We propose a stellar locus outlier (SLOT) method to determine the binary fraction of main-sequence stars statistically. The method is sensitive to neither the period nor mass ratio distributions of binaries and is able to provide model-free estimates of binary fraction for large numbers of stars of different populations in large survey volumes. We have applied the SLOT method to two samples of stars from the Sloan Digital Sky Survey (SDSS) Stripe 82, constructed by combining the recalibrated SDSS photometric data with the spectroscopic information from the SDSS and LAMOST surveys. For the SDSS spectroscopic sample, we find an average binary fraction for field FGK stars of 41% ± 2%. The fractions decrease toward late spectral types and are 44% ± 5%, 43% ± 3%, 35% ± 5%, and 28% ± 6% for stars with $g - i$ colors in the range 0.3–0.6 mag, 0.6–0.9 mag, 0.9–1.2 mag, and 1.2–1.6 mag, respectively. A modest metallicity dependence is also found. The fraction decreases with increasing metallicity. For stars with [Fe/H] between −0.5 and 0.0 dex, −1.0 and −0.5 dex, −1.5 and −1.0 dex, and −2.0 and −1.5 dex, the inferred binary fractions are 37% ± 3%, 39% ± 3%, 50% ± 9%, and 53% ± 20%, respectively. We have further divided the sample into stars from the thin disk, the thick disk, the transition zone between them, and the halo. The results suggest that the Galactic thin and thick disks have comparable binary fractions, whereas the Galactic halo contains a significantly larger fraction of binaries. Applying the method to the LAMOST spectroscopic sample yields consistent results. Finally, other potential applications and future work with the method are discussed.

Key words: binaries: general – stars: formation – stars: general – stars: statistics – surveys
photometric (Ivezić et al. 2007; Yuan et al. 2015a) and spectroscopic data of the SDSS Stripe 82, Yuan et al. (2015b, hereafter Paper I) build a large, clean sample of MS stars with accurate colors (about 1%) and well-determined metallicities (about 0.1 dex) to investigate the metallicity dependence and intrinsic widths of the SDSS stellar color loci. By fitting to the \(u - g\), \(g - r\), \(r - i\), and \(i - z\) colors as a function of the \(g - i\) color and [Fe/H] with two-dimensional polynomials, they obtained for the first time the metallicity-dependent stellar loci in the SDSS colors and find that the fit residuals can be fully accounted for by the uncertainties in photometric measurements, metallicity determinations, and calibration, suggesting that the intrinsic widths of the loci are at maximum of a few millimagnitudes if not zero. More interestingly, the distributions of residuals are asymmetric, pointing to the presence of a significant population of binaries. As we shall show in the current work, by modeling the distributions of residuals it is possible to reveal the binary fraction of field MS stars.

In this companion paper we propose a stellar locus outlier (SLOT) method to determine the binary fractions statistically. Compared with the previous techniques, the SLOT method is solely based on color deviations relative to the metallicity-dependent stellar loci and thus is independent of the separations (orbital periods) of binaries if one neglects the small fraction of spatially resolved wide binaries (semimajor axes \(\geq 100\) AU; Chanamé 2007). The method is also insensitive to the assumed mass ratio distribution. Because binaries of intermediate mass ratios contribute most of the observed deviations in the color–color space, those with close to unity or very small mass ratios contribute little. With modern photometric surveys (providing accurate colors) and large-scale spectroscopic surveys such as the SDSS and LAMOST (providing robust estimates of metallicity and surface gravity), the SLOT method is capable of providing model-free estimates of the binary fraction for large samples of stars of different populations. When combining results from other techniques, the method can also provide strong constraints on the distributions of orbital periods and mass ratios of binary stars.

In this paper we introduce the SLOT method, apply the method to stars of the SDSS Stripe 82 spectroscopically targeted by the SDSS and LAMOST, and present a model-free estimate of the binary fraction for field FGK stars. The variations of binary fraction with spectral type, metallicity, and population of the binary faction for field FGK stars. The variations of binary fraction with spectral type, metallicity, and population of the binary faction for field FGK stars. The variations of binary fraction with spectral type, metallicity, and population of the binary faction for field FGK stars. When combining results from other techniques, the method can also provide strong constraints on the distributions of orbital periods and mass ratios of binary stars.

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2. METHOD

As shown in Paper I, the intrinsic widths of the metallicity-dependent stellar loci of the SDSS colors for MS stars are essentially zero, i.e., the colors \(u - g\), \(g - r\), \(r - i\), and \(i - z\) of a single MS star are fully determined by its \(g - i\) color and [Fe/H]. To show how the SLOT method works, let us consider binary systems composed of two MS single stars. The colors of the binary systems will deviate from what is predicted by the metallicity-dependent loci (of single stars), as illustrated in Figure 1. The deviations can be simulated for all possible combinations of primary stars with \(g - i\) color ranging from 0.3 to 1.6 mag and secondary stars with \(g - i\) color ranging from 0.3 to 3.0 mag, with the condition that the color of the secondary is no bluer than that of the primary. For the simulation, we assume that both the primary and secondary stars have the same [Fe/H] of –0.8 dex. For a given binary of given \(g - i\) colors of the two component stars and metallicity [Fe/H], the absolute \(r\)-band magnitudes, \(M(r)\), of the two stars are computed using the photometric parallax relation of Ivezić et al. (2008, Equation (A7)), and their \(u - g\), \(g - r\), \(r - i\), and \(i - z\) colors are computed using the metallicity-dependent stellar loci determined in Section 3.1.2 of the current paper for \(g - i \leq 1.6\) mag and using the stellar loci of Covey et al. (2007) for \(g - i > 1.6\) mag. The absolute \(r\)-band magnitudes and \(u - g\), \(g - r\), \(r - i\), and \(i - z\) colors are then used to derive the absolute magnitudes in \(u\), \(g\), \(i\), and \(z\) bands. Given the absolute magnitudes in \(u\), \(g\), \(r\), \(i\), and \(z\) bands of both stars, the combined magnitudes and colors of the binary are calculated. The differences between the combined \(u - g\), \(g - r\), \(r - i\), and \(i - z\) colors of the system and those predicted by the metallicity-dependent stellar loci (of single stars) for a combined \(g - i\) color (of the binary system) are then deduced and plotted in Figure 2 as a function of the \(g - i\) colors of the primary and secondary stars, respectively. The deduced differences between the \(g - i\) color of the system and that of the primary star are shown in Figure 3.

Figure 2 shows the following. (1) Colors of binary systems generally do not follow the metallicity-dependent stellar loci of single MS stars. Rather, they deviate from the loci in a systematic way. Colors of simulated binaries are always redder in \(u - g\) and \(g - r\) and bluer in \(r - i\) and \(i - z\) than values predicted for single stars. Such systematic deviations are actually the underlying cause of the asymmetric distributions of residuals of colors with respect to the stellar loci, as reported in Paper I. (2) The deviations are very small. The maximum deviations are about 0.15, 0.036, –0.036, and –0.036 mag in the \(u - g\), \(g - r\), \(r - i\), and \(i - z\) colors, respectively. The deviations are nearly zero when the primary and secondary stars are of close to unity or very small mass ratios. They reach the maximum values at certain colors of the primary and secondary. Except for \(u - g\), the absolute values of maximum deviation in colors \(g - r\), \(r - i\), and \(i - z\) occur at large \(g - i\) colors of the primary. This implies that binary stars consisting of an early-type primary are more

![Figure 1. Plot illustrating the SLOT method. The line denotes the stellar locus of MS single stars of a given metallicity. The purple, blue, and red stars denote locations of a binary system and its primary and secondary stars, respectively. The colors of the binary system deviate from the stellar locus, as indicated by the arrow.](image-url)
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Figure 2. Distributions of differences between the combined $u-g$ (top left), $g-r$ (top right), $r-i$ (bottom left), and $i-z$ (bottom right) colors of binary systems and those predicted by the metallicity-dependent stellar loci for the combined $g-i$ color of the systems, as a function of the $g-i$ colors of the consisting primary and secondary stars. The binaries are assumed to be composed of two MS single stars with $[\text{Fe/H}] = -0.8$ dex. A color bar is overplotted by the side in each case. The initial set of stellar loci determined in Section 3.1.2 of the current paper and those of Covey et al. (2007) are used for $g-i \leq 1.6$ and $>1.6$ mag, respectively.

