Computational manipulation of a radiative MHD flow with Hall current and chemical reaction in the presence of rotating fluid

To cite this article: Subbu Alias Suba and R Muthucumaraswamy 2018 J. Phys.: Conf. Ser. 1000 012144

View the article online for updates and enhancements.

Related content
- A System of Linear Algebraic Equations for Giving Dark Multi-Soliton Solutions of the mKdV Equation
  Nian-Ning Huang and Zhong-Zhu Liu
- Effect of unsteady oscillatory MHD flow through a porous medium in porous vertical channel with chemical reaction and concentration
  M Chitra and M Suhasini
- Chemical reaction and radiation effects on MHD flow past an exponentially stretching sheet with heat sink
  Noran Nur Wahida Khalili, Abdul Aziz Samson, Ahmad Sukri Abdul Aziz et al.
Computational manipulation of a radiative MHD flow with Hall current and chemical reaction in the presence of rotating fluid

Subbu Alias Suba†, R Muthucumaraswamy

Department of Applied Mathematics, Sri Venkateswara College of Engineering, Sriperumbudur Taluk, Kanchipuram District, Tamilnadu, India-602117
E-mail: †suba@svce.ac.in

Abstract. A numerical analysis of transient radiative MHD(MagnetoHydroDynamic) natural convective flow of a viscous, incompressible, electrically conducting and rotating fluid along a semi-infinite isothermal vertical plate is carried out taking into consideration Hall current, rotation and first order chemical reaction. The coupled non-linear partial differential equations are expressed in difference form using implicit finite difference scheme. The difference equations are then reduced to a system of linear algebraic equations with a tri-diagonal structure which is solved by Thomas Algorithm. The primary and secondary velocity profiles, temperature profile, concentration profile, skin friction, Nusselt number and Sherwood Number are depicted graphically for a range of values of rotation parameter, Hall parameter, magnetic parameter, chemical reaction parameter, radiation parameter, Prandtl number and Schmidt number. It is recognized that rate of heat transfer and rate of mass transfer decrease with increase in time but they increase with increasing values of radiation parameter and Schmidt number respectively.

Key Words: Hall effect, MHD, radiation, first-order chemical reaction, rotation

1. Introduction

From the literature, an information is gained that numerical analysis of transient radiative MHD natural convective flow of a viscous, incompressible, electrically conducting and rotating fluid along a semi-infinite isothermal vertical plate has not been carried out taking into consideration Hall current and first order chemical reaction using Crank-Nicholson implicit finite difference scheme. Hence, in the current study, a computational manipulation has been performed using implicit finite difference scheme to investigate transient radiative MHD natural convective flow along a semi-infinite isothermal vertical plate in the presence of rotating fluid with Hall current and first order chemical reaction.

The physical sketch of the problem is outlined in Fig.1. We consider a vertical hot plate which is semi-finite in nature. We take the upward direction along the plate to be \( x \), the direction normal to its surface be \( y \) and the direction along its width be \( z \). We assume the natural convection flow to be unsteady, laminar, radiative, MHD and the fluid to be Newtonian, incompressible and rotating. In addition, Hall effect and first order chemical reaction are employed. The fluid moves upward along the plate. The plate and the fluid are maintained at the same temperature \( T = T_{\infty} \) and the concentration level everywhere in the fluid is \( C = C_{\infty} \). At time \( t' > 0 \), an impulsive motion is given to the plate in the upward
direction with constant velocity \( u_0 \) and the temperature of the plate is raised to \( T'_W \) and the concentration near the plate is also raised to \( C'_W \). Along \( y \)-axis, a magnetic field of strength \( B_0 \) is applied. The fluid is assumed to be gray which is emitting and absorbing but non-scattering medium. The change in the component of radiative flux in the stream-wise direction is negligible when compared to that in the direction normal to the plate.

