Characteristic Function of Three-Parameter Generalized Exponential Distribution

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Abstract. This research is about the characteristic function of three-parameter generalized exponential distribution. Three-parameter generalized exponential distribution has three parameters which are μ as a location parameter, λ as a scale parameter, α as a shape parameter. The characteristic function is retrieved from the expectation of \( e^{itx} \), whereas an imaginary number. Then, the characteristic function of three-parameter generalized exponential distribution was able to be determined by using definition and trigonometric expansions. Based on those two methods this study got the same results and then will be continued proving the fundamental properties of the characteristic function of three-parameter generalized exponential distribution.

1. Introduction

In Distribution Generalized Exponential was first introduced by Gupta and Kundu in 1999. This distribution is taken from one of the density functions cumulative used in the mid-19th century (Gompertz Verhulst) for compare death tables and general population growth rates where one of the three parameters is standardized into one. The distribution of the three-parameter generalized exponential has three parameters, namely the shape parameter (α), location parameter (μ), and scale parameter (λ). Many researchers have discussed the distribution of three parameters [1-3].

The characteristic function is one type of transformation that is often used in opportunity and statistical theory. The characteristic function can be used to determine the distribution function of a random variable known as the inversion theorem of the characteristic function. The inversion theorem of the characteristic function is one of the characteristics that characterize the characteristic function. The characteristic function also has basic properties. However, as far as the search that the author has done, it hasn’t been found an explanation of the characteristic function, especially the characteristic function of three-parameter exponential distribution.

In this study, the author will discuss more deeply the characteristics function, and prove the basic properties of the characteristic function of the distribution three-parameter exponential (1). According to Gupta and Kundu (1999), the distribution of the three-parameter generalized exponential has a density opportunity-shaped function:

\[
f(x; \alpha, \lambda, \mu) = \frac{\alpha}{\lambda} \left(1 - e^{-\frac{(x-\mu)}{\lambda}}\right)^{\alpha-1} e^{-\frac{(x-\mu)}{\lambda}}, \quad x > \mu, \alpha > 0, \lambda > 0
\]

(1)

Where α is a form parameter, λ is a scale parameter and μ is a location parameter, denoted as GE (α, λ, μ).

Based on the background described in this study, the study will be discussed Characteristic Functions three-parameters generalized exponential Distribution.”
2. Research Method
The characteristic function of the distribution of three-parameters generalized exponential can be obtained from the moment generating function by adding i as an imaginary part and trigonometric expansion. In this study, two ways are used to find the characteristic function of the distribution three-parameter exponential. Furthermore, we will prove the basic properties of the characteristics function three-parameter exponential distribution [4].

3. Result and Discussion
Distribution Generalized Three-Parameter Exponential
A random variable $X$ is said to have a distribution of three-parameters generalized exponential, with the parameters $\alpha$, $\lambda$ and $\mu$, if and only if the probability density function of $X$ is

$$f(x; \alpha, \lambda, \mu) = \frac{\alpha}{\lambda} \left(1 - e^{-\frac{\lambda}{\lambda+\mu} x} \right)^{\alpha-1} e^{-\frac{\lambda}{\lambda+\mu} (x-\mu)} ; x > \mu, \alpha > 0, \lambda > 0$$

where $\alpha$ are both parameter shapes, $\lambda$ is the scale parameter and $\mu$ is Location parameters

Graph Simulation Probability density function distribution Three-Parameter Generalized Exponential
In this section, we will discuss the graph simulation of the probability density function from the distribution of the three-parameter generalized exponential using Matlab software R2010b.

(1) Simulation graph of the probability density function of a distribution three-parameter generalized exponential with a value of $\alpha$ fixed, $\lambda$ increases, $\mu$ fixed

![Graph 1](attachment:image1.png)

**Figure 1** Graph The probability density function of the distribution three-parameter generalized exponential with $\alpha=2$, $\lambda=(1,2,3)$, $\mu=-2$.

