and the evaluation of interacting arguments which may represent information of various kinds, especially beliefs or goals. Argumentation can be used for modeling some aspects of reasoning, decision making, and dialogue. For instance, when an agent has conflicting beliefs (viewed as arguments), a (nontrivial) set of plausible consequences can be derived through argumentation from the most acceptable arguments for the

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agent. Argumentation has become an important subject of research in Artificial Intelligence and it is also of interest in several disciplines, such as Logic, Philosophy and Communication Theory [3, 4].

Many theoretical and practical developments build on Dung’s seminal theory of argumentation. A Dung’s Abstract Argumentation Framework (AF) or Abstract Argument System is a directed graph consisting of a set of arguments and a binary conflict-based attack relation among them [3, 4]. The sets of arguments to be considered are then defined under different semantics, where the choice of semantics equates with varying degrees of scepticism or credulousness. The main issue for any theory of argumentation is the selection of acceptable sets of arguments, based on the way arguments interact. Intuitively, an acceptable set of arguments must be in some sense coherent and strong enough (e.g., able to defend itself against all attacking arguments).

In this paper we propose ConArg (i.e., “Argumentation with Constraints”), a Java-based tool that can find all the classical extensions proposed by Dung [5], i.e., conflict-free, admissible, complete, stable, preferred and grounded, other successively ideated extension as semi-stable [6], stage [7] and ideal [8], and it can also solve the hard problems related to Weighted Argumentation Frameworks (WAF), which have been presented in [9, 2]. An example of these hard problems that ConArg is able to solve, is, given a WAF, a set of arguments and inconsistency budget $\beta$ [9, 2], checking whether $\beta$ is minimal or not. This specific problem is co-NP-complete [9, 2].

As the core of our solver we decide to use Constraint Programming (CP) [10], which is a powerful paradigm for solving combinatorial search problems that draws on a wide range of techniques from artificial intelligence, computer science, databases, programming languages, and operations research. Constraint programming is currently applied with success to many domains, such as scheduling, planning, vehicle routing, configuration, networks, and bioinformatics [10]. Constraint solvers search the solution space either systematically, as with backtracking or branch and bound algorithms, or use forms of local search that may be incomplete. An instance of a Constraint Satisfaction Problem (CSP) [10] as formally presented in Section 3, describes a problem in terms of constraints.

To solve problems related to WAFs we use semiring-based Soft Constraint Programming [11, 12] instead. The key idea behind this formalism is to extend the classical notion of constraint by adding a structure representing its level of satisfiability (or preference/cost), that is a semiring-like structure (see Section 3.1).

Even finding all the classical Dung’s extensions is not “easy”: the number of these extensions, which in practice are subsets of the $\mathcal{A}_{rgs}$ set of arguments, may explode for large $\mathcal{A}_{rgs}$ (the powerset of $\mathcal{A}_{rgs}$ has $2^{|\mathcal{A}_{rgs}|}$ elements). Therefore, it is important to use techniques to tackle this inherent complexity, as those ones adopted in CP. This is particularly important with conflict-free extensions, which represent the “least constrained” extensions.

To model all the introduced problems with constraints, we adopt Java Con-
Stringent Programming solver\footnote{http://www.jacop.eu} (JaCoP), a Java library that provides the Java user with a Finite Domain Constraint Programming paradigm\footnote{http://www.jacop.eu}. With ConArg, the user can import an interaction graph as a textual description file, or he can generate the input according to two different kinds of small-world networks: Barabasi\cite{13} and Kleinberg\cite{14} graphs. We suppose that interaction graphs, where nodes are arguments and edges are attacks (see Section\ref{sec:background}), represent in this case a kind of social network, and consequently show the related small-world properties\cite{13,16}. A practical example can be the study of discussion fora, where the users post their arguments that can attack other users’ arguments\cite{16,17}.

This work details, integrates, and extends with a description of the ConArg tool the research line previously proposed in\cite{18,19,20}. The remainder of this paper is organized as follows. In Section\ref{sec:background} we report the theory behind AFs and, in Section\ref{sec:background:waf} about the WAF formalism presented in\cite{9,2}. In Section\ref{sec:cp} we summarize the background on CP\cite{10} and on its soft extension proposed in\cite{11,12}. In Section\ref{sec:afscp} we show the mapping from AFs to CSPs, which is at the heart of ConArg: by solving the CSP, we find a solution of the related AFs (e.g., all the conflict-free extensions).

Section\ref{sec:waf} revises the unifying WAF formalism we originally proposed in\cite{18}, which is based on the notion of semiring structure\cite{11,12}. Afterwards, in Section\ref{sec:implementation} we show how we have implemented in ConArg the two WAFs respectively presented in Section\ref{sec:waf} and, in Section\ref{sec:advancedwaf}, the WAF advanced in Section\ref{sec:advancedwaf} and briefly reported in Section\ref{sec:background:waf}.

In Section\ref{sec:conarg} we describe the main features of ConArg by also showing some screenshots of the the application we developed, while in Section\ref{sec:performance} we report the performance in time of our constraint-based search; in Section\ref{sec:comparison}, we also show a performance comparison between our solution and the ASPARTIX system\cite{1}. A comparison with related work is given in Section\ref{sec:relatedwork} instead. Finally, Section\ref{sec:conclusion} draws the conclusive remarks and outlines future work.

2. Background on Argument Systems

In\cite{5}, the author has proposed an abstract framework for argumentation in which he focuses on the definition of the status of arguments. For that purpose, it can be assumed that a set of arguments is given, as well as the different conflicts among them. An argument is an abstract entity whose role is solely determined by its relations to other arguments.

\textbf{Definition 1.} An Argumentation Framework (AF) is a pair \((A_{rgs}, R)\) of a set \(A_{rgs}\) of arguments and a binary relation \(R\) on \(A_{rgs}\) called the attack relation. \(\forall a_i, a_j \in A_{rgs}, a_i R a_j\) means that \(a_i\) attacks \(a_j\). An AF may be represented by a directed graph (the interaction graph) whose nodes are arguments and edges represent the attack relation. A set of arguments \(B\) attacks an argument \(a\) if \(a\) is
attacked by an argument of $B$. A set of arguments $B$ attacks a set of arguments $C$ if there is an argument $b \in B$ which attacks an argument $c \in C$.

The “acceptability” of an argument depends on its membership to some sets, called extensions. These extensions characterize collective “acceptability”.

![Diagram](image)

Figure 1: An example of Dung Argumentation Framework; e.g., $c$ attacks $d$.

In Figure 1, we show an example of AF represented as an interaction graph: the nodes represent the arguments and the directed arrow from $c$ to $d$ represents the attack of $c$ towards $d$, that is $c R d$. Dung gave several semantics to “acceptability”. These various semantics produce none, one or several acceptable sets of arguments, called extensions. In Def. 2 we define the concepts of conflict-free and stable extensions:

**Definition 2.** A set $B \subseteq Ads$ is conflict-free iff no two arguments $a$ and $b$ in $B$ exist such that $a$ attacks $b$. A conflict-free set $B \subseteq Ads$ is a stable extension iff for each argument which is not in $B$, there exists an argument in $B$ that attacks it.

The other semantics for “acceptability” rely upon the concept of defense:

**Definition 3.** An argument $b$ is defended by a set $B \subseteq Ads$ (or $B$ defends $b$) iff for any argument $a \in Ads$, if $a$ attacks $b$ then $B$ attacks $a$.

An admissible set of arguments according to Dung must be a conflict-free set which defends all its elements. Formally:

**Definition 4.** A conflict-free set $B \subseteq Ads$ is admissible iff each argument in $B$ is defended by $B$.

Besides the stable semantics, three semantics refining admissibility have been introduced by Dung:

**Definition 5.** A preferred extension is a maximal (w.r.t. set inclusion) admissible subset of $Ads$. An admissible $B \subseteq Ads$ is a complete extension iff each argument which is defended by $B$ is in $B$. The least (w.r.t. set inclusion) complete extension is the grounded extension.

A stable extension is also a preferred extension and a preferred extension is also a complete extension. Stable, preferred and complete semantics admit multiple extensions whereas the grounded semantics ascribes a single extension to a given argument system.
The definitions of stage \[7\] and semi-stable \[6\] semantics are based on the idea of prescribing the maximization not only of the arguments included in the extension (as for the preferred extension) in Def. \[5\] but also of those attacked by it:

**Definition 6.** Given a set \(B \subseteq A_{rgs}\), the range of \(B\) is defined as \(B \cup B^+\), where \(B^+ = \{a \in A_{rgs} : a\) attacks \(B\}\). \(B\) is a stage extension iff \(B\) is a conflict-free set with maximal (w.r.t. set inclusion) range. \(B\) is a semi-stable extension iff \(B\) is a complete extension with maximal (w.r.t. set inclusion) range.

Ideal semantics \[8\], defined in Def. \[7\] provides a unique-status approach allowing the acceptance of a set of arguments possibly larger than in the case of the grounded extension.

**Definition 7.** A set \(B \subseteq A_{rgs}\) is ideal iff \(B\) is admissible and for each preferred extensions \(E\), then \(B \subseteq E\). The ideal extension is the maximal (w.r.t. set inclusion) ideal set.

### 2.1. Weighted Argumentation Frameworks and Related Hard Problems

In ConArg we also solve hard problems related to WAFs \[9, 2\]. Formally, a WAF is a triple \((A_{rgs}, R, w)\) where \(A_{rgs}\) is a Dung-style abstract argument system, and \(w : R \rightarrow \mathbb{R}^+\) is a function assigning real valued weights to attacks.

A key idea presented in \[9, 2\] is the inconsistency budget, \(\beta \in \mathbb{R}^+\), which the authors use to characterise how much inconsistency they are prepared to tolerate. The intended interpretation is that, given an inconsistency budget \(\beta\), we would be prepared to disregard attacks up to a total weight of \(\beta\) \[9, 2\]. Conventional AFs implicitly assume an inconsistency budget of 0. In Section \[5\] we focus on WAFs either, by considering a semiring-based constraint programming framework: the solution of these representations is implemented in ConArg as well.

As shown in \[9, 2\], while the the problem of finding the weighted version of the classical extensions (e.g., stable or admissible) is not computationally harder than the original problem, there are some important problems related to weighted grounded extensions that are very difficult to solve. The concept of inconsistency budget \(\beta\) has been introduced in Section \[2\].

