Modeling Neutrino Quasielastic Cross Sections on Nucleons and Nuclei

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Abstract. We calculate the total and differential quasielastic cross sections for neutrino and antineutrino scattering on nucleons using up to date fits to the nucleon elastic electromagnetic form factors $G_p^E, G_n^E, G_p^M, G_n^M$, and $F_A$ and pseudoscalar form factors. We compare predictions of the cross sections for nucleons and nuclei to experimental data. (Presented by Arie Bodek at CIPANP2003, New York City, NY 2003)

1. INTRODUCTION

Experimental evidence for oscillations among the three neutrino generations has been recently reported [1]. Since quasielastic (QE) scattering forms an important component of neutrino scattering at low energies, we have undertaken to investigate QE neutrino scattering using the latest information on nucleon form factors.

Recent experiments at SLAC and Jefferson Lab (JLab) have given precise measurements of the vector electromagnetic form factors for the proton and neutron. These form factors can be related to the form factors for QE neutrino scattering by conserved vector current hypothesis, CVC. These more recent form factors can be used to give better predictions for QE neutrino scattering.

The hadronic current for QE neutrino scattering is given by [2]

$$< p(p_2) | J_+^\nu | n(p_1) > =$$

$$\bar{u}(p_2) \left[ \gamma_\lambda F_V^1(q^2) + \frac{i \sigma_\lambda \nu q^\nu \xi F_V^2(q^2)}{2M} + \gamma_\lambda \gamma_5 F_A(q^2) + \frac{q_\lambda \gamma_5 F_P(q^2)}{M} \right] u(p_1),$$

where $q = k_\nu - k_\mu$, $\xi = (\mu_p - 1) - \mu_n$, and $M = (m_p + m_n)/2$. Here, $\mu_p$ and $\mu_n$ are the proton and neutron magnetic moments. We assume that there are no second class currents, so the scalar form factor $F_V^3$ and the tensor form factor $F_A^3$ need not be included.

Using the above current, the cross section is

$$\frac{d \sigma^{\nu, \nu}}{d q^2} = \frac{M^2 G_v^2 \cos^2 \theta_c}{8 \pi E^2_\nu} \times \left[ A(q^2) \mp \frac{(s-u)B(q^2)}{M^2} + \frac{C(q^2)(s-u)^2}{M^4} \right],$$

where

$$A(q^2) = \frac{m^2 - q^2}{4M^2} \left[ 4 - \frac{q^2}{M^2} \right] |F_A|^2$$
\[ \nu + n \rightarrow p + \mu^-, \text{BBA-2003 Form Factors, } m_A=1.00 \]

**FIGURE 1.** The QE neutrino cross section along with data from various experiments. The calculation uses \( M_A=1.00 \text{ GeV}, g_A=-1.267, M_2^2=0.71 \text{ GeV}^2 \) and BBA-2003 Form Factors. The solid curve uses no nuclear correction, while the dashed curve [5] uses a Fermi gas model for carbon with a 25 MeV binding energy and 220 Fermi momentum. The dotted curve is the prediction for Carbon including both Fermi gas Pauli blocking and the effect of nuclear binding on the nucleon form factors [8]. The data shown [3] are from FNAL 1983, ANL 1977, BNL 1981, ANL 1973, SKAT 1990, GGM 1979, LSND 2002, Serpukov 1985, and GGM 1977.

\[
- \left( 4 + \frac{q^2}{M^2} \right) |F_1|^2 - \frac{q^2}{M^2} |\xi F_2|^2 \left( 1 + \frac{q^2}{4M^2} \right) - \frac{4q^2}{M^2} ReF_1^{1*} \xi F_2^2, \right)
\]

\[
B(q^2) = - \frac{q^2}{M^2} ReF_1^*(F_1 + \xi F_2), \quad C(q^2) = \frac{1}{4} \left( |F_A|^2 + |F_1|^2 - \frac{q^2}{M^2} \left| \xi F_2^2 \right|^2 \right).
\]

Although we have not shown terms of order \((m_i/M)^2\), and terms including \(F_P(q^2)\) (which is multiplied by \(\left( m_i/M \right)^2\), these terms are included in our calculations [2].) The form factors \(F_1^V(q^2)\) and \(\xi F_2^V(q^2)\) are given by:

\[
F_1^V(q^2) = \frac{G_E^V(q^2) - \frac{q^2}{4M^2} G_M^V(q^2)}{1 - \frac{q^2}{4M^2}}, \quad \xi F_2^V(q^2) = \frac{G_M^V(q^2) - G_E^V(q^2)}{1 - \frac{q^2}{4M^2}}.
\]

