Probing a gravitational cat state: Experimental Possibilities

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Abstract. This is a progress report on a preliminary feasibility study of experimental setups for preparing and probing a gravitational cat state [1].

1. Introduction
This is a follow-up to the theoretical studies of [1], exploring the best suited schemes for preparing and probing a gravitational cat state, within the currently available experimental proposals. In the nature of a progress report, we aim to share our thoughts for further discussions, leaving plenty of room for improvements and collective wisdom.

2. Experimental proposals
One of us (MD) considered several possible ways of experimentally implementing the setup suggested by Anastopoulos and Hu in [1], that is, the cat state gravitationally interacting with a classical or quantum probe (Figure 1). Let us consider these sequentially.

3. Preparing a GravCat state
The most promising experimental proposal for the preparation of a gravitational cat state appears to be Romero-Isart et al.’s superconducting microsphere trapped in a harmonic potential created by a magnetic quadrupole field [2]. They suggest that this trapping method should make it possible to isolate a lead (Pb) microsphere of mass $M \sim 10^{14}$ amu and radius $R = 2$ microns from various sources of environmental decoherence, to a degree sufficient that it should be possible to place the microsphere in a coherent superposition of two position eigenstates. The
Figure 1. Force on a probe/detector exerted by a massive particle in a gravitational cat state, $c_+|+ > + c_-|->$, between locations $\pm L/2$.

The protocol they propose for creating this microsphere 'cat state' is parametric coupling to a qubit state.

How do Romero-Isart et al. obtain $M \approx 10^{14}$ amu and $R = 2$ microns? They suppose that the (superconducting) microsphere is trapped in a 3-D harmonic oscillator potential of the form (Figures 2-b and 2-c)

$$V_{\text{trap}} = \frac{M}{2} \left[ \omega^2 x^2 + \omega^2 (y^2 + z^2) \right],$$

where the trapping frequency

$$\omega_t \approx (1.05) \left( \frac{\sqrt{\mu_0/\rho}}{l^2} \right) I,$$

and $\omega_\perp = \frac{\omega_t}{\sqrt{2}}$, because the potential is created by a quadrupole magnetic field that traps the microsphere via the Meissner effect.

Here $M$ is the mass of the microsphere and $\rho$ is its mass density (assuming a homogeneous microsphere). The parameter $l$ is the radius of and separation between the anti-Helmholtz coils surrounding the microsphere in the setup illustrated in Figure 2-a, and $I$ is the current through the coils. In order for the microsphere to be trapped, it is required that the magnetic field at any point of the sphere be smaller than the critical field, $B_{\text{crit}}$, in order to allow superconductivity.
Figure 2. Microsphere trap setup in [2]: (a) Scheme of anti-Hemholtz coils (AHC) with a pickup coil. (b) Quadrupole field created by AHC. (c) Field expelled by microsphere in Meissner state.

Figure 3. Scheme for parametric coupling of microsphere to qubit in [2]: (a) The state $|+\rangle = |0\rangle$ prepped at $t = 0$ and recovered at $t = 2t^*$. (b) The superposition state Eq. (4) created at $t = t^*$.

This yields an upper bound on the radius of the sphere as

$$R < R_{\text{max}} \simeq 0.98 \frac{B_{\text{crit}}}{\omega_1 \sqrt{\mu_0 \rho}}$$

(3)

They also require that $R \gg \lambda, \xi$, where $\lambda$ is the sphere’s penetration depth and $\xi$ is its coherence length (at temperature $T = 0$). For Figure 2-a, they assume $l \gg R$, although this does not appear to be a mandatory condition. To determine the size of the sphere, they choose the parameters $\rho = 11.360 \frac{kg}{m^3}, \lambda = 30.5nm, \xi = 96nm$ (at $T = 0$), $B_{\text{crit}} = 0.08T$, $l = 25\mu m$, $I = 10A$ (assuming materials with a critical current density of $J_{\text{crit}} = 7 \times 10^{11} \frac{A}{m^2}$), $\omega_1 \simeq 2\pi \times 28kHz$, and $R_{\text{max}} = 3.7\mu m$. Thus they consider a microsphere of $R = 2\mu m$, which implies a microsphere mass of $M = \rho \frac{4}{3} \pi R^3 \simeq 10^{14} amu \simeq 10^{-13} kg$.

