Realization of the Dicke model using cavity-assisted Raman transitions

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We realize an open version of the Dicke model by coupling two hyperfine ground states using two cavity-assisted Raman transitions. The interaction due to only one of the couplings is described by the Tavis-Cummings model and we observe a normal mode splitting in the transmission around the dispersively shifted cavity. With both couplings present the dynamics are described by the Dicke model and we measure the onset of superradiant scattering into the cavity above a critical coupling strength.

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Ultracold atoms coupled to a high-finesse optical cavity have become a versatile tool for studying many-body physics in dissipative-driven systems [1]. For example, a theoretical proposal [2] suggested the use of cavity-assisted Raman transitions to realize an open version of the Dicke model [3] in order to study the properties of the associated superradiant phase transition [4, 5]. Following this idea, the process of self-organization [6-8] of a Bose-Einstein condensate coupled to a high-finesse cavity has been mapped to the Dicke model [9] and the corresponding phase-transition has been observed [9]. Together with subsequent work [10-13] this experiment has in turn led to interesting theoretical studies into the properties of the Dicke model [14-19] and the dynamics of a wider class of non-equilibrium models [20-22]. However, mapping to the process of self-organization constrains the range of accessible parameter regimes. In contrast, by implementing the original proposal [2] all parameters of the Dicke model are independently tunable. In addition, the original idea has been extended in ways which allow the study of more complex many-body systems such as the Lipkin-Meshkov-Glick model [23, 24] and spin glasses [25, 26], as well as the effects of modulating the parameters of the Dicke model [27, 28]. Studies of the non equilibrium Dicke models as well as their extensions will profit from the flexibility inherent in the original proposal.

In this letter, we realize an effective Dicke model using two cavity-assisted Raman transitions. Coupling due to one Raman transition alone creates a situation described by the Tavis-Cummings model [30] and we measure the normal mode splitting present in the transmission spectrum of the cavity, which allows us to characterize the effective atom-cavity coupling strength. With the second Raman coupling present, the dynamics are governed by the Dicke model and we observe the onset of superradiant scattering into the cavity above a critical coupling.

Our experimental scheme follows closely the original proposal [2], with a slightly altered level scheme, which was recently considered for the case of a single atom [31]. The details of our experimental setup have been described previously [32]. We trap N rubidium 87 atoms, indicated by the filled ellipses, in an equal mixture of hyperfine states inside a high finesse optical cavity. Two laser beams with Rabi frequencies Ωr and Ωs, indicated by solid arrows, are far detuned from the excited state but near resonant with the cavity-assisted Raman transition, indicated by dashed arrows. (b) With only one beam present, we probe the cavity transmission and observe a normal mode splitting. (c) With both beams present, we observe emission into the cavity above the critical coupling.

FIG. 1. Schematic representation of the experiment. (a) We trap N rubidium 87 atoms, indicated by the filled ellipses, in an equal mixture of hyperfine states inside a high finesse optical cavity. Two laser beams with Rabi frequencies Ωr and Ωs, indicated by solid arrows, are far detuned from the excited state but near resonant with the cavity-assisted Raman transition, indicated by dashed arrows. (b) With only one beam present, we probe the cavity transmission and observe a normal mode splitting. (c) With both beams present, we observe emission into the cavity above the critical coupling.

\[
\begin{align*}
\Delta_s & = \Delta_r = \Omega_p \\
\Delta_s & = \Delta_r = \Omega_r \\
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\end{align*}
\]
tice used to trap the atoms during the experiment has a waist of 70 µm and is actively stabilized to a trap depth of 230 µK. Near 780 nm the interaction of the atoms with the cavity is described in terms of the cavity QED parameters \((g, \kappa, \gamma) = 2\pi \times (1.1, 0.07, 3.0)\) MHz, where 2\(g\) is the single photon Rabi frequency for the \(|F = 2, m_F = 2\rangle\) to \(|F' = 3, m_{F'} = 3\rangle\) cycling transition, and \(\kappa\) and \(\gamma\) are the half-width-half-maximum linewidths of the cavity and atomic transition respectively. Due to birefringence, the two linear polarization modes of the cavity are split by 0.29 MHz. We align the magnetic field to one of the polarizations, so that the two modes couple to \(\pi\) and \(\perp\) transitions involving \(|\Gamma\rangle\), respectively, where both \(\eta\) and \(\zeta\) are small frequency offsets on the order of several megahertz. In this regime, there is a small differential Stark shift

