Novel Robust Stability Criteria of Uncertain Systems with Interval Time-Varying Delay Based on Time-Delay Segmentation Method and Multiple Integrals Functional

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Received 18 September 2020; Revised 27 October 2020; Accepted 18 November 2020; Published 10 December 2020

Interval time-varying delay is common in control process, e.g., automatic robot control system, and its stability analysis is of great significance to ensure the reliable control of industrial processes. In order to improve the conservation of the existing robust stability analysis method, this paper considers a class of linear systems with norm-bounded uncertainty and interval time-varying delay as the research object. Less conservative robust stability criterion is put forward based on augmented Lyapunov-Krasovskii (L-K) functional method and reciprocally convex combination. Firstly, the delay interval is partitioned into multiple equidistant subintervals, and a new Lyapunov-Krasovskii functional comprising quadruple-integral term is introduced for each subinterval. Secondly, a novel delay-dependent stability criterion in terms of linear matrix inequalities (LMIs) is given by less conservative Wirtinger-based integral inequality approach. Three numerical comparative examples are given to verify the superiority of the proposed approach in reducing the conservation of conclusion. For the first example about closed-loop control systems with interval time-varying delays, the proposed robust stability criterion could get MADB (Maximum Allowable Delay Bound) about 0.3 more than the best results in the previous literature; and, for two other uncertain systems with interval time-varying delays, the MADB results obtained by the proposed method are better than those in the previous literature by about 0.045 and 0.054, respectively. All the example results obtained in this paper clearly show that our approach is better than other existing methods.

1. Introduction

Many dynamic model systems in the real world contain very significant time delays in the transmission of data and materials, in automatic robot control system, the acquisition and transmission of sensor signals, and the calculation of controller and the drive of brake may lead to time delay. In many kinds of time-delay types, the interval time-varying delay is more representative. The lower bound of its time delay is not necessarily zero, and the time delay is within a changing interval. It is common in practical application of engineering, especially in chemical reactors, internal combustion engines, and network control [1, 2]. Consequently, the stability analysis about interval time-delay systems has attracted wide attention in these years.

Generally, aiming to analyze the stability of time-delay system, the most common method is to construct an appropriate LK functional (Lyapunov–Krasovskii functional, LKF) in time domain and combine it with linear matrix inequalities (LMIs). In general, the free weight matrix method, the time-delay segmentation method, the integral inequality method, the interactive convex combination method, and so forth are used to analyze its stability. Augmented functional method [3–5] can make full use of the system’s time-delay information to reduce the conservativeness of conclusion, but the introduction of matrix variables inevitably burdens the theoretical analysis and engineering calculation. Zhang et al. and Shen et al. [6, 7] obtain a conservative less stable stability criterion for linear systems with time-varying delays by constructing LKF with
triple integral functional terms and optimize the stability conditions of time-delay systems. The integral inequality method has the characteristics of simple form and few matrix variables, which can promote the stability analysis of time-delay systems. Gu [8] first introduced Jensen’s inequality into the stability analysis of time-delay systems, and then Ramakrishnan [9, 10], Zhang [11], and Gouaisbaut [12] further promoted Jensen’s inequality, resulting in different and novel forms. In various forms, we have obtained effective conclusions of different conservation. As an innovative method, the interactive convex combination method [13, 14] can solve the stability problems of systems with interactive convex combination. Wu et al. [15] studied the issue of robust stability analysis for a sort of uncertain neutral system with mixed time-varying delays, and a novel discrete and neutral delay-dependent stability criterion based on linear matrix inequalities was given, which could greatly reduce the complexity of theoretical derivation and computation. Li et al. [16] deal with a set of positive great reduce the complexity of theoretical derivation and conservatism caused by dealing with the functional derivatives, this paper attempts to study the robust stability problem of uncertain systems with interval time-varying delay by constructing a novel LKF and realizing less conservative integral inequalities. The main contributions of this paper include the following:

In this paper, the robust stability criterion is proposed based on the time-delay segmentation method. Specifically, the time-delay interval is divided into N equal parts. Then, a new LKF with quaduple integral term is constructed for different subintervals.

The constructed LKF is augmented with single integral terms and multiple integrals terms, which can make more connections among different vectors and then eliminate the redundant conservatism arising from estimating the interval time-varying delay. Moreover, in addition to the single integral, the double integral, and the triple integral, the quaduple integral is used as a term to construct the integral functional, which would make full use of more information about the upper and lower bounds of the time delay existing in the systems. The Wirtinger-based integral inequality and interactive convex combination technique are used to give conclusion in the form of LMIs without any extra parameters.

\( \mathbb{R}^n \) denotes n-dimensional Euclidean space. \( \mathbb{R}^{n \times m} \) denotes the set of all \( n \times m \) real matrices. * denotes symmetric terms in symmetric matrices. \( I \) denotes the identity matrix with proper dimensions. \( M = M^T > 0 \) denotes that \( M \) is a symmetric matrix. \( e_i \) denotes block input matrix with proper dimensions; for instance, \( e_6^T = [0 \quad 0 \quad 0 \quad 0 \quad 1 \quad 0 \quad 0 \quad 0] \).