(2) Simulation graph of the probability density function of the distribution of three-parameters generalized exponential with a value of $\alpha$ fixed, $\lambda$ increases, $\mu$ increases

![Graph 2](attachment:image2.png)

**Figure 2** Graph of probability density function of distribution three-parameter generalized exponential with $\alpha=2$, $\lambda=(1,2,3)$, $\mu=(-2,-1,0)$.

(3) Simulation of the graph of the probability density function from the distribution of three-parameters generalized exponential with a value of $\alpha$ fixed, $\lambda$ decreases, $\mu$ fixed
Figure 3 Graph of probability density functions of distribution three-parameter generalized exponential with $\alpha = 2$, $\lambda = (3,2,1)$, $\mu = -2$.

(4) Simulation graph of the probability density function from the distribution of three-parameters generalized exponential with a value of $\alpha$ fixed, $\lambda$ decreases, $\mu$ increases.

Figure 4 Graph of probability density function of distribution three-parameter generalized exponential with $\alpha = 2$, $\lambda = (3,2,1)$, $\mu = (-2,-1,0)$.

(5) Simulation graph of the probability density function of the distribution of three-parameters generalized exponential with $\alpha$ increasing, $\lambda$ increases, $\mu$ fixed.

Figure 5 Graph of probability density function of distribution three-parameter generalized exponential with $\alpha = (2,3,4)$, $\lambda = (1,3,5)$, $\mu = -2$.

(6) Simulation graph of the probability density function of the distribution of three-parameters generalized exponential with $\alpha$ increasing, $\lambda$ increasing, $\mu$ increasing.

Figure 6 Graph probability density function for distribution three-parameter generalized exponential with $\alpha = (2,3,4)$, $\lambda = (1,3,5)$, $\mu = (-2,-1.0)$. 
(7) Simulation graph of the probability density function of the distribution of three-parameters generalized exponential with value $\alpha$ increasing, $\lambda$ decreases, $\mu$ fixed

(8) Simulation graph of the probability density function of the distribution of three-parameters generalized exponential with value $\alpha$ increasing, $\lambda$ decreasing, $\mu$ decreasing

(9) Simulation graph of the probability density function of the distribution of three-parameters generalized exponential with values $\alpha$ increasing, $\lambda$ fixed, $\mu$ fixed

(10) Simulation graph of the probability density function of the distribution of the three-parameter generalized exponential with a value $\alpha$ increasing, $\lambda$ fixed, $\mu$ increased
Figure 10 Graph of probability density function of distribution three-parameter generalized exponential with $\alpha = (3,5,7)$, $\lambda = 1$, $\mu = (-1,0,1)$.

(11) Simulation graph of the probability density function of the distribution of the three-parameter generalized exponential with a value $\alpha$ decreasing, $\lambda$ fixed, $\mu$ fixed

Figure 11 Graph of probability density function of distribution three-parameter generalized exponential with $\alpha = (6,4,2)$, $\lambda = 1$, $\mu = 0$.

(12) Simulation graph of the probability density function of the distribution of three-parameters generalized exponential with a value $\alpha$ decreasing, $\lambda$ increases, $\mu$ fixed

Figure 12 Graph of probability density function of distribution three-parameter generalized exponential with $\alpha = (6,4,2)$, $\lambda = (1,2,3)$, $\mu = -2$

(13) Simulation graph of the probability density function of the distribution of the three-parameter generalized exponential with a value $\alpha$ decreasing, $\lambda$ decreases, $\mu$ fixed

Figure 13 Graph of probability density function of distribution three-parameter generalized exponential with $\alpha = (6,4,2)$, $\lambda = (3,2,1)$, $\mu = -2$. 
From the graph figure the probability density function of the three-parameter generalized exponential distribution can be seen that $\lambda$ is a scale parameter, $\alpha$ is a form parameter so that the value of decreases the $\lambda$ smaller the scale of the graph formed as a result of the graph increasing or tapering and vice versa, the increasing value of $\lambda$ the greater the scale of the graph formed as a result of the graph decreasing or sloping. Whereas $\mu$ is the location parameter so it can be seen in the picture that the graph will always start from $\mu$.

