First Predicted Cosmic Ray Spectra, Primary-to-Secondary Ratios, and Ionization Rates from MHD Galaxy Formation Simulations

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ABSTRACT
We present the first simulations evolving resolved spectra of cosmic rays (CRs) from MeV-TeV energies (including electrons, positrons, (anti)protons, and heavier nuclei), in live kinetic-MHD galaxy simulations with star formation and feedback. We utilize new numerical methods including terms often neglected in historical models, comparing Milky Way analogues with phenomenological scattering coefficients \( \nu \) to Solar-neighbourhood (LISM) observations (spectra, \( B/C, e^+ / e^- \), \( p/p \), \( ^{10}\text{Be}/^9\text{Be} \), ionization). We show it is possible to reproduce observations with simple single-power-law injection and scattering coefficients (scaling with rigidity \( R \)), similar to previous (non-dynamical) calculations. We also find: (1) The circum-galactic medium in realistic galaxies necessarily imposes a \( \sim 10 \) kpc CR scattering halo, influencing the required \( \nu(R) \). (2) Increasing the normalization of \( \nu(R) \) re-normalizes CR secondary spectra but also changes primary spectral slopes, owing to source distribution and loss effects. (3) Diffusive/turbulent reacceleration is unimportant and generally sub-dominant to gyroresonant/streaming losses, which are sub-dominant to adiabatic/convective terms dominated by \( \sim 0.1 – 1 \) kpc turbulent/fountain motions. (4) CR spectra vary considerably across galaxies; certain features can arise from local structure rather than transport physics. (5) Systematic variation in CR ionization rates between LISM and molecular clouds (or Galactic position) arises naturally without invoking alternative sources. (6) Abundances of CNO nuclei require most CR acceleration occurs around when reverse shocks form in SNe, not in OB wind bubbles or later Sedov-Taylor stages of SNe remnants.

Key words: cosmic rays — plasmas — methods: numerical — MHD — galaxies: evolution — ISM: structure

1 INTRODUCTION
The propagation and dynamics of cosmic rays (CRs) in the interstellar medium (ISM) and circum/inter-galactic medium (CGM/IGM) is an unsolved problem of fundamental importance for space plasma physics as well as star and galaxy formation and evolution (see reviews in Zweibel 2013, 2017; Amato & Blasi 2018; Kachelrieß & Semikoz 2019). For decades, the state-of-the-art modeling of Galactic (Milky Way; MW) CR propagation has largely been dominated by idealized analytic models, where a population of CRs is propagated through a time-static MW model, with simple or freely-fit assumptions about the “halo” or thick disk around the galaxy and no appreciable circum-galactic medium (CGM)’s “escape” (as a leaky box or flat halo-diffusion type model) outside of some radius (Blasi & Amato 2012a; Strong & Moskalenko 2001; Vladimirov et al. 2012; Gaggero et al. 2015; Guo et al. 2016; Johansson et al. 2016; Cummings et al. 2016; Korsmeier & Cuoco 2016; Evoli et al. 2017).

These calculations generally ignore phase structure or inhomogeneity in the ISM/CGM, magnetic field structure (anisotropic CR transport), streaming, complicated inflow/outflow/fountain and turbulent motions within the galaxy, and time-variability of galactic structure and ISM phases (although see e.g. Blasi & Amato 2012b; Johansson et al. 2016; Liu et al. 2018; Giacinti et al. 2018), even though, for example, secondary production rates depend on the local gas density which varies by several orders of magnitude in both space and time (even at a given galacto-centric radius) as CRs propagate through the ISM. Likewise, the injection itself being proportional to e.g. SNe rates is strongly clustered in both space and time and specifically related to certain ISM phases (see Evans 1999; Vázquez-Semadeni et al. 2003; Mac Low & Klessen 2004; Walch et al. 2015; Fielding et al. 2018), and other key loss terms depend on e.g. local ionized vs. neutral fractions, magnetic and radiation energy densities – quantities that can vary by ten orders of magnitude within the MW (Wolfire et al. 1995; Evans 1999; Draine 2011). And these static models cannot, by construction, capture non-linear effects of CRs actually modifying the galaxy/ISM structure through which they propagate. This in turn means that most inferred physical quantities such as CR diffusivities, residence times, re-acceleration efficiencies, and “convective” speeds (let alone their dependence on CR energy or ISM properties) are potentially subject to order-of-magnitude systematic uncertainties. That is not to say these static-Galaxy models are simple, however: their complexity focuses on evolving an enormous range of CR energies from \( \lesssim \text{MeV} \) to \( \gtrsim \text{PeV} \), including a huge number of different species, and incorporating state-of-the-art nuclear networks for detailed spallation, annihilation, and other reaction rates (recently, see Liu et al. 2018; Amato & Blasi 2018).

Meanwhile, simulations of galaxy structure, dynamics, evolution, and formation have made tremendous progress incorporating and reproducing detailed observations of the time-dependent, multi-phase complexity of the ISM and CGM (Hopkins et al. 2012a; Kim & Ostriker 2017; Grudic et al. 2019; Benincasa et al. 2020; Keating et al. 2020; Gurvich et al. 2020), galaxy inflows/outflows/fountains (Narayanan et al. 2006; Hayward & Hopkins 2017; Muratov et al. 2017; Anglés-Alcárrez et al. 2017; Hafen et al. 2019a;b; Hopkins et al. 2020a; Ji et al. 2020b), and turbulent motions (Hopkins 2013a,b; Guszejnov et al. 2017b; Escala et al. 2018; Guszejnov et al. 2018; Sarazin et al. 2019; Hopkins et al. 2020).

The term “halo” is used differently in CR and galaxy literature. In most CR literature, the “halo” is generally taken to have a size \( \sim 1 – 10 \) kpc, corresponding to the “thick disk” or “disk-halo interface” region in galaxy formation/structure terminology. In the galaxy community, the gaseous “halo” usually refers to the circum-galactic medium (CGM), with scale-lengths \( \sim 20 – 50 \) kpc and extent \( \sim 200 – 500 \) kpc (Tumlinson et al. 2017).

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2018; Rennehan et al. 2019), magnetic field structure and amplification (Su et al. 2017, 2018a, 2019; Hopkins et al. 2020b; Guszejnov et al. 2020c; Martin-Alvarez et al. 2018), dynamics of mergers and spiral arms and other gravitational phenomena (Hopkins et al. 2012a, 2013a; Fitts et al. 2018; Ma et al. 2017b; Garrison-Kimmel et al. 2018; Moreno et al. 2019), star formation (Grudić et al. 2018a; Orr et al. 2018, 2019; Grudić et al. 2019; Garrison-Kimmel et al. 2019b; Wheeler et al. 2019; Ma et al. 2020a; Grudić et al. 2020), and stellar “feedback” from supernovae (Martizzi et al. 2015; Gentry et al. 2017; Rosdahl et al. 2017; Hopkins et al. 2018a; Smith et al. 2018; Kawakatu et al. 2020), stellar mass-loss (Wiersma et al. 2009; Conroy et al. 2015; Höfner & Olofsson 2018), radiation (Hopkins et al. 2011; Hopkins & Grudić 2019; Hopkins et al. 2020d; Wise et al. 2012; Rosdahl & Teyssier 2015; Kim et al. 2018; Emerick et al. 2018), and jets (Bürzle et al. 2011; Öffner et al. 2011; Hansen et al. 2012; Guszejnov et al. 2017a), resolving the dynamics of those feedback mechanisms interacting with the ISM. However, these calculations (including our own) treat the high-energy astro-particle physics in an incredibly simple fashion. Most ignore it entirely. Even though there has been a surge of work in recent years arguing that CRs could have major dynamical effects on both the phase (temperature-density) structure and dynamics (inflow/outflow rates, strength of turbulence, bulk star formation rates) of galaxies (see Jubelgas et al. 2008; Ullig et al. 2012; Booth et al. 2013; Wiener et al. 2013b; Hanasz et al. 2013; Salem & Bryan 2014; Salem et al. 2016; Chen et al. 2016; Simpson et al. 2016; Girichidis et al. 2016; Pakmor et al. 2016; Salem et al. 2016; Wiener et al. 2017; Ruszkowski et al. 2017; Butsky & Quinn 2018; Faber et al. 2018; Jacob et al. 2018; Girichidis et al. 2018), essentially all of these studies have treated CRs with a “single-bin” approximation, evolving a single fluid representing “the CRs” (recently, see Salem et al. 2016; Chan et al. 2019; Butsky & Quinn 2018; Su et al. 2020; Hopkins et al. 2020e; Ji et al. 2020a; Butsky & Zweibel 2020). Even if one is only interested in the dynamical effects of CRs on the gas itself, so assumes the CR pressure is strongly dominated by ~GeV protons, this could be inaccurate in many circumstances. For example, certain terms which should “shift” CRs in their individual energies or Lorentz factors and therefore change their emission/loss/transport properties instead simply rescale “up” or “down” the CR energy density in single-bin models, effectively akin to “creating” new CRs.

More importantly, even if “single-bin” models allow for a reasonable approximate estimation of bulk CR pressure effects on gas, a “single-bin” CR model precludes comparing to the vast majority of observational constraints. Essentially, it restricts comparison to a handful of galaxy-integrated ~GeV γ-ray detections in nearby star-forming galaxies (Lacki et al. 2011; Tang et al. 2014; Griffin et al. 2016; Fu et al. 2017; Wojczyński & Niedźwiecki 2017; Wang & Fields 2018; Lopez et al. 2018), which in turn means that theoretical CR transport models are fundamentally under-constrained (see Hopkins et al. 2020b). Because the γ-rays constrain a galacticISM-integrated quantity over a narrow range of CR energies, different physically-motivated models which reproduce the same γ-ray luminosity can predict qualitatively different CR transport in the ISM/CGM/IGM (depending on how they scale with properties as noted above), as well as totally different effects of CRs on outflows, accretion, and galaxy formation (Hopkins et al. 2020c). This also precludes comparing to the enormous wealth of detailed Solar system CR data covering a huge array of species, as well as the tremendous amount of spatially-resolved synchrotron data from large numbers of galaxies spanning the densest regions of the ISM through the diffuse CGM, and all galaxy types.

In this manuscript, we therefore generalize our previous explicit CR transport models from previous studies to a resolved CR spectrum of electrons, positrons, protons, anti-protons, and heavier nuclei spanning energies ~MeV to ~TeV. This makes it possible to explicitly forward-model from cosmological initial conditions quantities including the CR electron and proton spectra, B/C and radiative isotope ratios, and detailed observables including synchrotron spectra, alongside Galactic magnetic field and halo and ISM structure. We show that for plausible injection assumptions the simulations can reproduce the observed Solar neighborhood values. We explicitly account for and explore the roles of a wide range of processes including: anisotropic diffusion and streaming, gyro-resonant plasma instability losses, “adiabatic” CR acceleration, diffusive/turbulent re-acceleration, Coulomb and ionization losses, catastrophic/hadronic losses (and γ-ray emission), Bremsstrahlung, inverse Compton (accounting for time- and space- varying radiation fields), and synchrotron terms. In § 2, we outline the numerical methods and treatment of spectrally-resolved CR populations, and describe our simulation initial conditions. In § 3 we summarize the qualitative results, and explore the effects of each of the different pieces of physics in turn. We also compare with observational constraints and attempt to present some simplified analytic models that explain the relevant scalings. We conclude in § 4.

2 METHODS

2.1 Non-CR Physics

The simulations here extend those in several previous works including Chan et al. (2019), Hopkins et al. (2020e) (Paper I), and Hopkins et al. (2020b) (Paper II), where additional numerical details are described. We only briefly summarize these and the non-CR physics here. The simulations are run with GIZMO (Hopkins 2015), in its meshless finite-mass MFM mode (a mesh-free finite-volume Lagrangian Godunov method). All simulations include magneto-hydrodynamics (MHD), solved as described in (Hopkins & Raives 2016; Hopkins 2018) with fully-anisotropic Spitzer-Braginskii conduction and viscosity (implemented as in Paper II; see also Hopkins et al. 2017; Su et al. 2017). Gravity is solved with adaptive Lagrangian force softening (matching hydrodynamic and force resolution). We treat cooling, star formation, and stellar feedback following the FIRE-2 implementation of the Feedback In Realistic Environments (FIRE) physics (all details in Hopkins et al. 2018b); as noted in § 3.2 our conclusions are robust to variations in detailed numerical implementation of FIRE. We explicitly follow the enrichment, chemistry, and dynamics of 11 abundances (H, He, Z, C, N, O, Ne, Mg, Si, S, Ca, Fe; Colbrook et al. 2017; Escala et al. 2018); gas cooling chemistry from ~10 − 106 K accounting for a range of processes including metal-line, molecular, fine-structure, photo-electric, and photo-ionization, including local sources and the Faucher-Giguère et al. (2009) meta-galactic background (with self-shielding) and tracking detailed ionization states; and star formation in gas which is dense (> 1000 cm−3), self-shielding, thermally Jeans-unstable, and locally self-gravitating (Hopkins et al. 2013b; Grudić et al. 2018a). Once formed, stars evolve according to standard stellar evolution models accounting explicitly for the mass, metal, momentum, and energy injection via individual SNe (Ia & II) and O/B or AGB-star mass-loss (for details see Hopkins et al. 2018a), and radiation (including photo-electric and photo-ionization heating and radiation pressure with a five-band radiation-hydrodynamic scheme; Hopkins et al. 2020d). Our initial conditions (see Fig. 1) are fully-cosmological “zoom-in” simulations, evolving a large box from redshifts z ≥ 100, with resolution concentrated in a ~1 − 10 Mpc co-moving volume centered on a “target” halo of

2 A public version of GIZMO is available at http://www.tapir.caltech.edu/~phopkins/Site/GIZMO.html
Figure 1. Mock images of the simulations studied here, selected as Milky Way (MW)-like galaxies near $z \approx 0$ from the FIRE cosmological simulation project. The three galaxies are $m12i$ (our “fiducial” galaxy, top), $m12f$ (middle), and $m12m$ (bottom), all broadly MW-like but different in detail with e.g. different extended outer gas/stellar disks, different bar/spiral arm strengths, and different detailed spatial distribution of gas & star formation within the disk. **Left:** Hubble Space Telescope-style $ugr$ composite image ray-tracing starlight (attenuated by dust in the simulations as Hopkins et al. 2004) with a log-stretch ($\sim 4$ dex surface-brightness range). **Middle:** Gas portrayed with a 3-band volume render showing “hot” ($T \gg 10^5$ K, red), “warm/cool” ($T \sim 10^4 - 10^5$ K, green), and “cold (neutral)” ($T \ll 10^4$ K, magenta) phases. **Right:** Gas again, but on larger scales, more clearly showing the continuing gas distribution well into the circum-galactic medium and “halo” up to $\gtrsim 100$ kpc beyond the galactic disk.

Interest. While there are many smaller galaxies in that volume, for the sake of clarity we focus just on the properties of the “primary” (i.e. best-resolved) galaxies in each volume.