2. Problem Description

The uncertain linear systems with interval time-varying delay are as follows:

\[
\begin{align*}
\dot{x}(t) &= (A + \Delta A(t))x(t) + (B + \Delta B(t))x(t - h(t)), \\
x(t) &= \varphi(t), & t \in [-h_M, 0],
\end{align*}
\]

where \( x(t) \in \mathbb{R}^n \) is the state vector of the system, \( A \) and \( B \) are system matrices with appropriate dimensions, \( h(t) \) is time-varying delay satisfying \( 0 \leq h_m \leq h(t) \leq h_M \), and \( \Delta A(t) \) and \( \Delta B(t) \) are unknown matrices with time-varying structure uncertainty. When \( \Delta A(t) \) and \( \Delta B(t) \) have norm bounded uncertainty, they can be described as follows:

\[
[\Delta A(t) \Delta B(t)] = DF(t)[E_a \ E_b],
\]

where \( D, E_a, \) and \( E_b \) are known matrices with appropriate dimensions, while \( F(t) \) is an uncertain matrix with measurable elements satisfying \( F(t)^T F(t) \leq 1, \forall t \), in which \( I \) represents the unit matrix of the appropriate dimension. When \( F(t) = 0 \), system (1) becomes a nominal system.
In this paper, assuming that $N$ is a positive integer greater than zero, $h_i (i = 1, 2, \ldots, N + 1)$ are scalars, and the time-delay interval $[h_m, h_M]$ can be averaged as follows:

$$h_m = h_1 < h_2 < h_3 < \cdots < h_N < h_{N+1} = h_M, \quad (3)$$

where $h_m = h_1$; $h_M = h_{N+1}$; then $h_1$ represents the length of subinterval $[h_0, h_{i+1}]$; namely, $h_0 = h_{i+1} - h_1 = (h_M - h_m)/N$.

To facilitate the proof of stability criteria, the following lemmas are summarized as follows:

**Lemma 1** (see [12]). Assuming any positive definite matrix $M = M^T > 0$, scalar $h > 0$, and continuous vector functions $x(t)$: $[0, h] \rightarrow \mathbb{R}^n$, the following inequality is established:

$$-h \int_{t-h}^{t} x^T(s) M x(s) ds \leq - \int_{t-h}^{t} x^T(s) ds M \int_{t-h}^{t} x(s) ds$$

$$- \left( \frac{h^2}{2} \right) \int_{t-h}^{t} \int_{t+\beta}^{t} x^T(s) M x(s) ds d\beta$$

$$\leq - \int_{t-h}^{t} \int_{t+\beta}^{t} x^T(s) ds d\beta M \int_{t-h}^{t} x(s) ds d\beta,$$  

$$- \left( \frac{h^2}{6} \right) \int_{t-h}^{t} \int_{t+\beta}^{t} x^T(s) M x(s) ds d\beta d\beta$$

$$\leq - \int_{t-h}^{t} \int_{t+\beta}^{t} x^T(s) ds d\beta M \int_{t-h}^{t} \int_{t+\beta}^{t} x(s) ds d\beta d\beta. \quad (5)$$

**Lemma 2** (see [17]). Assuming any positive definite matrix $M = M^T > 0$, scalar $h > 0$, and continuous vector functions $x(t)$: $[0, h] \rightarrow \mathbb{R}^n$, the following inequality is established:

$$-h \int_{t-h}^{t} x^T(s) M x(s) ds \leq - [x(t) - x(t - h)]^T$$

$$M [x(t) - x(t - h)] - 3\Theta^T \Theta, \quad (4)$$

where $\Theta = x(t) + x(t - h) - (2/h) \int_{t-h}^{t} x(s) ds$.

**Lemma 3** (see [17]). Assuming any positive definite matrix $M = M^T > 0$, scalars $0 \leq \alpha, \varepsilon \leq 1$, $\alpha = ((h(t) - h_0)/(h_{i+1} - h_1))$, $\varepsilon = ((h(t))^2 - h^2)/(h_{i+1}^2 - h_1^2)$, $h_i \leq h(t) \leq h_{i+1}$, and vector functions $x(t)$: $[0, h] \rightarrow \mathbb{R}^n$, the following inequality is established:

$$-(h_{i+1} - h_i) \int_{t-h_{i+1}}^{t-h_i} x^T(s) M x(s) ds \leq - \zeta^T(t) \left( e_7^T M e_7^T + e_8^T M e_8^T \right) \zeta(t)$$

$$- \alpha \zeta^T(t) e^T_7 M e_7 \zeta(t) - (1 - \alpha) \zeta^T(t) e_8^T M e_8 \zeta(t), \quad (6)$$

$$- \left( \frac{(h_{i+1}^2 - h_i^2)}{2} \right) \int_{t-h_i}^{t-h_{i+1}} \int_{t+\beta}^{t} x^T(s) M x(s) ds d\beta$$

$$\leq - \zeta^T(t) \left( e_9^T M e_{10}^T + e^T_8 M e_8 \right) \zeta(t) - e \zeta^T(t) e_{10}^T M e_{10} \zeta(t) - (1 - e) \zeta^T(t) e_8^T M e_8 \zeta(t),$$

where

$$\zeta^T(t) = x(t)x(t-h(t))x(t-h_1)x(t-h_{i+1}) \int_{t-h_i}^{t-h_{i+1}} x(s) ds \int_{t-h_i}^{t} x(s) ds \int_{t-h_{i+1}}^{t-h_i} x(s) ds \int_{t-h_i}^{t} x(s) ds \int_{t-h_{i+1}}^{t} x(s) ds d\beta$$

$$\cdot \int_{t-h_{i+1}}^{t-h_i} x(s) ds d\beta \int_{h_i}^{t} x(s) ds d\beta \int_{t-h_i}^{t-h_{i+1}} x(s) ds d\beta.$$

$$\int_{t-h_{i+1}}^{t-h_i} x(s) ds d\beta \int_{t-h_i}^{t} x(s) ds d\beta \int_{t-h_{i+1}}^{t} x(s) ds d\beta \int_{t-h_i}^{t-h_{i+1}} x(s) ds d\beta.$$
3. Main Results

In this section, the stability of the system is discussed in two steps. First, the stability criterion of the nominal system is given, and then the stability of the uncertain system is analyzed. The nominal system of system (1) is as follows:

\[
\begin{align*}
\dot{x}(t) &= Ax(t) + Bx(t - h(t)), \\
x(t) &= \varphi(t), & t \in [-h_M, 0].
\end{align*}
\]

(8)

For nominal systems (8), a new quadruple integral term L-K functional containing more time-delay information is constructed in each subinterval. The following conclusions are obtained by combining Lemmas 1–3.

