Gyrochronology Model Evaluation Method

Tomomi Otani  
*Embry-Riddle Aeronautical University*, otanit@erau.edu

Ted von Hippel  
*Embry-Riddle Aeronautical University*, vonhippt@erau.edu

Terry D. Oswalt  
*Embry-Riddle Aeronautical University*, oswaltt1@erau.edu

Alexander Stone-Martinez  
*Embry-Riddle Aeronautical University*, stonemaa@my.erau.edu

Derek Buzasi  
*Florida Gulf Coast University*

Follow this and additional works at: [https://commons.erau.edu/publication](https://commons.erau.edu/publication)

Part of the [Stars, Interstellar Medium and the Galaxy Commons](https://commons.erau.edu/publication)

**Scholarly Commons Citation**

Otani, T., von Hippel, T., Oswalt, T. D., Stone-Martinez, A., & Buzasi, D. (2021). Gyrochronology Model Evaluation Method. *The Astrophysical Journal*, (). Retrieved from [https://commons.erau.edu/publication/1570](https://commons.erau.edu/publication/1570)

This Article is brought to you for free and open access by Scholarly Commons. It has been accepted for inclusion in Publications by an authorized administrator of Scholarly Commons. For more information, please contact commons@erau.edu.
GYROCHRONOLOGY MODEL EVALUATION METHOD

Tomomi Otani,1 Ted von Hippel,1 Derek Buzasi,2 T. D. Oswalt,1 and Alexander Stone-Martinez1,3

1Department of Physical Sciences, Embry-Riddle Aeronautical University, 1 Aerospace Blvd, Daytona Beach, FL 32114, United States
2Department of Chemistry and Physics, Florida Gulf Coast University, 10501 FGCU Blvd S, Fort Myers, FL 33965, United States
3Department of Astronomy, New Mexico State University, 1780 E University Ave, Las Cruces, NM 88003, United States

ABSTRACT

Accurate stellar ages are essential for our understanding of the star formation history of the Milky Way, Galactic chemical evolution, and to constrain exoplanet formation models. Gyrochronology, an empirical relationship between stellar rotation and age, appears to be a reliable age indicator for main sequence stars over the mass range 0.6 to 1.0 $M_\odot$. Under the assumption that wide binaries contain two coeval stars, we selected pairs from the Kepler, K2, and TESS surveys in order to evaluate three Gyrochronology models (Barnes 2007; Angus et al. 2015; Mamajek & Hillenbrand 2008). In this first paper in a planned series, we describe a Monte Carlo approach to assess the precision and accuracy of age derived from these models. We use this technique to demonstrate that current gyrochronology models achieve $\sim 25\%$ precision in ages when $\sigma_P/P = 0.1$ and $\sigma_{B-V} = 0.02$. We also outline ways to improve this to $\sim 10\%$.

Corresponding author: Tomomi Otani
otanit@erau.edu
1. INTRODUCTION

Ages are among the most difficult stellar properties to determine, yet they provide key constraints on problems ranging from the formation and habitability of exoplanets to the Galaxy’s chemical evolution and star formation history. For decades it has been known that among main sequence (MS) stars of spectral type F through early M, rotation rates decrease with age as angular momentum is lost via winds (Skumanich 1972; Pallavicini et al. 1981; Soderblom 1985). An empirical framework for measuring stellar ages from rotation periods, gyrochronology, was developed over a decade ago (Barnes 2001, 2003, 2007, 2010; Barnes & Kim 2010). Gyrochronology works well for solar-type stars in open clusters (Meibom & Mathieu 2009; Meibom et al. 2011a,b, 2015) and is potentially one of the most precise stellar chronometric techniques for 0.6–1.0 M\(_\odot\) field stars (Barnes 2003; Soderblom 2010; Epstein & Pinsonneault 2014; van Saders 2016).

Among lower MS stars down to early M-type, the rotational shear between the convective and radiative zones is believed to generate magnetic fields (Guerrero et al. 2019). The resulting spots modulate the light curve as the star rotates, providing a convenient way to measure rotation periods. Although open clusters confirm that the gyrochronology paradigm is valid for the Sun and lower mass stars, there is ongoing concern about how its age precision is affected by such factors as the initial range of rotation rates (Barnes 2010; Matt et al. 2012), the choice of spin-down model (Aigrain et al. 2015), changing magnetic morphology (Buzasi 1997; Garraffo et al. 2018), and how mass loss rate may change for stars older than the Sun (van Saders 2016; Metcalfe & Egeland 2019). For example, some field stars with asteroseismic ages have been found to exhibit faster rotation rates than prescribed by gyrochronology (van Saders 2016). In addition, evidence is accumulating that the rate of rotational spin-down may temporarily stall for stars somewhat older than \(\sim 1\) Gyr (Curtis et al. 2020). An independent measure of ages can help identify the cause(s) of such discrepancies.

