Geometric quantum discord and non-Markovianity of structured reservoirs

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The reservoir memory effects can lead to information backflow and recurrence of the previously lost quantum correlations. We establish connections between the direction of information flow and variation of the geometric quantum discords (GQDs) measured respectively by the trace distance, the Hellinger distance, and the Bures distance for two qubits subjecting to the bosonic structured reservoirs, and unveil their dependence on a factor whose derivative signifies the (non-)Markovianity of the dynamics. By considering the reservoirs with Lorentzian and Ohmic-like spectra, we further demonstrated that the non-Markovianity induced by the backflow of information from the reservoirs to the system enhances the GQDs in most of the parameter regions. This highlights the potential of non-Markovianity as a resource for protecting the GQDs.

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I. INTRODUCTION

Quantum correlations occupy an important position in understanding fundamental characteristics of a quantum system. For this reason, they remain the research focus of people from the early days of quantum mechanics to now. Today, when we mention to quantum correlations, we know that in addition to entanglement [1], the concept of quantum discord constitutes another representative class of quantum correlation measure [2]. The related studies on this subject are mainly carried out around its quantification, its particular behaviors, and the control of it in various quantum systems [3]. Particularly, there has been an increasing interest of people on quantifying quantum discord from different perspectives, and to date there are a number of discord-like correlation measures being proposed [2, 4–9]. On the other hand, the behaviors of quantum discord in the spin chain [10], the two-level atoms [11], and the NMR system [12] have also been studied extensively.

From an applicative point of view, quantum discord is an invaluable resource for implementing many quantum tasks [13–18]. But it is very fragile, and the unavoidable interaction of a realistic system with its environment leads to irretrievable deterioration of the correlations in most cases [19–24]. This makes understanding of the connection between the environmental effects and evolution of quantum discord a vital problem. In fact, many studies have already been performed in this respect, and there were evidence indicating that sometimes the non-Markovian character of an environment may serve as a resource for protecting quantum discord from being destroyed completely [25–28]. It has also been observed that with elaborately chosen spectrum of the reservoir, the quantum discord can be frozen for an interval of time [22, 29] or be frozen permanently [30].

Although it is evident that sometimes the non-Markovianity can be used to enhance quantum discord of a system to some extent, we must to say that this is not always the case [31, 32]. Searching a general connection between non-Markovian character of an environment and the variation tendency of quantum discord is still an open subject in the quest for reliable ways to protect them. Toward that end, in this paper we establish an explicit dependence of the geometric quantum discords (GQDs) [6–9] of a two-qubit system on non-Markovianity of the zero-temperature bosonic structured reservoirs, and unveil the connections between the direction of information flow and enhancement of the GQDs for different initial states. Actually, with the rapid developments of the reservoir engineering technique [33–35], nowadays it is feasible to adjust experimentally frequency distribution of a reservoir to the desired regime such that the decay time for the quantum discord can be prolonged, provided that we know the explicit dependence of it on spectral density distribution of the reservoir.

The structure of this paper is arranged as follows. In Sec. II we recall briefly measures of the GQDs, while in Sec. III the model for the system-reservoir coupling is presented. Sec. IV is devoted to a derivation of the GQDs and their dependence on a reservoir-determined factor. Then in Sec. V we illustrate via two explicit examples our main findings. Finally, Sec. VI is devoted to a summary.

II. MEASURES OF THE GQD

There are many discord measures being proposed until now. We recall here three measures of the GQD. They are defined, respectively, by the trace distance, the Hellinger distance, and the Bures distance for different initial states. Actu-
and for the two-qubit states $\rho^X$ with the X-shaped matrix form (i.e., $\rho^X$ contains nonzero elements only along the main diagonal and anti-diagonal), the TDD can be obtained analytically. Particularly, for a restricted subset of $\rho^X$ with elements $\rho_{4,41} = 0$, we have

$$D_T(\rho^X) = 2|\rho_{23}|.$$  \hspace{1cm} (2)

