Development of rock fracturing model considering mineral composition and distribution and its application to coupled Thermal-Hydraulic-Mechanical-Chemical (THMC) simulator

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ABSTRACT

A numerical simulation model, considering the distribution of minerals in intact rock, was proposed to predict the tensile strength as well as the processes for generating and propagating a fracture in radial compression tests. The objective rock sample was modeled by considering the mineral distribution obtained through an image analysis. The proposed model accurately estimated the tensile strength and the tensile stress-vertical strain relation. In addition, using the damage variable, the generation and propagation of the fracture were also estimated by the proposed model.

Keywords: tensile strength, radial compression, fracture, simulation

1 INTRODUCTION

When excavating tunnels and underground structures with high overburdens, it is thought that the areas of tensile stress appear locally around the excavated structures and that tensile cracks and fractures are generated and develop in the rock masses. Several testing methods are available for estimating the tensile strength of intact rock, such as uniaxial tensile tests, biaxial tensile tests and radial compression tests. According to the ISRM Suggested Method (ISRM Testing Committee, 1978), the stress-strain relation can be accurately obtained through uniaxial tensile tests (Fairhurst, 1961). However, due to several difficulties encountered with uniaxial tensile tests, such as the testing control, the making of the specimen and the contact condition between the pedestal and the specimen, radial compression tests have usually been employed. Several researchers have carried out comparisons between the results of uniaxial tensile tests and radial compression tests and have described a good correlation between them (Lin, et al., 2008; Perrais and Diederichs, 2014). On the other hand, it may not be theoretically possible to control the cracks in radial compression tests due to the heterogeneity of the mineral array in the intact rock and the unstable stress condition at the loading points.

The heterogeneity of intact rock is strongly affected by the generated and developed cracks. The rock is generally heterogeneous due to the mineral distribution, the mineral component and the initial micro-crack. It is well known that they strongly affect the mechanical properties and the fracture behavior (Fujii, et al., 2005; Cowie and Walton, 2018; Osada, et al., 1999; Nara, et al., 2004; Takekura and Oda, 2002; Přikryl, 2001). Tang and Kaiser (1998) developed a crack-generated and crack-developed model by applying the damage theory with the Weibull distribution of the mechanical properties. However, it has been difficult to estimate several of the input parameters for the Weibull distribution through experimental work (Zhu and Tang, 2006). On the other hand, Mahabadi, et al. (2012) applied the FDEM (Finite-Discrete Element Method) to the simulation of radial compression tests by considering the heterogeneity of the mineral distribution in granite samples. They were able to successfully explain the shape of a propagating crack in radial compression tests. However, they could not obtain a good correlation for the tensile strength.

In this study, the heterogeneity of intact rock is considered in terms of the mineral distribution. Then, a numerical simulation method, describing the tensile behavior and the tensile strength, is developed. Radial compression tests on granite are carried out. By observing the tensile behavior, the proposed simulation method is applied to the experimental results. Then, the validity of the proposed simulation method is confirmed.

2 OUTLINE OF RADIAL COMPRESSION TESTS AND THEIR RESULTS

2.1 Outline of radial compression tests

In this paper, radial compression tests are carried out on a granite sample and the stress-strain relation is
measured. In addition, the generation and the propagation of the fracture are recorded. Fig. 1 shows the outline of the radial compression tests in this research work. Three cylindrical specimens, with a diameter of 50 mm and a length of 50 mm, were made from a granite block sample. In order to measure the strain on the radial plane of the specimen, a strain gauge was installed in the center of the radial plane, as shown in Fig. 1. A loading speed of 100 N/s was employed in each case. By obtaining the stress-strain relation until the destruction of the specimen, the generating and propagating processes of the fracture were recorded by a high speed camera whose frame size is 256 by 256 pixels and frame rate is 25000 fps.

