Chaos and isospin symmetry breaking in rotational nuclei

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Abstract

For nuclei with $N \approx Z$, the isospin degree of freedom is important and, for deformed systems, rotational bands of different isospin may be expected at low excitation energies. We have investigated, in a simple model space, the influence of the isospin-breaking Coulomb interaction on the degree of chaoticity of these rotational bands. The statistical measures used rely on an analysis of level-spacing distributions, which are extremely difficult to measure experimentally. We show, however, that the overlap integrals between states of similar frequency reflect well the degree of chaoticity. This quantity is closely related to the experimentally more accessible $\gamma$-decay “spreading width”.

1 Introduction

Random matrix theory (RMT) [1] has played an important role in our understanding of quantum many-body problems. In classical mechanics the underlying nature of a physical system is revealed [2] by the behaviour of its trajectories in phase-space. For regular motion, these trajectories are stable against small perturbations of their initial conditions, whereas in the case of chaotic motion the trajectories show hyperbolic instability and tend to fill the whole of the available phase space. In quantum-mechanics, it is not possible to define such trajectories since the position and momenta coordinates cannot be defined simultaneously. Consequently it is not possible to determine the motion of the system in the same way. RMT provides a valuable connection between the spectral properties of a quantum system and its corresponding classical motion. It has been demonstrated that the spectral distributions of few-body quantum systems which follow the predictions of RMT have chaotic
motion in the corresponding classical phase-space, whereas distributions which follow a Poissonian behaviour display regular classical motion. This is referred to as the Bohigas-Giannoni-Schmit conjecture [3] and is now accepted as a generic feature of quantum many-body systems.

The underlying motion of a classical system depends on its symmetries or, in other words, on the integrals of motion. For a system with a number of integrals of motion equal to the number of degrees of freedom, the classical motion is completely regular [2]. In quantum mechanics, the quantum number is the analogue of the integral of motion and it has been shown that symmetry breaking leads to chaotic motion. This has been clearly demonstrated in the interacting boson model (IBM) [4]. This model has three dynamical symmetry limits and the restriction to a particular dynamical symmetry corresponds to regular motion. It is the interpolation among the three relevant symmetry groups which gives rise to chaos.

The purpose of the present work is to investigate how the spectral distribution is modified by the inclusion of a two-body Coulomb force which leads of course to isospin symmetry breaking. Such an analysis has already been carried out for spherical nuclei [5–7]. In the present work, however, we shall study deformed nuclei and investigate the effects of the isospin symmetry breaking as a function of rotational frequency, following the recent observation of rotational bands with good isospin for $N \approx Z$ [8]. In the absence of a Coulomb force the states may be labelled by the isospin $T$ and we shall see that each individual set of states of a given $T$, displays chaotic dynamics. The introduction of the Coulomb force, however, breaks this symmetry and the entire set of states must now be considered together. The final dynamics depends on the strength of the Coulomb interaction relative to the cranking and deformation energies. For relatively small deformations and low rotational frequencies, chaotic dynamics are restored to the full set of states. However, for large deformations and high frequencies a more regular dynamics may persist.

We have performed the statistical analysis of eigen-values using the measures of Brody [9], Berry-Robnik [10] and the spectral-rigidity of Dyson and Mehta [11]. A similar analysis has been carried out in ref. [12] for a single type of nucleon. Of course such analysis demand data which are impossible to obtain. We have, therefore, investigated the spreading of an eigen-state at one frequency over the many other eigen-states to which it can $\gamma$ decay at a lower frequency. This quantity, which is closely related to the $\gamma$-decay spreading width is indeed shown to reflect the degree of chaoticity evaluated by the above statistical analysis.

The cranking hamiltonian is briefly described in section II and the theoretical approaches used to investigate the statistical distributions are given in section III. The spectral analysis of the eigen-values is presented in section IV, overlap
integrals in section V and the results are summarized in section VI.

