Estimator’s Property of Spatially Corrected Blundell-Bond on Dynamic of Spatial Durbin Panel Model Using Monte Carlo Simulation

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Abstract - This research discussed the properties of Spatially Corrected Blundell-Bond (SCBB) in overcoming the problem of endogeneity and spatial dependence that occur in dynamic Spatial Durbin Model (SDM) panels. The properties of the estimator tested were unbiased and normality. The properties test of the estimator was carried out using the Monte Carlo simulation approach. From the results of this research, it finds that the SCBB estimator has unbiased properties and follows a normal distribution. Based on the property of the estimator obtained, the SCBB parameter estimation method in the dynamic SDM panel model works well in overcoming endogeneity and spatial dependence problems.

Keywords: estimator’s property, Spatially Corrected Blundell-Bond, panel dynamics, Monte Carlo simulation

I. INTRODUCTION

In some studies, the response variable is not only determined by the predictor variable at the same time. However, it is also determined by the response variable in the previous time. For example, in the case, it is inflation. In the Phillips Curve theory, it states that one of the factors that influence the magnitude of the inflation rate in a given period is the inflation rate of the previous period (De Pedro, 2014).

In the dynamic model, the lag of the response variable as a predictor causes endogeneity problems. Endogeneity is the occurrence of a correlation between predictor variables and errors so that a method to overcome endogeneity problems is needed. One of them is Blundell and Bond System Generalized Method of Moment (GMM) Estimator, also known as BB-GMM. In overcoming endogeneity, the BB-GMM principle is to combine variable instrumental matrices that come from the moment of sample conditions in the form of first difference and form level. Instrumental variables are formed to overcome the correlation between predictor variables and errors that are the cause of endogeneity (Blundell & Bond, 1998).

In order to obtain more specific information, a study can be carried out in every area in a certain period. The possibility of inter-regional relations is called spatial dependence. Spatial dependence shows the function of the relationship between events at one point in a particular place and those around them (Haining, 1990). To overcome this problem, spatial econometrics is needed.

According to Elhorst (2014), in the spatial econometric model, there are three effects of spatial interactions. Those are the interaction between response variables, interactions between predictor variables, and the interaction effect between errors. Based on the interaction effect, one of the spatial models is Spatial Durbin Model (SDM). SDM is a spatial model that contains the effects of spatial interactions in response variables and predictor variables.

Elhorst (2014) explained that, if the problems faced are dynamic and contain spatial interaction effects, modeling is needed to overcome this problem, namely the dynamic spatial panel model. Dynamic panel spatial model is spatial modeling with lag time from the response variable. Dynamic panel models are characterized by a lag of response variables among predictor variables. Dynamic SDM panel models can be formed by adding lags of response variables in time as predictor variables in ordinary SDM equations. In the economic field, there are several examples of cases that need to be modeled using dynamic SDM panels. It includes economic growth and inflation.

In the dynamic SDM panel model, spatial lag of response variables as predictor variables, and the predictor variables causes the possibility of spatial dependence. To overcome this problem, a method that can overcome the problem of spatial endogeneity and dependence is Spatially Corrected Blundell-Bond (SCBB). This method is an extension of the dynamic BB-GMM panel data method due to spatial linkages in predictor variables and errors (Cizek, Jacobs, Ligthart, & Vrijburg, 2015).

Research on dynamic spatial panel modeling with
the SCBB method has been carried out by Jacobs, Ligthart, and Vrijburg (2009) in Spatial Error Model (SEM). Cizek et al. (2015) also conducted research in estimating SCBB parameters on spatial lag and errors. Research using non-dynamic SDM panels has been carried out. However, in the dynamic SDM panel model, Baltagi and Li (2006) compared non-dynamic SDM and dynamic SDM in the 46 cigarette demand models in the United States. Estimated parameters were carried out by the QML method. Next, Lee and Yu (2010) considered spatial autoregressive panel models with spatial autoregressive disturbances. Lee and Yu (2014) researched the efficiency of GMM estimators of spatial dynamic panel data models with fixed effects. They applied GMM estimation methods on dynamic spatial panel models with multiple spatial lags. Then, Baltagi, Fingleton, and Pirrotte (2014) performed estimation and forecasting parameters using a dynamic spatial panel with the GMM approach. Yu, De Jong, and Lee (2012) performed dynamic panel data models that allow for spatial interactions with QML and GMM.

