The galaxy H I–(sub)halo connection and the H I spatial clustering of local galaxies

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ABSTRACT

We extend the local stellar galaxy–(sub)halo connection to the atomic hydrogen (H I) component by seeding semi-empirically galaxies into a large N-body dark matter (DM) simulation. The main input to construct the mock galaxy catalogue are: our constrained stellar mass-to-(sub)halo circular velocity relation, assuming a scatter independent of any galaxy property, and the empirical $M_{H I}$ conditional probability distributions given $M_*$ for central and satellite galaxies. We find that the $\langle \log M_{H I} \rangle - \log M_{DM}$ relation is not a monotonic increasing function. It increases with mass up to $M_{DM} \sim 10^{12} M_\odot$, attaining a maximum of $\langle \log (M_{H I}/M_*) \rangle \sim 9.2$, and at higher (sub)halo masses, $\langle\log (M_{H I})\rangle$ decreases slightly with $M_{DM}$. The scatter around it is also large and mass dependent. The bivariate $M_{H I}$ and $M_{DM}$ distribution is broad and bimodal, specially at $M_{DM} \gtrsim 10^{12} M_\odot$, which is inherited from the input $M_{H I}$ conditional distributions. We also report the total (central+satellites) H I gas mass within halos, $M_{H I}^{\text{tot}}$, as a function of $M_{DM}$. The mean $M_{H I}^{\text{tot}}/M_{DM}$ relation is an increasing monotonic function. The galaxy spatial clustering increases weakly as the $M_{H I}$ threshold increases. Our H I mock galaxies cluster more in comparison to the blind H I ALFALFA (Arecibo Fast Legacy ALFA) survey but we show that it is mainly due to the selection effects. We discuss the implications of our results in the light of predictions from semi-analytical models and hydrodynamics simulations of galaxy evolution.

Key words: galaxies: evolution – galaxies: formation – galaxies: halos – galaxies: luminosity function, mass function – galaxies: statistics – cosmology: dark matter

1 INTRODUCTION

According to the current scenario of cosmic structure formation, galaxies formed and evolved within cold dark matter (CDM) halos from accreted and cooled gas within them. Depending on the different astrophysical processes and on the halo gravitational potential, a fraction of the cold accreted gas is deposited into the galaxy, part of this gas is transformed into stars while other part is heated or expelled by feedback processes due to stars and active galactic nuclei (AGNs). Therefore, the cold gas content that present-day galaxies have, given their halo and stellar masses ($M_*$ and $M_*$, respectively), provides important information to constrain the different processes of galaxy evolution (see e.g., Crain et al. 2017, and more references therein). The cold (neutral) gas in the interstellar medium of galaxies is composed by atomic hydrogen (H I), molecular hydrogen (H$_2$), helium, and metals, being the dominant in mass the H I component (e.g., Fukugita & Peebles 2004; Read & Tretham 2005; Rodríguez-Puebla et al. 2020).

Much work has been done in studying the H I content of galaxies in the nearby Universe. Among the largest efforts in this direction, we highlight the completion of the blind radio surveys H I Parkes All-Sky Survey (HIPASS; Meyer et al. 2004) and Arecibo Fast Legacy ALFA Survey (ALFALFA; Giovanelli et al. 2005; Haynes et al. 2011, 2018). While blind radio H I surveys do not impose any selection criteria on the target sample, their shallow flux detection limits, H I line width thresholds, and the small volumes surveyed bias the samples in other galaxy properties than the H I mass. For example, it is clear that they are biased to H I-rich, spiral blue galaxies (e.g., Baldry et al. 2008; Haynes et al. 2011; Catinella et al. 2012; Papastergis et al. 2012; Maddox et al. 2015; Chauhan et al. 2019). Moreover, these biases are mass dependent (see e.g., Huang et al. 2012; Maddox et al. 2015). From these sur-
vey, it was determined the local HI mass function (MF; Zwaan et al. 2005; Martin et al. 2010; Papastergis et al. 2012; Jones et al. 2018). While the total HI MF seems robust against the biases mentioned above (down to the masses allowed by the 21-cm flux limit of the used radiotelescopes), this is not the case for the HI-to-stellar mass relation (see e.g., Maddox et al. 2015), the projected two-point correlation function (2PCF) at different HI mass thresholds (Papastergis et al. 2013; Guo et al. 2017) and the HI velocity function (Chauhan et al. 2019). As a result of the biases, the HI 2PCFs measured, for instance from the HI ALFALFA survey, are expected to be lower than for all galaxies (Martin et al. 2012). Indeed, the measurements show that for HI-selected galaxies, the clustering amplitude is low and, depending on how the corrections to the bia-

ses are applied, different dependencies of clustering on HI mass are found (Basilakos et al. 2007; Meyer et al. 2007; Martin et al. 2012; Papastergis et al. 2013; Guo et al. 2017). By means of a (sub)halo1 abundance matching model (SHAM), Guo et al. (2017) attempted to reproduce the clustering measurements of the ALFALFA sample by selecting only haloes that formed relatively late. This result is interpreted as an intrinsic assembly bias effect on the HI gas content of galaxies. However, recent semi-analytical models of galaxy evolution do not find assembly age to be strongly correlated with the HI content of galaxies (Spinelli et al. 2020; Chauhan et al. 2020).

In the future, facilities such as the the five-hundred-meter Aperture Spherical radio Telescope (Nan et al. 2011; Li et al. 2013), the Square Kilometre Array (SKA; Blyth et al. 2015; Carilli 2015), or precursor instruments such as the Australian SKA Pathfinder (Johnston et al. 2008) and the outfitted Westerbork Synthesis Radio Telescope, will bring extragalactic HI studies more in line with optical/infrared surveys. The blind HI surveys will then be much deeper and for larger volumes than the current ones, helping to reduce the strong sample selection effects. By now, the HI-to-stellar mass and other correlations have to be constrained from radio follow-up observations of optically selected galaxy samples (e.g., Catinella et al. 2010, 2013, 2018; Wei et al. 2010; Saintonge et al. 2011; Papastergis et al. 2012; Kannappan et al. 2013; Boselli et al. 2014; Eckert et al. 2015; Stark et al. 2016). In Calette et al. (2018, hereafter Paper I) we undertook the task of compiling and homogenizing from the literature as much as possible galaxy samples, in the spirit of the ones listed above (including most of them), with the additional information of information on the galaxy morphology because the HI gas content of galaxies strongly depends on morphology. For determining the HI mass distributions, we have taken into account the reported upper limits for the radio non-detections: after homogenizing them, in particular those from the distant galaxies of the GASS sample, we have applied a survival analysis. As a result, we were able to constrain not only the mean $M_{HI} - M_*$ and $M_{HI} - M_*$ relations and their standard deviations for late- and early-type galaxies (LTG and ETG, respectively) down to $M_* > 10^7 M_\odot$, but the respective full conditional probability density function distributions (PDFs) of the HI- and H2-to-stellar mass ratios for a given $M_*$ (for previous attempts to constrain these distributions in the case of H1 see e.g., Lemonias et al. 2013).

An important step in Paper I was i) to analyze separately late- and early-type galaxies given their very different gas distributions, ii) to take into account and treat adequately the $M_{HI}$ upper limits, which dominate in number for the massive ETGs, and iii) to constrain the complete $M_{HI}$ and $M_{HI}$ conditional PDFs given $M_*$. From these distributions, it is possible to calculate any statistics, for instance, the mean logarithmic or arithmetic $M_{HI} - M_*$ and $M_{HI} - M_*$ relations.

Furthermore, in Rodriguez-Puebla et al. (2020, hereafter Paper II) we derived and used a well-constrained Galaxy Stellar Mass Function (GSMF) for all, late- and early-type galaxies down to $\sim 10^7 M_\odot$, and combined them with the HI and H2 conditional PDFs to generate the bivariate or joint ($M_*, M_{HI}$) and ($M_*, M_{HI}$) distribution functions. By projecting these bivariate distributions into the HI and H2 axes, we obtained the HI and H2 MFs, for LTGs and ETGs, as well as for all galaxies. In Paper II it was shown that our empirical HI and H2 MFs agree well with those measured from blind radio surveys or optically selected radio samples corrected for volume incompleteness. In particular, we have shown that the HI MF from blind radio surveys, like HIPASS and ALFALFA, are not affected by their selection effects. We concluded that our empirical statistical description of the local galaxy population regarding the stellar and HI and H2 gas contents integrates and extends well a large body of observational information.

1.1 The stellar–HI–dark matter mass connection

The bivariate $M_*$ and $M_{HI}$ distribution of the local galaxy population described above can be used to extend the empirical galaxy-halo connection to the HI gaseous component of galaxies. There were some previous attempts to set the galaxy-halo connection for $M_{HI}$, by means of the Halo Occupation Distribution (HOD) model and SHAM (e.g., Papastergis et al. 2012, 2013; Guo et al. 2017; Padmanabhan et al. 2017; Paul et al. 2018; Obuljen et al. 2019), or from physically motivated models that employed results from semi-empirical models (see e.g., Popping et al. 2015). The results are quite diverse, both regarding the HI-to-halo mass relation and its scatter, and the HI galaxy clustering. A potential shortcoming in applying the SHAM for $M_{HI}$ is that the key assumption of this technique - the existence of a monotonic and tight relation between $M_{HI}$ and $M_*$ (or any other halo property) - is probably not obeyed. On the other hand, the use of the measured HI 2PCFs in the HOD models could lead to incorrect HI-to-halo mass relations given the strong biases that the measured HI 2PCFs suffer due to the sample selection effects mentioned above. There are also some recent studies aimed at determining the HI gas content of halos selected from optical galaxy group catalogues and by measuring the HI stacked spectra of the entire groups (Ganguly et al. 2020) or by using the measured HI masses of the galaxy members from blind HI surveys (Ai & Zhu 2018; Tramonte & Ma 2020; Lu et al. 2020).

The results from the semi-empirical studies mentioned above may be affected by several selection and confusion effects in the observational data that they use. On the other hand, they determine the total galaxy HI mass within haloes but not the central (satellite) galaxy HI mass in haloes (subhaloes). Thus, there is still a lack of full and self-consistent empirical determinations of the galaxy stellar-HI-(sub)halo mass connection, which is crucial for constraining theoretical models (Obreschkow et al. 2016; Romeo 2020; Romeo et al. 2020) or for comparing with predictions from semi-analytical models and hydrodynamics simulations (e.g., Kim et al. 2017; Villaescusa-Navarro et al. 2018; Diemer et al. 2019; Baugh et al. 2019; Spinelli et al. 2020; Chauhan et al. 2020).

In order to establish the above-mentioned self-consistent connection, in this paper we take a different approach with respect
to previous ones. First, we perform a non-parametric SHAM with scatter in a large N-body cosmological simulation. The SHAM is applied to the stellar mass of galaxies and the maximum circular velocity of halos, $V_{\text{max}}$, and peak circular velocity of subhalos, $V_{\text{peak}}$; the use of halo/subhalo velocities instead of masses allows us to reproduce the dependence of the 2PCF on stellar mass. The SHAM result is used to assign stellar masses to the halos and subhalos in the simulation, and given $M_*$, the HI mass is drawn from the empirical HI-to-stellar mass distributions from Paper II. For the latter step, the HI-to-stellar mass distributions of LTGs and ETGs are allowed to differ between centrals and satellites by using the functions constrained in Calete et al. (2021) based on the recent extended GALEX Arecibo SDSS Arecibo survey (xGASS；Catinella et al. 2018), where galaxies were separated into centrals and satellites. In this way, we construct an empirically based mock (sub)halo-galaxy catalogue that allows us to predict the HI-to-(sub)halo mass relation for central and satellite galaxies as well as the HI spatial clustering.

