Effects of process parameters on tool vibration and force transmissibility in high-speed micro-milling machine

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Abstract
High-speed micro-milling is an emerging technology used to produce micro and miniaturized products with smooth surface finish and high dimensional precision. However, tool vibration is a major problem in micro-milling as it directly affects product accuracy, surface quality, and tool life. Inappropriate selection of process parameters increases radial and axial thrust as well as force transmitted to structure during micro-machining which results in rapid tool vibration. This work focuses on the experimental investigation of process parameters (cutting speed and depth of cut) in order to reduce tool vibration due to axial and radial thrust in a newly developed (Das et al. in Int J Adv Manuf Technol, 2021) high-speed micro-milling machine and to achieve stable machine operating conditions. The tool used in this experiment is a 2-flute end mill cutter (1 mm cutter diameter), and the workpiece is a commercially pure titanium (CpTi) plate. The operation is performed at different depths of cut and varying cutting speeds keeping the chip load constant. Vibration signals are acquired and processed to obtain the vibration thrust of the tool and the force transmitted to the structure due to vibration. Additionally, statistical analysis is performed on the experimental results using ANOVA, and regression equations are developed for the response functions. Eventually, optimization of the response functions has been performed using desirability analysis. The results indicate that as the depth of cut and cutting speed increases, both axial as well as radial thrust decreases leading to lower vibration amplitude of the cutting tool and reduction in force transmitted to the machine structure.

Keywords High-speed micro-milling · Tool vibration · Digital signal processing · Vibration analysis · Vibration amplitude · Force transmissibility

1 Introduction
The extended demand for micro and miniaturized components has improved the growth of micromachining technology, especially micro-milling. High-speed micro-milling is an adaptable technology, capable to produce complex features over difficult-to-machine materials with mirror surface finish and dimensional precision. A design approach for the ultra-precision high-speed micro-milling machine has been presented by Huo et al. [2–4] in their work. The authors reviewed state-of-the-art ultra-precision machines with micro-machining capability and discussed the considerations and specifications of a five-axis ultra-precision micro-milling machine. Parameters like motion accuracy, dynamic stiffness, and thermal stability were used to formulate the design approach for the machine. The authors also proposed an approach to analyse and optimize the dynamic performance of the machine at the early design stage. Babu et al. [5] used tech learning-based optimization (TLBO) to optimize the process parameters and tool parameters in order to reduce the dead metal zone (DMZ) and improve the machine performance during micro-milling. The authors developed a finite element model for DMZ and mechanistic models for cutting and thrust forces. These models were integrated to evaluate the cutting and thrust forces. Experimental investigations were carried out to verify the theoretical results and good agreement was found between the two. Pansare and Sharma [6, 7] presented an experimental advent to predict surface roughness using a mathematical model. The authors used the equivalence variance test [6] to verify the model.
and the ant colony optimization algorithm [7] for chip load-responsive optimization. Das et al. [1] also developed a high-speed vibration-free micro-milling machine. Certain aspects of micro-milling comprising recent advances and future trends have been reviewed by Balazs et al. [8] in their study. However, tool vibration in high-speed micro-machining is a crucial problem that significantly affects the dimensional accuracy and surface finish. Due to low stiffness, the micro-milling cutters are subjected to severe vibration under the action of cutting force and thrust force which also resulted in tool damage. Therefore, minimization of tool vibration is a challenging issue for high-speed micro-milling. Appropriate parametric optimization during high-speed micro-milling is highly essential in order to reduce tool vibration.

