Transition form factors of B decays into p-wave axial-vector mesons in the perturbative QCD approach

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The $B_{u,d,s} \to V,A$ form factors are studied in perturbative QCD approach ($V,A$ denote a vector meson and two kinds of p-wave axial-vector mesons: $^3P_1$ and $^1P_1$ states, respectively). The form factors are directly studied in the large recoiling region and extrapolated to the whole kinematic region within the dipole parametrization. Adopting decay constants with different signs for the two kinds of axial-vectors, we find that the two kinds of $B \to A$ form factors have the same sign. The two strange mesons $K_{1A}$ and $K_{1B}$ mix with each other via the SU(3) symmetry breaking effect. In order to reduce the ambiguities in the mixing angle between $K_{1A}$ and $K_{1B}$, we propose a model-independent way that utilizes the B decay data. Most of the branching fractions of the semileptonic $B \to Al\bar{\nu}$ decays are of the order $10^{-4}$, which still need experimental tests in the on-going and forthcoming experiments.

I. INTRODUCTION

In rare charmless $B$ decays, the main experimental observables are branching ratios and CP asymmetries. To predict these quantities, one needs to compute the hadronic decay amplitudes. Since hadronizations are involved in these decay channels, predictions on these observables are often polluted by our poor knowledge of the non-perturbative QCD. But fortunately, it has been shown that in $m_b \to \infty$ limit, the decay amplitudes are under control. For example, if the recoiling meson in the final state moves very fast, a hard gluon is required to kick the soft light quark in B meson into a collinear one and then the process is calculable. Keeping quarks’ transverse momentum, the perturbative QCD (PQCD) approach \cite{1} is free of endpoint divergence and the Sudakov formalism makes it more self-consistent. A bigger advantage is that we can really do the form factor calculation and the quantitative annihilation type diagram calculation in this approach. The importance of annihilation diagrams is already tested in the predictions of direct CP asymmetries of $B^0 \to \pi^+\pi^-$, $K^+\pi^-$ decays \cite{1,2} and in the explanation of $B \to \phi K^*$ polarization problem \cite{3,4}.

In the quark model, the possible quantum numbers $J^{PC}$ for the orbitally excited axial-vector mesons are $1^{++}$ or $1^{-+}$, depending on different spin couplings of the two quarks. In the SU(3) limit, these mesons can not mix with each other; but since the $s$ quark is heavier than $u,d$ quarks, $K_1(1270)$ and $K_1(1400)$ are not purely $^3P_1$ or $^1P_1$ states. These two mesons are believed to be mixtures of $K_{1A}$ and $K_{1B}$, where $K_{1A}$ and $K_{1B}$ are $^3P_1$ and $^1P_1$ states, respectively. Analogous to $\eta$ and $\eta'$, the flavor-singlet and flavor-octet axial-vector meson can also mix with each other. In general, the mixing angles can be determined by experimental data, but unfortunately, there is not too much data on these mesons which leaves the mixing angles much free. The $B$ meson decays offer a promising opportunity to investigate these axial-vector mesons. Since the observation of the $B \to J/\psi K_1$ \cite{5} and $D^*a_1(1260)$ \cite{6} decays, there are more and more experimental studies on B meson decays involving a p-wave axial-vector meson in the final state \cite{7}. In the present work, we use the PQCD approach to study the $B \to A$ form factors and semileptonic $B \to Al\bar{\nu}$ decays. As a byproduct, we also update the predictions on $B \to V$ form factors in the PQCD approach. In the large recoiling region, the $B \to A$ form factors are directly predicted using the most recent inputs evaluated in the QCD sum rules \cite{8,9}. We also extrapolate the form factors to the whole kinematic region by adopting the dipole parametrization to investigate the semileptonic $B \to Al\bar{\nu}$ decays. Using the $B^0 \to D^+K_{1A}$ and $B^0 \to D^+\pi^-$ decays, we also propose a model-independent method to remove the ambiguity in the mixing between the two strange...
axial-vector mesons.

This paper is organized as follows: In section II, we give the input quantities, including wave function of the B-meson, light-cone distribution amplitudes of the light vector mesons and light axial-vector mesons and input values of the various mesonic decay constants. In section III we give the factorization formulae and the numerical results for the \( B \to V \) and \( B \to A \) form factors, discuss the mixing between the strange axial-vector mesons and make the predictions on the semileptonic \( B \to A l \bar{l} \) decays. Our summary is given in the last section. Appendix A contains various functions that enter the factorization formulae in the PQCD approach.

II. FORMALISM OF THE PQCD APPROACH AND INPUTS

We will work in the rest frame of the B meson and use light-cone coordinates. In the heavy quark limit the mass difference between b quark and B meson is negligible: \( m_b \approx m_B \). Masses of axial-vector mesons are very small compared with the b quark mass, we keep them up to the first order. Since the light(vector/axial-vector) meson in a heavy-light system, whose light-cone matrix element can be decomposed as:

\[
P_{B(\gamma)} = \frac{m_{B(\gamma)}}{\sqrt{2}} (1, 1, 0, 0), \quad P_2 = \frac{m_{B(\gamma)}}{\sqrt{2}} (\eta, \frac{r_2^2}{\eta}, 0, 0),
\]

where \( r_2 \equiv \frac{m_V/A}{m_{B(\gamma)}} \) as the mass of the vector or axial-vector meson. For the momentum transfer \( q = P_{B(\gamma)} - P_2 \), there exists \( \eta \approx 1 - q^2/m_{B(\gamma)}^2 \). The momentum of the light antiquark in \( B(\gamma) \) meson and the quark in light mesons are denoted as \( k_1 \) and \( k_2 \) respectively (see Fig. II):

\[
k_1 = \left( 0, \frac{m_{B(\gamma)}}{\sqrt{2}} x_1, k_{1 \perp} \right), \quad k_2 = \left( \frac{m_{B(\gamma)}}{\sqrt{2}} x_2 \eta, 0, k_{2 \perp} \right).
\]

In the course of the PQCD calculations, the light-cone wave functions of the mesons are required. The B meson is a heavy-light system, whose light-cone matrix element can be decomposed as:

\[
\int_0^1 \frac{d^4 x}{(2\pi)^4} e^{-i k_{1 \perp} \cdot z} \langle 0 | b_\beta (0) \bar{q}_\alpha (z) | B(\gamma) (P_{B(\gamma)}) \rangle = \frac{i}{\sqrt{2N_c}} \left\{ (P_{B(\gamma)} + m_{B(\gamma)}) \gamma_5 \left[ \phi_{B(\gamma)} (k_1) + \frac{\not{q} - \not{r}}{\sqrt{2}} \bar{\phi}_{B(\gamma)} (k_1) \right] \right\} \beta \alpha,
\]

where \( n = (1, 0, 0, \not{r}) \) and \( v = (0, 1, 0, \not{r}) \) are light-like unit vectors. There are two Lorentz structures in B meson light-cone distribution amplitudes, and they obey the normalization conditions:

\[
\int \frac{d^4 k_1}{(2\pi)^4} \phi_{B(\gamma)} (k_1) = \frac{f_{B(\gamma)}}{2\sqrt{2N_c}}, \quad \int \frac{d^4 k_1}{(2\pi)^4} \bar{\phi}_{B(\gamma)} (k_1) = 0,
\]

with \( f_{B(\gamma)} \) as the decay constant of \( B(\gamma) \) meson. In principle, both the \( \phi_{B(\gamma)} (k_1) \) and \( \bar{\phi}_{B(\gamma)} (k_1) \) contribute in B meson transitions. However, the contribution of \( \bar{\phi}_{B(\gamma)} (k_1) \) is usually neglected, because its contribution is numerically small. So we will only keep the term with \( \phi_{B(\gamma)} (k_1) \) in equation \( 3 \). In the momentum space the light-cone matrix of B meson can be expressed as:

\[
\Phi_{B(\gamma)} = \frac{i}{\sqrt{6}} (P_{B(\gamma)} + m_{B(\gamma)}) \gamma_5 \phi_{B(\gamma)} (k_1).
\]

Usually the hard part is independent of \( k^+ \) or \( k^- \), so we integrate one of them out from \( \phi_{B(\gamma)} (k^+, k^-, k_{\perp}) \). With \( b \) as the conjugate space coordinate of \( k_{\perp} \), we can express \( \phi_{B(\gamma)} (x, k_{\perp}) \) in b-space by

\[
\Phi_{B(\gamma), \alpha \beta} (x, b) = \frac{i}{\sqrt{2N_c}} \left[ (P_{B(\gamma)} \gamma_5 + m_{B(\gamma)} \gamma_5) \right]_{\alpha \beta} \phi_{B(\gamma)} (x, b),
\]
two transverse polarization vectors \( \epsilon \) for the longitudinal polarization and transverse polarizations, respectively. Here \( \omega \).

Neutral vector meson’s longitudinal decay constants can be determined by their electronic decay widths through Table I. 

The longitudinal decay constants of charged vector mesons can be extracted from the decay \( \tau^- \rightarrow (\rho^-, K^{*-})\nu_T \). Neutral vector meson’s longitudinal decay constants can be determined by their electronic decay widths through \( V^0 \rightarrow e^+e^- \) and the results are given in Table I. Transverse decay constants are mainly explored by QCD sum rules, which are also collected in Table I.

The vector meson polarization vectors \( \epsilon \), which satisfy \( P \cdot \epsilon = 0 \), include one longitudinal polarization vector \( \epsilon_L \) and two transverse polarization vectors \( \epsilon_T \). The vector meson distribution amplitudes up to twist-3 are defined by:

\[
\langle V(P_2, \epsilon_L) | q_2(0) | 0 \rangle = \frac{1}{\sqrt{2N_c}} \int_0^1 dx e^{ixP_2z} \left[ m_V \phi_L^T(x) + \phi_L^T P_2 \phi_L^T(x) + m_V \phi_L^T(x) \right]_{\alpha \beta},
\]

\[
\langle V(P_2, \epsilon_T) | q_2(0) | 0 \rangle = \frac{1}{\sqrt{2N_c}} \int_0^1 dx e^{ixP_2z} \left[ m_V \phi_T^T(x) + \phi_T^T P_2 \phi_T^T(x) + m_V i \epsilon_{\mu \nu \rho \sigma} \gamma^\mu \epsilon_{\nu}^\sigma n^\rho \phi_T^T(x) \right]_{\alpha \beta},
\]

for the longitudinal polarization and transverse polarizations, respectively. Here \( x \) is the momentum fraction associated with the \( q_2 \) quark. \( n \) is the moving direction of the vector meson and \( v \) is the opposite direction. These distribution amplitudes can be related to the ones used in QCD sum rules by:

\[
\phi_V(x) = \frac{f_V}{2\sqrt{2N_c}} \phi_{||}(x), \quad \phi_V^T(x) = \frac{f_V^T}{2\sqrt{2N_c}} h_{||}^{(t)}(x),
\]

\[
\phi_T(x) = \frac{f_V}{4\sqrt{2N_c}} \frac{d}{dx} f_{||}^{(s)}(x), \quad \phi_T^T(x) = \frac{f_V^T}{2\sqrt{2N_c}} \phi_{||}(x),
\]

\[
\phi_V(x) = \frac{f_V}{2\sqrt{2N_c}} g_{||}^{(v)}(x), \quad \phi_V^T(x) = \frac{f_V}{8\sqrt{2N_c}} \frac{d}{dx} g_{||}^{(a)}(x).
\]

