On the temperature dependence of ballistic Coulomb drag in nanowires

M I Muradov and V L Gurevich

A F Ioffe Institute of Russian Academy of Sciences, 194021 St Petersburg, Russia

E-mail: muradov@mail.ioffe.ru

Received 7 December 2011, in final form 1 February 2012
Published 9 March 2012
Online at stacks.iop.org/JPhysCM/24/135304

Abstract
We have investigated within Fermi liquid theory the dependence of Coulomb drag current in a passive quantum wire on the applied voltage $V$ across an active wire and on the temperature $T$ for any values of $eV/k_BT$. We assume that the bottoms of the 1D minibands in both wires almost coincide with the Fermi level. We conclude that: (1) within a certain temperature interval the drag current can be a descending function of the temperature $T$; (2) the experimentally observed temperature dependence $T^{-0.77}$ of the drag current can be interpreted within the framework of Fermi liquid theory; (3) at relatively high applied voltages the drag current saturates as a function of the applied voltage; and (4) the screening of the electron potential by metallic gate electrodes can be of importance.

Coulomb drag predicted by Pogrebinskii [1] (see also Price [2]) is a phenomenon directly associated with Coulomb interaction of the electrons in a semiconductor. In a structure of two semiconductors separated by an insulating layer there would be a drag of electrons in semiconductor 1 (drag current) due to the direct Coulomb interaction with the electrons in semiconductor 2 (drive current) where an electric current flows. Continued advances in the semiconductor lithography technique have provided many examples of this effect between 2D electron layers separated by hundreds of angstroms and have encouraged interest in this field (see, e.g., [3], where a number of papers dealing with the Coulomb drag in two 2D layers are discussed).

The Coulomb drag effect for two parallel quantum wires in the ballistic regime has been investigated by Gurevich et al [4] for $eV \ll k_BT$. This case may be called linear as the drag current is a linear function of the applied voltage $V$. The authors of the present paper treated in [5] a nonlinear case where $eV \gtrsim k_BT$. In both cases the Fermi energy $\mu$ was assumed to be much larger than $k_BT$. As is well known, under this condition the transport phenomena are determined by a stripe of width $k_BT$ near the Fermi level. This means that the drag current $J_{\text{drag}}$ should have a maximum provided these stripes in both quantum wires overlap so that interwire electron collisions are possible. For identical wires (the case treated in [4, 5]) this requirement means coincidence of the bottoms of 1D bands of transverse quantization. Under these conditions $J_{\text{drag}}$ increases with temperature.

One encounters an entirely different situation if the Fermi level is near the bottom of a 1D band. If this is the case for two bands in both wires all the electrons of these bands can take part in electron–electron collisions. The very effect of drag is strongly dependent on the transferred (quasi)momentum $p_t$ in the course of interwire Coulomb scattering of electrons. The number of electrons involved decreases with $p_t$, whereas the interaction responsible for the drag increases and its influence is predominant for $J_{\text{drag}}$. Our purpose is to investigate this situation. In other words, we assume that

$$|\mu_n| \lesssim k_BT, \quad \mu_n \equiv \mu - \varepsilon_n(0)$$

where $\varepsilon_n(0)$ is the position of the bottom of the $n$th 1D band (a result of transverse quantization) and $\mu$ is the chemical potential. In our discussion of the situation we will use the experimental findings of [6] and [7] (see also the review paper [8]). As is seen in these works, the drag voltage peaks occur just where the quantized conductance of the drive (active) wire rises between the plateaus. This means that the maximum of the drag effect occurs provided the 1D bands of the two quantum wires are aligned (or, in other words, their bottoms coincide within the accuracy of $\lesssim k_BT$) and at the same time Fermi quasimomenta are small. The first two peaks are well pronounced. The first of them corresponds to alignment of the two ground 1D bands in both wires. The...
second one corresponds to alignment of the ground (first) 1D band of the passive (drag) wire and the second 1D band of the drive wire.

