Precision Theory of Electroweak Interactions

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ABSTRACT

As a part of the celebration of 50 years of the Standard Model of particle physics, I present a brief history of the precision theory of electroweak interactions. I emphasize in particular the theoretical preparations for the LEP program and the prediction of $m_t$ and $m_h$ from the electroweak precision data.

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1 Introduction

The Standard Model of the weak interaction went through three distinct stages of development. The first was the era of confusion, speculation, and, eventually, insight that ended with the 1967-8 papers of Weinberg and Salam [1,2]. The second, initiated by the discovery of 't Hooft and Veltman that non-Abelian gauge theories are renormalizable [3], was an experimental program that narrowed down the gauge group of the model to $SU(2) \times U(1)$ and verified the predictions of the model at the 10% level. The stage ended with the measurement of parity violation in deep inelastic electron scattering at SLAC [4]. At that point, the evidence for the Standard Model had become sufficiently compelling to motivate the award of the 1979 Nobel Prize to Sheldon Glashow, Abdus Salam, and Steven Weinberg. It is worth remembering that this prize was given before the actual discovery of the $W$ and $Z$ bosons in 1981 by the UA1 experiment [5].

Still, another stage was needed. The Standard Model had not yet been stress-tested with measurements of high precision. The accuracy of the measurements was not sufficient even to convince some that the weak interactions were based on an exact, not an approximate, gauge symmetry [6–9]. Beyond this, once the community began to consider the Standard Model as established, high-precision tests of this model would become a method to search for additional fundamental physics interactions hidden at very short distances.

In the 1990’s, the LEP and SLC experiments realized this promise, verifying the predictions of the Standard Model at the level of fractions of a percent. Alain Blondel describes the experimental achievements in his contribution to this volume [10]. This program also required an unprecedented theoretical effort to produce calculations of the predictions of the Standard Model at a level matched to the experimental precision. The purpose of this contribution is to review that theoretical achievement, the creation of a precision electroweak theory.

It is just an accident that I was asked to give the presentation on this topic at the symposium. Bryan Lynn, an important contributor to this field, was scheduled for this talk but could not attend the symposium. I have never done a high-precision electroweak calculation. But I hope that I can represent this history appropriately in the discussion to follow.

2 Earliest steps

The history of precision electroweak calculation actually begins before the concept of the electroweak theory and, to some extent, even before the idea of the $W$ boson. In 1957, Toichiro Kinoshita and Alberto Sirlin were inspired by the new $V-A$ theory
of charge-changing weak interactions [11, 12] and worked to make the predictions of that theory more precise. In advance of the postulate of $V-A$, Louis Michel had put forward a formalism for testing the form of the weak interaction using measurements on muon decay [13]. In a series of papers, Kinoshita and Sirlin computed the radiative corrections to the Michel formula to first order in $\alpha$ [14, 15]. Their results exposed many theoretically interesting features that would appear again in further precision quantum field theory calculations. The calculations contain infrared divergences that cancel in a way that is reminiscent of the cancellations in pure QED. But they also contain a logarithmic ultraviolet divergence due to the non-renormalizable nature of the 4-fermion interaction. This term needed to be absorbed into a new phenomenological parameter. This ultraviolet problem could not be resolved without a deeper underlying theory.

Two decades later, after the invention and the first evidence for the $SU(2) \times U(1)$ electroweak theory, it became possible to move this program forward. Martinus Veltman and Alberto Sirlin picked up again the dream of turning the theory of weak interactions into one buttressed by high precision calculation.

Veltman and Giampiero Passarino took the first major step in this direction. In 1978, they computed the complete set of 1-loop radiative corrections to the cross section for $e^+e^- \rightarrow \mu^+\mu^-$ in the electroweak theory [16]. In the process, they provided methods that made more general 1-loop electroweak calculations feasible. In particular, Passarino and Veltman presented a method to reduce all of the integrals that appear in this calculation to a small set of “master integrals”. Once these integrals have been evaluated, the work of computing radiative corrections is reduced to pure — although very complex — algebraic manipulation. The general formulae for the 3- and 4-point master integrals are not straightforward to obtain. But, fortunately,
Veltman could ask his former graduate student Gerard ’t Hooft to take time off from theoretical theory to help in finding the appropriate tricks. The resulting paper, which completely solved the problem of the evaluation of 1-loop Feynman integrals with massive denominators, is also foundational in this enterprise [17].

