Prediction of turning performances using an equivalent oblique cutting model

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Abstract
The main purpose of this paper is to predict the performances of the turning process using an equivalent oblique cutting model. Based on the real tool, an equivalent cut geometry is performed considering the effects of nose and edge radii. Edge direction and normal cutting angles, uncut chip thickness, and depth of cut were redefined by their equivalent values and then used as new inputs. Turning performances, such as cutting force components, cutting temperatures, and tribology parameters at the tool/chip interface, were predicted over a wide range of cutting conditions. The position of the maximum temperature at the tool/chip interface and its value are determined by solving the heat equation in the chip using the Finite Difference Method. Different assumptions were concluded, and the thermal problem is simply resolved using Laplace transform. It was determined that the maximum tool/chip interface temperature is situated far from the cutting edge about $0.317\ell_c$. It was also found that the partition coefficient is strongly related to sliding speed and it decreases about 20\% when chip velocity increases from 1 to 5 m/s. Acceptable agreement was concluded between experimental cutting force components and those predicted from the equivalent oblique cutting model. It can thus be concluded that the equivalent model of cut is highly recommended to predict turning performances.

Keywords Turning performances · Tool/chip interface temperature · Heat equation · Finite Difference Method (FDM) · Laplace transform · Partition coefficient

1 Introduction

Turning represents an essential process in manufacturing components in which tool geometry performs material removal. It can be used for a variety of work materials to generate different shapes with a better workpiece quality such as roughness, geometric and dimensional specifications, and surface finish aspects.

Machined materials are subject to plastic deformations. More than 90\% of mechanical work applied to workpiece material transforms to thermal energy which is spread in tool, workpiece, and chip [1]. The percentage of cutting heat flowing into the cutting tool varies from 2 to 18\%, into the workpiece from 1 to 20\%, and into the chip from 74 to 96\% [2]. According to Puls et al. [3], elevated tool/chip interface temperature accelerates tool wear and causes a thermal expansion of cutting tool which causes cutting edge displacement and worsens machined surface quality. On the other hand, it is well known that the direct measurement of tool/chip interface temperature in machining is very difficult. Huang et al. [4] have developed an on-line inverse technique to estimate the temperature field at the tool/chip interface of a turning tool using temperatures measured at some sensor-accessible locations. According to Kus et al. [5], it is necessary to use different measuring techniques to obtain consistent temperature data. They have estimated tool temperature by simultaneous measurement employing both a K-type thermocouple and an infrared radiation pyrometer.

Predicting performances, such as cutting forces, tool/chip interface temperature, heat partition between workpiece, tool and chip, tribology parameters, and tool wear, are highly recommended by the machining community. The models commonly used by researchers are analytical, numerical, and mechanistic. The analytical approach allows to understand the physical phenomenon which occurs when metal cutting
and predicts thermomechanical parameters over a wide range of cutting conditions. On the other hand, the numerical solutions provide a viable alternative to understand the machining process; however, its limitation consists in its ability to assimilate the complexity of machining problem.

Merchant was the pioneer who introduced the concept of shear angle [6]. Then, Lee and Shaffer have proposed an analytical model based on the slip line theory [7]. Later, Oxley introduced the first model based on the thermo-mechanical approach for the orthogonal cutting [8]. However, it was limited to a few industrial applications such as grooving and pipe cutting. For this reason, the oblique cutting modeling was highly recommended to study metal cutting with a complex geometry for each couple workpiece-tool (CWT).

The most recent thermomechanical approaches for oblique cutting were developed by Moufki et al. [9], Li et al. [10], and Abdellaoui and Bouzid [11]. However, the effects of edge and nose radii are not explicitly considered. In fact, nose radius has a significant effect on the geometry of uncut chip area, uncut chip thickness, cutting forces, chip geometry, heat generation, tool wear, and surface finish during the turning process [12, 13]. On the other hand, cutting edge radius has a critical role in machining performances for small uncut chip thickness. Besides, when turning with a larger tool nose radius, a large part of the uncut chip area will have a chip thickness less than the minimum uncut chip thickness [14]. Recently, Khlifi et al. [15] have developed a thermo-mechanical approach which leads to determine the equivalent geometry of the tool when considering tool nose radius. Roughing and finishing turning operations were considered for validation and predicted results were close to experimental data. Then, effects of tool nose and edge radii were considered by Khlifi et al. [16] during longitudinal turning. The main idea consists in considering that the equivalent geometry induces the same cutting force as the real tool. Thus, equivalent cutting angle, equivalent direction angle, equivalent uncut chip thickness, and equivalent depth of cut are defined for the new tool geometry.

Within the last 70 years, there have been improvements of the analytical models and simulation techniques on the one hand and a strong machining application on the other. The main focus for machining community consists in establishing a relevant single modeling method which permits to predict machining performances over a wide range of cutting conditions. In particular, accurate turning performances prediction such as the value and position of the maximum temperature at the tool/chip interface still remains a challenge due to the complexity of contact phenomena and tool geometry. To fill this gap, it is proposed in the present research work a predictive model which permits to predict all thermomechanical parameters such as cutting force components, maximum tool/chip interface temperature, and its position at the rake face and tool/chip contact parameters (contact length, temperature evolution, pressure, friction coefficient).

This paper is organized as follows: in Sect. 2, the equivalent model is presented, and its parameters are defined. In Sect. 3, two algorithms are presented. Algorithm 1 permits to determine equivalent geometry of the tool ($\kappa_{r}^{eq}$ and $\sigma_{n}^{eq}$) and the equivalent cutting parameters ($\alpha_{p}^{eq}$ and $t_{c}^{eq}$), and the second one predicts all equivalent thermomechanical parameters. Besides, in Sect. 4 is presented and three approaches are used. Oxley’s models, applied to equivalent oblique cutting, predict the average tool/chip interface temperature. Then, a Finite Difference Method (FDM) is presented to solve the heat equation in the chip. Based on the numerical results, the thermal problem was simplified and solved using Laplace transform. The analytical solution permits to predict the maximum tool/chip interface temperature and its position in the rake face. In the Sect. 5, experimental tests are presented, and acceptable agreement was noted between experimental force components and those predicted by equivalent model. Finally, tribological parameters were investigated in the Sect. 6.

### 2 Equivalent cutting for turning process

The real tool’s geometry is characterized by many parameters such as nose radius $r_{n}$, edge radius $r_{p}$, normal cutting angle $\alpha_{n}$, and the direction angle $\kappa_{r}$. Moreover, the turning operation is defined by cutting parameters which are depth of cut $a_{p}$, feed rate $f$, and cutting speed $V_{c}$. Considering all these parameters makes the study more and more complex. Thus, working with the equivalent tool geometry permits to avoid this complexity, mainly that of the nose and edge radii. The equivalent geometry is defined while conserving the same uncut chip area and values of the cutting force components compared to real cutting geometry [16].