Ages derived from wide non-interacting binary stars offer several advantages. First, components of each pair are coeval and should exhibit the same rotation age. Thousands of pairs are known for which high precision astrometric, photometric, and spectroscopic data are now available, and majority of these pairs are nearer than most clusters. Additionally, these pairs span a well-sampled range of ages, extending beyond the time most open clusters have lost most low mass members. Finally, unlike asteroseismology, which with current technology is limited to a relatively narrow range of masses, wide pairs contain stars that span the entire range of masses over which gyrochronology is believed to apply.

Gyrochronology requires measured rotation periods. The Kepler, K2, and TESS missions, with their continuous monitoring of selected star fields, have obtained data useful for deriving the rotation periods for thousands of stars. Unfortunately, the vast majority of Kepler and K2 stars do not have \(B-V\) color indices, the most common proxy for mass upon which most current gyrochronology models are based. For about 100 wide pairs in the original Kepler field, Janes (2017) constructed an empirical \(B-V\) vs. \(g-K\) relation. However, this approach imposed large uncertainties on the age estimates, especially for bluer stars where gyrochrones converge. Using \(ugriz\) and astrometric data from Zacharias (2015) to construct color-magnitude and reduced proper motion diagrams, we found over 50% of the Kepler wide pair components in Janes (2017) lie outside the \(g-r\) and \(r-i\) colors corresponding to the 0.4 \(< B-V < 1.5\) range over which gyrochronology applies.

A more recent search for wide pairs in the Kepler field was conducted by Godoy-Rivera & Chaname (2018). This study was the first to use Gaia DR2 astrometry, parallaxes, radial velocities and metallicities to identify candidate pairs. However, of the 55 pairs found, only 15 had rotational modulation and \(B-V\) data for both components. Moreover, about half the components have \(B-V\) \(\simeq 0.4\) where gyrochrones converge. In addition, two pairs in this sample have evolved components for which the gyrochronology paradigm does not apply.

In a pilot study, about 200 MS+MS pairs in the Washington Double Star Catalog Mason et al. (2001) were selected from K2 Campaign 5 (Oswalt, Buzasi, & Otani 2017). The selection criteria were: (1) good quality astrometric data; (2) \(V < 18\); and (3) separations \(10 < s < 60''\). In this field, only a few percent of the pairs showed rotational modulation in both components and have \(B-V\) colors in the MAST archive. We culled \(ugriz\) and astrometric data from Zacharias (2015) and Flewelling et al. (2016) and constructed color-magnitude and reduced proper motion diagrams in an attempt to identify and remove nonphysical pairs and unresolved third components.

Our period analysis determination began by iterative 4\(\sigma\) clipping extreme outliers, linearly detrending individual sectors, and PCHIP spline interpolating to fill small gaps in the light curve. Then three different algorithms were applied to estimate rotation periods: the Lomb-Scargle periodogram (LS) (Scargle 1998; VanderPlas 2018), the autocorrelation function (ACF) (McQuillan et al. 2013), and the wavelet transform (Wavelet) (Bravo et al. 2014). In the latter case we...
used the Morlet wavelet with $k = 6$. In each case we estimated uncertainties in the resulting period through a Monte Carlo procedure which maps the local amplitude noise in the periodogram (or corresponding function) to resulting changes in the location of the peak.

Figure 1. Rotation period vs. $B - V$ color index (a mass proxy, with low mass stars toward the right) for MS+MS wide binaries in several *Kepler K2* fields (Oswalt, Buzasi, & Otani 2017) and *TESS*. Colored curves are gyrochrone models using Angus et al. (2015), Barnes (2007), and Mamajek & Hillenbrand (2008) with ages as indicated in Myr. Several wide binaries may have components outside the color (mass) range for which gyrochronology applies and/or inconsistent implied gyrochronology ages.

Figure 1 compares the resulting sample of several dozen high confidence physical pairs to the three empirical gyrochronology models of Angus et al. (2015), Barnes (2007), and Mamajek & Hillenbrand (2008) (hereafter A15, B07, and MH08, respectively) using all *K2* and *TESS* cycle 1 wide binaries in which both primary and secondary rotation light curve modulations were detected. However, several pairs were obviously inconsistent with gyrochronology (i.e., those with near vertical or negative slopes). These outliers may be caused by several reasons:

- one or both members of the pair suffer from exceptionally large errors in color or rotation period,
- one or both members of the pair lie outside the region of parameter space in which gyrochronology is applicable,
- undetected tertiary components modified the evolutionary history, mass, or rotation period of the observed star,
- blending of the target star with another star along the line of sight,
- the presence of multiple stellar spots or groups yielding an incorrect rotation period,
- spots measured at different latitudes and/or differential rotation,
- pulsation periods masquerading as rotation periods, and
- problems with the gyrochronology models like uncertainties, fitting parameters inaccuracies, and missing physics in Gyrochronology model.