Several researchers tried to determine a stable machining condition in terms of low surface roughness and high tool life. Chen et al. [9] investigated that tool vibration increases with feed rate and depth of cut in precision milling. A stable machining condition was established at a higher spindle speed. Therefore, a better surface finish was found at low depth of cut, low feed rate, and high spindle speed. Similar effects were determined by Bhogal et al. [10], in which it was found that tool vibration increases at high cutting speed, feed rate, and depth of cut. However, in the case of micro-milling peculiar effects were observed. Singh et al. [11] used frequency response function and finite element modal analysis for the prediction of chatter stability in high-speed micro-milling of Ti-6Al-4V alloy. Experimental modal analysis was performed to verify the FEM results. Good agreement was found between the simulation and experimental results. In another work, Singh et al. [12] presented a mechanistic force model based on the incorporation of velocity and chip load cutting coefficient into the analytical model for the prediction of cutting forces in the micro-milling of Ti-6Al-4V alloy. Experimental investigations were performed to validate the analytical results. The outcomes of the comparison revealed that the developed model had lower prediction errors as compared to the existing models. Zhou et al. [13] performed CNC milling experiments on Al/SiC metal matrix composites and used this data for the prediction and control of the surface roughness of the composites using an artificial neural network. The authors proposed a feed-forward multi-layered ANN roughness prediction model, using the Levenberg–Marquardt backpropagation algorithm to evaluate the mathematical relationship between the cutting parameters and average surface roughness. The results revealed that the developed model was capable of predicting the average surface roughness with a mean error of 2.08%. A similar application of the backpropagation neural network for spindle vibration-based tool wear monitoring in the micro-milling process was presented by Hsieh et al. [14] in their work.

Rahman et al. [15] determined that the tool life enhances with an increase in depth of cut and reduction in cutting speed, however, feed force and radial force were found to increase with an increase in feed rate and depth of cut. At a low feed rate, due to elastic recovery, significant vibration is induced and may be further minimized by reducing the depth of cut [16]. The optimization of proper machining parameters is highly essential in micro-milling to reduce tool wear, tool vibration, and cutting and thrust force. To achieve this objective, Malekian et al. [17] developed several tool wear monitoring technologies for micro-milling in order to maintain product accuracy and tool life. Feed rate is the most important parameter for cutting force in micro-milling. An increase in feed rate is desirable for stable machining conditions. Additionally, low feed rate results in ploughing effect, and elastic recovery of the work material, and hence, chattering is observed [18]. Park and Rahnama [19] developed a chatter stability model based on parametric changing in order to reduce chatters in micro-milling. Wang et al. [20] demonstrated a better process damping effect at lower cutting speed (Spindle speed below 20,000 rpm), however, at a higher speed, low depth of cut was found to be favourable in order to achieve low vibration in micro-milling. Proper selection of feed rate and depth of cut is a much-needed criterion to reduce built-up edge and improve surface quality [21]. Ghoreishi et al. [22] investigated the effect of factors such as feed rate, cutting speed, depth of cut, and the presence of cryogenic coolant on tool vibration in high-speed machining of metal matrix composites. The authors concluded that the tool wear in the machining of metal matrix composites is decreased by the use of a cryogenic coolant.

Lu et al. [23] observed that increase in feed rate and depth of cut enhances the surface roughness of the machined product. In order to minimize tool vibration, low depth of cut is favourable for lower spindle speed (40,000–70,000 rpm). However, for machining with a higher depth of cut, the machine needs to be operated at a higher spindle speed (above 70,000 rpm) [24]. Zhang et al. [25] investigated that tool vibration is a significant source of reduction in cutting force in micro-milling. Therefore, the feed rate should be low in micro-milling in order to minimize cutting and thrust forces during micro-milling for reduced tool vibration [26]. Similarly, cutting force increases with cutting speed. However, this phenomenon is observed only during low-speed operation (spindle rotational speed below 5000 rpm). Liu et al. [27] performed milling with and without ultrasonic vibration to demonstrate the effects of vibration and cutting parameters on tool wear, cutting force, and wear mechanism. Authors also used statistical methods like Taguchi and ANOVA [28] during machining to perform strain rate analyses.

