Active steering control for vehicle rollover risk reduction based on slip angle estimation

Ke Shao1, Jinchuan Zheng2, Bin Deng3, Kang Huang4, Han Zhao3

1Tsinghua Shenzhen International Graduate School, Tsinghua University, Shenzhen 518055, People’s Republic of China
2School of Software and Electrical Engineering, Swinburne University of Technology, Hawthorn, VIC 3122, Australia
3School of Automotive and Transportation Engineering, Hefei University of Technology, Hefei 230009, People’s Republic of China
4School of Mechanical Engineering, Hefei University of Technology, Hefei 230009, People’s Republic of China

E-mail: hanzhao@hfut.edu.cn

Abstract: In this study, a robust controller is proposed for rollover risk suppression in automatically-driven vehicles by reducing the lateral acceleration through a steer-by-wire system equipped on the vehicles. First, since the slip angle is difficult to measure directly, a sliding mode observer combining an add-on switching term with a linear observer is developed to provide more robust estimation of slip angle in the presence of system uncertainties. Second, an adaptive sliding mode control method is proposed, in which the control gain is adaptively tuned to compensate for the system uncertainties. Lastly, simulation is conducted and the results verify that the proposed estimator and controller can reduce the vehicle rollover risk efficiently and robustly.

Nomenclature

Parameter Description

$\beta$ slip angle of vehicle
$\delta$ front wheel steering angle
$\delta_d$ steering wheel angle
$\phi$ roll angle of sprung mass
$\alpha_r$ vehicle lateral acceleration
$C_{nr}, C_{ar}$ cornering stiffness of tyre
$d$ track width
$F_{hr}, F_{re}$ vertical force of tyre
$F_{yf}, F_{yr}$ lateral force on tyre
$g$ gravity of earth
$h$ roll radius of sprung mass
$I_z$ yaw moment of inertia of vehicle
$l_f, l_r$ distance from axle to vehicle centre
$m$ vehicle mass
$r$ yaw rate of vehicle
$v_x$ vehicle forward velocity

1 Introduction

Active vehicle safety has attracted many researchers in recent years such as active obstacle-avoided system, rear-end avoidance, rollover prevention and so on. Although not occurring very often, vehicle rollover leads to a larger proportion of seriously injured occupants and occupant fatalities than general traffic accidents since it is often too late for drivers to respond when an emergency rollover is perceived [1, 2]. To improve the rollover stability and assist vehicle rollover prevention, active anti-rollover control is investigated by many academic institutions and automotive enterprises.

To prevent rollover, a single strategy of active braking, active steering or active suspension, or a combination of them is investigated by several researchers [3]. Although active braking can prevent rollover fast, it is often with an emergency shocking due to the sharp increasing of braking force [4, 5]. Active suspension can reduce the rollover risk by adjusting roll angle, but the rollover risk will not yet be eliminated as long as the vehicle speed or cornering curvature is large enough [6–8]. Steer-by-wire (SbW) system is firstly used to improve the vehicle handling characteristics [9, 10]. Since the steering angle of front wheels is flexibly adjusted by the controller based upon the driver’s steering command, active steering can prevent rollover in a smoother way while maintaining the riding comfort and driving safety [11]. Based on active steering, Solmaz et al. [12] designed a state feedback controller, in which the load transfer ratio (LTR) is efficiently limited. Imine et al. [13] developed an active steering system based on sliding mode control to avoid the rollover of heavy vehicles. Combined with an electronic stability control (ESC) system, Yim and Park [14] compared different linear quadratic controllers for rollover prevention. In [15], a mass-centre-position metric is proposed to generate the desired lateral acceleration which is then tracked by the active steering based rollover preventer system.

Another problem is the estimation of vehicle slip angle. Vehicle slip angle is an important variable in vehicle dynamic control but is usually difficult to measure directly. In [9], GPS and inertial navigation system are combined to measure the slip angle indirectly. Since yaw rate, lateral acceleration, individual wheel speed and steering angle can be directly obtained from ESC system or anti-lock system (ABS), various observers are designed to estimate the slip angle by using these available measurements, which are systematically investigated in Limroth’s work [16]. Linear observer is mostly used because it is easy to design and implement [17].

