Life–Space Foam: a Medium for Motivational and Cognitive Dynamics

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**Abstract**

General stochastic dynamics, developed in a framework of Feynman path integrals, have been applied to Lewinian field–theoretic psychodynamics [1,2,13], resulting in the development of a new concept of *life–space foam* (LSF) as a natural medium for motivational and cognitive psychodynamics. According to LSF formalisms, the classic Lewinian life space can be macroscopically represented as a smooth manifold with steady force–fields and behavioral paths, while at the microscopic level it is more realistically represented as a collection of wildly fluctuating force–fields, (loco)motion paths and local geometries (and topologies with holes). A set of least–action principles is used to model the smoothness of global, macro–level LSF paths, fields and geometry. To model the corresponding local, micro–level LSF structures, an adaptive path integral is used, defining a multi–phase and multi–path (multi–field and multi–geometry) transition process from intention to goal–driven action. Application examples of this new approach include (but are not limited to) information processing, motivational fatigue, learning, memory and decision–making.

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**1 Introduction**

One of the key challenges in modelling complex human behavior is combining enough detail with sufficient scope in a given representation. A trade–off between detail and scope seems inevitable: individual models are either broad or detailed – but almost never both. However, most recent advances in understanding complex biopsychosocial phenomena have been associated with multilevel approaches, such as ascertaining the genetic contribution to developmental psychopathology through interactive analysis of “precisely measured...
microenvironments and well-specified macroenvironments" [3]. The present article develops a formalism intended to capture the complexity of motivated human behavior without sacrificing either detail or scope. We will discuss nonlinear stochastic methods (as a generalization of conventional statistics) leading to a more sophisticated conception of quantum probability in a framework of Feynmanian path integral applications. Lewin’s force-field theory [1,2,13] will be used to illustrate how the new approach can be applied.

Applications of Nonlinear Dynamic Systems (NDS) theory in psychology have been encouraging, if not universally effective [4]. Its historical antecedents can be traced back to Piaget’s [5] and Vygotsky’s [21] interpretations of the dynamic relations between action and thought, Lewin’s theory of social dynamics and cognitive-affective development [2], and [23] theory of self-adjusting, goal-driven motor action.

Characteristically, one of the most productive applications of NDS to date is in the area of motor development (see [6,7,8,9]), with its subject matter lending itself to NDS treatment by possessing the required properties of an NDS entity: even a relatively simple skill, such as walking, is characterized by a stable, adaptive and self-organizing pattern, different from its component skills and capable of developing, from a wide range of starting points, to a well identified, stable attractor (e.g., most children, including those with impediments, eventually master a skill recognizable as “walking”) [10]. Cognitive development seems to be characterized by similar dynamical patterns [11,8]. For example, speaking in sentences is qualitatively different from its component skills of remembering words and voice production; and most children are eventually capable of creating sentences “on the fly”, fluently adapting to the constraints of their native language.

One important conclusion: in order for the powerful formalisms of NDS to be effective in an application, its subject matter should possess NDS-relevant properties such as stable, adaptive and self-organizing patterns of sufficient complexity, non-reducible to their components and capable of developing to stable attractors.

In the light of the above requirement, current conceptualizations of decision making (see [12,16]) don’t seem to be appropriate for NDS treatment. In particular, in the context of the two levels of analysis proposed below, most phenomena captured by the Decision Field Theory (DFT) [14], including its multi-alternative version (MDFT) [15], occur at a macroscopic level, while their microscopic underpinnings are either not known or of little consequence. Decision making, in essence, is about choice from finite alternatives – e.g., among competing courses of action (COA). However, once a certain COA is chosen (i.e., decided upon), implementing that decision (i.e., conducting the chosen action) introduces such levels of complexity, even in relatively simple
environments, that microscopic level of analysis becomes critical. And that’s where both the classical NDS and its generalization in the path–integral form, show their advantage.

As a simulation example of this new approach, we propose a path–integral multialternative decision–field model, as a generalization to the abstract linear discrete–time stochastic model of [15], operating on a single level of a linear multiattribute space. Our path–integral approach proposes a nonlinear, hybrid (i.e., both continuous and discrete–time), stochastic sum–over–histories mechanism, based on the quantum probability concept, operating at two distinct levels of the Lewinian ‘life space’.

1.1 Lewinian Life Space

Both the original Lewinian force–field theory in psychology (see [1,2,13]) and modern decision–field dynamics (see [14,15,16]) are based on the classical Lewinian concept of an individual’s life space. As a topological construct, Lewinian life space represents a person’s psychological environment that contains regions separated by dynamic permeable boundaries. As a field construct, on the other hand, the life space is not empty: each of its regions is characterized by valence (ranging from positive or negative and resulting from an interaction between the person’s needs and the dynamics of their environment). Need is an energy construct, according to Lewin. It creates tension in the person, which, in combination with other tensions, initiates and sustains behavior. Needs vary from the most primitive urges to the most idiosyncratic intentions and can be both internally generated (e.g., thirst, hunger or sex) and stimulus–induced (e.g., an urge to buy something in response to a TV advertisement). Valences are, in essence, personal values dynamically derived from the person’s needs and attached to various regions in their life space. As a field, the life space generates forces pulling the person towards positively–valenced regions and pushing them away from regions with negative valence. Lewin’s term for these forces is vectors. Combinations of multiple vectors in the life space cause the person to move from one region towards another. This movement is termed locomotion and it may range from overt behavior to cognitive shifts (e.g., between alternatives in a decision–making process). Locomotion normally results in crossing the boundaries between regions. When their permeability is degraded, these boundaries become barriers that restrain locomotion. Life space model, thus, offers a meta–theoretical language to describe a wide range of behaviors, from goal–directed action to intrapersonal conflicts and multi–alternative decision–making.

