Democratic Mass Matrices Induced by Strong Gauge Dynamics and Large Mixing Angles for Leptons

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We consider the dynamical realization of the democratic type of Yukawa coupling matrices as the Pendelton-Ross infrared fixed points. Such fixed points of the Yukawa couplings become possible through the introduction of many Higgs fields, which are made superheavy, but one massless mode. Explicitly, we consider a strongly coupled supersymmetric GUT based on $SU(5) \times SU(5)$, in which rapid convergence to the infrared fixed point generates sufficiently large mass hierarchy for quarks and leptons. In particular, it is found that the remarkable difference between mixing angles in the quark and lepton sectors can be explained as a simple dynamical consequence. We also discuss a possible scenario leading to a realistic mass spectra and mixing angles for quarks and leptons. In this scheme, the Yukawa couplings not only for top but also for bottom appear close to their quasi-fixed points at low energy and, therefore, $\tan \beta$ should be large.

§1. Introduction

Yukawa couplings are not direct observables, since they depend on the choice of the flavor basis. What we know in principle are the hierarchical masses of quarks and leptons, and the mixing matrices. Recent neutrino oscillation experiments strongly indicate that neutrinos are massive and lepton flavors are mixed. It is noteworthy that the mixing angles ($\sin^2 2\theta_{\text{sun}} \sim 0.84$ and $\sin^2 2\theta_{\text{atm}} \sim 1.0$) are as large as nearly bi-maximal in sharp contrast with the small mixing angles among quarks.

There have been proposed various phenomenological models explaining the origin of the Yukawa couplings allowing for such features by assuming certain flavor symmetries. The most commonly used one is the Froggatt-Nielsen mechanism based on abelian flavor symmetries. With this model, the hierarchical structures among the Yukawa couplings are nicely explained. However, the abelian symmetry itself cannot fix the $O(1)$ ambiguity of the parameter values. Therefore the structure of the mixing angles in the lepton sector as well as the neutrino masses are explained in no more detail than simply that those parameter values are not hierarchical. The flavor differences are also just input into the abelian flavor charges a priori.

In contrast to this situation, non-abelian flavor symmetries can impose precise relations among some elements of the Yukawa matrices. Therefore various non-
abelian symmetries have been considered to explain the mixing angles in the lepton sector as well as the hierarchical masses simultaneously. The democratic Yukawa matrices are an attractive possibility of this kind. The rigid democratic matrix, in which all elements are identical,* may be explained by assuming, e.g., $S_3$ flavor symmetries for three generations of quarks and leptons. It is interesting that one of the mass eigenvalues is much larger than the others, though there is no a priori difference among flavors. The $S_3$ flavor symmetry is assumed to be broken slightly to give rise to small masses for the first and second generations. It has been known for some time that Fritzsch-type mass matrices for quarks are realized by introducing a simple form of $S_3$ breaking parameters. Moreover, the lepton sector mixing angles uncovered by neutrino oscillation experiments may be explained in the democratic framework rather well.

However, no comprehensive explanation has been given for the origin of the flavor symmetry breaking terms in this framework. Moreover, the $S_3$ flavor symmetry allows the mass matrix to be proportional to the identity matrix, with the exception of the democratic-type matrix for the neutrino sector. The mixing matrix for the lepton sector can be obtained only if we assume the neutrino mass matrix to be almost diagonal. In other words, the democratic part must be small with some other reasons than the flavor symmetry. The same problem exists in the see-saw context, where both of the Yukawa couplings and the mass matrix of the right-handed neutrinos should be almost diagonal.

Besides, we are faced with a more difficult problem in considering grand unified theories. In, e.g., the $SU(5)$ GUT, the matter fields belong to a $10$ and a $\bar{5}$. However, $S_3$ symmetry allows an identity matrix for the Yukawa couplings among $10$s, which is strictly forbidden for the quark mass hierarchy. Thus the phenomenologically postulated forms for the democratic Yukawa matrices are not explained only by flavor symmetries.

Indeed, it is not a unique way to introduce flavor symmetries and their small breakings in order to explain the hierarchy among various couplings. For example, a large mass hierarchy may be generated by overlapping of the wave functions in extra dimensions. Renormalization with large anomalous dimensions induced by strong dynamics can also generate a mass hierarchy. Similarly, power-law running of the Yukawa couplings in the extra dimensions has also been utilized.

In this paper we consider models in which the democratic type of Yukawa matrices are realized as infrared attractive fixed point couplings. It has been known for some time that the ratio of the Yukawa coupling and the gauge coupling rapidly approaches to the so-called Pendelton-Ross fixed point (PRFP) in the infrared direction. If all elements of the Yukawa coupling matrix have an identical PRFP, then the democratic type matrix may be realized dynamically. In order to make this possible, we need to introduce many Higgs fields. In addition, a special kind of mass terms for the multiple Higgs fields are assumed, so that they acquire masses of the GUT scale, except for one massless mode, which is identified with

* Phenomenologically, the so-called extended democratic mass matrices are equally interesting. However, we restrict ourselves to rigid cases in this paper.
Higgs in the low energy theory.

Indeed, this idea is not new. Abel and King\textsuperscript{18} considered it several years ago, but they did not apply to the neutrino sector. In this paper, we stress that this mechanism is free from the difficulty in realizing democratic Yukawa matrices by the flavor symmetry. Moreover, we show that the typical difference between the mixing angles of quark and lepton sectors can be explained as a simple dynamical consequence in this framework. Our mechanism predicts very small neutrino Yukawa couplings, and therefore relatively light right handed neutrinos in the see-saw models.

Explicitly, we treat a supersymmetric GUT model based on the $SU(5) \times SU(5)$ gauge group, in which the doublet-triplet problem is solved nicely.\textsuperscript{19,20} There, one $SU(5)$ gauge interaction coupled with quarks and leptons is assumed to be strong, and another is weak just as in the conventional $SU(5)$ GUT. Then the renormalization group (RG) from the Planck scale to the GUT scale shows that the Yukawa couplings in the GUT model are aligned with a very good accuracy due to strong dynamics. After diagonalizing the Yukawa matrix, only one coupling is found to be much larger than the others. Further, we show that realistic Yukawa coupling matrices can be obtained within a simple framework.

In this kind of scenario, all of the Yukawa couplings, except for those in the neutrino sector, are found to be quite large at high energy, because the Yukawa coupling and the gauge coupling are of the same order at the PRFP. However, heavy top quark mass (178 GeV) indicates that the top Yukawa coupling is close to the so-called quasi-fixed point.\textsuperscript{21} This means that top Yukawa coupling can be fairly large at the GUT scale, which is a favorable feature of our scenario. However, the Yukawa couplings not only of the top quark but also of the bottom quark and the tau lepton are all large. Therefore a large value of $\tan \beta$ is predicted in this kind of scenarios.