\[
\omega_d \approx \frac{1}{6} \left( \frac{\Omega_r^2}{\Delta_r} + \frac{\Omega_s^2}{\Delta_s} - \frac{\Omega_r^2}{\Delta_r} - \frac{\Omega_s^2}{\Delta_s} \right)
\]

between the \(|1\rangle\) and the \(|0\rangle\) states that must be taken into account for the experiments presented here. In Eq. (1) and throughout this work we follow the convention that Rabi frequencies are calculated from the dipole element for the \(|F = 2, m_F = 2\rangle\) to \(|F' = 3, m_{F'} = 3\rangle\) cycling transition \([33]\).

As all detunings, \(\Delta_r, \Delta_s = (\omega_r - \omega_1) - \omega_s\), and \(\Delta_r - \omega_1\), are much larger then the hyperfine splittings of the excited state manifold \([33]\), the Raman rate for transitions involving \(\perp\) and \(\perp'\) is negligible, as these connect to states with different nuclear spin components. Only two Raman transitions, involving \(\pi\) and \(\perp'\), are near to Raman resonance. The first takes the atom from the \(|0\rangle\) to the \(|1\rangle\) state via absorption of a photon from the laser beam labeled by \(s\) and emission into the cavity (and its reverse), while the second takes the atom from the \(|0\rangle\) to the \(|1\rangle\) state via absorption of a photon from the cavity and emission into the laser beam labeled by \(r\) (and its reverse). We note in particular that the Raman transition taking the atom from the \(|0\rangle\) to the \(|F = 2, m_F = 0\rangle\) state is detuned by \(2\omega_2 \approx 2\pi \times 8.0\) MHz from Raman resonance. Taking into account the relevant transition strengths, the cavity couplings for the two nearly resonant Raman processes are identical.

After performing the adiabatic elimination of the excited states \([2]\) and neglecting any off resonant transitions \([31]\), the system, in a suitable rotating frame defined in the supplemental material, is described by the master equation (\(\hbar = 1\))

\[
\dot{\rho} = -i[H, \rho] + \mathcal{L}\rho \tag{4}
\]

with

\[
H = \omega a^\dagger a + \omega_0 (J_z + \frac{\delta}{N_\lambda} a^\dagger a J_z) + \frac{\lambda_r}{\sqrt{N_\lambda}} (a J_+ + a^\dagger J_-) + \frac{\lambda_s}{\sqrt{N_\lambda}} (a^\dagger J_+ + a J_-), \tag{5}
\]

and

\[
\mathcal{L}\rho = \kappa (2a^\dagger \rho a - a^\dagger a \rho - \rho a^\dagger a) \tag{6}
\]

Here, \(a (a^\dagger)\) is the annihilation (creation) operator for the cavity mode,

\[
J_+ = \sum_{j=1}^{N_\lambda} |1\rangle_j \langle 0|, \quad J_- = \sum_{j=1}^{N_\lambda} |0\rangle_j \langle 1| \quad \text{and} \quad J_z = \frac{1}{2} \sum_{j=1}^{N_\lambda} (|1\rangle_j \langle 1| - |0\rangle_j \langle 0|) \tag{7}
\]

