Knowledge model: a method to evaluate an individual’s knowledge quantitatively

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As the quantity of human knowledge increasing rapidly, it is harder and harder to evaluate a knowledge worker’s knowledge quantitatively. There are lots of demands for evaluating a knowledge worker’s knowledge. For example, accurately finding out a researcher’s research concentrations for the last three years; searching for common topics for two scientists with different academic backgrounds; helping a researcher discover his deficiencies on a research field etc. This paper first proposes a method named knowledge model to evaluate a knowledge worker’s knowledge quantitatively without taking an examination. It records and analyzes an individual’s each learning experience, discovering all the involved knowledge points and calculating their shares by analyzing the text learning contents with topic model. It calculates a score for a knowledge point by accumulating the effects of one’s all learning experiences about it. A preliminary knowledge evaluating system is developed to testify the practicability of knowledge model.
1 Introduction

The amount of human knowledge is rapidly increasing. Almost every discipline has been subdivided into lots of sub-disciplines. In information age, humans, especially knowledge workers, need to keep learning during their whole lives. There are a lot of demands for knowledge workers to estimate their knowledge quantitatively. The following are some examples:

- A computer engineer wants to estimate how much he has obtained the collection of concepts and algorithms of the curriculum “Information Retrieval”;

- A researcher wants to predict how much he will understand the contents of a lecture just from its poster, a subsequent decision of whether to attend it will be made based on the prediction;

- Two researchers with different academic backgrounds want to find out the set of knowledge points on which they both have a solid understanding, these knowledge points can serve as the starting point of an academic communication.

- A scientist wants to have a quantitative evaluation of his research concentrations for the last three years.

Most of an individual’s knowledge is obtained from postnatal learning. By recording and analyzing one’s learning history, it is possible to estimate his knowledge quantitatively.

1.1 Classification of an individual’s activities

To analyze one’s learning history, an individual’s daily activities are classified into two categories: learning activities and non-learning activities. Learning activities are those which are related to at least one piece of knowledge. The definitions of knowledge and non-knowledge
will be explained in section 1.4.1. Examples of learning activities are reading books, taking courses, discussing with someone about a piece of knowledge etc.

1.2 Capturing the text learning contents

Most learning processes can be associated with a piece of learning material. For example, reading a book, the book is the learning material. Taking a course or having a discussion, the course and discussion contents can be regarded as the learning material. Some of the learning materials are text or can be converted to text. For example, discussing about a piece of knowledge with others. The discussion contents can be converted to text by exploiting speech recognition technologies. Similarly, if one is reading a printed book, the contents of the book can be recorded by a camera like Google Glass (1), then converted to text by utilizing Optical Character Recognition (OCR) technology. If the book is an electronic one, no conversion is needed, text can be extracted directly.

1.3 Analyzing the text content with topic models

Having the extracted or converted text, with topic models, the main ideas of the text can be obtained in a quantitative manner (2). With probabilistic topic models, the main ideas of a piece of text can be computed as a distribution over a series of topics. Each topic is expressed as a word distribution over a vocabulary set. With the calculated topic distribution and word distribution, further analyzing of knowledge model is available.

1.4 Analyzing the learning history with knowledge model

Knowledge model can quantitatively evaluate an individual’s knowledge based on his learning history.
1.4.1 Organization of human knowledge

In knowledge model, all the knowledge pieces are organized in a tree structure. Every node of the knowledge tree can be referenced by a name. A branch node represents a discipline or sub-discipline of knowledge, such as math, computer science, and information retrieval etc. A leaf node represents a concrete piece of knowledge, which is explicit defined and has been widely accepted by the academic society, such as Bayes’ theorem, Mass-energy equivalence, Expectation-maximization algorithm etc. A leaf node of the knowledge tree is called a knowledge point. A branch node of it is called a knowledge branch.

The knowledge tree can be constructed and maintained empirically by a group of experts of each discipline. Fig. 1 is an example of the knowledge tree based on a classification of Wikipedia (3). To keep it simple, other nodes of the tree are omitted.

