INTRODUCTION

High angular resolution imaging of circumstellar disks have shown that these disks may not be coplanar. For example, the disk of Beta Pictoris exhibits a warp; the outer disk at radii greater than 50AU is tilted between 3-5 degrees from the disk interior (Burrows et al. 1993; Heap et al. 2000). Models for the scattered optical light of the bowed disk of AU Microscopii suggest that this disk is also warped, with a small tilt of about 3° (Krist et al. 2003). While warped disks are easiest to identify in nearly edge-on systems such as Beta Pictoris and AU Microscopii, less inclined or nearly face-on disks may also not be coplanar. This leads us to search for features in less inclined disks that might also be explained with a warped disk model. As proposed by Mouillet et al. (1997); Augereau et al. (2001b), the twistedness of Beta Pictoris’s warped disk could be a result of precession induced by unseen planets or planetesimals residing in the disk. Consequently by probing the geometry of the warp, we can constrain the properties of the unseen planetary system.

THE WARPED CIRCUMSTELLAR DISK OF HD100546

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ABSTRACT

We propose that the two armed spiral features seen in visible Hubble Space Telescope images of scattered light in HD100546’s circumstellar disk are caused by the illumination of a warped outer disk. A tilt of 6-15 degrees from the symmetry plane can cause the observed surface brightness variations providing the disk is very twisted (highly warped) at radii greater than 200 AU where the spiral features are seen. Dust lanes are due in part to shadowing in the equatorial plane from the inner disk within a radius of 100 AU. HD100546’s outer disk, if viewed edge-on, would appear similar to that of Beta Pictoris. A disk initially misaligned with a planetary system, becomes warped due to precession induced by planetesimal bodies and planets. However, the twistedness of HD100546’s disk cannot be explained by precession during the lifetime of the system induced by a single Jovian mass planet within the clearing at ∼ 13 AU. One possible explanation for the corrugated disk is that precession was induced by massive of bodies embedded in the disk at larger radius. This would require approximately a Jupiter mass of bodies well outside the central clearing at 13 AU and within the location of the spiral features or at radii approximately between 50-200 AU.

Subject headings:
viewer.

The outer disk is illuminated by starlight that may pass through an inner absorbing disk. Consequently starlight illuminating the outer disk should be attenuated in the equatorial plane. A warped disk can be described as a series of rings. Each ring crosses the equatorial plane at two points. The shadow from the inner disk could cause two spiral dustlanes or shadows in the outer warped disk, resulting in the appearance of two spiral features.

Here we consider the possibility that the disk of HD100546 is warped, and that the warp is responsible for the observed spiral features. We first consider the case where the outer disk is isotropically illuminated by the central star. If the disk at large radii is warped and optically thick then we would expect the optical images would exhibit only one spiral arm. Only the high side of the disk would be illuminated whereas the opposite side would be in shadow. If the disk is optically thin and warped at radii greater than 2" or 200AU, then we expect to see two spiral features. Regions of high inclination with respect to the viewer would appear brighter and would be located on opposite sides of the star.

We now consider the situation of a warped disk which is not isotropically illuminated. We expect that starlight illuminating the outer disk should be highly attenuated in the equatorial plane. At each radius an inclined disk crosses the equatorial plane at two opposing points. Consequently the shadow from the inner disk could cause two spiral dustlanes or shadows in the outer warped disk, resulting in the appearance of two spiral features. If the spiral features are due to an equatorial shadow then the outer disk could either be optically thin or thick. If the disk is optically thick then one would expect 2 spiral shadows and one bright spiral feature, whereas if the disk is optically thin then one would expect 2 spiral shadows with brighter spiral features lying between them.

