Abstract

We develop a general set of methods for computing the oblique electroweak parameters $S$ and $T$ for bottom-up and top-down motivated scenarios in which the Higgs weakly mixes with a superconformal extra sector. In addition to their utility in phenomenological studies, the observables $S$ and $T$ are also of purely theoretical interest as they are defined by correlation functions of broken symmetries. We show that in the limit where the extra sector enjoys an approximate custodial symmetry, the leading contributions to $S$ and $T$ can be recast as calculable data of the theory in the conformal phase. Using this result, we also obtain model-independent bounds on the sign and size of oblique electroweak corrections from unitary superconformal theories.
1 Introduction

The discovery of a Higgs-like resonance near 125 GeV at the LHC [1,2] signals the start of a dynamic new phase in particle theory and experiment. Needless to say, measuring the detailed properties of this resonance will provide crucial insights into the nature of electroweak symmetry breaking as well as possible clues to beyond-the-Standard Model (BSM) physics.

By far the simplest and best motivated interpretation is that this resonance is indeed a Higgs boson. There are two ways in which properties of the Higgs can provide information about BSM physics. First, the measured mass of the Higgs implies that the quartic coupling in the Higgs potential is not very small. This provides a simple way to discriminate between various models of electroweak symmetry breaking. Additionally, BSM physics, especially in “natural” theories, will modify the Higgs self-couplings as well as couplings to SM gauge bosons and fermions in general, thereby affecting production and decay modes.

These two effects are intricately related in natural theories. For example, in the minimal supersymmetric Standard Model (MSSM), the quartic coupling at tree level is determined by
the $SU(2)$ and $U(1)_Y$ gauge couplings, and is extremely small. Hence, additional contributions to the quartic coupling are needed. A natural way to allow for such contributions (i.e., without introducing fine tuning in the Higgs potential) is by coupling the Higgs to states beyond the MSSM. Since these states couple to the Higgs, they can also change production and decay of the Higgs relative to the SM and MSSM, for example via the dimension-five operators for gluon-fusion production ($gg \rightarrow h^0$) and diphoton decay ($h^0 \rightarrow \gamma\gamma$) [3].

Although in principle one could imagine a wide variety of states beyond the SM (or MSSM), precision electroweak measurements impose constraints on such extra sectors. In particular, the parameters known as $S$ and $T$ are the dominant contributions in the oblique approximation [4,5]. These parameters describe certain dimension-six operators that can appear in the effective theory below the scale of new physics. Present constraints on $S$ and $T$ allow for order 0.1 deviations at the 68% confidence level [6]. Contributions beyond this level are already difficult to accommodate. Indeed, even before the direct observation of a Standard Model-like Higgs, precision electroweak fits already indicated a good fit for the hypothesis of a weakly coupled Standard Model Higgs near 100 GeV [7].

Independent of its connection to phenomenology, the calculation of the parameters $S$ and $T$ is also quite interesting for purely formal reasons. This is because these observables involve correlation functions of broken symmetry generators:

$$S = -16\pi \frac{\Pi_{3Y}(M_Z^2) - \Pi_{3Y}(0)}{M_Z^2}$$

$$T = 4\pi \frac{\Pi_{11}(0) - \Pi_{33}(0)}{s_W^2 c_W^2 M_Z^2}$$

where $s_W$ and $c_W$ stand for $\sin \theta_W$ and $\cos \theta_W$, respectively, and $\Pi_{ij}(q^2)$ denotes the scalar factor of the vacuum polarization amplitude:

$$\langle J_i^\mu(q) J_j^\nu(-q) \rangle = i\eta^{\mu\nu} \Pi_{ij}(q^2) + q^\mu q^\nu \text{ terms}$$

for Fourier transformed symmetry currents $J_i^\mu$ evaluated at a reference four-momentum $q^\mu$. The subscripts on $\Pi_{ij}$ and $J_i^\mu$ refer to either the components of weak $SU(2)$ (for $i = 1, 2, 3$) or the hypercharge (for $i = Y$). Polarization amplitudes for unbroken symmetry generators are highly constrained by gauge invariance. Moreover, in supersymmetric gauge theories, it is often possible to calculate the leading behavior of these amplitudes by relating them to the computation of the NSVZ beta function [8]. The contributions $\Delta S$ and $\Delta T$ from an extra sector are even more informative as they are sensitive to details of symmetry breaking and the mass spectrum of new states.

In spite of their important role in both formal theory and phenomenology, it has proven notoriously difficult to extract quantitative estimates for $\Delta S$ and $\Delta T$ in strongly coupled extra sectors. This is because such computations appear to require detailed knowledge of the mass spectrum. For example, in order to estimate $\Delta S$ in technicolor theories, one has

\[ \Delta S \]
to make assumptions about the spectrum of strongly coupled bound states. One possibility is to assume the UV behavior resembles that of QCD, although other possibilities such as “walking” in an approximate conformal phase can also occur.

In this paper, we develop a set of techniques to compute $\Delta S$ and $\Delta T$ when the Higgs sector mixes with an additional sector which becomes superconformal at high energies. From the perspective of the extra sector, we will take the gauge group of the Standard Model to be a flavor symmetry. We couple the supersymmetric two Higgs doublet sector $H_u \oplus H_d$ to operators $O_u$ and $O_d$ of the extra sector via:

$$
\mathcal{L}_{\text{eff}} \supset \int d^2 \theta \left( \lambda_u H_u O_u + \lambda_d H_d O_d \right) + \text{h.c.}.
$$

(1.4)

Scenarios involving such mixing terms have been considered in \[9\]-\[16\].

The bottom-up motivation for this class of couplings is that compared with the MSSM, they can raise the mass of the lightest Higgs \[13\], and also allow a broader class of mixing angles in the (supersymmetric) two-Higgs-doublet model (s2HDM). The simplest way to generate a large correction to the Higgs mass is by increasing the size of the Yukawas $\lambda_u$ and $\lambda_d$, which also pushes the mass of the extra states higher than the weak scale. To avoid issues with low-scale Landau poles, it is therefore natural to envisage a situation in which the extra sector enters an approximate (super)conformal phase at higher energies.

Such extra sectors are also well-motivated in various string constructions. For example, they can arise from D3-branes probing a stack of intersecting seven-branes \[17\]. These D3-brane sectors also typically couple to the third generation of MSSM states, providing a simple mechanism to alter the masses of third generation superpartners relative to first and second generation superpartners \[18\]. From this perspective, constraints on $S$ and $T$ provide valuable information on possible stringy extensions of the Standard Model.

The phenomenology of such extra sectors is potentially quite rich, though also very model dependent. In principle, extra sector states could be charged under just $SU(2)_W \times U(1)_Y$, or may also include colored states as well. There could also be hierarchies between the colored and uncolored states, as well as additional gauge singlets which could facilitate hard-to-detect decay modes. Our aim here will be to derive constraints which do not involve such highly model dependent details. Combining this with earlier work on the dimension-five operators responsible for gluon fusion production and diphoton decays \[15\], this provides a more complete picture of the low energy consequences of mixing between the Higgs and a superconformal extra sector.

The basic outline of the present work will be to study the contributions to $S$ and $T$ when the Higgs vevs are the sole source of mass for the states of the extra sector. Of course, in realistic model building applications it is often necessary to introduce an additional vector-like mass for states of the extra sector, in which case there is an overall suppression by a factor of $v^2/\Lambda^2_{\text{vec}}$. To evaluate the leading contributions to $S$ and $T$ in the absence of such vector-like
masses we shall approximate the correlation functions by two-point functions evaluated in the superconformal phase. To this end, we shall assume that there is an approximate custodial symmetry so that the flavor symmetry of the extra sector is $SU(2)_W \times SU(2)_{\tilde{W}} \times U(1)_\chi$, with a custodial $SU(2)_{\text{diag}} \subset SU(2)_W \times SU(2)_{\tilde{W}}$, where $SU(2)_W$ denotes the weak sector gauge group (this symmetry could, however, be broken by the Higgs vevs). In this language, the hypercharge $Y$ is given by $Y = T_{\tilde{W}} + T_\chi$, where $T_{\tilde{W}}$ denotes the $\sigma(3)$ generator in $SU(2)_{\tilde{W}}$. The assumption of a custodial $SU(2)$ appears to be fairly mild, since we know $T_{\tilde{W}}$ to be small.

Using this, we characterize the contributions to both the $S$ and $T$ parameters in terms of calculable quantities in the conformal theory.

We show that the calculation of $\Delta T$ can be recast as a computation of wave-function renormalization of the Higgs field Kähler potential [14]. The resulting contribution to $\Delta T$ can be expressed as a change in the ratio of the $W$- and $Z$-boson masses, leading to the expression:

$$\Delta T = \frac{1}{\alpha_{EM}} \cos^2 2\beta \times \delta$$

(1.5)

where $\tan \beta = v_u/v_d$ denotes the ratio of the Higgs vevs and $\delta = \Delta - 1$ denotes the excess dimension of the Higgs, viewed as a field of the conformal sector. The scaling dimension of the Higgs can often be extracted by determining the infrared R-symmetry of the superconformal theory, via a-maximization [19].

The calculation of $\Delta S$ involves additional details of the extra sector. We find, however, that to leading order in the operator perturbations of (1.4), this contribution is independent of the details of the mass spectrum and is given by:

$$\Delta S_{(0)} = \frac{b_{AA}}{4\pi}$$

(1.6)

where $b_{AA}$ is a calculable anomaly coefficient:

$$b_{AA} = -3\text{Tr}(R_{IR}J_AJ_A).$$

(1.7)

Here, $R_{IR}$ is the R-symmetry of the CFT, and $J_A$ is the axial current $J_W - J_{\tilde{W}}$, formed from the $U(1)_W \times U(1)_{\tilde{W}}$ subalgebra of $SU(2)_W \times SU(2)_{\tilde{W}}$.