2.2 CR Physics & Methods

2.2.1 Overview & Equations Solved

Our CR physics implementation essentially follows the combination of Paper II & Hopkins (2021) with Girichidis et al. (2020). We explicitly evolve the CR distribution function (DF): $f = f(x, p, t, s, ...)$, as a function of position $x$, CR momentum $p$, time $t$, and CR species $s$. We assume a gyrotrropic DF for the phase angle $\phi$ and evolve the first two pitch-angle ($\mu \equiv p \cdot b$) moments of the focused CR transport equation (Isenberg 1997; le Roux et al. 2001), to leading order in $O(u/c)$ (where $u$ is the fluid velocity) for an arbitrary $f = f(p, \mu, ...)$. From Hopkins (2021), this gives the
equations solved:
\[
D_f \partial_t \nabla \cdot (\nu \mathbf{b} \partial_t \mathbf{f}_j) = j_0 + D \cdot \nabla \mathbf{u} \left( \frac{3j_0 + p \partial \mathbf{f}_0}{\partial \mathbf{p}} \right)
\]
(1)
\[
D_f \partial_t \mathbf{f}_j + \nabla \cdot (\mathbf{b} \partial_t \mathbf{f}_j) = \frac{1}{\nu^2} \frac{\partial}{\partial \mathbf{p}} \left[ \nu^2 \left( S_{f_j} \mathbf{f}_0 + D_{pp} \mathbf{f}_j + D_{pp} \frac{\partial \mathbf{f}_0}{\partial \mathbf{p}} \right) \right]
\]
(2)
where \(j_0 \equiv (\mathbf{v}^2 \mathbf{f}_0) / \nu^2\) is the n’th pitch-angle moment (so e.g. \(j_0\) is the isotropic part of the DF, and \(j_1 = (\mathbf{v}_1 \mathbf{f}_0)\), \(D \equiv \partial \mathbf{X} + \nabla \cdot (\mathbf{u} \mathbf{X}) \equiv \rho d \mathbf{X} / \rho\) is the conservative co-moving derivative, \(v = \gamma \beta m_c e\) the CR momentum, \(\mathbf{b} \equiv B / |B|\) is the unit magnetic field vector, \(j_0\) represent injection & catastrophic losses, \(S_{f_j}\) represents continuous loss processes described below, \(\nu\) is Alfven speed, the coefficients \(D\) are defined in terms of the scattering rate \(\nu = \nu_v + \nu_p\), the signed \(\nu \equiv (\nu_v - \nu_p) / (\nu_v + \nu_p)\), and the operator \(D(\mathbf{q}) \equiv \mathbf{b} \cdot \nabla (\mathbf{q} + \mathbf{v} \cdot \{1 - 3 \nu \mathbf{q} \mathbf{b}\}.)\) and Eddington tensor \(\mathbf{D} \equiv \hat{\mathbf{e}} (1 + 3 \nu \mathbf{b} \mathbf{b})\) defined in terms of \(\mathbf{e}\):
\[
\mathbf{e} \equiv \frac{1}{2} - \frac{1}{2} \left[ 1 - \frac{\mathbf{f}_j}{\mathbf{f}_0} \right]
\]
(3)
where \(\langle \mu \rangle \equiv \langle \mathbf{f}_j / \mathbf{f}_0\rangle\) and the moments hierarchy for \(\mathbf{f}_j\) is closed by the interpolated M1-like relation \((\langle \mu \rangle \approx (3 + 4 \langle \mu \rangle \sqrt{1 + 2 \sqrt{2} - 3 \langle \mu \rangle^2}) / 5\rangle\), which captures the exact behavior in both the “free streaming” or weak-scattering and isotropic-DF or strong-scattering limits (Hopkins 2021). All of the variables above should be understood to be functions of \(\mathbf{x}\) and \(t\). The CRs act on the gas-radiation field as well: the appropriate collisional/radiative terms are either thermalized or added to the total radiation or magnetic energy, and the CRs exert forces on the gas in the form of the Lorentz force (proportional to the perpendicular CR pressure gradient) and parallel force from scattering, as detailed in Paper II and Hopkins (2021). Note as defined therein the CR pressure tensor \(P = f d^4p \langle \rho \mathbf{v} \mathbf{v} \rangle\) is anisotropic following \(D\).

2.2.2 Spatial Evolution & Coupling to Gas
Operator-splitting (1) spatial evolution, (2) momentum-space operations, and (3) injection, the spatial part of Eqs. 1-2 can be written as a normal hyperbolic/conservation law for \(\mathbf{f}_0\); \(D_f \mathbf{f}_0 = -\nabla \cdot (\nu \mathbf{b} \mathbf{f}_0)\), and Eq. 2 for the flux \(\mathbf{f}_j\). That is discretized and integrated on the spatial mesh defined by the gas cells identically in structure to our two-moment formulation for the CR number density or energy and their fluxes from e.g. Paper II and Chan et al. (2019); Hopkins (2021), and solved with the same finite-volume method. Because the detailed form of the scattering rates \(\nu\) are orders-of-magnitude uncertain (see review in Paper II), we neglect details such as boundary flux terms and differences in diffusion coefficients for number and energy across the finite width of a momentum bin (i.e. use the “bin centered” \(\nu\)). At this level, the spatial equations for \(\mathbf{f}_0, \mathbf{f}_j\) are exactly equivalent to the two-moment equations for e.g. \((n, E_n)\) or \((E, \mathbf{F})\) (where \(F\) is the flux of \(q\) in Hopkins (2021), integrated separately for each \(j, n, s\). Per Paper II and Hopkins (2021), it is convenient to write the CR forces on the gas in terms of “bin integrated” variables, which can then be integrated into the Reimann solver or hydrodynamic source terms. Performing the relevant integrals within each bin \(n\) for species \(s\), to obtain the total energy \(e_n = \int_\nu 4\pi p^2 \rho^2 dE(p)\), total energy flux \(F_{E,n,s} = \int_\nu 4\pi p^2 \rho \mathbf{v} dE(p)\mathbf{v}\) and scalar isotropic-equivalent pressure \(P_{E,n,s} = \int_\nu 4\pi p^2 dE(p)/\nu\), the force on the gas can then be represented as a sum over all bins:
\[
D_i (\mathbf{p} \mathbf{u}) + \ldots = \sum_s \sum_n \left[ -\left[ \nu \mathbf{D}_{n,s} \right] \right] \cdot \left[ \nabla \cdot \left( \mathbf{D}_{n,s} \mathbf{f}_n \right) \right]
\]
(6)
similar to the procedure above, but remove the losses rather than transferring them to the neighboring bin.

In this paper we consider spectra of protons, nuclei, electrons, and positrons, with 11 intervals/bins for each lepton species and 8 intervals for each hadronic. For leptons these intervals span rigidities (1e-3 - 5.62e-3, 5.62e-3 - 1.78e-2, 1.78e-2 - 5.62e-2, 5.62e-2 - 1.78e-1, 1.78e-1 - 5.62e-1, 5.62e-1 - 1.78, 1.78 - 5.62, 5.62 - 178, 178 - 1000) GV. For hadronic species the ranges are identical but we do not explicitly evolve the three lowest-R intervals because these contain negligible energy and are highly non-relativistic. This corresponds to evolving CRs with kinetic energies over a nearly identical range for nuclei and leptons from < 1 MeV to > 1 TeV.

2.2.4 Injection & First-Order Acceleration

By definition our treatment of the CRs averages over gyro orbits (assuming gyro radii are smaller than resolved scales), so first-order Fermi acceleration cannot be resolved but is instead treated as an injection term \( j \). Algorithmically, injection is straightforward and treated as in Paper II, generalized to the spectrally-resolved method here: sources (e.g. SNe, AGN) inject some CR energy and number into neighbor gas cells alongside radiation, mechanical energy, metals, etc. We simply assume an injection spectrum (and ratio of leptons-to-hadrons injected, and use it to calculate exactly the \( \Delta E_{\text{inj}} \) and \( \Delta N_{\text{inj}} \) injected in a cell given the desired total injected CR energy \( \Delta E_{\text{inj}} = \sum \Delta E_{\text{inj}} \).

The relative normalization of the injection spectra for heavier species \( s \) (relative to \( p \) or \( e^- \)) is set by assuming the test-particle limit, given the abundance of that species \( N_s \) within the injection shock, e.g. \( dN_s/d\beta = (N_s / N_{\text{ISMF}}) dN/d\beta \). This is only important for CNO, as the primary injection of other species we follow (beyond \( p \) and \( e^- \)) is negligible. Because the acceleration is un-resolved, to calculate the ratio of heavy to light particle (nuclei \( N_s / N_{\text{ISMF}} \)), we need to make some assumption about where/when most of the acceleration occurs: for example, for pure core-collapse SNe ejecta (averaging over the IMF), \( N_{s/1}/N_{\text{ISMF}} \sim 0.015 \), while for the ISM at Solar abundances \( N_s/\text{ISM}/N_{\text{H}/\text{ISM}} \sim 0.0005 \). For initial injection mass \( M_{\text{ej}} \), if we assume most of the acceleration occurs at some time when the swept-up ISM mass passing through the shock (which increases rapidly in time) is \( M_{\text{ISMF}} \), then \( N_s \) \( \approx (N_s / N_{\text{ISMF}} M_{\text{ISMF}})/(N_{\text{ISMF}} M_\odot + N_s / N_{\text{ISMF}} M_{\text{ISMF}}) \) (where \( N_s \) \( \equiv dN_s/dM \) is the number of species \( s \) in the ejecta or ISM, per unit mass).

Equivalently we could write this in terms of the shock velocity relative to its initial value, assuming we are somewhere in the energy-conserving Sedov-Taylor phase. In either case, \( N_{s/1} \) and \( N_s \) are given by the abundances of the stellar ejecta and the ISM gas cell into which the CRs are being injected, which follow the detailed FIRE stellar evolution and yield models and reproduce extensive metallicity studies of galactic stars and the ISM (Ma et al. 2016, 2017b;; Muratov et al. 2017; Escala et al. 2018; Bonaca et al. 2017; de Voort et al. 2018; Wheeler et al. 2019).

2.3 CR Loss/Gain Terms Included

Our simulations self-consistently include adiabatic/turbulent/convective terms, diffusive re-acceleration, “streaming” or gyro-resonant losses, Coulomb, ionization, catastrophic/hadronic/fragmentation/pionic and other collisional, radioactive decay, annihilation, Bremsstrahlung, inverse Compton, and synchrotron terms, with the scalings below.

\[ \text{2.3.1 Collisional & Radiative Losses} \]

For protons and nuclei, we include Coulomb and ionization losses, catastrophic/collisional/fragmentation losses, and radioactive decay. Coulomb and ionization losses scale essentially identically with energy as \( T(p) = -3.1 \times 10^{-7} \text{ eV cm}^2 \text{ s}^{-1} N_{\text{neut}} / [n_e + 0.57 N_{\text{neut}}] \) (the difference being whether they operate primarily in ionized or neutral gas; Gould 1972), where \( n_e \) is the free (thermal) electron density (the Coulomb term), and \( N_{\text{neut}} \) the neutral number density (ionization term). For protons, we take the total pion/catastrophic loss rate to be \( f_s(p) = -6.37 \times 10^{-16} \text{ cm}^{-3} \text{ s}^{-1} N_{\text{neut}} \Theta(T[p]) - 0.28 \text{ GeV} f_s(p) \) (Mannheim & Schlickeiser 1994; Guo & Oh 2008), where \( n_e \) is the nuclear number density (\( \approx \rho / m_p \), \( \Theta(x) = 0 \) for \( x < 0 \), \( x = 1 \) for \( x \geq 1 \)). For heavier nuclei, we take the total fragmentation/catastrophic loss rate to be \( f_s(p) = -n_e \beta c \sigma_s(p) \) with \( \sigma_1 = 4.5^{1.1} (1 + 0.016 \sin(1.3 - 2.631 n_A)) \) mb (with \( A \) the atomic mass number) at \( \geq 2 \text{ GeV} \) and \( \sigma_r = \sigma_r(> 2 \text{ GeV}) = 1 - 0.62 \exp(-T/0.2 \text{ GeV}) \sin(1.57553[T/\text{GeV}]) \) at \( < 2 \text{ GeV} \) from Mannheim & Schlickeiser (1994), with the cross-sections for secondary production of various relevant species described below. For radioactive species, the loss rate scales as \( f_s(p) = f_s(p)/(\gamma_{\text{t/2}} \text{, } ln/2) \), where \( \gamma_{\text{t/2}} \) is the rest-frame half-life of the species.

For electrons and positrons, we include Bremsstrahlung, ionization, Coulomb, inverse Compton, and synchrotron losses. At our energies of interest we always assume electrons/positrons are relativistic for the calculation of loss rates. For Bremsstrahlung we take \( p = -(3/2 \pi) \alpha_{\text{e}} \sigma_{\text{e}} \gamma_1 n_e \text{ln}(2 \gamma_1^2 + 1)/\alpha_{\text{e}} \) (Gould & Burbridge 1965), while for Coulomb we have \( p = -(3/4 \pi) m_e c^2 \sigma_{\text{n}} \beta^2 \text{ln}(m_e c/\beta \sqrt{1 - \hbar \omega_e}) + \text{ln}(2)^2 (\beta^2 / 2 + 1/\gamma_1) + 1/2 + (\gamma_1 - 1)^2 / (16 \gamma_1^2) \) with the plasma frequency \( \omega_p \equiv 4 \pi e^2 n_e / m_e \) (Gould 1972). Ignoring Klein-Nishina corrections (unimportant at the energies of interest), for inverse Compton and synchrotron we have \( p = -(4/3 \pi) \gamma_1 n_e \alpha_{\text{u}} (u_{\text{a}} + u_{\text{b}}) \) (e.g. Rybicki & Lightman 1986), where \( u_{\text{a}} + u_{\text{b}} \) are the local radiation energy density and magnetic field energy density (given self-consistently from summing all five [ionizing, FUV, NUV, optical/NIR, IR] bands followed in our radiation-hydrodynamics approximation in-code, plus the un-attenuated CMB, and from our explicitly-evolved magnetic fields). Positron annihilation is treated akin to the catastrophic loss terms above with \( f_s(p) = -n_e \beta c \sigma_{\gamma e} f_s(p) \) with the Dirac \( \sigma_{\gamma e} \) \( \approx \pi n_e \gamma_1 (\gamma_1^2 - 1) (\gamma_1^2 + 4 \gamma_1 + 1) (\gamma_1^2 - 1)^{-1} \text{ln}(\gamma_1 + \sqrt{\gamma_1^2 - 1}) - (\gamma_1 + 3)(\gamma_1^2 - 1)^{-1/2} \) where \( \gamma_1 \) is the positron Lorentz factor in the electron frame and \( n_e \) is the classical electron radius.

Following Paper II and Guo & Oh (2008), the Coulomb losses and a factor \( = 1/6 \) of the hadronic losses (from thermalized portions of the cascade) are thermalized (added to the gas internal energy), while a portion of the ionization losses are thermalized corresponding to the energy less ionization potential. Other radiative and collisional losses are assumed to go primarily into escaping radiation.

\[ \text{2.3.2 Secondary Products: Fragmentation & Decay} \]

Our method allows for an arbitrary set of primary species, each of which can decay to an arbitrary set of secondary species: energy and particle number are transferred bin-to-bin in secondary-producing reactions akin to the bin-to-bin fluxes within a given species described above. For computational reasons, however, it is imprac-
tactical to integrate a detailed extended chemical network like those in codes such as GALPROP or DRAGON on-the-fly. We therefore adopt an intentionally highly-simplified network, intended to capture some of the most important secondary processes. In our “extended” networks, we evolve spectra for \( e^−, e^+, \bar{\nu}, \) and nuclei for H (protons), B, CNO, stable Be (\( ^7\)Be + \( ^{10}\)Be) and unstable \( ^{11}\)Be. For radioactive decay, we consider \( ^{10}\)Be, \( ^{10}\)B with \( t_{1/2} = 1.51\) Myr. We consider secondary \( e^− \) and \( e^+ \) produced by protons via pion production, with standard branching ratios (\( \sim 1/3 \) to each) and because our spectral “bins” are relatively coarse-grained assume the (local power-law) energy distribution of the injected leptons from a given proton bin traces the distribution in the proton bin shifted by the expected mean factor \( \approx 0.12 \) (the weighted mean given by integrating over the spectra of secondary energies at the scales of interest; see Moskalenko & Strong 1998; Di Bernardo et al. 2013; Reintert & Winkler 2018). We include a small systematic correction for production via heavier nuclei, which is generally sub-dominant to uncertainties in cross-sections. We similarly treat the production rate for \( \bar{\nu} \) from p and heavier nuclei using \( \sigma_{\bar{\nu}p} \approx 1.5 \text{ mb} \) (\( \approx 107.9 + 29.43x - 1.655x^2 + 189.9e^{-x/3} \) (with \( x \equiv \ln(R/GV) \)) with a again weighted-mean energy \( \approx 0.1 \) of the primary (di Mauro et al. 2014; Winkler 2017; Korsmeier et al. 2018; Evoli et al. 2018, and references therein).

The vast majority of B and Be stem from fragmentation of C, N, and O. Rather than follow C, N, and O separately, since their \( \sim 10^7 \) scales of interest; see Moskalenko & Strong 1998; Di Bernardo et al. 2013; Reintert & Winkler 2018). We include a small systematic correction for production via heavier nuclei, which is generally sub-dominant to uncertainties in cross-sections. We similarly treat the production rate for \( \bar{\nu} \) from p and heavier nuclei using \( \sigma_{\bar{\nu}p} \approx 1.5 \text{ mb} \) (\( \approx 107.9 + 29.43x - 1.655x^2 + 189.9e^{-x/3} \) (with \( x \equiv \ln(R/GV) \)) with a again weighted-mean energy \( \approx 0.1 \) of the primary (di Mauro et al. 2014; Winkler 2017; Korsmeier et al. 2018; Evoli et al. 2018). The number flux into the relevant secondary network using \( \sigma_{B\rightarrow\gamma\nu e} \approx 12T^{-0.024} \) mb and \( \sigma_{B\rightarrow\gamma\nu e} \approx 12T^{-0.018} \) mb.