However, the effect of process parameters (especially depth of cut) over the axial and radial thrust force is still undetermined for high-speed micro-milling operation. The thrust forces result in tool vibration in the axial and radial
directions. This deteriorates the surface finish and causes tool wear during high-speed machining of hard materials. Studying tool vibration during experiments gives the option to adjust the parameters in order to reduce the vibration and hence enhance the tool life and provide a better surface finish. This work is focused on the experimental investigation of process parameters (cutting speed and depth of cut) in order to reduce tool vibration due to axial and radial thrust in high-speed micro-milling. A 2-flute end milling cutter (1 mm cutter diameter) was used to cut a commercially available pure titanium plate of 5 mm thickness. The operation was performed at different depths of cut and varying cutting speeds keeping the chip load constant. Vibration signals were acquired and processed to obtain the axial vibration of the tool. Statistical analysis has been performed on the experimental results by ANOVA, and regression equations have been developed for the response functions. Eventually, optimization of the response functions has been performed using desirability analysis.

This paper provides a brief introduction to axial and radial thrust due to tool vibration in the first section. The methodology used has been discussed in the second section. The third section provides the results obtained from experiments and their discussions; the comparative study of the obtained results; the statistical model and the optimization of response functions. Finally, the conclusions have been provided in the fourth section.

2 Methodology

This work is based on the experimental vibration and force analysis of a two-flute milling cutter due to variation in depth of cut. During machine operation, vibrations are caused due to high spindle speed. Due to these vibrations, some amount of force is also transmitted to the structure of the machine. These vibrations and transmitted force may lead to unstable machining conditions and poor surface finish and feature generation on the workpiece. To study and analyse the best suited conditions with low vibrations and low force transmissibility, experimental investigations and analysis have been carried out.

The experiments were performed on CNC operated high-speed micro-milling centre with a two-flute milling cutter mounted on the spindle. The maximum rotational speed of the spindle is 60,000 rpm. For the experiments, a micro-milling cutter was mounted on the spindle of the micro-milling setup as shown in Fig. 1. Machining was performed over a plate of commercial pure titanium (CpTi) with a cross section of 25 mm × 25 mm and 5 mm thickness. The tool used was a 1 mm diameter 2 flutes milling cutter of AlTiN coated tungsten carbide made by Axis microtools. The operating conditions were varied by varying the operating frequency and depth of cut. The operating frequency was varied from 200 to 700 Hz at intervals of 100 Hz for three different depths of cut for each set of experiments. The depth of cut was 50, 75, and 100 µm. The cutting length for each set of operations was fixed at 20 mm.
The chip load was kept constant at 2 µm for each experiment. The chip load can be calculated from the formula, \( L = f/NZ \) = 2 µm. Where \( Z \) is the number of flutes; \( f \) is the feed rate and \( N \) is the rpm of the spindle. Therefore, with an increase in RPM, the feed rate is also increased to maintain the chip load as constant. For a particular cutting speed, the vibration signals have been taken for different depths of cut.

A spindle speed of 200 Hz was selected because it is the resonance condition of the system under the action of cutting force. The vibration data at 200 Hz signify the effect of resonance on the axial and radial vibration of the tool.

The vibration signals were acquired using two uniaxial accelerometers mounted in axial and radial directions on the spindle. A force transducer was used to acquire the signals of force transmitted to the structure. The accelerometers were mounted as close to the tool as possible to acquire the accurate readings of vibration. Similarly, the force transducer was mounted closest to the spindle on the structure to acquire accurate data. The position of the transducers affects the measurement results, but the trend of the results is expected to remain the same at all points. Moreover, mounting the sensors close to the tool will lead to accurate measurements of vibration data. The data were acquired using Bruel and Kjaer RT Photon pro 4 channel analyser. The accelerometers used were Bruel and Kjaer Type 4507 piezoelectric CCLD accelerometer with frequency range of 0.3–6000 Hz and sensitivity of 10.14 mV/ms^-2. The force transducer used was Bruel and Kjaer Type 8230–002 deltatron force transducer with a range of ±2000 N and sensitivity of 2.27 mV/N.

