A reference level of the Universe vacuum energy density and the astrophysical data

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(Dated: December 24, 2019)

An extended framework of gravity, in which the first Friedmann equation is satisfied up to some constant due to violation of gauge invariance, is tested against astrophysical data: Supernovae Type-Ia, Cosmic Chronometers, and Gamma-ray bursts. A generalized expression for the Friedmann equation including the possible vacuum contributions is suggested and two particular cosmological models with two independent parameters each are considered within this framework, compared on the basis of likelihood analysis. One of the models considered includes contribution of the residual vacuum fluctuations to the energy density and places limit on the UV cutoff scale as $k_{\text{max}} = 12.43^{+0.9}_{-1.6}[M_p/\sqrt{2} + N_{\text{sc}}]$, where $N_{\text{sc}}$ is the number of minimally coupled scalar fields. Model comparison performed using Akaike information criteria shows a preference for the conventional $\Lambda$CDM, over the extended models. A more general model with three parameters is considered within which an anti-correlated behavior between the dynamical vacuum fluctuations contribution and a negative cosmological constant was observed, which results in an upper limit at 95% C.L. of $\Omega_\Lambda \lesssim -0.14$.

I. INTRODUCTION

It is well-known that the reference level of energy density in the Minkowski space-time could be chosen arbitrarily as it is not immediately related to the observable data [1]. Thus, there is no immediate problem with the appearance of large values of the mean energy density of a vacuum state. Formally, the vacuum energy density could be nullified by some renormalization procedure [2], such as the Pauli-Villars [3], dimensional [4, 5], or point-splitting regularizations [6]. This situation is more acute in the general relativity (GR) because, though the space-time in GR looks locally like the Minkowski one, the uniform energy density and pressure have to result in the expansion of the universe. The vacuum energy density, calculated with the UV cut-off at a Planck level, would lead to a very high expansion rate of the universe in contradiction to the astrophysical observations [7].

Several approaches were proposed to avoid this issue, among which the first is to assume that a large value of vacuum energy does not exist in reality, implying that the renormalization procedure is not a “technical trick” but has physical meaning. However, investigations of numerous quantum mechanical systems, such as two-atom molecules [8] or atomic nuclei [9] insist for a reality of the ground state energy of the quantum oscillator. In such a scenario, the field oscillators are not principally different from the description above. Besides, an expectation of the “good” quantum theory suggests absence of infinities, as in the case of string theory [10]. Let us imagine that the (super)string theory [11] will be able to produce masses of all particles and all the fundamental interactions after successful compactification from 10 to 4 dimensions. In which case, no renormalization will be finally needed, due to the extended nature of a string. From the point of view of an ordinary field theory, it will be seen as a number of “sum rules” that have to be satisfied [12].

This leads to an alternate possibility: which is to assume that the vacuum energy density indeed exists, but the contributions from different fields compensate each other mutually. An example of the same is the exact supersymmetry in which the number of bosonic and fermionic degrees of freedom are equal, and additionally masses of particles and their super-partners are also equal. At present, no supersymmetric particles have been found [13], nevertheless, an idea of mutual compensation is not directly related or specific to the supersymmetry and was first suggested far before supersymmetry was discussed in [14]. If the numbers of fermionic and bosonic degrees of freedom are different, then there is no compensation of the main part vacuum energy density $\rho_v \sim M_p^4$, and one needs an alternate explanation for the
fact that the main part of the vacuum energy does not contribute to gravity [15–17].

Requiring a nongravitating vacuum energy, together with the fact that there is no invariant vacuum state, one may infer hints for a modification of the theory of gravity. A general transformation of coordinates in GR [6], could be a great gift for theoreticians pointing in the direction of a modification to GR, namely, the violation of the gauge invariance and a possibility to choose an arbitrary energy density level. A version of such a theory was suggested in [18], where the Friedmann equation is satisfied up to some(arbitrary) constant and takes the form

$$-\frac{1}{2}M_p^2 (a'^2 + Ka^2) + \rho a^4 = \text{const}, \quad (1)$$

where, prime denotes the derivative of the scale factor $a$ w.r.t the conformal time, and the energy density $\rho$ includes all kinds of matter. As in GR, three types of uniform spacial curvatures $K = \{-1, 0, 1\}$ are possible, but they are not related to the critical density. One has to note, that the Equation (1) has been deduced in some particular gauge [18]

$$ds = a^2(dt^2 - e^{4\chi} dr^2 - e^{-2\chi}r^2(\sin^2 \theta d\phi^2 + d\theta^2)), \quad (2)$$

