Primordial gravitational waves and the $H_0$-tension problem

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We analyse the $H_0$-tension problem in the context of models of the early universe that predict a blue tilted spectrum of primordial gravitational waves (GW’s). By considering the GW’s contribution, $N_{GW}^{\text{eff}}$, to the effective number of relativistic species, $N_{\text{eff}}$, and assuming standard particle physics, we discuss the effects of $N_{GW}^{\text{eff}}$ on the background expansion, especially the constraints on the Hubble parameter $H_0$. We analyse three scenarios which take into account the contribution of $N_{GW}^{\text{eff}}$ and perform a statistical study using recent data of cosmic microwave background, baryon acoustic oscillation, the latest measurement of the local expansion rate, along with the LIGO constraints on the tensor to scalar ratio, $r$, and the tensor index, $n_T$. For the models explored, we show that an additional contribution from the primordial GW’s background to $N_{\text{eff}}$ does not solve but alleviate the current $H_0$-tension problem.

I. INTRODUCTION

The search for gravitational waves (GW’s) is one of the main goals of modern cosmology. Along with current Cosmic Microwave Background (CMB) data, the results from experiments such as LIGO [13] and VIRGO [4] are able to provide constraints not only on models of the primordial universe, imposing limits on the tensor to scalar ratio, $r$, and on the tensor spectral index, $n_T$ [1], but also on the physics of the late-time cosmic acceleration (see, e.g., [7] [8]). Currently, some of these experiments have allowed to test the viability of several models of the early universe, and the present constraints are compatible with both the simplest slow-roll scenarios of inflation [9] and some alternative scenarios [10,11,12].

In this regard, it is worth mentioning that the recent CMB polarization data have excluded a much larger range of red tensor tilt values than of blue ones [17,18]. A possible detection of a blue tensor tilt in the future would rule out a large class of models of inflation, since the so-called consistency relation would not be fulfilled in the standard scenario without violating the null energy condition. Future experiments like Hanford-Livingston Virgo (HLV) network [19] (and also the Hanford-Livingston Virgo-Japan-India (HLVJ) network), the future space-based detector DECIGO [20], satellite missions such as LiteBIRD [21] and ground based experiments such as CMB-S4 [22], are expected to give a great improvement to the current limits on primordial GW’s (see also [23,24]).

The effects of the GW’s background on the CMB and matter power spectra are similar to those of massless neutrinos, unless the initial density-perturbation amplitude for the GW’s gas is non-adiabatic [27,28]. The primordial GW’s also affect the expansion of the universe. Since they are relativistic degrees of freedom, they add to the effective number of relativistic species, $N_{\text{eff}} + N_{GW}^{\text{eff}}$, which increases the radiation energy density and decreases the redshift of matter-radiation equality, according to the relation: $1 + z_{eq} = \Omega_{m}/\Omega_{r} = \Omega_{m}/\Omega_{r}(1 + 0.2271N_{\text{eff}})$. In standard inflationary models, which predicts a small red tensor tilt, the contribution from $N_{GW}^{\text{eff}}$ is negligible. However the same is not generally true for models that predict a primordial spectrum with a blue tensor tilt (see, e.g., [13,16,17]).

Another important aspect worth considering is that a higher contribution of $N_{eff}$ in the early universe leads to a faster expansion and hence a smaller value of the sound horizon at recombination, $r_s$, and a consequent higher value of the current expansion rate, $H_0$ [33]. For this reason, higher values of $N_{\text{eff}}$ have been considered as an attempt to alleviate the so-called $H_0$-tension problem [33,34], i.e., the discrepancy on the Hubble parameter value predicted in the context of the $\Lambda$CDM model using the Cosmic Microwave Background (CMB) data, i.e., $H_0 \approx 66.93 \pm 0.62 \text{ km.s}^{-1}\text{Mpc}^{-1}$ [37] and the value obtained from different geometric distance calibrations of Cepheids using observations of the Hubble Space Telescope (HST), $H_0 = 73.52 \pm 1.62 \text{ km.s}^{-1}\text{Mpc}^{-1}$ [38,39] (see also [40] and references therein for a overview of the discussion). While previous data from the Planck collaboration prefer larger values of $N_{\text{eff}}$ [31,32], the latest data released [41] show a preference for smaller values. Despite of that, the current data still allow values of $N_{\text{eff}}$ higher than the one predicted by standard model ($N_{\text{eff}} = 3.046$). Although extra neutrinos species are not predicted by the standard particle physics, primordial GW’s with a blue tensor spectrum can naturally provide a significant contribution for $N_{\text{eff}}$ and, therefore, may alleviate the current $H_0$-tension problem.

