Mathematical modelling for energy beam additive manufacturing

G M Mladenov¹², E G Koleva¹²³ and D N Trushnikov⁴

¹Institute of electronics, Bulgarian Academy of Sciences, 72, Tzarigradsko shose, Sofia 1784, Bulgaria
²Technological Center on Electron Beam and Plasma Technologies and Techniques – TC EPTT Ltd., Bulgaria
³University of Chemical Technology and Metallurgy – Sofia, 8, Kliment Ohridski blvd., Sofia 1756, Bulgaria
⁴Perm National Research Polytechnic University, Russian Federation

e-mail: eligeorg@abv.bg

Abstract. A normalized processing diagram for powder bed additive manufacturing by concentrated energy beams was constructed on the base of simple analytical model simulating the heating of the powder layers by a moving linear heat source. The distance of the processing regime points below the line of the dependency of the maximal dimensionless temperature (or of the maximal thermal efficiency) on the dimensionless beam velocity gives insight on the processing parameters range and on the possibility for prognosticated optimization of the utilized parameters of the energy beam at selective powder layer melting. The diagram provides useful reference and methodology to aid the selection of appropriate processing parameters during the early development stages of this perspective technology.

1. Introduction

Additive manufacturing (AM) is defined as a process of producing three-dimensional objects additively in a layer-by-layer manner. AM is a rapidly emerging fabrication process with significant potential to reduce the manufacturing cost, the energy consumption and the environmental impact. Concentrated energy beams implemented for additive manufacturing can be an electron beam, a laser beam or plasma flow.

Despite the benefits in the design and the cost, the metal based AM is in a relatively initial stage of development due to the limited understanding of the complex physical processes (more complicated than these during the conventional production processes) such as energy beam/material interactions, heat transfer and molten metal flow, phase transformations, thermal stresses and distortions. All these factors influence the final building quality and the material properties [1-3] of the built structures.

A comprehensive review of the additive manufacturing of metallic components is given in [4]. Many existing software packages, that are easy to implement, can help to understand some process features [5-8]. In [5] a heat conduction and finite element methods are applied. In [6] the computer simulations are based on a heat transfer and fluid flow algorithm. The work [7] utilizes the level set method. In [8] the volume of the fluid is simulated using the finite difference method. The application of the mixed lattice Boltzman method and arbitrary Lagrangian – Eulerian is presented in [9]. The presence of various assumptions: (i) not including some physical phenomena in simulations and (ii)...
lack of data on some details (as the temperature gradients on the melting pool surface, the role of the alloy impurities on the surface tension, viscosity and various instabilities in the molten pool flows, etc.) often makes it difficult the validation and prognostication implementation of the computed results.

In this paper a simple analytical model of heating samples, covered by a powder layer is discussed, as well as the use of known dimensionless parameters, which can reduce the total number of considered variables. In this way important understanding is provided that the dependencies between single process variables are unable to do. During the AM, the molten pool is small and the temperature gradients in the solid sample (powder layer and base material) determine the dimensions of the molten pool. These temperature gradients due to the thermal conduction are accountable for the spreading of the rest of the absorbed energy that is not utilized for melting. In the case of the selective melting of sequentially placed powder layers for building the produced structures, the powder bed is heated too. The temperature of the selectively melted area is defined by the increase of the powder bed temperature to the fusion temperature of the treated powder.

2. Thermal model

The solution of the thermal balance equation at heating the powder layer with thickness \( L \) by a linear moving thermal source with constant intensity \( P \), velocity \( V \) (see figure 1) and assuming no phase changes in the sample during heat transfer, at known material physical parameters: thermal conductivity \( \lambda \), thermal diffusivity \( a \) \( (a = \lambda/(C\cdot\rho)) \), where \( C \) is the specific heat and \( \rho \) is the powder layer density, \( T \) is the temperature of the studied point and \( T_0 \) is the temperature of the powder bed (the initial temperature), will take the form [20]:

\[
T = \frac{P}{2\pi\lambda L} \exp \left( -\frac{Vx}{2a} \right) K_0 \left( \frac{Vr}{2a} \right) + T_0
\]

where \( x \) is the distance from the moving center, \( r \) is the radius vector, moving together with the heat source, \( K_0 \) is the modified Bessel function of second kind of order zero.

