Unified model of Sepic converter in continuous and discontinuous conduction modes

Yongxiang Shang\textsuperscript{1} and Zian Tang
School of Automation and Electrical Engineering, Lanzhou Jiao Tong University, Lanzhou, Gansu, China
\textsuperscript{1}E-mail: 97552589@qq.com

Abstract. Aiming at the problems of the modelling methods for Sepic converter, they were described as in inexact, large amount of calculation and complicated expressions. A suitable hybrid switching model for both continuous conduction mode (CCM) and discontinuous conduction mode (DCM) is proposed. Firstly, according to the working process of the Sepic converter, the switching conditions of Sepic converter between different working conditions are formulated. Secondly, by introducing the logic proposition and the equivalent translation principle, the switching conditions of the converter in different modes can be transformed into mixed logical inequalities, moreover then, a unified model of Sepic converter in CCM and DCM modes is obtained. Finally, the simulation experiment of Sepic converter operating in CCM, BCM (boundary conduction mode) and DCM modes were conducted, and compared with the state space averaging method, the validity of effectiveness of the proposed approach was shown.

1. Introduction
The model of Sepic converter has a major significance for researchers working in control strategy analysis of Sepic converters. It has been well known that Sepic converters have hybrid features that exhibit both continuous and discrete dynamic behavior. Due to the discrete switching, the system structure of the converter is constantly changing. Different models are needed for its performance analysis. Therefore, it is significance to establish a general and unified model for Sepic converters \cite{1}.

At present, the main modeling methods of DC-DC converters are state space averaging method (SSA) \cite{2}, average switching method \cite{3}, and improved average modeling method \cite{4}. Though averaging techniques were used to make the characteristics of circuit more legible, in the derivation of the extended averaging model, the averaging state variables, instead of the transient-state variables, are used over an entire switching cycle or local switching subinterval. Consequently, the averaging model is an approximate model. In \cite{5}, the model of the high-frequency DC-DC converter was analyzed. However, these models are represented as the equivalent configuration of the practical circuit, while it fails to provide the general state equations. In \cite{6}, the network representation of average electrothermal model of the diode–transistor switch was presented. The model can be used for the electrothermal analysis of the converters in both CCM and DCM. However, the model represented as a unified states pace equation is helpful to analyze the converter.

The hybrid modeling methods have been used to build the accurate model of the DC–DC converters due to its powerful abilities \cite{7-9}. In \cite{7}, the Boost converters were represented as hybrid automatons which could operate in both CCM and DCM. But the control algorithm obtained by the
approximate state variables was difficult to apply to other converters. The mixed logical dynamical (MLD) model generalized a wide set of systems, including the linear hybrid systems and nonlinear dynamical systems. However, the main disadvantage in the MLD modeling is that the dynamical constraints make the model incredibly complicated to analyze and calculate [8]. In [9] the switching model of DC-DC converter was established, but it is only suitable for CCM mode.

In this paper, a hybrid switching modeling method, which can be applied to Sepic converters were presented. Firstly, on the basis of retaining all dynamic characteristics of Sepic converters, including transient and steady-state processes, the proposed modeling method can describe the operation of Sepic converters in CCM and DCM as a unified hybrid switching model. Second, the proposed modeling method adopts inequality to describe the switching constraints between different modes of Sepic converter, and the unified expression of the proposed model is associated with working mode which makes the conciseness of the proposed model. Third, the proposed modeling method does not have any hypotheses, which ensure the accuracy of the proposed model. The simulation results verify the validity of the proposed model.

2. Switching conditions of Sepic converters between different modes

The topologies of Sepic converters with ideal components are depicted in Figure 1. The PWM control scheme is shown in Figure 2.

![Figure 1. Sepic converter.](image1)

![Figure 2. Control scheme based on modulation signal and carrier signal.](image2)

The specific working process of Sepic can be divided into two operations, three modes and five working states, according to the switching on and off of switch $S$ and diode $D$, combined with the size of inductance current $i_{L1}$ and $i_{L2}$. The details are shown in Table 1.

| Operation | Modes   | States  | Switch, Currents | Inductor Currents |
|-----------|---------|---------|-----------------|-------------------|
| CCM       | Mode1   | State1  | ON, OFF         | $i_{L1}=0$, $i_{L2}=0$ |
|           | Mode2   | State2  | OFF, ON         | $i_{L1}=0$, $i_{L2}=0$ |
|           |         | State3  | OFF, ON         | $i_{L1}=0$, $i_{L2}=0$ |
| DCM       | Mode3   | State4  | OFF, OFF        | $i_{L1}=0$, $i_{L2}=0$ |
|           |         | State5  | OFF, OFF        | $i_{L1}, i_{L2}=0$ |
If $v_{\text{con}}$ is the modulation signal of the switch $S$, $v_{\text{ramp}}$ is the carrier signal. According to the control mode of switch $S$, it can be seen that when $v_{\text{con}} - v_{\text{ramp}} \geq 0$, $S$ is ON and $D$ is OFF; when $v_{\text{con}} - v_{\text{ramp}} < 0$ $S$ is OFF and $D$ is ON. The switching conditions of Sepic converters in different working conditions are shown in Figure 3.