Characteristics Function Distribution of Three-Parameter Generalized Exponential

(1) Characteristics Function Three-Parameter Generalized Exponential Distribution by definition

In this section, we will describe the characteristics function of distributions three-parameter generalized exponential. The characteristic function of a distribution three-parameter generalized exponential can be obtained from the function of the moment generator by adding $i$ as an imaginary part.

If $X$ is the distribution of the random variable distribution of the three-parameter generalized exponential with the parameters $\lambda$, $\alpha$, and $\mu$, then the characteristic function of $X$ is as follows:

$$\phi_x(t) = E(e^{itx}) = \int_{-\infty}^{\infty} e^{itx} f(x)dx$$

(2)

Therefore the $x$ limit of the probability density function of the distribution three-parameter exponential $x > \mu$, and based on a graphical simulation of the probability density function of the distribution of the three-parameters generalized exponential obtained by a plot graph which always starts from $\mu$, then

$$\phi_x(t) = \int_{\mu}^{\infty} e^{itx} f(x)dx$$

$$= \int_{\mu}^{\infty} e^{i(t(x-\mu)+\mu)} f(x)dx$$

$$= e^{i\mu} \int_{\mu}^{\infty} e^{it(x-\mu)} f(x)dx$$

$$= e^{i\mu} \sum_{n=0}^{\infty} \frac{(it)^n}{n!} \int_{\mu}^{\infty} e^{-\frac{(x-\mu)}{\lambda}} \frac{(x-\mu)^{n-1}}{\lambda^{n-1}} dx$$

(3)

for example

$$u = e^{-\frac{(x-\mu)}{\lambda}}$$

$$\ln u = \ln e^{-\frac{(x-\mu)}{\lambda}}$$

$$\ln u = -\frac{(x-\mu)}{\lambda}$$

$$(x-\mu) = -\lambda \ln u$$

so

$$du = -\frac{1}{\lambda} e^{-\frac{(x-\mu)}{\lambda}} dx \Rightarrow -du = \frac{1}{\lambda} e^{-\frac{(x-\mu)}{\lambda}} dx$$

The limits

$$x = \mu \Rightarrow y = 1$$

$$x = \infty \Rightarrow y = 0$$
so that equation (11) becomes the following form

$$\phi_x(t) = e^{it\mu} \int_0^1 e^{it(1-u)x} \alpha (1-u)^{\alpha-1} (-du)$$

$$= e^{it\mu} \alpha \int_0^1 e^{it(1-u)x} (1-u)^{\alpha-1} du$$

$$= e^{it\mu} \alpha \int_0^1 u^{-\alpha} (1-u)^{\alpha-1} du$$

Because $B(\alpha, \beta) = \int_0^1 x^{\alpha-1}(1-x)^{\beta-1} dx$, then

$$\phi_x(t) = e^{it\mu} \alpha B(1-\alpha, \alpha)$$

(4)

Transform the beta function into the form of a gamma function

Because $B(\alpha, \beta) = \frac{\Gamma(\alpha)\Gamma(\beta)}{\Gamma(\alpha+\beta)}$, it forms the transformation

$$B(1-\alpha, \alpha) = \frac{\Gamma(1-\alpha)\Gamma(\alpha)}{\Gamma(1-\alpha+\alpha)}$$

so that Equation (12) becomes

$$\phi_x(t) = e^{it\mu} \alpha \frac{\Gamma(1-\alpha)\Gamma(\alpha)}{\Gamma(1-\alpha+\alpha)}$$

$$= e^{it\mu} \alpha \frac{\Gamma(1-\alpha)\alpha\Gamma(\alpha)}{\Gamma(1-\alpha+\alpha)}$$

$$= e^{it\mu} \alpha \frac{\Gamma(1-\alpha)\Gamma(\alpha+1)}{\Gamma(1-\alpha+\alpha)}$$

So the characteristic function of the three-parameter generalized exponential distribution is

$$\phi_x(t) = e^{it\mu} \frac{\Gamma(1-\alpha)\Gamma(\alpha+1)}{\Gamma(1-\alpha+\alpha)}$$

