Two-electron photoionization of endohedral atoms

M. Ya. Amusia

Racah Institute of Physics, The Hebrew University, Jerusalem 91904, Israel; A. F. Ioffe Physical-Technical Institute, St. Petersburg, 194021, Russia

E. Z. Liverts and V. B. Mandelzweig

Racah Institute of Physics, The Hebrew University, Jerusalem 91904, Israel

Abstract

Using $\text{He}^@\text{C}_{60}$ as an example, we demonstrate that static potential of the fullerene core essentially alters the cross section of the two-electron ionization differential in one-electron energy $d\sigma^{++}(\omega)/d\varepsilon$. We found that at high photon energy prominent oscillations appear in it due to reflection of the second, slow electron wave on the $\text{C}_{60}$ shell, which ”dies out” at relatively high $\varepsilon$ values, of about 2÷3 two-electron ionization potentials. The results were presented for ratios $R_{\text{C}_60}(\omega,\varepsilon) \equiv d\sigma^{++}(\omega,\varepsilon)/d\sigma^{a++}(\omega,\varepsilon)$, where $d\sigma^{a++}(\omega,\varepsilon)/d\varepsilon$ is the two-electron differential photoionization cross section. We have calculated the ratio $R_{i,\text{ful}} = \sigma_i^{++}(\omega)/\sigma_i^{a++}(\omega)$, that accounts for reflection of both photoelectrons by the $\text{C}_{60}$ shell. We have calculated also the value of two-electron photoionization cross section $\sigma^{++}(\omega)$ and found that this value is close to that of an isolated $\text{He}$ atom.

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I. INTRODUCTION

Elimination of two electrons by a single photon from an atom or multi-atomic formation, such as cluster or fullerene, can take place only if the inter-electron interaction is taken into account.

The desire to study the manifestation of this interaction stimulates extensive experimental and theoretical investigation of the process that has particularly intensified during last ten-fifteen years (see, e.g., [1, 2]). Although a number of atoms were studied, the primary attention was given to Helium. At this moment theoretical and experimental investigations cover the frequency region from the near threshold region up to photon energy $\omega$ much higher than the two-electron ionization potential $I^{++}$.

Very close to threshold the two-electron photoionization cross section is determined by so-called Wannier regime [3] with both electrons strongly repulsing each other that results in acquiring almost the same energy and moving in opposite directions.

With increase of $\omega$ for $\omega$ considerably more than $I^{++}$, so-called shake-off (SO) mechanism [4] became dominant, in which one electron leaves the atom carrying away almost all energy $\omega$, while the second is removed due to alteration of the field acting upon it after the first electron emission. With energy growth so called quasi-free (QF) mechanism [5] becomes increasingly important. This mechanism accounts for almost equal sharing of photon energy between photoelectrons, where their interaction on the way out of the ionized atom is inessential.

Obviously, the cross section of two-electron photoionization $\sigma^{++}(\omega)$ is determined by initial and final state wave function of the considered object - target atom at the beginning and ion with two continuous spectrum electrons at the end. The situation is simplified considerably at high enough $\omega$, where cross sections and other characteristics of the two-electron ionization are expressed via the initial state wave function only.

In fact, one has to have in mind that two-electron ionization is a pure three-body problem only for two-electron atom and ions. This process for any other, more complicated objects is determined by more sophisticated wave functions. This is why most of the attention is given usually to studies of two-electron photoionization of $H^{-}$, $He$ and helium-like atoms. At high $\omega$ the $\sigma^{++}$ and $\sigma^{+}$ have similar $\omega$ dependence [6]. Therefore it is convenient to characterize the process by the ratio $R(\omega) = \sigma^{++}(\omega)/\sigma^{+}(\omega)$. 
The SO leads for $\omega >> I^{+}$ to $R(\omega) = R_{SO}$ (see [7] and references therein). Inclusion of QF increases $R(\omega)$ considerably but at high enough $\omega$ the ratio again reaches its $\omega$-independent value, $R_{QF} > R_{SO}$ [8].

During last time a lot of attention is given to photoionization of not isolated atoms, but atoms encapsulated into the fullerene, mainly $C_{60}$ shell (see, e.g. [9, 10]). The research in this area until now is pure theoretical. But we are positive, that it will become an object of experimental studies in not too distant future. It is well known, that $C_{60}$ radius $R$ is much bigger than that of the atoms staffed inside the so-called endohedral atom, $A@C_{60}$.