In this paper, we are interested in making predictions of the stellar-HI-halo mass connection for all the population of galaxies (LTGs+ETGs). In that regards, we take into account two relevant aspects: that the empirical HI conditional distributions given $M_*$ are different for central and satellite galaxies, and that to compare the results with the observational surveys, their biases must be considered. In particular, we explore the impact of these two aspects on the HI spatial clustering and its comparison with the ALFALFA measurements. We find that in order to reproduce the (biased) measurements of the ALFALFA HI clustering it will be necessary to implement more complex models to assign stellar and HI masses to the (sub)haloes (see e.g., Papastergis et al. 2013；Guo et al. 2017). This exploration will be performed in a forthcoming paper. However, it is important to note that our prediction of the $M_{\text{HI}}$ distribution in (sub)haloes for all galaxies is insensitive to these modifications.

The outline of this paper is as follows. In Section 2 we present our methodology and the inputs needed to construct a mock catalogue with galaxies (halos) information. In Section 3 we show our results from the mock catalogue generated such as the extended galaxy-(sub)halo connection, the HI spatial clustering, and some consistency tests. We discuss these results in Section 4 and compare some of them with theoretical predictions. Finally, we give our conclusions and future work in Section 5.

## 2 THE GALAXY-(SUB)HALO CONNECTION

This section describes our procedure to attain the galaxy-(sub)halo connection by including both the empirical stellar and HI mass distributions, and to obtain also predictions on the spatial clustering as a function HI mass. The procedure implies the construction of a mock galaxy catalogue based on the outcome of a large cosmological N-body simulation. Figure 1 presents an illustrative scheme of all the ingredients of our procedure, and it refers to the subsections where the details can be consulted. In short, we begin by specifying the halo catalogue of the N-body simulation that we employed, Section 2.1. Then, we describe how we assign stellar masses to every halo and subhalo in the simulation, Section 2.2, and their corresponding late- or early-type morphologies, Section 2.3. Once we have specified the above properties for our mock galaxies, in Section 2.4 we assign the HI mass from empirical distributions for central and satellite galaxies. Finally, every (sub)halo in the simulation ends with an assigned stellar and HI mass. As a result, the full distribution of $M_{\text{HI}}$ as a function of the (sub)halo circular velocity or mass is obtained and the HI 2PCF can be measured. We can also apply to the mock catalogue cuts to emulate selection effects in the observational surveys for comparison (Section 2.5).

### 2.1 The simulation

We use the snapshot $z = 0$ of the cosmological $N$-body SmallMultiDark-Planck (SMDP) simulation (Klypin et al. 2016) for (sub)halo properties and the spatial distribution of galaxies. The SMDP is a simulation with $3840^3$ particles and mass resolution of $9.63 \times 10^8 M_\odot / h$ in a box of $L = 400 h^{-1}$ Mpc on a side. The adopted cosmology is based on the flat ΛCDM model with $\Omega_{\Lambda} = 0.693$, $\Omega_\Lambda = 0.307$, $\Omega_B = 0.048$, $h = 0.678$, and $\sigma_8 = 0.829$, compatible with Planck 15 results (Planck Collaboration et al. 2016). Dark matter (sub)haloes were identified using the ROCKSTAR halo finder (Behroozi et al. 2013). Masses or $V_{\text{max}}$ for distinct haloes were defined using spherical overdensities according to the redshift-dependent virial overdensity $\Delta_{\text{vir}}(z)$ given by the spherical collapse model; for the selected cosmology, $\Delta_{\text{vir}} = 333$ at $z = 0$. For subhaloes, which is haloes that survived as bound entities inside the distinct haloes, we used the highest mass or $V_{\text{max}}$ reached along the main progenitor branch of the subhaloes, see below. The (sub)halo catalogues are entirely downloadable\(^2\) and their statistical properties were presented in detail in Rodriguez-Puebla et al. (2016).

### 2.2 Stellar masses

For every halo and subhalo in the SMDP simulation a galaxy stellar mass is assigned via a non-parametric SHAM technique. SHAM is a simple statistical approach for connecting a (sub)halo property ($M_*$, $V_{\text{max}}$, etc.) to that of a galaxy property (luminosity, $M_*$, etc.). In its simple form, SHAM assumes that there is a unique monotonic relation between the selected (sub)halo and galaxy properties. The above clearly implies that central and satellite galaxies have identical galaxy-(sub)halo relationships. In the past, several authors have tested the effects of SHAM assumptions on the resulting galaxy clustering (see e.g., Yang et al. 2009；Neistein et al. 2011；Rodriguez-Puebla et al. 2012, 2013；Campbell et al. 2018). These studies have shown that using identical relationships between centrals and satellites leads to inconsistent results as compared to the observed spatial clustering of galaxies. In particular, Rodriguez-Puebla et al. (2012, see also Rodríguez-Puebla et al. 2013) suggested that these relationships should be different in order to not only reproduce the observed clustering of galaxies but also the observed abundance of galaxies in groups of different halo masses. Nonetheless, other studies have found that the assumptions from SHAM can still be valid when instead of (sub)halo mass is used, for instance, $V_{\text{max}}$ or the highest $V_{\text{max}}$ reached along the main progenitor branch of the (sub)halo’s merger tree, $V_{\text{peak}}$ (Reddick et al. 2013；Hearin et al. 2013；Campbell et al. 2018；Dragomir et al. 2018；Wechsler & Tinker 2018, and more references therein). Following Dragomir et al. (2018), here we assume that the halo property that correlates best with $M_*$ and reproduces the observed galaxy spatial clustering is

$$V_{\text{DM}} \equiv \left\{ \begin{array}{ll} V_{\text{max}} & \text{for distinct haloes}, \\ V_{\text{peak}} & \text{for subhaloes}. \end{array} \right.$$  

(1)

Typically, authors use SHAM in its simplest form, that is, the

\(^2\) https://www.cosmosim.org
cumulative halo and subhalo velocity function and the cumulative GSMF are matched in order to determine the $M_* - V_{\text{DM}}$ relation. The above assumes that there is no scatter around the mean $M_* - V_{\text{DM}}$ relation. In reality, it is expected that this relation has some scatter since the properties of the galaxies might be determined by different (sub)halo properties and/or some environmental factors. For example, in the case of the $M_* - M_h$ relation, analysis of large galaxy group catalogues (Yang et al. 2009; Reddick et al. 2013), the kinematics of satellite galaxies (More et al. 2011), galaxy clustering (Shankar et al. 2014; Rodríguez-Puebla et al. 2015), as well as galaxy clustering combined with galaxy lensing (Zu & Mandelbaum 2015), it is found that this dispersion is of the order of $\sigma \sim 0.15 - 0.20$ dex. To take this into account, SHAM should be modified to include a dispersion around the $M_* - V_{\text{DM}}$ relationship. Thus, in this paper we use a more general procedure for SHAM as described below.

We define $\mathcal{H}(M_* | V_{\text{DM}})$ as the conditional probability distribution function that a halo with velocity in the range $\log V_{\text{DM}} \pm d \log V_{\text{DM}}/2$ hosts a galaxy with stellar mass in the range $\log M_* \pm d \log M_*/2$. We assume that $\mathcal{H}(M_* | V_{\text{DM}})$ is a lognormal function,

$$
\mathcal{H}(M_* | V_{\text{DM}}) = \frac{1}{\sqrt{2\pi} \sigma} \exp \left[ -\frac{(\log M_* - \mu_{\text{M}})^2}{2\sigma^2} \right],
$$

where $\mu_{\text{M}} = \langle \log M_* (V_{\text{DM}}) \rangle$ is the mean logarithmic stellar mass-to-$V_{\text{DM}}$ relationship and $\sigma$ is the scatter, assumed to be independent of $V_{\text{DM}}$. Then the connection between the stellar mass and halo $V_{\text{DM}}$ is given by the following equation:

$$
\phi_g(M_*) = \int \mathcal{H}(M_* | V_{\text{DM}}) \phi_{\text{DM}}(V_{\text{DM}}) d \log V_{\text{DM}},
$$

Figure 1. Schematic figure for the five steps taken to implement our mock galaxy catalogue: 1) we use the halo catalogues of the SMDP simulation; 2) stellar masses are assigned using the $V_{\text{DM}} - M_*$ relation assuming a total scatter of $\sigma = 0.15$ dex using the Rodríguez-Puebla et al. (2020) GSMF; 3) an early- or late-type morphology is assigned based on the observed abundance in the SDSS DR7 galaxy groups catalogues (Yang et al. 2012) and the Huertas-Company et al. (2011) morphologies; 4) HI masses are assigned depending on whether the galaxy is central or satellite and whether the galaxy is central or satellite based on the Calette et al. (2021) PDFs, and 5) our HI-halo connection is totally defined. Selection effects or biases that emulate those of the observational surveys can be introduced in the mock catalogue.
where
\[
\phi_{\text{DM}}(V_{\text{DM}}) = \phi_{b}(V_{\text{max}}) + \phi_{\text{sat}}(V_{\text{peak}})
\]  
(4)

is the (sub)halo \( V_{\text{DM}} \) function and the units for \( \phi_{b}, \phi_{\text{DM}}, \phi_{b} \) and \( \phi_{\text{sat}} \) are in Mpc\(^{-3}\) \( \text{dex}^{-1} \).

In this paper we use the GSF, \( \phi_{b}(M_{*}) \), reported in Paper II, which ranges from dwarf galaxies, \( M_{*} \sim 10^{7} \text{M}_\odot \), to massive galaxies at the centres of galaxy clusters, \( M_{*} \sim 3 \times 10^{12} \). We note that the GSF from Paper II was corrected for surface brightness incompleteness. For the total scatter around the \( M_{*}-V_{\text{DM}} \) relationship, we assume \( \sigma = 0.15 \) dex in order to be consistent with the results discussed above for the \( M_{*}-M_{\text{DM}} \) relation (we notice that the scatter around the \( M_{*}-V_{\text{DM}} \) relation is expected to be lower than around the \( M_{*}-M_{\text{DM}} \) relation). We derive a non-parametric mean \( M_{*}-V_{\text{DM}} \) relationship by numerically deconvolving Equation (3) as described in appendix D from Paper II.

The second panel of Figure 1 shows our obtained SHAM relationship between \( M_{*} \) and \( V_{\text{DM}} \); see also Section 3 below. Similarly to previous works, our relationship increases steeply with \( V_{\text{DM}} \) for velocities below \( V_{\text{DM}} \sim 160 \) km/s and it is shallower at higher velocities. Finally, Figure 2 shows that, as expected, our resulting mock galaxy catalogue reproduces well the observed SDSS DR7 projected 2PCFs at different stellar mass bins from Yang et al. (2012). For the 2PCFs, we assumed the plane parallel approximation with an observer along the \( z \) direction. This is estimated by means of the Landy & Szalay (1993) estimator and we integrate over the line of sight from \( r_{w} = 0 \) to \( r_{w} = 40 \) h\(^{-1}\) Mpc.

Note that this scatter is composed by the physical (intrinsic) scatter and random errors in the stellar mass determination. Estimates of random errors are of the order of 0.1 dex (e.g., Tinker et al. 2017, Paper II). The above implies that our assumed intrinsic scatter is then 0.11 dex.

