An Einstein-Podolsky-Rosen (EPR)-like argument using events separated by a time-like interval strongly suggests that measuring the polarization state of a photon of an entangled pair changes the polarization state of the other distant photon. Through a very simple demonstration, the Wigner-D’Espagnat inequality is used to show that in order to prove Bell’s theorem neither the assumption that there is a well-defined space of complete states \( \lambda \) of the particle pair and well-defined probability distribution \( \rho(\lambda) \) over this space nor the use of counterfactuals is necessary. These results reinforce the viewpoint that quantum mechanics implicitly presupposes some sort of nonlocal connection between the particles of an entangled pair. As will become evident from our discussion, relinquishing realism and/or free will cannot solve this apparent puzzle.

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I. INTRODUCTION

As emphasized by Schrödinger, entanglement is the characteristic trait of quantum mechanics.\(^1\) Einstein, Podolsky, and Rosen (EPR) used entangled states to try to prove that this theory is incomplete,\(^2\) and Bell made things clearer by showing that no local hidden variable theory can mimic quantum mechanics.\(^3\) Apparently, measuring the state of a particle can instantaneously change the state of another particle that can be arbitrarily distant from the first. But no superluminal telegraph can be devised using this phenomenon. The correlations become evident only when the results, gathered at two different spatial regions, are compared. In principle, one observer cannot know what kind of experiment the other is performing. That is, there is no detectable contradiction with special relativity. Bell’s theorem has been extended to real situations,\(^4\) proofs have been introduced that do not rely on inequalities,\(^5\) long-distance experimental tests of entanglement have been performed,\(^6\) and the use of entangled particles for cryptographic purposes has been proposed.\(^7\) However, the mystery remains,\(^8\) and even conflicting points of view on the conceptual significance of Bell’s theorem have been presented.\(^9\) Here I will advocate the viewpoint that quantum mechanics is an intrinsically nonlocal theory, that is, it implicitly presupposes that measuring the state of a distant particle of an entangled pair can instantaneously affect the other’s state.

II. THE WEIRDNESS OF QUANTUM ENTANGLEMENT

Let us consider the following situation: a source \( S \) emits pairs of entangled photons, \( \nu_1 \) and \( \nu_2 \), in the state

\[
|\psi\rangle = \frac{1}{\sqrt{2}} \left( |a, k\rangle |a, -k\rangle + |a, k\rangle |a, -k\rangle \right)
= \frac{1}{\sqrt{2}} \left( |b, k\rangle |b, -k\rangle + |b, k\rangle |b, -k\rangle \right)
= \frac{1}{\sqrt{2}} \left( |c, k\rangle |c, -k\rangle \right) = \ldots,
\]

where \( |a, k\rangle \) \( (|a, -k\rangle \) represents a photon with polarization parallel (perpendicular) to \( a \) following direction \( k \) \( (-k) \), and so on.

From (1) we see that quantum mechanical formalism does not allow us to assign definite polarization states to the photons. This can only be done when a polarization measurement is performed. For example, if photon \( \nu_1 \) (\( \nu_2 \)) is detected in state \( |a\rangle \), then we immediately know that the other photon of the pair, \( \nu_2 \) (\( \nu_1 \)), has been “forced” into the same state \( |a\rangle \). We also see that (1) does not allow us to predict the outcome of the measurement—state \( |a\rangle \) or \( |a, k\rangle \).
for example. This strongly suggests that quantum mechanics implicitly presupposes some sort of superluminal—actually, infinite-speed—interaction: measuring the state of one of the two particles instantaneously changes the state of the other. According to Bell, “in these EPR experiments there is the suggestion that behind the scenes something is going faster than light,” and Bohm declared: “I would be quite ready to relinquish locality; I think it is an arbitrary assumption.” An important point to be emphasized is that the correlations become evident when one reads the results that have been automatically registered. The observer’s consciousness does not seem to play any role in the process. If this were not so, mind states of distant observers would have to communicate to reproduce the quantum correlations.

