Lossy Gradient Compression: How Much Accuracy Can One Bit Buy?

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**Abstract**—In federated learning (FL), a global model is trained at a Parameter Server (PS) by aggregating model updates obtained from multiple remote learners. Generally, the communication between the remote users and the PS is rate-limited, while the transmission from the PS to the remote users are unconstrained. The FL setting gives rise to the distributed learning scenario in which the updates from the remote learners have to be compressed so as to meet communication rate constraints in the uplink transmission toward the PS. For this problem, one wishes to compress the model updates so as to minimize the loss in accuracy resulting from the compression error. In this paper, we take a rate-distortion approach to address the compressor design problem for the distributed training of deep neural networks (DNNs). In particular, we define a measure of the compression performance under communication-rate constraints—the per-bit accuracy—which addresses the ultimate improvement of accuracy that a bit of communication brings to the centralized model. In order to maximize the per-bit accuracy, we consider modeling the DNN gradient updates at remote learners as a generalized normal distribution. Under this assumption on the DNN gradient distribution, we propose a class of distortion measures to aid the design of quantizers for the compression of the model updates. We argue that this family of distortion measures, which we refer to as “$M$-magnitude weighted $L_2$” norm, captures the practitioner’s intuition in the choice of gradient compressor. Numerical simulations are provided to validate the proposed approach for the CIFAR-10 dataset.

**Index Terms**—Federated learning; Gradient compression; Gradient sparsification; DNN gradient modelling.

**I. INTRODUCTION**

Federated learning (FL) holds the promise of enabling the distributed training of large models over massive datasets while preserving data locality, guarantying scalability, and also preserving data privacy. Despite the great advantages promised by the FL, the communication overhead of distributed training poses a challenge to present-day network. As the size of the trained models and the number of devices participating to the training is ever increasing, the transmission from remote users to the parameter server (PS) orchestrating the training process becomes the critical performance bottleneck [1], [2]. In order to address this issue, the design of an effective gradient compressor is of fundamental importance. In this paper, we propose the *per-bit accuracy* as a rate-distortion inspired approach to the design of an efficient gradient compressor for FL. We also introduce a class of distortion measures for the design of gradient compression for the distributed DNN training.

**Literature Review:** FL consists of a central model which is trained locally at the remote clients by applying Stochastic Gradient Descent (SGD) over a local dataset. The local gradients are then communicated to the central PS for aggregation into a global model. Since this aggregation model does not require data centralization, it provides substantial advantages in terms of scalability, robustness, and security. For these reasons, in the recent years, there has been a significant interest in developing effective and efficient FL algorithms [3]–[5].

A natural constraint in distributed and decentralized optimization is with respect to transmission rates between nodes and its relationship to the overall accuracy [6]. Common in these approaches is the effort put forth in reducing the dimensionality of the gradients. Following [7], we shall refer to these dimensionality reduction schemes as *gradient compressors*. Gradient compressors can be divided in two classes: (i) gradient sparsification, [3]–[5], and (ii) gradient quantization [8]–[13]. These compression schemes improve the communication efficiency by reducing the dimensionality of the message being transmitted from the users to the PS through (i) sparsifying the transmitted model update and (ii) stochastic scalar quantization, i.e. non-subtractive dither quantization. In some literature, the dimensionality-reduction is performed on the whole gradient vector: for instance the authors of [10] introduce vector quantization for SGD.

**Contribution:** In the following, we shall consider the problem of rate-efficient gradient compression for distributed training. We consider, in particular, the FL training of DNNs and present relevant numerical simulations for this case. We introduce the *per-bit accuracy* as the relevant performance measure for distributed training under communication constraints. The per-bit accuracy corresponds to the improvement in accuracy that a gradient compressed within $R$ bits can provide to a...
as a generalized normal distribution (GenNorm). We argue that
For (ii), we propose a set of distortion measures between the
original and the compressed gradient in which the $L_2$ loss
is further weighted by the magnitude of the gradient to the
power of $M$. The paper concludes with numerical simulation
which show that a compressor designed through this approach
outperforms other compressors considered in the literature,
such as the gradient quantization and sparsification.

**Notation:** In the following, lower case boldface letters (eg.
z) are used for column vectors and uppercase boldface letters
(e.g. M) designate matrices. The all-zero vector of dimension $d$
is indicated as $\mathbf{0}_d$. We also adopt the shorthands $[m:n] \triangleq \{m, \ldots, n\}$ and $[n] \triangleq \{1, \ldots, n\}$. The $p$-norm of the vector $x$ is indicated as $\|x\|_p$. Calligraphic scripts are used to denote sets (e.g. $\mathcal{A}$) and $|\mathcal{A}|$ is used to denote its cardinality.

