Targeted False Data Injection Attack against DC State Estimation without Line Parameters

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Abstract—A novel false data injection attack (FDIA) model against DC state estimation is proposed, which requires no network parameters and exploits only limited phasor measurement unit (PMU) data. The proposed FDIA model can target specific states and launch large deviation attacks using estimated line parameters. Sufficient conditions for the proposed method are also presented. Different attack vectors are studied in the IEEE 39-bus system, showing that the proposed FDIA method can successfully bypass the bad data detection (BDD) with high success rates of up to 95.3%.

Index Terms—DC state estimation, false data injection attacks, stochastic process, phasor measurement unit

I. INTRODUCTION

The state estimator, a core component at the control room for monitoring and control, is subject to cyber attacks as measurement units are widely dispersed and communication networks transmitting measurements are vulnerable. It has been shown in [1] that an adversary can stealthily distort the state estimation results without being detected by the bad data detection (BDD), if the adversary has the full knowledge of system topological information including line admittance.

Inspired by [1], many subsequent works were devoted to studying the false data injection attack (FDIA) models against state estimation [2]–[10]. Particularly, in contrast to AC state estimation that is iterative and computationally harder to solve [11], DC state estimation model is commonly used owing to its simplicity and linearity. In [2], the authors proposed two different FDAs targeted at pre-specified state variable or meter, respectively. In [3], a method to transform FDIA into a load distribution attack based on DC model is proposed. An FDIA method that selects the meters to manipulate aiming to cause the maximum damage can be found in [4]. The authors of [5] exploited the concept of attacking region to design an FDIA model that requires only the line admittance and topology inside the attacking region. An efficient strategy to determine the optimal attacking region was later discussed in [6]. In [7], the authors showed that an adversary can launch FDIA on a bus or superbus only if the susceptance of every transmission line connected to that bus or superbus is known. Despite important advancement towards relaxing the pre-knowledge of network information, these methods still require partial information of a transmission network, which may still be unobtainable as the system model is critically protected by utilities.

Recently, the authors of [8] proposed an FDIA model requiring no line parameters, yet the method enforces an assumption that the target bus has only one line connected to the outside, which may be rare, limiting its applications in practice. The independent component analysis (ICA) and the principal component analysis (PCA) have been applied in [9] to design FDIA against DC state estimation purely from measurements, without any grid topology and line parameters. However, these data-driven FDIA may not target specific states since the exact Jacobian matrix cannot be obtained.

In this paper, we propose a novel FDIA model against DC state estimation that can target specific states without knowing any line parameters. Compared to network topology information critically protected and rarely transmitted, phasor measurement unit (PMU) measurements are easier to acquire through communication networks or software supply chain attack between PMU vendors and customers (e.g., the SolarWinds attack [12]). Hence, we, from the viewpoint of an adversary, exploit only PMU data collected at the buses inside the attacking region to estimate the line parameters of interest and design a stealthy FDIA. In contrast to previous works that require measurements from remote terminal units (RTUs), the proposed attack model requires only PMU measurements for designing the FDIA and can target specific states without any pre-knowledge on line parameters.

II. DC STATE ESTIMATION AND ATTACKING REGION

The power system state vector $x$ to be estimated consists of voltage phasors of all buses, while the measurement vector $z$ consists of power injections and line flows. The relationship between $z$ and $x$ can be formulated in a DC model:

$$z = Hx + e$$

(1)

where $e$ denotes the vector of measurement error, which is typically assumed to be an independent Gaussian random vector; $H$ represents the measurement Jacobian matrix.

To get the estimated state vector $\hat{x}$, the weighted least squares estimation problem is formulated and solved as:

$$\hat{x} = (H^TWH)^{-1}H^T Wz$$

(2)

where $W$ is a weight matrix.

For the detection of bad data, the residual-based BDD is typically used [1]. The residual between the measurement and the estimated state vector is defined as $r = \|z - H\hat{x}\|_\infty$. Only
if $r$ is smaller then a threshold value $\gamma$, the measurement $z$ is regarded as normal without bad data or malicious data.