The second GQD measure is the HDD, which is a modified version of the earliest proposed GQD. It reads

$$D_L(\rho) = 2 \min_{\Pi^A} \| \sqrt{\rho} - \Pi^A(\sqrt{\rho}) \|_2^2,$$ \hspace{1cm} (3)

where the minimum is taken over $\Pi^A = \{\Pi^A\}$, with

$$\Pi^A(\sqrt{\rho}) = \sum_k (\Pi^A_k \otimes I_B) \sqrt{\rho}(\Pi^A_k \otimes I_B),$$ \hspace{1cm} (4)

and $I_B$ is the identity operator in $\mathcal{H}_B$. Particularly, if we are restricted to the $(2 \times n)$-dimensional $\rho$, Eq. (3) yields

$$D_L(\rho) = 1 - \lambda_{\max}\{W_{AB}\},$$ \hspace{1cm} (5)

where $\lambda_{\max}\{W_{AB}\}$ is the maximum eigenvalue of the matrix $W_{AB}$ whose elements are given by

$$(W_{AB})_{ij} = \text{Tr}\{\sqrt{\rho}(\sigma_i^A \otimes I_B)\sqrt{\rho}(\sigma_j^B \otimes I_B)\},$$ \hspace{1cm} (6)

with $\sigma_{1,2,3}^S = A, B$ the three Pauli operators.

Finally, we recall the GQD measure of BDD, which is defined as

$$D_B(\rho) = \sqrt{(2 + \sqrt{2})[1 - \max_{\chi \in \Omega_0} F(\rho, \chi)]},$$ \hspace{1cm} (7)

with $F(\rho, \chi) = |\text{Tr}(\sqrt{\rho}(\sigma_i^A \otimes I_B)\sqrt{\rho}(\sigma_j^B \otimes I_B))|^2$, and for the special case of $(2 \times n)$-dimensional state $\rho$, $F_{\max}(\rho, \chi) = \max_{\chi \in \Omega_0} F(\rho, \chi)$ simplifies to

$$F_{\max}(\rho, \chi) = \frac{1}{2} \max_{||\chi|| = 1} \left(1 - \text{Tr}A + 2 \sum_{k=1}^{n_B} \lambda_k(A)\right),$$ \hspace{1cm} (8)

with $\lambda_k(A)$ being the eigenvalues of $A = \sqrt{(\sigma_i^A \otimes I_B)\sqrt{\rho}(\sigma_j^B \otimes I_B)}\sqrt{\rho}$ arranged in non-increasing order, $n_B = \text{dim } \mathcal{H}_B$, and $\vec{u}$ a unit vector in $\mathbb{R}^3$.

III. THE MODEL

We consider in this paper two noninteracting qubits denoted by $S = A$ and $B$. Each of them coupled locally to their independent zero-temperature bosonic reservoir. The Hamiltonian of the “qubit plus reservoir” subsystem reads

$$\hat{H} = \omega_0 \sigma_+ \sigma_- + \sum_k \omega_k b_k^\dagger b_k + \sum_k (g_k b_k \sigma_+ + \text{H.c.}),$$ \hspace{1cm} (9)

with $\omega_0$ being the transition frequency of the qubit, and $\sigma_{\pm}$ the raising and lowering operators. $b_k$ and $b_k^\dagger$ are the annihilation and creation operators for the field mode $k$ with frequency $\omega_k$ and the system-reservoir coupling constant $g_k$.

When the initial state of each qubit with its reservoir is in a product form, the evolution of the reduced density matrix for qubit $S$ is known to be described by

$$\dot{\rho}^S(t) = -i\frac{\Omega(t)}{2}[\sigma_+ \sigma_- , \rho^S(t)] + \frac{\Gamma(t)}{2}[2\sigma_- \rho^S(t)\sigma_+ - \{\sigma_+ \sigma_- , \rho^S(t)\}],$$ \hspace{1cm} (10)

where the time-dependent factors $\Gamma(t)$ and $\Omega(t)$ are as follows

$$\Gamma(t) = -2\text{Re} \left[\frac{\dot{\rho}(t)}{\rho(t)}\right], \Omega(t) = -2\text{Im} \left[\frac{\dot{\rho}(t)}{\rho(t)}\right],$$ \hspace{1cm} (11)

