Magnetic properties of charged spin-1 Bose gases with ferromagnetic coupling

Jihong Qin, Xiaoling Jian and Qiang Gu

Department of Physics, University of Science and Technology Beijing, Beijing 100083, People’s Republic of China

E-mail: jhqin@sas.ustb.edu.cn

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Abstract
The magnetic properties of a charged spin-1 Bose gas with ferromagnetic interactions are investigated within mean-field theory. It is shown that a competition between paramagnetism, diamagnetism and ferromagnetism exists in this system. It is shown that diamagnetism, being concerned with spontaneous magnetization, cannot exceed ferromagnetism in a very weak magnetic field. The critical value of reduced ferromagnetic coupling of the paramagnetic phase to ferromagnetic phase transition \(\bar{I}_c\) increases with increasing temperature. The Landé-factor \(g\) is introduced to describe the strength of the paramagnetic effect which comes from the spin degree of freedom. The magnetization density \(\bar{M}\) increases monotonically with \(g\) for fixed reduced ferromagnetic coupling \(\bar{I}\) as \(\bar{I} > \bar{I}_c\). In a weak magnetic field, ferromagnetism makes an immense contribution to the magnetization density. On the other hand, at a high magnetic field, the diamagnetism tends to saturate. Evidence for condensation can be seen in the magnetization density at a weak magnetic field.

1. Introduction
The magnetism of Fermi gases has always received considerable attention in solid-state physics, such as localized and itinerant electrons. In contrast, the magnetic properties of Bose gases has been less studied. However, since the realization of Bose–Einstein condensation (BEC) in ultracold atomic gases [1], more interest has been cast to this system. The Bose gases play an important role in understanding some exotic quantum phenomena, such as superconductivity and superfluidity. The ideal charged bosons were used originally to describe the superconductivity. It has been shown by Schafroth [2], Blatt and Butler [3] that an ideal gas of charged bosons exhibits the essential equilibrium features of a superconductor. Although the Bardeen–Cooper–Schrieffer (BCS) theory [4] explained the microscopic nature of conventional superconductivity, the charged Bose gas exhibits strong diamagnetism at a low temperature, which can be attributed to the Meissner effect. In recent years, the normal-state diamagnetism of high temperature cuprate superconductors has been explained by real-space charged bosons [5]. This also recasts new research interest in charged Bose gases.

Experimentally, since the realization of spinor BEC in optical traps [6, 7] the magnetic properties of spinor Bose gases has received considerable attention. Moreover, an ultracold plasma can be created by photoionization of laser-cooled neutral atoms [8]. The temperatures of electrons and ions can reach as low as 100 mK and 10 \(\mu\)K, respectively. The ions can be regarded as charged bosons if their spins are integers. The Landé-factor for different magnetic ions could also be different.

It is known that paramagnetism is from the spin degree of freedom of particles. On the other hand, charged spinless Bose gases can exhibit strong diamagnetism, similar to the Meissner effect, which comes from the orbital motion of the charge degree of freedom in a magnetic field. Theoretically, both the paramagnetism [9, 10] in neutral spin-1 Bose gases and the diamagnetism of the charged spinless Bose gases [11, 12] have been studied. Moreover, we [13] have discussed the competition of paramagnetism and diamagnetism in charged spin-1 Bose gases in an external magnetic field, using the Landé-factor \(g\) to evaluate the strength of the paramagnetic (PM) effect. It is shown that the gas exhibits a shift from diamagnetism to paramagnetism as \(g\) increases.
The ferromagnetism and superconductivity are not compatible in conventional physical models. The Meissner-Ochsenfeld effect shows that the conventional superconductor cancels all magnetic field inside when the temperature is below the superconducting transition temperature, which means they become perfectly diamagnetic. The discovery of several ferromagnetic (FM) superconductors in experiments [14–16] stimulates the research interest in the exotic magnetic properties of FM superconductors. The state of the Cooper pairs in the FM superconductors has been widely studied [14–18]. A stronger spin–orbit interaction in UGe2 results in an abnormal huge magnetocrystalline anisotropy [14–16]. Monthoux et al. [18] indicate that the favorite superconducting pairing type of this anisotropy is the triplet. Although the exact symmetry of the paired state has not yet been identified, a spin–triplet pairing is more likely than the spin–singlet pairing in these superconductors [14–16]. These behaviors are somewhat like charged spin-1 bosons. Thus, the charged spin-1 boson model helps in the understanding of the exotic magnetic properties observed in such materials.

