Nonthermal production of gravitinos and inflatinos

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We explicitly calculate nonthermal gravitino production during the preheating period in the inflationary Universe. Contrary to earlier investigations, we consider a two-field model to separate the mechanisms of supersymmetry breaking and inflation. We show that the superpartner of the inflaton is significantly generated, while the gravitino production is considerably smaller. Nonthermal production of gravitinos seems thus less worrisome than recently claimed.

I. INTRODUCTION

Inflationary models, when embedded in particle-physics motivated schemes, are usually constrained by the requirements of consistency with the phenomenology of the later evolution of the Universe. In particular, in the context of supergravity theories the parameters of the inflationary sector have to be restricted in order to avoid the thermal overproduction of gravitinos at reheating. These constraints, in the case of models with gravity-mediated supersymmetry breaking, can be summarized as an upper limit on the reheating temperature of the order of $10^{10}$ GeV (for a review, see [1]). In the last years it was shown that, during the stage of coherent oscillations of the inflaton condensate right after inflation, preheating can lead to an efficient nonthermal production of fermions [2]. In particular, it was recently argued [3–6] that the nonthermal production of 1/2–helicity component of gravitinos can in some cases be much more efficient than the thermal one, thus worsening the gravitino problem. The investigation that led to the present letter was actually triggered by this last statement. Explicit calculations of the amount of gravitinos produced at preheating were performed so far only in models with one superfield and without supersymmetry breaking in the vacuum. Therefore, the conclusions about the production of longitudinal gravitinos in these models could be misleading, since there is no longitudinal gravitino in the vacuum of the theory. Thus, one might wonder whether preheating could actually lead to a production of harmless inflatinos rather than of dangerous gravitinos.

In order to discriminate between inflatino and gravitino production it is necessary to consider more realistic schemes. The simplest possibility is to consider two separate sectors, one of which drives inflation, while the second is responsible for supersymmetry breaking today. In the present analysis we study the situation in which the two sectors communicate only gravitationally. The calculation of nonthermal gravitino production in this scheme requires substantial work. First, one has to develop a new formalism, to be able to clearly define and compute the production in systems with several coupled fields. Subsequently, an extended numerical investigation has to be carried out to obtain reliable results. In fact, this procedure led us to conclusions that do not confirm earlier analytical estimates [7].

The paper is structured as follows. First, we define the specific model considered, analyzing the evolution of the scalar fields. Then, we discuss how the definition of the gravitino varies in time according to the evolution of the background. We finally compute the spectrum of the gravitinos produced at preheating, concluding that what in the previous works was believed to be gravitino overproduction should rather be regarded as inflatino production.

II. GRAVITINO PRODUCTION IN A TWO FIELD MODEL

The system we are considering has the matter content of two superfields $\Phi$ and $S$, with superpotential

$$ W = \frac{m_\phi}{2} \Phi^2 + \mu^2 (\beta + S) $$ (1)

and with minimal Kähler potential $\mathcal{K} = \Phi^\dagger \Phi + S^\dagger S$. The field $\phi$ (that is, the scalar component of $\Phi$) acts as the inflaton. As it is known, the corrections from the Kähler potential to the scalar potential are very relevant when $\langle \phi \rangle \gtrsim M_p$. This is a common problem for supersymmetric theories of inflation, where $F$–terms usually spoil the flatness of the inflaton potential during inflation (for a review, see [8]). This is in particular true for the above superpotential (1), so that additional contributions – possibly additional scalar fields (as for example in $D$ term inflation [8]) – must be relevant during inflation. However, we are interested in the dynamics of the system after inflation, when $\langle \phi \rangle \lesssim M_p$, and supergravity corrections are not important. We then assume that in the reheating stage only one of the fields which were...
driving inflation is still relevant, and we refer to it as the inflaton. We also assume $m_\phi \sim 10^{13}$ GeV, as required by the COBE normalization of the CMB fluctuations for the “usual” chaotic inflation.