Figure 3. Distributions of differences between the combined $g-i$ colors and those predicted by the metallicity-dependent stellar loci of the sample are obtained as in Paper I. Two sets of Monte Carlo (MC) simulations are performed; one assumes that all stars in the sample are single, while the other assumes that all stars are binaries, in order to mimic the distributions of the observed sample in the color–color diagrams. For the first set of simulations, for a star of color $g-i$ and metallicity $[\text{Fe/H}]$, its $u-g$, $g-r$, and $i-z$ colors are computed as predicted by the metallicity-dependent stellar loci if $g-i \leq 1.6$ mag. For $g-i > 1.6$ mag, the stellar loci of Covey et al. (2007) are used. For the other set of simulations of binary stars, for a star of color $g-i$ and metallicity $[\text{Fe/H}]$, the mass of the primary is derived from the Dotter et al. (2008) isochrones (Figure 4) by linear interpolation. The mass of the secondary star is then generated by MC simulation for an assumed mass ratio distribution of binary stars. We assume that the mass ratio distribution follows a power law of index $\gamma = 0.3$ for mass ratios between 0.05 and 1 (Duchêne & Kraus 2013). If the mass of the secondary is smaller than $0.08 M_\odot$, its effects are then neglected. The $g-i$ color of the secondary is then derived from the same set of isochrones. The combined colors of the binary system are then calculated as described in the beginning of this section. For both sets of simulations, when calculating the predicted colors, the effects of uncertainties of metallicity determinations and photometric measurements, as well as of errors of photometric calibration, are fully taken into account using MC simulations.

Considering the very small color deviations we are dealing with, essentially all sources of errors must be reliably determined and propagated in order for the SLOT method to work. By adjusting the relative fraction of stars in the two sets of simulations to fit the

easily detected in the blue $u-g$ color, while those consisting of a late-type primary are more easily detected using red colors. Note that the artifacts at the secondary $g-i$ colors of 1.6 mag are caused by the different sets of stellar loci used below and above the value. Note also that the deviations in $g-r$ and $r-i$ colors are exactly equal but opposite. Figure 3 shows that colors of binary systems composed of two MS stars are slightly redder by 0.0–0.15 mag than those of the consisting primary stars. The

combined colors of such binary systems are obviously dominated by their primary stars.

Given a sample of MS stars with accurate photometric colors and spectroscopically determined metallicities, the metallicity-dependent stellar loci of the sample are obtained as in Paper I. Two sets of Monte Carlo (MC) simulations are performed; one assumes that all stars in the sample are single, while the other assumes that all stars are binaries, in order to mimic the distributions of the observed sample in the color–color diagrams. For the first set of simulations, for a star of color $g-i$ and metallicity $[\text{Fe/H}]$, its $u-g$, $g-r$, and $i-z$ colors are computed as predicted by the metallicity-dependent stellar loci if $g-i \leq 1.6$ mag. For $g-i > 1.6$ mag, the stellar loci of Covey et al. (2007) are used. For the other set of simulations of binary stars, for a star of color $g-i$ and metallicity $[\text{Fe/H}]$, the mass of the primary is derived from the Dotter et al. (2008) isochrones (Figure 4) by linear interpolation. The mass of the secondary star is then generated by MC simulation for an assumed mass ratio distribution of binary stars. We assume that the mass ratio distribution follows a power law of index $\gamma = 0.3$ for mass ratios between 0.05 and 1 (Duchêne & Kraus 2013). If the mass of the secondary is smaller than $0.08 M_\odot$, its effects are then neglected. The $g-i$ color of the secondary is then derived from the same set of isochrones. The combined colors of the binary system are then calculated as described in the beginning of this section. For both sets of simulations, when calculating the predicted colors, the effects of uncertainties of metallicity determinations and photometric measurements, as well as of errors of photometric calibration, are fully taken into account using MC simulations. Considering the very small color deviations we are dealing with, essentially all sources of errors must be reliably determined and propagated in order for the SLOT method to work. By adjusting the relative fraction of stars in the two sets of simulations to fit the
residual distributions with respect to the metallicity-dependent stellar loci of the observed sample, the binary fraction of the sample is determined. A minimum $\chi^2$ technique is used in the fit. Here $\chi^2$ is defined as

$$\chi^2 = \sum_{i=1}^{M} \frac{(N_{\text{obs}}^i - N_{\text{mod}}^i)^2}{\sigma_i^2 (M - 2)},$$

$$\sigma_i = (N_{\text{obs}}^i + N_{\text{mod}}^i)^{1/2},$$

where $N_{\text{obs}}^i$ and $N_{\text{mod}}^i$ are the observed and predicted numbers of stars in the $i$th bin, respectively. $\sigma$ is the Poissonian error associated with the number of stars $N_{\text{obs}}^i$ and $N_{\text{mod}}^i$. We vary $f_b$ from 0.0 to 1.0 at steps of 0.01 and calculate the $\chi^2$ value for each $f_b$. The minimum $\chi^2$ value, $\chi_{\text{min}}$, is determined the associated best-fit value of $f_b$, and $\sigma$ uncertainty are then determined. The $\sigma$ uncertainty corresponds to the difference of $f_b$ values at $\chi_{\text{min}}^2$ and $\chi_{\text{min}}^2 + 1$ (Avni 1976; Wall 1996).

For small observed samples, simulations can be carried out multiple times to reduce the random errors of the simulations. For the single-star set of simulations, the resultant distributions in $g - i$ and [Fe/H] are essentially the same as those of the observed sample. For the second set of simulations for binaries, the resultant distributions in $g - i$ of the simulated sample may differ slightly with respect to those of the observed sample. To account for possible variations of the binary fraction as a function of stellar color, for each $g - i$ bin of 0.1 mag width, the number of stars in the second set of simulations is adjusted to match that of the observed sample by duplicating or removing some randomly selected targets.

The determination of binary fraction for an observed sample relies on the residual distributions in determining the metallicity-dependent stellar loci for the sample. On the other hand, fitting the metallicity-dependent stellar loci for the sample will no doubt be affected by the possible presence of binaries in the sample. Therefore, iterations are needed. An initial set of metallicity-dependent stellar loci are first derived by $\sigma$ clipping when fitting the data in order to reduce the effects of binary stars as in Paper I. With the initial set of loci, an estimate of the binary fraction is obtained. The effects of binary stars are then estimated by comparing the initial set of stellar loci with those given by the simulations for the binary fraction derived above. The differences are corrected for. Note that when combining the two sets of simulations for single and binary stars, the possible variations of the binary fraction in individual bins of stellar colors are considered. With the revised metallicity-dependent stellar loci and updated residual distributions that result, a new estimate of the binary fraction is obtained. The above process is repeated until a convergence is achieved. The corrections to stellar loci with respect to the initial set are very small, at a level of a few millimagitudes. Given that the residual distributions with respect to the adopted metallicity-dependent stellar loci for the simulated samples of single or binary stars are barely affected by the small revisions of the metallicity-dependent stellar loci used, their effects on the residual distributions of the simulated samples are ignored. The corrections only affect the residual distributions of the observed sample.