In the following propositions, i.e., Proposition \[1\], Proposition \[2\] and Proposition \[3\] we show three complex problems proposed in \[9, 2\]. As for preferred extensions, we say an argument is credulously accepted if it forms a member of at least one weighted grounded extension, and sceptically accepted if it is a member of every weighted grounded extensions \[3, 4\]. Since there are multiple \(\beta\)-grounded extensions \[9, 2\], we can consider credulous and sceptical variations of the problem, as with preferred extensions. In Proposition \[1\] we consider the credulous case first:

**Proposition 1 \([9, 2]\).** Given a weighted argument system \((A_{rgs}, R, w)\), an inconsistency budget \(\beta\), and argument \(a \in A_{rgs}\), the problem of checking whether \(\exists L \in \text{wge}(A_{rgs}, R, w, \beta)\) such that \(a \in L\) is NP-complete.
In Proposition 2 we consider the “sceptical” version of the problem.

**Proposition 2 ([9, 2]).** Given a weighted argument system \( \langle A_{rgs}, R, w \rangle \), an inconsistency budget \( \beta \), and an argument \( a \in A_{rgs} \), the problem of checking whether, for all \( L \in wge(A_{rgs}, R, w, \beta) \), we have \( a \in L \) is co-NP-complete.

Suppose now we have a weighted argument system \( \langle X, A, w \rangle \) and a set of arguments \( S \). Then, what is the smallest amount of inconsistency would we need to tolerate in order to make \( S \) a solution? When considering conflict-free and admissible extensions, the answer is easy: we know exactly which attacks we would have to disregard to make a set of arguments admissible or consistent. However, when considering weighted grounded extensions, the answer is not so easy. There may be multiple ways of getting a set of arguments into a weighted extension, each with potentially different costs; we are thus typically interested in solving the problem expressed by Proposition 3:

**Proposition 3 ([9, 2]).** Given a weighted argument system \( \langle A_{rgs}, R, w \rangle \), a set of arguments \( L \subseteq A_{rgs} \), and an inconsistency budget \( \beta \), checking whether \( \beta \) is minimal w.r.t. \( \langle A_{rgs}, R, w \rangle \) and \( L \) is co-NP-complete.

3. Constraint Programming

A Constraint Satisfaction Problem (CSP) [10] is defined as a triple \( P = \langle V, D, C \rangle \), where \( X \) is set of variables \( V = \{x_1, x_2, \ldots, x_n\} \), \( D \) is a set of domains \( D = \{D_1, D_2, \ldots, D_n\} \) such that \( x_i \in D_i \), \( C \) is a set of constraints \( C = \{c_1, c_2, \ldots, c_t\} \). A constraint \( c_j \) is a pair \( \langle R_{O_j}, O_j \rangle \) where \( R_{O_j} \) is a relation on the variables in \( O_j = \text{scope}(c_j) \). In other words, \( R_i \) is a subset of the Cartesian product of the domains of the variables in \( O_j \). A solution to the CSP \( P \) is an \( n \)-tuple \( T = \langle t_1, t_2, \ldots, t_n \rangle \) where \( t_i \in D_i \) and each \( c_j \) is satisfied in that \( R_{O_j} \) holds on the projection of \( T \) onto the scope \( O_j \). In a given task one may be required to find the set of all solutions, \( \text{Sol}(P) \), to determine if that set is non-empty or just to find any solution, if one exists. If the set of solutions is empty the CSP is unsatisfiable. This simple but powerful framework captures a wide range of significant applications in fields as diverse as artificial intelligence, operations research, scheduling, supply chain management, graph algorithms, computer vision and computational linguistics [10].

One of the main reasons why constraint programming quickly found its way into applications has been the early availability of usable constraint programming systems, as JaCoP, which we will use in the implementation and solution of the AFs. Various generalizations of the classic CSP model have been developed subsequently. One of the most significant is the Constraint Optimization Problem (COP) for which there are several significantly different formulations, and the nomenclature is not always consistent [10]. Perhaps the simplest COP formulation retains the CSP limitation of allowing only hard Boolean-valued constraints but adds a cost function over the variables, that must be minimized. A weighted constraint \( \langle c, w \rangle \) is just a classical constraint \( c \), plus a weight \( w \) (over
natural, integer, or real numbers). The cost of an assignment $t$ of the variable is the sum of all $w(c)$, for all constraints $c$ which are violated by $t$ [10].

Then, the overall degree of satisfaction (or violation) of the assignment is obtained by combining these elementary degrees of satisfaction (or violation). An optimal solution is the complete assignment with an optimal satisfaction/violation degree. Therefore, choosing the operator used to perform the combination and an ordered satisfaction/violation scale is enough to define a specific framework. Capturing these commonalities in a generic framework is desirable, since it allows us to design generic algorithms and properties instead of a myriad of apparently unrelated, but actually similar properties, theorems and algorithms. In Section 3.1 we show the semiring-based framework [11] that we will adopt in Section 5 in order to parametrize WAFs.

### 3.1. Semiring-based Soft Constraints

A semiring [11] $S$ is a tuple $\langle A, +, \times, 0, 1 \rangle$ where $A$ is a set with two special elements $0, 1 \in A$ (respectively the bottom and top elements of $A$) and with two operations $+$ and $\times$ that satisfy certain properties: $+$ is defined over (possibly infinite) sets of elements of $A$ and is commutative, associative and idempotent; it is closed, $0$ is its unit element and $1$ is its absorbing element; $\times$ is closed, associative, commutative and distributes over $+$, $1$ is its unit element and $0$ is its absorbing element. The $+$ operation defines a partial order $\leq_S$ over $A$ such that $a \leq_S b$ if $a + b = b$; we say that $a \leq_S b$ if $b$ represents a value better than $a$. Moreover, $+$ and $\times$ are monotone on $\leq_S$, $0$ is its min and $1$ its max, $\langle A, \leq_S \rangle$ is a complete lattice and $+$ is its lub. Some practical instances of semirings are the Weighted semiring $\langle \mathbb{R}^+ \cup \{\infty\}, \min, +, \infty, 0 \rangle$ ($+$ is the arithmetic plus operation, to distinguish it from the generic semiring definition of $+$), the Fuzzy semiring $\langle [0..1], \max, \min, 0, 1 \rangle$, the Probabilistic semiring $\langle [0..1], \min, \times, 0, 1 \rangle$ ($\times$ is the arithmetic times operation, to distinguish it from the generic semiring definition of $\times$) and the Boolean semiring $\langle \{true, false\}, \lor, \land, false, true \rangle$, which can be used to model classical crisp CSPs.

Given $S = \langle A, +, \times, 0, 1 \rangle$ and an ordered set of variables $V$ over a finite domain $D$ (to simplify, we consider all the variables as defined on the same domain), a soft constraint is a function which, given an assignment $\eta : V \rightarrow D$ of the variables, returns a value of the semiring. Using this notation $C = \eta : V \rightarrow A$ is the set of all possible constraints that can be built starting from $S$, $D$ and $V$. Any function in $C$ depends on the assignment of only a finite subset of $V$. For instance, a binary constraint $c_{x,y}$ over variables $x$ and $y$, is a function $c_{x,y} : V \rightarrow D \rightarrow A$, but it depends only on the assignment of variables $\{x,y\} \subseteq V$ (the scope, of the constraint). Note that $c\eta[v := d_1]$ means $c\eta'$ where $\eta'$ is $\eta$ modified with the assignment $v := d_1$. Notice that $c\eta$ is the application of a constraint function $c : V \rightarrow D \rightarrow A$ to a function $\eta : V \rightarrow D$; what we obtain is a semiring value $c\eta = a$. Given the set $C$, the combination function $\otimes : C \times C \rightarrow C$ is defined as $(c_1 \otimes c_2)\eta = c_1\eta \times c_2\eta$ [11] [12]. The $\otimes$ builds a new constraint which associates with each tuple of domain values for such variables a semiring element which is obtained by multiplying the elements associated by the original constraints to the appropriate sub-tuples. Given a constraint $c \in C$ and a variable $v \in V$, the
projection \footnote{11} \footnote{12} of \( c \) over \( V \{v\} \), written \( c \downarrow (V \{v\}) \) is the constraint \( c' \) such that \( c' \eta = \sum_{d \in D} \eta[v := d] \). Informally, projecting means eliminating some variables from the scope.

A SCSP \footnote{12} is defined as \( P = \langle V, D, C, S \rangle \), where \( C \) is the set of constraints defined over variables in \( V \) (each with domain \( D \)), and whose preference is determined by semiring \( S \). The best level of consistency notion is defined as \( \text{blevel}(P) = \text{Sol}(P) \downarrow_\emptyset \), where \( \text{Sol}(P) = \bigotimes C \). A problem \( P \) is \( \alpha \)-consistent \footnote{12} if \( \text{blevel}(P) = \alpha \). \( P \) is instead simply “consistent” iff there exists \( \alpha >_{S} 0 \) such that \( P \) is \( \alpha \)-consistent. \( P \) is inconsistent if it is not consistent.

![Figure 2: A SCSP based on a Weighted semiring.](image)

**Example 3.1.** Figure 2 shows a weighted SCSP as a graph: the Weighted semiring is used, i.e. \( (\mathbb{R}^+ \cup \{\infty\}, \min, +, \infty, 0) \) (\( + \) is the arithmetic plus operation). Variables and constraints are represented respectively by nodes and arcs (unary for \( c_1 \)-\( c_3 \), and binary for \( c_2 \)); \( D = \{a, b\} \). The solution of the CSP in Figure 2 associates a semiring element to every domain value of variables \( X \) and \( Y \) by combining all the constraints together, i.e. \( \text{Sol}(P) = \bigotimes C \). For instance, for the tuple \( \langle a, a \rangle \) (that is, \( X = Y = a \)), we have to compute the sum of 1 (which is the value assigned to \( X = a \) in constraint \( c_1 \)), 5 (\( (X = a, Y = a) \) in \( c_2 \)) and 5 (\( Y = a \) in \( c_3 \)): the value for this tuple is 11. The blevel is 7, related to the solution \( X = a, Y = b \).

4. Mapping AFs to CSPs

In this section we propose the mapping from AFs to CSPs, which is at the hearth of ConArg. Given an AF \( \langle A_{rgs}, R \rangle \), we define a variable for each argument \( a_i \in A_{rgs} \) \((V = \{a_1, a_2, \ldots, a_n\})\) and each of these argument can be taken or not, i.e., the domain of each variable is \( D = \{1, 0\} \), 1 if taken in the extension, 0 if not taken.

