Suggestion on using the hexapod robot as a coordinate measuring machine in digitizing data and measuring space surfaces

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Abstract. Testing or digitizing the machined surface in inverse manufacturing always requires the use of modern, interconnected measuring devices, and these devices share tool control programs with probe trajectory to check 3D surfaces through CAM software. Coordinate measuring machines with a large workspace need very rigid mechanical structures and parallel structures used instead are one of the solutions to this problem. Despite the extremely high stiffness, the characteristics of the parallel structure are of compound design and kinematics. One of the typical difficulties is that the forward and inverse kinematic problems have many solutions. When used as a measuring machine, solving the kinematic problem is often required. If there are not enough bases to uniquely identify the endpoint coordinates, the application of robots such as hexapods to make CMM machines will fail. This paper introduces a technical solution to achieve a unique solution when solving a forward kinematic problem using a hexapod device. The sensors are mounted on the device to recognize the generalized coordinates and the auxiliary parameters at the same time, then the unique solution of the problem will be determined. The illustrative application on the parallel structure of the SRS and SPS hexapod configuration shows the correctness of the research idea.

1. Introduction

It is not difficult to see that so many medical applications have used parallel hexapod robot structure [1,2]. For example, denture load testing devices, Dental and bone sampling devices serve as dentures and stretch the tibia or orthopedics, rehabilitation [2,3,9]. These devices have sufficient precision and force to generate chewing pressure of up to 40 kg/cm² as a jaw due to their stiffness, which is an inherent advantage of the parallel structure [3].

The use of a hexapod structure leads to a complicated problem in solving the forward kinematic problem. For each set of extrapolation parameters included in the problem, there can be up to 216 solutions. Therefore, the coordinates of the measuring point cannot be determined [4]. However, if it is possible to specify some additional parameters explicitly, the forward kinematic problem will determine the unique solution. Our idea in this regard is to use the rotation angle sensors to determine the direction cosine parameters of robot limbs in the forward kinematic problem. The mathematical model and optimization algorithm using the Generalized Reduced Gradient (GRG) method will prove that the end-point is always uniquely determined in that situation, including SPS or SRS type.
2. Kinematic model of hexapod robot

2.1. Kinematic modelling of a hexapod robot in SPS type
The vector loop passing through one limb with the prismatic actuator type has a quadrilateral shape (as shown in Figure 1). Because the vector \( b_i \) and \( p_i \) are determined on two different frames of reference, the base frame and the moving frame, respectively, an \( R_{RPP} \) matrix is needed to add in the vector equation. The closed-loop equation on the quadrilateral vector is written as follows:

\[
\vec{l}_i = -\vec{b}_i + i + R_{RPP} \vec{p}_i \quad i = 1,..,6
\]

where, \( i \) presents the \( i^{th} \) limb of the robot.

![Figure 1. Parallel robot with the actuator of P-type and \( i^{th} \) vector loop of the robot](image)

A parallel structure will have both active and passive joints. For example, the \( P \) joint in the diagram above is an active joint that has one degree of freedom (DOF) joint and corresponds to a generalized coordinate. These active joints create the generalized coordinates with \( (i = 1,..,6) \) (Figure 3). Passive joints are two or three DOF joints. They do not create motion but only receive force and propagate through some components while eliminating unwanted components. Passive joints that produce auxiliary parameters, for example \( u_i \) and \( v_j \) in Figure 2, are the additional parameters those serve orientation of \( i^{th} \) limb of robot. To describe the coordinates of node \( B_i \), it is necessary to know the value of the generalized coordinate \( l_i \) immediately, the additional parameters and its direction cosine.

![Figure 2. The parameters describe the state of \( A_iB_i \) limb.](image)

So the detailed equation of the \( i^{th} \) limb is written as follows:
Retrieve six limbs to have a complete kinetic equation system of 18 equations in real size robot.

