The profile of students conceptual understanding and procedural knowledge in solving geometry problems based on van hiele levels

N I Laily¹, Sunardi¹, N Yuliati¹, E N Imamah¹, M P Sari¹
¹Departement of Mathematics Education Postgraduate, University of Jember, Indonesia

Email: novilaily19@gmail.com

Abstract. This study aimed to describe the conceptual understanding and procedural knowledge of students in problem solving polyhedral in terms of the level of visualization, analysis and informal deduction and to investigate the correlation between conceptual understanding and procedural knowledge to problem solving. The method used was a mixed method that combines quantitative and qualitative methods. 297 Junior High School students from five different schools in Jember were selected as samples and three students were selected as the subject. That Determine the subject chosen by the van hiele test. The data were collected through tests and interviews. The results of the test van hiele obtained visualization level students as much as 44.5%, students' level of analysis as much as 34.2%, informal deduction level students as much as 15.5% and 5.8% were classified at the level of pre-visualization. The results of the geometry test show that students at the visualization level meet some indicators of conceptual understanding but none of them meet the indicators of procedural knowledge, students at the level of analysis meet some indicators of conceptual understanding and procedural knowledge, while students with a level of deduction meet all indicators of conceptual understanding and procedural knowledge. The Results of the test showed that the higher the level of geometrical thinking students, the better the conceptual understanding and procedural knowledge. The results also showed that there was a significant relationship between conceptual understanding and procedural knowledge to solving the problem of 82.3% shown by the person correlation coefficient is 0.515 with a confidence level of 95% and a significant level $\alpha = 0.05$. The better the conceptual understanding and procedural knowledge the better the ability to solve the problem.

1. Introduction
Mathematics is one of sciences which has a great influence in the science and technology development so that mathematics is said as the universe of sciences. However, in fact, there are still many students who do not like mathematics, the students’ dislike of mathematics has a quite great impact because mathematics has a strong relation between its concepts so it allows students to be skilled in thinking to face problems. The students experience difficulties in mathematics learning because they do not understand the concept of mathematics, they think that by memorizing the formula, then all the mathematical problems can be solved, however when they memorize the formula, they do not know the concept and how to operate it. Therefore, it needs the conceptual understanding and procedural knowledge.

The definition of conceptual understanding by Hievert & Lefevre, [1] “knowledge that is rich in relationships. It can be thought of as a connected web of knowledge, a network in which the linking
relationships are as prominent as the discrete pieces of information. Relationships pervade the individual facts and propositions so that all pieces of information are linked to some network”. Crooks and Alibali, [2] divide conceptual knowledge into two sides, namely knowledge of general principles and principles that underlie procedures. While, the definition of procedural knowledge by Hiebert & Lefevre, [1] “One kind of procedural knowledge is a familiarity with the individual symbols of the system and with the syntactic convention for acceptable configurations of symbols. The second kind of procedural knowledge consists of rules or procedures for solving mathematical problems. Many of the procedures that students possess probably are chains of prescriptions for manipulating symbol”. The students who have poor procedural ability will experience difficulties in deepening their understanding dealing with mathematical ideas as well as in solving the mathematical problems, and vice versa. The students who have poor procedural knowledge will also influence their procedural knowledge. The dimension of knowledge consists of four types, namely factual, conceptual, procedural, and metacognitive [3]. Whereas, in particular, knowledge that students must have in mathematics learning are conceptual understanding and procedural knowledge. Suratman Research, [4] revealed that the students class VII on Linear Inequality in One Variable conceptual understanding were still low and the students’ procedural knowledge mostly belonged to a very low level. Then, the research by Surif et al, [5] also showed that the students’ have weak procedural and conceptual knowledge and both of these knowledge have a significant relationship in problem solving. One branch of mathematics which is difficult for students is Geometry.