In the following explanation, notice that \( b \) attacks \( a \) means that \( b \) is a parent of \( a \) in the interaction graph, and \( c \) attacks \( b \) attacks \( a \) means that \( c \) is a grandparent of \( a \). We need to define different sets of constraints:

1. **Conflict-free constraints.** Since we want to find the conflict-free sets, if \( R(a_i, a_j) \) is in the graph we need to prevent the solution to include both \( a_i \) and \( a_j \) in the considered extension: \( \neg(a_i = 1 \land a_j = 1) \). The other possible assignment of the variables \( (a = 0 \land b = 1), (a = 1 \land b = 0) \)
and \((a = 0 \land b = 0)\) are permitted: in these cases we are choosing only one argument between the two (or none of the two) and thus, we have no conflict.

2. **Admissible constraints.** For the admissibility, we need that if child argument \(a_i\) has a parent \(a_p\), but \(a_i\) has no grandparent \(a_g\) (parent of \(a_p\)), then we must avoid to take \(a_i\) in the extension because it is attacked and it cannot be defended by any grandparent: this can be expressed with a unary constraint \(a_i = 0\).

Moreover, if \(a_i\) has several grandparents \(a_{g_1}, a_{g_2}, \ldots, a_{g_k}\) and a parent \(a_p\) (which is the child of \(a_{g_1}, a_{g_2}, \ldots, a_{g_k}\)), we need to add a \(k + 1\)-ary constraint \(\neg (a_i = 1 \land a_{g_1} = 0 \land \cdots \land a_{g_k} = 0)\). The explanation is that at least one grandparent must be taken in the admissible set, in order to defend \(a_i\) from its parent \(a_p\). Notice that, if an argument is not attacked (i.e., it has no parents), it can be taken or not in the admissible set.

3. **Complete constraints.** To compute a complete extension \(\mathcal{B}\), we impose that each argument \(a_i\) which is defended by \(\mathcal{B}\) is in \(\mathcal{B}\), except those \(a_i\) that, in such case, would be attacked by \(\mathcal{B}\) itself [22]. This can be enforced by imposing that for each \(a_i\) taken in the extension, also all its \(k\) grandchildren \(a_{s_1}, a_{s_2}, \ldots, a_{s_k}\) (i.e., all the arguments defended by \(a_i\)) whose parents are not taken in the extension, must be in \(\mathcal{B}\). Formally, we enforce the assignments \((a_i = 1 \land a_{s_1} = 1 \land \cdots \land a_{s_k} = 1)\) only for those \(a_{s_i}\) for which it stands that \((a_{p_1} = 0 \land a_{p_2} = 0 \land \cdots \land a_{p_h} = 0)\), where \(a_{p_1}, a_{p_2}, \ldots, a_{p_h}\) are the \(h\) parents of \(a_{s_i}\).

4. **Stable constraints.** If we have an argument \(a_i\) with \(k\) parents \(a_{p_1}, a_{p_2}, \ldots, a_{p_k}\), we need to add the constraint \(\neg (a_i = 0 \land a_{p_1} = 0 \land \cdots \land a_{p_k} = 0)\). In words, if an argument is not taken in the extension (i.e., \(a_i = 0\)), then it must be attacked by at least one of the taken arguments: at least one parent of \(a_i\) needs to be taken in the extension (i.e., \(\exists j \in 1..k. a_{p_j} = 1\)). Moreover, if an argument \(a_i\) has no parent in the graph, it has to be included in the stable extension; notice that \(a_i\) cannot be attacked by arguments inside the extension, since it has no parent. The corresponding unary constraint is \(\neg (a_i = 0)\).

The following proposition states the equivalence between solving an \(AF_S\) and its related CSP.

**Proposition 4 (Solution equivalence).** Given an \(AF = \langle \mathcal{A}_{rgs}, R \rangle\), the solutions of the related CSP (see Section 3) \(P = \langle \mathcal{A}_{rgs}, \{0, 1\}, C \rangle\) correspond to all the

- conflict-free extensions by using \(C = \{\text{conflict-free}\}\) constraints,
- admissible extensions by using \(C = \{\text{conflict-free} \cup \text{admissible}\}\) constraints,
• complete extensions by using $C = \{\text{conflict-free} \cup \text{admissible} \cup \text{complete}\}$ constraints,

• stable extensions by using $C = \{\text{conflict-free} \cup \text{stable}\}$ constraints.

**Grounded and preferred extensions.** Concerning the other two classical semantics of Dung, i.e., the grounded and preferred ones, the solutions are obtained through two different steps: i) first, all the complete (for the grounded case) and admissible (for the preferred case) extensions are obtained by solving the corresponding constraints given in Proposition 4, ii) then these solutions are copied into a second CSP, where each variable has JaCoP type `SetVar`, that is defined as an ordered collection of integers. For instance, given $\mathcal{A}_{\text{args}} = \{a, b, c, d, e\}$, the admissible extension $\{a, c, d\}$ is translated into a `SetVar` variable $\{1, 0, 1, 1, 0\}$. Then we add a constraint $\text{AinB}(X, Y)$ for each couple of these variables, which checks if variable $X$ is contained into variable $Y$; if this is true for at least one $Y$, then $X$ cannot be a preferred extension, otherwise, it corresponds to a preferred extension. Viceversa, if we first find all the complete extensions and then we impose a constraint $\text{AinB}(X, Y)$ for each couple of variables, if $X$ is contained in each $Y$, this means that $X$ is a grounded extension.

**Hard problems related to preferred extensions.** An interesting problem is determining whether a set of arguments $T$ is a preferred extension, which is a co-NP-complete [22] problem. In ConArg we explicitly offer to the user the opportunity to solve this problem as a CSP, which is made of less constraints than the one that searches for all the preferred extensions. In this particular case, i) we still find all the admissible extensions, ii) but after this we impose a constraint $\text{AinB}(T, Y)$ for each admissible solution $Y$.

**Semi-stable, stage and ideal extensions.** The solution of these three extensions involves the computation of, respectively, all the complete, conflict-free, and admissible/preferred extensions. For instance, to find semi-stable extensions, we need to add conflict-free, admissible, and complete constraint classes to the problem, as defined in Proposition 4. Furthermore, we need to add the constraints limiting an extension $\mathcal{E}$ according to its range, defined as $\mathcal{E} \cup \mathcal{E}^+$, where $\mathcal{E}^+ = \{a \in \mathcal{A}_{\text{args}} : \mathcal{E} \text{ attacks } a\}$ (see Section 2). In order to find the range of an extension, we add $|\mathcal{A}_{\text{args}}|$ new `Integer` variables, which are set to 1 if the represented argument is attacked by at least one argument taken in the complete extension. This is achieved by using the JaCoP conditional constraint `IfThenElse`, whose guard is represented by an `Or` constraint (true if one of the parents is taken in the complete extension), and which sets the value of the new variables to 1 or 0 by using the `XeqC` constraint (variable equals to constant value). Then, all the obtained solutions are translated into `SetVar` variables, and maximality (w.r.t. set inclusion) is treated as for the preferred case, i.e., by solving a second CSP.

The same procedure is used for stage extensions as well, this time using admissible extensions as groundwork, instead of complete ones. Concerning the ideal semantics, we
• first find all the admissible extensions, and afterwards, by elaborating on these results, we find all the preferred extensions through the second step of the same CSP (preferred extensions are also admissible).

• Subsequently, we define a second CSP to check the precondition of the ideal semantics, that is if an admissible extension is subset of all the preferred extensions (see Def. 7 in Section 2). This second CSP receives the admissible and preferred extensions as input, obtained in the first CSP. In this second CSP, we impose that an admissible extension cannot be considered if it has an argument that is not taken in all the preferred extensions: in this way, we select only the admissible extensions that are subsets of the intersection of all the preferred extensions. To achieve this, we impose conflict-free and admissible constraint classes in order to find admissible solutions (see Proposition 4), but we also impose conditional IfThenElse constraints to exclude admissible extensions that contain an argument which is not included in the intersection of all preferred extensions: And constraints are used as guards, being “false” if an argument is not set to 1 in all the preferred extensions. If this happens, a XeqC forces the exclusion of that argument from the solution (i.e., it is set to 0).

• Eventually, we deal with maximality (w.r.t. set inclusion) by translating the results of the second CSP into a third CSP, and applying the same solution adopted above for preferred/semi-stable/stage extensions. The solution of this third CSP corresponds to the ideal semantics.

Additional user-defined constraints. Notice that we can easily impose further requirements on the sets of arguments which are expected as extensions, like "extensions must contain argument a when they contain b" or "extensions must not contain one of c or d when they contain a but do not contain b" [23]. For example, with JaCoP it is straightforward to model this kind of side-requirements with conditional constraints as IfThen(c1, c2), where constraint c2 (e.g., extension contains argument a, that is a = 1) must be satisfied if c1 is satisfied (e.g., extension contains argument b, that is b = 1). For the second example above, c1 corresponds to a = 1 ∧ b = 0 and c2 corresponds to (c = 1 ∨ d = 1) ∧ ¬(c = 1 ∧ d = 1).

To conclude, we remind that ConArg can solve all the problems presented in this section, among others. In Section 5, we propose a general parametrical framework where to express WAFs [18].

5. Expressing Weighted AFs with Semirings

There have been a number of proposals for extending Dung framework [5] in order to allow for more sophisticated modeling and analysis of conflicting information. A common theme among some of these proposals is the observation that not all arguments are equal, and that the relative strength of the arguments needs to be taken into account somehow [24, 25, 2, 26, 4, 27]. WAFs extend
Dung-style abstract argumentation systems by adding numeric weights to every edge in the attack graph, intuitively corresponding to the strength of the attack, or equivalently, how reluctant we would be to disregard it. In literature, we can find preferences directly associated with arguments [4] or, more frequently, with attacks [25, 24, 2, 26, 27]. In this work we focus on weights associated with the attack relationships.

For example, in Figure 3 we represent a weighted interaction graph with three contradictory arguments about weather forecasts announced by BBC and CNN:

\( a_1 \): Today will be dry in London since BBC forecast sunshine.
\( a_2 \): Today will be wet in London since CNN forecast rain.
\( a_3 \): BBC is more accurate than CNN.

