Simplified mathematical model and comparative analysis of possible ways of realizing the residual energy resource when decommissioning a channel-type reactor

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Abstract. We consider the problem of realization of the residual energy resource of fuel during decommissioning of the High-Power Channel-Type reactor. A mathematical model of the fuel assemblies spectrum for energy production is provided. Two methods of reactor decommissioning are compared. The first possible way to preserve the criticality of the reactor during decommission is the collapse of the core, when fuel assemblies with high energy production are discharged. The second possible way of decommissioning is formation of a two-zone loading: fresh fuel assemblies are put in the centre, burnt out on the periphery. It is shown that the second method gives a greater gain in power and unprocessed energy resources. The results are obtained on the point model of the fuel assemblies spectrum dynamics in the linear approximation of the with linear dependences of the power and the multiplication factor of FAs from energy production and indicate only the possibility of optimizing the fuel use in the process of removing the RBMK from service.

1. Introduction

The problem of nuclear power plant decommission is a complex task, and one of the aspects of which is optimal utilization of the fuel assemblies (FA) of decommissioned reactors. In relation to High-Power Channel-Type reactors (HPCTR), the problem of optimal use of shutdown reactors residual energy resources was considered in [1]. It has been shown that the re-use of fuel from a shutdown reactor in other nuclear power units can save a significant amount of fresh fuel, from tens to hundreds and even thousands of FAs. At the same time, the process of decommissioning of nuclear power plants from several power units was optimized. However, it is possible to use the fuel resource inside each power unit of nuclear power plants through internal permutations.

The physical and structural features of the HPCTR [2] (possibility of FA rearrangements during operation) make it possible to implement various approaches to the formation of a critical charge without fuelling, for example, removing the most burnt fuel ("collapsing" reactor) or forming a multi-zone (in particular, two-zone) charge, when the burnt fuel is placed on the periphery. That is, there is a principal possibility to prolong the operation of the decommission reactor being due to the appropriate structuring of core charge and thereby use the energy resource of unburnt fuel.
The purpose of this article is to use a simple model to pay attention to the physical possibility and efficiency of using the residual nuclear fuel resource due to internal core restructuring with a decrease in power during decommissioning.

2. Mathematical model of the FA spectrum dynamics in energy production.

To conduct research on the use of energy resources of the decommissioned reactor, the following mathematical model of the FA spectrum dynamics for energy production is proposed:

\[
\frac{\partial n(E,t)}{\partial t} = -\frac{\partial q(E,t)}{\partial E} - S(E,t) + F(E,t),
\]

where: \( E \) - energy production of FAs, MW \( \cdot \) day / FA, \( n(E,t)dE \) - the number of FAs with energy production in the interval from \( E \) to \( E + dE \) at time \( t \); \( S(E,t)dE \) is the number of FAs that have energy production in the range from \( E \) to \( E + dE \) discharged from the reactor at time \( t \) (unloading rate); \( F(E,t)dE \) is the number of FAs with energy production in the range \( E \) to \( E + dE \), loaded into the reactor at time \( t \) (loading rate); \( q(E,t) \) is the number of FAs increasing their energy production per level \( E \) per unit time at time \( t \) (burn-up flow).

Equation (1) is derived from the following balance considerations: the rate of change in the number of FAs \( \frac{\partial n}{\partial t}dE \) whose energy production lies in the interval from \( E \) to \( E + dE \) at time \( t \), is determined by the rate of decrease due to \( q(E + dE,t) - q(E,t) = \frac{\partial q}{\partial E}dE \), the unloading rate \( S(E,t)dE \), and the loading rate due to possible loading other reactors \( F(E,t)dE \).

The connection between the fuel burnup flow \( q(E,t) \) and the FA number \( n(E,t) \) is determined from the following considerations: for a unit of time, the FA energy production changes by the value \( W(E) \cdot 1 \), where \( W(E) \) is the power of the FA with energy production \( E \). Thus, all FAs with energy production ranging \( E \) to \( E + E \), loaded into the reactor at time \( t \) will cross the energy production level \( E \), that is, \( q(E,t) = n(E,t) \cdot W(E) \).

Then we can reduce equation (1) to the first-order partial differential equation for the function \( q(E,t) \):

\[
\frac{1}{W(E)} \frac{\partial q(E,t)}{\partial t} = -\frac{\partial q(E,t)}{\partial E} + S(E,t) + F(E,t).
\]

The initial condition is: \( q(E,0) = n(E,0)W(E) \).

