Power Tracking and State-of-Energy Balancing of an Energy Storage System by Distributed Control

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ABSTRACT
This paper addresses the power control problem for an energy storage system consisting of multiple energy storage units with dual objectives. On one hand, the power output of the energy storage system should track its reference. On the other hand, the state-of-energy of all the energy storage units should be balanced so as to maintain the maximum power capacity of the energy storage system. To achieve these two control objectives simultaneously, a novel distributed control scheme is proposed, which has three features. First, the proposed control scheme is distributed, which enables a more flexible and scalable configuration of the communication network. Second, the proposed control scheme allows online switch-on and switch-off operations for energy storage units, which makes the energy storage system more efficient for preplanned check and more resilient to unexpected fault. Third, the state-of-energy balancing mechanism can ensure the maximum power capacity of the energy storage system. Comprehensive numerical case studies are provided to show the effectiveness of the proposed control scheme.

INDEX TERMS
Distributed control, energy storage system, power tracking, state-of-energy balancing.

I. INTRODUCTION
The benefits of microgrid have been demonstrated in many aspects, such as renewable energy integration, transmission and distribution cost reduction, power quality improvement, and so on [1], [2]. The control of microgrids can be depicted in two dimensions. Vertically, the control of microgrids usually employs a hierarchical structure with multiple control levels according to different control objectives and system time scales, such as the primary-secondary-tertiary control structure [3], [4]: the primary control guarantees the basic voltage and frequency stabilization; the secondary control restores the deviated voltage and frequency resulted from the primary control; the tertiary control addresses the optimal power flow issues among microgrids and the main grid. Horizontally, on each level, the control of microgrids can be realized by centralized control, decentralized control or distributed control. The centralized control utilizes a two-way communication between the control center (CC) and each distributed generator (DG) [5], [6]. Since the CC takes all the communication and computation load, the centralized control could be costly to implement for large scale microgrid. The decentralized control uses the system frequency and bus voltage as the coordinating signals and thus requires no communication among DGs [7], [8]. While, though the decentralized control is cost-effective as no communication is needed, the control performance might adversely be less satisfactory, such as the inaccurate reactive power sharing by the conventional droop control. The distributed control is somehow in-between the centralized control and the decentralized control [9]–[11]. On one hand, similar to the centralized control, the distributed control utilizes communication among DGs to enhance the system performance, but in contrast to the two-way communication configuration of the centralized control, the communication network for the distributed control is neighbor-based, which can be made more economic, flexible and scalable. On the other hand, similar to the decentralized control, the control decisions for DGs in the distributed control are made locally, which would greatly reduce the computational load for the CC.

Energy storage system (ESS), which consists of multiple energy storage units (ESUs), plays an important role in the modern power grid [12], [13]. As a single entity of the grid, the ESS should regulate its power output to fulfill...
the buffering tasks, such as to mitigate the intermittence of renewable energy sources [14]–[16], or to stabilize grid frequency and voltage [17], [18]. The power output of the ESS should track its reference scheduled by some upper level control [19]–[21]. In [19], the optimal power flow for ESS in microgrids was investigated taking into account the ESU/voltage/current/power limits and the comprehensive electrical network model. In [20], a flywheel energy storage system is implemented to smooth the net power injected into the main grid by the wind turbines. A high-level energy management control algorithm was proposed to compensate the turbulent components of the wind power, which determines the power output set point for the low-level flywheels control. In [21], a hybrid energy storage system composed of a vanadium redox battery and a supercapacitor bank is used to smooth the power output of a 1-MW grid-connected photovoltaic (PV) system. The set points for the vanadium redox battery and the supercapacitor bank are decided according to the PV power output, the grid power demand, and the power sharing algorithm.

Besides working together as a single entity to achieve power tracking, the ESUs should share the power demand in a proper way such that the entire ESS can operate in full function. In particular, the energy level of each ESU should be balanced to keep the maximal power capacity of the ESS. Otherwise, if an ESU reaches critically high or low energy level, it will be forced off-line for safety protection, which will in turn reduce the power capacity of the ESS. For battery energy storage systems, the balancing of state-of-charge (SOC), which is defined as the ratio of the remaining charge and the charge capacity of a battery cell/pack, has attracted much attention [22]–[26]. In [22]–[24], the SOC balancing of islanded DC microgrids was considered. Adaptive droop control approaches were proposed in [22] and [23] in the way that the droop coefficient is set inversely proportional to the SOC in order to drive all the SOCs to the balanced state. In [24], a sliding mode controller was proposed which generates a control signal that determines its level of participation in the droop control to achieve SOC balancing. Reference [25] considered the SOC balancing of a grid-connected AC battery energy storage system. For each battery pack, the reference power output is determined by a global reference together with the SOCs of its neighboring battery packs. In [26], the batteries are classified as follower batteries and a leader battery. The SOCs of the follower batteries will track the SOC of the leader battery.

