Elastic Buckling Behavior of Thin Plate with Circle Holes under Axial Compression

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Abstract. The stability of members will be affected because the stress concentration resulted from the holes in thin plates. Elastic buckling performance of the perforated circle plate under axial compression is analyzed using a general finite element software ABAQUS in this paper. Results show that the buckling stability coefficient of plate is significantly low when the hole locates at the short side of the plates. The buckling stability coefficient of plate will decrease with the increase of the dimension of holes when the hole size is less than a certain value. The buckling mode of plates will change because of the dimension of holes. Finally recommended calculate formula of buckling stability factor k of perforated circle plate under axial compression is presented based on parameter analyse.

Keywords: Axial compression; Perforated circle plate; Elastic buckling; Buckling stability coefficient.

1. Introduction
With the rapid development of engineering materials and cold-formed technology, the cold-formed steel members are widely used in residential and industry building because of thin wall, high strength and complicated section forms of cold-formed steel members [1]. In order to facilitate the passage of auxiliary equipment and pipelines, personnel passage, and reduce structural dead weight, etc., holes are often made on the plates. The stress concentration will occur around the hole, which makes it is easy to enter the yield state and decrease its stability bearing capacity.

The research on the load-bearing factors of the thin plate under axial compression shows that the buckling coefficients of the plate are affected by different factors. Axial member research [2] showed that it had a little effect on the ultimate bearing capacity of the plate when the hole position was outside the effective width of the plate, while the bearing capacity decreased significantly with the increase of the width of the hole when the hole position located at the effective width of the plate. Li [3] gave the effect of opening hole rate and hole spacing on the bearing capacity of the plate analyzed by finite element software. The research results [4-5] showed that the hole position and the length to width ratio of the opening hole plate had a great impact on the capacity of the plate. When the opening moved from the relatively short edge to the center of the plate, the capacity of the plate first increased and then decreased. The influence of width, thickness and the porosity of the plate on the bearing capacity of the axial compression plate was analyzed [6], and an approximate calculation formula of the bearing capacity of the plate was proposed. Sawy [7] investigated the effects of the position of the opening hole along the long side and the aspect ratio of the plate on the buckling strength of an
eccentric opening rectangular plate subjected to axial uniform pressure. Wang [8] presented the equivalent thickness method to calculate the ultimate capacity of opening hole plate.

The elastic buckling behavior of thin plate with circle holes under axial compression will be investigated in this paper. The parameters analyzed include the plate aspect ratio, hole size, hole spacing and location, hole quantity. The calculate formula of buckling stability factor $k$ of perforated circle plate under axial compression is presented based on these parameter analyse.

2. Establishment of Finite Element Model of Thin Plate with Circle Hole

2.1. Definition of Basic Parameters of Section of Perforated Thin Plate

The plate opening form and size definition is shown in Figure 1, where $a$, $b$, $t$, $d$ are the length of plate, width of plate, thickness of plate, and diameter of hole, respectively. $S$ and $n$ are the hole spacing and the number of circle holes. $x_d$ is the end distance of hole.

![Figure 1. Definition of thin plate dimension.](image)

2.2. Establishment of Thin Plate Finite Element Model

The analysis model and mesh of thin plate with circle hole using finite element program ABAQUS is shown in Figure 2. Elastic modulus $E$ and poison’s ratio $\nu$ are equal to 206 GPa and 0.3, respectively. The plastic shell element which has four nodes and has six degrees of freedom for every node are used. The displacement of all edges of plate along $Y$ direction are constrained. The displacements of unloaded edges are constrained along $X$ direction. It can be verified as a four-edge simply supported plate. The uniform axial compression loads are added at the two end edges along $Z$ direction.

![Figure 2. Finite element model of thin plate with holes.](image)
3. Elastic Buckling of Axial Compression Thin Plate with a Single Circle Hole

The buckling mode and buckling stability coefficient of the axial compression plates with single circular hole are analyzed by using finite element method, in which the aspect ratio of the plate changes from 1 to 8, the width and thickness ratio of the plate was 100, the diameter of the circle hole to width of plate ratio changes from 0.1 to 0.8, and the position of the circle hole moves at the center of the plate along the X-axis. The buckling mode is shown in Figure 3.

![Figure 3. Buckling mode of plate with a single circular.](image)

The comparison on the buckling coefficient $k$ value is shown in Figure 4 when the hole position is moved from the plate end to the center of the plate along the longitudinal direction with different opening sizes ($d/b$) and different aspect ratio ($a/b$). As shown in Figure 4, the hole position has little influence on the buckling coefficient when the hole is small, such as $d/b=0.1$, especially when the plate has a larger aspect ratio. The buckling behavior of rectangular plates with the aspect ratio of $a/b=2,3,4,8$ is similar to that of square plates with $a/b=1$. When the hole is close to the plate end, which means $x_d$ decreases, the $k$ value of the buckling coefficient would be generally low. When the hole position is away from the plate end along longitudinal direction and $x_d/b$ is greater than or equal to 0.9, the hole size ($d/b$) and the opening position ($x_d/b$) has little influence on the buckling coefficient of the axial compression plate, and $k$ value tends to be stable. There is an unfavorable region for the plate, in which the buckling coefficient of the plate with hole will obviously decline, and the influence degree will be greater with the increase of the hole size. The range of opening position ($x_d/b$) is less than or equal to 0.9.
At the same time, in order to verify the influence of the width ratio \( b/t \) on the buckling coefficient of the axial compression plate with hole, the length and width ratio of the analyzed plate is 3 and the hole size is equal to 0.4. The buckling coefficients of plates with hole are analyzed for different width-thickness ratio \( (b/t=25, 100, 200) \). The analysis results are shown in Figure 5. As shown in Figure 5, the width-thickness ratio has little affection for the buckling coefficient of the thin plate with hole.

