NUCLEAR SUPERSYMMETRY: NEW TESTS AND EXTENSIONS

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Extensions of nuclear supersymmetry are discussed, together with a proposal for new, more stringent and precise tests that probe the susy classification and specific two-particle correlations among supersymmetric partners. The combination of these theoretical and experimental studies may play a unifying role in nuclear phenomena.

1 Introduction

Nuclear supersymmetry (n-susy), first proposed by Franco Iachello more than two decades ago [1], is a composite-particle phenomenon linking the properties of bosonic and fermionic systems, framed in the context of the Interacting Boson Model of nuclear structure [2]. Composite particles, such as the α-particle are known to behave as approximate bosons. As He atoms they become superfluid at low temperatures, an under certain conditions can also form Bose-Einstein condensates. At higher densities (or temperatures) the constituent fermions begin to be felt and the Pauli principle sets in. Odd-particle composite systems, on the other hand, behave as approximate fermions, which in the case of the IBFM are treated as a combination of bosons and an (ideal) fermion. In contrast to the theoretical construct of supersymmetric particle physics, where susy is postulated as a generalization of the Lorentz-Poincaré invariance at a fundamental level, n-susy has been subject to experimental verification [3]. Nuclear supersymmetry should not be confused with fundamental susy, which predicts the existence of supersymmetric particles, such as the photino and the selectron, for which no evidence has yet been found. If such particles exist, however, susy must be strongly broken, since large mass differences must exist among superpartners, or otherwise they would have been already detected. Competing susy models give rise to diverse mass predictions and are the basis for current superstring and brane theories [4].

Nuclear supersymmetry, on the other hand, is a theory that establishes precise links among the spectroscopic properties of certain neighboring nuclei. Even-even and odd-odd nuclei are composite bosonic systems, while odd-A nuclei are fermionic. It is in this context that n-susy provides a theoretical framework where bosons and fermions are treated as members of the same supermultiplet [5]. the mass difference between superpartners is thus of the order of 1/A, but the theory goes much further and treats the excitation spectra and transition intensities of the different systems as arising from a single Hamiltonian and a single set of transition operators. Originally framed as a symmetry among pairs of nuclei [1,2,5], it was subsequently extended to nuclear quartets or “magic squares”, where odd-odd nuclei could be incorporated in a natural way [6]. Evidence for the existence of n-susy (albeit possibly significantly broken) grew over the years, specially for the quartet...
of Fig. 1, but only recently more systematic evidence was found. This was achieved by means of one-nucleon transfer reaction experiments leading to the odd-odd nucleus $^{196}$Au, which, together with the other members of the susy quartet ($^{194}$Pt, $^{195}$Au and $^{195}$Pt) is considered to be the best example of n-susy in nature [6,7,8,9].

We should point out, however, that while these experiments provided the first complete energy classification for $^{196}$Au (which was found to be consistent with the theoretical predictions [6,7,8,9]), the reactions involved ($^{197}$Au($\vec{d},t$), $^{197}$Au($p,d$) and $^{198}$Hg($\vec{d},\alpha$)) did not actually test directly the supersymmetric wave functions, as we shall discuss below. Furthermore, whereas these new measurements are very exciting, the dynamical susy framework is so restrictive that there was little hope that other quartets could be found and used to verify the theory [6,7,8,9]. The purpose of this paper is two-fold. On the one hand we report on an ongoing investigation of one- and two-nucleon transfer reactions [10] in the Pt-Au region that will more directly analyze the supersymmetric wave functions and measure new correlations which have not been tested up to now. On the other hand we discuss some ideas put forward several years ago, which question the need for dynamical symmetries in order for n-susy to exist [11]. We thus propose a more general theoretical framework for nuclear supersymmetry. The combination of such a generalized form of supersymmetry and the transfer experiments now being carried out [12], could provide remarkable new correlations and a unifying theme in nuclear structure physics.