Although the discussion has been centered on the nonlocal aspects of quantum mechanics, the important question is knowing if a measurement performed on one of the photons of an entangled pair can change the state of the other. To examine this problem, it is preferable to consider time-like events. Let us imagine that the path followed by \( \nu_2 \) is modified so that the first observer, Alice (A), after measuring the state of \( \nu_1 \), can inform the second observer, Bob (B), about her result before he detects \( \nu_2 \). Naturally, assuming that the detection of \( \nu_1 \) cannot change the polarization state of \( \nu_2 \), it is irrelevant whether we consider space-like or time-like events. On the other hand, if there is some sort of nonlocal connection between the photons, it may be more illuminating to discuss situations in which there is no doubt about which one has been detected first. An aleatory sequence of photons in states \(|a\rangle\) and \(|a_\perp\rangle\) is indistinguishable from another aleatory sequence of photons in states \(|b\rangle\) and \(|b_\perp\rangle\), or \(|c\rangle\) and \(|c_\perp\rangle\), and so on. Therefore, the detection of photons \( \nu_2 \) provides us no information about the orientation of the polarizer on which photons \( \nu_1 \) are impinging. That is, without using a classical communication channel, A cannot use entangled states to send information to B. But time-like events allow us to try to clarify the following question: What does it really mean when we say—in agreement with (1)—that measuring the state of the first photon forces the second into a well-defined polarization state? When we are dealing with space-like events, it is not possible to assign an objective and well-defined polarization state to \( \nu_2 \) just before it impinges on the polarizer, since the question “Which photon was really detected first?” is meaningless. On the other hand, in the case of time-like events, A can send a message informing about the state of \( \nu_2 \), and B can then perform a measurement to check if the information is correct. It seems that EPR’s criterion—“If, without in any way disturbing a system, we can predict with certainty (i.e., with probability equal to unity) the value of a physical quantity, then there exists an element of physical reality corresponding to this physical quantity.”—is perfectly valid here. Therefore, depending on the orientation of the polarizer on which \( \nu_1 \) impinges, \( \nu_2 \) will reach its polarizer in two possible states: \(|a\rangle\) and \(|a_\perp\rangle\), or \(|b\rangle\) and \(|b_\perp\rangle\), or \(|c\rangle\) and \(|c_\perp\rangle\), etc. However, assuming that the detection of \( \nu_1 \) has no influence on the polarization state of \( \nu_2 \), there is no reason for \( \nu_2 \) to be found only in the states \(|a\rangle\) and \(|a_\perp\rangle\), for instance. On the other hand, if there is an influence, it still must be present when space-like events are considered, since the very same correlations can be observed. From this point of view, there must be a nonlocal connection between \( \nu_1 \) and \( \nu_2 \), unless we assume that a photon can somehow be in different polarization states simultaneously. We can also consider the following reasoning: If the outcome of the first measurement is aleatory, but that of the second becomes well determined (after we know the result of the first), then, considering the symmetry of the situation, either (a) the first measurement was not aleatory or (b) there must be some connection between \( \nu_1 \) and \( \nu_2 \). In other words, if quantum mechanics is complete, it must be nonlocal.

III. BELL’S THEOREM

The above notions, however, clash with the spirit of relativity, and we may conjecture that some still unknown local theory exists that perfectly mimics quantum theory. Bell’s theorem shows that this cannot be so. As a consequence, it seems that our conceptions of space and time must be revised; specifically, the concept of distance does not seem to be valid for a system of entangled particles. As has been emphasized, one way to demonstrate Bell’s inequality is to assume “that there is a well-defined space of complete states \(|\lambda\rangle\) of the particle pair and a well-defined probability distribution \([\rho(\lambda)]\) over this space when an experimental procedure for specifying an ensemble of pairs is given.” Although this may sound like a reasonable assumption, it can be considered an unnecessarily restrictive one, and arguments for dispensing with it have been presented, but they have used counterfactuals, thereby raising some serious criticisms. Counterfactual reasoning is based on the assumption that we could have performed another measurement on one of the particles of an entangled pair (instead of the measurement actually performed, for example) without changing the outcome of the measurement performed on the other, distant particle (locality assumption). But it can be argued that in this case we would be in another Universe, which invalidates the counterfactual-local reasoning. In particular, assuming that the present is rigidly determined by the past, two physical events, even separated by a space-like interval, can be interconnected in such a way that we cannot change one without changing the other. The introduction of free will, although very reasonable for some, only makes the argument more disputable. Many different versions of Bell’s theorem have been presented, but they either introduce \(\rho(\lambda)\) or use counterfactuals. Therefore, there seems to be no compelling theoretical evidence that entanglement implies that quantum mechanics
is inherently nonlocal. However, as we will see, to derive Bell’s theorem we do not need to assume that there is a well-defined $\rho(\lambda)$, or, alternatively, that counterfactual reasoning is valid.