In the present research work, equivalent parameters are defined using the work of Khlifi et al. [16]. Both effects of nose and edge radii are considered. Thus, the main equivalent cutting model is defined by four parameters which are the equivalent edge direction angle $\kappa_{r}^{eq}$, the equivalent depth of cut $a_{p}^{eq}$, the equivalent uncut chip thickness $t_{c}^{eq}$, and the equivalent normal rake angle $\alpha_{r}^{eq}$ (Figs. 1 and 2).

### 3 Thermo-mechanical modeling of equivalent cutting

Thermomechanical parameters are defined and determined using the work of Abdellaoui and Bouzid [11, 13] and the work of Khlifi et al. [16]. Two algorithms are used to determine cutting forces, cutting temperatures, and chip geometry (Fig. 3). The first one allows to determine the equivalent
geometry of the tool ($k_{eq}$ and $a_{eq}$) and the equivalent cutting parameters ($a_{eq}$ and $t_{1}^{eq}$). The second algorithm leads to determine the average tool/chip interface temperature and all other thermomechanical parameters for the equivalent cutting.

4 Study of the thermal problem in the chip

4.1 Oxley’s models applied to equivalent oblique cutting

Oxley has proposed a model which permits to determine the average temperature at the tool/chip interface $\tilde{T}_{int}^{eq}$ [8]. This model is described by the following equation:

$$\tilde{T}_{int}^{eq} = T_{sh}^{eq} + \psi \Delta T_{chip}^{max}$$  \hspace{1cm} (1)

where $\Delta T_{chip}^{max}$ is the maximum temperature rise into the chip, which occurred at the tool–chip interface and $\psi = 0.7$ is a factor that allows for $\tilde{T}_{int}$ being an average value.

The equivalent shearing temperature ($T_{sh}^{eq}$) is obtained from the equivalent shearing power ($P_{sh}^{eq}$) which is evaluated through the two following expressions:

$$P_{sh}^{eq} = F_{sh}^{eq} V_{sh}^{eq} = \rho c a_{eq}^{eq} t_{1}^{eq} V_{c} (T_{sh}^{eq} - T_{0})$$  \hspace{1cm} (2)

where $c$ is the work material specific heat ($J/kg \circ C$), $\rho$ is the work material density ($kg/m^3$), $F_{sh}^{eq}$ is the shearing force, and $V_{sh}^{eq}$ is the shearing velocity.

Based on the above equation, the shearing temperature $T_{sh}^{eq}$ is given by the following equation:

$$T_{sh}^{eq} = T_{sh}^{eq} + \Delta T_{chip}^{max}$$
According to Oxley [8], not all the plastic work of chip formation has occurred at the primary shear band. Moreover, according to Boothroyd [17], not all heat generated in the primary shear band is conducted to chip. Thus, the equivalent shearing temperature, based on the above assumptions, is given as follows:

$$T_{eq} = T_0 + \frac{F_{sh}^{eq} \cos \alpha_{n} \cos \lambda_s}{\rho c a_p t_1^{eq} \cos \eta_{sh} \cos (\phi_{n}^{eq} - \alpha_{n}^{eq})}$$

(3)

$$T_{sh} = T_0 + \eta(1 - \lambda)\frac{F_{sh}^{eq} \cos \alpha_{n} \cos \lambda_s}{\rho c a_p t_1^{eq} \cos \eta_{sh} \cos (\phi_{n}^{eq} - \alpha_{n}^{eq})}$$

(4)

where $\eta = 0.7$ is the factor that allows the fact that not all the plastic work of chip formation has occurred at the primary shear zone.

The parameter $\lambda (0 \leq \lambda \leq 1)$ is the proportion of heat conducted into the workpiece which is determined using the following equation:

$$\lambda = \begin{cases} 0.5 - 0.35 \log_{10}(R_T \tan \phi_n^{eq}) & \text{if } 0.04 \leq R_T \tan \phi_n^{eq} \leq 10 \\ 0.3 - 0.15 \log_{10}(R_T \tan \phi_n^{eq}) & \text{if } R_T \tan \phi_n^{eq} > 10 \\ \frac{\rho c a_p V}{k} & \end{cases}$$

(5)

where $k (W m^{-1} K^{-1})$ is the thermal conductivity of workpiece material.

Based on the research work done by Weiner [18], Rapier [19], and Boothroyd [17], Oxley [8] has represented Boothroyd’s results by the following equation:

$$\log_{10}\left(\frac{\Delta T_{\text{max}}^{\text{chip}}}{\Delta T_{\text{chip}}}\right) = 0.06 - 0.195\delta \sqrt{\frac{R_T t_2^{eq}}{t_c^{eq}}} + 0.5 \log_{10}\left(\frac{R_T t_2^{eq}}{t_c^{eq}}\right)$$

(6)

where $\delta$ is the ratio of the plastic zone thickness to the chip thickness ($t_2^{eq}$).

The average temperature rise in the chip ($\Delta \hat{T}_{\text{chip}}$) is determined from the equivalent friction power ($P_{\text{eq}}$) generated at the tool–chip interface:

$$P_{\text{eq}} = T_{(c/\Omega)} V_{\text{chip}} = \rho c a_p t_1 V \Delta \hat{T}_{\text{chip}}$$

(7)

where $V_{\text{chip}}$ is the chip flow velocity and $T_{(c/\Omega)}$ is the tool–chip interfacial force which is determined from the tool–chip interface pressure distribution ($p(z_{\text{sh}})$) and the average friction coefficient ($\bar{f}$). Hence:

$$T_{(c/\Omega)} = \int_{0}^{1} \frac{\bar{f} a_p p(z_{\text{sh}})}{\cos \lambda_s} d_{z_{\text{sh}}} = \frac{\bar{f} a_p p_{\text{max}}}{3 \cos \lambda_s}$$

(8)

Based on the above equations, the average temperature rise in the chip is given as follows:
\[ \Delta T_{\text{chip}} = \frac{j_l p_{\text{max}} \sin \phi_n}{3 \rho c t_1 \cos (\phi_n - \alpha_n)} \]  (9)

### 4.2 Numerical approach using Finite Difference Method (FDM)

To predict the position and the maximum value of the tool/chip interface temperature, a numerical simulation is developed based on the Finite Difference Method (FDM). Only the chip is considered and is supposed to have a rectangular form which is defined by its equivalent tool/chip contact length \( l_{eq} \) and equivalent chip thickness \( t_{eq} \). However, effects of equivalent normal shear angle \( \phi_{eq}^{\nu,n} \), equivalent normal cutting angle \( \phi_{eq}^{\nu,n} \), and equivalent chip flow angle \( \eta_{eq}^{\nu} \) are considered in the calculus of the boundary conditions and loadings.

Figure 4 corresponds to the thermal loadings and boundary conditions used to solve the heat equation which governs in the chip. The shearing plane is supposed to be at the same equivalent shearing temperature \( T_{eq}^{sh} \). Surfaces \( S_1 \) and \( S_2 \) are subject to convection fluxes. The tool/chip interface undergoes a heat flux \( Q_{eq}(z_f) \) which is generated by friction when the chip slides.

