Study on the decision considering imperfect maintenance of multiple components based on the exponential distribution

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Abstract. The real-time monitoring and the high reliability system of multiple components are studied in the mechanical system as background information. The characteristics of copula among multiple component of different systems are considered. The information of affected components is monitoring as covariate. With the prediction model of remaining useful life (RUL) based on stochastic filtering theory, and the strategy of opportunity maintenance, the maintenance decision model of dynamic is studied on the prediction information of remaining useful life. Lastly, the relevant theories are verified by simulation.

1. Introduction

In mechanical system, the correlation characteristics between components affect the degradation and the changes situation of degradation, which directly or indirectly affect remaining useful life distribution. Therefore, it is very important to study the prediction method of remaining useful life based on this in the process of operation system.

Recently, the state has provided great policy support in the life prediction of key products and major facilities. The outline of the national program for medium and long-term scientific and technological development (2006-2020), and manufacturing technology in the field of advanced manufacturing technology of national 863 plans, mainly support major products and major facility life prediction technology as cutting-edge technologies and one of the special topics [1].

Many researchers at home and abroad have conducted in-depth research on this issue. Do et al. [2] studied the situational decision model between perfect and imperfect maintenance. Nakagawa et al. [3] researched the maintenance decision model considering the limitation of maintenance time. Qureshi et al. [4] proposed to apply Proportional Intensity Model (PIM) for situational maintenance decision, which PIM relaxes the hypothesis condition of PHM and widen the application fields. Kvam et al. [5] studied the randomly correlated maintenance strategies of multiple parallel components. They assumed that all components had the same fault distribution function and fault rate function, and analysed the causes of the failure, and if one part failed, the failure rate of other components changed linearly. Nicolai et al. [6] considered the maintenance strategy of multiple components and the random and economic correlation, assuming that failure of one component affects the failure rate of other components. Koochaki et al. [7] studied the group maintenance strategy of a three-part series system. By adjusting the number of components in the group and degradation threshold of preventive replacement, the system of maintenance cost was reduced. Zhu [8], considering the economic dependence, studied the situational maintenance strategy of all components, which replaced preventive using the degradation value more
than preventive replacement threshold, and established the minimum fee model using decision variables including the maintenance time and the degradation threshold.

In conclusion, this paper takes into account the correlation theory between continuous degradation states among mechanical components mechanical system. Based on pair copula function and random filtering theory, the predictive model of remaining useful life of components is set up that it usually satisfies the character of the exponential distributions.

2. Reliability Analysis of Components Satisfied the Exponential Distribution

Exponential distribution includes many common distributions, such as Weibull distribution, Gamma distribution, Lognormal distribution, etc. It is of general significance to study the reliability of components by using related theories of exponential distribution. In particular, the mixed distribution of exponential distribution includes multiple models and the life of actual components. In this paper, the residual life of multiple components satisfying exponential distributions and dependency is analysed.

2.1. The residual life function of the exponential distribution

Suppose \( z_i \) is the real-time monitoring data received by a component at \( t_i \), and its remaining life is

\[
S_i = \{ T - t_i | T > 0, S_i \} 
\]

\( S_i \) is present information and historical monitoring information of one component at \( t_i \). That is,

\[
S_i = \{ z_i \leq Z_i \leq z_i + \Delta z_i, S_{i-1} \} 
\]

\[
= \{ z_1 \leq Z_1 \leq z_1 + \Delta z_1, z_2 \leq Z_2 \leq z_2 + \Delta z_2, \cdots, z_i \leq Z_i \leq z_i + \Delta z_i \} 
\]

In which, \( \Delta z_1, \Delta z_2, \cdots, \Delta z_i \to 0 \), \( S_i = \{ z_1, z_2, \cdots, z_i \} \). Therefore, at \( t_i \), the probability density function of one component satisfying the exponential distribution can be expressed as

\[
f_i(t, \theta | S_i) = g_i(\theta) \prod_{i\in L} \exp[h_i(t, \theta)] = g_i(\theta) \exp \left[ \sum_{i\in L} h_i(t, \theta) \right] 
\]

The function of remaining useful life is expressed as

\[
R_i(t, \theta) = \int_{t}^{+\infty} f_i(t, \theta | S_i) dt = \int_{t}^{+\infty} g_i(\theta) \exp \left[ \sum_{i\in L} h_i(t, \theta) \right] dt 
\]

In which, \( g_i(\theta) > 0 \) (\( i = 1, 2, \ldots, n \)) is the parameter function. \( h_i(t, \theta) \) is the inner second derivative in \( (0, +\infty) \). \( \theta \) is a constant variable.

