Particle-antiparticle in 4D charged Einstein-Gauss-Bonnet black hole

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Abstract

We study the charge of the 4D-Einstein-Gauss-Bonnet black hole by a negative charge and a positive charge of a particle-antiparticle pair on the horizons $r_-$ and $r_+$, respectively. We show that there are two types of the Schwarzschild black hole. We show also that the Einstein-Gauss-Bonnet black hole charge has quantified values. We obtain the Hawking-Bekenstein formula with two logarithmic corrections, the second correction depends on the cosmological constant and the black hole charge. Finally, we study the thermodynamics of the EGB-AdS black hole.

Keywords: Black hole, Entropy, Einstein-Gauss-Bonnet gravity.

1 Introduction

It’s well known that the Gauss-Bonnet gravity is introduced only in case $D > 4$ or more. In four-dimensional spacetime, the GB term does not make contributions to the gravitational dynamics, which makes the 4-dimensional minimally coupled GB gravity is hard to obtain. Recently, D. Glavan and C. Lin [1] proposed a novel 4-dimensional Einstein-Gauss-Bonnet (EGB) gravity, which has attracted great attention. An intriguing idea of D. Glavan and C. Lin is to multiply the GB term by the factor $1/(D - 4)$ before taking the limit. Offers a new 4-dimensional gravitational theory with only two dynamical degrees of freedom by taking the $D \rightarrow 4$ limit of the Einstein-Gauss-Bonnet gravity in $D > 4$ dimensions [2], which is in contradiction with Lovelock theorem.

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The four-dimensional symmetrical static and spherical black hole solution in EGB gravity were obtained \[4\], also, solutions of a charged black hole \[5\]. There has been lots of discussions about the self-consistency of the 4D EGB gravity. It was shown in several papers that perhaps the \(D > 4\) limit is not clearly defined, and several ideas have been proposed to remedy this inconsistency \[6, 7, 8\]. Many researchers have studied particles in the geometry of a black hole, precisely, the EGB black hole \[9, 10\]. Juan Maldacena and Leonard Susskind devised a theory linking two phenomena both discovered by Einstein: “Einstein-Rosen bridges” (or wormholes) and quantum entanglement \[11\]. According to them, if we move the two entangled particles apart would amount to digging an ER bridge around a single particle which would manifest its properties in several places in space-time. This theory sheds light on the problem of the EPR paradox which highlights the non-locality of quantum mechanics, which he opposes to the principle of the locality which is the basis of the theory of relativity. However, this \(ER = EPR\) correspondence is only demonstrated in a very simplified universe model, where gravity is generated in the absence of mass. The Hawking radiation of a black hole is a scrambled cloud of radiation entangled with the black hole. The connection between the laws of black hole mechanics with the corresponding laws of ordinary thermodynamic systems has been one of the remarkable achievements of theoretical physics \[12\]. In fact, the consideration of a black hole as a thermodynamic system with a physical temperature and entropy provides a deep insight to understand its microscopic structure. The study of EGB gravity becomes very important because it provides a broader setup to explore many conceptual questions related to gravity. This theory of gravity similar to Einstein’s gravity only benefits from the first and second order derivatives of the metric function in field equations. there is also a great connection between EGB gravity and the AdS / CFT demonstration correspondence \[13\].

Throughout the paper, we use the unit system where the speed of light \(c = \) the gravitational constant \(G_N = \) the vacuum permittivity \(4\pi\varepsilon_0 = 1\).

## 2 Charged Einstein-Gauss-Bonnet black hole

Consider now the charged Einstein-Gauss-Bonnet theory in D-dimensions with a negative cosmological constant \[14, 15\]

\[
I = \frac{1}{16\pi} \int d^Dx \left( R - 2\Lambda + \frac{\alpha}{D-4}G - F_{\mu\nu}F^{\mu\nu} \right) \tag{2.1}
\]

where \(\alpha\) is a finite non-vanishing dimensionless GaussBonnet coupling have dimensions of \([\text{length}]^2\), that represent ultraviolet (UV) corrections to Einstein theory, \(F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu\) is the Maxwell tensor and \(l\) is the AdS radius and

\[
G = R^2 - 4R_{\mu\nu}R^{\mu\nu} + R_{\mu\nu\rho\sigma}R^{\mu\nu\rho\sigma} \tag{2.2}
\]

\[
\Lambda = - \frac{(D - 1)(D - 2)}{2l^2} \tag{2.3}
\]
by solving the field equation we obtain the black hole solution

\[
\begin{align*}
  ds^2 &= -f(r)dt^2 + \frac{1}{f(r)}dr^2 + r^2 \left( d\theta^2 + \sin^2 \theta d\phi^2 \right) \\
  \text{(2.4)}
\end{align*}
\]

