Generating compressed broadband terahertz pulses using aperiodically poled electro-optic crystals

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We introduce a general technique to generate compressed broadband terahertz pulses based on difference frequency generation of optical pump radiation using aperiodically poled structures. The pump pulse format and poling of the crystals conceived are such that the emergent terahertz pulse is compressed. The approach circumvents pump pulse distortions that result from non-collinear approaches and the need for external compression. It is particularly efficient for the generation of pulses of intermediate durations of few to tens of cycles. For instance, conversion efficiencies in the few percent range are attainable in cryogenically cooled lithium niobate.

INTRODUCTION

High-field terahertz radiation are important probes for a variety of fundamental scientific investigations [1–3]. In addition, they are also touted to be invaluable to future particle accelerator technology [4–8], which could transform a wide swath of applications encompassing microscopy [9–11], X-ray generation and medical therapy [12].

While various methods to generate terahertz(THz) radiation, ranging from photoconductive switches [12] to Gyrotrons [13] and free-electron lasers [14] exist, they have been limited in one or more of frequency, peak power or bandwidth. With the proliferation of advanced solid-state laser technology, methods based on nonlinear optical frequency conversion of high power infrared or far-infrared radiation have gained ground. Certainly, they have produced the best optical-to-terahertz conversion efficiencies and peak electric fields to date [13]. In addition laser-driven approaches offer precise synchronization possibilities, which are instrumental for spectroscopic experiments.

However, despite this remarkable process, drawbacks persist with various nonlinear optical frequency conversion schemes. In organic crystals, terahertz generation is close to being perfectly phase matched, purely by virtue of material properties. Crystals shorter than the coherence length may thus be utilized [15–16] to generate broadband terahertz radiation > 1 THz efficiently. However, in general, phase-matching needs to be enforced by developing a suitable mechanism for efficient terahertz generation.

A general scheme for broadband phase-matching is via the use of tilted-pulse-fronts (TPF) [17] in non-collinear geometries. This method, more amenable for broadband terahertz generation of single-cycle terahertz pulses, has yielded very high conversion efficiencies and pulse energies in the < 1 THz in regime in lithium niobate [18–22]. However, the approach is ultimately limited by a spatio-temporal break-up of the optical pump pulse upon generating terahertz radiation at high conversion efficiencies [23–26]. This issue may be alleviated (but presumably not eliminated) significantly using stair-step echelons [27–28]. Alternatively, attempts to eliminate the drawbacks of using TPFs in lithium niobate by utilizing semiconductors as the nonlinear material, which have smaller pulse-front-tilt angles, are being pursued [29].

Quasi-phase-matching based on periodic inversion of the second order nonlinearity [30–35] addresses the issues posed by non-collinear phase-matching geometries but is more suited to the generation of narrowband terahertz pulses. Recently, we suggested the use of aperiodically poled structures [36] to generate broadband terahertz pulses. Such structures are advantageous in that spatio-temporal distortions of the terahertz pulse are significantly abated in comparison to non-collinear phase-matching techniques while also opening up the possibility of re-using the optical pump pulse in subsequent stages. However, they require an external device to compress the chirped terahertz pulses, which increases the complexity of practical implementation.

In this paper, we introduce an approach which utilizes aperiodic structures to generate broadband terahertz pulses which emerge already compressed. The approach is to use chirped crystals in conjunction with a pair of chirped pulses (with different relative chirp rates). Various terahertz frequencies are phase-matched at different locations in the crystal. However, the relative chirp between the optical pump pulses is set such that terahertz frequency $\Omega_1$ generated at location $z_1$ arrives exactly in phase as the terahertz frequency $\Omega_2$ generated at location $z_2$. As a consequence, the terahertz pulse emerges compressed upon exiting the crystal. The collinear geometry addresses the issues of pump pulse distortion suffered by
TPF(s) while circumventing the need for external compression demanded by prior proposals of aperiodic structures. It is found that the technique presented here is particularly suitable for generating terahertz pulses of few to tens of cycles, an intermediate regime which has received relatively less attention. As a specific example, the use of cryogenically cooled lithium niobate at 80 K pumped by 1 µm lasers are predicted to yield optical-to-terahertz energy conversion efficiencies of a few percent with free-space field strengths of 50 MV/m. Similar performance with materials such as Potassium Titanyl Phosphate (KTP) may also be envisaged. The use of chirped ≈100 ps pump pulse durations enables significant energy loading of demonstrated cm² crystals, resulting in focused field strengths of 500 MV/m and terahertz pulse energies of several mJ. Such high-field, few-cycle pulses could be transformative for terahertz driven electron acceleration, resulting in electron bunches with unprecedented properties, potentially disruptive for electron microscopy and X-ray generation.