FIG. 2. Normal-mode splitting in the Tavis-Cummings model. For a single coupling beam present, the cavity transmission spectrum shows a normal mode splitting around the dispersively shifted resonance. We vary the atom number for a fixed \(\lambda_r\) and \(\zeta\), record the cavity transmission (\(\langle n\rangle\)) normalized by the empty cavity transmission (\(\langle n_{\text{empty}}\rangle\)) and average traces corresponding to dispersive shifts in 10 kHz bins. Gray lines indicate the bare energies \(\omega_0 = -2\pi \times 12.02\) MHz - \(\zeta\) and \(\omega_0 = \omega_d\). Black lines indicate the eigenenergies of the coupled system for a coupling of \(\lambda_r = 2\pi \times 0.17\) MHz for a dispersive shift of \(\omega_d = -2\pi \times 0.5\) MHz. Transmission in excess of 1 is due to shot noise exceeding the amplitude of the Lorentzian used to fit the empty cavity transmission.
are the collective atomic operators satisfying the commutation relations \([J_+, J_-] = 2J_z\) and \([J_z, J_\pm] = \pm J_\pm\),
\[
\omega = \alpha \frac{1}{3} N \left( \frac{g^2}{\Delta} + \frac{g^2}{\delta} \right) - \alpha \frac{1}{3} N_\lambda \left( \frac{g^2}{\Delta} - \frac{g^2}{\delta} \right) - \eta,
\]
(9)
\[
\omega_0 = \omega_\text{dis} - (\omega_\text{hf} + \zeta - \omega_1),
\]
(10)
\[
\delta = \alpha \frac{2}{3} N_\lambda \left( \frac{g^2}{\Delta} - \frac{g^2}{\delta} \right),
\]
(11)
\[
\lambda_\text{r} = \beta \frac{\sqrt{3}}{12} \sqrt{N_\lambda} g \Omega_\text{r}, \quad \lambda_s = \beta \frac{\sqrt{3}}{12} \sqrt{N_\lambda} g \Omega_s,
\]
(12)
where \(N_\lambda \approx N/3\) is the number of atoms in the coupled states \(|1\rangle\) and \(|0\rangle\), \(N_\lambda = N - N_\lambda\), and \(\alpha \approx 0.06\) and \(\beta \approx 0.78\) are averaging factors taking into account the spatial averaging of the cavity coupling due to thermal motion. The averaging is described in more detail in the supplemental material, together with a derivation of Eq. (5) and an account of how we generate the necessary Raman couplings.

To realize the Dicke model we set \(\Omega_\text{r} = \Omega_s\). For our detuning from the excited state the difference between the two Raman couplings is small, \((\lambda_\text{s} - \lambda_\text{r})/(\lambda_\text{s} + \lambda_\text{r}) \approx 0.028\), so we set \(\lambda_\text{r} \approx \lambda_\text{s} = \lambda\) and arrive at
\[
H = \omega a^\dagger a + \omega_0 J_z + \delta N_\lambda \frac{\lambda}{\sqrt{N_\lambda}} (a + a^\dagger) (J_+ + J_-).
\]
(13)

Similarly, we can describe the situation of a single Raman coupling by setting \(\Omega_\text{s} = 0\) and \(\Omega_\text{r} > 0\). Assuming the atoms remain in the \(F = 1\) hyperfine ground state manifold, we combine \(\omega\) and the constant term proportional to \(\delta\) and arrive at the Tavis-Cummings model [20]
\[
H = (\omega_\text{d} - \eta) a^\dagger a + \omega_0 J_z + \frac{\lambda_\text{r}}{\sqrt{N_\lambda}} \left( a J_+ + a^\dagger J_- \right),
\]
(14)
where
\[
\omega_\text{d} = \alpha \frac{2}{3} N \frac{g^2}{\Delta}.
\]
(15)
is the dispersive shift of the cavity resonance due to \(N\) atoms in the lower hyperfine ground state manifold.

We start our experiments by forming a magneto-optical trap 15 mm above the cavity. At the end of the magneto-optical trapping phase we pump the atoms into the \(F = 1\) hyperfine manifold and load up to \(5 \times 10^6\) atoms into a single-beam optical dipole trap at 1064 nm. The beam forming the dipole trap is moved down by 15 mm over one second by a translation stage. Upon arrival in the cavity, we adiabatically lower the power in the 1064 nm trap and transfer the atoms into the intra-cavity optical lattice. By varying the number of atoms in the magneto-optical trap, we control the number of atoms delivered to the intra-cavity trap, up to a maximum of \(2 \times 10^5\). After loading, we non-destructively determine the atom number by measuring the dispersive shift \(\omega_\text{d}\), with an accuracy of approximately 5 kHz. To do this, we sweep the frequency of a weak probe beam across the dispersively shifted cavity. The Rabi frequency of the probe beam, \(\Omega_\text{p}\), has been adjusted to yield an average intra-cavity photon number on resonance of \(\langle n \rangle \approx 40\), to allow for sufficient signal to noise. After measuring the dispersive shift, we either turn on the Tavis-Cummings coupling and measure the transmission through the cavity, or we ramp up the strength of the Dicke coupling and observe the onset of superradiance by monitoring the scattering into the cavity. Both experiments are described in more detail below. At the end of the experimental sequence we remeasure the dispersive shift to determine atom loss during the experiment and we repeat the cycle. During the experiment, we detect the output light of the cavity which is coupled into a single mode fiber and directed onto a single photon counting module (SPCM).