A linear vehicle model is often adopted in dynamic control since the 1950s. In a linear model, the lateral force of tyre is treated as a value linearly related to the slip angle [18]. However, such an approximation is only valid in a linear region while the non-linearity will increase hugely out of that [19]. In case of linear observers, since the vehicle is involved with model uncertainties and disturbances due to model linearisation, physical parameter variation, road disturbance and so on, the linearly estimated slip angle will generate large deviation and weak robustness under certain conditions.

Adaptive control is widely used to solve engineering problems because of its effectiveness to deal with time-varying parameters in a vehicle model [20]. Compared to a linear observer, sliding mode observer (SMO) is characterised by its better robustness against bounded system errors, and rapid convergence to actual values [21, 22]. In this paper, the SbW system is employed to solve the rollover risk suppression problem. A non-linear SMO is designed for the estimation of slip angle, which is proved to possess better robustness against system uncertainties compared to a linear observer in the presence of disturbances. To compensate for the
2 Load transfer ratio

The rollover stability of a vehicle can be evaluated by the LTR. LTR is defined as the vertical load transfer index between the left and right side [18], given by

$$\text{LTR} = \frac{F_{r,w} - F_{l,w}}{F_{r,w} + F_{l,w}} \quad (1)$$

where $F_{r,w}$ and $F_{l,w}$ are the vertical forces at the left and right side, respectively, as illustrated in Fig. 1, which are both positive. Note that (1) is only valid before one side of the tyres lifts off the road. Since the total vertical tyre forces equal to the vehicle gravity, we have $F_{r,w} + F_{l,w} = mg$. It is obvious that $-1 \leq \text{LTR} \leq 1$. Specifically, $|\text{LTR}| < 1$ at a safe cornering condition and once the inner tyres lift off the road, $|\text{LTR}| = 1$, i.e. the vehicle rolls over as defined in this paper. An LTR value closer to zero will indicate a safer condition. Therefore, the rollover states of a vehicle can be defined by the range of LTR value as follows:

$$|\text{LTR}| = 1$$ rollover boundary \quad (2)

$$|\text{LTR}| < 1$$ rollover - stable region \quad (3)

Fig. 1 describes the lateral motion of the vehicle. As shown in Fig. 1, when a vehicle turns a corner, a lateral acceleration $a_l$ is generated. The lateral acceleration will further produce $\text{LTR}$ from the middle line of the vehicle. It is the roll centre and is commonly closed to the road. Both of the lateral acceleration and roll angle lead to the vertical load transfer between left and right wheels. From the torque balance along the roll centre $R$, LTR can be simply described by the following equation [23]:

$$\text{LTR} = \frac{2h}{d} \left( \frac{a_l}{g} + \phi \right) \quad (4)$$

It is obvious that LTR can be seen as the sum of the lateral acceleration $a_l$ due to the lateral force and the roll angle $\phi$, which reflects the load transfer due to the change of the position of the gravity centre of sprung mass. Lateral force will significantly change the distribution of the vertical load of left and right tyres by applying a principal rollover torque, and the position variation of the gravity centre of sprung mass will further increase the rollover risk.

At small roll angles, the impact of roll angle on LTR can be ignored for simplification. Equation (4) reduces to a simpler form as follows:

$$\text{LTR} \approx \frac{2h}{dg} a_l \quad (5)$$

To prevent rollover, $|\text{LTR}| < 1$ should be retained for all time. In practice, the rollover prevention should be activated before the inner tyres lift off the road. Therefore, in many literatures, a threshold value $\text{LTR}_\text{th}$ is often selected as a switching signal for the controller to detect the rollover risk when the LTR exceeds the threshold value. It is clear that, on the one hand, a smaller $\text{LTR}_\text{th}$ indicates better rollover stability but may intervene the driver's original steering command too frequently. On the other hand, a large $\text{LTR}_\text{th}$ may provide insufficient time for the controller to respond before the vehicle rolls over. In practice, vehicle roll angle is often difficult to measure directly due to the impact of the banked road [24]. Since the roll angle contributes much less to $\text{LTR}$ compared to lateral acceleration [23], we use the simplified LTR in (5) as the switching signal for the control system and the original LTR in (4) will be presented in the simulation results to demonstrate the rollover control performance. In this paper, $\text{LTR}_\text{th} = 0.8$ is chosen.

