Direct detection of electroweak dark matter

Ramtin Amintaheri

School of Physics, The University of Sydney, Physics Road, Camperdown, NSW 2006, Australia

1 Introduction

Astronomical measurements from sub-galactic to large cosmological scales require the existence of huge amount of obscure non-luminous matter in the universe which is not contained in the Standard Model (SM) of particles [1,2]. Dark matter (DM) constitutes great majority of the total mass density of the cosmos [3], and takes a key role in the characteristics of large and small astrophysical structures [4]. Among various hypotheses concerning the nature of DM and its interactions, it is compelling to couple the dark sector to the SM in a such minimal way that no new gauge field is introduced. In this method, the known features of the Standard model including the symmetry group, fundamental

Abstract TeV-scale dark matter is well motivated by notions of naturalness as the new physics threshold is expected to emerge in the TeV regime. We generalise the Standard Model by including an arbitrary SU(2) multiplet of dark matter particles in non-chiral representation. The pseudo-real representations can be viable DM candidates if one considers a higher-dimensional operator which induces mass-splitting, and avoids the tree-level inelastic scattering through Z-boson exchange. These effective operators give rise to sizable contributions from Higgs mediated dark matter interactions with quarks and gluons. A linear combination of the effective couplings named $\lambda$ is identified as the critical parameter in determining the magnitude of the cross-section. When $\lambda$ is smaller than the critical value, the theory behaves similar to the known renormalisable model, and the scattering rate stays below the current experimental reach. Nevertheless, above the criticality, the contribution from the higher-dimensional operators significantly changes the phenomenology. The scattering amplitude of pseudo-real models will be coherently enhanced, so that it would be possible for next generation large-exposure experiments to fully probe these multiplets. We studied the parameter space of the theory, taking into account both indirect astrophysical and direct search constraints. It is inferred that multi-TeV mass scale remains a viable region, quite promising for forthcoming dark matter experiments.

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Among various hypotheses concerning the nature of DM and its interactions, it is compelling to couple the dark sector to the SM in a such minimal way that no new gauge field is introduced. In this method, the known features of the Standard model including the symmetry group, fundamental

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a e-mail: Ramtin.Amintaheri@gmail.com (corresponding author)
interactions, quantum numbers, and spontaneous symmetry breaking are preserved; and no new forces are added to the physics. Since weak bosons are the only fundamental force carriers that can mediate interactions with dark matter, this construction can be realised simply through introducing a non-trivial electroweak multiplet.

We generalise the Glashow–Weinberg–Salam (GWS) Theory by an extra fermionic \(n\)-tuplet of \(SU(2) \times U(1)_Y\) symmetry group in the non-chiral representation, whose neutral component is referred to as electroweak dark matter (EWDM). The important feature of the fermionic version is that it limits the renormalisable interactions of the dark sector with the SM only through the electroweak gauge bosons. This property allows the limited number of free parameters of the model to be determined robustly; so that the theory can provide accurate phenomenological predictions for a wide range of DM experiments. It also avoids emergence of dangerous decay operator that make the system unstable.\(^1\)

Lower dimensional electroweak multiplets i.e. doublet and triplet have been used in different extensions of the Standard Model including supersymmetry \([5]\), little Higgs model [6], inert Higgs models [7,8], neutrino mass generation [9], and Kaluza–Klein theory [10]. In addition, multiplets of higher-dimension have been studied in more recent literature like inert Higgs doublet-septuplet [11], exotic Higgs quintuplet [12], quintets in neutrino mass mechanism [13,14], scalar and fermionic multiplets in SO(10) and \(E_6\) grand unified theories protected by a remnant discrete symmetry [15–17]; and finally fermionic quintuplet and scalar septuplet in minimal dark matter model. The latter is stabilised as a result of an accidental symmetry in the SM gauge groups and Lorentz representations which does not allow for any renormalisable decay channel for DM \([18,19]\).

**Direct detection** (DD) as one of the primary means in the search for dark matter, looks for nuclear recoil as the signal of an exotic particle collision with the detector. DM interactions with visible matter are so weak that there is a small probability to detect dark matter scattering off nucleus in the target volume. Nevertheless, if such a rare event is observed, DD experiment would reveal important properties of dark matter including its mass and coupling strength with the SM.

The past literature on direct detection mainly considered the real representations of the electroweak dark matter, due to the more straight forward phenomenological behaviour compared to the complex models. In addition, the effect of the mass-splitting and higher-dimensional couplings on the DD cross-section has been overlooked in the past.

Dark matter studies are predominantly classified into two broad categories. In the top-bottom scenario, dark matter appears in many BSM theories where the lightest neutral particle is preserved from decay by a given symmetry. These models, especially those which are designed to address the hierarchy problem, usually propose a DM candidate with electroweak interactions [20,21]. However, such UV-complete theories are meant to deal with other issues in physics such as supersymmetry [22,23] or extra dimensions [24,25], and are not primarily built to solve DM problem. The emergence of the dark matter candidate is sporadic and a matter of accident, so this method cannot help to study the dark matter physics in a structured manner.

In contrast, in the bottom-top scenario, interactions of the dark matter with the known fields are described using the effective field formalism [26,27]. This is particularly useful when the mediators of the reactions are much heavier than the DM mass scale, and therefore can be safely integrated out \([28,29]\). The only active degrees of freedom are the DM and SM particles, and the couplings between them are formulated in terms of higher-dimensional contact operators \([30]\). Although this method allows for a model-independent approach to obtain bounds from experiments, it cannot provide a comprehensive insight into dark matter physics, and does not offer a conceptual framework to develop the theoretical aspects.

Here we combine the two above mentioned approaches to put forward a new method to study the BSM physics in a more complete and fundamental way but only using a limited number of free parameters. In the first step, the researcher investigates a specific issue of interest, through identifying the simplest possible model of the dark sector. In our case, we intend to examine the interactions of the DM with the SM particles through one of the known forces. As discussed, this question can be addressed by introducing an additional electroweak multiplet. However, we remain agnostic about the new physics beyond the dark sector scale. So, in the next step, the UV theory is parametrised in terms of the higher-dimensional operators which respect Lorentz and gauge symmetries. So, such a hybrid approach allows an accurate classification of the EWDM models in a systematic manner, while it stays close to the solid realisation of the theory.

In this work, an Effective Field Theory (EFT) approach is employed to describe the non-renormalisable interactions of dark sector with the SM at low energies. We include the lowest-dimensional effective operators that are allowed by the symmetries of the electroweak theory. These operators encapsulate the effects of heavy particles, non-perturbative contributions and ultraviolet completions at higher energies. The coefficients of these operators can be constrained by the

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\(^1\) As an example, consider the scalar extension of the electroweak sector of the Standard Model. Due to the unity mass dimension of the scalar dark matter \(\Phi\), in this theory, it can couple to the pair of Higgs bosons through renormalisable quartic interactions. The scalar fields have a potential and can develop a vacuum expectation value \(v_\Phi\). Such a VEV will break any \(Z_2\) symmetry of the Lagrangian. In this case the DM can decay through renormalisable operators of the form \(v_\Phi \phi hh\) after the symmetry breaking.
available data from today’s relic density, indirect searches, direct detection and collider experiments.

These effective terms in the Lagrangian break the initial U(1)D symmetry of the dark sector down to Z2. Introducing new off-diagonal components to the mass matrix, they split the pseudo-Dirac dark matter into two Majorana fermions. This mechanism eliminates the dangerous tree-level DM coupling to the nucleon and therefore recovers the pseudo-real representations of EWDM theory, which otherwise would have been ruled out by current constraints.

In the previous work [31], the thermal masses of all possible EWDM models were computed using freeze-out mechanism. We also studied the gamma-ray probes of the theory in a variety of astronomical sources including the Milky Way’s black hole, inner Galaxy and dwarf satellites for continuum and line spectra.

In the current paper, we include the non-renormalisable interactions in the full theory to study the impact of the higher-dimensional operators on the effective scattering of EWDM off nucleus. The coupling of SU(2) multiplet to Higgs boson induced by five-dimensional operators, gives rise to new scattering diagrams. We evaluate the Wilson coefficients corresponding to these processes, and analyse the behaviour of the spin independent (SI) cross-section with respect to the changes in the higher-dimensional coupling constants. Finally, the phenomenological results are confronted with the latest experimental data as well as projected sensitivities of the future DD experiments.

This paper is organised as follows:

In the next Sect. 2, we generalise the electroweak dark matter prescription by including a dimension five operator which removes the elastic coupling to the Z-boson. This chapter finishes with a detailed analysis of the mass-splitting between different components and the phenomenological consequences.

After a brief review of the low-energy effective interactions of electroweak dark matter with nuclei in Sect. 3, we evaluate matrix elements of effective operators at parton level, and discuss the elastic spin-independent cross-section for the scattering process. In Sect. 4, using Feynman diagram matching, we provide a detailed calculation of the Wilson coefficients for both quark and gluon interactions, and explain the scale dependence of these coefficients. Section 5 is devoted to numerical computation of the spin-independent cross-section for EWDM scattering off nuclei. The results will be compared with current experimental bounds and projected sensitivities. Next, we combine the constraints from direct detection with astronomical indirect probes to explore the parameter spaces for real and complex models. Finally, we conclude the study in Sect. 6.

Details of the interaction Lagrangian of the dark sector and relevant Feynman rules are presented in the Appendix A. In the Appendix B, we introduce the mathematical framework for solving the loop integrals which emerge in the calculations of the Wilson coefficients of the effective theory.

## 2 EWDM theory

In this section, we study the extension of the SM by adding an arbitrary fermionic n-tuplet transforming under SU(2) × U(1)γ. A fourth chiral generation is severely disfavoured by electroweak precision [32,33] and Higgs production experiments [34,35], so it is conceivable that dark matter belongs to a non-chiral representation of the electroweak sector. We will briefly review the pseudo-real representations of this theory, investigate the mass-split between different components, and explain the phenomenological consequences.2

### 2.1 Non-chiral dark matter

By definition, the right and left chirality components of a Non-chiral or vector fermion are charged in the same manner under gauge transformation [36,37]. Consequently, a gauge-invariant mass term, $\tilde{\chi} \chi$, is permitted. The masses of these fermions are not constrained as they are not acquired by means of electroweak symmetry breaking (EWSB) [38]. They couple completely vectorial to the $Z$ and $W^\pm$ electroweak bosons $\tilde{\chi} y^\mu \chi$, which means that the left and right currents are treated equally. The electroweak precision measurements cannot constrain the vector fermions, and a degenerate non-chiral multiplet does not contribute to the oblique parameters $S, T$ and $U$ [39].

The dimension of the representation and the hypercharge $(n, y)$ can label each multiplet. We adopt the normalisation convention $q = y + \nu^{(3)}$, and the electric charge of the $i$th component in the multiplet can be written as:

$$q_i = \frac{1}{2} (n + 1) - i + y.$$  

(1)

For an electrically neutral DM, its weak isospin should equal the hyper charge but with opposite sign $-i^{(3)} = y$. A general electroweak n-tuplet can be represented as follows:

$$X = \left( \chi^{t+y}, \ldots, \chi^q, \ldots, \chi^0, \ldots, \chi^{-t+y} \right)^T,$$

(2)

where $\chi^q$ shows the field with electric charge $q$. The actual dark matter candidate $\chi^0$ is the $\frac{1}{2} (n + 1) + y$ component.

As a result, for odd-dimensional multiplets, the hypercharge has integer values, whereas n representations of odd dimension, $y$ can only be a half-integer.

2 The content of this section has already been presented in the previous work [31]. Nevertheless, since the theoretical background is crucial to understanding the current direct detection study, we provide a review of the literature in this paper.
The Standard Model is considered a consistent theory up to the Plank scale $M_{pl}$. Consequently, the gauge couplings are expected to stay perturbative up to this cut-off scale. Incorporating additional exotic SU(2) multiplets can accelerate the running of these marginal couplings towards the non-perturbative domain. This acceleration may result in the emergence of a Landau pole (LP) before reaching the Planck scale. It is believed that LP is linked to new physics dynamics that break the accidental symmetries of the Standard Model. We require the Landau pole to be situated above $M_{pl}$ threshold, which imposes the upper limit $n \leq 5$ on the dimensionality of the fermionic EWDM multiplet [18].

Non-chiral dark matter is typically categorised into real and complex representations due to distinct theoretical properties and phenomenological implications. For the purposes of this study, we will concentrate on pseudo-real models.

2.2 Pseudo-real representation

In the case of dark matter in $SU(2) \times U(1)$, pseudo-real representation, the hypercharge is non-zero. In this scenario, all the field content of the dark sector, including the neutral dark matter candidate should be Dirac fermions.

The dark sector Lagrangian for EWDM model can be written as:

$$\mathcal{L}_D = \bar{X} (i \slashed{D} - m) X - \frac{\lambda_0}{\Lambda} (H^\dagger T^0 H) (\bar{X} T^0 X)$$

$$- \frac{\lambda_0}{\Lambda} (H^\dagger T^0 H^c) (\bar{X} T^0 X)$$

$$- \frac{\lambda_c}{\Lambda} (H^c T^0 H) (\bar{X} T^0 X^c).$$

(3)

It includes terms with $\Lambda$ as the mass scale, $\lambda_0$ and $\lambda_c$ as coefficients, and $T^a$ and $T^a = \sigma^a / 2$ representing the $SU(2)$ generators respectively for the $n$-dimensional and fundamental representations. The covariant derivative is given by $D_\mu = \partial_\mu + i y g_s B_\mu + i g_w W^a_\mu T^a$.

We define the conjugate representation as $X^c \equiv CX^c$ where $X^c$ is meant to be the multiplet having charge conjugated components. The matrix $C$ is anti-symmetric and off-diagonal with ±1 alternate elements. This choice of normalisation returns $-i \sigma^2$ in the fundamental representation, such that $H^c = -i \sigma^2 H^*$. More explicitly one can write:

$$C_{i,j} \equiv \delta_{i,n+1-j} (-1)^{n+1-i-j}.$$

(4)

Therefore, the conjugate of the generic multiplet (2) is given by:

$$X^c = (-1)^{-r+y} (\psi^{-r+y})^c, \ldots (\psi^0)^c,$$

$$\ldots (-1)^y (\psi^y)^c, \ldots (-1)^{-r+y} (\psi^{-r+y})^T.$$

(5)

The crucial result is that, in order for the adjoint vectors $X^c T^0 X$ to be electrically neutral, the operators in the second line of Lagrangian limit the value of hypercharge to $y = \frac{1}{2}$, (refer to Eq. (8)). This restriction also leaves only the scenario of even-dimensional multiplets for EWDM model. Note that other terms in the first line of (3) are invariant under a $U(1)_D$ symmetry transformation $X \rightarrow e^{i\theta} X$ with $\theta$ being an arbitrary real parameter. However, the mentioned scalar products of the adjoint vectors explicitly violate the invariance of the theory down to a $Z_2$ symmetry under which dark sector fields are odd $X \rightarrow -X$.

