Bound-states of D-branes in L-R asymmetric superstring vacua

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Abstract

We discuss bound-states of D-branes in truly L-R asymmetric and thus non-geometric Type II vacuum configurations with extended supersymmetry. We argue for their stability as a result of residual supersymmetry and coupling to R-R potentials surviving in the massless spectrum. We then identify the open string excitations of these L-R asymmetric BPS D-branes. Finally, we briefly comment on possible applications and extensions.
1 Introduction

D-branes represent the best known class of non-perturbative states in String Theory since they admit a world-sheet description in terms of open strings \([1, 2]\). They couple minimally to R-R potentials \([2, 3]\) and break half of the original supersymmetries of Type II superstrings.

D-branes have been more or less explicitly considered in a variety of geometric \([4, 5, 6, 7, 8]\) and non-geometric \([9, 10, 11, 12, 13, 14, 15, 16]\) contexts whereby the parent Type II theory admits some involution exchanging Left and Right movers \([18, 20, 19, 21, 22, 23, 24, 25]\).

Very recently the existence and properties of D-branes in genuinely L-R asymmetric and thus non geometric Type II vacuum configurations has started being investigated \([26, 27]\). These include asymmetric orbifolds \([28]\), free fermion constructions \([29, 30, 31]\) and covariant lattices \([32]\). As pioneered by Ferrara and Koumas \([33]\), these constructions allow the embedding of \(D = 4\) extended supergravities with \(\mathcal{N} = \mathcal{N}_L + \mathcal{N}_R\) supersymmetries in Type II superstrings. While all \(\mathcal{N}\) even cases except \(\mathcal{N} = 6\) admit, but by no means require, L-R symmetric descriptions, all odd cases and \(\mathcal{N} = 6\) require non L-R symmetric descriptions.\(^1\) Up to \(\mathcal{N}_L \leftrightarrow \mathcal{N}_R\) interchange, the list of possibilities reads:\(^2\)

\[
\begin{align*}
\mathcal{N} = 8 & \quad \leftrightarrow \quad \mathcal{N}_L = 4, \quad \mathcal{N}_R = 4 \\
\mathcal{N} = 6 & \quad \leftrightarrow \quad \mathcal{N}_L = 2, \quad \mathcal{N}_R = 4 \\
\mathcal{N} = 5 & \quad \leftrightarrow \quad \mathcal{N}_L = 1, \quad \mathcal{N}_R = 4 \\
\mathcal{N} = 4 & \quad \leftrightarrow \quad \mathcal{N}_L = 2, \quad \mathcal{N}_R = 2 \quad \text{or} \quad \mathcal{N}_L = 0, \quad \mathcal{N}_R = 4 \\
\mathcal{N} = 3 & \quad \leftrightarrow \quad \mathcal{N}_L = 1, \quad \mathcal{N}_R = 2 \\
\mathcal{N} = 2 & \quad \leftrightarrow \quad \mathcal{N}_L = 1, \quad \mathcal{N}_R = 1 \quad \text{or} \quad \mathcal{N}_L = 0, \quad \mathcal{N}_R = 2 \\
\mathcal{N} = 1 & \quad \leftrightarrow \quad \mathcal{N}_L = 0, \quad \mathcal{N}_R = 1
\end{align*}
\]

When \(\mathcal{N}_L = 0\) or \(\mathcal{N}_R = 0\), all R-R fields are massive and we expect no BPS bound state of D-branes. Stable non-BPS D-branes with torsional K-theory charges may exist though \([34, 35, 36]\). In all other cases, massless R-R fields survive that must couple to some kind of D-branes. Since some extended supergravities can be obtained by spontaneous supersymmetry breaking in the presence of internal closed string fluxes \([37, 38]\), given

\(^1\)Excluding for the time being the possibility of enhanced supersymmetry (extra massless gravitini) from twisted sectors

\(^2\)\(\mathcal{N} = 7\) is equivalent to \(\mathcal{N} = 8\).
the uniqueness of the theories with $\mathcal{N} \geq 5$ and the rather rigid structure of
the theories with $\mathcal{N} = 3, 4$, it is tempting to conjecture some form of duality
between (D-branes in) non-geometric backgrounds and (D-branes in) geo-
metric flux compactifications. In fact one can turn the argument the other
way around. Given our limited knowledge as how to quantize string theory in
flux backgrounds \cite{39, 40, 41, 42}, one can use the non geometric world-sheet
construction as an equivalent ‘dual’ definition of the latter. Indeed duality
between geometric fluxes ($H_{ijk}$ 3-form and $T_{ij}^k$ torsion) and non geometric
ones ($Q_{ijk}$ and $R^{ijk}$) has been proposed and supported by some evidence
\cite{43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57}.

Clearly, once the correspondence has been established in highly supersym-
metric contexts as those we focus on in this note, it is natural to argue that
it hold in lower or non supersymmetric configurations, albeit with massless
R-R fields. The fate of L-R asymmetric D-branes in the absence of massless
R-R fields is an interesting problem that we will not address here.

The plan of this note is as follows. In section 2 we discuss the simplest
non-trivial case, $\mathcal{N} = 6$ supergravity, and identify the surviving massless
R-R vectors and the L-R asymmetric D-branes they couple to. We extend
the analysis to other L-R asymmetric realizations of extended supergravi-
ties in Section 3. Then, in Section 4, we construct the invariant boundary
states and discuss the relevant open string excitations. Section 5 contains
our conclusions and comments.

