A generalization of port-based teleportation and controlled teleportation capability

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Abstract: As a variant of the original quantum teleportation, the port-based teleportation has been proposed, and its various kinds of useful applications in quantum information processing have been explored. Two users in the port-based teleportation initially share an arbitrary pure state, which can be represented by applying one user’s local operation on a bipartite maximally entangled state. If the maximally entangled state is a 2M-qudit state, then it can be expressed as M copies of a two-qudit maximally entangled state, where M is the number of the ports. We here consider a generalization of the original port-based teleportation obtained from employing copies of an arbitrary bipartite (mixed) resource instead of copies of a pure maximally entangled one. By means of the generalization, we construct a concept of the controlled port-based teleportation by combining the control teleportation with the port-based teleportation, and analyze its performance in terms of several meaningful quantities such as the teleportation fidelity, the entanglement fidelity, and the fully entangled fraction. In addition, we present new quantities called the control power and the minimal control power for the new controlled version on a given tripartite quantum state.

I. INTRODUCTION

Quantum teleportation, proposed by Bennett et al. [1], is a fundamental and innovative way to transmit an unknown quantum information from a sender to a remote receiver by exploiting a prior distributed entanglement [2, 3]. According to its potential applications, this scheme has been studied by using various methods in experimental regimes as well as in theoretical ways [4–8].

As a variant of the quantum teleportation, the port-based teleportation (PBT) has been suggested [9–15]. While the standard teleportation requires a receiver’s recovering operation at the end of the protocol, the PBT scheme does not need such a correction operation, and only requires the receiver to choose a port depending on the classical information related to the sender’s measurement outcome. It has been shown from the feature of the PBT scheme that the PBT can have several kinds of applications about quantum information processing such as universal programmable quantum processor [9], instantaneous non-local quantum computation [10], quantum-channel discrimination [11], and quantum telecloning protocols [12, 13].

In the PBT protocol, two users initially prepare an arbitrary 2M-qudit pure state shared between them. The state can be decomposed as an arbitrary operation on one user’s system and a pure maximally entangled state, which can be represented as M copies of a two-qudit maximally entangled state. In this paper, we first employ copies of an arbitrary bipartite (mixed) state as the generalized resource instead of copies of the maximally entangled state in the PBT protocol, and analyze the generalized PBT.

We can also find another kind of the teleportation scheme over any tripartite pure states, called the controlled teleportation (CT), which is a modification of the splitting and reconstruction of the pre-shared quantum information [24, 25]. In the CT scheme, a controller’s assistance through a local measurement can improve the teleportation fidelity between the sender and the receiver, and hence it is natural to take into account for the controller’s power on the teleportation procedure. Recently, the control power has been rigorously investigated on the perfect CT scheme including higher dimensional cases [23, 24] as well as the general CT ones [25].

By combining the two different kinds of teleportation above, we here suggest a concept called the controlled port-based teleportation (CPBT), and analyze its performance by manipulating our generalized PBT.

This paper is organized as follows. In Sec. II we describe the definitions of several meaningful quantities which we here deal with such as the teleportation fidelity, the entanglement fidelity, and the fully entangled fraction with illustrating the relations between the teleportation fidelity and the other two fidelities. Furthermore, we briefly introduce the main idea of the PBT (see Subsec. II B). In Sec. III we provide a generalization of the original PBT, and derive the results related to the performance of our generalized PBT. In Sec. IV we provide a concept of the quantities for the new teleportation capabilities, and analyze its properties. Moreover, we study the controller’s control power (also minimal control power) for the CPBT on a given tripartite quantum state. Finally, discussions and remarks are offered in Sec. V and some open questions are raised for future work.

