Connectivity-Based Localization in Ultra-Dense Networks: CRLB, Theoretical Variance, and MLE

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ABSTRACT Performance analysis of connectivity-based geolocation in ultra-dense networks (UDNs) is a very important task. Although several performance analyses have been presented for range-free localization, determining the best achievable positioning accuracy of range-free localization remains an open problem. In this paper, we first derive the Cramer-Rao lower bound (CRLB) for the performance evaluation of range-free localization. All the current performance analyses in the literature for range-free localization are used to evaluate the real performance of a given algorithm, whereas the proposed CRLB provides a benchmark to evaluate the performance of any unbiased range-free location algorithm and determines the physical impossibility of the variance of an unbiased estimator being less than the bound. To the best of our knowledge, this is the first time in the literature that the CRLB for range-free localization has been derived. Second, the theoretical variance of centroid-based localization (CL) with an arbitrary node distribution is derived in this paper. In contrast to the existing theoretical variance of CL for uniform node distribution, the proposed theoretical variance can be used to evaluate the performance of CL in the case of an arbitrary node distribution. Additionally, characteristics of the proposed CRLB and theoretical variance are given in this paper. Finally, an optimal estimator based on a maximum likelihood estimator (MLE) is proposed to improve positioning accuracy. Since both prior information on the spatial node distribution and the connectivity property are effectively utilized in our algorithm, the proposed method performs better than the CL method and can asymptotically attain the CRLB.

INDEX TERMS Connectivity-based localization, Cramer-Rao lower bound (CRLB), theoretical variance, maximum likelihood estimator (MLE), centroid-based localization (CL).

I. INTRODUCTION

With the increasing number of networked devices, location estimation of a blind node (BN) in ultra-dense network (UDN) systems has gained considerable attention in recent years [1]. Wireless location as an important public safety feature has created many potential applications in future wireless communication systems, such as location-sensitive billing, fraud protection, person/asset tracking, fleet management, mobile yellow pages, wireless network design, radio resource management, and intelligent transportation systems [2], [3]. Although global navigation satellite systems (GNSSs), such as global positioning systems (GPSs), can provide location services with high positioning accuracy, their limitations, including high power consumption and degraded performance in outdoor rich-scattering scenarios and urban canyons, prevent GNSSs from being applied in complex urban and indoor environments.

Broadly, localization techniques in wireless communication systems can be divided into two categories: (1) range-based and (2) range-free methods (also known as connectivity-based localization).

Several range-based localization techniques, including time-of-arrival (TOA), time-difference-of-arrival (TDOA), angle-of-arrival (AOA), received signal strength (RSS)-based methods and hybrid location methods, are used for wireless localization. Range-based localization first builds explicit geometric relations among a BN and reference nodes (RNs) using the bearing angle and the absolute or relative distance, which are estimated from the AOA, TOA, RSS, and TDOA measurements. Subsequently, the BN’s position can be obtained based on the geometric model. Due to their high location precision,
range-based location methods have been widely studied in the literature [4]–[25]. Both closed-form solutions and iterative algorithms were investigated for the line-of-sight (LOS) environment [4]–[9]. For non-line-of-sight (NLOS) propagation, geometric constraint conditions [10]–[12] and machine learning theory [13]–[16] were developed to mitigate NLOS error. Most of these studies are based on a single path of measurements. To further improve positioning accuracy, AOA-based location algorithms using multiple antenna arrays were proposed in [17], [18] for multipath propagation environments. Additionally, many geometric dilutions of precision and Cramer-Rao lower bounds (CRLBs) were derived in literature [19]–[25].

Compared with range-based localization using the TOA, TDOA, RSS, and AOA measurements, the range-free method makes no assumption about the availability or validity of such information and only utilizes connectivity information to locate a BN. For a UDN, such as a wireless sensor network (WSN) or Internet of Things (IoT), nodes are usually low-cost and low-power and have no ability to obtain high-precision measurements of the TOA, TDOA, RSS, or AOA [26], [29]. Because of the hardware limitations and power constraints of nodes, range-free localization is often a preferred solution for UDNs [27]. Additionally, the high-density nodes in a UDN can further improve the performance of the range-free method. Several range-free methods have been addressed in the literature [26]–[31]. A well-known method of range-free localization is centroid-based localization (CL) [28], where the centroid of the RNs detecting a BN is estimated as the BN’s position. A location method based on multidimensional scaling (MDS) was proposed for range-free localization in WSNs [29]. Based on the connectivity matrix, the MDS method provides a relative position estimate for the WSN. Subsequently, localization and tracking methods [27], [30], [31] that consider the connectivity information and motion model of a BN were designed for mobility-assisted WSNs. In addition to the above location schemes, some theoretical analyses for performance evaluation on range-free localization were presented in [26], [27], [31]. The authors in [27] derived the spatially averaged area of the location region and the localization error probability assuming that the RNs are scattered according to a Poisson point process. The maximal localization error of mobility-assisted WSNs was addressed in [31]. A theoretical variance was presented in [26] to provide a performance analysis for the CL method based on the uniform distribution of RNs’ positions.

In this paper, we analyze the performance of range-free localization in terms of CRLB and theoretical variance. The corresponding optimal estimator based on a maximum likelihood estimator (MLE) is also proposed in this paper. The main contributions of this paper are as follows:

(1) This paper derives a CRLB for range-free localization in a UDN with randomly distributed RNs. Although several performance analyses have been presented for range-free localization [26], [27], [31], determining the best achievable positioning accuracy of range-free localization remains an open problem. All the current performance analyses [26], [27], [31] of range-free localization are used to evaluate the real performance of a given algorithm, whereas the proposed CRLB provides a benchmark to evaluate the performance of any unbiased range-free location algorithm and determines the physical impossibility of the variance of an unbiased estimator being less than the bound. To the best of our knowledge, this is the first time in the literature that the CRLB for range-free localization has been derived.

(2) The theoretical variance of the CL method with an arbitrary node distribution is derived in the paper. It should be noted that the theoretical variance of the CL method in [26] is derived for a uniform node distribution. The proposed theoretical variance can be used to evaluate the performance of CL in the case of an arbitrary node distribution. Moreover, characteristics of the proposed CRLB and theoretical variance are provided in this paper.

