Axial anomaly generation by domain wall motion in Weyl semimetals

Julia D. Hannukainen,1 Yago Ferreiros,1,2 Alberto Cortijo,3 and Jens H. Bardarson1

1Department of Physics, KTH Royal Institute of Technology, Stockholm, 106 91 Sweden
2IMDEA Nanociencia, Faraday 9, 28049 Madrid, Spain
3Instituto de Ciencia de Materiales de Madrid, CSIC, Cantoblanco, 28049 Madrid, Spain

A space-time dependent node separation in Weyl semimetals acts as an axial vector field. Coupled with domain wall motion in magnetic Weyl semimetals, this induces axial electric and magnetic fields localized at the domain wall. We show how these fields can activate the axial (chiral) anomaly and provide a direct experimental signature of it. Specifically, a domain wall provides a spatially dependent Weyl node separation and an axial magnetic field \( B_5 \), and domain wall movement, driven by an external magnetic field, gives the Weyl node separation a time dependence, inducing an axial electric field \( E_5 \). At magnetic fields beyond the Walker breakdown, \( E_5 \cdot B_5 \) becomes nonzero and activates the axial anomaly that induces a finite axial charge density—imbalance in the number of left- and right-handed fermions—moving with the domain wall. This axial density, in turn, produces, via the chiral magnetic effect, an oscillating current flowing along the domain wall plane, resulting in a characteristic radiation of electromagnetic waves emanating from the domain wall. A detection of this radiation would constitute a direct measurement of the axial anomaly induced by axial electromagnetic fields.

**Introduction.**—The smallest number of Weyl fermions realizable as quasiparticles in a crystal is two \([1, 2]\)—one left-handed and one right-handed. In the presence of inversion symmetry, we can choose the origin of momentum space such that one Weyl fermion resides at \( b \) and the other at \(-b\). Since time reversal does not change the handedness of a Weyl fermion, such a minimal Weyl semimetal necessarily breaks time-reversal symmetry \([3, 4]\). The Weyl node splitting \( 2b \) is then induced by the time-reversal breaking and can be thought of as a magnetization. Such a magnetic Weyl semimetal was recently realized in \( \text{EuCd}_2\text{As}_2 \) at intermediate temperatures—at very low temperatures the magnetic moments of the Eu atoms give rise to long-range magnetic order and a gap, but above the ordering-temperature magnetic fluctuations are still large enough to induce a splitting of a Dirac node into two Weyl fermions \([5, 6]\)—and in \( \text{EuCd}_2\text{Sb}_2 \) in an external magnetic field \([7]\); several further Weyl states in magnetic materials were experimentally observed \([8–11]\).

The electronic response of the Weyl fermions to external electromagnetic fields is fundamentally influenced by the chiral anomaly \([12, 13]\). The handedness of the Weyl fermions is not generally conserved and the axial density \( n_5 = n_L - n_R \), the difference in density of left- and right-handed Weyl fermions, instead satisfies the homogeneous anomaly equation \([14]\)

\[
\partial_t n_5 = \frac{e^2}{2\hbar^2\pi^2} \left( E \cdot B + \frac{1}{3} E_5 \cdot B_5 \right). \tag{1}
\]

Here \( E \) and \( B \) are the usual electric and magnetic fields, while \( E_5 \) and \( B_5 \) are so-called axial electric and magnetic fields \([15, 16]\). These have the property that they point in the opposite direction for the two chiralities. Direct experimental signatures of the anomaly have proven hard to come by. While negative magnetoresistance is a consequence of the chiral anomaly \([17, 18]\) it is not an unambiguous signature of it \([19–23]\). Axial fields are also challenging to realize as they may require systematic and significant straining of materials \([24–26]\); obtaining an axial electric field \( E_5 \) is particularly hard, as this requires controllable time-dependent strain. The reason for this is that the Weyl node separation \( b \) couples to the Weyl fermions as an axial vector potential and strain gives it a space-time dependence as \( b \to b(r, t) \). This then gives rise to axial fields through \( B_5 = \nabla \times b \) and \( E_5 = -\partial_t b \), in analogy with how electromagnetic fields are obtained from a vector potential \([27]\).

