QUARK MATTER IN COMPACT STARS: ASTROPHYSICAL IMPLICATIONS AND POSSIBLE SIGNATURES

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After a brief non technical introduction of the basic properties of strange quark matter (SQM) in compact stars, we consider some of the late important advances in the field, and discuss some recent astrophysical observational data that could shed new light on the possible presence of SQM in compact stars. We show that above a threshold value of the gravitational mass a neutron star (pure hadronic star) is metastable to the decay (conversion) to an hybrid neutron star or to a strange star. We explore the consequences of the metastability of “massive” neutron stars and of the existence of stable compact “quark” stars (hybrid neutron stars or strange stars) on the concept of limiting mass of compact stars, and we give an extension of this concept with respect to the classical one given in 1939 by Oppenheimer and Volkoff.

1. A brief history of quark matter in compact stars

In 1964 Murray Gell-Mann\(^1\) and George Zweig\(^2\) proposed that the proton, the neutron, and all the other hadrons (i.e. particles which feel the strong interaction) are not elementary, but are composed of smaller (point-like) constituents which were called quarks.\(^3\) In the original model there are three fundamental quarks nicknamed up (u) down (d) and strange (s). The premiere evidence for the existence of quarks came already at the end of 1967 from the first high energy electron-proton scattering experiment\(^3\) performed at the Stanford Linear Accelerator Center (SLAC), and a growing body of direct evidence for their reality\(^4\) was accumulated in the following years in numerous experiments in different laboratories all around the world.

It was then natural to speculate that when nuclear matter is compressed to densities so high that nucleons substantially overlap, a new phase of matter, in which quarks are the relevant degrees of freedom, could be formed. Already in 1965, Ivanenko e Kurdgelaidze\(^4\) suggested the possible existence of a “quarkian core” in very massive stars.

Two years later, at the end of 1967, Jocelyn Bell discovered\(^5\) the first pulsar. Pulsars were soon interpreted\(^6\) as strongly magnetized rotating neutron stars, the compact stellar remnants of supernova explosions hypothesized\(^7\) in 1934 by Baade and Zwicky\(^7\) soon after the Chadwick’s discovery of the neutron. The first calculation of the structure on a neutron star was performed in 1939 by Oppenheimer and

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\(^{a}\)Gell-Mann borrowed this name from a line of the novel Finnegans Wake by James Joyce.

\(^{b}\)Isolated quarks have never been observed. This has led to the quark confinement hypothesis, as one of the basic features of Quantum Chromodynamics (QCD), which is the fundamental theory of strong interactions.

\(^{c}\)In 1937 Landau suggested the existence of a “neutron core” (neutron matter core) in normal stars, to explain the energy source of stars as due to the release of gravitational energy via matter accretion into the stellar neutron core.
Volkoff. They assumed the star to be composed by pure neutron matter described as a non-interacting relativistic Fermi gas. More sophisticated calculations, with the inclusion of the effects of nuclear interactions on the equation of state (EOS) of neutron matter, were available at the end of 1950s. A more refined model for the internal composition of neutron stars was introduced in 1960 by Ambartsumyan and Saakyan, which suggested the possible presence of hyperons ($\Lambda$, $\Sigma^-$, $\Sigma^0$, $\Sigma^+$, $\Xi^-$, and $\Xi^0$ particles) in the inner core of neutron stars. All these early models gave, with a reasonable accuracy, the gross properties (i.e., maximum mass, radii, and central densities) of neutron stars. Particularly, it was clear that the central density of the maximum mass configuration could be as high as 10 times the central density ($\rho_0 = 2.8 \times 10^{14}\text{g/cm}^3$) of heavy atomic nuclei. Thus it was quite evident at the end of the 1960s and in the early 1970s, that neutron stars were the best candidate in the universe where quark matter could be found. It was in those years that the idea of pure quark stars11 or hybrid hadronic-quark stars12 (termed “baryon-quarkian stars” by the authors of ref.12) was conceived. The earliest studies, however indicated that it was unlikely that quark matter could be found in stable neutron stars, or to have a third family of quark compact stars. This conclusion was mainly due to some simplification in the treatment of the quark-deconfinement phase transition, which was considered as one at constant pressure. Further investigations have established that, since neutron star matter is a multicomponent system with two conserved “charges” (i.e., electric charge and baryon number), the quark-deconfinement phase transition proceeds through a mixed phase over a finite range of pressures and densities according to the Gibbs' criterion for phase equilibrium. These and subsequent studies have established that neutron stars may, very likely, contain quark matter in their interiors. Neutron stars which possess a quark matter core either as a mixed phase of deconfined quarks and hadrons or as a pure quark matter phase are called hybrid neutron stars or shortly hybrid stars (HyS). The more conventional neutron stars in which no fraction of quark matter is present, are currently referred to as pure hadronic stars (HS).

A further crucial step in the study of quark matter in astrophysical contest was made by Arnold R. Bodmer in 1971. In his pioneering work,20 Bodmer suggested the possibility of collapsed nuclei. He conjectured that collapsed nuclei could have a lower energy than normal (i.e., made of protons and neutrons) nuclei, and he speculated that normal nuclei are very long-lived isomers against collapse because of a “saturation” barrier between normal and collapsed nuclei. Among other possibilities, Bodmer discussed collapsed nuclei as composite states of $3\Lambda$ $u$-$d$-$s$ quarks (being $\Lambda$ the baryon number of the nucleus). These many-body $u$-$d$-$s$ quark states are what we call, with modern terminology, strangelets or quark nuggets. Bodmer also speculated that collapsed nuclei may have been copiously produced in the initial extremely hot and dense stages of the universe, and proposed that collapsed nuclei

\[\text{see however ref.s}\] for later studies on the mixed hadron-quark phase, where the effects of Coulomb, surface energies, and charge screening are taken into account.
may condensate as peculiar very compact massive “black” objects being possible candidates for dark matter.

The feasible stability of multi-quark systems was also investigated by Hidezumi Terazawa, who termed them super-Hypernuclei. However, these ideas brought a wide attention by the scientific community only after Edward Witten published, in 1984, his widely known paper on the “cosmic separation of phases”, where he examined the cosmological and astrophysical consequences of the absolute stability of strange quark matter (SQM)\(^a\), and calculated the bulk properties of Strange Stars within a simple model for SQM inspired to the MIT bag model for hadrons. In the following years the properties of SQM and the properties of strange stars were calculated by numerous groups.\(^23\)–\(^25\)

A recent valuable advance in the comprehension of quark matter properties, that is having a substantial impact on the study of neutron star physics, is the discovery of color superconductivity. Superconductivity, as it is well known, is a general feature of degenerate Fermi systems, which become unstable if there exist any attractive interaction at the Fermi surface. As recognized by Bardeen, Cooper and Schrieffer (BCS) this instability leads to the formation of a condensate of Cooper pairs and to the appearance of superconductivity. In Quantum Chromodynamics (QCD) the quark-quark (qq) interaction is strongly attractive in many channels. This will lead to quark pairing and color superconductivity. This possibility was already pointed out in the mid 1970s (see also ref.\(^28\)).