The inclusion of the $C_{60}$ shell can affect the photoionization atom in four directions. At first, and this is usually taken into account, the photoelectron emitted by $A$ is reflected by the $C_{60}$ shell that "works" as a static potential. At second, the $C_{60}$ shell has its internal degrees of freedom [11] that can be excited thus modifying the photoionization of the atomic shells in $A@C_{60}$ in a similar way that one shell affects the photoionization of the other in isolated atoms [12]. Among the $C_{60}$ degrees of freedom the most prominent is the giant resonance that persists not only in $C_{60}$ [11], but in its ions as well [13]. Real or virtual excitations of the giant resonance can modify the $A$ atom photoionization cross section from almost complete screening at $\omega \rightarrow 0$ to prominent enhancement, at $\omega$ in the vicinity of the giant resonance maxima [14, 15]. Note, that at high $\omega$ this screening becomes mainly inessential.

Other two directions of $C_{60}$ influence upon the $A$ atom photoionization is the direct knock out of $C_{60}$ electrons by the photoelectron from $A$ and the participation of $C_{60}$ electrons in the decay, both radiative and non-radiative, of the vacancy, created in $A$ after photon absorption. Note that this mechanism is similar to one of the frequently discussed channels of the molecular vacancy decay. The corresponding two possibilities in application to $C_{60}$ are not yet studied at all.

In this paper we consider a more complex process, namely two-electron photoionization of $He@C_{60}$. We will concentrate on the high $\omega$ region. Of course, to investigate this process experimentally is more complicated than one-electron photoionization. However, there are no indications whatsoever that such an investigation is either impossible (or even extraordinary difficult) or uninteresting.

The role of $C_{60}$ shell in two-electron photoionization is more complex than in the single-electron, since outgoing electrons can be either both from the inner atom $A$, or both from the
$C_{60}$, or one from $A$ and the other from $C_{60}$. However, by measuring two outgoing electrons in coincidence we can distinguish all the different processes.

We will consider mainly high $\omega$. Having in mind that our aim is study the $C_{60}$ influence upon double ionization, we have to consider the SO mechanism only. Indeed, the QF leads to two fast electrons, which are not affected by $C_{60}$, while in SO one of the electrons is slow. Its probability to leave $He@C_{60}$ can be therefore strongly affected.

Our attention in this paper will be given to differential in energy cross sections $d\sigma^{++}(\omega)/d\varepsilon_s$, where $\varepsilon_s$ is the ”slow” photoelectrons energy. It is implied that the ”fast” electron energy $\varepsilon_f$ is given by the conservation law $\varepsilon_f = \omega - \varepsilon_s - I^{++}$. Atomic system of units is accepted in this paper: $e = m_e = \hbar = 1$, where $e$ and $m$ are electron charge and mass, respectively.

We will assume that both electrons are removed from the $He$ atom, thus intensionally neglecting a process that one can call ”$A@C_{60}$ shake off”, in which after ”fast” electron leaves $A$, the ”slow” electron is emitted by the $C_{60}$ shell instead of located inside atom $A$. The ”$A@C_{60}$ shake off” is potentially very important due to big number of available for removing electrons. Therefore, it is possible that ”$He@C_{60}$ shake off” is much more probable than the ordinary one. It is possible, that an important role can be played by multiple ”$A@C_{60}$ shake of”, in which not one but several electrons can be removed from $C_{60}$ after instant creation of a vacancy in $A$. One has to have in mind that for an isolated atom $A$ the photoionization with excitation cross section $\sigma^{++}(\omega)$ can be of the order or even bigger than $\sigma^{++}(\omega)$. It is essential to have in mind that the $\sigma^{++}(\omega)$ for $A@C_{60}$ can be converted into $\sigma^{++}(\omega)$, since the excitation energy of $A^+$, particularly in the case of $He$, is bigger than the ionization potential of $C_{60}$. We plan, however, to concentrate on ”$A@C_{60}$ shake off” in another paper.

