I. INTRODUCTION

In recent years, we have studied a number of nuclei around doubly magic $^{100}$Sn, $^{132}$Sn, and $^{208}$Pb within the framework of the shell model employing realistic effective interactions derived from the meson-theoretic Bonn-A nucleon-nucleon ($NN$) potential \cite{1,4,22}. We have focused attention on nuclei with few valence particles or holes, since they provide the best testing ground for the basic ingredients of shell-model calculations, especially as regards the matrix elements of the two-body effective interaction. The main motivation for carrying out this extensive program of calculations was to try to assess the role of realistic effective interactions in the shell-model approach to the description of nuclear structure properties. The results of our calculations have so far turned out to be in remarkably good agreement with experiment for all the nuclei considered, providing evidence that realistic effective interactions are able to describe with quantitative accuracy the spectroscopic properties of complex nuclei. In this connection, it is worth noting that these results are considerably better than those obtained in earlier works for the light $s$-$d$ nuclei \cite{1,2,11,12}.

While in the $^{132}$Sn and $^{208}$Pb regions we have studied nuclei with both valence particles and holes, around $A = 100$ we have only considered the light Sn isotopes, namely we have not gone below the 50-82 shell. The study of nuclei lacking few nucleons with respect to $^{100}$Sn, which is the heaviest $N = Z$ doubly magic nucleus, is of course of great relevance from the shell-model point of view. Nuclei of this kind, however, lie well away from the valley of stability and experimental information on their spectroscopic properties is still very scanty. In this context, the proton-rich $N = 50$ isotones are of special interest. In fact, while the development of radioactive ion beams opens up the prospect of spectroscopic studies of a number of $^{100}$Sn neighbors, use of large multidetector $\gamma$-ray arrays is already providing new experimental data for these singly magic nuclei. In particular, four excited states in $^{98}$Cd, two proton holes from $^{100}$Sn, have been recently identified in an in-beam spectroscopy experiment \cite{13}.

On the above grounds, we found it very interesting to include in our program of calculations the proton-rich $N = 50$ isotones $^{98}$Cd, $^{97}$Ag, and $^{96}$Pd (some preliminary results have already been presented in Ref. \cite{4}). Actually, the motivation for the present study is twofold. On the one hand, these nuclei with two, three, and four holes in the $Z = 28 - 50$ shell offer the opportunity to test our realistic effective interaction for nuclei below $^{100}$Sn. On the other hand, the success achieved by our previous calculations on medium- and heavy-mass nuclei encourages us to make predictions which may stimulate, and be helpful to, future experiments.

The $N = 50$ isotones have long been the subject of theoretical interest. In most of the shell-model calculations performed in the last two decades \cite{1,2,4,5,6,7,10,13,21} (earlier references can be found in Ref. \cite{14}), however, attention has been focused on the lighter isotones up to mass 95 \cite{14,15,16,17} or 96 \cite{18,19,20}. In this context, we may mention the extensive study of the $N = 50$ isotones from $^{82}$Ge up to $^{96}$Pd performed some ten years ago by Ji and Wildenthal \cite{15,16}. In that work $^{78}$Ni was considered as a closed core and an empirical effective Hamiltonian was obtained by fitting the two-body matrix elements and the single-particle energies to approximately 170 experimental data. Actually, the low-energy spectra of $^{98}$Cd and $^{97}$Ag have been predicted only in the work of Ref. \cite{4}, where the protons were assumed to fill solely the $0g_{9/2}$ and $1p_{1/2}$ levels.

In all previous calculations empirical two-body matrix elements have been used, an exception being the work of Ref. \cite{17}, where the effective interaction was derived from the Sussex interaction \cite{21}. To our knowledge, the present calculations are the first ones where the two-body effective interaction has been derived from a modern free nucleon-nucleon potential.
Before closing this section we should remark that, at variance with our previous calculations in the $^{132}$Sn and $^{208}$Pb regions, we had to face here the problem of choosing a set of single proton-hole energies without much guidance from experiment. In fact, while no spectroscopic data are yet available for the single-hole valence nucleus $^{99}$In, only little relevant information is provided by the observed spectra of $^{98}$Cd and $^{97}$Ag. We will come back to this important point later.

