Aging Exponents for Nonequilibrium Dynamics following Quenches from Critical Point
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Via Monte Carlo simulations we study nonequilibrium dynamics in the nearest-neighbor Ising model, following quenches to points inside the ordered region of the phase diagram. With the broad objective of quantifying the nonequilibrium universality classes corresponding to spatially correlated and uncorrelated initial configurations, in this paper we present results for the decay of the order-parameter autocorrelation function for quenches from the critical point. This autocorrelation is an important probe for the aging dynamics in far-from-equilibrium systems and typically exhibits power-law scaling. From the state-of-the-art analysis of the simulation results we quantify the corresponding exponents (λ) for both conserved and nonconserved (order parameter) dynamics of the model, in space dimension d = 3. Via structural analysis we demonstrate that the exponents satisfy a bound. We also revisit the d = 2 case to obtain more accurate results. It appears that irrespective of the dimension, λ is same for both conserved and nonconserved dynamics.

INTRODUCTION

Following a quench from high temperature disordered phase to a point inside the ordered region, when a homogeneous system evolves towards the new equilibrium, several quantities [1–7] are of importance for the understanding of the nonequilibrium dynamics. Structure of a system is usually characterized by the two-point equal time correlation function [1] or by its Fourier transform, S, the structure factor, the latter being directly accessible experimentally. This correlation function, C(r,t), is defined as (r = |r|)

\[ C(r,t) = \langle \psi(\vec{r},t)\psi(\vec{0},t) \rangle - \langle \psi(\vec{r},t)\rangle \langle \psi(\vec{0},t) \rangle, \]  

(1)

where \( \psi(\vec{r},t) \) is a space \( (\vec{r}) \) and time \( (t) \) dependent order parameter. During a ‘standard’ nonequilibrium evolution, \( C(r,t) \) exhibits the scaling behavior [1]

\[ C(r,t) \equiv \tilde{C}(r/\ell(t)), \]  

(2)

with \( \ell \), the characteristic length scale, measured as the average size of the domains rich or poor in particles of specific type, typically growing as [1]

\[ \ell \sim t^\gamma. \]  

(3)

While understanding of the scaling form in Eq. (2) and estimation of the growth exponent \( \eta \) in Eq. (3) have been the primary focus [1,2] of studies related to kinetics of phase transitions, there exist other important aspects as well [3]. For example, during the evolution of the ferromagnetic Ising model, corresponding nearest neighbor \( (ij) \) version of the Hamiltonian being defined as [1]

\[ H = -J \sum_{(ij)} S_i S_j; \ S_i = \pm 1; \ J > 0, \]  

(4)

one is interested in the time dependence of the fraction of unaffected spins \( (S_i) \). This quantity also exhibits a power-law decay with time, \( t^{-\theta} \), the exponent \( \theta \) being referred to as the persistence exponent [2]. Furthermore, in an evolving system the time translation invariance is violated, implying different relaxation rates when probed by starting from different waiting times \( (t_w) \) or ages of the system. Such aging property [3–20] is often investigated via the two time order-parameter autocorrelation function [6]

\[ C_{ag}(t,w) = \langle \psi(\vec{r},t)\psi(\vec{r},t_w) \rangle - \langle \psi(\vec{r},t) \rangle \langle \psi(\vec{r},t_w) \rangle, \]  

(5)

with \( t > t_w \). Despite different decay rates for different \( t_w \), \( C_{ag}(t,w) \) in many systems exhibits the scaling property [2]

\[ C_{ag}(t,w) \sim (t/\ell(t))^{-\lambda}, \]  

(6)

where \( \ell \) and \( \ell_w \) are the characteristic lengths at \( t \) and \( t_w \), respectively.

For the understanding of universality in coarsening dynamics, it is important to study all these properties. Note that universality [1] in nonequilibrium dynamics depends upon the mechanism of transport, space dimension \( (d) \), symmetry and conservation of order parameter, etc. In addition, in each of these cases the functional forms or the values of the power-law exponents for above mentioned observables may be different for correlated and uncorrelated initial configurations [17,21,22]. I.e., there may be different universality classes depending upon whether a system is quenched to the ordered region with perfectly homogeneous configuration, say, for the Ising model from a starting temperature \( T_0 = \infty \), with equilibrium correlation length [22] \( \xi = 0 \), or from the critical point with \( \xi = \infty \).

