Analysis of angular stability of the synchronous machine by biparametric bifurcations

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Abstract. In this paper we present the qualitative solution techniques of dynamic systems applied to the second order model of the synchronous machine (swing equation), with the aim of achieving a deeper analysis about the stability of the electric power system. Initially, the dynamics of the electrical power system are briefly described, where the elements that interact in this and the stability classes to be analyzed are explained. Next, the mathematical model of the synchronous machine is formulated in detail from Newton's second law, in order to specify the theory of dynamic systems and the qualitative solutions that are applied to the study model. Finally, the results of the implementation of the qualitative solution techniques of the dynamic study system are presented, concluding with the stability zones of the synchronous machine.

1. Introduction

These guidelines, the power electrical system (PES) can be defined as the interconnection between sources of generation and loads through transmission lines, its main objective is to ensure the adequate delivery of electrical energy. Despite the simplicity with which this task is named, it is certainly one of the most complex tasks that exist in the field of engineering because it encompasses a large amount of manpower and abundant studies about its operation. Among the many areas of study of the PES is the study of stability, which starts from the need to keep the operation in parallel of all the generators, in such a way that the energy produced is balanced with that consumed by the loads plus the losses of the system, however, in the PES occur unusual events such as load connections and system failures that can affect the stability by altering the synchrony of the machines and producing some other modifications in the generators [1].

Due to the aforementioned it is necessary to go to the origin of the energy production where it is concluded that the responsibility of maintaining stability is of the synchronous generator, for which the stability analysis will focus entirely on the dynamics of its operation, which will be expressed through a mathematical model indicated in differential equations, explicitly; a dynamic system of second order. For the analysis of the synchronous generator model, the theory of dynamic systems poses different forms of solution, from analytical techniques, which give as a solution a mathematical expression that in most cases can be too complex to find and in some cases impossible, adding to these drawbacks that the mathematical expression found does not provide enough details of the behavior of the dynamic system. Another solution technique is the qualitative analysis, here it is intended to observe the behavior of the system in more detail and with respect to any initial condition you may have, because the behavior of dynamic systems has a great dependence on intrinsic parameters is simple observe the regions of stability of the system and the subsequent change of these [2].
Thus, it can be seen that the analysis of the synchronous generator model is carried out by qualitative techniques, identifying the points of equilibrium of the system and observing its stability dynamics before the variation of the characteristic parameters of the model, giving a wide margin of knowledge of the synchronous generator stability.

2. Stability of the electric power system
The operation of the PES includes the interaction of three subsystems: sources of active power and reactive power, loads or consumptions connected at the different points, connection network between generation systems and consumption centers. When disturbances occur in the operation, such as unplanned disconnection of generating units or transmission circuits, the balance of generation and consumption is lost. The concept of stability reflects fitness that after a disturbance has occurred the system recovers to a new equilibrium position. In general terms, stability can be classified into angular stability and tension stability. As the PES is characteristically non-linear, its stability depends on the initial conditions and the size of its perturbations, then the types of stability can be distinguished in other types according to their temporality [3].

Studying the stability of the PES becomes too extensive a task in which discoveries are currently being made, so it is necessary to focus on a single category to obtain specific results. In this work, the angular stability of the synchronous generator is studied. The angular stability is related to the aptitude of an electrical power system so that the synchronous machines maintain their synchronism in the absence of disturbances and recover to an equilibrium position after being subjected to any disturbance. The concept of synchronism means that the angular difference between the electrical axes of the rotors of the synchronous machines tends to a constant value, after the disturbance is eliminated, as well as that the frequency reaches a constant value after a damped oscillatory period [4,5].

3. Synchronous machine
The synchronous generator consists of a rotor which is driven by a turbine with multiple stages through a common transmission shaft, the latter can be modelled by a series of rotational masses, to represent the inertia of each stage of the turbine, connected by means of springs to represent the torsional rigidity of the transmission shaft and the coupling between the stages (Figure 1) [6,7].

