Hybrid dynamical type theories for navigation

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Abstract—We present a hybrid dynamical type theory equipped with useful primitives for organizing and proving safety of navigational control algorithms. This type theory combines the framework of Fu–Kishida–Selinger for constructing linear dependent type theories from state-parameter fibrations with previous work on categories of hybrid systems under sequential composition. We also define a conjectural embedding of a fragment of linear-time temporal logic within our type theory, with the goal of obtaining interoperability with existing state-of-the-art tools for automatic controller synthesis from formal task specifications. As a case study, we use the type theory to organize and prove safety properties for an obstacle-avoiding navigation algorithm of Arslan–Koditschek as implemented by Vasilopoulos. Finally, we speculate on extensions of the type theory to deal with conjugacies between model and physical spaces, as well as hierarchical template-anchor relationships.

I. Introduction

The problem of composing complex systems from simpler subcomponents is common to all branches of engineering. A general approach to specify interfaces between components – both logical and physical. For logical interfaces, arguably the most advanced specification language is dependent type theory, a formal system capable of expressing the entirety of constructive mathematics [29]. On the other hand, physical interfaces must deal with finite resources, leading naturally to linear logic, a simultaneous refinement of intuitionistic and classical logic that emphasizes the role of formulas as resources [18]. Combining linear logic with dependent types is still an active research area, but much progress has been made in recent years [10, 25, 30, 27]. We are particularly interested in the categorical framework of [15], in which the nonlinear dependent types can only depend on the “shape” of a linear term. Roughly speaking, this means that type dependency lives on the parameter level and cannot inspect state-level information of linear terms.

For applications to robotics, we desire a type system capable of encoding sequential and parallel compositions, as well as safety constraints. The former two forms of composition fit naturally within the framework of linear logic, particularly via its incarnation as the internal language of symmetric monoidal categories. Indeed, a weak version of such a category of directed hybrid systems compatible with sequential and independent parallel composition has been defined in [13]. On the other hand, safety constraints have a natural formulation in the internal language of presheaves over an agent’s sensorium, a setting particularly well-suited to formulating local conditions in the presence of uncertainty.

As we note below, both of these categories—directed hybrid systems and presheaves over the sensorium—can be connected via a state-parameter fibration, the primary input to the linear dependent type theory construction of [16]. Although that paper focuses primarily on a symmetric monoidal category of quantum circuits and a corresponding state-parameter fibration from a formal completion of the circuit category to the category of sets, their construction applies equally well to state-parameter fibrations from other symmetric monoidal categories to other locally cartesian closed categories. In our setting, we replace quantum circuits with directed hybrid systems and the base category \( \text{Set} \) with \( \text{PSh}(\Sigma) \), the category of presheaves over a sensorium \( \Sigma \).

With a state-parameter fibration and its corresponding linear dependent type in hand, we are ready to give types to hybrid controllers used by real engineers. We first provide a conjectural translation from a fragment of linear-time temporal logic (LTL) into the type theory with the goal of interfacing with existing state-of-the-art methods for controller synthesis from formal task specifications [22, 34]. Then as a case study, we type various components of a navigational controller designed by Arslan–Koditschek [2] and implemented by Vasilopoulos [37].

II. Related work

The closest related work within robotics is the body of literature on controller synthesis from temporal logic specifications. In 1977, Amir Pnueli first proposed linear-time temporal logic as a specification language for formal verification of computer programs [32]. In 1996, Antoniotti and Mishra used computation tree logic, a branching-time temporal logic developed by Clarke and Emerson to analyze concurrent programs [14], to generate a supervisory controller for a walking robot [1]. In the mid 2000s, Kress-Gazit, Fainekos, and Pappas developed a procedure for automatic controller synthesis from an robot model, class of admissible environments, and LTL task specification [22]. Subsequently as the field of robotics has grown in importance, temporal logic-based specification has remained the dominant paradigm for formal verification of controllers [28].
The main difference between our type-theoretic formalization and temporal logic-based approaches is ease of composition. Whereas temporal logics are close to natural human language [23], their basic connectives (e.g., “and,” “or,” and classical implication) are a less convenient setting for sequential and parallel composition of dynamical-system-based controllers than the linear logic corresponding to the symmetric monoidal category of directed systems [13].

