Effective Theories of Gamma-ray Lines from Dark Matter Annihilation

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Abstract

We explore theories of dark matter in which dark matter annihilations produce mono-energetic gamma rays (“lines”) in the context of effective field theory, which captures the physics for cases in which the particles mediating the interaction are somewhat heavier than the dark matter particle itself. Building on earlier work, we explore the generic signature resulting from $SU(2) \times U(1)$ gauge invariance that two (or more) lines are generically expected, and determine the expected relative intensities, including the possibility of interference between operators.

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I. INTRODUCTION

While the existence of dark matter is now secure, its nature remains elusive. Many experiments are searching for evidence of non-gravitational dark matter interactions through direct detection of its scattering off heavy nuclei, or by direct production in colliders. Yet another approach to search for dark matter indirectly is by looking for signals produced by its annihilation to Standard model (SM) particles. In particular, the Universe is transparent to $\sim 100$ GeV energy gamma rays on galactic distance scales, which allows one to use their distribution in the sky as well as in energy as handles to try to sift the signal from the (often poorly understood) astrophysical backgrounds.

Among the most striking potential signals one can imagine from dark matter annihilation is a mono-energetic “line” of gamma rays. Such a process occurs when two (non relativistic) dark matter particles annihilate into a two-body final state, one of which is a photon. The most canonical of such signals would be annihilation into two photons, whose energies are expected to be very close to the mass of the dark matter particle. While such a process is usually suppressed compared to the continuum of gamma rays that result from dark matter annihilations into charged or hadronic particles, the signature is distinctive and difficult for more conventional astrophysics to mimic.

In fact, recent analysis of data obtained by the Fermi-LAT collaboration [1, 2] has found tentative indications of such a line at an energy of about 130 GeV [3–5], originating from close to the galactic center [3–5]. Such a signal is tantalizing, and the presence of what may be a fainter secondary line in the data whose energy is consistent with annihilation into a $\gamma Z$ final state [5, 6] lends some credence to an interpretation in terms of dark matter annihilation. On the other hand, searches for signs of a signal in targets away from the galactic center have yielded results which are confusing at best [7–11], there are significant limits on a continuum signal associated with the regions of the sky where the line appears brightest [12], and most seriously, there seems to be a hint of a feature in photons arriving from the direction of the Earth’s limb [5] raising the possibility that the feature in the data is the result of a subtle instrumental effect [13–15]. Perhaps less likely, the signal could also correspond to more prosaic astrophysical processes [16–18]. Despite these potential issues, the feature at 130 GeV is very interesting and worthy of investigation.

In this article, we examine dark matter annihilations as a source of multiple lines, using the powerful language of effective field theory developed in Ref. [6]. While we are inspired by the feature currently observed in the Fermi data, we will be more concerned with the generic systematics of multiple line signals, which can be applied both to the currently observed feature and to future searches.
II. EFFECTIVE FIELD THEORY

We will take the dark matter (denoted by $\chi$) to be a singlet under all SM interactions, which implies that it couples to the electroweak gauge bosons (including the photon) through higher dimensional operators that result from integrating out electroweakly charged massive fields. We work at the level of this effective field theory containing the dark matter and the SM itself. We impose a $\mathbb{Z}_2$ symmetry under which the dark matter is odd and the SM is even in order to insure that the dark matter is stable. We are interested in operators containing at least one photon, so as to result in an observable gamma ray line signal. The operators are organized by the energy dimension of the field content, since one generically expects operators corresponding to higher dimensions to be less relevant in low energy processes, being more suppressed by the masses of the heavy particles that were integrated out to produce them.

This last issue raises an important question – does one expect that effective field theory can capture the physics of dark matter annihilation into photons at all? Like any effective theory, our theory is valid at very low momentum transfer, but fails to capture the physics of high energy processes, for which the complete theory in the ultra-violet is required. For (non-relativistic) dark matter annihilation, the characteristic momentum transfer is of order the mass of the dark matter itself, and so this assumption boils down to the requirement that the particles mediating the interaction between the dark matter and electroweak bosons are heavier than $m_\chi$. Since such mediator particles must be charged, their masses are bounded in general to be $\gtrsim 100$ GeV by LEP (or more by the LHC if they are stable on collider timescales [19]). However, in many theories the loop process connecting the dark matter to the weak bosons contains a mixture of SM as well as heavy mediator particles. For example, the line signal resulting from dark matter whose primary interaction is with SM light quarks is considered in Ref. [20].

