A New Gauge for Computing Effective Potentials
in Spontaneously Broken Gauge Theories

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Abstract

A new class of renormalizable gauges is introduced that is particularly well suited to compute effective potentials in spontaneously broken gauge theories. It allows one to keep free gauge parameters when computing the effective potential from vacuum graphs or tadpoles without encountering mixed propagators of would-be-Goldstone bosons and longitudinal modes of the gauge field. As an illustrative example several quantities are computed within the Abelian Higgs model, which is renormalized at the two-loop level. The zero temperature effective potential in the new gauge is compared to that in $R_\xi$ gauge at the one-loop level and found to be not only easier to compute but also to have a more convenient analytical structure. To demonstrate renormalizability of the gauge for the non-Abelian case, the renormalization of an SU(2)-Higgs model with completely broken gauge group and of an SO(3)-Higgs model with an unbroken SO(2) subgroup is outlined and renormalization constants are given at the one-loop level.
1 Introduction and Summary

The effective potential (EP) of a relativistic quantum field theory (QFT) \[1\] is a useful tool for investigating several questions of physical interest such as the vacuum structure of the theory, inflationary cosmology and finite temperature phase transitions (see e.g. \[2\] and references therein). Recently for instance there has been renewed interest in the details of the electroweak phase transition in the early universe since its nature is crucial in deciding whether the baryon asymmetry of the universe was created at that time. The availability of a good approximation scheme for the EP of the Higgs field is considered to be critical in that context by many authors (see e.g. \[3\] and references therein).

Although the EP of spontaneously broken gauge theories (SBGT) is frequently employed, there is no convenient gauge that at the same time

(i) allows the EP to be computed from graphs with no or one external line (from here on to be called vacuum graphs and tadpoles, respectively),

(ii) avoids awkward-to-use mixed propagators of would-be-Goldstone bosons and the longitudinal modes of the gauge field and

(iii) keeps at least one free gauge parameter.

Usually Landau gauge (i.e. \(R_\xi\) gauge with \(\xi = 0\)) is used because then points (i) and (ii) are fulfilled. However in this case there is no free gauge parameter left. Since in general the EP itself is gauge dependent, it would be important to see if physical results extracted from it are indeed gauge invariant.

In this paper a class of quadratic renormalizable gauges very similar to the (linear) \(R_\xi\) gauges is introduced that fulfills all of the above requirements. The price one has to pay is the existence of two or more gauge parameter of which a subset has to be set to unity to avoid mixed propagators. This causes no problems as long as no use is made of the renormalization group which would make all gauge parameters running and again give rise to the presence of mixed propagators.

The paper is structured as follows: In section 2 why and how vacuum graphs (or tadpoles) can be used to compute the EP of a QFT is briefly reviewed. In section 3 the new class of gauges is introduced for a general SBGT. Its features are described and its BRST invariance is written down. Section 4 is devoted to BRST and anti-BRST invariance. It is shown how the number of gauge parameters can be reduced by imposing anti-BRST invariance additionally to BRST invariance. In section 5 the Abelian Higgs model is used as an illustrative example. Its Feynman rules in the new class of gauges are given, it is renormalized at the two-loop level and the physical Higgs and gauge boson masses are computed at the one-loop level and their gauge independence checked. The one-loop contribution to the EP is compared to that in \(R_\xi\) gauge and found not only to be easier to
compute but also to have a more desirable analytical structure. The two-loop contribution to the EP is given. To demonstrate the renormalizability of the gauge in more complicated cases, in sections 3 and 4 the renormalization of an SU(2)-Higgs model with completely broken gauge group and of an SO(3)-Higgs model with an unbroken SO(2) subgroup is sketched with explicit results given at the one-loop level. Since the evaluation of the EP proceeds in close analogy to the Abelian case it is not considered anew.

2 The EP and 1PI n-Point Functions

In this section the well-known connection between the EP of a QFT and vacuum graphs [1, 2], tadpoles [3, 4] and higher order functions in a shifted theory is presented in a very compact way for further reference.

The EP of a QFT is the generator for the one particle irreducible (1PI) n-point functions at zero external momenta. If we expand the EP $V(\phi)$ about some point $\omega$, we get

$$V(\phi) = -\sum_{n=0}^{\infty} \frac{1}{n!} \Gamma_n(\omega, p_i = 0)(\phi - \omega)^n,$$

(1)

where $\Gamma_n(\omega, p_i)$ is the 1PI n-point Greens function in the modified theory where the field $\phi$ has been shifted by $\omega$. Taking the derivative of this equation with respect to $\omega$ and observing that the EP is independent of the point we expand it about, we arrive at the recursion relation

$$\Gamma_{n+1}(\omega, p_i = 0) = \frac{\partial}{\partial \omega} \Gamma_n(\omega, p_i = 0).$$

(2)

Setting $\phi = \omega$ in (1) gives

$$V(\omega) = -\Gamma_0(\omega, p_i = 0)$$

(3)

and therefore the n-th derivative of the EP is given by

$$V^{(n)}(\omega) = -\Gamma_n(\omega, p_i = 0).$$

(4)

Thus instead of summing over $n$ as in [1] we can use n-point functions in a shifted theory and integrate $n$ times. However, if higher order functions than tadpoles are used, the choice of integration constants is a non-trivial problem if no additional information is available. Throughout this paper vacuum graphs (i.e. $n = 0$) are used so that no integration over $\omega$ is needed.

It is crucial for the above derivation to hold that the only place where $\omega$ enters the modified Lagrangian is through the shifted scalar field as already noted in [3]. Otherwise [1] does not hold.
3 The New Gauge

Within the scalar sector of a gauge field theory with gauge group $G$ let $\Phi$ be a field that is expected to get a vacuum expectation value (vev) so that $G$ is spontaneously broken. Assume that $\Phi$ has been put into a real multiplet on which a homogeneous linear and unitary representation acts and that, for simplicity, $G$ is simple and the representation irreducible. (The generalization to non-simple groups and reducible representations is straightforward.)

First recall the class of $R_{\xi}$ gauges \[8]\: The gauge fixing term here is

$$L_{gf} = -\frac{1}{2\xi} \left( \partial_{\mu} A_{\mu}^{a} + ig v_{0}^{T} T_{a} \Phi' \right)^{2}, \quad (5)$$

where $\Phi = v_{0} + \Phi'$, $v_{0}$ being the tree-level vev of $\Phi$, $g$ the gauge coupling and $T_{a}$ the (imaginary and antisymmetric) gauge group generators. With this gauge fixing it is not possible to satisfy all three requirements (i)-(iii) simultaneously: If the Higgs field is shifted by some $\omega$ (additionally to $v_{0}$), then point (ii) is violated. If we modify $v_{0}^{T}$ in the gauge fixing term so as to avoid that, then the complete Lagrangian does no longer depend only on the sum of shifted field and shift, but on those two quantities separately which invalidates the derivation in section 2 and point (i) is no longer fulfilled. If we set $\xi = 0$, it turns out that (i) and (ii) are satisfied (most easily seen by the fact that the gauge to be introduced subsequently has effectively the same Feynman rules in the corresponding limit), but (iii) is violated, i.e. we have no check of gauge independence of physical quantities anymore.

Now choose a unit vector $\hat{v}$ in $\Phi$ space and consider the gauge fixing term

$$L_{gf} = -\frac{1}{2} \sigma_{ab} F_{a} F_{b}, \quad (6)$$

with

$$F_{a} = \partial_{\mu} A_{\mu}^{a} + ig \Theta_{ab} \phi^{T} \hat{v} \hat{v}^{T} T_{b} \phi + g R_{abc} \bar{\eta}_{b} \eta_{c} + g S_{abc} A_{b \mu} A_{c}^{\mu}, \quad (7)$$

where $\sigma_{ab}$ and $\Theta_{ab}$ are real and symmetric, $\bar{\eta}_{a}, \eta_{a}$ are anti-commuting ghost fields and repeated Latin indices are summed over all generator indices of the gauge group. This term necessitates the further addition of the ghost term

$$L_{gh} = (\partial_{\mu} \bar{\eta}_{a}) (D_{a}^{\mu} \eta_{a}) - g^{2} \Theta_{ac} \phi^{T} (\hat{v} \hat{v}^{T} T_{c} T_{b} - T_{b} \hat{v} \hat{v}^{T} T_{c}) \phi \bar{\eta}_{a} \eta_{b}$$
$$+ \frac{1}{4} g^{2} R_{abc} f_{ecd} \bar{\eta}_{a} \bar{\eta}_{b} \eta_{c} \eta_{d} - 2 g S_{abc} \bar{\eta}_{a} A_{b \mu} A_{c}^{\mu} D_{\mu} \eta_{c} \quad (8)$$

with

$$D_{a}^{\mu} \eta_{a} = \partial_{\mu} \eta_{a} + g f_{abc} \eta_{b} A_{c}^{\mu}, \quad (9)$$

where the $f_{abc}$ are the completely antisymmetric structure constants of our gauge group with normalization $[T_{a}, T_{b}] = if_{abc} T_{c}$. 