On the other hand, the relative simplicity of LTL allows for automatic controller synthesis once one restricts to a computationally tractable class of formulas [8]. Thus, the strengths of the type-theoretic and temporal logic approaches are in fact complementary. We provide a conjectural translation between the two logical systems below.

On the type theory side, many have worked on integrating linear logic with non-linear intuitionistic logic in general, and dependent type theory in particular. The idea of using a linear-nonlinear adjunction, i.e. a pair of symmetric monoidal functors between a symmetric monoidal closed category and a cartesian closed category, to relate a linear type theory to the simply typed lambda calculus originates with Benton [7]. Adding dependent types is significantly harder since it is not immediately clear what it means for a type to depend on a linear term. In particular, if $u : L$ is a linear term and $T_x$ is a type family depending on $L$, then both terms $x_u$ of type $T_u$ and the type $T_u$ itself generally reference the term $u$—a violation of linearity, the axiom that linear terms must be used exactly once.

One approach to resolving this contradiction is to have two separate contexts—one for linear data, and one for intuitionistic data—and prescribe how types are allowed to depend on linear terms. This is the approach taken by Cervesato and Pfenning in their Linear Logical Framework combining linear logic with LF [10]. In their system, types may not depend on linear terms at all, only intuitionistic terms.

A significant further advance was McBride’s use of indices $k \in \{0, 1, \omega\}$ to decorate the typing annotations $x : A$ to denote the number of times the variable $x$ is used. When $k = \omega$, his dependent types correspond to those of Cervesato and Pfenning. The index $k = 0$ corresponds to terms that occur in types, and $k = 1$ corresponds to terms evaluated at runtime. This allowed much more granular control over variable usage, enabling non-trivial type dependence on linear terms.

Recently, Licata, Shulman, and Riley vastly generalized this work in their fibrational framework for substructural and modal logics [27], parameterizing a type theory by an underlying mode theory. Categorically, their framework corresponds to a functor between 2-dimensional cartesian multicategories, a powerful but very abstract setting.

In the current paper, we employ the framework of Fu–Kishida–Selinger [16] for constructing linear dependent type theories from state-parameter fibrations. Their approach uses McBride’s approach to variable count indexing, but also shares the fibrational approach to categorical semantics of Licata–Shulman–Riley in a less abstract setting (monoidal category theory, rather than higher category theory).

III. A STATE-PARAMETER FIBRATION

The primary datum required in the construction of the linear dependent type theory defined in [16] is a state-parameter fibration, a categorical fibration from a symmetric monoidal closed category (a setting for linear logic) to a locally cartesian closed category (a setting for dependent type theory) satisfying some additional compatibility axioms.

To construct such a fibration, we start with a symmetric monoidal category of “generalized circuits,” in our case directed hybrid systems as defined in [13]. In brief (using the notation of [13], where full details can also be found), a hybrid system $H$ consists of a directed graph $G(H)$, whose vertices index a set of continuous modes $\{I_v\}_{v \in G(H)}$, and whose edges index a set of reset maps $\{r_e\}_{e \in G(H)}$ between these modes. We can then define a hybrid semiconjugacy $\alpha : H \to K$ of hybrid systems to be a graph morphism on the underlying graphs together with a collection of smooth maps $\alpha_v : I_v(H) \to I_{\alpha(v)}(K)$ restricting to classical smooth semiconjugacies of the continuous flows and respecting the discrete jumps in the sense that the squares

\[
\begin{array}{ccc}
I_v(H) & \xrightarrow{r_e} & I_{\alpha(v)}(K) \\
\alpha_v \downarrow & & \downarrow \alpha_{\alpha(v)} \\
I_{\alpha(v)}(K) & \xrightarrow{r_{\alpha(e)}} & I_{\alpha(\alpha(v))}(K)
\end{array}
\]

commute for each reset map $r_e : I_v \to I_{\alpha(v)}$. These basic constructions then allow us to define a directed hybrid system to be a cospan of hybrid semiconjugacies

$H_i \to H \leftarrow H_f$

such that (i) both legs are embeddings, (ii) each component map of the right leg is a diffeomorphism, (iii) the image of the graph $G(H_f)$ in $G(H)$ is a sink, and (iv) for every $\epsilon, T > 0$ and state $x \in I(H) = \sqcup_{v \in G(H)} I_v(H)$, there exists an $(\epsilon, T)$-chain from $x$ to the image of $H_f$ in $H$. Roughly speaking, such a system forms the ambient substrate for a “funnel” leading its initial subsystem $H_i$ into its final subsystem $H_f$.

