Multiple Point Model and Phase Transition Couplings
in the Two-Loop Approximation of Dual Scalar Electrodynamics

Larisa V. Laperashvili, Dmitri A. Ryzhikh
Institute of Theoretical and Experimental Physics, B.Cheremushkinskaya 25,
Moscow 117 259, Russia
and
Holger Bech Nielsen
Niels Bohr Institute, Blegdamsvej 17-21,
Copenhagen 0, Denmark

The simplest effective dynamics describing the confinement mechanism in the pure gauge lattice U(1) theory is the dual Abelian Higgs model of scalar monopoles [1-3].

In the previous papers [4-6] the calculations of the U(1) phase transition (critical) coupling constant were connected with the existence of artifact monopoles in the lattice gauge theory and also in the Wilson loop action model [6]. In Ref.[6] we (L.V.L. and H.B.N.) have put forward the speculations of Refs.[4,5] suggesting that the modifications of the form of the lattice action might not change too much the phase transition value of the effective continuum coupling constant. In [6] the Wilson loop action was considered in the approximation of circular loops of radii \( R \geq a \). It was shown that the phase transition coupling constant is indeed approximately independent of the regularization method: \( \alpha_{\text{crit}} \approx 0.204 \), in correspondence with the Monte Carlo simulation result on lattice [7]: \( \alpha_{\text{crit}} \approx 0.20 \pm 0.015 \).

But in Refs.[2,3] instead of using the lattice or Wilson loop cut–off we have considered the Higgs Monopole Model (HMM) approximating the lattice artifact monopoles as fundamental pointlike particles described by the Higgs scalar field.

1. The Coleman-Weinberg effective potential for the Higgs monopole model

The dual Abelian Higgs model of scalar monopoles (shortly HMM), describing the dynamics of confinement in lattice theories, was first suggested in Ref.[3], and considers the following Lagrangian:

\[
L = -\frac{1}{4g^2} F_{\mu\nu}^2(B) + \frac{1}{2} |(\partial_\mu - i B_\mu) \Phi|^2 - U(\Phi), \quad \text{where} \quad U(\Phi) = \frac{1}{2} \mu^2 |\Phi|^2 + \frac{\lambda}{4} |\Phi|^4
\]  

(1)

is the Higgs potential of scalar monopoles with magnetic charge \( g \), and \( B_\mu \) is the dual gauge (photon) field interacting with the scalar monopole field \( \Phi \). In this model \( \lambda \) is the self–interaction constant of scalar fields, and the mass parameter \( \mu^2 \) is negative. In Eq.(1) the complex scalar field \( \Phi \) contains the Higgs (\( \phi \)) and Goldstone...
(χ) boson fields:

\[ Φ = φ + iχ. \] (2)

The effective potential in the Higgs Scalar ElectroDynamics (HSED) was first calculated by Coleman and Weinberg [8] in the one–loop approximation. The general method of its calculation is given in the review [9]. Using this method, we can construct the effective potential for HMM. In this case the total field system of the gauge \((B_μ)\) and magnetically charged \((Φ)\) fields is described by the partition function which has the following form in Euclidean space:

\[ Z = \int [DB][DΦ][DΦ^+] e^{-S}, \] (3)

where the action \(S = \int d^4xL(x) + S_{gf}\) contains the Lagrangian [1] written in Euclidean space and gauge fixing action \(S_{gf}\). Let us consider now a shift: \(Φ(x) = Φ_b + \hat{Φ}(x)\) with \(Φ_b\) as a background field and calculate the following expression for the partition function in the one-loop approximation:

\[ Z = \int [DB][D\hat{Φ}][D\hat{Φ}^+] \exp\{-S(B, Φ_b) - \int d^4x \frac{δS(Φ)}{δΦ(x)}|_{Φ=Φ_b} \hat{Φ}(x) + h.c.\}\]

\[ = \exp\{-F(Φ_b, g^2, µ^2, λ)\}. \] (4)

Using the representation [2], we obtain the effective potential:

\[ V_{eff} = F(φ_b, g^2, µ^2, λ) \] (5)

given by the function \(F\) of Eq.(4) for the constant background field \(Φ_b = φ_b = \text{const}\). In this case the one–loop effective potential for monopoles coincides with the expression of the effective potential calculated by the authors of Ref.[8] for scalar electrodynamics and extended to the massive theory (see review [9]). As it was shown in Ref.[8], the effective potential can be improved by consideration of the renormalization group equation (RGE).

