How does the mass and activity history of the host star affect the population of low-mass planets?

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ABSTRACT
The evolution of the atmospheres of low and intermediate-mass planets is strongly connected to the physical properties of their host stars. The types and the past activities of planet-hosting stars can, therefore, affect the overall planetary population. In this paper, we perform a comparative study of sub-Neptune-like planets orbiting stars of different masses and different evolutionary histories. We discuss the general patterns of the evolved population as a function of parameters and environments of planets. As a model of the atmospheric evolution, we employ the own framework combining planetary evolution in MESA with the realistic prescription of the escape of hydrogen-dominated atmospheres. We find that the final populations look qualitatively similar in terms of the atmospheres survival around different stars, but qualitatively different, with this difference accentuated for planets orbiting more massive stars. We show that a planet has larger chances of keeping its primordial atmosphere in the habitable zone of a solar mass star compared to M or K dwarfs and if it starts the evolution having a relatively compact envelope. We also address the problem of the uncertain initial temperatures (luminosities) of planets and show that this issue is only of particular importance for planets exposed to extreme atmospheric mass-losses.

Key words: Hydrodynamics – Planets and satellites: atmospheres – Planets and satellites: physical evolution

1 INTRODUCTION
The evolution of planetary hydrogen-dominated atmospheres is controlled by the thermal evolution of the planet (e.g., Rogers & Seager 2010; Nettelmann et al. 2011; Miller & Fortney 2011; Valencia et al. 2013; Lopez, Fortney, & Miller 2012; Lopez & Fortney 2014), its atmospheric mass loss (e.g., Watson, Donahue, & Walker 1981; Lammer et al. 2003, 2016; Erkaev et al. 2007, 2016; Lecavelier des Etangs et al. 2004; Owen & Wu 2016; Salz et al. 2016; Kubyshkina et al. 2018a; Gupta & Schlichting 2019, 2020), and is strongly dependent on the stellar environment around the planet. This includes the heating by the host star and the amount of high-energy (X-ray and extreme ultraviolet, XUV) radiation received by the planet (e.g., Kubyshkina et al. 2018a). Both depend on the type of the host star (in particular, its mass and temperature) and the orbital distance of the planet. In addition, the XUV luminosity of the star of a given mass changes through time not univocally. Instead, the X-ray (and hence, XUV) luminosities of stars with similar masses in young clusters show a large spread up to about an order of magnitude due to the difference in the initial rotation rates of stars (e.g., Wright et al. 2011; Tu et al. 2015; Magaudda et al. 2020; Johnstone, Bartel, & Güdel 2020). This uncertainty in the stellar XUV holds up to about 1 Gyr, i.e., for the period when stars are most active and planets are hot and inflated after the dispersal of the protoplanetary disk, which is also when atmospheric escape is strongest (e.g., Owen & Wu 2016; Kubyshkina et al. 2018a; Gupta & Schlichting 2019, 2020). Thus simultaneous consideration of stellar and planetary evolution becomes important, and the population of evolved planets can look essentially different around stars of different types.

Another issue that may play a role in the atmospheric evolution is the initial state of a planet at the protoplanetary disk dispersal when atmospheric evaporation begins. Except for the parameters that can often be considered nearly unchanged after this moment (as the total mass of the planet and its orbit), other important initial parameters remain unknown, such as the initial atmospheric mass fraction and post-formation luminosity (hence, temperature) of a planet. Both of these parameters can be predicted by planetary formation and accretion models (e.g., Mizuno 1980; Mordasini et al. 2012; Mordasini, Marleau, & Morbidelli 2017; Leconte, Forget, & Lammer 2015; Morbidelli et al. 2009; Morbidelli & Raymond 2016; Morbidelli 2020). Their specific values for a given planet, however, depend on the formation scenario of a planet and on the properties of a protoplanetary disk, which are in general unknown, and on the assumptions of the specific model. This leads to large uncertainties in the prediction of some
initial planetary parameters (see, e.g., Ikoma & Hori 2012 for atmospheric masses and Mordasini, Marleau, & Mollière 2017 for initial luminosities).

In spite of this, certain initial parameters can be constrained by the present-day parameters of a planet. For the initial atmospheric mass fraction, this can be done by fitting the observed parameters of a planet, considering the evolution of the planetary atmosphere in a statistical way. This requires, however, both planetary radius and mass to be measured with sufficient accuracy, and the planet should not be exposed to extreme atmospheric escape throughout the evolution (Kubyshkina et al. 2019b). Considering initial luminosities (temperatures) of planets, Owen (2020) has shown that the post-formation luminosities can be constrained for young planets using escape argument, but only if the planets are not older than a few tens of Myr. We further address the issue of unknown post-formation luminosities here.

In the present paper, we focus on how the evolution of atmospheres of low-mass planets (with masses smaller than Neptune) is affected by the mass and activity history of the host stars. It was shown by Cloutier & Menou (2020), that the population of planets does not look the same around M dwarfs in comparison to heavier stars. In particular, they show that the relative occurrence rate of rocky to non-rocky planets increases ~6–30 times around mid-M dwarfs compared to mid-K dwarfs, and the slope of the radius valley bears the opposite sign. This can suggest that the atmospheric photoevaporation (the part of atmospheric mass loss related to the stellar XUV irradiation) may be more effective around low-mass stars. Here, we simulate planets evolving around M (0.5 $M_\odot$), K (0.75 $M_\odot$) and G (1.0 $M_\odot$) dwarfs, accounting for the possible variations in their activity history. We find that atmospheric escape of planets with the same equilibrium temperature ranges occurs more efficiently around lower-mass stars despite their luminosities being smaller. This indirectly supports the findings of Cloutier & Menou (2020). Also, for low-mass stars, we find the maximum differences in atmospheric mass loss caused by various activity histories are smaller than those caused by different stellar masses.

For our model, we use the upgraded version of the framework developed in Kubyshkina et al. (2020), which combines the thermal evolution of planetary atmospheres simulated by MESA (Modules for Experiments in Stellar Astrophysics, Paxton et al. 2018) with the realistic prescription of the atmospheric mass loss from Kubyshkina et al. (2018b). We now employ more sophisticated models to describe an input from the host star, i.e., the heating of planets and XUV irradiation. While in the previous version we considered a single model assuming solar-like host star with the XUV luminosity changing with time according to the power-law approximation by Ribas et al. (2005), in the present version, we employ the set of stellar models by Johnstone, Bartel, & Güdel (2020). The latter allows to consistently model stellar XUV luminosity throughout the evolution for different stellar masses and allows to include various stellar activity histories.

This paper is organized as follows. In Section 2 we describe the modeling approach employed in this paper and the stellar models we use. In Section 3, we derive what is the possible range of initial (post formation) planetary luminosities and the possible effects this poorly known parameter has on the final properties of evolved planets. In Section 4, we present and discuss the results of the simulation for a set of planets evolving in different stellar environments. In Section 4.1 we give an overview of the results, and in Section 4.2 we discuss the survival timescales of the atmospheres of planets with specific initial parameters. In Section 5 we propose an analytical approximation of the relation between the atmospheric mass fraction and planetary parameters. Finally, in Section 6 we summarize our conclusions.

2 THE MODELLING APPROACH.

2.1 Thermal evolution and atmospheric escape in MESA.

The basic approach employed in this work was described in detail in Kubyshkina et al. (2020). In brief, we use the ‘Modules for Experiments in Stellar Astrophysics’ (MESA) (Paxton et al. 2011, 2013, 2018) to couple the thermal evolution of a planet (lower atmosphere) with the realistic prescription of the atmospheric escape. The former is computed through the own planetary framework of MESA (Paxton et al. 2018), and as the latter we use the hydro-based approximation (HBA) (Kubyshkina et al. 2018b) based on the grid of hydrodynamical models of planetary upper atmospheres (Kubyshkina et al. 2018a).