With sufficient additional global symmetries, we can say even more, and compute subleading contributions to $\Delta S$ which are sensitive to the mass spectrum of the theory. In situations where the states of the extra sector get their mass dominantly from either $H_u$ or $H_d$, but not both, the flavor symmetry is effectively doubled, leaving us with a different $U(1)_W \times U(1)_{\tilde{W}} \times U(1)_\chi$ acting on each set of states independently. From a bottom-up perspective, this assumption is well-motivated, as it suppresses contributions to flavor changing neutral currents in a two Higgs doublet model. From a top-down perspective, this assumption is also satisfied in SQCD-like models and some string/F-theory constructions; we will describe this further in section [7]. We denote the “up-type” symmetries by $W_u, \tilde{W}_u, \chi_u,$ and
similarly for the “down-type” symmetries. In this case, we can evaluate a further anomaly:

\[ b_{W_uY_u} = -3 \text{Tr}(R_{IR}J_{W_u}J_{Y_u}). \]  

(1.8)

Including this contribution allows us to take into account further (logarithmic) contributions which are sensitive to the breaking of the \( SU(2)_{\text{cust}} \) symmetry:

\[ \Delta S = \frac{b_{AA}}{4\pi} - \frac{4b_{W_uY_u}}{2\pi} \log \frac{m_u}{m_d} \]  

(1.9)

where \( m_u = \lambda_u v_u \) and \( m_d = \lambda_d v_d \) define characteristic mass scales for the up- and down-type states.

Even without calculating the precise values of \( \Delta S \) and \( \Delta T \) from a conformal sector, we can still extract valuable model-independent information about the overall sign and relative size of the various contributions to these parameters. We find that in unitary theories where the Standard Model gauge groups remain weakly coupled, the sign of \( \Delta T \) is fixed, and is non-negative. Moreover, we find that the non-logarithmic contribution \( \Delta S(0) \geq 0 \). A priori, however, \( \Delta S \) can have either sign depending on the overall hierarchy between \( m_u \) and \( m_d \).

The rest of this paper is organized as follows. First, in section 2 we review some general properties of \( S \) and \( T \), and their evaluation in weakly coupled models. The striking simplicity of these results motivates our expectation that a similar result is available for more general extra sectors. In section 3 we discuss our setup in more detail, and show in sections 4 and 5 how to calculate the leading oblique electroweak corrections from a superconformal extra sector. In section 6 we determine some model-independent bounds on the sign and size of \( \Delta S \) and \( \Delta T \), and in section 7 we present some example computations. Section 8 contains our conclusions.

2 Review of Precision Electroweak Parameters

In this section we review the definitions of the oblique electroweak parameters \( S \) and \( T \). In preparation for our later discussion, we also present a schematic calculation of \( S \) and \( T \) for weakly coupled states. In subsequent sections we will aim to show how these contributions generalize to a (possibly strongly coupled) extra sector.

Let us begin with the definitions of the oblique electroweak parameters. In extensions of the Standard Model, new states can in principle alter the masses \( M_W \) and \( M_Z \) of the \( W \)- and \( Z \)-bosons, as well as the value of the Fermi decay constant \( G_F \), the weak mixing angle \( \theta_W \), and the gauge couplings \( g \) and \( g' \) for \( SU(2)_W \) and \( U(1)_Y \). An efficient way to parameterize such deviations is in terms of precision electroweak observables, connected with vacuum polarization amplitudes for the electroweak sector. We will assume throughout this work that the oblique approximation is valid, meaning that the primary contributions to precision
observables are captured by $S$ and $T$. These are defined by equations (1.1) and (1.2), which we reproduce here for convenience of the reader:

$$S = -16\pi \frac{\Pi_{3Y}(M_Z^2) - \Pi_{3Y}(0)}{M_Z^2}$$

$$T = 4\pi \frac{\Pi_{11}(0) - \Pi_{33}(0)}{s_W^2 c_W^2 M_Z^2}.$$  

In the limit where $M_Z$ is much smaller than the characteristic mass of new states, $S$ is specified by the derivative of $\Pi_{3Y}$ with respect to $q^2$, the square of the momentum transfer:

$$S = -16\pi \frac{\Pi'_{3Y}(0)}{M_Z^2}.$$  

In this sense, $S$ and $T$ involve the IR behavior of the vacuum polarization amplitudes.

Our primary interest is in contributions from new physics, which we denote by $\Delta S$ and $\Delta T$. In the models of interest to us here, the extension consists of two portions. First, there are the possible contributions from extending the Standard Model to the MSSM. This includes contributions from extending the Higgs sector to a supersymmetric two Higgs doublet model (s2HDM), as well as the other superpartners of the MSSM. Additionally, there are the contributions from the extra sector. In the limit where the Higgs sector only weakly mixes with an extra sector, we can treat its contribution to $S$ and $T$ independently from the s2HDM and MSSM contributions.

After the states of the extra sector pick up a mass of order $\Lambda$, we can integrate them out. For example, the higher dimension operators which contribute to $S$ and $T$ are:

$$O_S = \left(c^{(u)}_S \frac{H_u^i \sigma^{(i)} H_u}{\Lambda^2} + c^{(d)}_S \frac{H_d^i \sigma^{(i)} H_d^\dagger}{\Lambda^2} + c^{(\text{mix})}_S \frac{H_u^i \sigma^{(i)} H_d^\dagger}{\Lambda^2}\right) W^{(i)}_{\mu\nu} B^{\mu\nu}$$

$$O_T = c^{(u)}_T \frac{|H_u^i D_{\mu} H_u|^2}{\Lambda^2} + c^{(d)}_T \frac{|H_d^i D_{\mu} H_d|^2}{\Lambda^2} + c^{(\text{mix})}_T \frac{|H_u^i D_{\mu} H_d|^2}{\Lambda^2}$$

where $\frac{\sigma^{(i)}}{2} W^{(i)}_{\mu\nu}$ denotes the $SU(2)$ field strength and $B_{\mu\nu}$ denotes the $U(1)_Y$ field strength.

Once the Higgs fields develop a non-zero vev, we obtain non-zero contributions to the gauge boson two point functions entering the definitions of $S$ and $T$. In other words, our task is to compute the “order one coefficients” denoted by the $c$’s.

In a weakly coupled setting, the contributions to $S$ and $T$ from extra states are fully calculable. For examples of such calculations, see [4, 5, 20–22], and [23] for a pedagogical discussion. Consider a weakly coupled left-handed fermion doublet $(\psi_1, \psi_2)$ with hypercharge $Y$, and its $SU(2)_W$ singlet partners $\psi_1^c$ and $\psi_2^c$, with masses $m_1, m_2$. These states pick up a mass once the Higgs fields develop vevs. In our conventions, $H_u$ and $H_d$ have $U(1)_Y$ hypercharge $+1/2$ and $-1/2$, respectively. The contributions $\Delta S$ and $\Delta T$ are then (see
\begin{equation}
\Delta S = \frac{1}{6\pi} \left( 1 - 2Y \log \frac{m_1^2}{m_2^2} \right) \tag{2.6}
\end{equation}

\begin{equation}
\Delta T = \frac{1}{8\pi s_W^2 c_W M_Z^2} \left( \frac{m_1^2 + m_2^2}{2} - \frac{m_1^2 m_2^2}{m_1^2 - m_2^2} \log \frac{m_1^2}{m_2^2} \right). \tag{2.7}
\end{equation}

Particularly suggestive is the contribution \( \Delta S \) for a collection of Dirac fermions \( \psi \) of mass \( m(\psi) \) with weak isospin \( t_L^{(\psi)}, t_R^{(\psi)} \) and hypercharge \( y_L^{(\psi)}, y_R^{(\psi)} \) for the left- and right-handed components:

\begin{equation}
\Delta S = \frac{1}{3\pi} \sum_\psi \left[ \left( t_L^{(\psi)} - t_R^{(\psi)} \right)^2 - 2 \left( t_L^{(\psi)} y_L^{(\psi)} + t_R^{(\psi)} y_R^{(\psi)} \right) \log \frac{m_2^{(\psi)}}{\mu(0)} \right] \tag{2.8}
\end{equation}

with \( \mu(0) \) a UV cutoff. Since the total \( t_L^{(\psi)} y_L^{(\psi)} \) and \( t_R^{(\psi)} y_R^{(\psi)} \) vanish for each \( SU(2)_W \) multiplet, \( \Delta S \) is independent of \( \mu(0) \), as it must be for a finite contribution.

