Determination of the angle $\gamma$ using multibody $D$ decays in $B^{\pm} \rightarrow DK^{\pm}$

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We describe a method for determining $\gamma$ using $B^{\pm} \rightarrow DK^{\pm}$ decays followed by a multibody $D$ decay. In the talk we focus on $K_S \pi^- \pi^+$ final state, but other modes such as $D \rightarrow K_S K^- K^+$ and $D \rightarrow K_S \pi^- \pi^- \pi^0$ can also be used. The main advantages of the method are that it uses only Cabibbo allowed $D$ decays, and that large strong phases are expected due to the presence of resonances. Since no knowledge about the resonance structure is needed, $\gamma$ can be extracted without any hadronic uncertainty.

1 Basic idea

The theoretically cleanest way of determining the angle

$$\gamma = \arg(-V_{ud} V_{ub}^*/V_{cd} V_{cb}^*), \quad (1)$$

is to utilize the interference between the $b \rightarrow c \bar{u}s$ and $b \rightarrow u \bar{c}s$ decay amplitudes [1–7], see Fig. 1. The salient feature of these transitions is that they involve only distinct quark flavors and therefore do not receive any penguin contributions. In the original idea by Gronau and Wyler (GW) [1] the $B^\pm \rightarrow D_{CP} K^\pm$ decay modes are used, where $D_{CP}$ represents a $D$ meson which decays into a CP eigenstate. The dependence on $\gamma$ arises from the interference between the $B^\pm \rightarrow D^0 K^\pm$ and $B^\pm \rightarrow \bar{D}^0 K^\pm$ decay amplitudes. The main advantage of the GW method is that, in principle, the hadronic parameters can be cleanly extracted from data, by measuring the $B^\pm \rightarrow D^0 K^\pm$ and $B^\pm \rightarrow \bar{D}^0 K^\pm$ decay rates.

![Figure 1](image-url)

**Figure 1.** The dependence on $\gamma$ arises from the interference between the $B^\pm \rightarrow D^0 K^\pm$ and $B^\pm \rightarrow \bar{D}^0 K^\pm$ decay amplitudes.

In practice, however, measuring $\gamma$ in this way is not an easy task. Due to the values of the CKM coefficients and color suppression, the ratio between the two interfering amplitudes, $r_b$ [see Eq. (4)], is expected to be small, of order $10\% \sim 20\%$. This reduces the sensitivity to $\gamma$, which is roughly proportional to the magnitude of the smaller amplitude. It also leads to experimental difficulties in measuring the color suppressed $B^- \rightarrow \bar{D}^0 K^-$ mode (and its charge conjugate) preventing a straightforward application of the GW method [2]. There exist a number of extensions of the original GW proposal which avoid the problem by not relying on the measurement of $B^- \rightarrow \bar{D}^0 K^-$ amplitude [2, 3, 6]. Instead several different decay modes of $D$ mesons such as quasi two-body $D$ decays with one particle a resonance (e.g. $D^0 \rightarrow K^+ \pi^-$ [2, 3]) are used. Since these are really three body decays (for instance in the example mentioned, $K^+ \rightarrow K^0 \pi^+ \pi^0$ or $K^0 \pi^0$) one can pose the following questions:

- Can one use the complete phase space of such three-body $D$ decays for $\gamma$ extraction?

- Is it possible to avoid fits to Breit-Wigner forms in doing the Dalitz plot analysis?

As we show in the following, the answer to both of these questions is positive. The first question was raised already in [3], however, most of the results and applications we present are new. For the sake of concreteness, we concentrate on the $D \rightarrow K_S \pi^- \pi^+$ decay mode, while an extension to a larger set of the decay modes can be found in [8]. The advantage of using the chosen decay chain is threefold. First, one expects large strong phases due to the presence of resonances. Second, only Cabibbo allowed $D$ decay modes are needed. Third, the final state involves only charged particles, which have a higher reconstruction efficiency and lower background than neutrals. The price one has to pay is that a Dalitz plot analysis of the data is needed. We describe how to do the Dalitz plot analysis in a model-independent way, and explore the advantages gained by introducing verifiable model-dependence. The final balance between the advantages and disadvantages depends on yet-to-be-determined hadronic parameters and experimental considerations. Finally, we mention that an equivalent formalism to the one we present below has been independently developed by Atwood and Soni in [9].

