Contributions of semi-hadronic states $P\gamma; S\gamma, \pi^+\pi^-\gamma$ to amm of muon, in frames of Nambu-Jona-Lasinio model.

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We calculate the contribution of semi-hadronic states with pseudoscalar $P = \pi^0, \eta$ and scalar ($\sigma(550)$) meson accompanied with real photon as an intermediate state of a heavy photon to the anomalous magnetic moment of muon. We consider the intermediate states with $\pi^0$ and $\sigma$ as a hadrons in frames of Nambu-Jona-Lasinio model. The contribution of $\pi^0\gamma$ state is in agreement with results obtained in previous theoretical considerations as well as with experimental data $a_\mu^{\pi^0\gamma} \approx 4.5 \times 10^{-10}$, besides we estimate $a_\mu^{\eta\gamma} = 0.7 \times 10^{-10}$, $a_\mu^{\sigma\gamma} \sim 1.5 \times 10^{-11}$, $a_\mu^{\pi^+\pi^-\gamma} \sim 3.2 \times 10^{-10}$.

We discuss as well the LbL mechanism with $a_\mu^{Lb} = 10.5 \cdot 10^{-10}$.

I. INTRODUCTION

One of modern precise test of Standard Model is the measurement of anomalous magnetic moment of muon (amm) $a_\mu$. The SM contributions are usually split into three parts:

$$a_\mu = a_\mu^{QED} + a_\mu^{EW} + a_\mu^{hadr}.$$ 

Contributions of hadrons associated with real photons (semi-hadrons ones) can be separated to two classes. One of them consist in diagrams of vertex type with heavy photon with insertion of hadronic vacuum polarization block (see Fig1). Another contained the light-by-light scattering block (LbL) will be discussed below (Fig.2). Using the dispersion approach first type of contributions can be written as

$$a_\mu^{P(S)\gamma} = \frac{1}{4\pi^2} \int_{m_{P(S)}}^\infty ds \cdot \sigma_{ee\rightarrow P(S)\gamma}(s) \cdot K\left(\frac{s}{M_\mu^2}\right).$$ 

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The analytic form of the kernel $K(\rho)$ \cite{1} is:

$$K(\rho) = \int_0^1 \frac{x^2(1-x)dx}{x^2 + (1-x)\rho};$$

$$K(\rho) = \frac{1}{2} - \rho + \frac{1}{2}\rho(\rho - 2) \ln \rho - \frac{(\rho^2 - 4\rho + 2)\rho}{2\sqrt{\rho(\rho - 4)}} \ln \frac{\sqrt{\rho} + \sqrt{\rho - 4}}{\sqrt{\rho} - \sqrt{\rho - 4}};$$

$$\rho = \frac{s}{M_{\rho}^2}; \quad K^{(1)}(\rho)|_{\rho \gg 1} = \frac{1}{3\rho}.$$  \hspace{1cm} (2)

The main part of contribution to $a^\mu_{hadr}$ of order 5004 (units $10^{-11}$ implied) arise from $\pi^+\pi^-$ channel annihilation of $e^+e^-$ pair (3 $\pi$: 438 \cdot 10^{-11}; 2K: 314 \cdot 10^{-11}$ \cite{4}).

Below we will consider the annihilation channels

$$e^+e^- \rightarrow \gamma^* \rightarrow P\gamma, \quad S\gamma; \quad P = \pi_0, \eta; \quad S = \sigma,$$

$$e^+e^- \rightarrow \gamma^* \rightarrow \pi^+\pi^-\gamma.$$  \hspace{1cm} (3)

During the recent years the papers with calculation of semi-hadron states where published \cite{2, 4}. Rather stable results was obtained for the $\pi_0\gamma$ state, whereas a contradictive results was obtained for contribution of $\sigma\gamma$ state \cite{3}. Below we obtain these contributions in frames NJL model \cite{9, 11}, both are consistent with modern experimental data \cite{5}.

The relevant part of chiral Lagrangian in U(3)× U(3) chiral NJL is \cite{6, 9}

$$L = \bar{q}[i\tilde{\partial} + m - eQ\hat{A} + g_\pi(\lambda_3\pi_0 + \lambda_+\pi_+ + \lambda_-\pi_-)\gamma_5 + g_\sigma\sigma \cdot \lambda_3 + g_k(\lambda_+K_+ + \lambda_-K_-) + \frac{g_\rho}{2}(\lambda_3\hat{\rho}_0 + \lambda_4\hat{\omega})]q,$$  \hspace{1cm} (4)

where $\sigma = \lambda_u\sigma_u + \sigma_s\lambda_s$, $\bar{q} = (\bar{u}, \bar{d}, \bar{s})$ where $u, d, s$ are the quark fields, $Q$=diag(2/3,-1/3, -1/3) is, the quark charge matrix, $\lambda_1 = \frac{1}{\sqrt{3}}(\sqrt{2}\lambda_0 + \lambda_8)$ where $\lambda_i$ are Gell-Mann matrices and $\lambda_0 = \sqrt{2}/3 \text{ diag}(1,1,1)$, $g_\rho=5.95$ is the $\rho \rightarrow 2\pi$ coupling constant, $g_\sigma \approx 3$.