These acquired data were then processed in ME-Scope software to obtain the vibration amplitude for each operating condition. The thrust force acts in the axial direction of the spindle. Hence the amplitude of tool vibration along the axial direction signifies the movement due to the action of thrust force. Any change in amplitude due to changing the depth of cut specifies the effect of depth of cut on axial thrust force during machining. The vibration amplitude in the radial direction signifies the lateral vibrations of the tool. These vibrations can lead to irregular features on the workpiece. Hence, it is essential that radial vibration should be low. The force transmitted to the support structure signifies the stability of the structure. Lower forces signify higher stability and hence, higher efficiency of machining and increased tool life.

### 3 Results and discussion

#### 3.1 Analysis of the experimental results

The vibration spectra for axial direction obtained from the experiments on the micro-milling machine are shown in Fig. 2. The variation in vibration amplitude due to changes in depth of cut and cutting speed are also represented.

Figure 2 represents the frequency spectrum of axial direction obtained at different cutting speeds and depths of cut. The peaks obtained in the spectrum represent the frequencies at which the signals were recorded. Figure 2a is the frequency spectrum for 50 µm depth of cut at varying operating frequencies of the micro-milling machine. Similar frequency spectrums for 75 µm depth of cut and 100 µm depth of cut are shown in Fig. 2b and c, respectively. The arrows on the graph represent the direction in which the measurement was taken. The larger peaks were mostly observed at 200 Hz frequency. Therefore, the resonance frequency can be situated adjacent to 200 Hz. It was observed that the axial tool vibration decreases with an increase in cutting speed. This is expected because the stable cutting condition is established at higher cutting speeds in micro-milling. As per the previous literature, cutting force reduces with an increase in cutting speed during high-speed micro-milling. Therefore, a smooth cutting condition will be developed at a higher operating frequency of the spindle, resulting in a reduction in axial thrust and axial tool vibration. In addition, the feed rate has been proportionally increased with cutting speed to maintain a constant chip load. The chatter stability is developed at a higher feed rate. This is responsible for low vibration amplitude at higher operating frequencies as well. It is confirmed that for the same depth of cut, the vibration amplitude is the maximum when the operating frequency is equal to the resonance condition of the system. It is also observed that the vibration is lowest for maximum operating frequency.

The vibration spectra for radial direction obtained from the experiments on the micro-milling machine are shown in Fig. 3. The variation in vibration amplitude due to changes in depth of cut and cutting speed are also represented.

Figure 3 represents the frequency spectrum of axial direction obtained at different cutting speeds and depths of cut. The peaks obtained in the spectrum represent the frequencies at which the signals were recorded. The trend of radial vibration is similar to that of axial vibration. The arrows on the graph represent the direction in which the measurement was taken. But it can be observed that the vibration amplitude in the radial direction is higher as compared to the axial direction. This is due to the fact that during machining, the main cutting force is acting in the radial direction tangentially while the feed force is acting in the radial direction as well. On the other way, only the axial thrust force is acting in the axial direction. The combination of the main cutting force and feed force in the radial direction invariably exceeds the thrust force during the machining operation. Additionally, the axial load is on the spindle and the spindle holder reduces its movement in the axial direction. Hence, the vibration amplitude in the axial direction is lower than the vibration amplitude in the radial direction. Moreover, higher vibration amplitude in the axial direction will lead to
an improper depth of cut and thus poor feature generation and surface finish on the workpiece. Hence, lower vibration amplitude in the axial direction is desirable.