(2) Characteristic Functions Three-Parameter Generalized Exponential Distribution with trigonometric expansion

In addition to using the method described earlier, this section will explain other ways to determine the function of the characteristics, as follows:

It will be shown that

$$\phi_x(t) = E(e^{itx}) = E[Cos(tx) + i Sin(tx)]$$

$$= E[Cos(tx)] + i E[Sin(tx)]$$
Because the x limit of the probability density function of the distribution of generalized exponential three-parameters is \( x > \mu \), and based on a graph simulation the probability density function of the distribution of three-parameters generalized exponential is obtained plot graph which always starts from \( \mu \), then

\[
\phi_x(t) = \int_{\mu}^{\infty} e^{itx} f(x)dx
\]

\[
= \int_{\mu}^{\infty} e^{i(x-\mu+t)} f(x)dx
\]

\[
= e^{it\mu} \int_{\mu}^{\infty} e^{ix} f(x)dx
\]

Describe \( e^{i(x-\mu)} \) become \( \cos(t(x-\mu)) + i \sin(t(x-\mu)) \), so it becomes

\[
\phi_x(t) = e^{it\mu} \left[ \int_{\mu}^{\infty} \left[ \cos(t(x-\mu)) + i \sin(t(x-\mu)) \right] f(x)dx \right]
\]

Next will be completed one by one as follows:

The first part

(i). be searched \( E[\cos(tx)] \int_{\mu}^{\infty} \cos(t(x-\mu)) f(x)dx \)

Because \( \cos t = \frac{1}{2} \left( e^{it} + e^{-it} \right) \)

then

\[
\int_{\mu}^{\infty} \cos(t(x-\mu)) f(x)dx = \frac{1}{2} \int_{\mu}^{\infty} \left( e^{i(x-\mu)} + e^{-i(x-\mu)} \right) f(x)dx
\]

\[
= \frac{1}{2} \left[ \int_{\mu}^{\infty} e^{i(x-\mu)} f(x)dx + \int_{\mu}^{\infty} e^{-i(x-\mu)} f(x)dx \right]
\]

by letting

\[
u = e^{\frac{(x-\mu)}{\lambda}}
\]

\[
\ln u = \ln e^{\frac{(x-\mu)}{\lambda}}
\]

\[
\ln u = -\frac{(x-\mu)}{\lambda}
\]

\[
(x-\mu) = -\lambda \ln u
\]

so
\[ du = -\frac{1}{\lambda} e^{\frac{(x-\mu)}{\lambda}} \, dx \quad \text{and} \quad -du = \frac{1}{\lambda} e^{\frac{(x-\mu)}{\lambda}} \, dx \]

The limits

\[ x = \mu \quad \Rightarrow \quad y = 1 \]
\[ x = \infty \quad \Rightarrow \quad y = 0 \]

Thus Equation (4) becomes,

\begin{align*}
\phi(t) &= \frac{1}{2} \left[ \int_0^1 e^{i\alpha u} \alpha(1-u)^{-\frac{1}{2}} (-du) + \int_1^0 e^{-i\alpha u} \alpha(1-u)^{-\frac{1}{2}} (-du) \right] \\
\phi(t) &= \frac{1}{2} \left[ -\int_0^1 e^{i\alpha u} \alpha(1-u)^{-\frac{1}{2}} du - \int_1^0 e^{-i\alpha u} \alpha(1-u)^{-\frac{1}{2}} du \right] \\
\phi(t) &= \frac{1}{2} \left[ \int_0^1 e^{-i\alpha u} \alpha(1-u)^{-\frac{1}{2}} du + \int_1^0 e^{i\alpha u} \alpha(1-u)^{-\frac{1}{2}} du \right] \\
\phi_x(t) &= -\frac{\alpha}{2} \left[ \int_0^1 u^{-\frac{1}{2}} (1-u)^{-\frac{3}{2}} du + \int_1^0 u^{-\frac{1}{2}} (1-u)^{-\frac{3}{2}} du \right] \\
\phi_y(t) &= \frac{\alpha}{2} \left( B(1-it\lambda, \alpha) + B(1+it\lambda, \alpha) \right) \\
\phi_z(t) &= \frac{\alpha}{2} B(1-it\lambda, \alpha) - \frac{\alpha}{2} B(1+it\lambda, \alpha) \\
& \quad \text{Because } B(a,b) = \int_0^1 x^{a-1}(1-x)^{b-1} \, dx \text{ then} \\
\phi_x(t) &= \frac{\alpha}{2} B(1-it\lambda, \alpha) + \frac{\alpha}{2} B(1+it\lambda, \alpha) \\
& \quad \text{(7)}
\end{align*}

