Evaluation of interior-point method in Scilab

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Abstract. The interior-point method is one of the best methods for solving linear programming problems. The interior-point method has a polynomial time complexity for solving linear programming problems. Next, Scilab as a free source software provides a function to solve linear programming based on interior-point method, which is called Karmarkar. In this paper, we evaluate some results of Scilab Karmarkar function in solving linear programming problems. We observe that at some cases the results of the Karmarkar function are not as expected.

1. Introduction

Optimization is the process of finding an optimal solution. Many of life problems are related to optimization, for example in generating star catalog for satellite attitude navigation [10], optimizing bus timetable [9, 16], text-independent speaker identification [3], and in more theoretical matters for the development of optimization itself, as in [8, 13, 14]. Linear programming (LP) is part of optimization problems where we concern about linear function in the objective function and in all of the constraints. LP becoming famous after World War II when Dantzig [2] introduced a way to solve LP problems. His method is called the simplex method. The feasible region of an LP problem is a polyhedron, and in finding an optimal solution the simplex method moves along the vertices of the polyhedron. Many types of research have been done to explore the simplex method, and according to the Journal of Computing in Science and Engineering, the simplex method is included in the top 10 algorithms of the twentieth century [5]. The success of the simplex methods has raised some questions. One of the questions is whether LP problems that require an exponential number of iterations exist. In 1972 Klee and Minty [7] showed an example of LP problem such that the simplex method may need an exponential number of iterations.

The problem of Klee-Minty in n-dimensional is as following:

$$\min x_n$$

subject to \(\alpha x_{k+1} \leq x_k \leq 1 - \alpha x_{k-1}, \quad k = 1, ..., n,$$

where \(\alpha\) is a small positive number and \(x_0 = 0\).

The weakness of the simplex method in terms of complexity encouraged researchers to search for other methods with polynomial time complexity. The ellipsoid method is the first polynomial method invented for LO problems. This method started in the Soviet Union. After that many researchers studied this method both in theory and applied aspects. The application results were not good. The rate of convergence of the ellipsoid method is quite slow compared to the simplex method.
Karmarkar in 1984 proposed a method called Karmarkar projective method [6]. This method is efficient in theory and also in application. This method passes through the interior of the domain in obtaining an optimal solution. This method becomes the initiator of the interior point method. The complexity of the Interior point method (IPM) is polynomial with an upper bound: \[ \lceil \sqrt{n} \ln \frac{n \mu^0}{\epsilon} \rceil, \]
where \( \mu^0 > 0 \) denotes the initial value of barrier-parameter, \( \epsilon \) is absolute accuracy of the objective function and \( n \) is the number of inequalities [12].

Scilab which is an open source software for numerical computations provides a function to solve LP based on IPM, which is called Karmarkar. This algorithm is based on a primal affine-scaling interior point algorithm which is invented by Dikin [4] and then renewed by Barnes [1] and Vanderbei et.al. [15]. In this paper, we evaluate the Karmarkar function provided by Scilab.

2. Methods
In evaluating Scilab Karmarkar function we use Scilab version 6.01 (64 bit) [8] and 2-dimensional Klee-Minty problem, with \( \alpha=1/3 \). Hence the Klee-Minty problem used is:

\[
\begin{align*}
\min x_2 \\
\text{subject to } & \frac{1}{3} x_1 \leq x_2 \leq 1 - \frac{1}{3} x_1, \\
& 0 \leq x_1 \leq 1.
\end{align*}
\]

The Karmarkar function is evaluated using the Klee-Minty problem by dividing the starting point of the problem in 4 cases:

1. Starting point inside the interior of the feasible region.
2. Starting point outside the interior of the feasible region.
3. Starting point at the border of the feasible region.
4. The starting point is calculated by the Karmarkar function.

3. Results and Discussion

3.1. Starting point inside the interior of the feasible region
The experiment has been done for 1000 different starting point. The results are as expected. Some results will be given. If the starting point \( x_0=(0.8, 0.5) \), after we run the function Karmarkar we get results:

iter = 200.
exit flag = 0.
fopt = 2.290D-60
xopt = 2.326D-15 2.290D-60.

The movement of points toward the optimal solution is displayed in Figure 1a. This means after the maximum number of iterations is reached, indicated by exitflag=0, where the default maximum number of iterations is 200, the optimum value of our example is 2.290D-60 which is happening at \( x=(2.326D-15, 2.290D-60) \).

Then the number of iteration was changed such that exitflag=1, which means the solution is convergent to the optimal solution. The following results were obtained, at the 541\textsuperscript{st} iteration, the problem is convergent.

iter = 541.
exit flag = 1.
fopt = 9.84D-163
xopt = 2.326D-15 9.84D-163.

Next, the starting point is (0.1, 0.9) (Figure 1b). The results of Karmarkar are as following:

iter = 55.
exitflag = 1.
fopt = 1.060D-14
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xopt = 1.449D-16 1.060D-14.

The solution is already convergent at 55\textsuperscript{th} iterations.

Then we run the Karmarkar function with the starting point (0.9, 0.6) as depicted in Figure 1c, it reaches the maximum number of iterations 200 with exitflag=0.

iter = 200.
extiflag = 0.
fopt = 9.559D-59
xopt = 1.805D-16 9.559D-59.