In the transient state, the general heat equation in the chip is given when two dimensional as follows:

\[
\frac{\partial}{\partial z_f} \left( k_{eq} \frac{\partial T}{\partial z_f} \right) + \frac{\partial}{\partial x} \left( k_{eq} \frac{\partial T}{\partial x} \right) - \rho^c c^c V_{eq}^{chip} \left( \frac{\partial T}{\partial z_f} \right) + Q_{eq}(z_f) + Q_1^{eq} + Q_2^{eq} = \rho^c c^c \frac{\partial T}{\partial t} \tag{10}
\]

where \( k^{eq}(mW/mm^\circ C) \), \( \rho^{eq}(ton/mm^3) \), \( c^{eq}(mJ/ton^\circ C) \), and \( t \) are respectively the equivalent thermal conductivity, the equivalent density, the equivalent mass heat capacity of the chip, and the time.

Seven terms were considered in the heat equation which are described as follows:

- \( \frac{\partial}{\partial z_f} \left( k_{eq} \frac{\partial T}{\partial z_f} \right) \): The thermal conduction term according to \( Z_{eq} \)
- \( \frac{\partial}{\partial x} \left( k_{eq} \frac{\partial T}{\partial x} \right) \): The thermal conduction term according to \( X_{eq} \)
- \( \rho^c c^c V_{eq}^{chip} \left( \frac{\partial T}{\partial z_f} \right) \): Advection term
- \( Q_{eq}(z_f) \): Heat flux generated in the tool/chip interface
- \( Q_1^{eq} \): A convective heat flow which is applied at the surface \( S_1 \)
- \( Q_2^{eq} \): A convective heat flow which is applied at the surface \( S_2 \)
- \( \rho^c c^c \frac{\partial T}{\partial t} \): Transient term

To solve the heat equation described by the above equation (Eq. (10)), terms \( Q_{eq}(z_f) \), \( Q_1^{eq} \), and \( Q_2^{eq} \) are determined in the following sections.

#### 4.2.1 Convective heat flows: \( Q_1^{eq} \) and \( Q_2^{eq} \)

Surfaces \( S_1 \) and \( S_2 \) of the chip are exposed, respectively, to convective heat flows \( Q_1^{eq} \) and \( Q_2^{eq} \). \( Q_1^{eq} \) is applied at surface \( S_1 \) and is given by the following equation:
\[ Q_{1}^{eq} = h_{1}^{eq}(T_{1} - T_{a}) \]  

(11)

where \( h_{1}^{eq}, \ T_{1}, \) and \( T_{a} \) are, respectively, the equivalent heat convection coefficient, the temperature in surface \( S_{1}, \) and the air temperature.

The convective heat flow \( Q_{2}^{eq} \) is applied at the surface \( S_{2} \) and is given by the following equation:

\[ Q_{2}^{eq} = h_{2}^{eq}(T_{2} - T_{a}) \]  

(12)

where \( h_{2}^{eq}, \ T_{2}, \) and \( T_{a} \) are, respectively, the equivalent heat convection coefficient, the temperature in surface \( S_{2}, \) and the air temperature.

The coefficients of convection \( h_{1}^{eq} \) and \( h_{2}^{eq} \) are expressed in \((\text{mW} / \text{mm}^{2} \text{C})\) and determined using the dimensionless numbers which are Nusselt’s number \((Nu)\), Reynold’s number \((Re)\), and Prandtl’s number \((Pr)\).

\[
\begin{align*}
Nu_{i}^{eq} &= \frac{h_{i}^{eq}d_{si}^{eq}}{k_{a}n_{i}^{eq}}, \\
Re_{i}^{eq} &= \frac{\rho_{a} \nu_{i} {\nu_{i}^{eq}}}{\kappa_{a}}, \\
Pr_{i}^{eq} &= \frac{\mu_{a} \kappa_{a}}{\nu_{i}^{eq}}.
\end{align*}
\]  

(13)

where:

- \( k_{a}, \ \rho_{a}, \ C_{a}, \) and \( \mu_{a} \) are respectively the thermal conductivity in \((\text{mW} / \text{mm} \cdot \text{C})\), the density in \((\text{ton} / \text{mm}^{3})\), the mass heat capacity in \((\text{mJ} / (\text{ton} \cdot \text{C}))\), and dynamic viscosity in \((\text{ton} / \text{mins})\) of the air.
- \( i: \) surface index 1 or 2.
- \( d_{si}^{eq} \) represents \( \ell_{2}^{eq} \) for \( S_{2} \) and \( \ell_{1}^{eq} \) for \( S_{1} \).

The thermal properties of air are resumed in Table 1.

Based on the above equations, the convection coefficient \( h_{1}^{eq} \) is determined using equations as follows:

\[
\begin{align*}
\hat{h}_{1}^{eq} &= \frac{2k_{a}(Re_{i}^{eq})^{0.8}(Pr_{i}^{eq})^{0.33}}{\mu_{a}^{eq}}, \quad \text{if} \ Re_{i}^{eq} < 3.10^{5} \\
\hat{h}_{1}^{eq} &= \frac{0.036k_{a}(Re_{i}^{eq})^{0.8}(Pr_{i}^{eq})^{0.33}}{\nu_{i}^{eq}}, \quad \text{if} \ Re_{i}^{eq} > 3.10^{5}
\end{align*}
\]  

(14)

## 4.2.2  Heat flux: \( Q^{eq}(z_{fl}) \)

The heat flux \( Q^{eq}(z_{fl}) \) is supposed to have an exponential evolution with respect to the chip flow direction and resembles the evolution of tool/chip interface normal pressure \( P^{eq}(z_{fl}) \) [11].

Thus, the heat flux \( Q^{eq}(z_{fl}) \) is given as follows:

\[ Q^{eq}(z_{fl}) = \hat{d}_{0}^{eq}\left(1 - \frac{z_{fl}}{\ell_{1}^{eq}}\right)^{2} \]  

(15)

The term \( \hat{d}_{0}^{eq} \) represents the equivalent maximum heat flux at the tool/chip interface. It is determined from the equivalent thermal power which is evaluated per unit of equivalent length of cut \((L_{cut}^{eq})\) and generated by friction at the tool/chip interface. Hence, the total thermal power is determined as follows:

\[ P^{eq}_{\text{tot}} = \frac{T^{eq}_{c/(c)}V^{eq}_{\text{chip}}}{L_{cut}^{eq}} \]  

(16)

where:

- \( T^{eq}_{c/(c)} \) is the equivalent tangential force acted by the tool on the chip in the interface. It is given by the equation as follows:

\[ T^{eq}_{c/(c)} = \int_{0}^{z_{fl}} t^{eq}_{c} z^{eq}_{cut} P^{eq}(z_{fl}) dz_{fl} = \frac{\int_{0}^{z_{fl}} z_{fl} t^{eq}_{c} L^{eq}_{cut} \rho^{eq}_{chip} d^{eq} \rho^{eq}_{chip} \rho^{eq}_{chip} \rho^{eq}_{chip}}{3} \]  

