A Lower Bound on sin 2\(\beta\) from Minimal Flavour Violation

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Abstract

We point out that there exists an absolute lower bound on \(\sin 2\beta\) in all models with minimal flavour violation (MFV), that do not have any new operators beyond those present in the Standard Model and in which all flavour changing transitions are governed by the CKM matrix with no new phases beyond the KM phase. This bound depends only on \(|V_{cb}|, |V_{ub}/V_{cb}|\) and the hadronic parameters \(\bar{B}_K, F_{B_d} \sqrt{\bar{B}_d}\) and \(\xi\) relevant for the CP-violating parameter \(\varepsilon\) and the \(B^0_{d,s} - \bar{B}^0_{d,s}\) mixings. Performing a simple scanning over the present ranges of these parameters we find \(\sin 2\beta \geq 0.34\).

We illustrate how this bound could become stronger when our knowledge of the parameters in question improves and when the upper bound on the \(B^0_s - \bar{B}^0_s\) mixing ((\(\Delta M\))\(_s\)) will be experimentally known. Provided the future accurate measurements of \(\sin 2\beta\) through the CP asymmetry in \(B^0_d(\bar{B}^0_d) \rightarrow \psi K_S\) will confirm the low values recently reported by BaBar and Belle, there is a likely possibility that this class of models will be excluded. This would firmly imply the necessity of new CP-violating phases and/or new effective operators in the weak effective Hamiltonians for \(K^0 - \bar{K}^0\) and \(B^0_{d,s} - \bar{B}^0_{d,s}\) mixings. We also point out that within the MFV models there exists also an absolute lower bound on the angle \(\gamma\). We find \(\sin \gamma \geq 0.24\). This lower bound could become stronger in the future.
1 Introduction

The recent measurements of the time dependent CP asymmetry $a_{\psi K_S}$ in $B_0^0(\bar{B}_d^0) \rightarrow \psi K_S$ decays by BaBar [1] and Belle [2] indicate that the value of the angle $\beta$ in the unitarity triangle could turn out to be substantially smaller than expected on the basis of the standard analysis of the unitarity triangle within the Standard Model (SM) and the CDF measurement [3] of $a_{\psi K_S}$ reported last year. Indeed the measurements

$$ (\sin 2\beta)_{\psi K_S} = \begin{cases} 0.12 \pm 0.37 \pm 0.09 & \text{(BaBar) [1]}, \\ 0.45 \pm 0.44 \pm 0.08 & \text{(Belle) [2]}, \\ 0.79 \pm 0.42 & \text{(CDF) [3]} \end{cases} $$

imply the grand average

$$ (\sin 2\beta)_{\psi K_S} = 0.42 \pm 0.24. $$

This should be compared with the results of global analyses of the unitarity triangle within the SM which dependent on the error estimates give

$$ (\sin 2\beta)_{\text{SM}} = \begin{cases} 0.75 \pm 0.06 & \text{[4]}, \\ 0.73 \pm 0.20 & \text{[5]}, \\ 0.63 \pm 0.12 & \text{[6]} \end{cases} $$

where the last two results represent 95% C.L. ranges. Similar results can be found in [7, 8]. Clearly, in view of the large spread of experimental results and large statistical errors in (1), the SM estimates in (3) are compatible with the experimental value of $(\sin 2\beta)_{\psi K_S}$ in (2). Yet the small values of $\sin 2\beta$ found by BaBar and Belle might indicate new physics contributions to $B_0^0 - \bar{B}_d^0$ and $K^0 - \bar{K}^0$ mixings. In particular as discussed recently in several papers [3, 10, 11, 12] new CP violating phases in $B_d^0 - \bar{B}_d^0$ mixing could be responsible for small values of $\sin 2\beta$ in (2). Indeed in this case the asymmetry $a_{\psi K_S}$ measures $\sin 2(\beta + \theta_{\text{new}})$ and choosing appropriately $\theta_{\text{new}}$ one can obtain agreement with the results of BaBar and Belle.

