Numerical modeling of free convection in a partially heated triangular enclosure with chamfer

H H Alaydamee¹*, M A Alomari² and H I Dawood³

¹,³ Department of chemical Engineering, University of Al-Qadisiyah, Al-Qadisiyah, Iraq
² Department of Mechanical Engineering, University of Al-Qadisiyah, Al-Qadisiyah, Iraq

hussein.hantoosh@qu.edu.iq

Abstract
Two-dimensional numerical modelling of water natural convection inside a chamfered triangular cavity, that is being partially heated from below has been investigated in this paper. The walls of the cavity are considered isothermally isolated except a source part of heat which is inserted at the bottom wall while the inclined wall is considered as cold wall. A COMSOL Multiphysics 5.5 has been used to solve the numerical solutions. This paper considered some variables such as Rayleigh number (Ra = 10³ to 10⁶), heating length ratio (HL= 0.2, 0.4, and 0.5) and the ratio of the chamfer’s radius (R= 0.1, 0.2, and 0.25) while all other physical parameters are considered constant such as the length of the cold wall and the adiabatic wall. The results have been validated with previous published work in order to ensure the accuracy of the current coding. The main results observed an increase in streamlines and isotherms with the rise of Rayleigh (Ra) number and the rise of heating length ratio; additionally, the average Nusselt number (Nu_avg) value increases with the rise of the Ra number and the length of the heating source. For example, average Nusselt number increased from 2.0443 at Ra=10³ to 7.2793 at Ra= 10⁶ for (R=0.25 and HL = 0.5).

Keywords: Triangular enclosure; natural convection, numerical simulation, chamfer, Partially heated
Nomenclature

- \( C_p \): Specific heat, J. kg\(^{-1}\).K\(^{-1}\)
- \( g \): Gravitational acceleration, m.s\(^{-2}\)
- \( HL \): Hot wall length
- \( K \): Thermal conductivity, W.m\(^{-1}\).K\(^{-1}\)
- \( L \): Cavity Length, m
- \( \text{Nu}_{\text{avg}} \): Average Nusselt number
- \( P \): Non-dimensional pressure
- \( \text{Pr} \): Prandtl number
- \( R \): Chamfer radius ratio
- \( \text{Ra} \): Rayleigh number
- \( T_h \): Hot wall dimensional temperature, K
- \( T_c \): Cold wall dimensional temperature, K
- \( U \): Non-dimensional velocity component X-direction
- \( V \): Non-dimensional velocity component Y-direction
- \( W \): Non-dimensional Length of the enclosure
- \( X \): Non-dimensional X-coordinates
- \( Y \): Non-dimensional Y-coordinates
- \( \theta \): Dimensionless temperature
- \( \mu \): Dynamic viscosity, kg. m\(^{-1}\).s\(^{-1}\)
- \( \alpha \): Thermal diffusivity, m\(^2\).s\(^{-1}\)
- \( \beta \): Thermal expansion coefficient, K\(^{-1}\)