This article is organized as follows. In §2 we give a brief summary of the democratic type of mass matrices and the $S_3$ flavor symmetry. In §3 we present the general ideas leading to the democratic Yukawa matrices at low energy by strongly coupled gauge dynamics. There, also, the superpotential for the multiple Higgs fields and their mass spectra are discussed. In §4 we present an explicit model based on the $SU(5) \times SU(5)$ GUT. Then it is seen that a sufficiently large hierarchy of the couplings can be generated by RG from the Planck scale to the GUT scale. We also discuss predictions of this kind of scenario and the consistency with particle masses. There we work out the Yukawa couplings for the top quark, bottom quark and tau lepton obtained at low energy scale in an explicit model. The neutrino Yukawa couplings are discussed in §5. There it is shown that the large mixing angles among leptons can be realized as a dynamical consequence, if the right-handed neutrinos have new Yukawa couplings to some other fields. In §6 we consider the minute structure of the democratic mass matrices producing realistic masses and mixing angles. Finally §7 is devoted to conclusions and discussions, including comments on the extra-dimensional framework and also on the soft supersymmetry breaking parameters.
§2. Democratic mass matrices and $S_3$ flavor symmetry

In this section we give a brief review of the democratic mass matrices for quarks and leptons from the viewpoint of $S_3$ flavor symmetry. The democratic mass matrices give a phenomenologically successful description. However, it is found necessary to further constrain the parameters allowed by the symmetry. Specifically, the $S_3$ symmetry is not sufficient to give viable mass matrices. Indeed, some attempts have been made in order to give physical reasonings leading to the viable democratic mass matrices so far.$^{12}$

The rigid democratic matrix $J$ is diagonalized as

$$ J = \frac{1}{3} \begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix} = A \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix} A^T, \quad (2.1) $$

by the diagonalization matrix

$$ A = \begin{pmatrix} 1/\sqrt{2} & 1/\sqrt{6} & 1/\sqrt{3} \\ -1/\sqrt{2} & 1/\sqrt{6} & 1/\sqrt{3} \\ 0 & -2/\sqrt{6} & 1/\sqrt{3} \end{pmatrix} \quad (2.2) $$

For the democratic mass matrix, two vanishing eigenvalues are regarded as masses for the first and the second generations in the first approximation. Their masses are given by small deviations from the rigid matrix. The interesting feature of this approach is the idea of flavor democracy, namely that there is no a priori difference between flavors.

In the standard model (SM), the democratic mass matrices are realized by assuming $S_3$ flavor symmetry for each kind of SM matter field. Hereafter, we consider the minimal supersymmetric case; (MSSM). The superpotential of the MSSM is given by

$$ W = Y^u_{ij} Q_i u_j H^u + Y^d_{ij} Q_i d_j H^d + Y^e_{ij} L_i e_j H^d + \frac{\kappa_{ij}}{2M_R} L_i L_j H^u H^u, \quad (2.3) $$

where $i, j = (1,2,3)$ represent generations. Each of the quark and lepton fields, $Q_i, u_i, d_i, L_i, e_i$, is assigned to a three dimensional representation of a distinct $S_3$. Then the Lagrangian is invariant under permutations of any set of the matter fields.

Let us start with the mass matrices for quarks. It has been known that realistic mass matrices are obtained by introducing small breaking parameters of the flavor symmetry $\epsilon_{u(d)}$ and $\delta_{u(d)}$, which satisfy the relations $\epsilon_{u(d)} \ll \delta_{u(d)} \ll 1$.$^8$

$$ M_q \propto J + \begin{pmatrix} -\epsilon_q & 0 & 0 \\ 0 & \epsilon_q & 0 \\ 0 & 0 & \delta_q \end{pmatrix} = A \begin{pmatrix} 0 & -\sqrt{1/3}\epsilon_q & -\sqrt{2/3}\epsilon_q \\ -\sqrt{1/3}\epsilon_q & (2/3)\delta_q & -(\sqrt{2/3})\delta_q \\ -\sqrt{2/3}\epsilon_q & -(\sqrt{2/3})\delta_q & 1 + (1/3)\delta_q \end{pmatrix} A^T, \quad (2.4) $$
where \( q = u, d \). The matrix in the second line is of the Fritzsch type. It offers a phenomenologically good representation.

The mass eigenvalues of these matrices are found to be

\[
m_1^q : m_2^q : m_3^q \sim \frac{\epsilon_q^2}{\delta_q} : \delta_q : 1.
\] (2.5)

Therefore the ratio of the quark masses can be represented by assuming breaking parameters of the following orders:

\[
\epsilon_u \sim 10^{-4}, \delta_u \sim 10^{-2};
\]
\[
\epsilon_d \sim 10^{-2}, \delta_d \sim 10^{-1}.
\] (2.6)

The absolute values of the top and bottom quark masses are not determined by the flavor symmetry, and, therefore, they must be parameterized phenomenologically. In our dynamical mechanism, we can predict these absolute values by evaluating the Yukawa couplings at low energy. This is discussed in a subsequent section.

The mixing angles turn out to be small, since the diagonalization matrix for \( M_u \) and \( M_d \) are both close to \( A \). Quantitatively the CKM matrix is given to good approximation by

\[
U_{\text{CKM}} \sim \begin{pmatrix}
1 & O(\epsilon_d/\delta_d) & O(\epsilon_d) \\
O(\epsilon_d/\delta_d) & 1 & O(\delta_d) \\
O(\epsilon_d) & O(\delta_d) & 1
\end{pmatrix}.
\] (2.7)

It is found that the above choice of parameters may also reproduce realistic mixing angles.

Next let us consider the lepton sector. The mass matrix for the charged leptons is given in a similar fashion:

\[
M_\ell \propto J + \begin{pmatrix}
-\epsilon_e & 0 & 0 \\
0 & \epsilon_e & 0 \\
0 & 0 & \delta_e
\end{pmatrix}.
\] (2.8)

Here, if we take the flavor symmetry breaking parameters as

\[
\epsilon_e \sim 10^{-2}, \quad \delta_e \sim 10^{-1},
\] (2.9)

then the ratios of the charged lepton masses are parameterized well. Again, the absolute value is undetermined in this approach, whereas it is determined in our scheme.

The situation for the neutrino mass matrix is somewhat different. The \( S_3 \) flavor symmetry allows the couplings \( \kappa_{ij} \) given by Eq. (2.3) to be proportional to the identity matrix also. Therefore, the mass matrix is given by

\[
M_\nu \propto I + rJ + \begin{pmatrix}
0 & 0 & 0 \\
0 & \epsilon_\nu & 0 \\
0 & 0 & \delta_\nu
\end{pmatrix},
\] (2.10)
where $I$ denotes the identity matrix.

It is noted that this mass matrix may explain the mixing angles in the lepton sector nicely, if the parameter $r$ in Eq. (2.10) is much smaller than 1. For vanishing $r$, the lepton mixing matrix (or the MNS matrix) is given by $A^T$. [The matrix $A$ is given explicitly in Eq. (2.2).] Therefore, we obtain the three mixing angles as

$$
\sin^2 2\theta_{12} \sim 1, \quad \sin^2 2\theta_{23} \sim 0.94, \quad U_{e3} = 0.
$$

This should be compared with the observation made in recent neutrino experiments,\(^1\)–\(^4\) which yield

$$
\sin^2 2\theta_{12} \sim 0.84, \quad \sin^2 2\theta_{23} \sim 1.0, \quad |U_{e3}| < 0.23.
$$

Thus, the lepton mixing angles are found to be almost explained already without parameter $r$ in Eq. (2.10). In other words, a nearly diagonal neutrino mass matrix is favorable including the flavor symmetry breaking part from the phenomenological point of view. However, we have found no reasons to explain why $r$ is so small, or why the flavor symmetry breaking parameters are input only to the diagonal elements.

This fine-tuning problem remains or becomes more curious in the see-saw mechanism. The superpotential for the neutrino sector is given by

$$
W = Y^\nu_{ij} L_i \nu_j H^u + \frac{1}{2} M_{Rij} \nu_i \nu_j,
$$

where $\nu_i$ denote the right-handed neutrinos. Here we need to assume that these right-handed neutrinos belong to the same representation of $S_3$ as the lepton doublets $L_i$. If we assume a distinct $S_3$ group for the right-handed neutrinos like other fields, the neutrino Yukawa coupling matrix $Y^\nu$ is restricted to a democratic form. Then, the mixing angles cannot be large. The neutrino Yukawa matrix and the right-handed neutrino mass matrix are now parameterized as

$$
Y^\nu = y^\nu_0 (I + rJ) + \Delta Y^\nu, \quad MR = MR_0 (I + r'J) + \Delta MR.
$$

It should be noted that both of $Y^\nu$ and $MR$ must be nearly diagonal in order to have large mixing angles. Therefore, the free parameters $r$ and $r'$ are constrained to be small. This is not explained by the flavor symmetry.

