Universality of a Critical Magnetic Field in a Holographic Superconductor

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(Received 11 October 2014)

We study aspects of holographic superconductors analytically in the presence of a constant external magnetic field. It is shown that the critical temperature and the critical magnetic field can be calculated at nonzero temperature. We detect the Meissner effect in such superconductors. A universal relation between black hole mass $M$ and critical magnetic field $H_c$ is proposed to be $\frac{M}{c^2} \leq 0.687365$. We also discuss some aspects of phase transition in terms of black hole entropy and Bekenstein’s entropy to the energy upper bound.

PACS: 74.62.Bf, 74.25.Ha, 11.25.Tq, 44.05.+e

High-temperature superconductors are phases of matter which are quite conductive at a temperature close to $T \sim 138$ K. It was proven that such superconductors could be described by using a gauge/gravity duality (for a quick review see Ref. [1]). The gauge/gravity mechanism is based on a one-to-one correspondence between the exact black hole solutions of a gravitational model on a Riemannian curved manifold of spacetime with a negative cosmological constant and a conformal quantum theory (in flat spacetime). In a quantum picture, the quantum objects are relevant quantum operators and the physical quantities are totally renormalizable.

All these quantum objects live on a flat spacetime (boundary). If we assume that the black hole horizon is thermal at temperature $T_{BH}$ and the conformal field theory (CFT) has an associated nonzero temperature $T_{CFT}$, then it is possible to consider two systems in thermodynamic equilibrium. This means that $T_{BH} = T_{CFT}$. This duality backs to Maldacena’s conjecture in string theory as follows.

Maldacena conjecture:[2] Type II of superstring theory in bulk with $AdS_5 \times S_5$ geometry is dual, to a specific gauge theory, $N = 4, SU(N_c)$ super-Yang–Mills theory in four dimensions.

Another alternative name for this conjecture is Anti-de Sitter/conformal field theory (AdS/CFT). It is proven that this conjecture plays an important role in describing the high temperature superconductor (for earlier works see Refs. [4–9]). Following the basic logic of gauge/gravity, we need to have a one-to-one correspondence between each dynamical quantity of the superconductor (temperature and condensation of the order parameter) and a set of the parameters for the black hole. Depending on the specific type of condensation, the different types of holographic superconductors were considered by several researchers.[10–43]

In the high temperature material, the superconductivity is a function of the critical value of a magnetic field and the subject was studied in several works.[44–58] The Meissner effect was investigated by analytical and numerical methods while the external magnetic field was made with some important effects on the basic parameters of superconductors.[15–46]

In this Letter, we investigate analytically the effect of a constant external magnetic field on the critical temperature of a type $s$ superconductor. Holographic superconductors with a constant external magnetic field were studied by other researchers. Especially in Ref. [49], a hairy numerical solution was found for any value of an external magnetic field $H$ in the range $0 \leq H \leq H_{max}$, where $H_{max} = 0.687365$. The significant observation in the value of $H$ was that the critical temperature $T_c$ in the presence of a magnetic field vanishes when $H$ approaches the critical value $H_c = H_{max}$.

In this work, we revisit analytically the basic properties of such hairy black hole solutions. We will treat the equations of motion as a system of coupled nonlinear differential equations which can be reduced to an eigenvalue-eigenfunction problem near the criticality $T = T_c$. We show that it is possible to find semi-exact solutions for equations approximately near the critical point. Using these semi-exact solutions, we obtain the critical temperature and the maximum of an external magnetic field. The appearance of the conformal dimensions and uniqueness of the expectation values of the dual CFT operators are verified. We observe that for a typical mass of the black hole ($M$), there is an upper bound for the external magnetic field and the size of the thermal horizon (in the critical point). Consequently we show that this maximum size of horizon leads to a maximum value of the critical temperature. We compute the difference of specific heat between the normal and superconducting phases, and investigate the stability issue. We prove that such superconductors remain stable. To make a holographic model for a high temperature superconductor we need to intro-