2.2. Remaining useful life model of multi-component system with copula theory

Suppose there are \( n \) components in a mechanical system, and some interact with each other. Therefore, the component can be divided into two categories in the system: the dependency relationship between components (Ip) and the independent relationship between components (I2 part). Ip refers to the change of the remaining life which is not only affected by the component itself degradation, but also influenced by other several degradation components. I2 part refers to the change of the residual life that is not affected by other degradation of several components, and do not affect the degradation of other components. Namely degradation among components is independent.

Considering the relationship between different components that exist in different degrees, on the basis of pair-copulas theory by K Aas, C Czado, A Frigessi (2007), the distribution function of corresponding multivariate copulas connection is built with the method of step by step. So the problem of the derivative calculation is solved by the marginal density function. At the same time, several different types of copulas connect distribution functions and greatly enhance model practicability. Therefore, the pair-copula method is used in this section.

Assuming \( k \) of Ip receives the real-time monitoring data \( z^{(k)}_i \) at \( t_i \).
\( f_i^{(k)}(t, \theta | s_i^{(k)}, s_i^{(j)}) \) is the probability density function of remaining useful life. \( T_i^{(k)} \) means the remaining useful life,

\[
T_i^{(k)} = \{ T^{(k)} - t \mid T^{(k)} > 0, S_i^{(k)}, S_i^{(j)}, j \in D_k \}.
\]

In which, \( S_i^{(k)} \) represents present moment and historical monitoring information of \( k \) at \( t_i \), that is,

\[
S_i^{(k)} = \{ z_{i1}^{(k)} \leq z_{i1}^{(k)} + \Delta z_{i1}^{(k)}, S_{i1}^{(k)} \}
\]

\[
= \{ z_{i1}^{(k)} \leq z_{i1}^{(k)} + \Delta z_{i1}^{(k)}, z_{i2}^{(k)} \leq z_{i2}^{(k)} + \Delta z_{i2}^{(k)}, \ldots, z_{i1}^{(k)} \leq z_{i1}^{(k)} + \Delta z_{i1}^{(k)} \}
\]

In which, \( \Delta z_{i1}^{(k)}, \Delta z_{i2}^{(k)}, \ldots, \Delta z_{i1}^{(k)} \rightarrow 0 \), \( S_i^{(k)} \) is the historical monitoring information of all components at \( t_i \).

According to Sklar’s theorem, there is a correlation among multiple components of \( D_k \) and \( k \) at \( t_i \).

The probability density function of remaining useful life at \( t_i \),

\[
f_i^{(k)}(t, \theta | s_i^{(k)}) = f_i'(t, \theta | s_i^{(k)}, s_i^{(j)}) \quad j \in D_k
\]

\[
\sum_{j \in D_k} \frac{\partial C(F_i(t, \theta | s_i^{(k)}), F_i(t, \theta | s_i^{(j)}))}{\partial F_i(t, \theta | s_i^{(j)})} f_i(t, \theta | s_i^{(j)}) dt = \int_0^\infty \sum_{j \in D_k} \frac{\partial C(F_i(t, \theta | s_i^{(k)}), F_i(t, \theta | s_i^{(j)}))}{\partial F_i(t, \theta | s_i^{(j)})} f_i(t, \theta | s_i^{(j)}) \prod_{j=1}^{n} f_i(t, \theta | s_i^{(j)}) ds_i^{(1)} ds_i^{(2)} \ldots ds_i^{(n)}
\]

In which, \( T_0 \) is the initial life, and the average remaining life at the initial moment of operation is

\[
E_0^{(k)}(\theta) = \int_0^\infty t_0 f_0^{(k)}(t, \theta | s_0^{(k)}) dt = \int_0^\infty t f_0^{(k)}(t, \theta | s_0^{(k)}, s_0^{(j)}, j \in D_k) dt
\]

The mean remaining useful life at \( t_i \) is

\[
R^{(k)}(t_i, \theta) = E[T_i^{(k)} > 0, S_i^{(k)}] = E[T_i^{(k)} > 0, S_i^{(k)}, S_i^{(j)}, j \in D_k] = \int_0^\infty t f_i^{(k)}(t, \theta | s_i^{(k)}) dt
\]

Assumes that the random effect between components is one-way, and independent components (II part) of the remaining useful life of change are not affected by other components degradation, based on random filter according to the way to solve the density function of remaining useful life.