3
The action is now invariant under the BRST transformation

\[
\begin{align*}
\delta \phi &= -i \kappa g \eta_a T_a \phi \\
\delta A_\mu &= \kappa D_\mu \eta_a \\
\delta \eta_a &= -\kappa \sigma_{ab} F_b \\
\delta \bar{\eta} &= -\frac{1}{2} \kappa g f_{abc} \eta_b \eta_c
\end{align*}
\]  

(10)

with anticommuting \( \kappa \), provided \( R_{abc} \) is antisymmetric in its first two indices and \( S_{abc} \) is symmetric in its last two indices, i.e.

\[
R_{abc} = -R_{bac}, \quad S_{abc} = S_{acb}.
\]  

(11)

Note that the terms “gauge fixing” and “ghost” Lagrangian are somewhat ambiguous since \( \mathcal{L}_{gf} \) already contains the ghost fields. This, as well as the appearance of quartic ghost terms, is quite generic to quadratic gauge fixing functions \( F_a \) and requires us to BRST-quantize the theory instead of following the Fadeev-Popov procedure. Note that if we ignore for a moment the quadratic gauge and ghost terms in the gauge fixing function (7) and set \( \Theta_{ab} = \sigma_{ab} = \delta_{ab} \), we could loosely say that the new gauge obtains from \( R_\xi \) gauge by promoting the quantity \( \hat{v}^T T_a \Phi \) in (5) to a full field including its quantum fluctuations.

We assume here that it is possible to choose the \( \sigma_{ab}, \Theta_{ab}, R_{abc} \) and \( S_{abc} \) in such a way that renormalization forces no new terms in the Lagrangian upon us. If the \( \hat{v}^T T_a T_b \Phi \) cannot be expressed by \( \hat{v}^T \Phi \) and the \( \hat{v}^T T_a \Phi \) that might not be true and additional terms like e.g. \( \gamma_{abcd} \Phi^T T_b \hat{v} \hat{v}^T T_c T_a \Phi \) have to be introduced into the gauge fixing function. Instead of trying to set up a general procedure to pick the \( \sigma_{ab}, \Theta_{ab}, R_{abc}, S_{abc}, \gamma_{abcd}, \ldots \), examples will be given in sections 5, 6 and 7. See however the last paragraph of section 4.

Now let us introduce some constant shift \( \varphi \) by

\[
\Phi = \varphi \hat{v} + \Phi'.
\]  

(12)

Then \( \mathcal{L}_{gf} \) and \( \mathcal{L}_{gh} \) still depend only on the sum of shift and remaining quantum field \( \Phi' \) and the derivation in section 4 is valid and thus point (i) is fulfilled.

On the other hand, it is easy to check that for any shift \( \varphi \) the mixing terms between would-be-Goldstone modes and longitudinal gauge modes vanish and point (ii) is also fulfilled, provided \( (\sigma \Theta)_{ab} = \delta_{ab} \) when \( a \) or \( b \) corresponds to a broken generator, i.e. \( T_a \hat{v} \neq 0 \). Although \( (\sigma \Theta)_{ab} \) generally gets renormalized, we can set its renormalized value among the broken generators equal to \( \delta_{ab} \) (in BPHZ renormalization) and point (ii) remains valid.

Point (iii) is clearly satisfied, too, since \( \sigma_{ab} \) and \( \Theta_{ab} \) contain gauge parameters. Replacing \( (\sigma, \Theta, R, S) \rightarrow (\sigma/\xi, \xi \Theta, \xi R, \xi S) \) and letting \( \xi \rightarrow 0 \) one regains effectively the Feynman rules for Landau gauge among the \( R_\xi \) gauges. This explains naturally why in this gauge vacuum graphs or tadpoles can be used to compute the EP. However it is not clear to the present author if also the effective potential will always go smoothly to that of Landau gauge if the limit \( \xi \rightarrow 0 \) is
taken after the regulator is removed. This will only happen if even for \( \varphi \neq v_0 \) all infrared singularities stemming from vanishing ghost and longitudinal gauge boson masses cancel. It is easily seen, however, that this problem does not appear up to two loops.

Setting \((\sigma \Theta)_{ab} = \delta_{ab}\) for broken generators is unproblematic though, since no eigenvalues of the mass matrix disappear and therefore there is no discontinuous change in the singularity structure of the theory to any given order in the loop expansion.

It is important to ensure that \( F_a \) does not receive an expectation value since otherwise BRST symmetry would be spontaneously broken and the gauge a bad one in the sense of [11]. As in other gauges, the easiest way to achieve this is to impose appropriate symmetries on \( L_{\text{eff}} \) as is the case in all examples in this paper.

The class of gauges introduced by (6) is quadratic and renormalizable for appropriate \( \sigma_{ab}, \Theta_{ab}, R_{abc} \) and \( S_{abc} \) (up to the issue of closing the algebra of scalar fields appearing in \( L_{gf} \) and \( L_{gh} \), see above and next section). Because of its similarity to the class of \( R_\xi \) gauges, it is called \( R_\xi \) from here on.

4 BRST and anti-BRST Invariance

In this section the gauge fixing and ghost Lagrangians (6) and (8) will be derived in a simple way. Then a similar procedure will be followed to find gauge fixing and ghost Lagrangians that are additionally anti-BRST invariant.

Define nil-potent BRST and anti-BRST operators \( s \) and \( \bar{s} \) by [12]

\[
\begin{align*}
s \phi &= -i g \gamma_{a} T_{a} \phi \\
s A_{a}^{\mu} &= D_{a}^{\mu} \eta_{a} \\
s \eta_{a} &= \frac{1}{2} g f_{abc} \bar{\eta}_{b} \eta_{c} \\
s \bar{\eta}_{a} &= B_{a} \\
s B_{a} &= 0
\end{align*}
\]

with

\[
\begin{align*}
D_{a}^{\mu} \eta_{a} &= \partial_{\mu} \eta_{a} + g f_{abc} \eta_{b} A_{c}^{\mu} \\
D_{a}^{\mu} \bar{\eta}_{a} &= \partial_{\mu} \bar{\eta}_{a} + g f_{abc} \bar{\eta}_{b} A_{c}^{\mu}
\end{align*}
\]

and where \( B_{a} \) are auxiliary fields, i.e. without kinetic term. Consider

\[
\begin{align*}
\mathcal{L}_{gf+gh} &= \frac{s}{2} [\tilde{\eta}_{a} (\partial_{\mu} A_{a}^{\mu} + i g \Theta_{ab} \phi^{T} \hat{v} \hat{v}^{T} T_{b} \phi + \frac{1}{2} g R_{abc} \bar{\eta}_{b} \eta_{c} + g S_{abc} A_{b\mu} A_{c}^{\mu} + \frac{1}{2} \sigma_{ab}^{-1} B_{b})] \\
&= \frac{1}{2} \sigma_{ab}^{-1} B_{a} B_{b} \\
&\quad + B_{a} [\tilde{\eta}_{a} (\partial_{\mu} A_{a}^{\mu} + i g \Theta_{ab} \phi^{T} \hat{v} \hat{v}^{T} T_{b} \phi + \frac{1}{2} g (R_{abc} - R_{bac}) \bar{\eta}_{b} \eta_{c} + g S_{abc} A_{b\mu} A_{c}^{\mu}] \\
&\quad + \partial_{\mu} \bar{\eta}_{a} D_{a}^{\mu} \eta_{a} - g \Theta_{ac} \phi^{T} (\hat{v} \hat{v}^{T} T_{c} T_{b} - T_{b} \hat{v} \hat{v}^{T} T_{c}) \phi \bar{\eta}_{a} \eta_{b} \\
&\quad + \frac{1}{2} g^{2} R_{abc} f_{edef} \eta_{a} \bar{\eta}_{b} \eta_{c} \eta_{d} - g (S_{abc} + S_{acb}) \bar{\eta}_{a} \eta_{b} A_{c}^{\mu} D_{a}^{\mu} \eta_{c}
\end{align*}
\]
with some real quantities $\sigma_{ab}, \Theta_{ab}, R_{abc}$ and $S_{abc}$. Clearly we can assume $R_{abc}$ to be anti-symmetric in its first two and $S_{abc}$ to be symmetric in its last two indices. Nil-potency of $s$ makes $\mathcal{L}_{gf+gh}$ automatically $s$-invariant. Integrating out $B_a$ of the generating functional or, equivalently, eliminating it from the Lagrangian by its equation of motion, one recovers the sum of (6) and (8).