(3) An optimal estimator based on an MLE is proposed to improve positioning accuracy. Since both the prior information on spatial node distribution and the connectivity property are effectively utilized in our algorithm, the proposed method performs better than the CL method and can asymptotically attain the CRLB.

This paper is organized as follows. The signal model and some basic notation are presented in section II. In section III, the paper first derives a CRLB for range-free localization in a UDN with randomly distributed RNs. Then, the theoretical variance of the CL method is derived for an arbitrary node distribution. Some characteristics of the proposed CRLB and theoretical variance are given at the end of this section. Section IV proposes an iterative method based on an MLE using both connectivity information and the RN distribution. Section V gives the performance evaluation of the proposed CRLB, the theoretical variance, and the location method. The conclusions of this paper are given in Section VI.

II. SYSTEM MODEL

The purpose of range-free localization is to locate a BN using connectivity information between the BN and RNs. In general, the CL algorithm consists of two phases: listening and positioning. During the listening phase, the BN tries to listen and communicate with RNs. A communication link is established when the received signal power exceeds the detection threshold. In the positioning phase, the BN’s location is approximated as the average of the positions of all the RNs (the centroid of the RNs) within its transmission range. Obviously, the performance of CL is affected by many factors, such as the node density and randomness, the wireless channel environment, and the location scheme.

The wireless channel environment plays a very important role in the performance of localization systems. This channel environment determines how many and which RNs can be detected and used for localization. Propagation models are usually built to describe the wireless channel situation and predict the average received signal strength at a given distance from the transmitter. Although several propagation models...
have been addressed in the literature [32], [33], the path loss normal shadowing model is selected in this paper, since it is widely used for communication and localization applications, and it has been confirmed by field measurements [32].

Assuming that \((x, y)\) is the position of a BN to be estimated, and the known coordinate of the \(i\)th RN in a system of \(N\) RNs is \((x_i, y_i)\), without loss of generality, the BN’s position can be set to be \((0,0)\). The true distance between the \(i\)th RN and the BN can be modeled as:

\[
 r_i = \sqrt{(x_i - x)^2 + (y_i - y)^2} \tag{1}
\]

Based on the path loss normal shadowing model, the measured received power \(P_i\) at RN \(i\) in decibel milliwatts (dBm) can be regarded as a log-normal variable [32]. Therefore, the relation between \(P_i\) and \(r_i\) becomes:

\[
 P_i = P_0 - 10\beta \log_{10} \left( \frac{r_i}{r_0} \right) + n_i \tag{2}
\]

where \(\beta\) is the path-loss exponent, which indicates the rate at which the path loss increases with distance; \(n_i\) is a zero-mean Gaussian random process with standard variance (std) \(\sigma\) in decibels (dB); and \(P_0\) is the reference power at reference distance \(r_0\), which depends on the transmit power. Typically, \(r_0 = 1\) m. For simplicity, \(r_0\) is set to 1 m in this paper. The path loss normal shadowing model is utilized here to derive the CRLB and MLE for range-free localization.

Node randomness is another principal element for the performance of the localization method. Various node distributions are proposed in the literature based on different assumptions. Uniform distribution was first addressed in the literature to build a node distribution model where sensor nodes are assumed to be uniformly distributed in a disk of radius \(R\) [26], [27], [34]. However, more recently, it has been recognized that the assumption of uniformly distributed nodes is rather implausible for real, deployed wireless networks [35], [36]. In fact, node spatial distribution relies on many factors, such as the deployment method, the surroundings of the nodes, node motion, and even the communication protocol. According to the central limit theorem, the actual node location will follow a Gaussian distribution [35], [36]. In this model, \(N\) RNs are placed around a BN according to a two-dimensional Gaussian spatial distribution with mean \([x, y]\) and covariance matrix \(\sigma_p^2 I\). The probability that an RN is located at \((x_i, y_i)\) can be described by the probability density function (PDF) [36]:

\[
 f(x_i, y_i) = \frac{1}{2\pi\sigma_p^2} \exp \left( -\frac{(x_i - x)^2}{2\sigma_p^2} - \frac{(y_i - y)^2}{2\sigma_p^2} \right) \tag{3}
\]

It should be noted that (3) is based on Cartesian coordinates. For polar coordinates, the PDF (3) can be written as:

\[
 f(r_i) = \frac{1}{\sigma_p^2} \exp \left( -\frac{r_i^2}{2\sigma_p^2} \right) r_i > 0
\]

\[
 f(\phi_i) = \frac{1}{2\pi} -\pi \leq \phi_i < \pi
\]

where \(r_i\) is the range between the \(i\)th RN and the BN, which was defined in (1). \(\phi_i = \cos((x - x_i)/r_i)\) is the azimuth angle of RN \(i\) with respect to the BN. Equation (4) shows that RNs will appear around the BN with equal probability in different directions, and \(f(x_i, y_i)\) depends only on the distance between the RN and BN. This also implies that an RN near the BN may have a higher probability than an RN at a greater distance.

CL is the simplest range-free location method, which merely requires binary connectivity information between the BN and adjacent RNs. The CL algorithm is based on the following assumptions [28]:

1. There is perfect spherical radio propagation
2. All radios have identical transmission range (power)
3. RNs are symmetrically distributed around a BN.
4. CL is only based on connectivity information collected from adjacent RNs (single-hop assumption).

Assumptions (1-3) ensure that the CL algorithm is an unbiased estimator, and the fourth assumption simplifies the process of localization. Since both simulation and experimental results prove that this model adheres quite well with outdoor radio propagation in uncluttered environments [28], this paper also follows these assumptions.

