A Greedy Heuristic for Crossing-Angle Maximization

Graph Drawing 2018

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Crossing-Angle Maximization

crossing angle $\text{cr-} \angle (e, f)$

crossing angle $\text{cr-} \angle (\Gamma)$ of a drawing:
smallest crossing angle of two crossing edges

A Lot of Theory

NP-Hardness
1, 2, 3 - bends per edge
Counting edges of RAC graphs

Practice before GD’17 contest
2 force-directed algorithms
[Argyriou et al., '13, Huang et al., '10]
Crossing-Angle Maximization

crossing angle $\text{cr-} \angle (e, f)$

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**Crossing-Angle Maximization**

Compute a straight-line drawing $\Gamma$ of $G$ that maximizes $\text{cr-} \angle (\Gamma)$

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**A Lot of Theory**

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... [Argyriou et al., Arikushi et al, Didimo et al., Djumović et al., ...]

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Our Heuristic
Motivation: Win Graph Drawing Contest

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Challenge: No Restriction on Input
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**Our Heuristic**

**Design Goals**
- Fast
- Generic
- Easy to implement
Our Heuristic

Initial Drawing

select a vertex $v$

Move $v$ to a better position

repeat
Our Heuristic

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Initial Drawing → select a vertex $v$ → Move $v$ to a better position

repeat
Our Heuristic

Initial Drawing

Select a vertex $v$

Repeat

$S$: set of random points in $\mathbb{R}^2$

Move $v$ to best position in $S$
Our Heuristic

Initial Drawing

- select a vertex $v$

- repeat

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Our Heuristic

Initial Drawing

select a vertex $v$

repeat

restrict $S$ to a square $R$ around $v$

$S$: set of random points in $R$

Move $v$ to best position in $S$

fine tune
Our Heuristic

Initial Drawing

- select a vertex $v$
- repeat

$S$: set of random points in $R$

- Move $v$ to best position in $S$
- restrict $S$ to a square $R$ around $v$

fine tune
Our Heuristic

Initial Drawing

- select a vertex $v$

Repeat

- restrict $S$ to a square $R$ around $v$
- $S$: set of random points in $R$
- Move $v$ to best position in $S$
- fine tune
Our Heuristic

Initial Drawing

select a vertex $v$

repeat

$S$: set of random points in $R$

Move $v$ to best position in $S$

restrict $S$ to a square $R$ around $v$

fine tune
Our Heuristic

Initial Drawing

1. Select a vertex $v$
2. Repeat
   - Restrict $S$ to a square $R$ around $v$
   - Move $v$ to best position in $S$
3. Fine tune

$S$: set of random points in $R$

\[ R \]
Evaluation
Test Instances

- **North**
  - collection of small real world graphs

- **Rome**
  - collection of small real world graphs

- **Community**
  - Resembles community structure

- **1-Planar**

- **Triangulation + X**
  - Triangulation + set of random edges

100 randomly selected graphs per class
Research Questions

Q: What is good parametrization of our algorithm?

Q: What is a good choice for an initial drawing?

Q: Does our heuristic improve the crossing angle?
Research Questions

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Good Initial Drawing

Initial Drawing Styles:

**Random:** random position per vertex

**Fr+Cos:** Force-Directed + Angle Max. Force [Huang et al. '14]

**Stress:** Stress Majorization [Gansner et al. '05, OGDF]

**cr-small:** Drawing with small number of crossings [R. et al.'18]

Observations

Random seems to be a bad choice

Tendence towards Fr+Cos
Good Initial Drawing

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Q: Is Fr+Cos a good initial drawing?

North  Rome  Comm.  1-Planar  Triang.
**Q:** Is Fr+Cos a good initial drawing?

### North

![North Diagram](image1.png)

### Rome

![Rome Diagram](image2.png)

### Comm.

![Comm. Diagram](image3.png)

### 1-Planar

![1-Planar Diagram](image4.png)

### Triang.

![Triang. Diagram](image5.png)
Good Initial Drawing

Q: Is Fr+Cos a good initial drawing?