Assuming an FDIA attack is launched by an adversary, the measurement vector after manipulation is $z_{bad} = x + a$, where $a$ is termed as the attack vector. Let $\hat{x}_{bad} = \hat{x} + c$ be the estimated states from the $z_{bad}$. It has been shown in [5] that if $a = Hc$, then the residual after the attack remains the same as that before the attack: $r_{bad} = \|z_{bad} - H\hat{x}_{bad}\| = \|z - H\hat{x} + a - Hc\| = r$, indicating that the attack can stealthily bypass the BDD without being detected. However, it is obvious that the adversary needs to know $H$, i.e., the entire network information (topology information and line parameters) of the power system, which is practically infeasible. Thus, many efforts (e.g., [5], [7]) have been made to propose FDIA models using only partial network information or manipulating partial measurements.

One effective method is to divide the system into the non-attacking region and the attacking region. If the adversary knows the topology information inside the attacking region and can manipulate line flow and power injection measurements, then the adversary can launch successful FDIA without being detected by the BDD [1]. Network information or meter measurements outside the attacking region are not required.

In this paper, we also utilize the concept of attacking region to design the PMU-based FDIA. Nevertheless, the proposed FDIA requires no line parameters, irrespective of whether the line is located inside or outside the attacking region.

III. Constructing the FDIA Model Without Line Parameter Information

To construct the FDIA, we follow the common assumption that generator measurements cannot be attacked as they are well protected, while the measurements of line flow and power injection at load buses can be attacked because of the wide deployment of load meters and variation of load powers [6].

Assuming the adversary intends to attack a load bus $t$ that is termed as the target bus, then the attacking region defined in this paper consists of the following: the target bus, all buses that have direct connection to the target bus as well as the transmission lines between these buses (see Fig. 1).

Information needed in the attacking region: The adversary needs to know: 1. which buses are connected to the target bus to determine the attacking region. Nevertheless, the topology information of other buses inside the attacking region (e.g., the line between bus 1 and bus 2 in Fig. 1) is not needed. 2. similar to [1], [7], voltage angles $\delta_i$, $\forall i \in \Omega_A$ where $\Omega_A = \{1, 2, ..., t, ..., k\}$ denotes the set of bus numbers inside the attacking region. 3. the current phasor measurement $I_r\angle \theta_i$ flowing from the target bus to the dynamic load, whereas any other line current measurements (e.g., $I_{1r}\angle \theta_{1t}, I_{2r}\angle \theta_{2t}, ..., I_{kt}\angle \theta_{kt}$) are not needed by the adversary. Note that in contrast to previous works [1], [3], [5], no RTU measurements containing power injections and line flows are needed by the adversary.

Measurements to be manipulated in the attacking region: To bypass the BDD of DC state estimation [5], the adversary needs to manipulate all active bus power injections (e.g., $P_i, \forall i \in \Omega_A$) and all active line flows (e.g., $P_{ij}, \forall j \in \Omega_A, j \neq i$). Buses directly connected to the generator buses cannot be attacked, because of the assumption that the adversary cannot manipulate generator measurements.

A. Designing the Attack Vector

As discussed in Section III, in order to launch a stealthy FDIA to bypass the BDD, the attack vector $a$ needs to satisfy $a = Hc$. More specifically, if the adversary wants the power operators to believe that the angle of the target node is $\hat{\delta}_i$ rather than the true value $\delta_i$, the adversary needs to manipulate the line flows and bus injections as follows:

- substituting $P_{ij}$ by the fake $\hat{P}_{ij}$ for $\forall j \in \Omega_A, j \neq t$:
  \[
  \hat{P}_{ij} = B_{ij}(\delta_i - \delta_j) \tag{3}
  \]
- substituting $P_i, P_j$ by the fake $\hat{P}_i, \hat{P}_j$:
  \[
  \hat{P}_i = \sum_{j \in \Omega_A} P_{ij} \tag{4}
  \]
  \[
  \hat{P}_j = P_j + \hat{P}_i - P_{it} \tag{5}
  \]