2 Rotational Model

We employ a deformed shell-model hamiltonian, consisting of a cranked deformed one-body term, \( h' \) and a scalar-isoscalar two-body delta-function residual nuclear interaction \([13–15]\). The one-body term is the familiar cranked Nilsson mean-field potential which takes account of the long-range part of the nucleon-nucleon interaction. Thus

\[
H' = h' - g\delta(\hat{R}_1 - \hat{R}_2) + V_c \tag{1}
\]

where,

\[
h' = -4\kappa\sqrt{\frac{4\pi}{5}}Y_{20} - \omega J_x. \tag{2}
\]

The strength of the two-body interaction is \( G = g \int R_n^4 r^2 dr \) and the deformation energy \( \kappa \) is related to the deformation parameter \( \beta \) through

\[
\kappa \approx 0.16\hbar\omega_0(N + 3/2)\beta, \tag{3}
\]

where \( \hbar\omega_0 \) is the harmonic oscillator frequency of the deformed potential and \( N \) the quantum number of the major shell. For the case of \( f_{7/2} \) shell, \( \kappa=1.75 \) approximately corresponds to \( \beta = 0.16 \). The last term in Eq. (1), \( V_c \) is the Coulomb interaction among the valence protons. We have employed the empirically obtained \([16]\) two-body Coulomb matrix-elements in the \( f_{7/2} \) shell and are given by \( V_c(J) = 0.578, 0.486, 0.374 \) and \( 0.330 \) MeV for \( J = 0, 2, 4 \) and \( 6 \), respectively. In order to solve the eigen-value problem exactly, we are limited to a small configuration space. As in the previous work, the model space in the present analysis consists of a single j-shell. We have diagonalized the hamiltonian (1) exactly for neutrons and protons in the \( f_{7/2} \) shell. As the strengths of nn-, pp- and np-parts are identical, the hamiltonian in the absence of \( V_c \) is invariant with respect to rotations in isospace, i.e.

\[
\mathcal{R}H'\mathcal{R}^{-1} = H', \tag{4}
\]

where \( \mathcal{R} \) defines a rotation in isospace, generated by the isospin operators \( T_x, T_y \) and \( T_z \). Furthermore, the hamiltonian (1) is invariant with respect to a spatial rotation about the x-axis by an angle of \( \pi \). As a consequence, the signature \( \alpha \) is a good quantum number \([17]\), which implies that the shell model solutions represent states with the angular-momentum \( I = \alpha + 2n \) (\( n \) integer).
3 Statistical methods

In this section, we shall briefly outline the statistical methods used in the present work. For more details on these methods the reader is referred to ref. [12] and the references therein.

The level spacing is defined by

\[ S_i = X_{i+1} - X_i \; ; \; i = 1, 2, \ldots \] (5)

where \( X_i \) are the unfolded energy levels

\[ X_i = \tilde{N}(E_i) \; i = 1, 2, \ldots N. \] (6)

Roughly speaking, \( S_i \) is the original level spacing \((E_{i+1} - E_i)\) in units of the local average level separation. The unfolding of the spectra can be carried out in a different way using

\[ S_i = (E_{i+1} - E_i)\tilde{\rho}(E_i) \; i = 1, 2, \ldots N - 1. \] (7)

This unfolding of the spectra ensures that the average spacing in the series \( X_i \) is unity. In this way the fluctuation properties of the spectra of different systems can be compared. In our investigations both types of unfolding procedures led qualitatively to the same outcome. The results in the rest of the paper were produced using the mapping (7).

The first spectral statistics we use is the nearest-neighbour distribution \( P(S) \). The quantity \( P(S)dS \) gives the probability that the nearest-neighbour of an arbitrarily selected level \( S_i \) lies in the interval \((S_i + S, S_i + S + dS)\). From the finite set of \( \{S_i\}_{i=1}^N \) only a histogram can be constructed as an NND. In order to have good statistics the bin size of the histogram is chosen to ensure that there are at least seven spacings in each bin. We considered the NND in the interval \( S \in (0, 2) \).

Spectral statistics show Poissonian or GOE forms for integrable and fully chaotic systems. For the case of mixed phase phase the analytic forms of these statistics are not known. In the following we shall employ three parametrizations of Berry-Robnik [10], Brody [9] and the spectral-rigidity of Dyson and Mehta [11]. The Berry-Robnik parametrization of NND is given

\[ P_{BR}(q, S) = q^2 \exp(-qS) \text{erfc}(\frac{1}{2}\sqrt{\pi qS}) \]
\begin{align*}
+ (2q\bar{q} + \frac{1}{2} \pi q S^3) \exp(-\bar{q} S - \frac{1}{4} \pi q S^2) \tag{8}
\end{align*}

where \(0 \leq q \leq 1\) is the measure of the chaotic region of the phase space and \(\bar{q} = 1 - q\).