where the function $\chi(r/R_0)$ is related with the comoving distance $\chi$ as $re^{-\chi(r/R_0)} = R_0\Phi_K(\chi/R_0)$,

$$\Phi_K(x) \equiv \begin{cases} 
\sin(x), & \text{for } K = +1 \\
x, & \text{for } K = 0 \\
\sinh(x), & \text{for } K = -1.
\end{cases} \quad (3)$$

and $6R_0^{-2}$ is the universes’ present spatial curvature. Using this preferred gauge implies violation of the gauge invariance. As will be shown below, the main part of the vacuum energy density scales as $a^{-4}$ in this case, and the constant on the right-hand side of Equation (1) could be compensated. Then, we converge to GR if the main part of the vacuum energy is compensated exactly, but some residual value could remain. Below we assume that the constant on right-hand side of Equation (1) is this residual value and, at the same time, $\rho$ on the left-hand side of Equation (1) is an energy density without the main part of vacuum contribution. Such residual constant is equivalent to some amount of “invisible” (i.e., unperturbable) radiation, which can be either positive or negative. In principle, one could determine the effective equation of state for all the universes’ content and the value of constant in this extended gravitational framework (1) from high quality observational astrophysical data.

The “high-redshift” observations of cosmic microwave background radiation (CMB) [19–21] have a very high constraining power on the cosmological parameters. However, the interpretation of CMB and baryon acoustic oscillations (BAO) [22] data stays, at least mildly, model-dependent, i.e., they are based on the standard GR framework and the interpretation of these data in the current extended framework (1) is not a trivial problem, which we intend for a future investigation.

Model-independent low-redshift supernovae (SN) dataset give rigorous constraints on the cosmological models, as well. SN datasets have undergone substantial improvements in the last decade, with more SN and more robust statistical methods to compile a homogeneous dataset that can be implemented to test the cosmological models. The most used compilation has been provided as the joint light-curve analysis (JLA) dataset in Betoule et al. [23], with 740 selected SN up to a redshift of $z \sim 1.4$, which has been recently updated to $\sim 1050$ SN in [24]. Gamma-ray bursts (GRB) are the most energetic explosions in the universe. They are detectable up to very high redshifts ($z \sim 8$), therefore can be used to study the universes’ expansion rate when the empirical correlations between the spectral and intensity properties are appropriately calibrated. We use the GRB dataset comprising of 109 observations compiled in [25] using the well known Amati relation [26]. These two datasets are complemented with cosmic chronometers (CC) [27], which provides the measurements of the expansion rate at different redshifts and is a useful observable for estimating the Hubble constant value $H_0$ [28–32].

But for all that, the quality of the aforementioned experimental data is not sufficient to consider the equation of the state for the overall energy density $\rho$ in Equation (1), so some concrete/simplified models with specified vacuum nature are needed, here we consider two particular models. The first $\Lambda$CDM model is a simple extension of the well-known $\Lambda$CDM on basis of Equation (1). The second one is the vacuum fluctuations domination model (VFD) [33] in which the residual vacuum energy density and pressure arising due to universe expansion is considered (elaborated in Section II).

The current paper is organized as follows: In Section II, we describe the theoretical framework and cosmological observables, in Section III we then briefly present the datasets and model-selection criteria implemented. In Section IV, we present our results for cosmological inferences, constraints on parameters and finally summarize our conclusions in Section V.

## II. RESIDUAL VACUUM ENERGY DENSITY AND PRESSURE

A scalar field ($\phi(x)$) is a simple and convenient candidate to study the vacuum energy density problem, for which the energy-momentum tensor could be written in the form [6]

$$T^\nu_\mu = \frac{1}{\rho} \left( \partial_\mu \phi \partial^\nu \phi + \delta^\nu_\mu m^2 \phi^2 \right) . \quad (4)$$

After quantization of the scalar field, estimating the vacuum mean value of Equation (4), this expression could be written in the hydrodynamic form of some resting (i.e.,...
having 4-velocity $u^{\mu} = \{1, 0, 0, 0\}$ medium:

$$\langle 0 | T_{\mu}^{\nu} | 0 \rangle = (\rho_v + p_v) \delta^{\mu}_\nu \delta_0^v - p_v \delta^{\nu}_\mu.$$  \hspace{1cm} (5)

Only a covariance could permit a substance with $p_v = -\rho_v$, however according to Equation (1) the quantity $\langle 0 | T_{\mu}^{\nu} | 0 \rangle$ is not a perfect tensor for two reasons: the non-invariance of the vacuum state $|0\rangle$ relative to a general coordinate transformations, and the non-covariant character of the UV cut-off.