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047401-1
duce an appropriate gravitational sector that is written in the ‘good given’ units as follows:[6]
\[ \mathcal{L}_g = R - \frac{6}{L^2} - \frac{1}{4} F_{\mu\nu} F^{\mu\nu}, \]
(1)
where \( R \) is the Ricci scalar, \( L \) is the AdS length, \( A_\mu \) is the gauge field, and \( F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu \). If only one external constant magnetic field \( H \) is present, the nonzero components of the \( F_{\mu\nu} \) tensor are given by \( F_{xy} = \frac{H}{r^2} \). We consider an imaginary sphere with radius \( r \). We obtain the total energy density stored in \( F_{\mu\nu} F^{\mu\nu} \sim H^2 \). The gravitational field solution is a magnetized Schwarzschild Anti-de Sitter black hole in planar coordinates, \((t, r, x, y)\)\[50\]
\[ ds^2 = -f(r)dt^2 + \frac{dr^2}{f(r)} + r^2(dx^2 + dy^2), \]
(2)
\[ f(r) \equiv f = r^2 - \frac{M}{r} + \frac{H^2}{r^2}. \]
The black hole has an horizon which is a real solution to the equation \( f(r_+) = 0 \). The function \( f \equiv f(r) \) can be rewritten as
\[ f = r^2 - \frac{r_+^2}{r} - \frac{H^2}{r r_+} + \frac{H^2}{r^2}. \]
(3)
The temperature of the black hole is defined by
\[ T = \frac{f'(r_+)}{4 \pi} = \frac{r_+(3 - h^2)}{4 \pi}. \]
(4)
We define the dimensionless magnetic field as \( h^2 = \frac{H^2}{r_+^2} \) and now we have built a fully gravitational sector. For the matter sector we use a Lagrangian similar to the equation (5)
\[ \mathcal{L}_m = -\frac{1}{2} \left( \frac{1}{4} F^{ab} F_{ab} + m^2 |\psi|^2 + |\nabla \psi - i A \psi|^2 \right). \]
(5)
Note that Eq. (5) is different from the original GL theory. First, the mass term is negative \( m_0^2 < 0 \). Secondly, there is no potential term \( |\psi|^4 \) here. For stabilization we need an AdS bulk geometry. Our guess is that a physical mechanism like the Higgs mechanism launches in the region outside of the horizon.[51] Due to the symmetry of the metric and gauge freedom we set \( A_\mu = \phi(r) \), \( \psi^\dagger (r) = \psi (r) \). Thus, the equations of motion may be recast to the forms
\[ g^{\mu\nu} (\nabla_\mu - i A_\mu) (\nabla_\nu - i A_\nu) \psi - m^2 \psi = 0, \]
(6)
\[ \nabla_\mu F^{\mu\nu} = i (\psi^\dagger \nabla_\mu \psi - \psi \nabla_\mu \psi^\dagger) + |\psi|^2 A_\nu. \]
(7)
The equations of motion in the case of metric (2) can be written as
\[ \psi'' + \left( \frac{f'}{f} + \frac{2}{r} \right) \psi' + \frac{\phi^2}{f^2} \psi - \frac{m^2}{f} \psi = 0, \]
(8)
\[ \phi'' + \frac{2 \phi'}{r} \phi' - \frac{2 \phi^2}{f} \phi = 0. \]
We set \( L=1 \) and note that the mass of a scalar field is \( m^2 > m_{BP}^2 = -\frac{3}{4} \).
It is convenient to rewrite the equations in terms of dimensionless coordinate \( z = \frac{r}{r_+} \).
\[ \psi''(z) + \frac{f'}{f} \psi'(z) + \frac{r^2}{z^2} \left( \frac{\phi^2}{f(z)^2} - \frac{m^2}{f(z)} \right) \psi(z) = 0, \]
(9)
\[ \phi''(z) + \frac{m^2 r^2 \psi(z)^2}{z^4 f(z)} \phi(z) = 0. \]
(10)
Now the metric function is \( f \equiv f(z) = r_+^2 (z^2 - h^2 z - z + h^2 z^2) \).
We would like to find the solutions of the nonlinear Eqs. (9) and (10). For \( \{\phi, \psi\} \), Eqs. (9) and (10) can be solved as
\[ \phi(z) = \phi_0(z) - m^2 r_+^2 \int_0^z \frac{(z - w) \psi(w) \phi(w) dw}{w^2 f(w)} \]
(11)
\[ \psi(z) = \psi^+(z) + \psi^-(z) \]
\[ + \left( - \psi^+(z) \int \frac{\psi^+(z) p(z) d z}{W(\psi^+(z), \psi^-(z)) d z} + \psi^-(z) \int \frac{\psi^+(z) p(z) d z}{W(\psi^+(z), \psi^-(z)) d z} \right), \]
(12)
\[ \frac{d^2}{dz^2} \phi_0(z) = 0, \]
(13)
\[ \left\{ \frac{d^2}{dz^2} + \frac{f'}{f} \frac{d}{dz} - \frac{m^2 r_+^2}{z^4 f(z)} \right\} \psi^\dagger (z) = 0, \]
(14)
\[ p(z) = \frac{r_+^2 \psi(z) \phi(z)^2}{z^4 f(z)^2}, \]
(15)
\[ W(\psi^+, \psi^-) = \psi^+(\psi^-)' - \psi^-(\psi^+)', \]
(16)
The AdS/CFT mechanism is focused on the behavior of the fields at the AdS boundary, i.e., \( z = 0 \), when \( f \sim r_+^2 z^{-2} \), we are able to approximate solutions to Eqs. (9) and (10) as follows:
\[ \phi_0^0 (z) = c_0 + c_1 z, \quad \psi^0 (z) = c_\perp z^{\Delta_\perp}, \]
(17)
where the conformal dimension is \( \Delta_\perp = \frac{3}{2} \pm \sqrt{m^2 + \frac{9}{4}} \).
At this precise expression, \( c_\perp = \frac{(2z^2)}{e_\perp^2} \). We conclude that the precise form of the Wronskian is
\[ W(\psi^+(z), \psi^-(z)) = - (2c_+ c_- \sqrt{m^2 + \frac{9}{4}}) z^2. \]
(18)
We need to estimate the following pair of integrals,
\[ I_1 = \int_0^z \frac{(z - w) \psi(w) \phi(w) dw}{w^2 f(w)}, \]
\[ I_\perp = \int \frac{\psi^+(z) p(z) d z}{W(\psi^+(z), \psi^-(z))}. \]
Following the above information we know
\[ p(z)|_{z=0} \sim -r_x^{-2} c_\pm z^{2}z_0^2. \]
Thus we have
\[ I_\pm = A_\pm \times z^{2+2}\sqrt{m^2+\frac{a^2}{2}}. \]
(19)
The most common technique for obtaining precise information about the form of solutions within an interval \( z \in [0, 1] \) is to evaluate \( \psi_{z=0} \),
\[ \psi = c_+ z^{A_+} + c_- z^{A_-} + (c_+ A_+ \times z^{1+\sqrt{m^2+\frac{a^2}{2}}} - c_- A_- \times z^{1-\sqrt{m^2+\frac{a^2}{2}}}) \]
(20)
Maybe integral \( I_1 \) can be adequately approximated by an additive integration whose output is a sum over several smaller dimensional expressions
\[ I_1 = \int_z^w (z-w)\psi^2(w)\phi(w)dw \]
\[ = z\left\{ \frac{\psi^2(w)\phi(w)}{w^4f(w)} \right\}|_{z=e} \]
\[ - \left\{ \frac{w^2\psi(w)\phi(w)}{w^4f(w)} \right\}|_{z=e}, \quad e \sim 0. \]
(21)
We place the three expressions individually and finally we are able to write \( I_1 \) as
\[ I_1 = \alpha_0 z + \beta_0, \]
(22)
where \( \alpha_0 = (c_+)^2 c_\pm^2 z^{3+2} - \frac{a^2}{2} \), \( \beta_0 = (c_+)^2 c_\pm^3 z^{3+1-1} \). By collecting results from the past, the solution gives the shape to the forms of fields in the vicinity of the AdS boundary,
\[ \phi(z) = \mu - \frac{\rho}{r_x} z, \quad \psi(z) = \frac{O^+_+}{r_x^4} z^{A_+}. \]
Moreover, our attempts to establish the form of solutions in the vicinity of the AdS boundary are also fruitless.