In the 2000s, the Passarino-Veltman method was enhanced by additional tricks due to Bern, Dixon, Dunbar, and Kosower [18] and Ossola, Pittau, and Papadopoulos [19]. These methods allow general reductions of diagrams for processes with arbitrarily many external particles. The combination of methods is amenable to automation, and this is now achieved in codes such as MadGraph5_aMC@NLO [20, 21]. Today, even experimenters can generate predictions at 1-loop accuracy!

3 What, precisely, is $\sin^2 \theta_w$?

Once one knows how to compute the diagrams, there is another important conceptual problem to be solved. This is the question of how to organize the renormalization of the $SU(2) \times U(1)$ model.

Sirlin also had dreams of completing the 1-loop analysis of weak interaction processes. His focus was much broader than high energy $e^+e^-$ reactions; it included the computation of radiative corrections for neutrino reactions and other low-energy probes, and also the idea, new in the 1970’s, of building precise tests of grand unified theories. In [22] and [23], Sirlin and William Marciano began a series of papers that addressed these questions. Their results called attention to many of the important physical effects of electroweak radiative corrections. In particular, the running of the QED coupling constant $\alpha$ has a surprising large influence, giving a 3% correction to the tree-level prediction for the $W$ boson mass and an order of magnitude shift in the prediction for the grand unification scale.

Computing radiative corrections for such a wide variety of processes requires a systematic approach to renormalization. At the tree level, the electroweak theory has three parameters on which all of its predictions depend—the $SU(2) \times U(1)$ coupling constants $g$ and $g'$ and the Higgs field vacuum expectation value $v$ [24]. The electroweak theory is renormalizable. This means that, at the 1-loop level, these three parameters receive divergent corrections, but, once these three parameters are adjusted back to finite values, the expressions for all other observables of the theory are also rendered finite. However, these finite corrections depend on the definitions chosen for $g$, $g'$, and $v$. There are many, many possibilities to define these quantities. Unless one makes a definite choice, the values of the radiative corrections are ambiguous and untestable.

Marciano and Sirlin introduced the procedure of defining the parameters of the electroweak theory by an “on-shell” renormalization method. They defined the un-
derlying parameters of the theory by giving a special role to the quantities,

$$\alpha(m_Z^2), \ G_F, \ \sin^2 \theta_w|_{M-S} \equiv 1 - m_W^2/m_Z^2,$$

(1)

the running value of the QED coupling at the Z mass scale, the Fermi constant defined from muon decay, and the ratio of the W and Z boson masses. These quantities are not precisely observables, but they are very closely connected to quantities measured in weak interaction experiments. Marciano and Sirlin defined the underlying parameters of the electroweak theory so that these quantities should receive zero radiative corrections after renormalization. This definition supplies a set of counterterms that can then be used to cancel the divergences in all other predictions of the theory. A straightforward way to implement this is to write 1-loop expressions for electroweak observables in terms of the parameters in (1). Then, renormalizability implies that these expressions must be finite.

The parameters $\alpha(m_Z^2)$ and $G_F$ were already accurately known before the start of the precision electroweak measurements at the Z pole. The best choice for the third parameter is less clear. Two other definitions of $\sin^2 \theta_w$ proved to be useful in interpreting the results of the Z measurements. These make use of two quantities that would be measured especially accurately in those experiments, the value $m_Z$ of the Z boson mass and the value $A_\ell$ of the polarization asymmetry of the Z-lepton couplings. Specifically,

$$\sin^2 \theta_0 \ \text{defined by} \ \sin^2 2\theta_0 \equiv \frac{\alpha(m_Z^2)}{\sqrt{2}G_Fm_Z^2}$$

$$\sin^2 \theta_* \ \text{defined by} \ A_\ell \equiv \frac{1/4 - \sin^2 \theta_*}{1/4 - \sin^2 \theta_* + 2\sin^4 \theta_*}.$$  

(2)

All three definitions of $\sin^2 \theta_w$ coincide at the tree level. Then, by the considerations of the previous paragraph, the difference between any two of these values of $\sin^2 \theta_w$ generated by 1-loop corrections is a finite quantity that is a prediction of the electroweak theory. In fact, $\sin^2 \theta_0$ and $\sin^2 \theta_*$ differ only at the part-per-mil level, and the use of either give values for the Z branching ratios and asymmetries at the tree level that are already within 1% of the complete expressions. The quantity $\sin^2 \theta_*$ was generalized by Dallas Kennedy and Bryan Lynn to a gauge-invariant running $\sin^2 \theta_*(Q^2)$ [25], which similarly accurately encodes the electroweak corrections to the differential cross sections for $e^+e^- \rightarrow f\bar{f}$ processes.