In order to formalize the Lewinian life–space concept, a set of action principles need to be associated to Lewinian force–fields, (loco)motion paths (rep-
resenting mental abstractions of biomechanical paths \((17)\) and life space geometry. As an extension of the Lewinian concept, in this paper we introduce a new concept of life–space foam (LSF, see Figure 1). According to this new concept, Lewin’s life space can be represented as a geometrical object with globally smooth macro–dynamics, which is at the same time underpinned by wildly fluctuating, non–smooth, local micro–dynamics, describable by sum–over–histories \(\int_{\text{paths}}\), sum–over–fields \(\int_{\text{fields}}\) and sum–over–geometries/topologies \(\int_{\text{geom}}\).

LSF is thus a two–level geometrodynamical object, representing these two distinct types of dynamics in the life space. At its macroscopic spatio–temporal level, LSF appears as a ‘nice & smooth’ geometrical object with globally predictable dynamics – formally, a smooth \(n\)–dimensional manifold \(M\) with Riemannian metric \(g_{ij}\) (compare with \([18,19]\)), smooth force–fields and smooth (loco)motion paths, as conceptualized in the Lewinian theory. To model the global and smooth macro–level LSF–paths, fields and geometry, we use the general physics–like principle of the least action.

Now, the apparent smoothness of the macro–level LSF is achieved by the existence of another level underneath it. This micro–level LSF is actually a collection of wildly fluctuating force–fields, (loco)motion paths, curved regional geometries and topologies with holes. The micro–level LSF is proposed as an extension of the Lewinian concept: it is characterized by uncertainties and fluctuations, enabled by microscopic time–level, microscopic transition paths, microscopic force–fields, local geometries and varying topologies with holes. To model these fluctuating microscopic LSF–structures, we use three instances of adaptive path integral, defining a multi–phase and multi–path (also multi–field and multi–geometry) transition process from intention to the goal–driven action.

We use the new LSF concept to develop modelling framework for motivational dynamics (MD) and induced cognitive dynamics (CD).

According to Heckhausen (see \([20]\)), motivation can be thought of as a process of energizing and directing the action. The process of energizing can be represented by Lewin’s force–field analysis and Vygotsky’s motive formation (see \([21,22]\)), while the process of directing can be represented by hierarchical action control (see \([23,24]\)).

Motivation processes both precede and coincide with every goal–directed ac-

\footnote{We use the peculiar Dirac’s quantum symbol \(\int\) to denote summation over ‘discrete spectrum’ and integration over ‘continuous spectrum’ of paths, fields and geometries in the microscopic level of the Lewinian life space.}
Fig. 1. Diagram of the life space foam: classical representation of Lewinian life space, with an adaptive path integral $\int$ (see footnote 1) acting inside it and generating microscopic fluctuation dynamics.

tion. Usually these motivation processes include the sequence of the following four feedforward phases [21,22]: (*)

1. **Intention Formation** $\mathcal{F}$, including: decision making, commitment building, etc.
2. **Action Initiation** $\mathcal{I}$, including: handling conflict of motives, resistance to alternatives, etc.
3. **Maintaining the Action** $\mathcal{M}$, including: resistance to fatigue, distractions, etc.
4. **Termination** $\mathcal{T}$, including parking and avoiding addiction, i.e., staying in control.

With each of the phases $\{\mathcal{F}, \mathcal{I}, \mathcal{M}, \mathcal{T}\}$ in (*), we can associate a transition propagator – an ensemble of (possibly crossing) feedforward paths propagating through the ‘wood of obstacles’ (including topological holes in the LSF, see Figure 2), so that the complete transition is a product of propagators (as well as sum over paths). All the phases–propagators are controlled by a unique Monitor feedback process.

In this paper we propose an adaptive path integral formulation for these motivational–transitions. In essence, we sum/integrate over different paths and make a product (composition) of different phases–propagators. Also, recall that modern stochastic calculus permits development of three alternative descriptions of general Markov stochastic processes:\footnote{Recall that Markov stochastic process is a stochastic (random) process characterized by a lack of memory, i.e., the statistical properties of the immediate future are uniquely determined by the present, regardless of the past [25]. This Markov assumption can be formulated in terms of the conditional probabilities $P(x^t, t_i)$: if the times $t_i$ increase from right to left, the conditional probability is determined}{2}
Fig. 2. Transition–propagator corresponding to each of the motivational phases \(\{F, I, M, T\}\), consisting of an ensemble of feedforward paths propagating through the ‘wood of obstacles’. The paths affected by driving and restraining force–fields, as well as by the local LSF–geometry. Transition goes from Intention, occurring at a sample time instant \(t_0\), to Action, occurring at some later time \(t_1\). Each propagator is controlled by its own Monitor feedback.

(1) Langevin rate equations [25],
(2) Fokker–Planck equations [25], and
(3) Path integrals [29,30,31,32].

Here we follow the most general, path integral approach, namely it is the general Chapman–Kolmogorov integro–differential equation, with its condition entirely by the knowledge of the most recent condition. The general, continuous + discrete Markov process is generated by a set of conditional probabilities whose probability–density evolution, \(P = P(x', t' | x'', t'')\), obeys the general Chapman–Kolmogorov integro–differential equation

\[
\begin{align*}
\partial_t P & = -\sum_i \frac{\partial}{\partial x^i} \{A_i[x(t), t] P\} \\
& \quad + \frac{1}{2} \sum_{ij} \frac{\partial^2}{\partial x^i \partial x^j} \{B_{ij}[x(t), t] P\} \\
& \quad + \int dx \left\{ W(x' | x'', t) P - W(x'' | x', t) P \right\},
\end{align*}
\]

including: deterministic drift (the first term on the right, called the Liouville equation), diffusion fluctuations (the second term on the right, called the Fokker–Planck equation) and discontinuous jumps (the third term on the right, called the Master equation).
tional probability density evolution, \( P = P(x', t'|x'', t'') \), that we are going to model by various forms of the Feynman path integral \( \int \), providing us with the physical insight behind the abstract probability densities.