The paper is organized as follows. In Sec. II we give an outline of our calculations and describe in detail how we have determined the single-hole energies. Our results are presented and compared with the experimental data in Sec. III, where we also comment on the results of Ref. [27]. Section IV presents a summary of our conclusions.

II. OUTLINE OF CALCULATIONS

Our effective interaction $V_{\text{eff}}$ was derived from the Bonn-A free nucleon-nucleon ($NN$) potential using a $G$-matrix formalism, including renormalizations from both core polarization and folded diagrams. Since we have assumed $^{108}$Sn as a closed core, protons are treated as valence holes, which implies the derivation of a hole-hole effective interaction. We have chosen the Pauli exclusion operator $Q_2$ in the $G$-matrix equation,

$$G(\omega) = V + VQ_2 \frac{1}{\omega - Q_2Q_2}Q_2G(\omega),$$

(1)

as specified [22,23] by $(n_1, n_2, n_3) = (11, 21, 55)$ for both neutron and proton orbits. Here $V$ represents the $NN$ potential, $T$ denotes the two-nucleon kinetic energy, and $\omega$ is the so-called starting energy. We employ a matrix inversion method to calculate the above $G$ matrix in an essentially exact way [24]. For the harmonic oscillator parameter $\hbar\omega$ we adopt the value 8.5 MeV, as given by the expression $\hbar\omega = 45A^{-1/3} - 25A^{-2/3}$ for $A = 100$. Using the above $G$ matrix we then calculate the so-called $\hat{Q}$ box, which is composed of irreducible valence-linked diagrams up to second order in $G$. These are just the seven one- and two-body diagrams considered in Ref. [25].

Since we are dealing with external hole lines, the calculation of the $\hat{Q}$-box diagrams is somewhat different from that usual for particles. For example, the familiar core-polarization diagram $G_{3\text{ph}}$ becomes

$$\langle a b; J|G_{3\text{ph}}|c d; J \rangle = \frac{\langle a b| c d \rangle}{J} \sum_{J' J''} \langle J' J'' a b |\hat{Q}| J'' J' c d \rangle \left[ ( -1)^{J_a + J_b + J_c + J_d} \right] \sum_{p h} \left[ ( -1)^{J_p - J_h} \left( j_c j_d J_j a b J J' J'' 0 \right) \right] \times \frac{1}{\omega - (\epsilon_p - \epsilon_h - \epsilon_b - \epsilon_c)} \langle p|G(\omega_1)|a p \rangle \langle d|G(\omega_2)|h b \rangle,$$

(2)

where $\hat{x} = (2x + 1)^{1/2}$ and the off-shell energy variables are: $\omega_1 = \omega + \epsilon_h + \epsilon_a + \epsilon_b + \epsilon_c$ and $\omega_2 = \omega + \epsilon_h + \epsilon_b + \epsilon_c + \epsilon_d$. The $\epsilon$’s are the unperturbed single-particle energies. $X$ is the standard normalized 9-$j$ symbol. The cross-coupled $G$-matrix elements, on the right side of Eq. (2), are related to the usual direct-coupled ones by a simple transformation, as in Ref. [22].

The effective interaction, which is energy independent, can be schematically written in operator form as:

$$V_{\text{eff}} = \hat{Q} - \hat{Q}' \int \hat{Q} + \hat{Q}' \int \hat{Q} - \hat{Q}' \int \hat{Q} \int \hat{Q} \int \hat{Q} \cdots,$$

(3)

where the integral sign represents a generalized folding operation [27]. $\hat{Q}'$ is obtained from $\hat{Q}$ by removing terms of first order in the reaction matrix $G$. After the $\hat{Q}$ box is calculated, $V_{\text{eff}}$ is then obtained by summing up the folded-diagram series of Eq. (3) to all orders using the Lee-Suzuki iteration method [28].

As regards the electromagnetic observables, these have been calculated by making use of effective operators [29,30] which take into account core-polarization effects. More precisely, by using a diagrammatic description as in Ref. [29], we have only included first-order diagrams in $G$. This implies that folded-diagram renormalizations are not needed [30].