On the other hand as stressed in [3] the SM estimates of $\sin 2\beta$ are sensitive to the assumed ranges for the parameters

$$ |V_{cb}|, \quad |V_{ub}/V_{cb}|, \quad \hat{B}_K, \quad \sqrt{\hat{B}_d F_{B_d}}, \quad \xi $$

that enter the standard analysis of the unitarity triangle. The parameter $\xi$ is defined in (18). While for “reasonable ranges” (see table 1) of these parameters, values of $\sin 2\beta \leq 0.5$ are excluded, such low values could still be possible within the SM if some of the parameters in (3) were chosen outside these ranges. In particular, for $|V_{ub}/V_{cb}| \leq 0.06$ or...
$\hat{B}_K \geq 1.3$ or $\xi \geq 1.4$, values lower than 0.5 for $\sin 2\beta$ could be obtained within the SM. We agree with these findings.

In the present letter we will assume the "reasonable ranges" for the parameters in (4) as given in table 1. The question then arises whether small values of $\sin 2\beta$ could still be obtained from an analysis of the unitarity triangle in the extensions of the SM which do not contain any new phases. In this context we would like to point out that there exists an absolute non-trivial lower bound for $\sin 2\beta$ in models with minimal flavour violation (MFV) [13, 14], that do not have any new operators beyond those present in the SM and in which all flavour changing transitions are governed by the CKM matrix [15] with no new phases beyond the KM phase. The SM, several versions of the Minimal Supersymmetric Standard Model (MSSM) and the Two Higgs Doublet Models I and II belong to this class. Interestingly, this absolute lower bound on $\sin 2\beta$, which basically follows from $\varepsilon$ and $(\Delta M)_d$, depends only on the parameters in (4) that are common to all models in this class. It can also be influenced significantly by the measurement of $(\Delta M)_s$.

Now, as pointed out recently in [14] there exists a universal triangle in this class of models that can be determined in the near future from the ratio $(\Delta M)_d/(\Delta M)_s$ and from $\sin 2\beta$ measured first through the CP asymmetry in $B^0_d \to \psi K_S$ [16] and later in $K \to \pi \nu \bar{\nu}$ decays [17]. Also suitable ratios of the branching ratios for $B \to X_{d,s} \nu \bar{\nu}$ and $B_{d,s} \to \mu^+ \mu^-$ and the angle $\gamma$ measured by means of CP asymmetries in B decays can be used for this determination [14]. In this context, the implications of the analysis presented below are fourth-fold:

- The present ranges for the parameters in (4) give a non-trivial lower bound for the angle $\beta$ in the universal unitarity triangle which reads

$$\sin 2\beta \geq 0.34 .$$

- This bound shows that even the central value in (2) can be accommodated in principle within the MFV models. No new CP violating phases are necessary.

- On the other hand, as illustrated below, the reduction in the uncertainties of the parameters in (4) and the measurement of $(\Delta M)_s$ may result in a much stronger lower bound on $\sin 2\beta$, that may turn out to be inconsistent with the future improved experimental values of $(\sin 2\beta)_{\psi K_S}$.

The implications of the latter possibility would be rather profound. With the measurement of $(\sin 2\beta)_{\psi K_S}$ alone one would be able to conclude that one has to go beyond the concept of the MFV and that new CP violating phases and/or new local operators in the weak
effective Hamiltonians for $K^0 - \bar{K}^0$ and $B_{d,s}^0 - \bar{B}_{d,s}^0$ mixings are necessary to describe the data.

Finally

- There exists an absolute lower bound on the angle $\gamma$ which reads

$$\sin \gamma \geq 0.24 .$$  \hspace{1cm} (6)

Also this bound could become stronger in the future.

## 2 The Lower Bound on $\sin 2\beta$

In order to demonstrate that a lower bound on $\sin 2\beta$ exists in the MFV models we use the Wolfenstein parametrization [18] of the CKM matrix and its generalization to include higher order terms in $\lambda$ [19]. Two of the Wolfenstein parameters, $\lambda$ and $A$, are determined from semi-leptonic K and B decays sensitive to the elements $|V_{us}|$ and $|V_{cb}|$ respectively:

$$\lambda = |V_{us}| = 0.22, \quad A = \frac{|V_{cb}|}{\lambda^2} = 0.826 \pm 0.041 .$$  \hspace{1cm} (7)

As the decays in question are tree level decays with large branching ratios this determination is to an excellent approximation independent of any possible physics beyond the SM.

