Infrared Propagators in MAG and Feynman gauge on the lattice

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Abstract

We propose to investigate infrared properties of gluon and ghost propagators related to the so-called Gribov-Zwanziger confinement scenario, originally formulated for Landau and Coulomb gauges, for other gauges as well. We present results of our investigation of SU(2) lattice gauge theory in the maximally Abelian gauge (MAG), focusing on the behavior of propagators in the off-diagonal (i.e. non-Abelian) sector. We also comment on our preliminary results for general linear covariant gauges, in particular for Feynman gauge.

1 Introduction

Important features of quark and gluon confinement in QCD are believed to be closely related to the behavior of gluon and ghost propagators in the infrared limit. One must notice, however, that the study of infrared properties of these propagators must be performed by nonperturbative methods and at a fixed gauge. The Gribov-Zwanziger confinement scenario \cite{1,2} — proposed for Landau and Coulomb gauges — provides predictions for gluon and ghost propagators in the infrared limit, which may be tested by lattice simulations and by nonperturbative analytic methods such as Dyson-Schwinger equations. In particular, a suppressed infrared gluon propagator \( D(p^2) \) is predicted, with \( D(0) = 0 \). The latter statement implies maximal violation of reflection positivity for the gluons, a result that may be viewed as an indication of gluon confinement. (Note that it suffices to have violation of reflection positivity, not necessarily maximal.) At the same time, the infinite-volume limit favors gauge configurations on the boundary region known as the first Gribov horizon, where the smallest nonzero eigenvalue \( \lambda_{\text{min}} \) of the Faddeev-Popov matrix \( \mathcal{M} \) goes to zero. As a consequence, the ghost propagator \( G(p^2) \) — which is obtained from \( \mathcal{M}^{-1} \) — should be infrared-enhanced, introducing long-range effects in the theory. These, in turn, would be responsible for the color-confinement mechanism.

The nonperturbative study of infrared propagators may be carried out from first principles in lattice simulations, taking into account that the true infrared behavior is however obtained only at large enough lattice volumes. Considerable effort has been dedicated to
investigations of the above predictions for the Landau gauge, considering very large lattice sizes (see e.g. [3]). The status of these studies is discussed in [4]. Here we propose to test similar predictions for the propagators as applied to the lattice implementation of other gauges, to try to gain a unified understanding of the mechanism of confinement and its manifestations. We consider the maximally Abelian gauge (MAG) and the linear covariant gauges, in particular Feynman gauge.

In the case of the linear covariant gauges, which include and generalize Landau gauge, some studies suggest that the Gribov-Zwanziger confinement mechanism may apply to the complete class of such gauges [5, 6]. (A recent study of Dyson-Schwinger equations for Feynman gauge has been presented in [7].) On the other hand, for MAG, the usual confinement scenario is based on the concepts of Abelian dominance and of dual superconductivity [8]. Nevertheless, one might argue that a modified Gribov-Zwanziger scenario would likely hold in MAG for the non-Abelian directions in gauge-configuration space. A study of the Yang-Mills Lagrangian restricted to the (MAG) Gribov region by addition of a horizon function with Gribov parameter $\gamma$ has recently been carried out for $SU(2)$ gauge theory in [9]. As pointed out in that reference and also by other groups, the infrared behavior of propagators in MAG may be modified by the presence of ghost and gluon condensates of mass dimension two. An example of such objects is the ghost condensate $v$ [10, 11], related to the breakdown of a global $SL(2, R)$ symmetry. This quantity is expected to modify the symmetric and anti-symmetric components of the (off-diagonal) ghost propagator. In particular, a nonzero value for $v$ corresponds to nonzero anti-symmetric components of the ghost propagator. In Section 2 we present results of our lattice studies of pure $SU(2)$ theory in MAG. (The implementation of gauge fixing for MAG on the lattice is straightforward.) We consider gluon and ghost propagators, the ghost condensate $v$ mentioned above and the smallest eigenvalue of the Fadeev-Popov matrix. Our preliminary results have also been presented in [12] and [13]. We note that the bounds recently introduced for studying gluon and ghost propagators on large lattices in Landau gauge [14, 15] may be written also for other gauges.

Contrary to the case of MAG, the technical aspect of fixing the linear covariant gauges on the lattice is still not a settled issue. We comment on our recent proposals for gauge-fixing methods for these gauges in Section 3.