Under a real driving condition, the vehicle is impacted by many uncertainties such as system uncertain parameters, non-linear tyre force and unknown road conditions. Therefore, we also introduce a coefficient $c = 0.7$ to take these factors into account. It can be seen from (5) that $\text{LTR}$ is a linear function of $a_l$, which implies that the LTR threshold can be equivalently translated into a threshold of $a_l$, i.e. $a_{\text{th}} = (d/2h)c\text{LTR}_\text{th} = 0.85g$. From now on, $a_l$ will be considered as the controlled variable for indirect regulation of LTR [13].

3 System modelling

Consider a vehicle illustrated in Fig. 2, whose symbols have been described in Nomenclature. Suppose no slipping at the tyres and ignore the effects of aerodynamics. For a specific speed, based on Newton's law, a linear bicycle model can be built from lateral and yaw force balances along the mass centre of the bicycle [18]. Note that in the bicycle model, the lateral tyre forces is approximated as a linear function with respect to its slip angle, i.e. $F_{r,f} = 2C_{\alpha_f} r\alpha_f$ and $F_{l,f} = 2C_{\alpha_l} \alpha_l$. The system state-space equation of the bicycle can be described by

$$\dot{x} = A(t)x + B(t)\delta \quad (6)$$

where the state variable $x = [r \ \delta]^T$,

$$A(t) = \begin{bmatrix}
\frac{2C_{\alpha_f} r \delta + 2C_{\alpha_l} \delta}{I_x} & -\frac{2C_{\alpha_l} r \delta - 2C_{\alpha_f} \delta}{I_z} \\
-1 & \frac{2C_{\alpha_l} r \delta - 2C_{\alpha_f} \delta}{I_z} - 2C_{\alpha_f} \alpha_f
\end{bmatrix}$$

$$B(t) = \begin{bmatrix}
\frac{2C_{\alpha_f} r \delta}{I_x} \\
\frac{2C_{\alpha_l} r \delta}{mv_y}
\end{bmatrix}$$

Remark 1: Equation (6) is a linear parameter varying system, where some parameters may vary in practice. For instance, the mass, the moment of inertia and the roll radius may vary due to various loads. The vehicle speed may change due to driver's command. The cornering stiffness may vary due to tyre characteristics.

Furthermore, let the system output $y = a_l$, which is the lateral acceleration. Then, according to (6), we can obtain

$$y = C(t)x + D(t)\delta \quad (7)$$

where

$$C(t) = \begin{bmatrix}
-\frac{2C_{\alpha_l} r \delta - 2C_{\alpha_f} \delta}{I_z} + 2C_{\alpha_f} \alpha_f
\end{bmatrix}$$

$$D(t) = \frac{2C_{\alpha_f} r \delta}{I_x}$$

In this paper, a set of nominal values of system parameters are used for the design of the rollover risk suppression control system.
addition, we assume the yaw rate is the only measurable output and denote \( z = \dot{r} \). Assumining the model uncertainties satisfy the matching condition [25], the overall system dynamics can thus be described by

\[
\begin{align*}
\dot{x} &= Ax + B\delta + B\xi, \\
y &= Cx + D\delta + \xi, \\
z &= Ex,
\end{align*}
\]  

(8)

where the values of \( A, B, C \) and \( D \) are of the form given in (6) and (7) with the nominal parameters and the measurement vector \( E = [1\ 0] \). \( \xi \) indicates a disturbance stemming from system uncertainties, un-modelled dynamics and road disturbances acting on the tyres, which is assumed to be bounded by \( |\xi| \leq \xi^* \). Similarly, \( \xi \), lumps the bounded system uncertainties acting on the system output, satisfying \( |\xi| \leq \xi^* \).

