Optomechanical steady-state entanglement induced by electrical interaction

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We propose a scheme for generating remote continuous steady-state entanglement of output light leaked from optomechanical system, in which two mechanical oscillators are coupled through long-range Coulomb interaction. We show that the entanglement of output light is affected by the detuning and the strength of the Coulomb interaction. We also demonstrate that two movable mirrors and two light beams can be entangled in the steady state. We suggest an experimental readout scheme to fully verify the characteristic of entangled state.

OCIS codes: (270.5580, 270.5570) Optomechanical system; steady-state entanglement; Coulomb interaction; entangled state.

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1. Introduction

The research of micro-mechanical resonators (MRs) has attracted considerable interests in both quantum mechanics and nano-technology in the past decade, because of the fact that MRs is an ideal candidate to search quantum properties on mesoscopic objects. These quantum properties not only provide insights into the fundamental physical principle in quantum regime [1], but also give potential applications of MRs, such as, optomechanical metrology [2], quantum information processing [3,4], biological sensing [5], and gravitational wave detection [6].

However, only few quantum properties on MRs can be achieved experimentally directly, since the quantum properties on MRs are too weak to be observed, and they are always covered by the thermal fluctuation. Moreover, limited by the ground cooling condition [7], only the MRs with high frequencies can be directly cooled to its ground state with an average phonon number $\langle n \rangle \ll 1$ [8], and the MRs with lower frequencies need to be cooled further to lower temperature. Therefore, it is desirable to develop observing more quantum properties on MRs. And the steady entanglement in MRs is one of this kind of quantum properties. Quantum entanglement is one of the most important features of quantum mechanics.

Until now, several entanglement schemes based on MRs have been proposed to achieve continuous variable entanglement. These entanglement schemes range from the cavity modes to MRs, including the entanglement for two MRs [9,10,14], the entanglement between one MR and a cavity mode in optomechanical cavity systems [19], entanglement of the output light with optomechanical systems [9], entanglement of the optical and microwave cavity mode with a MR [17]. However, this scheme is rarely involved methods long-range entanglement[17].

The aim of our work is to generate the entanglement between the output light fields leaking out of two sides of an optomechanical system with two charged movable mirrors in it. The Coulomb interaction between the two charged MRs in such a system will set up the entanglement of the MRs and the modes in the cavities, As the system reaches steady state, the output fields will be entangled.

Our scheme that the entanglement between the two movable oscillators and between the two beams leaked from the two cavities created by coulomb interaction is quite different from the conventional optomechanical system [9,10] which the entanglement between two movable mechanical resonators is generated by the inner cavity modes or induced by the external atoms. So our scheme belongs to a kind of new structure for gen-
eration continuous entangled light. Contrast to the conventional methods, the coulomb interaction belong to long-range interactions [11, 12]. Furthermore, when the leaking beams are in entangle state, the two mirrors are cooled at the same time, so the influence of external noise is small. The entanglement can be keep a longer time coherence [13].

2. Model and Hamiltonian

As it is sketched in Fig. 1, we consider the model is composed of two spatially separated optomechanical cavities with a distance $r_0$. Each opto-mechanical cavities consists of one fixed mirror and one charged MR. When the distance between the two charged MRs is much large than the small oscillations of the charged MRs $r_0 \gg q_m$, the Coulomb interaction between charged MRs can be written as $V = \lambda q_1 q_2 \left[ \frac{e^2}{4\pi\epsilon_0} \frac{1}{r_0} \right]$, where $\lambda = \frac{2\epsilon_0 \epsilon_m \mu_0}{\pi}$. $k$ is the electrostatic force constants, $Q_m$ is the net charge for the MR $m (=1, 2)$. After redefined the equilibrium position and ignored the frequency shift caused by the Coulomb interaction, the motion of the MRs can be given by $H_{MR} = \sum_{m=1}^{2} \hbar \omega_m p_m^2 + \frac{\hbar \omega_{m}}{2} q_m^2$, where $p_m$ and $q_m$ are the position and momentum operators of MR $m$ with a frequency $\omega_m$. The energy for the optomechanical cavities can be described as $H_{c} = \sum_{m=1}^{2} \hbar \omega_{c,m} c_m^\dagger c_m$ with $\omega_{c,m}$ being the frequency for the cavity mode $c_m$. After each optomechanical cavities is driven by its corresponding laser fields in the frequency $\omega_{p,m}$ with an input power $P_m$, and a strength $e_{p,m} = \sqrt{2P_m/\hbar \omega_{c,m}}$, the Hamiltonian is given as:

$$H_T = H_{c} + H_{MR} + V + H_R + H_d;$$

$$H_R = \sum_{m=1}^{2} \hbar \omega_{m} q_m c_m^\dagger c_m;$$

$$H_d = \sum_{m=1}^{2} \hbar \left( e_{p,m} e^{-i\omega_{p,m} t} - H.C. \right),$$

where $H_R$ is for the radiation pressure couplings between the MR and its corresponding cavity mode, where $\chi_m = \hbar \omega_{c,m}/\omega_{m}$ is the strength of the radiation pressure coupling with a cavity length $L_m$. The last item is describe the optomechanical cavity driven by the external laser fields.

For simplify, we suppose both the optomechanical cavities and driven laser fields are identical, then we can get $e_{p,m} = e_p\left(\omega_{p,m} = \omega_p\right)$, and $\omega_m = \omega_M$. In the frame rotating with the driving frequency $\omega_p$, we can rewrite the Hamiltonian as:

$$H_T = \sum_{m=1}^{2} \left[ \hbar \Delta_0 c_m^\dagger c_m + \frac{\hbar e_{p,m}}{2} \left( p_m^2 + q_m^2 \right) \right]$$

$$+ \sum_{m=1}^{2} \left[ \left(-1\right)^m \hbar \chi_m c_m^\dagger c_m \right] + \lambda q_1 q_2$$

$$+ \sum_{m=1}^{2} \hbar \left( e_{p,k}^\dagger - H.C. \right),$$

with $\Delta_0 = \omega_c - \omega_p$ being the detuning from the cavity to the laser field. Fig. 1.

3. Quantum Langevin equations

A proper analysis of the system must include photon losses in the cavity and the Brownian noise acting on the mirror. Substituting The total Hamiltonian in Eq. 2 into that differential equation the Heisenberg equations of motion and adding the corresponding damping and noise terms, we obtain the quantum Langevin equations as follows:

$$\dot{q}_m = \omega_m p_m$$

$$\dot{p}_1 = -\omega_m q_1 + \chi_1 c_1 + \lambda q_2 - \gamma_1 p_1 + \xi_1$$

$$\dot{p}_2 = -\omega_m q_2 + \chi_2 c_2 + \lambda q_1 - \gamma_2 p_2 + \xi_2$$

$$\dot{c}_1 = -\left( \kappa_1 \right) c_1 + i \chi_1 q_1 + \xi_{1}\in' e^{-i\omega_1 t'} + \sqrt{\kappa_1} \xi_{1}\text{in}$$

$$\dot{c}_2 = -\left( \kappa_2 \right) c_2 + i \chi_2 q_2 + \xi_{2}\in' e^{-i\omega_2 t'} + \sqrt{\kappa_2} \xi_{2}\text{in}$$

The quantum Brownian noise $\xi_1$ and $\xi_2$ are from the coupling of the movable mirrors to their own environment. We suppose the correlation function at temperature $T$

$$\langle \xi_j(t) \xi_k(t') \rangle = \frac{\delta \lambda \gamma_m}{\omega_m} \int \frac{d\omega}{2\pi} e^{-i\omega(t-t')} \omega \left[ 1 + \coth \left( \frac{\hbar \omega}{2k_B T} \right) \right]$$

$$j, k = 1, 2$$

the mirror Brownian noise $\xi_1$ and $\xi_2$ are not Markovian and therefore can’t be described by delta correlated function. But the non-Markovian effects are achievable only in the case that the oscillators are working in a large mechanical quality factor $Q_m = \omega_m/\gamma_m \gg 1$. Hence, $\xi_1$ and $\xi_2$ become delta correlated:

$$\langle \xi_j(t) \xi_k(t') + \xi_k(t') \xi_j(t) \rangle / 2 \approx \gamma_m \left( 2\tilde{n} + 1 \right) \delta(t-t')$$

here $\tilde{n} = \exp \left( \hbar \omega_m/k_B T \right)$ the two cavity modes decay at the same rate $\kappa_1 = \kappa_2 = \kappa$, and $a_{1\text{in}}\left( a_{2\text{in}} \right)$ is the vacuum radiation input noise with the correlation relations which are given by

$$\langle a_{1\text{in}}(t) a_{1\text{in}}^\dagger(t') \rangle = N \delta(t-t')$$

$$\langle a_{1\text{in}}(t) a_{2\text{in}}^\dagger(t') \rangle = (N+1) \delta(t-t')$$

Fig. 1. Schematic description of the experimental system, including two cavity. Each cavity with the length $L$ is driven by a classical light fields. $r_0$ is the distance between the two movable mirrors in the absence of the radiation pressure and the Coulomb force.
steady-state equations plus an additional fluctuation operator with zero-mean value, \(\alpha = q_s + \delta q_s\), \(p = p_s + \delta p_s\). When we insert these expressions into the Eqs of a set of nonlinear algebraic equations for the steady state values and a set of quantum Langevin equations for the fluctuation operators\[19, 30\] can be calculated analytically. The values for the set of steady state equations are read:

\[
\begin{align*}
p_{1s} &= p_{2s} = 0 \\
q_{1s} &= \frac{\beta^2}{\lambda^2} q_s + \frac{1}{\lambda^2} q_{1s} \\
c_{1s} &= \frac{\beta^2}{\lambda^2} c_s + \frac{1}{\lambda^2} c_{1s}
\end{align*}
\]

From Eq (6) we have a third-order nonlinear equations array for \(|c_{m,s}\rangle\) and \(|q_{m,s}\rangle\). Unfortunately, for the exact expression is too cumbersome and will not be reported here. When radiation pressure coupling is strong, significant optomechanical entanglement is achieved\[13\]. For high finesse cavities and enough driving power, the system is characterized by a semiclassical steady state with the cavity mode in a coherent state with significant optomechanical entanglement \[19\].

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As the quantum noise and \(c_{m,in}\), \(m = 1, 2\) are zero-mean quantum Gaussian noise and the dynamics is linearized, the quantum steady state for the fluctuations can be rewrite as the follow expression:

\[
\dot{f} = Af + b
\]

Where \(A\) is drift matrix\[9\] describing full character of the quantum steady state for the fluctuations.

\[
\begin{align*}
A &= \begin{pmatrix}
0 & -\omega_m & 0 & 0 & 0 & 0 & 0 \\
-\omega_m^2 & 0 & -\lambda & -\gamma_m & -F_1 & 0 & 0 \\
-\lambda & 0 & -\omega_2 & -\gamma_m & 0 & -F_2 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & G_1 \\
0 & 0 & 0 & 0 & 0 & 0 & G_2 \\
0 & 0 & -F_2 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & G_2
\end{pmatrix}
\end{align*}
\]

When all of the eigenvalues of matrix \(A\) have a negative real parts, the system is running in a stable and reaches states. Under the help of Routh-Hurwitz criterion\[22\], we can get the following three nontrivial conditions on the system parameters:

\[
\frac{\lambda^2 - m_1 m_2 \omega_1^2 \omega_2^2}{2F_m/\hbar \omega_{0,m}} > 0
\]

\[
G_1 \star (m_2 F_1^2 \omega_2^2 - G_1 \lambda^2 + G_1 m_1 m_2 \omega_1^2 \omega_2^2) > 0
\]

\[
G_1 G_2 (G_1 \lambda^2 - m_2 F_1^2 \omega_2^2 - G_1 m_1 m_2 \omega_1^2 \omega_2^2) > 0
\]

The formal solution of Eq (5) is

\[
f(t) = M(t) f(0) + \int_0^t M(s) b(t-s) dt
\]

where \(M(t) = \exp(At)\).