First we characterize our system by measuring the normal mode splitting present in the Tavis-Cummings model. To do so, we set \(\eta = 0\), \(\zeta \neq 0\) and pulse \(\Omega_\text{r}\) on for 1 ms, with a power of 18(1) mW in the coupling beam. Simultaneously, we pulse the probe beam on for 1 ms and sweep its detuning relative to the empty cavity, \(\Delta_\text{p} = \omega_\text{p} - \omega_\text{r}\), from \(\Delta_\text{p} = -2\pi \times 1.4\) MHz to \(\Delta_\text{p} = -2\pi \times 0.1\) MHz. In the presence of the coupling beam, the system shows an avoided crossing in the transmission spectrum around \(\omega = \omega_0\) and the size of the splitting is given by \(2\lambda_\text{r}\).

Experimentally, we vary the atom number for a fixed \(\zeta\), which changes both \(\omega\) and \(\lambda_\text{r}\), and leads to the transmission spectra shown in Fig. 2. From the normal mode splitting we infer \(\omega_1 + \omega_\text{dis} - \omega_\text{hf} = -2\pi \times 12.02(32)\) MHz and \(\lambda_\text{r} = 2\pi \times 0.173(15)\) MHz at a dispersive shift of \(\omega_\text{d} = -2\pi \times 0.50(1)\) MHz.

A central feature of the Dicke model is a phase transition into a superradiant state once the coupling reaches a critical value, which for \(\omega_0, \omega > 0\) is given by [21]
\[
\lambda_\text{c} = \frac{1}{2} \sqrt{\frac{\omega_0}{\omega - \delta/2}} (\kappa^2 + (\omega - \delta/2)^2),\]
(16)
We note that the critical coupling depends on the power in
the coupling beams because $\omega_0$ varies with the differential stark shift $\omega_{\text{DS}}$. To observe the phase transition for a particular $\eta \neq 0$ and $\zeta \neq 0$, we start with both beams at a low power such that the coupling is well below the critical value and then linearly increase the total power, $P$, over 1 ms from 3.6(2) mW to 36(2) mW. We identify the critical coupling by a rapid increase in the cavity output as shown in Fig. 3. Experimentally, we again vary the atom number, which changes both $\omega$ and $\lambda$, resulting in the observed threshold powers shown in Fig. 4.

The observed thresholds are higher than expected from simple theory, Eq. (16). There are four possible mechanisms for this: thermal motion of the atoms, atom loss during the experiment, spontaneous emission and delays in the onset of super-radiance. Thermal motion has already been taken into account by the averaging factor $\beta$ and does not have a strong dependence on temperature. Atom loss is less then 10% and is thus unlikely to be a significant contributing factor. Spontaneous emission, however, is significant and we estimate a scattering rate of 0.55 ms$^{-1}$ for a power of 18 mW in each Raman beam. We also note that, for a fixed atom number $N_\chi$ and cavity coupling $g$, the Raman coupling $\lambda$ fixes the amount of population in the excited state since $P_c \propto \Omega_1^2/\Delta_2^2 \propto \lambda^2/(N_\chi g^2)$, independent of the chosen detuning from the excited state. Spontaneous emission will tend to depump the atoms from the collective spin state and thus decrease the effective coupling strength. This effect is also present in the Tavis-Cummings model, which we use to infer the effective coupling. Thus, if the power in each of the Raman beams for the threshold measurement was the same as for the splitting measurement, we could expect the same coupling. However, the total power at the beginning (end) of the Dicke experiment is 0.2 (2.0) times the power in Fig. 2. A corresponding decrease (increase) in spontaneous emission leads to a larger (smaller) coupling than what we would have expected based on the ratio of powers. Repeating the measurement of Fig. 2 at 0.2 (2.0) times the power we measure a splitting that deviates by 1.2 (0.89) from that expectation. The blue shaded region in Fig. 4 takes the corresponding uncertainty in estimating the coupling from the splitting measurement into account. There is a remaining discrepancy in the observed thresholds. We believe this is due to delays in the onset of superradiance. When ramping the power in finite time, there is a delay between crossing the threshold and observing light from the cavity. This leads to a systematic overestimation of the threshold. To account for this we require extending previous theoretical frameworks [21] to include spontaneous emission and is an interesting avenue for future research.

