BEC and the particle mass

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We derive new features of Bose-Einstein correlations by means of Quantum Field Theory at finite temperature, supplemented by operator-field evolution approach. The origin of the dependence of the measured two-particle correlation function (as well as the so-called correlation radius) on the particle mass has received its correct explanation. The lower bound on the particle emitting source size is estimated.

I. INTRODUCTION

Over the past few decades, a considerable number of studies have been made on the phenomena of multiparticle correlations induced by collisions between elementary particles and heavy ions. It is understood that the studies of correlations between produced particles, the effects of coherence and chaoticity, an estimation of particle emitting source size play the important roles in high energy physics. Notice that produced particles affect the time evolution of the system. What does this mean? The effect of quantum correlations between particles is essentially a statistical mechanical phenomenon, and therefore cannot be fully described only within quantum field theory (QFT) in closed form. This problem becomes manifest when we define the basic operators in the QFT at finite temperature, and when we derive the evolution of them. The common feature is the coexistence of quantum and statistical fluctuations in one system composed particles.

In our previous papers [1,2], we established the master equation in the form of the field-operator evolution (Langevin-like [3]) equation, which allows one to gain a better understanding of possible coherent behavior of an emitting source of elementary particles. The clear shapes of both two-particle Bose-Einstein and Fermi-Dirac correlation functions were observed in the LEP experiments ALEPH [4], DELPHI [5] and OPAL [6] which also indicated a dependence of the measured correlation radius on the hadron (π, K - mesons) mass. Recently, the ZEUS Collaboration at HERA reported [7] the results of Bose-Einstein correlations (BEC) between kaons (charged and neutral). The radii of production volume for neutral and charged kaons turned out similar. There is no yet the definite explanation of the origin of mass dependence above mentioned (see, e.g., [8]). Notice, that the LUND model [9] does not predict such a dependence. One of the aims of this paper is to clarify this question and to give an explanation of the origin of the dependence of the radius of two-particle source emitter on a particle mass.

We focused [2,1] on two specific features of the Bose-Einstein correlations clearly visible when the latter are presented in the language of the QFT, supplemented by the operator-field evolution. The features we discussed were:  
(i) How the possible coherence of the hadronizing (or deconfined) system (modeled here by the external stationary force $P$ that appears in the Langevin-like equations) influences the 2-particle BEC function $C_2(Q = \sqrt{(p_\mu - p'_\mu)^2})$, in which two particles are characterized by their four-momenta $p_\mu$ and $p'_\mu$ ?  
(ii) What is the true origin of the experimentally observed $Q$-dependence of the $C_2(Q)$ in the approach used in [2,1]?

The only physical meaning of the memory term $\hat{K}(p_\mu)$, the noise spectral function $\psi(p_\mu)$, and the so-called coherence function $\alpha = \alpha(m^2, \bar{p}^2, \beta)$ related to them

$$\alpha(m^2, \bar{p}^2, \beta) = \frac{M_{\text{ch}}^3 \bar{p}^2(\omega, \epsilon) P^2}{|\omega - \hat{K}(p_\mu)|^2 n(\omega, \beta)} \quad \text{as} \quad \epsilon > 0 \quad \text{and} \quad Q^2 \to 0, \quad (1)$$

have not as yet been investigated carefully in [1]. In formula (1), $\hat{K}(p_\mu) = \psi(p_\mu) \hat{c}(p_\mu)$ with $[\hat{c}(p_\mu), \hat{c}(p'_\mu)] = \delta(p_\mu - p'_\mu)$, where $p_\mu = (\omega, \vec{p})$; $n(\omega, \beta) = \{\exp[(\omega - \mu)\beta] - 1\}^{-1}$ is the number of Bose-particles in the reservoir characterized by the parameter $\mu$ (the chemical potential) and the inverse temperature $\beta$. We have already established [1] that there is at $p_\mu - p'_\mu \neq 0$ a finite volume $\Omega_0(r)$ for a two-particle emitter source, and there is a correlation picture. The space in which the massive fields settle has its own characteristic length $L_{\text{ch}} \sim M_{\text{ch}}^{-1}$, in the same sense that the massive field with mass $m_\epsilon$ for the Compton length $\lambda_{\epsilon} = m_\epsilon^{-1}$ is served. The appearance of the $\delta$-like distribution $\rho(\omega, \epsilon) = \int dx e^{i(\omega - \epsilon)x} \to \delta(\omega)$ as $\epsilon \to 0$ is a consequence of the finite volume of produced particles (formally, $\epsilon$ is given
Thus, the equation of motion (5) becomes
\[ J \] and the propagator satisfies the following equation:
\[ \text{(or the deconfinement region). We suggest the following form:} \]
\[ J \] follow from Eq. (2) with other hand, any distortion that disturbs this system for any reason (the coherence function \( P \) which means there is no distortion (under the constant force \( F \)) acting on the system of produced particles. On the other hand, any distortion that disturbs this system for any reason (the coherence function \( \alpha \) acting on the system of produced particles (\( M_{ch} \neq 0 \)). Rather, a strong distortion, \( \alpha \to \infty \), satisfies the point-like region of the particle source emitter.