More recently, the Particle Data Group has chosen to quote the value of $\sin^2 \theta_w$ using $\overline{\text{MS}}$ subtraction, with the parameters set by the best fit to the global corpus of electroweak data [26].
4 The Z lineshape

The LEP and SLC experiments were designed to sit on the Z resonance and measure $e^+e^- \to Z$ production and individual Z decays, hopefully with the highest statistics possible. This brought up another important question that required an improved conceptual understanding: What is the precise form of the Z resonance line-shape as a function of the $e^+e^-$ center of mass energy? In particular, where is the peak of the $e^+e^- \to Z \to f\bar{f}$ cross section relative to the position $m_Z$ of the Z boson pole, and how does the peak cross section compare to that in the simplest approximation?

At leading order, the Z is described as a Breit-Wigner resonance with width $\Gamma_Z$ computed from the Z-fermion couplings. However, this does not give a line-shape even close to the observed one. The line-shape is distorted by the effect of initial state radiation of photons from the incoming electrons and positrons, shifting the position of the peak and producing a long tail of the resonance toward higher values of the CM energy. The usual figure of merit for the magnitude of initial-state radiation is

$$\beta = \frac{2\alpha}{\pi} \left( \log \frac{s}{m_e^2} - 1 \right) = 0.108 \text{ at } s = m_Z^2 .$$

(3)

However, since the Z is narrow, the relative distortion of the resonance is larger, of order

$$-\beta \log \frac{m_Z}{\Gamma_Z} = -40\% .$$

(4)

On the other hand, in order to test the electroweak prediction for the Z width, the line-shape of the resonance needs to be known to part-per-mil accuracy.

This could be done using an approach introduced in 1987 by Victor Fadin and Eduard Kuraev. They argued that multiple photon emissions could be accounted by viewing the hard electrons and photons as partons of the electron and using the formalism for parton evolution in QCD (in this context, the Gribov-Lipatov equation [27]) to sum over real and virtual emissions systematically [28, 29]. Since the electron is an elementary particle, one could also obtain the initial condition for the parton evolution equation from perturbation theory and thus have a complete solution. The Z line-shape could then be obtained from an overlap of these parton distributions, convolved with a hard matrix element that included higher-order diagrams with virtual W and Z bosons. Using the Fadin-Kuraev solution as a starting point, later analyses added higher-order photon resummation and the complete finite order $\alpha^2$ contributions [30, 31].
5 The Yellow Book

With all of the conceptual elements in place, it was still necessary to carry out the hard work of turning these ideas into precise theoretical predictions. Too many people contributed to this effort to list them all in this short review, but I would like to call attention to the major “schools” that contributed, those of Manfred Böhm in Wurzburg (whose students include Wolfgang Hollik and Ansgard Denner) [32,33], Frits Berends in Leiden (whose students include Ronald Kleiss and Wim Beenakker) [34,35] and Dmitri Bardin in Dubna (whose students include Tord Riemann) [36,37]. Over the past 15 years, these calculations have been extended to full 2-loop order in the electroweak interactions by Ayres Freitas, Tord Riemann, and their collaborators; see [38,39] and references therein.

A milestone in the progress of the precision theory was the LEP Yellow Book “Z Physics at LEP 1”, edited by Guido Altarelli, Ronald Kleiss, and Claudio Verzegnassi [40]. Altarelli marshalled the efforts of the whole European theory community to ensure that all aspects of the $Z$ resonance physics would be worked out to 1-loop precision and the results explained in detail.
As the experiments began to take data, these theory predictions needed to be presented as event generators whose output could be directly compared to the observed distributions. The two most influential of these were ZFITTER, developed by a group at Dubna and DESY-Zeuthen led by Bardin [41, 42], and KORALZ, developed by Stanislaw Jadach, Bennie Ward, and Zbigniew Was [43]. Figure 3 shows the ZFITTER team in a relaxed moment.