In the present work, we test and update the initial conditions for the newborn planets as described in Section 3. The grid of considered planetary masses and initial atmospheric mass fractions remains the same as in the previous work (Kubyshkina et al. 2020): we study planets in mass range of 5–20 Earth mass ($M_\oplus$) and initial atmospheric mass fractions of 0.5–30%. Different from the previous work, the range of planetary masses corresponds to the total masses of planets instead of the core masses. This change does not have significant effects in the results and was introduced to unify the masses of planets with different initial atmospheric mass fractions at the beginning of the evolution, when the atmospheric mass loss is strongest. In the same mass bin, we compare therefore the planets with equivalent masses instead of those differing up to one third of their mass (between 0.5 and 30% of the envelope around the same core), as it was in the previous version of the model.

These planets evolve at different orbits around stars with different masses (0.5, 0.75, and 1.0 solar mass) and activity histories as described in 2.2. We consider the orbital separations between 0.03 and 0.5 AU, excluding the orbits where equilibrium temperatures are below 300 K, which is the restriction of our atmospheric escape models (Kubyshkina et al. 2018a,b).

The basic algorithm we employ to set up the model planet in MESA consists of five general steps: (1) creation of a coreless planet with mass of 0.1 Jupiter mass; (2) adding the solid core (i.e., setting the lower boundary conditions); (3) reducing the atmosphere to the desired value; (4) standardizing the energy budget through setting the initial entropy (i.e., adjusting the lower boundary conditions to stay consistent within the considered set of planets); (5) setting up the stellar heating (i.e., setting the upper boundary conditions) and letting the planet evolve for 5 Myr without atmospheric escape.

Except for step 4, all the other steps were described in depth in Kubyshkina et al. (2020). The initial entropy there was, however, set more or less arbitrarily, as this parameter is poorly constrained for young planets. For this reason, we explore this issue in more details in Section 3. With minor proviso we confirm that the particular choice of the initial entropy is of less importance for the final state of our simulated planets, thus the readers who are more interested in the post-evolution planetary distributions can skip Section 3.

2.2 Different stellar masses and evolutionary paths.

The rotation of the star and its high-energy radiation (X-ray plus extreme ultraviolet, XUV) are closely connected, with faster rotating stars being XUV brighter, and both rotation and XUV luminosity
decaying with time. This evolution though does not follow a unique path and depends on the stellar parameters, in particular stellar mass and the initial rotation rate; the rotation rates at the young ages (∼earlier than 1 Gyr) present a wide spread, and at the later stage the evolutionary tracks converge to the same path for a specific stellar mass (e.g., Tu et al. 2015; Johnstone, Bartel, & Güdel 2020).

In the present work we consider three stellar masses of 0.5, 0.75 and 1.0 solar mass (M⊙). To model their rotation and XUV evolution, we employ the models developed by Johnstone, Bartel, & Güdel (2020). This model represents the physical rotation model constrained by observations in young stellar clusters. To describe the internal stellar physics, it employs the evolutionary models by Spada et al. (2013) and use observations to derive empirical relations between stellar XUV luminosity and rotation for different stellar masses between 0.1 and 1.25 M⊙. To account for the whole range of possible rotation histories of stars, for each of stellar masses we consider the ‘fast’ and ‘slow rotator’ scenarios, assigning the rotation periods at 150 Myr to 0.5/1 and 7 days, respectively. We take for the fast rotator 0.5 day for stellar masses of 0.5 and 0.75 M⊙ and 1 day for 1.0 M⊙ where the latter is dictated by the model restrictions. These periods correspond to the short and long period wings in the distribution observed in young clusters (of ∼150 Myr old) (e.g., Johnstone et al. 2015; Johnstone, Bartel, & Güdel 2020). When further in the paper we address ‘fast’ and ‘slow’ rotating stars, we mean these exact models.

The resulting XUV luminosity (L_{XUV}) and corresponding bolometric luminosities (L_{bol}) are presented in Figure 1. The largest XUV radiation, as well as the largest difference between the slow and the fast rotators, are seen for the solar mass star. For each stellar mass, the XUV evolution for different rotators starts with nearly the same L_{XUV} in the saturation regime (defined by the saturation rotation period). The slow rotators drop out of the saturation regime earlier (with average saturation time being longer for low-mass stars), creating a bifurcation, and the tracks converge again after ∼1 Gyr.

To stay consistent with the new XUV model, we change our prescription of the stellar heating (previously based on Choi et al. 2016) to the equilibrium temperatures that are calculated from the stellar evolution tracks from Spada et al. (2013).

3 INITIAL CONDITIONS: POST-FORMATION LUMINOSITY OF YOUNG PLANETS

While other stellar and planetary parameters are measurable, the initial (post-formation) luminosity of the planet, hence entropy, is uncertain. Its value depends on the planetary mass and on a range of processes throughout the formation stage, as pebble and planetesimal accretion, gas accretion onto the core, contraction of the envelope, decay of radioactive elements in the core, etc, and also on the chosen formation scenario. These processes have been studied extensively for hot Jupiters (see, e.g., Bodenheimer, Hubickyj, & Lissauer 2000; Fortney et al. 2005, 2008; Marley et al. 2007; Spiegel & Burrows 2012; Mordasini et al. 2012; Mordasini 2013; Mordasini, Marleau, & Mollière 2017; Owen & Menou 2016; Berardo, Cumming, & Marleau 2017; Marleau et al. 2017). Although these processes are relatively well known, depending on the particular formation history, the final post-formation luminosity of the core of a specific mass may vary by up to about two orders of magnitude.

For sub-Neptune-like planets, this question was not addressed a lot so far. Owen (2020) showed that, after the age of ∼100 Myr, one can not put a tight constraint on the initial luminosity of the planet based on its present parameters. This suggests that the effect of the uncertain post-formation luminosity on planetary radius is short-living and therefore its choice is not critical for the study of the evolved population of planets. To look into this effect in more detail, we consider here three planets: the very young AU Mic b (∼22 Myr) and K2-33 b (∼9 Myr), and slightly older Kepler-411 d (∼212 Myr), and see how their initial luminosities affect their evolution. The main planetary and stellar parameters are summarized in Table 1. For K2-33 b, only the conservative upper limit for the mass is available. The size of the planet, however, suggests that the true mass is likely to be somewhat smaller than Jupiter (∼318 M⊕). In the present work, we follow the approach used by Kubyshkina et al. (2018c) and consider in our simulations the upper mass limit of 40 M⊕, which is the maximum mass allowing for usage of HBA for the atmospheric mass loss. As we show later, this upper limit is sufficient for the current task. We further exclude

\footnote{The MORS (MOdel for Rotation of Stars) code is available at https://ColinPhilipJohnstone/Mors.}
from consideration masses smaller than $10M_\oplus$; as we have shown in Kubyshkina et al. (2018c) by direct hydrodynamic modeling, such low mass would not allow K2-33 b to sustain any atmosphere at the age of the system.

For a given planet with a known mass, the radius is defined by the atmospheric mass fraction and the temperature profile of the atmosphere, where the latter is controlled by the internal luminosity of the planet and the stellar heating. For younger planets, AU Mic b and K2-33 b, we fit the observed parameters (i.e., their radii for the given planetary masses and positions relative to the host stars) at a given age using the MESA framework. To do so, we set various atmospheric mass fractions keeping the mass of the planet constant, then relaxed stellar irradiation to the level provided by observations and models by Johnstone, Bartel, & Güdel (2020), and finally adjust the central (core) entropy of the planet (hence luminosity) to fit the observed radius. In the following, we will in general refer to the entropy of the planet rather than luminosity. This is due to the entropy being a more general parameter and remaining in a similar range for planets considered here, while the more intuitive planetary luminosity varies significantly. Those two parameters are related with each other and other planetary parameters through the equation of state (Saumon, Chabrier, & van Horn 1995), and we show these relations in Appendix A.

In the present consideration, the lower mass limits for AU Mic b and K2-33 b are nearly the same, and the upper mass limit for K2-33 b is about twice the one for AU Mic b. The ages of the two systems are similar. However, the EUV luminosity of K2-33 is about 5 times higher than the one of AU Mic (which appears to have relatively low activity in comparison to other stars of a similar age, see, e.g., measurements in young clusters in Johnstone, Bartel, & Güdel 2020). Due to this, and the slightly closer orbital separation, K2-33 b experiences a much stronger atmospheric escape for a very similar range of masses and atmospheric mass fractions.