It was found that the temperature dependence of the drag current can be described by the law $\sim T^{-0.77}$ in the temperature interval from 60 mK to 1 K. The authors of these papers [6–8] highlighted this temperature dependence, claiming that the power-law temperature dependence of the drag resistance is a signature of the Luttinger liquid state. The authors of [9] evidently share the same opinion, claiming that for coupled Fermi liquid systems the drag resistance is always an increasing function of temperature.

In this paper we will argue that the situation is not so simple, and the temperature dependence observed in experiments can be explained within the Fermi liquid approach (see, for instance [10]).

The experimentally found magnitudes of the drag resistance are of the order of hundreds of ohms or even less and one should provide a special explanation for the weakness of the interwire electron–electron interaction. We believe that it is due to the screening of Coulomb interaction by the gates. Such screening has not been taken into consideration so far. In [5] the following equation has been derived for the drag current (see equation (13) in [5])

$$J_{\text{drag}} = -2e \sinh \left( \frac{eV}{2k_BT} \right) \frac{2\pi mL}{2\pi h} \left( \frac{2L}{2\pi h} \right)^2 \frac{e^2}{\kappa L} \times \sum_{n\geq 0} \int_0^\infty dp \int_0^\infty dp' B_{\text{mix}}(p+p') Q, \quad (2)$$

where

$$Q = \exp \frac{\epsilon dp - \mu}{k_B T} - \exp \frac{\epsilon dp' - \mu}{k_B T} - f(\epsilon_{np} - \mu)f(\epsilon_{np'} - \mu)\right) \right) $$

Here $f(\epsilon - \mu)$ is the Fermi function and $\kappa$ is the dielectric susceptibility.

The unscreened Coulomb interaction matrix element squared $g_{\text{int}}(p+p')$ can be written as $[K_0(d(p+p')/\hbar)]^2$ provided the widths of the wires are much smaller than the interwire distance $d$. Here $K_0(s)$ is the MacDonald function. Using the random phase approximation, one can straightforwardly take into consideration the screening by the gates as well as by the quantum wires themselves. However, as the resulting equation is too cumbersome, we will take into account the screening only by the gates, treating them as a single plane. As for the contribution of 1D wires to the screening, we will neglect it. As a result, we get for the screened Coulomb interaction

$$U_s(\omega, q_x) = \int \frac{dq_x}{(2\pi)^2} C_n(q_x) U(q_x) C_l(-q_x)$$

$$+ \int \frac{dq_y}{(2\pi)^2} C_n(q_y) U(q_y) \int \frac{dq_z}{2\pi} C_l(-q_y, -q_z) U(q_y, q_z) \times \frac{\Pi_{\text{mix}}^B(q_y, q_z)}{1 - \Pi_{\text{mix}}^B(q_y, q_z)} U(q_y, q_z), \quad (4)$$

where $C_n(q_x) = |n\rangle e^{iq_x\cdot r_x}|n\rangle$ and $C_l(q_x) = |l\rangle e^{iq_x\cdot r_l}|l\rangle$. Here $|n\rangle$ and $|l\rangle$ are the transverse wavefunctions of the first and second quantum wires. Precisely,

$$|n\rangle = \phi_{\text{ph}}(r_\perp), \quad r_\perp \longleftrightarrow y, z$$

is the wavefunction describing the transverse quantization. $U_q = 4\pi e^2/xq^2$ is the Fourier transform of the 3D Coulomb potential, $U(q_x, q_y) = \int dq_z U(q_x, q_y)$. The polarization operator $\Pi^B(\omega, q_x, q_y)$ for a 2D layer can be found in [11]. We assume that the gate electrodes are made of a metal where the period of plasma vibration is much shorter than any characteristic time of a semiconductor. Therefore we will deal only with the static as well as the long wave limit of this operator. In this limit it is reduced to the 2D electron density of states.

