**Abstract**

The effect of negative magneto-resistance for thin superconducting wire was considered in a simple model. These phenomena originated from competition of two mechanisms: fluctuations of the order parameter and quasiparticles charge imbalance which accompanies each phase slip event. First process results in conventional positive magnetoresistance while the second mechanism gives the negative contributions. Simple analytical formula is obtained for the negative magnetic resistance caused by both the thermodynamics (TAPS) and quantum fluctuations. Theoretical results are compared with experimental data and demonstrate good agreement between theory and experiment.

**Keywords:** superconducting wire, fluctuations, negative magnetic resistance

**Introduction**

It is well established that superconducting fluctuations play a very important role in reduced dimension. Above the critical temperature $T_c$ such fluctuations yield an enhanced conductivity [1-2]. Below $T_c$, fluctuations are known to destroy the long-range order in low-dimensional superconductors. It was first pointed out by Little [3] that one dimensional wires made of a superconducting material can acquire a finite resistance below $T_c$ of the bulk material due to thermally activated phase slips (TAPS) that destroy superconductivity locally by thermal fluctuations. In this process, the superconducting phase $\varphi(t)$ flips by $2\pi$ across those points of the wire where the order parameter is "temporarily" destroyed. According to the Josephson relation $h(\partial \varphi / \partial t) = 2eV$; consequently, such phase slips cause a nonzero voltage drop $V$ and, hence, dissipative currents inside the wire. A theory of the TAPS phenomenon was developed in Refs. [4],[5]. The TAPS mechanism is effective only in the vicinity of $T_c$. At low temperatures, well below $T_c$, phase slips can be triggered through quantum tunneling processes (QPS). First theoretical studies of the QPS effects [6] were performed within a simple approach based on the time dependent Ginzburg-Landau (TDGL) equations. Later, in Refs. [7-9], a microscopic theory of QPS processes was developed with the aid of the imaginary time effective action technique [8] which properly accounts for nonequilibrium, dissipative and electromagnetic effects during a QPS event. One of the main
conclusions reached is that the QPS probability is considerably larger than it was predicted previously [9].

The principal difference between TAPS and QPS is that in the former the required energy for the fluctuation is provided by the "classical" term energy of the order of $T$, while in QPS the relevant energy scale is $\sim \hbar \Gamma_Q$ where $\Gamma_Q = \langle \partial \varphi / \partial t \rangle$ is the rate of fluctuations.

Experimentally, TAPS is observed as broadening of the normal to superconducting transition whereas the QPS is borne out as a weak temperature dependent resistance at low temperatures [10-12]. Most of the experiments were performed in superconducting nanowires in which the cross section of the wire has dimensions comparable to the superconducting coherence length of the material.

An interesting phenomenon is observed if the superconducting wire is exposed to external magnetic field. In this case, a negative magneto-resistance (NMR) [13-16] is found in both the TAPS and QPS regimes. As of today, the origin of this effect especially in a short wires is not yet fully understood [15]. In fact, in a long wires, the NMR revealed in Ref. [10-12] could be caused by the suppression of the rate of activated phase slips as a consequence of the decrease of the charge-imbalance length $l_Q$ with magnetic field [15]-[16]. In this case (long wires) the wire length $L$ exceeds $l_Q$. Despite a number of theoretical works carried out in this approximation, analytical formulas have not yet been obtained that made it possible to compare this theory with existing experiments. The aim of this work is to obtain a simple analytical theory describing NMR in the long filament approximation and to compare the results with the experiment performed on amorphous indium oxide wires.

**Theoretical Background**

1. **TAPS Regime**

We start with a brief analysis of the effect of magnetic field, H, on the resistance in the TAPS regime. It has been shown [1] that the rate of TAPS, $\Gamma_T$, at a bias DC current, I, is:

$$\Gamma_T = \Omega \exp \left( -\frac{\Delta F}{T} \right)$$

(1)

where $\Delta F = N(\varepsilon_F) \Delta^2 (H,T) \xi(T) S/2$, $\Delta(H)$ is the superconducting gap, $\xi$ is the coherence length, $S$ is the cross-section of the wire, $k_B$ is the Boltzmann constant (here $k_B = 1$), $N(\varepsilon_F)$ is the density of electron states, while the pre-factor $\Omega$ has the McCurmy-Halperin form [2]. Taking for simplicity superconducting gap in the approximation form $\Delta^2 = \Delta_0^2 (1 - b^2 - t^2)$, [17], one obtains

$$\Omega = \frac{L}{\zeta} \left( \frac{\Delta F}{T} \right)^{1/2} \tau_{GL}^{-1} \xi_0 \frac{\pi \hbar}{8 T_c \sqrt{1 - b^2 - t^2}}$$

