Diffraction of entangled particles by light gratings

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Abstract

We analyze the diffraction regime of the Kapitza-Dirac effect for particles entangled in momentum. The detection patterns show two-particle interferences. In the single-mode case we identify a discontinuity in the set of joint detection probabilities, associated with the disconnected character of the space of non-separable states. For Gaussian multi-mode states we derive the diffraction patterns, providing an example of the dependence of the light-matter interaction on entanglement. When the particles are identical, we can explore the relation between exchange and entanglement effects. We find a complementary behavior between overlapping and Schmidt’s number. In particular, symmetric entanglement can cancel the exchange effects.

Keywords: Diffraction by light gratings; Two-particle interference; Entangled states; Exchange effects; Discontinuous processes

1 Introduction

Entanglement leads to physical effects unattainable for product states. One of the best known examples of such effects is multi-particle interference, which has been extensively studied in both the theoretical and experimental realms [1, 2]. In general these studies are within the framework of quantum optics. It would be interesting to consider similar schemes for massive particles. One natural candidate is the interaction of massive particles with light gratings. In particular, the Kapitza-Dirac effect, proposed long ago [3, 4], has been experimentally verified for atoms [5] and electrons [6].

We shall consider in this paper two-particle diffraction by light gratings with the particles entangled in momentum, showing the presence of two-particle interference effects absent for factorizable states. The arrangement reflects how the light-matter interaction is modified by entanglement. Other examples of the dependence of the interaction on entanglement can be found in the literature [7, 8].
Similar interference effects have been already discussed for the other type of non-product states, the (anti)symmetrized states of identical particles [9]. In the case of single-mode states our work bears many mathematical resemblances to that paper, but here we shall consider some aspects of the problem not discussed there. We shall show the existence of a discontinuity in the set of joint detection distributions when the two momenta are equal. This property reflects the discontinuous nature of the space of entangled states.

For multi-mode states our approach departs from the elementary qualitative treatment given in [9]. Here, we consider entangled Gaussian states and we can derive exact analytical expressions for the joint detection patterns. These patterns, corresponding to the sum of a large number of Gaussian terms, can accurately be described by a single Gaussian.

The main physical novelty in the multi-mode case occurs for identical particles. Then we can observe at work three quantum processes (diffraction and exchange and entanglement effects) at once. Our first finding in this scenario is the existence of a type of complementarity between overlapping (the variable ruling the intensity of the exchange interaction) and entanglement degree. When one of them increases the other must decrease. To evaluate the entanglement degree we use the Schmidt number [10, 11], which is a measure well-suited for continuous variable problems. Another consequence of the physical connection between particle identity and entanglement is the cancellation of the exchange effects for symmetric entangled states.

2 Single-mode states

We consider in this section single-mode states. First we describe the arrangement and the equations describing the light-matter interaction. In the second subsection we derive the interference patterns in the position representation. Finally, we discuss the singular behavior of the system when the initial momenta are equal.

2.1 The arrangement and fundamental equations

A sketch of the arrangement can be seen in Fig. (1). A source generates pairs of particles in entangled states. Each particle interacts with a light grating. The gratings, two standing light waves formed by counter-propagating lasers with different wavelengths, are denoted as L and R. After the gratings we place detectors working in coincidence to determine the joint probabilities.

The only relevant variables in the problem are those parallel to the grating [4], reducing the description of the system to a two-dimensional one (one variable for each particle). The initial entangled state in momentum is

$$|\phi_0> = \frac{1}{\sqrt{2}}(|p>_{L}|q>_{R} + |q>_{L}|p>_{R})$$ (1)
where \( p \) and \( q \) denote the longitudinal momenta of the particles and the subscripts \( L \) and \( R \) refer to the grating with which they interact. Next, we must describe the interaction of the particles with the gratings. The process is ruled by the lightshift potential \( V = V_0 \cos^2 Kx \), with \( K \) the wavenumber of the laser light and \( x \) the spatial coordinate in the direction parallel to the grating [4].