The second part

(ii). be searched \( i E[\sin(tx)] = i \int_\mu^{\infty} \sin(t(x-\mu)) f(x) \, dx \)

Because \( \sin t = \frac{1}{2i} \left( e^{it} - e^{-it} \right) \)

Then

\begin{align*}
& \int_\mu^{\infty} \sin(tx-\mu) f(x) \, dx - i \int_\mu^{\infty} \left( \frac{1}{2i} \left( e^{i(t-x)} - e^{-i(t-x)} \right) \frac{d}{dx} \right) \left( 1 - e^{-\frac{(x-\mu)}{\lambda}} \right) e^{-\frac{(x-\mu)}{\lambda}} \, dx \\
& = \frac{1}{2} \int_\mu^{\infty} \left( e^{i(t-x)} \frac{d}{dx} \right) e^{-\frac{(x-\mu)}{\lambda}} \, dx - \int_\mu^{\infty} \left( e^{i(t-x)} \frac{d}{dx} \right) \left( 1 - e^{-\frac{(x-\mu)}{\lambda}} \right) e^{-\frac{(x-\mu)}{\lambda}} \, dx \\
& \quad \text{by assuming} \\
& u = e^{\frac{(x-\mu)}{\lambda}} \\
& \ln u = \ln e^{\frac{(x-\mu)}{\lambda}} \\
& \ln u = -\frac{(x-\mu)}{\lambda} \\
& \frac{x-\mu}{\lambda} = -\lambda \ln u \\
& \text{so that} \\
\end{align*}
\[ du = \frac{1}{\lambda} e^{-\frac{(x-\mu)}{\lambda}} \, dx \text{ then } -du = \frac{1}{\lambda} e^{-\frac{(x-\mu)}{\lambda}} \, dx \]

The limits
\( x = \mu \quad \Rightarrow \quad y = 1 \)
\( x = \infty \quad \Rightarrow \quad y = 0 \)

So that Equation (16) becomes,
\[ \phi_1(t) = \frac{1}{2} \left[ \int_0^t e^{-it\ln u} (1 - u)^{\alpha-1} \, du - \int_0^t e^{it\ln u} (1 - u)^{\alpha-1} \, du \right] \]
\[ \phi_2(t) = \frac{1}{2} \left[ \int_0^t e^{-it\ln u} (1 - u)^{\alpha-1} \, du + \int_0^t e^{it\ln u} (1 - u)^{\alpha-1} \, du \right] \]
\[ \phi_3(t) = \frac{\alpha}{2} \left[ \int_0^t u^{it\lambda} (1 - u)^{\alpha-1} \, du - \int_0^t u^{it\lambda} (1 - u)^{\alpha-1} \, du \right] \]
\[ \phi_4(t) = \frac{\alpha}{2} \left[ \int_0^t (1 - u)^{\alpha-1} \, du - \int_0^t (1 - u)^{\alpha-1} \, du \right] \]

Because \( B(\alpha, \beta) = \int_0^1 x^{\alpha-1} (1 - x)^{\beta-1} \, dx \) then

\[ \phi_1(t) = \frac{\alpha}{2} \left\{ B(1 - it\lambda, \alpha) - B(1 + it\lambda, \alpha) \right\} \]
\[ \phi_2(t) = \frac{\alpha}{2} \left( B(1 - it\lambda, \alpha) - \frac{\alpha}{2} B(1 + it\lambda, \alpha) \right) \]

(9)