Sequential composition corresponds to the usual notion of composition of cospans via pushouts: the directed system $H' \diamond H$ obtained by composing the systems $H : H_i \to H_f$ and $H' : K \to H'_f$ is given by

\[
\begin{array}{ccc}
H & \xrightarrow{K} & H' \\
\downarrow & & \downarrow \\
H & \xrightarrow{H'} & H' \diamond H
\end{array}
\]

an operation defined only up to isomorphism [6] [19]. Thus, in order to get a strict category, we will consider directed systems up to conjugacy; that is, up to isomorphisms with
respect to hybrid semi-conjugacy. Using the categorical product with respect to semi-conjugacy defined in [13] as the monoidal product, we then have a symmetric monoidal category \( \text{DH} \) of directed systems up to conjugacy under sequential composition.

Following the recipe of [15], our first step towards constructing a corresponding linear dependent type theory is to embed \( \text{DH} \) into a symmetric monoidal closed category with products \( \overline{\text{DH}} \) by taking the Yoneda embedding. This procedure formally extends the category with exponential operations move types and terms from the state-level to the parameter level and vice versa, while ensuring that parameters can only depend on the shape of linear terms.

The second step is to form a category \( \overline{\text{DH}} \) of parameterized circuits. In their case, the spaces of parameters are sets; in ours, pre-sheaves over a topological space \( \Sigma \), which we call the sensorium. An object \( A \) of this category consists of a pair \( (A, (A_x)_{x \in A}) \) where \( A \) is a pre-sheaf over the sensorium \( \Sigma \), and \( (A_x) \) is an indexed family of objects of \( \overline{\text{DH}} \), one for each element of \( A \) (here, an element of a pre-sheaf is a morphism \( \ast \to A \), i.e. a choice of set-element over every open set of \( \Sigma \)). An arrow \( f : A \to B \) consists of a pair \( (f, (f_x)_{x \in A}) \) where \( f : A \to B \) is a map of pre-sheaves, and \( (f_x : A_x \to B_{f(x)}) \) is an indexed family of morphisms of \( \overline{\text{DH}} \).

One can check, following the proof of Fact 2.4 in [16], that the functor \( \overline{\text{DH}} \to \text{PSh}(\Sigma) \) given by projecting to the first component forms a state-parameter fibrations.

### IV. TYPE SYSTEM

For the purposes of our navigational case study, we focus on three families of simple types: \( \text{See}(n) := \forall x \in X. \text{At}(x) \in X \) where \( X \) is a two-dimensional manifold (the “workspace”), and \( \text{Safe}(c) \) where \( c \) is any term. Figure 1 provides their semantics as objects in \( \overline{\text{DH}} \), fixing the sensorium \( \Sigma := C(S^1, \mathbb{R}_{\geq 0}) \) corresponding to distance measurements from a limited range line-of-sight sensor. For example, the hybrid system associated to \( \text{See}(n) \) consists of \( n \) object centers and radii with a 0 vector field, while its pre-sheaf consists of a proof that for every sensor measurement \( f \) in an open set of uncertainty \( U \), there exist \( n \) connected components of distance readings less than a maximum range \( M \).

With the exception of the simple types, the syntax of the corresponding type theory shown in Figure 2 is identical to [16], which we refer to for the corresponding inference rules. The most interesting types are the linear dependent sum and linear dependent function. Just as with regular dependent types, a term of a linear dependent sum \( (x : A) @ B[x] \) is a pair \( (x, b_x) \) of types \( A \) and \( B[x] \), respectively, and a term of a linear dependent function \( (x : A) \Rightarrow B[x] \) is a function that returns a term of type \( B[x] \) for every term \( x : A \). However, linear (i.e. state-level) variables can only be used at most once with one exception—their “shape” can be used arbitrarily many times in parameter types. Roughly speaking, the shape of state-level data corresponds to replacing every simple type in its definition by the \( \text{Unit} \) parameter type.