3. DATA AND ANALYSIS

3.1. SDSS

3.1.1. Data

The SLOT method requires a sample of MS stars with accurate photometric colors and well-determined metallicities. The repeatedly scanned equatorial Stripe 82 (decl. $|< 1^\circ$.266, 20°34′ < R.A. < 4°00′) in the SDSS has delivered very accurate photometry for about one million stars in u, g, r, i, z bands (Ivezic et al. 2007). The data have been further calibrated by Yuan et al. (2015a) using an innovative spectroscopy-based stellar color regression method, achieving an accuracy of about 0.005, 0.003, 0.002, and 0.002 mag in colors $u - g$, $g - r$, $r - i$, and $i - z$, respectively. In addition, over 40,000 stellar spectra in the region have been released in the SDSS Data Release 9 (DR9; Ahn et al. 2012), along with the basic stellar parameters (radial velocity, effective temperature, surface gravity, and metallicity) deduced with the Sloan Extension for Galactic Understanding and Exploration (SEGUE; Yanny et al. 2009) Stellar Parameter Pipeline (SSPP; Lee et al. 2008a, 2008b). By combining the spectroscopic information with the recalibrated photometry of Stripe 82, we have constructed a large, clean sample of MS stars with well-determined metallicities and accurate colors and determined the metallicity-dependent stellar loci in the SDSS colors (Paper I).

The sample of Paper I includes stars selected with a variety of specific criteria in the SEGUE program (Yanny et al. 2009). Many stars in the sample are selected from the SDSS color–color diagrams as candidates of, for example, AGB (asymptotic giant branch stars), KG (K giants), MP (metal-poor stars), MSWD (main-sequence white dwarf binaries), and QSO. The specific selection algorithms may thus cause over- or underselection of binary stars. To avoid such effects, targets selected based on the color–color diagrams are excluded. The sample of the current work is the same as in Paper I except that only stars selected as candidates of BHB (blue horizontal branch stars), FG (F or G dwarfs), GD (G dwarfs), HHV (halo high-velocity stars), HOT (hot standard stars), HVS (hypervelocity stars), LKG (low-latitude K giants), MKD (M or K dwarfs), PHO (photometric standard stars), and RED (reddening standard

\footnote{See https://www.sdss3.org/dr9/algorithms/segue_target_selection.php#SEGUEets1 for details.}
stars) are included. Note that owing to its possible dependence on spectral types and metallicities, the binary fraction yielded by a given sample of stars, depending on the distributions of the sample in color and metallicity, may not reflect the characteristics of the whole Milky Way stellar population.

In total, 14,650 stars are selected; most are candidates of BHB, GD, MKD, PHO, and RED. Their distribution in the \( g - i \) and \([\text{Fe}/\text{H}]\) plane is shown in Figure 5. Note that in this work all colors refer to the dereddened values. The reddening corrections are performed using the extinction coefficients of Yuan et al. (2015a), derived using the star pair technique (Yuan et al. 2013). The average spectral signal-to-noise ratios (S/Ns) of the sample stars plotted against the \( g \) and \([\text{Fe}/\text{H}]\) plane are shown in Figure 6. The photometric errors in \( u \) band increase rapidly with the \( u \) magnitudes. The errors are about 0.01 mag at \( u = 19.0 \) mag and 0.1 mag at \( u = 22 \) mag. The photometric errors in \( g, r, i \) bands are dominated by the calibration uncertainties and essentially constant, at the level of 0.006 \pm 0.001, 0.005 \pm 0.001, and 0.005 \pm 0.001 mag, respectively. The photometric errors in \( z \) band increase modestly with the \( z \) magnitudes, 0.01 mag at \( z = 18.2 \) mag and 0.02 mag at \( z = 19 \) mag. A test of error estimates by Ivezić et al. (2007) shows that the magnitude errors computed by the photometric pipeline are reliable for the \( g, r, i, \) and \( z \) bands. For the \( u \) band, the errors may have been slightly underestimated, by about 10%. As a consequence, we have multiplied the photometric errors in the \( u \) band by 1.1 in the current work.

As described in the previous section, the SLOT method also requires robust estimates of errors of \([\text{Fe}/\text{H}]\). To estimate the random errors of \([\text{Fe}/\text{H}]\) yielded by the SSPP pipeline, 13,270 duplicate observations of comparable spectral S/Ns of stars that fall in the parameter ranges of the current sample (\( 4300 \) K \( \leq T_{\text{eff}} \leq 7000 \) K, \( \log g \geq 3.5 \) dex, \( -2.0 \) dex \( \leq [\text{Fe}/\text{H}] \leq 0.0 \) dex) are selected from the SDSS DR9 samples. Given the relatively narrow range of effective temperature of the sample stars, the random errors of \([\text{Fe}/\text{H}]\) are fitted as a function of S/N and \([\text{Fe}/\text{H}]\) of the following form:

\[
\sigma_{\text{ran}}([\text{Fe}/\text{H}]) = a_0 + a_1 \times [\text{Fe}/\text{H}] + a_2 \times ([\text{Fe}/\text{H}]^2 + a_3 \times S/N + a_4 \times S/N \times [\text{Fe}/\text{H}] + a_5 \times S/N^2. \quad (4)
\]

Only stars with S/N between 10 and 50 are used in the fitting. The resultant fit coefficients \( a_0-a_5 \) are 0.17, -0.066, -0.0088, -0.0063, \( 7.0 \times 10^{-4} \), and \( 6.8 \times 10^{-5} \), respectively. When assigning random errors of \([\text{Fe}/\text{H}]\) for stars with S/Ns higher than 50, the values given by the above equation for an S/N of 50 are used. For S/N \( \leq 10 \), the random errors of \([\text{Fe}/\text{H}]\) are about 0.11, 0.16, and 0.19 dex for \([\text{Fe}/\text{H}] = 0, -1, \) and \( -2 \) dex, respectively. For S/N = 50, the errors are 0.02, 0.04, and 0.05 dex for \([\text{Fe}/\text{H}] = 0, -1, \) and \( -2 \) dex, respectively. For the systematic errors, we have simply adopted the relation

\[
\sigma_{\text{sys}}([\text{Fe}/\text{H}] = 0.03 - 0.05 \times [\text{Fe}/\text{H}], \quad (5)
\]

based on the analysis of the dispersions of \([\text{Fe}/\text{H}]\) of member stars of clusters (Lee et al. 2008b; Smolinski et al. 2011), where \([\text{Fe}/\text{H}]\) refers to the SSPP adopted values. Note that \([\text{Fe}/\text{H}]\) values yielded by the SSPP may be affected by the binarity, i.e., the presence of a secondary star. However, the effects are likely to be insignificant in most cases (Schlesinger et al. 2010). The maximum additional uncertainty caused by binary contamination at S/N = 10 is about 0.17 dex in \([\text{Fe}/\text{H}]\) (Schlesinger et al. 2010). The uncertainties of \([\text{Fe}/\text{H}]\) error estimates affect mainly the \( u - g \) color. Their effects on the \( g - r \) and \( r - i \) colors are negligible.

3.1.2. Results

Using the sample selected above, we have carried out a global two-dimensional polynomial fit to the \( u - g, g - r, r - i, \) and
$i-z$ colors as a function of color $g-i$ and metallicity $[\text{Fe/H}]$ to determine an initial set of stellar color loci. As in Paper I, a fourth-order polynomial with 15 free parameters is adopted for color $u-g$ and a third-order polynomial of 10 free parameters is used for the other three colors. Two-sigma clipping is performed during the fitting process. The resultant fit coefficients are listed in the upper part of Table 1. Figure 7 compares this initial set of stellar color loci obtained here with those of Paper I. The differences are less than 0.03 mag in $u-g$ and 0.01 mag in other colors.

