ON THE SIZE DISTRIBUTION OF CLOSE-IN EXTRASOLAR GIANT PLANETS

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ABSTRACT

The precisions of extrasolar planet radius measurements are reaching the point at which meaningful and discriminatory comparisons with theoretical predictions can be made. However, care must be taken to account for selection effects in the transit surveys that detect the transiting planets for which radius measurements are possible. Here I identify one such selection effect, such that the number of planets with radius $R_p$ detected in a signal-to-noise ratio limited transit survey is proportional to $R_p^a$, with $a \sim 4–6$. In the presence of a dispersion $\sigma$ in the intrinsic distribution of planet radii, this selection effect translates to bias $b$ in the radii of observed planets. Detected planets are, on average, larger by a fractional amount relative to the mean radius $\langle R_p \rangle$ of the underlying distribution. I argue that the intrinsic dispersion in planetary radii is likely to be in the range $\sigma = (0.05–0.12)\langle R_p \rangle$, where the lower bound is that expected theoretically solely from the variance in stellar insolation, and the upper bound is the 95% c.l. upper limit from the scatter in observed radii. Assuming an arbitrary but plausible value of $\sigma(\langle R_p \rangle) \sim 10\%$, and thus $b \sim 6\%$, I infer a mean intrinsic radius of close-in massive extrasolar planets of $\langle R_p \rangle = (1.01 \pm 0.03)\langle R_\star \rangle$. This value reinforces the case for HD 209458b having an anomalously large radius, and may be inconsistent with coreless models of irradiated giant planets.

Subject heading: planetary systems — planets and satellites: general — stars: individual (HD 209458)

Online material: color figures

1. INTRODUCTION

After many years of effort, transit searches for extrasolar planets have recently come to fruition, with the detection of six extrasolar planets via the transit method. Most of these planets were discovered in deep Galactic field surveys by the OGLE collaboration (Udalski et al. 2002a, 2002b, 2002c, 2003), and subsequently confirmed via radial velocity follow-up (Konacki et al. 2003a, 2003b, 2004, 2005; Bouchy et al. 2004, 2005; Pont et al. 2004, 2005; Moutou et al. 2004). One was discovered in a shallow, wide-angle field survey by the TrES collaboration (Alonso et al. 2004). Along with the transiting planet HD 209458b (Henry et al. 2000; Charbonneau et al. 2000), originally discovered via radial velocity surveys (Mazeh et al. 2000), seven transiting planets are currently known, with many more sure to follow.

The photometric light curve of a star with a transiting planet, when combined with detailed spectroscopic follow-up including precise radial velocities, yields the planet radius $R_p$, mass $M_p$, and semimajor axis $a$, along with the mass $M_\star$, radius $R_\star$, and effective temperature of the host star. Of the routinely observable properties of transiting extrasolar planets, the radius is the primary diagnostic for testing theoretical models. Currently, the radii of the seven known transiting planets are determined to precisions of $\Delta R_p/R_p \equiv \delta R_p \sim 5\%–10\%$. A number of different sources of uncertainty contribute to the total error, but ultimately the uncertainty in the radius of a transiting planet is limited by the uncertainty in the primary mass, such that $\delta R_p \sim \frac{1}{2}\delta M_\star$. Stellar masses can be inferred with precisions of $\sim 10\%–20\%$, and thus in the future we can expect to be able to measure planetary radii to $\sim 3\%–7\%$.

The steadily increasing number and precision of planet radius measurements, along with improvements in the theoretical models of irradiated giant planets, has led many authors to present detailed comparisons between measured radii and model predictions. Other than the mysterious case of HD 209458b, whose anomalously large radius continues to elude a satisfying explanation (Guillot & Showman 2002; Burrows et al. 2003; Baraffe et al. 2003; Deming et al. 2005; Winn et al. 2005), these comparisons have generally resulted in agreement between theory and observations (Bodenheimer et al. 2003; Burrows et al. 2004; Chabrier et al. 2004; Guillot 2005; Laughlin et al. 2005). Although detailed inferences about the ensemble properties of giant close-in planets are not yet possible, the expected precisions should be sufficient to distinguish between, e.g., models with and without a large solid core.