We assume that the potential intensity is the same in both sides but the laser wavenumbers, \( K_L \) and \( K_R \), differ. In the diffraction regime of the Kapitza-Dirac effect the Raman-Nath approximation, neglecting the kinetic part of the motion, holds [4]. Then, as the initial wave function for the mode \( p \) in the position representation is \( \psi_0 = e^{ipx/\hbar} \), the wave function after the interaction will be \( \psi_\tau = e^{-iV\tau/\hbar}\psi_0 \), with \( \tau \) the interaction time. The exponential can be rewritten using the expression \( e^{iz \cos \phi} = \sum_n J_n(z)e^{in\phi} \), with \( J_n \) the Bessel function of order \( n \). Using simple trigonometric relations we obtain \( \psi_\tau = \sum_n b_n e^{i(2n\hbar KL + p)x/\hbar} \) with \( b_n = i^n e^{-iw}J_n(-w) \) and \( w = V_0\tau/2\hbar \).

Figure 1: Arrangement for two-particle Kapitza-Dirac diffraction. The source \( S \) produces pairs of entangled particles that interact with the light gratings \( L \) and \( R \). The continuous, dashed and dotted lines after the gratings correspond respectively to particles with \( n = 0, \pm 1 \) and \( \pm 2 \).

Returning to the momentum picture this evolution can be expressed as \( |p >_L \rightarrow \sum_n b_n|p + 2n\hbar K_L >_L \). The final state of the complete system is

\[
|\phi > = \frac{1}{\sqrt{2}} \sum_{n,m=-\infty}^{\infty} b_n b_m (|p + 2n\hbar K_L >_L |q + 2m\hbar K_R >_R \\
+ |q + 2n\hbar K_L >_L |p + 2m\hbar K_R >_R) \tag{2}
\]

From this expression we can derive the joint detection probabilities. The prob-
ability of measuring the system in the state \( |p + 2n\hbar K_L >_L |q + 2m\hbar K_R >_R \) is

\[
P(n^L_p, m^R_q) = |L < p + 2n\hbar K_L |R < q + 2m\hbar K_R |\phi >|^2 = \frac{1}{2} |b_n b_m \delta(0)\delta(0) + \sum_{n', m' = -\infty}^{\infty} b_{n'} b_{m'} \delta(p + 2n\hbar K_L - q - 2m'\hbar K_L) \delta(q + 2m\hbar K_R - p - 2m'\hbar K_R)|^2 \tag{3}
\]

where we have used the orthogonality of the momentum states \(< p|p_\phi >= \delta(p - p_\phi)\). The above expression must be interpreted in terms of distributions where, for instance, \(b_n \delta(0) = b_n < p + 2n\hbar K_L |p + 2n\hbar K_L >\). When the two conditions

\[
2(n - n')\hbar K_L = q - p \tag{4}
\]

and

\[
2(m - m')\hbar K_R = p - q \tag{5}
\]

simultaneously hold for some values of \(n, m, n'\) and \(m'\) we have interference effects. In effect, when these conditions are not fulfilled, \((n - n')K_L \neq (m' - m)K_R\), the detection probability is \(P(n^L_p, m^R_q) = |b_n b_m|^2 / 2\) with no interference effects. In contrast, when the conditions hold the probability becomes

\[
P(n^L_p, m^R_q) = \frac{1}{2} |b_n b_m|^2 + \frac{1}{2} |b_{n'} b_{m'}|^2 + \text{Re}(b_n^* b_m^* b_{n'}^* b_{m'}) \tag{6}
\]

The third term on the r. h. s. corresponds to the interference term between the alternatives (i) \(n\) photon absorptions in \(L\) and \(m\) in \(R\), and (ii) \(n'\) absorptions in \(L\) and \(m'\) in \(R\). The indistinguishability of both alternatives implies that their probability amplitudes must add, giving rise to interference effects.

We note that, for given \(K_L\) and \(K_R\), only in some cases do we have interference effects. The two above conditions lead to the relation \((n - n')K_L = (m' - m)K_R\) that can be only fulfilled when \(K_L / K_R\) is a rational number as, for instance, in the case \(K_L = K_R\). On the other hand, for each pair of values \(p\) and \(q\) (with the exception of, see later, \(p = q\)) there are always pairs of laser wavevectors showing interference. For instance, taking \(n = m = 0, n' = -1\) and \(m' = 2\) there are interference effects for \(K_L = (q - p) / 2\hbar\) and \(K_L = 2K_R\).