3. Preventive of Maintenance Strategy of Multi-Component

3.1. The remaining useful life function of the exponential distribution

Each maintenance activity produces a certain maintenance cost. Usually the cost of imperfect maintenance is less than the cost of perfect maintenance. Assume that the imperfect repair is carried out on \( k \) at \( t_i \), and the repair gain is \( G_i^{(k)}(t, p) \), and \( 0 \leq G_i^{(k)}(t, p) \leq R_i^{(k)}(t) \). In which, \( R_i^{(k)}(t) \) is the average remaining life of system at \( t_i \), and the maintenance cost based on the improvement factor of degradation quantity is

\[
C_i^{(k)}(t) = C_i^{(k)}(0) \left( \frac{G_i^{(k)}(t, p)}{R_i^{(k)}(t)} \right)^\lambda
\]
Among them, $C_i^{(k)}(0)$ is the maintenance cost that incurred when the system is restored to the initial state by imperfect maintenance, which is less than the cost of perfect maintenance. $\lambda$ is different maintenance costs and non-negative.

If $\lambda = 0$, $C_i^{(k)}(t) = C_i^{(k)}(0)$. If $0 < \lambda < 1$, the imperfect maintenance cost function $C_i^{(k)}(t)$ is the concave function that the growth rate of maintenance cost is faster than the growth rate of maintenance income. If $\lambda = 1$, the imperfect maintenance cost is a linear function, and the maintenance cost is proportional to the maintenance efficiency. If $\lambda > 1$, the imperfect maintenance expense is a convex function, and the growth rate of maintenance expense is slower than the growth rate of maintenance income.

3.2. Optimal maintenance time
Considering preventive maintenance and post-fault maintenance strategies, the maintenance and repair costs required for system operation are $C^{(k)}(t)$ in $[0, t]$, and the average cost of equipment operation is $C^{(k)}_c$. The average cost ratio $C^{(k)}_c$ of individual equipment operation can be obtained by updating theorem as

$$
C^{(k)}_c = \lim_{t \to \infty} \frac{C^{(k)}(t)}{E[T]} = \frac{C_0 + C_i^{(k)}(\alpha, t)[1 - \mathbb{P}(T_i^{(k)} < t_R^{(k)p} - t, s_i^{(k)})] + C_i^{(k)}(\beta, t)\mathbb{P}(T_i^{(k)} < t_R^{(k)p} - t, s_i^{(k)})]}{t + (t_R^{(k)p} - t_i)} + \int_{t}^{t_R^{(k)p} - t_i} f_i^{(k)}(t, \theta)s_i^{(k)}(\alpha, t) = (\beta, t)\mathbb{P}(T_i^{(k)} < t_R^{(k)p} - t, s_i^{(k)}) dt}
$$

In where, $E[C^{(k)}(t)]$ is the average operating cost of the component life cycle. $E[T]$ is the average life cycle of $k$. $t_R^{(k)p}$ is the optimized preventive maintenance time of $k$. $t_i$ is the monitoring point. $C_0$ is the cost of downtime loss. $C_i^{(k)}(\alpha, t)$ is the cost of preventive maintenance at $t_i$. $C_i^{(k)}(\beta, t)$ is the cost of replacement after failure at $t_i$, and $C_i^{(k)}(\alpha, t) < C_i^{(k)}(\beta, t)$. The expression of the minimum average operating cost of components can be obtained as,

$$
C_{\min} = \min C^{(k)}_c = \min \left\{ \frac{C_0 + C_i^{(k)}(\alpha, t)[1 - \mathbb{P}(T_i^{(k)} < t_R^{(k)p} - t, s_i^{(k)})] + C_i^{(k)}(\beta, t)\mathbb{P}(T_i^{(k)} < t_R^{(k)p} - t, s_i^{(k)})]}{t + (t_R^{(k)p} - t_i)} + \int_{t}^{t_R^{(k)p} - t_i} f_i^{(k)}(t, \theta)s_i^{(k)}(\alpha, t) = (\beta, t)\mathbb{P}(T_i^{(k)} < t_R^{(k)p} - t, s_i^{(k)}) dt} \right\}
$$

In the process of real-time prediction, due to the change of load and external environment during the operation of equipment, the optimal maintenance time $t_R^{(k)p}$ which is predicted at different monitoring points keeps changing, so the final decision time window $N_w$ is determined by adding time window.