Su and Yang (2015) examined QML estimation of dynamic panel data models with spatial effects in the errors. Then, Shi and Lee (2017) applied dynamic spatial panel data models with interactive fixed effects using QML. Bai, Zhou, and Fan (2018) compared QML and GMM by using Monte Carlo simulation.

Based on several studies conducted on dynamic spatial panels using SCBB described previously, no one has discussed the nature of the SCBB estimator in the dynamic SDM panel model. A good estimator is an estimator that fulfills the characteristics that are unbiased, consistent, and efficient. The validity of a parameter estimator can be evaluated from the estimator’s bias value. The estimator, having the smallest bias, has the greatest validity.

Besides unbiased characteristics, the properties of normality are also very important for estimators. In statistics, the normality test is used to test whether a random variable is normally distributed or not. However, this required assumption is rarely perfectly fulfilled on real data. One of the frequently found facts is the existence of data with outlier values on the side. It fails to fulfill the assumption of error normality. The normal distribution is needed to use several statistical tools, such as regression analysis, t-test, f-test, or Analysis of Variance (ANOVA). The normality test is needed to answer the question of whether the representative sample requirements are met or not. Thus, the results of the study can be generalized to the population or can represent the population. The consequence of estimator that does not spread normally will cause under a prediction (underestimate) or above a prediction (overestimate) decision of the actual level of the predetermined trial.

Therefore, the researchers discuss the properties of the SCBB estimator on the dynamic SDM panel. It is to overcome the problem of endogeneity and spatial dependence. The purpose of this study is to analyze the properties of estimators. The researchers use simulations to Monte Carlo simulations. Simulation is useful for knowing various estimation characteristics in several different conditions. Monte Carlo simulations are also known as sampling simulation or Monte Carlo sampling technique. This simulation uses existing data (historical data) which is used for other purposes. The researchers look at the estimating properties of the SCBB method. The concept of the Monte Carlo method is random number generation representing the observed uncertainties or risks. In this research, the simulations are carried out. It is because there is no theoretical explanation about how the estimator properties generated by the SCBB estimator on the dynamic panel SDM.

II. METHODS

The data to analyze the property of SCBB estimator method is about the inflation rate in Indonesia from the second quarter of 2013 to the fourth quarter of 2017 in 34 provinces in Indonesia. Inflation data is used because it is one of the dynamic economic problems. inflation is currently influenced by inflation in the previous period (lag inflation), it is can cause endogeneity problems (Blundell & Bond, 1998). Besides, inflation observed in several regions will lead to spatial dependence (Elhorst, 2014).

There are several predictor variables in this study. First, it is economic growth (X1). Data on economic growth is obtained from Product (PDRB). Constant Domestic Regional Bruto (DRB) is used to determine real economic growth that is not influenced by price factors. In this study, the PDRB is based on constant prices in percent.

Second, it is Sertifikat Bank Indonesia (SBI) or interest rates (X2). The nominal interest rate is the benchmark interest rate (BI rate). The interest rate is the real interest rate obtained by the Fisher equation. The expected inflation rate reduces the nominal interest rate according to the equation of \( r = i - \pi \) or \( i = r + \pi \). The \( r \) is the real interest rate, \( i \) is the nominal interest rate, and \( \pi \) is the inflation rate. The unit is in the form of a percent.

Third, it is the money supply (X3). The amount of money in circulation used is the money supply in the narrow sense (M1). It consists of currency and demand deposits.

Fourth, it is administered prices index (X4). This index comes from the price of goods or services regulated by the government. Those are fuel prices, basic electricity, and water rates. This index describes the average price changes of fuel oil, basic electricity, and water consumed by households in a certain period.

Meanwhile, the response variable data is the inflation rate (Y). Then, the analysis is done using R software version 3.4.1. Because the SCBB package is not yet available, coding is done manually based on the parameter estimation steps and the formula of the SCBB method.