To investigate the outliers and thereby potentially improve gyrochronology as a tool, objective criteria are required to distinguish which binary systems fit the models and which do not. Once outliers are identified, a range of techniques to find unresolved binaries, chance line-of-sight blends, etc. can be applied, including SED fitting and spectroscopy.

In this paper we develop a Monte Carlo strategy for propagating uncertainties in the observed stellar parameters and theoretical model coefficients in order to quantify the resulting age uncertainties. We employ this Monte Carlo strategy to evaluate the level of precision that each of three gyrochronology models can achieve for realistic stellar period and color uncertainties. As a first application of this strategy, we chose three recent empirical gyrochronology models (Barnes 2007; Angus et al. 2015; Mamajek & Hillenbrand 2008). The model evaluation method is discussed in Section 2.3. Application of the method using the three Gyro models and an evaluation of the age precision of those models are discussed in Section 3. The conclusions drawn from these experiments are described in Section 4.

2. MODEL EVALUATION METHOD

2.1. Gyrochronology Models

Barnes (2003) first proposed an empirical relationship between the rotation period, color, and age of the form:
\[ P = A^n \times a \ (B - V - c)^b \]  

(1)

where \( P \) is the rotation period in days, \( A \) is the age of the star in Myr, \( B - V \) is the color index, and \( a, b, c, \) and \( n \) are dimensionless free parameters. Since then, three other empirical models that used this simple relationship have been published (B07, A15, & MH08). The parameters used by each of these empirical models are shown in Table 1.

To estimate the age of a star, \( P \) and \( B - V \) must be obtained from observations. Nearly continuous photometry from space telescopes (Kepler, K2, and TESS) has provided rotation periods for thousands of stars with higher precision than is possible from ground-based observations. The majority of \( B - V \) color indices for these stars have come from ground-based photometry. While more complex empirical rotation-age relations have been introduced in recent years (see e.g. Meibom & Mathieu (2009); Meibom et al. (2011a,b); Barnes & Kim (2010); Lanzafame & Spada (2015); Spada & Lanzafame (2020)) the above three analytical relations are simple and readily amenable to testing our basic approach to using wide binaries to test gyrochronology models that is outlined below. In a subsequent paper we will examine these newer models.

| coefficients | Angus | Barnes | Mamajek+Hillenbrand |
|--------------|-------|--------|---------------------|
| a            | 0.4   | 0.7725 | 0.407               |
| + \( \sigma_a \) | 0.3   | 0.011  | 0.021               |
| - \( \sigma_a \) | 0.05  | 0.011  | 0.021               |
| b            | 0.31  | 0.601  | 0.325               |
| + \( \sigma_b \) | 0.05  | 0.024  | 0.024               |
| - \( \sigma_b \) | 0.02  | 0.024  | 0.024               |
| c            | 0.45  | 0.4    | 0.495               |
| + \( \sigma_c \) | N/A   | N/A    | 0.01                |
| - \( \sigma_c \) | N/A   | N/A    | 0.01                |
| n            | 0.55  | 0.5189 | 0.566               |
| + \( \sigma_n \) | 0.02  | 0.007  | 0.008               |
| - \( \sigma_n \) | 0.09  | 0.007  | 0.008               |

Solving Equation 1 for stellar age, adding a subscript to age, \( A_g \), to acknowledge that this is not necessarily the true age of the star, but rather the age provided via a gyrochronology relation, we have

\[
D(A_g) = \left\{ \frac{G(P)}{D(a) \times [G(B - V) - D(c)]^{D(b)}} \right\}^{\frac{1}{D(n)}},
\]

(2)

where \( G(B - V) \) and \( G(P) \) indicate that these observable parameters have uncertainties characterized by Gaussian distributions using the observed values and one-sigma uncertainties and \( D(a, b, c, n) \) indicates that the model parameter uncertainty distributions may not be Gaussian.

2.2. Uncertainty Sensitivity for Individual Stars

To propagate errors, we employed a Monte Carlo approach to evaluating Equation 2, allowing us to examine the age precision achievable with these empirical models throughout the color-period diagram. In this approach, we drew
values of $B-V$ color and rotation period $P$ in grid points throughout the color-period parameter space and then computed uncertainties in ages consistent with the obtained data, as well as values of the gyrochronology model coefficients ($a$, $b$, $c$, and $n$) consistent with their published values and uncertainties.

Figure 2 presents the ranges of expected age precision, $\sigma_{\text{age}}$, equivalent to a normalized age uncertainty, for the three gyrochronology models. The fractional rotation period uncertainties, $\sigma_P$, for the primary and secondary were both set at either 5% or 10% of the observed period. These are typical rotation period uncertainties obtained from Kepler, K2, and TESS for high quality data (Aigrain et al. 2015; Reinhold & Hekker 2020; Canto Martins et al. 2020). The $B-V$ uncertainty of 0.02 is appropriate for high quality ground-based photometry (Tonry et al. 2012). For the coefficient uncertainties, the published values listed in Table 1 were used. The uncertainties in model coefficients, observed rotation period, and colors were propagated via this Monte Carlo approach to derive uncertainties in the estimated ages.