In Figure 2 (top panel), we show which values of the entropy are required to reproduce the observed parameters (i.e., the radius of the planets for a given mass, orbit, and age of the planet, etc.) of AU Mic b and K2-33 b for a given atmospheric mass fraction. As expected, the smaller the atmospheric mass fraction is, the hotter the planet (and higher the entropy) should be to match the observed radii of the planets. We initially consider the central entropy to the interval of 6.5-9.0 k_{baryon}; to further restrict this interval, we compare our results to the predictions of two more approximations. First, we consider the analytical approximation of the post-formation entropy based on the formation models given in Mordasini, Marleau, & Mollère (2017). It gives the estimation of the entropy as a function of the absolute mass of the atmospheres; this approximation is, however, based on the calculations focused on hot Jupiters (up to 50 M_{Jup}). As both luminosity and entropy grow with increasing planetary mass, we, therefore, consider this approximation as an upper limit for the central entropy in the planetary mass range considered here. In Figure 2 (dashed lines), we show the predictions of this approximation given for different atmospheric mass fractions assuming the mass of the planets being the upper boundary of the observational uncertainty (and equal to 40M_\oplus in case of K2-33 b). The second estimation we show here (dashed-dotted lines) is the one suggested as the initial luminosity by Malsky & Rogers (2020)

\[
S_0 = 7.0 + \frac{M_{pl}}{25.0M_\oplus} \cdot \frac{k_b}{baryon}
\]  

By construction, this value is a lower limit of the central entropy expected from the formation models (see, e.g., Marleau & Cumming 2014). We plot it in Figure 2 assuming for each planet the mass at the lower limit predicted by observations (and 10M_\oplus in case of K2-33 b). We will highlight further the interval of initial entropies outlined by these two estimations as the most realistic values, but keep showing in plots the whole range of simulated $S_0$ to help illustrate the general trends.

After setting up the model planets reproducing parameters of AU Mic b and K2-33 b, we evolve them for 5 Gyr to see how the choice of the initial entropy could affect the physical properties of an evolved planet. The choice of the final age is of minor importance here, as all the major changes to the planet’s state occur during the first Gyr of their lifetime. In Figure 3 (top panel), we show the distribution of the final radii against the initial entropy of the planet. It appears that the radii of the evolved planets show a clear
correlation with initial entropy. This effect, however, as we will demonstrate below, is a consequence of the degeneracy between the initial entropy and the atmospheric mass fraction (Figure 2, top panel). Therefore, for both planets, the final radii correlate with the initial atmospheric mass fractions.

This dependence is rather narrow in the case of AU Mic b due to the relatively fine mass constraint and low atmospheric escape: it loses no more than ~4% of its atmosphere for the lowest planetary mass considered. As the radius of a planet depends on the atmospheric mass fraction rather than the total mass of a planet (see, e.g., Lopez & Fortney 2014; Kubyshkina et al. 2020), the final radius, in this case, is defined by the initial atmospheric mass fraction and the cooling path of the planet adjusted by its mass. We can, therefore, predict that the radius of the evolved AU Mic b will be in the range of ~2.9−4.1\(R_\oplus\), which implies that the planet will safely remain in the category of sub-Neptune-like planets. The constraint becomes more narrow if we consider only the "realistic" values of the initial entropy (~7.5−9\(k_B/\)baryon, as described above), giving the range of radii of ~2.9−3.6\(R_\oplus\). Better constraint on mass, however, would not improve this estimate significantly: at 5 Gyr we can see only a weak correlation between the radius and the mass of the planet.

In the case of K2-33 b, however, this dependence is largely spread due to the larger planetary mass interval, and, more importantly, the consequent wide range of atmospheric escape rates. K2-33 is an active star, which means that only massive enough planets can keep most of their envelope throughout the evolution at the orbit of K2-33 b. For the considered range of planetary masses, this is the case for mass bins of 30 and 40 \(M_\oplus\), which lose a maximum of about 2.5 and 0.8% of their envelopes. At the low-mass end of our considered model planets, however, the atmospheric mass loss can reach about 90% of the initial envelope. This creates a wide spread in radii of evolved planets between ~2.4 and 6.7 \(R_\oplus\). In the top panel of Figure 3, the points corresponding to the final radii of K2-33 b group into 6 distinct lines corresponding to 6 mass bins we considered (10, 12.5, 15, 20, 30, and 40 \(M_\oplus\)), with the distribution of lines getting denser at the larger radii (hence larger masses). This suggests that to put a tight constraint on the future evolution of the young planet orbiting an active star, one needs a good observational constraint on the planetary mass. In the bottom panel of Figure 3, we show how the radius at 5 Gyr depends on the mass of the planet. The predicted radius grows steeply with the mass in the mass range below ~30 \(R_\oplus\), and its growth slows down above this mass. Therefore, extending the mass range considered here to heavier planets would not change the overall result essentially.

The third planet we consider here, Kepler-411 d, is about 10 times older than AU Mic b. The age of the system is estimated to be 212 ± 31 Myr (Sun et al. 2019), which is too old for applying the same approach as for two other planets. Based on the conclusions made in Owen (2020), we expect the post-formation luminosity to be dissipated at this point in time. To test this statement, we use the following scheme. We first set up an initial grid of planets, consid-
ering planetary masses in the range of the observational uncertainty split into 10 bins with the step of about 1.1 $M_\oplus$, atmospheric mass fractions in range 1-10% (this range of $f_{at,0}$ allowed reproducing parameters of AU Mic b, which has nearly the same mass) with a 1% step, and the initial entropies in the same range as for other planets. We then set the initial age to 5 Myr (the average lifetime of protoplanetary disk Mamajek 2009) and evolve each planet in this grid for 5 Gyr, and check if the evolutionary track of planetary radius coincides at 212 ± 51 Myr (the age of the system) with the measured radius of Kepler-411 d.

In Figure 2 (bottom panel), we show the relation between the initial atmospheric mass fraction and the initial entropy of the planet for all model planets (empty black circles) and mark the ones which allow reproducing parameters of Kepler-411 d at the present time (green filled symbols). Note that these can be reproduced only for the narrow range of initial atmospheric mass fractions, in particular, if considering only the ‘realistic’ initial entropies of the planet. Given the resolution of the grid, in the latter case only $f_{at,0}$ of 4 and 5% are allowed. At the same time, the radii of evolved model planets at 5 Gyr show no visible correlation with the initial entropy, and the radii at the present age of the system show only a very weak correlation (we do not show these plots here). This confirms the statement formulated at the beginning of this section, that the initial entropy of the planet has no or only minor effect on the final radius distribution of the evolved planets (which is consistent with conclusions made in Owen 2020).

To demonstrate the mechanism, we show in Figure 4 (bottom panel) the evolutionary tracks of planetary radii for planets with the mass of 15.8 $M_\oplus$ (the only mass in our grid allowing us to reproduce Kepler-411 d with more than one $f_{at,0}$ for the ‘realistic’ range of entropies), initial atmospheric mass fractions of 4% (solid lines) and 5% (dashed lines) and different initial entropies (color-coded), denoting the age and radius of the planet by the black dashed rectangle. For both atmospheric mass fractions, at the initial stage, the various entropies of the planet create a spread in radii of about a factor of 1.8. However, as the heat-transfer rate is proportional to the temperature difference with the environment (hence, to the temperature gradient in the atmosphere given that the temperature at the upper boundary is fixed and is defined by the stellar irradiation), the spread in radii diminishes with time, and all the evolutionary tracks for the specific initial atmospheric mass fraction converge before 1 Gyr. At the age of the system, ~212 Myr, the uncertainty of the radius measurements (which is about 3%, i.e., rather small, in case of Kepler-411 d) is too large to distinguish between the different initial entropies of the planet.

