We construct the supersymmetric version of a model based on the gauge group $SU(3)_c \times SU(3)_L \times U(1)$. We discuss the mechanism of baryon number violation which induces nucleon decay, and derive bounds on the relevant couplings. We point out a new mechanism for nucleon decay which can be present in R-violating MSSM as well.

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The standard model of particle interactions, based on the gauge group $SU(3)_c \times SU(2)_L \times U(1)_Y$ is very successful experimentally. But it does not answer or address some important theoretical questions. One of these, for example, is the question of the number of generations of fermions. At present, we know of three generations, but the standard model does not explain why this number has to be three.

This question obtains a natural answer in an interesting extension of the standard model, based on the gauge group $SU(3)_c \times SU(3)_L \times U(1)_N$. In these theories, the fermion spectrum is extended, by including new quark-type fields, in such a way that chiral anomalies do not cancel in any single generation of fermion fields. The generations, on the other hand, are not exact replicas of each other, and all gauge anomalies cancel when all three generations are taken into account. Thus, this model requires the number of generations to be 3, or any multiple of 3. Another interesting feature of this model is that the Peccei-Quinn symmetry, necessary to solve the strong-CP problem, follows naturally from the particle content in these models. The aim of this paper is to construct a supersymmetric version of this model and discuss what sort of baryon-number-violating processes arise from such an extension.

We start with introducing the chiral superfields of this model. We start with the superfields that contain the quarks and leptons. We write only the left-handed fields throughout this paper, and omit any subscript $L$ for them.

$$\begin{align*}
\Psi_a : & \quad (1, 3, 0) \\
Q_1 : & \quad (3, 3, 2/3) \\
Q_i : & \quad (3, \bar{3}, -1/3) \\
U_c^a : & \quad (\bar{3}, 1, -2/3) \\
D_c^a : & \quad (\bar{3}, 1, 1/3) \\
T_1^c : & \quad (3, 1, -5/3) \\
B_c^i : & \quad (\bar{3}, 1, 4/3) \, .
\end{align*}$$

Here, $a$ is a generation index that runs from 1 to 3. The other generation index, $i$, runs only from 2 to 3. Thus, the field content of the first generation is different from that in the other two.

The electric charge generator is defined as

$$Q_{em} = \lambda_{3L} + \sqrt{3} \lambda_{8L} + N \, ,$$

where $\lambda_{3L}$ and $\lambda_{8L}$ are the two diagonal generators of $SU(3)_L$. In the fundamental representation, these are given by

$$\begin{align*}
\lambda_{3L} & = \frac{1}{2} \text{diag} (1, -1, 0) \, , \\
\lambda_{8L} & = \frac{1}{2\sqrt{3}} \text{diag} (1, 1, -2) \, ,
\end{align*}$$

and $N$ is the generator of $U(1)_N$ whose quantum numbers are given in Eq. (1). Using these formulas, we
see that the charges of the members of $\Psi_a$ should be $+1, 0, -1$, and the fermionic components of these fields can be identified with the left-chiral projection of the antilepton, the neutrino and the lepton in a given generation. The right-handed neutrino field, unknown so far from experiments, have not been introduced in this model, so there is no left-handed antineutrino field.

Looking now at the components of $Q_1$, we find that the two lower components have the charges of the $u$ and the $d$ quarks, and the fermionic fields for these components are identified with these quarks. The uppermost component has now a charge $5/3$, which is one of the exotic quarks in this model. Let us call it $T_1$. Its right-handed counterpart is the antiparticle of the field $T_1^c$ given in Eq. (1). Similarly, in $Q_i$, we have the fields known in the standard model, plus extra quark fields with charge $-4/3$, to be called $B_i$. The right-handed counterparts of these fields are conjugates of $B_i^c$ which appear in Eq. (3), where we also have the conjugates of the right-handed counterparts of the up-type and down-type quark fields present in the standard model.

So far, we have used only the quark and lepton fields present in the non-supersymmetric version of the model, and their superpartners. For the Higgs superfields, however, we must do something more. In the non-supersymmetric version, it was argued that the following Higgs multiplets can break the symmetry and give reasonable masses to the quarks and the leptons:

$$
\begin{align*}
\chi &: (1, 3, -1), \\ 
\rho &: (1, 3, 1), \\ 
\eta &: (1, 3, 0), \\ 
S &: (1, 6, 0).
\end{align*}
$$

In a supersymmetric version, the superpartners of these fields will give rise to chiral anomalies. Thus, to cancel them, we need other fields. The most obvious choice is a set of fields which are exactly in the complex conjugate representation of the gauge group. Let us call these fields $\chi^c$, $\rho^c$, $\eta^c$ and $S^c$. Thus, for example, $\chi^c$ would transform like $(1, 3, 1)$, and so on.

With this field content, we now write down the superpotential of the model. This is:

$$
W = h^{(1)} Q_1 T_1^c \chi^c + h^{(2)} Q_1 D_1^c \chi + h^{(3)} Q_1 D_1^c \rho + h^{(4)} Q_1 U_1^c \rho^c + h^{(5)} Q_1 U_1^c \eta + h^{(6)} Q_1 D_1^c \eta^c + h^{(7)} \Psi_a \Psi_b \Psi_c + h^{(8)} \Psi_a \Psi_b \Psi_c S^c + \lambda_{abc} \Psi_a \Psi_b \Psi_c + \lambda'_{abc} U_1^c D_1^c D_2^c + \lambda''_{abc} U_1^c D_1^c D_2^c + \kappa_{abc} T_1^c T_1^c B_1^c + \ldots
$$

because of the exchange symmetry of the $D^c$-fields. The same argument dictates that the couplings with the term $\Psi_a \Psi_b \Psi_c$ is totally antisymmetric in the generation indices. So, for three generations, there is only one such independent coupling, which we will denote simply by $\lambda$.