In this work our objective is to estimate \( \lambda \) for initial configurations with \( \xi = \infty \) in the three-dimensional Ising model as well as revisit the \( d = 2 \) case. We consider two
cases, viz., kinetics of ordering in uniaxial ferromagnets \[1,2\] and that of phase separation in solid binary \((A + B)\) mixtures \[1,2\]. For the former, the spin values \(\pm 1\) in Eq. \[4\] represent, respectively, the up and down orientations of the atomic magnets. In the second case, different values of \(S_i\) stand for an \(A\) or a \(B\) particle. During ordering in a magnetic system, the volume-integrated order parameter (note that \(\psi\) is equivalent to the spin variable) does not remain constant over time \[1\]. On the other hand, for phase separation in binary mixtures this total value is independent of time \[1\], i.e., conserved.

Even for such simple models and technically easier case of \(\xi = 0\), estimation of \(\lambda\) remained difficult, particularly for the conserved order-parameter case \[9,28\]. For quenches from the critical point, additional complexity is expected in computer simulations. In the latter case there exist two sources of finite-size effects \[29\]. First one is due to non-accessibility of \(\xi = \infty\) in the initial correlation \[24\] and the second is related to the fact \[30,31\] that \(\ell < \infty\), always. Nevertheless, via appropriate method of analysis \[26\] in each of the cases we estimate the value of \(\lambda\) quite accurately. It transpires that the obtained numbers are drastically different from those \[18\] for \(\xi = 0\). This is despite the fact that the growth exponent \(n\) does not depend upon the choice of initial \(\xi\).

The results are discussed in the background of available analytical information \[7,8,17\]. It is shown that the numbers obey a bound, obtained by Yeung, Rao and Desai (YRD) \[8\],

\[
\lambda \geq \frac{d + \beta}{2}.
\]

Here \(\beta\) is an exponent related to the power-law behavior of the structure factor at the waiting time \(t_w\) in the small wave vector \((k)\) limit \[32\]:

\[
S(k, t_w) \sim k^\beta.
\]

The rest of the paper is organized as follows. In Section II we provide details of the model and methods. Results are presented in Section III. Section IV concludes the paper with a brief summary and outlook.

**MODEL AND METHODS**

Monte Carlo (MC) simulations of the nearest neighbor Ising model \[33,34\], introduced in the previous section, are performed by employing two different mechanisms, viz., Kawasaki exchange \[36\] and Glauber spin-flip \[37\] methods, on a simple cubic or square lattice, with periodic boundary condition in all directions. For this system the value \[34\] of critical temperature in \(d = 3\) is \(T_c \simeq 4.51 J/k_B\), \(J\) and \(k_B\) being the interaction strength and the Boltzmann constant, respectively. Corresponding number in \(d = 2\) is \(\simeq 2.27 J/k_B\) \[34\]. Given that in computer simulations the thermodynamic critical point is not accessible, we have quenched the systems from \(T_s = T_c^d\), the finite-size critical temperature for a system of linear dimension \(L\) (see next section for a more detailed discussion on this) \[26,29\]. The final temperature was set to \(T_f = 0.6T_c\) starting composition always having 50% up and 50% down spins. Below we set \(J, k_B\) and \(\alpha\), the lattice constant that is chosen as the unit of length, to unity.

In Kawasaki exchange Ising model (KIM), a trial move consists of the interchange of particles between randomly selected nearest neighbor sites. For the Glauber Ising model (GIM), a trial move is a flip of an arbitrarily chosen spin. In both the cases we have accepted the trial moves by following the standard Metropolis algorithm \[33–35\]. KIM and GIM mimic the conserved and nonconserved dynamics, respectively. In our simulations, one Monte Carlo step (MCS), the chosen unit of time, is equivalent to \(L^d\) trial moves.