Footnote 5 continued

3 This upper bound remains valid when renormalisation group equations are calculated up to the two-loop level [40].

4 Extra information about the real modules can be found in the Appendix B of the reference [31].

5 We devised EWDM model as a plausible explanatory theory originating from unknown new physics at larger scales. So, it is also interesting to study the higher energy physics that generates the effective operators in the Lagrangian (3) to realise a UV-complete theory. We can show that such low energy terms can be produced through one and two-loop processes in the electroweak dark matter model. The left (right) figure below, depicts the 1-loop (2-loop) diagram connecting the pairs of dark particles $\bar{X}X$ ($X^c \bar{X}$) to the SM Higgs bosons $H^\dagger H$ ($H^c \bar{H}$) leading to $H^\dagger T^0 H \bar{X} T^0 X$ ($H^c T^0 H^c \bar{X} T^0 X$) operator, at lower energy scales:

6 A conjugate multiplet by definition has opposite hypercharge $-y$. It can be seen that the reflected $n + 1 - i$ element, in the conjugate multiplet, has the opposite electric charge of $-(n + 1) / 2 + i - y$. So, the conjugate multiplet can be considered as the reversed order multiplet with components having opposite charges. As an example compare the dark matter quadruplet $(\chi^+, \chi^+, \chi^0, \chi^-)$ with hypercharge 1/2 with the conjugate one $(\chi^-^c, (\chi^c)^0, (\chi^c)^-, (\chi^c)^+)$ with $y = -1/2$. 

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2.3 Mass-splitting

The effective operator with coefficient $\lambda_c$ induces the mass-splitting between the neutral and charged fields at tree-level. After EWSB, $H \rightarrow (0, (h + v)/\sqrt{2})^T$, the mass-splitting takes the form:

$$\Delta_q^{(\ell)} = m_q^{(\ell)} - m_0^{(\ell)} = -\frac{\lambda_c}{4\Lambda} v^2 q,$$

(6)

where $m_q^{(\ell)}$ denotes the tree-level mass of the particle $\psi^q$, and $m_0^{(\ell)} = m + \frac{\lambda_c}{4\Lambda} v^2 y$.

Furthermore, radiative corrections due to electroweak interactions with the gauge vectors, generate loop-induced mass-splitting between the charged and neutral components which can be written as [19]:

$$\Delta_q^{(\ell)} = m_q^{(\ell)} - m_0^{(\ell)} = q \left( q + \frac{2}{c_w} y \right) \Delta,$$

(7)

where $\Delta_q^{(\ell)}$ indicates the mass of $\psi^q$ component produced at 1-loop level, and the factor $\Delta$ is defined through $\Delta = \alpha_w m_w \sin^2(\theta_w/2) \approx 166 \text{ MeV}$ [41]. We adopt the notation $s_w \equiv \sin \theta_w$, and $c_w \equiv \cos \theta_w$, with $\theta_w$ being the weak mixing angle.

The interactions between dark matter (DM) and gauge fields are described by the kinetic part in the Lagrangian (3). After electroweak symmetry breaking, the covariant derivative can be expressed as $D_\mu = \partial_\mu + i e Q A_\mu + i g_X (T^3 - s_w^2 Q) Z_\mu + i g_Z (W_\mu^- T^- + W_\mu^+ T^+)$, where $g_Z \equiv g_w/c_w$.

Due to DM vector coupling to the neutral weak gauge field at tree-level, it can scatter coherently off the nuclei by $Z$-boson exchange [42]. The resultant cross-section is so large that the pseudo-real electroweak dark matter would be excluded, based on current experimental data [43].

However, as discussed in [31], the effective operator proportional to $\lambda_0$, removes the sizeable coupling to the Z-vector and resurrect the pseudo-real EWDM scenario. It breaks the would-be U(1)$_D$ symmetry, and splits the Dirac neutral state $\psi^0$. After EWSB, this term reduces to:

$$-\frac{\lambda_0}{\Lambda} (H^\dagger T^a H^c) (\bar{X}^a T^a X) + \text{cc}$$

$$= -\frac{\lambda_0}{\Lambda} \left( h + v \right)^2 \sum_{q=-1}^{+1} \frac{(-1)^q \sqrt{n^2 - 4q^2}}{2} (\bar{\psi}^- q \psi^-)^c \psi^q + \text{cc}. $$

(8)

The Higgs vacuum expectation value (VEV) introduces the extra term $\delta_0 = n v^2 \lambda_0/8\Lambda$ to the mass matrix of the neutral fields:

$$\left( (\bar{\psi}^q_c \psi^q) \right) \left( \begin{array}{cc} m_q^{(0)}/\Delta_1 q \frac{\delta_0^*}{\Delta_0} \\ m_0^{(0)} \end{array} \right) \left( \begin{array}{c} \psi^0 \\bar{\psi}^0_c \end{array} \right)$$

$$= \frac{1}{2} \left( \begin{array}{cc} \bar{\chi}^0 & 0 \\ 0 & \chi^0 \end{array} \right) \left( \begin{array}{c} \psi^0 \\bar{\psi}^0_c \end{array} \right).$$

(9)

As a result, this operator splits the neutral component into two Majorana fields $\chi^0$ and $\bar{\chi}^0$. Given $|\delta_0| \ll m$, these mass-eigenstates up to zeroth order in $O(\Lambda_0/\delta_0/m)$ can be cast as:

$$\chi^0 = \frac{1}{\sqrt{2}} (\psi^0 + (\psi^0)^c), \bar{\chi}^0 = \frac{1}{\sqrt{2}} (\psi^0 - (\psi^0)^c),$$

(10)

with masses $\bar{m}_0 = m_0^{(0)} + 2\Re \delta_0$, and $m_0 = m_0^{(0)} - 2\Re \delta_0$, respectively. For simplicity, we assume that the imaginary field $\chi^0$ is the lightest odd particle, without any loss of generality.

Apart from this, $\lambda_0$ term provides an off-diagonal contribution $\delta_0 \equiv (-1)^q v^2 \sqrt{n^2 - 4q^2} (\lambda_0/8\Lambda)$, resulting in mixing of $\psi^q$ and $(\psi^q)^c$ charged particles.7

By utilising the relations $\bar{\psi}^- q \psi^- q = (\bar{\psi}^- q (\psi^- q)^c$ and $(\bar{\psi}^- q (\psi^- q)^c)^c q$, one can limit the spectrum to positively charged fields $q > 0$. After diagonalising the mass matrix, we obtain:

$$\left( \begin{array}{c} \bar{\psi}^q \ (\psi^- q)^c \end{array} \right) \left( \begin{array}{cc} m_q + d_q & 2\delta_q \\ 2\delta_q & m_q - d_q \end{array} \right) \left( \begin{array}{c} \psi^q \ (\psi^- q)^c \end{array} \right)$$

$$= \left( \begin{array}{cc} X^q_2 & \chi^q_1 \\ \chi^q_1 & X^q_2 \end{array} \right) \left( \begin{array}{cc} m_q^{(2)} \ 0 \\ 0 \ m_q^{(1)} \end{array} \right) \left( \begin{array}{c} \chi^q_2 \ \bar{\chi}^q_1 \end{array} \right),$$

(11)

where we define $m_q \equiv m_0^{(0)} + \Delta R_q$, and $d_q \equiv \left( 2\Re - 1 \Delta \right. \left. - (\lambda_c - 4\Lambda) v^2 \right) q$. The real radiative mass-splitting takes the form $\Delta R_q \equiv m_q - m_0 = q^2 \Delta$ [18]. The new eigenstates can be cast as:

$$\left( \begin{array}{c} \chi^q_2 \\ \bar{\chi}^q_1 \end{array} \right) = \left( \begin{array}{cc} e^{i\frac{\phi_q}{2}} c_q & e^{-i\frac{\phi_q}{2}} s_q \\ -e^{i\frac{\phi_q}{2}} s_q & e^{-i\frac{\phi_q}{2}} c_q \end{array} \right) \left( \begin{array}{c} \psi^q \\ (\psi^- q)^c \end{array} \right),$$

(12)

where the hat notation in the exponent should be interpreted as the argument $\hat{\lambda}_0 \equiv \arg \lambda_0$. The mixing angle $\phi_q$ satisfies the condition $\sin \phi_q = |\delta_q|/\sqrt{|\delta_q|^2 + d_q^2}/4$. To simplify the notation, we introduce $c_q \equiv \cos(\phi_q/2)$ and $s_q \equiv \sin(\phi_q/2)$. The corresponding masses can be written as:

7 It is important to highlight that for a Dirac particle, $\psi^q$ and $(\psi^- q)^c$ are not equivalent.
Fig. 1 Left a: Mass-splitting between the dark matter candidate \( \chi^0 \) and the singly charged states \( \chi^\pm_1 \) (in blue) and \( \chi^\pm_2 \) (in red) as a function of the non-renormalisable coupling \( \lambda_c/\Lambda \) for the complex quadruplet. The figure shows mass-splitting for two values of the other non-renormalisable coefficient \( \lambda_0/\Lambda \). The curves corresponding to \( \lambda_0/\Lambda = 10^{-6} \) GeV\(^{-1} \) are in solid lines, while those at the minimum \( \lambda_0/\Lambda_{\text{min}} = 10^{-8} \) GeV\(^{-1} \) (24) are plotted in dashed-dotted style. The light (dark) blue shaded regions are excluded at minimum \( (10^{-6} \) GeV\(^{-1} \) value of \( \lambda_0/\Lambda \) as the theory cannot be considered as a dark matter model \( \Delta_t^{(1)} \approx 0 \) according to (14) (15). Right b: The right panel illustrates the mass-splitting of these fields \( \Delta_t^{(1)} \) (blue) and \( \Delta_t^{(2)} \) (red) with respect to the effective coupling \( \lambda_0/\Lambda \) for the same quadruplet. The solid lines show the result in case the other coupling has \( \Delta_\lambda/\Lambda = 1.1 \times 10^{-5} \) GeV\(^{-1} \) deviation from the mean value, and the dashed-dotted curves correspond to the average value of \( \lambda_0/\Lambda_{\text{av}} = 1.2 \times 10^{-5} \) GeV\(^{-1} \). In the lighter blue shaded region, the \( \chi^+_1 \) field becomes the lightest particle \( \Delta_t^{(1)} < 0 \) as shown in (16), when \( \lambda_c \) deviates from the mean, beyond the range specified in (14). The grey shaded area is ruled out by DD experiments due to inelastic scattering of DM off target nucleus via exchange of Z boson at tree-level (24).

Note that in both figures, the blue coloured regions still remain a valid beyond the Model (BSM) proposal.

As the other coupling \( \lambda_0 \) becomes bigger, this acceptable range \( \delta \lambda_c \) increases.

The plot Fig. 1a depicts the changes in mass-splitting between the fields \( \chi^+_1 \) and \( \chi^+_2 \) as a function of the non-renormalisable coupling \( \lambda_c \), in the case of a pseudo-real quartet. The observed trend is that the difference between the mass of the similarly charged particles \( \Delta^{(2)}_t - \Delta^{(1)}_t \) grows larger while the coupling deviates from its average value. The mass difference reaches a minimum value of \( 4|\delta_q|^2 \) at the mean coupling strength \( \lambda_{c,\text{av}} \) and the states with the same charge \( q \) become degenerate \( m^{(1)}_q = m^{(2)}_q = \Delta^{(2)}_q \) if \( \lambda_0 \) vanishes.

If \( \lambda_c \) surpasses the given range (14), it will establish a lower limit on the strength of the neutral coupling.

\[
\frac{\lambda_0}{\lambda} > \frac{1}{2} \sqrt{-n \Delta + \sqrt{(n^2 - 4) \Delta^2 + 4d^2}}. 
\] (16)

The left panel Fig. 1b illustrates how the mass-splitting of these particles changes as the coupling \( \lambda_0 \) varies. It is evident that mass difference between the states becomes greater when the coupling strengthens. Also, the minimum permissible value for \( \lambda_0 \) increases as the other coupling \( \lambda_c \) moves further from the mean.

The highest weight particle \( \chi^n/2 \) is distinct and possesses a mass of:

\[
m^n_2 = m^{(t)}_0 + \Delta^{(t)}_2 + \Delta^{(t)}_2 = m^{(t)}_0 + \Delta^{R}_2 + d^2.
\] (17)

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The condition that $\chi^{n/2}$ must be lighter than the DM candidate $\chi^0$ necessitates:

$$\frac{\lambda_c}{\Lambda} - 2\frac{\lambda_0}{\Lambda} < \frac{8}{n\nu^2} \Delta^{(1)}_0.$$  \hspace{1cm} (18)

This sets an upper limit on the strength of $\lambda_c$. If the charged coupling is below the threshold $\lambda_c/\Lambda < 4 (c_w^{-1} + 1) \Delta/v^2 \approx 2.3 \times 10^{-5}$ GeV$^{-1}$, there are no limitations on the value of the other coefficient $\lambda_0$. Should the charged coupling $\lambda_c$ surpasses this limit, the neutral coupling $\lambda_0$ will have a lower bound.

In all the complex scenarios except the doublet case, the bounds on the non-renormalisable couplings from the lighter charged state (15), (16) prove to be stronger than that of the highest weight (18). So in practice, the conditions above is only applicable to C2 model.

Figure 2a plots the mass-splitting of the $\chi^+$ state as a function of the charged coupling in the pseudo-real doublet model. It can be seen that the maximum allowed value of $\lambda_c$ increases with strength of the other coupling $\lambda_0$.

The right panel of this Fig. 2b illustrates the changes in the mass-splitting of the highest weight field with respect to the neutral coupling $\lambda_0$. The lower bound on $\lambda_0$ goes up as $\lambda_c$ gets further from its mean value.

### 3 EWDM–nucleon scattering

The scattering event rate in direct detection experiment is very sensitive to the actual form of coupling between dark matter particle and parton constituents of the hadron. At the beginning of this section, we provide a brief review on inelastic scattering process, and the restrictions imposed on the coupling strength. Then, we consider the full set of relativistic operators that contribute to the effective interactions of EWDM with quarks as well as gluons at the leading order. Finally, the matrix elements of these operators are evaluated at different scales, and the spin-independent scattering cross-section will be discussed.

#### 3.1 Inelastic scattering

By expanding the fermionic Lagrangian (3) in terms of the mass eigenstates, one can obtain the interactions of the neutral odd particles, carried out by exchange of gauge bosons:

$$L_{\text{int}}^0 = \frac{i}{2} g_\chi \overline{\chi^0} \gamma^\mu \chi^0 Z^\mu + \frac{g_w}{4} \left[ n e^{-\frac{1}{2} \lambda_0} c_+ - \sqrt{n^2 - 4} e^{\frac{1}{2} \lambda_0} s_+ \right] \overline{\chi^0} \gamma^\mu \chi^+_2 \right]
+ i \left[ n e^{-\frac{1}{2} \lambda_0} c_+ + \sqrt{n^2 - 4} e^{\frac{1}{2} \lambda_0} s_+ \right] \overline{\chi^0} \gamma^\mu \chi^+_2
- \left( n e^{-\frac{1}{2} \lambda_0} s_+ + \sqrt{n^2 - 4} e^{\frac{1}{2} \lambda_0} c_+ \right) \overline{\chi^0} \gamma^\mu \chi^+_1$$

A similar discussion is already provided in Ref. [31]. Since the current paper focuses on the direct detection of EWDM, it is necessary to review the inelastic scattering process in this work.
dark matter $\chi_0$ will not undergo up-scattering to the excited state $\tilde{\chi}_0$. For large DM mass, this proves to be equivalent to the condition:

$$\Delta_0 > \frac{1}{2}m_{N^\prime}v_\chi^2.$$  

In other words, if the mass-splitting is large enough, $\Delta_0 > \mathcal{O}(100)$ keV, then the inelastic nucleonic scattering will be forbidden due to kinematic restrictions, and the tree-level coupling mediated by Z vector is fully prevented.

