Using strong gravitational lensing to probe the post reionization H\textsc{i} power spectrum

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ABSTRACT

Probing statistical distribution of the neutral hydrogen (H\textsc{i}) using the redshifted 21-cm hyperfine-transition spectral line holds the key to understand the formation and evolution of the matter density in the universe. The two point statistics of the H\textsc{i} distribution can be estimated by measuring the power spectrum of the redshifted 21-cm signal using visibility correlation. A major challenge in this regard is that the expected signal is weak compared to the foreground contribution from the Galactic synchrotron emission and extragalactic point sources in the observing frequencies. In this work, we investigate the possibility of detecting the power spectrum of the redshifted 21-cm signal by using strong gravitational lensing of the galaxy clusters. This method has the advantage that it only enhances the H\textsc{i} signal and not the diffuse galactic foreground. Based on four simple models of the cluster potentials we show that the strong lenses at relatively lower redshifts with more than one dark matter halo significantly enhances the 21-cm signal from the post reionization era. We discuss the merits and demerits of the method and the future studies required for further investigations.

Key words: cosmology: dark ages, reionization, first stars; cosmology: large-scale structure of the Universe; galaxies: clusters: general; gravitational lensing: strong; radio lines: general; technique: interferometric

1 INTRODUCTION

It is now well established that the small perturbations in the initial matter density in the universe grew under gravitational instabilities and eventually formed the first luminous objects (Peebles, 1980; Shandarin & Zeldovich, 1989; Peacock, 1999; Padmanabhan, 2002; Bernardou et al., 2002). The universe in its earlier stage was predominantly filled with H\textsc{i}, which got ionised by the radiation from the first luminous objects during the epoch of reionization. Observational evidence suggests that by the redshift of \sim 6 most of the universe is completely ionised with traces of H\textsc{i} left in places with baryonic overdensities (see for example, Fan et al., 2006 and the references therein). These are also the places where the structures in the post reionization universe grew and the present galaxies and galaxy clusters originated. In this work, we focus our attention on probing the structure formation in the post reionization universe.

Sunyaev & Zeldovich (1972, 1975) suggested using the redshifted 21-cm hyperfine structure transition of H\textsc{i} to study the distribution of baryonic matter in the post reionization era. Bharadwaj et al. (2001); Wytis & Loeb (2001; 2003) uses the power spectrum of the redshifted 21-cm intensity fluctuation to probe the statistical distribution of H\textsc{i}. Bharadwaj et al. (2001) develops a methodology to estimate this H\textsc{i} power spectrum by using the radio interferometric observations. They show that correlating the directly observed quantity visibility by the interferometers in nearby baselines, it is possible to measure the H\textsc{i} power spectra unbiasedly. Several efforts are made to model the H\textsc{i} power spectra using analytic, semi analytic models and simulations (Bharadwaj & Srikant, 2004; Wyithe & Loeb, 2009; Bagla et al., 2010; Alonso et al., 2014 including the H\textsc{i} bias (Guha Sarkar et al., 2012; Sarkar et al., 2016; Castorina & Villaescusa-Navarro, 2017; Mao et al., 2018; Sarkar & Bharadwaj, 2018) and the redshift space distortion effects (Jennings et al., 2012; Hikage & Yamamoto, 2016; Sarkar & Bharadwaj, 2019; Modi et al., 2019). These studies collectively provide a fair description of the expected redshifted 21-cm signal from the post reionization era.

All these models predict a fairly weak amplitude for the visibility correlation signal of less than 1 mJy\textsuperscript{2} in the detection range of the existing telescopes like the GMRT, the MWA or the LOFAR (Bagla et al., 2014). Observationally, one aims to detect these signals by choosing a radio-quiet part of the sky. However, contribution from the diffuse Galactic synchrotron emission and uncertainties from foregrounds makes the signal from the post reionization era to be relatively weak compared to the foreground contribution. A way forward is to look for the signal from the redshifted H\textsc{i} hyperfine transition using gravitational lenses. This method has the advantage that it only enhances the H\textsc{i} signal and not the diffuse galactic foreground. Based on four simple models of the cluster potentials we show that the strong lenses at relatively lower redshifts with more than one dark matter halo significantly enhances the 21-cm signal from the post reionization era. We discuss the merits and demerits of the method and the future studies required for further investigations.

