On the perturbation of the luminosity distance by peculiar motions.

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ABSTRACT
We consider some aspects of the perturbation to the luminosity distance $d(z)$ that are of relevance for SN1a cosmology and for future peculiar velocity surveys at non-negligible redshifts.

1) Previous work has shown that the correction to the lowest order perturbation $\delta d/d = -\delta v/cz$ has the peculiar characteristic that it appears to depend on the absolute state of motion of sources, rather than on their motion relative to that of the observer. The resolution of this apparent violation of the equivalence principle is that it is necessary to allow for evolution of the velocities with time, and also, when considering perturbations on the scale of the observer-source separation, to include the gravitational redshift effect. We provide an expression for $\delta d/d$ that provides a physically consistent way to compute the impact of peculiar motions for SN1a cosmology and peculiar velocity surveys.

2) We then calculate the perturbation to the redshift as a function of source flux density, which has been proposed as an alternative probe of large-scale motions. We show how the inclusion of surface brightness modulation modifies the relation between $\delta z(m)$ and the peculiar velocity, and that, while the noise properties of this method might appear promising, the velocity signal is swamped by the effect of galaxy clustering for most scales of interest.

3) We show how, in linear theory, peculiar velocity measurements are biased downwards by the effect of smaller scale motions or by measurement errors (such as in photometric redshifts). Our results nicely explain the effects seen in simulations by Koda et al. 2013.

We critically examine the prospects for extending peculiar velocity studies to larger scales with near-term future surveys.

Key words: Cosmology: theory, observations, distance scale, large-scale structure; galaxies: distances and redshifts

1 INTRODUCTION
The perturbation to the luminosity distance $d(z)$ caused by cosmological structure is of interest for two reasons; on one hand it can degrade the precision of cosmological parameters inferred from the Hubble diagram obtained from standard candles such as SN1a (e.g. Riess et al. 1998; Perlmutter et al. 1999) while on the other it can provide a useful probe of structure.

At very low redshift the distance is, in the absence of structure, simply linearly proportional to the redshift and the dominant effect of density perturbations comes from the peculiar velocities associated with the growth of structure. These velocities cause a change to the redshift through the first order non-relativistic Doppler effect with the result that if a source has a positive radial peculiar velocity $\delta v$ then its distance is lower than for an object of the same redshift in an unperturbed universe; i.e. it has a peculiar displacement $\delta d = -\delta v/H$ and a corresponding fractional apparent luminosity enhancement of $2\delta v/Hr$.

Exploiting these measurable perturbations to the luminosity as a function of redshift as a probe of structure has a long and rich history. In the past these have been limited to very low redshift and in such studies it is legitimate to adopt a Newtonian analysis and consider only the effect of the motions – the effects here being the same for motions associated with the growth of structure or for hypothetical motions of sources relative to the cosmic frame that are not associated with density perturbations (we shall refer to such motions as “unsupported”). But to understand the effect on supernova cosmology, or to exploit motions as a probe of structure us-
ing data from deeper future surveys, a more detailed treatment is needed that allows for the non-negligible redshift of the sources. For one thing, one cannot simply apply the non-relativistic velocity addition formula to relate velocity and distance perturbations. And for finite redshifts other effects come into play. These include the gravitational redshift (Sachs & Wolfe 1967); the effect of aberration which modifies both the angular size and also the surface brightness of sources; and the effect of weak gravitational lensing, an effect distinct from aberration, which modifies sizes of galaxies leaving their surface brightness unaffected. The purpose of this paper is to elucidate some features of the observable effects of peculiar velocities at non-negligible redshifts.

The structure of the paper is as follows: In the following section (S2) we provide a review of previous work. We briefly describe the history of low-redshift peculiar velocity observational studies (S2.1) and we also contrast these with other probes of cosmic motions such as the kinematic SZ effect (S2.2.1); the observer motion induced dipole anisotropy of isotropic background radiation fields and the dipole in the source counts of very distant galaxies (S2.2.2); the effect of motions at low-redshift on structures in redshift space (S2.2.3) – sometimes called the ‘rocket-effect’; and the weak lensing effect (S2.3). In S2.4 we review recent theoretical developments. These are mostly based on the pioneering work of Sasaki and co-workers who, starting in 1987, provided a framework for describing the effects of linear density perturbations on the luminosity distance that includes the effect of peculiar velocities, gravitational lensing and the gravitational redshift (including the integrated Sachs-Wolfe effect). Many of the more recent studies isolated the terms that depend only on the peculiar motions and have applied the resulting simplified formulae to supernovae cosmology and to make forecasts for how future surveys can provide constraints on large-scale structure (coining the terms ‘Doppler-lensing’ and ‘anti-lensing’ to describe these effects, though, as we note, these are not particularly novel probes). We show, however, that the modifications to the normal Newtonian analysis that become non-negligible at finite redshift have a puzzling feature in that they appear to depend on the absolute motion of the sources relative to the cosmic frame.

In S3 we address the apparent violation of the equivalence principle in the above studies. We show how these apparent violations are banished once one allows for the decay of the peculiar velocity in the case of unsupported motions and, in the case of motions associated with structure, once one includes the Sachs-Wolfe effect.

In S4 we consider a novel proposal to measure peculiar velocities which is to try to measure the perturbation to the mean redshift as a function of flux density (rather than the perturbation to the flux-density as a function of redshift). We show that there is a non-trivial observable influence from the modulation of the surface brightness in this method. But we argue that there is a problem with this method in that, for most scales of interest, the observable is swamped by the effect of galaxy clustering. We then conclude with a discussion (S5) including a critical analysis of the prospects for extending peculiar velocity studies to larger scales with future surveys, and in appendix A we show how measurements of large-scale motions are biased by smaller scale structure.

2 PROBES OF PECULIAR VELOCITIES AND DISTANCES

2.1 Distance perturbation from standard candles

Exploiting $\delta d/dn$ as a probe of structure – so called ‘peculiar velocity’ or ‘cosmic-flow’ studies – has a long and rich history. Rubin et al. (1976) found a dipole moment of flux densities of galaxies with $cz$ in the range 3,500-6,500 km/s, indicating a motion of the Sun with respect to this shell at a speed of $\approx 600$ km/s and also a substantial motion of galaxies in this shell with respect to the cosmic microwave background frame. This was a most surprising and unexpected result since, outside of the local group, the Hubble flow appeared to be remarkably cold and close to linear. In a similar study, but with a more careful treatment of biases from selection effects, Tamman, Yahil & Sandage (1979) compared magnitudes of galaxies in the Virgo cluster with those of galaxies at the same redshift scattered around the sky to provide an impressively precise estimate of peculiar motion associated with the growth of the local super-cluster. About the same time it was realised that using HI velocity widths for spirals (Tully & Fisher, 1977; hereafter TF) gave distances with $\approx 20\%$ fractional distance uncertainty. Central velocity dispersions for ellipticals (Faber & Jackson, 1976) provided somewhat less precise distances but the discovery of the ‘fundamental plane’ in surface brightness, velocity dispersion and magnitude (Djorgovski and Davis, 1987; Dressler et al 1987) allowed the use of the $D_n - \sigma$ relation which uses velocity dispersion and surface brightness to predict the proper size of the galaxy and which provides distance precision similar to the TF relation. Further peculiar motion data have been obtained from SN1a (e.g. Turnbull et al. 2012), which have much higher precision of $\sim 8\%$ fractional distance error per object, but which are, as yet, a sparser sample of the velocity field than TF or FP samples. Other methods include using the tip of the red giant branch (TRGB) as a standard candle (Lee, Freedman & Madore, 1993) and surface-brightness fluctuations for early type galaxies which were pioneered by Tonry et al. (2000) and show generally good agreement with FP distances (Blakeslee et al. 2002). All of these techniques are, one way or another, directly measuring the perturbation to the luminosity distance caused by peculiar motions.