Taking the limit \( D \to 4 \), we obtain the exact solution in closed form

\[
- g_{00} = f(r) \approx 1 + \frac{r^2}{2\alpha} \left( 1 - \sqrt{1 + 4\alpha \left( \frac{2M}{r^3} - \frac{Q^2}{r^4} - \frac{1}{l^2} \right)} \right) \\
\text{(2.5)}
\]

this last solution solution could be obtained directly from the derivation done in \([16]\). In the large \( r \) limit, for two branches of solutions. In the limit of vanishing mass and charge \([17]\), for \((l^{-2} \sim 0)\) we find

\[
- g_{00} = 2 \frac{r^2 - 2Mr + Q^2 + \alpha}{r^2 + 2\alpha + \sqrt{r^4 + 4\alpha (2Mr - Q^2)}} \\
\text{(2.6)}
\]

with \(2M\) is the Schwarzschild radius. From this expression, we notice that, in the limit \( \alpha \to 0 \), the metric reduce to the Reissner–Nordström black hole solution. If \( \alpha \ll 0 \) the solution is still an AdS space, if \( \alpha \gg 0 \) the solution is a de Sitter space, \([15]\). The event horizon in spacetime can be located by solving the metric equation: \( g_{00} = 0 \). To find the black hole horizon we have to solve this equation

\[
- g_{00} = 2 \frac{r^2 - 2Mr + Q^2 + \alpha}{r^2 + 2\alpha + \sqrt{r^4 + 4\alpha (2Mr - Q^2)}} \\
\text{(2.7)}
\]

The solutions show that the event horizon is located at:

\[
r_{\pm} = M \pm \sqrt{M^2 - Q^2 - \alpha} \\
\text{(2.8)}
\]

we notice that the solution behaves like the Reissner-Norström (RN) solution. The black hole event horizon is the largest root of the equation above, \( r_+ \) is the black hole horizon \([1]\). However, the radius \( r_- \) represents a horizon, which can be a horizon mirage or virtual horizon. Therefore, to explain the presence of two horizon \( r_{\pm} \), we assume that we can represent the horizon \( r_- \) as a reflection symmetric of the horizon \( r_+ \) \([11]\). There is only one case where the horizon \( r_+ \) corresponds to the horizon \( r_- \), this case is equivalent to the degenerate solution into a singularity when \(|Q| = \sqrt{M^2 - \alpha}\), which corresponding to an extremal black hole, and this type of black hole which has a very low mass. We choose the same charge to describe particle-antiparticle pair in the EGB black hole horizons. In this paper, we propose that the two extremal entangled black holes are particles and antiparticles which are placed just on the horizon of the EGB black hole. The particle located near horizon \( r_+ \). On the other hand, the antiparticle is the reflection of the particle on the horizon \( r_- \). If the particle and antiparticle are connected by a bridge (by an Einstein-Rosen bridge), then the area of the bridge smaller than the area of the black hole horizon. First, we define two charges of a particle-antiparticle pair in the horizon
\[ q_{\pm} = \pm \sqrt{M^2 - \alpha} \]  \hspace{1cm} (2.9)

we will show later with a particular condition that the proposition of the charges \( q_{\pm} \) leads to an EGB entropy already obtained by other models. Every particle on the horizon has a charge which depends directly on the black hole mass; if the black hole mass larger, the charge of a horizon particle will be more important. Each particle of the charge \( q_+ = + \sqrt{M^2 - \alpha} \), is entangled with another antiparticle of the charge \( q_- = - \sqrt{M^2 - \alpha} \). The particle and antiparticle have opposite electric charges \( q_+ \) and \( q_- \), i.e. CPT anticommutes with the charges. If the number of particles \( N \) on the horizon \( r_+ \) is limited, the horizon charge will be \( Q_H = 0 \). If we take that the number of horizon particles \( N \rightarrow \infty \), the charge of the black hole is \( Q_H \neq 0 \). From Eqs.(2.8,2.9)

