Theory of the Josephson effect in superconductor / ferromagnet / superconductor junctions

Y. Tanaka  
Department of Applied Physics, Nagoya University, Nagoya, 464-8603, Japan.

S. Kashiwaya  
Electrotechnical Laboratory, Umezono, Tsukuba, Ibaraki 305-8568, Japan.  
(March 1, 1999)

Abstract

To reveal the influence of the exchange interaction on the Josephson effect, the d.c. Josephson current in unconventional-superconductor / ferromagnet / unconventional-superconductor junctions is studied. When the two superconductors have $d$-wave symmetry, the Josephson current is drastically suppressed with the increase of the exchange interaction. On the other hand, when the two superconductors have different parities, e.g. $s$-wave and $p$-wave, an enhancement of the Josephson current is obtained with the increment of the exchange interaction due to the break-down of SU(2) symmetry in the spin space.
The Josephson effect in superconductor / ferromagnet / superconductor (S/F/S) junctions has been theoretically studied since long time ago. The most interesting phenomena obtained in these works is the oscillating behavior of the Josephson current under the influence of the exchange interaction. In some cases, $\pi$-junction is attainable by choosing a proper thickness of the ferromagnet and the magnitude of the exchange interaction. However, all existing theories are intended to conventional superconductors and there has been presented no theory treating unconventional superconductors whose pair potentials have internal phases.

Recent tunneling theories for unconventional superconductors have revealed several novel features which are not expected for the conventional BCS superconductor junctions. First of all, the formation of the zero energy states (ZES) at the surfaces of $d$-wave superconductor has been predicted theoretically. The appearance and the disappearance of the ZES at the surfaces of high-$T_c$ superconductors have been studied in several tunneling spectroscopy experiments, and the consistency between theory and experiments has been checked in details. As for the d.c. Josephson effect in anisotropic singlet superconductors, we have presented a general formula (referred to as TK formula) which fully takes account of the internal phase of the pair potential including the ZES formation at the insulator.

The maximum Josephson current $I_C(T)$ is shown to have an anomalous temperature $T^{-1}$ dependence, that is, $I_C(T)$ becomes proportional to $T^{-1}$ at low temperatures when the ZES are formed at both interfaces.

On the other hand, the influences of the exchange interaction on the Andreev reflection have been analyzed based on the Bogoliubov equation. The theory has been extended to $d_{x^2-y^2}$-wave and $p$-wave symmetries recently, and the charge- and spin-transport properties between ferromagnet and superconductors have been revealed. In these theories, it has been shown that the amplitude of the Andreev reflection and the influence of the ZES are suppressed with the increase of the exchange interaction when the Cooper pairs are formed between two electron with antiparallel spins. This effect mainly originates from the breakdown of the retro-reflectivity in the Andreev reflection process.

At this stage, it is an interesting topic to clarify how the above-mentioned properties affect the Josephson current. This view point is really important, because a recent progress in thin film technology makes it possible to fabricate hybrid structures containing both ferromagnets and unconventional superconductors. In this paper, we present a formula for the d.c. Josephson current in S/F/S junctions of which superconductors are in unconventional pairing states. Although the formula can be applied for arbitrary symmetries, we focus on the two cases, i.e. $d$-wave superconductor / ferromagnet / $d$-wave superconductor ($d/F/d$) junctions and $s$-wave superconductor / ferromagnet / $p$-wave superconductor ($s/F/p$) junctions.