Note that \( P \) is the absorbed energy input (the beam energy \( P_b \) after correction for the energy losses by back scattered and secondary electrons in the case of an electron beam; the surface absorptivity or the coupling coefficient in the case of a laser beam) of the energy beam. In order to minimize the effect of the temperature dependence of the thermal constants, the values of \( \lambda \), \( C \) and \( a \) are taken for an intermediate (between the initial \( T_0 \) and melting \( T_m \) temperatures) temperature and the heat conduction process is assumed to be independent of the temperature. From the equality to zero of the first derivative of the thermal cycle of a sample point, it can be seen that all the points from the moving coordinate system with maximal temperatures are given by:

\[ T = \frac{P}{2\pi\lambda L} \exp \left( -\frac{Vx}{2a} \right) K_0 \left( \frac{Vr}{2a} \right) + T_0 \]
\[ \cos \varphi = -K_0(rV/2a)/K_1(rV/2a) \]  \hspace{1cm} (2)

or from the geometry conditions:

\[ \sin \varphi = \left\{ 1 - \left[ K_0(rV/2a)/K_1(rV/2a) \right]^2 \right\}^{-1/2} \]  \hspace{1cm} (2a)

where \( K_1(rV/2a) = K_0'(rV/2a) \) is the modified Bessel function of the second kind of order one and \( rV/2a \) is Péclet number (dimensionless) of the distances to the heat source, which has values proportional to the movement velocity. Then from (1) and (2) one can find the dimensionless maximum temperature \( \theta_m \):

\[ \theta_m = 2\pi \lambda L(T_m - T_0)/P = K_0(rV/2a) \times \exp\left[ (rV/2a) \times K_0(rV/2a) / K_1(rV/2a) \right] \]  \hspace{1cm} (3)

The function of the dimensionless parameters \( \theta_m (rV/2a) \) is more practical. It can be found by iterations from (3), taking into account (2a) and that \( y = b/2 = r \sin \varphi \), where \( b \) is the molten and solidified layer’s width. This function is given in figure 2 for a range of \( Vy/2a \), which is appropriate for beam heating at small distances from a moving linear heating source [21].

![Figure 2. The dependences of the maximum dimensionless temperature \( \theta_m \) on the dimensionless distances \( Vy/2a \); and the tilted on 90° dimensionless coordinates \( Y=Y(X) \).](image)

Another presentation \( Y(X) \) of that relation is given also in figure 2, where the tilted dimensionless coordinates are \( X=P/L.\lambda/\left(T_m-T_0\right) \) and \( Y=V.b/2a \).

The dimensionless thermal efficiency \( \eta_t \) of the heating process of the powder layer by a concentrated energy beam, which is in our case the ratio of the energy needed just to melt the material to the total energy delivered by the beam (in \( P \) should be taken into account the electron efficiency \( \eta_e \) or the laser beam coupling coefficient) is calculated as:

\[ \eta_t = VFS/P \]  \hspace{1cm} (4)

where \( S \) is the heat content of the melted material in the weld at temperature elevation between the initial temperature \( T_0 \) and the melting temperature \( T_m \):

\[ S = C(T_m - T_0) + H_f \approx 1.5. C(T_m - T_0) \]  \hspace{1cm} (5)

Here \( C \) is the mean specific heat for the temperature range between these temperatures and \( H_f \) is the heat of fusion.
It is easy to see that the thermal efficiency $\eta_t$ can be evaluated as the ratio $Y/X$.

In [21] this approach was applied successively for prognostication of the optimal regimes at electron beam welding.

3. Experimental data

In table 1, the assumed thermo-physical parameters of the powder layers for the simulation of the AM parameter points of some more important industrial metal alloys are given. In table 2, some experimental data for the regimes of selective electron beam and laser melting are presented. There:

$$X = (0.6 \rho)/(L\lambda(T_m - T_0))$$

The surface absorptivity (the electron efficiency for electron beams or the coupling coefficient for laser beams) was assumed 0.6. The maximal dimensionless temperature is:

$$\theta_m = (2\pi)/X$$

In table 1, the values of thermal conductivity $\lambda$ for powders are of order of 10% from the values for the bulk material, while the values of the density $\rho$ of the powder layers are assumed as 67% - 75% from the density of the bulk alloy.