The boundary condition: \( q(0,t) = q_0(t) \).

Equation (3) is defined on the interval \( (0,E_m) \). The physical formulation of the problem assumes that when the maximum depth of energy production is reached, \( E_m \) FA is extracted from the core. In the "ideal" mode of steady overloads, when only fresh FAs are loaded and only FAs with maximum energy output \( E_m \) are unloaded, the mathematical model (2-4) looks like:

\[
0 = -\frac{\partial q(E,t)}{\partial E}
\]

\[
q(0) = q_0.
\]

Then the fuel distribution in the energy production is:

\[
n(E) = \frac{q_0}{W(E)}.
\]

Power of the FA has a linear dependency on energy production and neutron flux density 
\[
W(E,r) = \frac{\phi(r)}{\phi_{max}}(W_0 - B \cdot E)
\]

in conditions of maximum flow. The distribution of FAs for energy production \( n(E) \) does not depend on the neutron flux density.

From physical considerations, it is clear that if fresh fuel is stopped, then the previously loaded fresh fuel will have some energy output \( E(t) = E_0 \) after a time \( t \), and the FA with an energy output less than \( E_0 \) in the reactor will not.

Let’s consider two ways of prolonging the reactor operation mentioned above due to the core restructuring. In this case, the estimates given assume a linear dependence of the multiplication factor and the power of the FA on energy production. A model was constructed with parameters close to those of the HPCTR [2].
\[ K(E) = K_0 - AE, K_{\infty} = 1.02, K_0 = 1.2, A = 1.1535 \times 10^{-4} \text{ FA} / (\text{MWt} \cdot \text{day}) \]

\[ W(E) = \frac{\phi(r)}{\phi_{\text{max}}} (W_0 - B \cdot E); W_0 = 3 \frac{\text{MWt}}{\text{FA}}, B = 5.3571 \times 10^{-4} \text{ day}^{-1} \]

The following physical values are used in calculations: the number of FAs in the core \( N = 1600 \); the square of the migration length \( M^2 = 0.04 \text{ m}^2 \); the area of the cell with FA \( a^2 = 0.06 \text{ m}^2 \).

3. The "collapsible" reactor
After cessation of feeding with fresh fuel as the burn-out proceeds, the left boundary of the spectrum shifts to the right and the right border to the left.

The distribution of the neutron flux density in such a reactor obeys the equation:

\[
\begin{align*}
\Delta \phi + \frac{K_{\infty}}{M^2} \phi &= 0, \\
\phi'(0) &= 0, \\
\phi(R) &= 0
\end{align*}
\]

(6)

where \( K_{\infty} = \int_{E_0}^{E_{\text{max}}} \frac{n(E)K(E)\, dE}{\int_{E_0}^{E_{\text{max}}} n(E)\, dE} \).

The radius of the reactor core is found from the relation \( \pi R^2 = a^2 \int_{E_0}^{E_{\text{max}}} n(E)\, dE \).

Criticality criterion for the reactor:

\[
\left( \frac{K_{\infty}}{K_{\text{eff}}} \right)^{-1} \left[ \frac{2.405}{R} \right]^2 = \left( \frac{2.405}{R} \right)^2
\]

(7)

It follows from this that for a given square of the migration length \( M^2 \), the effective multiplication factor \( K_{\text{eff}} \), the cell area, and the known spectrum, \( n(E) = \frac{q_o}{W(E)} \), it is possible to find the dependence of the energy output of the unloaded fuel \( E_{\text{max}}(t) \) on the energy production \( E_0(t) \) consequently, on the time of operation of the reactor without recharging with fresh fuel.

At the same time, it has been obtained that as fuel burns out it is necessary to extract fuel with less energy production in order to maintain criticality. Finally, there comes a point where maintaining criticality becomes impossible. Thus, the concept of a "collapsing" reactor assumes the extraction of fuel assemblies with less than the regular energy production \( E_m \).