(17)

- \( V^{eq}_{\text{chip}} \) represents the equivalent chip flow speed and is given as follows:

\[ V^{eq}_{\text{chip}} = \frac{V_{c} \sin \phi^{eq}_{n} \cos \lambda_{s}}{\cos \eta^{eq}_{s} \cos(\phi^{eq}_{n} - \alpha_{n})} \]  

(18)

Based on the above equations, the total thermal power per unit of equivalent length of cut \((L_{cut}^{eq})\) is given by the following equation:

\[ P^{eq}_{\text{tot}} = \frac{\int_{0}^{z_{fl}} z_{fl} t^{eq}_{c} L^{eq}_{cut} P^{eq} V^{eq}_{\text{chip}} \sin \phi^{eq}_{n} \cos \lambda_{s}}{3 \cos \eta^{eq}_{s} \cos(\phi^{eq}_{n} - \alpha_{n})} \]  

(19)

According to Norouzifard and Hamedi [1], a proportion \( \beta = 70\% \) of the equivalent thermal power \( P^{eq}_{\text{tot}} \) generated at the tool/chip interface is transmitted to the chip. Thus, the total thermal power per unit of equivalent length of cut \((L_{cut}^{eq})\) transmitted to the chip \( P^{eq}_{\text{chip}} \) is represented by the following relation:

---

**Table 1 Physical properties of air [20]**

| Dynamic viscosity, \( \mu_{a} \) (ton / mm / s) | Density, \( \rho_{a} \) (ton / mm / s) | Thermal conductivity, \( C_{a} \) (mJ / mm / C) | Mass heat capacity, \( K_{a} \) (mW / mm / C °C) |
|---|---|---|---|
| 1.850.10^{-11} | 1.177.10^{-12} | 0.0262 | 1006.10^{6} |
On the other hand, $P_{eq\ chip}$ can be determined by the calculus of the integral of the function $Q^eq(z)\ (z)$ which describes the heat flux distribution along the tool/chip contact. That way, $P_{eq\ chip}$ is given as follows:

$$P_{eq\ chip} = \int_0^{t^eq} Q^eq(z)\ dz = \frac{q^eq_0 t^eq}{3}$$  \hspace{1cm} (21)

Based on Eqs. (20) and (21), the term representing the equivalent maximum heat flux at the tool/chip interface is described by the following relation:

$$q^eq_0 = \frac{\beta^eq p^eq_0 V_c \sin \phi^eq_0 \cos \lambda_s}{3 \cos \eta^eq_0 \cos(\phi^eq_0 - \alpha_n)}$$  \hspace{1cm} (22)

### 4.2.3 Boundary conditions

The boundary conditions are as follows:

$$\begin{aligned}
T(z, 0, x) &= T^eq for 0 \leq x \leq t^eq_1 \\
Q^eq(z) &= -k^eq \frac{dT}{dz}|_{z=0} for 0 \leq z \leq t^eq \\
Q^eq_1 &= h^eq_1(T_1 - T_n) = -k^eq \frac{dT}{dx}|_{x=t^eq_1} for 0 \leq x \leq t^eq_2 \\
Q^eq_2 &= h^eq_2(T_2 - T_n) = -k^eq \frac{dT}{dx}|_{x=t^eq_2} for 0 \leq x \leq t^eq_2
\end{aligned}$$  \hspace{1cm} (23)

4.2.4 Discretization of the heat equation using Finite Difference Method (FDM)

The chip is discretized into elements: $(M - 1)$ elements along the equivalent tool/chip contact $t^eq_1$ and $(N - 1)$ elements along the equivalent chip thickness $t^eq_2$. Thus, it results in $(M \times N)$ nodes (Fig. 5).

The dimensions of each element are defined as follows:

$$\begin{aligned}
\Delta z &= \frac{t^eq}{M-1} \\
\Delta x &= \frac{t^eq}{N-1} \hspace{1cm} (24)
\end{aligned}$$

The increment time is determined by the following relation:

$$\Delta t = \frac{t^eq}{(N_t - 1)V^eq_{chip}}$$  \hspace{1cm} (25)

where $M, N$, and $N_t$ are chosen under the condition of stability as follows:

$$\frac{\Delta t}{\Delta x^2} \left( \frac{1}{\Delta x^2} + \frac{1}{\Delta z^2} \right) \leq \frac{1}{2}$$  \hspace{1cm} (26)

In the present research work, thermal conductivity $k^eq$, mass heat capacity $c^eq$, and density $\rho^eq$ of the chip material are estimated at the equivalent shearing plane temperature $T_{eq\ sh}$.

At $j$th time, the equations in discrete forms are given by
Based on the boundary conditions and loading, equations in discrete forms, heat equation, and the principle of equilibrium at nodes \((M, N)\) and \((M, 1)\), temperatures in all nodes are determined by equations as follows:

- **Nodes inside chip:** \(2 \leq m \leq M - 1\) and \(2 \leq n \leq N - 1\)

\[
T^{j+1}_{m,n} = \left( 1 - 2a^q \Delta t \left( \frac{1}{\Delta x^2} - \frac{1}{\Delta z^2} \right) \right) T^{j}_{m,n} + \frac{a^q \Delta t}{\Delta x^2} T^{j}_{m+1,n} + \frac{a^q \Delta t}{\Delta z^2} T^{j}_{m,n+1} + \left( \frac{a^q \Delta t}{\Delta z^2} - \frac{\Delta T_{\text{chip}}}{2\Delta z} \right) T^{j}_{m+1,n} + \left( \frac{a^q \Delta t}{\Delta z^2} - \frac{\Delta T_{\text{chip}}}{2\Delta z} \right) T^{j}_{m,n+1}
\]

- **Nodes at surface \(S_1: 2 \leq n \leq N - 1\)**

\[
T^{j+1}_{m,N} = \left( 1 - 2a^q \Delta t \left( \frac{1}{\Delta x^2} + \frac{1}{\Delta z^2} \right) \right) T^{j}_{m,N} + \frac{a^q \Delta t}{\Delta x^2} T^{j}_{m+1,N} + \frac{2a^q \Delta t}{\Delta z^2} T^{j}_{m,N-1} + \frac{2a^q \Delta t}{\Delta z^2} T^{j}_{m+1,N} + \frac{2a^q \Delta t}{\Delta z^2} T^{j}_{m,n+1}
\]

- **Nodes at tool/chip contact:** \(2 \leq m \leq M - 1\)

\[
T^{j+1}_{m,1} = \left( 1 - 2a^q \Delta t \left( \frac{1}{\Delta x^2} + \frac{1}{\Delta z^2} \right) \right) T^{j}_{m,1} + \frac{2a^q \Delta t}{\Delta x^2} T^{j}_{m,2} + \frac{2a^q \Delta t}{\Delta z^2} T^{j}_{m,1} + \left( \frac{a^q \Delta t}{\Delta z^2} - \frac{\Delta T_{\text{chip}}}{2\Delta z} \right) T^{j}_{m+1,1} + \left( \frac{a^q \Delta t}{\Delta z^2} - \frac{\Delta T_{\text{chip}}}{2\Delta z} \right) T^{j}_{m-1,1}
\]