We will also attempt to demonstrate the utility of the same LSF–formalisms in representing cognitive functions, such as memory, learning and decision making. For example, in the classical Stimulus encoding \( \rightarrow \) Search \( \rightarrow \) Decision \( \rightarrow \) Response sequence [33,34], the environmental input–triggered sensory memory and working memory (WM) can be interpreted as operating at the micro–level force–field under the executive control of the Monitor feedback, whereas search can be formalized as a control mechanism guiding retrieval from the long–term memory (LTM, itself shaped by learning) and filtering material relevant to decision making into the WM. The essential measure of these mental processes, the processing speed (essentially determined by Sternberg’s reaction–time) can be represented by our (loco)motion speed \( \dot{x} \).

2 Six Facets of the Life Space Foam

The LSF has three forms of appearance: paths + fields + geometries, acting on both macro–level and micro–level, which is six modes in total. In this section, we develop three least action principles for the macro–LSF–level and three adaptive path integrals for the micro–LSF–level. While developing our psycho–physical formalism, we will address the behavioral issues of motivational fatigue, learning, memory and decision making.

2.1 General Formalism

At both macro– and micro–levels, the total LSF represents a union of transition paths, force–fields and geometries, formally written as

\[
\text{LSF}_{\text{total}} = \text{LSF}_{\text{paths}} \cup \text{LSF}_{\text{fields}} \cup \text{LSF}_{\text{geom}}. \tag{1}
\]

Corresponding to each of the three LSF–subspaces in (1) we formulate:

(1) The least action principle, to model deterministic and predictive, macro–level MD and CD, giving a unique, global, causal and smooth path–field–geometry on the macroscopic spatio–temporal level; and

(2) Associated adaptive path integral to model uncertain, fluctuating and probabilistic, micro–level MD and CD, as an ensemble of local paths–fields–geometri–es on the microscopic spatio–temporal level, to which the
global macro–level MD and CD represents both time and ensemble average (which are equal according to the ergodic hypothesis).

In the proposed formalism, transition paths \( x^i(t) \) are affected by the force–fields \( \varphi^k(t) \), which are themselves affected by geometry with metric \( g_{ij} \).

2.1.1 Global Macro–Level of LSF_{total}

In general, at the macroscopic LSF–level we first formulate the total action \( S[\Phi] \), the central quantity in our formalism that has psycho–physical dimensions of \( \text{Energy} \times \text{Time} = \text{Effort} \), with immediate cognitive and motivational applications: the greater the action – the higher the speed of cognitive processes and the lower the macroscopic fatigue (which includes all sources of physical, cognitive and emotional fatigue that influence motivational dynamics). The action \( S[\Phi] \) depends on macroscopic paths, fields and geometries, commonly denoted by an abstract field symbol \( \Phi^i \). The action \( S[\Phi] \) is formally defined as a temporal integral from the initial time instant \( t_{ini} \) to the final time instant \( t_{fin} \),

\[
S[\Phi] = \int_{t_{ini}}^{t_{fin}} \mathcal{L}[\Phi] \, dt,
\]

with Lagrangian density given by

\[
\mathcal{L}[\Phi] = \int d^n x \, \mathcal{L}(\Phi_i, \partial_{x^j} \Phi^i),
\]

where the integral is taken over all \( n \) coordinates \( x^j = x^j(t) \) of the LSF, and \( \partial_{x^j} \Phi^i \) are time and space partial derivatives of the \( \Phi^i \)–variables over coordinates.

Second, we formulate the least action principle as a minimal variation \( \delta \) of the action \( S[\Phi] \),

\[
\delta S[\Phi] = 0,
\]

which, using techniques from the calculus of variations gives, in the form of the so–called Euler–Lagrangian equations, a shortest (loc)omotion path, an extreme force–field, and a life–space geometry of minimal curvature (and without holes). In this way, we effectively derive a unique globally smooth transition map

\[
F : INTENTION_{t_{ini}} \longrightarrow ACTION_{t_{fin}},
\]

performed at a macroscopic (global) time–level from some initial time \( t_{ini} \) to the final time \( t_{fin} \).

In this way, we get macro–objects in the global LSF: a single path described by Newtonian–like equation of motion, a single force–field described by Maxwellian–
like field equations, and a single obstacle–free Riemannian geometry (with
global topology without holes).

For example, in 1945–1949 Wheeler and Feynman developed their action–
at–a–distance electrodynamics [26], in complete experimental agreement with
the classical Maxwell’s electromagnetic theory, but at the same time avoiding
the complications of divergent self–interaction of the Maxwell’s theory as well
as eliminating its infinite number of field degrees of freedom. In Wheeler–
Feynman view, “Matter consists of electrically charged particles,” so they
found a form for the action directly involving the motions of the charges only,
which upon variation would give the Newtonian–like equations of motion of
these charges. Here is the expression for this action in the flat space–time,
which is in the core of quantum electrodynamics:

\[
S\left[ x; t_i, t_j \right] = \frac{1}{2} \sum_i m_i \int (\dot{x}^i_\mu)^2 \, dt_i + \frac{1}{2} \sum_{i \neq j} e_i e_j \int \int \delta(I^{2}_{ij}) \dot{x}^i_\mu(t_i) \dot{x}^j_\mu(t_j) \, dt_i \, dt_j
\]

with

\[
I^{2}_{ij} = \left[ x^i_\mu(t_i) - x^j_\mu(t_j) \right] \left[ x^i_\mu(t_i) - x^j_\mu(t_j) \right],
\]

where \( x^i_\mu = x^i_\mu(t_i) \) is the four–vector position of the \( i \)th particle as a function
of the proper time \( t_i \), while \( \dot{x}^i_\mu(t_i) = dx^i_\mu/\,dt_i \) is the velocity four–vector. The
first term in the action (5) is the ordinary mechanical action in Euclidean
space, while the second term defines the electrical interaction of the charges,
representing the Maxwell–like field (it is summed over each pair of charges;
the factor \( \frac{1}{2} \) is to count each pair once, while the term \( i = j \) is omitted to
avoid self–action; the interaction is a double integral over a delta function of
the square of space–time interval \( I^2 \) between two points on the paths; thus,
interaction occurs only when this interval vanishes, that is, along light cones
[26]).