Now consider the $s$- and $\bar{s}$-invariant gauge fixing and ghost Lagrangian

$$\mathcal{L}_{s\bar{s}} = \frac{1}{2} s \bar{s} \left( \omega \phi^T \dot{\bar{v}}^T \phi + \Gamma_{ab} \bar{\eta}_a \eta_b + \omega \Delta_{ab}^{-1} A_{a\mu} A_{b}^{\mu} \right)$$

with real $\omega$ and real and symmetric $\Gamma_{ab}$ and $\Delta_{ab}$. Applying the rules for $s$ and $\bar{s}$, integrating out $B_a$, rescaling $\bar{\eta}_a \rightarrow -\omega^{-1} \Delta_{a\bar{b}} \bar{\eta}_b$ and defining $\sigma_{ab} = \omega^2 (\Delta \Gamma \Delta)_{ab}^{-1}$, one gets

$$\mathcal{L}_{s\bar{s}} = -\frac{1}{2} \sigma_{ab} F_a F_b + (\partial_{\mu} \bar{\eta}_a) (D^\mu \eta_a) - g^2 \Delta_{abc} \phi^T (\dot{\bar{v}} \dot{T}_c T_b - T_b \dot{\bar{v}} \dot{T}_c) \phi \eta_a \eta_b + \frac{1}{2} g^2 R_{abc} f_{e\bar{d}c} \eta_a \eta_b \eta_e \eta_d - 2 g S_{abc} \eta_a A_{b\mu} D^\mu \eta_c$$

with

$$F_a = \partial_{\mu} A_a^\mu + ig \Delta_{ab} \phi^T \dot{\bar{v}}^T T_b \phi + g R_{abc} \bar{\eta}_b \eta_c + g S_{abc} A_{b\mu} A_c^{\mu}, \quad (19)$$

$$R_{abc} = \frac{1}{2} [\Delta_{ad} \Delta_{be} \Delta_{cf}^{-1} \sigma_{fg}^{-1} \Delta_{gh}^{-1} f_{deh} - (\sigma_{ad}^{-1} \Delta_{bf} - \sigma_{bd}^{-1} \Delta_{af}) \Delta_{ac}^{-1} f_{cej}]. \quad (20)$$

$$S_{abc} = -\frac{1}{2} \Delta_{ac} (f_{ebd} \Delta_{d\bar{c}}^{-1} + f_{e\bar{d}c} \Delta_{b\bar{a}}^{-1}). \quad (21)$$

Thus, given $\mathcal{L}_{gf}$ and $\mathcal{L}_{gh}$ by (6)-(8), $\mathcal{L}_{gf} + \mathcal{L}_{gh}$ is not only BRST but also anti-BRST invariant, if there exists $\Delta_{ab}$ such that

$$\Theta_{ab} = \Delta_{ab} \quad \text{for} \quad T_b \dot{v} \neq 0, \quad (22)$$

i.e. for $b$ corresponding to broken generators, and such that $R_{abc}$ and $S_{abc}$ fulfill (20) and (21). In the examples of sections 6 and 7 we will check at the one-loop level that the resulting conditions on the gauge parameters are stable under renormalization.

There is a subtlety about unbroken Abelian subgroups. In this case it can be consistent to have an invertible $\sigma_{ab}$ but to set $\Delta_{ab}$ and $\Delta_{ab}^{-1}$ to zero when $a, b$ refer to that subgroup such that they are their mutually inverses only for the restriction to the rest of indices. It is easily seen that then the ghost fields corresponding to the Abelian subgroup under consideration effectively drop from the theory. An example of this case is the SO(3)-Higgs model with its unbroken SO(2) subgroup discussed in section 4.

Again it needs to be noted that renormalization might force new terms in the Lagrangian on us, if $\dot{v}^T T_a T_b \Phi$ cannot be expressed through $\dot{v}^T \Phi$ and the $\dot{v}^T T_a \Phi$. Then one can try to introduce additional terms like e.g. $s \bar{s} (\Upsilon_{ab} \Phi^T T_a \dot{v}^T T_b \Phi)$ into $\mathcal{L}_{s\bar{s}}$ until the algebra of scalar fields appearing in $\mathcal{L}_{s\bar{s}}$ closes.

To close this section we can sketch a method of obtaining the desired renormalizable effective Lagrangian even when renormalization forces terms upon us.
that are not taken care of by (6)-(8): Given some field content (including ghost and auxiliary fields) as well as gauge group generators and structure constants, write down the most general dim \( \leq 4 \) Lagrangian invariant under the \( s \)-symmetry of (13) (possibly extended to include more fields, e.g. fermions). Impose other global symmetries (discrete and/or continuous, e.g. anti-BRST) to prevent the resulting \( F_a \) from getting an expectation value, to possibly eliminate other unwanted terms and to diminish the number of gauge parameters. However keep \( L_{\text{eff}} \) general enough such that when computing the effective potential a suitable subset of gauge parameters can be adjusted to the effect that unwanted mixings of fields disappear and still at least one gauge parameter is left.

5  Application I: The Abelian Higgs Model

In this section the use of \( T_\xi \) gauge is illustrated within the Abelian Higgs model. It is given by the Lagrange density

\[
L = \frac{1}{2} (D_\mu \Phi)^T (D^\mu \Phi) - \frac{1}{4} F_{\mu \nu} F^{\mu \nu} - \frac{1}{2} m^2 \Phi^T \Phi - \frac{\lambda}{4} (\Phi^T \Phi)^2
\]  

(23)

with

\[
F_{\mu \nu} = \partial_\mu A_\nu - \partial_\nu A_\mu , \quad (24)
\]

\[
D_\mu \Phi = (\partial_\mu + ig T_1 A_\mu) \Phi , \quad (25)
\]

\[
T_1 = \tau_2 = \begin{pmatrix} 0 & i \\ -i & 0 \end{pmatrix} , \quad (26)
\]

\[
\Phi^T = (\phi_1, \phi_2) . \quad (27)
\]

Here \( \phi_1 \) and \( \phi_2 \) are real scalar fields and with the scalar self-coupling \( \lambda > 0 \) and \( m^2 < 0 \) the U(1) gauge symmetry is spontaneously broken. \( s \)-invariance forces \( R_{abc} = 0 \). Imposing the symmetries \( (A_\mu, \phi_1) \rightarrow (-A_\mu, -\phi_1) \) and \( (A_\mu, \phi_2) \rightarrow (-A_\mu, -\phi_2) \) of \( \mathcal{L} \) also on \( \mathcal{L}_{\text{gf}} + \mathcal{L}_{\text{gh}} \), we get \( S_{abc} = 0 \).

If we choose \( \hat{\nu}^T = (1, 0) \), \( \sigma_{11} = \sigma/\xi \) and \( \Theta_{11} = \xi \) we have

\[
\mathcal{L}_{\text{gf}} = -\frac{\sigma}{2\xi} (\partial_\mu A^\mu + \xi g \phi_1 \phi_2)^2 , \quad (28)
\]

\[
\mathcal{L}_{\text{gh}} = \partial_\mu \bar{\eta} \partial^\mu \eta - \xi g^2 (\phi_1^2 - \phi_2^2) \bar{\eta} \eta . \quad (29)
\]

\( \mathcal{L}_{\text{eff}} = \mathcal{L} + \mathcal{L}_{\text{gf}} + \mathcal{L}_{\text{gh}} \) can also be obtained by the following procedure: Take the most general dim \( \leq 4 \) Lagrangian with given field content \( (\phi_1, \phi_2, A_\mu, \bar{\eta}, \eta, B) \). Impose the nil-potent \( s \)-symmetry of (13) with \( T_1 \) given above and \( f_{abc} = 0 \) and the discrete symmetries \( (A_\mu, \phi_1, B) \rightarrow (-A_\mu, -\phi_1, -B) \) and \( (A_\mu, \phi_2, B) \rightarrow (-A_\mu, -\phi_2, -B) \). Integrate out \( B \). Up to total divergencies and trivial changes of variables, \( \mathcal{L}_{\text{eff}} \) with the parameters given above is the result. Therefore \( \mathcal{L}_{\text{eff}} \) is renormalizable in any regularization scheme that observes the symmetries, e.g. dimensional regularization.
Since $\mathcal{L}_{\text{eff}}$ is now invariant under $(\bar{\eta}, \eta) \rightarrow (-\eta, \bar{\eta})$ and since $f_{abc} = 0$, and thus this operation relates $s'$- and $\bar{s}$-invariance, $\mathcal{L}_{\text{eff}}$ is also $s$-invariant.

Now we are ready to break the U(1) symmetry by shifting $(\phi_1, \phi_2) = (\varphi + \phi'_1, \phi'_2)$. For $\sigma = 1$, there is no mixing between would-be-Goldstone and longitudinal gauge field modes and $\mathcal{L}_{\text{eff}}$ becomes the sum of the following four pieces, ordered by their dimension:

\begin{align}
\mathcal{L}_0 &= -\frac{1}{4} \lambda \varphi^4 - \frac{1}{2} m^2 \varphi^2, \\
\mathcal{L}_1 &= - (\lambda \varphi^3 + m^2 \varphi) \phi'_1, \\
\mathcal{L}_2 &= \frac{1}{2} (\partial_{\mu} \phi'_1)^2 - \frac{1}{2} (3 \lambda \varphi^2 + m^2) \phi'_1^2 + \frac{1}{2} (\partial_{\mu} \phi'_2)^2 - \frac{1}{2} [(\lambda + \xi g^2) \varphi^2 + m^2] \phi'_2^2 \\
&\quad - \frac{1}{2} F_{\mu \nu} F^{\mu \nu} - \frac{1}{2} (\partial_{\mu} A^\mu)^2 + \frac{1}{4} g^2 \varphi^2 A_{\mu} A^\mu + \partial_\mu \bar{\eta} \partial^\mu \eta - \xi g^2 \varphi^2 \bar{\eta} \eta, \\
\mathcal{L}_{3,4} &= - \frac{1}{4} \lambda \phi'^4 - \lambda \varphi \phi'^3 - \frac{1}{2} (\lambda + \xi g^2) \phi'^2 \phi'_2 - (\lambda + \xi g^2) \varphi \phi'_2 - \frac{1}{4} \lambda \phi'^4 \\
&\quad + g^2 \varphi \phi'_1 A_{\mu} A^\mu + 2 g \phi'_2 A_{\mu} \partial^\mu \phi'_1 + \frac{1}{2} g^2 \phi'_1 A_{\mu} A^\mu + \frac{1}{2} g^2 \phi'_2 A_{\mu} A^\mu \\
&\quad - 2 \xi g^2 \varphi \phi'_1 \bar{\eta} \eta - \xi g^2 \phi'_2 \bar{\eta} \eta + \xi g^2 \phi'^2 \bar{\eta} \eta.
\end{align}