It is difficult to derive both \( \tilde{K}(p_{\mu}) \) and \( \psi(p_{\mu}) \) using the general properties of QFT. It is worth stressing that knowledge of one of these functions leads to an understanding of the other, due of the necessary condition [2]

\[ \int_{-\infty}^{+\infty} \frac{d\omega}{2\pi} \left| \frac{\psi(p_{\mu})}{\omega - K(p_{\mu})} \right|^2 = 1, \] (2)

which is nothing other than the generalized fluctuation-dissipation theorem. The spectral properties
\[ |\psi(p_{\mu})|^2 \to \frac{2}{\omega} \sin(\beta\omega)|\omega - \tilde{K}(p_{\mu})|^2, \ 0 < \beta < \infty \]
follow from Eq. (2) with
\[ \lim_{\beta \to \infty} \frac{1}{\pi} \frac{\sin(\beta\omega)}{\omega} = \delta(\omega). \] (3)

II. GREEN’S FUNCTION

In this paper, we would like to focus on the role of the particle mass, which influences the correlations between particles. To solve this problem, one must derive the memory term \( \tilde{K}(p_{\mu}) \) using the general properties of QFT.

We suppose that we are working with fields that correspond to a thermal field \( \Phi(x) \) with the standard definition of the Fourier transformed propagator \( F[\tilde{G}(p)] \)
\[ F[\tilde{G}(p)] = G(x - y) = Tr \{ T[\Phi(x)\Phi(y)]\rho_{\beta} \}, \] (4)
with \( \rho_{\beta} = e^{-\beta H}/Tr e^{-\beta H} \) being the density matrix of a local system in equilibrium at temperature \( T = \beta^{-1} \) under the Hamiltonian \( H \).

We consider the interaction of \( \Phi(x) \) with the external scalar field given by the potential \( U \). In contrast to an electromagnetic field, this potential is a scalar one, but it is not a component of the four-vector. The Lagrangian density can be written
\[ L(x) = \partial_{\mu}\Phi^*(x)\partial^{\mu}\Phi(x) - (m^2 + U)\Phi^*(x)\Phi(x) \]
and the equation of motion is
\[ (\nabla^2 + m^2)\Phi(x) = -J(x), \] (5)
where \( J(x) = U\Phi(x) \) is the source density operator. A simple model like this allows one to investigate the origin of the unstable state of the thermalized equilibrium in a nonhomogeneous external field under the influence of source density operator \( J(x) = U\Phi(x) \). For example, the source can be considered as \( \delta \)-like generalized function, \( J(x) = \tilde{\mu}\rho(x,\epsilon)\Phi(x) \), in which \( \rho(x,\epsilon) \) is a \( \delta \)-like succession giving the \( \delta \)-function as \( \epsilon \to 0 \) (where \( \tilde{\mu} \) is some massive parameter). This model is useful because the \( \delta \)-like potential \( U(x) \) provides the model conditions for restricting the particle emission domain (or the deconfinement region). We suggest the following form:
\[ J(x) = -\Sigma(i\partial_{\mu})\Phi(x) + J_R(x), \]
where the source \( J(x) \) decomposes into a regular systematic motion part \( \Sigma(i\partial_{\mu})\Phi(x) \) and the random source \( J_R(x) \). Thus, the equation of motion (5) becomes
\[ (\nabla^2 + m^2 - \Sigma(i\partial_{\mu}))\Phi(x) = -J_R(x), \]
and the propagator satisfies the following equation:
\[ -p^2 + m^2 - \hat{\Sigma}(p)\hat{G}(p) = 1. \quad (6) \]

The random noise is introduced with a random operator \( \eta(x) = -m^{-2}\Sigma(i\partial_x) \), for that the equation of motion looks like:

\[ \{\nabla^2 + m^2[1 + \eta(x)]\} \Phi(x) = -J_R(x). \quad (7) \]

We assume that \( \eta(x) \) varies stochastically with the certain correlation function (CF), e.g., the Gaussian CF

\[ \langle \eta(x) \eta(y) \rangle = C \exp(-z^2\mu^2_{ch}), \quad z = x - y, \]

where \( C \) is the strength of the noise described by the distribution function \( \exp(-z^2/L_{ch}^2) \) with \( L_{ch} \) being the noise characteristic scale. Both \( C \) and \( \mu_{ch} \) define the influence of the (Gaussian) noise on the correlations between particles that "feel" an action of an environment. The solution of Eq. (7) is

\[ \Phi(x) = -\int dy G(x, y) J_R(y), \quad (8) \]

where the Green’s function obeys the Eq.

\[ \{\nabla^2 + m^2[1 + \eta(x)]\} G(x, y) = \delta(x - y). \]

The final aim might having been to find the solution of Eq. (8), and then average it over random operator \( \eta(x) \). Note that the operator \( M(x) = \nabla^2 + m^2[1 + \eta(x)] \) in the causal Green’s function

\[ G(x, y) = \frac{1}{M(x) + i\sigma} \delta(x - y) \]

is not definitely positive. However, we shall formulate another approach, where the random force influence are introduced on the particle operator level.

We now introduce the general non-Fock representation in the form of the operator generalized functions

\[ b(x) = a(x) + R(x), \quad (9) \]

\[ b^+(x) = a^+(x) + R^+(x), \quad (10) \]

where the operators \( a(x) \) and \( a^+(x) \) obey the canonical commutation relations (CCR):

\[ [a(x), a(x')] = [a^+(x), a^+(x')] = 0, \]

\[ [a(x), a^+(x')] = \delta(x - x'). \]

The operator-generalized functions \( R(x) \) and \( R^+(x) \) in (9) and (10), respectively, include random features describing the action of the external forces.