Major reviews of the status of peculiar velocity studies in the mid '90s were provided by Dekel (1994) and by Strauss and Willick (1995) and a good snapshot of the state of the subject at the turn of the century is given by Hudson (1999) and by Courteau and Dekel (2001), highlighting the results presented at the famous conference in Victoria, B.C. in 1999. After something of a lull, the subject is now undergoing a revival, with major advances in the data available, including the ‘CosmicFlows-2’ compilation of $\approx 8,000$ mostly TF distances (Tully et al 2013; incorporating the SFI++ TF catalog of Springob et al. 2007 as well as SN1a distances) and with significant results coming from the 6dFGS survey with an impressive $\sim 10,000$ Fundamental Plane distances (Springob et al. 2013). In addition, there has been something of a revival in using the individually less precise galaxy flux densities as distance estimators (Nusser, Branchini and Davis 2011; Branchini, Davis and Nusser 2012) exploiting the larger sample size and excellent photometric accuracy of the 2MASS redshift survey (Huchra, 2012). Nusser, Bra-
chini and Feix (2013) have discussed how this might be extended to massive photometric surveys (≈ 15, 000 square degrees with 10−σ multi-passband optical through near-IR photometry for galaxies to to m_v ~ 24.5) that will be provided by the Euclid satellite along with the anticipated ground-based support photometry (Laureijs et al. 2011).

2.2 Other observable effects of peculiar motions

There are other potential probes of velocity that are related to the above but are not exactly equivalent to measurement of the luminosity distance.

2.2.1 The Kinematic Sunyaev Ze'davich effect

One is the kinematic SZ effect on the cosmic microwave background (CMB) which has been attempted with clusters of galaxies with WMAP by Kashlinsky et al. 2010 and Kashlinsky, Atrio-Barandela, & Ebeling 2011 and with Planck (Planck Collaboration et al. 2014) and with galaxies (Lavaux, Ashordi, & Hudson, 2013). This method has the distinguishing characteristic that the observable is largely independent of distance – unlike standard candle techniques that have poor sensitivity at large distance – and should in principle provide the most powerful constraints on peculiar velocities at large distances.

This method is also quite different from the other probes considered here in that it provides a remote measurement of the CMB dipole as would have been seen by observers in the clusters, providing essentially unique constraints on departures from homogeneity – as opposed to more easily measured departures from isotropy – of our Universe.

2.2.2 Dipoles of isotropic backgrounds and source counts

Another technique is to exploit the dipole induced in otherwise isotropic radiation backgrounds or distant source counts by our motion. This was originally applied to cosmic rays by Compton & Getting (1935) but is best known, and most accurately measured, from the dipole of the cosmic microwave background (e.g. Hinshaw et al. 2009) which indicates a solar motion of 369 km/s. The dipole moment of the X-ray background has been measured by Plionis and Georgantopoulos (1999) at 1.5keV and by Scharf et al. (2000) in the 2-10keV energy range, both of whom find results broadly compatible with the CMB motion. Lahav, Piran and Treyer (1997) have shown that in these bands the effects of clustering of sources and that induced by our motion are expected to be comparable.

A closely analogous effect is the dipole of counts of discrete sources. Ellis & Baldwin (1984) showed that the perturbation to the flux density of a distant source with spectral index α caused by our motion is $ΔS/S = (1 + α) \times (n \cdot v/c)$ with n the direction to the source. So, for unresolved sources with $N(> S) \sim S^{-α}$, and allowing for aberration, the amplitude of the dipole of the counts is $D = (2 + (1 + α)x) \times (v/c)$.

Baleisis et al. (1998) have attempted to measure this with the Green Bank (87GB) (Gregory & Condon 1991) and Parkes - MIT - NRAO (PMN) catalogues (Griffith & Wright 1993), but with limited success; they attribute the very large apparent signal to calibration errors. The dipole has been determined in the NVSS survey (Condon et al. 1998) by Blake & Wall (2002) and more recently by Singal (2011) and Rubart & Schwarz (2013), the last including sources from the Westerbork WENSS survey (Rengelink et al. 1997). These studies have found generally quite good agreement of the dipole direction with that of the CMB dipole, but, puzzlingly, finding something like 2-4 times the expected amplitude. Gibelyou & Huterer (2012) have measured dipoles of various galaxy redshift surveys and also gamma-ray bursts as well as the NVSS radio sources; they find only the latter to be discrepant with theoretical expectations and suggest the NVSS dipole may be corrupted by systematic errors.

Mertens et al. 2013 have suggested that the Milky Way motion would induce a dipole signal in the magnification measured in large-area weak lensing surveys such as LSST or DES. But the quantity that they consider is the perturbation to the angular size of objects at a constant distance, which is not directly comparable to what is actually observed.

The kinematic SZ and source count dipole effects are physically distinct from methods that measure the perturbation to the luminosity distance using flux densities of sources – or features in the distribution of luminosities like the ‘knee’ in the galaxy luminosity function – that can be considered to be ‘standard candles’. These different methods are nonetheless supposed to be measuring the same physical quantity.

2.2.3 Distortion of galaxy clustering in redshift space

Another manifestation of the perturbation to the distance is the distortion of the appearance of cosmological structure in redshift space (Kaiser 1987). There are two aspects to this. The most well known, and most highly exploited, is the distortion of the correlation function; a combination of smearing along the line of sight from small scale motions and a coherent squashing associated with large scale motions (see e.g. Hamilton 1997; Percival et al. 2011 for reviews). A separate effect, less widely considered, but more closely related to the subject here, is the distortion of the density contrast $δ(r)$ inferred from the density of galaxies as this is estimated by dividing the counts in a region by the selection function $φ(r)$; this is a function of the distance but is usually evaluated in redshift space, resulting in a biased picture of local structure. Sometimes referred to as the ‘Rocket-effect’, this effect, like peculiar velocity measurements, is most important at low redshifts.

In one sense one can consider this to be another probe of peculiar motions, but the difficulty is that the velocity induced density perturbation is superposed on the real-space clustering pattern, and decoupling these is not easy. The distance measurement methods as described above, in contrast, provide distances, and hence peculiar velocities, that are not biased or otherwise affected by perturbations to the galaxy density. Such methods give a clean determination of the velocity, though subject to the assumption that the luminosity function of the sources being used is universal (i.e. unperturbed by local density or other environmental influences). This decoupling of density and velocity is achieved at the cost of requiring that the estimator, in the case of galaxy-based measurements, may only use information contained in the shape of the flux density (or angular size) distribution of those objects that survive the selection criteria; they may not use the actual number of detected
objects. In the local universe the effect of velocities on the galaxy density field is not so much a useful probe of structure as an impediment to attempts to determine the local peculiar gravity field for comparison with the velocity field with the aim of determining the growth rate of structure (Nusser, Davis & Branchini 2014).

2.3 Perturbation to the luminosity distance from gravitational lensing

The fractional perturbation to the luminosity distance from peculiar motions falls rapidly with redshift, but at larger redshift another effect comes into play which is the modulation of sizes of objects by gravitational lensing. This was first considered theoretically by Zel’dovich (1964) and by Gunn (1967a; citing unpublished work of Richard Feynman presented at a Caltech colloquium in 1964). The essential question is: given a space-time with inhomogeneity and metric fluctuations, how is the angular size of distant objects modified by the structures along the line of sight. This can be obtained by propagating a narrow bundle of rays back from the observer to a distant source plane at some specified distance, which provides the 2x2 matrix giving the mapping from source plane positions to sky-plane angles whose determinant is the amplification (the off-diagonal and asymmetric parts of the diagonal components being the shear and the mean of the diagonal terms being conventionally written as 1 + κ with κ being the convergence). At linear order this may be calculated in the Born approximation; the first order relative deviation of a pair of null rays is calculated using tides evaluated along the unperturbed paths.

The above papers focused on the fluctuations in the convergence caused by structures along the line of sight. Also of interest is the mean amplification which will be non-zero if the paths to observed objects tend to avoid foreground objects. This was considered by Gunn (1967b) and by Dyer and Roeder (1972, 1974).

The lensing effect is a cumulative one; the amplification of a background object depends on an integral of the transverse tidal field along the line of sight. Consequently the convergence changes quite slowly with distance. Liouville’s theorem tells us that the surface brightness depends only on the redshift, so the fractional perturbation to the luminosity is, at linear order, just the fractional change in the solid angle, and therefore the luminosity distance perturbation is related to the convergence by \( \delta d/d = -\kappa(z) \).