We will use the matrix element of sub-process $\gamma^*(q, \mu) \rightarrow P(p)\gamma(k, \nu)$

$$M^{\gamma^*\rightarrow P\gamma} = \frac{\alpha}{\pi f_\pi} F_P(q^2)\epsilon^\mu_{\alpha\beta\gamma}q^\alpha k^\beta e^{\nu}(k)e^{\nu}(q), \quad f_\pi = 93 MeV,$$  \hspace{1cm} (5)

with condition $F(0) = 1$. We remind the current algebra expression for the pion decay width

$$\Gamma_{\text{exp}}^{\pi^0 \rightarrow 2\gamma} \approx 7.3 \text{ eV}.$$
NJL result is
\[ \Gamma_{N\text{JL}}^{\pi_0 \rightarrow 2\gamma} = \alpha^2 M_\pi^3 / (64\pi^3 f_\pi^2) \approx 7.1 \text{ eV}. \]

The similar expression for \( \gamma^*(q, \mu) \rightarrow S(p) \gamma(k) \):
\[ M_{\gamma^* \rightarrow S\gamma} = \frac{\alpha}{\pi f_\pi} F_S(q^2)(g_{\mu\nu} \cdot kq - q_{\mu} \cdot k_{\nu})e^\mu(k)e^{\nu}(q). \] (6)

Total cross sections of creation \( P \gamma, S\gamma \) in electron-positron annihilation are:
\[ \sigma_{\text{theor}}^{e^+e^- \rightarrow P\gamma} = 8\pi\alpha \frac{M_P^2}{3s} \Gamma_{P\gamma} \left( 1 - \frac{M_P^2}{s} \right) \frac{M_P^4}{(s - M_P^2)^2 + M_P^2\Gamma_P^2}, \] (7)

In the same way for scalar particles we obtain
\[ \sigma_{\text{theor}}^{e^+e^- \rightarrow S\gamma} = 8\pi\alpha \frac{M_S^2}{3s} \Gamma_{S\gamma} \left( 1 - \frac{M_S^2}{s} \right) \frac{M_S^4}{(s - M_S^2)^2 + M_S^2\Gamma_S^2}, \] (8)

The gauge invariant provide convergence of the loop momentum integral for \( a_\mu \) so the application of such a low-energy models as Nambu-Iona-Lasinio (NJL) one \cite{8} for description of processes of conversion of a virtual photon to light mesons and in particular to mesons and a real photons, NJL permits to calculate constant of strong coupling \( g_\pi, g_\rho, g_\sigma \).

Calculations leads to
\[ (g - 2)_{\mu}^{\pi\gamma} \approx 4.5 \cdot 10^{-10}, \]
\[ (g - 2)_{\mu}^{\eta\gamma} \approx 0.7 \cdot 10^{-10}, \]
\[ (g - 2)_{\mu}^{\sigma\gamma} \approx 0.15 \cdot 10^{-10}. \]

The contribution out the experimentally accessed region \( 0.6 < \sqrt{s} < 1.03 \text{ GeV} \) was obtained \cite{5}
\[ a_\mu(\pi_0\gamma, \sqrt{s} < 1.03\text{GeV})^{\text{exp}} = (4.5 \pm 0.15) \times 10^{-10}; \]
\[ a_\mu(\eta\gamma, 0.69 < \sqrt{s} < 1.33\text{GeV})^{\text{exp}} = (0.73 \pm 0.03) \times 10^{-10}. \] (9)

The contribution from the region below the experimentally accessible region is
\[ a_\mu(\pi_0\gamma, \sqrt{s} < 0.6\text{GeV}) = (0.13 \pm 0.01) \times 10^{-10}. \] (10)

The contribution of radiative processes with charged pions production and 2 neutral ones was found \cite{4, 5} to be is
\[ a_\mu(e^+e^- \rightarrow \pi^+\pi^-\gamma, \sqrt{s} < 1.2\text{GeV}) = (38.6 \pm 1.0) \times 10^{-11}. \] (11)
Note that $P \gamma$ we use only quark loops, whereas for $S \gamma$ besides quark loops the triangle loops with pions and kaons as well are relevant. Total contribution $\pi_0 \gamma; \eta \gamma; 2 \pi \gamma$ is

$$a^{\gamma}_{\mu}(e^+e^- \rightarrow hadr + \gamma) = (93 \pm 1.0) \times 10^{-11}. \quad (12)$$

For process $e^+e^- \rightarrow \sigma \gamma$ S. Narison had obtained [3] starting from QCD sum rules, two different results one of the is:

$$a_{\mu}(\sigma(600)\gamma) = 0.1 \cdot 10^{-10}, \quad (13)$$

which is in agreement with our NJL approach.

As for $\gamma^* \rightarrow \rho \rightarrow \sigma \gamma$ the quark loops as well as loops with $\pi_\pm, K_\pm$ must be taken into account, and, besides the imaginary part of meson loops amplitudes must be taken into account, whereas for quark loops only real part must be considered (naive confinement).

Both component of $\sigma$ meson $\sigma = \sigma_u \cos \alpha + \sigma_s \sin \alpha$ contribute, besides $\sigma_s$ do not contain quarks and pion loops. Main contribution arise from $\sigma_u$. For the case $\sigma \gamma$ main contributions arises from light quarks and from the pion loop with constructive interference, resulting $\Gamma_{\sigma \rightarrow 2\gamma} = 4.3 \text{ KeV}$ [8, 9].

In NJL approach we obtain for $e^+e^- \rightarrow \sigma(550)\gamma$:

$$a_{\mu}(\sigma(550)) = 0.16 \times 10^{-10}. \quad (14)$$

We use the $\sigma$-meson mass $m_\sigma = 550 \text{ MeV}$ as well calculated in [15] and agreement with experiment [16]. Calculating $\gamma^* \rightarrow \pi^+\pi^-\gamma$ we use Born approximation and the experimental pion form-factor [19]

$$(g - 2)_{\mu}^{\pi^+\pi^-\gamma} = \frac{1}{4\pi^3} \int_{4m_\pi^2}^{\infty} \sigma^{\pi^+\pi^-\gamma}(s)K(s)ds. \quad (15)$$

We use here [17, 18]

$$\sigma^{e^+e^-\rightarrow \pi^+\pi^-\gamma}(s) = \frac{2\alpha}{\pi} \sigma_B(s) \cdot \Delta(s);$$

$$\sigma_B(s) = \frac{\pi \alpha^2 \beta^3}{3s}|F_\pi(s)|^2; \quad \beta = \sqrt{1 - \frac{4m_\pi^2}{s}};$$

$$\Delta = \frac{3}{4\beta^2}(1 + \beta^2) - 2\ln \beta + 3\ln \frac{1 + \beta}{2} +$$

$$\frac{1}{8\beta^3}(1 - \beta)(-3 - 3\beta + 7\beta^2 - 5\beta^3)L_\beta + \frac{1 + \beta^2}{2\beta}F(\beta);$$

$$F(\beta) = -2Li(\beta) + 2Li(-\beta) - 2Li(1 + \beta) + 2Li(1 - \beta) +$$

$$3Li\left(\frac{1 + \beta}{2}\right) - 3Li\left(\frac{1 - \beta}{2}\right) + 3\xi_2, \quad \xi_2 = \frac{\pi^2}{6}. \quad (16)$$
Figure 1: Contributions from state $\gamma^* \to P; S; \pi^+\pi^-; \gamma$. Where $H = \pi^0; \eta; \pi^+\pi^-$. 

As a result, with $|F_\pi|^2 = 1$, we obtain $(g - 2)_{\mu}^{\pi^+\pi^-\gamma} = 0.7 \times 10^{-10}$, but with real form-factor $^{[19]}$, $(g - 2)_{\mu}^{\pi^+\pi^-\gamma} = 3.13 \times 10^{-10}$, agreement with contribution of nonresonance channel $^{[5]}$.

Analog of semi hadronic contributions is the light by light (L-b-L) scattering mechanism with intermediate states with scalar and pseudo scalar mesons (Fig 2). Convergence of different recent model calculations lead to the result $^{[13]}$ (see $^{[14]}$, A. Nyfeller talk and references there in)

$$a_{\mu}^{L-b-L} = (10.5 \pm 2.6) \times 10^{-10}.$$  

We put below the definite contributions (we follow the paper $^{[13]}$):

$$\pi_0 : 81.8 \times 10^{-11}; \eta : 5.62 \times 10^{-11}; \eta' : (8 \pm 1.7) \times 10^{-11};$$

$$\sigma(600) : 11.67 \times 10^{-11}; a_0(980) : 0.62 \times 10^{-11},$$

with the total contribution

$$a_{\mu}^{LbL} = (107.74 \pm 16.81) \times 10^{-11}.$$  

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Figure 2: Contributions type L-b-L mechanism with intermediate state $P; S$

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