- **Nodes at surface \(S_2: 2 \leq m \leq M - 1\)**

\[
T^{j+1}_{m,N} = \left( 1 - 2a^q \Delta t \left( \frac{1}{\Delta x^2} - \frac{1}{\Delta z^2} \right) \right) T^{j}_{m,N} + \frac{a^q \Delta t}{\Delta x^2} T^{j}_{m+1,N} + \frac{2a^q \Delta t}{\Delta z^2} T^{j}_{m,N-1} + \frac{2a^q \Delta t}{\Delta z^2} T^{j}_{m+1,N} + \frac{2a^q \Delta t}{\Delta z^2} T^{j}_{m,n+1}
\]

Equations (28) to (34) can be written with matrix writing as follows:

\[
[T^{j+1}] = [A][T^j] + [B]
\]

Solving the above equation permits to determine temperatures in all nodes at every \(j^{th}\) increment of time.
4.3 Hybrid analytical–numerical algorithm

The analytical algorithm allows to determine all equivalent thermo-mechanical parameters for the turning process such as cutting temperatures, cutting forces, and tribological parameters. To determine the position and the value of the maximum tool/chip interface temperature, a hybrid analytical–numerical algorithm is developed with respect to the assumptions presented in the above sections. The equivalent tool/chip interface temperature $\tilde{T}_{\text{eq}}^{\text{int}}$, the equivalent tool/chip contact length $\tilde{l}_{\text{c}}^{\text{eq}}$, the equivalent chip thickness $\tilde{t}_{\text{eq}}^{2}$, the equivalent chip flow velocity $\tilde{V}_{\text{chip}}^{\text{eq}}$, and the maximum of the equivalent of normal pressure $\tilde{p}_0^{\text{eq}}$ at tool/chip interface are determined by the analytical algorithm and, thus, they represent the inputs of the numerical algorithm. Besides, the equivalent convection coefficients $\tilde{h}_{\text{eq}}^{1}$ and $\tilde{h}_{\text{eq}}^{2}$ are estimated and the spatial and temporal discretization are done. Finally, Eq. (35) is resolved and isotherms are known for each time increment. Temperatures for nodes along the tool/chip contact permit to determine the position and the value of the maximum temperature. Figure 6 corresponds to the hybrid flowchart used in the present research work.

4.4 FDM results and discussion

Figures 7 and 8 present results predicted from the numerical simulation by Finite Difference Method (FDM). Oblique cutting with a single cutting edge was considered and four cutting speeds were used ($V_c = 180, 300, 420,$ and $540 \text{ m/min}$).

The workpiece material used in the simulation is AISI304, and its thermal properties and Johnson–Cook parameters are presented in Table 2. Normal cutting angle $\alpha_n = -6^\circ$, inclination angle $\lambda_s = -6^\circ$, edge direction angle $k_e = 75^\circ$, depth of cut $a_p = 1 \text{ mm}$, and uncut chip thickness $t_1 = 0.1 \text{ mm}$.

Figure 7 corresponds to the temperature distribution in the chip when steady state is reached. It is underlined that the maximum tool/chip interface temperature $T_{\text{int}}^\text{max}$ in the chip increases with respect to cutting speed. However, tool/chip contact length $l_c$ and chip thickness $t_2$ decrease.

Figure 8 presents the evolution of the tool/chip interface temperature along the tool/chip contact length $l_c$ with respect to $z_f$. It can be concluded that maximum temperature is situated approximately at $z_f$ and approaches the cutting edge as cutting speed increases. Moreover, it is highlighted that the temperature at the chip’s top is kept at the shearing temperature $T_{\text{sh}}$ and a slight decrease of shearing temperature is noted with respect to cutting speed. Figure 9 corresponds to the evolution of both the position and the maximum tool/chip interface temperature. It is shown that uncut chip thickness significantly affects the position and value of the maximum tool/chip interface temperature. In fact, this temperature increases with uncut chip thickness and its position moves away from the cutting edge.

Figure 10 presents the evolution of the maximum tool/chip interface temperature and its position with respect to time. Two cutting speeds are considered ($V_c = 180$ and $420 \text{ m/min}$) and two states were underlined: transient and steady state. The simulation period used represents the sliding time of the...
chip on the rake face. The transient regime takes about half of this period, then the steady state settles down. Evolution of the position ($z_{fl}^{\text{max}}$) and the maximum value of tool/chip interface temperature ($T_{\text{chip}}^\text{max}$) with respect to time is drawn in the same curve (Fig. 11). It is shown that steady state took place rapidly (about 0.25 ms for cutting with $V_c = 180$ m/min and 0.06 ms for cutting with $V_c = 420$ m/min). Both position and value of $T_{\text{chip}}^\text{max}$ vary when transient regime happened.

Figure 12 corresponds to the evolution of the temperature in the chip when fixing $z_{fl}$ at $z_{fl}^{\text{max}}$ and varying $x$ along chip thickness $t_2$. It is found that the top part of the chip (more than the half) is at the shearing temperature $T_{sh}$. This remark permits to neglect the convection in the chip’s top and consider it at the shearing temperature. Hence, boundary condition of the thermal problem can be changed and the Dirichlet condition (imposed temperature) can be used instead of the Neuman condition (imposed flux).

4.5 Analytical approach of thermal problem for equivalent cutting using Laplace transform method

Based on results found in the above section and when neglecting conduction in the direction of motion of the chip (along $z_{fl}$), thermal problem and boundary conditions can be drawn in Fig. 13. In this section, thermal problem will be solved using Laplace transform method.

Based on the above considerations and adapting steady state, the heat equation in the chip can be described by the following equation:

$$\frac{\partial T}{\partial t} = \alpha \frac{\partial^2 T}{\partial x^2}$$

where $T$ is the temperature, $t$ is time, $x$ is the position along the chip thickness, and $\alpha$ is the thermal diffusivity.
Table 2: Johnson–Cook and thermal parameters of AISI304 [13]

| Temperature (°C) | 110 | 1700 | 1800 |
|------------------|-----|------|------|
| Thermal properties | $\rho (\text{kg/m}^3)$ | $c (\text{Jkg}^{-1}\text{K}^{-1})$ | $k (\text{Wm}^{-1}\text{K}^{-1})$ |
| $\text{Max}$ | 1500 | 0.36 | 0.014 |
| $\nu$ | 1 | 0.54 |
| $T_0$ | 1694 | 1.54 |