In order to explore the binary fraction of field stars as a function of metallicity, the sample has been divided into four bins of metallicity: $-2.0 \text{dex} \leq [\text{Fe/H}] < -1.5 \text{dex}$, $-1.5 \text{dex} \leq [\text{Fe/H}] < -1.0 \text{dex}$, $-1.0 \text{dex} \leq [\text{Fe/H}] < -0.5 \text{dex}$, and $-0.5 \text{dex} \leq [\text{Fe/H}] \leq 0.0 \text{dex}$. The fit residuals in colors $u-g$, $g-r$, and $i-z$ as a function of $g-i$ for the entire SDSS sample and subsamples of different metallicity ranges are shown in Figure 8. The fit residuals of color $r-i$ are not shown because they degenerate with those of color $g-r$. As already pointed out in Paper I, there is an excess of stars whose colors, when compared with what is predicted by the fits, are redder in $u-g$ and $g-r$ but bluer in $i-z$. The asymmetries are most prominent for stars of red colors. The asymmetric residuals are fully consistent with the presence of binary stars in the sample in terms of the ranges and directions of the offsets as shown in Figure 2.

For the SDSS sample, two sets of MC simulations are performed: one assumes that all sample stars are single, while the other assumes that all sample stars are binaries. The detailed procedures have been described in the previous section. To reduce the random errors of the simulations, each set of simulations is carried out 10 times and then combined. Figure 9 shows the offsets of colors $u-g$, $g-r$, and $i-z$ relative to those predicted by the metallicity-dependent stellar loci as a function of $g-i$ for the simulated entire sample and subsamples of different metallicity ranges. Only in 10 randomly selected targets is shown. By adjusting the fractions of stars in the two sets of simulations to fit the observed residual distributions with respect to the metallicity-dependent stellar loci, the binary fractions of the entire SDSS sample and subsamples can be determined. In this process, the entire sample and subsamples of different metallicity bins are further divided into bins of $g-i$ color. The binary fractions of individual bins of color and metallicity are determined in order to investigate the possible dependence of binary fraction on spectral type and metallicity. As described in the previous section, several iterations are performed to obtain the final set of stellar loci devoid of contamination of binary stars in the observed sample and convergence of the estimate of the binary fraction. The final set of stellar loci derived from the SDSS sample are plotted in Figure 7. Compared with the initial ones, the differences in colors $u-g$, $g-r$, $r-i$, and $i-z$ are within the ranges of $-30$ to $25.4$ mag, $-0.5$ to $3.5$ mag, $-3.5$ to $0.0$ mag, and $-3.4$ to $-0.4$ mag, respectively, at $[\text{Fe/H}] = -0.5$ dex, and within the ranges of $-7.8$ to $21.6$ mmag, $0.0$ to $2.1$ mmag, $-2.1$ to $0.0$ mmag, and $-2.3$ to $-1.0$ mmag, respectively, at $[\text{Fe/H}] = -1.5$ dex. Considering the possible maximum offsets produced by the binary stars in the sample or caused by various potential

### Table 1

| Coeff. | $u-g$ | $g-r$ | $r-i$ |
|--------|-------|-------|-------|
| $a_0$  | 1.5862| 0.0548| -0.0548| -0.0806|
| $a_1$  | 0.2102| 0.0313| -0.0313| -0.0116|
| $a_2$  | 0.4032| 0.0208| -0.0208| -0.0059|
| $a_3$  | 0.2020| 0.0039| -0.0039| -0.0024|
| $a_4$  | 0.0356| 0.6244| 0.3756| 0.1780|
| $a_5$  | -3.5203| -0.3242| -0.0324| -0.0494|
| $a_6$  | 0.7480| -0.0005| 0.0005| -0.0170|
| $a_7$  | -0.1826| 0.1329| -0.1329| 0.0322|
| $a_8$  | -0.0711| -0.0158| 0.0158| 0.0092|
| $a_9$  | 8.3834| -0.0546| 0.0546| -0.0172|
| $a_{10}$ | -0.5626| |
| $a_{11}$ | -0.0070| |
| $a_{12}$ | -5.5546| |
| $a_{13}$ | 0.0606| |
| $a_{14}$ | 1.2053| |

### Notes.

$^a$ $f(x, y) = a_0 + a_1 x + a_2 y + a_3 x^2 + a_4 y^2 + a_5 x y + a_6 x^3 + a_7 y^3 + a_8 x^2 y + a_9 y^2 x + a_{10} x y^2 + a_{11} y x^2 + a_{12} x^2 y^2 + a_{13} x y^2 x + a_{14} y x^2 y + a_{15} x y^3 + a_{16} x^2 y^3$, where $x$ denotes color $g-i$ and $y$ denotes metallicity $[\text{Fe/H}]$.

$^b$ $f(x, y) = a_0 + a_1 x + a_2 y + a_3 x^2 + a_4 y^2 + a_5 x y + a_6 x^3 + a_7 y^3 + a_8 x^2 y + a_9 y^2 x + a_{10} x y^2 + a_{11} y x^2 + a_{12} x^2 y^2 + a_{13} x y^2 x + a_{14} y x^2 y + a_{15} x y^3 + a_{16} x^2 y^3$, where $x$ denotes color $g-i$ and $y$ denotes metallicity $[\text{Fe/H}]$. 


Figure 8. From top to bottom, fit residuals of colors $u - g$ (left), $g - r$ (middle), and $i - z$ (right) as a function of $g - i$ for the entire SDSS sample and subsamples of different metallicity ranges. The metallicity ranges are labeled on the top of each panel. The horizontal lines denote zero residuals.

Sources of error, in the current work the fit ranges of residuals for colors $u - g$, $g - r$, and $i - z$ in the current work are set at $-0.1$ to $0.2$ mag, $-0.03$ to $0.045$ mag, and $-0.05$ to $0.02$ mag, with bin sizes of $0.01$, $0.0025$, and $0.005$ mag, respectively.

Figures 10, 11, and 12 show the distributions of the observed residuals and of the best-fit models for the entire SDSS sample and subsamples of individual bins of $g - i$ color and metallicity, for colors $u - g$, $g - r$, and $r - i$, respectively. The ranges of color and metallicity of the bin, the resultant binary fraction and error, and the associated value of minimum $\chi^2$ are marked for each panel. The fits are reasonably good. The typical minimum $\chi^2$ values are in the range $0.8$–$3.0$, $0.8$–$2.0$, and $1.0$–$5.0$ for colors $u - g$, $g - r$, and $r - i$, respectively. The binary fractions derived from the data of individual colors are listed in Table 2 and plotted in Figure 13 as a function of color for the individual bins of metallicity. The weighted means by combining the results from all three colors are also listed and plotted, with weights determined by the corresponding uncertainties. Considering the smaller photometric errors of $g, r, i$ bands compared with those of $u$ and $z$ bands, the weaker dependence of color $g - r$ on metallicity compared with that of $u - g$, and the better residual fits in color $g - r$ compared with those in $u - g$ and $i - z$, we have increased the weights of results yielded by color $g - r$ by a factor of two.
The results from the individual colors agree well. For example, the binary fractions for the entire sample inferred from colors $u-g$, $g-r$, and $r-i$ are 40% ± 5%, 40% ± 5%, and 43% ± 4%, respectively, leading to a weighted mean of 42% ± 2%. The results are consistent with those determined for stars in the solar neighborhood (e.g., Raghavan et al. 2010). The current study also shows that the binary fraction of field FGK dwarfs decreases toward redder colors. The corresponding binary fractions are 44% ± 5%, 43% ± 5%, 35% ± 5%, and 28% ± 6% for stars with $g-i$ colors in the range 0.3–0.6 mag, 0.6–0.9 mag, 0.9–1.2 mag, and 1.2–1.6 mag, respectively. The trend is consistent with the finding of previous studies for FGK stars that blue, massive stars seem more often to have a companion than red, less massive ones (e.g., Eggleton & Tokovinin 2008; Raghavan et al. 2010; Gao et al. 2014). The trend is also consistent with the overall trend that the binary fraction decreases from the early-type OBA stars (e.g., Mason et al. 1998, 2009; Kobulnicky & Fryer 2007; Kouwenhoven et al. 2007) to the very late M dwarfs and brown dwarfs (e.g., Allen et al. 2007; Burgasser et al. 2003; Fischer & Marcy 1992; Henry & McCarthy 1990; Joergens 2008; Maxted et al. 2008; Siegler et al. 2005). This observed trend of increasing binary fraction with increasing primary mass is well reproduced by radiative hydrodynamical simulations (Bate 2014).

