Positronium oscillations to Mirror World revisited

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Abstract

We present a calculation of the branching ratio of orthopositronium decay into an invisible mode, which is done in the context of Mirror World models, where ordinary positronium can disappear from our world due to oscillation into its mirror twin. In this revision we clarify some formulas and approximations used previously, correct them at some places, add new effects relevant for a feasible experiment and finally perform a combined analysis. We include into consideration various effects due to external magnetic and electric fields, collisions with cavity walls and scattering off gas atoms in the cavity. Oscillations of the Rydberg positroniums are also considered. To perform a numerical estimates in a realistic case we wrote computer code, which can be adopted in any experimental setup. Its work is illustrated with an example of a planned positronium experiment within the AEgIS project.

1 Introduction

Since discovery of parity violation in particle physics in 1957 [1] its origin in Nature remains a mystery. An idea for how to retain the mirror symmetry based on new particles of opposite
chiralities was put forward long ago \[2\] and developed to the idea of two coexisting parallel Worlds in \[3\] (see also \[4\]), where mirror transformation can be realized as the composition of CP and particle↔mirror particle transformations.

The idea of a Mirror World has a long and interesting history nicely recounted in the review \[5\]. Today the Mirror World model is one of the most attractive possible extensions of the Standard Model of particle physics. This theory is based on the assumption that in addition to the Standard Model its mirror image exists and corresponding new particles interact very weakly with SM fields. Thus, the theory is based on the gauge group \( G \times G' \), where \( G = SU(3) \times SU(2)_L \times U(1)_Y \), \( G' = SU(3)' \times SU(2)'_R \times U(1)'_Y \). As it was pointed out before, originally the deep theoretical motivation for this extension was originally a restoration of the mirror symmetry: \( P \)-transformation changes ordinary left particles to mirror right ones \[2\]. If this symmetry is exact then every particle has its mirror partner (twin) with the same mass and lifetime but is charged under different gauge groups.

Certainly, if mirror particles exist they inevitably interact with ordinary particles gravitationally. But in general these models allow for two kinds of direct renormalizable interactions — portals — between the Standard Model (SM) and the Mirror World particles. One of them, called the Higgs portal \[6\], can be probed in high energy collider experiments (see e.g. Ref. \[7\]). Here we discuss implications of the second type of interactions between SM and Mirror World particles, dubbed as the Abelian portal,

\[
\mathcal{L}_Y = \frac{\epsilon}{2 \cos \theta_W \cos \theta'_W} B'_{\mu\nu} B^{\mu\nu},
\]

where \( B_{\mu\nu} \) and \( \theta_W \) are the \( U(1)_Y \) field strength tensor and the weak mixing angle, respectively, and \( B'_{\mu\nu} \) and \( \theta'_W \) are the same quantities in the mirror sector. Studying of this interaction calls for another sort of experiment.

The most sensitive way to search for the Mirror World in a laboratory is looking for the disappearance of ordinary orthopositronium (oPs) resulting from oscillation to the mirror twin and its subsequent decay to mirror photons. The oscillation becomes possible due to a photon - mirror photon mixing hidden in Eq. (1). Similar processes for bound states of quarks (vector mesons) are much less sensitive to the Abelian portal (1).

A new experiment \[8\] is proposed to search for the oPs disappearance with the sensitivity to branching ratio at the level of \( 10^{-7} - 10^{-8} \). This refers to the value of the mixing parameter entering (1) \( \epsilon \sim 10^{-9} \). Quite remarkably, it is the first time when experimentalists have a chance to enter a region of parameter space which is both phenomenologically viable and interesting for cosmological applications.
Indeed, the mirror matter can play very important role in cosmology and astrophysics (see, e.g. Refs [9, 10, 11]). Mirror matter can behave like dark matter on the astrophysical and cosmological scales. With Mirror symmetry (slightly) violated (say, by different vacuum expectation values of our Higgs field and Mirror Higgs field) the phenomenology becomes more fascinating. Indeed, photon-paraphoton mixing from the Abelian portal (1) implies the presence of millicharged particles: mirror particles effectively carry a tiny charge with respect to electromagnetism. Thus, mirror matter particles can be directly produced in collider experiments [12], be tested with electromagnetic precision measurements [13], change the CMB anisotropy picture due to participation in primordial plasma dynamics at recombination [14], contribute to supernovae explosions [15]. The idea of Mirror World naturally explains the presence of light sterile neutrino(s) at the eV-scale, as suggested by neutrino anomaly announced by the LSND experiment [16] and is now in agreement with the combined analysis of cosmological data [17, 18, 19]. Indeed, since we have sub-eV neutrinos in our sector, which within the SM acquire masses from dimension-5 interaction terms, then mirror (sterile) neutrino masses are of the same order. Mirror symmetry provides similar dimension-5 operators which give masses to mirror neutrinos and may yield mixing with neutrinos from our sector.

In this paper we revise positronium oscillations into its mirror twin. The paper is organized as follows. We start in Section 2 with the discussion of positronium oscillations in a vacuum. Further, in Section 3 we describe the relevant parameters of the proposed experiment [8] (Section 3.1), present the combined analysis of positronium scattering off the gas atoms (Section 3.2) and the cavity walls (Section 3.3), and take into account the interaction with electric and magnetic fields (Section 3.4). Numerical estimates are given for a particular setup of the proposed experiment [8]. Section 4 is devoted to the study of oscillations of the Rydberg (highly excited) positroniums. Section 5 contains conclusions and a discussion of open problems.