## 2 Infrared propagators in MAG

On the lattice, for the $SU(2)$ case, the MAG is obtained (see e.g. [16]) by minimizing the functional

$$ S = -\frac{1}{2dV} \sum_{x,\mu} Tr \left[ \sigma_3 U_\mu(x) \sigma_3 U_\mu^d(x) \right]. $$

At any local minimum one has that the Faddeev-Popov matrix, defined as

$$ \sum_{b y} M^{ab}(x, y) \gamma^b(y) = \sum_\mu \gamma^a(x) [V_\mu(x) + V_\mu(x - e_\mu)] + 2 \{ \gamma^a(x - e_\mu) [1 - 2(U_\mu^0(x))^2] 
- 2 \sum_b \gamma^b(x - e_\mu) [\epsilon_{ab} U_\mu^0(x) U_\mu^d(x) + \sum_{cd} \epsilon_{ad} \epsilon_{bc} U_\mu^d(x) U_\mu^c(x)] \}, $$

(2)
is positive-definite. Here the color indices take values 1, 2 and we follow the notation
\( U_\mu(x) = U_\mu^0(x) \mathbb{1} + i \sigma^a U_\mu^a(x) \) and \( V_\mu(x) = (U_\mu^0(x))^2 + (U_\mu^a(x))^2 - (U_\mu^1(x))^2 - (U_\mu^2(x))^2 \),
where \( \sigma^a \) are the 3 Pauli matrices. Notice that (as in Landau gauge \([17]\)) this matrix is
symmetric under the simultaneous exchange of color and space-time indices. Using the
relation \( U_\mu(x) = \exp[-ia_0 A_\mu(x)] \) one finds (in the formal continuum limit \( a \to 0 \)) the
standard continuum results \([18]\) for the stationary conditions above and for \( M^{ab}(x, y) \).

We have considered four values of \( \beta (2.2, 2.3, 2.4, 2.512) \) and lattice volumes up to \( 40^4 \).
Data for a larger lattice, of volume \( 56^4 \), have also been recently produced, but are not fully
analyzed yet. We include data for the ghost propagator at this volume in Fig. 2 below, for
comparison.

Our results for the gluon propagators are in agreement with the study by Bornyakov et
al. \([16]\): we see a clear suppression of the off-diagonal propagators compared to the diagonal
(transverse) one, supporting Abelian dominance. We have fitted our data for the various
 gluon propagators (at all values of \( V \) up to \( 40^4 \) and for \( \beta = 2.2 \)), obtaining the following
behaviors. For \( D(p^2) \) (transverse) diagonal, our data favor a Stingl-Gribov form

\[
D(p^2) = \frac{1 + dp^2}{a + bp^2 + cp^4},
\]
with a mass \( m = \sqrt{a/b} \approx 0.72 \text{GeV} \). Note that the above equation corresponds to
a pair of complex-conjugate poles \( z \) and \( z^* \). We can thus write \( z = x + iy \) with \( x = b/(2c) \approx 0.32 \text{GeV}^2 \) and \( y = \sqrt{a/c - x^2} \approx 0.47 \text{GeV}^2 \). Let us recall that in the case of
a Gribov-like propagator these two poles are purely imaginary. For \( D(p^2) \) transverse off-
diagonal our best fit is of Yukawa type, i.e. \( D(p^2) = 1/(a + b p^2) \), with a mass \( m = \sqrt{a/b} \approx 0.97 \text{GeV} \).
Finally, the longitudinal off-diagonal gluon propagator is best fitted by
\( D(p^2) = 1/(a + b p^2 + c p^4) \) (i.e. also of Yukawa type) with a mass \( m = \sqrt{a/b} \approx 1.25 \text{GeV} \).
As expected from Abelian dominance, the mass is larger in the off-diagonal case.

In Fig. 1 (left) we show our data for the ghost propagator \( G(p^2) \), as a function of an
improved momentum \( p \) (see Ref. \([19]\)). The data show little volume dependence at small
\( p \). (Note that, contrary to Landau gauge, here we can evaluate the ghost propagator at zero momentum.) We see no sign of an enhanced IR propagator. We have fitted our
data (at \( \beta = 2.2 \)), obtaining a behavior of the type \([13]\) above with \( a = 0.45(1) \text{GeV}^2 \),
\( b = 1.1(3) \), \( c = 0.73(30) \text{GeV}^{-2} \), \( d = 2.1(9) \text{GeV}^{-2} \). Thus, we see a Stingl-Gribov fit with
mass \( m \approx 0.6 \text{GeV} \) and complex poles given by \( x \approx 0.75, y \approx 0.22 \).

We next consider (see Fig. 1 right) the smallest eigenvalue \( \lambda_{\text{min}} \) of the Faddeev-Popov
matrix. We have looked at \( \lambda_{\text{min}} \) for several lattice volumes and values of \( \beta \) as a function
of \( 1/L \). The data are fitted to \( a \,(1/L)^b \) with \( b = 1.6(1) \), showing that \( \lambda_{\text{min}} \) vanishes more
slowly than \( (1/L)^2 \) (Laplacian). This may explain why we do not see a diverging ghost
propagator at zero momentum even at rather large lattice volumes \([15]\).