2.2 Results of radial compression tests

Fig. 2 shows the stress-strain relation for the three specimens and Table 1 shows the tensile strength. Fig. 3 displays the propagating process of the fracture obtained through the high speed camera. It is confirmed in this figure that the fracture was generated in the center of the radial plane and then propagated from the center to both loading points. Moreover, the fracture is seen to be located on the vertical center line of the radial specimen in the loading direction.

![Fig. 1. Outline of radial compression tests (Brazilian tests).](image)

![Fig. 2. Tensile stress-vertical strain relation obtained through radial compression tests.](image)

![Fig. 3. Fracture propagating process during radial compression tests (Br-1).](image)

| Specimen no. | Tensile strength (MPa) |
|--------------|------------------------|
| Br-1         | 6.21                   |
| Br-2         | 6.71                   |
| Br-3         | 5.52                   |
| **Average**  | **6.15**               |

3 PROPOSED NUMERICAL SIMULATION METHOD

3.1 Damage theory

Assuming the plane stress condition, the stress and strain have been calculated in this research work based on the equilibrium condition of forces and the elastic theory. In addition, the damage theory is applied in this simulation and the generating and propagating processes of the fracture are both simulated.

Based on the stress distribution in the objective rock, it is judged whether or not tensile damage or shear damage appeared (Zhu and Tang, 2004; Wang, et al., 2018). In particular, an objective area in the rock is divided into different areas of Representative Elementary Volume (REV). Then, the mean stress in each area of REV is calculated by the simulation. Finally, the damage in each area is calculated using both the tensile stress conditional equation in Eq. (1) and Mohr-Coulomb’s failure criterion.

$$
\begin{align*}
F_1 & \equiv -\sigma_3 - f_{t0} = 0 \\
F_2 & \equiv \sigma_1 - \frac{1+\sin\theta}{1-\sin\theta} \sigma_3 - f_{c0} = 0
\end{align*}
$$

(1)

where $F_1$, $F_2$, $\sigma_1$, $\sigma_3$, $f_{t0}$, $f_{c0}$ and $\theta$ are the condition of the tensile damage, the condition of the shear damage, the maximum principle stress, the minimum principle stress, the uniaxial tensile strength, the uniaxial compressive strength and the internal friction angle, respectively. In the damage theory, it is assumed that the elastic modulus of the damaged REV, $E$, decreases with the fracture propagation, as follows:

$$
E = (1 - D)E_0.
$$

(2)

where $E_0$ and $D$ present the elastic modulus without damage and the damage variable, respectively. Damage variable $D$ ranges from zero to one. $D = 0$ represents undamaged material and $D = 1$ represents failure. At the condition of $D > 0$, the fracture is being generated. Then, $D$ is defined as follows (Zhu and Tang; 2004):
\[ \sigma_i = \frac{E}{1+\nu} \left( \varepsilon_i + \frac{\nu}{1-2\nu} \varepsilon_v \right) \quad (i = 1, 2, 3), \quad (2) \]

where \( \varepsilon_i \ (i = 1, 2, 3) \) are the maximum principal strain, the intermediate principal strain and the minimum principal strain, respectively, \( \varepsilon_v \) is the volumetric strain and \( \nu \) is Poisson’s ratio. Based on Eq. (2) and Mohr-Coulomb’s failure criterion, strain \( \varepsilon_t \) and strain \( \varepsilon_c \), which are employed to estimate damage variable \( D \), are defined as follows:

\[ \bar{\varepsilon}_1 = \frac{1}{1+\nu} \left( \varepsilon_1 + \frac{\nu}{1-2\nu} \varepsilon_v \right) \quad (3) \]

\[ \bar{\varepsilon}_3 = \frac{1}{1+\nu} \left( \varepsilon_3 + \frac{\nu}{1-2\nu} \varepsilon_v \right) = \varepsilon_t \quad (4) \]

\[ \varepsilon_c = \bar{\varepsilon}_1 - \frac{1+\sin\theta}{1-\sin\theta} \bar{\varepsilon}_3. \quad (5) \]