The remaining two parameters, $\bar{\varrho}$ and $\bar{\eta}$, describe the unitarity triangle. In particular, the apex of this triangle, as shown in fig. 1, is given by [19]

$$\bar{\varrho} = \varrho (1 - \frac{\lambda^2}{2}) , \quad \bar{\eta} = \eta (1 - \frac{\lambda^2}{2}) .$$  \hspace{1cm} (8)

The lengths CB, CA and BA are equal respectively to

$$1, \quad R_b \equiv \sqrt{\bar{\varrho}^2 + \bar{\eta}^2} = (1 - \frac{\lambda^2}{2}) \frac{1}{\lambda} \frac{|V_{ub}|}{|V_{cb}|} , \quad R_t \equiv \sqrt{(1 - \bar{\varrho})^2 + \bar{\eta}^2} = \frac{1}{\lambda} \frac{|V_{td}|}{|V_{cb}|} .$$  \hspace{1cm} (9)

Now, the experimental value for the CP violating parameter $\varepsilon$ combined with the theoretical calculation of box diagrams describing $K^0 - \bar{K}^0$ mixing gives the constraint for $(\bar{\varrho}, \bar{\eta})$ in the form of the following hyperbola [1]

$$\bar{\eta} \left[(1 - \bar{\varrho}) A^2 \eta_2 F_{tt} + P_c(\varepsilon)\right] A^2 \hat{B}_K = 0.226 .$$  \hspace{1cm} (10)

Here $\hat{B}_K$ is a non-perturbative parameter and $P_c(\varepsilon) = 0.31 \pm 0.05$ [20] summarizes charm–charm and charm–top contributions in the SM. The new physics contributions to $P_c(\varepsilon)$ are negligible within the class of the MFV models considered here [14]. Most important for our considerations is the function $F_{tt}$ that in the SM results from box diagrams with top
Figure 1: Unitarity Triangle.

Next, the measurement of the $B^0_d - \bar{B}^0_d$ mixing (the mass difference $(\Delta M)_d$) determines $R_t$ in the unitarity triangle of fig. 1 through

$$R_t = 1.26 \frac{R_0}{A} \frac{1}{\sqrt{F_{tt}}}$$

where

$$R_0 = \sqrt{\frac{(\Delta M)_d}{0.47/\text{ps}}} \left[ \frac{200 \text{ MeV}}{F_{B_d} \sqrt{F_{tt}} \hat{B}_d} \right] \sqrt{0.55} \frac{\eta_B}{\eta_B}.$$  

Here $F_{tt}$ is the function present also in (10), $\hat{B}_d$ is a non-perturbative parameter analogous to $\hat{B}_K$, $F_{B_d}$ is the $B_d$ meson decay constant and $\eta_B$ is the short distance QCD factor, calculated within the SM in [21, 22].

The most important feature of the formulae (10) and (11) relevant for the discussion below is that in the context of the standard analysis of the unitarity triangle the different MFV models can be characterized by the value of the function $F_{tt}$. Moreover, as explained below, the new physics effects cancel in the ratio $\eta_2/\eta_B$.

The fact that new physics effects in $\varepsilon$ and $B^0_{d,s} - \bar{B}^0_{d,s}$ mixings within the MFV models can be described by a single function has been stressed in particular by Ali and London [5], who introduced the quantity $f$ related to $F_{tt}$ through