II. MAIN FORMULAS

The two-electron photoionization cross section of an atom $A$ in initial state $i$, $\sigma_i^{++}(\omega)$, can be presented by the following expressions:

$$\sigma_i^{++}(\omega) = \int_0^{\omega-I^{++}} \frac{d\sigma_i^{++}(\omega, \varepsilon)}{d\varepsilon} d\varepsilon,$$

(1)

where at high $\omega$, $\omega >> I^{++}$, the differential in energy cross section $d\sigma_i^{++}(\omega, \varepsilon)/d\varepsilon$ is
given by the expression \[16\]:

\[
\frac{d\sigma^{a++}(\omega, \varepsilon)}{d\varepsilon} = \frac{32\sqrt{2}Z^2\pi^3}{3c\omega^{7/2}} |\int \Psi_i(0, r)\varphi_\varepsilon(0)\, dr|^2.
\] (2)

Here \(\Psi_i(0, r)\) is the \(r_1 = 0\) value of \(\Psi_i(r_1, r_2)\), which is in our case the initial state wave function of atomic He. The "slow" outgoing electron is described by a pure Coulomb wave function that describes an \(s\)-wave electron that moves in the field of a nucleus with charge \(Z\).

Similar to Eq.(1), the cross-section of ionization with excitation \(\sigma^{\ast\ast}(\omega)\) is given by the following expression:

\[
\frac{d\sigma^{\ast\ast}(\omega, \varepsilon)}{d\varepsilon} = \frac{32\sqrt{2}Z^2\pi^3}{3c\omega^{7/2}} \sum_{n>0} |\int \Psi_i(0, r)\varphi_{n0}(0)\, dr|^2.
\] (3)

Here \(n\) is the principal quantum number of the second electron excitation level.

Now let us consider \(He@C_{60}\) two-electron photoionization and take into account the \(C_{60}\) shell. It is clear that \(C_{60}\) potential does not affect the "fast" electron, which remains to be described by a plane wave. The \(C_{60}\) shell generates potential field that is almost uniform at atomic distances \(r_a\), where the initial state of \(He\) is located. Therefore, for the initial state the embedding of an atom \(A\) into \(C_{60}\) can lead only to the shift of the energy scale, i.e., to modification of the ionization threshold. This feature is inessential for us, since \(\omega >> I^{\ast\ast}\).

Thus, we have to take into account the action of \(C_{60}\) upon the slow outgoing electron. The fullerene is a very complex structure. Therefore, to take into account its action upon photo-electron essential simplifications are necessary. We will follow here the approach developed in a number of papers (see \[17, 18\] and references therein), that substitutes the complicated \(C_{60}\) multi-atomic structure by a very simple so-called bubble potential \(V(r) = V_0\delta(r - R)\), where \(V_0\) is chosen to reproduce the affinity energy of the negative ion \(C_{60}\), and \(\delta(r - R)\) is Dirac delta function.

As it was demonstrated in \[18\] and \[19\] in connection to the one-electron photoionization, the influence of the bubble potential can be taken into account analytically, by constructing the outgoing wave function as a superposition of regular \(\varphi_{el}(r)\) and irregular \(\chi_{el}(r)\) (singular at \(r \to 0\)) solutions of the Schrödinger equation for an electron with energy \(\varepsilon\). Inside the potential bubble the photoelectron wave function \(\psi_{el}(r)\) differs from \(\varphi_{el}(r)\) only by a normalization factor \(D_l(\varepsilon) : \psi_{el}(r) = D_l(\varepsilon)\varphi_{el}(r)\). The factor \(D_l(\varepsilon)\) depends upon the
photoelectron energy $\varepsilon$, with $l$ being its electron angular momentum. Outside the $\delta$-sphere the function $\psi_{\varepsilon l}(\mathbf{r})$ is a linear combination of $\varphi_{\varepsilon l}(\mathbf{r})$ and $\chi_{\varepsilon l}(\mathbf{r})$. The coefficients of the linear combination are defined by the matching conditions of the wave functions on the spherical shell, i.e., at $r = R$.