2 Warming-up task: orthopositronium oscillations in vacuum

We begin with a simplified picture of vacuum oscillations. Hereafter if not stated otherwise we consider a model with the Mirror World, where the mirror symmetry is exact: values of all mirror masses and mirror coupling constants coincide with those of the corresponding
originals. Positronium couples to its mirror twin due to photon-paraphoton mixing emerging from the Abelian $U(1)_Y$-portal (1),

$$\mathcal{L}_{\text{int}} = \frac{\epsilon}{2} F'_\mu \epsilon \phi_{\epsilon}^\mu,$$

where $F_{\mu\nu}$ and $F'_{\mu\nu}$ are electromagnetic and paraelectromagnetic field strength tensors, respectively. Orthopositronium (oPs) can oscillate to its twin oPs' via virtual photon mixed with virtual paraphoton due to coupling (2). The corresponding transition matrix element reads

$$\langle \text{oPs} | \mathcal{L}_{\text{int}} | \text{oPs'} \rangle = 2\pi\epsilon f = \frac{\delta}{3},$$

where parameter

$$f \approx 8.7 \times 10^4 \text{ MHz}$$

is determined [20] by the one-photon annihilation diagram involving orthopositronium. Experimental constraints on mixing parameter $\epsilon$ have been recently reviewed in Ref. [10]. The first experimental limits on $\epsilon$ from oPs physics were discussed in Refs. [21, 22]. Searches for an invisible decay of orthopositronium have a rather long history [23, 24, 25]. Nonobservation of positronium disappearance $\text{oPs} \rightarrow \text{invisible}$ places a direct limit on the mixing parameter [26]

$$\epsilon < 1.55 \times 10^{-7}.$$

The strongest indirect limit on $\epsilon$ in a model with exact mirror symmetry comes from cosmology [10],

$$\epsilon < 3 \times 10^{-10}.$$  

(4)

It ensures that our and Mirror Worlds have never come to equilibrium in the early Universe; then with somewhat colder mirror plasma the mirror baryons serve as cold dark matter in the Universe. Remarkably, the limit (4) is not far from the estimate [27]

$$\epsilon \sqrt{\xi_{O'}} \approx (7 \pm 3) \times 10^{-10},$$

(5)

required to explain the annual modulation signal, in direct dark matter searches performed by CoGeNT and DAMA experiments. Here $\xi_{O'}$ is a mass fraction of the mirror oxygen in dark matter particles. Strictly speaking, there is no overlap between the two ranges (5) and (4). The gap grows on account of the fact that observation of the matter distribution in the Bullet cluster (1E 0657-558) can be reconciled with the Mirror dark matter only if the largest fraction of dark matter particles are confined in compact macroscopic objects (mirror
asteroids, stars, etc) [28]. Then, the dark matter particle flux should be lower than what was supposed in [27] and hence the required DAMA/CoGeNT signal interval of the mixing parameter is higher than (5). At the same time, the analyses of new data on primordial Nucleosynthesis [29] and cosmic microwave background anisotropy [30] suggest a somewhat higher rate of the Universe expansion which can be explained by a contribution of mirror matter. This relaxes the cosmological limit (4) to some extent.

In the nonsymmetric model, the cosmological bound is generally much weaker. For instance, let the only source of asymmetry be different values of the Higgs boson vacuum expectation values, $\langle H \rangle \neq \langle H' \rangle$. Then in the hierarchical case $\langle H \rangle \ll \langle H' \rangle$ one obtains [10]

$$\epsilon < 3 \times 10^{-9} \times \sqrt{\frac{\langle H' \rangle}{\langle H \rangle}},$$

from the successfulness of the Big Bang Nucleosynthesis.

To summarize the limits discussed above, in what follows we adopt

$$\epsilon = 1 \times 10^{-9} \quad (6)$$

as a reference number for numerical estimates. This particular value is of considerable interest, since it has been argued [31] that the DAMA/LIBRA periodic signal [32] and observations by CoGeNT [33] can be explained within the Mirror World model with a mixing of the order (6) by galactic mirror baryons, while other relevant experiments [34] were blind to this dark matter. In any case, if mirror matter plays the role of dark matter particles, the direct limit on mirror mixing should be of order (6). Moreover, it has been argued that studies of positronium in the proposed experiment [8] will be sensitive right up to the values of mixing parameter. Thus, for the first time the experiment enters into the phenomenologically interesting and cosmologically allowed region of the model parameter space.

For the reference mixing (6) the value of matrix element responsible for oPs $\leftrightarrow$ oPs$'$ oscillations (3) is

$$\delta \approx 1090 \text{ s}^{-1} \approx 7.3 \times 10^{-13} \text{ eV}, \quad (7)$$

which is much lower than the rate of orthopositronium decay (mostly decay to three photons) [35, 36] (see Ref. [37] for more precise and recent theoretical calculations),

$$\gamma \approx 7.040 \times 10^6 \text{ s}^{-1} \approx 4.634 \times 10^{-9} \text{ eV}. \quad (8)$$

Hereafter we work in the "flavor" basis: the evolution of the system oPs-oPs$'$ in a vacuum can be described by the Schrödinger equation for the doublet wave function $\Psi =$
where the Hamiltonian in the positronium rest frame accounts for the positronium and mirror positronium decays and their mutual oscillations,

\[
H = \begin{pmatrix}
E - i\gamma/2 & \delta/2 \\
\delta/2 & E - i\gamma/2
\end{pmatrix},
\]

where \( E \) is the kinetic energy of oPs and oPs'. When solving Equation (9) for the wave function one obtains the probability to have mirror positronium instead of initial positronium by the elapsed time \( t \),

\[
P(\text{oPs} \rightarrow \text{oPs}') = e^{-\gamma t} \sin^2 \frac{t\delta}{2}.
\]

Mirror orthopositronium decays mainly into three mirror photons and the probability of oPs disappearance is given by [38]

\[
\text{Br}(\text{oPs} \rightarrow \text{invisible}) = \gamma \int_0^\infty P(\text{oPs} \rightarrow \text{oPs}') dt = \frac{\delta^2}{2} \frac{1}{\gamma^2 + \delta^2} \approx \frac{\delta^2}{2\gamma^2},
\]

since \( \delta \ll \gamma \), cf. Eqs. (7) and (8).