$^a$Talk given by J. Zupan, based on [8].
2 Model independent determination of $\gamma$

Let us focus on the following cascade decay \(^2\)

$$B^- \to DK^- \to (K_S \pi^- \pi^+) D K^-,$$  \hspace{1cm} (2)

and define the amplitudes

$$A(B^- \to D^0 K^-) \equiv A_B,$$  \hspace{1cm} (3)

$$A(B^- \to D^+ K^-) \equiv A_B r_B e^{i(\delta_B - \gamma)}.$$  \hspace{1cm} (4)

The same definitions apply to the amplitudes for the CP conjugate cascade $B^+ \to DK^+ \to (K_S \pi^+ \pi^-) D K^+$, with the change of weak phase sign $\gamma \to -\gamma$ in (4). We have set the strong phase of $A_B$ to zero by convention, so that $\delta_B$ is the difference of strong phases between the two amplitudes. For the CKM elements, the usual convention of the weak strong phases has been used. The value of $|A_B|$ is known from the measurement of the $B^- \to D^0 K^-\,\text{decay width}$ using flavor specific decays of $D^0$. The amplitude $A(B^- \to D^0 K^-)$ is color suppressed and cannot be determined from experiment in this way \([2]\). The color suppression together with the experimental values of the ratio of the relevant CKM elements leads to the theoretical expectation $r_B \sim 0.1 - 0.2$ (see recent discussion in \([7]\)).

For the three-body $D$ meson decay we define

$$A_D(s_{12}, s_{13}) \equiv A_{12,13} e^{i\delta_{12,13}} \equiv A(D^0 \to K_S (p_1) \pi^- (p_2) \pi^+(p_3)),$$  \hspace{1cm} (5)

$$A(D^+ \to K_S (p_1) \pi^+(p_2) \pi^-(p_3)),$$

where $s_{ij} = (p_i + p_j)^2$, and $p_1, p_2, p_3$ are the momenta of the $K_S, \pi^-, \pi^+$ respectively. We also set the magnitude $A_{12,13} \geq 0$, such that $\delta_{12,13}$ can vary between 0 and $2\pi$. In the last equality the CP symmetry of the strong interaction together with the fact that the final state is a spin zero state has been used. With the above definitions, the amplitude for the cascade decay is

$$A(B^- \to (K_S \pi^- \pi^-)^D K^-) =$$

$$A_B \mathcal{P}_D (A_D(s_{12}, s_{13}) + r_B e^{i(\delta_B - \gamma)} A_D(s_{13}, s_{12})),$$

where $\mathcal{P}_D$ is the $D$ meson propagator. Next, we write down the expression for the reduced partial decay width

$$d\tilde{\Gamma}(B^- \to (K_S \pi^- \pi^-)^D K^-) =$$

$$\left(\frac{A_{12,13} + r_B A_{13,12}}{4} + 2r_B Re\left[A_D(s_{12}, s_{13}) A_D^*(s_{13}, s_{12}) e^{i(\delta_B - \gamma)}\right]\right) dp,$$  \hspace{1cm} (7)

where $dp$ denotes the phase space variables, and we used the extremely accurate narrow width approximation for the $D$ meson propagator.

\(^2\)In the following discussion we neglect $D^0 - D^+$ mixing, which is a good approximation in the context of the Standard Model, for more details see app. A of \([8]\).
bins are indexed with $i$. The $i$-th bin is obtained by mirroring the $i$-th bin over the axis of symmetry. The variables $c_i, s_i$ of the $i$-th bin are related to the variables of the $i$-th bin by
\begin{equation}
  c_i = c_i, \quad s_i = -s_i, \tag{9}
\end{equation}
while there is no relation between $T_i$ and $T_j$. Note that had one used $12 \leftrightarrow 13$ symmetric bins centered on the symmetry axis, one would have had $s_i = 0$.

Together with the information available from the $B^+$ decay, we arrive at a set of 4k equations
\begin{align}
  \hat{\Gamma}^+_i &\equiv \int d\Gamma(B^+ \to (K_S^0 \pi^- \pi^+)_{2D}K^-) = \\
  &T_i + r_B T_i + 2r_B[\cos(\delta_B - \gamma)c_i + \sin(\delta_B - \gamma)s_i], \tag{10a}
\end{align}
\begin{align}
  \hat{\Gamma}^-_i &\equiv \int d\Gamma(B^- \to (K_S^0 \pi^- \pi^+)_{2D}K^-) = \\
  &T_i + r_B T_i + 2r_B[\cos(\delta_B - \gamma)c_i + \sin(\delta_B - \gamma)s_i], \tag{10b}
\end{align}
\begin{align}
  \hat{\Gamma}^+_i &\equiv \int d\Gamma(B^+ \to (K_S^0 \pi^- \pi^+)_{2D}K^+) = \\
  &T_i + r_B T_i + 2r_B[\cos(\delta_B + \gamma)c_i + \sin(\delta_B + \gamma)s_i], \tag{10c}
\end{align}
\begin{align}
  \hat{\Gamma}^-_i &\equiv \int d\Gamma(B^- \to (K_S^0 \pi^- \pi^+)_{2D}K^+) = \\
  &T_i + r_B T_i + 2r_B[\cos(\delta_B + \gamma)c_i + \sin(\delta_B + \gamma)s_i]. \tag{10d}
\end{align}