Johnson–Cook law: 

$$\tau_{sh} = \frac{1}{\sqrt{3}} \left( A + B \left( \frac{\dot{\varepsilon}}{\dot{\varepsilon}_0} \right)^n \right) \left( 1 + m \ln \left( \frac{\dot{\varepsilon}_0}{\dot{\varepsilon}} \right) \right) \left( 1 - \left( \frac{\tau_{eq}}{\tau_{sh}} \right)^{\frac{1}{n}} \right)$$

With $\omega = \sqrt{\frac{\rho}{\nu^2}}$ and $a^* = \frac{a_0^*}{V_{chip}}$

Based on the boundary conditions, the solution of the heat equation in the Laplace domain is given as follows:

$$\theta(p, x) = \frac{q_0}{k_0} \left[ \frac{1}{\rho \sqrt{\frac{p}{\omega}}} e^{-\omega x} \right] - \frac{2q_0}{k_0 \rho \sqrt{q_0^c}} \left( \frac{1}{p \sqrt{\frac{p}{\omega}}} e^{-\omega x} \right) + \frac{2q_0}{k_0 \rho \sqrt{q_0^c}} \left( \frac{1}{p \sqrt{\frac{p}{\omega}}} e^{-\omega x} \right)$$

Based on the table of selected Laplace transforms, the equivalent temperature at the tool/chip interface is determined, when considering $x = 0$, and is given by the following equation:

$$T_{eq}^c(z_h, 0) = \tilde{T}_{eq}^c + \frac{2q_0}{k_0 \rho \sqrt{q_0^c}} \left( \frac{1}{p \sqrt{\frac{p}{\omega}}} e^{-\omega x} \right)$$

**Fig. 9** Evolution of the position and the maximum temperature at the tool/chip interface with respect to cutting speed: oblique cutting of AISI304L: $a_t = -6^\circ$, $a_r = -6^\circ$, $a_p = 1\text{mm}$; $k_t = 75^\circ$.
where: $q_0^{eq} = \frac{\beta f_\theta \rho V \sin \phi_n \cos \lambda_i}{\cos \phi_n \cos (\phi_n - \alpha_i)}$.

The average temperature at the tool/chip interface is determined as follows:

$$\bar{T}_{int} = \frac{1}{c} \int_0^c T^{eq}(z, 0) \, dz \approx \bar{T}_{sh}^{eq}$$

$\bar{T}_{sh}^{eq} = \frac{4}{l_c} \sqrt{\frac{\pi k^{eq}(r_{sh}^{eq}) \rho(r_{sh}^{eq}) c(r_{sh}^{eq}) V_{chip}^{eq}}{q_0^{eq}}}$

The position of the highest temperature at the tool/chip interface is determined from the Eq. (41) and is given as follows:

$$z_{\beta}(T_{max}^{eq}) = \frac{1}{4} l_c^{eq} \approx 0.317 l_c^{eq} \quad (43)$$

The highest temperature at the tool/chip interface is calculated from the following equation:
Based on the same cutting conditions used in the above Sect. 4.4, a comparison between results found with FDM and the analytical approach using Laplace transform (Eq. (41)) is done (Fig. 14). Steady state was considered and very well agreement was found between the two methods. These results are very encouraging and lead to simplifying the thermal problem in metal cutting. Hence, tool/chip interface temperature can be described by Eq. (41) which also permits to determine the highest one (Eq. (44)) and its position (Eq. (43)).

Effects of uncut chip thickness $t_1$ and cutting speed $V_c$ were studied on the value of maximum tool/chip temperature $T_{\text{max}}^{\text{int}}$ and its position $z_{fl}(T_{\text{max}}^{\text{int}})$ (Fig. 15). In fact, $T_{\text{max}}^{\text{int}}$ increases with cutting speed and uncut chip thickness; however, its position moves away from the cutting edge when $t_1$ or/and $V_c$ increase. According to Coromant [21] manufacturer, it is recommended to machine stainless steel at a cutting speed of less than 300 m/min. In fact,
in the recommended machining zone for stainless steel as highlighted in Fig. 15 (green zone), the maximum tool/chip interface temperature is less than 1400 °C and local melting does not appear in the chip. However, the local melting in the chip begins at \( V_c = 500 \text{m/min} \) when \( t_1 = 0.1 \text{mm} \) and at \( V_c = 360 \text{m/min} \) when \( t_1 = 0.2 \text{mm} \).

Average tool/chip interface temperature \( T_{int} \) is determined using the approaches presented in this section: Oxley’s theory adapted for oblique cutting; Finite Difference Method (FDM) and Analytical approach using Laplace transform. Figure 16 presents a comparison between these three approaches in terms of \( T_{int} \). Good agreement is noted between the three approaches and the maximum difference is about 50 °C.

Based on the equalities between the expressions of average tool/chip interface temperature \( T_{int} \) given by Oxley’s theory (Eq. (1)) and that found with Laplace transform (Eq. (42)), the partition coefficient \( \beta \) is given by the following equation:

\[
\beta \approx 1.225 \frac{\Delta T_{max}^{chip}}{f_0 \rho_0} \sqrt{\frac{\pi k \rho c}{l_c V_{chip}}} \tag{45}
\]

where \( \Delta T_{max}^{chip} \) is evaluated by Eqs. (6) and (9) and \( p_0 \) is the maximum tool/chip interface pressure.

Figure 17 corresponds to the evolution of partition coefficient \( \beta \) with respect to sliding velocity (\( V_{chip} \)). It is shown that \( \beta \) decreases with respect to \( V_{chip} \) and it is represented, following a linear regression, by the equation as follows:

\[
\beta = 0.765 - 0.026 V_{chip} \tag{46}
\]

5 Experimental results and discussion

For the validation, experimental data for oblique turning with 304 stainless steels conducted by Abdellaoui and Bouzid [11] are used. The tool holder is PCBNL2525M12 (\( k_r = 75^\circ \), \( \alpha_n = -6^\circ \), \( \lambda_e = -6^\circ \)). The cutting inserts are referenced CNMG120408 and CNMG120412 (\( r_e = 0.8 \text{mm} \) and \( r_p = 1.2 \text{mm} \)). According to Sela et al. [22], the cutting-edge radius \( r_p = 24 \mu \text{m} \). The cutting force components were recorded on TRANSMAB 450TD lathe machine using a triaxial force dynamometer KISTLER 9257A. Experimental cutting conditions were conducted using Taguchi array L9, as shown in Table 3.

When running algorithm 1 (Fig. 3), results of equivalent parameters were determined and resumed in Table 4. It is noted that the equivalent normal cutting angle has become approximately twice the initial value (\( \alpha_n = -6^\circ \)). However, the edge direction angle \( \kappa_r \) decreases and has reached 32° for test 6 when using CNMG120412. Moreover, it decreases with tool nose radius \( r_e \).