For the model of the calculation, we consider two-dimensional S/F/S junctions in the clean limit. We assume a situation that the ferromagnet, of which thickness is $d$, is inserted between two semi-infinite superconductors (see Fig. 1). The insulators located at the F/S interfaces ($x = 0$ and $x = d$) are described by potential $V(x) \{V(x) = H[\delta(x) + \delta(x - d)],$ where $\delta(x)$ $[\delta(x - d)]$ and $H$ are the $\delta$-function and its amplitude, respectively. The effective mass $m$ is assumed to be equal both in the ferromagnet and in the superconductors. For the model of the ferromagnet, we adopt the Stoner model where the effect of spin polarization is described by the one-electron Hamiltonian with an exchange interaction similarly to the case.
of Ref. 15. For the description of the unconventional superconductors, we apply the quasi-classical approximation where the Fermi-energy $E_F$ in the superconductor is much larger than the pair potential amplitude following the model by Bruder. The assumed spatial dependence of the pair potential for the quasiparticle injected from the left superconductor with spin index $\sigma$ (see Fig. 1) is described by

$$\Delta_\sigma(x, \theta) = \begin{cases} \Delta_{L\sigma}(\theta) \exp(i\varphi_L), & (x < 0) \\ \Delta_{R\sigma}(\theta) \exp(i\varphi_R), & (x > d) \end{cases} \quad (1)$$

with $\Delta_{L(R)\sigma}(\theta) = \Delta_0(T)f_{L(R)\sigma}(\theta)$, where $f_{L(R)\sigma}(\theta)$ is a form factor which specifies the symmetry of the pair potentials. The quantity $\theta$ and $\varphi_{L(R)}$ denotes an injection angle of the quasiparticles and the external phase of the pair potential of left [right] superconductor measured from the normal to the interface, respectively. The wave-vector of quasiparticles in the ferromagnet for the spin-up [down] and in the superconductor are expressed as $k_{N,\uparrow} = \sqrt{\frac{2m}{\hbar^2}(E_F + U)}$ [$k_{N,\downarrow} = \sqrt{\frac{2m}{\hbar^2}(E_F - U)}$], and $k_S = \sqrt{\frac{2mE_F}{\hbar^2}}$ respectively. The temperature dependence of $\Delta_0(T)$ is assumed to obey the BCS relation with transition temperature $T_C$.

In the following, we calculate the Josephson current in $d/F/d$ junctions where form factors are described by $f_{L,\uparrow}(\theta) = -f_{L,\downarrow}(\theta) = \cos[2(\theta - \alpha)]$ and $f_{R,\uparrow}(\theta) = -f_{R,\downarrow}(\theta) = \cos[2(\theta - \beta)]$. The quantity $\alpha$ [$\beta$] is the angle between the normal to the interface and the crystal axis of the left [right] superconductors [see Fig. 1(b) of Ref. 11(b)]. Similarly to a previous theory, the Josephson current is described by the coefficients of the Andreev reflection obtained by solving the Bogoliubov equation. To reduce the complexity, we choose asymmetric junction ($\alpha = -\beta$) configurations. After straightforward manipulations, the Josephson current is expressed by

$$R_N I(\varphi) = W \int_{-\pi/2}^{\pi/2} d\theta \sum_{\omega_n} \text{Real} \left\{ \frac{4B_2 \Gamma_L \tilde{\Gamma}_L \eta \sin \varphi \cos \theta}{(1 + \Gamma_L \tilde{\Gamma}_L)^2 B_1 + C_+ - C_- \Gamma_L^2 \tilde{\Gamma}_L^2 + 2B_2 \Gamma_L \tilde{\Gamma}_L \cos(\varphi)\eta} \right\} \quad (2)$$

$$B_1 = [(1 - \lambda_1 - iZ)(1 + \lambda_1 + iZ) - (1 + \lambda_1 - iZ)(1 - \lambda_1 + iZ) \exp(2ip_\uparrow^+ d)]$$

$$\times [(1 - \lambda_2 + iZ)(1 + \lambda_2 - iZ) - (1 + \lambda_2 + iZ)(1 - \lambda_2 - iZ) \exp(-2ip_\downarrow^- d)] \quad (3)$$

$$B_2 = 16\lambda_1\lambda_2$$

$$C_\pm = \pm 2(\lambda_1 + \lambda_2)\{(1 \pm \lambda_1 \pm iZ)(1 \pm \lambda_2 \mp iZ) - (1 \mp \lambda_1 \pm iZ)(1 \mp \lambda_2 \mp iZ) \exp[2i(p_\uparrow^+ - p_\downarrow^-)d]\}$$