In other words, if the designed deviation $c = (\hat{\delta}_i - \delta_i)^T$, then the attack vector $a = [\hat{P}_{ij} - P_{ij}, \hat{P}_i - P_i]^T, \forall j \in \Omega_A, j \neq i, \forall i \in \Omega_A$. Obviously, the adversary needs to know accurate values of $B_{ij}$ according to (3). Nevertheless, line parameters are critically protected and cannot be easily acquired. In the next sections, we will draw upon intrinsic load dynamics and the regression theorem of the Ornstein-Uhlenbeck (OU) process to estimate the line parameters $B_{ij}$ connected to the target bus $t$ and exploit the estimation result to design the attack vector.

B. The Load Dynamics Inside the Attacking Region

Inside an attacking region, the system can be represented by differential-algebraic equations:
\[
\begin{align*}
\dot{x} &= f(x, z) \tag{6} \\
z &= Hx \tag{7}
\end{align*}
\]
where (6) represents the load dynamics and (7) represents the relationship between the states and power injections as well as power flow relationship. It should be noted that only (7) is considered in former FDIA works, whereas the dynamics described by (6) are neglected by assuming that the system is in normal operating state. Nevertheless, we will show that the adversary can acquire essential information about physical systems, such as line parameters, by exploiting the load dynamics described by (6). For any bus $i$ inside the attacking region $\Omega_A$, the stochastic dynamic load model with DC power flow assumption can be represented as:
\[
\begin{align*}
\delta_i &= \frac{1}{\gamma_i}(P_i^a(1 + \sigma_i^a \xi_i) - P_i) \tag{8} \\
P_i &= \sum_{j \in \Omega_i} B_{ij}(\delta_i - \delta_j) \tag{9}
\end{align*}
\]
where $\delta_i$ is the voltage angle of bus $i$; $P_i$ is the active power injection of bus $i$; $P_i^a$ is the static active power injection of
bus \( i \); \( \Omega_i \) is the set of buses connected to bus \( i \); \( B_{ij} \) is the susceptance between bus \( i \) and \( j \); \( \tau_{pi} \) is the active power time constant of bus \( i \); \( \xi^p_i \) is a standard Gaussian random variable; \( \sigma^p_i \) describes the noise intensity of load variations.

This dynamic model can represent various loads including thermostatically controlled loads, induction motors, loads controlled by load tap changers, static loads, etc. in ambient conditions \([13]\). The difference among various loads is reflected by time constants, i.e., the relaxation rates of loads, which may range from 0.1s to 300s \([14], [15]\). Dynamic load models similar to \((8)-(9)\) have also been proposed and used in previous works \([13], [16], [17]\) for stability analysis. In addition, since load powers are constantly varying in practice, we make the common assumption \([13], [17]\) that static load powers are perturbed by independent Gaussian noises, i.e., \( P^i_t(1 + \sigma^P_i \xi^P_i) \).

Around steady state, \((3)\) can be linearized. By looking at \((8)\), one can easily observe that the Jacobian matrix \( J_{P, \delta} = \frac{\partial P}{\partial \delta} \) required for linearization carries crucial information of line parameters, as \( \frac{\partial P}{\partial \delta} = \sum_{j \in \Omega_i} B_{ij} \) and \( \frac{\partial P}{\partial \delta} = -B_{ij} \), for \( i \neq j \).