In this work we discuss how both of these difficulties—...
the generation of axial fields and detection of the axial anomaly—are overcome by studying domain wall motion [29, 30] in Weyl semimetals. Indeed, in a magnetic Weyl semimetal a space-time variation in the Weyl node separation is naturally realized at domain walls in the magnetization [31]. Such domain walls have been indirectly observed, for example, in the magnetic nodal semimetal CeAlGe [8]. Domain wall motion has also been studied in related systems such as junctions of ferromagnets and topological insulators [32–36].

Consider a magnetic domain wall along the x-direction, pointing in the ±z-direction deep in the bulk, as depicted in Fig. 1. For concreteness, we assume the easy-axis of the magnetic anisotropy to be in the z-direction and the hard-axis anisotropy to lie in the y-direction, making the xz-plane the easy-plane. The domain wall can be described in terms of two collective coordinates, the position $X(t)$ of the center of the wall and the internal angle $\phi(t)$, which measures the angle of the magnetization out of the easy-plane. $X(t)$ and $\phi(t)$ describe zero modes of fluctuations around the domain wall arising from translation invariance along the x-direction and rotation invariance around the z-axis, respectively [37]. While the existence of a hard-axis anisotropy formally breaks the rotational invariance, $X(t)$ and $\phi(t)$ are still good collective coordinates in the limit of weak anisotropy. There are two special configurations of the domain wall, the Néel wall for which $\phi = 0$ where the domain wall is situated in the easy plane, and the Bloch wall, for which $\phi = \pi/2$, where the angle out from the easy plane is maximal. The Bloch wall is illustrated in Fig. 1. Since $b$ rotates from $-b \hat{z}$ to $b \hat{z}$ an axial magnetic field localized at the domain wall is obtained. This is similar to the $B_5$ obtained at the surface of Weyl semimetals [38], except that it is not constrained to a definite location in space.

When the domain wall moves the magnetization becomes time-dependent, generating an axial electric field $E_5$. A controllable way of moving a domain wall is by a magnetic field $B = B_5 \hat{z}$. This results in a rigid shift of the domain wall center $X(t)$ with an average velocity that increases linearly with $B$ up until a critical value $B_c$, at which the internal angle starts rotating, and the velocity decreases—this is called the Walker breakdown [39]. The axial electric field generated in this movement is a function of both the rotation and the velocity of the domain wall. However, as we show, the axial anomaly (which depends on $E_5 \cdot B_5$) is only activated once the internal angle starts rotating, for magnetic fields larger than $B_c$. Once it is activated an axial density $n_5$, localized at the domain wall, builds up and an oscillating current is induced, via the chiral magnetic effect [40]. This results in electromagnetic radiation which is a direct signature of the axial anomaly induced by axial pseudo-fields.

**Domain wall dynamics.**—We take the Weyl node separation in a domain wall to define a unit magnetization $m$ as $b(r, t) = \Delta/(e v_F) m(r, t)$, where $e$ is the elementary charge, $v_F$ the Fermi velocity, and $\Delta$ an effective exchange coupling between the electrons and the magnetization. The variation of $b$ with $r$ and $t$ is slow enough, compared to typical electronic time and length scales, that the interpretation of it as a Weyl node separation in momentum space still makes sense. Expressed in the collective coordinates,

$$m = \left( \frac{\cos[\phi(t)]}{\cosh(\frac{x-X(t)}{\lambda})}, \frac{\sin[\phi(t)]}{\cosh(\frac{x-X(t)}{\lambda})}, -q \tanh(\frac{x-X(t)}{\lambda}) \right),$$

