FLuID: A Meta Model to Flexibly Define Schema-level Indices for the Web of Data

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ABSTRACT

Schema-level indices are vital for summarizing large collections of graph data. There is a large variety in how existing schema-level indices capture the schema of data instances. Each index has its value for a particular application scenario or information need. However, existing indices define only a single, fixed data structure that is tailored to a specific application scenario. Thus, the indices cannot be easily adapted or extended to changing requirements. In order to address these shortcomings, we propose a formal, parameterized meta model called FLuID (Flexible schema-Level Index model for the web of Data) that allows to quickly define, adapt, and compare different schema-level indices for distributed graph data. We conduct an extensive study of related works and abstract from the features of the existing index models to FLuID. In addition, FLuID provides novel features such as aggregation of instances over owl:sameAs. We conduct a detailed complexity analysis and show that indices defined with FLuID can be efficiently computed on average in $\Theta(n)$ w.r.t. $n$ being the number of triples in the input data graph. Furthermore, we implemented the FLuID meta model following an existing stream-based schema computation approach for the Web of Data. We empirically analyze different index models for different application scenarios, types of queries, datasets, and space requirements. This provides for the first time in-depth insights for understanding the influence of design choices of the index models and their usefulness in different scenarios.

1 Introduction

Summarizing data instances helps to efficiently manage huge amounts of data [1, 2]. For many application scenarios, indices are available that summarize data instances differently to address specific information needs. For example, search engines for the Web of Data summarize data instances in different ways to offer search and exploration on different levels [3, 4, 5, 6, 7, 8]. In general, we can distinguish instance-level indices and schema-level indices. Instance-level indices focus on finding specific data instances [9, 1, 3], e.g., searching for a specific book by its author such as the “Great Britain Central Electricity”. In contrast, schema-level indices (SLI) support structural queries, e.g., searching for bibliographic metadata using the type bibo:book and the property dct:creator [4]. Given a structural query, such as the one shown in the example in Listing 1, an SLI can return data source URIs of data sources containing bibliographic metadata. This is illustrated in step 1 in Fig. 1. Subsequently, one can access these data sources to download the actual data instances.

Listing 1: SPARQL query to find books that have a creator that is an Agent.

```
SELECT ?ds
WHERE {
  ?ds rdf:type bibo:book ;
    dct:creator foaf:Agent .
}
```
Figure 1: Finding data sources containing relevant information using a schema-level index (SLI). First, a structural query is issued to an SLI to identify relevant data sources (1). Second, access the data sources to harvest the data instances (2).

Finding relevant data sources on the Web is one application scenario for SLIs. For example, search systems like LODatio [4], LODeX [5, 6], Loupe [7], and LODatlas [8] rely on SLIs to offer a search for relevant data sources or exploration of data sources. Besides data source search and exploration, there are various other applications for SLIs. For example, the SLI SemSets [10] was developed to improve keyword-based retrieval by capturing sets of semantically related entities and TermPicker [11] uses an SLI to recommend vocabulary terms to ontology engineers.

In the past, various different SLIs have been developed for different purposes and data types [12, 10, 13, 5, 6, 7, 14, 11, 15, 16]. The problem is that each SLI defines its own, proprietary data structure that is designed for solving only a specific application scenario or answering only a specific information need. This is unfortunate since our observation is that the different application scenarios and information needs cannot be sufficiently supported by a single SLI. For graph databases, a common practice is to offer multiple instance-level indices [1]. In order to offer multiple and easily changeable schema-level indices, we need to be able to generalize from the existing SLI’s features into a high-level declarative language based on a handful of formally defined operators. In order to fill this gap, we conduct an in-depth analysis of existing SLIs with respect to the schema structures they capture. Furthermore, we provide a real-world use case that requires to perform scalable data search on the Web of Data. Based on the abstraction from the features of the existing indices as well as our data search scenario, we developed the formal meta model FLuID (short for: Flexible schema-Level Index model for the web of Data) to flexibly define arbitrary SLIs.

We analyze the space and time complexity for the computation of an arbitrary SLI defined with FLuID. We show that indices can be computed on average in $\Theta(n)$ with $n$ being the number of triples in the data graph. Furthermore, we define seven different SLIs from the related work using FLuID. We empirically compare the SLIs in three variants considering different sub-graphs of $0$, $1$, and $2$ hops lengths. The empirical evaluation is twofold. First, we evaluate the storage requirements of the computed indices. Second, we evaluate the quality of a stream-based computation of the indices for large datasets obtained from the Web of Data. The stream-based approach ensures scalability to graphs of arbitrary sizes by observing the data over a window but induces inaccuracies by potentially extracting incomplete schema structures [18, 13]. Finally, we introduce a formal grammar of FLuID’s metamodel, which is specified in Extended Backus–Naur Form (EBNF). The EBNF allows to quickly define and compute various SLIs in our schema computation framework. In summary, our contributions are the following:

(I) We introduce the first formal meta model FLuID to flexibly define and adapt arbitrary SLIs. We present how FLuID covers the functionalities of the existing index models.

(II) We show that any index defined with FLuID can be computed on average in $\Theta(n)$ with $n$ being the size of the input graph.

(III) We define and empirically compare seven index models using a single, generic index computation pipeline. We empirically evaluate the indices on two real-world datasets. We show for the first time how the different indices perform very differently depending on factors like the kind of queries applied and the characteristics of the
datasets. This supports our hypothesis that one needs to be able to provide different SLIs to different application scenarios.

The remainder of this paper is structured as follows. We present a data search scenario based on a real-world use case in Section 2. The scenario is used to derive top-down requirements to our meta model FLuID. We discuss related works in Section 3 and derive bottom-up requirements to FLuID. Subsequently, we introduce the key concepts and functionalities of FLuID in Section 4. A formal definition of FLuID’s schema elements and parameters is provided in Section 5. Based on our formal definitions, we conduct a complexity analysis of the amortized space and build-time of indices defined with FLuID in Section 6. We empirically evaluate seven selected index models defined with our meta model FLuID in Section 7. Finally, in Section 8, we describe the declarative language of FLuID and outline how our framework computes and queries arbitrary SLIs, before we conclude.

2 Data Search Scenario for the Web of Data

Our scenario is based on the use cases and requirements considered in the interdisciplinary EU H2020 project MOVING\(^1\). Goal of the MOVING project is to enable young researchers, decision makers, and public administrators to employ and use machine learning and data mining tools to search, organize, and manage large-scale information sources on the web. As content for our MOVING platform\(^2\), we consider scientific publications, videos of research talks, and social media posts.

To increase the variety, quantity, and quality of the scientific content, we automatically harvest bibliographic metadata from the Web of Data. The harvested metadata is then ingested into the MOVING backend and made available to the users through the MOVING platform’s search and exploration functionalities. The automatically harvested bibliographic metadata from the Web of Data is of great value to the users for three reasons: First, a search solely relying on the title can provide sufficiently good search results [19]. Second, the metadata often have links to thesauri concepts, which can complement existing metadata. Third, it greatly benefits training machine learning models to further improve the document search [20].

Thus, we aim to find data sources providing bibliographic metadata records that have a title, concepts, and an abstract etc. by querying an SLI computed for the Web of Data. To this end, we developed the Data Integration Service (DIS), that semi-automatically discovers and harvests bibliographic metadata (Fig. 2). To develop the Data Integration Service, we needed to address different challenges.

The first challenge is that the Web of Data is an enormous linked graph that offers billions of data instances.

\(^1\)http://moving-project.eu/description/  
\(^2\)http://platform.moving-project.eu/

Figure 2: Data Integration Service (DIS) for the Web of Data: 1. We formulate a structural query to find matching data sources, e.g., containing bibliographic metadata. 2. The Schema-level index returns a list of matching data sources and optionally (dotted line) a list matching data sources based on inferrable information. 3. The DIS accesses all data sources and validates the contained data instances. 4. The DIS harvests the relevant data, convert it into our data model, and integrates it into our database.
Figure 3: The three data instances identified by the URIs \(i_1, i_2, i_3\) use different combinations of types (\{Book\}, \{Proceedings\}, \{Book, Proceedings\}). Merging the types of \(i_1\) and \(i_2\) following the owl:sameAs provides equal type sets for all three data instances.

**Example 1.** The LODLaundromat\(^3\) \cite{lodlaundromat} is an example of a very large dataset obtained from the Web of Data with more than 38 billion statements. It requires about 6.2TB disk space. Since the Web of Data is growing and (polite) crawling can take several months\(^4\), a scalable index computation approach is needed.

**R1: Scalability.** The meta model needs to allow efficiently computing indices for graphs of arbitrary size, e.g., in linear time.

The second challenge is that despite bibliographic metadata records share common structural components such as title, author, keywords etc., their schema structures on the Web of Data can vary a lot \cite{bibliographicMetadataVary}. Therefore, we need multiple index models in order to find all relevant data sources, i.e., data sources containing bibliographic metadata.

**Example 2.** Springer\(^5\) uses a star-shaped schema structure with most information provided as literals. The British National Library\(^6\) uses more complex schema structures with various nodes further structuring the information. Furthermore, incoming properties can be of interested. If a person is linked via publisherOf to another data instance, this indicates that this other data instance is a publication.

**R2: Flexibility.** The meta model needs to support multiple index models and different schema structures.

In addition to structural variations, also various combinations of vocabularies can be used to model similar or even the same information \cite{vocabCombination}. Thus, the third challenge is to be able to exploit implicit information derived from the data to ease query building.

**Example 3.** Let us consider the example illustrated in Fig. 2, where we want to search for bibliographic metadata using a structural query. Manually crafting good structural queries requires a lot of work and time. With such a SPARQL query, however, one only finds the data sources explicitly using the type bibo:Book. Explicitly querying for each possible type from all vocabularies requires a lot of manual effort. In contrast, exploiting implicit information derived from the data can be done automatically with low effort. In our example, the data source providing “Proceedings” is also of interest to us since bibo:Proceedings is a rdfs:subClassOf bibo:Book\(^7\). For our Data Integration Service, we want to exploit ontology inferencing when using an SLI to automatically discover, harvest, and convert related data sources.

**R3: Support for RDFS inferencing.** The meta model needs to support summarizing data instances not only based on the explicitly available information in the data, i.e., the actually observed and crawled triples, but also on implicit information that can be inferred using RDFS.

On the Web of Data, implicit information can not only be derived from ontologies but also from special properties like owl:sameAs. The owl:sameAs property is commonly used to state that two data instances describe the same real world entity.

**Example 4.** As illustrated in Fig. 3, the three data instances identified by the URIs \(i_1, i_2, i_3\) use a distinct set of types. However, we can exploit that we know that \(i_1\) models the same real world entity as \(i_2\) (denoted by owl:sameAs). Merging the types of \(i_1\) and \(i_2\) provides equal type sets for all three data instances.

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\(^3\)http://lodlaundromat.org/

\(^4\)http://km.aifb.kit.edu/projects/btc-2014/

\(^5\)http://lod.springer.com/wiki/bin/view/Linked+Open+Data/Data+Description

\(^6\)http://www.bl.uk/bibliographic/pdfs/bldatamodelbook.pdf

\(^7\)http://lov.okfn.org/dataset/lov/vocabs/bibo
R4: Support for owl:sameAs inferencing. The meta model needs to support summarizing data instances based on the owl:sameAs closure.