$$F_{tt} = S_0(m_t) (1 + f).$$

Here $S_0$ is the Inami-Lim function [23] resulting within the SM from box diagrams with top quark exchanges. In the SM $f = 0$. While Ali and London argued that the universality
of the quantity \( f \) is approximate, we would like to stress that in fact in each order of perturbation theory \( f \) and consequently \( F_{tt} \) must be exactly the same for \( \varepsilon, B_d^0 - B_d^0 \) mixing and \( B_s^0 - \bar{B}_s^0 \) mixing. This follows from the fact that at scales \( \mathcal{O}(M_W) \) and higher scales, at which new particles are integrated out, Wilson coefficients of the relevant \( \Delta F = 2 \) operators proportional to \((V_{ts}^*V_{td})^2, (V_{tb}^*V_{td})^2 \) and \((V_{tb}^*V_{ts})^2 \) are exactly the same. The differences between these three cases show up only in the hadronic parameters in (14), which are not included in \( F_{tt} \) but factored out as seen in (10) and (11). Similarly the QCD corrections \( \eta_2 \) and \( \eta_B \) differ only from each other by different renormalization group evolutions below the scales \( \mathcal{O}(M_W) \) and consequently the ratio \( \eta_2/\eta_B \) does not depend on new physics contributions.

In view of these remarks it will be convenient in what follows to use the SM values \( \eta_2 = 0.57 \) \([21]\), \( \eta_B = 0.55 \) \([21, 22]\) and absorb all QCD corrections related to new physics contributions into \( F_{tt} \).

We are now ready to demonstrate the existence of the lower bound on \( \sin 2\beta \) in question. Noting that

\[
\sin 2\beta = \frac{2\bar{\eta}(1 - \bar{\rho})}{R_t^2}
\]

and combining (14) and (11) one finds [19]

\[
\sin 2\beta = \frac{1.26}{R_t^2 \eta_2} \left[ \frac{0.226}{A^2 B_K} - \bar{\eta} P_c(\varepsilon) \right]
\]

whereby the first term in the parenthesis is larger than the second term by a factor of 2–3. This dominant term is independent of \( F_{tt} \) and involves the QCD corrections only in the ratio \( \eta_2/\eta_B \). Consequently it is independent of \( m_t \) and the new parameters in the extensions of the SM. The dependence on new physics is only present in \( \bar{\eta} \) entering the second term that would be absent if the charm contribution to \( \varepsilon \) was negligible. In particular for \( \bar{\rho} > 0 \), the value of \( \bar{\eta} \) decreases with increasing \( F_{tt} \). In principle new physics could also contribute to \( P_c(\varepsilon) \), but in all known examples of MFV models such contributions are negligible [14].

In spite of the sensitivity of the second term in (15) to new physics contributions, there exists an absolute lower bound on \( \sin 2\beta \) in the MFV models, simply because for \( B_K > 0 \) the unitarity of the CKM matrix implies

\[
0 \leq \bar{\eta} \leq R_b.
\]

At first sight one would think that the lower bound for \( \sin 2\beta \) is attained for \( \bar{\eta} = R_b \), but this is clearly not the case as \( \bar{\eta} \) depends on the values of the parameters in (14). Consequently there is a correlation between the values of the two terms in (15). Moreover, not
arbitrary values of $A$, $\hat{B}_K$, $\sqrt{B_q F_{B_d}}$ and $F_{tt}$ are simultaneously allowed by the constraints (10) and (11). For instance for given values of $A$, $R_b$ and $F_{tt}$ an approximate lower bound for $\hat{B}_K$ follows from (10) [24]:

$$\hat{B}_K \geq \left[A^2 R_b (2.6 A^2 F_{tt} + 1.4)\right]^{-1}.$$  \hspace{1cm} \text{(17)}$$

Thus while the discussion presented above demonstrates that an absolute lower bound on $\sin 2\beta$ in the MFV models exists, $(\sin 2\beta)_{\text{min}}$ for given values of the parameters in (4) can only be found numerically by using the constraints (10) and (11) and scanning $F_{tt}$ in the full range allowed by these constraints, the unitarity of the CKM matrix $(1 - R_b \leq R_t \leq 1 + R_b)$ and the size of the $B_s^0 - \bar{B}_s^0$ mixing. In the latter case $R_t$ can be determined by measuring $(\Delta M)_s$ and using

$$R_t = 0.82 \xi_{\text{eff}} \left[\frac{(\Delta M)_d}{0.47/\text{ps}}\right], \quad \xi_{\text{eff}} = \xi \left[\frac{14.6/\text{ps}}{(\Delta M)_s}\right], \quad \xi = \frac{F_{B_s} \sqrt{B_{B_s}}}{F_{B_d} \sqrt{B_{B_d}}}.$$  \hspace{1cm} \text{(18)}$$