From Equations (15) and (17) substituted to equation (13), it is obtained

\[ \phi_1(t) = e^{it\alpha} \left\{ \frac{\Gamma(\alpha)\Gamma(\beta)}{\Gamma(\alpha + \beta)} \right\} B(1 - it\lambda, \alpha) \]

(10)

Transform the beta function in the form of the gamma function

Because \( B(\alpha, \beta) = \frac{\Gamma(\alpha)\Gamma(\beta)}{\Gamma(\alpha + \beta)} \) of the shape of its transformation

\[ B(1 - it\lambda, \alpha) = \frac{\Gamma(1 - it\lambda)\Gamma(\alpha)}{\Gamma(1 - it\lambda + \alpha)} \]

so that Equation (18) becomes

\[ \phi_1(t) = e^{it\alpha} \left( \frac{\Gamma(1 - it\lambda)\Gamma(\alpha)}{\Gamma(1 - it\lambda + \alpha)} \right) \]

(11)
Thus the characteristic function of the distribution of the three-parameter generalized exponential is
\[
\phi_\chi(t) = e^{it\mu} \frac{\Gamma(1-it\lambda)\Gamma(\alpha+1)}{\Gamma(1-it\lambda+\alpha)}
\]

Based on the translation of determining the characteristic function uses the definition and use trigonometry expansion obtained the same results. So that it can be concluded the characteristic function of the distribution is three-parameter generalized exponential are:
\[
\phi_\chi(t) = e^{it\mu} \frac{\Gamma(1-it\lambda)\Gamma(\alpha+1)}{\Gamma(1-it\lambda+\alpha)}
\]

Proof Basic Properties Characteristic Function of the Characteristic Function Distribution Three-Parameter Generalized Exponential

In this section, we will discuss the basic properties characteristic function of the characteristic function of distribution three-parameter exponential.

It will be shown that the characteristic function of the generalized three-parameter exponential distribution fulfills the characteristics of the characteristic function. Properties 1. Suppose that the function characteristic of the distribution three-parameter generalized exponential
\[
\phi_\chi(t) = e^{it\mu} \frac{\Gamma(1-it\lambda)\Gamma(\alpha+1)}{\Gamma(1-it\lambda+\alpha)}
\]

Then \( \phi_\chi(0) = 1 \).

Proof:

Note that
\[
\phi_\chi(t) = e^{it\mu} \frac{\Gamma(1-it\lambda)\Gamma(\alpha+1)}{\Gamma(1-it\lambda+\alpha)}
\]

Put \( t = 0 \), then we get
\[
\phi_\chi(0) = e^{0} \frac{\Gamma(1-0)\Gamma(\alpha+1)}{\Gamma(1-0+\alpha)} = 1
\]
\[
\phi_\chi(0) = e^{0} \frac{\Gamma(1)\Gamma(\alpha+1)}{\Gamma(1+\alpha)} = 1
\]
\[
\phi_\chi(0) = 1
\]
\[
\phi_\chi(0) = 1.1
\]
\[
\phi_\chi(0) = 1
\]

Properties 2. \( |\phi_\chi(t)| \leq 1 \)

Proof:

By definition, it is known that:
\[
e^{it\alpha} = \cos(t\alpha) + i\sin(t\alpha)
\]
and \( i = \sqrt{-1} \), so
\[ \left| e^{ix} \right|^2 = \cos^2(tx) + \sin^2(tx) = 1 \]
\[ \left| e^{ix} \right| = \sqrt{1} = 1 \]
be obtained,

\[ |\phi_e(t)| = \int_{-\infty}^{\infty} e^{ix} dF \]
\[ |\phi_{ie}(t)| = \int_{-\infty}^{\infty} e^{ix} dF \]
\[ \leq \int_{-\infty}^{\infty} |e^{ix}| dF \]
\[ = \int_{-\infty}^{\infty} 1 dF \]
\[ = \int_{-\infty}^{\infty} dF \]
\[ = \int_{-\infty}^{\infty} f(x) dx \]
\[ = \int_{\mu}^{\infty} \alpha \left( 1 - e^{-\frac{(x-\mu)}{\lambda}} \right)^{n-1} e^{-\frac{(x-\mu)}{\lambda}} dx \]