On the other hand, this effect ensures that in case of the relatively low atmospheric mass losses (which are, in the case of Kepler-411 d, comparable to those of AU Mic b) one can use the radius of the planet older than about 100 Myr (assuming other parameters of the system to be known with sufficient accuracy) to put a constraint on the initial atmospheric mass fraction of the planet, and therefore further inform planetary formation models. The narrow constraint on the initial atmospheric mass fraction also allows to track the planetary evolution into the future and provide a strong prediction on the radius of the planet at the ages older than 1 Gyr. In the top panel of Figure 4, we show the dependence of the planetary radius at 5 Gyr against the initial atmospheric mass fraction. The designations are the same as in Figure 2, i.e., the points allowing to reproduce Kepler-411 d are shown with green circles. We additionally highlight the points with ‘realistic’ initial entropies with the green ellipse. One can see, that the radius of Kepler-411 d at 5 Gyr lies in the narrow range of ~2.9-3.2$R_\oplus$, thus, as in the case of AU Mic b, the planet will likely remain in the category of sub-Neptunes.

In case of strong atmospheric mass loss, both tasks discussed above (i.e., resolving the initial atmospheric mass fraction and/or predicting the radius of the evolved planet) will be more complicated. The former one we have discussed in detail in Kubyshkina et al. (2019b). The issue is that with increasing atmospheric mass, the total mass of the planet increases only slightly, while its radius can increase by a few times its core radius. The atmospheric escape increases with the radius as at least $\sim R_{pl}^3$ (see, e.g., Kubyshkina et al. 2018b), which leads to the effect visually similar to the one illustrated in the bottom panel of Figure 4, but different by nature: the larger is the excess of the atmospheric mass fraction (hence radius), the larger becomes the atmospheric mass loss rate. Thus, the evolutionary tracks of the planets with initial atmospheric mass fractions above a certain value will converge with those that
started with smaller atmospheres after a few tens of Myr. Therefore, in the case of intensively escaping atmospheres, one can often constrain only the lower boundary of the possible initial atmospheric mass fraction from present time parameters for planets older than 1 Gyr, or even be unable to put any substantial constraint in case of low planetary masses. Among the planets simulated here, it is the case for the lower mass bin of K2-33 b (see the lower line-shaped group of cyan points in Figure 3), where the spread in the final radius is about 7%, which is comparable to the average observational uncertainty for close-in low-mass planets.

The second task (putting a constraint on the radius of the evolved planet for planets as old as a few tens to a hundred Myr) becomes also more complicated in case of strong atmospheric mass loss. As we have shown here for K2-33 b, the strong escape leads to the larger spread in the predicted radius distribution, and, in the case of the poor constraint on the planetary mass, can make it statistically insignificant.

To conclude, we have confirmed here that the initial entropies of planets have a minor (or absent) effect on the population of the evolved planets older than ~1 Gyr, and only affects it at the younger ages. In general, when setting the planet in MESA, the initial central entropy is set up more or less arbitrarily (see details in Paxton et al. 2013). For the range of planets considered here, this value is ~ 8.5 k_b/baryon throughout the grid. In our MESA models, we enforce that the initial entropy assigned by MESA is in between the estimations given by Mordasini, Marleau, & Mollière (2017) and Malsky & Rogers (2020) (i.e., in the ‘realistic’ range).

4 EVOLVED POPULATION: DEPENDENCIES ON ORBITAL DISTANCES AND HOST STAR PARAMETERS

To outline how a particular path of the stellar rotational (hence, activity) evolution and the orbital semi-major axis of the specific planet affects its final parameters, we consider here the following grid of planets. We consider in total 6 model stars with the masses of 0.5, 0.75, and 1.0 M_☉, each of them evolving as a fast or a slow rotator. The particular models were described in Section 2.2. For each star we consider the following grid of orbital distances: 0.03, 0.04, 0.05, 0.075, 0.1, 0.15, 0.2, 0.3, and 0.5 AU. We further adjust this grid for lower mass stars to exclude the orbits with equilibrium temperatures of the planet lower than 300 K, as these temperatures are not covered by the grid of hydrodynamical upper atmosphere models (Kubyshkina et al. 2018a) on which our estimations of the atmospheric escape rates are based. This allows only the following orbital ranges for lower mass stars: ≤ 0.15 AU for 0.5 M_☉, and ≤ 0.3 AU for 0.75 M_☉. At the outer boundary of these orbital separation ranges, however, most of the planets considered here are going to preserve most of their primordial atmosphere, thus their evolution is going to be dominated by pure thermal evolution (i.e., cooling and contraction).

We consider the planetary masses of 5.0, 7.5, 10.0, 12.5, 15.0, and 20.0 M_⊕. These are the total masses of planets, already including the atmospheric masses, which is done for the convenience of comparison to the observations, where one knows the total mass of the planet but not the atmospheric mass fraction. If the latter is more than ~10% of the planetary mass, the mass of the atmosphere can not be treated as insignificant relative to the core mass anymore. In the present work, we consider the atmospheric mass fractions of 0.5, 1, 2, 3, 5, 7, 10, 20, and 30%.

4.1 Overview of the results

In Figure 5 we show the ratio of the atmospheric mass that survived over the 5 Gyr of evolution as a function of planetary mass and the initial atmospheric mass fraction. The distributions of atmospheric mass fractions and planetary radii at 5 Gyr are available for each orbital separation and each stellar model described above and can be found in Appendices B and C. To outline the main features, here we show the distributions for planets orbiting the 0.5 M_☉ star evolving as fast (first column) or slow (second column) rotator at 0.03 and 0.05 AU, and for planets orbiting the solar mass star, fast (third column) or slow (fourth column) rotator, at 0.05 and 0.1 AU. We can see that the general behavior is similar in all cases: the amount of the atmosphere that survived the evaporation throughout the evolution increases towards larger planetary masses, larger orbital distances, and lower stellar activity (hence larger initial stellar rotation periods); i.e., the heavy planets receiving smaller amounts of stellar radiation are less subject to atmospheric escape.

As was demonstrated in, e.g., Chen & Rogers (2016), Kubyshkina et al. (2020) for a certain orbital separation (0.1 AU), the planets starting their evolution with relatively compact envelopes preserve a larger fraction of their initial atmospheres throughout the evolution. In Kubyshkina et al. (2020), the optimal initial atmospheric mass fraction was found to be in between 3-5% of the total mass of the planet for the whole planetary mass range (same as in the present paper). In Figure 5, this effect can be clearly seen by the white contour lines: for the planet of specific mass (y-axis) the relation f_{at}/f_{at,0} maximizes in a region of f_{at,0} where the line peaks. One can see that this effect remains at different orbits around different stars; moreover, the value of the optimal initial atmospheric mass fraction remains approximately the same everywhere, with only the maximum fraction of the preserved atmosphere changing according to the higher/lower atmospheric escape rates for a specific star and orbit. The effect diminishes at large orbital separations, where the majority of the planets considered here preserve most of their atmospheres. We will return to the discussion of this effect in the next section.

In Figure 5, one can see that at the same orbital separation (see 0.05 AU), the larger fraction of the atmosphere survives for planets orbiting lower mass star rather than higher mass star due to the lower XUV flux received (see Figure 1) and lower equilibrium temperature. The simulated planets orbiting the solar mass star at 0.05 AU (the third and the fourth panels in the top row) essentially lose all of their atmospheres, while at the same separation around the star half as heavy as Sun (the first and the second bottom panels) they preserve up to 90% of their envelopes.

When it comes to habitability studies, however, more reasonable is to compare the planets having the same equilibrium temperature rather than the same orbital distance. By definition

$$T_{eq} = \left( \frac{L_{bol}(1 - A_b)}{16\pi\sigma_{SB}R_p^2} \right)^{1/4},$$

(2)

where A_b is the albedo of the planet assumed to be 0 in this study, and σ_{SB} is the Stephan-Boltzmann constant. In Figure 6 we show the absolute values of the planetary atmospheric mass fractions at the age of 5 Gyr against the initial atmospheric mass fraction (y-axis) and the equilibrium temperature (x-axis). The temperature values
are given by the stellar models (Johnstone, Bartel, & Güdel 2020; Spada et al. 2013) at a given age. We show the distributions for two planets with masses of 12.5 and 20 $M_\odot$ (left and right columns, respectively), and stars of 0.5, 0.75, and 1.0 $M_\odot$ (rows from top to bottom) evolving as slow rotators. As the temperature ranges covered for different stellar masses are not the same, we mark the equilibrium temperature of 600 K with a white line throughout the panels for easy comparison.

