Constraining alternative theories of gravity using GW150914 and GW151226

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The recently reported gravitational wave events GW150914 and GW151226 caused by the mergers of binary black holes [1–3] provide a formidable way to set constraints on alternative metric theories of gravity in the strong field regime. In this paper, we develop an approach where an arbitrary theory of gravity can be parametrised by an effective coupling $G_{\text{eff}}$ and an effective gravitational potential $\Phi(r)$. The standard Newtonian limit of General Relativity is recovered as soon as $G_{\text{eff}} \to G_N$ and $\Phi(r) \to \Phi_N$. The upper bound on the graviton mass and the gravitational interaction length, reported by the LIGO-VIRGO collaboration, can be directly recast in terms of the parameters of the theory which allows an analysis where the gravitational wave frequency modulation sets constraints on the range of possible alternative models of gravity. Numerical results based on published parameters for the binary black hole mergers are also reported. Comparison of the observed phase of the GW150914 and GW151226 with the modulated phase in alternative theories of gravity does not give reasonable constraints due the large uncertainties in the estimated parameters for the coalescing black holes. In addition to these general considerations, we obtain limits for the frequency dependence of the $\alpha$ parameter in scalar tensor theories of gravity.

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I. INTRODUCTION

In September and December 2015, the LIGO-VIRGO collaboration has reported on the direct detection of gravitational-wave (GW) signals from coalescing binary black hole (BH) systems [1, 4]. This has opened new opportunities in gravity research and began the era of gravitational-wave astronomy. In particular, this achievement can be considered as the first direct probe of metric theories of gravity in the regime of strong fields and relativistic velocities. The individual masses of the merging BHs at the beginning of the collision were $29^{+8.4}_{-4.5} M_\odot$ and $36^{+5.4}_{-2.5} M_\odot$ for the September signal and $14.2^{+4.2}_{-2.6} M_\odot$ and $7.5^{+1.6}_{-2.3} M_\odot$ for the December signal. Specifically, the GW150914 signal was emitted by a rapidly evolving dynamical binary that merged in a fraction of a second with an observed variation of the period $P_b$ ranging from $\sim -0.1$ at $f_{GW} \sim 30$ Hz to $\sim -1$ at $f_{GW} \sim 132$ Hz. The frequency and amplitude of the GW151226 signal was observed over 55 cycles spanning a range in frequency from 35 to 450 Hz. Using the templates created from numerical relativity, the data is consistent with the merger of two compact objects into a merged black hole with masses of $\sim 65.3^{+4.1}_{-3.4} M_\odot$ and $\sim 21.8^{+5.9}_{-6.3} M_\odot$, respectively. In this process, the energy emitted in the form of gravitational waves (GW) amounts to $3.0^{+0.5}_{-0.4} M_\odot$ and $1.0^{+0.1}_{-0.2} M_\odot$ and the velocity $v$ reached the value $\sim 0.5c$ at the time of the merger. In particular, the signal from GW150914 exhibits the typical behaviour predicted by the coalescence of compact systems where inspiral, merger and ring down phases are traversed [5]. The LIGO-VIRGO collaboration has analyzed the three regimes adopting a parametrized analytical family of inspiral-merger-ringdown waveforms [6–11]. The signal is divided in terms of frequency: the early to late inspiral regime from $\sim 20$ Hz to $\sim 55$ Hz; the intermediate region from $\sim 55$ Hz to $\sim 130$ Hz and the merger-ringdown region from $\sim 130$ Hz until the end of the waveform. The simplest and fastest parameterized waveform model that is currently available [12] sets bounds on the physical effects based on the inspiral phase only, where a calibrated post-Newtonian (PN) treatment is sufficient. For the later phases, phenomenological coefficients fitted to Numerical Relativity (NR) waveforms are used. In this paper, we discuss the possibility to set constraints on extended theories of gravity via the modified inspiral phase.