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\textsuperscript{1} GMRT: Giant Meterwave Radio Telescope, Pune, India
\textsuperscript{2} MWA: Murchison Widefield Array, Australia
\textsuperscript{3} LOFAR: Low-Frequency Array, Netherlands
extra galactic compact sources add up to the redshifted 21-cm signal in the observing frequencies. These are usually referred to as foreground contamination to the signal and have several orders of magnitude higher amplitude (Trol [2016] compared to the signal itself. A majority of works [Datta et al. 2014; Ghosh et al. 2017; Choudhuri et al. 2015; Choudhuri et al. 2016] over the last decade towards the H I power spectrum estimation focuses on better detection, mitigation and avoidance of these foreground contaminations [Choudhuri et al., 2016; Chakraborty et al., 2019].

Galaxy clusters act as spatially distributed strong gravitational lenses (see [Kneib & Natarajan 2011] for a review) and hence may be used to detect the spatial statistics of the H I lenses (see Kneib & Natarajan (2011) for a review) and hence may avoid these foreground contaminations (Choudhuri et al., 2016). Deane et al. (2015) estimate the possibility of detecting strong lenses. Deane et al. (2015) estimate the possibility of detecting HI emission from strongly lensed individual galaxies using existing and future radio interferometers. The possibility of detecting post-reionization HI statistics using weak gravitational lensing is discussed in Poursaids & Metcalfe (2015). Blecher et al. (2019) gives the first observational effort to detect individual strongly lensed galaxies using the GMRT.

In this work, we investigate the possibility of probing the redshifted 21-cm power spectrum using its enhancement by strong gravitational lensing of galaxy clusters. We analytically calculate the modification in the visibility correlation signal in the presence of strong lensing. To access this effect in terms of observations, we consider four simple lensing models. We use a detailed model of gravitational lensing of galaxy clusters. We analytically calculate the correlation. The expected lensed visibility correlation signal from the GMRT is arranged in the following way. In section 2 we provide the analytical formalism to calculate the modified visibility correlation. The expected lensed visibility correlation signal from our four simple lens models are shown in section 3. We discuss the merits and demerits of this technique in section 4 and conclude.

2 FORMALISM

2.1 Visibility function of lensed H I emission

Specific intensity of H I emission at the observer \( I_{obs}(\vec{\theta}, z_s, z_L) \), originated from a direction \( \vec{\theta} \) in the sky (with respect to the centre of the field of view of observation) at the redshift \( z_s \) and modified by a strong gravitational lens at the redshift \( z_L \), can be written as

\[ I_{obs}(\vec{\theta}, z_s, z_L) = \int d\vec{\theta} \left[ I_0 + \delta I(\vec{\theta}, z_s) \right] G_L(\vec{\theta} - \vec{\theta}', z_s, z_L), \]

where \( G_L(\vec{\theta}, z_s, z_L) \) is the point spread function of the gravitational lens. Redshift dependence on the specific intensity at the observer arises from (i) the redshifted frequency of the 21-cm emission and (ii) the redshift dependence of the point spread function of the gravitational lens \( G_L \). The point spread function of the gravitational lens also depends on the angular direction in the sky. We shall discuss these dependencies in detail in a later section. The observed visibility by radio interferometers of such lensed H I emission can be written as

\[ V(\vec{U}, z_s, z_L) = \int d\vec{\theta} \int d\vec{\theta}' \delta I(\vec{\theta}', z_s) A(\vec{\theta}, z_s) G_L(\vec{\theta} - \vec{\theta}', z_s, z_L) \exp(-i2\pi \vec{U} \cdot \vec{\theta}). \]  

