Improved soft-wall model with a negative dilaton

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Abstract

I propose to change the sign of the dilaton in the infrared (IR) soft-wall AdS/QCD model, in order to implement confinement. The deformed model exhibits interesting properties, especially in describing chiral symmetry breaking. The expectation value of the scalar field $X$, which determines the quark mass and condensate, approaches a constant in the IR limit, rather than blows up in the original model. In contrast to the estimate in Ref. [1], this kind of solution will not lead to chiral symmetry restoration for highly-excited states, due to the property of the harmonic-oscillator equation. Instead, it will guarantee the Regge behavior of the axial meson spectrum, and also the pseudoscalar mesons. The value of the condensate can be fixed by requiring that the pion be massless in the chiral limit, but only under some approximation in the present model. We also find that, by relaxing the IR boundary conditions, the unphysical massless state in the vector channel can be eliminated.

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I. MOTIVATION

The holographic approach towards QCD, or AdS/QCD, has made significant progress in describing low energy hadron dynamics recent years. Except for predicting various low energy parameters with an unexpected precision, it joins together many traditional approaches to describe strong interactions, such as nonlinear chiral symmetry realization, hidden local symmetry, QCD sum rule, light-cone perturbative theory, etc. However, it also suffers from many problems, one of which is the description of the experimentally well established Regge trajectories of hadron spectrum. To overcome this, people propose to select a special background, namely a quadratically increasing dilaton together with a five dimensional anti de Sitter (AdS) space-time \([2]\). Fields propagating in this background satisfy harmonic-oscillator type equations, and give exactly Regge trajectories for various mesons. The model based on this background is often called soft-wall model, to be compared with the hard-wall model \([3]\), where the background is a slice of AdS space-time.

While Regge behavior is well described in this model, we now run into new troubles when trying to incorporating chiral symmetry breaking. If we introduce a tachyonlike scalar field \(X\) to break chiral symmetry as in the hard-wall model, the solution for the expectation value of \(X\) now diverges exponentially in the infrared (IR) limit. This is unreasonable, since every physical mode should be regular in the bulk region according to the standard AdS/CFT prescription \([4]\). Practically, this solution adds an exponentially increasing term to the equation for the axial modes, and the previously obtained Regge behavior was ruined. There has been some attempt to overcome this problem by adding higher dimensional terms to the potential of \(X\) \([5]\), see also \([6]\). A simpler way is to change the sign of the dilaton, and push the divergence into the background. This may seem a little rude, but it does regularize the divergent solution. Meantime, it gives the same Regge trajectories for the vector mesons, which has already been pointed out in Ref. \([2]\). Moreover, explicit calculations show that various mesons, including axial and pseudoscalar ones, will fall on Regge trajectories with the same slope.

Another motivation comes from the consideration of confinement property of the background. The original soft-wall background is in some sense equivalent to adding a Gaussian factor \(e^{-z^2}\) to the AdS metric. In Ref. \([7]\) an alternative deformation factor \(e^{z^2}\) was shown to result confining potential between heavy quarks. It is just this increasing exponential which
guarantees the confining property, according to the general criterion in Ref. [8]. Therefore, in order to obtain linear Regge trajectories in a confining background, one is naturally led to consider a negative dilaton $\Phi \sim -z^2$. Since the dilaton contributes to the action in the form $e^{-\Phi}$, which now increases quickly in the IR region, this kind of background includes actually a quasi-hard wall, not a soft wall any more.

The paper is organized as follows. In the next section, I show that in the modified background chiral symmetry breaking can be naturally incorporated. The Regge trajectories of various mesons, including the axial and pseudoscalar mesons, can be reproduced in the improved model. Some comments on the confining property of the background is given in Sec.III. In the last section I give a short summary.

