Tensorial Central Charges and New Superparticle Models with Fundamental Spinor Coordinates

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Abstract

We consider firstly simple $D = 4$ superalgebra with six real tensorial central charges $Z_{\mu\nu}$, and discuss its possible realizations in massive and massless cases. Massless case is dynamically realized by generalized Ferber-Shirafuji (FS) model with fundamental bosonic spinor coordinates. The Lorentz invariance is not broken due to the realization of central charges generators in terms of bosonic spinors. The model contains four fermionic coordinates and possesses three $\kappa$-symmetries thus providing the BPS configuration preserving $3/4$ of the target space supersymmetries. We show that the physical degrees of freedom (8 real bosonic and 1 real Grassmann variable) of our model can be described by $OSp(8|1)$ supertwistor. The relation with recent superparticle model by Rudychev and Sezgin is pointed out. Finally we propose a higher dimensional generalization of our model with one real fundamental bosonic spinor. $D = 10$ model describes massless superparticle with composite tensorial central charges and in $D = 11$ we obtain 0-superbrane model with nonvanishing mass which is generated dynamically.

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1 Introduction

Recently it became clear that in supersymmetric theories besides scalar central charges, which are present in conventional D=4 Poincaré supersymmetry scheme of Haag, Lukaszański and Sohnius [1] one should consider also nonscalar generalized central charges: tensorial [2, 3] or even spinor ones [7, 8]. Such generalized central extension of standard $N = 1$ D–dimensional supersymmetry algebra \{\(Q_{\alpha}, Q_{\beta}\)\} = i\Gamma_{\alpha\beta}^m P_m can be written in the form

\[ \{Q_{\alpha}, Q_{\beta}\} = Z_{\alpha\beta}, \]  

(1.1)

where \(Z_{\alpha\beta}\) is the most general symmetric matrix of Abelian generalized central charges.

The tensorial central charges \(Z_{m_1...m_p}\) appear through decomposition of the symmetric matrix \(Z_{\alpha\beta}\) on the basis defined by the products \(\Gamma^{(p)} \equiv \Gamma^{m_1...m_p} = \Gamma^{[m_1}...\Gamma^{m_p]}\) of \(D\)-dimensional \(\Gamma^m\) matrices.

\[ Z_{\alpha\beta} = (\Gamma_m C)_{(\alpha\beta)} P^m + \sum_{\text{symmetric}} (\Gamma_{m_1...m_p} C)_{(\alpha\beta)} Z_{m_1...m_p}, \]  

(1.2)

It should be noted that, as only symmetric matrices are really involved on the right hand side of (1.2), for particular dimensions and signatures one can also consider the superalgebras (1.2) without the momenta \(P_m\) described by the term linear in \(\Gamma_m\).

Main aim of this paper is to discuss the appearance of the tensorial central charges in the case of ‘physical’ \(D = 4\) \((D = 1 + 3)\) supersymmetry. If, for simplicity, we consider \(N = 1\) supersymmetry, one can generalize the standard \(D = 4\) superalgebra as follows

\[ \{Q_A, Q_B\} = Z_{AB}, \quad \{\bar{Q}_{\dot{A}}, \bar{Q}_{\dot{B}}\} = \bar{Z}_{\dot{A}\dot{B}}, \]  

(1.3)

\[ \{Q_A, \bar{Q}_{\dot{B}}\} = P_{\dot{A}B}. \]

where \((Q_A)^* = \bar{Q}_{\dot{A}}, (P_{\dot{A}B})^* = P_{\dot{B}A}, (Z_{AB})^* = \bar{Z}_{\dot{A}\dot{B}}\) and six real commuting central charges \(Z_{\mu\nu} = -Z_{\nu\mu}\) are related to the symmetric complex spin-tensor \(Z_{AB}\) by \footnote{For two-component \(D = 4\) Weyl spinor formalism see e.g. [4]. We have \((\sigma_{mn})_A^B = \frac{1}{2i}((\sigma_\mu)_A^B \sigma_\nu^\rho B - (\sigma_\nu)_A^B \sigma_\mu^\rho B)_1 - \frac{i}{2} \epsilon_{\mu\nu\rho\lambda}(\sigma^{\rho\lambda})_A^B = [(\sigma_\mu^\rho B)_A^B].\}

\[ Z_{\mu\nu} = \frac{i}{2} (\bar{Z}_{\dot{A}\dot{B}} \sigma_{\mu\nu}^{\dot{A}\dot{B}} - Z_{AB} \sigma_{\mu\nu}^{AB}). \]  

(1.4)

Thus the spin-tensors \(Z_{AB}\) and \(\bar{Z}_{\dot{A}\dot{B}}\)

\[ Z_{AB} = \frac{i}{4} Z_{\mu\nu} \sigma_{\mu\nu}^{AB}, \quad \bar{Z}_{\dot{A}\dot{B}} = -\frac{i}{4} Z_{\mu\nu} \tilde{\sigma}_{\mu\nu}^{\dot{A}\dot{B}} \]

represent the self-dual and anti-self-dual parts of the central charge matrices.