Summary In order to efficiently cope with the aforementioned variations, a flexible approach to define schema-level indices is needed. With a data search engine supporting multiple schema structures (R2) that can be efficiently computed for graphs of arbitrary size (R1) and supporting semantic features such as inferencing over RDFS ontologies (R3) and following the owl:sameAs-closure (R4), we are expecting to find more relevant data sources.

3 Related Work

A graph index is a specific data structure computed over a graph. Approaches for indexing graph data can be divided in instance-level indices and schema-level indices (SLI). Below, we first discuss a selection of instance-level indices and second discuss a selection of schema-level indices.

3.1 Instance-level Indices

Instance-level indices store information about the actual data instances and their statements [1]. They allow fast retrieval of elements from the graph or answering queries regarding reachability, distance, and shortest path between nodes [23].

The instance-level indices of Hexastore [24] are based on the possible ordering of the three elements in a statement, the so-called subject, predicate, and object. Other instance-level indices use frequent pattern mining [25] and approximative graph summaries [26]. The latter executes queries over the graph summary instead of the original graph. Yuan et al. [27] propose an iterative subgraph mining algorithm to mine frequent and discriminative features in subgraphs for the purpose of index optimization. While the original algorithm required multiple runs over the index [27], a later extension only needed a single pass [28]. Furthermore, asynchronous update strategies have been proposed to boost the performance for incremental updates on the graph while maintaining correctness of the updates [29, 30].

Summary Instance-level indices are well studied and highly efficient to retrieve nodes from graph databases. However, they are not designed to efficiently handle structural queries like the example given in Listing 1. Nevertheless, we can exploit the efficiency of available instance-level indices when we store the computed schema-level index (SLI) as a graph.

3.2 Schema-level Indices

Schema-level indices (SLIs) support to efficiently execute structural queries over distributed graph data. Structural queries are queries that focus on how resources are described, i.e., which combinations of types and properties are used to model the resources (see example Listing 1).

There are different possibilities how to define SLIs and different definitions of SLIs allow capturing different schemas. In the following, we present an overview of SLIs with emphasis on their schema structure, their application scenario, and how they were defined.

Characteristic Sets [12], illustrated in Fig. 4a, were defined as sets of data instances using a first-order-logic expression over triples. They were evaluated with respect to the accuracy of cardinality estimations for queries in RDF databases. Characteristic Sets summarize data instances along common incoming properties and outgoing properties. Therefore, we need to be able to distinguish incoming and outgoing properties, as reflected in the requirement below.

R5: Support for incoming and outgoing properties. The meta model needs to support summarizing data instances based on outgoing properties and incoming properties.

SemSets [10], illustrated in Fig. 4b, were defined using set operators following their own “Property Graph Data Model”. SemSets were developed to discover semantically similar sets of data instances in knowledge graphs in order to improve keyword-based ad-hoc retrieval. SemSets summarize data instances that share the same outgoing properties, which are linked to a common resource. In conclusion, we need to be able to remember which objects were linked with which properties.

R6: Support for property-object pairs. The meta model needs to support summarizing data instances based on the same objects linked via the same outgoing properties.

As illustrated in Fig. 4c, ABSTAT [14], LODeX [5, 6], and Loupe [7] summarize data instances based on a common set of RDF types and properties linking to resources with the same set of types. Therefore, we need to be able to treat types
and properties separately. Furthermore, we need to be able to remember which types were linked with which properties. This leads to the next two requirements.

**R7: Support for types and properties.** *The meta model needs to support summarizing data instances based on the same types and/or based on the same outgoing properties.*

**R8: Support for property paths.** *The meta model needs to support summarizing data instances based on the same types and based on the same outgoing properties that link to data instances with the same types.*

In addition to the schema structure illustrated in Fig. 4c, the index models of ABSTAT, LODeX, and Loupe have certain characteristics that distinguish them from each other. ABSTAT’s schema structure is only informally defined in a textual description. ABSTAT proposes so-called minimal patterns. For each extracted schema structure, they select the minimal number of types from the schema structures’ types such that all removed types are super-classes of the selected types.

Cebiric et al. [15] also take sub-properties as well as domain and range into account. Therefore, we need to be able to support RDF Schema inferencing (see R3 above).

Loupe’s schema structures are informally specified in a textual description, which for some aspects leaves room for interpretation. Loupe offers Class, Property, and Triple Inspector that allow different kind of queries. The Class Inspector considers only set of types, the Property Inspector only set of properties, and the Triple Inspector a combination of types and properties. Class and Property Inspector can be considered as sub-schema structures, e.g., only the type sets for the Class Inspector (which is covered already by R7). The Triple Inspector extracts triples of the form `<subjectType, predicate, objectType>` from the input graph. Thus, the captured schema structure is equivalent to ABSTAT and LODeX. In addition, LODeX clusters the RDF types and selects a representative type per cluster. Thus, they can comprehensively visualize several datasets hosted on DataHub[^8].

[^8]: https://datahub.io/
Another SLI is TermPicker [11], illustrated in Fig. 4d. TermPicker’s goal was to make data-driven recommendations of vocabulary terms. TermPicker summarizes data instances based on a common set of types, a common set of properties, and a common set of types of all property-linked resources. Thus, in order to model TermPicker one needs to be able to aggregate the information of neighboring data instances, which leads to the next requirement.

**R9: Support for property sets.** The meta model needs to support summarizing data instances based on the same types, based on the same outgoing properties, and based on the same types of all data instances linked via outgoing properties.

Several SLIs are defined using a stratified $k$-bisimulation [31, 13]. Bisimulation operates on state transition systems and defines an equivalence relation over states [32]. Two states are equivalent (or bisimilar) if they change into equivalent states with the same type of transition. Interpreting a labeled graph as a representation of a state transition system allows applying bisimulation on graph data in order to discover structurally equivalent parts. SchemEX [13] is one such SLI defined using a stratified $k$-bisimulation. SchemEX has a schema structure like ABSTAT, Loupe, and LODeX: based on a common set of types and properties linking to resources with a common set of types. SchemEX was evaluated on snapshots of the Web of Data using a stream-based schema extraction approach. Like Loupe, also sub-schema structures are supported. This means, for example, we can either query for types or for properties, or for a combination of both (see R7).

Tran et al. [33] model a label parameterized and height parameterized index. With label parameterization, only specific properties are considered. Height parameterization limits the maximum path-length of the subgraphs stored in the index. These resemble our next two requirements.

**R10: Support for label parameterization.** The meta model needs to support summarizing data instances based on the same sub-set of outgoing properties that are included in a Label Parameter set of properties.

**R11: Support for height-parameterization.** The meta model needs to support summarizing data instances based on an equivalent subgraph. The height parameter limits the the maximum path-length of that subgraph.

Čebirić et al. [15, 16] introduce compact graph summaries for RDF graphs using quotient graphs. Quotient graphs summarize data instances based on equivalence relations. They propose so-called Property Cliques and Type Cliques formalized as equivalence relations, which are able to capture different schema structures. Property Cliques summarize data instances based on the co-occurrence of related properties. In their work, two properties are related, if they co-occur in a data instance. Thus, being related is defined as a transitive relation.

**R12: Support for Transitivity Related Properties.** The meta model needs to support summarizing data instances based on transitively related properties, i.e., properties co-occurring in any data instance.

Furthermore, Čebirić et al. differentiate between incoming and outgoing properties. They propose so-called “strong equivalence” relations, where the related incoming and outgoing properties are the same, and so-called “weak equivalence” relations, where the incoming or the outgoing properties are the same (see R5). They also offer support of RDF Schema inferencing to compute their summaries (see R3).

Although their approach is formally defined using equivalence relations, they do not consider arbitrary index models. Essentially, every equivalence relation is supported, but the existing index model definitions are rather inflexible. For example, the quotient graph approach relies on the assumption that it always makes sense to transitively include related properties (co-occurring data instances throughout the dataset). This feature restricts every property occurring in the dataset to be in exactly one schema structure, which ensures a high data compression. This high compression comes with the price that many data instances may be summarized by the same schema structure even though they do not have common properties. In addition, their graph summaries do not support different features of payload, i.e., making it not suitable for different application scenarios. Thus, a more flexible and generic approach is needed that allows to quickly change and adapt such features.

### 3.3 Summary

In summary, we observed that instance-level indices provide efficient means to tasks such as reachability or shortest path queries. However, instance-level indices are not designed to handle structural queries. But, we can exploit the efficiency of available instance-level indices when we store the computed schema-level index (SLI) as a graph. Having the schema-level index graph stored in a scalable graph database, instance-level indices can index the SLI to improve the retrieval of payload elements. Therefore, to ensure the scalability, we decided to store the SLIs again as a graph and make them available using instance-level indices (see R1 on scalability).
Figure 5: The meta model FLuID (M2) is used to define index models (M1), for example Characteristic Sets. Index models (M1) are used to compute a concrete index (M0) for a given data graph.

SLIs are suited to handle structural queries. However, existing SLIs summarize data instances based on a variety of different schema structures, are developed for various different application scenarios, and are often defined informally in a textual description or only explained by examples [10, 7, 14, 11]. In summary, there exists no fully parameterized model that formally describes arbitrary SLIs and supports all the above mentioned requirements. A meta model is needed that allows flexibly defining arbitrary SLIs. Below, we first introduce the key concepts of our meta model FLuID, before we formally define its features.

4 Introduction to the Key Concepts of the Meta Model FLuID

In order to address the requirements derived in Sections 2 and 3, a flexible approach to define schema-level indices is needed. In this section, we introduce the key concepts for our meta model FLuID that cover all requirements. In Section 5, we provide a formal definition of FLuID.

As illustrated in Fig. 5, FLuID is a meta model to define schema-level index models (M2) [34]. These specific index models (M1) define how data instances are summarized, i.e., which schema structures are considered equivalent. Finally, computing a concrete schema-level index for a given data graph is implementation level (M0). In Section 4.1, we define the concept of data instances in a data graph. In Section 4.2, we formalize instance summarization using equivalence relations and define the concept of parameterized schema elements. In Section 4.3, we define the concept of payload to implement different application scenarios. Finally, in Section 4.4, we formally define a schema-level index (SLI) as combination of data graph, equivalence relation, and payload.

4.1 Instances in a Data Graph

A data graph $G$ is defined by $G \subseteq V_{UB} \times P \times (V_{UB} \cup L)$, where $V_{UB}$ denotes the set of URIs and blank nodes, $P$ the set of properties, and $L$ the set of literals. Please note, literals cannot be in subject position. A triple is a statement about a resource $s \in V_{UB}$ in the form of a subject-predicate-object expression $(s, p, o) \in G$. We define data instances $I_s \subseteq G$ to be a set of triples, where each triple shares a common subject URI $s$. The subset $V_C \subseteq V_{UB}$ contains all RDF classes. A RDF class $c \in V_C$ is a resource, where there exists a triple $(s, \texttt{rdf:type}, c) \in G$. Thus, the data instance $I_s$ with the subject URI $s$ is of type $c$. The properties $P$ can be divided into disjoint subsets $P = P_{type} \cup P_{rel}$, where $P_{type}$ contains the properties denoting type information and $P_{rel}$ contains the properties between data instances in the data graph. In the context of Linked Data, $P_{type}$ contains only $\texttt{rdf:type}$ and $P_{rel}$ all $p \in P \setminus P_{type}$.