Moreover, it turns out that this sort of fine-tuning must be required much stronger in the GUT models. In the $SU(5)$ GUT, the quark and lepton fields in one generation are combined into a $10 + \bar{5} + 1$ representation. The superpotential of their Yukawa interactions is given by

$$
W = Y^u_{ij} 10_i 10_j H(5) + Y^d_{ij} 10_i \bar{5}_j H(\bar{5}) + Y^\nu_{ij} \bar{5}_i^* 1_j H(5).
$$

We note that the Yukawa couplings satisfy $Y^e = (Y^d)^T$ at the GUT scale. Then the $S_3$ symmetry for $10_i$ allows the Yukawa coupling matrix $Y^u = y^u_0 (J + r''I)$ in general. However, the mass hierarchy of up-sector quarks cannot be obtained unless $r''$ is suppressed to $O(10^{-5})$. 
Thus the approach of the democratic mass matrix is an attractive possibility phenomenologically, but the flavor symmetries do not support the origin of the assumed Yukawa couplings. In the next section, we consider the dynamical realization of the democratic Yukawa matrices without recourse to any flavor symmetries. There it is seen that we are no longer troubled with the fine-tuning problems.

§3. Flavor democracy through strong gauge dynamics

Our basic assumption is that the Yukawa couplings are neither hierarchical nor aligned, but may be even somewhat anarchy at the fundamental scale. The purpose of this section is to show that the Yukawa couplings can be aligned into a rigid democratic form quite minutely at lower energy scales, owing to the strong gauge dynamics. The realization of realistic matrices is discussed in a subsequent section. Also, we consider such a mechanism in GUT models, since the problem of flavor symmetry is very severe, especially in GUT.

3.1. IR fixed point for a single Yukawa coupling

Before studying the democratic Yukawa matrix, let us consider the IR fixed point in the manner of Pendelton and Ross\(^{16}\) in the case of a single Yukawa coupling. In general, the RG equations for the gauge coupling \(g\) and the Yukawa coupling \(y\) are given at the one-loop level as

\[
\frac{d\alpha_g}{d\mu} = -b\alpha_g^2, \quad (3.1)
\]

\[
\frac{d\alpha_y}{d\mu} = (a\alpha_y - c\alpha_g)\alpha_y, \quad (3.2)
\]

where we define \(\alpha_g = g^2/8\pi^2\) and \(\alpha_y = |y|^2/8\pi^2\). The coefficients depend on the field content and the Yukawa interaction, but note that \(a\) and \(c\) are positive constants. Then the ratio \(x = \alpha_y/\alpha_g\) satisfies the RG equation

\[
\mu \frac{dx}{d\mu} = [ax - (c - b)]\alpha_g x. \quad (3.3)
\]

This beta function has a non-trivial fixed point at \(x = x^* = (c - b)/a\), which is IR attractive.

The convergence behavior of the RG flows around the fixed point can be found with a linear analysis. The deviation from the fixed point, \(\Delta x = x - x^*\), satisfies the equation

\[
\mu \frac{d\Delta x}{d\mu} = (c - b)\alpha_g \Delta x. \quad (3.4)
\]

If the gauge coupling does not change rapidly, then the flow of \(x\) around the IR fixed point can be evaluated as

\[
\Delta x(\mu) \sim \left( \frac{\mu}{\Lambda} \right)^{(c-b)\alpha_g} \Delta x(\Lambda). \quad (3.5)
\]
We see that the conditions for strong convergence are as follows; (i) The gauge coupling is large. (ii) The coefficient \((c - b)\) is positive. Also a larger \((c - b)\) makes the convergence stronger. The second condition implies that the coefficient \(b\) should not be very large, and therefore that asymptotically free gauge theories with rapidly running gauge couplings are excluded.\(^\star\)

Such a fixed point appears in gauge theories in space-times of greater than four dimensions as well.\(^1\) In \(4 + \delta\) dimensions, the RG equations for the gauge coupling and the Yukawa coupling at the one-loop level are given by

\[
\frac{d\alpha_g}{d\mu} = \delta\alpha_g - b\alpha_g^2, \tag{3.6}
\]

\[
\frac{d\alpha_y}{d\mu} = \delta\alpha_y + (a\alpha_y - c\alpha_g)\alpha_y, \tag{3.7}
\]

where \(\alpha_g\) and \(\alpha_y\) are dimensionless couplings. Then the RG equation for \(x = \alpha_y/\alpha_g\) is found to be identical to Eq. (3.3). We should note some particular features of the extra-dimensional theories. First, the running of these couplings does not take a logarithmic form, but power-law form with respect to the renormalization scale. Also, the gauge coupling becomes strong in the ultraviolet direction independently of the sign of \(b\). For these reasons, convergence to the IR fixed point is found to be very strong in general.

3.2. Models with 3 flavors

If we extend the Yukawa interaction to the case of multiple flavors naively, then IR fixed points for the Yukawa matrices may exist, but they are proportional to the identity matrix. Thus, a democratic Yukawa coupling matrix cannot be obtained through the renormalization effect in MSSM. However, it turns out that this is possible once we admit multiple Higgs fields.

First, let us consider only the up-type Yukawa couplings in the superpotential given by Eq. (2.3) or Eq. (2.16). We introduce nine elementary Higgs superfields, \(H_{ij}(i, j = 1, 2, 3)\), with the same properties, and extend the Yukawa interactions as

\[
W = \sum_{i,j=1,2,3} Y_{ij} Q_i u_j H_{ij}. \tag{3.8}
\]

In the \(SU(5)\) GUT, we may take \(Q = u = 10\) and \(H = H(5)\). In the supersymmetric theories the beta functions for the Yukawa couplings \(Y_{ij}\) are written in terms of the anomalous dimensions of \(Q_i, u_i\) and \(H_{ij}\). This is due to the so-called non-renormalization of superpotentials. Explicitly, these anomalous dimensions can be written as

\[
\gamma_{Q_i} = \left[ a_Q (\alpha_{g i1} + \alpha_{g i2} + \alpha_{g i3}) - c_Q \alpha_g \right], \tag{3.9}
\]

\[
\gamma_{u_i} = \left[ a_u (\alpha_{y i1} + \alpha_{y i2} + \alpha_{y i3}) - c_u \alpha_g \right], \tag{3.10}
\]

\[
\gamma_{H_{ij}} = \left[ 3a_H \alpha_{y ij} - c_H \alpha_g \right]. \tag{3.11}
\]

\(^\star\) In asymptotically non-free gauge theories, it is easier to have strong convergence, since the gauge couplings become large at high energy, and \(b\) is negative.\(^2\)
where we have defined \( \alpha_{y_{ij}} = |Y_{ij}|^2/(8\pi^2) \) as well. Note that the above interactions in Eq. (3.8) induce anomalous dimensions only in the diagonal elements, which are given above. Then, the RG equation for \( \alpha_{y_{ij}} \) is simply given by

\[
\frac{d\alpha_{y_{ij}}}{d\mu} = \left( \gamma_{Q_i} + \gamma_{u_j} + \gamma_{H_{ij}} \right) \alpha_{y_{ij}}. \tag{3.12}
\]

It is easy to see the existence of a non-trivial fixed point by using the beta function given by Eq. (3.12). The RG equations for \( x_{ij} = \alpha_{y_{ij}}/\alpha_g \) are given by

\[
\mu \frac{dx_{ij}}{d\mu} = \left[ (b - c) + \left( \hat{\gamma}_{Q_i} + \hat{\gamma}_{u_j} + \hat{\gamma}_{H_{ij}} \right) \right] \alpha_g x_{ij}, \tag{3.13}
\]

where we have defined \( c = c_Q + c_u + c_H \) and also

\[
\hat{\gamma}_{Q_i} = a_Q(x_{i1} + x_{i2} + x_{i3}), \tag{3.14}
\]

\[
\hat{\gamma}_{u_i} = a_u(x_{1j} + x_{2j} + x_{3j}), \tag{3.15}
\]

\[
\hat{\gamma}_{H_{ij}} = 3a_H x_{ij}. \tag{3.16}
\]

Therefore, the condition for the non-trivial fixed point is that the combinations in the square braket in the Eq. (3.13) vanish for all sets of \((i, j)\). The solution is unique and is found to be

\[
x_{ij}^* = x^* = \frac{c - b}{3a}, \tag{3.17}
\]

where \( a = a_Q + a_u + a_H \).

