Isospin Breaking and $\omega \rightarrow \pi^+\pi^-$ Decay

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We study $\omega \rightarrow \pi^+\pi^-$ decay up to including all orders of the chiral expansion and one-loop level of mesons in formalism of chiral constituent quark model. This G-parity forbidden decay is caused by $m_u \neq m_d$ and electromagnetic interaction of mesons. We illustrate that in the formalism both nonresonant contact interaction and $\rho$ resonance exchange contribute to this process, and the contribution from $\rho$ resonance exchange is dominant. We obtain that transition matrix element is

$$< \rho | H_{\rho\omega} | \omega > = \left[ -(3956 \pm 280) - (1697 \pm 130)i \right] \text{MeV}^2,$$

and isospin breaking parameter is $m_d - m_u = 3.9 \pm 0.22 \text{MeV}$ at energy scale $\mu \sim m_\omega$.

12.39.-x,12.40.Vv,13.25.Jx,12.15.Ff.

I. INTRODUCTION

The light current quark masses are basic input of quantum chromodynamics (QCD). The inequality of the light quark masses, especially, $m_u \neq m_d$, breaks the isospin symmetry or charge symmetry [1]. This breaking of isospin symmetry induces various measurable physics processes such as $\pi^0 - \eta$, $\Lambda - \Sigma^0$ mixing and $\omega \rightarrow \pi^+\pi^-$ decay etc. In this paper, we will focus on the $\omega \rightarrow \pi^+\pi^-$ decay, which is considered as the important source of charge symmetry breaking in nuclear physics.

In ref. [2] (we will quote it as I hereafter) we have shown that the chiral expansion at vector meson energy region converges slowly. Therefore, a well-defined effective field theory describing the physics in this energy region must be available for calculation on high order terms of the chiral expansion and meson loops. Obviously, method of chiral perturbative theory (ChPT) [3] is impractical to capture the high order term contribution because the number of free parameter increases very rapidly as perturbative order rising. In I, following the spirit of Manohar-Georgi (MG) model [4], we have constructed chiral constituent quark model (ChCQM) including lowest vector meson resonances. The advantages of this approach are that high order contribution of the chiral expansion and $N_c^{-1}$ expansion can be calculated consistently, and only fewer free parameters are required. Low energy limit and unitarity of the model are also examined successfully. In particular, it is a attractive property that although the leading order theoretical prediction does not match with experimental data, larger contribution from high order of the chiral expansion and pseudoscalar meson one-loop corrects theoretical prediction close to data very much. It is just the characteristic of the chiral expansion in this energy region. Therefore, in this the present paper, we also need to calculate $\omega \rightarrow \pi^+\pi^-$ decay to include all high order contribution and pseudoscalar meson one-loop correction.

This research is also motivated by the following reasons:

i) In the most recent references [5–7], the $\omega \rightarrow \pi^+\pi^-$ decay was treated as being dominant via $\rho$-resonance exchange, and the direct $\omega\pi^+\pi^-$ coupling is neglected. It has been pointed out in ref. [3] that the neglect of $\omega$ “direct” coupling to $\pi^+\pi^-$ is not valid. It can be naturally understood since $\pi\pi$ can make up of vector-isovector system, whose quantum numbers are same to $\rho$ meson. Thus in an effective lagrangian based on chiral symmetry, every $\rho$ field can be replaced by $\pi\pi$ and does not conflict with symmetry. Although authors of ref. [3] also pointed out that the present experimental data still can not be used to separate “direct” $\omega\pi\pi$ coupling from $\omega - \rho$ mixing contribution in model-independent way, it is very interesting to perform a theoretical investigation on “direct” $\omega\pi\pi$ coupling contribution. We will show that the contribution from interference of “direct” $\omega\pi\pi$ coupling and $\omega - \rho$ is about 15%. Thus “direct” $\omega\pi\pi$ coupling can not be neglected indeed.

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ii) The present study involves the investigation of \( \rho^0 - \omega \) mixing, which has been an active subject[5–14]. The mixing amplitude for on-mass-shell vector mesons has been observed directly in the measurement of the pion form-factor in the time-like region from the process \( e^+ e^- \to \pi^+ \pi^- \). For roughly twenty years, \( \rho^0 - \omega \) mixing amplitude was assumed constant or momentum independent even if \( \rho \) and \( \omega \) have the space-like momenta, far from the on-shell point. Several years ago, this assumption was firstly questioned by Goldman et. al. [1], and the mixing amplitude was found to be significantly momentum dependent within a simple quark loop model. Subsequently, various authors have argued such momentum dependence of the \( \rho^0 - \omega \) mixing amplitude by using various approaches [12,13]. In particular, the authors of ref. [13] has pointed out that \( \rho^0 - \omega \) mixing amplitude must vanish at \( q^2 = 0 \) (where \( q^2 \) denotes the four-momentum square of the vector mesons) within a broad class of model. This point will be also examined in ChCQM.

iii) It has been known that \( \omega \to \pi^+ \pi^- \) decay amplitude receive the contribution from two sources: isospin symmetry breaking due to \( u - d \) quark mass difference and electromagnetic interaction. In I we have shown that VMD in meson physics is natural consequence of the present formalism instead of input. The vector \( e^+ e^- \) decays are also predicted successfully. Therefore, the dynamics of electromagnetic interactions of mesons has been well established, and the calculation for \( \omega \to \pi^+ \pi^- \) decay from the transition \( \omega \to \gamma \to \rho \to \pi \pi \) and “direct” \( \omega \to \pi^+ \pi^- \) is straightforward. In this the present paper, we will pay our attention to isospin breaking due to \( m_u \neq m_d \). It is another purpose of this paper to determine isospin breaking parameter \( \delta m_q \equiv m_d - m_u \) via \( \omega \to \pi^+ \pi^- \) decay. This parameter is urgently wanted by determination of light quark mass ratios.

The contents of the paper are organized as follows. In sect. 2 we review the basic notations of the chiral constituent quark model with the lowest vector meson resonances. In sect. 3, the tree level effective lagrangian, which including all order contribution of the chiral expansion, is obtained. The pseudoscalar meson one-loop corrections are calculated in sect. 5. In sect. 6, the formulas and numerical results of \( \omega - \rho^0 \) mixing amplitude and \( \omega \to \pi^+ \pi^- \) are given. The sect. 7 is devoted to a brief summary.

II. CHIRAL CONSTITUENT QUARK MODEL WITH VECTOR MESON

The simplest version of chiral quark model which was originated by Weinberg [17], and developed by Manohar and Georgi [1] provides a QCD-inspired description on the simple constituent quark model. In view of this model, in the energy region between the chiral symmetry spontaneously broken (CSSB) scale and the confinement scale (\( \Lambda_{QCD} \sim 0.2 - 0.3GeV \)), the dynamical field degrees of freedom are constituent quarks (quasi-particle of quarks), gluons and Goldstone bosons associated with CSSB (these Goldstone bosons correspond to lowest pseudoscalar octet). In this quasiparticle description, the effective coupling between gluon and quarks is small and the important interaction is the coupling between quarks and Goldstone bosons. In I we have further included the lowest vector meson resonances into this formalism. At chiral limit, this model is parameterized by the following chiral constituent quark lagrangian

\[
\mathcal{L}_X = i\bar{q}(\bar{\gamma} + gA\gamma_5 - i\mathcal{V})q - m\bar{q}q + \frac{F^2}{16} < \nabla_\mu U \nabla^\mu U^\dagger > + \frac{1}{4}m_0^2 < V_\mu V^\mu > .
\]

Here \( < \ldots > \) denotes trace in SU(3) flavour space, \( \bar{q} = (\bar{q}_u, \bar{q}_d, \bar{q}_s) \) are constituent quark fields. \( V_\mu \) denotes vector meson octet and singlet, or more convenience, due to OZI rule, they are combined into a singlet “nonet” matrix

\[
V_\mu(x) = \lambda \cdot \mathbf{V}_\mu = \sqrt{2} \left( \begin{array}{ccc}
\rho^0 + \omega^0 & \rho^+ & K^{+} \\
\rho^- & -\rho^0 + \omega^0 & K^{0} \\
K^{*+} & K^{*0} & \phi^0
\end{array} \right).
\]

The \( \Delta_\mu \) and \( \Gamma_\mu \) are defined as follows,

\[
\Delta_\mu = \frac{1}{2}\{\xi^\dagger(\partial_\mu - ir_\mu)\xi - \xi(\partial_\mu - il_\mu)\xi^\dagger\},
\]

\[
\Gamma_\mu = \frac{1}{2}\{\xi^\dagger(\partial_\mu - ir_\mu)\xi + \xi(\partial_\mu - il_\mu)\xi^\dagger\},
\]

and covariant derivative are defined as follows

\[
\nabla_\mu U = \partial_\mu U - ir_\mu U + iU_l_\mu = 2\xi^\dagger\Delta_\mu \xi,
\]

\[
\nabla_\mu U^\dagger = \partial_\mu U^\dagger - il_\mu U^\dagger + iU^\dagger r_\mu = -2\xi^\dagger\Delta^\dagger \xi.
\]
where \( l_\mu = v_\mu + a_\mu \) and \( r_\mu = v_\mu - a_\mu \) are linear combinations of external vector field \( v_\mu \) and axial-vector field \( a_\mu \). \( \xi \) associates with non-linear realization of spontaneously broken global chiral symmetry introduced by Weinberg [8]. This realization is obtained by specifying the action of global chiral group \( G = SU(3)_L \times SU(3)_R \) on element \( \xi(\Phi) \) of the coset space \( G/SU(3)_v \):

\[
\xi(\Phi) \rightarrow g_R \xi(\Phi) g_L^\dagger(\Phi) = h(\Phi) \xi(\Phi) g_L^\dagger, \quad g_L, g_R \in G, \quad h(\Phi) \in H = SU(3)_v.
\]