The tensorial central charges (1.4) commute with fourmomenta \(P_\mu = \frac{1}{2} \bar{\sigma}_{\mu}^{\dot{A}B} P_{\dot{A}B}\) and transform as a tensor under the Lorentz group with generators \(M_{\mu\nu} = -M_{\nu\mu}\) (\(\eta_{\mu\nu} = \text{diag}(1, -1, -1, -1)\))

\[ [M_{\mu\nu}, Z_{\rho\lambda}] = -i(\eta_{\mu\rho} Z_{\nu\lambda} - \eta_{\nu\rho} Z_{\mu\lambda} - \eta_{\mu\lambda} Z_{\nu\rho} + \eta_{\nu\lambda} Z_{\mu\rho}). \]  

(1.5)

In this paper we shall consider two aspects of the appearance of tensorial central charges in \(D = 4\) superalgebra:
i. Their impact on the superalgebraic representations – in Section 2. We consider separately massive \((P_\mu P^\mu > 0)\) and massless \((P_\mu P^\mu = 0)\) cases. We recall that in the massless case half of the fermionic degrees of freedom can be eliminated what leads to the shortening of the massless supermultiplets and, hence, only \(N = 1/2\) supersymmetries acts nontrivially. It appears that the presence of particular form of the central charge

\[
Z_{\mu \nu} = K_\mu P_\nu - K_\nu P_\mu
\]  

(1.6)

provides the additional shortening of the massless supermultiplet (with \(N = 1/4\) supersymmetry realized nontrivially).

ii. Their dynamical consequences – in Section 3. The formula (1.6) for tensorial central charge can be derived from the generalized Ferber–Shirafuji (FS) model \([10, 11]\) with fundamental spinor coordinates \(\lambda_\alpha = (\lambda_A, \lambda^A)\) and additional central charge coordinates \(z^{AB} = z^{BA} = [z^{\dot{A}\dot{B}}]^*\). In such a model three Grassmann degrees of freedom out of four are pure gauge and can be gauged away by \(\kappa\)–transformations \([12]\). In the language of brane physics \([3, 4, 6, 13]\) such model corresponds to BPS configuration preserving \(3/4\) of the target space supersymmetry. Such configurations were not known before.

The model can be reformulated in terms of two Weyl spinors \(\lambda_A, \mu_A\) and one real Grassmann variable \(\zeta\) expressed by the generalization of supersymmetric Penrose–Ferber relations \([14, 16, 15]\) between supertwistor and superspace coordinates. Such reformulation is described by \(OSp(8|1)\) invariant free supertwistor model with the action

\[
S = -\frac{1}{2} \int d\tau Y_A G^{AB} \dot{Y}_B
\]  

(1.7)

where \(Y_A = (y_1, \ldots, y_8; \zeta) \equiv (\lambda_\alpha, \mu^\alpha, \zeta)\) is the real \(SO(8|1)\) supertwistor (see e.g. \([14]\)) and

\[
G^{AB} = \begin{pmatrix}
\omega^{(8)} & 0 \\
0 & 2i
\end{pmatrix} = \begin{pmatrix}
0_2 & I_2 & 0_2 & 0_2 \\
-I_2 & 0_2 & 0_2 & 0_2 \\
0_2 & 0_2 & 0_2 & I_2 \\
0_2 & 0_2 & -I_2 & 0_2 \\
0 & 0 & 0 & 0
\end{pmatrix}
\]  

(1.8)

is the \(OSp(8|1)\) supersymplectic structure with bosonic \(Sp(8)\) symplectic metric \(\omega^{(8)} = -(\omega^{(8)})^T\). It should be mentioned therefore that due to the presence of tensorial central charges the standard \(SU(2,2|1)\) supertwistor description \([10, 11, 15, 16, 17, 18]\) of the Brink–Schwarz (BS) massless superparticle \([19]\) with one complex Grassmann coordinate is replaced by a model with \(OSp(8|1)\) invariance and one real Grassmann degree of freedom.

It should be stressed that by the use of spinor coordinates in the presence of tensorial central charges

- we do not increase the initial number of spinor degrees of freedom (four complex or eight real components) in comparison with the model without tensorial central charges;
- we keep the manifest Lorentz invariance despite the presence of tensorial central charges.
In fact, when we use our formulae (see Section 3)

\[ P_{AB} = \lambda_A \bar{\lambda}_B, \quad Z_{AB} = \lambda_A \lambda_B, \quad \bar{Z}_{\dot{A}\dot{B}} = \bar{\lambda}_\dot{A} \bar{\lambda}_\dot{B} \]  

we find that, in comparison with standard FS model \((P_{AB} = \lambda_A \bar{\lambda}_B, \ Z_{AB} = \bar{Z}_{\dot{A}\dot{B}} = 0)\), only the phase of spinor \(\lambda_A\) becomes an additional physical bosonic degree of freedom.

We also show that our model can be related with generalized superparticle model of Rudychev and Sezgin \[20\]. In Section 4 we will describe the Rudychev–Sezgin model for \(D = 1 + 3\) and find the general solution of the BPS constraint \[20\] in terms of two bosonic spinors \((\lambda_A, \mu_A)\). It appears that by putting \(u_A = 0\) and fixing one normalization factor we arrive at our model.

In Section 5 we propose a generalization of our model for \(D > 4\) with one real fundamental spinor. In \(D = 10\) the model describes a massless superparticle with composite tensorial central charges. In \(D = 11\) we get the 0-superbrane model with mass generated dynamically in a way analogous to the brane tension generation \[21\].

In Section 6 we present final remarks.

2 On representations of \(N = 1, D = 4\) superalgebra with tensorial central charge

In order to describe the supersymmetry multiplets for the algebra \((1.3)\) we shall consider supercharges \(Q_A, \bar{Q}_\dot{A}\) in a particular Lorentz frame. We shall consider separately the massive \(P_\mu P^\mu = M^2 > 0\) and massless \(P_\mu P^\mu = 0\) cases.

\(M^2 > 0\):

We choose the rest frame for the fourmomentum, i.e. \(P_\mu = (M, 0, 0, 0)\). In such a way we obtain the algebra \((1.3)\) in the following \(U(2)\) invariant form

\[ \{Q_A, Q_B^\dagger\} = M \delta_{AB}, \]  

(2.1)

Further we use the \(U(2)\) transformations (space rotations \(SO(3) = SU(2)\) plus internal \(U(1)\)) to transform the central charge matrices to the form \(U \in U(2); \ \text{see} \ [24]\)

\[ Z = UZ^{(0)}U^+, \quad Z^{(0)} = \begin{pmatrix} a & 0 \\ 0 & b \end{pmatrix} = xI + y\sigma_3 \]  

(2.2)

where \(a\) and \(b\) are real and positive, \(x = \frac{1}{2}(a + b), \ y = \frac{1}{2}(a - b)\).

