BSW process of the slowly evaporating charged black hole

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Abstract

In this paper, we study the BSW process of the slowly evaporating charged black hole. It can be found that the BSW process will also arise near black hole horizon when the evaporation of charged black hole is very slow. But now the background black hole does not have to be an extremal black hole, and it will be approximately an extremal black hole unless it is nearly a huge stationary black hole.

1 Introduction

Bañados, Silk and West (BSW) process was first pointed out in [1] that a rotating black hole may accelerate a particle to arbitrarily high energy when two particles collide near the horizon. As a remarkable way to extract energy from blackholes, it evokes great interest and argument [1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31].

Soon after the article [1] appeared, BSW process was extended to the charged black hole [24] that particle accelerator may appear near the horizon of a charged black hole without rotating. Not as the rotating black holes, only few charged black holes have been studied [22, 23, 24, 25]. It may be an important reason that a charged black hole will reduce to a stationary uncharged black hole rapidly.

Because of the existence of time variable, until now the BSW process of the evaporating charged black hole has not been studied in literatures. In this paper we will make a try to discuss this problem. For the difficulty of time variable, we will avoid studying the fast evaporating black hole, but turn to discuss the situation that the evaporation of the charged black hole is very slow. [30] suggested that the extremal Kerr black hole is linearly unstable in BSW process. In other words, if one of the test particles with an arbitrarily small mass which can be described by a linear perturbation has the fine-tuned angular momentum (or charge) such that the BSW process works well, the center of mass (CM) energy will deviate perturbed state and be magnified till as large as the mass energy of the background rotating black hole (or the charged black hole). The occurrence of this linearly instability must satisfy two conditions:

1. The black hole is extremal.
2. The particles have the correct angular
momentums (or charges). According to the above ideas, we describe the decay of the charge as a small perturbation if the evaporation of the background black hole is very slow. So we can deal with the collision process of two particles by the usual method. Under this ansatz the BSW process will occur just when the perturbation is invalid, i.e., the black hole is linearly unstable, if the energy of the test particles matches a correct value in the collision process.

This paper is organized as follows. The general geodesic motion and the CM energy near the horizon of the slowly evaporating charged black hole are given respectively in the following two sections. In section 4 we discuss the classification of critical particles in BSW process. The last section is the conclusion of our paper.

2 general geodesic motion

The line element of the charged evaporating black hole is

\[ ds^2 = -e^{2\psi} f(r) dv^2 + 2e^{\psi} dv dr + r^2 (d\theta^2 + \sin^2 \theta d\phi^2), \]

(1)

where

\[ f(r) = 1 - \frac{2M}{r} + \frac{Q^2}{r^2}. \]

(2)

Here black holes mass \( M = M(r) \), charge \( Q = Q(r) \) and \( \psi = \psi(r) \) are the hidefunction of \( v \). The event horizon of the black hole is

\[ 2e^{-\psi} \dot{r} = f(r), \]

(3)

where \( \dot{r} = \frac{\partial}{\partial v}, \dot{r} = \frac{\partial}{\partial r} \).

The Hamiltonian for the geodesic motion is

\[ H[x^\alpha, p_\beta] = \frac{1}{2} g^{\mu\nu} p_\mu p_\nu, \]

(4)

where \( p_\mu \) is the conjugate momentum to \( x^\mu \). Let \( S = S(\lambda, x^\alpha) \) be the action of the parameter \( \lambda \) and coordinates \( x^\alpha \). The conjugate momentum \( p_\alpha \) is described by \( p_\alpha = \partial S/\partial x^\alpha - qA \). The Hamilton-Jacobi equation reads

\[ -\frac{\partial S}{\partial \lambda} = \frac{1}{2} g^{\mu\nu} \left( \frac{\partial S}{\partial x^\mu} - qA_\mu \right) \left( \frac{\partial S}{\partial x^\nu} - qA_\nu \right), \]

(5)

where \( dA_\nu = -Q/r dv \).

For charged evaporating black hole, the action can be written as

\[ S = \frac{1}{2} m^2 \lambda - Ev + L\phi + S_r(r) + S_\theta(\theta), \]

(6)

where \( m, E \) and \( L \) are the rest mass, energy and angular momentum respectively. If the evaporation of black hole is very slow, \( f \to 0 \)(i.e. \( \dot{r} \to 0 \)), we can separate variables by usual method. Substituting Eq. (6) into Eq. (5)

\[ - m^2 = 2e^{-\psi} \left( -E + \frac{Q}{r} \right) \frac{\partial S}{\partial r} + f \left( \frac{\partial S}{\partial v} \right)^2 + \frac{1}{r^2} \left( \frac{\partial S}{\partial \theta} \right)^2 + \frac{1}{r^2 \sin^2 \theta} L^2, \]