The binary fraction of field FGK stars is also found to increase with decreasing metallicity. For stars with [Fe/H] in the range
Figure 10. Distributions of the observed residuals (black lines) in $u-g$ color and the best-fit models (red solid lines) for the SDSS sample of individual bins of color and metallicity. The ranges of color and metallicity for each column and row are marked on the top and to the left of the figure, respectively. The two red dashed lines in each panel denote the contributions from the two simulated samples, of single and binary stars, respectively. The resultant binary faction, error, and corresponding value of minimum $\chi^2$ are labeled in each panel. The cyan and purple lines denote the fits plus and minus 1 $\sigma$ uncertainties. The vertical dashed lines represent zero residuals.

$-0.5$ to $0.0$ dex, $-1.0$ to $-0.5$ dex, $-1.5$ to $-1.0$ dex, and $-2.0$ to $-1.5$ dex, the inferred binary fractions are $37\% \pm 3\%$, $39\% \pm 3\%$, $50\% \pm 9\%$, and $53\% \pm 20\%$, respectively. The trend is likely stronger for red than blue stars. In spite of the very small fraction of metal-poor stars in the solar neighborhood, a number of studies have investigated the possible dependence of binary fractions on metallicity. Some studies find a lower binary fraction for subdwarfs when compared with corresponding MS stars (e.g., Riaz et al. 2008; Jao et al. 2009; Lodieu et al. 2009; Rastegaev 2010). However, some of those results are based on samples biased against binary stars. Yet some other studies find no obvious dependence of the binary fraction on metallicity (e.g., Latham et al. 2002; Chanamé & Gould 2004). Our results, based on a large stellar sample and an innovative, model-free method, agree with those found by Grether & Lineweaver (2007), Raghavan et al. (2010), and Gao et al. (2014) that metal-poor stars are more likely to have a companion than their metal-rich counterpart. If the trend is true, it may imply that lower-metallicity clouds are more liable to fragmentation and the formation of binary stars, as has been suggested by some numerical simulations (e.g., Machida et al. 2009). Alternatively, the initial binary fractions of stars formed under different metallicities are the same, but stars of different Galactic populations (e.g., the disk and halo populations) have evolved differently because of the different environments that eventually lead to the observed trend.

3.2. LAMOST

3.2.1. Data

There are 4580 stars in Stripe 82 targeted by the LAMOST and released in the LAMOST DR1 (Luo et al. 2012; Z. R. Bai et al., in preparation). Their basic stellar parameters have been determined with the LAMOST stellar parameter pipeline (LASP; Wu et al. 2014). The LAMOST Galactic surveys (Deng et al. 2012; Liu et al. 2014) target stars randomly selected on the color–magnitude diagrams (Carlin et al. 2012; Yuan et al. 2015c). Consequently, all those stars can be used for binary fraction determinations. By combining the LAMOST spectroscopic information and the recalibrated SDSS photometry of Stripe 82, we have constructed a sample of 3827 stars with well-determined metallicities and accurate colors from the LAMOST DR1. The selection criteria are similar to those used for the SDSS sample, except that we require a spectral S/N > 6.
Figure 11. Same as Figure 10, but for the $g - r$ color.

and $-1.0 \text{ dex} \leq [\text{Fe/H}] \leq 0.5 \text{ dex}$. Their distribution in the $g - i$ and [Fe/H] plane is shown in Figure 14. The spectral S/Ns of the sample stars plotted against the $r$-band magnitudes, as well as the photometric errors of the individual bands plotted against the magnitudes of the corresponding band, are shown in Figure 15. Owing to the relatively low spectral S/Ns of the LAMOST targets, the above S/N cut has led to the exclusion of a significant fraction of stars. This may introduce some bias in favor of binaries since they tend to be brighter compared with their counterpart of similar spectral type. Compared with the SDSS sample, the LAMOST sample contains more metal-rich stars from the Galactic disks.

To estimate the random errors of [Fe/H] yielded by the LASP pipeline, 11,685 duplicate observations of comparable spectral S/Ns of stars that fall in the parameter ranges of the current sample ($4300 \text{ K} \leq T_{\text{eff}} \leq 7000 \text{ K}, \log g \geq 3.5 \text{ dex}, -1.0 \text{ dex} \leq [\text{Fe/H}] \leq 0.5 \text{ dex}$) are selected from the LAMOST DR1. The random errors of [Fe/H] are fitted as a function of the S/N and [Fe/H] using Equation (4). Only stars with S/Ns lower than 50 are used in the fitting. The resultant fit coefficients $a_0 - a_5$ are $0.14, -0.063, 0.028, -0.0039, 0.0013,$ and $4.4 \times 10^{-5}$, respectively. When assigning the random errors of [Fe/H] for stars with S/Ns higher than 50, the values given by the above fit for an S/N of 50 are used. At S/N = 10, the random errors of [Fe/H] are about 0.10 and 0.18 dex for [Fe/H] = 0 and $-1 \text{ dex}$, respectively. At S/N = 50, the random errors are about 0.05 and 0.08 dex for [Fe/H] = 0 and $-1 \text{ dex}$, respectively. For systematic errors, we have assumed

$$\sigma_{\text{sys}}([\text{Fe/H}]) = 0.05 - 0.05 \times [\text{Fe/H}].$$

The values are assigned to be 0.05 dex when [Fe/H] $\geq 0.0$ dex.

3.2.2. Results

To account for possible systematic differences in [Fe/H] delivered by the SDSS DR9 and LAMOST DR1, we have derived the initial set of metallicity-dependent stellar loci separately for the LAMOST sample, using the same algorithm as for the SDSS sample. The resultant fit coefficients are listed in the upper part of Table 3. Figure 16 compares the initial sets of stellar loci obtained for the SDSS and LAMOST samples. The maximum differences are about 0.1 mag in $u - g$ and about 0.01 mag in other colors.

Similarly, two sets of MC simulations are performed for the LAMOST sample. Each set of simulations is carried out 10 times and then combined. When determining the binary fraction, the LAMOST sample has been divided into three bins of metallicity: $-1.0 \text{ dex} \leq [\text{Fe/H}] < -0.5 \text{ dex}, -0.5 \text{ dex} \leq [\text{Fe/H}] < 0.0 \text{ dex},$ and $0.0 \text{ dex} \leq [\text{Fe/H}] < 0.5 \text{ dex}$. For each bin of metallicity,
the sample is further divided into three bins of color \( g - i \): 0.3 mag \( \leq g - i < 0.6 \) mag, 0.6 mag \( \leq g - i < 0.9 \) mag, and 0.9 mag \( \leq g - i < 1.6 \) mag.

