The modulation and demodulation module of a high resolution MOEMS accelerometer

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Abstract. A MOEMS accelerometer with high precision based on grating interferometer is demonstrated in this paper. In order to increase the signal-to-noise ratio (SNR) and accuracy, a specific modulator and an orthogonal phase-lock demodulator are proposed. Phase modulation is introduced to this accelerometer by applying a sinusoidal signal to a piezoelectric translator (PZT) amounted to the accelerometer. Phase demodulation module consists of a circuit design and a digital design. In the circuit design, the modulated light intensity signal is converted to a voltage signal and processed. In the digital part, the demodulator is mainly composed of a Band Pass Filter, two Phase-Sensitive Detectors, a phase shifter, and two Low Pass Filters based on virtual instrument. Simulation results indicate that this approach can decrease the noise greatly, and the SNR of this demodulator is 50dB and the relative error is less than 4%.

1. Introduction
Accelerometer plays an important role in inertial navigation systems[1-2], biomedical area, military fields and our daily life, such as detecting the respiratory rate[3-6], measuring the Earth's surface and the distribution of the gravity field of the earth[7]. The accelerometers based on traditional methods suffer from low sensitivity, bad reliability, great complexity of signal processing, etc. However, optical accelerometers are able to achieve high resolution and sensitivity compared to the traditional ones. As acceleration is related to the displacement definitely, acceleration can be calculated accurately by measuring the displacement with high precision. A displacement sensor based on a grating interferometer with nanometer-scale displacement resolution has been demonstrated in [8]. Therefore, the optical accelerometer based on grating interferometer is able to achieve high resolution and sensitivity compared to the traditional ones. Furthermore, the MOEMS accelerometer which combines MEMS technology to optical accelerometer is of great significant due to the advantages of small size and light weight.

To further improve the SNR and suppress the noise, phase modulation and demodulation techniques are introduced in fiber-optic communication system[9,10]. In this paper, phase modulation is introduced to this accelerometer by applying a sinusoidal signal to a piezoelectric translator (PZT) amounted to the grating, which can increase the signal-to-noise ratio (SNR) and accuracy dramatically by demodulation. Meanwhile, the frequency of the signal is 500 Hz which is away from the main frequency ranges of noise signal. To demodulate the modulated signal, an orthogonal phase-lock amplifier technique based on virtual instrument is applied. The reference signal is a sinusoidal wave with 500Hz frequency and the orthogonal phase-lock demodulator is mainly composed of a Band Pass Filter, two Phase-Sensitive Detectors, a phase shifter, and two Low Pass Filters.
2. Principle
In this section we discuss the principle of the MOEMS accelerometer based on grating interferometer, then demonstrate how the sensor works with the modulation and demodulation module.

2.1. Sensing principle
As shown in Figure 1, the MOEMS accelerometer is based on a grating interferometer. The incident coherent laser beam is diffracted on the surface of an amplitude grating with duty ratio of 0.5. A part of light is directly reflected and serves as the diffracted beam 1; the other part reaches the mirror bonded to the mass, then is reflected back to the grating and becomes the diffracted beam 2. Both acceleration and displacement have a linear relationship, when acceleration is applied to the mass, the displacement changes correspondingly. Then acceleration of the mass can be measured by detecting the change of the brightness of the interference fringes. Since the intensity becomes weaker when the number of interference order increases and the 0th order is hard to measure due to its position, the selected order is \( \pm 1 \text{st} \). The relationship between the intensity of the first interference orders and the acceleration can be described as:

\[
I_{\pm 1} = \frac{2I_{in}}{\pi^2} (1 - \cos 2kd) = \frac{2I_{in}}{\pi^2} [1 - \cos \frac{4\pi}{\lambda} (d_0 + \Gamma a)]
\]

where \( a \) is the acceleration, \( I_{\pm 1} \), \( \lambda \), \( d_0 \), \( \Gamma \) are the intensity, the wavelength of the incident coherent laser beam, the initial distance between the grating and proof mass, the scaling factor, respectively.

![Figure 1. The schematic diagram of the MOEMS accelerometer](image)

2.2. Phase modulation principle
For slow frequency signal or DC signal, the weak signal will be buried, as random drift and 1/f noise brought by ambient environment will be amplified by DC amplifier[11]. To eliminate the impact and decrease the signal distortion, phase modulation is applied. The basic principle of phase modulation introduced in the grating interferometer can be described as follows: output a sinusoidal signal to a power-driven module to drive a PZT, then the length of the PZT is modulated by the signal.
Meanwhile, the distance between the grating and mirror is modulated by the sinusoidal signal and the optical path of diffracted beam 2 is changed. Since the phase shift between beam 1 and beam 2, the modulated intensity of the first interference orders received by the PD could be described as:

\[
I_{s1} = \frac{2I_{in}}{\pi^2}(1 - \cos 2kd) = \frac{2I_{in}}{\pi^2}[1 - \cos \left(\frac{4\pi}{\lambda}(d_0 + \Gamma a) + M \sin \omega_0 t\right)]
\]

where \(M\) is the modulation depth and \(\omega_0\) is the modulation frequency. The modulated phase can be expressed as \(M \sin \omega_0 t\).