Simplified the switching conditions between different working states, in order to simplify the modeling of Sepic converter. Since the state equation of the converter is the same in the same mode, if $i_L = i_{L1} + i_{L2}$, by added self-constraints of each mode, the switching conditions of the Sepic converter between different modes can be expressed as shown in Figure 4.

The general state equation of converters operating in mode $k(t)(k(t)=1, 2, 3)$ can be expressed as:

$$\dot{x}(t) = A_k x(t) + B_k v_{in} \quad (k = 1, 2, 3)$$

where $x(t)=[i_{L1}(t), i_{L2}(t), v_{C1}(t), v_{C2}(t)]^T$, $v_{C2}(t) = v_0(t)$ For briefness, $x(t) = [i_{L1}(t), i_{L2}(t), v_{C1}(t), v_{C2}(t)]^T$ is written as $x = [i_{L1}, i_{L2}, v_{C1}, v_{C2}]^T$ and $k(t)$ as $k$. $A_k$ and $B_k$ are summarized in Table 2.

![Figure 3. The state transition diagram of Sepic converter.](image)

![Figure 4. The switching conditions of Sepic converter between different modes.](image)

**Table 2. Coefficient matrices of Sepic.**

| Mode $k$ | $A_1$ | $B_1$ | $A_2$ | $B_2$ | $A_3$ | $B_3$ |
|---------|-------|-------|-------|-------|-------|-------|
| $k=1$   | \[0 0 1 0 \] | \[0 1 \] | \[0 0 1 0 \] | \[0 1 \] | \[0 0 1 0 \] | \[0 1 \] |
| $k=2$   | \[0 0 1 0 \] | \[0 1 \] | \[0 0 1 0 \] | \[0 1 \] | \[0 0 1 0 \] | \[0 1 \] |
| $k=3$   | \[0 0 1 0 \] | \[0 1 \] | \[0 0 1 0 \] | \[0 1 \] | \[0 0 1 0 \] | \[0 1 \] |
3. Hybrid switching modelling of Sepic converters

To obtain a unified model of Sepic converters operating in CCM and DCM, the logical variable \( \delta \in \{0, 1\} \) and continuous variable \( p(x) \) are applied to represent discrete switching variables and continuous state variables. Continuous variables are associated with logical variables and described as:

\[
\begin{align*}
[p_1(x) \geq 0] & \leftrightarrow [\delta = 1] \\
[p_2(x) < 0] & \leftrightarrow [\delta = 1]
\end{align*}
\]

(2)

For briefness, if \( \varepsilon \) is an infinitesimal positive number, then \( p(x) > 0 \) can be expressed as \( p(x) > \varepsilon \), and Equation (2) is equivalent to:

\[
\begin{align*}
p_1(x) & \geq \varepsilon \quad \text{and} \quad \delta = 1 \\
p_1(x) & < 0 \quad \text{and} \quad \delta = 0 \\
p_2(x) & < 0 \quad \text{and} \quad \delta = 1 \\
p_2(x) & \geq \varepsilon \quad \text{and} \quad \delta = 0
\end{align*}
\]

(3)

If proposition (2) is true, it can be obtained by logical rule [10]:

\[
\begin{align*}
p_1(x) & \geq -m_1(\delta - 1) \\
p_1(x) & \leq \delta (M_1 + \varepsilon) - \varepsilon \\
p_2(x) & \geq m_2 \delta \\
p_2(x) & \leq M_2 - \delta (\varepsilon + M_2)
\end{align*}
\]

(4)

where \( M \) and \( m \) are the maximum and minimum values of \( p(x) \).