6 Precision electroweak measurements at the $Z$

Alain Blondel’s article in this volume describes the experiments from viewpoint of one of the participants [10]. However, it seems appropriate here to highlight some of the most impressive comparisons between theory and experiment. In general, in this section, when I quote experimental values of observables, these are taken from the 2005 summary paper of the LEP Electroweak Working Group [44]. When I quote theoretical predictions, these are taken from the Standard Model best-fit values given in the article of Erler and Freitas in the Review of Particle Physics [26].

In Section 4, I emphasized the subtlety of predicting the $Z$ resonance line shape. The result of the LEP measurements of the lineshape are shown in Figs. 4 and 5. Figure 4 shows the point-by-point hadronic cross sections measured by the OPAL experiment, compared to the high-order theory [45]. Figure 5, which represents the composite measurements by the four LEP experiments ALEPH, DELPHI, L3, and OPAL as a 7-point scan [44], also shows more clearly the effect of initial-state radiation. The line is noticeably shifted to higher energies, to account for the energy of the photons radiated before the $e^+e^-$ annihilation. More notably, the peak height of
Figure 4: Measurements of the $Z$ boson lineshape, and comparison to theory; figure courtesy of T. Mori based on [45].
Figure 5: Composite figure representing measurements of the Z line shape by the four LEP experiments ALEPH, DELPHI, L3, OPAL [44]. The experimental errors have been increased by a factor 10 to make them visible. The dotted curve shows the ideal resonance shape before inclusion of initial-state photon radiation.
the resonance is decreased by 30%.

The width of the underlying Breit-Wigner resonance is affected by QCD and electroweak corrections and also, possibly, by the decay of the $Z$ to new light particles. The width extracted from the analysis of LEP data is $2.4955 \pm 0.0023$ MeV. (This value reflects very recent updates concerning the measurements of the LEP luminosity and the 2-loop calculation of Bhabha scattering to which these measurements are compared [47,48].) The Standard Model prediction is 2.4965 MeV.

A particularly interesting component of the $Z$ width is the decay to bottom quarks. The $b_L$ is the $SU(2)$ partner of the $t_L$, so in models in which the top quark interacts with new particles outside the Standard Model, the $b_L$ will usually also feel some of these effects. Even within the Standard Model, there is a significant radiative correction, due to the diagrams in Fig. 6, that gives a $-2\%$ correction to the $Z \rightarrow b\bar{b}$ partial width [37]. The observable

$$R_b = \frac{\Gamma(Z \rightarrow b\bar{b})}{\Gamma(Z \rightarrow \text{hadrons})}.$$  

(5)

offers a chance to observe these effects. From the lowest-order couplings without radiative corrections, one would expect $R_b = 0.220$. The full Standard Model prediction is 0.21562. The composite value from the four LEP experiments is $R_b = 0.21629 \pm 0.00066$. This is in excellent agreement with the Standard Model and puts a strong constraint on models of new interactions specific to the top quark.

As I have noted in Section 3, the polarization asymmetries of the quark and lepton couplings to the $Z$ are affected only slightly by radiative corrections. However, the overall pattern of these asymmetries is a striking qualitative prediction of the Standard Model. The polarization asymmetries are defined as

$$A_f = \frac{\Gamma(Z \rightarrow f_L\bar{f}_R) - \Gamma(Z \rightarrow f_R\bar{f}_L)}{\Gamma(Z \rightarrow f_L\bar{f}_R) + \Gamma(Z \rightarrow f_R\bar{f}_L)}.$$  

(6)

Within the Standard Model, the asymmetries depend strongly on the electroweak quantum numbers,

$$A_f = 0.15 \text{ for } \ell , \quad 0.63 \text{ for } u , \quad 0.94 \text{ for } d.$$  