Although the ferromagnetism [19–24] in a chargeless spinor Bose gas has also been involved in theory, it is little discussed when the FM interaction exists in a charged spin system. Accordingly, the magnetic behavior will become more complex in charged spin systems with FM interactions, where diamagnetism, paramagnetism and ferromagnetism compete with each other in such case.

In this paper, the magnetic properties of a charged spin-1 Bose gas with FM interactions are studied via mean-field theory. Alexandrov et al. found that the Coulomb or any other scattering may make charged Bose gases superconducting below a critical field [25] with a specific vortex matter [26]. Superconductivity is not obtained in our paper, probably because we used the mean-field approximation to deal with the FM interaction. In spite of this, the mean-field theory is still effective to point out the main physics of the magnetism, especially the FM transition [21]. The remainder of this paper is structured as follows. In section 2, we construct a model including Landau diamagnetism, Pauli paramagnetism and the FM effect. The magnetization density is obtained through the analytical derivation. In section 3, the results are obtained and discussed. A summary is given in section 4.

2. The model

The spin-1 Bose gas with FM couplings is described by the following Hamiltonian:

\[ H = \mu N = D_L \sum_{j,k,\sigma} (\epsilon^{\text{L}}_{jk} + \epsilon^{\text{Z}}_{\sigma} + \epsilon^{\text{m}}_{\sigma} - \mu)n_{jk,\sigma}, \]  

where \( \mu \) is the chemical potential and the Landau levels of bosons with charge \( q \) and mass \( m^* \) in the effective magnetic field \( B \) are

\[ \epsilon^{\text{L}}_{jk} = \left(j + \frac{1}{2}\right) \hbar \omega + \frac{\hbar^2 k^2}{2m^*}, \]  

where \( j = 0, 1, 2, \ldots \) labels different Landau levels and \( \omega = qB/(m^*c) \) is the gyromagnetic frequency. The energy level is degenerate with degeneracy

\[ D_L = \frac{qB \ell_L L_y}{2\pi \hbar}, \]  

where \( L_x \) and \( L_y \) are the length in the \( x \) and \( y \) directions of the system, respectively. The intrinsic magnetic moment associated with the spin degree of freedom leads to the Zeeman energy levels split in the magnetic field,

\[ \epsilon^{\text{Z}}_{\sigma} = -\frac{\hbar q}{m^* \sigma} B, \]  

where \( g \) is the Landé-factor and \( \sigma \) denotes the spin-\( z \) index of the Zeeman state \( | F = 1, m_F = \sigma \rangle \) (\( \sigma = 1, 0, -1 \)). The contribution to the effective Hamiltonian from the FM couplings is

\[ \epsilon^{\text{m}}_{\sigma} = -2I \sigma (m + \sigma n_\sigma), \]  

where \( I \) denotes FM coupling and spin polarization \( m = n_1 - n_{-1} \). The grand thermodynamic potential is expressed as

\[ \Omega_T \neq 0 = -\frac{1}{\beta} \ln \text{Tr} e^{-\beta(H - \mu N)} \]

\[ = \frac{1}{\beta} D_L \sum_{j,k,\sigma} \ln[1 - e^{-\beta(\epsilon^{\text{L}}_{jk} + \epsilon^{\text{Z}}_{\sigma} + \epsilon^{\text{m}}_{\sigma} - \mu)}], \]  

where \( \beta = (k_B T)^{-1} \). Through converting the sum over \( k_z \) to a continuum integral, we obtain

\[ \Omega_T \neq 0 = \frac{\omega m^* V}{(2\pi)^2 \hbar \beta} \sum_{j=0}^{\infty} \sum_{\sigma} \int dk_z \ln \]

\[ \{1 - e^{-\beta((j + \frac{1}{2}) \hbar \omega + \frac{\hbar^2 k^2}{2m^*} - g \frac{\hbar q}{m^* \sigma} B - 2I \sigma (m + \sigma n_\sigma) - \mu)}\}, \]  