Before celebrating the agreement between the theoretical predictions and observational constraints on extrasolar planetary radii, we should be certain that all relevant biases in the observations and/or theoretical predictions have been identified. Burrows et al. (2003) identified one such bias, such that the planet radius as inferred from a transit light curve is $\sim 3\%–10\%$ larger than the photospheric radius predicted by models, due to the fact that rays from the primary passing perpendicular to the planet radius vector suffer a longer path length through the atmosphere. The purpose of this Letter is to point out that there exists an additional selection effect on the radii of planets detected in S/N-limited field transit surveys. This selection effect is such that the number of detected planets is proportional to $R_p^a$, with $a \sim 4–6$. In the presence of intrinsic scatter in the distribution of planetary radii, this translates directly into a bias in the mean radius of detected planets relative to the intrinsic population. This bias can affect comparisons of theoretically predicted radii with the ensemble distribution of observed planet radii. It will also affect the interpretations of individual systems if there exist unaccounted-for dependences of the radius on unobservable parameters of the system, such as the migration timescale or an indeterminate amount of internal heating. Note that the bias identified here operates in the same sense as the Burrows et al. (2003) “transit radius” effect. Therefore, these two biases can easily combine to yield $\sim 10\%$ differences from the usual naive comparisons. These biases are easy to understand. However, they must be acknowledged and considered when drawing conclu-
sions about the agreement (or lack thereof) between theory and observations.

2. SELECTION EFFECTS IN FIELD TRANSIT SURVEYS

Field surveys for transiting planets are subject to a number of selection effects, which can lead to observed distributions of planetary parameters that are biased with respect to the underlying intrinsic distributions. These selection effects have been discussed by Gaudi et al. (2005) and Pont et al. (2005), although these studies were primarily concerned with biases in the periods of detected extrasolar planets and did not consider the bias with respect to planetary radius in any detail. Pepper et al. (2003) and Gaudi et al. (2005) present simple scaling relations for the number of planets detected in a S/N-limited field transit survey, as a function of the parameters of the star and planet. I review the basic steps here but refer the reader to these papers for a more detailed derivation of these relations.

The total signal-to-noise ratio (S/N) of the photometric detection of a planetary transit is

$$S/N = N_u^{1/2} \left( \frac{\delta}{\sigma_{ph}} \right) ,$$

where \( N_u \) is the total number of points in transit, \( N_{\text{int}} \) is the total number of data points in the light curve, \( \delta = (R_p/R_*)^2 \) is the depth of the transit, and \( \sigma_{ph} \) is the single-measurement relative photometric precision. In terms of fundamental parameters of the planet and star, the S/N scales as

$$S/N \propto R_p^{-3/2} T_{eq}^2 R_*^2 a^{1/2} d^{-1} ,$$

where \( T_{eq} \) is the equilibrium temperature of the planet, and I have assumed that \( \sigma_{ph} \) is limited by Poisson noise, such that \( \sigma_{ph} \propto L_u^{-1/2} d \), where \( L_u \) and \( d \) are the luminosity and distance of the host star, respectively.