In a detailed decoherence analysis, they argue that trapping a microsphere this way makes
negligible common sources of environmental decoherence (clamping losses, scattering of photons, damping created by background gas, blackbody radiation, and internal vibrational modes). They also characterize other sources of decoherence specific to their setup (damping due to hysteresis losses in the superconducting anti-Helmholtz coils, fluctuations in the trap frequency, and fluctuations in the trap center) and argue that they do not undermine their proposal.

Then, they show that parametric coupling to a qubit state puts the microsphere in a spatial superposition (Figure 3) described by the wave function

\[ |\Psi_s> = \frac{1}{\sqrt{2}} \left[ \hat{T}(-2\chi)|\uparrow, 0> + \hat{T}(2\chi)|\downarrow, 0> \right], \]  

(4)
after a time \( t^* = \pi/\omega_t \), where \( \chi \) is a dimensionless parameter that characterizes the parametric coupling, and \( \hat{T}(\ldots) \) is the usual translation operator. The two superposed wave packets are separated by a distance of \( l_s = 4\chi x_{zp} \), where \( x_{zp} \) is the zero-point motion of the microsphere in the trap, with overlap given by \( <0|\hat{T}(-2\chi)\hat{T}(2\chi)|0> = \exp[-l_s^2/(8\alpha^2)] \) and the requirement that \( 8\alpha^2 < l_s^2 \). This allows for an \( l_s \) of the order of 100 nm or more [3].

4. Probing a GravCat state

One of us (MD) has examined the leading state-of-the-art proposals in the past five years for ultrasensitive force measurement. A survey of which is contained in the Appendix.

We separate the consideration of a classical versus a quantum probe.

4.1. Classical Probe

For the role of the classical probe, the most promising experimental proposal appears to be Reinhardt et al.'s trampoline resonator [4], with effective mass 4.5 ng, diameter 100 microns (Figure 4-i), and projected force sensitivity of \( \sim 14 \text{ zN} \) at cryogenic temperatures (14 mK).

Assuming an experimental configuration between the resonator and the microsphere like that in Figure 1, and using Eq. (62) of [1] to calculate the resonator-microsphere Newtonian gravitational force we have:

\[ f = \frac{GM_{res}M_{sphere}L}{2(y^2 + L^2/4)^{3/2}} = \frac{(6.67 \times 10^{-11} \text{ units})(4.5 \times 10^{-12} \text{ kg})(3.8 \times 10^{-13} \text{ kg})}{[(10^{-6} \text{ m})^2 + (5 \times 10^{-7} \text{ m})^2]^{3/2}} (5 \times 10^{-7} \text{ m}) = 4.1 \times 10^{-23} \text{ N}, \]  

(5)

which is only around 3 orders of magnitude away from the projected force sensitivity range of the resonator. The specific values of \( L \) and \( y \) used in Eq. (5) were imported from Bahrami et al. [5], who proposed a cat state experiment very similar to that suggested by Anastopolous and Hu. Bahrami et al. indicated that for micromechanical systems (which the microsphere would be), reasonable values are those used in Eq. (5).

Two ways we might be able to boost the resonator-microsphere gravitational interaction to cover this three-orders-of-magnitude gap are:

(1) We might be able to change one or more of the non-mass parameters in the force calculation (within plausibility). For example, rather than a resonator-microsphere spatial separation \( y \) of one micron, we might use half a micron. This already increases the gravitational force to \( 1.6 \times 10^{-22} \text{ N} \), which is only 88 times smaller than 14 zN.

(2) We can try increasing the mass of the microsphere, consistent with the constraints of the Romero-Iar et al. setup.