**Figure 4.** Comparison on buckling coefficient \( k \) value of thin plate with circle hole.
4. Elastic Buckling Behavior of Axial Compression Thin Plate with Multiple Circle Holes

The buckling behavior of thin plate with multiple circle holes under axial compression is investigated on the basis of the study of thin plate with a simple circle holes under axial compression. The distance of holes is same for every analyzed model. The number of holes includes 3, 5, and 7. The buckling coefficients of thin wall with holes at different plate end distance are shown in Figure 6.

Figure 6 shows that the buckling coefficient $k$ of plate with multiple holes is not stable when the end distance of hole $x_d/b$ is less than 2.0 and the buckling coefficient $k$ will increase with the increase of end distance. the buckling coefficient $k$ of plate with multiple holes will be stable and greater than and equal to 3.5 when the plate end distance $x_d/b$ is more than 2.0. When the holes on both sides of the plate are too close to the relatively short edge of the plate, the buckling coefficient of the plate will be significantly reduced.

According to the analysis on end distance of hole $x_d/b$, the end distance of hole is set as a constant 2 in the later analysis. the buckling coefficients of thin plate with multiple holes are shown in Figure 7 with different hole size and holes distances, where the diameter of hole to width of plate ratio $d/b$ changes from 0.2 to 0.8, hole spacing $(S/d)$ changes from 2 to 25, and holes number is equal to 3. Figure 7 illustrates the diameter of hole to width of plate ratio has the large effect on the bucking coefficient and the hole spacing has litter effect on buckling coefficient when the hole spacing $(S/d)$ is large 5.

Figure 5. Buckling coefficients $k$ for plate with different width-thickness ratio.

Figure 6. $k$ for different plate end distance.
The influence of the number of holes on the buckling of plate is shown in Figure 11. As shown in Figure 8, the number of holes $n$ has little effect on the buckling coefficient of the plate when the hole spacing ($S/d$) is equal to 2 and hole end distance is equal to 2. So the number of holes had little influence on $k$ value when the plate hole end distance and hole spacing are taken as a safe value.

The buckling modes for thin plate with three holes and the diameter of the hole changing from 0.1 to 0.8 are shown in Figure 9 when the hole spacing ($S/d$) is equal to 2 and hole end distance is equal to 2. Figure 9 shows that the buckling mode is the local buckling including hole when the diameter of the hole to width of plate ratio is less than and equal to 0.4, while the buckling mode is the localized-local mode interaction when the diameter of the hole to width of plate ratio is more than 0.4.
The buckling modes for thin plate with different number of holes and the diameter of the hole being 0.4 are shown in Figure 10 when the hole spacing ($S/d$) is equal to 2 and hole end distance is equal to 2. Figure 10 shows that the buckling mode is the local buckling including hole. The number of holes had little influence on buckling mode when the plate hole end distance and hole spacing are taken as a safe value.

5. Suggestions for Elastic Buckling Stability Coefficient Formula of Thin Plate with Circle Holes

The finite element analysis shows that the main factors affecting the buckling coefficient and buckling mode of the thin plate with circle holes is the diameter of the hole to width of plate ratio when the plate hole end distance and hole spacing are taken as a safe value. Figure 11 plots the buckling stability coefficient and the diameter of the hole to width of plate ratio when aspect ratio is equal to 8, which shows that the buckling stability coefficient and the diameter of the hole to width of plate ratio is bilinear relationship.

Based on the results of finite element analysis, the elastic buckling coefficient $k$ of the thin plate under axial compression with circle holes can be calculated by the following formula (1):

\[
 k_i = \begin{cases} 
 4.00 - 0.6 \frac{d}{b} & d/b \leq 0.4 \\
 3.50 + 0.65 \frac{d}{b} & 0.4 \leq d/b \leq 0.8 
\end{cases}
\]

(1)
6. Conclusion

The effect factors on the elastic buckling behavior and buckling coefficient $k$ of rectangular plate with circle holes under axial compression are studied in this paper. The following conclusions can be drawn:

1) The buckling coefficient $k$ of the thin plate with simple circle hole under axial compression is greater than or equal to 3.5 when the hole end distance is greater than or equal to 0.9. The buckling coefficient $k$ tends to be stable when the hole end distance is equal to 2.

2) The buckling coefficient $k$ of the thin plate with multiple circle holes under axial compression is greater than or equal to 3.5 and tends to stable when the hole spacing ($S/d$) is equal to 2 and hole end distance is equal to 2.

3) The buckling mode is the local buckling including hole when the diameter of the hole to width of plate ratio is less than and equal to 0.4, while the buckling mode is the localized-local mode interaction when the diameter of the hole to width of plate ratio is more than 0.4.

3) The buckling stability coefficient $k$ and the diameter of the hole to width of plate ratio is bilinear, the formula of the elastic buckling coefficient $k$ is presented.

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