For faster generation of the equilibrium configurations at \(T_c^d\), Wolff algorithm \[38\] has been used. There a randomly selected cluster of identical spins/particles has been flipped. This way the critical slowing down \[39,43\] has been avoided.

The average domain lengths of a system during evolution have been calculated as \[31\],

\[
\ell(t) = \int P(\ell_d, t)\ell_d d\ell_d.
\]

Here \(P(\ell_d, t)\) is a domain-size distribution function, which is obtained by calculating \(\ell_d\), the distance between two successive interfaces, by scanning the lattice in all directions. Quantitative results are averaged over a minimum of 100 independent initial configurations for both KIM and GIM. To facilitate extrapolation of the results for aging in the thermodynamically large size limit, we have performed simulations with different system sizes. In \(d = 3\) the value of \(L\) varies between 24 and 128 for KIM and between 64 and 300 for GIM. In \(d = 2\), we have studied systems with \(L\) lying in the range \[64,512\] for KIM and \[64,1024\] for GIM. We have acquired structural data for both types of dynamics for fixed values of \(L\) in each of the dimensions. For this purpose, in \(d = 3\) we have considered \(L = 512\) and the results for \(d = 2\) were obtained with \(L = 1024\). Given that these system sizes are large we did not simulate multiple values of \(L\) in this case. Details on the statistics and the system sizes for the calculations of \(T_c^d\) can be found in the next section. Note that most of the results are presented from \(d = 3\). We revisit the \(d = 2\) case to improve accuracy so that certain conclusions on the dimension dependence can be more safely drawn.
RESULTS

As already mentioned, in computer simulations finite-size effects lead to severe difficulties in studies of phenomena associated with phase transitions. In kinetics of phase transitions, $\ell$ never reaches $\infty$, due to the restriction in the system size $[26]$. This is analogous to the fact that in critical phenomena $[24]$ one always has $\xi < \infty$. There, of course, exist scaling methods to overcome the problems in both equilibrium and nonequilibrium contexts $[14, 18, 26, 29, 30, 34, 40]$. For studies of coarsening phenomena starting from the critical point $[23, 26]$, difficulties due to both types of effects are encountered. Nevertheless, via construction of appropriate extrapolation method $[26]$ we will arrive at quite accurate conclusions.

In critical phenomena the true value of $T_c$ cannot be realized for $L < \infty$. In such a situation, for reaching conclusions in the $L = \infty$ limit, one defines $T_c^L$, pseudo critical temperature for a finite system, and relies on appropriate scaling relations $[41, 44]$. $T_c^L$ is expected to exhibit the behavior $\nu$

$$T_c^L - T_c \sim L^{-1/\nu},$$

(10)

where $\nu$ is the critical exponent corresponding to the divergence $[27, 45]$ of $\xi$ at $T_c$. In Fig. 1 we have presented results for $T_c^L$, as a function of $1/L$, from $d = 3$. The solid line there is a fit to the scaling form in Eq. (10) by fixing $\nu = 0.63$ and $\xi = 4.51$, respectively. The quality of fit confirms the validity of Eq. (10) as well as the accuracy of the estimations. We use the amplitude ($\nu = 4.4$) obtained from the fit to extract $T_c^L$ for $L$ larger than the presented ones. This number is $\nu \approx 3.9$ in $[27, 46, 47]$ $d = 2$.

The results in Fig. 1 were obtained by using the Glauber as well as the Wolff algorithms $[34, 37]$, exploiting the following facts. The fluctuation in the number of spins or particles of a particular species during simulations provides temperature dependent probability distributions for the corresponding concentration. These distributions functions are double peaked in the ordered region $[26, 34]$. On the other hand, above criticality one observes single peak character. The temperature at which the crossover from double to single peak shape occurs is taken as the $T_c^L$ for a particular choice of $L$. In the inset of Fig. 1 we show the distributions, $p(n_u)$ (see caption for the details of the notation), from two different system sizes. For both the system sizes the temperature is the same. It is seen that for the larger value of $L$ there is only one peak while the distribution for the smaller system has two peaks. This is expected in the present set up and is consistent with Eq. (10). Note that the crossover between single-peak and double-peak structures occur in a continuous manner. Thus, extremely good statistics is needed to identify this. The probability distribution close to $T_c^L$ were, thus, obtained, for each $L$, after averaging over a minimum of 500 independent runs. Only because of this our results in the main frame of Fig. 1 are accurate, the error bars being less than the size of the symbols. $T_c^L$ can also be estimated from the locations of the maxima, with the variation of temperature, in the thermodynamic functions like susceptibility and specific heat.