Therefore, the linearized load dynamics inside the attacking region can be described in a matrix form as follows:

\[
\begin{bmatrix}
\dot{\delta}_1 \\
\vdots \\
\dot{\delta}_k \\
\end{bmatrix} = \begin{bmatrix}
\frac{\tau_{p1}}{\tau_{p1}} & \cdots & 0 & 0 \\
0 & \ddots & \vdots & \vdots \\
0 & \cdots & \frac{\tau_{pk}}{\tau_{pk}} \\
0 & \cdots & \cdots & \frac{\tau_{pk}}{\tau_{pk}} \\
\end{bmatrix}
\begin{bmatrix}
\delta_1 \\
\vdots \\
\delta_k \\
\end{bmatrix}
+ \begin{bmatrix}
0 \\
\vdots \\
0 \\
\end{bmatrix}
\begin{bmatrix}
P^l_{\Delta t} \\
\vdots \\
P^l_{\Delta t} \\
\end{bmatrix}
+ \begin{bmatrix}
\sigma^p_1 \\
\vdots \\
\sigma^p_k \\
\end{bmatrix}
\begin{bmatrix}
\xi^p_1 \\
\vdots \\
\xi^p_k \\
\end{bmatrix}
\]  \hspace{1cm} (10)

where \( \sigma^p_i \) and \( \xi^p_i \) are standard Gaussian random variables; \( \tau_{pi} \) is the time constant of \( \delta^i \);

Equation \((10)\) can be written in the following compact form:

\[
\dot{\delta} = A\delta + S\xi^p \]  \hspace{1cm} (11)

from which it can be observed that \( \delta \) is a multi-dimensional OU process \([19]\) that is Markovian and Gaussian. In addition, as seen from \((10)\), the system state matrix \( A \) contains important information of the time constants and the Jacobian matrix, from which the line parameters can be extracted. In the next sections, a method to estimate matrix \( A \) purely from PMU measurements will be introduced. Particularly, the method enables the adversary to estimate the line parameters \( B_{ij} \) connected to the target bus \( t \) and build the attack vector for launching FDIA without requiring any other information other than the available PMU data within the attacking region.

C. The Regression Theorem for the OU Process

Considering the multi-dimensional OU process in \((11)\), if the system state matrix \( A \) is stable, which is satisfied in normal operating state, it can be shown that the \( \tau \)-lag correlation matrix of \( \delta \) satisfies a differential equation \([19]\):

\[
\frac{d}{dt} [C(\tau)] = AC(\tau) \]  \hspace{1cm} (12)

which is termed as the regression theorem of the multi-dimensional OU process. Particularly, the \( \tau \)-lag correlation matrix of \( \delta \) is defined as:

\[
C(\tau) = E[(\delta_{t+\tau} - E[\delta_t])(\delta_t - E[\delta_t])^T] \]  \hspace{1cm} (13)

where \( E[\cdot] \) denotes the expectation operator.

We can therefore estimate \( A \) from the \( \tau \)-lag correlation matrix of \( \delta \) by solving \((12)\):

\[
A = \frac{1}{\tau^2} \ln[C(\tau)C(0)^{-1}] \]  \hspace{1cm} (14)

Equation \((14)\) provides a way of estimating the system state matrix \( A \) that carries significant physical system information purely from the statistical properties of PMU measurements. In practical applications, the \( \tau \)-lag correlation matrix \( C(\tau) \) can be estimated by the sample correlation matrix obtained from a finite amount of PMU data as below:

\[
\hat{\mu}_s = \frac{1}{N} \sum_{i=1}^{N} \delta^{(i)} \]  \hspace{1cm} (15)

\[
\hat{C}(0) = \frac{1}{N-1} \sum_{i=1}^{N} [\delta^{(i)} - \hat{\mu}_s][\delta^{(i)} - \hat{\mu}_s]^T \]  \hspace{1cm} (16)

\[
\hat{C}(\Delta t) = \frac{1}{N-M-1} \sum_{i=1+M}^{N} [\delta^{(i)} - \hat{\mu}_s][\delta^{(i-M)} - \hat{\mu}_s]^T \]  \hspace{1cm} (17)

where \( N \) is the sample size of voltage angles, \( M \) is the number of samples that corresponds to the selected time lag \( \Delta t \), \( \delta^{(i)} = [\delta^{(i)}_1, \ldots, \delta^{(i)}_j, \ldots, \delta^{(i)}_k]^T \) represents the \( i \)th measurement of all voltage angles in the attacking region. Thus, the adversary can obtain the estimated matrix \( \hat{A} \) as follows:

\[
\hat{A} = \frac{1}{\Delta t} \ln[\hat{C}(\Delta t)\hat{C}(0)^{-1}] \]  \hspace{1cm} (18)