1.4.2 Learning sessions

An individual’s learning activities can be separated into a series of learning sessions based on some specific standards, such as intervals between activities or topics of activities. Details of how to discriminate learning sessions will be discussed in section 3. Table 1 illustrates some
Table 1: Some examples of learning sessions

| Date      | Activities                  | Duration(S) | Captured text contents                                                                 |
|-----------|-----------------------------|-------------|----------------------------------------------------------------------------------------|
| 2016-03-13| Started reading a document  | 3610        | ... Probabilistic models, such as hidden Markov models or Bayesian networks, are commonly ... |
| 2016-03-13| Stopped reading the document|             |                                                                                        |
| 2016-03-13| Started attending a class   | 2710        | ... how does the expectation maximization algorithm work ...                             |
| 2016-03-13| Stopped attending the class |             |                                                                                        |
| 2016-03-13| Started a discussion        | 930         | ... I think your understanding of Bayes’ theorem is wrong ...                            |
| 2016-03-13| Stopped the discussion      |             |                                                                                        |

examples of learning sessions.

1.4.3 An individual’s learning history

Each individual has a knowledge tree which records his learning history about each knowledge node. Each node of the tree has a data structure which records the individual’s every learning experience about the corresponding knowledge point or knowledge branch. Each recorded learning experience has the following 4 attributes:

- **Learning sequence ID**
  
  Recording the sequence ID of the learning experience.

- **Stop time**
  
  Recording when the learning session stopped.

- **Duration**
  
  Recording the duration time of a learning session.
Table 2: A subject’s learning history of the knowledge point “Bayes' rule”

| Learning sequence ID | Learning stop time     | Duration(S) | Proportion |
|----------------------|------------------------|-------------|------------|
| 1                    | 2/27/2016 18:41        | 1171        | 1.22%      |
| 2                    | 2/27/2016 18:47        | 220         | 2.12%      |
| 3                    | 2/29/2016 16:08        | 2523        | 1.17%      |
| 4                    | 2/29/2016 16:55        | 330         | 0.66%      |
| 5                    | 3/3/2016 16:21         | 1710        | 1.17%      |

- Proportion

  Recording the knowledge point’s share of the learning contents. The calculation of the proportion is based on results of topic model analysis, details of calculation will be discussed in section 3.

Table 2 is an example of learning history, it is a snippet of a subject’s learning history of the knowledge point “Bayes' rule”.

1.4.4 Calculation of an individual’s familiarity measure about a knowledge point

With an individual’s learning history of a knowledge point, it is possible to measure the individual’s familiarity of the knowledge point. There is no unanimous agreement of how previous learning experiences affect an individual’s current understanding of a knowledge point exactly. Therefore, there are many choices of calculating the familiarity measure. Details of calculation will be discussed in section 3.

Figure 2 illustrates a flowchart of using topic model and knowledge model to estimate an individual’s knowledge quantitatively. Each hexagon of the diagram indicates a step of processing, the following rectangle indicates the results of the processing.

A preliminary system of evaluating an individual’s knowledge is implemented in section 3.
Figure 2: A flowchart to estimate an individual’s knowledge quantitatively.
2 Related works

Recording an individual’s learning history is vital for knowledge model. Bush envisioned the memex system in which individuals could compress and store all of their personally experienced information, such as books, records, and communications (4). Inspired by memex, Gemmell et al. developed a project named ‘MyLifeBits’ to store all of a person’s digital media, including documents, images, sounds, and videos (5). Knowledge model shares a similar idea with memex and ‘MyLifeBits’ of recording an individual’s digital history, but with a different intention. Memex and ‘MyLifeBits’ are mainly for re-finding or review of personal data, knowledge model is for quantitatively evaluating a knowledge worker’s knowledge.

Probabilistic topic model. Probabilistic topic model is used to analyze the topics of a collection of text documents. Each topic is represented as a multinomial distribution of words over a vocabulary set. Each document is represented as a distribution over the topics (2, 6). Probabilistic latent semantic analysis (PLSA) (7) and Latent Dirichlet Allocation (LDA) (8) are two representative probabilistic topic models. PLSA models each word of a document as a sample from a mixture model. It has a limitation that parameterization of the model is susceptible to over-fitting. In addition, it cannot provide a straightforward way to make inferences about new documents (9). LDA is an unsupervised algorithm that models each document as a mixture of topics. It addresses some of PLSA’s limitations by adding a Dirichlet prior on the per-document topic distribution.

Forgetting curve. Human memory declines along time. In 1885, Hermann Ebbinghaus hypothesized the exponential nature of forgetting (10). Ebbinghaus found Equation 1 can be used to describe the proportion of memory retention after a period of time, \( t \) is the time in minutes counting from one minute before the end of the learning, \( k \) and \( c \) are two constants which equal
Figure 3: The percentage of memory retention in time calculated by Equation 1.