**Theorem 1.** For given scalars \(h_m, h_M, \) and \(\lambda_1, \lambda_2 (\lambda_1 > \lambda_2), \) it is asymptotically stable for the nominal system (8), if there exist positive definite symmetric matrices \(P_i (i = 1, 2, 3, 4, 5), Q_1, Q_2, U_1, U_2, X, R_j (j = 1, 2, 3, 4), \) such that the following linear matrix inequalities (LMIs) hold:

\[
\Phi = (\Phi_{i,j})_{10 \times 10} < 0,
\]

where

\[
\begin{align*}
\Phi_{11} &= P_1 A + A^T P_1 + Q_1 + h_\Delta^2 X_1 + h_\Delta^2 A^T X_2 A - X_2 + h_\Delta^2 X_3 + h_\Delta^2 A^T X_4 A + \left(\frac{h_\Delta^4}{4}\right) R_1 - h_\Delta^2 R_2 \\
&\quad + \left(\frac{h_\Delta^4}{4}\right) R_3 - 2h_\Delta^2 R_4 + \left(\frac{h_\Delta^2 - h_\Delta^2}{4}\right) A^T R_4 A - \left(\frac{h_\Delta^4}{4}\right) U_1 + \left(\frac{h_\Delta^2}{36}\right) A^T U_1 A \\
&\quad + \left(\frac{h_\Delta^2 - h_\Delta^2}{4}\right) U_2 - \left(\frac{h_\Delta^2 - h_\Delta^2}{4}\right) A^T U_2 A,
\end{align*}
\]

\[
\begin{align*}
\Phi_{12} &= P_1 B + h_\Delta^2 A^T X_2 B + h_\Delta^2 A^T X_4 B + \left(\frac{h_\Delta^4}{4}\right) A^T R_2 B \\
&\quad + \left(\frac{h_\Delta^4}{4}\right) A^T R_4 B + \left(\frac{h_\Delta^4}{36}\right) A^T U_1 B + \left(\frac{h_\Delta^2 - h_\Delta^2}{4}\right) A^T U_2 B,
\end{align*}
\]

\[
\begin{align*}
\Phi_{13} &= X, \Phi_{14} = 0, \Phi_{15} = P_2 + h_\Delta R_2, \Phi_{16} = \Phi_{17} = h_\Delta R_4,
\end{align*}
\]

\[
\begin{align*}
\Phi_{18} &= h_\Delta P_4 + \left(h_\Delta^2/2\right) U_1, \Phi_{19} = \Phi_{110} = h_\Delta P_5 + \left(\frac{h_\Delta^2 - h_\Delta^2}{2}\right) U_2,
\end{align*}
\]

\[
\begin{align*}
\Phi_{22} &= h_\Delta^2 B^T X_2 B - 2X_4 + h_\Delta^2 B^T X_4 B + \left(\frac{h_\Delta^4}{4}\right) B^T R_2 B + \left(\frac{h_\Delta^4}{36}\right) B^T U_1 B \\
&\quad + \left(\frac{h_\Delta^4 - h_\Delta^4}{4}\right) B^T R_4 B + \left(\frac{h_\Delta^4 - h_\Delta^4}{36}\right) B^T U_2 B,
\end{align*}
\]

\[
\begin{align*}
\Phi_{23} &= \Phi_{24} = X_4, \Phi_{25} = \Phi_{26} = \Phi_{27} = \Phi_{28} = \Phi_{29} = \Phi_{210} = 0,
\end{align*}
\]

\[
\begin{align*}
\Phi_{33} &= -Q_1 + Q_2 - X_2 - X_4, \Phi_{34} = 0, \Phi_{35} = -P_2, \Phi_{36} = \Phi_{37} = P_3,
\end{align*}
\]

\[
\begin{align*}
\Phi_{38} &= \Phi_{39} = \Phi_{310} = 0, \Phi_{44} = -Q_2 - X_4, \Phi_{45} = 0, \Phi_{46} = \Phi_{47} = -P_3,
\end{align*}
\]

\[
\begin{align*}
\Phi_{48} &= \Phi_{49} = \Phi_{410} = 0, \Phi_{55} = -X_1 - R_2, \Phi_{56} = \Phi_{57} = 0, \Phi_{58} = -P_4,
\end{align*}
\]

\[
\begin{align*}
\Phi_{59} &= \Phi_{510} = 0, \Phi_{66} = -X_3 - R_4, \Phi_{67} = \Phi_{68} = 0, \Phi_{69} = \Phi_{610} = -P_5,
\end{align*}
\]

\[
\begin{align*}
\Phi_{60} &= -X_3 - R_4, \Phi_{78} = 0, \Phi_{79} = \Phi_{210} = -P_5, \Phi_{88} = -R_1 - U_1,
\end{align*}
\]

\[
\begin{align*}
\Phi_{89} &= \Phi_{810} = 0, \Phi_{99} = -R_3 - U_2, \Phi_{910} = U_2, \Phi_{1010} = -R_3 - U_2,
\end{align*}
\]

\[
\begin{align*}
h_\Delta &= h_{i+1} = \frac{(h_M - h_m)}{N}, h_i = h_1 + \frac{(i - 1)(h_M - h_m)}{N}.
\end{align*}
\]
Proof. For the sake of simplicity, Theorem 1 holds when \( h(t) \in [h_2, h_3] \) first; and then Theorem 1 is generalized to be established when \( h(t) \in [h_i, h_{i+1}] (i = 1, 3, \ldots, N) \).