2.4 More recent theoretical developments

The early theoretical papers cited above considered universes containing point masses or used ‘swiss-cheese’ models. A significant advance in understanding of the magnitude-redshift relation in more realistic perturbed FLRW models with metric fluctuations and motions given according to the prediction for the spectrum of fluctuations in \( \Lambda \)CDM or similar models was provided by Sasaki (1987) and elucidated by Futamase and Sasaki (1989) and by Sugiyama, Sugiyama and Sasaki (1999; hereafter S99). These papers provided a unified treatment valid at both low and high redshift and revealed clearly how the lensing and peculiar velocity effects come to dominate at high and low redshifts respectively (they also included other, typically subdominant, effects such as the integrated Sachs-Wolfe (1967) effect).

Ignoring the effects of the peculiar gravitational potential, one finds from S99’s equation 3.15 that the effect of peculiar velocities of sources ‘S’ and the observer ‘O’ induce a fractional change in luminosity distance (which is to say minus half the fractional change in flux density at constant observed redshift)

\[
\delta d \bigg|_{z} = -a \frac{\delta a}{a'\chi} (v_S \cdot n - v_O \cdot n) + v_S \cdot n. \tag{1}
\]

Here \( a \) is the scale factor, \( \chi = \int dz/H \) is conformal distance, prime denotes derivative with respect to conformal time and \( n \) is a unit vector from the observer along the line of sight back to the source. Velocities here and below are in units such that the speed of light \( c = 1 \).

At low redshift, \( a/a'\chi = 1/(aH\chi) \approx 1/z \) so the first term in (1) is dominant and is simply the peculiar displacement \( \delta d = -\delta v/H \). The last term can be safely ignored for all of the observations described above, but becomes significant at finite redshift. Interestingly, and indeed rather puzzlingly, this term depends only on the motion of the source, unlike the leading order low-redshift effect that depends on the relative peculiar motion of the source with respect to that of the observer. Thus (1) might appear to say that in a situation where the observer and the sources share the same common motion, perhaps because they have been accelerated by some very distant mass excess, there would be an observable dipole perturbation to the luminosity distance at constant \( z \) from which we can infer our motion relative to the ‘cosmic frame’. But that would violate the equivalence principle (EP) which dictates that for sources and observer in free fall the only locally observable effect of distant masses is the tidal distortion, which would show up as a quadrupole.

The velocity of the observed region as a whole with respect to the state of motion of more distant matter cannot be determined by local measurements. Yet for \( v_S = v_O \) equation (1) seems to provide an absolute cosmic speedometer.

Subsequently, many papers have repeated this kind of analysis. Pyne & Birkinshaw (2004) also computed \( \delta d/d \) in perturbed FLRW models. They focussed mostly on gravitational lensing effects at high \( z \) but the velocity dependent terms in e.g. their equations 32 & 23 are equivalent to (1). Their analysis makes clear that (1) does not rely on the assumption that the velocities are purely growing mode linear perturbations.

Hui & Greene (2006) performed a similar perturbation analysis, and they also showed how (1) can be understood.
heuristically in the case of an unperturbed FRW model in which sources and the observer have peculiar velocities (i.e. what we are calling ‘unsupported’ motions). They argue that on all relevant scales equation (1), augmented by the usual gravitational lensing effect, provides an adequate description. Hui & Greene (2006), Cooray & Caldwell (2006) and Davis et al. (2011) have all used (1) to calculate the impact of peculiar velocities on the precision of SN1a cosmology; these papers calculate the extra contribution to the error covariance matrix over and above the diagonal contribution from uncorrelated measurement errors and the corresponding increase in the uncertainty in the derived dark energy properties.

Hui & Greene make the important point that even small velocities can have a big impact because the errors they induce are strongly spatially correlated. Since the primary goal of SN1a is to determine the dark energy ‘equation of state’, what is relevant is the monopole of the distance perturbation. For an isotropic uniformly sampled full sky survey the observer motion would be irrelevant as it generates only a dipole. But for realistic current or near term future surveys the sky coverage is strongly non-uniform and so there is coupling of the dipole and other low order moments into the monopole. So for real surveys, the motion of the observer – which couples to all of the errors at some level – is the most critical. Yet all of the papers cited above only compute the extra covariance arising from source motions. The solution, as suggested by Hui and Greene, is to calculate the correlated motions of SN hosts, conditional on the motion of the Local Group motion with respect to the CMB. They find that this makes little difference for most current SN1a surveys, but may be important for measurements that use lots of low-z SN to pin down the current expansion rate more accurately.

Bonvin, Durrer and Gasparini (2006) also calculated $\delta d/d$ in linearly perturbed FRW models, again reproducing the dependence on velocities of $S^99$ above (see their equation 59, and noting that in their notation $(\eta_0 - \eta_0)H_S$ is the same as $a'\chi/a$ in (1)). They suggest that the power spectrum of $\delta d/d$ provides a new observational tool which can be used to determine cosmological parameters. Second order perturbations to the distance have been considered by umeh, Clarkson & Maartens (2012, 2014).

Bonvin (2008) calculated the effect of peculiar velocities on weak gravitational lensing convergence in perturbed FRW models. The version of the paper in the journal has an expression for the velocity induced lensing convergence that, unlike (1), appears to depend only on the relative source and observer velocities, but this has been emended in a revised version on the arXiv.

Following this, Boleko et al. 2013 have noted that the standard lensing convergence effect (that produced by the transverse deflection of rays) “is swamped at low redshifts by a relativistic Doppler term that is typically neglected”. They have dubbed this ‘anti-lensing’. The same group (Bacon et al. 2014) have followed this with a more detailed analysis estimating how well this should be measurable with deeper future redshift surveys and have coined the phrase ‘Doppler-lensing’ to describe the effect of peculiar velocities.

There is clearly a bit of a disconnect between two disparate communities here. The cosmological perturbation theorists are evidently unaware that the probes of cosmology that they are proposing have a long and productive history of practical application going back about 4 decades as described in S2.1 above. The use of power-spectrum or auto-correlation analysis was also well developed early on (Gorski, 1988; Kaiser, 1988; Gorski et al. 1989: Groth, Juszkiewicz & Ostriker 1989). Somehow the equivalence of the physical quantities being discussed by the different communities here has been lost somewhere along the line. The confusion may have been exacerbated by the description of the velocity induced effects as ‘weak lensing convergence’. There is no lensing here, at least in the commonly used sense of deflection of light rays by cosmological structure (there is a deflection of rays caused by aberration associated with motion of the observer – but that is largely ignored in the papers above), and therefore no convergence of rays; all that is happening is that velocities perturb the relation between observed redshift and angular diameter distance.

As already mentioned, the asymptotic $z \to 0$ behaviour of equation (1) can be easily understood without recourse to relativity or cosmological perturbation theory; if a source has a peculiar velocity (relative to any peculiar velocity of the observer) then it has a peculiar displacement, in redshift space, that is simply $\delta r = -\delta v / H$. For most of the measurements described above, this simple notion provides a reasonable model for interpreting the observations. What is interesting about the formalism of Sasaki and colleagues (and the more recent theoretical papers cited above) is not just that it provides a nice unified treatment of peculiar motions and gravitational lensing, but that it should be valid to describe the effect of peculiar velocities at all redshifts, not just in the limit $z \ll 1$. Future redshift surveys will probe to larger redshifts and at even at $z \sim 0.1$ the next order correction to the dominant $\sim v / cz$ term – a contribution to $\delta d/d \sim v / c$ which is independent of redshift – is a 10% effect. Similarly, it would seem prudent to get this right when computing the impact of flows on scales of a few hundred Mpc on SN1a cosmology, given how much is riding on the result. But, as we have noted, the formalism used exhibits seemingly unphysical behaviour.

In the following section we show that in the case of ‘unsupported’ motions – that is to say motions of observers or sources considered as massless test particles in an unperturbed cosmology – the resolution of the puzzle of the apparent violation of the EP is that the velocities are decaying, so a spatially uniform flow at a given instant of cosmic time will appear non-uniform on the past light cone of the observer, and it is light-cone variables that appear in the formula (1). The finite redshift term can be thought of as the correction so that the total effect is determined by the relative velocities at constant cosmic time. For finite matter density and velocities associated with growth of structure the situation is a little more complicated, and the resolution is that the EP violating dipole from the velocity field alone is cancelled in a consistent treatment by the gravitational redshift effect. In the process we obtain a somewhat improved expression for the perturbation to the luminosity distance that properly accounts for the motion of the observer. This can be used to better predict the effect of structure on the precision of the SN1a cosmological parameter estimation.