Geometry learning required teachers to see the stages of the students’ thinking ability [6]. This is in line with the theory of Van Hiele which states that students experience five levels of thinking stages in geometry learning namely visualization level, analysis level, informal deduction level, formal deduction level and rigor level [7]. The results of the research related to the students’ thinking levels state that mostly students in schools are in the first third level. This is because the students are not capable to associate between the concepts of geometry existing in their mind. Yudianto, [8] conducted a research to 78 students the results of the research showed that the respondent percentage in the levels of visualization, analysis, informal deduction, deduction and rigor consecutively were 11.54%; 5.13%; 1.28%; 2.56%; and 2.56%; while 76.93% students were classified into transition level. The research conducted by Sunardi, [9] to 576 students from 13 classes of 13 SLTPN in Jember revealed that the respondent percentage in the levels of visualization, analysis, informal deduction, deduction and accuracy consecutively were 44.62%; 34.55%; 6.77%; 0.17%; and 0%. The respondents who could not be classified were 14.40%. Based on the explanations above, it can be concluded that most of the results of research in the first third level of van Hiele theory were visualization, analysis and informal deduction levels. The success of the students’ thinking process was influenced by the planning of solving the problems.

Problem solving is activities in solving a story problem, non-routine problem, applying mathematics in everyday life and proving the theorem [10]. Problem solving according to Polya, [11] consists of 4 stages, namely understanding problems, determining the plan of problem solving strategy, solving the strategy of problem solving and re-checking the answers obtained. One of geometry materials learned in SMP/MTs is Polyhedral.

The aim of this research was to describe a profile of the students’ conceptual understanding and procedural knowledge and analyze the relationship of them in solving polyhedral based on the geometry thinking levels of visualization, analysis and informal deduction.

2. Methods
The method used in this research was mixed methods according to [12] is the research method that combining qualitative and quantitative methods. The qualitative method is the method used to describe the profile of conceptual understanding and procedural knowledge in solving geometry problems based on van hiele levels. While the quantitative method is the method used to investigate the correlation between conceptual understanding and procedural knowledge on the geometry problem.
This research was conducted at 5 SMP/MTs Negeri in Jember Regency. The research samples were as many as 297 students. The research subjects were three IX grade students based on test of geometry thinking ability of van Hiele. namely visualization, analysis and informal deduction levels. First, the researcher compiled and validated the research instruments covering the tests and interview guidelines. The instrument validation was carried out by three validators who were two Mathematics Education lecturers from Jember University and teachers. The purpose of validation is to check the validity of the instrument before it is used in research. After that, the researcher coordinated with mathematics teacher to determine the sample and research schedule. The level of thinking ability test in geometry was quoted from Sunardi, [9] which covered 25 questions within 80 minutes as time allocation. The geometry thinking ability test was carried out to determine the students’ level of thinking based on Van Hiele. The geometry test was used to determine the profile of conceptual understanding and procedural knowledge of Van Hiele. The scores obtained from geometry test were used to analyze the relationship of conceptual understanding and procedural knowledge. Before being analyzed by using SPSS 20.0, the null hypothesis and the alternative hypothesis needed to be determined.

**Figure 1.** The model of Mixed Method

**Figure 2.** Geometry test about polyhedral
3. Research Findings

3.1 Results

In this research, The assessment of instrument research is done by three valuators consisting of two lecturers and a mathematics teacher. These are the result of the validation analysis of interview guidelines and geometry problem test from three validators as follow:

![Figure 3. Validators assessment result](image)

Based on the result of the experts’ assessment presented in the Chart 1, it can be concluded that the average total value of the validator (Va) in the interview guide was 2.85 and the geometry problem test was 2.92.

The first stage in this research was giving the test of Van Hiele geometry thinking to the students. The geometry thinking test was given to 129755 students of grade IX at five schools located in Jember regency. The students’ geometry thinking level test included 25 multiple choice questions with 5 options and the students must choose one of the choices from A, B, C, D, or E by crossing the answer that he thought was correct. Each of five questions represented the level of students’ geometry thinking. Students were classified into a certain level if they answer were 3 of 5 items correctly. If the students failed at a certain level, then it was considered as a failure at the next level. After being given a geometry thinking test, then given a geometry test about polyhedral to find out conceptual understanding and procedural knowledge.

| Table 1. Conceptual understanding of research subjects |
| --- | --- | --- | --- |
| No. | Indicator | SV | SA | SD |
| 1. | restating a concept | ✔ | ✔ | ✔ |
| 2. | classifying an object based on certain characteristics in accordance with the concept | ✔ | ✔ | ✔ |
| 3. | presenting concepts in various forms of mathematical representation | ✔ | ✔ | |
| 4. | developing the necessary conditions or sufficient conditions of a concept | ✔ | ✔ | |
| 5. | using, utilizing, and choosing certain procedures or operations | ✔ | | |
| 6. | applying a concept or algorithm | | | ✔ |