Now, from the point of view of Lewinian geometrical force–fields and (loco)mo-
tion paths, we can give the following life–space interpretation to the Wheeler–
Feynman action (5). The mechanical–like locomotion term occurring at the
single time \( t \), needs a covariant generalization from the flat 4–dimensional
Euclidean space to the \( n \)–dimensional smooth Riemannian manifold, so it
becomes (see e.g., [27,28],)

\[
S[x] = \frac{1}{2} \int_{t_{ini}}^{t_{fin}} g_{ij} \dot{x}^i \dot{x}^j \, dt,
\]  

(summation convention is always assumed)

where \( g_{ij} \) is the Riemannian metric tensor that generates the total ‘kinetic
energy’ of (loco)motions in the life space.

The second term in (5) gives the sophisticated definition of Lewinian force–
fields that drive the psychological (loco)motions, if we interpret electrical
charges \( e_i \) occurring at different times \( t_i \) as motivational charges – needs.

### 2.1.2 Local Micro–Level of LSF

After having properly defined macro–level MD & CD, with a unique transition map \( F \) (including a unique motion path, driving field and smooth–manifold geometry), we move down to the microscopic LSF–level of rapidly fluctuating MD & CD, where we cannot define a unique and smooth path–field–geometry. The most we can do at this level of fluctuating uncertainty, is to formulate an adaptive path integral and calculate overall probability amplitudes for ensembles of local transitions from one LSF–point to the neighboring one. This probabilistic transition micro–dynamics is given by a multi–path (field and geometry, respectively) and multi–phase transition amplitude \( \langle \text{Action}|\text{Intention} \rangle \) of corresponding to the globally–smooth transition map (4). This absolute square of this probability amplitude gives the transition probability of occurring the final state of \( \text{Action} \) given the initial state of \( \text{Intention} \),

\[ P(\text{Action}|\text{Intention}) = |\langle \text{Action}|\text{Intention} \rangle|^2. \]

The total transition amplitude from the state of \( \text{Intention} \) to the state of \( \text{Action} \) is defined on \( \text{LSF}_{\text{total}} \)

\[ \langle \text{Action}|\text{Intention} \rangle_{\text{total}} : \text{INTENTION}_{t_0} \longrightarrow \text{ACTION}_{t_1}, \quad (6) \]

given by adaptive generalization of the Feynman’s path integral \([35,36,37,38]\). The transition map (6) calculates the overall probability amplitude along a multitude of wildly fluctuating paths, fields and geometries, performing the microscopic transition from the micro–state \( \text{INTENTION}_{t_0} \) occurring at initial micro–time instant \( t_0 \) to the micro–state \( \text{ACTION}_{t_1} \) at some later micro–time instant \( t_1 \), such that all micro–time instants fit inside the global transition interval \( t_0, t_1, ..., t_s \in [t_{\text{ini}}, t_{\text{fin}}] \). It is symbolically written as

\[ \langle \text{Action}|\text{Intention} \rangle_{\text{total}} := \int \mathcal{D}[w\Phi] e^{iS[\Phi]}, \quad (7) \]

where the Lebesgue integration is performed over all continuous \( \Phi^i_{\text{con}} = \text{paths}+\text{fields}+\text{geometries} \), while summation is performed over all discrete processes and regional topologies \( \Phi^j_{\text{dis}} \). The symbolic differential \( \mathcal{D}[w\Phi] \) in the general path integral (7), represents an adaptive path measure, defined as a weighted product

\[ \mathcal{D}[w\Phi] = \lim_{N \to \infty} \prod_{s=1}^{N} w_s d\Phi^i_s, \quad (i = 1, ..., n = \text{con} + \text{dis}), \quad (8) \]

which is in practice satisfied with a large \( N \) corresponding to infinitesimal temporal division of the four motivational phases (*).
Now, since Feynman’s invention of the path integral [35], a lot of research has been done to make the real–time Feynman path integral mathematically rigorous (see e.g., [41,42,43,44,45,46]).

In the exponent of the path integral (7) we have the action $S[\Phi]$ and the imaginary unit $i = \sqrt{-1}$ (i can be converted into the real number $-1$ using the so–called Wick rotation, see next subsection).

In this way, we get a range of micro–objects in the local LSF at the short time–level: ensembles of rapidly fluctuating, noisy and crossing paths, force–fields, local geometries with obstacles and topologies with holes. However, by averaging process, both in time and along ensembles of paths, fields and geometries, we can recover the corresponding global MD and CD variables.

