On Channel Estimation for Rician Fading with the Phase-Shift in Cell-Free Massive MIMO System

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Abstract
Channel estimation (CE) is a crucial phase in wireless communication systems, especially in cell-free (CF) massive multiple input multiple output (M-MIMO) since it is a dynamic wireless network. Therefore, this work is introduced to study CE for the CF M-MIMO system in the uplink phase, wherein the performance of different estimators are evaluated, discussed, and compared in various situations. We assume the scenario in which each access point has prior knowledge of the channel statistics. The phase-aware-minimum mean square error (PA-MMSE) estimator, the non-phase-aware-MMSE (NPA-MMSE) estimator, and the least-squares estimator are the three estimators which are exploited in this work. Besides, we consider the Rician fading channel in which the line-of-sight path is realized with a phase-shift that models the users’ mobility where the considered phase-shift follows a uniform distribution. On the other hand, the mean-squared error metric is employed in order to evaluate the performance of each estimator, where an analytical and simulated result is provided for the PA-MMSE estimator and the NPA-MMSE estimator in order to assert our numerical results.

Keywords Channel estimation · Massive MIMO · Cell-free · Rician fading · Phase-shift

Abbreviations
5G Fifth generation.
AP Access point
CB Coherence block
CF Cell-free
CE Channel estimation
CSI Channel state information

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Introduction

The employment of a large number of antennas at the base stations knows, in the literature of communication systems as the massive multiple input multiple output (M-MIMO) technology [28] [1, 2]. Using this technology, each BS can offer good service to many users even if in the worst scenario, in which many users are served simultaneously [3, 4]. In addition, M-MIMO technology is one of the most widely used technologies through the 5G wireless communication system [5]. Furthermore, the literature of M-MIMO wireless networks addresses two types of antenna arrays: co-located and distributed antenna arrays. The co-located antenna scenarios have been defined in the literature as being located in a compacted array and have low backhaul demands [6]. On the other hand, with the distributed configuration of M-MIMO systems, the antennas are distributed over a wide geographical area [7, 8, 14]. This latter configuration is known in the literature as cell-free (CF) M-MIMO [9, 10]. CF M-MIMO offers a significant gain for the fifth generation (5G) wireless communication systems, in which an enormous number of access points (APs) serve a relatively small number of users in the same time-frequency resources. The CF rely on the front-haul connection between APs [14] and using channel-based MIMO schemes to get a local channel state information (CSI) [11].

CF M-MIMO is an innovative technology that enables such a system to operate with a higher potential to support the 5G wireless communication using only a straightforward signal processing technique. In addition, this technology can offer energy efficiency (EE), substantial throughput, and high precision. In its basic canonical design, CF M-MIMO technology uses a large number of APs, assumed to be randomly located in a large area and serving a reasonably small number of users which are also assumed to be randomly distributed [12–14]. Thus, depending on the user’s location, some APs experience higher levels of interference than others.

In CF M-MIMO, the whole network’s APs cooperate to serve all users. Additionally, the CF M-MIMO has a high potential to spatially multiplexing users and regulates the interference depending on the network design and architecture [7]. The distributed M-MIMO configuration (i.e., CF) affords an enhanced macro-diversity that contributes to a better EE and coverage probability contrary to the located M-MIMO configuration. Thereby, due to
the rising demand for connection and the need to provide excellent service to users, wireless communication systems are expected to benefit fully from the distributed M-MIMO design (i.e., Cell-free M-MIMO). As a consequence, CF M-MIMO has garnered significant attention from researchers.

2 Related Works

The purpose of this section is to present the related works that have been addressed the CF M-MIMO. Besides, some work denominated this system as a distributed antennas system [15–18]. In [19], the authors investigate the spectral efficiency (SE) for a CF M-MIMO system that operates in both uplink (UL) and downlink (DL) using a Rician fading channel. A deterministic component is introduced to reflect the line-of-sight (LoS) propagation environment. Besides, a phase is joined with the deterministic component to draw the users’ mobility. The authors propose a decoding strategy based on a two-layer approach to overcome interference in the UL phase, i.e., coherent and incoherent interferences. On the other hand, in the DL stage, two distinct transmission manners are investigated, in which they have named these transmissions as incoherent and coherent transmission.

In [20], the authors propose a beamforming-based technique for CF M-MIMO systems that operates under time division duplex protocol which obeys the constraint defined by short-term average power for this network type (i.e., CF network). Furthermore, the impact of imperfect channel knowledge is considered. Analytical and numerical results are presented to compare the proposed beamforming scheme and the traditional beamforming. The authors conclude that using the proposed beamforming technique, a better result in terms of achievable rate is achieved. In [21], the authors deal with network MIMO, where a large number APs serve a relatively small number of users, which is recently called CF M-MIMO, where it provides a much improvement for wireless communication systems. The authors have introduced an innovative idea, namely, the scalability of a CF network. In addition, the CF M-MIMO consider being scalable, even though in a situation with an infinite number of serving users, we can guarantee that the complexity can be finite. This concept is investigated using a dynamic cooperation cluster that has been treated previously for network MIMO. Furthermore, The pilot assignment and the formation of clusters are studied under the scalability concept and in the same way for channel estimation (CE), combining, and precoding schemes. Moreover, a distributed and centralized processes are studied, whereas the authors concluded that both modes provide the same CE quality since the backhaul is considered without loss.

The authors in [22] have addressed the hardware weakness, and their influence over CF M-MIMO since the antenna quality confronts guaranteeing good communication between transceivers. The SE and EE are investigated for both links (i.e., UL and DL), where a closed-form expression is driven for each stage. However, under the goal of maximizing the user rate. The power control algorithm, i.e. max-min, is adopted. Finally, the authors provide analytical and numerical results for the sake of affirming the theoretical study.

