Adaptive Detection of Dim Maneuvering Targets in Adjacent Range Cells

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Abstract—This letter addresses the detection problem of dim maneuvering targets in the presence of range cell migration. Specifically, it is assumed that the moving target can appear in more than one range cell within the transmitted pulse train. Then, the Bayesian information criterion and the generalized likelihood ratio test design procedure are jointly exploited to come up with six adaptive decision schemes capable of estimating the range indices related to the target migration. The computational complexity of the proposed detectors is also studied and suitably reduced. Simulation results show the effectiveness of the newly proposed solutions also for a limited set of training data and in comparison with suitable counterparts.

IndexTerms—Adaptive detection, dim maneuvering targets, range cell migration, radar, sonar, Model Order Selection rules, generalized likelihood ratio test.

I. INTRODUCTION

A DAPTIVE detection is a task of primary concern in radar and sonar systems [1], [2]. As a matter of fact, in the last decades, a large number of architectures have been developed for the detection of target echoes competing against noise and clutter interference by means of array of sensors. The common aspect for most of these contributions is the assumption that the target is point-like and located in the cell under test (CUT) only at a given range.

However, there exist at least three cases where the above assumption may be no longer valid. Specifically, the first situation concerns high-resolution radars [3] and sonars [4] which can resolve a target into several scattering centers occupying several consecutive range cells. In fact, a large amount of detection algorithms for range-spread target can be found in the open literature (see [3]–[7] and references therein).

The second case is related to the spillover of target energy between consecutive matched filter samples which makes a point-like target extended in range [8] yielding a detection performance degradation when only one sample is processed. In the seminal paper [9], the authors propose a detection architecture that jointly processes adjacent range cells to take advantage of the spillover limiting the aforementioned degradation.

This work was in part supported by the National Natural Science Foundation of China under Grant 61971412.

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The third situation arises from the need of increasing the signal-to-interference-plus-noise ratio (SINR) in the case of dim targets to guarantee reliable detection performance and high-quality target parameter estimates. To this end, radar systems transmit long bursts of pulses and integrate the corresponding backscattered energy. However, dim maneuvering targets can move through more than one range cell within the integration time interval [10]. As a consequence, it prevents conventional decision schemes from exploiting all the backscattered energy, since they are fed by the range bin under test only and, hence, do not account for the target migration to the contiguous range bin. Therefore, methods to cope with range cell migration (RCM) become of primary importance. A widely used tool for RCM compensation is the Keystone transform which has been applied in several fields as, for instance, radar detection [10] to mitigate target RCM due to radial velocity and acceleration, synthetic aperture radar imaging [11], [12] where the RCM is caused by linear range walk and range curvature. In [13], an alternative method relying on adjacent correlation function and Lv’s transform is devised to detect the maneuvering targets with radial jerk motion. More recently, in [14], innovative one-step and two-step detection architectures are conceived for dim maneuvering targets with and without estimating the slow-time index of the target signal in the CUT and based upon the generalized information criterion [15]. Remarkably, such architectures can overcome conventional detectors as the generalized adaptive matched filter (GAMF) [16] at the price of an increased computational complexity.

In this letter, we focus on the detection of dim maneuvering targets in the presence of RCM and further improve the results of [14] by devising innovative robust (with respect to the amount of training samples) architectures. To this end, we do not consider any possible phase/amplitude relationships between consecutive pulses and exploit, at the design stage, the Bayesian information criterion (BIC) rule [15], which is an asymptotic approximation of the optimal maximum a posteriori rule, to identify the pulse echoes containing target components over two consecutive range cells. Then, we conceive two-step architectures (TSA) and one-step architectures (OSA) relying on GLRT-based design criteria, where GLRT stands for generalized likelihood ratio test. The contributions of the present letter can be summarized as follows: 1) unlike [14], all the samples from two consecutive range cells occupied by the target are processed to increase the detection performance; 2) the samples free of signal components are exploited for the estimation of the interference covariance.
matrix (ICM) lending new architectures a robustness to the training set size; 3) the proposed architectures are designed to avoid a continuous computation of inverse matrices saving computational resources.