In subsequent sections, we provide rigorous simulations as well as a general analysis to elucidate the physics governing the proposed approach.

GENERAL PHYSICAL PRINCIPLE

Consider a situation where terahertz generated in the beginning of a section of length catches up with terahertz radiation generated at the end. This lends itself to a constraint that requires the net “arrival” time of all terahertz frequency components to be exactly equal. If terahertz radiation of a certain frequency $\Omega_g$ is phase-matched at a certain point $z$ in the crystal, then the corresponding delay at the end of a crystal of length $L$, extending between $z = \pm L/2$ would be given by:

$$\frac{(L/2 - z)n_{THz}}{c} + \frac{(z + L/2)n_g}{c} + \Delta t(z) = \text{constant} \tag{1}$$

In Eq. (1), $n_{THz}$ is the terahertz phase refractive index and $n_g$ is the group refractive index of the optical pump pulse. The first term in Eq. (1) corresponds to the time of arrival of the terahertz frequency generated at $z$ at the end of the crystal. The second term in Eq. (1) corresponds to the time of arrival of the optical pulse at $z$. $\Delta t(z)$ is the relative time delay (in the frame of the pump) at which the given terahertz frequency $\Omega_g(z)$ is generated. For $z = 0$, the left hand side of Eq. (1) is $L(n_{THz} + n_g)c^{-1}/2$ as shown in Fig. 1. Consequently, the constraint reduces to a spatially dependent $\Delta t(z)$ given by the following:

$$\Delta t(z) = \frac{z(n_{THz} - n_g)}{c} = \frac{z\Delta n}{c} \tag{2}$$

Alternatively, this may be viewed that the terahertz frequency to be phase-matched at $z$, must be generated at the delay $\Delta t(z)$ in the frame of the pump, as shown in Fig. 1. Now, consider a pump pulse format comprised of a broadband pulse centered at frequency $\omega_1$, chirped at a rate $b$, with a pulse duration $\tau$ and a narrowband pulse centered at $\omega_2$, also with duration $\tau$. Naturally, the generated terahertz frequency is merely the difference of the instantaneous frequencies of the two pulses given by $\Omega_g = \omega_1 + b\Delta t - \omega_2$. However, only the frequency components corresponding to $\Delta t(z)$ from Eq. (2) shall be phase-matched, which yields the following relationship for the terahertz generated as a function of propagation distance $\Omega_g(z)$:

$$\Omega_g(z) = \Omega_0 + \frac{b \Delta n}{c} \tag{3}$$

In fact, numerical simulations precisely depict this as plotted in Fig. 2. As can be seen, the optical intensity profile (in blue) is of the form described above and depicted schematically in Fig. 1. A single-cycle terahertz field which grows in peak field, while staying compressed can be seen to evolve.