The maximum amplitudes determined in different cutting operations have been analysed by the Taguchi method. Lower vibration amplitude was desirable in the experiment. Therefore, the “smaller is better” condition was applied during Taguchi analysis. Figure 4 shows the mean effect plots of SN ratios for the maximum amplitudes in axial and radial vibration. The lowest negative SN ratio indicates better results in terms of lower vibration amplitude. It is clear from Fig. 4 that the maximum amplitudes have been reduced at higher operating frequency (cutting speed) in both cases, i.e. axial and radial vibration. Meanwhile, the amplitude of tool vibration decreases along the axial and radial direction at a higher depth of cut. In both cases, the reduction of vibration amplitude was significant when the depth of cut increased from 50 to 75 µm. At higher depths of cut, a large amount of heat accumulation takes place in the cutting zone. This is attributed to larger involvement of the cutting tool in the workpiece leading to high frictional force. The low thermal conductivity of titanium restricts the dissipation of the accumulated heat. Therefore, a localized heating zone is generated over the workpiece surface resulting in thermal softening of the workpiece at the cutting area. Additionally, for a larger depth of cut, the cutting tool penetrates the surface which results in a better grip of the tool on the workpiece surface with less amount of chatter. This condition develops a stable machining condition with lower vibration amplitude. On the contrary, for lower depths of cut, the tool is just ploughing over the surface depending upon the cutting edge radius and results in chatter formation during machining. This effect increases the radial and axial thrust on the cutting tool, resulting in higher tool vibration. Eventually, no significant change was observed in vibration amplitude when the depth of cut was increased from 75 to 100 µm. This was evident as a stable machining condition was developed at a higher depth of cut. However, the amplitude of vibration

Fig. 2 Spectra for axial vibration for depth of cut of a 50 µm; b 75 µm; and c 100 µm
slightly increased when the depth of cut was increased from 75 to 100 µm. This is due to higher thrust force at a higher depth of cut.

The spectra for force transmitted to the structure are obtained from the experiments on the micro-milling machine and presented in Fig. 5. The variation in force transmitted due to changes in depth of cut and cutting speed are also represented.

Figure 5 represents the frequency spectrum for force transmitted to the structure of the micro-milling machine at different cutting speeds and depths of cut. The peaks obtained in the spectrum represent the frequencies at which the signals were recorded. It was observed that the transmitted force decreased with the increase in the depth of cut. This is attributed to thermal softening of the workpiece due to elevated heat accumulation with increasing depth of cut. Moreover, at higher depths of cut, material removal takes place as a result of shearing phenomena. However, the effect of ploughing is significant at a lower depth of cut rather than shearing which enhances the force requirement in the cutting phenomenon. Even though the shear force is higher at higher depths of cut, it is lower than the combination of shear force and ploughing force acting at lower depths of cut. Hence, lower force is transmitted to the structure at higher depths of cut. Apart from a peak at 200 Hz, two peaks at 400 Hz and 600 Hz are also observed. These peaks can occur due to the probable occurrence of second and third-order natural
frequencies of the system. However, this has not been confirmed in the present study.

Figure 6 depicts the mean effect plots for SN ratios of the transmitted force achieved in different machining conditions determined in the Taguchi analysis. “Smaller is better” condition has been applied during the analysis to minimize the output. Figure 6 clearly shows that the maximum amplitude of transmitted force is influenced significantly by the operating frequencies rather than the higher order frequencies. Therefore, a complete correlation has not been achieved between the maximum amplitude of transmitted force and operating frequencies. However, the transmitted force is observed to decrease at higher depths of cut substantiating the fact presented in the spectrum.

3.2 Comparative study

A comparative study of the variation in axial and radial vibrations and the force transmitted to the machine structure is presented in this section.

Figure 7 represents a comparative analysis of the variation of amplitude in axial vibrations with different depths of cut and cutting speed.

Figure 8 represents a comparative analysis of the variation of amplitude in radial vibrations with different depths of cut and cutting speed. It is shown in Figs. 7 and 8 that the tool vibration decreases along the axial as well as radial direction with increasing depth of cut. As the depth of cut increases, shearing action becomes dominant over the
Fig. 5 Spectra for force transmitted for depth of cut of a 50 µm; b 75 µm; and c 100 µm

Fig. 6 Mean effects plot for SN ratios of the force transmitted
This may increase the chatter and the amplitude of axial and radial vibration resulting in breakage of the cutting tool.