The final set of stellar loci for the LAMOST sample are plotted in Figure 16. Compared with the initial set, the differences in colors \( u - g \), \( g - r \), \( r - i \), and \( i - z \) are within the ranges of \(-0.1 \) to \(31.4\) mmag, \(-1.7\) to \(3.1\) mmag, \(-3.1\) to \(1.7\) mmag, and \(-3.2\) to \(-0.1\) mmag, respectively, for \([\text{Fe}/\text{H}] = 0.0\) dex, and within the ranges of \(-10.6\) to \(49.3\) mmag, \(-1.7\) to \(2.1\) mmag, \(-2.1\) to \(1.7\) mmag, and \(-3.8\) to \(-1.9\) mmag, respectively, for \([\text{Fe}/\text{H}] = -1.0\) dex.

Figures 17, 18, and 19 show the distributions of the observed residuals and of the best-fit models for the entire LAMOST sample and subsamples of individual bins of \( g - i \) color and metallicity, for colors \( u - g \), \( g - r \), and \( r - i \), respectively. The ranges of color and metallicity of the bin, the resultant binary fraction and error, and the associated value of minimum \( \chi^2 \) are marked. The fits are reasonably good. The typical minimum \( \chi^2 \) values are in the range \(1.2\) to \(3.0\), \(0.7\) to \(2.0\), and \(0.4\) to \(5.0\) in colors \( u - g \), \( g - r \), and \( r - i \), respectively. Similar to the SDSS sample, the fits in \( g - r \) color are the best. The binary fractions derived from the data of individual colors are listed in Table 4 and plotted in Figure 20 as a function of color for the individual bins of metallicity. The weighted means are also listed and plotted, using the same weighting method for the SDSS sample.

The results from the individual colors are consistent. The binary fractions of the LAMOST sample inferred from colors \( u - g \), \( g - r \), and \( r - i \) are \(36\% \pm 9\%\), \(36\% \pm 7\%\), and \(46\% \pm 7\%\), respectively, yielding a weighted mean of \(39\% \pm 4\%\).

Similar to the SDSS sample, the binary fraction of field FGK dwarfs is also found to decrease toward redder colors. The corresponding fractions are \(57\% \pm 12\%\), \(42\% \pm 6\%\), and \(24\% \pm 5\%\) for stars with \( g - i \) colors in the range \(0.3\) to \(0.6\) mag, \(0.6\) to \(0.9\) mag, and \(0.9\) to \(1.6\) mag, respectively. Similarly, the binary fraction for field FGK dwarfs is found to decrease with increasing metallicity. For stars with metallicities in the range \(-0.5\) to \(0.0\) dex, \(-0.9\) to \(-0.5\) dex, and \(-1.0\) to \(-0.5\) dex, the inferred binary fractions are \(29\% \pm 7\%\), \(36\% \pm 5\%\), and \(48\% \pm 11\%\), respectively.

Figure 21 compares the binary fractions deduced from the SDSS and LAMOST samples. The results are consistent within the uncertainties. Both samples show the same trend: the fraction of binaries is smaller for stars of redder colors and higher metallicities.
Earlier studies of the binary fraction are often limited to samples of the solar neighborhood, consisting mostly of metal-rich stars from the Galactic thin disk. It is thus difficult to use the results to explore the binary fractions for different stellar populations, which are very important for understanding the formation and evolution of binaries in different environments. The samples analyzed with the SLOT method in the current work probe a much larger and deeper volume than previous studies, from a few hundred parsecs to over 10 kpc, thus providing us a good opportunity to explore the fractions of binary stars in different stellar populations.
Table 3  
Fit Coefficients of the Stellar Color Loci for the LAMOST Sample

| Coeff. | \( u - g^\alpha \) | \( g - r^\beta \) | \( r - i^\gamma \) | \( i - z^\delta \) |
|--------|------------------|------------------|------------------|------------------|
| Initial set  |
| \( a_0 \) | 1.2509 | 0.0399 | -0.0399 | -0.0698 |
| \( a_1 \) | -0.4639 | -0.0137 | 0.0137 | 0.0038 |
| \( a_2 \) | -0.3676 | 0.0054 | -0.0054 | -0.0200 |
| \( a_3 \) | -0.2515 | 0.0044 | -0.0044 | -0.0164 |
| \( a_4 \) | -0.0599 | 0.6546 | 0.3454 | 0.1635 |
| \( a_5 \) | -2.1854 | 0.0858 | -0.0858 | -0.0364 |
| \( a_6 \) | 2.1414 | 0.0072 | -0.0072 | -0.0048 |
| \( a_7 \) | 6.2080 | -0.0411 | 0.0411 | -0.0303 |
| \( a_8 \) | -1.8420 | | | |
| \( a_9 \) | -0.3554 | | | |
| \( a_{10} \) | -4.1650 | | | |
| \( a_{11} \) | 0.4834 | | | |
| \( a_{12} \) | 0.9218 | | | |

| Final set  |
| \( a_0 \) | 1.2979 | 0.0314 | -0.0314 | -0.0675 |
| \( a_1 \) | -0.4330 | -0.0205 | 0.0205 | 0.0116 |
| \( a_2 \) | -0.3489 | 0.0022 | -0.0022 | -0.0144 |
| \( a_3 \) | -0.2457 | 0.0064 | -0.0064 | -0.0150 |
| \( a_4 \) | -0.0595 | 0.6832 | 0.3168 | 0.1635 |
| \( a_5 \) | -2.5446 | 0.0993 | -0.0993 | -0.0515 |
| \( a_6 \) | 2.0669 | 0.0140 | -0.0140 | -0.0126 |
| \( a_7 \) | 7.0989 | -0.0364 | 0.0364 | -0.0284 |
| \( a_8 \) | -1.7862 | | | |
| \( a_9 \) | -0.3094 | | | |
| \( a_{10} \) | -4.9581 | | | |
| \( a_{11} \) | 0.4808 | | | |
| \( a_{12} \) | 1.1548 | | | |

Notes.  
\[ f(x, y) = a_0 + a_1 y + a_2 x y^2 + a_3 y^3 + a_4 x y^4 + a_5 x^2 + \ldots \]  
where \( x \) denotes color \( g - i \) and \( y \) denotes metallicity \([\text{Fe/H}]\).  
\[ f(x, y) = a_0 + a_1 x y + a_2 x^2 y + a_3 x^2 y^2 + a_4 x^2 y^3 + a_5 x^3 + \ldots \]  
where \( x \) denotes color \( g - i \) and \( y \) denotes metallicity \([\text{Fe/H}]\).  

Figure 14.  
Distribution of the selected LAMOST DR1 stellar spectroscopic sample of Stripe 82 in the \( g - i \) and \([\text{Fe/H}]\) plane.  

Figure 15.  
Spectral S/Ns plotted against the \( r \)-band magnitudes for the selected LAMOST DR1 stellar spectroscopic sample of Stripe 82 (top left panel). The remaining panels show the photometric errors of the individual bands plotted against the magnitudes of the corresponding band.  

Figure 16.  
Initial (black) and final (red) sets of stellar color loci of \([\text{Fe/H}] = 0.0 \text{ dex} \) (solid lines) and \(-1.0 \text{ dex} \) (dashed lines), obtained from the LAMOST sample. For comparison, the initial sets of loci deduced from the SDSS sample (cyan) are also overplotted.  

We have divided the SDSS sample stars into four populations based on their locations in the \([\text{Fe/H}] - [\alpha/\text{Fe}]\) plane (Lee et al. 2011): thin-disk stars \(([\text{Fe/H}] > -0.6 \text{ dex}, [\alpha/\text{Fe}] \leq 0.2 \text{ dex})\), thick-disk stars \(([\text{Fe/H}] > -1.0 \text{ dex}, [\alpha/\text{Fe}] > 0.3 \text{ dex})\),
stars in the transition zone between the thin and thick disks ([Fe/H] > -0.6 dex, 0.2 dex < [\alpha/Fe] < 0.3 dex), and halo stars ([Fe/H] \leq -1.0 dex). Like [Fe/H], here the [\alpha/Fe] are those yielded by the SSPP pipeline. To ensure robust estimates of [\alpha/Fe], an additional spectral S/N cut of higher than 20 is imposed for the selection of stars except for halo stars. Since the binary fraction has been found to vary with color, to minimize the color effects, only stars with g - i color bluer than 0.9 mag are included. The resultant distributions of stars thus selected for the four populations in the g - i - [Fe/H] and [Fe/H] - [\alpha/Fe] planes are shown in Figure 22.

