Correlations between transfer reactions in nuclear supersymmetry

Roelof Bijker
Instituto de Ciencias Nucleares, Universidad Nacional Autónoma de México, A.P. 70-543, 04510 México, D.F., México
E-mail: bijker@nucleares.unam.mx

Abstract. Nuclear supersymmetry establishes precise correlations among the spectroscopic properties of different nuclei. It is shown that these correlations can be derived in terms of $SU(3)$ isoscalar factors by developing the concept of generalized $F$-spin. An application to one-neutron transfer reactions shows that the mixed symmetry $L = 2$ state is predicted to be excited more strongly than the first excited $L = 2$ state.

1. Introduction
Symmetries have played an important role in nuclear physics. Examples are isospin symmetry, the Wigner supermultiplet theory, special solutions to the Bohr Hamiltonian, the Elliott model, pseudo-spin symmetries and, more recently, dynamical symmetries and supersymmetries of the interacting boson model and its extensions.

Nuclear supersymmetry was proposed more than twenty five years ago [1] in the context of the interacting boson model (IBM) and the interacting boson-fermion model (IBFM) which have proved remarkably successful in providing a unified framework of even-even [2] and odd-even nuclei [3], respectively. One of its most attractive features is that it gives rise to a simple algebraic description, in which dynamical symmetries and supersymmetries play a central role, both as a way to improve our basic understanding of the importance of (super)symmetry in nuclear dynamics, and as a starting point for more precise calculations. Nuclear supersymmetry provides a theoretical framework in which different nuclei are treated as members of the same supermultiplet and whose spectroscopic properties are described by a single Hamiltonian and a single set of transition and transfer operators.

Aside from their esthetic appeal, (super)symmetries provide energy formula, selection rules and closed expressions for electromagnetic transition rates and transfer strengths which can be used as benchmarks to study and interpret the experimental data, even if these symmetries may be valid only approximately. In contrast to the situation in particle physics in which experimental evidence for the existence of supersymmetric particles is yet to be found, nuclear supersymmetry has been verified experimentally.

The purpose of this contribution is to study correlations between different one- and two-nucleon transfer reactions. It is shown that these correlations can be obtained in an explicit and elegant way in the framework of generalized $F$-spin. The ideas are illustrated by an application to one-neutron transfer reaction intensities.
This group chain corresponds to the decomposition of the neutron orbits with $j$ into a pseudo-orbital part with $l$ by supersymmetry [9, 10, 11, 12, 13, 14], in which the odd neutron is allowed to occupy the $3p$ and among bosons, also transformations that change a boson into a fermion and itself from other symmetries in that it includes, in addition to transformations among fermions $3p_{194}$ is provided by the neutron and an odd-odd nucleus. The best experimental evidence of a supersymmetric quartet of freedom [8]. In this case, a supermultiplet consists of an even-even, an odd-proton, an odd-even and odd-even nuclei form the members of a supermultiplet which is characterized a Lie algebra $U(6)$, $\sum j(2j + 1)$ is the dimension of the fermion space [1, 5, 6, 7]. In this framework, even-even and odd-even nuclei form the members of a supermultiplet which is characterized by $N = N + M$, i.e. the total number of bosons and fermions. Supersymmetry distinguishes itself from other symmetries in that it includes, in addition to transformations among fermions and among bosons, also transformations that change a boson into a fermion and vice versa (see Table 1).

The concept of nuclear SUSY was extended in 1985 to include the neutron-proton degree of freedom [8]. In this case, a supermultiplet consists of an even-even, an odd-proton, an odd-neutron and an odd-odd nucleus. The best experimental evidence of a supersymmetric quartet is provided by the $^{194,195}$Pt and $^{195,196}$Au nuclei as an example of the $U_n (6/12) \otimes U_\pi (6/4)$ supersymmetry [9, 10, 11, 12, 13, 14], in which the odd neutron is allowed to occupy the $3p_{1/2}$, $3p_{3/2}$ and $2f_{5/2}$ orbits of the 82-126 shell, and the odd proton the $2d_{3/2}$ orbit of the 50-82 shell. This supermultiplet is characterized by $N_n = 5$ and $N_\pi = 2$. The interpretation of these four nuclei as members of a supersymmetric quartet made it possible to predict [8] the properties of $^{196}$Au almost 15 years before they were measured experimentally [9, 11].