$$\pm 2(\lambda_1 - \lambda_2)$$

$$\times [(1 \mp \lambda_1 \pm iZ)(1 \pm \lambda_2 \mp iZ) \exp(2ip_\uparrow^+ d) - (1 \pm \lambda_1 \pm iZ)(1 \mp \lambda_2 \mp iZ) \exp(-2ip_\downarrow^- d)] \quad (4)$$

$$\Gamma_L = \frac{\Delta_{L\sigma}(\theta_+)}{\Omega_{L,+} + \omega_n}, \tilde{\Gamma}_L = \frac{\Delta_{L\sigma}(\theta_-)}{\Omega_{L,-} + \omega_n}, \Omega_{L,\pm} = \text{sgn}(\omega_n)\sqrt{\omega_n^2 + |\Delta_{L\sigma}(\theta_\pm)|^2}$$
\[ p_+^\pm = \sqrt{\frac{2m}{\hbar^2}} (i\omega_n + E_F \cos^2 \theta + U), p_-^\pm = \sqrt{\frac{2m}{\hbar^2}} (-i\omega_n + E_F \cos^2 \theta - U), \]

with \( \eta = \exp[i(p_+^\pm - p_-^\pm)k d] \), \( \theta_+ = \theta, \theta_- = \pi - \theta \), \( W = \pi R_N k_B T/e, Z = Z_0/\cos \theta \), and \( Z_0 = 2m\hbar/(\hbar^2 k_S) \). The normal resistance \( R_N \) of the junction is given by

\[ R_N^{-1} = \frac{1}{2} \sum_{j=1,2} \int_{-\pi/2}^{\pi/2} \cos \theta (1 - | r_j |^2) d\theta \]

(5)

\[ r_j = \frac{e^{2ikF \cos \theta} d_{\lambda_j} [1 - (\lambda_j - iZ)^2] - [1 - (\lambda_j + iZ)^2]}{1 - U/(E_F \cos^2 \theta) - \lambda_1 = \sqrt{1 + U/(E_F \cos^2 \theta)} \text{ and } \lambda_2 = \sqrt{1 - U/(E_F \cos^2 \theta)}. \]

Similar to the results by the TK formula, the Josephson current contains various \( \theta \) components, hence the properties of the junction is determined by the integration on \( \theta \). In order to visualize the \( \pi \)-junction formation, we define \( I_p(T) = I(\varphi_M) \), where the magnitude of the Josephson current shows its maximum in \( 0 < \varphi_M < \pi \). Figure 2 shows \( X \)-dependence of \( I_p(T) \) for \( d/F/d \) junction. The sign of \( I_p(T) \) changes from positive (negative) to negative (positive) for curve a (b). These periodic changes between \( 0 \)-junction and \( \pi \)-junction are quite similar to those obtained in the conventional ones. The origin of the periodic change can easily be checked analytically for larger magnitude of \( Z \). In such a case, the Josephson current is rewritten as

\[ R_N I(\varphi) = W \int_{-\pi/2}^{\pi/2} d\theta \sum_{\omega_n} \text{Real} \left\{ \frac{16\lambda_1 \lambda_2 \Gamma_L \Gamma_L \cos \theta \sin \varphi}{Z^4 \sin(p_+ d) \sin(p_- d)(1 + \Gamma_L \Gamma_L)^2} \right\} \]

(7)