2. Governing Equations and Boundary Conditions

2.1. Governing Equations

With the assumptions made above and under Boussinesq’s approximation, the flow model can be entered into a numerical formulation of the governing flow equations as presented below:

Continuity Equation:
For an incompressible flow, the rate of change of density is negligible, which leads to the simplification of the continuity equation as follows:

\[
 u_x + v_y = 0
\]  

(1)

Momentum Equations:

\[
 u_{t'} + uu_x + vv_y = \nu u_{yy} + 2\Omega w - \frac{\sigma'B_0^2(u + mw)}{\rho(1 + m^2)} + g(T' - T'_\infty)\beta + g(C' - C'_\infty)\beta^*
\]  

(2)
2.3. Dimensionless parameters

Using Eqn.(7), Eqns.(1)-(5) are reduced to

\[ w_{\nu} + u w_x + v w_y = \nu w_{yy} - 2\Omega u - \frac{\sigma'B_0^2(w - mu)}{\rho(1 + m^2)} \]  \hspace{1cm} (3)

where \( \sigma', m \) are electric conductivity and Hall parameter respectively.

Energy Equation:

In an incompressible flow, the energy dissipation can be small and hence it could be neglected in the energy equation as given below:

\[ T'_{\nu} + uT'_{x} + vT'_{y} = \frac{k}{\rho C_p} T'_{yy} + \frac{1}{\rho C_p} 16\alpha^* \sigma T_\infty^3 (T'_{\nu} - T') \] \hspace{1cm} (4)

where \( \alpha^* \) is the local radiant absorption coefficient and \( \sigma \) is Stefan-Boltzman constant.

Diffusion Equation:

\[ C'_{\nu} + uC'_{x} + vC'_{y} = DC'_{yy} - K'(C' - C'_\infty) \] \hspace{1cm} (5)

In all the above equations, \( u_x \) and \( u_{xx} \) denote the first order partial derivative and second order partial derivative of \( u \) with respect to \( x \) respectively. Similar notations are followed to denote the partial derivatives of the other flow field variables.

2.2. Boundary Conditions

The appropriate conditions at initial time and at boundary are specified below:

\[ \begin{align*}
 I.C : & \quad u = v = w = 0, \quad T' = T'^\infty, \quad C' = C'^\infty \\
 B.C : & \quad u = u_0, \quad v = w = 0, \quad T' = T'_W, \quad C' = C'_W \quad \text{when} \quad y = 0 \\
 & \quad u = v = w = 0, \quad T' = T'^\infty, \quad C' = C'^\infty \quad \text{when} \quad x = 0 \\
 & \quad u, v, w \to 0, \quad T' \to T'^\infty, \quad C' \to C'^\infty \quad \text{when} \quad y \to \infty
\] \hspace{1cm} (6)

2.3. Dimensionless parameters

Using non-dimensionalization, we can reduce the number of parameters in the problem. While doing experiment for one particular geometrical situation, if there is a need to repeat it for some other situation, if non-dimensionalization is used, the experiment need not be repeated for different situations. Results obtained for one situation/condition can be related with ease for another condition.

The following non-dimensional quantities are utilized here in order to reduce the number of parameters:

\[ \begin{align*}
 X = \frac{x u_0}{\nu}, & \quad Y = \frac{y u_0}{\nu}, & \quad \Omega = \frac{2\Omega u_0}{\nu^2}, & \quad T = \frac{(T' - T'^\infty)}{(T'_W - T'^\infty)}, \\
 U = \frac{u}{u_0}, & \quad V = \frac{v}{u_0}, & \quad M = \frac{\sigma'B_0^2 \nu}{\alpha_0^2 \rho}, & \quad C = \frac{(C' - C'^\infty)}{(C'_W - C'^\infty)}, \\
 t = \frac{\nu u_0^2}{\nu}, & \quad Pr = \frac{\nu}{\alpha}, & \quad Sc = \frac{\nu}{D}, & \quad Gr = \frac{\nu^3 (C'_W - C'^\infty)}{u_0^2 (T'_W - T'^\infty)} \\
 W = \frac{w}{u_0}, & \quad K = \frac{K' u_0}{\nu}, & \quad R = \frac{16\alpha^* \sigma^2 T_\infty^3}{\nu u_0^2}, & \quad Gc = \frac{\nu^3 (C'_W - C'^\infty)}{u_0^2}
\] \hspace{1cm} (7)

