Project-based learning in mathematics: A literature review

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Abstract. This study is a literature study with a descriptive qualitative research with library research that attempts to describe project-based learning in mathematics learning. In this literature study research the author uses various written sources such as articles, journals and documents that are relevant to the study in this study. This study focuses on the characteristics of project-based learning, steps of project-based learning, and examples of project-based learning assessments, Differences in Project-Based Learning Emphasis and Traditional Learning, Examples of the application of project-based learning in mathematics. The next research that is interesting to be developed as research is project-based learning in increasing High Order Mathematical Thinking (HOTs).

1. Introduction
Real-life problems require an individual to choose strategies and make decisions for solutions in every situation they face. Mathematics is one of the important things that underlies the development of modern technology, has an important role in various disciplines and develops human cognitive abilities [1]. Mathematics needs to be given to all students starting from elementary school to give them the ability to think logically, analytically, systematically, critically, and creatively, the ability to solve various problems in life, and the ability to work together [2–4]. The National Council of Teachers of Mathematics establish that the empowerment program from preschoolers through grade 12 must enable students to: (1) Build new mathematical knowledge solving mathematical problems; (2) Solve problems that arise in mathematics and in other contexts; (3) Implement and adjust various strategies that are appropriate for solving problems; (4) Monitor and reflect on the process of solving mathematical problems [5].

Student cognitive growth is the best maintenance in a social environment where students participate actively where they are helped to reflect on what they learn. For teachers, by developing a reflective classroom where students build new knowledge and learn to manage their own learning, they of course still need teachers to expand class discussions [6]. Another factor is that teaching mathematics at the elementary level, one of which is the delivery strategy [1,7]. The delivery strategy is a component for conducting the teaching process. There are two things that are the focus, which is a way of delivering instructional content to students and how to provide information and materials needed by the students. The teaching delivery strategy used by elementary school teachers has so far been less innovative [8]. In line with the above findings, Al-Hadad explained that the reality in the field was not in accordance with what was expected, mathematics learning still tends to be oriented towards textbooks [9]. Students can only work on math problems based on what the teacher exemplifies [10], if given different problems they will have difficulty in solving them. For this reason, a learning model is needed to be able to provide solutions to problems that occur through an innovative learning model.
Three competency domains that have different acquisition path (psychological processes), including: attitudes, knowledge and skills [11]. Attitudes are obtained through the activities of "accepting, executing, appreciating, experiencing, and practicing". Knowledge is gained through the activities of "remembering, understanding, applying, analyzing, evaluating, and creating". Skills are obtained through the activities of "observing, asking, trying, reasoning, presenting, and creating". Competency characteristics and differences in acquisition pathways also influence the standard characteristics of the process.

To strengthen the scientific approach (scientific), integrated thematic (thematic between subjects), and thematic (in a subject) need to be applied to learning based on disclosure / research (discovery / inquiry learning). To encourage students' ability to produce contextual work both individually and in groups it is strongly recommended to use a learning approach that produces project-based learning (PBL). Correspondingly, Nielson, Du, & Kolmos theorize that learning is not only about acquiring knowledge, but also about creating new knowledge collaboratively when handling complex problems; this requires interdisciplinary knowledge and innovative thinking [12]. Students must now be prepared cognitively and emotionally to collaboratively solve complex challenges that arise in their professional lives. Struyven, Dochy, and Janssens explain that student-focused teaching is based on constructivism teaching methods. The constructivist perspective views learning as an experiential process. Active learning strategies involve and demand students through discovery, this approach is proven to be able to encourage students in meaningful learning and produce quality grades Stryven [12–15].