We then turn attention to the prospects for measuring peculiar velocities from the perturbation to the redshift as a function of flux density, as proposed by Nusser, Branchini &
Feix (2013). We show that this has the expected asymptotic behaviour at very low redshift that \( \delta z / (1 + z) \cong n \cdot (v_S - v_O) \) but that, as with \( \delta d/d \), there are finite-redshift modifications; here caused in part by relativistic beaming effect modifying the surface brightness. But the challenge with this method is that the proposed measurable is swamped for most scales of interest by fluctuations in the mean redshift caused by large-scale structure.

In the Discussion section (S5) we consider the extension of peculiar velocity studies to larger scales with future redshift surveys.

In Appendix A we present a linearised analysis of the bias in the velocity field measurement caused by small scale motions or measurement errors and we show that this accounts nicely for the damping found empirically in the numerical studies of Koda et al. 2013.

3 PERTURBATION TO FLUX DENSITIES AT CONSTANT REDSHIFT

First let’s recall the origin of (1). Hui and Greene (2006) have given a very nice heuristic derivation of this. They compute the effect of source and observer velocities in a homogeneous FRW model; i.e. they consider the effect of what we are calling ‘unsupported’ motions. They calculate the perturbation to the object flux density and to the redshift at fixed distance, and then combine these to obtain the perturbation to \( d \) at fixed observed redshift. This is legitimate, but is somewhat roundabout as the perturbation to the flux densities involves consideration of relativistic beaming in which the redshift perturbation modulates the surface brightness of the sources (see e.g. the discussion in Davis et al. 2011). A more direct approach is simply to compute the perturbation to the angular size and redshift to obtain the ‘convergence’ or equivalently minus \( \delta d/d \).

Consider, for simplicity, a spatially flat background cosmology with metric \( ds^2 = a^2 (\eta) (-d\eta^2 + d\chi^2 + \chi^2 d\Omega^2) \) where the scale factor is dimensionless and is equal to unity at present: \( a(\eta_0) = 1 \) and with an observer at the origin of the spatial coordinate system \( \chi = 0 \).

In the absence of source or observer motions the redshift of a source at conformal distance \( \chi = \chi(z) = \int dz/H \) (where \( H \) is the Hubble parameter) and the distance is a function of redshift: \( \chi = \chi(z) = \int dz/H \). The angular diameter of a small spherical standard source of physical diameter \( l \) at this distance is \( \theta = \theta(\chi) = l/a\chi = (1 + \chi)l/\chi \), which varies with redshift as

\[
\frac{d\theta}{dz} = \frac{l}{\chi} \left( 1 - \frac{a}{a'\chi} \right). \tag{2}
\]

As is well known, this vanishes (at about \( z \approx 1.6 \) in the standard ΛCDM model) where \( \chi(z) \) is stationary owing to the focussing of rays by the smoothly distributed matter.

An observer moving with peculiar velocity \( v_O \) perceives a standard source at distance \( \chi \) to have angular size

\[
\theta = \theta(\chi)(1 - \mathbf{n} \cdot v_O + \ldots) \tag{3}
\]

because of special relativistic aberration – as is easily shown by applying a Lorentz boost and linearising in the assumed small velocity \( v_O \) – and the redshift is

\[
z = \chi(\chi) + \delta z \tag{4}
\]

where, in the present circumstances \( \delta z / (1 + z) \) is the fractional perturbation to the wavelength caused by the peculiar velocities of the source and observer.

The angular size that a source of this redshift would subtend in the absence of peculiar velocities is

\[
\theta_0 = \theta(\chi) = \theta(\chi(\chi) + \delta z) = \theta(\chi) + (\delta \theta / \delta z) \delta z + \ldots \tag{5}
\]

So, subtracting (5) from (3), and dividing by \( \theta \) gives the first-order fractional perturbation to the angle at this redshift \( z \) from peculiar motions as

\[
\frac{\delta \theta}{\theta} |_z = -\frac{d\theta}{dz} \frac{\delta z}{\theta} - \mathbf{n} \cdot v_O = \left( \frac{a}{a'\chi} - 1 \right) \frac{\delta z}{1 + z} - \mathbf{n} \cdot v_O. \tag{6}
\]

This is (minus) the perturbation to the angular diameter distance, but as this is at constant redshift this is the same as minus the perturbation to the luminosity distance, as the surface brightness depends only on the redshift. In the situation described here of unsupported peculiar motions in a homogeneous background cosmology the perturbation to the redshift is \( \delta z / (1 + z) = n \cdot (v_S - v_O) \) which in (6) is equivalent to (1).

More generally, the fractional wavelength perturbation \( \delta z / (1 + z) \) in (6) will also include contributions from the perturbation to the potential; the peculiar gravity. But note that for perturbations on scales much less than the Hubble scale, which is what is relevant here as the perturbations to the velocity on the horizon scale is on the order of the primary CMB anisotropy or about \( 10^{-5} \), which is only a few km/s, the perturbation to the redshift is well described by Newtonian theory, so the only significant ‘relativistic’ effect here is the aberration caused by the observer motion. And it is this aberration – or ‘beaming’ – that is the source of the asymmetry between observer and source velocities in (1).

Returning to the effect of unsupported peculiar motions, equation (6) can be written as

\[
\frac{\delta \theta}{\theta} |_z = \frac{a}{a'\chi} \left[ \left( 1 - \frac{a'}{a} \right) \mathbf{n} \cdot v_S - \mathbf{n} \cdot v_O \right]. \tag{7}
\]

This might appear, at face value, to have a zeroth order (in redshift) non-zero value in the case that the observer and source share the same motion. But that is illusory. The velocities appearing in (7) are at different times, and in the situation described here the velocities are decaying as (e.g. Peebles 1980) \( v \propto 1/\eta = 1 + z \), so \( v_S(\eta_S) = v_S(\eta_0)(1 + z) \), and

\[
\frac{\delta \theta}{\theta} |_z = \frac{a}{a'\chi} \left[ \left( 1 - \frac{a'}{a} \right) (1 + z) \mathbf{n} \cdot v_S - \mathbf{n} \cdot v_O \right] \tag{8}
\]

where the velocities are both at the time of observation \( \eta_0 \). But \( 1 - a'/a = 1 - z + O(z^2) \) and consequently at low redshift the perturbation to the angular (or luminosity) distance depends only on the relative motion of the source and observer (with corrections that are quadratic in the redshift):

\[
\frac{\delta \theta}{\theta} |_z \approx \frac{a}{a'\chi} \mathbf{n} \cdot (v_S - v_O) \eta_0 \tag{9}
\]

where, by ‘\( \approx \)’, we mean equal up to fractional corrections of order \( z^2 \). Thus the apparent EP violation in (1) appears because, in that equation, the source and observer velocities are not evaluated at the same time. When they are, then to lowest order the EP violation disappears.
Luminosity distance perturbations from peculiar motions

We are more interested in the case that the motions are associated with the growth of structure, in which case the velocities do not simply decay as $v \propto 1 + z$, rather the equation of motion for the peculiar velocity is

$$\frac{dv}{dt} = -Hv + g$$  \hspace{1cm} (10)

where $g$ is the gravity sourced in Poisson’s equation by the matter density perturbation $\delta \rho$ (Peebles 1980). Consequently $\delta z / \theta = (1 + z) \delta (\eta z) - g \Delta t$ which, in (7) would seem to give a zeroth order (in redshift) contribution to $\Delta \theta / \theta$ of $-n \cdot g \Delta t / cz \simeq -n \cdot g / H$. This is a dipole which, if observable, would allow one to determine the acceleration from distance structures from local observations, again in violation of the EP. But if one is going to incorporate the peculiar gravity then, to be consistent, one should use, following S^3\theta, 0\,000\,000 RAS, MNRAS following S\textsuperscript{3}\theta, 0\,000\,000 RAS, MNRAS, for $\delta z / (1 + z)$ in (7) not just the Doppler shift, but one should also include the gravitational redshift $\psi_S - \psi_O \simeq \int d\chi \cdot g(\chi)$ which, when multiplied by the large pre-factor $\sim 1 / z$ gives an effect of the same order for structures on the scale of the source-observer separation. We then have

$$\frac{\delta \theta}{\theta} \bigg|_z \simeq \frac{a}{a' \chi} \frac{n \cdot (v_S - v_O)_{\eta O} - n \cdot \frac{g_S}{H} + \frac{1}{z} \int d\chi \cdot g(\chi)}{x_O}.$$  \hspace{1cm} (11)

For the case of a distant attractor the gravity $g$ will be nearly spatially constant within the observed region and since $n = (\chi_S - \chi_O) / (\chi_S - \chi_X)$ the last two terms cancel and the gravity from distant matter is therefore unobservable from local measurements, again in accord with the EP.

