Continuous precedence relations for better modelling overlapping activities

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Abstract

Theoretically proper modeling of overlapping activities with traditional precedence relationships is impossible. This is due to the fact that traditional precedence relations create logic between the endpoints of activities. In contradiction, overlapping can be defined as a ‘continuous’ relation using time or location/production units between all the points of the predecessor and all the points of the successor activities. (E.g. at least 50 m safety distance is necessary between activity B and A during execution.) Recently developed point-to-point relations give a practically better solution, as they let planners form connections among as many internal points of the related activities as seemingly necessary, and all these points can be controlled during the execution phase. However, a theoretically proper solution is only available if an infinite number of point-to-point relations are used between two overlapping activities, but this is obviously impossible. The novelty of this paper is the definition of a new type of precedence relation for the Precedence Diagram Method (PDM), the so-called continuous relation. One continuous relation can be used instead of infinite point-to-point relations in order to perfectly model activity overlapping. Relation lags can be defined using either time or volume units. Another main advantage of this development is that continuous relations enable the definition of non-linear activities, while original PDM assumes continuous linear activities only.

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1. Introduction and literature review

Precedence Diagram Method, the prevailing network technique of our time has been with us – almost unchanged – for almost sixty years. The results of Fondahl [1]; Roy [2], [3]; IBM [4] and many others have led to the present form of PDM. The technique has hardly changed over the decades in spite of the critiques it has received about its modeling capabilities, especially the problem of modeling overlapped activities [5] [6]. Different authors proposed the connection of the internal points of the activities as a solution for modeling overlapped activities instead of using traditional end-point relations (SS, FF, FS, SF). These point-to-point relations have been developed in at least four parallel ways: the chronographic approach [7], [8], the Bee-line Diagramming Method [9], [10], the Relationship Diagramming Method, [11], and the Graphical Diagramming Method [12]. Complete and mathematically correct description of point-to-point relations with standardized nomenclature, mathematical model and algorithm handling both minimal and maximal relationships can be found in Hajdu’s work [13]. Point-to-point relations are acceptable from a practical point of view, but they are still not able to provide a theoretically perfect solution, because they cannot control all the points of the connected activities, which is required for perfect modelling of overlapping activities [6]. A theoretically perfect solution can be the use of an infinite number of point-to-point relations between overlapping activities – thus controlling all the points – but this is obviously impossible.

A new precedence relation is defined in this paper, the so-called continuous relation. Continuous relations form logical relations between all the points of the connected activities. Instead of the infinite number of point-to-point relations, one continuous relation using time or volume lag is enough to define overlapping in a theoretically proper way. Another advantage of continuous relations is that they can be defined between non-linear activities, so the very strict hypothesis of continuous, linear activities of the original PDM model is not a prerequisite in case of continuous relations.

The need for modeling overlapping in a better way has been with us for a long time. Attempts to model time and location distances in a continuous way between non-linear and non-continuous activities can be traced back to the early sixties when the organization methods used for assembly lines were used in the construction industry to model repetitive works [14]. Recent renaissance of time location diagrams and the extensive research of them have also led to some mathematical formulation of the spatial and temporal logic between activities (e.g. with singularity functions) [15], [16]. First attempts of defining such kind of relations using network technique is a result of Mubarak [17], who introduced the Dynamic Minimum Lag relation that can be used (only!) during the project control, allows any shape for the predecessor (execution can give different shapes for the time-location function) while successors are assumed to be linear. Continuous relations defined in this paper are more: they can be used as a precedence relation, they can be used between non-linear activities and they can be applied during the planning and the execution phases as well.

2. The mathematics of continuous relations

2.1. Notations

The following notations are used throughout the text:

- An activity \( i = 1 \ldots n \)
- The duration of activity \( d_i \)
- The length/volume of activity \( l_i = l_j = L \)
- The start of activity \( S_i \)
- The finish of activity \( F_i \)
- The continuous minimal relation with \( t_{ij} \) time lag between activity \( i \) and \( j \)
- The continuous minimal relation with \( l_{ij} \) volume lag between activity \( i \) and \( j \)
2.2. Activities

A function $f(t)$ is used to define activity $i$ as a function of time in the Cartesian coordinate system, where the horizontal axis represents time and the vertical axis represents location/production. For the sake of simplicity, it is assumed, that

- $f(t)$ is one-to-one, monotonically increasing or decreasing within its domain (and hence invertible), and
- the first and the second derivatives exist

Figure 1 shows some examples: the traditional linear activity, an activity that speeds up following a learning curve, and an activity with descending speed due to e.g. a continuously deepening trench.