The existing lower bound on $(\Delta M)_s$ implies an upper bound on $R_t$ that is much stronger than the unitarity bound $R_t \leq 1 + R_b$. Using the values of the parameters in table 1 we find $R_t^{\text{max}} = 1.03$. This bound eliminates the solutions with $\gamma \geq 90^\circ$. The range for $F_{tt}$ consistent with (11), (18) and $1 - R_b \leq R_t$ is then given by

$$\left[\frac{1.26 R_0}{A R_t^{\text{max}}}\right]^2 \leq F_{tt} \leq \left[\frac{1.26 R_0}{A (1 - R_b)}\right]^2.$$  \hspace{1cm} \text{(19)}$$

In practice the range for $F_{tt}$ consistent also with (10) is smaller than given in (19) as the upper limit in (19) corresponds to $\bar{q} = 0$.

Before presenting our numerical results we would like to recall that the weak sensitivity of $\sin 2\beta$ extracted from $\varepsilon$ and $B_s^0 - \bar{B}_s^0$ mixing within the SM to the value of the top quark mass has been stressed long time ago by Rosner [25]. The analytic formula for $\sin 2\beta$ in (15), exhibiting this feature, has been presented in [19]. Recently the weak dependence of $\sin 2\beta$ on new physics contributions in a number of SUGRA models belonging to the class of MFV models has been emphasized by Ali and London [5]. The formula (15), together with the range $0 \leq f \leq 0.75$ considered in [5], gives the explanation of their findings.

In the MSSM without any "SUGRA-constraints" the range $0 \leq f \leq 1.13$ is still allowed [26]. Moreover, it is conceivable that MFV models can be constructed for which $F_{tt}$ or $f$ are very different from those considered in [3] and [26]. In fact the bounds in (13) together with the values in table 1 give $1.3 \leq F_{tt} \leq 15.3$, or equivalently $-0.4 \leq f \leq 5.5$. In spite of this, our numerical analysis below demonstrates that $\sin 2\beta$ exhibits a rather moderate dependence on $F_{tt}$ in the full range allowed by (10), (11) and (19) and that an absolute non-trivial lower bound on $\sin 2\beta$ exists in the MFV models.
3 Numerical Analysis

In our numerical analysis we have used the standard parametrization of the CKM matrix [27] which is slightly more accurate than the improved Wolfenstein parametrization of [19]. In table [1], we list two ranges for the relevant input parameters corresponding to the present uncertainties and possible future reduced uncertainties. The relevant references to table [1] can be found in [27, 28]. The value of the running current top quark mass \( m_t \) defined at \( \mu = m_t^{\text{Pole}} \) is given here for completeness in order to allow the translation from \( F_{tt} \) to \( f \). With \( m_t = 165 \pm 5 \text{ GeV} \) one has

\[
F_{tt}^{\text{SM}} = S_0(m_t) = 2.35 \pm 0.11 .
\]  

(20)

We prefer to use \( F_{tt} \) instead of \( f \) as the small error on \( m_t \) can then be taken automatically into account. We are aware of the fact that other authors would possibly use slightly different ranges for the input parameters. Still, table [1] is representative for the present situation.

Finally, although \( |V_{ub}/V_{cb}| \) does not enter the formulae (10), (11) and (15) explicitly, its value enters our analysis in the same manner as in the standard analysis of the unitarity triangle [7]. See for instance (17) and (19).

