Viscous Ricci Dark Energy

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We investigate the viscous Ricci dark energy (RDE) model by assuming that there is bulk viscosity in the linear barotropic fluid and the RDE. In the RDE model without bulk viscosity, the universe is younger than some old objects at some redshifts. Since the age of the universe should be longer than any objects in the universe, the RDE model suffers the age problem, especially when we consider the object APM 08279 + 5255 at $z = 3.91$, whose age is $t = 2.1$ Gyr. In this letter, we find that once the viscosity is taken into account, this age problem is alleviated.

Recent observations like CMB anisotropy, supernovae and galaxies clustering have strongly indicated that our universe is spatial flat and there exists an exotic cosmic fluid called dark energy with negative pressure, which constitute about two thirds of the total energy of the universe. The dark energy is characterized by its equation of state $w$, which lies very close to $-1$, probably being below $-1$ indicated by the present data.

Many candidates including the cosmological constant, quintessence, phantom, quintom, holographic dark energy, etc., have been proposed to explain the acceleration. However, people still do not understand what’s dark energy so far. Ricci dark energy, which is a kind of holographic dark energy [1] taking the square root of the inverse Ricci scalar as its infrared cutoff, has been proposed by Gao et al. [2], and this model is also phenomenologically viable. Assuming the black hole is formed by gravitation collapsing of the perturbation in the universe, the maximal black hole can be formed is determined by the casual connection scale $R_{CC}$ given by the “Jeans” scale of the perturbations. For tensor perturbations, i.e. gravitational perturbations, $R_{CC}^{-2} = Max(\dot{H} + 2H^2, -\dot{H})$ for a flat universe, where $H = \dot{a}/a$ is the Hubble parameter, and according to the ref. [3], only in the case of $R_{CC}^{-2} = H + 2H^2$, it could be consistent with the current cosmological observations when the vacuum density appears as an independently conserved energy component. As we know, in flat FRW universe, the Ricci scalar is $R = 6(\dot{H} + 2H^2)$, which means the $R_{CC} \propto R$ and if one choices the casual connection scale $R_{CC}$ as the IR cutoff, the Ricci dark energy model is also obtained. For recent progress on Ricci dark energy and holographic dark energy, see ref. [4, 5, 6]. The energy density of RDE in flat universe reads

$$\rho_R = \frac{\alpha}{2} \frac{\dot{R}}{R} = 3\alpha \left( \dot{H} + 2H^2 \right),$$

where we have set $8\pi G = 1$ and $\alpha$ is a dimensionless parameter which will determine the evolution behavior of RDE.

Dissipative processes in the universe including bulk viscosity, shear viscosity and heat transport have been conscientiously studied [7]. The general theory of dissipation in relativistic imperfect fluid was put on a firm foundation by Eckart [8], and, in a somewhat different formulation, by Landau and Lifshitz [9]. This is only the first order deviation from equilibrium and may has a causal problem, the full causal theory was developed by Israel and Stewart [10], and has also been studied in the evolution of the early universe [12]. However, the character of the evolution equation is very complicated in the full causal theory. Fortunately, once the phenomena are quasi-stationary, namely slowly varying on space and time scale characterized by the mean free path and the mean collision time of the fluid particles, the conventional theory is still valid. In the case of isotropic and homogeneous universe, the dissipative process can be modeled as a bulk viscosity $\zeta$ within a thermodynamical approach, while the shear viscosity $\eta$ can be neglected, which is consistent with the usual practice [13]. Works on viscous dark energy models see ref. [14].

The bulk viscosity introduces dissipation by only redefining the effective pressure, $p_{eff}$, according to $p_{eff} = p - 3\zeta H$ where $\zeta$ is the bulk viscosity coefficient and $H$ is the Hubble parameter. The condition $\zeta > 0$ guarantees a positive entropy production, consequently, no violation of the second law of the thermodynamics [15]. In this letter, we are interested when the universe is dominated by a usual fluid and the RDE, and both of them have a bulk viscosity. The case $\zeta = \sqrt{3r} H$, implying the bulk viscosity is proportional to the fluid’s velocity vector, is physical natural, and has been considered earlier in a astrophysical context, see the review article of Grøn [10].

