Effects of radial electric field on ion temperature gradient field driven mode stability

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The local stability of ion-temperature gradient driven mode (ITG) in the presence of a given radial electric field is investigated using gyrokinetic theory and ballooning mode formalism with toroidal effect accounted for. It is found that, zero frequency radial electric field induced poloidal rotation can significantly stabilize ITG, while the associated density perturbation has little effect on ITG stability due to the modulation of finite-orbit-width effect. However, the parallel mode structure is slightly affected due to the evenly symmetric density modulation of ZFZF.

I. INTRODUCTION

Drift waves (DWs) turbulence [1], driven by free energy associated with plasma pressure gradients, are considered as candidates for inducing anomalous plasma transport and degradation of confinement in magnetically-confined fusion (MCF) devices. Ion-temperature gradient driven mode (ITG) is one of the most intensively studied DWs due to its potential role in causing anomalous ion thermal transport, which is much concerned in future fusion reactors. ITG has two branches, i.e., a slab branch by the coupling of ion parallel compression and diamagnetic drift, and a toroidal branch by the coupling of diamagnetic drift with the unfavored curvature in the weak field side [2, 3]. In-depth understanding of the mechanisms for ITG linear stability, nonlinear evolution and eventual saturation, is needed for quantitative understanding of plasma confinement in future tokamaks. Excitation of zonal flows (ZF), is considered as an important route for ITG self-regulation, and the regulation is achieved via nonlinear excitation of ZFs by ITG via modulation instability as ITG amplitude exceeds the threshold induced by frequency mismatch, which in turn, scatters ITG into the linearly stable short radial wavelength regime [4, 5].

ZF frequencies are typically meso-scale radial corrugations with toroidally symmetric (n = 0), and predominately poloidally symmetric (m = 0) scalar potential fluctuation, and consist of zero-frequency ZF (ZFZF) [6] and its finite frequency counter-part, geodesic acoustic mode (GAM) [5, 7]. Here, m/n are the poloidal/toroidal mode numbers of the torus. The nonlinear interaction of ITG with ZFs are observed in experiments [8, 9], as well as in large scale simulations [10, 11] where ITG and the associated transport are suppressed, and it is also found the threshold on pressure gradient on ITG stability is up-shifted as nonlinear effects are taken into account [12]. Furthermore, the radial electric field E∗ associated with large-scale mean flow, as well as its gradient, is also observed to be related to turbulence suppression and confinement improvement, and possibly the formation of transport barrier, as well as transition from low-

to high-confinement regime.

Several models are proposed to investigate the mechanism of ITG suppression by ZFs, among which are E × B shear effect and radial envelope modulation. In the E × B shearing model, a two-point nonlinear theory is proposed to understand flow shear suppression of ITG turbulence in cylindrical and toroidal geometry, and both high/low-frequency component of the radial electric field is considered [13, 14]. It is found that a significant reduction of turbulence activity occurs when the shearing rate ωs, which is proportional to ∂(vθ/γ)/∂n, with vθ = E∗/B being the radial electric field induced E × B drift velocity, exceeds the decorrelation rate of the ambient turbulence. Additionally, compared to the low-frequency E∗, the high-frequency component of E∗ typically plays less significant role in turbulence suppression due to its oscillating nature. On the other hand, the radial envelope modulation model may be referred to as the scattering process, i.e., the potential well associated with E∗ scatters turbulence into linearly stable short radial wavelength domain. The thresholds on the ITG amplitude for the ZFZF and/or GAM generation, however, are comparable to each other [5]. Global descriptions are needed in both models, which have previously been based on the local description [20]. More specifically, the modulational instability describing nonlinear interaction between ITG and ZFZF requires the ITG linear dispersion relation with finite-θk modification to the local one. Here, θk ≡ k∗/(n∂q) with k∗ being the radial envelope wavenumber, and q being the safety factor. Investigation of the interaction between turbulence and radial electric field in terms of the parallel mode structure is not found in the literature to date. Typically, this issue is treated in the gyrokinetic framework with toroidal effects neglected [21, 22].