The quantization of the relevant operators has been investigated and in some cases a very good agreement with the CFT results is found, \( O_- = 0 \Rightarrow c_- = 0 \). Here \( O_+ \) is an operator of dimension \( A_+ \) in the dual field theory and \( \langle O_+ \rangle \) corresponds to the vacuum expectation value of it. We will introduce a set of the boundary conditions\(^{51,52}\)
\[ \phi(1) = 0, \quad (h^2-3)\psi'(1) = m^2\psi(1). \]
(23)
The best advice is to use these continuously and to always expand the fields somewhere in the AdS boundary and horizon.\(^{54}\) The conventional computation method only provides facilities for exact matching; a more adaptive approach is required to support numerical aggregation. Both fields expand smoothly across a surface with location \( z = z_m \in [0, 1] \).\(^{55-58}\) We can conclude that the precise evaluation of expectation values of relevant operators is required for proper integration of actin into counters,
\[ \int_{r_x^+}^\infty \frac{\psi(t)dt}{t^{A_++1}}. \]
(24)
The vacuum expectation values of relevant operators are shown on the boundary, with no bulk term. At finite temperature we have \( h \neq 3 \). The analytical solutions in the AdS sector, in the vicinity of \( z=0 \), computed temporally as follows:
\[ \phi(z) = a \left[ 1 - (1-z) - \frac{m^2 r_x b^2}{2} (1-z)^2 \right], \]
\[ \psi(z) = b \left[ 1 - \frac{m^2}{h^2-3} (1-z) \right. \]
\[ \left. + (1-z)^2 \left( - \frac{a^2}{2} \frac{32\pi^2 T^2}{h^2-3} + \frac{3m^2(1-h^2)^2}{(h^2-3)^2} \right) \right]. \]
(26)
where \( a = -\phi'(1) \), and \( b = \psi(1) \). The process by which one obtains coefficients from the equations of motion in the presence of boundaries \( z = 0 \) and \( z = 1 \) is called the matching method,
\[ a \left( 1 - \frac{1}{2} \frac{m^2 r_x b^2}{T^2} \right) = \mu - \frac{\rho}{2r_x}, \]
\[ b \left( 1 - \frac{1}{2} \frac{m^2}{h^2-3} - \frac{1}{256} \frac{a^2}{\pi^2 T^2} + \frac{3m^2(1-h^2)^2}{8(h^2-3)^2} \right) \]
\[ = \left( \frac{O_+}{r_x^4} \right)^{\alpha} \frac{1}{r_x^4}, \]
(28)
\[ a \left( 1 - \frac{1}{2} \frac{m^2 r_x b^2}{T^2} \right) = -\frac{\rho}{r_x}, \]
\[ b \left( \frac{m^2}{h^2-3} + \frac{1}{64} \frac{a^2}{\pi^2 T^2} - \frac{3m^2(1-h^2)^2}{2(h^2-3)^2} \right) \]
\[ = 2A(\frac{O_+}{r_x^4})^{\Delta} \left( \frac{\Delta}{r_x^4} \right), \]
(30)
For comparison of the analytical with the numerical result,\(^{61}\) relative value \( m^2 = -2 \) was used. This system (Eqs. (27) and (29)) gives
\[ a = 4\mu - \frac{3\rho}{r_x^4}, \quad b^2 = \frac{8\pi T}{m^2} \left[ \frac{1}{r_x^4} - \frac{\rho}{ar_x^2} \right] \]
\[ \Rightarrow ar_+ c = \mu \Leftrightarrow \frac{(O_+)}{r_x^4} (r_x^4 - r_+^4) = 0. \]
(31)
According to the definition of dual quantities, we know
\[ r_{+c} = \frac{\rho}{\mu} \Rightarrow r_+^4 - M r_+ r_{+c} + H_c^2 = 0. \]
In the critical mode, the definition of critical temperature is given, followed by the abbreviation in parentheses,
\[ T_c = \frac{3r_{+c}^2 - H_c^2}{4\pi r_{+c}^4}. \]
(32)
Equally, it is clear that the elimination of \( r_c \) in \( T_c \) will lead to
\[
256T_c^4 \pi^4 H_c^4 + 96T_c^2 \pi^2 M^2 H_c^2 - 27M^4 + 64T_c^3 \pi^3 M^3 + 256H_c^6 = 0, \tag{33}
\]
where \( T_c(H) \) and \( H_c \) in Eq. (32) or (33) are defined as a critical point in the presence of the external magnetic field and the critical magnetic field that turns the superconducting state into a normal state below the \( T_0 = \frac{4 \pi r_c}{3} \) (with no magnetic field). The maximum of the magnetic field \( H_{\text{max}} \) is obtained at the minimum critical temperature \( T_c(H) = 0 \), hence to obtain the maximum magnetic field \( H_{\text{max}} \), we simply let the temperature in Eq. (34) vanish and obtain
\[
H_{\text{max}} = \frac{\pi r_c^2}{L^2}, \tag{34}
\]
which is exactly the numerical result of Ref. [49].

