Influence of true triaxial stress paths on mechanical properties of marble

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Abstract. The rock brittle failure is the result of the stress redistribution caused by the excavation of engineering construction, which can be estimated by cyclic loading tests. In this study, a series of true triaxial tests with different stress paths were conducted to investigate the influence of the variations in three principal stresses on the mechanical properties of Jinping marble. The complete stress-strain curves can be obtained due to the strain control in the monotonous loading tests whereas only the pre-peak stress-strain curves can be acquired using the stress control under cyclic loading conditions. In the true triaxial cyclic loading tests, the irreversible strains can be obtained accurately and used to describe the damage evolution of Jinping marble. Test results show that the irreversible strains have the different change trends in the different stress paths and their changes are most sensitive to the minimum principal stress. In addition, Jinping marble in the cyclic loading of the minimum principal stress test has the largest failure angle and numbers of the failure fragments, which indicates that Jinping marble becomes more brittle under this condition.

1. Introduction

In deeply-buried tunnels, rock mass is usually subjected to the true three-dimensional stress state ($\sigma_1 > \sigma_2 > \sigma_3$). Excavation of underground openings can cause in-situ stress redistribution\cite{1-2}. Therefore, rock mass experiences complex stress paths before failure. A better knowledge of the response of rock mass to the variation in different principal stresses is significant to design and construct the deep engineering safely and efficiently.

Influence of confining pressure on strength, deformability and failure of hard rock has been studied extensively\cite{3-6}. Additionally, a large number of researchers\cite{7-10} have investigated that $\sigma_2$ can play a vital role in mechanical properties of hard rock. However, available true triaxial test results are mainly associated with the loading path, that is, loading $\sigma_1$ until failure under constant $\sigma_2$ and $\sigma_3$. After excavation of deep tunnels, brittle fracturing of hard rock, such as spalling and rockburst, is primarily due to unloading of $\sigma_2$ or $\sigma_3$. Thus, it is essential to investigate the influence of different true triaxial stress path tests on the deformation and failure process of hard rock.

In this study, Jinping marble specimens, taken from the Chinese Jinping underground laboratory project (CJPL-II)\cite{11}, are tested under three different true triaxial cyclic loading paths. The main purpose is to investigate the influence of the change in $\sigma_1$, $\sigma_2$ and $\sigma_3$ on deformability and failure of Jinping marble. To validate the test results in these loading and unloading tests, the true triaxial monotonous loading tests are also conducted under corresponding stress states.

2. Test methodology

A series of true triaxial tests in this study were carried out using Northeastern University’s true triaxial apparatus\cite{12-13}. Jinping marble\cite{10, 14} with rectangular prismatic size of 50 × 50 × 100 mm$^3$ can be loaded
maximally to 1,200 MPa ($\sigma_1$), 1,200 MPa ($\sigma_2$) and 100 MPa ($\sigma_3$), respectively. Two mini linear variable differential transformers (LVDTs) and a beam strain gauge are used to measure $\varepsilon_1$, $\varepsilon_2$ and $\varepsilon_3$, respectively.

Test program in true triaxial monotonous loading tests can be divided into three steps, as shown in Figure 1(a). First, the hydraulic stresses ($\sigma_1 = \sigma_2 = \sigma_3$) are loaded at a stress rate of 0.5 MPa/s to the predetermined $\sigma_3$. Then, while keeping $\sigma_3$ constant, the biaxial stresses ($\sigma_1 = \sigma_2$) are increased at a stress rate of 0.5 MPa/s to the predefined $\sigma_2$. Finally, while keeping $\sigma_2$ and $\sigma_3$ constant, $\sigma_1$ is loaded using a strain rate of $5 \times 10^{-6}$/s for $\varepsilon_3$ until the residual stage. In addition, the predefined $\sigma_2$ and $\sigma_3$ in the true triaxial monotonous loading tests are 60 MPa and 30 MPa, which are approximate to in-situ stresses that are measured by the stress relieving method at CJPL-II.

