Modified gravitational search algorithm and its application to structural damage detection

Huiyong Guo, Jiaheng Wei
School of civil Engineering, Chongqing University, Chongqing 400045, P. R. China
ghy267@hotmail.com

Abstract—Gravitational search algorithm (GSA) is a novel intelligent optimization algorithm, which can be used in damage detection field. Considering it is difficult for the traditional GSA to avoid the problems of early precocity and low computational efficiency, a modified GSA is proposed to improve the detection efficiency and accuracy. First, Newton’s gravity law is described. Then, a gravitational search algorithm is introduced for damage detection and a modified gravitational search algorithm is proposed to enhance the search efficiency and accuracy. Finally, the objective function based on modal flexibility is proposed, and the modified gravitational search algorithm combined with the model updating technique is presented to detect the damage location and severity. A numerical example is given to verify the effectiveness of this algorithm. The calculated results show that the modified gravitational search algorithm can effectively detect the damage location and severity.

1. Introduction
In recent decades, plenty of industrial and civil buildings have been built. As time goes on, the performance of old buildings will decline, and the ultimate load-carrying capacity of the building also will be reduced [1-4]. In the service period, the durability of building material will gradually change, the mechanical properties will be weaken, and the building material may be corroded. Moreover, these buildings may encounter some serious natural disasters such as typhoons, flood or earthquakes. These will cause various damages to the building, which will have a serious effect on the comfort and safety of the building and threaten the property and lives safety of the people. So, structural damage detection has become an important research field at home and abroad. Currently, structural damage detection based on structural dynamic characteristics analysis is an effective method [5][6], which has wide applicability and accurate recognition results.

Gravitational search algorithm is a novel heuristic intelligent optimization algorithm, which possesses higher computation efficiency and smaller parameters than other heuristic optimization algorithms [7][8]. Gravitational search algorithm has been analyzed by many researchers and utilized for solving complex problems in scientific research and industrial application fields. In addition, the finite element model updating technique based on structural dynamic characteristics analysis is also an effective damage detection method, which can be combined with the heuristic optimization algorithms[9][10]. Considering that the conventional optimization algorithms easily suffer from the low efficiency and premature, a modified gravitational search algorithm is proposed to enhance the search efficiency. The modified gravitational search algorithm combined with the model updating technique is presented to diagnose structural damage.
2. law of gravity
There are interactions between objects in nature, so Newton presented the law of universal gravitation. The calculation formula of the law is written as follows:

\[ F_y = G \frac{M_{ai}M_{pj}}{R^2} \]  

(1)

where, \( M_{ai} \) is the \( i \)th active gravitational mass, \( M_{pj} \) is the \( j \)th passive gravitational mass, \( R \) is the Euclidean distance between objects, and \( G \) is the gravitational constant. Based on Newton's second law, the acceleration formula caused by gravitation is given by

\[ a_i = \frac{F_y}{M_i} \]  

(2)

where, \( M_i \) is the Inertial mass. In addition, gravitation has attenuation in nature, so the gravitational constant \( G \) will change with the age of the universe. The attenuation formula can be written as:

\[ G(t) = G(t_0) \times \left( \frac{t_0}{t} \right)^\beta, \beta < 1 \]  

(3)

where, \( G(t) \) is the gravitational constant at time \( t \), \( G(t_0) \) is the gravitational constant at time \( t_0 \), and \( \beta \) is the attenuation parameter. From equation (3), it can be seen that the gravitational constant \( G \) decreases as the time increases.

3. modified Gravitational search algorithm

3.1 Gravitational search algorithm
According to the law of universal gravitation and Newton's second law, a gravitational search algorithm is proposed as optimization search algorithm[8].