II. IMPROVED SOFT-WALL MODEL

The five dimensional AdS space-time can be given in the Poincaré coordinate as:

$$g_{MN} dx^M dx^N = \frac{1}{z^2} \left( \eta_{\mu\nu} dx^\mu dx^\nu - dz^2 \right),$$

where $\eta_{\mu\nu} = \text{Diag } (1, -1, -1, -1)$ and $\mu, \nu = (0, 1, 2, 3)$, $M, N = (0, 1, 2, 3, z)$. The dilaton is now chosen to be

$$\Phi(z) = -\Lambda^2 z^2,$$

which has an opposite sign to that in the original soft-wall model [2]. This kind of background was shown to exist in some tachyon-dilaton-gravity system [9], where the close string tachyon plays the role of the dilaton field here. The nontrivial dilaton profile violates scale invariance explicitly, and introduces a scale $\Lambda$ into the boundary theory. We choose to set $\Lambda$ to be 1 and recover it from dimensional analysis when needed. As in Ref. [3], one introduces two sets of gauge fields, $A_L$ and $A_R$, to represent the gauged chiral symmetry. Then this symmetry is broken both explicitly and spontaneously by employing a tachyonic scalar field $X$. With the dilaton turned on, the action reads:

$$S = \int d^5 x \sqrt{g} \ e^{-\Phi} \ \text{Tr} \left[ (D^M X)^\dagger (D_M X) + 3 X^\dagger X - \frac{1}{4g^2} (F^{MN}_{LM} F_{LMN} + F^{MN}_{RM} F_{RMN}) \right],$$

where $A = A^a t^a$, $F_{MN} = \partial_M A_N - \partial_N A_M - i [A_M, A_N]$, $DX = \partial X - i A_L X + i X A_R$. The generators of the gauge group are normalized as $\text{Tr } t^a t^b = \delta^{ab}/2$. 


A. Vacuum Solution

A nonzero expectation value $X_0$ of $X$ will breaks the $SU(N_f)_L \times SU(N_f)_R$ symmetry to the vector part. So one first takes $A_L$ and $A_R$ to be zero and solves the equation of motion for $X$. Denote the $v(z) = 2X_0$, then $v(z)$ satisfies the following equation:

$$\partial_z \left( \frac{1}{z^3} e^{-\Phi(z)} \partial_z v(z) \right) + 3 \frac{1}{z^6} e^{-\Phi(z)} v(z) = 0 \quad (4)$$

The solution is given by

$$v(z) = C_1 v_1(z) + C_2 v_2(z), \quad (5)$$

with $v_1(z) = z e^{-z^2} U(-1/2, 0; z^2)$ and $v_2 = z^3 e^{-z^2} \, _1F_1(1/2, 2; z^2)$. Here $U$ is the Tricomi function, and $\, _1F_1$ the Kummer function. Since $U(-1/2, 0; z^2) \to 1/\sqrt{\pi}$ and $\, _1F_1$ goes to unity as $z \to 0$, one can identify $C_1/\sqrt{\pi}$ as the quark mass and $C_2$ the condensate so that $v(z) \sim m_q z + \sigma z^3$. When $z \to \infty$, $v_1(z)$ vanishes as $z^2 e^{-z^2}$ and $v_2(z) \to 1/\sqrt{\pi}$, while in the original model one solution goes to a constant and the other diverges like $z^2 e^{z^2}$. So the divergence of $v(z)$ is successfully absorbed into the background, and now $v(z)$ becomes regular in the bulk. This will be important in guaranteeing the Regge trajectories of the axial and pseudoscalar mesons, as shown in the following.

B. Vector Sector

Now consider the fluctuations around the vacuum, which will correspond to the bound states of various mesons. First we consider the fluctuations from the vector combination $V = (A_L + A_R)/2$. Taking the axial gauge $V_z = 0$ and performing Fourier transformation in the $x$ direction, the transverse components $V^T_\mu$ then satisfy the following equation:

$$\partial_z \left( e^{-B} \partial_z V^T_\mu(q, z) \right) + q^2 e^{-B} V^T_\mu(q, z) = 0, \quad (6)$$