Furthermore, this places a restriction on the smallest testable value of the coefficient of the neutral pseudo-Dirac splitting term:

$$9\sqrt{\lambda_0/\Lambda} > 10^{-8} \text{ GeV}^{-1}.$$  

This also means that the scale of the new physics that violates $U(1)_D$ symmetry is likely to be about $\Lambda/\lambda_0 \sim 10^8$ GeV.

In the following discussion, we focus our study on elastic direct detection processes.

### 3.2 Effective interactions

The effective Lagrangian describing EWDM–nucleon scattering at parton level is composed of two parts: $\mathcal{L}_R$ which is constructed only from the renormalisable interactions of the UV theory, while $\mathcal{L}_{NR}$ allows for non-renormalisable couplings of DM to the SM Higgs (72):

$$\mathcal{L}_{\text{eff}} = \mathcal{L}_R + \mathcal{L}_{NR}. $$

The low-energy Lagrangian of the electroweak dark matter composed of renormalisable couplings, has already been studied in the literature. We provide a review in the Appendix C to calibrate our results with previous publication.

The effective Lagrangian which includes non-renormalisable interactions, containing DM bilinear operators of up to dimension-4, in leading order, is given by $^{11}$:

$$\mathcal{L}_{NR} = \sum_{q=d}^{h} m_q \left( C_{q}^{h3} + C_{q}^{h4} \right) \bar{q}^0 \chi^0 \chi^0 \bar{q}^0 + \frac{\alpha_s}{\pi} \left( C_{q}^{h3} + C_{q}^{h4} \right) \bar{q}^0 \chi^0 \chi^0 G_{\mu\nu}^a G^{\mu\nu}. $$

where $q$ and $G_{\mu\nu}^a$ denote quark field and gluon field strength tensor. $C_{q}^{h3}$ $C_{q}^{h4}$ and $C_{q}^{h4}$ are respectively Wilson coefficients for EWDM–quark and EWDM–gluon scatterings that include non-renormalisable DM-Higgs cubic (78) (quartic (79)) coupling.

As will be explained, the dominant loop momenta in the Feynman diagrams of the full theory are generally around

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$^{10}$ If the coupling constant $\lambda_0$ is not purely real, then there exists a tree-level inelastic scattering process that is mediated by scalar Higgs field (c.f. coupling (78)). The amplitude for this reaction is, nevertheless, suppressed by $(m_q/\Lambda)(\Im \lambda_0/\alpha_w)$ compared with Z vector induced diagram.

$^{11}$ We only use those operators which are not velocity suppressed in leading contribution. Also, recall that vector and tensor bilinears vanish for a Majorana DM.
the weak scale. So, we set the UV scale at $\mu_{\text{UV}} \approx m_z$, and therefore consider an effective theory with $N_f = 5$ active quarks lighter than this characteristic mass $m_q < \mu_{\text{UV}}$, that are $d, u, s, c, \text{and } b$.

Since the quark scalar operator breaks the chiral symmetry of QCD, it is suppressed by quark mass $m_q$. Electroweak dark matter scatters off gluons at loop level, therefore the effective interactions are suppressed by strong structure constant $\alpha_s \equiv g^2_s/4\pi$. As will be discussed later, in order to make the quark and gluon Wilson coefficients $C_q$ and $C_g$ comparable, we multiply them by factors of $m_q$ and $\alpha_s/\pi$ respectively.\(^{12}\)

All the operators in the Lagrangian above, are scalar type, and thus only generate Spin-Independent (SI) interactions.

### 3.3 Matrix elements

Nucleon indeed obtains the bulk of its mass $M_N$ through spontaneous chiral symmetry breaking, even in the limit of vanishing quark masses. However, a small fraction is attributed to the quark $\sigma$-terms which cause explicit breaking of the chiral symmetry. The contribution of valance and sea quarks to the nucleon mass is parametrised by the quark mass fraction:

$$ f_{Tq} = \frac{m_q}{m_N} \langle \bar{q}q \rangle, \quad \text{(27)} $$

where the matrix element $\langle \bar{q}q \rangle \equiv \langle N_i | \bar{q}q | N_f \rangle$, evaluates the scalar operator between the initial $|N_i\rangle$ and final $|N_f\rangle$ nucleon states.

For light quarks, this quantity can be determined experimentally from $\sigma$ terms of the nucleon–pion scattering (for up and down) and kaon–nucleon scattering (for strange). The pion–nucleon sigma term $\sigma_{\pi N} = (m_u + m_d) (\bar{u}u + \bar{d}d)/2$, can be read off $\pi - N$ scattering amplitude at Cheng–Dashen point \([45]\) using dispersive analysis \([46,47]\).

Alternatively, we can use chiral perturbation theory ($\chi$PT) where $\sigma_{\pi N}$ depends on a set of low energy constants which can be determined by fitting to the experimental $\pi - N$ scattering data \([48]\). Different extensions of this theory to the baryonic sector have been studied for this purpose, including heavy baryon $\chi$PT \([49,50]\), infrared $B\chi$PT \([51–53]\), and covariant $B\chi$PT with extended-on-mass-shell scheme \([54]\).

Determination of strangeness content of the nucleon is theoretically more involved \([55]\). One can input the value of $\sigma_{\pi N}$ to $B\chi$PT, and use the relationship between SU(3) flavour violation parameter and strangeness fraction to obtain $f_{Tq}$ \([56,57]\).

For heavy quarks, neither a theoretical framework nor any phenomenological experiment exist, and hence a need for lattice QCD non-perturbative simulations. Lattice calculations are performed using two different techniques \([58]\).

In the indirect method, the quark matrix element is obtained from variation of the nucleon mass with respect to the quark mass \([59,60]\) $\langle \bar{q}q \rangle = \partial M_N/\partial m_q$ through the Feynman–Hellmann theorem \([61,62]\).

In an alternative way, one can obtain the scalar matrix element from the ratio of the three-point to two-point functions of the nucleon. The 3-point function arises from two types of diagrams. The connected part contains the propagator of the valence up and down quarks. On the other hand, in the disconnected diagram, the sea quarks form a vacuum blob. The $\sigma_{\pi N}$ receives contribution from both parts, but for the sigma term of other quarks, only the disconnected piece is present \([63–65]\).

In this work, we use the quark mass fractions listed in Table 1 which are computed using lattice QCD simulation by \([44]\). It can be observed that heavier flavours have larger fraction of the nucleon mass.

Finally, worth noting is that scalar quark operator is independent of the scale to all orders $(\partial/\partial \mu) m_q \langle \bar{q}q \rangle = 0$.

QCD symmetric and gauge-invariant energy–momentum tensor (EMT) can be derived from the Noether current as associated with space-time translation invariance \([66]\):

$$ \Theta^{\mu\nu} = -G^{a\mu\lambda} G^a_{\lambda\nu} + \frac{1}{4} \eta^{\mu\nu} G^{a\lambda\rho} G^a_{\lambda\rho} + \sum_q \frac{i}{2} \bar{q} \left( \gamma^\mu D^\nu + \gamma^\nu D^\mu \right) q. \quad \text{(28)} $$

This tensor gives rise to the same physical momentum and generators of the Lorentz symmetry as the canonical one does \([67]\). It can be shown that $\Theta_{\mu\nu}$ is identified with derivative of the dilatation current which corresponds to the scaling transformation \([68]\).

Like any rank-two tensor, energy–momentum tensor can be decomposed into traceless and trace parts, in $d$ dimensions as \([69,70]\):

$$ \Theta^{\mu\nu} = \Theta^{\mu\nu} + \hat{\Theta}^{\mu\nu} \quad \text{(29a)} $$

$$ \equiv \left( \Theta^{\mu\nu} - \frac{\eta^{\mu\nu}}{d} \Theta^d \right) + \frac{\eta^{\mu\nu}}{d} \Theta^d \hat{\Theta}^d \quad \text{(29b)} $$

\(^{12}\) Note that any such factors as $m_q$ and $\alpha_s/\pi$ will be compensated when matching the effective and full theories.

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Table 1 Mass fraction parameter for different quark flavours \([44]\). The digits in parentheses show the statistical uncertainty

| $f_{Tq}$ | Proton | Neutron |
|---------|--------|---------|
| $f_{Td}$ | 0.0234(23) | 0.0298(23) |
| $f_{Tu}$ | 0.0149(17) | 0.0117(15) |
| $f_{Ts}$ | 0.0440(88) | 0.0440(88) |
| $f_{Tc}$ | 0.085(22) | 0.085(22) |
where the symmetric traceless parts [71]:

\[ O_{\mu
u}^q = \frac{i}{2} \theta \left( \gamma_\mu D_\nu + \gamma_\nu D_\mu - \frac{1}{2} \eta_{\mu\nu} \mathcal{D} \right) q, \]

\[ O_{\mu
u}^g = G^a_{\mu\nu} G^a_{\rho\sigma} - \frac{1}{4} \eta_{\mu\nu} G^a_{\rho\sigma} G^{a\rho\sigma} \]

are known as spin-2 twist-2 operators for quark and gluon respectively. Here, twist is defined as the difference between mass-dimension and spin. The QCD covariant derivative reads \( D_\mu = \partial_\mu + ig_\alpha A_\mu^\alpha \), where \( A_\mu^\alpha \equiv A_\mu^a \gamma^a / 2 \) with \( \gamma^a \) meant to be Gell-Mann matrices here.\(^{13}\)

Using equation of motion, the classical trace of EMT simplifies to \( \Theta_\mu^\mu = \sum_q m_q \bar{q} q \), which vanishes in the limit of zero quark mass. This, in fact, indicates that dilatation current is conserved and QCD is therefore scale invariant at classical level.

However, scale symmetry is broken due to running of the coupling constant which is an intrinsic quantum effect. Working in \( d = 4 - \epsilon \) dimensions, the quantised \( \Theta_\mu^\mu \) differs from the classical version by a divergent term [73] that should be renormalised \( -\epsilon / 4g^2 \)

\[ G_{\mu\nu}^q, G_{\mu\nu}^g = (\beta / 4\alpha_s^2)(G_{\mu\nu}^q, G_{\mu\nu}^g)_0 - \gamma_m \sum_q m_q \bar{q} q. \]

This is known as trace anomaly and is composed of contributions proportional to beta-function \( \beta \), and mass anomalous dimension \( \gamma_m \) [74].

Therefore, the renormalised trace of the full energy momentum can be expressed as [75]:

\[ O_{\mu
u}^q = \frac{\beta}{4\alpha_s} G_{\mu\nu}^q, G_{\mu\nu}^g + (1 - \gamma_m) \sum_q m_q \bar{q} q, \]

where the beta function and quark mass anomalous dimension to leading order are given by:

\[ \beta(\alpha_s) \equiv \frac{\partial \alpha_s}{\partial \ln \mu}, \]

\[ \gamma_m(\alpha_s) \equiv \frac{\partial \ln m_q}{\partial \ln \mu} = -\frac{3}{4} \frac{N_c}{N_f} \]

with \( N_c = 3 \) being the number of colours.

Either by applying the virial theorem in a stationary state [76,77], or using Poincaré invariance in four-momentum eigenstates [78], one can show that matrix element of the trace of energy–momentum tensor generates the nucleon mass (\( \Theta_\mu^\mu \)) is mN [79].

Taking the expectation value of the operator expression (31) within the nucleon state, for \( N_f = 3 \) flavours, and to the leading order \( O(\alpha_s^0) \), we get:

\[ -\frac{9}{8} \left( \alpha_s G_{\mu\nu}^g G^{\mu\nu} \right) + \sum_q (m_q \bar{q} q) = m_N. \]

It can be seen that the terms on the left hand side has order of \( O(m_N) \), hence factors of \( m_q \) and \( \alpha_s / \pi \) for quark and gluon scalar operators in the effective Lagrangian (26).

Moreover, as mentioned before, the quark scalar operator \( m_q \bar{q} q \) is renormalisation group invariant. The nucleon mass \( m_N \) is a physical quantity and hence scale independent. As a consequence, only using the current choice of factors, the gluon operator \( (\alpha_s / \pi) G_{\mu\nu}^g G^{\mu\nu} \) can be scale-invariant to the leading order of \( O(\alpha_s^0) \).

Therefore, the Gluon matrix element can be expressed as:

\[ \left( \frac{\alpha_s}{\pi} G_{\mu\nu}^g G^{\mu\nu} \right) = \frac{8}{9} m_N f_{\text{Tg}}, \]

where \( f_{\text{Tg}} \equiv 1 + \sum_q a_{\text{ud}, s} \text{f}_{\text{Tq}}. \)

By differencing the trace anomaly expression (31), the matrix element of the heavy quark \( Q \) can be shown to induce scalar gluon interactions of the form [80]:

\[ \langle m_Q \bar{Q} Q \rangle = -\frac{1}{12} \frac{\alpha_s}{\pi} G_{\mu\nu}^g G^{\mu\nu} \]

which is independent of the heavy flavour mass. This is equivalent to closing the heavy quark external loops in the scattering diagrams, and replacing them by one-loop coupling to gluons.

In case of charm quark, we can see that the theoretical prediction of (35) is close to the numerical value shown in Table 1, which is computed in by lattice QCD.\(^{14}\)

3.4 Scattering cross section

The non-renormalisable interactions only contribute to the scalar amplitude at the nucleon level:

\[ \mathcal{L}_{\text{NR}} = f_N^{\text{NR}} \bar{N} \kappa^0 N. \]

The effective non-relativistic amplitude for dark matter–nucleon scattering is derived from evaluation of the effective Lagrangian between initial and final nucleonic states:

\[ f_N^{\text{NR}} = \langle \mathcal{L}_{\text{NR}} \rangle = m_N \left[ \sum_q \left( C_{q}^{h3} + C_q^{h4} \right) f_{\text{Tq}} \frac{8}{9} \left( C_{q}^{h3} + C_q^{h4} \right) f_{\text{Tg}} \right]. \]

\(^{13}\) In general one can define spin-n twist-2 operators for quark and gluon by symmetrised traceless relations [72]: \( O_{\mu_1...\mu_n}^{\mu_1...\mu_n} \equiv \bar{q} \left( T_{\mu_1} D_{\mu_2} \cdots i D_{\mu_n} \right) q \).