(2)

Here $L$ is the wire length, $\xi(b,t) = \xi_0 / \sqrt{1 - b^2 - t^2}$, $\tau_{GL}$ is the Ginzburg-Landau relaxation time, $b = H / H_{c2}(0)$ and $t = T / T_c$ are the reduced magnetic field and the temperature correspondingly, $H_{c2}(0)$ is the second critical field at zero temperature. Substituting

$$\Gamma_{TAPS}(b,t) = \frac{L}{\xi_0 \sqrt{\tau_{GL}}} \left( \frac{1 - b^2 - t^2}{\tau_{GL}} \right)^{5/4} \exp \left( -\frac{\sqrt{1 - b^2 - t^2}}{t} \right).$$

(3)
Here \( \gamma = \frac{N(eq)\Delta^2}{2T_c} \) and \( \tau_{gl}^0 = \pi\hbar/8T_c \).

Using the Josephson relation and the rate of fluctuation, one can get the time-averaged voltage created by the phase slips \( V_{PS} = h\Gamma_{PS}/2e \). However, the fluctuation-governed resistive state of a 1D superconductor is, in reality, a dynamic process consisting of discrete events (phase slips) repeated on average with the rate \( \Gamma_{PS} \). During each phase slip of duration \( \sim h/\Delta \) inside the PS core region \( \sim \xi \), the magnitude of the order parameter goes to zero and the phase slips by \( 2\pi \). In this case each such fluctuation gives rise to a 'quasi-normal' region where a conversion from non-equilibrium quasiparticles to equilibrium Cooper pairs takes place. The charge imbalance is pinned to each phase slip and decays in time and space. The energy dissipation inside this quasi-normal region should be taken into account to obtain the total effective resistance.

The energy dissipation inside this quasi-normal region should be taken into account to obtain the total effective resistance [16]:

\[
R_{tot} = V_{PS}/I + R_Q.
\]

(4)

where \( R_Q = \frac{\rho_N}{S} A_Q \tau_0 \Gamma_{PS} \) is the additional resistance caused by non-equilibrium quasiparticles inside the phase sleep which is valid under condition \( \tau_0 \ll \tau_E \ll \Gamma_{PS}^{-1} \), here \( \tau_0 = 2T_c h/\pi\Delta^2 \), is the superconducting response time ('life time' of the PS), \( \rho_N \) is the resistivity of the PSC,

\[
A_Q = (D \tau_Q)^{1/2}
\]

(5)

is the charge imbalance decay length (normal domain pinned by the phase slip).

The relaxation of the charge imbalance \( Q^* \) in a 1-D superconducting channel can be described by the equation [18-21]:

\[
D \tau_Q \ddot{Q}^* + \tau_Q Q^* = \tau_0 \tau_E \dot{Q}^* + (\tau_0 + \tau_E) Q^*.
\]

(6)

Here \( D \) is the electron diffusion coefficient, \( \tau_Q \) is the charge imbalance decay time:

\[
\tau_Q^{-1} = \frac{\pi\Delta}{T_c} \left( 1 + \frac{2\tau_0}{\tau_S} \right)^{1/2}; \quad \tau_S^{-1}(H) = \frac{1.76T_c}{h} \frac{H^2}{H_c^2(0)};
\]

(7)

where \( \tau_E^{-1} = T^3/h\omega_D^2 \) is the electron phonon inelastic scattering time, \( \omega_D \) is the Debye frequency, \( \tau_S \) is the pair breaking time due to magnetic field in the limit of a small bias current \( I \ll I_c \).