2. Renormalization group equations in the Higgs monopole model

The RGE for the effective potential means that the potential cannot depend on a change in the arbitrary parameter — renormalization scale \(M\):

\[ \frac{dV_{eff}}{dM} = 0. \] (6)

The effects of changing it are absorbed into changes in the coupling constants, masses and fields, giving so–called running quantities.

Considering the RG improvement of the effective potential [8,9] and choosing the evolution variable as

\[ t = \log(\phi^2/M^2), \] (7)

we have the following RGE for the improved \(V_{eff}(φ^2)\) with \(φ^2 \equiv φ_b^2 [10]: \)

\[ (M^2 \frac{∂}{∂M^2} + \beta_λ \frac{∂}{∂λ} + \beta_g \frac{∂}{∂g^2} + \beta(µ^2) \frac{∂}{∂µ^2} - γφ^2 \frac{∂}{∂φ^2})V_{eff}(φ^2) = 0, \] (8)
where $\gamma$ is the anomalous dimension and $\beta_{(\mu^2)}$, $\beta_{\lambda}$ and $\beta_{g}$ are the RG $\beta$–functions for mass, scalar and gauge couplings, respectively. RGE \textsuperscript{[5]} leads to the following form of the improved effective potential \textsuperscript{[8]}:

$$V_{\text{eff}} = \frac{1}{2} \mu_{\text{run}}^2(t)G^2(t)\phi^2 + \frac{1}{4} \lambda_{\text{run}}(t)G^4(t)\phi^4.$$  \tag{9}$$

In our case:

$$G(t) = \exp\left[\frac{-1}{2} \int_0^t dt' \gamma(g_{\text{run}}(t'), \lambda_{\text{run}}(t'))\right].$$  \tag{10}$$

A set of ordinary differential equations (RGE) corresponds to Eq.\textsuperscript{[8]}:

\begin{align*}
\frac{d\lambda_{\text{run}}}{dt} &= \beta_{\lambda}(g_{\text{run}}(t), \lambda_{\text{run}}(t)), \tag{11} \\
\frac{d\mu_{\text{run}}^2}{dt} &= \mu_{\text{run}}^2(t)\beta_{(\mu^2)}(g_{\text{run}}(t), \lambda_{\text{run}}(t)), \tag{12} \\
\frac{dg_{\text{run}}^2}{dt} &= \beta_{g}(g_{\text{run}}(t), \lambda_{\text{run}}(t)). \tag{13}
\end{align*}

So far as the mathematical structure of HMM is equivalent to HSED, we can use all results of the scalar electrodynamics in our calculations, replacing the electric charge $e$ and photon field $A_\mu$ by magnetic charge $g$ and dual gauge field $B_\mu$.

The one–loop results for $\beta_{\lambda}(1)$, $\beta_{(\mu^2)}(1)$ and $\gamma$ are given in Ref.\textsuperscript{[8]} for scalar field with electric charge $e$, but it is easy to rewrite them for monopoles with charge $g = g_{\text{run}}$:

$$\gamma^{(1)} = -\frac{3g_{\text{run}}^2}{16\pi^2}. \tag{14}$$

\begin{align*}
\frac{d\lambda_{\text{run}}}{dt} &\approx \beta_{\lambda}^{(1)}(g_{\text{run}}^4 + 10\lambda_{\text{run}}^2 - 6\lambda_{\text{run}}g_{\text{run}}^2), \tag{15} \\
\frac{d\mu_{\text{run}}^2}{dt} &\approx \beta_{(\mu^2)}^{(1)} = \frac{\mu_{\text{run}}^2}{16\pi^2}(4\lambda_{\text{run}} - 3g_{\text{run}}^2), \tag{16} \\
\frac{dg_{\text{run}}^2}{dt} &\approx \beta_{g}^{(1)} = \frac{g_{\text{run}}^4}{48\pi^2}. \tag{17}
\end{align*}

