Dynamical next-to-next-to-leading order parton distributions and the perturbative stability of $F_L(x, Q^2)$

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It is shown that the previously noted extreme perturbative instability of the longitudinal structure function $F_L(x, Q^2)$ in the small Bjorken-$x$ region, is a mere artefact of the commonly utilized ‘standard’ gluon distributions. In particular it is demonstrated that using the appropriate – dynamically generated – parton distributions at NLO and NNLO, $F_L(x, Q^2)$ turns out to be perturbatively rather stable already for $Q^2 \geq O(2 – 3 \text{ GeV}^2)$.

A sensitive test of the reliability of perturbative QCD is provided by studying the perturbative stability of the longitudinal structure function $F_L(x, Q^2)$ in the very small Bjorken-$x$ region, $x \lesssim 10^{-3}$, at the perturbatively relevant low values of $Q^2 \gtrsim O(2 – 3 \text{ GeV}^2)$. For the perturbative–order independent rather flat function to be relevant at the perturbative–order independent rather flat

Concerning the perturbative stability of $F_L(x, Q^2)$ in the low $Q^2$ region, $Q^2 \lesssim 5 \text{ GeV}^2$, within the framework of the dynamical parton model [6,7]. For this purpose, following [5], we repeat our previous [3] ‘standard’ evaluation of the NLO and NNLO distributions within the dynamical approach where the parton distributions at $Q > 1 \text{ GeV}$ are QCD radiatively generated from valence–like (positive) input distributions at an optimally determined $Q_0 \equiv \mu < 1 \text{ GeV}$ (where ‘valence–like’ refers to $\alpha_J > 0$ for all input distributions $f(x, \mu^2) \sim x^{\alpha_J(1 - x)^{\alpha_F}}$). This more restrictive ansatz, as compared to the standard approach, implies of course less uncertainties concerning the behavior of the parton distributions in the small–$x$ region at $Q > \mu$ which is entirely due to QCD dynamics at $x \lesssim 10^{-2}$. The valence–like input distributions at $Q_0 \equiv \mu < 1$ are parametrized according to [5,8]

$$xq_v(x, Q_0^2) = N_q x^{\alpha_{q_v}}(1 - x)^{b_{q_v}}(1 + c_{q_v} \sqrt{x} + d_{q_v} x^{1.5}),$$

$$xw(x, Q_0^2) = N_w x^{\alpha_{w}}(1 - x)^{b_w}(1 + c_w \sqrt{x} + d_w x)(1)$$

for the valence $q_v = u_v, d_v$ and sea $w = \bar{q}, g$ densities, and a vanishing strange sea at $Q^2 = Q_0^2$, $s(x, Q_0^2) = s(x, Q_0^2) = 0$. All further theoretical details relevant for analyzing $F_2$ at NLO and NNLO in the \( \overline{\text{MS}} \) factorization scheme have been presented in [8]. The heavy flavor (dominantly charm) contribution to $F_2$ is taken as given by fixed–order NLO perturbation theory [9,10] using $m_c = 1.3 \text{ GeV}$ and $m_b = 4.2 \text{ GeV}$ as implied

