Strangeness counting in high energy collisions

F. Becattini

Università di Firenze and INFN Sezione di Firenze
Largo E. Fermi 2, I-50125 Firenze (Italy)
e-mail: becattini@fi.infn.it

Abstract

The estimates of overall strange quark production in high energy $e^+e^-$, pp and $p\bar{p}$ collisions by using the statistical-thermal model of hadronisation are presented and compared with previous works. The parametrization of strangeness suppression within the model is discussed. Interesting regularities emerge in the strange/non-strange produced quark ratio which turns out to be fairly constant in elementary collisions while it is twice as large in SPS heavy ion collision.

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1 Introduction

The interest for strangeness production in high energy collisions has been strengthened by the possibility of its use as a probe of the Quark Gluon Plasma formation [1]. An enhanced strange particle production in heavy ion collisions with respect to nucleon-ion collisions have actually been observed [2, 3]. In order to understand the relevance of such observations it is very important to study strangeness production in high energy elementary collisions like $e^+e^-$ or pp, as they provide a natural baseline to gauge the results in more complex systems.

The comparison between strange and non-strange particle production in whatever kind of collision can hardly be accomplished by using only experimental data without any input from models. In fact, not all particle species yields are practically measurable; moreover, even if an experiment was able to measure all stable (disregarding weak decays) hadrons species, strong decays following the primary hadron production modify the number of $u$, $d$, and (to a less extent) $s$ valence quarks, thereby affecting the quantitative estimation of their actual production in the hadronisation process. The direct experimental measurement of strange to non-strange quark production therefore would require the determination of the inclusive yields of all hadron species, an exceedingly difficult experimental task. As a consequence, a model of hadronisation is necessary to estimate the production of a large number of unmeasured hadrons and resonances. Of course a prerequisite of any estimate is the agreement between the model and the existing data with a fairly small number of free parameters in order not to spoil its predictive power.

In this paper I will discuss strangeness production by using a statistical-thermal model of hadronisation which turned out to be in good agreement with high energy collision data by using only three free parameters [4, 5].

2 Statistical model and strangeness suppression

The statistical-thermodynamical model of hadronisation has been described and discussed in detail elsewhere [5, 6]. The main idea of the model is the existence of a set of pre-hadronic clusters (or fireballs) converting into hadrons according to the equiprobability of multi-hadronic phase space states (Gibbs postulate), where phase space is locally defined by the mass and the rest frame volume of each cluster. This assumption entails the possibility of expressing overall hadron multiplicities (i.e. measured in full phase space) by means of thermodynamical formulae within a canonical framework, that is with exact quantum number conservation.

The model, whose basic parameters are the temperature $T$ of the clusters/fireballs and the sum $V$ of their proper volumes, is supplemented with an extra strangeness suppression factor, which is formally beyond a pure statistical hadronic phase space model. This factor $\gamma_S$ is introduced in the partition function used to derive hadron multiplicities, as a fugacity related to the valence strange quark content of the hadron [7]. The canonical partition function supplemented with $\gamma_S$ reads [5]:

\[
Z(Q^0) = \frac{1}{(2\pi)^5} \int_0^{2\pi} d^5 \phi \ e^{iQ^0 \cdot \phi} \exp \left[ V \sum_j \frac{(2J_j + 1)}{(2\pi)^3} \int d^3 p \ \log \left( 1 \pm \gamma_S^{s_j} e^{-\sqrt{p^2 + m_j^2/T - i p_j \bar{q}_j} \phi} \right) \right].
\]

(1)

where $Q^0 = (Q, N, S, C, B)$ is a five-dimensional vector containing the initial electric charge $Q$, baryon number $N$, strangeness $S$, charm $C$ and beauty $B$; $J_j$ and $m_j$ are the spin and
the mass of the $j$th hadron species; $q_j$ is its quantum number vector $(Q_j, N_j, S_j, C_j, B_j)$ and $s_j$ its number of valence strange+antistrange quarks. The hadron average multiplicities resulting from eq. (1), in the Boltzmannian limit, reads:

$$< n_j > = \gamma_s S_j (2J_j + 1) \frac{VT}{2\pi^2} m_j^3 K_2\left(\frac{m_j}{T}\right) \frac{Z(Q^0 - q_j)}{Z(Q^0)} \delta^{\sum_j n_j \mathbf{q}_j, \mathbf{Q}^0},$$

(2)

where the ratio $Z(Q^0 - q_j)/Z(Q^0)$ is defined as chemical factor [5] and, unlike an ordinary fugacity, is not an intensive quantity for it depends on the system size.