with $\text{Re}[x]$ and $\text{Im}[x]$ representing, respectively, the real and imaginary parts of $x$, while $\rho(t)$ obeys the integro-differential equation

$$\dot{\rho}(t) + i\omega_0 \rho(t) + \int_0^t f(t - t_1) dt_1 = 0,$$ \hspace{1cm} (12)

where the correlation function $f(t - t_1)$ is related to the spectral density $J(\omega)$ of the reservoir via

$$f(t - t_1) = \int d\omega J(\omega)e^{-i\omega(t-t_1)}.$$ \hspace{1cm} (13)

From Eq. (10) one can show that the reduced density matrix $\rho^S(t)$ for qubit $S$ takes the form

$$\rho^S(t) = \left(\rho_{11}^S(0)|p(t)|^2 \rho_{10}^S(0)p(t) - \rho_{11}^S(0)p(t)|p(t)|^2\right);$$ \hspace{1cm} (14)

with $\rho_{11}^S(0) = \langle i|\rho^S(0)|j\rangle$, and $\{|1\rangle, |0\rangle\}$ the standard basis.

If we further define $q(t) = |p(t)|^2$, then the decay rate $\Gamma(t)$ in Eq. (11) turns out to be

$$\Gamma(t) = -\frac{\dot{q}(t)}{q(t)},$$ \hspace{1cm} (15)

whose sign is determined solely by the slope of $q(t)$, namely, by $\dot{q}(t) = \partial q(t)/\partial t$.

The sign of $\Gamma(t)$ is also intimately related to the direction of information flow between the system and the reservoir. If $\Gamma(t)$ is always positive, i.e., $\Gamma(t) > 0$ in the whole time region, the evolution process is said to be Markovian, and the information flows from the system into the reservoir. On the other hand, it is non-Markovian if $\Gamma(t)$ takes on negative values within certain time intervals, and now there are temporary information backflow from the reservoir to the system. By the way, as the energy $\varepsilon(t) = \text{Tr} [\rho^S(t) H_S]$ ($H_S = \omega_0 \sigma_+ \sigma_-)$ for qubit $S$ was given by $\varepsilon(t) = \omega_0 \rho_{11}^S(0) q(t)$, the information backflow is also accompanied by the energy backflow.

In this paper, we will show that the direction of information flow between the system and the reservoir is also intimately related to the variation tendency of the GQDs. Particularly, the non-Markovianity of the dynamics can be detected efficiently by tracking the evolution of the GQDs.
IV. CONNECTION BETWEEN ENHANCEMENT OF GQDS AND DIRECTION OF INFORMATION FLOW

We consider two qubits being prepared initially in pure state of the following form

\[ |\Phi\rangle = \alpha|10\rangle + \sqrt{1 - \alpha^2}|01\rangle, \quad (16) \]

for which the two-qubit density matrix \( \rho(t) \) is given by

\[ \rho(t) = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & \alpha^2 q(t) & \alpha \beta g(t) & 0 \\ 0 & \alpha \beta g(t) & \beta^2 q(t) & 0 \\ 0 & 0 & 0 & 1 - q(t) \end{pmatrix}, \quad (17) \]

where \( \beta = \sqrt{1 - \alpha^2} \). Clearly, \( \rho(t) \) maintains the X form, and the GQDs are determined by the time-dependent factor \( q(t) \). Moreover, one can show that the three GQDs are independent of the sign of \( \alpha \), so we consider in the following only the initial state \( |\Phi\rangle \) with \( \alpha \geq 0 \).

A. The case of TDD

For \( \rho(t) \) in Eq. (17), the TDD can be obtained as

\[ D_T(\alpha) = 2\alpha \beta q, \quad (18) \]

which behaves as a monotonically increasing function of \( q \), except the trivial cases of \( \alpha^2 = 0 \) and 1.