In its most general form, the Hamiltonian is written in terms of the generators of the graded Lie algebra $G = U_\nu (6/12) \otimes U_\pi (6/4)$ which can be reduced to the rotation group as

$$U_\nu (6/12) \otimes U_\pi (6/4) \supset U^{B*} (6) \otimes U^{F*} (12) \otimes U^{B*} (6) \otimes U^{F*} (4) \supset U^B (6) \otimes U^{F*} (6) \otimes U^{F*} (2) \otimes U^{F*} (4) \supset U^{BF*} (6) \otimes U^{F*} (2) \otimes U^{F*} (4) \supset SO^{BF*} (6) \otimes SU^{F*} (4) \otimes U^{F*} (2) \supset Spin(6) \otimes U^{F*} (2) \supset Spin(5) \otimes U^{F*} (2) \supset Spin(3) \otimes SU^{F*} (2) \supset SU(2).$$

(2)

This group chain corresponds to the decomposition of the neutron orbits with $j = 1/2, 3/2, 5/2$ into an pseudo-orbital part with $l = 0, 2$ and a pseudo-spin part $s = 1/2$. Since the pseudo-

| Model  | Generators | Invariant | Symmetry         |
|--------|------------|-----------|-----------------|
| IBM    | $b_1^i b_j$ | $N$       | $U(6)$          |
| IBFM   | $b_1^i b_j$, $a_1^i a_j$ | $N, M$   | $U(6) \otimes U(\Omega)$ |
| SUSY   | $b_1^i b_j$, $a_1^i a_j$, $b_1^j a_j$, $a_1^j b_j$ | $N$       | $U(6/\Omega)$ |

2. Nuclear supersymmetry

Dynamical supersymmetries were introduced in nuclear physics in the context of the Interacting Boson Model (IBM) and its extensions [1]. The IBM describes collective excitations in even-even nuclei in terms of a system of interacting monopole ($s^\dagger$) and quadrupole ($d^\dagger$) bosons [2]. The bosons are associated with the number of correlated proton and neutron pairs, and hence the number of bosons $N$ is half the number of valence nucleons. For odd-mass nuclei the IBM was extended to include single-particle degrees of freedom [4]. The ensuing Interacting Boson-Fermion Model (IBFM) has as its building blocks $N$ bosons ($b_1^i$) with $l = 0, 2$ and $M = 1$ fermion ($a_1^j$) with $j = j_1, j_2, \ldots$ [3]. The IBM and IBFM can be unified into a supersymmetry (SUSY)

$$U(6/\Omega) \supset U(6) \otimes U(\Omega),$$

where $\Omega = \sum j(2j + 1)$ is the dimension of the fermion space [1, 5, 6, 7]. In this framework, even-even and odd-even nuclei form the members of a supermultiplet which is characterized by $N = N + M$, i.e. the total number of bosons and fermions. Supersymmetry distinguishes itself from other symmetries in that it includes, in addition to transformations among fermions and among bosons, also transformations that change a boson into a fermion and vice versa (see Table 1).

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(2)
orbital part of the odd neutron has the same values of angular momentum as the bosons, the boson and fermion chains can be combined at the level of $U(6)$.

