Abstract

We construct R-charged adS bubbles in $D = 5, \mathcal{N} = 8$ supergravity. These bubbles are characterised by four parameters. The asymptotic boundary of these solutions are deSitter times a circle. By comparing boundary energies, we study the possibility of a transition from certain class of black holes to these bubbles below a critical radius of the boundary circle. We argue that this may occur when four parameters of the bubble satisfy a constraint among themselves.

*Electronic address: anindyab, tanay, mukherji@iopb.res.in
In this Letter, we first construct the bubble solutions of $D = 5$, $\mathcal{N} = 8$ gauged IIB supergravity \cite{1, 2}. These bubbles carry the R-charges that correspond to the three $U(1)$ Cartan subalgebra of the gauge group $SO(6)$. At asymptotic boundary, these solutions approach $S^1 \times dS^3$. This is similar to the adS-Schwarzschild bubbles \cite{4, 5}. We then compute various components of the boundary stress-energy tensors associated with these R-charged bubbles. One of the main motivation for us to study these bubbles is an issue pointed out in \cite{8}. It was found earlier that a certain class of five dimensional black holes \cite{9, 10} (which are higher dimensional analogue of BTZ black holes) had same asymptotic boundary metric as that of the adS-Schwarzschild bubbles. As far as the boundary geometry is concerned, the only difference is that, for the black hole, the boundary radius of the $S^1$ (say parametrised by $\chi$) has no restriction. However, for the bubble, the size of $S^1$ has an upper bound (say $\Delta \chi_c$). Otherwise, for $\Delta \chi > \Delta \chi_c$, the space-time becomes singular\footnote{We will always assume antiperiodic boundary conditions of the fermions along $\chi$ for both the orbifold and the bubble.}. Furthermore, it was found in \cite{11} that the components of the boundary stress tensor of these holes are exactly same as that of a bubble in the limit of zero bubble radius. In general, however, the energy of the bubble space-time is lower than that of a black hole. Now suppose that we start with a black hole with large $\Delta \chi$ which parametrises the period of $S^1$ at the boundary. As we tune $\Delta \chi$ below the critical value $\Delta \chi_c$, the bubble solution for the bulk becomes available whose energy is less than that of a black hole. One would then expect a transition from the black hole to the bubble configuration. In the case of R-charged bubble, we also find a possibility of such a transition. Here the transition is infact from the same adS orbifold (with some radiation of massless matter) to R-charged bubble below certain critical radius of the boundary circle. In the following we construct the bubbles and then discuss the possibility of such a transition. In particular, this happens when the parameters specifying the solution satisfy certain relation among themselves. Due to the conjectured dS/CFT correspondence, one would then expect to see the signature of such a transition in the gauge theory on the boundary. One would also like to understand this phenomenon better in terms of string theory in the bulk. Neither of these issues are clear to us at this moment. We only offer some possibilities at the end of this Letter.

The bubble solution in $D = 5$, $\mathcal{N} = 8$ gauged type IIB supergravity has the following form\footnote{This bubble metric can be obtained by analytic continuation $t \to i\chi, \theta \to \frac{\pi}{2} + i\tau$ and $\beta_i \to i\beta_i$ of the $R$-charged black holes constructed in \cite{11, 12}. In literature, there are other analytic continuations of black holes in adS space giving rise to magnetic flux branes, see for example \cite{12}.}:

\[
\begin{align*}
    ds^2 &= H^{-2/3}f d\chi^2 + H^{1/3}(f^{-1}dr^2 - r^2d\tau^2 + r^2 \cosh^2 \tau d\Omega_2^2), \\
    H_i &:= 1 - q_i/r^2, \\
    H &= \prod_{i=1}^{3} (1 - q_i/r^2), \\
    f &= 1 - \frac{\mu}{r^2} + \frac{r^2}{l^2}H.
\end{align*}
\]

(1)

where

\[
\begin{align*}
    H_i &= 1 - q_i/r^2, \\
    H &= \prod_{i=1}^{3} (1 - q_i/r^2), \\
    f &= 1 - \frac{\mu}{r^2} + \frac{r^2}{l^2}H.
\end{align*}
\]

The three gauge field potentials $A^i_\mu$ are of the form

\[
A^i_\chi = \frac{\tilde{q}_i}{r^2 - q_i} - \frac{\tilde{q}_i}{r_+^2 - q_i}.
\]