The Feynman rules can immediately be read off and are given in table $\Box$. For comparison also the Feynman rules in generalized $R_\xi$ gauge, i.e.

\begin{align}
\mathcal{L}_{\text{eff}} &= -1/(2\xi) (\partial_{\mu} A^\mu + \xi g \varphi \phi'_2)^2, \\
\mathcal{L}_{\text{gh}} &= \partial_{\mu} \bar{\eta} \partial^\mu \eta - \xi g^2 \varphi (\phi'_1 + \varphi) \bar{\eta} \eta,
\end{align}

are given (‘generalized’ since the gauge parameter $\varphi$ is not necessarily the vev of the Higgs field). Note that as promised in Landau gauge the Feynman rules of both classes of gauges become identical (the remaining difference in the Higgs-gauge-Goldstone vertex is immediately seen to be irrelevant because now the gauge propagator is transverse) and therefore $\varphi$ in generalized $R_\xi$ Landau gauge can serve as the argument of the EP. It can be shown that the Feynman rules effectively coincide also for unitary gauge ($\xi \rightarrow \infty$) in both classes of gauges.

Up to an unphysical constant term the effective Lagrange density can be renormalized as

\begin{align}
\mathcal{L}_{\text{eff}} &= \frac{1}{2} (\partial_{\mu} \phi_{1B} + g_B A_{B\mu} \phi_{2B})^2 + \frac{1}{2} (\partial_{\mu} \phi_{2B} - g_B A_{B\mu} \phi_{1B})^2 - \frac{1}{4} F_{B\mu \nu} F^{B\mu \nu} \\
&\quad - \frac{1}{2} m^2 (\phi_{1B}^2 + \phi_{2B}^2) - \frac{1}{4} \lambda_B (\phi_{1B}^2 + \phi_{2B}^2)^2 - \frac{\sigma_B}{2 \xi_B} (\partial_{\mu} A^\mu_B + \xi_B g_B \phi_{1B} \phi_{2B})^2 \\
&\quad + \partial_{\mu} \bar{\eta}_B \partial^\mu \eta_B - \xi_B g^2_B (\phi_{1B}^2 - \phi_{2B}^2) \bar{\eta}_B \eta_B
\end{align}

with

\begin{align}
\phi_{1B} &= Z_{\phi_1}^B \phi_{1R}, \quad \phi_{2B} = Z_{\phi_2}^B \phi_{2R}, \quad A_{B\mu} = Z_{A}^B A_{\mu R}, \quad \bar{\eta}_B \eta_B = Z_{\bar{\eta} \eta} R \eta_R, \\
m_B &= Z_{m} R, \quad \lambda_B = Z_{\lambda} \lambda_R, \quad g_B = Z_{g} g_R, \quad \xi_B = Z_{\xi} \xi_R, \quad \sigma_B = Z_{\sigma} \sigma_R,
\end{align}
where renormalization constants $Z_x$ have been introduced and “$B$” and “$R$” denote bare and renormalized quantities, respectively. The Ward identity requires $Z_g Z_A = 1$. Using dimensional regularization [13] and the MS scheme [14] (as for all calculations in this paper) I have computed the $Z_x$ at the two-loop level for $\sigma_R = 1$. The result is given in the appendix.

As a check on the consistency of the gauge (6) and as an illustration for its use, the one-loop radiative corrections to the Higgs and physical gauge boson masses are computed in the next two paragraphs.

With the tree-level Higgs field vev $v_0^2 = -m^2/\lambda$ the tree-level Higgs mass obtains as $m_{H_0}^2 = 3\lambda v_0^2 + m^2 = -2m^2$. Its one-loop correction is given by [15] (let $k_\mu$ be the momentum flowing through the graphs)

$$m_{H_1}^2 = -m^2 \left[ \left(6\sqrt{3}\pi - 28\right)\lambda + 10g^2 - 6\frac{g^4}{\lambda} \right] + \left(8\lambda - 6g^2\right) \ln \frac{-2m^2}{\bar{\mu}^2}$$

$$+ \left(2\lambda - 6g^2\right) \ln \frac{g^2}{2\lambda} + 4 \left(\lambda - 2g^2 + 3\frac{g^4}{\lambda}\right) \sqrt{\frac{2g^2}{\lambda} - 1} \arctan \frac{1}{\sqrt{\frac{2g^2}{\lambda} - 1}} \right],$$

which as physical quantity is gauge independent as expected [15, 16, 17].

The determination of the physical gauge boson mass proceeds in close analogy:
Its tree-level value is given by $m_{A_0}^2 = g^2 v_0^2 = -m^2 g^2 / \lambda$. If we write
\[-i \left[ \mu \cdot \nu \left( \text{1-loop} \right) - i \left( \text{1-loop} \right) \right] = \left( g_{\mu\nu} - \frac{k_\mu k_\nu}{k^2} \right) A + \frac{k_\mu k_\nu}{k^2} B, \tag{42} \]
where in $\overline{R}_\xi$ gauge
\[-i \left[ \mu \cdot \nu \left( \text{1-loop} \right) - i \left( \text{1-loop} \right) \right] = \frac{\lambda^2}{g^2} - 4\lambda \ln \frac{g^2}{2\lambda} + \left( \frac{8}{3} \lambda^2 \left( g^2 - 2 \right) - \ln \frac{-g^2 m^2}{\lambda \bar{\mu}^2} \right) \ln \left( \frac{2g^2}{\lambda} - 1 \right) \arctan \left( \frac{2g^2}{\lambda} - 1 \right), \tag{43} \]
and truncate the external legs, then the one-loop correction to the physical gauge boson mass is easily seen to be given by
\[m_{A_1}^2 = A \big|_{k^2 = m_{A_0}^2, \varphi^2 = v_0^2}. \tag{45} \]

Using again the rules in tables [1] and [2], one gets
\[m_{A_1}^2 = -m^2 \left[ \left( -\frac{4}{3} \lambda + 10g^2 - \frac{62}{9} g^4 + \frac{g^6}{\lambda^2} \right) - \left( 6g^2 - \frac{10}{3} \frac{g^4}{\lambda} + \frac{3g^6}{\lambda^2} \right) \ln \frac{-g^2 m^2}{\lambda \bar{\mu}^2} \right. \]
\[- \left( \frac{4}{3} \ln \frac{g^2}{\lambda} + \left( \frac{8}{3} \lambda \left( g^2 - 2 \right) - \ln \frac{2g^2}{\lambda} + 8g^2 \right) \right) \left( \frac{2g^2}{\lambda} - 1 \right) \arctan \left( \frac{2g^2}{\lambda} - 1 \right) \right], \tag{46} \]
which also is gauge independent.

Now we proceed to compute the EP at the one-loop level in both $R_\xi$ and $\overline{R}_\xi$ gauges. The tree-level potential is just $V_0 = \frac{1}{4} \lambda \varphi^4 + \frac{1}{2} m^2 \varphi^2$. In $R_\xi$ gauge I have computed its one-loop correction by summing up graphs with all numbers of external lines (i.e. using [1] at $\omega = 0$) as well as from vacuum graphs as in [3, 17, 18], but as stated earlier the price one pays in the latter case is the use of mixed propagators between longitudinal gauge boson modes and would-be-Goldstone bosons. The result is of course the same:
\[V_{1,R_\xi}(\varphi) = \frac{1}{4(4\pi)^2} \left[ m_H^4 \left( \ln \frac{m_H^2}{\mu^2} - \frac{3}{2} \right) + m_A^4 \left( 3 \ln \frac{m_A^2}{\mu^2} - \frac{5}{2} \right) \right. \]
\[\left. + m_a^4 \left( \ln \frac{m_a^2}{\mu^2} - \frac{3}{2} \right) + m_b^4 \left( \ln \frac{m_b^2}{\mu^2} - \frac{3}{2} \right) - 2m_c^4 \left( \ln \frac{m_c^2}{\mu^2} - \frac{3}{2} \right) \right], \tag{47} \]
with (the tree-level values of the squares of the Higgs mass $m_H^2 = 3\lambda \varphi^2 + m^2$ and the physical gauge boson mass $m_A^2 = g^2 \varphi^2$ and with $m_{a,b}^2 = \frac{1}{2}(\lambda \varphi^2 + m^2) + \xi g^2 \varphi v_0 \pm \frac{1}{2} \sqrt{(\lambda \varphi^2 + m^2)(4\xi g^2 \varphi(\varphi - v_0))}$ and $m_c^2 = \xi g^2 \varphi v_0$.\]

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In $\overline{\kappa}_\xi$ gauge, we simply have

$$V_{1,\overline{\kappa}_\xi}(\varphi) = i \left( \text{1-loop} \right) \left( \frac{1}{1\text{PI}} \right)$$

$$= i \left[ \begin{array}{c} \bigcirc \bigcirc \\ \bigcirc + \bigcirc \\ \bigcirc + \bigcirc \\ \bigcirc + \bigcirc \\ \bigcirc + \bigcirc \\ \bigcirc + \bigcirc \\ \bigcirc + \bigcirc \\ \bigcirc + \bigcirc \end{array} \right]$$

$$= \frac{1}{4(4\pi)^2} \left[ m_H^4 \left( \frac{\ln m_H^2}{\mu^2} - \frac{3}{2} \right) + m_A^4 \left( 3 \frac{\ln m_A^2}{\mu^2} - \frac{5}{2} \right) + m_G^4 \left( \frac{\ln m_G^2}{\mu^2} - \frac{3}{2} \right) - m_{gh}^4 \left( \frac{\ln m_{gh}^2}{\mu^2} - \frac{3}{2} \right) \right]$$

with the would-be-Goldstone mass $m_G^2 = (\lambda + \xi g^2)\varphi^2 + m^2$, the ghost (and longitudinal gauge boson) mass $m_{gh}^2 = \xi g^2\varphi^2$ and $m_H^2$ and $m_A^2$ as above. Clearly, in $\overline{\kappa}_\xi$ gauge $V_1$ is not only easier to compute but also has a more convenient analytical structure due to the simple expressions for the appearing masses which contain no awkward square roots anymore.