In addition to being blind to super-survey size sources of gravity, expression (11) has the desirable characteristic that it is independent of the choice of ‘background’ that we consider the actual universe to be a perturbation about. For example, were one to consider the observable region to be a perturbation to a background with the “wrong” density and therefore the wrong value for the Hubble parameter, say, this would change the peculiar velocities. But the corresponding spatially constant density perturbation would produce a spatially constant tidal field which would compensate so that the physical observable $\delta d / d$ is invariant.

Finally, in linear theory the gravity and velocity are proportional to one another, with $v = (2f(\Omega_m) / \Omega_m) \times g / H$, where the perturbation growth rate factor $f(\Omega_m) \equiv d \ln \delta / d \ln a \simeq \Omega_m^{1.5}$, and we can express the perturbation of the luminosity distance entirely in terms of velocities as

$$\frac{\delta d}{d} \bigg|_z \simeq \frac{a}{a' \chi} \frac{n \cdot (v_S - v_O)}{x_O} + \frac{3 \Omega_m}{2f(\Omega_m)} \left( n \cdot v_S - \frac{H}{z} \int \frac{d\chi \cdot v(\chi)}{x_O} \right).$$  \hspace{1cm} (12)

where the velocities are all to be understood to be at the time of observation.

This formula (12) provides, at least in the context of linear theory, a physically consistent way to allow for the effect of peculiar motions on supernova cosmology. Given an assumed power spectrum for the density, and hence velocity, perturbations, it can be used as in the studies referenced above to compute the extra contribution to the covariance function. Also, given some estimate of the actual structure in the nearby universe, it can be used to correct the distances of SN for the associated motion, with the additional covariance from larger scale motions being calculated from the conditional probability distribution for the residual motions.

4 PERTURBATION TO THE REDSHIFT AT CONSTANT FLUX DENSITY

Nusser, Branchini and Feix (2013) have suggested that useful measurements of large scale motions could be obtained using the photometric redshifts from Euclid (Laureijs et al. 2011) which will be obtained from a combination of ground-based surveys (providing at least three of the $g, r, i$ and $z$ bands) and the Euclid photometry in the broad VIS band and in $Y, J$ & $H$. This is expected to provide $\sim 10^9$ photometric redshifts with better than $\sim 5\%$ precision in $1 + z$ to mean redshift $\bar{z} \simeq 1$ and covering 15,000 square degrees. The ground based surveys will perform photometry with $\sim 10$-sigma detections limits deeper than 24th magnitude and may by themselves provide useful photometric redshifts in advance of Euclid. Indeed some of these data in the Southern sky are already being collected by the DES\textsuperscript{2} dark energy survey which is planned to cover $\sim 5,000$ square degrees.

The Nusser, Branchini and Feix (2013) proposal differs from conventional peculiar velocity measurements in that rather than the considering the perturbation to the distance, and therefore the flux densities of objects, at a given redshift they are proposing to measure the perturbation to the redshift, as a function of direction, relative to the mean redshift for sources of a given flux density. To this end they define a redshift perturbation per galaxy

$$\Theta_i = \frac{z_i - z_{\cos}(m_i)}{1 + z_i},$$  \hspace{1cm} (13)

where $z_i$ is the redshift of the $i$th galaxy and $z_{\cos}(m)$ is the mean redshift for galaxies of apparent magnitude $m$. This, they argue is dominated, for sub-horizon scale structures, by the Doppler shift, so that the average of $\Theta_i$ over galaxies lying in some solid angle on the sky provides an estimate of an appropriately weighted average of $n \cdot (v_S - v_O)$ along the line of sight. What they propose is to use the angular power spectrum of this quantity to constrain the power spectrum of matter fluctuations.

The motivation for this is that they claim that this would provide measurements of large-scale motions with a precision corresponding to a fractional distance error of approximately 30% per galaxy from photometry and relatively low resolution spectroscopy. This is not very much worse than the precision obtained from the considerably more expensive TF or $D_n - \sigma$ techniques.

However, this technique differs from the normal method in several important respects: One is that the finite-redshift behaviour is somewhat different from that for the luminosity distance. A second is that by using only the angular variation of the line-of-sight averaged peculiar velocity this method discards useful information about the variation of motions along the line of sight (this is significant since it

\footnote{http://www.darkenergysurvey.org/}
is the angular variations that are most susceptible to systematic errors in the photometric zero-point). But the most important distinction is that the estimator tend will to be heavily influenced by large-scale structure in the galaxy spatial distribution. In fact, as we will show, the signal from peculiar motions will be swamped by fluctuations from galaxy clustering.

The assumption underlying the idea that an average of \( \Theta \) from structure will not be shown to be only dependent on relative velocities at constant proper time (plus corrections of order \( z^2 \)) for the case of unsupported motions and can be cast into a physically consistent form in the case of motions associated with growing structure with the inclusion of the Sachs-Wolfe term.

This is an idealised model. The assumption of standard candles means that observations at some \( m \) probe only a thin shell at the corresponding redshift. Allowing a distribution function for the absolute magnitude and surface brightness results in a distribution \( N(z|m) \) but with the same general dependence on redshift. As mentioned, the details also depend on galaxy SEDs and filters transmission functions.

This looks quite promising, but this estimator for motions also differs from conventional luminosity distance estimators in that \( \Theta \) is not unbiased by the density of galaxies. This has negative consequences. In a universe with no structure, but with moving sources and observers, the average of \( \Theta \) would provide a probe of motions with the relatively good precision indicated from the width of \( N(z) \). But in reality, even in the absence of peculiar velocities, there will be additional fluctuations in the mean redshift along each line of sight caused by density perturbations.

If one were to compute the average of \( \Theta_\ell \) for all of the galaxies in a cone of small solid angle \( d\Omega \) and in some modest range of magnitudes \( dm \) around \( m \) as \( \overline{\Theta} = N^{-1}\sum \Theta_\ell \) then the cosmological signal would be essentially just the mean of the line of sight source velocity weighted by the density of galaxies (and the finite redshift terms). But with large scale structure there will be an additional contribution

\[
\overline{\Theta} = (1 + \overline{\eta}(m))^{-1} \int d\Omega dz z^2 n(z) (z - \overline{\eta}(m)) / N. \tag{18}
\]

The integral here will, on average, be zero, but there will be fluctuations arising from the fluctuations in the number density of galaxies \( n(z) = \overline{\eta}(z)(1 + \delta(z)) \). In a simple model of ‘blobs’ with some characteristic scale \( \Delta z \) and rms density contrast \( \delta \) then the rms contribution to \( \overline{\Theta} \) from structure will be equal to \( \overline{\eta}(m) \delta \sqrt{\overline{\epsilon}} \) times some factor of order unity, where \( n_s \sim \overline{\eta} / \Delta z \) is the number of random structures along the line of sight. The ‘signal’, on the other hand, is on the order \( \overline{\Theta} \sim \delta \Delta z / \sqrt{\overline{\epsilon}} \). So for structures much smaller than the depth of the survey – which for deep surveys means much less than the Hubble scale – the signal will be swamped by the noise from large-scale structure.

Of course the density of galaxies in redshift space is measurable, so one can try to correct for this bias. But this leads one back to an estimator for the luminosity distance perturbation as a function of redshift where one has to infer the distance from the shape of the distribution of fluxes of objects above the flux limit (perhaps sub-divided by distance independent properties), and the statistical precision will not be the same as that of \( \Theta \).