Besides the main part of the vacuum energy $\rho_v \sim M_p^4$ there could be other vacuum contributions:

i) due to masses of the particles and condensates, which depend on the particular mechanism of particle mass generation,

ii) due to curvature of space-time, i.e., depending on the universe expansion rate.

For type i), which equivalently exists in the Minkowski space-time, one has [34]:

$$\rho_v = \frac{1}{4\pi^2} \int_0^{k_{max}} k^2 \sqrt{k^2 + a^2 m^2} dk \approx \frac{1}{16\pi^2} \left( \frac{k_{max}^4}{a^4} + \frac{m^2 k_{max}^2}{a^2} \right)$$

$$+ \frac{m^4}{8} \left[ 1 + 2 \ln \left( \frac{m^2 a^2}{4 k_{max}^2} \right) \right],$$

where $k_{max} \sim M_p$ as it has been suggested in [34]. The term of $\rho_v \sim k_{max}^4 a^{-4}$ scales with $a$ as an invisible radiaton and is compensated by the arbitrary const in the Equation (1) up to its residual value $\Omega_i$. The next term in Equation (6) corresponds to the substance with the equation of state $p_v = -\frac{1}{3} \rho_v$, which is widely discussed [35, 36], but they consider it as overall (total) equation of state without discussion of its origin. Fermions gives the contribution of the opposite sign and according to experimental observations (absence of the fast universe expansion) it is expected [12] that the mutual contributions compensate each other to the accuracy of the order of the critical density. The last term $\rho_v \sim m^4$ roughly corresponds to the cosmological constant [37], with the logarithmic accuracy. It also has to be almost compensated, while taking into account contribution of the condensates which are also of the order of $\sim m^4$.

For type ii), which expressed through time derivatives of the universe scale factor, one has [33, 34]:

$$\rho_v = \frac{a^2}{2a^6} M_p^2 S_0,$$  \hspace{1cm} (7)

The corresponding pressure is

$$p_v = \frac{M_p^2 S_0}{a^6} \left( \frac{1}{2} a'^2 - \frac{1}{3} a'' a \right),$$  \hspace{1cm} (8)

where

$$S_0 = \frac{k_{max}^2}{8\pi^2 M_p^2},$$

is determined by the ultra violet (UV) cut off of the comoving momenta, and $M_p = \sqrt{\frac{\pi}{\kappa}}$ is the reduced Planck mass. Accounting for additional scalar fields the quantity $k_{max}$ in Equation (9) should be replaced by $k_{max}^2 \rightarrow (N_{sc} + 2) k_{max}^2$, where it takes into account two degrees of freedom corresponding to the gravitational waves [34]. In a minimal variant of the Standard Model, there exists a SU(2) duplet of complex scalar fields [38], i.e., four scalar degrees of freedom $N_{sc} = 4$ prior to the spontaneous symmetry violation (so-called “elecroweak transition”). At the present time, after symmetry violation, only a single Higgs field is to contribute, but Pauli sum rules insist on existence of some new bosons [12].

The above definitions clearly follow the continuity equation,

$$\rho'_v + 3\frac{a'}{a} (\rho_v + p_v) = 0.$$  \hspace{1cm} (10)

Substituting of i) and ii) Equations (6) and (7) vacuum contributions into Equation (1) and adding a pressureless matter ($\Omega_m$) results in a very general cosmological model:

$$E(a)^2 = \frac{(1 - S_0 - \Omega_m - \Omega_L - \Omega_k) a^{-4} + \Omega_m a^{-3} + \Omega_k a^{-2} + \Omega_L + \Omega_L \ln a}{1 - S_0 a^{-2}},$$  \hspace{1cm} (11)

where $E(a) = H(a)/H_0$. From Equation (1), Equation (11) holds for an arbitrary signature of the space metric Equation (3). The “curvature” term $\Omega_k a^{-2}$ originates as from both the real spatial curvature and the vacuum contribution looking like the fluid with the equation of state $p_v = -\frac{1}{3} \rho_v$. The total amount of radiation is $1 - S_0 - \Omega_m - \Omega_L - \Omega_k$. Please note that the “invisible” radiation given by residual value of const in Equation (1) is indistinguishable from the real radiation, but at low redshifts the real radiation contribution is very low. Although a number of the contributions turn out to be possible in Equation (11) due to violation of the gauge invariance, we will consider below only two particular models with two independent parameters and besides assume a flat universe $K = 0$ for simplicity.