If the Hamiltonian is expressed in terms of Casimir invariants of the subgroups appearing in the reduction shown in Eq. (2) a dynamical supersymmetry arises. For example, the Hamiltonian

$$H = AC_{2U} + B C_{2SO} + C_{2Spin} + D C_{2SU} ,$$

(3)
describes the excitation spectra of a quartet of nuclei characterized by $N_e$ and $N_\pi$ [8]. The quartet consists of an even-even nucleus with $N_e = N_\nu$, $M_\nu = 0$ and $N_\pi = N_\pi$, $M_\pi = 0$, an odd-proton nucleus with $N_\nu = N_\nu$, $M_\nu = 0$ and $N_\pi = N_\pi - 1$, $M_\pi = 1$, an odd-neutron nucleus with $N_e = N_e - 1$, $M_e = 1$ and $N_\pi = N_\pi$, $M_\pi = 0$, and an odd-odd nucleus with $N_\nu = N_\nu - 1$, $M_\nu = 1$ and $N_\pi = N_\pi - 1$, $M_\pi = 1$. The energy spectra of the four nuclei belonging to the supersymmetric quartet are described by a single energy formula in terms of the eigenvalues of the Casimir operators

$$E = A [N_1(N_1 + 5) + N_2(N_2 + 3) + N_1(N_1 + 1)]
+ B \left[ \Sigma_1(\Sigma_1 + 4) + \Sigma_2(\Sigma_2 + 2) + \Sigma_3^2 \right]
+ C \left[ \tau_1(\tau_1 + 3) + \tau_2(\tau_2 + 1) \right]
+ D L(L + 1) + E J(J + 1).$$

(4)
The coefficients $A$, $B$, $B'$, $C$, $D$, and $E$ can be determined from the experimental excitation energies.

Recently, the structure of the odd-odd nucleus $^{194}$Ir was investigated by a series of transfer and neutron capture reactions [15]. In particular, the new data from the polarized $(\vec{d}, \alpha)$ transfer reaction provided crucial new information about and insight into the structure of the spectrum of $^{194}$Ir which led to significant changes in the assignment of levels as compared to previous work [16]. Fig. 1 shows the negative parity levels of $^{194}$Ir in comparison with the theoretical spectrum in which it is assumed that these levels originate from the $3p_{3/2}$, $3p_{3/2}$, $2f_{5/2} \otimes 2d_{5/2}$ configuration. The theoretical energy spectrum is calculated using the energy formula of Eq. (4) with $A = 26.3$, $B = 8.7$, $B' = -33.6$, $C = 35.1$, $D = 6.3$, and $E = 4.5$ (all in keV). Given the complex nature of the spectrum of heavy odd-odd nuclei, the agreement is remarkable. There is an almost one-to-one correlation between the experimental and theoretical level schemes [15].

The successful description of the odd-odd nucleus $^{194}$Ir opens the possibility of identifying a second quartet of nuclei in the $A \sim 190$ mass region with $U_{\nu}(6/12) \otimes U_{\pi}(6/4)$ supersymmetry. The new quartet consists of the nuclei $^{192,193}$Os and $^{193,194}$Ir and is characterized by $N_\nu = 5$ and $N_\pi = 3$ [17].

3. Correlations

The nuclei belonging to a supersymmetric quartet are described by a single Hamiltonian, and hence their wave functions are strongly related. Moreover, inspection of the wave functions with $U_{\nu}(6/12) \otimes U_{\pi}(6/4)$ supersymmetry shows that there is a large similarity between the quantum numbers of the even-even and the odd-neutron nuclei

$$|ee\rangle = |N_\nu, N_\pi; [N - f, f], \alpha, L \rangle ,$$
$$|on\rangle = |N_\nu - 1, N_\pi; [N - 1 - f, f], [1]; [N - f - g, f + g - h, h], \alpha, L, \frac{1}{2}, J \rangle .$$