In summary, we have demonstrated the ability to use cavity-assisted Raman transitions in order to create tunable atom-photon interactions. We have both studied the Tavis-Cummings and the Dicke model in this setup and shown that the effective cavity and spin frequencies are easily varied. Being able to both weakly probe the system and to dynamically change the couplings put our setup in an ideal situation to investigate the dynamics and steady state properties of the Dicke model and its generalizations. Furthermore, the scheme presented in this work could be easily extended to include additional couplings, more atomic states or additional cavity modes [34], for example by making use of the second birefringent mode present in our setup. In this work we have explored the regime where the effective atom-photon coupling exceeds the cavity decay rate and residual spontaneous emission can not be neglected. Future studies could explore a regime where the critical coupling is much reduced in order to avoid this problem, or add incoherent re-pumping to explore the regime of steady-state superradiance [35].

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Realization of the Dicke model using cavity-assisted Raman transitions: supplemental material

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In this Supplemental Material we give a detailed derivation of the Hamiltonian presented in the main body and how the cavity coupling is spatially averaged. In addition we present how we generate the necessary Raman couplings in the experiment.

I. DERIVATION OF THE DICKE MODEL

The full Hamiltonian, describing the atoms coupled to the two laser beams and the cavity mode, driving transitions in the D2 line, can be written

$$H_{D2} = H_0 + Q + Q^\dagger,$$

where

$$H_0 = \omega_c a^\dagger a + \sum_{j=1}^N \left\{ \sum_{F=1}^2 \sum_{m=-F}^{F} \omega_{F,m} \left| F,m \right\rangle \left\langle F,m \right| + \sum_{F'=0}^{3} \sum_{m=-F'}^{F'} \omega_{F',m} \left| F',m \right\rangle \left\langle F',m \right| \right\},$$

$$Q = \frac{1}{\sqrt{2}} \left( \frac{\Omega_r}{2} e^{-i \omega_r t} + \frac{\Omega_s}{2} e^{-i \omega_s t} \right) \times \sum_{j=1}^N \sum_{F'=0}^{3} \left( A_{F,F'}^{(+1)j} - A_{F,F'}^{(-1)j} \right) + g a \sum_{j=1}^N \sum_{F'=0}^{3} A_{F,F'}^{(0)j}.$$ (2)

Here we have introduced the notation $\left| i, m \right\rangle$ for the state $\left| F = i, m = m \right\rangle$ of atom $j$, and similarly $\left| i', m \right\rangle$ for the excited state $\left| F' = i, m = m \right\rangle$. The ground state frequencies $\omega_{F,m}$ and excited state frequencies $\omega_{F',m}$ are defined relative to the level $|1,1\rangle$, so that $\omega_{1,1} = 0$. Furthermore, we will assume large detunings from the excited states, so that the hyperfine splitting of the excited states is not resolved, and we therefore approximate $\omega_{F',m} = \omega_a$ for all the excited levels.

In Eq. (3) we have introduced atomic raising operators for atom $j$, connecting levels $F$ and $F'$:

$$A_{F,F'}^{(q)j} = \sum_{m=-F}^{F} c(F,m \rightarrow F',m+q) \left| F',m+q \right\rangle \left\langle F,m \right|.$$ (4)

where $c(F,m \rightarrow F',m')$ is the relative strength of the transition $|F,m\rangle \rightarrow |F',m'\rangle$, normalized such that $c(2,2 \rightarrow 3,3) = 1$. Also note that the factor $1/\sqrt{2}$ in Eq. (3) is due the fact that the classical beams couple to $\Delta' = (\sigma^+ - \sigma^-) / \sqrt{2}$ transitions.