A non-trivial but important matter is whether or not this fixed point is really IR attractive. This can be determined using linear analysis of the RG equations around the fixed point (3.17). The deviations from the fixed point \( \Delta x_{ij} \) satisfy the equation

\[
\mu \frac{d\Delta x_{ij}}{d\mu} = \alpha_g x^* \left( \begin{array}{cccccc}
& a' & a_Q & a_Q & a_Q & a_Q & a_Q & a_Q & a_Q & a_Q & a_Q & a_Q \\
& a_Q & a' & a_Q & a_Q & a_Q & a_Q & a_Q & a_Q & a_Q & a_Q & a_Q \\
& a_Q & a_Q & a' & a_Q & a_Q & a_Q & a_Q & a_Q & a_Q & a_Q & a_Q \\
& a_Q & a_Q & a_Q & a' & a_Q & a_Q & a_Q & a_Q & a_Q & a_Q & a_Q \\
& a_Q & a_Q & a_Q & a_Q & a' & a_Q & a_Q & a_Q & a_Q & a_Q & a_Q \\
& a_Q & a_Q & a_Q & a_Q & a_Q & a' & a_Q & a_Q & a_Q & a_Q & a_Q \\
& a_Q & a_Q & a_Q & a_Q & a_Q & a_Q & a' & a_Q & a_Q & a_Q & a_Q \\
& a_Q & a_Q & a_Q & a_Q & a_Q & a_Q & a_Q & a' & a_Q & a_Q & a_Q \\
& a_Q & a_Q & a_Q & a_Q & a_Q & a_Q & a_Q & a_Q & a' & a_Q & a_Q \\
& a_Q & a_Q & a_Q & a_Q & a_Q & a_Q & a_Q & a_Q & a_Q & a' & a_Q \\
& a_Q & a_Q & a_Q & a_Q & a_Q & a_Q & a_Q & a_Q & a_Q & a_Q & a'
\end{array} \right) \Delta x_{ij}, \tag{3.18}
\]

where \( a' = a_Q + a_u + 3a_H \). The eigenvalues of this matrix are \( 3a_H, 3a_H, 3a_H, 3a_H, 3(a_Q + a_H), 3(a_Q + a_H), 3(a_u + a_H), 3(a_u + a_H) \) and \( 3a \). It should be noted that all of these are positive, which ensures the IR attractiveness of the fixed point. Thus all Yukawa couplings can be aligned to the same value through the renormalization effect.\(^{*}\)

\(^{*}\) To be precise, what is aligned at low energy is not a Yukawa coupling but it is absolute value. The complex phase is not controlled by the dynamics. In this article, we treat the Yukawa couplings as if real, and we do not consider the complex phases.
It is possible also to reduce the number of Higgs to as few as three by assuming a $Z_3$ symmetry as follows. We assign the $Z_3$ charge $q_i = 2\pi/3i$ to $(Q_i, u_i, H_i)(i = 1, 2, 3)$, respectively. Then the $Z_3$ symmetry restricts the Yukawa interactions to

$$W = \sum_{i,j=1,2,3} Y_{ij} Q_i u_j H_{3-i-j}, \quad (3.19)$$

where the indices are defined modulo 3. With these interactions, the anomalous dimensions are diagonal. Explicitly, they are found to be

$$\gamma_{Q_i} = [aQ(\alpha_{y_1} + \alpha_{y_2} + \alpha_{y_3}) - cQ\alpha_g], \quad (3.20)$$

$$\gamma_{u_i} = [a_u(\alpha_{y_1} + \alpha_{y_2} + \alpha_{y_3}) - c_u\alpha_g], \quad (3.21)$$

$$\gamma_{H_i} = [aH(\alpha_{y_1} + \alpha_{y_2} + \alpha_{y_3}) - cH\alpha_g]. \quad (3.22)$$

The RG equations for $x_{ij} = \alpha_{y_{ij}}/\alpha_g$ are given by

$$\mu \frac{d\gamma_{ij}}{d\mu} = \left[(b-c) + \left(\gamma_{Q_i} + \gamma_{u_j} + \gamma_{H_{3-i-j}}\right)\right] \alpha_g x_{ij}, \quad (3.23)$$

where $c = cQ + c_u + c_H$ and $\hat{\gamma} = \gamma/\alpha_g$ again. It is immediately seen that couplings $x^*_ij = x^* = (c-b)/3a (a = aQ + a_u + a_H)$ give a set of fixed point solutions. However, it is found that this is not a unique solution. A linear perturbation about the fixed point $\Delta x_{ij}$ satisfies the differential equation,

$$\mu \frac{d\Delta x_{ij}}{d\mu} = \alpha_g x^* \left( \begin{array}{cccccccc} a & aQ & aQ & a_u & 0 & a_H & a_u & 0 \\ aQ & a & aQ & a_H & a_u & 0 & 0 & a_u \\ aQ & aQ & a & 0 & a_H & a_u & 0 & a_u \\ a_u & a_H & 0 & a & aQ & a_Q & 0 & a_H \\ 0 & a_u & a_H & 0 & a & aQ & a_Q & 0 \\ a_H & 0 & a_u & aQ & a & 0 & a_H & a_u \\ a_u & 0 & a_H & a_u & a_H & 0 & a & aQ \\ a_H & a_u & 0 & 0 & a_u & a_H & 0 & aQ \\ 0 & a_H & a_u & a_H & 0 & a_u & aQ & a \\ \end{array} \right) \quad (3.24)$$

where $a = aQ + a_u + a_H$. The eigenvalues of this matrix are found to be 0, 0, 2$aQ$, 2$a_u$, 2$a_H$, 2$a_H$ and 2$a$. The existence of two zero modes, namely constant modes, implies that the fixed point solution found above is not IR attractive. In other words, the Yukawa couplings obtained at low energy depend on their initial values, which are now assumed to be disordered.

However, additional flavor symmetry may solve this problem. Let us assume, e.g., the following discrete symmetry among matter fields;

$$Q_i \rightarrow Q_{i+1}, \quad u_i \rightarrow u_{i-1}, \quad H_i \rightarrow H_i. \quad (3.25)$$

This symmetry imposes additional relations on the Yukawa couplings, which are given by

$$x_{ij} = x_{(i+1)(j-1)} = x_{(i-1)(j+1)}. \quad (3.26)$$
Then, it is found that the zero modes in Eq. (3.24) are forbidden by this discrete symmetry. Also, a linear perturbation shows that the fixed point is unique and IR attractive. Thus, we have seen that the number of Higgs fields can be reduced by assuming some flavor structures in the fundamental theory. However, we do not consider this possibility further in this article.

3.3. Flavor democratic Higgs

To this point, we have seen that the gauge dynamics can align the Yukawa couplings to the same value in the IR limit, once many Higgs fields are introduced. However, this does not lead to a democratic mass matrix immediately. Only one mode composed of many Higgs fields, which is identified with the Higgs field in the MSSM, should be massless, and all others should be superheavy and decouple at low energy. Moreover, every fundamental Higgs field must contain the same amount of the massless mode, because otherwise the Yukawa couplings would differ from the democratic ones.