Figure 2 shows how age uncertainties depend on the uncertainties in models, color, and rotation period. Because the coefficient uncertainties of the A15 model (see Table 1) were significantly larger than the coefficient uncertainties of the other two models, this model yielded larger age uncertainties. This does not mean that the A15 model is of lower quality. Rather, A15 might simply be incorporating more realistic uncertainties. We remind the reader that none of the three models we are studying here are the latest models from these groups and therefore that they are not representing the most up-to-date gyro chronology models. We have chosen these models for our first paper in the series because their analytical form allows us to develop and demonstrate our Monte Carlo approach to the gyrochronology models. One of the reasons that this model obtained large coefficient uncertainties is that it relied on two clusters, Coma Ber (age = 0.5 ± 0.1 Gyr) and the Hyades (age = 0.625 ± 0.05 Gyr), which have similar ages yet substantially different color-period relations. For stars with $\sigma_P/P = 0.10$, the resulting age uncertainties were about 20 to 25% for both B07
and MH08 models. For stars with $\sigma_P/P = 0.05$, the age uncertainties were about 10 to 20% depending on the color and rotation period of the star. Although such age uncertainties are relatively large for research requiring absolute ages, they may be sufficient for research making use of relative ages. We also expect that further improvements to gyrochronology models will increase both the precision and accuracy of gyrochronological ages. Using these models with high quality rotation rates ($\sigma_P/P = 0.05$) will keep relative age uncertainties under 20%.

Figure 3 explores the limits of age determination using these three gyrochronology models. The plots are the same as Figure 2, but with smaller model coefficient uncertainties (top two rows: $\sigma_{\text{coeff},\text{used}}/\sigma_{\text{coeff},\text{published}} = 0.5$. This might be achieved by including many more stars in the empirical gyrochronology models with a broader range of known ages. In the bottom two rows, $\sigma_{\text{coeff}} = 0.0$ was used to show the age uncertainties resulting purely from the uncertainties in the observed color indices and rotation periods. For the results using $\sigma_{\text{coeff}} = 0.0$, if $\sigma_P = 10\%$, the expected age uncertainties can be as good as 15 to 20%. If $\sigma_P = 5\%$, the expected age can be as good as 5 to 10%, except at the high mass end (small $B-V$) for the B07 model. We conclude that the ultimate limiting age precision available from gyrochronology is likely to be 5% - 10%.
Figure 3. The age uncertainty, $\sigma_{\text{age}}$, for each model under different model coefficient uncertainties. Period uncertainties of $\sigma_P = 5\%$ and $10\%$ and $B-V$ uncertainties of $\sigma_{B-V} = 0.02$ were used. The panels in the third row suggest that ages could be obtained to 5-10\% precision if the uncertainties in model coefficients are negligible. Other combinations of coefficient, period, $B-V$ uncertainties are presented in Appendix A.
In this paper, we employed three simple empirical models based on Equation 1, without considering complications such as the slow-rotator sequence, metallicity, or other factors that may affect the intrinsic gyrochronology relations. This mathematical simplification allows us to compare these three published models directly to assess their internal precision and their sensitivity to the input colors and rotation periods. In a subsequent paper we will apply these lessons to more up-to-date and complex gyrochronology models.

2.3. Model Evaluation Method using Wide Binaries

In a wide binary system, it is safe to assume the two components are coeval (Kraus & Hillenbrand 2009). Assuming Equation 1 applies, the rotation periods of the primary and secondary components of a binary system are

\[ P_p = (A_{g,p})^n \times a[(B - V)_p - c]^b \]

and

\[ P_s = (A_{g,s})^n \times a[(B - V)_s - c]^b \]

where a, b, c, and n are the parameters listed in Table 1, \( P_p \) and \( P_s \) are the primary and secondary rotation periods, and \( [B - V]_p \) and \( [B - V]_s \) are the primary and secondary \( B - V \) colors.

Although the model parameters (a, b, c, n) have uncertainties, for any given value of those parameters the ages of the two stars must be the same (e.g. \( A_p = A_s \)). Taking the ratio of equations 3 and 4 yields

\[ \frac{P_p}{P_s} = \left( \frac{A_{g,p}}{A_{g,s}} \right)^n \left( \frac{(B - V)_p - c}{(B - V)_s - c} \right)^b . \]

Solving for the age ratio yields

\[ \frac{A_{g,s}}{A_{g,p}} = \left\{ \left[ \frac{(B - V)_p - c}{(B - V)_s - c} \right]^b \left( \frac{P_p}{P_s} \right)^b \right\}^{1/n} . \]

which reduces the number of parameters as the coefficient a drops out of the ratio. Therefore, we do not test the absolute value of the age parameter, only the consistency of the ages derived for the two components. Nominally, we expect \( \frac{A_{g,s}}{A_{g,p}} = 1 \). Yet because these ages are derived from an imperfect model under development, \( \frac{A_{g,s}}{A_{g,p}} \) is rarely identically unity, though it is typically in the vicinity.