Very recently the study of color superconductivity has become a research subject full of activity, since the realization that the typical superconducting energy gaps in quark matter may be of the order of \(\Delta \sim 100 \text{ MeV}\), thus much higher than those predicted in the early works.

The phase diagram of QCD has been widely analyzed in the light of color superconductivity.\(^29\)–\(^37\) Quarks, unlike electrons, have different flavors (\(u, d, s\) for the quark matter relevant for neutron star physics) and have color as well as spin degrees of freedom, thus many different quark pairing schemes are expected. At asymptotic densities (in the asymptotic freedom regime where perturbative QCD is valid) the ground state of QCD is the so called color-flavor locked (CFL) phase, in which all three quark flavors participate to pairing symmetrically.\(^f\) In the density regime relevant for neutron star physics (well below the asymptotic density regime) the ground state of quark matter is uncertain and many possible patterns for color superconductivity have been predicted.

\section{Hybrid stars}

A neutron star has a characteristic layered structure. The existence of various layers is a consequence of the onset of many different regimes (\(i.e.,\) particle species and phases) of dense matter, which are expected in the stellar interior. The details

\(^a\)\textit{see next section for the definition of strange quark matter.}

\(^f\)This phase of SQM is electrically charge neutral without any need for electrons.
of the stellar internal structure are sensitive to the particular high density EOS employed in the calculations. Very schematically, in the case of an hybrid star with sufficiently high mass (central density), these layers can be identified as follows:

1. **Surface** ($\rho_{Fe} = 7.9 \text{ g/cm}^3 < \rho < 10^6 \text{ g/cm}^3$). Matter in the neutron star’s surface is composed by a lattice of $^{56}\text{Fe}$ nuclei (the endpoint of thermonuclear burning). As the density increases, more and more electrons detach from their respective nuclei, and at $\rho \sim 10^4 \text{ g/cm}^3$, one has complete ionization.

2. **Outer crust** ($10^6 \text{ g/cm}^3 < \rho < \rho_n^{drip}$). The outer crust is a solid layer consisting of a Coulomb lattice of heavy nuclei ($A > 56$) in $\beta$–equilibrium with a degenerate relativistic electron gas. Going deeper into the crust (to higher density regions) nuclei in the lattice become more and more neutron rich, because the electron capture processes $e^- + p \rightarrow n + \nu_e$, lower the total energy of the system. When the density reaches the value $\rho_n^{drip} \approx 4.3 \times 10^{11} \text{ g/cm}^3$ (neutron drip density), nuclei are so neutron rich that neutron states in the continuum begin to be filled.

3. **Inner crust** ($\rho_n^{drip} < \rho < \rho_{core}$). Above the neutron drip density, the crust consist of a lattice of neutron rich nuclei embedded in an ultra–relativistic electron gas and in a neutron gas. Due to the nuclear pairing force, neutrons forms Cooper pairs and are expected to be in a $^1S_0$ superfluid state. As the density of matter increases and approaches that of uniform nuclear matter ($\rho_{NM} \approx 2.8 \times 10^{14} \text{ g/cm}^3$), one has a crust-core transition layer consisting of a mixed phase (the so called “nuclear pasta” phase) of “exotic nuclei” and a neutron–electron gas. First one encounters a stratum composed of spherical blobs of nuclear matter (neutron rich super-heavy nuclei) embedded in the neutron and electron gas. With increasing density, the nuclear matter spherical blobs, first turn in rod-like structures (“spaghetti”), and then to plate-like ones (“lasagna”). For higher densities one has the complementary...
nuclear shapes, i.e., the so called “anti-spaghetti”, and “Swiss cheese” (a phase in which balloons filled by neutron and electron gases are surrounded by nuclear matter). The appearance of nuclear pasta phase in the inner crust, is due to finite size effects (particularly the surface and Coulomb energies) which settle the minimum of the the local total energy per baryon of these competing exotic geometrical nuclear structures between each others and uniform $\beta$-stable nuclear matter.

(4) outer core (pure hadronic layer) ($\rho_{\text{core}} < \rho < \rho_{\text{mix}}$). When the density reaches a value $\rho_{\text{core}} = 0.5 - 0.8 \rho_{\text{NM}}$, nuclei merge together and a phase transition to nuclear matter takes place. In this region (nuclear matter layer) the star consist of asymmetric nuclear matter in $\beta$-equilibrium with $e^-$ and $\mu^-$. Neutron–neutron pairs form in a $^3P_0$ superfluid state and proton–proton pairs are in a $^1S_0$ superconducting state. Going deeper in the outer core, other hadronic constituents are expected, as hyperons, or possibly a Bose-Einstein condensate of negative kaons ($K^-$).

(5) mixed hadron–quark layer ($\rho_{\text{mix}} < \rho < \rho_{\text{QM}}$). At the critical density for the onset of the quark deconfinement phase transition, one encounters a layer consisting of a mixed phase between deconfined quarks and hadrons. On the top of this layer one has a Coulomb lattice of quark matter droplets embedded in a sea of hadrons and in a roughly uniform sea of electrons and muons. Moving toward the stellar center, various geometrical shapes (rods, plates) of the less abundant phase immersed in the dominant one are expected. This structured mixed phase (quark pasta phases) is analogous to nuclear pasta phase in the inner crust. It is important to stress that a number of recent studies have reexamined the physical conditions for the occurrence of this structured mixed phase in a compact star. It has been shown that the formation of the mixed phase could be inhibited (or it can occur in a “thin” radial region of the star) for large values of the Coulomb and surface energies, or due to the effects of charge screening.

(6) quark matter core ($\rho > \rho_{\text{QM}}$). Finally, in the more massive stars, one could find a pure strange quark matter core. Different color superconducting phases are expected in this region of the star.

In Fig. 2, we show the typical internal composition of a hybrid star. The cross section of the star has been obtained using a relativistic mean field EOS (model GM3) and is relative to the maximum mass configuration ($M = M_{\text{max}} = 1.448M_\odot$, with radius $R = 10.2$ km, and central density $\rho_c = 28.1 \times 10^{14}$ g/cm$^3$). This star has a pure QM core which extend for about 2.5 km, next it has a hadron-quark mixed phase layer with a thickness of about 5.5 km, followed by a nuclear matter layer about 2 km thick. On the top we have the usual neutron star crust. The presence of quarks makes the EOS softer with respect to the corresponding pure hadronic matter EOS. The stellar sequence associated to the latter EOS has the maximum mass configuration: $M_{\text{max}} = 1.552 M_\odot$, $R = 10.7$ km, $\rho_c = 25.4 \times 10^{14}$ g/cm$^3$.