In practice, we can construct the mass terms for the Higgs fields that ensure the above properties.\(^{18}\) We return to the 9 Higgs model given by Eq. (3.8). In the case of \(SU(5)\) GUT, an example of the superpotential for the Higgs fields may be given by

\[
W = M \sum_i (H(5)_{i+1,j} - H(5)_{ij})(\bar{H}(\bar{5})_{i+1,j} - \bar{H}(\bar{5})_{ij}) \\
+ M \sum_j (H(5)_{i,j+1} - H(5)_{ij})(\bar{H}(\bar{5})_{i,j+1} - \bar{H}(\bar{5})_{ij}).
\]

(3.27)

Then, the mass terms for the Higgs scalar fields,

\[
M^2 \sum_i |H(5)_{i+1,j} - H(5)_{ij}|^2 + M^2 \sum_j |H(5)_{i,j+1} - H(5)_{ij}|^2 + \cdots,
\]

(3.28)

give rise to just one massless mode. Explicitly, the massless mode \(H\) is given by

\[
H = \frac{1}{3} \sum_{i,j} H_{ij}.
\]

(3.29)

The presence of the massless mode is ensured by the shift symmetry with respect to transformations,\(^*\)

\[
H(5)_{i,j} \rightarrow C, \quad \bar{H}(\bar{5})_{i,j} \rightarrow \bar{C},
\]

(3.30)

where \(C\) and \(\bar{C}\) are constants. Other modes acquire a mass of order \(M\), which gives the decoupling scale. In a later section, we take \(M\) to be the GUT scale.

The massless modes composed of \(H(5)\) and \(\bar{H}(\bar{5})\) are identified with the Higgs fields \(H^u\) and \(H^d\) in the MSSM, respectively. It is important to note that the Higgs fields \(H_{ij}\) in both sectors contain the zero mode \(H\) with same factor,

\[
H_{ij} = \frac{1}{3} H + \cdots,
\]

(3.31)

\(^*\) This structure is common to the deconstruction models\(^{23}\) where the massless mode is generated as a Nambu-Goldstone boson.
for all \((i, j)\). Therefore, the Yukawa matrix appearing in the MSSM is democratic, once the Yukawa couplings have been sufficiently aligned by the strong gauge dynamics.

It may be said that the basic principle in our consideration is flavor democracy. We do not assume any hierarchical structures in the initial Yukawa couplings. In addition, the superpotential for the Higgs field does not have a flavor difference. Therefore, there is no a priori essential difference among flavors.

§4. A model based on the \(SU(5) \times SU(5)\) GUT

Next we consider the mass hierarchy between the first two generations and the third generation. The small masses for the first and second generations are generated by slight deviations from the rigid democratic mass matrix. From our standpoint of flavor democracy, these deviations should be traces of disorder in the initial couplings. The ratio of the up and top quark masses is more than \(1 : 10^5\). On the other hand, the ratio of the GUT scale and the Planck scale, which we assume to be the fundamental scale, is only about \(1 : 10^3\). Therefore, we need very strong gauge interaction in order to realize such a hierarchy. We can roughly estimate the strength of the gauge coupling by assuming that it is not running. It is seen from Eq. (3.5) that the gauge coupling must satisfy

\[
g^2 \left(\frac{4\pi}{2}\right)^2 > \frac{5}{6(c-b)}. \tag{4.1}\]

This condition implies that the gauge coupling must be non-perturbatively large.

Naively, the gauge coupling constant of the \(SU(5)\) GUT is thought to be rather weak, since unification of the three gauge couplings in the MSSM occurs in a rather weak coupling regime. Therefore, it seems unlikely that the gauge coupling of GUT will become very strong. Actually, however, this is not the case. The gauge coupling unification is not destroyed in the presence of any extra heavy fields belonging to \(SU(5)\) multiplets. However, corrections resulting from the extra matter fields enlarge the unified gauge coupling. Thus, one way to realize strong unification is to assume a suitable number of extra heavy fields in the MSSM.\(^{18}\)

Here we consider another scenario, which may be more attractive.\(^*\) Suppose that the gauge group is given by a product of two \(SU(5)\) groups and is broken spontaneously to their diagonal subgroup at the unification scale. We represent the gauge couplings for the two groups \(SU(5)'\) and \(SU(5)''\) by \(g'\) and \(g''\), respectively. Here, we assume that all of the matter fields and the Higgs fields are charged under \(SU(5)'\) but form a singlet under \(SU(5)''\) and, moreover, that \(g'\) is non-perturbatively strong but \(g''\) is weak. Then, the RG for the Yukawa couplings is driven by the \(SU(5)'\) gauge interaction, and it may be attracted to their IR fixed point very rapidly. Also, the gauge coupling \(g\) for the diagonal subgroup, which is given by \(1/g^2 = 1/g'^2 + 1/g''^2\), is weak.

\(^*\) One may expect also that the power law running of the gauge coupling in the extra dimensions offers the possibility of models with strong convergence as well. However, we do not pursue this point because of a problem discussed in §7.
The superpotential to be considered here is the same as that given in Eq. (2.16), except that the Higgs field is extended to multiple fields:

$$W = Y^u_{ij} \, 10_i \, 10_j H(5)_{ij} + Y^d_{ij} \, 10_i \, \bar{5}_j H(\bar{5})_{ij} + Y^\nu_{ij} \, \bar{5}_i \, 1_j H(5)_{ij}. \quad (4.2)$$

To be explicit, the anomalous dimensions for the matter fields and the Higgs fields are given at the one-loop level as

$$\gamma_{10_i} = -\frac{36}{5} \alpha_g' + 3 \sum_{j=1}^{3} \alpha_y^{u}_{ij} + 2 \sum_{j=1}^{3} \alpha_y^{d}_{ij}, \quad (4.3)$$

$$\gamma_{\bar{5}_i} = -\frac{24}{5} \alpha_g' + 4 \sum_{j=1}^{3} \alpha_y^{d}_{ji} + \sum_{j=1}^{3} \alpha_y^{\nu}_{ij}, \quad (4.4)$$

$$\gamma_{1_i} = 5 \sum_{j=1}^{3} \alpha_y^{\nu}_{ji}, \quad (4.5)$$

$$\gamma_{H(5)_{ij}} = -\frac{24}{5} \alpha_g' + 6 \alpha_y^{u}_{ij} + \alpha_y^{\nu}_{ij}, \quad (4.6)$$

$$\gamma_{H(\bar{5})_{ij}} = -\frac{24}{5} \alpha_g' + 4 \alpha_y^{d}_{ij}, \quad (4.7)$$

where \( \alpha_g = g'^2/8\pi^2 \) and \( \alpha_y^{a}_{ij} = |Y_a^{ij}|^2/8\pi^2 \) for \( a = u, d, \nu \). The RG equations for the Yukawa couplings \( Y^u_{ij}, Y^d_{ij} \) and \( Y^\nu_{ij} \) are given in terms of these anomalous dimensions as

$$\mu \frac{d\alpha_y^{u}_{ij}}{d\mu} = \left( \gamma_{10_i} + \gamma_{10_j} + \gamma_{H(5)_{ij}} \right) \alpha_y^{u}_{ij}, \quad (4.8)$$

$$\mu \frac{d\alpha_y^{d}_{ij}}{d\mu} = \left( \gamma_{10_i} + \gamma_{\bar{5}_j} + \gamma_{H(\bar{5})_{ij}} \right) \alpha_y^{d}_{ij}, \quad (4.9)$$

$$\mu \frac{d\alpha_y^{\nu}_{ij}}{d\mu} = \left( \gamma_{1_i} + \gamma_{1_j} + \gamma_{H(5)_{ij}} \right) \alpha_y^{\nu}_{ij}. \quad (4.10)$$

In the next section, we consider a dynamical mechanism compatible with large mixing angles in the lepton sector. There, all of the neutrino Yukawa couplings \( Y^\nu_{ij} \) are caused to decrease to very small values. For this reason, we consider RG behavior of other Yukawa couplings by ignoring the neutrino Yukawa couplings in this section.