ANALYTIC FORMULATION

Proof of functionality

In this section, we will analytically show that the approach proposed in the previous section based on physical reasoning, indeed functions as suggested. We may formally delineate the electric field envelope in time ($t$) of the broadband chirped pulse by the phasor $E_1(t) = E_1 e^{j\omega_1 t + jbt^2/2} e^{-t^2/\tau^2}$ and the narrowband pulse by $E_2(t) = E_2 e^{j\omega_2 t + jbt^2/2} e^{-t^2/\tau^2}$. This assumption is without loss of generality; the overarching conclusions hold good even for differing pulse durations and chirp rates for the two pulses. As per Eq. (2), $\Omega_0 = \omega_1 - \omega_2$. The overall envelope is given by $E(t) = E_1(t) + E_2(t)$. The terahertz spectral component with angular frequency $\Omega$ is given by $E_{THz}(\Omega) = A_{THz}(\Omega)e^{-j\Delta t}\chi_{THz}(\Omega)z$, where $A_{THz}(\Omega) = F(E(t))$ and $\chi_{THz}$ corresponds to the Fourier transform between temporal and spectral domains. The corresponding nonlinear evolution of $A_{THz}$ in the undepleted limit is given as follows:

$$\frac{dA_{THz}(\Omega, z)}{dz} = \frac{-j\Omega}{2cn_{THz}} \chi_{THz}^{(2)}(z)e^{j\Omega n_{zc}} F(|E(t)|^2) - \frac{\alpha(\Omega)}{2} A_{THz}(\Omega, z) \tag{4}$$

In Eq. (4), $\Delta n = n_{THz} - n_g$ represents the difference between the terahertz refractive index $n_{THz}$ at $\Omega_0$ and optical group refractive index $n_g$, $\chi_{THz}^{(2)}(z)$ is the $z$ dependent second-order nonlinear coefficient and $\alpha(\Omega)$ is the terahertz absorption coefficient. Equation (4) maybe reduced
FIG. 1.
Schematic of outlined approach: The optical intensity profile appears as a sequence of pulses with decreasing spacing and duration. Lower frequencies are generated at earlier times, \( t_{rel} \) relative to the pulse frame of reference. Simultaneously, the nonlinearity is such that these frequencies are phase-matched earlier in the crystal. The total arrival time of the terahertz is thus conserved for various frequencies, thus resulting in the emergence of compressed pulses at the output.

FIG. 2.
Numerical simulations depicting a chirped nonlinearity profile along with the intensity profile of the optical pump and terahertz field. The continuous evolution of a compressed terahertz pulse is observed.

to the form below:

\[
A_{THz}(\Omega, L) = \frac{-j\Omega \mathcal{F}[|E(t)|^2]\chi_0^{(2)}}{2cn_{THz}} \\
\cdot \int_{-L/2}^{L/2} e^{-j\Delta k z} \chi^{(2)}(z)e^{\alpha(z-L)/2}dz
\tag{5a}
\]

\[
A_{THz}(\Omega, L) = \frac{-j\Omega \mathcal{F}[|E(t)|^2]}{2cn_{THz}}\chi_0^{(2)} \cdot \mathcal{F}_{\Delta k}\left\{\text{rect}\left[\frac{z - L/2}{L}\right]\chi^{(2)}(z)e^{\alpha(z-L)/2}\right\}
\tag{5b}
\]

The integral in Eq.\( 5a \) may represented as a Fourier transform between the spatial and \( \Delta k = -\Delta n \Omega c^{-1} \) domains as shown in Eq.\( 5b \). This is possible by casting the situation as one where the nonlinearity \( \chi^{(2)}(z) \) is non-zero for \(-L/2 < z < L/2\) and vanishes everywhere else in the interval \([-\infty, \infty]\) as evident by the use of the \text{rect} function.

One obtains a different perspective when looking at terahertz generation through the lens of Eq.\( 5b \). For instance, consider the case of a perfectly periodically poled structure. In such a case, \( \chi^{(2)}(z) = \chi_0^{(2)}e^{-j\Omega_0 z} \), where \( \chi_0^{(2)} \) is some constant value, proportional to the bulk nonlinearity \( \chi_b^{(2)} \). In the case of a periodically poled structure, \( \chi_0^{(2)} = 2\pi \chi_b^{(2)} \). The finite length of the crystal (i.e the \text{rect} function in Eq.\( 5b \)) acts as a filter that produces the all familiar \text{sinc} response that characterizes phase-matching in a nonlinear process.