### 2.3.3 Adiabatic and Streaming/Gyro-Resonant/Re-Acceleration Terms

From the focused-transport equation and quasi-linear scattering theory, there are three “re-acceleration” and/or second-order Fermi (Fermi-II) terms, all of which we include: the “adiabatic” or “convective” term \( D : \nabla u \), the “gyro-resonant” or “streaming” loss term \( \propto D_{pp} \) and the “diffusive” or “micro-turbulent” reacceleration term \( \propto D_{pp} \). As shown in Hopkins (2021), these can be re-written in momentum space after gyro and pitch-angle averaging:

\[
\frac{\dot{p}}{p} = -D : \nabla u - \langle \mu D_{pp} \rangle + 2(1 + \beta^2) D_{pp} \frac{\dot{p}}{p^2} \tag{7}
\]

The physical nature and importance of these is discussed below and in detail in Hopkins (2021), but briefly, this includes all “re-acceleration” terms to leading order in \( \mathcal{O}(\alpha/\epsilon) \), and generalize the expressions commonly seen for these. The “adiabatic” (non-intertial frame) term reduces to the familiar \( -\nabla(B/\mu) \) as the DF becomes isotropic (\( \chi \rightarrow 1/3, D \rightarrow 1/3 \)), but extends to anisotropic DFs and is valid even in the zero-scattering limit. The \( D_{pp} \) term produces a positive-definite momentum/energy gain \( \dot{p}/p = 2\chi(1 + \beta^2)v^2/\nu^2 \), for the commonly adopted assumptions that give rise to the isotropic strong-scattering Fokker-Planck equation for CR transport we would have \( \dot{D}_{pp} \rightarrow \nu p^2/9D_{pp} \), and recover the usual “diffusive re-acceleration” expressions, but again the term here is more general, accounting for finite \( \beta \) and weak-scattering/anisotropic-\( f(\mu) \) effects. The \( D_{pp} \) term is often ignored in historical MW CR transport models (which implicitly assume \( \dot{\nu} = \nu = \) 0) but this gives rise to the “gyro-resonant” or “streaming” losses (Wiener et al. 2013b,a; Ruszkowski et al. 2017; Thomas & Pfrommer 2019). Specifically, since gyro-resonant instabilities/perturbations are excited by the CR flux in one direction (and damped in the other), if these contribute non-negligibly to the scattering rates then generally \( \dot{\nu}_\perp \gg \nu_v \) or \( \nu_\parallel \ll \nu_v \), so \( \nu_i \approx \nu_v \), in which case the \( D_{pp} \) term is always almost dominant over the \( D_{pp} \) term dimensionally. In this regime (i.e. if \( \dot{\nu}_\perp \ll \nu_v \)), then in flux-steady-state (\( D_{f\dot{f}} = 0 \)) the combined \( D_{pp} \), and \( D_{pp} \) term in Eq. 7 becomes negative-definite with \( \dot{p}/p \approx -\nu_\perp/3\nu_\parallel \) where \( \nu_\parallel \equiv \nu_v/|\nabla \nu_v| \).

Given the CR energies of interest, in our default simulations we will assume self-confinement contributes non-negligibly (or other effects prevent \( \dot{\nu}_\perp, \nu_\parallel \) cancellation; see § 3.3) so \( \nu_i = \nu_v/|\nabla \nu_v| \), self-consistently including all terms in Eq. 7. We run and discuss alternate tests with \( \nu_\perp \rightarrow 0 \) and different expressions for \( D_{pp} \) or the “diffusive reacceleration” terms but generically find none of these change our conclusions.

### 2.4 Default Input Parameters (Model Assumptions)

We vary the physics and input assumptions in tests below, but for reference, the default model inputs assumed are as follows.

#### 2.4.1 Injection

By default we assume all SNe (Types Ia & II) and fast (OB/WR) winds contribute to Fermi-I acceleration with a fixed fraction \( e_{vf} \sim 0.1 \) of the initial (pre-shock) ejecta kinetic energy going into CRs (and a fraction \( e_s \sim 0.02 \) of that into leptons). We adopt a single-power-power-law injection spectrum in momentum/rigidity with \( j(R) \propto R^{-s} \) and \( \psi_{inj} > 4.2 \) (i.e. a “canonical” predicted injection spectrum; discussed in detail in § 3.1). We will assume most acceleration happens at early stages after a strong shock forms, when the shocks have their highest velocity/Mach number and the dissipation rates are also highest – this occurs after the reverse shock forms, roughly when the swept-up ISM mass is about equal to the ejecta mass (\( M_{\text{ejecta}} \approx M_{\text{SN}} \)). Equivalently, given that most of the shock energy injected into the ISM, and therefore CR energy, comes from core-collapse SNe, we obtain nearly-identical results if we instead assume that the injection is dominated by shocks with velocity \( \geq 2000\text{km/s} \). We show below that slower (e.g. ISM or AGB, or late-stage Sedov/snowplow SNe) shocks cannot contribute significant Fermi-I acceleration of the species followed.

#### 2.4.2 Scattering Rates

In future studies we will explore physically-motivated models for scattering rates for all of local plasma properties, pitch angle, gyro-radius, etc. But in this first study we restrict to simple phenomenological models, where we parameterize by default the (pitch-angle-weighted) scattering rates as a single power-law \( \dot{\nu} = \nu_0 |\epsilon|/\epsilon (R/R_0)^{-\delta} \) with \( R_0 \equiv 1\text{Gyr} \) (e.g. \( \nu \propto 1/\ell_{\text{scattering}} \) where \( \ell_{\text{scattering}} \propto \mathcal{R} \) is some characteristic length). In the strong-scattering flux-steady-state limit, this gives a parallel diffusivity \( \kappa_p = v^2/3\dot{\nu} \) or, in the isotropic Fokker-Planck equation \( \partial^2 f = \nabla \cdot (D_\nu \nabla f) \), \( D_\nu = v^2/9\dot{\nu} \), so reduces to the common assumption in phenomenological Galactic CR models that \( D_\nu = \beta D_\nu(R/R_0)^\delta \). Here our default models take \( \dot{\nu}_0 \approx 10^{-8}s^{-1}, \delta = 0.5 \), equivalent to \( D_\nu(R = 1\text{Gyr}) \approx 10^{26}\text{cm}^2\text{s}^{-1} \).
Figure 2. Example CR spectra and secondary-to-primary ratios from our full live galaxy-formation simulation at redshift $z = 0$ in a “working model” with a single-power-law injection spectrum (with all abundance ratios at injection determined in-code) and fiducial scaling of the scattering rates $\rho \sim 10^{-3} s^{-1}$, see § 3.1 for details. Top: CR intensity (left) and kinetic energy density (right) spectra, for different species: protons $p$, anti-protons $\bar{p}$, electrons $e^-$, positrons $e^+$, B, C, radioactive ($^{10}$Be) and stable ($^{7}$Be+$^{9}$Be) Be. Lines show the median (dashed) and mean (solid) values in the simulation (small-scale “features” are artifacts of our spectral sampling), and shaded range shows the $\pm 1 \sigma$ range, for ISM gas in the Solar circle ($r = 7 - 9 kpc$) with local density $n = 0.3 - 3 \text{cm}^{-3}$. Points show observations (colors denote species), from the local ISM (LISM) from Voyager (circles; Cummings et al. 2016), AMS-02 (squares; Aguilar et al. 2018, 2019a,b, and references therein), and compiled from other experiments including PAMELA, HEAO, BESS, TRACER, CREAM, NUCLEON, CAPRICE, Fermi-LAT, CALET, HESS, DAMPE, ISOMAX (pentagons; Engelmann et al. 1990; Shikaze et al. 2007; Boezio et al. 2000; Obermeier et al. 2011; Adriani et al. 2014; Abdollahi et al. 2017; Boezio et al. 2017; H. E. S. Collaboration et al. 2017; Yoon et al. 2017; DAMPE Collaboration et al. 2017; Adriani et al. 2018; Atkin et al. 2019). For the non-Voyager data we omit observations at energies where the Solar modulation correction is estimated to be important (see Bindi et al. 2017; Bisschoff et al. 2019, and references therein). Middle Left: $^{10}$Be/$^{9}$Be ratio, in the same ISM gas; dark purple (light cyan) shaded range shows the $\pm 1 \sigma (\pm 2 \sigma)$ range, lines show median (dashed) and mean (solid). Points show compiled AMS-02 and other-experiment data (references above). Middle Right: $B/C$ ratio. For the Voyager data, we show both the directly observed values and the “modulation-corrected” value from Strong et al. (2007) who consider models where modulation could still be important for V1 data (note this would also reduce the value of $B/C$ observed at $\sim 1 \text{GeV}$). Bottom Left: $\mu/p$ ratio. Note the value at the highest-energies is significantly affected by our spectral upper boundary (we do not evolve $p$ or heavier ions with rigidity $\gtrsim 1000 \text{GV}$). Bottom Right: $e^+ / (e^- + e^+)$ ratio. We stress that we have not marginalized over parameters or “fit” to any of these observations, but simply survey a few model choices and show one which gives overall agreement.
2.5 Initial Conditions

In a follow-up paper, we will present full cosmological simulations from \( z \approx 100 \), as in our previous single-bin CR studies (see Hopkins et al. 2020a,b,c; Ji et al. 2020a,b and Paper II). These, however, are (a) computationally expensive, and (b) inherently chaotic owing to the interplay of N-body+hydrodynamics+stellar feedback (Su et al. 2017, 2018b; Keller et al. 2019; Genel et al. 2019), which makes it difficult if not impossible to isolate the effects of small changes in input assumptions (e.g. the form of \( v(R) \)) and to ensure that we are comparing to a “MW-like” galaxy. Because we focus on Solar-neighborhood observations, we instead in this paper adopt a suite of “controlled restarts” as in Orr et al. (2018); Hopkins et al. (2018b); Angles-Alcazar et al. (2020). We begin from a snapshot of one of our “single-bin” CR-MHD cosmological simulations from Paper II, which include all the physics here but treat CRs in the “single-bin” approximation from § 1. Illustrations of the stars and gas in these systems are shown in Fig. 1. Per § 1, these initial conditions have been extensively compared to MW observations to show that they broadly reproduce quantities important for our calculation like the Galaxy stellar and gas mass in different phases (El-Badry et al. 2018b; Hopkins et al. 2020e; Gurvich et al. 2020), molecular and neutral gas cloud properties and magnetic field strengths (Guszejnov et al. 2020b; Benincasa et al. 2020), gas disk sizes and morphological/kinematic structure (El-Badry et al. 2018a; Garrison-Kimmel et al. 2018), SNe and star formation rates (Orr et al. 2018; Garrison-Kimmel et al. 2019b), \( \gamma \)-ray emission properties (provided reasonable CR model choices; Chan et al. 2019; Hopkins et al. 2020b), and circumb-galactic medium properties in different gas phases (Faucher-Giguere et al. 2015; Ji et al. 2020b), suggesting they provide a reasonable starting point here. We take galaxy \( m_{12I} \) (with the initial snapshot from the “\( CR+(3 = 3e29) \)” run in Paper II) as our fiducial initial condition, though we show results are similar for galaxies \( m_{12F} \) and \( m_{12M} \).

We re-start that simulation from a snapshot at redshift \( z \approx 0.05 \), using the saved CR energy in every gas cell to populate the CR DF for all species, assuming an initially isotropic DF with the initial spectral shape and relative normalization of different species all set uniformly to fits to the local ISM (LISM) spectra (Bisschoff et al. 2019). The spectra are re-normalized to match the snapshot CR energy density\(^4\) before beginning, to minimize any initial perturbation to the dynamics. We then run for \( \approx 500 \) Myr to \( z = 0 \). This is more than sufficient for all quantities in the local ISM (LISM), which we use interchangeably with warm-phase ISM gas at Solar-like galactocentric radii and densities \( \sim 0.1 - 1 \) cm\(^{-3} \) to reach their quasi-steady-state values – only in the further CGM at \( > 10 \) kpc from the galaxy do CR transport timescales exceed \( \sim \) Gyr.\(^5\)

3 RESULTS & DISCUSSION

3.1 Working Models

3.1.1 Parameters: Single Power-Law Injection & Diffusion Can Fit the Data

The first point worth noting is that it is actually possible to obtain reasonable order-of-magnitude agreement with the Solar neighborhood CR data, as shown in Fig. 2. This may seem obvious, but recall that the models here have far fewer degrees of freedom compared to most historical Galactic CR population models: the Galactic background is entirely “fixed” (so e.g. Alfven speeds, magnetic geometry, radiative/Coulomb/ionization loss rates, convective motions, re-acceleration, etc. are determined, not fit or “inferred” from the CR observations); we assume a universal single-power-law injection spectrum (with just two parameters entirely describing the injection model for all species) and do not separately fit the injection spectra for different nuclei but assume they trace the injection of protons given their \( a \) priori abundances in the medium; and we similarly assume a single power-law scattering rate \( v(R) \) as a function of rigidity to describe all species.

In our favored model(s), the injection spectrum for all species is a single power-law with \( d j \propto \rho^{-1.2} d \rho \) (with all heavy species relative abundance following their actual shock abundances), with \( \approx 10\% \) of the shock energy into CRs and \( \approx 2\% \) of that into leptons, and the scattering rate scales as \( \dot{v} \approx 10^{-9} s^{-1} \beta R_{CR}^{0.5 - 0.6} \).

Under the assumptions usually made to turn the CR transport equations into an isotropic Fokker-Planck diffusion equation, this corresponds to \( D_{\nu} \approx D_{\nu} R_{\nu}^{0.5 - 0.6} \), with \( \delta \) in the range \( 0.5 - 0.6 \) and \( D_{\nu} \approx 10^{3} \) cm\(^2\) s\(^{-1}\). Briefly, we note in Fig. 2 that the largest statistical discrepancy between the simulations and observations appears to be between the flat values of \( \dot{p}/p \) at high energies \( \sim 10 - 300 \) GV, where our model \( \pm 1\sigma \) predictions continue to rise by another factor \( \sim 2 - 3 \). This is generically the most difficult feature to match, of those we compare, while simultaneously fitting all other observations, and we will investigate in more detail in future work. It is interesting in particular because it runs opposite to the recent suggestion that reproducing \( \dot{p}/p \) alongside B/C requires some “additional,” potentially exotic (e.g. decaying dark matter) source of \( p \) (Heising 2020). But we caution against over-interpretation of our result for several reasons: (1) the systematic detection/completeness corrections in the data and (2) the physical \( \dot{p} \) production cross-sections at these energies remain significant sources of uncertainty (Cuoco et al. 2019; Heising et al. 2020); (3) the observations still remain within the \( \pm 2\sigma \) range, so the LISM may simply be a \( \sim 2\sigma \) fluctuation; (4) recalling that the energy of a secondary \( \dot{p} \) is \( \sim 10\% \) the primary \( p \), most of this discrepancy occurs at such high energies that it depends sensitively on the behavior of our highest-energy \( p \) and C bins – i.e. our “boundary” bins; and (5) we are only exploring empirical models with a constant (in space and time) scattering rate, while almost any physical model predicts large variations in \( \dot{v} \) with local ISM environment, which can easily produce systematic changes in secondary-to-primary ratios at this level (Hopkins et al. 2020b).

3.1.2 Comparison to Idealized, Static-Galaxy Analytic CR Transport Models

The “favored” parameters (those which agree best with the observations) above in § 3.1.1 are completely plausible. The injection spectrum (\( \nu_{inj} \approx 4.2 \)) is essentially identical to the “canonical” theoretically-predicted injection spectrum and efficiency for first-order Fermi acceleration in SNe shocks (Bell 1978; Malkov & Drury 2001; Spitkovsky 2008; Caprioli 2012). Considering how different the models are in detail, the favored scattering rate in § 3.1.1 and its dependence on rigidity are remarkably similar to the values inferred from most studies in the past decade which have fit the CR properties assuming a simple toy model analytic Galaxy model and isotropic Fokker-Planck equation model for CR transport, provided they allow for a CR “scattering halo” with size \( \sim 5 - 10 \) kpc (Blasi & Amato 2012a; Vladimirov et al. 2012; Gaggero et al. 2015; Guo et al. 2016; Johannesson et al. 2016; Cummings et al. 2016; Korsmeier & Cuoco 2016; Evoli et al. 2017; Amato & Blasi 2018). Consider e.g. de la Torre Luque et al. 2021, who compare the

\(^4\) Throughout this paper, when we refer to and plot the CR “energy density” \( e_{CR} \), we will follow the convention in the observational literature and take this to be the kinetic energy density (not including the CR rest mass energy), unless otherwise specified.