\(^{14}\) Because charm flavour mass is close to QCD scale, the effect of higher-dimensional operators should be taken into account in lattice simulations. These operators are suppressed by powers of \( m_c \), and can provide a correction up to a few percent [81].
As discussed, the scalar matrix elements are evaluated at hadronic scale where only the three light quarks $d$, $u$ and $b$ are active.

Pseudo-real EWDM scattering off nuclei proceeds through two kinds of interactions. $f_N^R$ amplitude only contains renormalisable couplings of DM and the SM, whereas $f_N^{NR}$ includes higher-dimensional coupling to Higgs boson:

$$f_N = f_N^R + f_N^{NR}. \tag{38}$$

The elastic cross-section for spin-independent interactions of EWDM and nucleon $N$ can be written as [5]:

$$\sigma_N = \frac{4}{\pi} m_N^2 |f_N|^2, \tag{39}$$

where $m_N = M_x m_N / (M_x + m_N)$ is dark matter – nucleon reduced mass.\(^{15}\)

## 4 Wilson coefficients

In what follows, the Wilson coefficients of EWDM–nucleon scattering will be computed in leading order of non-renormalisable couplings $\lambda_0$ and $\lambda_c$. Since the relic has a very slow speed, only a small fraction of the incident DM momentum is transferred to the target nuclei. Therefore we also assume a zero momentum transfer.

In order to fix the value of Wilson coefficients, we use Feynman diagram matching. It involves computing the scattering amplitude corresponding to each diagram in the full theory, and then comparing them with the same amplitude in the effective Lagrangian at UV scale. We find the coefficients for both effective operators of quark and gluon by integrating out the mediators in the full EWDM–quark and EWDM–gluon scattering processes.

Our understanding of the nuclear physics matrix element of the scalar operator is restricted to the hadronic scale. However, the Wilson coefficients of the quark and gluon scattering are scale independent to leading order in the strong coupling constant, so there is no need to evolve them down to $\mu_{\text{had}}$ using the renormalisation group equations (RGE’s). Notwithstanding, as we cross the heavy flavour masses when integrating them out, the threshold corrections should be included in the gluon Wilson coefficient. The quark and gluon coefficients are matched by making a comparison between the two effective theories at ultraviolet and nucleon scales at $\mathcal{O}(\alpha_s^2)$ [82]:

$$c_q^{\text{h}} = c_q^{\text{h}}(\mu_{\text{had}}), \quad q = d, u, s \tag{40a}$$

$$c_g^{\text{h}}(\mu_{\text{had}}) = c_g^{\text{h}}(\mu_{\text{uv}}) - \frac{1}{12} \sum_{q=c, b, t} c_q^{\text{h}}(\mu_{\text{uv}}). \tag{40b}$$

Electroweak DM mixes with the SM Higgs through dimension-five operators introduced in (3). Since these interactions include both cubic and quartic DM-Higgs couplings, we need to consider the leading order diagrams which contain these two types of coupling.

### 4.1 EWDM–quark scattering

For cubic interactions (78), dark matter couples to quarks at tree level through exchange of Higgs boson (Fig. 3a). After integrating the scalar mediator out, the effective coefficient is derived as:

$$c_q^{\text{h}} = \frac{\lambda}{4 m_h^2}, \tag{41}$$

where we have defined the linear combination of the non-renormalisable constants as:

$$\lambda \equiv y \frac{\lambda_c}{\Lambda} - n \Re \frac{\lambda_0}{\Lambda}. \tag{42}$$

The effective interaction of EWDM with quarks induced by quartic Higgs coupling is generated at one loop level in leading order as depicted in Fig. 3b. This diagram gives rise to the effective coefficient:

$$c_q^{\text{h}} = \frac{m_q}{v^2} C(x^0, x^0, h, h) \int \frac{1}{(\ell + k)^2 - m_q^2} \ell - m_q^2 \frac{d^4\ell}{(2\pi)^4} = -i \frac{3}{32} g_w^2 \lambda \frac{m_q^2}{m_w^2} \left( B_0^{(2,1)} + B_1^{(2,1)} \right) \left( m_q^2 / m_h, m_q \right), \tag{43}$$

where $k$ denotes momentum of quark, and the vertex $C(x^0, x^0, h, h)$ is defined in (79). The two point functions $B_0^{(2,1)}$ and $B_1^{(2,1)}$ are evaluated in (86a) and (97). After performing the loop integration, we arrive at the Wilson coefficient:

$$c_q^{\text{h}} = \frac{\alpha_w}{28 \pi \frac{m_w^2}{m_q^2}} g_q(h), \tag{44}$$

where $\alpha_w \equiv g_w^2 / 4 \pi$ is the weak structure constant, $h \equiv (m_q / m_h)^2$, and the quark mass function is defined as:

$$g_q(x) = 2 + (1 - x) \ln x + \frac{1 - 3x}{x} K(x), \tag{45}$$

where $K$-function is defined in (93). Figure 4 illustrates the changes in the absolute value of the mass function with respect to the scaling variable $h$. It also shows the data-points for the masses of different flavours. It can be noticed that in the limit of vanishing quark mass $h \to 0$ which corresponds to the active light flavours, $g_q(h)$ approaches zero. Therefore $C_q^{\text{h}}$ cannot make a significant contribution to the total EWDM–nucleon scattering amplitude.
Fig. 3 Feynman diagrams which generate the effective coupling of electroweak dark matter with quarks at leading order. The EWDM-Higgs cubic and quartic vertices are represented by dot (●) and square (□) respectively.

Fig. 4 The absolute value of quark mass function $g_q$ in dependence of the parameter $h ≡ (m_q/m_h)^2$. For $h > 1/4$, this function is not defined. The data points correspond to the values at quark mass scales, namely up (light blue), down (dark blue), strange (light green), charm (dark green) and bottom (red).

4.2 EWDM–gluon scattering

Since by definition, electroweak DM is colourless under SU(3)$_c$, it cannot scatter off gluon fields at tree-level. The interactions of dark matter with gluon is therefore loop-induced.

Although gluon loop-level interactions generate a factor of $\alpha_s$, as discussed, due to order counting, it will be absorbed into the definition of the gluon operator $(\alpha_s/\pi) G_{\mu\nu} G^{\mu\nu}$. Consequently, these loop diagrams are not only suppressed, but can even dominate over the DM–quark reactions.

In general, the loop momentum is characterised by the masses of the virtual particles running in the loop as well as the external momenta. Accordingly, we classify the DM–gluon scattering diagrams into two types. If the momentum scale is dominated by mass of heavy particles like EWDM, gauge vectors and Higgs, then the process is referred to as short distance. On the other hand, long distance contributions arise from loop integrals whose momenta are governed by the quark masses [83].

Short distance diagrams should be evaluated explicitly using perturbative quantum chromodynamics machinery. This is also true of the long distance integrals involving the top quark. However, when light quarks i.e. up, down and strange run in the long-distance loops, both mass and momentum are below the QCD scale, so the process is characterised by the confinement dynamics. This contribution is already included in the quark scalar matrix element $\langle N|\bar{q}q|N \rangle$ when computing the quark mass fraction $f_T^q$. Charm and bottom flavours are also close to $\Lambda_{QCD}$, so the strong coupling is still large and non-perturbative effects are significant at their mass scale. Therefore, the contributions from softer quarks $d$, $u$, $s$, $c$ and $b$ should not be incorporated in the long-distance gluon Wilson coefficient $C_g$, otherwise we would count them twice in the calculations [84].

$$C_g = \sum_{q=\text{All}} C_{gq} \left( m_q \right) + C_g^{LD}(m_t).$$  \hspace{1cm} (46)

Note that $C_g$ is of leading order of strong structure constant $O(\alpha_s^0)$ in power counting, despite loop-suppression of the DM–gluon scattering [85]. As discussed, this is due to the factor of $\alpha_s/\pi$ for the gluon scalar operator in the effective Lagrangian.

Computation of the effective interactions of gluonic operators is a tedious task that requires constructing the tensor structure of the gluon field strength. When the field is weak which means the external momentum is much smaller than the characteristic scale of the process, then the gluon field strength can be treated as background field. In this case, it is more convenient to choose the Fock–Schwinger gauge [86,87]:

$$x^\mu A_\mu^q = 0. \hspace{2cm} (47)$$

In this gauge, the Wilson coefficient for gluon interactions can be easily extracted, since the coloured propagators are already defined in terms of the background field strength tensors. The origin is singled out in relation (47), and the gauge condition is not translational invariant. Accordingly, one
should be careful when computing gauge dependent quantities like propagators, because for example, forward $S(x,0)$ and backward $S(0,x)$ propagation have different forms, as will be shown explicitly. However, the translation symmetry will be restored in gauge-independent physical quantities like scattering amplitudes [88].

The most important property is that the gauge field can be directly replaced by the field strength tensor [89]:

$$A_\mu^a = \frac{i}{2} (2\pi)^4 \, G^a_{\mu\nu} \, \delta_{k\delta} \, \delta^{(4)}(k),$$

(48)

where the higher order derivative terms are irrelevant and therefore neglected. As a result, the gluon field strength bilinear $G^a_{\mu\nu} G^a_{\nu\mu}$ will appear in the amplitude of the effective scalar interactions with EWDM. The gluon scalar operator can be easily projected out of the field strength bilinear, using the identity $G^a_{\mu\nu} G^a_{\nu\mu} = \frac{1}{3} \left( \eta_{a\beta} \eta_{\mu\nu} - \eta_{a\nu} \eta_{\beta\mu} \right) G^a_{\rho\sigma} G^{a\rho\sigma} + \cdots$, where other terms in ellipsis are not relevant and thus omitted.

The DM–gluon interactions involving the cubic coupling are generated by the triangle-loop diagram of Fig. 5a. Clearly the triangle loop in this diagram where virtual quarks are circulating will only give rise to a long-distance integral.

To compute the scattering amplitude in this gauge, we need the EWDM two-point function in the gluon background. This requires computing the contribution of Higgs tadpole $\Gamma_q$ in the gluon external field. Higgs tadpole has the form:

$$\Gamma_q = - \text{tr} \int S^{(2)}_q(\ell) \, \frac{d^4 \ell}{(2\pi)^4}$$

$$= -i \frac{g_s}{4\pi^3} \, G^a_{\rho\sigma} \, G^{a\rho\sigma} m_a \left( A^{(3)}_0(m_q) - m_q^2 A^{(4)}_0(m_q) \right).$$

(49)

The scalar one-point functions $A^{(3)}_0$ and $A^{(4)}_0$ are defined in (80a). The second order correction to the propagator of a coloured fermion when two external gluons are inserted reads [90]:

$$S^{(2)}_q(\ell) = S_q^{(0)}(\ell) \int \left( -ig_s A(k_1) \right) S_q^{(0)}(\ell - k_1)$$

$$\times \int \left( -ig_s A(k_2) \right) S_q^{(0)}(\ell - k_1 - k_2) \frac{d^4k_1}{(2\pi)^4} \frac{d^4k_2}{(2\pi)^4}.$$  

(50)

$$S_q^{(0)}(\ell) = i/(\ell - m_q)$$ is the usual fermionic Feynman propagator in the vacuum. Since the gluon field contains derivative of Dirac delta function, the integration over the background field momenta can be carried out trivially. In practice, it reduces to differentiation of the propagators located after the vertex.

Using the relation $C^{h3}_q = i (\pi/\alpha_s) C(\bar{\chi}^0, \chi^0, h) m_t \Gamma_t/(\sqrt{m_t^2},$ with the vertex factor $C(\bar{\chi}^0, \chi^0, h)$ defined in (79), the amplitude of Feynman diagram 5a gives rise to the following coefficient:

$$C^{h3}_q = -\frac{\lambda}{48 m_h^2}.$$  

(51)

It can be checked that DM–gluon coupling arising from the hard triangle loop is related to the top quark contribution through $C^{h3}_q = -C^{t3}_q/12$. This behaviour can be explained by heavy quark expansion of the trace anomaly of the energy–momentum tensor [91]. In short distances of order $1/m_t$, one can expand the virtual top state in powers of $m_t^{-2}$. To the first order in $\alpha_s$, top scalar operator converts to the gluon operator as $m_t \tilde{t} \rightarrow (-\alpha_s/12\pi) \, G^a_{\mu\nu} G^{a\mu\nu} [92].$

Now, we move on to the EWDM-gluon interactions that include the quartic non-renormalisable Higgs coupling. As shown in Fig. 5, the diagrams are generated at two-loop level. At first step, one needs to evaluate the quantum corrections to the Higgs self energy $\Pi_q$ induced by virtual quarks, in the gluon external field.

When each quark propagator emits one gluon which is the case for diagram Fig. 5b (right), the self energy can be written as:

$$\Pi_q^{(1)}(q^2) = \left( \frac{g_w}{2 m_w} \right)^2 \frac{m_q^2}{2} \int S_q^{(1)}(\ell + q) \tilde{S}_q^{(1)}(\ell) \, \frac{d^4 \ell}{(2\pi)^4}$$

$$= - \left( \frac{g_w g_s}{2 m_w} \right)^2 G^a_{\mu\nu} G^{a\mu\nu} m_q^2 \left( 3m_q^2 B_0^{(2,2)} + q^2 B_0^{(2,1)} \right) (q^2 | m_q, m_q).$$  

(52)

The loop integrals $B_0^{(1,2)}$ and $B_0^{(2,2)}$ are explicitly defined in (86b) and (87b). The first order correction to the fermionic propagator which corresponds to one gluon field in the background, is given as:

$$S_q^{(1)}(\ell) = S_q^{(0)}(\ell) \int \left( -ig_s A(k_1) \right) S_q^{(0)}(\ell - k_1) \frac{d^4k_1}{(2\pi)^4}.$$  

(53)

Due to violation of the translation invariance in the gauge condition (47), propagation of an antiparticle in the opposite direction has a different form [88]:

$$\tilde{S}_q^{(1)}(\ell) = \int S_q^{(0)}(\ell + k_2) \left( -ig_s A(k_2) \right) \frac{d^4k_2}{(2\pi)^4} S_q^{(0)}(\ell).$$  

(54)

It can be seen that the quark masses and the external momentum $q$ contribute on equal footing to the loop momenta in (52). Since the dominant value of the external momentum is at Higgs mass scale, the quark box receives short-distance contributions, and therefore all quark flavours should be taken into account.
Executing the integrals, we finally find:

\[
\Pi_1^{(1)}(y) = -\frac{i\alpha_s\alpha_t}{4M_w^2} \frac{y}{1 - 4y} \left( \frac{1}{(2\pi)^4} \right) \times \left[ 8y (1 - 3y) K(y) + (1 - 6y) \right],
\]

(55)

where \(y \equiv (m_q/q)^2\), and \(K\)-function is defined in (93). When the momentum is smaller than twice quark mass \(y > 1/4\), one can use the identity (93) to avoid root of negative numbers and logarithm with complex arguments.