Therefore, we consider the following AF: \( \mathcal{A}_{rgs} = \{ a_1, a_2, a_3 \}, a_1 Ra_2, a_2 Ra_1 \) and \( a_3 Ra_2 \). In Figure 3 each of these three attack relationships is associated with a fuzzy weight (in \([0,1]\)) representing the strength of the attack, where 0 represents the strongest possible attack, and 1 the weakest one.

In the following we report how some works in literature can be cast into the same parametrical semiring-based framework presented in Section 3.1.

An argument can be seen as a chain of possible events that makes the hypothesis true [27]. The credibility of a hypothesis can then be measured by the total probability that it is supported by arguments. To solve this problem we can use the Probabilistic semiring \([0,1], \text{max}, \hat{\times}, 0, 1\) (see Section 3.1), where the arithmetic multiplication (i.e., \(\hat{\times}\)) is used to compose the probability values together (assuming that the probabilities being composed are independent). In [27] the authors associate probabilities with arguments and defeats. Then, they compute the likelihood of some set of arguments appearing within an arbitrary argument framework induced from the probabilistic framework. Weights can be also interpreted as subjective beliefs [9, 2]. For example, a weight of \( w \in (0,1) \) on the attack of argument \( a_1 \) on argument \( a_2 \) might be understood as the belief that (a decision-maker considers) \( a_2 \) is false when \( a_1 \) is true. This belief could be modeled using probability [9, 2] as well.

The Fuzzy Argumentation approach presented in [28] enriches the expressive power of the classical argumentation model by allowing to represent the relative strength of the attack relationships between arguments, as well as the degree to which arguments are accepted. In this case, the Fuzzy semir-
In addition, the Weighted semiring \(\langle \mathbb{R}^+ \cup \{\infty\}, \min, \hat{+}, \infty, 0 \rangle\) (where \(\hat{+}\) is the arithmetic plus) can model a generic “cost” for the attacks: for example, the number of votes in support of the attack \([9, 2]\), which consequently need to be minimized. Other possible interpretations of models that use the Weighted semiring are provided in \([9, 2]\): for instance, to rank the strengths of attacks in a relative way.

With the Boolean semiring \(\langle\{true, false\}, \lor, \land, false, true\rangle\) (see Section 3.1), we can cast the classic AFs originally defined by Dung [5] in the same semiring-based framework. Therefore, with a single parametrical semiring-based framework, we can capture the semantics of the different metrics used in literature by independent models. This leads to an unifying modeling framework, supported also by the solving techniques provided by (soft) Constraint Programming.

In the following of this section we rephrase all the classical definitions given in Section 2 in order to parametrize them with the notions of semiring and weighted attacks. We call these new extensions as \(\alpha\)-extensions, because they tolerate a level \(\alpha\) of attack-strength within the extension, while they attack the arguments outside the coalition with more strength. This is the philosophy we used in designing these \(\alpha\)-extensions.

The following definition rephrases the notion of WAF into semiring-based AF, called \(\mathcal{AF}_S\):

**Definition 8. (semiring-based AF)** A semiring-based Argumentation Framework \(\mathcal{AF}_S\) is a quadruple \(\langle \mathcal{A}_{rgs}, R, W, S \rangle\), where \(S\) is a semiring \(\langle A, +, \times, 0, 1 \rangle\), \(\mathcal{A}_{rgs}\) is a set of arguments, \(R\) the attack binary relation on \(\mathcal{A}_{rgs}\), and \(W : \mathcal{A}_{rgs} \times \mathcal{A}_{rgs} \rightarrow A\) is a binary function called the weight function. Given \(a, b \in \mathcal{A}_{rgs}\), \(\forall (a, b) \in R, W(a, b) = s\) means that \(a\) attacks \(b\) with a strength level \(s \in A\), the set of preference values of the semiring \(S\).

In Figure 4 we provide an example of a weighted interaction graph describing the \(\mathcal{AF}_S\) defined by \(\mathcal{A}_{rgs} = \{a, b, c, d, e\}\), \(R = \{(a, b), (c, b), (c, d), (d, c), (d, e), (e, e)\}\) with \(W(a, b) = 7, W(c, b) = 8, W(c, d) = 9, W(d, c) = 8, W(d, e) = 5, W(e, e) = 6\) and \(S = \langle \mathbb{R}^+ \cup \{\infty\}, \min, \hat{+}, \infty, 0 \rangle\) (i.e., the Weighted semiring).
Therefore, each attack function is associated with a semiring value that represents the “strength” of the attack between two arguments. We can consider the weights in Figure 4 as votes supporting the associated attack. A semiring value equal to the top element of the semiring 1 (e.g., 0 for the Weighted semiring) represents a no-attack relationship, not represented in Figure 4 to have a light notation. As a consequence of this, the bottom element of the semiring, i.e., 0 (e.g., ∞ for the Weighted semiring), represents the strongest attack possible.

In Def. 9 we define the strength of attack for a set of arguments that attacks an argument or another set of arguments; in the following, we will use the product symbol \( \prod \) in order to apply the \( \times \) operator of the semiring \( S \) on a sequence of semiring values:

**Definition 9. (attacks for sets of arguments)** Given an \( AF_S = (A_{rgs}, R, W, S) \), a set of arguments \( B \) attacks an argument \( a \) with a weight of \( k \), that is \( W(B, a) = k \), if \( \prod_{b \in B} W(b, a) = k \). A set of arguments \( B \) attacks a set of arguments \( D \) with a weight of \( k \), that is \( W(B, D) = k \), if \( \prod_{b \in B, d \in D} W(b, d) = k \).

In Def. 10 we redefine the notion of conflict-free set: conflicts can be now part of the solution until a cost threshold \( \alpha \) is met, and not worse: they are now called as \( \alpha \)-conflict-free solutions.

**Definition 10. (\( \alpha \)-conflict-free extensions)** Given an \( AF_S = (A_{rgs}, R, W, S) \), a subset of arguments \( B \subseteq A_{rgs} \) is \( \alpha \)-conflict-free iff \( W(B, B) \geq_S \alpha^2 \).

With respect to the \( AF_S \) in Figure 2 while the set \( \{a, b, c\} \) is not conflict-free in the crisp version of the problem because it includes the attacks between \( a \) and \( b \) and between \( c \) and \( b \), \( \{a, b, c\} \) is instead 15-conflict-free because \( W(a, b) \times W(c, b) = 15 \).

We now define two propositions that derive from Definition 10 and the properties explained in Section 3.1.

**Proposition 5.** If an extension is \( \alpha_1 \)-conflict-free, then the same extension is also \( \alpha_2 \)-conflict-free if \( \alpha_2 <_S \alpha_1 \).

For instance, \( \{a, b, c\} \) is also a 17-conflict-free because it is a 15-conflict-free and 17 < 15 in the Weighted semiring.

Definition 11 proposes the Dung’s stable extensions revisited in the semiring-based framework.

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\(^2\)In case of a partially ordered semiring, the \( \geq_S \) is replaced by \( \neq_S \). Similar considerations hold for the inequalities in the following of the text.
Definition 11. \((\alpha\text{-stable extensions})\) Given an \(AF_S = (\mathcal{A}_{rgs}, R, W, S)\), an \(\alpha\)-conflict-free set \(B\) is an \(\alpha\)-stable extension iff for each argument \(c \notin B\), \(W(B, c) <_S \alpha\).

For example, considering the problem in Figure 4 as unweighted (i.e., as a classical Dung AF), the set \(\{a, d\}\) corresponds to the only stable extension. This set is also a 4-stable extension, because it is 4-conflict-free since it is 0-conflict-free (see Proposition 5), and \(b, c, e\) are attacked by an element in \(\{a, d\}\) with a strength worse than 4, that is \(W(\{a, d\}, b) = 7\), \(W(\{a, d\}, c) = 8\), and \(W(\{a, d\}, e) = 5\). The extension \(\{a, d, e\}\) is instead 11-stable instead, since it is 11-conflict-free and the other arguments \(b\) and \(c\) are attacked by at least one argument in \(\{a, d, e\}\), i.e., \(aRb\) and \(dRc\).

Like in Section 3, the other \(\alpha\)-extensions rely upon the concept of defense, in this case, weighted defense:

Definition 12. \((\text{weighted-defense})\) Given an \(AF_S = (\mathcal{A}_{rgs}, R, W, S)\), an argument \(b \in \mathcal{A}_{rgs}\) is defended by a set \(B \subseteq \mathcal{A}_{rgs}\) (or, \(B\) defends \(b\)) iff \(\forall a \in \mathcal{A}_{rgs}\) such that \(aRb\), we have that \(W(a, b) > S W(B, a)\).

The set \(\{c\}\) in Figure 4 defends \(c\) because \(dRc\) and \(W(d, c) > S W(c, d)\), i.e., \((8 > S 9)\)\(^3\).

An \(\alpha\)-admissible set of arguments must be an \(\alpha\)-conflict-free set that weighted-defends all its elements. Formally:

Definition 13. \((\alpha\text{-admissible extension})\) Given an \(AF_S = (\mathcal{A}_{rgs}, R, W, S)\), an \(\alpha\)-conflict-free set \(B \subseteq \mathcal{A}_{rgs}\) is \(\alpha\)-admissible iff each argument in \(B\) is weighted-defended by \(B\).

Not considering weights in Figure 4 the admissible sets are: \(\{a\}, \{c\}, \{d\}, \{a, c\}, \{a, d\}\). The 1-admissible extensions are \(\{a\}, \{c\}\), and \(\{a, c\}\) instead: \(\{a\}\) because is not attacked by any other argument, \(\{c\}\) and \(\{a, c\}\) because \(c\) is able to weighted-defend itself from the attack performed by \(d\), i.e., \(W(d, c) > S W(c, d)\). As a further example, \(\{a, b, c\}\) is 15-admissible because it is 15-conflict-free, and \(c\) weighted-defends himself from \(d\), as explained before. All the 15-admissible extensions are \(\emptyset, \{c\}, \{c, e\}, \{a\}, \{a, c\}, \{a, c, e\}\), and \(\{a, b, c\}\).

Besides the \(\alpha\)-stable semantics, three semantics refining \(\alpha\)-admissibility can be introduced:

Definition 14. \((\alpha\text{-preferred, \(\alpha\text{-complete and \(\alpha\text{-grounded extensions})})\})\) An \(\alpha\)-preferred extension is a maximal (w.r.t. set inclusion) \(\alpha\)-admissible subset of \(\mathcal{A}_{rgs}\). An \(\alpha\)-admissible \(B \subseteq \mathcal{A}_{rgs}\) is an \(\alpha\)-complete extension iff each argument which is weighted-defended by \(B\) is in \(B\). The least (w.r.t. set inclusion) \(\alpha\)-complete extension is the \(\alpha\)-grounded extension.

