QCD sum rules and radial excitations of light pseudoscalar and scalar mesons

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ABSTRACT

The calculations of masses and decay constants of the radial excitations of light pseudoscalar and scalar mesons within QCD sum rules method are briefly reviewed. The predictions are based on the $1/N_c$-supported model spectra, which consist of infinite number of infinitely narrow resonances, and on the assumption that the ground states of light scalar mesons may be considered as the $\bar{q}q$-bound states. The results of the studies are compared with the existing experimental data and with the predictions of other theoretical approaches.
1 Introduction

Despite the fact that QCD is an indisputable theory of strong interactions, there are still the number of important open areas, where using QCD methods, one can arrive to different conclusions which can serve as various alternative descriptions of the results of concrete experimental studies.

The status of QCD predictions for the properties of light hadronic bound states is among opened and intriguing problems of modern phenomenology. The main puzzle is that the long-awaited glueball states have not yet well-identified candidates even in the most prominent scalar channel. Indeed, different phenomenological works indicate that they can mix with low-lying scalar mesons with the mass of over 1 GeV (see, e.g., [1], for a review). It is known that this sector is rich in different scalar resonances, like $I = 0 \delta$ or $f_0(600)$, $f_0(980)$, $f_0(1370)$, $f_0(1500)$, $I = 1/2 K^*(1430)$, $I = 1 a_0(980)$ and $a_0(1450)$ (see the most recent Particle Data Group report [2]). This list is minimal and does not include all actual resonances, which can manifest themselves, say, as the radial excitations of low-lying scalar hadronic states, systematised, e.g., in [3].

Moreover, there are different points of view on the nature of even low-lying scalar hadrons, such as $a_0(980)$ particle. Indeed, it is described by different authors either as the 4-quark state [4], $\bar{q}q$-quark mixed structure [5], or the $\bar{q}q$-bound state. The latter point of view is supported by the studies of the works [6] and [7].

The important logical check of the assumption that scalar mesons may be considered as the $\bar{q}q$-bound states is based on the investigations of the possibilities to predict masses and decay constants of their radial excitations and on the comparisons of the predictions of different methods. It is also interesting to study the results of applications of these methods in the pseudoscalar channel, where low-lying $\bar{q}q$ states are well-identified as $\pi$ and $K$ mesons.

Quite recently several approaches were developed, which have the aim to describe the properties of radial excitations of light mesons in various channels. Among others we can mention the QCD–string–inspired methods of the works [8], [9], the methods of the Effective Chiral Lagrangians [10] and the large-$N_c$ expansion–motivated approaches (see, e.g., [9]).

Let us remind that within pure $1/N_c$ expansion, originally proposed in [11], all quark antiquark mesons are becoming infinitely narrow resonances. The spectrum of the theory in the large-$N_c$ limit consists of an infinite number of these resonances, which belong to flavor nonets [12]. Of course, in the real world we have $N_c = 3 \neq \infty$ and the families of resonances with non-zero widths. However, the $1/N_c$-motivated spectrum is often used in various concrete physical considerations in four-dimensional QCD. Indeed, the phenomenological spectra, modeled by the comb of infinite number of infinitely narrow resonances was used, e.g., in [9] [13] and in the older works [14]-[16], which attracted definite interest again only recently (see, e.g., the comparison of the results obtained in [14]-[16] with the ones from [9] and [10]).

These works, like some other similar analyses, are based on the concept of duality,
proposed within the context of QCD in [17, 18] some time ago, tested in two-dimensions within 1/N_c-expansion in [19], and studied in detail in the review of [20]. The method of the QCD finite-energy sum rules (FESRs) [21], which are analogous to the dual sum rules in the theory of strong interactions [22], is another important technical tool, applied for theoretical investigations of the properties of radial excitations of mesons, performed in [14]-[16].

Since quite recently definite interest to the studies of the predictions for the spectra of radial excitations of light hadronic states was observed which is motivated in part by the experimental programs of the collaboration COMPASS at CERN and CEBAF Jefferson Laboratory facility, we decided to remind some basic theoretical results, obtained in the beginning of 90’s in the Theoretical Division of Institute for Nuclear Research, concentrating on the considerations of the pseudoscalar and scalar sectors.