One way to understand this result is to work in terms of a weakly coupled basis of left-handed Weyl-fermions \( \psi_L \) and \( \psi_R^c \) with effective Lagrangian:

\begin{equation}
L_{(\psi)} = -i\psi_L^c \overline{\sigma}^\mu \partial_\mu \psi_L - i\psi_L^c \overline{\sigma}^\mu \partial_\mu \psi_R^c - m(\psi) \psi_L^c \psi_R^c - \mu(\psi) \psi_L^c \psi_R^c \tag{2.9}
\end{equation}

where we have suppressed the implicit sum over the full \( SU(2)_W \) multiplet, since this symmetry is in general broken. Our main interest is the abelian symmetries \( U(1)_W \times U(1)_Y \), specified by weak isospin and hypercharge. To account for the appearance of left-handed and (conjugate) right-handed states, it is convenient to introduce an additional \( U(1)_{\tilde{W}} \) symmetry which encodes the \( U(1)_W \) charge of the parity conjugate state. Each field of the theory then carries a specific charge under \( U(1)_W \times U(1)_{\tilde{W}} \times U(1)_Y \):

\begin{equation}
\begin{array}{|c|c|c|}
\hline
\text{Field} & U(1)_W & U(1)_{\tilde{W}} & U(1)_Y \\
\hline
\psi_L & +t_L & +t_R & +Y \\
\psi_R^c & -t_R & -t_L & -Y \\
m(\psi) & -(t_L - t_R) & -(t_R - t_L) & 0 \\
\hline
\end{array} \tag{2.10}
\end{equation}

where weak hypercharge is given by the generator \( Y = T_{\tilde{W}} + T_Y \). We can also introduce the vector and axial combinations \( T_V = T_W + T_{\tilde{W}} \) and \( T_A = T_W - T_{\tilde{W}} \). From this perspective, we see that the mass term is charged under the axial \( U(1)_A \), but neutral under \( U(1)_V \times U(1)_Y \).

Evaluation of the S-parameter can now be understood by considering the \( \psi_L \) and/or \( \psi_R^c \) running in the loop. There are two ways to generate a non-zero contribution to \( S \): either from a loop with both \( \psi_L \) and \( \psi_R^c \) and a mass insertion, or a loop with either \( \psi_L \) or \( \psi_R^c \) and no mass insertion.

\footnote{We thank P. Langacker for helpful discussions.}
It is straightforward to see that when only $\psi_L$ or $\psi_R^c$ run in the loop, the contribution is proportional to $t_L^{(\psi)} y_L^{(\psi)}$ or $t_R^{(\psi)} y_R^{(\psi)}$, times a logarithmic factor. This is the second contribution in (2.8).

To get a non-zero contribution when both $\psi_L$ and $\psi_R^c$ appear in a loop, we must allow two mass insertions. Each mass insertion is neutral under $U(1)_V$ but carries axial charge proportional to $t_L - t_R$. Thus, the overall contribution from mass insertions is proportional to $(t_L - t_R)^2$, as appropriate for the two-point function for the axial current. This corresponds to the first term in equation (2.8).

3 Mixing with a Superconformal Extra Sector

In preparation for our discussion in subsequent sections, in this section we provide more details on our setup, as well as the tools available for studying SCFTs. As mentioned in the Introduction, our setup consists of a supersymmetric two Higgs doublet sector which couples to a superconformal extra sector via the F-terms:

$$L_{\text{eff}} \supset \int d^2 \theta \left( \lambda_u H_u O_u + \lambda_d H_d O_d \right) + \text{h.c.} \quad (3.1)$$

where $O_u$ and $O_d$ are operators of the extra sector. See [9, 11–16] for some examples of Higgs/extra sector mixing.

More precisely, we first imagine a “UV theory” (close to the GUT scale) specified by a superconformal fixed point, where the couplings $\lambda_u$ and $\lambda_d$ have been switched off and the Higgs fields have dimension one. To give an unambiguous meaning to the Yukawas $\lambda_u$ and $\lambda_d$, we assume that the two-point functions for the $O$’s are normalized so that at the UV fixed point, prior to mixing with the Higgs, we have:

$$\langle O^\dagger(x) O(0) \rangle = \frac{1}{16\pi^4 x^{2\Delta_O}} \quad (3.2)$$

where $\Delta_O$ is the scaling dimension of $O$.

We couple the Higgs to the extra sector by turning on $\lambda_u$ and $\lambda_d$, driving the theory to a new infrared fixed point, the “IR theory”. More precisely, we assume that near the scale $4\pi v$ with $v \simeq 246$ GeV, the theory has flowed to an approximate conformal phase, which can be viewed as the IR of the CFT. From the perspective of electroweak symmetry breaking, however, this scale serves as a UV cutoff. To keep the distinction clear, we will reference whether we are discussing the IR of the CFT or the electroweak sector. We denote the scale of conformal symmetry breaking as $M_{\text{CFT}}$. Below this scale, we can expect the CFT to develop a mass gap. More precisely, we can expect a collection of massive states as we pass from the CFT breaking scale down to the infrared of the electroweak scale. These thresholds will occur at various mass scales, and in general will depend on highly model
dependent details of a given conformal theory. Nevertheless, one of our aims will be to show that approximate conformal symmetry still constrains the behavior of two-point functions in the infrared of the electroweak theory. We shall also assume as in [15] that the mass spectrum is approximately supersymmetric, an assumption which is well-justified in various mediation scenarios.

A number of properties of the new fixed point are specified by the infrared R-symmetry of the mixed CFT. For example, the scaling dimension of any chiral primary operator $O$ is:

$$\Delta_{IR}(O) = \frac{3}{2}R_{IR}(O).$$

(3.3)

To couple the visible sector to the CFT, we view the Standard Model gauge group as a global symmetry of the CFT. The objects that show up in our computation of $\Delta S$ and $\Delta T$ will then be two-point functions of the currents that couple to these weakly gauged symmetries.

The infrared R-symmetry of the CFT also shows up in the calculation of two-point functions of symmetry currents. Although we are interested in the behavior of these two-point functions below the energy scale of conformal symmetry breaking, it turns out that many of these properties can be recast in terms of properties of the short-distance physics. This behavior is in turn related to the operator product expansion (OPE) of the currents. In more detail, recall that we need to calculate the behavior of the vacuum polarization amplitude $\Pi_{ij}^{\mu\nu}(q^2)$:

$$\Pi_{ij}^{\mu\nu}(q^2) = \langle J_i^\mu(q) J_j^{\nu}(-q) \rangle$$

(3.4)

in the infrared of the electroweak theory. Here, $J_i^\mu(q)$ denotes the (Fourier transform) of the flavor symmetry current, with flavor indices $i, j$ and Lorentz indices $\mu, \nu$. In a supersymmetric theory, this current sits inside a supermultiplet $J_i$, with

$$J_i(x, \theta, \bar{\theta}) = J_i(x) - \bar{\theta}\sigma_\mu \theta J_i^\mu(x) + ...$$

(3.5)

See for example [24–26] for further discussion of current supermultiplets. While the behavior of the correlation function in the infrared of the electroweak theory involves many details about the mass spectrum of the extra sector in the broken symmetry phase, the short distance behavior in the CFT is tightly constrained by the operator product expansion (OPE) for the symmetry currents. The OPE of the bosonic components $J_i$ is (see e.g. [27]):

$$J_i(x)J_j(0) = b_{ij} \frac{1}{16\pi^4 x^4} + d_{ijk} \frac{J_k(0)}{16\pi^2 x^2} + w_{ij} \frac{K(0)}{4\pi^2 x^{2-\gamma_K}} + c_{ij}^l \frac{O_l(0)}{x^{4-\Delta_l}} + \text{descendants.}$$

(3.6)

Here, $b_{ij}$ is an anomaly coefficient:

$$b_{ij} = -3\text{Tr}(R_{IR} J_i J_j).$$

(3.7)

This trace is over the various species of Weyl fermions in the theory, and is calculable via ‘t
Hooft anomaly matching as long as we know the infrared R-symmetry of the CFT. The terms with \( i = j \) correspond to the numerator of the NSVZ beta function. Contributions when \( i \neq j \) can be viewed as including the effects of kinetic mixing between different abelian symmetry factors. For non-abelian symmetry currents such off-diagonal contributions vanish.

The remaining terms in equation (3.6) include the operators \( \mathcal{O}_i(0) \) of dimension \( \Delta_i \), which specify real conformal primaries. \( K(0) \) corresponds to the vev of the Konishi operator (i.e. Kähler potential for weakly coupled fields), with anomalous dimension \( \gamma_K \). As discussed in [27,28], the OPE is useful even if the flavor symmetries of the currents are spontaneously broken.

Clearly, a number of properties of visible/extra sector mixing are specified by the IR R-symmetry of the new fixed point. Thankfully, this turns out to be calculable in many superconformal theories of interest. In general, the IR R-symmetry of the CFT is given by a linear combination of the UV R-symmetry \( R_{UV} \) with all infrared flavor symmetries \( F_i \):

\[
R_{IR} = R_{UV} + \sum t_i F_i \tag{3.8}
\]

with the coefficients \( t_i \) fixed by a-maximization [19]. An important remark is that the terms on the right-hand side of (3.8) can always be grouped in terms of conserved currents. Thus, another way to organize equation (3.8) is in terms of a putative infrared R-symmetry, and all possible linear combinations of conserved abelian flavor symmetries. The procedure of a-maximization determines the coefficients \( t_i \) so that the resulting R-symmetry is the unique one inside the same supermultiplet as the stress tensor. As stated above, finding this R-symmetry is equivalent to finding the scaling dimensions of chiral primary fields. When the Higgs fields develop small anomalous dimensions, we can treat the mixing between the Higgs and the extra sector as small.