The first $\Lambda$CDM-model corresponds to $\Omega_k = 0, S_0 = 0, \Omega_L = 0$ in Equation (11). It has independent parameters $\Omega_m$ and $\Omega_\Lambda$. In contrast to the well-known $\Lambda$CDM,
the matter density and the cosmological constant are not constrained by the relation $\Omega_\Lambda + \Omega_m = 1$ by virtue of Equation (1).

The second model is the vacuum fluctuations domination model (VFD), corresponding to the $\Omega_\Lambda = 0, \Omega_m = 0$, $\Omega_L = 0$ with the non-zero parameters $S_0$ and $\Omega_m$. Due to the denominator in Equation (11), the parameter $S_0 > 1$ is strictly required, which otherwise would imply that we have already witnessed a Big Rip. The VFD implies that among all the substances filling the universe the residual energy density and pressure of vacuum are the most significant. This is point of view to the key issue of the nature of dark energy and pressure. Without taking into account a “correct” vacuum contribution, we would lack an appropriate footing to add some exotic substances. Vacuum energy considerations would be an alternative to the quintessence dark energy models, were in fact shown to perform better or at least equivalent to $\Lambda$CDM with various datasets [39–43]. While we do not discuss extensively the effective equation of state (EoS), $w(z)$ (see [44] for details), it is interesting to note that the VFD model predicts $w(z) \rightarrow 1/3$ and drops to negative values, however larger than $-1$, at low-redshifts. This behavior in VFD is, in fact, equivalent to the hybrid expansion law implemented in [45] from very distinct physical considerations. While the EoS is similar, the two models predict distinct features at higher redshifts as VFD asymptotically behaves as a costing model, and later model retains deceleration without a matter-dominated regime. Finally, we also test a further extension of the VFD model by allowing for the $\Omega_\Lambda \neq 0$ component (hereafter V$\Lambda$CDM), which is, in fact, interesting to assess the nature of a cosmological constant, which could also originate from the vacuum fluctuations.

### III. DATASET AND METHODS

We utilize a combination of astrophysical datasets, essentially requiring them to be independent of any cosmological assumptions in obtaining the observables. A summary of which is presented here:

**Supernovae (SN):** The Pantheon compilation of $\sim 1050$ SNe observations presented in [24] has improved the statistical precision and the highest redshift ($z \sim 2$) to which the distances have been measured. We take advantage of these data set, which already remains a mild improvement over previous dataset [23]. The later dataset eases the analysis over the earlier dataset by marginalizing the supernovae standardization parameters a priori in a model-independent way.

**Cosmic Chronometers (CC):** Differential dating of galaxies was proposed as the means to estimate the Hubble constant in a model-independent way [27]. These estimations are related to the synthetic spectra of simple stellar populations and the models of stellar evolution. Recently a very robust characterization of the differential aging has been tested [46], and it provides an estimation of $H(z)$ at 6% accuracy (see also [47] for the recent review on the framework of differential dating). In this work we adopted the measurements provided by [28, 46, 48–50], listed in table 2 of [32], which comprises 31 measurements of $H(z)$ over the redshift interval $z \in (0.0798, 1.965)$. It has to be noted that the CC dataset presents with an intrinsic systematic effect due to the assumption of the stellar evolution models, whose effect on the estimation of $H_0$, was evaluated to be an additional systematic error of $\sigma_{sys} \sim 2.5$ in [32].

**Gamma ray bursts (GRB):** GRBs are observed in a wide range of spectroscopic and photometric redshifts, up to $z \sim 8$, and can be used to probe the high-$z$ Universe. As GRBs are not standard candles, they are standardized by utilizing the SN distance modulus in the overlapping redshift range and can provide insights, higher-redshift evolution of the cosmological models. We implement the GRB likelihood, as was earlier utilized in [51] (please refer for further details of likelihood construction). The current dataset comprising of 109 GRBs has been compiled in [25] utilizing the Amati relation [26]. The dataset has 50 GRBs at $z < 1.4$ and 59 GRBs at $z > 1.4$, in a total range of 0.1 $z < 8.1$ [25].