III. RESULTS AND DISCUSSIONS

The first step taken in carrying out the analysis in this study is to form a spatial weighting matrix. The spatial weight matrix (W) is an important part of modeling involving spatial data. The \( W_{ij} \) of the matrix for \( i=1,\ldots,N \) and \( j=1,\ldots,N \) is positive if locations \( i \) and \( j \) are considered as neighbors and zero otherwise. The spatial weight matrix is a non-negative size of \( N \times N \) matrix that presents a set of relationships between units of spatial observations. The spatial weight matrix used is inverse distance. The spatial weight matrix by inverse distance method is based on the actual distance between locations in the field. Nearby locations get a greater weight value. The inverse distance is in Equation (1).

\[
W_{ij} = \begin{cases}
\frac{1}{d_{ij}} & \text{if } i \neq j \\
0 & \text{if } i = j
\end{cases}
\]  

(1)
The measure of distance used in determining the weight size and the coordinates of the cartesian location are Euclidean distance (Brunsdon, Fotheringham, & Charlton, 1996). Euclidean distance can be obtained by Equation (2).

$$d_{ij} = \sqrt{(u_i - u_j)^2 + (v_i - v_j)^2}$$ (2)

It shows that $d_{ij}$ is a euclidean distance between $i$-th location and $j$-th location of the $u_i$ observation. Then, $u_i$ is a latitude coordinates at location $i$, is a latitude coordinates at location $j$, and $v_i$ is a longitude coordinate at location $i$. Last, $u_j$ is a longitude coordinate at location $j$.

The calculation of weight with inverse distance method is obtained from inverse distance results. Then, it is normalized. Row normalization is shown in Equation (3).

$$w_{ij} = \frac{w_{ij}}{\sum w_{ij}}$$ (3)

After the spatial weight matrix has been obtained, a spatial dependence test is carried out. Spatial dependence shows the function of the relationship between variables at one point in a particular location. It is with the same variable in the surrounding location, which causes non-autocorrelation assumptions not to be met. The test of spatial dependence test is carried out. Spatial dependence is normalized. Row normalization is shown in Equation (3).

$$I_t = n \sum_{i=1}^{n} \sum_{j=1}^{n} w_{ij} (x_{it} - \bar{x}) (x_{jt} - \bar{x})$$ (4)

It shows that $n$ is several observation locations. Next, $x_{i}$ is an observation value of location $i$ ($i = 1, 2, ..., n$) at time $t$, $x_{j}$ is an observation value of location $j$ ($j = 1, 2, ..., n$) at time $t$, and $\bar{x}$ is an average observation. The test statistics uses Equation (5) and variance in Equation (6).

$$Z(I) = \frac{I - E(I)}{\sqrt{\text{Var}(I)}}$$

$$\text{Var}(I) = \frac{n \sum_{i=1}^{n} \sum_{j=1}^{n} w_{ij}^2 (x_{it} - \bar{x})^2 + 2(\sum_{i=1}^{n} \sum_{j=1}^{n} w_{ij} x_{it} - \bar{x})^2 - n \sum_{i=1}^{n} (x_{it} - \bar{x})^2}{n^2 (\sum_{i=1}^{n} \sum_{j=1}^{n} w_{ij})^2}$$ (5-6)

The range of the values from the Moran’s I equation test statistic is -1 to 1. If the value of Moran’s I equation is in the range of -1 ≤ I ≤ 0, there is negative spatial autocorrelation. Then, if the value of Moran’s I equation is in the range of 0 < I ≤ 1, there is positive spatial autocorrelation. If the value is I = 0, there is no spatial autocorrelation.

