Analysis of the Doubly Heavy Baryons in the Nuclear Matter with the QCD Sum Rules

Zhi-Gang Wang

Department of Physics, North China Electric Power University, Baoding 071003, P. R. China

Abstract

In this article, we study the doubly heavy baryon states $\Xi_{cc}^+$, $\Omega_{cc}$, $\Xi_{bb}$ and $\Omega_{bb}$ in the nuclear matter using the QCD sum rules, and derive three coupled QCD sum rules for the masses, vector self-energies and pole residues. The predictions for the mass-shifts in the nuclear matter $\Delta M_{\Xi_{cc}} = -1.11$ GeV, $\Delta M_{\Omega_{cc}} = -0.33$ GeV, $\Delta M_{\Xi_{bb}} = -3.37$ GeV and $\Delta M_{\Omega_{bb}} = -1.05$ GeV can be confronted with the experimental data in the future.

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1 Introduction

In 2002, the SELEX collaboration reported the first observation of a signal for the doubly charmed baryon state $\Xi_{cc}^+$ in the charged decay mode $\Xi_{cc}^+ \to \Lambda_c^+ K^- \pi^+$ in the charm hadro-production experiment (E781) at Fermilab [1], and confirmed later by the same collaboration in the decay mode $\Xi_{cc}^+ \to p D^- K^{-}$. And there have been several theoretical approaches to calculate the doubly heavy baryon masses [3]. In Ref.[4], we study the $J^P = \frac{1}{2}^+$ doubly heavy baryon states $\Omega_{QQ}$ and $\Xi_{QQ}$ by subtracting the contributions from the corresponding $\frac{1}{2}^-$ doubly heavy baryon states with the QCD sum rules, and make reasonable predictions for their masses.

The QCD sum rules are a powerful theoretical tool in studying the ground state hadrons both in the vacuum and in the nuclear matter, and have given many successful descriptions of the hadron properties [5]. In Ref.[6], we study the $\Lambda$-type and $\Sigma$-type heavy baryon states in the nuclear matter using the QCD sum rules, and obtain three coupled QCD sum rules for the masses, vector self-energies and pole residues in the nuclear matter. In this article, we extend out previous works [4, 6] to study the properties of the doubly heavy baryon states $\Xi_{cc}$, $\Omega_{cc}$, $\Xi_{bb}$ and $\Omega_{bb}$ in the nuclear matter. The mass-shifts of the heavy mesons, heavy baryons and doubly heavy baryons differ greatly from the corresponding ones of the light mesons and light baryons due to the appearance of the heavy quarks, the full propagators of the heavy quarks in the nuclear matter undergo much slight modifications compared with that of the light quarks, the gluon condensates are slightly modified in the nuclear matter. The upcoming FAIR (facility for antiproton and ion research) project at GSI (heavy ion research laboratory) provides the opportunity to extend the experimental studies of the hadron properties in the nuclear matter into the charm sector [7], the present predictions can be confronted with the experimental data in the future. The ground state light-flavor hadrons in the nuclear matter have been studied extensively with the QCD sum rules [8, 9, 10], the heavy quarkonia $J/\psi$, $\eta_c$ in the nuclear matter have also been studied with the QCD sum rules [11], while the works on the heavy mesons in the nuclear matter are few [12].

The article is arranged as follows: we study the doubly heavy baryon states $\Xi_{QQ}$ and $\Omega_{QQ}$ in the nuclear matter with the QCD sum rules in Sec.2; in Sec.3, we present the numerical results and discussions; and Sec.4 is reserved for our conclusions.

1 E-mail, wangzgyiti@yahoo.com.cn.
2 The doubly heavy baryons in the nuclear matter with QCD sum rules

We study the doubly heavy baryon states $\Xi_{QQ}$ and $\Omega_{QQ}$ in the nuclear matter with the two-point correlation functions $\Pi(p)$,

$$\Pi(p) = i \int d^4x e^{ip \cdot x} \langle \Psi_0 | T \{ J_q(x), \bar{J}_q(0) \} | \Psi_0 \rangle,$$

$$J_q(x) = e^{ik} \bar{Q}_i(x) C \gamma^\mu q_j(x),$$

(1)

where the $i, j, k$ are color indexes, $Q = c, b$, the $C$ is the charge conjunction matrix, and the $| \Psi_0 \rangle$ is the nuclear matter ground state. We use the current $J_q(x)$ to interpolate the $\Xi_{QQ}, \Omega_{QQ}$, and obtain the current $J_q(x)$ to interpolate the $\Omega_{QQ}$ with a simple replacement $q \rightarrow s$. The correlation functions $\Pi(p)$ can be decomposed as

$$\Pi(p) = \Pi_s(p^2, p \cdot u) + \Pi_p(p^2, p \cdot u) \hat{p} + \Pi_u(p^2, p \cdot u) \hat{p},$$

(2)

according to Lorentz covariance, parity and time reversal invariance \cite{8, 9}. In the limit four-vector $u_\mu = (1, 0)$, the component $\Pi_s(p^2, p \cdot u)$ reduces to $\Pi_i(p_0, \vec{p})$, where $i = s, p, u$.

