Are Horned Particles the Climax of Hawking Evaporation?

T. Banks\(^1\), A. Dabholkar\(^2\), M. R. Douglas\(^3\), M. O’Loughlin\(^4\)
Dept. of Physics and Astronomy
Rutgers University
Piscataway, NJ 08855-0849

We investigate the proposal by Callan, Giddings, Harvey and Strominger (CGHS) that two dimensional quantum fluctuations can eliminate the singularities and horizons formed by matter collapsing on the nonsingular extremal black hole of dilaton gravity. We argue that this scenario could in principle resolve all of the paradoxes connected with Hawking evaporation of black holes. However, we show that the generic solution of the model of CGHS is singular. We propose modifications of their model which may allow the scenario to be realized in a consistent manner.

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\(^1\) (banks@physics.rutgers.edu)
\(^2\) (atish@physics.rutgers.edu)
\(^3\) (mrd@physics.rutgers.edu)
\(^4\) (ologhlin@physics.rutgers.edu)
1. Introduction

The apparent paradoxes of the Hawking process of quantum mechanical decay of a black hole have been a source of confusion and inspiration to theoretical physicists since Hawking’s groundbreaking papers in the mid seventies [1]. Hawking proposed that these paradoxes might lead to a generalization of quantum mechanics in which pure states can evolve into mixed states. Alternative scenarios in which coherence is preserved have often revolved around the idea that the endpoint of black hole decay would be some sort of exotic elementary particle. The apparent validity of Hawking’s analysis down to black hole masses where quantum gravitational or string theoretic effects become important provides strong constraints on such a scenario.

Many authors have expressed discomfort with the apparent information loss in the decay of a black hole into a stable elementary particle. This somewhat vague argument can be sharpened in those cases in which one deals with a theory containing global conserved quantum numbers such as baryon number. It is easy to construct models in which Hawking’s analysis is valid until the black hole is so light that it cannot emit enough ordinary particles to carry its baryon number. While the force of this argument is somewhat vitiated in the real world by current theoretical prejudices against the existence of global conserved quantum numbers, it is still a valid objection to many theoretical models. In such models one must postulate an infinite spectrum of almost degenerate elementary particles representing the hypothetical endpoints of the evolution of black holes with different values of baryon number. This leads to apparent paradoxes with thermodynamics and with many of the usual rules of quantum field theory, unless one postulates that the coupling between ordinary particles and these black hole remnants is extraordinarily weak for some unspecified reason.

There has for a long time been a widespread feeling that these problems would not be resolved until the correct short distance theory of gravity was discovered. Einstein’s gravitational theory certainly needs modification at small length scales if it is to be made compatible with quantum mechanics. It is hoped that the correct modification would also resolve the paradoxes of Hawking radiation, either at the classical or quantum mechanical level. There is, it seems to us, one bit of hope that the understanding of Hawking radiation
will not have to await the perhaps distant day when a consistent nonperturbative quantum theory of gravity is discovered. This is related to the fact that extreme Reissner Nordstrom (RN) black holes have zero Hawking temperature. The geometry of an extreme RN hole of large charge is smooth and varies on length scales much larger than the Planck scale. Perturbing this solution with a small amount of neutral matter one obtains a black hole of small Hawking temperature. It is plausible to conjecture that the evaporation of this black hole proceeds back to the extreme Reissner Nordstrom solution and then stops. Since all the physics takes place at large length scales, we should be able to describe this process without recourse to the short distance modifications of Einstein’s theory.\footnote{This idea has occurred to a large number of authors over the past fifteen years. As far as we know, it has been independently arrived at by one of us (T.B.), C.Callan, D. Gross, E. Martinec, J. Preskill, L. Susskind, F. Wilczek, and E. Witten. Probably the list is much longer. The only published versions of these ideas of which we are aware are\footnote{Either in general relativity or stringy dilaton gravity.}} This idea looks even more attractive when one considers the extremal black hole of the version of dilaton gravity that follows from string theory\footnote{The authors of this paper present their results in the context of the two dimensional effective}. When interpreted in terms of the correct metric variable, this is a completely nonsingular classical spacetime geometry with no event horizons. The idea that Hawking evaporation of black holes of large charge might terminate at this configuration is extremely compelling.

Alas, things are not so simple. Generic classical perturbations of the extreme RN solution\footnote{Either in general relativity or stringy dilaton gravity.}, including those caused by dropping in a bit of extra mass, lead to singularities. In the neighborhood of the singularities curvatures become infinite and one might imagine that the short distance theory becomes relevant. On the other hand, it seems a bit implausible that one would have to include high mass string modes to describe the evolution of a large smooth classical geometry. One might hope that quantum mechanical fluctuations of long wavelength gravitational fields were sufficient to tame these singularities. The question is how to isolate the relevant degrees of freedom and quantize them in a manner consistent with general covariance.

Recently, major progress towards a resolution of this problem has been made in a beautiful paper by Callan, Giddings, Harvey, and Strominger (CGHS)\footnote{They considered...}. They considered...
the extremal charged black hole derived from string theory in the work of Gibbons and Maeda, and Garfinkle et. al. The solution is spherically symmetric and a generating section of its spatial geometry is shown in fig. 1. It is an infinite horn stuck onto flat space, and has two asymptotic regions, “down the horn” and “out at infinity.” Nonextremal perturbations of this solution have horizons and singularities, but for small perturbations, the horizons are down the horn and the singularities are behind them. It is thus plausible that we can isolate the degrees of freedom involved in the quantum mechanical decay of the hole, and the possible elimination of both the singularity and the horizon, by constructing an effective two dimensional field theory for massless fields in the horn. CGHS were thus led to study a model of two dimensional gravity coupled to a scalar (dilaton) field and several conformally coupled scalar matter fields, which may be thought of as dimensionally reduced Ramond-Ramond fields from the type II superstring. The classical equations of motion of this theory were exactly soluble and the solutions included the extremal black hole (whose two dimensional representation is flat space time with a linear dilaton), as well as solutions in which Ramond Ramond matter collapses onto the extremal hole to form a singular hole of larger mass.