4.2 Schema Structure: Equivalence Relations over Instances

As discussed in Section 3, SLIs summarize data instances based on their schema, i.e., the combination of types and properties used. Each data instance uses a defined set of types and properties, thus, has exactly one schema. Since each data instance has a defined schema, SLIs partition the data graph into disjoint subsets of data instances. Equivalence relations describe any graph partitioning in a formal way. For a given set $X$, an equivalence relation on $X$ is a subset $EQR \subseteq X \times X$ that is reflexive, symmetric, and transitive. When $(x, y) \in EQR$, we say that $x$ is equivalent to $y$ or $x \sim y$. For any $y \in X$, the subset of $X$ that consists of all $x$ that are equivalent to $y$ is called the equivalence class of $y$, denoted $[y]_{EQR}$. 
Any two equivalence classes either are disjoint or coincide. This means that any equivalence relation on $X$ defines a partition (decomposition) of $X$, and vice versa [35]. Furthermore, it can be shown that the intersection of two equivalence relations over $X$ is also an equivalence relation.

For FLuID, we define equivalence relations over data instances. Our definition of data instance assigns each triple in $G$ to exactly one data relation (see Section 4.1). Thus, data instances define a partition over the data graph $G$. Consequently, the assignment of triples to data instances qualifies as an equivalence relation. Since data instances partition the data graph, a partitioning of data instances is again a partition of the data graph [35].

For FLuID, we call equivalence relations over data instances as well the corresponding equivalence classes schema elements. These form the abstract schema level [36] (M1) and define the schema structure in the index model, e.g., taking only properties into account. The entity mapping level [36] (M0) consists of instantiated schema elements. Instantiated schema elements have data instances mapped to them, e.g., there exists a data instance in the data graph $G$ using a certain set of properties.

In subsequent two sections, we describe FLuID’s schema elements as well as our parameterizations of these schema elements. These (parameterized) schema elements (SE) can be used to form equivalence relations over data instances.

### 4.2.1 Schema Elements

**Simple Schema Elements** summarize data instances $I_{s-1} \subseteq G$ and $I_{s-2} \subseteq G$ by comparing all triples $(s-1, p-1, o-1) \in I_{s-1}$ and all triples $(s-2, p-2, o-2) \in I_{s-2}$. For simple schema elements, we distinguish property cluster (PC), object cluster (OC), and property-object cluster (POC). Each simple schema element compares triples following a different strategy, i.e., comparing only the properties, only the objects, or both.

**Example 5.** As illustrated in Fig. 6, the property cluster (PC) compares only the properties of all outgoing triples of the two data instances $I_{s-1}$ and $I_{s-2}$. Therefore, although the data instances have different titles and abstracts, they are summarized by the same instantiated schema element (pc-1).

**Complex Schema Elements** partition the data graph by summarizing data instances based on three equivalence relations $\sim^s$, $\sim^p$, and $\sim^o$. Therefore, complex schema elements can be defined as 3-tuple $CSE := (\sim^s, \sim^p, \sim^o)$. While the simple schema elements summarize data instances by comparing triples using the identity equivalence “$\equiv$”, the complex schema elements compare triples using arbitrary equivalences $\sim^s$, $\sim^p$, and $\sim^o$, e.g., simple schema elements. Thus, they can be considered as containers to combine any number of simple schema elements.

**Example 6.** Let us consider, $CSE-1 := (T, =, =)$ and $CSE-2 := (T, =, PC)$, with the tautology $T$ (i.e., the information on subject position is always considered true). $CSE-1$ resembles the simple schema element Property-Object Cluster since $T$ considers all subjects equal and the identity equivalence “$\equiv$” is applied to the property and the object. With $CSE-2$, we apply the Property Cluster (PC) equivalence on the objects. Thus, in contrast to $CSE-1$, two data instances $I_{s-1}$ and $I_{s-2}$ are not considered equivalent if the objects are identical but if for each neighboring data instance (data instance with the object as subject URI) of $I_{s-1}$, there is a neighboring data instance of $I_{s-2}$ that has the same set of properties.

Please note, we formally define all introduced schema elements as equivalence relations in Section 5.1. In the subsequent section, we introduce five parameterizations on our schema elements, which enables us to model all index models from the related work.
Table 1: Overview of the FLuID meta model schema elements and parameterizations.

| Schema Elements (SE) | Description | Details |
|----------------------|-------------|---------|
| Simple Schema Element (SSE) | Triple based summarization of instances | Definitions 2 to 4, Figs. 7b to 7d |
| Complex Schema Element (CSE) | Summarization of instances using combinations of SSEs | Definition 5, Figs. 8 and 9 |

| Parameterizations on Schema Elements | Description | Details |
|--------------------------------------|-------------|---------|
| Chaining parameterization \( cp(SE, k) \) (short: \( SE^k \)) | Recursively repeat base-configuration for each connected instance up to \( k \) hops | Definition 6 |
| Label parameterization \( lp(SE, P) \) (short: \( SE_P \)) | Ignore existence of properties not in \( P \) | Definition 7 |
| Inference parameterization \( op(G, SG) \) (short: \( G_{SG} \)) | Include ontology reasoning by inferring additional triples from a schema graph \( SG \), e.g., using \( RG_{DFS} \) | Definition 8 |
| Direction parameter \( dp(SE, \delta) \) (short: \( \delta-SE \)) | Summarize instances following the direction parameter \( \delta = \{ o, i, u \} \): based on outgoing properties (o), incoming properties (i), or outgoing and incoming properties (u) | Definition 9 |
| Instance parameter \( ip(SE, \Delta) \) (short: \( SE[\Delta] \)) | Transitivity include instances in a property-based network | Definition 10 |

4.2.2 Parameterizations of Schema Elements

Parameterizations further specify our schema elements. There are five parameterizations defined in FLuID. **Chaining Parameterization** determines the size of the considered sub-graph of a complex schema element \( CSE \) of up to \( k \)-hops, and is denoted by \( CSE^k \). **Label Parameterization** allows restricting the SLI to consider only specific properties and can, e.g., be used to define type cluster \( OC_{type} \), where the object cluster only compares objects connected over the properties in \( P_{type} \). **Inference Parameterization** \( op \) is applied on the data graph \( G \) and enables ontology reasoning using a schema graph \( SG \). In practice this means that a schema graph \( RG_{DFS} \) is constructed, which stores all hierarchical dependencies between types and properties denoted by RDFS properties found in the data graph \( G \). **Direction Parameterization** \( dp \) is applied on schema elements to consider either only outgoing properties (o), only incoming properties (i), or both (u). Finally, **Instance Parameterization** \( \sigma \) allows merging equivalent data instances, e.g., data instances linked with \( owl:sameAs \).

Please note, we formally define all introduced parameterizations on schema elements in Section 5.2.

4.3 Application-Specific Information: Payload Elements

When instantiating (parameterized) schema elements, application specific information can be attached by using the notion of payload [4]. The payload comprises information about the actual data, e.g., number of data instances summarized or a reference to their data source. We do not pose any restrictions on what can be attached as payload. This enables us to adapt the index model to any possible application scenario. To incorporate payload into our FLuID meta model, we introduce the concept of **Payload Elements**. Payload elements are attached to schema elements and contain information about the summarized data instances.

**Example 7.** The data search engine LODatio stores data source URIs as payload, i.e., where the summarized data instances were found. Characteristic Sets [12] stores only the number of summarized data instances as payload to enable cardinality estimates of queries in Database Management Systems. As an extreme variant, LODatlas [8, 15, 16] uses a payload that stores all summarized data instances.

We define the payload \( PAY \) as a \( n \)-tuple of mapping functions, where each function maps the schema elements to one payload element. For each type of payload, a mapping function needs to be defined. One possible mapping function is the data source mapping function \( ds \), which maps a schema element to the data sources of all summarized data instances.
For different applications, e.g., cardinality estimation [12], one may define mapping functions, which return payload elements containing information about the number of data instances summarized by one instantiated schema element.

4.4 Instance Summarization with Schema-level Indices

FLuID provides parameterized simple and complex schema elements, which summarize data instances in a given data graph. This enables the desired high-level declarative language to model arbitrary schema structures defined by various index models. When we attach payload elements to schema elements, we are able to implement different application scenarios.

Thus, schema-level indices (SLIs) can be defined with the data graph $G$, an equivalence relation $EQR$, and an $n$-tuple of payload functions $PAY$. This leads to our first formal definition.

**Definition 1 (Schema-level Index).** A schema-level index is a 3-tuple $(G, EQR, PAY)$, where $G$ is the data graph which is indexed, $EQR$ is an equivalence relation over data instances in $G$, and $PAY$ is an $n$-tuple of payload functions, which map data instance information to equivalence classes in $EQR$.

4.5 Summary

We defined a schema-level index (SLI) as 3-tuple of data graph, equivalence relation, and payload mapping functions. We introduced four simple and complex schema elements as well as five parameterizations on these elements. These elements and parameterizations can be flexibly combined to model arbitrary SLIs.

5 Formal Definition of the FLuID Meta Model and its Features

In this section, we formally define the elements and parameterizations of FLuID introduced above. In Section 6, we conduct a detailed complexity analysis of the computation of schema-level indices based on the meta model FLuID. Readers may safely skip these sections, if they are merely interested in the practical application and empirical evaluation of FLuID and proceed to Section 7.

This remainder of this section is organised as follows: First, we define simple schema elements and complex schema elements in Section 5.1. Second, we define five parameterizations to further specify schema elements in Section 5.2. Third, we define two payload elements, which can be attached to schema elements to implement a specific application scenario in Section 5.3.

5.1 Schema Elements

In the following, we define simple and complex schema elements that abstract from commonly found schema structures.
5.1.1 Simple Schema Elements

Simple schema elements partition the data graph by summarizing data instances \( I_{s-1} \subseteq G \) and \( I_{s-2} \subseteq G \) by comparing all triples \((s-1, p-1, o-1) \in I_{s-1}\) and all triples \((s-1, p-1, o-1) \in I_{s-2}\). For simple schema elements, we distinguish property cluster (PC), object cluster (OC), and property-object cluster (POC). Each simple schema element compares triples following a different strategy. We use the data graph visualized in Fig. 7a as an example to explain the different strategies.

**Definition 2** (Property-Object Cluster). The Property-Object Cluster (POC) partitions the data graph by comparing the triples based on common properties and common objects. Thus, the equivalence relation POC holds true, if for each triple \((s-1, p-1, o-1) \in I_{s-1}\) there exists a triple \((s-2, p-2, o-2) \in I_{s-2}\) such that \(p-1 = p-2\) and \(o-2 = o-2\), and vice versa.

Analogously to the POC, we define the PC that compares triples only based on the same properties \((p-1 = p-2)\) and the OC that compares triples only based on the same objects \((o-1 = o-2)\).

**Definition 3** (Property Cluster). The Property Cluster (PC) partitions the data graph by comparing the triples based on common properties. Thus, the equivalence relation PC holds true, if for each triple \((s-1, p-1, o-1) \in I_{s-1}\) there exists a triple \((s-2, p-2, o-2) \in I_{s-2}\) such that \(p-1 = p-2\), and vice versa.