In this paper, we consider the power control problem for an ESS aiming at both power tracking and energy balancing. Motivated by the fact that the concept of SOC is not applicable to all types of ESUs, such as flywheels which store kinetic energy, an absolute energy balancing problem was studied in [27], which was formulated as a leaderless consensus problem and solved using the simultaneous eigenvalue placement technique [28]. While, the method of [27] only applies to the ESUs with the same energy capacity. In this paper, we further generalize the absolute energy balancing problem considered in [27] to a relative energy balancing problem by defining a new concept called state-of-energy (SOE), which is defined as the ratio of the remaining energy and the energy capacity of an ESU. The concept of SOE is equivalent to SOC when applied to batteries since the remaining energy of the batteries is in proportion to the remaining charge, i.e., \( \Delta E = UI \Delta t = U \Delta Q \), but it also applies to other types of ESUs. To achieve SOE balancing, the ESS is modeled as a multiagent system with each ESU being viewed as an agent and the SOE balancing problem is formulated as a leaderless consensus problem. To achieve power tracking, a command generator (CG) is designed, which together with the ESS constitutes an extended multiagent system and the power tracking problem is formulated as an interconnected leader-following consensus problem. These two consensus problems are coupled with each other and form a hybrid consensus problem, which, to the best of our knowledge, has not been encountered before. To overcome the technical difficulties, in this paper, a novel distributed consensus control algorithm is proposed which combines an integral-type consensus control law and a distributed observer for the CG. Rigorous Lyapunov analysis is conducted to prove the stability of the closed-loop system. Numerical case studies show strong evidence of the robustness, effectiveness and feasibility of the proposed control in real world applications. The advantages of the proposed control scheme are summarized as follows:

1) the proposed control scheme is distributed, which only relies on neighbor-based information exchange and thus enables a more economic, flexible and scalable configuration of the communication network;
2) the proposed control scheme allows online switch-on and switch-off operations for ESUs, which makes the ESS more efficient for preplanned check and more resilient to unexpected fault;
3) the SOE balancing mechanism can drive the SOEs of all the ESUs to the balanced state. As a result, the power capacity of the ESS can reach and remain maximum.

The rest of this paper is organized as follows. Section II introduces the system dynamics and the problem formulation. The main results are given in Section III and the proposed control approaches are illustrated by numerical case studies in Section IV. Finally, Section V concludes this paper.

II. PROBLEM FORMULATION
Consider a physically concentrated ESS consisting of \( N \) ESUs as shown in Fig. 1. Suppose all the ESUs are operating in the power control mode [29], [30]. For the \( i \)th ESU, \( i = 1, \ldots, N \), let \( E_i(t), E_{ci} \), and \( P_i(t) \) denote the remaining energy, energy capacity and power output, respectively. \( P_i(t) > 0 \) \( (P_i(t) < 0) \) indicates energy release (storage). Let

\[
\dot{x}_i(t) = \frac{E_i(t)}{E_{ci}} \tag{1}
\]

denote the SOE of the \( i \)th ESU. It follows that

\[
\dot{x}_i(t) = -1 \frac{1}{E_{ci}} P_i(t). \tag{2}
\]
Let $E_c = \sum_{i=1}^{N} E_{ci}$ and $P_{ess}(t) = \sum_{i=1}^{N} P_i(t)$ denote the energy capacity and power output of the entire ESS, respectively. Note that $P_{ess}(t)$ can be measured directly at the coupling point as shown in Fig. 1. Let $P_{ref} \in \mathbb{R}$ denote the reference power output for the ESS, which is assumed to be a piecewise constant signal scheduled by some upper level control. Let

$$P_e(t) = P_{ref} - P_{ess}(t)$$

denote the power tracking error for the ESS. Note that in this paper, we assume that the ESS is physically concentrated, such as the container ESS system. Therefore, the power loss over the transmission lines is neglected. The control objectives considered in this paper are given as follows.

1) (Power Tracking) the power output of the ESS should satisfy

$$P_e(t) = P_{ref} - P_{ess}(t) = 0.$$  \hspace{1cm} (4)

2) (SOE Balancing) the SOEs of all the ESUs should be balanced, i.e.,

$$x_i(t) - x_j(t) = 0, \quad i, j = 1, \ldots, N.$$  \hspace{1cm} (5)

By (4) and (5), the desired value of $P_i(t)$, denoted by $P_i^*$, can be calculated as follows. By (5), for $i, j = 1, \ldots, N$,

$$\dot{x}_i(t) = \dot{x}_j(t)$$

and thus by (2), $P_i^*/E_{ci} = P_j^*/E_{cj}$. For $i = 1, \ldots, N$, let $P^* = P_i^*/E_{ci}$, and thus $P_i^* = E_{ci}P^*$. As a result, by (4),

$$P_{ref} = P_{ess}(t) = \sum_{i=1}^{N} P_i(t)$$

$$= \sum_{i=1}^{N} P_i^* = \sum_{i=1}^{N} E_{ci}P^* = E_c P^*.$$  \hspace{1cm} (7)

Therefore,

$$P^* = P_{ref}/E_c$$

and hence,

$$P_i^* = \frac{E_{ci}P_{ref}}{E_c}.$$  \hspace{1cm} (9)

First, we propose a centralized control scheme, illustrated by Fig. 2, to achieve the control objectives (4) and (5) in the following way:

1) the CC collects $E_c$ of all the ESUs and calculates $E_c$;
2) the ESUs should be initialized such that $x_i(0) = x_j(0)$ for $i, j = 1, \ldots, N$;
3) the CC receives $P_{ref}$ from the upper level control, calculates $P_i^*$ by (9) for all the ESUs and then sends $P_i^*$ to each ESU individually.