In the previous section, in Eq.\( 3 \), the terahertz frequencies \( \Omega_g \) generated at various locations in the crystal were deduced for the proposed mechanism. Naturally, the phase-matching conditions for these frequencies must be fulfilled at their corresponding locations. This is possible by generating a nonlinear profile \( \chi^{(2)}(z) = \chi_0^{(2)}e^{-j\Omega_g z} \), wherein \( \Omega_g \) is given by Eq.\( 3 \). Practically, such a profile may be realized by a poling profile of the form of \( \chi^{(2)}(z) = \chi_0^{(2)}\text{sgn} \left[\cos(\Omega_g \Delta n z)\right] \), where \text{sgn} represents the signum function. As a limiting case, the impact of the \text{rect} function in Eq.\( 5b \) may be omitted, thereby resulting in \( A_{THz} \) being directly proportional to the Fourier transform of the nonlinearity profile. This approximation may also be justified by the fact that due to the finite bandwidth of the input pump pulses, the contributions of the crystal beyond the interval \([-L/2, L/2]\) are less significant. Upon explicit evaluation of \( \mathcal{F}[|E(t)|^2] \), we obtain the generated spec-
terial component $A_{THz}(\Omega, L)$ in Eq. (6) below.

$$A_{THz}(\Omega, L/2) = \frac{\pi \Omega x_0^{(2)} E_1 E_2 e^{-\frac{(\Omega - \Omega_0)^2 L^2}{4\Delta n^2}}}{2b \Delta n n_{THz}} \left( e^{-\frac{j\Omega(\Omega - \Omega_0)^2}{8b^2 \Delta n^2}} e^{\frac{-j\Delta n^2 L^2}{4\Delta n^2}} \right)$$

(6)

Inspecting the first exponential term in Eq. (6), we notice that the terahertz spectrum $A_{THz}(\Omega, L)$ is centered about the angular frequency $\Omega_0$ with bandwidth proportional to $(8\tau^{-2} + 2\tau_{TL}^{-2})^{1/2}$. In the limit that the chirped pulse duration is much greater than the transform limited bandwidth, i.e., $\tau >> \tau_{TL}$, the terahertz spectral bandwidth is proportional to that of the broadband pump pulse $\tau_{TL}^{-1}$. This corresponds to the range of possible beat frequencies between the two input pulses and is consistent with expectations. The terms enclosed within brackets in Eq. (6) correspond to the various phase terms that accrue. Given that the chirp rate $b = (\tau\tau_{TL})^{-1}$ and that the chirped pulse duration is significantly larger than the transform limited duration, i.e., $\tau >> \tau_{TL}$, the factor $e^{\frac{j\Omega(\Omega - \Omega_0)^2}{8b^2 \Delta n^2}}$ is well approximated by $e^{\frac{-j\Delta n^2 L^2}{4\Delta n^2}}$. In fact, one may achieve exact compensation by adjusting the chirp rate of the crystal to $b' = b\sqrt{1 + 4\tau^{-2}b^{-2}}$ which would result in exact phase compensation. In addition, the third term within brackets should be small for reasonable values of absorption. Thus, it has been analytically proven that the proposed approach produces a perfectly compressed terahertz pulse in the ideal limit. The qualitative trends deduced above are well justified by calculations employing the exact closed form expression based on imaginary error functions (erfi) below.