When \( h(t) \in [h_2, h_3] \), the L-K functional is constructed as follows:

\[
V_2 (x(t)) = V_{21} (x(t)) + V_{22} (x(t)) + V_{23} (x(t)) + V_{24} (x(t)) + V_{25} (x(t)),
\]

where

\[
V_{21} (x(t)) = x^T (t) P_1 x(t) + \int_{t-h_2}^{t} x^T (s) ds + \int_{t-h_3}^{t-h_2} x^T (s) ds \int_{t-h_3}^{t} x(s) ds
\]

\[
+ \int_{t-h_3}^{t} x^T (s) ds + \int_{t-h_3}^{t} x^T (s) ds + \int_{t-h_3}^{t} x^T (s) ds + \int_{t-h_3}^{t} x(s) ds d\beta
\]

\[
V_{22} (x(t)) = x^T (t) Q_1 x(t) ds + \int_{t-h_2}^{t} x^T (s) Q_2 x(s) ds,
\]

\[
V_{23} (x(t)) = h_2 \int_{t-h_2}^{t} x^T (s) X_1 x(s) ds + h_2 \int_{t-h_3}^{t} x^T (s) X_2 x(s) ds + (h_3 - h_2) \int_{t-h_3}^{t} x^T (s) X_3 x(s) ds d\beta
\]

\[
V_{24} (x(t)) = \left( \frac{h_3^2}{2} \right) \int_{t-h_3}^{t} x^T (s) R_1 x(s) ds + \left( \frac{h_2^2}{2} \right) \int_{t-h_2}^{t} x^T (s) R_2 x(s) ds d\beta
\]

\[
+ \left( \frac{(h_2^2 - h_3^2)}{2} \right) \int_{t-h_3}^{t} x^T (s) R_3 x(s) ds d\beta
\]

The derivative of L-K functional \( V(t) \) along the nominal system (8) is calculated as follows:

\[
\dot{V}_2 (t) = \dot{V}_{21} (t) + \dot{V}_{22} (t) + \dot{V}_{23} (t) + \dot{V}_{24} (t) + \dot{V}_{25} (t),
\]
where

\[
\dot{V}_{21}(t) = 2x^T(t)A^TP_1x(t) + 2x^T(t - h(t))B^TP_1x(t) + 2x^T(t)P_2 \int_{t-h_2}^{t} x(s)ds \\
- 2x^T(t - h_2)P_2 \int_{t-h_2}^{t} x(s)ds + 2x^T(t - h_2)P_3 \int_{t-h_2}^{t} x(s)ds \\
- 2x^T(t - h_3)P_3 \int_{t-h_3}^{t} x(s)ds - 2 \int_{t-h_2}^{t} x^T(s)dsP_4 \int_{t-h_2}^{t} x(s)dsd\beta \\
+ 2h_2x^T(t)P_4 \int_{t-h_2}^{t} x(s)dsd\beta + 2(h_3 - h_2)x^T(t)P_5 \int_{t-h_2}^{t} x(s)dsd\beta \\
- 2 \int_{t-h_3}^{t} x^T(s)dsP_5 \int_{t-h_3}^{t} x(s)dsd\beta,
\]

\[
\dot{V}_{22}(t) = x^T(t)Q_1x(t) - x^T(t - h_2)Q_1x(t - h_2) + x^T(t - h_2)Q_2x(t - h_2) - x^T(t - h_3)Q_2x(t - h_3),
\]

\[
\dot{V}_{23}(t) = h_3^2x^T(t)X_1x(t) - h_2 \int_{t-h_2}^{t} x^T(s)X_1x(s)ds - h_2 \int_{t-h_2}^{t} x^T(s)X_2x(s)ds \\
+ h_2^2x^T(t)X_2x(t) + (h_3 - h_2)^2x^T(t)X_3x(t) + (h_3 - h_2)^2x^T(t)X_4x(t) \\
- (h_3 - h_2) \int_{t-h_3}^{t} x^T(s)X_3x(s)ds - (h_3 - h_2) \int_{t-h_3}^{t} x^T(s)X_4x(s)ds, \\
\]

\[
\dot{V}_{24}(t) = \left(\frac{h_3^4}{4}\right)x^T(t)R_1x(t) - \left(\frac{h_2^4}{2}\right) \int_{t-h_2}^{t} x^T(s)R_1x(s)dsd\beta \\
+ \left(\frac{h_3^4}{4}\right)x^T(t)R_2x(t) - \left(\frac{h_2^4}{2}\right) \int_{t-h_2}^{t} x^T(s)R_2x(s)dsd\beta \\
+ \left(\frac{(h_3^2 - h_2^2)^2}{4}\right)x^T(t)R_3x(t) - \left(\frac{(h_3^2 - h_2^2)^2}{2}\right) \int_{t-h_3}^{t} x^T(s)R_3x(s)dsd\beta \\
+ \left(\frac{(h_3^2 - h_2^2)^2}{4}\right)x^T(t)R_4x(t) - \left(\frac{(h_3^2 - h_2^2)^2}{2}\right) \int_{t-h_3}^{t} x^T(s)R_4x(s)dsd\beta,
\]

\[
\dot{V}_{25}(t) = \left(\frac{h_3^6}{36}\right)x^T(t)U_1x(t) + \left(\frac{(h_3^2 - h_2^2)^2}{36}\right)x^T(t)U_2x(t) \\
- \left(\frac{h_3^5}{6}\right) \int_{t-h_2}^{t} x^T(s)U_1x(s)dsd\beta - \left(\frac{(h_3^2 - h_2^2)^2}{6}\right) \int_{t-h_2}^{t} x^T(s)U_2x(s)dsd\beta.
\]
From Lemmas 1 and 2, we can obtain the following:

\[
-h_2 \int_{t-h_2}^{t} x^T(s) x(s) ds \leq -\zeta^T(t) e_x X_1 e_T \zeta(t),
\]

\[
-h_2 \int_{t-h_2}^{t} x^T(s) \dot{x}(s) ds \leq -\zeta^T(t) (e_1 - e_2) X_2 (e_1^T - e_2^T) \zeta(t) - 3 \zeta^T(t) \left( e_1 + e_3 - \left( \frac{2}{h_2} \right) e_2 \right) X_2 (e_1^T - e_3^T) \zeta(t),
\]

where \( \zeta(t) \) is consistent with \( i = 2 \) in Lemma 3.