| Dimension 6 Operators | B1+B2 | $\frac{1}{\Lambda_{D1}}\chi^*\chi B_{\mu\nu}B^{\mu\nu} + \frac{1}{\Lambda_{D2}}\chi^*W_{\mu\nu}^aW^{a\mu\nu}$ | $\gamma\gamma, \gamma Z$ |
|-----------------------|--------------------------|--------------------------|
| B3+B4 | $\frac{1}{\Lambda_{D3}}\chi^*\chi B_{\mu\nu}\tilde{B}^{\mu\nu} + \frac{1}{\Lambda_{D4}}\chi^*W_{\mu\nu}^a\tilde{W}^{a\mu\nu}$ | $\gamma\gamma, \gamma Z$ |

| Dimension 8 Operators | D1+D2 | $\frac{1}{\Lambda_{D1}}(\chi\partial_\mu\chi - \chi^*\partial_\mu\chi)B^{\mu\alpha}\Phi^\dagger D_\alpha \Phi + \frac{1}{\Lambda_{D2}}(\chi\partial_\mu\chi - \chi^*\partial_\mu\chi)\Phi^\dagger W_{\mu\alpha}^a T^a \Phi D_\alpha \Phi$ | $\gamma\gamma, \gamma Z$ |
|-----------------------|--------------------------|--------------------------|
| D3+D4 | $\frac{1}{\Lambda_{D3}}(\chi\partial_\mu\chi^* - \chi^*\partial_\mu\chi)\tilde{B}^{\mu\alpha}\Phi^\dagger D_\alpha \Phi + \frac{1}{\Lambda_{D4}}(\chi\partial_\mu\chi^* - \chi^*\partial_\mu\chi)\Phi^\dagger \tilde{W}_{\mu\alpha}^a T^a \Phi D_\alpha \Phi$ | $\gamma\gamma, \gamma Z$ |

TABLE I: List of effective interactions for complex scalar dark matter and the type of line signals ($\gamma\gamma, \gamma Z$, and/or $\gamma h$) that they produce.
Nonetheless, even if there are light SM particles participating, the structure remains tightly constrained by $SU(2) \times U(1)$ electroweak gauge invariance. The presence of multiple lines thus remains generic and (provided the non-SM heavy mediators are sufficiently heavy compared to $m_\chi$), the relative rates of various line processes such as $\gamma\gamma$ and $\gamma Z$ are not likely to show large deviations from an effective theory description, although the precise mapping of the EFT coefficients to the UV theory parameters becomes more murky.

Our effective field theory is constructed as the Standard Model, plus a dark matter particle $\chi$ which we allow to be either a complex scalar or Dirac fermion. The real and Majorana cases are simply related to our results. The interactions of interest contain at least one $SU(2)$ $W^3_\mu$ or hyper charge $B^\mu$ gauge field, which will become a photon after rotating to the mass eigenbasis,

\begin{align}
B_\mu &= A_\mu \cos \theta_W - Z_\mu \sin \theta_W \\
W^3_\mu &= A_\mu \sin \theta_W + Z_\mu \cos \theta_W
\end{align}

where $A_\mu$ and $Z_\mu$ are the photon and $Z$ boson fields respectively, and $\theta_W$ is the electroweak mixing angle.

We consider the effective vertices shown in Tables I and II (for scalar and fermionic dark matter, respectively), which are built out of the dark matter fields, the field strengths $W_\alpha^{\mu\nu}$ and $B^{\mu\nu}$ (and their duals $\tilde{W}$ and $\tilde{B}$), the SM Higgs doublet $\Phi$, and its covariant derivative

\begin{equation}
D_\alpha \Phi = \partial_\alpha \Phi - ig_2 T_\alpha W_\alpha^{\mu\nu} \Phi - \frac{i}{2} g_1 B_\alpha \Phi,
\end{equation}

where $T_\alpha$ are the generators of the doublet representation of $SU(2)$, and $g_1$ and $g_2$ are gauge couplings. We define $\gamma^{\mu\nu} \equiv [\gamma^\mu, \gamma^\nu]$. For our purposes, it will be sufficient to work in the unitary gauge, for which we may take,

\begin{equation}
\Phi \rightarrow \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ V + h(x) \end{pmatrix},
\end{equation}

where $V \simeq 246$ GeV is the Higgs vacuum expectation value and $h(x)$ is the physical Higgs field.