(5)

with $N = N_\nu + N_\pi$ and $0 \leq h \leq g \leq 1$. The square brackets denote $U(6)$ representations and the coefficients $\alpha$ represent all labels in the reduction from $U^{BF}(6)$ to the rotation group. In the case of the odd-neutron nucleus, the angular momentum $L$ is coupled with the pseudo-spin $s = 1/2$ of the neutron orbits to the total angular momentum $J$. We note, that for $h = 0$
The wave functions share the quantum numbers $[\mathcal{N} - f, f], \alpha, L$ (for the odd-neutron nucleus the label $f$ is replaced by $f + g$). The main difference is the way in which the different $U(6)$ representations are coupled. Similar observations hold for the wave functions of the odd-proton and odd-odd nuclei

$$|{\text{op}}\rangle = |[\mathcal{N}_\nu], [\mathcal{N}_\nu - 1]; [\mathcal{N} - 1 - f, f], \alpha, L\rangle,$$

$$|{\text{oo}}\rangle = |[\mathcal{N}_\nu - 1], [\mathcal{N}_\nu - 1]; [\mathcal{N} - 2 - f, f], [1]; [\mathcal{N} - 1 - f - g, f + g - h, h], \alpha, L, \frac{1}{2}; J\rangle.$$

The correspondence between the quantum numbers of the wave functions of nuclei belonging to a supersymmetric quartet suggests that the correlations between transition rates and transfer intensities can be obtained in an explicit manner. In this section, I show how this can be achieved in the framework of the concept of generalized $F$-spin and discuss an application to one-neutron transfer reactions.

### 3.1. Generalized $F$-spin

The correlations between different transfer reactions can be derived in an elegant and explicit way by a generalization of the concept of $F$-spin which was introduced in the neutron-proton IBM (or IBM-2) [18] in order to distinguish between proton and neutron bosons. The eigenstates of the $U(6/12)_\nu \otimes U(6/4)_\pi$ supersymmetry are characterized by the irreducible representations $[N_1, N_2, N_3]$ of $U^{BF}_{\nu}(6)$ which arise from the coupling of three different $U(6)$ representations,
Table 2. \(SU(3)\) quantum numbers of the creation and annihilation operators

| \((\lambda, \mu)\) | \(F\) | \(F_z\) | \(Y\) |
|----------------|------|------|------|
| \(b^1_N\) (1, 0) | \(\frac{1}{2}\) | \(\frac{1}{2}\) | \(\frac{1}{3}\) |
| \(b^1_F\) (1, 0) | \(-\frac{1}{2}\) | \(-\frac{1}{2}\) | \(-\frac{1}{3}\) |
| \(a^1_F\) (1, 0) | 0 | 0 | \(\frac{2}{3}\) |

\([N_\nu]\) for the neutron bosons, \([N_\pi]\) for the proton bosons and \([N_\rho]\) for the pseudo-orbital angular momentum of the odd neutron (\(N_\rho = 0\) for the even-even and odd-proton nuclei of the quartet, and \(N_\rho = 1\) for the odd-neutron and the odd-odd nuclei). In analogy with the three quark flavors in the quark model (u, d and s), also in this case there are three different types of identical objects (\(b^1_N, b^1_F\) and \(a^1_F\)), which can be distinguished by \(F\)-spin and hypercharge \(Y\). The two kinds of bosons form an \(F\)-spin doublet, \(F = 1/2\), with charge states \(F_z = 1/2\) for protons (\(b^1_N\)) and \(F_z = -1/2\) for neutrons (\(b^1_F\)) [18]. In addition, the bosons carry hypercharge \(Y = 1/3\). The pseudo-orbital part of the angular momentum of the odd neutron (\(a^1_F\)) has \(F = F_z = 0\) and \(Y = -2/3\) [19, 20]. The three creation operators span the fundamental triplet representation of \(SU(3)\) with \((\lambda, \mu) = (1, 0)\), and the corresponding annihilation operators the antitriplet with \((\lambda, \mu) = (0, 1)\) (see Table 2).