(5)

Explicit form of \( \xi(\Phi) \) is usual taken

\[
\xi(\Phi) = \exp \{i\lambda^a \Phi^a(x)/2\}, \quad U(\Phi) = \xi^2(\Phi),
\]

(6)

where the Goldstone boson \( \Phi^a \) are treated as pseudoscalar meson octet. The compensating \( SU(3)_v \) transformation \( h(\Phi) \) defined by eq.(3) is the wanted ingredient for a non-linear realization of G. In practice, we shall be interested in transformations of \( \Delta_\mu, \Gamma_\mu \) and constituent quark fields under \( SU(3)_v \). The \( q, \bar{q} \) transform as matter fields of \( SU(3)_v \),

\[
q \rightarrow h(\Phi) q, \quad \bar{q} \rightarrow \bar{q} h^\dagger(\Phi).
\]

(7)

The vector meson fields transform homogeneously under \( SU(3)_v \)

\[
V_\mu \rightarrow h(\Phi) V_\mu h^\dagger(\Phi),
\]

(8)

which was suggested by Weinberg [8] and developed further by Callan, Coleman et. al. [10]. Since under local G, the explicit transformations of external vector and axial-vector fields are

\[
l_\mu \equiv v_\mu - a_\mu \rightarrow g_L(x) l_\mu g_L^\dagger(x) + ig_L(x) \partial_\mu g_L(x), \\
r_\mu \equiv v_\mu + a_\mu \rightarrow g_R(x) r_\mu g_R^\dagger(x) + ig_R(x) \partial_\mu g_R(x),
\]

(9)

\( \Delta_\mu \) is \( SU(3)_v \) invariant field gradients and \( \Gamma_\mu \) transforms as field connection of \( SU(3)_v \)

\[
\Delta_\mu \rightarrow h(\Phi) \Delta_\mu h^\dagger(\Phi), \quad \Gamma_\mu \rightarrow h(\Phi) \Gamma_\mu h^\dagger(\Phi) + h(\Phi) \partial_\mu h^\dagger(\Phi).
\]

(10)

Thus the lagrangian(1) is invariant under \( G_{global} \times G_{local} \).

The several remarks are need here. 1) Note that there is no kinetic term for vector meson fields in \( \mathcal{L}_\chi \). Therefore, in this formalism the vector mesons are treated as composites of constituent quarks instead of fundamental fields. The dynamics of vector meson resonances will be generated via loop effects of constituent quarks. 2) Note that there is kinetic term of pseudoscalar mesons in \( \mathcal{L}_\chi \). This is different from some other chiral quark models, in which there is no such term. Existing of this kinetic term is consistent with basic assumption of our model, because in this energy region, the dynamical field degrees of freedom are both constituent quarks and Goldstone bosons associated with CSSB. 3) In \( \mathcal{L}_\chi \) the parameter \( g_A \simeq 0.75 \) is determined by \( \beta \) decay of neutron. It has been pointed out in [1] that this value has included effects of intermediate axial-vector and radiative effects exchanges at low energy. In addition, the constituent quark mass parameter \( m \simeq 480 \text{MeV} \) has been fitted via low energy limit of the model. Such large value is required by convergence of chiral expansion at vector meson energy scale.

In this paper, we must go beyond chiral limit for obtaining isospin breaking results. The light current quark matrix

\[
\mathcal{M} = \text{diag}\{m_u, m_d, m_s\}
\]

(11)

can be usually included into external scalar fields, i.e., \( \tilde{x} = s + ip \), where \( s = s_{ext} + \mathcal{M}, \ s_{ext} \) and \( p \) are scalar and pseudoscalar external fields respectively. The chiral transformation for \( \tilde{x} \) is

\[
\tilde{x} \rightarrow g_R \tilde{x} g_L^\dagger.
\]

Thus together with \( \xi \) and \( \xi^\dagger, \tilde{x} \) and \( \tilde{x}^\dagger \) can form \( SU(3)_v \) invariant scalar source \( \xi^\dagger \tilde{x} \xi + \xi \tilde{x}^\dagger \xi \) and pseudoscalar source \( (\xi^\dagger \tilde{x} \xi^\dagger - \tilde{x}^\dagger \xi \xi^\dagger) \gamma_5 \). Then current quark mass dependent lagrangian is written

\[
-\frac{1}{2} \bar{q}(\xi^\dagger \tilde{x} \xi + \xi \tilde{x}^\dagger \xi) q - \frac{\kappa}{2} \bar{q}(\xi^\dagger \tilde{x} \xi^\dagger - \tilde{x}^\dagger \xi \xi^\dagger) \gamma_5 q.
\]

(12)

The above lagrangian will return to QCD lagrangian \( \bar{q} \mathcal{M} \bar{q} \) in absence of pseudoscalar mesons at high energy. So that there is a free parameter \( \kappa \) which can not be determined by symmetry alone. From viewpoint of phenomenology, this ambiguity is similar to Kaplan-Manohar ambiguity in ChPT [20]. It will be studied at elsewhere so that we do not further discuss it here. In next section, rigorous calculation will show that our results in this paper is independent of \( \kappa \).
To conclude this section, the ChCQM lagrangian including the lowest vector meson resonances and light current quark masses is

\[
\mathcal{L}_\chi = i\bar{q}i\mu + g_A \mathbf{A}\gamma_5 - i\mathcal{V} q - m\bar{q}q - \frac{1}{2}\bar{q}(\xi^\dagger \chi\xi^\dagger + \xi\bar{\chi}\xi)q - \frac{K}{2}\bar{q}(\xi^\dagger \bar{\chi}\xi^\dagger - \xi\bar{\chi}\xi)\gamma_5 q \\
+ \frac{F^2}{16} < \nabla_\mu U\nabla^\mu U^\dagger > + \frac{1}{4}m_0^2 < V^\mu V^\mu > .
\]

(13)

The effective lagrangian describing interaction of vector meson resonances will be generated via loop effects of constituent quarks.

III. LEADING ORDER EFFECTIVE LAGRANGIAN IN $N_C^{-1}$ EXPANSION

In this section, we will derive relevant effective lagrangian via calculating one-loop diagrams of constituent quarks, which is at the leading order in $N_C^{-1}$ expansion. In our calculation, the light current quark masses will be treated as perturbation and be expanded to the leading order. Pseudoscalar mesons which are localized in external line should satisfy soft pion theorem, i.e., $k^2 \rightarrow 0$(where $k^2$ denotes the four-momentum square of pseudoscalar mesons). However, $q^2$ is the four-momentum square of vector mesons, and obviously it is not very small comparing with chiral symmetry spontaneously broken scale. Therefore, the higher order terms of $q^2$ in the chiral expansion, have significant contribution which can not be neglected. Or in the other words, the chiral expansion at vector meson energy scale converge slowly. We will rigorously calculate all $q^2$-dependent term contribution.

We start with constituent quark lagrangian(1), and define vector external source $\bar{V}_\mu^a(a = 0, 1, \cdots, 8)$, axial-vector external source $\Delta^a_{\mu}$, scalar external source $S^a$ and pseudoscalar external source $P^a$ as follows

\[
\bar{V}_\mu^a = \frac{1}{2} < \lambda^a V_\mu + i\Gamma_\mu > , \quad \Delta^a_{\mu} = \frac{1}{2} < \lambda^a \Delta_{\mu} > ,
\]

\[
S^a = \frac{1}{4} < \lambda^a (\xi^\dagger \chi\xi^\dagger + \xi\bar{\chi}\xi) > , \quad P^a = \frac{K}{4} < \lambda^a (\xi^\dagger \bar{\chi}\xi^\dagger - \xi\bar{\chi}\xi) > \tag{14}
\]

(where $\lambda^1, \cdots, \lambda^8$ are SU(3) Gell-Mann matrices and $\lambda^0 = \sqrt{2}/3$). Then in lagrangian(1), the terms associating with constituent quark fields can be rewritten as follow

\[
\mathcal{L}_\chi^{\text{eff}} = \bar{q}(i\mu - m)q + \bar{V}_\mu^a \gamma^\mu \lambda^a q + ig_A \Delta_{\mu}^a \gamma^\mu \lambda^a \gamma_5 q - S^a \bar{q}\lambda^a q - P^a \bar{q}\lambda^a \gamma_5 q . \tag{15}
\]

The effective action describing meson interaction can be obtained via integrating over degrees of freedom of fermions

\[
e^{-iS_{eff}} = \int DqD\bar{q}e^{i\int d^4x\mathcal{L}_\chi(x)} = <\text{vac}, \text{out}|\text{in}, \text{vac}>_{\bar{\mathcal{V}}, \mathcal{A}, \mathcal{S}, \mathcal{P}} , \tag{16}
\]

where $<\text{vac}, \text{out}|\text{in}, \text{vac}>_{\bar{\mathcal{V}}, \mathcal{A}, \mathcal{S}, \mathcal{P}}$ is vacuum expectation value in presence external sources. The above path integral can be performed explicitly, and heat kernal method [21,22] has been used to regulate the result. However, this method is extremely difficult to compute very high order contributions in practice. In I we have provided an equivalent and efficient method to evaluate the effective action via calculating one-loop diagrams of constituent quarks directly. This method can capture all high order contributions of the chiral expansion.