\footnote{This bubble solutions in flat space-time were constructed and analysed in \cite{6, 7}.}
where $q_i$ and $\tilde{q}_i$ are given by

$$ q_i = \mu \sin^2 \beta_i, \quad \tilde{q}_i = \mu \sin \beta_i \cos \beta_i $$

(6)

and $r_+$ is given by the largest positive root of the equation

$$ 1 - \frac{\mu}{r^2} + \frac{r^2}{l^2} \left(1 - \frac{q_1}{r^2}\right) \left(1 - \frac{q_2}{r^2}\right) \left(1 - \frac{q_3}{r^2}\right) = 0. $$

(7)

Note that we have added a constant term in the gauge potential such that it vanishes at $r = r_+$. Furthermore, we notice from (5) that the gauge fields become singular at $r = \sqrt{q_i}$. However, if we choose

$$ \mu \geq q_i, $$

(8)

the singularity occurs at values of $r$ which is always less than $r_+$. This is indeed the case as can be seen from (6). We also have three scalars $X_i$ associated with the configuration. They are given by:

$$ X_i = H^{-1}_i H^\frac{1}{2}_i - (H^{-1}_i H^\frac{1}{2}_i)_{\text{at } r=r_+}. $$

(9)

As $H$ goes to unity at large $r$, by scaling $l^2/r^2$, the boundary metric can be written as

$$ ds^2 = d\chi^2 + l^2(-d\tau^2 + \cosh^2 \tau d\Omega_2^2) $$

(11)

This is $S^1 \times dS^3$. The radius of the circle is given by $\frac{\Delta \chi}{2\pi}$. On the other hand, at $r = r_+$, the circle, parametrised by $\chi$, collapses. However, the two sphere approaches a finite size $r_+ \cosh^2 \tau$. This solution, therefore, corresponds to a bubble of radius $r_+$ of $d = 5$, $\mathcal{N} = 8$ gauged supergravity. Note that the above configuration reduces to the adS-Schwarzschild bubble of [3] for $q_i = 0^*$. We note here that even though the R-charged adS black holes can have supersymmetric limit (where mass and the charges are related in a specific way), the analytically continued bubbles are inherently non-supersymmetric. Supersymmetry is broken here due to the antiperiodic boundary condition on the fermions along $\chi$.

Now we will study the bubble solution for three distinct cases: (1) $q_1 = q$, $q_2 = q_3 = 0$, (2) $q_1, q_2 \neq 0$, $q_3 = 0$ and (3) $q_1, q_2, q_3 \neq 0$.

**Case 1:** $q_1 = q$, $q_1 = q_2 = 0$

Defining $k_1 = 1 - q/l^2$, we get the radius of the single charged bubble is given by

$$ r_+^2 = -\frac{k_1 l^2}{2} + \frac{l}{2} \sqrt{k_1^2 l^2 + 4 \mu} $$

(12)

Since $\mu \geq q$, $r_+ \geq \sqrt{q}$ and the periodicity is

$$ \Delta \chi_1 = \frac{2\pi l^2 \sqrt{(r_+^2 - q)}}{2r_+^2 + (l^2 - q)}. $$

(13)
For later use, we now discuss the nature of $\Delta \chi_1$ as a function of the bubble radius $r_+$. First consider $q \leq l^2$. $\Delta \chi_1$ starts from zero at $r_+ = \sqrt{q}$ and goes to zero for large $r_+$. It has a maximum at

$$r_+ = r_c = \sqrt{\frac{l^2 + 3q}{2}},$$

with

$$\Delta \chi_{1c} = \frac{\pi l^2}{\sqrt{2(l^2 + q)}}.$$ \hspace{1cm} (15)

The behaviour of $\Delta \chi_1$ is shown in the following figure.

![Plot of $\Delta \chi_1$](image)

Figure 1: Plot of $\Delta \chi_1$ vs. $r_+$ for $l = 1$, and $q = 0.5$

If $q > l^2$, denominator of $\Delta \chi_1$ changes sign at $r_+ < \sqrt{q}$, but this is not allowed value. So the behaviour of $\Delta \chi_1$ is same for all cases.