In the $\delta$-potential approximation the differential two-electron photoionization of $He@C_{60}$ is thus given by formula similar to (1), that includes, however the factor $D_l(\varepsilon)$:

$$
\frac{d\sigma_{i}^{++}(\omega, \varepsilon)}{d\varepsilon} = \frac{32\sqrt{2}Z^2\pi}{3c\omega^{7/2}} |D_0(\varepsilon)|^2 \int \Psi_i(0, \mathbf{r}) \varphi_{\varepsilon 0}(\mathbf{r}) d\mathbf{r}|^2.
$$

(4)

The expression for $|D_l(\varepsilon)|^2$ is derived in [19, 20] and is presented as

$$
|D_l(\varepsilon)|^2 \equiv \frac{(k/\Delta L)^2}{[u_{kl}(R) v_{kl}(R) - k/\Delta L]^2 + u_{kl}^2(R)}.
$$

(5)

Here $u_{kl}(R)$ and $v_{kl}(R)$ are functions connected to the radial parts of regular $\varphi_{\varepsilon l}(\mathbf{r})$ and irregular $\chi_{\varepsilon l}(\mathbf{r})$ functions by relations $u_{kl}(r) = r\varphi_{\varepsilon l}(r)$ and $v_{kl}(r) = r\chi_{\varepsilon l}(r)$; $k = \sqrt{2\varepsilon}$, $\Delta L$ is the discontinuity of the logarithmic derivative of the wave function at $r = R$, connected to the fullerene radius $R$ and the electron affinity $I_f$ of the empty $C_{60}$ through the expression

$$
\Delta L = -\beta(1 + \coth \beta R),
$$

(6)

where $\beta = \sqrt{2I_f}$. The formula obtained are valid for low enough energies of the "slow" photoelectron. Namely, its wave length should be much bigger than the thickness of the fullerene shell.

At first one should calculate the ratio of differential cross sections (4) to (2) $R_{C_{60}}(\varepsilon)$ that according to Eqs.(3) and (4) is independent upon $\omega$

$$
R_{C_{60}}(\varepsilon) \equiv \frac{d\sigma_i^{++}(\omega, \varepsilon)}{d\sigma_i^{++}(\omega, \varepsilon)} = |D_0(\varepsilon)|^2,
$$

(7)

and then turn to the ratio $R_{i,ful}$ of cross sections $\sigma^{++}(\omega)$ and $\sigma^{a++}(\omega)$ that are determined using Eq.(1) with $d\sigma_i^{++}(\omega, \varepsilon)/d\varepsilon$ given by Eq.(4) and Eq.(2), respectively.

$$
R_{i,ful} = \sigma_i^{++}(\omega)/\sigma_i^{a++}(\omega)
$$

(8)

It is evident from (7) that determined in this way $R_{i,ful}$ is $\omega$-independent at high $\omega$. 
Since \( \varphi_{0}(r) \) and \( \chi_{l}(r) \) are pure Coulomb functions, the functions \( u_{kl} \) and \( v_{kl} \) (from Eqs. (6,7)) are expressed via the regular \( F_{l}(\eta, \rho) \) and irregular \( G_{l}(\eta, \rho) \) Coulomb wave function, respectively. Namely, one has \( u_{kl}(r) = F_{l}(-\frac{Z}{k}, kr) \) and \( v_{kl}(r) = G_{l}(-\frac{Z}{k}, kr) \), i.e. regular and irregular Coulomb wave functions that can be found in [20].

Let us note that the formula (7) can be extended to lower \( \omega \). Indeed, the interaction between two ionized electrons is particularly essential near ionization threshold \( \omega \geq I^{++} \), where Wannier regime prevails. It is known that already several eV above threshold the Wannier expressions are no more valid. Therefore, it is reasonable to assume that at about 10eV the interaction between outgoing electrons is inessential. One of them is represented by an \( s \)-wave, while the other by a \( p \)-wave, in order to preserve the total initial state angular momentum that is zero for \( He@C_{60} \) and \( l = 1 \) for the photon. Due to action of the \( C_{60} \) shell the second electrons wave function is modified in the same way as \( \varphi_{0}(r) \) in (2). Thus, it is reasonable to expect that to take this action into account one has to introduce the factor \(|D_{1}(\tau)|^{2} \) into (7). As a result, one obtains instead of Eq.(7) the following expression:

\[
R_{C_{60}}(\omega, \varepsilon) \equiv \frac{d\sigma^{++}_{i}(\omega, \varepsilon)}{d\sigma^{a++}_{i}(\omega, \varepsilon)} = |D_{0}(\varepsilon)|^{2}D_{1}(\omega - I^{++} - \varepsilon)|^{2}. \tag{9}
\]

Since for low and medium energies the cross section \( d\sigma^{a++}_{i}(\omega, \varepsilon)/d\varepsilon \) cannot be calculated using Eq.(2), we have to use other sources of absolute values of it in order to obtain \( R_{C_{60}}(\omega) \).