As shown in Fig. 2, to install a new ESU to the ESS, a direct communication channel between the CC and the new ESS should be implemented first. The energy capacity of the new ESS should be reported to the CC so that the CC can update the references for all the ESUs. If the existing ESU needs to be switched off, the CC also needs to be informed in advance so that the references for the other ESUs can be updated according to the new $E_c$.

There are several limits of this centralized control scheme. First, the CC undertakes all the communication and computation load, which requires high performance processing units to deal with big data sets and thus could be costly to implement. Second, since both $E_c$ and $E_{ci}$ need to be known to determine $P_i^*$ by (9), every time when new ESUs need to be installed or existing ESUs need to be switched off, the CC needs to be informed in advance and take actions accordingly, which is of low efficiency. Third, since there is no SOE balancing mechanism, the difference between the SOEs of different ESUs will accumulate as time goes on. Thus, the power capacity of the ESS may not always remain maximum.

### III. DISTRIBUTED CONTROL SCHEME

To break through the limits associated with the centralized control scheme, in this section, a distributed control scheme is proposed. First, we design a CG in the following form

$$\dot{\eta}_0(t) = \alpha P_e(t)$$ \hspace{1cm} (10)

where $\eta_0(t) \in \mathbb{R}, \alpha > 0$ is a gain parameter.

The ESS together with the CG can be viewed as a multigent system, whose communication network is represented by a digraph $\hat{G} = (\hat{V}, \hat{E})$ with $\hat{V} = \{0, 1, \ldots, N\}$ and $\hat{E} \subseteq \hat{V} \times \hat{V}$. Here, the node 0 is associated with the CG and the node $i, i = 1, \ldots, N$, is associated with the $i$th ESU.

1See Appendix for a summary of graph notation.
For $i = 0, 1, \ldots, N$, $j = 1, \ldots, N$, $(i, j) \in \mathcal{E}$ if and only if the $i$th ESU can receive the information from the CG or the $j$th ESU. Furthermore, we define a graph $\mathcal{G} = (\mathcal{V}, \mathcal{E})$ for the ESS where $\mathcal{V} = \{1, 2, \ldots, N\}$ and $\mathcal{E} = \{\mathcal{V} \times \mathcal{V}\} \cap \mathcal{E}$. In the following, let $\bar{A} = [a_{ij}] \in \mathbb{R}^{(N+1) \times (N+1)}$ be the weighted adjacency matrix of $\mathcal{G}$, $\bar{L} \in \mathbb{R}^{N \times N}$ be the Laplacian of $\mathcal{G}$ and $H = L + \text{diag}\{a_{10}, \ldots, a_{N0}\}$. Throughout this paper, we assume that the graph $\mathcal{G}$ contains a spanning tree with the node 0 as its root and the graph $\mathcal{G}$ is connected and undirected. Physically, this assumption means that the information of the CG can be transmitted to each ESU through a communication path and all the ESUs are connected together through the communication network. Mathematically, the power tracking and SOE balancing problem can be formulated as follows.

**Problem 1:** Given systems (2), (10) and the communication network described by $\mathcal{G}$, find $P_i(t)$ such that

$$\lim_{t \to \infty} P_a(t) = 0$$

and for $i, j = 1, \ldots, N$,

$$\lim_{t \to \infty} (x_i(t) - x_j(t)) = 0.$$  

To solve Problem 1, for the $i$th ESU, $i = 1, \ldots, N$, the control law is given as follows

$$\dot{\xi}_i(t) = \sum_{j=1}^{N} a_{ij} (x_j(t) - x_i(t))$$

$$\dot{\eta}_i(t) = \sum_{j=0}^{N} a_{ij} (\eta_j(t) - \eta_i(t))$$

$$P_i(t) = -\kappa_x \sum_{j=1}^{N} a_{ij} (x_j(t) - x_i(t)) - \kappa_\xi \xi_i(t) + \kappa_\eta \eta_i(t)$$

where $\kappa_x, \kappa_\xi, \kappa_\eta > 0$ are gain parameters. The block diagram of the control law (13) is shown in Fig. 3. The main result is given as follows.

**Theorem 1:** Given systems (2), (10) and the communication network described by $\mathcal{G}$, there exist gain parameters $\alpha, \kappa_x, \kappa_\xi, \kappa_\eta$ such that Problem 1 is solvable by the control law (13).