We insert a complete set of intermediate doubly heavy baryon states with the same quantum numbers as the current operators $J_q(x)$ into the correlation functions $\Pi(p)$ to obtain the hadronic representation \cite{9}, then isolate the ground state contributions of the doubly heavy baryons $\Xi_{QQ}$, and use the dispersion relation to recast the three components $\Pi_i(p_0, \vec{p})$ into the following form:

$$\Pi_i(p_0, \vec{p}) = \frac{1}{2\pi i} \int_{-\infty}^{\infty} d\omega \frac{\Delta \Pi_i(\omega, \vec{p})}{\omega - p_0},$$

(3)

where

$$\Delta \Pi_s(\omega, \vec{p}) = -2\pi i \frac{\lambda_{\Xi_{QQ}}^2 M_{\Xi_{QQ}}^2}{2E_p^*} \left[ \delta(\omega - E_p) - \delta(\omega - \bar{E}_p) \right],$$

$$\Delta \Pi_p(\omega, \vec{p}) = -2\pi i \frac{\lambda_{\Xi_{QQ}}^2}{2E_p^*} \left[ \delta(\omega - E_p^*) - \delta(\omega - \bar{E}_p) \right],$$

$$\Delta \Pi_u(\omega, \vec{p}) = +2\pi i \frac{\lambda_{\Xi_{QQ}}^2 \Sigma_{\Xi_{QQ}}^u}{2E_p^*} \left[ \delta(\omega - E_p^*) - \delta(\omega - \bar{E}_p) \right],$$

(4)

$E_p^* = \Sigma_{\Xi_{QQ}}^u + \bar{E}_p = \Sigma_{\Xi_{QQ}}^u - E_p^*, \bar{E}_p = \sqrt{M_{\Xi_{QQ}}^2 + p^2}$, the $M_{\Xi_{QQ}}$, $\Sigma_{\Xi_{QQ}}^u$, and $\lambda_{\Xi_{QQ}}$ are the masses, vector self-energies and pole residues of the doubly heavy baryon states $\Xi_{QQ}$ respectively in the nuclear matter.

We carry out the operator product expansion in the nuclear matter at the large space-like region $p^2 \ll 0$, and obtain the spectral densities at the level of quark-gluon degrees of freedom, then take the limit $u_\mu = (1, 0)$, and obtain the three components $\Pi_i(p_0, \vec{p})$ of the correlation function $\Pi(p)$ \cite{9}:

$$\Pi_i(p_0, \vec{p}) = \sum_n C_n^i(p_0, \vec{p}) \langle \mathcal{O}_n \rangle_{\rho N},$$

(5)

where the $C_n^i(p_0, \vec{p})$ are the Wilson coefficients, $\langle \mathcal{O}_n \rangle_{\rho N} = \langle \Psi_0 | \mathcal{O}_n | \Psi_0 \rangle = \langle \mathcal{O}_n \rangle + \rho_N \langle \mathcal{O}_n \rangle_N$ at low nuclear density in the linear approximation, the $\langle \mathcal{O}_n \rangle$ and $\langle \mathcal{O}_n \rangle_N$ denote the vacuum condensates and the nuclear matter induced condensates, respectively. We can obtain the imaginary parts of the QCD spectral densities through the formula

$$\Delta \Pi_i(\omega, \vec{p}) = \lim_{t \rightarrow 0} [\Pi_i(\omega + i\epsilon, \vec{p}) - \Pi_i(\omega - i\epsilon, \vec{p})].$$

(6)
We match the hadronic spectral densities with the QCD spectral densities, and multiply both sides with the weight function \((\omega - E_p)e^{-\frac{\omega^2}{T^2}}\), perform the integral \(\int_{-\omega_0}^{\omega_0} d\omega\),

\[
\int_{-\omega_0}^{\omega_0} d\omega \Delta\Pi_t(\omega, p_\perp)(\omega - E_p)e^{-\frac{\omega^2}{T^2}},
\]

to exclude the negative quasi-particle contributions, and obtain the following three QCD sum rules:

\[
\lambda^{s^2} - \frac{s^2}{\pi^2} \int_{r_0}^{s_0} ds p^E_{\Xi QQ}(s)e^{-\frac{s^2}{T^2}},
\]