First, let us consider a simple case when the universe is dominated by RDE only, then the Friedmann equation reads:

$$H^2 = \frac{\rho_R}{3} = \alpha \left[ \frac{(H^2)'}{2} + 2H^2 \right],$$

where prime denotes the derivative with respect to $x \equiv \ln a$ and hereafter we set $a_0 = 1$. The solution of the above equation is $H^2 = H_0^2 e^{-2(2-x)}$ and the energy
density of RDE is \( \rho_R = 3H_0^2e^{-2(2-\frac{\lambda}{\alpha})x} \). By using the conservation equation

\[
\rho'_R + 3\left(\rho_R + p_R - 3\zeta H\right) = 0
\]

we obtain the pressure of viscous RDE: \( P_R = (1 - \frac{2}{\alpha}) H_0^2 e^{-2(2 - \frac{\lambda}{\alpha})x} + 3\zeta H \) and the equation of state:

\[
w = -\frac{1}{3}\left(\frac{2}{\alpha} - 1\right) + \frac{\zeta}{H}.
\]

If we choose \( \zeta = \tau \rho^x \), then

\[
w = -\frac{1}{3}\left(\frac{2}{\alpha} - 1\right) + 3^x\tau H_0^{2\alpha - 1} e^{-(2\alpha - 1)(2 - \frac{\lambda}{\alpha})x}.
\]

In the special case of \( \zeta(t) = \zeta_0 = \text{const}, \) i.e. \( s = 0 \), \( w \) would be very large at very early time \( (\alpha < 1/2) \) or in the later time \( (\alpha > 1/2) \). In particular, for the case \( s = 1/2 \), it requires \( \alpha < 2/(2 + \sqrt{3}\gamma) \) to accelerate the universe \( (w < -1/3) \).

If the universe also contain another component with viscous RDE, the Friedmann equation and the corresponding equation of motion can be written as:

\[
H^2 = \frac{1}{3}(\rho_R + \rho_s)
\]

\[
\rho'_s = -3(\rho_s + p_s - 3\zeta_s H)
\]

\[
\rho'_R = -3(\rho_R + p_R - 3\zeta_R H)
\]

where \( \rho_s \) is the density of fluid with a barotropic equation of state \( p_s = (\gamma - 1)\rho_s \), and \( 0 \leq \gamma \leq 2 \) to satisfy the dominant energy condition (DEC). So, the equation of state parameter \( \omega = \frac{p}{\rho} = \frac{\gamma - 1}{\gamma} \). Here we will choose \( \zeta_R = \tau_R \sqrt{\rho} = \sqrt{3}\tau_R H \) and \( \zeta_s = \tau_s \sqrt{\rho} = \sqrt{3}\tau_s H \), where \( \rho = \rho_s + \rho_R \). Then the Friedmann equation \( (6) \) becomes

\[
\alpha y'' + (4 \alpha + 3 \alpha \gamma - 2) y' + 6 \left( -\gamma + 2 \alpha \gamma + \sqrt{3}\tau_s \right) y = 0
\]

where we have defined \( y = E^2 \equiv H^2/H_0^2 \). The general solution of eq. \( (9) \) is

\[
y = C_1 e^{-\lambda^+ x} + C_2 e^{-\lambda^- x}
\]

where \( \lambda^\pm = 2 + \frac{3}{2} \alpha - \frac{1}{\alpha} \pm \Delta \) and \( C_1, C_2 \) are integration constants. Here we have defined

\[
\Delta = \frac{1}{2\alpha} \sqrt{(2 - 4\alpha + 3\alpha \gamma)^2 - 24\sqrt{3}\alpha \gamma}.
\]

From eq. \( (10) \), the background evolution only depends on \( \tau_s \) and does not depend on the viscosity of RDE. By the definition of \( y \), we have \( C_2 = 1 - C_1 \), and the energy density of the fluid is

\[
\rho_\gamma = \frac{3}{4}H_0^2 e^{-\lambda^+ x} \left( C_1 \Sigma^+ + C_2 \Sigma^- e^{2\Delta x} \right),
\]

where \( \Sigma^\pm = 2 - 4\alpha + 3\alpha \gamma \pm 2\alpha \Delta \). Since definition of density parameter for the fluid is \( \Omega_{\gamma,0} \equiv \rho_\gamma/(3H_0^2) \), then

\[
C_1 = 1 - C_2 = \frac{\Omega_{\gamma,0}}{\alpha \Delta} - \frac{\Sigma^-}{4\alpha \Delta}.
\]