Assuming that the dimension of search space is \( d \), the space contains \( N \) particles, the population which contains \( N \) particles is \( X \), and the position expression of the \( i \)th particle in the population is \( \mathbf{X}_i = (x_{i1}, x_{i2}, ..., x_{id}) \). \( x_{ik} \) is the position component of the particle \( i \) in the \( k \)th dimensional space. The mass calculation formula is given by

\[ m_i(t) = \frac{fit_i(t) - worst(t)}{best(t) - worst(t)} \]  

(4)

\[ M_i(t) = \frac{m_i(t)}{\sum_{j=1}^{N} m_j(t)} \]  

(5)

where, \( fit(t) \) is the fitness value of the \( i \)-th particle at time \( t \), \( worst(t) \) is the worst fitness value in the population at time \( t \), \( best(t) \) is the best fitness value in the population at time \( t \). For the minimum value optimization problem, \( best(t) \) is the minimum value in the population and \( worst(t) \) is the maximum value in the population.

The gravitation between the particle \( i \) and the particle \( j \) is given by

\[ F_{ij} = G(t) \frac{M_{ij}M_{ji}}{R_{ij}(t) + \epsilon} \left( x_{ij}^k(t) - x_{jk}^k(t) \right) \]  

(6)

where \( G(t) \) is the gravitational constant with attenuation, \( R_{ij}(t) = \| \mathbf{x}_{ij}^k(t) - \mathbf{x}_{jk}^k(t) \|_2 \), \( \epsilon \) is a small constant. Here, an exponential attenuation formula is utilized as follow:

\[ G(t) = G_0 e^{-\alpha \frac{t}{T}} \]  

(7)

where \( G_0 \) is the initial attenuation value, \( \alpha \) is the attenuation coefficient, and \( T \) is the maximum iteration number. Thus, the total force of the particle \( i \) is
\[
F_i^k(t) = \sum_{j=A_{i}}^{N} \text{rand}_j \times F_j^k(t) \quad k = 1,2,\ldots,d; \quad (8)
\]

where \( \text{rand}_j \) is a random number. So, the acceleration caused by the total force is

\[
a_i^k(t) = \frac{F_i^k(t)}{M_i(t)} \quad \text{(9)}
\]

Therefore, the velocity and location of the particle \( i \) will change. The velocity and location formulas are written as follows:

\[
v_i^k(t+1) = \text{rand}_i \times v_i^k(t) + a_i^k(t) \quad \text{(10)}
\]

\[
x_i^k(t+1) = x_i^k(t) + v_i^k(t+1) \quad \text{(11)}
\]

From the location update information, we can look for the optimization solution and realize the exploration ability of the algorithm.

### 3.2 Modified gravitational search algorithm

In order to avoid the premature convergence and enhance the search efficiency, a modified gravitational search algorithm is proposed in this paper. A memory particle strategy is introduced into the gravitational search algorithm. A memory particle is provided to record the optimal solution in each generation and the memory particle will also carry out the gravitational motion. If the generated optimal solution in the next generation is better than the old one, the memory particle information will be replaced by the generated solution.

The acceleration of the particle \( i \) caused by the memory particle \( h \) in the \( k \)th dimensional space is

\[
a_{hi}^k(t) = \text{rand} \times G(t) \times M_h(t) \times \frac{x_h^k(t) - x_i^k(t)}{R_{hi}(t) + \varepsilon} \quad \text{(12)}
\]

The velocity of the particle \( i \) caused by the memory particle \( h \) is

\[
v_{hi}^k(t+1) = \text{rand}_i \times v_i^k(t) + a_{hi}^k(t) \quad \text{(13)}
\]

Thus, the total velocity of the particle \( i \) can be corrected to

\[
v_i^k(t+1) = (1-w)v_i^k(t+1) + wv_{hi}^k(t+1) \quad \text{(14)}
\]

where \( w \) is a weighted parameter. The location of the particle \( i \) will be changed according to the modified velocity as follow:

\[
x_i^k(t+1) = x_i^k(t) + v_i^k(t+1) \quad \text{(15)}
\]

Therefore, the algorithm is called as modified gravitational search algorithm.

### 4. Damage detection based on GSA

Gravitational search algorithm can utilize the model updating method to identify structural damage. Therefore, it belongs to the model updating technique in damage detection field. In order to detect damage location and extent, an effective objective function need to be established. The gravitational search algorithm with the objective function can assess structural damage state.