\[
\lambda^{s^2} - \frac{s^2}{\pi^2} \int_{(2m_Q + m_x)^2} ds p^E_{\Xi QQ}(s)e^{-\frac{s^2}{T^2}},
\]

\[
\lambda^{s^2} - \frac{s^2}{\pi^2} \int_{(2m_Q + m_x)^2} ds p^u_{\Xi QQ}(s)e^{-\frac{s^2}{T^2}},
\]

\[
\rho^n_{\Xi QQ}(s) = \frac{3}{8\pi^4} \int_{\alpha_1}^{\alpha_f} d\alpha \int_{1 - \alpha}^{1 - \alpha} d\beta \alpha \beta (1 - \alpha - \beta) (s - \tilde{E}_Q^2)(5s - 3\tilde{E}_Q^2)
\]

\[
+ \frac{3m_0^2}{8\pi^4} \int_{\alpha_1}^{\alpha_f} d\alpha \int_{1 - \alpha}^{1 - \alpha} d\beta (1 - \alpha - \beta) (s - \tilde{E}_Q^2)
\]

\[
- \frac{m_0^2}{12\pi^2} \frac{\alpha_0 G}{\pi} \int_{\alpha_1}^{\alpha_f} d\alpha \int_{1 - \alpha}^{1 - \alpha} d\beta (1 - \alpha - \beta) \left[\alpha + \beta + \frac{1}{\alpha^2} + \frac{1}{\beta^2}\right] \delta(s - \tilde{E}_Q^2)
\]

\[
+ \frac{m_0^4}{48\pi^4 T^2} \frac{\alpha_0 G}{\pi} \int_{\alpha_1}^{\alpha_f} d\alpha \int_{1 - \alpha}^{1 - \alpha} d\beta (1 - \alpha - \beta) \left[\frac{1}{\alpha^2} + \frac{1}{\beta^2}\right] \delta(s - \tilde{E}_Q^2)
\]

\[
+ \frac{1}{32\pi^4} \frac{\alpha_0 G}{\pi} \int_{\alpha_1}^{\alpha_f} d\alpha \int_{1 - \alpha}^{1 - \alpha} d\beta \left[3 + (1 + \alpha + \beta)\tilde{m}_Q^2\delta(s - \tilde{E}_Q^2)\right]
\]

\[
+ \frac{m_0^4}{3\pi^2} \int_{\alpha_1}^{\alpha_f} d\alpha \delta(1 - \alpha) \left[2 + (3\tilde{m}_Q^2 - 8\tilde{E}_Q^2)\delta(s - \tilde{E}_Q^2)\right]
\]

\[
- \frac{\langle q^i D_{i0} q\rangle_{PN}}{2\pi^2} \int_{\alpha_1}^{\alpha_f} d\alpha \delta(1 - \alpha) \left[2 + (3\tilde{m}_Q^2 - 8\tilde{E}_Q^2)\delta(s - \tilde{E}_Q^2)\right]
\]

\[
- \frac{\langle q^i D_{i0} q\rangle_{PN}}{2\pi^2} \int_{\alpha_1}^{\alpha_f} d\alpha \delta(1 - \alpha) \left[2 + (3\tilde{m}_Q^2 - 8\tilde{E}_Q^2)\delta(s - \tilde{E}_Q^2)\right]
\]