3 Realistic consideration

3.1 Realistic experimental setup

We proceed with the study of complications in the positronium description arisen in a real experiment. Indeed, the realistic case of positronium oscillations is much more involved. In our analysis we consider the general setup of experiments proposed in Ref. [39] and then further developed in Ref. [8] with more reliable estimates of the sensitivity to the mixing parameter.

In the experiment [8] oPs states will be formed in a thin nanoporous SiO\textsubscript{2} target placed on the bottom of the vacuum cavity (10 cm in diameter and 10 cm in height). After the formation a significant fraction of the oPs can escape inside the cavity and can become almost completely thermalized with the average temperature equal to the temperature of the target [35]. The cavity is surrounded by an almost hermetic 4\( \pi \) electromagnetic calorimeter to detect annihilation photons. The experimental signature of the oPs-oPs' oscillations is the absence of energy deposition in the calorimeter expected from the ordinary positronium
decays. The typical residual gas pressure is above $10^{-5}$ Torr. There is an optional external electric field $E \lesssim 100$ kV/cm and magnetic field $B \lesssim 10^3$ G (exceptionally up to $B \lesssim 10^5$ G).

Note that the experiment [8] can also be preformed, e.g., in the framework of the AEgIS project (Antihydrogen Experiment: Gravity, Interferometry, Spectroscopy) at CERN [40, 41]. The main goal of AEgIS experiment is a mass production of antihydrogen. In this experiment orthopositroniums are a source of slow positrons that recombined with antiprotons. There is a proposed technology of adopting Rydberg (highly excited) positroniums instead of the usual ground-level orthopositroniums: the former live longer and positroniums move slower inside. Thus, in the AEgIS experiment, not only ground states will by confined within the cavity, but also excited oPs with the Rydberg numbers $n \lesssim 30$.

### 3.2 Oscillations in gas

Let us consider the role of positronium interaction with gas. Generally, the time-evolution of the system oPs-oPs$'$ is described by density matrix

$$
\rho(t) = \int d^3 x \, \Psi \Psi^\dagger = \int d^3 x \begin{pmatrix} \psi_{oPs} \psi_{oPs}^* & \psi_{oPs}^* \psi_{oPs}' \\ \psi_{oPs} \psi_{oPs}' & \psi_{oPs}' \psi_{oPs}'^* \end{pmatrix} .
$$

(13)

The density matrix (13) solves the following equation [38]

$$
\frac{d \rho}{dt} = -i \mathcal{H} \rho + i \rho \mathcal{H}^\dagger + 2 \pi n v \int d \cos \theta \, F(\theta) \rho F^\dagger(\theta) ,
$$

(14)

with the Hamiltonian, cf. (10),

$$
\mathcal{H} = \begin{pmatrix} -\frac{2\pi}{k} n v f(0) + E - i \gamma/2 & \delta/2 \\ \delta/2 & E' - i \gamma'/2 \end{pmatrix} ,
$$

(15)

where $E$ and $E'$ are the positronium and mirror positronium energies, $\gamma$ and $\gamma'$ are their widths\(^1\), $k$ refers to the value of the positronium 3-momentum, and $v$ is the mean relative velocity between the positronium and gas molecules. The positronium scattering off gas is described by the matrix

$$
F(\theta) = \begin{pmatrix} f(\theta) & 0 \\ 0 & 0 \end{pmatrix} ,
$$

(16)

\(^1\)Different notations for oPs and oPs$'$ are used ($E$, $E'$ for energies and $\gamma$, $\gamma'$ for widths) because corresponding quantities can differ from one another in external electromagnetic field and for the case of broken mirror symmetry.
where $\theta$ is a scattering angle; in the cavity *mirror positronium* does not scatter off *ordinary matter* given the small mixing (6). The first term in the Hamiltonian (15) accounts for the presence of gas in the system, which changes the energy of the positronium state. There $n$ stands for the number density of gas particles, so that the frequency of the positronium scatterings off gas $w$ is determined by the cross section $\sigma$ or (with help of the optical theorem) by the forward scattering amplitude $f(0)$ as

$$w \equiv \sigma n v = \frac{4\pi}{k} n v \text{Im} f(0).$$  \hfill (17)

In what follows, we resort to the case of slow scatterings, where the scattering amplitude does not actually depend on the scattering angle $\theta$: *i.e.* scattering is saturated in $s$-wave. The real part of the scattering amplitude is positive\(^2\) and by analogy with Eq. (17), can be conveniently parametrized as

$$w_{\text{Re}} \equiv \frac{4\pi}{k} n v \text{Re} f(0).$$ \hfill (18)

In this case the problem is solved analytically as follows. Once positronium is produced (at $t = 0$), the system is in the pure flavor state,

$$\Psi(0) = \left( \begin{array}{c} 1 \\ 0 \end{array} \right), \quad \text{and hence} \quad \rho(0) = \left( \begin{array}{cc} 1 & 0 \\ 0 & 0 \end{array} \right).$$ \hfill (19)

We parametrize the density matrix at arbitrary moment as

$$\rho \equiv \left( \begin{array}{cc} \rho_1 & x + iy \\ x - iy & \rho_2 \end{array} \right),$$ \hfill (20)

then Eq. (14) yields

$$\frac{d\rho_1}{dt} = -\gamma \rho_1 - \delta y$$ \hfill (21a)

$$\frac{d\rho_2}{dt} = \delta y - \gamma' \rho_2$$ \hfill (21b)

$$\frac{dy}{dt} = \frac{\delta}{2} (\rho_1 - \rho_2) + \left( \frac{w_{\text{Re}}}{2} - \Delta \right) x - \frac{1}{2} (\gamma + \gamma' + w) y$$ \hfill (21c)

$$\frac{dx}{dt} = -\frac{1}{2} (\gamma + \gamma' + w) x + \left( \Delta - \frac{w_{\text{Re}}}{2} \right) y$$ \hfill (21d)

where $\Delta \equiv E' - E$.\(^2\)

\(^2\)At large distances $r$ positronium is attracted by an atom, *i.e.*, it propagates in the negative potential $U(r) < 0$, hence in the Born approximation for nonrelativistic particles we obtain positive value for the amplitude (see, *e.g.* [42]), $f \approx -2m \int_0^\infty U(r)r^2 dr > 0$. 