The RG $\beta$–functions for different renormalizable gauge theories with semisimple group have been calculated in the two–loop approximation and even beyond. But in this paper we made use the results of Refs.\textsuperscript{[11]} and \textsuperscript{[12]} for calculation of $\beta$–functions and anomalous dimension in the two–loop approximation, applied to the HMM with scalar monopole fields. The higher approximations essentially depend on the renormalization scheme. Thus, on the level of two–loop approximation we have for all $\beta$–functions:

$$\beta = \beta^{(1)} + \beta^{(2)}, \tag{18}$$

where

$$\beta_{\lambda}^{(2)} = \frac{1}{(16\pi^2)^2}(-25\lambda^3 + \frac{15}{2}g^2\lambda^2 - \frac{229}{12}g^4\lambda - \frac{59}{6}g^6). \tag{19}$$
and

\[ \beta^{(2)}_{(\mu^2)} = \frac{1}{(16\pi^2)^2} \left( \frac{31}{12} g^4 + 3\lambda^2 \right). \]  

(20)

The gauge coupling \( \beta_g^{(2)} \)–function is given by Ref.[11]:

\[ \beta_g^{(2)} = \frac{g^6}{(16\pi^2)^2}. \]  

(21)

Anomalous dimension follows from calculations made in Ref.[12]:

\[ \gamma^{(2)} = \frac{1}{(16\pi^2)^2} \frac{31}{12} g^4. \]  

(22)

In Eqs.(18)–(22) and below, for simplicity, we have used the following notations: \( \lambda \equiv \lambda_{\text{run}}, g \equiv g_{\text{run}} \) and \( \mu \equiv \mu_{\text{run}}. \)

3. The phase diagram in the Higgs monopole model

Now we want to apply the effective potential calculation as a technique for the getting phase diagram information for the condensation of monopoles in HMM. If the first local minimum occurs at \( \phi = 0 \) and \( V_{\text{eff}}(0) = 0 \), it corresponds to the Coulomb–like phase. In the case when the effective potential has the second local minimum at \( \phi = \phi_{\text{min}} \neq 0 \) with \( V_{\text{eff}}^{\min}(\phi_{\text{min}}^2) < 0 \), we have the confinement phase. The phase transition between the Coulomb–like and confinement phases is given by the condition when the first local minimum at \( \phi = 0 \) is degenerate with the second minimum at \( \phi = \phi_0 \). These degenerate minima are shown in Fig.1 by the curve 1. They correspond to the different vacua arising in this model. And the dashed curve 2 describes the appearance of two minima corresponding to the confinement phases.

The conditions of the existence of degenerate vacua are given by the following equations:

\[ V_{\text{eff}}(0) = V_{\text{eff}}(\phi_0^2) = 0, \]  

(23)

\[ \frac{\partial V_{\text{eff}}}{\partial \phi}|_{\phi=0} = \frac{\partial V_{\text{eff}}}{\partial \phi}|_{\phi=\phi_0} = 0, \quad \text{or} \quad V_{\text{eff}}'(\phi_0^2) \equiv \frac{\partial V_{\text{eff}}}{\partial \phi^2}|_{\phi=\phi_0} = 0, \]  

(24)

and inequalities

\[ \frac{\partial^2 V_{\text{eff}}}{\partial \phi^2}|_{\phi=0} > 0, \quad \frac{\partial^2 V_{\text{eff}}}{\partial \phi^2}|_{\phi=\phi_0} > 0. \]  