**Proof:** Let $\xi_{\text{sum}}(t) = \sum_{i=1}^{N} \xi_i(t)$. Since $\mathcal{G}$ is undirected, $a_{ij} = a_{ji}$, and thus it follows that

$$\dot{\xi}_{\text{sum}}(t) = \sum_{i=1}^{N} \sum_{j=1}^{N} a_{ij} (x_j(t) - x_i(t)) = 0.$$  

Therefore, $\xi_{\text{sum}}(t) = \xi_{\text{sum}}(0) \triangleq \xi_{\text{sum}}^*$ for all $t \geq 0$. Moreover, $\sum_{i=1}^{N} P_i(t)$

$$= \sum_{i=1}^{N} \left( -\kappa_x \sum_{j=1}^{N} a_{ij} (x_j(t) - x_i(t)) - \kappa_\xi \xi_i(t) + \kappa_\eta \eta_i(t) \right)$$

$$= -\kappa_\xi \xi_{\text{sum}}^* + \kappa_\eta \sum_{i=1}^{N} \eta_i(t).$$

Substituting (15) into (10) gives

$$\dot{\eta}_0(t) = \alpha \left( P_0(t) - \sum_{i=1}^{N} P_i(t) \right)$$

$$= \alpha \left( P_0(t) + \kappa_\xi \xi_{\text{sum}}^* - \kappa_\eta \sum_{i=1}^{N} \eta_i(t) \right).$$

Let

$$\tilde{P}_{\text{ref}} = P_{\text{ref}} - \kappa_\xi \xi_{\text{sum}}^*$$

$$\tilde{\eta}_0(t) = \eta_0(t) - \tilde{P}_{\text{ref}} / (\kappa_\eta N)$$

$$\tilde{\eta}_i(t) = \eta_i(t) - \eta_0(t)$$

$$\tilde{\eta}_i(t) = \tilde{\eta}_i(t) + \tilde{\eta}_0(t) = \eta_i(t) - \tilde{P}_{\text{ref}} / (\kappa_\eta N).$$

Then it follows

$$\dot{\tilde{\eta}}_0(t) = \dot{\tilde{\eta}}_0(t)$$

$$= \alpha \left( \tilde{P}_{\text{ref}} - \kappa_\eta \sum_{i=1}^{N} \eta_i(t) \right)$$

$$= \alpha \left( \tilde{P}_{\text{ref}} - \kappa_\eta \left( \eta_0(t) + \tilde{\eta}_i(t) \right) \right)$$

$$= \alpha \left( \tilde{P}_{\text{ref}} - \kappa_\eta \eta_0(t) - \kappa_\eta \sum_{i=1}^{N} \tilde{\eta}_i(t) \right)$$

$$= -\alpha \kappa_\eta \eta_0(t) - \alpha \kappa_\eta \sum_{i=1}^{N} \tilde{\eta}_i(t).$$

Moreover, by (13) and (18), we have

$$\ddot{\tilde{\eta}}_0(t) = \sum_{j=0}^{N} a_{ij} (\eta_j(t) - \eta_0(t))$$

$$+ \alpha \kappa_\eta N \eta_0(t) + \alpha \kappa_\eta \sum_{j=1}^{N} \tilde{\eta}_i(t)$$

$$= \sum_{j=0}^{N} a_{ij} (\tilde{\eta}_j(t) - \tilde{\eta}_0(t))$$

$$+ \alpha \kappa_\eta N \tilde{\eta}_0(t) + \alpha \kappa_\eta \sum_{j=1}^{N} \tilde{\eta}_i(t).$$

Let $\tilde{\eta}(t) = \text{col}(\tilde{\eta}_1(t), \ldots, \tilde{\eta}_N(t))$. Then

$$\begin{bmatrix} \ddot{\tilde{\eta}}_0(t) \\ \ddot{\tilde{\eta}}_0(t) \end{bmatrix} = \begin{bmatrix} -H + \alpha \kappa_\eta 1_N 1_N^T \\ -\alpha \kappa_\eta 1_N^T \end{bmatrix} \begin{bmatrix} \tilde{\eta}(t) \\ \tilde{\eta}_0(t) \end{bmatrix}$$

where $1_N \triangleq \text{col}(1, \ldots, 1) \in \mathbb{R}^N$. By Lemma 5 of [31] or Lemma 1 of [32], there exist $\alpha, \kappa_\eta > 0$ such that the origin of system (20) is exponentially stable. As a result, for $i = 0, 1, \ldots, N$,

$$\lim_{t \to \infty} \tilde{\eta}_i(t) = 0.$$  

\^For $x_i \in \mathbb{R}^{n_i}, i = 1, \ldots, m$, col($x_1, \ldots, x_m$) $\triangleq (x_1^T, \ldots, x_m^T)^T$. \n
\n
exponentially, and hence for \( i = 1, \ldots, N \),
\[
\lim_{t \to \infty} \hat{\eta}_i(t) = 0
\]
(22)

exponentially. By (15), it follows that
\[
P_e(t) = P_{ref} - P_{ext}(t) = P_{ref} - \sum_{i=1}^{N} P_i(t)
\]
\[
= P_{ref} + \kappa_\xi \tilde{x}_{sum} - \kappa_\eta \sum_{i=1}^{N} \eta_i(t)
\]
\[
= \bar{P}_{ref} - \kappa_\eta \sum_{i=1}^{N} \eta_i(t)
\]
\[
= \kappa_\eta \left( \sum_{i=1}^{N} \frac{P_{ref}}{\kappa_\eta N} - \sum_{i=1}^{N} \eta_i(t) \right)
\]
\[
= -\kappa_\eta \sum_{i=1}^{N} \tilde{\eta}_i(t)
\]
(23)

and thus
\[
\lim_{t \to \infty} P_e(t) = 0
\]
(24)

exponentially.