Figure 3 presents the function $\Theta(V_{rb}/2a)$ and the experimental points, representing the regimes for selective melting of some powder layers of industrial alloys. The tilted straight line on figure 3 represents the maximal thermal efficiency of 48.45% [20]. It is the theoretical limit for the ratio of the quantity of the utilized for melting of the material (at heating the samples by moving linear heating source) absorbed energy toward the total absorbed energy.

![Figure 3](image_url)

**Figure 3.** Normalized processing diagram of the function $\Theta (V_{rb}/2a)$ and the experimental data for selective melting of: ‘•’ Ti-6Al-4V; ‘*’ SS 316L; ‘x’ Inconel 625; ‘+’ CM247 and ‘o’ FeCoCrNi. The cited references (the origin of these data) are marked with numbers.
Table 1. Thermo-physical parameters for the powder layers from the alloys, used for selective energy beam melting.

| Powder         | \( \lambda \) [W/mK] | C [J/kgK] | P [kg/m³] | \( a=\frac{\lambda}{C\rho} \) [m²/s] | \( T_m \) [K] |
|----------------|------------------------|-----------|-----------|--------------------------------------|---------------|
| Ti6Al4V        | 0.63                   | 530       | 3000      | 0.396 \times 10^{6}                  | 1604          |
| SS316          | 1.63                   | 500       | 6000      | 0.543 \times 10^{6}                  | 1370          |
| Inconel625     | 2.52                   | 620       | 6000      | 0.677 \times 10^{6}                  | 1605          |
| CM247          | 1.54                   | 320       | 6000      | 0.802 \times 10^{6}                  | 1612          |
| FeCoCrNi       | 1.8                    | 450       | 5800      | 0.69 \times 10^{6}                   | 1600          |

Table 2. Experimental data for the AM of metallic parts in the powder bed.

