Opportunities at the Mathematics/Future Cities Interface

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Abstract

We make the case for mathematicians and statisticians to stake their claim in the fast-moving and high-impact research field that is becoming known as Future Cities. After assessing the Future Cities arena, we provide some illustrative challenges where mathematical scientists can make an impact.

More than half of the world’s population lives in a city, and this proportion is estimated to reach 60% by 2030 and 70% by 2050 [World Health Organization, Urban Population Growth, July, 2014]. See Figure 1 for a graphic showing our current “megacities.” Thanks to the proliferation of smart devices and interconnected services, cities are gushing with data, much of which relates to human behavior. City life generates data streams around on-line social media, telecommunication, geo-location, crime, health, transport, air quality, energy, utilities, weather, CCTV, wi-fi usage, retail footfall and satellite imaging. Viewing urban centres as “Living Labs” is a powerful new concept that is inspiring novel research leading to improved wellbeing and economic growth. We argue here that mathematicians can make an impact at the

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heart of this emerging interdisciplinary field, where hypotheses about human behavior must be quantified and tested against vast data sets and where decisions and interventions should be based on quantitative, testable predictions. Further, the rapid growth of large-scale, disparate, multi-resolution data sets is driving new research challenges for applied and computational mathematicians, drawing on hot topic areas such as dynamic and multiplex networks (see Figures 2 and 3) [5, 10], multiscale modelling and simulation [2], uncertainty quantification [16] and sparse tensors [13, 14].

Since the terms are relatively new and open to interpretation, we are using “Future Cities” here as a catch-all to cover topics that may equally well be classified under “Smart Cities” or “Urban Analytics.” However, for reference, we note that Batty et al. [4, page 481] define a smart city to be “a city in which ICT [Information and Communication Technologies] is merged with traditional infrastructures, coordinated and integrated using new digital technologies” and the recent report by Arup [12] makes an attempt to distinguish between smart and future versions: “There is still confusion in the market as to the distinction between Smart city solutions and Future city solutions. Future city solutions are innovative physical projects which are often but not exclusively associated with low carbon economies. Smart
Figure 2: Multiplex visualization of population density, housing costs, level of deprivation and level of drug misuse across the city of Glasgow, UK. Credit: c.LUSTlab/Institute for Future Cities, University of Strathclyde. Reproduced with permission.
Figure 3: A screen-shot extract from the data streams in Figure 2, showing reported level of drug misuse across the city of Glasgow. After discretization, for example based on city regions, combining the levels in Figure 2 naturally leads to a three-dimensional tensor, where two dimensions represent spatial coordinates and the third dimension indexes the data sources. Time-dependency in the data would add a fourth dimension. Extracting commonalities and differences, and summarizing patterns, can be cast in terms of tensor factorization—for example, generalizing the well known matrix-level Singular Value Decomposition (SVD). Note, however, that those four dimensions are not comparable—any results should be insensitive to the order in which we label the data streams, but, for most purposes, we should not reorder points in time or space. Credit: c.LUSTlab/Institute for Future Cities, University of Strathclyde. Reproduced with permission.
city solutions apply digital technologies to address social, environmental and economic goals.” The Future City research arena we envisage is inherently interdisciplinary, covering the physical and social sciences, engineering, business, law, and, in particular, dealing with issues of privacy and ethics. At the risk of buzzword overload, we also note that Future Cities is a topic that has strong overlaps with other big picture themes, including Data Science, Big Data, Complexity, Planet Earth, Digital Economy, Internet of Things and Computational Social Science.

Many urban centers across the world are becoming active in the Future Cities space, with governments and funding agencies showing strong support for these developments. Focussing on the authors’ home institutions, Glasgow City Council beat 30 other cities to win a £24M Future Cities Demonstrator competition, funded by the UK government’s innovation agency, the Technology Strategy Board; within this award, the Institute for Future Cities at the University of Strathclyde is developing a Digital Observatory that will allow public domain access to data generated in Glasgow and elsewhere. Future Cities is also one of the four strategic themes for Strathclyde’s £89M Technology and Innovation Centre, a hub for academic research and industrial collaboration, and the university offers a Masters degree in Leadership for Global Sustainable Cities. The University of Oxford’s Engineering and Physical Sciences Research Council (EPSRC) Centre for Doctoral Training in new Industrially Focused Mathematical Modelling has a strong data/analytics/technology component and its Said Business School hosts the Institute for New Economic Thinking. The University of Warwick, which has designated Sustainable Cities as one of its Global Priority Programmes, houses the Warwick Institute for the Science of Cities, and offers an EPSRC Centre for Doctoral Training in Urban Science. The University of Warwick is also a partner in the Centre for Urban Science and Progress (CUSP), a public-private research collaboration using New York City as a laboratory and classroom [Link to Sidebar 1], and in its recently announced branch ‘CUSP London’.

Sidebar 1

Quoting from CUSP’s website at http://cusp.nyu.edu/about/
“CUSP will instrument New York City and use existing data from a network of agencies to transform the city into a living laboratory and classroom. It will make sense of the vast amount of data it collects to help cities around the world become more productive, more livable, more equitable, and more resilient.”

Looking further afield, Horizon 2020, the biggest ever European Union research and innovation programme, chose Societal Challenges as one of its three pillars, in which a 100 Million Euro call for research projects is listed under the theme Smart Cities and Communities. The UK’s science and engineering research council, EPSRC, released a draft Strategic Plan in July 2014 that listed “designing and building future cities” as one of seven key challenges for the global economy, and the Technology Strategy Board chose Future Cities for one of its seven Catapult Centres [Link to Sidebar 2].