IV. THE WIGNER-D’ESPAGNAT INEQUALITY

Our purpose is to answer the question: can a local theory mimic the predictions of quantum mechanics? In other words, we are interested in situations represented by (1). This expression shows us, in an ideal situation, what correlations must be observed if the polarizations are measured. That is, we are considering “latent” probabilities, so to speak. Actually, there may be no polarizers, that is no experiment to determine the polarization of the photons. What we want to know is whether a local theory may have states with the same latency as the states represented by (1). Initially, let us imagine a situation in which two-channel polarizers I and II, on which $\nu_1$ and $\nu_2$ impinge, respectively, have the same orientation. According to (1), whenever $\nu_1$ is transmitted (reflected), $\nu_2$ must also be transmitted (reflected). Therefore, assuming locality (i.e., what happens to $\nu_1$ cannot affect $\nu_2$, and vice versa), whether $\nu_1$ and $\nu_2$ will be transmitted or reflected is already determined before they impinge on the polarizers; otherwise, we could have a situation in which one photon is transmitted and the other is reflected. In other words, perfect correlations and locality imply a strong form of determinism. Assuming that the source has no information about the orientations of the polarizers, each photon pair has to be emitted with “instructions,” so to speak, for all possible orientations. For instance, transmission in case of orientation a, reflection in case of orientation b, and so on. Or, put another way, the outcome of a potential experiment is determined by the photon’s hidden variable state and the polarizer orientation, and nothing else. As we will see, it is impossible to mimic the predictions of quantum mechanics in this case.

It is important to mention that in 1982 Itamar Pitowsky published a paper in which the EPR-Bell “paradox” was supposedly solved. His point was that the derivation of Bell’s inequality was valid only when a well-defined $\rho(\lambda)$ could be introduced, which was not the case for his model. But, as shown by Alan Macdonald, in the particular case of the Pitowsky model, there is another and very simple way to obtain these inequalities. Actually, a similar derivation had already been presented by Bernard D’Espagnat in his article on quantum theory and realism in Scientific American, and before that by Wigner. It is perhaps the simplest and most satisfactory proof of Bell’s inequality, but seldom presented or mentioned; probably because it is only valid for perfect correlations. Although it is already very simple, it still can be simplified even further, as we will see.

Let us assume that $N_0$ pairs of photons are emitted. Now let $N(a, b, c) [N(a_\perp, b, c)]$ represent the number of photon pairs in which the photons are “prepared” to be transmitted when impinging on a polarizer oriented parallel to b or c, and to be transmitted [reflected] if oriented parallel to a. Then, we must have

$$N(a, b, c) + N(a_\perp, b, c) = N(b, c),$$  \hspace{1cm} (2)

where $N(b, c)$ represents the number of photon pairs prepared to be transmitted when impinging on a polarizer oriented parallel to b or c. We also must have

$$N(a, c) \geq N(a, b, c)$$  \hspace{1cm} (3)

and

$$N(a_\perp, b) \geq N(a_\perp, b, c).$$  \hspace{1cm} (4)

From (2), (3) and (4) we obtain

$$N(a, c) + N(a_\perp, b) \geq N(b, c),$$  \hspace{1cm} (5)

which is the Wigner-D’Espagnat inequality.

According to quantum mechanics, we must have $N(a, c) = (N_0/2) \cos^2(a, c)$, $N(a_\perp, b) = (N_0/2) \sin^2(a, b)$, and $N(b, c) = (N_0/2) \cos^2(b, c)$. Thus, choosing $(a, b) = (b, c) = 30^\circ$, and $(a, c) = 60^\circ$, we obtain $0.5 \geq 0.75$, violating inequality (5).

