Application of the conformal mapping for state analysis of track circuits

M M Sokolov, S A Lunev
Omsk State Transport University, Omsk, Russia

Abstract. In the article, we present the results of using conformal mapping for state analysis of track circuits in various mode of operation. The work shows that the state of a track circuit can be derived from its input resistance. The article shows nomograms of input resistances when the rail line characteristics change. Authors regards that the input resistance areas for the control and shunt modes of the track circuit are completely separable over the entire range of variation of the rail line parameters. Nomograms of input resistances can be used in systems for diagnosing and monitoring track circuits.

1. Introduction
Information about tracks occupation and clearance is fundamental for railway safety systems [1]. Various methods of track rolling stock detection are used nowadays: spot wheel detectors, linear wheel and axle detectors, electromagnetic detection systems, video control systems and others. Track circuits act as the most common devices for detection of track section occupation on railroads in the Russian Federation. One of the significant advantages of these devices in comparison with alternative methods is the ability to control the integrity of the track circuit. This feature allows to avoid incidents caused by tracks damage and increases railway safety overall.

Russian Railway’s infrastructure development according to the Strategy of Rail Transport Development in the Russian Federation up to 2030 suggests modernization of existing signaling and interlocking systems in order to increase freight and passenger trains speed. Currently track circuits are frequently changed or supplemented with other devices in order to increase the discreteness of train location detection. The main reason for that is that at high speeds it is necessary to have more detailed information about the movement of the train: exact coordinates of train’s head and tail and real train’s speed. Existing detection systems are only able to give information about the number of occupied track circuits. For instance, when the train, which length is more than the length of the blocking segment of the way, passes this segment, it occupies three or more track circuits. This drastically reduces the railways capacity.

Currently, determining the coordinates of the location of a mobile unit has found wide use on classification yard tracks. The coordinates are determined with the following devices: axle counting systems, track circuits, inductive wheel sensors, pulse sensing devices and others. Numerous limitations in use of these devices and difficulties associated with their operation explain the need for finding alternative solutions. Consider the continuous rail line state monitoring with the use of existing tonal frequency track circuits devices.

Current tonal frequency track circuit, which is widespread on Russian Railways, consists of tonal frequency generator, matching devices for power supply side, rail line, matching devices for relay side and receiver (Figure 1).
The existing approach of signal level monitoring allows to determine if the track circuit is functioning and occupied or not. Therefore, the receiver is unable to give information, which can be used to explain why the parameters of the signal on its input do not represent normal operation state. At the same time, signal’s level drop can be caused by:

- rail break, which means that train is not allowed to enter the rail line; simple development and configuration; reliability and fault tolerance of the system, which are achieved due to backups and independent redundancy;
- changes in state of resistance of insulation of track line. In such cases train movement can be allowed with several restrictions.

2. Methods

When analyzing the state of the track circuit, it can be convenient to use the mathematical apparatus of the theory of two-port networks because track circuit can be represented as a cascade connection of equivalent circuits of its elements. In this case every element of equivalent circuit is described as a two-port network [2].

Authors suggest using the mathematical theory of conformal mappings in order to clarify the state of the track circuit when the level on the input of the track receiver is low. The use of this theory allows to convert the load resistance area to the input resistance area [3].

The result of conformal transformation of the right half-plane values of load impedances ($Z_n$) for linear fractional transformation (1) is the set of input resistance values. This set has the form of a circle or a sector lying in the right half-plane of complex plane. The radius and center coordinates of these circles can be calculated using formulas (2) and (3) [4].

$$Z_{ih} = \frac{A \cdot Z_n + B}{C \cdot Z_n + D}$$  \hspace{1cm} (1)

$$\alpha_b = \frac{A \cdot \overline{D} + B \cdot \overline{C}}{2 \cdot \text{Re}(C \cdot \overline{D})}$$  \hspace{1cm} (2)
\[
\rho = \frac{A}{C} - \alpha_b = \frac{(A \cdot D - B \cdot C)}{2 \cdot \text{Re}(C \cdot D)}
\]

(3)

where \( A, B, C, D \) – A-parameters of equivalent circuits including matching devices on the power supply side, rail line and matching devices on the relay side (ES further on);

\( \bar{C}, \bar{D} \) – complex conjugate values.

Consider the nature of the change of the obtained areas of the input resistances of ES in different track circuit operation states.

We consider a rail circuit 500 meters long with a signal current frequency of 420 Hz as a subject of analysis. Transformers POBS-2G(M) (Track Single-phase Armored Dry) with a transformation ratio \( n=38 \) are used as relay and supply transformers.

The nomograms presented below were obtained in the mathematical modeling environment MathCad based on the expressions (1)–(3) and equivalent schemas of track circuit in normal, shunt and control operational states [5].

3. Results

3.1. Normal state

The normal state of operation of the track circuit is characterized by its unoccupied and well-functioning condition. It should be carried out in a wide range of variations in the resistance of rail line insulation.

The changes in the area of the input resistances of the ES in the normal state of operation of the track circuit with different insulation resistances \( R_i \) are shown in Figure 2.

Figure 2. The areas of the input resistances of the ES in the normal state of operation of the track circuit.
The figure shows that the area of the input resistances of the ES in the normal state of operation of the track circuit depends on the insulation resistance of the rail line as follows: when insulation resistance increases, the center of the circle shifts towards increasing resistance, and the radius of the area itself increases.