Unfortunately it is not straightforward to estimate the disk optical depth and width of HD100546’s disk as a function of radius. Based on the near-IR scattered light surface brightness, Pantin et al. (2000) estimated that the disk has a high normal optical depth, $\tau \sim 1$, at radii greater than 1" or 100AU. However, Augereau et al. (2001) estimated a normal optical depth 40-50 times lower implying that the disk was optically thin at optical and near-IR wavelengths outside of $\sim 80$ AU. The observed infrared spectral energy distribution of HD100546 implies that the luminosity of emission from dust is large, 50% of the stellar photospheric luminosity. Most of the disk luminosity, 70%, is emitted in the mid-infrared and so from a radius $\sim 10 - 20$ AU (Bouwman et al. 2003), well within the location of the spiral features at $\sim 250$ AU in scattered light. The estimated radius of the mid-infrared emission based on the spectral energy distribution is approximately consistent with mid-infrared high angular resolution observations (Lin et al. 2003). The infrared spectral energy distribution can be explained with a disk containing an inner, puffed up region absorbing and re-radiating half of the stellar light, accounting for the large mid-infrared flux. Because of its large covering angle the puffed inner disk edge shades the outer disk (Bouwman et al. 2003; Dullemond et al. 2001; Dullemond & Dominik 2004). However, this model implies that the disk greater than 1" or 100AU which has been observed in scattered light is illuminated by the starlight that must pass first through the inner puffed up region of the disk. This situation makes it challenging to estimate the optical depth of the disk from the scattered light at radii greater than 1".

In this paper we search for warped disk models which can account for the spiral features seen in HD100546’s disk. Warped disk models have some advantages over other models. Because planetary systems are nearly Keplerian, orbit orientations vary extremely slowly. Consequently a warp can be maintained for many rotation periods. This eliminates one of the problems of the previously proposed transient spiral structure model. Edge-on systems such as Beta Pictoris which are clearly warped, suggest that less highly inclined systems such as HD100546 could also be warped. In section 2 we describe how we represent the geometry of a warped disk. We then explain how we synthesize model surface brightness images which can be compared to the observations. In section 3 we discuss dynamical or physical models for the disk warp based on the geometrical model which best matches the observed disk morphology. A summary and discussion follows.

2. Warped Disk Models

2.1. Warp Geometry

We first discuss our notation for describing the orientation of the planetary system with respect to the viewer. The orientation of a coplanar planetary system requires 2 angles to describe; $\chi$, corresponding to the position angle (counter clockwise from North) of the axis of disk rotation on the sky, and an inclination angle, $\theta$, which describes the tilt of this axis with respect to the line of sight. If $\theta$ is zero, then the planetary system is viewed face-on. For a system that is not coplanar, $\chi$ and $\theta$ refer to the orientation of the rotation axis corresponding to the total angular momentum of the system’s disk and planets.

We describe the warped disk with respect to the rotation axis of the planetary system. A warped disk undergoing circular motion can be described as a series of rotating tilted rings, each with a different radius, $r$. The orientation of each ring is specified by two angles, a precession angle, $\Omega(r)$, similar to the longitude of the accenting node, and an inclination angle, $i(r)$. These angles are given with respect to the rotation axis of the planetary system and the direction of the line of sight. We measure the angle $\Omega$ from a reference line in the system’s ecliptic. As viewed on the sky, this line lies on the rotation axis of the system, but is projected onto the system’s ecliptic plane. Our precession angle is the angle (measured at the star) between this line and the point at which the ring crosses the system’s ecliptic. This is similar to the longitude of the ascending node which is measured with respect to the vernal equinox and the point at which the orbit crosses the ecliptic.

2.2. Constructing a model scattered light image

To produce an model image of the optical scattering light, all reflecting and absorbing regions along the line of sight at each position on the sky must be considered. When the disk is optically thin, multiple scattering events and absorption can be neglected. In this case each photon originates from the star and is then reflected from
a single spot on the disk. At each position on the sky we can sum the reflected light at each location in the disk along the line of sight. We have restricted this modeling effort to an optically thin outer disk, consistent with the estimates by [Augereau et al. 2001a] at radii outside of 80AU. However we must keep in mind that future modeling efforts may need to consider outer disks with higher optical depth.

We begin by randomly sampling \( x, y \) positions in the plane perpendicular to the disk rotation axis. At each position we compute a disk plane \( z \) coordinate based on our assumed function for the precession and inclination angles \( \Omega(r) \) and \( i(r) \). To account for the disk thickness we add a vertical offset to \( z \) which was randomly chosen from a Gaussian distribution function. For the vertical structure of the disk we assume a normal distribution

\[
\rho(z) \propto \exp(-z^2/2h^2)
\]

where \( h \) is the standard deviation of the distribution. The FWHM of this distribution is 2.35\( h \). We adopt a disk aspect ratio, \( h/r \), that is independent of radius.