By adding a large number of RR fields to their Lagrangian, CGHS obtained a model whose quantum mechanics could be systematically studied in the $\frac{1}{N}$ expansion. They studied the semiclassical large N equations of motion and argued that the collapsing solutions of the classical equations became nonsingular, horizon free solutions of the modified equations. The collapsing matter forms a “zero energy bound state” concentrated in the asymptotic region “down the horn”. There are an infinite number of such states, and they carry zero ADM energy as measured by an observer in the other asymptotic region. They can however carry global conserved charges. We will argue in the last section of this paper that if the CGHS results are correct they suggest a description of the final state of black hole evaporation in terms of quantum fields interacting with “horned particles” that carry an infinite number of degrees of freedom. The concept of a “horned particle” or
cornucopion\textsuperscript{4} is based on the classical geometry of the extremal dilaton black hole\textsuperscript{3}, and seems to eliminate the paradoxes associated with scenarios in which black hole evaporation ends in a stable object.\textsuperscript{5} From the point of view of an external observer the black hole shrinks until it occupies a very small region in space. However, the infinite horn of the GHS solution is a repository for an infinite number of quantum states that are degenerate in ADM energy but very difficult to excite with an external probe. Following CGHS, we will argue that this property of the horn makes it a perfect answer to the conventional conundrums of black hole evaporation.

It thus seems critically important to understand the semiclassical equations studied by CGHS and verify that they have the sort of solutions that the arguments of those authors suggest that they have. This is what we have attempted to do in this paper. Unfortunately, we find instead that the collapsing solutions all have singularities. The classical Lagrangian used by CGHS becomes degenerate in the region of dilaton field associated with strong string coupling. When the string coupling becomes strong, the kinetic energy of the fields vanishes. The effect of large N quantum corrections is to add a term (the Liouville term) to the kinetic energy which does not vanish in this region of field space. However, if the conformal gauge Lagrangian is viewed as a nonlinear sigma model in two dimensional target space, the metric on this target space is degenerate along a line where the dilaton field takes on a certain finite value $\phi_0$. The effect of the Liouville term has been to move the degeneracy to a finite place in field space. We are then able to show that if the dilaton field attains the value $\phi_0$ at some point in spacetime, a generic solution of the equations will have singular curvature at this point. In particular, this occurs for the solutions corresponding to collapsing shock waves, which were studied by CGHS. This analysis is presented in the next section.

The only solutions of the CGHS equations which pass through $\phi_0$ without encountering a singularity, are singular conformal transforms of the linear dilaton solution. We can find

\textsuperscript{4} This name combines a description of the classical dilaton black hole geometry, with the notion that the infinite horn of the hole is full of unexpected information.

\textsuperscript{5} The suggestion that infinitely degenerate particles could resolve the unitarity puzzle of black holes was apparently first made in\textsuperscript{3}
such a solution of the equations for every incoming $f$-wave. The resulting spacetimes are flat but geodesically incomplete. We believe that they represent the “zero energy bound states” or cornucopions, envisioned by CGHS as the asymptotic state of gravitational collapse. However, their geodesic incompleteness, and the singular nature of solutions with shock wave boundary conditions, imply that within the context of the two dimensional theory, there is no way to connect these solutions on to those obeying classical collapse boundary conditions. We speculate that this may be possible in the context of the higher dimensional theory in which the CGHS Lagrangian is embedded.

Finally, in the last section of the paper we disregard the difficulties that we have uncovered and present an effective description of the interactions of ordinary quantum fields with a single cornucion. Our description is completely compatible with general covariance and with quantum mechanics.

2. Degenerate Lagrangians With Singular Solutions

Review of the work of CGHS

Quantum gravity coupled to a large number $N$ of free scalar fields, in the limit $\hbar \to 0$ with $N\hbar$ fixed, is a candidate for a system in which the leading order of a controlled perturbative expansion is a quantum theory of matter in classical geometry. If we integrate out the matter, the path integral has the form

$$Z = \int [dg] \exp - \frac{1}{\hbar} (S_G[g] + \frac{N\hbar}{2} \log \det(\Delta[g] + m^2)) \quad (2.1)$$

and we have an effective action describing the back reaction of matter and Hawking radiation on the geometry, which we can hope to treat classically. This theory is problematic in four dimensions; the effective action for the matter is non-local, and the classical equations of motion appear ill-defined. The effective action has divergent terms proportional to invariants formed from the squares of curvatures. To renormalize the theory one has to admit these terms in the classical action, and they lead to instability in classical gravity.
CGHS have exploited the fact that these counterterms do not arise in two dimensions.

For the classical gravitational action they use a theory of “dilaton gravity” with the action

\[ S = \frac{1}{2\pi} \int d^2 x \sqrt{-g} \left[ e^{-2\phi} (R + 4(\nabla \phi)^2) - \frac{1}{2}(\nabla f)^2 \right] \]  

(2.2)

Here \( g, \phi \) and \( f \) are the metric, dilaton, and matter fields, respectively, and \( \lambda^2 \) is a cosmological constant.

In two dimensions, integrating out conformally coupled matter fields gives the Polyakov action \([8]\). This action consistently describes Hawking radiation and its back reaction on the metric\([9]\). The full effective action in conformal gauge is then\([9]\):

\[ S_{\text{eff}} = \int d^2 e^{-2\phi} (-2\partial_+ \partial_- \rho + 4\partial_+ \phi \partial_- \phi - \lambda^2 e^{2\rho} + \kappa \partial_+ \rho \partial_- \rho - \frac{1}{2} \partial_+ f \partial_- f) \]  

(2.3)

where \( \kappa \equiv N\hbar/12 \).