A simple joint likelihood of these datasets is constructed as

$$\mathcal{L}(y|\Theta) = \mathcal{L}_{SN} \times \mathcal{L}_{CC} \times \mathcal{L}_{GRB}, \quad (12)$$

which is utilized to perform a Bayesian analysis through MCMC sampling. For this purpose, we use the emcee\(^1\) [52] package, which implements an affine invariant Metropolis-Hastings sampler. We also utilize the getdist\(^2\) package to analyze the chains and obtain posteriors. A complete Bayesian analysis has been performed here, as a simple Frequentist approach might not suffice the need of the strongly non-Gaussian posterior that the VFD model predicts. We implement flat/uniform priors on the parameters, as are summarized in Table 1.

| parameter | VFD | $\Lambda$CDM | $\Lambda$CDM | $\Lambda$CDM |
|-----------|-----|---------------|---------------|---------------|
| $H_0$     | [50.0,100.0] | [50.0,100.0] | [50.0,100.0] | [50.0,100.0] |
| $\Omega_m$ | [0.0,3.0] | [0.0,1.0] | [0.0,1.0] | [0.0,3.0] |
| $S_0$     | [1.0,10.0] | 0.0 | 0.0 | [1.0,5.0] |
| $\Omega_\Lambda$ | 0.0 | [0.0,1.0] | - | [-3.0,1.0] |

It is often convenient to take the conventional $\Lambda$CDM as a reference model for comparison and establish a corresponding statistical criteria. We implement the widely

\(^1\) [http://dfm.io/emcee/current/](http://dfm.io/emcee/current/)
\(^2\) [https://getdist.readthedocs.io/](https://getdist.readthedocs.io/)
used Akaike information criteria (AIC) [53] for model selection [54]. AICc, corrected for number of data points to the second-order is written as

$$\text{AICc} = -2 \log L^{\text{max}} + 2N_p + \frac{2N_p(N_p + 1)}{N_d - N_p - 1}, \quad (13)$$

where $N_p$ is the number of parameters and $N_d$ is the number of data points. The model preference is estimated by evaluating $\Delta \text{AICc}$, as a difference in the AICc value of the model in comparison to the reference model ($\Lambda$CDM). A positive value of $\Delta \text{AICc}$ indicates that the reference model is preferred over the model in comparison.\(^3\) The strength of the preference/rejection of a model is often gauged in terms of the Jeffrey’s scale as was discussed and implemented in earlier works [57–59].

**IV. RESULTS**

In Table II, we present the 1σ constraints obtained from our joint analysis. As can be clearly seen, also from the confidence regions shown in Figure 2, $\Omega_m$ is far less constrained in the VFD model in comparison to $\Lambda$CDM and $\Lambda$CDM, allowing for $\Omega_m \sim 1$. That is due to the strong anti-correlation between the $\Omega_m$ and $S_0$ parameters, which provides the upper limit on $\Omega_m$, obeying the lower limit of $S_0 > 1.0$ prior (see Section II). Given the very large uniform prior parameter space allowed for the $S_0$ parameter we find that the data is, in fact, able to constrain it, which however could benefit from additional data that can refine the posteriors on $\Omega_m$ (see for example [60]). In fact, $\Omega_m - S_0$ parameter space indicates the improvement of the fit over $S_0 = 0$, which corresponds simply to an Einstein-de Sitter like universe with $\Omega_m \sim 1$ [61]. Also, following Equation (9), we can immediately translate the constraint on $S_0$ to obtain the constraint on the UV cut-off scale $k_{\text{max}} = 12.43^{+0.9}_{-1.6}[M_p/\sqrt{2 + N_{sc}}]$, in the units of the reduced Planck mass.

In our analysis, the constraints on $H_0$ are driven by the CC dataset and are consistent with the earlier reported model-independent estimates in [31, 32]. However, we notice a small shift towards higher values ($\Delta H_0 \sim 1$) in the VFD model, which is in accordance with the faster acceleration rates as shown in the left panel of Figure 3. While we do not intend any implications in the context of the well-known $H_0$-tension [62–65], the extended formalism as in Equation (11) provides interesting possibilities which we intend to investigate in a future communication, also with an anticipation that the interpretation of high-$z$ observables, such as CMB will be taken into account.