We have assumed here that the angular extent of the gravitational lens is small and the flat sky approximation is valid. The interferometers usually do not observe at zero baselines, hence in absence of lensing, the first term in the integral of eqn. (1) is not measured. Strong lensing, however, modifies the first term by the point spread function of the lens \( G_L \). However, since lensing conserves the surface brightness, contribution through the first term in eqn. (1) in visibility remains the same and is not measured by the interferometers. The antenna beam pattern \( A(\vec{\theta}) \) of the telescope depends on the source redshift through the observing frequency and is related to the aperture function \( \tilde{a}(\vec{U}) \) as

\[ A(\vec{\theta}) = \int d\vec{U} \tilde{a}(\vec{U}) e^{i2\pi \vec{U} \cdot \vec{\theta}}. \]  

In this work, we investigate the effect of strong lensing on intensity mapping of the redshifted 21-cm emission from the neutral hydrogen. 21-cm intensity mapping is discussed in Bharadwaj et al. (2001) and Bharadwaj & Ali (2008). Bharadwaj & Ali (2008) define the 21-cm radiation efficiency \( \eta_{HI}(\vec{x}) \) as

\[ \eta_{HI}(\vec{x}, z_s) = \frac{\rho_{HI}(\vec{x}, z_s)}{\bar{n}_H(z_s)} \left( 1 - \frac{T_e(z_s)}{T(z_s)} \right) \left[ 1 - \frac{1 + z_s}{H(z_s)} \frac{dv}{dt} \right], \]

where \( \rho_{HI}(\vec{x}, z_s) \) is the density of H I at the point \( \vec{x} \) in the source redshift, \( \bar{n}_H(z_s) \) is the mean density at \( z_s \), \( T_e \) and \( T_s \) are the mean CMB and spin temperature at the source redshift and \( v \) give the peculiar velocity. The brightness temperature fluctuation in 21-cm emission is related to the 21-cm radiation efficiency \( \eta_{HI}(\vec{x}) \) as

\[ \delta T(\vec{\theta}, z_s) = \frac{\partial B}{\partial T} \bar{T}(z_s) \eta_{HI}(\vec{x}, z_s), \]

where \( B \) is the Plank’s function and \( \bar{T}(z_s) \) can be calculated for a background cosmology as

\[ \bar{T}(z_s) = 4.0mK(1 + z_s)^2 \left( \frac{3H_0^2}{0.02} \right) \left( \frac{0.7}{h} \right) \left( \frac{H_0}{H(z_s)} \right) \].

We define the lensing sampling function \( S_L(\vec{U}, z_s, z_L) \) as

\[ G_L(\vec{\theta}, z_s, z_L) = \int d\vec{U} S_L(\vec{U}, z_s, z_L) e^{i2\pi \vec{U} \cdot \vec{\theta}}. \]

The observed (lensed) visibility can be written as

\[ V(\vec{U}', z_s, z_L) = \bar{T}(z_s) \frac{\partial B}{\partial T} \int \frac{d\vec{k}}{(2\pi)^3} \eta_{HI}(\vec{k}) S_L \left( \frac{r_v k_z}{2\pi} \cdot z_s, z_L \right) \tilde{a} \left( \vec{U} - \frac{r_v k_z}{2\pi} \right) e^{i2\pi \vec{k} \cdot \vec{x}}. \]  

where the quantity \( \tilde{a}(\vec{k}) \) gives the 21-cm radiation efficiency in the Fourier space as

\[ \eta_{HI}(\vec{x}, z_s) = \int \frac{d\vec{k}}{(2\pi)^3} \tilde{a}(\vec{k}). \]

We have used \( \vec{k} \cdot \vec{x} = r_v k_z + \vec{k}_z \cdot \vec{\theta} \) with \( || \) denoting the component of the vector \( \vec{k} \) along the line of sight of observation and \( \perp \) the components in the plane of the sky, \( \vec{x} \), the comoving position vector. The quantity \( r_v \) denotes the comoving distance to the redshift, where for the observing frequency \( v \) and rest frequency of 21-cm emission \( v_0, z_s = v_0/v - 1 \).
2.2 Power spectrum for the lensed $H\text{ I}$ emission