After the selected length of $N_w$, the remaining useful life distribution of $k$ is predicted at $t_{i-n}$. The optimal preventive maintenance time $t_R^{(k)p}$ is obtained, but it falls out of the time window $N_w$. So preventive maintenance is not carried out at $t_R^{(k)p}$.

New monitoring information is received in real time to continue the real-time monitoring and life prediction of $k$. Until the optimal predictive maintenance time $t_R^{(k)p}$ of $k$ is predicted at $t_i$, and it falls
within the time window $N_w$. The remaining useful life of $k$ is stopped to make the maintenance decision, and the preventive maintenance of $k$ is determined at $t^{(k)}_R$.

4. Simulation and Analysis

Hypothesis that the system includes $k_1$ and $k_2$ of I part and $k_3$ of II part. Its degradation function and the initial life distribution accord with Weibull distribution. The density function is

$$f(x) = \rho \lambda (\rho x)^{\lambda - 1} \exp\left(- (\rho x)^{\lambda}\right)$$

The AIC is used to determine the relationship and the optimal expression of the remaining useful life is obtained by applying the maximum likelihood estimation parameters.

$$E(z^{(k)}) = \frac{1}{\rho} \Gamma\left(1 + \frac{1}{\lambda}\right) = \frac{1}{\rho} \int_0^\infty \frac{1}{y^2} e^{-y^\lambda} dy \propto \frac{1}{\rho} = h(t, z) = A + Be^{Ct} + D\exp(z)$$

The average remaining useful life of $k_1$ prediction is based on copula theory, and the average life prediction considering component dependency are shown in figure 1. It can be intuitively seen that the average remaining useful life considering component dependency is more suitable to the actual situation than that without considering component dependency.

![FIG. 1 comparison of the average residual life prediction of $k_1$ at different monitoring points](image)

On this basis, if the cost of downtime loss, preventive maintenance and replacement of components after failure is taken into account, then the optimal preventive maintenance time is analysed. Through the relationship between average cost ratio and decision variables determined, the optimal maintenance time is solved by PSO.

As can be seen from table 1, due to the change of downtime costs, other variables change differently, and the number of components that can be repaired is reduced from 3 to 2. In this way, the number of components that can be repaired by chance is optimized, so that the average maintenance expense ratio saved after opportunity maintenance is the maximum. The best opportunity to repair the area can be balanced between the loss caused when the parts deviate from their optimal replacement time and the loss caused by downtime.

| Situation Outage Cost | Part | Component | Time Window | Preventive Maintenance Cost | Minimum Average Cost | Optimal Preventive Maintenance Threshold | Opportunity Maintenance Area Threshold | Sum | Maintenance component |
|-----------------------|-----|-----------|-------------|----------------------------|----------------------|----------------------------------------|--------------------------------------|-----|-----------------------|
| 280                   | I   | $k_1$     | 0.05        | 310                        | 0.00909              | 0.0864                                 | 0.0211                               |     | ✓                     |
|    |   |   |   |   |   |
|----|---|---|---|---|---|
| $k_2$ | 400 | 0.00589 | 0.1357 | 0.0833 | √ |
| $k_3$ | 350 | 0.00701 | 0.1142 | 0.0714 | √ |
| $k_1$ | 310 | 0.00948 | 0.0711 |   |   |
| 180 | $k_2$ | 0.05 | 400 | 0.00628 | 0.0795 | 0.0333 | 0.0195 | √ |
| $k_3$ | 350 | 0.00475 | 0.1332 | 0.0286 |   |   |
| $k_1$ | 310 | 0.00956 | 0.0623 |   |   |
| 100 | $k_2$ | 0.05 | 400 | 0.00643 | 0.0485 | 0.0154 | 0.0168 | √ |
| $k_3$ | 350 | 0.00352 | 0.1026 | 0.0126 |   |   |

5. Conclusion
This paper mainly aims at the mechanical system under the condition of real-time monitoring. On the basis of analysing the existing dependence characteristics of multiple components, and the monitoring information affecting the components introduced as co-variable, the real-time residual life prediction model of multiple components is established with copula. At the same time, according to the function of predicted residual life, preventive maintenance strategy of real-time dynamic opportunities is given. The maintenance decision model is set up that the objective function is the maximization chance to repair. The prediction method of remaining useful life, the feasibility and effectiveness of maintenance decision model is verified.

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