2 The $\mathcal{N} = 6$ case with $\mathcal{N}_L = 2$ and $\mathcal{N}_R = 4$

In order to illustrate our point let us start with the simplest non trivial case,
$\mathcal{N} = 6$ supergravity with 24 supercharges $\cite{33, 58}$. The highest dimension
where the classical theory can be defined is $D = 6$. However the resulting
$\mathcal{N} = (2, 1)$ supergravity is anomalous and thus inconsistent at the quantum
level \cite{59}. So we are led to consider $D = 5$. One starts with a toroidal
compactification and quotients it by a chiral $Z_2$ twist of the L-movers (‘T-
duality’ on four internal directions)

\[ X^i_L \to -X^i_L, \quad \Psi^i_L \to -\Psi^i_L, \quad i = 6, 7, 8, 9 \]  

\[ (2.1) \]

\footnote{String theory prevents quantum inconsistencies thanks to the presence of new massless
‘chiral’ (twisted) states whenever modular invariance or tadpole cancellation is imposed.}
accompanied by an order two shift compatible with modular invariance in order to make twisted states massive. In Type II theories, chiral supersymmetric twists such as (2.1) are anyway level-matched by themselves. For definiteness, we consider the Type IIB case. In the untwisted sector the one-loop torus partition function reads

$$T_u = \frac{1}{2} \{(Q_o + Q_v)\bar{Q}\Lambda_{5,0}^{[0]} + (Q_o - Q_v)(X_o - X_v)\bar{Q}\Lambda_{1,5}^{[0]}\} \tag{2.2}$$

where $X_o - X_v = 4\eta^2/\theta_2^2$ encodes the effect of the $\mathbb{Z}_2$ projection on four internal L-moving bosons,$$
\Lambda_{l,r}^{[\alpha]} = \sum_{p_L,p_R} e^{i\pi[\alpha_L p_L - \alpha_R p_R]} q^{\frac{1}{8}(p_L + \frac{1}{2}h_L)^2} q^{\frac{1}{8}(p_R + \frac{1}{2}h_R)^2} \tag{2.3}$$

are (shifted) Lorentzian lattice sums of signature $(l, r)$ and $Q = V_8 - S_8$, $Q_o = V_4O_4 - S_4S_4$, $Q_v = O_4V_4 - C_4C_4$ with $O_n, V_n, S_n, C_n$ representing the characters of $SO(n)$ at level $\kappa = 1$. By modular transformations $S$ and then $T$ one finds the twisted sector

$$T_t = \frac{1}{2} \{(Q_s + Q_c)(X_s + X_c)\bar{Q}\Lambda_{1,5}^{[1]} + (Q_s - Q_c)(X_s - X_c)\bar{Q}\Lambda_{1,5}^{[1]}\} \tag{2.4}$$

where $X_s + X_c = 4\eta^2/\theta_3^2, X_s - X_c = 4\eta^2/\theta_3^2, Q_s = O_4S_4 - C_4O_4$ (‘massless’), $Q_c = V_4C_4 - S_4V_4$ (‘massive’) a.

Due to the shift, the massless spectrum receives contribution only from the untwisted sector. In $D = 5$ notation with $SO(3)$ little group one finds

$$(V_3 + O_3 - 2\Sigma_3) \times (\bar{V}_3 + 5\bar{O}_3 - 4\bar{\Sigma}_3) \rightarrow \tag{2.5}$$

$$(g + b + \phi)_{NS-NS} + 6A_{NS-NS} + 5\phi_{NS-NS} + 8A_{R-R} + 8\phi_{R-R} - \text{Fermi}$$

The hidden non-compact symmetry is $SU^*(6)$. After dualizing all massless 2-forms into vectors, the $15 = 7_{NS-NS} + 8_{R-R}$ vectors transform according to the antisymmetric tensor of $SU^*(6)$. The $14 = 1_{NS-NS} + 5_{NS-NS} + 8_{R-R}$ scalar moduli parameterize the space $M_{D=5}^{N=6} = SU^*(6)/Sp(6)$.

Reducing to $D = 4$ on another circle with or without further shifts, the massless spectrum is given by

$$(V_2 + 2O_2 - 2S_2 - 2C_2) \times (\bar{V}_2 + 6\bar{O}_2 - 4\bar{S}_2 - 4\bar{C}_2) \rightarrow \tag{2.6}$$

$$(g + b + \phi)_{NS-NS} + 8A_{NS-NS} + 12\phi_{NS-NS} + 8A_{R-R} + 16\phi_{R-R} - \text{Fermi}$$

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$^4$For $n$ odd $S_n$ coincides with $C_n$ and will be denoted by $\Sigma_n$. 
The hidden non-compact symmetry is $SO^\ast(12)$. After dualizing all massless 2-forms into axions, the $30 = 2_{NS-NS} + 12_{NS-NS} + 16_{R-R}$ scalar moduli parameterize the space $\mathcal{M}_{N=6}^{D=4} = SO^\ast(12)/U(6)$. The $16 = 8_{NS-NS} + 8_{R-R}$ vectors together with their magnetic duals transform according to the 32 dimensional chiral spinor representation of $SO^\ast(12)$.

In order to identify the conserved charges coupling to the surviving R-R and NS-NS graviphotons, it is convenient to first consider maximal $\mathcal{N} = 8$ supergravity in $D = 4$ obtained by compactification on $T^6$. The 12 NS-NS graviphotons couple to windings and KK momenta. Their magnetic duals to wrapped NS5-branes (H-monoples) and KK monopoles. The 32 R-R graviphotons (including magnetic duals) couple to (6) D1-, (6) D5- and (20) D3-branes.

The chiral $\mathbb{Z}_2$ projection from $\mathcal{N} = 8$ to $\mathcal{N} = 6$ eliminates 4 NS-NS vectors coupling to $p_L$ along the 4 twisted directions (so that $p'_L = 0$ implies $n'_i = m'_i$ along the directions $i = 6, 7, 8, 9$) and 8 R-R vectors thus ‘identifying’ different kinds of D-branes. Our aim is to make this statement more precise, i.e. to identify the bound states of D-branes allowed by the ‘T-duality’ quotient. Our proposal for the surviving $16_{R-R} = 2_{(1|5)} + 4_{(1|3)} + 6_{(3|3)} + 4_{(5|3)}$ D-brane charges is\footnote{We henceforth use an intuitive notation whereby the subscript indicates which D-branes appear in the bound-state carrying a particular R-R charge.}

\begin{equation}
q_1^a + \frac{1}{4!}\varepsilon_{ijkl}q_5^{aijkkl}, \quad q_1^i + \frac{1}{3!}\varepsilon_{jkl}q_3^{ijkl}, \quad q_3^{aij} + \frac{1}{2!}\varepsilon_{ijkl}q_5^{ijkl}, \quad q_5^{abijk} + \varepsilon_{ijk}q_3^{abl} (2.7)
\end{equation}

where $q_p^{\ldots}$ denote the ‘elementary’ Dp-brane charges.