To facilitate appropriate analysis of the autocorrelation data we will perform quenches from $T_c^L$ for different values of $L$. For each $L$, value of $\lambda$, to be referred to as $\lambda_L$, will be estimated. Finally, the thermodynamic limit number will be obtained from the convergence of $\lambda_L$ in the $L = \infty$ limit. In addition to the $L$-dependence, there will be other effects as well. These we will discuss in appropriate places.

In Fig. 2 we present two-dimensional cross-sections of the snapshots, taken during the evolution of both types of systems, from $d = 3$. For the sake of completeness we have compared the snapshots for the critical starting temperature with the ones for quenches with $\xi = 0$, i.e., from $T_s = \infty$. The upper frames are for conserved order-parameter dynamics and the lower ones are for the nonconserved case. In each of the cases the structure for quenches from the critical point appears different from that for $T_s = \infty$. Note that all the presented pictures are from simulations with $L = 128$ and the results for the critical point correspond to quenches from $T_c^L$, as men-
versus $k$ of $s$ponds to the existence of domain boundaries. This value in aging. Even though the results in Fig. 3 are from effects and onset of scaling in the structure as well as tive results with appropriate understanding of finite-size bound.

$S(k) \sim k^{-2}$, \hspace{1cm} (11)
given that in $d = 3$ the critical exponent $\eta$ ($\approx 0.036$, as opposed to 0.25 in $d = 2$), the Fisher exponent, that characterizes the power-law factor of the critical correlation as $r^{-d+2+\eta}$, has a small value. Typically in most of the coarsening systems scaling in the decay of autocorrelation function [cf. Eq. (6)] starts from a reasonably large value of $t_w$. By then the structure is expected to have changed from that at the beginning. Thus, the exponent $\sim -2'$ in Eq. (11) should be verified before being taken as the value of $\beta$ in the YRD bound for understanding of results following quenches from $T_c$. Furthermore, for $T_s = T_c$, one may even ask about the validity of a stable $\beta$. This is related to the question whether there exists a scaling regime or the structure is continuously changing. Keeping this in mind, in Fig. 3 we present plots of $S(k, t_w)$ versus $k$ for large enough values of $L$ and $t_w$, from $d = 3$. Results from both the dynamics are included. In fact $\beta$ appears to be stable at $\sim -2'$ even though character of structure changes at large $k$, e.g., an appearance of the Porod law $\sim k^{-4}$) is clearly visible that corresponds to the existence of domain boundaries. This value of $\beta$, i.e., $-2$, will be used later for verifying the YRD bound.

Note here that in Fig. 3 we have presented representative results with appropriate understanding of finite-size effects and onset of scaling in the structure as well as in aging. Even though the results in Fig. 3 are from $L = 512$, simulating this size for long enough time, a necessity in studies of aging phenomena, in $d = 3$ is very time consuming, particularly for the conserved dynamics. So, for aging the presented data are from smaller values of $L$ and the conclusions in the thermodynamic limit is drawn via appropriate extrapolations.

For the sake of completeness, in the inset of Fig. 3 we presented analogous results for $d = 2$. Here also the small $k$ behavior remains unaltered from that in the initial configuration, i.e., we have $\beta = -7/4$. In this dimension the Porod law demands $S(k) \sim k^{-3}$. In the rest of the paper, all figures will contain results for $d = 3$ only, except for the last one.

First results for $C_{agg}(t, t_w)$ are presented in Fig. 4 versus $t/t_w$, on a log-log scale. In part (a) we have shown data for the nonconserved dynamics, by fixing the system size, for a few different values of $t_w$. The observations are the following.