\(^5\) To confirm this, we have also re-started simulations with zero initial CR energy in all cells. This produces a more pronounced initial transient owing to the loss of CR pressure but converges to the same equilibrium after somewhat longer physical time, and all our conclusions are identical.
Figure 3. Comparison of CR spectra as Fig. 2 from the same galaxy initial condition, in gas with $n = 0.3 – 3 \text{ cm}^{-3}$, with different parameters of the injection spectrum $J_{inj} \propto p^{-\psi_{inj}}$ and assumed (universal) CR scattering rate $\nu = \nu_0 \beta R_\text{cr}^{-\delta}$. To reduce clutter, lines+shaded range show just the mean+1σ range for each model, we do not overplot the observations, and for the CR kinetic energy density $d\rho_{\nu}/d\ln T$, we show just $p$ (thick) and $e^+$ (thin). Note the “reference” parameters (about which we vary) are those from our best-fit in Fig. 2 (Appendix A considers variation about an alternative “reference” model). Left: Injection slope $\psi_{inj}$. As expected this shifts CR spectral slopes, but it also shifts secondary-to-primary ratios in a manner not trivially predicted by leaky-box type models. Middle: Scattering rate normalization $\nu_0$. These shift secondary-to-primary ratios qualitatively as expected, but often non-linearly; more surprising, higher $\nu_0$ (lower effective diffusivity) clearly produces systematically harder/shallower CR spectra. Right: Dependence of scattering rate on rigidity $\beta$. This shifts the spectral shape and secondary-to-primary dependence on $T$ roughly as expected, though again slightly non-linearly.

most recent best-fit models from both GALPROP and DRAGON, which both favor a CR scattering halo of scale-height $\sim 7 \text{ kpc}$ with a very-similar $D_{\text{scr}} \sim 0.6 \times 10^{29} \text{ cm}^2 \text{ s}^{-1}$ for $\sim 1 \text{ GV}$ protons and $\delta \sim 0.4 – 0.5$. This is also consistent with a number of recent studies using “single-bin” $\sim \text{ GeV-CR}$ transport models in cosmological galaxy formation simulations of a wide range of galaxy types (Chan et al. 2019; Su et al. 2020; Hopkins et al. 2020b,e,c; Busard & Zweibel 2020), compared to observational constraints from $\gamma$-ray detections and upper limits showing all known dwarf and $L^*$ galaxies lie well below the calorimetric limit (Lacki et al. 2011;
Fu et al. 2017; Lopez et al. 2018), which inferred that a value of $\tilde{\nu} \approx 10^{-5} \text{s}^{-1}$ at $E \approx 1 \text{GeV}$ was required to reproduce the $\gamma$-ray observations.

This is by no means trivial, however. Some recent studies using classic idealized analytic CR transport models have argued that features such as the “turnover” in B/C at low energies or minimum in $e^+ / e^-$ require strong breaks in either the injection spectrum or dependence of $\tilde{\nu}(p)$ (e.g. favoring a $D(p)$ which is non-monotonic in momentum $p$ and rises very steeply with lower-$p$ below $\sim 1 \text{GeV}$; Strong et al. 2011), or artificially strong re-acceleration terms (much larger than their physically-predicted values here) which would imply (if true) that most of the CR energy observed actually comes from diffusive reacceleration, not SNCC or other shocks (Drury & Strong 2017), or some strong spatial dependence of $\tilde{\nu}$ in different regions of the galaxy (Liu et al. 2018). Other idealized analytic transport models (Maurin et al. 2010; Trotta et al. 2011; Blasi 2017; Yuan et al. 2020) have argued for $\delta$ in the range ~0.3 – 1 and some for $\tilde{\nu}$ as large as $\sim 10^{-7} \text{s}^{-1}$ at $\sim 1 \text{GeV}$ (equivalent to $D_0 \sim 10^{27} \text{cm}^2 \text{s}^{-1}$). These go far outside the range of models which we find could possibly reproduce the LISM observations.

The fundamental theoretical uncertainty driving these large degeneracies in previous studies is exactly what we seek to address in this study: the lack of a well-defined galaxy model. In the studies cited above, quantities like the halo size, source spatial distribution, Galactic magnetic field structure and Alfvén speeds, key terms driving different loss processes (ionization, Coulomb, synchrotron, inverse Compton), adiabatic/convective/large-scale turbulent re-acceleration, are all either treated as “free” parameters, or some ad-hoc empirical model is adopted. For example, it is well-known that if one neglects the presence of any “halo”/CGM/thick disk (and so effectively recovers a classic “leaky box” model with sources and transport in a thin $\lesssim 200 \text{pc}$-height disk), then one typically infers a best-fit with much lower $D_0 \sim 10^{27} \text{cm}^2 \text{s}^{-1}$ and “Kolmogorov-like” $\delta \sim 0.3$ (Maurin et al. 2010). At the opposite extreme, assuming the “convective” term has the form of a uniform vertical disk-perpendicular outflow everywhere in the disk (neglecting all local turbulent/fountain/collapse/inflow/bar/spiral and other motions, and assuming a vertically-accelerating instead of decelerating outflow) – the inferred $\delta$ can be as large as ~1 (Maurin et al. 2010). Similarly, in these analytic models one can make different loss and/or re-acceleration terms as arbitrarily large or small as desired by assuming different Alfvén speeds, densities, neutral fractions, etc., in different phases; so e.g. models which effectively ignore or artificially suppress ionization & Coulomb losses will require a break in the injection or diffusion versus momentum $p$, to reproduce the correct observed spectra.

3.1.3 On the Inevitability of the “Halo” Size

One particular aspect requires comment here: in cosmological galaxy formation models, a very large “halo” is inevitable. Indeed, as noted in §1, in modern galaxy theory and observations, the region within < 10 kpc above the disk would not even be called the “halo” but more often the thick disk or corona or disk-halo interface. It is well-established that most of the cosmic baryons associated with galaxies are located in the CGM reaching several hundred kpc from galaxy centers, distributed in a slowly-falling power-law-like (not exponential or Gaussian) profile with scale lengths $\sim 20 – 50 \text{kpc}$ (Maller & Bullock 2004; Steidel et al. 2010; Martin et al. 2010; Werk et al. 2014; Sravan et al. 2016; Tumlinson et al. 2017). This is visually obvious in Fig. 1.

Thus, from a galaxy-formation point of view, it is not at all surprising that models with a large “CR scattering halo” are observationally favored and agree better with realistic galaxy models like those here. What is actually surprising, from the galaxy perspective, is how small the best-fit halo sizes in some analytic Galactic CR transport models (e.g. $\sim 7 – 8 \text{kpc}$, in the references above) actually are. These $\sim 5 – 10 \text{kpc}$ sizes are actually much smaller than the scale length for the free-electron density or magnetic field strength inferred in theoretical and observational studies of the CGM (see references above and e.g. Lan & Prochaska 2020). However, there is a simple explanation for this: as parameterized in most present analytic models for CR transport, the “halo size” does not really represent the scale-length of e.g. the magnetic energy or free-electron density profile; rather, “the halo size” in these models is more accurately defined as the volume interior to which CRs have an order-unity probability of scattering “back to” the Solar position. In the CGM (with sources concentrated at smaller radii), for any spatially-constant diffusivity, the steady-state solution for the CR kinetic energy density is a spherically-symmetric power-law profile with $\epsilon_{\delta,K} \propto \frac{1}{1 + \nu} (\text{Hopkins et al. } 2020)$, so the characteristic length-scale for scattering “back into” some $r = r_0$ is $\approx r_0$. In other words, in any slowly-falling power-law-like medium with spatially-constant diffusivity, the inferred CR scattering “halo scale” length at some distance $R_0 \approx 8 \text{kpc}$ from the source center (e.g. the Solar position) will always be $L_{\text{halo}} \approx R_0$ to within a factor of $\sim 2$ depending on how the halo and its boundaries are defined (and indeed this is what models infer), more or less independent of the actual CGM $n_0$ or B-field scale-length (generally $\gg R_0$).

3.2 Effects of Different Physics & Parameters

We now briefly discuss the qualitative effects of different variations on CR spectra, using tests where we fix all parameters and physics but then “turn off” different physics or adjust different parameters each in turn, with resulting spectra shown in Figs. 3, 4, & 5. Here, our “reference” model is that in Fig. 2. We have considered a set of simulations varying other parameters simultaneously, and in Appendix A, we repeat the exercise in Figs. 3, 4, & 5, but for variations with respect to a different reference model with larger scattering rate and different dependence of scattering on rigidity. This allows us to confirm that all of our qualitative conclusions here are robust.

It is useful to define some reference scalings, by reference to a toy leaky-box type model: if the CR injection rate in some $p$ interval were $d_j = j_0 \left( \frac{p}{p_0} \right)^{-\psi_{\text{inj}}} \frac{d^3p}{p}$, and the CR “residence time” (or escape time) were $\Delta t_{\text{res}} = \Delta t_{\text{res}} \left( \frac{p}{p_0} \right)^{-\psi_{\text{inj}}}$, then the observed number density would scale as $n_{\text{obs}} \propto \Delta t_{\text{res}} \propto \Delta t_{\text{res}} j_0 \left( \frac{p}{p_0} \right)^{-\psi_{\text{res}}} \frac{d^3p}{p}$.

For the more usual units of intensity we have $dI \propto \Delta N/dtd\Delta T \propto p^{\psi_{\text{inj}}} \propto \psi_{\text{inj}} + \frac{p_{\text{res}}}{\psi_{\text{inj}}} - 2$.

- Injection Spectra: As expected, the CR spectral shapes scales with the injection spectrum, shown in Fig. 3. However, the scaling is not perfectly linear as the above toy model would imply: changing the injection $\psi_{\text{inj}}$ by some $\Delta \psi_{\text{inj}}$, we obtain $\Delta \psi_{\text{obs}} \approx (0.7 – 0.9) \Delta \psi_{\text{inj}}$, depending on the CR energy range, species, etc. The issue is that part of the change in assumed slope is offset by (a) losses, (b) non-linear effects of CRs on the medium, and (c) non-uniform source distributions (where e.g. the effective “volume” of sources in a realistic disk sampled by a given $\Delta t_{\text{res}}$, is not $p$-independent, so one needs to convolve over the source distribution at each $p$). Shallower slopes ($\psi_{\text{inj}}$) produce a B/C ratio which is shallower (drops off more slowly) at low energies. More dramatically, in e.g. $e^+ / e^-$, because $e^+$ and $p$ are injected with the same slope and the $e^+$ secondaries have energy $\sim 0.1$ times their $p$ progenitors, a steeper $\psi_{\text{inj}}$ gives a lower value of $e^+/e^-$ at a given $R$ or $E$, and a sharper “kink” in the distribution, while shallower $\psi_{\text{inj}}$ gives a higher $e^+/e^-$ (rising more continuously to low-$E$). The CR kinetic energy density (normalization of the spectra) is slightly sub-linear in the injected CR fraction $\epsilon_{\delta,K}$, as lower CR pressure allows slightly more rapid gas collapse and star formation, raising the
Figure 4. CR spectra (as Fig. 3), removing different loss processes to see their effects. We simplify by focusing on just the spectra, B/C, and \( e^+ / (e^+ + e^-) \), which summarize the key effects. We consider removing, each in turn: **First Row**: (1) the “adiabatic” \( \bar{D} \); \( \nabla u \) gain/loss term in Eq. 1; or (2) catastrophic losses (still allowing secondary production, but no primaries are “destroyed” in the process; **Second**: (3) Bremsstrahlung; (4) inverse Compton; (5) synchrotron; **Third**: (6) Coulomb; (7) ionization; (8) Coulomb and ionization; **Fourth**: (9) the “diffusive reacceleration” \( D_{pp} \) term in Eq. 1; (10) the “streaming loss” \( D_{ps} \) term in Eq. 1. We also consider (11) arbitrarily increasing the diffusive re-acceleration term to a value much larger than physical. **Fifth**: Altering the “streaming speed” \( v_s \) to (a) \( v_s = v_{s, \text{ideal}} = (|B|^2/4\pi\rho)^{1/2} \) (our default, the ideal MHD Alfvén speed), (b) \( v_s = v_{s, \text{ion}} = (|B|^2/4\pi\rho_{\text{ion}})^{1/2} \) (the “ion Alfvén speed,” much faster in mostly-neutral gas, and favored in self-confinement models), and (c) \( v_s = 0 \) (assumed in older extrinsic turbulence models). All these changes are discussed in § 3.2. Generically inverse Compton+synchrotron alter high-energy lepton spectra, Coulomb+ionization alter low-energy spectra, and the effect of the “re-acceleration” terms are modest.
SFR and CR injection rate (see Hopkins et al. 2020e). The lepton-to-hadron ratio injected translates fairly closely to the $e^-/p$ ratio at $\sim 1 - 10$ GeV, for realistic diffusivities where losses are not dominant at $\sim 1$ GeV.

- Scattering Coefficients: Parameterizing the scattering coefficient as: $\nu = \nu_0 \beta R_{\text{GV}}$, recall this corresponds to $D_x \approx \beta D_{\nu} R_{\text{GV}}$ (with $D_0 \approx e^2/9 \nu_0$) in the often-assumed isotropic strong-scattering flux-steady-state negligible-streaming limit. Our preferred model has (in cgs units) $\nu_0 \sim 10^{-7}$ ($D_0 \approx 10^9$), $\delta \sim 0.5 - 0.6$. As shown in Fig. 3, lowering $\delta$ produces a “flatter” (nearly energy-independent) B/C ratio and systematically higher $e^-/e^+$ and $p/e$ ratio at energies $\gtrsim 1$ GeV, as well as flatter CR spectral slopes $\psi_{\nu,0}$ for high-$E$ hadrons (where the residence time is primarily determined by diffusive escape), as expected. Larger $\delta$ has the opposite effects (as expected), but also large-enough $\delta \gtrsim 1$ (see also Appendix A) at low energies increases B/C and makes hadronic and leptonic slopes more shallow, by increasing the effect of losses via slower transport. Even a modestly-lower $\delta \sim 0.3$ is strongly disfavored, given the fact that we cannot “remove” the halo here to compensate for the flatter B/C predicted. A much-higher $\delta > 1$ is also clearly ruled out, and these limits are robust even after marginalizing over the assumed injection spectra.

Changing the normalization $\nu_0$ has the obvious effects of e.g. increasing/decreasing the secondary-to-primary ratio and normalization of the spectra, but more interestingly also has a strong effect on the shape of the CR spectra (and scaling of secondary-to-primary ratios with $p$), where larger $\nu_0$ (lower diffusivity) produces shallower slopes for hadrons. This arises from the non-linear competition between the various loss terms (which become stronger at lower-$\nu_0$ and escape), and is generally a larger effect for hadrons

6 Briefly, at lower $\nu_0$ in steady-state with all else equal, the CR energy density should increase $\propto \nu_0^{-1}$. But as we decrease $\nu_0$ (1) the size of the CR scattering halo also decreases (making this dependence weaker) and (2) losses become important even for $\sim$ GeV protons, so the CR energy density cannot continue to increase.

where the loss timescales are systematically shorter at lower-$E$) as compared to leptons (except for the very lowest $\nu_0$ considered, e.g. $D_0 \lesssim 10^{23} \text{cm}^2 \text{s}^{-1}$).

- Ionization & Coulomb Losses: Fig. 4 shows that if we artificially disable ionization losses, the low-$E$ spectra of $p$, $e^-$, CNO, and many other species are significantly more shallow, and the B/C ratio also becomes flatter below $\sim 100 - 200$ MeV (in conflict with the Voyager data). At these densities and diffusivities, the effect of disabling Coulomb losses alone is relatively weak compared to ionization, however if we either consider the spectrum in much more tenuous gas (a poorer match to observations overall) or higher diffusivities, then the relative role of Coulomb losses increases until both are comparable. The Coulomb or ionization loss time at low energies is $\sim 1 \text{Myr} (T/10 \text{MeV}) (n/\text{cm}^{-3})^{-1} Z^{-2}$, so this is easily shorter than CR diffusive lifetimes at low-$E$, and they (Coulomb & ionization losses) scale almost identically, the only difference is whether they act in neutral or ionized gas. So if CRs are spread uniformly in volume (e.g. owing to efficient diffusion) then the ratio of losses integrated over CR trajectories or volume is just the ratio

7 It is worth commenting on the behavior of $^{10}\text{Be}/^{8}\text{Be}$ with varying $\nu$ in Fig. 3 (and Appendix A). Naively we would expect that, all else equal, $^{10}\text{Be}/^{8}\text{Be}$ should decrease with increasing CR “residence time” between secondary production and arrival at the Solar system, hence be lower for higher $\nu$. And at low CR energies, we often see behavior consistent with this (but the effects are weak and somewhat non-linear, owing to the non-zero effects of streaming and losses controlling the residence time, instead of diffusive escape). At high-energies, however, we clearly see $^{10}\text{Be}/^{8}\text{Be}$ increase with $\nu$ (either from increasing $\nu_0$, or increasing $\delta$ at $T \gg \text{GeV}$). While some of this owes to lower-$\nu$ runs sampling an effectively smaller CR scattering halo and source region, most of the effect owes to the fact that the runs with larger $\nu$ also produce much higher B/C at these energies. At $\sim 100$ GeV, for B/C $\gtrsim 0.3$ (much higher than observed, but predicted in these models if we artificially increase $\nu$), B actually dominates over C in producing $^{10}\text{Be}$ (Moskalenko & Mashnik 2001), with a significantly higher ratio of $^{10}\text{Be}$ to $^{8}\text{Be}$ production factors. So what we see is effectively that tertiary $^{10}\text{Be}$ production from B becomes important (though we caution that many of the relevant cross sections are not well-calibrated at these energies).
of total ISM+inner CGM gas mass in ionized vs neutral phases (see e.g. Hopkins et al. 2020b, for a derivation of this), which is $O(1)$ in the ISM (with modestly more gas in neutral phases, but not by a large factor). However as shown below, the lowest-energy CRs are not infinitely-diffusive, so the CR energy density and loss rates at low rigidities are higher in denser gas, which tends to be neutral (explaining why ionization losses have a larger integrated effect at low rigidities than Coulomb losses). In either case, for low-energy hadrons (with $\gamma \sim 1$, i.e. not ultra-relativistic), this gives $\Delta\nu_{\text{rec}} \propto p^2$, giving $\nu_{\text{obs}} \sim 0$ (assuming the usual injection spectrum), i.e. a "flat" intensity, as observed, and for $e^-$ (with $\beta \equiv 1$ and $\gamma \gg 1$) this gives $\Delta\nu_{\text{rec}} \propto p^3$, so $\nu_{\text{obs}} \sim 1$, as also observed.