It is straightforward to determine the two-loop contribution to the EP as

$$V_{2,\overline{\kappa}_\xi}(\varphi) = i \left( \text{2-loop} \right) \left( \frac{1}{1\text{PI}} \right)$$

$$= i \left[ \begin{array}{c} \bigcirc \bigcirc \\ \bigcirc + \bigcirc \\ \bigcirc + \bigcirc \\ \bigcirc + \bigcirc \\ \bigcirc + \bigcirc \\ \bigcirc + \bigcirc \\ \bigcirc + \bigcirc \\ \bigcirc + \bigcirc \end{array} \right]$$

$$= \frac{g^2}{(4\pi)^4 m_A^2} \left\{ -\frac{3}{4} m_{H\overline{G}g\overline{h}}^4 K_{HHH} - \frac{1}{4} (m_{H\overline{G}g\overline{h}}^2 + 2m_{g\overline{h}}^2) K_{HGG} \\
- \frac{1}{4} [(m_H^2 - 2m_A^2)^2 + 8m_A^2] K_{HAA} - \frac{1}{4} [(m_H^2 - 2m_A^2)^2 - 8m_A^2] K_{Hg\overline{h}g\overline{h}} \\
+ \frac{1}{2} T_{H\overline{A}g\overline{h}} K_{H\overline{A}g\overline{h}} - \frac{1}{2} T_{H\overline{G}A} K_{H\overline{G}A} + \frac{1}{2} m_{H\overline{G}g\overline{h}}^4 K_{H\overline{G}g\overline{h}} \\
+ \frac{1}{8} m_{H\overline{G}g\overline{h}}^4 \left[ 3(L_H^2 + L_G^2) + 2L_H L_G \right] + \frac{1}{2} m_A^2 [L_H L_G + 2(L_H + L_G) L_A] \\
+ \frac{1}{4} m_A^2 (L_A - L_g)^2 \\
+ \frac{1}{4} [(m_A^2 - m_{H\overline{G}g\overline{h}}^2) L_H + (m_H^2 - m_G^2) L_G - (m_A^2 L_A - m_{g\overline{h}}^2 L_{g\overline{h}})] (L_A - L_g) \\
+ m_A^4 (3L_H + L_G) + m_A^2 (m_H^2 + m_G^2 + \frac{2}{3} m_A^2 - m_A^2) L_A \\
- m_A^4 (m_H^2 + 2m_A^2) \right\}$$ (49)
where
\begin{align}
T_{xyz} &\equiv m_x^4 + m_y^4 + m_z^4 - 2m_x^2m_y^2 - 2m_x^2m_z^2 - 2m_y^2m_z^2, \\
\mu_{HGgh}^2 &\equiv m_H^2 - m_G^2 + m_{gh}^2 = 2\lambda\varphi^2, \\
L_x &\equiv m_x^2[\ln(m_x^2/\bar{\mu}^2) - 1], \\
K_{xyz} &\equiv K(m_x^2, m_y^2, m_z^2)
\end{align}
(50)

\begin{align}
\lim_{\epsilon \to 0} \left\{ (4\pi)^4 \int \frac{d^dp}{(2\pi)^d} \frac{d^dq}{(2\pi)^d} \frac{\mu^2\epsilon}{(p^2 - m_x^2)(q^2 - m_y^2)(p^2 + q^2 - m_z^2)} \\
+ \sum_{n=x}^3 m_n^2 \left[ \frac{2}{\epsilon^2} - \frac{2}{\epsilon} \left( \ln \frac{m_n^2}{\bar{\mu}^2} - \frac{3}{2} \right) + \frac{1}{2} \left( \ln \frac{m_n^2}{\bar{\mu}^2} - 1 \right)^2 + \frac{\pi^2 + 6}{12} \right] \right\}.
\end{align}
(53)

(For an evaluation of $K_{xyz}$ see e.g. [19] $V_{1,\overline{R}_\xi}$ and $V_{2,\overline{R}_\xi}$ are only slightly more complicated than the corresponding expressions in Landau gauge which one gets back by letting $\xi \to 0$ in our results.

Since the EP is not itself a physical quantity it can be and indeed is gauge dependent. However, its value at points where $V' = 0$ is a physical energy density and should therefore be gauge independent [16, 17, 20]. The energy density at the symmetry breaking solution of $V' = 0$ is given by

\begin{align}
V(v) = i \left\{ \cdot + \begin{array}{c} \text{1-loop} \\ \text{1PI} \end{array} + \begin{array}{c} \text{2-loop} \\ \text{1PI} \end{array} + \begin{array}{c} \text{1-loop} \\ \text{1PI} \end{array} - \begin{array}{c} \text{1-loop} \\ \text{1PI} \end{array} + \text{higher} \\ \text{loops} \end{array} \right\}_{\varphi = v_0},
\end{align}
(54)

where
\begin{align}
\begin{array}{c}
\text{1-loop} \\ \text{1PI}
\end{array} = \begin{array}{c}
\text{1-loop} \\ \text{1PI}
\end{array} + \begin{array}{c}
\text{2-loop} \\ \text{1PI}
\end{array} + \begin{array}{c}
\text{1-loop} \\ \text{1PI} \end{array} - \begin{array}{c}
\text{1-loop} \\ \text{1PI} \end{array} + \text{higher} \\ \text{loops}
\end{array}
\end{align}
(55)

The one-loop contribution is gotten by substituting $v_0$ into (47) and (48) and one finds $V_{1,\overline{R}_\xi}(v_0) = V_{1,\overline{R}_\xi}(v_0)$ = gauge independent as expected although the one-loop correction to the Higgs field vev, given by $v_1 = V'_1(v_0)/(2\mu^2)$, turns out to be gauge dependent as is expected for the location of the vev [19]. In $\overline{R}_\xi$ gauge it is easy to determine the two-loop contribution to (54) and it also turns out to be gauge independent. The same is true for the other stationary point, i.e. $\varphi = 0$ (however, the loop expansion is a bad approximation scheme here due to infrared divergences).

6 Application II: An SU(2)-Higgs Model

For the Abelian Higgs model $R_{abc}$ and $S_{abc}$ vanish. To demonstrate some non-trivial appearance of $R_{abc}$ the renormalization of an SU(2)-Higgs model will be
sketched in this section. This model can also be regarded as a truncated version of the standard electroweak model with vanishing weak mixing angle and no fermions.

We start from the Lagrangian

$$\mathcal{L} = \frac{1}{2} (D_\mu \Phi)^T (D^\mu \Phi) - \frac{1}{4} F_{a\mu\nu} F_a^{\mu\nu} - \frac{1}{2} m^2 \Phi^T \Phi - \frac{\lambda}{4} (\Phi^T \Phi)^2$$  \hspace{1cm} (56)

with

$$F_{a\mu\nu} = \partial_\mu A_a^\nu - \partial_\nu A_a^\mu - g \epsilon_{abc} A_b^\mu A_c^\nu ,$$ \hspace{1cm} (57)

$$D^\mu \Phi = (\partial^\mu + ig T_a A_a^\mu) \Phi.$$ \hspace{1cm} (58)

Here

$$\Phi^T = (\phi_0, \phi_1, \phi_2, \phi_3)$$ \hspace{1cm} (59)

is a collection of real scalar fields and with the scalar self-coupling \(\lambda > 0\) and \(m^2 < 0\) the SU(2) gauge symmetry is spontaneously broken down completely.