After testing the spatial dependence, a spatial model is formed. The model used in this study is the dynamic SDM panel model. According to Elhorst (2014), the dynamic SDM panel model is by Equation (7).

$$Y_t = \beta Y_{t-1} + \rho W Y_t + \delta W Y_{t-1} + \beta X_t + \lambda W X_t + u_t$$ (7)

The $Y_t$ is an $N \times 1$ vector of response variable of each spatial unit – location ($i = 1, ..., N$) at time $t$, $Y_{t-1}$ is an $N \times 1$ vector of response variable for every location ($i = 1, ..., N$) at time $t - 1$, $X_t$ is an $N \times m$ matrix of $m$ predictor variables for every location ($i = 1, ..., N$) at time $t$ ($t = 1, ..., T$), $W$ is an $N \times N$ matrix of spatial weight matrix. Transformation of a dynamic SDM panel model on the Equation (4) is in Equation (8).

$$Y_t = [I - \rho W]^{-1} [\tau Y_{t-1} + \delta W Y_{t-1} + \beta X_t + \lambda W X_t + u_t]$$ (8)

Based on the Equation (7) by substituting variables, the dynamic SDM panel can be defined in Equation (9).

$$Y_t = \tau y_{1,t-1} + \rho W y_{1,t} + \delta W y_{1,t-1} + \beta_1 x_{1,t} + \beta_2 x_{1,t} + \beta_3 x_{1,t} + \beta_4 W x_{1,t} + \lambda_1 W x_{1,t} + \lambda_2 W x_{1,t} + \lambda_3 W x_{1,t} + \lambda_4 W x_{1,t} + u_t$$ (9)

After the dynamic SDM panel model is formed, the parameter estimation is performed using the SCBB parameter estimation method. It is to overcome the endogeneity and spatial dependence problems. The SCBB estimator method is an extension of the dynamic BB-GMM panel data method due to spatial linkages in predictor variables and errors. According to Jacobs et al. (2009), the SCBB estimation method can be derived through Equation (10). Then, it can briefly be written as Equation (11). The $y_{1,t}$ is a $2N \times 1$ vector of the response variable.

$$\begin{bmatrix} Y_{1,t} \\ \Delta Y_{1,t} \end{bmatrix} = \begin{bmatrix} \Delta z_{1,t} \\ \Delta u_{1,t} \end{bmatrix} \theta + \begin{bmatrix} u_{1,t} \\ \Delta u_{1,t} \end{bmatrix}$$ (10)

$$y_{1,t} = \Delta z_{1,t} + \Delta u_{1,t}$$ (11)

The number of observations on the SCBB model is $2N (T-1)$ which will increase the efficiency of the estimation.

The number of observations on the SCBB model is $2N(T-1)$. It will increase the efficiency of the estimation. In the SCBB method, an instrument matrix is needed consisting of matrix instruments in the first difference equation and instrument variable matrix in the level equation. It is to overcome endogeneity. The instrument variable is said to be valid if it does not correlate with $\Delta u_t$ and correlates with the variable $\Delta Y_{t-1}$.

The instrument matrix in the first difference (HD) equations is based on the moment of Equation (12).

$$E[y_{1,t} \Delta u_{1,t}] = 0; E[(W y_{1,t} \Delta u_{1,t}) \Delta u_{1,t}] = 0;$$

$$E[H_{1,t} \Delta u_{1,t}] = 0$$

With $H_{1,t} = [W y_{1,t}] [W y_{1,t}] W x_0 x_0$. (12)

Suppose T is 5, the instrument matrix in the first difference ($H_{1,t}$) equation is Equation (13).

$$H_{1,t} = \begin{bmatrix} y_{1,1} & 0 & 0 & 0 & 0 & H_{1,3} - H_{1,2} \\ 0 & y_{1,1} & 0 & 0 & 0 & H_{1,4} - H_{1,3} \\ 0 & 0 & \ddots & \ddots & \ddots & \ddots \\ 0 & 0 & \ddots & \ddots & 0 & H_{1,T} - H_{1,T-1} \end{bmatrix}$$ (13)
The instrument matrix on the equation level ($H_i$) is based on Equation (14).

$$E[u'_o \Delta y_o] = 0; E[u'_o \Delta W_y] = 0;$$
$$E[u'_o \Delta H_o] = 0$$  \hspace{1cm} (14)

The instrument-level model matrix ($H_i$) is in Equation (15).

$$H_i = \begin{bmatrix}
\Delta y_{1,2} & 0 & 0 & \ldots & 0 & h_{1,3} \\
0 & \Delta y_{1,3} & 0 & \ldots & 0 & h_{1,4} \\
\vdots & \vdots & \ddots & \vdots & \vdots & \vdots \\
0 & 0 & 0 & \ldots & \Delta y_{t-1} & h_{t-1} \\
0 & 0 & 0 & \ldots & 0 & h_{t} 
\end{bmatrix}$$  \hspace{1cm} (15)