The letter is organized as follows: Section II contains the system model and the problem formulation. In Section III, TSAs and OSAs are devised including suitable modifications of them. Section IV is devoted to the numerical analysis and discussion. Finally, Section V concludes this letter outlining future research tracks.

II. SYSTEM MODEL AND PROBLEM STATEMENT

Let us consider a (radar or sonar) system equipped with a linear array of $N_a$ identical and uniformly distributed sensors (the inter-element spacing $d$ is half of the operating wavelength, $\lambda$ say, to avoid spatial aliasing). Moreover, denote by $N_p$ pulses belonging to the transmitted pulse train. Then, for a point-like target, the signal received by the $n$th antenna element can be written as

$$x_m(t) = R\left\{ \alpha \sum_{n=0}^{N_p-1} p(t - nT - \tau_0 + \frac{2\nu_n}{c} nT) \times e^{j2\pi(f_c + f_d)T} e^{j2\pi(m-1)\nu_n} \right\}, \quad (1)$$

where $\alpha \in \mathbb{C}$ accounts for target and channel effects, $T > 0$ is the pulse repetition time (PRT), $p(t)$ is an unit-energy pulse waveform, $\tau_0$ is the round-trip delay of the target, $v_t$ is the target radial velocity, $c$ is the waveform velocity of propagation, $f_c$ is the carrier frequency, $f_d$ is the target Doppler frequency, and $\nu_n = \frac{dn}{c} \tau$ is the spatial frequency with $\psi$ the nominal target angle of arrival (AOA). After matched filtering and digital sampling, for the $g$th range cell (fast time) which is the target location at $t = 0$ we obtain the data sequence as

$$y_n(g, g) = \alpha e^{j2\pi f_d T_g (g-1) X_p (\tau_0 - (g-1) T_p)} + (g-1) \frac{2\nu_n}{c} T_g e^{j2\pi(m-1)\nu_n} \triangleq \alpha(g, g) e^{j2\pi(m-1)\nu_n}, \quad (2)$$

where $g \in \Omega_p = \{1, \ldots, N_p\}$ indexes the slow time, $(g-1) \frac{2\nu_n}{c} T_g$ accounts for the range migration, $X_p (\cdot, \cdot)$ is the ambiguity function of $p(t)$, $T_g$ is the one-sided mainlobe width of the zero-Doppler cut of $X_p$, and $\alpha(g, g) = \alpha e^{j2\pi f_d T_g (g-1) X_p (\tau_0 - (g-1) T_p)} + (g-1) \frac{2\nu_n}{c} T_g f_d$. Equation (2) highlights that, in the case of maneuvering targets and for large values of $g$ (and, hence, of $N_p$), target response (ambiguity function value) associated with the $g$th range bin can decrease to zero implying that the mainlobe of the ambiguity function has migrated to the next contiguous range bin, namely the RCM has occurred. In what follows, for simplicity and without loss of generality, we set $g = 1$ and, hence, the next contiguous range bin is indexed by $g = 2$. Now, let us define by $Z_i = [z_{1,i}, z_{2,i}, \ldots, z_{N_a,i}] \in \mathbb{C}^{N_a \times N_p}$, $i = 1, 2$, the data matrix corresponding to the $i$th range bin whose columns contain the returns from the $N_a$ spatial channels. Then, if we assume that the $\tilde{g}$th echo from range bin $1$ contains target components and that the same echo from range bin $2$ is representative of interference only, we can write $z_{1,\tilde{g}} = [y_{1,1}, \ldots, y_{N_a,1}]^T + n_{1,\tilde{g}} \triangleq \alpha(1, \tilde{g}) v + n_{1,\tilde{g}}$ and $z_{2,\tilde{g}} = n_{2,\tilde{g}}$, where $v = [1, e^{j2\pi \nu_1}, \ldots, e^{j2\pi(N_a-1)\nu_1}]^T \in \mathbb{C}^{N_a \times 1}$ is the nominal spatial steering vector depending on $\psi$ and the $n_{i,\tilde{g}}$’s are the interference components. When the RCM occurs for some pulse index $\tilde{g}$, previous situation changes, namely $z_{1,\tilde{g}} = n_{1,\tilde{g}}$ is representative of interference only whereas $z_{2,\tilde{g}} = \alpha(2, \tilde{g}) v + n_{2,\tilde{g}}$ also contains target components. A pictorial description of the RCM is shown in Fig. 1 where the blue squares denote data with target components and white squares denote data free of useful signal echoes. In the first $l$ pulses, the target is in the first cell, then, it moves to the second range cell.