There exist sharp departures of the data sets from each other at large $t/t_w$. Higher the value of $t_w$ the departure occurs earlier from the plot for a smaller $t_w$. This is related to ‘standard’ nonequilibrium finite-size effects. With the increase of $t_w$ a system has less effective size available to grow or age for. This fact can be stated in the following way as well. Note that for a fixed system size the final value of $t$ is fixed. Thus, with the increase of $t_w$, i.e., of $t_w$, the value of the scaled variable $t/t_w$ decreases. Naturally, when the latter is chosen as abscissa variable, the finite-size effects start appearing earlier. Furthermore, even in the small $t/t_w$ region the collapse of the data set for $t_w = 10$ with those for the
larger $t_w$ values is rather poor. This, we believe, is due to the fact that in the scaling regime the structure is different \cite{32} from the initial configuration \cite{28}. (Also note that the scaling structure for $T_c = T_s$ is different from that for $T_s = \infty$.) During this switch-over to the scaling behavior the extraction of $\ell$ is also ambiguous, due to continuous change in the structure that, thus, lacks the property of Eq. (2). If we believe that by $t_w = 50$ the scaling regime has arrived (see the reasonably good collapse of data sets for $t_w = 50$ and $100$ in the small $\ell/\ell_w$ regime), the corresponding decay is consistent with $\lambda = 0.5$, a value that was predicted theoretically \cite{17}. Nevertheless, given the complexity of finite-size and other effects, further analysis is required, before arriving at a conclusion with confidence.

In part (b) of Fig. 4 we present similar results for the conserved dynamics. Here the system size is smaller than in (a). Note that due to slower dynamics in the conserved case ($n = 1/2$ for nonconserved case \cite{11,46}, whereas $n = 1/3$ for the conserved dynamics \cite{31,47,49} and these numbers are true irrespective \cite{17,21,50} of $T_s$) the convergence to the scaling regime has not happened even by $t_w = 100$. For the same reason the onsets of finite-size effects for different $t_w$ values are not so dramatically separated from each other in this case.

For both the dynamics, one gets an impression that the exponent has a tendency to increase with the increase of $t_w$. The phenomenon of convergence, however, is more complex and requires systematic study involving both $t_w$ and $L$. This we will perform in the rest of the paper.

Next we examine the effects of system size on the “scaling” regime. We remind the reader that there exist another type of finite-size effect related to $\xi < \infty$. Due to this, with changing system size the exponent will differ in “the scaling regime” as well. Related results are presented in Fig. 5. For the sake of brevity, here, we show data only for the conserved case.

In Fig. 5(a) we show $C_{\text{ag}}(t,t_w)$, for different values of $L$, versus $\ell/\ell_w$, on a log-log scale, by fixing $t_w$ to 20. In addition to the delayed appearance of late time finite-size effects, with the increase of system size the decay exponent shows the tendency of shifting towards smaller value \cite{20}. To pick the stable power-law regime appropriately, by discarding the finite-size affected and early transient regimes, in Fig. 5(b) we plot the instantaneous exponent \cite{14,18,31,48,49}

$$\lambda_i = \frac{d \ln C_{\text{ag}}(t,t_w)}{d \ln x}; \quad x = \frac{\ell}{t_w},$$

as a function of $\ell/\ell_w$, for a few values of $L$ with $t_w = 20$. From the flat parts we extract $L$-dependent exponent $\lambda_i$. We have performed this exercise for multiple values of $t_w$, for each type of dynamics.