It turns out to be clever to choose the generators as

$$T_{cAB} = -\frac{i}{2} \eta_{ABc},$$ \hspace{1cm} (60)

where the \(\eta_{ABc}\) are ’t Hooft symbols \[21]\]

$\eta_{ABc} = \epsilon_{ABc}$ for \(A, B, c = 1, 2, 3\)

$\eta_{A0c} = \delta_{Ac}$ for \(A, c = 1, 2, 3\)

$\eta_{0Bc} = -\delta_{Bc}$ for \(B, c = 1, 2, 3\)

$\eta_{00c} = 0$ for \(c = 1, 2, 3\), \hspace{1cm} (61)

and the direction of symmetry breaking as

$$\hat{v} = (1, 0, 0, 0).$$ \hspace{1cm} (62)

Then \(\Phi\) is nothing but a complex doublet transforming under the fundamental representation of SU(2), written in terms of its real components. The normalization is such that \(f_{abc} = \epsilon_{abc}\). Since there is no unbroken subgroup, we try

$$\sigma_{ab} = \sigma \delta_{ab}/\xi, \quad \Theta_{ab} = \xi \delta_{ab}.$$ \hspace{1cm} (63)

From the structure of the one-loop divergences for the quartic ghost coupling for \(R_{abc} = S_{abc} = 0\) it is then easy to guess as simplest admissible form

$$R_{abc} = \alpha \xi \epsilon_{abc}, \quad S_{abc} = 0.$$ \hspace{1cm} (64)

By considering all divergent one-loop diagrams we can now check renormalizability to this order. Indeed everything works out and with

$$\phi_{0B} = Z_{HR}^{\frac{1}{2}} \phi_{0R}, \quad \phi_{aB} = Z_{G}^{\frac{1}{2}} \phi_{aR}, \quad \alpha_{B} = Z_{a} \alpha_{R},$$ \hspace{1cm} (65)
$a = 1, 2, 3$, and otherwise the same definitions as in (37) the resulting effective Lagrangian $\mathcal{L}_{\text{eff}} = \mathcal{L} + \mathcal{L}_{gf} + \mathcal{L}_{gh}$ keeps its form under renormalization. The one-loop results for the $Z_a$ can be found in appendix B. To enjoy the absence of mixed would-be-Goldstone-gauge propagators we would set $\sigma_R = 1$.

$L_{\text{eff}}$ can also be obtained by the following procedure: Take the most general $\dim \leq 4$ Lagrangian with given field content $(\phi_0, \phi_a, A^\mu_a, \bar{\eta}_a, \eta_a, B_a)$, $a = 1, 2, 3$. Impose the nil-potent $s$-symmetry of (13) with $T_a$ and $f_{abc}$ given above, the discrete symmetry $(\phi_0, \phi_a) \rightarrow (-\phi_0, -\phi_a)$ and a global SO(3) symmetry, under which $\phi_0$ is a scalar and $\phi_a, A^\mu_a, \bar{\eta}_a, \eta_a, B_a$ are vectors. Integrate out the $B_a$. Up to total divergencies and trivial changes of variables, $\mathcal{L}_{\text{eff}}$ with the parameters given above is the result. Therefore $\mathcal{L}_{\text{eff}}$ is renormalizable in any regularization scheme that observes the symmetries, e.g. dimensional regularization.

To impose $\bar{s}$-invariance for this case we define

$$\Delta_{ab} = \xi \delta_{ab}$$

so that (24) is fulfilled. Using (20) and (21) it follows

$$R_{abc} = -\frac{\xi}{2\sigma} \epsilon_{abc}, \quad S_{abc} = 0$$

and therefore $\bar{s}$-invariance is equivalent to

$$\alpha = -\frac{1}{2\sigma}.$$ 

Stability of this condition under renormalization can easily be checked at the one-loop level using the results for $Z_\sigma$ and $Z_a$ given in appendix B.

7 Application III: An SO(3)-Higgs Model

For both examples considered so far, $\sigma_{ab}$ is proportional to $\delta_{ab}$ and $S_{abc} = 0$. To present a non-trivial appearance of both $\sigma_{ab}$ and $S_{abc}$, now the renormalization of an SO(3)-Higgs model with an unbroken SO(2) subgroup will be sketched. Consider again the Lagrangian (56), with $F^\mu_{\nu a}$ and $D^\mu \Phi$ as in (57) and (58), but now with a triplet of real scalar fields

$$\Phi^T = (\phi_1, \phi_2, \phi_3)$$

and the generators given by

$$T_{ijk} = -i \epsilon_{ijk}.$$ 

Again the normalization is such that $f_{abc} = \epsilon_{abc}$.

With the direction of symmetry breaking chosen as

$$\hat{v} = (0, 0, 1)$$
there is an unbroken SO(2) subgroup of rotations in field space around the 3-axis. While \( \Theta_{ab} \) contributes only for broken generators and we can set

\[
\Theta_{ab} = \xi \delta_{ab},
\]

we have to treat broken and unbroken generators differently in \( \sigma_{ab} \), \( R_{abc} \) and \( S_{abc} \). For \( \sigma_{ab} \) it is natural to try

\[
\sigma_{ab} = \frac{\sigma_w}{\xi} (\delta_{ab} - \delta_{a3}\delta_{b3}) + \frac{\sigma_1}{\xi} \delta_{a3}\delta_{b3}.
\]

In order to minimize the amount of complication involved we choose \( R_{abc} \) and \( S_{abc} \) to be non-zero only if \( \{a, b, c\} \) is a permutation of \( \{1, 2, 3\} \). Together with the symmetry properties (11) we get therefore

\[
R_{abc} = \xi \alpha \epsilon_{abc} \delta_{a3} + \xi \beta (\delta_{a3}\epsilon_{b3c} - \delta_{b3}\epsilon_{a3c}),
\]

\[
S_{abc} = \frac{1}{2} \xi \gamma (\epsilon_{abc} \delta_{a3} + \epsilon_{ac3} \delta_{b3}).
\]

Considering again all divergent one-loop diagrams and with

\[
\phi_{1,2B} = Z_G^1 \phi_{1,2R}, \quad \phi_{3B} = Z_H^1 \phi_{3R}, \quad A_{1,2B}^\mu = Z_W^1 A_{1,2R}^\mu, \quad A_{3B}^\mu = Z_A^1 A_{3R}^\mu,
\]

\[
\bar{\eta}_{1,2B} = Z_{\eta_a}^1 \bar{\eta}_{1,2R}, \quad \bar{\eta}_{3B} = Z_{\eta_a}^1 \bar{\eta}_{3R}, \quad \eta_{1,2B} = Z_{\eta_a} \eta_{1,2R}, \quad \eta_{3B} = Z_{\eta_a} \eta_{3R},
\]

\[
\alpha_B = Z_{a\alpha} \alpha_R, \quad \beta_B = Z_{\beta} \beta_R, \quad \gamma_B = Z_{\gamma} \gamma_R, \quad \sigma_{WB} = Z_{\sigma_{WB}} \sigma_{WR}, \quad \sigma_{AB} = Z_{\sigma_{AB}} \sigma_{AR},
\]

and otherwise the same definitions as in (77) the resulting effective Lagrangian keeps its form under renormalization. The one-loop results for the \( Z_x \) can be found in appendix C. To enjoy the absence of mixed would-be-Goldstone-gauge propagators we would set \( \sigma_{WR} = 1 \).

\( \mathcal{L}_{\text{eff}} \) can also be obtained by the following procedure: Take the most general \( \text{dim} \leq 4 \) Lagrangian with given field content \( \psi_a \equiv (\phi_a, A_a^\mu, \bar{\eta}_a, \eta_a, B_a) \), \( a = 1, 2, 3 \). Impose the nil-potent \( s \)-symmetry of (13) with \( T_a \) and \( f_{abc} \) given above, the discrete symmetries \( (\phi_1, \phi_2, \phi_3) \rightarrow (-\phi_1, -\phi_2, -\phi_3) \) and \( (\psi_1, \psi_3) \rightarrow (-\psi_1, -\psi_3) \) and a global SO(2) symmetry, under which \( \psi_3 \) are scalars and \( \psi_a \), \( a = 1, 2 \) are vectors. Integrate out the \( B_a \), \( a = 1, 2, 3 \). Up to total divergencies and trivial changes of variables, \( \mathcal{L}_{\text{eff}} \) with the parameters given above is the result. Therefore \( \mathcal{L}_{\text{eff}} \) is renormalizable in any regularization scheme that observes the symmetries, e.g. dimensional regularization.

To impose \( s \)-invariance for this case we define

\[
\Delta_{ab} = \xi (\delta_{ab} - \delta_{a3}\delta_{b3}) + \xi \Delta_{a3}^{-1} \delta_{a3}\delta_{b3}
\]

so that (22) is fulfilled. Using (20) and (21) it follows that

\[
R_{abc} = \left( \frac{\Delta_{a3}^2}{2\sigma_3} - \frac{1}{\sigma_w} \right) \xi \epsilon_{abc} \delta_{c3} - \frac{\Delta_{a3}}{2\sigma_3} \xi (\delta_{a3}\epsilon_{b3c} - \delta_{b3}\epsilon_{a3c}),
\]

\[
S_{abc} = \frac{1}{2} (1 - \Delta_a) (\epsilon_{abc} \delta_{c3} + \epsilon_{ac3} \delta_{b3})
\]
and therefore
\[ \alpha = \frac{\Delta_A^2}{2\sigma_A} - \frac{1}{\sigma_w}, \quad \beta = -\frac{\Delta_A}{2\sigma_A}, \quad \gamma = \frac{1 - \Delta_A}{\xi}. \] (80)

Eliminating the newly introduced parameter \( \Delta_A \) from these equations we get
\[ \alpha = \frac{(\gamma \xi - 1)^2}{2\sigma_A} - \frac{1}{\sigma_w}, \quad \beta = \frac{\gamma \xi - 1}{2\sigma_A}. \] (81)

In the case at hand we can diminish the number of gauge parameters by imposing
\[ \beta = 0 \] (82)
alternatively or additionally to \( \bar{s} \)-invariance. This condition can be shown to be stable under renormalization. Note that if we impose (82) additionally to \( s \)- and \( \bar{s} \)-symmetry, \( \bar{\eta}_3 \) and \( \eta_3 \) effectively drop from the theory. As already noted in section \( \mathbb{II} \) this is due to the fact that an unbroken Abelian subgroup does not always require a ghost field. Also, since now \( \Delta_A = 0 \), we loose invertibility of \( \Delta_{ab}^{-1} \); only the restriction of \( \Delta_{ab}^{-1} \) to \( a, b = 1, 2 \) can be inverted which turns out to be sufficient at this stage. We have
\[ \alpha = -\frac{1}{\sigma_w}, \quad \beta = 0, \quad \gamma = \frac{1}{\xi}, \] (83)
which also means that \( S_{abc} \) is no longer renormalized. Stability of (81), (82) and (83) under renormalization can easily be checked at the one-loop level using the results given in appendix C.