4 Slip angle observation

In this section, a SMO will be designed to estimate the slip angle \( \beta \) based on the only measurable yaw rate \( \dot{r} \) and front wheel steering angle \( \delta \). To design an SMO, the following assumptions are made:

(i) There exists a vector \( L \in \mathbb{R}^{2\times1} \) such that \( A - LE \) has stable eigenvalues.

(ii) There exists a Lyapunov pair \( (P, Q) \) such that for a scalar \( F \) the following structural constraint is satisfied:

\[
E^T F^T = PB. 
\]  

(9)

Based on the above assumptions, the observer is designed in the following form:

\[
\dot{x} = A\dot{x} + B\delta + L(z - E\xi) + Bu. 
\]  

(10)

where

\[
\nu = \begin{cases} 
-\rho \frac{F(E\dot{\xi} - z)}{|F(E\dot{\xi} - z)|}, & |F(E\dot{\xi} - z)| \neq 0 \\
0, & \text{otherwise}. 
\end{cases}
\]  

(11)

Define the state estimation error \( \hat{z} = \dot{\hat{x}} - x \). It is obvious that \( E\hat{\xi} - z = E\dot{\xi} \). The sliding mode gain \( \rho \) satisfies

\[
\rho \geq \xi^* + \eta, 
\]  

(12)

for a positive scalar \( \eta \).

To prove the stability of the proposed observer, from (6) and (10), we have

\[
\dot{\hat{x}} = (A - LE)\dot{x} - B\xi + Bu. 
\]  

(13)

Let \( A_0 = A - LE \) and choose the Lyapunov function \( V = \dot{\hat{x}}^2P\dot{x} \), and then differentiate it once with respect to time. From (14), (16) and (18), we get

\[
\begin{align*}
\dot{V} &= \dot{\hat{x}}^2P\dot{x} + \dot{\hat{x}}^2P\dot{\hat{x}} \\
&= \dot{\hat{x}}^2(PA_0 + A_0P)\dot{x} - 2\dot{\hat{x}}^2PB\xi + 2\dot{\hat{x}}^2PBu \\
&\leq -\dot{\hat{x}}^2Q\dot{x} - 2\dot{\hat{x}}^2E^T\xi^* - 2\dot{\hat{x}}^2P(\dot{\hat{x}} - z) \\
&\leq -\dot{\hat{x}}^2Q\dot{x} - 2\dot{\hat{x}}^2E^T\xi^* - 2\dot{\hat{x}}^2P(\dot{\hat{x}} - z) \\
&\leq -\dot{\hat{x}}^2Q\dot{x} - 2\dot{\hat{x}}^2P(\dot{\hat{x}} - z) \\
&\leq -\dot{\hat{x}}^2Q\dot{x} - 2\dot{\hat{x}}^2P(\dot{\hat{x}} - z)
\end{align*}
\]  

(14)

where \( Q > 0 \) because \( A_0 \) is a Hurwitz matrix. Therefore, the observation error \( \dot{\hat{x}} \) will converge to zero under the proposed observer.

The method of the selection of \( F \) can follow [25]. In this research, we select the scalar \( F = 10 \) and the sliding mode gain \( \rho = 0.75 \), which guarantees that the observer is stable. The linear feedback gain \( L = [8.8204 \ 1.0362]^T \) is chosen by pole placement to obtain the poles of \(-7 \pm 5i\).