Based on Table 1, it shows that subjects with visualization level only met two indicators of the six indicators of conceptual understanding. Subjects with analysis level were able to fulfill four indicators from six indicators of conceptual understanding. Whereas, the subjects with informal deduction level were able to meet all indicators of conceptual understanding.
Table 2. Procedural knowledge of research subjects

| No. | Indicator                                                                 | Subject |
|-----|----------------------------------------------------------------------------|---------|
| 1.  | using the procedure correctly                                             | ✔       |
| 2.  | communicating algorithm processes into problem situations                 | ✔       |
| 3.  | modify procedures to deal with factors in problem solving                 | ✔       |

Table 2 shows that only SD subjects met all indicators of procedural knowledge while none of SV met procedural knowledge indicators.

3.1.1 Geometry thinking levels on SV, SA and SD

Based on the results of geometry thinking test, it was found that 24 students were classified at the level of informal deduction, 53 students were classified at the level of Analysis, and 69 students were classified at the visualization level and 9 students were classified at the level of pre-visualization. There capitulation of the results of geometry thinking test can be seen on Figure 4.

Figure 4. Percentage of respondents thought level geometry

This study was in line with the study conducted by [8][9], states that most of the research results are still at the first three stages of van Hiele's theory, namely the level of visualization, analysis and informal deduction. From the results of geometry thinking test, 3 students were chosen as the subjects of this research since they represented each level of geometry thinking.

Table 3. Research Subjects

| Geometry Thinking Levels | Subject |
|--------------------------|---------|
| Visualization            | SV      |
| Analysis                 | SA      |
| Informal Deduction       | SD      |

3.1.2 The conceptual understanding and procedural knowledge with visualization level

In the problem given, SV was not able to describe the concept used. It occurred since he did not write what was known and asked from a simple problem on his answer sheet. SV also got misperception about the height of shape. The height written on the answer was the hypotenuse. Here is the SV's answer.
Figure 5. SV’s answer

It was also shown in the interview test. The following are the interview result that the researcher did with SV.

Q : After reading the questions, did you understand the problem given?
SV : Hmm InshaAllah, I understand Mam.

Q : How many times did you read the questions?
SV : 2 times Mam, hehehe.

Q : What information did you get and how did you relate it?
SV : The question showed that the shape is a parallelogram, so I wrote it directly. The area of the base was \(20 \times 20\), 1 kg of sand = 1000 \(\text{cm}^3\) and the height was 35 cm.

P : Try to look at the picture, where is 35 cm located? What is the height?
SV : This one Mam. (while pointing at the picture). It is the sloping height, Mam.

At the stage of completion plan, SV did not understand the problem very well. SV did not understand the shape of a given polyhedral. In fact, if you made a polyhedral visualized in a 2-dimensional figure, a square or rectangular base would look like a parallelogram. SV also did not know the volume formula of prism. It can be seen in the answer sheet and the following interview result.

Q : From that information, what sort of completion pattern did you think of?
SV : by using the volume formula of prism, Mam.

Q : Why did you use the formula?
SV : It was a prism.

Q : What was the formula for your prism?
SV : This one, Mam, \(V = \frac{1}{2} \times \text{base area} \times \text{height}\). The base is a parallelogram, Mam. So the base was replaced by the parallelogram formula which is \(a \times h\).

The result of the interview, it was clear that SV did not understand the concept of prism, so he was not able to solve the problem properly. SV’s completion plan was incorrect which made the result was incorrect as well. Based on the Van Hiele-based grouping level test, SV was at level 0 namely visualization. SV did not understand the characteristics of prism he observed.

3.1.3 The conceptual understanding and procedural knowledge with analysis level

On the given question, SA was able to describe the concept that is going to be used well. This can be seen from the answer sheet that he was able to write down what was known and asked rom the question very well. SA planned and did the problem solving very well and detail. The following was the SA’s answer.
This was also showed on the interview. The following was the interview results between the researcher and SA.