An UL stage for CF M-MIMO has been investigated in [23], where the authors have considered the most realistic scenario in which the channels are correlated. Besides, considering the Rician fading channel. On the other hand, SE is studied using various estimators. The authors presented Results to assert theoretical expressions. Conclude that the correlation among channels is beneficial for SE. Moreover, the performance deterioration is obtained using a linear MMSE estimator as it has no phase knowledge.
The user-centric strategy, wherein each user is served only by a set of APs that provides the most suitable channel conditions for that user [24]. However, as this technique is executed sequentially, i.e., for each user, it takes more time to serve all users in the network. Besides, the serving APs group changes from one user to another, i.e., if the kth user is served through a particular number of APs, the ith user can be served by a different number of APs that serve the kth user. In addition, we can discover in such a network that an AP serves multiple users, i.e., an AP can belong to different groups of APs as each APS group is made to offer a higher quality of service to the user. It is crucial to note that, since the users and APs distribution is uniform, therefore, in most situations, these groups of APs are overlapped (i.e., it is not practical to divide the network into non-overlapping groups) since the selected group is determined by the user’s position. Alternatively, in the scenario where different users are to be served by the cluster, the APS cooperates to provide the users with the best quality of service.

In [25], the authors address the conventional network with spatially correlated channels. In addition, the authors assume Rician fading channels, in which the channel gain is composed of two components (i.e., deterministic and non-deterministic components). Diverse estimators are utilized, and the channel estimate obtained is used for maximum-ratio combining. Moreover, a closed-form SE formula for UL and DL is given, with an asymptotic response is achieved when different estimators are utilized. Furthermore, a closed-form SE formula for UL and DL is provided, and an asymptotic response is obtained for each estimator. The authors conclude that the higher SE is attained using the minimum mean square error (MMSE) estimator relative to the other estimators.

After presenting the previous works to the best of our knowledge on the CF M-MIMO. The work organization is presented below

### 2.1 Organization

The rest of this work is structured in the following form: The CF system model is presented in Sect. 3, where the Rician fading channel is introduced. Sect. 4 investigates UL CE using various channel estimators (i.e., (least-squares) LS, phase-aware-minimum mean square error (PA-MMSE), non-phase-aware-minimum mean square error (NPA-MMSE)), where the mean-squared error (MSE) expression for each estimator is given. The numerical results are presented in Sect. 5, with which we assert our analytical expressions given in the previous sections. Sect. 6 presents the conclusion and future work.

### 2.2 Contributions of this Work

The principal contributions of this work are outlined as follows

1. Studying CE using the Rician fading channel in which the LoS path is used in each communication link (i.e., each communication that links a particular user to a particular AP).
2. Investigate and evaluate the influence of previous knowledge of the phase-shift over the CE using the PA-MMSE estimator. In addition, evaluate CE quality without any previous knowledge using the NPA-MMSE estimator.
3 Cell-Free System Model

In this section, we present the system model used, as shown in Fig. (1) below, where all users and all APs are considered outfitted with a single antenna. It is important to note that the multi-antenna AP situation can be constructed using a single antenna AP by handling each antenna of a multi-antenna AP as an independent AP. We studied and evaluated the uncorrelated channel model in which the small-scale fading (SSF) coefficients are not correlated, as indicated in [26]. On the other hand, we assume that the channel is flat fading over each coherence block (CB) that comports $\tau_c$ frequency (i.e., $\tau_c$ indicates the CB length). The CB length is affected by many factors, such as the users’ velocity, number of wavelengths, and spreading environment [27]. The channel coefficient that links the $k_{th}$ user to the $m_{th}$ AP is formulated as follows

$$g_{m,k} = \overline{g}_{m,k} e^{j\chi_{m,k}} + \Theta_{m,k}$$

Here, $\Theta_{m,k}$ symbolizes the channel gain that corresponds to the Non-LoS path between the $k_{th}$ user and the $m_{th}$ AP. In addition, this channel gain can be formulated as $\Theta_{m,k} \sim \mathcal{C}(\rho_{m,k})$, where $\rho_{m,k}$ signifies the large-scale fading (LSF) coefficient that encompassing the shadow fading and path-loss. On the other hand, the gain $\overline{g}_{m,k}$ is a positive coefficient (i.e., $\overline{g}_{m,k} \geq 0$) that denotes the LoS path components, and $\chi_{m,k} \sim \mathcal{U}[-\pi, \pi]$ symbolizes de phase-shift. It is important to highlight that the channel gain indicated in Eq. (1) follows the Rician distribution and is called the Rician fading model because $|\overline{g}_{m,k}|$ is Rice distributed. Furthermore, $\overline{g}_{m,k}$ does not follow a Gaussian distribution as in numerous studies that overlook the phase-shift. Assuming that the channel is considered an independent random variable for each user-AP link and the channel realizations over various CBs are independent and identically distributed.

![Fig. 1 Typical representation of a CF M-MIMO grid](image-url)
4 Uplink Channel Estimation

The CB is the time-frequency interval over which the channel that connects a user to his serving AP is considered unchanged. This interval length is $\tau_c$, which defines the number of frequencies used in each CB. Each CB is assumed to be divided into portions. One of these portions is the UL Pilot Sequence (PS) which has the length $\tau_p$ and must obey the following constraints $\tau_c > \tau_p$. In the slot $\tau_p$, all users transmit their PSs for the sake of communicating with its serving APs (where a very small interval comes after transmitting the pilot, namely, the CE process as mentioned in [28]). Thereby, because of the limited CB length, which also depends on the many environmental factors. In addition, we use the frequency reuse technique in order to obey this constraint where this frequency reuse leads to a known phenomenon in the literature of wireless communication systems, namely, pilot contamination (PC).