In the formal type theory, variable usage is encoded by the index \( k \) in \( x \vdash A \), and the inference rules enforce state-parameter usage constraints. The “!” lift, and force(‘) operations move types and terms from the state-level to the parameter level and vice versa, while ensuring that parameters can only depend on the shape of linear terms.

We also have a conjectural embedding of a fragment of linear temporal logic (LTL) [12] into the type theory shown in Figure 3. We translate the atomic propositions directly into hybrid dynamical types whose pre-sheaves enforce the invariant; for example, inhabitation in a partition element corresponds to a pre-sheaf only supported on that element. We translate “or” as the disjoint union of hybrid systems, and “and” as the parallel composition. The “Eventually” diamond corresponds to a directed system with codomain corresponding to the argument of the diamond operator. We simply translate the “Always” box operator as a directed system whose corresponding pre-sheaf always validates the argument proposition.

### V. NAVIGATION EXAMPLE

As a case study, we formulate the navigation algorithm of [12] as a composition of typed hybrid systems and proofs of correctness. Under an assumption on the curvature of the
obstacle boundaries (satisfied by circles, for example), the main theorem of the paper is essentially
\[(d(O_i, O_j) > 2R \land d(x_0, O_i) > 2R) \rightarrow \exists j, (d(O_i, O_j) \geq 2R) \lor (\Diamond (At(g))),\]
where \(x_0\) is the initial position of the robot, \(R\) is a safety radius. Rearranging things slightly, we want
\[d(x_0, O_i) > 2R \rightarrow \exists j, (d(O_i, O_j) \geq 2R) \lor (\Diamond (At(g))) \land (\Box d(O_i) > R).\]

Under the translation shown in Figure 3, our task is to construct a controller of type
\[(d(x_0, O_i) > R) \rightarrow (O_i, O_j : \text{Obstacle}) \land (d(O_i, O_j) \geq 2R) \land (c : (\text{Unit} \rightarrow \text{At}(g) \land (d(O_i, O_j) \leq 2R)) \land \text{Safe}(c)).\]

That is, if the initial position of the robot is safe, then we have a safe controller that reaches the goal unless the obstacle-separation assumption is violated.

The types shown in Figure 5 provide some Haskell-style typing declarations for the functions involved in such a controller. For example, the “go” function’s typing declaration in Figure 5 has the following meaning—for every natural number \(n\) and goal location \(g\), given the data of a Free navigational state and \(n\) visible objects, this controller will either end up at the goal while still seeing \(n\) objects or it will be interrupted. The Interrupt type says that the only interrupts correspond to seeing a new object, forgetting an object, or noting that the object separation axiom is violated.

The “controller” function (Figure 6) is defined by looping over the MPG (move-to-projected-goal) function as follows. The result of one application of the MPG function is either arriving at the projected goal or an interrupt. If you have arrived at the projected goal, check to see if it is the final goal point, in which case you are done. Otherwise, check to see if you have detected an obstacle separation violation. If so, pass the violation along and stop. In all other cases, perform MPG again with your current location and sensor readings. In addition, one would like a proof of safety — i.e. a term of the Safe type taking the controller definition as an argument. This corresponds to formalizing Proposition 3 of [2], which states that the robot’s free space is positively invariant.

VI. Future work

This paper has outlined a type theory useful for organizing and proving safety properties for navigational tasks — a baby step towards our broader goal of a functional programming language for physical work. On the practical side, the next step is to integrate some form of type checking with physical robot platforms. There are two parts to this task — (i) integrating the Robot Operating System (ROS) [33] with better support for functional programming in general and linear dependent types in particular, and (ii) completing the formalization of the algorithm in Figure 6. For the first task, one approach would be to use ideas from RosHask [12] to call ROS from Haskell [21], and either embed a linear dependent language in Haskell or follow Agda’s approach to foreign function interface to Haskell [31]. For the second task, the main obstacle is the reliance of the proof of safety on basic Euclidean geometry which requires some effort to formalize. Fortunately, it may be possible to leverage GeoCoq [9], a dependently typed formalization of Tarski’s axiomatic system [35] in the Coq proof assistant [11] [4], previously used to formalize the first book of Euclid’s Elements [5] [20].