2 The pseudoscalar and scalar channel: preliminaries

Let us first introduce the (pseudo)scalar quark currents

\[ \partial_\mu J^{(5)}_\mu = i m_q^{(+)} \bar{q} (\gamma_5) u , \]  

which are proportional to the divergence of (axial)vector currents:

\[ J^{(5)}_\mu = \bar{q} \gamma_\mu (\gamma_5) u , \]

where \( m_q^{(+)} = m_q \pm m_u \) are the sum and difference of the current quark masses with \( q = d, s \). The two-point functions of the (pseudo)scalar quark currents can be defined as:

\[ \Pi^{(P)S}(Q^2) = i (8 \pi^2) \int e^{i q x} \langle 0 | \partial_\mu J^{(5)}_\mu(x) \partial_\mu J^{(5)}_\mu(0) | > d^4 x . \]

where \( Q^2 = -q^2 \) is the Euclidean momentum transfer, and indexes \( P \) and \( S \) are labeling the pseudoscalar and scalar quark channels. These two-point functions have the following imaginary part

\[ R^{(P)S}_{\overline{\text{MS}}}(s/\mu^2) = 3 (m_q^{(+)} - (\mu^2))^2 s \left[ 1 + \frac{\alpha_s(\mu^2)}{\pi} \left( \frac{17}{4} C_F - \frac{3}{2} C_F \ln(s/\mu^2) \right) + \ldots \right] , \]

where \( s \) is the Minkowskian variable and \( \mu^2 \) is the normalisation point of the \( \overline{\text{MS}} \)-scheme. Here \( C_F = (N_c^2 - 1)/(2N_c) \) is one of the Casimir operators and we retain the 1-loop massless perturbative QCD correction only. The result for this correction can be extracted from the original \( \overline{\text{MS}} \)-scheme calculation of [23]. The spectral density of Eq. (4) enters in the following Euclidean function [24]

\[ D^{(P)S}(Q^2) = Q^2 \int_0^\infty \frac{R^{(P)S}_{\overline{\text{MS}}}(s/\mu^2)}{(s + Q^2)^3} ds , \]

which obeys the renormalisation group equation with anomalous mass dimension term, namely

\[ \left( \frac{\partial}{\partial \ln(\mu^2)} + \beta(a_s) \frac{\partial}{\partial a_s} + 2 \gamma_m(a_s) \frac{\partial}{\partial m_q^{(+)}} \right) D^{(P)S}(Q^2/\mu^2) = 0 . \]
Here $a_s(\mu^2) = \alpha_s(\mu^2)/(4\pi)$ and $m_q^{(+)}(\mu^2)$ are related to the renormalised coupling constant and quark masses, which depend from the normalisation point $\mu^2$. The QCD $\beta$-function and the anomalous dimension $\gamma_m$ of quark mass $m_q$ are defined as

$$\beta(a_s) = \frac{d a_s(\mu^2)}{d \ln \mu^2} = - \sum_{n \geq 0} \beta_n \left( \frac{\alpha_s}{4\pi} \right)^{n+2}, \quad (7)$$

$$\gamma_m(a_s) = \frac{d \ln m_q(\mu^2)}{d \ln (\mu^2)} = - \sum_{n \geq 0} \gamma_n \left( \frac{\alpha_s}{4\pi} \right)^{n+1}. \quad (8)$$

In the class of the $\overline{\text{MS}}$-schemes the analytical expressions for the first two coefficients of the QCD $\beta$-function and the anomalous dimension function $\gamma_m$ read:

$$\beta_0 = \left( \frac{11}{3} C_A - \frac{2}{3} N_F \right), \quad (9)$$

$$\beta_1 = \left( \frac{34}{3} C_A^2 - 2 C_F N_F - \frac{10}{3} C_A N_F \right), \quad (10)$$

$$\gamma_0 = 3 C_F, \quad (11)$$

$$\gamma_1 = \left( \frac{3}{2} C_F^2 + \frac{97}{2} C_F C_A - 5 C_F N_F \right), \quad (12)$$

where $C_F$ was introduced previously and $C_A = N_c$. Within the class of gauge-independent schemes the coefficient $\gamma_1$ is scheme dependent, while the coefficients $\beta_0$, $\beta_1$ and $\gamma_0$ do not depend from the choice of the subtraction scheme. Note that the general analytical 4-loop expressions for the QCD $\beta$-function and the $\gamma_m$-function were calculated in the works of $[25]$ and $[26]$ respectively. The results from $[26]$, expressed in terms of $N_c$, are in agreement with the outcomes of independent calculation of $[27]$. As to the perturbative corrections to the function of Eq. $(5)$, we will limit ourselves by the consideration of the following 1-loop expression:

$$D^{(P)S}(Q^2) = 3(m_q^{(+)}(Q^2))^2 \left[ 1 + \frac{11}{3} C_F \left( \frac{\alpha_s(Q^2)}{\pi} \right) + \ldots \right], \quad (13)$$

where within the $\overline{\text{MS}}$-scheme the running quark masses are related to the invariant quark masses $\hat{m}_q^{(+)}$ as

$$m_q^{(+)}(Q^2) = \hat{m}_q^{(+)} \exp \left[ \int_0^{\alpha_s(Q^2)} \frac{\gamma_m(x)}{\beta(x)} dx - \int_0^{2 \beta_0} \frac{\gamma_0}{\beta_0 x} dx + 2 \frac{\gamma_0}{\beta_0} \ln(2 \beta_0) \right]$$

$$= \hat{m}_q^{(+)} \left( \beta_0 \alpha_s(Q^2) / 2 \pi \right)^{\gamma_0 / \beta_0} \left[ 1 + \left( \frac{\gamma_1}{\beta_1} - 2 \frac{\gamma_0}{\beta_0} \ln(2 \beta_0) \right) \left( \frac{\alpha_s}{4\pi} \right) + \ldots \right], \quad (14)$$

and the 2-loop expression for the QCD coupling constant in the $\overline{\text{MS}}$-scheme, which corresponds to $N_F = 3$ numbers of active flavours, reads

$$\alpha_s(Q^2) = \frac{4\pi}{\beta_0 \ln(Q^2/\Lambda^2)} \left[ 1 - \frac{\beta_1 \ln \ln(Q^2/\Lambda^2)}{\beta_0^2 \ln(Q^2/\Lambda^2)} \right], \quad (15)$$

$$\Lambda = \Lambda_{\overline{\text{MS}}}^{(N_F=3)}. \quad (16)$$
Note that the relation between the running and the invariant quark masses was originally chosen in [23] in the way of Eq. (14) to fix the following scale-dependence of the quark running mass:

\[ m_q(\mu) = \hat{m}_q \left( \frac{1}{\ln(Q/\Lambda)} \right)^{\gamma_0/\beta_0} . \] (16)

Notice, that the application of the \(1/N_c\)-expansion is used in QCD for the choice of model phenomenological spectral function only. Of course, one can be more consistent, expanding all Casimir operators \(C_F\) and \(C_A\) in powers of \(N_c\) and keeping the leading terms of this expansion. In this case one may use the following formulae:

\[ \frac{m_q^{(+)}(Q^2)}{m_q^{(+)}(\mu^2)} = \left( \frac{\alpha_s(Q^2)}{\alpha_s(\mu^2)} \right)^{11N_c^2/2} , \] (17)

\[ \frac{\alpha_s(Q^2)}{\pi} = \frac{12\pi}{11N_c \ln(Q^2/\Lambda^2)} . \] (18)

However, we will avoid applications of these formulae in the concrete considerations, which will be discussed below.

It should be stressed, that the convention of choosing the renormalisation scale of running quark masses at \(Q^2 = 1\ \text{GeV}^2\) moved in our times to \(2\ \text{GeV}^2\). Moreover, since partly unknown QCD corrections of order \((\alpha_s/\pi)^4\) to Eq. (4) and Eq. (13) (the available results of the total calculations of this term, which are now continuing, see in [24] and [28]) may affect the precision of the determinations of light quark masses from the scalar and pseudoscalar quark channels, we will not present here any concrete results both for the running and invariant quark masses.