The value of the “invisible radiation”,

$$\Omega_i = -\text{const}/\left(\frac{1}{2}M_p^2H_0^2\right), \quad (14)$$

with the $\text{const}$ from Equation (1) is presented in the last row of Table II and in Figure 2. It indicates that $\Lambda$CDM insists on the GR framework of $\Omega_i = 1 - \Omega_m - \Omega_{\Lambda} \approx 0.016$, while VFD is clearly not a GR-based theory having $\Omega_i = 1 - S_0 - \Omega_m \approx -1.48$. The $\Omega_i$ freedom in $\Lambda$CDM model aids to a larger uncertainty of $\Omega_m$ in comparison to $\Lambda$CDM, while keeping the low-redshift dynamics indistinguishable. The effects are in fact noticeable only as a larger dispersion at the higher redshifts (see Figures 1 and 3). Equation (14), in turn allows us to place limits $\text{const} = 0.81^{+0.11}_{-0.05}[M_p^2H_0^2]$ on the constant of Equation (1) in the units of $[M_p^2H_0^2]$.

As shown in the left panel of Figure 1, all the models seem almost identical, while showing variation for $z > 1.5$. It is interesting that the relatively small dispersion of $\Omega_i$ for $\Lambda$CDM leads to visible effect in the $aH(a)$ dynamics. The deceleration parameter $q(a)$, shown in Figure 3 at the face-value appears to be inconsistent with the deceleration-acceleration transition redshift constrained in [32] (see also [66]). We find that the VFD approaches the coasting-like behavior at similar

\(^3\) Please refer to [55, 56] for more details.
TABLE II. 68% C.L. constraints for the VFD, ΛCDM, ΩCDM and VACDM models obtained using the SN+CC+GRB datasets. In evaluating the information criteria presented in the last row, ΛCDM is taken to be the reference model. Here * indicates a derived quantity and for unconstrained parameters we report the 95% C.L. limits.

| parameter | VFD | ΛCDM | ΩCDM | VACDM |
|-----------|-----|------|------|-------|
|           | b.f | 1σ   | b.f  | 1σ   | b.f  | 1σ   | b.f  | 1σ   |
| $H_0$     | 69.8| 69.8 ± 1.9 | 68.9 | 68.7 ± 1.8 | 68.9 | 68.9 ± 1.8 | 69.6 ± 1.8 |
| $S_0$     | 1.60| 1.98 ± 0.24 | 0   | –     | 0   | –     | > 2.16 |
| $\Omega_m$| 0.87| 0.64 ± 0.34 | 0.79 | 0.276 ± 0.090 | 0.302 | 0.303 ± 0.020 | > 0.594 |
| $\Omega_\Lambda$ | 0   | 0.703 | 0.708 ± 0.041 | 0.698* | 0.697 ± 0.020* | < −0.14 |
| $\Omega_\Lambda^*$ | −1.48| −1.62 ± 0.22 | 0.007 | 0.016 ± 0.037 | 0   | < −1.90 |
| $\chi^2_{b,f}$ | 1215.34 | 1213.24 | 1213.26 | 1214.16 |
| $\Delta$AICc | 4.02 | 1.92 | 0   | 4.86 |

FIG. 2. Confidence regions for the VFD, ΛCDM and ΩCDM model obtained using the CC+SN+GRB dataset, corresponding to the constraints presented in Table II.

not support the claim [51, 76], also within a cosmology-independent analysis [31, 32, 77, 78]. In this respect, VFD model provides an interesting possibility with an early linear coasting-like behavior and a late acceleration with $q(a = 1) = −0.74 ± 0.16$. More recently in [60], the authors study the primordial Nucleosynthesis\(^4\) with a linear coasting background and claim that the observed helium content can be reconciled when having a larger baryon density and no dark matter. Yet additional confirmation of this fact with some alternate nucleosynthesis analysis is very much desirable.

It is interesting to use $\Omega_m$ diagnostic [84],

$$
\Omega_m(z) = \frac{E^2(z) - 1}{(1 + z)^3 - 1}
$$

(15)