\(^3\)In the Weighted semiring, \(>_S\) is equivalent to \(<\) over the Real numbers, in the Probabilistic and Fuzzy ones, \(>_S\) corresponds to \(>\) over the Real numbers in the interval \([0,1]\) (see Section 3.1).
Note that now, if we can disregard consistency at will, we can always take the whole \( A_{rgs} \) set as an admissible and then preferred extension: \( \{a, b, c, d, e\} \) in Figure 4 the 43-admissible extension of course maximal, i.e., it is also preferred.

In Def. 15 we redefine also the semi-stable semantics as proposed in [4]. According to [4], given \( a \in A_{rgs} \) and \( B \subseteq A_{rgs} \) we define \( a^+_{\alpha} \) as \( \{c \mid W(a, c) < S_{\alpha}\} \) and \( B^+_{\alpha} \) as \( \{c \mid W(B, c) < S_{\alpha}\} \).

**Definition 15.** (\( \alpha \)-semi-stable extension) Given \( AF_S = \langle A_{rgs}, R, W, S \rangle \) and \( B \subseteq A_{rgs} \), \( B \) is called an \( \alpha \)-semi-stable extension iff \( B \) is \( \alpha \)-complete and \( B \cup B^+_{\alpha} \), called the \( \alpha \)-range of \( B \), is maximal w.r.t. set inclusion.

Some classical [6, 5] properties still hold among the new \( \alpha \)-extensions:

**Theorem 5.1.** Given \( AF_S = \langle A_{rgs}, R, W, S \rangle \), with a semiring \( S = \langle A, +, \times, 0, 1 \rangle \), and an \( \alpha \in A \), then

1. every \( \alpha \)-complete is also \( \alpha \)-admissible.

2. every \( \alpha \)-preferred extension is also \( \alpha \)-complete.

3. an \( \alpha \)-grounded extension is contained in every \( \alpha \)-preferred one.

4. every \( \alpha \)-stable extension is also \( \alpha \)-semi-stable.

5. every \( \alpha \)-semi-stable extension is also \( \alpha \)-preferred.

**Proof.** 1) is trivially proved by definition (see Def. 13 and Def. 14). For point 2), if \( B \) is the maximal set such that each argument in \( B \) is weighted-defended by \( B \), then each argument which is weighted-defended by \( B \) is in \( B \) (i.e., \( B \) is \( \alpha \)-complete). 3) derives from 1) and from the definition of \( \alpha \)-grounded extension, which is the minimal (w.r.t. set inclusion) \( \alpha \)-complete extension. Concerning 4), by definition an \( \alpha \)-stable extension maximizes the \( \alpha \)-range (see Def. 15), and, therefore, it is also \( \alpha \)-semi-stable. To prove 5), let \( B \) be an \( \alpha \)-semi-stable extension. Suppose \( B \) is not an \( \alpha \)-preferred extension, then there exists a set \( B' \supseteq B \) such that \( B' \) is an \( \alpha \)-complete extension. It follows that \( B'^+ \supseteq B^+ \). Therefore, \( (B' \cup B^+)^+ \supseteq (B \cup B^+)^+ \). But then \( B \) would not be an \( \alpha \)-semi-stable extension, since \( B \cup B^+ \) would not be maximal, and this leads to a contradiction.

Corollary 5.2. The following general inclusion relationships hold between \( \alpha \)-extensions: \( \alpha \)-stable \( \subseteq \alpha \)-semi-stable \( \subseteq \alpha \)-preferred \( \subseteq \alpha \)-complete, and \( \alpha \)-grounded \( \subseteq \alpha \)-complete.

Theorem 5.3 relates the new \( \alpha \)-extensions to their counterpart in the classical Dung’s framework [5].
Theorem 5.3. Given a classical AF = $\langle A_{rgs}, R \rangle$ as defined in Def. 1 and any possible related $\alpha$-version of it AF$_S$ = $\langle A_{rgs}, R, W, S \rangle$, then

1. $1$-conflict-free extensions in AF$_S$ correspond to conflict-free ones in AF.
2. $1$-admissible extensions in AF$_S$ are a subset of admissible ones in AF.
3. $1$-complete extensions in AF$_S$ are a subset of complete ones in AF.
4. $1$-semi-stable extensions in AF$_S$ are equivalent to semi-stable ones in AF.
5. $1$-stable extensions in AF$_S$ correspond to stable ones in AF.
6. $1$-grounded extensions in AF$_S$ are a subset of grounded ones in AF.
7. $1$-preferred extensions in AF$_S$ are a subset of preferred ones in AF.

Proof. Concerning 1), a semiring value equal to the top element of the semiring (i.e., 1) represents a no-attack relationship, so $\alpha$-conflict-free extensions do not include any attack among their arguments (i.e., they are conflict-free [5]). 2) and 3) hold because the notion of weighted-defense (see Def. 12) implies the classical notion of defense (see Def. 3). 4) and 5) hold because, if the taken arguments attack all, or maximize, the arguments outside with a strength greater that 1 (i.e., they are $\alpha$-stable and $\alpha$-semi-stable), it means that they respectively are stable and semi-stable according to the not-weighted semantics [6, 5] hold. 6) and 7) can be respectively proved after 3) and 2). □

At last, note that the cartesian product of two semirings is still a semiring [11, 12], and this can be fruitfully used to describe multi-criteria constraint optimisation problems.

6. Mapping Weighted AF$_S$ to a SCSP.

In this section we propose a mapping from semiring-based AF, that is the AF$_S$ presented in Section 5, to semiring-based SCSPs (see Section 3.1), as we do in Section 4 for not-weighted AF [5]: in this way, we can find all the $\alpha$-extensions described in Section 5 as a solution of the corresponding SCSP.

Given an AF$_S$ = $\langle A_{rgs}, R, W, S \rangle$ over a semiring $S$ = $\langle A, +, \times, 0, 1 \rangle$ (see Section 5), we define a variable for each argument $a_i \in A_{rgs}$, that is $V =$
\{a_1, a_2, \ldots, a_n\} and each of these argument can be taken or not as an element of one of the \(\alpha\)-extensions; i.e., the domain of each variable is \(D = \{1, 0\}\): 1 when the element belongs to the \(\alpha\)-extension, 0 otherwise. Parent and child relationships among arguments are used in the following formulation as in Section [4] that is considering the corresponding weighted interaction-graph. To compute the different \(\alpha\)-extensions we need to define distinct sets of constraints:

1. \(\alpha\)-conflict-free constraints. Since we want to find \(\alpha\)-conflict-free extensions, if \(W(a_i, a_j) = s <_S 1\) \((s \in A)\) we need assign a \(s\) “cost” to the solution that includes both \(a_i\) and \(a_j\) in the considered \(\alpha\)-conflict free extension: \(c_{a_i, a_j}(a_i = 1, a_j = 1) = s\). For the other possible variable assignments \((i.e., (a = 0, b = 1)(a = 1, b = 0)\) and \((a = 0, b = 0))\), \(c_{a_i, a_j} = 1\), since no conflict is introduced in the extension.

2. \(\alpha\)-admissible constraints. For the admissibility, we need that, if a child argument \(a_i\) has a parent \(a_p\), but \(a_i\) has no grandparent \(a_g\), then we must avoid to take \(a_i\) in the extension because it is attacked and cannot be defended by any grandparent: this can be expressed with a binary constraint, \(c_{a_p, a_i}(a_p = 0, a_i = 1) = 0\), which is equal to 1 for the other assignments of \(a_p\) and \(a_i\). Note that, differently from crisp admissible constraints in Section [4] here the assignment \(c_{a_p, a_i}(a_f = 1, a_i = 1)\) is allowed (it has a preference value of 1) because we tolerate attacks inside an \(\alpha\)-extension.

Moreover, we need to add a \(k+1\)-ary constraint \(c_{a_1, a_2, \ldots, a_k}(a_1 = 1, a_{g_1} = X_1, \ldots, a_{g_k} = X_k)\) among an argument \(a_i\) and its \(k\) grandparents \(a_{g_i}\), where each \(X_i \in D = \{0, 1\}\), that is each grandparent can be taken in the \(\alpha\)-admissible set or not (0/1 respectively). The preference for this constraint is equal to 0 if

\[
\prod_{g_i \text{ for } i=1..k, \text{s.t. } X_i=1} W(a_{g_i}, a_p) >_S W(a_p, a_i)
\]

, or equal to 1 otherwise \((i.e., \text{if } \leq_S)\). In words, the constraint has a preference value of 0 if the composition of the attack-weights of the taken grandparents towards a parent \(a_p\) of \(a_i\) is weaker than the attack of \(a_p\) towards \(a_i\). This because, as defined in Definition [13] this composition has to be stronger or equal, according to the preference-ordering of the adopted semiring (concept of weighted-defense, see Definition [12]).

3. \(\alpha\)-complete constraints. To compute a complete extension \(B\), we impose that each argument \(a_i\) that is defended by \(B\) is in \(B\), except those \(a_i\) that, in such case, would be attacked by \(B\) itself [22]. This can be enforced by imposing that for each \(a_i\) taken in the extension, also all its \(k\) grandchildren \(a_{s_1}, a_{s_2}, \ldots, a_{s_k}\) \((i.e., \text{all the arguments defended by } a_i)\) whose parents are not taken in the extension, must be in \(B\). Formally, \(c_{a_i, a_{s_1}, \ldots, a_{s_k}}(a_i = 1, a_{s_1} = 1, \ldots, a_{s_k} = 1) = 1\) only for those \(a_{s_i}\) for which it stands that \((a_{p_1} = 0, a_{p_2} = 0, \ldots, a_{p_h} = 0)\), where \(a_{p_1}, a_{p_2}, \ldots, a_{p_h}\) are
the $h$ parents of $a_i$; otherwise, $c_{a_i, a_{i_1}, \ldots, a_{i_h}} = 0$. Notice that the condition of weighted-defense for $\alpha$-complete extensions is granted by imposing also $\alpha$-admissible constraints in the problem (see Proposition 6).