Introduction

Due to its wide applications around the world, natural convection study areas have attracted numerous researches. A convective heat transfer in closed containers study has a significant importance for several industries such as aerospace, cooling of electronic equipment, airplanes cabin insulation, and wide range of other industries[1-5]. Different geometries have been studied over the last years such as squared[6], trapezoidal[7], rectangular[2], triangular cavity[8-10]..etc. Amir H. Mahmoudi, Ioan Pop, and Mina
Shahi studied different cases of a triangular cavity that is being heated partially with the inclined wall being the cold in two dimensional mode using finite volume technique [3]. They used a wide range of $Ha$, $Ra$, $\Theta$ ($104 < Ra < 10^7$, $0 < Ha < 10^0$, $0 < \Theta < 0.05$) and found that the highest reduction of the Nusselt number occurs when using $Ra=106$. When the heat source becomes closer to the diagonal, the circulation cells strength becomes less [3]. A different triangular configurations (inverted, straight, and tilted) were studied by Tanmay Basak, R. Anandalakshmi, and Monisha Roy [11]. For the value of $Ra=1000$, they stated that the conduction process is the main process of heat flow distribution in the inverted and the straight cavity configurations while the convection heat flow is the most influential for the tilted cavity configure due to the buoyancy force [11]. Similar triangular cavity configuration, of different heating wall positions from the previous studies, was discussed by Suvash C. Saha, and Y.T. Gu showing three various obvious stages. The three stages described the flow development in the cavity using the numerical simulations [5]. Additionally, another configuration were further discussed by Pratibha Biswal, and Tanmay Basak such as studying the natural convection in the concave/convex walls within right angled triangular enclosures[10]. Also, two connected cavities were analyzed for heat transfer considering the cold wall is the connection interface of the triangular enclosures[12]. Furthermore, several researches have taken into account studying the effect of natural convection in a triangular enclosure by considering different locations of the hot, cold, and the adiabatic wall in the same triangular enclosure[4], comparing experimental studies to the simulation ones[13], and studying the multiple cases where both squared and triangular cavities are considered for researching the higher rate of heat transfer. For example, Debayan Das, Leo Lukose, Tanmay Basak have looked at the multiple discrete heaters in order to get the lowest generation of entropy within the process of natural convection for the fluid occupied inside the triangular and squared enclosures[9]. Other researches also looked at many parameters, such as nanofluids, MHD, inclination angle of the cavity and magnetic field, that could affect the process of heat transfer and examined their influences on heat transfer[7, 14]. A recent study done by Nidal H. Abu-Hamdeh, Hakan F. Oztop, and Khalid A. Alnafaie examined the influential of various parameters on mixed convection inside lid-driven cavity with single-opened side[15]. Some of the studied parameters, such as moving lid, and open side wall resulted in a complicated process of heat transfer and flow field within the cavity[15]. Partially heated cavity walls were also considered for researching to reveal the possible effects on the heat transfer process[16]. However, none of the mentioned researches has further studied the effects of heat transfer of water filled in a chamfered triangular cavity where bottom wall is being partially heated. The present study is important in studying the effect of changing the length of the partially heated wall when keeping the inclined wall as the cold wall. Focusing on the natural convection behavior in triangular cavity has a high importance because of its valuable applications in wide areas as it can be seen in natural convection in a reservoir sidearms, equipment cooling, solar collectors and so on[17, 18]

Geometry Description

The considered problem is handled using the Finite Element Method. The problem can be shown in figure 1. The bottom wall of the triangular cavity is being partially and un-uniformly heated while the inclined wall is being considered as the cold wall and keeping the right vertical wall insulated. The length of the heated wall is considered to be 0.2, 0.4, and 0.5. A chamfer is considered to be 0.1, 0.2, and 0.25. The triangular enclosure is filled with water. Water thermo-physical properties are mentioned in table 1. The present paper aims to study the impact of some parameters such as $Ra$, chamfer radius, and heated wall
length on the flow behavior and transfer of heat. Figure 2 shows the used mesh (11956 mesh) for the numerical modelling for the present study.

![Figure 1. Physical description of the geometry; R=chamfer radius (0.1,0.2,0.25), HL= Hot wall length (0.2,0.4,0.5).](image)
Figure 2. Mesh used (11956 mesh) for the numerical modelling for the present study.

The fluid, water, enters the geometry from the left to the right. The heated wall is considered to be the bottom and the cold wall is considered to be the inclined wall while all other channel walls are maintained to be thermally isolated including chamfer. The following equations from literature are used to solve the prescribed problem[8]:

\[ \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \]  

(1)

\[ U \frac{\partial u}{\partial x} + V \frac{\partial u}{\partial y} = - \frac{\partial p}{\partial x} + Pr \left( \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} \right) \]  

(2)

\[ U \frac{\partial v}{\partial x} + V \frac{\partial v}{\partial y} = - \frac{\partial p}{\partial y} + Pr \left( \frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} \right) + Ra Pr \theta \]  