The atmospheric mass fraction forms a π-shaped 2D distribution, where the right edge is shaped by the atmospheric escape. The position of this edge is, as expected, not the same for different planetary masses, and also not the same for different stellar masses. For the 12.5 $M_\odot$-planet, the position of the edge corresponding to ~3% atmospheric mass fraction moves from about 650 K at the 0.5 $M_\odot$ star to about 900 K at the 1.0 $M_\odot$ star. That means that, at the same equilibrium temperature, a planet around a more massive star preserves more of its atmosphere, than around the low-mass star. This is due to the larger orbital separation, hence lower $F_{\text{XUV}}$ level despite the larger stellar activity. Though being trivial, this fact is often overseen when discussing the habitability of M-stars (Johnstone, Bartel, & Güdel 2020).

**4.2 Survival timescales of atmospheres: lucky parameters for prohibited areas**

As we have discussed in 4.1, the planets with different initial atmospheric mass fraction have different probabilities of keeping their primordial envelope throughout the evolution. Considering the relative amount of the atmosphere (i.e., $f_{\text{at}}/f_{\text{at},0}$, Figure 5) preserved after Gyrs, the optimal initial atmospheric mass fraction lies around 3-5% of the planetary mass for all simulated planets and all orbital separations and XUV levels. This can be seen by the shape of the isocontours, which peak at this region of $f_{\text{at},0}$.

Another parameter allowing to quantify this effect is the atmospheric survival timescale $\tau = M_{\text{atm}}/M$ at the early ages, where $M_{\text{atm}}$ is the current mass of the atmosphere and $M$ is the atmospheric escape rate at the same moment. In Figure 7 we show $\tau$ at the age of 10 Myr for planets with masses of 7.5, 12.5, and 20 $M_\odot$ orbiting the solar mass star evolving as a fast rotator at various orbits. We restrict our consideration to the orbital distances smaller than 0.1 AU, as this region is the most affected by atmospheric evaporation, and for most planets in our sample, the border between lost/preserved atmospheres lies within it. In terms of the atmospheric survival timescale at 10 Myr, the lost/preserved border is at $\tau \sim 100$-200 Myr, which roughly corresponds to the 2.25 line in Figure 7 (in logarithmic scale): for a given planetary mass, planets to the right of this line (towards orange/yellow contours) will certainly keep some of their atmospheres.

In line with what was shown before for relative atmospheric mass fractions of the evolved planets in Section 4.1, the dependence of the atmospheric survival timescale on the initial atmospheric mass fraction is not monotonic: $\tau$ maximizes at the value of $f_{\text{at}} \sim 3\%$. The shape of $\tau(f_{\text{at},0})$, therefore, suggests that at certain close-in orbital separations a planet of specific mass would be capable of keeping part of its atmosphere only for a restricted interval of initial atmospheric mass fractions (see, e.g., the 0.07 AU for 12.5 $M_\odot$ planet or 0.05 AU for 20 $M_\odot$ planet in Figure 7). Thus, at 5 Gyr, the absolute value of the atmospheric mass fraction of 12.5 $M_\odot$-planet at 0.07 AU will be non-zero only if it started with $f_{\text{at},0}$ in range of $\sim 1 - 7\%$.

This might seem contradictory at first. For the small initial atmospheres, this effect is clear – smaller atmospheres are lost faster. For the larger atmospheres, the radius of the planet grows with $f_{\text{at}}$ and consistently grows the escape rate, which leads to larger atmospheres escaping faster at the early stages of evolution. However, one would expect that after reaching the same amount of the atmosphere as the planet that started with smaller $f_{\text{at},0}$, the planet with the larger initial envelope would proceed along the same path (similar to the effect shown in Figure 4, bottom panel, for the (nearly pure) thermal evolution of Kepler-411 d). Thus two planets that started with compact and extended atmospheres are naively expected to end up having the same atmospheric mass fraction after a few Gyrs of evolution. In the present framework, we find, however, that the planets with larger initial atmospheric mass fractions under some con-
Figure 6. The absolute atmospheric mass fraction against the equilibrium temperature and initial atmospheric mass fraction for the slowly rotating stars ($\tau_{\text{rev}} \approx 150 \text{ Myr}$) = 7.0 days) of 0.5 $M_\odot$ (top row), 0.75 $M_\odot$ (middle row), and 1.0 $M_\odot$ (bottom row), and two planets of 12.5 $M_\oplus$ (left column) and 20.0 $M_\oplus$ (right column).
In Figure 7, we show the atmospheric mass fractions in % of the total planetary mass against the planetary mass and the initial atmospheric mass fraction for planets orbiting four different stars at three different orbital separations for each of them. These are the stars of 0.75 (the two left columns) and 1.0 $M_\odot$ (the two right columns) evolving as fast (the first and the third columns) and slow rotators (the second and the fourth), and orbital separations of 0.03, 0.05, and 0.075 AU for 0.75 $M_\odot$ star (rows from top to bottom), and 0.05, 0.075, and 0.1 AU for the solar mass star. In general, more massive planets with smaller initial atmospheric mass fractions tend to retain most of their atmospheres (orange-yellow contours). Note also that initial atmospheres are better preserved for planets orbiting at further out distances and around slowly rotating stars. One can see that, as predicted by $\tau$ distributions, at the closest-in orbits only a narrow region of initial atmospheric mass fractions allows to keep the atmosphere around the planet of a given mass. With increasing orbital separation, this region moves towards the larger initial atmospheric mass fractions and becomes broader, and finally, the effect diminishes at the large orbital separations (at least for the range of $f_{at,0}$ considered here).

Between the cases of the fast and slow rotating star, the quantitative difference is quite large (about an order of magnitude at the closest in orbits and up to about twice for the larger separations). Qualitatively, though, the distributions look very similar, i.e., $f_{at}$ maximizes in the same region, and the range of “lucky” $f_{at,0}$ does not change much (it might not look so at closest-in separations, but it is due to the maximum $f_{at}$ being close to zero in the case of fast rotating star). Thus, despite the effect of atmospheres surviving at certain initial atmospheric mass fractions is clearly caused by the atmospheric mass loss, the shape of the distribution (i.e., the concrete interval of $f_{at,0}$ for a given mass of the planet) is to some extent controlled by the thermal effects.

To compare these distributions at different host stars, but at similar equilibrium temperatures, we highlight the two panels with orbital separations corresponding to the similar temperatures ($\sim 1100$ K) at 0.75 and 1.0 $M_\odot$ stars with the green frames. These are 0.03 AU for 0.75 $M_\odot$ star, and 0.075 AU for 1.0 $M_\odot$ star (Figure 8). As was shown before in Figure 6, at the similar $T_{eq}$, the planet orbiting the more massive star preserves more of the initial atmosphere in comparison to the one orbiting the less massive star. In turn, the effect of the isolated “lucky” initial atmospheric mass fractions is less pronounced. Therefore, it exists in the very restricted range of close-in orbital separations and is more relevant for low-mass stars.

### 4.2.1 The source of the effect

To understand the source of this peculiar effect we consider two model planets as an example: both orbiting fast rotating 0.75 $M_\odot$ star at 0.03 AU orbital separation, and starting their evolution as 20 $M_\oplus$ planet with 20% (planet A) and 7% (planet B) of the mass in the atmosphere. Planetary masses are 7.5 (top panel), 12.5 (middle panel), and 20 $M_\oplus$ (bottom panel). The survival timescale of the atmospheres at the age of 10 Myr for planets orbiting 1.0 $M_\odot$ star evolving as a fast rotator ($P_{rot} (150 \text{ Myr}) = 1.0$ days) against the orbital separation and initial atmospheric mass fraction. Planetary masses are 7.5 (top panel), 12.5 (middle panel), and 20 $M_\oplus$ (bottom panel).