A comparison was made between experimental cutting force components (\( F_{c,exp} \), \( F_{f,exp} \), and \( F_{r,exp} \)) and those predicted by simulation (\( F_{c,eq} \), \( F_{f,eq} \), and \( F_{r,eq} \)) using the equivalent model and two inserts were considered: CNMG120408 in Fig. 18 and CNMG120412 in Fig. 19. In the present research work, an equivalent tool geometry was used. Effects of tool nose and edge radii were considered. Acceptable agreement is underlined between experimental cutting force components and those predicted from the equivalent oblique cutting model. Predicted results are in an approximation of 80% to the experimental results. This mean deviation is due to the measurement precision which is about ±50 N [20], and to the values of cutting angles which are estimated for the equivalent tool. In fact, for the real tool, the cutting angle \( \alpha_n = -6^\circ \); however, for the equivalent tool, it is about −12°. According to Mustafa et al. [23], the main cutting force increases with increased negative cutting angle. The results found are of great importance because they will allow us to study the equivalent tribological parameters and deduce the equivalent turning performances.
6 Prediction of tribological parameters

Based on the above sections, turning performances can be predicted and tribological parameters will be determined for the same experimental cutting conditions. Table 5 presents the tribological parameters which are tool/chip contact length, friction coefficient at the tool/chip interface, and the maximum tool/chip interface temperature and its position. All these parameters were determined from simulation, for all tests conducted by the two inserts (CNMG120408 and CNMG120412). Besides, a similarity criterion, namely, the Po-criterion \( P_o \), according to Astakhov [24], is determined. This dimensionless parameter is defined as the ratio of tool/chip contact length \( l_c \) to the uncut chip thickness \( t_1 \) (\( P_o = \frac{l_c}{t_1} \)). According to Astakhov [24] and Abdellaouï and Bouzid [13], cutting performances such as tribology parameters are mainly controlled by this parameter.

When analyzing the results in Table 5, it is underlined that \( T_{\text{max}} \) is between 1200 and 1500 °C for all tests. The highest one was determined for test 9 and the lowest one for test 1. This result was confirmed with two inserts. Moreover, it can be highlighted that \( T_{\text{max}} \) decreases with tool nose radius \( r_n \) and this result was confirmed by the work of Lefi et al. [14] and Kishawy [12]. This decrease seems to be due to the increase of the tool–workpiece contact surface which promotes the thermal exchange. However, the equivalent oblique cutting considered in the present research work agrees well with this result.

The evolutions of maximum tool/chip interface temperature \( T_{\text{int}} \) and average tool/chip friction coefficient \( \tilde{f} \) are given in Fig. 20. It is revealed that \( T_{\text{max}} \) increases with the dimensionless number \( P_o \); however, the friction coefficient increases.

According to Iqbal et al. [25], tool/chip contact length \( l_c \) decreases with cutting speed \( V_c \). However, when cutting speed \( V_c \) increases, the maximum tool/chip interface temperature \( T_{\text{int}} \) increases [26, 27]. Therefore, the maximum tool/chip interface temperature \( T_{\text{int}} \) decreases with respect to the dimensionless parameter \( P_o \). Based on results given by Fig. 20, \( T_{\text{max}} \) is described by the following linear regression:

\[
T_{\text{int}}^{\text{max}} = 1667.6 - 63.15 P_o = 1667.6 - 63.15 \frac{l_c}{t_1} \tag{47}
\]

According to Abdellaouï and Bouzid [13], Mustapha et al. [28], and Puls et al. [29], the average friction coefficient \( \tilde{f} \)
decreases with respect to the average tool/chip interface temperature $\bar{T}_{\text{int}}$. Abdellaoui and Bouzid [13] have represented the average friction coefficient $\bar{f}$ when machining stainless steel AISI304 with CNMG120408 and CNMG120412 inserts by the following equation:

$$\bar{f} = \alpha_1 \left( 1 - \left( \frac{\bar{T}_{\text{int}}(K)}{T_f(K)} \right)^{\frac{\alpha_2}{\alpha_3}} \right) + \alpha_3$$

(48)

where $\bar{T}_{\text{int}}(K)$ and $T_f(K)$ are average tool–chip interface temperature and work-material melting temperature, respectively. Constants $\alpha_1 = 0.5$, $\alpha_2 = 0.4$, and $\alpha_3 = 0.2$.

In summary, when the dimensionless number $P_{o \text{-criterion}} = \frac{l_c}{t_1}$ increases, the average tool/chip interface temperature $\bar{T}_{\text{int}}$ decreases; therefore, the average friction decreases.

![Fig. 17](image-url) Evolution of partition coefficient $\beta$ with respect to chip velocity (sliding speed). Results from simulation of oblique cutting of AISI304

**Table 3** Cutting conditions used in the experiments

| Tests | 1  | 2  | 3  | 4  | 5  | 6  | 7  | 8  | 9  |
|-------|----|----|----|----|----|----|----|----|----|
| $V_c$ (m/min) | 180 | 180 | 180 | 250 | 250 | 250 | 400 | 400 | 400 |
| $f$ (mm/rev) | 0.1 | 0.15 | 0.2 | 0.1 | 0.15 | 0.2 | 0.1 | 0.15 | 0.2 |
| $a_p$ (mm) | 0.5 | 1 | 2 | 1 | 2 | 0.5 | 2 | 0.5 | 1 |

**Table 4** Equivalent parameters of oblique turning

| Tests | 1  | 2  | 3  | 4  | 5  | 6  | 7  | 8  | 9  |
|-------|----|----|----|----|----|----|----|----|----|
| Insert CNMG120408: $r_c = 0.8$ mm, $r_\beta = 24 \mu$m |
| $d_h^e$ (mm) | 0.50 | 1.00 | 2.00 | 1.00 | 2.00 | 0.50 | 2.00 | 0.50 | 1.00 |
| $t_1^e$ (mm) | 0.07 | 0.13 | 0.18 | 0.08 | 0.14 | 0.13 | 0.09 | 0.10 | 0.17 |
| $\alpha_n^e$ (°) | -13.37 | -12.42 | -11.55 | -14.25 | -12.85 | -11.52 | -15.15 | -13.13 | -12.09 |
| $\alpha_{e1}^e$ (°) | 41.98 | 57.35 | 65.81 | 58.14 | 66.27 | 39.44 | 66.71 | 40.74 | 56.52 |
| Insert CNMG120412: $r_c = 1.2$ mm, $r_\beta = 24 \mu$m |
| $d_h^e$ (mm) | 0.50 | 1.00 | 2.00 | 1.00 | 2.00 | 0.50 | 2.00 | 0.50 | 1.00 |
| $t_1^e$ (mm) | 0.06 | 0.11 | 0.18 | 0.08 | 0.13 | 0.11 | 0.09 | 0.08 | 0.15 |
| $\alpha_n^e$ (°) | -13.08 | -12.29 | -11.52 | -14.08 | -12.85 | -11.52 | -15.15 | -13.02 | -12.05 |
| $\alpha_{e1}^e$ (°) | 33.88 | 49.65 | 62.02 | 50.31 | 62.44 | 31.97 | 62.84 | 32.94 | 48.98 |

**Fig. 18** Comparison between experimental and predicted cutting force components: AISI304L, CNMG120408, $\alpha_n = -6^\circ$, $\alpha_t = -6^\circ$
Fig. 19 Comparison between experimental and predicted cutting force components: AISI304L, CNMG120412, $\alpha = -6^\circ$, $\lambda = -6^\circ$.