Based on the principle of equivalence transformation, the switching conditions of Sepic converters between three operating modes can be correlated with logical variables.

\[
\begin{align*}
[v_{\text{con}} - v_{\text{ramp}} \geq 0] & \leftrightarrow [\delta_1 = 1] \\
[i_L(v_{\text{con}} - v_{\text{ramp}}) < 0] & \leftrightarrow [\delta_2 = 1] \\
v_{\text{con}} - v_{\text{ramp}} < 0 & \leftrightarrow [\delta_3 = 1]
\end{align*}
\]

(5) (6) (7)

According to (4), (5) (6) (7) can be written as:

\[
\begin{align*}
v_{\text{con}} - v_{\text{ramp}} & \geq -m_1(\delta_1 - 1) \\
v_{\text{con}} - v_{\text{ramp}} & \leq \delta_1 (M_1 + \varepsilon) - \varepsilon \\
i_L(v_{\text{con}} - v_{\text{ramp}}) & \geq m_2 \delta_2 \\
i_L(v_{\text{con}} - v_{\text{ramp}}) & \leq M_2 - \delta_2 (\varepsilon + M_2)
\end{align*}
\]

(8) (9)

Where \( M_1 = \max \{v_{\text{con}}, v_{\text{ramp}}\} \), \( m_1 = \min \{v_{\text{con}}, v_{\text{ramp}}\} \), \( M_2 = \max \{i_L(v_{\text{con}} - v_{\text{ramp}})\} \), \( m_2 = \min \{i_L(v_{\text{con}} - v_{\text{ramp}})\} \).

According to the actual operation of Sepic, \( M_1 > 0 \), \( m_1 < 0 \), \( M_2 > 0 \) and \( M_2 < 0 \) can be obtained. (8) (9) (10) can be expressed as:

\[
\begin{align*}
\frac{v_{\text{con}} - v_{\text{ramp}} + \varepsilon}{M_1 + \varepsilon} & \leq \delta_1 \leq 1 - \frac{v_{\text{con}} - v_{\text{ramp}}}{m_1} \\
\frac{i_L(v_{\text{con}} - v_{\text{ramp}})}{m_2} & \leq \delta_2 \leq \frac{i_L(v_{\text{con}} - v_{\text{ramp}}) - M_2}{-(\varepsilon + M_2)} \\
\frac{v_{\text{con}} - v_{\text{ramp}}}{m_1} & \leq \delta_3 \leq \frac{v_{\text{con}} - v_{\text{ramp}} - M_1}{-(\varepsilon + M_1)}
\end{align*}
\]

(11) (12) (13)

And expressed as:
When $\delta_k = 1 (k=1,2,3)$, the converters operate in mode $k$. When $\delta_1 = 0$, the converters operate in mode 2. The converter is switched to the next mode. Since $k (k=1,2,3)$ cannot be 1 at the same time, except is $\delta_1 + \delta_2 + \delta_3 = 1$. So, order:

\[ F(x,k) = f_k(x) \]
\[ G(x,k) = g_k(x) \]

Furthermore, the unified model of Sepic operating in CCM and DCM modes can be expressed as follows:

\[ F(x,k) = h_k(x) + f_k(x) \]
\[ G(x,k) = h_k(x) + g_k(x) \]

where $h_k(x) = 0.5k^2 - 2.5k + 3$ makes $F(x,k) = f_k(x)$, when $\delta = 1$. $h_k(x) = 0.5k^2 - 1.5k + 1$ makes $G(x,k) = g_k(x)$.

The calculation process of model (16) is as follows:

Step 1 Set initial variable $x$, sampling time $\Delta t$, and calculating time $T_c$. Let $k = 1$.
Step 2: Get the function values of $F(x,k)$ and $G(x,k)$.
Step 3: Calculate the value of $\delta$ from Equation (16) and get the value of $k$. Substitute it into the differential equation of the model and get the value of the output state variable $x$.
Step 4: If $\Delta t < T_c$, then go to step 2; otherwise output $x$, and the whole operation process ends.

4. Simulation results

A unified model of Sepic converter in CCM and DCM mode can be expressed by (16). To testify the validity and accuracy of the model, the simulation parameters of the Sepic converter are chosen as:

$\nu_{con} = 0.2$, $C_1 = 0.7 \mu F$, $C_2 = 30 \mu F$, $v_{in} = 30 \text{ V}$, $L_1 = 1.92 \text{ mH}$, $L_2 = 1.92 \text{ mH}$, $T_c = 0.08 \text{ s}$, $\Delta t = 10^{-8} \text{ s}$, $\varepsilon = 10^{-9}$, $f = 10 \text{ KHz}$.