In order to give further support to the above identification of R-R charges, we would like to show that a bound state of a D5 wrapped around the 4 twisted directions and one of the two untwisted directions and a D1 wrapped around the same circle preserve 1/3 of the susy of the background. The unbroken susy of $\mathcal{N} = 6$ are the ones satisfying

\begin{equation}
Q_L = \Gamma_{6789}Q_L (2.8)
\end{equation}

with no conditions on $Q_R$. For D5 wrapped along the twisted $T^d$ and one of the two circles, e.g. the one along the 4\textsuperscript{th} direction, the condition is

\begin{equation}
Q_R = \Gamma_{04}\Gamma_{6789}Q_L (2.9)
\end{equation}
using (2.8) one finds

$$Q_R = \Gamma_{04} Q_L$$

(2.10)

which is nothing but the condition for the supersymmetry preserved by a D1 along the ‘untwisted’ direction of the D5. Adding the D1 does not reduce supersymmetry any further: the bound-state is 1/3 BPS wrt the unbroken supersymmetry in the L-R asymmetric Type II background. It preserves 8 supercharges out of 24 surviving supercharges, since the 8 $Q_L$ completely determine the $Q_R$. Similar considerations apply to the other surviving bound states of D-branes. Each one preserves 1/3 of the 24 SUSY charges of $\mathcal{N} = 6$ supergravity.

A slightly different analysis applies to the BPS states carrying charges in the NS-NS sector. The $16_{NS-NS} = 8_{(11)}^e + 8_{(55)}^m$ surviving charges are

$$m^a_I, n^a_I, p^i_R = m^i_I + n^i_I ; \quad m^5_a, n^5_a, \quad \check{P}_{Ri} = m^5_i + n^5_i$$

(2.11)

where $n^I$ and $m^I$ denote windings and KK momenta with $I = (a, i) = 4, ..., 9$ and $a = 4, 5, i = 6, ..9$, while $n_{I,5}$ and $m_{I,5}$ denote H-monopoles (wrapped 5-branes) and KK momenta. In particular the two gravitini that together with their superpartners are rendered massive by the freely acting $\mathbb{Z}_2$ projection form a complex 1/2 BPS multiplet with mass equal to the KK momentum for $R_5 > \alpha'$.

There are many other superstring realizations of $\mathcal{N} = 6$ supergravity in $D = 4$. Given the uniqueness of the low-energy theory, they all share the same massless spectrum. One possibility is to break half of the L-moving supersymmetries by means of a $\mathbb{Z}_n$ chiral projection acting on 4 supercoordinates as

$$(Z^1, Z^2)_L \rightarrow (\omega Z^1, \omega^{-1} Z^2)_L, \quad (\Psi^1, \Psi^2)_L \rightarrow (\omega \Psi^1, \omega^{-1} \Psi^2)_L$$

(2.12)

with $\omega^n = 1$. In order to avoid massless twist with an order $n$ shift along the ‘untwisted’ directions $(Z^3_L, Z^i_R)$. The surviving NS-NS charges (8 electric and 8 magnetic) are as before, see (2.11). The surviving R-R charges (16 including both electric and magnetic, since a net separation cannot be made for them) are less intuitive to visualize. Later on we will offer a boundary state description. For the moment suffice it to say that they are bound states of different kinds of D-branes obtained one from the other by the action of the $\mathbb{Z}_n$ twists (chiral rotations) and shifts (non-geometric translations).

Yet another realization of $\mathcal{N} = 6$ supergravity in $D = 4$ has been proposed in [60]. It corresponds to a Type II compactification on the maximal
torus of $SU(3)^3$ with chiral $Z_3$ projection and no shift. The untwisted sector yields $\mathcal{N} = 5$ supergravity while the twisted sector produces the extra massless gravitino multiplet to complete the spectrum of $\mathcal{N} = 6$ supergravity. In this case, the untwisted sector produces only 6 NS-NS and 4 R-R graviphotons together with 8 NS-NS and as many R-R (pseudo)scalars. The missing massless states are contributed by the twisted sector and its conjugate. The 2 twisted NS-NS and 4 twisted R-R graviphotons couple to somewhat exotic charges. The former to twisted L-moving strings and 5-branes and the latter to bound states of L-R asymmetric ‘fractional’ D-branes.

3 Bound-states of D-branes and R-R Charges

We would now like to extend the previous analysis for $\mathcal{N} = 6$ to lower supersymmetric cases in $D = 4$. For simplicity, we start with models obtained by successive $Z_2$ chiral projections. Later on we will describe how to deal with $Z_n$ chiral twists and order $n$ shifts with $n \neq 2$. As already mentioned, we will not consider here cases with $\mathcal{N}_L = 0$ or $\mathcal{N}_R = 0$, typically but not necessarily involving $(-)^F L/R$ projections, since no massless R-R fields survive in these cases.