This set of interactions is complete up to terms of dimension 8. A similar list was studied, including bounds from indirect and direct detection and LHC searches, in Ref. [21].

### III. LINE CROSS SECTIONS

Since dark matter is expected to be highly non-relativistic (with velocity dispersion $v \sim 10^{-3}$) in the galactic halo, dark matter annihilation into photons may be simplified as an
Thus also unlikely to contribute to a large line signal. Operators $A_1$ and $A_2$ (which correspond to magnetic/electric operators $C_1$ – $C_4$ and $D_1$ – $D_4$) lead to vanishing cross sections, and thus are unlikely to lead to any observable line signal. The operators $C_7$ and $C_8$ lead to vanishing cross sections, and thus are unlikely to lead to any observable line signal. We will denote $p_1, p_2$ to be the incoming dark matter particle momenta, $p_3$ will be a photon, and $p_4$ is either another photon, $Z$ boson, or higgs boson. The differential cross section is written
\[
\frac{d\sigma}{d\Omega} = \frac{E_3}{256\pi^2} \frac{E_3}{E^3} v |\mathcal{M}|^2
\]
where $E = m_\chi + \mathcal{O}(v^2)$ is the energy of each dark matter particle, $v$ is the dark matter velocity, and $E_3 = |\vec{p}_3|$ is the energy of the outgoing line photon. $|\mathcal{M}|^2$ is the matrix element

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**Dimension 5 Operators**

| Operators | Expression | Signals |
|-----------|------------|---------|
| $A_1 + A_2$ | $\frac{1}{\Lambda_{A_1}} \bar{\chi} \gamma^{\mu \nu} \chi B_{\mu \nu} + \frac{1}{\Lambda_{A_2}} \bar{\chi} \gamma^{\mu \nu} \chi \vec{B}_{\mu \nu}$ | $\gamma \gamma, \gamma Z$ |

**Dimension 7 Operators**

| Operators | Expression | Signals |
|-----------|------------|---------|
| $C_1 + C_2$ | $\frac{1}{\Lambda_{C_1}} \bar{\chi} \gamma^\mu \chi B_{\mu \nu} B^\nu + \frac{1}{\Lambda_{C_2}} \bar{\chi} \gamma^\mu \chi W^a_{\mu \nu} W^{aq\nu}$ | $\gamma \gamma, \gamma Z$ |
| $C_3 + C_4$ | $\frac{1}{\Lambda_{C_3}} \bar{\chi} \gamma^\mu \chi B_{\mu \nu} \vec{B}^\nu + \frac{1}{\Lambda_{C_4}} \bar{\chi} \gamma^\mu \chi W^a_{\mu \nu} \vec{W}^{aq\nu}$ | $\gamma \gamma, \gamma Z$ |
| $C_5 + C_6$ | $\frac{1}{\Lambda_{C_5}} \bar{\chi} \gamma^5 \chi B_{\mu \nu} B^{\mu \nu} + \frac{1}{\Lambda_{C_6}} \bar{\chi} \gamma^5 \chi W^a_{\mu \nu} W^{aq\mu \nu}$ | $\gamma \gamma, \gamma Z$ |
| $C_7 + C_8$ | $\frac{1}{\Lambda_{C_7}} \bar{\chi} \gamma^5 \chi B_{\mu \nu} \vec{B}^{\mu \nu} + \frac{1}{\Lambda_{C_8}} \bar{\chi} \gamma^5 \chi W^a_{\mu \nu} \vec{W}^{aq\mu \nu}$ | $\gamma \gamma, \gamma Z$ |
| $C_9 + C_{10}$ | $\frac{1}{\Lambda_{C_9}} \bar{\chi} \gamma^{\mu \nu} \chi B_{\mu \alpha} \Phi^d_{\nu \alpha} + \frac{1}{\Lambda_{C_{10}}} \bar{\chi} \gamma^{\mu \nu} \chi W^a_{\mu \nu} T^a_\Phi$ | $\gamma h$ |
| $C_{11} + C_{12}$ | $\frac{1}{\Lambda_{C_{11}}} \bar{\chi} \gamma^{\mu \nu} \chi B_{\mu \nu} |\Phi|^2 + \frac{1}{\Lambda_{C_{12}}} \bar{\chi} \gamma^{\mu \nu} \chi \Phi^d_{\nu \alpha} T^a_\Phi$ | $\gamma h$ |
| $C_{13} + C_{14}$ | $\frac{1}{\Lambda_{C_{13}}} \bar{\chi} \gamma^{\mu \nu} \chi \vec{B}_{\mu \nu} |\Phi|^2 + \frac{1}{\Lambda_{C_{14}}} \bar{\chi} \gamma^{\mu \nu} \chi \Phi^d_{\nu \alpha} T^a_\Phi$ | $\gamma h$ |