Table 1: The ranges of the input parameters.

| Quantity         | Central | Present | Future |
|------------------|---------|---------|--------|
| \(|V_{cb}|      | 0.040   | ±0.002  | ±0.001 |
| \(|V_{ub}/V_{cb}| | 0.090   | ±0.018  | ±0.009 |
| \(\hat{B}_K\)   | 0.85    | ±0.15   | ±0.07  |
| \(\sqrt{\hat{B}_{k}F_{B_d}}\) | 200 MeV | ±40 MeV | ±20 MeV |
| \(m_t\)         | 165 GeV | ±5 GeV  | ±2 GeV |
| \((\Delta M)_{d}\) | 0.471/ps | ±0.016/ps | ±0.008 ps\(^{-1}\) |
| \((\Delta M)_{s}\) | > 14.6/ps  | > 16.6/ps  |
| \(\xi\)        | 1.16    | ±0.07   | ±0.04  |
| \(m_c(m_c)\)   | 1.30 GeV | ±0.05 GeV | ±0.05 GeV |

In fig. 2 we show \((\sin 2\beta)_{\text{min}}\) as a function of \( F_{tt} \) for the two choices of uncertainties in the input parameters given in table [1]. To this end we have scanned independently all the input parameters within the ranges of this table. We observe that the dependence of \((\sin 2\beta)_{\text{min}}\) on \( F_{tt} \) is rather weak. For \( F_{tt} \geq 13.5 \) (7.8) in the case of the present (future)
scanning there are no solutions for $\sin 2\beta$ in the ranges of the parameters considered. The absolut lower bound for $\sin 2\beta$ is found to be

$$
(sin 2\beta)_{\text{min}} = \begin{cases} 
0.34 & \text{Present}, \\
0.48 & \text{Future}.
\end{cases}
$$

We would like to emphasize that these bounds should be considered as conservative. After all they have been obtained by scanning independently all parameters in question. Had we used the error analysis of [4], the bounds in (21) would have been considerably stronger.

From fig. 2 one can extract the lower bounds for particular MFV models by setting $F_{tt}$ to the appropriate values. A number of supersymmetric MFV models has been reviewed by Ali and London [5], where references to the original literature can be found. Using the results of [5] we find as characteristic values $F_{tt} = 3.0$, $F_{tt} = 3.4$, $F_{tt} = 4.3$ for minimal SUGRA models, non-minimal SUGRA models and non-SUGRA models with EDM constraints respectively. Setting in addition $F_{tt} = 2.46$ and $F_{tt} = 5.2$ for the SM
and the MSSM version of [20] respectively, we obtain

$$(\sin 2\beta)_{\text{min}} = \begin{cases} 
0.57 (0.64) & \text{SM}, \\
0.53 (0.62) & \text{MSUGRA}, \\
0.50 (0.59) & \text{NMSUGRA}, \\
0.44 (0.54) & \text{NSUGRA}, \\
0.40 (0.49) & \text{MSSM}.
\end{cases}$$

The numbers in parentheses correspond to the future ranges of the input parameters. Our results for the models analyzed in [5] are compatible with the ranges for $\sin 2\beta$ presented there. We observe that the present $(\sin 2\beta)_{\text{min}}$ in the NSUGRA models and in the MSSM are in the ball park of the grand average in (2). The anatomy of the bound is given in table 2, where we show $(\sin 2\beta)_{\text{min}}$ as a function of $\hat{B}_K$ and $|V_{cb}|$ with all the remaining parameters scanned within the present and future ranges. There is no solution for the set of parameters corresponding to the last entry in table 2. The numbers in parentheses are discussed below. On the basis of this table and additional numerical analysis we find the following features in accordance with (15):

- $(\sin 2\beta)_{\text{min}}$ decreases with increasing $\hat{B}_K$ and $|V_{cb}|$ and decreasing $|V_{ub}/V_{cb}|$ and $F_{B_d}\sqrt{\hat{B}_d}$.

- In the ranges considered, the dependence of $(\sin 2\beta)_{\text{min}}$ on $\hat{B}_K$, $|V_{cb}|$ and $F_{B_d}\sqrt{\hat{B}_d}$ is stronger than on $|V_{ub}/V_{cb}|$. This is evident from (15) in which $|V_{ub}/V_{cb}|$ is not explicitly present but affects the bound only through the value of $\bar{\eta}$ in the sub-leading term and through its impact on the allowed ranges of the remaining parameters.

- One can check that the dependence of $(\sin 2\beta)_{\text{min}}$ on $\hat{B}_K$ and $A$ can be approximately given by a single variable $\tau = A^2\hat{B}_K$. This is clear from (15). The observed small departure from this regularity is caused by the correlations between various parameters as discussed below equation (15).

