Cosmological neutrino bounds for non-cosmologists

Max Tegmark

1 Dept. of Physics, Massachusetts Institute of Technology, Cambridge, MA 02139;
2 Department of Physics, University of Pennsylvania, Philadelphia, PA 19104, USA;
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I briefly review cosmological bounds on neutrino masses and the underlying gravitational physics at a level appropriate for readers outside the field of cosmology. For the case of three massive neutrinos with standard model freezeout, the current 95% upper limit on the sum of their masses is 0.42 eV. I summarize the basic physical mechanism making matter clustering such a sensitive probe of massive neutrinos. I discuss the prospects of doing still better in coming years using tools such as lensing tomography, approaching a sensitivity around 0.03 eV. Since the lower bound from atmospheric neutrino oscillations is around 0.05 eV, upcoming cosmological measurements should detect neutrino mass if the technical and fiscal challenges can be met.

I. INTRODUCTION

In the last few years, an avalanche of new cosmological data has revolutionized our ability to measure key cosmological parameters. Measurements of the cosmic microwave background (CMB), galaxy clustering, gravitational lensing, the Lyman alpha forest, cluster abundances and type Ia supernovae paint a consistent picture where the cosmic matter budget is about 5% ordinary matter, 25% dark matter, 70% dark energy, less than 1% curvature and less than 5% massive neutrinos [1, 2, 3, 4, 5]. The cosmic initial conditions are consistent with approximately scale-invariant inflation-produced adiabatic fluctuations, with no evidence yet for primordial gravitational waves [1, 2, 3, 4, 5].

How precisely do such cosmological observations give us information about neutrino masses, and how should the current limits be interpreted? For detailed discussion of post-WMAP astrophysical neutrino constraints, see [1, 2, 3, 4, 5] and in particular the excellent and up-to-date reviews [12, 13]. For a review of the theoretical and experimental situation, see [14]. The purpose of this symposium contribution is merely to provide a brief summary of the constraints and the underlying physics at a level appropriate for readers outside of the field of cosmology.

II. THE PHYSICS UNDERLYING COSMOLOGICAL NEUTRINO BOUNDS

Why do cosmological observations place strong bounds on neutrino masses? The short answer is that neutrinos affect the growth of cosmic clustering and this clustering can be accurately measured (Figure 1).

The CMB tells us that the Universe used to be almost perfectly uniform spatially, with density variations from place to place only at the level of $10^{-5}$. Gravitational instability caused these tiny density fluctuations to grow in amplitude into the galaxies and the large-scale structure that we observe around us today. The reason for this growth is simply that gravity is an attractive force:

$$\delta \propto a,$$

if the density at some point exceeds the mean density by some relative amount $\delta$, then mass will be pulled in from surrounding regions and $\delta$ increases over time. A classic result is that if all the matter contributing to the cosmic density is able to cluster (like dark matter or ordinary matter with negligible pressure), then fluctuations grow as the cosmic scale factor $a$ [17].
i.e., fluctuations double in amplitude every time the Universe doubles its linear size $a$.

If some fraction of the matter density is gravitationally inert and unable to cluster, the fluctuation growth will clearly be slower. If only a fraction $\Omega_\ast$ can cluster, then equation (1) is generalized to (10):

$$\delta \propto a^p,$$

(2)

where

$$p = \frac{\sqrt{1 + 24Ω_\ast}}{4} - 1 \approx Ω_\ast^{3/5}$$

(3)

and the approximation in the last step is surprisingly accurate. Such gravitationally inert components can include dark energy and (on sufficiently small scales) photons and neutrinos. Early on, the cosmic density was completely dominated by photons, so $p \approx 0$ and fluctuations essentially did not start growing until the epoch of matter-domination (MD). At recent times, the cosmic density has become dominated by dark energy $\Lambda$, causing fluctuations to gradually stop growing after a net growth factor of about $a_{\text{AD}}/a_{\text{MD}} \approx 4700$.3

Massive nonrelativistic neutrinos are unable to cluster on small scales because of their high velocities. Between matter domination and dark energy domination, they constitute a roughly constant fraction $f_\nu = 1 - Ω_\ast$ of the matter density. Equation (2) therefore gives a net fluctuation growth factor

$$\left(\frac{a_{\text{AD}}}{a_{\text{MD}}}\right)^p \approx 4700^p \approx 4700(1-f_\nu)^{3/5} \approx 4700 e^{-4M_\nu}$$

(4)

from matter-domination (MD) until today, where we have assumed $f_\nu \ll 1$ in the last step. We see that the basic reason that a small neutrino fraction has a large effect is simply that 4700 is a large number, so that a small change in the exponent $p$ makes a noticeable difference.

A key point to remember is that what mattered above was the neutrino density, specifically the fractional contribution $f_\nu$ of neutrinos to the total density. To translate observational constraints on the neutrino mass density into constraints on neutrino masses, we need to know the neutrino number density. Assuming that this number density is determined by standard model neutrino freezeout gives a number density around 112/cm$^3$ and 1.3

$$f_\nu \approx \frac{M_\nu}{\omega_m \times 94.4\,\text{eV}} \approx \frac{M_\nu}{14\,\text{eV}},$$

(5)

where

$$M_\nu \equiv \sum_{i=1}^{3} m_i^\nu$$

(6)

is the sum of the three neutrino masses and $\omega_m = h^2\Omega_m \approx 0.15$ is the measured matter density in units of 1.8788 $\times 10^{-26}$kg/m$^3$.

The power spectrum $P(k)$ shown in Figure 1 is the variance of the fluctuations $δ$ in Fourier space, so massive neutrinos suppress it by the same factor as it suppresses $δ^2$, i.e., by a factor 23:

$$\frac{P(k; f_\nu)}{P(k; 0)} \approx e^{-8f_\nu}.$$  

(7)

This means that a neutrino mass sum $M_\nu = 1\,\text{eV}$ cuts the power roughly in half on the small scales where neutrinos cannot cluster.

The length scale below which neutrino clustering is strongly suppressed is called the neutrino free-streaming scale, and roughly corresponds to the distance neutrinos have time to travel while the Universe expands by a factor of two. Intuitively, neutrinos clearly will not cluster in an overdense clump so small that its escape velocity is much smaller than the typical neutrino velocity. On scales much larger than the free-streaming scale, on the other hand, neutrinos cluster just as cold dark matter cannot.

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### III. WHAT ARE THE CONSTRAINTS?

Cosmological observations have thus far produced no convincing detection of neutrino mass, but strong upper limits as illustrated in Figure 2. This figure shows that the WMAP CMB-measurements alone 24 tell us almost nothing about neutrino masses and are consistent with neutrinos making up 100% of the dark matter. Rather, the power of WMAP is that it constrains other cosmological parameters so strongly that it enables scale structure data to measure the small-scale $P(k)$-suppression that massive neutrinos cause. Combining WMAP with Sloan Digital Sky Survey (SDSS) galaxy clustering measurements 26 gives the most favored value $M_\nu = 0$ and the 95% upper limit $M_\nu < 1.7\,\text{eV}$.

Including information about SDSS or 2dFGRS galaxy bias tightens this bound to $M_\nu < 0.6 - 0.7\,\text{eV}$ 1 28. Including SDSS measurements of the so-called Lyman α forest (intergalactic gas backlit by quasars) further tightens the bound to $M_\nu < 0.42\,\text{eV (95%)}.~$
These upper limits are complemented by the lower limit from neutrino oscillation experiments. Atmospheric neutrino oscillations show that there is at least one neutrino (presumably mostly a linear combination of $\nu_e$ and $\nu_\tau$) whose mass exceeds a lower limit around 0.05 eV \cite{14,21}. Thus the atmospheric neutrino data corresponds to a lower limit $\omega_\nu \gtrsim 0.0005$, or $f_\nu \gtrsim 0.004$. The solar neutrino oscillations occur at a still smaller mass scale, perhaps around 0.008 eV \cite{14,22,24}. These mass-splittings are substantially smaller than 0.42 eV, suggesting that all three mass eigenstates would need to be almost degenerate for neutrinos to weigh in near our upper limit. Since sterile neutrinos are disfavored from being thermalized in the early universe \cite{31,32}, it can be assumed that only three neutrino flavors are present in the neutrino background; this means that none of the three neutrinos can weigh more than about 0.42/3 = 0.14 eV. The mass of the heaviest neutrino is thus in the range 0.05 – 0.14 eV.

A caveat about non-standard neutrinos is in order. As mentioned above, the cosmological constraints to first order probe only the mass density of neutrinos, $\rho_\nu$, which determines the small-scale power suppression factor, and the velocity dispersion, which determines the scale below which the suppression occurs. For the low mass range we have discussed, the neutrino velocities are high and the suppression occurs on all scales where SDSS is highly sensitive. We thus measure only the neutrino mass density, and our conversion of this into a limit on the mass sum assumes that the neutrino number density is known and given by the standard model freeze-out calculation. In more general scenarios with sterile or otherwise non-standard neutrinos where the freezeout abundance is different, the conclusion to take away is an upper limit on the total light neutrino mass density of $\rho_\nu < 4.8 \times 10^{-28}$ kg/m$^3$ (95%). To test arbitrary non-standard models, a future challenge will be to independently measure both the mass density and the velocity dispersion, and check whether they are both consistent with the same value of $M_\nu$.

IV. OUTLOOK

Although cosmological neutrino bounds have recently improved dramatically, there is ample room for further improvement in the near and intermediate future. The basic reason for this is that the weakest link in current constraints is their dependence on other cosmological parameters. For instance, the galaxy clustering constraints cannot directly exploit the dramatic effect of neutrinos on the amplitude of the small-scale power spectrum shown in Figure 1 merely the slight change in its shape, and this shape change can be partially mimicked by changing other cosmological parameters such as the spectral index from inflation which effectively tilts the $P(k)$. The reason for this shortcoming is that galaxy surveys do not measure the clustering amplitude of matter directly, merely the clustering amplitude of luminous matter (galaxies), which is known to differ by a constant factor that is measured empirically.

Weak gravitational lensing bypasses this shortcoming. Light from distant galaxies or CMB patterns is deflected in a measurable way by the gravitational pull of all intervening matter, regardless whether it is luminous or dark, baryonic or non-baryonic, allowing the matter power spectrum $P(k)$ to be measured in a clean assumption-free way. This booming field has the potential to attain a neutrino mass sensitivity of 0.03 eV or better \cite{32,33,34}. Since the lower bound from atmospheric neutrino oscillations is around 0.05 eV, upcoming cosmological measurements should detect neutrino mass if the technical and fiscal challenges can be met.

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