These equations are related to each other through $12 \leftrightarrow 13$ and/or $\gamma \leftrightarrow -\gamma$ exchanges. All in all, there are $2k + 3$ unknowns in (10),
\begin{equation}
  c_i, \quad s_i, \quad r_B, \quad \delta_B, \quad \gamma, \tag{11}
\end{equation}
so that the 4k relations (10) are solvable for $k \geq 2$. In other words, a partition of the $D$ meson Dalitz plot to four or more bins allows for the determination of $\gamma$ without hadronic uncertainties. This is our main result.

When $c_i = 0$ or $s_i = 0$ for all $i$, some equations become degenerate and $\gamma$ cannot be extracted. However, due to resonances, we do not expect this to be the case. Degeneracy also occurs if $\delta_B = 0$. In this case, $\gamma$ can still be extracted if some of the $c_i$ and/or $s_i$ are independently measured, as discussed in the following sections.

### 3 Improved Measurement of $c_i$ and $s_i$

So far, we have used the $B$ decay sample to obtain all the unknowns, including $c_i$ and $s_i$, which are parameters of the charm system. We now show that the $c_i$ and $s_i$ can be independently measured at a charm factory [10–12]. This is done by running the machine at the $\psi(3770)$ resonance, which decays into a $D\bar{D}$ pair. If one $D$ meson is detected in a CP eigenstate decay mode, it tags the other $D$ as an eigenstate of the opposite CP eigenvalue. The difference between the two decay widths gives [8]
\begin{equation}
  c_i = \frac{1}{2} \left[ \int d\Gamma(D^+_{12} \to K_S(p)\pi^-(p_2)\pi^+(p_3)) - \int d\Gamma(D^+_{13} \to K_S(p)\pi^-(p_2)\pi^+(p_3)) \right]. \tag{12}
\end{equation}

where we have defined $D^+_{12} = (D^0 + D^0)/\sqrt{2}$. As stated above, obtaining these independent measurements reduces the error in the measurement of $\gamma$ by removing $k$ of the $2k + 3$ unknowns.

In addition, if one of the $D$ mesons decays into a non-CP eigenstate, we are sensitive to the $s_i$ variables as well. Consider for instance a $\psi(3770)$ decaying into a $D\bar{D}$ pair, of which one decays into $K_S\pi^+\pi^-$ and the other decays into some general state $g$. The partial decay width corresponding to the $i$–th bin of the $K_S\pi^+\pi^-$ Dalitz plot and the $j$–th bin of the $g$ final state’s phase space is
\begin{equation}
  \Gamma_{i,j} \propto T_{i,j}^b T_{i,j}^s - 2(c_i c_j + s_i s_j), \tag{13}
\end{equation}
where $T_{i,j}^b, c_{i,j}, s_{i,j}$ are defined as in (8). In particular, if one chooses $g = K_S^0 \pi^- \pi^+$ and $j = i$ (or $j = i$) one measures $s_i^2$. If, on the other hand, $g$ is a CP even (odd) eigenstate, $s_i^2 = 0$, $T_{i,j}^b = \pm c_i$ and equation (13) reduces to (12).

### 4 Assuming Breit-Wigner dependence

If the functional dependence of both the moduli and the phases of the $D^0$ meson decay amplitudes $A_D(s_{12},s_{13})$ were known, then the analysis would be simplified. There would be only three variables, $r_B$, $\delta_B$, and $\gamma$, that need to be fit to the reduced partial decay widths in Eq. (7). A plausible assumption about their forms is that a significant part of the three-body $D^0 \to K_S^0 \pi^- \pi^+$ decay proceeds via resonances. These include decay transitions of the form $D^0 \to K_S^0 \pi^0$ or $D^0 \to K^- (892) \pi^+ \pi^- \to K_S^0 \pi^- \pi^+$, as well as decays through higher resonances, e.g., $f_0(980)$, $f_2(1270)$, $f_0(1370)$ or $K^*_3(1430)$. An important feature is that there exists an overlap region of Cabibbo allowed $D^0 \to K^- (892) \pi^+$, $D^0 \to K_S^0 \pi^0$ decays, where the variation of strong phase will be large, allowing for the extraction of $\gamma$.