From Lemma 3, we can obtain the following:

\[
-(h_3 - h_2) \int_{t-h_2}^{t} x^T(s) x(s) ds \leq -\zeta^T(t) e_x X_1 e_T \zeta(t) - \zeta^T(t) (e_3 - e_2) X_4 (e_3^T - e_2^T) \zeta(t) - (1 - \alpha) \zeta^T(t) e_x X_3 e_T \zeta(t).
\]

Similarly, according to Lemma 3, we can obtain the following:

\[
-(h_3 - h_2) \int_{t-h_2}^{t} x^T(s) \dot{x}(s) ds \leq -\zeta^T(t) (e_3 - e_4) X_4 (e_3^T - e_4^T) \zeta(t) - (1 - \alpha) \zeta^T(t) (e_3 - e_2) X_4 (e_3^T - e_2^T) \zeta(t)
\]

\[
- \left( \frac{h_2^2}{2} \right) \int_{-h_2}^{0} \int_{t+\beta}^{t} x^T(s) R_1 x(s) ds d\beta \leq -\zeta^T(t) e_x R_1 e_T \zeta(t),
\]

\[
- \left( \frac{h_3^2 - h_2^2}{2} \right) \int_{-h_2}^{0} \int_{t+\beta}^{t} x^T(s) R_2 \dot{x}(s) ds d\beta \leq -\zeta^T(t) (h_2 e_1 - e_3) R_4 (h_2 e_1^T - e_3^T) \zeta(t),
\]

\[
- \left( \frac{h_3^2 - h_2^2}{2} \right) \int_{-h_2}^{0} \int_{t+\beta}^{t} x^T(s) R_3 x(s) ds d\beta \leq -\zeta^T(t) e_1 R_4 e_1^T \zeta(t) - \zeta^T(t) e_x R_3 e_T \zeta(t) - (1 - \varepsilon) \zeta^T(t) e_x R_3 e_T \zeta(t) - \zeta^T(t) (h_2 e_1 - e_3) R_4 (h_2 e_1^T - e_3^T) \zeta(t)
\]

\[
- \left( \frac{h_3^2 - h_2^2}{2} \right) \int_{-h_2}^{0} \int_{t+\beta}^{t} x^T(s) R_4 \dot{x}(s) ds d\beta \leq -\zeta^T(t) (h_3 - h_2) e_1 - e_2) R_4 ((h_3 - h_2) e_1^T - e_2^T) \zeta(t) - \zeta^T(t) (h_3 - h_2) e_1 - e_2) R_4 ((h_3 - h_2) e_1^T - e_2^T) \zeta(t) - \zeta^T(t) (h_3 - h_2) e_1 - e_2) R_4 ((h_3 - h_2) e_1^T - e_2^T) \zeta(t)
\]

\[
- \left( \frac{h_3^2 - h_2^2}{2} \right) \int_{-h_2}^{0} \int_{t+\beta}^{t} x^T(s) R_4 \dot{x}(s) ds d\beta \leq -\zeta^T(t) (h_3 - h_2) e_1 - e_2) R_4 ((h_3 - h_2) e_1^T - e_2^T) \zeta(t) - \zeta^T(t) (h_3 - h_2) e_1 - e_2) R_4 ((h_3 - h_2) e_1^T - e_2^T) \zeta(t) - \zeta^T(t) (h_3 - h_2) e_1 - e_2) R_4 ((h_3 - h_2) e_1^T - e_2^T) \zeta(t)
\]

\[
- \left( \frac{h_3^2 - h_2^2}{2} \right) \int_{-h_2}^{0} \int_{t+\beta}^{t} x^T(s) R_4 \dot{x}(s) ds d\beta \leq -\zeta^T(t) (h_3 - h_2) e_1 - e_2) R_4 ((h_3 - h_2) e_1^T - e_2^T) \zeta(t) - \zeta^T(t) (h_3 - h_2) e_1 - e_2) R_4 ((h_3 - h_2) e_1^T - e_2^T) \zeta(t) - \zeta^T(t) (h_3 - h_2) e_1 - e_2) R_4 ((h_3 - h_2) e_1^T - e_2^T) \zeta(t)
\]
expressed as follows:

\[ V_2(x(t)) \leq \zeta^T(t) \left[ a\Gamma_1 + (1-a)\Gamma_2 + \varepsilon\Gamma_3 + (1-\varepsilon)\Gamma_4 \right] \zeta(t), \]

Substituting (15) ~ (24) into (13), \( V_2(x(t)) \) can be expressed as follows:

\[ V_2(x(t)) \leq \zeta^T(t) \left[ a\Gamma_1 + (1-a)\Gamma_2 + \varepsilon\Gamma_3 + (1-\varepsilon)\Gamma_4 \right] \zeta(t), \]

where

\[
\begin{align*}
\Gamma_{11} &= \left( \Phi_2 - e_7X_3e_7^T - (e_2 - e_4)X_4(e_2^T - e_4^T), \\
\Gamma_{12} &= \left( \Phi_2 - e_6X_3e_6^T - (e_3 - e_4)X_4(e_3^T - e_4^T), \\
\Gamma_{13} &= \left( \Phi_2 - e_{10}R_3e_{10}^T - ((h_{i+1} - h_i)e_1 - e_7)R_i((h_{i+1} - h_i)e_1 - e_7^T), \\
\Gamma_{14} &= \left( \Phi_2 - e_9R_3e_9^T - ((h_{i+1} - h_i)e_1 - e_6)R_i((h_{i+1} - h_i)e_1 - e_6^T). \\
\end{align*}
\]