P: After reading the question, were you able to understand the problem given?
SA: Yes, Mam.
P: How many times did you read the question?
SA: I forgot, maybe 3 times, Mam.
P: From the question, what information did you get and how to elaborate it?
SA: It was known that side 1 is this (pointing), that was 20 cm, side 2 was this (pointing), it was 20 cm. Then, what was known was the hypotenuse, so I needed to find out the height by using phytagoras, then the height $\sqrt{1125}$ cm, Mam. Hmm… how could the results was in square root?
P: No problem. Do you think the height can be simplified?
SA: Yes, Mam, but I do not know how.

In the stage of planning the solution, SA looked understand the question. SA understood how to solve the problem however he did not know how to simplify the square root form. Therefore, the answer was more complicated and can not solve problems coherently and precisely. The following was the results of the interview between the researcher and SA.

P: From the information, what kind of solving pattern did you think?
SA: I used the formula of prism volume, Mam. But usually prism shape was not like this.
P: Yes, you were right, What is the formula of prism? How does prism usually look like?
SA: the base area multiplied by the height, Mam. Usually this one is vertical, but it was tilted.
P: So, what kind of shape is this?
SA: I do not know, Mam. It must be a prism but its side is tilted.

From the result of the interview, it seemed that SA understood the problem given through the question and planned to obtain solution. SA also understood that the shape given in the question was a prism with tilted side but the language he used to explain was different. However, SA was not able to simplify the results obtain from the calculation of height so that the answer of volume was inaccurate.

Based on Van Hiele-based grouping test, SA was categorized into level 1 namely analysis. SA was certain and understands the shape in the question. In this case, he also recognized the characteristics of prim he observed.
3.1.4 The conceptual understanding and procedural knowledge with informal deduction level

On the given question, SD was able to describe the concept used. This could be seen from the answer sheet showing that he was able to write down what was known and what was asked from the question well. Arrange and plan the solution well. It could be seen from the SD’s answer sheet. The following was the answer of SD. It was revealed from the following interview excerpt.

![Figure 7. SD’s answer](image)

The following was the answer of SD. It was revealed from the following interview excerpt.

P: After reading the question, have you understood the problem given?
SD: Yes, I have, Mam. (Answer certainly).
P: How many times did you read the question?
SD: 3 times, Mam.

The results of the interview revealed that SD faced difficulties in what was asked by the question. Based on the result of the observation, SD did not face any difficulties in solving the problem. This can be seen in SD’s answer which was good and thorough.

P: From the information, what kind of solving pattern did you think?
SD: Using the formula of prism volume, Mam. Usually I meet a question that used upright prism, but this question used tilted prism. So, I looked for the height first, Mam.
P: Yes, you are right. What is the formula of prism? What kind of shape is that? Please explain.
SD: The volume of prism is base area multiplied by height. The base area is a square that is side multiplied by side. What is it? I am sure that this shape is prism because the cover and base are the same and parallel but the side is tilted, Mam.

From the results of the interview, it seemed that SD understood the question given starting from arranging the solving plan up to doing the execution. SD also understood that the shape given in question was a prism with tilted side and his answer was also correct and detail. Based on Van Hiele-based grouping test, SD was in the level 2 namely informal deduction. SD was certain and understood the shape on question. In this case, SD also recognized the characteristics of prism he observed.

3.1.5 The relationship of conceptual understanding and procedural knowledge
Data were analyzed using SPSS 20.0, to determine the relationship between conceptual understanding and procedures by examining the person correlation coefficient. Now it is time to analyze the result on geometry test by using the inferential statistic. We start by analyzing the normality test.

**Table 4. The Results of Normality Test**

| Normal Parameters | Asymp. Sig. (2-tailed) |
|-------------------|------------------------|
| N                 | 297                    |
| Normal Parameters |                        |
| Mean              | 0E-7                   |
| Std. Deviation    | 136903258              |
| Absolute          | .036                   |
| Most Extreme Differences |                |
| Positive          | .035                   |
| Negative          | -.036                  |
| Kolmogorov-Smirnov Z | 831                   |
| Asymp. Sig. (2-tailed) | 831                   |

Normality test results obtained asymp.sig (2-tailed) significance value of 0.831 is greater than 0.05. Then according to the basis of decision making in the Kolmogorov-Smirnov normality test above, it can be concluded that the data are normally distributed. The next is the Linearity test that can be seen in table 5.