(11)

by letting
\[ u = e^{-\frac{(x-\mu)}{\lambda}} \]
\[ \ln u = \ln e^{-\frac{(x-\mu)}{\lambda}} \]
\[ \ln u = -\frac{(x-\mu)}{\lambda} \]
\[ (x-\mu) = -\lambda \ln u \]
So that
\[ du = -\frac{1}{\lambda} e^{\frac{(x-\mu)}{\lambda}} dx \]

The limits
\[ x = \mu \Rightarrow y = 1 \]
\[ x = \infty \Rightarrow y = 0 \]
Thus Equation (19) becomes,

\[ \phi_e(t) = \int_{1}^{0} \alpha (1-u)^{n-1} (-du) \]
\[ \phi_{ie}(t) = -\int_{1}^{0} \alpha (1-u)^{n-1} du \]

Letting \( y = 1-u \) then \( dy = -du \)

The limits
\begin{align*}
  u &= 1 \implies y = 0 \\
u &= 0 \implies y = 1
\end{align*}

So the equation becomes

$$
\phi_\lambda(t) = \int_0^\alpha y^{\alpha - 1} dy
$$

$$
\phi_\alpha(t) = \alpha \cdot \frac{1}{\alpha} y^\alpha |_0^1
$$

$$
\phi_\lambda(t) = y^\alpha |_0^1
$$

\phi_\lambda(t) = 1

Properties 3. The suppose \( \phi_\lambda(t) = e^{i\omega} \cdot \frac{\Gamma(1 - it\lambda) \Gamma(\alpha + 1)}{\Gamma(1 - it\lambda + \alpha)} \) d characteristic function of the random variable \( X \) of distribution three-parameter generalized Exponential. Then the characteristic function of the random variable \( -X \) is \( \bar{\phi}_\lambda(t) \)

Proof:

For example \( \bar{\phi}_\lambda(t) \) a complex conjugate of \( \phi_\lambda(t) \), it will be shown that the characteristic function \( (-t) \) is \( \bar{\phi}_\lambda(t) \), then:

$$
\phi_\lambda(-t) = e^{-i\omega} \cdot \frac{\Gamma(1 + it\lambda) \Gamma(\alpha + 1)}{\Gamma(1 + it\lambda + \alpha)}
$$

$$
\phi_\alpha(-t) = e^{-i\omega} \cdot \frac{\Gamma(1 + it\lambda) \Gamma(\alpha + 1)}{\Gamma(1 + it\lambda + \alpha)}
$$

= \bar{\phi}_\lambda(t)

So, the characteristic function \( (-t) \) of the distribution of three-parameters generalized exponential is

$$
\phi_\lambda(-t) = e^{-i\omega} \cdot \frac{\Gamma(1 + it\lambda) \Gamma(\alpha + 1)}{\Gamma(1 + it\lambda + \alpha)}
$$

From the proof of the three characteristics of the characteristic function, it can be concluded that the characteristic function of the distribution three-parameter exponential fulfills the basic properties of the characteristic function.

4. Conclusion

From the graph figure the probability density function of the distributions three-parameter generalized exponential can be seen that \( \lambda \) is a scale parameter, \( \alpha \) is a form parameter so that the value of decreases the \( \lambda \) smaller the scale of the graph formed as a result of the graph increasing or tapering and vice versa, the increasing value of \( \lambda \) the greater the scale of the graph formed as a result of the graph decreasing or sloping. Whereas \( \mu \) is the location parameter so it can be seen in the picture that the graph will always start from \( \mu \). The characteristic function of the distribution of three-parameters generalized exponential is the

$$
\phi_\lambda(t) = e^{i\omega} \cdot \frac{\Gamma(1 - it\lambda) \Gamma(\alpha + 1)}{\Gamma(1 - it\lambda + \alpha)}
$$

Characteristics function of the distribution three-parameter generalized exponential fulfilling the basic properties of characteristic functions

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