Both \( b^+ \) and \( b \) obviously define the CCR representation. For each function \( f \) from the space \( S(\mathbb{R}_\infty) \) of smooth decreasing functions, one can establish new operators \( b(f) \) and \( b^+(f) \)

\[ b(f) = \int f(x)b(x)dx = a(f) + \int f(x)R(x)dx, \]

\[ b^+(f) = \int \bar{f}(x)b^+(x)dx = a^+(f) + \int \bar{f}(x)R^+(x)dx. \]

The transition from the operators \( a(x) \) and \( a^+(x) \) to \( b(x) \) and \( b^+(x) \), obeying those commutation relations as \( a(x) \) and \( a^+(x) \), leads to linear canonical representations. If both \( b(f) \) and \( b^+(f) \) create the Fock representation of the CCR, one can then find the operator \( \hat{U} \) that obeys the following conditions:

\[ \hat{U}a(f)\hat{U}^{-1} = b(f), \]
\[ \hat{U}a^+(f)\hat{U}^{-1} = b^+(f). \]

We now move to a simple physical pattern. Let us define the differential evolution (in time) equation, where the sharp and chaotically fluctuating function obeying this equation is the main object. Since we are dealing with continuous time, we can formulate the stochastic differential equation applied to each (analytical) function under the distortion of a random force. In classical mechanics, the stochastic processes in a dynamic system are under the weak action of a "large" system [10]. "Small" and "large" systems are understood to mean that the number of the states of freedom of the former is less than that of the latter. We do not exclude even the possibility of interplay between these systems. When the "large" system is in an equilibrium state (e.g., a thermostat state), our method allows one to describe the evolution equation in an integral form that reveals the effects of thermalization.

III. EVOLUTION EQUATION

Following the idea of classical Brownian motion [11,3] of a particle with a unit mass, a charge \( g \), and velocity \( v(t) \) in the external, let us say, electric field \( E \), one can write in the following form the formal equation describing the evolution (in real time) of this particle:

\[ \partial_t v(t) = \gamma v(t) + F + gE, \]

where \( F \) stands for a random force subject to the Gaussian white noise, and \( \gamma \) is a friction coefficient.

Referring to [2,1] for details, let us recapitulate here the main points of our approach in the quantum case: the collision process produces a number of particles, out of which we select only one (we assume for simplicity that we are dealing only with identical bosons) and describe it by stochastic operators \( b(\vec{p},t) \) and \( b^+(\vec{p},t) \), carrying the features of annihilation and creation operators, respectively. The rest of the particles are then assumed to form a kind of heat bath, which remains in an equilibrium characterized by a temperature \( T \) (one of our parameters). We also allow for some external (relative to the above heat bath) influence on our system. The time evolution of such a system is then assumed to be given by a Langevin-type equation [2,1] for the new stochastic operator \( b(\vec{p},t) \)

\[ i\partial_t b(\vec{p},t) = A(\vec{p},t) + F(\vec{p},t) + P \tag{11} \]

(and a similar conjugate equation for \( b^+(\vec{p},t) \)). We assume an asymptotic free undistorted operator \( a(\vec{p},t) \), and that the deviation from the asymptotic free state is provided by the random operator \( R(\vec{p},t) \): \( a(\vec{p},t) \rightarrow b(\vec{p},t) = a(\vec{p},t) + R(\vec{p},t) \).

This means, e.g., that the particle density number (a physical number) \( \langle n(\vec{p}) \rangle_{ph} = \langle n(\vec{p}) \rangle + O(\epsilon) \), where \( \langle n(\vec{p}) \rangle_{ph} \) means the expectation value of a physical state, while \( \langle n(\vec{p}) \rangle \) denotes that of an asymptotic state. If we ignore the deviation from the asymptotic state in equilibrium, we obtain an ideal fluid. One otherwise has to consider the dissipation term; this is why we use the Langevin scheme to derive the evolution equation, but only on the quantum level. We derive the evolution equation in an integral form that reveals the effects of thermalization.

Equation (11) is supposed to model all aspects of the hadronization processes (or deconfinement). The combination \( A(\vec{p},t) + F(\vec{p},t) \) in the r.h.s of (11) represents the so-called \textit{Langevin force} and is therefore responsible for the internal dynamics of particle emission, as the memory term \( A \) causes dissipation and is related to stochastic dissipative forces [2]

\[ A(\vec{p},t) = \int_{-\infty}^{+\infty} d\tau K(\vec{p},t-\tau)b(\vec{p},\tau) \]

with \( K(\vec{p},t) \) being the kernel operator describing the virtual transitions from one (particle) mode to another. At any dependence of the field operator \( K \) on the time, the function \( A(\vec{p},t) \) is defined by the behavior of the system at the precedent moments. The operator \( F(\vec{p},t) \) is responsible for the action of a heat bath of absolute temperature \( T \) on a particle in the heat bath, and under the appropriate circumstances is given by

\[ F(\vec{p},t) = \int_{-\infty}^{+\infty} \frac{d\omega}{2\pi} \hat{c}(p_\mu)\hat{c}(p_\mu)e^{-i\omega t}. \]

The heat bath is represented by an ensemble of coupled oscillators, each described by the operator \( \hat{c}(p_\mu) \) such that \( [\hat{c}(p_\mu),\hat{c}^+(p_\mu')] = \delta^4(p_\mu - p_\mu') \), and is characterized by the noise spectral function \( \psi(p_\mu) \) [2,1]. Here, the only statistical assumption is that the heat bath is canonically distributed. The oscillators are coupled to a particle, which is in
turn acted upon by an outside force. Finally, the constant term \( P \) in (11) (representing an external source term in the Langevin equation) denotes a possible influence of some external force. This force would result, e.g., in a strong ordering of phases leading therefore to a coherence effect.