In this work, a local model is proposed to figure out the local properties of turbulence suppression by given E∗ and study the “linear” stability of ITG in the presence of E∗. Here, “local” means the ITG eigenmode equation is solved along the magnetic field lines, with toroidal effects and parallel compression properly accounted for, while physics associated with radial envelope is neglected systematically. Technically, this is achieved by deriving an ITG governing eigenmode equation in the existence of the radial electric field induced density modulation as well as poloidal rotation, which is then solved in ballooning space

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for the ITG local dispersion relation and mode structure. For simplicity of discussion and because of the temporal scale separation, ZFZF-type $E\nu$ on ITG local stability is investigated as an example. Analogous to the ZFZF, finite-frequency GAM effects on ITG stability may be treated similarly if time scale separation between ITG and GAM is satisfied, which is typically the case as we discuss in the final section. Finally, our model may also shed light on turbulence suppression by the mean flow, whose mechanism is not yet fully understood.

The rest of the paper is organized as follows. In section II, the ITG eigenmode equation in the presence of a given radial electric field is derived using gyrokinetic theory and ballooning mode representation. In section III, the ITG stability is investigated assuming a radial electric field with zero frequency, i.e., that of ZFZF, both in the short- and long-wavelength limit. Summary and discussions are given in Section IV.

II. GENERAL FORMALISM

For simplicity of discussion while focusing on the main scope of the present paper, we consider a tokamak with axisymmetric concentric circular magnetic surface and straight field line, and a left-handed coordinate $(r, \theta, \phi)$ is adopted, with $r$, $\theta$ and $\phi$ being the minor radius, poloidal and toroidal angles of the torus, respectively. The equilibrium magnetic field is given as $B = B_0[(1 - \epsilon \cos \theta) e_\theta + c e_\theta / q]$, where $\epsilon \equiv r/R$ is the inverse aspect ratio, $R$ is the on-axis major radius and $q \equiv r B_0 / (R B_\theta)$. ITG modes generally have ballooning structure with high mode numbers, and the characteristic scale of equilibrium profile is generally much larger than the distance between neighbouring mode rational surfaces. Consequently, the perturbed quantity can be expressed as

$$\delta \phi = e^{i n \phi - i \omega t} \sum_j \Phi(s - j) e^{-i (m_0 + j) \theta}. \quad (1)$$

Here, $s \equiv (r - r_0) / \Delta r = n q - m_0, r_0$ denotes the reference rational surface with $n q(r_0) - m_0 = 0, \Delta r = 1/\langle n \partial q / \partial r \rangle$ is the distance between neighboring mode rational surfaces, and $|j| \ll m_0$ is an integer.

The gyrokinetic equation is used to investigate the stability of ITG turbulence in the presence of a given radial electric field. Following Ref. [3], we take the flat density gradient limit to focus on effects of ion temperature gradient, i.e., assuming $n_\perp = L_{ni} / L_{Ti} \to \infty$, with $L_{ni} = -v_{i \parallel} / \partial n_\perp / \partial r$ and $L_{Ti} = -T_i / \partial (T_i / \partial r)$ being the characteristic scale length of ion density and temperature nonuniformity, respectively. The gyrokinetic equation for ion response to ITG can be written as

$$(\omega - k_\parallel v_\parallel + \omega_D + k_\theta v_\theta) \delta H_I = e \frac{T_i}{T_e} J_0(\omega + \omega_{\perp i}) F_0 \delta \phi_I$$

$$-i \frac{e}{B_0} b \times \nabla \delta \phi_I \cdot \nabla \delta H_E. \quad (2)$$