The equations of motion and constraint equations in this gauge are:

\[ 0 = \frac{\delta S}{\delta \phi} = e^{-2\phi} (4\partial_+ \partial_- \rho + 8\partial_+ \phi \partial_- \phi - 8\partial_+ \partial_- \phi + 2\lambda^2 e^{2\rho}) \]  

(2.4)

\[ 0 = \frac{\delta S}{\delta \rho} = G_{++} = 2e^{-2\phi} (2\partial_+ \partial_- \phi - 4\partial_+ \phi \partial_- \phi - \lambda^2 e^{2\rho} - 2\kappa \partial_+ \partial_- \rho) \]  

(2.5)

\[ 0 = \frac{\delta S}{\delta g_{++}} = T_{++} - G_{++} = \frac{1}{2}(\partial_+ f)^2 + e^{-2\phi} (4\partial_+ \phi \partial_+ \rho - 2\partial_+^2 \phi) - \kappa ((\partial_+ \rho)^2 - \partial_+^2 \rho + t_+(x^+)) \]  

(2.6)

(resp. \( T_{--} \)).

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6 As we intimated in the introduction this form is motivated by the low energy limit of string theory. It is our hope that a regime exists in which semiclassical, field-theoretic corrections eliminate a singularity of the classical theory. If this hope fails in the theory motivated by classical string theory, all is not lost. One could imagine broadening the class of two dimensional Lagrangians under consideration, to the more general theories of \([7]\). Even starting from string theory, one might be able to justify such an extension of the class of Lagrangians. The difficulty that we will encounter in the CGHS theory occurs at moderate values of the (large \( N \) scaled) string coupling. Stringy quantum corrections might easily give the effective low-energy action a more complicated dilaton dependence in this regime.

7 We note in passing that if we rescale the metric by a conformal factor which is a function of the dilaton, the effect is to produce one of the more general actions described in \([6]\).
The non-locality of the Polyakov action before gauge fixing has a remnant in the constraint equations— in conformal gauge, these are determined locally by conservation, but only up to two arbitrary functions $t_+(x^+)$ and $t_-(x^-)$. These functions must be determined by boundary conditions, in particular by the choice of incoming and outgoing quantum states. Furthermore they are not components of a tensor, but rather transform in a way which compensates the anomalous transformation law of the term $(\partial_+ \rho)^2 - \partial_+^2 \rho$ in the Liouville stress tensor. We note that the choice of $t_{\pm}$ is not a full specification of the quantum state of the system. The matrix element of the quantum stress tensor between any two states of a conformally invariant field theory is determined by two functions each of which depends on only one of the light cone coordinates.

The general solution of these equations with $\kappa = 0$ was obtained in [4]. With $f = 0$, we have a one-parameter family of static solutions, which in a certain coordinate system are

$$e^{-2\phi} = \frac{M}{\lambda} - \lambda^2 x^+ x^-$$

$$= e^{-2\rho} \quad (2.7)$$

For $M = 0$ the metric is flat and the dilaton linear in $\log x^+ x^-$, while $M > 0$ are “black hole” solutions. Since the $f$ matter is massless, it propagates freely along null geodesics,

$$f = f_+(x^+) + f_-(x^-). \quad (2.8)$$

The black hole solution can be obtained by sending in matter from the weak coupling region. The simplest case to consider is an $f$ shock-wave: as in [4]; take its stress tensor to be

$$\frac{1}{2} \partial^+ f \partial f = a \delta(x^+ - x^+_0); \quad (2.9)$$

the solution is then

$$e^{-2\rho} = e^{-2\phi} = -a(x^+ - x^+_0) \Theta(x^+ - x^+_0) - \lambda^2 x^+ x^-.$$ \quad (2.10)

For $x^+ < x^+_0$, this is simply the linear dilaton vacuum while for $x^+ > x^+_0$ it is identical to a black hole of mass $a x^+_0 \lambda$ after shifting $x^-$ by $a/\lambda^2$. The two solutions are are joined along the $f$-wave.
The $f$ shock wave will suffice to illustrate the properties of the $\kappa \neq 0$ system. In fact, by rescaling $\lambda$ and the coordinates, we can transform an $f$ wave of any finite extent to have arbitrarily small extent, so a smooth, non-singular solution must have a shock wave limit with non-singular metric and dilaton.

Ignoring back reaction, one finds Hawking radiation, with energy flux approaching a non-zero constant at late times. Clearly the back reaction must drastically modify this, and it becomes important when $e^{-2\phi} \approx \kappa$, comfortably before the radiated energy would have equaled the incoming energy.

The primary questions at this point are: is the solution of these equations of motion with initial data along the shock wave non-singular, and if so, what does it look like at late times? An attractive picture is given by [4]. If in the system at late times, the dilaton behaves as it did in the vacuum, quantum corrections will be negligible in the weak coupling asymptotic region, while the strong coupling region will be controlled by the Liouville action. This allows only flat space as a vacuum solution, and shock wave perturbations are described by patching together flat solutions. This region can be thought of as a “remnant object” which contains the information and conserved charges of the incoming $f$ wave.

_Singular Shock Waves and Incomplete Cornucopions_

All this assumes the existence and non-singular nature of the solution. A glance at our effective Lagrangian reveals a potential problem with the CGHS scenario. In conformal gauge the model resembles a nonlinear $\sigma -$ model Lagrangian with two dimensional target space. The target space fields are the Liouville field $\rho$ and the dilaton $\phi$. The metric on the target space is defined by the kinetic term in the Lagrangian, and has determinant $4e^{-2\phi}(\kappa - e^{-2\phi})$. For unitary matter $\kappa > 0$ and this degenerates at a physical value of the dilaton, $e^{-2\phi} = \kappa$. This degeneration of the target space metric can cause space time singularities in solutions of the field equations. Consider for example trying to do weak field perturbation theory in the amplitude of the $f$ wave in order to solve the full field equations with $\kappa \neq 0$. This requires us to invert the kinetic term in the linear dilaton background, and its degeneration causes this perturbation theory to be divergent [4].