The superfield $S$ leads to the breaking of supersymmetry in the true vacuum owing to its “Polonyi” superpotential $W$. By imposing $\beta = (2 - \sqrt{3}) M_p$, one can indeed break supersymmetry while retaining a vanishing cosmological constant in the true vacuum, where the fields $s$ (the scalar component of the superfield $S$) has expectation value of the order of $M_p$. The gravitino mass in the vacuum is of the order $\mu^2 / M_p$. In order to have a gravitino mass of about 100 GeV (that is the expected value for the gravitino mass in gravity–mediated supersymmetry breaking models), $\mu \sim 10^{10}$ GeV is required.

The superpotential $W$ describes a system where supersymmetry is broken in a “hidden” sector and is transmitted gravitationally to a “visible” one, to whom the inflaton belongs. This separation between the two sectors is suggested by the two different mass scales which characterize inflation (inflaton mass $\sim 10^{13}$ GeV) and supersymmetry breakdown (gravitino mass $\sim 10^{2-3}$ GeV).

Right after inflation the field $\phi$ is oscillating about the bottom of its potential with frequency proportional to $m_\phi$. The time dependent expectation value of $\phi$ acts as an effective mass for the Polonyi scalar, which has vanishing expectation value at this stage. In this initial period the (time dependent) expectation value of $\phi$ is the main source of supersymmetry breaking. The amplitude of the oscillations of the field $\phi$ eventually decreases, due to the expansion of the Universe, and for times of order of $m_{3/2}^{-1}$ the Polonyi scalar starts rolling down towards its true minimum and then oscillates about it.

The system is thus governed by two time scales. At “early” times, of the order of $m_{3/2}^{-1}$, the only relevant dynamics is the one of the inflaton sector, that is also the main source of supersymmetry breaking. At “late” times, much larger than $m_{3/2}^{-1}$, the system behaves as if it was in its true vacuum, and supersymmetry is broken by the Polonyi sector. To be more specific, we define the dimensionless parameter $\hat{\mu}^2 \equiv \mu^2 / (m_\phi M_p) \sim m_{3/2} / m_\phi$, that gives the ratio of the two time scales in the system. If supersymmetry is supposed to solve the hierarchy problem, $\hat{\mu}^2$ should be of the order of $10^{-11}$. Such a small parameter implies a very large difference between the two time scales of the problem, which is a source of technical difficulties in the numerical computations. As a consequence, we could not study the evolution of the system for such a small value of $\hat{\mu}^2$. Thus, we kept it as a free parameter and we studied how a variation of $\hat{\mu}^2$ affects the scaling of the relevant quantities.

To make the difference in the two time scales that govern the system manifest, we show in fig. 1 the evolution of the inflaton and the Polonyi scalar for the case $\hat{\mu}^2 = 10^{-2}$.

![Fig. 1. Evolution of the two scalar fields $\phi$ and $s$ for $\hat{\mu}^2 = 10^{-2}$.](image)

We can indeed see the two different stages of the scalar evolution, the first of which is characterized by the inflaton oscillations, while the second is governed by the dynamics of the Polonyi field. While $\langle s \rangle$ gives the supersymmetry breaking in the vacuum, both scalar fields contribute to break supersymmetry during their evolution. In particular, both their kinetic and potential energies contribute to the breaking, as emphasized in $\hat{\mu}^2$. Following $\hat{\mu}^2$, we define

$$f_i^2 \equiv m_i^2 + \frac{1}{2} \left( \frac{d\phi_i}{dt} \right)^2,$$

with $m_i = \exp \left( K M_p^{-2} / 2 \right) \left[ \partial_i W + M_p^{-2} \partial_i K W \right]$. The quantities $f_i$ give a “measure” of the size of the supersymmetry breaking provided by the $F$ term associated with the $i$–th scalar field. More precisely, we will be interested in the normalized quantities $r_i \equiv f_i^2 / (f_1^2 + f_2^2)$, which indicate the relative contribution of the two scalars.

In fig. 2 we show the evolution of $r_\phi$ and $r_s$ for the specific case $\hat{\mu}^2 = 10^{-2}$. As it occurs for all values of $\hat{\mu}^2$, in the initial stages $r_\phi \simeq 1$, while $r_s \simeq 1$ at the end. The regime of equal contribution is around $t m_\phi = \hat{\mu}^{-2}$.

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*In general, the presence of the Polonyi sector will affect the inflationary dynamics (as discussed in [1]). Again, since we are not interested in the inflationary expansion itself, we will not consider this issue here.