Length and frequency behavior

In Eqs. (7a)-(7c) below, we present an exact solution to Eq. (6).

$$A_{THz}(\Omega, z) = \frac{-j\pi \Omega x_0^{(2)} E_1 E_2 e^{-\frac{(\Omega - \Omega_0)^2 L^2}{4\Delta n^2}}}{4\sqrt{bb'} \Delta n n_{THz}} \left( e^{-\frac{j\Omega(\Omega - \Omega_0)^2}{8b^2 \Delta n^2}} e^{\frac{-j\Delta n^2 L^2}{4\Delta n^2}} \right) G(\gamma, \theta)$$

(7a)

$$G(\gamma, \theta) = e^{-\gamma \theta} \left( j + \text{erfi}\left[\sqrt{2}(\gamma - \frac{j\theta}{2})\right] \right)$$

(7b)

$$\gamma = \frac{z}{z_0} + \frac{\Omega - \Omega_0}{b \sqrt{\tau}}; \quad \theta = \frac{\alpha z_0}{2}; \quad z_0 = \frac{c}{b \sqrt{\tau} \Delta n}$$

(7c)

In Eq. (7a), the dependence of the terahertz spectral component $A_{THz}$ on length $z$, terahertz frequency detuning $\Omega - \Omega_0$ from the central frequency $\Omega_0$, and terahertz absorption coefficient $\alpha$ are captured by the function $G(\gamma, \theta)$ which is delineated in Eq. (7b). The variables $\gamma, \theta$ are normalized variables which capture various dependencies as depicted in Eq. (7c). In Eq. (7b), the prefactor $e^{-\gamma \theta}$ corresponds to the loss terms contained in Eq. (6). Note that $\gamma$ captures propagation and bandwidth effects, while $\theta$ predominantly relates to the effect of absorption. An important physical quantity here is the distance parameter $z_0 = \frac{c}{b \sqrt{\tau} \Delta n}$. Since the chirp rate $b' \approx (\tau\tau_{TL})^{-1}$, $z_0 = c \Delta n^{-1} \sqrt{\tau\tau_{TL}}$. It may be therefore viewed as some sort of cut-off interaction length beyond which terahertz generation becomes significant.

This becomes evident upon examining Fig. 3 where the function $G(\gamma, \theta)$ is plotted. The surface plot shows the variation of the magnitude $|G|$. The inset on top shows the phase $\angle G$, while the inset at the bottom represents slices of the surface plot as a function of $\gamma$ at various $\theta$ (or absorption) values. The behavior of $G$ is step-like with its magnitude abruptly increasing before saturating. Naturally, with increasing $\theta$, an exponential drop in $|G|$ due to absorption effects is evident.

Since $\gamma$ is proportional to $z/z_0$ and since $|G|$ saturates on average beyond a critical value of $\gamma$, it implies that $z_0$ is a critical length or ‘interaction length’ parameter. Both magnitude and phase behavior show rapid oscillations for smaller $\gamma$, but an eventual damping of oscillations for larger $\gamma$. The stabilization of phase over the scale of the critical propagation distance $z_0$ indicate that the compression is not instantaneous but occurs over a certain distance. Since $|G|$ varies with frequency and absorption coefficient, ripples in the spectral magnitude may also be expected. Furthermore, oscillations in spectral phase are also evident from the behavior of $\angle G$. For the
same $z$, $\gamma$ values increase with $\Omega$. In conjunction with the factor $\Omega$ in Eq. (7a), the spectral amplitude would show a ramp-like shape. However, the falling edge of the spectrum shall be also influenced by the exponential dependence on $e^{-2(\Omega - \Omega_0)^2\tau^2 L}$ in Eq. (7a). Finally, since $\Omega_0$ decreases with both $\tau$, $\tau_{TL}$, the optimal interaction lengths shall be expected to be shorter for smaller $\tau$, $\tau_{TL}$. These deductions are supported by numerical simulations in the next section.