We assume that the gates are in the plane $z = 0$, two quantum wires parallel to the plane (and oriented along the $x$-axis) are displaced by the same distance $z_0$, the interwire distance being $d$. For the electrons with coordinates $x, 0, z_0$ and $x', d, z_0$, belonging to two wires

$$U_s(\omega, x-x') = e^2 \int \frac{e^{-iq_xd - i\omega(x-x')}}{2\pi \sqrt{q_x^2 + q_y^2}} dq_x dq_y$$

$$+ e^2 \int \frac{e^{-iq_xd - i\omega(x-x')}}{2\pi \sqrt{q_x'^2 + q_y^2}} dq_x dq_y \times \frac{2\pi e^2 \Pi_{\text{mix}}^B(q_x, q_y)}{1 - 2\pi e^2 \Pi_{\text{mix}}^B(q_x, q_y) / \sqrt{q_x^2 + q_y^2}}. \quad (5)$$

In the static case we arrive at a simple result

$$U_s(x-x') = \frac{e^2}{\sqrt{(x-x')^2 + d^2}} - \frac{e^2}{\sqrt{(x-x')^2 + d^2 + (2z_0)^2}}, \quad (6)$$

the second term here describing the action of an ‘image’ (we assume that $z_0$ is bigger than the Bohr radius).

Therefore we get for the drag current instead of (2)

$$J_{\text{drag}} = -\frac{2e^2 mL}{\pi^2 h^2 \kappa^2} \left( \frac{d}{d} \right)^4 \left( \frac{d}{d} \right)^2 \sinh \frac{eV}{2k_BT} \times \sum_{n\geq 0} \int_0^\infty dp \int_0^\infty dp' (p+p')K_1^2(d(p+p')/\hbar) Q, \quad (7)$$

where now

$$Q = [1] \cosh[p_x^2 - \epsilon_{nl}^2]/4m|k_B T| \cosh[p_x^2 - \epsilon_{nl}^2 + mV]$$

$$- 2m\epsilon_{nl}/4m|k_B T| \right]^{-1} \times [1] \cosh[p_x^2 - \epsilon_{nl}^2]/4m|k_B T|$$

$$\times \cos[p_x^2 - \epsilon_{nl}^2 - mV - 2m\epsilon_{nl}/4m|k_B T|]^{-1}$$

and

$$\epsilon_{nl} = \epsilon_n - \epsilon_l.$$
i.e. the spacing $d$ between the quantum wires is larger than their distance $z_0$ to the gates.

Now

$$K_0\left(\frac{d^2p^2}{h}\right) - K_0\left(\frac{1+(2z_0/d)^2d^2p^2}{h}\right) \approx 2\left(\frac{z_0}{d}\right)^2d^2p^2 K_1\left(\frac{d^2p^2}{h}\right). \quad (9)$$

The scale of variation of $Q$ as a function of $p$ and $p'$ is the thermal momentum $\sqrt{4mk_BT}$. At the same time $(p + p')K^2_1(d(p + p')/h)$ is a rapidly decreasing function, the scale of its variation is $h/d$. For

$$h/d \ll \sqrt{4mk_BT}$$

one can take out of the integral all the slowly varying functions keeping as the integrand only $(p + p')K^2_1(d(p + p')/h)$. For $p_n \geq h/d$ in the case of $1D$ band alignment in two wires

$$\epsilon_n \approx \epsilon_l$$

we can retain just the contribution of these $1D$ bands in the sum and get

$$J_{drag} = J_0[(\sinh[eV/2k_BT])\cosh^2[p^2_n/4mk_BT]\cosh[(p^2_n - meV)/4mk_BT]\cosh[(p^2_n + meV)/4mk_BT]]^{-1}, \quad (10)$$

where

$$J_0 = - \frac{3e^5mL}{16\hbar^3 k^2}\left(\frac{z_0}{d}\right)^4. \quad (11)$$

Here we have made use of the equation

$$\int_0^\infty dx\int_0^\infty dy (x+y)K_1^2(x+y) = \frac{3\pi^2}{32}. \quad (12)$$

The temperature dependence in the considered temperature region as well as the dependence on the applied voltage is given by equation (10). Within a comparatively big temperature interval the drag is a descending function of temperature. At smaller temperatures it reaches a maximum.