1 The contribution of $N_{GW}^{\text{eff}}$ to the radiation content of the universe also affects the predictions of the primordial nucleosynthesis (BBN) [31,32].
A careful analysis of some possible effects of a large positive tilt on the background expansion was performed in Ref. [6] and in Ref. [27]. Here, we focus on the impact of $N_{GW}^{UV}$ on the constrains on $H_0$ in light of the recent Planck + HST results. We concentrate on models that predict a primordial tensor spectrum with a blue tilt, since in this case a significant contribution of $N_{GW}^{eff}$ is expected. We also discuss the sensitivity of the results with respect to the choice of the ultra-violet (UV) frequency cutoff, $k_{UV}$. We organized this paper as follows. In section II we review the theoretical framework and discuss the basic assumptions considered. The method, observational dataset and priors used in the computational analysis are discussed in section III. In section IV we present our results and in section V we summarize our conclusions.

II. THEORY

The spectrum of cosmological perturbations is a fundamental quantity to connect the physics of the early universe with observations. The primordial GW’s are described by the dimensionless tensor power spectrum $P_t(k)$,

$$P_t(k) = A_t(k_s) \left( \frac{k}{k_s} \right)^{n_t(k_s)},$$

where the spectral index $n_t$ parametrizes the dependence of $P_t(k)$ on the comoving wavenumber $k$. This follows in analogy to the standard scalar spectrum parametrization, in which $A_s$ and $n_s$ describes the scalar amplitude and the scalar spectral index respectively, and the tensor to scalar ratio, $r = A_t/A_s$, is the quantity that relates the amplitude of both the tensor and the scalar spectrum. The consistency relation of standard inflation, $r = -8n_t$, implies that $n_t$ is always negative in these models. However the same is not true in the case of other early universe scenarios [13][16][18].

The primordial tensor spectrum can also be written as a function of the frequency $f$ in the following form,

$$P_t(f) = rA_s \left( \frac{f}{f_s} \right)^{n_t} = rA_s \left( \frac{f/Hz}{1.6 \times 10^{-17}} \right)^{n_t},$$

for a pivot scale $k_s = 0.01$ Mpc$^{-1}$ and $f/Hz = 1.6 \times 10^{-17} / k$/Mpc$^{-1}$. The energy density of gravitational waves is given by

$$\rho_{GW} = \int_0^{k_{UV}} d\log k \frac{P_t(k)}{32\pi G a^2} \left[T'(k, \eta)\right]^2,$$

where $k_{UV}$ is an ultra-violet cutoff whose values will be discussed later. By performing this integral using the conformal time derivative of transfer function during the radiation domination it is possible to show that [27],

$$\rho_{GW} = \frac{A_s r}{32\pi G} \left( \frac{k_{UV}}{k_s} \right)^{n_t} \frac{1}{2n_t(a\eta)^2} = \frac{A_s r}{24n_t} \left( \frac{k_{UV}}{k_s} \right)^{n_t} \rho_{tot},$$

where in the second equality we considered that during this period $1/(a\eta)^2 = H^2 = 8\pi G \rho_{tot}/3$. At this epoch the total energy density is given by the sum of photons, neutrinos and GW’s energy densities, $\rho_{tot} = \rho_\gamma + \rho_\nu + \rho_{GW}$, which can be rewritten as

$$\rho_{tot} = \rho_\gamma \left( 1 + \frac{7}{8} \left( \frac{4}{11} \right)^{4/3} N_{eff} \right).$$

By substituting the above expression in the r.h.s. of Eq. [4] we obtain

$$\rho_{GW} = \frac{A_s r}{24n_t} \left( \frac{k_{UV}}{k_s} \right)^{n_t} \left( 1 + \frac{7}{8} \left( \frac{4}{11} \right)^{4/3} N_{eff} \right) \rho_\gamma = \frac{7}{8} \left( \frac{4}{11} \right)^{4/3} \left( N_{eff} - 3.046 \right) \rho_\gamma.$$