2.3 Morphologies

In this subsection, we describe how we assign early and late-type morphologies to the galaxies in our mock catalogue. Notice, however, that we assume that the scatter around the \( M_{*}-V_{\text{DM}} \) relation is independent of morphology, that is, the \( P(M_{*}|V_{\text{DM}}) \) distribution is assumed to be the same regardless of morphology. The reason for this is that in the literature there is not yet a consensus on whether the galaxy-halo connection depends on galaxy properties such as morphology, colour, or star formation rate. For example, some authors find that at a fixed halo mass the stellar masses of blue/star-forming galaxies are on average larger (e.g., More et al. 2011; Rodriguez-Puebla et al. 2015) than red/quenched galaxies, while other authors have found opposite results (e.g., Moster et al. 2018). Others simply do not find significant differences (Zu & Mandelbaum 2016; Behroozi et al. 2019). Nonetheless, all of them agree that at fixed stellar mass, red/quenched galaxies are more likely to reside in more massive haloes than blue/star-forming galaxies, consistent with weak lensing studies (e.g., Mandelbaum et al. 2006, 2016). Since colour/star formation rate correlate with galaxy morphology, similar trends are expected when morphology is used instead. In any case, assuming that \( P(M_{*}|V_{\text{DM}}) \) is independent of galaxy properties like morphology is actually irrelevant for our results since we are interested here in predictions of the H-I-stellar-(sub)halo connection for the whole galaxy population. However, the introduction of galaxy morphology is just a necessary step in order to assign H1 masses in our mock catalogue because the empirical \( M_{\text{HI}} \) distributions as a function of \( M_{*} \) that we use are given separated into ETGs and ETGs.

Morphologies are assigned based on the observational fractions of ETGs for centrals, \( f_{\text{cen}}^{\text{ETG}} \), and satellites, \( f_{\text{sat}}^{\text{ETG}} \), as a function of stellar mass. To do so, we first generate a random number \( U_{T} \) uniformly distributed between 0 and 1. Then, for a given galaxy
with mass \( M_* \), the morphology \( T \) is assigned as
\[
T = \begin{cases} 
\text{ETG} & \text{if } U_T < f_E(M_*), \\
\text{LTG} & \text{otherwise.} 
\end{cases}
\]

The index \( i \) refers either to a central or satellite galaxy. The fractions of ETGs for centrals and satellites were calculated from SDSS DR7 in Calette et al. (2021), based on the galaxy group catalogue from Yang et al. (2012), the photometric catalogues of Meert et al. (2015) and Meert et al. (2016), morphologies from Huertas-Company et al. (2011), and the volume corrections from Rodríguez-Puebla et al. (2020).

### 2.4 HI masses

At this point, every halo and subhalo in the SIMP simulation at the snapshot \( z = 0 \) has been assigned a stellar mass \( M_* \) and a galaxy morphology \( T \) depending on whether the galaxy is central or satellite. We use now these two galaxy properties to assign HI masses.

In Paper I we found that the \( M_{HI} \) (or \( R_{HI} \equiv M_{HI}/M_* \)) conditional PDFs given \( M_* \) for LTGs are well described by a Schechter function while ETGs are well described by a Schechter function plus a top-hat function. The function parameters for the \( R_{HI} \) distributions reported in Paper I were slightly improved in Paper II. As mentioned in the introduction, our determinations homogenize and integrate the observational data from various local galaxy samples, and, when necessary, allow us to estimate the \( R_{HI} \) distributions to values lower than the H I flux limits of some samples. From these distributions, we may calculate the logarithmic \( \log \langle R_{HI} \rangle \) relation and its scatter for LTGs, ETGs, and all galaxies, see solid lines surrounded by shaded bands in Figure 3. In general, our \( \log \langle R_{HI} \rangle \) relation for all galaxies is consistent with previous, more limited, determinations (see Paper I and Calette et al. 2021 for some comparisons; for the LTG and ETG relations there are not other works with which to compare). For example, the right lower panel of Figure 3 shows the \( \log \langle R_{HI} \rangle \) relation as presented in Catinella et al. (2018) for the xGASS survey. The largest differences are found at the high-mass end, where ETGs dominate. This is due to what is in Catinella et al. (2018), as in other previous studies, the H I mass of non-detections were set to their upper limit values, which leads to overestimating the means.

In other studies, the authors report arithmetic means instead of logarithmic ones or use the technique of stacking the H I spectra to increase the signal-to-noise ratio of non-detections; the stacking technique is equivalent to average arithmetically. As is well known, for distributions that are asymmetric, with a populated distribution on their low value side (as is the case with ETGs), these low values contribute little to the arithmetic mean in such a way that this mean is larger than the logarithmic one. From our full \( R_{HI} \) conditional PDFs, we can calculate also the arithmetic mean \( \langle R_{HI} \rangle \) relations. By comparing our \( \log \langle R_{HI} \rangle \) relations (dotted lines in the upper panels of Figure 3) with those presented in Brown et al. (2015) we find a reasonable agreement, as seen in Figure 3. These authors cross-match the ALFALFA and SDSS surveys and use the H I spectral stacking technique in \( M_* \) and colour bins. Their \( \langle R_{HI} \rangle \) relation for red galaxies agrees very well with our arithmetic mean \( \langle R_{HI} \rangle \) relation for ETGs, showing that the arithmetic means are highly skewed towards the high values of \( R_{HI} \), which correspond to detections. Since ALFALFA is biased towards H I gas-rich and blue galaxies, it is expected that the \( \langle R_{HI} \rangle \) relation for blue galaxies of Brown et al. (2015) should be above our relation for LTGs (note also that colour is not the same as morphology).

The \( R_{HI} \) conditional PDFs for LTGs and ETGs presented in Papers I and II do not separate between centrals and satellites. Sampling H I masses without taking into account centrals and satellites separately can be problematic as it will be equivalent to ignoring any environmental processes in satellite galaxies, such as strangulation and gas stripping. At the same time, the H I 2PCFs will be overestimated, specially at small distances, that is, at the 1-halo term. To overcome this problem, here we use the prescription presented in Calette et al. (2021) to obtain the \( R_{HI} \) conditional PDFs for LTGs and ETGs separated into central and satellite galaxies.

Briefly, in Calette et al. (2021) we used the xGASS survey (Catinella et al. 2018), which is a homogeneous sample that extends \( \text{xGASS} \) (Catinella et al. 2010, 2013) down to \( M_* \sim 10^9 \text{ M}_\odot \). The \( \text{xGASS} \) detection limits in \( M_{HI} \) are such that \( R_{HI} > 0.015 \) for \( M_* > 3 \times 10^{10} \text{ M}_\odot \) and increases as \( M_* \) decreases up to \( R_{HI} > 0.02 \) for \( M_* = 10^9 \text{ M}_\odot \). For non-detected galaxies in H I, the upper limits of \( M_{HI} \) are reported. Galaxies are classified as satellites or centrals using the SDSS DR7 Group Yang et al. (2007) catalogue and corrected for stripping (Janowiecki et al. 2017). To assign the morphology \( T \), we use the Huertas-Company et al. (2011) classification, mentioned above. In Calette et al. (2021) we applied Paper I and applied to the \( \text{xGASS} \) sample the upper-limit corrections and the statistical procedures described there. Thus, we obtained the \( R_{HI} \) conditional distributions given \( M_* \) for LTGs and ETGs, separated into centrals and satellites. Then, we applied a continuous and joint fit to all these distributions; for more details we refer the reader to Calette et al. (2021).
The fits to the $R_{\text{HI}}$ distributions for all LTGs and ETGs from xGASS, which is not separating into centrals and satellites, are roughly consistent with those from Paper I. However, there are three reasons why we cannot directly use the $M_{\text{HI}}$ distributions determined from xGASS only to separate into centrals and satellites: (1) xGASS extends down only to $M_* \sim 10^8 M_\odot$, (2) the GASS survey is just a subsample of all the compiled data set in Paper I, and (3) we want to be consistent with the distributions reported in Paper I and updated in Paper II. To consider the above and at the same time holding the differences in the $R_{\text{HI}}$ distributions of central and satellite found from xGASS, Calette et al. (2021) computed the $R_{\text{HI}} = M_{\text{HI}}/M_*$ distributions separated into centrals and satellites as follows:

$$P_i(> R_{\text{HI}}| M_*) = \frac{P_i(> R_{\text{HI}}| M_*)_{\text{xGASS}}}{P_i(> R_{\text{HI}}| M_*)_{\text{xGASS}}} \times P_i(> R_{\text{HI}}| M_*),$$

(6)

where $i$ refers to either LTG or ETG, $j$ to either central or satellite galaxy, and the sub-index xGASS refers to the respective distributions of xGASS galaxies as constrained in Calette et al. (2021). The distributions $P_i$ correspond to the ones reported in Paper II.

The ratio of the $R_{\text{HI}}$ distributions from xGASS in Eq. (6), should be considered as the factor needed to project the total $R_{\text{HI}}$ distributions of LTGs and ETGs into their corresponding distributions of centrals and satellite galaxies. The above warrants that our total distributions are consistent with the ones reported in Paper II. Since xGASS extends only down to $M_* \sim 10^8 M_\odot$, Calette et al. (2021) extrapolated to lower masses the fitted functions entering in the first term of the right side of Eq. (6), see for details that paper. The fourth row panels of Figure 1 show what the $R_{\text{HI}}$ conditional PDFs of centrals and satellites (both for LTGs and ETGs) look like as a function of stellar masses. Figure 3 shows the $(\log M_{\text{HI}}) - \log M_*$ relations and the standard deviations around them as calculated from the distributions from Paper II for LTGs, ETGs, and all galaxies (solid lines), as well as the corresponding mean relations for central and satellite galaxies after using Eq. (6) (dashed and dot-dashed lines, respectively).

To assign $M_\text{HI}$ masses to our mock central or satellite galaxies, we first generate a random number $U_{\text{HI}}$ uniformly distributed between 0 and 1. Then, for a given central or satellite galaxy with mass $M_*$ and morphology $T$, the $M_\text{HI}$ mass is given by solving the following equation for $R_{\text{HI}}$:

$$U_{\text{HI}} = P_i(> R_{\text{HI}}| M_*),$$

(7)

where $i$ refers to morphology $T$ (ETG or LTG), and $j$ to either central or satellite galaxy. Since the $M_\text{HI}$ masses are assigned following robust empirical $R_{\text{HI}}$ conditional PDFs given $M_*$ and the mock galaxy catalogue reproduces the observed GSF by construction (see above), then one expects that the catalogue should reproduce also the observed $M_\text{HI}$ MF (see Paper II). Figure 4 confirms this expectation. In this figure are also the H1 MFs of central and satellite galaxies separately. The former dominate by far the H1 gas content of galaxies at all masses. Notice that the downturn below $M_{\text{HI}} \sim 10^8 M_\odot$ both for all and central galaxies is due to the $M_{\text{HI}}$ completeness limit in our mock catalogue, conditioned by the completeness limit of $\sim 10^7 M_\odot$ in stellar mass (see Paper II).

Finally, our mock galaxy catalogue contains three properties both for central and satellite galaxies: stellar mass, $M_*$, morphology, $T$, and H1 mass, $M_{\text{HI}}$. In other words, we have implicitly determined the multivariate conditional distribution function $P(M_{\text{HI}}, T, M_*, | V_{\text{DM}})$ that describes the connection between haloes and galaxies. Due to the assumptions used in this paper the above multivariate distribution is $P(M_{\text{HI}}, T, M_*, | V_{\text{DM}}) = P(M_{\text{HI}}| T, M_*) P(M_*, | V_{\text{DM}})$.

2.5 ALFALFA sample biases

The goal of this paper is to extend the galaxy-(sub)halo connection to the H I component by means of the methodology described above. To check the robustness of our predicted results on the H I-(sub)halo connection it is key to compare the observed spatial clustering of galaxies as traced by their H I mass with the respective clustering measured in our mock catalogue.

To date, the largest survey in H I, where the projected 2PCF has been measured for several $M_{\text{HI}}$ thresholds, is the ALFALFA radio survey (Papastergis et al. 2013; Guo et al. 2017). However, a direct comparison between the resulting clustering of our mock galaxy catalogue and the ALFALFA survey is not trivial. To do so, we need to consider in our mock catalogue the biases that the ALFALFA survey introduces to the galaxy population. As discussed in the introduction, ALFALFA is a blind radio survey without any selection on target sample but its shallow $W_{500}$-dependent H I flux detection limit introduces strong biases in some galaxy properties of the surveyed population other than $M_{\text{HI}}$.