For \( 0 \leq a, \varepsilon \leq 1 \), according to convex combination technique, the following inequality is established:

\[
\begin{align*}
& a(\Gamma_1 + \lambda_1I) + (1-a)(\Gamma_2 + \lambda_1I) < 0, \\
& \varepsilon(\Gamma_3 + \lambda_2I) + (1-\varepsilon)(\Gamma_4 + \lambda_2I) < 0. \\
\end{align*}
\]

Namely,

\[
\begin{align*}
& a\Gamma_1 + (1-a)\Gamma_2 < -\lambda_1I, \\
& \varepsilon\Gamma_3 + (1-\varepsilon)\Gamma_4 < -\lambda_2I. \\
\end{align*}
\]

As a result of \( \lambda_1 > \lambda_2 \), combining (28) and (29), the following formula is available:

\[
\begin{align*}
& a\Gamma_1 + (1-a)\Gamma_2 + \varepsilon\Gamma_3 + (1-\varepsilon)\Gamma_4 < (\lambda_2 - \lambda_1)I < 0. \\
\end{align*}
\]

If \( a\Gamma_1 + (1-a)\Gamma_2 + \varepsilon\Gamma_3 + (1-\varepsilon)\Gamma_4 < 0 \), according to L-K stability theorem, there exists a sufficient small positive number \( \delta_2 \) for \( V_2(t) < -\delta_2\|x(t)\|^2 \) to hold, and then the nominal system (8) is asymptotically stable.

Without losing generality, when \( h(t) \in [h_i, h_{i+1}] (i = 1, 3, \ldots, N) \), the L-K function is constructed as follows:

\[
\begin{align*}
V_i(x(t)) &= V_{i1}(x(t)) + V_{i2}(x(t)) + V_{i3}(x(t)) + V_{i4}(x(t)) + V_{i5}(x(t)), \\
\end{align*}
\]

where
\[ V_{i1}(x(t)) = x^T(t)P_1x(t) + \int_{t-h_i}^{t} x^T(s)dsP_2 \int_{t-h_i}^{t} dx(s)ds + \int_{t-h_i}^{t} x^T(s)dsP_3 \int_{t-h_i}^{t} x(s)ds + \int_{t-h_i}^{t} x^T(s)dsP_4 \int_{t-h_i}^{t} x(s)dsd\beta \]

\[ + \int_{t-h_i}^{t} x^T(s)dsP_5 \int_{t-h_i}^{t} x(s)dsd\beta, \]

\[ V_{i2}(x(t)) = \int_{t-h_i}^{t} x^T(s)Q_1x(s)ds + \int_{t-h_i}^{t} x^T(s)Q_2x(s)ds, \]

\[ V_{i3}(x(t)) = h_1 \int_{-h_i}^{0} \int_{t-h_i}^{t} x^T(s)X_1x(s)dsd\beta + h_1 \int_{-h_i}^{0} \int_{t-h_i}^{t} x^T(s)X_2\dot{x}(s)dsd\beta \]

\[ + (h_{i+1} - h_i) \int_{h_i}^{h_i} \int_{t-h_i}^{t} x^T(s)X_3x(s)dsd\beta + (h_{i+1} - h_i) \int_{h_i}^{h_i} \int_{t-h_i}^{t} x^T(s)X_4\dot{x}(s)dsd\beta, \]

\[ V_{i4}(x(t)) = \left( \frac{h_i^2}{2} \right) \int_{-h_i}^{0} \int_{t-h_i}^{t} x^T(s)R_1x(s)dsd\lambda d\beta + \left( \frac{h_i^2}{2} \right) \int_{-h_i}^{0} \int_{t-h_i}^{t} x^T(s)R_2\dot{x}(s)dsd\lambda d\beta \]

\[ + \left( \frac{h_{i+1}^2 - h_i^2}{2} \right) \int_{h_i}^{h_i} \int_{t-h_i}^{t} x^T(s)R_3x(s)dsd\lambda d\beta + \left( \frac{h_{i+1}^2 - h_i^2}{2} \right) \int_{h_i}^{h_i} \int_{t-h_i}^{t} x^T(s)R_4\dot{x}(s)dsd\lambda d\beta, \]

\[ V_{i5}(x(t)) = \left( \frac{h_i^3}{6} \right) \int_{-h_i}^{0} \int_{t-h_i}^{t} \int_{t-\phi}^{t} x^T(s)U_1x(s)dsd\phi d\lambda d\beta + \left( \frac{h_{i+1}^3 - h_i^3}{6} \right) \int_{h_i}^{h_i} \int_{t-h_i}^{t} \int_{t-\phi}^{t} x^T(s)U_2\dot{x}(s)dsd\phi d\lambda d\beta, \]

where the definition of \( \zeta(t) \) is the same as that in Lemma 3. \( P_i (i = 1, 2, 3, 4, 5), Q_1, Q_2, Q_3, U_1, U_2, X_1, X_2, \) and \( R_j (j = 1, 2, 3, 4) \) are the matrices defined in the same formula (9). The same method is available. The following conclusions can be reached by the same method:

\[ \dot{V_i}(x(t)) \leq \zeta^T(t) \Gamma_i \zeta(t), \]

where

\[ \Gamma_i = \left( \frac{\Phi_0}{2} - \epsilon_2 X_4 \epsilon_7^T - (\epsilon_2 - \epsilon_4) X_4 (\epsilon_2^T - \epsilon_4^T), \right) \]

\[ \Gamma_{i2} = \left( \frac{\Phi_0}{2} - \epsilon_2 X_4 \epsilon_6^T - (\epsilon_3 - \epsilon_2) X_4 (\epsilon_3^T - \epsilon_2^T), \right) \]

\[ \Gamma_{i3} = \left( \frac{\Phi_0}{2} - \epsilon_{10} R_3 \epsilon_{10}^T - ((h_{i+1} - h_i) e_1 - \epsilon_2) R_4 ((h_{i+1} - h_i) e_{1}^T - \epsilon_2^T), \right) \]

\[ \Gamma_{i4} = \left( \frac{\Phi_0}{2} - \epsilon_{30} R_3 \epsilon_{30}^T - ((h_{i+1} - h_i) e_1 - \epsilon_0) R_4 ((h_{i+1} - h_i) e_{1}^T - \epsilon_0^T). \right) \]

