Simulation of Short-Pulse Generation from A Dynamically Detuned IR-FEL Oscillator and Pulse Stacking at An External Cavity

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Abstract. At the LEBRA facility of Nihon U., we have an IR-FEL oscillator to generate radiations in the range of wavelengths 1 – 6 µm for various experiments. A research program has been established to explore the application of the IR-FEL to generate attosecond UV and X-ray pulses through the high harmonic generation (HHG), where the IR-FEL pulses must have a high-peak power and a short-pulse duration. For the suitable FEL pulse generation within a limited macro-pulse length at the LEBRA LINAC, we apply a dynamical modulation to the electron bunch repetition, that is equivalent to a dynamical detuning of the FEL cavity length. We illustrate the potential performance of the IR-FEL with the dynamical detuning by time-dependent 3D FEL simulations. We also evaluate the enhancement of the FEL pulses by an external cavity stacking for the sake of the HHG application.

1. Introduction
The oscillator FEL is a compact and well-established device, that is widely used to generate monochromatic bright radiations at various wavelengths ranging from IR to THz. The generated pulses are highly affected by the condition of lasing, that changes by the cavity length detuning against the separation of electron bunches as well as the small signal gain and the cavity loss. Especially, when the cavity length is perfectly synchronized with the separation of electron bunches, the generated pulses become to have a high peak power and a few cycle pulse duration [1, 2]. However, a longer macro-pulse length of a milli-second scale is required to reach the saturation at the perfect synchronization.

To generate pulses at a higher peak power and a shorter pulse duration at the LEBRA facility where the macro-pulse length is 20 µs, we employ the dynamic desynchronization (dynamical modulation) that switches from a desynchronization to the perfect synchronization dynamically [3, 4] (see also [5, 6]). When we apply the modulation to the middle of electron pulse train acceleration, we can start from the desynchronization of the FEL-cavity and change to the perfect synchronization, thanks to the change of electron bunch separation during the pulse train. At the first stage, the light pulse is accumulated earlier to have enough small-signal gain. Then the interaction is switched so that the light pulses starts growing to reach a higher saturation level, respecting the perfect synchronization lasing.
For the enhancement, one may also consider to use an external cavity to stack pulses generated from the FEL oscillator. There were less attempts regarding on this direction compared with the laser cavity studies based on solid-state mode locked lasers. In the previous work [7], the accumulation of more than 75 times the energy of pulses was observed experimentally. Although the milli-second scale was required for this accumulation, it is interesting to investigate an efficient pulse stacking even within the short pulse train of 20 µs. See also [8] for a recent study.

Such the enhanced optical pulses with a few cycle duration opens a new direction to generate attosecond radiations ranging from UV to X-ray through the high-harmonic generation (HHG) in noble gasses. An advantage of the accelerator based light source includes a high-repetition generation rate of MHz to GHz. The possibility of HHG application was proposed together with the carrier-envelope phase stabilization in [9]. Motivated by this proposal, a research program has been established.

In this study, we will show some simulation results of the FEL oscillator, including the detuning property and the lasing in the presence of dynamical modulation, to see the potential performance of the LEBRA facility. We will also illustrate a pulse stacking enhancement at an external cavity toward HHG application by using the generated complex radiation fields. In our calculation, we perform a time-dependent 3D simulation, together with GENESIS 1.3 code [10] at the undulator part of simulation, that is commonly used to simulate the self-amplified spontaneous emissions (SASE).

2. FEL oscillator simulation
The lasing in the FEL oscillator grows by having iterative interactions between the generated light pulse and the electron beam. To have a better understanding of the FEL oscillator, we should include the detail of such the iterative process, i.e. the light pulse piling up inside the FEL-cavity consisting of two concave mirrors. However, if we try to include all the detail of the FEL-cavity, quite a lot of computational cost is required. So here we use a strategy of computation as shown in Fig. 1 to reduce the calculation cost while keeping the reasonable property of the FEL oscillator. See also [11, 12, 13] for the related studies.

In the FEL oscillator, the cavity length is an important parameter, that changes the interaction with the electron beam. We first illustrate the simulation result of the lasing with a variety of the cavity lengths. Here we consider a low charge operation mode at LEBRA facility using the parameters in Tab. 1, which is the ‘full-bunch mode operation’ of a S-band LINAC with a DC thermionic electric gun. The FEL-cavity is designed such that the single light pulse interacts with the electron bunches coming at 22.3 MHz frequency (every 128th RF wave antinodes).

We show the simulation result in Fig. 2, where the pulse energy growth during the electron pulse train is illustrated in the left figure, while the micro-pulse structure obtained at the end of pulse train (20 µs) is shown in the right figure. Here we define the cavity detuning values as the optical path length difference per single round trip, in the unit of the radiation wavelength.
Table 1. A set of parameters for the initial condition at undulator entrance.

| $K$  | $\lambda_u$ | period | $\lambda_r$ |
|------|-------------|--------|--------------|
| 2.02 | 48 mm       | 50     | 4020 nm      |

| $z_{\text{Rayleigh}}$ | $z_{\text{waist}}$ | FEL cavity loss | coupling |
|------------------------|---------------------|-----------------|---------|
| 1.47 m                 | 1.2 m               | 3%              | 1%      |

| $Q$ | $E_k$ | $\Delta E/E$ | repetition |
|-----|-------|--------------|------------|
| 40 pC | 68.4 MeV | 0.4 %        | 2.856 GHz   |

| $\sigma_z$ | $\sigma_{x,y}$ | $\varepsilon_{n,x,y}$ | pulse train |
|------------|----------------|------------------------|-------------|
| 0.3 ps     | 0.4 mm         | 15 $\pi$ mm mrad       | 20 $\mu$s   |

Figure 2. A macro-pulse waveform of accumulation without the dynamical detuning (left figure), and the corresponding pulse structures (right figure). Each number displayed suggests the cavity detuning length in the unit of radiation wavelength $\lambda_r$ respectively.