We next change to a rotating frame defined by the unitary transformation $U(t) = \exp(-iH_r t)$, with

$$H_r = \frac{1}{2} (\omega_a + \omega_r) a^\dagger a + \frac{1}{2} (\omega_a - \omega_r) \sum_{j=1}^N [2,2]_j \langle 2,2 \rangle$$ (5)

Assuming large detunings,

$$|\Delta_c|, |\Delta_r|, |\Delta_s| \gg |g|, |\Omega_r|, |\Omega_s|, \kappa, \gamma,$$ (6)

where the detunings are defined as

$$\Delta_c = \omega_s - \omega_a \simeq \Delta_r + \omega_2,2,$$

$$\Delta_r = \omega_r + \omega_2,2 - \omega_a \simeq \Delta_c,$$

$$\Delta_0 = \omega_c - \omega_a,$$ (7, 8, 9)

we can adiabatically eliminate the excited atomic states. This leaves us with an effective Hamiltonian coupling different ground states. We will furthermore neglect all off-resonant Raman transitions. Due to the Zeeman shifts induced by the magnetic field, the only near-resonant Raman transitions are those coupling the two levels $|1,1\rangle \equiv |0\rangle$ and $|2,2\rangle \equiv |1\rangle$. The laser beams will induce light shifts to the other ground states as well, but this does not influence the dynamics of the two-level subspace. The other ground states will, however, induce shifts to the cavity frequency, which must be accounted for. This leaves us with the following effective Hamiltonian:
\[ H = \sum_{j=1}^{N} \left\{ \frac{1}{6} \left( \frac{\Omega_s^2}{\Delta_s + \omega_1} + \omega_1 \right) |0\rangle_j \langle 0| + \frac{1}{6} \left( \frac{\Omega_s^2}{\Delta_s + \omega_1} + \Omega_r^2 \right) |1\rangle_j \langle 1| \right. \\
+ \frac{\sqrt{3}}{12} \left( \frac{g\Omega_r}{\Delta_r} |0\rangle_j \langle 1| a^\dagger + \frac{g\Omega_r}{\Delta_s} |1\rangle_j \langle 0| a^\dagger + \text{H.c.} \right) \right\} \\
+ \sum_{j=1}^{N} \frac{2g^2}{3} \left\{ \frac{1}{\Delta_r} ( |0\rangle_j \langle 0| + |1, 0\rangle_j \langle 1, 0| + |1, -1\rangle_j \langle 1, -1| ) \\
+ \frac{1}{\Delta_s} \left( |0\rangle_j \langle 0| + |1, 0\rangle_j \langle 1, 0| + |1, -1\rangle_j \langle 2, -1| + |2, -2\rangle_j \langle 2, -2| \right) a^\dagger a \\
- \eta a^\dagger a - (\zeta + \omega_{hf} - \omega_1) |1\rangle_j \langle 1| \right. \] \\
(10)

Here we have inserted the relevant coefficients \( c(F, m \rightarrow F', m') \), listed in Table (I). The coefficients for the cavity induced light shift to a ground level (\( \pi - \pi \) transition) add up to \( 2/3 \) for any level. We have also introduced the parameters

\[ \eta = \frac{1}{2}(\omega_s + \omega_r) - \omega_c \] \\
(11)

\[ \zeta = \frac{1}{2}(\omega_s - \omega_r) - \omega_{hf} \] \\
(12)

Note that we have chosen to define \( \zeta \) in terms of the hyperfine splitting, \( \omega_{hf} \), as opposed to the Zeeman shifted value \( \omega_1 \), because now \( \zeta \) is a parameter that is set directly in experiment (see Section IV).

At the beginning of each experimental run, the atoms are assumed to be in a mixture with equal population of the \( F = 1 \) ground levels. We can therefore effectively describe the system as consisting of two “species of atoms”:

\[ N_\lambda = N/3 \] atoms in the two-level subspace, \( \{ |0\rangle, |1\rangle \} \), and \( N_\lambda = N - N_\lambda \) atoms in the orthogonal subspace.