D. Estimation of Time Constants and Line Parameters

Once \( \hat{A} \) is calculated from the collected PMU data through \((13)\), the adversary can apply the least-squares estimation (LSE) optimization formulation to estimate the time constant \( \tau_{pi} \) and finally extract the desired line parameters for the attack vector. To this end, \((3)\) can be re-written in its discrete form:

\[
\begin{bmatrix}
\frac{\tau_{p1}}{\tau_{p1}} & \frac{\tau_{p1}}{\tau_{p1}} & \cdots & \frac{\tau_{p1}}{\tau_{p1}} \\
\cdots & \cdots & \cdots & \cdots \\
\frac{\tau_{pk}}{\tau_{pk}} & \frac{\tau_{pk}}{\tau_{pk}} & \cdots & \frac{\tau_{pk}}{\tau_{pk}} \\
\end{bmatrix}
\begin{bmatrix}
\frac{P^l_{\Delta t} - P^l_{(1)}}{\tau_{p1}} \\
\frac{P^l_{\Delta t} - P^l_{(n-1)}}{\tau_{pn}} \\
\end{bmatrix}
= \begin{bmatrix}
\frac{1}{\tau_{p1}} \hat{P}^l \\
\frac{1}{\tau_{pn}} \hat{P}^l \\
\end{bmatrix} \]  \hspace{1cm} (19)

where \( n \) is the sample size, \( P^l_{(i)} \) is the \( i \)th observation of active power of the target bus \( t \), \( i = 1, \ldots, n; \) \( \hat{P}^l \) denotes the sample mean of \( P^l_{(i)}; \) \( \delta^{(i)}_t \) represents the \( i \)th observation of voltage angle of bus \( t \). Therefore, the time constant can be estimated as \( 1/\tau_{pi} = (X^TX)^{-1}X^TY \) by the LSE method. Note that the real power injection at bus \( t \) can be computed from the PMU measurements of voltage and current phasors at bus \( t \).

Once \( \tau_{pi} \) is obtained, the adversary can use the result together with the estimated \( \hat{A} \) from \((13)\) to estimate the line parameters \( B_{ij} \). The proposed FDIA model against DC state estimation without pre-knowledge of line parameters is summarized in Algorithm 1 below.

**Remarks:**

- In this paper, 300s emulated PMU measurements with a sampling rate of 60 Hz are used for Step 2, while only a few milliseconds e.g., 0.0041s, are needed for Steps 3-6.
- The proposed FDIA against DC state estimation needs only a small number of PMU measurements inside the attacking region, whereas no additional measurements (i.e., line flows and line parameters) are required to carry out the attack.
- The adversary is aware of the network connectivity, which is a reasonable assumption in many works \([5], [7], [8], [20]\). Yet, the exact line parameters that may vary in practice are not required, highlighting the superiority of the proposed method compared to topology dependent methods.
The proposed FDIA model against DC state estimation without line parameters

1. Choose one target bus \( t \) and determine attacking region \( \Omega_A \).
2. Collect \( N \) voltage measurements for \( k \) buses inside attacking region and \( n \) current phasor measurements only for bus \( t \) from PMUs; Calculate the real power injections at bus \( t \).
3. Calculate the estimated matrix \( \hat{A} \) through (15)-(18).
4. Estimate the time constants \( \tilde{\tau}_{p15} \) through (19).
5. Estimate the line admittance \( \hat{B}_{1j} \) from \( A \) and \( \tilde{\tau}_{p15} \).
6. Given the target malicious phasor \( \delta_t \), design the attack vector \( \alpha \) through (3)-(5).