Next, submitting (13c) into (2) gives
\[
\hat{x}_i(t) = \frac{1}{E_{ci}} \kappa_\xi \sum_{j=1}^{N} a_{ij}(x_j(t) - x_i(t)) + \frac{1}{E_{ci}} \kappa_\xi \bar{\xi}(t) \left( \frac{P_{ref}}{\kappa_\xi \eta N} \right) + \frac{1}{E_{ci}} \kappa_\eta \eta_i(t).
\]
(25)

Let \( x(t) = \text{col}(x_1(t), \ldots, x_N(t)) \), \( \bar{\xi}(t) = \text{col}(\bar{\xi}_1(t), \ldots, \bar{\xi}_N(t)) \), \( \bar{\eta}(t) = \text{col}(\tilde{\eta}_1(t), \ldots, \tilde{\eta}_N(t)) = \eta(t) - \frac{P_{ref}}{\kappa_\eta N} 1_N \) and \( \Gamma = \text{diag}\{\frac{1}{E_{ci}}, \ldots, \frac{1}{E_{ci}}\} \). Then
\[
\hat{x}(t) = -\kappa_\xi \Gamma \bar{L}(x(t)) + \kappa_\xi \Gamma \bar{\xi}(t) - \kappa_\eta \Gamma \eta(t).
\]
(26)

Furthermore, let \( \hat{\tilde{\xi}}(t) = \tilde{\xi}(t) - \frac{P_{ref}}{\kappa_\xi N} 1_N \). Then we have
\[
\hat{x}(t) = -\kappa_\xi \Gamma \bar{L}(x(t)) + \kappa_\xi \Gamma \tilde{\xi}(t) - \kappa_\eta \Gamma \eta(t)
\]
\[
= -\kappa_\xi \Gamma \bar{L}(x(t)) + \kappa_\xi \Gamma \tilde{\xi}(t) + \kappa_\xi \Gamma \bar{\xi}(t) + \kappa_\eta \Gamma \eta(t)
\]
\[
= -\kappa_\xi \Gamma \bar{L}(x(t)) + \kappa_\xi \Gamma \tilde{\xi}(t) + \kappa_\xi \Gamma \bar{\xi}(t) - \kappa_\eta \Gamma \eta(t)
\]
\[
= -\kappa_\xi \Gamma \bar{L}(x(t)) + \kappa_\xi \Gamma \tilde{\xi}(t) - \kappa_\eta \Gamma \eta(t)
\]
(27)

and
\[
\hat{\tilde{\xi}}(t) = \tilde{\xi}(t) - \bar{L}(x(t)) - \bar{\xi}(t).
\]
(28)

By Lemma 1.1 of [33], \( \bar{L} \) is semi-positive definite. Let
\[
V(t) = \frac{1}{2} x(t)^T \bar{L}(x(t)) + \kappa_\xi \Gamma \tilde{\xi}(t) + \kappa_\eta \Gamma \eta(t).
\]
(29)

Then noting that both \( \Gamma \) and \( \bar{L} \) are symmetric gives
\[
\dot{V}(t) = -\kappa_\xi x(t)^T \bar{L}(x(t)) + \kappa_\xi \Gamma \tilde{\xi}(t) - \kappa_\xi \Gamma \bar{\xi}(t) - \kappa_\eta \Gamma \eta(t)
\]
\[
= -\kappa_\xi x(t)^T \bar{L}(x(t)) - \kappa_\xi \Gamma \bar{\xi}(t) - \kappa_\eta \Gamma \eta(t).
\]
(30)

Let \( y_{min} = \min\{\frac{1}{E_{ci}}, \ldots, \frac{1}{E_{ci}}\} \) and \( y_{max} = \max\{\frac{1}{E_{ci}}, \ldots, \frac{1}{E_{ci}}\} \). Then
\[
V(t) \leq -\kappa_\xi \min y_{min} ||\bar{L}(x(t))||^2 + \kappa_\eta \max y_{max} ||\bar{L}(x(t))|| \cdot ||\bar{\xi}(t)||. \] (31)