Applying Eq. (1) and the constitutive model described in Fig. 4, \( D \) can be obtained by the following equation:

\[
D = \begin{cases} 
0 & F_1 < 0 \text{ and } F_2 < 0 \\
1 - \left( \frac{\varepsilon_t}{\varepsilon_c} \right)^n & F_1 = 0 \text{ and } \Delta F_1 > 0 \\
1 - \left( \frac{\varepsilon_c}{\varepsilon_t} \right)^n & F_2 = 0 \text{ and } \Delta F_2 > 0 
\end{cases} \quad (6)
\]

where \( \varepsilon_{t0}, \varepsilon_{c0} \) and \( n \) are the critical tensile strain, the critical compressive strain and the constant parameter, respectively. In this research, \( n = 2 \) is employed.

The conditions of \( F_1 = 0 \) and \( \Delta F_1 > 0 \) in Eq. (6) are satisfied with the tensile damage condition and the loading condition. Thus, tensile failure occurs under these conditions. On the other hand, the conditions of \( F_2 = 0 \) and \( \Delta F_2 > 0 \) in Eq. (6) are also satisfied with the shear damage condition and the loading condition. Thus, shear failure occurs under these conditions. In this research, it is assumed that recovery is not possible if damage occurred.

3.2 Modeling of heterogeneity in mineral distribution

Assuming that the heterogeneity of intact rock causes the mineral components and their distribution, a simulation model of an objective rock is proposed in this research by considering the mineral distribution.

First, the image of the cross section of the objective rock specimen is obtained in Fig. 5(a). The obtained image data, that is, three minerals, Quartz, Feldspar and Biotite, are classified through the image analysis. After identifying the three minerals distinctly, the simulation model is made by considering the mineral distribution. Fig. 5(b) shows the model based on the mineral distribution (MMD).

3.3 Other criterion

As mentioned above, the criterion formula for the damage has been represented in Eq. (1). Eq. (1) is considered under the stress condition. On the other hand, several researchers (Zhu and Tang, 2006; Lu, et al., 2013) have given numerical simulations of fracture generation and fracture development by applying the following criterion:

\[
F_1 \equiv -\varepsilon_3 - \frac{\nu}{\nu_c} = 0 \\
F_2 \equiv \varepsilon_1 - \frac{1}{\nu_c} \left[ \varepsilon_o + \frac{\nu + \sin\theta}{1-\sin\theta} \varepsilon_3 - \nu (\sigma_2 + \sigma_3) \right] = 0 \quad (7)
\]

In this research, the numerical simulations have been carried out using two criterions (Eqs. (1) and (7)), and the validity of each criterion will herein be discussed.

3.4 Condition of numerical simulation

In this research, radial compression tests have been simulated and the validity of the proposed numerical method and the model will be confirmed. The geometry, boundary conditions and numerical mesh are shown in Fig. 6. The applied loading speed in the numerical simulation is 0.002 mm/step. Okubo, et al. (1993) reported that the tensile strength does not depend on the loading rate in the experiment. Therefore, the loading speed has been set by considering the numerical stability. 25134 triangle meshes are employed in this simulation.

Table 2 shows the parameters used in the simulation. Poisson’s ratio was obtained through uniaxial compression tests on the granite specimens. The internal
friction angle of the granite, the initial elastic modulus and the tensile strength, \( f_0 \), were estimated using the results of Savanick and Johnson (1974), Mavko et al. (2009) and Zhang (2017). The tensile strength of Biotite could not be found in any reports. Thus, referring to the analytical research by Mahabadi et al. (2012), 70% of Feldspar’s tensile strength has been applied here. The compressive strength of each mineral could not be found in any reports either. Thus, referring to the Griffith theory, 8 times the tensile strength of each mineral has been employed.