North | Rome | Comm. | 1-Planar | Triang.
---|---|---|---|---
stress
cr-small
random
**Good Initial Drawing**

**Q:** Is Fr+Cos a good initial drawing?

North | Rome | Comm. |
--- | --- | --- |

1-Planar | Triang.

| stress | cr-small | random |
| --- | --- | --- |
| 90 | 90 | 90 |
| 60 | 60 | 60 |
| 30 | 30 | 30 |
| 0 | 0 | 0 |
| 90 | 90 | 90 |

Need for a tool to compare paired drawings.
Tool to Compare Paired Drawings

**Input:** Ground set of Graphs \( G = \{G_1, G_2, \ldots, G_n\} \)

Two sets of drawings of \( G \)  
\( \{\Gamma[G_i] \mid G_i \in G\} \)  \( \{\Pi[G_i] \mid G_i \in G\} \)
Tool to Compare Paired Drawings

Input: Ground set of Graphs $\mathcal{G} = \{G_1, G_2, \ldots, G_n\}$

Two sets of drawings of $\mathcal{G}$: $\{\Gamma[G_i] | G_i \in \mathcal{G}\}$ and $\{\Pi[G_i] | G_i \in \mathcal{G}\}$

Q: Do the drawings $\Gamma$ have a larger crossing angle than $\Pi$?

for all $G_i$: $\text{cr-}\angle(\Gamma[G_i]) > \text{cr-}\angle(\Pi[G_i])$
Tool to Compare Paired Drawings

**Input:** Ground set of Graphs $\mathcal{G} = \{G_1, G_2, \ldots, G_n\}$

Two sets of drawings of $\mathcal{G}$

\[
\{\Gamma[G_i] \mid G_i \in \mathcal{G}\} \quad \{\Pi[G_i] \mid G_i \in \mathcal{G}\}
\]

A number $p \in [0, 1]$

**Q:** Do the drawings $\Gamma$ have a larger crossing angle than $\Pi$?

**Is there:** a subset $\mathcal{G}' \subseteq \mathcal{G}$, $|\mathcal{G}'| > p \cdot |\mathcal{G}|$ such that

for all $G_i \in \mathcal{G}'$:

\[
\text{cr-}\angle(\Gamma[G_i]) > \text{cr-}\angle(\Pi[G_i])
\]
Tool to Compare Paired Drawings

**Input:** Ground set of Graphs \( \mathcal{G} = \{ G_1, G_2, \ldots, G_n \} \)

Two sets of drawings of \( \mathcal{G} \) \( \{ \Gamma[G_i] \mid G_i \in \mathcal{G} \} \) \( \{ \Pi[G_i] \mid G_i \in \mathcal{G} \} \)

A number \( p \in [0, 1] \), \( \Delta > 0 \)

**Q:** Do the drawings \( \Gamma \) have a larger crossing angle than \( \Pi \)?

**Is there:** a subset \( \mathcal{G}' \subseteq \mathcal{G}, |\mathcal{G}'| > p \cdot |\mathcal{G}| \) such that

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Graphs and histograms showing the distribution of crossing angles and related measurements.
Good Initial Drawing

**Q:** Is Fr+Cos a good initial drawing?

North  Rome  Comm.  1-Planar  Triang.

- **stress**
- **cr-small**
- **random**
**Good Initial Drawing**

**Q:** Is Fr+Cos a good initial drawing?

| North | Rome | Comm. |
|-------|------|-------|

**fr-cos vs stress**

**fr-cos vs cr-small**
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Improvement of the Crossing Angle

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![Graph showing the improvement of the crossing angle for different datasets and parameters.](image-url)
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Move $v$ to best position in $S$
restrict $S$ to a square $R$ around $v$

fine tune
Our Heuristic

Initial Drawing

select a vertex \( v \)

repeat

What is a good next candidate?

\[ S: \text{ set of random points in } R \]

Move \( v \) to best position in \( S \)

restrict \( S \) to a square \( R \) around \( v \)

fine tune
Running Time

**Task** Find edges \( e,f \) s.t. \( \text{cr-} \angle (\Gamma, e, f) = \text{cr-} \angle (\Gamma) \)

Possibility *Sweep*: Sweep-Line Algorithm

Possibility *Bucket*:
- sort edges into buckets according to slopes
- edges of adjacent buckets form \( \text{cr-} \angle (\Gamma) \)

**Time to move a single vertex**
Running Time

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Time to move a single vertex
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Possibility **Sweep**: Sweep-Line Algorithm

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Time to move a single vertex

![Graph showing time to move a single vertex vs. #vertices for different sweep types]
Conclusion

a simple heuristic for Crossing-Angle Maximization
easy to implement
generic

Future Work

Drawings are not necessarily readable
Let $R$ be a region that ensure some properties of $v$
Optimize position of $v$ within $R$
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GD Contest duplicated the number of applied papers on Cr. Angle Max ;-) 

Future Work

Drawings are not necessarily readable
Let $R$ be a region that ensure some properties of $v$
Optimize position of $v$ within $R$
Thank you.
Good Parameter

Configurations:
- Sloppy: Fast and *inaccurate*
- Medium: Trade of between speed and accuracy
- Precise: Slow and *accurate*

**Time Limit**: $n$ seconds for an $n$-vertex graph
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allows fair comparison
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There is no obvious difference between the configurations
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There is no obvious difference between the configurations?