At small applied voltages we have

$$J_{drag} = J_0 \frac{eV}{2k_BT} \frac{1}{\cosh^2[p_n^2/4mk_BT]}, \quad (13)$$

a linear dependence on $V$, for bigger voltages the drag current saturates at

$$J_{drag} = 2J_0 \frac{1}{\cosh^2[p_n^2/4mk_BT]}, \quad (14)$$

We wish to emphasize that the screening has nothing to do with the temperature dependence. The small factor $(z_0/d)^4$ indicates that the screening can be important, as it can explain the magnitude of the effect (without regard for the screening, the theory would have given values for the the drag current that were too large).

Thus we have come to conclusion that the experimentally observed temperature dependence can be understood within the Fermi liquid approach. The temperature dependence is shown in figure 1 (for a linear case $eV \ll T$ on the left of the figure and for large applied voltages $eV \gg T$ on the right) where $T_n = p_n^2/2m$. The thin lines correspond to a $T^{-0.77}$ law and are given for the comparison. It is clearly seen that our curves can also be approximated by the $T^{-0.77}$ dependence, although the authors of [6] and [7] regard this dependence as evidence of Tomonaga–Luttinger (TL) liquid behaviour of the quantum wires. Indeed, they argued that the increase of the drag with decreasing temperature in a characteristic power-law fashion is in sharp contrast with the prediction of Fermi liquid theories and, therefore, may serve as a signature of the TL behaviour.

The Fermi liquid result (see figure 1) can be visualized as follows. At very low temperatures there is Fermi degeneracy and therefore the drag current as a function of temperature goes up. At higher temperatures the degeneracy is lifted while the average electron energy increases with temperature. This results in a decrease of the drag current.

For large values of $n$ and $l$ one can be sure of the applicability of the Fermi liquid approach. In our opinion, it would be of great importance to investigate by experiment and theory the physical conditions (including the reservoir influence) that would bring about transition from the Fermi
to Luttinger liquid behaviour for small values of $n$ and $l$. This problem seems not to be simple since such an investigation should take into account the influence of the number of 1D bands in the quantum wire, the vicinity of reservoirs, the electron–phonon interaction, and, of course, the role of temperature.

We would like to point out some outcomes of our theory. First, the interwire influence can be of importance for scaled down devices. According to our theory, to minimize an undesired influence of this sort one should avoid the alignment of 1D bands. Second, we note, that as the effect has a maximum as a function of the temperature, this fact also provides some degree of freedom to change such an influence. On the other hand, the effect can be used as a probe in the spectral analysis of nanostructures since it is very sensitive to the alignment of 1D bands. Finally, the effect can be important for direct investigation of Coulomb scattering in nanostructures.

References

[1] Pogrebinskii M B 1977 Sov. Phys. Semicond. 11 372
[2] Price P J 1983 Physica B 117 750
[3] Rojo A G 1999 J. Phys.: Condens. Matter 11 R31
[4] Gurevich V L, Pevzner V B and Fenton E W 1998 J. Phys.: Condens. Matter 10 2551
[5] Gurevich V L and Muradov M I 2000 Zh. Eksp. Teor. Fiz. Pis’ma Red. 71 164
[6] Debray P, Vasilopoulos P, Raichev O, Perrin R, Rahman M and Mitchell W C 1999 Physica E 6 694
[7] Debray P, Zverev V, Raichev O, Klesse R, Vasilopoulos P and Newrock R S 2001 J. Phys.: Condens. Matter 13 3389
[8] Debray P, Gurevich V, Klesse R and Newrock R S 2002 Semicond. Sci. Technol. 17 R21
[9] Peguiron J, Bruder C and Trauzettel B 2007 Phys. Rev. Lett. 99 086404
[10] Abrikosov A A 1988 Fundamentals of the Theory of Metals (Amsterdam: North-Holland)
[11] Stern F 1967 Phys. Rev. Lett. 18 546