**Dimension 8 Operators**

| Operators | Expression | Signals |
|-----------|------------|---------|
| $D_5 + D_6$ | $\frac{1}{\Lambda_{D_5}} \bar{\chi} \gamma^\mu \chi B^{\mu \alpha} \Phi^d_{\alpha \Phi} + \frac{1}{\Lambda_{D_6}} \bar{\chi} \gamma^\mu \chi \Phi^d_{\nu \alpha} T^a_\Phi$ | $\gamma Z, \gamma h$ |
| $D_7 + D_8$ | $\frac{1}{\Lambda_{D_7}} \bar{\chi} \gamma^\mu \chi \vec{B}^{\mu \alpha} \Phi^d_{\alpha \Phi} + \frac{1}{\Lambda_{D_8}} \bar{\chi} \gamma^\mu \chi \Phi^d_{\nu \alpha} T^a_\Phi$ | $\gamma Z, \gamma h$ |
| $D_9 + D_{10}$ | $\frac{1}{\Lambda_{D_9}} \bar{\chi} \gamma^5 \chi B^{\mu \alpha} \Phi^d_{\alpha \Phi} + \frac{1}{\Lambda_{D_{10}}} \bar{\chi} \gamma^5 \chi \Phi^d_{\nu \alpha} T^a_\Phi$ | $\gamma Z$ |
| $D_{11} + D_{12}$ | $\frac{1}{\Lambda_{D_{11}}} \bar{\chi} \gamma^5 \chi \vec{B}^{\mu \alpha} \Phi^d_{\alpha \Phi} + \frac{1}{\Lambda_{D_{12}}} \bar{\chi} \gamma^5 \chi \Phi^d_{\nu \alpha} T^a_\Phi$ | $\gamma Z$ |

**TABLE II:** List of effective interactions for Dirac fermion dark matter and the type of line signals ($\gamma \gamma, \gamma Z$, and/or $\gamma h$) that they produce.

expansion in $v^2$. We retain only the leading ($v$-independent) terms. In this limit, the operators $C_1$ – $C_4$ and $D_1$ – $D_4$ lead to vanishing cross sections, and thus are unlikely to lead to any observable line signal. Operators $A_1$ and $A_2$ (which correspond to magnetic/electric dipole moments for the dark matter) are strongly constrained by direct detection [22], and thus also unlikely to contribute to a large line signal. We leave consideration of all of these unpromising cases for future work.

An inelastically scattering dark matter particle with dipole interactions can evade direct detection constraints and might even explain the DAMA signal, see Refs. [20, 23, 26].
\( \mathcal{M} \) averaged over initial dark matter spins (if any) and summed over final state particle spins.