By (22), suppose \( ||\bar{\xi}(t)|| \leq \delta_\eta e^{-\beta_\eta t} \) for some \( \delta_\eta, \beta_\eta > 0 \). Then
\[
\dot{V}(t) \leq -\kappa_\xi y_{min} ||\bar{L}(x(t))||^2 + \kappa_\eta y_{max} ||\bar{L}(x(t))|| \cdot \delta_\eta e^{-\beta_\eta t}
\]
\[
= -\kappa_\xi y_{min} ||\bar{L}(x(t))|| \left( ||\bar{L}(x(t))|| - \frac{\kappa_\eta y_{max} \delta_\eta}{\kappa_\xi y_{min}} e^{-\beta_\eta t} \right)
\]
\[
\leq -\kappa_\xi y_{min} ||\bar{L}(x(t))|| \left( ||\bar{L}(x(t))|| - \delta_\eta e^{-\beta_\eta t} \right)
\]
(32)

where \( \delta_\eta = \frac{\kappa_\eta y_{max} \delta_\eta}{\kappa_\xi y_{min}} \). Note that for all \( t \geq 0 \), we have
\[
-||\bar{L}(x(t))|| \left( ||\bar{L}(x(t))|| - \delta_\eta e^{-\beta_\eta t} \right) \leq \frac{\delta_\eta^2}{4} e^{-2\beta_\eta t}
\]
(33)

where the equality holds if and only if \( ||\bar{L}(x(t))|| = \frac{\delta_\eta e^{-\beta_\eta t}}{2} \). Therefore, we have
\[
\dot{V}(t) \leq \kappa_\xi y_{min} \delta_\eta^2 e^{-2\beta_\eta t} \leq \delta_\eta e^{-2\beta_\eta t}
\]
(34)
where \( \delta_\eta = \kappa_\eta y^{\text{min}} \tilde{\delta}_\eta^2 / 4 \). Thus

\[
V(\infty) - V(0) \leq \frac{\delta_\eta}{2\beta_\eta}
\]

which in turn implies that \( V(t) \) is bounded. Since \( \mathcal{L} \) is positive semi-definite and \( \Gamma \) is positive definite, we have \( \mathcal{L}^{1/2} x(t) \) and \( \mathcal{X}(t) \) are both bounded, where \( \mathcal{L}^{1/2} \) denotes the square root of \( \mathcal{L} \). As a result, \( \mathcal{L} x(t) \) is bounded. Suppose \( \|\mathcal{L} x(t)\| \leq \rho \) for all \( t \geq 0 \). Let

\[
U(t) = \int_0^t \kappa_\eta x(\tau)^T \mathcal{L} \Gamma \tilde{\eta}(\tau) d\tau.
\]

then

\[
\|U(t)\| \leq \int_0^t \|x(\tau)^T \mathcal{L} \Gamma \tilde{\eta}(\tau)\| d\tau
\]

\[
\leq k_\eta \int_0^t \|\mathcal{L} x(\tau)\| \cdot \|\tilde{\eta}(\tau)\| d\tau
\]

\[
\leq k_\eta y^{\text{max}} \rho \int_0^t \delta_\eta e^{-\beta_\eta \tau} d\tau.
\]

Therefore, \( U(t) \) is bounded. Let

\[
W(t) = V(t) + U(t).
\]

Then, \( W(t) \) is lower bounded. Moreover, we have

\[
\dot{W}(t) = \dot{V}(t) + \dot{U}(t) = -\kappa_\eta x(t)^T \mathcal{L} \Gamma \mathcal{L} x(t) \leq 0.
\]

Since \( \mathcal{L} x(t) \), \( \mathcal{X}(t) \) and \( \tilde{\eta}(t) \) are all bounded, by (27), \( \dot{x}(t) \) is bounded. Therefore, \( \dot{W}(t) \) is bounded. Then, by Barbalat’s Lemma,

\[
\lim_{t \to \infty} \dot{W}(t) = 0.
\]

Since \( \Gamma \) is positive definite, it follows \( \lim_{t \to \infty} \mathcal{L} x(t) = 0 \), which by Remark 1.1 of [33] implies that

\[
\lim_{t \to \infty} (x_i(t) - x_j(t)) = 0
\]

for \( i, j = 1, \ldots, N \).

The diagram for the distributed control scheme is shown in Fig. 4. In contrast to the centralized control scheme, the distributed control scheme has the following advantages. First, different from the CC, the CG only undertakes limited communication and computation load. In particular, it updates its state based on \( P_{\text{ref}} \) and \( P_{\text{ess}}(t) \), and sends its state to its neighboring ESUs over the communication network. It does not need to acquire the energy capacities of the ESUs. Second, it suffices to set up the communication links such that the communication network for ESUs are connected when new ESUs need to be installed to the ESS or the existing ESUs need to be switched off from the ESS. No action is required for CG. Third, the term \( -\kappa_\xi \sum_{j=1}^N a_i(x_i(t) - x_j(t)) - \kappa_\xi \mathcal{X}(t) \) in the control law (13c) guarantees the SOE balancing of all the ESUs, which in turn keeps the maximum power capacity of the ESS for all the time.