The coordinates of the \( x, y, z \) position (in coordinates defined by the disk rotation axis) are then rotated using \( \chi \) and \( \vartheta \) to account for the orientation of the system rotation axis with respect to the viewer. To produce each model image, 200,000 points in the disk are sampled. Scattered starlight from these points are summed along each line of sight to produce a surface brightness image on the sky. The brightness of each point in the disk depends on the assumed albedo times the normal optical depth of that portion of the disk multiplied by the flux from the star at that position. The scattering amplitude of the reflected light from each point in the 3D model was modified by the Henyey-Greenstein scattering phase function. The scattering asymmetric parameter, \( g \), describes the scattering anisotropy (\( g = 0 \) corresponds to isotropic scattering, \( g = 1 \) to fully forward scattering). However, we do not vary \( g \) but instead adopt a fixed value of \( g = 0.5 \). We chose a representative value for \( g \) because of the uncertainty in the dust grain distribution and composition and because of the broadness of the filter used for the HST observations.

We take into account absorption of starlight from the inner disk by attenuating the starlight reaching the outer disk. We use an axisymmetric attenuation function that depends on the spherical coordinate, \( \theta \), where \( \sin \theta = z/r \). We assume that the opacity of the inner disk

\[
\tau_d(\theta) = \tau_0 \exp(-|\theta|/\theta_c)
\]

where \( \tau_0 \) is the opacity in the equatorial plane, and \( \theta_c \) describes an angular scale length. Approximately 50% of the total stellar luminosity is absorbed by the inner disk. An opaque torus blocking 50% of the light would cover angles \(-30^\circ < \theta < 30^\circ \). Because the star is an B9.5V star its spectrum is quite blue. Consequently we expect \( \tau_d \geq 1 \) at broad band optical wavelengths for angles a few times smaller than the \( \theta \sim 30^\circ \) required to block 50% of the total stellar luminosity.

Because the disk surface brightness is observed to drop \( \propto r^{-3} \) within 2.5" of the star [Grady et al. 2001; Augereau et al. 2001a], the disk normal optical depth must depend on radius. Taking into account the \( r^{-2} \) flux from the star, we match the observed radial drop with a \( -1 \) radial power in the normal optical depth [Augereau et al. 2001a]. The normal optical depth times the albedo is taken to be a power law function of radius, \( \propto r^{-1} \). We note that if there are large variations in the disk orientation then a different radial function may provide a better fit to the surface brightness profile of scattered light.

2.3. The precession angle

If the warp is due to the tidal force of a planet interior to the disk, then the precession rate of the longitude of the ascending node is approximately

\[
\Omega(r) \approx \frac{3}{4} n \left( \frac{M_p}{M_*} \right) \left( \frac{D}{r} \right)^2
\]

due to\, [Monillet et al. 1997]. This approximation is appropriate for particles at low inclinations and in nearly circular orbits. Here \( n \) is the mean motion of the disk at radius \( r \) (equivalent to the angular rotation rate for a circular orbit), \( D \) is the semi-major axis of the planet, \( M_p \) is the mass of the planet, and \( M_* \) is the mass of the central star. This precession rate is appropriate in the limit for \( D \ll r \) and is independent of the disk inclination. This precession rate also neglects the self-gravity of the disk and so is only appropriate when the disk is low mass.

After a time \( \Delta t \), an initially flat disk will have a precession angle

\[
\Omega(r) = \Omega_0 + A_0 \left( \frac{r}{r_0} \right)^{-\beta} - 1
\]

where \( \Omega_0 \) is the precession angle at a reference radius, \( r_0 \). When the precession is due solely to a single distant planet, the constant

\[
A_0 = -\frac{3\sqrt{GM_*D^2\Delta t}}{4r_0^3} \left( \frac{M_p}{M_*} \right)
\]

and \( \beta = 3.5 \), corresponding to precession angle \( \Omega \propto r^{-7/2} \). However if there are multiple planets in the disk or the disk itself contains mass, one may consider more general laws or power laws with \( \beta < 3.5 \). In our numerical exploration we have explored variations in \( \Omega_0, \beta \) and \( A_0 \) to match the observed morphology of HD100546’s disk.