1.84 and 1.25 separately (II).

\[ b = \frac{k}{((\log t)^c + k)} \]  

(1)

Figure 3 shows the percentage of memory retention in time calculated by Equation 1. Averell and Heathcote proposed other forms of forgetting curves. There is no unanimous agreement of how human memory declines. Psychologists have debated the form of the forgetting curve for a century (I2).

\section{A preliminary knowledge evaluating system}

A preliminary knowledge evaluating system is developed to test the feasibility of knowledge model. Because of the complexity of human learning activities and the workload of programming, it is impractical to handle all the learning situations once and for all. Therefore, it only handles the situation that a user is reading Portable Document Format (PDF) documents. Other document formats and learning methods like listening and discussing will be considered in fur-
ther research.

A plug-in for the Adobe Acrobat Reader application is developed. With the plug-in, the system can detect an individual’s PDF reading activities, then divides them into a sequence of learning sessions. Meanwhile, it extracts the text contents of each learning session, then uses topic model to analyze the topics of the text contents, and then selects the topics which are knowledge points, finally, it updates the individual’s learning histories of related knowledge points. With the learning histories, the individual’s familiarity measure of each knowledge point at time $t$ can be calculated with knowledge model.

3.1 An algorithm to discriminate an individual’s learning sessions

Discriminating learning sessions is critical to knowledge model, because it is essential to know how many times and how long for each time the individual has learned a knowledge point. Further analyses are based on these results. To detect learning sessions, the algorithm periodically checks what the individual is doing. Opening of a document indicates a learning session has started. If either of the following 4 conditions is satisfied, a learning session is assumed terminated.

1. If the foreground window has switched to another application (APP) from the Adobe Acrobat Reader application;

2. 1 has not happened, but the computer has idled for a certain period of time without any mouse or keyboard inputs detected, the individual is assumed having left for other things;

3. Both 1 and 2 have not happened, but all the opening documents have been closed, there is no active document;

4. All of the above events have not happened, but the current active document is different from the last one, that implies the individual has switched from one document to another.
It is assumed there is a learning session switch too.

Algorithm 1 shows how learning sessions are discriminated. The algorithm periodically checks the current active document and current active page, and compares with the last active document and page, a switch of documents indicates a switch of learning sessions. DocIsActive is flag variable which indicates whether there is an opening learning session. CheckInputInterval is a function which calculates the interval between current time and the individual’s last mouse or keyboard input time. The threshold of the input interval can be estimated by analyzing the individual’s input patterns (13). DocActivate, DocDeactivate, and PageActivate are three functions which record the dates of corresponding actions into a database. Function Sleep keeps the plug-in procedure silent for a while. The duration of a learning session equals the interval between its start and stop time. Recording of page numbers is for extracting learning contents, which will be analyzed with topic models.

Figure 4 shows some examples of discriminated learning sessions. Attribute “did” means document ID, which indexes the documents uniquely. Attribute “actiotype” indicates the type of an action. “Doc Act” means a document has been activated. “Page Act” is defined similarly. “Doc DeAct” means a document has been deactivated. That is to say, a learning session has stopped. Attribute “page” indicates a page number. Attribute “duration” records how long a page has been activated in seconds. If two learning session’s interval is less than a certain threshold, such as 30 minutes, and their learning material is the same, for example, the same document, they are merged into one session. Therefore, “Session2” and “Session3” are merged into one session.

3.2 Analyzing the learning contents with topic model

When discriminating learning sessions, text learning contents are also extracted. Because Algorithm 1 can record the accurate set of pages the individual has read during a learning session,
Algorithm 1 An algorithm to discriminate one’s learning sessions.

DocIsActive = false, ActiveDoc = LastDoc = LastPage = NULL;
while PDF Reader APP is running do
    CheckForegroundWindow();
    if PDF Reader APP is the foreground window then
        CheckInputInterval();
        if InputInterval < IntervalThreshold then
            CurrentDoc = AppGetCurrentDoc();
            if CurrentDoc != NULL then
                ActiveDoc = CurrentDoc;
                CurrentPage = AppGetCurrentPage();
                if ActiveDoc == LastDoc then
                    if DocIsActive == false then
                        DocActivate(ActiveDoc);
                        PageActivate(ActiveDoc,CurrentPage);
                        DocIsActive = true;
                    else
                        if CurrentPage != LastPage then
                            PageActivate(ActiveDoc,CurrentPage);
                            LastPage = CurrentPage;
                        end if
                    end if
                else
                    if (DocIsActive == true) && (LastDoc != NULL) then
                        DocDeactivate(LastDoc);
                    end if
                    DocActivate(ActiveDoc);
                    PageActivate(ActiveDoc,CurrentPage);
                    DocIsActive = true;
                    LastDoc = ActiveDoc;
                end if
            end if
        end if
    end if
    if (DocIsActive == true) && (ActiveDoc != NULL) then
        DocDeactivate(ActiveDoc);
        DocIsActive = false;
    end if
    Sleep(T); //T is set 10 seconds currently
    continue;
end while