The twist-2 distribution amplitudes can be expanded in terms of Gegenbauer polynomials \( C_n^{3/2} \) with the coefficients called Gegenbauer moments \( a_n \):

\[
\phi_{||, \perp}(x) = 6x(1-x) \left[ 1 + \sum_{n=1}^{\infty} a_n C_n^{3/2}(t) \right],
\]
TABLE II: Input values of the decay constants (absolute values) for the axial-vector mesons (in MeV). The transverse decay constants for \( ^1P_1 \) are evaluated at \( \mu = 1 \) GeV.

| Decay constant                  | Value         |
|---------------------------------|---------------|
| \( f_{a_1(1260)} \)            | 238 ± 10      |
| \( f_{f_1(1350)} \)            | 245 ± 13      |
| \( f_{a_2(1320)} \)            | 239 ± 13      |
| \( f_{K_{1A}} \)               | 250 ± 13      |
| \( f_{f'_1(1235)} \)           | 180 ± 8       |
| \( f_{f_1(1119)} \)            | 180 ± 12      |
| \( f_{K_{1B}} \)               | 190 ± 10      |
| \( f_{K_{1B}} \)               | 190 ± 10      |

where \( t = 2x - 1 \). The Gegenbauer moments \( a_i^{\|,\perp} \) are mainly determined by the technique of QCD sum rules. Here we quote the recent numerical results \([14, 15, 16, 17]\) as

\[
\begin{align*}
\alpha_i^{\|} \quad & = \quad 0.03 \pm 0.02, \quad \alpha_i^{\perp} \quad & = \quad 0.04 \pm 0.03, \\
\alpha_i^{\|} \quad & = \quad 0.15 \pm 0.07, \quad \alpha_i^{\perp} \quad & = \quad 0.14 \pm 0.06, \\
\alpha_i^{\|} \quad & = \quad 0.11 \pm 0.09, \quad \alpha_i^{\perp} \quad & = \quad 0.10 \pm 0.08, \\
\alpha_i^{\|} \quad & = \quad 0.18 \pm 0.08, \quad \alpha_i^{\perp} \quad & = \quad 0.14 \pm 0.07,
\end{align*}
\]

where the values are taken at \( \mu = 1 \) GeV.

Using equation of motion, two-particle twist-3 distribution amplitudes are related to the twist-2 LCDAs and the three-particle twist-3 LCDAs. But in some \( B \to VV \) decays, there exists the so-called polarization problem. A reasonable way has been suggested to resolve this problem in the PQCD approach: one needs to adopt the asymptotic LCDAs. As in Ref. \([18]\), we use the asymptotic forms for the twist-3 LCDAs:

\[
\begin{align*}
h_\parallel^{(t)}(x) \quad & = \quad 3t^2, \\
h_\parallel^{(s)}(x) \quad & = \quad 6x(1 - x), \\
g_\perp^{(a)}(x) \quad & = \quad 6x(1 - x), \\
g_\perp^{(e)}(x) \quad & = \quad \frac{3}{4}(1 + t^2). 
\end{align*}
\]

For the axial-vectors, the longitudinal and transverse decay constants are defined by:

\[
\langle A(P_2, \epsilon)|q_2\gamma_\mu\gamma_5 q_1|0\rangle \quad = \quad if_A m_A \epsilon_\mu^*, \quad \langle A(P_2, \epsilon)|q_2\sigma_{\mu\nu}\gamma_5 q_1|0\rangle \quad = \quad f_A^{T}(\epsilon_\mu P_{2\nu} - \epsilon_\nu P_{2\mu}).
\]

In the SU(2) limit, due to G-parity invariance, the longitudinal[transverse] decay constants vanish for the non-strange \( ^1P_1[^3P_1] \) states. This will affect the normalization for the corresponding distribution amplitudes which will be discussed in the following. For convenience, we take \( f_{V_0} \equiv f \) \([f_{P_0} (\mu = 1 \text{ GeV}) \equiv f]\) as the “normalization constant”. The decay constants of axial vector mesons shown in Table II are taken from Refs. \([18]\).

Distribution amplitudes for axial-vectors with quantum numbers \( J^{PC} = 1^{++} \) or \( 1^{-+} \) are defined by:

\[
\begin{align*}
\langle A(P_2, \epsilon^L)|\bar{q}_2\beta(x)q_1\alpha(0)|0\rangle \quad & = \quad -i \sqrt{2N_c} \int_0^1 dx e^{ix\gamma_5 - m_A(\hat{n}_L\phi_A(x) - \hat{n}_L\phi^T\phi^L_A(x) - m_A\gamma_5\phi^T_A(x))}_{\alpha\beta}, \\
\langle A(P_2, \epsilon^T)|\bar{q}_2\beta(x)q_1\alpha(0)|0\rangle \quad & = \quad -i \sqrt{2N_c} \int_0^1 dx e^{ix\gamma_5 - m_A(\hat{n}_T\phi^T(x) - \hat{n}_T\phi^T\phi^T_A(x) - m_A\gamma_5\phi^T_A(x))}_{\alpha\beta}.
\end{align*}
\]

Besides the factor \( i\gamma_5 \) from the left hand, axial-vector mesons’ distribution amplitudes can be related to the vector ones by making the following replacement:

\[
\phi_V \rightarrow \phi_A, \quad \phi_V^\dagger \rightarrow \phi_A^\dagger, \quad \phi_V \rightarrow \phi_A^\dagger, \\
\phi_V^T \rightarrow \phi_A^T, \quad \phi_V^\dagger \rightarrow \phi_A^T, \quad \phi_V \rightarrow \phi_A^T.
\]
for convenience, we formally define symmetry, are non-zero for only strange mesons. We normalize the distribution amplitude \( \phi_1 \) for 1\(^3\)P\(_1\) and 1\(^1\)P\(_1\) states, and 0\(_0\) states so that we can use Eq. (25) as the normalization condition. In the isospin limit, \( \phi_{\parallel}, g_{\perp}^{(s)} \) and \( g_{\perp}^{(a)} \) are antisymmetric [symmetric] for non-strange 1\(^3\)P\(_1\) [1\(^1\)P\(_1\)] states. In the above, we have taken \( f^T_{P_1} = f_{P_1} = f_{P_1}(\mu = 1 \text{ GeV}) = f \), thus we have

\[
\begin{align*}
\langle 1^3 P_1(P, \epsilon) | q_1 \gamma_{\mu} q_2 | 0 \rangle &= f^T_{P_1} a_0^{1,3 P_1} (\epsilon^*_{\mu} P_{\nu} - \epsilon^*_{\nu} P_{\mu}), \quad (22) \\
\langle 1^1 P_1(P, \epsilon) | q_1 \gamma_{\mu} q_2 | 0 \rangle &= i f_{P_1} a_0^{1,1 P_1} m_{P_1} \epsilon^*_{\mu}, \quad (23)
\end{align*}
\]

where \( a_0^{1,3 P_1} \) and \( a_0^{1,1 P_1} \) are the Gegenbauer zeroth moments. Then the normalization conditions of the distribution amplitudes are given by

\[
\int_0^1 dx \phi_{\perp} (x) = a_0^\perp 
\]

for 1\(^3\)P\(_1\) states and

\[
\int_0^1 dx \phi_{\parallel} (x) = a_0^\parallel 
\]

for 1\(^1\)P\(_1\) states. The zeroth Gegenbauer moments \( a_0^{1,3 P_1} \) and \( a_0^{1,1 P_1} \), characterizing the breaking of flavor SU(3) symmetry, are non-zero for only strange mesons. We normalize the distribution amplitude \( \phi_{\parallel} \) [\( \phi_{\perp} \)] of the 1\(^3\)P\(_1\) [1\(^1\)P\(_1\)] states as

\[
\int_0^1 dx \phi_{\parallel} (x) = 1 \left[ \int_0^1 dx \phi_{\perp} (x) = 1 \right]. 
\]

For convenience, we formally define \( a_0^\parallel = 1 \) for the 1\(^3\)P\(_1\) states so that we can use Eq. (25) as the normalization condition. Similarly, we also define \( a_0^\perp = 1 \) for 1\(^1\)P\(_1\) states so that \( \phi_{\perp} (x) \) has a correct normalization.
FIG. 1: Feynman diagrams for transition of $B$ meson to a vector or axial vector meson. The crosses represent Lorentz structures of the currents.

Up to conformal spin 6, twist-2 distribution amplitudes for axial-vector mesons can be expanded as:

$$\phi_{\parallel}(x) = 6x\bar{x}\left[a_{0}\parallel + 3a_{1}\parallel t + a_{2}\parallel \frac{3}{2}(5t^2 - 1)\right],$$

$$\phi_{\perp}(x) = 6x\bar{x}\left[a_{0}\perp + 3a_{1}\perp t + a_{2}\perp \frac{3}{2}(5t^2 - 1)\right],$$

where the Gegenbauer moments are calculated in Refs. [8, 9] shown in table III. From the results in table III, we can see that there are large uncertainties in Gegenbauer moments which can inevitably induce large uncertainties to form factors and branching ratios. We hope the uncertainties could be reduced in future studies in order to make more precise predictions.

As for twist-3 LCDAs, we use the following form:

$$g_{\perp}^{(v)}(x) = \frac{3}{4}a_{0}\parallel (1 + t^2) + \frac{3}{2}a_{1}\perp t^3, \quad g_{\perp}^{(a)}(x) = 6x\bar{x}(a_{0}\parallel + a_{1}\perp t),$$

$$h_{\parallel}^{(t)}(x) = 3a_{0}\parallel t^2 + \frac{3}{2}a_{1}\perp t(3t^2 - 1), \quad h_{\parallel}^{(s)}(x) = 6x\bar{x}(a_{0}\parallel + a_{1}\perp t).$$

In the following analysis, we will use $a_1$ to denote $a_1(1260)$, $b_1$ to denote $b_1(1235)$ for simplicity. It is also similar for $K_1$ and $f_1, h_1$.