Here, $k_\parallel \equiv (n q - m) / (q R)$ is the parallel wavenumber, with $\omega_D = 2 m c_\perp^2 / (x_\perp^2 + x_\parallel^2)$ is the magnetic drift frequency, with $\omega_d = k_\theta c T_i / (e B_\theta)$, $x_\perp = v_\perp / v_\parallel$, and $x_\parallel = v_\parallel / v_\parallel$ being the ion perpendicular/parallel velocities normalized by thermal velocity $v_\parallel$ is related to the curvature with $k_\parallel$ and $k_\theta = m_0 / r_0$ being the radial/poloidal mode numbers. $\delta H_I$ is the nonadiabatic ion response to ITG, $J_0(k_\parallel, r_\parallel)$ is the Bessel function of zero-index accounting for Finite Larmor radius (FLR) effects, $\rho_s = m v_{\perp i, s} / (e B_\perp)$ is the Larmor radius of species $s$, $F_0$ is the equilibrium ion distribution function, and $\omega_{\perp i} = \omega_{Ti} (x_\perp^2 + x_\parallel^2 - 3/2)$ is the ion diamagnetic frequency in the flat density limit, with $\omega_{Ti} = k_\theta c T_i / (e B L_{Ti})$. Furthermore, the last term on the right hand side accounts for the Doppler shift from the radial electric field induced poloidal rotation, with $v_\theta = -e b \times \partial \phi / B_0$, while the last term on the right hand side represents the perturbed diamagnetic term associated with the density perturbation induced by the radial electric field [20]. The last term, in fact, can be combined with the term proportional to $\delta H_E$, considering $F_0 + \delta H_E$ to be the renormalized equilibrium in the existence of the radial electric field. It is worth noting that the two additional terms, i.e., radial electric field induced poloidal rotation of ITG $k_\theta v_\theta B_\theta$, and variation along the magnetic field line induced by the density perturbation associated with the radial electric field $\propto \delta \phi_I \delta H_E$, can also be obtained from the perpendicular nonlinear term in nonlinear gyrokinetic equation [27], and thus, will be called “nonlinear terms” in the following discussion for convenience, though the radial electric field can also originate from linear effects, such as large scale mean flow. Here, subscripts “E” and “I” represent quantities associated with radial electric field and ITG, respectively. The dispersion relation can be derived from charge quasi-neutrality condition

$$\frac{e N_0 \delta \phi}{T_e} + \langle \delta H_e J_0 \rangle = -\frac{e N_0 \delta \phi}{T_i} + \langle \delta H_i J_0 \rangle, \quad (3)$$

with $e N_0 \delta \phi/T_e$ and $-e N_0 \delta \phi/T_i$ being adiabatic responses of electron and ion, respectively, and $\langle \cdots \rangle$ representing velocity space integration. The derivation follows closely the procedure of Ref. [28]. For typical ITG fluctuation with $k_\parallel v_\parallel, c_\parallel \gg \omega_d, k_\theta v_\theta$, electrons respond adiabatically, i.e., $\delta H_{1, e} = 0$. The nonadiabatic ion response can be derived as

$$\delta H_{1, i} \approx \frac{\Lambda}{\omega} \left[ -\left( \frac{e}{T_e} + \frac{e}{T_i} \right) F_0 \delta \phi_I + \delta H_E \right] \delta \phi_I$$

$$+ \frac{e}{T_i} J_0 F_0 \left( 1 + \frac{\omega_{\perp i}}{\omega} \right) \left( 1 + \frac{k_\parallel v_\parallel}{\omega} + \frac{k_\parallel^2 v_\parallel^2}{\omega^2} - \omega_D / \omega \right) \delta \phi_I. \quad (4)$$