Table 2. Mechanical properties of minerals (Savanick and Johnson (1974), Mavko et al. (2009) and Zhang (2017))

| Mineral     | Quartz | Feldspar | Biotite |
|-------------|--------|----------|---------|
| Elastic modulus, \( E_0 \) (GPa)  | 76.9   | 39.6     | 33.9    |
| Tensile strength, \( f_0 \) (MPa)  | 10.3   | 10.5     | 7.3     |
| Compressive strength, \( f_c \) (MPa) | 82.5   | 83.6     | 58.6    |
| Internal friction angle, \( \phi \) (°) | 50     | 50       | 50      |
| Poisson’s ratio, \( \nu \) (-)       | 0.19   | 0.19     | 0.19    |

4 SIMULATION RESULTS

4.1 Tensile stress-vertical strain relation

Fig. 7 shows a comparison between the numerical and the experimental results for the tensile stress-vertical strain relation in specimen BR-1. From this figure, it is seen that the results of the proposed simulation agree well with those of the experiment. It is also confirmed that the mechanical response from the initial phase to the yielding point can be represented in the simulation. After the yielding point in the experiment, the tensile stress-vertical strain relation cannot be followed since brittle behavior is observed. On the other hand, after the yielding point in the numerical simulation, no brittle behavior is observed.

The loading points cannot be reproduced in the numerical simulation. In the numerical simulation, the area without a propagating fracture covers the load, and the stress reduction after the yielding point appears in the tensile stress-vertical strain relation shown in Fig. 7.

4.2 Strain Criterion

The numerical simulation was carried out using the strain criterion of Eq. (7). The mineral distribution was used in Fig. 5. Fig. 9 shows the tensile stress-vertical strain relation, and Fig. 10 shows that the propagating process of the fracture is similar to that shown in Fig. 8. From Fig. 9, the tensile stress is seen to increase in the initial phase and to reach the yield point. After the yield point, the tensile stress decreases. In comparing Fig. 7 and Fig. 9, it can be easily observed that the yield point in Fig. 7 is larger than that in Fig. 9. On the other hand, the fracture arrives at one side of the specimen wall in Fig. 10. Fig. 10 actually represents the propagating fracture just like in the experiment.

Consequently, the strain criterion model cannot explain the mechanical behavior in the experiment of the radial compression tests.

5 CONCLUSIONS

In order to estimate the tensile behavior and the tensile strength of intact rock through radial compression
tests, a numerical simulation model has been proposed in this research that considers the distribution of minerals. In addition, the influence of two criterions, defined as the stress field and the strain field, has been compared. The proposed mineral distribution (MMD) and the criterion defined as the stress field have represented the mechanical behavior in the tensile stress-vertical strain relation. Moreover, the generation and propagation of a fracture have also shown a good correlation with the results of the experiment. However, at the near loading points, the propagating fracture was not represented well due to the stress compression field. In addition, comparing the stress and strain field criterions, the numerical simulation employing the stress field criterion showed a better correlation with the experimental results than that employing the strain field criterion.

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REFERENCES

1) Cowie, S. and Walton, G. (2018): The effect of mineralogical parameters on the mechanical properties of granitic rocks, *Engineering Geology*, 240, 204-225, doi:10.1016/j.enggeo.2018.04.021.

2) Fairhurst, C. (1961): Laboratory measurements of some physical properties of rock, *Proceedings of the Fourth U.S. Symposium on Rock Mechanics*, Document ID: ARMA-61-105.

3) Fujii, Y., Takemura, T., Takahashi, M., Lin, W. and Akaia, S. (2005): The Feature of Uniaxial Tensile Fractures in Granite and Their Relation to Rock Anisotropy, *Journal of the Japan Society of Engineering Geology*, 46(4), 227-231, doi: 10.5110/jjseg.46.227 (in Japanese).

4) ISRM Testing Committee (1978): Suggested methods for determining tensile strength of rock materials, *International Journal of Rock Mechanics and Mining Science & Geomechanics Abstract*, 15(3), 99 – 103, doi:10.1016/0148-9062(78)90003-7.

5) Lin, W., Takahashi, M., Nakamura, T. and Fujii, Y. (2008): Tensile strength and deformability of Inada granite and their anisotropy: Comparison between uniaxial tension test and Brazilian test, *Japanese Geotechnical Journal*, 3(2), 165 - 173, doi:10.3208/jgs.3.165 (in Japanese).