Let us now come to the single-hole (SH) energies. As already mentioned in the Introduction, no spectroscopic information on $^{99}$In is available. To obtain information on the location of the SH levels we have therefore resorted to an analysis of the spectra of the lighter $N = 50$ isotones. Of course, most relevant to this analysis are those states which are predominantly of one-hole nature. Actually, $^{91}$Nb is the first isotope where a state of this kind has been unambiguously identified for each of the four SH levels. More precisely, no states with a firm $\frac{1}{2}^-$ or $\frac{3}{2}^-$ assignment
are reported for the heavier isotones, while at least one \( \frac{1}{2}^- \) and one \( \frac{3}{2}^+ \) state have been identified up to \(^{95}\text{Rh}\). In \(^{97}\text{Ag}\) there is only one \( \frac{3}{2}^+ \) state, which is the ground state.

From the above it is clear that, if one wants to determine the SH energies by reproducing the observed one-hole states, calculations up to nine valence holes have to be carried out. It is to be expected, however, that a set of SH energies determined in this way may not be the most appropriate for calculations where few valence holes are considered. In fact, as is well known, significant changes in the nuclear mean field may occur when moving away from closed shells. In addition, an effective two-hole interaction derived by considering \(^{108}\text{Sn}\) as a closed core may not be quite adequate for systems with several valence holes as, in these cases, many-body correlations are likely to come into play.

In this situation, we have tried to determine the SH energies \( \epsilon_{p_1/2}, \epsilon_{p_3/2}, \) and \( \epsilon_{f_5/2} \) relative to the \( g_{9/2} \) level, which has long been recognized to be the lowest-lying one, from an analysis of the spectra of the \(^{108}\text{Sn}\) neighbors \(^{98}\text{Cd},^{97}\text{Ag}, \) and \(^{96}\text{Pd}\), with two to four proton holes in the \( N = 28-50 \) shell. We have found that (i) the energies of all the excited levels in \(^{98}\text{Cd}\) and \(^{97}\text{Ag}\), which have an experimental counterpart, are quite insensitive to the position of the \( p_{3/2} \) and \( f_{5/2} \) orbits; (ii) the ground-state energies of all three nuclei, as well as the seniority-two states \( J^\pi = 2^+, 4^+, 6^+ \), and \( 8^+ \) in \(^{96}\text{Pd}\), depend practically only on the sum of the energies of these two levels, \( \epsilon = \epsilon_{p_{3/2}} + \epsilon_{f_{5/2}} \). It turns out that all the considered experimental spectra are well described overall by fixing \( \epsilon \) at 5.2 MeV. More precisely, only the \( 2^+ \) states in the two even isotones and the \( \frac{3}{2}^+ \) and the \( 10^+ \) states in \(^{97}\text{Ag}\) and \(^{96}\text{Pd}\) show a rather large discrepancy. To eliminate this discrepancy a much larger value of \( \epsilon \) should be used, namely about 10 MeV. This value, however, would produce a significant downshift of all other levels. In addition, as we shall see in the following, it would be at variance with an empirical analysis of the one-hole states in \( N = 50 \) isotones. It may also be mentioned that the energies of the \( 2^+ \) states, as well as those of the \( \frac{3}{2}^+ \) and \( 10^+ \) states, are all strongly dependent on the two-body matrix element \( \langle g_{9/2}^2 \rangle J^\pi = 2^+ | V_{\text{eff}} | g_{9/2}^2 J^\pi = 2^+ \rangle \). In this context, we shall recall that also for the light Sn isotopes our calculations with the Bonn-A potential produced \( 2^+ \) excitation energies somewhat higher than the observed values [1].

As for the \( p_{1/2} \) level, two states are sensitive to its position. They are the \( \frac{11}{2}^- \) and \( 5^- \) states in \(^{97}\text{Ag}\) and \(^{96}\text{Pd}\), respectively. We find that their experimental energies are very well reproduced by our calculations for \( \epsilon_{p_{1/2}} = 0.7 \) MeV. We have verified that this choice is rather independent of the value of \( \epsilon \). For instance, increasing \( \epsilon \) by about 2 MeV brings \( \epsilon_{p_{1/2}} \) up to only 0.9 MeV.