### 3.1 Symmetry Assumptions

From the perspective of the extra sector, the weakly coupled gauge group of the Standard Model is essentially a flavor symmetry. Throughout this work, we shall assume that for an appropriate range of couplings and Higgs vevs, the extra sector also enjoys an approximate custodial \( SU(2) \) symmetry, \( SU(2)_{\text{cust}} \). To this end, we assume throughout that the extra sector has a flavor symmetry \( SU(2)_W \times SU(2)_{\tilde{W}} \times U(1)_\chi \) with \( SU(2)_{\text{cust}} = SU(2)_{\text{diag}} \subset SU(2)_W \times SU(2)_{\tilde{W}} \). In this limit, the Higgs fields \( H_u \) and \( H_d \) assemble into a \( 2 \times 2 \) matrix transforming as a bi-doublet \( (2,2) \) of \( SU(2)_W \times SU(2)_{\tilde{W}} \), which decomposes as \( 3 \oplus 1 \) under \( SU(2)_{\text{cust}} \). The electroweak-breaking vacuum will then preserve \( SU(2)_{\text{cust}} \) when the vevs \( v_u \) and \( v_d \) are equal.

The Standard Model hypercharge and electric charge embed in \( SU(2)_W \times SU(2)_{\tilde{W}} \times U(1)_\chi \).
in the standard way:

\[ Y = T_{\tilde{W}} + T_\chi \quad \text{and} \quad Q = T_W + T_{\tilde{W}} + T_\chi. \]  

(3.9)

where \( T_{\tilde{W}} \) and \( T_W \) denote the \( \sigma(3) \) direction of the respective \( SU(2) \) factors. It will prove convenient to introduce two additional linear combinations, which we refer to as the vector and axial symmetry generators:

\[ T_V = T_W + T_{\tilde{W}} \quad \text{and} \quad T_A = T_W - T_{\tilde{W}}. \]  

(3.10)

The generator \( T_V \) embeds as the \( \sigma(3) \) direction of \( SU(2)_{\text{cust}} \). Let us emphasize that in our terminology the notion of “vector” and “axial” generators only makes reference to the algebraic structure of the symmetry. In a parity symmetric theory this is equivalent to introducing currents which act on left- and right-handed states differently. Since we will be interested in theories where a weakly coupled description may not be available, we shall adhere to the conventions just outlined.

Even without introducing a mass scale into the CFT, we can break electroweak symmetry by introducing the explicit \( SU(2)_W \) breaking term which preserves \( U(1)_A \) and \( U(1)_V \):

\[ \mathcal{L}_{\text{eff}} \supset \int d^2 \theta \left( \lambda_u H_u^{(0)} O_u^{(0)} + \lambda_d H_d^{(0)} O_d^{(0)} \right) + h.c. \]  

(3.11)

so that only the electrically neutral components of the operators mix. This leads to a distinct fixed point, but is often quite close to the original fixed point if the Higgs dimension remains close to its free field value.

We shall also assume that there is a \( \mathbb{Z}_2 \) symmetry under which the axial current is odd and the vector current and \( U(1)_\chi \) current are even. Once we break electroweak symmetry, we can identify this \( \mathbb{Z}_2 \) symmetry with the standard parity of a vector-like supersymmetric theory. Since the two-point function \( \Pi_{3Y} \) is given by \( \frac{1}{2} (\Pi_{VV} + \Pi_{AA}) \) while the two-point function for \( \Pi_{AY} \) is \( -\frac{1}{2} \Pi_{AA} \), the full two-point function \( \Pi_{3Y} \) can be written as (see e.g. [5]):

\[ \Pi_{3Y} = \frac{1}{4} (\Pi_{VV} - \Pi_{AA}). \]  

(3.12)

This presentation of the two-point function will prove quite helpful when we study mass-independent contributions to the \( S \)-parameter.

\(^2\)Let us note that in the literature, it is common to refer to these symmetries as \( SU(2)_L \times SU(2)_R \times U(1)_\chi \), even though a left- or right-handed state could be charged under both non-abelian factors. To minimize confusion, we shall avoid this terminology in what follows.
4 Wave Function Renormalization and the $T$-Parameter

Having discussed our general assumptions about conformal symmetry breaking and flavor symmetries of the superconformal extra sector, we now compute the contribution $\Delta T$ from the states in the extra sector. We follow the same strategy outlined in [14]. Rather than directly evaluate the current two-point functions, we shall instead consider the net effect the extra sector has on the masses of the $W$- and $Z$-bosons, as reflected in the $\rho$-parameter

$$\rho \equiv \frac{M_W^2}{M_Z^2} e_W^2.$$  

This is in turn related to $T$ via (see e.g. [5]):

$$\rho = \frac{1}{1 - \alpha_{EM} T} \simeq 1 + \alpha_{EM} T$$  \hspace{1cm} (4.1)

Hence, to compute $\Delta T$, it is enough to compute the change in the masses of the gauge bosons from coupling to the extra sector.

To calculate this contribution, we view the Higgs vevs as a supersymmetric modulus which introduces an effective mass into the superconformal extra sector. The net response of the extra sector on the Higgs is then dictated by a change in the effective Kähler metric on the moduli space of the Higgs vevs. In a supersymmetric theory, these corrections show up in the Coleman-Weinberg [29] correction to the Kähler potential:

$$\delta K = -\frac{1}{32\pi^2} \text{Tr} \left( M^\dagger M \log \frac{M^\dagger M}{\mu^2(0)} \right).$$  \hspace{1cm} (4.2)

where $M$ denotes the mass matrix of the theory, and $\mu(0)$ denotes a UV cutoff scale.

To estimate the corrections to the Kähler potential, we work to leading order in conformal perturbation theory, that is, we restrict to contributions involving two Higgs fields. We assume the existence of an approximate custodial $SU(2)_{\text{cust}}$ symmetry. This means the Higgs fields $H_u$ and $H_d$ can be assembled into a $2 \times 2$ matrix $\Phi$ which transforms in the adjoint of $SU(2)_{\text{cust}}$, with a common dimension:

$$\Delta = 1 + \delta$$  \hspace{1cm} (4.3)

where $\delta$ denotes the excess Higgs dimension, once these fields mix with the CFT. The two real flavor neutral bilinears we can form are $\det \Phi + h.c.$ and $\Phi^\dagger \Phi$. However, since terms entering our Kähler potential need to be invariant under the infrared R-symmetry of the CFT, it follows that to leading order in conformal perturbation theory, the Kähler potential is constrained to be $(\Phi^\dagger \Phi)^{1/\Delta}$. In more formal terms, $\Phi^\dagger \Phi$ denotes the term of the OPE proportional to the identity operator. Since $\delta$ is taken to be small, we can take this operator to have scaling dimension $\Delta_{\Phi^\dagger \Phi} \simeq 2 \Delta_{\Phi}$.

Expanding out, we learn that the Kähler potential for the Higgs fields is, in the approx-
imate custodial limit:
\[ K = (\Phi^\dagger \Phi)^{1/\Delta} = (H_u^1 H_u + H_d^1 H_d)^{1/\Delta}. \] (4.4)

This expression is independent of the Yukawas because we are working in the special limit where the ratio \( \lambda_u/\lambda_d \approx 1. \)

This Kähler potential modifies the kinetic term for the Higgs doublet:
\[ L_{\text{kin}} = -g_i^j (D^{\mu} \Phi)^i (D^{\mu} \Phi)^{\dagger j}. \] (4.5)

As computed in [14], the resulting change in the mass of the W- and Z-bosons is then:
\[ M_W^2 = \frac{g^2}{2\Delta} (v_u^2 + v_d^2)^{1/\Delta} \quad \text{and} \quad M_Z^2 = \frac{M_W^2}{c_W^2} \times \frac{(v_u^2 + v_d^2)^2 + 4v_u^2 v_d^2 (\Delta - 1)}{\Delta (v_u^2 + v_d^2)^2}. \] (4.6)

Expanding to leading order in \( \delta \), we have:
\[ \rho = 1 + \left( 1 - \frac{4v_u^2 v_d^2}{(v_u^2 + v_d^2)^2} \right) \delta = 1 + \cos^2 2\beta \times \delta. \] (4.7)

Using the further relation \( \rho = 1/(1 - \alpha_{EM} T) \), we obtain the estimate:
\[ \Delta T = \frac{1}{\alpha_{EM}} \cos^2 2\beta \times \delta \] (4.8)

which vanishes in the custodial limit \( \tan \beta \to 1 \). Using a-maximization, it is possible to compute \( \delta \). In many cases distinct from SQCD-like theories, \( \delta \) can be small (see [18] for some examples). Summarizing, we have shown that in this limit, \( \Delta T \) is actually calculable, and is proportional to the excess dimension of the Higgs.

5 Anomalies and the S-Parameter

We now turn to an evaluation of the S-parameter in scenarios where the Higgs mixes with a conformal sector. From the perspective of the conformal theory, this effect is more subtle, as it cannot be recast in terms of a wave-function renormalization of the Higgs fields. Nevertheless, we will argue that enough data is typically available in a conformal theory to estimate the leading contributions to \( \Delta S \) in terms of various calculable anomalies of the conformal

\[ 3^{\text{As a brief digression, it is of course tempting to extend this to more general values of the Yukawas by rescaling the Higgs fields. Indeed, from the perspective of the conformal sector, the only thing the CFT “knows about” is the overall mass scale } m_u = \lambda_u v_u \text{ and } m_d = \lambda_d v_d, \text{ so we can assemble the corrections of } \text{equation } \text{(4.2)} \text{ into a single } 2 \times 2 \text{ matrix. However, what spoils the argument is that the overall coefficient of this term relative to the original } H^\dagger H \text{ terms is no longer constrained, since they respect different custodial } SU(2) \text{ symmetries. Indeed, this is in accord with the general expectation that away from the custodial } SU(2) \text{ limit, there will be a non-trivial dependence on the Yukawa couplings.} \]
\[ \Delta S = \frac{b_{AA}}{4\pi} - \frac{4b_{WY}}{2\pi} \log \frac{m_u}{m_d}, \]  

with \( b_{ij} \) an appropriate anomaly coefficient similar to that in equation (3.7).