Many possible astrophysical signals for the presence of a quark core in neutron stars have been proposed. Particularly, pulse timing properties of pulsars have attracted much attention since they are a manifestation of the rotational properties
Fig. 2. The internal composition of an hybrid star with a mass $M = M_{\text{max}} = 1.448 \, M_\odot$. The GM3 EOS\cite{38} has been used for the hadronic phase, and the bag model EOS, with $B = 136.6 \, \text{MeV/fm}^3$ and $m_s = 150 \, \text{MeV}$, for the quark phase.

of the associated neutron star. The onset of quark-deconfinement in the core of the star, will cause a change in the stellar moment of inertia.\cite{39} This change will produce a peculiar evolution of the stellar rotational period ($P = 2\pi/\Omega$) which will cause large deviations\cite{39,40} of the so called pulsar braking index $n(\Omega) = (\Omega \ddot{\Omega}/\dot{\Omega}^2)$ from the canonical value $n = 3$, derived within the magnetic dipole model for pulsars\cite{41} and assuming a constant moment of inertia for the star. The possible measurement of a value of the braking index very different from the canonical value (i.e. $|n| >> 3$) has been proposed\cite{39} as a signature for the occurrence of the quark-deconfinement phase transition in a neutron star. However, it must be stressed that a large value of the braking index could also results from the pulsar magnetic field decay or alignment of the magnetic axis with the rotation axis.\cite{42}

3. Strange Quark Matter

Strange quark matter (SQM) is an infinite deconfined mixture of $u$, $d$ and $s$ quarks in a color singlet state (colorless), together with an appropriate number of electrons to guarantee electrical neutrality. The compositions of SQM, for any given value of the total baryon number density $n$, is determined by the requirement of charge neutrality and equilibrium with respect to the weak processes:

\begin{align*}
    u + e^- &\rightarrow d + \nu_e, \\
    u + e^- &\rightarrow s + \nu_e, \\
    d &\rightarrow u + e^- + \bar{\nu}_e, \\
    s &\rightarrow u + e^- + \bar{\nu}_e, \\
    s + u &\rightarrow d + u
\end{align*}

which can be expressed in terms of the various particles chemical potentials $\mu_i$ as

$$
    \mu_d = \mu_s = \mu = \mu_u + \mu_e
$$

in the case neutrino–free matter ($\mu_{\nu_e} = \mu_{\bar{\nu}_e} = 0$).
The charge neutrality condition requires:

\[
\frac{2}{3}n_u - \frac{1}{3}n_d - \frac{1}{3}n_s - n_e = 0 \tag{2}
\]

where \(n_i\ (i = u, d, s, e)\) is the number density for the different particle species. Equations (1) and (2) implies that there is only one independent chemical potential, which we call the quark chemical potential \(\mu\). Due to the well known difficulties in solving the QCD equations on the lattice at finite density, various models (which incorporates the basic features of QCD) have been developed to get the equation of state of SQM. Along these lines, numerous studies (see e.g. ref.\textsuperscript{34,43–46} and references therein quoted) of SQM have been done using different variants of the Nambu–Jona-Lasinio model.\textsuperscript{47} In some recent work\textsuperscript{48–51}, nonstrange quark matter has been described using a more elaborate approach based on a nonlocal covariant chiral quark model. With this EOS various properties of hybrid stars have been calculated.\textsuperscript{51}

Here, we outline a schematic model\textsuperscript{14,52,53} for the equation of state of SQM, which is inspired to the MIT bag model for hadron. Despite this model is applicable at asymptotic densities (where perturbative QCD is valid), it has become very popular in the study of SQM in astrophysics.\textsuperscript{17,54–56} The basic idea of the model is to suppose that quarks are confined within a spherical region (the bag) of the QCD vacuum. Inside the bag, quarks interact very weakly (perturbatively) each other. The vacuum inside the bag (perturbative vacuum) is considered as an excited state of the true QCD vacuum outside the bag. Perturbative vacuum is characterized by a constant energy density \(B\), the bag constant, which accounts in a phenomenological way of nonperturbative aspects of QCD. This gives rise to an inward pressure \(P_B = -B\) on the surface of the bag, which balances the outward pressure originating from the Fermi motion of quarks and from their perturbative interactions. Thus in the MIT bag model for SQM, the essential phenomenological features of QCD, \textit{i.e.} quark confinement and asymptotic freedom, are postulated from the beginning. The short range qq interaction can be introduced in terms of a perturbative expansion in powers of the QCD structure constant \(\alpha_c\). The \textit{up} and \textit{down} quarks are assumed to be massless \((m_u = m_d = 0)\), and the \textit{strange} quark to have a finite mass, \(m_s\), which is taken as a free parameter.\textsuperscript{8}

The grand canonical potential \(\Omega\) per unit volume of SQM, up to linear terms in \(\alpha_c\) can be written as\textsuperscript{14,17,52,53}

\[
\Omega = \Omega^{(0)} + \Omega^{(1)} + B + \Omega_e. \tag{3}
\]

\(\Omega^{(0)}\) is the contribution of a non–interacting \(u,d,s\) Fermi gas

\[
\Omega^{(0)} = \Omega^{(0)}_u + \Omega^{(0)}_d + \Omega^{(0)}_s \tag{4}
\]

\textsuperscript{8}The value of the current quark mass, as reported by the Particle Data Group (http://pdg.lbl.gov/) are the following: \(m_u = 1–3\) MeV, \(m_d = 3–7\) MeV, \(m_s = 95 \pm 25\) MeV.
\[ \Omega^{(0)}_q = -\frac{1}{(\hbar c)^3} \frac{1}{4\pi^2} \mu_q^4, \quad (q = u, d) \]  
\[ \Omega^{(0)}_s = -\frac{1}{(\hbar c)^3} \frac{1}{4\pi^2} \left\{ \mu_s \mu_s^* \left( \frac{\mu^2_s}{2m_s^2} - \frac{5}{2} \right) + \frac{3}{2} m_s^4 \ln \left( \frac{\mu_s + \mu_s^*}{m_s} \right) \right\} \] 
with \( \mu_s^* \equiv \left( \mu_s^2 - m_s^2 \right)^{1/2} \). The perturbative contribution, \( \Omega^{(1)} \), to the grand canonical potential density up to linear terms in \( \alpha_c \) is
\[ \Omega^{(1)} = \Omega^{(1)}_u + \Omega^{(1)}_d + \Omega^{(1)}_s, \]  
\[ \Omega^{(1)}_q = \frac{1}{(\hbar c)^3} \frac{1}{4\pi^2} \frac{2\alpha_c}{\pi} \mu_q^4, \quad (q = u, d) \]  
\[ \Omega^{(1)}_s = \frac{1}{(\hbar c)^3} \frac{1}{4\pi^2} \frac{2\alpha_c}{\pi} \left\{ 3 \left[ \mu_s \mu_s^* - m_s^2 \ln \left( \frac{\mu_s + \mu_s^*}{m_s} \right) \right]^2 - 2\mu_s^4 \right\} \]  
\[ - 3m_s^4 \ln \left( \frac{m_s}{\mu_s} \right) + 6 \ln \left( \frac{\rho_{ren}}{\mu_s} \right) \left[ \mu_s \mu_s^* m_s^2 - m_s^4 \ln \left( \frac{\mu_s + \mu_s^*}{m_s} \right) \right] \] 
where \( \rho_{ren} \) is the so called renormalization point (see ref.\(^5\(^3\)). In the case of massless \( u \) and \( d \) quarks a standard choice is \( \rho_{ren} = 313 \) MeV. \( \Omega_e \) is the electron contribution to \( \Omega \). Electrons are treated as a massless (since in compact star interiors \( \mu_e >> m_e \)) non-interacting Fermi gas, thus their contribution to \( \Omega \) is given by Eq. (5) in terms of \( \mu_e \).