The BN localizes itself to the region that coincides with the intersection of the connectivity regions of the set of RNs, which is defined by the centroid of the RNs [28]:

\[
 (\bar{x}, \bar{y}) = \left( \frac{1}{M} \sum_{i=1}^{M} x_i, \frac{1}{M} \sum_{i=1}^{M} y_i \right) \tag{5}
\]

where \(M \leq N\) is the number of RNs that actually take part in the positioning process. It can be seen from (5) that the CL algorithm only averages the coordinates of the RNs to estimate the position of the BN using equal weight. To further improve positioning accuracy, several weighted CL (WCL) algorithms [37]–[48] have been proposed in the literature. The early WCL algorithm in [37] used the link quality indication as the weights and was applied in ZigBee-based sensor networks. Subsequently, various weight selection strategies [38]–[41] were proposed in the literature to provide higher positioning accuracy. In addition to the weight selection strategy, the choice of different RNs used in WCL [42], [43] is another way to improve system performance. Some low complexity WCL methods were presented in [44], [45] to reduce the computational burden. In addition, several studies reported work on the performance analysis of WCL [46]–[48]. WCL algorithms use the RSS, which is known to be a function of link distance, as a weighting factor to average the locations of neighboring RNs. The location estimate for the WCL algorithms can be written as:

\[
 (\bar{x}, \bar{y}) = \left( \frac{\sum_{i=1}^{M} w_i x_i}{\sum_{i=1}^{M} w_i}, \frac{\sum_{i=1}^{M} w_i y_i}{\sum_{i=1}^{M} w_i} \right) \tag{6}
\]
where the weight \( w_i \) is a function of the RSS \( P_i \). Since the existing WCL algorithms require RSS measurements, these are range-based localization techniques that go beyond the scope of our research. This paper focuses on range-free localization techniques using only single-hop connectivity information.

### III. THEORETICAL ANALYSIS OF THE CONNECTIVITY-BASED LOCALIZATION TECHNIQUE

This section derives the CRLB and theoretical variance to investigate the performance of the range-free localization technique. The CRLB is very important for parameter estimation, since it provides a benchmark to evaluate the performance of any unbiased estimator, while theoretical variance is used to evaluate the real performance of a given algorithm.

#### A. CRLB

For the range-free location technique, the BN’s location is estimated using the positions of the detected RNs. Therefore, the measurement vector is \( s = [x_1, y_1, \ldots, x_M, y_M]^T \), and the parameter vector \( \theta \) to be estimated is \( [x, y]^T \).

It is assumed that the PDF satisfies the “regularity” conditions:

\[
E \left[ \frac{\partial \ln f (s; \theta)}{\partial \theta} \right] = 0 \text{ for all } \theta
\]  

(7)

where the expectation is taken with respect to \( f (s; \theta) \). Then, the CRLB matrix is defined as the inverse of the Fisher information matrix (FIM) \( J_\theta \):

\[
J_\theta = \begin{bmatrix} J_{xx} & J_{xy} \\ J_{yx} & J_{yy} \end{bmatrix} = E \left[ \frac{\partial \ln f (s; \theta)}{\partial \theta} \left( \frac{\partial \ln f (s; \theta)}{\partial \theta} \right)^T \right]
\]

(8)

The joint conditional PDF \( f (s; \theta) \) can be written as:

\[
f (s; \theta) = \prod_{i=1}^{M} f (x_i, y_i; \theta)
\]

(10)

where

\[
f (x_i, y_i; \theta) = \frac{\Phi (r_i) f (x_i, y_i)}{\gamma}
\]

(11)

\( f (x_i, y_i) \) describes the node spatial distribution probability, which is defined in (3). \( \Phi (r_i) \) is the detection probability, which indicates the probability that the received signal power at RN \( i \) exceeds the detection threshold \( P_{th} \). \( \gamma \) is a normalization constant:

\[
\gamma = \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} \Phi (r_i) f (x_i, y_i) \, dx_i \, dy_i = \int_{0}^{+\infty} \Phi (r) f (r) \, dr
\]

(12)

where \( f (r) \) is the PDF of range \( r \), which can be obtained from (4). Since there is no closed-form solution for \( \gamma \), numerical integration methods, such as the MATLAB function “integral” can be used to solve \( \gamma \).

Based on the channel model in (2), the PDF of RSS \( P_i \) can be written as:

\[
f (P_i) = \frac{1}{\sqrt{2\pi\sigma}} \exp \left( -\frac{1}{2} \frac{(P_i - P_0 + 10\beta \log_{10} (r_i))^2}{2\sigma^2} \right)
\]

(13)

From (13), \( \Phi (r_i) \) can be calculated as:

\[
\Phi (r_i) = \frac{1}{P_{th}} \int_{P_{th}}^{+\infty} f (P_i) \, dP_i
\]

\[
= \int_{P_{th}}^{+\infty} \frac{1}{\sqrt{2\pi\sigma}} \exp \left( -\frac{1}{2} \frac{(P_i - P_0 + 10\beta \log_{10} (r_i))^2}{2\sigma^2} \right) \, dP_i
\]

(14)

Using the substitution method, (14) can be simplified as:

\[
\Phi (r_i) = \int_{\varphi(r_i)}^{+\infty} \frac{1}{\sqrt{2\pi}} \exp \left( -\frac{v^2}{2} \right) dv
\]

(15)

where

\[
\varphi (r_i) = \frac{P_{th} - P_0 + 10\beta \log_{10} (r_i)}{\sigma}
\]

(16)

and \( \Phi (r_i) \) can be computed using the MATLAB function “erfc”:

\[
\Phi (r_i) = 0.5 \text{erfc} \left( \frac{\varphi (r_i)}{\sqrt{2}} \right)
\]

(17)

Equation (17) shows that \( \Phi (r_i) \) only depends on the distance \( r_i \) for a given system.

Substituting (10)-(17) into (9), it can be derived as shown in the Appendix that the CRLB for the connectivity-based localization technique is:

\[
\text{CRLB} = \text{tr} \left[ J^{-1} \right] = \frac{4}{\tilde{\psi}^2 M}
\]

(18)

where \( \tilde{\psi}^2 \) can be obtained from (78):

\[
\tilde{\psi}^2 = \frac{1}{\gamma} \int_{0}^{+\infty} \left( \frac{1}{\Phi (r)} \frac{b}{r} \exp \left( -\frac{\varphi (r)^2}{2} \right) + \frac{r}{\sigma_r^2} \right)^2 \times \Phi (r) \frac{1}{\sigma_r^2} \exp \left( -\frac{r^2}{2\sigma_r^2} \right) \, dr
\]

(19)

\[
b = 10\beta / (\ln 10 \sqrt{2\pi\sigma})
\]

(20)

Since \( M \) is the number of RNs detecting the BN and is determined by the total number of RNs \( N \) and the channel transmission model (2), it may change in each positioning process. To evaluate the average performance, the average CRLB is defined as:

\[
\text{CRLB}_{\text{average}} = E \left[ \text{CRLB} \right] = E \left[ \frac{4}{\tilde{\psi}^2 M} \right] = \frac{4}{\tilde{\psi}^2} E \left[ \frac{1}{M} \right]
\]