![Figure 1. Examples for activity functions](image)

During the calculations the inverse functions of the activities will also be used. Equations 1 and 2 show the function and the inverse function of the predecessor activity, Equations 3 and 4 show the function and inverse function of the successor activity. ($c$ defines a right shift for function $f_j(t-c)$. See Fig. 2)

\[
\begin{align*}
y &= f_i(t) \quad & 0 \leq t \leq d_i & 0 \leq y \leq L \\
y^{-1} &= f_i^{-1}(t) \quad & 0 \leq t \leq L & 0 \leq y^{-1} \leq d_i \\
g &= f_j(t - c) \quad & 0 + c \leq t \leq d_j + c & 0 \leq y \leq L \\
g^{-1} &= f_j^{-1}(t + c) \quad & 0 \leq t \leq L & 0 + c \leq y^{-1} \leq d_j + c
\end{align*}
\]

2.3. Definition of minimal continuous relation with time lag

Two types of lags will be defined: time and volume lag.

The definition of the continuous relation with minimal time lag is the following:

**Definition #1 (continuous relation with minimal time lag):** Every point of activity $j$ is delayed by at least $t_{ij}$ time compared to every point with the same $i$ of activity $i$. (See Fig. 2).

It is obvious that the mathematical task is to define the necessary right shift for function $g$, that is to define the value of $c$. Mathematically: the minimum difference between the two inverse functions has to be at least $t_{ij}$ (See Eq 5.)

\[
\min\{g^{-1} - y^{-1}\} \geq t_{ij}
\]
Continuous relations using time lags are useful if technology requires some time to elapse between overlapping activities. (E.g. painting can start after the surface becomes dry.)

2.4. Definition of minimal continuous relation with location/volume lag

Continuous relation with minimal volume/location lag can be defined as follows:

Definition #2 (continuous relation with minimal volume lag): Every point of activity $j$ lags behind every point with the same $t$ of activity $i$ with at least $l_{ij}$ units (See Fig. 3)
It is obvious that the mathematical task is to define the necessary right shift for function \(g\), that is to define the value of \(c\). (See Eq 6.)

\[
\min\{y - g\} \geq l_{ij}
\]  

(6)

It is also obvious that only the time range when both activities are in progress has to be investigated. Let’s note that \(\min(S_i)=T\), where \(T\) denotes the time that belongs to \(l_i\) of activity \(i\). (\(f(T)=l_i; T=f^{-1}(l_i)\)) Also, if \(l_i<L\) then \(S_i<F_i\) and the upper limit of the time range to be investigated is \(F_i\).

Continuous relations with minimal volume lag are especially useful if a given safety distance is prescribed between the overlapped activities. (E.g. at least 50 meter safety distance must be ensured between earthwork and pipe laying.)

3. Mathematical solution

3.1. Calculations with continuous relations using time lag

The value of \(c\) must be defined if the minimum of the difference of the two inverse functions is set to at least \(t_{ij}\). For this, the minimum point of the difference of the two inverse functions has to be defined within the given domain. (Eq. 7)

\[
(g^{-1} - y^{-1})' = 0
\]  

(7)

If there is a \(t_{opt}\) that satisfies Eq. 7, or there are more \(t_{opt}\), then the second derivatives must be checked. If \((g(t_{opt})^{-1}-y(t_{opt})^{-1})''>0\), then the function has its minimum or local minimum in \(t_{opt}\). If the absolute minimum is in the given domain, then we have the optimal solution. In any other case, the ends of the domain must be checked as one of them (or both) can also provide the minimum value within the given domain. Once \(t_{opt}\) is defined, the value of \(c\) can be calculated by solving the equation below. (Eq. 8)

\[
g(t_{opt})^{-1} - y(t_{opt})^{-1} = t_{ij}
\]  

(8)

3.2. Calculations with continuous relations using volume lag

The value of \(c\) must be defined when the minimum of the difference of the activity functions has to be at least \(l_i\). For this the minimum point of the difference of the two inverse functions has to be defined within the domain of \(T; F_i\) (Fig.3) (Eq. 9)

\[
(y - g)' = 0
\]  

(9)

If there is a \(t_{opt}\) that satisfies Eq. 9, or there are more \(t_{opt}\), then the second derivatives must be checked. If \((y(t_{opt})-g(t_{opt}))''>0\), then the function has its minimum or local minimum in \(t_{opt}\). If the absolute minimum is in the given domain, then we have the optimal solution. In any other case, the ends of the domain must be checked as one of them (or both) can also provide the minimum value within the given domain. Once \(t_{opt}\) is defined, the value of \(c\) can be calculated by solving the equation below. (Eq. 10)

\[
y(t_{opt}) - g(t_{opt}) = l_{ij}
\]  

(10)
4. Illustration of the algorithm

4.1. Calculations with continuous relations using time lag

Two following activities have to be executed in a 100 meter long trench: A and B. Activity A starts slower and later – as the crew learns and becomes more experienced – speeds up according to a \( y = t^2 \) function.