\[
- \frac{\langle q^i g_s G q\rangle_{PN}}{12\pi^2} \int_{\alpha_1}^{\alpha_f} d\alpha \delta(s - \tilde{E}_Q^2),
\]
\[
\rho^{\alpha}_{\Xi Q Q}(s) = \frac{3m_q}{8\pi^4} \int_{\alpha_i}^{\alpha_f} d\alpha \int_{\beta_i}^{1-\alpha} d\beta \alpha \beta (s - \tilde{E}_Q^2)(2s - \tilde{E}_Q^2) + \frac{3m \tilde{m}_Q^2}{4\pi^4} \int_{\alpha_i}^{\alpha_f} d\alpha \int_{\beta_i}^{1-\alpha} d\beta (s - \tilde{E}_Q^2)
\]
\[
- \frac{m \tilde{m}_Q^2}{8\pi^2} \frac{\alpha \tilde{m}_Q \tilde{g} G}{\pi} \int_{\alpha_i}^{\alpha_f} d\alpha \int_{\beta_i}^{1-\alpha} d\beta \left[ \frac{\alpha}{\beta^2} + \frac{1}{\beta^3} \right] \left[ 1 + \frac{\tilde{m}_Q^2}{T^2} \right] \delta(s - \tilde{E}_Q^2)
\]
\[
- \frac{m \tilde{m}_Q^2}{24\pi^2 T^2} \frac{\alpha \tilde{m}_Q \tilde{g} G}{\pi} \int_{\alpha_i}^{\alpha_f} d\alpha \int_{\beta_i}^{1-\alpha} d\beta \left[ \frac{1}{\alpha^3} + \frac{1}{\beta^3} \right] \delta(s - \tilde{E}_Q^2)
\]
\[
+ \frac{m \tilde{m}_Q^2}{2\pi^2} \frac{\alpha \tilde{m}_Q \tilde{g} G}{\pi} \int_{\alpha_i}^{\alpha_f} d\alpha \int_{\beta_i}^{1-\alpha} d\beta \left[ \frac{1}{\alpha^2} + \frac{1}{\beta^2} \right] \delta(s - \tilde{E}_Q^2)
\]
\[
- \frac{m \tilde{m}_Q^2}{16\pi^2} \frac{\alpha \tilde{m}_Q \tilde{g} G}{\pi} \int_{\alpha_i}^{\alpha_f} d\alpha \int_{\beta_i}^{1-\alpha} d\beta \left[ 1 + \frac{\tilde{m}_Q^2}{2} \delta(s - \tilde{E}_Q^2) \right]
\]
\[
- \frac{m \tilde{m}_Q^2}{2\pi^2} \int_{\alpha_i}^{\alpha_f} d\alpha \alpha(1 - \alpha) \left[ 3s + 2\tilde{m}_Q^2 - 2\tilde{E}_Q^2 \right]
\]
\[
+ \frac{m \tilde{m}_Q^2}{2\pi^2} \int_{\alpha_i}^{\alpha_f} d\alpha \alpha(1 - \alpha) \left[ \frac{3}{2} + \left( \frac{5\tilde{m}_Q^2}{2} - \tilde{E}_Q^2 + \frac{\tilde{m}_Q^2 \hat{E}_Q^2}{T^2} \right) \delta(s - \tilde{E}_Q^2) \right]
\]
\[
+ \frac{m \tilde{m}_Q^2}{2\pi^2} \int_{\alpha_i}^{\alpha_f} d\alpha \alpha(1 - \alpha) \left[ 4\tilde{m}_Q^2 - 4\tilde{E}_Q^2 + \frac{4\tilde{m}_Q^2 \hat{E}_Q^2}{T^2} \right] \delta(s - \tilde{E}_Q^2)
\]
\[
- \frac{m \tilde{m}_Q^2}{2\pi^2} \int_{\alpha_i}^{\alpha_f} d\alpha \alpha(1 - \alpha) \left[ 1 + \frac{\tilde{m}_Q^2}{2} \delta(s - \tilde{E}_Q^2) \right]
\]
\[
\]
simultaneous iterations. With the simple replacements of the corresponding parameters, such as $m_\eta \to m_\eta$, $\langle q\bar{q}\rangle_{\rho N} \to \langle s\bar{s}\rangle_{\rho N}$, $\langle q\bar{q}\rangle_{\rho N} \to \langle s\bar{s}\rangle_{\rho N}$, etc, we can obtain the corresponding three QCD sum rules for the doubly heavy baryon states $\Omega_{QQ}$.

### 3 Numerical results and discussions

The input parameters are taken as $\langle q\bar{q}\rangle_{\rho N} = \frac{7}{9}\rho N$, $\langle s\bar{s}\rangle_{\rho N} = 0$, $\langle q\bar{q}\rangle_{\rho N} = \langle q\bar{q}\rangle_{\rho N} + \frac{25}{9}\rho N$, $\langle s\bar{s}\rangle_{\rho N} = \langle s\bar{s}\rangle_{\rho N} + y\frac{G_{\rho N}}{G_{\rho N}} = 0.33 \text{GeV}^4$, $\langle q\bar{q}\rangle_{\rho N} + \frac{1}{2}\langle g_sG_{\rho N}\rangle_{\rho N} = 0.031 \text{GeV}^2\rho N$, $\langle s\bar{s}\rangle_{\rho N} = 0.3 \text{GeV}^2\rho N$, $(\hat{s}\bar{D}\rho N)_{\rho N} = 0.3 \text{GeV}^2\rho N$, $\langle \bar{s}g_sGq\rangle_{\rho N} = 0.3 \text{GeV}^2\rho N$, $\langle q\bar{q}\rangle_{\rho N} = -(0.23) \text{GeV}^3$, $\langle s\bar{s}\rangle = (0.8) \text{GeV}^2$, $m_\rho = (0.11) \text{GeV}^3$, $y = 0.3 \pm 0.3$, $m_{\rho} = 6 \text{MeV}$, $m_\phi = 1.35 \text{GeV}$ and $m_\phi = 4.7 \text{GeV}$ at the energy scale $\mu = 1 \text{GeV}$.