The instrument variabole matrix in SCBB ($H_{SBB}$) is a combination of $H_o$ dan $H_i$ matrix in Equation (16).

$$H_{SBB} = \begin{bmatrix}
H_o & 0 \\
0 & H_i 
\end{bmatrix}$$  \hspace{1cm} (16)

Based on instrument variables used in the instrument variable matrix, a number of instrument variables in SCBB is $(T-1)(T-2)$. After obtaining the instrument variable matrix, the estimation method by SCBB uses the GMM principle to obtain consistent expectations. Under the assumption of instrument variables, $\theta$ vector is a unique solution for the moment of the condition of the population in Equation (17). It corresponds with the moment of sample conditions in Equation (18).

$$E(g(\theta)) = E(H_{SBB} u_{BB}) = E(H_{SBB} (y_{BB} - z_{BB} \theta)) = 0$$  \hspace{1cm} (17)

$$\hat{g}(\hat{\theta}) = N^{-1} \sum_{i=1}^{N} H_{SBB} (y_{BB} - z_{BB} \hat{\theta})$$  \hspace{1cm} (18)

Using the GMM principle, the SCBB estimator can be obtained by minimizing the weighted squared number from the sample condition. Based on the approach of Kelejian and Prucha (2010), the one-step estimator of SCBB is in Equation (19).

$$\hat{\theta} = [z'_{BB} H_{SBB} A_{BB} H'_{SBB}]^{-1} = [z'_{BB} H_{SBB} \hat{A}_{SBB} H'_{SBB} y_{BB}]$$  \hspace{1cm} (19)

In the BB-GMM principle, at this stage of selecting $\hat{A}_{SBB}$, it will not affect the consistency of the predicted results. However, by choosing the optimum weight of $\hat{A}_{SBB}$ it will be estimated efficiently. To obtain optimal weight, the BB-GMM principle is used to adapt $\hat{\theta}$. It is obtained in the first stage by using Equation (20).

$$\hat{A}_{SBB} = \hat{A}_{SBB}^{-1} = \left(N^{-1} \sum_{i=1}^{N} (H'_{SBB} \hat{u}_i u'(H_{SBB}))\right)^{-1}$$  \hspace{1cm} (20)

By substituting the optimal weight $\hat{A}_{SBB}$ SSBB, it obtains Equation (21).

$$\hat{\theta} = [z'_{BB} H_{SBB} \hat{A}_{SBB} H'_{SBB} y_{BB}]^{-1} [z'_{BB} H_{SBB} \hat{A}_{SBB} H'_{SBB} y_{BB}]$$  \hspace{1cm} (21)

The $z_{BB}$ is an matrix predictor variable. Then, $y_{BB}$ matrix response variable is an $H_{SBB}$ It is an instrument variable matrix of SCBB. The $\hat{A}_{SBB}$ optimal weighting of SCBB is under the BB-GMM principle.

After the parameters are obtained, the validity of an instrument variable is tested from the model obtained. To find out whether the instrument used is valid, the researchers do a test. Sargan test is used to determine the validity of the instrument variables. Its numbers exceed the estimated number of parameters (overidentifying conditions). Instrument variables are said to be valid if they are not correlated with errors. The Sargan test hypothesis is as follows.

$$H_0: E(H_{SBB}' \hat{u}) = 0$$ \hspace{1cm} (valid instrument)
$$H_1: E(H_{SBB}' \hat{u}) \neq 0$$ \hspace{1cm} (valid instrument)

The test statistics is in Equation (22).

$$S = (\hat{u}' H_{SBB}) \hat{A} (H_{SBB} \hat{u}) \sim \chi^2_{m}$$  \hspace{1cm} (22)

Then, $H_{SBB}$ is a matrix of instrument variable. Next, $\hat{u}$ is error component of the expected result, $l$ is a number of instruments, and $m$ is the number of parameters estimated.