Additionally, it can be observed that the vibration amplitude in the radial direction at any particular frequency and depth of cut is higher than the vibration in the axial direction. This is due to the action of the main cutting force along the radial direction which exceeds the thrust force along the axial direction. In addition, the axial load on the spindle is absorbed by the spindle holder and the machine structure. This restricts the translational movement in the axial direction. This results in higher stability along the Z axis. Moreover, the rotation of the spindle causes slightly higher vibration amplitude along the radial direction due to a slight amount of spindle runout. Lower axial vibrations lead to better finish and feature generation in the workpiece. Hence, lower axial vibration is desirable.

It is shown in Fig. 9 that the force transmitted to the machine structure decreases with an increase in the depth of cut. Higher depth of cut led to lower vibration amplitude, and thus lower force has been transmitted to the machine structure.

Lower tool vibration and low force transmission lead to higher stability, higher tool life, better surface finish, and less noisy operation. Therefore, in order to achieve a quality surface finish with improved tool life, the machining should be done at high speed. Additionally, a considerably higher depth of cut depending upon the cutting edge radius of the milling cutter during high-speed micro-milling operation is recommended based on this study.

### 3.3 Statistical analysis and optimization by desirability analysis

Analysis of variance (ANOVA) has been performed on the experimental values of the amplitudes of axial vibration,
amplitudes of radial vibration, and the transmitted force to check the significance of the input parameters. Two variable process parameters i.e. operating frequency, and depth of cut are selected as the source factors. For each analysis, a 5% level of significance has been defined. Table 1 depicts the ANOVA models for all three output parameters.

It is shown in Table 1 that the operating frequency (Spindle speed) is the most contributing process parameter affecting all three responses followed by the depth of cut. In all cases, the operating frequency is significant as the P-value is less than the level of significance (0.05). However, the depth of cut is significant for the amplitudes of radial vibration and transmitted force. It is not significant for the amplitudes of axial vibration. Additionally, the contribution of the depth of cut is very less for the three responses as compared to the operating frequency.

Additionally, regression equations have been developed for all three responses to predict the output values. Poisson’s regression model is found to be appropriate for the amplitudes of axial and radial vibration. For both cases, the interactions between operating frequency and depth of cut are taken into consideration. Equation 1 and Eq. 2 describe the regression model for the amplitudes of axial and radial vibration.

\[
Y_a = \exp \left( Y'_1 \right)
\]

\[
Y'_1 = 11.092 - 0.01426f_n - 0.0911a_p + 0.000008f_n^2 + 0.000467a_p^2 + 0.000029f_n \times a_p
\]  (1)

\[
Y_r = \exp \left( Y'_2 \right)
\]

\[
Y'_2 = 10.992 - 0.01226f_n - 0.0914a_p + 0.000008f_n^2 + 0.000497a_p^2 + 0.000009f_n \times a_p
\]  (2)

Here, \(Y_a\) and \(Y_r\) represent the amplitude of axial and radial vibration in nm, respectively. The operating frequency is defined by \(f_n\) and \(a_p\) denotes the depth of cut. Additionally, the regression equation of transmitted force has been determined by the general linear model which is found to be the most suitable fit for it. Equation 3 defines the regression equation for transmitted force.

\[
F_t = 37.39 + 7.28f_{n1} - 18.72f_{n2} - 5.72f_{n3} - 10.39f_{n4} + 34.28f_{n5} - 6.72f_{n6} + 6.78a_{p1} + 1.61a_{p2} - 8.39a_{p3}
\]  (3)

Here, \(F_t\) represents the transmitted force. The subscripts on the input parameters in Eq. 3 indicate the level of the parameters. Each numeric coefficient corresponding to the process parameter indicates the constant values of the equation for the particular process parameters. This is a conditional equation where a set of equations can be generated based on the variation of the process parameters. For example, the transmitted force for a particular set of input parameters \((f_n = 200 \text{ Hz} \ (f_{n1}), \ a_p = 75 \mu\text{m} \ (a_{p1}))\) can be calculated from Eq. 3 as, \((F_t = 37.39 + 7.28 + 1.61 = 46.28 \text{ mN})\).