**A. Dimension 6 Operators**

Operators B1 – B4 and C1 – C18 all have the form \( X F^{\mu \nu} F_{\mu \nu} \) or \( X F^{\mu \nu} \tilde{F}_{\mu \nu} \), where \( F^{\mu \nu} = B^{\mu \nu} \) or \( W^{\mu \nu} \). For the \( X F^{\mu \nu} F_{\mu \nu} \) operators, the matrix element will have the form \( \mathcal{M} = -2Y(p_3 \cdot p_4 \epsilon_3 \cdot \epsilon_4) \) where \( Y \) is whatever the Feynman rules of \( X \) yield, and depends on the spin of the dark matter. If \( p_4 \) corresponds to another photon, we find

\[
\sum_{\epsilon_3, \epsilon_4} |\mathcal{M}|^2 = 16YY^\dagger(p_3 \cdot p_4)^2.
\]  

(5)

To obtain \( |\mathcal{M}|^2 \), one averages this result over the dark matter spin states. In the case where \( p_4 \) corresponds to a massive gauge boson, we find:

\[
\sum_{\epsilon_3, \epsilon_4} |\mathcal{M}|^2 = 12YY^\dagger(p_3 \cdot p_4)^2.
\]  

(6)

For an operator of the form \( X F^{\mu \nu} \tilde{F}_{\mu \nu} \) the matrix element will have the form:

\[
|\mathcal{M}|^2 = YY^\dagger(p_3 \cdot p_4)^2.
\]  

(7)

For the (complex) scalar dark matter dimension six operators, \( YY^\dagger = \frac{1}{\Lambda^2} \) and the average over spin is trivial. The operators B1 and B2 interfere with one another, but are separate from B3 and B4. The resulting cross sections are:

\[
|v|\sigma_{B1,2}(\chi \chi^* \rightarrow \gamma \gamma) = \frac{2m^2}{\pi} \left( \frac{\cos^4 \theta_W}{\Lambda_{B1}^4} + \frac{2 \cos^2 \theta_W \sin^2 \theta_W + \sin^4 \theta_W}{\Lambda_{B1}^2 \Lambda_{B2}^2} \right),
\]  

(8)

\[
|v|\sigma_{B1,2}(\chi \chi^* \rightarrow \gamma Z) = \frac{3 \cos^2 \theta_W \sin^2 \theta_W (4m^2 - m_Z^2)^3}{64\pi m^4_{\chi}} \left( \frac{1}{\Lambda_{B1}^4} - \frac{2}{\Lambda_{B1}^2 \Lambda_{B2}^2} + \frac{1}{\Lambda_{B2}^4} \right),
\]  

(9)

and

\[
|v|\sigma_{B3,4}(\chi \chi^* \rightarrow \gamma \gamma) = \frac{4m^2}{\pi} \left( \frac{\cos^4 \theta_W}{\Lambda_{B3}^4} + \frac{2 \cos^2 \theta_W \sin^2 \theta_W + \sin^4 \theta_W}{\Lambda_{B3}^2 \Lambda_{B4}^2} \right),
\]  

(10)

\[
|v|\sigma_{B3,4}(\chi \chi^* \rightarrow \gamma Z) = \frac{\cos^2 \theta_W \sin^2 \theta_W (4m^2 - m_Z^2)^3}{8\pi m^4_{\chi}} \left( \frac{1}{\Lambda_{B3}^4} - \frac{2}{\Lambda_{B3}^2 \Lambda_{B4}^2} + \frac{1}{\Lambda_{B4}^4} \right),
\]  

(11)

respectively.
B. Dimension 7 Operators

A Dirac fermion can annihilate into $\gamma\gamma$ and $\gamma Z$ through the dimension seven operators $C5 – C8$ (recall that $C1 – C4$ vanish at zero velocity). The matrix elements are identical to $B1 – B4$ as far as the final state, and the only difference is the average over initial WIMP spins. The resulting cross sections are,

$$|v|\sigma_{C5,6}(\chi\bar{\chi} \rightarrow \gamma\gamma) = \frac{4m_\chi^4}{\pi} \left( \frac{\cos^4 \theta_W + 2\cos^2 \theta_W \sin^2 \theta_W}{\Lambda_{C5}^6} + \frac{\sin^4 \theta_W}{\Lambda_{C6}^6} \right),$$

(12)

$$|v|\sigma_{C5,6}(\chi\bar{\chi} \rightarrow \gamma Z) = \frac{3(4m_\chi^2 - m_Z^2)^3\cos^2 \theta_W \sin^2 \theta_W}{32\pi m_\chi^2} \left( \frac{1}{\Lambda_{C5}^6} - \frac{2}{\Lambda_{C5}^3 \Lambda_{C6}^3} + \frac{1}{\Lambda_{C6}^6} \right),$$