Recall that a necessary condition for the microsphere to be trapped is that the magnetic field at any point on the sphere be smaller than the critical field, \( B_{crit} \), in order to keep the Meissner
state. This condition limits the size of the microsphere, as can be seen from the interaction Hamiltonian between the microsphere and the magnetic field gradient \[3\]

\[
H_{int} = -\frac{1}{2} \mathbf{m} \cdot \mathbf{B} = \frac{3V}{4\mu_0} |\mathbf{B}|^2 = \frac{\rho V}{2} \omega_t^2 x^2,\tag{6}
\]

which implies that the trapping frequency is given by

\[
\omega_t = b \sqrt{\frac{3}{2\rho\mu_0}} \sim 2\pi \times \sqrt{2b[T/m]} Hz,\tag{7}
\]

where \(b\) is the magnetic field gradient, and \(|\mathbf{B}| = B = bx\). Now, if we assume that the center of mass of the sphere is at the position of \(B = 0\), the field at the surface will just be \(bR\). This implies that the maximum field gradient that can be used is

\[
b_{\text{max}} = \frac{B_{\text{crit}}}{R},\tag{8}
\]

and a maximum trapping frequency of

\[
\omega_{max} = b_{\text{max}} \sqrt{\frac{3}{2\rho\mu_0}}.\tag{9}
\]
Now, in order to cool the center of mass to the ground state, we need trap frequencies of at least tens of kHz, or, equivalently, a field gradient of $10^4 \frac{T}{m}$. For a Pb microsphere, we have $B_{T_c}^{\text{crit}} = 0.08T$, $\rho_{Pb} = 11,360 \frac{kg}{m^3}$, and $T_{T_c}^{\text{crit}} = 7.2K$. For $R_{\text{max}}^{\text{Pb}}[\mu m] = 3.7$, these values give

$$b_{\text{max}}^{Pb} = \frac{B_{T_c}^{\text{crit}}}{R_{\text{max}}^{\text{Pb}}} = \frac{8}{R_{\text{max}}^{\text{Pb}}[\mu m]} \times 10^4\frac{T}{m} = 2.16 \times 10^4\frac{T}{m},$$

(10)

and

$$\omega_{\text{max}}^{Pb} \sim 2\pi \times \frac{131}{R_{\text{max}}^{\text{Pb}}[\mu m]} kHz = 2\pi \times 35.4 kHz,$$

(11)

as required. This is why the Pb microsphere size is bounded to a few microns.

What about other superconducting elements? To investigate this, one of us (MD) examined the transition temperatures and critical fields of all known superconducting elements [6, 7]. He found that the most promising element for increasing the microsphere size and mass, consistent with the above constraints, is Technetium (Tc), a Type-II superconductor. A Tc microsphere would have $B_{T_c}^{\text{crit}} = 0.141T$, $\rho_{Tc} = 11,000 \frac{kg}{m^3}$, and $T_{T_c}^{\text{crit}} = 7.8K$. If in Eqs. (10)-(11) we use $B_{T_c}^{\text{crit}}$ and $R_{\text{max}}^{Tc}[\mu m] = 10$, we get

$$b_{\text{max}}^{Tc} = \frac{B_{T_c}^{\text{crit}}}{R_{\text{max}}^{Tc}} = \frac{14.1}{R_{\text{max}}^{Tc}[\mu m]} \times 10^4\frac{T}{m} = 1.41 \times 10^4\frac{T}{m},$$

(12)

and

$$\omega_{\text{max}}^{Tc} \sim 2\pi \times \frac{234}{R_{\text{max}}^{Tc}[\mu m]} kHz = 2\pi \times 23.4 kHz,$$

(13)

as required. This allows a Tc microsphere of mass

$$M_{\text{max}}^{Tc} = \rho_{Tc} \frac{4}{3} \pi (R_{\text{max}}^{Tc})^3 = 4.6 \times 10^{-11} kg = 2.77 \times 10^{16} amu.$$

(14)

