QCD Scales in Finite Nuclei

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Abstract

The role of QCD scales and chiral symmetry in finite nuclei is examined. The Dirac-Hartree mean-field coupling constants of Nikolaus, Hoch, and Madland (NHM) are scaled in accordance with the QCD-based prescription of Manohar and Georgi. Whereas the nine empirically-based coupling constants of NHM span thirteen orders of magnitude, the scaled coupling constants are almost all *natural*, being dimensionless numbers of order one. We speculate that this result provides good evidence that QCD and chiral symmetry apply to finite nuclei.
Although QCD is widely believed to be the underlying theory of the strong interaction, a direct description of nuclear properties in terms of the natural degrees of freedom of that theory, quarks and gluons, has proven elusive. The problem is that at sufficiently low energy, the physical degrees of freedom of nuclei are nucleons and (intranuclear) pions. Nevertheless, QCD can be mapped onto the latter Hilbert space and the resulting effective field theory is capable in principle of providing a dynamical framework for nuclear calculations. This framework is usually called chiral perturbation theory (\(\chi PT\)).

Two organizing principles govern this \(\chi PT\): (1) (broken) chiral symmetry (which is manifest in QCD) and (2) an expansion in powers of \(Q/\Lambda\), where \(Q\) is a general intranuclear momentum or pion mass, and \(\Lambda\) is the generic QCD large-mass scale \(\sim 1\) GeV, which in a loose sense indicates the transition region between the two alternative sets of degrees of freedom indicated above (that is, quark-gluon versus nucleon-pion). Typically, one constructs Lagrangians (that is, interactions) that display (broken) chiral symmetry and retains only those terms with exponents less than or equal to some fixed power of \((1/\Lambda)\). The chiral symmetry itself provides a crucial constraint: a general term has the structure \(\sim (Q/\Lambda)^N\) and \(N \geq 0\) is mandated. This guarantees that higher-order constructions in perturbation theory (viz., loops) will have even higher (not lower) powers of \((Q/\Lambda)\). The price one pays for this mapping from natural to effective degrees of freedom is an infinite series of interaction terms, where coefficients are unknown and must be determined from experiment.

To date only a few nuclear calculations have been performed within this framework. The seminal work of Weinberg [1] highlighted the role of power counting and chiral symmetry in weakening N-body forces. That is, two-nucleon forces are stronger than three-nucleon forces, which are stronger than four-nucleon forces, \(\cdots\) . This chain makes nuclear physics tractable. Van Kolck and collaborators [2] developed a nuclear potential model, including one-loop (two-pion exchange) contributions. Friar and Coon [3] developed non-adiabatic two-pion-exchange forces, while van Kolck, Friar and Goldman [4] examined isospin violation in the nuclear force. Rho, Park, and Min [5] were the first to treat external electromagnetic and weak interactions with nuclei. Essentially all of this work was focused on few-nucleon
systems, where computational techniques are sophisticated. Only the work of Lynn on (nuclear) chiral liquids was specifically directed at heavier nuclei and, more recently, Gelmini and Ritzi have calculated nuclear matter properties using lowest order nonlinear chiral effective Lagrangians.

Is there any evidence for chiral symmetry or QCD scales in finite nuclei? The tractability and astonishing success of the recent few-nucleon calculations of $^2$H, $^3$H, $^3$He, $^4$He, $^5$He, $^6$He, $^6$Li, and $^6$Be with only a weak three-nucleon force and no four-nucleon force confirms Weinberg’s power-counting prediction and yields strong but indirect evidence for chiral symmetry. The work of Lynn established a procedure for going beyond few-nucleon systems. Nuclear (N-body) forces either have zero range or are generated by pion exchange. Following Manohar and Georgi we can scale a generic Lagrangian component as

$$\mathcal{L} \sim -c_{lmn} \left[ \frac{\bar{\psi} \psi}{f_\pi^2 \Lambda} \right]^l \left[ \frac{\bar{\pi} \pi}{f_\pi} \right]^m \left[ \frac{\partial^\mu, m_\pi}{\Lambda} \right]^n f_\pi^2 \Lambda^2$$