\[(r_{\pm} - M)^2 = q_{\pm}^2 - Q^2 \]  \hspace{1cm} (2.10)

according to this formula, the charges \( q_+ \) are located on the real horizon, on the other hand, the charges of antiparticles \( q_- \) are located on a virtual horizon near the singularity. We also notice that \( q_{\pm}^2 \geq Q^2 \). In the framework of the gravitational repulsion between matter and antimatter, Eq.(2.10) behaves like virtual gravitational dipoles \[18\] in a black hole. If we assume that there is a complete disappearance of an AdS black hole \( (\alpha < 0) \), we obtain the position of the two charges

\[ r_\pm = q_\pm = \pm \sqrt{-\alpha} \]  \hspace{1cm} (2.11)

the problem of the negative radius \( r_- \) indicates a disappearance of the antiparticles with the disappearance of the black hole singularity, on the other hand, the particles escape the singularity. This aspect is equivalent to the position of the two horizons; the horizon \( r_- \) exists on the singularity and \( r_+ \) is the edge of the black hole. Which agrees with the violation of CP symmetry between matter and antimatter \[19\]. This complete disappearance of the antiparticles looks like a scenario of the disappearance of antimatter after the big bang. The relation (2.11) is valid only for an AdS black hole, this may indicate that the disappearance of the antiparticles is done on the second copy of AdS in other dimensions. The electric potentials \( \Phi_+ \) and \( \Phi_- \) arising from the charge \( q_+ \) and \( q_- \) respectively, at a distance \( r \) from the charge given by

\[ \Phi_+(r) = \frac{q_+}{|r - r_+|} \hspace{1cm} \Phi_-(r) = \frac{q_-}{|r - r_-|} \]  \hspace{1cm} (2.12)

\( \Phi_+ \) and \( \Phi_- \) are the conjugate (gauge independent) potentials for the electric (and magnetic) \( U(1) \) charges. Two opposite charges \( q_{\pm} \) have a potential of the electric dipole \( \Phi_+ + \Phi_- \) from

\[ \Phi_+(r) = \frac{\sqrt{M^2 - \alpha}}{|r - M - \sqrt{M^2 - Q^2 - \alpha}|} \hspace{1cm} \Phi_-(r) = \frac{-\sqrt{M^2 - \alpha}}{|r - M + \sqrt{M^2 - Q^2 - \alpha}|} \]  \hspace{1cm} (2.13)

In the extremal case \( M^2 = Q^2 + \alpha \), we obtain \( \Phi_+(r) = \Phi_-(r) \). The two potential is canceled out for \( M^2 = \alpha \).
3 Electric potential of EGB black hole

Last papers proposed to study a charged particles near the Schwarzschild black hole, or charged particle motion around magnetized Schwarzschild black holes [20, 21, 22, 23]. We are also interested in studying the particles (2.9) in the Schwarzschild horizon. We know that the Schwarzschild metric is a solution for a black hole without electric charge and angular momentum. In what follows we will use the previous results to show that the Schwarzschild black hole is not charged ($Q = 0$).

Assume that the radius $r_+$ is equal to the Schwarzschild radius $2M$, given by

$$q_\pm = \pm M$$

(3.1)

in this case, according to Eqs.(2.10,2.9) one can obtain

$$r_- = M \pm M$$

(3.2)

according to the two expressions (3.1,3.2), we can represent the Schwarzschild black hole by two visions:

If $r_- = 0$, the Schwarzschild black hole consists of a negatively charged singularity ($q_- = -M$) and a positively charged horizon ($q_+ = M$), therefore, if the number of particle and antiparticle is limited, the total Schwarzschild black hole charge is zero, this case is similar to an antimatter atom. If $r_- = 2M$ (the horizon degenerates), the Schwarzschild black hole consists of a neutral singularity and positive and negative charges on the horizon ($q_- = -M$; $q_+ = M$), therefore, for the Schwarzschild horizon to be neutral; the number of positive charges must be equal to the number of negative charges on the horizon. This result shows that the Schwarzschild black hole, behaves like the neutral atom, but it contains positive and negative charges. The Schwarzschild black hole contains a negatively charged singularity $q_-$ and a positively charged horizon $q_+$. We can express the electric potentials of negative and positive charges in the Schwarzschild black hole by

$$\Phi_+(r) = \frac{M}{|r| |1 - 2M/r|} \quad \Phi_-(r) = -\frac{M}{|r|}$$

(3.3)

if we take $\alpha = 0$ in Eq.(2.13), we also find the same potentials above, what was mentioned by [24].