(7)
The value of the leptonic asymmetry $A_\ell$, which is relatively modest, can be measured in several different ways. First, it can be measured from the forward-backward asymmetry in $e^+e^- \rightarrow f\bar{f}$ on the $Z$ resonance. At the tree level, this is related to $A_f$ in a relatively simple way,

$$A_{FB}(e^+e^- \rightarrow f\bar{f}) = \frac{3}{4} A_e A_f .$$

One should note that the very small predicted value of this asymmetry in $e^+e^- \rightarrow \mu^+\mu^-(1.7\%)$ is increased by about a factor of 2 due to radiative corrections. In the process $e^+e^- \rightarrow \tau^+\tau^-$, the final-state polarization of the $\tau$ can be measured from the weak-interaction asymmetries in the $\tau$ decays. At central values of the production angle $\cos \theta$, the observed polarization reflects $A_\tau$; however, at forward and backward values of $\cos \theta$, the observed asymmetry reflects the forward-backward asymmetry induced by $A_e$. More generally, the $\tau$ polarization is given at tree level by [46]

$$P_{-\tau}(\cos \theta) = -\frac{A_\tau(1 + \cos^2 \theta) + \frac{3}{4} A_e \cos \theta}{(1 + \cos^2 \theta) + \frac{3}{8} A_{FB} \cos \theta} ,$$

or, since $A_{FB}^\tau$ is small,

$$P_{-\tau}(\cos \theta) \approx -\left( A_\tau + 2A_e \frac{\cos \theta}{(1 + \cos^2 \theta)} \right) .$$

Thus, this observable separately measures $A_e$ and $A_\tau$. In the LEP measurements, the two values turned out to be compatible, as expected from Standard Model lepton universality,

$$A_\tau = 0.1439 \pm 0.0043$$
$$A_e = 0.1498 \pm 0.0049 .$$

Finally, experiments at the SLC using polarized $e^-_L$ beams could measure the ratio of the $e^-_L$ and $e^-_R$ couplings directly from the relative rates of the production of hadronic $Z$ events. The value of $A_e$ inferred from this technique was

$$A_e = 0.1516 \pm 0.0021 .$$

The overall consistency of the various $A_\ell$ measurements is shown in Fig. 7 [44]. The various measurements are expressed as values of $\sin^2 \theta_*$, defined by (2). Much ink has been spilled over the 3 $\sigma$ difference in the measurements from $A_\ell(SLD)$ and $A_{FB}^b$. This difference does not influence the overall confirmation of the Standard Model. Still, it would be good to measure the $A_\ell$ even more accurately. That can be done at next-generation $e^+e^-$ colliders [50, 51].

In contrast to $A_\ell$, the value of $A_d$ or $A_b$ in the Standard Model is expected to be almost maximal. This prediction can be tested using polarized beams. Maximality
Figure 7: Comparison of the measurements of $A_\ell$ from a variety of precision electroweak measurements [44]. The measurements are expressed in terms of the effective electroweak mixing angle $\sin^2 \theta_{\text{eff}}$. 

$\chi^2$/d.o.f.: 11.8 / 5

$A_{\ell b}^{0,l} \quad 0.23099 \pm 0.00053$

$A_{\ell}(P_\tau) \quad 0.23159 \pm 0.00041$

$A_{\ell}(\text{SLD}) \quad 0.23098 \pm 0.00026$

$A_{\ell b}^{0,b} \quad 0.23221 \pm 0.00029$

$A_{\ell b}^{0,c} \quad 0.23220 \pm 0.00081$

$Q_{\ell b} \quad 0.2324 \pm 0.0012$

Average $0.23153 \pm 0.00016$

$\Delta \alpha_{\text{had}} = 0.02758 \pm 0.00035$

$m_t = 178.0 \pm 4.3$ GeV
implies that the distribution of $b$ quarks in $e^-e^+ \rightarrow b\bar{b}$ should be strongly forward-peaked for $e^-_L$ beams and strongly backward-peaked for $e^-_R$ beams. The results of measurements at the SLC with polarized beams confirm this prediction, as shown in Fig. 8 [52].

The overall compatibility of the measurements of electroweak observables with the values predicted by the best-fit Standard Model is shown in Fig. 9 [44]. The bars on the right show the deviations of each measurement from the Standard Model expectation, in $\sigma$.