We define the $H\text{ I}$ power spectrum at any redshift as

$$<n_{H\text{ I}}^*(\vec{k})n_{H\text{ I}}(\vec{k})> = (2\pi)^3 \delta(\vec{k} - \vec{k}') P_{H\text{ I}}(\vec{k}).$$  \hspace{1cm} (10)

Bharadwaj et al. (2001) and Bharadwaj & Ali (2005) show that the visibility correlation at nearby baselines $V_2(\vec{U}, \vec{U}', \nu, \Delta \nu) = \langle \nu \rangle = \langle V^*(\vec{U}, \nu)V(\vec{U}', \nu + \Delta \nu) \rangle$ has the information of the redshifted 21-cm power spectrum. Here we estimate the observed visibility correlation for a given 21-cm power spectrum at redshift $z$, modified by an intervening strong gravitational lens at redshift $z_l$. We denote the visibility correlation with lensing as $V_{2L}$, where

$$V_{2L}(\vec{U}, \vec{U}', \nu, \Delta \nu) = C'(z) \int \xi \gamma \frac{d\xi}{(2\pi)^2} e^{-i\vec{v}_l \cdot \Delta \vec{v}} \int \frac{d\xi}{(2\pi)^2} e^{-i\vec{v}_l \cdot \Delta \vec{v}}$$

$$S_L(\frac{r_v}{2\pi}, z_s, z_L) \frac{r_v}{2\pi}, z_s, z_L) \tilde{a}^*(\vec{U} - \frac{r_v}{2\pi}) \tilde{a}(\vec{U}' - \frac{r_v}{2\pi}) P_{H\text{ I}}(\vec{k}).$$  \hspace{1cm} (11)

Here $\nu_1 = \nu + \Delta \nu$, $C'(z) = \langle T(z_s) \frac{d\xi}{(2\pi)^2} e^{-i\vec{v}_l \cdot \Delta \vec{v}} \rangle$, $r_v$ is the comoving distance at frequency $\nu + \Delta \nu$

$$r_v = r(\nu) + \frac{\partial r_v}{\partial \nu} \Delta \nu = r_\nu + r_\nu' \Delta \nu.$$  \hspace{1cm} (12)

The aperture function of a telescope depends on the dipole pattern as well as the antenna dimensions. For the purpose of this calculation, we assume $A(\vec{r}) = \exp(-\frac{\vec{r}^2}{\theta_0^2})$, where the parameter $\theta_0$ quantifies the field of view of the observation. The functions $S_L$ and $\tilde{a}$ depends weekly on $r_v$, where we assume $S_L(r_v, k_z, 2\pi, z_s, z_L) = S_L(r_\nu, k_z, 2\pi, z_s, z_L)$ and $a(\tilde{U} - r_v k_z/2\pi) = \tilde{a}(\tilde{U}' - r_\nu k_z/2\pi)$. For $k_z >> \frac{1}{r_\nu}$, the visibility correlation can be written as

$$V_{2L}(\vec{U}, \vec{U}', \nu, \Delta \nu) = C(z) |S_L(\vec{U}, z_s, z_L)|^2 \int dk_\parallel e^{-ik_\parallel \nu} \Delta \nu$$

$$\times \exp \left( -\frac{(\vec{U} - \vec{U}')^2}{(2\pi)^2} \right) \tilde{P}_{H\text{ I}}(\sqrt{k_\parallel^2 + \frac{2\vec{U}^2}{r_v^2}}) \right).$$  \hspace{1cm} (13)

where $C(z) = C'(z) \theta^2/2 \pi^2$.

Bharadwaj et al. (2001) considered the visibility correlation at nearby baselines to estimate the H I power spectrum from the observed visibilities. The nearby baseline correlation reduces the noise bias that appears by correlating the visibilities in the same baselines and introduces the factor $\exp \left( -\frac{(\vec{U} - \vec{U}')^2}{(2\pi)^2} \right)$. In this work, we do not consider the effect of measurement noise in the visibilities and correlate the visibilities at the same baselines. Furthermore, the visibility correlation is also done at the same frequencies. Hence eqn (15) simplifies to

$$V_{2L}(\vec{U}, \nu) = C(z)|S_L(\vec{U}, z_s, z_L)|^2 \int dk_\parallel \tilde{P}_{H\text{ I}}(\sqrt{k_\parallel^2 + \frac{2\vec{U}^2}{r_v^2}}).$$  \hspace{1cm} (14)

Clearly, the effect of gravitational lensing is a magnification of the visibility correlation by a factor of $|S_L|^2$.