Group theoretically, the generalized \(F\)-spin is defined by the reduction

\[
\begin{align*}
U(18) & \supset U(6) \otimes U(3) \\
\downarrow & \downarrow \\
[N] & [N_1, N_2, N_3] \\
\downarrow & \downarrow \\
[N_\nu, N_\pi, N_\rho] & [N_\nu, N_\pi - i, i, [N_\rho], [N_1, N_2, N_3]]
\end{align*}
\] (7)

with \(N = N_1 + N_2 + N_3\). Here \(U(6)\) is to be identified with the \(U^{BF}\) of the group reduction of Eq. (2) which is the result of first coupling the bosons at the level of \(U(6)\), followed by coupling the orbital part

\[
[[N_\nu], [N_\pi]; [N_\nu + N_\pi - i, i], [N_\rho]; [N_1, N_2, N_3]] .
\] (8)

This sequence of \(U(6)\) couplings can be described in a completely equivalent way by the three-dimensional index group \(U(3)\) of Eq. (7) which can be reduced to

\[
\begin{align*}
U(3) & \supset SU(3) \supset [SU(2) \supset SO(2)] \otimes U(1) \\
\downarrow & \downarrow \\
| [N_1, N_2, N_3] & \, (\lambda, \mu) \, , \\
\downarrow & \downarrow \\
& F \, , \, F_z \, , \, Y \}
\end{align*}
\] (9)

The relation between the two sets of quantum numbers is given by

\[
\begin{align*}
(\lambda, \mu) &= (N_1 - N_2, N_2 - N_3) , \\
F &= \frac{1}{2} (N_\pi + N_\nu - 2i) , \\
F_z &= \frac{1}{2} (N_\pi - N_\nu) , \\
Y &= \frac{1}{3} (N_\pi + N_\nu - 2N_\rho) .
\end{align*}
\] (10)
The one-to-one correspondence between the $U(6)$ couplings and the generalized $F$-spin basis shown in Eq. (10) makes it possible to reexpress the wave functions of Eqs. (5) and (7) in the $SU(3)$ basis of Eq. (9). As a result, one finds

\begin{align}
|ee\rangle &= \left(\mathcal{N} - 2f, f, \frac{N\pi - N\nu - N^2}{2}, \frac{N^3}{3}; \alpha, L, \frac{1}{2}; J\right), \\
|on\rangle &= \left(\mathcal{N} - 2f - 2g + h, f + g - 2h, \frac{N - 1 - 2f, N\pi - N\nu + 1, N - 3}{2}, \frac{N^3}{3}; \alpha, L, \frac{1}{2}; J\right),
\end{align}

for the even-even and the odd-neutron nuclei, and

\begin{align}
|op\rangle &= \left(\mathcal{N} - 1 - 2f, f, \frac{N\pi - N\nu - 1 - N - 1}{2}, \frac{N^3}{3}; \alpha, L\right), \\
|oo\rangle &= \left(\mathcal{N} - 1 - 2f - 2g + h, f + g - 2h, \frac{N - 2 - 2f, N\pi - N\nu, N - 4}{2}, \frac{N^3}{3}; \alpha, L, \frac{1}{2}; J\right),
\end{align}

for the odd-proton and odd-odd nuclei.

Matrix elements of $SU(3)$ tensor operators can now be obtained by applying the Wigner-Eckart theorem in generalized $F$-spin space, and correlations between different matrix elements can be expressed in terms of ratios of $SU(3)$ isoscalar factors.

### 3.2. One-neutron transfer

As an example of correlations in nuclear supersymmetry, let us consider one-neutron transfer reactions between the even-even and odd-neutron nuclei of the same supermultiplet, which are relevant for the reactions $^{194}\text{Pt} \leftrightarrow ^{195}\text{Os}$ and $^{192}\text{Pt} \leftrightarrow ^{193}\text{Os}$. These transfer reactions are described by the fermionic generators of the graded Lie algebra. In a study of the $^{194}\text{Pt} \leftrightarrow ^{195}\text{Os}$ stripping reaction, it was found [21] that one-neutron $j = 3/2$, $5/2$ transfer reactions can be described by the operator

\begin{equation}
\hat{P}_j = \frac{\alpha_j}{\sqrt{2}} \left[ \left( \hat{s}_\nu \times a_{\nu, j}^\dagger \right)^{(j)} - \left( \hat{d}_\nu \times a_{\nu, -j}^\dagger \right)^{(j)} \right] = \alpha_j T^{(1,1)}_{(2,0,0),(1,0,2),\frac{1}{2}, j},
\end{equation}