2 New experiments

The quartet of nuclei of Fig. 1 was classified by means of the $U_\nu(6/12) \times U_\pi(6/4)$ dynamical supersymmetry, obtained by combining the $SO^{BF}(6)$ symmetry limit for the odd neutron ($^{195}$Pt) and the $Spin(6)$ symmetry limit for the odd proton ($^{195}$Au) [6]. The excitation spectra of the nuclei $^{194}$Pt, $^{195}$Au and $^{195}$Pt was used...
to determine the Hamiltonian and subsequently the spectra of the odd-odd partner \(^{196}\text{Au}\) was predicted, for which at the time little or no experimental data was available. One should note, however, that the great majority of tests carried out for the supersymmetric framework have involved one-nucleon transfer experiments leading to the nuclei in figure 1 through reactions coming from outside the quartet, such as \(^{197}\text{Au}(\vec{d}, t)^{196}\text{Au}\) and \(^{196}\text{Pt}(\vec{d}, t)^{195}\text{Pt}\) that, in first approximation, are formulated using a transfer operator of the form \(a^\dagger b\). The latter reactions are useful to measure energies, angular momenta and parity of the residual nucleus and in principle provide information about the systems wave functions. However, they cannot test correlations present in the quartet’s wave functions and thus in the susy classification scheme as is the case for one-nucleon transfer reactions inside the supermultiplet. These reactions do provide a direct test of the fermionic sector of the graded Lie Algebras \(U_\nu(6/12)\) and \(U_\pi(6/4)\). These operators are related to the nondiagonal elements of the product:

\[
U_\nu(6/12) \otimes U_\pi(6/4) : \begin{pmatrix} b^\dagger b_b & b^\dagger b_{\nu} a_{\nu} \\ a^\dagger a_b & a^\dagger a_{\nu} a_{\nu} \end{pmatrix} \oplus \begin{pmatrix} b^\dagger b_{\pi} & b^\dagger b_{\pi} a_{\pi} \\ a^\dagger a_{\pi} & a^\dagger a_{\pi} a_{\pi} \end{pmatrix}.
\] (1)

New experimental facilities and detection techniques \([7, 8, 9]\) offer a unique opportunity for analyzing the supersymmetry classification in greater detail \([12]\). In reference \([13]\) we pointed out a symmetry route for the theoretical analysis of such reactions, via the use of tensor operators of the algebras and superalgebras. An alternative route is the use of a semi-microscopic approach where projection techniques starting from the original nucleon pairs lead to specific forms for the operators \([14, 15]\) which, however, are only strictly valid in the generalized seniority regime \([16]\). The former and latter routes may be related by a consistent-operator approach, where the Hamiltonian exchange operators are made to be consistent with the one-nucleon transfer operator implying that the exchange term in the boson-fermion Hamiltonian can be viewed as an internal exchange reaction among the nucleon and the nucleon pairs.

In addition to these experiments, we are also exploring the possibility of testing susy through new transfer reactions. The two-nucleon transfer \((\alpha, \vec{d})\) reaction probes \(n – p\) correlations in the nuclear wave function and constitutes a very stringent test of the supersymmetry classification. Note also that the \(^{194}\text{Pt}(\alpha, \vec{d})^{196}\text{Au}\) reaction corresponds to a combination of the single-nucleon transfers going either through \(^{195}\text{Pt}\) or through \(^{195}\text{Au}\), and that the corresponding operator is thus a product of the fermionic components in equation (1), as schematically indicated in Fig. (2). Likewise, the reaction \(^{195}\text{Pt}(^3\text{He}, t)^{195}\text{Au}\), is again expressible in terms of the superalgebra fermionic operators in (1) and in this case is associated to the beta-decay operator \([17]\). These reactions and their relation to single-nucleon transfer experiments raise the exciting possibility of testing direct correlations among transfer reaction spectroscopic factors in different nuclei, predicted by the supersymmetric classification of the magic quartet. A preliminary report on these analyses is presented in Ref. \([10]\).
3 Susy without Dynamical Symmetry