### 2.1.3 Infinite–Dimensional Neural Network

The adaptive path integral (7) incorporates the local learning process according to the basic formula (see e.g., [48,49])

\[
\text{new value}(t + 1) = \text{old value}(t) + \text{innovation}(t)
\]

The general weights $w_s = w_s(t)$ in (8) are updated by the MONITOR feedback during the transition process, according to one of the two standard neural learning schemes, in which the micro–time level is traversed in discrete steps, i.e., if $t = t_0, t_1, ..., t_s$ then $t + 1 = t_1, t_2, ..., t_{s+1}$:

1. A self–organized, unsupervised (e.g., Hebbian–like [47]) learning rule:

\[
w_s(t + 1) = w_s(t) + \frac{\sigma}{\eta}(w_s^d(t) - w_s^a(t)), \tag{9}
\]

where $\sigma = \sigma(t), \eta = \eta(t)$ denote signal and noise, respectively, while superscripts $d$ and $a$ denote desired and achieved micro–states, respectively; or

2. A certain form of a supervised gradient descent learning:

\[
w_s(t + 1) = w_s(t) - \eta \nabla J(t), \tag{10}
\]

where $\eta$ is a small constant, called the step size, or the learning rate, and $\nabla J(n)$ denotes the gradient of the ‘performance hyper–surface’ at the $t$–th iteration.
Both Hebbian and supervised learning\footnote{Note that we could also use a reward–based, reinforcement learning rule [50], in which system learns its optimal policy: \[ \text{innovation}(t) = |\text{reward}(t) - \text{penalty}(t)|. \]} are naturally used for the local decision making process (see below) occurring at the intention formation fase $\mathcal{F}$.

In this way, local micro–level of $LSF_{\text{total}}$ represents an infinite–dimensional neural network. In the cognitive psychology framework, our adaptive path integral (7) can be interpreted as semantic integration (see [51,34]).

### 2.2 Pathways of (Loco)Motion and Decision Making in $LSF_{\text{paths}}$

On the macro–level in the subspace $LSF_{\text{paths}}$ we have the (loco)motion action principle

\[ \delta S[x] = 0, \]

with the Newtonian–like action $S[x]$ given by

\[ S[x] = \int_{t_{\text{ini}}}^{t_{\text{fin}}} dt \left[ \frac{1}{2} g_{ij} \dot{x}^i \dot{x}^j + \varphi^i(x^i) \right], \quad (11) \]

where $\dot{x}^i$ represents motivational (loco)motion velocity vector with cognitive processing speed. The first bracket term in (11) represents the kinetic energy $T$,

\[ T = \frac{1}{2} g_{ij} \dot{x}^i \dot{x}^j, \]

generated by the Riemannian metric tensor $g_{ij}$, while the second bracket term, $\varphi^i(x^i)$, denotes the family of potential force–fields, driving the (loco)motions $x^i = x^i(t)$ (the strengths of the fields $\varphi^i(x^i)$ depend on their positions $x^i$ in LSF, see $LSF_{\text{fields}}$ below). The corresponding Euler–Lagrangian equation gives the Newtonian–like equation of motion

\[ \frac{d}{dt} T_{\dot{x}^i} - T_{x^i} = -\varphi^i_{x^i}, \quad (12) \]

(subscripts denote the partial derivatives), which can be put into the standard Lagrangian form

\[ \frac{d}{dt} L_{\dot{x}^i} = L_{x^i}, \quad \text{with} \quad L = T - \varphi^i(x^i). \]
Now, according to Lewin, the life space also has a sophisticated topological structure. As a Riemannian smooth $n$–manifold, the LSF–manifold $M$ gives rise to its fundamental $n$–groupoid, or $n$–category $\Pi_n(M)$ (see [39,40]). In $\Pi_n(M)$, 0–cells are points in $M$; 1–cells are paths in $M$ (i.e., parameterized smooth maps $f : [0,1] \to M$); 2–cells are smooth homotopies (denoted by $\simeq$) of paths relative to endpoints (i.e., parameterized smooth maps $h : [0,1] \times [0,1] \to M$); 3–cells are smooth homotopies of homotopies of paths in $M$ (i.e., parameterized smooth maps $j : [0,1] \times [0,1] \times [0,1] \to M$). Categorical composition is defined by pasting paths and homotopies. In this way, the following recursive homotopy dynamics emerges on the LSF–manifold $M$ (**)
0-cell: \( x_0 \in M \); in the higher cells below: \( t, s \in [0, 1] \);

1-cell: \( x_0 \xrightarrow{\text{f}} x_1 \) \( f : x_0 \simeq x_1 \in M \);
\( f : [0, 1] \rightarrow M \), \( f : x_0 \mapsto x_1, x_1 = f(x_0), f(0) = x_0, f(1) = x_1 \);
e.g., linear path: \( f(t) = (1 - t)x_0 + tx_1 \) or Euler–Lagrangian \( f - \) dynamics with endpoint conditions \((x_0, x_1)\):
\[
\frac{d}{dt} f_x^i = f_{x^i}, \quad \text{with} \quad x(0) = x_0, \quad x(1) = x_1, \quad (i = 1, \ldots, n);
\]

2-cell: \( x_0 \xrightarrow{\text{h}} x_1 \) \( h : f \simeq g \in M \);
\( h : [0, 1] \times [0, 1] \rightarrow M \), \( h : f \mapsto g, g = h(f(x_0)) \),
\( h(x_0, 0) = f(x_0), h(x_0, 1) = g(x_0), h(0, t) = x_0, h(1, t) = x_1 \);
e.g., linear homotopy: \( h(x_0, t) = (1 - t)f(x_0) + tg(x_0) \) or homotopy between two Euler–Lagrangian \((f, g) - \) dynamics with the same endpoint conditions \((x_0, x_1)\):
\[
\frac{d}{dt} f_x^i = f_{x^i}, \quad \text{and} \quad \frac{d}{dt} g_{x^i} = g_{x^i} \quad \text{with} \quad x(0) = x_0, \quad x(1) = x_1;
\]

3-cell: \( x_0 \xrightarrow{\text{j}} x_1 \) \( j : h \simeq i \in M \);
\( j : [0, 1] \times [0, 1] \times [0, 1] \rightarrow M \), \( j : h \mapsto i, i = j(h(f(x_0))) \),
\( j(x_0, t, 0) = h(f(x_0)), j(x_0, t, 1) = i(f(x_0)) \),
\( j(x_0, 0, s) = f(x_0), j(x_0, 1, s) = g(x_0) \),
\( j(0, t, s) = x_0, j(1, t, s) = x_1 \);
e.g., linear composite homotopy: \( j(x_0, t, s) = (1 - t)h(f(x_0)) + ti(f(x_0)) \);
or, homotopy between two homotopies between above two Euler–Lagrangian \((f, g) - \) dynamics with the same endpoint conditions \((x_0, x_1)\).