The right-hand side of the equation shows the tensorial character under generalized $F$-spin $(\lambda, \mu)$, $F$, $F_z$, $Y$ (upper indices) and under the groups appearing in Eq. (2) in the reduction of $U^{BFv}(6)$ to the rotation group $(\sigma_1, \sigma_2, \sigma_3), (\tau_1, \tau_2)$, $L$, $S$, $J$ (lower indices). Since the quantum numbers of the ground state in the even-even nucleus are given by

\begin{equation}
\left(\mathcal{N}, 0, \frac{N\pi - N\nu}{2}, \frac{N^3}{3}; (\mathcal{N}, 0, 0), (0, 0), 0\right),
\end{equation}

the only states in the odd-neutron nucleus that can be populated by the transfer operator of Eq. (13) are the ones with $f = h = 0$, $g = 0, 1$, $(\tau_1, \tau_2) = (1, 0)$, $L = 2$ and $J = j$. The corresponding matrix elements can be derived by first decoupling the pseudo-spin part of the neutron orbitals and then applying the $SU(3)$ Wigner-Eckart theorem which gives rise to an expression in terms of a $SU(3)$ isoscalar factor and a $SU(3)$ reduced matrix element

\begin{align}
ME_1 &= \left< (\mathcal{N} - 2g, g), \frac{N - 1}{2}, \frac{N\pi - N\nu + 1}{2}, \frac{N^3}{3} - 1; (\sigma_1, \sigma_2, \sigma_3), (1, 0, 2), \frac{1}{2}; j \right| \\
&\quad \times T^{(1,1)}_{(2,0,0),(1,0,2),\frac{1}{2}, j} \left| (\mathcal{N}, 0, \frac{N\pi - N\nu}{2}, \frac{N^3}{3}; (\mathcal{N}, 0, 0), (0, 0), 0) \right> \\
&= -\sqrt{\frac{2j + 1}{5}} \left< \mathcal{N} - 2g, g; (\sigma_1, \sigma_2, \sigma_3), (1, 0, 2) \bigg| T^{(1,1)}_{(2,0,0),(1,0,2)} \left( \mathcal{N}, 0, \frac{N\pi - N\nu}{2}, \frac{N^3}{3} - 1; (\mathcal{N}, 0, 0), (0, 0), 0 \right) \\
&\quad \times (\mathcal{N} - 2g, g); (\sigma_1, \sigma_2, \sigma_3), (1, 0, 2) \right>.
\end{align}
The inverse reaction is described by the transfer operator

\[ P_j^\dagger = \frac{\alpha_j}{\sqrt{2}} \left[ (s_\nu^j \times \tilde{a}_{\nu,j})^{(j)} - (d_\nu^j \times \tilde{a}_{\nu,j})^{(j)} \right] = \alpha_j T_{(2,0,0),(1,0),2,\frac{1}{2},j}^{(1,1),\frac{1}{2},-\frac{1}{2},1}. \]  

(16)