where \( V \) is the volume of the system. Equation (7) can be evaluated by Taylor expansion, and then performing the integral over \( k_z \). We get

\[ \Omega_T \neq 0 = -\frac{\omega V}{\hbar^2} \left(\frac{m^*}{2\pi \beta}\right)^{3/2} \times \sum_{i=1}^{\infty} \sum_{\sigma} \frac{1}{\sigma} \frac{e^{-\beta((j + \frac{1}{2}) \hbar \omega + \frac{\hbar^2 k^2}{2m^*} - g \frac{\hbar q}{m^* \sigma} B - 2I \sigma (m + \sigma n_\sigma) - \mu)}}{1 - e^{-\beta\hbar \omega}}. \]  

For the sake of convenience, we introduce some compact notation for the class of sums. It can be defined as

\[ \Sigma_{\alpha \sigma} [\alpha, \delta] = \sum_{i=1}^{\infty} \frac{\beta^2/2e^{-\beta(\alpha + \delta)}}{(1 - e^{-\beta \alpha})^\delta}, \]  

where \( x = \beta \hbar \omega \) and \( \mu - \epsilon^{\text{Z}}_{\sigma} - \epsilon^{\text{m}}_{\sigma} = (\frac{1}{2} - \varepsilon) \hbar \omega \). Within this notation, equation (8) can be rewritten as

\[ \Omega_T \neq 0 = -\frac{\omega V}{\hbar^2} \left(\frac{m^*}{2\pi \beta}\right)^{3/2} \sum_{\sigma} \Sigma_{1\sigma} [-D, 0]. \]
with $D = 3$. The particle density $n = N/V$ can be expressed as

$$n_{T \neq 0} = -\frac{1}{V} \left( \frac{\partial \Omega_{T \neq 0}}{\partial \mu} \right)_{T,V}$$

$$= x \left( \frac{m^+}{2\pi \beta h^2} \right)^{3/2} \sum_{\sigma} \Sigma_{1_\sigma} [2 - D, 0]. \quad (11)$$

The magnetization density $M$ can be obtained from the grand thermodynamic potential,

$$M_{T \neq 0} = -\frac{1}{V} \left( \frac{\partial \Omega_{T \neq 0}}{\partial B} \right)_{T,V}$$

$$= \frac{\hbar}{m^+ c} \left( \frac{m^+}{2\pi \beta h^2} \right)^{3/2}$$

$$\times \sum_{\sigma} \left\{ \Sigma_{1_\sigma} [-D, 0] + x \left( g\sigma - \frac{1}{2} \right) \right.$$  

$$\times \Sigma_{1_\sigma} [2 - D, 0] - x \Sigma_{2_\sigma} [2 - D, 1] \left. \right\}. \quad (12)$$

The relation among effective magnetic field $B$, external magnetic field $H$ and magnetization density $M$ is formally expressed as

$$B = H + 4\pi M. \quad (13)$$

For computational convenience, some dimensionless parameters are introduced below. $t = T/T^*$, $M = m^+ c M/(\hbar c)$, $\tilde{\omega} = \hbar \omega/(k_B T^*)$, $\tilde{I} = I n/(k_B T^*)$, $\tilde{\mu} = \mu/(k_B T^*)$, $\tilde{m} = m/n$, $\tilde{n}_\sigma = n_\sigma/n$ and $h = h\omega/(m^+ c k_B T^*)$, and then $x = \tilde{\omega}/t$, where $T^*$ is the characteristic temperature of the system, which is given by $k_B T^* = 2\pi \hbar^2 n^2 / m^*$. The mean-field self-consistent equations are derived,

$$\tilde{n}_1 = \tilde{\omega}^{1/2} \Sigma_{1_\sigma = 1}^\prime [2 - D, 0], \quad (14a)$$

$$1 = \tilde{\omega} t^{1/2} \sum_{\sigma = 1,0,-1} \Sigma_{1_\sigma}^\prime [2 - D, 0], \quad (14b)$$