**IV. CASE STUDIES**

In this section, based on Matlab, a series of numerical case studies are conducted to examine the performance of the proposed distributed control scheme. An ESS consisting of six ESUs is studied in the following case studies. The communication network for the ESS is shown in Fig. 5. In Section II, to enable a rigorous proof of Theorem 1, the ideal situation is considered: 1) there is no power loss during energy conversion; 2) there is no power loss in the transmission lines; 3) there is no power output limit for the ESU. In the subsequent case studies, the following practical situation will be considered: 1) the SOE dynamics (2) are modified as

\[
\rho_{ri} \dot{x}_i = -\frac{1}{E_{ci}} P_i \quad \text{(energy release)}
\]

\[
\dot{x}_i = \frac{1}{E_{ci}} \rho_{si} P_i \quad \text{(energy storage)}
\]

where \( \rho_{ri} \) and \( \rho_{si} \) represent the energy conversion efficiencies during the energy release and storage processes, respectively; 2) there is 5% power loss in the transmission lines; 3) the power output of the ESU is limited by the following saturation function

\[
P_i = \begin{cases} P_{i,\text{max}} & P_i \geq P_{i,\text{max}} \\ P_i & P_{i,\text{max}} < P_i < P_{i,\text{max}} \\ -P_{i,\text{max}} & P_i \leq -P_{i,\text{max}} \end{cases}
\]

where \( P_i \) is calculated by (13c), \( P_{i,\text{max}} \) is the power output limit, and \( P_{i}^s \) is the saturated power output. The specifications
of the ESUs are given in Table 1. The gain parameters of the CG and the control law (13) are selected to be $\alpha = 1$, $\kappa_x = 10^5$, $\kappa_{\xi} = 40$, $\kappa_{\eta} = 0.1$. In what follows, a series of case studies will be shown to test the effectiveness and robustness of the proposed control scheme.

A. POWER TRACKING AND SOE BALANCING

In this case, we consider the start phase of the ESS. Let $P_{\text{ref}} = 10 \text{ kw}$ and suppose the initial SOEs of the ESUs are given by $x_1 = 0.87$, $x_2 = 0.86$, $x_3 = 0.85$, $x_4 = 0.84$, $x_5 = 0.83$, $x_6 = 0.82$. The initial state of the control law (13) are selected to be $\eta_i(0) = 0$, $\xi_i(0) = 0$. The initial state of the CG is $\eta_0(0) = 0$. The system response is shown in Fig. 6. It can be seen that: 1) the SOE balancing has been achieved, 2) the power tracking has been fulfilled, 3) the power outputs of the ESUs are within the limits, and 4) the power loss in the transmission lines is about $(10.53 - 10)/10.53 = 5\%$.

B. ABRUPT CHANGE IN $P_{\text{ref}}$

In this case, we examine the system response subject to abrupt change in $P_{\text{ref}}$. Suppose at $t = 0 \text{ h}$, the ESS is in steady state with $x_i = 0.7$ for $i = 1, \ldots, 6$ and $P_{\text{ref}} = 10 \text{ kw}$; at $t = 1 \text{ h}$, $P_{\text{ref}}$ is changed to $P_{\text{ref}} = -10 \text{ kw}$. The system response is shown in Fig. 7. It can be observed that the SOE balancing has been successfully kept, and the reference power output has been tracked very fast. When $P_{\text{ref}} = -10 \text{ kw}$, the power loss in the transmission lines is also approximately $(10 - 9.524)/9.524 = 5\%$.

C. ONLINE ESS RECONFIGURATION

In this case, we consider the online switch-on and switch-off of ESUs. Let $P_{\text{ref}} = 10 \text{ kw}$. Suppose at $t = 0 \text{ h}$, $x_i = 0.7$ for $i = 1, \ldots, 6$ and the system is at steady state; at $t = 1 \text{ h}$, ESU 3 is switched off from the ESS; at $t = 1.5 \text{ h}$, ESU 3 is switched on to the ESS. The system response is shown in Fig. 8. The SOE balancing has been kept during the switch-off operation, and has been recovered during the switch-on operation. The power tracking has been quickly achieved after both the switch-on and the switch-off operations.
D. COMMUNICATION DELAY AND BREAKDOWN

In this case, we consider the situation where the communication network is imperfect. First, we consider the communication delay issue. Note that the ESUs in an ESS are usually physically concentrated [34]. Therefore, the communication delay will be typically on the time scale of $10^{-7}$ s (0.5 µs per 100 m [35]). In the simulation result, we consider the same problem setup as in Case A but the information exchange among agents is suffered from a constant time delay of 10 µs. The system response is shown in Figs. 9(a) and 9(b).

It can be seen that the system response is not much different from that in Case A, which shows the robustness of the proposed control scheme against communication delay. Second, we consider the communication breakdown issue. Suppose there are two operation modes for the communication network. In mode $M_1$, the communication link $a$ breaks down, while $b$ works normally; in mode $M_2$, the communication link $b$ breaks down, while $a$ works normally. The communication network switches between mode $M_1$ and $M_2$ every 1 second.