(25)

The first equation (23) applied to Eq.(6) gives:

\[ \mu_{\text{run}}^2 = -\frac{1}{2} \lambda_{\text{run}}(t_0) \phi_0^2 G^2(t_0), \quad \text{where} \quad t_0 = \log(\phi_0^2/M^2). \]  

(26)

It is easy to find the joint solution of equations

\[ V_{\text{eff}}(\phi_0^2) = V_{\text{eff}}'(\phi_0^2) = 0. \]  

(27)

Using RGE (11), (12) and Eqs.(24)–(27), we obtain:

\[ V_{\text{eff}}'(\phi_0^2) = \frac{1}{4} (-\lambda_{\text{run}} \beta(\mu^2) + \lambda_{\text{run}} + \beta_\lambda - \gamma \lambda_{\text{run}}) G^4(t_0) \phi_0^2 = 0, \]  

(28)
\[ \beta_\lambda + \lambda_{\text{run}}(1 - \gamma - \beta_{(\mu^2)}) = 0. \]  
\begin{equation} \tag{29} \end{equation}

Substituting in Eq. (29) the functions \( \beta_\lambda^{(1)}, \beta_{(\mu^2)}^{(1)} \) and \( \gamma^{(1)} \) given by Eqs. (14)–(17), we obtain in the one–loop approximation the following equation for the phase transition border:

\[ g_\text{PT}^4 = -2\lambda_{\text{run}} \left( \frac{8\pi^2}{3} + \lambda_{\text{run}} \right). \]  
\begin{equation} \tag{30} \end{equation}

The curve (30) is represented on the phase diagram (\( \lambda_{\text{run}}; g_{\text{run}}^2 \)) of Fig. 2 by the curve ”1” which describes the border between the Coulomb–like phase with \( V_{\text{eff}} \geq 0 \) and the confinement one with \( V_{\text{eff}}^{\text{min}} < 0 \). This border corresponds to the one–loop approximation.

Using Eqs. (14)–(22), we are able to construct the phase transition border in the two–loop approximation. Substituting these equations into Eq. (29), we obtain the following phase transition border curve equation in the two–loop approximation:

\[ 3y^2 - 16\pi^2 + 6x^2 + \frac{1}{16\pi^2} (28x^3 + \frac{15}{2} x^2 y + \frac{97}{4} xy^2 - \frac{59}{6} y^3) = 0, \]  
\begin{equation} \tag{31} \end{equation}

where \( x = -\lambda_{\text{PT}} \) and \( y = g_{\text{PT}}^2 \) are the phase transition values of \( -\lambda_{\text{run}} \) and \( g_{\text{run}}^2 \). Choosing the physical branch corresponding to \( g^2 \geq 0 \) and \( g^2 \to 0 \), when \( \lambda \to 0 \), we have received curve 2 on the phase diagram (\( \lambda_{\text{run}}; g_{\text{run}}^2 \)) shown in Fig. 2. This curve corresponds to the two–loop approximation and can be compared with the curve 1 of Fig. 2, which describes the same phase transition border calculated in the one–loop approximation. It is easy to see that the accuracy of the 1–loop approximation is not excellent and can commit errors of order 30%.

According to the phase diagram drawn in Fig. 2, the confinement phase begins at \( g^2 = g_{\text{max}}^2 \) and exists under the phase transition border line in the region \( g^2 \leq g_{\text{max}}^2 \), where \( e^2 \) is large: \( e^2 \geq (2\pi/g_{\text{max}})^2 \) due to the Dirac relation:

\[ eg = 2\pi, \quad \text{or} \quad \alpha \tilde{\alpha} = \frac{1}{4}. \]  
\begin{equation} \tag{32} \end{equation}