The measure of fit \(R^2\) for the regression equations Eq. 1, Eq. 2, and Eq. 3 has been evaluated as 97.69%, 97.77%, and 91.78%, respectively. Here, \(R\) is the Pearson Correlation Coefficient. The normal probability plot of axial vibration amplitudes is presented in Fig. 10a. The figure revealed that the residuals fall on a straight line implying the normal distribution of the errors. It indicates the residues are following the central limit theorem. This implies that the model is adequate and there is no reason to suspect any violation of the independence or constant variance assumption. The relation between the fitted values and the experimental values of axial vibration amplitudes is depicted in Fig. 10b. The fitted values almost coincide with the experimental ones. A similar phenomenon has

| Source                  | DF | Seq SS  | Adj SS  | Adj MS  | F-value | P-value | Contribution (%) | Responses                  |
|-------------------------|----|---------|---------|---------|---------|---------|------------------|----------------------------|
| Operating frequency     | 5  | 46,838  | 46,838  | 9367.5  | 13.38   | 0.00    | 80.17            | Amplitudes of axial vibration |
| Depth of cut            | 2  | 4588    | 4588    | 2293.9  | 3.28    | 0.08    | 7.85             | Amplitudes of radial vibration |
| Error                   | 10 | 6999    | 6999    | 699.9   |         |         |                  | Force transmitted            |
| Total                   | 17 | 58,425  |         |         |         |         |                  |                            |
| Operating frequency     | 5  | 60,718  | 60,718  | 12,143.7| 14.14   | 0.00    | 77.69            | Amplitudes of radial vibration |
| Depth of cut            | 2  | 8854    | 8854    | 4426.9  | 5.16    | 0.029   | 11.33            | Force transmitted            |
| Error                   | 10 | 8585    | 8585    | 858.5   |         |         |                  |                            |
| Total                   | 17 | 78,158  |         |         |         |         |                  |                            |
| Operating frequency     | 5  | 5292.9  | 5292.9  | 1058.59 | 19.68   | 0.00    | 80.88            | Force transmitted            |
| Depth of cut            | 2  | 713.4   | 713.4   | 356.72  | 6.63    | 0.015   | 10.90            |                            |
| Error                   | 10 | 537.9   | 537.9   | 53.79   |         |         |                  |                            |
| Total                   | 17 | 6544.3  |         |         |         |         |                  |                            |
been observed for radial vibration as shown in Fig. 10c and d. From the figure, it can be seen that most of the points are close to the centre line and hence this empirical model provides reliable prediction.

Additionally, a similar phenomenon has been observed for transmitted force as shown in Fig. 11. The residues are following the central limit theorem and the errors are distributed normally. In this case, the errors are slightly larger (deviated from the mean) which indicates a little amount...
of deviation of the prediction model from the experimental values. This is owing to the fact that the experimental values have not followed any particular trend. Eventually, the predicted values are almost close to the experimental values implying a reliable prediction model.

The analysis of the response variables can be explained through two-dimensional contour plots. The contour plots of axial vibration amplitude, radial vibration amplitude, and transmitted force are shown in Fig. 12. The contour plots are based on the predicted values generated from the regression model. The values of the response variables can be predicted from these plots for a particular operating frequency and depth of cut. It is clear that the amplitudes of axial and radial vibration are reduced at higher operating frequencies and depth of cut. However, the transmitted force does not follow any particular trend. In addition, it can be seen that the transmitted force is lower at higher depths of cut. These plots clearly concurred with the experimental analysis of this study.

Eventually, the response functions have been optimized using Desirability Function. Each predicted output response \( Y_i(x) \) for a given operating condition \( x \) has an own desirability function \( D_i \). The desirability function \( D_i \) lies between 0 and 1. \((D_i = 1)\) indicates completely desirable response. Hence, the target of the optimization operation is to increase the desirability function towards 1. In this analysis, all the three parameters need to be minimized. For this purpose, the particular desirability function has been defined as Eq. 4. In this Equation, \( U_i \) and \( T_i \) represents the upper value and the target value of the response function \( Y_i(x) \), while \( s \) indicates the exponent of importance to achieve the target value.

