Research Article

Feng Qiu, Junxia Qiu*, Heng Feng, Huajie Wang, Hongliang Qian, Xiaofei Jin, Kaiyuan Wang, and Feng Fan

Calculation method of stability bearing capacity of transmission tower angle steel considering semi-rigid constraint

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Abstract: The angle steel member is the most commonly used component form of the transmission tower structure. Considering its connection characteristics, we must deal with its stability analysis under semi-rigid constraint conditions for the proper study of the overall structure’s mechanical performance. Therefore, in order to establish a simple and high-precision method suitable for the ultimate bearing capacity analysis of the transmission tower, we build the refined finite element models of typical steel tower joints, analyze its moment-rotation curve and utilize simulation technique of spring elements to acquire the calculation method of its single angle stability bearing capacity, which is considering initial imperfection and residual stress. Furthermore, we analyze its bearing capacity under different constraint conditions such as rigid, semi-rigid and articulated connection. The results show that it is necessary to consider joint stiffness in the bearing capacity analyses. Finally, it’s confirmed that the calculation results of this method agree well with the experimental data, which validates its high accuracy. Therefore, the method provides technical support for high efficient component stability simulation in overall stability analyses of the transmission steel tower.

Keywords: single angle; stability bearing capacity; moment-rotation curve; semi-rigid joint; transmission tower

1 Introduction

As the lifeline engineering, the transmission tower is an important part of the power line system so its secure research has important significance. However, its complicated load state is very likely to cause structural damage and lead to the interruption of the power transmission, which will not only directly cause high economic losses, but also badly affect people’s normal life. The production activities are shown in Figure 1. Related engineering accidents [1–3] show that the failure mode of the transmission tower is generally caused by the instability of the angle steel members. Therefore, it is significant to study the ultimate bearing capacity of high-voltage transmission towers and improve the calculation accuracy of its ultimate bearing capacity from the angle of steel members to guarantee the normal operation of the power system.

At present, the calculation of the ultimate bearing capacity of the angle steel transmission tower mainly takes the joint as an ideal hinge or rigid connection [4–7]. Kennedy and Madugula [8] carried out an experimental study on 72 single-angle presser bars with rigid joint and articulated joint connections at the ends, in order to provide verification of the theoretical solutions for flexural, torsional-flexural, and plate buckling. Baton [9] and Liu [10] studied the compression test of a lot of equal-edge and non-equal-edge angle steel and the end connection was articulated by spherical hinge bearings, by which the ultimate compressive load capacity of single steel angles subjected to eccentrically applied axial load was investigated. Cao et al. [11] restrained the rotation of angle around
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In the actual project, the constraint action of the end of angle steel also has the characteristics of semi-rigid. To simulate the influence of the joint's actual stiffness accurately and restore the real constraint conditions of the end of angle steel, KeKe [14] and Kettler et al. [15] built the entity-unit models of bolts and gusset plate at the end of angles, which investigated the shear lag effect on the behavior and ultimate tensile capacity of high strength steel (HSS) tension angles with bolted and welded connections [14], and focused on the presentation of the influence of realistic end support conditions at the gusset plates on the member capacity in compression [15]. But in this method, each entity-unit model of angle with different slenderness ratio should be built, which leads to huge calculation expense, and the method using entity-unit model can't be used in whole tower structure analyses. So, it's necessary to establish a simple and high-precision method of angle capacity calculation, which is also suitable for ultimate bearing capacity analyses of transmission tower.

In this paper, the moment-rotation curve of the angle joint is gained by building a refined finite element model of a typical steel tower joint. And the practical calculation method for establishing curves is proposed with the Kish-Chen power model [15] which expressed the general quantitative relationship between bending moment and rotation of semi-rigid connections. The accuracy of the method is verified by the numerical simulation. Then based on the moment-rotation curve obtained by the calculation method, spring elements are applied to both ends of angle members to simulate the semi-rigid constraint form joints, the extreme point of the load-displacement curve is taken as the stable bearing capacity. In this way, the calculation method of the single angle stability bearing capacity is presented and its accuracy is verified by related experimental data, which provides technical support for the refined simulation calculation of the overall ultimate bearing capacity of the transmission tower structure.
2 Calculation method for moment-rotation curve of joint