Before proceeding to the main part of this work let us emphasise the essential role of the \(\langle m_\bar{q}q \rangle\) and \(\langle \beta(\alpha_s)/\alpha_s \rangle C^a_{\mu \nu} G^{a\mu \nu}_a \rangle\) condensates [29] in the description of the properties of the ground states of hadrons. Indeed, together with instanton effects (see, e.g., [30, 31]), these nonperturbative contributions should be mostly important for the calculations of the hadronic ground states masses and decay width coupling constant using the OPE technique and the Borel sum rules approach [29]. Note, that the infrared renormalon calculus, rediscovered in QCD in [32], supports the importance of consideration of the condensates with dimension \(d \geq 4\). Moreover, this approach favors the application of the dispersion relation of Eq. (5) in the scalar and pseudoscalar channels. Due to the theoretical arguments, given in [24], the renormalon calculus indicate that the ill-defined dispersion relation

\[ \mathcal{D}^{P(S)}(Q^2) = Q^2 \int_0^{\infty} \frac{R^{P(S)}_{\overline{\text{MS}}}(s)}{s(s + Q^2)^2} ds , \] (19)

contains \((\Lambda^2/Q^2)\)-correction, which is not consistent with the general structure of the standard massless operator-product expansion (OPE) technique. This fact indicates that in the process of the concrete phenomenological studies it is more consistent to consider the \(D\)-function of Eq. (4) [24] (in fact it is proportional to the double-differentiated dispersion

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relation, originally defined and used in [23]). Indeed, like in the vector channel, the standard OPE expansion of Eq. (5) is starting from the terms of order $O(\Lambda^4/Q^4)$. Thus, following the scalar-channel consideration of [24], we conclude that in the $\overline{\text{MS}}$-scheme the infrared renormalon (IRR) calculus is rather useful tool for the investigation of the general structure of the standard OPE formalism. Note, however, that the concept of the IRR-contributions is scheme-dependent. In the schemes with the frozen coupling constant (see, e.g., [33]–[35]) the IRR contributions are absent. The question of the existence in this case of the ultraviolet renormalon contributions seems to be opened. Another problem, which we are going to concentrate in this presentation, is related to the possibility to estimate the properties of radial excitations, namely their masses and decay width coupling constants, using the QCD sum rules method and the duality approach. In the next sections we will concentrate on the the analysis of this problem in the pseudoscalar and scalar quark channels.

3 Radial excitations of light pseudoscalar mesons

In the works of [15, 16] the properties of radial excitations of light pseudoscalar mesons were studied with the help of the following FESR

$$ M_k^{\text{th}} = \int_{s_{n-1}}^{s_n} R_{\text{th}}^P(s) s^k ds = M_k^{\text{ph}} = \int_{s_{n-1}}^{s_n} R_{\text{ph}}^P(s) s^k ds \quad , $$

(20)

where $s_n$ are the duality intervals, which will be defined below, the theoretical (th) spectral density of FESR is calculated in four dimensional QCD, while the phenomenological (ph) model for the spectral function of FESR is chosen in the form of $1/N_c$-motivated model

$$ R_{\text{ph}}^P(s) = \sum_{l=1}^{\infty} 2f_P^2(l) m_P^4(l) \delta(s - m_P^2(l)) \quad . $$

(21)

Index $P$ labels the sets of masses $m_P(l)$ and decay constants $f_P(l)$ of the radial excitations of light pseudoscalar mesons, namely $\pi$ and $K$-mesons, which are considered as massless particles.

Neglecting the slight $\alpha_s$-dependence, which comes from the leading-order terms of Eq. (4), the authors of [15, 16] obtained the following sum-rule :

$$ \frac{M_0^{\text{th}}(s_n)}{M_{n-1}^{\text{th}}(s_n)} = \frac{1}{2}(s_n + s_{n-1}) = \frac{M_0^{\text{ph}}(s_n)}{M_{n-1}^{\text{ph}}(s_n)} = m_P^2(n) \quad . $$

(22)

As the next step the bounds of integration in Eq. (20) where chosen as

$$ s_n = \frac{1}{2} \left[ m_P^2(n) + m_P^2(n+1) \right] \quad , $$

(23)

$$ s_0 = \frac{m_P(1)}{2} \quad , $$

(24)
where $m_P(1)$ is the mass of the first radial excitation of $\pi$ mesons, namely the $\pi'$ state. The choice of these duality intervals is supported by the studies of possibilities to combine the $1/N_c$-motivated spectrum with the duality approach in two and four dimensions [19].

As the results of iterative solution of the system of Eqs. (22)-(24) the following mass formula for the radial excitations of the $\pi$ meson was obtained [15, 16]:

$$m^2_\pi(l) = m^2_\pi' l, \quad l \geq 1.$$  

(25)

In the work [16] these considerations were generalized to the case of the $K$ meson radial excitations and the identical expression for the mass spectrum

$$m^2_K(l) = m^2_K' l, \quad l \geq 1$$  

(26)

was derived.