To organize the various contributions to the \( S \)-parameter, it is helpful to review some general features of dispersion relations for two-point functions. As explained in [5], this provides a way to relate the \( S \)-parameter to the short distance behavior of the correlation function. By definition, the contribution \( \Delta S \) from states of the CFT comes from evaluating \(-16\pi\Pi_{3Y}'(0)\), where we implicitly subtract off the contribution from the Standard Model and supersymmetric 2HDM. We can recast evaluation of \( \Pi_{3Y}'(0) \) as the residue integral of an analytic function \( \Pi_{3Y}'(s) \) in the complex \( s = q^2 \) plane:

\[ \Pi_{3Y}'(0) = \frac{1}{2\pi i} \oint ds \frac{s}{s} \Pi_{3Y}'(s). \]  

At a sufficiently large radius for the contour, we encounter poles and branch cuts in \( \Pi_{3Y}'(s) \), indicative of the mass spectrum of the theory. For example, in the case of a weakly coupled free field of mass \( m \), there is a pole at \( s = m^2 \) and a branch cut at \( s = 4m^2 \), when the spectrum enters a continuum. More generally, we can expect a complicated set of contributions, depending on the details of the mass spectrum of the CFT after breaking conformal symmetry.

Assuming suitable convergence properties of \( \Pi_{3Y}'(s) \), we can take the residue integral to extend out to a circle of large radius. The contour integral can then be replaced by contours encircling the poles in the \( s \)-plane, and the branch cut. The main approximation we are going to make throughout this work is that there is a branch cut which starts at \( s = M_{CFT}^2 \), and that we can introduce a single contour to encircle all of the poles. Contributions from the poles correspond to a given resonance, and are manifestly finite contributions to \( \Delta S \). Contributions from crossing a branch cut lead to effects which depend logarithmically on the mass scales of the theory. These branch cut effects are more subtle, as an individual logarithm will superficially appear to involve some dependence on a cutoff. Of course, the net contribution to \( \Delta S \) must be cutoff independent.

Motivated by these general considerations, we shall parameterize the net contribution to the \( S \)-parameter by grouping all of the poles into one contribution, and the branch cut contributions into another:

\[ \Delta S = \Delta S(0) - \frac{\delta b}{2\pi} \log f(m_u, m_d) \]  

where \( f(m_u, m_d) \) is a general function of \( m_u \) and \( m_d \) which can be viewed as summing up the thresholds of masses in the theory. We refer to \( \Delta S(0) \) as the mass-independent contribution, since to leading order these details only appear in the logarithmic term. In our parametrization, these extra terms vanish as \( f \to 1 \) in the custodial limit \( m_u \to m_d \).
The rest of this section is organized as follows. First, we show that the leading order contribution to $\Delta S(0)$ is a calculable anomaly coefficient. Next, we show that the leading contributions to the logarithmic terms can also be calculated, provided the extra sector enjoys some additional well-motivated symmetries. Finally, we also discuss the limit where the Higgs vevs are a subdominant contribution to the mass of the extra sector. In this limit we can also relate $\Delta S$ to the Higgs decay rates into $\gamma\gamma$ and $Z\gamma$.

5.1 The Mass-Independent Contribution

In this section we turn to the calculation of the mass-independent contribution $\Delta S(0)$ in equation (5.3). Rather than working directly in terms of the value of the two-point function in the infrared of the electroweak theory, we shall instead attempt to approximate $\Delta S(0)$ based on its high energy behavior.

The short-distance behavior of the current correlation functions is dictated by the OPE reviewed in equation (3.6):

$$J_i(x)J_j(0) = b_{ij} \frac{1}{16\pi^4 x^4} + d_{ijk} \frac{J_k(0)}{16\pi^2 x^2} + w_{ij} \frac{K(0)}{4\pi^2 x^2 - \gamma \kappa} + c_{ij} \frac{O_l(0)}{x^{4-i}} + \text{descendants.} \quad (5.4)$$

The analogue of mass insertions in a weakly coupled model correspond to the terms proportional to $c_{ij}O_l(0)$, with $O_l(0)$ real primaries of the superconformal theory. The operators which get a non-zero vev are $H_u^\dagger \sigma(l) H_u$ and $H_d^\dagger \sigma(l) H_d^1$, which have dimension close to two when the Higgs/extra sector mixing is small. This is in accord with the general discussion near equation (2.4). What we are going to do is recast these insertions as the first term in the OPE of an axial current two-point function.

Now, in the custodial $SU(2)$ limit, the Higgs field is a diagonal matrix proportional to the identity, and thus leaves unbroken $U(1)_Y \times U(1)_X$. In other words, insertions of the Higgs fields will only generate a contribution for the broken symmetry generator $U(1)_A$. This means that in the two-point function of equation (3.12):

$$\Pi_{3Y} = \frac{1}{4} (\Pi_{VV} - \Pi_{AA}) \quad (5.5)$$

it is enough to track the behavior of $\Pi_{AA}$.

To approximate the behavior of $\Pi_{AA}$, we treat the effects of the massive states as a single threshold correction, controlled by an overall beta function coefficient $b_{AA}$. Matching the asymptotic behavior of the CFT expression to that of the single threshold approximation, we will extract the mass-independent contribution $\Delta S(0)$. In other words, we approximate the net contribution of the CFT to the two-point function as if it were a single particle of mass $M$, with coupling to the gauge fields specified by its net effect on the beta functions. To this end, it is helpful to recall the behavior of a general threshold correction to a weakly
gauged flavor symmetry. This appears in the vacuum polarization amplitude:

\[ \Pi_{\mu\nu}^{XX}(q^2) = i \left( \eta^{\mu\nu} q^2 - g^\mu q^\nu \right) \tilde{\Pi}_{XX} (q^2, \mu^2_{(0)}, M^2) \]  

(5.6)

In the regimes \(|q| \gg M\) and \(|q| \ll M\), \(\tilde{\Pi}_{XX} (q^2, \mu^2_{(0)}, M^2)\) behaves as:

\[ \tilde{\Pi}_{XX} (q^2, \mu^2(0), M^2) = \frac{b_{XX}}{8\pi^2} \left\{ \begin{array}{ll} \log \frac{M}{\mu^2(0)} & \text{for } |q| \ll M \\ \log \frac{M}{|q|} & \text{for } |q| \gg M \end{array} \right. \]  

(5.7)

where \(b_{XX} = -3\text{Tr}(R_{IR}J_X J_X)\) is the beta function coefficient. The assumption that we can treat all of the states as a single threshold means that the overall coefficient \(b_{XX}\) tracks the mass-independent contribution to \(\Pi_{XX}^\prime(0)\).

Turning to our case, the single threshold approximation means that the mass-independent contribution to the \(S\)-parameter is given by an anomaly coefficient for the axial symmetry:

\[ \Delta S_{(0)} \simeq \frac{1}{2} \times -16\pi \times -\frac{1}{4} \times \frac{b_{AA}}{8\pi^2} = \frac{b_{AA}}{4\pi} \]  

(5.8)

where:

\[ b_{AA} = -3\text{Tr}(R_{IR}J_A J_A). \]  

(5.9)

The overall factor of \(1/2\) in equation (5.8) is due to the fact that in our \(Z_2\) symmetric theory, we must not double count the contribution from a Dirac fermion.

It is interesting to note that in the models we are considering where the CFT enjoys an \(SU(2)_W \times SU(2)_{\tilde{W}} \times U(1)_\chi\) symmetry, the anomaly coefficient \(b_{AA}\) is numerically the same as \(b_{VV}\), the contribution from the vector current. This is because after summing over all components of a multiplet, the net contribution from \(J_W J_{\tilde{W}}\) vanishes. Note, however, that we can also extend our discussion to cover the case detailed in equation (3.11), where we break by hand the non-abelian symmetry. In this case, we can still compute \(b_{AA}\), and now it could differ from \(b_{VV}\).

Finally, it is also instructive to compare our computation with the case of technicolor-like theories, and other models where the Higgs boson is a composite operator. This corresponds to allowing a possibly large correction to the scaling dimension of the Higgs once it mixes with the SCFT. In such cases, there is no sense in which there is a “mass insertion” by the Higgs fields. Indeed, this is reflected by the fact that the vector and axial two-point functions both make sizable contributions to the \(S\)-parameter, as for example in the estimate of \([5]\) for QCD-like technicolor theories. Moreover, this leads to a rather different behavior for the vacuum amplitude which is often well-approximated by a “vector-dominance” model of the scaled up QCD-like theory. The situation here is reversed, and is more in line with the weakly coupled analysis. Here, the dominant contribution is from the two-point function for the axial current. Observe, however, that the behavior of the vector and axial two-point functions in the UV regime of the CFT are the same, in accord with the fact that numerically,
\( b_{AA} = b_{VV} \) in models with \( SU(2)_W \times SU(2)_{\tilde{W}} \times U(1)_{\chi} \) symmetry.

### 5.2 The Logarithmic Contribution

In addition to the mass-independent contribution \( \Delta S(0) \), the general behavior of the two-point function will depend on the mass spectrum of the extra sector states. This is reflected in the mass threshold function \( f(m_u, m_d) \) in equation (5.3):

\[
\Delta S = \Delta S(0) - \frac{\delta b}{2\pi} \log f(m_u, m_d).
\]  

(5.10)

In general, we cannot hope to estimate this in full detail, as it involves unrealistic expectations on the amount of information we have about the CFT. However, in special cases, we can obtain an accurate approximation as follows.