3 SIMULATING LENSED 21-CM POWER SPECTRUM

In this section, we discuss the methodology to simulate lensed 21-cm power spectrum from the post reionization universe. We first model the 21-cm power spectrum from the expected dark matter distribution at post reionization redshifts, a scale-dependent $H\text{ I}$ bias and redshift space distortion effects. We use four simple models for the lensing potential based on strong lensing by galaxy clusters and calculate the corresponding lensing sampling functions.

\[\text{Figure 1. Expected visibility correlation with the GMRT as a function of baselines.}\]
the dark matter power spectra modified by a scale dependent bias. At wave-numbers of $> 0.4$ Mpc$^{-1}$ the redshift space distortion produced by peculiar velocity becomes important. At the highest $k \sim 10$ Mpc$^{-1}$ the H I power spectra probes the finger of god effects in individual over-density regions. Thus, strong lensing may provide a good probe of the scale dependent bias and the redshift space distortion effects and their evolution over redshift. Note that, for this choice of baseline ranges, for some redshifts, the visibility correlations plotted here requires extrapolation from the ranges of wave numbers used in Sarkar & Bharadwaj (2018).

### 3.2 Model for lensing potential

Strong lensing by galaxy clusters are studied very widely in literature (Richard et al., 2003; Limousin et al., 2008; Richard et al., 2010; Jauzac et al., 2015) where multiple images of lensed galaxies are used to reconstruct the gravitational potential of the lens. Many different approaches to model the projected lensing potential exists. Burkana (1998) use a Soften Power-law Elliptical Potential (SPEP) model to develop a computational algorithm for estimating the lensing potential. A semi analytical model of dark matter halo with different profiles like NFW (Li & Ostriker, 2002), soft and singular isothermal potential are discussed in Li & Ostriker (2002). The basic idea behind the reconstruction of the lensing potential in most of the methods is to start with a parametric model of the lensing potential and reconstruct the source structure given the lensed images (Killedar et al., 2012). The parameter of the model is then tuned, mostly using an MCMC variant, to find the best approximation to the lensing potential (Jullo et al., 2007; Birrer et al., 2015).

A typical cluster potential has several components, one or two large dark matter halos along with smaller components arising from the gravitational potential of the cluster galaxies. The larger halos, however, are the most important elements for the overall lensing (Elssadotter et al., 2007; Kassiola & Kovner, 1993) have used the Pseudo Isothermal Elliptical Mass Distribution (PIEMD) to model the individual cluster potentials. The PIEMD model is one of the widely used parametric models for the dark matter halo potentials of the galaxy clusters.

The projected potential for the PIEMD can be written as a function of the angular coordinates from the centre of the field of view of observations as

$$\psi(\theta_x, \theta_y) = 6\pi D_{ls} \theta_a \sigma_v^2 \int f(\theta) d\theta,$$

where $D_{ls}$ and $D_s$ are the distance between lens and source and the source and the observers respectively. The parameters $\theta_a$ and $\theta_0$ are the cut and the core radius, $\sigma_v$ gives the dark matter velocity dispersion to gravitationally support the halo. The angular variable $\theta$ depends on the angular coordinates $\theta_x$ and $\theta_y$ as

$$\theta^2 = \left[ \frac{\theta_x \cos \chi + \theta_y \sin \chi}{1 + \epsilon} \right]^2 + \left[ \frac{-\theta_x \sin \chi + \theta_y \cos \chi}{1 - \epsilon} \right]^2,$$

where $\epsilon$ is the ellipticity of the halo and $\chi$ gives the angle position of the ellipse. Each of the PIEMD component in the dark matter halo of the galaxy cluster is defined by seven parameters.