4. Entanglement of the output field

In order to analyze the nature of linear quantum correlations among the two MRs and among the two beams output field, the steady state of the correlation matrix of quantum fluctuations in this multipartite system can be considering. The noises from the phonon bath and photon bath are both zero mean quantum Gaussian noise, so the steady state of the system is a zero-mean multipartite Gaussian state.

4.A. Entanglement of the two mechanical oscillators interacted by Coulomb force

We use the definiton \(V_{ij} (\infty) = \frac{1}{2} (\{f_i (\infty) f_j (\infty) + f_j (\infty) f_i (\infty)\})\) which is the element of the covariance matrix. The information of entanglement of the two mirrors or two beams leaked from two sides of the cavities can be obtain with the help of the covariance matrix. The literature\[26, 27\] had proposed two criteria of the continuous variable entanglement. Here we used the Duan’s
criterion proposed in [27] and developed by , a state is
evertheless if the summation of the fluctuations in the
two EPR-like operators $X$ and $Y$ satisfy the following inequality: $(\Delta X)^2 + (\Delta Y)^2 < 2$. Here $X_m = Q_1 + Q_2$, $Y_m = P_1 - P_2$. We focus on the entanglement of the four possible bipartite subsystems of the four-body system that can be formed by traceless the others degree of freedom[15], such that we can obtain a reduced $4 \times 4$ CM $\tilde{V}$ from $V$.

$$ V_{ij} = \sum_{k,l} \int_0^\infty ds \int_0^\infty ds' M_{ik}(s) M_{jl}(s') \Phi_{kl}(s-s') $$

(12)

Here $\Phi_{kl}(s-s') = \langle \langle b_k(s) b_l(s') + b_l(s') b_k(s) \rangle \rangle / 2 = D_{kl} \delta(s-s')$ is the matrix of the station-
ary noise correlation functions. Here $D_{kl} = \text{Diag}(0, \gamma_m (2\bar{n}_m + 1), 0, \gamma_m (2\bar{n}_m + 1)$,
$k (2\bar{n}_m + 1), \kappa (2\bar{n}_m + 1), \kappa (2\bar{n}_m + 1), \kappa (2\bar{n}_m + 1))$ is a
 diagonal matrix and $\bar{n}_m = 0, \bar{n}_c = 0$. If we neglect the
frequency dependence The the frequency domain treat-
ment is same to the time domain and the Correlation
matrices have the same form. Under the stability condi-
tions, the following equations for the steady-state can be
obtained:

$$ M(\infty) = 0 $$

(13)

$$ AV + VA^T = -D $$

(14)

Eq(13) is named Lyapunov equation which is equivalent
to the Eq(12) for the steady-state. The linear equation
for $V$ can be straight forwardly solved used Eq(13) but
the exact expression is too complex to reported in the
article. We used the logarithmic negativity $E_N$ as a
measure of entanglement.[28, 30, 31]

$$ E_N = \max [0, -\ln 2\eta^-] $$

(15)

here $\eta^- = \sqrt{2} \left\{ \sum (V) - \left[ \sum (V)^2 - 4 \det V \right]^{1/2} \right\}^{1/2}$,
and $\sum (\tilde{V}) = \det B + \det C - 2 \det E$. $B, C$ and $E$ is
the a $2 \times 2$ block form of the $V$:

$$ V \equiv \begin{pmatrix} B & E \\ E^T & C \end{pmatrix} $$

The results are shown in Fig.2, where we study the entanglements of the two mechanical oscillators at the
steady state of the system versus the detuning and for
different values of the $\kappa, \gamma$. Here we find a parameter re-
region close to that of recently performed optomechanical experiments[32], and for simplicity, choose all the pa-
rameters of the two mirrors, two lasers and the two cav-
ties to be the same. $\omega/2\pi = 10 MHz$, $\kappa_1 = \kappa_2 = 0.8\omega_b$, $\gamma_m/2\pi = 100Hz$, $T = 300mK$, $\Delta_0 = \omega_b$Figure 1
demonstrate

$$ m = 20ng, \text{wavelength} = \frac{2\pi}{\omega_c} = 1064nm, \omega_c/2\pi = 2.8 \times 10^{14} Hz, C = 27.5nF, U = 1V, k_q = 8.897N m^2/C^2$$