If a planet internal to the disk is initially taken out of the plane containing the disk then the tilt angle \( i \) (with respect to the plane containing the planet) decreases with increasing radius; the situation considered by [Mouillet et al. 1997]. However if the disk itself is tilted via tidal forces from an external stellar encounter then \( i \) would increase with increasing radius. Consequently we allow the disk tilt angle, \( i(r) \), to vary slowly with radius. At small radii where structure in the disk is difficult to resolve in the presence of scattered light from the star, models of the form given by Equation (4) predict extremely tight corrugations. We let \( i \) drop to zero at small radii, so that tight corrugations in the inner region were removed from the model images. This allowed us to keep a simple powerlaw form for the precession angle. We allowed the inclination to vary with radius smoothly by using a spline function specified at 4 different radii \( r = 0, 100, 200 \) and 400AU. The inclination at \( r = 0 \) was set to 0°.
Common parameters for models are listed in Table 1. Parameters varied for individual models are listed in Table 2. We first attempt to match the morphology of the disk with a geometric model and then discuss physical models which can account for the observed geometry.

3. MODEL DISKS

In Figure 1 we show a model warped disk in comparison to the STIS image HD100546 from Figure 1e by Grady et al. (2001). The parameters used to describe this model are listed in Table 1 and as model #MA in Table 2. The rings comprising the disk are projected onto the sky in Figure 2. From a comparison between Figure 2 and Figure 4, we see that locations where the rings are in close proximity correspond to regions of higher surface brightness. These are regions where the disk slope or surface gradient (with respect to the line of sight) is high. A shadow from the inner disk is seen along the ring minor axes.

Surface brightness profiles for Model #MA along the major and minor axes, at position angles 127 and 37 degrees, are shown in Figure 3. We compare our model surface brightness profiles with the major axis profile shown in Figure 5a by Grady et al. (2001) of the STIS image. In both our model and the observed disk one spiral feature corresponds to a bump in the major axis surface brightness profile on the northwestern side at about 300AU from the nucleus. The bump in the surface brightness profile is about 0.2 in the log above a smoothly dropping curve; approximately consistent with that seen in the observed profile. The increase in surface brightness corresponding to the opposite spiral feature is much less prominent on the southeastern side in both model and observed profile than that on the northwestern side. Dustlanes are not prominent in the major axis surface brightness profile (in both model and observed profile), however they are deep in the model minor axis profile. The dips in the major axis profile exhibited by our model in Figure 3c, corresponding to dustlanes, are deeper than those observed. Equatorial shadowing in the model is more extreme than that observed. We discuss this problem in more detail below. The southern side of the disk is brighter than the north side because we allowed the scattering to be anisotropic and have taken the southern side to be nearer the viewer than the northern side. Some anisotropy in the scattering is consistent with the excess surface brightness in the near-infrared seen on the southern side at $r \sim 3''$ reported by Augereau et al. (2001).

We find that a warped model such as model #MA shown in Figures 1b, 2b can provide a good explanation for the spiral features observed in the disk of HD100546. The warp causes apparent surface brightness variations along two tightly wound spiral features, with darker regions within them. The darker regions were described as lanes by Grady et al. (2001). Two effects can account for regions of lower surface brightness: these regions are less inclined or more nearly perpendicular to the viewer (more nearly face-on), or they lie in the equatorial plane and so can be in the shadow of the inner disk. Higher surface brightness regions are those that rise above the midplane shadow. They also correspond to regions that have a steeper surface gradient or slope in the disk due to corrugations in the disk. Low inclination evenly illuminated warps can exhibit large spatial variations in the scattered light surface brightness, however this only occurs if the disk is highly corrugated or twisted and the disk is optically thin.

In Figure 4, lower right-hand panel, we show a nearly edge-on model #ME, with parameters listed in Tables 1 and 2. This model is identical to model #MA, shown in Figures 1-3 except the system is more highly inclined. Model #ME can be compared to the edge-on disk of Beta Pictoris. Beta Pictoris is less twisted in its outer region compared to this model and does not have a shadow in its midplane. The one planet model of Mouillet et al. (1997) would lead us to expect that the disk is more highly twisted at smaller radii. In future we may consider the possibility that some of the unresolved structure in Beta Pictoris’s or AU Microscopii’s inner disks at visible wavelengths might be explained with a warp that extends to smaller radii. We note that asymmetries between emission from one side of the disk compared to the opposite side would be introduced because the optical depth could be high at folds, and because of scattering asymmetry.