We analyze the pair \( (H_c, T_c) \) carefully. First, when there is no magnetic field \( H = 0 \), horizon size tends towards zero \( r_c = 0 \). It is also worth remembering that there is a famous entropy of a black hole which becomes simple in our case,
\[
S = \frac{\pi r_c^2}{L^2}. \tag{35}
\]
We know that \( S \geq 0 \). At the normal phase, where there is no entropy, this time external magnetic field disappears altogether. Thus from \( S \) we can see that the entropy change of a system during a normal process is zero. An example of its success is in determining the normal phase of a holographic superconductor. When things tend to move toward disorder we call this the second order phase transition from the normal to the superconducting phase. For the large enough external magnetic field, equilibration occurs initially at large values of \( H_c \) and proceeds away from Bekenstein’s entropy bound \( \frac{S}{T} < 2 \pi R \). [99] We show how gravitational entropy can be defined in general for a holographic superconductor. Does the critical system tend toward the maximum of entropy production and maximum \( H_c \)? The answer is yes. If it becomes more disordered (superconducting), its entropy and \( H_c \) increase. The external magnetic field \( H_c \) of the maximum property is derived from Bekenstein’s entropy bound principle. We list \( H_{\text{max}} \) and \( r_{c, \text{max}} \) for different values of masses in Table 1.