From the above findings it appears that the SH energies \( \epsilon_{p_{3/2}} \) and \( \epsilon_{f_{5/2}} \) cannot be determined individually from the experimental data for \(^{98}\text{Cd},^{97}\text{Ag}, \) and \(^{96}\text{Pd}\) presently available. To obtain an estimate for these two \( \epsilon \)'s, we have made a linear extrapolation of the energies of the \( \frac{7}{2}^- \) and \( \frac{5}{2}^- \) one-hole states observed in \(^{89}\text{Y},^{91}\text{Nb}, \) and \(^{93}\text{Tc}\). Actually, states of this kind have been unambiguously identified only in \(^{89}\text{Y}\) and \(^{91}\text{Nb}\). In particular, in the latter nucleus two \( \frac{3}{2}^- \) states have been observed which exhaust almost all the \( p_{3/2} \) strength. In our extrapolation, however, we have also included the experimental data relative to \(^{93}\text{Tc}\), according to the indications of Ref. [1]. In this work the level at 2.1 MeV is identified as an \( l = 3, J = \frac{5}{2}^- \) state while plausible arguments are given favoring the \( \frac{7}{2}^- \) assignment to the two states observed at 1.5 and 1.8 MeV. The above procedure yields the values of about 2 and 3 MeV for the \( \frac{3}{2}^- \) and \( \frac{5}{2}^- \) SH energies in \(^{99}\text{In}\). Owing to the uncertainty inherent in such a derivation, these values should be taken only as a reasonable estimate. In support of this procedure, however, speaks the fact that for the \( p_{1/2} \) level it yields \( \epsilon_{p_{1/2}} = 0.8 \) MeV.

On the above grounds, we have adopted for the SH energies the following values (in MeV): \( \epsilon_{g_{9/2}} = 0.0, \epsilon_{p_{1/2}} = 0.7, \epsilon_{p_{3/2}} = 2.1, \) and \( \epsilon_{f_{5/2}} = 3.1 \). It should be pointed out that these values are quite different from those adopted by other authors. In particular, the SH energies determined in Ref. [17] are higher than ours, the difference ranging from more than 1 MeV for \( \epsilon_{p_{1/2}} \) and \( \epsilon_{p_{3/2}} \) to 3.2 MeV for \( \epsilon_{f_{5/2}} \).

III. RESULTS AND COMPARISON WITH EXPERIMENT

We present here the results of our calculations for \(^{98}\text{Cd},^{97}\text{Ag}, \) and \(^{96}\text{Pd}\). They have been obtained by using the OXBASH shell-model code [12]. The experimental [33] and theoretical spectra are compared in Figs. 1, 2, and 3, where we report all the experimental levels, except the \( 13^+, 15^+ \) states observed at 6.7 and 7.0 MeV in \(^{96}\text{Pd}\), which cannot be constructed in our model space. In the calculated spectra only those yrast states which are candidates for the observed levels are reported. A complete list of excitation energies up to 5, 3, and 4 MeV is given in Tables I-III for \(^{98}\text{Cd},^{97}\text{Ag}, \) and \(^{96}\text{Pd}, \) respectively.

From Figs. 1-3 we see that our results are in very good agreement with experiment. A measure of the quality of
the agreement is given by the rms deviation $\sigma$, whose values are 107, 108, and 122 keV for $^{98}$Cd, $^{97}$Ag, and $^{96}$Pd, respectively. As was already discussed in Sec. II, the main point of disagreement is the position of the 2$^+$ state in both the even isotones, as well as that of the $^{13}$+$ and 10$^+$ states in $^{97}$Ag and $^{96}$Pd. In fact, the discrepancy between theory and experiment for the energies of these four states goes from 140 to 263 keV while it is less than 100 keV for all other states.