7 Prediction of $m_t$

In Section 3, I noted that the largest radiative correction to electroweak predictions comes in the renormalization of the value of $\alpha$ from $1/137$ at $Q^2 = 0$ to $1/129$ at off-shell momenta of order $m_Z^2$. There is another source of relatively large corrections: Since the top quark is a heavy quark with mass much greater than the $W$ mass, loops containing the top quark can give corrections of order

$$\frac{\alpha_w}{16\pi} \frac{m_t^2}{m_W^2}. \quad (13)$$

The Standard Model predicts quite significant shifts of 0.7% in the $W$ boson mass and 1.3% in the $Z$ boson width due to this effect.
| Measurement          | Fit  | \( \frac{\text{O}^{\text{meas}} - \text{O}^{\text{fit}}}{\sigma^{\text{meas}}} \) |
|---------------------|------|---------------------------------|
| \( \Delta \alpha_{\text{had}}^{(5)}(m_Z) \) | 0.02758 ± 0.00035 | 0.02767 |
| \( m_Z \ [\text{GeV}] \) | 91.1875 ± 0.0021 | 91.1874 |
| \( \Gamma_Z \ [\text{GeV}] \) | 2.4952 ± 0.0023 | 2.4965 |
| \( \sigma^0_{\text{had}} \ [\text{nb}] \) | 41.540 ± 0.037 | 41.481 |
| \( R_l \) | 20.767 ± 0.025 | 20.739 |
| \( A_{t fb}^{0,l} \) | 0.01714 ± 0.00095 | 0.01642 |
| \( A_l(P_\tau) \) | 0.1465 ± 0.0032 | 0.1480 |
| \( R_b \) | 0.21629 ± 0.00066 | 0.21562 |
| \( R_c \) | 0.1721 ± 0.0030 | 0.1723 |
| \( A_{t fb}^{0,b} \) | 0.0992 ± 0.0016 | 0.1037 |
| \( A_{t fb}^{0,c} \) | 0.0707 ± 0.0035 | 0.0742 |
| \( A_b \) | 0.923 ± 0.020 | 0.935 |
| \( A_c \) | 0.670 ± 0.027 | 0.668 |
| \( A_l(\text{SLD}) \) | 0.1513 ± 0.0021 | 0.1480 |
| \( \sin^2\theta_{\text{eff}}(Q_{fb}) \) | 0.2324 ± 0.0012 | 0.2314 |
| \( m_W \ [\text{GeV}] \) | 80.425 ± 0.034 | 80.389 |
| \( \Gamma_W \ [\text{GeV}] \) | 2.133 ± 0.069 | 2.093 |
| \( m_t \ [\text{GeV}] \) | 178.0 ± 4.3 | 178.5 |

Figure 9: Compilation of precision electroweak measurements, and comparison to the predictions of the Standard Model using the best-fit parameters [44].
These shifts in precisely measured electroweak parameters were needed for the success of the Standard Model fit. Conversely, if one assumed the validity of the Standard Model without additional heavy particles, the electroweak fit put limits on the value of the top quark mass. In the early 1990’s, as the CDF and DO experiments at Fermilab raced to discover the top quark, the Standard Model electroweak fit predicted an increasingly narrow range in which the top quark should be found. To give one example, a 1993 paper by the ALEPH collaboration interpreted their Standard Model electroweak fit as a measurement [53]

\[ m_t = 156 \pm \frac{22}{25} \pm \frac{17}{22} \text{ GeV}, \] (14)

where the second error is the uncertainty associated with the unknown value of the Higgs boson mass [53]. A 1994 update of this analysis by Martinez gave [54]

\[ m_t = 156^{+22+17}_{-25-22} \text{ GeV}, \] (15)

an estimate quite comparable to that given by the CDF Collaboration in 1994 at the “evidence for” stage of the direct search for the top quark [55],

\[ m_t = 156 \pm 10^{+13}_{-12} \text{ GeV}, \] (16)

It is much more difficult to obtain strong constraints on the mass of the Higgs boson. Loop diagrams containing the Higgs boson depend on the Higgs boson mass only as

\[ \frac{\alpha_w}{16\pi} \log \frac{m_h^2}{m_W^2}. \] (17)

This is a much weaker dependence on the unknown mass and also gives shifts about 5 times smaller for the actual mass values. However, it became clear in the late 1990’s that values of the Higgs boson mass above about 200 GeV would give results inconsistent with the observed values of the electroweak observables. Figure 10, produced by the LEP Electroweak Working Group in 2011, shows the contemporary 68% CL contour in the plane of \((m_h, m_t)\) [56]. The constraint from the known value of the top quark mass clearly indicated a value of the Higgs boson mass below 200 GeV. This constraint did assume the validity of the Standard Model. Theories with physics beyond the Standard Model could allow a heavy Higgs boson, but only in specific scenarios with well-defined and observable consequences [57]. We now know that these scenarios did not play out and that the prediction based on the Standard Model was correct.