where $B = \Phi(z) + \log z$. The normalizable solutions $v_n$, obtained at $q^2 = m^2_{v_n}$, are then considered to be dual to vector mesons. Applying a Bogoliubov transformation $v_n = e^{B/2}\psi_n^v$, one obtains a Schrödinger equation:

$$-\psi_n''' + V_v(z)\psi_n^v = m^2_{v_n} \psi_n^v, \quad (7)$$

with the potential

$$V_v(z) = (B')^2/4 - B''/2 = z^2 + 3/(4z^2). \quad (8)$$
Now the equation for $\psi_n^v$ is exactly the same as in the original soft-wall model, and the regular solutions are:

$$\psi_n^v(z) = e^{-z^2/2} z^{3/2} \sqrt{\frac{2}{n+1}} \mathcal{L}_1^1(z^2),$$

with $\mathcal{L}_m^m$ the associated Laguerre polynomials and

$$m^2_{vn} = 4(n + 1), \ n = 0, 1, 2, \ldots$$

One can then use the experimental value for $m_\rho$ to determine the parameter $\Lambda$, which gives $\Lambda = m_\rho/2 \simeq 0.388$GeV. The form of corresponding gauge fields reads

$$v_n(z) = e^{-z^2} z^2 \sqrt{\frac{2}{n+1}} \mathcal{L}_1^1(z^2).$$

An extra exponential $e^{-z^2}$ now appears in $v_n$ which forces it to vanish at the IR boundary, while in Ref. [2] $v_n$ diverges as $z^{2n+2}$. The normalization condition is of the same form:

$$\int_0^\infty \frac{e^{-\Phi}}{z} v_m v_n \, dz = \delta_{mn}.$$ 

(12)

One can also find the non-normalizable solution to Eq. (7) at spacelike momentum $q^2 < 0$. This kind of solution can be separated into the source $V^0_\mu(q)$ and the bulk-to-boundary propagator $V(q, z)$, as $V^T_\mu(q, z) = V(q, z) V^0_\mu(q)$. The general solution for $V(q, z)$ is given by

$$V(q, z) = A e^{-z^2} U(-\frac{q^2}{4}, 0, z^2) + B z^2 e^{-z^2} _1F_1(1 - \frac{q^2}{4}, 1/2, z^2).$$

(13)

At large $z$, $U(-\frac{q^2}{4}, 0, z^2) \sim (z^2)^{q^2/4}$ and $_1F_1(1 - \frac{q^2}{4}, 1/2, z^2) \sim e^{z^2} (z^2)^{-1-q^2/4}$. When $q^2 < 0$, the second part of the solution diverges as $z \to \infty$, and thus should be discarded according to the standard “regularity in the bulk” condition [4]. Then $A$ can be determined from the boundary condition $V(q, 0) = 1$ to be $A = \Gamma(1 - q^2/4)$. From the solution of $V(q, z)$, one can derive the two-point function, which turns out to be

$$\Pi_V(q^2) = \frac{e^{z^2} V(q, z) \partial_z V(q, z)}{g_s^2 q^2} \bigg|_{z=\epsilon} = \frac{1}{2g_s^2} \psi_1^v \left(1 - \frac{q^2}{4}\right) - \frac{2}{g_s^2 q^2} + \text{Constant}.$$ 

(14)

Comparing to our previous result [10], an additional term $-\frac{2}{g_s^2 q^2}$ appears, which seems to correspond to a “pion” pole in the vector mode. This unphysical mode has already been
pointed out in the pioneer work of Karch et al [2]. It also affects the three point functions, e.g., the electromagnetic (EM) form factors of the vector mesons. Following the derivation in Ref. [11], one find that the EM form factors are given by

$$F_{nk}(q^2) = \int_0^\infty \frac{dz}{z} e^{z^2} V(q, z) v_n(z) v_k(z).$$

(15)

At $q^2 = 0$ the off-diagonal transitions do not vanish and the diagonal ones do not normalize to unity, which signal a violation of charge conservation.