Here, in order to emulate the "selection function" of the ALFALFA survey, we use the $R_{\text{HI}}$-$M_*$ correlation reported in Maddox et al. (2015, see also Huang et al. 2012). We assume that this correlation is lognormal distributed, and we interpolate the median and the dispersion as a function of $M_*$ as tabulated in their Table 1. We define this distribution as $S(> R_{\text{HI}}| M_*)$. Thus, for a given galaxy from our mock catalogue with properties $M_*$ and $M_{\text{HI}}$, we generate a random number $U_S$ uniformly distributed between 0 and 1, and we determine whether the galaxy will be observed by the ALFALFA survey if $S(> R_{\text{HI}}| M_*) \geq U_S$, otherwise it is not observed by ALFALFA. Note that the above is independent of the galaxy morphology or whether the galaxy is central or satellite. As we will show below, applying the ALFALFA-like
selection function will decrease the fraction of ETGs, as expected. This criterion will be applied to our catalogue when comparing the HI spatial clustering with ALFALFA in §§3.2.

3 RESULTS

Section 2 describes how we generate a mock (sub)halo-galaxy catalogue in the 3D N-body simulation. By construction the mock catalogue follows the observed total GSMF as well as the empirical HI mass conditional PDFs of galaxies given their $M_*$. It is expected that the mock galaxy catalogue reproduces the observed stellar projected 2PCFs and the HI mass function. Indeed, in Figures 2 and 4 we showed that this is the case. Next, we present our results on the galaxy-(sub)halo connection including HI mass measured from our mock catalogue.

In the upper panels of Figure 5, we show the mean $\langle \log M_\star(V_{\text{DM}}) \rangle$ and $\langle \log M_\star(M_{\text{DM}}) \rangle$ for all galaxies as well as the respective standard deviations. Recall that the $M_\star-V_{\text{DM}}$ relation was assumed to be the same for central and satellite galaxies. However, in the case of the $M_\star-M_{\text{DM}}$ relation (right upper panel) satellite galaxies segregate from centrals towards lower subhalo masses for a given stellar masses (see e.g., Rodriguez-Puebla et al. 2012). As shown by these authors (see also Rodriguez-Puebla et al. 2013), SHAM reproduces the observed spatial clustering of galaxies only when the stellar-to-(sub)halo relations of centrals and satellites are different in the direction shown in Fig. 5. In our SHAM, this segregation emerges because $V_{\text{DM}}$ is the halo property assumed to correlate better with $M_\star$ instead of $M_{\text{DM}}$, and, as discussed in §§2.2, this form of SHAM is expected to generate results consistent with the spatial clustering of galaxies (see also, Reddick et al. 2013, and Dragomir et al. 2018). Finally, note that the scatter around the $M_\star-M_{\text{DM}}$ relation increased with respect to the $M_\star-V_{\text{DM}}$ relation, specially at low masses, where the segregation between centrals and satellites is large.

Once we have shown that our mock galaxy catalogue is quite realistic, we can proceed to explore the predictions from this catalogue for (i) the galaxy-(sub)halo connection for the HI gas content (§§3.1), and (ii) the HI projected 2PCFs for different $M_{\text{HI}}$ thresholds (§§3.2). These are the main goals of this paper.

3.1 The galaxy HI-(sub)halo connection

The left and right lower panels of Figure 5 show the full bivariate $R_{\text{HI}}-V_{\text{DM}}$ and $R_{\text{HI}}-M_{\text{DM}}$ distributions, $\Phi(R_{\text{HI}}|V_{\text{DM}})$ and $\Phi(R_{\text{HI}}|M_{\text{DM}})$, respectively. The colour isocontours cover a range of four orders of magnitudes. Note that the distributions at a given $V_{\text{DM}}$ or $M_{\text{DM}}$ are bimodal. This is inherited from the empirical $R_{\text{HI}}$ conditional distribution given $M_\star$. We also plot the logarithmic mean, $\langle \log M_{\text{HI}} \rangle$, and its corresponding standard deviation for all galaxies (black solid line and shaded region) as well as the logarithmic mean for central (violet dashed line) and satellite (red dashed-dotted line) galaxies. The insets show the values of the respective standard deviations as a function of $V_{\text{DM}}$ and $M_{\text{DM}}$. In dex. Tables A1 and A2 in Appendix A provide the numerical values of, respectively, $\langle \log M_{\text{HI}}(V_{\text{DM}}) \rangle$ and $\langle \log M_{\text{HI}}(M_{\text{DM}}) \rangle$ along with their scatters.

On average, $M_{\text{HI}}$ for all galaxies increases roughly as $M_{\text{HI}} \propto V_{\text{DM}}^3$ or $M_{\text{HI}} \propto M_{\text{DM}}$ up to a peak at $V_{\text{DM}} \approx 160$ km/s or $M_{\text{DM}} \approx 10^{12}$ $M_\odot$, respectively; these trends are dominated by central galaxies, while satellite galaxies follow roughly similar trends but with lower values of $M_{\text{HI}}$. For large values of both $V_{\text{DM}}$ and $M_{\text{DM}}$, the correlation becomes very weak. If any, $(\log M_{\text{HI}})$ slightly decreases while the scatter strongly increases. The decrease of $(\log M_{\text{HI}})$ is mainly due to centrals, while for satellites the mean relation keeps constant. At the largest velocities or masses, $(\log M_{\text{HI}})$ is constant with $V_{\text{DM}}$ or $M_{\text{DM}}$. Our results show that the $M_{\text{HI}}-V_{\text{DM}}$ and $M_{\text{HI}}-M_{\text{DM}}$ relations are neither monotonically increasing nor tight. In §§4.1, we discuss the implications of the above for the traditional form of SHAM, and in §§4.3 we compare our results with those from semi-analytical models and hydrodynamics simulations, some of which have predicted that the $M_{\text{HI}}-M_{\text{DM}}$ relation is non monotonic. In particular, the change of behavior of this relation at $M_\star \sim 10^{12}$ $M_\odot$ is linked to the onset of AGN feedback (Kim et al. 2017; Baugh et al. 2019; Chauhan et al. 2020).

The stellar-HI-(sub)halo connection found here can be understood as the combination of the stellar mass conditional distribution given $V_{\text{DM}}$, $P(M_\star|V_{\text{DM}})$, and the empirical HI mass conditional distribution given $M_\star$, $P(M_{\text{HI}}|M_\star)$. The former was assumed to be independent of galaxy morphology and environment, while the latter strongly segregates by galaxy morphology and weakly by environment (defined by their central or satellite nature, see Fig. 3 and Calette et al. 2021). By analysing the two distributions mentioned above, we can interpret the results shown in Figure 5. For $V_{\text{DM}} \lesssim 160$ km/s or $M_{\text{DM}} \lesssim 10^{12}$ $M_\odot$, the corresponding stellar masses are below $M_\star \sim 10^{10}$ $M_\odot$. The galaxy population at these masses is completely dominated by LTGs, see Section 3.2 (also Figure A1 in Calette et al. 2021). For these galaxies, as seen in Figure 3, on average $R_{\text{HI}} \propto M_\star^{-0.4}$, that is $M_{\text{HI}} \propto M_\star^{0.4}$. On the other hand, from Figure 5, we see that $M_\star \propto V_{\text{DM}}$ at low velocities. Therefore, roughly $M_{\text{HI}} \propto V_{\text{DM}}^{3/4}$, which is close to what it is seen in Fig. 5 for $V_{\text{DM}} \lesssim 160$ km/s. For $M_\star > 10^{10}$ $M_\odot$, on average $R_{\text{HI}}$ for all galaxies strongly decreases with $M_\star$ due to the increasing fraction of ETGs as $M_\star$ increases, which have much lower values of $R_{\text{HI}}$ than LTGs at fixed $M_\star$, see Fig. 5 in Rodriguez-Puebla et al. (2020). The inferred dependence is $R_{\text{HI}} \propto M_\star^{-1.3}$ or $M_{\text{HI}} \propto M_\star^{-0.2}$, combined with $M_\star \propto V_{\text{DM}}^{1.7}$ for large velocities (Fig. 5), we expect $M_{\text{HI}} \propto V_{\text{DM}}^{0.3}$ in rough agreement with what it is seen in Figure 5 for $V_{\text{DM}} \gtrsim 160$ km/s. For the largest velocities, the total $R_{\text{HI}}-M_\star$ relation flattens (see Fig. 5 in Rodriguez-Puebla et al. 2020), hence the total $M_{\text{HI}}-V_{\text{DM}}$ relation flattens, too.

The scatter distribution around the total $M_{\text{HI}}-V_{\text{DM}}$ relation is large. For $V_{\text{DM}} \lesssim 160$ km/s, the standard deviations increase from $\sim 0.5$ to 0.9 dex. For central galaxies, the scatter is lower, while the opposite applies for satellites. For larger velocities, the standard deviation increases up to 1.2 dex and at the largest $V_{\text{DM}}$ values it slightly decreases. A large scatter is expected due to the broadness of the total $M_{\text{HI}}$ conditional PDFs at the stellar masses corresponding to these velocities. The total PDFs are broad because of (i) the strong difference between LTGs and ETGs in the distributions of HI masses given $M_\star$, see Fig. 3, and (ii) the large scatter around the mean $R_{\text{HI}}-M_\star$ relations of LTGs and ETGs, as seen in Fig. 3. An expected consequence of the former is that the scatter around the mean $M_{\text{HI}}-V_{\text{DM}}$ relation at the $\sim 160-400$ km/s range should be significantly segregated by morphology, with LTGs lying above the mean relation and ETGs below it. In a forthcoming paper we will present these results for the LTG and ETG populations separately, and under different assumptions regarding the covariance of the
scatter around the $V_{DM}$-$M_*$ relation with morphology or colour. According to Fig. 5, around the $M_{HI}$-$V_{DM}$ relation there is also a (weak) segregation by central and satellite galaxies. Up to $\sim 400$ km/s, the former lies on average slightly above the mean $M_{HI}$-$V_{DM}$ relation and the latter below it. However, since this segregation is small, it is not expected to be a primary source of the large scatter around the $M_{HI}$-$V_{DM}$ relation.

It is worth mentioning that the scatter around the $M_{HI}$-$V_{DM}$ relation is not only large but also with a highly asymmetric distribution, and with even a bimodality at $V_{DM} > 160$ km/s as seen in Figure 5. As a result of the broad asymmetric distribution, different statistical estimators taken as representative of the population will significantly differ among them. Figure 6 shows our predicted galaxy HI-(sub)halo connection using the geometrical mean ($log(M_{HI})$) (as in Fig. 5, black line), the arithmetic mean ($\langle M_{HI} \rangle$) (blue line), and the median $\bar{M}_{HI}$ (red line). Up to $V_{DM} \sim 160$ km/s, where the scatter is relatively small and symmetric, the differences among the three statistical estimators are relatively small. For higher (sub)halo velocities, since the distribution of $M_{HI}$ presents a long tail towards low velocities, the arithmetic mean results much higher than the logarithmic mean and the median. The median differs from the geometric mean due to the increasing relevance at high velocities of the second peak in the $M_{HI}$ distribution at its low-velocity end.

The explanations given above about the shape of the mean logarithmic $M_{HI}$-$V_{DM}$ relation and its scatter apply also to the shape of the mean logarithmic $M_{HI}$-$M_{DM}$ relation and its scatter (lower right panel of Fig. 5). The difference is that in the latter case, instead of $P(M_{HI}|V_{DM})$, we have $P(M_{HI}|M_{DM})$, which segregates by environment (see upper right panel of Fig. 5). The segregation of the $M_{HI}$-$M_{DM}$ relation between centrals and satellites produces a weaker segregation of the $M_{HI}$-$M_{DM}$ relation between centrals and satellites than in the $M_{HI}$-$V_{DM}$ relation due to a compensation.
Figure 6. The galaxy H I (sub)halo connection using three different statistical estimators: logarithmic mean, arithmetic mean, and median as labelled. The relations are different because the join distributions are asymmetric and even bimodal (see Fig. 5).