Figure 9. We present the evolutionary tracks of the atmospheric mass fraction, planetary radius, atmospheric escape rate and the temperature of the cooling core for these two planets. Simply
Figure 8. The atmospheric mass fractions of the planets at 5 Gyr around different stars against planetary masses and initial atmospheric mass fraction. Different columns correspond to different stellar models. First column: \( M_* = 0.75 M_\odot \), fast rotator \( (P_{\text{rot}}(150 \text{ Myr}) = 0.5 \text{ days}) \). Second column: \( M_* = 0.75 M_\odot \), slow rotator \( (P_{\text{rot}}(150 \text{ Myr}) = 7.0 \text{ days}) \). Third column: \( M_* = 1.0 M_\odot \), fast rotator \( (P_{\text{rot}}(150 \text{ Myr}) = 1.0 \text{ days}) \). Fourth column: \( M_* = 1.0 M_\odot \), slow rotator \( (P_{\text{rot}}(150 \text{ Myr}) = 7.0 \text{ days}) \). Indicative equilibrium temperatures for these stellar masses and orbital separations are indicated in each panel.

Figure 9. Evolutionary tracks of the 20 \( M_\oplus \) planet that started the evolution with 7 (black solid lines) and 20% (red dotted lines) of its mass in the atmosphere. Planetary parameters are presented in the following order. Top left: the atmospheric mass fraction. Bottom left: the central (core) temperature of the planet. Top right: the radius of the planet. Bottom right: the atmospheric escape rate. The vertical green line represents the time when the two atmospheric mass fractions of two planets become equal.

from the formulation (where we include the mass of the atmosphere into planetary mass at the beginning of the simulation, i.e., planetary mass changes with time), it is clear that for these two planets having the same atmospheric mass fraction does not mean having the same total mass. Instead, \( M_{\text{pl}A} = 0.8 M_0 + M_{\text{atm}A} \), and \( M_{\text{pl}B} = 0.93 M_0 + 1.16 M_{\text{atm}A} \), where \( M_0 \) is the total initial mass of the planet (which is the same for A and B), and \( M_{\text{atm}A} \) is the mass of the atmosphere of planet A. Therefore, at the time when \( f_{\text{at},A} = f_{\text{at},B} \) (green vertical line in Figure 9), \( M_{\text{pl}A} = 16.93 M_\oplus \), and \( M_{\text{pl}B} = 19.67 M_\oplus \). This alone is enough for the atmospheric escape rate of planet A to be \( \sim 3 \) times larger than the one of planet B.

However, we noticed one further difference between the two planets. Though two planets start their evolution having the same core luminosity, they evolve differently through the “disk phase”, i.e., through the 5 Myr period that follows the preparation stage in our simulations and precedes the main evolution phase, when the planet cools down without atmospheric escape. The planet with the larger atmosphere (A) cools down more slowly than the one with the thinner atmosphere (B), and therefore starts the evolution after the “disk dispersal” (the mass-loss switches on) having a larger core temperature. After that moment, the temperature of the core of planet A drops down faster compared to planet B, due to the larger atmospheric escape, but not fast enough to reach the same temperature to the point when the two atmospheric mass fractions are equal (see bottom left panel of Figure 9); the picture does not change qualitatively if considering the actual mass of the atmosphere instead of the atmospheric mass fraction. The larger temperature
results in the larger radius corresponding to the same \( f_{\text{es}} \) (top right panel), which in turn increases further the atmospheric escape rate of planet A (bottom right panel), intensifying the effect we consider here.

To split between the ‘mass’ and ‘temperature’ parts of the effect, we have reconsidered the planets in question employing the approach where the mass of the core is treated as the main part of the planetary mass and the atmospheric mass as a minor addition (which is the most conventional approach and was used in Kubyshkina et al. 2020). Thus, in the example considered before, planets A and B start with the core masses of 20 \( M_{\oplus} \) with the corresponding additions of the atmospheric mass on top of this. In this formulation, at the moment when \( f_{\text{es},A} = f_{\text{es},B} \) (coming \( \sim 80 \) Myr later than in the example above), the masses of planets are equal. The temperature difference, however, remains and leads to the difference of a factor of about two in the atmospheric escape rates. Thus, for planet B this difference in approach does not change the evolutionary track a lot: it preserves slightly more of the initial atmosphere due to the 10% larger mass. Planet A, at the same time, preserves its atmosphere longer (up to \( \sim 1.2 \) Gyr instead of 566 Myr), but is yet unable to keep it. Therefore the same “restricted zone” effect can be observed also with the alternative formulation of the problem, being caused purely by the difference in core temperatures of the planets.

Changing the duration of the “disk phase” and the initial luminosities of planets also change the overall picture quantitatively, but not qualitatively. However, it is important to note that we do not consider here the actual planet formation or the realistic protoplanetary disk phase. As we discussed before in Section 3, the formation models suggest that for different planets the post-formation luminosity (at the time of a disk dispersal) is indeed not the same. The approximation by Mordasini, Marleau, & Mollière (2017), which we considered in Section 3, predicts larger initial luminosity for a larger mass of the atmosphere. However, the detailed study of the effect requires the actual application of the formation models to set up the initial conditions of the evolving planets and the thorough investigation of their assumptions. This detailed investigation lies outside the scope of the present work and is left for a future study.

In light of the discussion presented here, this effect puts some constraints on the conclusions we made in Section 3, namely that the initial luminosity does not affect the final state of the evolved planet and therefore the overall planetary demographics. This may not be true for planets subject to extreme atmospheric mass loss, i.e., for the low to intermediate mass planets at short orbits. Having certain initial parameters (atmospheric mass fraction and core temperature) can affect the survival timescales of the atmospheres, and let the planetary atmosphere survive through the initial stage of the young and active host star. If the planet keeps its atmosphere through the first Gyr of evolution, it is most likely that the planet keeps it up to the average age of 5-10 Gyr.

This also suggests that the simultaneous consideration of the atmospheric mass loss and the thermal evolution of the atmosphere becomes particularly important in these regions (at orbits within \( \sim 0.05 \) AU). Additionally, the existence of a “lucky interval” in the initial parameters of the planet could explain the existence of some peculiar planets, whose parameters seem to be unlike for their environment, as, e.g., LTT 9779 b (Jenkins et al. 2020), or TOI-132 b (Diaz et al. 2020).

### 5 ANALYTIC RELATION BETWEEN PLANETARY RADII AND ATMOSPHERIC MASS FRACTION

The simultaneous consideration of the thermal evolution of the atmosphere and the atmospheric escape is being important, in particular for the planets subject to the strong escape. However, the relation between the atmospheric mass fraction of the planet and its radius for a specific set of planetary parameters can be approximated with a relatively simple function. We have derived an analytical function in Kubyshkina et al. (2020) for the planets in the mass range of 5-20 \( M_{\oplus} \) orbiting a solar mass star at 0.1 AU. In the present work, we extend such parameterization to a wider range of physical parameters, such as different stellar masses and orbital separation. In Kubyshkina et al. (2020), we used as a basic approximation function the cubic polynomial. Here, we employ the following function, which allows reducing the average mean square error throughout the whole parameter space.

\[
f_{\text{es}}(R_{\oplus}) = C_1(R_{\oplus} - R_c) C_2 \tanh(C_3(R_{\oplus} - R_c))
\]

This function has as the argument \( (R_{\oplus} - R_c) \), where \( R_c \) is the core radius of the planet, i.e., the thickness of the atmosphere. This ensures that when \( f_{\text{es}} = 0 \) the radius of the planet equals its core radius\(^3\). For convenience, in approximation we considered \( R_c = M_{\oplus}^{0.27\pm0.02} \), which describes well the core radii adopted in the MESA simulations. As a final value of the exponent we adopt 0.25, as it allows to minimize the approximation errors. \( C_1, C_2, \) and \( C_3 \) are free parameters of the approximation, which we fit. In Figure 10 (top panel) we illustrate the typical dependence of the function and how changes in each of the coefficients affect its shape.

The main parameters of the evolving planets (except of the argument and the function in Equation 3, \( R_{\oplus} \) and \( f_{\text{es}} \)) are planetary mass, orbital separation, equilibrium temperature and received XUV flux (\( F_{\text{XUV}} \)). For the star, these are stellar mass and XUV luminosity. All these parameters change with the age of the system and are not independent of each other. In particular, the equilibrium temperature and the XUV flux received by the planet depend on the orbital separation and the bolometric and XUV luminosities of the star, while the luminosities depend on the age of the system and the stellar mass, and the XUV luminosity, in turn, depends on the \( P_{\text{rot}}(\text{age}) \). To rule out degeneracies we consider the following.