Therefore, we have:

\[
\begin{align*}
g_{\text{crit}}^2 &= g_{\text{max}1}^2 \approx 18.61, & \tilde{\alpha}_{\text{crit}} &= \frac{g_{\text{crit}}^2}{4\pi} \approx 1.48, & \alpha_{\text{crit}} &= \frac{1}{4\tilde{\alpha}_{\text{crit}}} \approx 0.17 & \text{in the one–loop approximation}, \\
g_{\text{crit}}^2 &= g_{\text{max}2}^2 \approx 15.11, & \tilde{\alpha}_{\text{crit}} &= \frac{g_{\text{crit}}^2}{4\pi} \approx 1.20, & \alpha_{\text{crit}} &= \frac{1}{4\tilde{\alpha}_{\text{crit}}} \approx 0.208 & \text{in the two–loop approximation}.
\end{align*}
\]  
\begin{equation} \tag{33} \end{equation}

Comparing these results, we obtain the accuracy of deviation between them of order 20%.

The last result (33) coincides with the lattice values obtained for the compact QED by Monte Carlo method [7]:

\[ \alpha_{\text{crit}} \approx 0.20 \pm 0.015, \quad \tilde{\alpha}_{\text{crit}} \approx 1.25 \pm 0.10. \]  
\begin{equation} \tag{34} \end{equation}

Writing Eq. (13) with \( \beta_\mu \) function given by Eqs. (17), (18), and (21), we have the following RGE for the monopole charge in the two–loop approximation:

\[ \frac{dg_{\text{run}}^2}{dt} \approx \frac{g_{\text{run}}^4}{48\pi^2} + \frac{g_{\text{run}}^6}{(16\pi^2)^2}, \quad \text{or} \quad \frac{d\log \tilde{\alpha}}{dt} \approx \frac{\tilde{\alpha}}{12\pi} (1 + 3 \frac{\tilde{\alpha}}{4\pi}). \]  
\begin{equation} \tag{35} \end{equation}
The values for $g_{crit} = g_{max1,2}$ indicate that the contribution of two loops described by the second term of Eq. (33) is about 0.3, confirming the validity of perturbation theory.

In general, we are able to estimate the validity of two–loop approximation for all $\beta$–functions and $\gamma$, calculating the corresponding ratios of two–loop contributions to one–loop contributions at the maxima of curves 1 and 2:

\[
\begin{align*}
\lambda_{crit} &= \lambda_{max1}^{run} \approx -13.16 \\
g_{crit}^2 &= g_{max1}^2 \approx 18.61 \\
\frac{\gamma^{(2)}}{\gamma^{(1)}} &\approx -0.0080 \\
\frac{\beta^{(2)}}{\beta^{(1)}_{\mu^2}} &\approx -0.0826 \\
\frac{\beta^{(2)}}{\beta^{(1)}_{\lambda}} &\approx 0.1564 \\
\frac{\beta^{(2)}_{g}}{\beta^{(1)}_{g}} &\approx 0.3536 \\
\lambda_{crit} &= \lambda_{max2}^{run} \approx -7.13 \\
g_{crit}^2 &= g_{max2}^2 \approx 15.11 \\
\frac{\gamma^{(2)}}{\gamma^{(1)}} &\approx -0.0065 \\
\frac{\beta^{(2)}}{\beta^{(1)}_{\mu^2}} &\approx -0.0637 \\
\frac{\beta^{(2)}}{\beta^{(1)}_{\lambda}} &\approx 0.0412 \\
\frac{\beta^{(2)}_{g}}{\beta^{(1)}_{g}} &\approx 0.2871 \\
\end{align*}
\]

Here we see that all ratios are sufficiently small, i.e. all two–loop contributions are small in comparison with one–loop contributions, confirming the validity of perturbation theory in the 2–loop approximation. The accuracy of deviation is worse ($\sim 30\%$) for $\beta_g$–function. But it is necessary to emphasize that calculating the border curves 1 and 2 of Fig. 2, we have not used RGE (13) for monopole charge: $\beta_g$–function is absent in Eq. (29). Therefore, the calculation of $g_{crit}$ according to Eq. (31) does not depend on the approximation of $\beta_g$ function. The above–mentioned $\beta_g$–function appears only in the second order derivative of $V_{eff}$ which is related with the monopole mass $m$ (see Refs. [2,3]).