(13)

and

$$|v|\sigma_{C7,8}(\chi\bar{\chi} \rightarrow \gamma\gamma) = \frac{8m_\chi^4}{\pi} \left( \frac{\cos^4 \theta_W + 2\cos^2 \theta_W \sin^2 \theta_W}{\Lambda_{C7}^6} + \frac{\sin^4 \theta_W}{\Lambda_{C8}^6} \right),$$

(14)

$$|v|\sigma_{C7,8}(\chi\bar{\chi} \rightarrow \gamma Z) = \frac{(4m_\chi^2 - m_Z^2)^3(\cos^2 \theta_W \sin^2 \theta_W)}{4\pi m_\chi^2} \left( \frac{1}{\Lambda_{C7}^6} - \frac{2}{\Lambda_{C7}^3 \Lambda_{C8}^3} + \frac{1}{\Lambda_{C8}^6} \right).$$

(15)

The remaining dimension seven operators lead to single lines. For $C9$ and $C10$, the antisymmetry of $\gamma_{\mu\nu}$ forces the $\chi\chi \rightarrow \gamma\gamma$ cross section to vanish identically, leaving only a $\gamma Z$ line:

$$|v|\sigma_{C9,10}(\chi\bar{\chi} \rightarrow \gamma Z) = \frac{(4m_\chi^2 - m_Z^2)^3(4m_\chi^2 + m_Z^2)\cos^2 \theta_W \sin^2 \theta_W}{16m_\chi^4 \pi} \left( \frac{1}{\Lambda_{C9}^6} - \frac{2}{\Lambda_{C9}^3 \Lambda_{C10}^3} + \frac{1}{\Lambda_{C10}^6} \right).$$

(16)

Whereas operators $C11 – C14$ result in a single $\gamma h$ line,

$$|v|\sigma_{C11,12}(\chi\bar{\chi} \rightarrow \gamma h) = \frac{(4m_\chi^2 - m_h^2)^3V^2}{64m_\chi^4 \pi} \left( \frac{\cos^2 \theta_W}{\Lambda_{C11}^6} - \frac{\cos \theta_W \sin \theta_W}{\Lambda_{C11}^3 \Lambda_{C12}^3} + \frac{\sin^2 \theta_W}{4\Lambda_{C12}^6} \right),$$

(17)

and

$$|v|\sigma_{C13,14}(\chi\bar{\chi} \rightarrow \gamma h) = \frac{(4m_\chi^2 - m_h^2)^3V^2}{16m_\chi^4 \pi} \left( \frac{\cos^2 \theta_W}{\Lambda_{C13}^6} - \frac{\cos \theta_W \sin \theta_W}{\Lambda_{C13}^3 \Lambda_{C14}^3} + \frac{\sin^2 \theta_W}{4\Lambda_{C14}^6} \right).$$

(18)

C. Dimension 8 Operators

Dimension eight operators could in principle contribute to line signals from scalar dark matter, but in practice these operators lead to cross sections which vanish in the zero velocity limit. Thus, we limit our discussion to the case where the dark matter is a Dirac fermion, for which there are potentially both $\gamma Z$ and $\gamma h$ final states. For $D5 – D8$, we have two sets
of interfering operators,

\[ |v| \sigma_{D5,6}(\chi \bar{\chi} \rightarrow \gamma Z) = \frac{(4m^2 - m_Z^2)^3(4m^2 + m_Z^2)V^4(g_2 \cos \theta_W + g_1 \sin \theta_W)^2}{4096 \pi m^4 \Lambda_{D5}^4} \times \left( \frac{\cos^2 \theta_W}{\Lambda_{D5}^8} - \frac{\cos \theta_W \sin \theta_W}{\Lambda_{D5}^4 \Lambda_{D6}^4} + \frac{\sin^2 \theta_W}{4 \Lambda_{D6}^8} \right), \quad (19) \]

\[ |v| \sigma_{D5,6}(\chi \bar{\chi} \rightarrow \gamma h) = \frac{(4m^2 - m_h^2)^3V^2}{1024 \pi m^4 \Lambda_{D5}^4} \left( \frac{\cos^2 \theta_W}{\Lambda_{D5}^8} - \frac{\cos \theta_W \sin \theta_W}{\Lambda_{D5}^4 \Lambda_{D6}^4} + \frac{\sin^2 \theta_W}{4 \Lambda_{D6}^8} \right), \quad (20) \]