It is therefore interesting to study this issue concerning the perturbative stability of $F_L(x, Q^2)$ in the low $Q^2$ region, $Q^2 \lesssim 5 \text{ GeV}^2$, within the framework of the dynamical parton model [6,7]. For this purpose, following [5], we repeat our previous [3] ‘standard’ evaluation of the NLO and NNLO distributions within the dynamical approach where the parton distributions at $Q > 1 \text{ GeV}$ are QCD radiatively generated from valence–like (positive) input distributions at an optimally determined $Q_0 \equiv \mu < 1 \text{ GeV}$ (where ‘valence–like’ refers to $\alpha_J > 0$ for all input distributions $f(x, \mu^2) \sim x^{\alpha_{J}(1 - x)^{\alpha_{F}}}$). This more restrictive ansatz, as compared to the standard approach, implies of course less uncertainties concerning the behavior of the parton distributions in the small–$x$ region at $Q > \mu$ which is entirely due to QCD dynamics at $x \lesssim 10^{-2}$. The valence–like input distributions at $Q_0 \equiv \mu < 1$ are parametrized according to [5,8] for the valence $q_v = u_v, d_v$ and sea $w = \bar{q}, g$ densities, and a vanishing strange sea at $Q^2 = Q_0^2$, $s(x, Q_0^2) = s(x, Q_0^2) = 0$. All further theoretical details relevant for analyzing $F_2$ at NLO and NNLO in the $\overline{\text{MS}}$ factorization scheme have been presented in [8]. The heavy flavor (dominantly charm) contribution to $F_2$ is taken as given by fixed–order NLO perturbation theory [9,10] using $m_c = 1.3 \text{ GeV}$ and $m_b = 4.2 \text{ GeV}$ as implied.
by optimal fits [7] to recent deep inelastic c– and b–production HERA data. Since a NNLO calculation of heavy quark production is not yet available, we have again used the same NLO $O(\alpha_s^2)$ result. Finally, we have used for our fit–analyses the same deep inelastic HERA–H1, BCDMS and NMC data, with the appropriate cuts for $F_{2}^{c,n}$ as in [8] which amounts to a total of 740 data points. The required overall normalization factors of the data turned out to be 0.98 for H1 and BCDMS, and 1.0 for NMC. We use here again solely deep inelastic scattering data since we are mainly interested in the small–$x$ behavior of structure functions. The resulting parameters of the NLO and NNLO fits are summarized in Table 1. The dynamical gluon and sea distributions, evolved to (1) at a common input scale $Q_0^2 = \mu^2 = 0.5 \text{ GeV}^2$ optimal at both perturbative orders.

Table 1: Parameter values of the NNLO and NLO QCD fits with the parameters of the input distributions referring to (1) at a common input scale $Q_0^2 = \mu^2 = 0.5 \text{ GeV}^2$ optimal at both perturbative orders.

|        | NNLO | NLO |
|--------|------|-----|
| $u_v$  | 0.6210 | 0.1911 |
| $d_v$  | 0.4393 | 0.2028 |
| $\bar q$ | 0.3055 | 0.4810 |
| $g$    | 0.5312 | 0.8678 |
| $u_v$  | 0.0741 | 0.3161 |
| $d_v$  | 6.5186 | 4.6906 |
| $\bar q$ | 2.8205 | 14.580 |
| $g$    | 0.9737 | 11.884 |
| $\chi^2$/dof | 1.037 | 1.073 |
| $\alpha_s(M_Z^2)$ | 0.112 | 0.113 |

Parameter values of the NNLO and NLO QCD fits with the parameters of the input distributions referring to (1) at a common input scale $Q_0^2 = \mu^2 = 0.5 \text{ GeV}^2$ optimal at both perturbative orders.

In LO, $c_{L,ns}^{(1)} = \frac{\alpha_s}{4\pi} x$, $c_{L,ps}^{(1)} = 0$, $c_{L,g}^{(1)} = 24x(1-x)$ and the singlet–quark coefficient function is decomposed into the non–singlet and a ‘pure singlet’ contribution, $c_{L,q}^{(n)} = c_{L,ns}^{(n)} + c_{L,ps}^{(n)}$. Sufficiently accurate simplified expressions for the
The gluonic contribution $F^g_L$ to $F_L$ in \ref{fig:gluonic_contribution} is shown in Fig.~\ref{fig:gluonic_contribution} at two characteristic low values of $Q^2$. Although the perturbative instability of the subdominant quark contribution $F^q_L$ as obtained in a ‘standard’ fit does not improve for the dynamical (sea) quark distributions \cite{5}, it is evident from Fig.~\ref{fig:gluonic_contribution} that the instability disappears almost entirely for the dominant dynamical gluon contribution already at $Q^2 \simeq 2$ GeV$^2$. This implies that the dynamical predictions for the total $F_L(x, Q^2)$ become perturbatively stable already at the relevant low values of $Q^2 \gtrsim \mathcal{O}(2 - 3$ GeV$^2$) as shown in Fig.~\ref{fig:dynamical_parton_model} in contrast to the ‘standard’ results in Fig.~\ref{fig:standard_parton_distributions}. In the latter case the stability has not been fully reached even at $Q^2 = 5$ GeV$^2$ where the NNLO result at $x = 10^{-5}$ is more than 20\% larger than the NLO one. A similar discrepancy prevails for the dynamical predictions in Fig.~\ref{fig:standard_parton_distributions} at $Q^2 = 2$ GeV$^2$. This is, however, not too surprising since $Q^2 = 2$ GeV$^2$ represents somehow a borderline value for the leading twist–