Following the analysis done in Landau gauge \([17]\), we consider the anti-symmetric off-
diagonal ghost propagator \( \langle \mid \epsilon_{ab} G^{ab}(p^2)/2 \mid \rangle \) rescaled by \( L^2/\cos (\pi \tilde{p}_\mu a/L) \), as a function
of the (unimproved) momentum \( p \) for all lattice volumes and \( \beta \) values considered. The
data show nice scaling for all cases considered. The data at \( V = 40^4 \) and \( \beta = 2.2 \) can
be fitted by \( \Phi(p) = (a + b p/L^2)(p^4 + v^2) \) with \( a = 0.0026(7) \text{GeV}^2 \), \( b = 32.6(7) \text{GeV}^{-1} \)
and \( v^2 = 1.7(1) \text{GeV}^4 \). We thus have a rather large ghost condensate \( v \approx 1.3 \text{GeV}^2 \), but
we cannot be sure that it survives in the infinite-volume limit, since the overall constant \( a \)
might be null. We can also fit data at several $V$’s and $\beta$’s for $\Phi(p^2)$ as a function of $p$ and $L$ (see Fig. 2 right). We obtain $\Phi(p) = (a + bp/L^2)(p^4 + v^2)$ with $a = 0.0033(6) GeV^2$, $b = 35.8(5) GeV^{-1}$ and $v^2 = 1.87(8) GeV^4$. We note that the fit parameters change little with the (physical) lattice volume. In fact, data obtained recently for a larger lattice volume, $56^4$, are seen to fall nicely on top of the fit done for the smaller volumes, as seen in Fig. 2 (right).

We have also investigated possible effects of Gribov copies on our results, by considering the difference between our standard gauge fixing (using the stochastic overrelaxation algorithm [20]) and the so-called smearing method [21]. The effects are found to be of the order of the statistical error.

3 Linear covariant gauges

As mentioned in the Introduction, gauge-fixing to linear covariant gauges (other than Landau gauge) on the lattice is a challenge. More precisely, the gauge condition is given by

$$\partial_\mu A_\mu^a(x) = \Lambda^a(x)$$

for real-valued functions $\Lambda^a(x)$. As opposed to the case of Landau gauge — for which $\Lambda^a(x) = 0$ and the gauge is fixed by minimizing a simple functional of the gauge-transformed links — in the general case no such functional exists [22]. The solution to this problem presented in [22], based on the consideration of a modified gauge-fixing condition for the minimizing functional, may be affected by spurious minima and it leads to an altered form of the Faddeev-Popov matrix. We propose to consider a class of gauges on the lattice that coincides with the perturbative definition of linear covariant gauges in the formal continuum limit. Our method is based on a three-step process. Instead of minimizing a functional of $\Lambda^a(x)$ directly, we first fix the gauge to Landau gauge, i.e. the transformed gauge fields.
Figure 2: Left: plot of the quantity $\Phi(p^2) = L^2 / \cos (\pi \tilde{p}_\mu a / L) \left| \epsilon_{\alpha \beta} G^{ab}(p^2) / 2 \right|$ as a function of $p$ for lattice volumes $V = 8^4$, $16^4$, $24^4$, $40^4$ and $\beta = 2.2$. Right: plot of $\Phi(p^2)$ as a function of $p$ and $L$. Data for a larger volume, $56^4$, are also included here.

satisfy $\partial_\mu A^a_\mu(x) = 0$. Then we determine a transformation $\phi^b(x)$ such that

$$A^a_\mu(x) \equiv A^a_\mu(x) + D^{ab}_\mu \phi^b(x) \quad (5)$$

satisfies Eq. (4). Finally, we repeat the procedure for several functions $\Lambda^a(x)$ with a Gaussian distribution of width $\sqrt{\xi}$. The case $\xi = 1$ corresponds to Feynman gauge. The resulting distribution of $\partial A^a_\mu(x)$ is shown for $\xi = 1$ in Fig. 3 in comparison with the original (Gaussian) distribution taken for $\Lambda^a(x)$. We see that the expected distribution is fairly well reproduced.

Our preliminary results were presented in [23]. We are currently investigating an alternative method for fixing these gauges.

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Figure 3: Distribution of $\partial A'^{\alpha}_\mu(x)$ (solid line) compared with a Gaussian of width $\sqrt{\xi}$ (dashed line).

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