II. SYSTEM MODEL

In the following, we consider the distributed training of a
machine learning (ML) model across $N$ sets (e.g. a
federated learning setting) and the communication between the remote device and the PS is
limited to $R$ bits per iteration. Let us begin by introducing the
optimization and the communication setting, followed by the
formal statement of our performance optimization problem.

**A. Distributed Optimization Setting**

Consider the scenario with $N$ clients each posses a local
dataset $D_n = \{(d_{nk}, v_n(k))\} \in \mathbb{D}_n$ for $n \in [N]$ and wish to minimize the
**loss function** $\mathcal{L}$ as evaluated across all the local datasets and over the choice of model $w \in \mathbb{R}^d$, that is

$$\mathcal{L}(w) = \frac{1}{|D_n|} \sum_{n \in [N]} \sum_{d_{nk} \in \mathbb{D}_n} \mathcal{L}(d_{nk}, w).$$  

(1)

For the loss function in (1), we assume that there exists a
unique minimizer $w^*$ of (1), that is,

$$w^* = \arg\min_{w \in \mathbb{R}^d} \mathcal{L}(w).$$  

(2)

A common approach for numerically determining the optimal value in (2) in the centralized scenario is through the
iterative application of (asynchronous) stochastic gradient
descent (SGD). In the (centralized) SGD algorithm, the learner
maintains a estimate of the minimizer in (2), $w_t$, for each time
$t \in [T]$. The final estimate of (2) is $w_T$. At each time $t \in [T]$,
the estimate $w_t$ is updated as

$$w_{t+1} = w_t - \eta_t g_t,$$  

(3)

for $w_0 = \mathbf{0}_d$, where $\eta_t$ is an iteration-dependent step size $\eta_t$
called **learning rate**, and where $g_t$ is the stochastic gradient of $\mathcal{L}$ evaluated in $w_t$, that is

$$E[g_t] = \sum_{d_{nk} \in \mathbb{D}} \nabla \mathcal{L}(d_{nk}, w_t).$$  

(4)

In (4), $\nabla \mathcal{L}(w_n)$ denotes the gradient of $\mathcal{L}(d_{nk}, w_t)$ in $w_t$
as evaluated over the dataset $\mathbb{D} = \bigcup_{n \in [N]} \mathbb{D}_n$.

In the FL setting, given that the datasets $\mathbb{D}_n$ are distributed
at multiple remote learners, the SGD algorithm as in (3)
has to be adapted as follows. First (i) the PS transmits
the current model estimate, $w_t$, to each client $n \in [N]$, then
(ii) each client $n \in [N]$ accesses its local dataset
$D_n = \{(d_{nk}, v_n(k))\} \in [\mathbb{D}_n]$ and computes the stochastic
gradient, $g_{nt}$, as in (4) and communicates it to the PS. Finally
(iii) the PS updates the model estimate as in (3) but where $g_t$
is obtained as

$$g_t = \frac{1}{N} \sum_{n \in [N]} g_{nt},$$  

(5)

and uses $g_t$ to update the model estimate as in (3). We refer to
the distributed version of SGD for the FL setting as **federated
averaging**.

**B. Federated Learning with Communication Constraints**

Customarily, in the FL setting, the communication is assumed to take place over some noiseless, infinity capacity
link connecting the PS and the remote users and vice-versa.
In a practical scenario, the users model wireless mobiles,
IoT devices, or sensors which have significant limitations in
the available power and computational capabilities. In
these scenarios, we can still assume that users rely on some
physical and MAC layers’ protocols that are capable of reliably
delivering a certain payload from the users to the PS.

For this reason, in the following, we assume that the
communication between each of the users and the PS takes
place over a rate-limited channel of capacity $dR$, where $d$
is the dimension of the model in Sec. II-A. In other words, each
user can communicate up to $dR$ bits for each iteration
$t \in [T]$. In the following, we refer to the operation of
converting the $d$-dimensional gradient vector $g_{nt}$ to a $dR$
binary vector as **compression** and is indicated though the
operator

$$\text{comp}_R : \mathbb{R}^d \rightarrow [2^{dR}].$$  

(6)

The reconstruction of the gradient is denoted by $\text{comp}_R^{-1}$. Note
that in (6), $R$ indicates the number of bits per model dimension.
For simplicity, we assume in the following that all users are
subject to the same communication constraint and all employ
the same set of compressors.