(21)
Clearly, numerical calculation can be directly used to compute \( E \{ \frac{1}{M} \} \). For further performance analysis, an analytic method is presented here. With a sufficiently large \( M \), the expected mean can be replaced with the sample mean [34], and the average CRLB is approximated as:

\[
CRLB_{\text{average}} \approx \frac{4}{\psi^2} \frac{1}{M} \tag{22}
\]

where

\[
\bar{M} = E \{ M \} = E \{ N \Phi (r) \}
= NE \{ \Phi (r) \} = N \int_{0}^{+\infty} \Phi (r) f (r) \, dr
= N \int_{0}^{+\infty} \Phi (r) \frac{1}{\sigma_p^2} \exp \left( - \frac{r^2}{2\sigma_p^2} \right) \, rdr
\tag{23}
\]

**B. THEORETICAL VARIANCE OF CENTROID LOCALIZATION WITH AN ARBITRARY NODE DISTRIBUTION**

Similar to CRLB, which provides a benchmark to evaluate the performance of any unbiased location algorithm, the theoretical variance is very important for performance analysis, since it is used to evaluate the real performance of a given algorithm. Although the theoretical variance of the CL algorithm has been proposed for uniform node distribution [26], the case of arbitrary node distribution is still an open issue.

This subsection derives a theoretical variance for the CL algorithm with an arbitrary node distribution.

The theoretical variance is defined as:

\[
\text{cov} (\theta) = tr \left\{ E \left( \left( \bar{\theta} - \theta \right) \left( \bar{\theta} - \theta \right)^T \right) \right\}
= E \left( \left( \bar{x} - x \right)^2 + \left( \bar{y} - y \right)^2 \right) \tag{24}
\]

Substituting the CL estimate (5) into (24) gives:

\[
\text{cov} (\theta) = E \left( \left( \frac{1}{M} \sum_{i=1}^{M} x_i - x \right)^2 + \left( \frac{1}{M} \sum_{i=1}^{M} y_i - y \right)^2 \right) \tag{25}
\]

The first term in the above equation can be written as:

\[
E \left( \left( \frac{1}{M} \sum_{i=1}^{M} x_i - x \right)^2 \right)
= E \left( \frac{1}{M} \sum_{i=1}^{M} (x_i - x)^2 \right)
= E \left( \frac{1}{M^2} \sum_{i=1}^{M} (x_i - x)^2 + \sum_{i=1}^{M} \sum_{i=1}^{M} (x_i - x)(x_j - x) \right) \tag{26}
\]

Note that \( x_i \) and \( x_j \) are independent in the case of \( i \neq j \), and it can be obtained from the RN node distribution (3) that \( E (x_i - x) = 0 \). Thus, (26) can be reduced to:

\[
E \left( \left( \frac{1}{M} \sum_{i=1}^{M} x_i - x \right)^2 \right) = E \left( \frac{1}{M^2} \sum_{i=1}^{M} (x_i - x)^2 \right) \tag{27}
\]

Similarly,

\[
E \left( \left( \frac{1}{M} \sum_{i=1}^{M} y_i - y \right)^2 \right) = E \left( \frac{1}{M^2} \sum_{i=1}^{M} (y_i - y)^2 \right) \tag{28}
\]

Substituting (27) and (28) into (25), the theoretical variance of the CL algorithm can be calculated as:

\[
\text{cov} (\theta) = E \left( \frac{1}{M^2} \sum_{i=1}^{M} r_i^2 \right) = E \left( \frac{1}{M} \bar{r}^2 \approx \frac{\bar{r}^2}{M} \right) \tag{29}
\]

Similar to (22), \( E \{ 1/M \} \) can be solved using a numerical method or approximation (23). \( \bar{r}^2 \) can be calculated as:

\[
\bar{r}^2 = \frac{1}{\gamma} \int_{0}^{+\infty} r^2 \Phi (r) f (r) \, dr \tag{30}
\]

where \( f (r) \) is the PDF of the range between the RNs and BN. Using different \( f (r) \), the theoretical variance of CL with an arbitrary node distribution can be calculated using (29) and (30).

For the Gaussian node distribution, \( f (r) \) can be obtained from (4):

\[
f (r) = \frac{1}{\sigma_p^2} \exp \left( - \frac{r^2}{2\sigma_p^2} \right) \quad 0 < r \tag{31}
\]

For a uniform node distribution, \( f (r) \) is:

\[
f (r) = \frac{2r}{R^2} \quad 0 < r < R \tag{32}
\]

where \( R \) is the distributed radius of the uniform node distribution.

**C. CHARACTERISTICS OF THE PROPOSED CRLB AND THE THEORETICAL VARIANCE**

The characteristics of the proposed CRLB and the theoretical variance are given in the following propositions.

**Proposition 1**: With a high received power, the proposed CRLB can be approximated as:

\[
CRLB_{\text{H}} \approx \frac{2\sigma_p^2}{N} \tag{33}
\]

**Proof**: Note that \( \Phi (r) \) is the detection probability. For high received power \( P_r \), \( \Phi (r) \rightarrow 1 \) and \( \varphi (r) \ll 0 \), which leads to

\[
\frac{1}{\Phi (r)} b \exp \left( - \frac{\varphi (r)^2}{2} \right) \rightarrow 0 \tag{34}
\]

\[
\gamma = \int_{0}^{+\infty} \Phi (r) f (r) \, dr \rightarrow 1 \tag{35}
\]
Substituting (34) and (35) into (19) gives:
\[
\tilde{\psi}^2 = \int_0^{+\infty} \frac{r^3}{\sigma_p^6} \exp\left(-\frac{r^2}{2\sigma_p^2}\right) dr
\]
\[
= -\int_0^{+\infty} \frac{r^2}{\sigma_p^4} d \exp\left(-\frac{r^2}{2\sigma_p^2}\right)
\]
\[
= -\int_0^{+\infty} \frac{2}{\sigma_p^3} \exp\left(-\frac{r^2}{2\sigma_p^2}\right) d r
\]
\[
= -\frac{2}{\sigma_p^2} \exp\left(-\frac{r^2}{2\sigma_p^2}\right) \bigg|_0^{+\infty} = \frac{2}{\sigma_p^2} \tag{36}
\]
Substituting \(\Phi (\gamma) \rightarrow 1\) into (23) gives:
\[
\bar{M} = N \int_0^{+\infty} \frac{1}{\sigma_p^2} \exp\left(-\frac{r^2}{2\sigma_p^2}\right) r dr = N \tag{37}
\]
Substituting (36) and (37) into (22) gives:
\[
CRLB_H \approx \frac{4 \sigma_p^2}{\tilde{\psi}^2 \bar{M}} = \frac{2\sigma_p^2}{N} \tag{38}
\]

**Remark 1:** Proposition 1 shows that the performance of the range-free method mainly depends on the node distribution of the RNs rather than the channel environment in the case of a high signal-to-noise ratio (SNR). This phenomenon occurs because all the RNs can link to a BN with a good channel environment.