In our analysis we assume that beyond photons and three families of standard model neutrinos, the radiation energy density is also made up of gravitational waves, with no extra relativistic degrees of freedom, i.e., $N_{eff} = 3.046 + N_{GW}^{eff}$. Solving for $N_{eff}$ we find

$$N_{eff} = \frac{8}{7} \left( \frac{11}{4} \right)^{4/3} \left[ \frac{A_s r}{24n_t} \left( \frac{k_{UV}}{k_s} \right)^{n_t} \right] + 3.046.$$

The above equations should only be trusted in the regime $\rho_{GW}/\rho_{tot} << 1$, which is the case considering the current observational constraints. We assume in our analysis that the power law form of the tensor power spectrum holds all the way up to the cutoff frequency $k_{UV}$ — see [42] for a discussion on different approaches. We also assume the standard thermal history with an instantaneous transition from the phase of the early universe responsible for producing the primordial spectrum to the phase of radiation domination.

By neglecting the contribution of gravitational waves to $\rho_{tot}$ in Eq. [4] one obtains the following approximation:

$$N_{eff} = 3.046 + \left[ 3.046 + \frac{8}{7} \left( \frac{11}{4} \right)^{4/3} \right] \frac{A_s r}{24n_t} \left( \frac{k_{UV}}{k_s} \right)^{n_t}.$$
FIG. 1. $N_{\text{eff}}$ as a function of $K_{\text{UV}}$ for some particular values of $n_T$.

The result of the above equation is illustrated in Fig. 1 for some selected values of the parameter $n_t$ with $r = 10^{-3}$ and $A_s = 2.2 \times 10^{-9}$. Clearly, there is a steep increase of $N_{\text{eff}}$ as a function of $k_{\text{UV}}$ showing that the bigger the value of $n_T$, the smaller the value of $k_{\text{UV}}$ at which this steep increase occurs.

Going back to Eq. (3), one needs to impose both IR and UV cutoffs to the integral in this equation. The only modes that contribute to the radiation energy density are those inside the horizon at a given time since those modes oscillate, propagating as massless modes. This implies that the IR cutoff is a time-dependent quantity. As a consequence, the contribution of $N_{\text{GW}}^{\text{eff}}$ that affects BBN is different from the contribution of $N_{\text{eff}}^{\text{GW}}$ that affects CMB, for example. However, in the case of blue tilted tensor spectrum the IR contribution to $\rho_{\text{GW}}$ is negligible compared to the UV contribution. Therefore, here we will focus only in the case of blue tilted spectrum since we are interested in analyzing the contribution of $N_{\text{eff}}^{\text{GW}}$, and such contribution is negligible in the case of red tensor tilt. We also consider that $N_{\text{eff}}^{\text{GW}}$ does not depend on time for a fixed UV cutoff.

For the UV cutoff several possibilities have been considered in the literature (see, e.g., [6, 27, 31]). Firstly, we perform a more general test, without assuming any specific value for the cutoff, in order to see how the results are sensitive to this choice. Looking at Fig. 2 we note that lower values of $k_{\text{UV}}$ allows higher values for $n_t$ and $r$. Due to this behavior, we choose to consider two very different values for the cutoff, which are both model independent. In the first one (hereafter Model 1), inspired by the approach of [31], we simply take it to be given by the GUT scale, i.e., $10^{16}$GeV. This corresponds to consider the ratio $k_{\text{UV}}/k_* \approx 10^{56}$ (or equivalently, $f_{\text{UV}} \approx 10^{40}$Hz). In a second case (hereafter Model 2), we consider the ratio $k_{\text{UV}}/k_* \approx 10^{24}$ ($f_{\text{UV}} \approx 10^{16}$Hz), as suggested in [27].

This latter choice follows from the assumption that the power law spectrum extends over $\approx 60$ e-folds, and is chosen not due to some particular mechanism for generating the gravitational waves in the early universe, but instead to correspond to the expected amount of expansion. This value, nevertheless, coincides approximately to the value chosen in [6] from inflationary arguments.