Using the same simulated SDSS samples as described in Section 3.1, the binary fractions for the four populations are determined. Figures 23, 24, and 25 show the distributions of the observed residuals and of the best-fit models for the four populations in individual bins of g - i color, for colors u - g, g - r, and r - i, respectively. The resultant distributions of stars thus selected for the four populations in the g - i - [Fe/H] and [Fe/H] - [\alpha/Fe] planes are shown in Figure 22.

Using the same simulated SDSS samples as described in Section 3.1, the binary fractions for the four populations are determined. Figures 23, 24, and 25 show the distributions of the observed residuals and of the best-fit models for the four populations in individual bins of g - i color, for colors u - g, g - r, and r - i, respectively. The resultant distributions of stars thus selected for the four populations in the g - i - [Fe/H] and [Fe/H] - [\alpha/Fe] planes are shown in Figure 22.

The binary fractions deduced for stars of populations of the Galactic thin and thick disks, of the transition zone between them, and of the halo are 39% ± 6%, 39% ± 5%, 35% ± 4%, and 55% ± 10%, respectively. The results suggest that the Galactic thin and thick disks have comparable binary fractions, whereas the Galactic halo contains a significantly larger binary fraction. The trend is the same for results obtained from the individual colors and from the individual color bin. The results indicate that halo stars are formed in an environment different from that of disk stars.

5. CONCLUSIONS AND DISCUSSIONS

In Paper I, by combining the spectroscopic information and recently recalibrated photometry of the SDSS Stripe 82, we have built a large, clean sample of MS stars with accurate colors and well-determined metallicities and demonstrated that the u - g, g - r, r - i, and i - z colors of MS stars are fully determined by their g - i colors and metallicities. In other words, the intrinsic widths of the metallicity-dependent stellar color loci are essentially zero. In this paper we have shown that MS binaries do not follow the metallicity-dependent stellar loci defined by single stars. Binaries can be distinguished from single stars by their small deviations from the loci. The deviations from the metallicity-dependent stellar loci of an observed sample of MS stars are contributed by (1) uncertainties in the color measurements and calibrations, (2) uncertainties in the metallicity
Figure 18. Same as Figure 10, but for the LAMOST sample and for residuals in $g - r$ color.

Table 4

|                  | 0.3 $\leq g - i < 1.6$ (mag) | 0.3 $\leq g - i < 0.6$ (mag) | 0.6 $\leq g - i < 0.9$ (mag) | 0.9 $\leq g - i < 1.6$ (mag) |
|------------------|-------------------------------|-------------------------------|-------------------------------|-------------------------------|
| Based on color $u - g$ |                               |                               |                               |                               |
| $-1.00 < [Fe/H] < -0.50$ dex | 0.42 ± 0.26                  | 0.64 ± 0.63                  | 0.55 ± 0.30                  | 0.13 ± 0.34                  |
| $-0.50 < [Fe/H] < 0.00$ dex | 0.34 ± 0.11                  | 0.52 ± 0.37                  | 0.33 ± 0.16                  | 0.22 ± 0.14                  |
| $-0.00 < [Fe/H] \leq 0.50$ dex | 0.32 ± 0.16                  | 0.25 ± 0.56                  | 0.31 ± 0.21                  | 0.22 ± 0.22                  |
| $-1.00 < [Fe/H] \leq 0.50$ dex | 0.36 ± 0.09                  | 0.59 ± 0.32                  | 0.38 ± 0.12                  | 0.23 ± 0.12                  |
| Based on color $g - r$ |                               |                               |                               |                               |
| $-1.00 < [Fe/H] < -0.50$ dex | 0.47 ± 0.19                  | 0.62 ± 0.38                  | 0.36 ± 0.31                  | 0.17 ± 0.21                  |
| $-0.50 < [Fe/H] < 0.00$ dex | 0.33 ± 0.09                  | 0.53 ± 0.28                  | 0.37 ± 0.16                  | 0.22 ± 0.11                  |
| $-0.00 < [Fe/H] \leq 0.50$ dex | 0.27 ± 0.11                  | 0.43 ± 0.56                  | 0.34 ± 0.19                  | 0.21 ± 0.16                  |
| $-1.00 < [Fe/H] \leq 0.50$ dex | 0.34 ± 0.07                  | 0.58 ± 0.23                  | 0.38 ± 0.12                  | 0.23 ± 0.09                  |
| Based on color $i - z$ |                               |                               |                               |                               |
| $-1.00 < [Fe/H] < -0.50$ dex | 0.52 ± 0.19                  | 0.55 ± 0.27                  | 0.51 ± 0.33                  | 0.41 ± 0.31                  |
| $-0.50 < [Fe/H] < 0.00$ dex | 0.46 ± 0.09                  | 0.59 ± 0.23                  | 0.59 ± 0.15                  | 0.28 ± 0.11                  |
| $-0.00 < [Fe/H] \leq 0.50$ dex | 0.34 ± 0.13                  | 0.15 ± 0.49                  | 0.40 ± 0.19                  | 0.17 ± 0.14                  |
| $-1.00 < [Fe/H] \leq 0.50$ dex | 0.46 ± 0.07                  | 0.55 ± 0.17                  | 0.53 ± 0.11                  | 0.28 ± 0.09                  |
| Combined          |                               |                               |                               |                               |
| $-1.00 < [Fe/H] < -0.50$ dex | 0.48 ± 0.11                  | 0.59 ± 0.22                  | 0.45 ± 0.16                  | 0.21 ± 0.14                  |
| $-0.50 < [Fe/H] < 0.00$ dex | 0.36 ± 0.05                  | 0.55 ± 0.15                  | 0.42 ± 0.08                  | 0.24 ± 0.06                  |
| $-0.00 < [Fe/H] \leq 0.50$ dex | 0.29 ± 0.07                  | 0.30 ± 0.27                  | 0.35 ± 0.10                  | 0.20 ± 0.09                  |
| $-1.00 < [Fe/H] \leq 0.50$ dex | 0.39 ± 0.04                  | 0.57 ± 0.12                  | 0.42 ± 0.06                  | 0.24 ± 0.05                  |
Figure 19. Same as Figure 10, but for the LAMOST sample and for residuals in $i - z$ color.

Figure 20. Binary fractions derived from the residuals in colors $u - g$ (top left), $g - r$ (top right), $i - z$ (bottom left), and the combined data of all three colors (bottom right) for field FGK stars of the LAMOST sample, plotted against $g - i$ color for the individual bins of color and metallicity. The typical error bars are marked in the top right corner of each panel.
The fractions are 44% ± 5%, 43% ± 3%, 35% ± 5%, and 28% ± 6%, for stars with $g - i$ colors in the range 0.1–0.6 mag, 0.6–0.9 mag, 0.9–1.2 mag, and 1.2–1.6 mag, respectively. The fraction is also found to increase with decreasing metallicity. The trend is likely stronger for stars of redder colors. For stars with [Fe/H] in the range $-0.5$ to $0.0$ dex, $-1.0$ to $-0.5$ dex, $-1.5$ to $-1.0$ dex, and $-2.0$ to $-1.5$ dex, the inferred binary fractions are $37\%$ ± $3\%$, $39\%$ ± $3\%$, $50\%$ ± $9\%$, and $53\%$ ± $20\%$, respectively. We have further divided the SDSS sample into stars from the Galactic thin and thick disks, from the transition zone between them, and from the halo by their locations in the [Fe/H]–[$\alpha$/Fe] plane. For the four populations, the binary fractions are $39\%$ ± $6\%$, $39\%$ ± $5\%$, $35\%$ ± $4\%$, and $55\%$ ± $10\%$, respectively, suggesting that thin- and thick-disk stars have similar binary fractions, whereas for halo stars, the fraction seems to be significantly higher. Applying the method to the LAMOST sample yields consistent results. The binary fraction of the entire LAMOST sample is $39\%$ ± $4\%$. The fractions for stars with $g - i$ colors in the range $0.3$–$0.6$ mag, $0.6$–$0.9$ mag, $0.9$–$1.2$ mag, and $1.2$–$1.6$ mag, respectively. For stars with [Fe/H] in the range $0.0$–$0.5$ dex, $-0.5$ to $0.0$ dex, and $-1.0$ to $-0.5$ dex, the inferred binary fractions are $29\%$ ± $7\%$, $36\%$ ± $5\%$, and $48\%$ ± $11\%$, respectively.