Here, $\Lambda \equiv i c_\perp / k_\theta B_0$, and will be used in the rest of the paper. The two terms in first bracket of equation [4] are the formal nonlinear terms, and represent the effects associated, respectively, with the potential and density
fluctuation of $\delta\Phi$. The quasi-neutrality condition of ITG is applied to simplify the first term. Substituting the ion and electron response into quasi-neutrality condition \[\text{c\textsuperscript{3}},\] one then has the ITG WKB dispersion relation

\[
\left\{ \frac{1}{\tau (1 + \omega sT_i / \omega)} + b_{\perp} - \frac{k^2_{\perp} \rho_i^2}{2} \omega_d + 2 \omega_d C \right\} \delta \phi_E - T_i \left( \frac{\delta H_{E,i}}{\varepsilon N_0} \right) \delta \phi = 0, \tag{5}\]

with $b_{\perp} \equiv k^2_{\perp} \rho_i^2 / 2$, and $k_{\perp}$ being the perpendicular wavenumber. The first terms of four equations \[\text{c\textsuperscript{4}}\] constitute the linear ITG dispersion relation, with the first three terms being respectively, adiabatic electron response, the FLR effect (polarization) and parallel compressibility, while the forth term related to magnetic drift peculiar in toroidal configuration, resulting in coupling of neighbouring poloidal harmonics. The last two terms are nonlinear modifications due to poloidal rotation and density modulation associated with the radial electric field, respectively. Noting $k^2_{\perp} = k^2_{\parallel} - \partial^2 / \partial \eta^2$, the eigenmode equation in real space for $j$-th poloidal harmonics can be derived as

\[
\left( b_0 s^3 \frac{d^2}{dz^2} - \frac{1}{\tau (1 + \omega sT_i / \omega)} - b_0 + \frac{k^2_{\perp} \rho_i^2}{2} \omega^2 \right) \Phi_z = \frac{\omega_d}{\omega} \left[ \Phi_{z+1} + \Phi_{z-1} + \hat{s} \frac{d}{dz} \left( \Phi_{z+1} - \Phi_{z-1} \right) \right] + \frac{\Lambda}{(\omega + \omega sT_i)} \left[ \left( 1 + \frac{1}{\tau} \right) \delta \phi_E - \left( \frac{T_i \delta H_{E,i}}{\varepsilon N_0} \right) \right] \Phi_z. \tag{6}\]

Here, $\hat{s} \equiv r (d \psi / dr) / q$ is the magnetic shear, $\tau \equiv T_e / T_i$, $b_0 \equiv k^2_{\perp} \rho_i^2 / 2$, $z \equiv s - j = nq - m$ is the normalized distance to the mode rational surface. The first term on the right-hand side of equation \[\text{c\textsuperscript{4}}\] comes from the curvature drift induced coupling between neighbouring poloidal harmonics. Moreover, the term proportional to $\delta H_{E,i}$ may also have poloidal-dependence, and causes additional toroidal coupling. For instance, GAM with $\omega \gg \omega_{tr,i}$, is characterised by up-down antisymmetric $(x \sin \theta)$ density fluctuation \[\text{c\textsuperscript{3}},\] while ZFZF with $\omega \ll \omega_{tr,i}$, has cos-type density fluctuation \[\text{c\textsuperscript{2}}\]. Here $\omega_{tr,i} \equiv \nu_{tr,i} / (qR)$ is the circulating ion transit frequency. Equation \[\text{c\textsuperscript{6}}\] can be analyzed using the ballooning mode formalism framework \[\text{c\textsuperscript{2}}\], which is accomplished by taking $\Phi(\eta)$ $\equiv$ $\int \Phi(z) \exp(-iz) dz$, with $\eta$ being the extended poloidal angle along the magnetic field lines. The ITG eigenmode equation in ballooning space reads

\[
\frac{d^2 \Phi(\eta)}{d\eta^2} + \Omega^2 b \left( \frac{\tau \Omega}{1 + \tau \Omega \varepsilon^{1/2} T_i} + b (1 + \hat{s}^2 \eta^2) \right) \frac{\delta \phi_E}{\delta \Phi} - \frac{2}{\Omega} \left( \cos \eta + \hat{s} \sin \eta \right) \left( \frac{T_i \delta H_{E,i}}{\varepsilon N_0} \right) \Phi(\eta) = 0, \tag{7}\]