For \( X = 0 \), since \( p_-^\pm = p_+^{\pm*} \) and \( \lambda_1 = \lambda_2 \) is satisfied, although an oscillatory change of \( I_p(T) \) due to the Friedel oscillation exists, \( I_p(T) \) is always positive [negative] for \( \alpha = 0 \) (curve a of Fig. 2) \([\alpha = \pi/4 \text{ (curve b of Fig. 2)}\] \). However, for finite \( X \), \( I_p(T) \) changes its sign thorough the sign-change of the factor \( \text{Real} \{\lambda_2/\sin(p_+ d) \sin(p_- d)\} \). On the other hand, as for the temperature dependence of the magnitude of \( I_p(T) \), a rapid increment at low-temperature is obtained for \( \alpha \neq 0 \) with larger \( Z \). This feature is similar to those obtained in the calculation of the TK formula, and originates from the ZES formed at the interface. The most interesting result, peculiar to the present model, is obtained for \( \sin^{-1}(k_{N,\downarrow}/k_S) < \theta < \pi/2 \) when the Andreev reflection at the \( S/F \) interface becomes an evanescent wave [the virtual Andreev reflection (VAR) process described in Ref. 17 as shown in Fig. 1. Since the ferromagnet behaves as if it were an insulator, the Josephson current shows an exponential dependence as a function of \( d \). Also the Josephson current is drastically suppressed as \( X \) becomes larger. This is because the angle region where the Josephson current is carried by the VAR process monotonically increases with the increase of \( X \). Such a Fermi surface effect on the d.c. Josephson current has been never discussed in previous theories.

Next, let us move to an \( s/F/p \) junction as a typical example for the singlet superconductor / ferromagnet / triplet superconductor junction where \( f_{R_\uparrow}(\theta_+) = f_{R_\downarrow}(\theta_+) = \exp(-i\theta) \) and \( f_{R_\uparrow}(\theta_-) = f_{R_\downarrow}(\theta_-) = -\exp(i\theta) \) are satisfied. Resulting Josephson current is given by
the temperature dependence of the transition temperature of the ferromagnet $T_X$.

The magnitude of the d.c. Josephson current once. As shown in curve a of Fig. 3, $I(\varphi)$ has a period of $\pi$ (not $2\pi$), since the lowest order Josephson coupling between the two superconductor diminishes due to the difference of the parity and the rotational symmetry, i.e., $SU(2)$ symmetry, in the spin space. As seen in Eq. (9), since $\lambda_2 = 1$ and $p_+^d = p_{-}^{d*}$ is satisfied for $X = 0$, $R_N I(\varphi)$ vanishes for larger $Z$. However, for finite $X$, $\text{Imag}(\lambda_2/\int \sin(p_{+}^d d) \sin(p_{-}^{d*} d))$ is non-zero as shown in Eq. (9). Then the cancellation between up spin injection and down spin injection becomes incomplete and the imaginary part of Eq. (9) also becomes finite.

As the result, the cancellation in the lowest order is lifted and the component with period $2\pi$ recovers as shown in curves b and c in Fig. 3. The recovery in the lowest order enhances the magnitude of the d.c. Josephson current once. As shown in curve d in Fig. 3, $I_p(T)$ is reduced with the further increment of $X$. This reduction is caused by the breakdown of the retroreflectivity in the Andreev reflection similar to the cases of $d/F/d$.

In the above, the temperature dependence of $X$ has not been taken into account. If the transition temperature of the ferromagnet $T_\theta$ is sufficiently larger than $T_C$, $X$ can be regarded as a constant for all temperatures below $T_C$. However, when $T_\theta \sim T_C$, $I_p(T)$ is expected to have a non-monotonous temperature dependence due to the competition between the pair potential amplitude enhancement (positive contribution) and the polarization enhancement (negative contribution). For simplicity, consider a $d/f/d$ junction and assume the temperature dependence of $X(T)$ as $X(T) = X_0 \sqrt{1 - T/T_\theta}$. As shown in Fig. 4, $I_p(T)$ changes drastically from 0-junction ($\alpha = 0$ curve a) to $\pi$-junction ($\alpha = \pi/4$ curve b) at $T = T_\theta$. Such an anomalous feature may be accessible in actual experiment.