III. $B \to V, B \to A$ FORM FACTORS AND SEMILEPTONIC $B \to A\bar{\nu}$ DECAYS

A. PQCD approach

The basic idea of the PQCD approach is that it takes into account the intrinsic transverse momentum of valence quarks. The decay amplitude, taking the first diagram in Fig. III as an example, can be expressed as a convolution of wave functions $\phi_B, \phi_2$ and hard scattering kernel $T_H$ with both longitudinal and transverse momenta:

$$M = \int_{0}^{1} dx_1 dx_2 \int \frac{d^2E_{1T} d^2E_{2T}}{(2\pi)^2 (2\pi)^2} \phi_B(x_1, \vec{k}_{1T}, P_B, t)T_H(x_1, x_2, \vec{k}_{1T}, \vec{k}_{2T}, t)\phi_2(x_2, \vec{k}_{2T}, P_2, t).$$

(31)

Usually it is convenient to compute the amplitude in coordinate space. Through Fourier transformation, the above equation can be expressed by:

$$M = \int_{0}^{1} dx_1 dx_2 \int d^2\vec{b}_1 d^2\vec{b}_2 \phi_B(x_1, \vec{b}_1, P_B, t)T_H(x_1, x_2, \vec{b}_1, \vec{b}_2, t)\phi_2(x_2, \vec{b}_2, P_2, t).$$

(32)

This derivation is mainly concentrated on tree level diagrams, but actually we have to take into account some loop effects which can give sizable corrections. The $O(\alpha_s)$ radiative corrections to hard scattering process $H$ are depicted...
in Fig. 2. In general, individual higher order diagrams may suffer from two types of infrared divergences: soft and collinear. Soft divergence comes from the region of a loop momentum where all its momentum components vanish:

\[ l^\mu = (l^+, l^-, \vec{l}_T) = (\Lambda, \Lambda, \vec{\Lambda}), \]

where \( \Lambda \) is the typical scale for hadronization. Collinear divergence originates from the gluon momentum region which is parallel to the massless quark momentum,

\[ l^\mu = (l^+, l^-, \vec{l}_T) \sim (m_B, \Lambda^2/m_B, \vec{\Lambda}). \]

In both cases, the loop integration corresponds to \( \int d^4l/l^4 \sim \log \Lambda \), thus logarithmic divergences are generated. It has been shown order by order in perturbation theory that these divergences can be separated from the hard kernel and absorbed into meson wave functions using eikonal approximation [19]. But when soft and collinear momentum overlap, there will be double logarithm divergences in the first two diagrams of Fig. 2. These large double logarithms can be resummed into the Sudakov factor whose explicit form is given in Appendix A.

Furthermore, there are also another type of double logarithm which comes from the loop correction for the weak decay vertex correction. The left diagram in Fig. 1 gives an amplitude proportional to \( 1/((1 - x_2^2)x_1) \). In the threshold region with \( (1 - x_2^2) \to 0 \) [(to be precise, \( (1 - x_2^2) \sim O(\Lambda_{QCD}/m_B) \)], additional soft divergences are associated with the internal quark at higher orders. The QCD loop corrections to the electro-weak vertex can produce the double logarithm \( \alpha_s \ln^2(1 - x_2) \) and resummation of this type of double logarithms lead to the Sudakov factor \( S_1(x_2) \). Similarly, resummation of \( \alpha_s \ln^2 x_1 \) due to loop corrections in the other diagram leads to the Sudakov factor \( S_1(x_1) \). These double logarithms can also be factored out from the hard part and grouped into the quark jet function. Resummation of the double logarithms results in the threshold factor [20]. This factor decreases faster than any other power of \( x \) as \( x \to 0 \), which modifies the behavior in the endpoint region to make pQCD approach more self-consistent. For simplicity, this factor has been parameterized in a form which is independent on channels, twists and flavors [21].

Combining all the elements together, we can get the typical factorization formulae in the pQCD approach:

\[
\mathcal{M} = \int_0^1 dx_1 dx_2 \int d^2\vec{b}_1 d^2\vec{b}_2 (2\pi)^2 \phi_B(x_1, \vec{b}_1, P_B, t) \\
\times T_H(x_1, x_2, Q_1, \vec{b}_1, \vec{b}_2, t) \phi_2(x_2, \vec{b}_2, P_2, t) S_1(x_2) \exp[-S_B(t) - S_2(t)].
\]

(35)
\textbf{B. } \bar{B} \to V \text{ form factors}

\begin{equation}
\langle \bar{B}(P_2, e^+)| \bar{q}\gamma^\mu b |\bar{B}(P_B) \rangle = -\frac{2V(q^2)}{m_B + m_V} \epsilon_{\mu\nu\rho\sigma} \epsilon^{* \rho}_a P_B P_{2\sigma},
\end{equation}

\begin{equation}
\langle \bar{B}(P_2, e^+)| \bar{q}\gamma^\mu \gamma_5 b |\bar{B}(P_B) \rangle = 2\text{im}_V A_0(q^2) \epsilon^{* \mu \cdot q}_a q^\mu + i(m_B + m_V) A_1(q^2) \left[ \epsilon^{* \mu \cdot q}_a q^\mu \right] - iA_2(q^2) \frac{\epsilon^{* \cdot q}_a q^\mu}{m_B + m_V} \left[ (P_B + P_2) - \frac{m_B^2 - m_V^2}{q^2} q^\mu \right],
\end{equation}

\begin{equation}
\langle \bar{B}(P_2, e^+)| \bar{q}\sigma^{\mu\nu} q_b b |\bar{B}(P_B) \rangle = -2iT_1(q^2) \epsilon_{\mu\nu\rho\sigma} \epsilon^{* \rho}_a P_B P_{2\sigma},
\end{equation}

\begin{equation}
\langle \bar{B}(P_2, e^+)| \bar{q}\sigma^{\mu\nu} \gamma_5 q_b b |\bar{B}(P_B) \rangle = T_2(q^2) \left[ (m_B^2 - m_V^2) \epsilon^{* \mu \cdot (P_B + P_2)} \right] + T_3(q^2) \left( \epsilon^{* \cdot q}_a q^\mu \right) \left[ \frac{q^2}{m_B^2 - m_V^2} (P_B + P_2) \right],
\end{equation}

where \( q = P_B - P_2 \), and the relation \( 2m_v A_0(0) = (m_B + m_V) A_1(0) - (m_B - m_V) A_2(0) \) is obtained in order to cancel the pole at \( q^2 = 0 \).

The factorization formulae are given as:

\begin{equation}
V(q^2) = 8\pi C_F m_B^2 (1 + r_2) \int_0^1 dx_1 dx_2 \int_0^\infty b_1 b_1 b_2 b_2 d_2 \phi_B(x_1, b_1)
\end{equation}

\begin{equation}
\times \left[ \phi^V_\alpha(x_2) - r_2 ((1 - x_2) \phi^V_\alpha(x_2) + \phi^V_\alpha(x_2)) + \frac{2}{\eta} \phi^\eta_\alpha(x_2) \right]
\end{equation}

\begin{equation}
\times h_e(x_1, (1 - x_2) \eta, b_1, b_2) \alpha_s(\mu t^2_e) \text{exp}[ - S_{ab}(t^2_e) ] S_t(x_2)
\end{equation}

\begin{equation}
- r_2 (\phi^V_\alpha(x_2) - \phi^\eta_\alpha(x_2)) h_e(x_1, (1 - x_2) \eta, b_1, b_2) \alpha_s(\mu t^2_e) \text{exp}[ - S_{ab}(t^2_e) ] S_t(x_1),
\end{equation}

\begin{equation}
A_0(q^2) = 8\pi C_F m_B^2 \int_0^1 dx_1 dx_2 \int_0^\infty b_1 b_1 b_2 b_2 d_2 \phi_B(x_1, b_1)
\end{equation}

\begin{equation}
\times \left[ \eta \phi^V_\alpha(x_2) - r_2 ((\phi^V_\alpha(x_2) + \phi^\eta_\alpha(x_2))(1 - x_2) \eta - 2\phi^\eta_\alpha(x_2)) \right]
\end{equation}

\begin{equation}
\times h_e(x_1, (1 - x_2) \eta, b_1, b_2) \alpha_s(\mu t^2_e) \text{exp}[ - S_{ab}(t^2_e) ] S_t(x_2)
\end{equation}

\begin{equation}
- 2r_2 \phi^\eta_\alpha(x_2) h_e(x_1, (1 - x_2) \eta, b_1, b_2) \alpha_s(\mu t^2_e) \text{exp}[ - S_{ab}(t^2_e) ] S_t(x_1),
\end{equation}

\begin{equation}
A_1(q^2) = 8\pi C_F m_B^2 (1 + r_2) \int_0^1 dx_1 dx_2 \int_0^\infty b_1 b_1 b_2 b_2 d_2 \phi_B(x_1, b_1)
\end{equation}

\begin{equation}
\times \left[ \phi^V_\alpha(x_2)(1 + \eta - x_2 \eta) - r_2 ((-3 + \frac{2}{\eta} + 2x_2) \phi^V_\alpha(x_2) + (1 - 2x_2) \phi^\eta_\alpha(x_2)) \right]
\end{equation}

\begin{equation}
\times h_e(x_1, (1 - x_2) \eta, b_1, b_2) \alpha_s(\mu t^2_e) \text{exp}[ - S_{ab}(t^2_e) ] S_t(x_2)
\end{equation}

\begin{equation}
- r_2 \phi^V_\alpha(x_2) h_e(x_1, (1 - x_2) \eta, b_1, b_2) \alpha_s(\mu t^2_e) \text{exp}[ - S_{ab}(t^2_e) ] S_t(x_1),
\end{equation}

\begin{equation}
T_1(q^2) = 8\pi C_F m_B^2 \int_0^1 dx_1 dx_2 \int_0^\infty b_1 b_1 b_2 b_2 d_2 \phi_B(x_1, b_1)
\end{equation}

\begin{equation}
\times \left[ \phi^V_\alpha(x_2)(1 + \eta - x_2 \eta) - r_2 ((-3 + \frac{2}{\eta} + 2x_2) \phi^V_\alpha(x_2) + (1 - 2x_2) \phi^\eta_\alpha(x_2)) \right]
\end{equation}

\begin{equation}
\times h_e(x_1, (1 - x_2) \eta, b_1, b_2) \alpha_s(\mu t^2_e) \text{exp}[ - S_{ab}(t^2_e) ] S_t(x_2)
\end{equation}

\begin{equation}
- r_2 (\phi^V_\alpha(x_2) - \phi^\eta_\alpha(x_2)) h_e(x_1, (1 - x_2) \eta, b_1, b_2) \alpha_s(\mu t^2_e) \text{exp}[ - S_{ab}(t^2_e) ] S_t(x_1),
\end{equation}
With terms suppressed by $r_2^2$ neglected, $V_2(q^2)$ can be expressed linearly by $V_0(q^2)$ and $V_1(q^2)$:

$$A_2(q^2) = \frac{1}{\eta} [(1 - r_2)^2 A_1(q^2) - 2 r_2 (1 - r_2) A_0(q^2)].$$

The definitions of the function $S_{ab}(t)$ in Sudakov exponent $\exp[-S_{ab}(t)]$, the factorization scales $t^*_i$s and hard functions $h_i$, are given in Appendix A.

The numerical results for the form factors at maximally recoil point are collected in table IV. The first error comes from decay constants and shape parameter $\omega_b$ of $B(s)$ meson; while the second one is from hard scales $t^*_i$s, the threshold resummation parameter $c = 0.4 \pm 0.1$ and $A_{QCD}((0.25 \pm 0.05)$GeV). To make a comparison, we also collect the results using other approaches [22, 23, 24, 25, 26]. From table IV we can see that most of our results are consistent with others within theoretical errors.