where $\Omega \equiv \omega / (\tau \sqrt{\omega sT_i / \omega q})$, $b \equiv \tau b_0 / \sqrt{\varepsilon T_i}$, $\epsilon_{T_i} \equiv L_{T_i} / R$ and $\Delta_E \equiv \Lambda / [\omega sT_i \sqrt{\varepsilon T_i} (1 + \tau \Omega / \varepsilon^{1/2} T_i)]$. Equation \[\text{c\textsuperscript{7}}\] is general and can be applied to study the nonlinear modification of any given radial electric field to ITG stability, with the nonlinear modifications by the radial electric field accounted for by the last two terms. In this work, as a proof of principle demonstration, we will consider ZFZF-type stationary radial electric field, while the effects of energetic particle induced GAM (EGAM)/GAM can be investigated straightforwardly following the same approach if the GAM/EGAM frequency is smaller than ITG growth rate. It is natural to take the dominant $m = 0, 1$ components of nonadiabatic ion response \[\text{c\textsuperscript{2}}\],

\[
\delta H_{E,i} = \frac{eF_0 \delta \phi_Z}{T_i} (1 + i \lambda \cos \theta), \tag{8}\]

and $m = 0$ component of $\delta \phi_Z$, i.e., $\overline{\delta \phi_Z}$, where \[\text{c\textsuperscript{7}}\] is $f_0^{2\pi} (\cdots) d\theta / 2\pi$ represents surface averaged quantity. The higher order $m = 1$ density perturbation of ZFZF is included, to account for its unique role in inducing periodic modification to the ITG eigenmode potential well along the magnetic field line, that determines the condition for ITG stability. Here, $\lambda = \omega_{Dr} / \omega_{tr,i}$ represents the finite drift orbit width effect, with $\omega_{Dr} = 2 \omega_d (x^2 / 2 + x^2_\parallel)$ being the magnetic drift frequency associated with normal curvature and $\omega_{dr} \equiv k_r \varepsilon T_i / (eBR)$.

III. EFFECTS OF ZERO-FREQUENCY $E_r$ ON ITG LINEAR STABILITY

With the specified expression of $\delta H_{E,i}$ presented in equation \[\text{c\textsuperscript{8}}\], equation \[\text{c\textsuperscript{7}}\] can be written as

\[
\frac{d^2 \Phi}{d\eta^2} + \Omega^2 b \left( \frac{\tau \Omega}{1 + \tau \Omega \varepsilon^{1/2} T_i} + b (1 + \hat{s}^2 \eta^2) \right) \frac{\delta \phi_E}{\delta \Phi} + \frac{2}{\Omega} \left( \cos \eta + \hat{s} \sin \eta \right) \left( \Delta_{E \delta \phi_Z} \cos \eta \right) \Phi = 0, \tag{9}\]

where $\Delta_{E \delta \phi_Z}$ represents modification due to the electrostatic potential, and $\Delta_{E \delta \phi_Z} \cos \eta$ results from the $m = 1$ density perturbation of ZFZF, with $\Delta_E \equiv -i \tau A (2 \omega_{Dr} / \omega_{tr,i}) \Delta_E$ and $A \equiv \left( F_0 (x^2 / 2 + x^2_\parallel / x_\parallel) \right)$. Note that, the velocity space integral $A$ vanishes for typical Maxwellian distribution, but finite value for non-even symmetric distribution, e.g., shifted-Maxwellian distribution with non-zero average parallel velocity due to auxiliary current drive. Equation \[\text{c\textsuperscript{10}}\] will be investigated in both the short- and long- wavelength limits, corresponding to strong and moderate ballooning cases, respectively, as investigated Refs. \[\text{c\textsuperscript{3}}, \text{c\textsuperscript{28}}\]. The two limiting parameter regimes, can be studied by taking $b \sim O(1)$ and $b \ll 1$, respectively, as we shown after equation \[\text{c\textsuperscript{11}}\] that, the mode width in ballooning space, is proportional to $1 / \sqrt{b}$ (and thus, $\sim O(\sqrt{b})$ radially).
A. Short-wavelength limit