which was developed as a diagnostic for ΛCDM being equivalent to $\Omega_m$. As one can see from Figure 3 (right panel), the relatively small dispersion of $\Omega_i$ leads to a considerable dispersion of $\Omega_m(z)$ at $z > 1$ for ΛCDM compared to the standard ΛCDM. For the VFD model this quantity could be interpreted as “effective” matter content, which is, certainly, very distinct from actual $\Omega_m$ constraints. In comparison, the earlier model-independent reconstruction in [32] (Fig. 6 therein) gives qualitatively similar predictions for $\Omega_m(z)$ at $z > 2$ as the VFD model (i.e, a downward folding of the curve is implied). It should be noted that this feature in [32]\(^4\) Please refer to [41, 79–83], references therein, for an extended discussion and contrasting arguments on the early-time (pre-recombination) behavior of linear coasting-like models. Also, please note that here we have bunched an incomplete list of works, which study linear-coasting behavior arising due to different physical considerations.
was primarily due to the Baryon Acoustic Oscillation observations at \( z \sim 2.4 \) [85, 86], which we have not utilized here. However, it is interesting to notice that in the standard GR-like phenomenology, such behavior indicates a negative energy density \((E(z) < E(z)_{\Lambda \text{CDM}})\), which was earlier noted in [87–90], and is in accordance with recent model-independence predictions utilizing various datasets in [32, 77, 91]. On the other hand, towards \( z \to 0 \) we notice a similar folding of the curve, which is associated with the ‘Big Rip’ [92] future, with \( w_{de} < -1.0 \) and phantom-like behavior in \( \Omega_m \) diagnostic [93]. This, in turn, asserts the interesting aspects of the VFD model, which, being different in formulation from the standard scenarios, can facilitate distinct phenomenological predictions through the modeling of a single parameter \( S_0 \).

In any case, as reported in the last row of Table II, the AICc statistics indicate a preference for the reference \( \Lambda \text{CDM} \) model w.r.t both \( \Lambda \text{CDM} \) and VFD. The comparison between the \( \Lambda \text{CDM} \) and VFD models with equal number of parameters boils down to the difference \( \Delta \chi^2 = 2.1 \), which is just about the moderate significance having a very different parameter space. While at face value, it might seem mildly discouraging for the VFD model, we emphasize the advantage of VFD being able to produce diverse phenomenology, while also having physical motivation, over the more restricted and yet only a phenomenological \( \Lambda \text{CDM} \).

Although increasing the number of free parameters makes constraints/predictions less stringent, we discuss a more general model including three independent parameters \( \Omega_m, \Omega_{\Lambda}, S_0 \). In this case we notice a multi-modal behavior of the likelihood determined by the prior regions of the parameters. The first one is, in fact, equivalent to \( \Lambda \text{CDM} \) in which \( S_0 \lesssim 1 \) is either small or negative (ignoring for the moment that from theoretical point of view \( S_0 \) has to be greater than unity). The second part parameter space corresponds to the VFD model extended by the cosmological constant term (V\( \Lambda \text{CDM} \)). As is shown in Figure 4, the theoretical prior of \( S_0 > 1 \) naturally gives rise to a negative value of \( \Omega_{\Lambda} \) (negative cosmological constant) and an highly degenerate scenario. A better calibration of \( \Omega_m \), e.g, aforementioned primordial nucleosynthesis, would help to break the degeneracy to provide upper and lower bounds for \( S_0 \) and \( \Omega_{\Lambda} \), respectively.

The low-redshift evidence for a negative cosmological constant was recently investigated in [94], finding motivation in the fact that string theory might not accommodate stable de Sitter (dS) vacua [95, 96] (see also [97]). Here a negative cosmological constant \((\Omega_{\Lambda} < 0)\) was modeled alongside a dark energy component \((\Omega_{\phi})\) with EoS \( w_{de} \neq -1 \). Having no detection of the same, a lower limit of \( \Omega_{cc} \gtrsim -14 \) at 95% C.L. was reported.

In our modeling however, we find a contrasting 95% C.L. upper limit of \( \Omega_{cc} \lesssim -0.14 \), while a lower limit remains degenerate with larger values of \( S_0 \) and \( \Omega_m \). This extension also modifies the limits on the UV cutoff to \( k_{\text{max}} > 12.9[M_p/\sqrt{2 + N_{sc}}] \) at 95% C.L., which differs mildly but remains in agreement with the constraints set without the inclusion of cosmological constant. It is worth stressing that the formalism considered is entirely different from that of [94], and that \( \Lambda \text{CDM} \) is not a part of the allowed parameter space in our analysis, due to the prior \( S_0 > 1 \). The better-fits to the data are shifted towards the higher limits of the priors, as reported in the last column of table I. We also find that this extension does not improve the AICc statistics over the VFD model within the assumed prior ranges, being disfavored w.r.t \( \Lambda \text{CDM} \) at \( \Delta \text{AIC} \sim 4.86 \). Incidentally, this is also very similar to the \( \Delta \text{AIC} \sim 4.8 \) reported in [94], while using different datasets. As a side note, we notice that the lower bound on \( \Omega_{cc} \) set in [94], is essentially accompanied by \( w_{de} \to -1 \) (consistent with \( w_{\phi} = -1 \) at 1\( \sigma \)) and the marginalized posterior is limited by the assumed lower limit of the prior range. This makes the parameter space equivalent to \( \Lambda \text{CDM} \) and in-turn speculate the lower bound, as they model \( \Omega_{\phi} \equiv 1 - \Omega_m - \Omega_{cc} \) and this would imply an extended linear degeneracy between \( \Omega_{\phi} \) (positive) and \( \Omega_{cc} \) (negative) for all lower values.
than the quoted limit, having well-constrained the combination $\Omega_\phi + \Omega_{cc} \sim 0.65$.