8
We are interested in the total probability for the orthopositronium to disappear, which under study reads
\[
Br \left( \text{oPs} \rightarrow \text{invisible} \right) = \frac{\int_0^\infty \rho_2 dt}{\int_0^\infty \rho_1 dt + \int_0^\infty \rho_2 dt}. \tag{22}
\]
One can integrate the system (21), with the initial condition (19), and introducing notations
\[
Y \equiv \int_0^\infty y dt, \quad X \equiv \int_0^\infty x dt, \quad P_i \equiv \int_0^\infty \rho_i dt, \quad i = 1, 2,
\]
arive at a linear system of algebraic equations:
\[
\begin{align*}
-\gamma P_1 - \delta Y & = -1 \\
\delta Y - \gamma' P_2 & = 0 \\
\frac{\delta}{2} (P_1 - P_2) + \left(\frac{w \text{Re}}{2} - \Delta\right) X - \frac{1}{2} (\gamma + \gamma' + w) Y & = 0 \\
-\frac{1}{2} (\gamma + \gamma' + w) X + \left(\Delta - \frac{w \text{Re}}{2}\right) Y & = 0
\end{align*} \tag{23}
\]
Then the oscillation probability (or probability to disappear) (22) is given by
\[
Br \left( \text{oPs} \rightarrow \text{invisible} \right) = \frac{P_2}{P_1 + P_2}. \tag{24}
\]
Let us introduce the following notations:
\[
\Gamma \equiv \frac{1}{2} (\gamma + \gamma' + w), \quad \Delta_{\text{Re}} \equiv \Delta - \frac{w \text{Re}}{2}. \tag{25}
\]
Taking into account the hierarchy \(\delta \ll \Gamma\), the solution of the system (23) can be simplified and we obtain \(P_1 + P_2 = \frac{1}{\gamma} \left(1 + \mathcal{O}\left(\frac{\delta^2}{\Gamma^2}\right)\right)\) and for the probability [38],
\[
Br \left( \text{oPs} \rightarrow \text{invisible} \right) = \frac{\delta^2 \Gamma}{2 \gamma' \Gamma^2 + \Delta_{\text{Re}}^2} \left(1 + \mathcal{O}\left(\frac{\delta^2}{\Gamma^2}\right)\right). \tag{26}
\]
Note that in the “flavor” basis, the case of an exact mirror symmetry implies equal energies and widths of positronium and its twin in the absence of external electromagnetic fields, hence \(\Delta = 0\) and \(\gamma' = \gamma\).

For the numerical estimate, we apply the obtained formula to a particular case of the experiment described in Section 3.1. Here the cavity is proposed to be filled with nitrogen \(N_2\) at room temperature \(T_0\), so that
\[
k_B T_0 = 0.025 \text{ eV}. \tag{27}
\]
The momentum transfer cross section of positronium on nitrogen was measured in Ref. [43] as follows:

\[ \sigma_m \equiv \int (1 - \cos \theta) \frac{d\sigma}{d\Omega} d\Omega = (35 \pm 8) \times 10^{-16} \text{cm}^2, \] (28)

and remains almost constant with temperature varying at \( k_B T_0 < 0.3 \text{eV} \). And if scattering amplitude does not depend on scattering angle we obtain for the total cross section after integrating over the angle variables \( \sigma \equiv \int \frac{d\sigma}{d\Omega} d\Omega \approx \sigma_m \). In accordance with the feasible experimental setup [8] we assume that orthopositroniums are emitted from the target with a temperature of \( T_{\text{oPs}} \sim T_0 \), close to the nitrogen temperature (27). Thus, the positronium average momentum is \( k = m_{\text{oPs}} v_{\text{oPs}} \) where positronium average velocity equals

\[ v_{\text{oPs}} = \sqrt{\frac{3k_B T_{\text{oPs}}}{m_{\text{oPs}}}} = 8.1 \times 10^6 \text{cm s}^{-1} \times \sqrt{\frac{k_B T_{\text{oPs}}}{0.025 \text{eV}}}. \] (29)

In principle, there may be processes of pick-off annihilation and ortho-to-parapositronium conversion. But the corresponding probabilities are negligibly small (see, e.g. Ref. [44]) at the positronium energies and the gas pressures relevant for the class of experiments in question [8].