Figure 7. Logarithmic mean of the $M_{HI}$-to-$M_*$ ratio as a function of (sub)halo mass, $M_{DM}$, for all galaxies (black solid line) as well as for only central (violet dashed line) and satellite (red dashed-dotted line) galaxies. The standard deviation of the relations for central and satellite galaxies are shown with the shaded regions.

Effect. Observations show that satellites have on average smaller H I masses at a given $M_*$ than centrals (see Fig. 3, and Calette et al. 2021) but at a given $M_{DM}$, satellites have on average larger $M_*$ than centrals. Hence, the difference in $M_{HI}$ at a given $M_{DM}$ between satellites and centrals becomes smaller than in the case of a given $V_{DM}$. Distinct halos contain on average more H I gas than subhalos of similar masses in the mass range of $\sim 10^{13} - 10^{13} M_\odot$, but the differences are small. In Figure 7 we plot the inferred $R_{HI-DM}$ relation for all galaxies along together with the relations of central and satellite galaxies. Note that the scatter around the mean H I-to-halo mass relation of satellites galaxies is larger than for centrals, specially at $M_{DM} \lesssim 10^{12} M_\odot$.

3.1.1 The total galactic H I gas content inside distinct haloes

Based on optical galaxy group catalogues for determining halo (group) masses, some authors have recently presented observational estimates of the total H I mass within these haloes, that is, the sum of H I masses of the central and satellite galaxies in the halo, $M_{HI}^{cen} + M_{HI}^{sats}$ (Ai & Zhu 2018; Obuljen et al. 2019; Guo et al. 2020; Tramonte & Ma 2020; Lu et al. 2020). The upper panel of Figure 8 shows the mean ($\langle M_{HI} \rangle$) and standard deviation$^6$ as a function of $M_*$ as measured from our mock catalogue. The mean for central galaxies, ($\langle M_{HI}^{cen} \rangle$), and the sum of all satellite H I masses within a given distinct halo, ($\langle M_{HI}^{sats} \rangle$), are also plotted with dashed and dot-dashed lines, respectively. The two sets of lines are for two minimum H I masses used to account for galaxies within the distinct halo, $M_{HI,min} = 10^7 M_\odot$, black lines, and $M_{HI,min} = 10^6$, red lines. Columns (8)-(9) of Table A2 in Appendix A present the respective numerical values for the case $M_{HI,min} = 10^8 M_\odot$.

Figure 8 shows that the dependence of $M_{HI}^{cen}$ on $M_*$ cannot be described by a double power law. In the $10^{12} \lesssim M_*/M_\odot \lesssim 10^{13}$ regime the relation flattens. This is because for haloes up to $\sim 2 \times 10^{12} M_\odot$, central galaxies dominate by far the H I gas content in the halos for which the relation flattens at these masses. For larger masses, the contribution of satellites increases and at $M_* \sim 10^{14} M_\odot$ the combined H I gas of satellites is on average $\sim 1$ dex larger than in the central galaxy. In the lower panel of Figure 8 we show the same as in the upper panel but for stellar mass. Two minimum stellar masses were used to sum up satellites, $10^7$ and $10^8 M_\odot$ (the data corresponding to the former limit are presented in columns 10–11 of Table A2 in Appendix A). The differences between both cases are negligible. Unlike $M_{HI}$, the dependence of $M_*$ on $M_*$ can be described by a double-power law. This is mainly because the dependence of $M_*$ on $M_*$ for centrals does not flattens or decreases as it is the case for $M_{HI}$.

We compare our semi-empirical inferences with previous ones in Figure 9. In this figure we show with isocountours the full bivariate $M_{HI}^{cen}$ and $M_*$ distribution. The total H I mass estimates using stacked H I spectra for galaxy groups by Guo et al. (2020) agree with our findings up to $M_* \sim 10^{13} M_\odot$ (we have transformed their halo masses defined at the 200$\rho_c$ radius to our virial masses). For larger halo masses, their total H I masses are lower than our determinations. As seen in Figure 9 the above is mainly due to the lower total H I mass they find for satellites, $M_{HI}^{sats}$, while the agreement with the H I mass of centrals at all halo masses is encouraging. The low values of $M_{HI}^{sats}$ may be due to the SDSS group catalogue not sampling the high halo mass end with enough statistics, due to the too low 21-cm line fluxes of many satellites as to contribute to the H I stacked spectra above the allowed instrumental signal-to-noise ratio or due to the use of a too narrow window in the allowed H I velocity in such a way that not all satellites are captured (Chauhan et al. 2021). On the other hand, the determinations for massive groups in Obuljen et al. (2019), roughly agree with our results at the high halo mass range. Unlike Guo et al. (2020), Obuljen et al. (2019) do not directly measure the H I content of haloes, but instead use empirical relations to derive it. To determine $M_{HI}^{sats}$ as a function of group (halo) mass they integrate the ALFALFA galaxy H I MFs at different group masses.

3.2 The H I galaxy spatial clustering

We have discussed already that when implementing the SHAM for $V_{DM}$ instead of $M_{DM}$, the projected 2PCFs, $\omega(r)$, from the SDSS DR7 galaxies in stellar mass bins are recovered, see Figure 2. Next,
we present our predictions for the projected 2PCFs as a function of HI mass.

Figure 10 shows the resulting HI projected 2PCFs, $\omega_p^{HI}(r)$, for various $M_{HI}$ thresholds from our mock galaxy catalogue. In general, the amplitude of $\omega_p^{HI}(r)$ is nearly independent of the $M_{HI}$ threshold. In more detail, however, we notice that the amplitude of the one-halo term increases with $M_{HI}$ while the two-halo term is almost independent of $M_{HI}$, and in consequence, the resulting projected 2PCFs are not power laws. The above can be broadly understood as the result of the differences between the $M_{HI} - V_{DM}$ (or $M_{HI} - M_{DM}$) relations of centrals and satellites and the flattening of both relations with similar values of the mean (log $M_{HI}$) at high halo masses. Note that for low $M_{HI}$ thresholds central galaxies contribute more to $\omega_p^{HI}(r)$ than satellites but for high $M_{HI}$ thresholds, centrals and satellites contribute equally to $\omega_p^{HI}(r)$. This behaviour of $\omega_p^{HI}(r)$ is very different to the one of $\omega_p^{HI}(r)$, as $\omega_p^{HI}(r)$ depends strongly on stellar mass.

There are other features related to the HI projected 2PCFs that are worth mentioning. We begin by emphasizing the fact that the 2PCFs mass thresholds used in Fig. 10 are usually above the mean $M_{HI} = V_{DM}$ and $M_{HI} = M_{DM}$ relations, see Fig. 5. In other words, the larger the HI mass threshold, the larger the distance from, for example, the (log $M_{HI}$) values. The above is interesting for various reasons. First, increasing the HI mass threshold implies that we are sampling increasingly rarer masses for a given $V_{DM}$ or $M_{DM}$. Secondly, similar mass thresholds have been used in the past to derive the HI galaxy-halo connection (e.g., Guo et al. 2017). Recall that HOD (and related) models assume that galaxy properties, in this case $M_{HI}$, are totally determined by a halo property such as $V_{max}$ or $M_h$. The lower panels of Figure 5 show that this is not the case. In view of the above, it is thus relevant to ask whether HOD (and similar) models are appropriate to constrain the real HI-to-halo mass relation. We will come back to this point in Section 4.

Figure 11 shows again the HI projected 2PCFs, $\omega_p^{HI}(r)$, for different $M_{HI}$ thresholds as indicated in the panels. The thick red lines correspond to the measurements from our mock catalogue. One of the key aspects in our mock catalogue is that we allowed the HI mass conditional distributions for a given $M_*$, both for LTGs

Figure 8. Mean arithmetic total HI mass (top panel) and total stellar mass (bottom panel) within a halo as a function of its virial mass (solid lines). The dashed and dot-dashed lines show the respective HI and stellar masses contained only in the central galaxy and the sum of all satellites within the halo, respectively. Results for two threshold HI/stellar masses are shown (see labels in the panels).

Figure 9. Same as top panel of Figure 8, but now we compare our results for $M_{HI}^{lim} = 10^8$ M$_\odot$ with Guo et al. (2020) and Obuljen et al. (2019) observational inferences. Note that Guo et al. (2020) inferred not only the total HI content within haloes but also the contributions from the central. The isocountours correspond to the bivariate $M_{HI}^{tot}$ and $M_h$ distribution.

Figure 10. Projected HI 2PCFs from the galaxy mock catalogue measured above four $M_{HI}$ thresholds as indicated by the labels.

$H_I$–(sub)halo connection and $HI$ clustering

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Figure 11 shows again the HI projected 2PCFs, $\omega_p^{HI}(r)$, for different $M_{HI}$ thresholds as indicated in the panels. The thick red lines correspond to the measurements from our mock catalogue. One of the key aspects in our mock catalogue is that we allowed the HI mass conditional distributions for a given $M_*$, both for LTGs
and ETGs, to be different for centrals and satellites as observations show, see Section 2.4. The black dashed lines in Figure 11 show the resulting $\omega_p^\mathrm{HI}(r)$ functions when assuming that the H I distributions are the same for centrals and satellites, that is, the ratio in Eq. (6) is set equal to 1. The fact that satellite galaxies have lower H I gas fractions than centrals clearly works in the direction of lowering $\omega_p^\mathrm{HI}(r)$ at small distances, i.e. at the 1-halo term.

In Figure 11, we compare our results to the corresponding $\omega_p^\mathrm{HI}(r)$ functions from the ALFALFA $\alpha.40$ survey (Haynes et al. 2011) as measured by Papastergis et al. (2013, pentagons) and from the ALFALFA $\alpha.70$ survey as measured by Guo et al. (2017, open circles). Beyond the differences between both studies, it is evident that our predicted H I 2PCFs are above both studies. At this point, it is important to recall that while the ALFALFA survey is blind in radio, it actually suffers from biases in galaxy properties different to $M_{\mathrm{HI}}$, see introduction. The main reason for these biases is related to the low detection limit in H I flux and its dependence on the H I line width (Haynes et al. 2011; Huang et al. 2012; Chauhan et al. 2019). As a result, the $R_{\mathrm{HI}} - M_*$ relation of ALFALFA is biased towards H I-rich galaxies, avoiding galaxies with intermediate to low values of $R_{\mathrm{HI}}$ (Huang et al. 2012; Maddox et al. 2015; see Figure 6 of Rodríguez-Puebla et al. 2020). H I-poor galaxies tend to be earlier types and reside in denser environments, hence, they are expected to be more clustered than H I-rich galaxies. Therefore, the observed low clustering of ALFALFA observations as compared to our predictions is most likely due to the lack of H I-poor highly clustered galaxies in the ALFALFA survey. The $\omega_p^\mathrm{HI}(r)$ functions from our mock galaxy catalogue showed in Figure 11 should be taken as predictions to compare with what will be measured in future deep and large H I surveys, not significantly affected by sample biases.

Section 2.5 describes a simple way to emulate the non-trivial selection effects of the ALFALFA survey. The violet lines in Figure 11 show the resulting $\omega_p^\mathrm{HI}(r)$ functions from our mock catalogue after applying this ALFALFA-like selection. The functions approximate those reported from ALFALFA, specially for the low $M_{\mathrm{HI}}$ threshold. Nonetheless, observations still show slightly lower amplitudes than our predictions.