It is worth noting that the existence of GWs confirms metric theories of gravity, among them General Relativity (GR), but there is ample room for other possibilities (see [1] for a detailed discussion). Any extended theory of gravity can be parameterized by means of a suitable post-Newtonian parametrization where the governing param-
eter is the effective gravitational coupling constant \( G_{\text{eff}} \) and the effective gravitational potential \( \Phi(r) \). Both these quantities are functions of the radial coordinate that influence the phase of the GW signal. In other words, the GW waveform could, in principle, single out the range of possible gravitational metric theories that are in agreement with the data.

The paper is organised as follows. In Sec. II, we discuss how different theories of gravity can be parametrized by the coupling constant and the gravitational potential. The main differences of these theories with respect to GR can be reduced to the effective dependence on the radial coordinate. It is then straightforward to obtain the corresponding phase modulation and we will exemplarily do so in Sec. III and compare with the observed data. Sec. IV is devoted to the discussion of Shapiro delay that can be modulated according to the parameters of the given theory. Discussion and conclusions are drawn in Sec. V.

II. EFFECTIVE GRAVITATIONAL CONSTANT IN EXTENDED THEORIES OF GRAVITY

Alternative theories of gravity are extensions of GR where higher order curvature invariants and/or additional scalar fields are taken into account in the Hilbert-Einstein gravitational action (see [13–16] for a comprehensive review on the subject). If the gravitational Lagrangian is nonlinear in the Ricci scalar or, more generally, in the curvature invariants, the field equations become higher than second order in the derivatives; it is for this reason that such theories are often called higher-order gravitational theories. In principle, one can take into account wide classes of higher-order-scalar-tensor theories of gravity in four dimensions [13].

With the emergence of the inflationary paradigm, these theories have gained heightened attention as they can provide solutions to the shortcomings of the standard cosmological model. These are for example: the horizon problem, the density fluctuation problem, the dark matter problem, the exotic relics problem, the thermal state problem the cosmological constant problem the singularity problem the timescale problem [18–22].

Furthermore, the presence of scalar fields is important also in multidimensional gravity, such as Kaluza-Klein theories and in the effective action of string theory. In this framework, the strength of gravity, given by the local value of the gravitational coupling, depends on time and location. For example, the Brans-Dicke theory, that is the most used scalar-tensor theory of gravity [23], includes the hypothesis suggested by Dirac of the variation of the gravitational coupling with time [24]. As a consequence, scalar-tensor theories do not satisfy the Strong Equivalence Principle as the variation of the gravitational constant \( G_{\text{eff}} \) – which is, in general, different from \( G_N \), the standard Newton gravitational constant – implies that local gravitational physics depends on the scalar field strength. Theories which present such a feature are called non-minimally coupled theories.

In these theories, the gravitational coupling is determined by the form of the Lagrangian. We can have two physically interesting situations which could be tested by experiments:

1. when \( G_{\text{eff}}(r) \rightarrow \infty \rightarrow G_N \), the Newton gravitational constant and GR are recovered.

2. The possibility that gravitational coupling is not asymptotically constant, i.e. \( G_{\text{eff}} \) is always varying with the epoch and \( G_{\text{eff}} / G_{\text{eff}} |_{\text{now}} \neq 0 \).

The variability of the gravitational coupling can be tested by three classes of experiments:

- Through observations of Solar System dynamics. In fact, several weak-field tests of GR are based on planetary motion and dynamics of self-gravitating objects nearby the Sun. Deviations from classical tests are possible probes for the variation of the gravitational coupling.

- Through binary pulsar systems. In order to obtain information from these systems, it has been necessary to extend the post-Newtonian approximation, which can be used only in the presence of a weakly gravitationally interacting n-body system, to strong gravitationally interacting systems. The estimation of \( G / G \) is \( 2 \times 10^{-11} \) per year [26, 27].

- Through gravitational lensing observations of distant galaxies [28].

Concerning the solar system tests, the most stringent limits are obtained by Lunar Laser Ranging (LLR) combined with accurate ephemeris of the solar system. LLR consists of measuring the round-trip travel time of photons that are reflected back to Earth from mirrors located on the Moon; the change of round-trip time contains information about the Earth-Moon system. The round-trip travel time has been investigated for many years and the best estimates for \( G / G \) range from \( 4.0 \times 10^{-11} \) to \( 10^{-11} \) [25, 26]. However, none of these tests probes the strong field regime which, up to now, could not be investigated at all.