**Definition 4** (Object Cluster). The Object Cluster (OC) partitions the data graph by comparing the triples based on common objects. Thus, the equivalence relation OC holds true, if for each triple \((s-1, p-1, o-1) \in I_{s-1}\) there exists a triple \((s-2, p-2, o-2) \in I_{s-2}\) such that \(o-1 = o-2\), and vice versa.

The POC defines the schema structure of SemSets (see Fig. 4b), which fulfills requirement R6. However, schema structures like TermPicker (see Fig. 4d) define schema structures considering properties independent of the object (R7). Thus, we need the PC and the OC.

5.1.2 Complex Schema Elements

FLuID provides three simple schema elements, which summarize data instances by comparing outgoing triples of data instances. However, we also need to consider schema structures that go beyond the scope of a single data instance (R11). Thus, we define the complex schema element as an extension of a simple schema element. The simple schema elements are combinations of equivalence relations by using the identity equivalence “\(\sim\)” on properties and/or objects. Complex schema elements extend on this concept and allow arbitrary equivalence relations over subjects, properties, and objects.

**Definition 5** (Complex Schema Element). A complex schema element partitions the data graph by summarizing data instances based on the three given equivalence relations \(\sim^s\), \(\sim^p\), and \(\sim^o\). We define it as 3-tuple \(CSE := (\sim^s, \sim^p, \sim^o)\). Two data instances \(I_{s-1}, I_{s-2}\) are considered equivalent by a CSE, if \(s-1 \sim^s s-2 \land p-1 \sim^p p-2 \land o-1 \sim^o o-2\) holds true for all triples in both data instances with \(s-1\), \(s-2\) being the subjects of \(I_{s-1}, I_{s-2}\) respectively.

5.2 Parameterizations of Schema Elements

In the following subsection, we further specify our simple and complex schema elements using our parameterizations.

5.2.1 Chaining Parameterization

Through complex schema element, one can consider two directly connected data instances. The chaining of \(k\)-many complex schema elements can consider a path length of \(k\)-many data instances (without circles) (R11). As discussed in the related work, this maximum path length of the matching subgraph structure can be formalized using a stratified \(k\)-bisimulation [37].

**Definition 6.** The chaining parameterization is a function \(cp(CSE, k)\), which takes a complex schema element \(CSE\) and a chaining parameter \(k \in \mathbb{N}\) as input and returns an equivalence relation \(CSE^k\). For \(CSE^k\), the equivalence relation \(CSE\) is recursively applied up to \(k\) hops. Two data instances \([s_1]\_I\) and \([s_2]\_I\) are equivalent according to \(cp(CSE, k)\) if three conditions hold true: (1) For \(k = 0\), the subject equivalence \(s_1 \sim^s s_2\). (2) For \(k > 0\), all three equivalences \(s_1 \sim^s s_2 \land p_1 \sim^p p_2 \land o_1 \sim^o o_2\). (3) For \(k > 0\), the recursion step \((o_1][, o_2][) \in cp(CSE, k - 1)\) is applied.

**Example 8.** Let us consider the following chained complex schema element \(cp((OC, =, OC), 2)\). According to the complex schema element \((OC, =, OC)\), all triples of the data instances \(I_{s-1}\) and \(I_{s-2}\) need have the same objects \((\sim^s := OC)\), all the properties need be same \((\sim^p := =)\), and all neighboring data instances of both data instances also need to have the same objects in all their triples \((\sim^o := OC)\). Chaining \(CSE^2\) from Example 6 with a parameter \(k = 2\)
means the PC equivalence is extended for another hop such that for each “equivalent neighbor” at distance 1 there has to be also an “equivalent neighbor” at distance 2.

With the chaining parameterized complex schema element \( CSE^k \), we can consider graph structures of arbitrary size \((R11)\).

### 5.2.2 Label Parameterization

The label parameterization, proposed by Tran et al. [33], applied on our schema elements, fulfills the two requirements \( R7 \) and \( R10 \). First, considering only subsets of properties in the index is a mandatory requirement \((R10)\). Second, considering different subsets of properties for different schema elements allows treating types and properties separately \((R7)\).

**Definition 7** (Label Parameterization). The label parameterization is a function \( lp(SE, P_r) \), which takes as input a schema element \( SE \) and a set of properties \( P_r \subseteq P \) and returns a schema element \( SE_{P_r} \). The returned schema element \( SE_{P_r} \) is a restriction of \( SE \) in terms that all assertions about the triples in \( SE \) only have to be true iff the property of the triple is included in the parameter property set \( P_r \).

Using the properties \( P_{type} \), the label parameterized object cluster \( lp(OC, P_{type}) \) summarizes data instances that have the same set of resources connected with the property \( rdf:type \), i.e., data instances with the same set of types. To ease notation, we simply denote this label parameterized object cluster as \( OC_{type} \). Since we can treat types and properties separately, we can define indices like TermPicker [11] and SchemEX [13] with FLuID.

**Example 9.** The example of TermPicker is illustrated in Fig. 8. TermPicker summarizes data instances that have the same type sets, the same property sets, and the same type set extracted from all neighbors.

\[
EQR_{TermPicker} = (OC_{type} \cap PC_{rel}, =_\emptyset, OC_{type})
\]  

To model TermPicker we make use of the intersection of the label parameterized object cluster \( OC_{type} \) and the label parameterized property cluster \( PC_{rel} \). Since complex schema elements naturally consider property paths, similar to the triple patterns of Loupe [7], they capture which property linked to which type set of a neighbor. For TermPicker, this is undesired, thus, we use \( =_\emptyset \) as predicate equivalence \( \sim^p \) in the complex schema element. The identity equivalence on the empty set is a tautology. This way, no path information is taken into account and basically all types of all neighbors are aggregated.

**Example 10.** The example of SchemEX [13] is illustrated in Fig. 9. SchemEX summarizes data instances that have the same type sets and the same properties linking to neighbors that have the same type sets.

\[
EQR_{SchemEX} = (OC_{type}, =_\text{rel}, OC_{type})
\]
To model SchemEX we combine the label parameterized object cluster $OC_{type}$ and the identity equivalence $=_{rel}$ in a complex schema element. Since complex schema elements naturally consider property paths, they capture which property linked to which type set of a neighbor.

By modeling TermPicker and SchemEX, we showed that FLuID is able to capture property paths (R8) as well as property sets (R9). With the functionalities of the meta model FLuID we cover the requirements R5 to R11 derived from the related work. In addition, we achieved the desired flexibility (R2) by allowing combinations of simple and complex schema structures.

ABSTAT [14], LODeX [5, 6], and Loupe [7] define the same schema structure as SchemEX. ABSTAT, however, uses additionally inferred information using a subtype schema graph. In the subsequent section, we address this requirement (R11).

5.2.3 Inference Parameterization

ABSTAT [14] incorporates additional information derived from an ontology using a subtype schema graph (R3). ABSTAT’s schema graph is constructed by extracting the schema assertions contained in the data graph. We extend the idea of the schema graph from ABSTAT. We include all RDF Schema\(^9\) properties in the schema graph (R3) and not only the type information. Thus, our RDFS schema graph contains hierarchical dependencies of $rdfs:subClassOf$ and $rdfs:subPropertyOf$ in a tree structure with further cross connections regarding $rdfs:range$ and $rdfs:domain$ [17]. Such a schema graph enables search for related types and properties.

Let $SG := (V_C \cup P, E)$ be an edge-labeled directed multigraph and $E \subseteq (V_C \cup P) \times (V_C \cup P)$. The set of nodes is the union of the set of RDF classes and properties. The edge-label function $\phi : E \rightarrow P$ assigns labels from a given set of possible properties $P$ to all edges $e \in E$. Please note, multigraphs allow parallel edges between nodes, thus multiple relationships between nodes are possible in $SG$.

We construct the RDFS schema graph by extracting all triples containing RDFS vocabulary terms, namely all properties $P_{RDFS} = \{rdfs:subClassOf, rdfs:subPropertyOf, rdfs:range, rdfs:domain\}$ and label the schema graph using the RDFS edge-label function $\phi_{RDFS}$. The schema graph constructed using the labeling function $\phi_{RDFS}$ is called $SG_{RDFS}$. With hierarchical dependencies of types and properties represented using our schema graph, additional triples can be inferred using our inferencing parameterization.

**Definition 8** (Inferencing Parameterization). The inferencing parameterization is a function $op(G, SG)$, which takes any data graph $G$ and schema graph $SG$ as input and, based on the entailment rules defined in the schema graph $SG$, returns a data graph $G_{SG}$, which additionally includes all inferred triples.

Please note, ABSTAT uses the subtype schema graph to select a minimal number of types such that all remaining types can be inferred at a later step. With our notion of inferencing, we can infer types and properties. We do not consider when the actual inferencing happens. Our schema graph can either be used to select minimal schema structures and then infer additional information when the index is queried or select maximum schema structures by inferring additional information at index computation time.

5.2.4 Direction Parameterization

Our schema elements $OC$, $PC$, $POC$, and $CSE$ only take outgoing properties into account. Schema structures like Characteristic Sets [12] (see Fig. 4a) consider also incoming properties. To address incoming properties (R5), a directed version of the three simple schema elements can be defined by additionally considering the incoming triples $(x, p, i) \in G$ with $i$ as the subject of the data instance being in object position.

**Definition 9** (Direction Parameterization). The direction parameterization is a function $dp(EQR, \delta)$, which takes as input a schema element $SE$ and one direction parameter $\delta \in \{i, o, u\}$ and returns a schema element $\delta-SE$, respectively. With $i$, we denote to consider only incoming properties, with $o$, we denote to consider only outgoing properties, and with $u$, we denote to consider incoming properties and outgoing properties. The returned schema element $\delta-EQR$ is a restriction of $SE$ in terms that all assertions about the triples in $SE$ are applied on the incoming edges only, the outgoing edges only, or on both.

**Example 11.** The Undirected Property Cluster ($u-PC$) partitions the data graph by comparing the triples based on common incoming and outgoing properties. Thus, the equivalence relation $u-PC$ holds true, iff for each triple $(s-1, p-1, o-1) \in I_{s-1}$ there exists a triple $(s-2, p-2, o-2) \in I_{s-2}$ such that $p-1 = p-2$, and vice versa, and for each triple $(x-1, p-3, s-1) \in G$ there exists a triple $(x-2, p-4, s-2) \in G$ such that $p-3 = p-4$, and vice versa.

\(^9\)https://www.w3.org/TR/2014/REC-rdf-schema-20140225/
5.2.5 Instance Parameterization

Instance Parameterization allows to include schema structures extracted from other data instances. The support for unions of data instances introduces another new functionality in FLuID beyond the state-of-art. We define the instance parameterization, which allows considering data instances that resemble the same real-world entity by exploiting the owl:sameAs property \((R4)\). In the context of Web of Data, the owl:sameAs property is of particular interest since it explicitly states the equivalence of two data instances. To take this information into account, we use the notion of SameAs Instances \([I]_\sigma\), which are equivalence classes of data instances, i.e., basically merged data instances.

Two data instances \(I_{i-1}, I_{i-2}\) are equivalent according to the equivalence relation \(\sigma\), iff there is a property-path in the data graph \(G\) labeled owl:sameAs from \(s-1\) to \(s-2\) [17]. Please note, the property-path is independent of the direction of the owl:sameAs relation.