(3)

\[ U \frac{\partial \theta}{\partial x} + V \frac{\partial \theta}{\partial y} = \left( \frac{\partial^2 \theta}{\partial x^2} + \frac{\partial^2 \theta}{\partial y^2} \right) \]  

(4)

\[ Ra = \frac{g \beta (T_h-T_c) L^3}{\nu \alpha}, \quad Pr = \frac{v}{\alpha} \]

Several boundary conditions were used as follow:

- Boundary conditions of the entry side: \( U = 1, V = 0, \) and \( \theta = 0 \)
- Boundary conditions of the outlet: \( \frac{\partial \theta}{\partial x} = 0, \quad P = 0 \)

Boundary conditions at the geometry wall, excluding the heated wall: \( U = 0, V = 0, \) and \( \frac{\partial \theta}{\partial y} = 0 \)

Heated wall boundary conditions: \( U=V=0, \quad \theta = 1 \)

The local and average Nusselt numbers for the heated surface are calculated as below:

\[ Nu_{loc} = - \frac{\partial \theta}{\partial y} |_{L}, \quad Nu_{av} = - \frac{1}{HL} \int_0^L \frac{\partial \theta}{\partial y} |_{HL} \, dX \], where \( (L) \) refers to the heater length.
Model Validation

To check for the present study validation, the present work is to be validated by comparing the resulted behaviors with a past published work performed by Qiqiang Wang, Junfeng Li, Reaihan E, Yucheng Ren, Jing Li, Jie Li, and Mingguo Ma.[19] which studied a natural convection comparison between rectangular and triangular enclosures using wide range of Rayleigh number between $10^3$ to $10^6$. The results met a high agreement with a very low percentage error. For example, figure 3 shows an excellent agreement at $Ra=10^6$.

![Figure 3. Comparison of current work isotherms (top part) and streamlines (bottom part) with the previous published paper (Comparison study of natural convection between rectangular and triangular enclosure) [19] at Ra=10^5.](image)
Results and Discussion

The considered variables in this work are a range of Ra number (Ra = 10^3, 10^4, 10^5, 10^6), heater length (HL = 0.2, 0.4, 0.5) and the chamfer radius (R = 0.1, 0.2, 0.25) while other parameters are considered constant. This section presents the numerical results as a contour of streamlines and isotherms to examine the fluid flow behavior and the temperature gradient. Also, the value of Nu_{avg} is computed for the hot side to see the convection enhancement inside the enclosure. Figure 4 presents the effect of the Ra number (Ra = 10^3 to 10^6) and the source heater length (HL=0.2, 0.4, and 0.5) on the streamlines at constant ratio of chamfer’s radius (R=0.1). At low Ra and small HL the flow circulation increases with the rise of the Ra number. Furthermore, the main vortex split into two vortices at high Rayleigh number, where these vortices flow in two opposite direction. At HL = 0.4 and 0.5, the present of the secondary vortex is not appearing where the fluid produces one dense vortex that enhances with the increase of the Ra. Figure 5 presents the isotherms contours at a range of Ra number (Ra= 10^3, 10^4, 10^5, 10^6), and HL ratio (HL= 0.2, 0.4, 0.5) while the R ratio constant. It is clear from the figure that the isotherms enhance with the increase of the HL ratio and Ra number. Figure 6 and figure 7 presents the influence of the Ra number and ratio of heating length (HL) on the contours of the streams and isotherms respectively at large arch of the chamfer (R= 0.25). It is observed that the increasing of R value a sensible effect on both the isotherms and the streamlines where the trends of the contours are similar to the previous figures which considered R=0.1. Figure 8 shows the average Nusselt number (Nu_{avg}) variation at wide values of Ra number (Ra = 10^3 to 10^6), heat length ratio (HL= 0.2, 0.4, and 0.5) and the ratio of the chamfer’s radius. At constant R value and HL ratio (R=0.1 and HL = 0.2), the value of the Nu_{avg} increases from 1.2049 at Ra=10^3 to 4.078 at Ra= 10^6 which indicates that the heat transfer enhances with the rise of the Ra number. On the other hand, the value of the Nu_{avg} rises as the ratio of the heating length increases, for example at constant Ra number (Ra= 10^6) Nu_{avg} increases from 4.08 to 7.3 at HL= 0.2 and 0.5 respectively. The physical explanation that the value of the Nu_{avg} increases with the rise of the Ra number and HL ratio is due to have increasing in the strength of the stream circulation with the increase of the Ra and HL and this circulation enhances the convection transfer of heat which means increasing in Nu_{avg} value. In contrast, the increasing of the R value has very small impact on the Nu_{avg}. According to the results shown in figure 8, chamfer radius ratio has negligible effect on the relationship between Nu_{avg} and Ra, that is the resulted data at R=0.1 were exactly the same for those at R=0.2 and 0.25.
| Ra  | HL=0.2  | HL=0.4  | HL=0.5  |
|-----|---------|---------|---------|
| $10^3$ | ![Streamline Image](image) | ![Streamline Image](image) | ![Streamline Image](image) |
| $10^4$ | ![Streamline Image](image) | ![Streamline Image](image) | ![Streamline Image](image) |
| $10^5$ | ![Streamline Image](image) | ![Streamline Image](image) | ![Streamline Image](image) |
| $10^6$ | ![Streamline Image](image) | ![Streamline Image](image) | ![Streamline Image](image) |