| Alloy        | Beam   | \( T_0 \) [K] | L [m] | \( r_b \) [m] | V [m/s] | P [W] | X | \( \frac{Vr_c}{2a} \) | \( \theta_m \) | Ref. |
|--------------|--------|---------------|-------|---------------|---------|-------|---|------------------------|--------------|------|
| Ti6Al4V      | e beam | 923           | 150 \times 10^{-6} | - | 600 | 71.54 | 11.85 | 0.0878 | [10]          |
| Ti6Al4V      | e beam | 923           | 150 \times 10^{-6} | - | 600 | 71.54 | 16.3 | 0.0878 | [10]          |
| Ti6Al4V      | e beam | 923           | 150 \times 10^{-6} | 0.2 | 60 | 559.87 | 37.87 | 0.0112 | [11]          |
| Ti6Al4V      | e beam | 923           | 50 \times 10^{-6} | 150 \times 10^{-6} | 6.4 | 1400 | 1306.38 | 212.21 | 0.048 | [11]          |
| Ti6Al4V      | laser  | 298           | 30 \times 10^{-6} | 52 \times 10^{-6} | 1.6 | 250 | 3505.94 | 104.51 | 0.0018 | [12]          |
| Ti6Al4V      | laser  | 298           | 20 \times 10^{-6} | 75 \times 10^{-6} | 0.8 | 150 | 1458.67 | 52.52 | 0.0043 | [13]          |
| Ti6Al4V      | laser  | 298           | 20 \times 10^{-6} | 75 \times 10^{-6} | 1.5 | 200 | 1944.9 | 142.04 | 0.00323 | [13]         |
| Ti6Al4V      | laser  | 498           | 30 \times 10^{-6} | 70 \times 10^{-6} | 0.7 | 175 | 2152.76 | 92.8 | 0.00292 | [14]          |
| Ti6Al4V      | laser  | 498           | 90 \times 10^{-6} | 120 \times 10^{-6} | 1.029 | 375 | 2690.76 | 92.8 | 0.00233 | [14]          |
| Ti6Al4V      | e beam | 730           | 86 \times 10^{-6} | 50 \times 10^{-6} | 1.6 | 160 | 3479.02 | 101 | 0.0018 | [25]          |
| Ti6Al4V      | e beam | 730           | 86 \times 10^{-6} | 200 \times 10^{-6} | 0.8 | 240 | 2609.26 | 202.02 | 0.0024 | [25]          |
| Ti6Al4V      | e beam | 730           | 86 \times 10^{-6} | 150 \times 10^{-6} | 1.6 | 320 | 3219.34 | 303.03 | 0.0027 | [25]          |
| Ti6Al4V      | e beam | 730           | 86 \times 10^{-6} | 200 \times 10^{-6} | 1.6 | 480 | 2609.26 | 404.04 | 0.0024 | [25]          |
| Ti6Al4V      | e beam | 730           | 86 \times 10^{-6} | 250 \times 10^{-6} | 1.6 | 640 | 2782.93 | 505.05 | 0.00225 | [25]         |
| Ti6Al4V      | e beam | 730           | 86 \times 10^{-6} | 200 \times 10^{-6} | 2.4 | 720 | 3913.9 | 606.06 | 0.0016 | [25]          |
| Ti6Al4V      | e beam | 730           | 86 \times 10^{-6} | 150 \times 10^{-6} | 4.0 | 800 | 5798.36 | 757.57 | 0.011 | [25]          |
| Ti6Al4V      | e beam | 730           | 86 \times 10^{-6} | 150 \times 10^{-6} | 5.6 | 1120 | 8117.71 | 1060 | 0.000774 | [25]         |
| Ti6Al4V      | e beam | 730           | 86 \times 10^{-6} | 150 \times 10^{-6} | 8.0 | 1600 | 11596.72 | 1515.15 | 0.000542 | [25]         |
| SS316L       | laser  | 298           | 30 \times 10^{-6} | 27 \times 10^{-6} | 0.5 | 150 | 1910.83 | 12.424 | 0.00329 | [15]          |
| SS316L       | laser  | 298           | 30 \times 10^{-6} | 27 \times 10^{-6} | 2.5 | 400 | 2123.14 | 62.12 | 0.00296 | [15]          |
| SS316L       | laser  | 298           | 50 \times 10^{-6} | 100 \times 10^{-6} | 0.2 | 97 | 333.71 | 18.4 | 0.019 | [16]          |
| Inconel625   | laser  | 298           | 100 \times 10^{-6} | 850 \times 10^{-6} | 0.11 | 900 | 209.43 | 69.26 | 0.03 | [17]          |
| CM247        | laser  | 298           | 20 \times 10^{-6} | 75 \times 10^{-6} | 0.4 | 100 | 395.26 | 18.7 | 0.016 | [18]          |
| CM247        | laser  | 298           | 20 \times 10^{-6} | 75 \times 10^{-6} | 2 | 200 | 790.51 | 93.52 | 0.00794 | [18]          |
| FeCoCrNi     | laser  | 298           | 20 \times 10^{-6} | 25 \times 10^{-6} | 0.33 | 200 | 3015.075 | 5.98 | 0.00208 | [19]          |
| FeCoCrNi     | laser  | 298           | 50 \times 10^{-6} | 25 \times 10^{-6} | 0.66 | 200 | 1205.545 | 11.96 | 0.005209 | [19]         |
4. Discussion
The normalized processing diagram on figure 3 utilises the dimensionless maximal temperature and the dimensionless velocity of the beam. Non-dimensional numbers are beneficial, because they reduce the total number of the variables to be studied and provide important understanding, which a single process variable is unable to provide.

The positions of the dimensionless processing parameter points must be chosen in the processing parameters window that is the area below the function $\Theta(Vr_b)/(2a)$ shown in figure 3. The region of the processing regime points below the line of the maximal dimensionless temperature (or the maximal thermal efficiency) gives the potential to prognosticate the possible optimal parameters utilized at energy beam selective melting. The optimal place of the point of the heating process regime of selective melting of powder layers by an energy beam is the position as close as possible to the limit value, given by the function $\Theta(Vr_b)/(2a)$. The points from figure 3 with regime dimensionless parameters situated above this line can be explained as follows. The points from [10] were calculated by the authors of this paper and there is a lack of data, which does not allow to discuss the reason for this discrepancies. The point for the regime of laser selective melting from [17] is presenting a process with too big beam spot. In this case, the heat model with moving linear heating source does not give good results. The explanation for the position of one point from the regimes of electron beam AM from [25], where $r_b$ is $250\times10^{-6}$ m can be similar. The processes in the liquid metal and the dimensions of the melting pool in these cases are more important for the obtained results (and for the thermal efficiency of the heating process).