**Definition 10** (Instance Parameterization). The instance parameterization \(ip\) is a function \(ip(SE, \Delta)\), which extends any schema element \(SE\) to additionally consider all connected data instances following the instance equivalence relation \(\Delta\). The returned schema element \(SE[\Delta]\) is an extension of \(SE\), which restricts the triples to be in \([I]_\Delta\).

**Example 12.** Fig. 11 shows an example graph to illustrate the instance parameterization. According to the object cluster definition, the data instances \(I_{i-1}, I_{i-2}\), and \(I_{i-3}\) are not equivalent. Merging \(I_{i-1}\) and \(I_{i-2}\) to a SameAs instance \([I]_\sigma\) leads to the equivalence of all three data instances \(I_{i-1}, I_{i-2}, I_{i-3}\).

Merging data instances also allows to model Property Cliques [16]. Property Cliques transitively check the co-occurrence of properties and summarize all data instances that have at least one property in common (\(R12\)). Instance parameterization also allows this by using an instance equivalence relation that considers all data instances equivalent if they share at least one property.

Formally, two data instances \(I_{i-1}, I_{i-2}\) are equivalent according to the equivalence relation \(\rho\), iff there is a property \(p-1\) with \(p-1 \in [I_{i-1}]_\rho\) and \(p-1 \in [I_{i-2}]_\rho\). Please note that \(\rho\) takes transitively co-occurring properties into account (see \(R12\)). Thus, \(\rho\) may summarize two data instances that do not share any property in the data graph.

Furthermore, Čebirić et al [16] distinguish “strong equivalence” and “weak equivalence”. The undirected Property Cluster using related properties \(\rho\) resembles the Property Cliques with “strong equivalence” [16].

\[
EQR_{strongPropertyClique} := u-PC[\rho]
\]  

The “weak equivalence” uses either incoming or outgoing edges [16]. Thus, we need to combine the \(PC[\rho]\) with the \(i-PC[\rho]\).
Please note, the intersection $A \cap B$ of two equivalence relations $A, B$ is again a equivalence relation. Furthermore, for all data instances $I_s \in A \cap B$ follows that $I_s \in A$ and $I_s \in B$. However, the same does not hold true for $A \cup B$. The transitivity of $A \cup B$ fails. Thus, if we want to combine equivalence relations to new equivalence relations, we cannot use the simple union of equivalence relations.

Therefore, we define the extended union operator $\cup_{ex}$ as binary relation between to equivalence relations.

**Definition 11 (Extended Union $\cup_{ex}$).** We define $A \cup_{ex} B := A \cup B \cup X$, where $X$ is the set of data instances that are transitively included. To compute $X$, we iteratively add all pairs of data instances $I_{i,1}, I_{i,2}$ to $X$, where there is a pair $I_{i,1}, I_{i,2} \in A$ and a $I_{i,2}, I_{i,3} \in B$, and vice versa.

This allows to model “weak equivalences” since they summarize data instances based on either incoming or outgoing properties using the related property instance parameterization $\rho$. This means, every data instance that shares at least one property is considered equivalent. The Property Cluster (PC) and incoming Property Cluster (i-PC) are computed for the merged data instances $[i_\rho x]$. For comparing either the incoming or the outgoing properties, we use the extended union $\cup_{ex}$ (see Definition 11).

$$ EQR_{weakPropertyClique} := i\text{-PC}[^\rho] \cup_{ex} PC[^\rho] $$  (4)

**Summary** In total, FLuID provides four schema elements and five parameterizations. This allows capturing all schema structures defined by index models found in the related work and beyond. In order to fully support all application scenarios, e.g., the data search scenario described in Section 2, we need to formally introduce the notion of payload to the FLuID meta model. In the following subsection, we define two possible payload elements.

### 5.3 Payload Elements

When instantiating (parameterized) schema elements, application specific information can be attached to the instantiated schema elements by using the notion of payload [4]. The payload comprises information about the summarized data instances. This information can be, e.g., a reference to the location where the data instances where discovered (the so-called data source) or the number of data instances summarized by the specific instantiated schema element.

There is a unlimited number of application scenarios with different requirements to the payload. Thus, we introduce the payload element as a generic element in our FLuID meta model that does not pose any restrictions on the content. These payload elements can be further specified using payload mapping functions. In this section, we define two such payload mapping functions. The resulting payload elements are presented in Fig. 12.

The payload element $ds-1$ is computed by the data source mapping function. It contains the two data source URIs, where the data instances summarized by the instantiated schema element where found. The payload element $ic-1$ is computed by the instance count mapping function and contains one integer value (68), i.e., the number of summarized data instances by the instantiated schema element. In the following, we formally define the two payload mapping functions.

#### 5.3.1 Data Source Payload

The first payload mapping function is the *Data Source Payload ds*. The Data Source Payload $ds$ maps a schema element to the set of data source URIs of all summarized data instances. Schema elements are defined as equivalence classes of bins.
data instances that are sets of triples. Therefore, we can treat an instantiated schema element as a set of data instances. For each data instance, we can extract the data source, e. g., by extracting the graph information of quads\[^{10}\]. Quads are often used by Linked Data Crawlers, e. g., the well known LDSpider [38].

This leads to the first definition of a payload mapping function:

**Definition 12** (Data Source Payload). A schema element \( SE \) is mapped to the data sources using a function \( ds \), which takes a schema element \( SE \) as input and returns all data source URIs, with \( ds(SE) := \bigcup_{I \in SE} \text{context}(I) \). The function \( \text{context} : \mathcal{P}(G) \rightarrow \mathcal{P}(V_G) \) returns all data source URIs of a data instance \( I \) [4].

### 5.3.2 Instance Count Payload

For different applications, e. g., cardinality estimation [12], one may define mapping functions, which return payload elements containing information about the number of data instances summarized by one schema element.

**Definition 13** (Instance Count Payload). A schema element \( SE \) is mapped to the number of summarized data instances using a function \( ic \), which takes a schema element \( SE \) as input and returns an integer, with \( ic(SE) := |SE| \).

### 5.4 Summary

We formally defined FLuID’s schema elements and parameterizations using first-order logic. Our (parameterized) schema elements are defined as equivalence relations over the data graph \( G \). Thus, they allow to flexibly define schema-level indices that define partitions over the data graph. We use our formal definition in the subsequent section to analyze the space and build-time complexity of index models defined with FLuID.

### 6 Complexity Analysis of Index Computations

In this section, we analyze the complexity of the computation process of SLIs defined with FLuID. As described in Sections 4 and 5, we can define various different schema-level index models using FLuID’s schema elements, payload elements, and parameterizations. Based on a specific model of a schema-level index, we can compute a concrete SLI for any given data graph.

In order to estimate the computational complexity, we conduct a space and time analysis of the computation process for the index models defined with FLuID. We primarily focus on the average (amortized) complexity.

To compute an SLI, as first step the data graph needs to be summarized. A data graph is set of unique triples (see Section 4.1). Each triple belongs to exactly one data instance, uniquely identified by the subject URI. To compute an SLI, we need to summarize these data instances to so-called instantiated schema elements. This means, we want to partition the data instances into disjoint subsets. This problem is a special case of the set union problem [39]. The set union problem is well known in complexity theory and states that to find partitions of elements in a set, you need \( \mathcal{O}(n \cdot m \cdot \alpha(m + n, n)) \), with \( \alpha \) as the functional inverse of Ackermann’s function [39]. It is generally accepted that in practice \( \alpha \leq 4 \) holds true [39]. Thus, the index computation can be done in linear time.

However, to compute SLIs, we can estimate the expected average complexity more precisely. Please note, we assume that we can compute the label of each partition in constant time since we can assume that we can access hash maps in constant time [40, 13]. Below, we analyze in detail the amortized average space and time complexity of the schema-level index computation.

Let us assume we have a data instance \( I_{s,1} := \{ (s-1, p-1, o-1), (s-1, p-2, o-2) \} \) and we want to compute a property cluster (PC). For each triple in \( I_{s,1} \), we extract the property to generate the property set, i. e., \( ps := \{ p-1, p-2 \} \). We instantiate the property cluster with the property set \( ps \) as instantiated schema element in the SLI. We can identify this particular instantiated schema element by storing it in a hash map, where the key is the hash value of the property set. The payload, e. g., the number of summarized data instances, is stored as value. For another data instance \( I_{s,2} := \{ (s-2, p-1, o-3), (s-2, p-2, o-4) \} \), the same property set is extracted, thus, the same instantiated schema element with the same hash value. The payload can be updated by increasing the data instance counter. Consequently, the instantiated schema element summarizes both data instances.

One can see that the computational expensive task is the extraction of the schema of each data instance, e. g., extracting the property set. However, we can exploit that every schema element in the FLuID model is defined as equivalence relation. In particular, for one data instance \( I_s \) we can instantiate exactly one (parameterized) schema element, e. g., one

---

\[^{10}\]https://www.w3.org/TR/n-quads/
property cluster (PC) and one label parameterized object cluster ($OC_{type}$) etc. We define complex index models by combining simple schema elements with complex schema elements, where we can include neighboring data instances in the schema structure. As one can see in Example 10, SchemEX uses type sets of neighboring data instances to compute the schema of the actual data instance. Therefore, one may assume that depending on the in-degree of a data instance, we have to extract the schema more than once. Instead, we store for each data instance the computed schema elements in a hash map. This avoids the expensive task of extracting the schema of data instances more than once.

**Corollary 1.** For each data instance $I_s \subseteq G$, we need to compute the schema according to each schema element in the index definition only once.

Let us consider the example illustrated in Fig. 13. For the data graph (a), the schema defined by SchemEX is computed (b) and (c). For each data instance $I_{s-1}$ to $I_{s-7}$, a object cluster is computed and instantiated. Since some data instances share the same schema according to the simple schema elements, they are summarized by the same instantiated simple schema element. For example, $I_{s-1}$ and $I_{s-5}$ share the same type Book and consequently are summarized by $pc_{type}$-1 (see Fig. 13c). When we know which instantiated schema element is computed from which data instance, the instantiated complex schema elements can re-use the instantiated simple schema elements without a need to compute the schema again. For the chained complex schema elements, the same principal applies.

For the space complexity of the index, the important factor is how well the schema elements can summarize the data instances, i.e., how many different schema elements need to be instantiated in the schema-level index. The worst case is that no two data instances are summarized by the same instantiated schema element, i.e., all instantiated schema elements in the index are singletons and only summarize exactly one instance. In this worst case, each instantiated schema element is a new entry for our hash map. Thus, the upper bound for the time complexity and the upper bound for the space complexity are identical.
6.1 Complexity of Schema Elements

Given a definition of concrete schema-level index model with out FLuID meta model, we denote with s the number of simple schema elements and with c the number of complex schema elements used in this index model. The parameterizations are applied on these simple and complex schema elements and pose further restrictions or relaxations. We analyze their impact on the complexity subsequently.

The data graph $G$ contains $n$ triple. Since data instances partition the data graph, each triple is assigned to exactly one data instance. Simple schema elements partition the data graph by summarizing data instances $I_{s,1} \subseteq G$ and $I_{s,2} \subseteq G$ by comparing only triples $(s-1, p-1, o-1) \in I_{s,1}$ to only triples $(s-2, p-2, o-2) \in I_{s,2}$. Thus, for each simple schema element in the index model, we only need one operation for each triple in the data graph.