| Mass \( M \) | 0.7 | 0.5 | 0.3 |
|-------------|-----|-----|-----|
| \( H_{\text{max}} \) | \( \geq 0.3 \) | \( \geq 0.4 \) | \( \geq 0.5 \) |
| \( r_{c, \text{max}} \) | \( \in [0.6,0.7] \) | \( \in [0.7,0.8] \) | \( \in [0.8,0.9] \) |

Table 1. The values of \( H_{\text{max}} \) and \( r_{c, \text{max}} \) for different values of masses.

To complete the discussion of the thermodynamic point of view, we refer to the Rutgers formula. This formula indicates the discontinuity in specific heat and it shows the difference between specific heat of pure metals at the critical temperature in the superconducting and normal phase[62,63]
\[
\Delta C = C_{\text{superconducting}} - C_{\text{normal}} = \mu_0 T \left( \frac{d H_c(T)}{d T} \right)^2 + \left( \frac{d H_c(T)}{d T} \right)^2. \tag{35}
\]
At the critical temperature \( T_c \) that is the critical point in the absence of the external magnetic field, and at \( H_c(T_c) = 0 \), the first term in the Rutgers formula Eq. (35) vanishes and becomes the form of
\[
\Delta C = \mu_0 T \left( \frac{d H_c(T)}{d T} \right)^2. \tag{36}
\]
From the Rutgers formula (36) at the critical point, one can apparently observe that \( \Delta C \) must be positive. It is adequate to study the system after an infinitesimal deviation from the Universal bound, when \( H_c = M^{2/3} u - \delta H_c \), we obtain
\[
\delta(\Delta C) = \Delta C |_{H_c=M^{2/3} u} - \Delta C |_{H_c=M^{2/3} u - \delta H_c} > 0. \tag{37}
\]
Thus the system is stable under this bound. We note that the phase transition studied here is type I to type II. The reason is that under the magnetic field \( H_c \), there is the Meissner effect. When the magnetic field \( H \) becomes stronger, we will observe the quantized vortices. The results show a rapid increase in \( H_c \) in the superconducting phase the more \( H \) increases through the normal phase system. Finally, we note that the case of \( T=0 \) which is equal to \( h = \sqrt{3} \) in Ref. [61] is

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studied and here is not checked. Holographic superconductors are a gravitational analogy of high temperature superconductors in condensed matter physics. Fundamental properties of this class of superconductors can be simulated to the properties of a black hole in the gravitational sector in this model.

In summary, we have reviewed the effect of a constant external magnetic field on a superconductor type. We show that the matching method can give a critical magnetic field and a critical temperature in the nonzero temperature. The study of the behavior of the critical temperature and the critical magnetic field shows that with the increasing critical magnetic field, the critical temperature decreases and approaches to zero at a maximum magnetic field. We also show that the superconducting state is consistent with the existence of a thermodynamic state with the maximal entropy. The universal relation between the magnetic field for $H_c$ and the mass of black hole $M$ is $\frac{H_c}{\sqrt{27}} \leq \frac{\sqrt{27}}{256} = 0.687365$.

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