Therefore, in order to account for possible range migration, it is reasonable to process the returns associated with (at least) two consecutive range cells. Summarizing, the detection problem at hand can be formulated in terms of the following multiple hypothesis test

$$H_{l,h} : \begin{cases} z_{1,l+1} = n_{1,l+1}, \ldots, z_{1,N_p} = n_{1,N_p}, \\ z_{2,1} = n_{2,1}, \ldots, z_{2,l} = n_{2,l} \\ z_{2,l+1} = n_{2,l+1}, \ldots, \\ z_{2,N_p} = n_{2,N_p}, \\ z_{1,1,\tilde{g}} = \alpha(1, \tilde{g}) v + n_{1,\tilde{g}} \\ z_{2,1,\tilde{g}} = \alpha(2, \tilde{g}) v + n_{2,\tilde{g}} \\ z_{2,l+1,\tilde{g}} = \alpha(2, l+1) v + n_{2,l+1,\tilde{g}} \\ z_{2,l+h,\tilde{g}} = \alpha(2, l+h) v + n_{2,l+h,\tilde{g}} \\ r_k = m_k, k = 1, \ldots, K, \\ z_{1,1} = n_{1,1}, \ldots, z_{1,N_p} = n_{1,N_p} \\ z_{2,1} = n_{2,1}, \ldots, z_{2,N_p} = n_{2,N_p}, \\ r_k = m_k, k = 1, \ldots, K, \\ \end{cases} \quad (3)$$

where $R = [r_1, \ldots, r_K]$ are the training data, $1 \leq l \leq N_p$, $0 \leq h \leq N_p - l$ are unknown integers indexing which vectors contain target components, $n_{1,i}, n_{2,i}, m_i \sim \mathcal{C}N_a(0, M)$ are statistically independent interference vectors. As for $\alpha(1, i), i = 1, \ldots, l$ and $\alpha(2, i), i = l+1, \ldots, l+h$, they are modeled according to the Swerling II model [17]. Finally, note that when $H_{l,h}$ is declared, the nominal target AOA $\psi$ can be used as a preliminary estimate of the actual target AOA.

III. DESIGN ISSUES

In this section, we devise two classes of architectures for problem (3). The first class pursues a natural approach which consists in estimating the pulse indices corresponding to the range transition (Subsection III.A) and then in applying decision schemes based upon such estimates (Subsection III.B). It follows that such architectures consist of two stages (TSA): the first stage solves the RCM problem whereas the second stage is responsible for target detection. The second approach (OSA)}
jointly performs the above operations using a penalized GLRT-based decision scheme [18] (Subsection III.C). Even though from a conceptual point of view these approaches share the same operations, from an operating point of view they can lead to different performance as shown in Section IV.