The system response is shown in Figs. 9(c) and 9(d). In comparison to the results of Case A, it takes longer time for the system to reach steady state. However, since in both mode $M_1$ and $M_2$ the communication network is connected, power tracking and SOE balancing can still be achieved.

E. TRANSIENT RESPONSE AND CONTROL GAIN SELECTION

The system transient response, such as the overshoot, oscillation and settling time, is closely related to the control gain selection. In this case, we will discuss how these control gains should be selected and how they may affect the system performance. First, in control law (13c), $\kappa_x$ is the gain for the proportional term. Note that the SOE differences $\sum_{j=1}^{N} a_i(x_j(t) - x_i(t))$ will normally be on the scale of $10^{-2} \sim 10^{-1}$, $\kappa_x$ should be chosen large enough such that the SOE of the ESUs can reach nearby the steady state quickly. Given that the power output of the ESU in this paper is on the
scale of $10^3$ w, $\kappa_\epsilon$ is chosen to be on the scale of $10^4 \sim 10^5$. Second, in control law (13c), $\kappa_\eta$ is the gain for the integral term, which mainly affects the system steady state performance. Large $\kappa_\epsilon$ can guarantee fast steady state convergence, but it will adversely result in big oscillation in system transient response. In this paper, we choose $\kappa_\epsilon$ to be on the level of $10^4$. Third, in control law (13c), $\kappa_\eta$ determines the changing rate of the global synchronized signal, which should be chosen small enough so that the ESU can have enough time to achieve local power output tracking. While, excessively small $\kappa_\eta$ would be unnecessary which would lead to extremely slow power output tracking for the entire ESS. We choose $\kappa_\epsilon$ to be on the level of $10^{-1}$. In the simulation results, the problem setup is the same as in Case A. While, the control gains are selected to be $\kappa_\epsilon = 2 \times 10^5$, $\kappa_\eta = 200$, $\kappa_\eta = 10$. The system response is shown in Fig. 10. As analyzed before, large control gains have led to big overshoot, big oscillation and shorter settling time.

V. CONCLUSION

In this paper, the power control problem is investigated for an ESS consisting of multiple ESUs. To achieve simultaneous power tracking and SOE balancing, a distributed control scheme is proposed based on a novel consensus control algorithm. In contrast to centralized control, the proposed control law relies on a distributed communication network, allowing the online switch-on and switch-off operations of ESUs and can maintain the maximum power output capacity of the ESS. While, there is still room for this work to be improved. In order to enable a rigorous proof of Theorem 1, the following ideal situation is assumed: 1) there is no power loss during energy conversion; 2) there is no power loss in the transmission lines; 3) there is no power output limit for the ESU; 4) there is no time delay in the communication network. So far, to provide a rigorous proof without these assumptions would be very hard since the closed-loop system without these assumptions would be a state-dependent switched time-delayed uncertain system which could be very difficult to analyze. Though it is shown by numerical simulation that the proposed control scheme works well without these assumptions, it is still meaningful to theoretically show the robustness and resiliency of the proposed control scheme.

APPENDIX

GRAPH NOTATION

A digraph $\mathcal{G}$ is defined as $\mathcal{G} = (\mathcal{V}, \mathcal{E})$, which consists of a finite set of nodes $\mathcal{V} = \{1, \ldots, N\}$ and an edge set $\mathcal{E} = \{(i,j), i,j \in \mathcal{V}, i \neq j\}$. An edge from node $i$ to node $j$ is denoted by $(i, j)$, and node $i$ is called the neighbor of node $j$. If the digraph $\mathcal{G}$ contains a sequence of edges of the form $(i_1, i_2), (i_2, i_3), \ldots, (i_k, i_{k+1})$, then, the set $\{(i_1, i_2), (i_2, i_3), \ldots, (i_k, i_{k+1})\}$ is called a path of $\mathcal{G}$ from node $i_1$ to node $i_{k+1}$, and node $i_{k+1}$ is said to be reachable from node $i_1$. A digraph is said to contain a spanning tree if there exists a node $i$ such that any other node is reachable from it and node $i$ is called the root of the spanning tree. A digraph is said to be connected if it contains a spanning tree. The edge $(i, j)$ is called undirected if $(i, j) \in \mathcal{E}$ implies $(j, i) \in \mathcal{E}$. The digraph $\mathcal{G}$ is called undirected if every edge in $\mathcal{E}$ is undirected. The weighted adjacency matrix $A = [a_{ij}] \in \mathbb{R}^{N \times N}$ of $\mathcal{G}$ is defined as $a_{ii} = 0$, and for $i \neq j$, $a_{ij} > 0 \iff (j, i) \in \mathcal{E}$ and $a_{ij} = 0$ otherwise. Moreover, $a_{ij} = a_{ji}$ if $(j, i)$ is an undirected edge. The Laplacian matrix $L = [l_{ij}] \in \mathbb{R}^{N \times N}$ of $\mathcal{G}$ is defined as $l_{ii} = \sum_{j=1}^{N} a_{ij}$ and $l_{ij} = -a_{ij}$ if $i \neq j$.

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