The overall instability mode of a single angle is complex. The principal axes of the angle steel section (z axis and w axis in Figure 3) are not parallel with the connected leg, by which the load is transferred. Temple and Sakla examine the load-carrying capacity and behavior of a single-angle compression member welded [16]. The angle steel is under biaxial bending and compression, and the buckling axis is the u axis which is located between x axis and y axis. Thus, the constraint forms of rotation around x axis and y axis of angle both have some influences on bearing capacity. In practical structures, the direction around x axis is outside the plane of the gusset plate, of which the flexural rigidity in this direction is weak, so the rotation in this direction can be treated as free. The end constraint of angles around y axis is mainly researched in this paper, and the hinged connection and rigid connection are both relative to this direction.

Figure 3: Instability axes of single angle

2.1 The finite element model of angle joint

As shown in Figure 4, a typical angle joint of 800v high voltage transmission tower is selected as an example model. The section of secondary angle is 45 × 5mm, and that of main angle is 75 × 6mm. The thickness of gusset plate is 6mm. Secondary angles are connected to the main angle by two M16 bolts.

The refined finite element model of angle joint is built by ANSYS. Angles, gusset plates and bolts are established by SOLID 185, which is often used to construct the three-dimensional solid structure. CONTA174 and TARGE 170 are used to simulate the contact surface and target surface when the components contact. The material of the secondary angle is Q235 and its Poisson’s ratio is 0.3. The main angle and the gusset plate are made of Q420 steel. Bolts in the model are 9.9 high-strength bolts. The bilinear kinematic constitutive model is adopted for all materials, the elastic modulus is 206000 Mpa, and the tangent modulus of the hardening section is 2% of the elastic modulus, which is 412 Mpa. The main angle is restrained on two ends. Uniform surface load q is applied to the top end of the left secondary angle so that the rotational stiffness of the joint could be studied. The bending moment of joint (M_y) is shown as Eq. (1) and Figure 5.

\[ M_y = qA \times L \]  

Where A is the sectional area of secondary angle, L is the distance between top end of the angle and center of lower bolt.

Figure 4: Typical angle joint of transmission tower

Figure 5: Finite element model of angle joint
2.2 Analysis of the semi-rigid performance of angle joint

The deformation graph of the joint model is shown in Figure 6. The secondary angle is bent by the loads. The maximum displacement occurred at the end of the angle, and it reached 4.3cm. From Figure 7, it is seen that the part of the angle is constrained by bolts rotated, which indicates the semi-rigid constraint of secondary angle from the angle joint. The von Mises stress contour of the left secondary angle is shown in Figure 8. Almost all the sections where bolt holes were located had been plastic areas, which lead to the disruption of the model.

As shown in Figure 9, line AB is selected as the reference line to calculate the rotation of the secondary angle. Point A’ and B’ are displaced to joint positions of point A and B. Line AB is translated to be line CB’, and then CA’ is the relative displacement from point A to B. Then the rotation angle $\theta$ can be calculated with Eq. (2).

$$
\theta = \frac{A'C}{A'B'} = \frac{\sqrt{(z_B - z_A)^2 + (x_B - x_A)^2}}{L_{AB}}
$$

Where $L_{AB}$ is the distance from point A to B, $z_A$ and $z_B$ are the $z$-direction displacement of A and B, $x_A$ and $x_B$ are the $x$-directional displacement of A and B.

The moment-rotation curve of the angle joint shown in Figure 10 is established based on $\theta$ and $M_y$. The slope of the curve at the origin is the initial rotation stiffness of the joint ($K$), which is calculated to be 259.17 kN·m/rad. And the ultimate moment ($M_u$) is 1.32 kN·m.
2.3 The influences of axial force and end constraint on moment-rotation curve

The bending moment is applied to the second angle when the semi rigidity is analyzed, and the top end of the right angle is completely free. In actual structures, the angle members mainly suffer from the axial force, and the right angle is restrained by the joint. In order to prove the correctness of the moment-rotation curve, the influences of the axial force and constraint in the right angle on moment-rotation curve are studied.