Introducing the fourdimensional Majorana spinor

\[ Q_\alpha = \begin{pmatrix} Q_A \\ Q_A^\dagger \end{pmatrix} \]  

(2.3)

one obtains for the matrix of commutator of supercharges

\[ S_{\alpha\beta} = \{Q_\alpha, Q_\beta\} = \begin{pmatrix} xI_2 + y\sigma_3 & M \\ M & xI_2 - y\sigma_3 \end{pmatrix} \]  

(2.4)
with
\[ \det S = (M^2 - a^2) (M^2 - b^2), \] (2.5)

The matrix \( S \) can be diagonalized and the number of supersymmetries acting nontrivially corresponds to the number of nonvanishing diagonal elements of the matrix \( S \) (compare e.g. with [25]). Thus we should consider the following three cases:

1. If \( M \neq \pm a \) and \( M \neq \pm b \) there are four nontrivial supersymmetries. The diagonal form of the matrix (2.4) is \( \text{diag}(a + M, a - M, b + M, b - M) \).

2. If \( M = \pm a \) or \( M = \pm b \), but \( a \neq b \), then one of the eigenvalues is equal to zero. Denoting the corresponding charge by \( Q_{\text{null}} \) one gets \( \{Q_{\text{null}}, Q_{\text{null}}^+\} = 0 \) and, if \( Q_{\text{null}}^+|0\rangle = 0 \) and \( Q_{\text{null}}^+|1\rangle = |1\rangle \), one obtains that \( |\langle 1|1\rangle|^2 = 0 \). Assuming that the representation space of our superalgebra is span by the positive norm states, we should discard \( Q_{\text{null}} \) as generating trivial representations.

3. If \( a = b \) and \( m = \pm a \) only two supercharges generate nontrivial representations, i.e. we obtain only \( N = \frac{1}{2} D = 4 \) supersymmetries.

\( M = 0 \):

In such a case, using the light–cone frame \( P_m = (p, 0, 0, p) \) (\( p_0 = |p| = p \) one gets)
\[ \{Q_A, Q_B^+\} = \delta_{A1} \delta_{B1} 2p , \] (2.6)

In our framework the supercharges \( Q_2 \) generate trivial representation space. If we assume that does exist a nontrivial Clifford vacuum \( (Q_2^+|0\rangle = 0, \ 2 < 0|0 \rangle = 1) \) then we obtain from \( 2 < 0|Q_2^+|0\rangle = Z_{22} \ 2 < 0|0 \rangle = 0 \) that \( Z_{22} = Z_{22} = 0 \).

We shall assume further that
\[ Z_{AB} Z_{AB} = \bar{Z}_{\dot{A}\dot{B}} \bar{Z}_{\dot{A}\dot{B}} = 0, \] (2.7)

what implies \( Z_{12} = 0 \). Thus we arrive at the algebra with all nontrivial relations being collected in \( Q_1, Q_1^\dagger \) sector
\[ \{Q_1, Q_1^\dagger\} = 2p, \quad Q_1^2 = Z_{11}, \quad Q_1^{12} = Z_{i\dot{i}}. \] (2.8)

Because the relations (2.8) are invariant under the phase transformations \( Q_1 \rightarrow e^{ia} Q_1 \), \( Z_{11} \rightarrow e^{2ia} Z_{11} \), one can fix the central charge \( Z_{11} \) to be real \( Z_{11} = \bar{Z}_{i\dot{i}} = r \). Introducing
\[ R_+ = \frac{1}{2} (Q_1 + Q_1^+), \quad R_- = \frac{1}{2} (Q_1 - Q_1^+), \] (2.9)

one gets
\[ \{R_+, R_+\} = M_+ = (p + r), \quad \{R_-, R_-\} = M_- = (p - r), \] (2.10)
\[ \{R_+, R_-\} = 0. \]

We can distinguish the following two cases:

1. \( r \neq p \). In such a case we have two nontrivial supersymmetries (as in the case \( r = 0 \)).
2. \( r = \pm p \). In such a case remains only one nontrivial supersymmetry. Such a case can be described in covariant way by the spinor ansatz \[ 1.3 \] \(^2\). It can be shown that the corresponding tensorial central charge can be expressed by Eq. \[ 1.6 \] where

\[
K_m = \frac{1}{2} \delta_m^{BA} K_{AB}, \quad K_{AB} = 2(\lambda_A \bar{\mu}_B + \bar{\mu}_A \lambda_B)
\]

(2.11)

and \( \lambda^A \bar{\mu}_A = 1 \). In our special coordinate frame we should choose \( K_m = (0, 1, 0, 0) \).

We see therefore that the presence of tensorial central charges can reduce the number of nontrivial supersymmetries to one. In the dynamical model this should be realized by the presence of additional third \( \kappa \)–symmetry. In the next section we consider the relations \[ 1.9 \], and, thus, \[ 1.6 \] and \[ 2.11 \], built in as the dynamical constraints.