The UL pilot length is $\tau_p$, i.e., the system can benefit from the $\tau_p$ orthogonal pilot (the $\tau_p$ users can use orthogonal pilots where the remaining users reuse the same pilots, which are already assigned to the first $\tau_p$ users). We consider that the $k_{th}$ user transmitted the $\psi_k \in \mathbb{C}^{\tau_p \times 1}$ that meets the following requirements $\|\psi_k\|^2 = \tau_p$. The obtained signal $y_m^p$ at the $m_{th}$ AP is expressed as

$$y_m^p = \sum_{k=1}^{K} \sqrt{q_k} g_{m,k} \psi_k + w_m^p$$ (2)

Here $y_m^p \in \mathbb{C}^{r \times 1}$ indicates the spread vector and $w_m^p \sim \mathcal{N}_C(0, \sigma^2 \|\psi_k\|^2)$ denotes the noise. In order to estimate the channel $g_{m,k}$, an adequate statistic is used, where the $m_{th}$ AP calculate the inner product of the obtained signal and the transmitted PS from the $k_{th}$ user. This statistic increases the degree of freedom and provides a signal without depending on the PS. The expression of the obtained signal is written as

$$y_{m,k}^p = \psi_k^H y_m^p = \sum_{j=1}^{K} \sqrt{q_j} g_{m,j} \psi_j^H \psi_j + \psi_k^H w_m^p$$ (3)

In CF M-MIMO, generally speaking, the number of users is large relative to the PS length (i.e., $K > \tau_p$). Thus, we consider a set $\mathcal{P}_k$ of users employing the same pilot as the $k_{th}$ user, counting itself as well. It is important to note that users with a different PS (i.e., orthogonal PS) have their product equal to zero (i.e., $\psi_k \psi_j = 0$, $\forall k \neq i$).

$$y_{m,k}^p = \sqrt{q_k} \tau_p g_{m,k} + \sum_{j \in \mathcal{P}_k \setminus \{k\}} \sqrt{q_j} g_{m,j} \psi_j^H \psi_j + \psi_k^H w_m^p$$ (4)

Here $\psi_k^H w_m^p \sim \mathcal{N}_C(0, \sigma^2 \tau_p)$. By relying on Eq. (4), we evaluate the quality of CE obtained through various estimators. Besides, we investigate the influence of the perfect knowledge regarding phase-shift using the MMSE estimator called PA-MMSE estimator and no information regarding the phase-shift knowledge using two estimators, namely the MMSE estimator with no phase-shift knowledge called NPA-MMSE estimator and the LS estimator with no knowledge of phase-shift.

In the literature on wireless communication systems, some algorithms are used to estimate the phase-shift, as mentioned in [29, 30]. These algorithms require a multi-antenna system and/or a large number of realizations, which is not the purpose of this work.
4.1 Phase-Aware MMSE Channel Estimation

4.1.1 Phase-Aware MMSE Channel Estimation in Coefficient Form

This estimator is considered to have perfect knowledge of $\tilde{g}_{m,k}$, $\beta_{m,k}$, and $\chi_{m,k}$ at the $m_{th}$ AP where the estimated channel assumed linked the $k_{th}$ user to the $m_{th}$ AP. As mentioned in [31], we can write the estimate channel coefficient based on PA-MMSE as

$$\hat{g}_{m,k}^{pa-mmse} = \tilde{g}_{m,k}e^{j\chi_{m,k}} + \frac{\sqrt{q_k} \beta_{m,k}(y_{m,k}^p - \tilde{y}_{m,k})}{\tilde{\gamma}_{m,k}}$$

(5)

Here $\tilde{\gamma}_{m,k} = \sum_{l \in T_k} q_k \tau_p \beta_{m,l} + \sigma^2$ and $\tilde{y}_{m,k} = \sum_{l \in T_k} \sqrt{q_k} \tau_p \tilde{g}_{m,k} e^{j\chi_{m,k}}$. It is important to highlight that $\chi_{m,k}$ and $\tilde{y}_{m,k}^p$ change in each CB. Hence, $y_{m,k}^p$ also changes in each CB. On the other hand, we assume that $\tilde{g}_{m,k}$ and $\beta_{m,k}$ are kept constant for a long-time since they have a relatively slow variation over a long duration. The mean and the variance of estimation error coefficient $\hat{g}_{m,k}^{pa-mmse} = \hat{g}_{m,k}^{pa-mmse} - \hat{g}_{m,k}$ are expressed as

$$\begin{align*}
\omega_{m,k} & = \text{Var}\{\hat{g}_{m,k}^{pa-mmse}\} = \beta_{m,k} - \frac{q_k \tau_p \beta_{m,k}}{\tilde{\gamma}_{m,k}}, \\
\mathbb{E}\{\hat{g}_{m,k}^{pa-mmse}\} & = \mathbb{E}\{\hat{g}_{m,k}^{pa-mmse} - \hat{g}_{m,k}\} = 0.
\end{align*}$$

(6)

The MSE of the PA-MMSE estimator is expressed as

$$\Delta_{pa-mmse} = \mathbb{E}\{|\hat{g}_{m,k}^{pa-mmse} - \hat{g}_{m,k}^{pa-mmse}|^2\} = \mathbb{E}\{|z_{m,k}^{pa-mmse}|^2\} = \text{Var}\{z_{m,k}^{pa-mmse}\}, = \omega_{m,k}$$

(7)

The estimated channel coefficient based on PA-MMSE exhibits the following properties

$$\mathbb{E}\{\hat{g}_{m,k}^{pa-mmse} | \chi_{m,k}\} = \tilde{g}_{m,k} e^{j\chi_{m,k}},$$

$$\text{Var}\{\hat{g}_{m,k}^{pa-mmse} | \chi_{m,k}\} = \beta_{m,k} - \text{Var}\{\hat{g}_{m,k}^{pa-mmse}\}.$$ 

(8)

(9)

It is worth noting that $\hat{g}_{m,k}^{pa-mmse}$ does not follow a Gaussian distribution. In addition, $\hat{g}_{m,k}^{pa-mmse}$ and $\hat{g}_{m,k}^{pa-mmse}$ are not correlated.

In this situation, the estimated channel coefficients are computed locally i.e., the CE process is not computed in the central processing unit as in [32] since each AP is computed the CE itself and does not share it with any other APs as in [7, 33, 34].

4.1.2 Phase-Aware MMSE Channel Estimation in vector form

As an extended version to vector mode, the estimated channel vector expressed as follows

$$\mathbf{g}_{k}^{pa-mmse} = \mathbf{y}_{k} \mathbf{\hat{g}}_{k} + \sqrt{q_k} \mathbf{R}_k \mathbf{\Xi}_k^{-1} (\mathbf{y}_{k}^p - \tilde{y}_{k}^p)$$

(10)

Here $\mathbf{g}_{k} = [\tilde{g}_{1,k}, \ldots, \tilde{g}_{M,k}]^T$, $\mathbf{y}_{k} = \text{diag}(e^{j\chi_{1,k}}, \ldots, e^{j\chi_{M,k}})$, $\mathbf{y}_{k}^p = [y_{1,k}^p, \ldots, y_{M,k}^p]^T$, $\tilde{y}_{k}^p = [\tilde{y}_{1,k}^p, \ldots, \tilde{y}_{M,k}^p]^T$, $\mathbf{\Xi}_k = \text{diag}(\xi_{1,k}, \ldots, \xi_{M,k})$, and $\mathbf{R}_k = \text{diag}(\beta_{1,k}, \ldots, \beta_{M,k})$.