Three stress paths for the true triaxial cyclic loading tests are controlled at a stress rate of 0.2 MPa/s. Specifically, the first stress path is loading and unloading $\sigma_1$ until failure under constant $\sigma_2$ and $\sigma_3$, as shown in Figure 1(b); the second stress path is loading and unloading $\sigma_2$ until failure under constant $\sigma_1$ and $\sigma_3$, as shown in Figure 1(c); the third stress path is loading and unloading $\sigma_3$ until failure under constant $\sigma_1$ and $\sigma_2$, as shown in Figure 1(d). The predefined $\sigma_2$ and $\sigma_3$ in the true triaxial cyclic tests are same as the value in the true triaxial monotonous loading tests. The predefined $\sigma_1$ in the true triaxial cyclic loading tests are determined based on the calculated crack damage stress $\sigma_{cd}$ in the abovementioned true triaxial monotonous loading tests.

![Figure 1](image-url) Schematic diagram of the true triaxial stress paths. (a) monotonous loading stress path and (b-d) three cyclic loading stress paths. The dash line represents the omitted unloading and reloading cycles. Modified from [1-2].

3. Test results

3.1. Stress-strain curves

Figure 2 shows the stress-strain curves of Jinping marble in the true triaxial tests. Due to the usage of the strain control, the complete stress-strain curves are obtained in the true triaxial monotonous loading tests (see Figure 2(a)). Jinping marble has the significant ductile deformability at $\sigma_2 = 60$ MPa and $\sigma_3 = 30$ MPa. In addition, Jinping marble experiences the multiple stress-drops at the post-peak stage, which indicates that the loss of the bearing capacity for Jinping marble is multistage. In the true triaxial cyclic
loading tests, only the pre-peak stress-strain relationships are acquired due to the stress-control (see Figure 2(b-d)). This behavior is due to the abrupt failure once the critical stress state occurs.

Figure 2 Stress-strain curves of Jinping marble under the true triaxial (a) monotonous and (b-d) cyclic loading stress conditions which correspond to stress paths in Figure 1 (a-d), respectively. Modified from [2].

3.2. Characteristics of the Irreversible strains

Rock progressive fracturing is actually the process of the crack development, that is, crack initiation, propagation and coalescent. The generation of cracks can produce irreversible strains. Thus, irreversible strains, according to the loading and unloading cycles, can be accurately calculated in the true triaxial cyclic tests to describe the damage evolution of Jinping marble.

To investigate the sensitivity of the irreversible strains to three principal stresses, the changes in three principal stresses and irreversible strains in three true triaxial cyclic tests are normalized as follows[15, 16]:

\[ D_1 = \frac{\sum \epsilon_{1}^{\text{irr}}}{\sum \epsilon_{1}^{\text{irr}}} \]
\[ D_2 = \frac{\sum \epsilon_{2}^{\text{irr}}}{\sum \epsilon_{2}^{\text{irr}}} \]
\[ D_3 = \frac{\sum \epsilon_{3}^{\text{irr}}}{\sum \epsilon_{3}^{\text{irr}}} \]
\[ D_v = \frac{\sum \epsilon_v^{\text{irr}}}{\sum \epsilon_v^{\text{irr}}} \]
\[ D_\sigma = \frac{\sum (\Delta \sigma)}{\sum (\Delta \sigma)} \]

where \( D_1, D_2, D_3 \) and \( D_v \) are the normalized irreversible strains in the maximum, intermediate and minimum principal stress and volumetric directions, respectively; \( D_\sigma \) is the normalized changes in the principal stresses; \( \epsilon_1^{\text{irr}}, \epsilon_2^{\text{irr}}, \epsilon_3^{\text{irr}}, \epsilon_v^{\text{irr}} \) and \( \Delta \sigma \) are the irreversible maximum, intermediate and
minimum principal strain, volumetric strain, and the changes in stress in an individual cycle, respectively; \(i\) and \(j\) are the number of cycles, and \(n\) is the total number of cycles.

Figure 3 shows the relationships between the normalized irreversible strains \((D_1, D_2, D_3\), and \(D_v\)) and changes in the principal stresses \((\Delta \sigma)\) for Jinping marble in three true triaxial cyclic loading tests. Generally, the values of the normalized irreversible strains are different, that is, \(D_1 > D_2 > D_3 > D_v\) except for the cyclic loading \(\sigma_3\) test \((D_2 > D_1 > D_3 > D_v)\). With increasing \(D_0\), the normalized irreversible strains in three different stress paths exhibit different change trends. More specifically, in the true triaxial cyclic loading \(\sigma_1\) and \(\sigma_2\) tests, \(D_1, D_2, D_3\) and \(D_v\) increase slightly and then rapidly in the last two cycles (nearly \(0.9D_0\)). The increase rates for \(D_1, D_2, D_3\) and \(D_v\) in the true triaxial cyclic loading \(\sigma_3\) test are relatively stable, and a small increase in \(D_0\) can cause an obvious increase in \(D_1, D_2, D_3\) and \(D_v\). This behavior indicates that the variations in the irreversible strains are more sensitive to changes in \(\sigma_3\) than other principal stresses.