3.1 $\mathcal{N} = 5 = 1_L + 4_R$ case

For $\mathcal{N} = 5 = 1_L + 4_R$ using $Z_2^L \times Z_2^R$ which acts by T-duality along $T^4_{6789}$ and $T^4_{4589}$ combined with order two shifts, that eliminate massless twisted states, it is easy to see that the surviving $12_{NS-NS} = 6^e_{(1)I} + 6^m_{(5)I}$ charges in the NS-NS sector are

$$p^I_R = m^I_1 + n^I_1 \quad \hat{p}_{RI} = m_{5I} + n_{5I}$$

(3.1)

In the R-R sector one finds $6_{R-R} = 6^{(1533)} + 2^{(3333)}$ charges

$$q^I_{(1335)} = q^I_1 + \frac{1}{4!}\varepsilon_{ijkl}q^I_5q^I_{ijl}q^I_{k} + \frac{1}{3!}\varepsilon^{IJKL'}q^I_3q^J_3q^K_3q^L_3 + \frac{1}{3!}\varepsilon^{IJKL'}q^I_3q^J_3q^K_3q^L_3$$

(3.2)

where $i_1, j_1, k_1, l_1$ run over the four directions orthogonal to $T^2_I$ while $K', L'$ and $K'', L''$ run over the two sets of two directions orthogonal to $T^2_I$ and

$$q^I_{(3333)} = q^I_3q^I_{2I3} + \frac{1}{2!}\varepsilon^{I2I3}J_3J_2J_3q^I_3J_2J_3 + \frac{1}{2!}\varepsilon^{I2I3}J_3J_2J_3q^I_3J_2J_3 + \frac{1}{2!}\varepsilon^{I1I2}J_1J_2J_3q^I_3J_2J_3$$

(3.3)
Bound states of D-branes carrying the above charges are 1/5 BPS since they preserve 4 supercharges out of the 20 supercharges present in the background, i.e. those satisfying with \( \mathcal{Q}_L = \Gamma_{6789} \mathcal{Q}_L \) and \( \mathcal{Q}_L = \Gamma_{4567} \mathcal{Q}_L \). For instance for the bound state with charge \( q^{4}_{1235} \) the 4 residual supercharges are those satisfying \( \mathcal{Q}_R = \Gamma_{04} \mathcal{Q}_L \).

As in the \( \mathcal{N} = 6 \) case, a different analysis applies to BPS states carrying KK momenta or windings or their magnetic duals. However, at variant with the \( \mathcal{N} = 6 \), the three massive gravitini cannot form a single complex 2/5 BPS multiplet. One of them, together with its superpartners, should combined with string states which are degenerate in mass at the special rational point in the moduli space where the chiral \( \mathbb{Z}_2 \times \mathbb{Z}_2 \) projection is allowed.

In [33], “minimal” \( \mathcal{N} = 5 \) superstring solutions have been classified into four classes which correspond to different choices of the basis sets of free fermions or inequivalent choices of shifts in the orbifold language.

Due to the uniqueness of \( \mathcal{N} = 5 \) supergravity in \( D = 4 \), all models have the same massless spectrum, contributed by the untwisted sector. In addition to the graviton \( g_{\mu\nu} \) and 5 gravitini \( \psi_{\mu} \), one has 10 graviphotons \( A_{\mu} \), 11 dilatini \( \chi \) and 10 scalars \( \phi \). The latter parameterize \( \mathcal{M}^{D=4, \mathcal{N}=5} = SU(5,1)/U(5) \). The graviphotons together with their magnetic duals transform according to the 20 complex (3-index totally antisymmetric tensor) representation of \( SU(5,1) \).

It would be interesting to explore how different massive spectra could affect higher derivative terms in the superstring effective action. Notice that the 8 R-R axions and the NS-NS axion, dual to \( b_{\mu\nu} \) decouple from perturbative amplitudes. Only non-perturbative effects, due to ‘non-geometric brane wrappings’, could induce dependence on these fields as well as on the dilaton.

Two alternative superstring constructions with \( \mathcal{N} = 5 \) supergravity in the massless spectrum have been proposed in [60]. The first consists in a \( \mathbb{Z}_7 \) asymmetric orbifold of the maximal tours of \( SU(7) \). A twist (chiral rotation) of the L-movers \( \theta_L = (\omega_7, \omega_2^R, \omega_4^R) \) is accompanied by a shift (chiral translation) of the R-movers such that \( 7\sigma_R = (1, 2, -3, 0, 0, 0, 0, 0, 0) \). The second consists in a \( \mathbb{Z}_3 \) asymmetric orbifold of the maximal tours of \( SU(3)^3 \). A twist (chiral rotation) of the L-movers \( \theta_L = (\omega_3, \omega_3, \omega_3) \) is accompanied by a shift (chiral translation) of the R-movers such as \( 3\sigma_R = (1, 1, -1, 0; 1, -1, 0; 1, -1, 0) \).

In general one expects to get \( \mathcal{N} = 5 \) supergravity by means of a chiral

\[ ^6 \text{Using a 7-dim notation with } \sum_i x_i = 0. \]

\[ ^7 \text{Using a 9-dim notation with } \sum_i x_i^I = 0 \text{ for } I = 1, 2, 3. \]
$Z_n$ twist $\theta_L = (\omega^a_n, \omega^b_n, \omega^c_n)$ with $a + b + c = 0 \pmod{n}$ on a suitable (Lie algebra) lattice combined with a R shift, satisfying level matching and not belonging to $I^*$, the dual to the lattice invariant under $\theta_L$, to avoid massless twisted sectors that can contribute extra gravitini unless specific choices of phases (discrete torsions) are made \cite{61}.

### 3.2 $\mathcal{N} = 4 = 2_L + 2_R$ case

The $\mathcal{N} = 4 = 2_L + 2_R$ case allows both geometric (L-R symmetric) and non-geometric (truly L-R asymmetric) descriptions. In particular one can start by considering the combined effect of two T-duality projections $\mathbb{Z}_2^L \times \mathbb{Z}_2^R$ on $T^4$. Depending on the choice of discrete torsion $\epsilon = \pm$ and shifts $\sigma_V$ one can have various cases. In the absence of shifts, the model, though non-geometric, turns out to be L-R symmetric \cite{13}. For $\epsilon = +$, the Type IIB compactification enjoys $\mathcal{N} = (2, 2)$ ‘enhanced’ supersymmetry since massless gravitino multiplets appear in the twisted sectors. The resulting non-compact symmetry is $SO(5,5)$. For $\epsilon = -$ the Type IIB compactification enjoys $\mathcal{N} = (2, 0)$ supersymmetry. In addition to the supergravity multiplet (with 5 self-dual antisymmetric tensors), one has 21 $\mathcal{N} = (2, 0)$ tensor multiplets, 5 from the untwisted sector and 16 from the twisted sectors. The hidden non-compact symmetry of the model is $SO(5,21)$. Though non-geometric this compactification is topologically equivalent to a compactification on K3. The torus partition function involves the ‘diagonal’ modular invariant rather than the ‘charge-conjugation’ modular invariant \cite{13, 14}.