In this paper, we present a formula of the Josephson current in $S/F/S$ junctions where the Cooper pair is formed between two electrons with antiparallel spins. In most cases, the break down of the retro-reflectivity of the Andreev reflection reduces the magnitude of the Andreev reflection and the resulting Josephson current is suppressed. However, in the case of Josephson junctions formed between two superconductors with different parities, the Josephson current can be enhanced by the exchange interaction due to the recovery of the first order Josephson coupling. Throughout this paper, the spin-orbit scattering is ignored. The essential conclusion may not be changed even if we take into account of this effect if the magnitude of the spin orbit coupling is small, because the above feature is governed by the parity. From an experimental viewpoint, it is possible to make a Josephson junction with $d$-wave superconductor and ferromagnet using Mn oxides compound and high-$T_c$ superconductors or Sr$_2$RuO$_4$. We hope anomalous properties predicted in this paper will be detected near future.

\[
R_N I(\varphi) = W \sum_{\omega_n} \int_{-\pi/2}^{\pi/2} d\theta \text{Imag} \left\{ \frac{4B_2 \Gamma_L \Gamma_R \sin \varphi \cos \theta}{\left[ (1 + \Gamma_L^2)(1 - \Gamma_R^2)B_1 + C_{+} - C_{-} \Gamma_L^2 \Gamma_R^2 \right] + 2iB_2 \eta \Gamma_L \Gamma_R \sin \varphi} \right\} \tag{8}
\]

with $\Gamma_R = \frac{\Delta_R(\theta_+)}{\Omega_{R,+}}$, $\Omega_{R,+} = \text{sgn}(\omega_n) \sqrt{|\Delta_R(\theta_+)|^2 + \omega_n^2}$, $f_{L,1}(\theta_+) = -f_{L,1}(\theta_+) = 1$. Only the lowest order coupling remains finite at the limit of large magnitude of $Z$, then $R_N I(\varphi)$ converges to

\[
R_N I(\varphi) = W \int_{-\pi/2}^{\pi/2} d\theta \sum_{\omega_n} \frac{16\lambda_1 \cos^2 \theta \sin \varphi \Gamma_L (1 - |\Gamma_R|^2) \frac{\lambda_2}{\sin(p_+^d d) \sin(p_+^{d*} d)}}{\left( 1 + \Gamma_L^2 \right) \left| 1 - \Gamma_R^2 \right|^2 \partial \frac{\pi}{2} \sum_{\omega_n} \left( \frac{4B_2 \Gamma_L \Gamma_R \sin \varphi \cos \theta}{\left[ (1 + \Gamma_L^2)(1 - \Gamma_R^2)B_1 + C_{+} - C_{-} \Gamma_L^2 \Gamma_R^2 \right] + 2iB_2 \eta \Gamma_L \Gamma_R \sin \varphi} \right) \right\} \tag{9}
\]
We would like to thank N. Yoshida, J. Inoue, M. Koyanagi, and M. R. Beasley for fruitful discussions. This work has been partially supported by the Core Research for Evolutional Science and Technology (CREST) of the Japan Science and Technology Corporation (JST) and a Grant in aid for Scientific Research from the Ministry of Education, Science, Sports and Culture.
REFERENCES