Since the study of the charged Einstein Gauss-Bonnet black hole shows a property similar to that of RN black hole, like Eq.(2.8), which shows that the parameter $\alpha$ creates a passage between the Schwarzschild black hole and RN black hole. We can rewrite the Schwarzschild metric as

$$ds^2 = \frac{\Phi_-}{\Phi_+} dt^2 - \frac{\Phi_+}{\Phi_-} d\tau^2 + r^2 (d\theta^2 + \sin^2 \theta d\phi^2)$$

(3.4)

the Schwarzschild metric depends on the potentials of particles and antiparticles. In the Schwarzschild black hole, $N$ is limited and we write

$$Q_S = q_+ \sum_{n=0}^{N-1} (-1)^n = 0$$

(3.5)
Table 1: The possible values of the EGB black hole charge as a function of the charge of a particle.

For these two relations above to be valid, the number \( N \) must be even, i.e. the number of particles equal to the number of antiparticles. But if \( N \) is odd, we get \( Q = q_+ \) or \( Q = q_- \), this is at odds with an uncharged Schwarzschild black hole. This shows that the number of particles on the horizon is equal to the number of antiparticles in the Schwarzschild black hole singularity, which corresponds exactly with the proposition (2.9) of entangled particles with antiparticles, one can’t have on black hole particles which are not entangled. Since the formula (2.8) is a generalization of the RN black hole horizons, this shows that the EGB black hole charge is the generalization of charge of RN black hole (for \( \alpha = 0 \)) and zero charges of the Schwarzschild black hole. In the EGB black hole, when \( N \to \infty \), the analytic continuation of the Riemann zeta function of 0 \( (\zeta(0) = 1/2) \) give

\[
Q_{EGB} = q_\pm \sum_{n=0}^{N-1} (-1)^n = \pm \frac{1}{2} \sqrt{M^2 - \alpha}
\]  

(3.6)

this charge corresponds exactly with a physical event horizon because \( Q_{EGB} \) check the condition of existence of the event horizon: \( 2Q_{EGB} < r_s \). We can also describe the charge of EGB black hole in the case of \( N \neq \infty \), with \( N \) being odd, i.e. \( Q_{EGB} = \pm q_+ \). Usually, we get

all values above of the black hole charge verify condition (2.10). In the case where the charge \( q_+ = 0 \) is zero, the EGB black hole is transformed into a Schwarzschild black hole. The values of the charge of \( Q_{EGB} \) are quantified according to the constant values of \( M \) and \( \alpha \). We represent this quantification by the total charge of an Einstein Gauss-Bonnet black hole is

\[
Q_{EGB} = m\sqrt{M^2 - \alpha} \quad m = \{-1, -1/2, 0, 1/2, 1\}
\]  

(3.7)

Substitute Eq. (3.7) into Eq. (2.8) and we obtain

\[
r_\pm = M \pm \sqrt{(1 - m^2)(M^2 - \alpha)}
\]  

(3.8)

the EGB black hole horizon is quantified according to the values of \( m \). We want to calculate the electric potential of the EGB black hole on the horizon \( (r = r_+) \) from its charge (3.7) in the horizon \( r_+ \) (3.8), we obtain

\[
\Phi_{EGB} = \frac{Q_{EGB}}{r_+} = \frac{m\sqrt{M^2 - \alpha}}{\sqrt{(1 - m^2)(M^2 - \alpha)} + M}
\]  

(3.9)

for the case where \( m = 0 \), this potential becomes zero, which corresponds exactly to the Schwarzschild black hole.
4 EGB-AdS black hole thermodynamics

4.1 The black hole first law

We define the pressure [25] of the cosmological constant (2.3) for $D \rightarrow 4$

$$8\pi P = 3l^{-2} \quad \Lambda = -3l^{-2}$$ (4.1)