![Figure 1](image1.png)

**Figure 1.** Physical modelling of the axis of the synchronous machine.

When the free-body rotation is considered (Figure 2), it can be assumed that the axis is rigid when the total inertia of the rotor \( J \) is simply the sum of the individual inertias. Any unbalanced torque acting on the rotor will result in the acceleration or deceleration of the rotor as a complete unit according to Newton's second law (Equation (1)).

\[
J \frac{d\omega_m}{dt} + \tau_d \omega_m = \tau_t - \tau_e,
\]

where \( J \) is the total moment of inertia of the turbine and generator rotor (kg m\(^2\)), \( \omega_m \) is the speed of the rotor axis (rad/s), \( \tau_t \) is the torque produced by the turbine (N m), \( \tau_e \) is the electromagnetic torque and \( \tau_d \) is the torque damping coefficient and is the sum of the rotational mechanical loss due to winding and friction [8,9].
In steady state the angular velocity of the rotor is the synchronous speed \( (\omega_{sm}) \) while the torque of the turbine is equal to the sum of the electromagnetic torque \( (\tau_e) \) and the damping torque \( (\tau_d \omega_{sm}) \), where in addition the net touch of the axis \( (\tau_m) \) is obtained by Equation (2),

\[
\tau_t = \tau_e + \tau_d \omega_{sm}, \quad \tau_m = \tau_t - \tau_d \omega_{sm} \tag{2}
\]

\[
\omega_m = \omega_{sm} + \Delta \omega_m = \omega_{sm} + \frac{d\delta_m}{dt} \tag{3}
\]

The position of the rotor with respect to the reference axis of synchronous rotation is defined by the power angle \( (\delta) \), the speed can be expressed in Equation (3), where \( d\delta_m/dt = \Delta \omega_m \), is the deviation of the speed in mechanical radians per second. Finally, substituting and multiplying by the synchronous speed \( (\omega_{sm}) \), Equation (4).

\[
J \omega_{sm} \frac{d^2 \delta_m}{dt^2} + \omega_{sm} \tau_d \frac{d\delta_m}{dt} = \omega_{sm} \tau_m - \omega_{sm} \tau_e \tag{4}
\]

\[
J \omega_{sm} \frac{d^2 \delta_m}{dt^2} + \omega_{sm} \tau_d \frac{d\delta_m}{dt} = \frac{\omega_{sm}}{\omega_m} P_m - \frac{\omega_{sm}}{\omega_m} P_e \tag{5}
\]

Equation (5) shows power expressed which is the product of the speed and torque of the axis, \( P_m \) is the net power of the mechanical axis and \( P_e \) is the electrical power in the air gap, later this expression is transformed into Equation (6), because during a disturbance the velocity of the synchronous machine is very close to the synchronous speed.

\[
\omega_m \approx \omega_{sm} 2H \frac{d^2 \delta_m}{dt^2} = P_m - P_e - \tau_d \frac{d\delta_m}{dt}, \quad H = \frac{1}{2} J \omega_{sm} \tag{6}
\]

The model proposed in Equation (6) is expressed as a system of second order, Equation (7).

\[
2H \frac{d \Delta \omega_m}{dt} = P_m - P_e - T_m \Delta \omega \quad \frac{d \Delta \omega_m}{dt} = P_m - \frac{E_G E_M \sin(\delta)}{2H x_T} \quad \frac{T_m}{2H} \Delta \omega \tag{7}
\]

3.1. Bi-parametric bifurcation

In most systems the parameters are redefined and condense into one, however, most of these analyses only reveal the behavior of the system conditioned to a single value, thus preventing a deeper vision. In order to observe more characteristics of the behavior of the system, we resort to the use of two parameters, this practice also allows us to visualize additional bifurcations that are not normally observed in a mono-parametric model [10,11].