C. $B \to A$ form factors

Following Ref. [32], the $B \to A$ form factors are defined by:

$$\langle A(P_2, \epsilon^* | q \gamma^\mu \gamma_5 q | b \bar{B}(P_B) \rangle = -\frac{2iA(q^2)}{m_B - m_A} \epsilon^{\mu\rho\sigma} \epsilon^*_\rho P_{B\rho} P_{2\sigma},$$

$$\langle A(P_2, \epsilon^* | q \gamma^\mu | b \bar{B}(P_B) \rangle = -2m_A V_0(q^2) \frac{\epsilon^* \cdot q}{q^2} q^\mu - (m_B - m_A) V_1(q^2) \left[ \epsilon^*_\mu - \frac{\epsilon^* \cdot q}{q^2} q^\mu \right]$$

$$+ V_2(q^2) \frac{\epsilon^* \cdot q}{m_B - m_A} \left[ (P_B + P_2)^\mu - \frac{m_B^2 - m_A^2}{q^2} q^\mu \right],$$

$$\langle A(P_2, \epsilon^* | q \sigma^{\mu\nu} \gamma_5 q | b \bar{B}(P_B) \rangle = -2T_1(q^2) \epsilon^{\mu\rho\sigma} \epsilon^*_\rho P_{B\rho} P_{2\sigma},$$

$$\langle A(P_2, \epsilon^* | q \sigma^{\mu\nu} q | b \bar{B}(P_B) \rangle = -i T_2(q^2) \left[ (m_B^2 - m_A^2) \epsilon^* \cdot q - (\epsilon^* \cdot q)(P_B + P_2)^\mu \right]$$

$$-i T_3(q^2)(\epsilon^* \cdot q) \left[ q^\mu - \frac{q^2}{m_B^2 - m_A^2} (P_B + P_2)^\mu \right],$$

with a factor $-i$ different from $B \to V$ and the factor $m_B + m_V (m_B - m_V)$ is replaced by $m_B - m_A (m_B + m_A)$. Similar to $B \to V$ form factors, the relation $2m_A V_0 = (m_B - m_A) V_1 - (m_B + m_A) V_2$ is obtained at $q^2 = 0$. In the PQCD approach, $B \to A$ form factors’ formulae can be derived from the corresponding $B \to V$ form factor formulas using the replacement in Eq. (20) with the proper change of the sign of the pre-factor $r_2$ in $V$ and $A_1$. The form factors in the large recoiling region can be directly calculated. In order to extrapolate the form factors to the whole kinematic region, we use the results obtained in the region $0 < q^2 < 10$GeV and we recast the form factors by adopting the dipole parametrization for the form factors

$$F(q^2) = \frac{F(0)}{1 - a(q^2/m_B^2) + b(q^2/m_B^2)^2}.$$


|          | $B \to \rho$ | $B \to K^*$ | $B \to \omega$ | $B_s \to K^*$ | $B_s \to \phi$ |
|----------|-------------|-------------|----------------|--------------|--------------|
| LFQM[23] | $V$         | 0.27        | 0.31           |              |              |
|          | $A_0$       | 0.28        | 0.31           |              |              |
|          | $A_1$       | 0.22        | 0.26           |              |              |
|          | $A_2$       | 0.20        | 0.24           |              |              |
| LCSR[24] | $V$         | 0.323       | 0.411          | 0.293        | 0.311        | 0.434        |
|          | $A_0$       | 0.303       | 0.374          | 0.281        | 0.360        | 0.474        |
|          | $A_1$       | 0.242       | 0.292          | 0.219        | 0.233        | 0.311        |
|          | $A_2$       | 0.221       | 0.259          | 0.198        | 0.181        | 0.234        |
|          | $T_2$       | 0.267       | 0.333          | 0.242        | 0.260        | 0.349        |
| LQCD[22] | $V$         | 0.35        |                |              |              |
|          | $A_0$       | 0.30        |                |              |              |
|          | $A_1$       | 0.27        |                |              |              |
|          | $A_2$       | 0.26        |                |              |              |
| SCET LCQM[26] | $V$ | 0.298       | 0.339          | 0.275        | 0.323        | 0.329        |
|          | $A_0$       | 0.260       | 0.283          | 0.240        | 0.279        | 0.279        |
|          | $A_1$       | 0.227       | 0.248          | 0.209        | 0.228        | 0.232        |
|          | $A_2$       | 0.215       | 0.233          | 0.198        | 0.204        | 0.210        |
|          | $T_1 = T_2$ | 0.260       | 0.290          | 0.239        | 0.271        | 0.276        |
|          | $T_3$       | 0.184       | 0.194          | 0.168        | 0.165        | 0.170        |

This work

| $V$ | $A_0$ | $A_1$ | $A_2$ | $T_1 = T_2$ | $T_3$ |
|-----|------|------|------|-------------|------|
| $0.21^{+0.05+0.03}_{-0.04-0.02}$ | $0.25^{+0.06+0.04}_{-0.05-0.02}$ | $0.19^{+0.04+0.02}_{-0.04-0.02}$ | $0.20^{+0.04+0.02}_{-0.04-0.02}$ | $0.26^{+0.05+0.03}_{-0.05-0.02}$ | $0.26^{+0.05+0.03}_{-0.05-0.02}$ |
| $0.25^{+0.06+0.04}_{-0.05-0.02}$ | $0.31^{+0.07+0.05}_{-0.07-0.03}$ | $0.23^{+0.05+0.03}_{-0.05-0.03}$ | $0.24^{+0.05+0.04}_{-0.05-0.03}$ | $0.31^{+0.06+0.05}_{-0.06-0.03}$ | $0.18^{+0.04+0.03}_{-0.03-0.02}$ |
| $0.16^{+0.04+0.02}_{-0.04-0.02}$ | $0.19^{+0.04+0.02}_{-0.04-0.02}$ | $0.15^{+0.03+0.02}_{-0.03-0.01}$ | $0.15^{+0.04+0.02}_{-0.04-0.02}$ | $0.18^{+0.04+0.03}_{-0.03-0.02}$ | $0.18^{+0.04+0.03}_{-0.03-0.02}$ |
| $0.13^{+0.03+0.02}_{-0.03-0.01}$ | $0.14^{+0.04+0.02}_{-0.03-0.01}$ | $0.12^{+0.03+0.02}_{-0.02-0.01}$ | $0.11^{+0.02+0.01}_{-0.02-0.01}$ | $0.12^{+0.03+0.02}_{-0.02-0.01}$ | $0.12^{+0.03+0.02}_{-0.02-0.01}$ |
| $0.19^{+0.04+0.03}_{-0.03-0.02}$ | $0.23^{+0.05+0.03}_{-0.05-0.02}$ | $0.18^{+0.04+0.02}_{-0.04-0.02}$ | $0.18^{+0.03+0.02}_{-0.03-0.01}$ | $0.23^{+0.05+0.03}_{-0.05-0.02}$ | $0.19^{+0.04+0.03}_{-0.03-0.02}$ |
| $0.17^{+0.04+0.02}_{-0.03-0.02}$ | $0.20^{+0.05+0.03}_{-0.04-0.02}$ | $0.15^{+0.04+0.02}_{-0.04-0.02}$ | $0.16^{+0.03+0.02}_{-0.03-0.01}$ | $0.16^{+0.04+0.03}_{-0.03-0.02}$ | $0.19^{+0.04+0.03}_{-0.03-0.02}$ |

The real physical states $K_1(1270)$ and $K_1(1400)$ are mixtures of the $K_{1A}$ and $K_{1B}$ states with the mixing angle $\theta_K$:

$$|K_1(1270)| = |K_{1A}|\sin\theta_K + |K_{1B}|\cos\theta_K,$$

$$|K_1(1400)| = |K_{1A}|\cos\theta_K - |K_{1B}|\sin\theta_K.$$  

(46)  

(47)

In the flavor SU(3) symmetry limit, these mesons can not mix with each other; but since $s$ quark is heavier than the $u, d$ quarks, $K_1(1270)$ and $K_1(1400)$ are not purely $1^3P_1$ or $1^3P_1$ states. Generally, the mixing angle can be determined by the experimental data. One ideal method is making use of the decay $\tau^- \to K_1\nu_\tau$, whose partial decay rate is given by

$$\Gamma(\tau^- \to K_1\nu_\tau) = \frac{m_\tau^3}{16\pi}G_F^2|V_{us}|^2f_A^2 \left(1 - \frac{m_A^2}{m_\tau^2}\right)^2 \left(1 + \frac{2m_A^2}{m_\tau^2}\right),$$

(48)  

with the measured results for branching fractions [12]:

$$BR(\tau^- \to K_1(1270)\nu_\tau) = (4.7 \pm 1.1) \times 10^{-3}, BR(\tau^- \to K_1(1400)\nu_\tau) = (1.7 \pm 2.6) \times 10^{-3}.$$  

(49)
TABLE V: $B \to a_1, b_1, K_{1A}, B, h_1, b_8, f_1, f_8$ form factors. $a, b$ are the parameters of the form factors in dipole parametrization. The errors are from: decay constants of $B$ meson and shape parameter $\omega_b$, $\Lambda_{QCD}$ and the scales $\xi_i, \xi_8$; Gegenbauer moments of axial-vectors' LCDAs.

\[<a_{172.1}^{\text{Table V}}=0.26^{+0.06}_{-0.05}+0.01+0.03>\]

\[<a_{172.1}^{\text{Table V}}=0.34^{+0.01}_{-0.07}+0.00+0.02>\]

\[<a_{172.1}^{\text{Table V}}=0.43^{+0.03}_{-0.01}+0.00+0.02>\]

\[<a_{172.1}^{\text{Table V}}=0.38^{+0.00}_{-0.00}+0.00+0.00>\]

\[<a_{172.1}^{\text{Table V}}=0.30^{+0.01}_{-0.00}+0.00+0.00>\]

\[<a_{172.1}^{\text{Table V}}=0.27^{+0.06}_{-0.05}+0.01+0.06>\]

\[<a_{172.1}^{\text{Table V}}=0.35^{+0.08}_{-0.07}+0.01+0.08>\]

\[<a_{172.1}^{\text{Table V}}=0.47^{+0.11}_{-0.09}+0.01+0.10>\]

\[<a_{172.1}^{\text{Table V}}=0.14^{+0.03}_{-0.01}+0.00+0.02>\]

\[<a_{172.1}^{\text{Table V}}=0.37^{+0.08}_{-0.07}+0.01+0.09>\]

\[<a_{172.1}^{\text{Table V}}=0.33^{+0.08}_{-0.07}+0.00+0.08>\]

\[<a_{172.1}^{\text{Table V}}=0.12^{+0.03}_{-0.02}+0.00+0.02>\]

\[<a_{172.1}^{\text{Table V}}=0.26^{+0.06}_{-0.05}+0.01+0.06>\]

\[<a_{172.1}^{\text{Table V}}=0.20^{+0.04}_{-0.03}+0.00+0.03>\]

\[<a_{172.1}^{\text{Table V}}=0.03^{+0.00}_{-0.00}+0.00+0.00>\]

\[<a_{172.1}^{\text{Table V}}=0.17^{+0.04}_{-0.03}+0.00+0.04>\]

\[<a_{172.1}^{\text{Table V}}=0.12^{+0.03}_{-0.02}+0.00+0.03>\]

\[<a_{172.1}^{\text{Table V}}=0.16^{+0.04}_{-0.03}+0.00+0.04>\]

\[<a_{172.1}^{\text{Table V}}=0.21^{+0.05}_{-0.04}+0.00+0.05>\]

\[<a_{172.1}^{\text{Table V}}=0.19^{+0.05}_{-0.04}+0.00+0.05>\]

\[<a_{172.1}^{\text{Table V}}=0.21^{+0.05}_{-0.04}+0.00+0.05>\]

\[<a_{172.1}^{\text{Table V}}=0.19^{+0.04}_{-0.03}+0.00+0.04>\]