A. First Stage of TSA: RCM Estimation

The preliminary stage of the TSAs is aimed at estimating parameters $l$ and $h$ using two BIC-based selection rules. More precisely, the first rule is devised according to the two-step design paradigm that consists in applying well-established parameters and, hence, the inversion of $S_{l,h}$ for each admissible pair $(l,h)$. To reduce the computational load of (7), we replace $S_{l,h}$ with $S$, which does not require to be updated. The reduced-complexity BIC rule is given by

$$
\min_{l \in \Omega_l} \min_{h : l + h \leq N_p} \left\{ -2 \ln \left( f_l(h, Z; \alpha_l, h, M, \hat{M}) + p_l(l, h) \right) \right\},
$$

where $Z = [Z_1, Z_2] \in \mathbb{C}^{N_h \times 2N_p}$, $\alpha_l, h, M \triangleq [\alpha_l M^{-1} z_{l+1} \ldots \alpha_l M^{-1} z_1 \ldots \alpha_l M^{-1} z_{2+h} \ldots \alpha_l M^{-1} z_{l+h}]^T \in \mathbb{C}^{(l+h) \times 1}$ is the maximum likelihood estimate (MLE) of $\alpha = [\alpha(1,1) \ldots \alpha(2, l + 1) \ldots \alpha(2, l + h)]^T$ for known $M$ [18]. $f_l(h, Z; \alpha, M)$ is the probability density function (PDF) of $Z$ under $H_{l,h}$, and $p_l(l, h) = (l + h) \ln (4N_h N_p)$ is the penalty term accounting for the number of unknown parameters ($\alpha$) and the volume of data. Finally, replacing $M$ with $S/K = RR^H/K$ to achieve adaptivity and neglecting the irrelevant constants, the final optimization problem is

$$
\min_{l \in \Omega_l} \min_{h : l + h \leq N_p} \left\{ -2K \Lambda_{l,h}(Z, S) + p_l(l, h) \right\},
$$

where $\Lambda_{l,h}(Z, S) = \sum_{i=1}^{l} |z_{i+1} - S^{-1} z_i|^2 + \sum_{i=1}^{l+h} |z_{i+1} - S^{-1} z_i|^2$.

The second selection rule consists in applying the BIC criterion over $Z$ and $R$ to obtain

$$
\min_{l \in \Omega_l} \min_{h : l + h \leq N_p} \left\{ -2 \ln \left[ f(R; \hat{M}_{l,h}) f_l(h, Z; \alpha_l, h, S_{l,h}, \hat{M}_{l,h}) \right] + p_2(l, h) \right\},
$$

where $S_{l,h} = RR^H + \sum_{i=1}^{N_p} z_{i+1} z_i^H + \sum_{i=1}^{l+h} z_{i+1} z_i^H + \sum_{b=l+1}^{N_p} z_{b+h} z_b^H$, $M_{l,h} = S_{l,h}/(2N_p + K)$, and $p_2(l, h) = (2l + 2h + N_p^2) \ln (4N_h N_p + 2N_h K)$ is the penalty term. It is possible to show that (6) is equivalent to

$$
\min_{l \in \Omega_l} \min_{h : l + h \leq N_p} \left\{ (4N_p + 2K) \ln \det (\hat{M}_{l,h}) + p_2(l, h) \right\},
$$

Notice that the above equation requires the computation of $\alpha_l h, S_{l,h}$ and $S_{l,h}$, respectively. Each admissible pair $(l,h)$ is selected according to (8) with $\alpha_l h, S_{l,h}$ replaced by $\hat{M}_{l,h}$ with $S$, which does not require to be updated. The reduced-complexity BIC rule is given by

$$
\min_{l \in \Omega_l} \min_{h : l + h \leq N_p} \left\{ (4N_p + 2K) \ln \det (\hat{M}_{l,h}) + p_2(l, h) \right\},
$$

where $\hat{M}_{l,h} = S_{l,h} + \sum_{i=1}^{l} (z_{i+1} - \hat{\alpha}_l h, s_i) (z_{i+1} - \hat{\alpha}_l h, s_i)^H + \sum_{b=l+1}^{N_p} (z_{b+h} - \hat{\alpha}_l h, s_b) (z_{b+h} - \hat{\alpha}_l h, s_b)^H$, $\hat{\alpha}_l h, s_b$ being the maximum likelihood estimate (MLE) of $\alpha = [\alpha(1,1) \ldots \alpha(2, l + 1) \ldots \alpha(2, l + h)]^T$. It is worth noticing that the price to be paid for the reduced computational load is a performance degradation especially when training data are limited as shown in Section IV.