By imposing the ALFALFA-like selection effect in our mock catalogue, the fraction of ETGs is decreased as expected, but when comparing our fraction of ETGs as a function of $M_*$ with ALFALFA, the latter shows lower ETG fractions even at high masses. Figure 12 shows the above comparison. This is because massive ETGs have both low H I content and large $W_{50}$ equivalent widths both of which act against detection in this and other H I blind surveys (see Obreschkow et al. 2013; Chauhan et al. 2019). Moreover, the volume of ALFALFA is relatively small, so massive galaxies, typically ETGs, are underrepresented due to cosmic variance. In order to explore an extreme case, in addition to the ALFALFA-like selection effect, we assume that ALFALFA observes LTGs only. The resulting measured $\omega_p^\mathrm{HI}(r)$ are plotted in Figure 11 with blue dotted lines. They are almost indistinguishable from the previous case.

Our analysis shows that while the selection effects on properties different to $M_{\mathrm{HI}}$, introduced by the shallowness of a blind H I survey, like ALFALFA, is not crucial for measuring the H I MF (our

![Figure 11](https://example.com/figure11.png) Projected H I 2PCFs measured from different $M_{\mathrm{HI}}$ thresholds in the galaxy mock catalogue. The solid red line is our prediction for all local galaxies. This is the H I spatial clustering expected from a large volume and deep H I blind survey. The black dashed line shows the projected 2PCFs from our approach but assuming the same H I gas content for central and satellite galaxies. The violet solid line corresponds to the projected 2PCFs measured in the catalogue after emulating the ALFALFA survey selection effect (see text), while the blue dotted line is when the condition of not taking into account ETGs is also included. The symbols with error bars correspond to observational measurements from ALFALFA, see labels in the right-hand panel.

![Figure 12](https://example.com/figure12.png) Fraction of ETGs as a function of $M_*$ as measured in the complete galaxy mock catalogue (solid line) and after imposing the ALFALFA-like selection (dashed line). The corresponding fractions measured in the ALFALFA survey in many stellar mass bins are showed with circles and error bars.
HI MF is consistent with ALFALFA, see Fig. 4), these selection effects become critical for measuring the HI spatial clustering (as well as for the H1-to-stellar mass correlation, see Papers I and II). In §§4.2 we discuss the caveats of our approach and how, by overcoming them, we could improve our comparison with the galaxy HI clustering from ALFALFA.

4 DISCUSSION

4.1 HI mass is not determined by the halo scale: implications for SHAM and HOD

The assumption that the halo mass/maximum circular velocity determines the properties of the galaxies is central to galaxy-halo models such as the HOD and SHAM. Previous works have shown that, this is a good approximation for galaxy properties such as luminosity and stellar mass (for a discussion, and references see Dragomir et al. 2018). As discussed in Section 3.2, when using $M_{HI}$, the central assumption of SHAM or HOD models is clearly flawed as can be seen in the lower panels of Fig. 5. Thus expecting to derive a $M_{HI}$–$V_{DM}$ (or $M_{HI}$–$M_{DM}$) relationship from these models under the assumptions above is fruitless. In particular, it is clear that, by construction, SHAM will tend to predict an increasingly monotonic mean relation between $M_{HI}$ and $V_{DM}$ (or $M_{DM}$); a relation that will fail to reproduce the observed clustering from HI-surveys as discussed in §§3.2. Here, we elaborate more on this argument by deriving some $M_{HI}$–$V_{DM}$ relations from SHAM and studying their resulting clustering in HI mass.

Figure 13 presents three different $M_{HI}$–$V_{DM}$ relations obtained by means of SHAM (see Section 2.2) using the empirical HI MF from Rodríguez-Puebla et al. (2020). We assume that the scatter around the mean logarithmic $M_{HI}$–$V_{DM}$ relation is normally distributed and use three different values for the scatter: $\sigma = 0.15$ (blue line), $0.40$ (green line), and $0.60$ (cyan line) dex. The black solid line and the shaded area reproduce our predicted mean $M_{HI}$–$V_{DM}$ relation and its scatter as shown in Fig. 5. It is clear that SHAM predicts that the average relations increase monotonically with $V_{DM}$. Notice, however, that by increasing the scatter, the SHAM results approach our $M_{HI}$–$V_{DM}$ relation. Nonetheless, by construction, SHAM will always introduce a monotonic correlation between $M_{HI}$ and $V_{DM}$ or $M_{DM}$, regardless of the scatter assumed.

Figure 14 shows the resulting HI clustering based on our SHAM experiments. We note that there are two potential flaws that are evident for SHAM. First, the fact that SHAM does not separate between centrals and satellites. As discussed in Section 2.4, this assumption is problematic as it is similar to ignoring environmental effects for satellite galaxies, so it is not surprising that the one halo-term is overestimated. Secondly, since SHAM introduces a monotonic correlation between $M_{HI}$ and $V_{DM}$, the amplitude of the two-halo term will always increase with $M_{HI}$, thus overestimating the two-halo term. Note that as we decrease the scatter, the tighter and stronger the correlation with $V_{DM}$ is, and thus the larger the amplitude of the 2PCFs. A similar result has been reported in Guo et al. (2017) and Stiskalek et al. (2021).

The left-hand panel of Figure 15 is as Figure 13 but for the halo mass $M_{DM}$. We used the tight relation between $V_{DM}$ and $M_{H}$ as measured in N-body simulations (Rodríguez-Puebla et al. 2016) to convert the $M_{HI}$–$V_{DM}$ to $M_{HI}$–$M_{H}$. We show in this plot a previous determination by Padmanabhan & Kulkarni (2017, green solid line) of the H1-to-halo mass relation with SHAM using the observed HI Mass Function. The SHAM implemented by these authors did not include scatter; this is why their H1-to-halo mass relation looks similar to our case with small scatter, blue line. As can be seen, this relation is far from what we actually found with our semi-empirical model.

In the past, some authors used in different ways the HI spatial clustering information provided by current blind HI surveys for constraining the galaxy H1-halo connection by means of HOD or SHAM models (e.g., Padmanabhan et al. 2017; Guo et al. 2017; Obuljen et al. 2019). As discussed in §§3.2, the HI projected 2PCFs are measured for galaxies with masses larger than a given $M_{HI}$ threshold, and the values of this threshold are typically above the mean H1-to-halo mass relation as determined here, implying that the measured 2PCFs correspond to a biased galaxy population. On the other hand, it is well known that HI blind surveys are strongly biased to gas-rich, blue galaxies (see introduction). Both shortcomings are related because the minimum H1 mass threshold is determined by the HI mass completeness limit of the survey, and the latter is partially related to the HI survey sensitivity, which introduces a bias against gas-poor, red galaxies (typically ETGs).

In Figure 15 we show the $M_{HI}$–$M_{DM}$ relation (left-hand panel) and its inverse, the $M_{DM}$–$M_{HI}$ relation (right-hand panel), as measured from our catalogue when imposing the ALFALFA-like selection, red dashed lines. Recall that with this selection, the $\omega_{p}^{H1}(r)$ function with a threshold of $M_{HI} = 10^{9} M_{\odot}$ resulted in a reasonably agreement with the ALFALFA measurements (Fig. 11). For comparison, the respective relations along with their standard deviations as measured from our galaxy mock catalogue without imposing any selection criterion, are also plotted in Figure 15. An implication of this figure and Fig. 11 is that by (roughly) reproducing the ALFALFA H1 spatial clustering, the galaxy H1-halo connection is biased to high $M_{HI}$ values at a given halo mass or to low halo masses at a given $M_{HI}$. In the latter case, the difference would be larger if the ALFALFA H1 clustering would be better reproduced.
by assuming that the scatter around the $M_*-V_{DM}$ relation depends on morphology (see §§4.2.3 below).

In the right-hand panel of Figure 15, the results from the SHAM model of Guo et al. (2017) for central galaxies are reproduced. The inverse $M_{HI}-M_{DM}$ relation in our case refers to haloes and subhaloes (central and satellite galaxies); however, we note that for distinct haloes (centrals) the results remain similar. Guo et al. (2017) found that their simple SHAM model predicts much higher H-I clustering than the measurements from ALFALFA. Then, they extended the model by introducing other halo parameters such as the halo formation time. When using the 2PCFs from ALFALFA to constrain their model, the authors found that H-I galaxies are more likely to reside in late-forming haloes (assembly bias).7 When applying the ALFALFA-like selection function to our mock catalogue, our $M_{DM}-M_{HI}$ relation (red dashed line) is more consistent with the one from the Guo et al. (2017) model, although our relation is slightly above it. Recall that the Guo et al. (2017) model matches the ALFALFA H-I clustering by construction. In section 4.2.3, we discuss potential modifications to our model that could help to reproduce better the ALFALFA H-I clustering.

Finally, the left-hand panel of Figure 15 presents the results based on the more physically motivated inferences of H-I mass from Popping et al. (2015) for star-forming galaxies. In order to infer H-I gas mass, the authors used the star formation histories from SHAM and the inverted star formation rate–surface density relations to infer galaxy HI masses. It is interesting how our $M_{HI}-M_{DM}$ relation for gas-rich galaxies (due to the imposed ALFALFA-like selection, on the H-I clustering explicitly as we do. It is likely that these effects are outweighed by the formation-time halo selection.

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7 Guo et al. (2017) have also applied to their mock catalogue the $W_{50}$-dependent flux detection limit of ALFALFA but did not present their effects.
4.2 Caveats and the galaxy HI clustering of ALFALFA

4.2.1 Morphological classification

In this paper we use the observed fractions of ETG for centrals and satellites based on the automatic morphological classification from Huertas-Company et al. (2011). As discussed in Section 2.3, we introduce galaxy morphology just as a necessary step in order to assign HI masses to our galaxies. Thus, a potential concern is the effects of using alternative morphological classifications in our methodology. To study the above, we use an alternative morphological classification for SDSS galaxies.

Based also on SDSS, Domínguez Sánchez et al. (2018) presented a new automatic morphological classification of galaxies. As shown in Calette et al. (2021), the Domínguez Sánchez et al. (2018) classification implies a higher fraction of both central and satellite ETGs than Huertas-Company et al. (2011) up to $M_\odot \sim 10^{11}$. When we use the fractions from Domínguez Sánchez et al. (2018) to assign HI masses, we note that, as expected, some of the results for the HI-halo connection change. Figure 16 shows the logarithmic mean $M_{\text{HI}}$--$M_{\text{DM}}$ relation for all galaxies and for centrals and satellites using both the Huertas-Company et al. (2011) (same line code as in Fig. 5) and Domínguez Sánchez et al. (2018, the lower extreme of the shades areas) morphological classifications. For the latter classification, the values of $\langle \log M_{\text{HI}} \rangle$ are lower than for the former classification, in particular for satellite galaxies. This is expected because, as mentioned above, Domínguez Sánchez et al. (2018) classify a larger fraction of ETGs, specially in the case of satellites, than Huertas-Company et al. (2011), and ETGs contain less HI than LTGs of the same stellar mass. The shaded areas in this figure can be considered as systematic in our inferences due to the uncertainty in the galaxy morphological classification. Similarly, in Figure 17, we plot the HI projected 2PCFs from our mock catalogue using both morphological classifications. Inferences using Huertas-Company et al. (2011) and Domínguez Sánchez et al. (2018) are plotted, respectively, with solid red and dashed cherry lines. The differences seen are actually smaller than those found when applying the ALFALFA selection, and hence they would be considered as second order effect. We conclude that our inferences may depend on the assumed morphological classification—as well as on the separation criterion for defining LTG and ETGs— but the main trends and conclusions are unaffected by this.