- Hadronic/Catastrophic/Spallation/Pionic/Annihilation Losses: Obviously, we cannot get the correct secondary-to-primary ratios if we do not include these processes; our question here is whether these processes strongly modify the primary spectrum. Annihilation serves to "cut off" the spectrum of $\bar{\nu}$ and $e^+$ around their rest-mass energies. Radioactive losses here only shape the $10^7$Be ratios. As for the spectra of CRs, at LISM conditions, the $e^-$ and $p$ population (as required by the $e^+/e^-$ and $\bar{\nu}/p$ ratios and $\gamma$-ray luminosity) is mostly primary, with relatively modest catastrophic losses, so we see in Fig. 4 that such losses do not dramatically reshape the spectra of these primaries (of course, they can do so in extreme environments like starbursts, which reach the proton calorimetric limit). Nonetheless removing the actual losses from e.g. pionic+hadronic processes does produce a non-negligible increase in the $p$ spectrum, and artificially boosts B/C owing to the "retained" primaries producing more B, and the lack of losses of B from spallation, which are actually significant under the conditions where B/C would normally be maximized.

- Inverse Compton & Synchrotron Losses: Fig. 4 also shows that if we disable inverse Compton (IC) & synchrotron losses, the high-$E$ $e^-$ and $e^+$ spectra become significantly more shallow, basically tracing the shape of the $p$ spectrum (set by injection+diffusion). The magnitude of the change to the spectrum therefore depends on the assumed $\nu(p)$ scaling (compare e.g. Appendix A, where we consider a reference model with $\delta \sim 1$, where the effect is somewhat smaller). For high-energy leptons, IC-synchrotron loss times are $\sim 1$ Myr ($T/100$ GeV)$^{-1}$ ($u_{\text{sh}} + u_{\text{rad}}$)/3 eV cm$^{-1}$, so shorter than diffusive escape times, and this $\Delta\nu_{\text{rec}} \propto p^1$ produces $\nu_{\text{obs}} \sim 3$, as observed. Since IC & synchrotron scale identically with the radiation & magnetic energy density, respectively, whichever is larger on average dominates (volume-weighted, since CR transport is rapid at these $p$).

Even in the MW, it is actually not always trivial that the synchrotron losses should be comparable to IC losses, since in many Galactic environments, $u_{\text{sh}} \ll u_{\text{rad}}$. Consider some basic observational constraints in different regions, noting $u_{\text{sh}} \approx 0.02 eV$ cm$^{-1}$ $B_{100G}^4$. First, e.g. the CGM, where $B \ll 1 \mu G$ (Farnes et al. 2017; Prochaska et al. 2019; Vernstrom et al. 2019; Lan & Prochaska 2020; Malik et al. 2020; O’Sullivan et al. 2020), but $u_{\text{rad}}$ cannot be lower than the CMB value $\approx 0.3$ eV cm$^{-1}$; or at the opposite extreme consider typical star-forming complexes or OB associations or superbubbles (where most SN events occur) with observed upper limits from Zeeman observations in e.g. Crutcher et al. (2010); Crutcher (2012) of $|B| \lesssim 10 \mu G$ ($n/300$ cm$^{-3}$)$^{1/2} \sim 5 \mu G$ ($M_{\text{GC}}/10^7 M_{\odot}$)$^{-1/2}$ (inserting the GMC size-density relation; Bolatto et al. 2008) compared to observed $u_{\text{rad}} \sim 300$ eV cm$^{-3}$ averaged over the entire regions out to $\sim 200$ pc and $\sim 10^8$ eV cm$^{-3}$ in the central $\sim 40$ pc (Lopez et al. 2011; Pellegrini et al. 2011; Barnes et al. 2020; Olivier et al. 2020). But the ratio $u_{\text{sh}}/u_{\text{rad}}$ is maximized in the WIM phases with $n \sim 0.1 - 1$ cm$^{-3}$, $u_{\text{rad}} \sim 1.3$ eV cm$^{-3}$ (the ISRF+CMB; Draine 2011) and $B \sim 1 - 10 \mu G$ ($u_{\text{sh}} \sim 0.02 - 2$ eV cm$^{-3}$) Sun & Reich 2010; Jansson & Farrar 2012; Havercorn 2015; Beek et al. 2016; Mao 2018; Ma et al. 2020b). In Fig. 6, we show a quantitative plot of this for the same simulation as Fig. 4, comparing the energy density in different (radiation, magnetic, CR, thermal) forms, as a function of local gas density, just for gas in the Solar circle; this agrees well with the broad observational constraints above, and indeed shows that $u_{\text{sh}}/u_{\text{rad}}$ is maximized in the WIM phases. The fact that this is a volume-filling phase, and that CRs diffuse effectively (so that the total synchrotron emission is effectively a volume-weighted integral) ensures the synchrotron losses are not much smaller than the inverse Compton in the integral, allowing for the standard arguments (e.g. Voelk 1989) to explain the observed far infrared (FIR)-radio correlation (at least within the > 1 dex observed 90% inclusion interval; Magnelli et al. 2015; Delhaize et al. 2017; Wang et al. 2019).

As a consequence of this, in Fig. 4, we see that the effect of removing synchrotron losses on the $e^-$ spectrum is generally comparable to the effect of removing IC losses, but the synchrotron losses are somewhat larger at energies $\lesssim 20$ GeV which contain most of the $e^-$ energy (thus in a "bolometric" sense synchrotron dominates over IC losses), while IC losses slightly dominate at even larger energies. This owes to the fact that higher-energy CRs (being more diffuse) sample an effectively larger CR scattering halo, therefore with loss rates reflecting lower-density CGM gas where $u_{\text{sh}} \gtrsim u_{\text{rad}}$.

- Re-Acceleration (Convective, Streaming/Gyro-Instability, and Diffusive): We discuss the different "re-acceleration" terms in detail below in § 3.3. In Fig. 4, we see that removing each of the three re-acceleration terms in turn has relatively small effects. The convective term can have either sign, while the "streaming" term is almost always a loss term, and the "diffusive re-acceleration" term is a gain term; on average for CRs we see the sum of the three (usually dominated by the convective term) results in a weak net loss term on average.

For the sake of comparison with historical Galactic CR transport models which usually only include the "diffusive re-acceleration" term with an ad-hoc or fitted coefficient, we run one more test ("Maximal Diffuse Reacceleration x10") where we artificially (1) turn off both the convective and streaming re-acceleration/loss terms, which are generally larger and have the opposite sign; (2) adopt $\nu_{\text{sh}} \sim 10^{-8} s^{-1}$ GeV$^{-1}$, so $D_{\nu_{\text{sh}}}$ is a factor $\sim 10 \times$ larger at $\sim$ GeV and $\sim 100 \times$ larger at $\sim$ MeV compared to our "preferred" values (closer to what would be inferred in a "leaky box" model with no halo); (3) further replace our expression for the $D_{\nu_{\text{sh}}}$ terms derived directly from the focused CR transport equation with the more adhoc expression $\nu_{\text{sh}}/p = (1/9) (\nu_{\text{sh}}/\nu_{\text{FIR}}) (\partial \ln f_{\nu_{\text{FIR}}} / \partial \ln p) \sim (4/9) \nu_{\text{sh}}^4/\nu_{\text{FIR}}$, about $5$ times larger than the value we would otherwise obtain. With this (intentionally un-realistic) case we find noticeable effects with a steeper low-$E$ slope and a more-peaked B/C, reproducing the very large implied role of diffusive re-acceleration for CR energy in some previous models.

- Streaming Terms (Non-Symmetric Scattering): Per § 2.2, we assume by default (motivated by SC models) that scattering is anisotropic in the fluid frame such that $\nu_{\perp} \neq \nu_{\parallel}$, giving $\nu_{\parallel} \approx \nu_{\perp} f_{\parallel}$. Although this is almost always expected, if somehow the scattering were perfectly isotropic in that frame and the Alfvén frame, we would have $\nu_{\parallel} \to 0$, so the $D_{\nu_{\perp}}$ term (which gives rise to CR "streaming" motion at $\nu_{\text{eff}} \to \nu_{\parallel}$ in the strong-scattering $\nu_{\parallel} \to \infty$ surface density and star formation efficiency and convolving with the IMF for a young SSP, see Lee & Hopkins (2020)).

8 This can also be derived taking the observed nearly-constant MW cloud
limit) and $D_{p,\alpha}$ term (the “streaming loss”; § 3.3) vanish. Since we do not predict $\tilde{p}$, here, in Fig. 4 we compare a run where we simply set $\tilde{v}_i = 0$. This makes only very small differences. Even for the scaling adopted in Fig. 4, $\tilde{v} \approx 10^{-8} \, c \, t^{1/2}_{\text{CR}}$, and reasonable $\tilde{v}_i \sim 10 \, \text{km s}^{-1}$, the streaming velocity only dominates over the diffusive velocity ($\sim \kappa \left| \nabla e \right| / \left| \mathbf{v} \right| \tilde{p} \left( \mathbf{v}_{\text{ionic}} \right)$) at $\tilde{E} \lesssim 100$ MeV. For our preferred model with smaller and more-weakly-$R$-dependent $\tilde{v} \approx 10^{-9} \, c \, t^{1/2}_{\text{CR}}$, streaming only dominates at $\lesssim \text{MeV}$.

Note, however, that in this study $\tilde{v}_i \equiv \left| \mathbf{B} \right|^2 / 4\pi \rho_{\text{ion}}$, which can be very large in molecular clouds with typical $\rho_{\text{ion}} \lesssim 10^{-7}$. If we simply use this everywhere, we find in Fig. 4 that it has a significant effect, making the low-energy slopes shallower in $p$ and $e$ and lowering the peak B/C, as the CRs escape neutral gas nearly immediately without losses. However properly treating this regime requires a self-consistent model for self-confinement including the damping terms acting on gyro-resonant waves, which we defer to future work.

**Numerics:** For extensive tests of the numerical implementation of the spatial CR transport, we refer to Chan et al. (2019); Hopkins et al. (2020b). We have also considered some pure-numerical variations here in Fig. 5, including: (1) replacing the generalized closure relation in Eqs. 1-2 with the simpler “isotropic” closure from Hopkins (2021), which assumes the CR DF is always close-to-isotropic, closer to e.g. the formulation in Thomas & Pfrommer (2019), or going further and using the older (less accurate) formulation of the CR flux equation from Chan et al. (2019). This makes very little difference, as CRs are indeed close-to-isotropic and the timescale for the flux equation to reach steady-state (where the formal differences in these formulations vanishes) is short ($\sim \nu^{-1}$) compared to other simulation timescales. (2) We have also considered the effect of simply assuming the ultra-relativistic limit always in the spatial transport terms including the “re-acceleration” terms, instead of correctly accounting for $\beta$. This is purely to test how the more accurate formulation alters the results; removing the $\beta$ dependence in these terms artificially makes the low-energy spectra more shallow and lowers the low-energy ($\ll 1$ GeV) B/C while raising $^{10}\text{Be}/^{9}\text{Be}$. So it is important to properly include these terms. (3) We have re-run our fiducial and several parameter-variation models with both the FIRE-2 and FIRE-3 (Hopkins et al., in prep.) versions of the FIRE code, which utilize the same fundamental physics and numerical methods, but differ in that FIRE-2 adopts somewhat older fits to quantities like stellar evolution tracks and cooling physics. This has no significant effects on any CR quantities we examine in this paper. Finally (4) we have tested various “reduced speed of light” (RSOL) values (which limit the maximum free-streaming speed of CRs to prevent extremely small timesteps). As extensively detailed in Hopkins (2021) our numerical formulation is designed so that when the system is in steady-state, the RSOL has no effect at all on solutions, so long as is faster than other relevant speeds in the problem. Our default tests here adopt an RSOL of $\tilde{c} = 10^3 \, \text{km s}^{-1}$, which is more than sufficient for convergence, but in several model variants including raising/lowering $\tilde{v}$ (1 GV) by $\pm 1$ dex, and changing the slope $\tilde{v} \sim R^{\alpha}$ from $\alpha = 0.3 - 1$, we have tested values $\tilde{c} = 300 - 3 \times 10^3 \, \text{km s}^{-1} (\approx 0.001 - 1)$. We find that at $\tilde{E} \approx 1$ GV, we can reduce $\tilde{c}$ as low as $\approx 500 - 1000 \, \text{km s}^{-1}$ and obtain converged results; but for the highest-energy CRs (which can reach diffusive speeds $\sim \kappa \left| \nabla e \right| / \left| \mathbf{v} \right| \sim 0.1 \, r$) we require $\tilde{c} \gtrsim 3000 \, \text{km s}^{-1}$ for converged results with this particular RSOL formulation (Eq. 50 from Hopkins 2021, as compared to the formulation from Eq. 51

![Figure 6](image_url)
actually determines the net effect on the CR spectrum and energy) is not actually \( p_{\text{ad}}/p \), but a mean (volume-averaged) time-integrated (over the CR travel time) \( p_{\text{ad}} \), which it is well-known from many galactic/ISM theoretical and observational studies (Stanimirovic et al. 1999; Elmegreen 2002a; Décam & Le Bourlot 2002; Mac Low & Klessen 2004; Block et al. 2010; Bournaud et al. 2010; Hopkins 2013a; Squire & Hopkins 2017) is dominated by the largest-scale modes which are coherent over \( \lambda \sim H \), the disk scale height. \(^{11}\) Briefly, this can be understood with a simple toy model. Since \( \nabla u \) has either sign, and modes on small scales \( \lambda \) compared to the total CR travel length \( l \) along \( b \) are un-correlated, then averaging over CR paths (assuming a diffusive 3D random walk in space with \( \lambda \ll l \) or averaging over volume \( dV \) (equivalent if the CRs are in steady-state or we assume ergodicity), the coherent effect of the modes is reduced by a factor of \( \sim N_{\text{mode}}^{-1/2} \sim (\ell/\lambda)^{-3/2} \). So for any realistic spectrum the largest coherent modes dominate the integral, and for any realistic disk structure these must have \( \lambda \sim \text{MIN}(\ell, H) \sim H \) (for the energies of interest), giving \( O(\nabla : \nabla u) \sim O(t_{\text{dyn}}^{-1}) \) with the disk dynamical time \( t_{\text{dyn}} \sim 35 \text{ Myr} \) at the Solar position.

The magnitude of the coherent effect of this term can then be estimated as \( \Delta p/p \sim O(\Delta t_{\text{rms}}/t_{\text{dyn}}) \). Since at \( R \gtrsim 1 \text{ Gyr} \) the residence time \( \Delta t_{\text{rms}} \) decreases with \( R \), this term is most important at lower energies, as expected. The sign is not \( a\text{-priori} \) obvious, however. But again note the averaging above: if CRs diffuse efficiently, so the CR density is not strongly dependent on the local gas density, then the CR travel time integral above is dominated by the largest \( \text{volume-filling} \) phases of the ISM and inner halo/corona traversed. These diffuse phases are the ones that are most strongly in outflow, so more often than not, the appropriately-weighted \( \nabla : \nabla u \) is small (for detailed discussion of how the adiabatic term depends on ISM phases, see Pfrommer et al. 2017; Chan et al. 2019), and the net effect of this term is usually to decrease CR energies. Because the effect is weaker at higher CR energies, this has the net effect of making the CR spectra more shallow (i.e. if \( D_{\text{pp}} \propto p^{-\alpha} \), this decreases \( \alpha \)).

3.3.2 “Streaming” Term

Next consider the \( D_{\text{pp}} \) or “streaming” term \( \dot{p} = p_{\text{ad}} = -(\nu_{\bot} / \nu_{\|}) D_{\text{pp}} \). Here \( \nu_{\|} \) is the CR energy flux and pressure in a narrow interval in \( p \). In SC-motivated models, as discussed in detail in Hopkins (2021) and noted above, the asymmetry in \( \nu_{\bot} \) and \( \nu_{\|} \) gives \( \nu_{\bot} / \nu_{\|} \sim 1 \), so the ratio of the “streaming” \( D_{\text{pp}} \) to “diffusive” \( D_{\text{pp}} \) terms is always \( \gtrsim \nu_{\|} / \nu_{\bot} \sim \nu_{\|} / \nu_{\bot} \) (where \( \nu_{\|} \) is the effective bulk transport speed of CRs) – i.e. it dominates whenever the CRs move at trans or super-Alfvénic speeds, which is usually true. Moreover, as shown in Hopkins (2021) or seen by plugging into Eq. 1–2, in flux-steady-state the sum of these two terms becomes \( \dot{p} = p_{\text{ad}} = -\nu_{\bot} f_{\|} / f_{\bot} v_{\text{rel}} / \nu_{\|} - 2 \nu (1 + \beta_{c}) v_{\text{rel}}^{2} \sim -\nu_{\bot} (\nu_{\|} / \nu_{\bot}) \approx 10Gyr / c_{\text{p}}^{2} \). The \( \text{adiabatic} \) term is thus negative, representing the “streaming losses” energy lost to gyro-resonant instability as the CRs move, and the leading term is \( \sim \nu_{\bot} / c_{\text{grad}}. \) This is \( \text{negative/representing the} \) “streaming” term, because (a) on galaxy scales the bulk turbulent and convective/fountain/inflow/outflow motions are super-Alfvénic (\( |u| \gtrsim v_{\text{rel}} \)).