$F = 0.88nN^{10}aNa$Fig.2

\begin{figure}[h]
\centering
\includegraphics[width=0.5\textwidth]{fig2}
\caption{Plot of the logarithmic negativity $E_N$ as a function of the normalized detuning $\Delta/\omega_m$ of the mechanical oscillator. Here the optical cavity of length $L = 25mm, P = 50mW$, the mechanical oscillator has a frequency $\omega_m/2\pi = 10 MHz$, a damping rate $\gamma_m/2\pi = 100 Hz$.}
\end{figure}

4.B. The continuous entanglement between two
beam outpus light induced by optomechanical resonators

In this section, we study the entanglement of the output field. The equation in Eqs(3) can be solved by the input-
output relation:

$$ \delta a_{out} = \sqrt{2}\kappa_j \delta a_j - \delta a_{jin} $$

(16)

The Eq(8) can be written as $\tilde{f}_j(\omega) = (-i\omega - A)^{-1} b$
in the frequency domain by Fourier transformation.

In order to study the nature of the output fields leaked from two sidebands of our system, one can solve the ana-
alyzed solution from the two parts the input-output relation
for the two-mode field is the equations of Eq(??) can be
solved in the frequency domain by Fourier transform-
lation with the solution. Where $\delta a_j(\omega)$ is the Fourier
transformation of $\delta a_j(\omega)$.

We only choose the part relevant to output fields from
In the interaction picture, $\omega$ represents the detuning from
the cavity frequency. Used the relation Eq(16) we
obtain the following linear equation:

$$ \delta f_{out}(\omega) = c (-i\omega - A)^{-1} b - e $$

(17)

Where the matrices have the follow forms:
$c = \text{diag}(0, 0, 0, 0, \sqrt{2}k, \sqrt{2}k, \sqrt{2}k, \sqrt{2}k)$, $e = [0, 0, 0, 0, \delta X_{1in}, \delta Y_{1in}, \delta X_{2in}, \delta Y_{2in}]^T$. The output
correlation matrix can be written as $V_{ij}^{out}(\omega) = \frac{1}{2} \left\{ \langle f_i^{out}(\omega) f_j^{out}(\omega') \rangle + \langle f_i^{out}(\omega') f_j^{out}(\omega) \rangle \right\}$. $D_{kl}$
$$ = \text{Diag}(\kappa (2\bar{n}_m + 1), \kappa (2\bar{n}_c + 1), \kappa (2\bar{n}_c + 1), \kappa (2\bar{n}_c + 1))$$

The squeezing spectrum which is defined in a frame of
Fourier transformation can be calculated from the
correlation matrix:

$$ S_{out}(\omega) = \frac{1}{4} \left\{ \delta X_f(\omega) \delta X_f(\omega') + \delta X_f(\omega') \delta X_f(\omega) + \delta Y_f(\omega) \delta Y_f(\omega') + \delta Y_f(\omega') \delta Y_f(\omega) \right\} $$

(18)
In order to measure logarithmic negativity, one has to measure all independent entries of the correlation matrix. We can use feasible experimental methods have been realized in to experimental detection of the generated entanglement of the output field. In our schematic, the measurement of the field quadratures of the output field leaked from cavity can be straightforwardly performed by homodyning the cavity output using a local oscillator with an appropriate phase.

5. Conclusion
We propose a scheme to generate steady-state continuous entanglement of two output beams which leaked from two sides of cavities induced by long-range Coulomb interaction. We show that the entanglement of output light is affected by the detuning and the strength of the Coulomb interaction. We also demonstrate that two movable mirrors and two light beams can be entangled in the steady state. We suggest an experimental readout scheme to fully verify the characteristic of entangled state. The results show that such optomechanical entanglement can persist for higher environment temperatures using parameters based on the existing experiment.

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