In the next subsection we use the micro–level implications of the action \( S[x] \) as given by (11), for dynamical descriptions of the local decision–making process.

On the micro–level in the subspace \( LSF_{\text{paths}} \), instead of a single path defined by the Newtonian–like equation of motion (12), we have an ensemble of fluctuating and crossing paths with weighted probabilities (of the unit total sum). This ensemble of micro–paths is defined by the simplest instance of our adap-
tive path integral (7), similar to the Feynman’s original sum over histories,

\[ \langle \text{Action}|\text{Intention} \rangle_{\text{paths}} = \oint D[w|x] e^{iS[x]}, \quad (13) \]

where \( D[w|x] \) is a functional measure on the space of all weighted paths, and the exponential depends on the action \( S[x] \) given by (11). This procedure can be redefined in a mathematically cleaner way if we Wick–rotate the time variable \( t \) to imaginary values, \( t \mapsto \tau = it \), thereby making all integrals real:

\[ \oint D[w|x] e^{iS[x]} \xrightarrow{\text{Wick}} \oint D[w|x] e^{-S[x]}, \quad (14) \]

Discretization of (14) gives the standard thermodynamic–like partition function

\[ Z = \sum_j e^{-w_j E_j / T}, \quad (15) \]

where \( E_j \) is the motion energy eigenvalue (reflecting each possible motivational energetic state), \( T \) is the temperature–like environmental control parameter, and the sum runs over all motion energy eigenstates (labelled by the index \( j \)). From (15), we can further calculate all thermodynamic–like and statistical properties (see e.g., Feynman, 1972) of MD and CD, as for example, transition entropy, \( S = k_B \ln Z \), etc.

From cognitive perspective, our adaptive path integral (13) calculates all (alternative) pathways of information flow during the transition \( \text{Intention} \rightarrow \text{Action} \).

In the language of transition–propagators, the integral over histories (13) can be decomposed into the product of propagators (i.e., Fredholm kernels or Green functions) corresponding to the cascade of the four motivational phases (*)

\[ \langle \text{Action}|\text{Intention} \rangle_{\text{paths}} = \oint dx^F dx^I dx^M dx^T K(F, I) K(I, M) K(M, T), \quad (16) \]

satisfying the Schrödinger–like equation (see e.g., [52])

\[ i \partial_t \langle \text{Action}|\text{Intention} \rangle_{\text{paths}} = H_{\text{Action}} \langle \text{Action}|\text{Intention} \rangle_{\text{paths}}, \quad (17) \]

where \( H_{\text{Action}} \) represents the Hamiltonian (total energy) function available at the state of \( \text{Action} \). Here our ‘golden rule’ is: the higher the Hamiltonian \( H_{\text{Action}} \), the lower the microscopic fatigue.

In the connectionist language, our propagator expressions (16–17) represent activation dynamics, to which our \text{Monitor} process gives a kind of backpropagation feedback, a common type of supervised learning (10).
2.2.1 Mechanisms of decision making under uncertainty

Now, the basic question about our local decision making process, occurring under uncertainty at the intention formation phase $\mathcal{F}$, is: Which alternative to choose? (see [15,49,34]). In our path–integral language this reads: Which path (alternative) should be given the highest probability weight $w$? Naturally, this problem is iteratively solved by the learning process (9–10), controlled by the monitor feedback, which we term algorithmic approach.

In addition, here we analyze qualitative mechanics of the local decision making process under uncertainty, as a heuristic approach. This qualitative analysis is based on the micro–level interpretation of the Newtonian–like action $S[x]$, given by (11) and figuring both processing speed $\dot{x}$ and LTM (i.e., the force–field $\varphi(x)$, see next subsection). Here we consider three different cases:

(1) If the potential $\varphi(x)$ is not very dependent upon position $x(t)$, then the more direct paths contribute the most, as longer paths, with higher mean square velocities $[\dot{x}(t)]^2$ make the exponent more negative (after Wick rotation (14)).

(2) On the other hand, suppose that $\varphi(x)$ does indeed depend on position $x$. For simplicity, let the potential increase for the larger values of $x$. Then a direct path does not necessarily give the largest contribution to the overall transition probability, because the integrated value of the potential is higher than over another paths.

(3) Finally, consider a path that deviates widely from the direct path. Then $\varphi(x)$ decreases over that path, but at the same time the velocity $\dot{x}$ increases. In this case, we expect that the increased velocity $\dot{x}$ would more than compensate for the decreased potential over the path.

Therefore, the most important path (i.e., the path with the highest weight $w$) would be one for which any smaller integrated value of the surrounding field potential $\varphi(x)$ is more than compensated for by an increase in kinetic–like energy $\frac{m}{2}\dot{x}^2$. In principle, this is neither the most direct path, nor the longest path, but rather a middle way between the two. Formally, it is the path along which the average Lagrangian is minimal,

$$< \frac{m}{2} \dot{x}^2 + \varphi(x) > \rightarrow \min,$$

i.e., the path that requires minimal memory (both LTM and WM, see LSF fields below) and processing speed. This mechanical result is consistent with the ‘filter theory’ of selective attention [53], proposed in an attempt to explain a range of the existing experimental results. This theory postulates a low level filter that allows only a limited number of percepts to reach the brain at any time. In this theory, the importance of conscious, directed attention is minimized. The type of attention involving low level filtering corresponds to the concept
of early selection [53].