The mean and the covariance of estimation error vector $\mathbf{g}_{k}^{pa-mmse} = \mathbf{g}_{k}^{pa-mmse} - \mathbf{g}_{k}$ are expressed as
The MSE of PA-MMSE in vector form can be written as

\[
\Pi_{\text{pa-mmse}} = \mathbb{E}\left\{ |\hat{g}_{k}^{\text{pa-mmse}} - \tilde{g}_{k}^{\text{pa-mmse}}|^{2}\right\} = \mathbb{E}\left\{ |\tilde{g}_{k}^{\text{pa-mmse}}|^{2}\right\} = \text{Cov}\left\{ \tilde{g}_{k}^{\text{pa-mmse}} \right\} = \text{Tr}(\Omega_{k})
\]

The estimated channel vector based on PA-MMSE exhibits the following properties

\[
\mathbb{E}\{\hat{g}_{k}^{\text{pa-mmse}} | \mathcal{Y}_{k}\} = \mathcal{Y}_{k}\hat{g}_{k},
\]

\[
\text{Cov}\{\hat{g}_{k}^{\text{pa-mmse}} | \mathcal{Y}_{k}\hat{g}_{k}\} = \mathbb{R}_{k} - \Omega_{k},
\]

4.2 Non-Phase-Aware MMSE Channel Estimation

4.2.1 Non-Phase-Aware MMSE Channel Estimation in Coefficient Form

The linear MMSE estimator or the NPA-MMSE estimator is an estimator that has complete knowledge of LSF \(\beta_{m,k}\) and \(\tilde{g}_{m,k}\) coefficients whereas, it has no information regarding phase shift \(\chi_{m,k}\). The NPA-MMSE estimate of the channel that links the \(k_{th}\) user to the \(m_{th}\) AP is expressed as follows:

\[
\hat{g}_{m,k}^{\text{npa-mmse}} = \sqrt{q_{k}q_{m,k}^{\prime}} \frac{\hat{\beta}_{m,k}^\prime}{\hat{\xi}_{m,k}^\prime}
\]

Here \(\hat{\beta}_{m,k}^\prime = \beta_{m,k} + \tilde{\tilde{g}}_{m,k}^2\) and \(\hat{\xi}_{m,k}^\prime = \sum_{l \in \mathcal{P}} q_{k} q_{m,k} \left( \beta_{m,k} + \tilde{\tilde{g}}_{m,k}^2 \right) + \sigma^2 = \sum_{l \in \mathcal{P}} q_{k} q_{m,k} \beta_{m,k}^\prime + \sigma^2\). The mean and the variance of estimation error coefficient \(g_{m,k}^{\text{npa-mmse}} = g_{m,k}^{\text{npa-mmse}} - \hat{g}_{m,k}^{\text{npa-mmse}}\) are expressed as

\[
\begin{align*}
\omega_{m,k}^\prime &= \text{Var}\left\{g_{m,k}^{\text{npa-mmse}}\right\} = \hat{\beta}_{m,k}^\prime - \frac{q_{k}q_{m,k}^{\prime}(\hat{\beta}_{m,k}^\prime)^2}{\hat{\xi}_{m,k}^\prime} \\
\mathbb{E}\{g_{m,k}^{\text{npa-mmse}}\} &= 0,
\end{align*}
\]

The MSE of NPA-MMSE is written as follows

\[
\Delta_{\text{npa-mmse}} = \mathbb{E}\left\{ |\hat{g}_{m,k}^{\text{npa-mmse}} - \hat{g}_{m,k}^{\text{npa-mmse}}|^{2}\right\} = \mathbb{E}\left\{ |\tilde{g}_{m,k}^{\text{npa-mmse}}|^{2}\right\} = \text{Var}\left\{\tilde{g}_{m,k}^{\text{npa-mmse}}\right\}, = \omega_{m,k}^\prime
\]

\[
= \hat{\beta}_{m,k}^\prime - \frac{q_{k}q_{m,k}^{\prime}(\hat{\beta}_{m,k}^\prime)^2}{\hat{\xi}_{m,k}^\prime} = \hat{\beta}_{m,k}^\prime (1 - \frac{q_{k}q_{m,k}^{\prime}(\hat{\beta}_{m,k}^\prime)}{\hat{\xi}_{m,k}^\prime})
\]

The estimated channel coefficient based on NPA-MMSE has the following properties

\[
\mathbb{E}\{\hat{g}_{m,k}^{\text{npa-mmse}}\} = 0, \quad \text{Var}\{\hat{g}_{m,k}^{\text{npa-mmse}}\} = \hat{\beta}_{m,k}^\prime - \omega_{m,k}^\prime.
\]
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4.2.2 Non-Phase-Aware MMSE Channel Estimation in Vector Form

The estimated channel vector based on Eq. (15) is expressed as follows

$$\hat{g}_{k}^{npa-mmse} = \sqrt{q_k} R_k (\Xi'_k)^{-1} y^p_k$$  \hspace{1cm} (19)

Here $y^p_k = [y^p_{1,k}, \ldots, y^p_{M,k}]^T$, $\Xi'_k = \text{diag}(\xi'_1, \ldots, \xi'_M)$, and $R'_k = \text{diag}(\beta'_1, \ldots, \beta'_M)$. The mean and the covariance of estimation error vector $\tilde{g}_{k}^{npa-mmse} = g_{k}^{npa-mmse} - \hat{g}_{k}^{npa-mmse}$ are expressed as

$$\begin{aligned}
\Omega'_k &= R'_k - q_k \tau_p \Xi'_k R'_k (\Xi'_k)^{-1} R'_k \\
\mathbb{E}\{\tilde{g}_{k}^{npa-mmse}\} &= 0_M
\end{aligned} \hspace{1cm} (20)