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only the related pages’ text contents are extracted. This strategy brings in less errors than extracting all the text contents of the whole document. Because a document may contain many pages, usually some of them are not read during a learning session, it is unreasonable to count them in.

The inputs of a probabilistic topic model are a collection of $N$ documents, a vocabulary set $V$, and the number of topics $k$. The outputs of a probabilistic topic model are the followings:

- $k$ topics, each is word distribution: $\{\theta_1, ..., \theta_k\}$;
- Coverage of topics in each document $d_i$: $\{\pi_{i1}, ..., \pi_{ik}\}$;
  \[\pi_{ij}\] is the probability of document $d_i$ covering topic $\theta_j$.

In the implementation, $N$ is set 1 because there is only one document during a learning session, $k$ is set 2 currently. The LDA analysis of learning contents is based on the implementation
of MeTA, which is an open source text analysis toolkit (14).

Before topic model analysis, the text learning contents are scanned to find out the word group which is a multi-word knowledge point, such as “inverse document frequency” (IDF). The word group is then merged into one word like inverse-document-frequency. After the merging of multi-word knowledge points, the text contents are analyzed with the unigram method of LDA.

### 3.3 Computation of a knowledge point’s share of the learning contents

Topic model can calculate each topic’s contribution to the learning contents and each term’s share of a topic. Each knowledge point can be allocated with a share based on its topic share. The share is an estimation of how much the learning contents concern the knowledge point. Only the top $m$ terms of each topic are considered. Each related topic term’s share is calculated with Equation 2. $\varphi_{ij}$ is the share of term $i$ of topic $j$, $\pi_j$ is topic $j$’s share of the learning contents, $p(t_i|\theta_j)$ is term $i$’s share of topic $j$. A knowledge point’s share equals its topic term share.

$$\varphi_{ij} = \frac{\pi_j p(t_i|\theta_j)}{\sum_{j=1}^{k} \sum_{i=1}^{m} \pi_j p(t_i|\theta_j)}$$ (2)

### 3.4 Computation of the familiarity measure of a knowledge point at a particular time

With the recognized learning sessions and the results of topic model analysis, an individual’s learning history of a knowledge point can be generated. Table 2 shows an example of an individual’s learning history of a knowledge point. With the learning history, there are many choices to calculate the individual’s familiarity measure of a knowledge point. The simplest method is just considering the cumulative learning time of each knowledge point, multiplied by its corre-
sponding share in each learning session. However, human brain works in a very complicated manner when learning. A lot of factors affect how effective an individual can learn a knowledge point. For example, human memory declines. There is much difference between learning a knowledge point yesterday and three years ago. Moreover, subsequent learning of a knowledge point will be associated with what have been learned previously. A simplified method of calculating familiarity measures is used in this preliminary implementation. The computation is based on the following hypotheses:

- Each learning experience of a knowledge point is independent from other learning experiences of it;
- The effect of each learning experience declines in time according to Ebbinghaus’ forgetting curve of Equation 1;
- The familiarity measure of a knowledge point is the additive effects of all the learning experiences of it.

Equation 3 is used to calculate an individual’s familiarity measure of knowledge point $k_i$ at a particular time $t$. The input is sequence of $n$ learning sessions. $d_j$ is session $j$’s duration in seconds; $\xi_{ij}$ is knowledge point $k_i$’s share in session $j$, it is calculated with Equation 2; $b_j$ is the proportion of memory retention of learning session $j$ at time $t$, it is calculated with Equation 1.

$$F_{k_i} = \sum_{j=1}^{n} d_j \cdot \xi_{ij} \cdot b_j$$ (3)