The concept of dynamical algebra (not to be confused with that of dynamical symmetry) implies a generalization of the concept of symmetry algebra. The latter is defined as follows: $G$ is the dynamical algebra of a system if all physical states considered belong to a single irreducible representation (IR) of $G$. (In a symmetry algebra, in contrast, each set of degenerate states of the system is associated to an IR). The best known examples of a dynamical algebra are perhaps $SO(4,2)$ for the hydrogen atom and the $U(6)$ IBM algebra for even-even nuclei. A consequence of having a dynamical algebra associated to a system is that all states can be reached using the algebra’s generators or, equivalently, all physical operators can be expressed in terms of these operators [9]. Naturally, the same Hamiltonian and the same transition operators are employed for all states in the system. To further clarify this point, it is certainly true that a single $H$ and a single set of operators are associated to a given even-even nucleus in the IBM framework, expressed in terms of the $U(6)$ (dynamical algebra) generators. It doesn’t matter whether this Hamiltonian can be expressed or not in terms of the generators of a single chain of groups (a dynamical symmetry).

In the same fashion, if we now consider $U(6/12)$ to be the dynamical algebra for the pair of nuclei $^{194}\text{Pt} - ^{195}\text{Pt}$, it follows that the same $H$ and operators (including in this case the transfer operators that connect states in the different nuclei) should apply to all states. It also follows that no restriction should be imposed on the form of $H$, except that it must be a function of the generators of $U(6/12)$ (the enveloping space associated to it). It should be clear that the concept of supersymmetry does not require the existence of a particular dynamical symmetry. Extending these ideas to the $\nu - \pi$ space of IBM-2 we can say that susy is equivalent to requiring
that a product of the form
\[ U_\nu(6/\Omega) \otimes U_\pi(6/\Omega') \] (2)
plays the role of dynamical (super) algebra for a quartet of even-even, even-odd, odd-even and odd-odd nuclei. Having said that, it should be stated that the quartet dynamical susy of references [6, 7, 8, 9] has the distinct advantage of immediately suggesting the form of the quartet’s Hamiltonian and operators, while the general statement made above does not provide a general recipe. For some particular cases, however, this can be done in a straightforward way. In reference [11], for example, the \( U(6/12) \) supersymmetry (without imposing one of the three dynamical IBM symmetries) was successfully tested for the Ru and Rh isotopes. In that case a combination of \( U^{B+F}(5) \) and \( SO^{B+F}(6) \) symmetries was shown to give an excellent description of the data, as shown in Figs. 3 and 4. The \( U(6/12) \) case is simple because, using a pseudospin decomposition, there are isomorphic \( U(6) \) algebras for the bosons and the fermion and any combination of the three dynamical IBM algebras can be considered \[ U(6/12) \]
\[ j = 1/2, 3/2, 5/2 \]
\[ \bar{l} = 0, 2 \quad s = 1/2 \]
\[ G^{BF}_l \equiv G^{B}_l + G^{F}_l \] (3)
and an arbitrary interaction expressed in terms of $G_{BF}$ implies explicit correlations between the boson-boson and boson-fermion interactions [18].

An immediate consequence of this proposal is that it opens up the possibility of testing susy in other nuclear regions, since dynamical symmetries are very scarce and have severely limited the study of nuclear supersymmetry.

4 Generic Susy

We have recently initiated a renewed search for supersymmetry in nuclei [12, 18]. We have yet to discover a general mechanism to generate all appropriate operators.
in the general case, but a set of guiding rules are the following:

1) The Hamiltonian should describe the members of the doublet or quartet.

2) The boson-Hamiltonian, plus the single-particle orbits, should essentially determine the boson-fermion interaction, for both the odd-proton and odd-neutron nuclei.

3) The combination of the previous three should give a prediction for the odd-odd Hamiltonian, and thus about the $p - n$ interaction.

Although the analysis is not concluded, our preliminary results for the $W$ and $Hf$ nuclei are quite encouraging [18]. The first calculation involves a mixture of $SU(3)$ and $SO(6)$ symmetries in $U(6/12)$. This calculation employs the $Q$-consistent formalism and a comparison between the experimental and calculated $BE(2)$ transitions and quadrupole moments is shown in tables 1 and 2. The agreement is very good, except for one transition in $^{183}W$. We also show an example of generic susy in $U(6/4)$. It corresponds to the supermultiplet composed of $^{174}Hf$ and $^{173}Hf$. In this case the hamiltonian uses a combination of Casimir operators of the $U(5)$ and $SO(6)$ groups and of their subgroups. In figure 5 we compare the experimental and calculated level energies in these two nuclei [18].