As regards the structure of the states having an experimental counterpart, we find that the positive-parity states in all three nuclei are dominated by the $g_{9/2}^{-}$ configuration, while the negative parity ones are practically of pure $g_{9/2}^{-}(n-1) p_{1/2}^{-1}$ character. In $^{98}$Cd and $^{97}$Ag only the ground states receive a significant contribution from configurations other than the dominant one, the percentage being about 20% in both nuclei. As for $^{96}$Pd, the wave functions of the ground state and the first four excited states are even less pure. In fact, the percentage of the $g_{9/2}^{-}$ configuration reaches at most 81% for the $4^+_1$, $6^+_1$, and $8^+_1$ states, being only 64% for the ground state. Note that in these states, as well as in the ground states of $^{98}$Cd and $^{97}$Ag, a significant percentage of the $g_{9/2}^{-}(n-2) p_{3/2}^{-2}$ and $g_{9/2}^{-}(n-2) f_{5/2}^{-2}$ is present. In particular, in the ground state of $^{96}$Pd the percentage of each of these two configurations is 9%.

To conclude this discussion, a further comment is in order. As it occurs for the other than the dominant one, the percentage being about 20% in both nuclei. As for $^{96}$Pd the percentage of each of these two configurations is 9%.

From Figs. 1-3 we see that rather little experimental information is presently available for $^{98}$Cd, $^{97}$Ag, and $^{96}$Pd. Much richer spectra, however, are predicted by the theory. It is therefore interesting to discuss in some detail our predictions, in the hope that they may verified in a not too distant future. As regards $^{96}$Cd, it may be seen from Table I that, just above the first four excited states having an experimental counterpart, we find three states with $J^\pi = 4^-, 5^-$, and $0^+$, the first two being the members of the doublet $g_{9/2}^{-} p_{1/2}^{-1}$ and the third one arising from the configuration $p_{1/2}^{-2}$. The position of the $5^-$ state is quite consistent with the experimental information available for the two lighter even isotones. In fact, in $^{97}$Cd and $^{98}$Ru a $5^-$ state has been observed at 2.65 and 2.62 MeV, respectively. Between 3.8 and 5 MeV we find all the members of the $g_{9/2} p_{3/2}^{-1}$ multiplet and the $7^-$ state arising from the $g_{9/2} f_{5/2}^{-1}$ configuration. In this energy interval is also located the $2^+$ state of the $p_{3/2}^{-1}$ configuration.

In Table II all the excitation energies up to 3 MeV are reported for $^{97}$Ag. Below this energy we find all the states arising from the configurations $g_{9/2}^{-3} p_{1/2}^{-2}$ and $g_{9/2}^{-2} p_{1/2}^{-1}$ as well as the two seniority-one states of the $g_{9/2} p_{3/2}^{-1}$ and $g_{9/2} f_{5/2}^{-1}$ configurations. In particular, we predict as first excited state a seniority-one $1^-$ state at about 0.5 MeV. This prediction is in agreement with the experimental findings for the lighter isotones $^{93}$. Furthermore, it should be mentioned that our first $2^+$ state is essentially a pure seniority-three $g_{9/2}^{-1} p_{1/2}^{-1}$ state while almost all the $f = 3$ one-hole strength is concentrated in the second one at 2.6 MeV. On the other hand, we find that the $p_{3/2}^{-1}$ strength is almost equally distributed between the first and second $f^{-}$ states.

As for $^{96}$Pd, only 2 out of the 25 states which we predict up to 4 MeV (see Table III) arise from configurations other than $g_{9/2}^{-4}$ and $g_{9/2}^{-3} p_{1/2}^{-1}$. They are the $J^\pi = 0^+$ and $3^-$ states with a $g_{9/2}^{-2} p_{1/2}^{-2}$ and $g_{9/2}^{-3} p_{3/2}^{-1}$ dominant component, respectively.

From the above discussion it is evident that some of our predictions are closely related to the values adopted for $\epsilon_{p_{1/2}}$ and $\epsilon_{f_{5/2}}$. For instance, as shown before, we find that the wave functions of several states in the three considered isotones contain non negligible components outside the $(g_{9/2} p_{1/2}^{-1})$ space. This indicates that a two-level model space would not be adequate even for the description of the heavier $N = 50$ isotones. We also predict the absence of a pronounced gap above the $0^+_2$ state in the spectrum of $^{98}$Cd as well as rather low-lying one-hole $3/2^-$ and $5/2^-$ states in $^{97}$Ag. This makes it clear that, in absence of a spectroscopic study of $^{99}$In, the discovery of new selected levels in $^{98}$Cd and $^{97}$Ag represents the best source of information on the SH spectrum.