One learning that creates an active, collaborative atmosphere, and can increase self-confidence in students is Project-based Learning [16–19] describes that the PBL is a learning model that focuses on the main concepts and principles of a discipline, engages students in problem solving and reasoning activities, gives students opportunities to work autonomously to construct their own learning, and produce products student work. PBL is part of the learning process that places a strong emphasis on problem solving as a collaborative effort [20,21]. The learning model of the PBL centered on a relatively timed process, focusing on problems, meaningful learning units by integrating concepts from a number of components of knowledge, or discipline, or field of study [21–23]. The focus of learning lies in the concepts and core principles of a study discipline, involving students in investigating problem solving and other meaningful tasks, giving students the opportunity to work autonomously to construct their own knowledge. Blumenfeld, et al. [24] for example, described the process of project-based science as follows:

_The presumption is that students need opportunities to construct knowledge by solving real problems through asking and refining questions, designing and conducting investigations, gathering, analysing, and interpreting information and data, drawing conclusions, and reporting findings._ (p. 150)

Blumenfeld et al. states that in addition to helping students gain an understanding of a concept, project-based learning also functions as a bridge between phenomena in the classroom and experiences in real life [22]. As described in the following statement.

_Proponents of project-based learning claim that as students investigate and seek resolutions to problems, they acquire an understanding of key principles and concepts. Project-based learning also places students in realistic, contextualized problem-solving environments. In so doing, projects can serve to build bridges between phenomena in the classroom and real-life experiences; the questions and answers that arise in their daily enterprise are given value and are shown to be open to systematic inquiry. Hence, project-based education requires active engagement of students' effort over an extended period of time. Project-based learning also promotes links among subject matter disciplines and presents an expanded, rather than narrow, view of subject matter._

2. Method
This study is a qualitative study with a descriptive analysis technique with a library research where this research seeks to describe existing phenomena, which are taking place now or in the past. This article highlights the concept of project-based learning in mathematics.
3. Result and Discussion

3.1 Project-based learning

The Buck Institute for Education states that project-based learning is a learning model that engages students in problem-solving activities and provides opportunities for students to work autonomously to construct their own learning, and ultimately produce valuable and realistic student work products.

Projects are complex tasks, based on challenging questions or problems, which involve students in design, problem solving, decision making, or investigative activities; giving students the opportunity to work independently for long periods of time; and lead to a realistic product or presentation [25]. Klein et al. explained that project-based learning is a learning strategy that empowers students to gain new knowledge and understanding based on their experiences through various presentations [11]. The characteristics of project-based learning are: (1) students investigate important ideas and ask questions; (2) students find understanding in the process of investigating, according to their needs and interests; (3) students produce and creative thinking, critical and skilled to investigate, concluded the material, as well as connecting with real-world problems, authentic and issues.

Olson explains that in project-based learning, students plan and carry out investigations on several topics or themes that use cross-subjects or cross-material [11]. In line with the statements above, Prabowo and Puadi, et al states that the PBL has advantages as a learning environment; (1) authentic contextual (goal-directed activities) that will strengthen the relationship between the activity and conceptual knowledge that underlies it; (2) promoting learning autonomy (self regulation) and the teacher as a guide and learning partner who will develop productive thinking skills; (3) collaborative learning that gives students opportunities to learn from each other which will improve conceptual understanding and technical skills; (4) realistic, oriented towards active learning to solve real problems, which contributes to the development of problem solving skills; (5) provide internal feedback that can sharpen thinking skills [26,27].

Thomas states that other advantages of project-based learning, namely: (1) increasing student motivation; (2) improve problem solving skills; (3) increase collaboration. The importance of group work in projects causes students to be able to develop and practice students' communication skills and scientific performance; and (4) improving resource management skills that are responsible for completing complex tasks [25]. Project-based learning has the characteristics that students can choose topics and/or project presentations/products, produce final products such as presentations, recommendations for solving problems related to the real world, involve various disciplines, vary in duration of time, display the teacher in roles facilitator [28] Learners explore, assess, interpret, synthesize, and information to produce various forms of learning outcomes. Project-based learning is a learning method that uses problems as a first step in gathering and integrating new knowledge based on experience in real activities. Through project-based learning, the inquiry process starts with raising guiding questions and guiding students in a collaborative project that integrates various subjects (material) in the curriculum.