Remark 1. Firstly, different from [15], in which the delay range was divided into two equidistant subintervals, new LKF comprising quadruple-integral term and quadratic forms of double-integral term was constructed. In this paper, for each subinterval, the time-delay interval is divided into N
equal parts by using the method of time-delay partitioning. A new LKF with four integral terms is designed for each partitioned interval, and the quadratic form of double integral is introduced, such as \( \int_0^\infty x^T(s)dsd\beta \). Although the double integral functional term \( \int_0^- h(t) x^T(s)dsd\beta \) is also used in [1, 10], it is not introduced into the definition of augmented vector. Secondly, the triple integral functional term integrand used in the new LKF contains the state vector \( x \), and the lower bound information of the delay interval is introduced. Thanks to the coexistence of the quadratic integral functional term and the quadratic term \( \int x^T(s)dsd\beta \), the conservativeness of the stability conclusion is significantly reduced.

**Remark 2.** In formula (9), the new stability criterion does not involve redundant free-weight matrices but skillfully uses Wirtinger-based integral inequality to define the cross terms generated by LKF derivatives and uses a few free matrices to represent the relationship between the relevant terms. Therefore, the complexity of theoretical derivation and computation is reduced, and the conservativeness of conclusions is reduced.

**Theorem 2.** For the scalars \( h_m, h_M \), and \( \mu, \lambda_1, \lambda_2 \) \( (\lambda_1 > \lambda_2) \), it is asymptotically stable for the nominal system (8), if there exist positive definite symmetric matrices \( P_i (i = 1, 2, 3, 4, 5) \), \( Q_1, Q_2, Q_3, U_1, U_2, X_j \), and \( R_j \) \( (j = 1, 2, 3, 4) \), such that the following LMIs hold:

\[
\Phi = (\Phi_{11})_{10\times 10} > 0, \tag{35}
\]

where \( \Phi_{11} = \Phi_{11} + Q_3 \) and \( \Phi_{22} = \Phi_{22} - \mu Q_3 \); other items in \( \Phi \) are defined the same as in \( \Phi \) Theorem 1.

Next, the robust stability of uncertain systems with interval time-varying delays (1) is considered.

**Theorem 3.** For the scalars \( 0 < h_m < h_M \) and \( \mu, \lambda_1, \lambda_2 \) \( (\lambda_1 > \lambda_2) \), it is asymptotically stable for the uncertain system (1), if there exist positive definite symmetric matrices \( P_i (i = 1, 2, 3, 4, 5) \), \( Q_1, Q_2, Q_3, U_1, U_2, X_j \), and \( R_j \) \( (j = 1, 2, 3, 4) \), the scalar \( \delta > 0 \), and the free matrices with suitable dimension \( T_1, T_2 \), such that the following LMIs hold:

\[
\begin{bmatrix}
\Phi & \Gamma_1 D T_1^T \\
\ast & -\delta I & 0 \\
\ast & \ast & -\delta I
\end{bmatrix} < 0, \tag{36}
\]

where

\[
\begin{align*}
\Gamma_1 &= \begin{bmatrix}
T_1^T & 0 & 0 & 0 & 0 & T_2^T
\end{bmatrix}, \\
\Gamma_2 &= \begin{bmatrix}
E_a & 0 & E_b & 0 & 0 & 0 & 0
\end{bmatrix}.
\end{align*}
\]

**Remark 4.** For the uncertain system (1), \( A \) and \( B \) in equation (9) are replaced by \( A + \Delta A \) and \( B + \Delta B \), respectively. According to the proof of Theorem 1, the asymptotic stability of system (1) is obtained. This fulfills the proof.

### 4. Numerical Examples

The following three numerical examples are used to compare the results of the existing literature with the method proposed in this paper. MADB (Maximum Allowable Delay Bound) is defined as the upper bound of the maximum allowable delay to ensure the stability of the system, and it is the most common criterion to compare the conservativeness of the stability conclusions of time-delay systems.

**Example 1.** First consider the following closed-loop control systems with interval time-varying delays:

\[
\dot{x}(t) = \begin{bmatrix}
0 & 1 \\
-1 & -2
\end{bmatrix} x(t) + \begin{bmatrix}
0 & 0 \\
1 & 1
\end{bmatrix} x(t - h(t)). \tag{38}
\]

For given \( h_m \) according to (35) in Theorem 2 and (9) in Theorem 1, Tables 1 and 2 give corresponding MADB from two aspects, \( \mu = 0.3 \) and \( \mu = \alpha \), respectively. It can be clearly seen from Tables 1 and 2 that the method proposed in this paper is obviously better than the conclusion in the existing literature.

To verify the validity of the results, given \( \mu = 0.3, h_m = 1 \), and \( h_M = 3.0796 \) and given initial condition \( x(t) = [2 -2]^T \), the state response curve of \( x(t) \) is shown in Figure 1. It can be seen that the state trajectory of the above-mentioned system can quickly reach a stable state under the action of the obtained MADB, which further verifies the correctness of the proposed stability criterion.

**Example 2.** Uncertain systems with interval time-varying delays are considered:

\[
\dot{x}(t) = \begin{bmatrix}
-2 + \lambda_1 & 0 \\
0 & -1 + \lambda_2
\end{bmatrix} x(t) \\
+ \begin{bmatrix}
-1 + \lambda_3 & 0 \\
1 & -1 + \lambda_4
\end{bmatrix} x(t - h(t)), \tag{39}
\]

where \( \lambda_1, \lambda_2, \lambda_3, \lambda_4 \) are unknown parameters satisfying \( |\lambda_1| \leq 1.6, |\lambda_2| \leq 0.05, |\lambda_3| \leq 0.1, |\lambda_4| \leq 0.3 \).