2.2 Kinematic modelling of a hexapod robot in SRS type

Figure 3. Developing the $i^{th}$ limb of the R type actuator parallel robot

In Figure 3, the point O is the centre of the fixed platform, and point P is the centre of the moving platform. The vectors $r$ and $h$ cross in space while $a$ and $b$ are coplanar due to the connection by the joint R (Figure 3).

$$\overrightarrow{OP} + R_{RPY} \cdot \overrightarrow{PC_i} - \overrightarrow{OA_i} = a_i \cdot B_i + B_i \cdot C_i$$

$$\Leftrightarrow \overrightarrow{OP} = a_i \cdot B_i + B_i \cdot C_i - R_{RPY} \cdot \overrightarrow{PC_i} + \overrightarrow{OA_i}$$

Detailed equation follow generalized variable has the form:

$$\begin{bmatrix}
    x_{Ai} \\
    y_{Ai} \\
    z_{Ai}
\end{bmatrix} =
\begin{bmatrix}
    a_c \cdot \theta_{1i} \cdot s \cdot \theta_{3i} + b_c \cdot (\theta_{1i} + \theta_{2i}) \cdot s \cdot \theta_{3i} \\
    a_c \cdot \theta_{1i} + b_c \cdot (\theta_{1i} + \theta_{2i}) \cdot c \cdot \theta_{3i} \\
    a_s \cdot \theta_{1i} + b \cdot s \cdot (\theta_{1i} + \theta_{2i})
\end{bmatrix}
+ \begin{bmatrix}
    x_{ci} \\
    y_{ci} \\
    z_{ci}
\end{bmatrix}$$

2.3 Scale analysis of forward and inverse problems in both structures

Figure 4. Relationship between SPS and SRS limb types
The SPS type: as Figure 4 can see, on the limb of SPS configuration l, has the role of generalized coordinates \((i = 1,..,6)\) and the parameters \(u_i, v_i (i = 1,..,6)\) are the auxiliary parameters that play direction cosine of the \(i^{th}\) limb. Thus, in the kinematic model, after retrieving the system of 6 equations such as equation (2), there are 18 equations, including:

In the inverse problem, given the parameters including \(p_x, p_y, p_z, \alpha, \beta, \gamma\) we need to find parameters including 6 generalized coordinates \(l_i (i = 1,..,6)\) (which are variables using active structural control), and 12 auxiliary parameters \(u_i, v_i (i = 1,..,6)\) . So the problem here is finding 18 hidden from 18 equations, this system solves.

In the forward kinematic problem, given the generalized variables \(l_i (i = 1,..,6)\) we need to find the parameters \(p_x, p_y, p_z, \alpha, \beta, \gamma\) and 12 auxiliary parameters \(u_i, v_i (i = 1,..,6)\) . The problem still has 18 parameters solved from 18 equations.

The SRS type: For the SRS limb structure, \(\theta_{2i} (i = 1,..,6)\) are generalized coordinates. The parameters \(\theta_{1i}, \theta_{2i}\) with \((i = 1,..,6)\) are auxiliary parameters with the role of direction cosine of the \(i^{th}\) limb. Thus, after retrieving equation (4) with \((i = 1,..,6)\) we will get a system of 18 equations. The institution of the forward and inverse problem as follows:

In the inverse problem, given parameters \(p_x, p_y, p_z, \alpha, \beta, \gamma\) we need to find \(\theta_{2i}\) parameters with \((i = 1,..,6)\) which are generalized variable using structural control proactively and parameters \(\theta_{1i}, \theta_{2i}\) with \((i = 1,..,6)\) are 12 auxiliary parameters. The determination of 18 parameters from the system of 18 equations (4) after the retrieval is unique.

For the forward problem, given the generalized variables \(\theta_{2i}\) with \((i = 1,..,6)\) find the parameters \(p_x, p_y, p_z, \alpha, \beta, \gamma\) and 12 parameters \(\theta_{1i}, \theta_{2i}\) with \((i = 1,..,6)\) . Finding 18 unknowns from the system of 18 equations is also uniquely determined.

With the institution for both types of limb: SRS and SPS, the forward problem has many solutions because the coordinates are given but the auxiliary parameters are not fixed. Each set of auxiliary parameters changes to create different endpoints. For a single forward kinematic solution, it is necessary to give both the generalized variable and the auxiliary parameters. These parameters are obtained by measuring them with sensors and then transferred to the forward problem.

3. Suggest to use of hexa structure as a CMM

3.1. Technical solution
If the parameters \(u_i, v_i (i = 1,..,6)\) (Figure 2) and the parameters \(\theta_{1i}, \theta_{2i} (i = 1,..,6)\) (Figure 3) receive a definite stop value, the problem always returns only one unique experience set. With the SPS structure, this set of values is the generalized coordinate values, while with the SRS structure, it is the generalized coordinate values \(\theta_{2i}\) . So the key problem here is how to identify the parameters \(u_i, v_i\) and parameters \(\theta_{1i}, \theta_{2i}\) respectively.