After we change the maximum number of iterations, the Karmarkar function converge (exitflag=1) at the iteration 547\textsuperscript{th}.

iter = 547.
extiflag = 1.
fopt = 2.78D-164
xopt = 1.805D-16 2.78D-164.

If the starting point is (0.9, 0.4), the Karmarkar function also attain its maximum number of iterations as follows (Figure 1d):

iter = 200.
extiflag = 0.
fopt = 5.789D-60
xopt = 6.229D-16 5.789D-60.

Then it is converge when it reaches 542\textsuperscript{nd} iteration with exitflag=1.

iter = 542.
extiflag = 1.
fopt = 9.41D-163
xopt = 6.229D-16 9.41D-163.

Hence, when the starting point in the interior of feasible region, from our experiments the Karmarkar function always converge to an optimal solution as long as the number of iteration is satisfied.

3.2. Starting point outside the interior of the feasible region

The starting point that we point out first here is \(x_0=(1,5)\). There is a comment as the result of running the Karmarkar function. And it is as expected.

Wrong value for input argument #4. \(x_0\) does not satisfy the inequality constraints.

Next when the starting point is \(x_0=(1;-5)\), the comment is also as expected as follows.

Wrong value for input argument #4. \(x_0\) is not positive.

3.3. Starting point at the border of the feasible region

We divide the behavior of the Karmarkar function at the border in two parts: at the border but not the vertex and then at the vertex.

At the border but not the vertex. At the border of the Klee-Minty problem that satisfied \(x_2=1-1/3\ x_1\), we obtain the results with exitflag=1 and the value of the objective function is 0.6666707 at \(x^*=(0.9999878, 0.6666707)\). This result is not true, as we can easily check that the optimal value is 0 at \(x^*=(0, 0)\). As an example when we start from (0.6, 0.8), the movement of points is displayed in Figure 2a, and the results are as following.

iter = 15.
extflag = 1.
fopt = 0.6666707
xopt = 0.9999878 0.6666707.
Figure 1. The movement of points toward the optimal solution from some starting points.

At the border with \( x_2=1 \), we obtain the result with exitflag=1, the value of the objective function \( 0.3333359 \) at \( x^*=(1., \ 0.3333359) \). This result is not true. As an example for the starting point \( (1,0.5) \) the results as follows and the movement of points is displayed in Figure 2b.

\[
\begin{align*}
\text{iter} & = 16. \\
\text{exitflag} & = 1. \\
\text{fopt} & = 0.3333359 \\
\text{xopt} & = 1. \quad 0.3333359.
\end{align*}
\]

Next for the border \( x_2=1/3 \ x_1 \), the result is true. The movement of points when the starting point is \( (0.6, 0.2) \) is showed in Figure 2c and the Karmarkar function results are as follows.

\[
\begin{align*}
\text{iter} & = 200. \\
\text{exitflag} & = 0. \\
\text{fopt} & = 1.962D-61 \\
\text{xopt} & = 2.497D-15 \quad 1.962D-61.
\end{align*}
\]

For the border \( x_1=0 \), the result also as expected as depicted in Figure 2d. The following are the results of Karmarkar function when the starting point \( (0, 0.9) \).

\[
\begin{align*}
\text{iter} & = 80. \\
\text{exitflag} & = 1. \\
\text{fopt} & = 2.582D-15 \\
\text{xopt} & = 0. \quad 2.582D-15.
\end{align*}
\]
Figure 2. The movement of points toward the optimal solution from some border but not the vertex.

At the vertex. At (0,1), the Karmarkar function cannot go to the optimal solution as showed by exitflag=3.
iter = 1.
exitflag = -3.
fopt = 1.
xopt = 0. 1.

At (1, 2/3) the result is not true, exitflag=1 but $x^* = (1.0219107, 0.6666667)$.
iter = 1.
exitflag = 1.
fopt = 0.6666667
xopt = 1.0219107 0.6666667.

At (1, 1/3) the Karmarkar function cannot go to the optimal solution as showed by exitflag=-3.
iter = 1.
exitflag = -3.
fopt = 0.3333333
xopt = 1. 0.3333333.

At (0,0) the Karmarkar result exitflag=-3, which means its search direction became zero.
iter = 1.
exitflag = -3.
fopt = 0.
xopt = 0. 0.
3.4. The starting point is calculated by the Karmarkar function. 
When the Karmarkar function automatically calculates a feasible initial guess the output is as follows and the movement of a point is depicted in Figure 3.

iter = 80.
exit flag = 1.
fopt = -2.643D-08
xopt = -4.763D-08 -2.643D-08.

The optimum value is (-4.763D-08 -2.643D-08), where it is outside the feasible region, and hence this is not true.

![Figure 3. The movement of points when the starting point is calculated by the Karmarkar function.](image)

4. Conclusion
The Karmarkar function in Scilab needs more improvement, because at some cases the results are not as expected, i.e. when the starting point at the border of feasible region (at the vertex and not the vertex) and when the starting point is calculated by the Karmarkar function.

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