Modeling tension-compression asymmetry and failure anisotropy in bending operations of a magnesium alloy

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Abstract. The deformation behavior of the magnesium sheet alloy AZ31 at room temperature is investigated using different standard and non-standard mechanical tests as well as respective finite-element simulations. In order to address the bendability, three-point bending tests were performed for different orientations of the sample with respect to the sheets axes of orthotropy. The elastic-plastic deformation was modeled using a two-yield-surface model, which is able to describe the evolving strength differential effect as well as the anisotropy of the material. For the prediction of material failure, a numerically efficient damage criterion based on transformation of strain rates is calibrated and applied. It is shown that this model is able to predict the observed edge failure of specimens with good accuracy.

1. Introduction

Despite the high strength-to-weight ratio of magnesium alloys in general, the application of rolled sheets and extruded profiles is still restricted. Challenges to face for applications are its planar anisotropy, the tension-compression asymmetry (the so-called strength differential effect, SD effect) [1] as well as its limited formability at room temperature. Strong and efficient prediction tools for forming operations, for structural responses to loads and for lifetime analyses are required on the macro scale to extend the use of magnesium alloys.

The tension-compression asymmetry requires special (not von Mises) yield functions to describe plastic deformation. Formulation of the yield function including the third stress invariant naturally account for this asymmetry, but are not necessarily convex. Formulations using linear transformations of the stress deviator lead to convex yield functions [2, 3]. For describing the evolution of anisotropy/SD effect, either the coefficients of anisotropy can be defined as functions of an equivalent strain [4, 5], or a two-surface model [6, 7] can be used.

In most industrial applications, the use of shell or membrane elements is preferred due to their computational efficiency and robustness. In these cases plane stress conditions are considered. Therefore, stress triaxiality and Lode angle depend on each other. This prevents the use of two-parameter approaches triggering the evolution of a damage variable to address material failure. Hence, common models applied in sheet metal forming to predict the forming limits are based on through-thickness necking of the materials, e.g. [8]. However, for some e.g. advanced high-strength steel (AHSS) and magnesium alloys, fracture occurs without preceding necking. In these cases, failure models waiving an evolution equation for damage appear suitable. These uncoupled damage models can be used to signpost a mechanical stage, for which either the overall forming is limited, or structural instability sets in. These
models are computationally more efficient compared to coupled damage models. The required damage parameters indicating critical stages are derived from the elasto-plastic behavior of the material by a hybrid approach involving mechanical testing and numerical analyses. Focusing on magnesium sheets in the following, successful predictions are reported by several authors, e.g. [9-11].

The method described in the following reproduces the direction-dependent irreversible deformation as well as the failure behavior. Alongside (smooth) tensile and notched samples, Arcan-type samples with emerging shear stress states are incorporated in the parameter optimisation strategy. Failure points are analysed as global quantities. Fracture is predicted in terms of crack initiation location and local mechanical state. An uncoupled failure model is proposed and exploited for this purpose. It is shown that regardless of its phenomenological character, the proposed model predicts the deformation and failure at monotonous loading of the magnesium sheets investigated.

2. Material and calibration tests

The material under investigation is the magnesium alloy AZ31 (Mg + 3%Al + 1%Zn) in its as-received condition. It was provided by POSCO in sheets of 1.17 mm thickness. The microstructure is fully recrystallized, evidencing an average grain size of 11 µm. The matrix of mechanical tests conducted comprises tensile test following the DIN standard along the rolling direction (RD), along the transverse direction (TD), and along 45° to the RD (DD), compression tests along the RD and TD as well as through-thickness (TT), Arcan-type samples cut along the RD and TD, with loading angles of 0°, 15°, 30°, and 45°, as well as notched bars with a notch radius of 2 mm (NB2) and 6 mm (NB6), respectively. Details of the tests are given in previous publications [12, 13]. Table 1 summarises the tests conducted.

|orientation| tension| compression| Ar 0°| Ar 15°| Ar 30°| Ar 45°| NB2| NB6|
|-----------|--------|------------|------|-------|-------|-------|----|----|
| RD        | 4      | 4          | 4    | 4     | 4     | 3     | 3  | 3  |
| TD        | 4      | 4          | 4    | 3     | 3     | 3     | 3  | 3  |
| DD        | 4      | 3          | 3    | 3     | 3     | 4     | 4  | 4  |
| RD        | 3      | 3          | 3    | 3     | 4     | 4     | 4  | 4  |
| TD        | 3      | 3          | 3    | 3     | 4     | 4     | 4  | 4  |

3. Models

3.1. Plasticity model

A phenomenological two-yield surface model [7] is adopted here to numerically describe the deformation behaviour. This model principally captures the tension-compression asymmetry and describes the evolution of the strength differential effect. A symmetric (for glide mechanisms) and an asymmetric yield surface (for twinning mechanisms) are combined by two interacting plastic potentials. Assuming the additive decomposition of the strain increment, the total strain increment consists of the elastic and parts related to glide and twinning, respectively:

\[ d\varepsilon = d\varepsilon^e + d\varepsilon^p = d\varepsilon^e + d\varepsilon^p_g + d\varepsilon^p_t. \] (1)