Even if we use a sphere radius slightly less than $R_{\text{max}}^{Tc}$, for example $R_{\text{max}}^{Tc}[\mu m] = 8$, we would still obtain

$$M_{\text{max}}^{Tc} = \rho_{Tc} \frac{4}{3} \pi (R_{\text{max}}^{Tc})^3 = 2.36 \times 10^{-11} kg = 1.42 \times 10^{16} amu.$$

(15)

So Tc seems to afford us a microsphere mass two orders of magnitude greater than Pb. Using a Tc microsphere of $R_{\text{max}}^{Tc} = 10$ microns in Eq. (5), we then obtain

$$f_{01} = \frac{GM_{\text{res}}^{Tc}M_{\text{sphere}}^{Tc}}{2(y^2 + \frac{L^2}{4})^{3/2}} = \frac{(6.67 \times 10^{-11} units)(4.5 \times 10^{-12} kg)(4.6 \times 10^{-11} kg)}{(5 \times 10^{-7} m)^2 + (5 \times 10^{-7} m)^2} = 4.94 \times 10^{-21} N,$$

(16)

which is very close to the 14 zN projected force sensitivity of the resonator. Furthermore, if we allow $y = 5 \times 10^{-7} m$, we get

$$f_{02} = \frac{GM_{\text{res}}^{Tc}M_{\text{sphere}}^{Tc}}{2(y^2 + \frac{L^2}{4})^{3/2}} = \frac{(6.67 \times 10^{-11} units)(4.5 \times 10^{-12} kg)(4.6 \times 10^{-11} kg)}{(5 \times 10^{-7} m)^2 + (5 \times 10^{-7} m)^2} = 1.95 \times 10^{-20} N,$$

(17)

which exceeds 14 zN.

It therefore appears that superconducting microspheres made of Tc are what’s needed to make our proposal feasible. However, since Tc is a Type-II superconductor, this complicates the theoretical feasibility of Romero-Isart et al.’s scheme for fashioning coherent microsphere cat states. Also, increasing the microsphere mass makes fluctuations in the trap center more appreciable as a source of decoherence. These issues are currently under study.
4.2. Quantum Probe
For the role of a quantum probe, Anastopoulos and Hu [1] propose to use a von-Neumann probe, i.e., a quantum harmonic oscillator coupled to a macroscopic detector. To model this, they specialize the Hamiltonian description of the quantum harmonic oscillator to a single-mode Jaynes-Cummings model and consider the deep strong-coupling limit. By treating the oscillator in the ground state and entangling it with the two-state spatial superposition that characterizes the gravitational cat state, they find that the reduced density matrix for the oscillator degrees of freedom has distinguishable off-diagonal terms only when

$$\omega_\text{osc}^3 \ll \left( \frac{f_0}{M_\text{osc}} \right)^2 M_\text{osc}. \quad (18)$$

Hence, a position measurement of the oscillator by a macroscopic detector will collapse the wave function of the joint system and find the oscillator in a neighborhood of phase space associated with the cat state collapsing to either $-L/2$ or $L/2$, with Born-rule relative frequencies. They also find that a continuous position measurement will lead to Rabi-type oscillations between the two neighborhoods of phase space, with transition probability $p(t) = \sin^2 \nu t$, where $\nu$ is the transition frequency.

In considering possible experimental implementations of this von-Neumann probe, one of us (MD) found that the most promising candidate seems to be the state-of-the-art optomechanical harmonic oscillator described in [8, 5]. Such an oscillator has a mass of 100 ng, and would experience a Newtonian gravitational force of $f_{01} = 9.07 \times 10^{-22}$ Newtons from a Pb microsphere cat state, using Eq. (5). And for a Tc microsphere, it would experience $f_{02} = 1.10 \times 10^{-19}$ Newtons. However, for a Tc microsphere, Eq. (18) gives

$$2.4 \times 10^{23} \frac{\text{rad}^3}{\text{sec}^3} \gg 1.21 \times 10^{-28} \frac{N^2}{kg}, \quad (19)$$

while for a Pb microsphere, we get

$$2.4 \times 10^{23} \frac{\text{rad}^3}{\text{sec}^3} \gg 8.23 \times 10^{-33} \frac{N^2}{kg}. \quad (20)$$

This means that, even with state-of-the-art optomechanical oscillators, we are still a long way off from experimentally realizing the quantum probe of a gravitational cat state.