### 2.3. Phase demodulation principle

Phase demodulation is used to extract the carrier signal of displacement which is related to the acceleration. Since the modulated intensity of the first interference orders is very weak and varies with time periodically. Thus, correlative detection technology is introduced in detection of feeble signal to improve the signal-noise ratio. The demodulation module consists of a signal processing circuit design and a digital design based on virtual instrument.

In the part of circuit design, a photodetector converts the light intensity to a voltage signal. Because the output electrical signals from the photodetector are pretty small, the modulated signals should be amplified by precise differential amplifier. The Band Pass Filter is applied to reduce the impact of noise signal. The power supply design is an adjustable voltage source which uses a LC-filter circuit and can provide variable types of voltages. The useful modulated signal could be described as:

\[
V(t) = A \times I_{in} \{1 - \cos \left[\frac{4\pi}{\lambda}(d_0 + \Gamma a) + M \sin \omega_0 t\right]\}
\]

where \(A\) is the circuit amplification factor.

Considering the impact of the noise, the modulated signal could be described as:

\[
V(t) = V_1(t) + n(t)
\]

\[
= A \times I_{in} \{1 - \cos \left[\frac{4\pi}{\lambda}(d_0 + \Gamma a) + M \sin \omega_0 t\right]\} + C \sin(\omega_0 t + \theta)
\]

where \(n(t), V_1(t), \omega_0, C, \theta\) are the noise signal, the useful signal, the frequency of useful signal, the amplitude and the initial phase of noise signal, respectively. Meanwhile, a reference signal whose frequency is the same as the frequency of the useful signal should be applied in the demodulation system. Through the circuit design, the phase modulated signal is obtained and used as the input signal for the digital design to demodulate.

According to the Bessel function,

\[
\cos(M \sin \omega_0 t) = J_0(M) + 2J_2(M) \cos 2\omega_0 t + \ldots
\]

\[
\sin(M \sin \omega_0 t) = 2J_1(M) \sin \omega_0 t + 2J_3(M) \sin 3\omega_0 t + \ldots
\]

Then equation (4) can be rewritten as:

\[
V(t) = A \times I_{in} \left\{1 - \cos \left[\frac{4\pi}{\lambda}(d_0 + \Gamma a)\right] \left[J_0(M) + 2J_2(M) \cos 2\omega_0 t + \ldots\right]\right\} + C \sin(\omega_0 t + \theta)
\]

In the digital design, phase-lock demodulator is mainly composed of a Band Pass Filter, two Phase-Sensitive Detectors (PSD), a phase shifter as well as two Low Pass Filters. One reference signal \(V_2(t)\) can be described as:

\[
V_2(t) = B \sin(\omega_0 t + \theta)
\]

where \(B\) is the amplitude of reference signal.

The phase sensitive detection (PSD) process is shown in figure 2. The dotted line describes the power spectral density of \(1/f\) noise and white noise which focus on low frequency. After phase modulation, the frequency spectrum of the useful signal removes to the frequency around 500Hz where the noise
signal is quite weak. Because of the PSD, the signal whose frequency is the same as the frequency of the reference signal will be changed to frequency-doubled signal and DC signal which can be described as:

\[ V_{PSDout} = R_{V_{out}2} + R_{V_{out}n} = V_1(t)V_2(t) + n(t)V_2(t) \]  

(7)

The \( R_{V_{out}2} \) can be described as:

\[
R_{V_{out}2} = ABI_{in} \left[ \sin(\omega_0 t + \theta) - \cos\left(\frac{4\pi}{\lambda}(d_0 + \Gamma a)\right)J_0(M)\sin(\omega_0 t + \theta) \\
+ J_2(M)\sin(3\omega_0 t + \theta) - \sin(3\omega_0 t - \theta)) + ... \\
+ J_4(M)\sin(5\omega_0 t + \theta) - \sin(3\omega_0 t - \theta)) + ... \\
+ J_5(M)\cos(4\omega_0 t + \theta) + \cos(4\omega_0 t - \theta)) + ... \right]
\]