B. TSA Architectures

The second (detection) stage of TSAs exploits the estimates of $l$ and $h$, denoted by $\hat{l}$ and $\hat{h}$, respectively, provided by the first stage and the following GAMF-like [10] decision rule

$$
\Lambda_{l,h}(Z, S) \overset{H_{1,l,h}}{\geq} \eta, \quad (9)
$$

where $\eta$ is the threshold$^2$ set according to the value of the probability of false alarm ($P_{fa}$) and $\Lambda_{l,h}(Z, S)$ has been defined after (5). Thus, we can obtain two architectures by cascading (9) with (5) (TSA-1) and (9) with (7) (TSA-2). In addition, the left-hand side of (9) can be suitably modified to make it less sensitive to the amount of secondary data by replacing $S$ with $S_{l,h}$ (see the definition after (6)), which exploits additional data drawn from those associated to the range cells under test. Therefore, the modified decision rule is

$$
\Lambda_{l,h}(Z, S_{l,h}) \overset{H_{1,l,h}}{\geq} \eta. \quad (10)
$$

The above decision rule can be coupled with (5) and (8) to obtain the modified TSA-1 (M-TSA-1) and modified TSA-2 (M-TSA-2), respectively.

C. OSA Architectures

The one-stage detection architectures rely on a “penalized generalized likelihood ratio test” [13], whose penalty term is borrowed from BIC rule, and jointly perform training and RCM estimation without intermediate steps. Again, we develop two OSAs that differ in the way secondary data are

$$
\sum_{i=1}^{l+h} (z_{2w} - \alpha_l h, s_{2w}(i)v) (z_{2w} - \alpha_l h, s_{2w}(i)v)^H/(2N_p + K) + \sum_{i=1}^{l+h} (z_{2w} - \alpha_l h, s_{2w}(i)v) (z_{2w} - \alpha_l h, s_{2w}(i)v)^H/(2N_p + K)
$$

$^2$Hereafter, we denote by $\eta$ the generic detection threshold.
incorporated into the decision statistic. This first architecture
(OSA-1) relies on the GAMF [16] and is given by

$$\max_{l \in \Omega_p, h : l + h \leq N_p} \left\{ \Lambda_{l,h}(Z,S) - \frac{p_1(l,h)}{2K} \right\} H_{l,h} \geq H_0 \eta. \tag{11}$$

The second architecture (OSA-2) is obtained by applying the
logarithm of the GLRT over both primary and secondary data, namely

$$\max_{l \in \Omega_p, h : l + h \leq N_p} \left\{ \ln[f(R; \hat{M}_{l,h})f_{l,h}(Z; \hat{\alpha}_{l,h}, \hat{M}_{l,h})] - \frac{p_2(l,h)}{2} \right\}$$

$$- \max_M \left\{ \ln[f(R; M)f_0(Z; M)] \right\} H_{l,h} \geq H_0 \eta, \tag{12}$$

where $f_0(Z; M)$ is the PDF of $Z$ under $H_0$. It is possible to
show that (12) can be recast as

$$\ln \det \left( \frac{S + ZZ^T}{2N_p + K} \right) + \max_{l \in \Omega_p, h : l + h \leq N_p} \left\{ \frac{1}{\det(\hat{M}_{l,h})} - \frac{p_2(l,h)}{4N_p + 2K} \right\} H_{l,h} \geq H_0 \eta. \tag{13}$$

IV. PERFORMANCE ASSESSMENT

In this section, we investigate the behavior of the pro-
posed architectures in terms of probability of detection ($P_d$),
computational complexity, and mean value of misclassified
pulses (MVMP) defined as the sum of the number of pulses
containing target components but classified as noise and
the number of noise-only pulses classified as target (this metric is
estimated only for the TSAs since OSAs inherit the selection
capabilities of BIC). The competitors are the likelihood ratio
estimator assuming perfect knowledge of $l$, $h$, and $M$ (clairvoyant
detector), the GAMF and the generalized adaptive subspace
detector (GASD) [16] both over data from two range cells, and
the best detector of [14] defined by (10) and (11) (2S-GIC) and
fed by data from the first range bin. Notice that the clairvoyant
detector represents an upper bound for the performance.