4. $\alpha$-stable constraints. If we have a child node $a_i$ with multiple parents $a_{f_1}, a_{f_2}, \ldots, a_{f_k}$, we need to add the constraint $c_{a_i, a_{f_1}, \ldots, a_{f_k}}(a_i = 0, a_{f_1} = 0, \ldots, a_{f_k} = 0) = 0$. In words, if a node is not taken in the extension (i.e., $a_i = 0$), then it must be attacked by at least one of the taken nodes, that is at least a parent of $a_i$ needs to be taken in the stable extension (that is, $a_{f_j} = 1$).

Moreover, if a node $a_i$ has no parent in the graph, it has to be included in the stable extension (notice $a_i$ cannot be attacked by nodes inside the extension, since he has no parent). The corresponding unary constraint is $c_{a_i}(a_i = 0) = 0$.

Proposition 6 shows how to find all the $\alpha$-extensions presented in Section 5 by using the proper classes of constraints to build the intended SCSP:

**Proposition 6. (Equivalence for $\alpha$-extensions)** Given a semiring-based Argumentation Framework $AF_S = \langle Args, R, W, S \rangle$ and the related SCSP (see Section 3.1) $P = \langle Args, \{0, 1\}, C, S \rangle$, the $\alpha$-consistent solutions of $P$ (see Section 3.1) corresponds to all the

- $\alpha$-conflict-free extensions by using $C = \{\alpha$-conflict-free$\}$ constraints,
- $\alpha$-admissible extensions by using $C = \{\alpha$-conflict-free $\cup \alpha$-admissible$\}$ constraints,
- $\alpha$-complete extensions by using $C = \{\alpha$-conflict-free $\cup \alpha$-admissible $\cup \alpha$-complete$\}$ constraints,
- $\alpha$-stable extensions by using $C = \{\alpha$-conflict-free $\cup \alpha$-stable$\}$ constraints.

These constraints have been implemented in JaCoP similarly as described for their not-weighted versions in Section 4. To deal with costs, differently from Section 4, we introduce a new $IntVar$ variable to represent the cost of an attack. In Figure 6 we present the JaCoP code we use to find $\alpha$-conflict-free extensions. The first IfThenElse constraint is used to specify the cost of an argument $a_i$ attacking $a_j$. This cost, which is saved in the new $IntVar$ variable $costArray[k]$, is equal to the cost of the attack between $a_i$ and $a_j$ (i.e., $attackCost[i, j]$) if both $a_i$ and $a_j$ are taken in the extension (i.e, the are both equal to 1). Otherwise (else branch) it is equal to the top preference of the semiring: in this case of Weighted semiring, it is equal to 0. The same constraint is repeated for each pair of attacking arguments in the weighted interaction graph. At last, the sum (i.e., $Sum$ constraint in Figure 6) of all these costs is computed in the variable $totalCost$, which is imposed to be less or equal to $\alpha$ (i.e., $XlteqC$ constraint Figure 6).
store.impose(new IfThenElse(new And(new XeqC(a[i], 1), new XeqC(a[j], 1)), new XeqC(costArray[k], attackCost[i,j]), new XeqC(costArray[k], 0)));

/* To impose that the total cost of the attacks is below threshold \( \alpha \) */
store.impose(new Sum(costArray, totalCost));
store.impose(new XlteqC(totalCost, alpha));

Figure 6: An example of JaCoP code to find \( \alpha \)-conflict-free extensions.

The conditions on the attack costs for the other classes of constraints, that is \( \alpha \)-admissible, \( \alpha \)-complete, and \( \alpha \)-stable ones, are managed in the same way: we add variables to represent the costs, and we constrain the value of the their sum to be less/equal than a threshold. As regards \( \alpha \)-grounded and \( \alpha \)-preferred extensions, minimality and maximality with respect set inclusion are solved as explained in Section 6 for their not-weighted version.

In ConArg we have implemented two different semirings from Section 3.1 that is the Weighted semiring \( \langle \mathbb{R}^+ \cup \{\infty\}, \min, \hat{+}, \infty, 0 \rangle \) and the Fuzzy semiring \( \langle [0..1], \min, \max, 0, 1 \rangle \). Therefore, it is possible find all the \( \alpha \)-extensions presented in this section according to these two different system of preferences. To conclude, we remind that ConArg can find all the \( \alpha \)-extensions in this section.

6.1. Weighted Grounded Extensions

ConArg is also able to solve hard problems related to the WAF formalism presented in [9, 2]. More precisely, we can find all the \( \beta \)-grounded extensions (see Section 2.1), and we can also give a solution to all the problems described in Proposition 1, Proposition 2, and Proposition 3 (see Section 2.1). We have decided to include also these problems because they are NP-complete (i.e., Proposition 1) or co-NP-complete (i.e., Proposition 2 and Proposition 3), and we can consequently take advantage of Constraint Programming [11] to tackle their inherent complexity.

In [9, 2], \( \beta \)-grounded extensions are computed as Dung’s classical grounded-extensions [5], but only after having removed from the WAF all the attacks whose strengths sum up to an inconsistency budget defined by a threshold \( \beta \). Therefore, from an original WAF and a given \( \beta \) we can obtain several derived WAFs, on which classical grounded extensions are computed.

As a result, in our implementation of ConArg we find \( \beta \)-grounded extensions exactly as described in Section 4 for classical grounded extensions, that is with complete, admissible and complete constraints, and then by checking the minimality with respect set inclusion. Threshold \( \beta \) is given as input from a user. To solve the problem explained in Proposition 1, we impose the value of the input argument \( a \) as equal to 1 (i.e., \( a \) must be present in the extension), by using JaCoP constraint \( XeqC \). Then we can state that the problem has a solution as soon as we find a \( \beta \)-grounded extension (containing \( a \)), or no solution otherwise. To solve the problem described in Proposition 2, we proceed in the same way as for Proposition 1 but we require that \( a \) is contained in each \( \beta \)-grounded
extension; in this case, the problem is positively solved. The problem in Proposition 3 is slightly more complex: to solve it, first we check that the input set \( L \) correspond to a \( \beta \)-grounded extension. Since we can solve minimality between a set and a set of sets (we do the same to find grounded extension, for example), afterwards we check if \( L \) w.r.t. all the other \( \beta \)-grounded extensions.

7. The Tool

In this section we briefly present the visual interface and all the options of ConArg\(^4\), our visual tool that generates interaction graphs and finds Dung’s extensions \(^5\) over it (see Section 2) by using Constraint Programming \(^10\). ConArg has been entirely programmed in the Java language using the NetBeans development environment\(^5\). ConArg can be downloaded as an archive file containing the .jar file of the project, and a directory with all the .jar files of the used third-party libraries.

To program and solve constraints we adopted the *Java Constraint Programming* library (JaCoP), which is a Java library that provides the user with *Finite Domain Constraint Programming* paradigm \(^10\). JaCoP provides different type of constraints: for example, the most commonly used primitive constraints, such as arithmetical constraints, equalities and inequalities, logical, reified and conditional constraints, combinatorial (global) constraints. It provides a significant number of (global) constraints to facilitate an efficient modeling. Finally, JaCoP defines also decomposable constraints, i.e., constraints that are defined using other constraints and possibly auxiliary variables. It also provides a modular design of search to help the user on specific characteristics of the problem being addressed.

\(^4\)Downloadable at https://sites.google.com/site/santinifrancesco/tools/ConArg.zip

\(^5\)http://netbeans.org/
The first window of the graphical interface of ConArg can be used to choose the interaction graph we want to adopt to solve our argumentation-related problems; it is depicted in Figure 7. To generate and work with these graphs we use the Java Universal Network/Graph Framework (JUNG) [31], a Java software library for the modeling, generation, analysis and visualization of graphs. With JUNG we are capable to generate directed graph, where nodes are considered as arguments, and edges as directed attacks.

It is possible to generate five different kinds of interaction graphs: from left to right in Figure 7 (top to bottom), it is possible to select:

i) a random Barabasi network with small-world properties [13]. In this case (and in ii)) it is also possible to select the desired number of arguments/nodes in the generated network. The JUNG library [31] implements a simple evolving scale-free random graph generator. At each time step, a new vertex is created and connected to existing vertices according to the principle of “preferential attachment” [13], whereby vertices with higher degree have a higher probability of being selected for attachment. At a given time-step, the probability $p$ of creating an edge between an existing vertex $v$ and the newly added vertex is $p = (\text{degree}(v) + 1)/(|E| + |V|)$. $|E|$ and $|V|$ are, respectively, the number of edges and vertices currently in the network.

ii) a random Kleinberg network with small-world properties [14]. Kleinberg adds a number of directed long-range random links to an $n \times n$ lattice network (vertices as nodes of a grid, undirected edges between any two adjacent nodes). Links have a non-uniform distribution that favors arcs to close nodes over more distant ones. In the implementation provided by JUNG [31], each node $u$ has four local connections, one to each of its neighbors, and in addition one or more long range connections to some node $v$, where $v$ is chosen randomly according to probability proportional to $d^\theta$ where $d$ is the lattice distance between $u$ and $v$ and $\theta$ is the clustering exponent, which can be specified by a user in the window in Figure 7. Note that the number of nodes, which can be selected in Figure 7, corresponds to $n$, leading to a total of $n \times n$ nodes in the final generated graph.

iii) the case-study interaction graph presented in [9, 2], in order to let a user compare the solutions of ConArg with the same solutions given in [9, 2].

iv) a textual description of the network, saved as a file with the .dl extension. In this way it is possible to import a user’s own network in ConArg. We decided to use the .dl extension because, in this way, we are capable to import examples generated and used in ASPARTIX [1]. This textual format is really easy to use, since, in its basic form, it only consists in a list of node and attack declarations: for example, the AF where $\mathcal{A}_{rgs} = \{a, b, c\}$ and $aRb, bRc$, consists in the file `arg(0).arg(1).arg(2).att(0,1).att(1,2)`.

v) the interaction graph represented in this paper in Figure 4, in order to check the correctness of the examples reported in Section 5.
Figure 8: The drop-down list shows all the possible problems that can be solved in ConArg.

All the generated graphs can be then also exported to the same .dl format used by ASPARTIX [2], but only in their not-weighted form. Therefore, it is possible to test ASPARTIX over the random graphs generated in case i) and ii). Since these two kinds of graph are randomly generated, successive generations with the same exact parameter result in different output networks.

