Modeling of the thermal stresses in the welded rails of the continuous welded track in the permafrost zone

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Abstract: The article focuses on the numerical methods that are used to solve heat conduction nonlinear problems with the help of the h-matrix to account for the relaxation of thermal stresses in continuously welded rails. A general mathematical model on the basis of the heat conduction nonlinear equation is presented to calculate the thermal field in an infinite unlimited plate. The developed mathematical model can be applied to choose the implementation modes of the continuously welded rails into the optimal thermal mode.

1. Introduction

The industrial land development in the Far North and Siberia implies an increase in the railway track length alongside with its active operation. The characteristic features of the roadbed with some sections of permafrost, marshiness, spreading of karst processes include the growth of track deformation, caused by cryogenic, karst-suffosion processes in roadbed soils.

At present time the replacement works of the jointed track structure with the wooden roadbed into the thermal-stressed structure with the reinforced concrete roadbed within the realization of the project «The Northern Latitudinal Railway». It should be mentioned that the sections included in the project «The Northern Latitudinal Railway» are located in the complicated geological permafrost conditions that lead to the subgrade deformation and the track structure in general [1-2].

According to Clause 2.2.1 of The Instruction for the construction, laying and repair of the continuous welded track [3], the subgrade must be strong and stable, it should have sufficient dimensions to place the ballast prism. It is not allowed to have heaves with the height more than 10 mm, as well as track settlements, embankment slope creeps and sloughed-offs and other deformations of the subgrade. The identified defects and other deformations of the subgrade must be eliminated in accordance with «The Rules for the terms and operation of the repair works and planned preventive alignment of the railway track» [4].

Realizing the track reconstruction along the route of «The Northern Latitudinal Railway», a continuous welded rail structure has been laid on the straight section with the deforming subgrade. Carrying out the complete overhauling of Level 1 track, the demands set due to the Instruction [3] were followed, however as the section was located in the permafrost zone a range of deformations (in the form of subgrade soil settlements presented in figure 1) began developing again in the thawing process of the permafrost soil.
Figure 1 shows that the repetitive development of the previously eliminated failures in the form of track settlements on the section are noticed again. According to the data of the track check in October 2017, the amount of deformation on Kilometers 2 169 was 260 mm per 79 meters. In 2019 the failures were completely eliminated, however the analysis of the graphic diagrams reveals the tendency of the repetitive development of the failures. To provide the train traffic safety on this section, it is needed to perform a periodic track raising that, in its turn, leads to the overrate of the side slope and the shoulder decrease. All these factors have a negative effect on the fail-safe work of the continuous welded railway track.

Figure 1. Graphic diagram of the diagnostic complex DCI after carrying out the complete overhauling of Level 1 track.

2. Methods and results
The calculating method used for the laying conditions of the continuous welded railway track [3] does not take into account the stresses in the continuously welded rails emerging in the process of the thermal track grading deformations of the subgrade soil. The resulting track grading deformations also cause significant alternate stresses in the continuously welded rails. Modeling of the thermal conductivity processes and their influence on the stress-strain state of a prestressed continuously welded rail structure will solve the problem of the thermal optimization of the continuously welded rail initial fixing and the forecast of pre-buckling state of continuously welded rails.

The Helmholtz equation is considered to model the propagation of the thermal phenomena in a solid object. It is proposed to develop a numerical algorithm without any saturation that allows to solve the spectral problem for the homogeneous Helmholtz equation, the boundary value problem for the non-homogeneous Helmholtz equation and the non-stationary heat conduction problem [5-7].

\[ \Delta \Phi(x, y, z) + \lambda^2 \Phi(x, y, z) = F(x, y, z), (x, y, z) \in \Omega \]

\[ \Phi_{\partial \Omega} = 0 \]

Here \( \Omega \) – an axis rotation body \( Oz \), \( \partial \Omega \) – its boundary. If \( F(x, y, z) \) is identically equal to zero, then the problem at eigen values is considered, otherwise, if \( \lambda^2 \) – is a non-eigen value, the boundary problem is solved. Since the right part is arbitrary, the problem under consideration is three-dimensional.

Let us introduce a curved coordinate system \((r, \theta, \varphi)\), connected with the Cartesian coordinate system \((x, y, z)\) relations

\[ x = v(r, \theta)cos\varphi, y = v(r, \theta)sin\varphi, z = u(r, \theta) \]

If the Cauchy-Riemann conditions are met:
then the coordinate system \((r, \theta, \phi)\) is orthogonal and in this coordinate system the Laplacian of the scalar function has the form:

\[
\Delta \Phi = \frac{r}{\nu \omega^2} \left[ \frac{\partial}{\partial r} \left( r^2 \frac{\partial \Phi}{\partial r} \right) + \frac{\partial}{\partial \theta} \left( \frac{\nu \partial \Phi}{r \partial \theta} \right) \right] + \frac{1}{\nu^2} \frac{\partial^2 \Phi}{\partial \varphi^2}, \quad w^2 = \left( \frac{\partial v}{r \partial \theta} \right)^2 + \left( \frac{\partial u}{\partial \theta} \right)^2
\]

(4)

It is useful to assume that \((r, \theta, \phi)\) are spherical coordinates, and the relations (3) specify the mapping of a sphere of unit radius to the interior of the considered body of rotation \(Q\). We use \(G\) to denote the area obtained by the meridional section of the body of rotation \(Q\) (i.e., the body \(Q\) is obtained by rotating the area \(G\) around the axis \(z\)). Let \(\psi = \phi(\xi)\), \(\psi = u + iv\), \(\xi = r \exp(\iota \theta)\) - conformal mapping of the sphere \(|\xi| \leq 1\) at the interiority of the area \(G\) [8].