3.1. Sensitivity to parameters

In Figure 4 we show the effect of varying some of the parameters used to describe the disk. Model #MA has a disk inclination that increases with increasing radius. When the inclination is held constant, as is true in Model #MC, shown on the top-right in Figure 4 the spiral arms do not extend as far to the south-west and north-east as is seen in the scattered light image. We find that the angular extent of each spiral feature is smaller than that observed if $i$ is held fixed. If the inclination with respect to the system axis increases with radius, then the spiral features extend over a larger range of angles, as shown in Model #MB on the upper left in Figure 4 Model #MB has an even higher outer inclination than Model #MA. Disks with larger tilts (larger $i$) tend to produce higher surface brightness variations in the spiral arms. However if $i$ is increased past 10 degrees then the disk can be folded with respect to the viewer and this reduces the contrast of the spiral features. Model #MA and #MB have regions where multiple folds of the disk are encountered along the line of sight (see Figure 2), and only the fold edges are regions of high surface brightness.

It is interesting to note that the apparent ends of the two spiral arms are about 180 degrees apart and oriented nearly along the major axis of the disk. Such a situation arises naturally from the warped models. In contrast, spiral arms that are due to spiral density waves are not expected to end at locations 180 degrees apart. The folds of the warped disk cause high surface gradient regions (corresponding to higher surface brightness regions) on either side of the nucleus. However along the major axis these folds are oriented along the line of sight. As a result, spiral features in the model scattered light images tend to end along the disk major axis. Higher resolution models show a nested series of self-similar spiral features inside the outermost ones. This follows since we have adopted a power law form for the precession angle.

Similar morphology to that observed is seen when the inner disk has a higher equatorial opacity $\tau_0$ but a shorter opacity angular scale length $\theta_\tau$. Model #MD shows such a model. This model has a slightly larger outer disk scale height which has the effect of smoothing the model sur-
face brightness image. Had we left the disk scale height similar to that of model #MA this model would have had extreme contrast in its dust lanes. We find that thinner disks have more sharply defined spiral features and deeper dustlanes. When the angular scale length of the equatorial shadow is shorter, ($θ_τ$ is smaller), the dustlane is more sharply defined. Higher equatorial inner disk opacity (large $τ_0$) causes deeper shadows.

As we commented above, we suspect that Model #MA has deeper dustlanes than observed. The equatorial shadow for Model #MA has an opacity of 1 (at visible wavelengths) at equatorial angles $θ = ±13°$. This opacity is sufficiently high that it is approximately consistent with the absorption of $~50\%$ of the stellar light from the inner disk. However this high opacity and larger angular scale length (in the inner disk opacity) implies that the outer disk is illuminated by visible light that has been significantly attenuated by the inner disk; much of the outer disk has a tilt lower than $13°$. Our model computes the surface brightness at one wavelength only corresponding to a central optical wavelength for the broad STIS image. It may be possible to improve the model image by integrating and summing images at different wavelengths. It is also possible that wings of the stellar point spread function have smoothed the appearance of the outer disk, reducing the actual surface brightness contrast. The depth of the dustlanes in our model images can be decreased by increasing the angular scale length of the shadow, $θ_τ$. However then the model midplane opacity must be reduced to reproduce the observed morphology. A midplane opacity below 1 would be unrealistic as we expect the midplane to be dense and optically thick at visible wavelengths. To improve the model we suspect that we would require a more complex function for the shadow that than that given by Equation (2). This function would necessarily be described by a larger number of parameters. A more complex model is difficult to constrained with the STIS image but could be constrained by future high quality and multi-wavelength images.

The twistedness of the disk is set by $A_0$. Higher $|A_0|$ corresponds to more highly wound spiral features. However, variations in $A_0$ can also change the radius of the spiral features if the radius $r_0$ is not simultaneously adjusted. Because the spiral features are tightly wound we could not place constraints on the parameter $β$ which sets the dependence of $α$ on radius, consequently we set $β = 3.5$, consistent with the model explored by Mouillet et al. (1997). The contrast between the surface brightness on the near side compared to that on more distant side is larger if the scattering is more anisotropic; (the scattering asymmetry parameter $q$ is larger).

Based on our exploration of models with different parameters we have found the following: Only models with highly twisted disks have tightly wound spiral features similar to those observed. Only models with inclination increasing with radius have spiral features that extended over a sufficiently large range of azimuthal positions. Only relatively thin disks, $h/r \lesssim 0.15$, have sufficiently sharp or fine features to be similar to the observations. We find a degeneracy between the disk inclination, the angular form of attenuation from the inner disk, and the disk thickness, because these parameters all affect the contrast or amplitude of the spiral features. We did not find models with morphology similar to that observed with disks higher than $i \sim 15°$ and lower than $i \sim 6°$. Low inclination disks failed to exhibit sufficiently high surface brightness variations and high inclination disks exhibited multiple folds along the line of sight, reducing the surface brightness variations. Models with small shadow angular scale lengths ($θ_τ \lesssim 5°$) have dustlanes that are excessively deep, suggesting that the upper layers of the inner disk have an opacity distribution (as a function of $θ$) with a moderately large angular scale length.