In the same window (see Figure 7) it is possible to select the weights we can assign to attacks as well. Consequently, it is possible to mix previous options i)-v) with following options a)-c). From left to right in Figure 7 we can:

a) randomly assign to arcs/attacks a weight in the interval $[1 \ldots \max]$, where $\max$ is selected by a user before the generation (see Figure 7). These weights are then interpreted in the Weighted semiring $\langle \mathbb{R}^+ \cup \{\infty\}, \min, +, \infty, 0 \rangle$ (see Section 3.1).

b) randomly assign to attacks a weight in the interval $[0 \ldots 1]$. These weights are then interpreted in the Fuzzy semiring $\langle [0..1], \min, \max, 0, 1 \rangle$ (see Section 3.1).

c) generate no weight for the attack, in order to model classical Dung’s AF [3].

After the generation of the interaction graph, which becomes visible in the ConArg window together with the weights on the arcs (if required during the generation), it is possible to select the desired problem we want to solve. This is illustrated in the drop-down list visualised in Figure 8. The problems are
Figure 9: The drop-down list shows all the possible problems that can be solved in ConArg.

grouped by related topic, i.e., classical extensions (see Section 4), coalitional extensions (described in [32], but out of the scope of this paper), $\alpha$-extensions (see Section 6), and problems related to $\beta$-grounded extensions (see Section 6.1). If the generated graph is weighted, then it is only possible to solve problems related to $\alpha$-extensions and $\beta$-grounded extensions, while if the graph is not weighted, it is only possible to solve classical problems [5].

By clicking on a specific problem is then possible to be asked for additional information, as for example an $\alpha$ threshold for $\alpha$-extensions (see Section 5), or a $\beta$ threshold for $\beta$-grounded extensions [9, 2]. After the solutions are computed, a user can graphically browse all of them, where arguments/nodes taken in the solution (i.e., in the corresponding extension) are filled with color gray to be distinguished from arguments outside the extension.

In Figure 9 we show the eighth solution (out of thirteen) after having asked to find all the $\beta$-grounded extensions with $\beta$ equal to 6. The arguments/nodes taken in the extension correspond to argument id-numbers 3, 4, 6 and 7. The considered graph has been obtained by removing arcs corresponding to the attacks between 6 and 3, and 4 and 0, represented by the dotted lines in Figure 9. This because in [9, 2] it is possible to tolerate an inconsistency in the solution up to threshold $\beta$. The tolerated inconsistency corresponds to the sum of the weights on the removed arcs, which, in the case of Figure 9, is equal to 2 (as reported also in the figure).
Argumentation and social networks. In order to test ConArg over sensibly wide interaction-graphs (see Section 8), our attention has turned to random networks with small-world properties, as Barabasi [13] and Kleinberg [14] networks.

The reason is that social networks usually show a structure typical of small-world graphs [15]. A practical example can be the study of discussion fora or discussion groups, where the users post their arguments that may attack other users’ arguments. Everyday examples are online social platforms, such as Facebook, e-commerce sites, such as Amazon, and technical fora, such as TechSupport Forum, which support the unfolding of informal exchanges in the form of debates or discussions, amongst several users. It is acknowledged (e.g., in [17]) that computational argumentation could benefit these online systems by supporting a formal analysis of the exchanges taking place therein [16].

As far as we know, no in-depth study has already been accomplished on describing the specific small-world, or, more in general, network properties of interaction graphs in Argumentation. As a result, in ConArg we support the generation of small-world graphs according to two generic well-know kinds of properties (i.e., Barabasi and Kleinberg), and we leave the suggested elaboration to future work.

8. Performance Tests

The main goal of this section is to test ConArg. All the following experiments are commented together at the end of this section, in order to give a panoramic view over them. To the best of our knowledge, these tests represent the first attempt to find and tests Argumentation extensions in small-world networks.

To solve all the following problems we adopt a Depth First Search (DFS) algorithm [10]: this algorithm searches for a possible solution by organising the search space as a search tree. In every node of this tree a value is assigned to a domain variable and a decision whether the node will be extended or the search will be cut in this node is made. The search is cut if the assignment to the selected domain variable does not fulfil all constraints. Each time during the search, we select the variable that has most constraints assigned to it, and we assign to it a random value from its current domain: we use Most-ConstrainedStatic as the variable selection heuristic and IndomainSimpleRandom as the value selection heuristic, both natively offered by JaCoP. Using the MostConstrainedStatic heuristic means that, since we test the tool with small-world/scale-free networks, we first select the hub nodes of the graph during the search: nodes with more links are inspected before the other ones. Moreover, we set a timeout of 180 seconds to interrupt the search procedure and to report the number of solutions found only within that time threshold.

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[1] http://www.facebook.com
[2] http://www.amazon.com
[3] http://www.techsupportforum.comforums/
Figure 10: A small-world network with 40 nodes, generated with JUNG by using the BarabasiAlbertGenerator class [31] [33]. The big hubs that lead to the small-world property are mainly nodes 0, 1 and 2.

For the first round of experiments we use Barabasi networks, whose properties are explained in item a) of Section 7. An example of such random graphs with 40 nodes is shown in Figure 10. These results are shown in Table 1 and they are averaged over 10 different random networks with respectively 10, 20, 30, 32, 37, 40, 60 and 100 nodes/arguments each. When the problem implies an exhaustive search we report the number of found extensions (i.e., conflict-free, admissible, complete and stable). In parentheses we also show the time (in milliseconds) needed to complete the search; when at least one of the 10 random instances for each class exceeds the time threshold of 180 seconds, we highlight this by using * within parentheses. “Grounded” column in Table 1 only reports the number of milliseconds used to find the single solution, while “Check if preferred” column shows the time needed to check if a candidate extension is preferred or not (results are average over 20 candidate extensions for each of the 10 instances).

In order to study our implementation on different networks, we have repeated the same tests Kleinberg networks, explained in item b) of Section 7. We set a clustering coefficient of 0.5 for all the tests over this kind of network. An example of such graphs is shown in Figure 11. In Table 2 we report the performance collected with the same methodology as for Table 1. The “Grounded” and “Check if preferred” columns are not reported in Table 2 since the obtained performance are similar to Table 1.

In the successive experiments we test how ConArg behaves over WAFs, that is argumentation frameworks with weights labelling the attacks (see Section 2.1). We have executed some tests concerning α-conflict-free extensions (see Section 5). The results are show in Figure 12 and they report the number of 1 up
Table 1: We show the tests on eight different Barabasi networks with respectively 10, 20, 30, 32, 37, 40, 60 and 100 nodes/arguments. # tags identify the number of elements (e.g., nodes or extensions). In parentheses we also report the number of milliseconds needed to find all the solutions; the * tag means that the search for some of the 10 random instances has been interrupted after the predefined threshold of 3 minutes. “Grounded” and “Check if preferred” columns clearly report the time to solve the problem only.

Figure 11: A small-world network with 36 nodes generated with JUNG by using the KleinbergSmallWorldGenerator class [31, 13]. Differently from the network in Figure 10 here the small-world property is achieved through a two-dimensional grid structure and few long-distance links between nodes. No big hubs are present.
Table 2: We show the tests on six different *Kleinberg* networks with respectively 9, 25, 36, 49, 64 and 100 nodes/arguments (as a remind, these networks have \( n \times n \) nodes). The meaning of \# and * tags is the same as in Table 1.

| #Nodes(edges) | #Conf-free (ms) | #Adm. (ms) | #Compl. (ms) | #Stable (ms) |
|---------------|----------------|------------|-------------|-------------|
| 9 (45)        | 21 (4.25)      | 13 (5.58)  | 10 (4.03)   | 7 (1.54)    |
| 25 (125)      | 6,986 (596)    | 13,966 (106) | 533 (91)   | 82 (12)    |
| 36 (180)      | 354,513 (36,290) | 63,560 (7,260) | 10,856 (1,966) | 541 (80) |
| 49 (245)      | 1,418,333 (*)  | 1,163,836 (*) | 273,330 (76,863) | 5,370 (1,151) |
| 64 (320)      | 1,620,483 (*)  | 931,105 (*) | 687,358 (*) | 73,315 (19.019) |
| 100 (500)     | 618,484 (*)    | 591,537 (*) | 495,050 (*) | 515,615 (*) |

Figure 12: We show the number of 1 (from left to right) up to 5-conflict-free extensions found in *Kleinberg* networks with 16 (grouped on the left) and 36 (grouped on the right) nodes. Results are averaged on 10 different networks each.

In the following we list the global conclusions we collect from this section on performance:

- As a first remark, we notice that the number of Dung’s extensions strongly depends on the topology of the considered interaction graph, even if these networks show the same small-world phenomenon. In particular, the most apparent feature of Barabasi networks is that they always show one complete and one stable extensions (which coincide), whatever the network size is (see Table 1). Moreover, they always show a high number of conflict-free
and admissible extensions, which grows very quickly with the number of nodes. On the contrary, Table 2 shows that Kleinberg networks are “more balanced” in this sense, since we can find less conflict-free and admissible extensions, and up to hundreds of thousands complete and stable extensions. This is the reason why we think a deep study on the features of real argumentation networks is really important: their differences sensitively impact on the feasibility of working with them in an effective way.

• The second issue concerns the feasibility of working with argumentation networks itself. From Table 1 and Table 2 we can see that the number of conflict-free and admissible extensions explodes between 32-40 nodes for both Barabasi and Kleinberg networks. Admissible extensions explode after 37 nodes (see Table 1). It is still possible to easily work with networks of 49 and 64 nodes, considering complete and stable extension respectively (see Table 2). These are the “attention thresholds” that should be taken into account when working with such networks.

• Constraint Programming performs extremely well on “yes/no” argumentation problems. For instance, checking if an extension is preferred is always solved almost instantaneously (see Table 1), even if, as a remind, it is co-NP-complete problem [22]. However, as one could expect, Constraint Programming performs worse when it deals with the exhaustive enumeration of all the possible solutions, especially when the problem is loosely constrained, as for conflict-free extensions. Admissible, complete and stable extension represent a progressive refinement of conflict-free ones, through the addition of further constraints. In case of less constrained problems,
propagation techniques are less effective, and the search space is consequently wider. Even if any complete search-method has to face this sudden state explosion, we are confident that improving the search with additional (maybe ad-hoc) heuristics can lead to better performance: for instance, we can detect and remove symmetries (Chapter 10 in [10]), or add global constraints (Chapter 6 in [10]) related the structure of the network. Note that these (and other possible) improvements strongly depends on the topology of small-world networks.