To summarize, the EOS of SQM (at zero temperature) in the approximation for the grand potential density we are considering is:
\[ P = -\Omega^{(0)} - \Omega^{(1)} - B - \Omega_e \]  
\[ \rho = \frac{\varepsilon}{c^2} = \frac{1}{c^2} \left\{ \Omega^{(0)} + \Omega^{(1)} + \sum_{f=u,d,s} \mu_f n_f + B + \Omega_e \right\}, \] 
with \( P = -\Omega \) being the pressure, \( \rho \) the (mass) density and \( \varepsilon \) the energy density.

The number densities for each particle species can be calculated using the thermodynamical relation:
\[ n_i = -\left( \frac{\partial \Omega_i}{\partial \mu_i} \right)_{TV}, \quad (i = u, d, s, e) \]  
and the total baryon number density is
\[ n = \frac{1}{3} (n_u + n_d + n_s) \] 

For pedagogical purpose, let us consider the limiting case of massless non-interacting quarks (\( m_s = 0, \alpha_c = 0 \)). In this case SQM is composed by an equal
number of $u, d, s$ quarks with no electrons (i.e. $n_e = 0, n_u = n_d = n_s$) and the EOS takes the following simple form:

$$\varepsilon = Kn^{4/3} + B, \quad P = \frac{1}{3} Kn^{4/3} - B, \quad K = \frac{9}{4}\frac{\pi^{2/3}}{\hbar c}.$$ \hspace{1cm} (14)

Eliminating the total baryon number density, one has:

$$P = \frac{1}{3} (\varepsilon - 4B).$$ \hspace{1cm} (15)

To recapitulate, the properties of SQM, within this model, depend on the values of the three parameters $B, m_s$ and $\alpha_c$. The EOS, in the form $P = P(\rho)$, is essentially determined by the value of the bag constant $B$. The net fraction of leptons ($e^-, e^+$) which neutralize the electric charge of the quark component of SQM, will mainly depend on the values of $m_s$ and $\alpha_c$. Most frequently (i.e. for the most plausible values of the model parameters$^{53}$), the quark component has a positive electric charge, thus electrons will be present in SQM to neutralize it (as it was assumed by tacit agreement in the definition of SMQ given at the beginning of this section$^b$).

### 3.1. The strange matter hypothesis

According to the so called strange matter hypothesis$^{20–22}$ (or Bodmer–Terazawa–Witten hypothesis), SQM is the true ground state of matter. In other words, the energy per baryon of SQM (at the baryon density where the pressure is equal to zero) is supposed to be less than the lowest energy per baryon found in atomic nuclei, which is about 930.4 MeV for the most bound nuclei ($^{62}$Ni, $^{58}$Fe, $^{56}$Fe).

If the strange matter hypothesis is true, then a nucleus with $A$ nucleons, could in principle lower its energy by converting to a strangelet. However, this process requires the simultaneous weak decay of about a number $A$ of $u$ and $d$ quarks of the nucleus into strange quarks. The probability for such a process is proportional to $G_F^2 A$, with $G_F$ being the Fermi constant. Thus, for a large enough baryon number ($A > A_{\text{min}} \sim 5$), this probability is extremely low, and the mean life time for an atomic nucleus to decay to a strangelet is much higher than the age of the Universe. In addition, finite size effects (surface, coulomb and shell effects) place a lower limit ($A_{\text{min}} \sim 10–10^3$, depending on the values of the model parameters) on the baryon number of a stable strangelet even if in bulk SQM is stable$^{53,54}$. On the other hand, a step by step production (i.e. at different times) of $s$ quarks will produce hyperons in the nucleus, that is to say, a system (hypernucleus) with a higher energy per baryon with respect to the original nucleus. Thus according to the strange matter hypothesis, the ordinary state of matter, in which quarks are confined within hadrons is a metastable state with a mean life time much higher than the age of the Universe.

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$^b$For small values of $m_s$ and large values of $\alpha_c$ the quark component of SQM has a negative charge$^{53}$ and thus positrons will be present to guarantee global charge neutrality.
The success of traditional nuclear physics, in explaining an astonishing amount of experimental data, provides a clear indication that quarks in a nucleus are confined within protons and neutrons. Thus the energy per baryon for a droplet of \(u, d\) quark matter (the so called nonstrange quark matter in the bulk limit) must be higher than the energy per baryon of a nucleus with the same baryon number.

These conditions in turn may be used to constrain the values of the parameters entering in models for the equation of state of SQM and to find the region in the parameter space where the strange matter hypothesis is fulfilled and nonstrange quark matter is unstable. For example,\(^{53,54}\) in the case of the bag model EOS, for non-interacting quarks (\(\alpha_c = 0\)) one has \(B \simeq 57–91\) MeV/fm\(^3\) for \(m_s = 0\), and \(B \simeq 57–75\) MeV/fm\(^3\) for \(m_s = 150\) MeV. Our present understanding of the properties of ultra-dense hadronic matter, does not allow us to exclude or to accept \textit{a priori} the validity of the strange matter hypothesis.

4. Strange stars

One of the most important and captivating consequences of the strange matter hypothesis is the possible existence of \textit{strange stars}, that is compact stars which are completely (or almost completely) made of SQM\(^1\). These stars, as we will see in a moment, have bulk properties (mass and radius) very similar to those of neutron stars (hadronic stars). Thus pulsars could be strange stars.

The structural properties of non-rotating compact stars are obtained integrating numerically the Tolman–Oppenheimer–Volkoff (TOV) equations.\(^8,17,41\) The basic input to solve these equations is the stellar matter EOS. In the case of strange stars, one has to use one of the various models for SQM.

The properties of the maximum mass configuration for strange star sequences obtained using a few models for the SQM equation of state are reported in table 1. In this table, the EOS labeled as \(B60_0\) refer to the bag model EOS (described in sect. 3) with \(B = 60\) MeV/fm\(^3\) and \(m_s = 0\); model \(B60_{200}\) is for the same value of the bag constant but taking \(m_s = 200\) MeV; model \(B90_0\) refer to the case \(B = 90\) MeV/fm\(^3\) and \(m_s = 0\) (\(\alpha_c = 0\) in all the three cases). The models denoted as \(SS1\) and \(SS2\) refer to stellar sequences obtained with the equation of state for SQM by Dey \textit{et al.}\(^{57}\)

In Fig. 3 we plot the mass-radius (MR) relation for strange stars (red curves) obtained using different model for SQM. For comparison, we plot in the same figure, the MR relations for hadronic stars (black curves): The curves labeled with \(BBB1\), \(BBB2\) (ref.\(^{58}\)), \(WFF\) (ref.\(^{59}\)) and \(KS\) (ref.\(^{60}\)) are for \textit{nucleon stars} (\textit{i.e.} neutron stars having a \(\beta\)-stable nuclear matter core), the curve labeled \textit{hyp} refers to an \textit{hyperon star} (ref.\(^{17,38}\)). As we can see, there is a striking qualitative difference between the mass-radius relation of strange stars with respect to that of neutron stars. For