Sidebar 2

A Catapult is a physical centre where businesses, scientists and engineers work together to develop ideas into new products and services. The London-based Future Cities Catapult was established in June 2013; see, https://futurecities.catapult.org.uk/. An illustrative project is Sensing London, which focusses on

**Data Collection**: deploying a range of air pollution sensors across Hyde Park, Brixton, Enfield and Elephant & Castle.

**Data Mashing**: overlaying and integrating data, and applying state-of-the-art algorithmics, modelling and visualisation to generate new insights.

**Trialling innovation**: for example, building a virtual “asthma-guard” to let asthmatics know in real time where it is safe to walk in the city.
The report [12], commissioned by the UK Department for Business, Innovation and Skills, considered opportunities for UK industry in smart city technology across five urban market verticals—energy, water, transport, waste and assisted living—estimating a global market of $408 Billion by 2020. As we prepared this article, Cisco announced plans to open a $30 Million Global Internet of Everything Centre in Barcelona, focusing on smart cities.

To be more concrete about opportunities for the mathematical sciences community we now focus briefly on recent developments and prospects in dynamical systems and in networks. Our discussion is, of course, biased towards our own research interests.

Macro-scale observations have revealed scaling laws that relate city population size to other attributes, such as energy consumption, household income and patent production, and important distinctions have been drawn between linear, sublinear and superlinear growth [1, 15]. Explanatory, micro-scale models based on “hidden” laws must of course be consistent with such observations. Long-time dynamics and stability are key issues in the modelling of complex urban systems, as are sensitivities to parameter choices, including thresholds due to resource limitations [6]. In principle, good mathematical models can be used to map out the ranges of possible behavior, helping us to understand whether we might be observing a phenomenon that is constrained within a single domain of attraction (whilst there are others as yet unseen) with a very low probability of breaking out, or else we might be observing a trajectory of a chaotic process, where the qualitative macroscopic behaviour is predictable but the quantitative evolution of specific individuals is not (due to sensitive dependence and instability driven disruptions). In modelling terms, it may not be appropriate to surgically extract the city from its surroundings, and an open model, subject to a range of external influences, may be more realistic. Phenomena of interest may then be subject to persistent cycling or boiling—never approaching quiescence [18].

Digital interactions in an urban setting can naturally be represented as graphs, or networks, but the links between nodes in the system typically have an important time-dependent feature: who just texted whom, who just logged in to which free wifi zone, who just reported a crime at which location? Two of us have written previously in SIAM News about how a dynamic view of classical concepts in graph theory has led to useful new algorithms [9]. However, alongside the data-driven issue of extracting and summarizing information from network observations, there is an equally compelling challenge to derive models that describe the underlying dynamics. Representing
a network as a time-dependent matrix, $A(t)$, whose $(i,j)$ element quantifies the current level of interaction between nodes $i$ and $j$, we can formalize concepts from the social sciences to derive suitable laws of motion [Link to Sidebar 3]. In an urban context, where dynamic interactions take place on many levels between a range of parties, it is natural to think of dynamic models that operate across many layers, with the dynamics on one layer (say, the evolution of attitudes towards healthy lifestyle) coupled to the dynamics on another (say, the reach of a social media campaign). Moreover, with the advent of smartphones and GPS, we can now monitor geographical location across time and hence test models of urban movement [11].

Sidebar 3

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Traag, Antonio and Van Dooren [17] looked at the concept of social balance (my friend’s friend is my friend, my enemy’s enemy is my friend, . . . ) to derive matrix-valued ordinary differential equations (ODEs) of the form $\dot{A}(t) = A(t) \times A(t)$ and $\dot{A}(t) = A(t) \times A(t)^T$. Given such an $A(t)$, [7] developed an accompanying ODE for the level of importance, or centrality, of the network nodes, showing that the matrix logarithm function arises naturally. An alternative concept from the social sciences, triadic closure (the more friends I have in common with somebody, the more likely I am to become their friend), was used in [8] to derive a stochastic birth and death model for link dynamics. There, a mean-field analysis agreed with simulations showing that the network can self-organize into either of two very different long-term behaviors.

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In the preamble to his recent text *The New Science of Cities* [3], Michael Batty, from The Bartlett Centre for Advanced Spatial Analysis, discusses three central principles that inform his “networks and flows” perspective of city science; each of which resonates strongly with the standpoint of this article. Batty’s first principle is that the relations between objects, not the intrinsic attributes of those objects, should condition our understanding; a viewpoint familiar to those of us who have been exposed to graph
theory or category theory. Second, we should aim to measure, categorize and look for universal scalings when we observe and compare city networks across space and time. Third, having gathered macrolevel observations we should seek to understand the micro-level principles that drive them—in the language of applied mathematics, we should aim for explanatory models, based on explicit modelling assumptions, with predictive power. Batty’s book makes use of concepts such as agent based modelling, flocking, graph theory, Markov chains, Markovian decision problems, optimization and self-similarity/fractals, and hence is an excellent starting-off point for mathematicians wishing to enter the field.

In the spirit of micro-level digital interactions, one of us has initiated a Linked-In group on *MSSC: Mathematical Sciences for Smart Cities* and interested readers are encouraged to join us.

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