where $\lambda$ is the domain wall width and $q = \pm 1$ is the topological charge [37]; we consider the case $q = -1$, as depicted in Fig. 1. The dynamics of the domain wall is encapsulated in a ferromagnetic action which considers the precession and exchange coupling of the magnetization [29]. The action describing the precession is given by a Berry phase term $S_B = \hbar/3 \int dt \int d^3x \sqrt{1 - \phi^2}$, with $\theta = 2 \tan^{-1}(\sqrt{1 - \phi^2})$ and $a$ is the lattice constant. The exchange coupling contributes the term $S_H = \int dt \int d^3x \frac{1}{\sqrt{2}} \lambda H_\parallel$, with $H_\parallel = -\frac{1}{2} J a^2 (m_\parallel^2 - K m_\perp^2 - K_\perp m_\perp^2)$ being the Heisenberg Hamiltonian in the continuous limit. Here $J$, $K$, and $K_\perp$ are all positive constants: $J$ is the exchange energy, and $K$ and $K_\perp$ are the easy- and hard-axis anisotropy energies. The contribution from an external magnetic field $B = B_\parallel \hat{z}$, applied in the direction of the easy-axis anisotropy, is included as a Zeeman term, $S_Z = \int dt \int d^3x H_\parallel$, with the energy $H_\parallel = -\hbar a^2 \int d^3x m \cdot \gamma B$, where $\gamma$ is the electron gyromagnetic ratio.

The collective coordinate description of the domain wall in terms of $X(t)$ and $\phi(t)$ is valid as long as there is translational invariance in the $x$-direction and rotational invariance around the $z$-direction. While the existence of a hard-axis anisotropy strictly speaking would deform the domain wall and break the rotational invariance, the deformation is negligible in the limit $K_\perp \ll K$, in which $X(t)$ and $\phi(t)$ are good collective coordinates [30]. While this is not an essential limit, it simplifies our discussion so we assume it from now on. The domain wall action in this limit, including the external magnetic field, is in terms of collective coordinates [29]

$$S_{FM} = -\frac{2 \hbar A}{a^2} \int dt \left( \dot{\phi} X + v_\perp \sin^2 \phi - \gamma B X \right).$$

Here $A$ is the cross-section of the sample in the $yz$-plane and $v_\perp = \lambda K_\perp/(2 \hbar)$, with $\lambda = \sqrt{J/K}$ the domain wall width. The first term in Eq. (3) is the Berry phase term, the second the contribution from the Heisenberg Hamiltonian and the last term is the Zeeman term. The time evolution of the collective coordinates is given by the action $S_{FM}$ together with damping, which takes into account the relaxation of the magnetization. Incorporating the damping as a dissipation function $W = -\hbar N_\alpha/2[(X/\lambda)^2 + \phi^2]$, where $\alpha$ is the Gilbert
angular frequency \( \omega \) increases with time with an oscillatory motion. The magnitude of the magnetic field therefore plays a role in how the domain wall moves, which has implications for the onset of the chiral anomaly. The anomaly equation, Eq. (1), (with \( E = 0 \)) is proportional to

\[
E_5 \cdot B_5 = \frac{\Delta^2}{e^2 \nu_F^2 \lambda^3} \frac{\phi \cos \phi}{\cosh^3 \left( \frac{x - X(t)}{\lambda} \right)},
\]

which is zero when \( B < B_c \), implying that the chiral anomaly only is activated in the Walker breakdown regime. The axial electric field, which contributes with the term \( \phi \), is also nonzero before the Walker breakdown, but is then orthogonal to the axial magnetic field.

**Measuring the anomaly.**—The axial fields generated by the domain wall motion give rise to the chiral anomaly, where the nonconservation of axial charge generates an axial chemical potential \( \mu_5 = (\mu_L - \mu_R)/2 \), with \( \mu_L \) and \( \mu_R \) the chemical potentials of left- and right-handed Weyl fermions, respectively. The anomaly equation, with regard to the axial fields, has the form \( \partial_t n_5 = \epsilon^2/(6\hbar^2 \pi^2) E_5 \cdot B_5 - n_5 / \tau \), where the second term takes into account inter-valley scattering between the two Weyl cones, with inter-valley scattering time \( \tau \) [42, 43] and where \( E_5 \cdot B_5 \) oscillates in time with period \( \tau_0 = 2\pi / \omega \).