†There is of course a possible moduli problem associated with these oscillations. However, we do not consider this issue here.
As a starting point, one has to diagonalize (at each time) the coupled $\theta-\Upsilon$ system. We denote the two fermionic mass eigenstates by $\psi_1$ and $\psi_2$. In fig. 3 we show the evolution of their masses for the specific case $\tilde{\mu}^2 = 10^{-2}$. The most relevant information which can be derived by this evolution is very clear: at late times the fields $\psi_1$ and $\psi_2$ have, respectively, the mass of the inflatino and of the gravitino field. That is, at late times we have the identification $\psi_1 \equiv \phi \equiv \Upsilon$, $\psi_2 \equiv \theta$ ($s = v = 0$, being the goldstino). This situation is orthogonal to the initial one, when $\psi_1 \equiv \theta$, $\psi_2 \equiv s \equiv \Upsilon$ ($\phi = v = 0$). At intermediate times, when supersymmetry is broken by both scalar fields, the gravitino is a mixture of $\psi_1$ and $\psi_2$.

To qualitatively appreciate the evolution of the gravitino occupation number, we may consider $N_{\theta} \equiv r_1 N_1 + r_2 N_2$ and the orthogonal combination $N_{\Upsilon} \equiv r_2 N_1 + r_1 N_2$, where $r_i$ are the relative contributions of the two scalars to supersymmetry breaking defined above.

Let us now consider the fermionic content. We denote the fermions of the two chiral multiplets by $\phi$ (the “inflatino”) and $s$ (the “Polonyino”). One linear combination of them is the goldstino $\nu$, while the one orthogonal to $\nu$ is denoted by $\Upsilon$. Initially, $\nu \equiv \bar{\phi}$, while $\nu \equiv s$ at late times. In addition, we have the gravitino field, whose longitudinal and transverse component are denoted by $\theta$ and by $\psi_T^I$, respectively. The transverse component is decoupled from the other fermion fields, and its quanta are produced only gravitationally. \textsuperscript{[3]}\textsuperscript{[4]}\textsuperscript{[12]}. It will not be considered in the remainder of this work. The longitudinal gravitino component, which in the super-higgs mechanism is provided by the goldstino $\nu$, is however coupled with $\Upsilon$. Equations of motion for the coupled system are given in [3]. The step from these equations to the occupation numbers of the fermions $\theta$ and $\Upsilon$ is far from trivial and requires a significant extension of the existing formalism for nonperturbative production in the one field case. We will present the details in a separate publication [3]. In the remainder of this work we will only present our results.

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{fig2}
\caption{Relative contribution of the two scalar fields $\phi$ and $s$ to the supersymmetry breaking during their evolution. As in fig. 1, $\tilde{\mu}^2 = 10^{-2}$.}
\end{figure}

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{fig3}
\caption{Evolution of the masses of the two fermionic eigenstates. As in fig. 1, $\tilde{\mu}^2 = 10^{-2}$. Notice the different normalizations for the two masses.}
\end{figure}

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{fig4}
\caption{Evolution of $N_{\theta}$ and $N_{\Upsilon}$ for $\tilde{\mu}^2 = 10^{-2}$ and $k = m_{\phi}$. See the text for details.}
\end{figure}

We show in fig. 3 the evolution of $N_1$, $N_2$, $N_{\theta}$, $N_{\Upsilon}$ for modes of comoving momentum $k = m_{\phi}$ and for $\tilde{\mu}^2 = 10^{-2}$. Notice that (by construction) $N_{\theta} \equiv N_1$ at early times, while $N_\theta \equiv N_2$ at late ones. We also see that $\psi_1$ is populated on time scales $m_{\phi}^{-1}$, while $\psi_2$ on time scales $\tilde{\mu}^{-2} m_{\phi}^{-1}$. This feature is common for all $\tilde{\mu}^2$ [3]. We remark that the identification $\theta \equiv r_1 \psi_1 + r_2 \psi_2$ should be taken only as a qualitative indication. However, the most relevant identification $\theta \equiv \psi_2$ at late times is a rigorous one, as should be clear from the above discussion.

We are now ready to present our most important result: the occupation number of $\Upsilon$ and $\theta$ at the end of the process. We show them in figs. 4 and 5 respectively.