Besides the variation of the gravitational coupling, it is well known that a wide class of these theories gives rise to Yukawa-like corrections, \( e^{-hm} / m^2 \) in the gravitational potential [15]. Here, the parameter \( m \) is an effective mass related to the additional degrees of freedom in the gravitational action. Specifically, an additional scalar field is introduced by the corresponding Klein-Gordon equation of the form \( \square \phi - dV(\phi) / d\phi = 0 \) that has to be added to the standard set of Einstein field equations. In the static case, the Klein-Gordon equation reduces to

\[
(\nabla^2 - m^2) \phi = 0, \tag{1}
\]

where the effective mass \( m \) is given by the minimum of the potential \( V(\phi) \). The solution of Eq.(1) is a Newtonian potential corrected by a Yukawa-like term that, as
in the Klein-Gordon case, disappears at infinity allowing to recover the Newtonian limit and Minkowski flat spacetime.

In general, most alternative gravities have a weak field limit that can be expressed in the form (see also [29])

$$\Phi(r) = -\frac{G_N M}{r} \left[ 1 + \sum_{k=1}^{n} \alpha_k e^{-r/r_k} \right] = -\frac{G_{\text{eff}} M}{r}, \quad (2)$$

where $G_N$ is the value of the gravitational constant as measured at infinity and $r_k$ is the interaction length of the $k$-th component of the non-Newtonian corrections (see also [30, 31]). See Ref. [30, 31], for a general discussion of this last equation containing the non-Newtonian corrections.

Clearly, the standard Newtonian potential is restored as soon as $G_{\text{eff}} \to G_N$, which means $e^{-r/r_k} \to 0$ at infinity. The amplitude $\alpha_k$ of each component is normalized to the standard Newtonian term and the signs of the $\alpha_k$ coefficients indicate if the corrections are attractive or repulsive [32].

For the simplicity of the estimations, one can truncate to the first term of the expansion series in Eq. (2)\(^1\).

One then obtains a potential for the form

$$\Phi(r) = -\frac{G_N M}{r} \left[ 1 + \alpha_1 e^{-r/r_1} \right], \quad (3)$$

where the influence of non-Newtonian terms can be parameterized through the constants $(\alpha_1, r_1)$. For asymptotically large distances, where $r \gg r_1$, the exponential term tends to 0 and consequently the gravitational coupling tends to the limiting value $G_N$. In the opposite case when $r \ll r_1$, the exponential term tends to unity, consequently, by differentiating Eq. (3) and comparing with the gravitational force measured in laboratory experiments, one can get

$$G_{\text{lab}} = G_N \left[ 1 + \alpha_1 \left( 1 - \frac{r}{r_1} \right) \right] \simeq G_N (1 + \alpha_1), \quad (4)$$

where $G_{\text{lab}} = 6.67 \times 10^{-8} \text{ g}^{-1}\text{cm}^3\text{s}^{-2}$ is the standard Newton gravitation constant precisely measured in Cavendish-like experiments and where $G_N$ and $G_{\text{lab}}$ are identically the same in the standard gravity. However, the inverse square law is asymptotically valid, but the measured coupling constant is different by a factor $(1 + \alpha_1)$.

For self-gravitating systems, any correction involves a characteristic length that acts at a certain scale. The range of the characteristic scale $r_k$ corresponds to the Compton’s length

$$r_k = \frac{\hbar}{m_k c} \quad (5)$$

is identified through the mass $m_k$ of a pseudo-particle. Accordingly, in the weak energy limit, fundamental theories attempting to unify gravity with other forces introduce extra particles with mass which may carry the further degrees of freedom of the gravitational force [33].