The solution of equation (11) is given in \( S(\mathcal{R}) \) by

\[
\tilde{b}(p_\mu) = \frac{1}{\omega - \tilde{K}(p_\mu)} [\tilde{F}(p_\mu) + \rho(\omega_P, \epsilon)],
\]

where \( \omega \) in \( \rho(\omega, \epsilon) \) was replaced by new scale \( \omega_P = \omega/P \). It should be stressed that the term containing \( \rho(\omega_P, \epsilon) \) as \( \epsilon \to 0 \) yields the general solution to Eq. (11). Notice, that the distribution \( \rho(\omega_P, \epsilon) \) indicates the continuous character of the spectrum, while the arbitrary small quantity \( \epsilon \) can be defined by the special physical conditions or the physical spectra. On the other hand, this \( \rho(\omega_P, \epsilon) \) can be understood as temperature-dependent succession (3), in which \( \epsilon \to \beta^{-1} \). Such a succession yields the restriction on the \( \beta \)-dependent second term in the solution (12), where at small enough \( T \) there is a narrow peak at \( \omega = 0 \).

From the scattering matrix point of view, the solution (12) has the following physical meaning: at a sufficiently outgoing past and future, the fields described by the operators \( \tilde{a}(p_\mu) \) are free and, the initial and the final states of the dynamic system are thus characterized by constant amplitudes. Both states, \( \varphi(-\infty) \) and \( \varphi(+\infty) \), are related to one another by an operator \( S(\mathcal{R}) \) that transforms state \( \varphi(-\infty) \) to state \( \varphi(+\infty) \) while depending on the behaviour of \( \mathcal{R}(p_\mu) \):

\[
\varphi(+\infty) = \mathcal{R} = S(\mathcal{R})\varphi(-\infty).
\]

In accordance with this definition, it is natural to identify \( S(\mathcal{R}) \) as the scattering matrix in the case of arbitrary sources that give rise to the intensity of \( \mathcal{R} \).

Based on QFT point of view, relation (9) indicates the appearance of the terms containing nonquantum fields that are characterized by the operators \( \tilde{a}(p_\mu) \). Hence, there are terms with \( \tilde{R} \) in the matrix elements, and these \( \tilde{R} \) cannot be realized via real particles. The operator function \( \tilde{R}(p_\mu) \) could be considered as the limit on an average value of some quantum operator (or even a set of operators) with an intensity that increases to infinity. The later statement can be visualized in the following mathematical representation [2]:

\[
\tilde{R}(p_\mu) = \sqrt{\alpha} \Xi(p_\mu, p_\mu), \quad \Xi(p_\mu, p_\mu) = \langle \tilde{a}^+(p_\mu) \tilde{a}(p_\mu) \rangle_\beta,
\]

where \( \alpha \) is the coherence function that gives the strength of the average \( \Xi(p_\mu, p_\mu) \).

In principal, interaction with the fields described by \( \tilde{R} \) is provided by the virtual particles, the propagation process of which is given by the potentials defined by the \( \tilde{R} \) operator function.

The condition \( M_{ch} \to 0 \) (or \( \Omega_0(r) \sim \frac{1}{M_{ch}} \to \infty \)) in the representation

\[
\lim_{p_\mu \to p'_\mu} \Xi(p_\mu, p'_\mu) = \lim_{Q^2 \to 0} \Omega_0(r) n(\tilde{\omega}, \beta) \exp(-q^2/2) \to \frac{1}{M_{ch}^4} n(\omega, \beta),
\]

with [2]

\[
\Omega_0(r) = \frac{1}{\pi^2} r_0 r_z r_t^2
\]

means that the role of the arbitrary source characterized by the operator function \( \tilde{R}(p_\mu) \) in \( \tilde{b}(p_\mu) = \tilde{a}(p_\mu) + \tilde{R}(p_\mu) \) disappears.

IV. GREEN’S FUNCTION AND KERNEL OPERATOR

Let us go to the thermal field operator \( \Phi(x) \) by means of the linear combination of the frequency parts \( \phi^+(x) \) and \( \phi^-(x) \)

\[
\Phi(x) = \frac{1}{\sqrt{2}} \left[ \phi^+(x) + \phi^-(x) \right]
\]
On the mass-shell, \( D \) applying the direct Fourier transformation to both sides of Eqs. (18) and (19) with the following properties of the propagating and interacting of the quantum fields with mass \( m \), which are transformed into new equations for the frequency parts \( \phi^+(x) \) and \( \phi^-(x) \) of the field operator \( \Phi(x) \):

\[
\phi^-(x) = \int \frac{d^3 \mathbf{p}}{(2\pi)^3 2(\mathbf{p}^2 + m^2)^{1/2}} \tilde{b}(p_\mu) e^{-ipx},
\]

\[
\phi^+(x) = \int \frac{d^3 \mathbf{p}}{(2\pi)^3 2(\mathbf{p}^2 + m^2)^{1/2}} \tilde{b}^+(p_\mu) e^{ipx}.
\]

One can easily find two equations of motion for the Fourier transformed operators \( \tilde{b}(p_\mu) \) and \( \tilde{b}^+(p_\mu) \) in \( S(\mathbb{R}_4) \):

\[
[\omega - \hat{K}(p_\mu)]\tilde{b}(p_\mu) = \tilde{F}(p_\mu) + \rho(\omega_P, \epsilon),
\]

\[
[\omega - \hat{K}^+(p_\mu)]\tilde{b}^+(p_\mu) = \tilde{F}^+(p_\mu) + \rho(\omega_P, \epsilon),
\]

which are transformed into new equations for the frequency parts \( \phi^+(x) \) and \( \phi^-(x) \) of the field operator \( \Phi(x) \):

\[
i\partial_0 \phi^+(x) - \int_{\mathbb{R}_4} K(x - y) \phi^+(y)dy = F(x) + P \partial_0 D(x) e^{-\epsilon t},
\]

\[
-i\partial_0 \phi^-(x) - \int_{\mathbb{R}_4} K^+(x - y) \phi^-(y)dy = F^+(x) + P \partial_0 D(x) e^{-\epsilon t}.
\]

Here, the field components \( \phi^+(x) \) and \( \phi^-(x) \) are nonlocalized under the effect of the invariant formfactors \( K(x - y) \) and \( K^+(x - y) \), respectively. In general, these formfactors can admit the description of locality for nonlocal interactions. The function \( D(x) \) in Eqs. (16) and (17) obeys the commutation relation

\[
[\Phi(x), \Phi(y)]_\epsilon = -iD(x)
\]

and can be written [12]

\[
D(x) = \frac{1}{2\pi} \epsilon(x^0) \left[ \delta(x^2) - \frac{m^2}{2x^2_\mu} \Theta(x^2) J_1 \left( m \sqrt{x^2_\mu} \right) \right],
\]

where \( \epsilon(x^0) \) and \( \Theta(x^2) \) are the standard unit and the step functions, respectively, while \( J_1(x) \) is the Bessel function.