The Sepic converter operates in BCM when $R$ is equal to [11]:

\[ R = R_C = \frac{2L_{eq}f}{(1-d)^2} = 30 \text{ \Omega} \quad (L_{eq} = \frac{L_1 + L_2}{L_1}) \]

where $R_C$ is the critical value of resistance $R$. Furthermore, the Sepic converter will operate in CCM and DCM when $R < R_C$ and $R > R_C$ respectively. Consequently, when $R = 15 \text{ \Omega}$, the Sepic converter operates in CCM and the output voltage is derived:

\[ v_o = \frac{\alpha}{1-\alpha} v_{in} = \frac{0.2}{1-0.2} \times 30 = 7.5 \text{ V} \]

When $R = 60 \text{ \Omega}$, the Sepic converter operates in DCM and the output voltage is derived:

\[ v_o = \frac{d}{\sqrt{k}} v_{in} = 10.606 \text{ V} \quad k = 2L_{eq}/RT \]

The resistance $R$ varies as follows:
Based on (16), the simulation results of the proposed model are shown in Figure 5.

In Figure 5, the inductor-current waveform shows that the Sepic converter can seamlessly operate in CCM, BCM, and DCM operations. When $R = 15\ \Omega$, the Sepic converter works in CCM and the output voltage $v_o=7.5\ \text{V}$. When $R = 30\ \Omega$, the Sepic converter works in BCM and $v_o$ is still $7.5\ \text{V}$. When $R = 60\ \Omega$, the Sepic converter works in DCM and the output voltage $v_o=10.606\ \text{V}$. The simulation waveforms of inductor current and output voltage are able to illustrate the effectiveness of the hybrid switching model.

The accuracy of the hybrid switching (HS) method is checked by a comparison of the inductor current and the output voltage among the state space averaging (SSA) model.

Based on the SSA model, the state equations of the Sepic converter operating in CCM and DCM are different. So, the simulations of the converter working in CCM and DCM are implemented by different models. The simulation results are depicted in Figure 6 and the enlarged views of the Sepic converter operating in CCM and DCM are shown in Figure 7.

For the ideal Sepic converter, the value of $i_L$ should not be negative. However, from the simulation results of the SSA model, sometimes the value of $i_L$ is negative. Consequently, the comparison of the simulation results indicates that the HS model is more accurate than SSA model.

**Figure 5.** Simulation results under load disturbance.

**Figure 6.** Comparisons of different models operate in (a) CCM (b) DCM.
5. Conclusions
In this paper, a unified hybrid switching model of Sepic converter in CCM and DCM modes is established. The proposed method based on the hybrid switching theory and characteristics of Sepic converter in actual working conditions, the switching conditions between different working modes of Sepic converter is transformed into inequalities by introducing the principle of equivalent switching, so that the modeling method can describe the different states of the converter in different working modes, thus establishing a unified model of Sepic converter. The proposed modeling method retains all dynamic processes of Sepic converter, including transient and steady-state processes. In the process of modeling, no approximate assumptions are made, which ensures the accuracy of the model. The accuracy and validity of the proposed model are verified by simulation experiments and comparison with SSA model.

References
[1] Masksimovic D and Cuk S 2002 A unified analysis of PWM converters in discontinuous modes IEEE Transactions on Power Electronics 6 476-490
[2] Mahdavi J and Emmaadi A 1997 IEEE Transactions on Circuits & Systems I Fundamental Theory & Applications 44 767-770
[3] Van Dijk E, Spruijt J N, O’ Sullivan D M and Klaassens J B 1995 PWM-switch modeling of DC-DC converters IEEE Transactions on Power Electronics 10 659-665
[4] Xu S, Li F, Yao Y, Lu S and Sun W 2015 A high-frequency model for a PCM buck converter IEEE Transactions on Power Electronics 30 2304-2312
[5] Aboushady A A, Ahmed K H, Finney S J and Williams B W 2013 Linearized large signal modeling, analysis, and control design of phase-controlled series-parallel resonant converters using state feedback IEEE Transactions on Power Electronics 28 3896-3911
[6] Krzysztof Górecki 2008 A new electrothermal average model of the diode–transistor switch Microelectronics Reliability 48 51-58
[7] Ji Q, Ruan X, Xie L and Ye Z 2015 Conducted EMI spectra of average-current-controlled boost PFC converters operating in both CCM and DCM IEEE Transactions on Industrial Electronics 62 2184-2194
[8] Zhang J, Xie Z Z and Yang G L 2012 Mixed logical dynamical modeling and constrained optimal PWM control of DC/DC converters Electric Machines and Control 16 106-112
[9] Li J F, Han J G and Tang T H 2011 Unified Modeling of Switching Converters Based on Switching System Journal of South China University of Technology (Natural Science Edition) 39 157-164
[10] Williams H P 2013 Model Building in Mathematical Programming 5th Edition News & Media 264 1132-1138
[11] Niculescu E and lancu E P 1999 Boundary between CCM and DCM in fourth-order PWM converters africon 711-714