This result indicates that if one can engineer spectral distribution of the structured reservoir such that the time-dependent factor \( q(t) \) is increased in certain time intervals, the TDD can be enhanced, and its maximum is achieved when \( q(t) \) reaches its maximum. Moreover, from Eq. (15) one can see that the increase of \( q(t) \) with time corresponds to the negative \( \Gamma(t) \). Therefore, for the present case the negative decay rate \( \Gamma(t) \), or equivalently, the backflow of information from the reservoirs to the system, is always favorable for enhancing the TDD.

B. The case of HDD

We now turn to consider the HDD. For \( \rho(t) \) in Eq. (17), the eigenvalues of \( W_{AB} \) can be derived as

\[ \lambda_{1,2} = 2\alpha^2 \sqrt{q(1-q)}, \quad \lambda_3 = 1 - 4\alpha^2 \beta^2 q, \quad (19) \]

the relative magnitudes of which depend on the parameters involved, and thus the analytical solution of \( D_L(\rho) \) is somewhat complex. We discuss it via the following two cases.

First, for \( \alpha^2 < 1/3 \), we have \( \lambda_{\max}(W_{AB}) = \lambda_3 \), thus

\[ D_L(\rho) = 4\alpha^2 \beta^2 q, \quad (20) \]

which increases with the increase of \( q \) except the trivial case of \( \alpha^2 = 0 \). See, for example, the plots of \( D_L(\rho) \) versus \( q \) for \( \alpha^2 = 0.1 \) and 0.3 showed in the left panel of Fig. 1.

Second, for \( \alpha^2 > 1/3 \), we have

\[ D_L(\rho) = \begin{cases} 1 - 2\alpha^2 \sqrt{q(1-q)} & \text{if } q \in [q_{c1}, q_{c2}], \\ 4\alpha^2 \beta^2 q & \text{if } q \notin [q_{c1}, q_{c2}], \end{cases} \quad (21) \]

where the parameters \( q_{c1} \) and \( q_{c2} \) are given by

\[ q_{c1,2} = \frac{(2 - \alpha^2) \pm \beta \sqrt{3\alpha^2 - 1}}{2\alpha^2(1 + 4\beta^2)}. \quad (22) \]

and \( q_{c1} \) takes the “ − ” sign, \( q_{c2} \) takes the “ + ” sign. One can check that for \( \alpha^2 \in (1/3, 0.5) \), both \( q_{c1} \) and \( q_{c2} \) are larger than 0.5, while for \( \alpha^2 \in (0.5, 1) \), we have \( q_{c1} \leq 0.5 \) and \( q_{c2} \geq 0.5 \). Moreover, for \( \alpha^2 = 0.5 \), we have \( q_{c1} = q_{c2} = 0.5 \).

From Eq. (21) one see that \( D_L(\rho) \) is a monotonically increasing function of \( q \) when \( q \in [q_{c1}, q_{c2}] \) with \( q_{c1} \leq 0.5 \), and when \( q \notin [q_{c1}, q_{c2}] \), while it is a monotonically decreasing function of \( q \) otherwise. By combining these with Eq. (22), we summarize the \( q \) dependence of \( D_L(\rho) \) as follows:

(i) If \( \alpha^2 \in (1/3, 0.5) \), \( D_L(\rho) \) always behaves as a monotonically increasing function of \( q \), as exemplified by the blue curve for \( \alpha^2 = 0.5 \) in the left panel of Fig. 1.

(ii) If \( \alpha^2 \in (0.5, 1) \), \( D_L(\rho) \) is a monotonically increasing function of \( q \) in the regions \( q \leq q_{c1} \) and \( q \geq 0.5 \), and a monotonically decreasing function of \( q \) in the region \( q \in (q_{c1}, q_{c2}) \). See, the exemplified plot for \( \alpha^2 = 0.7 \) and 0.9 showed in Fig. 1.

From the above discussion we see that for the initial states \( |\Phi\rangle \) with \( \alpha^2 \leq 0.5 \), the HDD can always be enhanced by the backflow of information from the reservoir to the system. For \( \alpha^2 > 0.5 \), however, the HDD is enhanced by the information backflow only when \( q \leq q_{c1} \) and \( q \geq 0.5 \), while it is enhanced with the information loss when \( q \in (q_{c1}, 0.5) \).