To conclude this discussion, a further comment is in order. As it occurs for the $13^+$ state in $^{97}$Ag and the $2^+$ and $10^+$ states in $^{96}$Pd, we expect that the calculated excitation energies of all other states in these two nuclei arising from the $2^+$ state of $^{98}$Cd may be somewhat overestimated (200-300 keV). This is the case, for instance, of the $(3/2^+)_1$, $(3/2^+)_2$, and $(5/2^+)_1$ states in $^{97}$Ag.

Let us now come to the electromagnetic observables. The effective operators needed for the calculation have been derived as described in Sec. II. Experimental information on electromagnetic properties in proton-rich $N = 50$ isotones is very scanty. The measured $E2$ transition rates $B\left(E2;\beta\rightarrow\alpha\right)$ are compared with the calculated values in Table IV, where we also report our predicted $B(E2)$ values for all the states having an experimental counterpart. As regards the $B(E2; 8^+ \rightarrow 6^+)$ in $^{98}$Cd, the two different measured values result from the experiments of Refs. [33].

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1We define $\sigma = \{(1/N_d) \sum |E_{exp}(i) - E_{calc}(i)|^2\}^{1/2}$, where $N_d$ is the number of data.
and $^{34}$, where this nucleus was produced by a fusion-evaporation reaction and by fragmentation of a $^{106}$Cd beam, respectively. While there are some doubts about both these values $^{34}$, $^{35}$, the one in Ref. $^{34}$, which corresponds to a proton effective charge fairly larger than $e$, is consistent with that measured for $^{96}$Pd. From Table IV we see that the agreement between experiment and theory for the $B(E2; 8^+ \rightarrow 6^+)$ and $B(E2; 6^+ \rightarrow 4^+)$ in $^{96}$Pd is quite satisfactory, the calculated values being only slightly smaller than the observed ones. As for $^{98}$Cd, the calculated $B(E2; 8^+ \rightarrow 6^+)$ value agrees with the result of Ref. $^{24}$ within the error bars. It is worth noting that our results do not differ significantly from those obtained using an effective proton charge $e_{\text{eff}}^p = 1.35e$.

As regards the magnetic observables, only the magnetic moment of the $8^+$ state in $^{96}$Pd is known. The measured value is $10.97 \pm 0.06$ nm $^{30}$, to be compared with the calculated one 10.54 nm.

We have already mentioned in the Introduction that several calculations have been performed to study the shell-model structure of the $N = 50$ isotones. We only comment here on the calculation of Ref. $^{17}$, which, assuming $^{108}$Sn as a closed core, the two-hole effective interaction was derived by using the Sussex matrix elements in a perturbation scheme up to second order without folded-diagram renormalization. As pointed out in Sec. II, the adopted SH energies, as determined from a least-squares fit to the energies of the $N = 50$, $37 \leq Z \leq 44$ nuclei, are much higher than ours. In that work, however, attention was focused on nuclei with $Z = 34 - 46$ and no results were given for $^{98}$Cd and $^{97}$Ag, for which experimental information has become available only in more recent times. We have therefore found it interesting to perform calculations for these two nuclei using the effective interaction and the SH energies of Ref. $^{17}$. We have also calculated a more complete spectrum of $^{96}$Pd than that given in $^{17}$. Hereafter we shall refer to these calculations as Sussex (SUX) calculations.