Reducing the model with $\epsilon = +$ to $D = 4$, one finds 4 untwisted NS-NS charges $n^i_1, m^i_1$ and their magnetic duals $n^a_3, m^a_3$. In the untwisted sector one finds only $8_{R-R} = 2_{(1|5)} + 6_{(3|3)}$ T-duality invariant R-R charges charges

$$q^a_3 + \frac{1}{4!} \varepsilon_{ijkl} q^{ijkl}_3, \quad q^{aij}_3 + \frac{1}{2!} \varepsilon_{ijkl} q^{ijkl}_3$$

(3.4)

In the absence of shifts, there are 16 additional R-R charges from the twisted sectors $q^f_3$. Including shifts, one can eliminate massless states in the twisted sectors and render the background genuinely asymmetric under the exchange of Left and Right movers.

Many more L-R asymmetric $\mathcal{N} = 4 = 2_L + 2_R$ models can be constructed \cite{33}. A systematic analysis is beyond the scope of this note.
### 3.3 $\mathcal{N} = 3 = 1_L + 2_R$ case

The simplest $\mathcal{N} = 3$ model has 3 matter vector multiplets and can be constructed in two steps \[33\].

The first steps consists in a ‘geometric’ $\mathbb{Z}_2$ freely acting orbifold (locally equivalent to $K^3 \times T^2$). The $\mathbb{Z}_2$ action combines a twist breaking $\mathcal{N} = 8 = 4_L + 4_R$ to $\mathcal{N} = 4 = 2_L + 2_R$ and a shift preventing new massless states from appearing in the twisted sector. The resulting massless spectrum consists in $\mathcal{N} = 4$ supergravity coupled to 6 matter vector multiplets. The scalar manifold is

$$
\mathcal{M}_{\mathcal{N}=4} = \frac{SO(6,6)}{SO(6) \times SO(6)} \times \frac{SL(2)}{U(1)}
$$

(3.5)

Using the notation introduced in Sect. 2, the one-loop partition function in the untwisted sector reads

$$
\mathcal{T}_{\mathcal{N}=4}^{\mathcal{N}=4} = \frac{1}{2} \left\{ |Q_o + Q_v|^2 \Lambda_{4,4} \Lambda_{2,2}^{[0]} + |Q_o - Q_v|^2 |X_o - X_v|^2 \Lambda_{2,2}^{[0]} \right\}
$$

(3.6)

Massless states are contributed by $|Q_o X_o|^2$ and $|Q_v X_o|^2$, that produce in all 12 massless vectors ($G_{\mu,4 \pm i5}$, $B_{\mu,4 \pm i5}^{(2)}$, $C_{\mu,4 \pm i5}^{(2)}$, $C_{\mu,4 \pm i5}^{(4)}$), while the 16 twisted sectors

$$
\mathcal{T}_{\mathcal{N}=4}^{\mathcal{N}=4} = \frac{16}{2} \left\{ |Q_s + Q_c|^2 |X_s + X_c|^2 \Lambda_{2,2}^{[1]} + |Q_s - Q_c|^2 |X_s - X_c|^2 \Lambda_{2,2}^{[1]} \right\}
$$

(3.7)

only contribute massive states because of the shift.

The second step consists in a non geometric (say Left-) projection combined with a shift along the orthogonal directions. The partition function in the untwisted sector reads

$$
\mathcal{T}_{\mathcal{N}=3}^{\mathcal{N}=3} = \frac{1}{2} \left\{ |Q|^2 \Lambda_{2,2}^{(4,5)} \Lambda_{2,2}^{(8,9)} + |Q_o - Q_v|^2 \Lambda_{2,2}^{(4,5)} \Lambda_{2,2}^{(8,9)} \right\}
$$

(3.8)

\[(Q_o - Q_v)^{(6,7)} \Lambda_{2,2}^{(8,9)} + (Q_o - Q_v)^{(8,9)} \Lambda_{2,2}^{(6,7)} \Lambda_{2,2}^{(8,9)} \right\}

where the superscript $(I, I + 1)$ denote the ‘untwisted’ directions in a given sector.

Twisted sectors only contribute massive states. The only massless states thus arise from

\[
(V_2 - S_2 - C_2) \times (\bar{V}_2 + 2\bar{S}_2 - 2\bar{C}_2) \rightarrow \\
(g + b + \phi)_{NS-NS} + 2A_{NS-NS} + 4\phi_{R-R} + 2A_{R-R} - \text{Fermi}
\]

(3.9)

9
and

\[(2O_2 - S_2 - C_2) \times (4\bar{O}_2 - 2\bar{S}_2 - 2\bar{C}_2) \rightarrow 8\phi_{NS-NS} + 4\phi_{R-R} + 2A_{R-R} - \text{Fermi} \tag{3.10}\]

and correspond to \(\mathcal{N} = 3\) supergravity coupled to three vector multiplets, as anticipated. In all, there are 2 NS-NS vectors and 4 R-R vectors. The scalar moduli parameterize \(\mathcal{M}^{D=4}_{\mathcal{N}=3} = SU(3,3)/SU(3) \times SU(3) \times U(1)\). Vector fields together with their magnetic dual transform according to the (complex) 6 of \(SU(3,3)\).

The surviving NS-NS charges are \(p^a_R\) with \(a = 4, 5\) and their magnetic duals \(\hat{P}^a_R\).