\(^1\)When the values of the EOS parameters are such that the strange matter hypothesis is not fulfilled, the possible compact stars containing deconfined quark matter are the hybrid stars.
Table 1. Properties of the maximum mass configuration for strange stars obtained from different equations of state of SQM (see text for details). $M$ is the gravitational stellar (maximum) mass in unit of the solar mass $M_\odot$, $R$ is the corresponding radius, $\rho_c$ the central density, $n_c$ the central number density in unit of the saturation density ($n_0 = 0.16$ fm$^{-3}$) of nuclear matter, $P_c$ is the central pressure.

| EOS  | $M/M_\odot$ | $R$ (km) | $\rho_c$ (g/cm$^3$) | $n_c/n_0$ | $P_c$ (dyne/cm$^2$) |
|------|-------------|----------|----------------------|-----------|---------------------|
| $B090_0$ | 1.964 | 10.71 | $2.06 \times 10^{15}$ | 6.94 | $0.49 \times 10^{36}$ |
| $B090_{200}$ | 1.751 | 9.83 | $2.44 \times 10^{15}$ | 7.63 | $0.54 \times 10^{36}$ |
| $B090_0$ | 1.603 | 8.75 | $3.09 \times 10^{15}$ | 9.41 | $0.73 \times 10^{36}$ |
| $SS1$ | 1.438 | 7.09 | $4.65 \times 10^{15}$ | 14.49 | $1.40 \times 10^{36}$ |
| $SS2$ | 1.324 | 6.53 | $5.60 \times 10^{15}$ | 16.34 | $1.64 \times 10^{36}$ |

strange stars with “small” (i.e. $M$ not to close to $M_{\text{max}}$) mass, $M$ is proportional to $R^3$. In contrast, neutron stars (hadronic stars) have radii that decrease with increasing mass. This difference in the MR relation is a consequence of the differences in the underlying interactions between the stellar constituents for the two types of compact stars. In fact, “low” mass strange stars are bound by the strong interaction, contrary to the case of neutron stars, which are bound by gravity. This can be demonstrated looking at the different contributions (gravitational and internal binding energy) to the stellar total binding energy (see for example Fig.s 1, 2 and 3 in ref. 55).

As it is well known, there is a minimum mass for a neutron star ($M_{\text{min}} \sim 0.1M_\odot$). In the case of a strange star, there is essentially non minimum mass. As the stellar central density $\rho_c \rightarrow \rho_{\text{surf}}$ (surface density) a strange star (or better to say a lump of SQM for low value of the baryon number) is a self-bound system, until the baryon number becomes so low that finite size effects destabilize it.

4.1. The surface: bare or crusted strange stars?

According to the standard view, a strange star has a very sharp boundary. In fact, the density drops abruptly from $\rho_{\text{surf}} \sim 4\times 10^{14}$ g/cm$^3$ to zero on a length scale typical of strong interactions, in other words the thickness of the stellar “quark surface” is of a few fermis (1 fm = $10^{-15}$m). This is of the same order of the thickness of the surface of an atomic nucleus. The density at the surface of a strange star, can be immediately calculated (in the limit $m_s \rightarrow 0$) using the simple

\footnote{As an idealized example, remember that pure neutron matter is not bound by nuclear forces (see e.g. ref. 61).}
Fig. 3. The mass-radius relation for different theoretical models of compact stars. Black curves refer to hadronic stars and red curves to strange stars. The green dashed curve gives the upper limit, extracted from observational data by X.-D. Li et al.,\textsuperscript{97} for the radius of the compact star in SAX J1808.4-3658. The green dashed straight line labeled \( R_{\text{Sch}} \) gives the Schwarzschild radius as a function of the stellar mass.

EOS given in Eq.s (14) and (15), and it is given by

\[
\rho_{\text{surf}} = \frac{4B}{c^2}, \quad \eta_{\text{surf}} = \left( \frac{3B}{K} \right)^{3/4}.
\]

Strange stars with this sort of exposed quark matter surface are known as bare strange stars.

Electrons are bound to the star by the electromagnetic force, thus they can extend for several hundreds of fermis above the “quark surface”. This thin layer is usually referred to as the \textit{electrosphere}. As a consequence of this charge distribution a very strong electric field is established at the stellar surface. This field has been estimated to be of about \( 10^{17} \) V cm\(^{-1} \) and directed outward.\textsuperscript{23} Such a huge electric field is expected to produce an intense emission of \( e^+e^- \) pairs\textsuperscript{64} and a subsequent hard X-ray spectrum, at luminosities well above the Eddington limit \( k \), as long as the stellar surface temperature is above \( \sim 5 \times 10^8 \) K. Thus a bare strange star should produce a striking characteristic signal, which differs both qualitatively and quantitatively from the thermal emission from a neutron star (hadronic star), and which could provide an observational signature for their existence.\textsuperscript{64–67}

The Eddington limit is the critical luminosity (due to matter accretion onto the star) above which the outward force due to the radiation pressure exceeds the inward gravitation force acting on the infalling material. In the case of steady spherical accretion of hydrogen, Thomson scattering, Newtonian gravity, the Eddington luminosity is given by\textsuperscript{41} \( L_{\text{Edd}} \approx 1.3 \times 10^{38}(M/M_{\odot}) \) erg/s. A super-Eddington luminosity \( (L > L_{\text{Edd}}) \) is allowed for a bare strange star, because at its surface SQM is bound by strong interaction rather than gravity.\textsuperscript{23,64}
special conditions (i.e. bare quark surface and high temperature) are realized in the case of “young” strange stars.\textsuperscript{68–70}

Older and “cold” ($T < 5 \times 10^8$ K) strange stars will likely form a crust of “normal” (nuclei and electrons) matter via mass accretion onto the star from the interstellar medium or from a companion star, or from the matter left in the supernova explosion which could have formed the strange star. In fact, the strong electric field at the stellar surface will produce a huge outward-directed force on any single positive ion (nucleus) of normal matter which is accreted onto the star. This force greatly overwhelms the force of gravity acting on the incoming positive ion. This accreted material will start to be accumulated on the top of the electrosphere. Thus a strange star will form a crust of normal matter,\textsuperscript{23} which is suspended above the quark surface by the tiny ($\sim 10^2$–$10^3$ fm) electrostatic gap, and will completely obscure the “quark surface”. This crust is similar in composition to the outer crust of a neutron star (i.e. a Coulomb lattice of heavy neutron rich nuclei plus an electron gas). In fact, when the density of matter at the base of this crust will reach the neutron drip density ($\rho_n^{\text{drip}} \approx 4.3 \times 10^{11}$ g/cm$^3$), the neutrons could freely enter in the SQM stellar core, and there they will be dissolved (their constituent quarks will be deconfined) due to the assumed absolute stability of SQM. Thus, the density at the base of the crust can not exceeds the value of the neutron drip density. This condition sets an upper limit for the mass of the stellar crust. For a strange star with a mass of $1.4 M_\odot$, the mass of its crust is of the order of $M_\text{crust} \sim 10^{-5} M_\odot$ and its thickness of the order of a few hundreds of meters.\textsuperscript{23,62}

Recently, the gap between the quark surface and the nuclear crust of strange stars has been investigated within a model\textsuperscript{63} which accounts for the detailed balance between electrical and gravitational forces and pressure, and in addition considers the effects of color superconductivity possibly occurring in the strange star core.