Inserting the above definitions, the total resistance has the form:

\[
R_{tot} = R_{tot} = R_{TAPS} + R_Q = \frac{h}{2eI} \left[ 1 + \frac{\beta}{t^{3/4}(1-b^2-t^2)^{5/4}(t^3+ab^2)^{3/2}} \right] \Gamma_{TAPS}
\]

(8)
where \( \alpha = \frac{3.52 \tau_e(T_c) T_e}{\hbar} \), \( \beta = I \frac{Aq_p N}{\Delta_0} \left( \frac{2D \tau_e(T_c)}{\pi \Delta_0} \right)^{1/2} \); \( A_Q = a(t - b^2 - t^2)(t^3 + ab^2)^{-1/4} \) and \( a = (0.56 \tau_e(T_c) D)^{1/2} \).

Defining a reduced resistivity in the form:

\[
r_{TAPS} = \frac{R_{tot(b)} - R_{tot(0)}}{R_{tot(0)}},
\]

(9)

and inserting Eqs. (7)-(8), one obtains:

\[
r_{TAPS}(b, t) = \frac{1 + \frac{\beta}{t^{3/4}(1 - b^2 - t^2)^{5/4}} [t^3 + ab^2]^{1/2}}{1 + \frac{\beta}{t^{3/2}(1 - t^2)^{5/4}}} \left( \frac{\Gamma_{TAPS(b,t)}}{\Gamma_{TAPS(0,t)}} \right) - 1
\]

(10)

where the ratio

\[
\frac{\Gamma_{TAPS(b,t)}}{\Gamma_{TAPS(0,t)}} = \left( \frac{1 - b^2 - t^2}{1 - t^2} \right)^{5/4} \exp \left( \gamma \frac{\sqrt{1 - t^2} - \sqrt{1 - b^2 - t^2}}{t} \right).
\]

(11)

Hence

\[
r_{TAPS}(b, t) = \left( \frac{1 - b^2 - t^2}{1 - t^2} \right)^{5/4} \left[ 1 + \frac{\beta}{t^{3/4}(1 - b^2 - t^2)^{5/4}} [t^3 + ab^2]^{1/2} \right] \frac{1}{\gamma} \exp \left( \gamma \frac{\sqrt{1 - t^2} - \sqrt{1 - b^2 - t^2}}{t} \right) - 1
\]

(12)

2. QPS Regime

Quantum fluctuations usually give an effect at low temperatures and in thin nano-wires (with width less than 100nm) where the effects of a decrease in the region of electron-hole imbalance in a magnetic field are poorly studied. In this case, the magnetoresistance has the form of Eqs. (7) (8) (10) [15] where, however [7],

\[
\Gamma_{QPS} = \frac{A(b,t) S_{QPS} L}{\hbar} \exp(-S_{QPS});
\]

where

\[
S_{QPS} = A \frac{R_Q L}{\rho N} \xi(t) \xi(t - \frac{1}{2})
\]

(13)
Here $S_{QPS}$ is the action, $\xi$ is the GL coherence length, $\xi_0$ is the coherence length at zero temperature and the magnetic field $b$, $A \sim 1$ is constant, $L$ is the wire length, $R_N$ is a wire resistance in a normal state resistance, $R_Q = \frac{\hbar}{4e^2}6.46$ kOhm.

Using Eq.(13) one obtains

$$R_{tot} = A^*(1 - b^2 - t^2)^{3/2} \left[ 1 + \frac{\beta}{t^{3/4}(1-b^2-t^2)^{5/4}[t^3 + \alpha b^2]^{3/2}} \right] \exp \left( -\gamma_{QPS} (1 - b^2 - t^2)^{1/2} \right)$$  \tag{14}

Where $A^* = \frac{A_0}{2e} A R_Q R_N L \xi \frac{R_Q}{R_N} \xi_0$; $\gamma_{QPS} = A R_Q L R_N \xi \xi_0$.