When gradients are compressed before transmissions, the
 evolution in (3) is rewritten as

$$\hat{w}_{t+1} = \hat{w}_t - \eta_t \hat{g}_t,$$  

(7)

for $\hat{w}_0 = \mathbf{0}_d$, where $\eta_t$ is an iteration-dependent step size $\eta_t$
called **learning rate**, and where $\hat{g}_t$ is the stochastic gradient of $\mathcal{L}$ evaluated in $\hat{w}_t$, that is

$$E[\hat{g}_t] = \sum_{d_{nk} \in \hat{\mathbb{D}}} \nabla \mathcal{L}(d_{nk}, w_T).$$  

(8)

**C. Compression Performance Evaluation**

In the following, we are interested in characterizing the compression performance in the terms of the loss of accuracy
as a function of the communication rate. More formally, give
the model estimate $\hat{w}_t$ and the gradient estimate $\hat{g}_t$, we wish
determine

$$G_R(\hat{w}_{t+1}) = \min_{\text{comp}_R, \text{comp}_R^{-1}} \mathcal{L}(\hat{w}_{t+1}),$$  

(8)
where \( \tilde{w}_{t+1} \) is obtained as in (7). In general, we are interested in determining the effect of compression through the SGD iterations, so that \( L(w_T) - G_R(\tilde{w}_T) \) corresponds to the overall loss of accuracy due to compression at the training horizon, \( T \).

The minimization in (8) is generally too complex, as the loss function \( L \) is generally non-convex in the model \( w \). Additionally, lacking a statistical description of the SGD process is available, one cannot properly resort to classical compression techniques from information theory. To address these difficulties, later in Sec. III we consider a rate-distortion approach in which we simplify the minimization in (8) for the case of DNN training by considering (i) a family of distortion measures which captures the loss of accuracy as a function of the gradient magnitude, and (ii) assume that the DNN gradients are iid draws from the GenNorm distribution. These two simplifications yield a compressor design which shows improved performance over other compressors considered in the literature.

### D. DNN training

While the problem formulation in Sec. III-C is rather general, in the remainder of the paper, we shall only consider the scenario of DNN training. More specifically, we consider a DNN for classification of the CIFAR-10 dataset. The DNN model we consider is specified in Table I; the model is trained using SGD with a learning rate 0.0001, and cross-entropy as loss. The images are passed through the model in mini batches of 32 at three epochs for every communication round.

### III. A RATE-DISTORTION APPROACH TO DNN GRADIENT COMPRESSION

In this section we introduce the a rate-distortion approach for the design of the optimal compressor for DNN gradients. Since (8) is generally intractable, we instead simplify the problem as follows. We choose (i) a certain distribution to approximate the gradient entries, and (ii) choose a distortion that correlates the loss in accuracy in (8) when we compress the original weights. Once these two elements have been selected – that a gradient distribution and a gradient distortion measure– the compressor in (8) is chosen as the the quantizer which mininizes the chosen distortion for the given gradient distribution, as in the classic [15].

#### A. DNN Gradient Distribution

As argued in [14], the gradient entries observed performing the DNN training are well approximated by the generalized normal distribution [16]. Note that the generalized normal distribution encompasses the Laplace and normal distribution as a special case: DNN gradients have been assumed to follow a Laplace distribution in [17] while they have been assumed to have a normal distribution in [18], [19]. We argue that the generalized normal assumption also partially reconciles these two (partially conflicting) research results.

In Fig. 1 we plot (i) the histogram of the DNN gradient updates, (ii) the Laplace fitting, (iii) the normal fitting and (iv) the GenNorm fitting. As it can observed, the gradient histogram is closer to the GenNorm distribution since it is more concentrated in zero and it has heavier tails than the normal distribution.

#### B. DNN Gradient Distortion

In choosing the distortion metric for our quantizer design, we keep two considerations in mind: (i) the practitioner perspective on effective sparsification and (ii) existing bounds on the accuracy loss from weight perturbation.

In the literature, top \( K \) sparsification consists in setting all but the \( K \) largest weights of the gradient entries to zero at each step of SGD. It is well known among the ML practitioners, that top \( K \) sparsification can be effectively used to reduce the dimensionality of the gradient updates while resulting in only a small loss in accuracy [7], [20], [21].