In the short-wavelength limit, i.e., \( b \sim O(1) \), the eigenfunction is strongly localized in ballooning space \( [3] \). Thus, strong coupling approximation can be adopted by taking \( \cos \eta \approx 1 - \eta^2/2 \) and \( \sin \eta \approx \eta \) \([24]\). Note that, the assumption underlying the above strong coupling approximation is that the mode is localized around \( \eta = 0 \), and the introduction of the \( \cos \eta \)-type periodic modulation does not affect the validity of the assumption due to the even-symmetric of it. The eigenmode equation then becomes

\[
\frac{d^2 \Phi}{d \eta^2} + q^2 \Omega^2 b \left( \frac{\tau \Omega}{1 + \tau \Omega \epsilon_{T_i}^L} + b + \frac{2}{\Omega} \Delta E \delta \phi_Z + \Delta' E \delta \phi_Z \right)
+ \left( b^2 s - \frac{2s - 1}{\Omega} - \frac{\Delta' E \delta \phi_Z}{2} \right) \eta^2 \Phi = 0,
\]

which can be rewritten as a standard Weber equation with the most unstable ground eigenmode being given by \( \delta \phi = \exp(-\sigma \eta^2) \) with

\[
\sigma = \frac{q^2 \Omega^2 b}{2} \left( \frac{\tau \Omega}{1 + \tau \Omega \epsilon_{T_i}^L} + b + \frac{2}{\Omega} \Delta E \delta \phi_Z + \Delta' E \delta \phi_Z \right).
\]

The half width of the lowest eigenmode in \( \eta \) space is proportional to \( 1/\sqrt{b} \). The corresponding dispersion relation is

\[
q^2 \Omega^2 b \left[ \frac{\tau \Omega}{1 + \tau \Omega \epsilon_{T_i}^L} + b + \Delta E \delta \phi_Z + \Delta' E \delta \phi_Z \right]^2
+ \left( b^2 s - \frac{2s - 1}{\Omega} - \frac{\Delta' E \delta \phi_Z}{2} \right) = 0.
\]

The dispersion relation is similar to corresponding linear result \([3]\), except terms proportional to \( \Delta E \) and \( \Delta' E \) originate from the contribution of radial electric field induced poloidal rotation and density fluctuation, respectively. The dependence of ITG growth rate and real frequency on the radial electric field are solved from the theoretical dispersion relation equation \([11]\), which are then compared with the numerical solution of equation \([8]\), and good agreement between analytical and numerical results are obtained, as shown in Fig. 1a and 1b respectively.

It is found in equation \([11]\) that the \( m = 1 \) density perturbation is of order \( k_r \rho_i q \) compared to \( m = 0 \) potential fluctuation. Thus, it is obvious that \( m = 1 \) density fluctuation affects the dispersion relation slightly. The ITG growth rate decreases significantly with increasing \( \delta \phi_Z \).