2.4. Dimensionless Flow Equations

Using Eqn.(7), Eqns.(1)-(5) are reduced to

\[ U_X + V_Y = 0. \] \hspace{1cm} (8)

\[ U_t + U U_X + V U_Y - \Omega W = U_{YY} + TGr + CGc - \frac{M(U + mW)}{1 + m^2} \] \hspace{1cm} (9)
\[ W_t + UW_X + VW_Y + \Omega U = W_{YY} - \frac{M(W - mU)}{1 + m^2} \]  
\[ T_t + UT_X + VT_Y = \frac{1}{Pr} T_{YY} - T \frac{R}{Pr} \]  
\[ C_t + UC_X + VC_Y = \frac{1}{Sc} C_{YY} - KC \]  

The respective conditions at initial time and at boundary in dimensionless form:

**I.C:** \( U = V = W = T = C = 0 \)

**B.C:** \( U = 1, V = W = 0, T = C = 1 \) when \( Y = 0 \)
\( U = V = W = T = C = 0 \) when \( X = 0 \)
\( U, V, W, T, C \to 0 \) when \( Y \to \infty \)

3. **Average Skin Friction Coefficient, Average Heat Transfer Rate and Average Mass Transfer Rate**

In dimensionless form, the average skin friction coefficient is given by

\[ \bar{C_f} = - \int_0^1 \left( \frac{\partial U}{\partial Y} \right)_{Y=0} dX \]  

The dimensionless average heat transfer rate, known as Nusselt number, is given by

\[ \bar{Nu} = - \int_0^1 \left( \frac{\partial T}{\partial Y} \right)_{Y=0} dX \]  

The dimensionless average mass transfer rate, known as Sherwood number, is given by

\[ \bar{Sh} = - \int_0^1 \left( \frac{\partial C}{\partial Y} \right)_{Y=0} dX \]  

4. **Numerical Technique**

The set of dimensionless Eqns. (8)-(12) are discretized and then solved in a coupled manner under the boundary conditions along with the initial conditions given in Eqn.(13) by Crank-Nicholson implicit finite difference scheme. The corresponding finite difference equations are reckoned as given below:

\[ \frac{U_{i,j}^{n+1} - U_{i-1,j}^{n+1} + U_{i,j}^{n} - U_{i-1,j}^{n}}{4\Delta X} + \frac{V_{i,j}^{n+1} - V_{i-1,j}^{n+1} + V_{i,j}^{n} - V_{i-1,j}^{n}}{2\Delta Y} = 0 \]  

\[ \frac{U_{i,j}^{n+1} - U_{i,j}^{n}}{\Delta t} + \frac{U_{i,j}^{n+1} - U_{i-1,j}^{n+1} + U_{i,j}^{n} - U_{i-1,j}^{n}}{2\Delta X} + \frac{V_{i,j}^{n+1} - V_{i,j-1}^{n+1} + V_{i,j}^{n} - V_{i,j-1}^{n}}{4\Delta Y} = 0 \]  

\[ \Omega W_{i,j}^{n} - M \frac{U_{i,j}^{n} + mW_{i,j}^{n}}{1 + m^2} \]
respectively. Hence, it is realized that radiation parameter and Schmidt number enhance the increase with increasing values of rotation parameter, radiation parameter and Schmidt number. Sherwood number or $Sc$ is noticed from Figs.12 and 13 that species concentration $W$ and $R$ parameters (difference) are significant near the hot plate. A reverse trend is observed for Fig.11 highlights the radiation effect. We grasp that $U$ increases whereas $W$ decreases on increasing $\Omega$. It is also inferred that the buoyancy effects (due to concentration and temperature difference) are significant near the hot plate. A reverse trend is observed for $W$ in Fig.9.

Since the energy equation is not coupled as seen from Eqn.(11) and as it only involves the parameters $Pr$ and $R$, the temperature profiles shown in Figs.10 and 11 show the effects of $Pr$ and $R$ only. Fig.11 highlights the radiation effect. We grasp that $T$ decreases as $R$ increases. It is noticed from Figs.12 and 13 that species concentration $C$ decreases on increasing either $K$ or $Sc$. This implies that chemical reaction tends to reduce species concentration whereas mass diffusion enhances species concentration which is obvious from Eqn.(7).