3 Generalization of Ferber–Shirafuji superparticle model: spinor fundamental variables and central charges

We generalize the model presented in \[ 11 \] as follows

\[
S = \int d\tau \left( \lambda_A \bar{\lambda}_B \Pi^{AB} + \lambda_A \lambda_B \Pi^{\dot{A}\dot{B}} + \bar{\lambda}_A \bar{\lambda}_B \Pi_\tau^{\dot{A}\dot{B}} \right),
\]

(3.1)

where

\[
\Pi^{AB} \equiv d\tau \Pi^{AB} = dX^{AB} + i (d\Theta^A \bar{\Theta}^B - \Theta^A d\bar{\Theta}^B),
\]

\[
\Pi^{\dot{A}\dot{B}} \equiv d\tau \Pi^{\dot{A}\dot{B}} = dz^{\dot{A}\dot{B}} - i \bar{\Theta}^{\dot{A}} d\Theta^{\dot{B}},
\]

(3.2)

and \( d\Theta^{(A} \bar{\Theta}^{B)} = \frac{1}{2} (d\Theta^A \bar{\Theta}^B + d\Theta^B \bar{\Theta}^A) \)

are the supercovariant one–forms in \( D = 4, N = 1 \) generalized flat superspace

\[
M^{(4+6|4)} = \{ Y^M \} \equiv \{ (x^{A\dot{A}}, z^{AB}, \bar{z}^{\dot{A}\dot{B}}, \Theta^A, \bar{\Theta}^{\dot{A}}) \},
\]

(3.3)

with tensorial central charge coordinates \( z^{mn} = (z^{AB}, \bar{z}^{\dot{A}\dot{B}}) \) (see \[ 1.2 \]). The complete configuration space of the model \[ 3.1 \] contains additionally the complex-conjugate pair \( (\lambda_A, \bar{\lambda}_A) \) of Weyl spinors

\[
\mathcal{M}^{(4+6+4|4)} = \{ q^M \} \equiv \{ (Y^M; \lambda^A, \bar{\lambda}^{\dot{A}}) \} = \{ (x^{A\dot{A}}, z^{AB}, \bar{z}^{\dot{A}\dot{B}}, \lambda^A, \bar{\lambda}^{\dot{A}}, \Theta^A, \bar{\Theta}^{\dot{A}}) \},
\]

(3.4)

Calculating the canonical momenta

\[
\mathcal{P}_M = \frac{\partial L}{\partial \dot{q}^M} = (P_{A\dot{A}}, Z_{AB}, \bar{Z}_{\dot{A}\dot{B}}; P^A, \bar{P}^{\dot{A}}, \pi^A, \bar{\pi}^{\dot{A}}),
\]

(3.5)

\(^2\)Gauge fixing corresponding to \[ 2.8 \] is given by

\[
\lambda_a = \sqrt{2p} \begin{pmatrix} 1 \\ 0 \end{pmatrix}
\]
we obtain the following set of the primary constraints

\[ \Phi_{AB} \equiv P_{AB} - \lambda_A \bar{\lambda}_B = 0, \] (3.6)
\[ \Phi_{AB} \equiv Z_{AB} - \lambda_A \lambda_B = 0, \] (3.7)
\[ \Phi_{\bar{A}\bar{B}} \equiv \bar{Z}_{\bar{A}\bar{B}} - \bar{\lambda}_{\bar{A}} \bar{\lambda}_{\bar{B}} = 0, \] (3.8)
\[ P_A = 0, \quad \bar{P}_\bar{A} = 0, \] (3.9)

\[ D_A \equiv -\pi_A + i P_{AB} \bar{\Theta}^B + i Z_{AB} \Theta^B = 0, \] (3.10)
\[ \bar{D}_{\bar{A}} \equiv \bar{\pi}_{\bar{A}} - i \Theta^B P_{\bar{B}\bar{A}} - i \bar{Z}_{\bar{A}\bar{B}} \bar{\Theta}^\bar{B} = 0. \] (3.11)

Because the action (3.1) is invariant under the world line reparametrization, the canonical Hamiltonian vanishes

\[ H \equiv \dot{q}^M \mathcal{P}_M - L(q^M, \dot{q}^M) = 0 \] (3.12)

It can be deduced that the set (3.6)-(3.11) of 14 bosonic and 4 fermionic first class constraints contains 6 bosonic and 3 fermionic first class constraints

\[ B_1 = \lambda^A \bar{\lambda}^B P_{AB} = 0, \] (3.13)
\[ B_2 = \lambda^A \bar{\mu}^B P_{AB} - \lambda^A \mu^B Z_{AB} = 0, \] (3.14)
\[ B_3 \equiv (B_2)^* = \bar{\mu}^A \bar{\lambda}^B P_{AB} - \bar{\lambda}^A \mu^B \bar{Z}_{\bar{A}\bar{B}} = 0, \] (3.15)
\[ B_4 = 2 \bar{\mu}^A \bar{\lambda}^B P_{AB} - \bar{\lambda}^A \mu^B Z_{AB} - \bar{\lambda}^A \mu^B \bar{Z}_{\bar{A}\bar{B}} = 0, \] (3.16)
\[ B_5 = \lambda^A \bar{\lambda}^B Z_{AB} = 0, \] (3.17)
\[ B_6 \equiv (B_5)^* = \bar{\lambda}^A \bar{\lambda}^B \bar{Z}_{\bar{A}\bar{B}} = 0, \] (3.18)
\[ F_1 = \lambda^A D_A = 0, \] (3.19)
\[ F_2 \equiv (F_1)^* = \bar{\lambda}^A \bar{D}_\bar{A} = 0, \] (3.20)
\[ F_3 = \bar{\mu}^A \bar{D}_\bar{A} = 0, \] (3.21)

where we assume that \( \lambda^A \mu_A \neq 0 \) and

\[ \bar{\mu}^A = \frac{\mu^A}{\lambda^B \mu_B}, \quad \bar{\lambda}^A = \frac{\bar{\mu}^A}{\lambda^B \mu_B}, \] (3.22)

i.e. \( \lambda^A \mu_A = \bar{\lambda}^A \bar{\mu} = 1 \). One can show \(^3\) that our first class constraints (3.13) - (3.21) can be chosen for any particular form of the second spinor \( \mu^A \) as a function of canonical variables \( (q^M, \mathcal{P}_M) \). Further we shall propose and motivate the choice for \( \mu^A, \bar{\mu}^A \).