The Brody distribution is given by
\begin{align*}
P_B(b, S) = (1 + b) AS^b \exp(-AS^{1+b}), \tag{9}
\end{align*}

where
\begin{align*}
A = \Gamma \left( \frac{2 + b}{1 + b} \right)^{1+b}, \tag{10}
\end{align*}

and \(\Gamma\) denotes the usual gamma function. For fully chaotic and ordered systems \(q = b = 1\) and \(q = b = 0\), respectively.

The spectral statistics \(\bar{\Delta}_3(L)\) measures the long-range correlation of the unfolded levels
\begin{align*}
\Delta_3(X, L) = \min \frac{1}{L} \int_X^{X+L} \left( N_u(E) - (AE + B) \right)^2 dE, \tag{11}
\end{align*}

where \(N_u\) is the cumulative level density of the unfolded levels \(X_i\). We average \(\Delta_3(X, L)\) over intervals \((X, X+L)\) to get \(\bar{\Delta}_3(L)\), as outlined in [3]. For ordered systems \(\bar{\Delta}_3(L) = L/15\) and for fully chaotic ones \(\bar{\Delta}_3(L) \approx \ln(L)/\pi^2 - 3/4\) for \(L \gg 1\). The explicit calculation of the spectral rigidity was done with the method described in [18].

The spectral rigidity for a mixed statistics [19] is given by
\begin{align*}
\Delta_3(q, L) = \Delta^P_3(qL) + \Delta^{GOE}_3((1 - q)L). \tag{12}
\end{align*}

The \(b\) and \(q\) parameters of the NND distributions and the spectral rigidity are determined by least-squares fits to the numerical results. The errors in the best-fit values of these parameters were estimated by the method of maximum likelihood and the use of constant chi-squared boundaries as a confidence limit [20].
4 Results and discussions

The cranked shell model calculations have been carried out for (4-neutrons + 4-protons) in the $f_{7/2}$ shell, $\langle f_{7/2} \rangle^4 \langle f_{7/2} \rangle^4$. We have used the $f_{7/2}$ shell for simplicity in carrying out the exact deformed shell model calculations. For this problem the dimensionality of the matrix to be diagonalized for $\alpha = 0$ (favoured configuration) is 2468. In order to check the deformation dependence of the statistical distributions, we have performed calculations at two deformation values with $\kappa = 1.75$ and 3.50. Since the cranking is a rather poor approximation at lower rotational frequencies, most of the statistical analysis will be provided at higher frequencies.

The nearest neighbour distribution is shown in Fig. 1 for $\bar{\hbar}\omega = 0.5$ MeV. The statistical analysis has been carried out for each isospin bin separately and the obtained NND clearly fits GOE for $T=0,1$ and 2 separately. For other isospin values ($T=3$ and 4), the number of states are very few and it is not possible to perform the statistical analysis. In fact for $T=4$, there is only one state. In Fig. 2, the statistical analysis is performed with the same parameters as in Fig. 1, but with all the 2468 states without considering the isospin quantum number. Fig. 1(a) shows NND with no Coulomb interaction, isospin conserved. The obtained distribution is closer to the Poisson than to Gaussian orthogonal ensemble. This feature of obtaining a Poission like distribution from the superposition of several GOE distributions has already been demonstrated in ref.[6]. This can be explained by noting that since there is no interaction between the states with good $T$, the near degeneracy of two eigen-values belonging to two different isospin quantum numbers is unavoidable and consequently gives rise to a more Poission-like distribution which favours energy levels with zero spacings.

The NND in the presence of the Coulomb interaction is shown in Fig. 2(b) and as is seen that the distribution becomes closer to GOE. The isospin mixing resulting from the Coulomb interaction is quite small and is shown in Table 1 for the lowest five-states and the 14th and the 15th states. We have obtained the isospin mixing by expanding the wavefunction in terms of the states with good $T$. The isospin mixing for the lowest state at $\bar{\hbar}\omega = 0.5$ MeV is about 0.02%. The mixing for the lowest three $T=0$ states is from the $T=1$ state and components of the other isospin states are negligible. We would like to mention here that isospin mixing for some of the states, for instance for the 14th and the 15th states is quite large and is due to the near degeneracy of the two-states, the energies of the two-states being -33.43 and -33.41, respectively. The isospin mixing in the present model analysis appears to be quite small and is due to the fact that we consider Coulomb interaction only among the valence protons. In principle, there should be also a contribution from the core protons.
In Fig. 2(c), the results are presented by artificially increasing the Coulomb matrix elements by a factor of two and the distribution turns out to be more close to GOE with substantially reduced chi-square. As one expects, for a strong symmetry breaking the distribution should converge to a single GOE \[6\]. Fig. 1 clearly demonstrates a strong dependence of the statistical distribution on the residual Coulomb interaction.