1. A. I. Buzdin, L. N. Bulaevskii, S. V. Panyukov, Pis'ma Zh. Eksp. Teor. Fiz. 35, 147 (1982) [JETP Lett. 35, 178 (1982)], L. N. Bulaevskii, A. I. Buzdin and S. V. Panyukov, Solid State Commun. 44, 539 (1982). A. I. Buzdin, B. Bujicic and M. Yu. Kupriyanov, Zh. Eksp. Teor. Fiz. 101, 231 (1992) [Sov. Phys. JETP 74, 124 (1992)].
2. A. I. Buzdin and M. Yu. Kupriyanov, Pis'ma Zh. Eksp. Teor. Fiz. 52, 1089 (1990) [JETP Lett. 52, 487 (1990)]; ibid. 53, 308 (1991) [53, 321 (1991)].
3. E. A. Demler, G. B. Arnold, and M. R. Beasley, Phys. Rev. B 55, 15174 (1997).
4. L. N. Bulaevskii, V. V. Kuzii, and A. A. Sobyanin, JETP Lett. 25, 290 (1977).
5. C. R. Hu, Phys. Rev. Lett. 72, 1526 (1994).
6. Y. Tanaka and S. Kashiwaya, Phys. Rev. Lett., 74, 3451 (1995); S. Kashiwaya, Y. Tanaka, M. Koyanagi, and K. Kajimura, 53, 2667, (1996).
7. S. Kashiwaya, Y. Tanaka, H. Takashima, M. Koyanagi, and K. Kajimura, Phys. Rev. B, 51, 1350, (1995); S. Kashiwaya, T. Ito, K. Oka, S. Ueno, H. Takashima, M. Koyanagi, Y. Tanaka and K. Kajimura, Phys. Rev. B. 57, 8680 (1998).
8. L. Alff, H. Takashima, S. Kashiwaya, N. Terada, H. Ihara, Y. Tanaka, M. Koyanagi and K. Kajimura, Phys. Rev. B, 55, 14757, (1997).
9. J. W. Ekin, Yixi Xu, S. Mao, T. Venkatesan, D. W. Face, M. Eddy, and S. A. Wolf, Phys. Rev. B 56, 13746 (1997).
10. J. Y. T. Wei, N.-C. Yeh, D. F. Garrigus, and M. Strasik, Phys. Rev. Lett. 81, 2542 (1998).
11. Y. Tanaka and S. Kashiwaya, (a)Phys. Rev. B, 53, 11957, (1996); (b)ibid., 56, 892, (1996).
12. Yu. S. Barash, H. Burkhardt, and D. Rainer, Phys. Rev. Lett. 77, 4070 (1996).
13. R. A. Riedel and P. F. Bagwell, Phys. Rev. B 57, 6084 (1998).
14. A. F. Andreev, Zh. Eksp. Teor. Fiz. 46, 1823 (1964). [Trans. Sov. Phys. JETP, 19, 1228].
15. M. J. M. de Jong and C. W. J. Beenakker: Phys. Rev. Lett. 74 (1995) 1657.
16. I. Zutic and O.T. Valls, cond-mat/9808282 [ibid., cond-mat/9902080].
17. S. Kashiwaya, Y. Tanaka, N. Yoshida and M. R. Beasley, cond-mat/9812160.
18. Jian-Xin Zhu, B. Friedman, and C.S. Ting, preprint.
19. N. Yoshida, Y. Tanaka, J. Inoue, and S. Kashiwaya, J. Phys. Soc. Jpn. 1999.
20. V. A. Vas’ko, K. R. Nikolaev, V. A. Karkin, P. A. Kraus, and A. M. Goldman, Appl. Phys. Lett. 73, 844 (1998).
21. C. Bruder, Phys. Rev. B, 41, 4017 (1990).
22. A. Furusaki and M. Tsukada, Solid State Commun. 78, 299, (1991).
23. Y. Maeno, H. Hashimoto, K. Yoshida, S. Nishizaki, T. Fujita, J. G. Bednorz, and F. Lichtenberg, Nature, 372, 532 (1994).
Figure Captions

Fig. 1 A schematic illustration of the reflection and transmission process at the interface of $S/F/S$ junction.

Fig. 2 $X$ dependence of $I_p(T)$ for $d/F/d$ junction for $dk_S = 5$ and $Z_0 = 5$ at $T = 0.1T_d$. a: $\alpha = 0$ b: $\alpha = \pi/4$.

Fig. 3 Current phase relation $I(\varphi)$ for $s/f/p$ junction with $dk_S = 1$ and $Z_0 = 5$ at sufficiently low temperatures. a: $X = 0$ b: $X = 0.1$ c: $X = 0.5$ and $X = 0.99$. Fig. 4 Temperature dependence of $I_p(T)$ for $d/f/d$ junction with $Z_0 = 5$, $dk_S = 5$, $X_0 = 0.5$, and $T_\theta = 0.5T_C$. a: $\alpha = 0$ b: $\alpha = 0.25\pi$. 
$eR_N I_p(T) / \Delta(0)$