We can express the ADM mass $M$ of the black hole in terms of $r_\pm$ by solving Eq.(2.7) for $r = r_+$ resulting in

$$M = \frac{l^{-2}r_+^4 + r_+^2 + Q_{EGB}^2 + \alpha}{2r_+}$$ (4.2)

The Hawking temperature is easy to give by calculating surface gravity at the horizon

$$T^2 = -\frac{1}{8\pi^2} \nabla_\mu \xi_\nu \nabla^\mu \xi^\nu = \frac{1}{16\pi^2} f'^2(r_+)$$ (4.3)

where $\xi^\mu$ is a killing vector, which for a static, spherically symmetric case takes the form $\xi^\mu = \partial^\mu_t$. $\xi_\mu$ satisfies the Killing equation

$$\nabla_\mu \xi_\nu + \nabla_\nu \xi_\mu = 0$$ (4.4)

The Hawking temperature of the black hole can be calculated as

$$T = \frac{3l^{-2}r_+^4 + r_+^2 - Q_{EGB}^2 - \alpha}{8\pi \alpha r_+ + 4\pi r_+^4}$$ (4.5)

if we suppose that the charges $q_+$ and $q_-$ are behave like a gas on the black hole. The black hole first law reads [25]

$$dM = TdS + \Phi_{EGB}dQ_{EGB} + VdP + Ad\alpha$$ (4.6)

The parameters $V$ and $A$ are the conjugate quantities of the pressure $P$ and GB coupling parameter $\alpha$, respectively.

If we fix $P$ and $\alpha$ the Hawking-Bekenstein formula is then given by

$$S = \int \frac{dM}{T} - \int \frac{\Phi_{EGB}}{T}dQ_{EGB}$$ (4.7)

to describe the entropy according to the charges of the particles and antiparticles present on the black hole, we have not fixed the charge $Q_{EGB}$ of the black hole. The solution of the first part of Eq.(4.7) already computed by [15, 25] as

$$\int \frac{dM}{T} = \int_0^{r_+} \frac{1}{T} \left( \frac{\partial M}{\partial r'} \right) dr' = \pi r_+^2 + 2\pi \alpha \log r_+^2 + S_0$$ (4.8)
where $S_0$ is an integration constant. Next, we use (3.9) to calculate the second term in Eq.(4.7). We write the potential $\Phi_{EGB}$ as a function of $Q_{EGB}$ for a fixed position $r = r_+$ we obtain

$$\int \frac{\Phi_{EGB}}{T} \, dQ_{EGB} = \int \frac{Q_{EGB}}{Tr_+} \, dQ_{EGB}$$

(4.9)

we substitute Eq.(4.5) into the last equation

$$\int \frac{\Phi_{EGB}}{T} \, dQ_{EGB} = \int \frac{(8\pi\alpha + 4\pi r_+^2) Q_{EGB}}{(3l-2r_+^4 + r_+^2 - \alpha) - Q_{EGB}^2} \, dQ_{EGB}$$

(4.10)

we consider that the expression $Q_{EGB}$ (3.7) is defined on a fixed horizon (3.8)

$$\int \frac{\Phi_{EGB}}{T} \, dQ_{EGB} = -2 \left(2\pi\alpha + \pi r_+^2\right) \log |Q_{EGB}^2 - (3l-2r_+^4 + r_+^2 - \alpha)| + S_1$$

(4.11)

with $S_1$ is an integration constant. Therefore, substitute Eqs.(4.8,4.11) in to Eqs.(4.7) we obtain

$$S = \frac{A}{4} + 2\pi\alpha \log \frac{A}{A_0} + \left(\frac{A}{2} + 4\pi\alpha\right) \log \left|\frac{m^2}{A_1} (M^2 - \alpha) - \frac{1}{4\pi A_1} (A - \Lambda A^2 - 4\pi\alpha)\right|$$

(4.12)

where $A_0$ and $A_1$ are some constants with units of area, $A \equiv 4\pi r_+^2$ is the area of the event horizon of the black hole. This equation generalizes the Hawking-Bekenstein (HB) formula by a two supplementary logarithmic term [26], instead of just one additional logarithmic form [15, 25]:

$$S = \frac{A}{4} + 2\pi\alpha \log \frac{A}{A_0}$$

(4.13)

If we fix the EGB black hole charge we find the entropy according to the model [15]. We remark that the second entropy correction contains the term $m$ which describes the states of the charge $Q_{EGB}$. We also remark that there is the presence of the cosmological constant $\Lambda$ in the second logarithmic term. Contrary to the model [15], the entropy above described also by the cosmological constant with the charges $Q_{EGB}$, this comes after varying $S$ with respect to $Q_{EGB}$. We calculated the electric potential of EGB black hole, and by using this potential we write the entropy with two logarithmic corrections.