Complex schema elements summarize based on simple schema elements or based on the identity equivalence (Definition 5). Thus, without parameterizations, we have linear space and time complexity in the order of $\Theta((s + c) \cdot n)$, with $n$ triples in the data graph.

6.2 Complexity of Direction Parameterization

With the direction parameterization, incoming and/or outgoing triples can be considered (Definition 9). The incoming property of data instance $I_{s,1} \subseteq G$ is the outgoing property of another data instance $I_{s,2} \subseteq G$. Thus, for undirected schema elements $u$-EQR, we may have to consider each triple twice. However, this is still linear w.r.t to the number triples $n$ in the data graph.

6.3 Complexity of Label Parameterization

The label parameterization reduces the number of considered objects and properties for each simple schema element by restricting the properties $p$ to be in the set $P$. Considering all excluded properties $P \setminus P_1$ and that each property can occur more than one time in the dataset, we can define a constant $l \geq |P \setminus P_1|$, which denotes the number of occurrences of excluded properties in the dataset. Thus, the space complexity is still linear, but we can find a new lower average complexity of $\Theta((s + c) \cdot (n - l))$. The time complexity is unchanged.

6.4 Complexity of Instance Parameterization

The instance parameterization aggregates data instances. Thus, it does not impact the overall size of the index since no new information is added. Aggregating data instances to unions can be done in principal in constant time like triples are aggregated to data instances in constant time using hash maps. As shown by Cebiric et al. [16], the number of merges of data instances can be bound by the number of distinct properties appearing in the data graph $G$. For sameAs instances $\sigma$, the number is 1. For transitively related property $\rho$, this factor can be growing linear in the size of the data set. This leads to a worst case build-time complexity of $O(n^2)$. However, in their experiments, the average complexity was shown to be close to a constant factor.

6.5 Complexity of Inference Parameterization

The inferencing parameterization requires additional space to store all inferred types and properties. As defined in Section 5.2.3, types and properties are only added to the schema graph, if there exists a triple in the dataset using a property in $P_{RDFS}$. In the input data graph $G$ of size $n$, we find $r$ many schema triples that use a property in $P_{RDFS}$. In general, we can assume that $r \ll n$ for real-world datasets. The complexity depends on the number of additional triples $t$ that can be inferred from the $r$ triples with $t \leq r$. Thus, we have an upper bound for space complexity using the inference parameterization in the order of $\Theta((s + c) \cdot (n - l) \cdot t)$.

For example, we have the triples $(s_1, p_1, o_1) \in G$, $(s_2, p_2, o_2) \in G$, and $(p_1, rdfs:subPropertyOf, p_2) \in G$. We add the RDFS statement about $p_1$ to the schema graph. This allows inferring the triple $(s_1, p_2, o_1)$ as well. However, the schema graph does not contain inferreable information about $p_2$. Therefore, for this triple, no new triple can be inferred.

Please note, with a linear dependency $g = f(n)$, we end up with a quadratic complexity. This means, when using inference parameterization, we have in the worst case a quadratic complexity $O(n^2)$. In the worst case, we extract a fully connected (complete) schema graph. That means, for each indexed triple, all $r$ possible triples in the schema graph are inferred. Furthermore, the worst case requires all triples in the data graph to use properties from $P_{RDFS}$. This is unrealistic for real-world datasets [17].
To support this claim, we analyzed in a preliminary experiment two datasets that were crawled from the Web of Data [17]. The first dataset, TimBL-11M\textsuperscript{11} with 11 million triples has 2.0\% RDFS properties. The second and larger dataset DyLDO-127M\textsuperscript{11} (127 million triples) has 0.6\% RDFS properties [17]. As the amount of RDFS properties is quite low, we can consider the parameter $g$ constant. Thus, there is no linear dependency of $g$ and $n$ and overall the average case complexity is linear in these two cases.

The schema graph can be implemented using hash maps, which guarantees constant time for lookup and addition operations. Inferencing operations are linear in the number of inferrable types and properties. Thus, we have the same time complexity as the space complexity. Please note, the triples that are added to the schema graph from the input data graph may be excluded from computing the index, i.e., triple using a property in $P_{RDFS}$ are excluded using the label parameterization.

### 6.6 Complexity of Chaining Parameterization

The chaining parameterization defines the data instances’ neighborhood up to a maximum path length of $k$. As illustrated in Fig. 13, chaining complex schema elements increases the number of instantiated complex schema elements that need to be stored and computed. However, according to Corollary 1, we can re-use already instantiated schema elements. Thus, the complexity grows linearly in $k$.

### 6.7 Summary

The space complexity of the index is in the order of $\Theta((s + c) \cdot k \cdot (n - l) \cdot t)$ and the overall time complexity is in the order of $\Theta((s + c) \cdot k \cdot n \cdot t)$ with $s$ simple schema elements and $c$ complex schema elements in the index model definition, the chaining parameter $k$, $l$ excluded properties in the data graph using the label parameterization, and $t$ inferrable triples using the inference parameterization. Thus, indices defined with FLuID can be on average computed in linear time and space with respect to the number of triples $n$.

We analyzed the amortized space complexity as well as the amortized build-time complexity. In principal, each index model can be computed on average in linear time using hash-maps.

However, growing linear in space and time can in practice still be unfeasible for graphs of arbitrary size if the constant factor is too large. Thus, we conduct an empirical evaluation in the subsequent section to estimate the storage footprint on real-world datasets. Furthermore, we conduct an evaluation of the quality of a stream-based computation approach.

### 7 Empirical Evaluation of Index Models defined with FLuID

In this section, we empirically investigate the scalability of the index computation approach (see R1). For the empirical evaluation, we focus on the data search application scenario outlined in Section 2. The search engine for the Web of Data aims to return data sources containing data instances that match a given information need, e.g., structural query for bibliographic metadata records. Therefore, the underlying index needs to scale for the size of the Web of Data. Our evaluation of the scalability is twofold. First, we evaluate the storage requirements of computed indices. Second, we evaluate the quality of our stream-based computation approach.

We run experiments with a total of seven different index models out of which five are directly taken from the related work. Furthermore, we evaluate two alternative implementations of the SchemEX+U+I index model. The SchemEX+U+I index model is a semantic version of SchemEX, where we include SameAs instances $\sigma$ and an RDFS schema graph $SG_{RDFS}$.

The first index model resembles Characteristic Sets and is defined using an undirected property cluster.

\[
\text{Characteristic Sets} := (G, u-PC, PAY_{ds})
\]  

The second index model resembles the Weak Property Cliques [15, 16]. The equivalence relation $EQR_{WeakPropertyClique}$ is formalized in Eq. (4).

\[
\text{WeakPropertyClique} := (G, EQC_{WeakPropertyClique}, PAY_{ds})
\]  

The third index models resembles SemSets and is defined using a property-object cluster.

\[
\text{SemSets} := (G, POC, PAY_{ds})
\]  

\textsuperscript{11}Detailed descriptions of the datasets in Section 7.1
The equivalence relations $EQR_{\text{TermPicker}}$ and $EQR_{\text{SchemEX}}$ of TermPicker and of SchemEX are formalized in Eq. (1) and in Eq. (2), respectively.

\[
\text{TermPicker} := (G, EQR_{\text{TermPicker}}, PAY_{ds})
\]

\[
\text{SchemEX} := (G, EQR_{\text{SchemEX}}, PAY_{ds})
\]

All five indices are also visualized in Fig. 4. The formal definition of SchemEX+U+I index using our meta model FLuID is:

\[
\text{SchemEX}+\text{U}+\text{I} := (G_{SG_{RDFS}}, (OC_{type} = \text{rel}, OC_{type})[\sigma], PAY_{ds})
\]

We change the complexity of these indices by additionally applying the chaining parameterization $cp$ on each of the seven SLIs. Reasonable values for the chaining parameter $k$ are 0, 1, and 2 [33]. The payload of each SLI comprises the data sources only. This is required for our data search application scenario outlined in Section 2.

### 7.1 Datasets

We use two datasets of the Web of Data with different characteristics. Although reasonably large, both datasets allow us to compute a gold standard. Please note that we did not perform any pre-processing despite removing invalid (not parsable) triples.

The first dataset is the TimBL-11M, which contains about 11 million triples (673 Thousand data instances) [13]. The crawl was conducted with a breadth-first search starting from the FOAF profile of Tim Berners-Lee and allows us to compare the approximation results to the previous work of Konrath et al. [13].

The second dataset is the DyLDO-127M, which contains about 127 million triples (7 million data instances) [41]. The Dynamic Linked Data Observatory (DyLDO) provides regular snapshots from the Web of Data. We use their first snapshot crawled from about 95,000 seed URIs. This crawl was done with a breadth-first search but limited to a crawling depth of two [41]. Although there are more recent DyLDO snapshots, they are decreasing in size. Thus, we decided to take the first and largest one.

### 7.2 Evaluation 1: Index Size

We evaluate the index size for indices computed over the two datasets when stored as an RDF graph.

To this end, we compare the number of triples in the index to the number of triples in the dataset (compression ratio). Furthermore, we compare the number of different schema elements in the index to the number of data instances in the dataset (summarization ratio). This ratio gives an idea of how well the defined schema structure can summarize data instances on the Web of Data. For the compression and summarization ratio, we use exact indices. This means, we loaded the complete data graph into the main memory, before we started the index computation process.

#### 7.2.1 Results

We present the compression ratio and summarization ratio in Table 2. As one can see, there is a huge variety in terms of how well indices compress and summarize the data. For the TimBL-11M dataset, SemSets’ compression ratio (with $k = 1$) is about 10 times larger than all other indices with the exception of Weak Property Cliques (only about 5 times larger). For the DyLDO-127M dataset, SemSets’ compression ratio (with $k = 1$) is up to 75 times larger. Additionally, there is no increase in index size from $k = 1$ to $k = 2$, but a more than ten-times increase from $k = 0$ to $k = 1$. The same phenomenon appears for the summarization ratio.

SemSets is the only index using only object clusters. In contrast, the other indices either ignore objects or consider their type information only. SemSets has a summarization ratio of $20\% - 25\%$, i. e., on average 4 – 5 data instances share the same schema structure. The smallest index, Characteristic Sets, has a summarization ratio of $0.3\%$, i. e., about 330 data instances share the same schema structure.

A notable exception is the Weak Property Clique. The indices using Weak Property Cliques result in the most condensed summarization (summarization ratio of less than $0.1\%$).
When considering the semantics of RDFS and owl:sameAs, an existing scalable index computation approach for SLIs is the stream-based approach, which was shown to work on very large datasets. The stream-based approach observes the data graph over a stream of triples using a window technique from stream databases [18]. This allows the index computation over graphs of arbitrary size. However, with the scalability of the approach come approximation errors since only a part of the data graph that fits into the window is kept simultaneously in the main memory. Thus, we potentially extract incomplete schema structures.