Four contrasting joint models are built based on the example model. Axial forces sized 20kN, 40kN and 60kN are applied to secondary angles of models 1-3. The right secondary angle of model 4 is restrained, and it can just deform axially. The calculation results of the initial rotation stiffness and the ultimate moment of contrasting models are shown in Table 1. With the increasing axial force of the secondary angle, the initial rotational stiffness decreases slightly. And the maximum decreasing amplitude is just 2.3%. The increased amplitude from the constraint of the right secondary angle is less than 1%. All of the five models’ ultimate moments are 1.32 kN·m. Based on the analysis above, it’s seen that the influences of the axial force and constraint in the right angle can be ignored when the moment-rotation curve is established.

2.4 Simplified calculation method for establishing moment-rotation curve

Since it’s complicated to gain the moment-rotation curve by building refined finite element model of the angle joint, the simplified calculation method of establishing the curve is of good value. Based on Kishi-Chen’s research, the Kishi-Chen power function model [17] is selected as the basic model to become the simplified calculation, and calculation equations of the model are shown in Eq. (3) and Eq. (4).

\[ M = \frac{K_i \theta}{1 + \left(\frac{\theta}{\theta_0}\right)^n}^{1/n} \quad (3) \]

\[ \theta_0 = \frac{M_u}{K_i} \quad (4) \]

Where \( K_i \) is the initial rotation stiffness of the joint, \( M_u \) is the ultimate moment, and \( n \) is the shape parameter of the moment-rotation curve.

With the Kish-Chen power model, the moment-rotation curve of angle joint can be established by \( K_i, M_u \) and \( n \), so it’s necessary to determine the relationships between these three parameters and the joint model parameters. The width and thickness of the secondary angle \( (b_1 \text{ and } t_1) \), the thickness of gusset plate \( (t) \), the width and thickness of the main angle \( (b_2 \text{ and } t_2) \), the steel strength of angle \( (f_y) \), the diameter, distance and number of bolts \( (d, s \text{ and } p) \) are chosen to be the model parameters, some of which are shown in Figure 11. In total 135 finite element models of angle joint are built and calculate \( K_i \), \( M_u \) of each model. The values of \( n \) of element models are fitted according to the moment-rotation curves with the least square methods. Parts of the results are shown in Table 2.

Zhao analyzed the influences of model parameters on \( K_i, M_u \) and \( n \), the parameters which have significant influences are selected to fit the formulas [18]. The form of fit formula is shown as Eq. (5).

\[ x = e^{k_1 b_1^{k_2 t_1^{k_3 d^{k_4 t_k^a b_2^{k_5 b_2^{k_7 f_y^{k_8 s^{k_9 p^{k_{10}}}}}}}}}} \quad (5) \]

| Models            | Axial force/kN | If the right angle is restrained | Initial rotational stiffness | Ultimate moment |
|-------------------|----------------|----------------------------------|-----------------------------|-----------------|
| Example model     | 0              | No                               | 259.17                      | 1.32            |
| Contrast model 2  | 20             | No                               | 257.77                      | 1.32            |
| Contrast model 3  | 40             | No                               | 255.83                      | 1.32            |
| Contrast model 4  | 60             | No                               | 253.28                      | 1.32            |
| Contrast model 5  | 0              | Yes                              | 261.00                      | 1.32            |
Table 2: Parameters and calculation results of angle joint models

| Number | $b_1$/mm | $t_1$/mm | $t$/mm | $p$ | $d$/mm | $s$/mm | $b_2$/mm | $t_2$/mm | $f_y$/MPa | $K_i$/kN-m/rad | $M_u$/kN-m | $n$ |
|--------|----------|----------|--------|-----|--------|--------|----------|----------|---------|----------------|------------|------|
| 1      | 50       | 4        | 6      | 2   | 16     | 70     | 75       | 6        | 235     | 267.18         | 1.20       | 4.615|
| 2      | 50       | 5        | 6      | 2   | 16     | 70     | 75       | 6        | 235     | 305.49         | 1.56       | 3.700|
| 3      | 50       | 6        | 6      | 2   | 16     | 70     | 75       | 6        | 235     | 337.52         | 1.92       | 3.194|
| 4      | 63       | 4        | 6      | 2   | 20     | 70     | 75       | 6        | 235     | 373.56         | 1.80       | 3.829|
| 5      | 63       | 5        | 6      | 2   | 20     | 70     | 75       | 6        | 235     | 423.13         | 2.40       | 3.071|
| ...    | ...      | ...      | ...    | ... | ...    | ...    | ...      | ...      | ...     | ...             | ...        | ...  |
| 131    | 90       | 8        | 8      | 3   | 16     | 70     | 140      | 12       | 235     | 1270.85        | 6.30       | 4.081|
| 132    | 90       | 8        | 8      | 3   | 24     | 70     | 140      | 12       | 235     | 1554.77        | 10.56      | 3.583|
| 133    | 100      | 10       | 8      | 3   | 16     | 70     | 140      | 12       | 235     | 1841.20        | 4.80       | 4.518|
| 134    | 100      | 10       | 8      | 3   | 24     | 70     | 140      | 12       | 235     | 1646.80        | 10.80      | 4.175|