The longitudinal decay constants (in MeV) can be straightly obtained:

\[|f_{K_1(1270)}| = 160^{19}_{21}, \quad |f_{K_1(1400)}| = 125^{74}_{125}. \quad \text{(50)}\]

In principle, one can combine the decay constants for $K_{1A}, K_{1B}$ evaluated in QCD sum rules with the above results to determine the mixing angle $\theta_K$. But since there are large uncertainties in Eq. (50), the constraint on the mixing angle is expected to be rather smooth:

\[-143^\circ < \theta_K < -120^\circ, \quad \text{or} \quad -49^\circ < \theta_K < -27^\circ, \quad \text{or} \quad 37^\circ < \theta_K < 60^\circ, \quad \text{or} \quad 113^\circ < \theta_K < 153^\circ, \quad \text{(51)}\]

where we have taken the uncertainties from the branching ratios in Eq. (49) and the first Gegenbauer moment $a_{1K_1}$ into account but neglected the mass differences as usual. In this paper, for simplicity, we use two reference values in...
TABLE VI: Same as Table V except $B_s \to h_s, b_1, f_s, f_1, K_{1A}, K_{1B}$.

| $F$ | $F(0)$ | $a$ | $b$ | $F$ | $F(0)$ | $a$ | $b$ |
|-----|-------|-----|-----|-----|-------|-----|-----|
| $A_{B, K_{1A}}$ | $0.25^{+0.05}_{-0.00}+0.00+0.00+0.05$ | $1.63^{+0.06}_{-0.06}+0.00+0.00+0.06$ | $0.18^{+0.04}_{-0.04}+0.00+0.00+0.04$ | $A_{B, K_{1B}}$ | $0.18^{+0.04}_{-0.04}+0.00+0.00+0.04$ | $1.63^{+0.06}_{-0.06}+0.00+0.00+0.06$ |
| $V_{B, K_{1A}}$ | $0.36^{+0.06}_{-0.06}+0.00+0.00+0.06$ | $1.63^{+0.06}_{-0.06}+0.00+0.00+0.06$ | $0.42^{+0.04}_{-0.04}+0.00+0.00+0.04$ | $V_{B, K_{1B}}$ | $0.42^{+0.04}_{-0.04}+0.00+0.00+0.04$ | $1.63^{+0.06}_{-0.06}+0.00+0.00+0.06$ |
| $V_{B, K_{1A}}$ | $0.43^{+0.06}_{-0.06}+0.00+0.00+0.06$ | $1.63^{+0.06}_{-0.06}+0.00+0.00+0.06$ | $0.33^{+0.04}_{-0.04}+0.00+0.00+0.04$ | $V_{B, K_{1B}}$ | $0.33^{+0.04}_{-0.04}+0.00+0.00+0.04$ | $1.63^{+0.06}_{-0.06}+0.00+0.00+0.06$ |
| $V_{B, K_{1A}}$ | $0.34^{+0.06}_{-0.06}+0.00+0.00+0.06$ | $1.63^{+0.06}_{-0.06}+0.00+0.00+0.06$ | $0.43^{+0.04}_{-0.04}+0.00+0.00+0.04$ | $V_{B, K_{1B}}$ | $0.43^{+0.04}_{-0.04}+0.00+0.00+0.04$ | $1.63^{+0.06}_{-0.06}+0.00+0.00+0.06$ |
| $V_{B, K_{1A}}$ | $0.30^{+0.06}_{-0.06}+0.00+0.00+0.06$ | $1.63^{+0.06}_{-0.06}+0.00+0.00+0.06$ | $0.34^{+0.04}_{-0.04}+0.00+0.00+0.04$ | $V_{B, K_{1B}}$ | $0.34^{+0.04}_{-0.04}+0.00+0.00+0.04$ | $1.63^{+0.06}_{-0.06}+0.00+0.00+0.06$ |
| $V_{B, K_{1A}}$ | $0.18^{+0.04}_{-0.04}+0.00+0.00+0.04$ | $1.63^{+0.06}_{-0.06}+0.00+0.00+0.06$ | $0.28^{+0.04}_{-0.04}+0.00+0.00+0.04$ | $V_{B, K_{1B}}$ | $0.28^{+0.04}_{-0.04}+0.00+0.00+0.04$ | $1.63^{+0.06}_{-0.06}+0.00+0.00+0.06$ |

TABLE VII: $B \to a_1, b_1$ form factors at maxima recoil and the results in the light-front quark model (LFQM) and light-cone sum rules (LCSR). The errors in this work are from: decay constants of $B$ meson and shape parameter $\omega_b$; $\Lambda_{QCD}$ and the scales $t_s, s$; Gegenbauer moments in axial-vectors’ LCDAs.

| $B \to a_1$ | This work | LFQM [23, 35] | LCSR [33] |
|-------------|------------|---------------|------------|
| $A$ | $0.26^{+0.06}_{-0.06}+0.00+0.00+0.06$ | $0.25$ | $0.48^{+0.09}$ |
| $V_0$ | $0.34^{+0.07}_{-0.07}+0.00+0.00+0.07$ | $0.13$ | $0.30^{+0.05}$ |
| $V_1$ | $0.33^{+0.07}_{-0.07}+0.00+0.00+0.07$ | $0.37$ | $0.37^{+0.07}$ |
| $V_2$ | $0.13^{+0.04}_{-0.04}+0.00+0.00+0.04$ | $0.18$ | $0.42^{+0.08}$ |
| $T_1(T_2)$ | $0.34^{+0.08}_{-0.08}+0.00+0.00+0.08$ | $0.30^{+0.07}_{-0.07}+0.00+0.00+0.07$ | $0.30^{+0.07}_{-0.07}+0.00+0.00+0.07$ |
| $B \to b_1$ | This work | LFQM [23, 35] | LCSR [33] |
|-------------|------------|---------------|------------|
| $A$ | $0.19^{+0.04}_{-0.04}+0.00+0.00+0.04$ | $0.10$ | $-0.25^{+0.05}$ |
| $V_0$ | $0.45^{+0.10}_{-0.10}+0.00+0.00+0.10$ | $0.39$ | $-0.39^{+0.07}$ |
| $V_1$ | $0.33^{+0.07}_{-0.07}+0.00+0.00+0.07$ | $0.18$ | $-0.20^{+0.04}$ |
| $V_2$ | $0.03^{+0.02}_{-0.02}+0.00+0.00+0.02$ | $-0.03$ | $-0.09^{+0.02}$ |
| $T_1(T_2)$ | $0.27^{+0.06}_{-0.06}+0.00+0.00+0.06$ | $0.18^{+0.04}_{-0.04}+0.00+0.00+0.04$ | $0.18^{+0.04}_{-0.04}+0.00+0.00+0.04$ |
TABLE VIII: $B_{u,d,s} \rightarrow K_i(1270), K_i(1400),$ $B_{u,d,s} \rightarrow h_1(1170),$ $h_1(1380)$ and $B_{u,d,s} \rightarrow f_i(1285),$ $h_1(1420)$ form factors for physical axial-vector mesons at maximally recoil point, i.e. $q^2 = 0$. Results in the first line of each form factor are calculated using $\theta_K = 45^\circ, \theta_P = 10^\circ$ or $\theta_P = 38^\circ$, while the second line corresponds to the angle $\theta_K = -45^\circ, \theta_P = 45^\circ$ or $\theta_P = 50^\circ$.

The errors are from: decay constants of $B_i(\psi)$ meson and shape parameter $\omega_0$; $\Lambda_{QCD}$ and the scales $\lambda_{C8};$ Gegenbauer moments in axial-vectors’ LCDAs.