\(^{11}\) For our purposes, the largest modes with \( \lambda \sim H \) where \( H \) is the disk scale-height and Tsoname length have, by definition in a trans or super-sonically turbulent ISM, \( \nabla u \sim V_{\text{f,obs}} / 10^{9} \text{km s}^{-1} \) where \( t_{\text{obs}} \) is the galactic dynamical time (Elmegreen & Efremov 1997; Gammie 2001; Hopkins 2013b; Hopkins & Christiansen 2013).
CR re-acceleration gains generally (1) require un-realistic ener-
gectics in this component (as cautioned by Drury & Strong 2017); (2) ignored both convective and streaming loss terms which, for any realistic model, offset the diffusive term as described above; (3) adopted unrealistic low diffusivities (high $\nu$) appropriate for e.g. leaky-box models but not models with a more realistic halo, so $D_{\nu}$ is much larger than it should be; and (4) treat the normalization of the diffusive-reacceleration as arbitrary, e.g. through “fitting” the value of $\nu_0$ that appears in $D_{\nu}$, and adopt order-of-magnitude larger values than physically allowed here. This can easily be seen from Eq. 7: the diffusive-only re-acceleration timescale is $\tau_{\nu} \sim (B/\mu G)^{-1} (\dot{\nu}/10^{-9} s^{-1})^{-1}$. Without invoking orders-of-magnitude lower $\dot{\nu}$ or larger $B$, this cannot compete with the streaming loss and adiabatic terms (timescale $\sim 10 - 100 $Myr, as noted above) let alone other CR loss/escape terms.

3.4 Where are Most CRs Accelerated?

There is an extensive literature using the abundance patterns of CRs to constrain their acceleration sites; most of these studies have argued that most of the $\sim$MeV-TeV CRs followed here must come from sites relatively near SNe, probably in “super-bubbles,” with $\sim 10\%$ of the initial ejecta kinetic energy ending up in CRs (see e.g. Higdon et al. 1998; Parizot et al. 2004; Becker Tjus & Merten 2020, and references therein). As illustrated in Fig. 7, we find the same, and below we argue this must be the case from abundance, energetic, and gravito-turbulent considerations.

3.4.1 Abundances: Most CRs Must Come From Initial SNe Shocks

It has been argued by a number of authors that significant CR Fermi-
I acceleration could occur in shocks with modest Mach number of just $\sim 5$ or even lower (Ryu et al. 2003; Schure et al. 2012; Vink & Yamazaki 2014; Guo et al. 2014). If this alone were sufficient to produce an order-unity fraction of the “new” CRs/Fermi-I acceleration to $\sim$ GeV (we stress that Fermi-II acceleration from shocks is included in our methods), then CRs could be continuously created almost everywhere throughout the ISM, as it is well-established ob-
servationally that the majority of all the gas in the ISM is super-
sonically turbulent, with e.g. typical Mach numbers in the mass-
ive atomic and molecular clouds that contain $\sim 1/2$ of the galaxy gas mass typically ranging from $\mathcal{M} \sim 10 - 100$ (Evans 1999; Mac Low & Klessen 2004; Elmegreen & Scalo 2004; McKee & Ostriker 2007). If these were the dominant sites of initial CR acceleration, then the primary CR O/H ratio would just trace the ISM abundance, with e.g. $N(O)/N(H) \sim 0.0005$ (Lodders 2019). But in the LISM, the ratio of $N(O)/N(H)$ in CRs (where almost all O is primary) is $\sim 0.01$ (Cummings et al. 2016). The qualitative discrepancy is the same if we consider any of CNO or any heavier species from Ne through Fe. Most other processes (re-acceleration, spallation, violations of the test-particle limit in acceleration) make the discrepancy more, not less, dramatic. This also rules out stellar winds as the source of most CR acceleration; while AGB winds (which are very low-energy) are mildly enhanced in CNO, even this is nowhere near sufficient, and faster OB/WR winds are so weakly enhanced that they give essent-
ially identical results to “pure ISM” acceleration.

If we assume acceleration near SNe, we can constrain the total ratio of “entrained” mass per SNe at the time of acceleration, to the initial ejecta mass. Again, if the only requirement for effi-
cient CR acceleration were a Mach number $\mathcal{M} \gtrsim 5$, then accel-
eration would be efficient throughout the end of the snowplow phase of remnant evolution – we test this directly in Fig. 7 by running a model where the swept-up mass $M_{\text{swept}}$ is given by the mass at the end of the Sedov-Taylor phase.

Figure 7. Top: Ratio of observed C/H in LISM CRs (at Solar-circle with gas $n = 0.3 - 3$ cm$^{-3}$) vs. CR energy in simulations (lines are median+shaded ranges $\pm 2\sigma$ and observations (points), for three different assumptions about where CRs are accelerated, which determines the injection abun-
dance ratios (see § 2.2.4). Our default ($M_{\text{swept}} \approx M_{\text{ej}}$) assumes CRs accel-
erate when the swept-up mass from SNe or stellar wind injection equals the ejecta mass (i.e. around reverse-shock formation or the onset of the Sedov-
Taylor phase). For SNe (which dominate the sources) this is similar to assum-
ing most acceleration occurs in shocks with velocity $\gtrsim 2000$ km s$^{-1}$. Blue/dashed line assumes efficient acceleration occurs through the end of the Sedov-Taylor phase such that most of the total CR energy is produced in the final stages when $M_{\text{swept}} \sim M_{\text{ej}}$ and the initial CR abund-

ces essentially trace the ambient ISM abundances (roughly Solar). This gives C/H an order of magnitude lower than observed. Green/dotted shows the result assuming acceleration with “pure” ejecta abundance ratios. The results for N/H and O/H are almost identical, and do not depend strongly on e.g. CR transport parameters. Bottom: Distribution of injection sites: PDF of ISM circum-SNe gas densities just before explosion in the outer disk (radii labeled), where most LISM CRs are accelerated. Despite stars forming at re-
solved densities $n \gtrsim 1000$ cm$^{-3}$ in these simulations, most SNe (by number) explode in evacuated (super)bubble-like regions.

In general, at the end of the Sedov-Taylor phase for a clustered group of $N_{\text{SN}}$, the shock velocity is $v_{\text{shock}} \sim 200$ km s$^{-1}$, and the swept-up ISM mass is $\sim 3000N_{\text{SN}}M_{\odot}$ (Cioffi et al. 1988; Welch & Naab 2015; Kim & Ostriker 2015a; Hopkins et al. 2018a). The initial ejecta metallicity has been completely diluted at this point, predicting $N(O)/N(H) \sim 0.0005$. Again, per § 2.2.4, if the acceleration occurs from a mix of ambient gas and ejecta when the shock has entrained a mass $M_{\text{swept}}$, then the CR $N(O)/N(H) \approx (N_{\text{ej}}/M_{\text{ej}} + N_{\text{ISM}}M_{\text{swept}})/(N_{\text{ej}}M_{\text{ej}} + N_{\text{ISM}}M_{\text{swept}})$. If we instead assume $M_{\text{swept}} \approx M_{\text{ej}}$ (our default model), then we obtain $N(O)/N(H) \sim 0.008$ for both core-collapse and Ia SNe, in excellent agreement with the CR observations, as shown for C/H in Fig. 7 (we do not show O/H but the conclusions are essentially identical). Theoretically, this is a
particularly interesting value, since it corresponds to the time when the reverse shock fully-forms and propagates through the ejecta, essentially to the “onset” of the shock and end of the ejecta free-streaming phase. In any model where the CR acceleration efficiency to \( \sim 5\text{ GeV} \) is an increasing function of Mach number or an increasing function of the shock kinetic energy dissipation rate, this will be the phase which dominates acceleration. \(^{12}\) For completeness, in Fig. 7 we also consider a model where CRs are accelerated with “pure ejecta” abundances, i.e. \( M_{\text{ejecta}} \rightarrow 0 \), which in contrast over predicts the abundance of intermediate elements.

### 3.4.2 Energetics: Most CR Energy Comes From SNe Energy

If we assume that CR acceleration imparts a constant fraction \( \epsilon \sim 0.1 \) of the thermalized/dissipated shock kinetic energy to CRs, then we have directly verified that in our simulations most of the CR energy comes from SNe, even if we allow CR injection at ISM shocks of arbitrarily low Mach number. This is expected: integrated over time and the stellar IMF, the kinetic energy input from stellar mass-loss (dominated by fast O/B winds) is \( \sim 10\% \) that of core-collapse SNe (Leitherer et al. 1999, 2014; Smith 2014; Rosen et al. 2014; Eldridge et al. 2017). The input from proto-stellar radiation and jets is only \( \sim 1\% \) of that from SNe, while the energy from winds accelerated around remnants (e.g. PWNe, XRBs, etc.) is even smaller still (Federrath et al. 2014; Bally 2016; Guszejnov et al. 2020a). From ISM shocks, our simulations reproduce the usual result that the ISM turbulent dissipation rate is \( \sim 1-5\% \) of the SNe energy input rate (Hopkins et al. 2012a; Faucher-Giguère et al. 2013; Kim & Ostriker 2015a; Martizzi et al. 2015; Orr et al. 2018), which also follows from the trivial order-of-magnitude expectation for super-sonic turbulence, \( \dot{E} \sim (1/2)M_{\text{ejecta}}v_{\text{shock}}^2/\sigma_{\text{dyn}} \), where \( v_{\text{shock}} \sim v_{\text{shock}}(H) = H/\sigma = L_{\text{SN}} \) for any disk with Toomre \( Q \sim 1 \), given canonical MW-like values for \( M_{\text{ejecta}} \sim 10^{9}M_{\odot} \) and \( \sigma_{\text{dyn}} \sim 100\text{ km s}^{-1} \) with \( L_{\text{SN}} = r/v_{\text{shock}} \) at the effective radius \( \sim 5\text{ kpc} \). What this cannot tell us is “how close” to SNe CRs are accelerated (e.g. at the onset or later in the shock), since by definition during e.g. the Sedov-Taylor phase the shock energy is conserved – for this we refer to the abundance argument.

### 3.4.3 Environment: Most SNe Explode in Super-Bubbles

If most CRs are accelerated “near” SNe (before they sweep up a mass \( \gg M_{\text{ejecta}} \)), it follows trivially in simulations like ours that most CRs are accelerated in super-bubble environments, simply because the majority of SNe explode in such environments. Note that this is weighted by number or energy, so most SNe come from \( \sim 10^{5}M_{\odot} \) stars that explode \( \sim 30\text{ Myr} \) after they reach the main sequence, well after more massive stars in the complex have exploded and destroyed their natal GCMs; see e.g. Grudie et al. 2018a. We have shown that the fact that most SNe energy goes into super-bubble type structures (overlapping SNe shocks during the energy-conserving phase) is true for FIRE simulations in a number of studies (Hopkins et al. 2012b, 2013c; Muratov et al. 2015; Escala et al. 2017). The input from proto-stellar radiation and jets is only \( \sim 1\% \) of that from SNe, while the energy from winds accelerated around remnants (e.g. PWNe, XRBs, etc.) is even smaller still (Federrath et al. 2014; Bally 2016; Guszejnov et al. 2020a). From ISM shocks, our simulations reproduce the usual result that the ISM turbulent dissipation rate is \( \sim 1-5\% \) of the SNe energy input rate (Hopkins et al. 2012a; Faucher-Giguère et al. 2013; Kim & Ostriker 2015a; Martizzi et al. 2015; Orr et al. 2018), which also follows from the trivial order-of-magnitude expectation for super-sonic turbulence, \( \dot{E} \sim (1/2)M_{\text{ejecta}}v_{\text{shock}}^2/\sigma_{\text{dyn}} \), where \( v_{\text{shock}} \sim v_{\text{shock}}(H) = H/\sigma = L_{\text{SN}} \) for any disk with Toomre \( Q \sim 1 \), given canonical MW-like values for \( M_{\text{ejecta}} \sim 10^{9}M_{\odot} \) and \( \sigma_{\text{dyn}} \sim 100\text{ km s}^{-1} \) with \( L_{\text{SN}} = r/v_{\text{shock}} \) at the effective radius \( \sim 5\text{ kpc} \). What this cannot tell us is “how close” to SNe CRs are accelerated (e.g. at the onset or later in the shock), since by definition during e.g. the Sedov-Taylor phase the shock energy is conserved – for this we refer to the abundance argument.

#### 3.5.1 Variation With Galactic Environment

In Fig. 8 we note that the variation in the CR spectra can be significant, and quantify some of these variations and their dependence on the local Galactic environment. We have examined how the spectra vary as a function of local ISM properties including: galacto-centric radius, height above the midplane, gas density, temperature, ionization fraction, local inflow/outflow velocity, magnetic energy density, radiation energy density, turbulent dissipation rate, star formation or SNe rate per unit volume, plasma \( \beta \), and other properties. For most of these, there is some correlation with CR spectra, but it is important to remember that all of these parameters are themselves mutually correlated within a galaxy; as a result most of the systematic variation with the properties above can be captured by the dependence on galacto-centric radius \( r \) and local gas density \( n \), shown explicitly in Fig. 8.

The dependence on galacto-centric \( r \) in Fig. 8 (even at fixed \( n, T, \) etc.) is easily understood: towards the galactic center, the source density is higher, and assuming CRs are efficiently escaping, steady-state requires that the kinetic energy density be some declining function of galacto-centric radius \( r \) (see Hopkins et al. 2020b). Specifically, \( n_{\text{eff}} \propto \pi/2 D_{\odot} \) for a spatially-constant isotropic-equivalent diffusivity \( D_{\odot} \propto r \) where \( r_{\text{eff}} \) is the injection rate, proportional to the SNe rate) if losses are negligible. This scaling provides a reasonable description of what we see outside of a few hundred pc, in fact, and the CR kinetic energy density therefore drops by an order of magnitude (or more) between the equivalent of the Galactic Central Molecular Zone (\( r \leq 400\text{ pc} \)) and Solar neighborhood/LISM (\( r \sim 8\text{ kpc} \)). The spectra towards the Galactic center are also shallower/harder in hadrons (at intermediate rigidities from \( \sim 0.01-10\text{ GV} \)), while being steeper/softer in leptons: this owes to the fractional importance of losses. The Galactic center has much higher neutral gas/nucleon/radiation/magnetic energy densities overall, so the loss rates are all enhanced: this makes the hadronic spectra (where loss rates increase at low energies) shallower/harder and leptonic spectra (where loss rates increase at high energies) steeper/softer.

#### 3.5.2 Energetics: High Energy Most SNe

At a given galacto-centric \( r \), e.g. at the Solar circle (\( r \sim \sim 8\text{ kpc} \)), there is much less variation, but there is still a significant

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\(^{12}\) For the usual definition of CR acceleration efficiency \( \eta(M) \), the flux of accelerated CRs in a strong shock is \( F_{\text{cr}} = \sim \eta(M)(1/2)\rho v^{3} \) (for shock velocity \( v_{\text{shock}} \) and upstream density \( \rho \)), so the contribution to the total CR energy in some time interval \( dt \) is \( dE_{\text{cr}} \sim F_{\text{cr}} dt \sim \eta(M)\rho v^{3} dV \sim \eta(M)\rho v^{3} dV/dx \) (with \( r_{\text{shock}} \) the shock radius). But in the Sedov-Taylor phase \( (\rho_{\text{shock}} \sim v_{\text{shock}}^{2} \sim 5/2) \) this is just \( \eta(M)M_{\text{shock}} r_{\text{shock}} \) with \( M_{\text{shock}} \sim r_{\text{shock}}^{-3/2} \). Thus, any model where \( \eta(M) \) is an even weakly increasing function of \( M \) (as expected qualitatively in most theories of diffusive shock acceleration, e.g. Blandford & Eichler 1987; Amato & Blasi 2005; Bell 2013) will produce most CR acceleration at the smallest \( r_{\text{shock}} \) possible (i.e. the onset of the Sedov-Taylor phase, when the reverse shock forms).