For given \( h_m \) according to (36) in Theorem 3, Table 3 gives corresponding MADB in the simulation. From the comparison results, it can be seen that, for this example, this method improves the conclusions of the existing literature.

Given the initial condition \( x(t) = [0.1 \ 0.2]^T \), the state response curve of \( x(t) \) is shown in Figure 2 when the constant of time-varying delay \( h(t) \) is 1.4723. When \( h(t) \) takes variable \( 1.51 + 0.51 \sin t \), the state response curve of \( x(t) \) is shown in Figure 3. It can be seen that \( x(t) \) can quickly reach a stable state under the action of the nonlinear disturbance and the obtained MADB, thus verifying the correctness of the proposed stability criterion.
Example 3. Consider another uncertain system with interval time-varying delays. The system parameters are as follows:

\[
A = \begin{bmatrix}
-0.4 & 0 \\
0 & -1
\end{bmatrix},
\]

\[
B = \begin{bmatrix}
-0.9 & 0 \\
-1 & -0.7
\end{bmatrix},
\]

\[
D = \begin{bmatrix}
1 & 0 \\
0 & 1
\end{bmatrix},
\]

\[
E_a = E_b = \begin{bmatrix}
0.2 & 0 \\
0 & 0.2
\end{bmatrix}.
\]

Similarly, according to (36) in Theorem 3, for given \( h_m \) and \( \mu = \text{any} \), Table 4 gives corresponding MADB in the simulation. As can be seen from Table 4, the robust stability theorem proposed in this paper enlarges the upper bound of the maximum allowable delay to guarantee the stability of the system. It has lower conservatism.

**Table 1:** In Example 1, MADB is simulated to be obtained for different \( h_m \) and different methods.

| \( \mu \) | Method | \( h_m = 1 \) | \( h_m = 2 \) | \( h_m = 3 \) |
| --- | --- | --- | --- | --- |
| 0.3 | Literature [24] | 2.4042 | 2.5870 | 3.4766 |
|  | Literature [18] | 2.4328 | 2.6322 | —— |
|  | Literature [11] (\( N = 2 \)) | 2.5278 | 3.0744 | 3.9136 |
|  | Literature [11] (\( N = 3 \)) | 2.7368 | 3.4836 | 4.2857 |
|  | Literature [25] | 3.16 | 3.50 | 4.32 |
|  | Theorem 2 | 3.0796 | 3.9064 | 4.4152 |

**Table 2:** In Example 1, when \( \mu = \text{any} \), MADB is simulated to be obtained for different \( h_m \) and different methods.

| Method | \( h_m = 0.3 \) | \( h_m = 0.5 \) | \( h_m = 0.8 \) |
| --- | --- | --- | --- |
| Literature [13] (\( N = 2 \)) | 1.1677 | 1.3078 | 1.5333 |
| Literature [13] (\( N = 4 \)) | 1.2043 | 1.3429 | 1.5633 |
| Literature [18] | 1.3531 | 1.4663 | 1.6592 |
| Literature [26] | 1.4347 | 1.5336 | 1.7140 |
| Literature [14] | 1.6837 | 1.8120 | 2.0209 |
| Literature [27] | 1.78 | 1.81 | 1.90 |
| Theorem 1 | 1.9236 | 2.1384 | 2.2473 |

**Table 3:** In Example 2, MADB is simulated to be obtained for different \( h_m \) and different methods.

| Method | \( h_m = 0.2 \) | \( h_m = 0.4 \) | \( h_m = 0.6 \) |
| --- | --- | --- | --- |
| Literature [10] (\( N = 2 \)) | 1.1337 | 1.1703 | 1.2123 |
| Literature [28] (\( N = 2 \)) | 1.1783 | 1.2123 | 1.2527 |
| Literature [28] (\( N = 4 \)) | 1.1871 | 1.2246 | 1.2686 |
| Literature [11] (\( N = 2 \)) | 1.3369 | 1.3571 | 1.3817 |
| Literature [11] (\( N = 3 \)) | 1.3809 | 1.4003 | 1.4216 |
| Theorem 3 | 1.4241 | 1.4413 | 1.4723 |

**Figure 1:** State response curve of \( x(t) \) when \( h(t) = 1.4723 \).
5. Conclusion

In this paper, we study the robust stability of a class of uncertain systems with interval time-varying delays. A new stability criterion based on LMI is proposed by constructing a new LKF containing a generalized term of quadruple integral. In order to improve the computational efficiency and simplify the conclusion, the criterion avoids the use of model transformation and free weight matrix definition techniques. Instead, Wirtinger-based integral inequalities and interactive convex combination techniques with tighter definition techniques are adopted, which make full use of the lower bound information of the delay and obtain a lower conservative conclusion. Finally, numerical simulations show that the proposed criterion enlarges the upper bound of the maximum delay allowed to guarantee the stability of the system and is more competitive than the existing methods.

However, the new stability criterion proposed in this paper mainly focuses on a class of linear systems with norm-bounded uncertainty and interval time-varying delay. How to get the similar conclusion for nonlinear system is another interesting topic and the next work for us; and some related researches are hopeful to supply reference to us [29, 30].

Data Availability

Three numerical examples are used to compare the results of the existing literature with the method proposed in this paper.

Conflicts of Interest

The authors declare that there are no conflicts of interest with respect to the research, authorship, and/or publication of this article.

Acknowledgments

This research was partially supported by the Young Scientists Fund of the National Natural Science Foundation of China (Grant no. 61503391), China Postdoctoral Fund under Grant no. 2017T100770, the Key Laboratory Fund under Grant no. 6142003190204, and the Special Scientific Research Program of Department of Education of Shaanxi Province (Grant no. 20JK0728).

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