(8)

\[ R_{V_{out}n} \] can be described as:

\[ R_{V_{out}n} = -C[\cos(2\omega_0 t + \theta_n) + \cos\theta_n] \]  

(9)

After LPF, frequency-doubled signal is filtered out. Then the demodulated signal becomes DC signal:

\[ V_{out} = -ABI_{in}\sin\left(\frac{4\pi}{\lambda}(d_0 + \Gamma a)\right)J_1(M)\cos\theta \]  

(10)

\[ S(\omega) \rightarrow \text{modulation} \rightarrow \text{PSD} \]

**Figure 2.** The phase sensitive detection process

When the phase shifter is applied to the reference channel, the orthogonal reference signal \( V_3(t) \) has the form:

\[ V_3(t) = B\cos(\omega_0 t + \theta) \]  

(11)

Then the output signals after LPF can be expressed as \( X \) and \( Y \):

\[ X = -ABI_{in}\sin\left(\frac{4\pi}{\lambda}(d_0 + \Gamma a)\right)J_1(M)\cos\theta \]  

(12)

\[ Y = -ABI_{in}\sin\left(\frac{4\pi}{\lambda}(d_0 + \Gamma a)\right)J_1(M)\cos(\theta + 90^0) \]

Finally, the amplitude of the output signal \( V_{out} \) can be expressed as:

\[ V_{out} = \left( X^2 + Y^2 \right)^{\frac{1}{2}} = \left| ABI_{in}\sin\left(\frac{4\pi}{\lambda}(d_0 + \Gamma a)\right)J_1(M) \right| \]  

(13)
where $V_{\text{out}}$ is the phase demodulated signal related to acceleration. Therefore, acceleration could be calculated by equation (13) through the measurement of the demodulated signal.

3. Simulation results and discussions

In this paper, modulation and demodulation module are designed and simulated to increase the performance of the high resolution MOEMS accelerometer.

![Figure 3. Signal generator based on LabVIEW](image)

![Figure 4. The front panel of the demodulation module](image)

![Figure 5. The block diagram of the demodulation module based on LabVIEW](image)

In the circuit design, PZT is driven by a power-driven module. The intensity of the first order is received by S8745-01 which is a photodetector (PD) with a preamplifier. The other PD detects the intensity of the ambient light. Then the two detectors convert light intensity signals to electrical signals. After a differential amplifier which contains AD620, the influence of the ambient light can be decreased. The modulated signal output from follower which is composed of an OPA277, is acquired by a DAQ (NI USB-6212). The circuit design of signal processing is shown in figure 3. The front panel and block diagram of the digital demodulation module are shown in figure 4 and figure 5. The whole simulation module contains the following parts: a signal generator, a reference channel, a Band Pass Filter, two PSD and Low Pass Filters. Real-time signal will be shown in the waveform graph of the front panel. The Lower and upper cut-off frequencies of the BPF are 450Hz and 550Hz. The cut-off frequency of the LPF is 1Hz. Better magnitude-frequency characteristics obtained when the type of BPF and LPF is Butterworth.
As is shown in figure 6, the signal generator based on LabVIEW generates the modulated signal which is not a standard sinusoidal signal as shown in figure 7. In order to obtain the real signal and eliminate the distortion, the sampling frequency is set to 10KHz, which is much higher than the frequency of the modulated signal.

The reference signal channel is divided into two parts. Both parts consist of a reference signal generator, but one of them contains a phase shifter. In the front panel, reference signal could be selected as sinusoidal signal or square signal. The frequency and amplitude of both two orthogonal reference signals are 500Hz and 1V respectively. The phase difference between the two reference signals is 90 degree. The demodulated signal is measured by the orthogonal component without phase adjustment between the modulated signal and reference signal. In this case, the precise demodulated signal is produced. The block diagram of reference signal channel is shown in figure 8.

In terms of the Band Pass Filter and the Low Pass Filter, the common hardware active filter which consists of operational amplifiers and R, C components, is easy to implement but the parameter modulation is difficult to conduct\cite{12}. For digital filter, there is no need to consider the impact caused by surrounding environment and stray capacitance. Also, it can provide different filtering types, orders and topologies such as Butterworth, Chebyshev, Bessel. The cut-off frequency of the BPF and LPF could also be changed in terms of different input signals. The block diagram of Band Pass Filter is shown in figure 9.