The S test statistic is asymptotically distributed by $\chi^2$ with a degree of freedom of the number of instruments minus the number of parameters used in the model (q). The test criteria are that $H_0$ is accepted if the value is $S \leq \chi^2_q$. It means that the instrument is valid. Instead, $H_1$ is rejected if the value is $S > \chi^2_q$. It means that the instrument is invalid.

Besides using the S value, the test criteria can also be done by looking at the p-value. The $H_1$ is accepted if it is $p$-value > $a$. It is rejected if it is $p$-value ≤ $a$. If $H_1$ is rejected, it is necessary to add instrument variables by making pair combinations with other predictor variables. Those are previously assumed to be strictly exogenous.

After obtained a valid instrument variable, the further analysis of the estimator’s property generated from the SCBB method is carried out. Estimator’s property of SCBB on SDM performed with Monte Carlo Simulation. Monte Carlo simulation method is a numerical computation method by involving the sampling with random numbers. The key to the Monte Carlo method lies in the generation of random numbers. It represents the observed uncertainties or risks.

According to Rubinstein and Kroese (2016), there are several steps of Monte Carlo simulation. First, it determines the value of the initial parameter as a fixed parameter. The parameter value is obtained from estimating parameters from the initial data using SCBB. Historical data from predictor variables are fixed data. It will be used in the estimation of the next parameter. The data generation process is carried out at the error as an input for further parameter estimation.
Second, it calculates the value of using the equation based on the parameters obtained in the first step and saves the value. Third, it generates the error values using a normal distribution. It is an error without being influenced by a time factor written with a time factor with \( v_i \sim \mathcal{N}(0, \sigma^2) \). It is also an individual-influenced error and time written with \( \varepsilon_{it} \sim \mathcal{N}(0, \sigma^2) \).

Fourth, it calculates the value of \( y_{ij} \) by substituting the error value generated in the third step and the fixed parameter and fixed predictor variable in the first step. Fifth, it estimates the parameters using the input value of \( y_{ij} \). It is obtained in this step and the fixed predictor variable in the first step. Sixth, it repeats the third to the sixth step around 500 times. Thus, 500 parameters are obtained. Seventh, it analyzes the nature of the unbiased estimator. It is based on the results of the simulation by calculating the average value of the parameters. Those are assumed based on the number of replications. To see the normality properties, the Anderson-Darling test, and the histogram of each parameter of each replication are used.

An estimator is said to be unbiased if the estimator approaches the actual value of the expected parameter. In this study, the parameter that will be assumed is \( \theta \). Then, the parameter estimator is said to be unbiased if \( E(\hat{\theta}) = \theta \). \( E(\hat{\theta}) \) is the average value of the number of replications in the simulation. It uses Equation (23).

\[
\hat{\theta} = \frac{1}{n_{iter}} \sum_{k=1}^{n_{iter}} \hat{\theta}_k
\]  

(23)

Besides unbiased characteristics, the nature of the estimator tested is normality. Normality test estimator is very important to test the significance of parameters. Testing the significance of the parameters is done based on the distribution of the estimator. Violation of normal assumptions will result in parameter estimators becoming biased. One normal test is to use Anderson-Darling test statistics with the hypothesis as follows.

\( H_0: \hat{\theta}_k \) is normally distributed
\( H_1: \hat{\theta}_k \) is not normally distributed

Anderson-Darling test statistics are in Equation (24).

\[
A^2 = - n - S
\]  

(24)

Where it is:

\[
S = \frac{1}{n} \sum_{i=1}^{n} (2i - 1) \log \frac{\phi(Z_i)}{\Phi(Z_n)} + \log(1 - \Phi(Z_{n+1-i}))
\]  

(25)

It shows that \( F(Z) \) is the normal standard cumulative distribution function of i. The \( H_0 \) is accepted if the value is \( A^2 \leq A^2_{table} \) or the p-value > \( \alpha(0,05) \). It means that the estimator is normally distributed. Before doing spatial modeling, the researchers test the spatial dependence. Testing spatial dependence uses the Moran Index test with the hypothesis as follows.