The axial chemical potential is solved adiabatically when \( \tau_0 \gg \tau \), and then \( n_5 = \epsilon^2 \tau/(6\hbar^2 \pi^2) E_5 \cdot B_5 \) at large times \( t \gg \tau \). In the opposite limit \( \tau_0 \ll \tau \), the domain wall oscillates faster than the inter-valley scattering and the number density takes the form \( n_5 = \epsilon^2/(6\hbar^2 \pi^2) \int_0^\infty ds E_5(x, s) \cdot B_3(x, s) \). The relevant limit depends on the size of the external magnetic field.

The axial chemical potential is considered to be space and time dependent, (which holds true in the limit where \( \tau_0 \) is larger than the intra-valley scattering time) and relates to the axial number density as \( 3\pi^2 \hbar^2 \nu_F^2 n_5 = \mu_5^2 + \mu_5 (\pi^2 k_B^2 T^2 + 3\mu_5^2) \), which apply in the limit of small magnetic fields, \( \hbar \dot{E} \ll \mu_5^2 / \nu_F \), where \( \mu = (\mu_L + \mu_R)/2 \) is the average chemical potential, \( T \) the temperature and \( k_B \) the Boltzmann constant [44]. \( \mu_5(x, t) \) oscillates in time, is located at the domain wall, and travels along the \( x \)-direction as \( X(t) \) evolves with time, as is depicted, in the limit \( \tau_0 \ll \tau \), in Fig. 2.

The axial chemical potential generates a current density [45] proportional to the external magnetic field through the chiral magnetic effect [44],

\[
J_z(x, t) = \frac{e^2}{2\pi^2 \hbar^2} \mu_5(x, t) B.
\]

This oscillating current density gives, in the instantaneous approximation, rise to an electric field outside the sample, \( E(r, t) = -1/(4\pi \varepsilon_0 c^2) \int dr' |r - r'|^{-2} \partial_t J(r', t) \), where \( \varepsilon_0 \) is the vacuum permittivity and \( c \) the speed of light in vacuum [46]. This form requires that \( L_z > \nu_F \tau_0 \), where \( L_z \) is the width of the sample in the \( z \)-direction (in the opposite limit accumulation of charge at the edge of the sample becomes important and must be taken into account).
FIG. 3. The electric field $E_z(x,t)$ of Eq. (9) as a function of time, normalized by $E$ defined such that $E_z(x,t)/E = \partial_t \mu_5(x,t)|_{x=X(t)}/(\mu \omega)$. The left inset features the corresponding frequency spectrum as a function of the frequency $\omega_n = n \omega$, $n \in \mathbb{Z}$ and the period time $\tau_0$ of the electric field. Only frequencies where $n$ is odd are non-zero (green). The polar plot in the right inset depicts the angular dependence on $\varphi$ of the amplitude of the electric field, due to two sources, Eq. (10), for $\theta = \pi/2$. The unit-less radius is given by $C_{\mu 5}/(\mu \omega) \cos \varphi$, where $C_{\mu 5}$ is the amplitude of $\partial_t \mu_5(x,t)|_{x=d/2-X(t)}$. For parameter values see Fig. 2 and [28].

account), which for realistic parameters [28] and an applied magnetic field $B \sim 1$ T holds true when $L_z \sim 10$ $\mu$m. Further approximating the spatial dependence of the current density as a Dirac delta function around the center of the domain wall, yields a far field electric field of the form

$$E_z(r,t) = \frac{e^2 BV}{8\pi^3 \hbar^2 \varepsilon_0 c^2} \frac{\partial_t \mu_5(x,t)|_{x=X(t)}}{\sqrt{(x-X(t))^2+y^2+z^2}} \quad (9)$$

where $V$ is the sample volume. The motion of the domain wall is negligible in the limit $x \gg X(t)$ or when $y^2+z^2 \gg (x-X(t))^2$, and the source can be considered to be fixed at the origin; the electric field in this limit is anti-periodic in $t$, $E_z(r,\tau_0/2-t) = -E_z(r,t)$, as is shown in Fig. 3. This means that the corresponding frequency spectrum, depicted in the left inset of Fig. 3, only has non-zero values at odd frequencies $\omega_n = 2\pi(2n+1)/\tau_0 = (2n+1)\omega$, $n \in \mathbb{Z}$, and peaks at $n = 0$; for the parameters in [28] the peak frequency is $\sim 0.2$ GHz.