It is worthwhile to note that, unlike the minimal supersymmetric standard model (MSSM), one cannot impose an $R$-parity to eliminate all baryon and lepton number violation in this model. The reason is that there are lepton number violating interactions even in the gauge sector of this model. Baryon number violation can be eliminated, for example, by introducing a discrete $Z_2$ symmetry under which all the superfields presented in Eq. (4) change sign, whereas those presented in Eq. (3) do not. But we take the
most general superpotential allowed by gauge symmetry and supersymmetry, which is the one in Eq. (8), and examine its consequences for nucleon decay.

In order to have proton decay, one needs lepton number violation in addition to baryon number violation. In this model, lepton number violation has two different sources. Even in the non-supersymmetric version, lepton number violation was present due to SU(3)$_L$ gauge interactions, since the charged antilepton is put in the same multiplet as the charged lepton and the neutrino. In the supersymmetrized version, additional lepton number violation comes in through various Yukawa terms which can be derived from the superpotential. We will see that these are the terms which give leading contributions to proton decay in this model.

One such contribution is shown in Fig. 1, which is mediated by a squark of charge $-1/3$. Such contributions to proton decay exist even in the MSSM [5, 6]. There is, however, one distinctive feature here. Notice that the coupling $\lambda'$ involves the multiplet $Q_i$, where $i$ can take only the values 2 or 3. Thus, the outgoing quark can belong only to the second or the third generation, barring contributions coming from intergenerational mixing. So the dominant decay mode arising out of this diagram would be

$$p \to K^+ \nu_\alpha,$$  \hspace{1cm} (7)

where the neutrino can be from any generation, depending on the relative strengths of the different couplings $\lambda''_{abj}$. The effective coupling for this decay will be given by

$$G_\beta \simeq \frac{\lambda''_{11j} \lambda''_{12j}}{m_{d_j}^2},$$  \hspace{1cm} (8)

where the generation index $j$, as mentioned before, can take only the values 2 or 3 in view of the antisymmetry property of the couplings $\lambda''$ mentioned in Eq. (8). This will lead to a lifetime of

$$\tau_p \simeq \left( m_0 G_\beta^2 \right)^{-1}.$$  \hspace{1cm} (9)

The experimental lower bound on these modes is about $10^{32}$ years. Using that, we obtain

$$\lambda''_{11j} \lambda''_{12j} < 10^{-24},$$  \hspace{1cm} (10)

assuming the superpartner masses in the range of 1 TeV.

If we consider the effects of intergenerational mixing, we can extend the bounds [7] to any product of the couplings of the form $\lambda'' \lambda'$. Of course, intergenerational mixings will suppress decays like $p \to \pi^+ \nu$, and therefore the bounds will be a little weaker.

Let us now look at a different mechanism for nucleon decay. The relevant diagram has been shown in Fig. 2. Here, one utilizes the couplings $\lambda''$ and $\lambda$. Again, the vertex at the right end of the figure exists in the MSSM in the form of the coupling $LL\bar{E}^c$ and so this kind of diagram exists even in the MSSM [8]. But the distinctive feature here is that there is only one coupling $\lambda$, which connects three different generations of lepton fields. Thus, the vertex at the right end of the diagram must have fields from all three different generations. Let us also assume that, to a first approximation, the couplings of the gaugino fields are flavor diagonal. In this case, the three outgoing leptonic fields belong to three different generations. Since the $\tau$-lepton is heavier than the nucleon, this means that the charged leptons available in the decay product must be $\mu^- e^-$ or $\mu^+ e^+$. In other words, we obtain the following decay modes at the quark level:

$$u^c d^c s^c \to \mu^\pm e^\mp \nu_\tau,$$  \hspace{1cm} (11)

taking into account that the couplings $\lambda''$ must be antisymmetric in the down-type quark indices. For the proton, it implies the decay mode

$$p \to K^+ \mu^\pm e^\mp \nu_\tau.$$  \hspace{1cm} (12)

Of course, we can also have $p \to \pi^+ \mu^\pm e^\mp \nu_\tau$ etc, but those will be suppressed by intergenerational mixings.

The effective operator here is a six-fermion one, with the effective coupling

$$G_\beta^\prime \simeq \frac{g^2 \lambda''_{112} \lambda'}{M_2 m_{d_j}^2 m_{\psi_i}^2}.$$  \hspace{1cm} (13)
This will give a lifetime
\[ \tau_p \simeq \left( \frac{m_p^{11} G_F^2}{\beta} \right)^{-1}. \] (14)

There are no direct experimental limits on the specific decay modes obtained here. But if we take the lower limit of \(10^{31}\) years as a benchmark value, we obtain
\[ \lambda''_{112} \lambda < 10^{-16}, \] (15)
assuming the masses of the supersymmetric particles to be of order 1 TeV.

In addition, as we mentioned earlier, lepton number violation can come from SU(3)\(_L\) gauge couplings. However, these cannot induce proton decay at the tree level. The reason is that in the quark sector, such couplings involve exotic quarks, which will have to be much heavier than the proton.

Notice that we have dealt with a specific version of SU(3)\(_C\) \(\times\) SU(3)\(_L\) \(\times\) U(1) models. One can apply similar ideas to a different model containing right-handed neutrinos \([9]\). The consequences are similar, so we do not discuss it in detail.

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