The equilibrium temperature can be defined through the bolometric luminosity as defined in Equation 2, i.e., it is proportional to \( \tau_{\text{bol}} \). The XUV flux at the planetary orbit is \( F_{\text{XUV}} = \frac{L_{\text{XUV}}}{4\pi d_0^2} \), where the stellar XUV luminosity depends on the bolometric luminosity and the rotation period of the star (see Figure 1). \( L_{\text{XUV}} \) is commonly considered as a sum of X-ray and EUV luminosities, where the latter can be expressed through the former using empirical relations (see, e.g., Sanz-Forcada et al. 2011; Johnstone, Bartel, & Güdel 2020). In turn, \( L_X \) can be described as

\[
\frac{L_X}{L_{\text{bol}}} = C \left( \frac{P_{\text{rot}}}{\tau_{\text{conv}}} \right)^\beta,
\]

where \( \tau_{\text{conv}} \) is the convective turnover time depending on the stellar mass, and constants \( C \) and \( \beta \) are different for the saturated

\(^3\) In MESA simulations when \( f_{\text{es}} \to 0 \) the radius of the planet tends to the value slightly larger than \( R_c \), which is likely connected to the complication of representing very thin atmospheres due to the interpolation within the parameter tables. We, therefore, focus on minimizing the error of approximation for the larger atmospheres \( f_{\text{es}} \geq 0.5\% \).
here these values are $\tau_{\text{conv}} = 31.7$, 60.4, and 135 days for 1.0, 0.75, and 0.5 $M_\odot$ stars, respectively (which leads to $r M_\odot^2 \approx \text{const}$). We therefore do not consider ages smaller than 100 Myr in our approximation. It serves two goals: simplification of $\tau_{\text{conv}}(\text{age})$ described above, and the exclusion of the saturation period in the stellar evolution, which allows to consider only one value of $\beta$ in Equation 4 within our approximation. Additional motivation to exclude the first 100 Myr when fitting the coefficients in Equation 3 is that the relation $f_{\text{at}}(R_{\text{pl}})$ at this time is affected by the initial luminosity of the planet, which is to a large extent model-dependent.

We therefore consider the following planetary/stellar parameters when parameterizing the coefficients $C_1$, $C_2$, and $C_3$ in Equation 3: planetary mass $M_{\text{pl}}$, normalised bolometric flux at the planetary orbit $L_{\text{bol}}(R_{\text{pl}})$, Rossby number of the star $R_{\text{pl}} = \frac{R_{\text{pl}}}{c_{\text{rs}}}$, and the age of the system. As the dependence on the stellar mass becomes pretty weak after inclusion of $\tau_{\text{conv}}$, we leave it outside of consideration.

With the parameters described above, we fit the coefficients in Equation 3 as follows

$$C_1 = (0.0464 \pm 0.0093) \times \left( \frac{L_{\text{bol}}}{d_0^2} \right)^{-0.09} \times R_{\text{o}}^{-0.04} \times \left( \frac{M_{\text{pl}}}{M_\odot} \right)^{0.07} \times \left( \frac{\text{age}}{100 \text{Myr}} \right)^{0.03} \left( \begin{array}{l} \text{(5)} \end{array} \right)$$

$$C_2 = (1.0296 \pm 0.1304) \times \left( \frac{M_{\text{pl}}}{M_\odot} \right)^{0.095} \times \left( \frac{R_{\text{o}}}{R_\odot} \right)^{0.01} \times \left( \frac{L_{\text{bol}}}{d_0^2} \right)^{0.01} \left( \begin{array}{l} \text{(6)} \end{array} \right)$$

$$C_3 = (1.0053 \pm 0.3647) \times \left( \frac{M_{\text{pl}}}{M_\odot} \right)^{-0.04} \times \left( \frac{R_{\text{o}}}{R_\odot} \right)^{0.02} \times \left( \frac{L_{\text{bol}}}{d_0^2} \right)^{-0.02} \times \left( \frac{\text{age}}{100 \text{Myr}} \right)^{-0.017} \left( \begin{array}{l} \text{(7)} \end{array} \right)$$

In Figure 10 (bottom panel), we compare the atmospheric mass fraction corresponding to the specific planetary and stellar parameters to the one predicted by approximation 3. The plot contains $\sim 5500$ points, covering the whole range of planetary and stellar parameters at 3 time snapshots: 100 Myr, 1 Gyr, and 5 Gyr; each black cross represents a snapshot for an individual planet. For atmospheric mass fractions above 0.5% of planetary mass, 99.7% of the $f_{\text{at}}/f_{\text{at,approx}}$ points, given by central values of coefficients $C_1$, $C_2$, and $C_3$, lie in between 0.5 and 2, and 88% of points lie in between 0.8 and 1.2. The spread decreases towards larger atmospheres, heavier planets, and older ages. Thus, for planets heavier than 10 $M_\odot$ and atmospheric mass fractions above 5%, ~82% of points lie between 0.9 and 1.1 (in comparison to 63% for the whole range of applicability). For $f_{\text{at}} < 0.3\%$, the approximation does not work anymore and the relation $f_{\text{at}}/f_{\text{at,approx}}$ shows clear systematic error (see the footnote at the beginning of this Section).

By analyzing the coefficients $C_1$, $C_2$, and $C_3$ and comparing it to the scheme in Figure 10 (top panel), one can see that the atmospheric mass fraction for a given radius is larger for larger coefficients, and thus for smaller bolometric luminosities (hence cooler planet) and slower rotation of the star (hence smaller XUV flux). For larger planetary mass (increase in $C_2$), the same atmospheric mass fraction results in slightly larger radii for a lighter planet, with this difference decreasing with increasing atmospheric mass. Finally, after ruling out the dependencies on the stellar bolometric flux and rotation, the remaining dependence on the age of the system (as a proxy of the cooling planetary core) is relatively weak and predicts a slightly larger $f_{\text{at}}$ for the same radii at the later age (and a cooler core). At the younger ages, not considered here,
however, the exponent in this dependence increases by about an order of magnitude.

6 CONCLUSIONS

The evolution of the atmospheres of low and intermediate mass planets is closely connected to the physical properties of their host stars. In this paper, we have modeled the evolution of a wide range of sub-Neptune-like planets orbiting low-mass stars (0.5-1 \( M_\odot \)), that follow different evolution paths. Our main conclusions can be summarised as follows.

- For all stellar masses, the distribution of the parameters of evolved planets shows similar patterns. Planets with larger masses or orbiting farther out from the star keep more of their primordial atmospheres than planets with smaller masses or orbiting closer-in. Less straightforward, planets that start their evolution having compact envelopes (\( \sim 3 - 5\% \) of their total mass) tend to keep a larger fraction of their primordial envelope for a given planetary mass and orbital separation.

- Additionally, planets evolve differently depending on the rotation rate of the host star (i.e., its activity level). This difference is more accentuated for planets orbiting more massive stars, as the difference in \( \Delta \chi_{UV} \) emitted by the slow and fast rotating stars increases with stellar mass (see Figure 1).

- A planet has larger chances to keep its primordial atmosphere in the habitable zone (or in any regions with equal equilibrium temperature) of a more massive star, even though luminosities of low mass stars are lower. This fact is particularly relevant for habitability studies.

- We have tested the influence of the initial (post-formation) planetary luminosities on the parameter distribution of the evolved planets by modeling three young planets, K2-33 b, AU Mic b, and Kepler-411 d. We came to the conclusion that the uncertain initial luminosity does not in general significantly affect the state of the evolved planet. It can, however, constrain the possibility to predict the future state of a young planet. To put a tight constraint on the future evolution of the young planet orbiting close to an active star, one needs a good observational constraint on the planetary mass.