The energy density of viscous RDE is

\[
\rho_R = \frac{3}{4}H_0^2 e^{-\lambda^+ x} \left[ C_1 \Pi^- + C_2 \Pi^+ e^{2\Delta x} \right],
\]

where \( \Pi^\pm = 2 + 4\alpha - 3\alpha \gamma \pm 2\alpha \Delta \). By using eq. \( (9) \), we obtain the equation of state for viscous RDE:

\[
w_R \equiv \frac{p_R}{\rho_R} = -1 + \frac{C_1(12\sqrt{3}\tau_R + \lambda^+ \Pi^-)}{3(C_1 \Pi^- + C_2 \Pi^+ e^{2\Delta x})} + \frac{C_2(12\sqrt{3}\tau_R + \lambda^- \Pi^+)}{3(C_1 \Pi^- + C_2 \Pi^+ e^{2\Delta x})} e^{2\Delta x}.
\]

Obviously, the present value of \( w_R \) is

\[
w_{R0} = -1 + \frac{12\sqrt{3}\tau_R + C_1 \lambda^+ \Pi^- + C_2 \lambda^- \Pi^+}{12(1 - \Omega_{\gamma,0})}
\]

where we have used eq. \( (10) \) and if \( \Delta \) is real, the past and future value of \( w_R \) is

\[
w_R(x \rightarrow -\infty) = -1 + \frac{\lambda^+}{3} + \frac{4\sqrt{3}\tau_R}{\Pi^-},
\]

\[
w_R(x \rightarrow \infty) = -1 + \frac{\lambda^-}{3} + \frac{4\sqrt{3}\tau_R}{\Pi^+},
\]

and one can see that the value of the equation of state parameter is determined by both the viscosity of the fluid and that of RDE and does not blow up neither in the past nor in the future.

In the following, we will consider the case of \( \gamma = 1 \), which corresponding the equation of state of dark matter \( w_m = 0 \). By assuming \( \tau_m \ll (2 - \alpha)^2/(24\sqrt{3}\alpha) \), we obtain

\[
\Delta \approx \frac{2 - \alpha}{2\alpha} - \tilde{\tau}_m,
\]

\[
\lambda^+ \approx 3 - \tilde{\tau}_m,
\]

\[
\lambda^- \approx 4 - \frac{\alpha}{2} + \tilde{\tau}_m,
\]

\[
\Sigma^+ \approx 4 - 2\alpha - 2\alpha \tilde{\tau}_m,
\]

\[
\Sigma^- \approx 2\alpha \tilde{\tau}_m,
\]

\[
\Pi^+ \approx 4 - 2\alpha \tilde{\tau}_m,
\]

\[
\Pi^- \approx 2\alpha + 2\alpha \tilde{\tau}_m,
\]

\[
C_1 = 1 - C_2 \approx \frac{2\Omega_{m,0}}{2 - \alpha} + \frac{\alpha \tilde{\tau}_m}{2 - \alpha} \left( \frac{4\Omega_{m,0}}{2 - \alpha} - 1 \right),
\]

where \( \tilde{\tau}_m \approx \frac{6\sqrt{3}\tau_m}{2 - \alpha} \). And the present equation of state is

\[
w_{R0} \approx -1 \left[ \frac{2}{\alpha} - \frac{1 + 3\sqrt{3}(\Omega_R + \Omega_m)}{1 - \Omega_{m,0}} \right],
\]
so it requires
\[ \alpha < \frac{2(1 - \Omega_{m0})}{2 - \Omega_{m0} + 3\sqrt{3}(\tau_R + \tau_m)} \]  
(28)
to accelerate the universe at present. The past and future values of the equation of state parameter are:
\[
w_R(x \to -\infty) \approx \frac{2\sqrt{3}}{\alpha} \left[ \tau_R - \frac{\alpha}{2 - \alpha} \tau_m \right], \quad (29)
\]
\[
w_R(x \to \infty) \approx -\frac{1}{3} \left( \frac{2}{\alpha} - 1 \right) + \frac{2\sqrt{3}}{\alpha} \left( \frac{\tau_R}{2} + \frac{\tau_m}{2 - \alpha} \right), \quad (30)
\]
where we have assumed \( \tau_R \) is the same order as \( \tau_\gamma \) and kept only linear terms of them.