In this case the reduced resistance has the form:

$$r_{QPS}(b, t) = 1 + \frac{\beta}{t^{3/4}(1-b^2-t^2)^{5/4}[t^3 + \alpha b^2]^{3/2}} \frac{\gamma_{QPS}(b, t)}{\gamma_{QPS}(0, t)} - 1$$ \tag{15}

Similarly, the ratio $\Gamma_{QPS}(b, t) / \Gamma_{QPS}(0, t)$ is:

$$\frac{\Gamma_{QPS}(b, t)}{\Gamma_{QPS}(0, t)} = \frac{(1-b^2-t^2)^{3/2}}{1-t^2} \exp \left( -\gamma_{QPS} [(1 - b^2 - t^2)^{0.5} - (1 - t^2)^{0.5}] \right)$$  \tag{16}

Substituting Eq. (16) into Eq.(15) one obtains for reduced resistance in QPS case:

$$r_{QPS}(b, t) = \frac{(1-b^2-t^2)^{3/2}}{1-t^2} \left( \frac{1+ \beta}{t^{3/4}(1-b^2-t^2)^{5/4}[t^3 + \alpha b^2]^{3/2}} \right) \exp \left( -\gamma_{QPS} [(1 - b^2 - t^2)^{0.5} - (1 - t^2)^{0.5}] \right) - 1$$ \tag{17}

**Applicability of the Model**

The theory is valid provided that the length of the superconducting wire significantly exceeds the characteristic size of the PS: $L \gg \Lambda_Q$. It immediately gives the condition for the wire length

$$\frac{L}{a} \gg \left[ t(1 - t^2 - b^2)(t^3 + \alpha b^2) \right]^{-1/4}$$  \tag{18}

Result is presented in Fig. 1
Fig. 1 Length of the wire in the units of diffusion length \(a = (0.56D \tau_{\epsilon_0})^{1/2}\) versus reduced magnetic field \(b\) for three different temperatures (from below to top \(t=0.75, 0.62\) and \(0.39\)). Below the line \(\frac{L}{a} = \text{const}\), the ranges of the magnetic field where the theory for infinite long wire is valid.

For typical parameters the characteristic diffusion length \(a \sim 50\text{nm}\), hence the length of the long wire must excite this magnitude.

**Comparing with Experiment.**

To compare our results with experiments we consider magneto-resistance data of the thin (\(w=24\text{ nm}\)) but long (\(L=0.2\mu m\)) wires of the amorphous InO material (coherence length \(\xi_0 = 50\text{nm}, T_c = 1.92K, H_{c2}(0) = 10T\)[22]). In this system below the superconducting critical temperature, the wires with thickness equal to or less than 100 nm, show negative magnetoresistance (NMR). Using Eqs. (12) and (17) the magnetic field dependence of the reduced resistance was compared with experimental NMR data of the Ref. [22] (fig. 6a). The results are presented in Fig.2 where the TAPS theory results are more relevant than the QPS.
NMR resistance as a function of reduced magnetic field for TAPS (Eq. (12), bold line) and experimental data from Ref. [22] (dots) (Red curve at Fig.6a). Material dependent dimensionless fitting parameters in Eq. (12) are chosen as \( g = 39, \alpha = 3800, \beta = 10, \tau = 0.62 \).

NMR resistance as a function of reduced magnetic field for QPS (Eq. (17), bold line) and experimental data from Ref. [22] (dots) (green curve at Fig.6a). Material dependent dimensionless fitting parameters in Eq. (12) are chosen as \( g = 42, \alpha = 1200, \beta = 0.8, \tau = 0.39 \).
The fitting parameters of the QPS theory are slightly different from that of TAPS. Strictly speaking, the theory of NMR is applicable only near the critical temperature however, one has use it at low temperature as a "toy model". It should be noted however that qualitatively the theory of NMR is applicable even in this case in low temperature range.

Conclusions

The effect of negative magnetoresistance for thin infinite superconducting wire was considered in a simple phenomenological model. Analytical formulas describing NMR in the model based on competition of two mechanism: fluctuations of the order parameter and quasiparticles charge imbalance which accompanies each phase slip event are obtained. First process results in conventional positive magnetoresistance while the second mechanism gives the negative contributions. Analytical formulas for NMR caused both by thermodynamic (TAPS) and quantum fluctuations are proposed. Theoretical result are compared with experimental data and demonstrate good agreement between theory and experiment. The theory is valid provided that the length of the superconducting wire significantly exceeds the characteristic size of the PS: \( L \gg \Lambda_Q \). Main results of the theory are presented in Eqs. (12), (17) and (18) and Figs. 1-3.

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