This way of regarding $\gamma_s$ is in fact a grand-canonical one, as fugacities can be defined only in a grand-canonical framework. On the other hand, it is possible to define $\gamma_s$ in a more general way which is independent of the adopted statistical formalism, i.e. microcanonical, canonical or grand-canonical.

Indeed, for a certain multihadronic state $\{n_1, \ldots, n_K\}$ ($n_i$ is the number of hadrons belonging to species $1$, ..., $n_K$ is the number of hadrons belonging to species $K$) originating from the collision, its phase-space probability is multiplied by a factor $\gamma_s$ powered to the number of strange+antistrange quarks whose creation out of the vacuum is needed in order to set up that state. According to this definition, in a canonical framework, the probability $P$ of realizing a multihadronic state $\{n_1, \ldots, n_K\}$ in a collision whose initial state does not have any strange quarks is:

$$P \propto \exp\left(-\frac{E}{T}\right) \gamma_s^{\sum^N_{i=1} n_i s_i} \delta^{\sum_j n_j \mathbf{q}_j, \mathbf{Q}^0},$$

(3)

where $s_j$ is the number of valence strange+antistrange quarks contained in the $j$th hadron. This probability can be worked out to calculate the canonical partition function leading to the same expression as in eq. (1). The above definition can be easily extendend to a microcanonical or a grand-canonical framework. Furthermore, it should be emphasized that this definition is more general and more appropriate for collisions with initial strange quarks (for instance K p) in which the use of the same partition function (1) obtained for colliding systems devoid of valence strange quarks would lead to odd results. In this special case, the probability (3) becomes:

$$P \propto \exp\left(-\frac{E}{T}\right) \gamma_s^{\sum^N_{i=1} n_i s_i - |S|} \delta^{\sum_j n_j \mathbf{q}_j, \mathbf{Q}^0},$$

(4)

where $|S|$ is the initial absolute strangeness, so that the canonical partition function will differ by a factor $1/\gamma_s^{\delta S}$ from the (1).

### 3 Results on strangeness production

The free parameters $(T, V$ and $\gamma_s$) of the statistical-thermal model are determined by a fit to the measured hadron multiplicities (or multiplicity ratios) by taking into account the decay chain following the primary production. The quality of the fits is good [5] and the multiplicity of up to 25-30 particles can be well reproduced. For heavy ion collision [8] one more free parameter is necessary (the baryon-chemical potential) as the number of participant nucleons, i.e. the total baryon number, is a measured quantity unlike in $e^+e^-$, pp and p$\bar{p}$ where it is known a priori. The extracted $\gamma_s$ values in elementary collisions at various centre of mass energy points are shown in fig. 1 along with those for heavy ion collisions at SPS energies [8]; also shown the result of a fit to AGS Si+Au hadronic ratios quoted in ref. [9] (see table 1). All $\gamma_s$’s turn
Table 1: Results of thermal model fit to particle ratios at AGS Si+Au collisions. The number of participant nucleons has been set to 108 and electric charge to 45 according to the estimation of ref. [9]. The fitted parameters are $T$, $VT^3 \exp[-0.7\text{GeV}/T]$, $\gamma_S$ and $\mu_B/T$ whilst $\lambda_S$ is derived from the fitted parameters and their errors. The left column is the set of parameters obtained by setting the error on the number of participants to 10, the right column to 0.1 and keeping the same total electric charge. Besides experimental errors, also the uncertainties on masses, widths and branching ratios of hadrons have been taken into account according to the procedure described in ref. [8]. The results are consistent with those in refs. [9, 10].

|                | Fit 1          | Fit 2          |
|----------------|---------------|---------------|
| $T$ (MeV)      | 118.4±11.6    | 132.8±11.3    |
| $VT^3 \exp[-0.7\text{GeV}/T]$ | 0.73±0.25 | 1.04±0.25 |
| $\gamma_S$     | 0.923±0.16    | 0.792±0.12    |
| $\mu_B/T$      | 4.41±0.49     | 3.89±0.34     |
| $\chi^2/\text{dof}$ | 21.1/6   | 22.5/6      |
| $V$ (fm$^3$)   | $\approx$ 1250 | $\approx$ 664 |
| $\lambda_S$    | 0.488±0.093   | 0.569±0.10    |