The parameters of the high-resolution radar and maneuver-
ning target are: $f_c = 10.0$ GHz, bandwidth 500 MHz, range
resolution 0.3 m, PRT = 1 ms, $N_p = 16$, $N_a = 8$ and $v_t = 30$ m/s. In this scenario, the point-like target will occupy
more than one range cell during a pulse integration interval.
The related curves of $P_d$ versus SINR (defined as in [14])
for $P_{fa} = 10^{-3}$, $K = 12 < 2N_a$ are shown in Fig. 2(a).
It turns out that M-TSA-1 overcomes the other competitors
with a gain of more than 9 dB over the GAMF at $P_d > 0.5$.

The TSA-2, OSA-2, and M-TSA-2 follow the M-TSA-1 and
with $P_d$ values contained in an interval of about 1 dB. The
2S-GIC experiences a loss of 4 dBs with respect to M-TSA-
2. The MVMP curves versus SINR are shown in Fig. 2(b).
Inspection of the figure highlights that for SINR values lower
than 10 dB, architectures based on [5] return better estimation
results than TSA-2 (rule [7]) and M-TSA-2 (rule [8]). When
SINR > 10 dB values, (7) and (8) slightly outperform (5).

The detection performances when the RCM does not occur
are shown in Fig. 2(c) for $P_{fa} = 10^{-3}$, $K = 12$, $l = N_p$, and
$h = 0$. The M-TSA-1 still guarantees superior performance
over the other detectors.

Finally, we compare the considered architectures from a
computational point of view using the usual Landau notation.
As expected, the GAMF is the architecture with the lowest
computational load since it does not involve the dis-
tant computation, data-dependent normalization, and discrete
search; its computational load is given by $O(N_a^2 + N_a^2K + 2N_aN_p)$. The GASD with complexity $O(N_a^2 + N_a^2(K + 2N_p))$ is
slightly more time demanding than the GAMF
due to the data-dependent normalization and shares a sim-
lar complexity with OSA-1 and TSA-1, that, in turn, are
$O(N_a^2 + N_a^2K + 2N_aN_p + \frac{1}{2}N_p^2)$. Proceeding in order of
increasing complexity, we obtain that M-TSA-1 and M-TSA-
2 are $O(2N_a^2 + N_a^2(K + 2N_p))$ and $O(\frac{1}{2}N_a^2N_p + \frac{1}{2}N_a^2N_p^2)$, respectively. The most complex architectures are OSA-2 and
TSA-2, which are $O(N_a^2N_p^2 + 2N_a^2N_p^2 + N_aN_a^3)$, and
2S-
GIC whose complexity is $O(N_a^2N_p^2 + \frac{3}{2}N_a^2N_p^2 + \frac{1}{2}N_a^2N_a^3)$. As a matter of fact, they require the computation of $\hat{M}_{l,h}$ and its
determinant for each $l \in \Omega_p$ and $h : l + h \leq N_p$.
Summarizing, the analysis singles out the M-TSA-1 as the
architecture that provides an excellent compromise between
detection/estimation performance and computational load.

V. CONCLUSION

This letter focused on the adaptive detection of dim maneu-
vering target in the presence of range migration. In this con-
text, data containing the returns from two adjacent range cells
have been exploited to conceive six different decision schemes
with different computational requirements that incorporate
the BIC rule to estimate the range migration indices. The
performance assessment pointed out that the M-TSA-1 can
ensure an excellent trade off between detection performance
and computational cost also for low volumes of training data.
Future research tracks may include the design of architectures
accounting for the spillover of target energy or heterogeneous
environments.
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