A relative familiarity measure can be calculated by dividing the familiarity measures with the mean value of them.
### Table 3: A subject’s statistics and familiarity measures of 5 randomly selected knowledge points

| Knowledge point name | Learning frequency | Cumulative learning time(S) | Latest learning date       | Familiarity measure |
|----------------------|--------------------|-----------------------------|----------------------------|--------------------|
| Bayes’ rule          | 5                  | 5954                        | 3/3/2016 16:21             | 15.14              |
| Conditional entropy  | 3                  | 6294                        | 2/24/2016 16:13            | 25.75              |
| Posterior distribution| 5                  | 4715                        | 3/5/2016 17:44             | 35.05              |
| Lagrange multiplier  | 1                  | 751                         | 2/27/2016 19:52            | 3.97               |
| Expectation-maximization algorithm | 12 | 11448                      | 3/3/2016 16:21             | 122.54             |

#### 3.5 Results

A subject’s 13 days (from 2/23/2016 to 3/6/2016) of PDF documents reading histories are recorded and analyzed. During the period of time, the subject has read 38 documents for 417 times. For the simplicity of calculation, pages on which the subject has spent less than 30 seconds are ignored; learning sessions which are less than 150 seconds are also ignored. After the filtering, there are a total of 43 learning sessions recognized, 69 knowledge points were captured. Table 3 illustrates the subject’s statistics and familiarity measures of 5 randomly selected knowledge points, the calculation time is 2016-03-29 19:24:00. The values of familiarity measures change over time, because human memory declines over time.

#### 4 Potential applications of knowledge model

With a quantitative evaluation of an individual’s knowledge, many decisions which were made empirically can now be considered based on a numerical analysis. The following are some examples:

##### 4.1 Searching common topics

As mentioned in section 1, knowledge model can be used to discover common topics efficiently for people with different education or cultural background. A discipline or sub-discipline they
both are interested in can be selected first, then find out the set of knowledge points with which they both are familiar based on the familiarity measures. These knowledge points can serve as the common topics of their conversation. This application can be extended to discover common topic terms which are not defined as knowledge points, such as a movie star’s name.

4.2 Selecting a lecture

It is common for a knowledge worker to take part in all kinds of academic lectures. It is frustrated and wasting of time that a lecture is too recondite to understand. To help a potential audience predict how much he can understand the contents of the lecture, the lecturer can list a set of knowledge points which are important to understand it on the poster, then the audience can check his familiarity measures of those knowledge points. A score of how much he can understand it can be calculated based on the familiarity measures.

4.3 Evaluating a scientist’s research concentrations in a period of time

Because an individual’s learning histories of all knowledge points have been recorded. It is convenient to extract fragments of the learning histories of a period of time, for example, the last three years, then use knowledge model to calculate the familiarity measures of that period. The set of knowledge points which have larger familiarity measures are the scientist’s research concentrations.

4.4 Selecting appropriate referees for a research paper

When a research paper is submitted for reviewing, choosing the optimal referees from a candidate set is a difficult problem. At present it is usually decided empirically. With knowledge model, an objective numerical analysis is possible. For example, each candidate referee’s research concentrations can be calculated, the submitted paper’s knowledge points and their corresponding shares can also be calculated, by matching these values, the optimal referee list can
be obtained.

4.5 Evaluating a knowledge worker’s expertise on a discipline or sub-discipline

With an individual’s familiarity measures of all the knowledge points, it is not hard to evaluate his expertise on a discipline or sub-discipline. The knowledge points are organized in a tree structure, each subtree represent a discipline or sub-discipline of knowledge. The evaluation can be made based on how many knowledge points the individual has mastered and the average familiarity measure of the subtree.

5 Conclusion

In this paper, a method named knowledge model which can quantitatively evaluate a knowledge worker’s knowledge is proposed. The main idea is to record an individual’s learning histories of each piece of knowledge, and then use the learning history as an input to calculate the individual’s familiarity measure of each knowledge point. A preliminary knowledge evaluating system is developed, it analyzes an individual’s PDF documents reading activities, then uses topic model and knowledge model to calculate the individual’s familiarity measures of captured knowledge points. An algorithm of discriminating learning sessions is devised. In addition, a method of calculating the individual’s familiarity measure of a knowledge point based on its learning history is proposed.

6 Discussion

6.1 Normalization among knowledge points

The calculation of familiarity measure mainly considers the individual’s time devotion to a knowledge point and its share of each learning content. However, the complexity levels of
knowledge points are usually different. For example, spending 20 minutes is sufficient for a normal knowledge worker to understand and remember the Pythagorean Theorem, but it is usually not enough to understand a complicated algorithm like LDA. Therefore, the familiarity measures should be normalized among knowledge points. Each knowledge point can be allocated with a complexity level. The familiarity measure can be multiplied by a factor, which is a function of the knowledge point’s complexity level. The complexity level of a knowledge point can be decided empirically by a group of experts when constructing the knowledge tree.