Now, the two kinds of Yukawa couplings \( \alpha_y^{u} \) and \( \alpha_y^{d} \) are coupled to each other in the RG equations. Despite of this complication, it is found that a non-trivial fixed point exists and is IR attractive. The coupling ratios \( x^{u}_{ij} = \alpha_y^{u}_{ij}/\alpha_g \) and \( x^{d}_{ij} = \alpha_y^{d}_{ij}/\alpha_g \) satisfy the following equations:

$$\mu \frac{dx^{u}_{ij}}{d\mu} = \left( -\frac{96}{5} + b + \sum_{k=1}^{3} \left( 3x^{u}_{ik} + 3x^{u}_{jk} + 2x^{d}_{ik} + 3x^{d}_{jk} \right) \right) \alpha_g' x^{u}_{ij}, \quad (4.11)$$

$$\mu \frac{dx^{d}_{ij}}{d\mu} = \left( -\frac{84}{5} + b + \sum_{k=1}^{3} \left( 3x^{u}_{ik} + 2x^{d}_{ik} + 4x^{d}_{kj} \right) \right) \alpha_g' x^{d}_{ij}. \quad (4.12)$$
Then, it is straightforward to find the non-trivial fixed point solution, which turns out to be
\[
x_{ij}^u = x_{ij}^{u*} = \frac{552 - 25b}{1050} \sim 0.53 - 0.02b, \tag{4.13}
\]
\[
x_{ij}^d = x_{ij}^{d*} = \frac{384 - 25b}{700} \sim 0.55 - 0.04b. \tag{4.14}
\]
Thus the Yukawa couplings are fixed at the GUT scale, once \( b \) is given. In any case, the Yukawa couplings become non-perturbatively large, accompanied by the gauge coupling. It is a rather tedious problem to verify the IR attractive nature of this fixed point by a linear perturbation. However, this is quite easy to see by solving the differential equations given by (4.11) and (4.12) numerically. In Fig. 1, the flow lines for \( x_{ij}^u \) and \( x_{ij}^d \) are displayed in the case \( b = 0, \alpha_{g'} = 1.0 \). Their initial conditions are chosen at random just for demonstration. It is seen that both couplings converge to their fixed point values very rapidly.

People have studied GUT models based on a product group for the purpose of avoiding the doublet-triplet splitting problem.\(^{19,20}\) Recently, models based on \( G = SU(5)' \times SU(5)'' \) also have been proposed in this context,\(^{20}\) where the gauge charge assignment for the Higgs fields is different from that above. It is assumed that \( H(\bar{5}) \) carries (1, \( \bar{5} \)) charges for the product group \( SU(5)' \times SU(5)'' \), while \( H(\bar{5}) \) carries (5, 1), as before. It is noted that the down-type Yukawa interactions in the superpotential given by Eq. (4.2) are no longer invariant. In order to generate the down-type Yukawa interactions, we introduce another field \( \Sigma \) belonging to (\( \bar{5}, 5 \)). We assume also that the spontaneous symmetry breaking \( SU(5)' \times SU(5)'' \to SU(5)_{\text{diag}} \) is induced by a vacuum expectation value of \( \Sigma \). Then, the invariant (but non-renormalizable) term
\[
\frac{Y_d^{ij}}{\Lambda} \ 10, \bar{5}_j H(\bar{5})_{ij} \Sigma \tag{4.15}
\]
can generate the down-type Yukawa couplings effectively after symmetry breaking. It seems rather difficult to investigate renormalization involved with non-renormalizable operators like this. However, we can also consider the corrections

Fig. 1. RG running of \( x_{ij}^u \) and \( x_{ij}^d \) for randomly chosen initial couplings. We assumed also \( b = 0 \) and \( \alpha_{g'} = 1.0 \). The parameter \( t \) represents the renormalization scale \( \mu \) by using \( \ln(\mu/M_{\text{pl}}) \), where \( M_{\text{pl}} \) is the Plank scale.
in the broken vacuum, where these operators play the role of the Yukawa interactions instead. Therefore, it is expected that the above analysis for the IR fixed point can be equally applied to this model.

As a general property of the present mechanism, the Yukawa couplings for the third generations are rather large at the GUT scale. The masses for the top quark, bottom quark and tau lepton are given in terms of the Yukawa couplings obtained after diagonalization. It is easy to find these Yukawa couplings renormalized at $O(100)$ GeV by solving the RG equations for the MSSM. The neutrino Yukawa couplings can be ignored in this analysis. Then, the low energy couplings are determined with respect to the strong gauge coupling $g'$ and the model parameter $b$. However, it is found that the resultant Yukawa couplings are practically insensitive to these parameters. This is because the initial Yukawa couplings are fairly large, and such flows converge to the so-called quasi-fixed point.\textsuperscript{21} To be explicit, the low energy couplings are found to be $Y_t = 1.00$, $Y_b = 0.94$ and $y_\tau = 0.62$ in the case of $\alpha g' = g'^2/8\pi^2 = 1.0$ and $b = 0$. It is seen first that $\tan \beta$ should be large as $O(50)$ in order to explain the mass ratio $m_t/m_b$, because $Y_t$ and $Y_b$ appear to be almost the same. Therefore the mass of the top quark is predicted to be the same as the vacuum expectation value of the neutral Higgs, 174 GeV. This is in a very good agreement with observation. We can also predict mass ratio of bottom quark and tau lepton, $m_b/m_\tau$. The above couplings yield $m_b/m_\tau = 1.52$, which is slightly smaller than the expected value.\textsuperscript{*}

\section{5. Neutrino sector}

In the previous section, we saw that the Yukawa couplings are aligned into democratic forms very well. The deviations from an exact democratic form are responsible for masses of the first two generations and mixing angles among quarks. However, the mixing matrix necessarily becomes close to the identity matrix for any small deviation. If we consider the neutrino Yukawa couplings, ignored in the previous section, into account, then these couplings are found to be attracted to an IR fixed point by strong dynamics as well as other Yukawa couplings. Obviously this is incompatible with the observed large mixing angles for leptons.

In this respect, it is phenomenologically favorable for the neutrino Yukawa matrix to be nearly diagonal at low energy. Suppose that the neutrino Yukawa couplings do not have a non-trivial fixed point for some reasons. Then these couplings cannot be aligned to a democratic form and become dependent on their initial couplings at the Planck scale. Also, we can find a sizable parameter region for the initial couplings consistent with the observed neutrino mass and mixings.

In practice, such a situation is realized with a simple assumption, since the right-handed neutrino is a singlet of $SU(5)$. In general, GUT models may contain a pair of superheavy vector-like fields, and so on. Then the right-handed neutrino is allowed to

\textsuperscript{*} In the case with large $\tan \beta$, we have to take into account large SUSY threshold corrections to the bottom mass.\textsuperscript{24}
have additional Yukawa interactions with them. These Yukawa couplings are driven to be very large, like the others, by the strong gauge interactions. Consequently the anomalous dimension of the right-handed neutrino is significantly enhanced. We note that the IR fixed point depends on a balance between the negative contribution of gauge interactions and the positive contribution of Yukawa interactions. The presence of extra Yukawa couplings may destroy this balance and cause the IR fixed point to move away from the neutrino Yukawa couplings. Explicitly, the coefficient $c$ in the fixed point equation (3.13) is made negative by the additional radiative corrections. Thus, the large difference between the mixing matrices of the quark and lepton sectors can be attribute to the dynamics of the right-handed neutrino.