Figure 11: Some of the model parameters

where $x$ is the parameter which need to be fitted, $e$ is the natural constant.

The fitting results of $K_i$, $M_u$ and $n$ are shown as Eqs. (6)–(8).

$$K = e^{-4.702540}b_1^{1.362327}t_1^{-0.391488}d^{0.233223}$$

(6)

$$M = e^{-14.199584}b_1^{1.709699}t_1^{0.786713}f_y^{0.788590}$$

(7)

$$n = e^{-0.846650}b_1^{0.150867}t_1^{-0.674903}d^{-0.162979}p^{0.134414}$$

(8)

Since the example model is not included in the 135 models, it can be used to verify the accuracy of these formulas. $K$ and $M_u$ of the example model is calculated by Eq. (6) and Eq. (7). The value of $K$ is 247.81 kN-m/rad, and the error between the formula result and the numerical simulation result is 4.4%. $M_u$ is calculated to be 1.30, and the error is 1.5%. The value of $n$ is calculated by Eq. (8). The moment-rotation curve of the example model is established with the results of formulas, and it is compared with the numerical simulation result, which is shown in Figure 12. It is seen that the curve by formula results agrees well with the numerical one. The average error of the data is 5.04%, and the accuracy of these equations is high. So the moment-rotation curve can be established directly by Eq. (3) to Eq. (8) with model parameters, which is much easier than the way of a building refined finite element model.
3 Calculation method of single angle bearing capacity

In order to improve computational efficiency, the single model is built by SHELL 181 (a kind of element which only needs to input thickness and does not need to generate solid for thin-walled structure) instead of SOLID 185. And COMBIN 39 (a one-way element with nonlinear function, which can input generalized force deformation curve) is applied to both ends of the angle steel model according to the moment-rotation curve, which is used to simulate the semi-rigid constraint.

The simplified model of single is shown in Figure 13. The restraint rotation around y axis is applied to nodes on the lines AB and DE, and the rotational stiffness of springs is defined with the moment-rotation curve of the joint. The displacement in y direction of the nodes in the middle of lines AB and DE is restrained, and the displacement in x direction of points B and E is restrained. The displacement constraint in z direction is applied to the nodes in the middle section of the angle steel.

The influence of initial imperfection and residual stress is considered when the bearing capacity is calculated. The first mode of eigenvalue analysis is selected as the shape of initial disfigurement, and the maximum lateral displacement of a single is changed to be 1/1000 of the single length by adjusting node coordinates. Zhou et al. established a distribution model of residual stress which is shown in Figure 14 [19]. And the uniform load $p$ is applied to the nodes on line AB and DE.

The section of example Q235 single angle model is 45 $\times$ 5mm (the material properties of the finite element model are shown in Section 2.1), and the slender ratio of the weak axis ($\lambda_z$) is 170. The contour graph of Von Mises stress when the single is damaged is shown in Figure 15. And the load-y directional displacement curve of the central node of the angle is shown in Figure 16. The extreme point of the curve is taken as the stable bearing capacity. When the load exceeds this point, the following curve will show a downward trend. This shows that the load corresponding to the extreme point is the maximum load that the component can withstand. If the component wants to maintain balance, it needs to reduce the end load, otherwise, the component will be in the unstable equilibrium state. It is seen that the single bucked overall when the load came up to 31.83kN. The maximum displacement in the y direction of the node is 16mm, and the maximum stress reached 300 MPa.
4 Accuracy verification of the method

Zhou et al. summed up the restraint conditions of the double-bolt connection end from a series of experiments about single angles by the Beijing Electric Power Research Institute [19]. The end of test angles was connected with the gusset plate by two M16 bolts, which is similar to the example single in this paper. And the end constraint in the test is shown in Figure 17. The capacities of the test angles are calculated with the method which is proposed in this paper, and the results are shown in Table 3. It’s seen that the calculation results agree well with test data. The maximum error is 10.3%, and the average one is 6.4%. Therefore, this method has high precision.