C. The case of BDD

When considering the BDD for the initial state \( |\Phi\rangle \), by writing the unit vector \( \vec{u} = (\sin \theta \cos \phi, \sin \theta \sin \phi, \cos \theta) \), one can...
derive the eigenvalues of $\Lambda$ analytically as
\[
\epsilon_{1,2} = 0, \\
\epsilon_{3,4} = \frac{1}{2} \left[ \chi \cos \theta \pm \sqrt{\xi \cos^2 \theta + 4\alpha^2 q(1 - q)} \right],
\]
where $\chi = 2\alpha^2 q - 1$, and $\xi = 4(1 - \alpha^2 q^2)q^2 - 4q + 1$.

Due to the parameters $\alpha^2$, $\cos \theta$, and $q$ involved, the $\epsilon_i$ cannot be ordered by magnitude in general. But as $\epsilon_3 \geq \epsilon_4$, solutions of $F_{\text{max}}$ in $D_B(\rho)$ can be obtained by separating $\alpha^2$ into the following three different regions.

First, for $\alpha^2 \leq 1/3$, we have
\[
F_{\text{max}} = 1 - \alpha^2 q,
\]
by combining of which with Eq. (7), one can note that except the trivial case $\alpha^2 = 0$, $D_B(\rho)$ always increases with the increase of $q$. See, e.g., the exemplified plots for $\alpha^2 = 0.1$ and 0.3 displayed in the right panel of Fig. 1.

Second, for $\alpha^2 \in (1/3, 0.5]$, we have
\[
F_{\text{max}} = \begin{cases} 
1 - \alpha^2 q, & \text{if } q \not\in [q_{c3}, q_{c4}], \\
\frac{1}{2} + \sqrt{\alpha^2 q(1 - q)}, & \text{if } q \in [q_{c3}, q_{c4}],
\end{cases}
\]
where the parameters
\[
q_{c3, c4} = \frac{2\alpha \pm \sqrt{3\alpha^2 - 1}}{2\alpha(1 + \alpha^2)},
\]
and $q_{c3}$ decreases from 0.75 to 1/3, while $q_{c4}$ increases from 0.75 to 1. Then, in the regions of $q < q_{c3}$ and $q > q_{c4}$, $F_{\text{max}}$ is decreased by increasing $q$. In the region of $q \in [q_{c3}, q_{c4}]$, however, the situation is somewhat complicated: if $q_{c3} \geq 0.5$, which corresponds to $\alpha^2 \in (1/3, 0.382]$, $F_{\text{max}}$ is decreased by increasing $q$; if $q_{c3} < 0.5$, which corresponds to $\alpha^2 \in (0.382, 0.5]$, $F_{\text{max}}$ is increased (decreased) by increasing $q$ when $q \in [q_{c3}, 0.5]$ ($q \in (0.5, q_{c4})$). Thus, the $q$ dependence of $D_B(\rho)$ are as follows:

(i) If $\alpha^2 \in (1/3, 0.382]$, $D_B(\rho)$ always increases with the increase of $q$.
(ii) If $\alpha^2 \in (0.382, 0.5]$, $D_B(\rho)$ increases (decreases) with the increase of $q$ when $q < q_{c3}$ and $q > 0.5$ ($q \in [q_{c3}, 0.5]$). See the blue curve for $\alpha^2 = 0.5$ in the right panel of Fig. 1.