The matrix elements of the operator of Eq. (16) for one-neutron transfer reactions from the ground state of the odd-neutron nucleus to the final even-even nucleus are given by

\[ ME_2 = \langle (N' - 2g, g), \frac{N' - 2g}{2}, \frac{N' - N_\nu}{2}, \frac{N'}{3}; (\sigma_1, \sigma_2, \sigma_3), (1, 0), 2 \parallel T_{(2,0,0),(1,0),2,\frac{1}{2},j}^{(1,1),\frac{1}{2},-\frac{1}{2},1} \nonumber \parallel (N', 0), \frac{N' - 1}{2}, \frac{N' - N_\nu + 1}{2}, \frac{N'}{3} - 1; (N, 0, 0), (0, 0, 0), 1, 2 \rangle \nonumber \]

\[ \left( N' - 2g, g \right); (\sigma_1, \sigma_2, \sigma_3), (1, 0), 2 \parallel T_{(2,0,0),(1,0),2}^{(1,1)} \parallel (N', 0); (N, 0, 0), (0, 0, 0), 0 \rangle. \]  

(17)

Since the reduced matrix elements appearing in Eqs. (16) and (18) are the same, the matrix elements for one-neutron stripping and pick-up reactions are strongly correlated. Their ratio is proportional to that of two SU(3) isoscalar factors

\[ \frac{ME_2}{ME_1} = \frac{\langle N', 0 \rangle}{\langle N, 0 \rangle} \left( \frac{N' - 2g, g}{2}, \frac{N' - N_\nu + 1}{2}, \frac{N'}{3} - 1; (1, 1), \frac{1}{2}, -\frac{1}{2}, 1 \right) \left( \frac{N' - 2g, g}{2}, \frac{N' - N_\nu + 1}{2}, \frac{N'}{3} - 1; (1, 1), \frac{1}{2}, -\frac{1}{2}, 1 \right). \]  

(18)

The isoscalar factors can be derived in the standard way from the recursion relations for the SU(3) step operators combined with the orthonormality conditions, De Swart’s phase convention [22] and Racah’s factorization lemma

\[ \left\langle \begin{array}{ccc} (\lambda_1, \mu_1) & (\lambda_2, \mu_2) & (\lambda, \mu) \\ F_1, F_{1z}, Y_1 & F_2, F_{2z}, Y_2 & F, F_z, Y \end{array} \right\rangle = \left\langle \begin{array}{ccc} (\lambda_1, \mu_1) & (\lambda_2, \mu_2) & (\lambda, \mu) \\ F_1, Y_1 & F_2, Y_2 & F, Y \end{array} \right\rangle \left\langle F_1, F_{1z}, F_2, F_{2z}; F, F_z \right\rangle. \]  

(19)

Table 3 shows the results for some relevant isoscalar factors. A proof by induction made it possible to obtain closed expressions as a function of \( k \) which are valid for all allowed values of \( F \)-spin and hypercharge \( Y \) [23].

The correlations between one-neutron stripping and pick-up reactions can be obtained in an explicit form by applying Racah’s factorization lemma to Eq. (18) and inserting the appropriate values of the SU(3) isoscalar factors from Table 3 and the SU(2) Clebsch-Gordan coefficients

\[ \frac{ME_2}{ME_1} = \begin{cases} -1 & g = 0 \\ \sqrt{\frac{N_\nu}{N_\nu(N - 1)}} & g = 1 \end{cases}. \]  

(20)

It is important to emphasize, that Eqs. (18) and (20) are parameter-independent predictions which are a direct consequence of nuclear supersymmetry and which can be tested experimentally. Tables 4 and 5 show the intensities of one-neutron transfer reactions between even-even and odd-neutron nuclei and vice versa.
The largest intensity is that of the final state with \( (\lambda, \mu) = (\lambda, \mu) = (N-1, 1, 0) \) with relative intensity [17, 21]

\[
I_\lambda(\text{ee} \rightarrow \text{on}) = \frac{(N-1)(N+1)(N+4)}{2(N+2)},
\]

which gives 29.3 for \(^{194}\text{Pt} \rightarrow \text{^{195}Pt} (N_\pi = 2)\) and \(N_\nu = 5\) to be compared to the experimental value of 19.0 for \( j = 5/2 \), and 37.8 for \(^{192}\text{Os} \rightarrow \text{^{193}Os} (N_\pi = 3)\) and \(N_\nu = 5\). The equivalent ratio for the inverse reactions is given by