![Figure 3](image)

**Figure 3** Relationships between the normalized irreversible strains and changes in principal stresses under true triaxial cyclic loading stress conditions which correspond to stress paths in Figure 1 (c-d), respectively.

To quantify the relationships between the irreversible strains and principal stress, an empirical formula which can reflect the stress dependency is proposed as follows:

\[
D = A_1 \exp \left( \frac{\Delta \sigma}{t_1} \right) + A_2 \exp \left( \frac{\Delta \sigma}{t_2} \right) \tag{6}
\]

where \(A_1, A_2, t_1\) and \(t_2\) are empirical parameters and their fitting results are shown in Table 1. Figure 3 and Table 1 show that the proposed formula has good fitting results with \(R^2 > 0.98\). In addition, the fitting lines in Figure 3 pass through two critical points: \((0, D_0)\) and \((1, 1)\). Here, \(D_0\) is defined as the initial damage value, which quantifies the damage level of Jinping marble before the loading and unloading cycles. Therefore, these fitting parameters can be written:

\[
D_0 = A_1 + A_2 \tag{7}
\]

\[
1 = A_1 \exp \left( \frac{1}{t_1} \right) + A_2 \exp \left( \frac{1}{t_2} \right) \tag{8}.
\]
In other words, the parameters $A_1$ and $A_2$ affect the initial damage level together. Although the shape of the fitting line is controlled by the four model parameters, it is mainly influenced by the value of the parameters $A_1$ and $t_1$ since the value of the parameter $A_2$ is extremely small.

### Table 1 Fitting parameters in the proposed model in three true triaxial cyclic loading tests.

| Stress path | Item | $A_1$   | $A_2$   | $t_1$   | $t_2$   | $R^2$ |
|-------------|------|---------|---------|---------|---------|-------|
| cyclic $\sigma_1$ (Figure 1(b)) | $D_1$ vs. $D_\sigma$ | 2.86E-03 | 6.34E-10 | 2.29E-01 | 4.78E-02 | 0.9969 |
| cyclic $\sigma_2$ (Figure 1(c)) | $D_2$ vs. $D_\sigma$ | 2.42E-03 | 5.37E-10 | 2.45E-01 | 4.72E-02 | 0.9976 |
| cyclic $\sigma_3$ (Figure 1(d)) | $D_3$ vs. $D_\sigma$ | 8.16E-04 | 4.00E-12 | 1.92E-01 | 3.84E-02 | 0.9993 |

### 3.3. Failure characteristics

Fig. 4 shows the three-dimensional failure images of Jinping marble in the true triaxial tests. The number of rock fragments is responsible for the failure intensity. As shown in Figure 4(a), only one macro failure plane passes through the rock specimen in the true triaxial monotonous loading tests. The true triaxial cyclic loading tests produce multiple macro failure planes, especially for the cyclic loading $\sigma_3$ test (see Figure 4 (d)). The cyclic loading $\sigma_3$ test also has the largest failure angle, which is defined by the angle between the average failure plane and the $\sigma_1$ applied plane. Therefore, the rock failure intensity is more sensitive to the changes in $\sigma_3$.

![Failure images of Jinping marble under the true triaxial (a) monotonous and (b-d) cyclic loading stress conditions which correspond to stress paths in Figure 1 (a-d), respectively.](image)

### 4. Conclusions

In the true triaxial monotonous loading tests, the complete stress-strain curves containing the post-peak stages can be obtained due to the strain control, and Jinping marble exhibits the obvious ductile deformability. In the true triaxial cyclic loading tests, the pre-peak stress-strain curves can only be acquired due to the stress control, and irreversible strains can be obtained accurately using corresponding cycles.

The irreversible strains have the different change trends in the different stress paths. The variations in the irreversible strains are most sensitive to the minimum principal stress. An empirical formula is proposed to describe the irreversible strains in the three principal stress and volumetric directions.

The marble specimen in the true triaxial cyclic loading $\sigma_3$ test has the largest failure angle and number of the failure fragments. This behavior shows that Jinping marble becomes more brittle in this stress path.
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