and

\[ |v| \sigma_{D7,8}(\chi \bar{\chi} \rightarrow \gamma Z) = \frac{(4m^2 - m_Z^2)^3(24m^2 + m_Z^2)V^4(g_2 \cos \theta_W + g_1 \sin \theta_W)^2}{4096 \pi m^4 \Lambda_{D7}^4} \times \left( \frac{\cos^2 \theta_W}{\Lambda_{D7}^8} - \frac{\cos \theta_W \sin \theta_W}{\Lambda_{D7}^4 \Lambda_{D8}^4} + \frac{\sin^2 \theta_W}{4 \Lambda_{D8}^8} \right), \quad (21) \]

\[ |v| \sigma_{D7,8}(\chi \bar{\chi} \rightarrow \gamma h) = \frac{(4m^2 - m_h^2)^3V^2}{256 \pi m^4 \Lambda_{D7}^4} \left( \frac{\cos^2 \theta_W}{\Lambda_{D7}^8} - \frac{\cos \theta_W \sin \theta_W}{\Lambda_{D7}^4 \Lambda_{D8}^4} + \frac{\sin^2 \theta_W}{4 \Lambda_{D8}^8} \right). \quad (22) \]

For operators D9 – D12, the \( \gamma h \) line vanishes in the limit of zero velocity, leaving a single bright \( \gamma Z \) line from each set of operators. The cross sections are,

\[ |v| \sigma_{D9,10}(\chi \bar{\chi} \rightarrow \gamma Z) = \frac{(4m^2 - m_Z^2)^3V^4(g_2 \cos \theta_W + g_1 \sin \theta_W)^2}{16384 \pi m^4 \Lambda_{D9}^8} \times \left( \frac{\cos^2 \theta_W}{\Lambda_{D9}^8} - \frac{\cos \theta_W \sin \theta_W}{\Lambda_{D9}^4 \Lambda_{D10}^4} + \frac{\sin^2 \theta_W}{4 \Lambda_{D10}^8} \right), \quad (23) \]

and

\[ |v| \sigma_{D11,12}(\chi \bar{\chi} \rightarrow \gamma Z) = \frac{(4m^2 - m_Z^2)^3V^4(g_2 \cos \theta_W + g_1 \sin \theta_W)^2}{4096 \pi m^4 \Lambda_{D11}^8} \times \left( \frac{\cos^2 \theta_W}{\Lambda_{D11}^8} - \frac{\cos \theta_W \sin \theta_W}{\Lambda_{D11}^4 \Lambda_{D12}^4} + \frac{\sin^2 \theta_W}{4 \Lambda_{D12}^8} \right). \quad (24) \]

IV. SUMMARY

We are now in a position to summarize the various possible annihilation modes for each operator class. The processes resulting from each operator which are not suppressed by the dark matter velocity are listed in in the third column of Tables I and II. As is evident from the table, any operator which can produce a \( \gamma \gamma \) line will (modulo interference between two operators) also result in a \( \gamma Z \) one, whereas some of the higher dimension operators are able to produce \( \gamma Z \) or \( \gamma h \) lines in isolation. Of course, a specific UV theory of dark matter may result in more than one operator being turned on. Typically one expects that relevant operators of the lowest dimension will dominate the size of each line with corrections from higher order terms being controlled by \( m_\chi / \Lambda_i \) to the appropriate power.
Our results are suggestive of new ways to interpret the results of line searches. Given a choice of dark matter mass and now that the Large Hadron Collider has measured the Higgs boson mass, the energy of each line is determined,

\[ E_{\gamma\gamma} = m_\chi \] (25)
\[ E_{\gamma Z} = \frac{4m^2_\chi - m^2_Z}{4m_\chi} \] (26)
\[ E_{\gamma h} = \frac{4m^2_\chi - m^2_h}{4m_\chi} \] (27)

where it should be clear that the energy in each case refers to the energy of the final state photon, and the label applies to the process which produced the gamma ray. Since multiple lines are a fairly generic feature, it would be interesting to recast single line searches into searches for multiple lines based on a given value of \( m_\chi \). For example, a search for lines related to a scalar dark matter particle could search simultaneously for two lines with energies \( E_{\gamma\gamma} \) and \( E_{\gamma Z} \) based on operators B1 and B2. At each putative dark matter mass, a bound can be placed in the \( \Lambda_{B1}-\Lambda_{B2} \) plane.