To write Eq. (10) on the form of a Dicke model Hamiltonian, we introduce collective atomic operators,

\[ J_+ = \sum_{j=1}^{N_\lambda} |1\rangle_j \langle 0|, \quad J_- = \sum_{j=1}^{N_\lambda} |0\rangle_j \langle 1| \] \\
(13)

\[ J_z = \frac{1}{2} \sum_{j=1}^{N_\lambda} ( |1\rangle_j \langle 1| - |0\rangle_j \langle 0| ) \] \\
(14)

where the index \( j \) now runs only over atoms in the two-level subspace. We also introduce an inversion operator \( J_z' = 1/2 \sum_{j'} ( |2, 1\rangle_{j'} \langle 2, 1| + |2, 0\rangle_{j'} \langle 2, 0| + |2, -1\rangle_{j'} \langle 2, -1| + |2, -2\rangle_{j'} \langle 2, -2| - |1, 0\rangle_{j'} \langle 1, 0| - |1, 1\rangle_{j'} \langle 1, 1| ) \) for the orthogonal subspace, where the index \( j' \) runs over the \( N_\lambda \) that are not in the two-level subspace. We can then rewrite Eq. (10), after dropping constant terms,

\[ H = \frac{1}{2} \left( \frac{\Omega_s^2}{\Delta_s + \omega_1} + \frac{\Omega_r^2}{\Delta_r} - \frac{\Omega_r^2}{\Delta_s - \omega_1} \right) J_z + \frac{\sqrt{3}}{12} \left( g\Omega_r J_- a^\dagger + \frac{g\Omega_s}{\Delta_s} J_+ a^\dagger + \text{H.c.} \right) \] \\
+ \frac{2g^2}{3} \left( \frac{1}{\Delta_s} - \frac{1}{\Delta_r} \right) J_z a^\dagger a + \frac{2g^2}{3} \left( \frac{1}{\Delta_s} - \frac{1}{\Delta_r} \right) J_z'^{\dagger} a^\dagger a \\
+ \frac{g^2}{3} \left( \frac{1}{\Delta_s} + \frac{1}{\Delta_r} \right) N_\lambda a^\dagger a + \frac{g^2}{3} \left( \frac{1}{\Delta_s} + \frac{1}{\Delta_r} \right) N_\lambda a^\dagger a \\
- \eta a^\dagger a - (\zeta + \omega_{hf} - \omega_1) |1\rangle_j \langle 1| \] \\
(15)

Since we assume that the population of the states outside the two-level subspace stays constant, we can replace \( J_z' = -N_\lambda / 2 \). This allows us to write an effective Hamiltonian for the dynamics of the \( N_\lambda \) atoms in the two-level

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TABLE I. The relative transition strengths for the two-level subspace, \( |1, 1\rangle, |2, 2\rangle \) [2].

| \( F' \) | \( m'_p \) | \( m'_p \) | \( c(1, 1 \rightarrow F', m'_p) \) |
| --- | --- | --- | --- |
| 2 | \( \sqrt{\frac{1}{3}} \) | \( -\frac{\sqrt{1}}{2} \) | \( \frac{\sqrt{1}}{2} \) |
| 1 | \( -\sqrt{\frac{2}{3}} \) | \( \sqrt{\frac{2}{3}} \) |
| 0 | \( \frac{\sqrt{1}}{2} \) |
| 3 | \( \sqrt{\frac{1}{2}} \) | \( -\frac{\sqrt{2}}{2} \) | \( \frac{\sqrt{2}}{2} \) |
| 2 | \( -\sqrt{\frac{1}{3}} \) | \( \sqrt{\frac{1}{3}} \) |
| 1 | \( \frac{\sqrt{1}}{2} \) |

superspace:

\[
H = \omega a^\dagger a + \omega_0 J_z + \frac{\delta}{N_\lambda} a^\dagger a J_z
\]

\[
+ \frac{\lambda_r}{\sqrt{N_\lambda}} (a J_+ + a^\dagger J_-) + \frac{\lambda_s}{\sqrt{N_\lambda}} (a^\dagger J_+ + a J_-), \tag{16}
\]

where

\[
\omega = \frac{N}{3} \left( \frac{g^2}{\Delta_s} + \frac{g^2}{\Delta_r} \right) - \frac{N_\lambda}{3} \left( \frac{g^2}{\Delta_s} - \frac{g^2}{\Delta_r} \right) - \eta, \tag{17}
\]

\[
\omega_0 = \frac{1}{6} \left( \frac{\Omega_r^2}{\Delta_s} + \frac{\Omega_s^2}{\Delta_s} - \frac{\Omega_r^2}{\Delta_r} - \frac{\Omega_s^2}{\Delta_r} - \zeta - \omega_1 \right), \tag{18}
\]

\[
\delta = 2 N_\lambda \left( \frac{g^2}{\Delta_s} - \frac{g^2}{\Delta_r} \right), \tag{19}
\]

\[
\lambda_r = \frac{\sqrt{3}}{12} \sqrt{N_\lambda g \Omega_r}, \tag{20}
\]

\[
\lambda_s = \frac{\sqrt{3}}{12} \sqrt{N_\lambda g \Omega_s}. \tag{21}
\]