II. PRELIMINARIES

A. Teleportation fidelity, entanglement fidelity, and fully entangled fraction

We shortly review the mathematical definitions and the relation between the teleportation fidelity, the entanglement fi-
The entanglement fidelity is defined as

\[ F(\Lambda_\varphi) = \text{Tr} \Phi^+ \left( (\Lambda_\varphi \otimes \mathbb{I}) \Phi^+ \right), \]

where \( \Phi^+ = \frac{1}{d^2} \sum_{i,j=0}^{d-1} |ii\rangle \langle jj| \) is a maximally entangled state with Schmidt rank \( d \). It is known that \( F(\Lambda_\varphi) = f(\varphi) \) if \( \Phi^+ \) is equal to a pure state \( |\varphi\rangle \langle \varphi| \) which attains the maximum in Eq. (2), that is, \( f(\varphi) = \langle \varphi | \varphi \rangle = \text{Tr} \Phi^+ \varphi = F(\Lambda_\varphi) \). Without loss of generality, we may assume that the entanglement fidelity \( F(\Lambda_\varphi) \) is equivalent to the fully entangled fraction \( f(\varphi) \) by taking \( \Phi^+ \) in Eq. (1) as \( |\varphi\rangle \langle \varphi| \) satisfying \( f(\varphi) = \langle \varphi | \varphi \rangle \).

We now observe that useful relations between the fidelities. The teleportation fidelity and the entanglement fidelity over \( \Lambda_\varphi \) takes a universal relation \([26,27]\) in the form of

\[ T(\Lambda_\varphi) = \frac{dF(\Lambda_\varphi) + 1}{d + 1}. \]

We remark that \( T(\Lambda_\varphi) > \frac{2}{d^2+1} \) (or \( F(\Lambda_\varphi) > \frac{1}{2} \)) if and only if \( \varphi \) is said to be meaningful for the teleportation, since it was shown that the classical teleportation can take the fidelity at most \( T(\Lambda_\varphi) = \frac{2}{d^2+1} \) (or \( F(\Lambda_\varphi) = \frac{1}{2} \)) \([27]\).

### B. Port-based teleportation

Suppose that Alice wants to teleport to Bob a \( d \)-dimensional unknown pure state, which Alice wishes to teleport to Bob. Let \( A = \{A_1, A_2, \ldots, A_M\} \) and \( B = \{B_1, B_2, \ldots, B_M\} \) denote the Alice’s and Bob’s total systems for \( M \) ports, respectively. To begin with, we assume that the sender Alice and the receiver Bob share a 2\( M \)-qudit pure state of the form

\[ \varphi_{AB} = (O_A \otimes I_B) \Phi^+_{A_1B_1} \otimes \cdots \otimes \Phi^+_{A_MB_M} (O_A^\dagger \otimes I_B), \]

where Alice’s operations \( O_A \) satisfying \( \text{Tr}[O_A O_A^\dagger] = d^M \). Let \( A_i := A \setminus \{A_i\} \) and \( B_i := B \setminus \{B_i\} \), then we can represent the PBT channel \( \Lambda_\varphi \) over the state \( \varphi_{AB} \) as

\[ \Lambda_\varphi(\psi_T) = \sum_{i,j=1}^M \text{Tr}_{A_i B_j} [\Pi_i^{(T)}(\varphi_{AB} \otimes \psi_T) \Pi_i^{(T)}]_{B_i \rightarrow B_j}, \]

where the positive operator-valued measurement elements are described by \( \{\Pi_i^{(T)}\}_{i=1}^M \) such that \( \sum_{i=1}^M \Pi_i^{(T)} = \mathbb{1}_A \), and

\[ c_{AB}^{(T)} = \frac{\text{Tr}_{B_j} [\Phi^+_{A_1B_1} \otimes \cdots \otimes \Phi^+_{A_MB_M} (O_A^\dagger \otimes I_B) \otimes \psi_T]}{d^{M-1}}. \]