**Efficiency trends**

The upper limit of Eq. (7a), is given by Eq. (6). Some general trends on conversion efficiency can thus be gleaned by utilizing Eq. (6). The conversion efficiency $\eta$ may be expressed as follows:

$$\eta = \pi \varepsilon_0 \int_0^\infty n_{THz} |A_{THz}(\Omega)|^2 d\Omega \quad (8a)$$

$$\eta \leq \frac{\pi^{1/2} 2 F_{pump}^2 \Omega^2 \tau_{TL}}{2 \varepsilon_0 n_{THz} \Delta n^2 n_0^2} \quad (8b)$$

Based on Eq. (8b), the proportionality to $\tau_{TL}$ delineates that the conversion efficiency is inversely proportional to the generated terahertz bandwidth. This is understandable given that a larger bandwidth, when apportioned over the same length translates to less interaction length per frequency. Thus, it is plausible that the conversion efficiency that is generated from such structures would be lesser than that of multi-cycle generation methods. On the other hand, it can be seen that despite this limitation, the conversion efficiency for moderate terahertz pulse durations breaches the percent level. Furthermore, the conversion efficiency is inversely proportional to the difference in terahertz phase and optical group refractive index $\Delta n$. In light of this, KTP with smaller values of $\Delta n$ and potentially larger damage fluence thresholds may be of similar if not superior performance in relation to lithium niobate.

**SIMULATION RESULTS**

In this section, we present simulation results incorporating the full effects of dispersion, absorption as well as pump depletion. While the simulation results are carried out for lithium niobate, the illustrated trends apply to other material systems. The material parameters and numerical model are the same as those utilized in [35]. The maximum pump fluence that may be impinged on the crystals, is assumed to scale as $F_{pump} = 0.85(\tau/100ps)^{0.5} J/cm^2$, where $\tau$ is the $e^{-2}$ pulse duration.

In Fig. 4, we study the dependence of the optical-to-terahertz energy conversion efficiency ($\eta$), the electric field (E) as well as optimal crystal lengths as a function of the transform limited bandwidth of the pump pulse $\tau_{TL}$, the chirped pulse duration $\tau$ (or equivalently the chirp rate $b$) and the terahertz frequency $f_{THz}$ using undepleted calculations. The bandwidth of the generated terahertz pulse, is limited by $\tau_{TL}$, or the range of beat frequencies between the two optical pump pulses. Since, the crystal is aperiodic, each terahertz frequency is generated at a designated region within the crystal. Therefore, the bandwidth of the generated terahertz pulse is also correlated to the chirp rate (w.r.t. length) or $b\Delta nc^{-1} L_{opt} \approx \tau_{TL}^{-1}$. However, since $b = (\tau \tau_{TL})^{-1}$, one obtains a condition for the optimal crystal length $L_{opt} \approx c\tau \Delta n^{-1}$, for a given chirped pulse duration $\tau$. However, the maximum interaction length is limited by the terahertz absorption coefficient. Thus, $L_{opt} \approx \alpha^{-1}$ and consequently $\tau_{opt} \approx \alpha^{-1} \Delta n c^{-1}$. As the bandwidth increases, the maximum absorption length reduces by virtue of spanning larger frequencies and hence both the optimal chirped pulse duration and crystal length drop. These aforementioned trends are evident in Fig. 4(a) which depicts $\eta$ as a function of $\tau$ and $\tau_{TL}$ for $f_{THz} = 0.5$ THz. Further-
more, in line with the efficiency relationship with $\tau_{TL}$ delineated in section, we see an increase in efficiency with increasing $\tau_{TL}$. Intuitively, this may be understood by the fact that when the same length is apportioned over a larger bandwidth, the coherent build up of each terahertz spectral component is limited. Therefore for single-cycle pulses, the efficiency is limited to a fraction of a percent but for few-cycle or tens of cycles long terahertz pulses, the conversion efficiencies breach the percent mark comfortably. In Fig. 3(b), the peak electric fields as a function of $\tau,\tau_{TL}$. Since the emergent terahertz pulses are compressed, the peak fields are proportional to $\eta/\tau_{TL}$. Therefore, for small $\tau_{TL}$, the peak electric fields are large despite the low conversion efficiencies. After a minor drop for intermediate values of $\tau_{TL}$, the peak fields tend to stabilize or saturate. The maximum field strengths for collimated beams in free space are on the order of 50 MV/m. However, if one uses cm$^2$ aperture crystals, focused field strengths of several hundred MV/m are indeed feasible.