### Table 2

| k | # | Characteristic Sets | Weak Property Clique | SemSets | SchemEX | TermPicker | SchemEX +U+I |
|---|---|---------------------|----------------------|---------|---------|------------|-------------|
| 0 | t | na (na)             | na (na)              | 0.3M (2.9%) | 0.3M (2.9%) | 0.3M (2.9%) | 0.4M (3.8%) |
| 0 | e | na (na)             | na (na)              | 2.8T (0.4%) | 2.8T (0.4%) | 2.8T (0.4%) | 3.1T (0.5%) |
| 1 | t | 0.7M (6.5%)         | 1.9M (17.9%)         | 7.6M (69.2%) | 0.8M (6.9%) | 0.7M (6.5%) | 0.8M (7.1%) |
| 1 | e | 9.6T (1.4%)         | 74 (< 0.1%)          | 13.9T (20.6%) | 12.0T (1.8%) | 10.8T (1.6%) | 11.3T (1.7%) |
| 2 | t | 1.6M (14.6%)        | 1.1M (9.9%)          | 7.6M (69.2%) | 1.4M (12.5%) | 1.8M (16.0%) | 1.8M (15.9%) |
| 2 | e | 37.2T (5.5%)        | 50 (< 0.1%)          | 13.9T (20.6%) | 27.7T (4.1%) | 37.3T (5.5%) | 31.0T (4.6%) |

For the TimBL-11M dataset and the DyLDO-127M dataset and for the chaining parameter \(k \in \{0, 1, 2\}\), we present \#t and \#e. \#t is the number of triples in millions (M) in the SLI and in brackets the ratio compared to the number of triples in the dataset (compression ratio). \#e is the number of schema elements in thousands (T) in the SLI and in brackets the ratio compared to the number of instances in the dataset (summarization ratio).

However, the combination of either incoming or outgoing related properties in Weak Property Cliques leads to considerably large compression ratio. Weak Property Clique indices are more than twice the size of Characteristic Sets indices.

When considering the semantics of RDFS and owl:sameAs in SchemEX+U+I the index size increases compared to SchemEX by about 3% more triples. Despite being a larger index in terms of the number of triples, for \(k = 1\) fewer schema elements are instantiated when including the semantics of RDFS and owl:sameAs. For \(k = 0\) and \(k = 2\), SchemEX+U+I instantiates more schema elements than SchemEX.

#### 7.2.2 Summary

Including the semantics of owl:sameAs and RDF Schema always increases the size of the index, it is independent to the number of instantiated schema elements that partition the data graph. Furthermore, combining schema elements using the extended union operator \(\cup_{ex}\) and property related instances \(\rho\) summarizes data instances into a handful of instantiated schema elements with a considerably large size. A more complex index definition does not automatically increase the index size compared to more simplistic index definitions.

#### 7.3 Evaluation 2: Quality of a Stream-based Computation Approach

An existing scalable index computation approach for SLIs is the stream-based approach, which was shown to work efficiently for large datasets requiring only limited computational power (e.g., single desktop computer) [13].

In the stream-based approach, the data graph is observed over a stream of triples using a window technique from stream databases [18]. This allows the index computation over graphs of arbitrary size. However, with the scalability of the approach come approximation errors since only a part of the data graph that fits into the window is kept simultaneously in the main memory. Thus, we potentially extract incomplete schema structures.

**Procedure**  To evaluate the SLIs with respect to the approximation quality, we compute the indices with a window size of 1k, 100k, and 200k data instances and compare them to a gold standard. Each gold standard is an exactly computed index on our server machine with 1TB main memory, i.e., we loaded the complete data graph into the main memory, before we started the index computation process (see also Section 7.2). Our (not optimized) implementation used for the evaluation requires about 40 times as much space for processing the gold standard in-memory than the raw disk space of the dataset.

For SchemEX+U+I, we evaluate two different implementations, i.e., SchemEX+U+oi and SchemEX+U+pl. In SchemEX+U+oi, the RDFS schema graph is computed on-the-fly and in SchemEX+U+pl, the RDFS schema graph is computed in a pre-processing step. This means for SchemEX+U+oi, each triple from the input data graph is either added to the schema graph or added to the data instances in the window. Thus, we start with an empty schema graph and add statements over time. For SchemEX+U+pl, we require two passes over the graph. In the first pass, we extract
all triples that use a property in $P_{RDFS}$ and built the schema graph. In the second pass, we compute the schema for the data graph using the previously computed schema graph.

**Queries** Following Konrath et al. [13], we evaluate if the payload, i.e., the data source, is assigned to the correct schema element during the index computation. To this end, we apply queries on the approximate index as well as on the gold standard.

We distinguish simple queries (SQ) and complex queries (CQ). Simple queries match (parameterized) object cluster, which can also be sub-schema structures in our experiments. These simplest queries appear to be the most used SPARQL queries in search scenarios [42]. The complex queries match the (parameterized) complex schema elements, thus, the complete schema structure as defined by the specific index model.

The queries are generated from the schema elements instantiated in the gold standard. To this end, we extract all combinations of types, properties, and resources from the gold standard schema elements. The resulting combinations of types, properties, and resources are used as queries in our experiments.

**Example 13.** This example illustrates, how the queries are constructed for our experiments. We compare two indices, the gold standard SLI and the approximate SLI, e.g., computed with a window size of 100,000 instances. We extract the type set $\{\text{OnlineAccount, UserAccount}\}$ from a label parameterized object cluster $SE_5$ instantiated in a gold standard SLI. We construct a query $q$ querying for all schema elements with these types (see example query Listing 2). This query also includes supersets, i.e., we also query for schema elements that have more than these two types. We issue the same query on both indices. For the returned schema elements, we compare the attached data sources.

Listing 2: Example of a simple query that is selecting data sources from object clusters, where the objects are “Online Account” (line 4) and “User Account” (line 6).

```sql
1 SELECT ?ds
2 WHERE {
3 ?tc fluid : has Attribute ?a .
4 ?a fluid : getLinkObject <http://xmlns.com/foaf/0.1/OnlineAccount> .
5 ?tc fluid : has Attribute ?b .
6 ?eqc fluid : has Subject Equivalence ?tc .
7 ?eqc fluid : has Payload ?pe .
8 ?pe fluid : payload ?ds .
9 }
```

**Metrics** The approximation quality is measured by comparing the set of data sources returned from each query using the harmonic F1-measure. Following Konrath et al. [13], we define precision as $Pr(q) = \frac{|D_{gold}\cap D_{win}|}{|D_{win}|}$ and recall as $Re(q) = \frac{|D_{gold}\cap D_{win}|}{|D_{gold}|}$, where $D_{gold}$ denotes the data sources from the gold standard and $D_{win}$ the data sources from the stream-based index computation. With precision and recall, we can compute the commonly known F1-score $F1(q) = 2 \cdot \frac{Pr(q) \cdot Re(q)}{Pr(q) + Re(q)}$.

**7.3.1 Results**

Table 3 shows the approximation quality in terms of F1-score for the seven indices with chaining parameter $k \in \{0, 1, 2\}$. Please note, for indices with a chaining parameter $k = 0$, the simple queries and the complex queries have the same complexity (Definition 6). Please also note, Characteristic Sets and Weak Property Cliques ignore connected resources (see Fig. 4a). Thus, simple queries are not available (na) since they query for parameterized object cluster. Furthermore, the complex queries for Characteristic Sets with $k = 1$ are comparable to the simple queries for the other indices, since the Characteristic Set is modeled using one simple schema element.

From the results of our experiments, we can state that simple queries consistently show higher F1-scores than complex queries. This is not surprising since less information is required to satisfy the information need.

TermPicker and Weak Property Cliques are the only indices that have a higher F1-score on the DyLDO-127M dataset than on the TimBl-11M dataset. As described in Section 5.1.2, TermPicker is the only index considering no property paths, but sets of properties independent for each hop. This restriction is the only difference in the schema structure compared to SchemEX. Still, TermPicker has a 50% lower F1-score than SchemEX on the TimBl-11M dataset.

Weak Property Clique is the only index that used property-related instances $\rho$ and the extended union $\cup_{ex}$. The F1-scores of Weak Property Cliques are the highest in our experiments. The difference in F1-scores compared to the other indices is the highest for $k = 2$. 

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We computed the Pearson and Spearman correlation coefficient for the three different cache sizes. We also observe an influence of the characteristics of the crawled dataset on the approximation quality. All indices have calculated a coefficient of 0.10 for the simple queries and 0.80 for the complex queries.

The approximation quality of an index computed in a stream-based approach depends on various factors. First, we observe a significant positive correlation between compression ratio and summarization ratio. Compression ratio, summarization ratio, and approximation quality depend on the queries as well as the characteristic of the dataset. However, we found a significant positive correlation between compression ratio and summarization ratio. From the results of our experiments, we conclude that schema-level indices perform very differently in terms of compression ratio and summarization ratio, and approximation quality depending on the queries as well as the characteristic of the dataset. However, we found a significant positive correlation between compression ratio and summarization ratio.

We also observe an influence of the characteristics of the crawled dataset. Second, simple queries consistently outperform complex queries. Third, a larger window size improves the quality often only marginally. Finally, more complex index structures, i.e., index models using more (parameterized) schema elements, do not indicate a low approximation quality. SemSets only have a small drop in F1-score from 1 k = 1 to k = 2. SchemEX+U+ol consistently has a lower F1-score than SchemEX. Since the schema graph is generated on-the-fly, i.e., while the triple stream is analyzed, the schema graph is incomplete until the last triple using a RDFS property is processed. SchemEX+U+pl uses a schema graph that was generated in a pre-processing step. Compared to SchemEX+U+ol, SchemEX+U+pl has consistently higher F1-scores. Furthermore, for window sizes 100k and 200k, SchemEX+U+pl has higher F1-scores than SchemEX (except for the case of k = 2 applied on TimBL-11M dataset).

We also observe an influence of the characteristics of the crawled dataset on the approximation quality. All indices have on average a 0.15 lower F1-score on the DyLDO-127M dataset compared to the TimBL-11M dataset. In particular, simple queries achieve much lower F1-scores. On average, simple queries have 0.25 lower F1-scores and complex queries have 0.04 lower F1-scores on the DyLDO-127M dataset compared to the TimBL-11M dataset.

Furthermore, larger window sizes consistently improve the F1-scores. In contrast, on-the-fly inferencing lowered the F1-scores in our experiments compared to no inferencing.

### 7.3.2 Summary

The approximation quality of an index computed in a stream-based approach depends on various factors. First, we observe a influence of the characteristics of the crawled dataset. Second, simple queries consistently outperform complex queries. Third, a larger window size improves the quality only marginally. Finally, more complex index structures, i.e., index models using more (parameterized) schema elements, do not indicate a low approximation quality. Below, we reflect on our key results and discuss them in detail.

### 7.4 Discussion

From the results of our experiments, we conclude that schema-level indices perform very differently in terms of compression ratio, summarization ratio, and approximation quality depending on the queries as well as the characteristic of the dataset. However, we found a significant positive correlation between compression ratio and summarization ratio. We computed the Pearson and Spearman correlation coefficient for the n = 32 SLIs reported in Table 2. For Pearson, we calculated an coefficient of 0.84 (p < 0.0001) and for Spearman an coefficient of 0.64 (p < 0.0001) with a degree of freedom df = 30.