I note that the various factors in equation (2) are unlikely to be independent. For example, it may be that planet properties are correlated with host star mass and thus \( R_* \). In addition, models of irradiated planets predict that their radii will likely depend on the amount of stellar isolation absorbed by the planet. Thus, \( R_p \) is expected to be correlated with \( T_{eq} \), although precise predictions for this correlation are hampered by an incomplete understanding of the physical processes involved with the absorption and redistribution of the energy from the incident stellar radiation. There is also observational evidence that the mass of close-in giant planets is correlated with \( a \), such that planets closer to their parent star are more massive (Gaudi et al. 2005; Mazeh et al. 2005). All else equal, this would lead to a correlation of \( R_p \) with \( a \). Laughlin et al. (2005) argue that such a correlation between \( R_p \) and \( a \) largely explains the apparent discrepancy between the period distributions inferred from radial velocity surveys and transit surveys. This is unlikely to be correct, first because the evidence for a correlation of \( R_p \) with \( a \) is weak, and second because even for the strength of the correlation quoted by Laughlin et al. (2005), the magnitude of resulting selection effect would be less than half that due to the inevitable selection effect arising from the direct dependence on \( a \) (Gaudi et al. 2005).

Although these correlations are interesting topics for future study, their contribution to the selection effects are unlikely to dominate over the direct \( R_p^2 \) term in equation (2). I will therefore ignore the other terms for the remainder of the discussion and assume \( S/N \propto R_p^2 d^{-1} \).

At a limiting \((S/N)_{\min}\), the maximum stellar distance out to which a transiting planet can be detected is therefore \( d_{\max} \propto R_p^2 \). The number of planets detected is proportional to the total volume over which a planet gives rise to a transit with \( S/N \geq (S/N)_{\min} \). Assuming a constant volume density \( n \) of stars and no dust, the number of planets that can be detected is therefore proportional to \( d_{\max}^3 \propto R_p^6 \). If the intrinsic distribution of planetary radii is \( f(R_p) \propto dn/dR_p \), the observed distribution of planetary radii \( f_0(R_p) \) will be

$$f_0(R_p) \propto f(R_p) R_p^\alpha ,$$

with \( \alpha = 6 \). I leave \( \alpha \) as a free parameter to allow for departures from the ideal case, as discussed below.

There are a number of assumptions that enter into the derivation of equation (3), some of which I have explicitly stated, and others of which are reviewed in Gaudi et al. (2005). The most relevant for the present discussion are the assumptions of a constant volume density of stars, no dust, Poisson noise–limited photometry, and a S/N-limited survey. All of these are violated to some extent in the actual field surveys that have detected transiting extrasolar planets. S. Dorsher et al. (2005, in preparation) calculate the expected scaling of the number of detected planets with \( R_* \) for the OGLE surveys, using a realistic Galactic model of the source star and dust distribution, including the joint distribution of host star luminosities and radii, and considering the actual error properties of the OGLE photometry. They find that the number of detected planets is a high power of \( R_* \), but with a somewhat smaller exponent than \( \alpha = 6 \). The range is \( \alpha \sim 4-6 \), with lower indices expected for planets with smaller \( a \). The properties of the TrES survey have not been described in detail, so I will assume for simplicity that they are similar to the OGLE survey.

3. THE RADIUS BIAS

It is clear from equation (3) that due to the \( R_p^2 \) selection effect in S/N-limited transit surveys, the observed distribution of planet radii will be a biased subset of the underlying intrinsic distribution. For example, the mean of the observed distribution will generally be larger than the mean of the underlying distribution. As a result, if this bias is not taken into account, incorrect inferences about the population of planets as a whole could be drawn from the properties of observed planets.

The magnitude of the bias will clearly depend on the intrinsic distribution of planets. To provide a quantitative estimate of the bias, I adopt a Gaussian form for the intrinsic distribution \( f \) of planet radii, with mean \( \langle R_p \rangle \) and standard deviation \( \sigma \). The mean \( \langle R_p \rangle \) and standard deviation \( \sigma \) of the observed planet distribution \( f_0 \) can then be calculated in the usual way. Although exact analytic expressions for \( \langle R_p \rangle \) and \( \sigma \) can be found, they are cumbersome and not particularly illuminating. The bias, defined here as the fractional difference between the mean of the observed distribution relative to the mean of the intrinsic distribution, \( b \equiv \langle R_p \rangle / \langle R_p \rangle \), is shown in Figure 1. For \( \sigma \langle R_p \rangle \ll 1 \), the observed distribution \( f_0 \) is approximately a Gaussian. Therefore, the mean of \( f_0 \) can be approximated by its maximum, which yields

$$\langle R_p \rangle_0 \approx \langle R_p \rangle \left[ \frac{1}{2} + \frac{1}{2} \left( 1 + 4 \alpha \frac{\sigma^2}{\langle R_p \rangle} \right)^{1/2} \right].$$