The MSE of NPA-MMSE in vector form is expressed as follows

$$\Pi_{npa-mmse} = \mathbb{E}\{|\tilde{g}_{k}^{npa-mmse}\|^2\} = \mathbb{E}\{|\tilde{g}_{k}^{npa-mmse}\|^2\} = \text{Cov}\{\tilde{g}_{k}^{npa-mmse}\} = \text{Tr}(\Omega'_k)$$

The estimated channel vector based on NPA-MMSE exhibits the following properties

$$\mathbb{E}\{\tilde{g}_{k}^{npa-mmse}\} = 0_M, \quad \text{Cov}\{\tilde{g}_{k}^{npa-mmse}\} = \Omega'_k - \Omega'_k,$$  \hspace{1cm} (22)

4.3 LS Channel Estimation

4.3.1 LS Channel Estimation in Coefficient Form

The traditional LS estimator, which is among the non-bayesian estimators that have no information concerning channel statistics (i.e., LSF $\beta_{m,k}, g_{m,k}$, and the phase-shift $\chi_{m,k}$) [31], where this estimator aims to minimize the difference between the received signal (i.e., the spread signal) and the desired signal as follows

$$y^p_{m,k} - \sqrt{q_k} \tau_p g_{m,k} = \sum_{j \in P_k \setminus \{k\}} \sqrt{q_j} g_{m,j} \psi^H_k \psi_j + \psi^H_k w^p_m$$ \hspace{1cm} (23)

where $P_k$ indicates the group/set of users that uses the same PS as the $k_{th}$ user, including itself. In order to minimize the aforementioned difference, the Eq. (23) can be rewritten as follows

$$|y^p_{m,k} - \sqrt{q_k} \tau_p g_{m,k}|^2 = |\sum_{j \in P_k \setminus \{k\}} \sqrt{q_j} g_{m,j} \psi^H_k \psi_j + \psi^H_k w^p_m|^2$$ \hspace{1cm} (24)

The CE expression using this estimator is written as follows

$$\hat{g}_{m,k}^{ls} = \frac{1}{\sqrt{q_k} \tau_p} y^p_{m,k}$$ \hspace{1cm} (25)

Where CE using the LS estimator exhibits the following properties

$$\mathbb{E}\{\hat{g}_{m,k}^{ls}\} = 0, \quad \text{Var}\{\hat{g}_{m,k}^{ls}\} = \frac{1}{q_k \tau^2_p} \xi^2_{m,k},$$ \hspace{1cm} (26)
The error of estimation is indicated by $\tilde{g}_{m,k}^{ls} = g_{m,k}^{ls} - \hat{g}_{m,k}^{ls}$ which exhibits the following properties

$$E\{\tilde{g}_{m,k}^{ls}\} = 0, \quad \text{Var}\{\tilde{g}_{m,k}^{ls}\} = \frac{1}{q_k \tau_p} \xi_{m,k} - \hat{\beta}_{m,k}',$$  \hfill (27)

### 4.3.2 LS Channel Estimation in Vector Form

The CE expression using LS estimator (in vector form) is written as follows

$$\mathbf{g}_{k}^{ls} = \frac{1}{\sqrt{q_k \tau_p}} \mathbf{y}_k^p$$  \hfill (28)

Here $\mathbf{y}_k^p = [y_{1,k}^p, \ldots, y_{M,k}^p]^T$ is a vector form, thereby the CE using the LS estimator exhibits the following properties

$$E\{\mathbf{g}_{k}^{ls}\} = \mathbf{0}_M, \quad \text{Cov}\{\mathbf{g}_{k}^{ls}\} = \frac{1}{q_k \tau_p} \Xi_k',$$  \hfill (29)

The error of estimation is indicated by $\tilde{g}_{k}^{ls} = g_{k}^{ls} - \hat{g}_{k}^{ls}$ which has the following properties

$$E\{\tilde{g}_{k}^{ls}\} = \mathbf{0}_M, \quad \text{Cov}\{\tilde{g}_{k}^{ls}\} = \frac{1}{q_k \tau_p} \Xi_k' - R_k',$$  \hfill (30)

After having given the MSE expressions for each estimator in both forms. The table below summarizes the MSE expressions that are obtained for all estimators using either the coefficient form or the vector form (Table 1).

### 5 Numerical Result and Discussions

The purpose of this section is to affirm our numerical expression provided in the previous sections. We treat the CF M-MIMO in which $M = 100$ APs (except for Fig. (4), where the number of APs is varied) and $K = 40$ users are deployed where the distribution of either APs and users is a uniform distribution within the area. Besides, this area is considered to be a square of dimension $1000 \times 1000$ m$^2$ (except for Fig. (2), where the area dimension is varied). Furthermore, we consider that the CB length is setting to $\tau_c = 200$ samples, the PS length

| Table 1 | The MSE expressions that are obtained for all estimator for coefficient and vector forms |
| --- | --- |
| Estimator | MSE expression in the case of channel coefficient | MSE expression in the case of channel vector |
| PA-MMSE Estimator | $\Delta^{\text{pa-mmse}} = \beta_{m,k} (1 - q_k \epsilon_{m,k}^{\gamma_k})$ | $\Pi^{\text{pa-mmse}} = Tr(\mathbf{R}_k - q_k \tau_p \mathbf{R}_k \Xi_k^{-1} \mathbf{R}_k)$ |
| NPA-MMSE Estimator | $\Delta^{\text{npa-mmse}} = \hat{\beta}_{m,k} (1 - q_k \epsilon_{m,k}^{\gamma_k})$ | $\Pi^{\text{npa-mmse}} = Tr(\mathbf{R}_k' - q_k \tau_p \mathbf{R}_k' (\Xi_k')^{-1} \mathbf{R}_k')$ |
| LS Estimator | $\Delta^l = \frac{1}{q_k \tau_p} \xi_{m,k} - \hat{\beta}_{m,k}'$ | $\Pi^l = \frac{1}{q_k \tau_p} \Xi_k' - R_k'$ |
is setting to $\tau_p = 5$ ($\tau_p < K$, as indicates in the previous section), and UL power is setting to $q = 100$ mW. On the other hand, the pilot assignment/allocation to each user is assigned in such a manner that this user receives lower interference. In other words, the first $\tau_p$ pilots are assigned randomly to the first $\tau_p$ users, whereas the remaining users assign with pilots in order to give the lowest interference levels to the users in the actual set of pilots (i.e., $P_k$), leading to the most common phenomenon in wireless communication systems called the PC problem, as it is considered a bottleneck for modern wireless systems.