The underlying reason for all these is that, $V(0, z) = e^{-z^2}$ is not a flat function as expected. One may ask, if we can relax the IR boundary condition\(^1\) to make $V(0, z)$ flat, at least approximately near the ultraviolet (UV) boundary? This sounds reasonable, since $V(q, z)$ does not describe a physical state, or the physical vacuum. To this end, we retain the IR divergent part in the general solution \(^{[13]}\) with $B$ a general function of $q^2$. Since this part is sub-leading in the UV region, it does not affect the determination of the coefficient $A$. Now we calculate the correlation function again. In general there will be additional contributions from the IR boundary, since now neither $V(q, z)$ or its derivative vanishes there. They are unwanted from the holographic point of view, and can be eliminated by adding suitable boundary terms. Then the correlation function can be derived to be:

$$\Pi_V(q^2) = \frac{e^{z^2} V(q, z) \partial_z V(q, z)}{g_s^2 q^2} \bigg|_{z=\epsilon} = -\frac{1}{2g_s^2} \psi \left(1 - \frac{q^2}{4}\right) + 2 \frac{B(q^2) - 1}{g_s^2 q^2} + \text{Constant.}$$

(16)

So the IR divergent part contributes to the $q^2 = 0$ pole too. To eliminate this unphysical pole, we are forced to choose $B(0) = 1$. Interestingly, with this choice $V(0, z)$ becomes

$$V(0, z) = e^{-z^2} + z^2 e^{-z^2} _1F_1(1, 2, z^2) = 1,$$

(17)

and charge conservation is restored. Since now $V(q, z)$ does not vanish at the IR boundary as the normalizable modes do, the decomposition formula \(^{[13]}\) of $V(q, z)$ does not exist any more. As a result, general vector meson dominance will be violated, and there will be direct couplings of the external fields to the mesons. The complete form of $V(q, z)$ and the phenomenological effects of it are still under investigation.

\(^1\) Such kind of modification has once been investigated in Ref. [12].
C. Axial Sector

The fluctuations of the axial combination $A = (A_L - A_R)/2$ are a little complicated. The axial field can be separated into the transverse and longitudinal parts as $A_\mu = A_{\mu \perp} + \partial_\mu \varphi$, where $\partial^\mu A_{\mu \perp} = 0$. The longitudinal part $\varphi$ will get entangled with the pseudoscalar fluctuations, so we decay to talk about them in the next subsection. The transverse part, after taking the axial gauge $A_z = 0$, satisfies the equations:

$$\left[ \partial_z \left( \frac{e^{-\Phi}}{z} \partial_z A_\mu^a \right) + \frac{q^2 e^{-\Phi}}{z} A_\mu^a - \frac{g_5^2 v(z)^2 e^{-\Phi}}{z^3} A_\mu^a \right]_{\perp} = 0,$$

(18)

Again one takes the Bogoliubov transformation to change them into the Schrödinger form, and obtain the potential:

$$V_a(z) = z^2 + \frac{3}{4z^2} + \frac{g_5^2 v(z)^2}{z^2}.$$  

(19)

From the IR behavior of $v(z)$, the potential $V_a(z)$ can be expanded as:

$$V_a(z) = z^2 + \frac{3}{4z^2} + \frac{g_5^2 v(z)^2}{z^2}$$

$$= z^2 + \left[ \frac{3}{4} + \frac{g_5^2 C_2^2}{\pi} \right] z^{-2} + O(z^{-4}).$$

(20)

Then we get the following spectrum approximately:

$$m_{a_n}^2 \approx 4n + 2 \sqrt{1 + \frac{g_5^2 C_2^2}{\pi} + 2}$$

(21)

