A unified simulation of low-to-high cycle fatigue failure effects for metals with efficient algorithm

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Abstract. Within the framework of a recently established elastoplasticity model incorporating fatigue failure effects into inherent response features, a new and efficient algorithm is proposed to simultaneously treat fatigue failure effects from low to high cycle cases. From the new model it is possible to derive an explicit algorithm with which the accumulated plastic work is directly calculated by means of a recursive scheme, thus bypassing very time-consuming procedures in carrying out numerical integrations of the elastoplastic rate equations for a large number of loading-unloading cycles. Comparisons of simulation results with fatigue failure data from low to high cycle cases are presented to demonstrate the efficacy of the new algorithm.

1. Introduction

The reliability and safety issues of key engineering components made of metals under cyclic loading conditions is a research hotspot. The central objective is to establish realistic constitutive models and efficient algorithms for simulating complicated effects induced by fatigue failure. Numerous analytical and numerical results in this respect have been suggested from micro-structural, phenomenological, and experimental standpoints. Reference may be made to the survey articles [1-3] for certain representative samples. Toward a direct and unified approach for simulating fatigue failure effects from low to high cycle cases, innovative elastoplasticity models have been most recently established in the previous work [4-6] and developed later in [7] toward the automatic incorporation of fatigue failure effects into inherent response features.

At each loading-unloading cycle, the elastoplastic response should be calculated by carrying out numerical procedures of integrating the elastoplastic rate equations. It is expected that the computation effort would be very time-consuming as the loading-unloading cycle repeats itself at so many times. That may be the case particularly in large-scale FEM computations with a great number of elements. As such, it is of much value to design an efficient algorithm for substantially expediting time-consuming numerical computations.

The main objective of this contribution is to present an efficient algorithm for simultaneously simulating low-to-high cycle fatigue failure effects within the framework of the innovative elastoplastic models in [4-7]. Toward this goal, a recursive numerical scheme will be designed to directly calculate the accumulated plastic work, thus bypassing usual time-consuming procedures in the foregoing. Comparison of simulation results with fatigue failure data from low- to high-cycle cases will be presented to demonstrate the efficiency of the new algorithm.
2. **New elastoplastic equations incorporating fatigue effects**

A reduced form of the self-consistent elastoplasticity model established in [4-6] is used for our purpose. In what follows, the following notations will be used: $D$ and $\tau$ are the stretching and the Kirchhoff stress; the deviatoric part of the latter is signified by $\bar{\tau}$; $D^e$ and $D^p$ are the elastic and the plastic part of $D$; $\dot{\tau}^{\log}$ is the logarithmic stress rate; $\rho$ is the plastic index. Details may be found in [4-6]. Below is the innovative elastoplasticity model:

\[
D = D^e + D^p, \quad (1)
\]

\[
D^e = \frac{1}{2G} \bar{\tau}^{\log} + \frac{\nu}{E}(\text{tr} \bar{\tau}^{\log}) I, \quad (2)
\]

\[
D^p = \rho \left( \frac{\bar{f}}{2h} + \frac{\bar{f}}{2} \frac{\partial f}{\partial \tau} \right),
\]

with von Mises yield function specified by

\[
\begin{cases}
    f = g - r, \\
    g = \frac{1}{2} J_2 = \frac{1}{2} \text{tr} \bar{\tau}^{2}, \quad r = \frac{1}{3} q^2 (k),
\end{cases} \quad (3)
\]

where $q$ is the stress limit dependent on the plastic work $k$:

\[
q = \frac{1}{2} \left( 1 + \frac{\lambda}{k_c} \right) \left( 1 - \tanh \beta \left( \frac{k}{k_c} - 1 \right) \right), \quad (4)
\]

\[
\kappa = \tau : D^p, \quad (5)
\]

and finally, $\bar{f}$ and $\bar{h}$ define the plastic modulus:

\[
\bar{f} = 2G \bar{\tau} : D, \quad \bar{h} = \frac{2}{3} J_2 \left( 3G + qq' (\kappa) \right), \quad (6)
\]

As in classical Hooke's law, here the elastic constants $G$, $\nu$ and $E$ are the shear modulus, Poisson's ratio and the Young modulus. However, unlike the usual flow rule, the plastic index $\rho$ here is no longer limited to the two values 1 and 0 for the loading and the unloading case, respectively, but allowed to smoothly take values from 0 to 1, as given below

\[
\begin{cases}
    \rho = \frac{1}{e^m - 1} (e^{mg/m} - 1) \\
    m = m_0 e^{ak/k_c}
\end{cases} \quad (7)
\]

In the above, $\lambda$, $\xi$, $m_0$, $a$ and $\beta$ are dimensionless positive material parameters, and $q_0$ and $k_c$ are positive material parameters with the dimension of stress.

3. **New elastoplastic equations incorporating fatigue effects**

The reduced forms of the elastoplastic equations in the foregoing are derived in the uniaxial loading-unloading cases and given as follows:
\[
\frac{dh}{d\tau} = \frac{1}{E} \frac{\rho}{3G(1-\rho) + s}\frac{s}{s'}
\]
\[
\frac{dh}{d\tau} = \rho \frac{\tau}{3G(1-\rho) + qq'}
\] (8)

Here, \(\tau\) and \(h\) are used to designate the axial stress and the axial logarithmic strain, respectively.