We implement LSF using the COST 321 Walfish-Ikegami model for the micro-cells scenario, as shown in [35]. In addition, we address the situation in which APs consider positioned at the height of 12.5 m whereas users consider positioned at the height of 1.5 m. Furthermore, we deem that only the LoS path that links each user to its serving AP. One can ask the question of why we have considered the LoS path between each user-AP pair. We have investigated the LoS path in order to represent the mean of the channel gain that is given in Eq. (1). Therefore, the adopted path-loss model is expressed as follows

$$\theta_{m,k} = -30.18 - 10\eta \log_{10}\left(\frac{d_{m,k}}{1m}\right) + \Gamma_{m,k}$$

(31)

Here $d_{m,k}$ indicates the distance between the $m_{th}$ AP and the $k_{th}$ user and $\eta$ indicates the path-loss exponent ( $\eta$ is setting to 2.6 for all numerical results except for Fig. (5), in which $\eta$ is varied ). The coefficient $\Gamma_{m,k}$ indicates the shadowing. The Rician parameter is a function of the distance $d_{m,k}$ as follows

$$\kappa_{m,k} = 10^{1.3-0.003d_{m,k}}$$

(32)
This work studied correlated-shadowing as reported in [7], where $\Gamma_{m,k}$ is expressed as follows

$$\Gamma_{m,k} = \sqrt{\delta \mu_m + \sqrt{1 - \delta} \nu_k},$$  \hspace{1cm} (33)

Here $\mu_m \sim N(0, \sigma_{sf}^2)$, and $\nu_k \sim N(0, \sigma_{sf}^2)$ symbolize the effect of blockage provided by the large obstacles in the neighborhood of the $m_{th}$ AP and the $k_{th}$ user, respectively. The parameter $\delta$ symbolizes the shadowing, which obeys the following inequality $0 \leq \delta \leq 1$. The covariance of users pair or APs pair is computed based on the following equations (i.e., Eqs. (34), (35))

$$\mathbb{E}\{\mu_m \mu_n\} = 2^{-d_{m,n} \over d_{dc}}$$  \hspace{1cm} (34)

Here $d_{m,n}$ indicates the distance between the $m_{th}$ AP and $n_{th}$ AP, where $d_{dc}$ indicates the decorrelation distance, which relies on the propagation environment.

$$\mathbb{E}\{\nu_k \nu_j\} = 2^{-d_{k,j} \over d_{dc}}$$  \hspace{1cm} (35)

Here $d_{k,j}$ indicates the distance between the $k_{th}$ user and $j_{th}$ user. For the decorrelation distance, we assume the same value for both situations (i.e., $d_{dc} = 100$ same decorrelation distance). We setting the factors $\sigma_{sf} = 8$ and $\delta = 0.5$. Besides, The LSF, and $\bar{g}_{m,k}$ are computed based on the Rician factor, $\kappa_{m,k}$ and the path loss model, $\theta_{m,k}$ as

$$\bar{g}_{m,k} = \sqrt{\kappa_{m,k} \over \kappa_{m,k} + 1} \sqrt{\theta_{m,k}}, \quad \beta_{m,k} = {1 \over \kappa_{m,k} + 1} \theta_{m,k}$$  \hspace{1cm} (36)

Before evaluating our simulation result, we bring to your attention that analytical and simulated results are introduced for the PA-MMSE and NPA-MMSE estimators whereas, analytical results are given for the LS estimator. Moreover, we evaluate the performance of each estimator using the MSE metric (i.e., given in Table 1).

In Fig. (2), the MSE is plotted against the area size, where the parameters’ value are previously defined (i.e., $M$, $K$, $q$,.....). Additionally, the Fig. (2) is provided to evaluate the performance of three estimators in terms of the area dimension. In terms of area size, the performance of the proposed PA-MMSE, NPA-MMSE, and LS estimators is compared. As expected, better performance is achieved through the PA-MMSE estimator since the PA-MMSE estimator has complete knowledge of the phase-shift, the LSF coefficient, and the channel average $g_c$. Whereas the performance obtained through the NPA-MMSE estimator is dependent on the area dimension used as when the area dimensions range is between [100 m, 700 m], the NPA-MMSE estimator provides the worst performance compared to the LS estimator. In addition, when the area dimension range is superior to 700 m, the NPA-MMSE estimator offers better performance than the LS estimator. As the area dimension increases, the performance provided using the NPA-MMSE estimator approaches the PA-MMSE estimator’s performance. Furthermore, the asymptotic behavior of the NPA-MMSE estimator is explained by the fact that the phase-shift impact decreases as the area dimension increases. In other words, when the area dimension increase, a large gap between each pair of users and each pair of APs is more expect to be formed. Hence, the effect of phase-shift knowledge is not important.

In Fig. (3), the MSE is plotted against the number of users, $K$. Additionally, this figure shows how the estimators perform when a different number of users are served
in the network. Besides, the performance of the PA-MMSE, NPA-MMSE, and LS estimators is evaluated regarding the number of users in the network. As expected, the best performance is obtained by the PA-MMSE estimator as the PA-MMSE estimator has complete knowledge of the phase shift, LSF coefficient, and the channel mean $\overline{g}_k$, while the performance obtained by the LS estimator is the worst. In addition, when the number of users in the network is equal to 5 (i.e., no pilots are reused, all used pilots are orthogonal to each other because $\tau_p = 5$), we notice that the PA-MMSE and NPA-MMSE perform similarly whereas, the LS provided the worst performance even if a small number of users in the network is served (i.e., the MSE values remain constant irrespective of the number of users). As the number of serving users increase, the NPA-MMSE estimator provides increased MSE values compared to PA-MMSE. This is due to the fact that as the number of users increases, a small gap between each pair of users is more likely to emerge (since the users are randomly distributed). Thus, the effect of the phase shift knowledge is an important contributor factor to the CE, as it has a considerable influence on obtaining a better CE.