Although we termed this ‘heuristic approach’ in the sense that we can instantaneously feel both the processing speed \( \dot{x} \) and the LTM field \( \varphi(x) \) involved, there is clearly a psycho-physical rule in the background, namely the averaging minimum relation (18).

From the decision making point of view, all possible paths (alternatives) represent the consequences of decision making. They are, by default, short-term consequences, as they are modelled in the micro-time-level. However, the path integral formalism allows calculation of the long-term consequences, just by extending the integration time, \( t_{fin} \rightarrow \infty \). Besides, this averaging decision mechanics – choosing the optimal path – actually performs the ‘averaging lift’ in the LSF: from micro-level to the macro-level.

2.3 Force-Fields and Memory in \( LSF_{fields} \)

At the macro-level in the subspace \( LSF_{fields} \) we formulate the force-field action principle

\[
\delta S[\varphi] = 0, \tag{19}
\]

with the action \( S[\varphi] \) dependent on Lewinian force-fields \( \varphi^i = \varphi^i(x) \) (\( i = 1, ..., N \)), defined as a temporal integral

\[
S[\varphi] = \int_{t_{ini}}^{t_{fin}} \mathcal{L}[\varphi] \, dt, \tag{20}
\]

with Lagrangian density given by

\[
\mathcal{L}[\varphi] = \int d^n x \mathcal{L}(\varphi_i, \partial_{x^j} \varphi^i),
\]

where the integral is taken over all \( n \) coordinates \( x^i = x^i(t) \) of the LSF, and \( \partial_{x^j} \varphi^i \) are partial derivatives of the field variables over coordinates.

On the micro-level in the subspace \( LSF_{fields} \) we have the Feynman-type sum over fields \( \varphi^i \) (\( i = 1, ..., N \)) given by the adaptive path integral

\[
\langle \text{Action}|\text{Intention} \rangle_{fields} = \oint \mathcal{D}[w \varphi] \, e^{iS[\varphi]} \frac{Wick}{\oint \mathcal{D}[w \varphi]} \, e^{-S[\varphi]}, \tag{21}
\]

with action \( S[\varphi] \) given by temporal integral (20). (Choosing special forms of the force-field action \( S[\varphi] \) in (21) defines micro-level MD & CD, in the LSF\_fields space, that is similar to standard quantum-field equations, see e.g., [54].) The corresponding partition function has the form similar to (15), but with field energy levels.
Regarding topology of the force fields, we have in place $n$–categorical Lagrangian–field structure on the Riemannian LSF manifold $M$,

$$\Phi^i : [0, 1] \rightarrow M, \Phi^i : \Phi^i_0 \mapsto \Phi^i_1,$$

generalized from (***) above, using

$$\frac{d}{dt} f^{x^i} = f^{x^i} \rightarrow \partial_\mu \left( \frac{\partial L}{\partial \Phi^i} \right) = \frac{\partial L}{\partial \Phi^i},$$

with

$$[x_0, x_1] \rightarrow [\Phi^i_0, \Phi^i_1].$$

2.3.1 Relationship between memory and force–fields

As already mentioned, the subspace $LSF_{fields}$ is related to our memory storage [34]. Its global macro–level represents the long–term memory (LTM), defined by the least action principle (19), related to cognitive economy in the model of semantic memory [55]. Its local micro–level represents working memory (WM), a limited–capacity ‘bottleneck’ defined by the adaptive path integral (21). According to our formalism, each of Miller’s 7 ± 2 units [56] of the local WM are adaptively stored and averaged to give the global LTM capacity (similar to the physical notion of potential). This averaging memory lift, from WM to LTM represents retroactive interference, while the opposite direction, given by the path integral (21) itself, represents proactive interference. Both retroactive and proactive interferences are examples of the impact of cognitive contexts on memory. Motivational contexts can exert their influence, too. For instance, a reduction in task–related recall following the completion of the task – the well–known Zeigarnik effect [57] – is one of the clearest examples of force–field influences on memory: the amount of details remembered of a task declines as the force–field tension to complete the task is reduced by actually completing it.

Once defined, the global LTM potential $\varphi = \varphi(x)$ is then affecting the locomotion transition paths through the path action principle (11), as well as general learning (9–10) and decision making process (18).

On the other hand, the two levels of $LSF_{fields}$ fit nicely into the two levels of processing framework, as presented by [58], as an alternative to theories of separate stages for sensory, working and long–term memory. According to the levels of processing framework, stimulus information is processed at multiple levels simultaneously depending upon its characteristics. In this framework, our macro–level memory field, defined by the fields action principle (19), corresponds to the shallow memory, while our micro–level memory field, defined by the adaptive path integral (21), corresponds to the deep memory.
On the macro–level in the subspace $LSF_{geom}$ representing an $n$–dimensional smooth manifold $M$ with the global Riemannian metric tensor $g_{ij}$, we formulate the geometric action principle

$$\delta S[g_{ij}] = 0,$$

where $S = S[g_{ij}]$ is the $n$–dimensional geodesic action on $M$,

$$S[g_{ij}] = \int d^n x \sqrt{|g_{ij}|} dx^i dx^j.$$  \hfill (22)

The corresponding Euler–Lagrangian equation gives the geodesic equation of the shortest path in the manifold $M$,

$$\ddot{x}^i + \Gamma^i_{jk} \dot{x}^j \dot{x}^k = 0,$$

where $\Gamma^i_{jk}$ are the Christoffel’s symbols of the affine connection on $M$, which is the source of the curvature of $M$, and at the same time is a geometrical description for noise (see [59,60]). The higher the local curvatures of the LSF–manifold $M$, the greater the noise in the life space. This noise is the source of our micro–level fluctuations. It can be internal or external; in both cases it curves our micro–LSF.