3.2. Shunt state

The shunt state of the rail circuit is characterized by its intact condition, in the case when the rail line is shunted at any point by resistance with a value of at least 0.06 Ohm.

The change in the area of the ES input resistances in the shunt state of the track circuit with different insulation resistances and shunt overlay coordinates within 0.06 Ohm ($L_{sh}$) is shown in Figure 3.

![Figure 3. The areas of the input resistances of the ESS in the shunt state of the track circuit.](image)

The figure shows that the area of the input resistances of the ES in the shunt state of the track circuit depends on the insulation resistance of the rail line and the coordinates of the overlay of the shunt as follows:

- the radius of the circle is much smaller than the radius of the circle characterizing the normal state;
- the coordinates of the center of the circle are much smaller than the values of the coordinates of the center of the circle characterizing the normal state;
- the radius of the circle is much less than the values of the coordinate of its center (therefore, instead of the circle in the figure, the cross marks its center);
- with increasing insulation resistance, the area shifts towards an increase in active resistance and towards a decrease in inductive resistance;
- as the location of shunt overlay moves from the supply end, the area shifts towards increasing the active and inductive resistances.

3.3. Control state
The control state of a track circuit is characterized by its unoccupied state and complete electrical break of the rail line at any point of the rail line. The change in the areas of the input resistances of the ES in the control state of the track circuit when combining different insulation resistances and coordinates of a complete electrical break of the rail line (L_C) is shown in the Figure 4.

![Figure 4. The areas of the input resistances of the ES in the control state of the track circuit.](image)

The figure shows that the areas of the input resistances of the ES in the control state of the track circuit depends on the insulation resistance of the rail line and the coordinates of the shunt overlap as follows:

- the radius of the circle is much smaller than the radius of the circle characterizing the normal state;
- the coordinates of the center of the circle are much larger than the values of the coordinates of the center of the circle characterizing the normal state;
- the radius of the circle is much less than the values of the coordinate of its center;
• with increasing insulation resistance, the area shifts towards increasing active resistance and increasing inductive resistance;
• as far as the place of complete electrical break of the rail line is moved from the feed end, the area is shifted towards increasing resistance.

Let us estimate how distinguishable are the areas of the input resistances of the ES in the shunt and control states of track circuit operation.

Figure 5 shows the mutual arrangement of areas of input resistances of the track circuit in shunt and control states.

4. Conclusions
As it can be seen from the figure, the areas of the input resistances of the ES are completely distinguishable in the whole range of variations of the insulation resistance of the rail line, the coordinates of the shunt overlap and the coordinates of the complete electrical break of the rail line. A graphical representation of the input resistance area allows you to identify the state in which the studied system is located.

The nature of the change in this area allows us to estimate the rate of change in the parameters of this state (insulation resistance, shunt overlay coordinate, complete electrical break of a rail line) and can be used as a diagnostic feature in rail track monitoring and diagnostics systems (eg [6], [7], [8]).

References

[1] Theeg G, Vlasenko S 2009 Railway Signalling & Interlocking: international Compendium: book (Hamburg: DVV Media Group GmbH) p 448
[2] Volkov E A, Sankovskiy E I, Sidorovich D Y 2005 *Teoriya linejnyh ehlektricheskikh cepej zhelezodorozhnoj avtomatiki, telemechaniki i svyazi* [Theory of linear electrical circuits of railway automatics, telemechanics and communication] (Moskow: Marshrut) p 507

[3] Lunev S A, Ayupov R Sh, Sokolov M M 2010 Control of the technical condition of the elements of the power supply system of non-hauling railways consumers *Nauchnye problemy transporta Sibiri i Dal'nego Vostoka.* (Novosibirsk) 1 pp 254-257

[4] Lavrent'ev M A, Shabat B V *Metody teorii funkciy kompleksnogo peremennogo* [Methods of the theory of functions of a complex variable] (Moscow; Lan') p 688

[5] Tarasov E M, ZHeleznov D V, Belonogov A S *Princip invariantnosti v sistemah kontrolya sostoyanj rel'sovyh linij* [The principle of invariance in the systems of state control of rail lines] (monograph) (Moscow; UMC ZHDT) p 213

[6] Pat. 123388 RF, MPK B 61 L 23/16, B 61 L 25/04. *Ustrojstvo dlja kontrolja sostojanija rel'sovoj linii i zapolnenija puti* [Device for monitoring the state of the rail line and filling the track] / Lunev S A, Seroshtanov S S, M M Sokolov (RF). Publ. 27.12.2012. Vol. 36

[7] Pat. 130942 RF, MPK B 61 L 25/00. *Ustrojstvo dlja kontrolja sostojanija rel'sovoj linii i zapolnenija puti* [Device for monitoring the state of the rail line and filling the track] / Lunev S A, Seroshtanov S S, Sokolov M M, Drevinskaja E S (RF). Publ. 10.08.2013. Vol. 22

[8] Pat. 141222 RF, MPK B 61 L 25/00. *Ustrojstvo dlja kontrolja sostojanija rel'sovoj linii i zapolnenija puti* [Device for monitoring the state of the rail line and filling the track] / Lunev S A, Seroshtanov S S, Sokolov M M (RF) Publ. 27.05.2014. Vol. 15