There have been several attempts to constrain $r_k$ and $\alpha_k$ (and hence $m_k$) by experiments on scales in the range $1 \text{ cm} < r < 10^9 \text{ cm}$, using a variety of independent and different techniques [34–36]. The expected masses for particles which should carry the additional gravitational force are in the range $10^{-15} \text{ eV} < m_k < 10^{-3} \text{ eV}$. Given these, one can obtain the following estimates for the parameters

$$|\alpha_1| \sim 10^{-2}, \quad r_1 \sim 10^4 - 10^5 \text{ cm}. \quad (6)$$

Assuming that the dilaton is an ultra-soft boson which carries the scalar mode of gravitational field, one obtains a length scale of $\sim 10^{22} - 10^{23} \text{ cm}$, if the mass range is $m \sim 10^{-27} - 10^{-28} \text{ eV}$. This length scale is necessary to explain the flat rotation curves of the spiral galaxies. Furthermore, Very Long Baseline Interferometry observations impose a limit of $\alpha \sim 1.4 \times 10^{-2}$ [37]. On the other hand, binary-pulsar data places a limit from $10^{-2}$ to $10^{-4}$ on $\alpha$ [38–41].

However, new limits from GW150914, reported in [1], give as an upper limit for the graviton mass $m_g \leq 10^{-22} \text{ eV}$ and $r_g \geq 10^{18} \text{ cm}$ for the related Compton length. We obtain the same limit also for GW151226. These experimental numbers open new interesting perspectives in the present debate as soon as the above $m_k$ and $r_k$ are interpreted. Below, we will discuss how $G_{\text{eff}}$ and $\Phi(r)$ could be constrained according to the GW150914 and GW151226 data. As we will see, such constraints can be interpreted, at fundamental level, as the above effective mass $m_k$ and interaction length $r_k$.

### III. CONSTRAINTING $G_{\text{eff}}$ AND $\Phi(r)$ BY GW150914 AND GW151226

Starting from the above considerations, it is possible to constrain $G_{\text{eff}}$ and $\Phi(r)$ by the GW parameters reported for the events GW150914 and GW151226. Before this, let us review the post-Newtonian approximation required to perform this kind of analysis. Specifically, let us compute the 3.5PN approximation that relevant for our analysis [42, 43]. In particular, PN waveform models at the 3.5PN order are developed e.g. in [44].

To compare the theoretical waveforms with experimental sensitivities, we write the Fourier transform of the two GW strains $h_+, h_\times$ as

$$h_+ = A e^{i\phi_+} \frac{c^3}{r} \left( \frac{G_{\text{eff}} M}{c^4} \right)^{\frac{5}{2}} \frac{1}{f^\frac{7}{2}} \left( 1 + \cos^2 \frac{i}{2} \right), \quad (7)$$

$$h_\times = A e^{i\phi_\times} \frac{c^3}{r} \left( \frac{G_{\text{eff}} M}{c^4} \right)^{\frac{5}{2}} \frac{1}{f^\frac{7}{2}} \cos i, \quad (8)$$

\(^1\) This assumption is not applicable in some cases where additional corrections are taken into account.
where \( i \) is the inclination angle of the line of sight and the constant \( A \) has the value
\[
A = \frac{1}{\pi^3} \left( \frac{5}{24} \right)^{\frac{1}{2}}.
\]
The phase \( \phi_+ \) is given as
\[
\phi_+(f) = 2\pi f \left( t_c + \frac{v_c}{c} \right) - \varphi_c - \frac{\pi}{4} + \frac{3}{4} \left( \frac{G_{\text{eff}} M}{c^3} \right) 8\pi f t_c,
\]
where \( \varphi_c \) and \( t_c \) are the value of the phase and the time at coalescence, respectively. Furthermore the phases of the two strains are directly related, \( \phi_+ = \phi_+ + \frac{\pi}{2} \).

An accurate computation of the phase going well beyond the Newtonian approximation, is crucial for discriminating the signal of a coalescing binary from the noise. Therefore one has to give the PN correction to the phase (10). In order to exploit the signal present in the detector, and thus detect sources at further distance, an accurate theoretical prediction on the time evolution of the waveform is required.