Otherwise, if instead we choose an $n$–dimensional Hilbert–like action (see [61]),

$$S[g_{ij}] = \int d^n x \sqrt{\det |g_{ij}|} R,$$  \hfill (23)

where $R$ is the scalar curvature (derived from $\Gamma^i_{jk}$), we get the $n$–dimensional Einstein–like equation

$$G_{ij} = 8\pi T_{ij},$$

where $G_{ij}$ is the Einstein–like tensor representing geometry of the LSF manifold $M$ ($G_{ij}$ is the trace–reversed Ricci tensor $R_{ij}$, which is itself the trace of the Riemann curvature tensor of the manifold $M$), while $T_{ij}$ is the $n$–dimensional stress–energy–momentum tensor. This equation explicitly states that psychology of the LSF is proportional to its geometry. $T_{ij}$ is an important quantity, representing motivational energy, geometry–imposed stress and momentum of (loco)motion. As before, we have our ‘golden rule’: the greater the $T_{ij}$–components, the higher the speed of cognitive processes and the lower the macroscopic fatigue.

The choice between the geodesic action (22) and the Hilbert action (23) depends on our interpretation of time. If time is not included in the LSF manifold $M$ (non–relativistic approach) then we choose the geodesic action. If time is included in the LSF manifold $M$ (making it a relativistic–like $n$–dimensional
space–time) then the Hilbert action is preferred. The first approach is more related to the information processing and the working memory. The later, space–time approach can be related to the long–term memory: we usually recall events closely associated with the times of their happening.

On the micro–level in the subspace $LSF_{geom}$ we have the adaptive sum over geometries, represented by the path integral over all local (regional) Riemannian metrics $g_{ij} = g_{ij}(x)$ varying from point to point on $M$ (modulo diffeomorphisms),

$$\langle Action|Intention\rangle_{geom} = \oint D[g_{ij}] e^{i S[g_{ij}]} \xrightarrow{Wick} \oint D[g_{ij}] e^{-S[g_{ij}]}, \quad (24)$$

where $D[g_{ij}]$ denotes diffeomorphism equivalence classes of metrics $g_{ij}(x)$ of $M$.

To include the topological structure (e.g., a number of holes) in $M$, we can extend (24) as

$$\langle Action|Intention\rangle_{geom/top} = \sum_{topol.} \oint D[g_{ij}] e^{i S[g_{ij}]}, \quad (25)$$

where the topological sum is taken over all connectedness–components of $M$ determined by the Euler characteristics of $M$. This type of integral defines the theory of fluctuating geometries, a propagator between $(n−1)$–dimensional boundaries of the $n$–dimensional manifold $M$. One has to contribute a meaning to the integration over geometries. A key ingredient in doing so is to approximate (using simplicial approximation and Regge calculus [61]) in a natural way the smooth structures of the manifold $M$ by piecewise linear structures (mostly using topological simplices $\Delta$). In this way, after the Wick–rotation (14), the integral (24–25) becomes a simple statistical system, given by partition function

$$Z = \sum_{\Delta} \frac{1}{C_{\Delta}} e^{-S_{\Delta}},$$

where the summation is over all triangulations $\Delta$ of the manifold $M$, while the number $C_T$ is the order of the automorphism group of the performed triangulation.

2.4.1 Micro–level geometry: the source of noise and stress in LSF

The subspace $LSF_{geom}$ is the source of noise, fluctuations and obstacles, as well as psycho–physical stress. Its micro–level is adaptive, reflecting the human ability to efficiently act within the noisy environment and under the stress conditions. By averaging it produces smooth geometry of certain curvature, which is at the same time the smooth psycho–physics. This macro–level geom-
etry directly affects the memory fields and indirectly affects the (loco)motion transition paths.

3 Discussion

We have presented a new psychodynamical concept of the life space foam, as a natural medium for both motivational dynamics and induced cognitive theory of learning, memory, information processing and decision making. Its macro–level has been defined using the least action principle, while its micro–level has been defined using adaptive path integral. The totality of six facets of the LSF have been presented: paths, fields and geometries, on both levels.

The formalisms proposed and developed in this paper, can be employed in generating a number of meaningful and useful predictions about motivational and cognitive dynamics. These predictions range from effects of fatigue or satiation on goal-directed performance, through general learning and memory issues, to the sophisticated selection between conflicting alternatives at decision making and/or sustained action stages.

For example, one of the simplest types of performance–degrading disturbances in the LSF is what we term motivational fatigue – a motivational drag factor that slows the actors’ progress towards their goal. There are two fundamentally different sources of this motivational drag, both leading to apparently the same reduction in performance: (a) tiredness / exhaustion and (b) satiation (e.g., boredom). Both involve the same underlying mechanism (the raising valence of the alternatives to continuing the action) but the alternatives will differ considerably, depending on the properties of the task, from self–preservation / recuperation in the exhaustion case through to competing goals in the satiation case.

The spatial representation of this motivational drag is relatively simple: uni–dimensional LSF–coordinates may be sufficient for most purposes, which makes it attractive for the initial validation of our predictive model. Similarly uncomplicated spatial representations can be achieved for what we term motivational boost derived from the proximity to the goal (including the well–known phenomenon of ‘the home stretch’): the closer the goal (e.g., a finishing line) is perceived to be, the stronger its ‘pulling power’ [1,2,22]. Combinations of motivational drag and motivational boost effects may be of particular interest in a range of applications. These combinations can be modelled within relatively simple uni–dimensional LSF–coordinate systems.

In their general form, the formalisms developed in this paper, are consistent with dynamic–connectionist models of decision making, such as decision field
theory [14,16] and related theory of memory retrieval [62]. In particular, our multi-path integrals are able, in principle, to account for multi-alternative preferential choice behaviors as conceptualized in the multi-alternative decision field theory [15], potentially leading to testable predictions, e.g., those concerning the effects of similarity, compromise and time pressure on decision quality.

Examining specific predictions of this type in various goal-oriented contexts is the focus of our current work.

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