\( H_0: \tau = 0 \) (no spatial autocorrelation)
\( H_1: \tau \neq 0 \) (there is spatial autocorrelation)

The simulation is done by generating an error value without being influenced by a time factor is written with \( v_i \sim \mathcal{N}(0, \sigma^2) \). Then, an unknown error from every individual \( (i=1,2,...,n) \) at time \( t \) is with \( \varepsilon_{it} \sim \mathcal{N}(0, \sigma^2) \). The simulation results obtain parameters of \( \tau, \rho, \delta, \beta_1, \beta_2, \beta_3, \beta_4, \lambda_1, \lambda_2, \lambda_3, \lambda_4 \), as 500, respectively.

In the dynamic SDM parameter values, the simulation results are carried out an average calculation. It is to see the nature of the irregularities and normality testing of the SCBB estimator based on the normality test along with the histogram formed. In this discussion, the bias values of each parameter will be presented based on the average value of estimate parameter result using the SCBB parameter estimation method with Monte Carlo simulation. It is done with 500 replications. The average value of the parameters is used to see the nature of the irregularities of SCBB estimators. The estimator is said to be unbiased if the average or expected value of the estimator is the same as the value of the parameter. A good estimator has a very small bias value. The bias value is obtained from the difference between the estimated value of the parameter and the actual parameter value. The bias values of each parameter are presented in Table 2.

Table 1 shows the results of the calculation of the Moran index using inverse weighting distance. The results of testing spatial dependence show that there is spatial dependence in this study. It can be seen from significant at \( \alpha = 5\% \) for the entire study period. After showing there is spatial dependence, the researchers conduct dynamic panel spatial regression analysis using the SCBB parameter estimation method.

The simulation is done by generating an error value using a normal distribution. An error from the individual \( i \) without being influenced by a time factor is written with \( v_i \sim \mathcal{N}(0, \sigma^2) \). Then, an unknown error from every individual \( (i=1,2,...,n) \) at time \( t \) is with \( \varepsilon_{it} \sim \mathcal{N}(0, \sigma^2) \). The simulation results obtain parameters of \( \tau, \rho, \delta, \beta_1, \beta_2, \beta_3, \beta_4, \lambda_1, \lambda_2, \lambda_3, \lambda_4 \), as 500, respectively.

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In Table 2, it can be seen that the parameter values generated using the SCBB parameter estimation method in the dynamic SDM model produce a very small bias value. It is even close to zero. This shows that the estimator of the SCBB parameter satisfies one of the properties of the

| Period  | I   | Z(I)   | P-Value |
|---------|-----|--------|---------|
| 2013q3  | 0,1275 | 4,5987 | 0,0000  |
| 2013q4  | 0,1177 | 4,7320 | 0,0000  |
| 2014q1  | 0,1142 | 4,2498 | 0,0000  |
| 2014q2  | 0,1104 | 4,1133 | 0,0000  |
| 2014q3  | 0,0981 | 3,7645 | 0,0001  |
| 2014q4  | 0,1078 | 4,0657 | 0,0000  |
| 2015q1  | 0,0975 | 3,7348 | 0,0002  |
| 2015q2  | 0,0965 | 3,7204 | 0,0001  |
| 2015q3  | 0,0904 | 3,5396 | 0,0004  |
| 2015q4  | 0,0847 | 3,3639 | 0,0007  |
| 2016q1  | 0,0768 | 3,1240 | 0,0018  |
| 2016q2  | 0,0838 | 3,3330 | 0,0008  |
| 2016q3  | 0,0844 | 3,3923 | 0,0006  |
| 2016q4  | 0,0890 | 3,5546 | 0,0003  |
| 2017q1  | 0,0926 | 3,6751 | 0,0002  |
| 2017q2  | 0,0802 | 3,3281 | 0,0008  |
| 2017q3  | 0,0625 | 2,8244 | 0,0047  |
| 2017q4  | 0,0572 | 2,6940 | 0,0071  |

Table 1 Spatial Dependence
predictor’s goodness which is not biased. This means that
the SCBB parameter estimation method works well in
the dynamic SDM panel model. Furthermore, the nature
of the estimator discussed is the nature of normality. The
test results using the Anderson Darling normality test are
presented in Table 3.