Figure 6a depicts the behaviour of the normalised Higgs self-energy \(\Pi_q \equiv (\alpha_w\alpha_s/4m_w^2)G_{\mu\nu}G^{\mu\nu} \tilde{\Pi}_q\), with respect to \(y\). The data-points corresponds to the values for different quark masses at the dominant momentum \(q \approx m_b\). The two-point function vanishes when the momentum is either much larger \(y \to 0\), or much smaller \(y \to \infty\) than the quark mass. It is finite at \(y = 1/4\) if approaching from below, but explodes when taking the limit from right.

In case two gluon fields are attached with the same propagator as in diagram Fig. 5b (left), the self energy is properly obtained as:

\[
\Pi_1^{(2)}(q^2) = \left( \frac{g_w}{2m_w} \right)^2 \frac{m_q^2}{m_w^2} \text{tr} \int S_q^{(0)}(\ell + q) S_q^{(2)}(\ell) \frac{d^4\ell}{(2\pi)^4}
\]

\[
= \left( \frac{g_wg_s}{2m_w} \right)^2 \frac{m_q^4}{m_w^4} \left( B_0^{(1,2)} - m_q^2 B_0^{(1,3)} + q^2 B_1^{(1,3)} - 2m_q^4 B_0^{(1,4)} - m_q^2 q^2 B_1^{(1,4)} \right) \times (q^2m_q, m_q).
\]

(56)

The loop integrals \(B_0^{(1,3)}\) and \(B_0^{(1,4)}\) are evaluated in (88b) and (89b). The fermionic propagator for the antiparticle emitting two gluons is given by:

\[
\tilde{S}_q^{(2)}(\ell) = \int S_q^{(0)}(\ell + k_1 + k_2) (-i\gamma_5 A(k_2))
\]

The dominant contribution to the box integral (56) is provided by the quark mass. As discussed before, when computing such long-distance diagrams, due to the implicit infrared cut-off at \(\Lambda_{QCD}\) in the loop momenta, only top quark should be considered.

Carrying out the integrals, we arrive at the following expression:

\[
\Pi_1^{(2)}(y) = -\frac{i\alpha_s\alpha_t}{8m_w^2} \frac{y}{1 - 4y} \left[ 2(1 - 2y) K(y) - 1 \right].
\]

(58)

For \(y < 1/4\), using the identity (93), the result can be expressed in terms of the inverse cotangent.

As illustrated in Fig. 6a, the Higgs two-point function in two gluon background field \(\Pi_1^{(2)}\) has the same asymptotic behaviour as that of \(\Pi_1^{(1)}\). It has only one a-sided limit at \(y = 1/4\) when approaching from left. It is also in clear view that we only need to take into account the input from the three heaviest quarks \(c, b\) and \(t\), in the calculations, and can safely ignore other lighter flavours.

The total Higgs self energy therefore can be written as:

\[
\Pi^h = \sum_{q=\text{All}} \Pi_1^{(1)} + \Pi_1^{(2)} = \sum_{q=c,b} \Pi_1^{(1)} + \Pi_1^{(2)},
\]

(59)

where \(\Pi_1^{(1)} + \Pi_1^{(2)}\).

At this stage, we need to evaluate the second loop using the Higgs correlator \(\Pi_1^h\) in order to find the Wilson coefficient of the gluon operator.

\[
\frac{\alpha_s}{\pi} C_{\mu\nu} G^{\mu\nu} C^{-1}_{gh} = i C(\chi^0, \chi^0, h, h)
\]
Fig. 6 (Left a) The normalised two-point function of Higgs boson in two gluon background fields with respect to $y = (m_q/q)^2$ with $q$ being the external momentum. The blue curve corresponds to $i\Pi^{(1)}_q$ correlation function, where each internal quark propagator emits one gluon (c.f. Fig. 5b-right). While the red curve represents $i\Pi^{(2)}_q$ function, in which two gluons are attached to the same internal quark propagator (c.f. Fig. 5b-left). The data-points present the value of the two-point functions at the mass of up (light blue), down (dark blue), strange (light green), charm (dark green), bottom (red) and top (black) quarks. (Right b) Changes in the gluon mass functions in dependence of $h = (m_q/m_h)^2$. The blue curves indicate the analytical and numerical gluon mass functions $g^{(q)}_u$ and $g^{(q)}_d$ where softer i.e. up, down, strange, charm and bottom flavours run in the loops. Whereas when top quark is the only virtual particle, the relevant mass functions $g^{(t)}_u$ and $g^{(t)}_d$ are plotted in red. Negative-valued functions are shown in dash-dotted style.

\[ g^{(t)}(x) = x \left[ \frac{-1}{4x-1} + \frac{8x}{(4x-1)^2} \left( 24x^2 - 12x + 1 \right) \times \ln 4x + 12x \ln(4x + 1) \right]. \]  

(63)

In addition, $I_0^{(q)}$ is expressed by the following integral:

\[ I_0^{(q)}(x) = 8x^2 \int_{0}^{\infty} \frac{l + 3x}{\sqrt{l} \left( l + 4x \right)^{\frac{3}{2}} (l + 1)^2} \times \ln \frac{\sqrt{l} + 4x + \sqrt{l}}{2\sqrt{x}} \, dl. \]  

(64)

This loop integral is also dominated by momenta at Higgs mass scale $q \approx m_h$. The mass function $g^{(q)}_u$ is given by:

\[ g^{(q)}_u(x) = x \left[ \frac{-1}{1 - 4x} + \frac{2x}{(1 - 4x)^2} \left( 1 - 24x + 48x^2 \right) \times \ln 4x - 6x \ln(1 + 4x) \right]. \]  

(65)

Figure 6b illustrates changes of the mass functions $I_0^{(q)}$, $g^{(q)}_u$, $g^{(q)}_d$ and $g^{(q)}_t$, involved in gluonic interactions through quartic coupling. The lighter flavour functions $I_0^{(q)}$ and $g^{(q)}_d$ vanish in the massless quark limit, and the top flavour ones $I_0^{(t)}$ and $g^{(t)}_d$ approach infinity in the heavy quark region. We can also safely ignore the non-analytical lighter quark terms $I_0^{(q)}$ in $C^{44}_g$.

It is noticed that the computational $I_0$ and analytical $g^{(q)}_d$ terms in the effective coupling (61) have the same order of magnitude, but opposite signs. This leads to an accidental
cancellation which decreases the contribution of $C^D_4$ coefficient to the effective amplitude by an order of magnitude.

By numerical evaluation of the Wilson coefficients for the loop-induced processes (51) and (61), it can be deduced that total amplitude arising from the loop diagrams, dominates over that of tree-level scattering (44) by a factor of about 3.

5 Direct detection constraints

Having derived all the required theoretical ingredients, in this section we numerically compute the EWDM–nucleon SI cross section, and compare our results with the latest direct search data and future sensitivities.

In the previous section, it was found that the effective amplitude for all the main channels have positive sign, therefore all the diagrams containing the non-renormalisable couplings contribute constructively to the total cross-section through (38).

At leading order, all the effective couplings induced by higher-dimensional operators have a factor of $\lambda$. We can therefore define the critical coupling constant $\lambda_{cr}$ where the non-renormalisable amplitude equates the renormalisable one $f^R_N = f^R_N$, so that:

$$\lambda_{cr} \equiv f^R_N / f^R_N / \lambda.$$  (66)

Figure 7a presents the critical coupling for the two pseudo-real EWDM representations. It can be observed that $\lambda_{cr}$ is almost independent of DM mass, particularly in TeV mass region. In addition, the value of the critical parameter in quartet model is about an order of magnitude above that of doublet dark matter. That is due to the factor of $\left( n^2 - 4y^2 + 1 \right)$ in the charged weak induced amplitude (107) and lack of the light charged EWDM mediated diagrams in $C_2$ model.

In fact the behaviour of the pseudo-real models of the electroweak dark matter crucially depends on the strength of the coupling $\lambda$. In order to further study this, in Fig. 7b, we compare the performance of the complex models with three indicatory values of the coupling.

Below the critical coupling, the direct search observables of the model behave similar to those of the renormalised theory, as shown for $\lambda = 10^{-6}$ GeV$^{-1}$ case. Since the coupling is less than the minimum possible critical value $\lambda < \lambda_{C_2}$, there are distinct spectra for different representations.

Obviously this can happen if both $\lambda_0$ and $\lambda_c$ are small. In addition, if the relationship between the two couplings is in direct proportion that is $\lambda_c / \lambda_0 = 2n$, then effect of the higher-dimensional operators cancel out each other. In such cases, the effective theory will produce the same signal for direct detection experiments as the renormalisable low-energy Lagrangian (101).

In the event that $2n \Re \lambda_0 > \lambda_c$, the coupling constant will take negative values $\lambda < 0$. It causes a measurable destructive interference between the renormalisable and non-renormalisable amplitudes, which leads to the suppression of the total SI cross-section. This is illustrated in the figure, for $\lambda = -10^{-5}$ GeV$^{-1}$ curve. At the critical point of $\lambda = -\lambda_{cr}$, the scattering amplitude will totally vanish in the leading order.

On the contrary, above $\lambda_{cr}$, the scattering amplitude is governed by the non-renormalisable terms. Since the effective coupling induced by the higher-dimensional operators is independent of the representation, the cross-section curves
of all pseudo-real models converge. This behaviour can be verified for the indicative value of \( \lambda = 10^{-4} \text{ GeV}^{-1} \). As the coupling to higher-dimensional operators strengthens, the non-renormalisable couplings enhance the scalar effective amplitude, and thus the total scattering cross-section of pseudo-real modules increases significantly.

The SI cross-sections for various representations of EWDM have been compared with current experimental data from XENON1T (2018) [43], PandaX-4T (2021) [93] and LUX-ZEPLIN (LZ) (2022) [94], and future projection for DARWIN detector [95] in Fig. 8, for two values of the coupling \( \lambda \).

In general, the SI cross-section has small dependence on EWDM mass, especially above TeV scale. That is due to the fact that the non-renormalisable effective couplings are totally independent of dark matter mass, and the renormalisable contribution becomes mass independent in the heavy dark matter limit (c.f. Sect. 1).

The solid lines illustrate the cross-section curves of the real theories as well as the complex models for \( \lambda = 10^{-6} \text{ GeV}^{-1} \) which is an indicative value for the coupling strength being below the criticality. In this region, the predicted SI cross-section is far below the present direct detection bound. The future DARWIN experiment might fully probe the multiplets of dimension \( n > 3 \) in TeV mass scale, although the higher mass range will remain unconstrained.

At very small cross-sections the potential DM signal would be saturated by the background atmospheric, solar, and diffuse supernova neutrinos colliding with target nuclei [96]. The discovery limit is defined as the cross-section where there is 90\% probability that experiment can detect the true DM with a minimum significance of 3-\( \sigma \) [97]. We refer to this lower limit as \( \text{neutrino floor} \) which is shown in Fig. 8 as the border of the yellow shaded area [98]. Within this parameter region, it would be difficult to discover dark matter events.

It can be seen that, below the critical value \( \lambda_{cr} \), scattering cross-section enhances with the dimension of representation. The reason is that \( W \)-boson mediated renormalisable interactions are proportional to \( \lambda/2[n^2 - (4y^2 + 1)] \) (c.f. (107),(110)). This factor increases when going either from a complex representation \( \mathbb{C} \) to a higher-dimensional real model \( \mathbb{R} (n+1) \) or from a real \( n \)-tuplet \( \mathbb{R} \) to a larger complex multiplet \( \mathbb{C} (n+1) \). The \( Z \)-boson induced interactions rise in proportion to \( y^2/4 \); however, the effective EWDM–Z coupling is comparatively smaller than \( W \) contribution.

In contrast, for \( \lambda = 10^{-4} \text{ GeV}^{-1} \), the coupling is above the maximum critical strength \( \lambda > \lambda_{cr} \). As discussed, the non-renormalisable operators, in this case, provide a constructive contribution to the scattering amplitude which could increase the total SI cross-section up to orders of magnitude. Therefore, for high values of non-renormalisable coupling, the pseudo-real EWDM can reach the detection limit, and produce signals that potentially could be observed by direct detection experiments.

The pseudo-real doublet has the smallest dimension of the SU(2) representations, and misses the light charged component. Consequently the scattering cross-section arising from renormalisable interactions falls below the neutrino background limit for this model. Nevertheless, the coupling to Higgs boson via non-renormalisable operators can increase the scattering rate to the current direct detection energy thresholds. As a result future experiments would be promising to detect the doublet EWDM scenario.

5.1 Parameter space

In this section, we update the observational bounds on the free parameters of the electroweak theory of DM, to include the constraints from both direct detection and indirect searches.

Figure 9 illustrates the valid values of EWDM mass for the real models, as well as the neutral coupling – mass \( (\lambda_0 - m_\gamma) \) plane for the complex representations. The vertical shaded patches are excluded by the three gamma-ray observations that are inner Milky Way, dwarf galaxies and monoenergetic lines. In addition, the top horizontal areas in the pseudo-real representations are disfavoured by elastic direct detec-
Fig. 9 (Upper panels) Summary charts plotting the viable values of EWDM mass as the free parameter for the real representations: triplet (left a) and quintuplet (right b). The vertical bars are excluded by gamma-ray observations from the inner Milky Way (green), satellite galaxies (blue), and photon line probes (red). We used Isothermal core density distribution for inner Galaxy searches and monochromatic line measurements. The vertical dash-line shows the freeze-out mass for each model which is determined by the thermal production. The vertical axis does not indicate any physical variable for the real models, and only makes the one-dimensional graphs comparable with 2D plots of the complex multiplets. (Lower panels) The mass-coupling \( m_\chi - \lambda_0 \) plane visualising the two-dimensional parameter space of the complex systems including doublet (left c) and quartet (right d). The top horizontal grey area is ruled out by elastic direct DM experiments. In addition, the horizontal region at the bottom is disfavoured by inelastic EWDM–nucleon interactions through Z-boson exchange.

Since nucleus recoiling constraints are weakened above 100 GeV scale, direct detection data favours TeV electroweak dark matter. Real models cannot be probed by present DD experiments, as their scattering amplitudes do not receive any contribution from the non-renormalisable operators.

As discussed, due to the effective interactions with Higgs boson which are induced by higher-dimensional operators, the scattering amplitude can be significantly enhanced in the pseudo-real dark matter representations. So, the direct search data will set an upper-limit for the values of non-renormalisable couplings. Apparently, regions of parameter space above \( \lambda_0 \approx 10^{-4} \text{ GeV}^{-1} \) are not accessible for complex models, with constraints on the pseudo-real doublet being slightly weaker.

For gamma-ray probes, we use the core density profile which is favoured by the electroweak theory of dark matter. Restrictions exerted by the indirect searches are noticeably representation dependant. For the real triplet \( (R3) \), the TeV mass interval including and above the thermal value is acceptable, although lower DM masses are ruled out. It can be observed that the real quintuplet \( (R5) \) is more strictly bounded by indirect detection, leaving only the thermal mass and higher ranges available. The complex quadruplet \( (C4) \) favours wider allowed areas of the parameter space particularly at larger masses and stronger scalar coupling. The pseudo-real doublet \( (C2) \) is not constrained by gamma-ray searches of EWDM.
To summarise, the multi-TeV and higher mass regions of the parameter space with an intermediate coupling strength are favoured by a combination of the direct and indirect experiments and could be explored further by the future probes.