There are many additional theoretical issues related to both the gravitational cat state and the probes, e.g., 1) how intact a gravcat state could remain, how long it can exist, in the presence of massive objects (such as the Earth); 2) why the gravitational force interaction with a classical probe should necessarily collapse the cat state wave function (in contrast to other massive bodies in nature, such as the Earth); 3) whether the collapse postulate is actually compatible with the hypothesis of a semiclassical gravitational source; 4) in the experimental schemes discussed above, the impact of motional decoherence and decoherence by vortices in Type II superconductors, to name just two. They are currently under investigation.

Appendix: Experimental schemes for classical probes of a GravCat State
One of us (MD) examined all the state-of-the-art proposals in the past five years for ultra-sensitive force measurements. He then identified a handful of proposals that seemed initially promising for our project, and did some back-of-the-envelope estimates on each. Among these, he was able to identify only one ultra-sensitive force measurement scheme that can play the role of a classical probe of sufficiently large mass and sufficiently high force-measurement sensitivity that, in combination with Romero-Isart et al.’s superconducting microsphere cat state proposal [2], might yield a feasible experimental scheme for detecting the gravitational field of a cat state.

Those handful of proposals were:
(i) Schreppler et al.’s scheme involving an ultra-cold atom cloud in a high finesse cavity [9] which, to date, produced the smallest externally applied force measured of 42 yN (yoctonewtons). 

(ii) Moser et al.’s scheme involving carbon nanotube mechanical resonators with quality factors greater than a million [10, 11], which yields force measurements on the zN (zeptonewton) scale. 

(iii) Tao et al.’s scheme using single-crystal diamond nanomechanical resonators with quality factors exceeding one million [12], which produces force sensitivities of a few hundred zN. 

(iv) Ranjit et al.’s scheme involving laser-cooled silica microspheres as force sensors in a dual beam optical dipole trap in high vacuum [13], which yields force measurement sensitivity at the aN (attonewton) scale. 

(v) Kleckner et al.’s [14] and Reinhardt et al.’s [4] schemes involving the use of optomechanical trampoline resonators, which yield projected maximum force sensitivities on the aN and zN scales, respectively. 

(vi) Wagner et al.’s use of state-of-the-art torsion balance pendulums to test for WEP violations with a precision of one part in $10^{13}$ [15].

Of all these, only proposal 5 seems promising for our purpose. To justify that, it will first be explained why the other proposals were deemed not viable. 

Proposal 1 yields the greatest force sensitivity, but the atom cloud used has a miniscule mass of only $1.8 \times 10^{-22}$ kg. Using the center of mass of this atom cloud in place of the resonator in Eq. (5) (keeping all other parameters the same), one obtains a gravitational force of only $10^{-34}$ N, which is eleven orders of magnitude smaller than the force sensitivity of Schreppler et al.’s scheme. In addition, the scheme requires that the measured force be an externally applied force that oscillates at the natural frequency of the center of mass motion of the atom cloud in the cavity ($\sim 12$ kHz). It is unclear how this could be done with the gravitational force from a cat state, even if it were somehow possible to fashion an atom cloud with a center of mass of, let’s say, 4.5 ng (which would yield a force magnitude that falls just within the sensitivity of Schreppler et al.’s scheme). 

Proposal 2 has a similar obstacle in that the mass of these carbon nanotube mechanical resonators is only $10^{-20}$ kg, which when used in place of the resonator in Eq. (5) yields a gravitational force of only $10^{-32}$ N, or eleven orders of magnitude smaller than the force sensitivity of the Moser et al. scheme. 

Proposal 3 used several different kinds of single-crystal diamond nanomechanical resonators, the largest of which has a mass of $10^{-12}$ kg (the mass isn’t given in the paper, but the estimated volume is given, and this was used in combination with the estimated mass density of single-crystal diamond given in [16]). When the largest resonator is used in place of the resonator in Eq. (5), it yields a gravitational force of $10^{-24}$ N. Since this resonator has a force sensitivity of only 540 zN, the expected gravitational force is around five orders of magnitude smaller. 