On the mass-shell, \( D(x) \) becomes

\[
D(x) \approx \frac{1}{2\pi} \epsilon(x^0) \left[ \delta(x^2) - \frac{m^2}{4} \Theta(x^2) \right].
\]

At this stage, it must be stressed that we have new generalized evolution Eqs. (16) and (17), which retain the general features of the propagating and interacting of the quantum fields with mass \( m \) that are in the heat bath (reservoir) and are chaotically distorted by other fields. For further analysis, let us rewrite the system of Eqs. (16) and (17) in the following form:

\[
i\partial_0 \phi^+(x) - K(x) * \phi^+(x) = f(x),
\]

\[
-i\partial_0 \phi^-(x) - K^+(x) * \phi^-(x) = f^+(x),
\]

where \( A(x) * B(x) \) is the convoluted function of the generalized functions \( A(x) \) and \( B(x) \), and

\[
f(x) = F(x) + P \partial_0 D(x) e^{-\epsilon t}.
\]

Applying the direct Fourier transformation to both sides of Eqs. (18) and (19) with the following properties of the Fourier transformation

\[
F[K(x) * \phi^+(x)] = F[K(x)]F[\phi^+(x)],
\]
we get two equations

\[-p^0 - \tilde{K}(p_\mu)\phi^+(p_\mu) = F[f(x)],\]  
\[\begin{align*}
[p^0 - \tilde{K}^+(p_\mu)]\phi^-(p_\mu) &= F[f^+(x)].
\end{align*}\]  

Multiplying Eqs. (20) and (21) by \(p^0 - \tilde{K}^+(p_\mu)\) and \(-p^0 - \tilde{K}(p_\mu)\), respectively, we find

\[-p^0 - \tilde{K}(p_\mu)\omega_{\mu
u}^\gamma(p_\mu) \omega^\gamma_{\mu
u} = T(p_\mu),\]  

where

\[T(p_\mu) = [p^0 - \tilde{K}(p_\mu)]F[f(x)] - [p^0 + \tilde{K}(p_\mu)]F[f^+(x)].\]

We are now at the stage of the main strategy: we have to identify the field \(\Phi(x)\) introduced in Eq. (4) and the field \(\Phi(x)\) (13) built up of the fields \(\phi^+\) and \(\phi^-\) as the solutions of generalized Eqs. (16) and (17). The next step is our requirement that Green’s function \(\tilde{G}(p_\mu)\) in Eq. (6) and the function \(\Gamma(p_\mu, \beta)\), that satisfies Eq. (22)

\[p^0 + \tilde{K}(p_\mu)\omega_{\mu
u}^\gamma(p_\mu) \omega^\gamma_{\mu
u} = 1,\]  

must be equal to each other, where [12]

\[\tilde{G}(p_\mu) \rightarrow G(p^2, g^2, m^2) \simeq 1 - g^2 \xi(p^2, m^2)\]

with \(g\) being the scalar coupling constant and the one-loop correction of the scalar field \(\xi \ll 1/m^2\) at \(1/4 \leq (m^2/p^2) \leq 1\). This means we define the operator kernel \(\tilde{K}(p_\mu)\) in (14) from the condition of the nonlocal coincidence of the Green’s function \(\tilde{G}(p_\mu)\) in Eq. (6), and the thermodynamic function \(\tilde{\Gamma}(p_\mu, \beta)\) from (23) in \(S(\mathbb{R}_4)\)

\[\lim_{x-x' \rightarrow \mathcal{O}(r)} F[\tilde{G}(p_\mu) - \tilde{\Gamma}(p_\mu, \beta)] = 0.\]

We can easily derive the kernel operator \(\tilde{K}(p_\mu)\) in the form

\[\tilde{K}(p_\mu) = (m^2 + \vec{p}^2)^\frac{1}{2}\left[1 + g^2 \xi(p^2, m^2)\left(1 - \frac{\omega^2}{m^2 + \vec{p}^2}\right)\right]^\frac{1}{2},\]  

where

\[\xi(m^2) = \frac{1}{96 \pi^2 m^2} \left(\frac{2 \pi}{\sqrt{3}} - 1\right),\]  

\[p^2 \simeq m^2,\]

and

\[\xi(p^2, m^2) = \frac{1}{96 \pi m^2} \left(i \sqrt{1 - \frac{4 m^2}{p^2} + \frac{\pi}{\sqrt{3}}} + \pi \right),\]  

\[p^2 \simeq 4m^2.\]

The ultraviolet behaviour at \(|p^2| > m^2\) leads to

\[\xi(p^2, m^2) \simeq -\frac{1}{32 \pi^2 p^2} \left[\ln \frac{|p^2|}{m^2} - \frac{\pi}{\sqrt{3}} - i \pi \Theta(p^2)\right].\]
V. CORRELATION FUNCTION