Consider the structure of joint S and joint U in the situation that changes positions as shown in Figure 5. Because rotation around the z-axis is not necessary for this structure, the S joint is replaced by the U joint and the two direction cosine \(u_i, v_i\) or \(\theta_{1i}, \theta_{2i}\) are easily measured by the corresponding encoders. These encoders are arranged on two U-axis perpendiculars to each other (measure motion Rx and Ry on Cardan joint by two encoders \(e_1\) and \(e_2\) as shown in Figure 5). These parameters are added to the above forward kinematic problem so that the results of the forward kinematic problem always have a unique solution. Thus, 18 encoders are needed to complete the hexapod CMM, 6 of which measure generalized coordinates, and 12 encoders measure auxiliary parameters. In this case,
the two SRS and the SPS configuration respectively become the URU and UPU configuration without affecting the kinematic function of the structure.

**Figure 5.** The sphere joint is replaced by the Cardan joint when $R_z$ movement is not needed.

### 3.2. Method and tool for the forward kinematic problem of a hexapod robot

The parallel robot kinematic problem corresponding to the equations (2) and (4) is proposed to be solved by GRG numerical methods such as [5,6,8]. The capacity of the GRG method has been confirmed following the Banana function form of the inverse kinematic problem according to the experiment in [7]. The main test required here is on the initialized mathematical model (2) or (4) with specific parameters, and auxiliary parameters are provided enough. When it is necessary to determine a point is included in the inverse problem, we can find it exactly out of the $2^{16}$ solutions generated in the forward problem or not.

For example, on the same Stewart type robot, solve the following inverse problem:

| $P_0$ | $P_1$ | $P_2$ | $P_3$ | $P_4$ | $P_5$ | $P_6$ | $P_7$ | $P_8$ | $P_9$ | $P_{10}$ | $P_{11}$ | $P_{12}$ | $P_{13}$ | $P_{14}$ | $P_{15}$ | $P_{16}$ |
|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|---------|---------|---------|---------|---------|---------|---------|
| (x1)  | (x2)  | (x3)  | (x4)  | (x5)  | (x6)  | (x7)  | (x8)  | (x9)  | (x10)| (x11)  | (x12)  | (x13)  | (x14)  | (x15)  | (x16)  | (x17)  |
|       |       |       |       |       |       |       |       |       |       |         |         |         |         |         |         |         |
| (y1)  | (y2)  | (y3)  | (y4)  | (y5)  | (y6)  | (y7)  | (y8)  | (y9)  | (y10)| (y11)  | (y12)  | (y13)  | (y14)  | (y15)  | (y16)  | (y17)  |
|       |       |       |       |       |       |       |       |       |       |         |         |         |         |         |         |         |
| (z1)  | (z2)  | (z3)  | (z4)  | (z5)  | (z6)  | (z7)  | (z8)  | (z9)  | (z10)| (z11)  | (z12)  | (z13)  | (z14)  | (z15)  | (z16)  | (z17)  |
|       |       |       |       |       |       |       |       |       |       |         |         |         |         |         |         |         |

### Figure 6.** Point taken into the inverse problem to then find in the forward problem

With the requirement to correctly establish the point included in the above inverse problem when solving the forward problem, it is necessary to determine the unique relationship when changing between $P_2$ point and $P_4$ point as shown in the Figure 7. With the help of sensors measuring the direction cosine angles as mentioned in section 3.1, these angles are shown as in the interface for solving the forward kinematic problem in Figure 8. The result is the point that was put into the
previous inverse kinematic problem. The point to search is distinguished from \((2^{16} - 1)\) remaining point very clearly. This is what needs to be proven, when adding the measured auxiliary parameters about the direction cosine of the robot limbs the forward problem has a unique solution as desired.

![Figure 7. Relationship between points and images when converting kinematic information](image)

**Figure 7.** Relationship between points and images when converting kinematic information

4. Conclusions
In a parallel structure-forward kinematic problem, if enough sensors are arranged to provide all generalized variables and auxiliary parameters simultaneously, the endpoint coordinates are always uniquely determined. This principle holds true for all parallel structures. And for use as a CMM machine, the arrangement of sensors enough are necessary. Through the illustrated example that has been shown, our ideas are completely correct. The point of inclusion in the previous inverse problem was recovered precisely through the forward problem when given the generalized variables and auxiliary parameters simultaneously. This is true for both the SRS hexapod configuration and the SPS hexapod configuration.

With the above results, the application of the Hexapod structure in one of two SRS or SPS configuration as the Coordinate Measuring Machine CMM is completely feasible.
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