We can recover the QCD sum rules in the vacuum by taking the limit $\rho_N = 0$, then differentiate the Eqs. (8-9) with respect to $\frac{1}{x}$ respectively, and eliminate the pole residues $\lambda_{\Xi_{QQ}}$ (here we smear the asterisk * to denote the pole residues in the vacuum), and obtain two QCD sum rules for the masses $\Xi_{QQ}$ with respect to the spinor structures $\hat{\rho}$ and 1, respectively. On the other hand, we can divide Eq. (9) by Eq. (8) to obtain the QCD sum rules for the masses $\Xi_{QQ}$. From Eq. (10), we can see that if we take the limit $\rho_N = 0$, $\Sigma_{\Xi_{QQ}} \neq 0$, due to appearance of the term $m_\eta\langle q\bar{q}\rangle_{\rho N}$, however, such terms are small enough to be neglected safely. The QCD spectral densities from the operator product expansion in the vacuum and in nuclear matter by taking the limit $\rho_N = 0$ have slight discrepancies, the discrepancies cannot result in remarkable differences on the masses and pole residues.

In Ref. [4], we study the doubly heavy baryon states $\Xi_{QQ}$ and $\Omega_{QQ}$ by subtracting the possible contributions from the corresponding negative-parity doubly heavy baryon states with the QCD sum rules. In that reference, we choose the spinor structure $1 + \gamma^0$, take the Borel parameters as $T_{\Xi_{QQ}} = 2.8 - 3.8 \text{GeV}^2$, $T_{\Xi_{QQ}} = 3.0 - 4.0 \text{GeV}^2$, $T_{\Omega_{QQ}} = 7.7 - 9.1 \text{GeV}^2$, $T_{\Omega_{QQ}} = 7.9 - 9.3 \text{GeV}^2$, the continuum threshold parameters as $s_0_{\Xi_{QQ}} = 4.2 \pm 0.1 \text{GeV}^2$, $s_0_{\Omega_{QQ}} = 4.3 \pm 0.1 \text{GeV}^2$, $b_0_{\Xi_{QQ}} = 10.8 \pm 0.1 \text{GeV}^2$, $b_0_{\Omega_{QQ}} = 10.9 \pm 0.1 \text{GeV}^2$, and obtain the masses $M_{\Xi_{QQ}} = (3.57 \pm 0.14) \text{GeV}$, $M_{\Omega_{QQ}} = (10.17 \pm 0.14) \text{GeV}$, $M_{\Omega_{QQ}} = (10.32 \pm 0.14) \text{GeV}$.

The predictions are consistent with the value from the SELEX collaboration $T_{\Xi_{QQ}} = (3518.9 \pm 0.9) \text{MeV}$ [1 2]. For the technical details, one can consult Ref. [4].

In this article, we choose the spin structures 1 and $\hat{\rho}$, see Eq. (2), take the continuum threshold parameters as $s_0_{\Xi_{QQ}} = (17.5 \pm 0.5) \text{GeV}^2$, $s_0_{\Omega_{QQ}} = (18.5 \pm 0.5) \text{GeV}^2$, $b_0_{\Xi_{QQ}} = (117.0 \pm 1.0) \text{GeV}^2$ and $b_0_{\Omega_{QQ}} = (119.0 \pm 1.0) \text{GeV}^2$ consulting Ref. [4], and vary the Borel parameters $T^2$ to reproduce almost the same masses as Ref. [2]. In calculations, the Borel parameters are taken as $T_{\Xi_{QQ}}^2 = (2.4 - 2.8) \text{GeV}^2$, $T_{\Xi_{QQ}}^2 = (2.4 - 2.8) \text{GeV}^2$, $T_{\Xi_{QQ}}^2 = (7.0 - 7.5) \text{GeV}^2$ and $T_{\Xi_{QQ}}^2 = (6.7 - 7.2) \text{GeV}^2$, respectively.

The interpolating currents $J_q(x)$ have nonvanishing couplings to both the positive- and negative-parity doubly heavy baryon states. For the QCD sum rules in the nuclear matter, we cannot choose the spinor structure $1 + \gamma^0$ to avoid the possible contaminations from the negative-parity doubly heavy baryon states. In this article, we choose the spinor structures 1 and $\hat{\rho}$ in stead of the spinor structure $1 + \gamma^0$ in the limit $\rho_N = 0$, the ratios between the negative- and positive-parity contributions are about $-9\%$ and $-8\%$ in the charm and bottom sectors, respectively [4]. For the technical details, one can consult Ref. [4]. The contaminations from the negative-parity doubly heavy baryon states are rather small. We can also take into account the contributions from the negative-parity doubly heavy baryon states explicitly as Ref. [15], and perform detailed analysis, the tedious analysis maybe our next work.
If we take the limit $m_u = m_d = m_q \to 0$ in the QCD sum rules for the nucleons, the hadronic parameters $M^\ast_N$ and $\Sigma^\ast_N$ in the nuclear matter can be approximated as

$$M^\ast_N = -\frac{8\pi^2}{T^2} \langle \bar{q}q \rangle_{PN},$$
$$\Sigma^\ast_N = \frac{64\pi^2}{3T^2} \langle q^4 \rangle_{PN},$$