Finally, for $\alpha^2 > 0.5$, we have
\[
F_{\text{max}} = \begin{cases} 
\frac{1}{2} + \sqrt{\alpha^2 q(1 - q)}, & \text{if } q \in [q_{c5}, q_{c6}], \\
\frac{1}{2}[1 + \sqrt{\gamma + 4\alpha^2 q(1 - q)}] & \text{if } q \not\in [q_{c5}, q_{c6}],
\end{cases}
\]
with the parameters $q_{c5}$ and $q_{c6}$ being given by
\[
q_{c5, c6} = \frac{1 \pm \alpha\beta}{2(1 - \alpha^2)^{3/2}},
\]
and $q_{c5}$ increases from 1/3 to 0.5, while $q_{c6}$ decreases from 1 to 0.5. Then, by combining this with Eq. (7) one can obtain that $D_B(\rho)$ is a monotonic increasing function of $q$ in the regions $q < q_{c5}$ and $q > 0.5$, and a monotonic decreasing function of $q$ in the region $q \in [q_{c5}, 0.5]$. See the exemplified plots for $\alpha^2 = 0.7$ and 0.9 in the right panel of Fig. 1.

We summarize the connections between the variation trend of the BDD and the direction of information flow between the system and the reservoir as follows: for $\alpha^2 \in (0, 0.382]$, the BDD can always be enhanced by the backflow of the previously lost information, while for $\alpha^2 \in (0.382, 0.5]$ ($\alpha^2 \in (0.5, 1)$), it is enhanced with the information leaking into the reservoir in the region of $q \in [q_{c3}, 0.5]$ ($q \in [q_{c5}, 0.5]$), and by the information backflow otherwise.

V. EXPLICIT EXAMPLES

In this section, we illustrate through two examples the main findings of this paper.

A. Lorentzian spectral density reservoirs

The first example we considered is the Lorentzian reservoir with spectral density of the following form [38]
\[
J(\omega) = \frac{1}{2\pi} \frac{\gamma_0 \lambda^2}{(\omega - \omega_0)^2 + \lambda^2},
\]
where $\lambda$ denotes spectral width of the reservoir and is related to the reservoir correlation time via $\tau_B \approx \lambda^{-1}$, while $\gamma_0$ denotes decay rate of the qubit’s excited state in the Markovian limit of flat spectrum and is related to the qubit relaxation time via $\tau_R \approx \gamma_0^{-1}$.

For this reservoir, the factor $q(t)$ is given by [39]
\[
q(t) = e^{-\lambda t} \left( \cosh \frac{dt}{2} + \frac{\lambda}{d} \sinh \frac{dt}{2} \right)^2,
\]
with $d = (\lambda^2 - 2\gamma_0 \lambda)^{1/2}$. Then from Eq. (15) we obtain
\[
\Gamma(t) = \frac{2\gamma_0 \lambda}{\lambda + d \coth \frac{dt}{2}}.
\]
It implies that there are temporary appearance of negative $\Gamma(t)$ only when $\lambda < 2\gamma_0$, for which the reservoir is said to be non-Markovian, and the previously lost quantum information may be fed back into the system again.

By choosing $\lambda/\gamma_0 \in [0.02, 1.98]$, we plotted in Fig. 2 the parameter regions in which the GQDs can be enhanced. For the TDD with $\alpha^2 \in (0, 1)$, HDD with $\alpha^2 \in (0, 0.5]$, and BDD with $\alpha^2 \in (0, 0.382)$, they can always be enhanced by the backflow information from the reservoir to the system. For the HDD with $\alpha^2 > 0.5$ and BDD with $\alpha^2 > 0.382$, one can see that although there are $(\lambda, t)$ regions (the orange shaded areas) in which they are degraded by the backflow information, and regions (the red shaded areas) in which they are enhanced with the information losing into the reservoir, they are in fact very narrow. In most of the $(\lambda, t)$ regions, the temporary flow of information from the reservoir back to the system can enhance the values of them.
the information flowing from the reservoir back to the system. In the following, by fixing \( s < 0 \) for which the HDD (b) and BDD (c) with \( \alpha^2 = 0.7 \) are enhanced, while the orange shaded regions correspond to negative \( \Gamma(t) \) but the HDD and BDD cannot be enhanced.