A close study of Tables.1-3 shows the respective effect of rotation, radiation and Schmidt number on the average skin friction coefficient $\overline{C_f}$, average Nusselt number $\overline{Nu}$ and average Sherwood number $\overline{Sh}$. It is identified that $\overline{C_f}$, $\overline{Nu}$ and $\overline{Sh}$ decrease with increase in time but increase with increasing values of rotation parameter, radiation parameter and Schmidt number respectively. Hence, it is realized that radiation parameter and Schmidt number enhance the heat transfer rate and mass transfer rate respectively.

$$\frac{[W_{i,j}^{n+1} - W_{i,j}^n]}{\Delta t} + U_{i,j}^n \left[ \frac{W_{i,j}^{n+1} - W_{i-1,j}^{n+1} + W_{i,j}^n - W_{i-1,j}^n}{2\Delta Y} \right] + V_{i,j}^n \left[ \frac{W_{i,j+1}^{n+1} - W_{i,j-1}^{n+1} + W_{i,j}^n - W_{i,j-1}^n}{2\Delta Y} \right] =$$

$$\frac{W_{i,j}^{n+1} - 2W_{i,j}^n + W_{i,j+1}^n + W_{i,j-1}^n - 2W_{i,j}^n + W_{i,j+1}^n}{(2\Delta Y)^2} - \Omega U_{i,j}^n - \frac{M}{1 + m^2} \left[ W_{i,j}^n - mU_{i,j}^n \right]$$

(19)

$$\frac{[T_{i,j}^{n+1} - T_{i,j}^n]}{\Delta t} + U_{i,j}^n \left[ \frac{T_{i,j}^{n+1} - T_{i-1,j}^{n+1} + T_{i,j}^n - T_{i-1,j}^n}{2\Delta X} \right] + V_{i,j}^n \left[ \frac{T_{i,j+1}^{n+1} - T_{i,j-1}^{n+1} + T_{i,j}^n - T_{i,j-1}^n}{2\Delta X} \right] =$$

$$\frac{1}{Pr} \left[ T_{i,j}^{n+1} - 2T_{i,j}^n + T_{i,j+1}^n + T_{i,j-1}^n - 2T_{i,j}^n + T_{i,j+1}^n \right] - \frac{R}{Pr} \left[ \frac{T_{i,j}^{n+1} + T_{i,j}^n}{2} \right]$$

(20)

$$\frac{[C_{i,j}^{n+1} - C_{i,j}^n]}{\Delta t} + U_{i,j}^n \left[ \frac{C_{i,j}^{n+1} - C_{i-1,j}^{n+1} + C_{i,j}^n - C_{i-1,j}^n}{2\Delta X} \right] + V_{i,j}^n \left[ \frac{C_{i,j+1}^{n+1} - C_{i,j-1}^{n+1} + C_{i,j}^n - C_{i,j-1}^n}{2\Delta X} \right] =$$

$$\frac{1}{Sc} \left[ \frac{C_{i,j}^{n+1} - 2C_{i,j}^n + C_{i,j+1}^n + C_{i,j-1}^n - 2C_{i,j}^n + C_{i,j+1}^n}{2(\Delta Y)^2} - \frac{K}{2} \left[ \frac{C_{i,j}^{n+1} + C_{i,j}^n}{2} \right] \right]$$

(21)

The grid points are identified by an index $i$ which runs in the $x$ direction and an index $j$ which runs in the $y$ direction.

The discretized Eqns.(17) - (21) are then solved numerically using time marching method and Thomas Algorithm[9]. The optimal step sizes are taken to be $\Delta X = 0.05$, $\Delta Y = 0.25$ and $\Delta t = 0.01$. A rectangular region is considered with sides $X_{max} = 1$ and $Y_{max} = 14$.

5. Results

Figs.2 and 3 show the effect of rotation on the fluid velocities. It is revealed from these two figures that $U$ and $W$ decrease on increasing $\Omega$.

Figs.4 and 5 exhibit the influence of Hall current on $U$ and $W$. It is communicated from Figs.4 and 5 that $U$ increases whereas $W$ decreases on increasing the Hall parameter $m$.

The influence of $M$ on $U$ and $W$ is depicted in Figs.6 and 7. These figures illustrate that the magnetic force opposes the motion in $x$-direction but supports the motion in $z$-direction.