These so called crusted strange stars will look very similar to neutron stars concerning their emission properties, which are mainly determined by the stellar surface composition.

Very recently, a new and alternative description of the surface region of a strange star has been proposed. According to the authors of ref.\textsuperscript{71}, the crust of a strange star could consist of an heterogeneous phase composed of a Coulomb lattice of positively charged strangelets immersed in an sea of electrons. At the border with the uniform SQM stellar core, the crust will consist of bubbles filled with an electron gas embedded in SQM. In between these two extreme regions, one will find various geometrical structures (rods, plates, etc) of one component (SQM/e$^-$ gas) into the other, similarly to the situation encountered in the crust-core transition layer of a neutron star or in the mixed hadron-quark layer in a hybrid star. This alternative possibility for the crust structure, descend from imposing global (rather than local) electric charge neutrality,\textsuperscript{16} and it could be realized when the finite-size energy cost (Coulomb, surface energies, etc) is less than the energy gain passing from the homogeneous to the hererogeneous phase. In the context of the bag model for SQM, it has been estimated\textsuperscript{72} that this strangelet crust could be formed when the quark
matter surface tension is less that a critical value given by
\[ \sigma_{\text{crit}} \simeq 12 \left( \frac{m_s}{150 \text{ MeV}} \right)^3 \frac{m_s}{\mu} \text{ MeV/fm}^2 \]  
where \( \mu \) is the quark chemical potential (\( \mu \approx 300 \text{ MeV} \) in the surface region). For the *strangelet crust* of a strange star with a total mass of \( 1.4M_\odot \) and total radius \( R = 10 \text{ km} \), the authors of ref.\textsuperscript{71} have found \( M_{\text{crust}} \simeq 6 \times 10^{-6}M_\odot \) and a radial extension \( \Delta R \approx 40 \text{ m} \). In the case of a *strangelet crust*, there is a much reduced density jump at the stellar surface, and a different radiation spectrum is expected with respect to the case of a *bare* strange star.

4.2. The core: role of color superconductivity

Until now, we have considered a very simple picture for the stellar core. The core is made of (locally) uniform SQM with a modest radial dependence of the density profile.\textsuperscript{23} However, the internal structure of a strange star could be much more complex. As we already mentioned in the introduction, a large number of color superconducting phases could be present in the range of densities spanned in the stellar core. Of particular astrophysical interest are cristalline phases of color superconducting SQM. The presence of these cristalline structures could constitute a basic prerequisite for modelling pulsar glitches in strange stars.

5. Neutron star observations

Many different kinds of astrophysical observations are currently used\textsuperscript{73–75} to uncover the true nature of neutron stars. In the preceding pages, we have already mentioned some of such observables. In addition, observations of the thermal radiation from isolated neutron stars, combined with measurements (estimates) of their ages, allow for the determination of the stellar cooling history. These data, when compared with theoretical cooling curves,\textsuperscript{70,76–79} provide important informations to get a realistic picture of the neutron star composition.

Binary stellar systems in which at least one component is a neutron star represent the most reliable way to measure the mass of the compact star. One of the most accurate mass determination is that of the neutron star associated to the pulsar PSR 1913 +16, which is a member of a tight (orbital period equal to 7 h 45 min) neutron star-neutron star system. The mass of PSR 1913 +16 is\textsuperscript{80} \( 1.4408 \pm 0.0003M_\odot \). Such impressive accuracy is made possible by measuring general relativistic effects, such as the orbital decay due to gravitational radiation, the advance of periastron, the Shapiro delay, *etc*. Neutron star masses in NS–NS binary systems lie in the range 1.18 to 1.44 \( M_\odot \) (see ref.\textsuperscript{81,82}). Recently Nice *et al.*\textsuperscript{83} have determined the mass of the neutron star associated to the millisecond pulsar PSR J0751 +1808, which is a member of a binary system with a helium white dwarf secondary. Measuring general relativistic effects, the authors of ref.\textsuperscript{83} have obtained for the mass of PSR J0751 +1808 the value \( 2.1 \pm 0.2 \) \( M_\odot \) at 68% confidence level \( (2.1^{+0.2}_{-0.5})M_\odot \) at
95% confidence level). Another important finding is the determination of a lower limit for the mass of the compact star in the pulsar I of the globular cluster Terzan 5 (Ter 5 I). At 95% confidence, the mass of Ter 5 I exceeds $1.68 \, M_{\odot}$. These large values of the measured masses of PSR J0751 +1808 and Ter5 I are very important, since they push up (with respect to the previous measured mass sample$^{81,82}$) the lower limit of the value of the Oppenheimer-Volkoff maximum mass $M_{\text{max}}$ of neutron stars. As it is well known, $M_{\text{max}}$ is directly related to the overall stiffness of the EOS.

Another quantity related to the EOS is the maximum rate of rotation ($\Omega_{\text{max}}$) sustainable by a compact star. Equilibrium sequences of rapidly rotating compact stars have been constructed numerically in general relativity by several groups.$^{85-88}$ The numerical results for $\Omega_{\text{max}}$ obtained for a broad set of realistic EOS can reproduced with a very good accuracy using simple empirical formulae which relate $\Omega_{\text{max}}$ to the mass ($M_{\text{max}}$) and radius $R_0$ of the non-rotating maximum mass configuration.$^{89}$

In the following part of this section, we discuss a few observational constraints for the mass-radius relation of neutron stars.

### 5.1. SAX J1808.4-3658

The transient X-ray source and Low Mass X-ray Binary (LMXB) SAX J1808.4-3658 was discovered in September 1996 by the Dutch-Italian BeppoSAX satellite.$^{90}$ Type-I X-ray bursts have been detected from this source in September 1996, April 1998, and October 2002. Very recently, a weak precursor event to the burst of October 19, 2002, has been identified in the observational data taken by the Rossi X-ray Timing Explorer satellite. Coherent X-ray pulsation with a period of $P = 2.49 \, \text{ms}$ was discovered in 1998 by Wijnands and van der Klis.$^{92}$ Lately, burst oscillations (i.e. millisecond-period brightness oscillations during X-ray bursts) have been detected for this source, at the frequency of $\sim 401 \, \text{Hz}$, i.e. at the same frequency of the coherent X-ray pulsation. This has confirmed the expectation that the burst oscillations frequency is equal to the pulsar frequency, and thus to the rotational spin frequency of the associated compact star.