| $B \rightarrow K_i(1270)$ | $B \rightarrow K_i(1400)$ | $B \rightarrow h_1(1170)$ | $B \rightarrow h_1(1380)$ | $B \rightarrow h_1(1420)$ |
|--------------------------|---------------------------|--------------------------|--------------------------|--------------------------|
| $A$                      | $0.35^{+0.08}_{-0.07}+0.07$ | $0.35^{+0.08}_{-0.07}+0.07$ | $0.35^{+0.08}_{-0.07}+0.07$ | $0.35^{+0.08}_{-0.07}+0.07$ |
| $B$                      | $0.05^{+0.06}_{-0.05}+0.06$ | $0.05^{+0.06}_{-0.05}+0.06$ | $0.05^{+0.06}_{-0.05}+0.06$ | $0.05^{+0.06}_{-0.05}+0.06$ |
| $V_0$                    | $0.63^{+0.14}_{-0.13}+0.14$ | $0.63^{+0.14}_{-0.13}+0.14$ | $0.63^{+0.14}_{-0.13}+0.14$ | $0.63^{+0.14}_{-0.13}+0.14$ |
| $V_1$                    | $0.12^{+0.03}_{-0.03}+0.03$ | $0.12^{+0.03}_{-0.03}+0.03$ | $0.12^{+0.03}_{-0.03}+0.03$ | $0.12^{+0.03}_{-0.03}+0.03$ |
| $V_2$                    | $0.10^{+0.01}_{-0.01}+0.01$ | $0.10^{+0.01}_{-0.01}+0.01$ | $0.10^{+0.01}_{-0.01}+0.01$ | $0.10^{+0.01}_{-0.01}+0.01$ |
| $T_1(T_2)$               | $0.46^{+0.01}_{-0.01}+0.01$ | $0.46^{+0.01}_{-0.01}+0.01$ | $0.46^{+0.01}_{-0.01}+0.01$ | $0.46^{+0.01}_{-0.01}+0.01$ |
| $T_3$                    | $0.33^{+0.07}_{-0.07}+0.07$ | $0.33^{+0.07}_{-0.07}+0.07$ | $0.33^{+0.07}_{-0.07}+0.07$ | $0.33^{+0.07}_{-0.07}+0.07$ |
| $B \rightarrow h_1(1270)$ | $B \rightarrow h_1(1380)$ | $B \rightarrow h_1(1170)$ | $B \rightarrow h_1(1380)$ | $B \rightarrow h_1(1420)$ |
| $A$                      | $0.13^{+0.02}_{-0.02}+0.02$ | $0.13^{+0.02}_{-0.02}+0.02$ | $0.13^{+0.02}_{-0.02}+0.02$ | $0.13^{+0.02}_{-0.02}+0.02$ |
| $B$                      | $0.07^{+0.08}_{-0.08}+0.08$ | $0.07^{+0.08}_{-0.08}+0.08$ | $0.07^{+0.08}_{-0.08}+0.08$ | $0.07^{+0.08}_{-0.08}+0.08$ |
| $V_0$                    | $0.30^{+0.07}_{-0.07}+0.07$ | $0.30^{+0.07}_{-0.07}+0.07$ | $0.30^{+0.07}_{-0.07}+0.07$ | $0.30^{+0.07}_{-0.07}+0.07$ |
| $V_1$                    | $0.23^{+0.01}_{-0.01}+0.01$ | $0.23^{+0.01}_{-0.01}+0.01$ | $0.23^{+0.01}_{-0.01}+0.01$ | $0.23^{+0.01}_{-0.01}+0.01$ |
| $V_2$                    | $0.26^{+0.01}_{-0.01}+0.01$ | $0.26^{+0.01}_{-0.01}+0.01$ | $0.26^{+0.01}_{-0.01}+0.01$ | $0.26^{+0.01}_{-0.01}+0.01$ |
| $T_1(T_2)$               | $0.15^{+0.04}_{-0.04}+0.04$ | $0.15^{+0.04}_{-0.04}+0.04$ | $0.15^{+0.04}_{-0.04}+0.04$ | $0.15^{+0.04}_{-0.04}+0.04$ |
| $T_3$                    | $0.13^{+0.03}_{-0.03}+0.03$ | $0.13^{+0.03}_{-0.03}+0.03$ | $0.13^{+0.03}_{-0.03}+0.03$ | $0.13^{+0.03}_{-0.03}+0.03$ |
| $B \rightarrow f_i(1285)$ | $B \rightarrow f_i(1420)$ | $B \rightarrow f_i(1285)$ | $B \rightarrow f_i(1420)$ | $B \rightarrow f_i(1420)$ |
| $A$                      | $0.15^{+0.04}_{-0.04}+0.04$ | $0.15^{+0.04}_{-0.04}+0.04$ | $0.15^{+0.04}_{-0.04}+0.04$ | $0.15^{+0.04}_{-0.04}+0.04$ |
| $B$                      | $0.04^{+0.09}_{-0.09}+0.09$ | $0.04^{+0.09}_{-0.09}+0.09$ | $0.04^{+0.09}_{-0.09}+0.09$ | $0.04^{+0.09}_{-0.09}+0.09$ |
| $V_0$                    | $0.20^{+0.05}_{-0.05}+0.05$ | $0.20^{+0.05}_{-0.05}+0.05$ | $0.20^{+0.05}_{-0.05}+0.05$ | $0.20^{+0.05}_{-0.05}+0.05$ |
| $V_1$                    | $0.33^{+0.07}_{-0.07}+0.07$ | $0.33^{+0.07}_{-0.07}+0.07$ | $0.33^{+0.07}_{-0.07}+0.07$ | $0.33^{+0.07}_{-0.07}+0.07$ |
| $V_2$                    | $0.32^{+0.06}_{-0.06}+0.06$ | $0.32^{+0.06}_{-0.06}+0.06$ | $0.32^{+0.06}_{-0.06}+0.06$ | $0.32^{+0.06}_{-0.06}+0.06$ |
| $T_1(T_2)$               | $0.26^{+0.05}_{-0.05}+0.05$ | $0.26^{+0.05}_{-0.05}+0.05$ | $0.26^{+0.05}_{-0.05}+0.05$ | $0.26^{+0.05}_{-0.05}+0.05$ |
| $T_3$                    | $0.23^{+0.05}_{-0.05}+0.05$ | $0.23^{+0.05}_{-0.05}+0.05$ | $0.23^{+0.05}_{-0.05}+0.05$ | $0.23^{+0.05}_{-0.05}+0.05$ |
Ref. \[9\]

\[\theta_K = \pm 45^\circ.\] (52)

Besides, the flavor-octet and the flavor-singlet also mix with each other:

\[|f_1(1285)| = |f_1|\cos \theta_1 P_1 + |f_8|\sin \theta_1 P_1, \quad |f_1(1420)| = -|f_1|\sin \theta_1 P_1 + |f_8|\cos \theta_1 P_1,\] (53)

\[|h_1(1170)| = |h_1|\cos \theta_1 P_1 + |h_8|\sin \theta_1 P_1, \quad |h_1(1380)| = -|h_1|\sin \theta_1 P_1 + |h_8|\cos \theta_1 P_1.\] (54)

The reference points are chosen as: \(\theta_3 P_1 = 38^\circ\) or \(\theta_3 P_2 = 50^\circ; \theta_1 P_1 = 10^\circ\) or \(\theta_1 P_1 = 45^\circ\). These reference points are very close to the ideal mixing angle \(\theta_1 P_1 = 35.3^\circ\). We should point out that if the mixing is ideal: \(f_1(1285)\) is made up of \(\bar{u}u + \bar{d}d\) while \(f_1(1420)\) is composed of \(\bar{s}s\). As a result, some of the form factors are very small, which leads to small production rates of this meson.

In Table \[V\] and \[VII\] results of the form factors at \(q^2 = 0\) for \(B_{u,d,s} \rightarrow a_1, f_1, f_8, K_{1A}, b_1, h_1, h_8\) and \(K_{1B}\) transitions are listed, together with the parameters \(a, b\), which are obtained within the dipole parametrization. The form factors for the \(B_{(s)}\) to physical states transitions are collected in Table \[VIII\]. In our calculation, minus values for decays constants of \(1 P_1\) mesons \(^1\) have been used. The errors in the results are from: decay constants of \(B_{(s)}\) mesons and shape parameters \(\omega_i; \Lambda_{QCD}((0.25 \pm 0.05)\text{GeV})\) and the scales \(t_s; \) Gegenbauer moments of axial-vectors’ LCDAs. As the quark contents (to be more precise, the mixing angles) of the axial-vectors \(K_1(f_1, h_1)\) have not been uniquely determined, we give two sets of results for form factors as in Ref. \[10\]: in Table \[VIII\] the results in the first line are obtained using \(\theta_K = 45^\circ, \theta_1 P_1 = 10^\circ\) and \(\theta_3 P_1 = 38^\circ\) while the second line using \(\theta_K = -45^\circ, \theta_1 P_1 = 45^\circ\) and \(\theta_3 P_1 = 50^\circ\).

A number of remarks on \(B \rightarrow A\) form factors are given in order.

1. The parameters \(a\) in most form factors are around \(1.7, \) but these parameters in \(V_1(q^2)\) and \(T_2(q^2)\) are around \(0.7). The situation is similar for the parameter \(b\). In most form factors, this parameter is close to \(0.7,\) while in \(V_1(q^2)\) and \(T_2(q^2)\) it’s close to \(-0.14).\n
\(^1\) Decay constants given in QCD sum rules \(\[8, 9\] are both positive for two kinds of axial-vectors, which will give negative values for \(B \rightarrow 1 P_1\) form factors. For non-strange \(1 P_1\) mesons, this minus sign will not give any physical differences as it can not be observed experimentally. But we should point out that the minus sign will affect the mixing between \(K_{1A}\) and \(K_{1B}\) by changing the mixing angle \(\theta\) to \(-\theta\).
2. Some of the form factors for the two kinds of axial-vector mesons are very different. As an example, we will give a comparison of the $B \rightarrow \rho$, $B \rightarrow a_1(1260)$ and $B \rightarrow b_1(1235)$ form factors. Form factors $V_0$, $V_1$, $T_1$ for $B \rightarrow A$ transition are larger than the corresponding $B \rightarrow V$ ones. It seems that the form factor $A^{B \rightarrow (a_1,b_1)}$ is somewhat equal to or even smaller than $V^{B \rightarrow \rho}$. But actually that is artificial: as in Eq. \((36)\), the pre-factor of $V_1(q^2)$ is $m_B + m_V$ while for $B \rightarrow A$ form factor $A_1(q^2)$, the factor becomes $m_B - m_A$. We take $A_0$ and $T_1$ as an example to explain the reason for the large $B \rightarrow A$ form factors. In Table \(\text{XXX}\) we give contributions from three kinds of LCDAs to $T_1$: $\phi^T$, $\phi^v$ and $\phi^a$. The contribution from $\phi^T$ is larger for $B \rightarrow a_1$, than the other two transitions only because the axial-vector $a_1$ decay constant is larger. Furthermore, larger axial vector meson mass induces larger contributions from twist-3 distribution amplitudes $\phi^v$, $\phi^a$ for both of $T_1^{B \rightarrow b_1}$ and $T_1^{B \rightarrow a_1}$.

3. Some $B \rightarrow A$ form factors strongly depend on mixing angles, which is obvious in Table \(\text{VIII}\). In our calculation for form factors involving $f_1$ mesons, we have used the mixing angle between the octet and singlet: $\theta = 38^\circ(50^\circ)$ which is very close to the ideal mixing angle $\theta = 35.3^\circ$. That implies the lighter meson $f_1(1285)$ is almost made up of $\frac{su+dd}{\sqrt{2}}$ while the heavier meson $f_1(1420)$ is dominated by the $\bar{s}s$ component. Thus $B \rightarrow f_1(1420)$ and $B_{a1} \rightarrow f_1(1285)$ form factors are suppressed by the flavor structure and are numerically small. The form factors involving $h_1$ are similar if the mixing angle is taken as $45^\circ$.

4. The SU(3) symmetry breaking effect between $B \rightarrow a_1$ and $B \rightarrow K_{1A}$ transition form factors is less than 10%. It is also similar for the $B \rightarrow 1P_1$ transition form factors.

5. In Table \(\text{V}\) we can see that the form factor $A^{B \rightarrow K_{1A}}$ is almost equal to $A^{B \rightarrow K_{1B}}$. But the physical states $K_1(1270)$ and $K_1(1400)$ are mixtures of $B \rightarrow K_{1A,1B}$. With the mixing angle $\theta_K = \pm 45^\circ$, the $B_{d,s} \rightarrow K_1(1270)(K_1(1400))$ form factors are either enhanced by a factor $\sqrt{2}$ or highly suppressed.

Up to now, there are many studies using some non-perturbative methods on the $B \rightarrow A$ form factors: the constituent quark-meson (CQM) model \[27\], ISGW \[28, 29\], QCD sum rules (QCDSR) and light-cone sum rules (LCSR) \[31, 32, 33, 34\] and light-front quark model (LFQM) \[23, 35\]. Results in LFQM and LCSR are collected in Table \(\text{VII}\) to make a comparison. These two approaches are very different with the PQCD in the treatment of dynamics of transition form factors, but at first we will analyze the differences caused by non-perturbative inputs. For $B \rightarrow a_1$ and $B \rightarrow K_{1A}$ form factors, most of our results (except $V_0$ and $T_{1,2}$) are slightly larger than (or almost equal to) those evaluated in LFQM, as slightly larger decay constants for $a_1$ and $K_{1A}$ are used. The $f_{K_{1A}}$ and $f_{K_{1B}}$ are $203$ MeV and $186$ MeV are used by LFQM). Small differences in $V_0$ and $V_1$ have induced a large difference in $V_2$, which could be reduced in future studies using more precise hadronic inputs. As the decay constant of $b_1$ is zero in the isospin limit, the shape parameter $\omega$ in LFQM can not be directly determined and the same value as that of $a_1$ is used. It is also similar to $K_{1B}$: they used the same shape parameter as that of $K_{1A}$ which predicts $f_{K_{1B}} = 11$ MeV. Comparing with the QCDSR results $f_{K_{1B}} = f_{K_{1B}}^\ast a_0^\ast$ given in Table \(\text{II}\) and \(\text{III}\) we can see: although they are consistent within large theoretical errors, the central value of $f_{K_{1B}}$ in QCDSR is larger than the prediction in the LFQM. Thus our predictions for $B \rightarrow 1P_1$ form factors (central values) are larger than those in LFQM. Compared with the recent LCSR results \[33\] collected in Table \(\text{VIII}\) our results differ from theirs in two points: one difference is that positive decay constants for $1^P_1$ mesons are adopted in LCSR, which leads to the minus sign of the form factors for $1^P_1$ mesons; the other difference is that form factor $A(0)$ in LCSR is larger than that in the PQCD approach.