Simulations are conducted to verify the proposed method. For comparisons, a linear Luenberger observer (LO) with the same feedback gain \( L \) is introduced and given by

\[
\dot{\hat{x}} = A\dot{\hat{x}} + B\delta + L(z - E\dot{\hat{x}}).
\]  

(15)

In addition, a kinematic-dynamic observer (KDO) developed in [26] is also compared which has the form as follows:

\[
\dot{\hat{\beta}} = \frac{\alpha_s}{v_s} - r + K(\alpha_s - \dot{\hat{a}}_s).
\]  

(16)
where the gain is selected as $K = -0.05$. $a_i$ is the measured lateral acceleration which is calculated by $a_i = v_i(\dot{\beta} + r)$ in the simulation. Further, from (6), the estimation $\hat{a}_i$ is obtained by

$$\hat{a}_i = -2C_{a,f}\frac{C_{a,f}}{m}\dot{\beta} - 2C_{a,\delta}j_x - \frac{C_{a,\delta}}{mv_j} + \frac{2C_{a,f}}{m}\delta. \quad (17)$$

The above equation is the dynamic model based injection term that enters (16) and used to modify the kinematic estimation. It can also be seen that the KDO requires additional measured $a_i$ for feedback, which may introduce measurement noise that affects the estimation accuracy.

Consider the steering angle as shown in Fig. 3 and a white noise with a power of 0.0005 is used to mimic the system uncertainty $\xi_i$. The simulation results for the designed observers are shown in Fig. 4. We can see from Fig. 4a that, even if the existence of uncertainties, the estimated slip angle $\beta$ under the proposed observer can track the actual vehicle slip angle accurately. However, the slip angle estimated by the linear observer (15) as well as the KDO (16) generates a larger deviation from the actual value due to their weak robustness. The comparison of estimation errors is illustrated in Fig. 4b. It is clear that the estimation error of the proposed observer is much smaller than both LO and KDO.

**Remark 2:** We can see from (10) and (15) that compared to a linear observer the proposed observer contains a sliding mode term. This compensatory switching term endures the observer better robustness against uncertainties than a linear observer. The term $\nu$ may cause chattering in the observer depending on the level of uncertainty. However, the chattering only occurs in the software, which only causes minor numerical problem.

5 **Active steering for rollover risk suppression**

In this section, based on (6) and (7) an adaptive robust controller is designed to reduce the rollover risk of vehicle in the presence of uncertainties. The control objective is to retain the lateral acceleration under its threshold value once a rollover risk is detected as described in Section 4. The stability of the control system is proved and the selection of the controller parameters is also discussed at the end of the section.

5.1 **Robust adaptive sliding mode controller**

First, introduce an auxiliary state $x_r$, satisfying

$$x_r = y - y_r, \quad (18)$$

$$\dot{y}_r = \frac{1}{\tau}(y_r - y). \quad (19)$$

where $y_r = a_{i,th}$ is the threshold value of lateral acceleration, $\tau$ is a positive constant. It is obvious that for a certain $y_r$, $\dot{y}_r$ generates a smooth reference command $y_r$. The design parameter $\tau$ will be selected to define the transient speed of $y_r$ in response to $y_r$. From (23), the state $x_r$ defines the integral of the tracking error of $y$. The control objective is to enable $x_r = 0$ for $t \to \infty$, that is, $y \to y_r$.

Next, since there exist a feed forward term $D\delta$ in system output (11), to avoid the control input derivative, we define the following integral form sliding variable:

$$s = x_r = \int (y - y_r)dt. \quad (20)$$

The measured yaw rate and the estimated slip angle are fed back to the controller. By neglecting the uncertainties with $\xi_i = 0$ and letting $\dot{\delta} = \delta_i$ in (11), solving $s = 0$ for $\delta_i$ leads to

$$\delta_i = \frac{m}{2C_{a,f}}(y_r - C\hat{x}). \quad (21)$$

where $\hat{x} = [r \ \dot{\beta}]^T$. $\delta_i$ is called the equivalent control, which maintains $s = 0$ once the sliding condition is reached. To tolerate system uncertainties, a compensatory switching control is designed by

$$\delta_i = \frac{m}{2C_{a,f}}(\hat{\delta}_a\text{sign}(s)). \quad (22)$$

where $\hat{\delta}_a$ is the sliding mode gain to be designed. Generally, it is essential that the bounds of uncertainties are known a priori such...
that a suitable constant gain can be designed to cover the uncertainties. However, in practice the bounds of uncertainties are sometimes difficult to evaluate directly. Under such a condition, a large gain has to be designed, however, the control performance may be defected by the increased control effort [27].