In what follows, we set \( T_{\text{oPs}} = T_0 \). One observes, that \( v_{\text{oPs}} \) is much higher than the velocity of nitrogen atoms, so one can replace \( v \) in Eq. (17) with \( v_{\text{oPs}} \) in order to estimate the collision frequency and, in particular, to check that for the cross section (28), the slow scattering approximation is valid indeed and the scattering amplitude does not depend on the scattering angle as we have assumed. For corresponding energies of colliding particles the cross section is mostly saturated by the real part of the amplitude, so

\[ |\text{Re} f(0)| \approx \sqrt{\sigma/4\pi}. \]

The nitrogen pressure \( P_{N_2} \) is related to the nitrogen number density \( n_{N_2} \) by \( P_{N_2} = n_{N_2} k_B T_{N_2} \). Obviously, the presence of gas suppresses oscillations (see Eq. (26)), if its density is high enough for positronium to scatter off atoms at least once. This effect can be understood as the loss of coherence due to a different angular distribution of the positronium and its mirror twin after collision, which is encoded in Eq. (16) (see also Ref. [45] for discussion of similar effects). For the reference temperature (27) this implies a pressure \( P_{N_2} \) of order \( 10^{-3} \) Torr or higher. We put all the formulas above into Eq. (26) with the reference value of the mixing parameter (6). When introducing dimensionless variables \( \mathcal{P} \equiv P_{N_2} / (10^{-3} \text{ Torr}) \) and \( \mathcal{T} \equiv k_B T_{N_2} / (0.025 \text{ eV}) \) we write for the invisible branching ratio

\[ \text{Br (oPs \rightarrow invisible)} = 1.2 \times 10^{-8} \times \left( \frac{\epsilon}{10^{-9}} \right)^2 \times \frac{1 + 0.067 \times \mathcal{P}/\mathcal{T}^{1/2}}{(1 + 0.067 \times \mathcal{P}/\mathcal{T}^{1/2})^2 + (0.29 \times \mathcal{P}/\mathcal{T})^2}. \] (30)
This dependence can be used [21] to check against the possible systematics once the signal is found.

### 3.3 Oscillations in a finite volume

In a laboratory orthopositronium is trapped in a cavity. Then if positronium velocities are high enough to reach the walls before decay, the interaction with wall material modifies the (possible) oscillations into mirror positronium. The reason is simple: positronium interacts with walls, while its twin does not and if produced in a cavity flies away freely.

Let us use the instant approximation for the positronium interaction with a wall. Then, between the scatterings off the wall, the oscillations proceed as in an infinite volume case, but when positronium hits the wall, nondiagonal elements of the density matrix (13) nullify. The oscillations stop at this point, since the mirror positronium flies away, while the positronium gets reflected, and hence their wave functions get separated in space, and the coherence gets lost. This is the main observation.

To account for this instant decoherence properly, let us consider the oscillations of a stable positronium. Then, the evolution of the density matrix elements is described by the following equations:

\[
\begin{align*}
\frac{d\rho_1}{dt} &= -\delta y \\
\frac{d\rho_2}{dt} &= \delta y \\
\frac{dy}{dt} &= \frac{\delta}{2} (\rho_1 - \rho_2)
\end{align*}
\]

and \( x \) remains zero with initial condition (19), which is the pure positronium state.

It is convenient to introduce the variable \( s \equiv \rho_1 - \rho_2 \) and parametrize the time interval between \( i-1 \)-th and \( i \)-th reflections as \( \tau_i \); the system starts to evolve at \( t = 0 \) being in the positronium state (19). Then before the first reflection the solution of the system (31) is

\[
t < \tau_1 : \quad s = \cos t\delta , \quad y = \frac{1}{2} \sin t\delta .
\]

At \( t = \tau_1 \) we nullify \( y(\tau_1) = 0 \), but the diagonal variable remains intact, \( s(\tau_1) = \cos \tau_1\delta \). Then, the evolution of the system between the first and second reflections proceeds as follows,

\[
\tau_1 < t < \tau_1 + \tau_2 : \quad s = \cos \tau_1\delta \cos (t - \tau_1)\delta , \quad y = \frac{1}{2} \cos \tau_1\delta \sin (t - \tau_1)\delta .
\]
It is straightforward to see that right after the \( n \)-th reflection the solution reads

\[
 s(\tau_n) = \prod_{k=1}^{n} \cos \tau_k \delta, \quad y(\tau_n) = 0.
\]

When introducing the average time between reflections

\[
 \langle \tau \rangle \equiv \frac{t}{n}
\]

one finds that the positronium velocity (29) and the typical size \( L \sim 10 \text{ cm} \) of the detector volume are related as \( \langle \tau \rangle \simeq L/v_{\text{OPS}} \). For the reference value of the oscillation rate (7), the average reflection rate obeys

\[
 \langle \tau \rangle \delta \ll 1.
\]

Then we can use the exponential approximation to obtain the smooth solution

\[
 s \approx \exp \left( \sum_{k=1}^{n} \log \left( 1 - \frac{(\tau_k \delta)^2}{2} \right) \right) \approx \exp \left( -\frac{\delta^2 n}{2} \frac{1}{n} \sum_{k=1}^{n} \tau_k^2 \right) = \exp \left( -\frac{\delta^2 \langle \tau^2 \rangle}{2 \langle \tau \rangle} t \right) \tag{32}
\]

Thus, collisions with walls result in the exponential suppression of oscillations.

One can mimic the suppression (32) by introducing an additional suppression to the Equation (31c),

\[
 \frac{dy}{dt} = \frac{\delta}{2} s - wy \tag{33}
\]

Then, at \( \delta/w \ll 1 \) one has

\[
 s = \exp \left( -\frac{\delta^2 t}{w} \right),
\]

and matching (32) is achieved with

\[
 w = \frac{2 \langle \tau \rangle}{\langle \tau^2 \rangle}. \tag{34}
\]

Thus, comparing (33) to (21c) one accounts for the collisions with walls by replacing (see (25))

\[
 \Gamma \to \Gamma + w
\]

in the formula for oscillations (26).

To support our approach, let us consider a limiting case when the time intervals between the collisions with the walls are all equal, \( \tau_{k+1} - \tau_k = \tau \), then \( \langle \tau^2 \rangle^{1/2} = \langle \tau \rangle = \tau \) and one obtains for the oscillation probability

\[
 \rho_2 = \frac{1}{2} \left( 1 - e^{-\frac{\delta^2 \tau t}{\tau}} \right) \approx \frac{1}{4} \delta^2 \tau t. \tag{35}
\]
This result is in agreement with Refs. [46, 47] for the neutron-antineutron and neutron-mirror
neutron systems where any neutron collides for many times. On the contrary, the obtained
result deviates from a similar one in Ref. [8] where $w = 1/\langle \tau \rangle$ instead of (34), hence it is
two times smaller. The origin of this term in the corresponding equations of Ref. [8] was not
explained in that paper. One can obtain this answer with our method provided the random
variable $\tau$ is distributed in accordance with the Poisson statistics, which we find unjustified
when the positronium trajectories are reasonably tractable.