6 Conclusion

The electroweak theory was generalised to incorporate a fermionic multiplet in non-chiral representation of the SU(2) × U(1)E symmetry group, ensuring that the successful properties of the Standard Model were minimally altered. The pseudo-real models are ruled out by direct detection experiments because they scatter off the nuclei at tree-level via Z-boson exchange. The complex EWDM formalism was expanded by including dimension-five UV operators that generate new couplings to the Higgs boson. This approach rejuvenated the pseudo-real representations as the aforementioned effective term divides the pseudo-Dirac DM into two Majorana fields, thereby removing the sizeable Z-mediated reaction with nuclei.

While EWDM does not scatter off the nucleon at tree-level, it does through non-renormalisable couplings, in addition to loop diagrams. In this paper, we studied the direct detection of electroweak dark matter as a suitable method to probe effects of ultraviolet operators on the pseudo real models at TeV scale. We formulated the effective scalar theory of EWDM–nucleon scattering at parton level which is quite useful in examining the non-renormalisable couplings in a systematic way.

All the diagrams that make a contribution to the EWDM–nucleon scattering at leading order of $O(\lambda^{-1})$ were taken into account. We evaluated the tree-level (one-loop) process that gives rise to the DM–quark collision through the non-renormalisable cubic (quartic) Higgs coupling, in addition to the one-loop (two-loop) processes generating interactions with gluons.

The effective amplitudes for the main scattering channels all have positive values which leads to a constructive contribution of the non-renormalisable interactions to the predicted detection rate.

In this framework, we studied the SI cross section of the electroweak DM arising from effective operators of the lowest dimension. The behaviour of the scattering cross-section across different pseudo-real models is determined by the (square of) parameter $\lambda$ which is a linear combination of the two non-renormalisable couplings (42).

There exist a critical value for this parameter at which the amplitudes for renormalisable and non-renormalisable effective couplings are the same. Below $\lambda_{cr}$, different EWDM representations have distinct spectra which lies well below the present direct detection constraints. However, as $\lambda$ gets stronger than the criticality value, the spectral curves for complex modules tend to become degenerate. The DD cross-section keeps increasing up to orders of magnitude, and therefore will be bounded from above by the measurement data.

If the charged and neutral couplings are proportional to each other, then $\lambda$ approaches zero. At this minimum limit, the effective theory discussed in this paper, will behave similar to the renormalisable model from the viewpoint of experimentally observable results.

The pseudo-real doublet as the least constraint EWDM model lies far below the neutrino floor. It is therefore difficult to detect this model in current direct detection searches. However, the non-renormalisable effects can raise the annihilation rate to the level that hopefully will be detectable for the next generation experiments.

Finally, we combined the astrophysical indirect search and direct detection bounds to find the allowed parameter space for all the electroweak dark matter representations. In general, pseudo-real theories support a wider range of viable values of the parameters. The higher mass intervals and intermediate coupling regime are overall favoured by a combination of constraints imposed by ID and DD data.

In conclusion, in case, the non-renormalisable coupling of DM to the SM through Higgs field is negligible, the resultant cross-section would stay below the current experimental constraints. However, if the new physics responsible for the mass-splitting of the dark species is closer to the electroweak scale, the effect of the higher-dimensional operators will escalate the scattering amplitude. This would potentially open the window for detection of electroweak dark matter in the near future experiments.

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Appendix A: Lagrangian and Feynman rules

The entire interaction Lagrangian that outlines the pseudo-real DM multiplet coupling with the SM fields, can be broken down as follows:\textsuperscript{16}:

\[ \mathcal{L}_{\text{int}} = \mathcal{L}_A + \mathcal{L}_Z + \mathcal{L}_W + \mathcal{L}_H. \]  

(67)

The couplings between the dark sector particles and gauge bosons are obtained from expanding the kinetic component in the overarching Lagrangian (3). The Lagrangian for electromagnetic interactions is as follows:

\[ \mathcal{L}_A = -e \left[ n_\mu^2 \chi_2 \gamma^\mu \chi_2 - \sum_{q=1}^{n/2-1} q \left( \chi_1^q \gamma^\mu \chi_1^q + \chi_2^q \gamma^\mu \chi_2^q \right) \right] A_\mu. \]

(68)

The dark particles can couple to the neutral weak gauges via:

\[ \mathcal{L}_Z = -\frac{g_2}{2} \left[ (n e_2^2 - 1) \chi_2^2 \gamma^\mu \chi_2^2 - \frac{1}{\sqrt{2}} \chi_1^2 \gamma^\mu \chi_1^2 \right] + \sum_{q=1}^{n/2-1} \left( [2e_2^2 q + \cos \phi_q] \chi_1^q \gamma^\mu \chi_1^q + [2e_2^2 q - \cos \phi_q] \chi_2^q \gamma^\mu \chi_2^q \right) Z_\mu. \]

(69)

The interactions that are induced through the charged weak boson can be expressed as:

\[ \mathcal{L}_W = -\frac{g_w}{2\sqrt{2}} \left[ 2 n_{\text{eff}} - 1 \right] e^{\frac{i}{2} \phi_0} \left[ c_{q-1}^{\text{eff}} + c_{q}^{\text{eff}} \right] \gamma^\mu \chi_2^q \]

\[ + \frac{1}{\sqrt{2}} \left( n e^{\frac{i}{2} \phi_0} c_+ - \sqrt{n^2 - 4 e^{\frac{i}{2} \phi_0} c_+} \sqrt{\chi_0^+ \gamma^\mu \chi_0^+} - \frac{i}{\sqrt{2}} \left( n e^{\frac{i}{2} \phi_0} s_+ + \sqrt{n^2 - 4 e^{\frac{i}{2} \phi_0} s_+} \sqrt{\chi_0^+ \gamma^\mu \chi_0^+} \right) + \frac{i}{\sqrt{2}} \left( n e^{\frac{i}{2} \phi_0} s_+ - \sqrt{n^2 - 4 e^{\frac{i}{2} \phi_0} s_+} \sqrt{\chi_0^+ \gamma^\mu \chi_0^+} \right) \right] \]

\[ + \sum_{q=2}^{n-1} \left( \sqrt{n^2 - 4(q-1)^2 s_{q-1}^q} - \sqrt{n^2 - 4q^2 c_{q-1}^q} \right) \times \chi_1^{-1} \gamma^\mu \chi_1^q \]

\textsuperscript{16} The full Lagrangian for EWDM theory has already been presented in reference [31]. However, because calculation of the Wilson coefficients crucially depends on the Feynman rules of the UV-theory, the content is repeated in this paper.

By utilising the interaction Lagrangian terms, we can determine the Feynman rules for the vertices connecting the pseudo-real EWDM to the electroweak gauge fields and Higgs scalar. Feynman rule for the electromagnetic interactions of the charged odd particles with photon can be cast as:

\[ C^{\mu}(\chi_1^q, \chi_2^q, A) = -\frac{i}{2} ne \gamma^\mu, \]

\[ C^{\mu}(\chi_1^q, \chi_1^q, A) = -ie q \gamma^\mu \delta_{ij}. \]

(73)

For vertices that include the neutral weak Z boson, the interaction is given by:

\[ C^{\mu}(\chi_1^q, \chi_2^q, Z) = -\frac{i}{2} g_z \left( n e_2^2 - 1 \right) \gamma^\mu, \]

\[ C^{\mu}(\chi_1^q, \chi_1^q, Z) = -\frac{i}{2} g_z \left( 2e_2^2 q + \cos \phi_0 \right) \gamma^\mu, \]

\[ C^{\mu}(\chi_2^q, \chi_2^q, Z) = -\frac{i}{2} g_z \left( 2e_2^2 q - \cos \phi_0 \right) \gamma^\mu. \]
factorising the root of unity out to the Feynman rules for the conjugate weak charged field.

For the pseudo-real doublet, the relations simplify to:

\[ C^\mu(\chi_1^q, \chi_2^q, Z) = -\frac{i}{2} g_w \sin \phi_q \gamma^\mu, \]
\[ C^\mu(\chi_1^q, \chi_1^{-q}, Z) = -C^\mu(\chi_1^{-q}, \chi_1^q, Z) = g_w \gamma^\mu. \] (74)

It should be noted that the Z boson can alter the flavour of the dark particles having the same electric charge.

Interactions of the fields with different charges are mediated by \( W^- \) gauge boson through:

\[ C^\mu(\chi_2^{-q/2-1}, \chi_2^q, W^-) = -\frac{i}{\sqrt{2}} \sqrt{n-1} e^{i\frac{2\lambda_0}{\lambda}} C_{-1}^\mu g_w \gamma^\mu, \]
\[ C^\mu(\chi_1^{-q/2-1}, \chi_1^q, W^-) = \frac{i}{\sqrt{2}} \sqrt{n-1} e^{i\frac{2\lambda_0}{\lambda}} C_{-1}^\mu g_w \gamma^\mu, \]
\[ C^\mu(\chi_2^{q/2-1}, \chi_1^{-q}, W^-) = \frac{i}{\sqrt{2}} \sqrt{n-1} e^{i\frac{2\lambda_0}{\lambda}} C_{-1}^\mu g_w \gamma^\mu, \]
\[ C^\mu(\chi_1^{q/2-1}, \chi_1^{-q}, W^-) = -\frac{i}{\sqrt{2}} \sqrt{n-1} e^{i\frac{2\lambda_0}{\lambda}} C_{-1}^\mu g_w \gamma^\mu, \]
\[ C^\mu(\chi_2^{-q/2-1}, \chi_2^q, W^-) = \frac{i}{\sqrt{2}} \sqrt{n-1} e^{i\frac{2\lambda_0}{\lambda}} C_{-1}^\mu g_w \gamma^\mu, \]
\[ C^\mu(\chi_1^{q/2-1}, \chi_1^q, W^-) = -\frac{i}{\sqrt{2}} \sqrt{n-1} e^{i\frac{2\lambda_0}{\lambda}} C_{-1}^\mu g_w \gamma^\mu, \]
\[ C^\mu(\chi_2^{q/2-1}, \chi_2^{-q}, W^-) = \frac{i}{\sqrt{2}} \sqrt{n-1} e^{i\frac{2\lambda_0}{\lambda}} C_{-1}^\mu g_w \gamma^\mu. \] (75)

For the pseudo-real doublet, the relations simplify to:

\[ C^\mu(\chi_0^q, \chi_0^{-q}, W^-) = -\frac{1}{2} g_w \gamma^\mu, \]
\[ C^\mu(\chi_0^q, \chi_0^-, W^-) = -\frac{i}{2} g_w \gamma^\mu. \] (76)

By replacing the vertex factor \( C \) with its conjugate, after factorising the root of unity out \( C^\mu = i C^\gamma \gamma^\mu \), one can reach to the Feynman rules for the conjugate weak charged field \( W^- \):

\[ C^\mu(\chi_1^{-q/2-1}, \chi_1^q, W^+) = i C^\mu(\chi_1^{-q/2-1}, \chi_1^q, W^-) \gamma^\mu. \] (77)

The non-renormalisable cubic couplings to the SM Higgs scalar follow the following rules:

\[ C(\chi_2^q, \chi_2^q, h) = -\frac{2i}{v} \Delta_{\frac{5}{2}(n-1)} = \frac{i}{2} (n-1) v \frac{\lambda_0}{\Lambda}, \]
\[ C(\chi_1^q, \chi_1^q, h) = -\frac{i}{v} \left( \Delta_{\frac{5}{2}(n-1)} + (-1)^{q+1} 2|\delta_q| \sin \phi_q \right) \]
\[ = \frac{i}{2} \left( \frac{q-1}{2} \right) \frac{\lambda_0}{\Lambda} \]
\[ + (-1)^q \sqrt{n^2 - 4q^2} \sin \phi_q \frac{\lambda_0}{\Lambda} \]}
\[ C(\chi_2^q, \chi_2^q, h) = -\frac{i}{v} \left( \Delta_{\frac{5}{2}(n-1)} + (-1)^q \frac{2}{2} |\delta_q| \sin \phi_q \right) \]
\[ = \frac{i}{2} \left( \frac{q-1}{2} \right) \frac{\lambda_0}{\Lambda} \]
\[ + (-1)^q \sqrt{n^2 - 4q^2} \sin \phi_q \frac{\lambda_0}{\Lambda} \]
\[ C(\chi_1^q, \chi_2^q, h) = C(\chi_1^q, \chi_2^q, h) \]
\[ = (-1)^{q+1} \frac{i}{v} \left| \delta_q | \cos \phi_q \right| \]
\[ + (-1)^q \frac{4}{v} \sqrt{n^2 - 4q^2} \sin \phi_q \frac{\lambda_0}{\Lambda} \]
\[ C(\chi_1^q, \chi_0^q, h) = -\frac{i}{v} \left( \Delta_{\frac{5}{2}(n-1)} - 2\Re q \delta_0 \right) \]
\[ = \frac{i}{2} \left( \frac{q-1}{2} \right) \frac{\lambda_0}{\Lambda} - n \left| \delta_q \right| \]
\[ C(\chi_0^q, \chi_0^q, h) = -\frac{i}{v} \left( \Delta_{\frac{5}{2}(n-1)} + 2\Re q \delta_0 \right) \]
\[ = \frac{i}{2} \left( \frac{q-1}{2} \right) \frac{\lambda_0}{\Lambda} + n \left| \delta_q \right| \]
\[ C(\chi_0^q, \chi_0^q, h) = C(\chi_0^q, \chi_0^q, h) \]
\[ = \frac{i}{2} \left( \frac{q-1}{2} \right) \frac{\lambda_0}{\Lambda}. \] (78)

The vertex for the non-renormalisable quartic interactions between two EWDM fields and two Higgs particles, take the following form:

\[ C(\chi_2^q, \chi_2^q, h, h) = -\frac{i}{v^2} \Delta_{\frac{5}{2}(n-1)} = \frac{i}{2} (n-1) v \frac{\lambda_0}{\Lambda}, \]
\[ C(\chi_1^q, \chi_1^q, h, h) = -\frac{i}{v^2} \left( \Delta_{\frac{5}{2}(n-1)} + (-1)^{q+1} 2|\delta_q| \sin \phi_q \right) \]
\[ = \frac{i}{2} \left( \frac{q-1}{2} \right) \frac{\lambda_0}{\Lambda} \]
\[ + (-1)^q \sqrt{n^2 - 4q^2} \sin \phi_q \frac{\lambda_0}{\Lambda} \]
\( C(\chi^q_2, \chi^q_1, h, h) = -\frac{i}{v^2} \left( \Delta_0 (q_1, q_2) + (-1)^q \frac{2}{2} |\phi_q| \sin \phi_q \right) \)

\[
\begin{align*}
&= i \left[ \frac{q}{2} - \frac{1}{2} \right] \frac{\lambda_c}{\Lambda} + (-1)^{q+1} \sqrt{n^2 - 4q^2} \sin \phi_q \left( \frac{\lambda_0}{\Lambda} \right), \\
&= -\frac{1}{2} \frac{\lambda_c}{\Lambda} - i \frac{\lambda_0}{\Lambda}, \\
&= i \frac{n}{4} \lambda_0 \Lambda.
\end{align*}
\]

**Appendix B: Loop integrals**

In order to compute the Wilson coefficients in the full theory of EWDM, we need to introduce and evaluate new loop integrals.