4.2.2 Errors in membership and central/satellite designation

As discussed in Section 2.4, we assign HI masses to halos and subhalos identified in the simulation by using the $R_{\text{HI}}$ conditional distributions derived in Paper II and separated into centrals and satellites following the prescription from Calette et al. (2021). For such prescription, in that paper we used the xGASS (Catinella et al. 2018) survey and the central/satellite designation tabulated for each galaxy. As shown in Campbell et al. (2015, see also Bravo-Alfaro et al. 2000), group finders may suffer from membership allocation and central/satellite designation errors. Therefore, a potential concern is the impact of errors from group finders in the central/satellite $R_{\text{HI}}$ conditional distributions reported in Calette et al. (2021). As is described in Campbell et al. (2015), errors from group finders tend to smooth out the differences between centrals and satellites, which leaves the possibility that the differences between their conditional distributions are larger to what was reported in Calette et al. (2021). We notice, however, that the xGASS team improved the Yang et al. (2007) group membership by visually inspecting false pairs and galaxy shredding (see Janowiecki et al. 2017). Thus, we expect that the Calette et al. (2021) H I conditional distributions will be only marginally affected by errors in central/satellite designation.

4.2.3 The comparison to the ALFALFA H I clustering

In Section 3.2, we showed that our mock galaxy catalogue tends to overpredict the observed H I clustering from the ALFALFA survey, even after imposing an ALFALFA-like selection effect and in the extreme case of assuming that galaxies in this survey consist only of late-types. Thus, a potential concern is in our method when assigning morphology to the galaxies seeded in the simulation haloes. As explained in Section 2.3, we assumed that the scatter around the $M_*-V_{\text{DM}}$ relation is independent of galaxy morphology. We are aware that this assumption implies that clustering properties of ETGs and LTGs are almost identical, at least for the two-halo term. Thus, by construction one expects that our mock catalogue will not recover the well known clustering properties of ETGs and LTGs as a function of stellar mass. This shortcoming in combination with the ALFALFA selection effects may work in the direction of overpredicting the H I clustering. There are at least three options that could help solve the above issue:

(i) The scatter around the $M_*-V_{\text{DM}}$ relation depends on morphology. In this assumption, ETGs and LTGs occupy haloes of different masses. In particular, if there is a strong correlation between halo and morphology in which haloes of ETGs are more massive than those of LTGs at fixed $M_*$, then the clustering of ETGs will...
be stronger due to the halo bias (see e.g., Rodríguez-Puebla et al. 2015; Correa & Schaye 2020).

(ii) Assembly bias. Halo masses of ETGs and LTGs could be identical but their properties are different. If galaxy morphology strongly correlates with halo’s assembly time, such as formation time, then the clustering of ETGs will be stronger due to the halo assembly bias (see e.g., Hearin & Watson 2013; Correa & Schaye 2020).

(iii) A combination of both.

Studying the above options is beyond the scope of this paper, in which we have focused on studying the galaxy population as a whole. While we will study in more detail the dependence of the galaxy-subhalo connection with galaxy morphology in a forthcoming paper, this is actually less important as here any dependence will be wash-out (averaged) when studying the whole population.

Finally, the $\omega_p^{HI}(r)$ functions for all galaxies measured in future large and deep H I surveys, which will not be significantly affected by selection effects, are expected to be similar to those reported in Section 3.2.

4.3 Comparisons to theoretical predictions

The new generation of SAM and cosmological hydrodynamics simulations, after post-processing, are able to predict the total galaxy H I gas content associated to haloes. In Figure 18 we show the median of the total H I mass, $\overline{M_{HI}}(r)$, as a function of $M_h$ for several SAMs (Kim et al. 2017; Baugh et al. 2019; Spinelli et al. 2020; Chauhan et al. 2020) and the Illustris-TNG100 simulation (Stevens et al. 2019; Chauhan et al. 2020). We have homogenized the halo masses to the virial mass. Our semi-empirical inference for the median $\overline{M_{HI}}-M_h$ relation is showed with the red thick line.

Our results show a relatively smooth increasing of $\overline{M_{HI}}$ with $M_h$ in the $3 \times 10^{12} - 10^{13} \ M_\odot$ mass range, similar to what is predicted by the GAEA SAM (Spinelli et al. 2020) and the Illustris-TNG100 simulation, but in tension with other SAMs: an old version of GALFORM (Kim et al. 2017), its new version (Baugh et al. 2019), and SHARK (Chauhan et al. 2020). The decrease of $\overline{M_{HI}}$ predicted by these models at the halo masses where the central dominates by far in the total H I content of haloes ($M_h \lesssim 2 \times 10^{12} \ M_\odot$), is mainly associated with the AGN feedback. The feedback, mainly mechanical, keeps the gas in the halo hot, preventing it from accreting into the galaxy. The major difference is with GALFORM (Baugh et al. 2019), even though we find excellent agreement with their total (central+satellites) stellar mass in haloes, see lower panel of Figure 18. The AGN feedback strength affects more drastically the H I gas content of galaxies than stellar masses in SAMs. Thus, the H I gas fraction of galaxies could be an important constrain for the way AGN feedback is included in models and simulations, together with the more traditionally used high-mass end of the GSMF.

Figure 19 compares our results (black lines and shaded areas) with those of Chauhan et al. (2020), SHARK and Spinelli et al. (2020, GAEA) regarding the median H I gas content of centrals, of all satellites, and of the sum of both as a function of $M_h$, from left to right respectively. The red lines are for the default SHARK model while the purple ones are for the case of no AGN feedback. The SHARK H I gas contents of centrals in haloes are only slightly lower than our semi-empirical determinations at the masses $M_h \lesssim 10^{12} \ M_\odot$. For larger masses, $\overline{M_{HI}}/M_h$ falls more sharply in the default SHARK model than our determinations, due to the effect of AGN feedback.
feedback as discussed above. As for the sum of HI gas in satellites within distinct haloes, SHARK predicts values significantly lower than our determinations, specially for $M_h < 10^{13} \, M_{\odot}$, and this contributes to the deep drop in the SHARK $M_{HI}/M_h$ ratio at halo masses in between $10^{12}$ and $5 \times 10^{12} \, M_{\odot}$. As shown in Chauhan et al. (2020), the ram-pressure stripping prescription (strangulation) has little effect on the HI gas content of satellites. On the other hand, Chauhan et al. (2020) assume that the gas that is ejected from satellites due to stellar feedback is transferred to the ejected gas reservoir of the central. Therefore, that gas cannot be reincorporated into the hot halo gas of the satellites. In the future, we will explore whether this assumption is or not relevant.

As for GAIA (dashed blue line), the $M_{HI}/M_h$ ratio of centrals agree well with our semi-empirical determinations. In particular, the drop at $M_h \gtrsim 10^{12} \, M_{\odot}$, related to the AGN feedback strength, is only slightly less drastic than our determinations. However, at $M_h \gtrsim 3 \times 10^{12} \, M_{\odot}$, the $M_{HI}/M_h$ ratio becomes constant instead of showing a continuous drop as in our case or in the case of SHARK. Regarding the HI contained in the satellites, GAIA predicts higher values than our determinations and much higher than those of SHARK. As a result, the total HI-to-halo mass ratio in haloes is slightly higher in GAIA at $M_h \gtrsim 10^{12} \, M_{\odot}$ than our determinations, but is in line with the trend, showing no depression.

5 SUMMARY AND CONCLUSIONS

In this paper we have presented a semi-empirical approach to link the HI gas content of galaxies to their stellar masses and DM (sub)haloes in the SNBD N-body cosmological simulation. Galaxies are initially linked to DM (sub)haloes via the $M_{DM}$-$V_{DM}$ relation derived from SHAM. We assume that galaxies are lognormally distributed around the $M_{DM}$-$V_{DM}$ relation with a dispersion of 0.15 dex. To every galaxy in the catalogue we assign either an early- or late-type morphology based on the observed fractions of ETCs for centrals and satellites from the SDSS. Yang et al. (2012) group catalogue and the Huertas-Company et al. (2011) morphological classification of SDSS galaxies. Finally, we assign a HI mass using the central/satellite $M_{HI}$ conditional PDFs given $M_h$ of ETGs and LTGs derived from observations in Paper I and in Calle et al. (2021). Thus, for every halo or subhalo in the SNBD simulation we have assigned a stellar mass, galaxy morphology (LTG or ETG), and HI mass. We emphasize that the morphology assignation is a necessary step for sampling the empirical $M_{HI}$ distributions, though in this paper, we are interested in studying the galaxy stellar-HI-halo connection for the galaxy population as a whole and not in its segregation by morphology. Our main results are as follows:

- The value of $\langle \log M_{HI} \rangle$ for the whole population as a function of $V_{DM}$ ($M_{DM}$) monotonically increases up to $V_{DM} \approx 160$ km/s ($M_{DM} \sim 10^{12} \, M_{\odot}$) where it reaches a maximum value of $\langle \log (M_{HI}/M_\odot) \rangle \approx 9.2$, Fig. 5. At higher (sub)halo velocities (masses) it decreases only slightly. This is in contrast to $\langle \log M_{DM} \rangle$ which increases monotonically as a function of $V_{DM}$ ($M_{DM}$).
- The scatter around $\langle \log M_{HI} \rangle$ increases with $V_{DM}$ ($M_{DM}$). At low (sub)halo velocities (masses), $V_{DM} \approx 80$ km/s ($M_{DM} \sim 10^{11} \, M_{\odot}$) this is ~ 0.5 dex. At $V_{DM} \gtrsim 160$ km/s ($M_{DM} \gtrsim 10^{12} \, M_{\odot}$) it increases rapidly reaching a maximum of ~ 1.2 dex.
- In general, the $M_{HI}$ conditional PDFs as a function of $V_{DM}$ ($M_{DM}$) are broad and highly asymmetric with a long tail towards lower values of $M_{HI}$. For $V_{DM} \gtrsim 160$ km/s ($M_{DM} \gtrsim 10^{12} \, M_{\odot}$), the PDFs are bimodal with a second peak appearing at lower values of $M_{HI}$, a behaviour simply inherited from the input $M_{HI}$ conditional PDFs as function of $M_h$. As a result, different statistical estimators as logarithmic mean, arithmetic mean, and median of $M_{HI}$ as a function of $V_{DM}$ ($M_{DM}$) differ among them (Fig. 6).
- There are not significant differences in $\langle \log M_{HI} \rangle$ as a function of $V_{DM}$ or $M_{DM}$ between centrals and satellites (recall that the velocity or mass of subhaloes is calculated at their peak value in the past). On average, satellites have slightly lower values of HI mass at a given $V_{DM}$ or $M_{DM}$ than centrals (Fig. 5). The scatter is broader for satellites than for centrals by 0.25 dex for $V_{DM} \lesssim 160$ km/s and 0.3 dex for $M_{DM} \lesssim 10^{12} \, M_{\odot}$.
- The HI projected 2PCFs from our mock catalogue increase only slowly in amplitude with increasing HI mass threshold (Fig. 10).
- Assuming identical $M_{HI}$ mass conditional PDFs for centrals and satellites results in 2PCFs that have higher amplitudes, especially

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8 Here we assumed that $V_{DM}$ corresponds to the maximum circular velocity for distinct haloes, whereas for subhaloes it is the peak maximum circular velocity reached along the halo’s main progenitor branch, see Eq. (1).
at the 1-halo term, than in our more realistic fiducial model, where satellites have lower HI gas fractions than centrals, see Figs. 7 and 10.

- Our predicted HI projected 2PCFs have higher amplitudes than the ones measured in the blind HI ALFALFA survey, see Fig. 11. When emulating ALFALFA selection effects in our catalogue, which selects gas-rich late-type galaxies, the projected 2PCFs are closer, but yet slightly above ALFALFA particularly for $M_{HI} > 10^{11} M_\odot$.

- The simple SHAM is unable to reproduce realistic 2PCFs (Fig. 14) as it assumes identical $(\log M_{HI}(V_{DM}))$ relations for centrals and satellites and predicts that it monotonically increasing.

- The total galactic (central + satellites) HI gas content in distinct haloes, $M_{HI}$, strongly depends on $M_\text{HI}$ and is completely dominated by centrals up to $\sim 10^{13} M_\odot$, while at higher masses the total contribution from satellites increases; at $M_\text{HI} \sim 10^{13} M_\odot$ both contributions are roughly equal and at $M_\text{HI} = 10^{14} M_\odot$ the total HI in satellites is larger by $> 1$ dex than the central galaxies contribution (Fig. 8). The inferred mean $M_{HI}^{1/3}$–$M_\text{HI}$ relation does not present a dip at the masses where AGN feedback is expected to be important.