In interaction picture, the equations (13) is rewritten as follow

\[
e^{iS_{eff}} = <0|\mathcal{T}_q e^{i\int d^4x\mathcal{L}_{\chi}^1(x)}|0> \nonumber \\
= \sum_{n=1}^{\infty} i \int d^4p_1 \frac{d^4p_2}{(2\pi)^4} \cdots \frac{d^4p_n}{(2\pi)^4} \Pi_n(p_1, \cdots, p_n) \delta^4(p_1 - p_2 - \cdots - p_n) \nonumber \\
= i\Pi_1(0) + \sum_{n=2}^{\infty} i \int \frac{d^4p_1}{(2\pi)^4} \cdots \frac{d^4p_{n-1}}{(2\pi)^4} \Pi_n(p_1, \cdots, p_{n-1}) , \tag{17}
\]

where $\mathcal{T}_q$ is time-order product of constituent quark fields, $\mathcal{L}_{\chi}^1$ is interaction part of lagrangian(13), $\Pi_n(p_1, \cdots, p_n)$ is one-loop effects of constituent quarks with $n$ external sources, $p_1, p_2, \cdots, p_n$ are four-momentas of $n$ external sources respectively and
\[ \Pi_n(p_1, \ldots, p_{n-1}) = \int d^4p_n \Pi_n(p_1, \ldots, p_n) \delta^4(p_1 - p_2 - \cdots - p_n). \]  

(18)

To get rid of all disconnected diagrams, we have

\[ S_{\text{eff}} = \sum_{n=1}^{\infty} S_n, \]

\[ S_1 = \Pi_1(0), \]

\[ S_n = \int \frac{d^4p_1}{(2\pi)^4} \cdots \frac{d^4p_{n-1}}{(2\pi)^4} \Pi_n(p_1, \ldots, p_{n-1}), \quad (n \geq 2). \]

(19)

Hereafter we will call \( S_n \) as \( n \)-point effective action.

The \( S_1 \) is tadpole-loop contribution of constituent quarks, which is independent of the purpose this paper. The two-point effective action \( S_2 \) has been evaluated in I,

\[ S_2 = \frac{F_0^2}{16} \int d^4x < \nabla_\mu U \nabla^\mu U^\dagger > + \int \frac{d^4q}{(2\pi)^4} \Pi_2(q), \]

\[ \Pi_2(q) = -\frac{1}{2} A(q^2)(q^2 \delta_{\mu\nu} - q_\mu q_\nu) < V^\mu(q)V^\nu(-q) >, \]

(20)

where kinetic term of pseudoscalar mesons has been renormalized, and \( A(q^2) \) is defined as follow

\[ A(q^2) = g^2 - \frac{N_c}{\pi^2} \int_0^1 dt \cdot t(1 - t) \ln (1 - \frac{t(1-t)q^2}{m^2}). \]

(21)

Here a universal constant of the model, \( g \), is defined to absorb logarithmic divergence from quark loop integral

\[ g^2 = \frac{8}{3} \frac{N_c}{(4\pi)^{D/2}} \left( \frac{m^2}{\mu^2} \right)^{D/2} \Gamma(2 - \frac{D}{2}). \]

(22)

In I we have fitted \( g = \pi^{-1} \sqrt{N_c/3} \) which satisfy the first KSRF sum rule [23] rigorously.

A. Three-point effective action

Up to the leading order of light current quark masses, there are three kinds of three-point effective action. They are made up of by external sources \( V\Delta\Delta \), \( VVS \) and \( V\Delta P \) respectively. The effective action with external source \( V\Delta\Delta \) has been obtained in I,

\[ S_3^{(1)} = -\frac{g_4^2}{2} \int d^4x \int \frac{d^4q}{(2\pi)^4} e^{iq\cdot x} B(q^2)q^\mu < \tilde{V}^\nu(q)|\Delta_\mu(x), \Delta_\nu(x)| >, \]

(23)

where

\[ B(q^2) = -g^2 + \frac{N_c}{2\pi^2} \int_0^1 dt_1 \cdot t_1 \int_0^1 dt_2(1 - t_1t_2)[1 + \frac{m^2}{m^2 - t_1(1 - t_1)(1 - t_2)q^2}] \ln (1 - \frac{t_1(1-t_1)(1-t_2)q^2}{m^2}), \]

(24)

Let us now calculate three-point effective action with external source \( V\tilde{V}S \). Note that since \( \tilde{V}_\mu = V_\mu + ieQ A_\mu + i\pi \partial_\mu \pi + \cdots \), for \( \omega - \rho^0 \) mixing or \( \omega - \pi \) coupling vertices, scalar external source \( S \) will reduce to constant matrix.

\[ iS_3^{(2)} = \frac{i}{2} \int d^4x d^4y d^4z \cdot \tilde{V}_\mu^a(x) V_\nu^b(y) S^c < 0|T \{ \bar{q}(x)\gamma^\mu \lambda^a q(x)\bar{q}(y)\gamma^\nu \lambda^b q(y)\bar{q}(z)\lambda^c q(z)\}|0 > \]

\[ = -i \int \frac{d^4q}{(2\pi)^4} \frac{d^4l}{(2\pi)^4} < \tilde{V}_\mu(q)\tilde{V}_\nu(-q)S > Tr_{c,L} \{ S_F(l)\gamma^\nu S_F^\dagger(l + q)\gamma^\mu \}, \]

(25)

where \( Tr_{c,L} \) denotes trace taking over color and Lorentz space, \( S_F(k) = i(k - m + ie)^{-1} \) is propagator of constituent quark fields in momentum space. The direct calculation will give
\[ S_3^{(2)} = \frac{N_c}{12\pi^2 m} \int d^4x \int \frac{d^4q}{(2\pi)^4} e^{iq} h_0(q^2)(q^2 \delta_{\mu\nu} - q_\mu q_\nu) < \bar{V}^\mu(q) \bar{V}^\nu(x) S(x) > \]
\[ = \frac{N_c}{24\pi^2 m} \int d^4x \int \frac{d^4q}{(2\pi)^4} e^{iq} h_0(q^2)(q^2 \delta_{\mu\nu} - q_\mu q_\nu) < \bar{V}^\mu(q) \bar{V}^\nu(x) (\xi^\dagger \xi + \xi \xi^\dagger) >, \]
where
\[ h_0(q^2) = \int_0^1 dt \frac{6t(1-t)}{1-t(1-t)q^2/m^2}. \]

Next, we calculate three-point effective action with external source \( \bar{V} \Delta P \).
\[ iS_3^{(3)} = -g_A \int d^4x d^4y d^4z \cdot \langle 0 | T \{ \bar{q}(x) \gamma^\mu \lambda^\alpha q(x) \bar{q}(y) \gamma^\nu \gamma_5 \lambda^\beta q(y) \bar{q}(z) \gamma_5 \lambda^\gamma q(z) \} | 0 \rangle > \bar{V}_\mu(x) \Delta_\nu(y) P^c(z) \]
\[ = g_A \int \frac{d^4q}{(2\pi)^4} \frac{d^4k}{(2\pi)^4} \frac{d^4l}{(2\pi)^4} \{ \bar{V}_\mu(q) \Delta_\nu(k) P(-q-k) > Tr_{(c,L)}[S_F(l)\gamma^\alpha \gamma_5 S_F(l-k)\gamma_5 S_F(l+q)\gamma^\mu] \]
\[ + < \bar{V}_\mu(q) P(-q-k) \Delta_\nu(k) > Tr_{(c,L)}[S_F(l)\gamma^\mu S_F(l-q)\gamma_5 S_F(l+k)\gamma^\nu \gamma_5] \}. \]
Due to \( \Delta_\mu \propto \partial_\mu \pi + \cdots \) and \( P \propto \pi + \cdots \), for purpose of this paper, the soft pion theorem tells us \( k^2 \to 0 \) and \( (k + q)^2 \to 0 \). In addition, we can find that \( \kappa \Delta^\mu(k) \Rightarrow k^2 \pi(k) \to 0 \). Then performing the loop-integral in eq.(28), and employing the above discussion in our calculation, we obtain
\[ S_3^{(3)} = -\frac{N_c m}{4\pi^2} g_A \int \frac{d^4q}{(2\pi)^4} \frac{d^4k}{(2\pi)^4} \frac{d^4l}{(2\pi)^4} \{ \alpha_1(q^2) < [\bar{V}_\mu(q), \Delta_\nu(k)] P(-q-k) > \]
\[ + \alpha_2(q^2) (q^2 \delta_{\mu\nu} - q_\mu q_\nu) < \bar{V}_\nu(q), \Delta_\mu(k) ] P(-k-q) \}, \]
where
\[ \alpha_1(q^2) = \left( \frac{4\pi\mu^2}{m^2} \right)^{1/2} (2 - D/2) \int_0^1 dt_1 \int_0^1 dt_2 \{ t_1 t_2 (1-t_1) q^2 \}
\[ + \frac{t_1 t_2 (1-t_1) q^2}{m^2 - t_1 (1-t_1)(1-t_2) q^2}, \]
\[ \alpha_2(q^2) = \int_0^1 dt_1 \int_0^1 dt_2 \frac{2t_1 t_2 (1-t_1)}{m^2 - t_1 (1-t_1)(1-t_2) q^2}. \]

The third three-point effective action \( S_3^{(3)} \) is \( O(m_q) \) and free parameter \( \kappa \)-dependent. However, if \( \bar{V}_\mu = \omega_\mu \), \( S_3^{(3)} \) vanish, and if \( \bar{V}_\mu = \rho_\mu \lambda^3 \), \( S_3^{(3)} \) provide an isospin conservation \( p\pi \pi \) vertex which is order \( m_u + m_d \) and much smaller that leading order vertex. Thus the contribution from \( S_3^{(3)} \) will be omitted in this paper.