The $\sigma$ value relative to the SUX calculations for $^{98}$Cd, $^{97}$Ag, and $^{96}$Pd turns out to be 84, 353, and 218 keV, respectively. More explicitly, the experimental position of the positive-parity states is well reproduced. In particular, the calculated energies of the $2^+$ states in both even isotopes, as well as those of the $\frac{13}{2}^-$ and $10^+$ states in $^{97}$Ag and $^{96}$Pd, come closer to the experimental values than those obtained from our calculations. For all other positive-parity states the agreement with experiment obtained from SUX and our calculations is comparable. On the other hand, the $\frac{17}{2}^-$ and $5^-$ states in $^{97}$Ag and $^{96}$Pd lie 704 and 560 keV above the experimental ones, respectively, and the excitation energies of the first $5^-$ and $\frac{1}{2}^-$ states in $^{98}$Cd and $^{97}$Ag are predicted to be about 3.5 and 1.4 MeV, which are not consistent with the experimental information available for the lighter isotones. Furthermore, for $^{97}$Ag the $p_{3/2}$ and $f_{5/2}$ strengths are concentrated in the second $\frac{3}{2}^-$ and the second $\frac{5}{2}^-$ states, which are predicted to lie at about 3 and 4 MeV, respectively. These values are more than 1 MeV higher than ours which come quite close to those obtained by extrapolating the energies of these states from the lighter isotones. From the above we feel that the values of the SH energies adopted in Ref. $^{17}$ are overestimated. As regards the $p_{1/2}$ level, this conclusion is strongly supported by the fact that, as mentioned above, the calculated energies of the $\frac{17}{2}^-$ and $5^-$ states in $^{97}$Ag and $^{96}$Pd, which are substantially dependent on the position of this level, are largely overestimated. On the other hand, we have verified that decreasing the values of SH energies is not sufficient to improve the agreement between theory and experiment on the whole. In fact, while this produces a lowering of the negative-parity states it moves the positive-parity states up to too high an energy. This latter effect, however, might be compensated by taking into account folded-diagram renormalization, which produces in general a compression of the spectra. In this context, we may mention that the authors of Ref. $^{17}$ say that the folded-diagram renormalization would have worked against the outcome of their calculations. Our preceding remarks indicate that this conclusion could have been turned round had they made a different choice of the SH energies.

IV. SUMMARY AND CONCLUSIONS

In this paper, we have performed a shell-model study of the $N = 50$ isotones $^{98}$Cd, $^{97}$Ag, and $^{96}$Pd employing a two-hole effective interaction derived from the Bonn-A nucleon-nucleon potential by means of a $G$-matrix folded-diagram approach. We have shown that all the experimental data available for these nuclei are well reproduced by our calculations. In addition, some of our predictions, namely the existence of a $5^-$ state in $^{98}$Cd and a $\frac{1}{2}^-$ state in $^{97}$Ag at 2.7 and 0.5 MeV, respectively, are strongly supported by the experimental information existing for the lighter isotones.

This work is framed in the context of an extensive program of calculations aimed at assessing just how accurate a description of nuclear structure properties can be provided by an effective interaction derived from the $NN$ potential. The quality of the results presented here turns out to be comparable to that obtained in the $^{132}$Sn and $^{208}$Pb regions where, however, there is a substantially larger body of experimental data with which to compare the calculated spectroscopic properties. In particular, we emphasize that the experimental information existing for the $N = 50$ isotones provides only little guidance to the choice of the SH energies, which renders it a difficult task. In this
situation, our choice has been based on an analysis of the spectra of $^{98}$Cd, $^{97}$Ag, and $^{96}$Pd and on the values of the experimental $\frac{\Delta}{\Delta}$ and $\frac{\delta}{\delta}$ single-hole energies in $^{89}$Y, $^{91}$Nb, and $^{93}$Tc. We feel, however, that the unavoidable uncertainty in the adopted SH energies should not be so severe as to make our findings questionable.

In conclusion, we are confident that the present work may be of stimulus and help towards new experimental studies of the proton-rich $N = 50$ isotones.

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FIG. 1. Experimental and calculated spectrum of $^{98}$Cd

FIG. 2. Experimental and calculated spectrum of $^{97}$Ag

FIG. 3. Experimental and calculated spectrum of $^{96}$Pd

| TABLE I. Calculated energy levels in $^{98}$Cd up to 5 MeV. |
|---------------------------------|-----------------|-----------------|
| $J^\pi$ | E(MeV) | $J^\pi$ | E(MeV) |
| 0$^+$ | 0.0 | 0$^+$ | 3.499 |
| 2$^+$ | 1.606 | 3$^-$ | 3.581 |
| 4$^-$ | 2.117 | 5$^-$ | 4.466 |
| 6$^+$ | 2.295 | 4$^-$ | 4.578 |
| 8$^+$ | 2.414 | 2$^+$ | 4.594 |
| 5$^-$ | 2.705 | 6$^-$ | 4.594 |
| 4$^-$ | 3.083 | 7$^-$ | 4.801 |