However, in general case, the wall collision rate is not a quantity which can be well-
de fined in any real experiment. For example, the authors of Ref. [8] argue that the number
of collisions in the proposed experiment is of order unity, i.e. $\tau \gamma \sim 1$. In this case, the
gas pressure, the geometry of cavity, and the properties of the oPs source (like energy and
angle distributions) become important and the wall collision rate becomes a largely uncertain
quantity (e.g., it depends on the direction of positronium propagation, etc.).

To take into account all those effects and make accurate predictions for the probability
of positronium disappearance, a numerical calculation should be performed. We apply the
Monte-Carlo approach to simulate the evolution of the oPs-oPs’ system in the following way.
The density matrix is calculated numerically by solving Eq. (21) with initial conditions (19).
We use the geometry of the positronium cavity described in Section 3.1. In our numerical
simulations, we suppose that the positroniums in the initial state have isotropic nonrela-
tivistic Maxwell velocity distribution with a temperature of $T_{oPs}$ (equal to the temperature
of gas), which is reasonably close to the realistic distribution observed in Ref. [35]. When
collision with the wall takes place we change the density matrix in the following way
\begin{equation}
\rho \rightarrow \begin{pmatrix}
\rho_{11} & 0 \\
0 & 0
\end{pmatrix},
\end{equation}
which corresponds to total disappearance of mirror positronium from the cavity. The deco-
herence due to gas collisions is already taken into account in Eq. (21). However, collisions
with gas also change the direction of positronium motion. So, we simulate time intervals
between the collisions according to its average value (17): the probability of the positronium
experiencing an interaction in small time interval $dt$ equals $P_{int} = \rho_{11} (1 - e^{-wdt})$. After the
collision with a gas atom, the direction of the positronium velocity changes according to the
isotropic distribution. We neglect small changes in the absolute velocity in the scattering off
gas atoms. To get feeling of the influence of wall collision let us consider the same topology
of experiment cavity as it is described in Section 3.1 but with different absolute size, i.e.
we take vacuum cavity of $L$ cm in diameter and $L$ cm in height. In Fig. 1 we present the
Figure 1: The dependence of the branching ratio on the characteristic size of experiment cavity $L$ at different values of gas pressure. We chose $\epsilon = 10^{-9}, k_B T_{\text{Ps}} = 0.025$ eV.

branching ratio dependence on the characteristic size $L$ of cavity where orthopositroniums decay.

The dependencies of the branching ratio of orthopositronium disappearance on gas temperature and gas pressure are presented in Fig. 2. To see clearly the effect of wall collisions we also plot the pressure dependencies obtained with Eq. (30) for comparison. Obviously, the cavity finite size effect as well as the gas effect are always in a reduction of positronium disappearance branching ratio: collisions prevent positronium from oscillations to its twin.

3.4 Oscillations in static electric and magnetic fields: Zeeman and Stark effects

It is well known that in an external static magnetic field $B$ the state of orthopositronium with $m = 0$ mixes with parapositronium [48, 49] and these states show Zeeman shifts in
Figure 2: The dependence of the branching ratio of the positronium disappearance on gas (here is nitrogen) pressure (left panel) and gas temperature (right panel) with (points) and without (solid lines, Eq. (30)) wall collisions. The positronium temperature equals nitrogen temperature and we set $\epsilon = 10^{-9}$. Note that $\text{Br(oPs} \rightarrow \text{invisible})$ scales as $\epsilon^2$. 

their energies and decay rates$^3$

$$
\Delta_\pm = \frac{1}{2} \left( -\Delta_{HFS} \pm \sqrt{\Delta_{HFS}^2 + (4\mu_0 B)^2} \right),
$$

$$
\Gamma_\pm = \frac{1}{2} \left( \gamma + \gamma_P \pm \frac{\gamma - \gamma_P}{\sqrt{\Delta_{HFS}^2 + (4\mu_0 B)^2}} \right),
$$

where $\Delta_{HFS} \approx 8.4 \times 10^{-4}$ eV is hyperfine energy splitting $^{[50]}$, $\gamma_P \approx 8.1 \times 10^9$ s$^{-1}$ is parapositronium decay rate and $\mu_0 \approx 5.79 \times 10^{-5}$ eV/T is the Bohr magneton. Here the conditions $\gamma, \gamma_P \ll \Delta_{HFS}$ were used and we neglected radiative corrections to gyromagnetic ratios of electron and positron.

If the magnetic field is strong, i.e. $4\mu_0 B \gg \Delta_{HFS}$ which corresponds to $B \gg 3.6$ T, then $\Gamma_\pm \approx \frac{1}{2} (\gamma + \gamma_P) \gg \gamma$. Hence, the state of orthopositronium with $m = 0$ quickly decays and its contribution to oPs–oPs$'$ oscillations is negligibly small. When the magnetic field is weak, $4\mu_0 B \ll \Delta_{HFS}$, the shifts in the energy levels and decay rates read

$$
\Delta_+ \approx \frac{(2\mu_0 B)^2}{\Delta_{HFS}}, \quad \Delta_- \approx -\Delta_{HFS} - \frac{(2\mu_0 B)^2}{\Delta_{HFS}},
$$

$$
\Gamma_+ \approx \left[ 1 - \left( \frac{2\mu_0 B}{\Delta_{HFS}} \right)^2 \right] \gamma + \left( \frac{2\mu_0 B}{\Delta_{HFS}} \right)^2 \gamma_P, \quad \Gamma_- \approx \left( \frac{2\mu_0 B}{\Delta_{HFS}} \right)^2 \gamma + \left[ 1 - \left( \frac{2\mu_0 B}{\Delta_{HFS}} \right)^2 \right] \gamma_P.
$$