We can avoid the use of perturbation theory by attempting to solve the equations of
motion directly. The shock wave problem has vacuum boundary conditions on $I_R^-$, and boundary conditions on the shock wave determined by continuity. We will set up a power series expansion in $x^+$ for the solution, and find that the first derivative of the metric and dilaton diverges on the shock wave.

It is convenient to use coordinates $\sigma^\pm$ in which the vacuum (linear dilaton) metric is $\eta_{ab}$, and

$$\phi_0 = \frac{\lambda}{2}(\sigma^- - \sigma^+).$$

(Weak coupling is $e^{-2\phi} \to \infty, \sigma^+ \to \infty, \sigma^- \to -\infty$). In these coordinates $t_+ = t_- = 0$.

Let the $f$ shock wave be at $\sigma_+ = 0$ with $T_+^f = a\delta(\sigma_+)$, and take $\phi = \phi_0 + \theta(\sigma^+)\tilde{\phi}$, $\rho = \theta(\sigma^+)\tilde{\rho}$, then for $\kappa = 0$ we have

$$\tilde{\phi} = -\frac{1}{2} \log\left(1 - \frac{a}{\lambda}e^{\lambda\sigma^-}(1 - e^{-\lambda\sigma^+})\right),$$

$$\tilde{\rho} = \tilde{\phi}.$$  \hspace{1cm} (2.12)

To set the boundary conditions for the equations of motion just after the shock wave, we require the metric and dilaton to be continuous and have finite derivatives, allowing us to write

$$\phi = \phi_0 + \phi_1(\sigma^-)\sigma^+ + \ldots,$$

$$\rho = \rho_1(\sigma^-)\sigma^+ + \ldots.$$  \hspace{1cm} (2.14)

The leading term in $\sigma^+$ of the equations of motion are then ($f'$ is $df/d\sigma^-$)

$$0 = T_{+-} = e^{-2\phi}(2\phi'_1 - 2\lambda\phi_1) - \kappa\rho'_1$$

$$0 = \delta S/\delta \phi = \rho'_1 + \lambda\phi_1 - 2\phi'_1$$  \hspace{1cm} (2.16)

and

$$a = \int^{\epsilon}_{-\epsilon} d\sigma^+ G_{++}$$

$$= \int d\sigma^+ (2e^{2\rho - 2\phi}\partial_+ e^{-2\rho} \partial_+ \phi - \kappa(\partial_+^2 \rho - (\partial_+ \rho)^2))$$

$$= 2e^{-2\phi}\phi_1 - \kappa\rho_1.$$  \hspace{1cm} (2.18)

For $\kappa = 0$ we have $\phi_1 = a e^{2\phi}/2$, $\rho_1 = \phi_1 + \text{const}$ which agrees with the exact solution.
This analysis suffices to give the boundary conditions for the equations of motion; continuing the series expansion should give a finite radius of convergence $R_c$ for any finite $\sigma^-$. However, this radius could go to zero for large $\sigma^-$, or for large values of the fields. In particular, we assumed that $\sigma^+$ was small compared to any function of $\sigma^-$. This means that we cannot see the singularity of the original classical solution at $a_+ e^{\lambda_+ \sigma^-} = 1$. Furthermore we assume $\rho_1 \sigma^+$ and $\phi_1 \sigma^+$ are small – if this breaks down, $R_c \rightarrow 0$.

For $\kappa \neq 0$, combine ((2.16)) and ((2.17)) to get

$$\phi' = \frac{\lambda}{2} \phi \left( \frac{2e^{-2\phi - \kappa}}{e^{-2\phi} - \kappa} \right)$$

with solution

$$\phi_1 = \frac{a}{2} e^{\lambda \sigma^-} \left( 1 - \kappa e^{\lambda \sigma^-} \right)^{-1/2};$$
$$\rho_1 = \frac{a}{\kappa} \left( (1 - \kappa e^{\lambda \sigma^-})^{-1/2} - 1 \right).$$

As $e^{-\lambda \sigma^-} \rightarrow \kappa$, these blow up, and the curvature $R = -2\partial_+ \partial_- \rho \sim \rho'_1$ blows up as well.

The radius of convergence of the series expansion $R_c$ goes to zero at this point, since $\rho_1 \sigma^+$ and $\phi_1 \sigma^+$ are not small.

Thus, an attempt to expand the solution just above the shock wave runs into a singularity at the spacetime point where the shockwave meets the timelike line where $1 = \kappa e^{2\phi(\sigma)} = \kappa e^{2\phi_0}$ in the linear dilaton vacuum. Note that by varying $x_0^+$ for fixed $a$ and $\kappa$, we can find solutions for which this new singularity is either behind or in front of the horizon of the classical black hole. To show that this singularity is not just a failure of the power series in $\sigma^+$ we study the general solution around a point where $\phi = \phi_0$. We attempt to solve the equation near this point by first writing $\phi = \phi_0 + \Delta$. By taking linear combinations of the trace and dilaton equations we obtain

$$(e^{2\Delta} - 2)\partial_+ \partial_- \rho = -2\partial_+ \partial_- \Delta$$

$$(e^{2\Delta} - 1)\partial_+ \partial_- \rho = \frac{3}{2} (4\partial_+ \Delta \partial_- \Delta + \lambda^2 e^{2\nu})$$

Both of these equations exhibit the potential for curvature singularities, but the second is much more dangerous. The right hand side depends only on first derivatives of the
field. If a solution takes on the value $\Delta = 0$ anywhere in spacetime, the right hand side of (2.22) must vanish at that point in order to avoid a curvature singularity. But the first derivatives of a solution at a given point generally change if we change the initial data or the driving term in the equation. Thus generic solutions that take on the value $\Delta = 0$ will have curvature singularities.