Out of many details (for which we refer to [2,1]) the 2-particle BEC function for identical particles is defined as

\[ C_2(Q) = \chi(N) \cdot \frac{\tilde{f}(p_\mu, p_\mu')}{f(p_\mu) \cdot f(p_\mu')} \Delta(r_f Q) \approx \chi(N) \cdot [1 + D(p_\mu, p_\mu')] (1 + \sum_{l \geq 1} (r_f \cdot Q)^l), \]  

(25)

where \( \tilde{f}(p_\mu, p_\mu') = \langle \hat{b}^+(p_\mu)\hat{b}^+(p_\mu')\hat{b}(p_\mu)\hat{b}(p_\mu') \rangle \) and \( \tilde{f}(p_\mu) = \langle \hat{b}^+(p_\mu)\hat{b}(p_\mu) \rangle \) are the corresponding thermal statistical averages with \( \hat{b}(p_\mu) \) being the corresponding Fourier transformed solution (12). The multiplicity \( N \) depending factor is equal to \( \chi(N) = (N(N - 1)) / (N)^2 \). We have introduced in (25) the factor \( \Delta(r_f Q) \approx 1 + \sum_{l \geq 1} (r_f \cdot Q)^l \), which is nothing other than the consequence of the Bogolyubov’s principle of weakening of correlations at large distances that is characterized by the parameter of weakening of correlations \( r_f \) relevant to the particle pair correlations. In the limit of infinitely small scale \( r_f \) the function \( C_2(Q) \) coincides with that obtained in [2]. The latter statement means that one returns to the ideal correlation pattern.

As was shown in [2], (note that operators \( \hat{R}(p_\mu) \) by definition commute with themselves and with any other operator considered here),

\[ \tilde{f}(p_\mu, p_\mu') = \tilde{f}(p_\mu) \tilde{f}(p_\mu') + \langle \hat{a}^+(p_\mu)\hat{a}(p_\mu)\rangle\langle \hat{a}^+(p_\mu')\hat{a}(p_\mu') \rangle + \langle \hat{a}^+(p_\mu)\hat{a}(p_\mu) \rangle \hat{R}(p_\mu) + \langle \hat{a}^+(p_\mu')\hat{a}(p_\mu') \rangle \hat{R}(p_\mu'), \]

\[ \tilde{f}(p_\mu) = \langle \hat{a}^+(p_\mu)\hat{a}(p_\mu) \rangle + |\hat{R}(p_\mu)|^2. \]  

(26)

This defines \( D(p_\mu, p_\mu') = \tilde{f}(p_\mu, p_\mu') / [\tilde{f}(p_\mu) \cdot \tilde{f}(p_\mu')] - 1 \) in Eq. (25) in terms of the operators \( \hat{a}(p_\mu) \) and \( \hat{R}(p_\mu) \) which in our case are equal to

\[ \hat{a}(p_\mu) = \frac{\tilde{F}(p_\mu)}{\omega - \hat{K}(p_\mu)} \quad \text{and} \quad \hat{R}(p_\mu) = \frac{\rho(\omega_p, \epsilon)}{\omega - \hat{K}(p_\mu)}. \]

This means, therefore, that the correlation function \( C_2(Q) \), as defined by Eq. (25), is essentially given in terms of \( \rho(\omega_p, \epsilon) \) and the following two thermal averages for the thermostat operators \( \hat{F}(\vec{p}, t) \) (for details see [13]):

\[ \langle F^+(\vec{p}, t)F(\vec{p}', t') \rangle = \delta^3(\vec{p} - \vec{p}') \int \frac{d\omega}{2\pi} |\psi(p_\mu)|^2 n(\omega, \beta)e^{i\omega(t-t')}, \]  

(27)

\[ \langle F(\vec{p}, t)F^+(\vec{p}', t') \rangle = \delta^3(\vec{p} - \vec{p}') \int \frac{d\omega}{2\pi} |\psi(p_\mu)|^2 [1 + n(\omega, \beta)]e^{-i\omega(t-t')}. \]  

(28)

Notice that with only delta functions present in Eqs.(27) and (28) we would have a situation in which the hadronizing (or deconfined) system would be described by some kind of colored noise only because of the presence of \( n(\omega, \beta) \) that carries the quantum properties. The integrals multiplying these delta functions and depending on (a) the momentum characteristic of a heat bath \( \psi(p_\mu) \) and (b) the assumed bosonic statistics of produced secondaries resulting in factors \( n(\omega, \beta) \) and \( 1 + n(\omega, \beta) \), respectively, bring the description of the system considered here closer to reality.

We now see easily that the existence of BEC (i.e., that \( C_2(Q) > 1 \)) is strictly connected with nonzero values of the thermal averages Eqs. (27) and (28). However, in the form presented there, they differ from zero only at one point, namely for \( Q = 0 \) (i.e., for \( p_\mu = p_\mu' \)). Actually, this is the price we pay for the QFT assumptions tacitly made here, namely for the infinite spatial extension and for the uniformity of our reservoir. However, we know from the experiments in, e.g., [14,4-7] that \( C_2(Q) \) reaches its maximum at \( Q = 0 \) and falls towards its asymptotic value of \( C_2 = 1 \) at large of \( Q \) (actually at \( Q \sim 1 \) GeV/c). To reproduce the same behaviour by means of our approach, we must replace the delta functions in Eqs. (27) and (28) by functions with supports larger than those limited to one point only. This means that these functions should not be infinite at \( Q_\mu = p_\mu - p_\mu' = 0 \) but remain more or less sharply peaked at this point, otherwise remaining finite and falling to zero at small but finite values of \( |Q_\mu| \) (actually identical to those at which \( C_2(Q) \) reaches unity)

\[ \delta(p_\mu - p_\mu') \implies \Omega_0 \cdot \exp[-(p_\mu - p_\mu')L^\mu\nu(r)(p_\nu - p_\nu')]. \]  

(29)

Here we replace the \( \delta \)-function with the smearing (smooth) dimensionless generalized function \( \Omega(q = Qr) = \exp[-(p_\mu - p_\mu')L^\mu\nu(r)(p_\nu - p_\nu')] \), where \( L^\mu\nu \) is the structure tensor of the space-time size and it defines the spherically-like domain of emitted (or produced) particles.