### 4.2 Pressure, volume and temperature

It is also possible to calculate the entropy (4.12) at finite temperature $T = \beta^{-1}$, by the use of expression (4.5)

which is also written as

$$S = \frac{A}{4} + 2\pi\alpha \log A + \frac{1}{2} (A + 8\pi\alpha) \log \frac{B}{A_1} + S_0 - S_1$$

(4.14)
we define a new area
\[ B = \left| \beta^{-1} r_+ (A + 8\pi \alpha) \right| \]  
(4.15)
we show that
\[ S = 2\pi \alpha \log A e^{\frac{A}{8\pi \alpha}} B \frac{A + 8\pi \alpha}{4\pi \alpha} + S_0 - S_1 \]  
(4.16)
we study two different types of charged EGB-AdS black holes, allow for a first order small-black-hole/large-black-hole (SBH/LBH), we choose for SBH a surface of the form \( A \ll 8\pi \alpha \), we choose for LBH a surface of the form \( A \gg 8\pi \alpha \), because the values of \( \alpha \) included in the interval \([-1, 1]\) [13].
First, we assume that \( A \ll 8\pi \alpha \), i.e. a space difference of AdS space (since \( \alpha \gg 0 \)). The entropy can be rewritten into the following simple form
\[ S = 2\pi \alpha \log A B^2 + S_0 - S_1 \quad \text{and} \quad B = 8\pi \alpha \beta^{-1} r_+ \]  
(4.17)
we obtain a case where the subadditivity inequality is saturated for \( S_0 \leq S_1 \): 
\[ S \equiv S(AB) \leq S(A) + S(B) \]  
(4.18)
where
\[ S(A) = 2\pi \alpha \log A \quad \text{and} \quad S(B) = 4\pi \alpha \log B \]  
(4.19)
Next, we assume that \( A \gg 8\pi \alpha \), from Eq.(4.16) the entropy can be rewritten into the following form
\[ S = \frac{A}{2} \log \frac{\sqrt{e} A r_+}{\beta A_1} + 2\pi \alpha \log \frac{A}{A_0} \]  
(4.20)
According to AdS/CFT correspondence, gravitational theories on AdS\(_{2+1}\) space of radius \( R \) are dual to CFT\(_2\).
for \( r_+ A = \beta A_1 \), we obtain the same entropy in the EGB black hole framework (4.13).
Next, we assume that \( A \sim 8\pi \alpha \)
\[ S = A \log \frac{2e^{1/4} A r_+}{\beta A_1} + \frac{A}{4} \log \frac{A}{A_0} \]  
(4.21)
for \( 2r_+ A = \beta A_1 \), we obtain the same entropy in the EGB black hole framework (4.13). According to the last two cases, we can transform the two entropies (4.20,4.21) into (4.13), which shows that there is a possibility of return entropy with two logarithmic corrections (4.12) to entropy with a single correction. Therefore, we can find the entropy (4.13) anyway by using the hypothesis (2.9) for the condition of \( A \gtrsim 8\pi \alpha \). This condition is also valid for an AdS-EGB black hole: \( \alpha \gtrsim 0 \).
Let us consider an asymptotically AdS-EGB black hole spacetime (\( \Lambda \ll 0 \)), the cosmological constant corresponds to thermodynamic pressure with \( 8\pi P = -\Lambda \), and the conjugate variable of \( P \) corresponds to thermodynamic volume [27]
\[ V = \left( \frac{\partial M}{\partial P} \right)_{S, Q, \alpha} = \frac{1}{3} r_+ A \]  
(4.22)
this conjugate variable was interpreted geometrically as an effective volume for the region outside the EGB-AdS black hole horizon \[28\]. For the static black holes, the thermodynamic volume is only a function of the event horizon. From (4.20,4.21) we can introduce this condition

\[ A \geq 8\pi\alpha \iff \frac{1}{6}\beta A_1 \leq V \leq \frac{1}{3}\beta A_1 \]  