As already mentioned, in the above statistical analysis, we have used all the eigen-values. It is also of interest to perform the statistical analysis of different excitation energy windows. In Fig. 3, the NND is plotted for three bins each consisting of 800 eigen-states, the top panel shows NND for the lowest 800 states. The middle panel from 801 to 1600 and the lower panel from 1601 to 2400. As is evident from this figure, a strong dependence of NND on excitation energy is obtained. This conforms with earlier studies that the degree of chaoticity increases with the excitation energy.

In Figs. 4, similar results as in Fig. 2 are presented but for \(\hbar \omega = 2.5\) MeV. As compared to \(\hbar \omega = 0.5\) MeV, the distributions now deviate more from GOE and can be understood from the fact that with increasing frequency the quasiparticles align along the axis of rotation and the motion tends to be more regular. In order to quantify the deviations from GOE statistics, we have analyzed the NND with Brody, Berry-Robnik and spectral-rigidity parametrizations. The results are presented in Fig. 5 with all the three measures. In Fig. 5(a), the parameters are presented with no Coulomb potential, including all the isospin states. The Brody parameter is small indicating that NND is regular for all the rotational frequencies studied. The Berry-Robnik and the spectral-rigidity parameters on the other hand are between those expected for pure GOE and Poission distributions. In the presence of the Coulomb potential, the results are given in Fig. 5(b) and as is apparent all the three measures give values close to 1 signifying that the underlying classical motion is chaotic. We have also calculated these statistical measures by considering the isospin bins separately and in Fig. 5(c), the results are given for T=0. It is noted from Fig. 5(c) that all the three measures give values very close to 1. Comparing the middle and the lower panels of Fig. 5, it is observed that the degree of chaoticity is larger for T=0 level statistics than with Coulomb interaction. This unexpected result can be understood in the following manner: The chaoticity for the T=0 states is determined by the residual interaction and in the present model includes the neutron-proton interaction. The combined residual interaction is quite strong and we obtain chaotic behaviour. On the other hand, in the statistical analysis of Fig. 5(b), the degree of chaoticity is determined by the Coulomb interaction among the good isospin states and in the present model this interaction is small and consequently the distribution is less chaotic. This would also explain the apparent contradiction with the results obtained in ref.[5]. It was shown in ref.[5] that the distributions are similar whether one separates the experimental levels into isospin bins or not. The realistic isospin
mixing is stronger than calculated in the present work and is the reason for the apparent discrepancy between the present work and the results presented in ref.[5].

The isospin mixing for $\hbar \omega = 2.5$ MeV for the lowest few eigen-states are given Table 1. As compared to $\hbar \omega = 0.5$ MeV, it is observed that isospin mixing is lower. This can be explained by noting that with increasing frequency the particle alignment occurs along the axis of rotation and there is a transition from the paired ($J = 0$) configuration to the aligned state ($J = 2j - 1$) at the first bandcrossing. The Coulomb energy is maximum for the paired state and is least for the aligned configuration [16] and this is the reason for the lowering of the isospin mixing with increasing rotational frequency.

The spectral-rigidity statistical measure is provided in Fig. 6 for the three cases studied earlier at $\hbar \omega = 0.5$ MeV. The results with no Coulomb interaction and considering all the states, shown in the top panel of Fig. 6 , are between Poission and GOE. The results with Coulomb interaction, shown in the middle panel, are close to GOE and agree with the other statistical measures presented earlier. The results for $T=0$ levels shown in Fig. 6(c) exactly follow the expected GOE spectral-rigidity.

The results with larger deformation, $\kappa = 3.5$ for Brody, Berry-Robnik and spectral-rigidity parametrizations are presented in Fig. 7. It is noted that the degree of chaoticity is slightly less as compared to $\kappa = 1.75$ and can be understood from the fact that with increasing deformation the collective effects dominate and the results tend to become more regular.