We therefore tacitly introduce a new parameter, \( r_\mu \), a 4-vector that has the dimension of length. This defines the region of nonvanishing particle density with the space-time extension of the particle emission source. Expression (29)
must be understood in the sense that Ω(Qr) is a function that, in the limit of \( r \to \infty \), strictly becomes a δ-function. With such a replacement, we now have

\[
D(p_\mu, p'_\mu) = \frac{\sqrt{\Omega(q)}}{(1 + \alpha)(1 + \alpha') \sqrt{\Omega(q) + 2\sqrt{\alpha\alpha'}}},
\]

where

\[
\tilde{\Omega}(q) = \gamma \cdot \Omega(q), \quad \gamma = \frac{n^2(\omega, \beta)}{n(\omega, \beta) n(\omega', \beta)},
\]

The coherence function \( \alpha \) is another very important one that summarizes our knowledge of other than space-time characteristics of the particle emission source. Notice that \( \alpha > 0 \) only when \( P \neq 0 \). For \( \alpha = 0 \), we actually find

\[
1 < C_2(Q) < \chi(N)[1 + \chi(Qr)](1 + r_f \cdot Q + ...),
\]

i.e., it is contained between the limits corresponding to very large (lower limit) and very small (upper limit) values of \( P \). Because of this, \( \alpha \) plays the role of a coherence parameter. Ignoring the energy-momentum dependence of \( \alpha \), and assuming that \( \alpha' = \alpha \), we get the expression

\[
C_2(Q) \simeq \chi(N) \left\{ 1 + \lambda_{\text{new}}(m, \beta) e^{-q^2} \left[ 1 + \lambda_{\text{corr}}(m, \beta) e^{q^2/2} \right] \right\} (1 + r_f \cdot Q + ...),
\]

where the new intercept function becomes as \( \lambda_{\text{new}} = \gamma(\omega, \beta)/(1 + \alpha)^2 \), and the new coherence correction in the brackets of Eq. (32) carries an additional intercept function \( \lambda_{\text{corr}} = 2 \alpha / \sqrt{\gamma(\omega, \beta)} \). In fact, since \( \alpha \neq \alpha' \) (because \( \omega \neq \omega' \) and, therefore, the number of states identified here with the number of particles with given energy \( n(\omega) \) is also different), we must use the general form Eq. (25) for \( C_2 \) with details given by Eqs. (30) and (31) and with \( \alpha = 1 \) depending on the particle mass and such characteristics of the emission process as the temperature \( T \) and chemical potential \( \mu \) occurring in the definition of \( n(\omega) \). Note that Eq. (32) differs from the usual empirical parameterization of \( C_2(Q) \) [15, 16, 4-7],

\[
C_2(Q) \sim (1 + \lambda e^{-q^2})(1 + \alpha Q + ...),
\]

which is nothing other but the Goldhaber parameterization [17] at \( a = 0 \) with \( 0 < \lambda < 1 \) being a free parameter adjusting the observed value of \( C_2(Q = 0) \), customary called a "coherence strength factor" or "chaoticity" of \( \lambda = 0 \) for fully coherent sources and \( \lambda = 1 \) for fully incoherent sources; \( a \) is a c-number, with \( \Omega(Qr) \) usually represented as Gaussian. Returning to the particle mass dependence of \( C_2(Q) \) (the correlation radius, in particular), we find that this dependence comes from \( \alpha \)-coherence function (1) containing the operator kernel \( \tilde{K}(p_\mu) \), defined correctly up to the second-order of the scalar coupling constant \( g^2 \) in Eq. (24) within the framework of the QFT.

The \( \alpha \)-representation in (1) must be clarified: In fact, \( M_{ch}^3 \) can be broken down into two parts: \( M_{ch}^3 \rightarrow M_{ch}^{(0)} \cdot M_{ch}^3 \), where \( M_{ch}^{(0)} \) is the small massive scale characterized by the time-like scale \( r^0 \), while \( M_{ch}^3 \) is the characteristic mass associated with the spatial inverse components \( r_x, r_y, M_{ch}^3 \sim (r_x r_y)^{-1} \). Taking into account the properties of the distribution \( [\rho(\omega, \psi)]^2 \), we can suggest the following replacement \( M_{ch}^3 [\rho(\omega, \psi)]^2 \rightarrow [M_{ch}^{(0)} \rho(0, \psi)] M_{ch}^3 \rho(\omega, \psi) \), where the first multiplier is of the order of \( O(1) \), while the second reflects the massive scale \( \mu_{ch} \) of the particles production region, i.e., \( M_{ch}^3 \rho(\omega, \psi) \sim O(\mu_{ch}^2) \), \( \mu_{ch} < m \). Thus,

\[
\alpha \sim O \left[ \frac{\mu_{ch}^2}{(\omega - \tilde{K}(p_\mu))^2 n(\omega, \beta)} \right].
\]