The remaining 8 bosonic and 1 fermionic constraints are the second class ones. They are

\[ \lambda^A \bar{\mu}^B P_{AB} + \lambda^A \bar{\mu}^B Z_{AB} = 0, \quad \mu^A \bar{\lambda}^B P_{AB} + \bar{\lambda}^A \mu^B \bar{Z}_{\bar{A}\bar{B}} = 0, \] (3.23)

\(^3\) We recall \([22]\) that the first class constraints are defined as those whose Poisson brackets with all constraints weakly vanish. Then one can show \([22]\) that the first class constraints form the closed algebra.
\[ \hat{\mu}^A \hat{\mu}^B Z_{AB} - 1 = 0, \quad \hat{\mu}^A \hat{\mu}^B \bar{Z}_{\dot{A}\dot{B}} - 1 = 0, \quad (3.24) \]
\[ P_A = 0, \quad \bar{P}_A = 0, \quad (3.25) \]
\[ S_F \equiv \hat{\mu}^A D_A - \hat{\mu}^A \bar{D}_\dot{A} = 0, \quad (3.26) \]

We see that the number \( \# \) of on-shell phase space degrees of freedom in our model is
\[ \# = (28B + 8F) - 2 \times (6B + 3F) - (8B + 1F) = 8B + 1F \quad (3.27) \]
in distinction with the standard massless superparticle model of Brink–Schwarz [19] or Ferber-Shirafuji [10, 11] containing \( 6B + 2F \) physical degrees of freedom.

In order to explain the difference in the number of fermionic constraints, let us write down the matrices of Poisson brackets for the fermionic constraints (3.10), (3.11). In our case it has the form
\[ C_{\alpha\beta} = \begin{pmatrix} \{D_A, D_B\}_P & \{D_A, \bar{D}_B\}_P \\ \{D_A, \bar{D}_B\}_P & \{\bar{D}_A, \bar{D}_B\}_P \end{pmatrix} = 2i \begin{pmatrix} \lambda_A \lambda_B & \lambda_A \bar{\lambda}_B \\ \bar{\lambda}_A \lambda_B & \bar{\lambda}_A \bar{\lambda}_B \end{pmatrix} \quad (3.28) \]
while for the standard FS model [10, 11] we obtain
\[ C_{\alpha\beta}^{FS} = 2i \begin{pmatrix} 0 & \lambda_A \bar{\lambda}_B \\ \bar{\lambda}_A \lambda_B & 0 \end{pmatrix} \quad (3.29) \]

Now it is evident that in our case the rank of the matrix \( C \) is one, while for FS model it is equal to two
\[ \text{rank}(C) = 1, \quad \text{rank}(C^{FS}) = 2. \]
Consequently, in our model there are three fermionic first class constraints generating three \( \kappa \)-symmetries, one more than in the FS model.

In order to clarify the meaning of the superparticle model (3.1) and present an explicit representation for its physical degrees of freedom, we shall demonstrate that it admits the supertwistor representation in terms of independent bosonic spinor \( \lambda^A \), bosonic spinor \( \mu^A \) being composed of \( \lambda^A \) and superspace variables
\[ \mu^A = (x^{AB} + i \Theta^A \bar{\Theta}^B) \bar{\lambda}_B + 2z^{AB} \lambda_B + i \Theta_A (\Theta^B \lambda_B), \quad (3.30) \]
\[ \bar{\mu}^{\dot{A}} = (x^{B\dot{A}} - i \Theta^B \bar{\Theta}^{\dot{A}}) \lambda_B + 2z^{\dot{A}B} \bar{\lambda}_B - i \bar{\Theta}^{\dot{A}} \bar{\Theta}^B \bar{\lambda}_B \quad (3.31) \]
and one real fermionic composite Grassmann variable \( \zeta \)
\[ \zeta = \Theta^A \lambda_A + \bar{\Theta}^{\dot{A}} \bar{\lambda}_{\dot{A}} \quad (3.32) \]
Eqs. (3.30) - (3.32) describe \( OSp(8|1) \)–supersymmetric generalization of the Penrose correspondence which is alternative to the previously known \( SU(2, 2|1) \) correspondence, firstly proposed by Ferber [10]. Performing integration by parts and neglecting boundary terms we can express our action (3.1) in terms of \( OSp(8|1) \) supertwistor variables as follows:
\[ S = - \int \left( \mu^A d\lambda_A + \bar{\mu}^{\dot{A}} d\bar{\lambda}_{\dot{A}} + id\zeta \zeta \right). \quad (3.33) \]
Eq. (3.33) presents the free $OSp(8|1)$ supertwistor action. It can be rewritten in the form (1.7) with real coordinates $Y^A = (\mu^\alpha, \lambda^\alpha, \zeta)$ where real Majorana spinors $\mu^\alpha, \lambda^\alpha$ are obtained from the Weyl spinors $(\mu^A, \bar{\mu}^{\dot{A}}), (\lambda^A, \bar{\lambda}^{\dot{A}})$ by a linear transformation changing for the $D = 4$ Dirac matrices the complex Weyl to real Majorana representation.