Another approach for gradient compression is uniform quantization with non-subtractive dithering [10], [22]–[24].
This approach finds its theoretical foundations in works such as [25, eq. (8)] which provides a bound on the loss in accuracy as a function of the $L_2$ perturbation of the DNN weights.

Given the two compression approaches in which either (i) only the magnitude of the gradients is considered or (ii) the $L_2$ norm of the gradients is used for the quantizer design, we propose a new class of distortion measures combining these approaches:

$$d_{M-L_2}(g, \hat{g}) = \frac{1}{d} \| g \|_M - \| \hat{g} \|_2 \approx \frac{1}{d} \sum_{j \in [d]} |g_j|^M \| g_j - \hat{g}_j \|_2,$$

where $g_j$ and $\hat{g}_j$ denote the $j$-th elements of $g$ and $\hat{g}$ (gradient and quantized vectors), respectively. In the following, we refer to $d_{M-L_2}$ as the $M$-magnitude weighted $L_2$ distortion.

C. Quantizer Design

Having substantiated our choice of gradient distribution and the gradient distortion measure, as in Sec. III-A and Sec. III-B respectively, we are now in position to argue for the proposed rate-distortion approach for the quantizer design. In particular, we argue that the quantizer minimizing the expected $M - L_2$ distortion in (9) under the GenNorm assumption provides a good performance in view of (8). In mathematical language,

$$G_R(\bar{w}_T) \approx \sum_{t \in [T]} \sum_{n \in [n]} d_{M-L_2}(g_{tn}, \bar{w}_{tn}).$$

Under the assumption in (10), the compressor in (6) can be designed as the classic rate distortion problem. In Sec. IV we will consider a scalar quantizer design minimizing such expected distortion and show its effectiveness in DNN training. We note that the $K$-means/LGB algorithm for the class of distortions in (10) takes a particularly simple form. In particular, let $c_k(i)/t_k(i)$ be the $i$-th centroid/threshold estimate at iteration $k$ in the scalar LGB algorithms, then

$$c_{k+1}(i + 1) = \frac{\int_{t_k(i)}^{t_k(i+1)} g^{M+1} \text{pdf}(g) \, dg}{\int_{t_k(i)}^{t_k(i+1)} g^{M} \text{pdf}(g) \, dg},$$

$$t_{k+1}(i + 1) = \frac{c_k(i + 1) + c_k(i)}{2},$$

for $i \in [2^R]$ where $2^R$ is the number of quantization levels, $\text{pdf}(g)$ denotes the distribution fitted to the gradient vector (in (11) we ignore the quantization boundaries due to limited space).

In Fig. 2 we plot the position of the positive centroids resulting from the $K$-means algorithm in (11) when applied to the GenNorm distribution with zero mean, unitary variance, and varying $\beta$. On the $x$-axis we have the values of $\beta \in [0.5, 1]$. On the $y$-axis we have the position of the positive centroids: since the distribution is symmetric, the centroids are also symmetric. On the various panels, we plot four combinations of the quantizer rate — $R \in \{2, 3\}$ and $M \in \{2, 4\}$. As we can observe, lower values of $\beta$ correspond to heavier tail of the distribution which, in turn, result in centroid position further away from zero. As $M$ increases, the spreading of the centroid toward large gradient values increases.

IV. NUMERICAL EVALUATIONS

In this section, we compare the performance of our proposed rate-distortion inspired compression with other conventional ML compression techniques, such as floating point conversion and top-$K$ sparsification.

A. Compression Strategies

Let us begin by describing in further detail the compression techniques we consider in our simulations.

- **top-K sparsification + scalar uniform quantization**: For the uniform quantizer with a given quantizer rate $R_u$, the $2R_u$ quantization centers are uniformly distributed between the minimum and maximum values of the samples in each layer and each iteration. The sparsification level $K_u$ is accordingly chosen such that

$$dR = \log \left( \frac{d}{K_u} \right) + R_u K_u.$$  

- **top-K sparsification + floating point representation**: In the context of (6), top-$K$ can be applied to meet the rate constraint only once a certain format for representing the gradient entries has been established. So, we consider a floating point (fp) representation of the entries with 16 and 8 bits. Accordingly, the number of the relationship between the sparsification
parameter $K$, the digit precision $p$ (in bits), and the rate constraint $R$ in (8) is

$$dR = \log \left( \frac{d}{K_{fp}} \right) + K_{fp} p. \quad (13)$$

- **topK sparsification + 2-magnitude weighted $L_2$:** For our proposed quantizer with a given rate $R_{mw}$, the $2^{R_{mw}}$ quantization centers at each layer and iteration are found by the K-means algorithm described in Section III-C for $M = 2$. We use a topK sparsification before our compressor where the sparsification level $K_{mw}$ satisfies the following:

$$dR = \log \left( \frac{d}{K_{mw}} \right) + K_{mw} R_{mw}. \quad (14)$$

B. Simulation Results

In Fig. 3 we plot the accuracy of the trained network for different quantization rates $dR = 664$ kbits and $dR = 996$ kbits. We use the network introduced in Table I. The compression strategies are as detailed in Sec. IV-A. For $dR = 664$ kbits, we choose the following parameters for each of the quantizers:

- **topK + uniform:** $R_u = 2, K_u = 331725$
- **topK + 16fp:** $p = 16, K_{fp} = 41466$
- **topK + 8fp:** $p = 8, K_{fp} = 82932$
- **topK + 2-mw $L_2$:** $M = 2, R_{mw} = 2, K_{mw} = 331725$

and for $dR = 996$ kbits, the following set of parameters are proposed:

- **topK + uniform:** $R_u = 3, K_u = 331725$
- **topK + 16fp:** $p = 16, K_{fp} = 62199$
- **topK + 8fp:** $p = 8, K_{fp} = 124398$
- **topK + 4-mw $L_2$:** $M = 4, R_{mw} = 3, K_{mw} = 331725$

The blue dotted curve shows the accuracy of our proposed quantizer versus the iteration number. The performances of the uniform compressor, topK-8fp and topK-16fp are shown in red squared, yellow starred, green circled curves, respectively.

We point out that the optimization over $M$ in the first two panels was only partial: further tuning of this parameter would result in further performance improvements. Also note that the quantizer for mw $L_2$ and uniform quantization is chosen adaptively. In other words, the quantizer is chosen as a function of the empirical gradient distribution at each iteration. For the case of mw $L_2$, this is attained by pre-calculating the centroid for different values of $\beta$ as in Fig. 2. At each iteration, then, the gradient vector is pre-processed to obtain a vector of zero mean and unitary variance which is then quantized using the pre-calculated scalar quantizer.

**Adaptive vs. non-adaptive mw $L_2$ quantization:** In the first two panels of Fig. 3 we consider an adaptive quantization strategies. That is, the quantizer is chosen at each iteration as a function of the empirical gradient distribution. In actuality, the GenNorm modelling of [14] produces rather stable estimates of the GenNorm parameters describing the empirical gradient distribution. For this reason, one can consider the performance when the quantizer is non-adaptive. The performance of such a non-adaptive quantizer, together with its adaptive counterpart, is depicted in the third panel of Fig. 3. The small loss of performance from a non-adaptive quantizer suggests that, in a practical scenario, one would do a first boot-up training phase in which the gradient parameters are estimated. The quantizer would then be chosen as a function of such estimates and used throughout the remaining iterations.

**GenNorm Modelling under mw $L_2$ quantization:** As a last numerical consideration, we would like to argue that the GenNorm modelling of the DNN gradients remain valid also under mw $L_2$ quantization. In other words, the choice of quantization strategy does not fundamentally alters the gradient distribution. Indeed, in Fig. 4 we plot the GenNorm parameters modelling the last layer of the DNN and observe the consistency of these estimate through the iteration.

The code for the numerical evaluations in this section is provided online at [https://github.com/sadausk/FL_RD](https://github.com/sadausk/FL_RD).

V. Conclusion

In this paper, the problem of efficient gradient compression for DNN has been considered. In particular, we approach this
problem from a rate-distortion perspective in which the quantizer used for compression is designed under an assumption on (i) the distribution of the gradient updates and (ii) the distortion measure which minimized the loss in accuracy. For (i), we assume that the gradient updates are in each DNN layer and at each iteration are identically and independently distributed according to a generalized normal distribution. For (ii), we assume that the distortion capturing the relationship between gradient perturbation and loss in accuracy is the $M$-magnitude weighted $L_2$ norm, that is the $L_2$ norm in which the error is further multiplied by the magnitude of the gradient to the power of $M$. We argue that this choice of distortion naturally bridges between two classical gradient sparsification approaches: $M = 0$ recovers uniform quantization, while $M \to \infty$ recovers top-K sparsification. In this work, both of these assumptions are validated through numerical evaluations: a more theoretical justification of these two assumptions will be investigated in our future research. Furthermore, in our simulation the optimization over the weighting parameter $M$ was only partial: a more detailed investigation over the role of this parameter is also left for future research.

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