We now demonstrate the above mechanism by examining the RG equations only for the neutrino Yukawa couplings. As a toy model with a single flavor, let us consider the superpotential given by

$$W = Y^\nu \bar{5} 1 H(5) + \kappa 1 \Phi \bar{\Phi}, \quad (5.1)$$

where the extra matter fields $\Phi$ and $\bar{\Phi}$ belong to the $SU(5)$ representations $R$ and $\bar{R}$, respectively. The RG equations for $\alpha_y = |Y^\nu|^2/8\pi^2$ and $\alpha_\kappa = |\kappa|^2/8\pi^2$ are found to be

$$\mu \frac{d\alpha_y}{d\mu} = [7\alpha_y + R\alpha_\kappa - 4C_2(5)\alpha_y] \alpha_y, \quad (5.2)$$

$$\mu \frac{d\alpha_\kappa}{d\mu} = [5\alpha_y + (R+2)\alpha_\kappa - 4C_2(R)\alpha_y] \alpha_\kappa, \quad (5.3)$$

where $C_2$ denotes the Casimir index of each representation. Also, these can be rewritten into equations in terms of $x_y = \alpha_y/\alpha_g$ and $x_\kappa = \alpha_\kappa/\alpha_g$ as

$$\mu \frac{dx_y}{d\mu} = [7x_y + Rx_\kappa - 4C_2(5) + b] \alpha_g x_y, \quad (5.4)$$

$$\mu \frac{dx_\kappa}{d\mu} = [5x_y + (R+2)x_\kappa - 4C_2(R) + b] \alpha_g x_\kappa. \quad (5.5)$$

There are four fixed points in the coupling space of $(x_y, x_\kappa)$, three of which are immediately seen from the above equations as

$$(x_y, x_\kappa) = (0, 0), \quad (5.6)$$

$$= \left( \frac{4C_2(5) - b}{7} , 0 \right), \quad (5.7)$$

$$= \left( 0, \frac{4C_2(R) - b}{R+2} \right). \quad (5.8)$$

The first two fixed points are what we have been studying to this point, and the third one is new. It is noted that the fourth one appears in the region of negative $\alpha_y$ for a large representation $R$. Therefore, this fixed point is unphysical, and also it is not IR attractive. Here we consider such a case.

$^*)$ $R$-parities for the extra fields may be assigned properly.
The RG flow diagram for \((x_y, x_\kappa)\) is shown in Fig. 2 in the case of \(b = 0\) and \(R = \text{adj}\) as an example. The marked points A, B and C are the fixed points given by (5.6), (5.7) and (5.8), respectively. It is seen that the fixed point B is now unstable in the direction of the new coupling, while C is IR attractive instead. Thus, the neutrino Yukawa coupling is found to decrease at low energy in the presence of \(\kappa\).

In Fig. 3 the evolution of the neutrino Yukawa coupling with respect to the scale is shown. Each line corresponds to a RG flow presented in Fig. 2. It is found that the Yukawa couplings decrease monotonically at low energy and are not aligned. It is expected that the neutrino Yukawa couplings for multiple flavors have the same properties as these flows, even though the other Yukawa couplings, \(Y_u\) and \(Y_d\), are aligned to their fixed points. We now add the terms

\[
Y_{ij}^\nu \bar{\nu}_5 \lambda (5)_{ij} + \kappa_i \phi \bar{\phi} + M_{Rij} \lambda \lambda_j,
\]

to the superpotential given by Eq. (4.2). It is easily found that ratio of the neutrino Yukawa couplings satisfies the RG equation

\[
\mu \frac{d}{d\mu} \ln \left( \frac{\alpha_{yik}^\nu}{\alpha_{yjk}^\nu} \right) = \left( \gamma_{5i}^* + \gamma_{1k} + \gamma_{H(5)} \right) - \left( \gamma_{5j}^* + \gamma_{1k} + \gamma_{H(5)} \right)
\]

\[
= a_5 \left( \sum_k \alpha_{yik}^\nu - \alpha_{yjk}^\nu \right) + 3a_H \left( \alpha_{yik}^\nu - \alpha_{yjk}^\nu \right),
\]

where the Yukawa couplings other than \(\alpha_{y_i}^\nu\) are assumed to take their fixed point values. Because the neutrino Yukawa couplings are suppressed, as seen above, the right-hand side of (5.10) becomes very small quickly. This shows that the ratio of the neutrino couplings is almost unchanged in the low energy region.

In the Frogatt-Nielsen mechanism, \(^6\) the neutrino Yukawa couplings are constrained except for the \(O(1)\) coefficient, and, therefore, the masses and mixing angles for neutrinos are not predicted. However, we find a sizable probability for the cou-
plings distributed in anarchy to be compatible with the observed masses and mixing angles.\(^{25}\)

Now, if three of the nine initial neutrino Yukawa couplings happen to be larger than the others to some extent, then they may be dominant in the low energy matrix also. Since there is no specific flavor basis, we may regard these couplings as the diagonal elements. Thus, it is not so special that the neutrino Yukawa coupling matrix is close to diagonal, and therefore the large lepton mixing angles are described well. We leave an explicit survey of such a parameter space for future works.

In our mechanism, the neutrino Yukawa couplings are strongly suppressed as seen in Fig. 3. However, this does not imply that the see-saw mechanism yield very small neutrino masses. Note that this suppression occurs because of enhancement of anomalous dimensions of the right-handed neutrinos. Then, their Majorana masses \(M_R\) are also suppressed by the anomalous dimensions. Consequently, it is found that the neutrino masses themselves are not suppressed through this mechanism. It is noted that the masses for the right-handed neutrinos are expected to appear at the intermediate scale in our scenario, even though the bare parameter for \(M_R\) is set to the GUT scale.

§6. Setup for realistic mass matrices

How can we describe the mass differences among the first and the second generations? In the phenomenological approach, the two flavor symmetry breaking parameters \(\epsilon\) and \(\delta\) are taken to be hierarchical as given in Eq. (2.6). In addition, it is necessary for these breaking parameters to be diagonal elements, which is not explained from the standpoint of flavor symmetry either.

Now, the problem to be considered is how to realize such a specific form of deviations from the democratic couplings in our dynamical framework. The first possibility is to generate such deviations away from the fixed point by assuming proper initial couplings. We consider this by examining the RG equations given by Eq. (3.13) again. Suppose, e.g., that one of the Yukawa couplings, say \(Y_{33}\), happens to be significantly smaller than the others at the Planck scale. Then \(x_{ij}\) for \(i, j = 1, 2\) converge to the fixed point value rapidly, because the beta functions for these couplings do not contain \(x_{33}\). The couplings \(x_{i3}\) and \(x_{3i}\) for \(i = 1, 2\) may be affected by \(x_{33}\). Therefore we can examine the RG flows while ignoring differences among the \(x_{ij}\) for \(i, j = 1, 2\) in the first step of the approximation. In this situation, the deviations from the fixed point \(\Delta x_{ij} = x_{ij} - x^*\) are found to satisfy

\[
\frac{d}{d\mu} [\Delta x_{13} - \Delta x_{23}] = \alpha_g x^*(a' - a_u) [\Delta x_{13} - \Delta x_{23}],
\]

\[
\frac{d}{d\mu} [\Delta x_{31} - \Delta x_{32}] = \alpha_g x^*(a' - a_Q) [\Delta x_{31} - \Delta x_{32}].
\]