**B. Four-point effective action**

There is only one four-point effective action relating to \( \omega \to \pi^+ \pi^- \) decay. It is made up of by four external source \( \bar{V} \Delta S \). As shown in the above subsection, here \( S \) reduces to a constant matrix.
\[ iS_4 = \frac{g^2}{2} \int d^4x d^4y d^4z d^4w \cdot \bar{V}_\mu(x) \Delta_\nu(y) \Delta_\sigma(z) S^d(w) \times [ \langle 0 | T \{ \bar{q}(x) \gamma^\mu \lambda^\alpha q(x) \bar{q}(y) \gamma^\nu \gamma_5 \lambda^\beta q(y) \bar{q}(z) \gamma^\sigma \gamma_5 \lambda^\gamma q(z) \bar{q}(w) \lambda^\delta q(w) \} | 0 \rangle > \]
\[ = g_A \int \frac{d^4q}{(2\pi)^4} \frac{d^4k}{(2\pi)^4} \frac{d^4l}{(2\pi)^4} \frac{d^4\ell}{(2\pi)^4} \{ \bar{V}_\mu(q) \Delta_\nu(k) \Delta_\sigma(-k-q) S > Tr_{(c,L)}[S_F^2(l)\gamma^\mu S_F^2(l-q)\gamma^\nu \gamma_5 S_F^2(l-k)\gamma^\sigma \gamma_5] \]
\[ + \bar{V}_\mu(q) \Delta_\nu(k) \Delta_\sigma(-k-q) S > Tr_{(c,L)}[S_F^2(l+q+k)\gamma^\mu S_F^2(l+k)\gamma^\nu \gamma_5 S_F^2(l+q)\gamma^\sigma \gamma_5] \]
\[ + \bar{V}_\mu(q) \Delta_\nu(k) \Delta_\sigma(-k-q) S > Tr_{(c,L)}[S_F^2(l+q)\gamma^\mu S_F^2(l)\gamma^\nu \gamma_5 S_F^2(l+k)\gamma^\sigma \gamma_5] \}. \]
To perform integral of four-momenta \( l \) in the above equation and employ indentities in Appendix to simplify result, we can obtain

\[
S_4 = - \frac{N_c}{8\pi^2g^2m^2} \int d^4x \int \frac{d^4q}{(2\pi)^4} e^{iqx} (\delta_{\mu\nu}q_\sigma - \delta_{\mu\sigma}q_\nu) \\
\times \{ h_1(q^2) < \{ \bar{V}^\mu(q), S(x) \} \Delta^\nu(x) \Delta^\sigma(x) > + h_2(q^2) < \bar{V}^\mu(q) \Delta^\nu(x) S(x) \Delta^\sigma(x) > \},
\]

where

\[
h_1(q^2) = \int_0^1 dt_1 \cdot t_1^2 \int_0^1 dt_2 (1 - t_2) \frac{3 - 2t_1^2 t_2 (1 + 2t_1)(1 - t_2)q^2/m^2}{[1 - t_1^2 t_2 (1 - t_2)q^2/m^2]^2},
\]

\[
h_2(q^2) = \int_0^1 dt_1 \cdot t_1^2 \int_0^1 dt_2 (1 - t_2) \frac{4(1 - t_1)[3 - 4t_1^2 t_2 (1 - t_2)q^2/m^2]}{[1 - t_1^2 t_2 (1 - t_2)q^2/m^2]^2}.
\]

C. Relevant effective vertices at tree level

In the following we will give all relevant effective vertice at tree level. Since one-loop correction of pseudoscalar mesons will be calculated, we also need to include four-pseudoscalar meson vertices, which will be derived from \( O(p^3) \) effective lagrangian of this formalism. The effective vertices involving electromagnetic interaction have been calculated up to one-loop level in \( \mathbf{I} \). We will quote them in sect. 5. Directly. All effective vertices can be divided into two part: one is isospin conservation and another is isospin broken. In addition, we should point out that, so far, all meson fields are still non-physical. The physical meson fields can be obtained via the following field rescaling which make kinetic terms of pseudoscalar mesons and vector mesons into standard form

\[
\rho_\mu \rightarrow \frac{1}{g} \rho_\mu, \quad \omega_\mu \rightarrow \frac{1}{g} \omega_\mu,
\]

\[
\pi \rightarrow \frac{2}{f_\pi} \pi, \quad K \rightarrow \frac{2}{f_\pi} K.
\]

Since in this paper \( K \)-mesons only appear as intermediate states in one-loop diagrams, for sake of convenience, we neglect the difference between \( f_\pi \) and \( f_K \) (since the results yielded by this difference are twofold suppressed by light current quark mass expansion and \( N_c^{-1} \) expansion).

The \( \rho^0 - \omega \) mixing vertex, which breaks isospin symmetry, is included in eq. (34)

\[
\mathcal{L}_{\rho^0 - \omega} = \frac{N_c}{12\pi^2g^2m^2} \int \frac{d^4q}{(2\pi)^4} e^{iqx} h_0(q^2) (q^2 \delta_{\mu\nu} - q_\mu q_\nu) \omega^\mu(q) \rho^{0\nu}(x) < \lambda^3 \xi M \xi + \xi^\dagger M \xi^\dagger) > \tag{35}
\]

\[
= \frac{N_c}{6\pi^2g^2m^2} \frac{m_u - m_d}{m} \int \frac{d^4q}{(2\pi)^4} e^{iqx} h_0(q^2) (q^2 \delta_{\mu\nu} - q_\mu q_\nu) \omega^\mu(q) \rho^{0\nu}(x) + \cdots. \tag{36}
\]

The isospin symmetry unbroken vector\( \to \phi \phi \) vertex is included in eqs. (26) and (23)

\[
\mathcal{L}_{\phi \phi}^{(\Delta I = 0)} = -2 \int \frac{d^4q}{(2\pi)^4} e^{iqx} b(q^2) \phi^\mu S(x) \phi^\nu \Delta^\mu(x) \Delta^\nu(x) >, \tag{37}
\]

where

\[
b(q^2) = \frac{1}{g f_\pi^2} [A(q^2) + g_A^2 B(q^2)]. \tag{38}
\]

The isospin symmetry broken vector\( \to \phi \phi \) vertex is include in eqs. (26) and (22)

\[
\mathcal{L}_{\phi \phi}^{(\Delta I = 1)} = - \frac{N_c}{12\pi^2m} \int \frac{d^4q}{(2\pi)^4} e^{iqx} (\delta_{\mu\nu}q_\sigma - \delta_{\mu\sigma}q_\nu) \\
\times \{ [h_0(q^2) + \frac{3}{4}g_A^2h_1(q^2)] < \{ \bar{V}^\mu(q), \xi M \xi + \xi^\dagger M \xi^\dagger \} \Delta^\nu(x) \Delta^\sigma(x) >
\]

\[
- \frac{3}{4} g_A^2 h_2(q^2) < \bar{V}^\mu(q) \Delta^\nu(x)(\xi M \xi + \xi^\dagger M \xi^\dagger) \Delta^\sigma(x) > \}. \tag{39}
\]
In particular, define

\[ s(q^2) = \frac{4}{gf_f^2} [h_0(q^2) + 3\frac{4}{3}g_A^2(h_1(q^2) - \frac{h_2(q^2)}{2})], \quad (40) \]

we have

\[
L_{\rho^0\pi\pi} = i \int \frac{d^4q}{(2\pi)^4} e^{iq\cdot x} b(q^2)(q^2 \delta_{\mu\nu} - q_{\mu}q_{\nu})\rho^{0\mu}(q)[\pi^+(x)\partial^\nu\pi^-(x) - \partial^\nu\pi^+(x)\pi^-(x)]
\]

\[
L_{\omega^\pi\pi} = -iN_c \frac{m_u - m_d}{m} \int \frac{d^4q}{(2\pi)^4} e^{iq\cdot x} s(q^2)(q^2 \delta_{\mu\nu} - q_{\mu}q_{\nu})\omega^\mu(q)
\times \left\{ \pi^+(x)\partial^\nu\pi^-(x) - \partial^\nu\pi^+(x)\pi^-(x) \right\}
\]

\[
L^{(\Delta I=0)}_{\rho^0KK} = \frac{i}{2} \int \frac{d^4q}{(2\pi)^4} e^{iq\cdot x} b(q^2)(q^2 \delta_{\mu\nu} - q_{\mu}q_{\nu})\rho^{0\mu}(q)
\times \left\{ \{K^+\partial^\nu K^-(x) - \partial^\nu K^+(x)K^-(x)\} - [K^0(x)\partial^\nu \bar{K}^0(x) - \partial^\nu K^0(x)\bar{K}^0(x)] \right\}
\]

\[
L^{(\Delta I=1)}_{\rho^0KK} = -iN_c \frac{m_u - m_d}{m} \int \frac{d^4q}{(2\pi)^4} e^{iq\cdot x} s(q^2)(q^2 \delta_{\mu\nu} - q_{\mu}q_{\nu})\rho^{0\mu}(q)
\times \left\{ \{K^+\partial^\nu K^-(x) - \partial^\nu K^+(x)K^-(x)\} + [K^0(x)\partial^\nu \bar{K}^0(x) - \partial^\nu K^0(x)\bar{K}^0(x)] \right\}
\]

\[
L^{(\Delta I=0)}_{\omega KK} = \frac{i}{2} \int \frac{d^4q}{(2\pi)^4} e^{iq\cdot x} b(q^2)(q^2 \delta_{\mu\nu} - q_{\mu}q_{\nu})\omega^\mu(q)
\times \left\{ \{K^+\partial^\nu K^-(x) - \partial^\nu K^+(x)K^-(x)\} + [K^0(x)\partial^\nu \bar{K}^0(x) - \partial^\nu K^0(x)\bar{K}^0(x)] \right\}
\]

\[
L^{(\Delta I=1)}_{\omega KK} = -iN_c \frac{m_u - m_d}{m} \int \frac{d^4q}{(2\pi)^4} e^{iq\cdot x} s(q^2)(q^2 \delta_{\mu\nu} - q_{\mu}q_{\nu})\omega^\mu(q)
\times \left\{ \{K^+\partial^\nu K^-(x) - \partial^\nu K^+(x)K^-(x)\} - [K^0(x)\partial^\nu \bar{K}^0(x) - \partial^\nu K^0(x)\bar{K}^0(x)] \right\}
\]