**Case 2: $q_1, q_2 \neq 0, q_3 = 0$**

As before, we define $k_2 = 1 - (q_1 + q_2)/l^2$, and get the radius of the double charged bubble. This is

$$r_+^2 = -\frac{k_2 l^2}{2} + \frac{l}{2} \sqrt{k_2^2 l^2 + 4\mu - \frac{4q_1 q_2}{l^2}}.$$ \hspace{1cm} (16)

Since $\mu \geq q_i$ then $r_+^2$ is always greater or equal to largest $q_i$. The periodicity is given by

$$\Delta \chi_2 = \frac{2\pi l^2 \sqrt{(r_+^2 - q_1)(r_+^2 - q_2)}}{2r_+^2 + r_+(l^2 - q_1 - q_2)}.$$ \hspace{1cm} (17)

As for $\Delta \chi_2$, when $(q_1 + q_2) \leq l^2$, we get the two different branches of periodicity where for small $r_+$, $\Delta \chi_2$ starts from infinity and goes zero at $r_+ = \sqrt{q_2} \ (q_2 < q_1)$ and the second branch starts from $r_+ = \sqrt{q_1}$ goes to maximum value and then reaches to zero for large $r_+$. Here we give the plot for the case where $q_1 = q_2 = q$. 

4
The zero in the plot is at \( r_+ = \sqrt{q} \) and there is a maximum at

\[
r_+ = r_c = \sqrt{\frac{l^2}{4} + q + \frac{l}{4}\sqrt{l^2 + 16q}},
\]

with

\[
\Delta \chi_2 c = \frac{\pi l^2 (l + \sqrt{l^2 + 16q})}{(\frac{l^2}{4} + q + \frac{l}{4}\sqrt{l^2 + 16q})^{\frac{3}{2}(3l + \sqrt{l^2 + 16q})}}.
\]

Note that the region below \( r_+ = \sqrt{q} \) is not an allowed region for the bubble space-time. This can be seen from (16) as \( \mu \geq q \). On the other hand if \( q_1 + q_2 > l^2 \), from (17), we see that \( \Delta \chi_2 \) becomes singular at some value of \( r_+ < \sqrt{q_1} \). This is again not an allowed region. The nature of \( \Delta \chi_2 \) in the allowed region is same as the earlier one.

**Case 3: \( q_1, q_2, q_3 \neq 0 \)**

Next we consider the bubbles for three non-zero charges. We find that \( \Delta \chi_3 \) has three zeros and there are two branches of periodicity. The zeroes and the maxima merge into a single zero and single maximum respectively by choosing the equality among all three charges. The nature of \( \Delta \chi_3 \) would then be same as \( \Delta \chi_1 \).

Before we proceed further, we would like to make few comments on the stability of R-charged bubble solution. In [6, 7], it was argued that the bubble solutions are quantum mechanically unstable while being classically stable. As for classical stability, we can explicitly show in our case that a massless scalar on this charged bubble background does not have a normalisable mode with negative mass square on the deSitter factor\( ^{\dagger} \). The quantum stability of these bubbles are more difficult to analyse. However, as in [5], we note that these bubbles can be embedded in

\[\begin{align*}
(f_{rr}^3)\partial_{r^2}\phi + \partial_r (r^3 f) \partial_r \phi &= -M^2 r \phi,
\end{align*}\]

\( \dagger \) The equation of motion of a massless scalar field in this background (with massless excitations along \( \chi \)) is given by
global adS space. Because of the periodicity along $\chi$, one needs to identify surfaces in the global adS space. This in turn makes the space non-smooth. This may prevent nucleation of additional bubbles.

We now compute the energy associated with this charged bubbles. Using counter term subtraction procedure of [13, 14], the boundary stress tensor can easily be evaluated. For generic values of charges, the components of the stress tensor are given by:

$$T_\chi = -\frac{3}{16\pi G_5 l_5^3} \left[ \mu + \frac{l^2}{4} - \frac{2}{3} (q_1 + q_2 + q_3) \right], \quad (21)$$

$$T_\tau = \frac{1}{16\pi G_5 l_5^3} \left[ \mu + \frac{l^2}{4} - \frac{2}{3} (q_1 + q_2 + q_3) \right], \quad (22)$$

$$T_\phi = T_\psi = \frac{1}{16\pi G_5 l_5^3} \left[ \mu + \frac{l^2}{4} - \frac{2}{3} (q_1 + q_2 + q_3) \right]. \quad (23)$$

These components reduce to adS-Schwarzschild bubble if we set the charges $q_i$ to zero. In general, it also follows that due to the presence of the charges, the energy of this bubble is larger than the adS-Schwarzschild one. This can be seen explicitly from (22).