V. DISCUSSION

As our discussion based on time-like events and entangled particles has made evident, quantum mechanics is intrinsically a nonlocal theory, and this conclusion is equally valid for space-like events. Measuring the polarization state of one of the photons of an entangled state, represented by (1), instantaneously forces the other, distant photon into a well-defined polarization state. Paradoxical as it may sound, this seems to be true independently of the Lorentz frame used to describe the events (it is important to remember, however, that special relativity is not necessarily incompatible with the idea of a preferred frame). In other words, if quantum mechanics is complete, it must be
definiteness was introduced in an attempt to avoid the use of hidden variable states. In the realm of imagined experiments, but as a consequence and not as a basic assumption. Actually, counterfactual photon hidden variable state and the orientation of the polarizer—may imply a sort of virtual contrafactualness, valid of determinism—as in the present paper, for example, in which the outcome of the experiment depends only on the counterfactual definiteness presupposes determinism, the converse is not necessarily true. Strictly speaking, some kinds of determinism—as in the present paper, for example, in which the outcome of the experiment depends only on the photon hidden variable state and the orientation of the polarizer—may imply a sort of virtual contrafactualness, valid in the realm of imagined experiments, but as a consequence and not as a basic assumption. Actually, counterfactual definiteness was introduced in an attempt to avoid the use of hidden variable states.¹⁸

A delicate point related to Bell’s inequalities involves realism. It was implicit in our assumption of hidden variable states. It is evident that if realism is abandoned, it becomes difficult to explain the predicted correlations assuming locality. Therefore, abandoning realism does not solve the EPR puzzle. Of course, solipsism is a logical alternative but very unsatisfactory as a predictive tool and difficult to maintain consistently in real life. It seems that physics has little to contribute to this longstanding philosophical debate.

Another delicate point is related to the use of a free-will assumption to derive Bell’s inequalities. According to Bell, “In the analysis [of EPR experiments] it is assumed that free will is genuine, and as a result of that one finds that the intervention of the experimenter at one point has to have consequences at a remote point, in a way that influences restricted by the finite velocity of light would not permit. If the experimenter is not free to make this intervention, if that is also determined in advance, the difficulty disappears.”¹⁹ As has become evident from our discussion, abandoning free will—which plays no role in our demonstration—is not a solution to EPR paradox.

In conclusion:

We have reexamined Bell’s theorem using a different approach and trying to answer the question: Can a local theory mimic the predictions of quantum mechanics? We have assumed that no information about the orientation of the polarizers is contained in the state of the emitted pair of photons. In other words, Nature is governed by physical laws; nothing that might sound like a sort of Big Conspiracy exists. Actually, there may be no polarizers, that is, no experiment to determine the polarization of the photons. We only know what correlations must be observed if the polarizations are measured. This is in agreement with expression (1), which only expresses potentialities. Strictly speaking, we are not discussing whether Nature is nonlocal or not—although experimental evidence strongly suggests it is—²⁰—but whether quantum mechanics is nonlocal or not. In our demonstration, neither the assumption that there is a well-defined space of complete states of the particle pair and a well-defined probability distribution over this space nor the use of counterfactuals is needed. This makes the conclusion that quantum mechanics is intrinsically nonlocal almost inescapable.