Let \(\bar{A}\) and \(A\) be the maximum and the minimum amplitude of the axial stress at each loading-unloading cycle. From the reduced forms above, it may be deduced by means of effective approximate evaluation of high accuracy that the plastic work produced in the \(i\)-th loading-unloading cycle from the minimum amplitude \(A\) to the maximum amplitude \(\bar{A}\), denoted as \(k_i\), may be calculated with the following direct recursive scheme:

\[
k_{i+1} = k_i + \phi \left( \frac{\phi - e^{m(k_i)}}{\phi - e^{m(k_n)}} + a \right) \psi, \quad n = 2, 3, \ldots, N,
\] (9)

with

\[
\phi = \frac{e^{m(k)}}{3G} \left( 3G + q(k)q'(k) - q(k)q'(k_n) \right), \quad \psi = \frac{q^2(k)}{6G},
\]

\[
a = \frac{\bar{A}}{q^2(k)} \text{sgn} R - \left( 1 + \frac{(-1)^n}{2} (R - 1) (1 - \text{sgn} R) \right) \frac{\bar{A}}{q^2(k)},
\] (10)

In the above, \(\text{sgn}\) denotes the sign function. It should be noted that the \(n\) above doubles the usual cycle number when \(R < 0\). When the total plastic work \(k\) approaches the critical plastic work \(k_c\), the stress limit \(q\) goes rapidly down to vanish and, as such, fatigue failure will emerge. Hence, a unified failure criterion for determining the cyclic number \(s\) to fatigue for loading-unloading procedures of varying amplitudes may be derived, as given below:

\[
\begin{cases}
k_n \leq k_c, \\
k_{n+1} \geq k_c,
\end{cases}
\] (11)

The new and direct criterion leads to a unified algorithm of high efficiency in simultaneously treating low-to-high cycle fatigue cases, as will be illustrated below.

| Table 1. Parameter values for CG301LN steel and Elbrodur-NIB copper alloy. |
|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|
| Material        | \(E\) (GPa)     | \(q_0\) (MPa)   | \(\kappa\) (MPa) | \(\nu\)          | \(\lambda\)    | \(\xi\)          | \(\beta\)       | \(m_0\)         | \(\alpha\)      |
| CG 301LN steel  | 103             | 820             | 640             | 0.3             | 1.41           | 1.2             | 8              | 20              | 2               |
| Elbrodur-NIB    | 53              | 520             | 240             | 0.3             | 0.49           | 1               | 8.5            | 0.05            |                 |
4. Numerical examples for model validation

For the purpose of model validation, we consider the failure data presented in [8] for CG 301LN steel and in [9] for Elbrodur-NIB copper alloy. The parameter values are listed in table 1 and simulation results with these parameter values are calculated by means of both usual integral procedures and the explicit recursive algorithm proposed. Simulation results and comparisons are depicted through figures 1–4 and listed in tables 2 and 3.

Table 2. Comparison of new and usual algorithms for low-to-ultrahigh cycle fatigue failure for CG 301LN steel.

| Amplitude (MPa) | 1,000 | 900  | 800  | 700  | 600  | 500  |
|----------------|-------|------|------|------|------|------|
| Cycle (usual)  | 3,559 | 10,944 | 44,553 | 285,537 | 2,916,454 | N/A |
| Cycle (new)    | 3,520 | 10,938 | 44,852 | 292,270 | 3,157,832 | 39,681,324 |
| Error (%)      | 1.09  | 0.05  | 0.67  | 2.36  | 8.27  | N/A |
| Time (usual, min) | 2.409 | 6.989 | 26.147 | 158.593 | 1,730.83 | >10000 |
| Time (new, min) | 0.009 | 0.027 | 0.113 | 0.736 | 7.954 | 99.947 |

Table 3. Comparison of new and usual algorithms for fatigue failure of Elbrodur-NIB copper alloy.

| Amplitude (MPa) | 550  | 500  | 450  | 400  | 350  | 300  |
|----------------|------|------|------|------|------|------|
| Cycle (usual)  | 2,524 | 7,272 | 19,488 | 49,829 | 125,100 | 316,488 |
| Cycle (new)    | 2,528 | 7,290 | 19,516 | 50,066 | 125,950 | 321,015 |
| Error (%)      | 0.19  | 0.25  | 0.14  | 0.48  | 0.68  | 1.43 |
| Time (usual, min) | 1.211 | 3.329 | 8.276 | 22.809 | 56.554 | 141.322 |
| Time (new, min) | 0.0059 | 0.0167 | 0.0421 | 0.1151 | 0.2895 | 0.6551 |

Figure 1. Comparison of simulation results with monotonic uniaxial tensile data for CG 301LN steel in [8].

Figure 2. Comparison of simulation results with fatigue data for CG 301LN steel in [8].
5. Conclusions

It has been shown that the computation time is greatly minimized with the new proposed algorithm. Particularly, less than 100 minutes are needed in calculating the S-N curve for a number of stress amplitudes in the high cycle fatigue case for CG 301LN steel, while $10^7$ cycles and more are involved. However, it is expected that more than 100 hours would be consumed with usual numerical integration procedures in this case.

The new direct algorithm of high efficiency is provided for the uniaxial case in the current study. It is of much significance to extend this algorithm to the broad case of multi-axial fatigue effects. Also, medium high and even low cycle fatigue effects need to be handled, in addition to high cycle fatigue cases treated here. That is particularly the case for multi-axial effects, which results will be reported in the follow-up studies.

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