**Table 5. The Result of Linearity Test**

| ANOVA Table | Sum of Squares | df | Mean Square | F | Sig. |
|-------------|----------------|----|-------------|---|------|
| Between Groups | (Combined) | 220264,631 | 16 | 1391,538 | 7.305 | .000 |
| Procedural ^ conceptual | Linearity | 20047,340 | 1 | 20047,340 | 105,247 | .000 |
| Deviation from Linearity | 2217,291 | 15 | 147,819 | .775 | .704 |
| Within Groups | Total | 53334,022 | 280 | 190,475 | 75008,653 | 296 |

Linearity test results obtained by the value of Deviation from Linearity Sig. is 0.704 greater than 0.05, it can be concluded that there is a significant linear relationship between the variables of procedural knowledge (X) with the conceptual understanding variable (Y). The final step is the correlation bivariate person.

**Table 6. Correlation of conceptual understanding and procedural knowledge**

| Correlations | Conceptual | procedural |
|--------------|------------|------------|
| Pearson Correlation | 1 | .515** |
| Sig. (2-tailed) | .000       | 1          |
| N            | 297        | 297        |
| Pearson Correlation | .515**  | 1          |
| Sig. (2-tailed) | .000       | 1          |
| N            | 297        | 297        |

**. Correlation is significant at the 0.01 level (2-tailed).**
The result showed that can be seen in the person correlation coefficient of 0.515. In addition, the value of sig. (2-tailed) 0.000 was less than the 0.05 significance level. Since the value was smaller than the significance level, so that there was a significant relationship between conceptual understanding and procedural knowledge in problem solving polyhedral.

3.2 Discussion
The results showed that SV profile of conceptual understanding in solving geometry problems was not able to meet six indicators of conceptual understanding, but he fulfilled two indicators, which were stating the concept and classifying an object according to certain characteristics according to the concept. SV still can write what is known and asked more correctly can be seen in Figure 3 which is incorrectly written height. SV is also wrong in compiling the planning of completion by incorrectly writing the prism volume formula so that it can be said that SV does not understand the concept of prism. SV profiles with procedural knowledge have not been able to meet the three indicators of procedural knowledge. That is because SV has not been able to use procedures correctly, has not been able to communicate the problem solving process correctly and also has not been able to check every step of problem solving and make improvements to the solutions that have been made if an error is encountered.

Next, the profile of SA with conceptual understanding in solving geometry problem fulfilled four out of six indicators of conceptual understanding. SA very understands of the questions given, so the SA can write what is known and asked well and coherently. SA also understands what is in the problem, it's just a different mention. SA is very confident with the answer, although there are a few answers that are not so simple that the resulting answer is quite complicated. SA cannot resolve the problem until the end. SA profile with procedural knowledge is only able to fulfill three indicators of procedural knowledge. SA can write the completion procedure correctly and also can make plans or strategies that are appropriate in solving problems in the form of mathematical models. The SA can also explain or communicate the reasons for each of the steps in resolving the problem of polyhedral. It's just that the SA has not been able to check every step of the problem solving and make improvements to the solutions that have been made if an error is encountered.

While SD’s profile with conceptual understanding in solving geometry problems met six indicators of conceptual understanding. SD very understands of the questions given. SD can also understand the concept of the parallelepipedum and can also know the structure given to the problem. SD can answer the questions well, correctly and coherently. The SD profile with procedural knowledge also fulfills all indicators of procedural knowledge. This can be seen from the answers given by students and the interview process. SD is very coherent and neat in completing completion. SD can write what is known and asked correctly, can make appropriate settlement strategies, can explain the reasons for each step in solving problems and is also able to make improvements to the algorithms that have been made. this study was in line with the study conducted by [13] showing that there are individual differences in conceptual and procedural knowledge. Research conducted by [4], states that the Conceptual Understanding and Procedural Knowledge of Linear Inequality Material of One Variable Class VII Middle School Students can be concluded that the conceptual understanding of Grade VII students is still very low and students' procedural knowledge is also mostly students at very low levels.