By exploiting Eq. (3), we can obtain the entanglement fidelity for the channel \( \Lambda_\varphi \) \([10,13]\) as

\[ F(\Lambda_\varphi) = \text{Tr}_{B_j} [\Phi^+_{A_1B_1} \otimes \cdots \otimes \Phi^+_{A_MB_M} (O_A^\dagger \otimes I_B) \otimes \psi_T]. \]

It was shown that if the PBT protocol is the deterministic standard one, that is, \( O \) is the identity operator and \( \{\Pi_i^{(T)}\}_{i=1}^M \) is the pretty good measurement then for sufficiently large \( M \gg 0 \) and any \( \epsilon > 0 \), the entanglement fidelity \( F(M) \) for the standard PBT protocol is given by \([9,10,13]\)

\[ F(M) = 1 - \frac{d^2 - 1}{4M} + O(M^{-\frac{3}{2}+\epsilon}). \]

As a corollary of the result, the teleportation fidelity \( T(M) \) of the protocol is given by

\[ T(M) = 1 - \frac{d(d-1)}{4M} + O(M^{-\frac{3}{2}+\epsilon}), \]

since (see Eq. (3))

\[ T(M) = \frac{dF(M) + 1}{d + 1}. \]

### III. OUR GENERALIZED PORT-BASED TELEPORTATION

In this section, we take into account copies of an arbitrary mixed state for a generalized PBT instead of copies of a maximally entangled state in the original PBT (See Figures 1). First,
we analyze the entanglement fidelity of the PBT for copies of a depolarized state. Note that any mixed state can always be transformed to a depolarized state with a parameter \( p \in [0, 1] \) by local operation and classical communication (LOCC). More precisely,

\[
\rho_{AB}^{(AB)}(\text{LOCC}) = p \Phi_{AB}^* + \frac{1 - p}{d^2} \mathbb{1}_{AB}.
\]

where \( \Phi_{AB}^* \) is the maximally entangled state and \( p \in [0, 1] \). As in the scenario of the PBT, let

\[
\phi_p^{(AB)} := (O_A \otimes I_B) \rho_p^M (O_A^* \otimes I_B)
\]

be the \( M \)-port initial setting for the depolarized state

\[
\rho_p = (D_p \otimes I_B)(\Phi_{AB}^*),
\]

where \( D_p \) is the depolarizing channel with noise rate \( 1 - p \), that is,

\[
D_p(\rho) = (1 - p) \frac{\mathbb{1}}{d} + p \rho.
\]

Then the depolarized teleportation channel \( \tilde{\Lambda}_{\phi_p} \) becomes

\[
\tilde{\Lambda}_{\phi_p}(\psi_T) = \sum_{r=1}^M \text{Tr}_{AB} \left[ \Pi_{AB}^{(T)} (\phi_p^{(AB)}(\psi_T)) \sqrt{\Pi_{AB}^{(T)}} \right] = \sum_{r=1}^M \text{Tr}_{AB} \left[ \Pi_{AB}^{(T)} (O_A \otimes I_B) \sigma_p^{(r;AB)} (O_A^* \otimes I_B) \otimes \psi_T \right],
\]

where \( \sigma_p^{(r;AB)} \) in Eq. (10) can be represented as follows.

**Lemma 1.** For any \( p \in [0, 1] \),

\[
\sigma_p^{(r;AB)} = \sum_{r=0}^M p^{M-r}(1 - p)^r \frac{1}{d^{M-1}} (M - 1) \frac{1}{r!} \Phi_{AB}^* \mathbb{1}_{AB}.
\]

**Proof.** Since

\[
\sigma_p^{(r;AB)} = \left[ \text{Tr}_{B_r} \left( f_p^{(AB)} \otimes \cdots \otimes f_p^{(AB)} \right) \right]_{B_r \rightarrow B},
\]

it is tedious but straightforward to obtain the equality in Eq. (11). \( \square \)

**Theorem 2.**

\[
F(\tilde{\Lambda}_{\phi_p}) = p F(\tilde{\Lambda}_{\phi_p}) + \frac{1}{d^2} (1 - p).
\]

where \( \phi \) is the state in Eq. (2), that is, \( \phi = \phi_1 \).