3.2 Characteristics of Project Based Learning

Project-based learning has characteristics that distinguish it from other learning models. The Buck Institute for mentions the characteristics of project-based learning, including: (1) The contents in Project-based learning are focused on students' ideas, namely in forming their own picture of working on relevant topics and student interests that are balanced with student experience, daily; (2) Conditions for encouraging students to be independent, that is in managing their tasks and study time; (3) Effective and interesting activities strategies, that is in seeking answers to questions and solving problems using skills; (4) Application of productive results in helping students develop learning skills and integrate in perfect learning, including strategies and the ability to use cognitive problem solving strategies.

3.3 Steps in Project Based Learning

The steps of learning in Project-based learning as developed by The George Lucas Educational Foundation consist of: (1) Beginning with essential questions; (2) The planning rules of the project; (3)
Make a schedule of activities; (4) Monitoring student project development; (5) Evaluation of students' work; (6) Evaluation of students' learning experiences.

3.4 Assessment Model

Project appraisal is an assessment of a task that must be completed within a certain period or time. The task is in the form of an investigation since the planning, data collection, organizing, processing and presenting data. Project appraisal can be used to clearly understand the understanding, the ability to apply, the ability to investigate and the ability of students to provide information about something that becomes their investigation. In project appraisal there are 3 (three) things to consider, namely: (1) Management ability, namely the ability of students to choose a topic if it has not been determined by the teacher, looking for information and managing the time of data collection and report writing; (2) Relevance, that is suitability with subjects by considering the stages of knowledge, understanding and skills in learning; (3) Authenticity, i.e. the projects undertaken by students must be the result of their work, taking into account the teacher's contribution in the form of instructions and support for student projects.

Project appraisal is carried out starting from the planning, the work process, to the final results of the project. To that end, educators need to determine the things or stages that need to be assessed, such as the preparation of designs, data collection, data analysis, and preparation of written reports. Task reports or research results can also be presented in the form of posters. Implementation of assessment can use assessment tools / instruments in the form of a checklist or rating scale.

Project-based learning design has several applications that must be met, including: (1) focused on questions or problems, which encourage students to undergo (with hard work) concepts and core principles or subject matter from a scientific discipline; (2) Investigation in the form of a design process, decision making, problem discovery, problem solving, discovery, or the process of building a model, the core activities of the project include the transformation and construction of student knowledge; (3) projects in project-based learning are not created by the teacher, written in the script, or bundled, but there is also the involvement of students in the selection of context and determination of work time; and (4) Project-based learning involves real life challenges, focusing on authentic (not simulative) questions or problems and their solutions have the potential to be applied in real fields of Bereiter & Scardamalia [16].

3.5 Examples of the application of project-based learning in mathematics

The application of the project-based learning model is material in 8th grade which is about numbers (1) Make generalizations of patterns in rows of numbers and rows of object configurations (2) Solve problems related to patterns in rows of numbers and object configurations. Packed in five (5) meetings with an additional one meeting related to project assignments with indicators of competency achievement are: (1) be able to determine the next terms of a number pattern; (2) 2. be able to determine the rules of a given number pattern; (3) be able to find elements in each number pattern; (4) be able to explain the understanding of arithmetic sequences and sequences; (5) be able to explain the understanding of geometric sequence and sequence; (6) be able to determine the next terms in arithmetic sequence; (7) be able to determine the next terms in the geometry row; (8) be able to find elements in arithmetic ranks; (9) be able to find elements in the geometric sequence; (10) be able to distinguish patterns, sequences and sequences; (11) solving problems related to number patterns.