SAX J1808.4-3658 is the first and the best studied member of the class of accretion-driven millisecond pulsars (see$^{94,95}$ and references therein quoted). These sources are believed to be the progenitors of millisecond radio pulsars by the way of spin up by mass transfer from the companion star in a LMXB.$^{96}$

Using the measured X-ray fluxes during the high- and low-states of the source in the time of the April 1998 outburst, and in addition the restrictions from modelling the observed X-ray pulsation at $P = 2.49 \, \text{ms}$, X.-D. Li et al.$^{97}$ have derived an upper limit for the radius of the compact star in SAX J1808.4-3658, which is given by the dashed curve in Fig. 3. The dashed line, labeled $R = R_{\text{Sch}}$, in this figure represents the Schwarzschild radius, which is the lower limit for the stellar radius. In fact, being a source of type-I X-ray burst and according to the current interpretation
of this phenomenon (thought to be produced by thermonuclear explosion on the surface of a neutron star), the compact object in SAX J1808.4-3658 can not be a black hole. Thus the allowed range for the mass and radius of SAX J1808.4-3658 is the region confined by these two dashed lines in Fig. 3. In the same figure, we report the theoretical mass-radius relations for different sequences for pure hadronic compact star (black curves) and strange stars. These models have been illustrated in the previous section. The results reported in Fig. 3, clearly indicate that a strange star model is more compatible with the semiempirical mass-radius relation for SAX J1808.4-3658, than an hadronic star model.

Recently Leahy et al., from the analysis of the light curves of SAX J1808.4-3658 during its 1998 and 2002 outburst, have obtained limits on the mass and radius for the compact star in this source. At the 99.7% confidence level they obtain $6.9 \text{ km} \leq R \leq 11.9 \text{ km}$, and $0.75 \leq M/M_\odot \leq 1.56$. These limits are compatible with strange stars models as well as with hybrid or pure hadronic stars (see the theoretical MR curves in Figs 3 and 4). For example, in the case of a non-rotating nucleonic stars within the BBB1 equation of state for a star with $M = 1.0M_\odot$, one has $R = 11.2 \text{ km}$. Rotation with a period of 2.5 ms, will not change appreciably the value of the stellar radius, since this rotation rate is still "far" from the mass shed limit for this EOS.
5.2. XTE J1739-285

In a recent paper Kaaret et al.\textsuperscript{99} have reported the discovery of burst oscillations at 1122 Hz in the X-ray transient XTE J1739-285. If the burst oscillation frequency in this source is coincident with the stellar spin rate, as in the case of SAX J1808.4-3658 (ref.\textsuperscript{93}), thus XTE J1739-285 contains the most rapidly rotating compact star discovered up to now, and the first with a submillisecond spin period ($P = 0.891$ ms). This discovery gives a model independent observational constraint to EOS dense stellar matter, and it has prompted a number of studies\textsuperscript{100–102} in this direction.

In one of these studies, Lavagetto et al.\textsuperscript{100} have derived the following upper limit

$$R < 9.52\left(\frac{M/M_\odot}{1} \right)^{1/3} \text{ km}$$  \hspace{1cm} (18)

for the radius of the compact star in XTE J1739-285, using its inferred spin period and a simple empirical formula\textsuperscript{103} which approximately gives the minimum rotation period $P_{\text{min}} = 2\pi/\Omega_{\text{max}}$ for a star of mass $M$ and non-rotating radius $R$. This upper limit is depicted in Fig. 4 by the green curve labeled XTE J1739-285. For comparison are reported in the same figure, the theoretical MR relations for nucleon stars (black curves), hyperon stars (brown curve), hybrid stars (blue curves) and strange stars (red curves). Some of these models have been already mentioned in the previous sections. Curve (L) is relative to neutrons stars with a pure neutron matter core.\textsuperscript{104} This stellar sequence has been included as an extreme example of a very stiff EOS. The nucleon star sequence labeled (GM1) has been obtained within a relativistic quantum field theory approach in the mean field approximation, for one of the parameter set (GM1) given by Glendenning and Moszkowski.\textsuperscript{38} The curve (Hyp) refers to hyperon stars calculated with the previous GM1 EOS when hyperons are include. Curves (Hyb1) and (Hyb2) represent hybrid star sequences calculated using the same GM1 EOS for hyperonic matter to describe the hadronic phase, and the bag model EOS to describe the quark phase. We took $B = 208.24$ MeV/fm$^3$ (Hyb1), $B = 136.63$ MeV/fm$^3$ (Hyb2) and $m_u = m_d = 0$, $m_s = 150$ MeV. The curve (Hyb3) refers to hybrid stars constructed using a different parametrization for the EOS by\textsuperscript{38} (GM3) for the hadronic phase and using $B = 80$ MeV/fm$^3$. The MR curve labeled (Hyb(CFL)) refers to hybrid stars\textsuperscript{105} whose core is a mixed phase of nuclear matter and color-superconducting CFL quark matter. Finally, we consider CFL strange stars\textsuperscript{107} (SS(CFL)) within the bag model EOS with $B = 70$ MeV/fm$^3$, $m_u = m_d = 0$ and $m_s = 150$ MeV and quark pairing gap $\Delta = 100$ MeV.

From the results in Fig. 4, we see that the constraint derived from the burst oscillations in XTE J1739-285 does not allow to discriminate among possible different types of compact stars. Anyhow, no one of the present astrophysical observations can prove or confute the existence of strange stars (or hybrid stars), \textit{i.e.} the presence of SQM compact stars.\textsuperscript{73–75,108}
6. Metastability of hadronic stars and their delayed conversion to quark stars

One of the most recent developments in studying the astrophysical implications of SQM in compact stars is the realization that pure hadronic compact stars above a threshold value of their mass are metastable.\textsuperscript{109–111} The metastability of hadronic stars originates from the finite size effects in the formation process of the first SQM drop in the hadronic environment.

In cold ($T = 0$) bulk matter the quark-hadron mixed phase begins at the static transition point defined according to the Gibbs’ criterion for phase equilibrium

$$
\mu_H = \mu_Q \equiv \mu_0, \quad P_H(\mu_0) = P_Q(\mu_0) \equiv P_0
$$

where $\mu_H = (\epsilon_H + P_H)/n_{b,H}$ and $\mu_Q = (\epsilon_Q + P_Q)/n_{b,Q}$ are the chemical potentials for the hadron and quark phase respectively, $\epsilon_H$ ($\epsilon_Q$), $P_H$ ($P_Q$) and $n_{b,H}$ ($n_{b,Q}$) denote respectively the total (i.e., including leptonic contributions) energy density, the total pressure and baryon number density for the hadron (quark) phase.

Consider now the more realistic situation in which one takes into account the energy cost due to finite size effects in creating a drop of deconfined QM in the hadronic environment. As a consequence of these effects, the formation of a critical-size drop of QM is not immediate and it is necessary to have an overpressure $\Delta P = P - P_0$ with respect to the static transition point. Thus, above $P_0$, hadronic matter is in a metastable state, and the formation of a real drop of QM occurs via a quantum nucleation mechanism. A sub-critical (virtual) droplet of deconfined QM moves back and forth in the potential energy well separating the two matter phases on a time scale $\nu_0^{-1} \sim 10^{-23}$ seconds, which is set by the strong interactions. This time scale is many orders of magnitude shorter than the typical time scale for the weak interactions, therefore quark flavor must be conserved during the deconfinement transition. We will refer to this form of deconfined matter, in which the flavor content is equal to that of the $\beta$-stable hadronic system at the same pressure, as the $Q^*$-phase. Soon afterwards a critical size drop of QM is formed the weak interactions will have enough time to act, changing the quark flavor fraction of the deconfined droplet to lower its energy, and a droplet of $\beta$-stable SQM is formed (hereafter the Q-phase).