Proposal 4 uses silica microspheres of 3 micron diameter, with an estimated mass of at most $10^{-12}$ kg (the mass isn’t given in the paper, but Barker [17] uses 10 micron diameter silica microspheres and gives this mass value). Thus these microspheres yield a gravitational force six to seven orders of magnitude smaller than the force sensitivity of Ranjit et al.’s scheme. 

Proposal 6 is, of course, a very different force measurement scheme than all the others in that the goal of a torsion balance is to detect a difference in the directions of the external force vectors applied to the test bodies, rather than the absolute magnitudes of the forces. The scheme described in Wagner et al. uses test bodies of masses 10 g each and has a force sensitivity of one part in $10^{13}$ (for the Eötvös parameter). If we supposed a gravitational force interaction between one of these test bodies and the Romero-Isart et al. microsphere cat state, keeping all the other
parameters the same in Eq. (5), the resulting force, \( \sim 10^{-14} \text{ N} \), yields a differential acceleration of \( \sim 10^{-12} \text{ m/s}^2 \) for one test body, which would be within this sensitivity range (the smallest differential acceleration their torsion balance was able to detect was \( \sim 10^{-15} \text{ m/s}^2 \)). However, for such macroscopic test bodies, it seems highly implausible that experimentalists could keep the other parameters the same; in particular, it seems doubtful that the spatial separation between a test body and the microsphere can be only one micron in practice; the experimental setup in Romero-Isart et al. [2] (Figure 2-a) involves surrounding the microsphere by an anti-Helmholtz coil configuration only 25 microns in width. Much more experimentally feasible, it seems, is a separation on the order of a centimeter or a millimeter. Using a one millimeter separation, the resulting gravitational force (\( \sim 10^{-23} \text{ N} \)) and differential acceleration (\( \sim 10^{-21} \text{ m/s}^2 \)) for a test body is significantly below the maximum sensitivity of the torsion balance.

Why then does proposal 5 seem promising? First, the largest trampoline resonator used in the Kleckner et al. proposal has a mass of 110 ng, diameter of 80 microns (see Figure 2 of [14]), and a projected force sensitivity on the aN scale at cryogenic temperatures. To be more precise about this last feature, Kleckner et al. write:

Trampoline resonators are also suitable for use as ultra-high resolution force sensors. Assuming the quality factor increase is also seen for the lowest frequency devices [i.e., the resonator with \( m = 110 \text{ ng} \)], it should be possible to obtain a thermal force noise in the aN/Hz regime at demonstrated [cryogenic] temperatures. This is comparable to or better than the single crystal Si resonators currently used in magnetic resonance force microscopy (MRFM) experiments [36, 37]. Furthermore, the rear side optical access can be used to provide extremely precise position sensitivity while leaving the front side free for surface modifications required for use as sensors. (Last page)

Now, the gravitational force between the 110 ng trampoline resonator and the microsphere, keeping all other parameters the same in Eq. (5), is equal to \( 1 \times 10^{-21} \text{ N} \). This is only around three orders of magnitude away from the projected force sensitivity range of their resonator. Second, the largest trampoline resonator used in the Reinhardt et al. proposal [4] has a mass of 4.5 ng, diameter of 100 microns (Figure 4-i), and a projected force sensitivity of \( \sim 14 \text{ zN} \) at cryogenic temperatures (14 mK). The gravitational force between the 4.5 ng resonator and the microsphere, keeping all other parameters the same in Eq. (5), is equal to \( 4.1 \times 10^{-23} \text{ N} \), which is also only around three orders of magnitude away from the projected force sensitivity range of their resonator. But because of the greater specificity of the Reinhardt et al. proposal (in regards to the magnitude and conditions of maximum force sensitivity), it was chosen as the most promising to combine with Romero-Isart et al.’s proposal.

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