Notice that due to the nonzero quark condensate $C_2$, the vector and axial spectrum are now split, with the leading order mass splitting independent of the excitation number $n$. This is in contrast to the estimation in Ref. [1], where a constant behavior of $X$ at large $z$ was believed to introduce only a $1/n$ deformation of the axial masses and lead to chiral symmetry restoration for highly-excited states. The asymptotic wave functions in the IR region are

$$a_n(z) = e^{-z^2} z^{m+1} \sqrt{\frac{2}{n+m}} L_n^m(z^2),$$

(22)

where $m = \sqrt{1 + \frac{g_5^2 C_2^2}{\pi}}$. These solutions cannot be extended naively to the small-$z$ region since they have the wrong $z \to 0$ behavior for general $m$. To obtain the explicit solutions in the whole region one must employ numerical methods. It is also interesting to study the non-normalizable mode. Because of the complicated form of $v(z)$, this is more involved than the vector one and we leave it to future work.
Now we turn to the fluctuations around the scalar field $X$, and parameterizes $X$ as $X = (X_0 + S) \exp(i2\pi)$ as in Ref. [14] and Ref. [15]. The equation for $S$ is easily derived:

$$\partial_z \left( \frac{1}{z^3} e^{-\Phi(z)} \partial_z S^n \right) + 3 \frac{1}{z^5} e^{-\Phi(z)} S^n - q^2 \frac{1}{z^3} e^{-\Phi(z)} S^n = 0 .$$  \hspace{1cm} (23)$$

The Schrödinger potential becomes $V_s(z) = z^2 + \frac{3}{4z^2} - 2$, and the corresponding spectrum is given by $m_{s,n}^2 = 4n + 2$. Comparing with the corresponding spectrum in Ref. [15], one sees that the deformation of the dilaton profile decreases the masses of the scalar mesons.

Finally, the pseudoscalar fluctuations $\pi$ and the longitudinal part of the axial field satisfy the coupled equations

$$\partial_z \left( \frac{e^{-\Phi}}{z} \partial_z \varphi^a \right) + \frac{g_5^2 v(z)^2 e^{-\Phi}}{z^3} (\pi^a - \varphi^a) = 0 ,$$  \hspace{1cm} (24)

$$-q^2 \partial_z \varphi^a + \frac{g_5^2 v(z)^2}{z^2} \partial_z \pi^a = 0 ,$$  \hspace{1cm} (25)

These two equations can be combined [16] to give the decoupled form equation:

$$\partial_z \left[ \Gamma(z) \partial_z y^a \right] + \Gamma(z) \left[ q^2 - \beta(z) \right] y^a = 0 ,$$  \hspace{1cm} (26)

where $y^a = [e^{-\Phi(z)}/z] [\partial_z \varphi^a]$, $\beta(z) = g_5^2 v(z)^2 / z^2$ and $\Gamma(z) = z / [\beta(z) e^{-\Phi(z)}]$. One can then change this equation into the Schrödinger form with the Bogoliubov transformation $y = e^{B/2}\tilde{y}$ with

$$\tilde{B}(z) = 2 \log v(z) - \Phi(z) - 3 \log z.$$  \hspace{1cm} (27)

The corresponding Schrödinger potential becomes:

$$V_p = \frac{\tilde{B}^2}{4} - \frac{\tilde{B}''}{2} + \frac{g_5^2 v(z)^2}{z^2} .$$  \hspace{1cm} (28)$$

By using the asymptotic behavior of $v(z)$ at large $z$, the potential can be expanded as:

$$V_p = z^2 + \left( \frac{g_5^2 C_2^2}{\pi} - \frac{9}{4} \right) z^{-2} - 4 + O(z^{-4})$$  \hspace{1cm} (29)$$

Then the spectrum is approximately given by

$$m_{p,n}^2 \approx 4n + 2 \sqrt{\frac{g_5^2 C_2^2}{\pi} - 2} - 2 .$$  \hspace{1cm} (30)$$
Since the quark mass $C_1$ does not appear, the lowest state must be massless. To satisfy this condition, one must choose $\frac{g_0^2 C_2}{\pi} = 3$ at this order, which then induces that $m_{p_n}^2 \approx 4n$ and $m_{a_n}^2 \approx 4n + 6$. However, when one further expand the potential to include higher powers of $z^{-1}$, variation of $C_2$ is not enough to ensure that the lowest state be a massless state. This has been confirmed by numerical calculation. It is interesting to study whether these higher powers can be eliminated by fine-tuning the detailed form of $\Phi(z)$ and the metric in the IR region. If this is true, we can always determine the quark condensate by requiring the lowest state being massless. A physical $\pi$ mass can then be obtained by taking the quark mass term as perturbation. Since all the normalizable modes are suppressed in the IR region as $z^n e^{-z^2}$, one can easily derive the Gell-Mann-Oakes-Renner (GOR) relation following the procedure of Refs. 17, 18.