Eqs. (33) give the following result:

\[
\alpha^{-1}_{crit} \approx 5,
\]

which is important for the phase transition at the Planck scale predicted by the Multiple Point Model (MPM).

4. Multiple Point Model and Critical Values of the U(1) and SU(N) Fine Structure Constants

Investigating the phase transition in HMM, we had pursued two objects: from one side, we had an aim to explain the lattice results, from the other side, we were interested in the predictions of MPM.

4.1. Anti-grand unification theory

Most efforts to explain the Standard Model (SM) describing well all experimental results known today are devoted to Grand Unification Theories (GUTs). The supersymmetric extension of the SM consists of taking the
SM and adding the corresponding supersymmetric partners. Unfortunately, at present time experiment does not indicate any manifestation of the supersymmetry. In this connection, the Anti–Grand Unification Theory (AGUT) was developed in Refs.[13-17, 4] as a realistic alternative to SUSY GUTs. According to this theory, supersymmetry does not come into the existence up to the Planck energy scale: \( M_{pl} = 1.22 \times 10^{19} \) GeV.

The Standard Model (SM) is based on the group SMG:

\[
SMG = SU(3)_c \times SU(2)_L \times U(1)_Y. \tag{38}
\]

AGUT suggests that at the energy scale \( \mu_G \sim \mu_{pl} = M_{pl} \) there exists the more fundamental group \( G \) containing \( N_{gen} \) copies of the Standard Model Group SMG:

\[
G = SMG_1 \times SMG_2 \times \ldots \times SMG_{N_{gen}} \equiv (SMG)^{N_{gen}}, \tag{39}
\]

where \( N_{gen} \) designates the number of quark and lepton generations (families).

If \( N_{gen} = 3 \) (as AGUT predicts), then the fundamental gauge group \( G \) is:

\[
G = (SMG)^3 = SMG_{1st\ generation} \times SMG_{2nd\ generation} \times SMG_{3rd\ generation}, \tag{40}
\]

or the generalized ones:

\[
G_f = (SMG)^3 \times U(1)_f, \quad \text{or} \quad G_{ext} = (SMG \times U(1)_{B-L})^3, \tag{41}
\]

which were suggested by the fitting of fermion masses of the SM (see Refs.[16]), or by the see–saw mechanism with right-handed neutrinos [17].

4.2. Multiple Point Principle

AGUT approach is used in conjunction with the Multiple Point Principle proposed in Ref.[4]. According to this principle Nature seeks a special point — the Multiple Critical Point (MCP) — which is a point on the phase diagram of the fundamental regularized gauge theory \( G \) (or \( G_f \), or \( G_{ext} \)), where the vacua of all fields existing in Nature are degenerate having the same vacuum energy density. Such a phase diagram has axes given by all coupling constants considered in theory. Then all (or just many) numbers of phases meet at the MCP. MPM assumes the existence of MCP at the Planck scale, insofar as gravity may be "critical" at the Planck scale.

The philosophy of MPM leads to the necessity to investigate the phase transition in different gauge theories. A lattice model of gauge theories is the most convenient formalism for the realization of the MPM ideas. As it was mentioned above, in the simplest case we can imagine our space–time as a regular hypercubic (3+1)–lattice with the parameter \( a \) equal to the fundamental (Planck) scale: \( a = \lambda_P = 1/M_{pl} \).