E. Analysis on the Performance of the Proposed FDIA Model

In this section, sufficient conditions for Algorithm 1 are presented as the theoretical basis for the proposed FDIA.

Theorem 1: If the number of independent measurements is equal to the number of states to be estimated, then the proposed algorithm is perfect, i.e., the residual remains the same before and after the attack.

Proof: The Jacobian matrix estimated by the adversary can be represented by \( H = H + \Delta H \) due to the estimation error in \( \hat{B}_{1j} \). Then the attack vector designed by the adversary takes the form \( \alpha = \hat{H}c = (H + \Delta H)c \), where \( c \) is the deviation intended to be injected to bus \( t \). The measurement to be manipulated is \( z_{bad} = z + \alpha = z + (H + \Delta H)c \). At the power operators’ side, if DC state estimation is exploited, the result of (2) is \( \hat{z}_{bad} = \hat{x} + c + (H^TWH)^{-1}H^TWH\Delta Hc \) Consequently, the residual after the FDIA is \( r_{bad} = \|z_{bad} - \hat{H}\hat{z}_{bad}\|_\infty = \|r + (I - H(H^TWH)^{-1}H^TWH)\Delta Hc\|_\infty \). It can be observed that if the number of independent measurements is the same as that of states to be estimated, i.e., \( H \) is a square matrix with full rank, then the FDIA is perfect as \( r_{bad} = r \).

However in real-life applications, the number of measurements is typically larger than that of states. If the matrix \( H \) is full rank but not a square matrix, the residual after the attack may not remain the same and depends on \( \Delta Hc \).

It will be demonstrated in Section 1V that the proposed FDIA can launch targeted large deviation attacks with a high success rate owing to the good estimation accuracy of \( \hat{H} \).

IV. CASE STUDIES

In this section, the proposed FDIA model is tested on a modified IEEE 39-bus 10-generator system. We consider the worst situation for the adversary, i.e., the power system is fully measured by RTUs for state estimation purposes at the control room, while the adversary can only collect limited PMU data within the attacking region. In this study, there are totally 85 RTU measurements containing 39 bus injections and 46 line flows used by the system operator for state estimation. All 85 measurements contain independent Gaussian noises described by \( \forall e_{15} \sim N(0,0.05^2) \) in (1). Note that these RTU measurements are not used by the adversary.

A. Constructing the Proposed FDIA without Line Parameters

Assuming the adversary intends to manipulate the voltage angle of bus 15, the attacking region includes bus 15 as well as buses 14 and 16 that are directly connected to bus 15, as defined in Section III and shown in Fig. 2. The time constants of the loads inside the attacking region are set arbitrarily as \([\tau_{p14}, \tau_{p15}, \tau_{p16}] = [0.1, 27.88, 206.01] \); the process noise intensities \( \sigma_i^2 \), \( i \in \{14, 15, 16\} \) in (8) describing power variations are set to be 1. As the proposed FDIA is purely measurement-based, its robustness against measurement noise should be tested. Following the approach in [21], PMU measurement noises with a standard deviation of 10% of the largest state changes between discrete time steps are added to the PMU data of voltages and currents utilized by the adversary. It will be shown that through the proposed FDIA method, the adversary can stealthily bypass BDD to manipulate the angle of bus 15.

By Algorithm 1, the adversary collects 300s, 60 Hz PMU data of voltage angle measurements for all the buses inside the attacking region (i.e., bus 14, 15, and 16) as well as current phasor measurements of the target bus 15, i.e., \( N = 18000 \) in (15) and \( n = 10 \) in (19), then the matrix \( A \) and the time constants \( 1/\tau_{p15} \) can be estimated by (13)-(19). With these estimations, the estimated line admittances \( \hat{B}_{114-15}, \hat{B}_{116-15} \) can be extracted. Table I presents the estimation results for the time constants and line admittances, showing relatively good estimation accuracy.