In order to calculate the PN corrections we write the equation of motion in a more general form
\[
\frac{dv^i}{dt} = - \frac{G_{\text{eff}} M}{r^2} \left[ (1 + \mathcal{A}) \frac{x^i}{r} + B v^j \right] + O \left( \frac{1}{c^6} \right),
\]
such that it has a term proportional to the relative separation \( x^i \) and a term proportional to the relative velocity \( v^i \) in the center of mass frame. Here, the effective gravitational constant is not given by the standard Newton constant, but by \( G_{\text{eff}} = G_N (1 + \alpha) \).

Explicit expressions for the functions \( \mathcal{A} \) and \( B \) are extremely long and are given in Ref. [45]. Here we address the issue of how to obtain constraints from GW150914 and GW151226 data in the framework of the frequency-domain waveform model [5]. We proceed by considering the following relation for the frequency-domain phase
\[
\phi = 2\pi f t_c - \varphi_c - \frac{\pi}{4} + \frac{3}{128\eta} \left( \frac{\pi M G_{\text{eff}}}{c^3} \right)^{\frac{1}{4}} \sum_{i=0}^{7} \varphi_i(\Theta) \left( \frac{\pi M G_{\text{eff}}}{c^3} \right)^{\frac{1}{4}},
\]
where \( \varphi_i(\Theta) \) are the PN expansion coefficients that are functions of the intrinsic binary parameters. The information on the spin \( \chi_i \) (with \( i = 1, 2 \)) is incorporated via the following relations
\[
\chi_s = \frac{\chi_1 + \chi_2}{2},
\]
\[
\chi_a = \frac{\chi_1 - \chi_2}{2}.
\]

The 3.5PN expansion coefficients are
\[
\varphi_0 = 1,
\]
\[
\varphi_1 = 0,
\]
\[
\varphi_2 = \frac{3715}{756},
\]
\[
\varphi_3 = -16\pi + \frac{113\pi\chi_a}{3} + \left( \frac{113}{3} - \frac{76\pi}{3} \right) \chi_s,
\]
\[
\varphi_4 = \frac{15293365}{508032} + \frac{27145\eta}{504} + \frac{3085\eta^2}{72} + \left( -\frac{405}{8} + 200\eta \right) \chi_a - \frac{405}{8} \delta \chi_a \chi_s + \left( -\frac{50}{2} \right) \chi_S,
\]
\[
\varphi_5 = \left[ 1 + \log \left( \frac{\pi G_{\text{eff}} M f}{c^3} \right) \right] \left[ \frac{3845\pi}{756} - \frac{65\pi\eta}{9} + \delta \left( -\frac{732985}{2268} - \frac{140\eta}{9} \right) \chi_a + \left( -\frac{732985}{2268} + \frac{24260\eta}{81} + \frac{340\eta^2}{9} \right) \chi_s \right],
\]
\[
\varphi_6 = \frac{1158321326351}{4694215680} - \frac{6848\gamma_E}{21} - \frac{64\pi^2}{3} + \left( \frac{15737765635}{3048192} + \frac{2255\pi^2}{12} \right) \eta + \frac{76055\eta^2}{1728} - \frac{127825\eta^3}{1296},
\]
\[
\varphi_7 = \frac{77996675}{254016} + \frac{378515\pi\eta}{1512} + \frac{7405\pi^2}{756} + \delta \left( -\frac{25150083775}{3048192} + \frac{26804935\eta}{6048} - \frac{1985\eta^2}{48} \right) \chi_a + \left( \frac{25150083775}{3048192} + \frac{1056665559\eta}{762048} - \frac{1042165\eta^2}{3024} + \frac{5345\eta^3}{36} \right) \chi_s.
\]
where \( \varphi_0, ..., \varphi_7 \) indicate the 0, ..., 3.5PN approximation, respectively and \( \gamma_E = 0.577 \) is the Euler-Mascheroni con-
the frequency is much higher. Consequently, the inspiral
Figure 2, due to the lower mass involved in the merger,
range \( f\) parameter analysis does not rule out
is shown as black solid line, while the extended theory is
provided by [3] and given in Table I. The GR evolution
\( \delta \phi \) constraints due to the
\( \delta \phi \) parameters.