(4.23)

the condition concerning the temperature is

\[ \frac{A_1}{3V} \leq T \leq \frac{A_1}{6V} \]  

(4.24)

the volume of AdS-EGB black hole checks the above relationship. Whether the black hole surface checks \( A \geq 8\pi\alpha \), the maximum and minimum values of the black hole volume depend on \( \beta \). Therefore, the temperature (4.5) is expressed as a function of the pressure (4.1) and \( V \)

\[ T = \frac{6PV}{A + 8\pi\alpha} + \frac{r_+^2 - Q_{EGB}^2 - \alpha}{(A + 8\pi\alpha)r_+} \]  

(4.25)

we substitute Eqs. (3.7,3.8) into the last equation, we get a corresponding fluid equation of state:

\[ PV = \frac{A + 8\pi\alpha}{6}T - \frac{1}{3}\sqrt{(1 - m^2)(M^2 - \alpha)} \]  

(4.26)

which leads to

\[ PV = \frac{A + 8\pi\alpha}{6}T - \frac{1 - m^2}{m^2 \frac{Q_{EGB}^2}{3(r_+ - M)}} \]  

(4.27)

this equation is in good agreement with the equation \( P = f(T,V) \) found by \[28\] for a charged AdS black holes (\( \alpha = 0 \)). We use the specific volume \( \upsilon = 2r_+l_P^2 \equiv 2r_+ = 6V/N \), where \( l_P = \sqrt{\hbar G/c^3} \equiv 1 \) is the Planck length and \( N = A/l_P^2 \) is the number of states associated with the horizon \[28\].

\[ P = \left(1 + \frac{8\pi\alpha}{A}\right)\frac{T}{\upsilon} - \frac{1 - m^2}{m^2 \left(1 - \frac{2M}{\upsilon}\right)} \frac{4Q_{EGB}^2}{\upsilon^2N} \]  

(4.28)

Note that for \( m = \pm 1 \), we obtain the ideal gas law. We can show the Van der Waals equation for \( A \gg 8\pi\alpha \):

\[ (P + \frac{a}{v^2})(v - b) = T \]  

(4.29)

These curves are interpreted as follows:

- for a temperature \( T > T_c \) the fluid is only stable under one phase: the supercritical fluid;
- for a temperature \( T < T_c \) the fluid is stable under a single-phase, gas or liquid, or present simultaneously in two phases in equilibrium, gas and liquid. the formula (4.29) yields

\[ a = \frac{4q_{+}r_+}{N}\sqrt{1 - m^2} \geq 0 \quad \frac{b}{\upsilon} = \frac{8\pi\alpha}{A} \]  

(4.30)

where the parameter \( a \) measures the attraction between particles and the parameter \( b \) corresponds to the volume of fluid of particles. We notice that \( b \) is less than \( \upsilon \) because \( A \gg 8\pi\alpha \). When \( N \rightarrow \infty \), the attraction between particles in the fluid will be zero \( a = 0 \). If there is a non-minimal coupling between particle-antiparticle pair, the attraction will be zero for \( N \rightarrow \infty \). For an EGB-AdS black hole with \( N \neq \infty \) (table 1), the parameter \( a \) will be zero.
5 Conclusion

In this paper, we have considered charged 4D EGB-AdS black holes as a working substance. We studied the relationship between positive and negative charges with the black hole in 4D EGB theory based on the work of Glavan & Lin [1]. We have assumed a particular shape of the charges present on the black hole by the degenerate solution. We calculated the electric potentials of particles and antiparticles for the EGB black hole, then we deduce the potentials of these charges for the Schwarzschild black hole. We have shown that the charge of a Schwarzschild black hole is zero. We have adopted that the EBG black hole is a generalization of the Schwarzschild black hole. We have found in this case a simplified charge of EGB black hole which takes district values. By using this potential we write the HB formula with two logarithmic corrections. The second correction depends on the discrete values of the EBG black hole charge also depends on the cosmological constant. For \( A \geq 8\pi \alpha \), we can obtain the HB entropy with a single correction from that with two corrections. In this case, we studied the AdS-EGB black hole thermodynamics, this study allows obtaining the Van der Waals equation.

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