We incorporated an additional parameter, \( \Delta \), as an age ratio “tolerance”. This parameter represents the degree to which a gyrochronology model is imperfect and yet still tolerably useful. Our goal was to use this age tolerance parameter to explore the required precision of the models and observational parameters in order to achieve useful gyrochronology ages. For the remainder of this paper, we use \( \Delta = 0.1 \), i.e. a 10% age difference between the primary and secondary that be tolerated and deemed consistent. Other values of \( \Delta \) could be chosen, based on the age precision required for a particular project.

We again assumed Gaussian or other distributions in the observed quantities and model parameters, as described above. Equation 6 is recast as

\[ D \left( \frac{A_{g,s}}{A_{g,p}} \right) = \left\{ \left[ \frac{G(B - V)_p - D(c)}{G(B - V)_s - D(c)} \right]^{D(b)} / \left[ \frac{G(P_p)}{G(P_s)} \right] \right\}^{1/D(n)} . \]

As an example of Equation 7, the age-comparison probability distribution using each model is shown in Figure 4. Color and period values for the binary KIC 7596937 + KIC 7596922, along with the models parameters, were drawn randomly from their quoted error distributions and applied to Equation 7 to calculate \( \frac{A_{g,s}}{A_{g,p}} \). This process was repeated 10,000 times in order to derive the age ratio distribution for this binary. The median and surrounding interval covering \( \pm 34.1\% \) of the probability distribution were taken as the age ratio and its uncertainty for the binary. (This interval would be consistent with the \( \pm 1\sigma \) uncertainty range for a Gaussian distribution.)

Figure 4 assumes \( \sigma_p = 0.05 \times P \) and \( \sigma_{B-V} = 0.02 \). For this binary, the probability distribution area of the B07 model is mostly within \( \frac{A_{g,s}}{A_{g,p}} = 1 \pm \Delta = 0.90 \) to 1.10. On the other hand, most of the probability distributions of the MH08 and A15 models yielded values larger than \( 1 + \Delta = 1.10 \). These results show that the estimated secondary age
of binary stars tend to be older than the primary age when MH08 and A15 models are used to calculate ages for this pair.

Figure 4. An example of the age-comparison probability distribution from Equation 7, using the models of B07, MH08, & A15. The Kepler binary system (KIC 7596937 + KIC 7596922) is used for this example with the observed periods and $B - V$ colors along with simulated period uncertainties, $\sigma_P = 0.05 \times P$, and color uncertainty, $\sigma_{B-V} = 0.02$. The vertical bar shows the region within $1 \pm \Delta$, where $\Delta$ has been chosen to be 0.1.

To evaluate the degree to which the primary and secondary stars are consistent within the tolerance $\Delta$, the fractions of the probability distributions within $1 \pm \Delta$ was obtained for each star, ratioed, and converted to a percentage as follows:

$$\text{Coeval probability} = 100 \times \left( \frac{\int_{\Delta=0.9}^{1+\Delta=1.1} \frac{A_{g,s}}{A_{g,p}}}{\int_{0}^{\infty} \frac{A_{g,s}}{A_{g,p}}} \right)$$  \hspace{1cm} \text{(8)}$$

This value corresponds to the likelihood that the binary pair is coeval within the adopted tolerance and assumed model. Larger values from this equation increase confidence in the consistency of the model and the veracity of the identified binary properties. If this value is low, it is likely that the stellar properties were not measured adequately or that there is a problem with the gyrochronology model itself. Follow-up observations can determine whether the former is the case. If not, adjustment to the empirical gyrochronology model may be called for. The threshold for age inconsistency was set at 99.7%, which is indicative of disagreement in the ages at $\geq 3\sigma$. In the following section, we use the same formalism to examine a related issue – the age precision possible with current and hypothetically improved uncertainties in the empirical model constants and the observed parameters.

3. RESULT AND DISCUSSION

Two wide binaries were selected to illustrate the model evaluation analysis described in Section 2.3. The colors and rotation periods of each component are listed in Table 3. Binary 1 is a case where the ages of the two components appear to be consistent using all three gyrochronology models. Binary 2 is a counterexample, in which the ages of the two components appear to be inconsistent. Binary 1 was observed by Kepler with 29.4 min exposures for 90 days and Binary 2 was observed by Kepler/K2 with 29.4 min exposures for approximately 80 days. Rotation periods were obtained using three methods (LS, ACF, and Wavelet) as described in Section 1. The $B - V$ colors were obtained from the TESS Input Catalog (Stassun et al. 2018). The rotation periods and colors of these wide binary stars are shown in the color-period diagrams in Figure 5 along with the gyrochrone grids of the B07, MH08, & A15 models.
The line connecting the components of Binary 1 is nearly parallel to a gyrochrone while that of Binary 2 is clearly not parallel to any gyrochrone. An eyeball assessment in this diagram provides a helpful age consistency check, though to determine quantitative age consistencies the observed and model uncertainties, the latter of which are not apparent in this diagram, must be incorporated.