The action (3.33) produces only the second class constraints

$$P^{(\lambda)}_A - \mu_A = 0, \quad P^{(\mu)}_A = 0,$$

$$\bar{P}^{(\lambda)}_{\dot{A}} - \bar{\mu}_{\dot{A}} = 0, \quad \bar{P}^{(\mu)}_{\dot{A}} = 0,$$

$$\pi^{(\zeta)} = i\zeta$$  

(3.34)  

(3.35)  

(3.36)

The Dirac brackets for the $OSp(8|1)$ supertwistor coordinates are

$$[\mu_A, \lambda_B]_D = \delta_A^B, \quad [\bar{\mu}_{\dot{A}}, \bar{\lambda}^{\dot{B}}]_D = \delta_{\dot{B}}^{\dot{A}},$$

$$\{\zeta, \zeta\}_D = -i$$  

(3.37)  

(3.38)

They can be also obtained after the analysis of the Hamiltonian system described by the original action (3.1). For this result one should firstly perform gauge fixing for all the gauge symmetries, arriving at the dynamical system which contains only second class constraints, and then pass to the Dirac brackets in a proper way (see [17] for corresponding analysis of the BS superparticle model). This means that the generalization of the Penrose correspondence (3.30), (3.31), (3.32) should be regarded as coming from the second class constraints (primary and obtained from the gauge fixing) of the original system and, thus, should be considered as a relations hold in the strong sense (i.e. as operator identities after quantization) [22]. Hence, after the quantization performed in the frame of supertwistor approach, the generalized Penrose relations (3.30), (3.31), (3.32) can be substituted into the wave function in order to obtain the $D = 4$ superspace description of our quantum system.

We shall discuss now the relation of Eq. (3.30), (3.31), (3.32), (3.33) with the known FS $SU(2,2|1)$ supertwistor description of the BS superparticle [10, 11, 15, 16, 17, 18]. The standard FS description is given by the action

$$S = -\int (\mu^A d\lambda_A + \bar{\mu}^{\dot{A}} d\bar{\lambda}_{\dot{A}} + i d\xi d\bar{\xi})$$

(3.39)

supplemented by the first class constraint

$$\mu^A \lambda_A - \bar{\mu}^{\dot{A}} \bar{\lambda}_{\dot{A}} + 2i\xi \bar{\xi} = 0$$

(3.40)

The $SU(2,2|1)$ supertwistor $(\lambda^A, \bar{\mu}^{\dot{A}}, \bar{\xi})$, contains complex Grassmann variable $\xi$ and the supersymmetric Penrose–Ferber correspondence is given by

$$\bar{\mu}^{\dot{A}} = (x^{B\dot{A}} - i \Theta^{B\dot{A}}) \lambda_B$$

(3.41)

$$\zeta = \Theta^A \lambda_A, \quad \bar{\xi} = \bar{\Theta}^{\dot{A}} \bar{\lambda}_{\dot{A}}.$$  

(3.42)

Comparing Eqs. (3.39) – (3.42) with our $OSp(8|1)$ supertwistor description (3.30) – (3.33) of the superparticle (3.1) with additional central charge coordinates, we note that
• Besides additional terms proportional to tensorial central charge coordinates \( z^{AB} \), 
\( \bar{z}^{AB} \), there is present in (3.31) the second term quadratic in Grassmann variables. 
This second term, however, does not contribute to the invariant \( \mu^A \lambda_A \).

• In our model we get
\[
\mu^A \lambda_A - \bar{\mu} \lambda_A = 2 \lambda_A \lambda_B z^{AB} - 2 \lambda_A \lambda_B \bar{z}^{AB} + 2i \Theta^A \lambda_A \bar{\Theta} \lambda_A
\]
and
\[
\bar{\lambda}^{\dot{A}} \dot{\lambda}_A = 2 \lambda_A \lambda_B \bar{z}^{AB} - 2 \lambda_A \lambda_B \bar{z}^{AB} + 2i \Theta^A \lambda_A \bar{\Theta} \lambda_A
\]
i.e. we do not have additional first class constraint generating \( U(1) \) symmetry (compare to (3.40) of the standard supertwistor formulation). Thus our action (3.33) is not singular in distinction to (3.39), where the first class constraint (3.40) should be taken into account, e.g. by introducing it into the action with Lagrange multiplier [18].

• The complex Grassmann variable \( \xi \) (3.42) of FS formalism is replaced in our case by the real one \( \zeta \) (3.32). This difference implies that in our supertwistor formalism the limit \( z^{AB} \to 0 \), \( \bar{z}^{AB} \to 0 \) does not reproduce the standard \( SU(2, 2|1) \) supertwistor formalism. Indeed, this is not surprising if we take into account that, from algebraic point of view, \( SU(2, 2|1) \) is not a subsupergroup of \( OSp(8|1) \).

4 \( D = 4 \) Rudychev-Sezgin model in spinor representation

Recently the most general superparticle model associated with space–time superalgebra (1.1) was proposed by Rudychev and Sezgin [20]. Introducing generalized real superspace \( (X^{\alpha \beta}, \Theta^\alpha) \) they consider the following action
\[
S = \int d\tau L = \int d\tau \left( P_{\alpha \beta} \Pi^{\alpha \beta} + \frac{1}{2} e_{\alpha \beta} P^{\alpha \gamma} C_{\gamma \delta} P^{\delta \beta} \right),
\]
where \( \Pi^{\alpha \beta} = \dot{X}^{\alpha \beta} - \dot{\theta}^{(\alpha \beta)} \) \( (\dot{\alpha} \equiv \frac{d\alpha}{d\tau}) \), \( C \) is the charge conjugation matrix and \( e_{\alpha \beta} \) is the set of Lagrange multipliers, generalizing einbein in the action for standard Brink-Schwarz massless superparticle [19].

Generalized mass shell condition, obtained by varying \( e_{\alpha \beta} \) in (4.1), takes the form
\[
P^{\alpha \gamma} C_{\gamma \delta} P^{\delta \beta} = 0.
\]

In [20] the model (4.1) was applied for exotic space–times with more then one time–like dimensions (for \( D = 4 \) there was considered the model with signature \( (2, 2) \)). However, it can be considered as well in the frame of one–time physics. If we choose for \( \alpha, \beta = 1, \ldots, 4 \) the charge conjugation matrix \( C_{\alpha \beta} \) and the Lagrange multiplier \( e_{\alpha \beta} \) to be antisymmetric, we obtain from (4.1) the \( D = (1 + 3) \)-dimensional model.