Solutions which avoid being singular when $\Delta$ vanishes must satisfy the restricted equations obtained by setting $\Delta = 0$ above: It follows that a linear combination of the Liouville field and dilaton must satisfy the massless wave equation,

$$\partial_+ \partial_-(2\Delta - \rho) = 0 \quad (2.23)$$

and that

$$4\partial_+ \Delta \partial_- \Delta + \lambda^2 e^{2\rho} = 0 \quad (2.24)$$

Thus

$$\Delta = \frac{\rho}{2} + \Delta_+(\sigma^+) + \Delta_-(\sigma^-) \quad (2.25)$$

Plugging the latter equation into the constraint equations we obtain

$$\frac{1}{2}(\partial_+ f)^2 + \kappa[(\partial_+ \rho)^2 + 4\partial_+ \Delta \partial_+ \rho - 2\partial_+^2 \Delta + t_+] = 0 \quad (2.26)$$

$$(\partial_- \rho)^2 + 4\partial_- \Delta \partial_- \rho - 2\partial_-^2 \Delta - t_- = 0 \quad (2.27)$$

Note that these are algebraic equations for the derivatives of the Liouville field with respect to each of the light cone coordinates, with coefficients which depend only on the corresponding coordinate. Thus the Liouville field must be the sum of two functions, each of which depends only on one of the light cone coordinates. We see that near the degeneracy point of the Lagrangian the metric must be flat, which also implies that the dilaton must satisfy the massless wave equation. The two pairs of functions $\Delta_\pm$ and $\rho_\pm$ must also satisfy the constraint (2.24).

In fact, there are exact solutions of the full nonlinear equations satisfying the constraint that the curvature is zero and the dilaton a massless free field. For these solutions, the dilaton and Liouville field are both solutions of the free wave equation ($\phi = \phi_+(\sigma^+) + \phi_-(\sigma^-)$ and $\rho = \rho_+(\sigma^+) + \rho_-(\sigma^-)$) satisfying the additional constraints

$$\partial_\pm^2 e^{-\rho_\pm} = -\tau_\pm e^{-\rho_\pm} \quad (2.28)$$
\[ \tau_\pm = t_\pm - \frac{1}{2\kappa}(\partial_\pm f)^2 \]  
(2.29)

\[ \partial_\pm \phi_\pm = \pm \lambda e^{2\rho_\pm} \]  
(2.30)

The metric of these solutions is of course locally flat, but our coordinates cover only a finite region of Minkowskian space. To see this, note that for a normalizable left moving \( f \)-wave the equation (2.28) has the form of a zero energy Schrödinger equation for \( e^{-\rho} \) with a potential which falls to zero at infinity. The asymptotic solutions of this equation are \( e^{-\rho} \to A_+\sigma^+ + B_\pm \) as \( \sigma^+ \to \pm\infty \). If we choose a flat metric in a Minkowski coordinate system in the region that the \( f \)-wave has not yet reached, then \( A_- = 0 \). For a generic potential (generic \( f \)-wave profile) the solution of the zero energy Schrödinger equation will have nonzero \( A_+ \). The coordinate in which the metric is Minkowski is related to \( \sigma^+ \), for large positive \( \sigma^+ \), by

\[ \frac{dy^+}{d\sigma^+} = e^{2\rho_+} \to \left( \frac{1}{A_+\sigma^+} \right)^2 \]  
(2.31)

and it is easy to see that \( \sigma^+ = \infty \) is a finite point in \( y^+ \) so that the original spacetime is geodesically incomplete.

It is plausible that these solutions represent the asymptotic “zero energy bound states” of CGHS, although their geodesic incompleteness is somewhat puzzling. What we have shown above is that any nonsingular solution must locally resemble one of these zero energy bound states near the point where the Lagrangian degenerates. Furthermore, the initial conditions defined by the shock wave solutions of CGHS do not evolve into something that satisfies this condition, but instead they define a solution which develops a singularity. This is indicated by our initial power series analysis. The existence of singularities in the generic solution of these equations is an assurance that the singularity we saw there is not an artifact of the breakdown of the series. The power series solution does not satisfy the crucial condition (2.30). While this argument is not completely rigorous, we feel confident that it is correct.

**Some Modified Proposals**

Thus, in the two dimensional field theory defined by CGHS, there is no way to connect the weak coupling region with the strongly coupled Liouville region without crossing the
singularity at \( 1 = \kappa e^{2\phi} \). One may attempt to avoid this difficulty by appealing to the higher dimensional theory of which the CGHS Lagrangian is a two dimensional approximation. Indeed, in the higher dimensional black hole solutions, the dilaton asymptotes to a constant value at infinity. Thus, at least for a sufficiently large value of the asymptotic coupling one might hope to stay below the degeneracy in field space over all of space time. It is somewhat peculiar to have to require that a coupling be sufficiently large in order for a semiclassical calculation to make sense. Note however that it is the scaled large N coupling \( g^2N \) which is required to be larger than some finite number. Quantum fluctuations of fields besides the \( f' \)s are still suppressed.

In order to write a classical two dimensional field theory which will have the correct asymptotics of the full four dimensional black hole, we need only include one extra scalar field in the theory. This is the two dimensional representation of a fluctuating radius of the two sphere. The four dimensional line element is

\[
ds^2 = g_{\mu\nu}d\sigma^\mu d\sigma^\nu - e^{2\Sigma(\sigma)}d\Omega^2
\]

(2.32)

Here \( g_{\mu\nu} \) is the two dimensional metric in the radius-time plane and \( \Omega \) is the three dimensional solid angle. The two dimensional Lagrangian for these variables is

\[
\sqrt{-ge^{-2\phi}} [e^{2\Sigma} [R + 2(\partial\Sigma)^2 + 4(\partial\phi)^2 - 8\partial\phi\partial\Sigma] - 2 + Q^2 e^{-2\Sigma}]
\]

(2.33)

where \( Q \) is the magnetic charge of the black hole.

Unfortunately, we cannot simply couple this theory to \( N \) scalar matter fields if we want to obtain a sensible system in which ordinary classical dilaton gravity is recovered at asymptotically large distances from the center of the black hole. If the scalar fields are massless all over the four dimensional spacetime their nonlocal effective action will be important even at infinity and they will modify the classical solutions there. The only way to prevent this would be to take the scaled coupling \( g^2N \) at infinity very small, in which case we would encounter the same singularity that we found before.\(^8\) Instead we must find a model in which the quantum fluctuations of the matter fields are naturally suppressed

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\(^8\) It is easy to verify that adding the \( \Sigma \) field to the large N action does not remove the degeneracy.
at large four dimensional distances, and are only important inside the horn of the black hole.