In Figs. 1 and 2 we have plotted the frequency
domain phase representation of GW150914 and GW151226
and show the effect of varying the \( \delta \phi_1 \) parameters as pro-
vided by the single parameter analysis of [2, 3, 5]. Note
that we follow their naming convention in introducing the
quantities \( \varphi^{bd} \) and \( \varphi^{bd} \) that contain the logarithmic
dependence with frequency. In addition, the variation with
the leading order deviation \( \alpha = \pm 10^{-2} \) is shown. The
single parameter analysis of [2] was performed by setting
all but the considered \( \delta \phi_1 \) to 0. In contrast, the multiple
parameter analysis was done by allowing all \( \delta \phi_1 \) to vary
freely. The latter leads to an error that is almost one or-
der of magnitude larger due to the additional degrees of
freedom. We do not consider GW150914 and GW151226
data for the multiple parameter analysis performed in
[2, 5] due to the large error bars in the \( \delta \phi_1 \) parameters.

For the masses, we have used the values as given in [3]
with \( m_1 = 36.2 M_\odot, m_2 = 29.1 M_\odot \) for GW150914, while
\( m_1 = 14.2 M_\odot, m_2 = 7.5 M_\odot \) for GW151226. The initial
spins were only constrained to be less than \( \chi_1 < 0.7, \chi_2 < 0.8 \)
[3, 4] and for our analysis we have taken the values of
\( \chi_1 = 0.7, \chi_2 = 0.8 \). Additionally, we adopted the values
\( t_c = 0.43 \text{s} \) for GW150914 and \( t_c = 1 \text{s} \) for GW151226
[1, 2, 5] and set \( \varphi_c = 0 \) for both.

Furthermore, we studied the sensitivity of our results
by varying the initial masses \( m_1, m_2 \) and initial spins
\( \chi_1, \chi_2 \) within the errors and ranges obtained by [4]. We
found that the resulting changes were mostly quantita-
tive, such as altering the slopes of the curves, and that the
qualitative behaviours of the curves, such as the width of
the constraints given by different \( \alpha \) remained unchanged.
Thus the results plotted in Figs. 1 and 2 are representa-
tive of the possible physical parameters reported in [4].

In Figure 1, we show the frequency-domain phase rep-
resentation for GW150914 and as shaded blue area the
constraints due to the \( \delta \phi_1 \) of the combined events, as
provided by [3] and given in Table I. The GR evolution
is shown as black solid line, while the extended theory is
marked in the range of \( \alpha \in [-10^{-2}, +10^{-2}] \) as red curves.
We report on the early inspiral range \( f \in [20, 90] \text{Hz} \) and zoom
in on the range \( f \in [80, 90] \text{Hz} \) in the inset. As
on can observe, at all the parameter orders the single
parameter analysis does not rule out \( |\alpha| < 10^{-2} \).

The data of the second event, GW151226 is shown in
Figure 2, due to the lower mass involved in the merger,
the frequency is much higher. Consequently, the inspiral
regime lasts until 450Hz and we show the phase in the
range \( f \in [40, 200] \text{Hz} \). Inspecting equations (20) and
(21), we see that the variation with \( \alpha \) is more pronounced
for objects with lower total mass \( M \). Hence, in principle
this event could yield tighter constraints on the value of
the allowed \( \alpha \).

In order to investigate the required tolerances, in Fig. 3
we plot two PN orders, 0 and 4 where, for GW151226,
we have decreased the variations by factors of 2, 5 and
10. For the PN terms of order 0, an increase by an order
of magnitude would be sufficient to set constraints on \( \alpha \).
More promising still are the higher order terms which,
even with a factor of 5 improvement, would be able to
set constraints on \( |\alpha| < 10^{-3} \).

We stress that these results are obtained from the sin-
gle parameter analysis, while a more correct treatment
would have to adopt the uncertainties of the multiple pa-
ter parameter analysis. Furthermore, we expect that as more
GWs are detected, the statistics on the combined poste-
density distributions [3] will improve and we will be
able to set stronger constraints on these types of alterna-
tive theories of gravity in the near future.