The Lagrange multiplier \( e_{\alpha \beta} = -e_{\beta \alpha} \) and the generalized momenta \( P_{\alpha \beta} = P_{\beta \alpha} \) can be decomposed as follows
\[
e_{\alpha \beta} = C_{\alpha \beta} \frac{1}{2} (e + \bar{e}) + (C \gamma_5)_{\alpha \beta} \frac{1}{2} (e - \bar{e}) + (C \gamma_5 \gamma_\mu)_{\alpha \beta} e^\mu,
\]
\[ P_{\alpha\beta} = (C^\gamma_{\mu})_{\alpha\beta} P^\mu + (C^{\sigma\mu\nu})_{\alpha\beta} Z_{\mu\nu}, \quad (4.4) \]

Using the decomposition (1.3), (1.4) and the Weyl spinor notations we can write the action for the general \( D = 4 \) Rudychev-Sezgin model as follows

\[
S = \int d\tau L = \int d\tau \left( \Pi_{AB} P_{\dot{A}\dot{B}} + \Pi_{\tau} Z_{AB} + \Pi_{AB} Z_{\dot{A}\dot{B}} \right) + \epsilon^{AB} \left( P_{AC} P_{\dot{C}} - Z_{AC} Z_{\dot{C}} \right) + \bar{e}^{\dot{A}B} \left( Z_{AC} P_{\dot{C}B} - P_{AC} Z_{\dot{C}B} \right),
\quad (4.5)
\]

The fermionic constraints are identical with the ones present in our model (3.10), (3.11) and the bosonic constraints are given by Eq. (4.2), which in the Weyl spinor notation reads

\[
P_{\dot{A}\dot{B}} P_{\dot{A}\dot{B}} = Z_{AB} Z_{\dot{A}\dot{B}} = \bar{Z}_{\dot{A}\dot{B}} \bar{Z}_{\dot{A}\dot{B}}, \quad (4.6)
\]

The spinorial formulation of the Rudychev–Sezgin model can be obtained by expressing \( P_{\dot{A}\dot{B}} \), \( Z_{AB} \) and \( \bar{Z}_{\dot{A}\dot{B}} \) in terms of spinor coordinates.

Using the technique of spinor Lorentz harmonics [23], one can show that the general solution of the constraints (4.6) has the form

\[
P_{\dot{A}\dot{B}} = \lambda_A \bar{\lambda}_B + u_A \bar{u}_B, \quad \bar{Z}_{\dot{A}\dot{B}} = Z \lambda_A \bar{\lambda}_B + \bar{Z} u_A \bar{u}_B \pm i (\lambda_A u_B + \lambda_B u_A) \sqrt{|Z|^2 - 1},
\quad (4.7)
\]

It is easy to see that

\[
P_{\dot{A}\dot{B}} P_{\dot{A}\dot{B}} = M^2 = |\lambda_A u_A|^2.
\quad (4.8)
\]

Thus, if \( \lambda_A u_A = 0 \) we obtain the model for massless superparticle. In such a case, because two Weyl spinors are proportional \( u_A \propto \lambda^A \), it is sufficient to consider only one spinor \( \lambda^A \). In such a case Eq. (4.7) acquires the form

\[
P_{\dot{A}\dot{B}} = \lambda_A \bar{\lambda}_B, \quad \bar{Z}_{\dot{A}\dot{B}} = Z \lambda_A \bar{\lambda}_B, \quad (4.9)
\]

what leads to the conditions

\[
P_\mu P^\mu = M^2 = 0, \quad Z_{AB} Z^{AB} = Z_{\dot{A}\dot{B}} Z^{\dot{A}\dot{B}} = 0,
\quad (4.10)
\]

and the covariant constraints (4.6) are certainly satisfied.

Our model (3.1) appears when \( Z = 1 \), while the standard FS model corresponds to \( Z = 0 \). It can be shown that if \( Z \neq 1 \), there are two fermionic first class constraints, and thus the model possesses two fermionic gauge symmetries (\( \kappa \)-symmetries). Only if \( Z = 1 \) we arrive at the model with three \( \kappa \)-symmetries, which, in the brane language, corresponds to the preservation of \( N = 3/4 \) target space supersymmetries.
5 Higher–dimensional Generalization.

It is quite interesting to consider a generalization of our model to $D > 4$. For any $D$, the extension of our generalization of the Cartan-Penrose representation \( (1.9) \) with one $D$–dimensional bosonic spinor $\lambda_A$ looks as follows

\[
P_{\alpha\beta} = \lambda_\alpha \lambda_\beta, \quad (\lambda_\alpha)^* = \lambda_\alpha, \quad \alpha = 1, \ldots, 2^k. \tag{5.1}
\]

where \( (1.9) \) is obtained if $k = 2$. The expression \( (5.1) \) solves the BPS condition $\det P_{\alpha\beta} = 0$ as well as more strong Rudychev-Sezgin BPS constraint \( (4.2) \) valid in the model \( (4.1) \) with antisymmetric charge conjugation matrix $C$ ($C_{\alpha\beta} = -C_{\beta\alpha}$).

Using \( (5.1) \) we get the multidimensional generalization of our action \( (3.1) \) which reads

\[
S = \int_{M^1} \lambda_\alpha \lambda_\beta \Pi^{\alpha\beta} \tag{5.2}
\]

\[
\Pi^{\alpha\beta} = dX^{\alpha\beta} - i d(\Theta^{(\alpha} \Theta^{\beta)}), \quad \alpha = 1, \ldots, 2^k
\]

and for $k = 2$ we get the action \( (3.1) \).