REFERENCES AND NOTES

¹E. Schrödinger, “Discussion of probability relations between separated systems,” Proc. Cambridge Phil. Soc. 31, 555-563 (1935).
²A. Einstein. B. Podolsky, and N. Rosen, “Can quantum mechanical description of physical reality be considered complete?,” Phys. Rev. 47, 777-780 (1935), reprinted in Quantum Theory and Measurement, edited by J. A. Wheeler and W. H. Zurek (Princeton University Press, Princeton, 1983).
³J. S. Bell, “On the Einstein-Podolsky-Rosen paradox,” Physics 1, 195-200 (1964), reprinted in Speakable and unspeakable in quantum mechanics (Cambridge University Press, Cambridge, 1987) and in Quantum Theory and Measurement, edited by J. A. Wheeler and W. H. Zurek (Princeton University Press, Princeton, 1983).
⁴J. F. Clauser, M. A. Horne, A. Shimony, and R. A. Holt, “Proposed experiment to test local hidden-variables theories,” Phys. Rev. Lett. 23, 880-884 (1969), reprinted in Quantum Theory and Measurement, edited by J. A. Wheeler and W. H. Zurek (Princeton University Press, Princeton, 1983), J. S. Bell, “Introduction to the hidden-variable question,” in Foundations of Quantum Mechanics, edited by B. D’Espagnat (Academic, New York, 1971), reprinted in Speakable and unspeakable in quantum mechanics (Cambridge University Press, Cambridge, 1987); J. F. Clauser and M. A. Horne, “Experimental Consequences of Objective Local Theories,” Phys. Rev. D 10, 526-535 (1974); L. C. Ryff, “Bell and Greenberger, Horne, and Zeilinger theorems revisited,” Am. J. Phys. 65, 1197-1199 (1997).
⁵D. M. Greenberger, M. Horne, and A. Zeilinger, “Going Beyond Bell’s Theorem,” in Bell’s Theorem, Quantum Theory, and Conceptions of the Universe, edited by M. Kafatos (Kluwer Academic, Dordrecht, 1989); D. M. Greenberger,
strict Einstein locality conditions," Phys. Rev. Lett. 81, 2924 (1988); G. Weihs, T. Jennewein, C. Simon, H. Weinfurter, and A. Zeilinger, "Violation of Bell's inequality under experiment using pairs of light quanta produced by optical parametric down-conversion, Phys. Rev. Lett. 49 (1982); A. Aspect, J. Dalibard, and G. Roger, "Experimental test of Bell's inequalities using time-varying analyzers," Phys. Rev. Lett. 49, 1297-1300 (1982).

6S. Fasel, N. Gisin, G. Ribordy, and H. Zbinden, “Quantum key distribution over 30 km of standard fiber using energy-time entangled photon pairs: a comparison of two chromatic dispersion reduction methods,” quant-ph/0403144 (2004); I. Marcikic, H. de Riedmatten, W. Tittel, H. Zbinden, M. Légré, and N. Gisin, “Distribution of time-bin entangled qubits over 50 km of optical fiber,” quant-ph/0404124 (2004); C-Z. Peng, T. Yang, X-H. Bao, J-Zang, X-M. Jin, F-Y. Feng, B. Yang, J. Yang, J. Yin, Q. Zhang, N. Li, B-L. Tian, and J-W. Pan, “Experimental Free-Space Distribution of Entangled Photon Pairs over a Noisy Ground Atmospheric of 13 km,” quant-ph/0412218 (2004).

7N. Gisin, G. Ribordy, W. Tittel, and H. Zbinden, “Quantum cryptography,” quant-ph/0101098 (2001); M. Aspelmeyer, T. Jennewein, A. Zeilinger, M. Pfennigbauer, and W. Leeb, “Long-Distance Quantum Communication with Entangled Photons using Satellites,” quant-ph/0305105 (2003); C. Elliot, D. Pearson, and G. Troxel, “Quantum Cryptography in Practice,” quant-ph/0307049 (2003); R. Kaltenbaek, M. Aspelmeyer, T. Jennewein, C. Brukner, A. Zeilinger, M. Pfennigbauer, and W. Leeb, “Proof-of-Concept Experiments for Quantum Physics in Space,” quant-ph/0308174 (2003); R. Thew, A. Acín, H. Zbinden, and N. Gisin, “Experimental Realization of Entangled Qutrits for Quantum Communication,” quant-ph/0307122 (2003); A. Poppe, A. Fedrizzi, T. Lorünser, O. Maurhardt, R. Ursi, H. Böhm, M. Peev, M. Suda, T. Jennewein, and A. Zeilinger, “Practical Quantum Key Distribution with Polarization Entangled Photons,” quant-ph/0404115 (2004); C. Elliot, A. Colvin, D. Pearson, O. Pikalo, J. Schlafer, and H. Yeh, “Current status of the DARPA Quantum Network,” quant-ph/0503058 (2005).

8N. Gisin, “How come the Correlations?,” quant-ph/0503007 (2005); L. C. Ryff, “Interference, distinguishability, and apparent contradiction in an experiment on induced coherence,” Phys. Rev. A. 52, 2591-2596 (1995); L. C. Ryff, “The Strange Behavior of Entangled Photons,” Found. Phys. Lett. 10, 207-220 (1997); L. C. Ryff, “Two-photon interference without intrinsic indistinguishability,” Quantum Semiclass. Opt. 10, 409-414 (1998); L. C. Ryff, “Interaction-Free Which-Path Information and Some of Its Consequences,” Z. Naturforsch. 56a, 155-159 (2001).