Next we would like to emphasize that the measurement of $B^0_s - \bar{B}^0_s$ mixing may significantly improve the lower bound on $\sin 2\beta$ considered here. Measuring $R_t$ by means of (18) could provide a lower bound on $R_t$ in addition to the known upper bound. This in turn would exclude high values of $F_{tt}$ as seen in (14). The numbers in the parentheses in table 3 show the impact of the $(\Delta M)_s$ measurement with $\xi_{\text{eff}} \geq 1.0$, or equivalently $(\Delta M)_s \leq 19.6/\text{ps}$ for $\xi = 1.16$. We observe that the impact of the measurement of $(\Delta M)_s$ is very large, in particular in the case of the present scenario. The values $\sin 2\beta \leq 0.55$ are excluded. With increasing $\xi_{\text{eff}}$ the impact becomes stronger. It is weaker for smaller
Table 2: Values of $(\sin 2\beta)_{\text{min}}$ in the MFV models for specific values of $\hat{B}_K$ and $|V_{cb}|$ with remaining parameters in the “present” and ”future” ranges. The values in the parentheses show the impact of the measurement of $(\Delta M)_s$ with $\xi_{\text{eff}} \geq 1.0$.

| $\hat{B}_K$ | $|V_{cb}|$ | Present | Future |
|------------|---------|---------|--------|
| 0.70       | 0.038   | 0.59 (0.62) | 0.70 (0.70) |
|            | 0.040   | 0.54 (0.58) | 0.65 (0.65) |
|            | 0.042   | 0.49 (0.55) | 0.59 (0.61) |
| 0.85       | 0.038   | 0.49 (0.62) | 0.60 (0.68) |
|            | 0.040   | 0.44 (0.58) | 0.54 (0.65) |
|            | 0.042   | 0.40 (0.55) | 0.49 (0.61) |
| 1.00       | 0.038   | 0.42 (0.62) | 0.51 (0.68) |
|            | 0.040   | 0.38 (0.58) | 0.46 (0.64) |
|            | 0.042   | 0.34 (0.55) | 0.42 (—) |

$\xi_{\text{eff}}$, but even for $\xi_{\text{eff}} = 0.90$, $\sin 2\beta \leq 0.41$ and $\sin 2\beta \leq 0.51$ are excluded in the present and future scenario respectively.

We note that the previously found dependence of $(\sin 2\beta)_{\text{min}}$ on $\hat{B}_K$ and $|V_{cb}|$ is strongly affected by the lower bound on $R_t$. In particular the dependence on $\hat{B}_K$ is negligible. This is related to the fact that with $\xi_{\text{eff}} \geq 1.0$, $R_t$ is confined to $0.82 \leq R_t \leq 1.03$ and $(\sin 2\beta)_{\text{min}}$ is governed by the values of $R_t$ and $R_b$. With decreasing $\xi_{\text{eff}}$ the $\hat{B}_K$ dependence becomes again visible.

One remark on the $\xi_{\text{eff}}$ dependence is in order. As we have seen $(\sin 2\beta)_{\text{min}}$ increases with increasing $\xi_{\text{eff}}$. This feature is valid only for $\gamma$ in the first quadrant. For $\gamma$ in the second quadrant $(\sin 2\beta)_{\text{min}}$ decreases with increasing $\xi_{\text{eff}}$, as already noticed in [9].

Finally we would like to point out that the absolute lower bound on $\sin 2\beta$ implies within the MFV models an absolute lower bound on the angle $\gamma$. We find

$$
(\sin \gamma)_{\text{min}} = \begin{cases} 
0.24 & \text{Present,} \\
0.39 & \text{Future,} 
\end{cases}
$$

(23)

with $\gamma$ in the first quadrant. The second quadrant in the MFV models is excluded through the lower bound on $(\Delta M)_s$. 