We use the CVC to determine \(G_E^V(q^2)\) and \(G_M^V(q^2)\) from the electron scattering form factors \(G_E^p(q^2), G_E^n(q^2), G_M^p(q^2),\) and \(G_M^n(q^2)\):

\[
G_E^V(q^2) = G_E^p(q^2) - G_E^n(q^2), \quad G_M^V(q^2) = G_M^p(q^2) - G_M^n(q^2).
\]
The axial form factor $F_A$ and the pseudoscalar form factor $F_P$ (related to $F_A$ by PCAC) are given by

$$F_A(q^2) = \frac{g_A}{\left(1 - \frac{q^2}{M_A^2}\right)^2}, \quad F_P(q^2) = \frac{2M^2F_A(q^2)}{M^2 - q^2}.$$  

In the expression for the cross section, $F_P(q^2)$ is multiplied by $(m_f/M)^2$. Therefore, in muon neutrino interactions, this effect is very small except at very low energy, below 0.2 GeV. $F_A(q^2)$ needs to be extracted from QE neutrino scattering. At low $Q^2$, $F_A(q^2)$ can also be extracted from pion electroproduction data.

Previously, people have assumed that the vector form factors are described by the dipole approximation.

$$G_D(q^2) = \frac{1}{\left(1 - \frac{q^2}{M_V^2}\right)^2}, \quad M_V^2 = 0.71 \text{ GeV}^2$$

$$G_E^p = G_D(q^2), \quad G_E^n = 0, \quad G_M^p = \mu_p G_D(q^2), \quad G_M^n = \mu_n G_D(q^2).$$

We refer to the above combination of form factors as ‘Dipole Form Factors’. It is an approximation that has been improved by us in a previous publication [3]. We use our updated form factors to which we refer as ‘BBA-2003 Form Factors’ (Budd, Bodek, Arrington). We also use our updated value [3] of $M_A 1.00 \pm 0.020$ GeV which is in good agreement with the theoretically corrected value from pion electroproduction [4] of $1.014 \pm 0.016$ GeV.

### 2. COMPARISON TO EXPERIMENTAL DATA

Figures 1, 2, and 3 show the QE cross section for $\nu$ and $\bar{\nu}$ with BBA-2003 Form Factors and $M_A=1.00$ GeV. The normalization uncertainty in the data is approximately 10%. The solid curve uses no nuclear correction, while the dashed curve [5] uses a NUANCE [6] calculation of a Smith and Moniz [7] based Fermi gas model for carbon. This nuclear model includes Pauli blocking (see Figure 4(a) ) and Fermi motion, but not final state interactions. The Fermi gas model was run with a 25 MeV binding energy and 220 MeV Fermi momentum. The dotted curve is the prediction for Carbon including both Fermi gas Pauli blocking, and the effect of nuclear binding on the nucleon form factors as modeled by Tsushima et al [8] (see Figure 4(b)). Note that this model is only valid for $Q^2$ less than $1 \text{ GeV}^2$, and that the binding effects on the form factors are expected to be very small at higher $Q^2$. Both the Pauli blocking and the nuclear modifications to bound nucleon form factors reduce the cross section relative to the cross section with free nucleons.

The updated form factors improve the agreement with neutrino QE cross section data and give a reasonable description of the cross sections from deuterium.

We plan to continue to study the nuclear corrections, adopting models which have been used in precision electron scattering measurements from nuclei at SLAC and
JLab. For example, we plan to study the Pauli blocking correction using an improved Fermi Gas model with a high momentum tail [9], as well as more sophisticated nuclear spectral functions. In addition, we will continue to update the extraction of $M_A$ from previous neutrino experiments, using the updated versions of the input parameters and electromagnetic form factors.

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\[ \bar{\nu} + p \rightarrow n + \mu^+, \text{ BBA-2003 Form Factors, } m_A=1.00 \]

**FIGURE 3.** The QE antineutrino cross section along with data from various experiments. The calculation uses \( M_A=1.00 \) GeV, \( g_A=-1.267 \), \( M^2_V=0.71 \) GeV\(^2\) and BBA-2003 Form Factors. The solid curve uses no nuclear correction, while the dashed curve \([5]\) uses a Fermi gas model for carbon with a 25 MeV binding energy and 220 MeV Fermi momentum. The dotted curve is the prediction for Carbon including both Fermi gas Pauli blocking and the effect of nuclear binding on the nucleon form factors \([8]\). The data shown are from SKAT 1990, GGM 1979, Serpukov 1985, and GGM 1977.

**FIGURE 4.** (a) The Pauli blocking suppression for a Fermi gas model for carbon with a 25 MeV binding energy and 220 MeV Fermi momentum. (b) The ratio of bound to free nucleon form factors for \( F_1, F_2, \) and \( F_A \) from ref \([8]\). Note that this model is only valid for \( Q^2 \) less than 1 GeV\(^2\), and that the binding effects on the form factors are expected to be very small at higher \( Q^2 \).