$^3$In the limit $B \to 0$ the state “+” goes to the orthopositronium state while “−” to parapositronium.
Thus, when the weak magnetic field is applied the oPs state becomes a superposition of “+” and “−” states of which “−” decays quickly, while “+” is approximately $m = 0$ state of the orthopositronium which acquires shifts in its energy and decay rate. Numerically we estimate

$$\Delta_+ \approx 2.5 \times 10^6 \text{ s}^{-1} \times \left( \frac{B}{100 \text{ G}} \right)^2.$$  (39)

The mixing between new state and the mirror twin of $m = 0$ state of the orthopositronium also gets corrections of order $(\mu_0 B/\Delta_{HFS})^2$ which can be neglected.

There is also the quadratic Zeeman effect in positronium [51]. The corresponding energy shift

$$\Delta_D = 2\alpha_0^2 \alpha_0^3 B^2 \approx 10^4 \text{ s}^{-1} \times \left( \frac{B}{100 \text{ G}} \right)^2,$$  (40)

where $a_0 \approx 0.1 \text{ nm}$ is the Bohr radius, applies equally to all four spin states of the lowest energy and can be considerable for large magnetic field.

In an external static electric field $E$ all three orthopositronium states get shifted equally due to the Stark effect [42],

$$\Delta_S = -\frac{1}{2}\alpha_0 \epsilon_0 E^2 = -1.85 \times 10^{-7} \text{ eV} \times \left( \frac{E}{100 \text{ kV/cm}} \right)^2 \approx -2.8 \times 10^8 \text{ s}^{-1} \times \left( \frac{E}{100 \text{ kV/cm}} \right)^2,$$  (41)

where $\alpha_0 = \frac{9}{2} 4\pi a_0^3$ determines polarizability of orthopositronium, and $\epsilon_0$ refers to the free space permittivity. With a reasonable field strength available in a laboratory (in particular, in the AEGIS experiment), $E \sim 100 \text{ kV/cm}$, the energy splitting due to the Stark effect (41) can be considerable.

Let us summarize the effect of the external electromagnetic field. For large magnetic field, $m = 0$ state immediately decays and does not contribute to the oscillations. Hence, in case of unpolarized orthopositroniums this results in decreasing of total oscillation probability by a factor $2/3$. This behavior gives a tool to check for possible systematics in the experiment, where magnetic fields as large as $10 \text{ T}$ are available (as, in particular, in the experiment [8]), which is quite useful especially if evidence for the oscillations is found. In the weak magnetic and electric fields the $m = 0$ oPs state acquires total energy shift

$$\Delta_0 = \Delta_+ + \Delta_D + \Delta_S,$$  (42)

and its decay width becomes $\gamma_0 = \Gamma_+$. The oPs states with $m = \pm 1$ get energy shift

$$\Delta_1 = \Delta_D + \Delta_S,$$  (43)
while their decay rates remain intact.

Averaging the disappearance probability over states with different angular momentum components

$$
Br \ (oPs \rightarrow \ invisible) = \frac{\int_0^\infty \rho_2^{m=0} dt + 2 \int_0^\infty \rho_2^{m=\pm1} dt}{\int_0^\infty (\rho_1^{m=0} + 2\rho_1^{m=\pm1} + \rho_2^{m=0} + 2\rho_2^{m=\pm1}) dt}, 
$$

one obtains

$$
Br \ (oPs \rightarrow \ invisible) = \frac{\delta^2}{2\gamma(1 + 2\Gamma_+)} \left( \frac{\Gamma_0}{\Delta_0^2 + \Gamma_0^2} + 2 \frac{\Gamma_+}{\gamma} \frac{\Gamma}{\Delta_1^2 + \Gamma^2} \right), 
$$

where $\Gamma_0 = \frac{1}{2}(\Gamma_+ + \gamma + w)$.

The results for invisible branching ratio of the orthopositronium in the presence of electric and magnetic fields with different values of gas pressure are presented in Figures 3 and 4, respectively. Here we also include the effect of the wall collisions in the geometry described

![Figure 3](image_url)

Figure 3: The dependence of the branching ratio of the positronium disappearance on the external electric field $E$ at different values of gas pressure. We chose $\epsilon = 10^{-9}, k_B T_{oPs} = 0.025$ eV and the magnetic field $B = 0$.

in Section 3.1. We see that generically presence of an external electromagnetic field leads to
Figure 4: The dependence of the branching ratio of the positronium disappearance on the magnetic field $B$ at different values of gas pressure. We chose $\epsilon = 10^{-9}$, $k_B T_{\text{op}} = 0.025$ eV and the electric field $E = 0$.

decreasing of the oscillation rate and thus to deterioration of the sensitivity of experiment to the Mirror World physics. Some nontrivial dependence on the magnetic field stems from the fact that both Zeeman effect contributions to the energy splitting have the sign which is opposite to the contribution of the coherent scattering (15). As we have already mentioned the dependence on the external parameters like electric or magnetic fields can be used to get rid of systematic uncertainties or (in case of positive signal) to get evidence that the observed effect is due to the oscillation nature of positronium disappearance rather than direct decay into some invisible particles.