Out to be definitely less than 1 except the AGS point which is consistent with 1, in agreement with the results of refs. [9, 10], though with a large uncertainty. This finding indicates that a strangeness chemical equilibrium at hadron level is not attained in most of the examined collisions. However, no apparent regularity emerges from the plot, as $\gamma_S$ has different values in $e^+e^-$ and heavy ion collisions with respect to pp and p$\bar{p}$ collisions. The situation drastically changes when computing the ratio $\lambda_S$:

$$\lambda_S = \frac{\langle s\bar{s} \rangle}{0.5(\langle u\bar{u} \rangle + \langle d\bar{d} \rangle)} ,$$

between newly produced valence $s\bar{s}$ pairs and half the sum of newly produced valence $u\bar{u}$ and $d\bar{d}$ pairs. As shown in fig. 2, $\lambda_S$ has a pretty constant value of around 0.2 for all examined $e^+e^-$, pp and p$\bar{p}$ collisions whereas it is twice as high in heavy ion collision both at SPS and AGS energies; the central $\lambda_S$ value at AGS is indeed larger than at SPS but the fit error is so large that a conclusion is not allowed. It should be emphasized that $\lambda_S$ is a strangeness suppression parameter related to the *quark content* of hadrons whereas $\gamma_S$ is related to the strange hadrons phase space. The computation of $\lambda_S$ proceeds as follows: firstly, the best fitted primary average multiplicities $< n_j >$ for all hadron species are used to count the total number of quarks $< Q_i >$ of a given flavour $i$:

$$< Q_i > = \sum_j < n_j > (T, V, \gamma_S) q_{ij}$$

where $q_{ij}$ is the number of valence quarks of flavour $i$ contained in the $j^{th}$ hadron. Secondly, the initial colliding quarks are subtracted so to count only the primarily newly produced ones. The errors on $\lambda_S$ are estimated by propagating the fit errors on $T$, $V$ and $\gamma_S$ onto the primary multiplicities $< n_j >$. Needless to say, the reliability of the estimate where only few hadron species were measured relies upon the agreement between model and data where a large number of measurements are available.

The reason for the different behaviour of $\gamma_S$ and $\lambda_S$ in elementary collisions is twofold. On one
hand, a hadron gas with fixed $\gamma_S$, $T$ and $V$ has an increasing value of $\lambda_S$ for an increasing total baryon number (see fig. 3); hence, if $\lambda_S$ is constant, $\gamma_S$ must be lower in pp collisions (baryon number 2) than in $e^+e^−$ and pP, provided that temperatures and volumes have similar values as it actually is [3]. The physical reason of the increase of $\lambda_S$ with $B$ in a hadron gas is the lower energy threshold for strange pair production in a baryon-rich environment where $\Lambda + K$ production is favoured in comparison with kaon pair production in a baryon-free environment. On the other hand, if $\lambda_S$ is constant, a decrease of $\gamma_S$ with increasing $V$ is expected, due to the so-called canonical strangeness enhancement [11] implying an increase of the strange hadronchemical factor (see eq. (2)) in increasingly large systems (see fig. 4). This effect accounts for the $\gamma_S$ difference between $e^+e^−$ and pP collisions as the multiplicities and the volumes of the latter are somewhat higher for the considered centre of mass energy points. Other smaller contributions arise from the anticorrelation between $\gamma_S$ and $T$ (outcoming temperatures in $e^+e^−$ are slightly lower in $e^+e^−$ than in pP) and from the relatively more abundant production of heavy flavours in $e^+e^−$-collisions which slightly affect the fit of $\gamma_S$.

If $\lambda_S$ is more fundamental than $\gamma_S$, the best way of parametrising strangeness suppression would be to fix the mean absolute value of strangeness (with possible superimposed fluctuations) instead of using $\gamma_S$. This would amount to extend the conservation laws from 5 ($Q, N, S, C, B$) to 6 ($Q, N, S, C, B, |S|$) in the calculation of canonical partition functions in eq. (1), where $|S|$ is now meant to be the overall number of strange quarks including both initial and newly produced ones. An interesting question, under current investigation, is why $\gamma_S$ has worked well anyhow in reproducing the strangeness suppression. This is not surprising for large systems because the demand of exact $|S|$ leads to a fugacity, namely $\gamma_S$ (see for instance ref. [13]), but it is a relevant question for elementary collisions where this is not possible; perhaps the replacement of $\gamma_S$ with $\lambda_S$ might lead to a further improvement of the fits.

The $\lambda_S$ value estimated by the statistical-thermal model of hadronisation in elementary collisions is in agreement with a previous estimate quoted in ref. [14] only for centre of mass energies $\sqrt{s} < 100$ GeV, whereas its rise in pP collisions at $\sqrt{s} > 100$ GeV claimed there is not observed. The reason of this discrepancy is the fact that $\lambda_S$ was estimated in ref. [14] by using only $K/\pi$ ratio as experimental input and two parametrizations of hadron multiplicities [13] which, unlike ours [3], do not satisfactorily reproduce all available measured multiplicities in pP collisions. As an example, for pP collisions at $\sqrt{s} = 546$ GeV, the parametrization in ref. [13] predicts a $\Lambda/K_0$ ratio of 0.49, by taking the $\lambda_S = 0.28$ value quoted in ref. [14], whereas the experimental value is 0.24±0.05 [17].