![Contour plots of response variables](image)

Fig. 12 Contour plots of response variables
The individual desirability function of each response has been calculated using Eq. 4. Further, the composite desirability (D) that defines the combined desirability of all the responses has been evaluated by the Eq. 5. In this case, the number of response function is three. Therefore, Eq. 5 provides the modified composite desirability function for the three responses.

\[
D = \sqrt{D_1 \cdot D_2 \cdot D_3}
\]

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\[
D_i = \begin{cases} 
1, & \text{if } Y_i(x) < T_i \\
\left(\frac{Y_i(x) - U_i}{T_i - U_i}\right)^8, & \text{if } T_i \leq Y_i(x) \leq U_i \\
0, & \text{if } Y_i(x) > U_i
\end{cases}
\]

Table 2 depicts the detail of the optimization performed on the three response functions along with the predicted optimized values. The composite desirability D has been evaluated as 0.906. Eventually, the set of the optimum input process parameters has been defined as operating frequency = 532 Hz, and depth of cut = 100 µm.

### Table 2

| Responses          | Axial vibration amplitude (nm) | Radial vibration amplitude (nm) | Force transmitted (mn) |
|--------------------|--------------------------------|---------------------------------|------------------------|
| Target value (T)   | 10                             | 13                              | 15                     |
| Upper value (U_i)  | 240                            | 280                             | 81                     |
| Desirability fuction (D_i) | 0.961                        | 0.971                           | 0.796                  |
| Predicted optimum values | 19.06                        | 20.6                            | 28.49                  |

4 Conclusion

The work was focused on the experimental investigation of process parameters (cutting speed and depth of cut) in order to reduce tool vibration and force transmitted to machine structure in high-speed micro-milling operation. A 2-flute end milling cutter (1 mm cutter diameter) was used to cut a commercially available pure titanium plate of 5 mm thickness. The operation was performed at different depth of cut and varying cutting speeds keeping the chip load constant. Vibration and force signals were acquired and processed to obtain the axial and radial vibration of the tool and the force transmitted to the machine structure. The signals have been studied and analysed. Additionally, statistical analysis has been performed by ANOVA and regression equations have been developed for the response functions. Eventually, optimization of the response functions has been performed using desirability analysis. Following conclusions can be drawn from the results of this study:

- The operating frequency (spindle speed) is the most significant parameter followed by depth of cut influencing the axial and radial vibration, and force transmissibility.
- The amplitudes of axial and radial vibration have been reduced with increasing the operating frequency (spindle speed). This is attributed to lower cutting force and less chatter at higher operating frequency (spindle speed).
- The amplitudes of axial and radial vibration have been reduced with increasing the depth of cut. This is due to the stable machining condition at higher depth of cut. The ploughing of the cutting tool has been reduced at higher depth of cut leading to less requirement of specific cutting energy. However, it further depends on the cutting edge radius of the milling cutter. Too much increase in depth of cut may result in increased axial and radial vibration.
- The amplitude of force transmissibility has not followed any particular pattern with the operating frequency. This may be due to the occurrence of higher order peaks on the spectrum. However, the force transmissibility has been reduced with increasing the depth of cut. This is again attributed to stable machining condition at higher depth of cut depending on the tool cutting edge radius.
- The set of the optimum input process parameters to minimize the axial and radial vibration amplitude, and the force transmissibility has been defined as operating frequency = 532 Hz, and depth of cut = 100 µm. In addition, it can be concluded that higher depth of cut (depending on tool cutting edge radius) at higher cutting speed develops a stable machining condition. This leads to a better surface finish, stable machining operation, less tool wear, increased tool life and reduced chatter.

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Declarations

Conflict of interest The authors declare they have no known competing interests or personal relationships that could have appeared to influence the work reported in this paper.
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