Up to \(O(p^4)\), the tree level four-pseudoscalar meson effective lagrangian has been derived in I. The isospin symmetry unbroken four-pseudoscalar vertex is included in the following lagrangian

\[
L^{(\Delta I=0)}_{4\varphi} = \frac{f_\pi^2}{16} < \nabla_\mu U^\dagger \nabla^\mu U > + \frac{N_c}{12(4\pi)^2} \frac{m}{f_\pi^2} < \nabla_\mu U^\dagger \nabla^\mu U > + \frac{N_c}{12(4\pi)^2} (1 - g_A^4) < \nabla_\mu U^\dagger \nabla^\mu U > , \quad (42)
\]

where we have used \(g^2 = N_c/(3\pi^2)\). The isospin symmetry broken four-pseudoscalar vertex is proportional to \(m_d - m_u\), which is included in the following lagrangian

\[
L^{(\Delta I=1)}_{4\varphi} = \frac{f_\pi^2}{8} B_0 < \mathcal{M}(U + U^\dagger) > + \frac{N_c m}{(4\pi)^2 g_A^2} < \nabla_\mu U^\dagger \nabla^\mu U > (\mathcal{M}U^\dagger + U\mathcal{M}) > . \quad (43)
\]

I can be found that the eqs.(52)-(53) are free parameter \(\kappa\) independent. Moreover, we can see that every vector \(\varphi\varphi\) in eq.(11) includes an antisymmetry factor \((q^2 \delta_{\mu\nu} - q_{\mu}q_{\nu})\) (where \(q\) denotes four-momenta of vector mesons). Thus the first term of eq.(12) does not contribute to \(\omega \to \pi^+\pi^-\) decay via pseudoscalar meson loops. This antisymmetry factor also constrains that the vertices with one of factors \(KK, \partial_\mu \bar{K}^0\partial^\mu K\) and \(K^0 K^0\partial_\mu \bar{K}^0\partial^\mu K\) do not contribute to \(\omega \to \pi^+\pi^-\) decay via pseudoscalar meson loops. Then the relevant four-pseudoscalar vertices can explicitly read as follows,
IV. ONE-LOOP CORRECTIONS OF PSEUDOSCALAR MESONS

In this section we calculate one-loop correction of mesons. Because of \( m_v^2 > m_K^2 > m_{\pi}^2 \), it can be expected that the dominant contribution comes from one-loop diagrams of pseudoscalar mesons. In addition, we can treat pion as massless particle but must take \( m_K^2 \neq 0 \). This difference is very important, since \( \pi \)-loop yields imaginary part of \( T \)-matrix but \( K \)-loop does not at \( m_\omega \) scale. In our calculation, the mass difference between \( K^\pm \) and \( K^0 \) is also neglected.

There are three kinds of one-loop diagrams correcting to “direct” \( \omega \pi \pi \) coupling and \( \omega - \rho^0 \) mixing(fig. 1 and fig. 2).

\[
\begin{align*}
K, \eta_8 & \quad \pi \quad (a) \\
\omega & \quad \pi \\
\pi & \\
\end{align*}
\]

FIG. 1. One-loop correction to “direct” \( \omega \pi \pi \) coupling.

\[
\begin{align*}
K, \eta_8 & \quad \rho \\
\omega & \quad \pi \\
\pi & \\
\end{align*}
\]

FIG. 2. One-loop correction to \( \omega - \rho^0 \) mixing.

It must be pointed out that, in \( T \)-matrices yielded by figure 1-(b) and figure 2-(b), the contribution of imaginary part is dominant. We have shown in I that it can not ensure unitarity of \( S \)-matrix if we only calculate figure 1-(b) and figure 2-(b). The unitarity can be ensured through summing over all diagrams in chain approximation(fig. 3 and fig. 4).

\[
\begin{align*}
\omega & \quad \pi \\
\pi & + \omega \quad \pi \\
\pi & \\
\pi & \\
\omega & \\
\pi & \\
\pi & + \cdots
\end{align*}
\]

FIG. 3. Chain approximation for “direct” \( \omega \pi \pi \) coupling.

\[
\begin{align*}
\omega & \quad \rho \\
\pi & + \omega \quad \pi \\
\pi & + \omega \quad \pi \\
\omega & \\
\pi & + \cdots
\end{align*}
\]

FIG. 4. Chain approximation for \( \omega - \rho^0 \) mixing.

A. Tadpole diagram

Since pion is treated as massless particle, the nonzero tadpole diagram contribution is yielded by \( K \) or \( \eta_8 \) mesons(fig. 1-(a) and fig. 2-(a)). For sake of convenience, here we assume \( m_{\eta_8} = m_K \).

A Correction to \( \omega - \rho^0 \) mixing

The tree level \( \omega \rho^0 \varphi \varphi \) is contained in eq.(35),
\[ \mathcal{L}_{\omega \rho \varphi} = -\frac{N_c}{12\pi^2 g^2 f_{\pi}^2 m} \int \frac{d^4q}{(2\pi)^4} e^{iq \cdot x} h_0(q^2)(q^2 \delta_{\mu \nu} - q_\mu q_\nu) \omega^\mu(q) \rho^{0\nu}(x) \times \lambda^3(\varphi^2 M + M \varphi^2 + \varphi M \varphi) > . \] (45)

In momentum space, the calculation on fig. 2-(a) is straightforward

\[ \mathcal{L}_{\omega \rho \varphi}^{(tad)} = -\frac{N_c}{12\pi^2 g^2 f_{\pi}^2 m} \int \frac{d^4q}{(2\pi)^4} e^{iq \cdot x} h_0(q^2)(q^2 \delta_{\mu \nu} - q_\mu q_\nu) \omega^\mu(q) \rho^{0\nu}(x) \times \lambda^3(\lambda^0 \lambda^a, M \lambda^a + \lambda^a M \lambda^a) > . \] (46)

The generators \( \lambda^a \) of SU(N) obey the completeness relations

\[ \sum_{a=1}^{N^2-1} \lambda^a A \lambda^a B = -\frac{2}{N} < AB > + 2 < A > < B > , \]

\[ \sum_{a=1}^{N^2-1} \lambda^a A > < \lambda^a B > = 2 < AB > - \frac{2}{N} < A > < B > . \] (47)

Then we have

\[ \sum_{a=1}^{8} \lambda^3(\lambda^0 \lambda^a, M \lambda^a + \lambda^a M \lambda^a) > = \frac{16}{3} < \lambda^3 M > = \frac{16}{3}(m_u - m_d) . \] (48)

Substituting eq. (48) into eq. (46) and performing loop integral, we obtain

\[ \mathcal{L}_{\omega \rho \varphi}^{(tad)} = -\frac{4}{3} \frac{N_c}{6\pi^2 g^2} \frac{m_u - m_d}{m} \int \frac{d^4q}{(2\pi)^4} e^{iq \cdot x} h_0(q^2)(q^2 \delta_{\mu \nu} - q_\mu q_\nu) \omega^\mu(q) \rho^{0\nu}(x) , \] (49)

where

\[ \zeta = \frac{2\lambda}{(4\pi)^2} \frac{m^2_{\pi}}{f_{\pi}^2} \quad \lambda = \left( \frac{4\mu^2}{m^2_{\rho}} \right)^{\epsilon/2} \Gamma(1 - \frac{D}{2}) . \] (50)

Here we define a constant \( \lambda \) to absorbe quadratic divergence from meson loop integral. Its value has been determined as \( \lambda = 2/3 \) in \( I \) by OZI rule.

**B Correction to “direct” \( \omega \pi \pi \) mixing**

The isospin symmetry broken \( \omega - 4 \varphi \) coupling vertex is included in eq. (33). Expanding eq. (33) to contain four pseudoscalar meson fields, we can obtain

\[ \mathcal{L}_{\omega \pi \pi}^{(tad)} = -\frac{N_c}{12\pi^2 f_{\pi}^2 m} \int \frac{d^4q}{(2\pi)^4} e^{iq \cdot x} s(q^2)(\delta_{\mu \nu} q_\sigma - \delta_{\mu \sigma} q_\nu) \omega^\mu(q) \int \frac{d^4k}{(2\pi)^4} k^2 - m^2_{\pi} + i\epsilon \]

\[ \times \sum_{a=1}^{8} \left\{ I_2 M(\partial^\nu \pi \lambda^a [\lambda^a, \partial^\sigma \pi] + \lambda^a [\lambda^a, \partial^\nu \pi] \partial^\sigma \pi) > + \frac{1}{2} I_2 \partial^\nu \pi \partial^\sigma \pi (\lambda^a \lambda^a M + M \lambda^a \lambda^a + \lambda^a M \lambda^a) \right\} \]

\[ = \frac{10}{3} \zeta \frac{iN_c}{12\pi^2} \frac{m_u - m_d}{m} \int \frac{d^4q}{(2\pi)^4} e^{iq \cdot x} s(q^2)(q^2 \delta_{\mu \nu} - q_\mu q_\nu) \omega^\mu(q) \]

\[ \times [\pi^+(x) \partial^\nu \pi^-(x) - \partial^\nu \pi^+(x) \pi^-(x) ] , \] (51)

where \( I_2 = \text{diag}\{1, 1, 0\}, s(q^2) \), to see eq. (40) and eq. (47) has been used.
B. $K$-loop contribution

Here $K$-loop denotes that one-loop diagrams in fig.1-(c) and fig.2-(c). In this subsection, $T$ will denote time-order product of $K$-meson field.