Let us now focus our attention on another class of metric. These are adS orbifolds—a version of BTZ black hole in five dimensions. This class of space-time was extensively studied in [9, 10]. The asymptotic boundary of these black holes are same as the bubbles that we have been discussing so far. The only difference as far as the boundary structures are concerned, unlike the bubbles, for the black holes, the period of $\Delta \chi$ can take any value. Boundary stress tensor of these holes were computed in [11]. They are given by the expressions (21) –(23) with $\mu, q_i = 0$. Let us now consider the boundary metric $S^1 \times dS^3$ for large $S^1$ along with some radiation matter given by $A^i$ and $X_i$. Since, for bubbles, $\Delta \chi$ has a maximum critical value, the only solutions that are available in the bulk for large $S^1$ (or in other words, for large $\Delta \chi$) are the black holes. However, as we shrink $S^1$ down to lower radius, the R-charged bubbles also become available below a critical radius given by the maximum of the expression in (10). Infact, the energy associated with the bubble is lower than the black hole if

$$\mu > \frac{2}{3} (q_1 + q_2 + q_3). \quad (24)$$

This follows from equation (22). One would therefore expect a transition in the bulk from the black hole to the bubble below a critical radius.

Two questions then immediately arise: (1) What triggers this vacuum to vacuum transition(if at all) in string theory formulated around the background with boundary metric $S^1 \times dS^3$? (2) Through dS/CFT correspondence, can we understand this transition in gauge theory? Neither of these issues are clear to us **. However, it is tempting to speculate that in string theory, it may arise due to instabilities coming from the winding modes along $S^1$. On the other hand, in gauge theory on $dS^3 \times S^1$, it may arise due to the condensation of an order parameter related to bubble radius in the bulk.

---

**Some understanding along this direction will appear in a forthcoming paper [15].**

---

where $f = 1 - \mu r^{-2} + r^2 l^{-2} H$, and $M$ satisfies $\nabla^2_{dS_5} \phi = M^2 \phi$. Now by studying this equation at $r \to \infty$ and $r \to r_+$, one easily checks that there is no normalisable mode for $M^2 < 0$.**
Acknowledgments: We wish to thank Balram Rai for explaining us some of his unpublished works. We have also benefited from discussions with Sumit Das and Ashoke Sen on issues related to this work.

References

[1] K. Behrndt, M. Cvetic and W. Sabra, Nucl. Phys. B553, (1999) 317, hep-th/9810227
[2] M. Cvetic and S.S. Gubser, JHEP 9904 (1999) 024, hep-th/9902195
[3] S.S. Gubser and J. Heckman, “Thermodynamics of R-charged black Holes in AdS(5) from effective strings”, hep-th/0411001
[4] D. Birmingham, M. Rinaldi, Phys. Lett. B544 (2002) 316, hep-th/0205246
[5] V. Balasubramanian and S.F. Ross, Phys. Rev. D66 (2002) 086002, hep-th/0205290
[6] E. Witten, Nucl. Phys. B195 (1982) 481.
[7] O. Aharony, M. Fabinger, G. Horowitz and E. Silverstein, JHEP 0207 (2002) 007, hep-th/0204158
[8] S. F. Ross and G. Titchener, “Time-dependent spacetime in AdS/CFT: bubble and black hole”, hep-th/0411128
[9] M. Banados, Phys. Rev. D57 (1998) 1068, gr-qc/9703040
[10] M. Banados, A. Gomberoff and C. Martinez, Class. Quant. Grav. 15 (1998) 3757, hep-th/9805087
[11] R.-G. Cai, Phys. Lett. B544 (2002) 176, hep-th/0206223.
[12] H. Lu, C.N. Pope and J.F. Vazquez-Poritz, Nucl. Phys. B709 (2005) 47, hep-th/0307001
[13] V. Balasubramanian and P. Kraus, Commun. Math. Phys. 208 (1999) 413, hep-th/9902121
[14] J.T. Liu and W. Sabra, “Mass in anti-deSitter space”, hep-th/0405171
[15] B. Rai, to appear.