Figure 3. A macro-pulse waveform of accumulation with the dynamical detuning (left figure), and the corresponding pulse structures (right figure). We start from the cavity detuning length $\Delta L = -0.75 \lambda_r$, and change to the zero-detuning at each arrow point respectively.
When the cavity detuning length is zero, the round trip light path is perfectly synchronized with the separation of electron bunches that interacts with the single light pulse. We see from the left figure that the saturation level near the perfect synchronization is higher although it needs more interactions to reach the saturation. The lasing at the perfect synchronization is far from the saturation within the 20\(\mu\)s pulse train. When the detuning value is near or equal to the perfect synchronization, the central part of light pulse interacts with the dense part of electron bunches iteratively, resulting in the higher-peak power and shorter pulse duration as shown in the right figure.

Although the high saturation at the perfect synchronization requires lots of interactions, there is a way to make the saturation level higher even within a short pulse train by using the dynamic desynchronization or the dynamical modulation [3, 4].

We simulate this dynamical modulation using the desynchronization mode at detuning \(-0.75\lambda_r\), where the earlier accumulation is obtained. We show the result in Fig. 3, where the the modulation to the perfect synchronization (\((-0.75\lambda_r \rightarrow 0)\) is applied at each arrow point of pulse train in the left figure. We see in the right figure that the generated pulse structure respects the property of perfect synchronization, including a higher peak power and a shorter pulse duration than that of detuning \(-0.75\lambda_r\) in Fig. 2.

Unfortunately, we understand that it is difficult to reproduce this behavior at LEBRA facility right now since we observed a significant demagnetization of the undulator. However, it is worth commenting that the result of the dynamical modulation behavior of FEL oscillator (Fig. 3) as well as the detuning property (Fig. 2) is qualitatively well agreed with the previous experimental studies obtained in [4]. This qualitative agreement supports our simulation strategy illustrated in Fig. 1.

3. Pulse stacking of FEL lights

Next, we proceed to the pulse stacking using the radiation complex fields generated from the iterative calculations of the FEL oscillator. Although the external cavity consists of many details of optical components, here we use a simplification so that we can capture the essential feature of the pulse stacking from the FEL oscillator, especially at a reasonable computational cost.

We plan to stack the pulses in the external cavity, where the repetition of the pulses is 44.6 MHz, in other words, the one round light path length is half the size of that in FEL-cavity. We assume 10\% injection transmittance, and 1% (transmittance) loss for the others, and do not include the other detail of mirrors and their diffractions for simplicity.

We show the result of pulse stacking in Fig. 4, where we use the radiation field data obtained at detuning \(-0.25\lambda_r\) in Fig. 2 as an illustration. Here we consider the pulse stacking from the pulses generated in the FEL-cavity at 22.3 MHz. Note that here we also define the detuning of external cavity to be the difference of the one round light path length, and the two round light path length at the perfect synchronization should be the same as the separation of the electron bunches that interact with the single light pulse in FEL-cavity. In the left figure, we scan through detuning values of the external cavity and estimate the maximal peak power accumulated along the pulse train. In the negative region of detuning values, the summation of the FEL-cavity detuning \((-0.25\lambda_r)\) and the double of detuning value at each peak position becomes a negative integer in the unit of wavelength, meaning that the resonance from the wavelength holds. On the other hand, in the positive region, the summation becomes a non-integer, while the double of separation of peak positions becomes an integer. The difference from the positive and negative region may be related with the fact that a pulse chirp is generated during the growth from the interaction with electron beam. This difference may be used to help finding the perfect synchronization point of the external cavity without installing the measuring devices at high accuracy. We also display the pulse structure at the maximal peak power (right figure) at a negative detuning and a positive detuning. The pulse shape at the negative detuning looks
Figure 4. An example of pulse stacking. The detuning scan (left) is performed at each $\lambda_r/16$. Each value displayed in the right figure corresponds to the detuning value in the unit of radiation wavelength.

similar to that of the original pulse, while the positive detuning modifies the pulse shape.

4. Discussion
We showed some simulation results regarding the FEL oscillator, the dynamical modulation lasing, and the pulse stacking at the external cavity. We started our simulation respecting the low charge operation mode (Tab. 1), since we would like to understand the essential property and behavior of the FEL oscillator and the generated pulses for the pulse stacking.

On the other hand, at the LEBRA facility, we also have a high charge mode operation where more than 10 times bunch charge can be transferred and used for the FEL lasing. So we will proceed to understand the property of the lasing from the high charge mode operation and its pulse stacking, where a much higher peak power can be expected, that is suitable for the HHG application.

Also, we have a plan to replace the undulator that is currently demagnetized significantly. After the replacement, our simulation result will be compared directly with the experiment.

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