Once the line admittances are estimated, the adversary can design the attack vector according to (3)-(5). Assuming the adversary launches an attack such that \( \delta_{15} = 10^\circ \). Table II presents a comparison between the estimated state values inside the attacking region through the DC state estimation before and after the attack, which validates the effectiveness of the proposed FDIA method.

**TABLE II: DC STATE ESTIMATION RESULTS BEFORE AND AFTER THE ATTACK \( \delta_{15} = 10^\circ \)**

| State Variables | Before attack (degree) | After attack (degree) |
|-----------------|------------------------|-----------------------|
| \( \delta_{14} \) | 2.8300 \(^\circ\)       | 2.8720 \(^\circ\)     |
| \( \delta_{15} \) | 2.4557 \(^\circ\)       | 9.5409 \(^\circ\)     |
| \( \delta_{16} \) | 4.0061 \(^\circ\)       | 3.9314 \(^\circ\)     |
B. Different Attack Vectors under DC State Estimation

As shown in Fig. 3, the estimation error of the line admittance $B_{ij}$ may affect the residual after the attack. To test the performance of the FDIA model under the estimation error of line parameters, different attacks are tested. Various Monte Carlo simulations using 1000 samples have been carried out.

Fig. 4 presents the probabilities of the FDIA attacks to bypass the BDD for different attacks. If we choose 95%-quantile of the residual before the attacks, i.e., $\gamma = 0.85011$ to be the BDD threshold, the probabilities to bypass the BDD for $\delta_{15} = 0^\circ, \delta_{15} = 10^\circ, \delta_{15} = 20^\circ, \delta_{15} = 26^\circ$ are 94.7%, 95.3%, 94.2% and 93.5%, respectively, showing that the adversary can successfully bypass the BDD even for big deviation attacks.

C. Different Attack Vectors under AC State Estimation

Although the proposed FDIA is based on DC model, we also test its performance if AC state estimation is implemented in the control room. As seen in Table III, the attack $\delta_{15} = 10^\circ$ can still be launched successfully. Meanwhile, the states of other buses inside the attacking region are almost unaffected.

Nevertheless, if the designed deviation $c$ becomes larger, the proposed FDIA method may fail as the adversary does not manipulate reactive power in the system due to the assumption of DC model. As presented in Fig. 4 the probabilities to bypass the BDD in AC state estimation for $\delta_{15} = 0^\circ, \delta_{15} = 10^\circ, \delta_{15} = 15^\circ, \delta_{15} = 20^\circ$ are 95.5%, 93.6%, 0% and 0%, respectively, if the 95-quantile of the residual in AC state estimation before the attacks, i.e., $\gamma = 0.86936$, is used. These results show that, if the adversary intends to launch large deviation attacks with higher success rates, future efforts on designing targeted FDIA attacks against AC state estimation are needed.

V. Conclusion

In this paper, a novel FDIA model against DC state estimation using PMU data has been proposed, which can target specific states without any pre-knowledge of line parameters. Sufficient conditions for the proposed FDIA model are also provided. Numerical results showed that the proposed model can launch targeted large deviation attacks with high probabilities, if DC model is exploited. Future works on designing FDIA against AC state estimation are needed for large attacks.

![Fig. 3](image1.png)

**Fig. 3:** (a) Probability to pass the BDD for different attacks if DC state estimation is applied; (b) Zoomed-in picture of (a).

**Fig. 4:** (a) Probability to pass the BDD for different attacks if AC state estimation is applied; (b) Zoomed-in picture of (a).