To achieve these indicators five (5) meetings. The first meeting is to teach the students associated with the pattern integer numbers including triangular pattern, a pattern square numbers, number patterns rectangle, with one model of learning. The second meeting is to teach the students related to pascal triangle pattern on the number, sequence and series of arithmetic and geometric sequence and series with one model of learning. The third meeting is to teach the students associated with the pattern in which the Fibonacci numbers on duty informed cover activities undertaken during the 4th and 5th meeting. Project Worksheets which were informed at the 3rd meeting as follows:
Leonardo da Pisa (1175 - 1250) or Leonardo Pisano, better known as Fibonacci, is an Italian mathematician known as the discoverer of Fibonacci numbers. Leonardo acted as the person who introduced the system of writing and calculating Arabic numbers to the European world. Father of Leonardo, Guilielmo (William) has a nickname Bonacci which means "good" or "simple". After he died, Leonardo was often referred to by the name of Fibonacci (from the word philius Bonacci, son of Bonacci).

William led a trading post (some accounts say he was a trade representative for Pisa) in Bugia, North Africa (now Named, Algeria).

As a young child, Leonardo traveled there to help his father. It was there that Leonardo learned about the Arabic number system. Seeing the Arabic number system is simpler and more efficient than Roman numbers, Fibonacci then traveled to the Mediterranean region to learn from the famous Arab mathematicians at that time. Leonardo just returned home around the 1200s. In 1202, at the age of 27, he wrote down the knowledge he had learned in the book Liber Abaci or the Book of Calculation. This book shows the practicality of the Arabic number system by applying it to commercial accounting, conversion of various sizes and weights, interest calculations, money exchanges, and various other applications. This book was welcomed by European scholars, and had an important impact on European thought, although its use was only widespread after the discovery of printing about the next three centuries.

1. Make a project implementation plan with a group team that has been formed, including the division of tasks of each group member, compile a schedule for completing the task, carry out the project, make the project results in the form of presentation presentations or wall magazines, make invitations for 8th grade representatives from other classes, teachers subject and mathematics Principals to attend project presentations, make presentations related to project results.

2. Make observations in the nature around us that have a pattern of Fibonacci numbers. Make a presentation or wall magazine related to Fibonacci Numbers. The presentation includes:
   a) A brief history of Fibonacci numbers.
   b) How to get the golden ratio (Golden Ratio) of Fibonacci numbers.
   c) Examples of objects in nature that have Fibonacci number patterns and / or gold comparisons (can take the example given by the teacher).

3. Present in front of your classmates and 8th grade representatives from other classes at your school.

4. The 4th meeting is to learn students related to solving problems in number patterns through project assignments.

5. The 5th meeting is to learn students related to solving problems in number patterns through project assignments.

Figure 1. Project Worksheet

Figure 1. shows that The steps of project-based learning are: (1) Determination of fundamental questions; (2) Design (plan) completion of the task, in this step design (plan) completion of project tasks with the group team that has been formed, which includes activities to be carried out, time allocation, arranging schedules and division of each task members of the group. The project implementation schedule is at the 4th meeting and for one week after the 4th meeting before the 5th meeting where during the one week students consult with the teacher about the completion of the project assignment while at the 5th meeting is the presentation of project results; (3) Making a project completion schedule; (4) Evaluating experience.

4. Conclusion
Project-based learning with an authentic learning experiences approach adapted from Thomas in [16] explains that learning is carried out in three stages, namely: (1) the preparation phase; (2) the PBL process stage; and (3) evaluation phase. The preparation stage is the standard stage of learning introduction to explore information and create a schedule of participants and direct students to introduce themselves to each other and collect expectations of the implementation of project activities. The stages
of the PBL process are the main stages of learning and consist of a number of activities regarding the preparation and important steps of project teaching. The evaluation phase is a form of activity in assessing the results of student project work. PBL learning process requires a teacher to prepare well the tools used by students through stages: what is the description of the school, challenging investigations, description of PBL learning, determining the technology used by students (email, googledoc, edmodo, glogster, schoology, etc.) then the last stage is how a teacher reflects on the assignments given to students.

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