A Correction to $\omega-\rho^0$ mixing

The $\omega-\rho^0$ effective action yielded by $K$-loop is follow

$$ iS_{\omega-\rho^0}^{(K\text{-loop})} = -\int d^4x d^4y <0|\mathcal{T}\{\mathcal{L}^{(\Delta I=0)}_{\omega KK}(x)\mathcal{L}^{(\Delta I=1)}_{\rho KK}(y) + \mathcal{L}^{(\Delta I=1)}_{\rho KK}(x)\mathcal{L}^{(\Delta I=0)}_{\omega KK}(y)\}|0> $$

$$ = \frac{N_c}{3\pi^2} \frac{m_u - m_d}{m} \int \frac{d^4q}{(2\pi)^4} \frac{d^4k}{(2\pi)^4} b(q^2)s(q^2)(q^2\delta_{\mu\nu} - q_\mu q_\nu)(q^2\delta_{\alpha\beta} - q_\alpha q_\beta) \times \omega^\mu(q)\rho^0\nu(-(q)(l + q))\Delta_K(l)\Delta_K(l + q), $$

(52)

where $\Delta_K(l) = i(l^2 - m_K^2 + i\epsilon)^{-1}$ is propagator of $K$-meson. Integrating over $l_\mu$ in the above equation and defining

$$ \Sigma_k(q^2) = \frac{1}{(4\pi)^2}\{\lambda(m_K^2 - q^2/6) + \int_0^1 dt[m_K^2 - t(1-t)q^2]\ln(1 - \frac{t(1-t)q^2}{m_K^2})\}, $$

(53)

we have

$$ iS_{\omega-\rho^0}^{(K\text{-loop})} = \frac{N_c}{6\pi^2} \frac{m_u - m_d}{m} \int \frac{d^4q}{(2\pi)^4} b(q^2)s(q^2)\Sigma_k(q^2)q^2(q^2\delta_{\mu\nu} - q_\mu q_\nu)\omega^\mu(q)\rho^0\nu(x). $$

(54)

The corresponding effective lagrangian reads

$$ \mathcal{L}_{\omega-\rho^0}^{(K\text{-loop})} = \frac{N_c}{6\pi^2} \frac{m_u - m_d}{m} \int \frac{d^4q}{(2\pi)^4} e^{i q x} b(q^2)s(q^2)\Sigma_k(q^2)q^2(q^2\delta_{\mu\nu} - q_\mu q_\nu)\omega^\mu(q)\rho^0\nu(x). $$

(55)

B Correction to “direct” $\omega\pi\pi$ coupling

The “direct” $\omega\pi\pi$ coupling effective action yielded by $K$-loop can be evaluated as follow

$$ iS_{\omega\pi\pi}^{(K\text{-loop})} = -\int d^4x d^4y <0|\mathcal{T}\{\mathcal{L}^{(\Delta I=0)}_{\omega KK}(x)\mathcal{L}^{(\Delta I=1)}_{\pi KK}(y) + \mathcal{L}^{(\Delta I=1)}_{\omega KK}(x)\mathcal{L}^{(\Delta I=0)}_{\pi KK}(y)\}|0> $$

$$ = - \frac{N_c}{\pi^2 f_\pi^2} \frac{m_u - m_d}{m} \int \frac{d^4q}{(2\pi)^4} \frac{d^4k}{(2\pi)^4} \frac{d^4l}{(2\pi)^4} [16m^2 f_\pi^{-2}b(q^2) - \frac{2}{3} s(q^2) - \frac{3}{18}\Delta_K(l)\Delta_K(l + q)] $$

$$ \times (q^2\delta_{\mu\nu} - q_\mu q_\nu)(k \cdot l)\omega^\mu(q)\pi^+(\nu - \kappa)\pi^-(\kappa)\Delta_K(l)\Delta_K(l + q) $$

$$ \simeq - \frac{N_c}{\pi^2 f_\pi^2} \frac{m_u - m_d}{m} \int \frac{d^4q}{(2\pi)^4} \frac{d^4k}{(2\pi)^4} [8m^2 f_\pi^{-2}b(q^2) - \frac{1}{3} s(q^2)(1 + q^2 N_c/4\pi^2 f_\pi^2)] \Sigma_k(q^2) $$

$$ \times (q^2\delta_{\mu\nu} - q_\mu q_\nu)k^\nu\omega^\mu(q)\pi^+(\nu - \kappa)\pi^-(\kappa), $$

(56)

where we have taken soft pion limit and $3 - g_A^4 \simeq 3$ due to $g_A^4 \ll 0.3$. The corresponding effective lagrangian reads

$$ \mathcal{L}_{\omega\pi\pi}^{(K\text{-loop})} = \frac{iN_c}{2\pi^2 f_\pi^2} \frac{m_u - m_d}{m} \int \frac{d^4q}{(2\pi)^4} e^{i q x} [8m^2 f_\pi^{-2}b(q^2) - \frac{1}{3} s(q^2)(1 + q^2 N_c/4\pi^2 f_\pi^2)] \Sigma_k(q^2) $$

$$ \times (q^2\delta_{\mu\nu} - q_\mu q_\nu)\omega^\mu(q)\pi^+(x)\partial^\nu\pi^-(x) - \partial^\nu\pi^+(x)\partial^\nu\pi^-(x). $$

(57)

C. Chain contribution of $\pi$-loop

Finally, we calculate chain approximation corrections of $\pi$-loop in fig. 3 and fig. 4.

A Correction to “direct” $\omega\pi\pi$ coupling
The effective action yielded by $\pi$-loop in fig. 1-(b) is evaluated as follows

$$iS^{(\pi\text{-loop})}_{\omega\pi} = -\int d^4xd^4y \langle 0 | T \{ \mathcal{L}_{\omega\pi}(x) \mathcal{L}_{\omega\pi}(y) \} | 0 \rangle$$

$$= \frac{4N_c}{3\pi^2 f_\pi^2} m_u - m_d \int \frac{d^4q}{(2\pi)^4} \frac{d^4k}{(2\pi)^4} \frac{d^4l}{(2\pi)^4} s(q^2) \left( 1 + \frac{q^2 N_c}{4\pi^2 f_\pi^2} \right)^2 \delta_{\mu\nu} - q_\mu q_\nu \right)$$

$$\times (l \cdot k)^\nu \omega^\mu(q) \pi^+(q) \pi^-(q) \Delta_+(l) \Delta_+(l + k), \quad (58)$$

where we have employed soft pion limit and $g_A^4 \ll 3$, and $\Delta_+(l) = i(l^2 + i\epsilon)^{-1}$ is propagator of pion. Integrating over $l$ in the above equation and defining

$$\Sigma_\pi(q^2) = \frac{q^2}{(4\pi)^2} \left\{ \frac{\lambda}{6} + \int_0^1 dt \cdot t(1 - t) \ln \frac{t(1 - t)q^2}{m^2} + \frac{i}{6} \text{Arg}(-1) \theta(q^2 - 4m^2_\pi) \right\}, \quad (59)$$

we have

$$S^{(\pi\text{-loop})}_{\omega\pi} = \frac{2N_c}{3\pi^2 f_\pi^2} m_u - m_d \int \frac{d^4q}{(2\pi)^4} \frac{d^4k}{(2\pi)^4} \frac{d^4l}{(2\pi)^4} s(q^2) \left( 1 + \frac{q^2 N_c}{4\pi^2 f_\pi^2} \right)^2 \Sigma_\pi(q^2)$$

$$\times (q^2 \delta_{\mu\nu} - q_\mu q_\nu) \omega^\mu(q) \pi^+(q) \pi^-(q) \left[ \pi^+(x) \partial^\nu \pi^-(x) - \partial^\nu \pi^+(x) \pi^-(x) \right]. \quad (60)$$

In eq. (59), $\text{Arg}(-1) = -\pi$ has been fitted in $I$ due to requirement of unitarity. The corresponding effective lagrangian reads

$$\mathcal{L}^{(\pi\text{-loop})}_{\omega\pi} = \frac{iN_c}{12\pi^2} m_u - m_d \int \frac{d^4q}{(2\pi)^4} e^{iq\cdot x} s(q^2) \left( 1 + \frac{q^2 N_c}{4\pi^2 f_\pi^2} \right)^2 \Sigma_\pi(q^2)$$

$$\times (q^2 \delta_{\mu\nu} - q_\mu q_\nu) \omega^\mu(q) \pi^+(x) \partial^\nu \pi^-(x) - \partial^\nu \pi^+(x) \pi^-(x). \quad (61)$$

Comparing eq. (59) and tree level vertex(11), we can find that every one-loop in fig.3 contributes a factor

$$- \Xi(q^2) = -4f_\pi^2 \left( 1 + \frac{q^2 N_c}{4\pi^2 f_\pi^2} \right)^2 \Sigma_\pi(q^2). \quad (62)$$

Thus summing over all diagrams in fig. 3, we obtain

$$\mathcal{L}_{\omega\pi} = \frac{iN_c}{12\pi^2} m_u - m_d \int \frac{d^4q}{(2\pi)^4} e^{iq\cdot x} s(q^2) \left( 1 + \frac{q^2 N_c}{4\pi^2 f_\pi^2} \right)^2 \Sigma_\pi(q^2)$$

$$\times (q^2 \delta_{\mu\nu} - q_\mu q_\nu) \omega^\mu(q) \pi^+(x) \partial^\nu \pi^-(x) - \partial^\nu \pi^+(x) \pi^-(x). \quad (63)$$