Table 3. Wide binaries used in the experiment presented in Figure 6. The ages of the Binary 1 components appear consistent with the three models, whereas the ages of the Binary 2 components appear inconsistent. For this experiment, three methods were used for stellar rotation period determinations, from which we determine the average rotation period and average rotation period uncertainties (in parentheses). $B - V$ colors were obtained from the TESS Input Catalog (Stassun et al. 2018). 1-σ uncertainties are in parenthesis.

| name       | LS       | ACF       | Wavelet | $B - V$ |
|------------|----------|-----------|---------|---------|
| binary 1   |          |           |         |         |
| KIC 7596937| 11.4 (0.6)| 10.4 (0.6)| 12.0 (0.6)| 0.75 (0.07) |
| KIC 7596922| 21.0 (0.4)| 21.1 (0.2)| 20.9 (0.6)| 1.39 (0.28) |
| binary 2   |          |           |         |         |
| EPIC 220673232| 14.3 (0.4)| 14.4 (0.3)| 14.6 (0.4)| 1.37 (0.09) |
| EPIC 220673503| 17.5 (0.4)| 17.5 (0.2)| 17.8 (0.5)| 1.39 (0.22) |

Figure 5. Color-period diagrams for two wide binaries (binary 1: KIC 7596937 + KIC 7596922 and binary 2: EPIC 220673232 + EPIC 220673503) in the model grids of A15 (left), B07 (middle) B07, and M08 (right). The age of each gyrochrone (in Myr) is listed in the legend. The crosses indicate the observed color and rotation period of each target along with their uncertainties. Because the components are coeval, the line connecting the two components of a binary should lie on a gyrochrone.
Figure 6 displays the corresponding age ratio probability distributions of the components of these two binary systems. To evaluate the contribution of the uncertainties in period and color to the overall age error budget, we examined the effects of a range of period and color uncertainties. For $B-V$ uncertainties, we chose $\sigma_{B-V} = 0.10, 0.05,$ and 0.02 mag. As stated above, the best ground-based observations typically achieve $\sigma_{B-V} \approx 0.02$ mag, yet some of the $B-V$ uncertainties listed in the TESS Input Catalog (Stassun et al. 2018) are surprisingly large, and more than 50% of our targets have uncertainties similar or even larger than 0.10 mag. For period uncertainties, we used $\sigma_P = 0.10, 0.05,$ and 0.01. The average period uncertainties that can be obtained by ground- and space-based observation are circa $\sigma_P = 0.10$ and 0.05, respectively. With high-quality space-based data and for stars with large and uniform amplitude light curves, the nominal period uncertainties can drop to $\sigma_P \leq 0.01,$ although differential rotation and the unknown spot latitude may place a limit on period precision even with these datasets.

For Binary 1, the age ratio distributions employing the B07 model peak near 1 ($\Delta = 0$). If improved observations were to find the exact same period and color for these two stars while the uncertainties in both parameters decreased, then the age consistency, as measured by the probability that $A_{g,s}/A_{g,p} = 1 \pm \Delta,$ would improve by the percentages shown in Figure 6. The age ratio distributions derived from the other two models are substantially offset from 1, and therefore if improved observations recovered the exact same period and color for these stars as the uncertainties in both parameters decreased, then the age consistency would decrease and could go to essentially zero. Note that this experiment does not imply that improved observations would yield this result, only that they could. Improved observations could find different enough colors or periods for one or both of these binary components that a different model would be favored or that all three would be inconsistent.

For Binary 2, as expected from Figure 5, the majority of the probability distribution is outside the $1 \pm \Delta$ range, and if those colors and periods remained the same as more precise measurements were obtained, the age inconsistency would become more and more significant. Specifically, for these values and with period uncertainties less than 0.05 P and $\sigma_{B-V} \leq 0.05,$ the probability that the both stars have the same ages is excluded at the 2$\sigma$ (95%) level. With these same values, uncertainties of P should be less than 0.01 P to exclude coevality at 3$\sigma$ (99.7%). Binary 2 is already a candidate for further examination to check physical association, contamination, or other issue. If improved observations increase the probability that the components have inconsistent ages based on gyrochronology models, the importance to gyrochronology of understanding such a system only grows.