One of our main interests is to apply the generic form of supersymmetry to the

\[
B(E2) \ (e^2l^2) \text{ and } Q \ (eb) \text{ in } ^{182}W
\]

| $J^+_i \to J^+_f$ | Exp. | Calc. |
|------------------|------|-------|
| $2^+_1 \to 0^+_1$ | 0.839(18) | 0.8422 |
| $4^+_1 \to 2^+_1$ | 1.201(61) | 1.1877 |
| $6^+_1 \to 4^+_1$ | 1.225(135) | 1.2777 |
| $2^+_2 \to 0^+_1$ | 0.021(1) | 0.0040 |
| $2^+_2 \to 2^+_1$ | 0.041(1) | 0.0072 |
| $2^+_3 \to 4^+_1$ | 0.00021(1) | 0.0006 |
| $2^+_3 \to 0^+_1$ | 0.006(1) | 0.0000 |
| $2^+_3 \to 0^+_2$ | 1.225(368) | 0.6840 |
| $2^+_3 \to 2^+_1$ | 0.0039(5) | 0.0001 |

| $Q$ | Exp. | Calc. |
|-----|------|-------|
| $2^+_1$ | $-2.00^{+0.04}_{-0.08}$ | -1.86 |
| $2^+_2$ | $1.94^{+0.30}_{-0.04}$ | 1.61 |

Table 1. Experimental and calculated reduced transition probabilities and quadrupole moments in $^{182}W$. Taken from [18].

\[
B(E2) \ (e^2l^2) \text{ in } ^{183}W
\]

| $J^+_i \to J^+_f$ | Exp. | Calc. |
|------------------|------|-------|
| $\frac{3}{2}^- \to \frac{1}{2}^-$ | 0.938(62) | 0.603 |
| $\frac{5}{2}^- \to \frac{1}{2}^-$ | 0.68(4) | 0.603 |
| $\frac{7}{2}^- \to \frac{3}{2}^-$ | 0.20(3) | 0.173 |
| $\frac{13}{2}^- \to \frac{9}{2}^-$ | 1.1(3) | 0.915 |
| $\frac{17}{2}^- \to \frac{13}{2}^-$ | 0.89(12) | 0.925 |
| $\frac{5}{2}^- \to \frac{1}{2}^-$ | 0.005(2) | 0.000 |
| $\frac{7}{2}^- \to \frac{5}{2}^-$ | 0.10(4) | 0.010 |
| $\frac{7}{2}^- \to \frac{5}{2}^-$ | 0.012(5) | 0.023 |
| $\frac{7}{2}^- \to \frac{1}{2}^-$ | 0.082(9) | 0.011 |
| $\frac{5}{2}^- \to \frac{3}{2}^-$ | 0.001(1) | 0.001 |
| $\frac{5}{2}^- \to \frac{5}{2}^-$ | 0.027(6) | 0.004 |
| $\frac{7}{2}^- \to \frac{5}{2}^-$ | 0.43(2) | 0.001 |
| $\frac{5}{2}^- \to \frac{3}{2}^-$ | 1.30(18) | 0.860 |

Table 2. Experimental and calculated reduced transition probabilities in $^{183}W$. Taken from [18].
Figure 5. Experimental and calculated positive-parity states in $^{174}$Hf and negative-parity states in $^{173}$Hf. The energy scale is in MeV. Taken from [18].

Pt-Au region and compare the results with the traditional scheme, particularly for the new transfer experiments [12]. In addition, we expect to find other examples of quartet supersymmetric behavior, once the constraints set by dynamical symmetry are dropped.

We continue to search for a more general theoretical framework that can encompass the particular cases that we can solve at this point.

We conclude by proposing that nuclear susy may be a more general phenomenon than was previously realized and that may yet play an important, unifying role in nuclei.

5 Dedication:

We dedicate this paper to Franco Iachello, who has managed to uniquely combine the Platonic ideal of symmetry with the down-to-earth Aristotelic ability to recognize these patterns in Nature.

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Figure 6. Detail of “The School of Athens” (Plato on the left and Aristoteles on the right), by Rafael.

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