**B.1 One-point function**

The scalar one-point function is given by [101]:

\[
A_0^{(n)}(m) = \mu^{4-d} \int \frac{1}{(k^2 - m^2)^n} \frac{d^dk}{(2\pi)^d} = \frac{(-1)^q i}{(4\pi)^{d/2}} \mu^{4-d} \frac{m^{d-2n}}{\Gamma(n - d/2)} \frac{\Gamma(n)}{\Gamma(n)}.
\] (80a)

In \( d = 4 \) dimensions, for \( n = 1, 2 \) the integral diverges. In dimensional regularisation scheme, the dimension is taken to be \( d - 4 \ll 1 \). The regularisation scale \( \mu \) fixes the dimension of measure at 4. Therefore, to \( O(d - 4) \) we get:

\[
\begin{align*}
A_0^{(1)}(m) &= \frac{i}{2} \frac{m^2}{\mu^2} \left( \Delta - 2 \ln \frac{m}{\mu} + 1 \right), \\
A_0^{(2)}(m) &= \frac{i}{4\pi} \left( \Delta - 2 \ln \frac{m}{\mu} \right). \tag{80b}
\end{align*}
\]

**B.2 Two-point function**

The scalar two-point function can be written as:

\[
B_0^{(n_2, n_1)}(p^2 | m_1, m_2) = \mu^{4-d} \int \frac{1}{(k^2 - m_1^2)^n_1 (k^2 - m_2^2)^n_2} \frac{d^dk}{(2\pi)^d}.
\] (84)

Note that \( B_0^{(n_2, n_1)}(p^2 | m_2, m_1) = B_0^{(n_1, n_2)}(p^2 | m_1, m_2) \), also \( B_0^{(0, n)}(p^2 | m_1, m_2) = B_0^{(n)}(m_1) \). For \( n_1 = n_2 = 1 \), the loop integral is divergent. To order \( O(d - 4) \) we are left with [102]:
B_0(p^2|m_1, m_2) = \frac{i}{(4\pi)^2} \left[ \Delta + 2 \ln \frac{m_1 m_2}{\mu^2} \\
+ \frac{1}{2} (x_1 - x_2) \ln \frac{x_2}{x_1} - \kappa^2 K(x_1, x_2) \right],
\text{(85a)}

B_0(p^2|m, m) = \frac{i}{(4\pi)^2} \left[ \Delta + 2 - 2 \ln \frac{m}{\mu} - \kappa^2 K(x) \right],
\text{(85b)}

where \(x_1 \equiv (m_1/p)^2, x_2 \equiv (m_2/p)^2, \kappa\) is the Kallen function \(\kappa(x_1, x_2) \equiv \sqrt{(1 - x_1 - x_2)^2 - 4x_1 x_2}\), and we define \(K\)-function in (91).

In the event of mass degeneracy in the propagators, we use the parameter \(\chi \equiv (m/p)^2\). The Kallen function reduces to \(\kappa(x) = \sqrt{1 - 4\chi}\), and the simplified \(K\)-function is defined in (93).

The following special two-point functions facilitate computation of the loop diagrams encountered in the theory of EWDM:

\[ B_0^{(1,2)}(p^2|m_1, m_2) = \frac{i}{(4\pi)^2 p^2} \left[ \frac{1}{2} \ln \frac{x_1}{x_2} + (1 + x_1 - x_2) K(x_1, x_2) \right], \]
\text{(86a)}

\[ B_0^{(1,2)}(p^2|m, m) = \frac{i}{(4\pi)^2 p^2} K(x). \]
\text{(86b)}

\[ B_0^{(2,2)}(p^2|m_1, m_2) = \frac{i}{2(2\pi)^2} \frac{1}{p^4 \kappa^2} \left[ (1 - x_1 - x_2) K(x_1, x_2) - 1 \right], \]
\text{(87a)}

\[ B_0^{(2,2)}(p^2|m, m) = \frac{i}{2(2\pi)^2} \frac{1}{p^4 \kappa^2} \left[ (1 - 2x) K(x) - 1 \right]. \]
\text{(87b)}

\[ B_0^{(1,3)}(p^2|m_1, m_2) = \frac{i}{2(4\pi)^2} \frac{1}{p^4 \kappa^2} \left[ 4x_1 K(x_1, x_2) - \frac{1}{x_2} (1 - x_1 - x_2) \right], \]
\text{(88a)}

\[ B_0^{(1,3)}(p^2|m, m) = \frac{i}{2(4\pi)^2} \frac{1}{p^4 \kappa^2} \left( 4x K(x) - \frac{1 - 2x}{x} \right). \]
\text{(88b)}

\[ B_0^{(1,4)}(p^2|m_1, m_2) = \frac{i}{6(4\pi)^2} \frac{1}{\kappa^4 m_1^4 p^2} \left[ 12x_1 x_2^2 (1 + x_1 - x_2) K(x_1, x_2) \\
+ (1 - x_1 - x_2)^3 - x_2 \left( (1 - x_2)^2 + 8x_1 - 9x_2^2 \right) \right], \]
\text{(89a)}

\[ B_0^{(1,4)}(p^2|m, m) = \frac{i}{6(4\pi)^2} \frac{1}{\kappa^4 m^4 p^2} \times \left[ 12x^3 K(x) + (1 - x) (1 - 6x) \right]. \]
\text{(89b)}

One can use the following identity:
\[ i \ln \frac{1 - x_1 - x_2 + \kappa}{2 \sqrt{x_1 x_2}} = \text{Arctg} \frac{|\kappa|}{x_1 + x_2 - 1}. \]
\text{(90)}

In order to redefine the \(K\)-function when \(\kappa^2 < 0\), where \(|\kappa|(x_1, x_2) \equiv -i\kappa(x_1, x_2) = \sqrt{4x_1 x_2 - (1 - x_1 - x_2)^2}\) is a positive real function:

\[ K(x_1, x_2) = \frac{1}{\kappa} \ln \frac{1 - x_1 - x_2 + \kappa}{2 \sqrt{x_1 x_2}} = -\frac{1}{|\kappa|} \text{Arctg} \frac{|\kappa|}{x_1 + x_2 - 1}. \]
\text{(91)}

In this way, the root of negative numbers and complex logarithms in the expressions for 2-point functions can be avoided.

In the case of equal masses \(m_1 = m_2 \equiv m\), the equality below, serves the same purpose:
\[ i \ln \frac{1 + \kappa}{2 \sqrt{\chi}} = \text{Arccot} |\kappa|. \]
\text{(92)}

here \(|\kappa(x)| = -i\kappa(x) = \sqrt{4\chi - 1}\). Note that this gives the useful identity:
\[ \frac{1}{2} K(x) = \frac{1}{\kappa} \ln \frac{1 + \kappa}{2 \sqrt{\chi}} = -\frac{1}{|\kappa|} \text{Arccot} |\kappa|, \]
\text{(93)}

The vector loop-integral
\[ B_0^{(n_1, n_2)}(p^2|m_1, m_2) \equiv \mu^{4-d} \int \frac{d^d k}{[(k + p)^2 - m_1^2]^{n_1} (k^2 - m_2^2)^{n_2}} \]
\text{(94)}

can be decomposed into external momentum \(p_\mu\) as a Lorentz covariant quantity:
\[ B_0^{(n_1, n_2)} = p_\mu B_1^{(n_1, n_2)}, \]
\text{(95)}

where the scalar coefficient function is given by:
\[ B_1^{(n_1, n_2)} = \frac{1}{2p^2} \left( B_0^{(n_1-1, n_2)} - B_0^{(n_1, n_2-1)} - f B_0^{(n_1, n_2)} \right), \]
\text{(96)}

where \(f \equiv p^2 + m_2^2 - m_1^2\). In case of single dominator factor \(n_i = 1\), one should use the related one-point function in the equation above. For example:
covariant quantities that are external momentum $p_\mu$ and metric $\eta_{\mu\nu}$:

$$ B_{\mu\nu}^{(n_1, n_2)} = \eta_{\mu\nu} B_{00}^{(n_1, n_2)} + p_\mu p_\nu B_{11}^{(n_1, n_2)}, $$

and the scalar coefficients read:

$$ B_{00}^{(n_1, n_2)} = \frac{1}{3} \left[ B_0^{(n_1, n_2-1)} + m_2^2 B_0^{(n_1, n_2)} \right], $$

$$ B_{11}^{(n_1, n_2)} = \frac{1}{3} \left[ B_0^{(n_1, n_2-1)} + m_2^2 B_0^{(n_1, n_2)} \right] + 2 \left( B_1^{(n_1, n_2-1)} + f B_1^{(n_1, n_2)} \right). $$

**Appendix C: Renormalisable couplings**

In this section, we revisit the computation of the electroweak dark matter–nucleon scattering cross-section through the usual renormalisable couplings. This allows us to calibrate our results with previous publication before studying the higher-dimensional operator extension of the electroweak theory of dark matter.

The SI effective interactions of EWDM with nucleon arising from a UV Lagrangian which only contains renormalisable operators, in leading order, can be written as:

$$ \mathcal{L}_R = \sum_{q=d} b \left( m_q C_q \bar{x}^0 x^0 \bar{u}^i u^i + \frac{C_{q1}}{M_X} \bar{x}^0 i \partial_\mu \gamma_\mu x^0 O_{\mu\nu}^{0q} + \frac{C_{q2}}{M_X} \bar{x}^0 i \partial_\mu i \partial_\nu x^0 O_{\mu\nu}^{0q} \right) + \frac{\alpha_s}{\pi} C_q \bar{x}^0 x^0 G_{\mu\nu}^a G^{\mu\nu a}, $$

where the Lagrangian is decomposed into a term representing interactions with quarks in the first line, and another term in the second line for coupling to gluon $[83]$. The order of loop momenta in the scattering diagrams of the renormalisable theory is again around the weak scale $[105]$. This agrees with the similar observation in case of the non-renormalisable extension where we took the factorisation scale at $\mu_{uv} \approx m_z$.

In addition to the scalar interactions, the renormalisable Lagrangian (101) apparently includes terms that couple to the quark twist-2 operators. The matrix element of twist operators are defined through the parton-distribution functions (PDF’s) of quarks and anti-quarks. PDF $q^{(N)}(x)$ expresses the probability for a given parton species $q$ to carry a portion $x$, called longitudinal fraction, of the total momentum of hadron $N$. Due to the symmetries linking proton and neutron, the distribution functions of neutron can be obtained from those of proton by interchanging the up and down quarks, e.g. $u^{(n)} = d^{(p)}$ etc.

The $n$th moment of the PDF can be defined as:

$$ q_n^{(N)} \equiv \int_0^1 x^{n-1} q^{(N)}(x) \, dx. $$

The probability distributions are normalised so that the first moments $q_1^{(N)}$ give the net number of the partons in the hadron:

$$ \int_0^1 (q^{(N)}(x) - \bar{q}^{(N)}(x)) \, dx = #_q. $$

The second moments $q_2^{(N)}$ return the averaged longitudinal fraction:

$$ \int_0^1 x (q^{(N)}(x) + \bar{q}^{(N)}(x)) \, dx = \langle x \rangle_q. $$

One can expand the operator product of two quark currents in the deep inelastic scattering process $e + N \to e + X$, where $X$ is a generic hadronic final state. The leading terms in OPE is given by twist-2 operators, and higher twist operators get suppressed by factors of inverse momentum transfer. By equating the dispersion integrals with contours lying in physical and unphysical regions of $1/x$ complex plane, we drive $[106, 107]$:}

$$ \langle O_q^{\mu_1 \ldots \mu_n} \rangle = \frac{1}{m_N} \left( p_{\mu_1} \ldots p_{\mu_n} - \text{trace} \right) \left( q_n^{(N)} + (-1)^n \bar{q}_n^{(N)} \right). $$

17 It should be noted that suppression by factors of $M_Z^{-1}$ in the twist-two coupling in (101), will be cancelled out by the derivative of the DM field when computing the dark matter matrix element. So, the effective amplitude arising from twist-2 operators can have the same order of magnitude as those for scalar operators.

18 The twist-2 operators of gluon does not contribute in leading order of $O(a_s^0)$ [104].
These equations are termed as **moment sum rules**, and relate the moments of the distribution functions to the matrix element of twist-2 operators. By including the radiative correction through operator rescaling, it can be shown that moments of PDF, and thus matrix elements of twist-2 operator are logarithmically scale dependent.\(^{19}\)

In the theory of EWDM, we are especially interested in the matrix element of spin-2 twist-2 operator which is obtained from the second moments of quark and anti-quark distributions:

\[
\langle \mathcal{O}_{\mu\nu}^a \rangle = \frac{1}{m_N} \left( \rho_\mu \rho_\nu - \frac{1}{4} m_N^2 \eta_{\mu\nu} \right) \left( \bar{q}_2^{(N)} + \bar{q}_2^{(N)} \right). \tag{106}
\]

The PDF’s used in calculation of the matrix element are available at different scales, so we only need to decide about the appropriate energy level to evaluate it. It turns out that due to asymptotic freedom, in lower energy regions the uncertainty arising from the perturbative expansion in \(\alpha_s\) increases [111]. Therefore, we evaluate the matrix element at the factorisation scale \(\mu_{fv} = m_z\).

We use the second moments of PDF’s for proton provided by [103] evaluated at \(\mu_{fv} = m_z\) which are presented in Table 2. It can be seen that 2nd moments of valence quarks are an order of magnitude larger than those of the sea quarks.

### C.1 Wilson coefficients

The Wilson Coefficients and mass functions have been actually computed by several research groups, but for some reason there are some discrepancies in the literature. We review the calculations to assess the accuracy of the published works, and present our results in this section for comparison.\(^{20}\)

| Table 2 | Second moments of the PDF’s for proton computed at \(\mu_{fv} = m_z\) [103]. The digits in parentheses represent the statistical uncertainty |
|---------|---------------------------------------------------------------|
| \(a_2^{(p)}\) | 0.118(3) |
| \(a_2^{(p)}\) | 0.223(3) |
| \(s_2^{(p)}\) | 0.0258(4) |
| \(c_2^{(p)}\) | 0.0187(2) |
| \(b_2^{(p)}\) | 0.0117(1) |

In contrast to the scalar bilinears, the twist operators are scale dependant even at leading order of \(\alpha_s\) [115]. However, there is no need to re-evaluate the twist-2 coefficients at low energy region. Since, as discussed, we adopted PDF’s at UV scale \(\mu_{fv} = m_z\) where the effective Lagrangian is matched with the full theory. This choice would also avoid the errors such as quark mass threshold, resulting from evolving down the Wilson coefficients.