Let us note in passing, that in a model with broken mirror symmetry where the positronium and its mirror twin obtain mass splitting larger than orthopositronium width, oscillations are absent because of the loss of coherence, and the branching ratio of an invisible mode is strongly suppressed (see Eq. (26)),

$$\text{Br}(\text{op} \rightarrow \text{invisible}) \simeq \frac{\delta^2 \Gamma}{2 \gamma \Delta^2} \frac{1}{\Delta^2}.$$  

However, with magnetic field tuned to cancel this mass splitting by using the Zeeman effect,
one still has a possibility of the Mirror World hunting via the positronium portal.

4 Rydberg states and models with vacuum mass splitting between twins

Let us proceed with discussion of oscillations of (highly) excited positronium with zero total momentum \( l = 0 \) and the large principal quantum number \( n \). Its annihilation rate into photons to the leading order in QED coupling reads

\[
\Gamma_{\mathsf{oPs}_n \rightarrow 3\gamma} = \gamma \frac{n + 1}{2n^3},
\]

thus numerically,

\[
\Gamma_{\mathsf{oPs}_n \rightarrow 3\gamma} \simeq 7 \times 10^6 \text{s}^{-1} \times \frac{n + 1}{2n^3}.
\]

At large \( n \) it scales as \( \propto n^{-2} \) and becomes much smaller than the decay rate of the orthopositronium ground state. Meanwhile, the fastest transition from the level \( ns \) is to the level \( 2p \), in which the rate is

\[
\Gamma_{ns \rightarrow 2p} = \frac{4}{3} \left[ \frac{m\alpha^2}{4} \left( \frac{1}{4} - \frac{1}{n^2} \right) \right]^3 \times |d|^2,
\]

where \( m \) is the electron mass, and the squared matrix element of the dipole moment [42] is given by

\[
|d|^2 = \frac{4}{m^2\alpha} \frac{2^{15}n^9(n - 2)^{2n-6}}{3(n + 2)^{2n+6}}.
\]

At a large \( n \) it approaches

\[
|d|^2 \approx \frac{4}{m^2\alpha} \frac{2^{15}\exp(-8)}{3n^3},
\]

and thus the transition rate (48) asymptotes to

\[
\Gamma_{ns \rightarrow 2p} \simeq \frac{2^7}{3^2} \cdot \frac{\alpha^5m}{\exp(8)} \frac{1}{n^3} \approx 7.5 \times 10^7 \text{s}^{-1} \times \frac{1}{n^3}.
\]

The estimates in (47) and (49) explain what happens to the Rydberg positroniums. Low excited levels of \( n < 20 \) decay into \( 2p \) state, which quickly (\( \Gamma_{2p \rightarrow 1s} = 3.1 \times 10^8 \text{s}^{-1} \)) decays further into \( 1s \), where finally positronium annihilates. At a larger \( n \) the direct annihilation of excited states dominates. Note that the matrix element of the positronium oscillation (3)
also depends on $n$. The estimate of the total oscillation probability of the excited positronium follows from the formula in (12) upon rescaling

$$\delta \rightarrow \delta \frac{n + 1}{2n^3}, \quad \gamma \rightarrow \Gamma_{oP\rightarrow\gamma_{n\rightarrow\gamma}} = \gamma \frac{n + 1}{2n^3}.$$ 

Hence, at large $n$ the invisible decay branching ratio of the Rydberg positronium coincides with that of the ground state.

Note that for the model with small vacuum splitting between positronium and its twin one can think of oscillations between states of different principal numbers $n$ and $n'$. Then both $E$ and $E'$, and $\gamma$ in two diagonal entries depend on these numbers and the formula for the disappearance branching ratio can be generalized to this case.

5 Conclusions and open problems

In this paper we have presented the complete analysis of orthopositronium oscillations into its twin within the Mirror World models. We took into account the relevant effects due to the possible scatterings off walls and gas atoms, and the influence of external electric and magnetic fields.

In a background-free case, the highest sensitivity to the mixing of a photon and paraphoton responsible for the oscillations in positronium sector, is achieved in the pure vacuum with an infinitely large experimental volume. At a given value of mixing parameter the invisible branching ratio of positronium (disappearing via oscillation to its twin, which subsequently decays into paraphotons) decreases when applying electric or magnetic fields, adding gas to the cavity and decreasing its size (or equivalently increasing positronium velocities). We have calculated this decrease as a function of the relevant physical parameters, which can be used to get rid of possible systematics in the experiment, if evidence of a positronium disappearance is found.

To perform numerical estimates, a computer code has been written, which allows us to account for a realistic geometry of the cavity, a realistic positronium spectrum (distribution over velocity), the details of scattering off the cavity walls and gas atoms, etc. To illustrate the code, the numerical results for the setup of positronium experiment [8] proposed within the AEgIS project have been presented. For the first time, the experiment will enter the phenomenologically interesting and cosmologically allowed region of the model parameter space. The code can be used for other setups and may be further improved by implementing
more details of positronium interactions with gas, walls, and magnetic and electric fields. The properly completed modification can be used in simulations of events in a real experiment.

The presented numerical results for the experimental setup [8] can be improved with the account of high-velocity positroniums. Their presence in a small number seems to slightly diminish the sensitivity to the mixing parameter: faster scatterings off gas atoms and walls spoil oscillations to the Mirror World. A similar effect — decreasing sensitivity to invisible mode — is expected for a more accurate realistic treatment of positronium scatterings including induced positronium annihilation, small blind spots in the detector, etc. Thus, we propose our numerical estimates to be used to place an upper limit on the sensitivity of a given experiment in the mixing parameter.

For the first time, we analyzed oscillations of excited positroniums — Rydberg positroniums — into their similarly excited twins. In the vacuum the branching ratio of a high-level Rydberg positronium decay into nothing is the same as that of the ground state (at the same mixing). Rydberg positroniums will be available, e.g., in the AEgIS experiment and can be used for better control over possible systematics. They might be of some interest in the models with a slightly violated Mirror symmetry, resulting in a small mass shift between electron and its twin.

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