Fig.8 depicts how the primary velocity is affected by chemical reaction. It is observed that the fluid velocity quickly increases adjacent to the plate and then decreases towards $U = 0$ as $Y \to \infty$. It is also inferred that the buoyancy effects (due to concentration and temperature difference) are significant near the hot plate. A reverse trend is observed for $W$ in Fig.9.
Figure 2. Primary velocity dissemination $U$ for different $\Omega$ where $M = 0.2$.

Figure 3. Secondary velocity dissemination $W$ for different $\Omega$ where $M = 0.2$. 
Figure 4. Primary velocity dissemination $U$ for different $m$

Figure 5. Secondary velocity dissemination $W$ for different $m$
Figure 6. Primary velocity dissemination $U$ for different $M$ where $\Omega = 0.4$.

Figure 7. Secondary velocity dissemination $W$ for different $M$ where $\Omega = 0.4$. 
Figure 8. Primary velocity dissemination $U$ for different $K$

Figure 9. Secondary velocity dissemination $W$ for different $K$
Figure 10. Temperature dissemination $T$ for different $Pr$

Figure 11. Temperature dissemination $T$ for different $R$
Figure 12. Concentration dissemination $C$ for different $K$

Figure 13. Concentration dissemination $C$ for different $Sc$
Table 1. Average skin friction coefficient $C_f$ for various rotation parameter $\Omega$

| $t$ | $\Omega = 1.5$ | $\Omega = 3.0$ | $\Omega = 4.0$ |
|-----|-----------------|-----------------|-----------------|
| 0.0 | 0               | 0               | 0               |
| 0.1 | -0.23774        | -0.29975        | -0.37991        |
| 0.2 | 4.29590         | 4.10701         | 3.86445         |
| 0.3 | 6.46980         | 6.11863         | 5.67245         |
| 0.4 | 7.69725         | 7.17043         | 6.50933         |
| 0.5 | 8.36198         | 7.67723         | 6.82671         |
| 0.6 | 8.66075         | 7.86152         | 6.87277         |
| 0.7 | 8.75572         | 7.88777         | 6.81203         |
| 0.8 | 8.77500         | 7.86752         | 6.73861         |
| 0.9 | 8.77757         | 7.84609         | 6.68349         |
| 1.0 | 8.77793         | 7.83123         | 6.64673         |

Table 2. Average heat transfer rate $Nu$ for various radiation parameter $R$

| $t$ | $R = 0.2$ | $R = 2.0$ | $R = 5.0$ |
|-----|-----------|-----------|-----------|
| 0.0 | 0         | 0         | 0         |
| 0.1 | 6.24547   | 7.53054   | 9.39848   |
| 0.2 | 4.8342    | 6.55976   | 8.88049   |
| 0.3 | 4.37848   | 6.33635   | 8.82388   |
| 0.4 | 4.21477   | 6.29341   | 8.83645   |
| 0.5 | 4.18461   | 6.31068   | 8.85954   |
| 0.6 | 4.21627   | 6.34426   | 8.87965   |
| 0.7 | 4.25692   | 6.37083   | 8.89213   |
| 0.8 | 4.2825    | 6.39429   | 8.89756   |
| 0.9 | 4.2953    | 6.38972   | 8.89938   |
| 1.0 | 4.30151   | 6.39188   | 8.89993   |

Table 3. Average mass transfer rate $Sh$ for various Schmidt number $Sc$

| $t$ | $Sc = 0.16$ | $Sc = 0.6$ | $Sc = 2.01$ |
|-----|-------------|-------------|-------------|
| 0.0 | 0           | 0           | 0           |
| 0.1 | 2.90347     | 5.73902     | 11.87694    |
| 0.2 | 2.23581     | 4.44861     | 8.78114     |
| 0.3 | 2.00147     | 4.00241     | 7.90361     |
| 0.4 | 1.90603     | 3.81437     | 7.55065     |
| 0.5 | 1.87352     | 3.74189     | 7.40164     |
| 0.6 | 1.87079     | 3.72711     | 7.35399     |
| 0.7 | 1.8764      | 3.73479     | 7.35538     |
| 0.8 | 1.88072     | 3.74367     | 7.37122     |
| 0.9 | 1.88308     | 3.74815     | 7.38431     |
| 1.0 | 1.88451     | 3.74969     | 7.39155     |
6. Concluding Remarks
Computed results are presented in the form of graphs and tables which interpret the following observations:

- The primary velocity dissemination shows an enhancing behavior initially and then decreases.
- The primary fluid looses its speed when the values of rotation parameter, magnetic parameter and chemical reaction parameter are increased whereas the trend is reversed in case of increasing Hall parameter.
- In the subsidiary direction, the fluid is hastened with increasing values of magnetic parameter and chemical reaction parameter but the rotation parameter and Hall parameter has a contrary effect.
- Viscosity and thermal radiation reduces temperature of the plate.
- Chemical reaction and momentum diffusion shortens concentration of the plate.
- $C_f$, $Nu$ and $Sh$ decrease with increase in time but they are increasing when $\Omega, R, Sc$ are increased respectively.

7. List of Symbols

$B_0$ - applied magnetic field
$C'$ - species concentration
$C'_W$ - concentration at the wall
$C'_\infty$ - concentration at infinity
$C$ - dimensionless concentration
$T'$ - temperature of the flow field
$T'_W$ - temperature of the fluid at the wall
$T'_\infty$ - temperature of the fluid at infinity
$T$ - dimensionless temperature of the flow field
$t'$ - time
$t$ - dimensionless time
$u_0$ - velocity of the plate
$X, Y, Z$ - dimensionless co-ordinate axes
$C_p$ - specific heat at constant pressure
$D$ - molecular diffusivity
$g$ - acceleration due to gravity
$Gr$ - thermal Grashoff number
$Gc$ - mass Grashoff number
$\kappa$ - thermal conductivity
$M$ - magnetic parameter
$Nu$ - dimensionless average Nusselt number
$Sh$ - dimensionless average Sherwood number
$Pr$ - Prandtl number
$Sc$ - Schmidt number
$\alpha$ - thermal diffusivity
$\beta$ - volumetric coefficient of thermal expansion
\( \beta^* \) - volumetric coefficient of expansion with concentration
\( \nu \) - kinematic viscosity
\( \rho \) - density of the fluid in the boundary layer
\( \Omega' \) - rotational velocity
\( \Omega \) - dimensionless rotation parameter

References
1. Das U N, Deka R K and Soundalgekar V M 1994 Effects of mass transfer on flow past an impulsively started infinite vertical plate with constant heat flux and chemical reaction *ForschungsIngenieurwesen* **60** 284-287
2. Seth G S, Hussain S M and Sarkar S 2014 Effects of Hall current and rotation on unsteady MHD natural convection flow with heat and mass transfer past an impulsively moving vertical plate in the presence of radiation and chemical reaction *Bulgarian Chemical Communications* **46** 704-718
3. Sinha S 2015 Effect of chemical reaction on an unsteady MHD free convective flow past a porous plate with ramped temperature *Proceedings of ICFM 2015 Gauhati University* **March** 26-28
4. Seth G S, Kumbhakar B and Sarkar S 2015 Soret and Hall effects on unsteady MHD free convection flow of radiating and chemically reactive fluid past a moving vertical plate with ramped temperature in rotating system *International Journal of Engineering, Science and Technology* **7** 94-108
5. England W G and Emery A F 1969 Thermal radiation effects on the laminar free convection boundary layer of an absorbing gas *Journal of Heat Transfer* **43** 37-44
6. Das S, Guchhait S K, Jana R N and Makinde O D 2016 Hall effects on an unsteady magneto convection and radiative heat transfer past a porous plate *Alexandria Engineering Journal* **55** 1321-1331
7. Rani H P and Reddy G J 2013 Soret and Dufour effects on transient double diffusive free convection of couple-stress fluid past a vertical cylinder *Journal of Applied Fluid Mechanics* **6** 545-554
8. Cowling T G 1957 *Magnetohydrodynamics* (New York : Interscience Publisher)
9. Carnahan B, Luther H A and James O 1990 *Applied Numerical Methods* (Florida : Krieger Publishing Company)
10. Sarma D and Pandit K K 2016 Effects of Hall current rotation and Soret effects on MHD free convection heat and mass transfer flow past an accelerated vertical plate through a porous medium *Ain Shams Eng J*. doi:10.1016/j.asej.2016.03.005
11. Brewster M Q 1992 *Thermal Radiative Trans.and Properties* (John Wiley and Sons)