**Figure 4.** Raleigh number (Ra) effects on streamlines at (R=0.1, HL=0.2,0.4, and 0.5).
Figure 5. Raleigh number (Ra) effects on isotherms at (R=0.1, HL=0.2,0.4, and 0.5).
| Ra   | HL=0.2 | HL=0.4 | HL=0.5 |
|------|--------|--------|--------|
| $10^3$ | ![Image](image1) | ![Image](image2) | ![Image](image3) |
| $10^4$ | ![Image](image4) | ![Image](image5) | ![Image](image6) |
| $10^5$ | ![Image](image7) | ![Image](image8) | ![Image](image9) |
| $10^6$ | ![Image](image10) | ![Image](image11) | ![Image](image12) |

**Figure 6.** Raleigh number (Ra) effects on isotherms at (R=0.25, HL=0.2,0.4, and 0.5).
Figure 7. Raleigh number (Ra) effects on isotherms at (R=0.25, HL=0.2,0.4, and 0.5).
Figure 8. Average Nusselt number ($N_{u_{avg}}$) of the bottom wall at various Rayleigh number ($Ra$) at ($R=0.1, 0.2$ and $0.25$, $HL=0.2, 0.4$ and $0.5$).

Table 1. Water thermo physical properties.

| Properties | Pure water |
|------------|------------|
| $C_p$ (J/kg K) | 4179 |
| $\rho$ (kg/m$^3$) | 997.1 |
| $\alpha$ (m$^2$/s) | 1.47x10$^{-7}$ |
| $k$ (W/m.K) | 0.613 |
| $\beta$ (1/K) | 21x10$^{-5}$ |
| $\mu$ (kg/m.s) | 8.9x10$^{-4}$ |

Conclusions

Numerical investigation of natural convection inside a closed cavity, triangular shape, occupied with water and heated from the bottom has been studied in this paper. The main results of this research can be concluded in the following points:

1. The streamlines strength increases with the rise of the Ra number and heating source length while the ratio of the chamfer radius has insensible effect of the streamlines contour.
2. The isotherms strength increases with the rise of the Ra number and the heating source length and has affectless for the chamfer’s radius ratio.

3. The value of the $\bar{N}u_{avg}$ increases with the rise of the Ra number and the ratio of the heating source length. For example, $\bar{N}u_{avg}$ rises from 2.0443 to 7.2793 when increasing Ra from $10^3$ to $10^6$ respectively at R=0.25 and HL=0.5.

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