Experimentally, the branching ratios of the color allowed tree-dominated processes $B^0 \rightarrow a_1^\pm \pi^\mp$ and $B^0 \rightarrow b_1^\pm \pi^\mp$ have been measured by the BaBar and Belle collaborations \[36, 37, 38\] and averaged by the heavy flavor averaging group \[2\]. These two channels can be used to extract the $B \rightarrow a_1$ and $B \rightarrow b_1$ form factors.\[39\]:

\[V_0^{B \rightarrow a_1} = (1.54 \pm 0.28 \pm 0.03) f_+^{B \rightarrow \pi} = 0.38 \pm 0.07 \pm 0.01, \]
\[V_0^{B \rightarrow b_1} = (1.45 \pm 0.36 \pm 0.03) f_+^{B \rightarrow \pi} = 0.35 \pm 0.03 \pm 0.01, \]
where the penguin contributions are neglected for the small Wilson coefficients. As we can see, \( V_0^{B_{d,s}} \) is consistent with our predictions within the errors, however \( V_0^{B_{d,s}} \) is smaller than our predictions.

### D. Semileptonic \( B \to A l \bar{\nu}_l \) decays

After integrating out the off shell \( W \) boson, one obtains the effective Hamiltonian for \( b \to ul \bar{\nu}_l \) transition

\[
\mathcal{H}_{eff}(b \to ul \bar{\nu}_l) = \frac{G_F}{\sqrt{2}} V_{ub} \bar{u} \gamma_{\mu} (1 - \gamma_5) b \bar{l} \gamma^\mu (1 - \gamma_5) \nu_l, \tag{57}
\]

where \( V_{ub} \) is the CKM matrix element. With the form factors at hand, the \( B \to A l \bar{\nu}_l \) decay widths are derived as:

\[
\frac{d\Gamma_L(B \to A l \bar{\nu}_l)}{dq^2} = \left( \frac{q^2 - m_l^2}{q^2} \right)^2 \frac{\sqrt{\lambda(m_B^2, m_A^2, q^2)} G_F^2 V_{ub}^2}{384 m_B^2 \pi^3} \times \frac{1}{q^2} \left\{ \frac{3m_l^2 \lambda(m_B^2, m_A^2, q^2) V_0^2(q^2) + }{m_B - m_A} \right\}^2, \quad (58)
\]

\[
\frac{d\Gamma_T(B \to A l \bar{\nu}_l)}{dq^2} = \left( \frac{q^2 - m_l^2}{q^2} \right)^2 \frac{\sqrt{\lambda(m_B^2, m_A^2, q^2)} G_F^2 V_{ub}^2}{384 m_B^2 \pi^3} \times \left\{ \frac{(m_l^2 + 2q^2) \lambda(m_B^2, m_A^2, q^2) A(q^2)}{m_B - m_A} + \frac{(m_B - m_A) V_1(q^2)}{\sqrt{\lambda(m_B^2, m_A^2, q^2)}} \right\}^2, \quad (59)
\]

where \( \lambda(m_B^2, m_A^2, q^2) = (m_B^2 + m_A^2 - q^2)^2 - 4m_B^2 m_A^2 \). \( L \) and \( T \) in the subscripts denote contributions from the longitudinal polarization and the two transverse polarizations, respectively. \( m_l \) represents the mass of the charged lepton, and \( q^2 \) is the momentum square of the lepton pair. Integrating over the \( q^2 \), one obtains the longitudinal and transverse decay width of \( B \to A l \bar{\nu}_l \) decays.

For the semileptonic \( B \to A l \bar{\nu}_l \) decays, physical quantities \( \text{Br}_L, \text{Br}_+, \text{Br}_-, \text{Br}_{\text{total}} \) and \( \text{Br}_L/\text{Br}_T = \Gamma_L/\Gamma_T \) are predicted, where \( \text{Br}_L = \text{Br}_+ + \text{Br}_- \) and \( \text{Br}_{\text{total}} = \text{Br}_L + \text{Br}_T \) with \( \text{Br}_L, \text{Br}_+ \) and \( \text{Br}_- \) corresponding to contributions of different polarizations to branching ratios. Results for the \( B \to A l \bar{\nu}_l \) \((l = e, \mu) \) and \( B \to A \tau \bar{\nu}_\tau \) decays are listed in Table [X] and [XI] respectively, with masses of the electron and muon neglected in the calculation. There are some remarks:

- Most of the branching ratios are of the order \( 10^{-4} \). Some of the branching ratios are sensitive to the mixing angles, especially for \( K_1(1270) \) and \( K_1(1400) \): one mixing angle gives constructive contributions and the other gives destructive contributions. Branching ratios for these two mixing angles are two-order different, just as mentioned in the discussions about the form factors. For the decays \( B_s \to K_1(1270) \) and \( B_s \to K_1(1400) \), ratios of the contributions from longitudinal polarization and transverse polarization are also much different with each other.

- The branching ratios of \( B \to A \tau \bar{\nu}_\tau \) decays are smaller than those of corresponding \( B \to A e \bar{\nu}_e \) decays, because the heavy \( \tau \) lepton brings a smaller phase space than the light electron.

- Except for the \( B_s \to K_1 l \bar{\nu}_l(l = e, \tau) \) decay channels, the ratios(\( \text{Br}_L/\text{Br}_T \)) in \( B \to 1^3 P_l l \bar{\nu}_l \) decays are about \( 1.0 \sim 1.2 \), while in \( B \to 1^3 P_l l \bar{\nu}_l \) decays, their values are roughly \( 2.0 \sim 2.5 \). The LCSR calculation [33] has similar ratios for \( B \to 1^3 P_l l \bar{\nu}_l \) decays. However, their ratios for \( B \to 1^1 P_l l \bar{\nu}_l \) decays are around 0.5. That means in these decays the contributions of transverse polarization are relatively larger in LCSR, which may be caused by their much larger form factor \( A \).
TABLE X: The total branching ratios for the $b \to u \bar{v}_l$ ($l = e, \mu$ and unit $10^{-4}$ for branching ratios). $Br_{L}$ and $Br_{+}$ are the longitudinally and transversely polarized contributions to the branching ratios. And $Br_T = Br_{+} + Br_{-}$. For the decays with a mixing meson, results in the first lines are calculated using $\theta_K = 45^\circ$, $\theta_{1P} = 10^\circ$ or $\theta_{3P} = 38^\circ$, while the second line corresponds to the angle $\theta_K = -45^\circ$, $\theta_{1P} = 45^\circ$ or $\theta_{3P} = 50^\circ$.

| & $Br_L$ & $Br_+$ & $Br_-$ & $Br_{Total}$ & $Br_L/Br_T$ |
|---|---|---|---|---|---|
| $B^0 \to a_1^+$ & $1.60^{+1.03}_{-0.82}$ & $0.04^{+0.02}_{-0.02}$ & $1.31^{+0.71}_{-0.56}$ & $2.96^{+1.74}_{-1.30}$ & $1.18^{+0.20}_{-0.22}$ |
| $B^0 \to b_1^+$ & $2.10^{+1.07}_{-0.86}$ & $0.03^{+0.02}_{-0.02}$ & $0.76^{+0.44}_{-0.36}$ & $2.88^{+1.54}_{-1.22}$ & $2.67^{+0.35}_{-0.35}$ |
| $B^- \to f_1^0(1285)$ & $0.93^{+0.66}_{-0.48}$ & $0.03^{+0.01}_{-0.01}$ & $0.73^{+0.39}_{-0.31}$ & $1.69^{+0.99}_{-0.79}$ & $1.22^{+0.21}_{-0.23}$ |
| $B^- \to f_1^0(1420)$ & $0.04^{+0.04}_{-0.03}$ & $0.01^{+0.02}_{-0.02}$ & $0.09^{+0.06}_{-0.05}$ & $1.25^{+0.27}_{-0.31}$ & $< 10^{-3}$ |
| $B^- \to h_1^0(1170)$ & $1.08^{+0.55}_{-0.44}$ & $0.02^{+0.01}_{-0.01}$ & $0.43^{+0.25}_{-0.20}$ & $1.53^{+0.80}_{-0.65}$ & $2.41^{+0.37}_{-0.31}$ |
| $B^- \to h_1^0(1380)$ & $1.38^{+0.70}_{-0.56}$ & $0.02^{+0.01}_{-0.01}$ & $0.54^{+0.31}_{-0.26}$ & $1.94^{+0.82}_{-0.68}$ & $2.42^{+0.32}_{-0.32}$ |
| $B_+ \to K_1^{*+}(1270)$ & $3.65^{+2.27}_{-1.87}$ & $0.08^{+0.05}_{-0.04}$ & $2.01^{+1.21}_{-1.00}$ & $5.75^{+3.52}_{-2.88}$ & $1.74^{+0.50}_{-0.30}$ |
| $B_+ \to K_1^{+}(1400)$ & $0.01^{+0.01}_{-0.00}$ & $0.01^{+0.00}_{-0.00}$ & $0.02^{+0.01}_{-0.00}$ & $0.02^{+0.01}_{-0.00}$ & $0.45^{+0.32}_{-0.30}$ |

TABLE XI: The same as Table X except $b \to u \tau \bar{v}_\tau$.