B Correction to $\omega - \rho^0$ mixing

If in fig. 2-(b), tree level vertex $\mathcal{L}_{\omega\pi}$ is replaced by $\mathcal{L}_{\omega\pi}$ which contains all diagram contribution in fig. 3, then summing tree diagram and fig. 2-(b) is just chain approximation correction to $\omega - \rho^0$ mixing. The effective action yielded by $\pi$-loop in fig. 2-(b) is evaluated as follow

$$iS^{(\pi\text{-loop})}_{\omega\rho} = -\int d^4xd^4y \langle 0 | T \{ \mathcal{L}_{\omega\pi}(x) \mathcal{L}_{\rho\pi}(y) \} | 0 \rangle$$

$$= \frac{N_c}{6\pi^2} m_u - m_d \int \frac{d^4q}{(2\pi)^4} b(q^2) s(q^2) \Sigma_\pi(q^2) \frac{q^2 \delta_{\mu\nu} - q_\mu q_\nu}{1 + \Xi(q^2)} \omega^\mu(q) \rho^\nu(-q). \quad (64)$$

Thus chain approximation of fig. 4 yield effective lagrangian as follow

$$\mathcal{L}^*_{\omega\rho} = \frac{N_c}{6\pi^2} m_u - m_d \int \frac{d^4q}{(2\pi)^4} e^{iq\cdot x} \left( g^{-2} h_0(q^2) - \frac{q^2}{(1 + \Xi(q^2))} b(q^2) s(q^2) \Sigma_\pi(q^2) \left( q^2 \delta_{\mu\nu} - q_\mu q_\nu \right) \omega^\mu(q) \right) \rho^\nu(x). \quad (65)$$
V. $\omega - \rho^0$ MIXING AND $\omega \rightarrow \pi^+\pi^-$ DECAY

Due to VMD, the electromagnetic interaction contributes to $\omega \rightarrow \pi^+\pi^-$ decay through $\omega \rightarrow \gamma \rightarrow \rho^0 \rightarrow \pi\pi$ and $\omega \rightarrow \gamma \rightarrow \pi\pi$. In I we have evaluated $\rho\pi\pi$ vertex, $\rho^0 - \gamma$ mixing vertex and “direct” $\gamma\pi\pi$ vertex up to one-loop level. The “direct” $\gamma\pi\pi$ vertex reads

$$L_{\gamma\pi\pi}^c = \int \frac{d^4q}{(2\pi)^4} e^{iq.x} \bar{F}_\pi(q^2) A_\mu(q) [\pi^+(x)\partial^\mu\pi^-(x) - \partial^\mu\pi^+(x)\pi^-(x)],$$

(66)

where $A_\mu$ is photon field, $\bar{F}_\pi(q^2)$ is nonresonant background part of pion form factor. Explicitly, $\bar{F}_\pi(q^2)$ reads

$$\bar{F}_\pi(q^2) = 1 + \frac{q^2b_\gamma(q^2)}{1 + \Sigma(q^2)},$$

(67)

where

$$b_\gamma(q^2) = \frac{b(q^2)}{2(1 + 3\zeta)} - D(q^2) - \frac{C(q^2)\Sigma_0(q^2)}{1 + 11\zeta/3},$$

$$C(q^2) = \frac{1}{2f_\pi^2}[A(q^2) + 2g_\Lambda B(q^2)],$$

$$\Sigma_0(q^2) = \frac{2}{f_\pi^2}[2\Sigma_\pi(q^2) - \Sigma_K(q^2)],$$

$$\Sigma(q^2) = [1 + \frac{q^2C(q^2)}{1 + 11\zeta/3}\Sigma_0(q^2)],$$

$$D(p^2) = \frac{1}{16\pi^2f_\pi^2}\{\lambda + \int_0^1 dx \cdot x(1 - x) \ln \left[\left(1 - \frac{x(1 - x)p^2}{m_K^2}\right)^2 \right] - \frac{2}{3}i\pi\theta(p^2 - 4m_\pi^2)\}.$$

The complete $\rho\pi\pi$ vertex reads

$$L_{\rho\pi\pi}^c = \int \frac{d^4q}{(2\pi)^4} e^{iq.x} g_{\rho\pi\pi}(q^2)(q^2\delta_{\mu\nu} - q_\mu q_\nu)\rho^{0\mu}(q)[\pi^+(x)\partial^\nu\pi^-(x) - \partial^\nu\pi^+(x)\pi^-(x)],$$

(69)

with

$$g_{\rho\pi\pi}(q^2) = \frac{b(q^2)}{(1 + 2\zeta)(1 + \Sigma(q^2))}. $$

(70)

Moreover, the complete $\rho - \gamma$ mixing vertex reads

$$L_{\rho\gamma}^c = -\frac{1}{2}\int \frac{d^4q}{(2\pi)^4} e^{iq.x} b_{\rho\gamma}(q^2)(q^2\delta_{\mu\nu} - q_\mu q_\nu)\rho^{0\mu}(q)A^\nu(x),$$

(71)

where

$$b_{\rho\gamma}(q^2) = \frac{A(q^2)}{\zeta A(q^2)} - f_\pi^2b_\gamma(q^2)[1 + \frac{q^2b_\gamma(q^2)}{1 + \Sigma(q^2)},$$

(72)

Thus due to VMD, the complete $\omega - \gamma$ mixing vertex can be obtained directly

$$L_{\omega\gamma}^c = -\frac{1}{6}\int \frac{d^4q}{(2\pi)^4} e^{iq.x} b_{\rho\gamma}(q^2)(q^2\delta_{\mu\nu} - q_\mu q_\nu)\rho^{0\mu}(q)A^\nu(x).$$

(73)

Eqs. (71) and (73) will lead to $\omega - \rho^0$ mixing at the order of $\alpha_{e.m.}$ through the transition process $\omega \rightarrow \gamma \rightarrow \rho^0$, which is

$$L_{\omega\rho}^{e.m.} = \frac{1}{12}\int \frac{d^4q}{(2\pi)^4} e^{iq.x} b_{\rho\gamma}(q^2)(q^2\delta_{\mu\nu} - q_\mu q_\nu)\omega^{\mu}(q)\rho^{0\nu}(x).$$

(74)
In addition, eqs. (60) and (73) also lead to “direct” $\omega\pi\pi$ coupling at the order of $\alpha_{c.m.}$ through the transition process $\omega \rightarrow \gamma \rightarrow \pi\pi$, which is

$$L_{\omega\pi\pi}^{c.m.} = -\frac{i}{\epsilon^2} \int \frac{d^4q}{(2\pi)^4} e^{iq\cdot x} \tilde{F}_\pi(q^2)b_{\rho\rho}(q^2)(q^2\delta_{\mu\nu} - q_\mu q_\nu)
\times \omega^{\mu}(q)[\pi^+(x)\partial^\nu\pi^-(x) - \partial^\nu\pi^+(x)\pi^-(x)].$$

(75)

Eq. (74) together with eqs. (49), (57) and (63) give the complete $\omega - \rho^0$ mixing vertex as follow

$$L_{\omega\pi\pi}^c = \int \frac{d^4q}{(2\pi)^4} e^{iq\cdot x} g_{\rho\pi\pi}(q^2)(q^2\delta_{\mu\nu} - q_\mu q_\nu)\omega^{\mu}(q)\rho^{\nu}(x),$$

(76)

where vector meson fields have been normalized to physical fields, and

$$\Theta_{\omega\rho}(q^2) = \frac{N_c}{6\pi^2} \frac{m_u - m_d}{m} \{g_2 h_0(q^2)(1 - \frac{4}{3} \zeta) + q^2 b(q^2) s(q^2)[\Sigma_K(q^2) - \frac{\Sigma_\pi(q^2)}{1 + \Xi(q^2)}] + \frac{2\alpha}{3} b_{\rho\rho}(q^2).$$

(77)

The complete “direct” $\omega\pi\pi$ vertex can be obtained via summing eqs. (51), (57), (63) and (75),

$$L_{\omega\pi\pi}^c = -i \int \frac{d^4q}{(2\pi)^4} e^{iq\cdot x} g_{\omega\pi\pi}(q^2)(q^2\delta_{\mu\nu} - q_\mu q_\nu)\omega^{\mu}(q)[\pi^+(x)\partial^\nu\pi^-(x) - \partial^\nu\pi^+(x)\pi^-(x)].$$

(78)

where all meson fields have been normalized to physical fields, and $g_{\omega\pi\pi}(q^2)$ is defined as follow

$$g_{\omega\pi\pi}(q^2) = \frac{N_c}{12\pi^2} \frac{m_u - m_d}{m} \left\{ s(q^2) \frac{1}{1 + \Xi(q^2)} - \frac{10}{3} \zeta \right\}
- 6 f_{\pi}^{-2} \Sigma_K(q^2)[8m^2 f_{\pi}^{-2} b(q^2) - \frac{s(q^2)}{3} (1 + \frac{q^2 N_c}{4\pi^2 f_{\pi}^2})] + \frac{2\alpha}{3} \tilde{F}_\pi(q^2)b_{\rho\rho}(q^2).$$

(79)

Thus G-parity forbidden $\omega \rightarrow \pi^+\pi^-$ includes a nonresonant background contribution, eq. (78), and $\rho$ resonance exchange contribution (eqs. (69) and (70)). The decay width on $\omega$ mass-shell is

$$\Gamma(\omega \rightarrow \pi^+\pi^-) = \frac{m_\omega^3}{48\pi} \frac{m_\pi^2 \Theta_{\omega\rho}(m_\omega^2)g_{\rho\pi\pi}(m_\omega^2)}{m_\omega^2 - m_\rho^2 + im_\rho \Gamma_\rho} - g_{\omega\pi\pi}(m_\omega^2)^2(1 - \frac{4m_\pi^2}{m_\omega^2})^{3/2}.$$ 