An even better exercise is to extract $\lambda_L$ from the plots of $\lambda_i$ versus $\ell_w/\ell$. This helps the extrapolation of $\lambda_i$ to the $x = \infty$ limit, thereby elimination of any corrections, if present for small $x$, via judicial identification of the trend of a data set. These plots are shown in Fig. 5(c). From Fig. 5(b) it was already clear that the corrections are weak in this case and so, in Fig. 5(c) also we observe flat behavior of the relevant region and obtain the same values of $\lambda_L$. Similar procedure is followed in the nonconserved case as well. The above mentioned flat behavior in the intermediate regime confirms that there exists power-law relationship between $C_{\text{ag}}(t,t_w)$ and $\ell/\ell_w$. The plots of Figs. 4(b) and 5(c) are expected to convey similar message. Nevertheless, the weak dependence of $\lambda_i$ on $x$,
FIG. 5. (a) Log-log plots of $C_{ag}(t, t_w)$ versus $\ell/\ell_w$, for $t_w = 20$ and different values of the linear dimension of the simulation box in $d = 3$. The solid lines represent power-laws. (b) Plots of the instantaneous exponents, $\lambda_i$, versus $\ell/\ell_w$, for $t_w = 20$ and two values of $L$ in $d = 3$. (c) Same as (b) but $C_{ag}(t, t_w)$ is plotted versus $\ell_w/\ell$. The dashed horizontal lines represent the estimated values of $\lambda_L$, the $L$-dependent aging exponent. All results are from the conserved dynamics.

FIG. 6. (a) Plots of $\lambda_L$ versus $1/L$, for the conserved order-parameter dynamics. Data from a few different values of $t_w$ are shown. The dashed lines are power-law fits to the simulation data sets. (b) Same as (a) but for the nonconserved order parameter dynamics. All results are from $d = 3$. In both the parts insets contain scaling plots of the autocorrelation function. The values of $L$ and $t_w$ are mentioned inside the frames.

Data for $\lambda_L$, for a particular type of dynamics, when plotted versus $1/L$, for multiple values of $t_w$, should provide a good sense of convergence. Corresponding number should be the value of $\lambda$ for a thermodynamically large system. This exercise has been shown in Fig. 6 for both conserved (a) and nonconserved (b) dynamics.
FIG. 7. Same as Fig. 6 but here the results are from $d = 2$.

The dashed lines there are fits to the form

$$\lambda_L = \lambda + AL^{-b},$$

where $A$ and $b$ are constants. For both KIM and GIM, fit to each of the data sets provides $\lambda$ value quite consistent with the others. In Fig. 7 we show analogous results for $d = 2$—part (a) for KIM and part (b) for GIM. Compared to Ref. [26], these results are obtained after averaging over larger number of initial realizations. In the insets of Fig. 6 and Fig. 7, we show scaling plots of the autocorrelation function. Given that we have chosen the largest simulated system sizes, the collapse of data from different $t_w$ values, in each of the cases, is good. The estimated values of $\lambda$, obtained after averaging over the convergences of the fittings, by considering different numbers of data points for each $t_w$, along with those for uncorrelated initial configurations [8, 14, 18], are quoted in Table I. All numbers in this table are from simulation studies. For the comparison of these numbers with the YRD bound, in Table I we have quoted the values of $\beta$ for 50 : 50 starting composition of up and down spins (see caption for more details) [26, 32]. For the uncorrelated case it is clear that the structures are different for the conserved and nonconserved cases. For the correlated initial configurations even though the $\beta$ values for the two types of dynamics appear same, the overall structures are different, as expected [1] (see Fig. 3).

TABLE I. List of values of $\lambda$ for the nearest neighbor Ising model. Here “Correlated” and “Uncorrelated” imply results for quenches from $T_s = T_c$ and $T_s = \infty$, respectively. For the GIM, we have quoted the theoretical predictions [8, 17, 21] inside the parentheses. For the values of the lower bounds [9] please see Table II.

| Model | $d = 2$ | $d = 3$ |
|-------|---------|---------|
|       | Correlated | Uncorrelated | Correlated | Uncorrelated |
| KIM   | 0.13±0.02 | 3.6±0.2  | 0.64±0.05 | 7.5±0.4  |
| GIM   | 0.14±0.02 | 1.32±0.04 | 0.57±0.07 | 1.69±0.04 |

For the sake of completeness, in Table II we list the values of the persistence exponent $\theta$ for the two universality classes in $d = 2$ and 3. Due to technical difficulty with the estimation in conserved case, for this quantity we quote only the values for the nonconserved dynamics. This table contains the values of fractal dimensionality ($d_f$) of the scaling structures formed by the persistent spins as well [23, 51, 52]. From the values of the quantities presented in Table III it is again clear that the universality for correlated and uncorrelated initial configurations are different. For the domain growth, of course, as previously mentioned, the value of $n$ does not differ between the correlated and uncorrelated initial configurations [17, 21, 50].