6.2 Normalization among knowledge workers

If knowledge model is used for self-evaluating, like most applications mentioned in section 4, it is not essential to normalize familiarity measures among knowledge workers. A relative value of dividing the familiarity measures by the mean value is sufficient. If knowledge model is used as a judgement of a competition, normalization of familiarity measures among knowledge workers is essential. For example, using knowledge model analysis as a substitution of an examination. The normalization can be made by multiplying the familiarity measures by the individual’s relative Intelligence Quotient (IQ). However, other intractable issues will be brought in too, such as how to obtain a convincing IQ value for an individual and how to avoid cheating.

6.3 Evaluation of knowledge model

Evaluation of the effectiveness of knowledge model is a huge project.

- Firstly, a knowledge worker’s almost all the learning activities should be recorded and analyzed, such as reading of all kinds of documents and web pages, attending of lectures, oral discussions etc. Each knowledge point’s complete learning history is obtained, its relative familiarity measure is also computed;
• Secondly, select a sample of knowledge points and group them according to their relative familiarity measures;

• Thirdly, let the individual take an examination which tests his understanding of the sample knowledge points;

• Fourthly, compare the results of the examination with the relative familiarity measures calculated by knowledge model;

• Finally, repeat the above procedures on other knowledge workers to reduce the randomness of the results.

A detailed evaluation will be considered in further research works.

### 6.4 Limitations of using Ebbinghaus’ forgetting curve

Ebbinghaus’ forgetting curve formula is used in the computation of familiarity measures. It depicts the decline of memory retention in time. Many research results have testified the soundness of the formula (15, 16).

However, other factors may affect the speed of memory decay as well. Such as how the information is presented and the physiological state of the individual. There are no unanimously accepted formulas of how these factors affect the speed of memory decay. In addition, it is difficult to obtain accurate values for these factors.

The calculation of the familiarity measures is based on the individual’s learning histories of a long range of time, usually several years or decades of years. In my opinion, when observing from a long time range, it can be hypothesized that the average presentation qualities and average physiological states among knowledge points are equivalent, so these factors can be ignored.
If other forms of forgetting curve formulas are proved to be better than Ebbinghaus’, it can be used as a substitution when calculating familiarity measures.

### 6.5 Privacy issues

Recording learning histories of each knowledge point will inevitably violate an individual’s privacy. To protect the privacy, the learning histories can be password protected or even encrypted. They are stored in the individual’s personal storage, and should not be revealed to other people. The only information the outside world can see is the individual’s familiarity measures of the knowledge points. The knowledge points which may involve the individual’s privacy are separated from other knowledge points, every output of their familiarity measures should be authorized by the owner.

### References and Notes

1. Here is an introduction of Google Glass: https://en.wikipedia.org/wiki/Google_Glass.

2. D. M. Blei, *Communications of the ACM* **55**, 77 (2012).

3. It can be found at https://en.wikipedia.org/wiki/Branches_of_science.

4. V. Bush, A. W. M. Think, *As we may think* **176**, 101 (1945).

5. J. Gemmell, G. Bell, R. Lueder, S. Drucker, C. Wong, *Proceedings of the tenth ACM international conference on Multimedia* (ACM, 2002), pp. 235–238.

6. M. Steyvers, T. Griffiths, *Handbook of latent semantic analysis* **427**, 424 (2007).

7. T. Hofmann, *Proceedings of the 22nd annual international ACM SIGIR conference on Research and development in information retrieval* (ACM, 1999), pp. 50–57.
8. D. M. Blei, A. Y. Ng, M. I. Jordan, *the Journal of machine Learning research* 3, 993 (2003).

9. Y. Lu, Q. Mei, C. Zhai, *Information Retrieval* 14, 178 (2011).

10. H. Ebbinghaus, *Memory: A contribution to experimental psychology*, no. 3 (University Microfilms, 1913).

11. It can be found at http://psychclassics.yorku.ca/Ebbinghaus/memory7.htm.

12. L. Averell, A. Heathcote, *Journal of Mathematical Psychology* 55, 25 (2011).

13. G. Liu, L. Feng, *arXiv preprint arXiv:1601.07273* (2016).

14. The package is available at https://meta-toolkit.org.

15. J. M. Murre, J. Dros, *PloS one* 10, e0120644 (2015).

16. D. C. Rubin, A. E. Wenzel, *Psychological review* 103, 734 (1996).