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{gluonic_contribution.png}
\caption{The gluonic contribution $F^g_L$ to $F_L$ in \ref{fig:gluonic_contribution} with $F^g_L = \frac{2}{3} x C_{L,g} \otimes g$ in the dynamical (dyn) and standard (std) parton approach at NNLO and NLO for two representative low values of $Q^2$. The standard parton distributions utilized in the lower panel are taken from \cite{8}.
}
\end{figure}

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{dynamical_parton_model.png}
\caption{Dynamical parton model NNLO and NLO predictions for $F_L(x, Q^2)$.
}
\end{figure}

NLO and NNLO coefficient functions $c^{(2)}_{L,i}$ and $c^{(3)}_{L,i}$, respectively, have been given in \cite{1}. It has been furthermore noted in \cite{1} that especially for $C_{L,q}$ both the NLO and NNLO contributions are rather large over almost the entire $x$–range. Most striking, however, is the behavior of both $C_{L,q}$ and $C_{L,g}$ at very small values of $x$: the vanishingly small LO parts ($xe^{(1)}_{L,i} \sim x^2$) are negligible as compared to the (negative) constant NLO 2-loop terms, which in turn are completely overwhelmed by the positive NNLO 3-loop singular corrections $xe^{(3)}_{L,i} \sim -\ln{x}$. This latter singular contribution might be indicative for the perturbative instability at NNLO \cite{1}, as discussed at the beginning, but it should be kept in mind that a small–$x$ information alone is insufficient for reliable estimates of the convolutions occurring in \cite{2} when evaluating physical observables.
Figure 3. As in Fig. 2 but for the common standard parton distributions as taken from [5].

2 contribution to become dominant at small $x$ values. This is further corroborated by the observation that the dynamical NLO twist–2 fit slightly undershoots the HERA data for $F_2$ at $Q^2 \simeq 2$ GeV$^2$ in the small–$x$ region (cf. Fig. 1 of [7]). The NLO/NNLO instabilities implied by the standard fit results obtained in [28] at $Q^2 \lesssim 5$ GeV$^2$ are even more violent than the ones shown in Fig. 3. This is mainly due to the negative longitudinal cross section (negative $F_L(x, Q^2)$) encountered in [28]. The perturbative stability in any scenario becomes in general better the larger $Q^2$, typically beyond 5 GeV$^2$ [128], as shown in Figs. 2 and 3. This is due to the fact that the $Q^2$–evolutions eventually force any parton distribution to become sufficiently steep in $x$.

To summarize, we have shown that the extreme perturbative NNLO/NLO instability of the longitudinal structure function $F_L$ at low $Q^2$, noted in [2–4], is an artefact of the commonly utilized ‘standard’ gluon distributions rather than an indication of a genuine problem of perturbative QCD. In fact we have demonstrated that these extreme instabilities are reduced considerably already at $Q^2 = 2 - 3$ GeV$^2$ when utilizing the appropriate, dynamically generated, parton distributions at NLO and NNLO. These latter parton distributions have been obtained from a NLO and NNLO analysis of $F^p,n_2$ data, employing the concepts of the dynamical parton model. It is gratifying to notice, once again, the advantage of the dynamical parton model approach to perturbative QCD.

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