TABLE II. List of $\beta$ values for the nearest neighbor Ising model. Validity of YRD bound can be checked by putting these numbers in Eq. (7) and comparing the outcome with the results quoted in Table I. While preparing this table, $\eta$ in $d = 3$ has been set to zero (see discussion in the context of Fig. 3). For the sake of convenience, we have put the values of the bounds [9] inside the parentheses.

| Model | $d = 2$ | $d = 3$ |
|-------|---------|---------|
|       | Correlated | Uncorrelated | Correlated | Uncorrelated |
| KIM   | -1.75 (0.125) | 4 (3)  | -2 (0.5) | 4 (3.5)  |
| GIM   | -1.75 (0.125) | 0 (1)   | -2 (0.5) | 0 (1.5)  |
TABLE III. List of values of the persistence exponent, $\theta$, and related fractal dimension ($d_f$) for the nonconserved Ising model.

| Exponent | $d = 2$ | $d = 3$ |
|----------|---------|---------|
|          | Correlated | Uncorrelated | Correlated | Uncorrelated |
| $\theta$ | 0.035 | 0.225 | 0.105 | 0.180 |
| $d_f$    | 1.92 | 1.53 | 2.77 | 2.65 |

CONCLUSION

Universality in kinetics of phase transition \[1\] is less robust compared to that in equilibrium critical phenomena \[27, 34, 43\]. In kinetics, the classes are decided \[1\] by transport mechanism, space dimension, order-parameter symmetry and its conservation, etc. In each of these cases there can be further division into universality classes \[17, 23, 26\] based on the range of spatial correlation in the initial configurations. In this paper, we have examined the influence of long range correlation on the decay of order-parameter autocorrelation function, a key quantity for the study of aging phenomena \[6, 7\] in out-of-equilibrium systems, by quenching the nearest neighbor Ising model \[27, 45\] from the critical point to the ordered region. We have investigated both conserved \[1\] and nonconserved \[1\] order-parameter dynamics.

In the nonconserved case our study mimics coarsening in an uniaxial ferromagnet. On the other hand, the conserved dynamics is related to the kinetics of phase separation in solid binary mixtures. Despite difficulty due to multiple sources of finite-size effects, we have estimated the exponents for the power-law fall of the autocorrelation function rather accurately. We observe that in both the cases the decays are significantly slower than those for the quenches from perfectly random initial configurations \[6, 8, 14, 18\].

Even though for quenches with $\xi = 0$ the values of $\lambda$ differ significantly in the two cases, for quenches from the critical point, i.e., for $\xi = \infty$, the exponents are practically same. This is irrespective of the space dimension. For the magnetic case there exist analytical prediction \[17\] and the numbers obtained from our simulations are in reasonable agreement with the former. The source of deviations that exist may have its origin in the estimation error for $T_c^L$ as well as in the statistical error in nonequilibrium simulations. The discrepancy in $d = 3$ may still be real given that KIM and GIM numbers from our analysis are quite close to each other.

In the literature of aging phenomena there exist lower bounds \[7, 8\] for the values of $\lambda$. Our results for both types of dynamics are consistent with one of these bounds. This we have checked via the analysis of structure, a property that is embedded in the construction of the bound.

This work, combined with a few others \[14, 17, 18, 23\], provides a near-complete information on the universality in coarsening dynamics in Ising model, involving “realistic” space dimensions, conservation property of the order parameter and spatial correlations in the initial configurations. Analogous studies in other systems should be carried out, by employing the methods used here, to obtain more complete understanding, e.g., of the influences of hydrodynamics on relaxation in out-of-equilibrium systems with long range initial correlations.

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