(14)

respectively. Such relations are simple and elegant, however, the predictions change monotonously with variations of the Borel parameter $T^2$. In the present case, if we take the limit $m_q \to 0$ and neglect other terms of minor importance, we can obtain the following relations,

$$M^\ast_{\Xi QQ} = f(T^2, m_Q, s^0_{\Xi QQ}) \langle \bar{q}q \rangle_{PN},$$
$$\Sigma^\ast_{\Xi QQ} = g(T^2, m_Q, s^0_{\Xi QQ}) \langle q^4 \rangle_{PN},$$

(15)

from Eqs.(8-13). The formal notations $f(T^2, m_Q, s^0_{\Xi QQ})$ and $g(T^2, m_Q, s^0_{\Xi QQ})$ are complex functions of the variables $T^2, m_Q$ and $s^0_{\Xi QQ}$. It is difficult to carry out the integrals over the variables $s, \alpha, \beta$ analytically due to the appearance of the heavy quark mass $m_Q$, we expect that the $f(T^2, m_Q, s^0_{\Xi QQ})$ and $g(T^2, m_Q, s^0_{\Xi QQ})$ are sensitive to the Borel parameter $T^2$, the numerical calculations confirm such conjecture.

Finally, we obtain the masses and pole residues in the vacuum $M_{\Xi cc} = (3.53 \pm 0.32)$ GeV, $M_{\Omega cc} = (3.61 \pm 0.29)$ GeV, $M_{\Xi bb} = (10.14 \pm 0.47)$ GeV, $M_{\Omega bb} = (10.27 \pm 0.46)$ GeV, $\lambda_{\Xi cc} = (0.117 \pm 0.046)$ GeV, $\lambda_{\Omega cc} = (0.128 \pm 0.047)$ GeV, $\lambda_{\Xi bb} = (0.287 \pm 0.162)$ GeV, $\lambda_{\Omega bb} = (0.359 \pm 0.211)$ GeV; the masses, pole residues and vector self-energies in the nuclear matter $M^\ast_{\Xi cc} = (2.43 \pm 0.27)$ GeV, $M^\ast_{\Omega cc} = (3.28 \pm 0.26)$ GeV, $M^\ast_{\Xi bb} = (6.77 \pm 0.38)$ GeV, $M^\ast_{\Omega bb} = (9.22 \pm 0.41)$ GeV, $\lambda^\ast_{\Xi cc} = (0.033 \pm 0.005)$ GeV, $\lambda^\ast_{\Omega cc} = (0.078 \pm 0.021)$ GeV, $\lambda^\ast_{\Xi bb} = (0.008 \pm 0.002)$ GeV, $\lambda^\ast_{\Omega bb} = (0.075 \pm 0.033)$ GeV, $\Sigma^\ast_{\Xi cc} = (0.139 \pm 0.005)$ GeV, $\Sigma^\ast_{\Omega cc} = (-0.006 \pm 0.001)$ GeV, $\Sigma^\ast_{\Xi bb} = (0.507 \pm 0.019)$ GeV, $\Sigma^\ast_{\Omega bb} = (0.009 \pm 0.001)$ GeV. The mass-shifts in the nuclear matter are $\Delta M_{\Xi cc} = -(1.11 \pm 0.04)$ GeV, $\Delta M_{\Omega cc} = -(3.37 \pm 0.09)$ GeV, $\Delta M_{\Xi bb} = -(1.05 \pm 0.06)$ GeV, where the mass-shifts $\Delta M$ defined by $\Delta M = M^\ast - M$ are the scalar self-energies. Come the uncertainties from the Borel parameters $T^2$.