The curves of $\tilde{m}$ versus $\tilde{I}$ in figure 1(b) are superposed for different Landé-factors ($g = 0.1, 0.3$ and 0.5). It suggests that $\tilde{m} = \tilde{n}_1 - \tilde{n}_{-1}$ is independent of the Landé-factor, so $\tilde{I}_c$ at a certain temperature are equal for any Landé-factor. Here, $\tilde{I}_c$ is the critical value of reduced FM coupling of the PM phase transition. $\tilde{I}_c \approx 0.19$ in this situation. When $\tilde{I} < \tilde{I}_c$, $\tilde{m}$ equals 0, and the value of $\tilde{m}$ increases with increasing $\tilde{I}$ while $\tilde{I} > \tilde{I}_c$ until saturation. In the region of $\tilde{I} > \tilde{I}_c$, the magnetization density $M$ increases with the Landé-factor for fixed $\tilde{I}$, which is attributed to the PM effect [13]. Diamagnetism, paramagnetism and ferromagnetism compete with each other.
reduced temperature in [13].

paramagnetism and diamagnetism has been discussed in a very weak magnetic field. The competition between spontaneous magnetization, cannot overcome ferromagnetism gases, which is due to the internal field induced by the in such a system. The diamagnetism of charged Bose gases, which is due to the internal field induced by the spontaneous magnetization, cannot overcome ferromagnetism in a very weak magnetic field. The competition between paramagnetism and diamagnetism has been discussed in [13].

Figure 2 plots the $I_c$ dependence of temperature at magnetic field $h = 0.00001$. The region below $I_c$ is the PM phase, while the region above it is the FM phase. As the temperature increases, $I_c$ increases monotonically. It is shown that it is hard for spontaneous magnetization to occur at high temperature, when the Bose statistics reduces to Boltzmann statistics.

It is assumed that $\bar{m}$ will reach to a nonzero equivalence at $\bar{I} = 0.2$ for an arbitrary value of the Landé-factor for the situation of figure 1. To further study the influence of FM coupling to spontaneous magnetization, figure 3 is plotted. It is shown when $\bar{I} < \bar{I}_c(\approx 0.19)$, the value of $\bar{m}$ will be zero for any Landé-factor value. Thus, the evolution of $\bar{m}$ with the Landé-factor $g$ is superposed and keeps zero for $\bar{I} = 0$ and $\bar{I} = 0.1$. For fixed $\bar{I}$ when $\bar{I} > \bar{I}_c(\approx 0.19)$, the magnetization density $\bar{M}$ increases monotonically with $g$. On the other hand, $\bar{m}$ maintains a constant in despite of $g$. Our results also show that diamagnetism gives little contribution to the magnetism in the weak magnetic field, while paramagnetism and ferromagnetism play significant roles in the magnetization density in the region for $\bar{I} > \bar{I}_c$.

The interaction between paramagnetism and ferromagnetism is intricate. The increase of $\bar{m}$ due to increasing the reduced FM coupling $\bar{I}$ will contribute to the paramagnetism.

Above we have discussed the very weak magnetic field situation; now we turn to investigate the magnetic properties of charged spin-1 Bose gases at a finite magnetic field, where diamagnetism will emerge clearly. The result of the dependence of the total magnetization density $\bar{M}$ and $\bar{m} = \bar{n}_1 - \bar{n}_{-1}$ with Landé-factor $g$ at a definite magnetic field $h = 0.1$ at reduced temperature $t = 0.1$ is shown in figure 4. At low temperature in the definite magnetic field, there is a competition among the paramagnetism, diamagnetism and ferromagnetism. It is shown that diamagnetism dominates in
the small $g$ region, and therefore the magnetization density exhibits a negative value. When $g > 0.45$, the system presents paramagnetism which is independent of reduced FM coupling $\bar{I}$. As seen from figure 4, the curves of $\bar{I} = 0.1, \bar{I} = 0.3$ and $\bar{I} = 0.5$ match together. It means that $\bar{m}$ tends to saturate if $\bar{I}$ is greater than a critical value. The increase of $\bar{I}$ after this critical value does not contribute to the magnetization density. Then the system exhibits a similar magnetization density at $\bar{I} = 0.1, \bar{I} = 0.3$ and $\bar{I} = 0.5$.