We then analyze the contribution of the radial electric field induced poloidal rotation and density modulation on ITG stability, by turning off the corresponding terms in equation \([8]\). It is shown in Fig. 2a that, when the \( E_r \)-induced poloidal rotation is kept while the density perturbation is turned off, the ITG growth rate is almost the same as that with both effects properly accounted for; while as only the \( E_r \)-induced density perturbation is kept, the ITG growth rate is slightly affected by the scalar potential. We thus conclude that the reduction of the growth rate is mainly due to the potential fluctuation (poloidal rotation). Besides, it is found that the ITG growth rate is of order \( \epsilon_{T_i}/L_T \), which is much larger than characteristic GAM/EGAM frequency \( \sim C_s/R \), hence our analysis can also be applied to modulation of ITG by EGAM/GAM. The mode structure is also shown in Fig. 3 and it is clearly seen that the mode structure peaks at \( \eta = 0 \) and the even symmetry is not broken, resulting from the even-symmetric period modulation introduced by the density fluctuation of ZFZF \((\propto \cos \theta)\) as denoted by \( \Delta E \). We note that, equation \([9]\) can be further simplified, by substituting the quasi-neutrality condition of ZFZF into equation \([7]\) to replace the last term proportional to \( (\delta H_{E,i}) \). This process will introduce \( O(k_r^2 \rho_i^2) \) uncertainty since it is \( \langle \delta H_{E,i} \rangle \) in the quasi-neutrality condition instead of \( (\delta H_{E,i}) \) in equation \([4]\). In this case, effects induced by the \( m = 1 \) density modulation of ZFZF cannot be larger than \( O(k_r^2 \rho_i^2) \), and is thus weak, as shown by our numerical results.

**FIG. 1**: The dependence of normalized growth rate (a) and real frequency (b) of ITG, which are normalized by \( C_s/L_T \), on the normalized ZFZF intensity \( \epsilon_{\delta \phi/Z} / T_i \).

The squares represent the theoretical result given by Eq. \([11]\) while circles are numerical result of Eq. \([8]\). Here, \( C_s^2 \equiv 2T_i/m_i \) is the sound speed, \( \epsilon_{T_i} = 0.2 \), \( b = 1 \) and \( A = 1 \).

**FIG. 2**: Numerical result of integrated and separated effects of potential and density fluctuation of ZFZF.
B. Long-wavelength limit

For typical tokamak plasmas, strong coupling approximation is usually a crude constraint. In more general cases, $b \ll 1$ (long-wavelength limit) is satisfied, and strong coupling approximation no longer holds. In the long-wavelength limit, there are two branches, i.e., toroidal branch and slab branch. We are more concerned about the toroidal branch, which is characterized by fast variation over connection length scale $(\eta \sim O(1))$ and a superimposed slowly varying envelope over secular scale. The self-consistent ordering is given by balancing parallel compressibility and adiabatic electron response, which results in $\Omega = O(b^{-1/3})$. Taking $\Phi(\eta) = C_0(\eta) \cos \eta/2 + S_0(\eta) \sin \eta/2$ with $\eta \equiv \bar{\eta}$, and $\bar{\epsilon} = b^{1/3}$ denoting slow variation in $\eta$, the eigenmode equations can be derived from vanishing coefficients of $\sin \eta/2$ and $\cos \eta/2$:

$$\frac{dS_0}{d\eta_1} = \frac{b^{2/3}q^2\Omega_0^3 \tau}{1 + \tau \Omega \epsilon_{Ti}^{1/2}} - \frac{1}{4b^{1/3}} + q^2 \Omega_0^2 b^{2/3} \Delta_0 \delta \phi_Z 
+ \frac{1}{2} q^2 \Omega_0^2 b^{2/3} \Delta_0 \delta \phi_Z \right] C_0 + q^2 \Omega_0 b^{1/3} \bar{\eta} S_0 = 0, \hspace{1cm} (12)$$

$$\frac{dC_0}{d\eta_1} = \left[ \frac{b^{2/3}q^2\Omega_0^3 \tau}{1 + \tau \Omega \epsilon_{Ti}^{1/2}} - \frac{1}{4b^{1/3}} + q^2 \Omega_0^2 b^{2/3} \Delta_0 \delta \phi_Z 
+ \frac{1}{2} q^2 \Omega_0^2 b^{2/3} \Delta_0 \delta \phi_Z \right] S_0 - q^2 \Omega_0 b^{1/3} \bar{\eta} C_0 = 0. \hspace{1cm} (13)$$