The results presented here offer new constraints on the empirical galaxy-(sub)halo connection both for stellar and HI masses. These results are relevant for calibrating and testing the predictions from new generation SAM and hydrodynamics simulations of galaxy evolution within the $\Lambda$CDM cosmology. We have presented some preliminary comparisons with some of these theoretical predictions and discussed that our semi-empirical results can be relevant to constrain the strength of AGN feedback and the environmental processes that satellite galaxies undergo.

Our results on the galaxy HI-(sub)halo connection show that $M_{HI}$ is not totally determined by the halo properties $V_{\text{max}}$ or $M_{DM}$. The above has deep implications as it confines the central assumptions in the galaxy-halo connection models such as the HOD and SHAM. Thus, alternative tools, like the ones used here, should be adopted in order to derive a realistic H I-to-halo mass relation. We also found that the HI conditional PDFs as a function of $V_{\text{max}}$ or $M_{DM}$ are highly asymmetric and even bimodal. In view of the above, to characterize the dependence of $M_{HI}$ on $M_{DM}$ at large masses is better to use the median of $M_{HI}$, as a function of $M_{DM}$ (or $V_{\text{DM}}$). For the median $M_{HI}$, we see evidence of a weak decrease of $M_{HI}$ with $M_{DM}$ for (sub)haloes larger than $M_{DM} \sim 10^{12} M_\odot$, suggesting that the galaxies in these (sub)haloes have exhausted most of their gas reservoir, have not accreted in a long time or have even ejected their gas, e.g. due to AGN feedback.

On the other hand, we presented predictions on the HI spatial clustering for the whole population of local galaxies. Currently, blind HI surveys, such as ALFALFA, are shallow and introduce strong selection effects to some properties of their optical hosts. We have shown that while these selection effects are not crucial for measuring the HI mass function (see Fig. 4), they become critical for measuring the HI spatial clustering (as well as for the H I-to-stellar mass correlation, see Papers I and II). In this sense, we showed that using the ALFALFA HI spatial clustering for constraining the HI galaxy-halo connection may lead to incorrect results, unless the biases described above have been introduced within the galaxy-halo connection model employed. The results on the HI spatial clustering presented here can be used for testing theoretical predictions as well as for estimates in the designing of larger and deeper HI surveys that will be completed with forthcoming radio telescopes such as SKA or its Pathfinder instruments.

Finally, the fact that our mock catalogue predicts (slightly) higher HI clustering than ALFALFA even after imposing the selection effects of ALFALFA, suggests that the scatter around the $M_{HI}$–$V_{DM}$ relation should depend on morphology in such a way that the clustering properties of LTGs and ETGs differ, as observations show. In a forthcoming paper we will explore the effects of introducing the above-mentioned dependence of the scatter on morphology. In any case, we highlight that the results presented here for the whole galaxy population are valid whether there is a dependence of the scatter around the $M_{HI}$–$V_{DM}$ relation on morphology or not.

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DATA AVAILABILITY

The data underlying this article will be shared on reasonable request to the corresponding author.

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APPENDIX A: TABLES FOR THE GALAXY-HALO CONNECTION

Table A1 presents the data related to the bottom-left panel of Figure 5. For different values of $\log(M_{DM}/\text{M}_\odot)$, we give the corresponding $\langle \log(M_{HI}/\text{M}_\odot) \rangle$ values and their standard deviations for all, central, and satellite galaxies.

Table A2 presents the data related to the bottom-right panel of Figure 5 and to Figure 8. For different values of $\log(M_{DM}/\text{M}_\odot)$ showed in column (1), we give the corresponding $\langle \log(M_{HI}/\text{M}_\odot) \rangle$ values and their standard deviations (in dex) for all, central, and satellite galaxies, columns from (2) to (7), respectively. Columns (8) and (9) are the mean arithmetic values of the total HI mass and the sum of the HI mass in satellites inside distinct haloes, respectively (upper panel of Fig. 8). Columns (10) and (11) are as columns (8) and (9) but for stellar masses, respectively (lower panel of Fig. 8).
### Table A1. Mean logarithmic $M_{\text{HI}}$–$V_{\text{DM}}$ relations and their standard deviations for all, central, and satellite galaxies as presented in Fig. 5.

| $\log V_{\text{DM}}$ | $\langle \log M_{\text{HI}} \rangle$ | $\sigma$ | $\langle \log M_{\text{HI}} \rangle_c$ | $\sigma_c$ | $\langle \log M_{\text{HI}} \rangle_s$ | $\sigma_s$ |
|----------------------|--------------------------------------|---------|--------------------------------------|---------|--------------------------------------|---------|
| 1.80                 | 8.09                                 | 0.59    | 8.30                                 | 0.43    | 7.54                                 | 0.57    |
| 1.89                 | 8.40                                 | 0.64    | 8.66                                 | 0.46    | 7.92                                 | 0.64    |
| 1.99                 | 8.77                                 | 0.68    | 9.02                                 | 0.50    | 8.35                                 | 0.73    |
| 2.08                 | 9.04                                 | 0.75    | 9.25                                 | 0.60    | 8.68                                 | 0.84    |
| 2.17                 | 9.15                                 | 0.88    | 9.32                                 | 0.77    | 8.85                                 | 0.99    |
| 2.27                 | 9.10                                 | 1.03    | 9.23                                 | 0.96    | 8.85                                 | 1.11    |
| 2.36                 | 9.01                                 | 1.11    | 9.10                                 | 1.08    | 8.81                                 | 1.15    |
| 2.45                 | 8.91                                 | 1.15    | 8.96                                 | 1.14    | 8.76                                 | 1.15    |
| 2.55                 | 8.84                                 | 1.15    | 8.87                                 | 1.15    | 8.74                                 | 1.12    |
| 2.64                 | 8.76                                 | 1.13    | 8.79                                 | 1.15    | 8.68                                 | 1.07    |
| 2.73                 | 8.75                                 | 1.10    | 8.77                                 | 1.12    | 8.64                                 | 1.01    |
| 2.83                 | 8.71                                 | 1.08    | 8.73                                 | 1.09    | 8.58                                 | 0.99    |
| 2.92                 | 8.72                                 | 1.07    | 8.74                                 | 1.09    | 8.60                                 | 0.86    |
| 3.01                 | 8.77                                 | 1.04    | 8.77                                 | 1.06    | 8.77                                 | 0.78    |
| 3.11                 | 8.77                                 | 0.94    | 8.77                                 | 0.95    | 8.78                                 | 0.66    |
| 3.20                 | 9.12                                 | 0.92    | 9.12                                 | 0.94    | 9.07                                 | 0.11    |

### Table A2. Mean logarithmic $M_{\text{HI}}$–$V_{\text{DM}}$ relations and their standard deviations for all, central, and satellite galaxies (columns 1–7), as plotted in Fig. 5, and mean arithmetic $M_{\text{HI}}^\text{tot}$–$M_h$, $M_{\text{HI}}^\text{sat}$–$M_h$, $M_{\text{HI}}^\text{tot}$–$M_h$, and $M_{\text{HI}}^\text{sat}$–$M_h$ relations (columns 8–11, respectively), as plotted in Fig. 8.

| $\log M_{\text{DM}}$ | $\langle \log M_{\text{HI}} \rangle$ | $\sigma$ | $\langle \log M_{\text{HI}} \rangle_c$ | $\sigma_c$ | $\langle \log M_{\text{HI}} \rangle_s$ | $\sigma_s$ | $\log (M_{\text{HI}}^\text{tot})$ | $\log (M_{\text{HI}}^\text{sat})$ | $\log (M_{\text{HI}}^\text{tot})$ | $\log (M_{\text{HI}}^\text{sat})$ | $\log (M_{\text{sat}}^\text{tot})$ | $\log (M_{\text{sat}}^\text{sat})$ |
|----------------------|--------------------------------------|---------|--------------------------------------|---------|--------------------------------------|---------|----------------------------------|----------------------------------|----------------------------------|----------------------------------|----------------------------------|----------------------------------|
| 10.5                 | 7.97                                 | 0.56    | 8.15                                 | 0.43    | 7.58                                 | 0.61    | 4.88                            | 8.16                            | 5.80                            | 6.10                            | 6.74                            | 9.57                            |
| 10.8                 | 8.26                                 | 0.56    | 8.38                                 | 0.46    | 7.95                                 | 0.69    | 8.67                            | 5.62                            | 8.27                            | 6.46                            | 9.27                            | 7.61                            |
| 11.1                 | 8.64                                 | 0.60    | 8.75                                 | 0.49    | 8.36                                 | 0.76    | 9.00                            | 6.51                            | 8.66                            | 7.91                            | 9.85                            | 8.32                            |
| 11.4                 | 8.97                                 | 0.66    | 9.07                                 | 0.53    | 8.67                                 | 0.86    | 9.32                            | 7.28                            | 9.27                            | 8.56                            | 10.32                           | 9.57                            |
| 11.7                 | 9.16                                 | 0.76    | 9.27                                 | 0.64    | 8.83                                 | 0.99    | 9.57                            | 7.91                            | 9.85                            | 8.32                            | 10.32                           | 9.57                            |
| 12.0                 | 9.20                                 | 0.91    | 9.30                                 | 0.82    | 8.85                                 | 1.09    | 9.74                            | 8.46                            | 10.32                           | 9.88                            | 10.32                           | 9.57                            |
| 12.3                 | 9.12                                 | 1.04    | 9.20                                 | 0.99    | 8.82                                 | 1.14    | 9.87                            | 8.94                            | 10.66                           | 9.57                            | 10.32                           | 9.57                            |
| 12.6                 | 9.01                                 | 1.11    | 9.07                                 | 1.10    | 8.77                                 | 1.15    | 9.99                            | 9.34                            | 10.92                           | 9.08                            | 10.32                           | 9.57                            |
| 12.9                 | 8.91                                 | 1.14    | 8.94                                 | 1.14    | 8.74                                 | 1.13    | 10.13                           | 9.70                            | 11.15                           | 10.52                           | 10.32                           | 9.57                            |
| 13.2                 | 8.82                                 | 1.15    | 8.85                                 | 1.16    | 8.68                                 | 1.08    | 10.30                           | 10.04                           | 11.38                           | 10.92                           | 10.32                           | 9.57                            |
| 13.5                 | 8.77                                 | 1.12    | 8.78                                 | 1.14    | 8.67                                 | 1.01    | 10.49                           | 10.34                           | 11.62                           | 11.28                           | 10.32                           | 9.57                            |
| 13.8                 | 8.72                                 | 1.11    | 8.75                                 | 1.12    | 8.52                                 | 0.98    | 10.74                           | 10.65                           | 11.87                           | 11.61                           | 10.32                           | 9.57                            |
| 14.1                 | 8.73                                 | 1.07    | 8.74                                 | 1.08    | 8.61                                 | 0.94    | 10.98                           | 10.93                           | 12.11                           | 11.92                           | 10.32                           | 9.57                            |
| 14.4                 | 8.81                                 | 1.09    | 8.80                                 | 1.11    | 8.83                                 | 0.88    | 11.24                           | 11.21                           | 12.36                           | 12.21                           | 10.32                           | 9.57                            |
| 14.7                 | 8.70                                 | 1.01    | 8.70                                 | 1.02    | 8.70                                 | 0.69    | 11.50                           | 11.47                           | 12.61                           | 12.49                           | 10.32                           | 9.57                            |
| 15.0                 | 8.95                                 | 0.89    | 8.96                                 | 0.91    | 8.77                                 | 0.35    | 11.70                           | 11.69                           | 12.80                           | 12.71                           | 10.32                           | 9.57                            |