(80)

Using the experimental data $B(\omega \rightarrow \pi^+\pi^-) = (2.21 \pm 0.30)\%$ [24] together with eq. (80), we obtain

$$m_u - m_d = -(3.9 \pm 0.22)\text{MeV}$$

(81)

at energy scale $\mu \sim m_{\omega}$. Here the error bar is from the uncertainty in branch ratio of the process $\omega \rightarrow \pi^+\pi^-$. In the standard way, the $\omega - \rho^0$ mixing amplitude is

$$\int \frac{d^4q}{(2\pi)^4} \Pi_{\omega\rho}(q^2) = <\omega|\int d^4x L_{\omega\rho}(x)|\rho> \Rightarrow \Pi_{\omega\rho}(q^2) = q^2 \Theta_{\omega\rho}(q^2).$$

(82)

The off-shell $\omega - \rho^0$ mixing amplitude is obviously momentum dependent, and vanished at $q^2 = 0$. This is consistent with the argument by O’Connell et. al. in ref. [13] that this mixing amplitude must vanish at the transition from time-like to space-like four momentum within a broad class of models. In addition, the value of isospin broken parameter (51) leads on $\omega$ mass-shell $\omega - \rho^0$ mixing amplitude as follow

$$\text{Re}\Pi_{\omega\rho}(m_\omega^2) = -(3956 \pm 280)\text{MeV}^2, \quad \text{Im}\Pi_{\omega\rho}(m_\omega^2) = -(1697 \pm 130)\text{MeV}^2.$$ 

(83)

In ref. [6], the on-shell mixing amplitude has extracted from the $e^+e^- \rightarrow \pi^+\pi^-$ experimental data in a model-dependent way. In eq. (74), the real part of on-shell mixing amplitude agree with result of ref. [6]. The imaginary part, however, is much larger than one in ref. [6] which is around $-300\text{MeV}^2$. It must be pointed out that, in ref. [6] the author’s analysis bases on a model without “direct” $\omega\pi\pi$ coupling. Therefore, it is insignificant to compare the value of on-shell mixing amplitude of this the present paper with one of ref. [6]. Fortunately, the ratio between $\omega \rightarrow \pi\pi$ decay amplitude and $\rho \rightarrow \pi\pi$ decay amplitude should be model-independent. This value can test whether a model is right or not. The on-shell mixing amplitude in ref. [6] yields
\[ R_{\omega\rho}^{\text{exp}} = \frac{\langle \pi^+\pi^- | \omega >}{\langle \pi^+\pi^- | \rho >} = -(0.0060 \pm 0.0009) + (0.0322 \pm 0.0050)i. \]  

The present paper predicts

\[ R_{\omega\rho} = \frac{m_\omega^2 \Theta_{\omega\rho}(m_\omega^2)}{m_\omega^2 - m_\rho^2 + im_\rho \Gamma_\rho} \frac{g_{\omega\pi\pi}(m_\omega^2)}{g_{\rho\pi\pi}(m_\omega^2)} = -(0.0084 \pm 0.0007) + (0.0331 \pm 0.0021)i. \]  

We can see that this theoretical prediction agree with experimental excellently.

Moreover, the follows have also been revealed in our studies of this paper:

i) If we take \( g_{\omega\pi\pi}(q^2) = 0 \) in eq. (80), we have \( B(\omega \rightarrow \pi^+\pi^-) = (2.56 \pm 0.34)\% \). So that the contribution from interference between “direct” \( \omega\pi\pi \) coupling and \( \omega - \rho^0 \) mixing is about 15%. The dominant contribution are from \( \rho \)-resonance exchange. This conclusion indicates all pervious studies which without “direct” \( \omega\pi\pi \) coupling are good approximation even though this neglect is an ad hoc assumption. However, in mechanism of \( \omega \rightarrow \pi^+\pi^- \) with “direct” \( \omega\pi\pi \) coupling, larger imaginary part of on-shell \( \omega - \rho^0 \) mixing amplitude is allowed, but it is not allowed in the mechanism without the direct coupling.

ii) If we do not consider the contributions from one-loop diagrams of pseudoscalar mesons, i.e. setting \( \Sigma_K(q^2) = \Sigma_\pi(q^2) = 0 \), we obtain \( B(\omega \rightarrow \pi^+\pi^-) = (2.86 \pm 0.47)\% \). Thus the contribution from one-loop of pseudoscalar mesons is about 30% and can not be omitted. This conclusion is consistent with I. In addition, in this case, the on-shell \( \omega - \rho^0 \) mixing amplitude is about \(-4700\text{MeV}^2\). So that we can see that the larger imaginary part of on-shell \( \omega - \rho^0 \) mixing amplitude is yielded by pseudoscalar meson loops. In I, we have shown that this larger imaginary part is required by the unitarity of this effective field theory.

VI. SUMMARY

In this paper, we study G-parity forbidden \( \omega \rightarrow \pi^+\pi^- \) decay up to one-loop level of mesons. This process is yielded by isospin symmetry breaking due to \( m_u \neq m_d \) and electromagnetic interaction of mesons. The decay amplitude contains two parts of contributions which are from “direct” \( \omega\pi\pi \) coupling and \( \omega - \rho^0 \) mixing respectively. In the previous studies, the “direct” \( \omega\pi\pi \) coupling is neglected. We show that the “direct” \( \omega\pi\pi \) coupling and its interference with \( \omega - \rho^0 \) mixing contribute to on-shell decay amplitude about 15% only. It also interprets why the previous studies are good approximations even without “direct” \( \omega\pi\pi \) coupling. We suggest that the decay amplitude ratio \( R_{\omega\rho} \) should be model-independent, and our prediction agree with experimental data excellently.

The formula of \( \omega - \rho^0 \) mixing amplitude is also obtained. Since our calculation is beyond the chiral expansion(including all orders contribution of the chiral expansion) and one-loop contribution of pseudoscalar mesons is considered, the momentum-dependence of the off-shell mixing amplitude is very complicated. However, the mixing amplitude also vanishes at \( q^2 = 0 \). For case of on \( \omega \) mass-shell, the mixing amplitude emerges larger imaginary which is from one-loop contribution of pion and is required by unitarity of this effective field theory.

In our calculation, all vertices are expanded to the leading order light current quark masses. At this order, the decay amplitude yielded by isospin broken is proportional to \( m_d - m_u \). The theoretical prediction of isospin breaking parameter is \( m_d - m_u = (3.9 \pm 0.22)\text{MeV} \) at energy scale \( \mu \sim m_\omega \). This value is important for determining light quark masses at vector meson energy scale.

Appendix

Here we provide some identities which are used in calculation on four-point effective action. In sect. 3.2 we have used \( q, k \) and \( -k - q \) to denote four-momentum square of external source \( \bar{V}_\mu \), \( \Delta_\nu \) and \( \Delta_\sigma \). For the purpose of this paper, \( \Delta_\rho(k) \) reduces to \( k_\rho \pi(k) \). So that due to soft pion theorem we have \( k^2 \rightarrow 0 \), \( (q + k)^2 \rightarrow 0 \) and \( k_\mu \Delta_\nu(k) \rightarrow 0 \). Moreover, due to space-like condition of vector meson fields, \( q^\mu V_\mu(q) = 0 \), we have

\[ (\delta_{\mu\sigma} q_\nu + \delta_{\mu\nu} q_\sigma) < \{ V_\mu(q), S \} \Delta_\nu^\sigma(-k - q) > = 0, \]

\[ (\delta_{\mu\sigma} q_\nu + \delta_{\mu\nu} q_\sigma) < V_\mu(q) \Delta_\nu^\sigma(k) S \Delta_\sigma^\nu(-k - q) > = 0. \]

\[ q_\nu q_\sigma k_\mu < \{ V_\mu(q), S \} \Delta_\nu^\sigma(k) \Delta_\sigma^\nu(-k - q) > \rightarrow \frac{-q^2}{2} q_\nu \delta_{\mu\nu} < \{ V_\mu(q), S \} \Delta_\nu^\sigma(k) \Delta_\sigma^\nu(-k - q) >, \]
\[ q_\nu q_\sigma k_\mu < V^\mu(q) \Delta^\nu(k) S \Delta^\sigma(-k-q) > \rightarrow -\frac{q_\nu^2}{2} q_\sigma \delta_{\mu\nu} < V^\mu(q) \Delta^\nu(k) S \Delta^\sigma(-k-q) >, \]

\[ k_\mu \delta_{\nu\sigma} < \{ V^\mu(q), S \} \Delta^\nu(k) \Delta^\sigma(-k-q) > \rightarrow -q_\nu \delta_{\mu\nu} < \{ V^\mu(q), S \} \Delta^\nu(k) \Delta^\sigma(-k-q) >, \]

\[ k_\mu \delta_{\nu\sigma} < V^\mu(q) \Delta^\nu(k) S \Delta^\sigma(-k-q) > \rightarrow -q_\sigma \delta_{\mu\nu} < V^\mu(q) \Delta^\nu(k) S \Delta^\sigma(-k-q) >, \]

The following integral identities are also used in our calculation

\[
\int_0^1 dx \cdot x^2 \int_0^1 dy (1-y) \frac{xy}{\left[ t^2 - m^2 + x y (1-x) q^2 \right]^4} = \int_0^1 dx \cdot x^2 \int_0^1 dy (1-y) \frac{(1-x)}{\left[ t^2 - m^2 + x y (1-x) q^2 \right]^4} = \int_0^1 dx \cdot x^2 \int_0^1 dy (1-y) \frac{(1-x)}{\left[ t^2 - m^2 + x y (1-x) q^2 \right]^4}.
\]

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