The mass modifications in the nuclear matter are about $-31\%, -33\%, -9\%$ and $-10\%$ for the doubly heavy baryon states $\Xi cc$, $\Xi bb$, $\Omega cc$ and $\Omega bb$, respectively, which are consistent with the quark condensates modifications in the nuclear matter $\frac{\langle \bar{q}q \rangle_{PN} - \langle \bar{q}q \rangle_{\Xi QQ}}{\langle \bar{q}q \rangle_{\Xi QQ}} \approx -41\%$, $\frac{\langle \bar{s}s \rangle_{PN} - \langle \bar{s}s \rangle_{\Xi QQ}}{\langle \bar{s}s \rangle_{\Xi QQ}} \approx -(15 \pm 15)\%$, where the uncertainty $\pm 15\%$ comes from the uncertainty of the parameter $\delta y = \pm 0.3$. There appear additional quark condensates associated with the light flavor quarks, such as $\langle \bar{q}q \rangle_{PN}, \langle \bar{q}^i D_{Dq} \rangle_{PN}, \langle \bar{q}^i D_{Dq} \rangle_{\Xi QQ}$, etc, their contributions are not large but cannot be neglected safely. The $\Xi QQ$ and $\Omega QQ$ baryons have two heavy quarks besides a light quark, the heavy quark interacts with the nuclear matter through the exchange of the intermediate gluons, and the modifications of the gluon condensates in the nuclear matter are mild, $\frac{\langle \bar{q}q \rangle_{\Xi QQ}}{\langle \bar{q}q \rangle_{\Xi QQ}} \approx 0.93(\pm 0.03)$. We expect that the mass modifications are smaller than that of the nucleons, which have three light quarks, $uud$ or $udd$, and the approximation in Eq.(15) makes sense.

If we take into account the uncertainties of the continuum threshold parameters, $\delta s^0_{\Xi cc} = \pm 0.5$ GeV, $\delta s^0_{\Xi bb} = \pm 0.5$ GeV, $\delta s^0_{\Omega cc} = \pm 1.0$ GeV, $\delta s^0_{\Omega bb} = \pm 1.0$ GeV, additional uncertainties $\delta \Delta M_{\Xi cc} = \pm 0.03$ GeV, $\delta \Delta M_{\Omega cc} = \pm 0.01$ GeV, $\delta \Delta M_{\Xi bb} = \pm 0.10$ GeV, $\delta \Delta M_{\Omega bb} = \pm 0.04$ GeV for the mass-shifts $\Delta M$ are introduced. The uncertainty $\delta y = \pm 0.3$ leads to the uncertainties $\delta \Delta M_{\Xi cc} = \pm 0.43$ GeV and $\delta \Delta M_{\Omega cc} = \pm 1.13$ GeV. The uncertainty $\delta \sigma_N = 10$ MeV leads to the uncertainties $\delta \Delta M_{\Xi cc} = \pm 0.37$ GeV, $\delta \Delta M_{\Omega cc} = \pm 0.10$ GeV, $\delta \Delta M_{\Xi bb} = \pm 1.05$ GeV and $\delta \Delta M_{\Omega bb} = \pm 0.27$ GeV, respectively.

We can also take into account the perturbative $\mathcal{O}(\alpha^\ast_s)$ corrections in the leading logarithmic approximation through multiplying the QCD spectral densities by the anomalous-dimension fac-
tors,
\[
\frac{\log(T^2/A_{QCD}^2)}{\log(\mu^2/A_{QCD}^2)} \left[ 1 - 2T_{J+1} \right],
\]
where the anomalous-dimensions of the interpolating currents are \( \Gamma_J = \frac{6}{13 - 2N_f} \), and the anomalous-dimensions of the local operators \( O \) are \( \Gamma_{O(pq)} = \frac{4}{13 - 2N_f} \), \( \Gamma_{m_q(pq)} = 0 \), \( \Gamma_{g_s^2 G G} = 0 \), and the \( N_f \) is the flavor number. The anomalous-dimension factors can lead to the uncertainties \( \delta M^\omega_{\Xi_{cc}} = (13 - 17)\% \), \( \delta M^s_{\Xi_{bb}} = (10 - 13)\% \), \( \delta E^\omega_{\Xi_{cc}} = (28 - 36)\% \), \( \delta E^s_{\Xi_{bb}} = (21 - 27)\% \), \( \delta M^\omega_{\Xi_{bb}} = (16 - 21)\% \), \( \delta M^s_{\Xi_{cc}} = (10 - 13)\% \), \( \delta E^\omega_{\Xi_{bb}} = (34 - 46)\% \), \( \delta E^s_{\Xi_{cc}} = (20 - 26)\% \) for the values \( \Lambda_{QCD} = (200 - 300) \) MeV.

The masses in the vacuum and in the nuclear matter are increased significantly, we can normalize the masses in the vacuum to the values \( M_{\Xi_{cc}} = 3.53 \) GeV, \( M_{\Xi_{bb}} = 3.61 \) GeV, \( M_{\Xi_{cc}} = 10.14 \) GeV, and estimate the uncertainties of the mass-shifts due to the energy scales as \( \frac{\delta E_{\Xi_{cc}}}{\Delta M_{\Xi_{cc}}} = -(5 - 7)\% \), \( \frac{\delta E_{\Xi_{bb}}}{\Delta M_{\Xi_{bb}}} = (4 - 5)\% \), \( \frac{\delta E_{\Xi_{cc}}}{\Delta M_{\Xi_{cc}}} = -(10 - 14)\% \), \( \frac{\delta E_{\Xi_{bb}}}{\Delta M_{\Xi_{bb}}} = (6 - 8)\% \).

We can refit the Borel parameters and threshold parameters to reproduce the experimental data approximately [2], the Borel parameters (or the effectively energy scales) and the threshold parameters have some correlations. In calculations, we observe that larger threshold parameters can lead to smaller masses and cancel out the enhanced factors induced by the non-zero anomalous-dimensions in Eq.(16), however, the contributions from the high resonances and continuum states are included in. So we can fix the threshold parameters and take larger Borel parameters to cancel out those enhanced factors, see Eqs.(14-15), then the uncertainties of the mass-shifts induced by the energy scales are about a few percents, and can be neglected safely. On the other hand, we can introduce Borel parameter dependent threshold parameters and study the systematic uncertainties [14], it is a hard work before the experimental data are enough and the precise values of the pole residues are known, we postpone those works in the future.

If we take the Ioffe current to interpolate the proton, the QCD sum rules indicate that there exists a positive vector self-energy \( \Sigma^v_N = (0.23 - 0.35) \) GeV with the typical values of the relevant condensates and other input parameters and a reasonable negative scalar self-energy with the suitable parameters [9]. There exists substantial cancelation between the scalar and vector self-energies, the self-energies \( \Sigma^v_N \) and \( \Sigma^s_N \), which correspond to the real energy-independent optical potentials \( S \) and \( V \), satisfy the relation \( \Sigma^v_N / \Sigma^s_N \approx -1 \) in the leading order approximation. While the mean-field models predicate that the typical self-energies of the nucleons in nuclear matter saturation density are \( \Sigma^v_N \approx -350 \) MeV and \( \Sigma^s_N \approx +300 \) MeV respectively, the effective non-relativistic central potentials \( S + V \) are about tens of MeV. In the present case, \( \Sigma^s_{\Xi_{cc}} + \Sigma^s_{\Xi_{bb}} = -0.97 \) GeV, \( \Sigma^s_{\Xi_{cc}} + \Sigma^s_{\Xi_{bb}} = -0.34 \) GeV, \( \Sigma^s_{\Xi_{cc}} + \Sigma^s_{\Xi_{bb}} = -2.86 \) GeV, \( \Sigma^s_{\Xi_{cc}} + \Sigma^s_{\Xi_{bb}} = -1.04 \) GeV, the net optical potentials \( S + V \) are large, as the self-energies \( |\Sigma^v| \ll |\Sigma^s| \). The present prediction of the mass-shift \( \delta M_{\Xi_{cc}} = -1.11 \) GeV can be confronted with the experimental data from the CBM and PANDA collaborations in the future [7], where the properties of the charmed baryons in the nuclear matter will be studied. The \( \Xi_{cc} \) have interesting properties, for example, the theoretical predictions of their lifetimes based on different quark models are in agreement with each other [13], but about one-order larger than the upper limit of the experimental data [1]. The precise measurement of the lifetime by the PANDA and LHCb collaborations in the future maybe shed light on the apparent discrepancy.

### 4 Conclusion

In this article, we extend our previous works on the \( \Lambda \)-type and \( \Sigma \)-type heavy baryon states to study the doubly heavy baryon states \( \Xi_{QQ} \) and \( \Omega_{QQ} \) in the nuclear matter using the QCD sum rules, and derive three coupled QCD sum rules for the masses, vector self-energies and pole residues in the
Figure 1: The masses from the QCD sum rules in the vacuum and in the nuclear matter versus the Borel parameter $T^2$, the $A$, $B$, $C$ and $D$ denote the $\Xi_{cc}$, $\Omega_{cc}$, $\Xi_{bb}$ and $\Omega_{bb}$ baryons, respectively.
nuclear matter, then take the limit $\rho_N = 0$ to recover the QCD sum rules in the vacuum, finally obtain the values of the masses and pole residues in the vacuum, and the masses, vector self-energies and pole residues in the nuclear matter. The numerical results indicate that the mass-shifts in the nuclear matter are about $\Delta M_{\Xi_{cc}} = -1.11$ GeV, $\Delta M_{\Omega_{cc}} = -0.33$ GeV, $\Delta M_{\Xi_{bb}} = -3.37$ GeV and $\Delta M_{\Omega_{bb}} = -1.05$ GeV, respectively, which can be confronted with the experimental data in the future.

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