\section{Constraints from the Shapiro delay}

In this section we obtain constraints from the Shapiro
delay using the relative time difference between observa-
tions at multiple frequencies. This allows one to infer
violations of the EP using the observed time delay from
astrophysical particle messengers like photons, gravitons
or neutrinos [46–48]. To date, the strongest constraints
on the frequency dependence of the PPN-\( \gamma \)-parameter
are obtained by observations of fast radio bursts (FRB’s)
yielding \( \Delta \gamma(f) \sim 10^{-9} \). In the case of FRB’s, the largest
uncertainty is the signal dispersion due to the poorly un-
derstood line-of-sight free electron population [48]. The
fact that this uncertainty is completely avoided for grav-
itational waves makes them an appealing messenger
to test EP violations.

The Shapiro gravitational time delay is caused by the
slowing passage of light as it moves through a gravita-
tional potential (3) as

\begin{equation}
\Delta t_{\text{grav}} = -\frac{1 + \gamma}{c^3} \int_{r_e}^{r_o} \Phi(r) dr ,
\end{equation}

where \( \gamma \) is the (theory dependent) PPN parameter and \( r_o \)
and \( r_e \) are the positions of the observer and the source
of emission. Let us conservatively assume a short burst of
emission, that is all wave-frequencies are emitted at the
same instant. Now given the observed signal duration for
GW150914 of \( \sim 0.2s \), we can obtain an estimate for the
frequency dependence of \( \gamma \), respectively \( \alpha \). In absence
of other dispersive propagation effects, e.g. due to Lorentz
invariance violation (see also [49, 50]), we obtain an upper
limit for \( \Delta \alpha/\Delta f \).

For example, in scalar tensor theories the \( \gamma \) PPN pa-
ter is expressed in terms of non minimal coupling
function of a scalar field, equivalently, in terms of the \( \alpha \)
Figure 1: Frequency-domain phase representation for GW150914 with masses $m_1 = 36.2 M_\odot$, $m_2 = 29.1 M_\odot$ and initial spins $\chi_1 < 0.7$, $\chi_2 < 0.8$ [4]. The solid black curve is GR prediction where $\alpha = 0$ and $\delta\varphi = 0$. The red lines are $\alpha = \pm 0.01$. The shaded blue area is the range allowed for $\delta\varphi$, parameter in accordance with Table 1 of [5]. From left to right: the first on the left column show the phase at 0PN order and the right one is for the 0.5PN. The second on the left column is 1PN while on the right there is the 1.5PN. The third on the left column represents the phase at 2PN. On the right column, the phase at 2.5PN is shown. Finally, the fourth on left column show the 3PN and on the right 3.5PN. Note that the error on the $\delta\varphi$ is so large that it falls outside the scale. The inset frequency ranges from 80 Hz to 90 Hz to be illustrate the curves at these frequencies.
Figure 2: Frequency-domain phase representation for GW151226 with masses $m_1 = 14.2 M_{\odot}, m_2 = 7.5 M_{\odot}$ and initial spins $\chi_1 < 0.7, \chi_2 < 0.8$ \cite{2,3}. The solid black curve is GR prediction where $\alpha = 0$ and $\delta \varphi_2 = 0$. The red lines are $\alpha = \pm 0.01$. The shaded blue area is the range allowed for $\delta \varphi_2$ parameter in accordance with Table 1 of \cite{5}. From left to right: the first on the left column show the phase at $0 \text{PN}$ order and the right one is for the $0.5 \text{PN}$ order. The second on the left column is $1 \text{PN}$ while on the right there is the $1.5 \text{PN}$. The third on the left column represents the phase at $2 \text{PN}$. On the right column, the phase at $2.5 \text{PN}$ is shown. Finally, the fourth on left column show the $3 \text{PN}$ and on the right $3.5 \text{PN}$. The inset shows the frequency from 180 to 190 Hz.
Figure 3: Two PN orders for GW151226, 0 (left) and 4 (right), with errors in $\delta \phi_i$ improved (from top to bottom) by factors of 2, 5, and 10. The scales have increased in order to more clearly show the curves.