However, while including the semantics of owl:sameAs and RDF Schema always increases the size of the index, it can reduce the number of instantiated schema elements that partition the data graph. Furthermore, there is a significant negative correlation between summarization ratio and approximation quality of a stream-based computation approach. We computed the Pearson and Spearman correlation coefficient for the three different cache sizes k = 1k, 100k, and 200k. We compared the reported F1-scores for the complex queries (for k = 0; we used the simple queries) for each cache size processed.

| TimBL-11M | Characteristic Sets | Weak Property Clique | SemSets | SchemEX | TermPicker | SchemEX+U+ol | SchemEX+U+pl |
|-----------|---------------------|----------------------|---------|----------|------------|--------------|--------------|
| k | Q | 100k | 200k | FL | ID: A M | m | FL | ID: A M | m |
| 0 | SQ | na | na | na | .94 | .97 | .98 | .94 | .97 | .98 | .85 | .91 | .93 | .92 | .97 | .98 |
| 1 | SQ | .60 | .77 | .78 | .73 | .76 | .76 | .73 | .75 | .76 | .22 | .25 | .29 | .65 | .70 | .71 | .71 | .74 |
| 1 | CQ | .04 | .18 | .21 | .42 | .44 | .45 | .26 | .32 | .33 | .11 | .24 | .26 | .04 | .05 | .07 | .06 | .18 | .19 | .16 | .22 | .23 |

| DyLDO-127M | Characteristic Sets | Weak Property Clique | SemSets | SchemEX | TermPicker | SchemEX+U+ol | SchemEX+U+pl |
|-----------|---------------------|----------------------|---------|----------|------------|--------------|--------------|
| k | Q | 100k | 200k | FL | ID: A M | m | FL | ID: A M | m |
| 0 | SQ | na | na | na | .56 | .57 | .58 | .56 | .57 | .58 | .56 | .57 | .58 | .44 | .47 | .47 | .57 | .59 | .60 |
| 1 | SQ | .68 | .71 | .72 | .89 | .92 | .89 | .31 | .32 | .33 | .16 | .17 | .18 | .12 | .14 | .15 | .13 | .13 | .14 | .39 | .41 | .43 |
| 1 | CQ | .03 | .13 | .15 | .51 | .58 | .58 | .23 | .25 | .25 | .08 | .09 | .09 | .04 | .04 | .04 | .06 | .07 | .08 | .07 | .08 | .09 |
We also observe an influence of the characteristics of the crawled dataset on the approximation quality. All indices have a lower summarization ratio can lead to a higher F1-score. This means, index structures that summarize well, i.e., we found a negative correlation coefficient with a p-value less than 0.05 (see Table 2), there is high probability that data instances are summarized correctly by chance and thus result in high F1-scores.  

One should note that the extreme summarization ratio of Weak Property Cliques also produces the highest F1-scores. This can point to another explanation for the observed correlation. With only a handful of schema elements in the index (see Table 2), there is high probability that data instances are summarized correctly by chance and thus result in high F1-scores.

We also observe an influence of the characteristics of the crawled dataset on the approximation quality. All indices have on average a .15 lower F1-score on the DyLDO-127M dataset compared to the TimBL-11M dataset. The TimBL-11M dataset was crawled starting from only one seed URI. In contrast, the DyLDO-127M dataset was crawled using more than 95,000 seed URIs from 652 different pay-level domains [41]. Therefore, there are a lot of different connected components in the DyLDO-127M dataset. The dataset characteristic also influences the size of the index. On average, the compression ratio of indices computed for the TimBL-11M dataset is 14.9% and for the DyLDO-127M dataset it is 10.5%. Furthermore, our results suggest that the object cluster does not summarize data instances well and particularly not over two hops.

Comparing the approximation quality results of the evaluation 2 (Table 3) to the results in the data instance summarization Table 2 brings additional insights into why TermPicker and Weak Property Cliques are the only index model that have consistently higher F1-scores on the DyLDO-127M dataset. TermPicker has larger schema elements (in terms of the number of data instances summarized) than SchemEX, which explains from the index configuration. When the queries do not correctly retrieve the data sources of a large schema element, it has more impact on the overall F1-score than when they do not correctly retrieve the data sources of a small schema element. However, Characteristic Sets have the largest schema elements, but not a generally lower F1-score. As noted above, for $k = 1$ the CQ should be compared to the SQ.

The presented results might generalize to other datasets and different index structures. To avoid bias, we chose two datasets with different characteristics. Although the largest dataset has more than 127 million triples, this resembles only a fraction of the Web of Data. For example, the LODLaundromat (LOD38B) dataset contains more than 38 billion triple that require more than 6.2TB disk space. However, the dataset size has to be limited to compute a gold standard index our server machine.

Furthermore, we empirically evaluated carefully selected, realistic schema-level indices. These were chosen from the related work covering four different application scenarios (see Section 3).

Furthermore, we investigated the possibility to natively support all OWL properties. However, we analyzed the TimBL11M dataset, the DyLOD127M dataset, the Billion Triple Challenge 2014 dataset, and the LODLaundromat dataset with respect owl:sameAs. Aggregated over all four datasets, we found that about 99% of all OWL properties are owl:sameAs. Thus, we do not expect a great impact on schema-level indices when including other OWL properties. Including other OWL properties as an extension of FLuID is possible if needed in the future, i.e., other owl properties become widely used.

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**Table 4: Results of correlation analysis between summarization ratio and approximation quality (F1-score).** Pearson and Spearman coefficients and respected p-values calculated for $n = 32$ SLIs for three different cache sizes with a degree of freedom $df = 30$.

| Cache Size | Pearson | Spearman |
|------------|---------|----------|
| 1k         | $-0.35$ | $p < 0.05$ | $-0.74$ | $p < 0.0001$ |
| 100k       | $-0.38$ | $p < 0.04$ | $-0.74$ | $p < 0.0001$ |
| 200k       | $-0.38$ | $p < 0.04$ | $-0.75$ | $p < 0.0001$ |

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[12] http://lodlaundromat.org/
8 Implementation of the FLuID Meta Model

We implemented a scalable index computation framework for the FLuID meta model. As foundation, we used the SchemEX framework [13]. Our FLuID framework computes schema-level indices defined with our FLuID meta model for graphs of arbitrary size and is available under an open source license\(^\text{13}\).

Our FLuID model is defined using first-order logic. Although the definitions are quite formal, they allow us to formulate our schema elements and parameterizations also in a high-level descriptive language, as shown by our grammar in Listing 4. Our FLuID Framework parses index model definitions formulated with our grammar and combines so-called `SchemaElementFactories` with `PayloadElementFactories`, accordingly. These factories resemble the schema elements and the payload elements from the FLuID model. They define how to compute and instantiate schema elements and payload elements for the SLI given the input data graph. Multiple index models can be defined that are computed in parallel for the same data graph. Each index model is separated by `&&` and stored in a different output directory or repository of a triple store.

For example, in Listing 3, we defined SchemEX and Characteristic Sets. Each (parameterized) schema element is translated into one `SchemaElementFactory`.

Listing 3: Sample Properties File to compute SchemEX and Characteristic Sets in parallel.

```
1 # Sample Properties File
2 schema=(OC_type, =_rel, OC_type) && u-PC
3 output=SchemEX&&CharSets
```

Furthermore, the FLuID Framework offers a selection of pre-processing functionalities, e.g., `NoisyDataFilter`, that can be combined in `BasicQuadPipeline`. Computed SLIs can be queried using our `FLuID Query Engine`. The query engine transforms SPARQL queries for types and properties into queries that can be executed over the index. The querying works for any index defined with FLuID. Please find more details in our GitHub repository\(^\text{14}\).

To implement the data search scenario described in Section 2, we extended the existing data source search engine LODatio \[^{4}\] to make use of additional features offered by FLuID. LODatio+\[^{15}\] uses a schema-level index to search for data sources with relevant information.

9 Conclusion

We have presented the novel, formal meta model FLuID to define arbitrary schema-level index models. We demonstrated how it covers all functionalities of existing works and beyond. The empirical evaluation revealed a lot of variation in terms of compression ratio, summarization ratio, and approximation quality for different datasets and different queries. However, they also indicate a correlation between summarization ratio and approximation quality of a stream based computation approach. Our results reinforce the hypothesis that there is no single schema-level index that fits for all application scenarios. Thus, clearly multiple index models for the Web of Data are needed. FLuID’s schema elements can be easily combined and efficiently computed. Thus, with FLuID it is easier to develop and compare schema-level indices.

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\[^{13}\]https://github.com/t-blume/fluid-framework
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\[^{15}\]http://lodatio.informatik.uni-kiel.de/
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10 Appendix

Listing 4: Syntax of the declarative FLuID language in EBNF.

%direction parameterization%
UNDIRECTED = "u−"
INCOMING = "i−"
OUTGOING = "o−"

%label parameterization%
REL_OP = "_"

%labeling parameterization%
BISIM_OP = "∗"

%instance parameterization%
INSTANCE_OP = "|"

%extended union%
UNION = " U "

%intersection%
INTERSECTION = " I "

%complex schema element%
CSE_SEP = ","
CSE_OPEN = "("
CSE_CLOSE = " )"

%simple schema elements%
OBJECT_CLUSTER = "OC"
PROPERTY_CLUSTER = "PC"
PROPERTYOBJECT_CLUSTER = "POC"

%basic equivalence relations%
TAUTOLOGY = "T"
IDENTITY = "≡"

%regularly used property sets%
TYPES = "type"
RELATIONS = "rel"
RDFS = "rdfs"

%instance sets for instance parameterization%
SAME_AS = "~"
RELATED_PROPERTY = "p"

NUMBER = << [0−9] + >>

%productions%
SCHEMA_ELEMENT = SIMPLE_SCHEMA_ELEMENT | COMPLEX_SCHEMA_ELEMENT | COMBINE_OP SIMPLE_SCHEMA_ELEMENT | COMPLEX_SCHEMA_ELEMENT | [INSTANCE_PARAM] | [BISIM_PARAM];

SIMPLE_SCHEMA_ELEMENT = [DIRECTION_OP] OBJECT_CLUSTER [LABEL_PARAM] | [DIRECTION_OP] PROPERTY_CLUSTER [LABEL_PARAM] | [DIRECTION_OP] PROPERTYOBJECT_CLUSTER [LABEL_PARAM] | BASIC_ELEMENTS;

COMPLEX_SCHEMA_ELEMENT = CSE_OPEN SIMPLE_SCHEMA_ELEMENT {COMBINE_OP SIMPLE_SCHEMA_ELEMENT} CSE_SEP SIMPLE_SCHEMA_ELEMENT {COMBINE_OP SIMPLE_SCHEMA_ELEMENT} CSE_SEP SCHEMA_ELEMENT CSE_CLOSE;

BASIC_ELEMENTS = TAUTOLOGY | IDENTITY [LABEL_PARAM];

COMBINE_OP = UNION | INTERSECTION;
DIRECTION_OP = UNDIRECTED | INCOMING | OUTGOING;
BISIM_PARAM = BISIM_OP NUMBER;
LABEL_PARAM = REL_OP PROPERTY_SET;
INSTANCE_PARAM = INSTANCE_OP INSTANCE_SET;

INSTANCE_SET = SAME_AS | RELATED_PROPERTY;
PROPERTY_SET = TYPES | RELATIONS | RDFS;