5 DISCUSSION

5.1 Summary of major results

To recapitulate the major results so far, in Section 3 we showed how the somewhat puzzling dependency of the finite-redshift corrections to the luminosity distance perturbations on the absolute motion of the source and observer is removed once allowance is made for the time evolution of the velocity field and for the gravitational redshift. We note that Sasaki and colleagues never separated the different contributions, so this does not cast any doubt on the validity of their results. The only question is regarding the legitimacy of using the peculiar velocity sourced effects alone. We obtained an
expression for $\delta d/d$ in terms of the peculiar velocity that is manifestly respectful of the equivalence principle and which provides a consistent way to allow for the motion of the source. This provides an improved way to estimate of the impact of motions on SN1a cosmology precision. But this is not a huge effect; for local perturbations on scale of a few hundred Mpc this is a 10% correction, and in the standard $\Lambda$CDM model there is not much effect from larger scales. Nor is this a very big effect for peculiar velocity measurements (or ‘Doppler-lensing’; we hope we have dispelled any notion that these are different) if one is measuring the effect of perturbations on scales much smaller than the typical observer-source separation, but again (12) should provide a more accurate result and so should be used in forecasting the performance of future surveys.

In Section 4 we considered the perturbation to the redshift for sources of a given flux density. We showed that one needs here to consider the effect of the motions on the surface brightness which results in a non-trivial dependence of the effect on the depth of the survey. But we also argued that the signal would, except for very large-scale modes, be swamped by galaxy clustering (as this measure of velocity is not unbiased with respect to galaxy number density).

5.2 Prospects for peculiar velocities in larger, future surveys

We turn now to the prospects for extending peculiar velocity studies to greater depth, with particular focus on the possibility of exploiting future photometric surveys, possibly augmented with low resolution spectroscopy to provide redshifts, but without the more expensive velocity width information required for TF or $D_n - \sigma$ measurements.

5.2.1 Scaling of FOM with depth

Extending peculiar velocity studies to large distances generally has a rather poor return on telescope time because the distance errors grow linearly with depth $D$. And surveys without velocity dispersions for early type galaxies or HI velocity widths for spirals have generally higher distance error per galaxy. But there is still potentially useful information. The number of regions of a given size of interest $L$ grows as $D^3$ so the net error on the variance of flows on scale $L$ does decrease (but only as $1/\sqrt{D}$ so the figure of merit (FOM) scales linearly with depth – or only as the cube-root of the number of galaxies surveyed). The measurement of Tammann, Yahil and Sandage (1979) using luminosities of galaxies as a distance indicator gave a measurement of the coherent peculiar motion on the ~10Mpc/h scale of the local supercluster with ~100 km/s precision. So scaling up to a survey volume with huge numbers of such structures tells us that obtaining significant measurements of the variance of flows on similar scales should be do-able. Plus, of course, in addition to making more precise estimates of the velocity power spectrum at fixed scale, deeper surveys are invaluable for measuring bulk motion on the scale of the entire survey volume. It might be hard to justify mounting a survey on these grounds, but if they are carried out for other reasons then there is no question that they should yield information on peculiar velocities.

5.2.2 Peculiar velocities vs. redshift space distortions

It is reasonable to ask whether the information thus gained will be competitive with or complementary to that garnered from redshift space distortion studies (RSD), for which the FOM scales as the volume. At the present, these two techniques are roughly equally statistically powerful for measurement of the growth rate parameter $\beta$, despite there being many, many more galaxies in the surveys used for RSD (~800,000) as compared to the samples of currently ~ a thousand distance measurements (Hudson & Turnbull 2012; Koda et al. 2013). The reason is that RSD measurements are an auto-correlation measurement and therefore suffer from cosmic variance whereas velocity based determinations exploit cross-correlation and are therefore free of cosmic variance. Future surveys such as TAIPAN (Koda et al. 2013) and WALLABY (Duffy et al. 2012) with tens of thousands of distances will yield significant improvement in peculiar velocity measurements (see e.g. Koda et al. 2013). But ultimately in the future the balance will shift in favour of RSD because distance-based velocities scale poorly with depth as noted above (kinetic SZ measurements, however, do not suffer this penalty).

A further argument for measuring peculiar velocities as well as RSD is that both are affected by small scale motions, but in somewhat different ways. Modelling RSD involves the pairwise velocity distribution as a function of projected separation which is difficult to predict. It is dangerous to assume a dispersion that is independent of projected separation as in e.g. the Landy, Szalay & Broadhurst (1998) method since it is expected that, qualitatively speaking, the dispersion at small projected separation is dominated by the ‘1-halo’ contribution which is heavily weighted to massive clusters while at larger separation one is dealing with pairs that are in separate virialised systems where the appropriate average is dominated by small systems. One can try to estimate the pairwise velocity dispersion from group and cluster catalogs, but the result from e.g. the Crook et al. (2007) group and cluster catalogue from 2MRS is small and seems to be at odds with what is needed to match the data. This, of course, may be simply because this simple modelling does not include the relative motions of groups and clusters.

These small-scale peculiar motions also cause a bias in measurements of larger-scale flows. Peculiar velocity measurements suffer are well known to be susceptible to biases of various kinds (see e.g. Faber et al. 1994; Strauss & Willick 1995) that are often described as ‘Malmquist’ bias. These biases depend on how the data are selected and analysed (e.g. whether one regresses magnitude onto line-width or vice versa) and on what quantity one tries to measure (e.g. peculiar velocity as a function of distance vs. peculiar displacement as a function of redshift; or in Strauss and Willick’s terminology method I vs method II).

Peculiar velocity measurements may be also be biased by environmental (i.e. density) dependence of the luminosities of galaxies. But in the context of measurements of large-scale linear structure these can in principle be measured by cross-correlating apparent peculiar velocity and density in Fourier space. The true peculiar velocity is 90 degrees out of phase with the density and so appears as an imaginary contribution to the cross-spectrum while linear environmen-
tial bias shows up in the real part. See McDonald 2009 for further discussion.

The most serious ‘Malmquist’-type effects can be avoided if one uses the flux densities, together possibly with auxiliary distance independent quantities such as velocity widths or dispersions, to estimate the distance to a collection of galaxies in a localised region of redshift space (i.e. method II). For example, if there is no selection on velocity width, as is often a reasonable approximation, and if residuals in log velocity width are approximately Gaussian, as is again often assumed, then an unbiased estimate of the peculiar displacement is obtained from the deviation of the mean log velocity width from that expected if the galaxies are at the distance indicated by their redshift. This is known, for historical reasons, as the ‘inverse’ method. More generally, if the joint distribution of absolute magnitude and velocity width, or other distance independent attributes, is known then one can obtain the likelihood of the distance under the assumption that all of the galaxies in that region lie at the same distance, and thereby obtain an unbiased maximum likelihood estimate for the distance.

These results rely critically on the assumption that the velocity field is locally ‘cold’, so the galaxies in question can be assumed to have a common distance. Motions associated with small and intermediate scale structure violate this assumption and this introduces a bias not dissimilar to that which effects RSD. The effect is easy to understand; if one is measuring the flow in a region of redshift space on the back of a supercluster where the density of galaxies will be falling with radius and if there is any dispersion in distance at a given redshift then in a region of redshift space there will tend to be more galaxies that have scattered away from us than have scattered towards us from behind. In the Appendix 1. we show that in linear theory, one would expect this to cause the measured velocity in redshift space to be biased downwards by a factor $1 - k^2 \sigma_v^2 / \beta H^2$.

In principle (though subject to having a viable model for the biasing of the space distribution of galaxies) this velocity bias, and hence $\sigma_v$ also, is measurable. So peculiar velocity measurements provide independent constraints on the distribution of relative motions of galaxies that is the major nuisance factor in the interpretation of RSD.

Another potentially useful difference between these probes is that RSD measures only that component of the large-scale velocity field that is associated with the large-scale density contrast. In conventional models for structure formation that is all there is, but in principle one could imagine that the actual cause of peculiar motions is not given by the gravity predicted by the density contrast either because of some kind of exotic ‘biasing’ (e.g. large-scale astrophysical modulation of galaxy creation) or some exotic source of gravity. It would be nice to be able to test this aspect of the prediction of gravitational instability directly.

5.2.3 Peculiar velocities without velocity widths: the ‘photometric plane’

Turning to the precision that might be obtained from future surveys without velocity-width information, Bacon et al. (2014) made forecasts for ‘Doppler-lensing’ experiments with DES, Euclid & SKA. The most conservative example they considered was the DES photometric survey with spectroscopic follow-up to obtain redshift for those galaxies at $0.1 < z < 0.3$. They assumed a surface density of 0.7 objects per square arc-minute, and that distances could be obtained with fractional error per galaxy of 30%. These numbers both seem a little optimistic.