For an adaptive controller, the controller parameters are adaptively updated to deal with the time variant uncertainties by the adaptation law, which guarantees the control performance compared to a constant parameter controller. In this research, the sliding mode gain is designed to be time varying to cover the uncertainty term, $\xi_2$, which is updated by the following adaptation law:

$$\dot{k} = \gamma \text{sign}(s),$$

(23)

and $\gamma \geq 1$ is a positive constant to be designed. It is clear that $\gamma$ determines the adaptation speed of the sliding mode gain.

Then, the following adaptive sliding mode controller is obtained:

$$\delta = \delta_k + \delta_s,$$

(24)

with $\delta_k$ and $\delta_s$ given by (26) and (27), respectively.

### 5.2 Stability analysis

To prove the system stability under the designed ASMC controller, choose the Lyapunov function

$$V = \frac{1}{2} \dot{x}^2 + \frac{1}{2} \xi_2^2,$$

(25)

where $\dot{k} = \dot{k} - k$, $k$ is assumed to be an unknown finite non-negative sliding mode gain satisfying $k \geq \xi_2 + \eta_2$, where $\eta_2$ is a positive real number. Evaluating the derivative of $V$ along the trajectories of the system in (25) yields

$$\dot{V} = ss + \dot{k} \dot{k} - k \dot{k},$$

(26)

Substituting (13), (28) and (29) to (31), we have

$$\dot{V} = s(\xi_2 - \dot{k} \text{sign}(s)) + (\dot{k} - k) \gamma \text{sign}(s).$$

(27)

Since $\gamma \geq 1$, $k \geq \xi_2 + \eta_2$, it is easy to verify $V < 0$. This implies that $s$ will converge to zero in the presence of the uncertainties leading to $y \rightarrow y_f$.

Thus far, the controller based on the model in (11) has been designed without the consideration of the dynamics in (6). Therefore, the stability of the system (6) under the designed control law should be investigated.

By integrating the adaptation law (28), we have

$$\dot{k} = \int \gamma \text{sign}(s) \, dt = \gamma \int \text{sign}(s) \, dt = \gamma \bar{t}. $$

(28)

Since $s$ converges to zero, we have

$$\int \gamma \, dt \leq \bar{t},$$

(29)

where $\bar{t}$ is the bound of the integration. Therefore, $\dot{k}$ is bounded by

$$\dot{k} \leq \gamma \bar{t}. $$

(30)

Assuming $\bar{y}_r = [0 \bar{y}]^T$, it is obvious that $\dot{x} = x + \bar{y}_r$. Further assume the upper bound of $\bar{y}_r$ is $\| \bar{y}_r \| \leq \bar{y}_r$. Substituting $\delta$ given by (29) into (6) yields

$$\dot{x} = Ax + B\dot{\delta} + B\xi_2^s,$$

(31)

where the exogenous inputs $\bar{y}_r$, $y_1$, $\dot{k}$ and $\xi_2$ are all bounded. $A$, $B$, $C$ and $D$ are all finite for a specific constant speed. It is easy to verify that the closed-loop system matrix $A - \frac{(m/2C_{\alpha f})}{B}$ is a Hurwitz matrix. Fig. 5 plots the system eigenvalues with respect to speed ranged from 30 to 120 km/h with an interval of 5 km/h, where the real parts of the eigenvalues are all located in the left half-plane, implying the stability of the closed-loop system.

### 5.3 Control implementation

The proposed control diagram is illustrated in Fig. 6. The front wheel steering angle $\delta$, the yaw rate $\dot{r}$ and vehicle lateral acceleration $a_y$ are assumed to be known since they can be detected by sensors or obtained directly from an ESC system or an ABS. Based on the proposed observer, the slip angle is estimated for the ASMC controller. Once the lateral acceleration $a_y$ exceeds its threshold of $a_{\delta}$, the controller is then activated and generates the control input $\delta$ to replace the driver's command. Otherwise, the controller is turned off in order to return the control priority to the driver.