4. Two-zone reactor
It is proposed to consider a model of a two-zone fuel loading reactor. In the central zone is fuel with energy production from \( E_0(t) \) to \( E_x(t) \) and in the peripheral from \( E_x(t) \) to \( E_m(t) \). As the fuel burns out, the left and right boundaries of the spectrum \( E_0(t) \) shift to the right, and the boundary between the zones \( E_x(t) \) to the left. The critical state of the reactor is described by a system of equations:

\[
\begin{align*}
\Delta \phi_1 + \frac{K_1}{M^2} \phi_1 &= 0, \\
\Delta \phi_2 + \frac{K_2}{M^2} \phi_2 &= 0
\end{align*}
\]

(8)

where for the inner zone:

\[
K_1 = \frac{\int_{E_0}^{E_x} N(E)K(E)dE}{\int_{E_0}^{E_x} N(E)dE}, \quad N_1 = \int_{E_0}^{E_x} N(E)dE - \text{FA number in the central zone}
\]

\[
\pi r_1^2 = a^2 N_1 - \text{the size of the first zone}
\]
And for the outer zone:

\[
\begin{align*}
K_2 &= \frac{\int_{E_x}^{E_m} E(E)K(E)dE}{\int_{E_x}^{E_m} E(E)dE} - \text{medium multiplication factor} \\
N_2 &= \int_{E_x}^{E_m} E(E)dE - \text{FA number in the zone} \\
\pi R^2 &= a^2(N_1 + N_2) - \text{the size of the zone}
\end{align*}
\]

For identical diffusion coefficients in the zones, the equations of uniqueness have the form:

\[
\begin{align*}
\phi_1'(0) &= \phi_2'(R) = 0 \\
\phi_1(r_1) &= \phi_2(r_1) \\
\phi_1'(r_1) &= \phi_2'(r_1)
\end{align*}
\]

The solution of the two-zone problem allows one to obtain a critical condition that will contain in addition to \(K_{\text{eff}}\), the parameters \(r_1, R, K_1, K_2\), which in turn depend on \(E_0(t), E_x(t), E_{\text{max}}(t)\). Thus, for a given \(K_{\text{eff}}\), one can find \(E_0(t)\) at which it is possible to maintain criticality in the reactor. At the same time, the concept of a two-zone reactor does not involve the extraction of FAs, but only their rearrangement to the periphery. If we assume that the reactor maintains a neutron flux density such that the power \(W_0\) is removed from the most stressed FA, then the reactor power can be calculated as follows:

\[
W = W_1 N_1 + W_2 N_2
\]

Where \(W_1, W_2\) is the average power of the FA in the center and at the periphery, respectively:

\[
\begin{align*}
W_1 &= \frac{W_0}{K_1^E K_1^\Phi} \\
W_2 &= \frac{W_0 \cdot \phi_1^{\text{max}}}{K_2^E K_2^\Phi}
\end{align*}
\]

\(K_1^E, K_1^\Phi, K_2^E, K_2^\Phi\) are the coefficients of unevenness in energy production and flux density in each zone.

### 5. Results

The simulation of the decommissioning of a HPCTR was carried out using the two strategies described above. The minimum permissible effective multiplication factor was assumed equal to 1.02, (considering the margin for axial leakage and the operational reserve of reactivity). It has been obtained that the "collapsing" reactor can, generate heat energy of about 170 GW per day until the critical loss is reached after fueling stop, however, its power is reduced to 25% of the nominal one. The residual resource (the number of FAs remaining in the core that could be used) after stopping the "collapsing" reactor was 366 FAs in terms of fresh fuel. This is 48% less than if the reactor had simply been stopped.

Two-zone charged the reactor can provide energy up to 400 GW by the time of total loss of criticality, but by the time it stops, power is reduced to 14% of the nominal. The amount of unprocessed resource decreases in comparison with the "collapsing" reactor - 311 FAs in terms of fresh fuel. This is 56% less than in the case of a simple stop.

These estimates are limiting until the complete exhaustion of the reactivity reserve and are obtained on the basis of a simplified model of a HPCTR with linear dependences of the power and the multiplication factor of FAs from energy production. In this case, the long operation of the reactor is explained by a significant (4-5 times) decrease in power. There may be reasons limiting the reactor’s decommissioning - technological and economic constraints. In general, it has been shown that by organizing the appropriate core charge, it is possible to effectively use energy resources in a HPCTR being decommissioned, using such simple operations as the FA rearrangements in the core. With the imposition of technical, economic and other constraints that do not allow the full utilization of the residual energy resources to be fully utilized with a reduction in reactor power, a known method of moving unburned FAs from the shutdown power unit to the operating one can be used [1].
References
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