\textsuperscript{1}From fig. 3 one may be tempted to identify $\psi_1 \equiv \tilde{\phi}$ and $\psi_2 \equiv \tilde{s}$. Although this identification is rigorous only at the beginning and at the end of the evolution, it can be used for an “intuitive” understanding of the system.

\textsuperscript{2}These spectra are shown at the time $t = 10 \tilde{\mu}^{-2} m_{\phi}^{-1}$. In
The time required for the numerical computation increases linearly with \( \tilde{\mu}^2 \), so that we could perform it up to \( \tilde{\mu} = 10^{-6} \). The realistic case \( \tilde{\mu}^2 = 10^{-11} \) is far from our available resources, so we kept \( \tilde{\mu}^2 \) as a free parameter. The case \( \tilde{\mu}^2 = 10^{-11} \) can be clearly extrapolated from the results we are going to discuss. Moreover, the case \( \tilde{\mu}^2 = 0 \) can be studied analytically, and it agrees with the limit \( \tilde{\mu} \to 0 \) that one deduces from the numerical results. For \( \tilde{\mu} = 0 \), only the inflatino is produced. The mass of the Polonyi fermion does instead vanish identically, so no quanta of this particle are produced at preheating. Notice that \( \tilde{\mu} = 0 \) corresponds to a situation with unbroken susy in the vacuum, and it reproduces the models with one single field studied so far. We stress that in this case only the inflatino is produced at preheating.

In fig. 6 the spectrum of \( \psi_1 \) in shown for \( \tilde{\mu}^2 = 10^{-2}, 10^{-3}, 10^{-4}, 10^{-5}, 10^{-6} \). It is apparent that the main features of the spectrum are independent of the value of \( \tilde{\mu}^2 \). The reason for this is that this eigenstate is associated to the inflatino, that is produced by the coherent oscillations of the inflaton. The inflaton dynamics occurs on time scales of the order of \( m_\phi^{-1} \), and is independent on \( \tilde{\mu}^2 \). The spectrum given in fig. 6 shows a more significant dependence on \( \tilde{\mu}^2 \). In particular, as in the case of the spectrum in fig. 5, the occupation number is of the order of unity if the comoving momenta are smaller than some cut–off \( k_* \). At variance with the spectrum of \( \psi_1 \), however, the cut–off depends on the value of \( \tilde{\mu}^2 \).

The total number of particles produced in this case is a increasing function of \( \tilde{\mu}^2 \).

It is possible to estimate in a very simple way how \( k_* \) scales with \( \tilde{\mu}^2 \). The cut–off in the spectrum (in terms of the comoving momentum) of particles produced at preheating is indeed generically proportional to \( \sqrt{\dot{m} a} \), where \( \dot{m} \) is the time–derivative of the effective mass of the produced particle, and \( a \) is the scale factor of the Universe at the time of preheating. In the case of the Polonyino, the typical mass scale is \( m_{3/2} \), while the typical time scale is \( m_{3/2}^{-1} \). As a consequence, \( \sqrt{\dot{m} a} \sim m_{3/2}^{-1/2} \). When the energy in the Universe is dominated by the sinusoidal oscillations of a massive scalar field, \( a(t) \sim t^{2/3} \sim (m_{3/2})^{-2/3} \). Collecting all these estimates, we obtain \( k_* \sim m_\phi \left( \tilde{\mu}^2 \right)^{1/3} \). Although this is quite a simple estimate, it shows very good agreement with the numerical results. Moreover, it agrees with the fact that \( N_\phi = 0 \) for \( \tilde{\mu}^2 = 0 \).

In conclusion, our calculation confirms that one fermionic eigenstate is efficiently produced at preheating. However, as we have initially remarked, in more realistic models (i.e. with more than one chiral superfield and supersymmetry broken in the vacuum) this fermionic field should be regarded as the “inflatino” rather than the “gravitino”. We have shown that gravitino production is significantly reduced if the sector responsible for supersymmetry breaking today is coupled only gravitationally to the one responsible for inflation. The reason for this small production is related to the smallness of the supersymmetry breaking scale (i.e. of \( \tilde{\mu}^2 \) with respect to the scale of inflation.

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