I am grateful to Roberto Peccei, Duncan Morris, Gerard ’t Hooft, Konstadinos Sfetsos and especially Bernard de Wit for valuable discussions and suggestions, to Jim Congleton for proofreading an early version of the manuscript and to the referee for crucial suggestions. This work was supported by Stichting FOM.

**Appendix A**

Using dimensional regularization and the MS scheme the \( Z_x \) up to two loops in the Abelian Higgs model are for \( \sigma = 1 \) (all quantities are the renormalized ones, \( \epsilon = 4 - d \), \( d \) = dimension of space-time):
\[ \mu^\epsilon Z_H = 1 + \frac{(6 + 2\xi)g^2}{(4\pi)^2 \epsilon} + \frac{-4\lambda^2 - 4\lambda \xi g^2 + (-\frac{10}{3} + 2\xi - 3\xi^2)g^4}{(4\pi)^4 \epsilon} + \frac{8\lambda \xi g^2 + (20 + 12\xi^2)g^4}{(4\pi)^4 \epsilon^2} \] (84)
\[
\mu' Z_G = 1 + \frac{(6 - 6\xi)g^2}{(4\pi)^2\epsilon} + \frac{-4\lambda^2 + 4\lambda\xi g^2 + \left(\frac{-10}{3} - 2\xi - 11\xi^2\right)g^4}{(4\pi)^4\epsilon} \\
+ \frac{-8\lambda\xi g^2 + (20 - 24\xi + 12\xi^2)g^4}{(4\pi)^4\epsilon^2} 
\]  
(85)

\[
\mu' Z_A = 1 - \frac{\xi g^2}{(4\pi)^2\epsilon} - \frac{4g^4}{(4\pi)^4\epsilon} 
\]  
(86)

\[
\mu' Z_\eta = 1 - \frac{2\xi^2 g^4}{(4\pi)^4\epsilon} 
\]  
(87)

\[
Z_m = 1 + \frac{8\lambda - 6g^2}{(4\pi)^2\epsilon} + \frac{-20\lambda^2 + 32\lambda g^2 + \frac{43}{3} g^4}{(4\pi)^4\epsilon} \\
+ \frac{112\lambda^2 - 96\lambda g^2 + 40g^4}{(4\pi)^4\epsilon^2} 
\]  
(88)

\[
\mu^- Z_\lambda = 1 + \frac{20\lambda - 12g^2 + 6g^4/\lambda}{(4\pi)^2\epsilon} + \frac{-120\lambda^2 + 56\lambda g^2 + \frac{158}{3} g^4 - \frac{104}{3} g^6/\lambda}{(4\pi)^4\epsilon} \\
+ \frac{400\lambda^2 - 360\lambda g^2 + 188g^4 - 32g^6/\lambda}{(4\pi)^4\epsilon^2} 
\]  
(89)

\[
\mu^- Z_g = 1 + \frac{\frac{2}{3}g^2}{(4\pi)^2\epsilon} + \frac{4g^4}{(4\pi)^4\epsilon} + \frac{\frac{4}{3} g^4}{(4\pi)^4\epsilon^2} 
\]  
(90)

\[
Z_\xi = 1 + \frac{4\lambda + \left(\frac{-20}{3} + 4\xi\right) g^2}{(4\pi)^2\epsilon} + \frac{-12\lambda^2 - 8\lambda g^2 + \left(\frac{1}{3} + 12\xi^2\right) g^4}{(4\pi)^4\epsilon} \\
+ \frac{48\lambda^2 + \left(\frac{-152}{3} + 16\xi\right) \lambda g^2 + \left(32 - \frac{80}{3} \xi + 4\xi^2\right) g^4}{(4\pi)^4\epsilon^2} 
\]  
(91)

\[
Z_\sigma = 1 + \frac{4\lambda + \left(-6 + 6\xi\right) g^2}{(4\pi)^2\epsilon} + \frac{-12\lambda^2 - 8\lambda g^2 + \left(\frac{13}{3} - 4\xi + 12\xi^2\right) g^4}{(4\pi)^4\epsilon} \\
+ \frac{48\lambda^2 + \left(-48 + 32\xi\right) \lambda g^2 + \left(28 - 48\xi + 24\xi^2\right) g^4}{(4\pi)^4\epsilon^2} 
\]  
(92)

Note that \(Z_G Z_A = 1\) up to terms of higher than two-loop order as required by the Ward identity.

The one- and two-loop counterterms can be reconstructed from the \(Z_x\) above. For illustrative purposes and because some of them are used in the text, the one-loop counterterms are given in table 2.

Appendix B

The \(Z_x\) for the SU(2)-Higgs model of section 6 in MS are for general \(\sigma\) at the one-loop level:

\[
\mu' Z_H = 1 + \frac{g^2}{(4\pi)^2\epsilon} \left(\frac{9}{2} + 3\xi - \frac{3\xi}{2\sigma}\right) 
\]  
(93)
\[ \mu^{-1} Z_G = 1 + \frac{g^2}{(4\pi)^2\epsilon} \left( \frac{9}{2} - \xi - \frac{3\xi}{2\sigma} \right) \] (94)

\[ \mu^{-1} Z_A = 1 + \frac{g^2}{(4\pi)^2\epsilon} \left( \frac{25}{3} - \frac{2\xi}{\sigma} \right) \] (95)

\[ \mu^{-1} Z_\eta = 1 + \frac{g^2}{(4\pi)^2\epsilon} \left( 3 - \frac{\xi}{\sigma} \right) \] (96)

\[ Z_m = 1 + \frac{g^2}{(4\pi)^2\epsilon} \left( \frac{12\lambda}{g^2} - \frac{9}{2} \right) \] (97)

\[ \mu^{-1} Z_\lambda = 1 + \frac{g^2}{(4\pi)^2\epsilon} \left( \frac{24\lambda}{g^2} - 9 + \frac{9g^2}{8\lambda} \right) \] (98)

\[ \mu^{-1} Z_g = 1 + \frac{g^2}{(4\pi)^2\epsilon} \left( -\frac{43}{3} \right) \] (99)

\[ Z_\xi = 1 + \frac{g^2}{(4\pi)^2\epsilon} \left( \frac{4\lambda}{g^2} + \frac{41}{6} - \xi + \frac{\xi}{\sigma} \right) \] (100)

\[ Z_\sigma = 1 + \frac{g^2}{(4\pi)^2\epsilon} \left( \frac{4\lambda}{g^2} - \frac{3}{2} - \xi + 4\alpha\xi + \frac{3\xi}{\sigma} + \frac{\sigma\xi}{2} + 4\alpha^2\sigma\xi \right) \] (101)

\[ Z_\alpha = 1 + \frac{g^2}{(4\pi)^2\epsilon} \left( -\frac{4\lambda}{g^2} + \frac{3}{2} + \xi + \frac{\xi}{4\alpha} + 2\alpha\xi - \frac{\xi}{\sigma} \right) \] (102)

In an actual application to compute the EP we would set \( \sigma = 1 \).

**Appendix C**

The \( Z_x \) for the SO(3)-Higgs model of section [7] in MS are for general \( \sigma_A \) and \( \sigma_W \) at the one-loop level:

\[ \mu^{-1} Z_H = 1 + \frac{g^2}{(4\pi)^2\epsilon} \left( 12 + 8\xi - \frac{4\xi}{\sigma_W} \right) \] (103)

\[ \mu^{-1} Z_G = 1 + \frac{g^2}{(4\pi)^2\epsilon} \left( 12 - 4\xi - \frac{2\xi}{\sigma_A} - \frac{2\xi}{\sigma_W} \right) \] (104)

\[ \mu^{-1} Z_A = 1 + \frac{g^2}{(4\pi)^2\epsilon} \left( 8 + 6\gamma\xi - \frac{2\xi}{\sigma_W} + \frac{2\gamma\xi^2}{\sigma_W} \right) \] (105)

\[ \mu^{-1} Z_W = 1 + \frac{g^2}{(4\pi)^2\epsilon} \left( 8 - 3\gamma\xi - \frac{\xi}{\sigma_A} - \frac{\xi}{\sigma_W} - \frac{\gamma\xi^2}{\sigma_A} \right) \] (106)

\[ \mu^{-1} Z_{\eta\lambda} Z_{\eta\lambda} = 1 + \frac{g^2}{(4\pi)^2\epsilon} \left( 3 - 2\alpha\xi + 2\beta\xi - 3\gamma\xi - \frac{\xi}{\sigma_W} - 2\beta\gamma\xi^2 - \gamma\xi^2 - \frac{\gamma\xi^2}{\sigma_W} \right) \] (107)

\[ \mu^{-1} Z_{\bar{\eta}} Z_{\eta W} = 1 + \frac{g^2}{(4\pi)^2\epsilon} \left( 3 + \alpha\xi - \beta\xi + \frac{3\gamma\xi}{2} - \frac{\xi}{2\sigma_A} - \frac{\xi}{2\sigma_W} + \beta\gamma\xi^2 + \frac{3\gamma^2\xi^2}{2} \right) \]
\[ -\frac{\gamma \xi^2}{\sigma_a} + \frac{3 \gamma \xi^2}{2 \sigma_w} - \frac{\gamma^2 \xi^3}{2 \sigma_a} \)  