It should be stressed that the $\pi'$ meson was observed by several experimental collaborations (see [2]). In the work [16] the result $m_{\pi'} = 1240$ MeV, obtained at Protvino accelerator by Dubna–Milan–Bologna Collaboration [36], was used. Substituting it into Eq. (25), it is easy to get the following predictions

$m_\pi(2) = 1753$ MeV, $m_\pi(3) = 2148$ MeV, $m_\pi(4) = 2480$ MeV [16]. These numbers are in good agreement with the linear trajectories, obtained in [3], and with the results of the recent OPE-based analysis of the work [9]. Note also, that the application of the Effective Chiral Lagrangians gives $m_\pi(2) = 1.98$ GeV [10], while the experimental number from [36] is $m_\pi(2) = 1.77 \pm 0.03$ GeV. The inclusion of the $\pi$-meson radial excitations in the studies of the QCD sum rule model for the pion wave-function gave the following prediction: $m_\pi(2) = 2.05 \pm 0.15$ GeV [37]. It is consistent with the result of [10], but is slightly higher than the prediction from [16], which is in surprisingly good agreement with the experimental result of [36].

Fixing now the experimental value of $m_K' = 1.46$ GeV [2] as the input parameter, it is possible to obtain the predictions for the masses of higher radial excitations of $K$ meson [16], namely $m_K(2) = 2.06$ GeV and $m_K(3) = 2.53$ GeV. The prediction for the mass of the second radial excitation is consistent with the result $m_K(2) = 2.1$ GeV obtained in [10], which is slightly higher than the experimental result $m_K(2) = 1.86$ GeV [2]. The comparison of the results of [16] in the $K$-meson channel with the results of other theoretical studies are really welcomed.

As the next step the FESR model for the decay constants of the radial excitations of light pseudoscalar mesons was estimated [15, 16]. Reminding that the duality interval of the ground state for the pseudoscalar particles can be defined as $s_0 = m^2_P(1)/2$, taking the ratio of the following FESRs

$$\frac{M^2_{1h}(s_n)}{M^2_{2h}(s_n)} = \frac{s_{n+1}^2 - s_n^2}{s_0^2} = \frac{f_P^2(n)m^4_P(n)}{f_P^2 m^4_P}$$  

(27)

and supplementing it with Eq. (23), it is possible to get the following “linear dual model” for the coupling constants of radial excitations of the light pseudoscalar mesons [15, 16]:

$$f_P(l) = 2\sqrt{2} \frac{m^2_P}{m_P(1)m_P(l)} f_P, \quad l \geq 1.$$  

(28)
Taking into account the concrete values for $m_\pi$, $m_K$, the values of the decay constants $f_\pi$ and $f_K$ and the expressions for the masses of radial excitations of the pseudoscalar mesons $m_\pi(l)$ and $m_K(l)$, one can get the FESR-inspired estimates for the decay constants of radial excitations of $\pi$ and $K$ mesons. It will be interesting to study the possible numerical uncertainties of this model using other approaches.

To conclude this section it is also worth mentioning that the similar “linear dual spectrum” in the vector channel [14] can be also obtained within Veneziano model [38].

4 Radial excitations of light scalar mesons

We are now ready to discuss the most intriguing part of the work [16], which is devoted to the derivation of “linear dual spectra” in the light scalar meson channel, whose ground state representatives $a_0(980)$ and $K^*(1430)$ will be considered as the massive $\bar{q}q$-bound states. The only difference with the discussions presented in Sect.3 is related to the redefinition of the ground-state duality interval from $(s_0)_P = m_\pi^2(1)/2$ to $(s_0)_S = 3m_S^2/2$, where $m_S$ are the masses of $a_0(980)$ and $K^*(1430)$ light scalar mesons. This definition of $(s_0)_S$ is coming from the following ratio of the FESRs:

$$\frac{M^1_{th}(s_0)}{M^0_{th}(s_0)} = \frac{2(s_0)_S}{3} = m_S^2. \quad (29)$$

Combining this new value of the duality interval with Eq. (22) and Eq. (23) the authors of [16] obtained the following scalar analog of the “pseudoscalar linear dual model” derived in the previous section [16]:

$$m_S(n) = (n + 1)m_S^2, \quad (30)$$

$$f_S^2(n) = \frac{1}{n + 1}f_S^2. \quad (31)$$

Thus, assuming the $\bar{q}q$-structure of $a_0(980)$ meson, we expected that the masses of its radial excitations may be estimated as $a_0(1) = 1380$ MeV, $a_0(2) = 1697$ MeV, $a_0(3) = 1960$ MeV. Note, that the work [16] was the first one, where the possibility of the existence of extra light scalar resonance near $a_0(1) = 1.4$ GeV was predicted. It is known now that there is $a_0(1450)$ meson in nature [2]. So, it may treated as the possible candidate for the first radial excitation of $a_0(980)$ meson. Another pleasant feature of the derived in [16] “linear dual spectrum” for the $a_0$-meson excitations is that it turns out to be in satisfactory agreement with the results from [9]. As to the strange light scalar particle, namely $K^*_0(1430)$ meson, within described above approach its possible radial excitations should have the following masses: $m_{K^*_0}(1) = 2022$ MeV and $m_{K^*_0}(2) = 2477$ MeV. Note, that the experimental data indicate the existence of $K^*_0(1950)$ meson, which following discussed above classification, can be considered as the candidate for the first radial excitation of $K^*_0(1430)$ meson. This meson may be the good candidate for the nonet partner of $a_0(1450)$ meson. However, to get better understanding on the structure of the scalar nonets and the nature of both $a_0(980)$ and $a_0(1450)$ mesons it is rather important to continue studies of the classification of hadrons in the light scalar sector using various approaches.
5 Conclusion

In this work the applications of the QCD FESR approach for the derivation of “linear dual spectra” of radial excitations in the pseudoscalar and scalar quark antiquark channel were reminded. In the process of considerations both nonperturbative and, probably more important in the investigation of this problem, perturbative QCD effects, were neglected. It is worth to emphasise that higher-order perturbative QCD corrections for the massless two-point function of pseudoscalar and scalar quark currents, calculated in the works of [39, 40], are more important, than the ones for the two-point function of the vector channel, calculated in the works [41, 42]. In view of this it can be of interest to take these calculated corrections into account in the studies of the properties of radial excitations of light mesons, based on the OPE approach.

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References

[1] F. E. Close and N. A. Tornqvist, J. Phys. G 28, R249 (2002) hep-ph/0204205.
[2] K. Hagiwara et al. (Particle Data Group Collab.), Phys. Rev. D 66, 010001 (2002)
[3] A. V. Anisovich, V. V. Anisovich, and A. V. Sarantsev, Phys. Rev. D 62, 051502 (2000) hep-ph/0003113.
[4] N. N. Achasov and V. V. Gubin, Phys. Rev. D 56, 4084 (1997) hep-ph/9703367.
[5] S. B. Gerasimov, in 12th International Conference on Selected Problems of Modern Physics (Blokhintsev 03), Dubna, Russia, 8-11 June 2003, hep-ph/0311080.
[6] T. Umekawa, K. Naito, M. Oka and M. Takizawa, hep-ph/0403032.
[7] S. Narison, Nucl. Phys. Proc. Suppl. 96, 244 (2001) hep-ph/0012235.
[8] A. M. Badalian, B. L. G. Bakker, and Y. A. Simonov, Phys. Rev. D 66, 034026 (2002) hep-ph/0204088.
[9] S. S. Afonin, A. A. Andrianov, V. A. Andrianov, and D. Espriu, JHEP 0404, 039 (2004) hep-ph/0403268.
[10] S. M. Fedorov and Y. A. Simonov, JETP Lett. 78, 57 (2003) [Pisma Zh. Eksp. Teor. Fiz. 78, 67 (2003)] hep-ph/0306216.
[11] G. ’t Hooft, Nucl. Phys. B 72, 461 (1974).
[12] E. Witten, Nucl. Phys. B 160, 57 (1979).
[13] M. Golterman, S. Peris, B. Phily, and E. De Rafael, JHEP **0201**, 024 (2002) [hep-ph/0112042].

[14] N. V. Krasnikov and A. A. Pivovarov, Phys. Lett. B **112**, (1982) 397.

[15] A. L. Kataev, N. V. Krasnikov, and A. A. Pivovarov, Phys. Lett. B **123**, (1983) 93.

[16] S. G. Gorishny, A. L. Kataev, and S. A. Larin, Phys. Lett. B **135**, 457 (1984).