These equations reveal that these differences shrink rapidly as the energy decreases and eventually the couplings satisfy the relations

\[ x_{13} = x_{23}, \quad x_{31} = x_{32}. \]
Moreover, the differences among \(x_{ij}\) for \(i, j = 1, 2\), which are ignored in the above argument, are found to satisfy

\[
\frac{d}{d\mu} [\Delta x_{11} - \Delta x_{21}] = \frac{d}{d\mu} [\Delta x_{12} - \Delta x_{22}] = \alpha_g x^* a_Q [\Delta x_{13} - \Delta x_{23}],
\]

\[
\frac{d}{d\mu} [\Delta x_{11} - \Delta x_{12}] = \frac{d}{d\mu} [\Delta x_{21} - \Delta x_{22}] = \alpha_g x^* a_u [\Delta x_{31} - \Delta x_{32}].
\]  

(6.4)

(6.5)

Therefore, it is seen that these differences are not affected by a large deviation of \(x_{33}\) and remain small. In Fig. 4, an example of the RG flows for the couplings \(x_{ij}\) obtained by numerical analysis is displayed. The bold line represents the flow of \(x_{33}\), whose initial coupling is assumed to be relatively smaller than the others. Note that the Yukawa coupling \(Y_{33}\) itself is not far from the others. The dashed lines represent the flows of \(x_{i3}\) and \(x_{3i}\) for \(i = 1, 2\), while the dotted lines represent the flow of \(x_{ij}\) for \(i, j = 1, 2\). Indeed, the behavior of these RG flows supports the above argument.

Similar forms of Yukawa coupling matrices can be obtained by introducing a weak interaction breaking the flavor democracy. For example, we assume that only one of the Higgs fields, say \(H_{33}\), has an extra superpotential, \(\lambda H_{33} \Phi_1 \Phi_2\), with very small coupling \(\lambda\). We consider a linear perturbation around the fixed point. The extra Yukawa interaction affects only the beta function of \(\Delta x_{33}\). To be explicit, the RG equations given in Eq. (3.18) are modified to

\[
\frac{d\Delta x_{ij}}{d\mu} = \alpha_g x^* [(M \Delta x)_{ij} + \delta \lambda x^* \delta_{i3} \delta_{j3}],
\]

(6.6)

where \(M\) denotes the matrix given in Eq. (3.18) and \(\delta \lambda x^* \sim |\lambda|^2/8\pi^2\).

When differences among the initial Yukawa couplings happen to be rather small already, Eqs. (6.1), (6.2), (6.4) and (6.5) hold as well.

In the end, the Yukawa couplings take the form

\[
Y = y_0 \begin{pmatrix}
1 + O(\epsilon) & 1 + O(\epsilon) & 1 + \delta' \\
1 + O(\epsilon) & 1 + O(\epsilon) & 1 + \delta'' \\
1 + \delta'' & 1 + \delta'' & 1 + \delta
\end{pmatrix},
\]

(6.7)

at low energy scales. Here, \(\epsilon\) represents the incompleteness of the strong convergence to the fixed point, which can be as small as \(10^{-5}\). Other parameters representing deviations from the fixed point, \(\delta, \delta'\) and \(\delta''\), are found to be of the same order.
Though these values depend on the initial Yukawa couplings, we can assume the relation \( \epsilon \ll \delta \ll 1 \). After transforming this matrix \( Y \) by using the diagonalizing matrix given by Eq. (2.2), we obtain

\[
A^T Y A = y_0 \begin{pmatrix}
O(\epsilon) & O(\epsilon) & O(\delta) \\
O(\epsilon) & O(\delta) & O(\delta) \\
O(\delta) & O(\delta) & 3 + O(\delta)
\end{pmatrix}.
\] (6.8)

This is very similar to the Fritzsch-type matrix and, therefore, offers us a phenomenologically favorable pattern for realistic masses and mixing angles. The ratios of the mass eigenvalues are given by \( O(\epsilon) : O(\delta) : O(1) \). We do not explicitly study the question of which initial couplings actually give viable mass matrices, because there is a strong model dependence.

§7. Conclusions and discussion

The approach of a democratic mass matrix seems to be attractive phenomenologically. The concept of flavor democracy offers an interesting viewpoint distinct from other approaches. Above all, it is noted that the lepton mixing angles, bi-large mixings and small \( U_{e3} \) are given simply by assuming a democratic mass matrix for the charged leptons.

However, it is also true that these phenomenologically favorable matrices are not explained by flavor symmetries. In this paper, we considered models in which the democratic type of Yukawa couplings are realized as an IR attractive fixed point.\(^{18}\) For this purpose, we introduced multiple Higgs fields, from which only one massless mode is allowed by a special type of mass terms. The large mass hierarchies are realized easily by using a strong gauge interaction for quarks and leptons as well as the Higgs fields. To be explicit, we considered an \( SU(5) \times SU(5) \) GUT model, in which one of the gauge couplings is non-perturbatively strong.

Indeed, the setup of the Higgs fields may be somewhat artificial. However, we found some advantages of this scenario. The democratic Yukawa couplings are realized even in GUT models without any adhoc assumptions. Hence, small mixing angles as well as large mass hierarchies in the quark sector are naturally obtained. In particular, the large lepton mixing angles can be explained through extra interactions of the right-handed neutrinos. It is seen that a simple assumption on the initial couplings may lead to low energy Yukawa couplings of the Fritzsch type. This kind of scenario also predicts a top quark mass in good agreement with experiment and a large \( \tan \beta \).

We are not concerned with the complex phases of the Yukawa couplings and simply assuming them to be real. Therefore, the origin of the \( CP \) violating phases in the quark and lepton sectors is not explained in our models. On the other hand, complex phases may be introduced by assuming complex flavor symmetry breaking parameters \( \delta \) and \( \epsilon \) in the phenomenological mass matrices. However, since we have not found comprehensive explanations for these parameters in the flavor symmetry approach, the origin of \( CP \) violation is also unclear. In our model, the phases of the Yukawa couplings are not aligned by the renormalization effect, as noted in §3.
Hence, explanation of the complex phase still seems to be difficult. A possible method of generating complex phases for the deviations from the fixed point couplings is to introduce small perturbations through complex couplings other than the Yukawa couplings. In any case, it is beyond the scope of this paper to explain the $CP$ violating phases as well.

In this article, we considered only supersymmetric models. However, the PR fixed point exists in non-supersymmetric theories as well. Therefore, the democratic mass matrices may also be realized in some non-supersymmetric models, though the beta functions for Yukawa couplings are complicated.

In §3 we mentioned that the PR fixed point appears also in extra dimensions and exhibits rather strong convergence in general. However, a naive extension with gauge and Higgs fields in the bulk of the extra dimensions is not effective. This is because the anomalous dimensions of the Higgs fields vanish due to the effective $N=2$ supersymmetry, and, therefore, the democratic-type fixed point is not realized. This does not rule out any possibility of extra-dimensional models, and we leave this for future study.

Finally, some comments on soft supersymmetry breaking parameters are in order. It is known that the PR fixed point induces peculiar relations among soft parameters also. Above all, $A$-parameters are found to be aligned in the present scenarios with the same dynamics for the Yukawa coupling alignment. The soft scalar masses for quarks and leptons become flavor universal at low energy due to large corrections by the democratic Yukawa couplings as well as the gauge couplings. These properties are very desirable for the flavor problem in supersymmetric extensions.

However, the strong gauge dynamics causes the soft scalar masses and the $A$-parameters to increase and become comparable with the strongly coupled gaugino mass. Therefore, the gaugino mass in the strongly coupled sector must be much smaller than the MSSM gaugino masses. In this respect, the extension to extra dimensions is also considerable, because FCNC and also $CP$ problems may be solved or ameliorated without assuming such a special case. Otherwise, it would be more natural in our scenarios that supersymmetry breaking is mediated below the GUT scale by, e.g., the gauge mediation mechanism. Further studies of the supersymmetry breaking parameters in our models will be discussed elsewhere.

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\footnote{Various flavor violating processes have been examined in the phenomenological approach of the democratic mass matrices.}
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