The R-R charges are the geometric charges of the parent \(\mathcal{N} = 4 = 2_L + 2_R\) theory left invariant by the double T-duality projection. Starting with

\[q_1^a, \quad q_5^0, \quad q_{3}^{aij}\] \(\tag{3.11}\)

one finds that the T-duality invariant combinations are simply

\[q_1^a + \frac{1}{3!} \varepsilon_{bcj} q_3^{bij}, \quad q_3^{aij} + \frac{1}{3!} \varepsilon_{bkl} q_5^{bijkl} \tag{3.12}\]

In \(\mathcal{N} = 3\) supergravity, there are no 1/2 BPS particle states. One can consider 1/3 BPS states, which descend from 1/2 or 1/4 BPS states in the ‘parent’ \(\mathcal{N} = 4\) theory \[63, 64\]. Bound-states of D-branes carrying the one of the above charges are 1/3 BPS the same is true for states carrying KK momenta, windings or their magnetic duals. Notice however that the gravitino which becomes massive in the breaking of \(\mathcal{N} = 4\) to \(\mathcal{N} = 3\) belongs to a long multiplet.

In \[33\] a complete classification of “minimal” \(\mathcal{N} = 3\) superstring solutions was given. Depending on the choice of fermionic sets, there are four classes with \(3 + 4K\) matter vector multiplets, with \(K = 0, 1, 2\), that give rise to some eleven sub-classes. Moreover with an extra chiral projection (ie splitting the geometric K3 into two chiral \(\mathbb{Z}_2\)) one can get models with \(1 + 2K\) matter vector multiplets, with \(K = 0, 1, 2\). In particular a model with only one vector multiplet, containing three complex scalars, is possible.

Another construction attributed to Narain in \[33\], is an asymmetric \(\mathbb{Z}_3\) projection with \(\theta = (\omega_3, \omega_3, \omega_3; 1, \omega_3, \omega_3^{-1})\) acting on the lattice of \(SU(3)^3\).
3.4 $\mathcal{N} = 2 = 1_L + 1_R$ case

We will be very brief about this case since, choosing geometric projections, it includes widely studied Calabi-Yau compactifications of Type II superstrings. In particular the geometric $T^6/\mathbb{Z}_2 \times \mathbb{Z}_2$ orbifold can be resolved to a CY 3-fold with Hodge numbers $h_{11} = 51$ and $h_{21} = 3$ or its mirror, related to one another by the choice of discrete torsion.

The $\mathcal{N} = 2 = 1_L + 1_R$ case can also be obtained by non-geometric $\mathbb{Z}_2^L \times \mathbb{Z}_2^R \times \mathbb{Z}_2^R$ projections, i.e. by adding a further $\mathbb{Z}_2^R$ projection onto the $\mathcal{N} = 3 = 1_L + 2_R$ case. One can either perform a specular projection that leads to a non geometric but L-R symmetric model that allows for further $\Omega$ projection [62] or perform a different $R$ projection leading to a genuinely L-R asymmetric model. Alternatively one can consider $\mathbb{Z}_2^L \times \mathbb{Z}_2$ with L-R asymmetric shifts acting on both K-K momenta and windings. With generic choices of the shifts, there are no massless twisted states and no NS-NS massless vectors. There are however 4 R-R vectors that together with their magnetic duals couple to the eight different kinds of D3-branes one can wrap around the internal ‘3-cycles’.

Another possibility is to consider $\mathbb{Z}_3^L \times \mathbb{Z}_3^R$ with L-R asymmetric shifts breaking directly $\mathcal{N} = 8 = 4_L + 4_R$ to $\mathcal{N} = 2 = 1_L + 1_R$. In this case only the R-R graviphoton survives in the untwisted sector and together with its magnetic dual it couples to D3-branes wrapped around the ‘3-cycles’ dual to the holomorphic and anti-holomorphic 3-forms.

Another interest class are the ‘magic’ supergravities recently constructed in [65, 61].

4 Open string excitations

A convenient vantage point for identifying the relevant open string excitations is to use the boundary state formalism [66, 67]. In this formalism D-branes are represented as coherent or rather ‘squeezed’ states of closed string harmonic oscillators. The transverse boundary-to-boundary (cylinder) amplitude reads

$$\hat{A}_{ab} = \langle B_a | \exp(-\pi \ell \mathcal{H}_a) | B_b \rangle,$$

where $a, b$ label the (different) boundary states.

---

8We put 3-cycles in quotes since the background is non-geometric.
For (obliquely) magnetized D9-branes [66, 68, 69, 70, 71, 72, 73] the contribution of the bosonic coordinates reads

\[
|B_a^{(X)} = \sqrt{\text{det}(G_a + F_a)} \exp(- \sum_{n>0} a_{-n}^i R_{ij} (F_a) \tilde{a}_{-n}^j)|0_a\rangle \tag{4.2}
\]

where

\[
R_a = \frac{1 - F_a}{1 + F_a} \tag{4.3}
\]
is the relative ‘rotation’ between Left and Right movers induced by the internal magnetic field. The zero-mode contribution is implicit in \( |0_a\rangle \) and consists in a sum over all \( p_L = -R_a p_R \). For non-compact directions \( p_L = p_R \).