[17] A. Bramon, E. Etim, and M. Greco, Phys. Lett. B **41**, 609 (1972).

[18] J. J. Sakurai, Phys. Lett. B **46**, 207 (1973).

[19] A. Bradley, C. S. Langensiepen, and G. Shaw Phys. Lett. B **102**, 180 (1981).

[20] M. A. Shifman, *in Boris Ioffe Festschrift “At the Frontier of Particle Physics/Handbook of QCD”,* (World Scientific, Singapore, 2001), Vol.3, p.1447 [hep-ph/0009131].

[21] K. G. Chetyrkin, N. V. Krasnikov, and A. N. Tavkhelidze, Phys. Lett. B **76**, 83 (1978).

[22] A. A. Logunov, L. D. Soloviev, and A. N. Tavkhelidze, Phys. Lett. B **24**, 181 (1967).

[23] C. Becchi, S. Narison, E. De Rafael, and F. J. Yndurain, Z. Phys. C **8**, 335 (1981).

[24] D. J. Broadhurst, A. L. Kataev, and C. J. Maxwell, Nucl. Phys. B **592**, 247 (2001) [hep-ph/0007152].

[25] T. van Ritbergen, J. A. M. Vermaseren, and S. A. Larin, Phys. Lett. B **400**, 379 (1997) [hep-ph/9701390].

[26] J. A. M. Vermaseren, S. A. Larin, and T. van Ritbergen, Phys. Lett. B **405**, 327 (1997) [hep-ph/9703284].

[27] K. G. Chetyrkin, Phys. Lett. B **404**, 161 (1997) [hep-ph/9703278].

[28] P. A. Baikov, K. G. Chetyrkin, and J. H. Kuhn, Phys. Rev. Lett. **88**, 012001 (2002) [hep-ph/0108197].

[29] M. A. Shifman, A. I. Vainshtein, and V. I. Zakharov, Nucl. Phys. B **147**, 385 (1979).

[30] V. A. Novikov, M. A. Shifman, A. I. Vainshtein, and V. I. Zakharov, Nucl. Phys. B **191**, 301 (1981).

[31] P. Nason and M. Palassini, Nucl. Phys. B **444**, 310 (1995) [hep-ph/9411246].

[32] V. I. Zakharov, Nucl. Phys. B **385**, 452 (1992).

[33] N. V. Krasnikov and A. A. Pivovarov, Mod. Phys. Lett. A **11**, 835 (1996) [hep-ph/9602272].
[34] D. V. Shirkov and I. L. Solovtsov, Phys. Rev. Lett. 79, 1209 (1997) [hep-ph/9704333].

[35] Y. A. Simonov, Phys. Atom. Nucl. 65, 135 (2002) [Yad. Fiz. 65, 140 (2002)] [hep-ph/0109081].

[36] G. Bellini et al., Phys. Rev. Lett. 48, 1697 (1982).

[37] A. V. Radyushkin, in “Continuous advances of QCD”, Minneapolis 1994, (River Edge, World Scientific, 1994) p.238 [hep-ph/9406237].

[38] G. Veneziano, Nuovo Cimento A 57, 190 (1968).

[39] S. G. Gorishny, A. L. Kataev, S. A. Larin, and L. R. Surguladze, Mod. Phys. Lett. A 5, 2703 (1990);
     S. G. Gorishny, A. L. Kataev, S. A. Larin, and L. R. Surguladze, Phys. Rev. D 43, 1633 (1991).

[40] K. G. Chetyrkin, Phys. Lett. B 390, 309 (1997) [hep-ph/9608318].

[41] K. G. Chetyrkin, A. L. Kataev, and F. V. Tkachov, Phys. Lett. B 85, 277 (1979);
     M. Dine and J. R. Sapirstein, Phys. Rev. Lett. 43, 68 (1979);
     W. Celmaster and R. J. Gonsalves, Phys. Rev. Lett. 44, 560 (1980).

[42] S. G. Gorishny, A. L. Kataev, and S. A. Larin, Phys. Lett. B 259, 144 (1991);
     L. R. Surguladze and M. A. Samuel, Phys. Rev. Lett. 66, 560 (1991) [Erratum-ibid. 66, 2416 (1991)];
     K. G. Chetyrkin, Phys. Lett. B 391, 402 (1997) [hep-ph/9608480].