3.2. Mass constraints

By matching the observed morphology of the disk with our model, we can estimate the extent that the disk is twisted. This is described by parameters $A_0$, and $β$ (see Equation 4). We consider here the hypothesis that the twist was caused by an unseen inner planet that is misaligned with the outer disk (e.g., Mouillet et al. 1997; Augereau et al. 2001b). We remind the reader that Equation (4) refers to a tilted disk with negligible mass which is perturbed by an inner planet. The parameter $A_0$ depends on the time since the disk was initially tilted, $Δt$, and the mass and semi-major axis of the hypothetical inner planet causing the precession. We can assume that $Δt$ is less than the age of the star or $Δt < 10^7$ years. This allows us to place a lower limit on the mass of the planet times the square of the planet semi-major axis. Using Equation (4) and replacing $Δt$ with the age of the system, $t_{age}$, we find

$$D^2 M_p > \frac{4|A_0|M_\star r_0^{7/2}}{3\sqrt{G M_\star t_{age}}}$$

Computing these quantities for our value of $A_0$ (in radians) and reference radius $r_0$ (listed in Table 2), we find

$$D^2 M_p \gtrsim 2.9 \times 10^4 AU^2 M_J$$

where $M_J$ is the mass of Jupiter. However, this constraint is impossible to satisfy for a single Jovian mass at small radius. This constraint cannot be satisfied for a Jovian mass planet within the lit edge of the disk at 13 AU (Gradv et al. 2003). A brown dwarf sized object at $D \sim 200$AU would open a gap in the disk that would be observable in the images. The simplest scenario of an initially misaligned low mass disk and and a single inner planet fails to account for the twistedness of the disk.

One way to account for the highly twisted disk would be if the disk itself contained significant mass. For example, a Jovian mass of planetesimals between 100-200 AU but misaligned with the outer disk could account for the twisted nature of the disk. If this mass is not confined to one body but extended then the radial power of the precession angle would be reduced. This might account for the somewhat better match of our model #MB with $β = 2.5$ instead of 3.5. HD100546’s disk appears to be more twisted than Beta Pictoris’s disk, however Beta Pictoris is 10-20 times older so the extent of the twist is less of a constraint on the planetary and disk system.

3.3. Leading or trailing arms

It may in future become possible to measure the sense of rotation of HD 100546’s disk. The disk rotation axis could be similar to the star’s rotation axis. Unfortunately the rotation axis of the star HD100546 is not known.
Clarke et al. (1999) attempted to measure this axis from polarization measurements, however these were affected by scattering from the surrounding dust. In spite of this, we now discuss the sense of the warp in comparison to the direction disk rotation. Future measurements of either the disk or stellar rotation could be used to support or refute dynamical explanations for the spiral morphology.

Equation 8 shows that the inner disk should precess faster than outer disk and in the retrograde sense. Consequently we expect the disk to should twist in the direction of rotation as the radius increases. If we differentiate Equation 8 we find that $d\Omega/dr$ is positive and so $\Omega$ increases in the direction of rotation. The spiral features would be leading instead of trailing. If the twist is a result of precession induced by bodies in the disk interior, we would predict that the disk is rotating clockwise on the sky. We expect the northwestern side to be blueshifted from the system center of mass, and the southeastern side to be redshifted. The southwestern side is probably closer to us, based on the nebulosity on the southern side seen in the STIS image Grady et al. (2001) and and because the excess seen in the scattered near-infrared light at $r \sim 3''$ on the southern side Augereau et al. (2001).