\(^{13}\) Note that the galactocentric radii in Fig. 8 are spherical radii, so some of...
Figure 8. Comparison of CR energy spectra (ρ, thick, and e, thin) vs. environmental properties, as Fig. 2. Top: Spectra vs. galacto-centric radius (allowing for the entire range of gas densities at each R). The CR spectra depend strongly on R, with higher normalization (tracing higher source density+proximity) as R → 0, but also harder intermediate-energy hadronic spectra & shifted leptonic, owing to more rapid losses. Middle: Spectra vs. gas density n at fixed galacto-centric radius R = 7 − 9 kpc. At fixed R, there is still significant dependence on n. High-energy CRs, which diffuse rapidly, exhibit weaker variations. Low-energy CRs exhibit dramatic systematic dependence, as their lower diffusivity leads to “trapping” in dense gas. Bottom: Spectra vs. gas temperature at fixed n = 0.1 − 1 cm⁻³ and R = 7 − 9 kpc. After controlling for n and R, there is little systematic dependence on temperature or other phase properties (ionization state, magnetic field strength), but there is still substantial variation in low-energy CR spectra, especially in hot gas, reflecting the stochastic nature of SNe super-bubbles.

Figure 9. Correlation between CR kinetic energy density in bins of CR energy/momentum/rigidity (dεₖᵢ /d ln T) and gas density (n), in gas at the Solar circle (R = 7 − 9 kpc). For CR protons in a narrow interval of energy at these radii, we fit the correlation dεₖᵢ /d ln T ∝ n^α (as in Fig. 6) to a power-law slope α, and show the best fit α (line) with the ±1σ range (shaded), as a function of CR energy T. This can also be considered an “effective adiabatic index” dεₖᵢ /d ln T ∝ n^α at each T. Low-energy CRs approach the adiabatic relativistic tight-coupling limit α → 4/3 (slightly lower, as losses also increase at high-n for low-energy protons, and offset the adiabatic increase in εₖᵢ). High-energy CRs being more diffusive become more spatially-uniform with α ∼ 0.

systematic dependence on the local gas density n, seen also in Fig. 8. Higher-density environments have higher CR energy density: again this is qualitatively unsurprising, since (a) in the “tight coupling” limit εₖᵢ ∝ n^7/3, (b) even without tight-coupling, the “adiabatic” re-acceleration term is typically positive in denser regions, and typically negative in lower-density regions, and (c) denser regions are positively correlated with CR sources (e.g. SNRs). Figure 9 quantifies how CR energy density/pressure scales with gas density, as a function of CR energy or rigidity, giving a quantitative indication of “how tightly coupled” CRs are to gas (which is also crucial for understanding how CRs do or do not modify thermal behaviors of the gas such as the classical thermal instability; see Butskey et al. 2020). At the highest energies for hadrons, the effect is negligible, owing to fast diffusion (the CR density is basically un-coupled from the gas density), while at the lowest energies the effect is strongest (the small diffusivity produces tight coupling). Thus the net effect is that lower-density regions at a given galacto-centric radius have slightly harder CR spectra with lower total energy density (opposite the effect with galacto-centric radius). The effect at the highest densities for leptons is a bit more complicated owing to the non-linear effects of IC and synchrotron losses. As discussed below, this has important implications for CR ionization.

Even controlling for galactocentric radius and gas density, there is still large variation in the total CR kinetic energy density, with the 90% inclusion interval (~2σ to +2σ range) spanning nearly ~ 3 dex (~ 0.7 dex ± σ range) at the lowest energies or for certain species (with smaller ~ 0.2 dex variation in the energy density of e.g. high-energy protons where diffusion is rapid and losses minimal). This scatter does not primarily come from a systematic dependence on a third variable that we can identify (e.g. any of the variables noted above) – the residual dependence on e.g. gas temperature, etc., in Fig. 8 is minimal. Rather, this appears to owe mostly to effectively stochastic variations in space and time: the combination of e.g. multiple second-order/weak correlations, how the variation with density reflects gas at different scale-heights; we consider in more detail the difference between the variation along the disk midplane cylindrical radius versus vertical height above the disk below in § 3.5.4.
3.5.2 Variation with Time

We can also examine the variations of CR spectra with time in a given galaxy. We can do this either by simply taking the CR spectra in different regions as above (§ 3.5.1) at different snapshots near \( z \sim 0 \) (separated by some \( \Delta t \)), or, because our code is quasi-Lagrangian, we can explicitly ask how the CR spectrum seen by an individual Lagrangian gas parcel varies in time. On sufficiently small timescales \( \Delta t \ll t_{\mathrm{dyn}} \), smaller than the galaxy dynamical time, the variations are small as expected: the galaxy and source distribution and bulk ISM properties by definition evolve on slower timescales and even if the CR escape/loss timescales are more rapid, they simply converge to quasi-steady-state. On very large timescales \( \Delta t \sim t_{\mathrm{H}} \gtrsim \) Gyr of order the Hubble time, we are asking an effectively different question (how CR spectra vary with cosmic time, or as a function of redshift), and the galaxy is fundamentally different: this is a key question for understanding e.g. the FIR-radio correlation but is qualitatively different from what we seek to understand here, so we defer this to future work. On intermediate timescales \( t_{\mathrm{dyn}} \lesssim \Delta t \lesssim \) Gyr, the variations in time at a given \( r, n \) are largely effectively stochastic, with a similar amplitude to that described above – i.e. the statistics of the CR spectra are effectively ergodic. In principle there are some events (e.g. a starburst, large galaxy merger, etc.) which could cause a substantial deviation from this but recall our galaxies our chosen for their MW-like properties which means we specifically selected systems without such an event between \( z \sim 0 - 0.1 \).

3.5.3 Variation Across MW-Like Galaxies

Next compare variation across different MW-like galaxies. Fig. 10 specifically compares our \( \text{m12i}, \text{m12f}, \text{m12m} \) halos, at the same cosmic time: these three halos produce arguably the three “most Milky Way-like” galaxies in the default FIRE suite, and all have been extensively compared to each other and to different MW properties in previous studies (see Ma et al. 2016; El-Badry et al. 2018a,b; Sanderson et al. 2018; Bonaca et al. 2017; Gurvich et al. 2020; Guszejnov et al. 2020b; Garrison-Kimmel et al. 2019b; Benincasa et al. 2020; Samuel et al. 2020). There are some non-negligible differences in detail between the systems, many apparent in Fig. 1: for example, in this particular set of runs, \( \text{m12f} \) has a slightly higher mass and more extended gas+stellar disk, with a hotter CGM halo; \( \text{m12i} \) has a slightly more rising star formation history in its outskirts (making them bluer) and a strong bar which induces a warp and a slight central “cavity” in the SFR, akin to the MW central molecular zone (see Orr et al. 2021); \( \text{m12m} \) has a less centrally-peaked rotation curve, and a pseudobulge driven by bar-buckling (Debattista et al. 2019). But given the large uncertainties in characterizing the MW’s star formation history and present-day spatial distribution of star formation over the entirety of the Galactic disk (let along the disk+halo gas & magnetic field structure across the entire galaxy, not just the Solar neighborhood), these are all broadly “equally-plausible” MW analogs (for more detailed discussion, see Sanderson et al. 2020).

We see systematic differences between the three galaxies which are comparable to the dispersion in CR spectra within a galaxy. Even though these are all very similar (relative to e.g. other \( \sim L_\odot \) galaxies of much earlier or later type), the detailed differences above produce some systematic differences in the spatial distribution and degree of clustering of SNe, the local PDF of gas and radiation and magnetic field energy densities (and therefore losses and secondary production rates), and other related quantities. This goes further to show that differences in the details of the local structure of the ISM are crucial for interpreting CR spectra at better than order-of-magnitude level.
3.5.4 Comparison with γ-ray Observations

Fig. 11 further compares the variation of CR properties with Galactic environment to constraints from diffuse Galactic γ-ray emission. First, we show the spatial dependence from Fig. 8 in more detail, specifically plotting the CR proton number or energy density in narrow bins of CR energy or rigidity as a function of Galactic position. We decompose the spherical radial trend from Fig. 8 into cylindrical coordinates, comparing the CR profile in the disk midplane versus cylindrical galactocentric radius, and at the Solar circle (≈ 8 kpc cylindrical) as a function of vertical height above the disk. We see the expected trend: owing to less-efficient diffusion, lower-energy CR protons have shorter radial and vertical scale-lengths. In the vertical direction (again, at the Solar circle), the CR profiles are approximately exponential ($n(R, z) \propto \exp (-|z|/h_{e_0})$) within a few kpc of the disk, and the vertical scale-height increases systematically with CR energy, from $\sim 0.5 - 1$ kpc at $\sim 1 - 5$ MeV to $\sim 2$ kpc at $\sim 1 - 3$ GeV to $\sim 6 - 10$ kpc at $\sim 0.3 - 1$ TeV. In the midplane radial direction the qualitative trends are similar but the profiles deviate more strongly from a single exponential and differ from one another more weakly as a function of energy, owing to the continuous distribution of sources populating the disk and much larger range of galactocentric scales considered. This confirms, however, that there is an extended CR halo, whose size increases as a function of CR energy. The dependence of profile shape on energy is generally weaker at low energies ($\lesssim$ GeV), owing to the fact that low-energy hadrons (with low diffusivity) have residence times increasingly dominated by losses.

Fig. 11 compares to observations of the inferred CR emissivity in γ-rays at energies $E_\gamma > 100$ MeV and $E_\gamma > 1$ GeV. We calculate the emissivity directly from the CR spectra (including pion production from protons and heavier nuclei, as well as $e^\pm$ and $\bar{\nu}$ annihilation), with the same cross sections used in code (see § 2.2 and e.g. Dermer et al. 2013). We caution that there can be spatial variations in emissivity from e.g. nearby clouds or Galactic structures even at a given galactocentric radius (see Ackermann et al. 2012 and note that detections and upper limits in Fig. 11 at the same distance.

14 In Fig. 11 we see that there is, especially at low energies, a steeper initial falloff at small $|z|$ followed by a somewhat shallower vertical profile. This reflects the fact that the quasi-exponential vertical profile transitions to a more quasi-spherical, power-law profile in the CGM, at sufficiently large radii where the disk/source geometry is no longer important.
range often differ in emissivity by much more than their statistical error bars), and that there are often very large distance uncertainties (which are themselves model-dependent) regarding where observed emission actually originates, so it is important to compare the observed points and uncertainty range to the full range (shaded area) predicted. With that in mind, the simulations agree quite well with both the radial and vertical trends, at a range of different $\gamma$-ray/CR energies. Comparing our three different galaxy models, we note that there are some appreciable differences in the profiles especially in the galactic nucleus (which is sensitive to the instantaneous state of the galaxy, e.g. whether there has been a recent nuclear starburst, while the Milky Way appears to be in a period of quiescence; Orr et al. 2021), and at large radii (where the less-extended star forming disk in m12i owing to its bar and warped disk structure noted in Fig. 1 leads to a more rapid falloff at $\gtrsim 15\text{kpc}$).

Note that the above applies to protons (other hadrons are similar). The electrons (leptons) behave differently, however, owing to the more complicated role of losses. Considering the electron vertical profiles at the Solar circle, for example, we see the $e^-$ scale-height decreases with increasing energy weakly (by $\sim 20\%$) from $1 \rightarrow 50\text{MeV}$ where diffusivities are very low so transport is dominated by advection and streaming, then increases with energy (by a factor of $\sim 2$ from $50\text{MeV}$ to $50\text{GeV}$) at intermediate energies where losses are not dominant (outside the disk) and diffusivity increases with energy, then at $\gtrsim 50 - 100\text{GeV}$ the scale height starts to drop significantly with increasing energy owing to rapid inverse Compton and synchrotron losses even in the halo.

We have also compared the variation of the mean predicted spectral index ($\alpha$, in $I \propto E^{-\alpha}$) of the emissivity as a function of midplane radius $R$ to FERMI observational estimates from Acero et al. (2016) and Yang et al. (2016). Interestingly, at energies $\sim 3 - 30\text{GeV}$ which dominate the fitted indices in those observational studies, our m12i run at $z = 0$ produces a trend very similar to that observed (wherein $\alpha$ peaks with a very steep/soft value at $\lesssim 2\text{kpc}$ in the galactic center, then falls rapidly by $\Delta\alpha_s \sim 0.4 - 0.6$ with increasing $R$ to shallower/harder values from $R \sim 2 - 6\text{kpc}$, then gradually increases/steeeps again by $\Delta\alpha_s \sim 0.3$ out to $R \sim 10 - 20\text{kpc}$). However, our m12m and m12f runs at $z = 0$ do not show the same trend (they show a weaker trend of $\alpha_s$ with $R$, with occasionally opposite sign); moreover analyzing different snapshots shows this varies in time, as well. This is because the $\gamma$-ray emission (scaling as $\sim n_{\text{gas}} R_0$) is sensitive to the densest emitting regions in each annulus, which have different spectral slopes at intermediate energies as shown in Fig. 8; moreover as discussed in Acero et al. (2016) this can also depend on variations in losses (as some dense regions reach calorimetric losses) and the structure of outflows (with advection modifying transport speeds). As such, these higher-order trends, while possible to reproduce, are sensitive to the instantaneous dynamical state of the galaxy.

### 3.5.5 Implications for the CR Ionization Rate

A number of studies have attempted to compare the CR ionization rate $\zeta$ inferred from the observed molecular line structure of GMCs, to the rate one would get from simply extrapolating the LISM CR spectrum. Although these inferences of $\zeta$ must be taken with some caution as the values are strongly model-dependent and have potentially large systematic errors, a number of independent studies in e.g. Indriolo et al. (2009); Padovani et al. (2009); Indriolo & McCall (2012); Indriolo et al. (2015); Cummings et al. (2016) have concluded that there must be some variation in $\zeta$ between nearby molecular gas in GMCs in the Solar neighborhood (where the inferred $\zeta \sim 10^{-16}\text{s}^{-1}$) and the (predominantly ionized, much-lower-density) LISM (which has an implied $\zeta \sim 10^{-17}\text{s}^{-1}$). Similarly, In-

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**Figure 12.** Comparison of CR ionization rates vs. environment. Top: Differential contribution to the total CR ionization rate $\zeta$ from CRs with different kinetic energies, from $p$ (thick) and $e$ (thin); all other species are sub-dominant) in different gas densities $n$ at the Solar circle ($R = 7 - 9\text{kpc}$). A broad range of energies contribute, with the lowest-energy CRs dominant in the most-dense gas. Usually $p$ dominate. Middle: Total ionization rate $\zeta$ (summing all CR energies and species) vs. gas density at fixed galacto-centric radius $R = 7 - 9\text{kpc}$. Enhanced ionization in GMC environments (high densities, compared to LISM $n \sim 0.1 - 1\text{cm}^{-3}$) arises naturally. Bottom: Ionization rate $\zeta$ vs. galacto-centric radius (weighted by total ionization rate of neutral gas). The enhanced nuclear CR densities lead to strongly-enhanced $\zeta$ as $R \rightarrow 0$. We compare observational estimates in Indriolo et al. (2015), from dense GMC molecular line tracers. Error bars show the range of distance and inferred $\zeta$ for each cloud; points show individual upper limits and detections without error bars treating each velocity sub-channel of each cloud as a separate system and using kinematic models to place it at its own distance.
driolo et al. (2015) showed there must be significant variation with galacto-centric radius (with larger ζ towards the Galactic center).

Recalling that CR ionization is dominated by low-energy CRs with $E \lesssim 100 \text{ MeV}$, the variations we have described above immediately provide a potential explanation for all of these observations. We can make this more rigorous by directly calculating the CR ionization rate from our spectra, as shown in Fig. 12. Following Indriolo et al. (2009)\(^{15}\) we take: $\zeta \equiv \sum_j 4\pi r_j^2 \frac{\jmath_{\text{ion}}}{J_{\text{ion}}} J_r(T) \sigma_s(T) dT$ with $\jmath_{\text{ion}} \approx 2 \text{ MeV}/n$, $\jmath_{\text{ion}} \approx 10^7 \text{ MeV}/n$, and $\sigma_s(T) \approx 7.63 \times 10^{-20} \text{ cm}^2 \cdot \text{s}^{-1} (1 - 0.067 \beta^2 - 0.20 \log_{10}(1/\beta))$, and calculate $\zeta$ in every cell in our simulations. We can then weight by the actual total ionization rate of molecular gas ($\approx \int d^3 \mathbf{r} \zeta(\mathbf{r}) n_\text{H}_2(\mathbf{r})$) to compare to the GMC observations which measure molecular indicators.

First, we examine which CRs contribute primarily to the ionization rate. As expected, these are low-energy CRs, primarily protons, but there is a broad range of energies which contribute similarly to the total ionization rate, and low-energy electrons are not at all negligible.