Equations (12) and (13) can be cast into a Weber equation for $C_0$ and $S_0$. The dispersion relation for the most unstable ground eigenstate is

$$\Omega^4 + \frac{\Lambda \delta \phi_Z}{\tau \omega_\perp \epsilon_{Ti}^{1/2}} \left[ 1 + \frac{i \tau \Delta_1 \rho q}{2} \right] \Omega^2 - \frac{\epsilon_{Ti}^{1/2}}{4bq^2} \Omega = \frac{1}{4bq^2\tau}. \hspace{1cm} (14)$$

Here, terms in the bracket of Eq. (14) come from the $m = 0$ component of radial electric field induced poloidal rotation and $m = 1$ density modulation, while other terms originate from linear dispersion relation $\Lambda$. It is noteworthy that the $m = 1$ component of density perturbation enters and affects the ITG dispersion relation, while the $\theta$-type density modulation of EGAM/GAM has no influence on the dispersion relation, possibly due to the odd symmetry of the density modulation of GAM/EGAM with $\omega_G > \omega_{tr,\perp}$. The dependence of ITG growth rate and real frequency on scalar potential of the radial electric field are solved from the theoretical dispersion relation, which are then compared with the numerical solution of Eq. (9), and good agreement are obtained, as shown in Fig. 4. An artificially small $b = 0.01$ is adopted to separate different scales, although this is not the most relevant parameter regime for ITG stability. As shown in Fig. 4, the $E_r$ induced poloidal rotation is the main reason for the reduction of the ITG growth rate, as clarified by our theoretical analysis; while its density perturbation has weak effect on ITG stability. The mode structure is shown in Fig. 6, which is wider than that in the short-wavelength limit, as demonstrated by our analysis.
IV. CONCLUSION AND DISCUSSION

In this paper, a governing equation is formulated to investigate the ITG “linear” stability in the presence of a given radial electric field, using gyrokinetic equation and ballooning mode representation. The effects of the radial electric field on ITG linear stability consist of $E_r$-induced poloidal rotation and density fluctuation, and their integrated and separated contribution to ITG stability are studied both theoretically and numerically.

Here, ZFZF is presented as an example for clarity of discussion. For the adopted ZFZF-like radial electric field with $\omega \ll \omega_{tr,i}$, we found that the poloidal rotation is the main reason for the significant reduction of the ITG growth rate both in the short- and long-wavelength limit. In contrast, the up-down symmetric density perturbation, which peaks at the un-favourable curvature region, have weak suppression effect on ITG turbulence in both short- and long-wavelength limit. The extension of our ITG stability analysis to include EGAM/GAM with frequency being much higher than ion transit frequency, and thus up-down anti-symmetric density perturbation ($\propto \sin \theta$), is straightforward, since in that case the GAM/EGAM frequency is smaller than ITG growth rate.

The present work, motivated to understand local properties and mechanism of ITG stability in the existence of a given $E_r$ in ballooning space, found that, the radial electric field always plays a stabilizing role on ITG. This model can also be applied to study the possible cross-scale interaction between ITG and AEs, with the former being microscopic and the latter being macro- or meso- scale, mediated by ZFZF. In such a two-predator system, effects of ZFZF generated by one turbulence can be considered as “passive” or “equilibrium” for another turbulence. Additionally, it is of great interest and importance to investigate the nonlinear modulation of ITG by EGAM/GAM, which can be excited by AEs (internally) and neutral beam injection (externally), and can act as an active control of ITG turbulence. But, in contrast to ZFZF, the time scale separation of EGAM/GAM with ITG is not necessarily always satisfied, and depends on specific experimental conditions. Hence, fully nonlinear process, i.e., DW-GAM nonlinear evolution, should be considered to account for comparable frequency between ITG and GAM, and will be investigated in a future publication.

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Data Availability

The data that support the findings of this study are available from the corresponding author upon reasonable request.
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