5 Overlap-integrals and the decay probabilities

In a regular regime, it is expected that the wavefunction should evolve smoothly as a function of the cranking frequency. This is, indeed, the case for the yrast wavefunction. The calculation of the overlap between the cranking yrast wavefunctions at the neighbouring frequency points is almost equal to one, except in the bandcrossing region. However, in the region of chaoticity, the overlaps are expected to have a wide spread. This is illustrated in Fig. 8, where the overlaps are plotted between $\hbar \omega = 0.55$ MeV and $\hbar \omega = 0.50$ MeV at three different energy regimes. This overlap is proportional to the transitions between the neighbouring spin states. In Fig. 8(a), the overlap is presented for the 10th state which is close to the yrast line and it is clear that it has a peak value for one state, for all other states the overlap is close to zero. The overlap for the 500th eigen-state is shown in Fig. 8(b) and the results indicate that the overlap for this state has a broad spectrum. This state lies in the regime of chaoticity and demonstrates that the state cannot be traced as a function
of the cranking frequency. In Fig. 8(c), the overlap-integral is plotted for the 1500th eigen-state and the state appears to have a much more spread as compared to Fig. 8(b). This can be easily explained by noting that the degree of chaoticity increases with the excitation energy.

It is also of interest to check the overlap-integral for the case where the statistical analysis shows regular motion at all excitation energies. In Fig. 9, the results are presented for $\kappa = 3.50$ and $\hbar \omega = 2.5$ MeV which has a more regular behaviour as is evident from Fig. 7(b). Fig. 9 shows that for all the three eigenstates considered at different excitation energies, the overlap-integral peaks for one particular bra eigen-state. From the comparison of Fig. 8 and 9, it is clear that the degree of chaoticity and overlap-integral are closely related to each other. In Fig. 9, where the statistical analysis shows chaotic features at high excitation energy, the overlap-integral for a state in that regime shows a very high degree of fragmentation. In comparison, the overlap-integral peaks for one particular state at the three chosen excitation energies. For this chosen deformation and cranking frequency parameters, the corresponding statistical analysis indicate a more regular motion at the three considered excitation energies.

6 Summary

In the present work, the statistical distributions of the rotational bands has been studied in the presence of the Coulomb potential. As already mentioned in the introduction, this is relevant to the recent discovery of the isospin rotational bands near $N=Z$. The analysis has been carried out using a simple cranked shell model approach. Although, this model is not very realistic to make a direct comparison with the experimental data, but it contains all the essential ingredients of a more realistic deformed rotational model. The advantage in the simple model is that it can be solved exactly and the effects of the Coulomb interaction can be evaluated precisely. In a realistic approach, Hartree-Fock or Hartree-Fock-Bogoliubov many-body techniques are employed. It is known that these approximations also contribute to the isospin mixing and it is not possible to obtain a true measure of the isospin mixing.

We have shown that NND is very sensitive to the Coulomb interaction and can provide a measure of the isospin mixing in nuclei. The NND with the inclusion of all the good isospin states (in the absence of the Coulomb potential) is closer to the Poissonian distribution. However, the investigation of the NND for each isospin bin separately, shows that the distribution is GOE. The degree of chaoticity for each isospin bin is determined by the two-body residual interaction which in the present work is very strong and we obtain NND which fits the distribution of GOE. The presence of the Coulomb interaction
among the good isospin states immediately gives rise to GOE-like distribution and it has been shown that the degree of chaoticity is quite sensitive to the strength of the Coulomb interaction.

It has been demonstrated that the degree of chaoticity depends on the deformation. The chaotic features appear more prevalent in normal deformed bands as compared to the superdeformed shapes.

Furthermore, it has been shown in section V that the overlap-integral between the cranking wavefunctions at neighbouring frequencies and the degree of chaoticity are related to one another. This overlap-integral is proportional to the transition probabilities between the adjacent states. It has been demonstrated that in a regime where the statistical analysis shows chaotic motion, the corresponding overlap-integral is highly fragmented and for the regular motion the overlap-integral peaks for one particular state.

In a more realistic problem, this overlap would have to be taken between states of neighbouring spin. This would be I and I-2 for stretched E2 transitions. We are presently calculating these transitions using the projected shell model [21] approach with a realistic interaction and model space.