The discussions above all focused on fixed magnetic field. Next we study the influence of magnetic field on magnetism. The evolution of the total magnetization density $\bar{M}$ and $\bar{m} = \bar{n}_1 - \bar{n}_{-1}$ with magnetic field at reduced temperature $t = 0.6$ with $g = 0.5$ is shown in figure 5. The gas always manifests paramagnetism no matter what the values of the $\bar{I}$ are. It indicates that in the case of $g = 0.5$, diamagnetism cannot overcome paramagnetism no matter how strong the magnetic field is. This behavior is qualitatively consistent with the result of charged spin-1 Bose gases [13]. In this region, the stronger ferromagnetism induces a larger $\bar{m}$, which will enhance paramagnetism. With increasing magnetic field, diamagnetism also increases. However, this will not change the paramagnetism of this system. Whether diamagnetism can increase infinitely with magnetic field is an important issue.

In order to manifest the paramagnetism and diamagnetism in detail, in figure 6 we study the dependence of the total magnetization density $\bar{M}$, the paramagnetization density $\bar{M}_p$, and the diamagnetization density $\bar{M}_d$ in reduced temperature $t = 0.6$ with $g = 0.5$ and $\bar{I} = 0.5$. $\bar{M}_d$ holds a constant since FM coupling is larger. $\bar{M}_d$ tends to saturate with magnetic field. It indicates that diamagnetism will not increase infinitely with magnetic field. This is why in figure 5 the gas preserves paramagnetism even though the magnetic field is large.

It is significant to evaluate the diamagnetic behavior at the high magnetic field limit. Without consideration of spin, the diamagnetization density,

$$\bar{M}_d = \frac{t^3/2}{h} \sum_{l=1}^{\infty} \frac{l^{-3/2} e^{-l(\bar{\omega}/2 - \bar{\mu})/t}}{(1 - e^{-l\bar{\omega}/t})} \left[ 1 + \bar{\omega} \left( -\frac{1}{2} - \frac{e^{-l\bar{\omega}/t}}{1 - e^{-l\bar{\omega}/t}} \right) \right],$$

when $\bar{\omega} \to \infty$, $\bar{M}_d$ can be reduced to,

$$\bar{M}_d^{\bar{\omega} \to \infty} = -\frac{1}{2} \bar{\omega}^{t/2} \sum_{l=1}^{\infty} \frac{l^{-1/2} e^{l\bar{\omega}/2}}{e^{l\bar{\omega}/2}},$$

from equation (14b), we can obtain,

$$1 = \bar{\omega}^{t/2} \sum_{l=1}^{\infty} \frac{l^{-1/2} e^{l\bar{\omega}/2}}{e^{l\bar{\omega}/2}}.$$

Substituting equation (18) into (17), $\bar{M}_d^{\bar{\omega} \to \infty} = -1/2$ can be obtained. This analytical result illustrates that the diamagnetization density $\bar{M}_d$ tends to a finite value at high magnetic field.

In order to investigate the magnetic properties of the charged spin-1 Bose gas in low temperature, we assume $\gamma = 0.1$. The evolution of the total magnetization density $\bar{M}$ and $\bar{m} = \bar{n}_1 - \bar{n}_{-1}$ with reduced temperature $t$ of charged spin-1 Bose gases with $\gamma = 0.1$ and $g = 1$, at magnetic field $h = 0.00001$. The reduced FM coupling $\bar{I}$ is chosen as:\n
$$\bar{I} = 0 (\text{solid line}), 0.1 (\text{dashed line}), 0.3 (\text{dotted line}), 0.5 (\text{dash dotted line}).$$

4. Summary

In summary, we study the interplay among paramagnetism, diamagnetism and ferromagnetism of a charged spin-1 Bose gas with FM coupling within the mean-field theory. In a very weak magnetic field, it is shown that the ferromagnetism is stronger than the diamagnetism, where the diamagnetism is related to spontaneous magnetization. The critical value of reduced FM coupling $\bar{I}_c$ of the PM phase to FM phase transition increases with increasing temperature. The Landé-factor $g$ is assumed as a variable to evaluate the strength of the PM effect. The gas exhibits a shift from diamagnetism to paramagnetism as $g$ increases at a finite magnetic field. Ferromagnetism plays an important role in the magnetization density in the weak magnetic field. Diamagnetism cannot increase infinitely with magnetic field at high magnetic field. Condensation is predicted to occur through studying the low temperature magnetic properties in a weak magnetic field.
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