The case $k = 4$ can be treated as describing spinorial $D = 10$ massless superparticle model with 126 composite tensorial central charges $Z_{m_1 \ldots m_5}$ (cf. with \[2, 3\]). Indeed, using the basis of antisymmetric products of $D = 10$ sigma matrices we obtain

\[
\lambda_\alpha \lambda_\beta \equiv P_{\alpha\beta} = P_m \sigma_{m}^{\alpha\beta} + Z_{m_1 \ldots m_5} \sigma_{m_1 \ldots m_5}^{\alpha\beta}, \tag{5.3}
\]

Contraction of this equation with $\sigma^{m\alpha\beta}$ produces the expression for momenta in terms of bosonic spinors

\[
P_m = \frac{1}{16} \lambda_\alpha \sigma_{m}^{\alpha\beta} \lambda_\beta \quad \Rightarrow \quad P_m P_m = 0. \tag{5.4}
\]

The mass shell condition $P_m P_m = 0$ appears then as a result of the $D = 10$ identity

\[
(\sigma_{m})_{(\alpha\beta}(\sigma^{m})_{\gamma)\delta) = 0.
\]

The action \( (5.2) \) for $k = 8$ can be treated as describing a 0–superbrane model in $D = 11$ superspace with 517 composite tensorial central charge described by 32 components of one real Majorana $D = 11$ bosonic spinor. In distinction to the above case such model does not produce a massless superparticle\( ^4 \). Indeed, decomposing \( (5.1) \) in the basis of products of $D = 11$ gamma matrices, one gets

\[
\lambda_\alpha \lambda_\beta = P_m \Gamma_m^{\alpha\beta} \quad \Rightarrow \quad P_m P_m = 0. \tag{5.5}
\]

The $D = 11$ energy-momentum vector is then given by

\[
P_m = \frac{1}{32} \lambda_\alpha \Gamma^{m\alpha\beta} \lambda_\beta \tag{5.6}
\]

and the $D = 11$ mass-shell condition reads

\[
M^2 = P_m P_m = \frac{1}{1024} (\lambda \Gamma^m \lambda) (\lambda \Gamma^m \lambda) \tag{5.7}
\]

\^4Note, that the $D = 11$ Green–Schwarz superparticle model does exist and was presented in \[26\].
Using the $D = 11$ Fierz identities one can prove that the mass shell condition acquires the form

$$M^2 = P_m P^m = 2 \, Z^{mn} Z_{mn} - 32 \frac{5!}{64} \lambda \Gamma_{mn} \lambda, \quad Z_{m_1 \ldots m_5} = \frac{1}{32 \, 5!} \lambda \Gamma_{m_1 \ldots m_5} \lambda. \quad (5.8)$$

If we take into consideration that the equations of motion for our model (5.2) imply that the bosonic spinor $\lambda^\alpha$ is constant ($d\lambda^\alpha = 0$), we have to conclude that (5.2) with $k = 8$ provides the $D = 11$ superparticle model with mass generated dynamically in a way similar to the tension generating mechanism, studied in superstring and higher branes in [21].

Performing the integration by parts we can rewrite the action (5.2) in the $OSp(1|2^k)$ (i.e. $OSp(1|16)$ for $D = 10$ and $OSp(1|32)$ for $D = 11$) supertwistor $Y^A = (\mu^\alpha, \zeta)$ components:

$$S = - \int (\mu^\alpha d\lambda_\alpha + i d\zeta \zeta), \quad \alpha = 1, \ldots, 2^k. \quad (5.9)$$

The generalized Penrose–Ferber correspondence between real supertwistors and real generalized superspace looks as follows

$$\mu^\alpha = X^{\alpha \beta} \lambda_\beta - i \Theta^\alpha (\Theta^\beta \lambda_\beta), \quad \zeta = \Theta^\alpha \lambda_\alpha. \quad (5.10)$$

More detailed discussion of higher dimensional case will be given in our subsequent publication.

6 Final remarks

In this paper we proposed and discussed in some detail a new $D = 4$ massless superparticle model with three $\kappa$–symmetries. These $\kappa$-symmetries correspond to target space supersymmetries preserved by the BPS configuration. The BPS configurations are identified usually with some supersymmetric branes or their intersections [4, 3, 6, 13]. The triviality of realization of a part of target space supersymmetry is explained by the presence of the corresponding number of fermionic gauge $\kappa$–symmetries. Thus our case corresponds to BPS configurations preserving $3/4$ of the target space supersymmetry. Such configurations were not known before, as the usual superbranes conserve not more then $1/2$ supersymmetries, while their intersections keep $1/4$ and less supersymmetries.

We would like also recall that in the 'M-theoretic' approach (see e.g. [27, 3, 6]) the tensorial central charges $Z_{m_1 \ldots m_p}$ are considered as carried by p-branes. Following such treatment, one should interpret e.g. in $D = 4$ central charges $Z_{\mu \nu}$ as an indication of presence of $D = 4$ supermembrane ($p = 2$). The relation of our superparticle model with such $D = 4$ membrane states is not clear now and can be regarded as an interesting subject for further study. Here we should only guess that there should be some singular point–like limit of supermembrane, which should keep the nontrivial topological charge and increase the number of preserved (realized linearly) $D = 4$ target space supersymmetries. Similar limiting prescription should be possible e.g. for 5–branes in $D = 10, 11$ leading to the $D = 10$ and $D = 11$ superparticle actions (5.2) with the relation (5.1) describing composite tensor charges.
At the end of the paper (see (5.9), (5.10)) we only proposed a generalized FS model for \( D > 4 \). We shall consider in more detail the cases of \( D = 10 \) and \( D = 11 \) (as well as \( D = 12 \) with two times \([1, 20]\)) in the nearest future.

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