We now show how to extract these contributions when the Hilbert space of states splits into up-type and down-type states:

\[
\mathcal{H} = \mathcal{H}_u \oplus \mathcal{H}_d.
\]  

(5.11)

More precisely, we consider the related system where we break by hand the \( SU(2)_W \) symmetry by omitting the fields \( H_u^{(+)} \) and \( H_d^{(-)} \) from the visible sector. In this limit, the flavor symmetry considered previously is \( U(1)_W \times U(1)_{\tilde{W}} \times U(1)_{\chi} \). The up-down approximation amounts to the further assumption that the flavor symmetry contains two copies of this symmetry, which we denote by \( G_{\text{up}} \times G_{\text{down}} \). We will denote the up-type symmetries by \( U(1)_{W_u} \times U(1)_{\tilde{W}_u} \times U(1)_{\chi_u} \), and the down-type symmetries by \( U(1)_{W_d} \times U(1)_{\tilde{W}_d} \times U(1)_{\chi_d} \). The two hypercharges are given by \( Y_{u,d} = T_{\tilde{W}_{u,d}} + T_{\chi_{u,d}} \).

Although this might seem like an ad hoc assumption, it is well-motivated from both bottom-up and top-down considerations. For example, in a generic model with two Higgs doublets, there can often be problems with flavor-changing neutral currents. Such additional symmetries can naturally suppress potentially dangerous contributions to flavor-changing neutral currents. This is the case in the MSSM, where couplings between the up-type states and \( H_d \) do not appear holomorphically. Additionally, from a top-down perspective, such a splitting occurs in many string/F-theory constructions, a feature we will study more in section 7.

Combining this with our previous discussion near equation (5.7), we deduce the further contribution to the two point function:

\[
\Delta S - \Delta S(0) = -\frac{b_{W_u} y_u}{\pi} \log \frac{m_u^2}{\mu^2(0)} - \frac{b_{W_d} y_d}{\pi} \log \frac{m_d^2}{\mu^2(0)}
\]  

(5.12)
where the relevant anomaly coefficients are:

\[
\begin{align*}
  b_{W_u Y_u} &= -3 \text{Tr}(R_{IR} J_{W_u} J_{Y_u}) \quad (5.13) \\
  b_{W_d Y_d} &= -3 \text{Tr}(R_{IR} J_{W_d} J_{Y_d}) \quad (5.14)
\end{align*}
\]

Since in the fully theory the states of the CFT fill out complete $SU(2)_W \times SU(2)_{\tilde{W}}$ multiplets, we also have $b_{W_u Y_u} = -b_{W_d Y_d}$. In other words, the full estimate for the $S$-parameter is:

\[
\Delta S = \frac{b_{AA}}{4\pi} - \frac{4b_{W_u Y_u}}{2\pi} \log \frac{m_u}{m_d} \quad (5.15)
\]

and all dependence on the UV cutoff $\mu(0)$ has dropped out, as it must. The factor of $-4$ is due to the factor of $-1/4$ appearing in equation (3.12). Since this contribution is not $\mathbb{Z}_2$ symmetric, there is also an additional factor of 2 relative to the mass-independent contribution, as per our discussion near equation (5.8).

In other words, we have reduced the computation of the $S$-parameter to the computation of two anomaly coefficients. This is clearly a significant simplification, and can often be extracted without detailed knowledge of the CFT.

### 5.3 Vector-Like Limit and Higgs Decays

The vector-like limit amounts to switching on large mass terms in the extra sector which preserve the symmetries $SU(3)_C \times SU(2)_W \times U(1)_Y$. For a characteristic mass scale $\Lambda_{\text{vec}}$, the contribution to the $S$-parameter decouples as $v^2/\Lambda_{\text{vec}}^2$. In this subsection we first specify the dimension six supersymmetric operator related to the $S$-parameter and then show that this is also related to dimension five operators which control Higgs decays to $\gamma\gamma$ and $Z\gamma$. These decays are particularly interesting as probes of new physics as they are only generated radiatively.

When supersymmetry is preserved, the kinetic terms for the gauge fields are:

\[
\mathcal{L}_{\text{eff}} = \text{Im} \int d^2\theta \frac{\tau_W}{8\pi} \text{Tr}_{SU(2)} \mathcal{W}^\alpha \mathcal{W}_\alpha + \text{Im} \int d^2\theta \frac{\tau_Y}{8\pi} \mathcal{B}^\alpha \mathcal{B}_\alpha. \quad (5.16)
\]

with $\mathcal{W}_\alpha$ and $\mathcal{B}_\alpha$ the superfield strengths for $SU(2)_W$ and $U(1)_Y$, respectively. Here, the holomorphic gauge coupling is:

\[
\tau(i) = \frac{4\pi i}{\theta(i)} + \frac{\theta(i)}{2\pi}. \quad (5.17)
\]

Integrating out the states of the extra sector, we read off $\Delta S$ from the single holomorphic dimension six operator which mixes $SU(2)_W$ and $U(1)_Y$:

\[
\mathcal{O}_{\text{HHWB}} = \text{Re} \int d^2\theta \frac{-b_{\text{mix}}}{8\pi} H_u \sigma^{(i)} H_d \mathcal{Y}_\alpha \mathcal{B}_\alpha \frac{H_u \sigma^{(i)} H_d \mathcal{Y}_\alpha \mathcal{B}_\alpha}{\Lambda_{\text{mix}}^2}. \quad (5.18)
\]
The energy scale $\Lambda_{\text{mix}}$ is roughly related to the mass of the extra states via $\lambda_u \lambda_d \Lambda_{\text{mix}}^2 \simeq M^2$. Comparing with our discussion near line (2.4), we have $b_{\text{mix}} \simeq 16\pi c_{\text{mix}}^{(\text{mix})}$. This specifies a different limit from the one where the Higgs is the sole source of mass. Indeed, with a conserved $SU(2)_W \times U(1)_Y$ flavor symmetry and approximate supersymmetry in the extra sector, the coefficients $c_u$ and $c_d$ in equation (2.4) are suppressed by a factor of $\mu_{\text{MSSM}}/\Lambda_{\text{mix}}$ relative to $c_S^{(\text{mix})}$. Switching on the Higgs vevs $\langle H_u^0 \rangle = v_u/\sqrt{2}$ and $\langle H_d^0 \rangle = v_d/\sqrt{2}$, we obtain:

$$\Delta S = \frac{4 s_W c_W}{\alpha_{\text{EM}}} \frac{b_{\text{mix}}}{16\pi} \frac{v_u v_d}{\Lambda_{\text{mix}}^2}. \quad (5.19)$$

In a CP-preserving supersymmetric two Higgs doublet model, $v_u v_d/\Lambda_{\text{mix}}^2 > 0$. So in other words, the sign of $b_{\text{mix}}$ fixes the sign of $\Delta S$.

The coefficient $b_{\text{mix}}$ is also closely connected with decays of the Higgs into $\gamma\gamma$ and $Z\gamma$. The other dimension six operators compatible with gauge invariance and holomorphy are:

$$O_{HHWW} = \text{Re} \int d^2\theta \frac{-b_W}{8\pi} \frac{H_u H_d \text{Tr}_{SU(2)_W} W^\alpha W^\alpha}{\Lambda_W^2} \quad (5.20)$$

$$O_{HHYY} = \text{Re} \int d^2\theta \frac{-b_Y}{8\pi} \frac{H_u H_d B^\alpha B^\alpha}{\Lambda_Y^2} \quad (5.21)$$

$$O_{HH\gamma\gamma} = \text{Re} \int d^2\theta \frac{-b_{\text{EM}}}{16\pi} \frac{H_u H_d E^\alpha E^\alpha}{\Lambda_{\text{EM}}^2} \quad (5.22)$$

with $E^\alpha$ the supersymmetric field strength for $U(1)_{\text{EM}}$, and $b_G = -3\text{Tr}(R_{IR} J_G J_G)$ calculable beta function coefficients. Assuming that there is only weak Higgs/SCFT mixing, we can ignore the distinction between the physical and holomorphic gauge couplings. Expanding the operator $O_{HH\gamma\gamma}$ in the 2HDM mass eigenstate basis yields the mixing angle dependence and sign of possible contributions to $h^0 \to \gamma\gamma$, $H^0 \to \gamma\gamma$ and $A^0 \to \gamma\gamma$ [15]. A similar analysis also applies to decays to $Z\gamma$ by adding up the contributions from $O_{HHWB}$, $O_{HHWW}$ and $O_{HHYY}$. This yields the same 2HDM mixing angle dependence found in [15].

Finally, switching on the Higgs vevs for the dimension six operators and matching the contribution of $O_{HHWB} + O_{HHWW} + O_{HHYY}$ to that of $O_{HH\gamma\gamma}$, we also learn that $\Delta S$ is related to the beta function coefficients:

$$\frac{b_{\text{mix}}}{\Lambda_{\text{mix}}^2} = \frac{b_{\text{EM}}}{\Lambda_{\text{EM}}^2} - \frac{b_Y}{\Lambda_Y^2} - \frac{b_W}{\Lambda_W^2}. \quad (5.23)$$

6 Unitarity Constraints

Having recast the contributions to $\Delta S$ and $\Delta T$ in terms of calculable anomaly coefficients, we now extract model-independent bounds present in any unitary conformal theory. This

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4 There can be further suppression if there is an approximate Peccei-Quinn symmetry in the Higgs sector.
enables us to impose constraints on the relative size, and sign of these contributions.