Two parameters are to be selected in the ASMC controller of (11) for the desired performance. The first parameter is the value of $r$, which is actually the time constant of the low pass filter of (19). For a smaller value of $r$, $y_1$ will converge faster to the reference command, i.e. the desired $a_\delta$ threshold. Since the control input $\delta$
we choose

the filter may saturate the steering input and make the control impractical. In this research, we choose control signal instead of a high-frequency turning on/off of the

To balance the impacts of both control smoothness and accuracy,

out that although the switching operation is prohibited, the sliding

can significantly impact the convergence rate of $y$. We can see clearly from (27) that a bigger $\gamma$ will drive both $y$ and $k$ to converge faster. However, from (22), a large $\gamma$ implies a large amplitude of the control input that may lead to system chattering. Since $\gamma \geq 1$, we choose $\gamma = 5$ to balance the convergence rate and control signal smoothness.

**Remark 3:** Commonly, the controller (11) will be activated once the lateral acceleration $|a_y| > a_{\text{th}}$ and be turned off if $|a_y| \leq a_{\text{th}}$. However, this switching operation is harmful for the vehicle safety and manoeuvrability. To avoid this phenomenon, (11) is approximately modified as

\[
\delta = \zeta(\delta_0 + \delta_1)
\]

where $\zeta$ is an automatically adjusted parameter given by

\[
\zeta = \frac{|a_y|}{l + |a_y|}
\]

where $l > 0$. Then, based on (38), the controller will be turned on only at the first time when $|a_y| > a_{\text{th}}$ and will provide a continuous control signal instead of a high-frequency turning on/off of the controller. It is clear that if $l$ is smaller, $\zeta \to 1$, the lateral acceleration will be driven to the threshold faster. To the contrary, if $l$ is larger, the desired control signal will be suppressed but will lead to a smoother manoeuvrability. However, it should be pointed out that although the switching operation is prohibited, the sliding mode invariance cannot be retained when the system state moves on the sliding surface that implies an increased tracking error [25]. To balance the impacts of both control smoothness and accuracy, we choose $l = 5$.

**6 Simulation results**

To verify the proposed robust controller for rollover risk suppression, simulation in MATLAB and CARSIM software environment is conducted in this section. The sport utility vehicle parameters used in the simulation are shown in Table 1. A J-turn condition is utilised to mimic an emergency steering for obstacle avoidance. In addition, to avoid possible overly large position deviation response to the control input, the steering angle variation is set to saturate at 80°.

First, Fig. 7 presents the profile of lateral acceleration $a_y$ under open-loop control in comparison with ASMC. We can see that the lateral acceleration $a_y$ exceeds the threshold value of 0.85 g at about 1.4 s. From Figs. 7 and 8, the lateral acceleration under open-loop control goes to zero at 3.5 s but its roll angle diverges quickly, both of which imply that the vehicle rolls over. It can also be seen from Fig. 7 that the lateral acceleration is effectively reduced compared to the open-loop control, which demonstrates the efficiency of the proposed control strategy for rollover risk suppression. As a result of the reduced lateral acceleration, the roll angle is simultaneously reduced (Fig. 8). Similarly, the yaw rate and slip angle of the vehicle are also reduced due to the modified trajectory as given in Figs. 9 and 10. Fig. 11 indicates that for the vehicle under ASMC control, the controller is activated several times whenever the vehicle lateral acceleration exceeds 0.85 g for rollover risk suppression. When the controller is activated, the steering angle is swiftly reduced, and then saturated until the lateral acceleration reduces to the threshold. After that, the steering angle restores to the driver's command to minimise the lateral position deviation. The path deviation of the controlled vehicle is shown in Fig. 12.