The plastic strain increments are calculated from the two yield functions, namely Bron & Besson [14] and CPB [2] using normality rule. Two coupled isotropic hardening functions are used:

\[ R_g(p_g, p_t) = R_0_g + H_g p_g + Q_{1g} \left( 1 - \exp \left[ -b_{1g} p_g \right] \right) + Q_{2g} \left( 1 - \exp \left[ -b_{2g} p_g \right] \right) + H_{gt} p_t, \] (2)

\[ R_t(p_g, p_t) = R_0_t + Q_{1t} \left( \exp \left[ -b_{1t} p_t \right] - 1 \right) + H_{tg} p_g. \] (3)

with \( p_g \) and \( p_t \) being the plastic multipliers of the glide and twinning mechanism, respectively. The determination of the numerous parameters was realized by running FE-simulations of the mechanical tests, constructing a target function from the differences between simulation and experimental results and minimizing the target function.
3.2. Failure model

An approach based on linear transformations of the plastic strain rate tensor referred in the orthotropic
material axes is chosen for modelling of directionality of failure. A local fracture criterion is formulated
as an integral over the loading history:
\[
D = \int_0^t f(\dot{\epsilon}_{pl}) dt = D_c
\]

(4)

In order to address the anisotropy in failure strains, the strain rate components in the material frame
need to be mapped to a “corresponding” strain rate tensor. This is done by a linear transformation
rendering a deviatoric deformation measure \(\dot{\epsilon}_{pl}^d\),
\[
\dot{\epsilon}_{pl}^d = L \cdot \dot{\epsilon}_{pl}
\]

(5)

with
\[
L = \begin{pmatrix}
\frac{1}{3}(L_{TT} + L_{SS}) & -\frac{1}{3}L_{SS} & -\frac{1}{3}L_{TT} & 0 & 0 & 0 \\
-\frac{1}{3}L_{SS} & \frac{1}{3}(L_{LL} + L_{SS}) & -\frac{1}{3}L_{LL} & 0 & 0 & 0 \\
-\frac{1}{3}L_{TT} & -\frac{1}{3}L_{LL} & \frac{1}{3}(L_{LL} + L_{TT}) & 0 & 0 & 0 \\
0 & 0 & 0 & L_{LT} & 0 & 0 \\
0 & 0 & 0 & 0 & L_{TS} & 0 \\
0 & 0 & 0 & 0 & 0 & L_{SL}
\end{pmatrix},
\]

(6)

which introduces six adjustable parameters \(L_{ij}\). A material point is assumed to be broken if
\[
D = \int_0^t (\dot{\epsilon}_{i}^d + \langle \dot{\epsilon}_{i}^d \rangle) dt = 1,
\]

(7)

with \(\langle . \rangle\) being the positive part function. The damage indicator \(D\) can be evaluated as a local quantity
in a post processing step of the FE-simulation for any forming process. By the relation equation (5) a
tensor transformation is applied, similar to what is common in stress tensor transformations used in yield
functions for anisotropic media.

3.3. Model simplifications and parameter calibration

Previous investigations [15] revealed that for the present material, the mechanical anisotropy is primarily
manifested by its basal texture, which renders a plastic deformation behaviour similar to planar
anisotropy. The failure strain quantified by tensile tests, however, differs significantly between RD, TD
and DD orientation. In order to reduce the models complexity, the following supposition is made:
Coefficients in the linear transforms of the plasticity model affecting the RD and TD orientation were
set to unity – RD and TD deformation behaviour become hence identical. Parameters affecting the out-
of-plane direction (plasticity and failure model) were set to unity, since testing in this orientation is not
possible for sheets. Isotropic hardening functions for glide- and twinning mechanisms are taken from
[13]. All other plasticity parameters were re-calibrated using the data provided in the test matrix shown
in table 1. The fits are visualised here using the force-elongation record of tensile and compression tests
as an example, see figure 1. The resulting yield functions are shown for the space of plane principal
stresses in figure 2. Note that due to the simplification, the simulated curves compromise RD and TD
behavior with respect to their plastic response.

The parameters of the failure model are determined using the global failure strains measured during
the tensile tests, see figure 3. Local strains recorded by digital image correlation are generally higher
due to strain localization, but show more scatter. The model is apparently able to capture the direction-
dependent fracture of tensile samples with model parameters \(L_{LL}=10.0, L_{TT}=1.3, L_{SS}=3.0,\) and \(L_{LT}=1.4.\)
Note that in this approach the anisotropy in failure is captured, although the anisotropy with respect to deformation is not.

![Figure 1. Stress-strain data for uniaxial tension and compression.](image1)

![Figure 2. Plane stress representation of the 2-yield-surface model.](image2)

![Figure 3. Verification of the failure model: predicted global failure strains compared to experimental counterparts.](image3)

4. Three-point bending

4.1. Experiments

For the three-point bending tests, rectangular strips with the as-received thickness of 1.17 mm, 60 mm in length and 10 mm in width are used. The tests are conducted with an in-house tool with a punch radius of 2 mm and circular supports with a radius of 5 mm, see figure 4. 15 samples each from orientations
along the RD, along the TD, and along the DD are fabricated and tested. The bending tests are performed with spans of 18 mm, 20 mm, and 25 mm between the supports. These dimensions are set with the help of an in-built scale on the support platform. Furthermore, the samples are coated with the use of a thin Teflon sheet between the supports and the beam. Figure 5 displays two bent strips of the 18 mm and 25 mm width configuration. Details are reported in [16].