III. ANALYSIS

We work with the above values for $k_{\text{UV}}$ in order to verify how sensitive the results are to changes on the cutoff. Therefore, we consider two cases of the $\Lambda$CDM $+r + n_t$ model with $k_{\text{UV}}/k_* \approx 10^{56}$ (Model 1) and $k_{\text{UV}}/k_* \approx 10^{24}$ (Model 2). In both the contribution $N_{\text{GW}}^{\text{eff}}$ is treated as a derived parameter which is a function of $r$ and $n_t$. This model is parametrized by the free parameters $r$ and $n_t$ in addition to the usual set of cosmological parameters: the baryon density, $\Omega_b h^2$, the cold dark matter density, $\Omega_c h^2$, the ratio between the sound horizon and the angular diameter distance at decoupling, $\theta$, the optical depth, $\tau$, the primordial scalar amplitude, $A_s$, and the primordial spectral index $n_s$. In a second analysis we also consider the model $\Lambda$CDM $+r + n_t + k_{\text{UV}}$ in which the cutoff frequency is treated as a free parameter. In all cases we
analyze the constraints on the derived parameter $H_0$. We work with flat priors and purely adiabatic initial conditions, also fixing the sum of neutrino masses to 0.06eV.

In our analysis we use the more recent release of the package CosmoMC [43]. We use the CMB data set from a recent release of the Planck Collaboration [37] (hereafter “PLC”), considering the high-$\ell$ Planck temperature data and the low-$\ell$ data by the joint TT,EE,BB and TE likelihood. We use B-mode polarization data from the BICEP2 Collaboration [45], [46] to constraint the parameters associated to the tensor spectrum, using the combined BICEP2/Keck-Planck likelihood, hereafter BKP. We perform our analysis for a scalar and tensor pivot scale $k_*=0.01\text{Mpc}^{-1}$, since at this value the BKP release data are most sensitive and it is close to the decorrelation scale between the tensor amplitude and slope for Planck and BKP joint constraints [17]. We also consider a Gaussian prior on the optical depth parameter $\tau = 0.055 \pm 0.009$ recovered from Planck HFI large angular scale polarization measurements [41, 47]. In order to complement our analysis, we use Baryon Acoustic Oscillation (BAO) data from the 6dF Galaxy Survey (6dFGS) [48], Sloan Digital Sky Survey (SDSS) DR7 Main Galaxy Sample galaxies [49], BOSS galaxy samples, LOWZ and CMASS [50].

We also use in all of the analysis data from GW direct detection experiments. The ground-based interferometers can probe the primordial gravitational wave spectrum in a range of frequencies from 1Hz to $10^4$Hz, providing us with upper limits on the parameters $r$ and $n_t$. Here, in particular, we use data from LIGO [1–3]. We call all these data sets as data set 1. Pulsars also can be used to constraint the tensor parameters. However, as shown in [6] the upper limits provided by LIGO are more constraining than the ones provided by pulsars. For this reason we do not include data from pulsars in our analysis. In a second analysis we add to the data set 1, the Riess et al. results on the local expansion rate, $H_0 = 73.52 \pm 1.62 \text{km.s}^{-1}\text{Mpc}^{-1}$ (68% C.L.), based on direct measurements made with the Hubble Space Telescope and Gaia [89]. Hereafter we denote this data set, which include Riess et al. results (HST), as dataset 2.

IV. RESULTS AND DISCUSSION

The main results of our analysis are shown in Fig. (3), where confidence regions in the plane $n_T - H_0$ of the Model 1 (blue), Model 2 (red) and $\Lambda$CDM with relaxed consistency relation (gray) using the dataset 1 (left) and dataset 2 (right).

In addition to the cosmological parameters, we also vary the nuisance foregrounds parameters [44].

FIG. 3. Confidence regions (68% and 95%) for the plane $n_T - H_0$ of the Model 1 (blue), Model 2 (red) and $\Lambda$CDM with relaxed consistency relation (gray) using the dataset 1 (left) and dataset 2 (right).
TABLE I. Confidence limits (68\%) for the $H_0$ and $n_t$ parameters using the two data sets discussed in the text.

|          | $\Lambda$CDM + $r$ + $n_t$ | $k_{UV} = 10^{22}$ | $k_{UV} = 10^{24}$ |
|----------|----------------------------|-------------------|-------------------|
| dataset 1| $H_0$                      | 67.50 ± 0.55      | 67.69^{+0.53}_{-0.69} | 67.60 ± 0.56      |
|          | $n_t$                      | 0.60 ± 0.41       | 0.14^{+0.27}_{-0.10} | 0.05^{+0.71}_{-0.41} |
| dataset 2| $H_0$                      | 68.04 ± 0.52      | 68.62^{+0.57}_{-1.10} | 68.39^{+0.49}_{-0.95} |
|          | $n_t$                      | 0.59 ± 0.42       | 0.22^{+0.24}_{-0.07} | 0.03^{+0.13}_{-0.05} |

In Table I we show the best fit values for the $H_0$ and $n_t$ parameters with 68\% confidence limits obtained from the two analyzed data sets. We note that the $H_0$-tension problem is slightly alleviated by the contribution $N_{GW}^{\nu}$. The best-fits and mean values are not in the limiting region corresponding to the steep increase of $H_0$ obtained in [31].