8 \( e^+e^- \rightarrow W^+W^- \)

I should not end this discussion of the successes the Standard Model in describing electroweak interactions without noting one other important LEP measurement, that of the cross section for the process \( e^+e^- \rightarrow W^+W^- \).
Figure 10: 68% confidence region of the $m_h$-$m_t$ plane allowed by precision electroweak measurements, as determined by the LEP Electroweak Working Group in 2011 [56]. The measurement of $m_t$ and the constraints on $m_h$ are from the Tevatron experiments CDF and DO.

Figure 11: Diagrams contributing to the amplitude for $e^+e^- \rightarrow W^+W^-$ at the tree level.
Thinking naively, the amplitude for the production of two longitudinally polarized $W$ bosons should be approximated by the cross section for the production of charged scalars, times the product of the $W$ polarization vectors, giving
\[ \frac{d\sigma}{d\cos \theta}(e^+e^- \rightarrow W^+_0 W^-_0) \sim \frac{d\sigma}{d\cos \theta}(e^+e^- \rightarrow w^+_0 w^-_0) \cdot |\epsilon^{*\mu}(W^+)\epsilon^\mu(W^-)|^2 \] (18)

However, the product of these polarization vectors becomes very large at high energy,
\[ \epsilon^{*\mu}(W^+)\epsilon^\mu(W^-) \rightarrow \frac{s}{2m_W^2} , \] (19)
leading to a prediction for the S-wave cross section that violates unitarity. In the Standard Model, the amplitude for $e^+e^- \rightarrow W^+W^-$ is given at the tree level by sum of the three diagrams shown in Fig. 11. Thus, there is the possibility that the unitarity-violating terms could cancel among these diagrams. At first sight, these seems unlikely. But, in fact, this cancellation is guaranteed by $SU(2) \times U(1)$ gauge invariance and its consequence, the Goldstone Boson Equivalence Theorem [58–60]. The tree-level analysis was done in the mid-1970’s by Flambaum, Khriplovich, and Sushkov and by Alles, Boyer, and Buras [61, 62], and their results displayed this cancellation explicitly.

Providing a theoretical prediction for the precision measurement of this cross section at LEP 2 made it necessary to solve additional problems. To treat $W$ bosons realistically, it is necessary to allow them to go off the mass shell, while retaining a sufficient level of gauge-invariance to allow the gauge cancellations to go through. Processes with off-shell $W$ bosons are in principle indistinguishable from general $e^+e^- \rightarrow 4$ fermion processes, and so these must also be modelled correctly in the event generator. These issues were addressed in the codes YFSWW, by Jadach, Placzek,
Figure 13: Measurements of the total cross section for $e^+e^- \rightarrow W^+W^-$ by the four LEP experiments, compared to theoretical predictions [68].
Skrypek, Ward, and Was [63,64], and RACOONWW, by Denner, Dittmaier, Roth, and Wackeroth [65–67].

The results of the four LEP 2 experiments, compared to the predictions of the event generators, are shown in Fig. 13. For illustration, the predictions of tree-level calculations that omit the $ZWW$ diagram and both of the $s$-channel diagrams in Fig. 11 are also shown. The cancellation required by gauge invariance is manifest. This cancellation takes place only if the triple gauge boson vertices are of the form required by Yang-Mills theory within an accuracy of a few percent.

9 Conclusions

The LEP/SLC program of precision electroweak measurement required an unprecedented theoretical effort to provide high-precision predictions of the properties of the electroweak bosons. The comparison of these theoretical and experimental efforts was a triumph that leaves no doubt that $SU(2) \times U(1)$ gauge invariance is actually a property of nature. Whatever lies beyond, we can now take the validity of electroweak gauge invariance as a foundation to rely on as we move forward.

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