(4)
The resulting bias using this approximation is shown in Figure 1; it agrees well with the exact result. For $\sigma/(\langle R_p \rangle) \ll (4\alpha)^{-1/2}$, 

$$b \approx \alpha \left( \frac{\sigma}{\langle R_p \rangle} \right)^2. \quad (5)$$

Thus, if the intrinsic distribution of planet radii has a standard deviation of 10%, the mean of the observed planets will be larger than that of the underlying population by $\sim$4%–6%. I note that the $R_p^*$ selection effect also tends to result in an observed dispersion that is smaller than the intrinsic dispersion. However, for $\sigma/(\langle R_p \rangle) \leq 25\%$, the intrinsic and observed dispersions differ by $\leq 10\%$ for $\alpha \leq 6$.

The magnitude of the bias is relatively insensitive to the form of the intrinsic distribution $f_r(R_p)$. As an example, consider the case in which the bulk of planets have the form assumed in the above analysis, but there exists an additional population of “inflated” planets. The inflated planets have the same dispersion as the other planets, but their mean radii are larger by 10% and are 10 times less common. Such a population might be indicated by the existence of HD 209458b, which appears to have an anomalously large radius. In this case, the bias is very similar, as shown in Figure 1.

4. SUMMARY AND DISCUSSION

I have argued that there exists a bias in the radii of extrasolar planets detected in field transit surveys. This bias arises from the fact that there exists a strong selection effect in these surveys, such that the number of planets detected is proportional to $R_p^*$, with $\alpha \sim 4$–6. The magnitude of this bias depends on the form of the intrinsic distribution of planet radii, and is $b \sim \alpha(\sigma/\langle R_p \rangle)^2$ for a Gaussian distribution with mean $\langle R_p \rangle$ and dispersion $\sigma$. Although the assumption of a Gaussian distribution of radii is reasonable, I stress that a bias exists for any intrinsic distribution with a finite dispersion.

One might be tempted to conclude that this bias only affects inferences about the ensemble distribution of planetary radii, and that point-to-point comparisons between calculations and measurements of the radii of individual systems would be immune to bias. This is not necessarily the case, however, because there may exist unobservable or poorly constrained parameters that effect the planetary radius, such as the migration timescale, age of the system, or an indeterminate amount of internal heating. These hidden parameter dependences may give rise to a dispersion in planetary radii at fixed values of the observable parameters. Thus, an observed planet is likely to have a larger radius than might be expected based on models incorporating typical values of these hidden parameters.

Although the primary purpose of this paper is to simply point out the existence of a radius bias, it is interesting to reconsider the comparison between models and data in light of this previously unaccounted-for effect. To do so, I use the predictions of models of irradiated giant planets by Bodenheimer et al. (2003) and Laughlin et al. (2005). Predictions of other models are similar, although there are discrepancies at the $\sim$10% level. I use these models simply because these authors present their predictions in an easily accessible numerical form, specifically as tables of predicted radii as a function of planet mass $M_p$ and $T_{eq}$ (Bodenheimer et al. 2003), and as predictions for specific systems (Laughlin et al. 2005). These models span planet
masses in the range $M_p = (0.11-3)M_J$, equilibrium temperatures in the range $T_{eq} = 113-2000$ K, and consist of models with and without solid cores of mass $M_s = M_p$, and $40 M_s$ for $M_p \geq M_s$. I increase all model radii by 5% to roughly account for the "transit radius" effect discussed by Burrows et al. (2003). This is somewhat larger than the magnitude expected for more massive planets such as OGLE-TR-56b (Burrows et al. 2004), and about half that expected for less massive planets like HD 209458b (Burrows et al. 2003). I augment the specific predictions of Laughlin et al. (2005) to include the recently confirmed planet OGLE-TR-10, using the stellar and planet parameters from Konacki et al. (2005), but accounting for the improved photometry of Holman et al. (2005). Linearly interpolating from Table 1 of Bodenheimer et al. (2003), the predictions for the radius of the planet $T_{eq} = 1196$ K are $R_p = 1.04 R_J$ and $R_p = 1.13 R_J$ for models with and without a core, as compared to the (preliminary) measured radius of $R_p = (1.06 \pm 0.06) R_J$.