As such, either experimental data or results of existing calculations of differential in energy \( \varepsilon \) cross section can be used to substitute into

\[
R_{C_{60}}(\omega) \equiv \int_{0}^{\omega - I^{++}} |D_{0}(\varepsilon)|^{2}|D_{1}(\omega - I^{++} - \varepsilon)|^{2}d\sigma^{a++}_{i}(\omega, \varepsilon) / \int_{0}^{\omega - I^{++}} d\sigma^{a++}_{i}(\omega, \varepsilon). \tag{10}
\]

In order to have a crude estimation of the role of both factors in (9), we have calculated in Eq.(10) the factor \( R_{C_{60}}(\omega) \) using Eq.(1) well outside its range of validity \( \omega \gg I^{++} \), which gives us \( R_{C_{60}}^{(h)}(\omega) \).

Since at \( \omega \) close to \( I^{++} \), \( d\sigma^{a++}_{i}(\omega, \varepsilon)/d\varepsilon \) is almost \( \omega \)-independent, as an estimation for \( R_{C_{60}}(\omega) \) at \( \omega \geq I^{++} \) can serve an approximate relation

\[
R_{C_{60}}^{(l)}(\omega) \equiv \int_{0}^{\omega - I^{++}} |D_{0}(\varepsilon)|^{2}|D_{1}(\omega - I^{++} - \varepsilon)|^{2}d\varepsilon/ (\omega - I^{++}). \tag{11}
\]
III. DETAILS OF CALCULATIONS

First of all, one should notice that the term $k/\Delta L$ in denominator of Eq.(5) was taken with the sign "minus", whereas there was "plus" in the Ref.\[18, 19\]. This is because the authors of the latter references used the irregular Coulomb functions $v_{kl}(r) = -v_{kl}(r)$, whilst we used the Coulomb functions $u_{kl}(r)$ presented in the Handbook \[20\].

The Eqs.(9-11) include the function $D_1(\varepsilon)$ at $\varepsilon = 0$, for $\varepsilon = \omega - I^{++}$. It is clear that the Coulomb functions $F_l(-\frac{Z}{r}, kr)$ and $G_l(-\frac{Z}{r}, kr)$, presented in the expression (5) for $D_2^l$, have singularities at zero energy, i.e. at $k = 0$ that corresponds to $\varepsilon = 0$. However, the presentations of Coulomb functions given in \[20\] enable to obtain the following limit relations:

$$\lim_{k \to 0} \frac{u_{kl}(r)}{\sqrt{k}} \equiv f_l(Z, r) = \sqrt{\pi r} J_{2l+1}(\sqrt{8Zr}),$$

$$\lim_{k \to 0} \frac{v_{k0}(r)}{\sqrt{k}} \equiv g_0(Z, r) = \frac{p(2Zr)}{\sqrt{2\pi Z}} - \sqrt{\frac{r}{\pi}} [2\gamma - 1 + \ln(2Zr)] J_1(\sqrt{8Zr}).$$

Here $\gamma$ is Euler’s constant, $J_{\mu}(x)$ is a usual Bessel function of the first kind, and $p(\alpha)$ is the following sum, which is infinite but rapidly convergent: $p(\alpha) = \sum_{j=0}^{\infty} P_j(\alpha)$, where $P_j(\alpha)$ could be obtained by the recurrence relations

$$P_j = -\frac{\alpha}{j(j-1)} \left[ (-1)^j \frac{(2j-1)j^{j-1}}{(j!)^2} + P_{j-1} \right]$$

with the initial values $P_0 = 1, \quad P_1 = 0$. Functions $g_l(Z, r)$ for $l > 0$ could be then obtained by using Eqs.(12)-(14) and the following recurrence relation

$$g_l(Z, r)f_{l-1}(Z, r) = g_{l-1}(Z, r)f_l(Z, r) + \frac{l}{Z}.$$  \hspace{0.5cm} (15)

The latter is a consequence of the Wronskian relation (see Eq.(14.2.5) in Ref.\[20\]) for the regular and irregular Coulomb functions. Note that Eq.(12) for the regular Coulomb functions could be found in numerous handbooks on non-relativistic quantum mechanics (see, e.g. Ref.(13)), within the accuracy of a normalization factor. However, we did not find in scientific literature the corresponding relations (13)-(15) for irregular Coulomb functions.