For the VACDM model implied here, the early-coasting and late-acceleration behavior of the VFD model is preserved and arrives at similar expectations for the quantities $O_m(a)$ and $q(a)$, in fact, with tighter constraint of $q(a = 1) = -0.76 \pm 0.10$. While this tighter constraint is contrary to intuition due to the additional parameter, it shows that the freedom to have a negative cosmological constant further aids the assertion of late-time acceleration. In this context, it is interesting to note that a very similar phenomenology based on dynamical symmetry breaking (see also [98]) was recently presented as a modified Friedmann’s cosmology in [99].

As was already stressed, we imply that the negative cosmological constant originates, in fact, from the vacuum fluctuations due to the Pauli sum rules violation [12]. In the string theory (more generally, M-theory [100–102] or the F-theory [103, 104]), a false vacuum is usually considered [96, 105, 106], which is analogous to the condensate arising in the Higgs mechanism of particle masses generation in the standard model [38]. In particular, the string compactification [107, 108] or bounding the strings by branes [109] results in a “landscape” [105] or “swampland”\(^{5}\) [95, 108, 114–117] of the effective field theories [118–120]. As for explicit consideration of the vacuum quantum fluctuations within string theory [121], to date, it is in an infantile state, because the theory is, in fact, the first-quantized one, whereas its second quantized version is not completely created yet [11, 122, 123] (however, see also [124]). This prevents considering the zero-point fluctuations consisting of the creation and annihilation of the strings. Only after reducing to an effective field theory, the zero-point energy could be taken into account.

One has also to note that in the context of the generalized framework considered here, a gauge violating version of the string theory should be developed. For instance, it could be not a single moving relativistic string, but a system of the relativistic strings coupled into a crystal-like lattice. The preferred reference frame will appear in which such a “crystal” is at rest “in toto”. At the same time, all linear perturbations of this system have to manifest Lorentz invariance.

\(^{5}\) While not a straightforward comparison, the $\sim 2\sigma$ significant result for a negative cosmological constant here, at a face value would be in contrast with the “swampland” conjecture/bound proposed to explain in-existence of (meta)stable dS vacua in string theory (see for e.g., [97, 110–113] and references therein).
degree of accuracy of the experimental data, a certain simpler model without a theoretical background allows describing the experimental data better than a more fundamental but complex model. The ΛCDM considered shows that, after compensating for the main part of vacuum energy, the gravity framework could still be close to GR, whereas the VFD manifests a non-GR framework explicitly. Within this framework we constrain the UV cut-off scale (Equation (9)) to be $k_{\text{max}} = 12.43^{+0.9}_{-1.61}[M_\odot/\sqrt{2 + N_{\text{sc}}}]$, where $N_{\text{sc}}$ number of the minimally coupled scalar fields.

The consideration of $\Omega_\Lambda$ in the VFD model strictly requires a negative value of the cosmological constant to agree with the astrophysical data, at a 95% C.L. upper limit of $\Omega_\Lambda < -0.14$, which contrasts with the lower limit reported in [94], whose authors imply that the string theory insists on the negative cosmological constant, i.e., an AdS space arises from compactification [96]. At first sight, these facts are unrelated from a theoretical point of view as string theory considers the problem classically and arrives at the GR framework after compactification. Moreover, leading the support by the Marie S.-Curie Cofund Multiply “MASTEDIS” Fellowship.

The VI. ACKNOWLEDGEMENTS

B.S.H acknowledges financial support by ASI Grant No. 2016-24-H.0. B.S.H acknowledges INFN Roma, Tor Vergata Computing Centre services (RMLab) and is thankful to Federico Zani for providing help with the same. B.S.H is thankful to Maurizio Firrotta for useful discussions and we thank Massimo Bianchi for constructive comments on the draft. V.L.K. acknowledges the support by the Marie S.-Curie Cofund Multiply “MASTEDIS” Fellowship.

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