**Proof.** By using Lemma 1 it can be shown that

\[
F(\tilde{\Lambda}_{\phi_p}) = \frac{1}{d^2} \sum_{r=1}^M \text{Tr}_{AB} \left[ (O_A \otimes I_B) \sigma_p^{(r;AB)} (O_A^* \otimes I_B) \right]
\]

\[
= F(\tilde{\Lambda}_{\phi_p}) \cdot \sum_{r=0}^M p^{m-r}(1 - p)^r \frac{1}{d^2} (M - 1)
\]

\[
+ \frac{1}{d^2} \sum_{r=0}^M p^{m-r}(1 - p)^r \frac{1}{d^2} (M - 1)
\]

\[
= F(\tilde{\Lambda}_{\phi_p}) \cdot \sum_{r=0}^M (1 - p)^r (1 - p) \frac{1}{d^2} (M - 1)
\]

\[
= p F(\tilde{\Lambda}_{\phi_p}) + \frac{1}{d^2} (1 - p),
\]

since

\[
\sum_{r=1}^M \text{Tr}_{AB} \left[ (O_A \otimes I_B) \mathbb{1}_{AB} (O_A^* \otimes I_B) \right] = 1.
\]

\( \square \)

By Theorem 2 we clearly obtain the following corollary.

**Corollary 3.**

\[
\Gamma(\tilde{\Lambda}_{\phi_p}) = \frac{d^2}{d^2 - 1} p F(\tilde{\Lambda}_{\phi_p}) + 1 + p \frac{1}{d^2 - 1}.
\]

We now note that the depolarized fully entangled fraction of \( \rho_p \) can be obtained by

\[
f(\rho_p) = \max_{\{\psi_1\} \in \{\phi_1\}} (\phi^* | \rho_p | \phi^*)
\]

\[
= p + \frac{1 - p}{d^2} = \frac{1}{d^2} (1 - p).
\]

we can thus have an identity \( p = \frac{\beta}{d^2} - \frac{1}{d^2} \).

For \( p \in [0, 1] \), let \( F(\rho) \) be the entanglement fidelity and the teleportation fidelity for the standard PBT protocol over the state \( \rho_p^M \), respectively. Then by Theorem 2 and Corollary 3 the \( F(\rho) \) and \( \Gamma(\rho) \) can be described as follows.

**Theorem 4.** For sufficiently large \( M \gg 0 \) and for any \( \epsilon > 0 \), we obtain that

\[
F(\rho) = f(\rho_p) - \frac{p(d^2 - 1)}{4M} + O(M^{-\frac{1}{2} + \epsilon}),
\]

\[
\Gamma(\rho) = \Gamma(\Lambda_p) - \frac{pd(d - 1)}{4M} + O(M^{-\frac{1}{2} + \epsilon}).
\]

Finally, we investigate the generalized teleportation fidelity and the entanglement fidelity for copies of an arbitrary mixed state shared between Alice and Bob.
Theorem 5. For any bipartite mixed state $\rho$ over Alice and Bob, i.e., mixed teleportation channel $\tilde{\Lambda}_{\rho^{AB}}$, the entanglement fidelity $F^\rho$ and the teleportation fidelity $F^T_\rho$ for the standard PBT on $\rho^{BM}$ are shown as
\[
F^\rho(M) = f(\rho) \left(1 - \frac{d^2 f(\rho) - 1}{4M} + \frac{d^2 f(\rho) - 1}{4M} + O(M^{-\frac{3}{2} + \epsilon})\right),
\]
\[
F^T_\rho(M) = \mathbb{F}_T(\Lambda_\rho) \left(1 - \frac{d^2}{4M} + \frac{d^2}{4M} + O(M^{-\frac{3}{2} + \epsilon})\right).
\]

Proof. Recall that if $\rho = d^2 f(\rho) - 1 = d^2 f(\rho) - 1$, since without loss of generality, we may assume that $\rho$ can be transformed to $\rho^\rho$ satisfying $f(\rho) = f(\rho^\rho)$ under LOCC. Thus from Theorem[3], we have
\[
F^\rho(M) = f(\rho) - \frac{d^2 f(\rho) - 1}{4M} - \frac{d^2 f(\rho) - 1}{4M} + O(M^{-\frac{3}{2} + \epsilon})
\]
\[
= f(\rho) \left(1 - \frac{d^2}{4M} + \frac{d^2}{4M} + O(M^{-\frac{3}{2} + \epsilon})\right)
\]
and
\[
F^T_\rho(M) = \frac{dF^\rho(M) + 1}{d + 1}
\]
\[
= \mathbb{F}_T(\Lambda_\rho) \left(1 - \frac{d^2}{4M} + \frac{d^2}{4M} + O(M^{-\frac{3}{2} + \epsilon})\right).
\]

This completes the proof. □

IV. CONTROLLED PORT-BASED TELEPORTATION AND ITS CONTROL POWER

In this section, we propose a new extended concept of the PBT, called the CPBT, which combines the well-known two ideas about CT and PBT. Under this consideration, Charlie can be endowed with assistance ability as a control power.

Let $\varphi_{ABC} := |\varphi\rangle|\varphi_{ABC}\rangle$ be a three-qudit pure state. For any $\alpha \in \{A, B, C\}$, let the maximal CPBT fidelity $F_{CT}^\varphi$ be the maximal teleportation fidelity (depending on the port $M$) of the resulting two-qudit state $\rho_\tilde{\Lambda}$ by the subsystem $\beta_\gamma$, where $\{\alpha, \beta, \gamma\} = \{A, B, C\}$, after the outcome measurement $\{M_i\}$ performed on the state $\rho_\alpha = T_{\beta_\gamma}\varphi_{ABC}$. In other words, it mathematically becomes
\[
F_{CT}^\varphi(M) = \max_{\{M_i\}} \sum_{i=0}^{d-1} \text{Tr}(M_\varphi \rho_\varphi) \mathbb{F}_T(\tilde{\Lambda}_\rho)(1 - \frac{d^2}{4M})
\]
\[
= \max_{\{M_i\}} \sum_{i=0}^{d-1} \text{Tr}(M_\varphi \rho_\varphi) \mathbb{F}_T(\tilde{\Lambda}_\rho)(1 - \frac{d^2}{4M}) + \frac{d^2}{4M} + O(M^{-\frac{3}{2} + \epsilon}),
\]
where the maximum is taken over all measurements $\{M_i\}$ on the subsystem $\alpha$, and $F_{CT}^\alpha$ is called the maximal CT fidelity, which is defined as the maximal teleportation fidelity of the resulting two-qudit state in the subsystem $\beta$ by the measurement on the system $\alpha$ as in Refs. [21, 23, 25], that is,
\[
F_{CT}^\alpha(\varphi_{ABC}) = \max_{\{M_\varphi\}} \sum_{i=0}^{d-1} \text{Tr}(M_\varphi \rho_\varphi) \mathbb{F}_T(\tilde{\Lambda}_\rho)(1 - \frac{d^2}{4M}) + O(M^{-\frac{3}{2} + \epsilon}).
\]

We notice that since $F_{CT}^{\varphi_{ABC}}(M) \geq F_{CT}^\rho(M)$, one can naturally define a concept of the control power for the CPBT on a given tripartite quantum state as the difference between the maximal CPBT fidelity and the teleportation fidelity without control.

For each $\alpha \in \{A, B, C\}$ and a sufficiently large port number $M > 0$, let the control power $F_{CT}^{\varphi_{ABC}}(M)$ of the state $\varphi_{ABC}$ be defined as
\[
F_{CT}^{\varphi_{ABC}}(M) = F_{CT}^{\varphi_{ABC}}(M) - F_{CT}^\rho(M),
\]
and the minimal control power $F_{CT}(M)$ of the state $\varphi_{ABC}$ be defined as
\[
F_{CT}(M) = \min_{\forall\alpha \in \{A, B, C\}} F_{CT}^{\varphi_{ABC}}(M).
\]

Then the following corollary can clearly be obtained.

Corollary 6. For each $\alpha \in \{A, B, C\}$, the control power of a state $\varphi_{ABC}$ can simply be expressed as
\[
F_{CT}^{\varphi_{ABC}}(M) = F_{CT}^\varphi(M) \left(1 - \frac{d^2}{4M}\right) + O(M^{-\frac{3}{2} + \epsilon}),
\]
where $F_{CT}^\varphi(\varphi_{ABC})$ is the control power for the CT proposed in Ref. [25], that is,
\[
F_{CT}^\varphi(\varphi_{ABC}) = F_{CT}^\varphi(\varphi_{ABC}) - \mathbb{F}_T(\tilde{\Lambda}_\rho).\]

Hence by exploiting Corollary 6 and the results in Ref. [25], we can readily compute the control powers for three-qubit pure states, especially, the extended GHZ state, $|\varphi_{GHZ}\rangle = a|000\rangle + b|111\rangle$, and the W-class state, $|\varphi_{W}\rangle = w_0|000\rangle + w_1|100\rangle + w_2|011\rangle + w_3|110\rangle$ with the coefficients $w_i \geq 0$ such that $\sum_i w_i^2 = 1$, that is,
\[
F_{CT}^\varphi(\varphi_{GHZ}) = \frac{2|a||b|}{3},
\]
\[
F_{CT}^\varphi(\varphi_{W}) = \frac{1}{6}(2w_0w_2 + 1 - \sqrt{W_a}),
\]
where
\[
W_a = \max \left\{w_1^2 + (\mp w_0 \pm w_3 + w_1)^2)w_1^2 + (w_0 \pm w_2 \pm w_3)^2\right\},
\]
the control powers for the CPBT on the extended GHZ state and the W-class state become
\[
F_{CT}(M) = F_{CT}^{\varphi_{GHZ}}(M) = \frac{2|a||b|}{3} \left(1 - \frac{1}{M}\right) + O(M^{-\frac{3}{2} + \epsilon}),
\]
and
\[
F_{CT}^{\varphi_{W}}(M) = \frac{1}{6}(2w_0w_2 + 1 - \sqrt{W_a}) \left(1 - \frac{1}{M}\right) + O(M^{-\frac{3}{2} + \epsilon}),
\]
respectively.
V. CONCLUSIONS

In this paper, we have introduced the new quantities for the controlled teleportation capability as a variant of teleportation capability for the original quantum teleportation (or the PBT and the CT), and have analyzed its control powers in terms of the port number $M$ representing how faithfully the PBT on a given tripartite state can be performed. Here, we have made use of a generalization technique to employ the PBT channel on copies of an arbitrary bipartite mixed state instead of the standard one on copies of a pure bipartite maximally entangled state. Furthermore, we have found explicit formulas and relations on the teleportation fidelity and the entanglement fidelity as well as the maximal teleportation fidelity, and thus we have derived the control powers for the CPBT on tripartite quantum states.

There are still some intriguing open questions on the PBT itself or beyond. First, we can imagine another variants of the PBT protocols or their communication capabilities in quantum communication. For example, a controlled dense-coding [31] and a remote state-preparation scheme [32]. Since quantum communication. For example, a controlled dense-coding [31] and a remote state-preparation scheme [32]. Since quantum communication. For example, a controlled dense-coding [31] and a remote state-preparation scheme [32]. Since quantum communication.

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