4. SUMMARY AND DISCUSSION

In this paper we have presented a new model for the spiral features seen in visible scattered light images of HD100546’s disk. We reproduce the two-armed spiral features with a highly twisted warped disk model. The disk inclination with respect to the system ecliptic is low, 6 to 15$^\circ$ degrees and increasing with radius. The disk is fairly thin with a ratio of FWHM/r of $\sim 0.15$. Surface brightness variations are due to a high surface gradient with respect to the line of sight in the folds of the disk. Dark lanes increase the contrast of the spiral features and are caused by equatorial shadowing from an inner unresolved disk. As a nearly Keplerian system can maintain a warp for long periods of time (secular timescales rather than rotational periods), this model has some advantages over transient spiral models. The observed spiral features end at positions approximately located on the disk major axis (as seen on the sky). This is a feature common to moderate inclination warped disk models that would not be exhibited by a spiral density wave model for the spiral features.

However, the morphology requires the disk to be so twisted that the tidal force from a Jovian mass object in the inner disk clearing cannot have induced the twist during the lifetime of the star. One possibility is that the disk warp could have been induced by a significant mass (Jovian mass) of objects that are inclined with respect to the outer disk, well outside the inner clearing, at intermediate radii between 100-200 AU. We now discuss proposed mechanisms for accounting for the tilt between the disk and objects in the inner disk. A stellar encounter could tidally induce a tilt in the outer disk. In this case we would expect the disk tilt or inclination would increase with radius (e.g., as calculated by Kobayashi & Ida, 2001). Our models better match the angular extension of the spiral feature when the tilt increases with radius, suggesting that this may be the case for HD100546’s disk. Such a stellar encounter, while unlikely in the field, could have been probable in a denser stellar environment such as the star’s birth cluster. An alternative scenario is that resonances between multiple planets cause an inclination increase in one of the planets (Thommes & Lissauer, 2003). In this case, the disk would have tilt or inclination that decreases with radius Mouillet et al. (1997). Also this model would likely require vertical resonances for planets at large semi-major axes to account for the highly twisted disk. The different dynamical scenarios predict different radial forms for the angles used to describe the warp, i and $\Omega$. Consequently better observations and accompanied modeling of the HD100546’s disk and other circumstellar disks will produce better constraints on the disk tilt and precession angle as a function of radius. These in turn will allow better tests of the dynamical models for the warp formation and evolution. The dynamical models have the capability of providing unique constraints on the mass distribution in the outer disk.

In this paper we have used a very simple model to take into account equatorial shadowing due to the opacity of the inner disk. Unfortunately we find a degeneracy in our models between the disk tilt, the angular form of attenuation from the inner disk, and the disk thickness. This is because these parameters all affect the surface brightness contrast or amplitude of the spiral features. Shadowing is likely to be a strong function of color, consequently multi-color imaging may be able to probe the structure of the inner unresolved disk as well as better constrain the geometry and structure of the outer disk. Warped disks seen at visible wavelengths should exhibit asymmetries due to optical depth variations, and asymmetries due to forward scattering. Both of these affects should be more severe at bluer wavelengths, providing a possible way to discriminate between geometrical models. We note that some asymmetries and perceived clumps in visible light in edge-on warped disks (e.g., as seen in AU Microscopii Liu, 2004), might in future be explained with a warped disk rather than a model containing eccentric rings. However asymmetries observed in thermal or mid-infrared emission, such as is observed in Beta Pictoris by Telesco et al. (2005), could not be explained via a warped disk model as at these wavelengths we expect the disks to be more nearly optically thin the emission should be nearly isotropic.

The observations of Eisner et al. (2004) suggest that BeBe stars are not highly warped. However, moderate inclination warps, such as found here would not violate the comparison of submillimeter and near-infrared position angles and ellipticities. It is possible that highly tilted warped disks may provide alternate explanations for excess far-infrared emission in some systems as such disks cover a larger solid angle and so can absorb more stellar flux at large radii than a coplanar disk of the same thickness.

I thank the Research School of Astronomy and Astrophysics of the Australian National University and Mount Stromlo Observatory for hospitality and support during Spring 2005. Support for this work was in part provided by National Science Foundation grant AST-0406823, and the National Aeronautics and Space Administration under Grant No. NNG04GM12G issued through the Origins of Solar Systems Program. Support was also provided by
Fig. 1.— The left-hand panel shows the log surface brightness of model warped disk, #MA with parameters are given in Tables 1 and 2. The middle panel shows log of the surface brightness of the HST image of HD100546 from Grady et al. (2001). Our model shows that a warped disk can exhibit surface brightness variations in a two armed spiral pattern. The surface brightness variations are due to the slope or gradient changes in the disk surface with respect to the line of sight and an equatorial shadow from an inner disk.

the National Science Foundation to the Kavli Institute for Theoretical Physics under Grant No. PHY99-07949.