Furthermore, the effect of the density \(\lambda\) on the proposed CRLB is provided in the following proposition.

**Proposition 2:** With a high received power, the proposed CRLB can be approximated using the density \(\lambda\) as:
\[
CRLB_H \approx \frac{1}{5.95 \pi \lambda} \tag{39}
\]

**Proof:** The density \(\lambda\) is defined as the number of RNs per unit area \(\lambda = N/D\), where \(D\) is the main distributed area. For a Gaussian node distribution, \(D\) is usually defined as a 3 std area that covers 99.74% of the nodes. From (4), the range PDF is a Rayleigh distribution. The left-tail probability of the Rayleigh distribution is
\[
\int_0^R f (r) dr = \int_0^R \frac{1}{\sigma_p^2} \exp\left(-\frac{r^2}{2\sigma_p^2}\right) r dr = -\exp\left(-\frac{R^2}{2\sigma_p^2}\right) \bigg|_0^R
\]
\[
= 1 - \exp\left(-\frac{R^2}{2\sigma_p^2}\right) = 0.9974 \tag{40}
\]
where \(R\) is the distributed radius and can be solved from the above equation:
\[
R = 3.45 \sigma_p \tag{41}
\]
The main distributed area is \(D = \pi R^2 = 11.9 \pi \sigma_p^2\), and the density \(\lambda\) is calculated as:
\[
\lambda = N/D = N/\left(11.9 \pi \sigma_p^2\right) \tag{42}
\]
Substituting (42) into (33), Proposition 2 holds.

Proposition 2 shows that the CRLB is inversely correlated with \(\lambda\). This means that a more densely distributed network will lead to higher positioning accuracy. Thus, the range-free method is a preferred solution for a UDN.

The relationship between the practical performance of CL and the proposed CRLB is provided in the following proposition.

**Proposition 3:** In the case of a Gaussian node distribution and high received power, the theoretical variance of centroid-based localization is equal to the CRLB:
\[
\text{cov} (\theta)_H = CRLB_H = \frac{2 \sigma_p^2}{N} \tag{43}
\]

**Proof:** Note that \(\Phi (\gamma) \rightarrow 1\) and \(\gamma \rightarrow 1\) in the case of high received power. Substituting into (30) gives:
\[
r^2 = \int_0^{+\infty} \frac{r^2}{\sigma_p^2} \exp\left(-\frac{r^2}{2\sigma_p^2}\right) dr \tag{44}
\]
Comparing (44) with (36), it is seen that:
\[
r^2 = \sigma_p^2 \tilde{\psi}^2 = 2 \sigma_p^2 \tag{45}
\]
Substituting (37) and (45) into (29) gives:
\[
\text{cov} (\theta)_H = \frac{r^2}{\bar{M}} = \frac{2 \sigma_p^2}{N} = CRLB_H \tag{46}
\]

**Remark 2:** Since the theoretical variance represents the practical performance of the CL method, it can be determined from Proposition 3 that the CL method can reach the CRLB in the high-SNR case. This can be explained by the fact that almost all the RNs around the BN can communicate with the BN in the case of a high SNR. Therefore, the joint PDF (11) reduces to a Gaussian node distribution (3) for \(\Phi (\gamma) \rightarrow 1\) and \(\gamma \rightarrow 1\). For the Gaussian PDF (3), the centroid estimate (5) is an MLE. It is well known that the MLE is asymptotically unbiased and can asymptotically attain the CRLB with sufficiently large measurements. It is asymptotically efficient and optimal [49]. Therefore, CL has optimal performance in the case of a high SNR. Proposition 3 also proves the effectiveness of the proposed CRLB. For a more practical channel with changing SNRs, a novel location method based on the MLE is proposed in the following section to improve performance.

**IV. MLE OF CONNECTIVITY-BASED LOCALIZATION**

The proposed CRLB in section III indicates the best achievable positioning accuracy of the range-free localization technique. This section proposes a novel location method based on an MLE that addresses another unsolved problem, namely, how to attain the CRLB using range-free localization.
As discussed in Proposition 3, the CL algorithm becomes the MLE and has optimal performance for the high SNR case. In a real channel situation, the SNR is not always high, which will degrade the performance of CL. This section derives an iterative method based on the MLE to obtain higher positioning accuracy.

The MLE is found by maximizing the PDF (10) or, equivalently, by maximizing the likelihood function.

\[
J(\theta) = \ln f(s; \theta) = \sum_{i=1}^{M} \ln f(x_i, y_i; \theta)
\]

\[
= \sum_{i=1}^{M} \ln \Phi(r_i) + \ln f(x_i, y_i) - \ln \gamma
\]

\[
= \sum_{i=1}^{M} \left( \ln \Phi(r_i) - \ln \frac{r_i^2}{2\sigma_p^2} - \ln \left( 2\pi\sigma_p^2 \right) - \ln \gamma \right)
\]

An iterative method of Newton-Raphson is developed here to make (47) solvable.