We have found one such model, though there is no reason to expect it to be unique. Add to the dilaton-graviton Lagrangian of low energy string theory an $SU(2)$ gauge theory with Higgs field in the adjoint representation, via the prescription $\mathcal{L}_{\text{flat}} \rightarrow e^{-2\phi} \mathcal{L}_{\text{min}}$ where $\mathcal{L}_{\text{flat}}$ is the flat space Lagrangian of the gauge-Higgs system and $\mathcal{L}_{\text{min}}$ is the same Lagrangian minimally coupled to the stringy metric. In addition, add $N$ fermions in the doublet representation of $SU(2)$ with Lagrangian

$$\bar{\psi} i e^\mu_m \gamma^m D_\mu - H^a \tau^a \psi$$

(2.34)

The fermions are minimally coupled to the stringy metric. If they really corresponded to low energy string excitations, they would also have a characteristic derivative coupling to the dilaton. We do not know whether this affects the discussion of zero modes that we will give below. Since at the moment we are only interested in finding one model that works, we can omit the stringy dilaton-fermion coupling if necessary.

In flat space, the Dirac equation in an 't Hooft Polyakov monopole background has exact normalizable zero energy modes which fall exponentially with the distance away from the monopole. As a consequence of the infinite length of the black hole horn, the Dirac equation in the extremal monopole background has not just isolated zero modes, but a continuum starting at zero corresponding to modifications of the zero mode wave function by multiplication with a two dimensional plane wave in the infinite horn. The effective action for the quantum fluctuations of these modes is just $N$ copies of the massless two dimensional Dirac equation. By bosonization, we recover the $f$ fields of CGHS. Note however that all of these modes are localized in four dimensional spacetime within a fermion Compton wavelength of the black hole. Thus, virtual fermions in these states do not interact with fields far from the hole, and the contribution to the effective action from quantum fluctuations of these modes will not make a substantial change in the equations of motion far from the black hole. Contributions to the effective action from massive fermion scattering states will have the usual derivative expansion in inverse powers of the fermion mass and will have a similarly innocuous effect on the large distance equations of
Imagine now a spherical shell of fermion matter collapsing inward from infinity onto the extremal dilaton black hole of this model. Take the ADM energy of this shell to be much smaller than the black hole mass, which is much larger than the Planck scale (they are all of order $N$). This will ensure that the formation of a horizon in the classical evolution occurs only when the collapsing matter has penetrated deep into the horn of the black hole. Take the coupling at infinity to be such that the coupling at the position of the black hole throat (in the static extremal solution) is larger than the critical value $e^{-2\phi_0} = \kappa$. In the four dimensional region far from the throat of the black hole the evolution of this system will follow the classical equations of motion with the stress tensor appropriate to the spherical shell of matter. As the collapsing shell approaches the throat of the black hole, our neglect of fermionic quantum corrections to the equations of motion is no longer justified, and there is a complicated region right around the throat which we are unable to analyze. It seems plausible to assume that an observer some distance “down the horn” from the throat of the black hole, will see most of the collapsing matter coming at him in the form of massless $f$-waves.

The problem of determining the further evolution of the system inside the horn is now very close to that posed by CGHS. The difference is that we now envision the two dimensional world of the effective theory to be only semi-infinite. The right half of the world (the side which points towards the throat and the four dimensional world outside it) terminates at a point where the coupling (varying according to the linear dilaton solution) is greater than the critical value. To the right of this point the two dimensional effective theory is not valid. A plausible set of boundary conditions for an $f$-shockwave is shown in fig. 2. Boundary conditions must be imposed along a segment of the shock stretching from left future null infinity to the intersection of the shock with the timelike line $T$ where $e^{-2\phi} = c < \kappa$, as well as along this timelike line itself. In the shaded region we have the linear dilaton vacuum, and this defines the boundary conditions on the shock. We do not understand the appropriate boundary conditions to put on the timelike line $T$. In some sense they should be static, i.e. all fields should be constant along this line, because we are attempting to describe the situation at times after the infalling matter has passed the point.
whose world line is $T$. Our uncertainty about these boundary conditions has prevented us from coming to definite conclusions about the evolution of the system. The analyses that we are able to do (such as power series expansion of the solution in the vicinity of the shock) reveal no singularities, but we have no complete argument that the dilaton does not pass through $\phi_0$ at some point in the interior of the unshaded region in fig. 2, thereby causing a spacetime singularity. If it were within our power, we would attempt to demonstrate that these solutions had a nonsingular time evolution which asymptotically approached the nonsingular solutions of the CGHS equations which we have identified with cornucopions. Since the latter solutions would no longer be thought of as describing a fully infinite two dimensional world, their geodesic incompleteness would no longer be a problem.

Our modification of the argument of CGHS depends crucially on re-embedding their two dimensional problem in the four dimensional problem from which it came. Another possible avenue of research is to study modifications of the two dimensional Lagrangian, within the general class of Lagrangians described in [7]. In the context of string theory we may think of this generalization as a way of incorporating higher orders in the string loop expansion. More generally we can think of these corrections as coming from integrating out modes of the higher dimensional theory which are effectively massive in the horn of the black hole. A typical example would simply be higher partial wave modes of the fermions in the model we described above.

We do not know if a Lagrangian can be found which satisfies all relevant criteria. It must reduce to the CGHS Lagrangian for very weak coupling. It must have static solutions corresponding to a one parameter family of black holes, with the extremal member of the family singularity and horizon free. When the theory is modified by appending the Liouville action to it, collapsing solutions should all be singularity free. The virtue of this approach when compared to our higher dimensional scenario is that there is no necessity to assume that the asymptotic coupling in four dimensions is greater than some critical value.9 It

9 In superstring theory the asymptotic coupling is a boundary condition that theorists hope will be fixed by nonperturbative quantum dynamics. Dine and Seiberg have argued that it cannot be fixed at a very small value. The possibility of fixing it depends on competition between terms in the effective potential which are a priori of different orders in the coupling. It is perhaps too bizarre to speculate that the nontrivial lower bound on the coupling that is required for nonsingular...
clearly deserves further investigation.