We remark on some differences between our result and the effective Hamiltonian derived in [1]: 1) There is a contribution to the cavity frequency, \( \omega \), coming from the population of the ground levels outside the two level subspace, 2) we have chosen to scale the \( \delta \) parameter by \( N_\lambda \) to make \( \delta \) directly comparable to \( \omega \) (in [1] this parameter was subsequently assumed to be zero), and 3) there was a sign error in \( \delta \) in [1] that we have corrected.

III. SPATIAL AVERAGING DUE TO THERMAL MOTION

The derivation so far has assumed that all atoms are equally coupled to the cavity. During the experiment the atoms are confined in an intra-cavity lattice at 1556 nm. Because the lattice spacing is very close to the wavelength of the scattered light at 780 nm, the trap sites at the center of the cavity are at anti-nodes of the cavity field [3, 4]. However, due to the thermal spread of the atoms the average coupling is slightly reduced. The relevant averages for processes proportional to \( g^2 \) and \( g \) are [5]

\[
\alpha = \int f^2(r, z) \rho_a(x) \, d^3 x \approx \frac{1 + e^{-4/\epsilon}}{2} + \frac{2}{1 + 2/\epsilon}, \tag{24}
\]

and

\[
\beta = \int f(r, z) \rho_a(x) \, d^3 x \approx \frac{1 - 1/\epsilon}{1 + 1/\epsilon}. \tag{25}
\]

respectively, where, \( f(r, z) = \cos(kz) \exp(-r^2/w^2) \) is the mode function of the cavity, \( \rho_a(x) \) is the atomic density and \( \epsilon \) is the ratio of the trap depth to temperature. In the last step we have used the harmonic approximation and assumed that the atoms a trapped at anti-nodes of the cavity mode [4, 5]. Knowing the trap depth of our intra-cavity lattice, we infer \( \epsilon \) from a temperature measurement done via absorption imaging after ballistic expansion. For our experiments \( \epsilon \approx 7 \) and we arrive at the values stated in the main body.

IV. GENERATION OF RAMAN COUPLING BEAMS

In order to generate the necessary Raman coupling beams we take light resonant with the empty cavity frequency \( \omega_c \), offset it by \( \eta \) via a double pass acousto-optical modulator (AOM), and modulate the phase of
the offset beam at a frequency of $\omega_{hf} + \zeta$ with the help of a broadband electro-optical modulator. The resulting positive and negative sidebands are at frequencies $\omega_s = \omega_c + \eta + \omega_{hf} + \zeta$ and $\omega_r = \omega_c + \eta - \omega_{hf} - \zeta$ respectively. To amplify the available power in the coupling beams we seed a tapered optical amplifier with the light containing the carrier and both sidebands.

To create the coupling desired for the Tavis-Cummings model, we direct the output of the tapered amplifier onto a cavity with a finesse of 1200 and a free spectral range (FSR) of $\text{FSR} = 17.22 \text{ GHz} \approx 3\omega_{hf}$. The cavity is actively locked to the negative sideband and thus only transmits the desired frequency component while strongly suppressing both the carrier and the positive sideband. The output of the cavity passes an AOM and mechanical shutter used for switching. Just before entering the experiment chamber, a calibrated pick-off is used to stabilize the power in the coupling beam by feeding back on the modulation strength of the AOM.

In order to generate the coupling necessary for the Dicke model, the cavity in the output path of the tapered amplifier is replaced by a cavity with the same finesse but with a $\text{FSR} = 2(\omega_{hf} + \zeta)$. This cavity lets through both sidebands and strongly suppresses the carrier. Deriving the Raman beams in this way allows us to set $\eta$ and $\zeta$ independently, and switch the power to either a single Raman coupling beam or both together, as well as to stabilize and ramp their power.

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