Figure 2 shows the radii of observed extrasolar planets, together with the predictions of Bodenheimer et al. (2003) for the range of $T_{eq}$ spanned by these systems, $T_{eq} = 900-1800$ K. As has been noted by numerous authors, the radius of HD 209458b is anomalously large. It is important to emphasize that this object is not subject to the bias discussed here, as it was originally discovered via radial velocity measurements. The average observed radius of the remaining six planets is $1.07 R_J$, with an error in the mean of 0.02 R_J and an rms of 0.05 R_J. The intrinsic dispersion is consistent with zero, however the true dispersion is likely considerably larger. Just from the variance in $T_{eq}$, the models of Bodenheimer et al. (2003) predict a dispersion of 3%-5%. Any additional variance in the properties of the systems that affect the planet radius (i.e., in the age, metallicity, or core mass) would give rise to a larger dispersion. The 95% c.l. upper limit on the intrinsic dispersion is $0.12 R_J$, respectively. Thus, the intrinsic dispersion is likely in the range $(0.05-0.12) R_J$.

Without detailed knowledge of the intrinsic scatter in the radii of giant planets, it is difficult to assess the effect of the bias on the current data set with any confidence. I will therefore proceed rather speculatively. For definiteness, I will adopt $\sigma = 0.1 R_J$. Although arbitrary, this value seems plausible, given expectations about the variance in planet properties and histories. Assuming $\alpha = 6$, this gives a bias of $b = 6\%$. Using this to "debias" the mean radius of the observed planets gives an estimate of the mean intrinsic radius of close-in massive extrasolar planets of $\langle R_p \rangle = (1.01 \pm 0.03) R_J$. In contrast, the models of Bodenheimer et al. (2003) and Laughlin et al. (2005) predict $\langle R_p \rangle = (1.16 \pm 0.01) R_J$ for coreless planets with the observed masses and effective temperatures. Provided that the models of Bodenheimer et al. (2003) span the full range of intrinsic planet properties, then this inferred value of the mean intrinsic radius implies that the majority of planets have a massive core. This also strengthens the case for HD 209458b having an anomalously large radius.

Although these results are tantalizing, I stress that the comparison between the measured radii and theoretical predictions presented here is only preliminary. A more detailed study should be performed using a full suite of theoretical predictions, and a more careful consideration of the observational and theoretical biases.

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Note added in proof.—Recently, Holman et al. (2005) have revised their measurement of the radius of OGLE-TR-10 from the preliminary value of $R_p = (1.06 \pm 0.06) R_J$ used in this Letter to the larger value $R_p = (1.16 \pm 0.05) R_J$. This revision has no qualitative impact on the conclusions presented here, although it does change some of the quantitative results. In particular, the average observed radius of close-in giant planets is $\langle R_p \rangle = (1.10 \pm 0.03) R_J$, the intrinsic dispersion in radii is likely in the range $\sigma = (0.05-0.13) R_J$, and the inferred average intrinsic radius is $\langle R_p \rangle = (1.03 \pm 0.03) R_J$. 