**Table 1 Parameters of a sport utility vehicle**

| Parameter | Value | Parameter | Value |
|-----------|-------|-----------|-------|
| $m$       | 1274 kg | $g$       | 9.81 m/s² |
| $N$       | 10   | $d$       | 1.639 m |
| $I_y$     | 1523 kgm² | $l_y$    | 1.016 m |
| $C_{n,f}$ | 27748 N/rad | $l_c$    | 1.462 m |
| $C_{n,r}$ | 54700 N/rad | $h$     | 0.54 m |

**Fig. 7** Lateral acceleration $a_y$ in J-turn condition (the blue curve under open-loop control terminates at about 3.5 s because rollover has occurred at that time and the simulation halts. This applies to the rest of the figures). The path deviation of the controlled vehicle is shown in Fig. 12.

**Fig. 8** Roll angle $\phi$ variations in J-turn condition.

**Fig. 9** Yaw rate $\psi$ variations in J-turn condition.
The vertical forces of both vehicles are provided in Fig. 13. It can be seen that the right side tyres of the open-loop vehicle lift off the road at around 2.5 s. As the steering angle maintains, all the tyres lift off the road at 3.5 s which indicates an absolute rolling over. For the controlled vehicle, although one of the right tyres lifts off the road at some time, the vehicle does not roll over under the whole J-turn condition. Fig. 14 plots the LTR variations of the vehicles according to (4). It is worth noting that it has a similar trend compared to the lateral acceleration in Fig. 7, which implies the validation of using the lateral acceleration as the switching signal instead of LTR.

**Remark 4:** The ending trajectory of the controlled vehicle in Fig. 12 slightly deviates from the desired trajectory. This is because in the simulation the path following algorithm is not used and the driver is assumed not to interfere the steering. It can be seen that the lateral position deviation is less than 0.5 m, which is trivial. In practice, the driver will also operate the vehicle to the desired trajectory after the rollover risk is cleared.

**7 Conclusions**

In this paper, the rollover risk reduction problem of a sport utility vehicle based on an SbW system is investigated. The rollover stability is described by LTR. Since the lateral acceleration of a vehicle impacts the rollover stability significantly, it is used as the system output for indirect control of LTR. To estimate the vehicle slip angle, an SMO is designed based on the only measurement of the front wheel steering angle and the vehicle yaw rate. The proposed SMO proves stronger robustness and faster convergence for the estimation of slip angle compared to a linear observer in the presence of system uncertainties. To reduce rollover risk, a robust active controller based on ASMC is proposed to consider system uncertainties based on the designed adaptation law. The stability of the controller is proved by Lyapunov stability analysis, and the selection of the controller parameters is also discussed. To prove the efficiency and robustness of the proposed ASMC controller, MATLAB-CARSIM simulation is conducted where a real full car model is utilised. Simulation results under a J-turn scenario verify that the lateral acceleration under the proposed ASMC controller can be reduced by 0.2 g as compared to open-loop control. In consequence, the proposed controller effectively drags the LTR.
value from $-1$ to $-0.8$ implying the success of rollover risk reduction.

The main contributions of this work lie in that the proposed SMO can achieve more accurate estimation of the slip angle than linear observers that are commonly used in automotive engineering. In addition, the proposed control strategy is useful for improving the safety for vehicle rollover prevention by reducing its lateral acceleration. For practical applications of the proposed controller, the lateral acceleration threshold is required to be pre-specified according to the physical model of the vehicle under control. This may need some tuning for best performance in practice. Besides, the proposed controller needs the feedback signals of steering angle, yaw rate and vehicle speed. Thus, extra sensor is required if any signal is not available. Lastly, the proposed controller should be integrated with the existing vehicle electronic control system and requires a fast real-time processor to swiftly switch it on once the tendency of rollover is detected.

To simultaneously improve the vehicle handling stability in rollover prevention procedure, it is essential to combine the active steering with other control strategies such as differential braking or active suspension. In addition, since the single active steering may cause a trajectory deviation, to avoid a possible accident such as a rear-end collision, the rollover prevention control system is required to be well coordinated with other control systems. Those issues will be discussed in our future work.

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9 References

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