(7)
then separating variables and defining a constant $K$

$$-m^2 r^2 - 2e^{-\psi} r^2 \left( -E + qQr \right) \frac{\partial S}{\partial r} + fr^2 \left( \frac{\partial S}{\partial r} \right)^2 = K = \left( \frac{\partial S}{\partial \theta} \right)^2 + \frac{1}{\sin^2 \theta} L^2,$$

the Hamilton-Jacobi function becomes

$$S = \frac{1}{2} m^2 \lambda - E\nu + L\phi + \delta_r \int Rdr + \delta_\theta \int^0 \sqrt{\Theta}d\theta,$$

where

$$R = \frac{\partial S}{\partial r} = \frac{-e^{-\psi} \left( -E + qQr \right) \pm \sqrt{e^{-2\psi} \left( -E + qQr \right)^2 - f \left( m^2 + \frac{K}{r^2} \right)}}{f},$$

$$\Theta = \left( \frac{dS_\theta}{d\theta} \right)^2 = \frac{K - \frac{1}{\sin^2 \theta} L^2}{f}.$$

With the aid of $p^\alpha = g^{\alpha\beta} p_\beta$, from Hamilton-Jacobi Eq.(7), we can obtain the motion equations

$$\frac{dv}{d\lambda} = e^{-\psi} \delta_r R,$$

$$\frac{dr}{d\lambda} = e^{-\psi} \left( -E + qQr \right) + f \delta_r R,$$

$$\frac{d\theta}{d\lambda} = \frac{1}{r^2 \sin^2 \theta} \sqrt{\Theta},$$

$$\frac{d\phi}{d\lambda} = \frac{1}{r^2 \sin^2 \theta} L,$$

and the effective potential is

$$\sqrt{\nu} = e^{-\psi} \left( -E + qQr \right) + f R.$$

### 3 CM energy near the horizon

The four momenta of two particles with rest masses $m_1$ and $m_2$ is $p_{(i)}^\alpha = m_{(i)} u_{(i)}^\alpha$, and the CM energy $E_{CM}$ of the two particles defines

$$E_{CM}^2 = m_1^2 + m_2^2 - 2g^{ab} p_{(1)a} p_{(2)b}.$$

Hence the CM energy of two colliding general geodesic particles in the evaporating charged black hole spacetime is

$$E_{CM}^2 = \left( m_1^2 + m_2^2 - 2\left( e^{-\psi} (-E_1 + q_1Q) \delta_{2r} R_2 + e^{-\psi} (-E_2 + q_2Q) \delta_{1r} R_1 \right) + f \delta_{1r} R_1 \delta_{2r} R_2 + \frac{1}{r^2} \delta_{1\theta} \sqrt{\Theta_1} \delta_{2\theta} \sqrt{\Theta_2} + \frac{1}{r^2 \sin^2 \theta} L_1 L_2 \right).$$
Unlike the stationary black hole, the evaporating black hole needs an additional condition that the evaporation of the black hole is very slow (i.e. \( \dot{r} \rightarrow 0 \)). Thus \( f(r) \rightarrow 0 \) can be seen as a small parameter in the near-horizon limit. The CM energy of two general geodesic particles in the near-horizon limit can be written as

\[
E_{CM}^2 = m_1^2 + m_2^2 - 2\left\{ \frac{E_2r_H - q_2Q}{2(E_1r_H - q_1Q)}\left( m_1^2 + \frac{\mathcal{K}_1}{r_H^2} \right) - \frac{E_1r_H - q_1Q}{2(E_2r_H - q_2Q)}\left( m_2^2 + \frac{\mathcal{K}_2}{r_H^2} \right) \right\} + \frac{f(r)}{r_H^2} \frac{r_H^2}{8} e^{-2\psi} \psi^2 - \frac{1}{r_H^2} \frac{\delta_{1\theta}\sqrt{\Theta_1} + \delta_{2\theta}\sqrt{\Theta_2}}{\sin^2 \theta L_1 L_2}. \]  

We can see that the \( E_{CM} \) in evaporating black hole is the same as the \( E_{CM} \) in stationary black hole \(^{24}\) if we omit the second order approximate term \( f_{evaporate}r_H^2/8e^{-2\psi} \) in evaporating black hole (It is \( f_{stationary}r_H^2/8 \) in stationary black hole). That is to say the evaporation of the charge has a little effect on the CM energy, but we can see it will make a subtle effect on the classification of critical particles from next section.