In Fig. 4, the MSE is plotted against the number of APs in the network, $M$. The performance of PA-MMSE, the PA-MMSE, and the LS estimators are evaluated with respect to the number of APs in the network. In addition, we emphasize that during this work, we assume single antenna APs. According to the figure, better performance is achieved by the PA-MMSE estimator since the PA-MMSE estimator has complete knowledge of the phase-shift, the LSF coefficient, and the channel average $\overline{g}_k$. Furthermore, the worst performance is obtained by the traditional LS estimator. Also, the
NPA-MMSE estimate outperforms the LS estimator in terms of performance (i.e., the NPA-MMSE estimator’s MSE values are lower than the LS estimator’s MSE values). Consequently, the obtained gap between the PA-MMSE estimator and the PA-MMSE estimator in terms of the MSE metric may be explained by the fact that the phase-shift knowledge strongly influences the CE process. In conclusion, phase shift knowledge is an important factor for the CE process.

In Fig. 5, the MSE is plotted against various path-loss exponent values, $\eta$. The performance provided by PA-MMSE, the PA-MMSE, and the LS estimators is compared with respect to various path-loss exponent values. As indicates in the figure, better performance is achieved by the PA-MMSE estimator where it provides the low MSE values. Resulting from the fact that the PA-MMSE estimator has complete knowledge of the phase-shift, the LSF coefficient, and the channel average $\bar{g}_k$. Furthermore, the worst performance is provided by the traditional LS estimator, where for $\eta \geq 3.5$, higher performance degradation is obtained. Consequently, the performance obtained by the NPA-MMSE estimator is much better than the LS estimator (i.e., the NPA-MMSE estimator’s MSE values are lower than the LS estimator’s MSE values). In other words, the NPA-MMSE estimator provides results comparable to the PA-MMSE estimator compared to LS estimator since the LS estimator provides a higher degradation result, which is obvious for high $\eta$ values even if when a large value of the path-loss exponent. Notice that, in the standard situation, in which $\eta$ is setting to 2.6, the LS and NPA-MMSE estimators perform similarly a little taken. On the other hand, the PA-MMSE estimator provided
better performance thanks to the phase-shift knowledge that strongly affects the CE process.

6 Conclusion

This work has studied the CE for CF M-MIMO in the UL phase and considered the Rician fading channel where the LoS path is realized with a phase-shift that models the users’ mobility wherein the considered phase-shift follows a uniform distribution. The performance of different estimators has evaluated, discussed, and compared in various situations. In addition, we have assumed the scenario in which each AP has prior knowledge of channel statistics. We have concluded that phase-shift knowledge is an important task for the CE process since the PA-MMSE estimator presents better performance in all studied situations (i.e., the performances have evaluated using the MSE metric) whereas the PA-MMSE estimator provides results relatively lower than the PA-MMSE estimator. Moreover, we have affirmed that when the area dimension increases, the phase-shift influence is reduced (as the area dimension increase, the NPA-MMSE estimator has provided asymptotically result to the PA-MMSE estimator). As the area dimension increases, a large gap between each users pair and each APs pair is more likely to occur. Hence, the impact of phase-shift knowledge diminishes. On the other
hand, analytical and simulated results have provided for the PA-MMSE estimator and the NPA-MMSE estimator, which have yielded similar results.

Appendix

A The NPA-MMSE Estimator’s Derivation

Using Eq. (3), where the CE relies on the received pilot signal, the NPA-MMSE estimator and MSE are defined in [31] as

$$
\hat{g}_{npa-mmse}^{m,k} = \frac{\mathbb{E}\{g_{m,k}(y_{m,k}^p)^*\}}{\mathbb{E}\{|y_{m,k}^p|^2\}} y_{m,k}^p
$$

(37)

Since the estimation error and the NPA-MMSE estimate are uncorrelated random variable, the variance of the estimation error vector (i.e., the MSE) expression is driven as

$$
\omega'_{m,k} = \text{Var}\{\hat{g}_{npa-mmse}^{m,k}\} = \mathbb{E}\{|\hat{g}_{npa-mmse}^{m,k}|^2\} - \mathbb{E}\{|\hat{g}_{npa-mmse}^{m,k}\|^2\}
$$

(38)

The NPA-MMSE estimator and the relative MSE are obtained by computing the following expectations and putting them into Eq. (37), as shown in Sect. 4.2:

$$
\mathbb{E}\{g_{m,k}(y_{m,k}^p)^*\} = \sqrt{q_k} \tau_p \mathbb{E}\{|g_{m,k}|^2\} + \mathbb{E}\{g_{m,k}(\psi_k^H w_m^p)^*\}
$$

$$
+ \sum_{j \in \mathcal{P}_k \setminus k} \sqrt{q_j} \tau_p \mathbb{E}\{g_{m,k} g_{m,j}^*\} = \sqrt{q_k} \tau_p (\beta_{m,k} + \tilde{g}_{m,k}^2) = \sqrt{q_k} \tau_p \beta_{m,k}'
$$

(39)

$$
\mathbb{E}\{|y_{m,k}^p|^2\} = \sum_{j \in \mathcal{P}_k} q_j \tau_p^2 \mathbb{E}\{|g_{m,j}|^2\} + \mathbb{E}\{|\psi_k^H w_m^p|^2\}
$$

$$
= \sum_{j \in \mathcal{P}_k} q_j \tau_p^2 (\beta_{m,j} + \tilde{g}_{m,k}^2) + \sigma_p^2 \tau_p
$$

$$
= \sum_{j \in \mathcal{P}_k} q_j \tau_p^2 (\beta_{m,j}') + \sigma^2 \tau_p
$$

(40)

Thus, by replacing the terms given in Eq. (38) with the expressions derived from (39) and (40), the Eq. (38) becomes
Finally, the mean and the variance regarding the NPA-MMSE estimator are expressed as

\[
\mathbb{E}\{\hat{g}_{m,k}^{\text{npa-mmse}}\} = \frac{\sqrt{q_k} (\beta'_{m,k})}{\xi_{m,k}} \mathbb{E}\{y_{m,k}\} = 0
\]  

\[
\text{Var}\{\hat{g}_{m,k}^{\text{npa-mmse}}\} = \mathbb{E}\{||\hat{g}_{m,k}^{\text{npa-mmse}}||^2\} = \frac{q_k (\beta'_{m,k})^2}{(\xi_{m,k})^2} \mathbb{E}\{y_{m,k}^2\} = q_k \tau_p (\beta'_{m,k})^2 (\xi_{m,k})^{-1}
\]  