In the part of PSD design, a digital multiplier is used instead of analog multiplier chip. After the Low Pass Filters, DC signals are obtained. SNR and relative deviation are displayed in the front panel. A series of output voltages is obtained when the input acceleration is changed. Demodulated signal is buried in the noise, whose amplitude is 0.1 V, the RMS (Root-Mean-Square), SNR (Signal-to-noise ratio), max relative deviation and absolute deviation are presented in Table1.
| Acceleration (g) | RMS (V) | Max relative deviation (%) | Absolute deviation (mm) | SNR (dB) |
|-----------------|---------|---------------------------|-------------------------|----------|
| 0.990           | 7.043   | 0.365                     | 0.260                   | 54.3     |
| 0.991           | 7.013   | 0.647                     | 0.450                   | 51.6     |
| 0.992           | 7.071   | 0.952                     | 0.670                   | 52.3     |
| 0.993           | 6.980   | 0.550                     | 0.380                   | 51.7     |
| 0.994           | 7.103   | 0.389                     | 0.276                   | 52.4     |
| 0.995           | 6.955   | 0.463                     | 0.322                   | 53.8     |

From table 1, we can see the max relative deviation of the output signal is below 1% and the signal-noise ratio reaches up to 50 dB when the amplitude of the noise signal is about 0.1V.

Figure 10 shows the relationship between the acceleration and output voltage after LPF when the acceleration is 0.992g. DC signal is reserved after LPF. Figure 11 represents the acceleration measurement result which is the absolute value of a sine wave signal. In this figure, there is little difference between the two curves and the output voltage is well matched to equation (13). Through the simulation of demodulation, it is clear that the measured value is close to the theoretical output value.

The simulation shows the max relative deviation is below 1% and the SNR of this demodulator is about 50dB. This design is quite fit for the phase modulated signal whose frequency is 500Hz.

4. Conclusions
In this paper, we have simulated a modulator and an orthogonal phase-lock demodulator to demodulate the modulated signal of a high resolution MOEMS accelerometer which is based on phase-sensitive detection technique. The theoretical analysis and corresponding LabVIEW simulation are presented in this paper. The simulation shows the max relative deviation of the acceleration is below 1% and the SNR of this demodulator is about 50 dB. This modulation and demodulation module is quite fit for this specific MOEMS accelerometer.

5. References
[1] Tan C W and Park S. 2005 Design of accelerometer-based inertial navigation systems IEEE. T. Instrum. Meas. 50 2520-30
[2] Hopkins R, Miola J, Setterlund R, Dow B and Sawyer W 2005 The Silicon Oscillating Accelerometer: A High-Performance MEMS Accelerometer for Precision Navigation and Strategic Guidance Applications Proceedings of the 61st Annual Meeting of The Institute of Navigation (Cambridge, MA, 27-29 June 2005) pp 1043-52

[3] Phan D H, Bonnet S, Guillemaud R, Castelli E and Pham Thi N Y 2008 Estimation of respiratory waveform and heart rate using an accelerometer Biomedical Imaging: From Nano to Macro, 2008. ISBI 2008. 5th IEEE International Symposium on (Paris, 14-17 May, 2008) pp 4916-19

[4] Dianne S, Ward, Kelly R, Evenson, Amber, Vaughn, Anne Brown, Rodgers, Richard P and Troiano 2005 Accelerometer use in physical activity: best practices and research recommendations Med. Sci. Sport Exer. 37 582-588

[5] Salarian A, Russmann H., Vingerhoets F J, Burkhard P R and Aminian K 2007 Ambulatory Monitoring of Physical Activities in Patients With Parkinson's Disease IEEE. Trans. Biomed Eng 54 2296-99

[6] Kau L J and Chen C S 2015 A Smart Phone-Based Pocket Fall Accident Detection, Positioning, and Rescue System IEEE. J. Biomed. Health. 19 44-56

[7] Shuangshuang Z, Juan Z, Changlun H, Jian B and Guoguang Y, 2012 Optical accelerometer based on grating interferometer with phase modulation technique Appl. Opt. 51 7005-10

[8] Shuangshuang Z, Changlun H, Jian B, Guoguang Y and Feng T 2012 Appl. Opt. 50 1413-16

[9] Winzer P J and Essiambre R J, 2006 Winzer P J, Essiambre R. Advanced Optical Modulation Formats Proceeding of the IEEE 94 952-985

[10] Lin Q, Chen L and Wu X 2010 A high-resolution fiber optic accelerometer based on intracavity phase-generated carrier (PGC) modulation Meas. Sci. Technol. 22 1-6

[11] Jinzhan G 2002 Detection of Weak Signals (Beijing: Tsinghua University Press) p 171

[12] Liu Y L and Zhang R 2012 AD630 Lock-in Amplifier Circuit for Weak Signal Adv. Mater. Res. 482-484 975-980