Using the limit relations (12)-(15), one could present Eq.(5) for the zero energy in the form:

$$D_2^l(0) = \frac{1}{[(\Delta L)g_l(Z, R)f_l(Z, R) - 1]^2 + (\Delta L)^2 f_l^4(Z, R)}.$$  \hspace{0.5cm} (16)
In particular, for the helium atom expression (16) yields: \( D_0^2(0) = 1.38590 \) and \( D_1^2(0) = 3.09129 \).

The function \( \Psi_i \) can be obtained using variational principle that minimizes the initial state energy. One has to have in mind, that, generally speaking, this procedure is able to reproduce initial state energy absolutely accurate, leading to wave function that satisfies the precise Schrodinger equation for two electrons in the field of a nucleus with the charge \( Z \). In its practical implementations, however, the variation wave functions reproduces the initial energy approximately and therefore is good on the average.

As to photoionization, since one of the outgoing electrons is fast, its wave function is a plane wave with a wavelength short in the atomic scale. Therefore, photoeffect is able to test the short range behavior of \( \Psi_i(r_1, r_2) \), namely, as it is seen from Eq.(1), \( \Psi_i(0, r) \).

This is why instead of a variational, we use a locally correct wave function \[22, 23, 24\], that describes with high accuracy the two-electron photoionization \[16\] and some average characteristics of the initial state.

IV. RESULTS OF CALCULATIONS

Here we presented the results of our calculations for the two-electron photoionization of \( He@C_{60} \). For this object the radius \( R = 6.64a.u. \) and the electron affinity of the empty \( C_{60} \) is equal to \( I_f = 2.65eV \). The initial state of \( He \) considered in this paper is \( i = 1s \), i.e. \( l = 0 \). The results for \( R_{C_{60}}(\varepsilon) \) from Eq.(7) are presented in Fig.1 together with the function \( D_1^2(\varepsilon) \) from the Eqs.(9-11). As it was expected and similar to the case of one-electron photoionization, the ratio \( R_{C_{60}}(\varepsilon) \) is a strongly oscillating function of \( \varepsilon \).

Fig.(2) depicts \( R_{C_{60}}(\omega, \varepsilon) \), given by Eq.(10) for several \( \omega \) as functions of \( \varepsilon \). It is not incidental, that the curves behavior at \( \varepsilon = 0 \) and \( \varepsilon = \omega - I^{++} \) are different. This is because one of the electron is represented by an \( s \)-wave while the other by \( p \)-wave.

Note that \( R_{C_{60}}(\omega, \varepsilon) \) becomes a prominently varying curve already for \( \omega - I^{++} \geq 20eV \). For \( \omega - I^{++} \geq 60eV \) it has many oscillations.

Fig.(3) presents the results for the ratio \( R_i,ful(\omega) \) determined by Eq.(8). As one could see, with \( \omega \) growth this ratio rapidly approaches the asymptotic value \( R_i,ful \rightarrow 1 \). Note, that in Eq.(8) we assumed that "slow" electron wave function is modified by the \( C_{60} \) shell. In fact, this is incorrect already for \( \omega \geq 160eV \).
FIG. 1: The reflection and refraction factors $D_L^2 (L = 0, 1)$ as a function of photoelectron energy $\varepsilon$ for $He@C_{60}$.

Fig.(4) presents the relations $R_{C_{60}} (\omega)$ given by Eqs. for both $\varepsilon$ distributions, that are valid respectively at high, and at low $\omega$. It is seen that both curves, $R_{C_{60}}^{(h)} (\omega)$ and $R_{C_{60}}^{(l)} (\omega)$, prominently oscillate up to $\omega \sim 150eV$ and are close to each other. It permits us to suggest that the curve $R_{C_{60}} (\omega)$ is valid for correct $\varepsilon$ distribution instead of only their limiting cases for high and low $\omega$. The strongest is the role of the fullerene shell at $\omega < 110eV$. 
FIG. 2: The cross sections ratio $R_{C_{\omega}}(\omega, \varepsilon)$ as a function of photoelectron energy $\varepsilon$ for different values of photon energy $\omega$.

FIG. 3: The cross sections ratio $R_{i,ful}$ as a function of photon energy $\omega$. 
FIG. 4: The cross sections ratios $R_{C^{60}}^{(h)}$ and $R_{C^{60}}^{(l)}$ as functions of $\omega$ for high and low photon energy, respectively.

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