The decay amplitude can then be fit to a sum of Breit-Wigner functions and a constant term. Following the notations of Ref. [13] we write
\begin{equation}
  A_D(s_{12},s_{13}) = A(D^0 \to K_S(p_1)\pi^-(p_2)\pi^+(p_3)) = \\
  a_0e^{i\delta_0} + \sum_\tau a_\tau e^{i\delta_\tau} s_\tau(s_{12},s_{13}), \tag{14}
\end{equation}
where the first term corresponds to the non-resonant term and the second to the resonant contributions with \( r \) denoting a specific resonance. The functions \( \phi_i \) are products of Breit-Wigner functions and appropriate Legendre polynomials that account for the fact that \( D \) meson is a spin 0 particle. Explicit expressions can be found in Ref. [13].

One of the strong phases \( \delta_i \) in the ansatz (14) can be put to zero, while others are fit to the experimental data together with the amplitudes \( a_i \). The obtained functional form of \( A_\rho(s_{12}, s_{13}) \) can then be fed to Eq. (7), which is then fit to the Dalitz plot of the \( B^\pm \to (K_S \pi^- \pi^+) p K^\pm \) decay with \( r_B \), \( \delta_B \) and \( \gamma \) left as free parameters.

5 Discussions

The observables \( \hat{\Gamma}^\pm \) defined in (10) can be used to experimentally look for direct CP violation. Explicitly,

\[
d^\pm_{ij} \equiv \hat{\Gamma}^\pm_{ij} - \hat{\Gamma}^\pm_{ji} = 4 r_B \sin \gamma [c_i \sin \delta_B \mp s_i \cos \delta_B],
\]

(15)

Nonzero \( a_{CP} \) requires non-vanishing strong and weak phases. Due to the resonances, we expect the strong phase to be large. Therefore, it may be that direct CP violation can be established in this mode even before the full analysis to measure \( \gamma \) is conducted. With more data, \( \gamma \) can be extracted assuming Breit-Wigner resonances (cf. section 4). Eventually, a model independent extraction of \( \gamma \) can be done (cf. section 2 and 3).

The above proposed method for the model independent measurement of \( \gamma \) involves a four-fold ambiguity in the extracted value. The set of equations (10) is invariant under each of the two discrete transformations

\[
P_\pi \equiv \{ \delta_B \to -\delta_B, \pi \to -\pi, s_i \to -s_i \};
\]

(16)

\[
P_- \equiv \{ \delta_B \to -\delta_B, \gamma \to -\gamma, s_i \to s_i \};
\]

(17)

The discrete transformation \( P_\pi \) is a symmetry of the amplitude (6) and is thus an irreducible uncertainty of the method. It can be lifted if the sign of either \( \cos \delta_B \) or \( \sin \delta_B \) is known. The ambiguity due to \( P_- \) can be resolved if the sign of \( \sin \delta_B \) is known or if the sign of \( s_i \) can be determined in at least some part of the Dalitz plot for instance by fitting a part of the Dalitz plot to Breit-Wigner functions. The \( r_B \) suppression present in the scheme outlined above can be somewhat lifted if the cascade decay \( B^- \to DX^- \to (K_S \pi^- \pi^+) p X^- \) is used [5, 7]. Here \( X^- \) is a multibody hadronic state with an odd number of kaons (for instance \( K^- \pi^- \pi^+, K^- \pi^0 \) or \( K_S \pi^- \pi^0 \)). The same formalism as outlined above applies also to this case with trivial changes [8]. In addition to using different \( B \) modes, statistics may be increased by employing various \( D \) decay modes as well. An interesting possibility are Cabibbo allowed, \( D \to K_S \pi^- \pi^- \pi^0, K^- K^+ K_S \), and Cabibbo suppressed, \( D \to K^- K^+ \pi^0, \pi^- \pi^+ \pi^0, K_S K^+ \pi^- \) decay modes, to which our formalism applies with very minor changes [8].

In conclusion, we have shown that the angle \( \gamma \) can be determined from the cascade decays \( B^- \to K^- (K_S \pi^- \pi^+) p \). The reason for the applicability of the proposed method lies in the presence of resonances in the three-body \( D \) meson decays that provide a necessary variation of both the phase and the magnitude of the decay amplitude across the phase space. The fact that no Cabibbo suppressed \( D \) decay amplitudes are used in the analysis is another advantage of the method and leads to a sensitivity on \( \gamma \) at order \( \mathcal{O}(r_B) \). However, it does involve a Dalitz plot analysis with possibly only parts of the Dalitz plot being practically useful for the extraction of \( \gamma \). In reality, many methods have to be combined in order to achieve the required statistics for a precise determination of \( \gamma \) [4].

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