The iterative method attempts to maximize the likelihood function (47) by finding a zero of the derivative function. Using \( \partial J(\theta) / \partial \theta = 0 \), we have:

\[
g_1(\theta) = \sum_{i=1}^{M} \kappa(r_i) (x - x_i) = 0
\]

\[
g_2(\theta) = \sum_{i=1}^{M} \kappa(r_i) (y - y_i) = 0
\]

where \( \kappa(r_i) = \psi(r_i) / r_i \) and \( \psi(r_i) \) can be obtained from (68). Assume that there is an initial guess for the solution to (48). Using the first-order approximation of the Taylor expansion, (48) becomes:

\[
g_1(\theta) \approx g_1(\theta_0) + \frac{\partial g_1(\theta)}{\partial x} \bigg|_{\theta=\theta_0} (x - x_0) + \frac{\partial g_1(\theta)}{\partial y} \bigg|_{\theta=\theta_0} (y - y_0) \approx 0
\]

\[
g_2(\theta) \approx g_2(\theta_0) + \frac{\partial g_2(\theta)}{\partial x} \bigg|_{\theta=\theta_0} (x - x_0) + \frac{\partial g_2(\theta)}{\partial y} \bigg|_{\theta=\theta_0} (y - y_0) \approx 0
\]

where

\[
\frac{\partial g_1(\theta)}{\partial x} = \sum_{i=1}^{M} \frac{\partial \kappa(r_i)}{\partial x} (x - x_i) + \kappa(r_i)
\]

\[
\frac{\partial g_1(\theta)}{\partial y} = \sum_{i=1}^{M} \frac{\partial \kappa(r_i)}{\partial y} (x - x_i)
\]

\[
\frac{\partial g_2(\theta)}{\partial x} = \sum_{i=1}^{M} \frac{\partial \kappa(r_i)}{\partial x} (y - y_i)
\]

\[
\frac{\partial g_2(\theta)}{\partial y} = \sum_{i=1}^{M} \frac{\partial \kappa(r_i)}{\partial y} (y - y_i) + \kappa(r_i)
\]

Substituting (68) into \( \kappa(r_i) \) gives:

\[
\kappa(r_i) = \psi(r_i) / r_i = -\frac{1}{\Phi(r_i) r_i^2} b \exp \left( -\frac{\psi(r_i)^2}{2} \right) + \frac{1}{\sigma_p^2} (52)
\]

Differentiating (52) with respect to \( r_i \) produces:

\[
\frac{\partial \kappa(r_i)}{\partial r_i} = b e^{\psi(r_i)^2/2} \frac{\partial \psi(r_i)}{\partial r_i} \Phi(r_i) r_i^2 + \frac{\partial \Phi(r_i)}{\partial r_i} = \frac{\partial \Phi(r_i)}{\partial r_i} r_i^2 + 2r_i \Phi(r_i) \cdot (55)
\]

Substituting (63) into (55) gives:

\[
\frac{\partial (\Phi(r_i) r_i^2)}{\partial r_i} = -b \exp \left( -\frac{\psi(r_i)^2}{2} \right) r_i + 2r_i \Phi(r_i) \cdot (56)
\]

Substituting (54) and (56) into (53) gives:

\[
\frac{\partial \kappa(r_i)}{\partial r_i} = r_i b \phi(r_i)
\]

where

\[
\phi(r_i) = \exp \left( -\frac{\psi(r_i)^2}{2} \right)
\]

Substituting (57) and (64) into (51) gives:

\[
\frac{\partial \kappa(r_i)}{\partial x} = b (x - x_i) \phi(r_i)
\]

\[
\frac{\partial \kappa(r_i)}{\partial y} = b (y - y_i) \phi(r_i)
\]

Substituting (59) into (50), \( \partial g_i(\theta) / \partial \theta_j \) can be obtained. Expressing (49) in matrix form gives:

\[
H(\theta_{k+1} - \theta_k) = -G\cdot (60)
\]

where \( \theta_k \) is the kth iterative estimate of \( \theta \).
Since $G$ is the derivative of the log-likelihood function, we find the MLE as:

$$\theta_{k+1} = \theta_k - \left( H^T H \right)^{-1} H^T G \quad (61)$$

Note that at convergence, $\theta_{k+1} = \theta_k$ and from (49), $g_1(\theta_k) = g_2(\theta_k) = 0$, as desired.

It is well known that the iteration of Newton-Raphson may not converge. This fact will be particularly evident when $H$ is a singular matrix. In this case, it is seen from (61) that the correction term may fluctuate greatly from iteration to iteration. To avoid divergence, a simple convergence test is provided here. Assume that $\theta_C = [x_C \ y_C]^T$ and $\theta_I = [x_I \ y_I]^T$ are the final estimates of the CL method (5) and the proposed method (61), respectively. The following condition can be used to determine whether the iterative method converges or diverges.

$$|\theta_C - \theta_I| \leq 3\sqrt{\text{cov}(\theta)}$$  \quad \text{for the convergence case}

$$|\theta_C - \theta_I| > 3\sqrt{\text{cov}(\theta)}$$  \quad \text{for the divergence case}

For the case of convergence, $\theta_I$ collected from the iterative method (61) is chosen as the final estimate. Otherwise, the proposed method selects $\theta_C$ (5) as the location result.

Other numerical iterative methods, such as the MATLAB functions ‘fminsearch’ and ‘fsolve’, can also be used to solve (47) and (48).

The contributions of the proposed algorithm are summarized here:

1. The cost functions (47) and (48) are first proposed here based on the MLE. Using the proposed cost functions (47) and (48), other numerical iterative methods can be developed to obtain the optimal performance.

2. Compared with standard numerical iterative methods, the proposed method gives analytic formulas that are more suitable for application in embedded systems. The simulation results in Section V show that the proposed method has the optimal performance and the shortest running time.

3. A convergence test is given here to avoid divergence.

V. PERFORMANCE EVALUATION

In this section, we evaluate the performance of the range-free localization in a UDN with randomly distributed nodes and verify the accuracy of the analytical results obtained. The root-mean-square errors (RMSEs) are defined as

$$\sqrt{E\left[ (x - \bar{x})^2 + (y - \bar{y})^2 \right]}$$

in units of m. The statistical results are obtained from the average of 30000 independent runs. The simulation parameters are set as follows: $P_{th} = -60\text{dBm}$, $N = 40$, $\sigma_p = 30\text{m}$, $\beta = 4$, and $\sigma = 4\text{dB}$. For each simulation, some simulation parameters may be different from the values listed above. We will specify the difference in every simulation.