More theoretically, the type theory still leaves much to be desired. Perhaps most urgent is the inclusion of template-anchor relationships, a form of controller hierarchy integral to much of robotic programming [17]. A first step towards this goal would be to extend the type theory to deal with hybrid semiconjugacies, the vertical morphisms in the double category of hybrid systems defined in [13], potentially leveraging work of New and Licata on double categorical semantics of gradual typing [26].

Another important direction is the exploration of more expressive categories of “funnel-like” systems closed under sequential composition. For example, since our category of directed systems is not monoidal closed, we need to use the Yoneda embedding to formally adjoin exponential objects. These formal exponentials are not hybrid systems, but rather spaces of recipes for producing hybrid systems. However, just as it is often useful to use the fact that the collection of functions between two finite sets is itself a set, or the collection of linear maps between two vector spaces is a vector space, we believe that putting dynamical structures on the space of “funnels” between two systems will prove useful. However, fundamental obstructions exist when considering even classical dynamical systems, and finding a convenient category in this setting requires considering alternative generalizations of manifolds such as Chen spaces, diffeological spaces, or Frölicher spaces [3] [15].

Thus, a more natural setting for exploring exponentials in categories of hybrid systems might be to consider more
topological perspectives such as locally preordered spaces [24]. Additionally, we might imagine a refinement procedure that takes a crude plan ("funnel") to get from an initial system to a final system, and iteratively improves the plan, incorporating a hierarchical notion of dynamics.

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REFERENCES

[1] Marco Antoniotti and Bud Mishra. “Discrete event models+ temporal logic= supervisory controller: Automatic synthesis of locomotion controllers”. In: Proceedings of 1995 IEEE International Conference on Robotics and Automation. Vol. 2. IEEE. 1995, pp. 1441–1446.

[2] Omur Arslan and Daniel E Koditschek. “Sensor-based reactive navigation in unknown convex sphere worlds”. In: The International Journal of Robotics Research 38.2-3 (2019), pp. 196–223.

[3] John Baez and Alexander Hoffnung. “Convenient categories of smooth spaces”. In: Transactions of the American Mathematical Society 363.11 (2011), pp. 5789–5825.

[4] Bruno Barras et al. “The Coq proof assistant reference manual: Version 6.1”. PhD thesis. Inria, 1997.

[5] Michael Beeson, Julien Narboux, and Freek Wiedijk. “Proof-checking Euclid”. In: Annals of Mathematics and Artificial Intelligence (Jan. 2019), p. 53. doi:10.1007/s10472-018-9606-x url: https://hal.archives-ouvertes.fr/hal-01612807

[6] Jean Bénabou. “Introduction to bicategories”. In: Reports of the midwest category seminar. Springer. 1967, pp. 1–77.

[7] P Nick Benton. “A mixed linear and non-linear logic: Proofs, terms and models”. In: International Workshop
[8] Roderick Bloem et al. “Synthesis of reactive (1) designs”. In: Journal of Computer and System Sciences 78.3 (2012), pp. 911–938.

[9] Pierre Boushy, Gabriel Braun, and Julien Narboux. “Formalization of the Arithmetization of Euclidean Plane Geometry and Applications”. In: Journal of Symbolic Computation. Special Issue on Symbolic Computation in Software Science 90 (2019), pp. 149–168. DOI: 10.1016/j.j.sc.2018.04.007 URL: https://hal.inria.fr/hal-01483457

[10] Iliano Cervesato and Frank Pfenning. “A linear logical framework”. In: Information and Computation 179.1 (2002), pp. 19–75.

[11] Thierry Coquand and Gérard Huet. “The calculus of constructions”. PhD thesis. INRIA, 1986.

[12] Anthony Cowley and Camillo J Taylor. “Stream-oriented robotics programming: The design of roshask”. In: 2011 IEEE/RSJ International Conference on Intelligent Robots and Systems. IEEE, 2011, pp. 1048–1054.

[13] Jared Culbertson et al. “Formal composition of hybrid systems”. In: Theory and Applications of Categories 35.45 (2020), pp. 1634–1682.

[14] E Allen Emerson and Edmund M Clarke. “Using branching time temporal logic to synthesize synchronization skeletons”. In: Science of Computer Programming 2.3 (1982), pp. 241–266.