parameter, that is (for more details see [51]):

$$\gamma - 1 = - \frac{(f'(\varphi))^2}{f(\varphi) + 2[f'(\varphi)]^2} = -2 \frac{\alpha^2}{1 + \alpha^2},$$

(26)

In this case, the delay (25) takes the form

$$\Delta t_{grav} = -(2 - \frac{2\alpha^2}{1 + \alpha^2})/c^3 \int_{r_a}^{r_o} \Phi_N(r)(1 + \alpha e^{-r/r_e})dr,$$

(27)
of which the most important contribution comes from the
We report the frequency dependence of each parameter of Figure 6. in [3], median and 90% credible regions. For each parameter we report the corresponding quantities for the combined signals of GW150914 and GW151226 analyses as in [3, 5].

| waveform regime | parameter $f-$dependence $\Delta$GW150914+GW151226 |
|-----------------|--------------------------------------------------|
| early-inspiral  | $\delta \varphi_0$ $f^{-5/4}$ $-0.05^{+0.08}_{-0.11}$ |
|                 | $\delta \varphi_1$ $f^{-4/3}$ $0.18^{+0.31}_{-0.29}$ |
|                 | $\delta \varphi_2$ $f^{-1}$ $-0.05^{+0.12}_{-0.21}$ |
|                 | $\delta \varphi_3$ $f^{-2/3}$ $0.11^{+0.66}_{-0.18}$ |
|                 | $\delta \varphi_4$ $f^{-1/3}$ $-0.6^{+0.26}_{-0.82}$ |
|                 | $\delta \varphi_{5l}$ log$(f)$ $0.27^{+0.49}_{-0.1}$ |
|                 | $\delta \varphi_{5d}$ $f^{1/3}$ log$(f)$ $-0.38^{+0.99}_{-0.72}$ |
|                 | $\delta \varphi_{6l}$ $f^{2/3}$ $2.66^{+3.53}_{-1.73}$ |

The constraints provided by GW150914 and GW151226 on GR and, in general, metric theories of gravity, are unprecedented due to the nature of the sources and the strong field regime. However they have not reached high enough precision to definitively discriminate among competing theories. Furthermore, in order to extract new physical effects, one would need a wide range of GW waveforms beyond the standard forms adopted for GR and allow for polarisations beyond the standard $+\times$ and $+\times$ modes [53].

Finally, more stringent bounds could be obtained by combining results from multiple GW observations [11, 54–56]. Given the rate of coalescence of binary black holes as inferred in Ref. [57], we are looking forward to the upcoming joint observation surveys from advanced LIGO and VIRGO experiments.

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Moreover, we have used the Shapiro delay of GW150914 to set an upper limit $|\alpha(250Hz) − \alpha(35Hz)| < 1.3 \times 10^{-9}$. Although this result was obtained for scalar tensor theories, this applies for all theories where the PPN-$\gamma$ is at least quadratic in $\alpha$.

It is evident that for $r_1 \gg r_e, r_o$ the value for $\Delta \alpha/\Delta f$ is just half the constraint that can be set on $\Delta \gamma/\Delta f$ using the usual Shapiro delay. Thus with the same assumptions for the potential encountered by the gravitons as [47] (corresponding to a Shapiro delay of 1800 days), we can set the limit $|\alpha(250Hz) − \alpha(35Hz)| < 1.3 \times 10^{-9}$. However, since, as we have demonstrated, extended theories of gravity give rise to an effective gravitational coupling constant $G_{eff}$, the post-Newtonian dynamics of any metric formalism can be obtained straightforwardly for the lowest order deviation parameter $\alpha = const$. The recently detected gravitational waveforms of GW150914 and GW151226 can thus give constraints on the theory.

Using the fact that the gravitational wave frequencies are modulated through $G_{eff}$, we have shown that this modulation will change the phase of the detected gravitational signal. Our conclusions are in agreement with [52] who found that that GW150914 and GW151226 do not
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