Then, instead of the problem (1) - (2), we have an internal problem in the sphere of unit radius, for the equation (4). Besides on its margin, the boundary condition \(\Phi = 0\) is set. Further we assume that the conformal mapping of the sphere of unit radius to the interiority of the area \(G\) is known.

The discrete Laplacian is obtained as an \(h\)-matrix [9]

\[
H = \frac{2}{L} \sum_{k=0}^{L} A_k \otimes h_k, \quad L = 2L + 1
\]

(5)

Here the prime mark denotes, that the summand at \(k=0\) is taken with the coefficient \(1/2\); the character \(\otimes\) the Kronecker product of matrices; \(h\) — the matrix of the size \(L \times L\) with elements:

\[
h_{kij} = \cos k \frac{2\pi(i - j)}{L}, (i, j = 1, 2, ..., L)
\]

\(A_k\) is the matrix of the discrete operator, corresponding to the differential operator

\[
\frac{r}{\nu \omega^2} \left[ \frac{\partial}{\partial r} \left( r^2 \frac{\partial \Phi}{\partial r} \right) + \frac{\partial}{\partial \theta} \left( \frac{\nu \partial \Phi}{r \partial \theta} \right) \right] - \frac{k^2}{\nu^2} \Phi, k = 0, 1, ..., l
\]

(6)

with the boundary condition: \(\Phi|_{\nu=1}=0\).

To discretize the differential operator (6), we select a grid according to \(\theta\) consisting of \(n\) knots [10]:

\[
\theta_v = \frac{\pi}{2} (y_v + 1), y_v = \cos \nu \cdot \theta_v = \frac{(2\nu - 1)\pi}{2n}, \nu = 1, 2, ..., n,
\]

we also apply the interpolation formula

\[
g(\theta) = \sum_{\nu=1}^{n} \frac{T_n(x) g_v}{n(-1)^{\nu-1} \sin \nu \cdot \theta_v (y-y_v)}, \quad y = \frac{1}{\pi} (2\theta - \pi),
\]

\[
g(\psi) = g(\theta_v), \nu = 1, 2, ..., n, \quad T_n(x) = \cos(n \arccos(x))
\]

(7)

The first and second derivatives of \(\theta\), included in the relations (6), we obtain by differentiating the interpolation formula (7).

According to \(r\), we select a grid consisting of \(m\) knots:

\[
r_v = \frac{1}{2} (z_v + 1), z_v = \cos \nu \cdot \chi_v = \frac{(2\nu - 1)\pi}{2m}, \nu = 1, 2, ..., m,
\]

we also apply the interpolation formula

\[
q(r) = \sum_{\nu=1}^{m} \frac{T_m(r-r_0) q_k}{m(-1)^{\nu-1} \sin \nu \cdot \chi_v (r-v_0)(z-z_v)}, \quad q_v = q(r_v), z = 2r - 1
\]

(8)
The first and second derivatives according to \( r \), included in the expression (6), we find by differentiating the interpolation formula (8).

Thus, the calculation of the eigen values, the considered boundary problem \((1) \rightarrow (2)\) is reduced to the calculation of the eigen values of the matrices \( \Lambda_k \), i.e. the problem is reduced to a two-dimensional one. The proposed problem formulation can be used for calculating the thermal impact not only when fastening and operating continuously welded rails, but also for the seasonal fluctuations in the temperature of the subgrade and the permanent was in general.

### 3. Conclusions

«The Instruction for the construction, laying and repair of the continuous welded track» does not fully reveal the operating conditions of the continuous welded track in terms of the likely development of subgrade deformations, in terms of determining the actual temperature of continuously welded rails fastening after performing mechanized track alignment.

In order to accurately determine the actual temperature of the continuously welded rails in the areas of the continuous welded track laying that tend to develop the repetitive subgrade deformations, it is necessary to make out a scheme for installing cross-sections marks outside the deformation boundaries.

Numerous cases of track raising when developing longitudinal profile deformations contribute to an increase in the steepness of the embankment slopes in course of time, that leads to the complete absence of the subgrade shoulder on the embankments, and, as a result, contributes to the shoulder sliding of the ballast prism. The presented nonlinear mathematical model makes it possible to apply scaling and adaptation procedures to various operating conditions of the railway track, as well as to consider various design characteristics of the permanent way. In general, the proposed model for calculating the dynamic behavior of the continuously welded rails will improve the calculation accuracy of thermal stresses and the optimal temperature of their fastening, and it can also be used in the integrated monitoring system of the pre-failure state of the continuous welded track.

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