To summarize the rather unfortunate situation: CGHS have proposed an extremely attractive quantum endpoint to the Hawking evaporation process. Their mechanism does not appear to work in the original model which they proposed. Instead, we have demonstrated that generic solutions of their equations, including solutions that correspond to matter collapsing on the extremal black hole, are singular. We have proposed modifications of their model in which respectively, the higher dimensional aspects of the problem (most importantly the fact that in the four dimensional extremal black hole solution the coupling is bounded from below) or higher order quantum corrections to the two dimensional action, remove the singularities. There is a plausible argument that the first of these mechanisms leads to singularity free evolution if the asymptotic value of the coupling is greater than some critical value. However, our inability to solve the full four dimensional problem with quantum corrections, or to understand the precise boundary conditions that it imposes on the effective two dimensional theory, makes this argument less than convincing.

3. Jumping to Conclusions

Let us suppose that the difficulties that we have discovered in the approach of CGHS can be overcome, and that a sensible quantum evolution of an extremal black hole perturbed by infalling matter can be achieved in some version of dilaton gravity. Suppose further that an analysis of uncharged black holes in the quantum regime comes to similar conclusions. What can we deduce about the description of the resulting stable quantum states, and how does this description avoid the problems with evolution of a black hole into a stable quantum particle that we reviewed in the introduction? The first general statement that we can make is that the extremal black hole will be a new kind of particle state, which we have termed a “horned particle” or cornucopion.

Indeed, to an observer in the asymptotically flat region, the extremal black hole with small charge looks like an essentially pointlike object. Nonetheless, in the context of evolution of black holes in our scenario might be related to the superstringy mechanism which fixes the value of the coupling.
quantum field theory, it is obvious that this particle has an infinite number of states. The semiclassical geometry is infinitely extended in the direction “inside the hole”, and quantum fields propagating in such a background will have an infinite number of states whose wave functions fall off exponentially as we move away from the location of the hole into the asymptotically flat region. Each of these states is thus another localized particle and we will argue below that all of these particles will have very close to the same ADM energy.

The existence of horned particles immediately resolves many of the apparent paradoxes of Hawking’s picture of black hole evaporation. Even if we believe the slightly fuzzy arguments about the impossibility of extracting the information lost to the black hole in the last few Planck times of its evaporation, we do not have to give up quantum mechanics, nor even resort to Dyson’s hypothesis that black hole evaporation ends with the creation of a baby universe[11]. The information lost in the formation of the black hole may be viewed as the information contained in the state of the horned particle. Any global quantum numbers that the theory might possess are likewise carried by horned particles. Given the semiclassical picture of a horned particle as a geometry with a semi-infinite tube sticking out of ordinary spacetime, the existence of almost degenerate states with arbitrarily large values of baryon number is no longer surprising. It is an obvious consequence of the fact that the field modes in the horn carry baryon number, and that states concentrated in the horn do not contribute to ADM energy.

The question that we must address is whether it is possible to find a description of such horned particles which is consistent with quantum mechanics, general covariance, and ordinary experience. The crucial issue is to understand why it is very difficult to produce such particles in ordinary circumstances, or to excite their infinite number of internal states. For example, the existence of an infinite number of almost degenerate particle states makes even the microcanonical partition function of statistical mechanics ill defined. Thus one must understand why it is impossible to produce cornucopions in ordinary circumstances even when the energy to create them is available. Systems of ordinary particles, even at

10 We will see below that there should be some degree of nondegeneracy of the ADM energies of different states confined to the horn.
energies much higher than the cornucopion mass, must not be able to come into equilibrium with most internal cornucopion states.

We approach this issue by first studying the path integral description of a single cornucopion interacting with quantum fields of wavelength much larger than the size of the cornucopion throat. The particle follows a world line $x^\mu(\tau)$ as a function of its proper time $\tau$. We assume that fluctuations of its internal geometry are small, and choose coordinates $\tau$ and $\eta$ to describe its internal structure. $\tau$, as mentioned above, is the proper time for the motion of the particle in the external spacetime, and $\eta$ is the proper distance from some fiducial point at the mouth of the classical geometry. Quantum fields propagating on the background geometry may be expanded in a complete set of functions of $\eta$. Internal fields will be associated with elements of this complete set with support inside the horn while external fields may be defined as those associated with functions which vanish at values of $\eta$ larger than some small fixed constant $d$. There is of course some arbitrariness in this distinction, and it is not a useful one for a description of processes going on near the mouth of the horn. However, for purposes of our effective field theory, this arbitrariness is immaterial. Internal fields $\phi_i(\eta, \tau)$ will carry a label $i$ which describes, among other things their properties under rotation of the underlying spherically symmetric classical geometry; they will be tensor spherical harmonics. External fields $\chi_A(x)$ will be taken to depend only on the external spacetime coordinates $x^\mu$. The label $A$ will describe, among other things, the particular function of compact support through which they probe the internal geometry. Thus, the interaction Lagrangian between external fields and the internal structure of the cornucopion takes the form:

$$\mathcal{L}_{int} = \int d\eta d\tau \mathcal{O}_i(\eta, \tau) f^i_A(\eta) F_M[x^\mu(\tau)] O^{A;M}(x^\mu(\tau)) \sqrt{-g(x(\tau))} \quad (3.1)$$