4.3. AGUT-MPM prediction of the Planck scale values of the U(1), SU(2) and SU(3) fine structure constants

The usual definition of the SM coupling constants:

\[
\alpha_1 = \frac{5}{3} \frac{\alpha}{\cos^2 \theta_{MS}}, \quad \alpha_2 = \frac{\alpha}{\sin^2 \theta_{MS}}, \quad \alpha_3 = \alpha_s = \frac{g_s^2}{4\pi}, \tag{42}
\]

where \( \alpha \) and \( \alpha_s \) are the electromagnetic and strong fine structure constants, respectively, is given in the Modified minimal subtraction scheme (\( \overline{MS} \)). Here \( \theta_{MS} \) is the Weinberg weak angle in \( \overline{MS} \) scheme. Using RGE with
experimentally established parameters, it is possible to extrapolate the experimental values of three inverse running constants $\alpha_i^{-1}(\mu)$ (here $\mu$ is an energy scale and $i=1,2,3$ correspond to U(1), SU(2) and SU(3) groups of the SM) from the Electroweak scale to the Planck scale. The precision of the LEP data allows to make this extrapolation with small errors (see [13]). Assuming that these RGEs for $\alpha_i^{-1}(\mu)$ contain only the contributions of the SM particles up to $\mu \approx \mu_{Pl}$ and doing the extrapolation with one Higgs doublet under the assumption of a "desert", the following results for the inverses $\alpha_{Y,2,3}(\mu)$ (here $\alpha_Y \equiv \frac{3}{5} \alpha_1$) were obtained in Ref.[4] (compare with [18]):

$$\alpha_{Y,1}(\mu_{Pl}) \approx 55.5; \quad \alpha_{2}(\mu_{Pl}) \approx 49.5; \quad \alpha_{3}(\mu_{Pl}) \approx 54.0.$$ (43)

The extrapolation of $\alpha_{Y,2,3}(\mu)$ up to the point $\mu = \mu_{Pl}$ is shown in Fig.3.

According to AGUT, at some point $\mu = \mu_G < \mu_{Pl}$ (but near $\mu_{Pl}$) the fundamental group $G$ (or $G_f$, or $G_{ext}$) undergoes spontaneous breakdown to its diagonal subgroup:

$$G \rightarrow G_{\text{diag.subgr.}} = \{g, g, g|\|g \in SMG\},$$ (44)

which is identified with the usual (low–energy) group SMG.

The AGUT prediction of the values of $\alpha_i(\mu)$ at $\mu = \mu_{Pl}$ is based on the MPM assumptions, and gives these values in terms of the corresponding critical couplings $\alpha_i, crit$ [13-15,4]:

$$\alpha_i(\mu_{Pl}) = \frac{\alpha_i, crit}{N_{\text{gen}}} = \frac{\alpha_i, crit}{3} \quad \text{for} \quad i = 2, 3,$$ (45)

and

$$\alpha_1(\mu_{Pl}) = \frac{\alpha_1, crit}{\frac{2}{3}N_{\text{gen}}(N_{\text{gen}}+1)} = \frac{\alpha_1, crit}{6} \quad \text{for} \quad U(1).$$ (46)

It was assumed in Ref.[4] that the MCP values $\alpha_i, crit$ in Eqs.(45) and (46) coincide with the triple point values of the effective fine structure constants given by the lattice SU(3)–, SU(2)– and U(1)–gauge theories.

If the point $\mu = \mu_G$ is very close to the Planck scale $\mu = \mu_{Pl}$, then according to Eqs.(43) and (46), we have:

$$\alpha_{1st \, gen.}^{-1} \approx \alpha_{2nd \, gen.}^{-1} \approx \alpha_{3rd \, gen.}^{-1} \approx \alpha_Y^{-1}(\mu_{Pl}) = \frac{\alpha_Y^{-1}(\mu_{Pl})}{6} \approx 9,$$ (47)

what is almost equal to the value:

$$\alpha_{\text{crit.,theor}}^{-1} \approx 8$$ (48)

obtained theoretically by Parisi improvement method for the Coulomb-like phase [4,6]. The critical value [18] is close to the lattice and HMM ones: see Eq.(37). This means that in the U(1) sector of AGUT we have $\alpha$ near the critical point, and we can expect the existence of MCP at the Planck scale.