In addition, the prior we consider for the optical spectrum can naturally provide a significant contribution to $N_{GW}^{\nu}$. In spite of this, Fig. (3) clearly shows that a contribution of $N_{GW}^{\nu}$ does have a role in alleviating the $H_0$-tension problem for models that predict $n_t$ in a certain range, i.e., $n_t \simeq O(10^{-1})$, with the specific values depending on $k_{UV}$.

As mentioned earlier, the GW's background energy density affects the expansion rate during the BBN. On the other hand, the BBN results imposes limits on the value of $N_{eff}$ which, from Eq. (8), implies constraints on $n_t$. Considering the cutoff choice $k_{UV} = 10^{22}\text{Mpc}^{-1}$ and a small value of the tensor to scalar ratio, e.g. $r \approx 10^{-3}$, we obtain from Eq. (8) that an upper limit of $N_{eff} < 5$ implies $n_t < 0.5$ whereas by considering the more restrictive limit suggested in [51], $N_{eff} < 3.56$, the constraint on $n_t$ changes only slightly to $n_t < 0.49$. On the other hand, in the same context, if we change the cutoff value to $k_{UV} = 10^{24}\text{Mpc}^{-1}$ we obtain $n_t < 0.2$, which is a very restrictive limit. Higher values of $r$ implies in more restrictive constraints on $n_t$, in agreement with the results obtained in [31].

Finally, it is worth mentioning that the BBN constraints on $n_t$ are particularly sensitive to the value of $k_{UV}$. This result can be understood from the behavior shown in Fig. (1), which illustrates how $N_{eff}$ varies with $k_{UV}$ for some particular values of $n_T$. Unlike the constraints on $n_T$ from LIGO [24], for example, BBN imposes constraints on the quantity $N_{eff}$, which involves an integral in $k$. In addition, as mentioned in [24], the BBN constraints depends on the value of both the primordial helium mass fraction and the baryonic density parameter.

V. CONCLUSION

A consistent analysis of the early universe should take into account the effects of primordial GW’s on the background expansion. A higher contribution of the effective number of relativistic species, $N_{eff}$, at early times leads to a faster pre-recombination expansion rate and a smaller value of the sound horizon at recombination. The resulting reduction in the angle subtended by the CMB acoustic peaks can be compensated by an increase in the value of the current expansion rate, $H_0$ [33]. While extra neutrinos species are not predicted by the standard particle physics, primordial GW’s with a blue tensor spectrum can naturally provide a significant contribution for $N_{eff}$ and, therefore, alleviate the current $H_0$-tension problem [40].

In this paper we have considered the extra contribution from the primordial GW’s background to $N_{eff}$ and studied its consequences on the expansion history of the universe, particularly on the current constraints on the $H_0$ parameter. Considering three scenarios and two data sets involving recent CMB and BAO observations along with priors from LIGO and the HST measurements of $H_0$, we have showed that models with a blue tensor spectrum ($n_T > 0$) allow a higher value of $H_0$ than does the standard $\Lambda$CDM model (see Table I). Our results, therefore, show that for the models explored an additional contribution of $N_{GW}^{\nu}$ to $N_{eff}$ does not solve the current $H_0$-tension problem but can alleviate it. Moreover, they reinforce the need to explore early universe models with blue tensor spectrum, as predicted by some non-inflationary scenarios. This is especially important in the current moment considering the great improvement in the constraints on $N_{eff}$ recently provided by the new polarization data from Planck and also the promise of a huge advance in the constraints on the tensor parameters with the next generation of experiments.

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4 Previous CMB data allowed even higher values of $N_{eff}$ and $H_0$ implying in a stronger preference for models that predict an extra contribution to $N_{eff}$ (see, e.g., [24] [33]).

5 Constraints from LIGO are more robust in the sense that they constrain directly $n_T$ instead of a quantity which can be degenerated to modifications on the neutrino sector.
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