This redshift range and surface density implies a space density of $8 \times 10^{-2} (\text{Mpc}/h)^{-3}$ but to reach that density requires observations of galaxies about 4 magnitudes fainter than $M_*$ or about 24th magnitude (i.e. about as deep as the 10-$\sigma$ photometric limits – though only a sub-set selected by photometric redshift to lie at this low redshift would need to be targeted). To obtain spectra this faint requires $\sim$ 1 hour integrations on a 10m telescope. The surface density is well matched to e.g. DEIMOS on Keck-II but the problem is that the field of view of this instrument is less than 100 square arc-minutes so to cover the area the size of the DES footprint would take $\sim 10^5$ hours, which is impractical.

The 30% fractional distance error estimate was based on the small scatter in sizes of objects in the HST COSMOS survey found by Schmidt et al. (2012) who devised a weak lensing convergence estimator based on the size and magnitude of these objects. But scatter in observed size does not simply equate to the distance precision as there are efficiency factors arising from incompleteness (Schmidt et al. 2012 attempted to estimate these).

If one were simply to use the flux densities of all galaxies in a flux limited survey taken willy-nilly without any auxiliary information the distance error is very much higher (and the figure of merit scales inversely as the square of this). An interesting question is whether one can do significantly better using information such as size (or surface-brightness), morphology and colour. In this vein Tammann, Yahil & Sandage divided their galaxies by morphologically determined ‘luminosity classes’ and similarly Nusser, Branchini & Davis (2011) sub-divided the 2MRS galaxies according to their classification as spirals or ellipticals. These two sub-samples have similar shaped LFs but $M_*$ differs by about 0.4 magnitudes so combining velocity estimates obtained from these separately has slightly better performance than taking them together and ignoring the difference in their LFs.

For elliptical galaxies it may be possible to improve the precision of photometric distances using other distance independent information. Kormendy (1977) showed that the size of elliptical galaxies is strongly correlated with their surface brightness, and applied this to measure distances to the Virgo cluster. The small scatter in this ‘Kormendy relation’ would indicate something like 30% fractional distance uncertainty. Adding colour information, de Carvalho & Djorgovski (1998) claimed distance precision of 25% and argued that it might be possible to get 15% distance uncertainty with more homogeneous data, which would be better even than the $D_L - \sigma$ relation.$^3$

This is all very encouraging, but subsequent studies suggest that the promised performance from the ‘photometric fundamental plane’ may be hard to achieve. Graham (2002) showed that for early type galaxies the Sersic index cor-

$^3$ Scodellio et al. (1997) found a still smaller scatter in the Kormendy relation for E and S0 galaxies in clusters, but their result is not directly comparable and, as they pointed out, does not provide a distance indicator.
relates with velocity dispersion giving another photometric method for distance estimation. He fit for log effective radius as a function of surface brightness and Sersic index. However, he found fractional rms distance errors of \( \sim 38\% \) (E-galaxies only) or 48\% (E + S0). This was using HST photometry at relatively low redshift, so that may be optimistic. La Barbera et al. (2005) performed the same kind of analysis for a pair of clusters at \( z \sim 0.3 \) and obtained a tighter relation with \( \sim 31\% \) fractional distance error. Huff and Graves (2014) fit for a photometric fundamental plane for early type galaxies (those with best-fit photo-z template being passive) in SDSS using surface brightness and concentration index (SDSS did not measure Sersic index). They used this to measure weak lensing convergence but if used as a distance indicator would have a precision of approximately 40%.

These studies therefore find relations with larger scatter – and therefore poorer distance estimate performance – than some of the earlier studies indicated might be achievable. And even these results are probably over-optimistic as most of them are affected by selection effects. For example, the Kormendy relation for ellipticals is that the surface brightness is decreasing function of size. There is no doubt that the absence of large galaxies with high surface brightness is real, but the absence of small galaxies of low surface brightness is in part at least a selection effect. This selection bias acts to artificially tighten up the relations, inflating the apparent distance precision.

A more realistic estimate of the precision that may be obtainable requires analysis of surveys with well defined selection criteria. Smith, Loveday and Cross (2009) have measured the bivariate luminosity-surface brightness distribution in the UKIDS LAS. They show that with a optical u – r colour split both blue and red subsamples are quite well described by Cholioniewski models (Cholioniewski, 1985). The red sub-sample has nearly luminosity independent surface brightness and has a faint end slope \( \alpha \) close to zero and much larger than the value of \( \alpha \approx -1 \) characteristic of the galaxy population as a whole. The bigger \( \alpha \) the better the precision per galaxy, and \( \alpha = 0 \) gives approximately 50% error in distance per galaxy, so this kind of precision should be obtainable for the red, primarily early-type, sub-sample, and the above studies suggest that including morphological information should improve this. But the downside is that a large \( \alpha \) means that one cannot exploit the large numbers of sub-\( L_* \) galaxies that are found for later types. For \( \alpha = 0 \) the space density rapidly converges to \( \phi_* \), which for these objects is \( \approx 1.1 \times 10^{-2}\,(\text{Mpc/h})^{-3} \) and one reaches a density of \( \sim \) half of this by going about half a magnitude below \( M_* \), or to about \( m_r \approx 21 \), and this would be a more reasonable goal for spectroscopic follow-up with the 400 fibre 2df/AAOmega spectrograph on the AAT\(^4\) perhaps augmented by the 1000 fibre WEAVE spectrograph on the WHT\(^5\) if this goes ahead, but it would still be a massive effort. If this were done, it would provide then \( \sim 50\% \) distance errors (rather than 30\%) for objects within \( z \sim 0.3 \) but these would have a space density of only \( \approx 6 \times 10^{-3}\,(\text{Mpc/h})^{-3} \) or a little over a factor 10 smaller than assumed by Bacon et al. 2014. So overall this results in a big decrease in the FOM.

To obtain high space density at \( z \sim 0.3 \) as anticipated by Bacon et al. 2014 would require advanced spectroscopes such as PFS\(^6\) or the Maunakea Spectroscopic Explorer (MSE)\(^7\) that combine very wide field with the reach of a 8-10m scale telescope, but of course there would be competing demands for these, and the currently proposed surveys for e.g. PFS would not be suitable. In the nearer term it will be possible to try to do peculiar velocity measurements with photometric redshifts using DES. But the challenge there is that with 5\% errors in \( 1 + z \) the effect of bias described in the Appendix is expected to be strong even for very large-scale perturbations, and the signal on smaller scales will be corrupted. In addition, there is the concern regarding the impact of catastrophic photometric redshift errors. An alternative to DES, though likely delivering data on a slightly longer time-scale, is the J-PAS\(^8\) survey with 56 narrow band filters that, it is claimed, will give photometric redshifts with \( \sim 0.3\% \) precision in \( 1 + z \) to \( \text{i}_{\text{AB}} \approx 22.5 \) over 8,000 square degrees.

In summary, the prospects for extension of peculiar velocity studies to larger scales using photometric surveys, perhaps augmented with low resolution spectroscopy, are interesting but will not deliver the kind of \( \sim 30\% \) fractional distance precision that has been suggested in some previous studies. The spatial sampling density will also be an issue, and it will be interesting to see what information can be gained from ‘photometric-plane’ studies of the more numerous late-type galaxies. And without spectroscopy, the measurements will be limited to very large spatial scales. Nonetheless, one can expect to obtain useful information as a spin-off from these surveys that are being carried out primarily for weak-lensing and for RSD/BAO measurements.

But more conventionally, we can expect to see good progress at lower redshift (where the signal per volume element is much stronger) with the TAIPAN spectrograph that is being developed for the UKST Schmidt telescope and which represents a significant advance on the current 6df instrument and which will provide FP distances as well as redshifts, and from the ASKAP WALLABY survey These will both extend the range of spatial scales that peculiar velocities probe and will also improve the signal to noise at lower redshift by increasing the density of sampling.