Let us return to the problem of the \( Q \)-dependence of the BEC. One more remark is in order here: The problem with the \( \delta(p_\mu - p'_\mu) \) function encountered in two particle distributions does not exist in the single particle distributions that are in our case given by Eq. (26), and which can be written as

\[
\tilde{f}(p_\mu) = (1 + \alpha) \cdot \Xi(p_\mu, p_\mu),
\]

where \( \Xi(p_\mu, p_\mu) \) is the one-particle distribution function for the "free" (undistorted) operator \( \tilde{a}(p_\mu) \), namely

\[
\Xi(p_\mu, p_\mu) = \langle \tilde{a}^+(p_\mu) \tilde{a}(p_\mu) \rangle = \Omega_0 \cdot \frac{\psi(p_\mu)}{\omega - \tilde{K}(p_\mu)}^2 n(\omega, \beta).
\]
Notice that the actual shape of \( \tilde{f}(p_\mu) \) is dictated by both \( n(\omega) \) (calculated for fixed temperature \( T \) and chemical potential \( \mu \) at energy \( \omega \), as given by the Fourier transform of field operator \( \hat{K} \) (24) and the shape of the reservoir in the momentum space provided by \( \psi(p_\mu) \)), and by the \( \delta \)-like distribution of external force \( \rho(\omega_p, \epsilon) \). On the other hand, it is clear from Eq. (35) that \( \langle N \rangle = \langle N_{ch} \rangle + \langle N_{coh} \rangle \), where \( \langle N_{ch} \rangle \) and \( \langle N_{coh} \rangle \) denote multiplicities of particles produced chaotically and coherently, respectively.

The \( m \)-dependence of \( C_2(Q, m) \) is essentially given by

\[
\alpha(m^2, \beta) \sim \frac{n(\omega, \beta)}{m^2(\omega, \beta) L_{ch}},
\]

which is nothing other but the effective number of Bose-particles in the plane phase-space with the size \( L_{ch} \) having the mean mass \( \bar{m}(\omega, \beta) = m n(\omega, \beta) \). Obviously, \( \alpha \to 0 \) as \( n(\omega, \beta) \to \infty \) (Goldhaber parameterization) and \( \alpha \to \infty \) as \( n(\omega, \beta) \to 0 \) (that is trivial result with \( C_2(Q) \approx 1 \)). Notice, that the condition \( \alpha(m, \beta) < 1 \) always yields at sufficiently strong temperature \( T \), and its validation would be very weak when \( T \to 0 \). In the latter case the heat bath is simply absent.

On the other hand, neglecting the energy-momentum dependence of \( \alpha(\alpha') \), the latter can be estimated within the formula

\[
\alpha \approx \frac{2 - \bar{C}_2(0)}{\bar{C}_2(0) - 1} \sqrt{\frac{1}{\chi(N)}},
\]

where \( \bar{C}_2(0) = C_2(0) / \chi(N) \), and we assume that the BEC function at \( Q = 0 \), \( C_2(0) \), and the mean multiplicity \( \langle N \rangle \) are known from the experiment. Within the formula (36) and the Eq. (32), it is evidently that an increasing of the particle mass \( m \) leads to an enhancement of the intercept function \( \lambda_{new}(m, \beta) \). Therefore in the case of the investigation of the correlations between heavy particles the chaotic coherence effects become negligible (\( \alpha \to 0 \)).

Finally, we present the lower bound on the particle emitting source size:

\[
r_{ch} \sim L_{ch} > \frac{1}{m \sqrt{n(\beta)}} \left( \frac{1}{\gamma(n)/\Delta C_2^{\max} - 1} \right)^{\frac{1}{2}},
\]

where \( \Delta C_2^{\max} = C_2^{\max} - 1, \bar{C}_2^{\max} = C_2^{\max} / \chi(N), \) and \( C_2^{\max} \) is the maximal value of the \( C_2 \)-function in the vicinity of \( Q = 0 \).

The lower bound on the particle emitting source size carries the dependence of:
- the particle mass \( m \) given by \( \alpha \) and \( \alpha' \) which are defined within (34) with the kernel operator \( \hat{K}(p_\mu) \) (24) calculated in the QFT;
- the mean multiplicity factor \( \chi(N) \) defined within formula (25);
- the maximal value of the BEC function \( \bar{C}_2 \);
- the absolute temperature of a heat bath and the chemical potential (the \( \gamma(n) \)-factor defined in Eq. (31)).

VI. CONCLUSION

To summarize: using the QFT, supplemented by Langevin-like evolution Eqs. (16) and (17) to describe hadronization (or deconfinement) processes, we have derived the two-particle BEC function in a form explicitly showing the origin of both the so-called coherence (and how it influences the structure of the BEC) and the \( Q \)-dependence of the BEC represented by the correlation function \( C_2(Q) \). The dynamic source of coherence is identified in our case with the existence of a constant external term \( P \) in the evolution equation. Therefore, for \( P \to \infty \), we have all phases aligned in the same way, and \( C_2(Q) = 1 \). This is because coherence has already been introduced on the level of a particle production source as a property of the fields or operators describing produced particles. It is therefore up to the experimenter to decide which proposition is followed by nature: the simpler formula (33) or the rather more complicated Eq. (25) in combination with Eq. (30).

With our approach, it is also clear that the form of \( C_2 \) reflects the distributions of the space-time separation between the two observed particles.

Finally, we would like to stress that our discussion is so far limited to only a single type of secondaries produced. This is enough to attain our general goals, i.e., to explain the possible dynamic origin of coherence in BEC, the origin of the specific shape of the correlation \( C_2(Q) \) functions, and to explain the dependence of the correlation radius on the particle mass due to coherence function \( \alpha \), as seen from the the QFT perspective. Actually, \( r \) decreases with the
mass $m$. It is then plausible that, in describing the general BEC effect, they should be combined somehow, especially if the experimental data indicate such a need.

The final note concerns to that the ZEUS at HERA resulted that $r$ values for pions and kaons are not so different and the effects comes from heavier particles. Therefore more precise measurements from different processes are necessary.

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