Consider first the contribution to $T$, given by equation (4.8):

$$\Delta T = \frac{1}{\alpha_{EM}} \cos^2 2\beta \times \delta. \quad (6.1)$$

Even if the Higgs dimension deviates from its free field value, $\Delta T$ is small when $\tan \beta \simeq 1$. The important feature for us is that $\cos^2 2\beta$ is a number between zero and one which is always non-negative. In principle, $\delta$ could be either positive or negative. However, in a unitary conformal theory where the Higgs is a chiral primary, the sign is fixed so that $\delta \geq 0$ (see e.g. [30, 31]). To get a negative sign for the excess Higgs dimension, one would need to either be in a non-conformal phase, or in a limit where the Higgs is not a gauge-invariant operator. This could happen if the Standard Model gauge group becomes strongly coupled, though this is rather antithetical to the set of assumptions made in this work. We therefore conclude that in most motivated situations, $\Delta T \geq 0$. This is also in accord with the weakly coupled expression in equation (2.7) near the custodial $SU(2)$ limit. Expanding $m_1 = m_2(1 + \varepsilon)$, one obtains a positive definite contribution to $\Delta T$ in this case as well.

Consider next the contribution to $S$:

$$\Delta S = \frac{b_{AA}}{4\pi} - \frac{4b_{W_uY_u}}{2\pi} \log \frac{m_u}{m_d} \quad (6.2)$$

which depends on the beta function coefficients $b_{AA}$ and $b_{W_uY_u}$. Even without knowing the precise value of these coefficients, we can still deduce some valuable information. First of all, we have that in a general unitary theory, the matrix $b_{ij}$ has non-negative eigenvalues, that is, it is non-negative definite [32,33]. In the present case with flavor symmetry $U(1)_W \times U(1)_{\tilde{W}} \times U(1)_\chi$, the $3 \times 3$ matrix of beta function coefficients:

$$b_{ij} = \begin{pmatrix} b_{VV} & b_{AA} \\ b_{AA} & b_{\chi\chi} \end{pmatrix} \quad (6.3)$$

is non-negative definite. All off-diagonal elements vanish due to our $\mathbb{Z}_2$ symmetry, and the fact that the vector and axial currents embed in non-abelian symmetries (and so do not mix with $U(1)_\chi$). From this, we deduce that $b_{AA} \geq 0$. Putting this together, we learn that when $m_u = m_d$, the contribution to $S$ is fixed to be positive. A simple way to evade this bound is to consider a non-conformal extra sector. This is related to the fact that spin 1 particles contribute to this two-point function with the opposite sign of spin 1/2 or 0 particles.

Consider next the logarithmic contributions to $\Delta S$. In contrast to the structure of equation (6.3), here, there can in principle be mixing terms between $U(1)_V$ and $U(1)_\chi$, because the non-abelian structure is no longer present, as we have projected down to just the up-type
states. In terms of the vector and axial two point functions, we have:

\[ b_{W_uV_u} = \frac{1}{4} \left( b_{V_uV_u} + 2b_{V_u\chi_u} - b_{A_uA_u} \right). \]  \hspace{1cm} (6.4)

which could have either sign. Unitarity imposes the milder constraints:

\[ b_{V_uV_u} \cdot b_{\chi_u\chi_u} \geq \frac{1}{4} (b_{V_u\chi_u})^2, \quad b_{V_uV_u} + b_{\chi_u\chi_u} \geq 0, \quad b_{A_uA_u} \geq 0. \]  \hspace{1cm} (6.5)

Putting these considerations together, we conclude that while the mass-independent contribution to \( \Delta S \) is non-negative definite, there can be deviations away from this value, corresponding to the corrections proportional to \( \log(m_u/m_d) \). These other contributions can in principle have either sign, providing a simple way to produce a smaller net value of \( \Delta S \). Of course, one must be careful about over-interpreting this result. If we work in a regime where the logarithmic correction starts to dominate over the constant shift from \( b_{AA} \), the threshold approximation adopted here is on the verge of breaking down. Nevertheless, it does indicate that in principle, either sign of \( \Delta S \) is possible.

7 Examples

In this section we discuss some examples, showing how to compute the leading order contributions to \( S \) and \( T \) from an extra sector. Even at weak coupling, our parametrization of \( S \) is helpful, as it provides a simple way to capture the leading order contributions from wave function renormalization. First, we consider a weakly coupled example to fix our conventions. After this, we discuss the analogue of this result when the extra sector involves SQCD-like dynamics. We then turn to some top-down examples motivated by string constructions.

Let us emphasize that our aim here is not to construct realistic models, but rather, to illustrate the utility of our computational framework. Indeed, contributions from the two Higgs doublet sector can, for appropriate mixing angles, counteract shifts from the extra sector (see e.g. [14,34]). A more complete study would also involve a combined analysis with present constraints on Higgs production and decays, which can also be computed [15]. It would be interesting to perform this more complete analysis.

7.1 Weakly Coupled Models

We first consider a weakly coupled example. In this case, the calculation of \( T \) is essentially the same as in the weakly coupled setting, so we restrict our discussion to \( \Delta S \). Our aim will be to show that our general analysis reproduces the expected weakly coupled result. To fix conventions, we consider a collection of chiral superfields \( N_{L,R} \) and \( E_{L,R} \) with quantum
numbers:

|   | \(U(1)_W\) | \(U(1)_{\tilde{W}}\) | \(U(1)_\chi\) |
|---|--------|--------|--------|
| \(N_L\) | +1/2  | 0     | \(+\chi\) |
| \(\tilde{E}_L\) | -1/2  | 0     | \(+\chi\) |
| \(N_R\) | 0     | -1/2  | \(-\chi\) |
| \(\tilde{E}_R\) | 0     | +1/2  | \(-\chi\) |

We denote by \(Y\) the hypercharge of the “left-handed doublet”. In this model, \(Y = \chi\). Note that here, the “right-handed” states are actually specified by chiral superfields, just as the left-handed states. This is always possible to do, by complex conjugation of a given field.

This toy model possesses the up-type and down-type splitting we want; we take the up-type fields to be \(N_{L,R}\) and down-type fields to be \(E_{L,R}\). The mass terms of the theory are:

\[
W_{\text{mass}} = m_d \tilde{E}_L E_R + m_u N_L N_R
\]  

(7.2)

which are generated by the Higgs vevs. In the case of a weakly coupled doublet, we can also set \(\chi = -1/2\), corresponding to an interaction term in the unbroken phase:

\[
W_{\text{int}} = \lambda_d H_d \tilde{E}_R E_R + \lambda_u H_u L N_R.
\]  

(7.3)

The overall contribution to \(S\) is therefore:

\[
\Delta S = \frac{1}{2\pi} \left( \frac{b_{AA}}{2} - 4b_{W_uY_u} \log \frac{m_u}{m_d} \right) = \frac{1}{2\pi} \left( \frac{1}{2} - 2Y \log \frac{m_u}{m_d} \right).
\]  

(7.6)

Compared to the weakly coupled formulae of equations (2.6) and (2.8) reviewed in section 2, the overall contribution is larger by a factor of 3/2. This is because each chiral multiplet contains a Weyl fermion, and a complex scalar.

Part of the point of our formalism is that it provides a simple way to incorporate the
effects of wave function renormalization for states of the extra sector. These effects are encoded in the anomalous dimension $\gamma$ of a field, which is related to the total dimension as:

$$\Delta = 1 + \frac{\gamma}{2}. \quad (7.7)$$

Now, in a chiral multiplet, the R-charge of the Weyl fermion is $R - 1$, where $R$ is the R-charge of the complex scalar. This is in turn related to the anomalous dimension via:

$$R - 1 = \frac{2}{3} \Delta - 1 = -\frac{1 - \gamma}{3}. \quad (7.8)$$

The resulting contribution to $S$ is given by plugging in the R-charge assignment for the Weyl fermions of the chiral multiplet:

$$\Delta S = \frac{1 - \gamma}{2\pi} \left( \frac{1}{2} - 2Y \log \frac{m_u}{m_d} \right). \quad (7.9)$$

We see that as the anomalous dimensions of the extra sector fields increase, $\Delta S$ decreases. While it might therefore seem possible to alter the sign of $\Delta S$ by increasing $\gamma$, the amount it can increase is bounded above. This is because in a unitary superconformal theory $b_{AA} \geq 0$. We will see an example of such a bound when we turn to SQCD-like extra sectors. Sharper upper bounds for scaling dimensions in SCFTs have been studied in [35, 36]. In the case where we take into account wave-function renormalization effects, we can also calculate the leading order shift to $T$:

$$\Delta T = \frac{1}{\alpha_{EM}} \cos^2 2\beta \times \frac{\gamma}{2}. \quad (7.10)$$

It is also of interest to ask how much tuning is required in this toy model to keep the contributions $\Delta S$ and $\Delta T$ smaller than an order 0.1 correction to the Standard Model reference value. Let us consider the special case $\lambda_u = \lambda_d$. In the idealized situation where $\Delta S = 0$, we can solve for $\tan \beta$ in terms of $Y$:

$$\tan \beta = \exp \left( \frac{1}{4Y} \right). \quad (7.11)$$

The value of $\Delta T$ is then:

$$\Delta T \simeq \frac{1}{\alpha_{EM}} \tanh^2 \left( \frac{1}{4Y} \right) \times \frac{\gamma}{2} \quad (7.12)$$

which can in principle be small. Of course, it is a matter of taste (or lack thereof) to decide how much tuning to accept in such situations.