Based on the concept of modal flexibility, the compliance confidence index can be obtained as

\[
ADCdiagFLEX_i = \frac{|2[\text{diag}F_{ci}^T] [\text{diag}F_{ci}^T]|}{|\text{diag}F_{ci}^T| |\text{diag}F_{ci}^T| + |\text{diag}F_{ci}^T| |\text{diag}F_{ci}^T|} \quad \text{(16)}
\]

in which \( F_{ci} = \frac{1}{\omega_{ci}^2} \Phi_{ci} \Phi_{ci}^T \), \( F_{ci} = \frac{1}{\omega_{ci}^2} \Phi_{ci} \Phi_{ci}^T \),

where \( \omega_{ci} \) and \( \Phi_{ci} \) are the \( i \)th measured frequency and mode shape, \( \omega_{ci} \) and \( \Phi_{ci} \) are the \( i \)th calculated frequency and mode shape, \( \text{diag}F_{ci}^T \) and \( \text{diag}F_{ci}^T \) are the diagonal vectors of the flexibility matrixes.
Considering the minimum optimization problem, we can establish the fitness function or objective function as follow:

$$fit = \eta (1 - \prod_{i=1}^{N_m} ADC_{diagFlEX_i})$$  \hspace{1cm} (17)$$

where $\eta$ is sensitivity factor, $N_m$ is the mode number. By using the objective function, gravitational search algorithm can identify structural damage. The location of a particle can be converted into damage information. Through the optimization search of the basic GSA or the modified GSA, we can find the damage location and extent.

5. Numerical Examples

![Euler-Bernoulli simple supported beam](image)

**Figure 1** Euler-Bernoulli simple supported beam

| Scenario 1 | Scenario 2 |
|------------|------------|
| Element    | Damage     | Element | Damage     |
| 2          | 20%        | 3       | 25%        |
| 6          | 10%        | 5       | 30%        |
| 9          | 20%        | 7       | 20%        |

An Euler-Bernoulli simple supported beam is utilized to verify the proposed gravitational search algorithm, which are shown in Figure 1. The size of the beam is given as follows: beam length is 6 m, element number is 10, node number is 11, each node includes two degrees of freedom, i.e. the vertical displacement and rotation angle, cross-sectional area $A$ is 0.005m$^2$, inertial moment $I$ is 1.67m$^4$, elastic modulus $E$ is 32GPa, density $\rho$ is 2500kg/m$^3$. The first three frequencies and mode shapes are extracted as measurement information. Generally, the measured mode shapes are often incomplete, and the rotation angle is difficult to be measured. Therefore, we only use the vertical displacement to detect structural damage. The damage scenarios of simply supported beam are listed in Table 1. The parameters of the GSA given as follows: population size $N$ is 50, operation parameter $N_k$ is 20, attenuation coefficient $\alpha$ is 5, the maximum iteration number $T$ is 100, weighted parameter $w$ is 0.5, sensitivity factor $\eta$ is $10^6$, and the number of mode $N_m$ is 3.