References

[1] T.Suzuki, Progr.Theor.Phys. B0, 929 (1988).
[2] L.V.Laperashvili and H.B.Nielsen, Int.J.Mod.Phys. A16, 2365 (2001).
[3] L.V.Laperashvili, H.B.Nielsen and D.A.Ryzhikh, Int.J.Mod.Phys. A16, 3989 (2001).
[4] D.L. Bennett and H.B. Nielsen, Int. J. Mod. Phys. A9, 5155 (1994).

[5] L.V. Laperashvili, Phys. of Atom. Nucl. 57, 471 (1994); ibid, 59, 162 (1996).

[6] L.V. Laperashvili and H.B. Nielsen, Mod. Phys. Lett. A12, 73 (1997).

[7] J. Jersak, T. Neuhaus and P.M. Zerwas, Phys. Lett. B133, 103 (1983); Nucl. Phys. B251, 299 (1985).

[8] S. Coleman and E. Weinberg, Phys. Rev. D7, 1888 (1973).

[9] M. Sher, Phys. Rept. 179, 274 (1989).

[10] C.G. Callan, Phys. Rev. D2, 1541 (1970); K. Symanzik, in: Fundamental Interactions at High Energies, ed. A. Perlmutter (Gordon and Breach, New York, 1970).

[11] D.R.T. Jones, Nucl. Phys. B75, 531 (1974); Phys. Rev. D25, 581 (1982).

[12] M.E. Machacek and M.T. Vaughn, Nucl. Phys. B222, 83 (1983); ibid, B249, 70 (1985).

[13] H.B. Nielsen, "Dual Strings. Fundamental of Quark Models", in: Proceedings of the XYII Scottish University Summer School in Physics, St. Andrews, 1976, p. 528.

[14] D.L. Bennett, H.B. Nielsen, I. Picek, Phys. Lett. B208, 275 (1988).

[15] C.D. Froggatt, H.B. Nielsen, Origin of Symmetries, Singapore: World Scientific, 1991.

[16] C.D. Froggatt, G. Lowe, H.B. Nielsen, Phys. Lett. B311, 163 (1993); Nucl. Phys. B414, 579 (1994); ibid B420, 3 (1994);
C.D. Froggatt, H.B. Nielsen, D.J. Smith, Phys. Lett. B358, 150 (1996);
C.D. Froggatt, M. Gibson, H.B. Nielsen, D.J. Smith, Int. J. Mod. Phys. A13, 5037 (1998).

[17] H.B. Nielsen, Y. Takanishi, Nucl. Phys. B588, 281 (2000); ibid, B604, 405 (2001); Phys. Lett. B507, 241 (2001); hep-ph/0011168, hep-ph/0101181, hep-ph/0101307.

[18] P. Langacker, N. Polonsky, Phys. Rev. D47, 4028 (1993); D49, 1454 (1994); D52, 3081 (1995).
Fig. 1 The effective potential $V_{\text{eff}}$: the curve 1 corresponds to the "Coulomb"-"confinement" phase transition; curve 2 describes the existence of two minima corresponding to the confinement phases.
Fig. 2 The one-loop (curve 1) and two-loop (curve 2) approximation phase diagram in the dual Abelian Higgs model of scalar monopoles.

Fig. 3 The evolution of three inverse running constants $\alpha^{-1}(\mu)$, where $i=1,2,3$ correspond to U(1), SU(2) and SU(3) groups of the SM. The extrapolation of their experimental values from the Electroweak scale to the Planck scale was obtained by using the renormalization group equations with one Higgs doublet under the assumption of a "desert". The precision of the LEP data allows to make this extrapolation with small errors. AGUT works in the region $\mu_G \leq \mu \leq \mu_{P^*}$. 