(1)

where $\psi$ and $\pi$ are nucleon and pion fields, respectively, $f_\pi$ and $m_\pi$ are the pion decay constant, 92.5 MeV, and pion mass, 139.6 MeV, respectively, $\Lambda \sim 1$ GeV has been discussed above, and $(\partial^\mu, m_\pi)$ signifies either a derivative or a power of the pion mass. Dirac matrices and isospin operators (we use $\vec{t}$ here rather than $\vec{\tau}$) have been ignored. Chiral symmetry demands

$$\Delta = l + n - 2 \geq 0 .$$

(2)

Thus the series contains only positive powers of $(1/\Lambda)$. If the theory is natural, the dimensionless coefficients $c_{lmn}$ are of order (1). Thus, all information on scales ultimately resides in the $c_{lmn}$. If they are natural, scaling works. Our limited experience with nuclear-force models suggests that natural coefficients are the rule.

Unfortunately, zero-range nuclear-force models are not widely used. However, a recent calculation has been performed using zero-range forces for an extended range of mass number $A$ and this work provides significant new information on QCD and chiral symmetry in nuclei. Nikolaus, Hoch, and Madland (NHM) used a series of zero-range interactions to perform
Dirac-Hartree calculations in mean-field approximation for a total of fifty-seven nuclei. Their Lagrangian [using their notation] is given by

\[ \mathcal{L} = \mathcal{L}_{\text{free}} + \mathcal{L}_{4f} + \mathcal{L}_{\text{hot}} + \mathcal{L}_{\text{der}} + \mathcal{L}_{\text{em}} , \]  

(3)

where \( \mathcal{L}_{\text{free}} \) and \( \mathcal{L}_{\text{em}} \) are the kinetic and electromagnetic terms, respectively, and

\[ \mathcal{L}_{4f} = -\frac{1}{2} \alpha_S (\bar{\psi} \gamma_\mu \psi)(\bar{\psi} \gamma_\mu \psi) - \frac{1}{2} \alpha_V (\bar{\psi} \gamma_\mu \psi)(\bar{\psi} \gamma_\mu \psi) - \frac{1}{2} \alpha_{TS} (\bar{\psi} \gamma_\mu \gamma_\nu \psi)(\bar{\psi} \gamma_\mu \gamma_\nu \psi) - \frac{1}{2} \alpha_{TV} (\bar{\psi} \gamma_\mu \gamma_\nu \psi)(\bar{\psi} \gamma_\mu \gamma_\nu \psi) , \]  

(4)

\[ \mathcal{L}_{\text{hot}} = -\frac{1}{3} \beta_S (\bar{\psi} \psi)^3 - \frac{1}{4} \gamma_S (\bar{\psi} \psi)^4 - \frac{1}{4} \gamma_V [(\bar{\psi} \gamma_\mu \psi)(\bar{\psi} \gamma_\mu \psi)]^2 , \]  

(5)

\[ \mathcal{L}_{\text{der}} = -\frac{1}{2} \delta_S (\partial_\mu \bar{\psi} \psi)(\partial^\mu \bar{\psi} \psi) - \frac{1}{2} \delta_V (\partial_\mu \bar{\psi} \gamma_\mu \psi)(\partial^\mu \bar{\psi} \gamma_\mu \psi) . \]  

(6)

In these equations, \( \psi \) is the nucleon field, the subscripts \( S \) and \( V \) refer to the isoscalar-scalar and isoscalar-vector densities, respectively, and the subscripts \( TS \) and \( TV \) refer to the isovector-scalar and isovector-vector densities, respectively, containing the nucleon isospin operator \( \vec{\tau} \). The nine coupling constants of the NHM Lagrangian were determined in a self-consistent procedure that solved the model equations for several nuclei simultaneously in a nonlinear least-squares adjustment algorithm with respect to measured ground-state observables (Table IV of Ref. [10]). The predictive power of the extracted coupling constants is quite good both for other finite nuclei and for the properties of saturated nuclear matter (see Tables VIII, IX, and XI of Ref. [10]).

\( \mathcal{L}_{4f} \) contains four two-nucleon–force terms corresponding to \( \Delta = 0 \), the first term of \( \mathcal{L}_{\text{hot}} \) is a three-nucleon–force term corresponding to \( \Delta = 1 \), whereas the remaining two terms are four-nucleon–force terms corresponding to \( \Delta = 2 \). Finally, \( \mathcal{L}_{\text{der}} \) contains two nonlocal two-nucleon–force terms, also corresponding to \( \Delta = 2 \). The derivative terms act on \( \bar{\psi} \psi \), rather than on just one of the fields, because the latter generate a factor \( E \cong M \), the nucleon mass, whereas the former generate an energy difference that is considerably smaller. The latter
terms would spoil the series in Eq. (1) since $M \cong \Lambda$. However, either by a transformation or by rearranging the series, this problem could in principle be eliminated [8].