By definition, since our “fiducial” model roughly reproduces the observed low-energy Voyager CR spectra of $e^-$ and $p$, it should also approximately reproduce the LIS-M inferred $\zeta \approx 10^{-15} \text{ s}^{-1}$ in diffuse ionized gas of the appropriate densities, and we see in Fig. 12 that this is indeed the case. But we also see immediately in Fig. 12 that this reproduces the observed $\zeta \approx 10^{-18} \text{ s}^{-1}$ in dense molecular gas at the Solar neighborhood. The reason is simply the combination of (1) the dependence of low-energy CR densities on gas density discussed in § 3.5.1 and shown explicitly in Fig. 12 for the ionizing spectra in different gas environments, together with (2) the fact that the observations are sensitive to a total-ionization-rate-weighted-mean in molecular gas, which will always (by definition) give a systematically higher value than the volume-weighted mean or median in a medium with variations. Likewise, for the same reasons detailed above, these models naturally reproduce the observed trend of $\zeta$ with Galacto-centric radius, again shown in Fig. 12. Here, to compare the simulations with observations even heuristically, we simply take a total actual-ionization-rate-weighted average, so weighted somewhat towards more dense gas, at various galacto-centric annuli, and compare with compiled observations of dense GMC cloud cores, attempting to account for the large systematic uncertainties in their distances.

We stress that this is occurring as described in the low-energy CRs: some of the CR ionization studies above assumed, based on γ-ray observations, that the CR background must be more smooth; however the observed γ rays are dominated by orders-of-magnitude higher-energy CRs, which as shown in § 3.5.1 are indeed distributed much more uniformly.

4 CONCLUSIONS

We have presented and studied the first live-MHD galaxy formation simulations to self-consistently incorporate explicitly-evolved CR spectra (as opposed to a single field simply representing the total CR energy). As such we explicitly follow the ISM+CGM gas dynamics (inflows, outflows, fountains, ISM turbulence, super-bubbles, etc), thermal phase structure of the gas, magnetic field structure, along with spectrally-resolved CR populations from $\sim \text{MeV}$ to $\text{TeV}$, including protons $p$, electrons $e^-$, anti-protons $\bar{p}$, positrons $e^+$, intermediate primaries like C, N, O, stable secondaries $^7\text{Be}$, $^8\text{Be}$, and radioactive secondaries $^{10}\text{Be}$, with a network that allows for all the major CR evolutionary processes. We also adopt a recently-developed detailed treatment of the CR transport equations which, unlike the commonly-adopted “isotropic Fokker Planck” equation does not impose any assumptions about strong scattering, isotropic scattering rates or magnetic field structures, etc, but instead includes all terms from the Vlasov equation to leading order in $O(u/c)$ (where $u$ is the Galactic fluid velocity), and gives much more general expressions for terms like the “re-acceleration” terms commonly invoked.

From the point of view of understanding CR transport physics, the fundamental advantage of these models as compared to traditional historical models which treat the Galaxy with a simplified time-static analytic model (ignoring or making different assumptions for e.g. inflows/outflows, turbulence, source distributions, etc), is that many terms which can, in principle, be freely adjusted or varied in those analytic models (e.g. presence or absence or “size” of the “halo,” presence of inflows/outflows, structure or absence of turbulent/fountain motions, structure or neglect of the magnetic field structure, inhomogeneous density/ionization/magnetic field strength/radiation energy density variations in the ISM and therefore rates of all loss terms, ratios of injected primaries, etc.) are determined here by the self-consistent cosmological evolution. This removes tremendous degeneracies and allows us to explore some crucial sources of systematic uncertainty in those models (which remain order-of-magnitude).

From the point of view of constraining CR models for application in Galaxy formation simulations, the models here allow us to calibrate CR transport assumptions in far greater detail and rigor than is possible with “single” bin models (where one can compare e.g. the total γ-ray emission from $p \rightarrow \pi^0$ production to observations, but this provides only a single, galaxy-integrated data point for a few galaxies). In the future, we will explore detailed synchrotron spectra from these models in external galaxies. This will allow us to explore the consequences of CRs in other galaxies with much greater fidelity.

4.1 Key Conclusions

We show the following:

(i) It is possible to (roughly) match Solar/LISM CR constraints with simple transport and injection models. Specifically assuming a single-power-law injection spectrum with a standard slope ($\sim 4.2$), single-power-law scaling of the CR scattering rate with rigidity $\nu \approx \beta R^{-4}$ and $\delta \approx 0.5 – 0.6$, following all the major loss/gain processes with their expected (locally-varying) rates. As expected, in the LISM, we show that the shape of the high-energy hadronic spectra are regulated by injection+escape (dependence of scattering rates on rigidity), while high-energy lepton+hadronic spectra are regulated (primarily) by ionization+Coulomb losses.

(ii) “Large” halo sizes are inevitable & favored. The normalization of the halo structure is not a free parameter in our models. Indeed, it is now well-established that a majority of the baryons and significant magnetic field strengths extend to hundreds of kpc around galaxies in the CGM, so it is un-avoidable that the “thin disk” or “leaky box” model would be a poor approximation. In terms of the idealized cylindrical CR scattering halos sometimes adopted analytically, in the limit of diffusive CRs, the correct “effective” halo size (defined as the region interior to which a CR has a non-negligible probability of scattering to Earth) will always be (up to an order-unity factor) the same as the Solar circle radius $r_\odot \sim 8 \text{kpc}$). This in turn means that relatively low CR scattering rates (giving relatively high effective diffusivities), compared to

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\(^{15}\) Note our definition is slightly different from Indriolo et al. (2009), who also multiply by the parameter $\xi = 0.15 \pm 0.23$ in ionized or atomic or molecular gas respectively, to account for secondary ionizations. We correct the observations by this factor so they can be compared directly: i.e. we define $\zeta$ such that an exactly identical CR spectrum will produce an identical $\zeta$, regardless of the ambient non-relativistic gas properties.

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decades-older “leaky box” models which ignored the halo+CGM, are required. Our inferred scattering rate at \( \sim 1 \text{GV}, \bar{\nu} \sim 10^{-2} \text{s}^{-1} \), is in fact in excellent agreement (within a factor of \( \sim 2 \), despite enormous differences in model details) with most recent analytic Galactic CR transport models, almost all of which have argued that a scattering halo\(^\text{16}\) with effective size \( \sim 5 - 10 \text{kpc} \) is required to match the LISM observations (Blasi & Amato 2012a; Vladimirov et al. 2012; Gaggero et al. 2015; Guo et al. 2016; Jóhannesson et al. 2016; Cummings et al. 2016; Korsmeier & Cuoco 2016; Evoli et al. 2017; Amato & Blasi 2018; de la Torre Luque et al. 2021).

(ii) Re-acceleration terms are not dominant, and obey a generic ordering. There are three terms which can act as “re-acceleration”: the “adiabatic” or non-inertial frame term \( \dot{p}_{\text{ad}} = -p \nabla \cdot \dot{\mathbf{u}} \), the “streaming loss” term \( \dot{p}_{\text{st}} = -\langle \dot{\nu} \rangle \nabla /c^2 \langle \mathcal{F} \rangle /c^2 \), and the “diffusive” or “micro-turbulent” re-acceleration term \( \dot{p}_{\text{diff}} = 2(1 + \beta^2)p^{-1} \dot{D}_{\text{pp}} \sim \langle \dot{\nu} \rangle (\gamma/c)^2 \). We show that for almost any physically-realistic structure of the ISM in terms of \( \nu, \mathbf{u}, \mathbf{v} \), etc. and allowed values of \( \bar{\nu} \), there is a robust ordering with \( |\dot{p}_{\text{ad}}| \gtrsim |\dot{p}_{\text{st}}| \gtrsim |\dot{p}_{\text{diff}}| \gtrsim \bar{\nu} \) at \( \gtrsim \text{GV} \), and that these terms have at most modest (tens of percent) effects on the total CR spectrum.

(iv) Most \( \lesssim \text{TeV} \) Galactic CRs are accelerated in SNe shocks, in super-bubbles, early in the Sedov-Taylor phase (after the reverse shock forms). Observed abundances of intermediate and heavy primary elements in CRs are all consistent, to leading order, with a universal single-power-law acceleration spectrum with all species tracing their in-situ abundances in the test particle limit if we assume CRs are accelerated with an efficiency \( \epsilon \sim 10\% \) of strong shock energy when the shock first forms – i.e. when the entrained mass of ambient ISM material is approximately equal to the initial ejects mass (\( M_{\text{eject}} \gg M_{\odot} \)). This is naturally predicted if CRs accelerate when the shock first forms, and the kinetic energy dissipation rate and Mach number are maximized. If instead acceleration occurred primarily in stellar wind/jet shocks, diffusive ISM shocks with Mach number \( \gg 1 \), or throughout the entire Sedov-Taylor phase of SNe remnants, then the abundances of CNO at \( \sim \text{MeV-TeV} \) would be under-predicted by factors of \( \sim 20 \). Given the favored conditions, most MW acceleration occurs in SNe shocks within super-bubble-type conditions.

(v) CR spectra vary significantly in time & space, both systematically and stochastically. With more realistic Galactic models, substantial variations are expected between and within Galaxies. CR energy densities decrease with increasing galacto-centric radius \( r \propto 1/r \), for constant scattering rates, over a range of radii) and spectra are harder in hadrons, softer in leptons towards the Galactic center, owing to differences in loss rates and source spatial distributions. We show this naturally reproduces Galactic \( \gamma \)-ray emissivity observations, though \( \gamma \)-ray-inferred variations in spectral shape can be sensitive to the local dynamical state of the dense \( \gamma \)-ray emitting gas in the Galactic center. At the Solar circle, the CR spectra still vary significantly with local environment and gas density \( n \), with e.g. CR kinetic energy density \( \propto n^{\alpha} \) (i.e. higher in more dense environments) – the effect is stronger at lower CR energies owing to tighter coupling with the gas, while becoming negligible at \( \gtrsim 1 \text{GeV} \). Even controlling for e.g. \( r, n \) and other variables (temperature, etc.), the scatter in particularly low-energy CR spectra from point-to-point in space or time or between galaxies can vary by orders of magnitude (\( \sim 90\% \) interval of \( \pm 1.5 \text{dex} \) at \( \lesssim 10 \text{MeV} \)), owing to the enormous inhomogeneities in local source distributions (clustered star formation & SNe), loss rates (orders-of-magnitude variation in local densities, ionization fractions, radiation energy densities, etc.), and local gas dynamics (e.g. local inflow/outflow, turbulence structure). This provides a natural explanation for observations which have inferred a different CR ionization rate in local molecular clouds (compared to the diffuse LISM observed by Voyager) and different ionization rates at different Galactic positions.

4.2 Future Work
This is only a first study and there are many different directions in which it can be extended. In future work, we will explore predictions for a wide range of galaxies outside of the MW, from dwarfs through starbursts and massive ellipticals, at both low and high redshifts. With the models here, we can make detailed forward-modeled predictions for spatially-resolved synchrotron spectra in these galaxies, to compare to the tremendous wealth of resolved extragalactic synchrotron studies. We can also forward-model the \( \gamma \)-ray spectrum, which provides a key complementary constraint, albeit only in a few nearby galaxies.

We stress that the extremely simple (and constant in space and time) scaling of the scattering rates adopted here, \( \bar{\nu} = \bar{\nu} \beta R_{\text{GV}} \), is purely heuristic/empirical. This is quite radically different, in fact, from what is predicted by either traditional “extrinsic turbulence” models for CR scattering or more modern “self-confinement” motivated models. In both of those model classes the local scattering rates (e.g. across different ISM regions at the Solar circle) can vary by many orders of magnitude in both space and time, on scales smaller than the CR residence time or disk/halo scale height, as a strong function of the local turbulence properties, plasma-\( \beta \), neutral fractions, magnetic field strength, gas density, and other parameters (Hogkins et al. 2020b). These effects simply cannot be captured in standard CR transport models which adopt simplified static analytic models for Galactic structure. The most interesting application of the new methods here, which attempt to combine more detailed CR propagation constraints with detailed, live galaxy simulations that explicitly evolve those parameters, is therefore likely to be exploring and making detailed predictions from those more physically-motivated CR transport/scattering models, in a way which was previously not possible.

It will also be particularly important, especially with a variable \( \bar{\nu} \), to investigate how local variations in plasma properties modify CR loss and other key timescales commonly assumed in analytic models for CR transport or observables such as the FIR-radio or \( \gamma \)-ray-SFR relations. For example, the “effective” or mean synchrotron loss timescale (\( M_{\text{synch}} \)) at some CR energy represents a complicated weighted average over different ISM regions, so can differ significantly from the synchrotron loss timescale estimated using just the volume-averaged mean magnetic field value. Exploring where and when these differences are important in detail will be an important subject for future study.

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APPENDIX A: ADDITIONAL PHYSICS & PARAMETER VARIATIONS

In § 3.2, Figs. 3, 4, & 5 compared the effects of different parameter and physics variations on CR observables, by turning on and off different physics or varying different parameters with respect to the “reference” or best-fit model in Fig. 2.

We have explored a number of other variations as well, as described in the main text, in order to identify robust trends and the best-fit model compared to observations. Figs. A1, A2, & A3 illustrate some of these. These are identical to Figs. 3, 4, & 5, except that we consider a different “reference” model as the baseline about which parameters and physics are varied. Specifically here we take a model with a fixed higher scattering rate (lower diffusivity) normalization $\bar{\nu}_0 = 10^{-8} \text{s}^{-1}$, which is then re-tuned (fitting $\delta$ and $\psi_{\text{inj}}$) to try and reproduce the spectra and B/C ratios as best as possible, giving $\bar{\nu} \sim 10^{-8} \beta R^{-1}_G$ (i.e. $\delta = 1$, with slightly-different $\psi_{\text{inj}} = 4.3$), as compared to main-text default $\bar{\nu}_0 = 10^{-9} \text{s}^{-1}$, $\delta = 0.5 \pm 0.6$, $\psi_{\text{inj}} = 4.2$. We stress that directly comparing this reference model to the observations as in Fig. 2 shows that even with “re-fitting” $\delta$ and $\psi_{\text{inj}}$ at this $\bar{\nu}_0$, the fit (comparing to Solar circle LISM data) is significantly more poor than our default main-text model: B/C is too flat between $\sim 0.3 \text{ – 100 GeV}$ (under-predicting B/C at $< 3 \text{ GeV}$ and over-predicting B/C at $> 3 \text{ GeV}$), $^{10}\text{Be}/^{9}\text{Be}$ is systematically too-high at $\sim 0.03 \text{ – 100 GeV}$, $e^+/(e^+ + e^-)$ is “too flat” (it does not feature the “curvature” observed from $\sim 0.5 \text{ – 300 GeV}$), and the spectra are too hard, under (over)-predicting the intensity of $e^-$ and $p$ at $< 100 \text{ GeV}$ ($> 100 \text{ GeV}$).

Nonetheless, this provides a useful reference case to consider the systematic effects of different physics and parameter variations in Figs. 3, 4, & 5. Because of non-linear interactions between the different physics, as described in the text, it is not totally obvious that changing one of the physics or assumptions would have the same systematic effect if we also change the “reference” model. For example, since the diffusivity at low CR energies is much lower here than in our main-text reference model, certain losses in dense ISM environments could be qualitatively more important, and this can order-of-magnitude change the ratio of e.g. diffusive reacceleration to streaming loss terms. Nonetheless, Figs. A1, A2, & A3 confirm that all of our qualitative conclusions in the text, regarding the systematic effects of these variations as well as their qualitative importance, appear to be robust.
Figure A1. As Fig. 3, comparing CR spectra in simulations with different injection slopes $\psi_{\text{inj}}$, scattering rate normalization $\bar{\nu}$, and dependence of scattering rates on rigidity $\delta$. The “reference” parameters here are different from Fig. 3: we vary about a “reference” model with $\bar{\nu}_{\text{inj}} = 2.0$, $\bar{\nu} = 0.3$, and $\delta = 10$. This model is a notably poorer fit to the observations in Fig. 2 compared to that $\bar{\nu}_{\text{inj}} = 0.3$, $\bar{\nu} = 1.0$, and $\delta = 2.0$. The “reference” parameters here are different from Fig. 3: we vary about a “reference” model with $\bar{\nu}_{\text{inj}} = 2.0$, $\bar{\nu} = 0.3$, and $\delta = 10$. This model is a notably poorer fit to the observations in Fig. 2 compared to that in the main text, but represents a model “re-tuned” to at least reasonably fit B/C with different parameters. Systematically varying the parameters about the reference model defaults allows us to see that all the qualitative conclusions from Fig. 3 regarding the systematic effects of these parameter variations are robust to the “reference” model or other parameter choices.
Figure A2. CR spectra with different loss terms and other physics disabled, as Fig. 4, but with the variations being with respect to the alternative “reference model” from Fig. A1. Again, despite the systematically different reference-model parameters, the systematic effects of these physics variations are consistent with Fig. 4.
Figure A3. CR spectra with different closure assumptions, arbitrarily re-normalized magnetic field strengths, and reduced-speed-of-light, as Fig. 5, with the variations being with respect to the alternative “reference model” from Fig. A1. As in Fig. A2, the systematic effects of these physics variations are consistent with Fig. 5.