(108) 

\[ \mu' Z_{\eta A} Z_{\eta W} = 1 + \frac{g^2}{(4\pi)^2 \epsilon} \left( 3 - 3 \gamma \xi - \frac{\xi}{\sigma_a} - 2 \beta \gamma \xi^2 - \frac{\gamma^2 \xi^2}{\sigma_w} \right) \)  

(109) 

\[ Z_m = 1 + \frac{g^2}{(4\pi)^2 \epsilon} \left( \frac{10 \lambda}{g^2} - 12 \right) \)  

(110) 

\[ \mu^{-\epsilon} Z_\lambda = 1 + \frac{g^2}{(4\pi)^2 \epsilon} \left( \frac{22 \lambda}{g^2} - 24 + \frac{12 \lambda^2}{\lambda} \right) \)  

(111) 

\[ \mu^{-\epsilon} Z_g = 1 + \frac{g^2}{(4\pi)^2 \epsilon} (-14) \)  

(112) 

\[ Z_\xi = 1 + \frac{g^2}{(4\pi)^2 \epsilon} \left( \frac{4 \lambda}{g^2} - 1 - 6 \gamma - 2 \xi - \alpha \xi + \beta \xi - \frac{3 \gamma \xi}{2} + \frac{\xi}{2 \sigma_a} + \frac{9 \xi}{2 \sigma_w} - \beta \gamma \xi^2 - \frac{3 \gamma^2 \xi^2}{2} - \frac{\gamma \xi^2}{\sigma_a} - \frac{3 \gamma \xi^2}{2 \sigma_w} + \frac{\gamma^2 \xi^3}{2 \sigma_a} \right) \)  

(113) 

\[ Z_{\sigma_a} = 1 + \frac{g^2}{(4\pi)^2 \epsilon} \left( \frac{4 \lambda}{g^2} - 9 - 6 \gamma - 2 \xi - \alpha \xi + 5 \beta \xi - \frac{15 \gamma \xi}{2} + \frac{\xi}{2 \sigma_a} + 4 \beta^2 \sigma_a \xi + \frac{13 \xi}{2 \sigma_w} - 5 \beta \gamma \xi^2 - \frac{3 \gamma^2 \xi^2}{2} - \frac{\gamma \xi^2}{\sigma_a} - \frac{7 \gamma \xi^2}{2 \sigma_w} + \frac{\gamma^2 \xi^3}{2 \sigma_a} \right) \)  

(114) 

\[ Z_{\sigma_w} = 1 + \frac{g^2}{(4\pi)^2 \epsilon} \left( \frac{4 \lambda}{g^2} - 9 - 6 \gamma - 2 \xi + \alpha \xi + 3 \beta \xi + \frac{3 \gamma \xi}{2} + \frac{3 \xi}{2 \sigma_n} + \frac{11 \xi}{2 \sigma_w} + 2 \sigma_w \xi + 4 \alpha \beta \sigma_w \xi + 6 \gamma^2 \sigma_w \xi + \beta \gamma \xi^2 + \frac{3 \gamma^2 \xi^2}{2} - \frac{3 \gamma \xi^2}{\sigma_a} + \frac{3 \gamma^2 \xi^2}{2 \sigma_w} + \frac{3 \gamma^2 \xi^3}{2 \sigma_a} \right) \)  

(115) 

\[ Z_\alpha = 1 + \frac{g^2}{(4\pi)^2 \epsilon} \left( -\frac{4 \lambda}{g^2} + 9 + 6 \gamma + 2 \xi + \frac{2 \xi}{\alpha} + \alpha \xi + \beta \xi - \frac{3 \gamma \xi}{2} + \frac{6 \gamma^2 \xi}{\alpha} - \frac{3 \xi}{2 \sigma_a} - \frac{7 \xi}{2 \sigma_w} + 3 \beta \gamma \xi^2 - \frac{3 \gamma^2 \xi^2}{2} + \frac{3 \gamma \xi^2}{\sigma_a} - \frac{3 \gamma \xi^2}{2 \sigma_w} + \frac{4 \beta \gamma \xi^2}{\alpha \sigma_w} - \frac{3 \gamma^2 \xi^3}{2 \sigma_a} \right) \)  

(116) 

\[ Z_\beta = 1 + \frac{g^2}{(4\pi)^2 \epsilon} \left( -\frac{4 \lambda}{g^2} + 9 + 6 \gamma + 2 \xi + 2 \alpha \xi + 3 \gamma \xi - \frac{5 \xi}{\sigma_w} + \frac{\gamma^2 \xi}{\sigma_w} \right) \)  

(117) 

\[ Z_\gamma = 1 + \frac{g^2}{(4\pi)^2 \epsilon} \left( -\frac{4 \lambda}{g^2} + \frac{11}{2} + \frac{\alpha}{2 \gamma} - \frac{\beta}{2 \gamma} + 6 \gamma + 2 \xi + \frac{\alpha \xi}{2} - \frac{3 \gamma \xi}{2} - \frac{\xi}{2 \sigma_a} + \frac{2 \xi}{\sigma_w} + \frac{\beta \gamma \xi^2}{\sigma_a} - \frac{\gamma \xi^2}{\sigma_w} - \frac{\gamma^2 \xi^3}{\sigma_a} \right) \right) \)  

(118)
In an actual application to compute the EP we would set $\sigma_W = 1$. Notice that because of ghost number conservation (i.e. $\bar{\eta}_x$ and $\eta_y$ appear always in pairs $\bar{\eta}_x\eta_y$) the $Z_{\bar{\eta}_W}$, $Z_{\eta_W}$, $Z_{\bar{\eta}_A}$ and $Z_{\eta_A}$ are not determined uniquely, but only the combinations $Z_{\bar{\eta}_A}Z_{\eta_A}$, $Z_{\eta_W}Z_{\bar{\eta}_W}$, $Z_{\bar{\eta}_A}Z_{\eta_W}$ and $Z_{\eta_W}Z_{\eta_A}$, of which only three are independent.

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constant: \[ r - i \left( \frac{1}{4} \lambda \varphi^4 + \frac{1}{2} m^2 \varphi^2 \right) \] same

tadpole: \[ -i(\lambda \varphi^3 + m^2 \varphi) \] same

propagators:

| Higgs: \[ \mu \nu \varphi \] | \[ \frac{i}{k^2 - m^2_H} \] | same |
| would-be-Goldstone: \[ \mu \nu \] | \[ \frac{i}{k^2 - m^2_G} \] | same |
| gauge: \[ \mu \nu \varphi \] | \[ -i \left[ \frac{g_{\mu \nu} - k_{\mu}k_{\nu}/k^2}{k^2 - m^2_A} + \frac{\xi k_{\mu}k_{\nu}/k^2}{k^2 - m^2_{gh}} \right] \] | same |
| ghost: \[ \mu \nu \varphi \] | \[ \frac{i}{k^2 - m^2_{gh}} \] | same |

vertices:

| \( \mathcal{T}_{\xi} \) | \( R_{\xi} \) | \( \mathcal{T}_{\xi} \) | \( R_{\xi} \) |
| \[ -6i \lambda \varphi \] | same | \[ 2ig^2 \varphi g_{\mu \nu} \] | same |
| \[ -6i \lambda \] | same | \[ 2ig^2 g_{\mu \nu} \] | same |
| \[ -6i \lambda \] | same | \[ 2ig^2 g_{\mu \nu} \] | same |
| \[ 2i(\lambda + \xi g^2) \varphi \] | same | \[ 2i(\lambda + \xi g^2) \varphi \] | same |
| \[ -2i(\lambda + \xi g^2) \varphi \] | same | \[ -2i\xi g^2 \varphi \] | same |
| \[ 2i(\lambda + \xi g^2) \] | same | \[ -2i\xi g^2 \] | — |
| \( k_1 \) \( k_2 \) | \[ 2gk_{1\mu} \] | \( g(k_1 + k_2)_{\mu} \) | \[ 2i\xi g^2 \] | — |

Table 1: Feynman rules for the Abelian Higgs model in \( \mathcal{T}_{\xi} \) gauge for \( \sigma_R = 1 \) and in generalized \( R_{\xi} \) gauge with \( m^2_H = 3\lambda \varphi^2 + m^2 \), \( m^2_G = (\lambda + \xi g^2)\varphi^2 + m^2 \), \( m^2_A = g^2 \varphi^2 \), \( m^2_{gh} = \xi g^2 \varphi^2 \). \( k_\mu \) is the momentum flowing through the propagators.
Table 2: One-loop counterterms in $R_{\xi}$ gauge for the Abelian Higgs model for $\sigma_R = 1$. $k_\mu$ is the momentum flowing through the two-point functions.