Figures 4(c) and (d) delineate the global optima (w.r.t to parameters $\tau, \tau_{TL}$ for conversion efficiencies and fields obtained for various central terahertz frequencies as well the corresponding optimal crystal lengths. Naturally, these represent the many-cycle limit, i.e. $\tau_{TL} = 50$ ps or the edge of the parameter space used for $\tau_{TL}$. The corresponding optimal values of $\tau \approx \alpha^{-1} \Delta n c^{-1} \approx 100$ ps. In Fig. 4(c), the conversion efficiency shows an increasing trend with frequency, in line with the trends suggested by Eq. 7(b) albeit only linearly and not quadratically owing to the effects of increasing absorption with terahertz frequency. The maximum electric fields also increase with frequency and are very close to the transform limited values, further depicting the efficacy of the presented mechanism in achieving compression. Figure 4(d) depicts the optimal interaction lengths as a function of frequency. Here, too, the trends of decreasing optimal lengths are consistent with increasing absorption coefficients with terahertz frequency.

**Spectra, phase and temporal formats**

In this section, we present the spectra, phase and temporal electric field traces for various values of $\tau_{TL}$ (or terahertz bandwidths). In each case, the terahertz frequency was $\approx 0.5$ THz and for optimal values of $\tau$ obtained from the previous simulations. In Fig. 5(a)-(c), we present the spectrum, phase and temporal format for $\tau_{TL} = 0.5$ ps. The spectral profile in 5(a) is clearly broadband spanning 0.2 to 0.8 THz. The increasing ‘ramp’ like shape and ripples are in line with the expectation from Eq. 7 and descriptions in section. Understandably, the transform limited duration of the corresponding terahertz pulse would be proportional to $\tau_{TL}$. For 0.5 ps, this corresponds to a single-cycle terahertz pulse as can be seen in Fig. 5(c). It is evident from the flat group delay (GD) (save for the ripples, also consistent with Eq. 7(c) in Fig. 5(b) that the transform limited trace (red) is very close to the obtained trace from the structure. Similarly, Figs. 5(d)-(f) and Figs. 5(g)-(i) represent the corresponding properties for $\tau_{TL} = 4$ ps and $\tau_{TL} = 25$ ps respectively, which both produce pulses (green) close to transform limited values (red).

**Cascading effects**

While previous sections addressed the terahertz pulse properties, here we present the influence of terahertz generation in such systems on the optical spectrum in Fig. 6. In Fig. 6(a), the optical spectrum corresponding to two pulses separated by a frequency of 0.5 THz, with parameters $\tau_{TL} = 25$ ps, and $\tau = 150$ ps. Since the energy in the narrowband pulse and broadband chirped pulse are equal, their relative strengths in the spectral domain are dissimilar as is evident via the inset in Fig. 6(a). Note, that inset depicts that the linewidths of the two lines are different, since they correspond to differing bandwidths. Furthermore, the conversion efficiency versus length is similar to that obtained via the undepleted calculations, albeit producing higher efficiencies due to the increase in pump intensity resulting from increased spectral bandwidth. Since, the strength in the two lines differ, the situation is similar to that of a cascaded terahertz parametric amplifier [37]. The spectrum shows an increasing red-shift with distance, accompanied also with sum frequency generation due to the symmetry in phase-matching owing to the small terahertz frequency, similar to that described in [37]. It is worthwhile to note that the situation that was imagined in the absence of depletion
is well justified by even the rigorous simulations.

CONCLUSION

In conclusion, a new method to generate broadband terahertz pulses using a combination of chirped pulses and aperiodically chirped crystals can produce compressed pulses with peak fields and high efficiencies. The approach is particularly efficacious for generating few-cycle to tens of cycles of pulses, yielding conversion efficiencies of ≈5%. This is a parameter regime, not addressed by prior work which has primarily paid attention to the very long pulse regime or very short pulse regime. The work is highly relevant to high field terahertz acceleration and coherent X-ray generation.

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