#### C.1.1 EWDM–quark scattering

Since no tree-level interactions are allowed via renormalisable couplings to gauge bosons, dark matter scatters off quarks at one loop level. As shown in Fig. 10, this happens through Higgs-exchange Penguin (10a) and Box (10b) diagrams [113].\(^{21}\)

The effective couplings are model dependent, and for a general n-tuplet with hyper-charge \(Y\) are given by:

\[
C_q^i = \frac{\alpha^2_w}{4m_w^2} \left[ \frac{1}{8m_w^4} (n^2 - 4y^2 - 1) g_h(w) + \frac{1}{2\alpha_w^4m_w^2} g_h(z) \right] + \frac{3\alpha_w^2}{c_w^4m_w^2} y^2(c_q^2 - c_{q_i}^2) g_{Box}(z), \tag{107a}
\]

\[
C_{q_i}^i = \frac{\alpha^2_w}{8m_w^4} (n^2 - 4y^2 - 1) g_h(w) + \frac{\alpha_w^2}{c_w^4m_w^2} y^2(c_q^2 + c_{q_i}^2) g_h(z), i = 1, 2, \tag{107b}
\]

where \(w \equiv (m_w/M)_2\) and \(z \equiv (m_z/M)_2\). Quark-Z boson vector and axial-vector couplings are \(c_{q_i}^i \equiv T_q^{(3)} - 2Q_q s_w^2\) and \(c_q^i = T_q^{(3)}\) respectively.

Footnote 20 continued

\(^{19}\) This result obtained from operator renormalisation analysis agrees with Dokshitzer–Gribov–Lipatov–Altarelli–Parisi (DGLAP) equations for evolution of the parton splitting functions [108–110].

\(^{20}\) Unless explicitly specified otherwise, our calculations agrees with the [112] results, including the factor of 2 correction for Z-boson contribution in [104]. More explicitly, we reached to a faintly different Higgs mass function \(g_h\) as explained in the footnote 22.

Regard [113], in the large DM mass limit \(m_v/M_\gamma \to 0\), both scalar and twist-2 quark coefficients give the same results as our calculation, although the mass functions \(f_{0/1}\) and \(f_{0/1}\) are formulated in a slightly different form. About gluon contribution, the 2-loop scattering diagrams are presented, but not evaluated explicitly. All the Wilson coefficients in their work has the same sign, therefore the accidental cancellation between different terms (c.f. Sect. 1 for details) does not occur. The higher-dimensional operator is explained, however it is not taken into account in computation of the scattering amplitude.

About Ref. [114], the scalar quark \(c_{1/2}^{(0)}\) and twist-2 \(c_{1/2}^{(2)}\) Wilson coefficients lead to the same outcome as our results, in the limit of heavy dark matter particle \(m_v/M_\gamma \to 0\), whilst the evaluated integrals take a different form. This is also true of gluon coefficients \(c_{q_i}^i\) except the top flavour contribution to the Z-bosons mediated interactions. It is the very last term in expression for \(c_{q_i}^i\) and does not match our calculations of \(c_q^i\) in the same limit.

As to [18], about the scalar operator, although the coefficient induced by the Box diagram is different, our results matches for the Higgs-mediated penguin diagram, up to an overall constant. The expression for the twist-2 operator is also different and has an opposite sign. The contribution from the gluon operator was not taken into account in this reference.

\(^{21}\) There are also other penguin diagrams with Z and γ mediators. It turns out that the amplitude for these diagrams vanish, and therefore they are not shown in the figure.
The terms proportional to $g_H$ in (107a) are generated by the penguin diagram Fig. 10a mediated by Higgs boson, and the rest of the terms in equations (107) are induced by the box diagrams Fig. 10b.

Scalar Higgs $g_h$, box $g_{box}$, twist-1 $g_{t1}$ and twist-2 $g_{t2}$ mass functions are defined as:

\[ g_h(x) = 2\sqrt{x}(1 - \ln x) \]
\[ - \frac{4}{\sqrt{4-x}} (3 - x) \text{Arctg} \sqrt{\frac{4}{x} - 1}, \quad (108a) \]
\[ g_{box}(x) = \frac{\sqrt{x}}{24} (2 - x \ln x) \]
\[ + \frac{1}{12 \sqrt{4-x}} (4 - 2x + x^2) \text{Arctg} \sqrt{\frac{4}{x} - 1}, \quad (108b) \]
\[ g_{t1}(x) = \frac{\sqrt{x}}{12} \left[ 1 - 2x - x(2 - x) \ln x \right] \]
\[ + \frac{1}{6} \sqrt{4-x} \left( 2 + x^2 \right) \text{Arctg} \sqrt{\frac{4}{x} - 1}, \quad (108c) \]
\[ g_{t2}(x) = -\frac{\sqrt{x}}{4} \left[ 1 - 2x - x(2 - x) \ln x \right] \]
\[ + \frac{x}{2 \sqrt{4-x}} \left( 2 - 4x + x^2 \right) \text{Arctg} \sqrt{\frac{4}{x} - 1}. \quad (108d) \]

In the limit of large dark matter mass $x \rightarrow 0$, the mass functions take the following values:

\[ g_h(x) \rightarrow -3\pi, \quad (109a) \]
\[ g_{box}(x) \rightarrow \frac{\pi}{12}, \quad (109b) \]
\[ g_{t1}(x) \rightarrow \frac{\pi}{3}, \quad (109c) \]
\[ g_{t2}(x) \rightarrow 0. \quad (109d) \]

Consequently, even if dark matter is considerably heavier than electroweak gauge bosons $M_\chi \ll m_w$, the quark Wilson coefficients except $C_{12}^\chi$ are not suppressed to leading order $O(\alpha^2)$.

### C.1.2 EWDM–gluon scattering

Vector fields cannot mediate the Penguin diagram of Fig. 5a, so at the level of renormalisable couplings, EWDM will only interact with gluons through two-loop processes.

Figure 11, presents the Feynman diagrams that yield the effective interactions of EWDM with gluons. The double-triangle diagram Fig. 11a obviously only gives rise to long-distance contributions as there is no other scale than mass of quarks circulating in the quark triangle integral. The same holds for double-box diagrams Fig. 11c where only one quark propagator emits the two gluon fields. That is because, the external momentum which is predominantly of order of the weak gauge boson mass cannot contribute to the quark box loop more than what virtual quark masses do. In contrast, the reaction Fig. 11b where each quark propagator is attached with one gluon field, yields short distance contribution. In this case, the mass of gauge bosons as the external momentum dominates the quark loop integral.

At first step, we need to evaluate the loop integral attached with gluon fields, in order to derive the polarisation functions of the neutral and charged week bosons in the gluon background. Due to current conservation, the longitudinal part of the two-point functions do not contribute [116]. Finally, the transversal polarisation function is used to compute the second loop, and thus to find the Wilson coefficient for the

---

\[ ^2 \text{The Higgs mass function } g_h \text{ is slightly different from } g_H \text{ in [112]. It originates from a sign difference in the expression for reduction of the vector integral to scalar correlation functions.} \]

\[ 2M_\chi^2 B_1^{(1,2)}(m_q, m_q) = A_0^{(0)}(m_q) - B_0 \pm \left( 2M_\gamma^2 - m_\gamma^2 \right) B_0^{(1,2)}, \]

where we used minus sign for the last term. In the large DM mass limit $M_\gamma \gg m_\gamma$, there is a discrepancy of a factor of a few, so the final results are not significantly affected.
C\text{gluon scalar operator which is given by:}
\begin{align*}
C_\text{g}^s &= -\frac{\alpha_s^2}{48 m_h^2} \left[ \frac{1}{8m_w} (n^2 - 4y^2 - 1) g_\text{h}(w) + \frac{1}{2c_w^4 m_w} y^2 g_\text{h}(z) \right] \\
&\quad + \frac{\alpha_s^2}{4} \left[ \frac{1}{8m_w} (n^2 - 4y^2 - 1) \right] 2g_\text{box}(w) + g_t(w, t) \\
&\quad + \frac{1}{2c_w^4 m_w^2} y^2 \sum_{q=u} h_\text{q} \left[ c_q^2 + c_q^2 \right] g_\text{box}(z) \\
&\quad + c_l^2 (g_v + I_v) (z, t) + c_t^2 \left[ g_u + I_u \right] (z, t) \right], \quad (110)
\end{align*}

where \( t \equiv (m_t/M_A)^2 \), and mass of all flavours except top quark has been ignored.

The first line of the expression (110) for the gluonic coefficient is induced by the two-triangle diagram Fig. 11a. It can be verified that this long distance contribution arising from the top quark loop is related to the effective coupling with the top flavour in diagram Fig. 10a through \( C_\text{g}^s = -C_l^l/12. \) The last two lines come from the two double-box diagrams Fig. 11b and c. In each case, the first term (the 2nd line) is mediated by W bosons, but the second one (3rd line) is generated by the Z-boson exchanges. For W-boson mediated interactions (2nd line) the first term comes from the 1st and 2nd generations, whereas the mass function \( g_t \) in the second term formulates the contribution from the 3rd generation of quarks:
\begin{align*}
g_t(x, y) &= -\frac{x^{3/2}}{12(y - x)} - \frac{x^{3/2} y^2}{24(y - x)^2} \ln y \\
&\quad + \frac{x^{5/2} (y - x)}{24(y - x)^2} \ln x \\
&\quad + \frac{x^{3/2} \sqrt{y} (2 + y) \sqrt{4 - y}}{12(y - x)^2} \frac{\ln 4 - x}{x} \\
&\quad + \frac{\ln 4 - x}{x} \right]. \quad (111)
\end{align*}

Concerning the Z-boson exchange reactions (3rd line), the first term is generated by all quarks lighter than top flavour. The contribution from top quark is further decomposed into vector mass functions with analytical and non-analytical form of:
\begin{align*}
g_v(x, y) &= -\sqrt{x} \left[ \frac{4y^2 - xy + x^2}{12(4y - x)^2} \right] + \frac{x^{3/2} (48y^3 - 20xy^2 + 12x^2 y - x^3)}{24(4y - x)^3} \ln x + \frac{x^{3/2} y^2 (4y - 7x)}{6(4y - x)^3} \ln 4y \\
&\quad - \frac{x^{3/2} \sqrt{y} (16y^3 - 4(2 + 7x) y^2 + 14(2 + x) y + 5x)}{12(4y - x)^3} \sqrt{y} \ln y \\
&\quad - \frac{48(x^2 - 2x + 4) y^3 - 4x (5x^2 - 10x + 44) y^2 + 12x^3 (x - 2) y - x^3 (x^2 - 2x + 4)}{12(4y - x)^3} \sqrt{4 - x} \ln \frac{4 - x}{x}. \quad (112a)
\end{align*}

\begin{align*}
I_v(x, y) &= -x^{3/2} \int_0^\infty \frac{I \left( \frac{2 - I}{2} \right)}{l (l + x)^2 (l + 4y)^{5/2}} \ln \frac{\sqrt{4x} + \sqrt{I}}{2 \sqrt{x}} \, dl, \quad (112b)
\end{align*}
and axial vector analytical and non-analytical mass functions:

\[ g_a(x, y) = \frac{\sqrt{x}(2y - x)}{4(4y - x)} + \frac{x^{3/2}}{2(4y - x)^2} \ln x \]

\[ -\frac{x^{3/2}y^2}{4(4y - x)^2} \ln 4y - \frac{x^{3/2}y}{4(4y - x)^2} \sqrt{y(2y - y - 1)} \times \arctan \sqrt{1 - y} \]

\[ -\frac{8x}{4(4y - x)^2} \sqrt{4 - x} \times \arctan \sqrt{\frac{4 - x}{x}}. \] (112c)

\[ I_a(x, y) = x^{3/2}y^2 \int_0^\infty \frac{(l + 4y)(2 - l)\sqrt{l + 4 + l\sqrt{l}}}{l (l + x)^2 (l + 4y)^{5/2}} \times \ln \frac{\sqrt{l + 4 + \sqrt{l}}}{2\sqrt{x}} dl. \] (112d)

In the high DM mass limit \( x, y \to 0 \), the mass functions reduce to:

\[ g_t(x, y) \to \frac{x}{12(1 + r^2)^2}. \] (113a)

\[ g_s(x, y) \to \frac{x}{24} \frac{1}{(1 - 4r^2)^3} \left(2 + 5r + 28r^3 - 88r^4 + 96r^6\right). \] (113b)

\[ I_t(x, y) \to -0.19. \] (113c)

\[ g_s(x, y) \to \frac{x}{4} \frac{1 - 2r - 2r^2 + 8r^4}{(1 - 4r^2)^2}. \] (113d)

\[ I_s(x, y) \to 0.36. \] (113e)

with \( r = \sqrt{x/y} \). Therefore, the Wilson coefficient for gluon operator will not be suppressed, in the limit of large dark matter mass.

C.2 Cross-section

The effective amplitude for EWDM–nucleon scattering is obtained form the S-matrix element of the scalar and twist interactions induced by \( \mathcal{L}_R \) in the non-relativistic limit:

\[ f^R_N = \langle \mathcal{L}_R \rangle = m^2 \left[ \sum_{q = d, u, s} C^t_q f_{tq} + \frac{3}{4} \sum_{q = d} \left( \bar{q}^{(N)}_2 + \bar{q}^{(N)}_\nu \right) \right] \times (C^u_q + C^s_q) - \frac{8}{9} g_s f_{tq}. \] (114)

Notice the difference in the number of active flavours for scalar and twist-2 quark contributions as the associated Wilson coefficients \( C^u_q \) and \( C^s_q \) are evaluated at different scales of \( \mu_{\text{had}} \) and \( \mu_{\text{uv}} \) respectively.

The spin-independent scattering cross-section with nucleon can be obtained from the effective amplitude:

\[ \sigma_N = \frac{4}{3} m^2 \chi^2 \left| f^R_N \right|^2. \] (115)

As discussed in the previous sections, the Wilson coefficients generated by renormalisable couplings depend on EWDM mass through the mass functions. However, when dark matter is much heavier than the electroweak gauge mediators \( M_Y \ll m_w \), these effective coefficients become independent of \( M_Y \). As a result, the spin-independent scattering cross-section will not be sensitive to EWDM mass.

It is observed that the effective coefficients generated by twist-2 operators are positive whereas other coefficients are negative. Additionally, their amplitudes are comparable with each other. This leads to an accidental cancellation among different contributions which reduces the scattering cross-section by an order of magnitude. As a consequence, the SI cross-section remains below the current direct detection bounds [85].

The complex doublet model has the lowest dimension of representation and thus receives the smallest W-boson contribution due to the factor of \( \frac{1}{2} [n^2 - (4y^2 + 1)] \). In addition, C2 multiplet lacks those diagrams that include the light charged dark propagator \( \chi_1^+ \) in the loops. As a result, the effective coupling significantly reduces, and the spectrum becomes more dependent on DM mass, especially below TeV scale. The scattering cross-section for the pseudo-real model, therefore stays well below the neutrino floor.

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