Activity B goes ahead with a constant 8 m/day speed. It is defined by the \( g = 8(t-c) \) function, where \( c \) stands for the necessary right shift of B (that is the relative start to A) in order to ensure the 2-day continuous relation between B and A in every section. (This is explained in Fig. 4)

The function and the inverse function of activity A is shown in (11) and (12). The same functions for activity B are shown in (13) and (14).

\[
\begin{align*}
    y &= t^2 & 0 \leq t \leq 10 & 0 \leq y \leq 100 \\
    y^{-1} &= \sqrt{t} & 0 \leq t \leq 100 & 0 \leq y^{-1} \leq 10 \\
    g &= 8(t-c) & 0 + c \leq t \leq 12.5 + c & 0 \leq y \leq 100 \\
    g^{-1} &= \frac{t}{8} + c & 0 \leq t \leq 100 & 0 + c \leq y^{-1} \leq 12.5 + c
\end{align*}
\]

First the minimum point has to be found by applying (7) for the inverse functions (12) and (14). This is shown in (15)

\[
(g^{-1} - y^{-1})' = \left(\frac{t}{8} + c - \sqrt{t}\right)' = \frac{1}{8} - \frac{1}{2\sqrt{t}} = 0 \quad \rightarrow \quad 2\sqrt{t} = 8 \quad \rightarrow \quad t = 16
\]
The second derivative is positive at $t=16$, so the minimum value has been found. (16)

$$(g^{-1} - y^{-1})'' = \left(\frac{t}{8} + c - \sqrt{t}\right)'' = \left(\frac{1}{8} - \frac{1}{2\sqrt{t}}\right)' = \frac{1}{4} t^{-\frac{3}{2}} = \frac{1}{4} 16^{-\frac{3}{2}} = 0.039 > 0 \quad (16)$$

The value of $c$ can be defined by substituting the solution of (15) (that is $t=16$) into (8). This can be seen in the equation below (17).

$$g\left(t_{opt}\right)^{-1} - y\left(t_{opt}\right)^{-1} = t_{ij} \rightarrow \frac{16}{8} + c - \sqrt{16} = 2 \rightarrow c = 4 \quad (17)$$

With this, we have reached the solution. ($c=4$) Activity B can start 4 days after the start of activity A.

**4.2. Calculations with continuous relations using volume lag**

In this case 8 meter continuous safety distance is prescribed between A and B. Fig. 4 shows a graphical explanation.

The functions of activity A and B are the same. (11) and (13).

First the minimum point has to be found by substituting functions (11),(13) into (9) This is shown in (18)

$$(y - g)' = (t^2 - 8(t - c))' = 2t - 8 = 0 \rightarrow t = 4 \quad (18)$$

The second derivative is positive at $t=4$, so the minimum value has been found. (19)

$$(y - g)'' = (t^2 - 8(t - c))'' = (2t - 8)' = 2 > 0 \quad (19)$$

The value of $c$ can be defined by substituting the solution of (16) (that is $t=4$) into (10). This can be seen in the equation below (20).

$$y\left(t_{opt}\right) - g\left(t_{opt}\right) = l_{ij} \rightarrow t^2 - 8(t - c) = l_{ij} \rightarrow 16 - 32 + 8c = 8 \rightarrow c = 3 \quad (20)$$

With this we have come to the solution. ($c=3$) Activity B can start 3 days after the start of activity A.

**5. Discussion, and further research**

In case of complicated activity functions, finding the optimal solutions using the above presented method might be difficult or impossible. For computer calculations some of the existing approximation methods must be applied.

Maximal continuous relations can also be defined using the same methodology presented in this paper.

Activities can take a form that cannot be described by an invertible and derivable function. (e.g. sudden change in productivity rate due to activity break or the application of a second machine etc.) These cases have not been investigated yet.
6. Conclusions

The introduction of continuous relations and non-linear activities to Precedence Diagramming Method affects the very fundaments of the technique, therefore all the developments (e.g. cost optimization, resource optimization etc.) based on traditional PDM must be checked and adapted to the developments presented in this paper.

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