The contribution of the fermionic coordinates to the boundary state is somewhat subtler. In the NS-NS sector, there are no fermionic zero-modes and one has

\[
|B_a, \eta\rangle_{\text{NS-NS}}^{(\psi)} = \exp(i \eta \sum_{n \geq 1/2} \psi^i_{-n} R_{ij} (F_a) \tilde{\psi}^j_{-n}) |\eta\rangle \tag{4.4}
\]

where \( \eta = \pm \) stands for possible GSO projections and choice of superghost picture. In the R-R sector, fermions admit zero-modes, that cancel the Born-Infeld action and replace it with the Wess-Zumino coupling

\[
|B_a, \eta\rangle_{\text{R-R}}^{(\psi, \beta, \gamma)} = \frac{1}{\sqrt{\text{det}(G_a + F_a)}} \exp(i \eta \sum_{n>0} \tilde{\psi}^i_{-n} R_{ij} (F_a) \psi^j_{-n}) |0_a, \eta\rangle \tag{4.5}
\]

where

\[
|0_a, \eta\rangle = U_{A\tilde{B}} (F_a) |A, \tilde{B}\rangle \tag{4.6}
\]

with

\[
U_{A\tilde{B}} (F_a) = \left[ A \text{Exp}(\frac{-F_{ij} \Gamma^{ij}}{2}) C \Gamma_{11} \frac{1 + \eta \Gamma_{11}}{1 + i \eta} \right]_{A\tilde{B}} \tag{4.7}
\]

where AExp means antisymmetrization of the vector indices of the \( \Gamma \) matrices.

Magnetized D-branes in L-R symmetric orbifolds have been considered in [74, 75]. The case of \( g_L = g_R^{-1} \), where \( g_L \) and \( g_R \) are the generators of the orbifold group \( \Gamma_{L,R} \) on L- and R- movers, was discussed in [14, 15, 16] and is equivalent to choosing a different (diagonal) modular invariant torus partition function. We would like to generalize the analysis to the case of
genuinely L-R asymmetric orbifolds in which $g_L \neq g_R$. The action of a L-R asymmetric rotation on the superstring coordinates imply
\[ g_L g_R |B, F_a⟩ = |B, F'_a⟩ \] (4.8)
where the action of the shift on the bosonic zero-modes is understood and the ‘transformed’ magnetic field $F'_a$ is such that
\[ R(F'_a) = R(g_L)R(F_a)R^t(g_R) \] (4.9)
For a $Z_{N_L}^L \times Z_{N_R}^R$ action, invariant boundary states would then be of the form
\[ |B, F⟩_g = \frac{1}{\sqrt{N_L N_R}} (1 + g_L + g_R + ... + g_L^{N_L-1} g_R^{N_R-1}) |B, F⟩ = \] \[ = \frac{1}{\sqrt{N_L N_R}} \sum_{l,r} |B, F_{(l,r)}⟩ \] (4.10)
where the ‘induced’ magnetic field $F_{(l,r)}$ is determined by the condition
\[ R(F_{(l,r)}) = R(g_L^l)R(F)R^t(g_R^r) \] (4.11)
Computing the amplitude
\[ \tilde{A}(F)_g = g \langle B, F | \exp(-\pi \ell \mathcal{H}_{cl}) | B, F⟩_g \] , (4.12)
one can easily identify the couplings of the invariant magnetized D-brane to the closed string states of the L-R asymmetric orbifold and check the BPS no-force condition. Performing a modular $S$ transformation one can then find the open string excitations. The typical term would be of the form
\[ \mathcal{A}_{g,h} = \Lambda(g,h)\mathcal{I}(g,h) \sum_{\alpha} c_{\alpha}^{CSO} \frac{\partial_\alpha (0)}{\eta^3} \prod_I \frac{\partial_\alpha (\epsilon_I(g,h)\tau)}{\vartheta_1 (\epsilon(g,h)\tau)} \] (4.13)
where $g = g_L g_R$, $h = h_L h_R$, $\Lambda(g,h)$ is the lattice invariant under $gh = g_L h_L g_R h_R$, while $\mathcal{I}(g,h)$ is the ‘intersection’ number counting the invariant discrete zero-modes and finally $\epsilon_I(g,h)$ are related to the eigenvalues of $gh = g_L h_L g_R h_R \rightarrow \text{diag}(e^{2i\epsilon_I(g,h)})$. The BPS condition in the transverse channel translates into the supersymmetry condition $\sum_I \epsilon_I = 0 \pmod{1}$.

Requiring integer multiplicities may put some additional constraints on the choice of $F$ and of the phases in the projection [14, 15, 16]. Due to the
relative ‘rotations’ among the various components in the invariant boundary state, both ‘twisted’ (non integer moded) and ‘untwisted’ (integer moded) open strings will appear in the spectrum of a single D-brane bound state. For L-R symmetric non geometric Type I compactifications this has already been observed in [12, 13, 17] and was anyway implicit in the systematic construction of [20, 21].

Let us illustrate the above procedure for the bound-state of D-branes in the \( \mathcal{N} = 5 \) model obtained by asymmetric \( \mathbb{Z}_3 \) projection on the torus of \( SU(3)^3 \) with shift along \( v = \alpha_1/3 \) which is not a lattice vector, while \( 3v \) is. Notice that prior to twists and shifts there are 27 boundary states associated to the 27 conjugacy classes of \( SU(3)^3 \). Let us denote these by \( \vec{r} = (r_1, r_2, r_3) \) with \( r_i = 0, 1, 2(\equiv -1 \text{mod} 3) \). The direct channel annulus amplitude between any two of these reads

\[
A_{\vec{r}, \vec{s}} = N_{\vec{r}, \vec{s}} \mathcal{X}_\vec{t}
\]

(4.14)

where \( \mathcal{X}_\vec{t} = (V_8 - S_8) \chi_{t_1} \chi_{t_2} \chi_{t_3} \) denote the super-characters, the only massless one being \( \mathcal{X}_0 = (V_8 - S_8) \chi_0 \chi_0 \chi_0 \). The fusion rule coefficients \( N_{\vec{r}, \vec{s}} \) simply reflect the \( \mathbb{Z}_3^3 \) selection rules of the center of \( SU(3)^3 \). The transverse channel amplitude is

\[
\tilde{A}_{\vec{r}, \vec{s}} = B_{\vec{r}} B_{\vec{s}}^{\dagger} \mathcal{X}_\vec{t}
\]

(4.15)

where the boundary reflection coefficients are simply given by

\[
B_{\vec{t}} = \frac{S_{\vec{t}}}{\sqrt{S_{\vec{t}}^0}} \rightarrow \frac{\omega_{\vec{t}}}{\sqrt{3^3}}
\]