5 Influence of joint stiffness on bearing capacity

In order to study the influence of the joint stiffness on the bearing capacity, the bearing capacities of a single angle under different constraint conditions such as rigid connection, connection and hinged connection are calculated. The example model is the angle with a semi-rigid connection whose schematic diagram is shown in Figure 13. The schematic diagram of a single angle model with a rigid connection is shown in Figure 18(a). All the freedom degrees except rotation around x axis of nodes on line AB are restrained. And the nodes on the line DE can just move in z direction. The uniform load is applied on line DE. The schematic diagram of a single angle model with a hinged connection is shown in Figure 18(b). The constraint conditions of the model are similar to that of the example model, except that there are no string elements at the ends of the angle.

Table 3: Comparison between calculation results of presented method and experimental data

| Number | Section | Slenderness ratio $\lambda_2$ | Yield strength $f_y$/MPa | Test results $N_1$/kN | Calculation results with method proposed in this paper $N_2$/kN | $\frac{N_2 - N_1}{N_1} \times 100\%$ |
|--------|---------|-------------------------------|--------------------------|-----------------------|------------------------------------------------|----------------------------------|
| 1      | $\angle 45 \times 5$ | 203                           | 280                      | 27.3                  | 26.4                                          | 3.2%                             |
| 2      | $\angle 45 \times 5$ | 170                           | 280                      | 27.3                  | 26.4                                          | 3.2%                             |
| 3      | $\angle 56 \times 5$ | 136                           | 280                      | 59.5                  | 53.4                                          | 10.3%                            |
| 4      | $\angle 63 \times 5$ | 120                           | 280                      | 66.2                  | 70.3                                          | 6.2%                             |
| 5      | $\angle 45 \times 5$ | 101                           | 280                      | 56.4                  | 53.0                                          | 6.0%                             |
| 6      | $\angle 56 \times 5$ | 90                            | 280                      | 77.6                  | 73.9                                          | 4.5%                             |
| 7      | $\angle 45 \times 5$ | 84                            | 280                      | 66.0                  | 59.6                                          | 9.7%                             |
| 8      | $\angle 56 \times 5$ | 67                            | 280                      | 79.3                  | 73.2                                          | 7.7%                             |
The bearing capacities of the singles whose sections are $45 \times 5\text{mm}$ under three constraint conditions are calculated, and the results are shown in Figure 19. The capacity of angle with semi-rigid constraint is between that of rigid angle and hinged angle. The capacity of rigid angle is 16.7% higher than that of semi-rigid constraint angle on average. And the capacity of the semi-rigid constraint angle is 14.7% higher than that of the hinged one. So, it is necessary to take into account joint stiffness in single angle bearing capacity analysis.

### 6 Conclusions

The calculation method of single angle bearing capacity is studied in this paper, and the conclusions are as follows:

The refined finite element model of angle joint is built, and the semi rigidity of the joint is analyzed. The practical formulas of establishing the moment-rotation curve of the angle joint are fitted. The curve can be gained with these equations instead of building a finite element model, which can improve computational efficiency.

According to the moment-rotation curve, the calculation method of single angle stability bearing capacity considering initial imperfection and residual stress is presented by utilizing the simulation technique of spring elements. And the accuracy of this method is verified with the related test data.

Based on the calculation results of the single angles with different constraint conditions, it is seen that the capacity of rigid angles is 16.7% higher than that of semi-rigid constraint angles on average. And the capacity of the semi-rigid constraint angles is 14.7% higher than the hinged one. So, it is necessary to take into account joint stiffness in single angle bearing capacity analysis.

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### Author contributions:

All authors have accepted responsibility for the entire content of this manuscript and approved its submission.

### Conflict of interest:

The authors state no conflict of interest.

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