Here, $M$ is an external spacetime tensor index, and $F_M$ is a function of $x^\mu$ and its proper time derivatives. The functions $f^i_A$ have compact support concentrated near the mouth of the horn. $\mathcal{O}_i$ and $O^{A;M}$ are complete sets of composite operators constructed out of the internal and external fields $\phi_i$ and $\chi_A$ respectively. Note that this expression is manifestly invariant under diffeomorphisms of the external spacetime, while it picks out a special coordinate system on the part of space time swept out by the world history of the
This interaction Lagrangian must be supplemented by terms in the action describing the internal dynamics of the horn, $S_I[\phi_i(\eta, \tau)]$, and the dynamics of the external fields $S_E[\chi_A(x)]$. In principal all three of these terms can be computed from the structure of $S_E[\chi_A(x)]$ alone by plugging in the decomposition of fields described above. For many choices of this fundamental action, the system will have a set of local and global conservation laws, and for all systems we have conservation laws associated with the stress energy tensor in the asymptotically flat external spacetime. To study these it is convenient to separate out a set of collective coordinates from the internal variables of the horn. We have already done this with the position coordinate $x^\mu(\tau)$.

Given the Lagrangian that we have written, it is very easy to understand how the system it describes could have an infinite number of states that are degenerate, or almost degenerate, in ADM energy. The ADM energy functional depends only on the values of the fields $\chi_A$ in the asymptotically flat region of spacetime. Through the interaction term $L_{\text{int}}$, the internal fields $\phi_i$ can act as sources for the $\chi_A$ and thereby influence the asymptotic ADM energy. However, there are many classical configurations of the $\chi_A$ for which the interaction Lagrangian is the same. Any change in $\chi_A$ which has support only for $\eta \geq d$ does not change $L_{\text{int}}$. More realistically, in quantum mechanics we can imagine that the wave functionals of many states will be concentrated on field configurations which vanish rapidly away from some finite place in the horn. These states will have an exponentially small leakage into the region $\eta \leq d$ where the interaction Lagrangian has its support. For example, such a state would be created by a wave packet of particles travelling down the horn. Excitations of the horn in these states will cause very tiny changes in the ADM energy. In the asymptotic limit in which the wave packet approaches the end of the horn, it will make no change at all. These are the zero energy bound states of CGHS.

The Lagrangian we have written above appears to describe the hypothetical CGHS endpoint for black hole evaporation in a manner that is completely consistent with both quantum mechanics and general relativity. It describes a system with the ability to absorb the “information” that has gone down the black hole during earlier stages of its evolution. In particular, although the cornucopion looks pointlike to an external observer, it has an
infinite number of degenerate states and can carry any value of global conserved quantum numbers. Paradoxes involving thermodynamics are avoided because the external observer finds it very difficult to excite most of the states of the horn. These states are not difficult to excite because they have high energy, but because they are far away “down the horn” from the external observer. Under normal circumstances they would never come into equilibrium with a gas of particles in the external spacetime, and will not lead to paradoxical infinities in statistical mechanics or quantum field theory.\footnote{This argument refers to equilibration of a system of external fields with an already existing cornucopion. The problem of pair creation of horned particles at sufficiently high energies and temperatures is also relevant to these thermodynamic questions. It is similar to questions about soliton creation in ordinary field theory. There, we would expect the amplitudes for these processes to be very tiny because the coupling is weak. For the case of cornucopions there might be additional suppression because of the infinite volume of space within the horn. It is even possible that this sort of infinitely extended object avoids the contraints of crossing symmetry altogether, and that the amplitude for quantum pair creation of horned particles is strictly zero, even though scattering amplitudes off already existing particles are finite. It seems reasonable that the most efficient way to create them is through classical gravitational collapse.}

It is reasonable to suppose that a unitary S-matrix exists connecting states on both asymptotic regions of the spacetime. In most practical circumstances, the external observer would not be in a position to obtain information about particles scattered down the horn, and would have to content himself with measuring inclusive cross sections and constructing a density matrix. However, there is no issue of principle here. In principle one could imagine constructing tiny detectors and anchoring them at some position deep in the hole\footnote{Remember that in the extremal CGHS black hole the spacetime inside the hole is flat. There would not seem to be a problem of constructing apparatus that could remain indefinitely at a fixed position in the horn.} to observe particles that come down the horn. At some later time these detectors could send a signal to the external observer, enabling him to construct the full quantum S-matrix.

In view of the fact that the CGHS mechanism leads to a scenario which can resolve all of the paradoxes of black hole evaporation, it is important to find models in which their mechanism actually works. Although we have not succeeded in this task in the present
paper, we feel that the possibility of finding a two dimensional Lagrangian for gravity coupled to one or more scalars that implements the CGHS proposal is still open. Of course, even if such a Lagrangian is found, and can be justified as a well defined approximation to a more realistic theory of quantum gravity, the CGHS proposal can only deal with very special situations in which the approximation of a classical geometry, that varies smoothly on scales much larger than the Planck length, makes sense. However, it can serve as a paradigm for a more general resolution of the “Hawking paradox”, which would apply to regimes which cannot be described by classical differential geometry. The important point about this mechanism is that it provides a concrete picture of how black hole evaporation could be compatible with ordinary quantum mechanics. We suspect that this picture will survive the details and difficulties of various models and will form the intuitive basis for the final understanding of black hole evaporation within the context of a complete theory of quantum gravity.\[13\]

\[13\] We should however note that even the successful construction of a model exhibiting the CGHS mechanism does not rule out the possibility that in some circumstances black hole evaporation does lead to the creation of a baby universe\[11\]. It would be interesting to develop an effective theory of such processes, similar to the one we have constructed for the hypothetical stable configurations proposed by CGHS.
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NOTE ADDED

After this work was completed we received a preprint by Russo et. al. (SU-ITP-92-4) in which substantially the same conclusions as ours about singularities in the CGHS equations were obtained. We thank L. Thorlacius for sending us a copy of this manuscript.
Figure Captions

Figure 1: A section of the three dimensional spatial geometry of the extremal dilaton black hole with fixed polar angle.

Figure 2: The Penrose diagram of the radius-time plane of a hypothetical nonsingular solution of the four dimensional large $N$ equations.
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