In the scenario proposed by the authors of ref.,\textsuperscript{109} a pure HS whose central pressure is increasing due to spin-down or due to mass accretion, \textit{e.g.}, from the material left by the supernova explosion, or from a companion star. As the central pressure exceeds the threshold value $P_0^*$ at the static transition point, a virtual drop of quark matter in the $Q^*$-phase can be formed in the central region of the star. As soon as a real drop of $Q^*$-matter is formed, it will grow very rapidly and the original HS will be converted to strange star or to an hybrid star, depending on whether or not the strange matter hypothesis is fulfilled.

To calculate the nucleation time $\tau$, \textit{i.e.}, the time needed to form the first critical droplet of deconfined QM in the hadronic medium, one can use the relativistic quantum nucleation theory\textsuperscript{112} (for more details, see\textsuperscript{109,110}). The nucleation time $\tau$, \textit{et al.}
Fig. 5. Nucleation time as a function of the gravitational mass of the hadronic star. Solid lines correspond to a value of $\sigma = 30$ MeV/fm$^2$ whereas dashed ones are for $\sigma = 10$ MeV/fm$^2$. The nucleation time corresponding to one year is shown by the dotted horizontal line. The different values of the bag constant (in units of MeV/fm$^3$) are plotted next to each curve. The hadronic phase is described with the GM1 model for the equation of state.

can be calculated for different values of the stellar central pressure $P_c$. In Fig. 5, we plot $\tau$ as a function of the gravitational mass $M_{HS}$ of the HS corresponding to the given value of the central pressure, as implied by the solution of the TOV equations for the pure HS sequences. Each curve refers to a different value of the bag constant and surface tension $\sigma$. As we can see, from the results in Fig. 5, a metastable hadronic star can have a mean-life time many orders of magnitude larger than the age of the universe $T_{\text{univ}} = (4.32 \pm 0.06) \times 10^{17}$ s. As the star accretes a small amount of mass (of the order of a few per cent of the mass of the sun), the consequential increase of the central pressure leads to a huge reduction of the nucleation time and, as a result, to a dramatic reduction of the HS mean-life time.

To summarize, pure hadronic stars having a central pressure larger than the static transition pressure for the formation of the Q*-phase are metastable to the “decay” (conversion) to a more compact stellar configuration in which deconfined QM is present (HyS or SS). These metastable HS have a mean-life time which is related to the nucleation time to form the first critical-size drop of deconfined matter in their interior. The critical mass $M_{cr}$ of the metastable HS is defined as the value of the gravitational mass for which the nucleation time is equal to one year: $M_{cr} \equiv M_{HS}(\tau = 1\ yr)$. Pure hadronic stars with $M_H > M_{cr}$ are very unlikely to be observed. $M_{cr}$ plays the role of an effective maximum mass for the hadronic branch of compact stars. While the Oppenheimer–Volkov maximum mass $M_{HS,max}$

\[ The\ actual\ mean-life\ time\ of\ the\ HS\ will\ depend\ on\ the\ mass\ accretion\ or\ on\ the\ spin-down\ rate\ which\ modifies\ the\ nucleation\ time\ via\ an\ explicit\ time\ dependence\ of\ the\ stellar\ central\ pressure.\]
is determined by the overall stiffness of the EOS for hadronic matter, the value of $M_{cr}$ will depend in addition on the bulk properties of the EOS for quark matter and on the properties at the interface between the confined and deconfined phases of matter (e.g., the surface tension $\sigma$).

In Fig. 6, we show the mass-radius (MR) curve for pure HSs within the GM1 model for the EOS of the hadronic phase, and that for hybrid or strange stars for different values of the bag constant $B$. The configuration marked with an asterisk on the hadronic MR curves represents the HS for which the central pressure is equal to $P_0^*$ and $\tau = \infty$. The full circle on the HS sequence represents the critical mass $M_{cr}$ configuration, in the case $\sigma = 30$ MeV/fm$^2$. The full circle on the HyS (SS) mass-radius curve represents the hybrid (strange) star which is formed from the conversion of the hadronic star with $M_{HS} = M_{cr}$. We assume that during the stellar conversion process the total number of baryons in the star (i.e. the stellar baryonic mass) is conserved. Thus the total energy liberated in the stellar conversion is given by the difference between the gravitational mass of the initial hadronic star ($M_{in} \equiv M_{cr}$) and that of the final hybrid or strange stellar configuration $M_{fin} \equiv M_{QS}(M_{cr})$ with the same baryonic mass: $E_{conv} = (M_{in} - M_{fin})c^2$.

The stellar conversion process starts to populate the new branch of quark stars (see Fig. 6). Long term accretion on the quark star can next produce stars with masses up to the maximum mass $M_{QS, max}$ for the QS sequence.
The possibility to have metastable hadronic stars, together with the feasible existence of two distinct families of compact stars, demands an extension of the concept of maximum mass of a “neutron star” with respect to the classical one introduced by Oppenheimer and Volkoff.\(^8\) Since metastable HS with a “short” mean-life time are very unlikely to be observed, the extended concept of maximum mass must be introduced in view of the comparison with the values of the mass of compact stars deduced from direct astrophysical observation. Having in mind this operational definition, the authors of ref.\(^110\) define as limiting mass of a compact star, and denote it as \(M_{\text{lim}}\), the physical quantity defined in the following way:

\[(a)\text{ if the nucleation time } \tau(M_{\text{HS,max}}) \text{ associated to the maximum mass configuration for the hadronic star sequence is of the same order or much larger than the age of the universe } T_{\text{univ}}, \text{ then} \]
\[M_{\text{lim}} = M_{\text{HS,max}}, \quad (20)\]
in other words, the limiting mass in this case coincides with the Oppenheimer–Volkoff maximum mass for the hadronic star sequence.

\[(b)\text{ If the critical mass } M_{\text{cr}} \text{ is smaller than } M_{\text{HS,max}} \text{ (i.e. } \tau(M_{\text{HS,max}}) < 1 \text{ yr}), \text{ thus the limiting mass for compact stars is equal to the largest value between the critical mass for the HS and the maximum mass for the quark star (HyS or SS) sequence} \]
\[M_{\text{lim}} = \max[M_{\text{cr}}, M_{\text{QS,max}}], \quad (21)\]

\[(c)\text{ Finally, one must consider an “intermediate” situation for which } 1\text{ yr} < \tau(M_{\text{HS,max}}) < T_{\text{univ}}. \text{ As the reader can easily realize, now} \]
\[M_{\text{lim}} = \max[M_{\text{HS,max}}, M_{\text{QS,max}}], \quad (22)\]
depending on the details of the EOS which could give \(M_{\text{HS,max}} > M_{\text{QS,max}}\) or vice versa.

The delayed stellar conversion process, described so far, represents the second “explosion” – the Quark Deconfinement Nova (QDN) – in the two-step scenario proposed in ref.\(^109,110\) to explain a possible delayed SN-GRB connection.

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