• The presence of weights, that is in case of WAF, brings more performance degradation when the goal is to enumerate all the solutions. The reason is that a certain amount of conflict is tolerated (see Section 2.1 and Section. 5), so that more solutions satisfy the relaxed problem. As we can see in Figure 12 and Figure 13 the number of weighted extensions quickly increases as we allow for more tolerance: for instance, 4-grounded extensions in networks with 64 nodes (see Figure 13) are more than 922% of the corresponding 3-grounded extensions. Even larger proportions hold between 3-grounded and 2-grounded, and 2-grounded and 1-grounded extensions. We can suppose than anytime we increase the tolerance threshold by one, the number of extensions augments by one order of magnitude, barely for all the cases in Figure 13. Figure 12 shows that $\alpha$-conflict-free extensions (see Section. 5) rapidly increase in large networks, since, for instance, 1-conflict-free extensions are three order of magnitude more in networks with 36 nodes than in networks with 16 nodes. Their number increases less, but still considerably in networks with 36 nodes, if the tolerance threshold is raised (e.g., 1 to 2-conflict-free): around 1000 more extensions for every threshold increase of one unit (see Figure 12).

All the performance have been collected using a MacBook with a 2.4Ghz Core Duo processor and 4Gb 1067Mhz DDR3 of RAM.

8.1. Comparison with ASPARTIX [1]

The ASPARTIX tool [1, 33], which is based on Answer Set Programming (ASP), can be considered as the most complete and advanced system in literature for solving AFs and WAFs. ASPARTIX can be used not only to compute the standard extensions for classical argumentation frameworks defined by Dung [5], but also for preference-based AF’s (PAF’s) [24], value-based AF’s (VAF’s) [25] and bipolar AF’s (BAF’s) [34]. In the latter case it is also possible to compute, save, and complete extensions, as well as distinguish between the classical d-admissible (following Dung), s-admissible (for stable) and e-admissible (for closed) extensions, for which also the respective preferred extensions are available. Furthermore, ASPARTIX is able to provide encodings for semi-stable and ideal semantics.

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www.dbai.tuwien.ac.at/proj/argumentation/systempage/
In order to execute it, it is required to use an ASP solver like Gringo/Clasp\cite{10}\ or DLV\cite{11}. Recent advances in ASP systems, in particular, the metasp optimization frontend for the ASP-package Gringo/ClaspD provides direct commands to filter answer sets satisfying certain subset-minimality (or -maximality) constraints \cite{33}. Since we decided to compare the two tools by considering only admissible, complete and stable extensions, we opted for the DLV system, because we do not need any minimality/maximality optimisation on the considered classes of problems.

We decided to compare ASPARTIX and ConArg on three different problems with the same (averaged on 10) random Kleinberg networks: \textit{i}) finding all admissible extensions using 36 nodes, \textit{ii}) finding all complete extensions using 49 nodes, and \textit{iii}) finding all stable extensions using 64 nodes. We have chosen these problems because, as we can notice in Figure\cite{11} the are computationally demanding, but still solvable within the threshold of 3 minutes in ConArg.

In Figure\cite{14} we compare the different execution times of these problems, for both ASPARTIX and ConArg. To measure the time of ASPARTIX, we have used the \textit{OS X} terminal command “\textit{time}”. We have summed User and Sys times: User is the amount of CPU time spent in user-mode code (outside the kernel) within the process. Sys is the amount of CPU time spent in the kernel within the process. As for all the other tests, performance have been collected using a MacBook with a 2.4Ghz Core Duo processor and 4Gb 1067Mhz DDR3 of RAM. As we can see from the bars in Figure\cite{14} ConArg outperforms ASPARTIX on all the three proposed problems. Performance in time are improved by respectively \textit{i}) 74\%, \textit{ii}) 65\%, and \textit{iii}) 72\%.

9. Related Work

As far as we know, few systems have been proposed in literature to study AFs and (especially) WAFs from the computational point of view. To the best of our knowledge, the results presented in \cite{20,19} are among the first ones proposed on large problems, and the first ones using random networks showing small-world properties. By using ASPARTIX \cite{1}, the only other tests have been proposed in \cite{33}, where graphs ranging from 20 to 110 arguments are randomly generated. Two methods are used: the first generates arbitrary AFs and inserts for any pair (\textit{a}, \textit{b}) the attack from \textit{a} to \textit{b} with a given probability \textit{p}. The other method generates AFs with a \textit{n} × \textit{m} grid structure. The tested extensions are the preferred, semi-stable, stage and resolution-based grounded semantics \cite{33}. However, in this paper we have opted for testing our tool on small-world networks, since, in general, they show to be the most appropriate topology to represent social networks (see Section\cite{7}). Moreover, we also propose tests on WAFs and hard problems presented in \cite{9,2}.

\footnotesize{\textsuperscript{10} \url{http://potassco.sourceforge.net} \textsuperscript{11} \url{http://www.dlvsystem.com/dlvsystem/index.php/Home}
Figure 14: A time performance comparison between ASPARTIX \cite{1} and ConArg over three different problems: from left to right, finding all admissible extensions (in 10 networks of 36 nodes), finding all complete extensions (in 10 networks of 49 nodes), and finding all stable extensions (in 10 networks of 64 nodes). On the vertical axis we report the time (in seconds) needed to solve the problems.

For example, in \cite{9,2}, one of the main inspiration sources of this work (at least for what concerns WAFs), no solving mechanism is proposed to solve the problems presented in the paper. The focus is rather in defining the computationally hard problems, and proposing the related complexity proofs.

In \cite{35} the authors present GORGIAS-C, which is a system implementing a logic programming framework of argumentation that integrates together preference reasoning and constraint solving. The system computes answers to queries asked on a logic program with priorities on rules, and domain constraints on variables. GORGIAS-C is implemented as a modular meta-interpreter for its logic programs on top of Logtalk\footnote{http://logtalk.org} using SWI-Prolog\footnote{http://www.swi-prolog.org} and its “Constraint Logic Programming (Finite Domain)” library and has successfully been used with ECLIPS\footnote{http://eclipseclp.org} with CLP over reals. No computational results on problems related to AFs have been yet presented for this tool. Moreover, the system proposed in \cite{35} appears to be a more general framework for reasoning on multi-agent system, while our solution is more focused on the computational point of view.

In \cite{22} the authors associates to each subset $S$ of arguments a formula in propositional logic; then, $S$ is an extension under a given semantics if and only if the formula is satisfiable (i.e., they solve the problem with SAT \cite{36}). An extensive survey of the difference between SAT and CP can be found in \cite{36}; summarizing, CP is more expressive in the modelling phase: this allows to find
more complex semantics (e.g., grounded or semi-stable \cite{6} ones) and further user-defined constraints on classical semantics \cite{22}. In addition, in CP the user has the possibility to inform the solver about problem specific information and then to appropriately tune it, while in SAT there is usually little room and need for this parametrization. The modeling in \cite{22} does not include preferred, grounded or weighted extensions \cite{18,9,2}; furthermore, the encoding presented in \cite{22} has no practical implementation and performance tests.

In a very recent paper \cite{37}, the authors present how to encode AFs as CSPs. They show how to represent preferences over arguments as a (partial or total) preorder. In this work we have decided to model quantitative preferences instead of qualitative ones, even if qualitative preferences can be clearly cast in our semiring-based framework either. Moreover, while in \cite{21} some of the authors of this paper model the preferences over arguments, in this paper we associate weights with attacks instead, as also proposed in \cite{18,9,2}. In addition to \cite{37}, we provide a practical implementation of the constraint modelling (i.e., the ConArg tool) and performance tests.

The tool\textsuperscript{15} described in \cite{38} provides a demonstration of a number of basic argumentation components that can be applied in the context of multi-agent systems. These components include algorithms for calculating argumentation semantics, as well as for determining the justification status of the arguments and providing explanation in the form of formal discussion games. Thus, even in this case the problem is not challenged from the computational point of view.

The \textit{ASPARTIX} system \cite{1,33} is a tool for computing acceptable extensions for a broad range of formalizations of Dung’s AFs and generalisations thereof, e.g., value-based AFs \cite{25} or preference-based \cite{21}. \textit{ASPARTIX} relies on a fixed disjunctive \textit{Datalog} program which takes an instance of an argumentation framework as input, and uses the Answer-Set solver DLV for computing the type of extension specified by the user. However, \textit{ASPARTIX} is not able to solve weighted AFs, as well as other ASP systems \cite{39}. Since \textit{ASPARTIX} appears to be the most complete and frequently updated tool among the others, we selected it to be compared against ConArg in Section \ref{sec:experiments} where time performance are shown and commented for both systems.

Finally, in \cite{32} some of the authors of this paper extend classical AFs \cite{5} in order to deal with coalitions of arguments. The initial set of arguments is partitioned into subsets, or coalitions. Each coalition represents a different \textit{line of thought}, but all the coalitions show the same property inherited by Dung, e.g., all the coalitions in the partition are admissible (or conflict-free, complete, stable). Even this kind of problems based on coalitions can be solved by ConArg.

10. Conclusions and Future Work

We have presented ConArg, a constraint-based tool programmed in Java, which can solve several problems related to AFs and WAFs. In this way, we have

\footnote{http://heen.webfactional.com}
proposed an unifying computational framework for Argumentation problems, with strong mathematical foundations and efficient solving heuristics. Thanks to AI-based techniques, ConArg is able to efficiently solve some computationally hard extensions, as, for instance, the problems presented in [9, 2] related to $\beta$-grounded extensions.

The inspiration behind this work has been to study AFs/WAFs from the computational point of view, by developing a general common framework, then implementing the related tool and, finally, studying the problem by exploring the difficulties in practically solving such instances.

In addition, a second goal has been to link AFs/WAFs to small-world networks (see Section 7): all the tests in Section 8 have been performed over two different kinds of small-world networks. As far as we know, this corresponds to the first attempt in this direction. A comparison with ASPARTIX (see Section 8.1) shows that constraint solving techniques prove to be able to efficiently deal with large-scale problems. Practical applications of our ConArg may consist, for example, in automatically studying discussion fora [10] or social networks [10] in general, where arguments may be rated by users leading to the definition of WAFs, with different strength values associated with the attacks.

For the future we have many open issues. We would like to investigate the properties of interaction graphs, in order to reproduce the tests we have presented in this paper on real-world cases (not only generated in a random way). Therefore, we would like to set up our AFs/WAFs from real social networks, using real data. Close to this topic, we would also like to study the topology of real AFs, in order to further improve the performance of the tool with ad-hoc heuristics depending on the topology of the adopted networks.

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