Given the inferred dependence of the binary fraction on stellar color and metallicity, the binary fraction inferred from samples of stars of a broad range of color and metallicity with the SLOT method, for example, the whole SDSS and LAMOST samples in the current work, will depend on the color and metallicity distributions of the sample stars. However, for (sub)samples of narrower color and metallicity ranges, for example, the SDSS and LAMOST subsamples of individual bins of color determinations, and (3) the presence of binaries. By modeling the observed deviations from the metallicity-dependent stellar loci, we propose the SLOT method to determine the binary fraction of MS stars statistically. The method is sensitive to neither the period nor mass ratio distributions of binaries and is applicable to large survey volumes. With accurate colors measured by modern photometric surveys and robust estimates of metallicities and surface gravities delivered by large-scale spectroscopic surveys, the SLOT method provides a promising tool to obtain model-free estimates of binary fractions for large numbers of stars of different populations.

![Figure 21](image1.png)

**Figure 21.** Binary fractions deduced from the SDSS (filled circles) and from the LAMOST (filled stars) samples plotted against $g - i$ color for the individual bins of color and metallicity. The typical error bars are marked.

![Figure 22](image2.png)

**Figure 22.** Distributions of stars of different stellar populations (black: thin disk; cyan: transition disk; red: thick disk; blue: halo) of the SDSS sample in the [Fe/H]–$g - i$ (top panel) and [$\alpha$/Fe]–[Fe/H] (bottom panel) planes.

| $0.3 \leq g - i < 0.9$ | $0.3 \leq g - i < 0.6$ | $0.6 \leq g - i < 0.9$ |
|-----------------------|-----------------------|-----------------------|
| Thin disk$^a$ | 0.36 ± 0.10 | 0.33 ± 0.18 | 0.46 ± 0.12 |
| Tran. disk$^b$ | 0.39 ± 0.10 | 0.34 ± 0.18 | 0.45 ± 0.11 |
| Thick disk$^c$ | 0.24 ± 0.07 | 0.29 ± 0.15 | 0.25 ± 0.08 |
| Halo | 0.52 ± 0.17 | 0.54 ± 0.18 | 0.57 ± 0.22 |

**Table 5.** Binary Fractions of Field FGK Stars in Galactic Disks and Halos

**Notes.**

$^a$ S/N ≥ 20, [Fe/H] > −0.6 dex, [$\alpha$/Fe] ≤ 0.2 dex.

$^b$ S/N ≥ 20, [Fe/H] > −0.6 dex, 0.2 dex < [$\alpha$/Fe] ≤ 0.3 dex.

$^c$ S/N ≥ 20, [Fe/H] > −1.0 dex, [$\alpha$/Fe] > 0.3 dex.

$^d$ S/N ≥ 10, [Fe/H] ≤ −1.0 dex.
Figure 23. Same as Figure 10, but for the SDSS sample of different stellar populations in $u-g$ color.

and metallicity in the current work, the derived binary fractions are barely affected by the color and metallicity distributions of the sample stars. The binary fractions derived depend on, but are insensitive to, the assumed mass ratio distribution of binary stars. In this work, we have adopted a power-law mass ratio distribution of index $\gamma = 0.3$. Tests with a completely different index of $\gamma = 1$ show that the inferred fractions decrease only by a few percent, whereas the trends of variations of the fraction with stellar color and metallicity remain unchanged. Although the SLOT method suffers from little bias, the samples could be biased. The samples used in this work include only stars with spectral S/N higher than a specific value; thus, they may introduce a Malmquist bias that favors brighter binaries. Given the good S/Ns and the wide range of magnitude of the SDSS sample stars, the effects of Malmquist bias for the SDSS sample are likely to be small. For the LAMOST sample, the effects could be somewhat larger.

In the current work, we have neglected binary stars composed of a normal star and a compact object, such as a white dwarf, a neutron star, or a black hole. However, the fraction of such systems should be negligible compared with binaries consisting of two normal stars. In the current work, we have also neglected wide binaries that are spatially resolved in the SDSS images. They would be missed by the method. However, given that the median distances of the SDSS and LAMOST sample stars are, respectively, about 2.0 and 1.0 kpc and that 90% of the targets are at least 1.0 and 0.6 kpc away, the numbers of such systems that have been left out are likely to be very small (e.g., Sesar et al. 2008). Superpositions of two unassociated stars by chance are also neglected, considering the high Galactic latitudes of Stripe 82. Systems consisting of triple or more stars are also neglected, which shall be regarded as binaries as the effects of the third- and higher-order companions on the combined colors are negligible.

The SLOT method proposed in this work is limited to stars spectroscopically targeted owing to the requirement of robust estimates of metallicity and surface gravity. Given the strong dependence of $u-g$ color on metallicity, it is worth exploring the dependence of colors $g-r$ and $i-z$ as a function of $g-i$ and $u-g$ in the future. With sophisticated modeling, it is possible to extend the method to estimate binary fractions for a complete sample of stars using photometric data alone.

The SLOT method is only applied to Stripe 82 in the current work, as Stripe 82 has extremely well calibrated photometry with colors accurate to a few millimagnitudes. Applying the method to all stars spectroscopically targeted by the SDSS and LAMOST surveys, with photometry from the SDSS, is also possible, once the SDSS colors have been further recalibrated using the stellar color regression method (Yuan et al. 2015a).

The SLOT method can be applied to star clusters using accurate photometric data alone, as cluster member stars presumably have the same metallicity. By analyzing the offsets in colors and
in brightness, one can constrain both the binary fraction and mass ratio distribution of cluster member stars. If accurate distances to individual field stars are available, for example, from the Gaia satellite (Perryman et al. 2001), the offsets in colors and brightness can also be used to constrain the binary fraction and mass ratio distribution for field stars.

The binary fraction for field stars can also be estimated by variations in stellar radial velocities from multiepoch spectroscopy (Gao et al. 2014). Thanks to the ongoing LAMOST surveys, multiepoch spectroscopic data have increased significantly. A significant fraction of stars targeted by the LAMOST has been observed twice or multiple times. By combining the offsets in colors and the variations in radial velocity, one can constrain both the binary fraction and orbital period distribution very well. In addition, a color offset–velocity variation diagram will provide a simple classification scheme to distinguish MS binaries, compact binaries with a luminous (hot) companion, and those with a faint companion, as shall be introduced in another paper of this series (H. Yuan et al. 2015, in preparation).

By searching for MS stars that exhibit large variations in radial velocity but zero or unusual color offsets, one may have the possibility of finding a large number of stellar mass black hole and neutron star binary systems in the Galaxy.

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Figure 25. Same as Figure 10, but for the SDSS sample of different stellar populations in $i-z$ color.

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