Fig. 1 compares the theoretical variances and the CL algorithm in the case with different reference powers. Three theoretical variances, including the proposed theoretical variances (29) for the uniform and Gaussian node distributions and the existing theoretical variance [26] derived for a uniform node distribution, are compared in the simulation. To compare the two node distributions under the same coverage area, the std of the Gaussian distribution is set to $\sigma_p = R/3.45$, where $R$ is the distributed radius of the uniform distribution. As discussed in Proposition 2, the distributed radius $R = 3.45\sigma_p$ covers 99.7% of the RNs in the Gaussian node distribution. In Fig. 1, the reference power $P_0$ varies from -15 dBm to 30 dBm, and at least one RN detects the BN. It can be seen from the figure that the three theoretical variances match well with the results obtained from the CL method. This result proves the effectiveness of the proposed theoretical variances (29). For a uniform node distribution, the proposed theoretical variance provides a more accurate evaluation of the CL method than the existing theoretical variance in [26]. This result occurs because the authors in [26] use the approximation method to obtain an analytical solution for uniform node distribution. With the same coverage area, the case of a Gaussian node distribution has a higher positioning accuracy than that of a uniform node distribution, as shown in Fig. 1. Comparing the PDFs between the uniform and Gaussian cases, more RNs are located around the BN in the case of the Gaussian node distribution than in that of the uniform node distribution, which leads to better performance in the Gaussian node distribution case. Fig. 1 also shows that there is a nonlinear relationship between the positioning error and the reference power $P_0$. Increasing transmit power will result in a higher location accuracy in range-based location techniques, such as RSS localization, whereas this conclusion may not hold in the case of range-free methods. In contrast to range-based localization, the range-free method cannot obtain the absolute distance between RNs and the BN. A larger $P_0$ in the range-free method may lead to a greater number of RNs detecting the BN. However, those RNs may be located farther from the BN, which may degrade performance. Of course, a value of $P_0$ that is too low may result in a high failure probability of localization, where no RN detects the BN, as shown in Fig. 2. It can be seen from the figure that the failure probability of localization increases.
as $P_0$ decreases. For $P_0 \leq -10dBm$, the failure probability for the uniform node distribution exceeds 25%. Obviously, such a high failure probability is not acceptable for a practical system. Thus, when choosing a proper reference power, we should first consider the failure probability of localization rather than the positioning accuracy. Fig. 2 also shows that the failure probability of the Gaussian node distribution is much smaller than that of the uniform node distribution. This phenomenon is caused by the fact that more RNs are located around the BN in the case of the Gaussian node distribution, as discussed below Equation (4).

Fig. 3 is shown to study the difference between the proposed CRLBs (21) and (22). (21) is an exact CRLB, while (22) approximates (21) in the case with a sufficiently large $N$. In the figure, $P_0 = 5dBm$ and the number of RNs varies from 5 to 55. It can be seen from the figure that the approximate CRLB (22) asymptotically attains the exact CRLB (21) as N increases. For N=10, the difference between the two CRLBs is less than 4%. Therefore, the proposed CRLBs (21) and (22) can be substituted for each other in a practical channel environment.

Fig. 4 shows two CRLBs and the theoretical variance comparison with different reference powers $P_0$. It can be seen from the figure that $CRLB_H$ (33) matches well with $CRLB_{average}$ (21) in the case of high reference power ($P_0 \geq 30dBm$), which proves the effectiveness of Proposition 1. The figure also shows that the theoretical variance (29) can asymptotically reach the CRLB as the reference power $P_0$ increases. Thus, the CL method has optimal performance for the case of high reference power, such as $P_0 \geq 20dBm$. For the case of low $P_0$, there still exists a performance gap between the CL method and CRLB. Fig. 4 conforms to the analysis results of Proposition 3.

The effect of density $\lambda$ on the positioning accuracy is analyzed in Fig. 5. In this figure, $P_0 = 30dBm$, and the density $\lambda$ varies from 0.0003 to 0.0119. Two CRLBs, (21) and (39), are compared in Fig. 5. It is observed that the approximate CRLB (39) and exact CRLB (21) are a good match for various densities $\lambda$ in the case of high reference power $P_0 = 30dBm$. According to (39), the proposed CRLB in a good channel environment depends completely on the density $\lambda$. Increasing $\lambda$ can effectively improve system performance.
Performance comparisons between the proposed method and the CL method (5) are recorded in the following figures. The proposed CRLB (21) and theoretical variance (29) are also included in the figures. Fig. 6 is shown to compare the performance of the methods in cases with different \( P_0 \). In Fig. 6, \( N = 30 \), \( \sigma_p = 10 m \), and \( P_0 \) varies from \(-15 \) dBm to 30 dBm. The figure shows that the proposed method based on the MLE provides a better performance than the CL method and can attain the CRLB in the case of different values of \( P_0 \). Thus, the proposed method has optimal performance in various channel environments.

Fig. 7 is shown to evaluate the performance of the two methods for various numbers of RNs when \( P_0 = 5dBm \) and the number of RNs varies from 30 to 90. Fig. 7 shows that the number of RNs can effectively reduce location error, and the proposed method provides a higher positioning accuracy than the CL method.

Performance comparisons between two methods in various channel environments are presented in Figs. 8 and 9. Fig. 8 shows performance comparison with different stds \( \sigma \) of \( P_i \). In the figure, \( P_0 = 15dBm \), \( N = 60 \), and the std \( \sigma \) of \( P_i \) varies from 2 dB to 6 dB. Performance comparisons with different path-loss exponents \( \beta \) are shown in Fig. 9. In the figure, \( P_0 = 15dBm \), \( N = 60 \), and \( \beta \) varies from 2 to 5. Both Figs. 8 and 9 show that the proposed method works well in various channel situations.

The following simulations are performed to compare the proposed iterative method with standard numerical iterative algorithms. Two MATLAB functions, ‘fminsearch’ and ‘fsolve’, are selected here. The former is used to minimize the cost function \( |J| \) (47), while the nonlinear Equation (48) is solved by ‘fsolve’. The simulation settings of Figs. 10 and 11 are the same as in Fig. 7. It can be seen from Fig. 10 that both the proposed method and ‘fsolve’ perform better than ‘fminsearch’. This improve performance is because the nonlinear Equation (48) provides a clearer description for the MLE than the cost function (47). Although the proposed method and ‘fsolve’ have the optimal performance, the average running time of the proposed method is only one third of that of ‘fsolve’, as shown in Fig. 11. Thus, the proposed method is more suitable for embedded systems.
VI. CONCLUSION

Because of the hardware limitations of nodes in a UDN, solutions in range-free localization are considered a cost-effective selection compared with more expensive range-based algorithms. This paper derives the CRLB and theoretical variance for performance analyses of range-free localization. The proposed CRLB provides a benchmark to evaluate the performance of any unbiased range-free location algorithm, and the real performance of CL with an arbitrary node distribution can be evaluated by the proposed theoretical variance. Theoretical analysis shows that CRLB is inversely correlated with the density of the UDN. Thus, a more densely distributed network will lead to higher positioning accuracy. The optimal estimator based on an MLE combining the RN selection compared with more expensive range-based algorithms.