- We anticipate that all three planets considered in Section 3 will likely remain in the category of sub-Neptune-like planets after Gyrs of evolution. For planets experiencing relatively low atmospheric mass loss we can constrain the radii at 5 Gyrs, that is \( \sim 2.9-3.6 \, R_\oplus \) and \( \sim 2.9-3.2 \, R_\oplus \) in the case of AU Mic b and Kepler-411 d, respectively. For K2-33 b, experiencing extreme atmospheric mass losses, we can only put a loose constraint of \( \sim 2.4-6.7 \, R_\oplus \) due to the weak observational constraint on planetary mass. The upper limit of the radius can further slightly increase if the planetary mass is higher than \( 40 \, M_\oplus \).

- In close-in orbits, where planets are exposed to extreme atmospheric mass losses, having relatively small initial atmospheric mass fractions (in range of \( \sim 5-15\% \)) enhances the chances for the planet to keep some of its primordial atmosphere. This conclusion should be further investigated by self-consistently including planetary formation models at the initial condition of the simulations. However, already at this point, we can conclude that certain combinations of initial atmospheric mass fraction and post-formation luminosity of a young planet at close-in orbit can allow it to keep a part of its primordial atmosphere while at other starting conditions it would not be possible (which could explain the presence of planets in so-called hot-Neptune desert). It is thus clear that the simultaneous modeling of the thermal evolution of the atmosphere and the atmospheric mass loss, as well as an accurate prescription of the starting parameters, is of crucial importance for very close-in planets.

- Finally, we constructed an approximation describing the relation between the atmospheric mass fraction and the radius of a planet and bounded its coefficients to some of the main physical parameters of planets and host stars. The analytical approximation shown in equations 3, 5, 6, and 7 can thus be used to quickly estimate the atmospheric mass fraction for a planet of a given mass, at a given age, orbiting a low-mass star at a given orbital separation.

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DATA AVAILABILITY

The data underlying this article will be shared on reasonable request to the corresponding author.

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APPENDIX A: THE RELATION BETWEEN PLANETARY LUMINOSITY AND ENTROPY

In general, the equation of state (EOS) defines univocally the parameters of the system (matter) as a function of density and temperature (see, e.g., Saumon, Chabrier, & van Horn 1995, which is the basic EOS used in MESA). The common approach in prescribing EOS is the minimization of the Helmholtz free energy of a system given as a function $F(V, T, \{N_i\})$ of volume, temperature, and particle numbers of specific species making up the “fluid” environment. In this formulation, the pressure and the entropy can be expressed as the first partial derivatives of free energy: $P(\rho, T) = -\frac{\partial F}{\partial \rho}$, and thermodynamical $(\rho, T)$ of volume, temperature, and particle numbers of specific species making up the “fluid” environment. In this formulation, the pressure and the entropy can be expressed as the first partial derivatives of free energy: $P(\rho, T) = -\frac{\partial F}{\partial \rho}$, and thermodynamical $(\rho, T)$. This pair of parameters dictate the mechanical $(P)$ and thermodynamical $(S)$ equilibrium of the system (in our case, planetary atmosphere), and appears explicitly in the EOS. Other parameters, such as specific heats, expansion coefficients, adiabatic gradients, etc., are defined as the secondary derivatives by specific variables.

In MESA, to create the initial model the solid core of a planet (or a star) is defined by setting up the lower boundary conditions. The user has to set up the mass $(M_*)$ underlying the minimum (core) radius in the simulation (thus setting the central pressure $P_c$ or, equivalently, density $\rho_c$), and the temperature of the core. For stars, the latter is a specific value (low enough to exclude the hydrogen burning). For planets, however, this approach does not work if the initial guess on parameters is not close enough to real ones (Paxton et al. 2013). As a function $F(V, T, \{N_i\})$ of volume, temperature, and particle numbers of specific species making up the “fluid” environment. In this formulation, the pressure and the entropy can be expressed as the first partial derivatives of free energy: $P(\rho, T) = -\frac{\partial F}{\partial \rho}$, and thermodynamical $(\rho, T)$. This pair of parameters dictate the mechanical $(P)$ and thermodynamical $(S)$ equilibrium of the system (in our case, planetary atmosphere), and appears explicitly in the EOS. Other parameters, such as specific heats, expansion coefficients, adiabatic gradients, etc., are defined as the secondary derivatives by specific variables.

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The initial planetary luminosity (top panel) and core temperature (bottom panel) of Kepler-411 d against the initial mass of the atmosphere as modelled in Section 3, for different levels of the initial entropy (color coded in the bottom panel).

Figure A1. The initial planetary luminosity (top panel) and core temperature (bottom panel) of Kepler-411 d against the initial mass of the atmosphere as modelled in Section 3, for different levels of the initial entropy (color coded in the bottom panel).

The specific entropy $S_{c,0}$, therefore, sets an “amplitude” for the temperature of the innermost cell in MESA simulations (here referred to as $T_{\text{core}}$) and the luminosity of the planet. In Figure A1, we show the values of the initial luminosity and core temperature of Kepler-411 d against the mass of the atmosphere for different initial entropy levels. We only show here the case of Kepler 411 d, as the grid of the model points for this planet is the most comprehensive.

APPENDIX B: POPULATION OF EVOLVED PLANETS: ATMOSPHERIC MASS FRACTIONS

Here we present the overall set of distribution of the atmospheric mass fractions at the 5 Gyr age against the mass of the planet and its initial atmospheric mass fraction, which has been outlined in Section 4.1. The results are shown in Figure B1, and organized after the host star models: stellar masses of 0.5, 0.75, and 1.0 $M_{\odot}$ (pairs of columns, from left to right), and rotation type (in each pair of columns, the left one corresponds to the fast, and the right one corresponds to the slow rotator). Different rows correspond to different orbital separations between 0.03 and 0.5 AU (from top to bottom). For lower mass stars (0.5 and 0.75 $M_{\odot}$), the upper limit of $d_0$ is defined by the condition that the equilibrium temperature should be above 300 K. For the solar mass star, we omit the distributions at 0.03 and 0.04 AU, as all the test planets lose their atmospheres entirely at these orbits, even in case of the slow rotator.

APPENDIX C: POPULATION OF EVOLVED PLANETS: RADII OF PLANETS

To illustrate how the distribution of the atmospheric mass fractions of evolved planets looks in terms of planetary radii, we show in Figure C1 the distributions of planetary radii against the initial atmospheric mass fractions (y-axis) and orbital separations (x-axis). The columns are organised by stellar models as in Figure B1, and the rows correspond to different planetary masses from 5 to 20 $M_{\oplus}$ (from top to bottom). The minimum of the color scale (deep blue) for each $M_{pl}$ corresponds to the core radius of the simulated planet. For low-mass planets, these distributions are mainly shaped by escape, thus the effect of more compact atmospheres surviving longer can be seen. The radius of a planet is mainly defined by the atmospheric mass fraction and the temperature of the planet. The core temperatures/luminosities of planets of specific mass are nearly the same after 5 Gyr of evolution (see the discussion in Section 3). Thus, in the parameter spaces where planets are weakly affected by the atmospheric mass loss (i.e., at large $M_{pl}$ or $d_0$) the contour lines are nearly horizontal, with a small decay towards larger orbital separations. The latter implies that the radii are mainly affected by the internal heat of the planet, while the external stellar heating has a minor effect.

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Figure B1. Atmospheric mass fraction at 5 Gyr of the test planets orbiting solar mass star evolving as the fast (left column) and the slow (right column) rotator. Orbital separations from top to bottom are 0.05, 0.075, 0.1, 0.15, 0.2, 0.3, and 0.5 AU. The 0.03 and 0.04 AU orbits are not shown, as all the test planets totally lose their envelopes at these distances.
Figure C1. Radii of the model planets at 5 Gyr in dependence on the initial atmospheric mass fraction and orbital separation. Distributions are shown for different planetary masses from 5 to 20 $M_⊕$ (lines from top to bottom as referred in plots), and different host stars: 0.5 $M_\odot$ (two left columns), 0.75 $M_\odot$ (two middle columns), and 1.0 $M_\odot$ (two right columns). For each pair of columns, the left corresponds to the star evolving as the fast rotator, and the right column to the one evolving as the slow rotator. The radii of the solid cores assumed for the given planetary masses are, from top to bottom, 1.54, 1.72, 1.86, 1.98, 2.08, and 2.25 $R_⊕$ (Rogers et al. 2011).