### B. Ohmic-like spectral density reservoirs

The second type of structured reservoir we considered has the Ohmic-like spectral density of the form

\[
J(\omega) = \eta \omega^s e^{-\omega/\omega_c},
\]

where \( \omega_c \) is the cutoff frequency, and \( \eta \) the dimensionless coupling constant. Their inverse are related to the Markovian character of the system dynamics, we showed that the variation trend of the three GQDs are intimately related to the direction of information flow between the system and the reservoir. We identified explicitly the family of two-qubit measures we adopted are the well-accepted TDD, HDD, and BDD. By solving analytically their dependence on a time-dependent factor \( q(t) \) whose derivative determines the non-Markovian character of the system dynamics, we showed that the trend variation of the three GQDs are intimately related to the direction of information flow between the system and the reservoir. We identified explicitly the family of two-qubit states for which the considered GQDs can be enhanced by the information backflow from the reservoirs to the system, and

**VI. SUMMARY AND DISCUSSION**

In summary, we have investigated evolutions of the GQDs for a pair of qubits interacting independently with their own zero-temperature bosonic structured reservoirs. The discord measures we adopted are the well-accepted TDD, HDD, and BDD. By solving analytically their dependence on a time-dependent factor \( q(t) \) whose derivative determines the non-Markovian character of the system dynamics, we showed that the variation trend of the three GQDs are intimately related to the direction of information flow between the system and the reservoir. We identified explicitly the family of two-qubit states for which the considered GQDs can be enhanced by the information backflow from the reservoirs to the system, and

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**FIG. 2:** (Color online) Comparison between the direction of information flow and enhancement of the GQDs in Lorentzian reservoirs. For (a), \( (\lambda, t) \) in the cyan shaded regions correspond to negative \( \Gamma(t) \) which is a signature of information backflow, and the TDD with \( \alpha^2 \in (0, 1) \), HDD with \( \alpha^2 \in (0, 0.5] \), and BDD with \( \alpha^2 \in (0, 0.382] \) can always be enhanced. For (b) and (c), the cyan (red) shaded regions correspond to negative (positive) \( \Gamma(t) \) for which the HDD (b) and BDD (c) with \( \alpha^2 = 0.7 \) are enhanced, while the orange shaded regions correspond to negative \( \Gamma(t) \) but the HDD and BDD cannot be enhanced.

**FIG. 3:** (Color online) Comparison between the direction of information flow and enhancement of the GQDs in super-Ohmic reservoirs with \( s = 3 \) and \( \omega_c = 2\omega_0 \). For (a), \( (\lambda, t) \) in the cyan shaded regions correspond to negative \( \Gamma(t) \) which is a signature of information backflow, and the TDD with \( \alpha^2 \in (0, 1) \), HDD with \( \alpha^2 \in (0, 0.5] \), and BDD with \( \alpha^2 \in (0, 0.382] \) can always be enhanced. For (b) and (c), the cyan (red) shaded regions correspond to negative (positive) \( \Gamma(t) \) for which the HDD (b) and BDD (c) with \( \alpha^2 = 0.7 \) are enhanced, while the orange shaded regions correspond to negative \( \Gamma(t) \) but the HDD and BDD cannot be enhanced.

It should be note that Fig. 3 is plotted with \( \omega_0 t \in [0, 10] \). When we extend it to a more wide region, we can find there are also temporary enhancement of the HDD and BDD with the information leaking into the reservoirs in the weak-coupling regime. For other values of \( \omega_c \), we also found similar connections between the direction of information flow and enhancement of the GQDs. For concise presentation, we did not plot the corresponding figures here.
states for which the GQDs are enhanced with the information leaking into the reservoirs.

Moreover, by considering two explicit structured reservoirs with the Lorentzian and Ohmic-like spectral density distributions, we showed that although there are regions in which the information backflow cannot enhance the magnitudes of HDD and BDD, and there are also regions in which the HDD and BDD are enhanced by an increase in the amount of information lost into the reservoirs, they are all very narrow. In most of the parameter regions, they are still enhanced by the information backflow. In this sense, non-Markovianity which signifies a backflow of information from the environments to the system, may be a potential resource deserved to be explored for designing schemes by which the GQDs of open quantum systems can be preserved or enhanced.

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