Simulation results show that the proposed method has better performance than the CL method and can asymptotically attain the CRLB. An analysis that incorporates multihop communication, mobility-assisted UDNs, and RN location error will be left for a future study.

APPENDIX

Substituting (14) into $\frac{\partial \Phi (r_i)}{\partial r_i}$ gives:

$$\frac{\partial \Phi (r_i)}{\partial r_i} = \frac{b}{r_i} \int_{P_{th}}^{+\infty} \left( -\frac{P_i - P_0 + 10\beta \log_{10}(r_i)}{\sigma^2} \right) \exp \left( -\frac{(P_i - P_0 + 10\beta \log_{10}(r_i))^2}{2\sigma^2} \right) dP_i \tag{62}$$

where $b = 10\beta / \ln 10 \sqrt{2\pi} \sigma$. Letting $v = (P_i - P_0 + 10\beta \log_{10}(r_i)) / \sigma$, we have:

$$\frac{\partial \Phi (r_i)}{\partial r_i} = \frac{b}{r_i} \int_{P_{th} - P_0 + 10\beta \log_{10}(r_i)}^{+\infty} \left( -\frac{v^2}{2} \right) dv \tag{63}$$

From (1), $\frac{\partial r_i}{\partial x}$ can be calculated as

$$\frac{\partial r_i}{\partial x} = \frac{x - x_i}{r_i} = \cos \phi_i \tag{64}$$

From (63) and (64), $\frac{\partial \ln \Phi (r_i)}{\partial x}$ can be calculated as

$$\frac{\partial \ln \Phi (r_i)}{\partial x} = \frac{1}{\Phi (r_i)} \frac{\partial \Phi (r_i)}{\partial r_i} \frac{\partial r_i}{\partial x}$$

$$= -\frac{1}{\Phi (r_i)} \frac{b}{r_i} \exp \left( -\frac{v^2}{2} \right) \cos \phi_i \tag{65}$$

Substituting (3) into $\frac{\partial \ln f (x_i, y_i)}{\partial x}$ gives:

$$\frac{\partial \ln f (x_i, y_i)}{\partial x} = -\frac{r_i}{\sigma_p^2} \cos \phi_i \tag{66}$$

Combining (65) and (66) gives:

$$\frac{\partial \ln f (x_i, y_i)}{\partial x} = \frac{\partial \ln \Phi (r_i)}{\partial x} + \frac{\partial \ln f (x_i, y_i)}{\partial x} - \frac{\partial \ln \gamma}{\partial x} = \psi (r_i) \cos (\phi_i) \tag{67}$$

where

$$\psi (r_i) = -\left( \frac{1}{\Phi (r_i)} \frac{b}{r_i} \exp \left( -\frac{v^2}{2} \right) + \frac{r_i}{\sigma_p^2} \right) \tag{68}$$

The expectation of $\frac{\partial \ln f (x_i, y_i; \theta)}{\partial x}$ can be obtained from (67) and (68):

$$E \left[ \frac{\partial \ln f (x_i, y_i; \theta)}{\partial x} \right] = E \left[ \psi (r_i) \right] \int_{-\pi}^{+\pi} \frac{\cos \phi_i}{2\pi} d\phi_i \tag{69}$$

Clearly, the second term of (69) equals 0. Thus,

$$E \left[ \frac{\partial \ln f (x_i, y_i; \theta)}{\partial x} \right] = 0 \tag{70}$$

From (10) and (70), the expectation of $\frac{\partial \ln f (s; \theta)}{\partial x}$ is

$$E \left[ \frac{\partial \ln f (s; \theta)}{\partial x} \right] = \sum_{i=1}^{M} E \left[ \frac{\partial \ln f (x_i, y_i; \theta)}{\partial x} \right] = 0 \tag{71}$$

Similarly,

$$\frac{\partial \ln f (s; \theta)}{\partial y} = \sum_{i=1}^{M} \frac{\partial \ln f (x_i, y_i; \theta)}{\partial y} \tag{72}$$
\[
\frac{\partial \ln f(x_i, y_j; \theta)}{\partial y} = \psi(r_i) \sin(\phi_i) \quad (73)
\]

Similarly,
\[
E \left[ \frac{\partial \ln f(s; \theta)}{\partial y} \right]^2 = \frac{M}{2} \psi^2 \quad (80)
\]

From (67) and (73), \( J_{xy} \) can be obtained:
\[
E \left[ \left( \frac{\partial \ln f(s; \theta)}{\partial x} \right) \left( \frac{\partial \ln f(s; \theta)}{\partial y} \right) \right] = E \left[ \sum_{i=1}^{M} \psi(r_i) \cos(\phi_i) \left( \sum_{j=1}^{M} \psi(r_j) \sin(\phi_j) \right) \right]
\]

where
\[
E \left[ \cos(\phi_i) \sin(\phi_i) \right] = \int_{-\pi}^{\pi} \cos(\phi_i) \sin(\phi_i) d\phi_i = \frac{\sin(\phi_i)}{2\pi} \bigg|_{-\pi}^{\pi} = 0 \quad (82)
\]

Substituting (82) into (81) gives:
\[
E \left[ \frac{\partial \ln f(s; \theta)}{\partial x} \right] \left( \frac{\partial \ln f(s; \theta)}{\partial y} \right) = 0 \quad (83)
\]

Substituting (79)-(81) into (9), the FIM becomes:
\[
J_\theta = \frac{M}{2} \bar{\psi}^2 I \quad (84)
\]

Finally, the proposed CRLB is derived as:
\[
CRLB = tr \left[ J_\theta^{-1} \right] = \frac{4}{\psi^2 M} \quad (85)
\]

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