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\(^5\) http://www.ing.iac.es/weave/
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\(^7\) http://www.cfht.hawaii.edu/en/news/MSE
\(^8\) http://j-pas.org/
One way to analyse cosmic-flow data would be to consider each galaxy to provide a measure of the peculiar velocity at a position in space which is the estimated distance. If one lived in a rich cluster then mapping velocity vs. estimated-distance space coordinates would make considerable sense. At cosmological distances distance space coordinates would make considerable sense. Therefore one way of analysing cosmic-flow data would be to consider each galaxy to provide a measure of the peculiar velocity at a position in space which is the estimated distance. If one lived in a rich cluster then mapping velocity vs. estimated-distance space coordinates would make considerable sense. At cosmological distances distance space coordinates would make considerable sense. Therefore one way of analysing cosmic-flow data would be to consider each galaxy to provide a measure of the peculiar velocity at a position in space which is the estimated distance.
Strauss & Willick 1995) then a lot of the bias problems are ameliorated.

But it is not completely free of bias. If you imagine galaxies as having random motions, much like molecules in a gas, (or large random measurement errors, as is the case for photometric redshifts) then, if you look in a localised region in redshift space (perhaps a thin shell within a cone of some solid angle) then, if there is, for instance, a radially outwards increasing density of galaxy density then there will be more galaxies in the selected region that are scattered in from greater distances than from the nearer side. That results in a positive bias in the estimated luminosity distance at the chosen redshift that is proportional to the radial gradient of the density of galaxies; this is opposite in sign to that produced by peculiar motions, which are proportional to (minus) the radial gradient of the potential.

The peculiar displacement is \( r = -v_{\text{ran}}/H \). If this has a probability distribution, independent of position, \( P(r) \) then the mean peculiar displacement of galaxies is \( \mathbf{r} = \int dr (1 + \delta_g(r)) P(r) / \int dr (1 + \delta_g(r)) P(r) \) where \( \delta_g(r) \) is the real-space galaxy density contrast. If there is only a large-scale density perturbation then one can locally expand \( \delta_g \) as a Taylor series and, since the mean of the distribution of \( r \) is taken to be zero, we have \( \mathbf{r} \propto \langle \delta_g/dr \rangle r^2 \) corresponding to a peculiar velocity \(-H\mathbf{r}\).

For a plane wave, \( \delta_g(r) = \delta_g \exp(ik \cdot r) \) the inferred peculiar velocity is

\[
v_{\text{mean}} = v_{\text{phys}} + v_{\text{bias}} = v_{\text{phys}} \times (1 - k^2 \sigma_v^2 / \beta H^2).
\]

So if the velocity dispersion is known, one can estimate, and correct for, the bias on any scale of interest (noting, of course, that wavelength and wave number are related by \( \lambda = 2\pi/k \)).

This bias – or ‘damping’ – of the measured peculiar velocity will tend to bias estimates of \( \beta \) from comparison of density and velocity fields. In the analysis of Willick & Strauss (1998) the effect of small-scale motions was included in the model.

It is worth noting that that a similar bias also affects peculiar velocities measured at a given estimated distance (Method I). In this case the bias is known as “inhomogeneous Malmquist bias” (Hudson 1994, Strauss & Willick 1995). It has the opposite sign as the bias considered above since uncertainties in the distances cause the estimated real-space positions of galaxies to scatter out of overdensities, leading to an apparent peculiar velocity signal that has the same sign as the infall signal. Usually this bias is eliminated by using the galaxy density field to correct individual distance estimates. However it is also possible in principle to make a statistical correction on a mode-by-mode basis. In this case, the bias is very similar to the one described above, but with opposite sign:

\[
v_{\text{mean}} = v_{\text{phys}} + v_{\text{bias}} = v_{\text{phys}} \times (1 + k^2 \sigma_v^2 / \beta).
\]

Note that for most realistic surveys \( \sigma_8 \gg \sigma_v / H \), hence this bias is larger than the Method II bias.

Returning to the case of peculiar displacement in redshift space, it is clear that the model for random motions discussed above is rather idealised. It may be a reasonable model for redshift measurement errors, but real galaxy motions are a complicated mix of virialised velocities within bound systems and streaming motions on larger scales. We would argue nonetheless that this model should be applicable – provided one is dealing with large-scale waves in the linear regime – to describe the effect of small \( \sim Mpc/h \) scale motions in virialised groups and clusters.

Whether one should include the effect of motions on intermediate scales depends rather on how the data are analysed. A very large scale density enhancement (the scale of interest) by itself causes a smooth deformation of the physical distance shell corresponding to the chosen constant \( z \). Perturbations on some smaller scale (which we’ll assume here is either linear or quasi-linear so position and redshifts are single-valued functions of one-another) will further corrugate the constant-\( z \) distance surface. If one makes an estimate of the very large scale velocity by averaging the flux densities of the galaxies then the intermediate scale perturbations will bias the result in the manner of (A1). This is because, again in the case that the large-scale mode has a positive radial density gradient, wherever the intermediate scale motions result in a positive peculiar displacement there will, on average, be a slightly higher density of galaxies and these being more distance will have a slightly higher Hubble velocity. But if one were to bin the measurements on the intermediate scale and create peculiar velocity estimates normalising by the local density of galaxies, then the intermediate scale motions would no longer couple to the larger-scale density perturbations.

In redshift space distortions, large-scale peculiar motions cause a ‘squishing’ of the clustering pattern, enhancing the apparent amplitude of modes with wave vector out of the plane of the sky. But the peculiar velocity from smaller scale motions tends to counteract that. In (A1) we see that large-scale motions are also biased. The velocity dispersion here is a ‘single-galaxy’ or ‘1-point’ statistic. In RSD we deal with the pairwise velocity dispersion. At small projected separation, where both pairs most probably reside in the same virialised halo the pairwise averaging gives a large variance as it tends to be dominated by large clusters. But for pairs with substantial projected separation the distribution function is the convolution of the distribution for two places which, as far as small-scale motions are concerned provide a velocity dispersion very similar to that in (A1). Thus it would seem that measurements of the velocity bias given by (A1) – which depends to some extend on assumptions about the galaxy density bias since \( \beta = f(\Omega)/b \) – could provide independent constraints on the distribution of small-scale motions that is the nuisance factor in RSD measurements.

In \( \Lambda \)CDM the pairwise line-of-sight velocity dispersion falls from about 700 km/s at \( \sim 1Mpc/h \) scales to about 500 km/s at a few Mpc/h and above (Jenkins et al. 1998). This is consistent with the idea that at small separation one is seeing pairs in the same halo, so one is measuring the velocity dispersion of clusters weighted by the square of the number of galaxies in those clusters, rather than weighting galaxies equally. The estimate from LCRS (Jing, Mo & Borner 1997) is flat at \( \sim 500 \) km/s out to a few Mpc (the uncertainty becoming large for larger separation). We interpret this to mean that at the these separations the pairwise velocity difference distribution is the convolution of the single-particle distribution with itself so that the appropriate value for the 1-particle RMS velocity is \( \sigma_v = 500/\sqrt{2} \simeq 350 \) km/s. With \( \beta \simeq 0.5 \) (i.e. \( b \simeq 1 \)) this suggests that the bias in (A1)
would be a ∼ 25% effect for \( k \sim 0.1h/\text{Mpc} \) (or wavelength \( \lambda \simeq 60\text{Mpc}/h \)).

Koda et al. 2013 have calculated the power spectrum of peculiar velocities in numerical cosmological simulations. They observed a damping at high frequencies of the velocity power as measured in redshift space that they found could be described by a ‘sinc’ function: 

\[
D_u = \frac{\sin(k\sigma_K/H)}{(k\sigma_K/H)}
\]

with \( \sigma_K \simeq 1300 \text{ km/s} \). Expanding this to lowest non-trivial order would give 

\[
D_u(k) \simeq 1 - k^2 \sigma_K^2 / 3H^2
\]

or equivalently 

\[
D_u(k) = 1 - k^2 \sigma_v^2 / \beta H^2
\]

with \( \sigma_v \simeq 375 \text{ km/s} \) (with \( \beta = 0.5 \)), in excellent agreement with our prediction. However, we would emphasise that this 1-particle velocity dispersion from simulations is considerably larger than the value seemingly preferred by the data of Willick & Strauss (1998) who found a ML value of \( \sigma_v \simeq 140 \text{ km/s} \).

Equation (A1) can also be used to estimate the bias of velocities caused by errors in photometric redshifts based on broad-band filter photometry. But the results, for conventional broad-band imaging surveys delivering ∼ 5% fractional errors in \( 1 + z \) are quite depressing. A wider range of scales can be probed using J-PAS with its 56 narrow band filters, which is designed to deliver much better precision of 

\[
\delta(1 + z) \simeq 0.003.
\]