Table 2 Parameter of Bias Value

| Parameters | Parameter fixed | Average parameters of simulation results | Bias Value |
|------------|----------------|-----------------------------------------|------------|
| τ          | -0.009216      | -0.009205                               | 1.12E-05   |
| ρ          | 0.999837       | 0.986808                                | 1.16E-03   |
| δ          | 0.008684       | 0.008674                                | 1.05E-05   |
| β₁         | 5.28e-10       | 5.28e-10                                | 6.65E-13   |
| β₂         | -1.003093      | -1.001751                               | 1.34E-03   |
| β₃         | -3.18e-11      | -3.18e-11                               | 3.99E-14   |
| β₄         | 0.000362       | 0.000362                                | 4.89E-07   |
| λ₁         | -2.12e-10      | -2.12e-10                               | 2.84E-13   |
| λ₂         | 1.003999       | 1.002643                                | 1.36E-03   |
| λ₃         | -1.95e-10      | -1.95e-10                               | 2.61E-13   |
| λ₄         | -0.000356      | -0.000356                               | 4.82E-07   |

Table 3 The Results of Anderson Darling Test Statistics

| parameters | Anderson Darling Test Statistics | p-value | Decision |
|------------|----------------------------------|---------|----------|
| τ          | 0.64081                          | 0.09418 | Unbiased |
| ρ          | 0.65452                          | 0.08711 | Unbiased |
| δ          | 0.65457                          | 0.08708 | Unbiased |
| β₁         | 0.53600                          | 0.16910 | Unbiased |
| β₂         | 0.67043                          | 0.07956 | Unbiased |
| β₃         | 0.51126                          | 0.19490 | Unbiased |
| β₄         | 0.62470                          | 0.10320 | Unbiased |
| λ₁         | 0.64954                          | 0.08961 | Unbiased |
| λ₂         | 0.62470                          | 0.10320 | Unbiased |
| λ₃         | 0.64986                          | 0.08945 | Unbiased |
| λ₄         | 0.62568                          | 0.10270 | Unbiased |

Based on the Anderson Darling test in Table 3, it
can be seen that all parameters generated using the SCBB
parameter estimation method in the dynamic panel SDM
model have a p-value > α(0.05). With a real level of 5%, it
can prove that SCBB estimators are normally distributed.
Normality of the estimator properties can also be seen based
on the histogram of each parameter. The histogram of each
parameter is presented in Figures 1-11.
Figure 4 Histogram of Beta1 Parameters

Figure 5 Histogram of Beta2 Parameters

Figure 6 Histogram of Beta3 Parameters

Figure 7 Histogram of Beta4 Parameters
Based on the histogram in Figure 1 to Figure 11, it can be seen that all parameter values of $\tau$, $\rho$, $\delta$, $\beta_1$, $\beta_2$, $\beta_3$, $\beta_4$, $\lambda_1$, $\lambda_2$, $\lambda_3$, $\lambda_4$ from monte carlo simulation results with 500 replications show normal distribution respectively. This is consistent with Anderson Darling’s test that the parameter values of the SCBB estimator following the normal distribution. The results of this study are in accordance with research conducted by Cizek et al. (2015) that the estimator is normal and Kukenova and Monteiro (2009) that the estimator has a small bias. The distribution center for the parameters $\tau$, $\rho$, $\delta$, $\beta_1$, $\beta_2$, $\beta_3$, $\beta_4$, $\lambda_1$, $\lambda_2$, $\lambda_3$, $\lambda_4$ is around the value of -0.009205; 0.986808; 0.008674; 5.28e-10; -1.001751; -3.18e-11; 0.000362; -2.12e-10; 1.002643; -1.95e-10; and -0.000356 respectively.

**IV. CONCLUSIONS**

The SCBB estimator on the dynamic SDM panel satisfies the properties of the estimator’s goodness. It is unbiased and has normal distribution. The bias value generated in each parameter is very small and close to zero. This means that SCBB estimators work well in overcoming problems with endogeneity and spatial dependence.

The researchers conclude that in the cases containing endogeneity and spatial dependence, the SCBB method can be used to handle it. In this research, the properties of other estimators have not been analyzed of such as efficiency and consistency. For further research, it is recommended to analyze the properties of other estimators.
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