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Fig. 2.—Projected circular disk rings at evenly spaced radii corresponding to the disk shown in figure 1 and described by model #MA. Regions where projected rings are in close proximity correspond to regions of bright surface brightness (see figure 1). This is because these regions have large slopes or surface gradients with respect to the line of sight. The equatorial shadow lies along the minor axes of the rings.

Fig. 3.—Major and minor axis surface brightness profiles for model #MA, that is also shown in Figures 1 and 2. Dips in the surface brightness profile are seen where there are dark dustlanes due to equatorial shadowing by the inner disk. Along the major axis at about 300 AU, an increase in the surface brightness is seen at a fold in the disk. A hole at $r = 13$ AU has been placed near the star, primarily to limit the flux range covered by the plot. For the major axis profile, positive $r$ refers to southeastern side of the disk. For the minor axis profile positive $r$ refers to the near or southwestern side of the disk. The $y$-axis shows $\log_{10}$ of the surface brightness normalized to the peak value.
Fig. 4.— The effect of varying warped disk parameters. These models are similar to that (Model #MA) shown in Figures 1-3. Varied parameters are listed in Table 2. On the top left is Model #MB which has a higher inclination at large radii than model #MA. The top right panel shows Model #MC which has constant inclination with radius. The bottom left panel shows Model #MD which has a higher equatorial opacity for inner disk (larger $\tau_0$) than Model #MA but a lower inner disk opacity angular scale height ($\theta_r$). On the lower right we show a nearly edge-on disk, model #ME, to illustrate the extent of the twist, and for comparison to edge-on systems such as Beta Pictoris.
TABLE 1
COMMON PARAMETERS DESCRIBING THE WARP MODELS

| Parameter | Value |
|-----------|-------|
| $\chi$   | $40^\circ$ |
| $A_0$    | $-260^\circ$ |
| $r_0$    | 250 AU |
| $\Omega_0$ | $230^\circ$ |
| $\beta$  | 3.5 |

Note. — The models shown in Figures 1, 2, 3 and 4 have these parameters in common. The position angle of the system rotation axis on the sky is denoted by $\chi$. The precession angle at the reference radius $r_0$ is $\Omega_0$. The reference radius $r_0$ is given in AU ($100\text{AU} \sim 1''$ on the sky for HD100546). The parameters $A_0$ and $\beta$ describe the sensitivity of the precession angle $\Omega$ with radius or the twistedness of the warp; see Equation (4). The angles $\chi$, $\Omega_0$ and $A_0$ are given in degrees.

TABLE 2
PARAMETERS DESCRIBING THE INDIVIDUAL MODELS

|     | #MA | #MB | #MC | #MD | #ME |
|-----|-----|-----|-----|-----|-----|
| $\vartheta$ | $52^\circ$ | $52^\circ$ | $52^\circ$ | $52^\circ$ | $89^\circ$ |
| $i$  | $3, 6, 15^\circ$ | $3, 5, 20^\circ$ | $6, 6, 6^\circ$ | $3, 6, 15^\circ$ | $3, 6, 15^\circ$ |
| $h/r$ | 0.05 | 0.05 | 0.05 | 0.08 | 0.05 |
| $\tau_0$ | 3.0 | 3.0 | 3.0 | 5.0 | 3.0 |
| $\theta_\tau$ | $12.0^\circ$ | $12.0^\circ$ | $12.0^\circ$ | $5.0^\circ$ | $12^\circ$ |

Note. — The inclination of the system rotation axis with respect to the viewer is $\vartheta$. The inclination of the warped disk with respect to the system rotation axis is $i$. The angles $\vartheta$, and $i$ are given in degrees. The disk aspect ratio, assumed to be constant with radius is denoted by $h/r$. The standard deviation of the vertical density distribution is $h$ and the FWHM is $2.35h$. Inclination angles are given at three radii, $r = 100, 200$ and $400\text{AU}$, and we set $i = 0$ at $r = 0$. A spline function was fit between these values. The opacity $\tau_0$ describes the opacity of the inner disk in the equatorial plane. The parameter $\theta_\tau$ describes the angular scale height of the inner disk’s opacity in degrees. Surface brightness profiles, projected rings and images for model #MA are shown in Figures 1, 2, 3. Images are shown in Figure 4 for Models #MB–#ME.