For scenario 1, the damages occur in elements 2, 6 and 9, respectively. In order to verify the validity and accuracy of the proposed algorithm, a basic GSA is also used to detect the damage with the same fitness function. The damage locations are unknown, so we need to search all 10 elements of the beam. The search precision is 0.0001, which means that the size of the search space is 10000$^{10}$. Therefore, an efficient search algorithm is necessary to find the optimization solution. The detection results are shown in Figure 2, and the convergence curves of two algorithms are drawn in Figure 3. In Figure 3, we utilize the logarithmic coordinates to draw the convergence curves, which can perfectly describe the fitness value. From the two figures, we can easily find that the modified GSA is superior to the basic one. The modified GSA has not only the better detection results but also the higher search accuracy than the basic one.
For scenario 2, the damages occur in elements 3, 5, 7 and 8, respectively. In order to verify the validity and accuracy of the proposed algorithm, a basic GSA is also used to detect the damage with the same fitness function. The damage locations are unknown, so we still need to search all 10 elements of the beam. The search precision still is 0.0001, which means that the size of the search space is 1000010. The sensitivity factor $\eta$ is 10^6, so the converted precision value is 1, which can improve the search sensitivity. The basic GSA and the modified GSA are used to search the optimum damage solution. The detection results are shown in Figure 4, and the convergence curves of two algorithms are drawn in Figure 5. In Figure 5, we utilize the logarithmic coordinates to draw the convergence curves, which can perfectly describe the fitness value. From the two figures, we can easily find that the modified GSA is superior to the basic one. The modified GSA has not only the better detection results but also the higher search accuracy than the basic one.
From the two scenarios, we can see that the gravitational search algorithm and the modified gravitational search algorithm possess the capacity of damage detection. The basic gravitational search algorithm can approximately find damage location, but the algorithm cannot precisely identify the damage severity. The modified gravitational search algorithm not only can find damage location, but also can identify the damage severity with good accuracy. In other words, the modified gravitational search algorithm is apparently superior to the basic gravitational search algorithm for structural damage detection.

6. Conclusions
In this paper, a gravitational search algorithm with model updating technique is used to detect the damage location and severity. First, Newton’s gravity law is described. Then, a gravitational search algorithm is introduced for damage detection and a modified gravitational search algorithm is proposed to enhance the search efficiency and accuracy. Finally, the objective function based on modal flexibility is proposed, and the modified gravitational search algorithm combined with the model updating technique is presented to detect the damage location and severity. The numerical examples show that the modified gravitational search algorithm is more effective than the basic gravitational search algorithm for damage detection.
Acknowledgment
This work was supported by the National Natural Science Foundation of China (Grant No. 51578094).

References
[1] A. Bagheri, H. A. Zare, P. Rizzo, “Time domain damage localization and quantification in seismically excited structures using a limited number of sensors,” Journal of Vibration and Control, Vol. 23, Issue 18, 2017, pp. 2942-2961.
[2] J. P. Amezquita-Sanchez, H. Adeli, “Structural damage localization from modal strain energy change,” Archives of Computational Methods in Engineering, Vol. 23, Issue 1, 2016, pp. 1-15.
[3] Y. L. Xu, J. F. Lin, “Multistage damage detection of a transmission tower: Numerical investigation and experimental validation,” Structural Control and Health Monitoring, Vol. 26, Issue 8, 2019, pp. 1-33.
[4] H. F. Lam, J, H, Yang, “Bayesian structural damage detection of steel towers using measured modal parameters,” Earthquakes & Structures, Vol. 8, Issue 4, 2016, pp. 935-956.
[5] P. J. Li D. W. Xu, J. Zhang, “Probability-based structural health monitoring through Markov chain Monte Carlo sampling,” International Journal of Structural Stability and Dynamics, Vol. 16, Issue 7, 2016, pp. 1-16.
[6] K. K. Nair, A. S. Kiremidjian, K. H. Law, “Time series-based damage detection and localization algorithm with application to the ASCE benchmark structure,” Journal of Sound and Vibration, Vol. 291, Issue 1, 2006, pp. 349-368.
[7] E. Rashedi, H. Nezamabadi-Pour, S. Saryazdi, “GSA: A Gravitational Search Algorithm,” Information Sciences, Vol. 179, Issue 13, 2009, pp. 2232-2248.
[8] S. Yazdani, H. Nezamabadi-Pour, S. Kamyab, “A gravitational search algorithm for multimodal optimization,” Swarm and Evolutionary Computation, Vol. 14, 2014, pp. 1-14.
[9] W. X. Ren, S. E. Fang, M. Y. Deng, “Response surface-based finite element model updating using structural static response,” Journal of Engineering Mechanics, Vol. 137, Issue 4, 2011, pp. 248-257.
[10] R. Perera, R. Torres, “Structural damage detection via modal data with genetic algorithms,” Journal of Structural Engineering, Vol. 132, Issue 9, 2006, pp. 1491-1501.