These results generalize to multiple domain walls. Adjacent domain walls have opposite topological charge $q = \pm 1$, and due to this they traverse in opposite directions under the influence of a magnetic field, where the direction depends on the sign of the magnetic field [30]: a domain wall with $q = -1$ travels in the positive (negative) $x$-direction, when $B > 0$ ($B < 0$). Consider, for example, a Weyl semimetal with two domain walls separated by a distance $d > \lambda$, and the domain wall with $q = -1$ is located at $x = -d/2$ and the domain wall with $q = 1$ at $x = d/2$. The collective coordinate describing the center of each domain wall is $X_q(t) = q(d/2 - X(t))$, where $X(t)$ is the solution of the equations of motion for a single wall with $X(0) = 0$. When a magnetic field $|B| > B_c$ is applied, the axial anomaly gives rise to a current density, $-q J_z(x,t)$, located at the center of each domain wall. This gives rise to an angle dependent electric field which, when calculated assuming the same conditions as for the single domain wall case, is of the form

$$E_z(r,t) = - \frac{e^2 BV}{4\pi^3 \hbar^2 \varepsilon_0 c^2} \frac{\partial_t \mu_5(x,t)|_{x=d/2-X(t)}}{r^2} \sin \theta \cos \varphi \quad (10)$$

Here $r$ is the radial distance from the sample and $\theta$, $\varphi$ are the polar and azimuthal angles respectively, defined with the origin between the two domain walls. This takes the form of a current dipole with amplitude dependence plotted as a function of $\varphi$ for $\theta = \pi/2$ in Fig. 3. The amplitude is zero when the the distance to the two sources is equal which happens for $\varphi = \pm \pi/2$, (as well as $\theta = 0$, $\theta = \pi$) and it is maximum for $\varphi = 0$ and $\varphi = \pi$. The radiated electric field might only exist during a finite length of time depending on the velocity of the domain walls, until the domain walls annihilate each other or until they reach the boundary of the sample. Pinning [30]—local enhancement of easy-axis anisotropy, due to for example impurities, confining the domain wall to a certain region—could modify the details of the radiation field, since adjacent domain walls could be prevented from annihilating one another and the electromagnetic radiation would come from a fixed location.

Discussion.—We have shown how field-driven motion of a domain wall in a magnetic Weyl semimetal leads to the activation of the axial anomaly. This results from the space and time dependent Weyl node separation emerging from the domain wall motion, which generates axial electromagnetic fields. The anomaly generates an axial chemical potential at the domain wall, which in turn results in an oscillating current and electromagnetic microwave radiation, detection of which would constitute a direct measurement of the axial anomaly. Experimental techniques to detect such microwave radiation are advanced and can even be done on-chip [47–50]. While we have made some simplifying approximations to highlight the fundamental physics, we expect the qualitative picture to be robust in realistic situations, and a general feature of any domain wall motion in Weyl semimetals. For example, current-driven domain wall motion will lead to the same axial anomaly-triggering mechanism as the one described here, but will allow for an electronic control of anomaly activation, which may be useful in designing experiments and applications. We have also worked in the limit of weak hard-axis anisotropy where a description
of the domain wall in terms of collective coordinates is sufficient. Deviations away from this limit will lead to a more complicated theory that needs to take into account modes beyond just the zero modes we include, but this is not expected to modify the qualitative description of the emergence of axial fields located at the domain wall.

We have focused our discussion on the use of domain wall motion for detecting anomaly physics. The other way around, namely the effects of the anomaly on the physics of domain walls and related spintronics phenomena is an interesting avenue for future studies.

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