The enhancement of $\lambda_S$ in heavy ion collision at SPS energies is indeed very interesting. An explanation of this effect by advocating the formation of a fully equilibrated hadron gas is not viable, as $\gamma_S = 1$ seems to be ruled out from recent analyses of the presently available data [8][13][18]. On the other hand, it is not possible either to explain the rise of $\lambda_S$ by invoking a canonical strangeness enhancement from pp to heavy ion collisions at fixed $\gamma_S$ just because a fixed $\gamma_S$ would not be able to account for the production of $\phi$ meson in pp collisions [12].

A fully equilibrated hadron gas seems to be able to account for AGS data at $\sqrt{s} = 5.4$ GeV, in agreement with the conclusions of refs. [3][10], though the outcome error on $\gamma_S$ is quite large. However, two caveats for AGS fit should be mentioned; firstly, the ratios used in the fit have been measured in limited rapidity intervals whereas the model calculations involve full phase space multiplicities. Secondly, the fit has been performed within a pure grand-canonical framework, while a canonical treatment should bring about a slight $\gamma_S$ rise because a strangeness suppression due to the finite volume is involved. However, for the large extracted $V$ values (see table 1), this is expected to be a minor correction.
4 Conclusions

A statistical-thermal model of hadronisation supplemented with a non-equilibrium strangeness suppression parameter $\gamma_S$ has been used to study primary strangeness production in elementary and heavy ion collisions at high centre of mass energy. The reliability of the model is based on its success in reproducing many particle multiplicities at several centre of mass energies, with only three free parameters. It is found that the ratio of newly produced strange to non strange quark $\lambda_S$ (see eq. (5)) is fairly constant for all elementary collisions. This constancy emerges as an universal feature of hadronisation along with the constancy of temperature [5]. Thus, while $u$ and $d$ quarks are produced in the same amount, strange quarks are produced in a fixed ratio with $u$, $d$, which is most likely related to their higher mass. All quark pairs generated before or during the hadronisation process eventually fill hadronic phase space according to a pure statistical law. Hence, the fact that $\gamma_S < 1$ in a modified hadron gas framework, simply reflects the underlying more fundamental $\lambda_S \simeq 0.2$ parameter. In order to emphasize the role of the hadronisation model in extracting the $\lambda_S$ regularity, it should be noticed that pure experimental observables such as the $K^+/\pi^+$ ratio does not show up such constancy (see fig. 5). In fact, kaon yield is enhanced in $e^+e^-$ collisions with respect to the examined $pp$ and $p\bar{p}$ collisions because of the contribution of heavy flavoured hadron decays.

In heavy ion collisions at SPS energies, $\lambda_S$ turns out to be twice as large. This enhancement cannot be accounted for a hadron gas explanation and it is a subject of reflection. There have been some attempts to explain it in terms of Quark-Gluon Plasma formation [12][13].

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Figure captions

Figure 1  Fitted $\gamma_S$ in $e^+e^-$, pp, p$\bar{p}$ and heavy ion collisions as a function of centre of mass energy (nucleon-nucleon centre of mass energy for heavy ions).

Figure 2  Estimated $\lambda_S$ in $e^+e^-$, pp, p$\bar{p}$ and heavy ion collisions as a function of centre of mass energy (nucleon-nucleon centre of mass energy for heavy ions). The two different estimates for p$\bar{p}$ collisions correspond to the two limiting cases of annihilation (lower) or survival (higher) of the 6 colliding valence quarks.

Figure 3  The $\lambda_S$ parameter in a hadron gas at fixed $T$, $V$ and $\gamma_S$ as a function of the number of initial protons (baryon number equal to electric charge).

Figure 4  Chemical factor $Z(0,0,1,0,0)/Z(0,0,0,0,0)$ of a neutral strange hadron (e.g. $K^0$) in a completely neutral hadron gas as a function of the volume for $\gamma_S = 0.5$ and three different values of temperature. The chemical factor approaches its grand-canonical limit, i.e. 1, for large volumes.

Figure 5  $K^+/\pi^+$ ratio in elementary collisions as a function of centre of mass energy.
Figure 1: 

Graph showing the dependence of $\gamma_s$ on $\sqrt{s}$ (GeV) for different collision types:
- $pp$ collisions (circles)
- $\bar{p}p$ collisions (triangles)
- $e^+e^-$ collisions (squares)
- A+B collisions (dots)
Figure 2:
Figure 3:

T=170 MeV
V=20 fm³
γ_S=0.5
Figure 4:
Figure 5: