A superconducting Fabry–Perot cavity for trapping cold molecules

Fatemeh S Tahsildaran F 1,2, Manish Vashishta 2, Amir Hossein Farahbod 3, Rasoul Malekfar 1,2, Pavle Djuricanin 2, Katsunari Enomoto 2,4 and Takamasa Momose 2,5,∗

1 Atomic and Molecular Physics Group, Department of Physics, Faculty of Basic Sciences, Tarbiat Modares University, Tehran, Iran
2 Department of Chemistry, The University of British Columbia, Vancouver, BC, Canada
3 Research School of Plasma Physics and Nuclear Fusion, Research Institute of Nuclear Sciences and Technologies, AEOI, Tehran, Iran
4 Department of Physics, University of Toyama, Toyama 930-8555, Japan
5 Department of Physics and Astronomy, The University of British Columbia, Vancouver, BC, Canada

E-mail: momose@chem.ubc.ca

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Abstract
A superconducting Fabry–Perot microwave cavity for a molecular trap was designed, constructed and characterized. The cavity was designed to create intense microwave fields that are sufficient to trap cold polar molecules by the AC Stark force. By coating the mirror surfaces with a superconducting material, an unloaded quality factor of up to $1.1 \times 10^6$ at 24.087 GHz was achieved at a temperature of 2.24 K. The field strength of 1.06 MV m$^{-1}$ obtained for a TEM$_{02}$ transverse mode with an input power of 10 W is sufficiently intense to trap ammonia molecules at a temperature of 85 mK, which is achievable by a conventional Stark molecular decelerator. We have also implemented an ion optics assembly for sensitive detection of molecules inside the microwave cavity by resonance enhanced multi-photon ionization.

Keywords: cold molecules, microwave trap, Fabry–Perot, superconducting

(Some figures may appear in colour only in the online journal)

1. Introduction
The field of cold and ultracold molecule research has been expanding rapidly in the last few decades because of their promising new possibilities in the study of a wide range of fundamental and practical problems in both physics and chemistry. The increase of interaction time between radiation and cold molecules due to their slow translation velocities would increase the precision in spectroscopy that can be applicable to explore various physics beyond the standard model [1]. Another important application in the field of chemistry is the control of cold collisions and reactions with well-defined states with long interaction times [2–6]. Reactions below 10 K are also relevant to chemical reactions in interstellar space, which still remains unexplained [7].

Manipulation of molecules by an optical dipole force is one of the key techniques that has to be developed for the precise control of molecular motion in any quantum state. Recent progress in laser cooling of molecules has reached a tremendous milestone with the ability of cooling molecules to the ultracold regime [8–10]. It has also been proposed that microwave (MW) and infrared (IR) radiation slightly detuned from resonance frequencies of rotational and rovibrational transitions provide forces to molecules by the AC Stark effect that can be used to decelerate and trap molecules [11–13]. An MW molecular focuser and decelerator has been demonstrated for NH$_3$ in the $J = 1, K = 1$ rotational excited state [14, 15].

In our previous work, we have also reported an MW lens effect for the rotational ground state of CH$_3$CN by combining...
Figure 1. (a) An overview of the experimental setup. It consists of a pulsed valve (NZ), a skimmer (SK), a microwave cavity (CV) and a micro-channel plate (MCP) detector. (b) A cross section view of the Fabry–Perot type MW cavity resonator and an ion optics system. M1, M2: Concave mirrors, HL: holes for molecular beams, AN: an MW loop antenna. The MW cavity is attached to the top of a cold head (CH). The ion optics system consists of two metallic plates (HV1: repeller (1600 V) and HV2: extractor (−300 V)).

A slow molecular beam generated by a pulsed counter rotating nozzle and an MW standing wave in a cylindrical waveguide cavity [16]. A transverse magnetic mode \( \text{TM}_{0,1,\rho} \) of an MW field slightly red-detuned from a rotational transition provides two-dimensional radial confinement harmonic potentials for molecules by the AC dipole force. Here, \( \rho \) is the number of longitudinal modes of the cylindrical waveguide cavity. The created field intensity of 195 kV m\(^{-1}\) was sufficiently strong to change the momentum of CH\(_3\)CN and acetone molecules with a velocity of 120 m s\(^{-1}\) created by a counter rotating nozzle [16]. We have also developed a superconducting cylindrical waveguide cavity [17], and demonstrated the focusing of a cold PbO molecular beam in a cylindrical waveguide cavity [18].

A cylindrical waveguide resonator would be useful for the deceleration and focusing of cold molecular beams, but it is not ideal for trap experiments as the optical access is significantly limited. In this article, we discuss the characterization of a newly designed Fabry–Perot type open MW resonator, aiming to trap cold molecules inside the resonator for optical detection. A normal conductor Fabry–Perot type MW resonator has been reported by Dunseith et al [19]. They have achieved a quality factor of \( 5.1 \times 10^4 \) at 14.5 GHz. In this study, we have investigated a superconducting Fabry–Perot resonator, which can have a much higher quality factor. We have focused on the resonance frequency corresponding to the tunneling transition of NH\(_3\) at 23.7 GHz. With our cavity, we have achieved a quality factor of \( 1.1 \times 10^6 \) at 2.24 K.

2. Microwave cavity

2.1. Design

The overall experimental setup is shown in figure 1(a). Pulsed molecular beams were introduced using a home-made pulsed valve (CRUCS valve), and skimmed by a skimmer with a 2 mm diameter (beam dynamics). The molecular beams enter the cavity through a hole located at the center of the cavity.

The Fabry–Perot MW resonator was composed of two spherical mirrors, each with a radius of curvature of \( R = 80 \) mm and a diameter of \( D = 90 \) mm. A cross section view of the cavity is shown in figure 1(b). The mirrors were made of oxygen free copper. The mirror surface was precisely machined with the surface roughness of 0.5 \( \mu \)m (rms). The inner surfaces of the mirrors were coated with a superconducting material, which was an alloy of lead and tin with a critical temperature of around 7–8 K [20]. The coating was done electrochemically using lead tetrafluoroborate and a lead/tin electrode. Both mirrors had a hole at their center with the diameter of \( H = 5 \) mm and the length of 6 mm for the transmission of molecular beams into the cavity. In addition to the transmission holes, one of the mirrors had a coupling hole in order to transmit MWs into the cavity by a loop antenna. The coupling hole with a diameter of 2.5 mm was located 15 mm below the transmission hole. The two mirrors were mounted on a copper block which was adjusted to the cold head of a closed cycle Gifford–McMahon (GM) refrigerator (SHI RDK-408D). The lowest temperature of the mirrors was 2.24 K, monitored by Si diode sensors (Lake Shore, DT-470).

The distance between the two mirrors was set around \( L = 60 \) mm at the center to achieve a resonant frequency close to 23.7 GHz (wavelength \( \lambda = 12.65 \) mm) which corresponds to the \( J = 1, K = 1 \) tunneling splitting of NH\(_3\) [14]. The distance of the mirrors was adjusted manually at room temperature by optimizing the resonant frequencies. The mirror parameter defined as \( g = 1 - L/R \) is 0.25 for this condition, which corresponds to a stable near confocal resonator that lies in the first quadrant in the resonator stability diagram. The mirror Fresnel number is \( D^2/4L\lambda = 2.67 \) and the hole Fresnel number is \( H^2/4L\lambda = 0.008 \).

For the detection of molecules inside the cavity, two metallic plates were attached at the bottom (HV1) and the top (HV2) of the cavity with an insulator. These plates provide the necessary electric fields to the molecules inside the cavity in order to extract the ions produced by pulsed laser ionization. The ions were extracted upwards and detected by a micro-channel plate.
(MCP) detector. All the MW field measurements described below were done with the complete cavity mounted, including the ion optics.

2.2. MW modes

Our cavity is a circularly symmetric open cavity, and therefore the electric field inside the cavity may be described by Laguerre–Gaussian modes [21]. However, due to the off-axis position of the coupling antenna, the degeneracy for transverse modes is slightly lifted. Therefore, in the following we use the field distribution written in Cartesian coordinates, with the z–axis along the molecular beam axis and x– and y–axes perpendicular to the beam axis. The electric field is described by Hermite–Gaussian modes [22].

For a Fabry–Perot cavity whose radii of curvature of the two mirrors are equal i.e. $R_1 = R_2 = R$, the resonant frequency of a TEM$_{0,n,p}$ mode is given by [23, 24]

$$\nu_{m,n,p} = \frac{c}{2L} \left[p + \frac{m + n + 1}{\pi} \arccos \left(1 - \frac{L}{R}\right)\right]. \quad (1)$$

Here, $m + 1$ and $n + 1$ correspond to the number of antinodes in the transverse directions (along the x– and y–axes, respectively), and $p$ is the number of antinodes along the axial direction (the z–axis). The symbol $c$ is the speed of light.

For a cavity with $R = 80$ mm and $L = 60$ mm, seven to nine antinodes exist along the axial direction ($p$) for MW frequencies around 20–24 GHz. Assuming a standard Gaussian beam, the radius of the beam waist of our cavity is calculated to be $w_0 = 12.5$ mm at the center of the cavity, with the Rayleigh range of $z_R = 38.6$ mm. The beam waist at the mirror surface is $w_M = 15.8$ mm.

2.3. Quality factor

The expected quality factor for a particular mode depends on various power loss mechanisms inside the cavity. In general, it is defined as

$$Q = \frac{2\pi\nu W}{P_{\text{loss}}} \quad (2)$$

where $\nu$ is the resonance frequency of the mode, $W$ is the stored energy in the cavity and $P_{\text{loss}}$ is the power loss in the cavity.

The quality factor can be obtained from the measurement of the full-width at half maximum (FWHM) of the reflected signal in the setup (see figure 2 below). The FWHM of the reflected signal, $\delta\nu$, at a frequency $\nu$ is related to the quality factor as

$$Q_t = \frac{\nu}{\delta\nu}. \quad (3)$$

The cavity quality factor determined from FWHM is called the loaded quality factor, $Q_t$, and is related to the unloaded quality factor, $Q_0$, as $Q_t^{-1} = Q_0^{-1} + Q_c^{-1}$, where $Q_c$ is the quality factor due to the input coupler. The coupling parameter $\beta$ is defined as the ratio, $Q_0/Q_c$. Thus,

$$Q_t = \frac{Q_0}{1 + \beta}. \quad (4)$$

The coupling parameter $\beta$ can be obtained experimentally by measuring the voltage reflection coefficient, $\Gamma$, which is the ratio of the amplitude of the reflection signal to the maximum amplitude of the reflection obtained. The relation between the voltage reflection coefficient, $\Gamma$, and the coupling parameter, $\beta$, is given by

$$|\Gamma|^2 = \left(1 - \frac{\beta}{1 + \beta}\right)^2. \quad (5)$$

At critical coupling, all the power is transmitted into the cavity and there is no reflection signal. In this case, the voltage reflection coefficient is null, $\Gamma = 0$, and therefore, $\beta = 1$ and $Q_t = Q_0/2$.

An alternative way to obtain the loaded quality factor, $Q_t$, is the measurement of the decay of stored energy in the cavity in the time domain. The decay of the stored energy in the cavity at a time $t$, $W(t)$, is related to the input energy, $W(0)$ as

$$W(t) = W(0)e^{-2\pi\nu t/Q_t}. \quad (6)$$

The curve fit of a decay curve with an exponential function provides the loaded quality factor directly.

In general, the loaded quality factor $Q_t$ is affected by various power losses such as the diffraction by a finite mirror size, surface roughness, surface resistance, and the existence of the center holes. Among them, the surface resistance is a dominant factor that determines the total quality factor at room temperature [19, 25–28]. The total loaded quality factor of our cavity at room temperature is expected to be on the order of $10^5$ to $10^6$.

3. Experiments

The electronics used for the characterization of MW fields inside the cavity is shown in figure 2. The MW signal was generated by a synthesizer (a) (Anritsu MG3693C) and amplified by a K-band amplifier (c) (MKU 2410 A, 30 dB gain, maximum output power 40 dBm, Kuhne Elektronic) to up to 10 W after an MW isolator (b).

The generated MW was fed into the cavity via a coaxial SMA cable (h) after an MW circulator (d) (SMC1826, 18–26 GHz, Sierra MW). The coupling antenna to the cavity was a loop-type (0.8 mm diameter) made of a semi-rigid cupro-nickel coaxial cable, which was inserted into the cavity through the coupling hole at an off-axis of the cavity (see figure 1(b)). The antenna was anchored to the cold shield to keep its temperature as close as the cavity mirror temperature.
Figure 3. (a) Reflection signals from the cavity between 18 GHz and 25 GHz observed at room temperature with an input power of 20 mW. The present MW detector provides negative values for MW signals. The $y$-axis (reflection signal) is set such that resonance signals appear as downward peaks. The assignment of $p$ of each resonance is given above each resonance. Only TEM$_{0,0}$, TEM$_{0,1}$, and TEM$_{0,2}$ modes are indicated for simplicity. (b) TEM$_{0,0}$ modes for $p = 7, 8$ and $9$. (c) TEM$_{0,1}$, TEM$_{1,0}$ modes for $p = 7, 8$ and $9$. (d) TEM$_{0,2}$, TEM$_{2,0}$, TEM$_{1,1}$ modes for $p = 7, 8$ and $9$.

In order to measure the coupling strength, $\beta$, the loaded quality factor, $Q_L$, and the resonance frequency, $\nu$, of various modes inside the cavity, reflected MW signals were separated from the input MW via circulator (d) and detected by a diode (f) (8473C; 0.01–26.5 GHz, Agilent) after an appropriate attenuator (e).

In this work, the loaded quality factors ($Q_L$) of the cavity were determined from equation (3) by measuring the resonance linewidths. Then, the loaded quality factors were converted to the unloaded quality factors ($Q_0$) using equations (4) and (5). In most cases, the position of the coupling antenna was adjusted to the critical coupling position, where MW reflection signals became null (i.e., $\beta = 1$). Then, the relation of $Q_0 = 2Q_L$ was used to obtain the unloaded quality factors.

4. Results

In this section, we discuss details of the observed MW fields and their quality factor in the cavity.

4.1. Resonance modes

Figure 3 shows the reflection signals from the cavity between 18 GHz and 25 GHz observed at room temperature. The distance between the mirrors was set to 61.69 mm for the reflection spectrum shown in figure 3. The resonance modes are identified as a sharp peak in the reflection spectrum that corresponds to a reduction of a reflected signal from the cavity due to the transmission of MW into the cavity. Table 1 lists the observed frequencies of TEM$_{m,n}$ for $m + n = 0, 1$ and $2$ with $p = 5–9$ at 298 K. The assignments are given as vertical sticks with the value of $p$ in figure 3(a).

As can be seen in equation (1), the modes with the same $m + n$ are degenerate. However, due to the asymmetry of the cavity such as the presence of the coupling hole and the deformation of the mirror curvatures, the degeneracy is slightly lifted such that the splitting of peaks is observed for each mode. An example is shown in figure 3(c) for the TEM$_{0,1}$, TEM$_{1,0}$ modes with $p = 7, 8, 9$. Since the degeneracy of these modes is two, a doublet was observed in each mode. The separation of the doublet is 53 MHz, 46 MHz and 41 MHz for $p = 7, 8, 9$, respectively.

Two sharp peaks were also observed for $m + n = 2$ as shown in figure 3(d). The degeneracy of these modes is three, corresponding to $(m, n) = (0, 2), (2, 0)$ and $(1, 1)$. There might be an additional weak broader resonance between the two sharp peaks (see figure 3(d)), but its existence is not well
Table 1. Observed resonance frequencies, $\nu_{\text{obs}}$, and their unloaded quality factor, $Q_0$, for TEM$_{m,n,p}$ at 298 K and 2.3 K for $p = 5$–9. Calculated frequencies obtained by the least-square-fitting using $R$ and $L$ as fitting parameters are shown as $\nu_{\text{calc}}$ for the frequencies at 298 K. Fitted parameters are $R = 79.11 \pm 0.39$ mm and $L = 61.77 \pm 0.02$ mm. For the fitting, the averaged values of each doublet (shown as *) were used.

| Mode $m + n$ | $p$ | $\nu_{\text{obs}}$ (GHz) | $\nu_{\text{calc}}$ (GHz) | $Q_0 \times 10^3$ | $\nu_{\text{obs}}$ (GHz) | $Q_0 \times 10^3$ |
|-------------|-----|-----------------|-----------------|---------------|-----------------|---------------|
| 0 0         | 5   | 13.171          | 13.176          |               |                 |               |
| 1 0         | 6   | 15.601          | 15.603          |               |                 |               |
| 1 1         | 5   | (14.222)$^a$    | 14.219          | 6.0           | 18.08           | 0.15          |
| 1 2         | 6   | (16.656)$^a$    | 16.645          | 5.2           | —               | —             |
| 2 0         | 5   | (15.247)$^a$    | 15.261          | 5.0           | —               | —             |
| 2 1         | 6   | (17.683)$^a$    | 17.688          | 13.6          | 19.17           | 1.9           |
| 0 1         | 7   | 18.022          | 18.029          | 15.7          | 19.12           | 4.9           |
| 1 2         | 8   | 20.447          | 20.456          | 19.054        | 15.7            | 4.9           |
| 1 3         | 9   | 22.869          | 22.883          | 22.869        | 15.7            | 4.9           |
| 2 2         | 7   | 20.058          | 20.126          | (19.081)$^a$  | 13.6            | 4.9           |
| 2 3         | 8   | 22.497          | 22.573          | (19.512)$^a$  | 13.7            | 4.9           |
| 2 4         | 9   | 24.935          | 25.018          | (19.943)$^a$  | 13.7            | 4.9           |

Identified at this moment. For $p = 7, 8, 9$, the two peaks were separated by 97, 87, 77 MHz, respectively, corresponding to about two times larger splitting than that for the $m + n = 1$ modes.

Since the exact values of $R$ and $L$ are not known, the observed resonance frequencies were also fitted with equation (1) using the parameters $L$ and $R$ as fitting parameters. For the doublets in $m + n = 1$ and 2, the averaged values were used in the fitting, which are shown in parenthesis in Table 1. The obtained fitted parameters of $R = 79.11 \pm 0.39$ mm and $L = 61.77 \pm 0.02$ mm reproduced the observed frequencies within 17 MHz ($\nu_{\text{calc}}$). The small standard deviations of the fitted parameters for both $R$ and $L$ support the present mode assignments.

4.2. Quality factors: mode dependence

Figure 4 shows the resonance frequencies of the TEM$_{0,0,p}$ mode and their quality factor for $p = 5$–9 at room temperature. The unloaded quality factor for TEM$_{0,0,5}$ was $1 \times 10^4$ and it monotonically reduced for larger $p$. The observed longitudinal $p$ dependence agrees well with the simulation that the diffraction loss of larger $p$ is more in a Fabry–Perot cavity [29].

The middle columns of Table 1 summarize the resonance frequencies and their quality factor of various TEM$_{m,n,p}$ modes at room temperature. At room temperature, the quality factor of the TEM$_{0,0,7}$ mode is a few times less than that of the TEM$_{1,0,7}$/TEM$_{0,1,7}$ modes. This is due to the existence of the
center holes for molecular beams. Since the TEM\(_{1,0,p}/\)TEM\(_{0,1,p}\)
modes have a node at the center axis of the cavity, the loss of
the quality factor due to the the center hole at the mirror
surfaces is minimized. It agrees well with the previous calcu-
lations, in which higher quality factors are predicted for
the TEM\(_{0,1}\) mode than that for the TEM\(_{0,0}\) mode [26–28].
It is also noted that the quality factors of the doublet in the
TEM\(_{2,0,p}/\)TEM\(_{0,2,p}/\)TEM\(_{1,1,p}\) modes are also a few times larger
than that of the TEM\(_{0,0,p}\) mode. For \(p = 8\) and \(9\), the quality
factors of the TEM\(_{2,0,p}/\)TEM\(_{0,2,p}/\)TEM\(_{1,1,p}\) modes are even
larger than that of the TEM\(_{1,0,p}/\)TEM\(_{0,1,p}\) modes.

For the trapping of molecules, it is desired to have the
largest MW field intensity along the beam axis (\(z\)-axis). The
TEM\(_{0,0}\) mode has the maximum field at the center axis. How-
ever, we have observed a low quality factor for the TEM\(_{0,0,p}\)
mode around 20–25 GHz with our cavity design. The lowest
transverse mode, TEM\(_{m,n,p}\) mode with \(m + n = 1\), has a
node along the center cavity axis, which is not useful for trap-
ping molecules. The next transverse mode, TEM\(_{m,n,p}\) mode
with \(m + n = 2\), has non-zero electric fields at the center for
TEM\(_{0,2,p}\) and TEM\(_{2,0,p}\). Since the quality factor of the TEM\(_{m,n,p}\)
mode with \(m + n = 2\) is relatively high, and this mode has
non-zero electric fields on or near the center axis of the cavity,
this mode would be useful for the trapping of molecules. In the
following, we focus on the characterization of the TEM\(_{m,n,p}\)
mode with \(m + n = 2\). In order to make the TEM\(_{m,n,9}\) mode
with \(m + n = 2\) near the target frequency of 23.7 GHz, a mir-
or distance of \(L = 64.50\,\text{mm}\) was used for the following
measurements.

4.3. Quality factors: temperature dependence

The key design of our cavity is the surface coating with a
superconducting material in order to increase the quality fac-
tor by minimizing the surface resistivity. Figure 5(a) shows
the frequency shift of the doublet in the TEM\(_{m,n,9}\) mode for
\(m + n = 2\) at different temperatures. The resonance frequency
changes by 75 MHz from the room temperature to 2.3 K. This
shift corresponds to the change of the cavity length by 0.22 mm
according to equation (1). This shift is caused by the thermal
contraction of copper. Two mirrors of our cavity are tightly
attached to the cold head of the refrigerator with a copper base
plate. The thermal expansion coefficient of copper between
300 K and 4 K was reported previously [30]. The change of
the length \(\Delta L\) per unit length from 293 K to 4 K was found to
be \(2.93 \times 10^{-3}\). Therefore, the distance change at 64.50 mm
corresponds to 0.19 mm, which matches to our shift of the
resonant frequency.

We have observed a narrowing of the resonance spectrum
by reducing the temperature. Figure 5(b) shows the quality
factor of the TEM\(_{m,n,9}\) mode for \(m + n = 2\) at different tem-
peratures. In order to measure the temperature dependence of
the quality factor, we first optimized the antenna position at
room temperature such that the MW reflection became mini-
mum (i.e. the critical coupling). Then the cavity was cooled
by the refrigerator under vacuum and the linewidth of the
resonance was measured to derive the loaded quality factor.
The unloaded quality factor is twice the loaded quality factor
under this condition, i.e. the coupling parameter, \(\beta = 1\). As the
temperature of the cavity decreases from room temperature
to 5 K, the quality factor increases gradually. Below 3 K,
there is a significant increase in the quality factor due to the
superconducting phase.

The unloaded quality factor increased from \(2.4 \times 10^4\) at
298 K to \(6.8 \times 10^4\) at 44 K for the peak at the higher fre-
quency side of the TEM\(_{m,n,9}\) mode for \(m + n = 2\). The gradual
increase of the quality factor from room temperature to
\(~50\,\text{K}\) is explained by the reduction of the surface resis-
tance of the mirror material at lower temperature. The sur-
face resistivity of a lead-tin alloy at room temperature is about
\(\rho(293\,\text{K}) = 3.5 \times 10^{-7}\,\Omega\,\text{m}\), while that at 50 K is about
\(\rho(50\,\text{K}) = 3 \times 10^{-8}\,\Omega\,\text{m}\) [31]. We expect that the quality factor is
increased by a factor of 3 by cooling the cavity from room
temperature to 50 K, which is consistent with our experimental
observations. Note that, in this temperature range, the surface
The peak at higher frequency. Black line: A theoretical curve of $G$ by equation (8). Note that equation (8) is mainly applicable to just for a reference. Estimated in a straightforward manner [32], figure 6 suggests the geometry factor of about 8.

The advantage of using the lead-tin surface coating is the superconductor transition around 7–8 K. For superconducting cavities, the quality factor can be expressed by [32, 33]

$$Q = \frac{G}{R_e}$$  \hspace{1cm} (7)

where $G$ is a geometry factor determined from the cavity geometry, and $R_e$ is the effective resistance of the lead-tin superconductor. The effective resistance has two components, $R_e = R_{BCS} + R_0$, where $R_{BCS}$ is given by BCS theory [32]

$$R_{BCS} = 6.85 \times 10^{-5} f^{1.5} \frac{1}{T} \exp \left( -\frac{15}{T} \right),$$  \hspace{1cm} (8)

and $R_0$ is the residual resistance. Here, the frequency $f$ is given in GHz and the resistance in $\Omega$. From this equation, it is seen that the surface resistance decreases almost 10 times from 4.5 K to 2.0 K, corresponding to the increase of the quality factor by a factor of 10 from 4.5 K to 2.0 K. This estimation agrees well with the observed unloaded quality factor for the TEM$_{m,n,9}$ mode with $m+n = 2$ below 10 K. The black line in figure 6 is a theoretical curve obtained from $G/R_{BCS} + A$ with $G = 8$ and $A = 70000$, where $R_{BCS}$ is calculated by equation (8). Although the geometry factor cannot be estimated in a straightforward manner [32], figure 6 suggests that the observed quality factor can be explained by assuming the geometry factor of about 8.

4.4. Quality factors: coupling strength

We have also measured the quality factor at different insertion depths of the coupling antenna. The result is shown in figure 7. The coupling antenna was inserted via the coupling hole whose diameter (2.5 mm) was smaller than the wavelength of MW radiations used in our cavity. Apart from diffraction losses due to the coupling hole, the antenna acts as a normal conductor inside the cavity causing the effective resistance of the cavity to increase as the insertion depth is increased. The insertion depth of the antenna was changed by rotating a nut located on a feedthrough through which the antenna was mounted inside the cavity.

The coupling parameter depends on the insertion depth. When the reflection signal is minimum, the coupling parameter (equation (5)) becomes 1, which corresponds to the critical coupling condition. Theoretically the critical coupling gives the highest electric field amplitude. The critical coupling was observed at an insertion depth of 1.12 mm in the scale shown in figure 7. The red and black points in figure 7 show the loaded quality factor calculated from the linewidth of the resonance. The critical coupling was observed at an insertion depth of 1.12 mm in the scale shown in figure 7. The critical coupling condition is shown as a broken line.

4.5. Time domain measurements

We have also measured the quality factor of the cavity using the ring down time of stored energy when a pulsed MW is applied. The MW frequency was tuned at the resonance of the higher frequency peak of the TEM$_{m,n,9}$ mode for $m+n = 2$. These measurements were done at the critical coupling condition ($\beta = 1$). Even after an MW pulse was switched off, non-zero signal was detected at the detector for some time, which corresponds to the ring down time of the stored energy in the cavity. The decay was on the order of several microseconds. The two traces shown in figure 8 have a decay time constant of 3.36 μs and 3.02 μs for the 100 ms and 1000 ms pulse.
Figure 8. Decays of the stored energy inside the cavity after MW pulses of 100 ms (blue) and 1000 ms (red) duration. The decay signals were normalized such that the maximum signal detected by the detector becomes 1. MW frequency was set to the higher frequency peak of the TEM$_{m,n,9}$ mode for $m+n=2$. The applied MW pulse was switched off at $t=0$ for each trace. The position of the antenna was set to the critical coupling condition such that the reflection signal was zero while MW pulses were applied due to the destructive interference between the incoming and outgoing MW fields. Once the applied MW pulse is off, only the outgoing MW fields from the cavity exists, which are detected by the detector.

durations, respectively. These decay constants correspond to unloaded quality factors of $1.0 \times 10^6$ and $9.1 \times 10^5$ with $\beta = 1$. The evaluated unloaded quality factors obtained from the ring down time are fairly consistent with the unloaded quality factors obtained from the linewidth of the resonance.

The ring down time is slightly shorter for longer MW pulses. This is due to the increase of the cavity temperature caused by the stored MW energy inside the cavity. Due to the non-zero resistivity of the surface, the MW energy is slightly dissipated to the cavity mirrors that causes the increase of the temperature. Since the quality factor drastically depends on the temperature of the cavity in the superconducting regime (figure 6), the increase of the cavity temperature was observed as a reduction of the quality factor.

Figure 9(a) shows the unloaded quality factor of the higher frequency peak of the TEM$_{m,n,9}$ mode for $m+n=2$ at different MW input powers. The quality factor was evaluated from the pulse decay data with a pulse duration of 100 ms. The temperature of the cavity was 2.3 K. At higher input power, the stored energy is higher. Therefore, the decrease of the quality factor may be expected at significantly higher input power. With an MW pulse duration of 100 ms, the observed quality factor was almost constant around $1 \times 10^6$ between 3 W and 10 W input power. The thermal coupling between the cavity and cold head was high and prevented large increases in temperature with high MW input power.

Figure 9(b) shows the unloaded quality factor of the TEM$_{m,n,9}$ mode for $m+n=2$ at different MW pulse widths. The input power was 10 W. The temperature of the cavity was 2.3 K. About 10–20% decrease of the unloaded quality factor was observed at a 1 s MW pulse relative to short (100 ms) pulses. The decrease of the quality factor with longer MW pulses is due to the MW power dissipation through the surface of the cavity mirrors, which raises the temperature of the cavity. The cooling power of the refrigerator we used is rated as 1 W at 4.2 K, which suggests that an average MW power of 50 mW would result in a temperature rise of about 0.1 K at 2.3 K. Although there are some decreases in the quality factor by applying longer MW pulses, figure 9(b) proves that the current design of the cavity maintains the high quality factor for more than 1 s, which is long enough to demonstrate the trapping of cold molecules inside the cavity.

5. Discussion

Our cavity is designed to trap NH$_3$ molecules in the upper level of the inversion doublet with a transition frequency of 23.7 GHz between the two levels [14]. The upper level of the inversion doublet is a low field seeking state such that it can be decelerated using a DC Stark decelerator.

Based on the characterization of our cavity described in the previous section, TEM$_{2,0,9}$/TEM$_{0,2,9}$ at a resonance frequency of around 23.7 GHz can be used for the trapping of NH$_3$ molecules. Figure 10 shows the electric field
Figure 10. Electric field distribution inside the cavity (a) TEM\(_{0,0,9}\), (b) TEM\(_{0,2,9}\) for the cases that the field distributions are described by perfect Hermite–Gaussian modes. The same stored energy inside the cavity was assumed for TEM\(_{0,0,9}\) and TEM\(_{0,2,9}\). (c) Electric field amplitude on the cavity axis (z) at x = y = 0. Black: TEM\(_{0,0,9}\). Red: TEM\(_{0,2,9}\). (d) Electric field amplitude along the y axis at x = z = 0.

Figure 11. Time of flight distribution of ammonia molecular beam.

distribution of TEM\(_{0,2,9}\) of perfect Hermite–Gaussian modes. The field distribution of TEM\(_{0,0,9}\) is also shown as a reference. The TEM\(_{0,2,9}/\text{TEM}_{2,0,9}\) modes have two nodes along x and y axes, respectively. The field intensity along the center axis of the cavity (x = 0 and y = 0) is non-zero. The magnitude of the field at the center for the TEM\(_{0,2,9}\) mode is about 70% of that of the TEM\(_{0,0,9}\) mode. However, as discussed above, the quality factor of the TEM\(_{0,2,9}\) mode is orders of magnitude larger than that of the TEM\(_{0,0,9}\) mode, the TEM\(_{0,2,9}\) mode is more appropriate for the use of trapping cold molecules.

The maximum unloaded quality factor obtained for this mode was slightly above 10⁶. The maximum electric field at the center of the cavity for the TEM\(_{0,0,9}\) mode is calculated by \(E_0 = (4PQ_0/\pi\epsilon_0\omega L)\), where P is the power input into the cavity. From this equation, we can calculate the electric field intensity of the TEM\(_{0,2,9}\) mode at the center of the cavity to be 1.06 MV m\(^{-1}\) at the unloaded quality factor of 10⁶ and the input power of \(P = 10\) W. By using the permanent electric dipole moment of NH\(_3\) of 1.47 Debye [14], the trap depth for NH\(_3\) is calculated to be 85 mK. This corresponds to the translational velocity of 9 m s\(^{-1}\), which is achievable by a conventional Stark decelerator [14]. We may also use a few anti-nodes of the electric fields between the entrance of the cavity and the center as a decelerator [12], which allows us to trap NH\(_3\) with an entrance velocity of 20 m s\(^{-1}\).

For the detection of trapped NH\(_3\) at the center of the cavity, a standard resonance enhanced multi-photon ionization (REMPI) technique can be applied. Although the cavity mirrors shield a part of the electric fields applied by ion optics outside the cavity, the electric field of the ion optics shown in figure 1 provides sufficient electric fields to extract ions produced by UV laser pulses. As an example, figure 11 shows a time of flight distribution of NH\(_3\) traveling through the cavity. A good signal-to-noise ratio was obtained. The signal of NH\(_3\) was detected even if we moved the position of the ionization UV pulses to ±1 mm along the molecular beam axis and ±1.5 mm along the transverse direction to the beam axis from the center of the cavity, which shows that the detection range is only limited by the distribution of the molecular beam inside the cavity.
6. Conclusions

In this paper, we report the characterization of a superconducting MW cavity designed for trapping cold NH$_3$ molecules. By cooling the cavity below 2.3 K, an unloaded quality factor of more than 10$^6$ was obtained for the TEM$_{m,n,2}$ mode with $m + n = 2$. With an input MW power of 10 W, we can achieve the electric field strength of 1.06 MV m$^{-1}$ inside the cavity, and maintained the quality factor for longer than 1 s. Such an AC electric field provides a trapping potential of 85 mK for NH$_3$ molecules, which is achievable by a conventional Stark decelerator. Trapping experiments are underway in our laboratory.

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ORCID iDs

Rasoul Malekfar https://orcid.org/0000-0001-5529-5983
Katsumi Enomoto https://orcid.org/0000-0002-4980-4216
Takamasa Momose https://orcid.org/0000-0001-8976-1938

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