III. COMMENTS ON THE CONFINEMENT PROPERTY OF THE MODEL

To explore the confining property of the background, one could compute the quark-antiquark potential through the prescription of Refs. 19, 20. Namely one choose a rectangle loop $C$ with one time direction, and calculate the expectation value of the Wilson loop from the proper area of a fundamental string worldsheet ending on $C$ at the boundary. When a nontrivial dilaton is turned on, the generalization is not so straightforward. One way to implement the dilaton was to work in the string frame rather than Einstein frame metric, as done in Ref. 21. However, there are still some ambiguities in doing so due to the complicated worldsheet coupling to the background dilaton field. To avoid this ambiguity, we choose to consider some phenomenologically equivalent background in which the dilaton is trivial. In the vector sector, the soft-wall model can also be described by adding a warp factor to the AdS metric 22:

$$g_{MN}dx^M dx^N = \frac{A(z)}{z^2} \left( \eta_{\mu\nu} dx^\mu dx^\nu - dz^2 \right),$$

with $A(z) = e^{-2\Phi}$. Such kind of metric has been studied first in Ref. 23, but only as a deformation of the hard-wall model. To calculate the quark-antiquark potential, one first defines the following functions:

$$f^2(z) = -g_{00}g_{\mu\nu}$$

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Thanks Oleg Andreev for pointing out this to me.
\[ g^2(z) = -g_{00}g_{zz} \] (32)

Generally, the string worldsheet will not pass the point \( z_m \) where \( f(z) \) has a minimum or \( g(z) \) diverges. When the distance \( L \) between the quark and antiquark is large enough, most of the worldsheet lies close to \( z_m \). This part gives the dominant contribution to the potential, \( E \approx f(z_m)L \). Thus confinement occurs if and only \( f(z_m) \neq 0 \), and the corresponding confining tension is given by \( f(z_m) \) [8]. For the above metric, one has \( f(z) = g(z) = e^{-2\Phi/z^2} \).

The criterion can only be fulfilled when \( \Phi(z) \) goes to \(-\infty\) as \( z \to \infty \). From this one easily concludes that the original background with \( \Phi(z) \sim z^2 \) in Ref. [2] is not confining. For the present choice \( \Phi(z) = -z^2 \), \( f(z) \) has a nonzero minimum \( f(z_m) = 2e \) at \( z_m = \sqrt{2} \), leading to a nonvanishing confining potential. The detailed calculation of the quark potential in such a background confirms the above estimation [7]. At finite temperature, the deconfinement phase transition indeed happens and exhibits interesting properties, see, e.g., [24, 25].

IV. SUMMARY

The properties of a “soft-wall” model with a negative dilaton are discussed. All the physical fluctuations become regular in the infrared region after the modification. The vacuum solution of the field \( X \), which is dual to the quark bilinear operator, tends to a constant in the IR limit. This ensures the Regge behavior of the spectrum of the pseudoscalar and axial mesons. At the same time, this does not lead to chiral symmetry restoration for highly-excited mesons. Under some assumptions, the value of the quark condensate can be determined by requiring the pion being massless in the chiral limit. We also point out that the present choice for the dilaton leads to confinement, while that of the original soft-wall model does not. The unphysical pion mode in the vector channel seems to result from the improper IR boundary condition. Once the boundary condition is relaxed, the unphysical state can be eliminated.

Note Added: While writing this paper, it was noted that Ref. [26] also proposes to modify the sign of the dilaton in the soft-wall model to obtain confinement.
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