The age of our universe at redshift \( z \) is given by \( t(z) = T(z)/H_0 \), where
\[
T(z) = \int_z^\infty \frac{dz'}{(1 + z')E(z')} = \int_{-\infty}^{-\ln(1+z)} \frac{dx}{E(x)}, \quad (32)
\]
is the so-called dimensionless age parameter. For a flat CDM universe dominated by matter \( (\Omega_{m0} = 1) \), \( t_0 = 2/(3H_0) \), and according to the observations of the Hubble Space Telescope Key project, the present Hubble parameter is constrained to be \( H_0 = 9.776h^{-1} \), \( 0.64 < h < 0.80 \), which is consistent with the conclusions arising from observations of the CMB and large scale structure. This gives \( t_0 \approx 8 \sim 10 \)Gyr, which does not satisfy the stellar age bound: \( t_0 > 11 \sim 12 \)Gyr, namely, the age of the universe should be longer than any of others in the universe. For ΛCDM model, in which \( \Omega_{m0} \approx 0.27 \), the age parameter is
\[
T(z) = \int_z^\infty \frac{dz'}{(1 + z')\left[\Omega_{m0}(1 + z)^3 + (1 - \Omega_{m0})\right]^{1/2}}, \quad (33)
\]
hence, it easily satisfied the constraint \( t_0 > 11 \sim 12 \)Gyr, because the value of \( \Omega_{m0} \) is smaller than that in CDM model, consequently, the integrand in (33) is bigger when \( z \) is large, and thus, the total age of the universe is going to be longer in ΛCDM model. However, the age of the universe at \( z = 3.91 \) in ΛCDM model is about \( t(3.91) \approx 0.12H_0^{-1} \approx 1.44 \sim 1.79 \)Gyr which is still younger than the old object APM 08279 + 5255 observed recently\(^{[17]}\) at the same redshift with age 2.1 Gyr. In the RDE model without viscosity, \( t(3.91) \approx 0.10H_0^{-1} \approx 1.26 \sim 1.57 \)Gyr with \( \alpha = 0.46 \), so the universe is also younger than APM 08279 + 5255.

We plot the total age and its evolution with different values of \( \alpha \) in Fig[1] and Fig[2], where three black points denote the age of some old objects in the universe: LBDS 53W091\(^{[18]}\) at \( z = 1.43 \) with age \( t = 3.5 \) Gyr, LBDS 53W069\(^{[19]}\) at \( z = 1.55 \) with age \( t = 4.0 \) Gyr and APM 08279 + 5255 at \( z = 3.91 \) with age \( t = 2.1 \) Gyr. It seems that matter was diluted so fast that makes the universe younger than these old objects in the RDE model.

However, this age problem can be alleviated in the viscous RDE model. We plot the evolution of age of the universe with different values of \( \alpha \) in this model, see Fig[3] and it indicates the viscosity could really alleviate the age problem. And actually, only the viscosity of matter affects the evolution of age of the universe, so it could alleviate the age problem in other cosmological models.

In conclusion, we have investigated the Ricci dark energy model when the bulk viscosity \( \zeta_R = \tau_R\sqrt{\rho} \) is taken into account in this letter. The energy conservation equations will have additional terms proportional to the bulk viscosity in this case. However, in this model, the evolution of the universe only depends on the bulk viscosity \( \zeta_R = \tau_R\sqrt{\rho} \) of ordinary fluids with equation of state \( p = (\gamma - 1)\rho \), and it does not depend on \( \zeta_R \). The RDE model suffers the age problem since the age of the universe should be longer than any other objects at any redshifts in the universe. It seems that the problem is caused by the fact that matter is diluted too fast. When one consider the viscosity of matter, it changes the energy conservation equation for the matter, consequently,
it makes matter diluted a little bit slower, and so the age problem is alleviated.

Considering the viscosity of fluid is a next step from the idea one to treat fluid more realized, since the real fluid should have the viscous properties when it flows, so it is very interesting and worth further studying.

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