**TABLE III: AC STATE ESTIMATION RESULTS BEFORE AND AFTER ATTACK WITH $\delta_{15} = 10^\circ$**

| State Variables | Before attack | After attack |
|-----------------|---------------|--------------|
| $V_{14} - \delta_{14}$ | 0.9383/0.23681$^\circ$ | 0.9346/3.1980$^\circ$ |
| $V_{15} - \delta_{15}$ | 0.9485/2.6828$^\circ$ | 0.9487/10.4592$^\circ$ |
| $V_{16} - \delta_{16}$ | 0.9594/4.1889$^\circ$ | 0.9589/4.2063$^\circ$ |

**REFERENCES**

[1] Y. Liu, P. Ning, and M. K. Reiter, “False data injection attacks against state estimation in electric power grids,” Proc. ACM Conf. Comput. Commun. Security, pp. 21–32, 2009.

[2] G. Liang et al., “False Data Injection Attacks Targeting DC Model-Based State Estimation,” IEEE PES General Meeting, Chicago, IL, USA, 2017.

[3] Y. Yuan et al., “Modeling load redistribution attacks in power systems,” IEEE Trans. Smart Grid., vol. 2, no. 2, pp. 382–390, 2011.

[4] X. Liu et al., “Modeling of Local False Data Injection Attacks With Reduced Network Information,” IEEE Trans. Smart Grid, vol. 6, no. 4, pp. 1686–1696, 2015.

[5] R. Deng and H. Liang, “False Data Injection Attacks with Limited Susceptance Information and New Countermeasures,” IEEE Trans. Ind. Inform., vol. 15, no. 3, pp. 1619–1628, 2019.

[6] X. Liu et al., “Modeling of Local False Data Injection Attacks Against Power System State Estimation,” Proc. American Control Conf., vol. 20–July, pp. 2987–2992, 2020.

[7] M. Esmaeilfalah et al., “Stealth false data injection using independent component analysis in smart grid,” IEEE SmartGridComm, pp. 244–248, 2011.

[8] Z. H. Yu and W. L. Chin, “Blind False Data Injection Attack Using PCA Approximation Method in Smart Grid,” IEEE Trans. Smart Grid, vol. 6, no. 3, pp. 1219–1226, 2015.

[9] X. Li et al., “2015 Ukraine Blackout: Implications for False Data Injection Attacks,” IEEE Trans. Power Syst., vol. 32, no. 4, pp. 3317–3318, 2017.

[10] L. Sobczak, “Major hack hits energy companies, U.S. agencies,” E&EE News, 2020.

[11] C. A. Cañizares, “On bifurcations, voltage collapse and load modeling,” IEEE Trans. Power Syst., vol. 10, no. 1, pp. 512–522, 1995.

[12] D. Karlsson and D. J. Hill, “Modelling and identification of nonlinear dynamic loads in power systems,” IEEE Trans. Power Syst., vol. 9, no. 1, pp. 157–166, 1994.

[13] X. Wang, “Estimating dynamic load parameters from ambient PMU measurements,” IEEE PES General Meeting, Portland, USA, 2018.

[14] G. Pierrou and X. Wang, “Analytical Study of the Impacts of Stochastic Load Fluctuation on the Dynamic Voltage Stability Margin Using Bifurcation Theory,” IEEE Trans. Circ. and Syst. I, vol. 67, no. 4, pp. 1286–1295, 2020.

[15] H. Mohammed and C. O. Nwankpa, “Stochastic analysis and simulation of grid-connected wind energy conversion system,” IEEE Trans. Energy Conv., vol. 15, no. 1, pp. 85–90, 2000.

[16] G. Pierrou and X. Wang, “Online PMU-Based Method for Estimating Dynamic Load Parameters in Ambient Conditions,” IEEE PES General Meeting, Montreal, Canada, 2020.

[17] C. Gardiner, “Stochastic Methods: A Handbook for the Natural and Social Sciences,” 2009.

[18] X. Liu and Z. Li, “Local load redistribution attacks in power systems with incomplete network information,” IEEE Trans. Smart Grid, vol. 5, no. 4, pp. 1665–1676, 2014.

[19] N. Zhou, D. Meng, and S. Lu, “Estimation of the dynamic states of synchronous machines using an extended particle filter,” IEEE Trans. Power Syst., vol. 28, no. 4, pp. 4152–4161, 2013.