\[
I_\lambda(\text{on} \rightarrow \text{ee}) = \frac{I_\lambda(\text{ee} \rightarrow \text{on})}{N_\pi(N-1)},
\]

which gives 1.96 for \(^{195}\text{Pt} \rightarrow \text{^{194}Pt} \) and 3.24 for \(^{193}\text{Os} \rightarrow \text{^{192}Os} \). It is important to note that, whereas the final state with \((\sigma_1, \sigma_2, \sigma_3) = (N-1, 1, 0)\) in the odd-neutron nucleus \(^{195}\text{Pt}\) represents the first excited doublet of states at an excitation energy of approximately 100 keV, in the even-even nucleus \(^{194}\text{Pt}\) this configuration corresponds to a highly excited \(L = 2\) state with mixed symmetry at an excitation energy of approximately 2.5 - 3.0 MeV [24]. Despite the reduction in relative intensity due to the correlation factor \(N_\pi/N_\nu(N-1)\), the mixed symmetry state is nevertheless predicted to be excited more strongly than the first excited \(L = 2\) state. This prediction could be tested in future transfer experiments.
Table 5. Intensities of one-neutron transfer reactions between odd-neutron and even-even nuclei

\[ I_n(\text{on} \rightarrow \text{ee}) = |\langle \text{ee} \parallel P_j \parallel \text{on} \rangle|^2. \]

| \( n \) | \( (\lambda, \mu) \) | \( (\sigma_1, \sigma_2, \sigma_3) \) | \( I_n(\text{on} \rightarrow \text{ee}) \) |
|---|---|---|---|
| 1 | \( (\mathcal{N}, 0) \) | \( (\mathcal{N}, 0, 0) \) | \( I_1(\text{ee} \rightarrow \text{on}) \) |
| 2 | \( (\mathcal{N}, 0) \) | \( (\mathcal{N} - 2, 0, 0) \) | \( I_2(\text{ee} \rightarrow \text{on}) \) |
| 3 | \( (\mathcal{N} - 2, 1) \) | \( (\mathcal{N} - 1, 1, 0) \) | \( I_3(\text{ee} \rightarrow \text{on}) \) |
| 4 | \( (\mathcal{N} - 2, 1) \) | \( (\mathcal{N} - 2, 0, 0) \) | \( I_4(\text{ee} \rightarrow \text{on}) \) |

4. Summary and conclusions

In this contribution, I discussed the concept of supersymmetry as used in nuclear physics. Nuclear supersymmetry establishes precise links among the spectroscopic properties of different nuclei belonging to a supermultiplet. Since the wave functions of the members of a supermultiplet are connected by symmetry, there exists a high degree of correlations between different one- and two-nucleon transfer reaction intensities. In order to discuss these correlations in an explicit manner, the concept of generalized \( F \)-spin was developed. As a consequence, the correlations can be expressed in an elegant way in terms of \( SU(3) \) isoscalar factors. The relevant \( SU(3) \) isoscalar factors were derived in a concise way by combining the usual methods based on recurrence relations with a proof by induction.

As an application, I discussed the correlations between one-neutron stripping and pick-up reactions. Nuclear supersymmetry predicts that the \( L = 2 \) mixed symmetry states in the even-even nuclei \(^{194}\text{Pt}\) and \(^{192}\text{Os}\) are excited much stronger (two to three times as strong) than the first excited \( L = 2 \) state. These predictions can be tested experimentally by combining \((\vec{d}, p)\) stripping and \((p, d)\) pick-up reactions.

Finally, it is important to note, that the technique of the generalized \( F \)-spin is valid for any system whose wave functions are described in terms of couplings of three different symmetric representations of \( U(6) \), as for example in nuclear supersymmetry (this contribution), in an algebraic description of the neutron skin in neutron-rich nuclei [25] and in an isospin-invariant extension of the IBM for light nuclei called IBM-3 [26].

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