Our Monte Carlo tests of these three gyrochronology relations underscored the importance of improved observational data. Although the best $B-V$ uncertainties that can generally be obtained from ground-based telescopes are $\sim 0.02$ mag (Tonry et al. 2012), most of the TESS Input Catalog (Stassun et al. 2018) entries have $\sigma_{B-V} \geq 0.05.$ On the other hand, colors obtained from Pan-STARRS and Gaia often have substantially smaller uncertainties. Pan-STARRS $g-r$ uncertainties are around 0.04 and Gaia $G_{BP}-G_{RP}$ uncertainties are 0.002 at $G < 13,$ and 0.01 at $G = 17$ (Tonry et al. 2012; Evans et al. 2018). Therefore, we suggest that the next generation of gyrochronology relations be based on photometry from Gaia and Pan-STARRS, or other large-field, high-quality surveys. Most of the original Kepler periods have reported uncertainties around $\sigma_P/P = 0.05$ (Aigrain et al. 2015). However, for K2 typical uncertainties are larger because the observation span is much shorter than the original Kepler data (see Fig. 4 of Reinhold & Hekker (2020)). That figure suggests that obtaining uncertainties of less than 5% of the observed period is possible for rotation period up to 25 days only if the light curve variation amplitude is large compared to the high-frequency noise level in the photometric light curve. However, it would be difficult to obtain rotation period uncertainties less than 5% of the observed periods for rotation periods longer than approximately 25 days without additional follow-up observations.
Figure 6. Comparison of age ratio probability distributions. Top: Binary 1 (KIC 7596937 + KIC 7596922), which approximately parallels a gyrochrone in Figure 5. Bottom: Binary 2 (EPIC 220673232 + EPIC 220673503), which does not parallel a gyrochrone in Figure 5. Different combinations of $B - V$ and period uncertainties were used in each panel and these are shown in the upper right of each panel. Note that the y-axis range is different in each row. The red, blue, and yellow distributions are based on the B07, MH08, & A15 models, respectively. The percentage of each distribution that is within the tolerance area is shown in parenthesis in the lower right in each panel. A ten percent tolerance ($\Delta = \pm 0.10$) in age agreement between components of such binary is illustrated by vertical black lines. In these test cases, the age of Binary 1 is most consistent with B07, whereas none of the three models yield an age consistent for Binary 2 ($\Delta \approx 0.5$) to better than $\sim 50\%$. 
4. CONCLUSION

For gyrochronology to achieve its full potential for stellar age determinations, it is important to quantify the effects of uncertainties in both the model parameters and the observed data. To that end, we developed a Monte Carlo approach to comparing the gyrochronology models of B07, MH08, and A15 and evaluated the age precision sensitivity to the input model and observed data uncertainties. We then studied the additional information that wide binaries provide. In principle, binary components are co-eval, which removes one model variable (a in Eq. 1). Removing this model variable also removes its uncertainty when propagating errors. A further advantage of applying gyrochronology to wide binaries is that objects which are outliers in the color-period diagram, i.e. binaries that appear to have components with inconsistent ages, provide an opportunity for observational follow-up that is not provided by single filed stars. Many of these cases may turn out to be hierarchical systems with an unresolved component, suffer line-of-sight blending with another star, or have some other issue that renders the models inappropriate or compromises the data. To improve the precision of gyrochronology ages, the prevalence of such issues should be determined, and methods to identify these problems should be developed.

We found that with the empirical gyrochronology models of B07, MH08, and A15, gyrochronology ages can be determined to approximately 10 to 20% if $\sigma_{B-V} \leq 0.02$ and $\sigma_P \leq 0.05$. To determine outliers in the color-period diagram, we suggest relying on data with $\sigma_{B-V} \leq 0.05$ and $\sigma_P \leq 0.05$. We also suggest that the empirical models, which are primarily based on open clusters, be reconstructed with the more widely available and precise color indices from the major sky surveys such as Pan-STARRS and Gaia.

This material is based upon work supported by the National Science Foundation under Grant No. AST-1715718, and AST-1910396, and by NASA under Grant No. NNX16AB76G and 80NSSC21K0245.

This paper includes data collected by the TESS mission, which are publicly available from the Mikulski Archive for Space Telescopes (MAST) and described in Jenkins et al. (2016). This paper also includes data collected by the Kepler and K2 missions. The Kepler/K2 light curves are archived at MAST.