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In analogy to (22) we find

\[
\sin \gamma \min = \begin{cases} 
0.68 (0.82) & \text{SM}, \\
0.58 (0.70) & \text{MSUGRA}, \\
0.53 (0.63) & \text{NMSUGRA}, \\
0.46 (0.54) & \text{NSUGRA}, \\
0.40 (0.47) & \text{MSSM}.
\end{cases}
\]  

The anatomy of \((\sin \gamma)_\min\) is given in table 3. As in the case of \((\sin 2\beta)_\min\) the impact of the measurement of \((\Delta M)_s\) is very significant.

Table 3: Values of \((\sin \gamma)_\min\) in the MFV models for specific values of \(\hat{B}_K\) and \(|V_{cb}|\) with remaining parameters in the “present” and ”future” ranges. The values in the parentheses show the impact of the measurement of \((\Delta M)_s\) with \(\xi_{\text{eff}} \geq 1.0\).

| \(\hat{B}_K\) | \(|V_{cb}|\) | Present | Future |
|----------------|----------------|---------|---------|
| 0.70           | 0.038          | 0.40 (0.77) | 0.62 (0.80) |
|                | 0.040          | 0.38 (0.76) | 0.58 (0.78) |
|                | 0.042          | 0.37 (0.74) | 0.54 (0.77) |
| 0.85           | 0.038          | 0.31 (0.77) | 0.45 (0.79) |
|                | 0.040          | 0.30 (0.76) | 0.43 (0.78) |
|                | 0.042          | 0.29 (0.74) | 0.42 (0.77) |
| 1.00           | 0.038          | 0.25 (0.77) | 0.37 (0.79) |
|                | 0.040          | 0.25 (0.76) | 0.35 (0.78) |
|                | 0.042          | 0.24 (0.74) | 0.34 (—) |

4 Conclusions

We have pointed out that there exists an absolute lower bound on \(\sin 2\beta\) in the MFV models, that do not have any new operators beyond those present in the Standard Model and in which all flavour changing transitions are governed by the CKM matrix with no new phases beyond the KM phase. This bound depends only on \(|V_{cb}|\), \(|V_{ub}/V_{cb}|\) and the non-perturbative parameters \(\hat{B}_K\), \(F_{Bd}\sqrt{\hat{B}_d}\) and \(\xi\) relevant for the CP-violating parameter \(\varepsilon\) and the \(B^0_{d,s} - \bar{B}^0_{d,s}\) mixings. The present ranges of these parameters imply \(\sin 2\beta \geq 0.34\).

We have illustrated how the lower bound on \(\sin 2\beta\) could become stronger when our knowledge of the input parameters in question improves and when the upper bound on
(ΔM)_s will be experimentally known. In particular, if the upper bounds on ̂B_K and
|V_{cb}| and lower bounds on |V_{ub}/V_{cb}|, F_{Bd}\sqrt{B_d} and ξ_{eff} in (18) will be improved, (sin 2β)_{min}
will be shifted above 0.5. Consequently, provided the future accurate measurements of
a_{ψK_S} will confirm the low values reported by BaBar and Belle, there is a likely possibility
that all MFV models will be excluded. This would firmly imply the necessity of new
CP-violating phases and/or new effective operators in the weak effective Hamiltonians for
K^0 - ̄K^0 and B_{d,s}^0 - ̄B_{d,s}^0 mixings.

We have also pointed out that the lower bound on sin 2β implies within the MFV
models an absolute lower bound on the angle γ. This provides an additional test of the
MFV models once precise measurements of γ will be available.

Clearly other measurements, in particular those of the rare decay branching ratios and
various CP asymmetries, will have an additional impact on the analysis presented here,
but this is a different story. For a very recent review see [29].

It will be exciting to watch the experimental progress in the values of a_{ψK_S} and (ΔM)_s
and the theoretical progress on ̂B_K, |V_{ub}/V_{cb}|, F_{Bd}\sqrt{B_d} and ξ. Possibly we will know
already next summer that new CP violating phases and/or new operators in the effective
weak Hamiltonians are mandatory.

We would like to thank Martin Gorbahn for useful discussions and critical comments
on the manuscript. This work has been supported in part by the German Bundesminis-
terium für Bildung and Forschung under the contract 05HT9WOA0 and by the Deutsche
Forschungsgemeinschaft under grant No. SFB 375.

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