| TABLE II. Calculated energy levels in $^{97}$Ag up to 3 MeV. |
|---------------------------------|-----------------|-----------------|
| $J^\pi$ | E(MeV) | $J^\pi$ | E(MeV) |
| 0$^+$ | 0.0 | 0$^+$ | 2.055 |
| 1$^+$ | 0.528 | 1$^+$ | 2.077 |
| 2$^+$ | 1.007 | 2$^+$ | 2.210 |
| 3$^+$ | 1.342 | 3$^+$ | 2.318 |
| 4$^+$ | 1.501 | 4$^+$ | 2.326 |
| 5$^+$ | 1.593 | 5$^+$ | 2.339 |
| 6$^+$ | 1.648 | 6$^+$ | 2.479 |
| 7$^+$ | 1.766 | 7$^+$ | 2.558 |
| 8$^+$ | 1.801 | 8$^+$ | 2.747 |
| 5$^+$ | 1.814 | 9$^+$ | 2.929 |
| 4$^+$ | 2.024 | 10$^+$ | 2.962 |

| TABLE III. Calculated energy levels in $^{96}$Pd up to 4 MeV. |
|---------------------------------|-----------------|-----------------|
| $J^\pi$ | E(MeV) | $J^\pi$ | E(MeV) |
| 0$^+$ | 0.0 | 0$^+$ | 3.422 |
| 2$^+$ | 1.678 | 2$^+$ | 3.559 |
| 4$^+$ | 2.201 | 3$^-$ | 3.569 |
| 6$^+$ | 2.397 | 3$^+$ | 3.585 |
| 8$^+$ | 2.507 | 0$^+$ | 3.619 |
| 5$^-$ | 2.618 | 3$^-$ | 3.744 |
| 4$^+$ | 2.805 | 6$^+$ | 3.762 |
| 4$^+$ | 2.991 | 7$^-$ | 3.827 |
| 2$^+$ | 3.048 | 8$^+$ | 3.877 |
| 6$^+$ | 3.085 | 3$^-$ | 3.916 |
| 0$^+$ | 3.161 | 6$^-$ | 3.920 |
| 5$^+$ | 3.354 | 10$^+$ | 3.924 |
| 7$^+$ | 3.413 |
TABLE IV. Calculated and experimental $E2$ transition rates (in W.u.) between yrast states in $^{98}$Cd, $^{97}$Ag, and $^{96}$Pd.

| Nucleus | $J_i^p \rightarrow J_f^p$ | $B(E2)_{\text{calc}}$ | $B(E2)_{\text{expt}}$ |
|---------|---------------------------|------------------------|------------------------|
| $^{98}$Cd | $2^+ \rightarrow 0^+$ | 3.71 | |
|           | $4^+ \rightarrow 2^+$ | 4.28 | |
|           | $6^+ \rightarrow 4^+$ | 2.98 | |
|           | $8^+ \rightarrow 6^+$ | 1.20 | 1.17$^{+0.14}_{-0.11}$, 0.54$^{+0.28}_{-0.13}$ |
| $^{97}$Ag | $13^+ \rightarrow 9^+$ | 4.23 | |
|           | $15^+ \rightarrow 13^+$ | 4.15 | |
|           | $9^+ \rightarrow 7^+$ | 2.42 | |
| $^{96}$Pd | $2^+ \rightarrow 0^+$ | 6.27 | |
|           | $4^+ \rightarrow 2^+$ | 0.78 | |
|           | $6^+ \rightarrow 4^+$ | 0.51 | 0.78$ \pm 0.11$ |
|           | $8^+ \rightarrow 6^+$ | 0.20 | 0.34$ \pm 0.05$ |
|           | $10^+ \rightarrow 8^+$ | 4.29 | |
|           | $12^+ \rightarrow 10^+$ | 3.28 | |

$^a$ Reference [34].  
$^b$ Reference [13].
$^{97}\text{Ag}$

- (21/2$^+$)
- (17/2$^-$)
- (17/2$^+$)
- (13/2$^+$)
- (9/2$^+$)

Expt. (9/2$^+$)  Calc. (9/2$^+$)