4.2. Modeling results
The simplified two-yield surface plasticity model is used to simulate the three-point bending tests by conducting 3D FE-simulations using twofold symmetry conditions. Frictionless contact between sheet, punch and support is assumed. Figure 6 shows the results in terms of global punch force as a function of displacement for all tested spans for the three orientations. The fit is excellent over the whole range of deformation and demonstrates the high predictive power of this model.

Failure during bending was evidenced in the experiments for the configurations W18 (TD and DD) and for W20 DD. Samples in RD orientation never failed. The same holds true for the wide configuration W25, where all samples remained intact. This result is captured by the failure model. Failure points are indicated by symbols in figure 6. They meet with the first occurrence of a crack in the sheets, which is associated with a load drop of the force-displacement record.

5. Discussion and Conclusion
The formability here is explored here by means of bending tests, in which commonly the deformation state is between uniaxial stress and plane strain deformation. While the deformation model is calibrated using test experiencing a variety of stress triaxialities ranging from -1/3 (uniaxial compression) to close to 2/3 (round notched bar tension), the failure model is tuned on the basis of tensile test only. This appears inconsistent. However, trials for finding elements of the failure transformation matrix, equation 6, meeting the failure point of all tests given in table 1 gave less satisfying results than those depicted in figure 6. A possible reason for this is the high ductility of tensile test along the RD, which is detailed in [13].

It can be concluded, that the simplified two yield surface model together with the phenomenological failure model provide a powerful tool for the prediction of formability of hcp metals, for which a strength differential effect is reported. For the prediction of bendability, uniaxial tests for calibration of the failure model appears sufficient as it provides accurate results.
Figure 6. Prediction of the deformation and failure behaviour during bending.

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[1] Hosford W F 1993 The mechanics of crystals and textured polycrystals (New York: Oxford University Press)
[2] Cazacu O, Plunkett B and Barlat F 2006 Orthotropic yield criterion for hexagonal closed packed metals Int. J. Plast. 22 1171-94
[3] Plunkett B, Cazacu O and Barlat F 2008 Orthotropic yield criteria for description of the anisotropy in tension and compression of sheet metals International Journal of Plasticity 24 847-66
[4] Yoon J, Cazacu O and Mishra R K 2013 Constitutive modeling of AZ31 sheet alloy with application to axial crushing Materials Science and Engineering: A 565 203-12
[5] Mekonen M N, Steglich D, Bohlen J, Letzig D and Mosler J 2012 Mechanical characterization and constitutive modeling of Mg alloy sheets Materials Science and Engineering: A 540 174-86
[6] Kim J H, Kim D, Lee Y-S, Lee M-G, Chung K, Kim H-Y and Wagoner R H 2013 A temperature-dependent elasto-plastic constitutive model for magnesium alloy AZ31 sheets International Journal of Plasticity 50 66-93
[7] Steglich D, Tian X and Besson J 2016 Mechanism-based modelling of plastic deformation in magnesium alloys European Journal of Mechanics - A/Solids 55 289-303
[8] Marciniak Z and Kuczynski K 1967 Limit strains in the processes of stretch-forming sheet metal International Journal of Mechanical Sciences 9 609-20
[9] Jia Y and Bai Y 2016 Ductile fracture prediction for metal sheets using all-strain-based anisotropic eMMC model International Journal of Mechanical Sciences 115-116 516-31
[10] Lee J-Y, Steglich D and Lee M-G 2018 Fracture prediction based on a two-surface plasticity law for the anisotropic magnesium alloys AZ31 and ZE10 International Journal of Plasticity 105 1-23
[11] Li F-F, Fang G and Qian L-Y 2019 Forming limit analysis of Mg-2Zn-1.2Al-0.2Ca-0.2RE alloy sheet using ductile fracture models International Journal of Damage Mechanics 29 1181-98
[12] Steglich D 2020 Mixed-Mode Deformation and Failure of a Magnesium Sheet Quantified using a Modified Arcan Fixture Experimental Mechanics 60 109-18
[13] Steglich D and Besson J 2021 Prediction of deformation and failure anisotropy for thin magnesium sheets under mixed-mode loading Mechanics of Materials 163 104064
[14] Bron F and Besson J 2004 A yield function for anisotropic materials: Application to aluminium alloys Int. J. Plast. 20 937-63
[15] Jeong Y and Steglich D 2020 Modelling-assisted description of anisotropic edge failure in magnesium sheet alloy under mixed-mode loading *International Journal of Mechanical Sciences* **181** 105680

[16] Nagendra A and Steglich D 2020 Experimental and Numerical Bendability Analysis of a 3rd Generation Magnesium Alloy *Procedia Manufacturing* **47** 1274-80