[15] Alfred Fröhlicher. “Categories cartésiennement fermées engendrées par des monoides”. In: Cahiers de topologie et géométrie différentielle catégoriques 21.4 (1980), pp. 367–375.

[16] Peng Fu, Kohei Kishida, and Peter Selinger. “Linear dependent type theory for quantum programming languages”. In: Proceedings of the 35th Annual ACM/IEEE Symposium on Logic in Computer Science. 2020, pp. 440–453.

[17] Robert J Full and Daniel E Koditschek. “Templates and anchors: neuromechanical hypotheses of legged locomotion on land”. In: Journal of experimental biology 202.23 (1999), pp. 3325–3332.

[18] Jean-Yves Girard. “Linear logic”. In: Theoretical computer science 50.1 (1987), pp. 1–101.

[19] Marco Grandis and Robert Paré. “Limits in double categories”. In: Cahiers de topologie et géométrie différentielle catégoriques 40.3 (1999), pp. 162–220.

[20] Thomas Little Heath et al. The thirteen books of Euclid’s Elements. Courier Corporation, 1956.

[21] Simon Peyton Jones. Haskell 98 language and libraries: the revised report. Cambridge University Press, 2003.

[22] Hadas Kress-Gazit, Georgios E Fainekos, and George J Pappas. “Temporal-logic-based reactive mission and motion planning”. In: IEEE transactions on robotics 25.6 (2009), pp. 1370–1381.

[23] Hadas Kress-Gazit, Georgios E Fainekos, and George J Pappas. “Translating structured english to robot controllers”. In: Advanced Robotics 22.12 (2008), pp. 1343–1359.

[24] Sanjeevi Krishnan. “A convenient category of locally preordered spaces”. In: Applied Categorical Structures 17.5 (2009), pp. 445–466.

[25] Neelakantan R Krishnaswami, Pierre Pradic, and Nick Benton. “Integrating linear and dependent types”. In: ACM SIGPLAN Notices 50.1 (2015), pp. 17–30.

[26] Daniel R Licata and Max S New. “Call-by-name Gradual Type Theory”. In: Logical Methods in Computer Science 16 (2020).

[27] Daniel R Licata, Michael Shulman, and Mitchell Riley. “A fibrational framework for substructural and modal logics”. In: 2nd International Conference on Formal Structures for Computation and Deduction (FSCD 2017). Schloss Dagstuhl-Leibniz-Zentrum fuer Informatik, 2017.

[28] Matt Luckuck et al. “Formal specification and verification of autonomous robotic systems: A survey”. In: ACM Computing Surveys (CSUR) 52.5 (2019), pp. 1–41.

[29] Per Martin-Löf. “Constructive mathematics and computer programming”. In: Studies in Logic and the Foundations of Mathematics. Vol. 104. Elsevier, 1982, pp. 153–175.

[30] Conon McBride. “I got plenty o’ nuttin’”. In: A List of Successes That Can Change the World. Springer, 2016, pp. 207–233.

[31] Ulf Norell. “Dependently typed programming in Agda”. In: International school on advanced functional programming. Springer, 2008, pp. 230–266.

[32] Amir Pnueli. “The temporal logic of programs”. In: 18th Annual Symposium on Foundations of Computer Science (sfcs 1977). IEEE, 1977, pp. 46–57.

[33] Morgan Quigley et al. “ROS: an open-source Robot Operating System”. In: ICRA workshop on open source software. Vol. 3. 3.2. Kobe, Japan. 2009, p. 5.

[34] Indranil Saha et al. “Automated composition of motion primitives for multi-robot systems from safe LTL specifications”. In: 2014 IEEE/RSJ International Conference on Intelligent Robots and Systems. IEEE, 2014, pp. 1525–1532.

[35] Alfred Tarski and Steven Givant. “Tarski’s system of geometry”. In: Bulletin of Symbolic Logic 5.2 (1999), pp. 175–214.

[36] Matthijs Vákár. “Syntax and semantics of linear dependent types”. In: arXiv preprint arXiv:1405.0033 (2014).

[37] Vasileios Vasilopoulos et al. “Reactive semantic planning in unexplored semantic environments using deep perceptual feedback”. In: IEEE Robotics and Automation Letters 5.3 (2020), pp. 4455–4462.