9A. Shimony, “An Analysis of Stapp’s ‘A Bell-type theorem without hidden variables’,” quant-ph/0404121 (2004), and references therein; H. P. Stapp, “Comments on Simonys’s Analysis,” quant-ph/0404169 (2004). Interviews with John Bell and David Bohm in The Ghost in the Atom, edited by P. C. W. Davies and J. R. Brown (Cambridge University Press, Cambridge, 1986). The first quotation is from page 49, the second from page 125.

10A. Shimony, “An exposition of Bell’s theorem,” in Search For A Naturalistic World View, vol. II (Cambridge University Press, Cambridge, 1993), p. 103.

11J. Pitowsky, “Resolution of the Einstein-Podolsky-Rosen and Bell Paradoxes,” Phys. Rev. Lett. 48, 1299-1302 (1982).

12J. Pitowsky, “Resolution of the Einstein-Podolsky-Rosen and Bell Paradoxes,” Phys. Rev. Lett. 49, 1215 (1982). I thank Prof. Macdonald for kindly calling my attention to his Comment.

13B. D’Espagnat, “The Quantum Theory and Reality,” Scientific American 241, 128-140 (1979).

14E. P. Wigner, “On hidden variables and quantum mechanical probabilities,” Am. J. Phys. 38, 1005-1009 (1970); E. P. Wigner, “Interpretation of Quantum Mechanics,” lectures given in the Physics Department of Princeton University during 1976, revised and printed in Quantum Theory and Measurement, edited by J. A. Wheeler and W. H. Zurek (Princeton University Press, Princeton, 1983).

15J. S. Bell, “Bertlman’s socks and the nature of reality,” Journal de Physique 42, 41-61 (1981), reprinted in Speakable and unspeakable in quantum mechanics (Cambridge University Press, Cambridge, 1987).

16J. S. Bell, “How to teach special relativity,” Progress in Scientific Culture, Vol. 1, No 2, summer 1976, reprinted in Speakable and unspeakable in quantum mechanics (Cambridge University Press, Cambridge, 1987).

17J. F. Clauser and A. Shimony, “Bell’s theorem: experimental tests and implications,” Rep. Prog. Phys. 41, 1881-1927 (1978).

18Interview with John Bell in The Ghost in the Atom, edited by P. C. W. Davies and J. R. Brown (Cambridge University Press, Cambridge, 1986). The quotation is from page 47.

19S. J. Freedman and J.F. Clauser, “Experimental test of local hidden-variable theories,” Phys. Rev. Lett. 28, 938-941 (1972), reprinted in Quantum Theory and Measurement, edited by J. A. Wheeler and W. H. Zurek (Princeton University Press, Princeton, 1983); A. Aspect, P. Grangier, and G. Roger, “Experimental realization of Einstein-Podolsky-Rosen-Bohm gedankenexperiment: a new violation of Bell’s inequalities,” Phys. Rev. Lett. 49, 91-94 (1982); A. Aspect, J Dalibard, and G. Roger, “Experimental test of Bell’s inequalities using time-varying analyzers,” Phys. Rev. Lett. 49, 1804-1807 (1982); Y. H. Shih and C. Alley, “New type of Einstein-Podolsky-Rosen-Bohm experiment using pairs of light quanta produced by optical parametric down-conversion, Phys. Rev. Lett. 61, 2921-2924 (1988); G. Weihs, T. Jennewein, C. Simon, H. Weinfurter, and A. Zeilinger, “Violation of Bell’s inequality under strict Einstein locality conditions,” Phys. Rev. Lett. 81, 5039-5043 (1998).

M. A. Horne, A. ShimONY, a& Zeilinger, “Bell’s theorem without inequalities,” Am. J. Phys. 58, 1131-1143 (1990); L. Hardy, “Nonlocality for two particles without inequalities for almost all entangled states,” Phys. Rev. Lett. 71, 1665-1668 (1993).