Alternately, if one has a particular UV theory in mind such that either one operator or the other (or some linear combination with a fixed ratio) is generated, one can improve the sensitivity by searching for two lines at correlated energies with a fixed intensity ratio for the two. In Table III, we list, for each operator, the strength of the first (lowest energy) and second line implied by each set of operators. For convenience, we have introduced the shorthand notation:

\[ f_1(\Lambda_1, \Lambda_2, n) \equiv \left( \frac{\cos^4 \theta_W}{\Lambda_1^{2n}} + \frac{2 \cos^2 \theta_W \sin^2 \theta_W}{\Lambda_1^2 \Lambda_2^n} + \frac{\sin^4 \theta_W}{\Lambda_2^{2n}} \right) , \] (28)
\[ f_2(\Lambda_1, \Lambda_2, n) \equiv \cos^2 \theta_W \sin^2 \theta_W \left( \frac{1}{\Lambda_1^{2n}} - \frac{2}{\Lambda_1^2 \Lambda_2^n} + \frac{1}{\Lambda_2^{2n}} \right) , \] (29)
\[ f_3(\Lambda_1, \Lambda_2, n, m) \equiv \left( \frac{\cos^2 \theta_W}{\Lambda_1^{2n}} - \frac{\cos \theta_W \sin \theta_W}{\Lambda_1^2 \Lambda_2^n} + \frac{\sin^2 \theta_W}{4 \Lambda_2^{2n}} \right) (g_2 \cos \theta_W + g_1 \sin \theta_W)^m . \] (30)

The operator groups D5+D6 and D7+D8 each predict a fixed ratio between the two lines, regardless of the specifics of the relative coefficients of the operators within each category. The ratios are:

\[ \frac{|v| \sigma_{D5,6}(\chi \chi \to \gamma Z)}{|v| \sigma_{D5,6}(\chi \chi \to \gamma h)} = \left( \frac{4m^2_\chi - m^2_Z}{4m^2_\chi - m^2_h} \right)^3 \frac{(4m^2_\chi + m^2_Z)V^2(g_2 \cos \theta_W + g_1 \sin \theta_W)^2}{4 m^2_\chi m^2_Z} , \] (31)
\[ \frac{|v| \sigma_{D7,8}(\chi \chi \to \gamma Z)}{|v| \sigma_{D7,8}(\chi \chi \to \gamma h)} = \left( \frac{4m^2_\chi - m^2_Z}{4m^2_\chi - m^2_h} \right)^3 \frac{(24m^2_\chi + m^2_Z)V^2(g_2 \cos \theta_W + g_1 \sin \theta_W)^2}{4 m^2_\chi m^2_Z} . \] (32)
and their relative intensities, are the consequence of the particles mediating the interactions between the dark matter and photons are much heavier than the dark matter itself, the central result that there are multiple lines, as to the possible responsible operators, and thus the first clues as to the nature of the dark matter.

V. OUTLOOK

Gamma ray line searches make up a crucial part of searches for the indirect detection of dark matter. We have studied using the tools of effective field theory the generic multi-line signatures of dark matter annihilation. While our specific analytic results apply only in the case where the particles mediating the interactions between the dark matter and photons are much heavier than the dark matter itself, the central result that there are multiple lines, and their relative intensities, are the consequence of $SU(2) \times U(1)$ gauge invariance, and thus rather generic.

We have examined the set of lowest operators which can contribute to $\gamma\gamma$, $\gamma Z$, and $\gamma h$ lines, for dark matter which is a complex scalar or Dirac fermion. Our results suggest that an interesting extension of the current suite of searches for photon lines at gamma ray telescopes would include the simultaneous search for two lines at fixed relative energies. Such a search should improve the sensitivity to specific UV theories of dark matter in many cases, which fix the ratio between the interfering operators of a given dimensionality. Should a set of lines be discovered, the energies and relative intensities of the set provide key information as to the possible responsible operators, and thus the first clues as to the nature of the dark matter responsible.
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