(4.16)

In the language of magnetized branes the three boundary states per each \( T_{SU(3)}^2 \) correspond to branes with magnetic quantum number \( (n, m) = (1, 0), (-1, 1), (0, -1) \), which are T-dual to rotated branes. Clearly one has

\[
\mathcal{A}_{a,b}^{SU(3)} = \langle B_a | q^H | B_b \rangle = N_{ab} \epsilon^c \chi_c^{SU(3)}
\]

(4.17)

We are now ready to discuss the effect of twists and shifts. Since, in this case, we can represent the original boundary states as magnetized (or rotated) brane states, the surviving ‘regular’ bound-states are simply given by

\[
|B_a\rangle_{\mathbb{Z}_3 \mathbb{Z}_3} = \frac{1}{\sqrt{3}} (|B_a\rangle + \theta_L \sigma_R |B_a\rangle + \theta^2 \sigma^2 R |B_a\rangle)
\]

(4.18)
or more explicitly

\[
|B_a\rangle_{\mathbb{Z}_3^{L\neq R}} = \frac{1}{\sqrt{3}} \left[ \sum_{p_L = -R a p_R} |B; R_a; p_L| R \rangle \right] + \sum_{R_L^P = R a p_R} e^{2\pi i v_R p_R} |B; R_L R a; p_L| R \rangle + \sum_{R_L^{-1} p_L = -R a p_R} e^{-2\pi i v_R p_R} |B; R_L^{-1} R a; p_L| R \rangle
\]

(4.19)

where \( R_L \) is the (chiral) rotation of \( 2\pi/3 \) and \( v_R \) parameterizes the shift of order three.

Computing the self-overlap of an invariant boundary state, yields the annulus amplitudes in the closed string ‘tree-level’ channel where the coupling to the 4 surviving R-R graviphotons can be extracted. Performing a modular \( S \) transformation yields the open string ‘loop’ channel that reads

\[
\mathcal{A}_{\mathbb{Z}_3^{L\neq R}} = \frac{1}{6} \sum_{a,b \in \mathbb{Z}_3} \Lambda(a,b) \mathcal{I}(a,b) \sum_{\alpha} c_{\alpha}^{GSO} \prod_I \frac{\vartheta_\alpha(a\tau + b)}{\vartheta_1(a\tau + b)}
\]

(4.20)

where \( \Lambda(a,b) \) is the shifted lattice sum, appearing only for \( a = b = 0 \) and depending on the choice of boundary state, and \( \mathcal{I}(a,b) \) are multiplicities representing the number of ‘chiral’ fixed points, \( i.e. \mathcal{I}(0,0) = 1, \mathcal{I}(\pm 1,0) = 3 \) and \( \mathcal{I}(0,\pm 1) = 3 \).

In addition to the ‘regular’ bound-states of D-branes, the \( \mathcal{N} = 5 \) model obtained by asymmetric \( \mathbb{Z}_3 \) projection on the torus of \( SU(3)^3 \) should admit ‘fractional’ ones that would couple to massive twisted fields.

The above procedure can be straightforwardly generalized and implemented in L-R asymmetric orbifolds of ‘geometric’ superstring vacua whenever D-brane boundary states are known or computable.

5 Outlook

In this note, we have discussed bound-states of D-branes in genuinely L-R asymmetric and thus non-geometric Type II vacuum configurations with extended supersymmetry. These L-R asymmetric D-branes couple to the R-R graviphotons surviving in the massless spectrum. We have also shown that they preserve a fraction of the supersymmetries of the background and described a procedure to identify the relevant open string excitations thereof.
Boundary state techniques apply to general (rational) CFT and thus one can envisage the possibility of extending the present analysis to Gepner models [76, 77, 78, 79] or other abstract CFT’s combined in a L-R asymmetric fashion, after some twist and shift.

In addition to being interesting in their own right, since L-R asymmetric backgrounds, though promising, are largely unexplored and above all since their very existence looks rather counter-intuitive, L-R asymmetric D-branes may find concrete applications in Black Hole attractor solutions and their microstate counting. In particular, the boundary state description we have adopted for the identification of the open string excitations can be easily adapted to cases with several stacks of ‘intersecting’ L-R asymmetric D-branes combined with (surviving) fundamental string windings and KK momenta. Moreover the analogue of fractional D-branes should be present also in genuinely L-R asymmetric backgrounds. Some simple instances have already been proposed [17, 27].

In this note we have only considered bound-states of D-branes that couple to R-R graviphotons and are thus point-like objects (particle states) along the non-compact directions. Since the L-R asymmetric projections only act in the compact ‘directions’, it is almost trivial to construct bound-states of D-branes that are extended along some non-compact direction. In particular, invariant bound states of (magnetized) D9-branes and other lower dimensional branes are an obvious possibility. The existence of invariant bound-states of ‘intersecting’ or differently magnetized D-branes invading the non-compact spacetime directions calls for the existence of L-R asymmetric Ω-planes with opposite R-R charge and ‘tension’. Indeed given the similarity between crosscap and boundary states one might be tempted to simply consider invariant combinations of Ω-planes. If the naive guess is correct as for the L-R asymmetric D-branes one could start building an entire new class of ‘unoriented’ vacuum configurations where the L-R asymmetric twists and shifts fix many if not all of the closed string moduli [80] and the ‘intersecting’ L-R asymmetric unoriented D-brane account for the massless gauge and matter fields.

Acknowledgements

The author would like to acknowledge P. Anastasopoulos, F. Marchesano, J.F. Morales, G. Pradisi, N. Prezas, A. Sen and especially S. Ferrara for...
illuminating discussions and to thank A. Sagnotti and C. Angelantonj for valuable comments on the manuscript. The work has been supported in part by the European Community Human Potential Program under contract MRTN-CT-2004-512194, by the INTAS grant 03-516346, by MIUR-COFIN 2003-023852, and by NATO PST.CLG.978785.

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