A similar analysis applies in cases where the corresponding chiral multiplet is charged.
under both $SU(2)_W$ and $SU(2)_{\overline{W}}$, as considered near equation (2.8):

\begin{array}{|c|c|c|c|}
\hline
 & U(1)_W & U(1)_{\overline{W}} & U(1)_\chi \\
\hline
\psi_L & +t_L & +t_R & +\chi \\
\psi^c_R & -t_R & -t_L & -\chi \\
m_{(\psi)} & -(t_L - t_R) & -(t_R - t_L) & 0 \\
\hline
\end{array} \tag{7.13}

Using our general formulae, we reproduce equation (2.8). Note that here we have adopted a convention in which all “right-handed” states have been conjugated to left handed states. Additionally, we have only written one component of a given $SU(2)_W \times SU(2)_{\overline{W}}$ multiplet. Observe that once we sum over a complete multiplet, we obtain $b_{AA} = b_{VV}$ in the conformal phase.

### 7.2 SQCD-like Sectors

We now turn to a more involved example where the states of the extra sector participate in an SQCD-like theory. To this end, consider a theory with gauge quantum numbers:

\begin{array}{|c|c|c|c|c|}
\hline
 & U(1)_W & U(1)_{\overline{W}} & U(1)_\chi & SU(N_c)_{\text{extra}} \\
\hline
N_L & +1/2 & 0 & +\chi & N_c \\
E_L & -1/2 & 0 & +\chi & N_c \\
N_R & 0 & -1/2 & -\chi & N_c \\
E_R & 0 & +1/2 & -\chi & N_c \\
\hline
\end{array} \tag{7.14}

where we take $g$ generations of each chiral superfield. In this case, the mixing between the superconformal sector and the Higgs is given by the analogue of equation (7.3). This interaction term now drives the theory to a new fixed point, mixing the Higgs and conformal sector. Weak mixing corresponds to a short flow to the new fixed point, where the Higgs field retains a scaling dimension close to its weakly coupled value.

First consider the behavior of the conformal theory in the absence of Higgs/extra sector mixing. The extra sector is then a supersymmetric version of QCD. For an appropriate number of flavors, this theory will be in the conformal window. The symmetries of this toy model imply that the chiral fields of the extra sector have the same R-charge, which we denote by $R$. The value of $R$ is fixed, since vanishing of the $SU(N_c)_{\text{extra}}$ beta function yields the condition:

\[
N_c + \frac{4g}{2} (R - 1) = 0 \tag{7.15}
\]

or:

\[
R = 1 - \frac{N_c}{2g} \tag{7.16}
\]

Once we introduce the coupling to the Higgs fields, the R-charge assignments in the infrared will consequently adjust. Since we are working in the limit where there is an ap-
proximate custodial $SU(2)$ symmetry, the fields $H_u$ and $H_d$ have the same R-charge, which we denote by $R_H$. Moreover, because the beta function condition is the same, the R-charge assignment of equation (7.16) is the same. Imposing the further condition that the superpotential has R-charge 2 in the infrared, we also have the condition:

$$R_H = \frac{N_c}{g}. \quad (7.17)$$

Let us now turn to the calculation of $\Delta S$ and $\Delta T$. First of all, the excess Higgs dimension is:

$$\delta_H = \Delta_H - 1 = \frac{3N_c - 2g}{2g}. \quad (7.18)$$

The regime of weak Higgs/SCFT mixing corresponds to $2g \simeq 3N_c$, which is near the top of the conformal window. Using equation (4.8), we therefore have:

$$\Delta T = \frac{1}{\alpha_{EM}} \cos^2 \frac{2\beta}{2} \times \left( 3N_c - \frac{2g}{2} \right). \quad (7.19)$$

Consider next $\Delta S$. We first evaluate the anomaly coefficient $b_{AA}$:

$$b_{AA} = -3\text{Tr} \left( R_{IR} J_A J_A \right) = -3 \left( R - 1 \right) \times \left( 4gN_c \cdot \left( \frac{1}{2} \right)^2 \right). \quad (7.20)$$

Using equation (7.16), we learn that the anomaly is:

$$b_{AA} = \frac{3}{2} N_c^2. \quad (7.21)$$

The other anomaly we need to evaluate is $b_{W_u Y_u}$:

$$b_{W_u Y_u} = -3\text{Tr} \left( R_{IR} J_{W_u} J_{Y_u} \right) = -3 \times (R - 1) \times \left( gN_c \left( \frac{1}{2} \right) \cdot \chi \right) \quad (7.22)$$

from which we obtain:

$$b_{W_u Y_u} = \frac{3\chi}{4} N_c^2. \quad (7.23)$$

Thus, the overall contribution to $S$ is:

$$\Delta S = \frac{3N_c^2}{4\pi} \left( \frac{1}{2} - 2Y \log \frac{m_u}{m_d} \right) \quad (7.24)$$

where we have used $Y = \chi$ in this model.
7.3 F-theory Inspired Scenarios

We now turn to some examples motivated from F-theory model building (for reviews see [37–40]). In F-theory models, the visible sector is obtained from intersecting seven-branes, while probe D3-branes can serve as superconformal extra sectors [17]. Visible sector matter fields descend from six-dimensional fields which propagate in our four spacetime dimensions as well as two compact dimensions of the internal geometry. When the D3-brane is close to an E-type Yukawa point, this leads to a strongly coupled $\mathcal{N} = 1$ superconformal theory. The flavor symmetry of the theory contains the Standard Model gauge group, embedded in $E_8 \supset SU(5)_{GUT} \times SU(5)_L$. These theories can be viewed as $\mathcal{N} = 1$ deformations of the $\mathcal{N} = 2$ supersymmetric Minahan-Nemeschansky theory [41,42]. Moreover, although they are $\mathcal{N} = 1$ theories, they share many features of their $\mathcal{N} = 2$ precursors, allowing for a quantitative study of some aspects of the theory [18,43,44].

In contrast to weakly coupled models, the way that the states of the D3-branes pick up a mass is more subtle because strong coupling effects play a role in generating mass terms. Even so, moving the D3-brane away from the E-type point can be modelled in terms of $(\overline{5} \oplus 5)'s$ which are coupled to a strong $U(1)_{D3}$ [17]. The effective number of these $(\overline{5} \oplus 5)'s$ is irrational, reflecting the fact that the anomalous dimensions of these states are non-trivial. By moving along the direction of one of the Higgs curves (say, the $H_u$), we can see that activating a vev for the $H_u$ leads to a pairing between the electroweak doublets of the $\overline{5}$ and corresponding singlets of the extra sector. In this limit, the colored triplets of the $\overline{5}$ do not pick up a mass, but since they are neutral under $SU(2)_W$, they do not enter into the electroweak parameters anyway. Combining this observation with anomaly matching considerations, we can compute $\Delta S$:

$$\Delta S = \frac{N_{\text{eff}}}{2\pi} \left( \frac{1}{2} + \log \frac{m_u}{m_d} \right)$$

(7.25)

Here, the effective number $N_{\text{eff}} = \delta b_{SU(5)}$ is the contribution to the $SU(5)$ beta function from the extra sector states. There can be additional shifts to this number from coupling to the Higgs fields (which explicitly breaks $SU(5)_{GUT}$) but these effects turn out to be subleading [18]. In typical F-theory examples, $\delta b_{SU(5)} \simeq 2-4$ (see e.g. [18]). The contribution to $\Delta T$ involves knowing $\delta$, the excess dimension of the Higgs. In typical examples, this can be on the order of 0.01 to 0.1 [18].

8 Conclusions

The detection of the Higgs boson opens a doorway to studying signatures of physics beyond the Standard Model. One well-motivated class of such scenarios involves mixing the Higgs with an extra sector, which in many cases of interest can be superconformal. In this work we have shown that it is possible to extract the leading-order contributions to $\Delta S$ and $\Delta T$
from calculable data of the extra sector such as anomaly coefficients. We have also provided model-independent bounds on the size and sign of possible contributions to $\Delta S$ and $\Delta T$, and illustrated in some examples the applicability of these results. In the remainder of this section we discuss potential directions of future investigation.

Since the coupling to the extra sector can affect the Higgs mass, a natural next step would be to calculate these contributions in conformal perturbation theory. It would be interesting to see how our estimates for various two-point functions feed into such calculations, and it may be possible to use the information we already have to compute these mass effects, perhaps along the lines of [14].

One can view our analysis as part of a more general program to characterize the possible higher-dimension operators involving Higgs fields. In previous work [15] an analysis in terms of anomaly coefficients was used to deduce the leading order contributions to gluon fusion and diphoton decays for Higgs/SCFT mixing. Combining this with our present analysis provides a way to impose further constraints, and to discover possibly favored corners of parameter space. A detailed numerical scan of such scenarios would no doubt be useful.

Finally, in addition to its potential relevance for phenomenological analyses, the methods used in this paper are also of interest from a purely formal standpoint. More broadly, one can introduce the analogue of the $S$ and $T$ parameters for a general superconformal theory. Note that since the leading order behavior of the correlation functions are calculable and rely only on information about global symmetries, it is possible to use our methods to study SCFTs without weakly-coupled UV descriptions, similar to the F-theory example discussed above. It would no doubt be interesting to calculate $\Delta S$ and $\Delta T$ in such situations.

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