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The expansion coefficients of the lowest few eigen-states with Coulomb-interaction in terms of the states with good isospin. This gives a measure of the isospin mixing. The calculations are presented with deformation parameter $\kappa = 1.75$. The results are also provided for the two lowest states which show very large isospin mixing and is due to the near degeneracy of the corresponding eigen-values.

| State | T=0 | T=1 | T=2 | T=3 | T=4 |
|-------|-----|-----|-----|-----|-----|
| $\bar{h}\omega = 0.5 \text{ MeV}$ |
| 1     | 0.99981 | 0.00015 | 0.00004 | 0.0 | 0.0 |
| 2     | 0.99797 | 0.00199 | 0.00003 | 0.0 | 0.0 |
| 3     | 0.99412 | 0.00587 | 0.00001 | 0.0 | 0.0 |
| 4     | 0.00573 | 0.99422 | 0.00005 | 0.0 | 0.0 |
| 5     | 0.99120 | 0.00836 | 0.00002 | 0.0 | 0.0 |
| 14    | 0.12958 | 0.87026 | 0.00015 | 0.0 | 0.0 |
| 15    | 0.87250 | 0.12740 | 0.00010 | 0.0 | 0.0 |
| $\bar{h}\omega = 2.5 \text{ MeV}$ |
| 1     | 0.99988 | 0.00011 | 0.00000 | 0.0 | 0.0 |
| 2     | 0.99951 | 0.00048 | 0.00000 | 0.0 | 0.0 |
| 3     | 0.99885 | 0.00115 | 0.00000 | 0.0 | 0.0 |
| 4     | 0.00223 | 0.99776 | 0.00000 | 0.0 | 0.0 |
| 5     | 0.99956 | 0.00044 | 0.00000 | 0.0 | 0.0 |
| 12    | 0.03497 | 0.96499 | 0.00004 | 0.0 | 0.0 |
| 13    | 0.95128 | 0.04870 | 0.00002 | 0.0 | 0.0 |

Fig. 1. The nearest neighbour distributions (NND) of the eigen-spectra for $\kappa = 1.75$ and $\bar{h}\omega = 0.5 \text{ MeV}$ with no Coulomb potential. The NND is calculated for each isospin bin separately. In all the three cases the NND fits the GOE distribution.

Fig. 2. The nearest neighbour distribution (NND) of the eigen-spectra for $\kappa = 1.75$ and $\bar{h}\omega = 0.5 \text{ MeV}$. (a) the results with no Coulomb interaction but considering all the good isospin states. The calculated NND is between that expected for GOE and Poissionian distributions. (b) Shows the results by including the Coulomb potential and the distribution becomes close to GOE. In (c), the Coulomb matrix elements have been artifically increased by a factor of 2 and NND fits the GOE distribution with a much reduced chi-square.
Fig. 3. The nearest neighbour distributions (NND) of the eigen-spectra for $\kappa = 1.75$ and $\hbar \omega = 0.5$ MeV with Coulomb potential. (a) the NND is calculated by including the lowest 800 eigen-values, (b) the eigen-values from 801 to 1600 are included and in (c) the eigen-values from 1601 to 2400 are used in the statistical analysis. The spectra become more chaotic with increasing excitation energy.

Fig. 4. This is same as Fig. 2 but for $\hbar \omega = 2.50$ MeV. The NND appears to deviate from GOE as compared to Fig. 2.

Fig. 5. The degree of chaoticity using the measures of Brody (denoted by open circle), Berry-Robnik (denoted by open square) and spectral-rigidity (denoted by filled diamond) for $\kappa = 1.75$. (a) Shows the results with no Coulomb potential but including all the eigen-values, (b) gives the results with Coulomb potential and (c) shows the results for T=0 states only.

Fig. 6. The spectral-rigidity parameter of Dyson and Mehta for $\kappa = 1.75$. (a) shows the results with no Coulomb potential but including all the eigen-values, (b) gives the results with Coulomb potential and (c) shows the results for T=0 states only.

Fig. 7. This is same as Fig. 5 but with $\kappa = 3.50$.

Fig. 8. The overlap-integral, $| \langle \Phi^j(\omega = 0.50) | \Phi^i(\omega = 0.55) \rangle |^2$ in (a), (b) and (c) for i=10, 500 and 1500 using the deformation $\kappa = 1.75$. The overlap of these three ket states are calculated with all the eigen-states on the bra side, j=1 to 2468. The overlap is maximum for j=i and are shown only around this maximum value. In (a), the overlap is almost equal to one for j=i and is expected for a regular spectrum. For the regimes (b) and (c) which are chaotic, the overlaps show a broad spectrum.

Fig. 9. The overlap-integral, $| \langle \Phi^j(\omega = 2.45) | \Phi^i(\omega = 2.50) \rangle |^2$ in (a), (b) and (c) for i=10, 500 and 1500 using the deformation $\kappa = 3.50$. The overlap of these three ket states are calculated with all the eigen-states on the bra side, j=1 to 2468. For all the three cases consided here, the overlap-integral peaks around j=i and is an indication of the regular motion at all excitation energies for the present deformation value.