Note that the same analytical model of heating was successively applied also in [22-24] at the evaluation of the weld depth at EB surface thermal modification and at EB welding of thin plates at regimes, resulting in spherical cross-sections of the welds. Using the values of known $\theta_m$ for given $P$ and $L$ (in these cases $L$ is the thickness of the welded thin plates or the scanned by the beam length), the curve in figure 2 gives the possibility to obtain the depth of heating of a specific material to some temperature (as an example - the melting temperature at welding or the temperature of phase-transformation at hardening).

5. Conclusions
The analytical model for heating solid samples by a moving linear heat source, which has already been outlined by the authors for electron beam welding [21], is adapted for samples, covered by a powder layer. A normalized processing diagram for the powder bed during AM by concentrated energy beams was constructed and discussed. In conjunction with the experimental parameter data available in the literature, the applicability of this diagram was investigated and processing parameters window was demonstrated.

It is intended that these processing diagrams will provide useful insight to practicing scientists and engineers for selecting the appropriate processing parameters for experimentation during the early stages of research and technology development programs.

Acknowledgements
The reported study was partially supported by the Government of Perm Region, research project № S-26/787 from 21.12.2017 as well as by grants from the Russian Foundation for Basic Research RFBR №18-08-01016 A.

References
[1] Korner C, Bauereiss A and Attar E 2013 *Modelling and Simulation in Materials Science and Engineering* 21(8) 085011
[2] Klassen A, Bauereiss A and Korner C 2014 *Journal of Physics D: Applied Physics* 47(6) 065307
[3] King W, Anderson A T, Ferencz R M, Hodge N E, Kamath C and Khairallah S A 2015 *Materials Science and Technology* 31(8) 957–968
DebRoy T, Wei H L, Zuback J S, Mukherjee T, Elmer J W, Milewski J O, Beese A M, Wilson-Heid A, De A and Zhang W 2018 Progress in Materials Science 92 112–224

Contuzzi N, Campanelli S and Ludovico A 2011 3D finite element analysis in the selective laser melting process Int. J. Simul. Model 10(3) 113–21

Manvatkar V, De A and DebRoy T 2015 Mater. Sci. Tech 31(8) 924–30

Lee Y, Nordin M, Babu S S and Farson D F 2014 Metall Mater. Trans. B 45(4) 1520–1529

Korner C, Attar E and Heiml P 2011 J. Mater. Process. Technol. 211(6) 978–987

Thomas M, Baxter G J and Todd I 2016 Acta Materialia 108 26–35

Juechter V, Scharowsky T, Singer R F and Korner C 2014 Acta Mater. 76 252–258

Vrancken B, Thijs L, Kruth J P and Humbeeck J V 2014 Acta Mater. 68 150–158

Qiu C, Adkins N J E and Attallah M M 2013 Mater. Sci. Eng. A 578 230–239

Xu W, Brandt M, Sun S, Elambasseril J, Liu Q, Latham K, Xia K and Qian M 2015 Acta Mater. 85 pp 74–84

Kamath C, El-dasher B, Gallegos G F, King W E and Sisto A 2014 Int. J. Adv. Manuf. Technol. 74 65–78

Ziołkowski G, Chlebus E, Szymczyk P and Kurzac J 2014 Archives Civ. Mech. Eng. 14 608–614

Cooper D E, Blundell N, Maggs S and Gibbons G J 2013 J. Mater. Process. Technol. 213 2191–2200

Carter L N, Martin C, Withers P J and Attallah M M 2014 J. Alloys Compd. 615 338–347

Brif Y, Thomas M and Todd I 2015 Scr. Mater. 99 93–96

Rikalin N 1951 Calculation of Welding Thermal Processes Mashgiz Publ. House (Moscow: 291)

Koleva E, Mladenov G and Vutova K 1999 Vacuum 53(1-2) 67–70

Dvorkin I, Ledovskoy V and Mladenov G 1972 Electronnaia Technica, Ser.4-Vacuum and gas discharge tubes 3 (In Russian) pp 54–68

Petrov P and Mladenov G 1991 Vacuum 42(1/2) 29–32

Koleva E and Mladenov G 2010 Process Parameter Optimization and Quality Improvement at Electron Beam Welding Welding: Processes, Quality, and applications Seria Mechanical Engineering Theory and Applications (Nova Sci. Publishers) ed R J Klein chapter 2 pp 101–166

Riedlbauer D, Schar T, Korner C and Mergheim J 2017 Int. J. Adv. Manuf. Technol. 88 DOI 10.1007/s00170-016-8819-6 pp 1309–1317