Various studies of strong lensing by galaxy clusters in optical wavelengths have estimated the lensing potentials. They show that a typical lensing potential has several PIEMD components and estimate the seven parameters of each of these components. In this work, we consider four simple models of the cluster potentials based on PIEMD parametrization. Though these choices of parameters do not reflect any particular observed galaxy cluster, the parameters are chosen to lie within the known ranges of the observed galaxy clusters (see for reference Smith et al. (2005); Johnson et al. (2014); Cerny (2018)). Here is a brief description of the four models we choose here:

- **M 1**: A single circular pseudo isothermal halo.
- **M 2**: A single pseudo isothermal elliptical halo.
- **M 3**: Two component PIEMD cluster halo with the same parameters except for the position of their centres.
- **M 4**: Two component PIEMD cluster halo with different parameters for the components.

The gravitational lens at a redshift $z_l$ only modifies the specific intensity from redshifts $z > z_l$. In this work, we consider all our lens models are at a redshift of 0.3 and $z_l = 1.0$. We first discuss the result with $z_l = 0.3$ in the next section.

### 3.3 Point spread function of the gravitational lens

Given a projected lensing potential $\psi(\theta_x, \theta_y)$, the matrix $A_{ij}(\theta_x, \theta_y)$ can be estimated at each angular positions as

$$A_{ij}(\theta_x, \theta_y) = \delta_{ij} - \frac{\partial^2 \psi}{\partial \theta_i \partial \theta_j},$$

where $(i, j)$ corresponds to the combinations $(x, x), (x, y), (y, x), (y, y)$ etc, $\delta_{ij} = 1$, if $(i, j) = (x, x)$ or $(y, y)$ and 0 otherwise. The lensing magnification function $\mu(\theta_x, \theta_y)$ can be calculated as the inverse modulus of the determinant of the matrix $A_{ij}$. Loutsenko (2018) show that the lensing magnification function can be approximated as the point spread function of the gravitational lens when the dimension of the lensing potential is much larger than the wavelength of the lensed radiation and interference effects can be neglected. Hence, we may write,

$$G_L(\theta_x, \theta_y) = \frac{1}{\det | A_{ij}(\theta_x, \theta_y) |}.$$

We generate a grid in the image plane to estimate the magnification function and hence the point spread function. We show the critical curve for a lens redshift of 0.3 and three different source redshifts for our four lens models in Figure 2. The critical curves for the circular model are circular. As the lensing models get complicated the critical curves also show interesting features. The magnification function of the strong gravitational lens is...
Figure 2. Critical curves generated by the four lensing models at the fiducial source redshifts of 1.25, 1.5 and 3.0.

Figure 3. Area in arcsec$^2$ with magnification over a certain threshold is plotted. Each panel shows for all four potential with the panels (a), (b) and (c) corresponding to source redshift of 1.25, 1.5 and 3.0 respectively. The lens redshift is 0.3.

Figure 4. Modulus square of the azimuthally averaged lensing sampling function is plotted against baselines. Each panel shows for all four potential with the panels (a), (b) and (c) corresponding to source redshift of 1.25, 1.5 and 3.0 respectively. The lens redshift is 0.3.
expected to be unity away from the critical curves. Near to the critical curve the magnification increases. An important aspect of the lensing models is the area in the image plane over which the magnification function has a value above a certain threshold \( S_{L} = 0.3 \). We plot this area in arc sec \(^2\) as a function of the magnification threshold in Figure 3. In each panel, we show results from four models with different panels corresponding to different source redshifts. Lens redshift is kept fixed at \( z_{L} = 0.3 \). Clearly, the single halo models have significantly lower magnifications compared to the double halo models. The area for a given model increases slowly with the redshifts. Within the two single and two double halo model we do not notice much difference in the effectiveness of the magnification reflected in these plots.