| & $Br_L$ & $Br_+$ & $Br_-$ & $Br_{Total}$ & $Br_L/Br_T$ |
|---|---|---|---|---|---|
| $B^0_+ \to a_1^+$ & $0.69^{+0.44}_{-0.36}$ & $0.03^{+0.01}_{-0.01}$ & $0.62^{+0.33}_{-0.27}$ & $1.34^{+0.78}_{-0.63}$ & $1.06^{+0.18}_{-0.20}$ |
| $B^0_+ \to b_1^+$ & $0.88^{+0.45}_{-0.36}$ & $0.02^{+0.01}_{-0.01}$ & $0.36^{+0.21}_{-0.17}$ & $1.26^{+0.66}_{-0.54}$ & $2.32^{+0.38}_{-0.39}$ |
| $B^-_+ \to f_1^0(1285)$ & $0.39^{+0.25}_{-0.20}$ & $0.02^{+0.01}_{-0.01}$ & $0.34^{+0.18}_{-0.14}$ & $0.74^{+0.43}_{-0.35}$ & $1.08^{+0.19}_{-0.21}$ |
| $B^-_+ \to f_1^0(1420)$ & $0.36^{+0.23}_{-0.19}$ & $0.02^{+0.01}_{-0.01}$ & $0.32^{+0.17}_{-0.13}$ & $0.70^{+0.40}_{-0.32}$ & $1.09^{+0.19}_{-0.21}$ |
| $B^-_+ \to h_1^0(1170)$ & $0.49^{+0.24}_{-0.20}$ & $0.01^{+0.01}_{-0.01}$ & $0.21^{+0.12}_{-0.10}$ & $0.70^{+0.37}_{-0.30}$ & $2.12^{+0.33}_{-0.27}$ |
| $B^-_+ \to h_1^0(1380)$ & $0.60^{+0.31}_{-0.25}$ & $0.02^{+0.01}_{-0.01}$ & $0.27^{+0.15}_{-0.13}$ & $0.89^{+0.42}_{-0.38}$ & $2.13^{+0.32}_{-0.27}$ |
| $B_+ \to K_1^{*+}(1270)$ & $3.69^{+2.27}_{-1.87}$ & $0.08^{+0.05}_{-0.04}$ & $2.01^{+1.21}_{-1.00}$ & $5.75^{+3.52}_{-2.88}$ & $1.74^{+0.50}_{-0.30}$ |
| $B_+ \to K_1^{+}(1400)$ & $0.01^{+0.01}_{-0.00}$ & $0.01^{+0.00}_{-0.00}$ & $0.02^{+0.01}_{-0.00}$ & $0.02^{+0.01}_{-0.00}$ & $0.45^{+0.32}_{-0.30}$ |

E. More Discussions on the Mixing between $K_{1A}$ and $K_{1B}$

As pointed out, the $B \to K_1(1270)$ and $B \to K_1(1400)$ form factors have either quite large or quite small values, for the mixing angles are $\pm 45^\circ$. Actually, these two values are just chosen for illustration, as the determination in $\tau$ decays are not stringent. There are some attempts to determine the mixing angles between the two $K_1$ mesons...
in $B$ meson decays. For example, the authors in Ref. [23] found that the mixing angle between $K_{1A}$ and $K_{1B}$ is two-fold: $\theta = 38^\circ$ or $\theta = 50^\circ$. However, their determination depends on the LFQM predictions on the $B \to A$ form factors, which is model-dependent. To reduce the uncertainties caused by the dynamics of strong interactions, we propose to use the $\bar{B}^0 \to D^+ K^-_1$ decay to extract the mixing angle between these two mesons. The dynamics of this charm decay is very similar to that of $\bar{B}^0 \to D^+ \pi^-$. Neglecting the higher power corrections, the decay amplitudes of $\bar{B}^0 \to D^+ M^-$ ($M^-$ denotes $\pi^-$ or $K^-$) can be factorized into the $B \to D$ form factor and a convolution of a hard kernel with the light-cone distribution amplitude of the emitted light meson $[40]$. To the leading order in $\alpha_s$, the convolution reduces to the decay constant of the emitted light meson. Then the factorization formula is proved to have the form:

$$A(\bar{B}^0 \to D^+ M^-) = \frac{G_F}{\sqrt{2}} m_B f_M F_0^{B\to D}(1 - r^2) V_{ud}^* V_{us} a_1, \quad (60)$$

where $r = m_D/m_B$. Due to the small value for the longitudinal decay constant $f_{K_{1B}}$, the decay amplitude of $\bar{B}^0 \to D^+ K_{1B}$ is very small. Thus the physical decay channels receive the leading contributions from $\bar{B}^0 \to D^+ K_{1A}$. Utilizing the $B \to D$ form factors which are well explored in the heavy quark effective theory, we can directly present our predictions on $\bar{B}^0 \to D^+ K_1$ decays, if the mixing angle is known. On the other hand, one can also obtain the mixing angle, if the experimental data on the branching ratio is provided. In practice, in order to reduce the uncertainty from the nonperturbative inputs, one can use the experimental data of the branching fraction of $\bar{B}^0 \to D^+ \pi^-$ instead of any theoretical model. The ratios of branching fractions are given as

$$R \equiv \frac{BR(\bar{B}^0 \to D^+ K^-_1)}{BR(\bar{B}^0 \to D^+ \pi^-)} = \frac{f_{K_1}^2 |V_{us}|^2}{f_{\pi}^2 |V_{ud}|^2}, \quad (61)$$

where $f_{K_1}$ is the decay constant for a physical state.

In the ratios for the two channels $\bar{B} \to D K_1(1270)$ and $\bar{B} \to D K_1(1400)$, the main uncertainties come from the decay constants of $K_1(1270)$ and $K_1(1400)$ which are combinations of the two decay constants $f_{K_{1A}}$ and $f_{K_{1B}}$. From the table II and III we can see the parameter $a_0^{||K_{1B}}$ has the largest uncertainty. In Fig[3] we plot the dependence on the mixing angle of the branching ratios utilizing the decay constants evaluated in the sum rules and we also take the uncertainty of $a_0^{||K_{1B}}$ into account for the error estimation: the left diagram (a) denotes the ratio $\frac{BR(\bar{B}^0 \to D^+ K^-_1(1270))}{BR(\bar{B}^0 \to D^+ \pi^-)}$, while the right diagram (b) denotes the ratio the ratio $\frac{BR(\bar{B}^0 \to D^+ K^-_1(1400))}{BR(\bar{B}^0 \to D^+ \pi^-)}$. The uncertainties caused by the Gegenbauer moment $a_0^{||K_{1B}}$ are shown in these diagrams: the red solid line denotes the central value, while the blue short-dashed (green long-dashed) line denotes the lower (upper) uncertainty. Once the experimental
data are available in the future, these two diagrams can be used to extract the mixing angles in model-independent way. As an illustration, we will give our predictions utilizing the decay constants extracted from the $\tau$ decays. The branching ratio of $\bar{B}^0 \rightarrow D^+\pi^-$ has been averaged as \[148\]:

\[
BR(\bar{B}^0 \rightarrow D^+\pi^-) = (2.65 \pm 0.15) \times 10^{-3},
\]

which gives the following predictions on the branching fractions:

\[
BR(\bar{B}^0 \rightarrow D^+ K^-_{1270}) = (2.1 \pm 0.5) \times 10^{-4},
\]

\[
BR(\bar{B}^0 \rightarrow D^+ K^-_{1400}) = (1.2^{+1.8}_{-1.2}) \times 10^{-4}.
\]

These results will be certainly tested on the future experiments and the measurements are very helpful to detect the internal structure of $\bar{K}_1(1270)$ and $\bar{K}_1(1400)$.

IV. SUMMARY

The PQCD approach is based on $k_T$ factorization where we keep the transverse momentum of valence quarks in the mesons to smear the endpoint singularity. $k_T$ resummation of double logarithms results in the Sudakov factor. Resummation of double logarithms from the threshold region leads to the jet function. Sudakov factor and jet function can suppress the contribution from the large $b$ region and small $x$ region, respectively. This makes the PQCD approach self-consistent. Inspired by the success of the PQCD approach in non-leptonic B decays \[71\], we give a comprehensive study on the charmless $B \rightarrow A$ transition form factors and the semileptonic $B \rightarrow A l\bar{\nu}$ decays in the PQCD approach. Semi-leptonic and radiative decays are somewhat simpler than non-leptonic decays as only one hadronic meson involved in the final state. In this case, the dominant amplitude can be parameterized into form factors. In order to make precise prediction and extract the CKM matrix elements, we have to know the behavior of form factors. In the PQCD approach, the final state meson moves nearly on the light-cone and a hard-gluon-exchange is required. Thus the dominant contribution is from the hard region which can be factorized. In section III, we have used the same input hadronic parameters with Ref. \[18\] and updated all the $B \rightarrow V$ decay form factors in the PQCD approach. Compared with the results evaluated from other approaches, we find that despite a number of theoretical differences in different approaches, all the numerical results of the form factors are surprisingly consistent with each other.

In section III we study $B \rightarrow A$ form factors. As the quark contents for the axial-vectors have not been uniquely determined, we give two different sets of results for the form factors according to different mixing angles. For the axial-vector mesons $f_1$, we have used the mixing angle between the octet and singlet: $\theta = 38^\circ(50^\circ)$ which is close to the ideal mixing angle $\theta = 35.3^\circ$. With this mixing angle, one can easily check that the lighter meson $f_1(1285)$ is made up of $\frac{\bar{u}u + \bar{d}d}{\sqrt{2}}$ while the heavier meson $f_1(1420)$ is composed of $\bar{s}s$. Thus partial decay widths of $B \rightarrow f_1(1420)l\bar{\nu}$ and $B_s \rightarrow f_1(1285)l\bar{\nu}$ are suppressed by the flavor structure.

The mixing angle between the two strange mesons $K_1(1270)$ and $K_1(1400)$ has large ambiguities. In order to reduce these ambiguities, we propose to use the $\bar{B}^0 \rightarrow D^+K^-_1$ decay to extract the mixing angle between these two mesons. Our method is model-independent which receives very small uncertainties. In Fig. B we show the strong dependence of the $\bar{B}^0 \rightarrow D^+K_1$ decay branching ratio on the mixing angle $\theta_K$. Our calculation can be used to constrain this mixing angle using experimental measurements. These studies of higher resonance production in $B$ decays can help us to uncover the mysterious structure of these excited states.

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in which

where the variables are defined by

with

where

the specific processes. To simplify the analysis, the following parametrization has been used [21]:

where \( H_0^{(1)}(z) = J_0(z) + iY_0(z) \).

The Sudakov factor from threshold resummation is universal, independent of flavors of internal quarks, twists, and the specific processes. To simplify the analysis, the following parametrization has been used [21]:

with \( c = 0.4 \pm 0.1 \). This parametrization, symmetric under the interchange of \( x \) and \( 1 - x \), is convenient for evaluation of the amplitudes. It is obvious that the threshold resummation modifies the end-point behavior of the meson distribution amplitudes, rendering them vanish at \( x \to 0 \) or 1.

Function \( S_{ab}(t) \) in Sudakov factors is given by

in which \( S_B(t) \) and \( S_2(t) \) are defined as

with the quark anomalous dimension \( \gamma_q = -\alpha_s/\pi \). The explicit form for the function \( s(Q, b) \) is:

where the variables are defined by

\[ \hat{q} \equiv \ln[Q/(\sqrt{2}\Lambda)], \quad \hat{b} \equiv \ln[1/(b\Lambda)], \]
and the coefficients $A^{(i)}$ and $\beta_i$ are

\[ \beta_1 = \frac{33 - 2n_f}{12}, \quad \beta_2 = \frac{153 - 19n_f}{24}, \]

\[ A^{(1)} = \frac{4}{3}, \quad A^{(2)} = -\frac{67}{9} - \frac{\pi^2}{3} - \frac{10}{27}n_f + \frac{8}{3}\beta_1 \ln \left( \frac{1}{2} e^{\gamma_E} \right), \]

(A9)

$n_f$ is the number of the quark flavors and $\gamma_E$ is the Euler constant. We will use the one-loop running coupling constant, i.e. we pick up only the four terms in the first line of the expression for the function $s(Q, b)$.

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