Facility: Kepler, TESS

REFERENCES

Aigrain, S. et al., 2015, MNRAS, 450, 3226
Angus, R., Aigrain, S., Foreman-Mackey, D., & McQuillan, A., 2015, MNRAS, 450, 1787
Barnes, S. A., 2001, ApJ, 561, 1095
Barnes, S. A., 2003, ApJ, 586, 464
Barnes, S. A., 2007, ApJ, 669, 1167
Barnes, S. A., 2010, ApJ, 722, 222
Barnes, S. A., & Kim, Y., 2010, ApJ, 721, 675
Epstein, Courtney R., & Pinsonneault, Marc H., 2014, ApJ, 780, 2
Bravo, J.-P., Roque, S., Estrella, R., Leão, I. C., & De Medeiros, J. R., 2014, A&A, 568, A34
Buzasi, D. L., 1997, ApJ, 484, 2
Canto Martins, B. L., Gomes, R. L., Messias, Y. S., De Lira, S. R., et al. 2020, ApJS, 250, 20
Curtis, J. L., Agüireos, M. A., Matt, S. P., Covey, K. R. et al. ApJS, 250, 20
Evans, D. W., Riello, M., De Angeli, F., Carrasco, J. M. et al., 2018, “Gaia Data Release 2: Photometric Content and Validation,” arXiv:1804.09368
Flewelling, H., Magnier, E., Chambers, K. et al. 2016, “The Pan-STARRS1 Database and Data Products,” arXiv:1612.05243
Godoy-Rivera, D., & Chaname, J. 2018, MNRAS, 479, 4440
Garraffo, C., Drake, J. J., Dotter, A., Choi, J. M., et al., 2018, ApJ, 862, 1
Guerrero, G., Zaire, B., Smolarkiewicz, P. K., de Gouveia Dal Pino, E. M., Kosovichev, A. G., & Mansour, N. N. 2019, ApJ, 880, 6
Janes, K., 2017, ApJ, 835, 75
Jenkins, J., Twicken, J., McCauliff, S., Campbell, J. et al., 2016, Proceedings of the SPIE, 99133E
Kraus, Adam, M., & Hillenbrand, Lynne, A., 2009, ApJ, 704, 531
Lanzafame, A. C., & Spada, F., 2015, A&A, 584, A30
McQuillan, A., Aigrain, S. & Mazeh, T., 2013, MNRAS, 432, 1203
Mamajek, E. E., & Hillenbrand, L. A., 2008, ApJ, 687, 1264
Mason, B., Wycoff, G., Hartkopf, W., Geoffrey, D., & Worley, C., 2001, AJ, 122, 3466
Matt, S., MacGregor, K., Pinsonneault, M., & Greene, T., 2012, ApJL, 754, 26
Metcalfe, T., & Egeland, R., 2019, ApJ, 871, 39
Meibom, S., & Mathieu, R., 2009, ApJ, 695, 679
Meibom, S., Mathieu, R., Stassun, K., Liebesny, P., & Saar, S., 2011, ApJ, 733, 115
Meibom, S. et al. 2011, ApJ, 733, 9
Meibom, S., Barnes, S., Platais, I., Gilliland, R., Latham, D., Mathieu, R. 2015, Nature, 517, 589
Oswalt, T., Buzasi, D., Otani, T. 2017, BAAS, 229, 240.26
Pallavicini, R., Golub, L., Rosner, R., Vaiana, G., Ayres, T., Linsky, J., 1981, ApJ, 248, 279
Reinhold, Timo, & Hekker, Saskia, 2020, A&A, 635, 43
Scargle, J. D., 1998, ApJ, 504, 405
Skumanich, A., 1972, ApJ, 171, 565
Soderblom, D.R., 1985, AJ, 90, 2103
Soderblom, D.R., 2010, ARA&A, 48, 581
Spada, F., & Lanzafame, A. C., 2020, A&A, 636, A76
Stassun, K. G., Oelkers, R. J., Pepper, J. Paegert, M. et al., 2018, AJ, 156, 3
Tonry, J. L., Stubbs, C. W., Lykke, K. R., et al., 2012, ApJ, 750, 99
van Saders, J., Ceillier, T., Metcalfe, T., Aguirre, V., Pinsonneault, M., García, R., Mathur, S., & Davies, G., 2016, Nature, 529, 181
VanderPlas, J. T., 2018, ApJS, 236, 16
Zacharias, N. et al., 2015, AJ, 150, 101
Figure A.1. The age uncertainty, $\sigma_{\text{age}}$, for each model under different model coefficient uncertainties which are not listed in Figure 3 are shown here. In this plot, $\sigma_{\text{coeff}} = \text{published}$ values, $\sigma \frac{P}{P} = 0.1$, and $\sigma_{B-V} = 0.1$, 0.05, and 0.02 are used.
Figure A.2. The age uncertainty, $\sigma_{\text{age}}$, for each model under different model coefficient uncertainties which are not listed in Figure 3 are shown here. In this plot, $\sigma_{\text{coeff}} = \text{published}$ values, $\sigma P/P = 0.05$, and $\sigma_{B-V} = 0.1, 0.05, \text{and } 0.02$ are used.
Figure A.3. The age uncertainty, $\sigma_{\text{age}}$, for each model under different model coefficient uncertainties which are not listed in Figure 3 are shown here. In this plot, $\sigma_{\text{coeff}} = \text{published values}$, $\sigma_{P/P} = 0.01$, and $\sigma_{B-V} = 0.1$, 0.05, and 0.02 are used.