3.4 Results

We calculate the Fourier transform of \( G_{L} \) and estimate the modulus square of the azimuth averaged lensing sampling function \( | S_{L} |^{2} \). Each panel of Figure 3 shows the variation of \( | S_{L} |^{2} \) with baselines for all four lens models for \( z_{S} = 0.3 \). Note that the point spread function of the gravitational lens and hence the lensing sampling function depends on both the source and the lens redshifts. Three panels (a), (b) and (c) corresponds to the three different source redshifts. The sharp increase of the magnification function near to the critical curves results in oscillation in the sampling function and can be seen clearly in all models at larger baselines. The single halo models (M 1 and M 2) show the almost constant value of the \( | S_{L} |^{2} \) at baselines corresponding to the Einstein radius of each model and then a sharp decrease. At higher baselines, the single halo models have strong oscillatory features. The double halo models, in general, have higher values of \( | S_{L} |^{2} \) compared to the single halo models. This is expected from the Figure 3 where the same models show a significantly large portion of the area over a certain high magnification threshold. The oscillations in the double halo models are also subdominant because their corresponding magnification function has larger values at different places in the image planes.

We use the eqn (14) to estimate the lensed visibility correlation for our four lens models. The azimuthally averaged visibility correlation in the presence of the four lens models are shown in Figure 5 for the lenses at a redshift of 0.3. Different panels (a), (b) and (c) shows the results for three different redshifts 1.25, 2.5 and 3.0 of the source 21-cm emission. The one halo models show the characteristics oscillations as in Figure 3 and have in general lower amplitude than the two halo models. The lensed visibility correlation for the two halo models is over ~ 100 \( \mu \text{Jy}^{2} \) for all the baseline ranges plotted here. They show lesser oscillations as expected and explained earlier. A major challenge in detecting the redshifted \( \text{H} \text{I} \) emission is the presence of strong foreground. The foreground contribution comes from the diffused galactic synchrotron emission (DGSE) as well as the extragalactic sources. Jelic et al. 2008, Trot 2016, Ram Marthi et al. 2017. Though there have been techniques to model and subtract the effect of compact source foregrounds in literature (Choudhuri et al. 2017), mitigating the DGSE is an outstanding challenge towards the detection of the redshifted \( \text{H} \text{I} \) emission. An advantage of using the strong gravitational lensing to enhance the redshifted \( \text{H} \text{I} \) signal is that the enhancement happens only for the \( \text{H} \text{I} \) signal and not for the DGSE. The grey solid lines in the three panels in Figure 5 show the DGSE contributions at the frequencies corresponding to the redshifted \( \text{H} \text{I} \) signal (Trot 2016). We observe that with the two halo models, at least for a baseline range of investigation, the strong lensing enhances the \( \text{H} \text{I} \) signal significantly and is expected to help in foreground removal.

All the results presented so far are for the lens redshift of \( z_{L} = 0.3 \). To investigate the effect of lens redshift, we calculate the visibility correlations in the presence of strong lenses at a redshift of 1.0 in Figure 3. All the other parameters of the models including the lenses are kept same. Clearly, in this case, the enhancement of the visibility correlation due to strong lensing for all the model is rather less.