The construction of the NHM Lagrangian was motivated by empirically-based improvements to a Walecka type scalar-vector model [11,12], but using contact (zero-range) interactions to allow treatment of the Fock (exchange) terms. It was not motivated either by power counting or by chiral symmetry. The pion degrees of freedom are ignored and the Lagrangian is not complete; additional operators in each order of $(1/\Lambda)^\Delta$ are possible. Specifically, the NHM Lagrangian, Eqs. (4)–(6), has four operators in order $(1/\Lambda)^0$, one operator in order $(1/\Lambda)^1$, and four operators in order $(1/\Lambda)^2$, constituting an incomplete mix of three different orders in $(1/\Lambda)$.

Nevertheless, a meaningful comparison can be made of the generic chiral Lagrangian given by Eqs. (1) and (2) and the NHM Lagrangian given by Eqs. (4)–(6), precisely because our test of naturalness does not care whether a specific $c_{lmn}$ coefficient is 0.5 or 2.0. Changing (refining) the model by adding terms would change all of the $c_{lmn}$, but the same test of naturalness still applies. Adding new terms would simply change a specific coefficient by an amount $\sim 1$ (or less).

The nine coupling constants of the NHM Lagrangian are shown in Table 1, both in dimensional and dimensionless form [the latter obtained by equating Eqs. (1) and (4)–(6), with $\Lambda = 1$ GeV, using isospin operators $\vec{t}$ in Eq. (1), and solving for $c_{lmn}$ in terms of $\alpha$, $\beta$, $\gamma$, and $\delta$]. In the former form they span more than thirteen orders of magnitude, while in the latter form six of the nine coupling constants can be regarded as natural. Only the very small $\alpha_{TS}$ and large $\gamma_S$ and $\gamma_V$ are unnatural. However, the sum of the latter appears to be natural, and we speculate that the difference may not be well determined in the least-squares adjustments to the measured observables. The unnaturally small $\alpha_{TS}$, if correct, would presuppose a symmetry to preserve its small value.

Although these results were not obtained as a test of chiral symmetry and QCD scales (NHM at that time were unaware of these developments) and hence are imperfect, they are conversely completely unbiased. This result is very indicative of the role of chiral symmetry
and QCD in finite nuclei. A systematic study of this approach is clearly indicated.
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### TABLE I. Optimized Coupling Constants for the NHM Lagrangian and Corresponding Dimensional Power Counting Coefficients and Chiral Expansion Order

| Coup. Const. | Magnitude       | Dimension | $c_{lmn}$ | Order |
|--------------|-----------------|-----------|-----------|-------|
| $\alpha_S$  | $-4.508 \times 10^{-4}$ | MeV$^{-2}$ | -1.93     | $\Lambda^0$ |
| $\alpha_{TS}$ | $7.403 \times 10^{-7}$ | MeV$^{-2}$ | 0.013     | $\Lambda^0$ |
| $\alpha_V$  | $3.427 \times 10^{-4}$ | MeV$^{-2}$ | 1.47      | $\Lambda^0$ |
| $\alpha_{TV}$ | $3.257 \times 10^{-5}$ | MeV$^{-2}$ | 0.56      | $\Lambda^0$ |
| $\beta_S$   | $1.110 \times 10^{-11}$ | MeV$^{-5}$ | 0.27      | $\Lambda^{-1}$ |
| $\gamma_S$  | $5.735 \times 10^{-17}$ | MeV$^{-8}$ | 8.98      | $\Lambda^{-2}$ |
| $\gamma_V$  | $-4.389 \times 10^{-17}$ | MeV$^{-8}$ | -6.87     | $\Lambda^{-2}$ |
| $\delta_S$  | $-4.239 \times 10^{-10}$ | MeV$^{-4}$ | -1.81     | $\Lambda^{-2}$ |
| $\delta_V$  | $-1.144 \times 10^{-10}$ | MeV$^{-4}$ | -0.49     | $\Lambda^{-2}$ |