6) Lu, Y. L., Elsworth, D. and Wang, L. G. (2013): Microcrack-based coupled damage and flow modeling of fracturing evolution in permeable brittle rocks, *Computers and Geotechnics*, 49, 226-244, doi: 10.1016/j.compgeo.2012.11.009

7) Mahabadi, O.K., Lisjak, A. Munjiza, A. and Grasselli, G. (2012): Y-Geo: New Combined Finite-Discrete Element Numerical Code for Geomechanical Applications, *International Journal of Geomechanics*, 12, 676-688,doi:10.1061/(ASCE)GM.1943-5622.0000216.

8) Mavko, G., Mukerji, T. and Dvorkin, J. (2009): *The Rock Physics Handbook*, doi:10.1017/CBO9780511626753 (2009).

9) Nara, Y., Ohno, Y., Imai, Y. and Kaneko, K. (2004): Anisotropy and Grain-Size Dependence of Crack Growth due to Stress Corrosion in Granite, *Shigen-to-Sozai*, 120, 25 - 31, doi:10.2473/shigentosozai.120.25 (in Japanese).

10) Okubo, S., Jin, F. and Akiyama, M. (1993): Loading-Rate Dependency of Uniaxial and Indirect Tensile Strength, *Shigen-to-Sozai*, 109(11), 865-869, doi:10.2473/shigentosozai.109.865.

11) Osada, M., Yamabe, T. and Yoshinaka, R. (1999): Initial Distribution of Microcracks in Inada Granite, *Journal of the Japan Society of Engineering Geology*, 39(6), 500-510, doi: 10.5110/jjseg.39.500 (in Japanese).

12) Perras, M. A. and Diedercihs, M. S. (2014): A review of the tensile strength of rock: Concepts and testing, *Geotechnical and Geological Engineering*, 32, 525 - 546, doi:10.1007/s10706-014-9732-0.

13) Přikryl, R. (2001): Some microstructural aspects of strength variation in rocks, *International Journal of Rock Mechanics & Mining Sciences*, 38, 671-682, doi:10.1016/S1365-1609(01)00031-4 (2001).

14) Savanick, G. A. and Johnson, D. I. (1974): Measurements of the Strength of Grain Boundaries in Rock, *International Journal of Rock Mechanics and Mining Sciences & Geomechanics Abstracts*, 11(5), 173-180, doi:10.1016/0148-9062(74)90884-5.

15) Takemura, T. and Oda, M. (2002): Three-dimensional Fabric Analysis of Microcracks Associated with Brittle Failure of Granitic Rocks, *The Journal of the Geological Society of Japan*, 108(7), 453-464, doi:10.5575/geosoc.108.453 (in Japanese).

16) Tang, C. A. and Kaiser, P. K. (1998): Numerical Simulation of Cumulative Damage and Seismic Energy Release During Brittle Rock Failure—Part I: Fundamentals, *International Journal of Rock Mechanics and Mining Sciences*, 35, 113-121, doi:10.1016/S0148-9062(97)00009-0.

17) Wang, J., Elsworth, D., Wu, Y., Liu, J., Zhu, W. and Liu, Y. (2018): The Influence of Fracturing Fluids on Processes: A comparison Between Water, Oil and SC-CO2, *Rock
18) Zhang, L. (2017): *Engineering Properties of Rocks* (Second Edition), doi:10.1016/B978-0-12-802833-9.00003-1.

19) Zhu, W. C. and Tang, C. A. (2004): Micromechanical model for simulating the fracture process of rock, *Rock Mechanics and Rock Engineering*, 37, 25 - 56, doi:10.1007/s00603-003-0014-z.

20) Zhu, W. C. and Tang, C. A. (2006): Numerical simulation of Brazilian disk rock failure under static and dynamic loading, *International Journal of Rock Mechanics & Mining sciences*, 43, 236-252, doi:10.1016/j.ijrmms.2005.06.008.