4 DISCUSSION AND CONCLUSION

In this work, we develop a formalism to estimate the power spectrum of the redshifted 21-cm specific intensity using strong gravitational lensing. Using the visibility correlation to measure the power spectrum, we show how the lensing sampling function enhances the visibility correlation signal. We choose four simple fiducial models of gravitational lenses to demonstrate the effect of strong lensing and the expected visibility correlation signal for lens redshifts of 0.3 and 1.0 and for signal redshifts of 1.25, 1.5 and 3.0. The fiducial lens models are chosen to represent the dark matter halo potential of galaxy clusters. We see that for the lenses at a redshift of 0.3 the visibility correlation signal is significantly magnified at the smaller baselines and hence the larger angular scales. At large baselines, the lensed visibility correlation for single halo models show a sharp drop in signal and become oscillatory in nature. Two double halo models, we use here show higher enhancement of the visibility correlation. Apparently, the required sensitivity for detecting the lensed \( \text{H} \text{I} \) signal is about a few tens of \( \mu \text{Jy} \) for visibilities (See Figure 5). The existing radio interferometers like the MeerKAT, the GMRT and future interferometers like the SKA mid can achieve this sensitivity with only a few tens of hours of observations at lower baselines to carry out studies using this formalism. We show here that the effect of strong lensing is a multiplication of the observed unlensed visibilities by the lensing sampling function \( S_{L} \). The sampling function can be seen as a modification of the baseline distribution of the telescope, where it down weights the measured visibilities at certain baselines and up weights some others. The uncertainty in the power spectrum estimates highly depend on the baseline distribution of a telescope. Hence, the sensitivity of a telescope and lens combination needs to be accessed together. We are presently working on a power spectrum estimator based on the basic idea presented in this paper and estimating the feasibility of using this method with the known strong lenses for observations with the GMRT, the MeerKAT and the SKA mid. For the lenses at a redshift of 1.0 the enhancement due to gravitational lensing is found to be less significant, limiting the effectiveness of this formalism for cluster lenses at higher redshifts.

A striking advantage of this approach is that the strong lensing by a lens at a redshift of \( z_{L} \) enhances the signal from the redshifted 21-cm emission at a redshift of \( z_{S} > z_{L} \), however, it does not enhance the diffused emission from the Galaxy, which acts as a foreground to the 21-cm signal. The major challenge with the foreground signals like DGSE is that they are inherently several orders of magnitude larger (Di Matteo et al. 2002, Oh & Mack 2003). Jelic et al. 2008, Ali et al. 2008, Ghosh et al. 2018, Paciga et al. 2013 than the redshifted 21-cm signal and hence effects like residual foregrounds after the subtraction, residual gain errors add to bias in the estimators of the 21-cm power spectra (Datta et al. 2010, Kumar et al. 2020). We observe that for the best cases discussed in this work, that is for the two double halo lens models at
a redshift of 0.3, the enhanced visibility correlation signal is still at only a few percent of the expected galactic foreground. The lensing effect, however, enhances the H\textsc{i} signal selectively and significantly. Hence, even though the lensed signal is less than the DGSE, the enhancement is still quite significant and would be quite important in subtraction of the continuum of DGSE signal.

Note that, the formalism developed here assumes that the gravitational potential of the lens galaxy clusters is already known. Gravitational lensing is well studied with optical images and several techniques have been developed over the years to estimate the lensing potential. At present, there exist several galaxy clusters with a model for lensing potentials (see for example Richard et al. (2014), Cerny (2018) etc.). In ongoing work, we are engaged in to survey all the existing cluster lens models in literature and find the best candidates to implement the formalism developed in this work. We note that the existing lensing models often have large uncertainties with larger uncertainties for the regions with high magnification (Richard et al., 2014). This may limit the effectiveness of the present formalism. We expect with the advent of better algorithms to estimate the lensing potential the uncertainties in the models will decrease and the lensing models would be accurate enough to be used.

The radio continuum signal from the galaxy clusters is expected to add to the lensed redshifted 21-cm radiation as an extra foreground element. At present, we have not estimated the contribution from the cluster continuum. Since the continuum signal from the cluster is not expected to vary over a small range of frequencies, we expect that a continuum subtraction based method can be used.
to mitigate the cluster continuum. We plan to investigate this in future work.

In summary, this work introduces a new visibility based method to estimate the redshifted 21-cm power spectrum. Based on the initial calculations with model cluster halo potentials, the method itself looks promising. However, in order to implement this method, we need to design an estimator of the power spectrum based on the lensed visibility correlation and access its bias and variance. It is to be noted that the variance of the estimator partly depends on the number of independent samples one uses for one estimate. Since the lensing sampling function enhances the visibility correlation at only limited regions in the visibility plane, the method described here by itself may be limited by sample variance. Furthermore, the sample variance depends on the actual lens model and the angular scale or baseline in concern. We are investigating the practical implementation of the lensed visibility correlation method and assessments of the reliability of the estimates combining the best known strong lens candidates from optical studies. This results will be presented in detail in future work.

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DATA AVAILABILITY

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