![Bar chart showing comparison between experimental and predicted cutting force components](image)

Table 5 Equivalent tribology parameters

| Tests | 1   | 2   | 3   | 4   | 5   | 6   | 7   | 8   | 9   |
|-------|-----|-----|-----|-----|-----|-----|-----|-----|-----|
| Insert| CNMG120408: $r = 0.8 mm, r = 24 \mu m$ |     |     |     |     |     |     |     |     |
| $T_{eq}$(°C) | 1227 | 1266 | 1281 | 1329 | 1365 | 1347 | 1429 | 1468 | 1508 |
| $T_{eq}$(°C) | 0.13 | 0.20 | 0.25 | 0.14 | 0.20 | 0.19 | 0.17 | 0.15 | 0.23 |
| $f_{eq}$(°C) | 0.42 | 0.63 | 0.78 | 0.45 | 0.64 | 0.59 | 0.53 | 0.46 | 0.72 |
| $f_{eq}$(°C) | 0.58 | 0.46 | 0.41 | 0.51 | 0.42 | 0.44 | 0.45 | 0.43 | 0.36 |
| $f_{eq}$(°C) | 6.00 | 4.85 | 4.33 | 5.62 | 4.57 | 4.53 | 5.88 | 4.60 | 4.23 |
| $f_{eq}$(°C) |       |     |     |     |     |     |     |     |     |
| Insert| CNMG120412: $r = 1.2 mm, r = 24 \mu m$ |     |     |     |     |     |     |     |     |
| $T_{eq}$(°C) | 1208 | 1246 | 1280 | 1325 | 1357 | 1321 | 1459 | 1437 | 1486 |
| $T_{eq}$(°C) | 0.12 | 0.18 | 0.25 | 0.15 | 0.19 | 0.17 | 0.15 | 0.13 | 0.21 |
| $f_{eq}$(°C) | 0.39 | 0.58 | 0.78 | 0.46 | 0.61 | 0.55 | 0.47 | 0.41 | 0.66 |
| $f_{eq}$(°C) | 0.61 | 0.50 | 0.41 | 0.51 | 0.43 | 0.47 | 0.44 | 0.47 | 0.38 |
| $f_{eq}$(°C) | 6.50 | 5.27 | 4.33 | 5.75 | 4.69 | 5.00 | 5.22 | 5.12 | 4.40 |

Fig. 20 Evolution of maximum tool/chip interface temperature and tool/chip interface coefficient with respect to Po-criterion (Po).

![Graph showing evolution of maximum tool/chip interface temperature and coefficient](image)
7 Conclusion

In the present paper, turning performances were determined using equivalent oblique cutting with a single cutting edge. Equivalent tool parameters and equivalent cutting conditions were evaluated considering the effects of tool nose and edge radii. The results are used in a thermomechanical algorithm which is controlled by one loop with respect to average tool/chip interface temperature. All equivalent thermomechanical parameters such as cutting force components, cutting temperatures, and tribology parameters were determined with a good accuracy.

To determine the maximum tool/chip interface temperature and its position, a hybrid analytical–numerical algorithm was used. The thermal problem in the chip is resolved in the first time using the Finite Difference Method (FDM) considering transient regime. It is concluded that a steady state takes place rapidly and a transient regime can be omitted in the resolution of the heat equation in the chip. Besides, the chip’s top was considered as a convection surface; however, numerical results show that it can be considered as a Dirichlet condition (fixed temperature) instead of a Neumann one (heat flux across the boundaries).

Based on the considerations drawn from numerical results, steady state and a fixed temperature in the chip’s top were considered. The thermal problem was resolved again using Laplace transforms. The evolution of tool/chip interface temperature was represented by an equation that permits to determine the highest temperature and its position at the tool/chip interface. Good agreement was underlined between numerical and experimental results using the equivalent oblique cutting model. Tribology parameters at the tool/chip interface as like maximum tool/chip interface temperature $T_{\text{max}}$ and its position, tool/chip contact length $l_c$, and average friction coefficient $\bar{f}$ were evaluated for all tests. It was concluded that $T_{\text{int}}$ decreases with $P_0$; however, $\bar{f}$ increases.

As a perspective, the equivalent oblique cutting model will be used to predict all turning performances over a wide range of cutting conditions. Crater wear and tool life will be studied with respect to tribological parameters for each couple workpiece-tool (CWT) in a future research work.

**Abbreviations**

- $\alpha_p$: Cutting depth [mm]; $a_p^{eq}$: Equivalent depth of cut [mm]; $c_p$: Equivalent work material specific heat [J kg$^{-1}$ K$^{-1}$]; $\bar{f}$: Feed rate [mm/rev]; $f^\ast$: Average coefficient of friction at tool–chip interface; $f_{eq}$: Equivalent average coefficient of friction at tool–chip interface; $F_{eq}$: Equivalent tangential force [N]; $F_{eq}^{sh}$: Equivalent shear force [N]; $F_{eq}^{int}$: Equivalent radial force [N]; $F_{eq}^{sh}$: Equivalent shearing force [N]; $k_{eq}$: Equivalent work material thermal conductivity [Wm$^{-1}$ K$^{-1}$]; $P_0$: Tool–chip contact length for element $i$ [mm]; $P_{eq}$: Equivalent of maximum normal stress at tool–chip contact [N/mm$^2$]; $r_b$: Cutting edge radius [mm]; $r_c$: Tool-nose radius [mm]; $t_1$: Uncut chip thickness [mm]; $T_{eq}$: Equivalent chip thickness [mm]; $T_f$: Workpiece melting temperature [K]; $T_{in}$: Initial workpiece temperature [K]; $T_{sh}$: Equivalent shearing temperature [K]; $T_p$: Average tool–chip interface temperature [K]; $V_c$: Cutting speed [m/min]; $V_{sh}$: Equivalent chip velocity [mm/s]; $\alpha_n$: Normal rake angle ['']; $\beta$: Secondary shear zone thickness ratio to chip thickness; $\delta$: Equivalent normal shear angle ['']; $\gamma_{sh}$: Equivalent shear strain; $\eta_{eq}$: Equivalent chip flow angle ['']; $\eta_{sh}$: Shearing direction angle ['']; $\lambda$: Edge direction angle ['']; $\kappa_{eq}$: Equivalent edge direction angle ['']; $\phi$: Edge inclination angle ['']; $\rho_{eq}$: Equivalent work material density [kg/m$^3$]; $\tau_{sh}$: Equivalent shear stress [N/mm$^2$]

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**Declarations**

**Ethics approval** The authors declare that there is no ethical issue applied to this article.
Consent to participate  The authors declare that all authors have read and approved to submit this manuscript to IJAMT.

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