4. Results

4.1. Dimensionless model of the synchronous machine

In order to start the analysis of the dynamic system (Equation (6)), it is necessary to study its dimensionless form, describing the parameters in Equation (8).

\[
\Delta \omega = x_{\delta}, \quad \delta = y, \quad \frac{p_m}{2H} = P, \quad \frac{T_m}{2H} = T, \quad \frac{E_G E_M}{2H x_T} = 1, \tag{8}
\]

where \( E_G \) and \( E_M \) are the voltages at the output of the synchronous machine and the load respectively, \( x_T \) is the reactance of the line, the previous parameters together with \( 2H \) are inherent in the construction.
of the machine and are taken to values in p.u. In order to simplify the analysis, we finally get the following system (Equation (9)):

\[
\begin{align*}
\dot{x} &= P - \sin(y) - Tx \\
\dot{y} &= x
\end{align*}
\]  

(9)

4.2. Fixed points

As can be seen in Figure 3, Equation (10) and Equation (11), only two fixed points will be obtained which are periodic each, later the analysis will be made only in the marked points and by extension the nature of the other periodic points to obtain will obey to the analysis of the points initially studied the system of (Equation (9)). Solving this system of equations, we get Equation (10) and Equation (11).

\[
\begin{align*}
\bar{x}_1^* \to (x^*, y^*)_1 &= (0, \sin^{-1}(P) + 2k\pi) \\
\bar{x}_2^* \to (x^*, y^*)_2 &= (0, \sin^{-1}(P) - (2k + 1)\pi), k \in \mathbb{Z}
\end{align*}
\]  

(10) (11)

4.3. Linearization of the system

From the two periodic fixed points obtained previously in the Equation (10) and Equation (11), the respective Jacobian matrices are obtained in Equation (12).

\[
\begin{align*}
J &= \begin{bmatrix} -T & -\cos(y) \\ 1 & 0 \end{bmatrix}, J_1 = J|_{x^*} = \begin{bmatrix} -T & \sqrt{1 - P^2} \\ 1 & 0 \end{bmatrix}, J_2 = \begin{bmatrix} -T & -\sqrt{1 - P^2} \\ 1 & 0 \end{bmatrix}
\end{align*}
\]  

(12)

4.4. Characteristic polynomials and stability

From the characteristic polynomials obtained and from the stability analysis, the fixed point 1 will be stable in nature as long as the value of the parameter $T$ is positive, while the fixed point 2 will be a saddle point because the value of the determinant will always be negative. From this analysis, it is formulated in Figure 4, which shows the stability of fixed point 1 [12].

The blue and red color of Figure 4 shows the values for which the fixed point 1 is stable and unstable respectively, in green color ($P = \pm 1$) the fixed points 1 and 2 are located in the same position and its stability is critical, so here we have a bifurcation, similarly in the yellow region the fixed point 1 will be a center, from a variation of the parameter we have a change in stability, so this strip reacts another bifurcation. When obtaining the trace and the detective of the Jacobian matrices, it is obtained Equation (13).

\[
\begin{align*}
J(\bar{x}_1^*) \to P_1: \lambda^2 + T\lambda + \sqrt{1 - P^2}, \text{Tr} = -T, \text{Det} = \sqrt{1 - P^2} \\
J(\bar{x}_2^*) \to P_2: \lambda^2 + T\lambda - \sqrt{1 - P^2}, \text{Tr} = -T, \text{Det} = -\sqrt{1 - P^2}
\end{align*}
\]  

(13)
5. Conclusion
The qualitative techniques of solution of dynamic systems are very useful in the analysis of physical systems, for the present case it was possible to analyze the model of the synchronous machine determining in depth the stability regions with respect to the variation of the parameters: mechanical power and shock absorber torque. Due to the dynamics of the rotary axis of the machine two periodic fixed points are produced that converge when the mechanical power is equal to the maximum magnitude of the electrical power, independently of the value of the shock absorber, this event occurs in the cases in which the power needed to feed the loads connected to the network exceeds a certain threshold and by means of control actions the mechanical energy supplied either by water or thermal means is greater than the nominal electric power generation, in these cases imbalances occur that can severely affect the physical structure of the machine. On the other hand, when you do not have a shock absorber (electrical power is equal to mechanical power) the machine goes into critical oscillation, an event that is only detrimental to large instants of time, for most conditions imbalance can be implemented control actions. Known, the final deduction is that the shock absorber torque must provide a loss of speed in the machine to maintain a fixed attractor point, that is, have a stable reference.

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