### 2.2 Interference in the position representation

In the position representation the initial entangled state is \(\Psi_0 = (\psi_L(x)\psi_R(y) + \psi_L(y)\psi_R(x))/\sqrt{2}\), with \(x\) and \(y\) the longitudinal spatial variables of the two particles. After the interaction this expression becomes

\[
\Psi(x, y) = \frac{1}{\sqrt{2}} (\varphi_L(x)e^{ipx/\hbar}\varphi_R(y)e^{ipy/\hbar} + \varphi_L(y)e^{ipy/\hbar}\varphi_R(x)e^{ipx/\hbar}) \tag{7}
\]
with \( \varphi_L(x) = \sum_n b_n \exp(i2nK_Lx) \) and a similar expression for \( \varphi_R \). The final two-particle detection probability, \( |\Psi_{\tau}(x,y)|^2 \), is

\[
P(x,y) = P_{\text{pro}}(x,y) + \Re(\varphi_L^*(x)\varphi_R(x)\varphi_R^*(y)\varphi_L(y)e^{i(q-p)(x-y)/\hbar})
\]

with \( 2P_{\text{pro}} = |\varphi_L(x)|^2|\varphi_R(y)|^2 + |\varphi_L(x)|^2|\varphi_R(x)|^2 \) the probability of the product state corresponding to a mixture of the initial product states \( \psi_L(x)\psi_R(y) \) and \( \psi_L(y)\psi_R(x) \) with equal weights \( 1/2 \).

The second term on the r. h. s. of this equation represents the two-particle interference effects, with the typical dependence on trigonometric functions of \( x - y \). At variance with momentum interference there are no constraints between the values of \( p,q,K_L,K_R \) and the \( n' \)'s. As a matter of fact, the spatial interference contains all the possible numbers of photon interchanges (all the \( n' \)'s) through \( \varphi \). Moreover, there is spatial interference for any value of the ratio \( K_L/K_R \). The above properties show the different qualitative behavior of interference in the position and momentum representations for the same initial entangled states.

The interference effects discussed in this subsection and the previous one are mathematically similar to those in [9] for identical particles. In that paper they were considered in detail and we shall not continue here that line of argumentation, which can be easily translated to our problem. Instead, we shall analyze another physical aspect of the problem that was not treated there, the existence of a discontinuity.

### 2.3 Discontinuity

Let us consider the particular case \( p = q \) in Eqs. (4) and (5), the conditions for the existence of interference effects. In this case the effects only exist for \( n = n' \) and \( m = m' \). However, the resultant patterns cannot be considered as genuine two-particle interference as the necessary condition to have interference is the existence of indistinguishable alternatives. When \( n = n' \) and \( m = m' \) there is only one alternative for photon absorption by the particles. In the case \( p = q \), at difference with the rest of values \( p \neq q \), there is not two-particle interference.

We obtain the same conclusion, in a little bit more intuitive way, in the position representation. For instance, in the particular case \( K_L = K_R = K \), the joint detection probability becomes

\[
P(x,y) = P_{\text{pro}}(x,y) + |\varphi_K(x)|^2|\varphi_K(y)|^2 \cos((q-p)(x-y))
\]

When the two momenta are equal the dependence on \( x - y \), characteristic of the spatial two-particle interference, vanishes.

This behavior can be understood in terms of the form of the initial state \( \phi_0 \), which transforms for \( p = q \) into \( \sqrt{2}\left| p >_L p >_R \right| \) that is a product state instead of an entangled one. Moreover, the normalization is incorrect: the normalized (in the single-mode sense) two-particle state is \( \left| p >_L p >_R \right| \). We
have a discontinuity in the set of entangled states. For any \( p \neq q \) you have the entangled state \( \phi_0 \). However, for \( p = q \) there is not an entangled state.

The structure of the space of entangled states (with equal coefficients \( 1/\sqrt{2} \)) is not \( \mathbb{R}^2 \) but \( \mathbb{R}^2 - \{p = q\} \). It is constituted by two disconnected parts separated by the line \( p = q \). The same conclusion holds for the general states \( \alpha|p >_L |q >_R + \beta|q >_L |p >_R \), where the mathematical space is \( S^2 \otimes \{\mathbb{R}^2 - \{p = q\}\} \) instead of \( S^2 \otimes \mathbb{R}^2 \), with \( S^2 \) the Bloch sphere.

This mathematical discontinuity translates into a physical discontinuity. If we evaluate the joint probability as the limit \( q \to p \) within the space of entangled states we have \( \lim_{q \to p} P(x, y) = 2P_{pro} \). This is the joint detection pattern for entangled momenta with \( q \approx p \) (although \( p \neq q \)). It doubles at any point the pattern \( P_{pro} \) of a product state. Consequently we have a discontinuity for equal values of the momenta. In its vicinity we have joint patterns with sharply different values. The transition between entangled and product states is not continuous.

3 Multi-mode states

We consider in this section the multi-mode case. In order to deal with analytical expressions we only consider Gaussian states, which have been extensively used in the literature (see [12] for non-Gaussian situations). The unnormalized initial state is

\[
\Phi_0(p, q) = e^{-p^2/Q^2} e^{-q^2/Q^*_2} e^{-pq/P^2}
\]

(10)

The coefficients \( Q, Q_2 \) and \( P \) denote the spread of each exponential.

When \( P^{-2} \neq 0 \) the above state is entangled. A good measure of the entanglement degree for continuous variable systems is the Schmidt number [10, 11]. For our problem it can be easily evaluated analytically when \( 4P^4 \geq Q^2Q^2_2 \), giving \( S = (1 - Q^2Q^2_2/4P^4)^{-1/2} \). The system is entangled when \( S > 1 \).

The unnormalized wave function after the interaction, \( \Phi_s(p, q) \), is evaluated using two times the Fourier transform (we omit some constant coefficients)

\[
\Phi_s(p, q) = \int dx \int dy e^{-ipx/h} e^{-iqy/h} \sum_{n,m} b_n b_m \int dp_0 \int dq_0 \Phi_0(p_0, q_0) \times e^{i(p_0 + 2n\hbar K_L)x/h} e^{i(q_0 + 2m\hbar K_R)y/h} = \sum_{n,m} b_n b_m \Phi_0(p - 2n\hbar K_L, q - 2m\hbar K_R)
\]

(11)

The normalized wave function is \( \Phi = N \Phi_s \), with \( N \) the normalization factor. It can be explicitly evaluated when \( 4P^4 \geq Q^2Q^2_2 \) and is given by

\[
N^{-2} = \sum_{n,m,r,s} b_n^* b_m^* b_r b_s \pi P^2 e^{-4(n^2 + r^2)\hbar^2K^2_2/Q^2} e^{-4(m^2 + s^2)\hbar^2K^2_2/Q^2} \times
\]

(12)
\[ e^{-4(mn+rs)\hbar^2 K_L K_R / P^2} e^{\alpha^2 Q^2 / 8} \exp \left( \frac{P^4 \left( \beta - \frac{\alpha Q^2}{2P^2} \right)^2}{2Q^2 \left( \frac{4P^4}{Q^2 Q_*^2} - 1 \right)} \right) \]  

with \( \alpha = 4\hbar K_L (n + r)Q^2 + 2\hbar K_R (m + s)P^{-2} \) and \( \beta = 4\hbar K_R (m + s)Q_*^2 + 2\hbar K_L (n + r)P^{-2} \).

Figure 2: \( P(o, q) \) in arbitrary units versus \( q \) in units \( Q = 1 \). The black, red and blue curves correspond respectively to \( P = \infty, 1.1 \) and 0.75.

We give next a graphical presentation of the detection patterns associated with these equations. We take as momentum unity \( Q = 1 \). In addition, we use the values \( Q_* = 0.9, \hbar K_L = 0.2 \) and \( \hbar K_R = 0.3 \). In order to see the variation of the patterns with the entanglement degree we consider three different values of \( P: \infty, 1.1 \) and 0.75 that correspond to a product and two entangled initial states. We represent in Fig. 2 the two-particle detection pattern for \( p = 0, P(0, q) = |\Phi(0, q)|^2 \). In the evaluation we only consider the terms \( n = 0, \pm 1, \pm 2 \).

We have that in the three cases the sum of multiple Gaussian terms (the terms in Eq.(11)) reduces in a very good approximation to a single effective Gaussian distribution: \( P_{eff}(0, q) = \sigma_{eff} \exp\left(-q^2/Q_{eff}^2\right) \), with the values \( \sigma_{eff} = 0.7, 0.65, 0.42 \) for \( P = \infty, 1.1, 0.75 \), and \( Q_{eff}^2 = 4000 \) for all the \( P \)'s. Note that these single distributions are not normalized (the normalization is only for the sum over \( p \) of all the \( P(p, q) \)).

\( Q_{eff} \) is a measure of the width of the distribution. More physically it quantifies to how many \( (p = 0, q) \)-modes effectively affect the diffraction. In all the
cases, for product states and for different degrees of entanglement, it is equal. On the other hand, $\sigma_{\text{eff}}$ represents the intensity of the diffraction process for the affected modes. In contrast with $Q_{\text{eff}}$, it depends on the entanglement degree. In our example it decreases for increasing Schmidt’s numbers. For other values of $p$ that behavior will be different (the total probability for all the $p$'s is the same in the three cases).

We have also studied the form of the two-particle diffraction patterns in the position representation. For the sake of shortness we do not present the complete analysis here. We only signal that, as in the momentum representation, a single Gaussian distribution fits well the multiple Gaussian terms sum.

4 Identical particles

In this section we study the multi-mode case when the two particles are identical. It is interesting because in addition to the interference and entanglement effects we must consider the exchange ones. The last effects are only relevant when the overlapping between the particles is large. Then the two particles must be diffracted by the same light grating (see Fig. 3).

Figure 3: The same as Fig. 1 but with only one light grating. The mother particle decays into two daughter ones that interact with the grating.

4.1 Entanglement and overlapping

First of all we must analyze the relation between entanglement degree and overlapping, the two measures that determine the intensity of the entanglement and exchange effects. The state of two identical particles (described by the entangled state $\Phi_0$ previous to the consideration of the identity conditions) is obtained via
the standard procedure of (anti)symmetrization:

$$\Phi_{ide}(p,q) = (2 \pm 2\theta)^{-1/2}(\Phi_0(p,q) \pm \Phi_0(q,p))$$

(13)

with $\theta$ the overlapping of the two particles

$$\theta = \int dp \int dq \Phi_0^*(p,q)\Phi_0(q,p) = QQ^* \left(\frac{4P^4 - Q^2Q_*^2}{P^4(Q^2 + Q_*^2)^2 - Q^4Q_*^4}\right)^{1/2}$$

(14)

The last expression is valid for $P^4 \geq Q^4Q_*^4/(Q^2 + Q_*^2)^2$ (for instance, for the values used in the previous section). When this condition does not hold the overlapping cannot be evaluated analytically. The signs $+$ and $-$ in $\pm$ refer respectively to bosons and fermions.

Through this paper we have not considered the spin (or electronic) part of the quantum state because it is not relevant in the diffraction dynamics. When the spin variables are taken into account the (anti)symmetrization refers to the complete state and, for example, the spatial wave function of fermions can be symmetrized. By assuming that the two particles are in a symmetrized spin state, $|s \rangle_1 |s \rangle_2$ or $(|s \rangle_1 | - s \rangle_2 + | - s \rangle_1 |s \rangle_2)/\sqrt{2}$, we do not need to consider these cases here. With this choice the spatial part of the state must be symmetrized for bosons and antisymmetrized for fermions. The extension to antisymmetrized spin states is immediate.

Although there is some controversy on the characterization of entanglement in systems of identical particles, following the criterion in [13] it is clear that $\Phi_{ide}$ is entangled. For instance, for fermions, the state is non-entangled if and only if $\Phi_{ide}$ is obtained by antisymmetrizing a factorized state [13]. Thus, for fermions (13) represents an entangled state. Moreover, in this approach one clearly sees that entanglement is associated with $\Phi_0$, the state that is (anti)symmetrized. Then it is natural to assume that the amount of entanglement in $\Phi_{ide}$ is the same of $\Phi_0$. As signaled in the previous section we use the Schmidt number to quantify the entanglement contained in $\Phi_0$.

Two interesting conclusions can be easily obtained:

(i) We fix $Q$ and $Q_*$ and vary $P$ over all the values showing entanglement, $(QQ_*/2)^{1/2} \leq P < \infty$. When $P \to (QQ_*/2)^{1/2}$ we have $\theta \to 0$ and $S \to \infty$. The overlapping is very small, whereas the entanglement tends to very high values. On the other hand, for $P \to \infty$ these limits are $\theta \to 2QQ_*/(Q^2 + Q_*^2)$ and $S \to 1$. It is simple to see that this value of the overlapping is maximum. Now the system tends to a factorized one whereas the overlapping is finite, reaching the maximum value compatible with $Q$ and $Q_*$. We observe a complementary behavior. The entanglement is maximum when the overlapping is null and vice versa.

(ii) Symmetric entangled states represent a special case in the relation between entanglement and overlapping. For symmetric entanglement, $Q = Q_*$, the overlapping reaches its maximum value $\theta = 1$ independently of $P$. This is
so for any type of symmetric entangled state, $\Phi_{\text{sym}}(p, q) = \Phi_{\text{sym}}(q, p)$, because $\int dp \int dq |\Phi_{\text{sym}}(p, q)|^2 = 1$ is due to the normalization. For symmetric entangled states we have maximum overlapping regardless of the particular entanglement degree of the state.

In addition, symmetric states have another relevant property. When we take $\Phi_0$ symmetric $\Phi_{\text{ide}}$ is undefined (it has the form $0/0$) for fermions. No pair of fermions can be prepared in a symmetric entangled state (with our choice of the spin part), a property that can be seen as a natural extension of Pauli’s exclusion principle from product to non-factorizable states. In effect, when the state is a product one, $\bar{\phi}(p)\phi_*(q)$, the symmetry condition implies $\bar{\phi} = \phi_*$, a relation that is forbidden by Pauli’s exclusion principle.

On the other hand, for bosons, we have $\Phi_{\text{ide}} = \Phi_{\text{sym}}^0(p, q)$ that corresponds to a state without symmetrization (in the sense of symmetrization for identical particles). The exchange effects vanish: in the presence of symmetric entanglement there are not exchange effects in two-boson systems. This result confirms a previous analysis in [14], where a similar behavior in the position representation was found for pairs of non-entangled identical bosons in a two-slit arrangement: in the cases where the overlapping between the two bosons is large (it is increased by the diffraction process to values close to unity) the two-boson diffraction pattern is indistinguishable from that of a product state and does not show the typical characteristics associated with exchange effects.

4.2 Diffraction patterns

After analyzing the relation between $S$ and $\theta$ in the initial state we evaluate the diffraction patterns. By similitude with Eq. (11) the final state is

$$\Phi_{\text{ide}}(p, q) = \mathcal{N} \sum_{n, m, r, s} b_n^* b_m^* b_r b_s \pi \xi^2 \left(1 - \frac{\xi^4}{P^4}\right)^{-1/2} e^{-4(n^2 + r^2)\hbar^2 K^2/Q^2} \times$$

$$e^{-4(m^2 + s^2)\hbar^2 K^2/Q^2} e^{-4(mn + rs)\hbar^2 K^2/P^2} e^{\mu^2 \xi^2/4} \exp \left(\frac{\xi^2 \left(P - \frac{\mu^2}{P}\right)^2}{4 \left(1 - \frac{\xi^4}{P^4}\right)}\right)$$

where $\mathcal{N}_s = \sum_{n, m, r, s}$ is the normalization factor. The calculation of the normalization factor $\mathcal{N}$ is a little bit more involved than in the previous section. In $|\Phi_{\text{ide}}|^2$ we have four terms, two direct and two crossed ones. The integration of each one of them over the full momentum space gives a contribution to the squared normalization factor. Moreover, we must take into account that the crossed terms must be added to the direct ones in the case of bosons, but subtracted in that of fermions. For instance, the contribution of the first term is
where $\xi^{-2} = Q^{-2} + Q_{*}^{-2}$, $\mu = 2\hbar K(2nQ_{*}^{-2} + 2rQ^{-2} + (s + n)P^{-2})$ and $\overline{\mu} = 2\hbar K(2sQ_{*}^{-2} + 2nQ^{-2} + (m + r)P^{-2})$. This expression is valid when $P^4 \geq \xi^4$. For other values of the parameters an analytical expression cannot be obtained.

It is simple to see that the two direct (D) terms are equal, and the crossed (C) ones also. Finally we have $N_{2} = 2N_{2}^{D} \pm 2N_{2}^{C}$. $N_{D}$ is given by $N_{*}$ and $N_{C}$ by a similar expression with obvious modifications in the order of the coefficients $n, m, r, s$.

Figure 4: The same as Fig. 2 with identical particles. The black, red and blue lines correspond to the cases $P = 200, 1.1$ and 0.75, whereas continuous and dashed ones represent respectively bosons and fermions.

Next, we represent these patterns. With the same values for $Q$ and $Q_{*}$ of Fig. 2, $K = 0.2$ and for the parameter $P$ the values 200 (negligible entanglement), 0.75 (small overlapping) and 1.1 (intermediate case) (see Fig. 4). We observe a clear difference between bosons and fermions. In the first case the distribution is peaked around the point $(0, 0)$, whereas in the second this point is forbidden by Pauli’s exclusion principle. Instead, we see a valley around this point with two peaks around. In addition to this general trend we see that the visibility (difference between maximum and minimum values) is strongly dependent on the entanglement degree of each curve.
5 Conclusions

We have shown that entangled states are diffracted by a light grating in a different way than product ones. This is the essence of multi-particle interference. In an alternative view, our results reflect the dependence of the light-matter interaction on the separability of the state. In our proposal one can easily visualize how the joint detection patterns change with the entanglement degree.

The discontinuity found in the single-mode approximation provides, in principle, a method to experimentally study the disconnected character of the space of entangled states. It would be also interesting to test the cancellation of exchange effects by symmetric entanglement described above. This result, which corroborates previous findings for non-entangled states [14], suggests the existence of a rich and unexplored relation between entanglement and particle identity. For instance, the complementarity between overlapping and Schmidt’s number reflects a type of competition between both effects.

In this paper we have restricted our considerations to thin light gratings. For thick ones we move to the Bragg regime of the Kapitza-Dirac effect. One expects to find similar effects to those described here, but this must be verified by an explicit calculation.

We have not discussed the experimental realization of the proposal. As the diffraction regime of the Kapitza-Dirac effect has already been tested [5, 6], we must concentrate on the sources of entangled particles for the arrangement. For two gratings, a decaying particle initially at rest, seems to be a good candidate. The single-mode case can be approximated by restricting the two generated beams to some particular directions using collimators. For only one grating, the photodissociation of molecules traveling towards the standing light wave, looks to be a more adequate process than the decay of unstable particles.

In the case of electrons a nanotip source can be interesting. The antisymmetrization properties of such a source have been studied in [15]. They lead to a dip in the joint detection probability. Although the state of electrons appears to be entangled in momentum (Eqs. (15) and (16) in [15]), it is not true entanglement: the state is obtained by (anti)symmetrization of two-particle product states (Eqs. (12) and (13) in [15]) and then, according to the criterion in [13], the entanglement is associated with the (anti)symmetrization procedure. However, due to the Coulomb interaction between the electrons, there can be true entanglement in the system. An extended analysis including that interaction would be necessary to evaluate the entanglement and to see the viability of an experimental test.

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