**B The Proposed PA-MMSE Estimator’s Derivation**

Assuming that, the channel coefficient is estimated in a coherence block wherein the phase-shift is considered constant, and the proposed PA-MMSE estimator has previous knowledge of the phase shift. Thus, using Eq. (3) and that the CE relies on the received pilot signal, the PA-MMSE estimator is expressed as

\[
\hat{g}_{m,k}^{\text{pa-mmse}} = \tilde{g}_{m,k} e^{jx_{m,k}} + \frac{\sqrt{q_k} \tilde{p}_{m,k} (y_{m,k}' - \bar{y}_{m,k}')}{\xi_{m,k}}
\]  

It is known that if \(A\) and \(B\) are constants, and as the expectation is a linear operator, the following equalities are true

\[
\mathbb{E}\{AX + B\} = A\mathbb{E}\{X\} + B
\]  

\[
\text{Var}\{AX + B\} = A^2 \text{Var}\{X\}
\]  

**Demonstration**

\[
\text{Var}\{AX + B\} = \mathbb{E}\{(AX + B)^2\} - (\mathbb{E}\{AX + B\})^2
\]

\[
= \mathbb{E}\{(A^2X^2 + ABX + B^2) - (A\mathbb{E}\{X\} + B)^2\}
\]

\[
= A^2 \mathbb{E}\{X^2\} + AB\mathbb{E}\{X\} + B^2 - (A^2 \mathbb{E}\{X\} + B)^2
\]

\[
= A^2 \mathbb{E}\{X^2\} - A^2 \mathbb{E}\{X\}^2
\]  

\[
\text{Var}\{AX + B\} = A^2 \text{Var}\{X\}
\]  

As the PA-MMSE estimator has complete knowledge of the phase-shift and based on Eqs. (45), (46), the estimated channel coefficient is written as \(\hat{g}_{m,k}^{\text{pa-mmse}} = AX + B\) (i.e., \(A = \frac{\sqrt{q_k} \tilde{p}_{m,k}}{\xi_{m,k}}\), \(X = (y_{m,k}' - \bar{y}_{m,k}')\), and \(B = \tilde{g}_{m,k} e^{jx_{m,k}}\)). Thus, the mean of the estimated channel coefficient is computed as
As the mean of the received channel vector is zero, i.e., \( \mathbb{E}\{X\} = \mathbb{E}\{(y_{m,k}^p - \tilde{y}_{m,k}^p)\} = 0 \).

According to Eq. (46), the variance of the estimated channel coefficient is computed as

\[
\text{Var}\{\hat{g}_{m,k}^{\text{MMSE}} | X_{m,k}\} = \mathbb{E}\{\hat{g}_{m,k}^{\text{MMSE}}\} = 0
\]

\[
\text{Var}\{\hat{g}_{m,k}^{\text{MMSE}} | X_{m,k}\} = A^2 \text{Var}\{X | X_{m,k}\},
\]

\[
= \frac{q_k \beta^2_{m,k}}{\xi^2_{m,k}} \text{Var}\{(y_{m,k}^p - \tilde{y}_{m,k}^p) | X_{m,k}\} = \frac{q_k \beta^2_{m,k}}{\xi^2_{m,k}} \tau_p \xi_{m,k} = \frac{q_k \beta^2_{m,k}}{\xi_{m,k}},
\]

\[
\text{Var}\{\hat{g}_{m,k}^{\text{MMSE}} | X_{m,k}\} = \beta_{m,k} - \xi_{m,k}.
\]

C The LS Estimator’s Derivation

According to Eq. (3), where the CE relies on the received pilot signal, the LS estimator and its corresponding MSE (the MSE determines the variance of the estimation error) are determined in [31] as

\[
\mathbb{E}\{\hat{g}_{m,k}^{\text{LS}}\} = \frac{1}{\sqrt{q_k \tau_p}} \mathbb{E}\{y_{m,k}^p\} = 0
\]

\[
\text{Var}\{\hat{g}_{m,k}^{\text{LS}}\} = \mathbb{E}\{||\hat{g}_{m,k}^{\text{LS}}||^2\} = \frac{1}{q_k \tau_p^2} \mathbb{E}\{||y_{m,k}^p||^2\} = \frac{1}{q_k \tau_p^2} \xi_{m,k}
\]

\[
\mathbb{E}\{\hat{g}_{m,k}^{\text{LS}}\} = \mathbb{E}\left\{g_{m,k} - \frac{1}{\sqrt{q_k \tau_p}} y_{m,k}^p\right\} = 0,
\]

As the mean of the estimation error is zero (i.e., \( \mathbb{E}\{\hat{g}_{m,k}^{\text{LS}}\} = 0 \)), the variance regarding the LS estimator is expressed as

\[
\text{Var}\{\hat{g}_{m,k}^{\text{LS}}\} = \mathbb{E}\{||\hat{g}_{m,k}^{\text{LS}}||^2\}
\]

\[
= \mathbb{E}\{|g_{m,k}|^2\} + \frac{1}{q_k \tau_p^2} \mathbb{E}\{||y_{m,k}^p||^2\} - \frac{1}{q_k \tau_p^2} \left\{\mathbb{E}\{g_{m,k}(y_{m,k}^p)^*\} - \mathbb{E}\{(g_{m,k})^* y_{m,k}^p\}\right\}
\]

\[
= (\beta_{m,k} + \bar{g}^2_{m,k}) + \frac{\xi_{m,k}}{q_k \tau_p} - 2(\beta_{m,k} + \bar{g}^2_{m,k})
\]

\[
= \frac{\xi_{m,k}}{q_k \tau_p} - (\beta_{m,k} + \bar{g}^2_{m,k}) = \frac{\xi_{m,k}}{q_k \tau_p} - \beta_{m,k}^2
\]

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