The value of geometry test results shows that the conceptual understanding and procedural knowledge are interconnected. One can not solve a problem with the right procedure because it does not have a conceptual understanding, and vice versa. This can be seen in the person correlation coefficient of 0.515. In addition, the value of sig. (2-tailed) 0.000 was less than the 0.05 significance level. Since the value was smaller than the significance level, H0 was accepted. This meant that there was a significant relationship between conceptual understanding and procedural knowledge in problem solving polyhedral. The data also showed that there was a positive relationship between conceptual understanding and students' procedural knowledge, which meant that if conceptual understanding was high, procedural knowledge was also high and vice versa. This study was in line with the study conducted by [5] showing that the level of procedural and conceptual knowledge of
students is weak and that there is a significant relationship between conceptual knowledge and procedural knowledge in problem solving. Thus, students must master both so that understanding and skills in solving mathematical problems can be improved.

4. Conclusions
Based on the research results and discussion, indicate that the level of visualization tends to not be able to understand the problem well, the level of analysis is quite capable of understanding the problem well, while the level of informal deduction is able to understand the problem well. This can be seen from the results of research that shows that the students' level of visualization fulfilled two of the six indicators of conceptual understanding and none of them meet the indicators of procedural knowledge. Students at the analysis level met four of the six indicators of conceptual understanding and one of the three indicators of procedural knowledge, while students with informal deduction level met all indicators of conceptual understanding and procedural knowledge. The higher the level of thinking geometry the better students are conceptual understanding and procedural knowledge. The results also showed that there was a significant relationship between conceptual understanding and procedural knowledge that was 0.515 and the sig (2-tailed) value of 0.000 was smaller than the significant level $\alpha = 0.05$ with a 95% confidence level. The researchers suggest students must master both so that understanding, knowledge and skills in solving mathematical problems can be improved. In addition, an appropriate learning strategy is also needed.

Based on the results of this study, we propose the following suggestions for further research:
1. What learning strategies can improve students' conceptual understanding and procedural knowledge based on each level of van hiele?
2. Can students with each level of van hiele apply the same learning strategy?

Acknowledgment
I would like to express my gratitude to Geometry research group, Postgraduate Program of Mathematics Education, Jember, Indonesia.

References
[1] Crooks N M and Alibali M W 2014 Defining and measuring conceptual knowledge in mathematics. Developmental Review, 34(4) pp.344-377
[2] Hiebert J and Lefevre P 1986 Conceptual and procedural knowledge in mathematics: An introductory analysis. Conceptual and procedural knowledge: The case of mathematics 2 pp.1-27
[3] Krathwohl D R and Anderson L W 2009 A taxonomy for learning, teaching, and assessing: A revision of Bloom's taxonomy of educational objectives Longman
[4] Suratman D 2011 Pemahaman Konseptual dan Pengetahuan Prosedural Materi Pertidaksamaan Linear Satu Variabel Siswa Kelas VII SMP (Studi Kasus di Mts. Ushuluddin Singkawang. Jurnal Cakrawala Kependidikan 9(2)
[5] Surif J, Ibrahim N H and Mokhtar M 2012 Conceptual and procedural knowledge in problem solving. Procedia-Social and Behavioral Sciences 56 416-425
[6] Abdussakir A 2012 Pembelajaran Geometri Sesuai Teori Van Hiele. Madrasah: Jurnal Pendidikan dan Pembelajaran Dasar, 2(1)
[7] Van de Walle J A, S Folk, K S Karp, dan J M Bay-Williams 2013 Elementary and Midle School Mathematics: Theaching Developmentally. 8th Ed. Boston: Pearson
[8] E Yudianto, T Sugirarti, and D Trapsilasivi 2018 The identification of van Hiele level students on the topic of space analytic geometry, J. Phys.: Conf. Ser. 983 012078
[9] Sunardi 2000 Tingkat Perkembangan Konsep Geometri Siswa Kelas 3 SLTPN di Jember. In Prosiding Komperensi Nasional X Matematika
[10] Sumarmo U 2005 Pembelajaran matematika untuk mendukung pelaksanaan kurikulum tahun 2002 sekolah menengah. In Makalah Seminar di FPMIPA Universitas Negeri Gorontalo
[11] Polya G 1973 How to Solve it Nueva Jersey, Estados Unidos
[12] Sugiyono 2017 Metode Penelitian Kombinasi (Mixed Method). Bandung, Indonesia: AlfaBeta.
[13] Hallett D, Nunes T, Bryant P and Thorpe C M 2012 Individual differences in conceptual and procedural fraction understanding: The role of abilities and school experience Journal of Experimental Child Psychology 113(4) 469-486