4 classification of critical particles

We denote

\[
F = e^{-2\psi}(-E + qQr)^2 - f(m^2 + \frac{\mathcal{K}}{r^2}),
\]  

then we have

\[
F' = -f' \left( \frac{\mathcal{K}}{r^2} + m^2 \right) + \frac{2Kf}{r^3} - 2e^{-2\psi} \psi' \left( \frac{qQ}{r} - E \right)^2 - \frac{2qQe^{-2\psi} \left( \frac{qQ}{r} - E \right)}{r^2},
\]

and

\[
F'' = -f'' \left( \frac{\mathcal{K}}{r^2} + m^2 \right) + 4Kf' \frac{1}{r^3} - 6Kf \frac{1}{r^4}
+ \frac{8qQe^{-2\psi} \psi'}{r^2} \left( \frac{qQ}{r} - E \right) - e^{-2\psi} \left( 4\psi'^2 - 2\psi'' \right) \left( \frac{qQ}{r} - E \right)^2
+ e^{-2\psi} \left[ \frac{2q^2Q^2}{r^4} + \frac{4qQ(\frac{qQ}{r} - E)}{r^3} \right],
\]  

where

4
where

\[ f' = -\frac{2M'}{r} + \frac{2M}{r^2} - \frac{2Q^2}{r^3} + \frac{2QQ'}{r^2}, \]

\[ f'' = -\frac{2M''}{r} + \frac{4M'}{r^2} - \frac{4M}{r^3} + \frac{6Q^2}{r^4} - \frac{8QQ'}{r^3} + \frac{2Q'^2 + 2QQ''}{r^2}. \]

Below we will discuss the classification of critical particles by using the idea of [31]:

(1) \( F = 0 \), it is

\[ e^{-2\psi} \left( -E + q\frac{Q}{r} \right)^2 - f \left( m^2 + \frac{K}{r^2} \right) = 0. \]

When \( r \to r_H \), according to Eq. (3), it becomes

\[ -E + q\frac{Q}{r} = 2e^\psi \dot{r} (m^2 + \frac{K}{r^2}). \]

If the evaporation of black hole is very slow, \( f \to 0 \) (i.e. \( \dot{r} \to 0 \)), it becomes

\[ -E + q\frac{Q}{r} = 0. \]

(2) \( F' = 0 \), it is

\[ -f' \left( \frac{K}{r^2} + m^2 \right) + \frac{2Kf}{r^3} - 2e^{-2\psi} \psi' \left( \frac{qQ}{r} - E \right)^2 - \frac{2qQe^{-2\psi} \left( \frac{qQ}{r^2} - E \right)}{r^2} = 0. \]

When \( F = 0 \) and \( r \to r_H \), it becomes

\[ -f' \left( \frac{K}{r^2} + m^2 \right) + \frac{2K^2 e^{-\psi} \dot{r}}{r^3} - 8\psi' \left[ \dot{r} \left( m^2 + \frac{K}{r^2} \right) \right]^2 - \frac{4qQe^{-2\psi} \dot{r} \left( m^2 + \frac{K}{r^2} \right)}{r^2} = 0. \]

If the evaporation of black hole is very slow, \( f \to 0 \) (i.e. \( \dot{r} \to 0 \)), it is

\[ -f' \left( \frac{K}{r^2} + m^2 \right) = 0. \]

From Eq. (8) we know \( K \geq 0 \). If \( m \neq 0 \) and \( m^2 + K/r^2 > 0 \), we have the description

\[ f' = -\frac{2M'}{r} + \frac{2M}{r^2} - \frac{2Q^2}{r^3} + \frac{2QQ'}{r^2} = 0. \]

From this equation we can see that the background black hole does not have to be an extremal black hole, and it will be extremal only if \( m', Q' \to 0 \). In this case it will be nearly a huge stationary black hole.

(3) \( F'' = 0 \), it is
When $F = 0$, $F' = 0$ and $r \to r_H$, its expression becomes very complex. If the evaporation of black hole is very slow $f(r) \to 0$ (i.e. $\dot{r} \to 0$), it becomes

\begin{equation}
-f'' \left( \frac{K}{r^2} + m^2 \right) + e^{-2\psi} \frac{2q^2Q^2}{r^4} = 0.
\end{equation}

In order to simplify the expression, we denote

$$C(r) = -f'' \left( \frac{K}{r^2} + m^2 \right) + e^{-2\psi} \frac{2q^2Q^2}{r^4},$$

According to the idea of the article [31], we summarize our result in the table:

| Class | $F(r)$ at $r = r_H$ | Scenario | Parameter region |
|-------|---------------------|----------|-----------------|
| I     | $F = F' = 0, F'' > 0$ | direct collision | $C(r) > 0$ |
| II    | $F = F' = F'' = 0$ | LSO collision | $C(r) = 0$ |
| III   | $F = F' = 0, F'' < 0$ | multiple scattering | $C(r) < 0$ |
| IV    | $F = 0, F' < 0$ | multiple scattering | $f' > 0$ |

5 conclusion

In this paper, we discuss the BSW of slowly evaporating charged black hole. When the evaporation of black hole is very slow, the CM energy is the same as the Reissner-Nordström black hole near the horizon. But now the black hole may be not extremal, and it may also be a near-extremal black hole unless $M'$ and $Q'$ of the black hole are both very small.

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