Direct extraction of the chiral quark condensate
and bounds on the light quark masses

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Abstract

We select sum rules from which one can extract directly reliable limits on the size of the chiral symmetry breaking light quark condensate $\langle \bar{\psi}\psi \rangle$. Combined results from the nucleon and $B^*-B$ mass-splitting sum rules give a result compatible with the standard value: $\langle \bar{\psi}\psi \rangle(1 \text{ GeV}) \approx (-229 \text{ MeV})^3$, through the determinations of the quark-gluon mixed condensate. The vector form factor of the $D \to K^*\ell\nu$ semi-leptonic decay leads to the range $0.6 \leq \langle \bar{\psi}\psi \rangle/(-229 \text{ MeV})^3 \leq 1.5$. The upper limit combined with the Gell-Mann-Oakes-Renner (GMOR) relation implies the interesting lower bound on the sum of light quark masses: $(m_u + m_d)(1 \text{ GeV}) \geq 9.4 \text{ MeV}$, which combined with the ratio of light quark masses from chiral perturbation theory leads to $m_s(1 \text{ GeV}) \geq (121 \pm 12) \text{ MeV}$. The lower limit combined with the positivity of the $m_s^2$ contribution to the GMOR relation leads to the upper bound : $(m_u + m_d)(1 \text{ GeV}) \leq 15.7 \text{ MeV}$, which is independent on the nature of chiral symmetry breaking.
1 Introduction

The chiral condensate of light quarks \( \langle \bar{\psi}\psi \rangle \) is one of the fundamental parameters of non-perturbative QCD and chiral symmetry. Therefore it is of prime interest to determine it, as directly as possible, from experimental data. It is related to the pion decay constant \( f_\pi = 93.3 \text{ MeV} \), the pion mass \( m_\pi \) and the sum of light quark mass \( (m_u + m_d) \) by the Gell-Mann Oakes Renner (GMOR) relation [13]:

\[
m^2 f^2 = -(m_u + m_d) \langle \bar{\psi}\psi \rangle + O(m_d^2)
\]  \( (1) \)

In the standard treatment of chiral symmetry breaking (chiral perturbation theory [2]), the light quark masses are very small \( (m_q \leq 10 \text{ GeV}) \) and therefore the \( O(m_q^2) \) terms are negligible. In other approaches (generalized chiral perturbation theory [3]) the quark masses are not so small and therefore the \( O(m_q^2) \) term might become important or even dominant. A precise determination of the condensate is therefore of great theoretical interest for clarifying the nature of the mechanism of chiral symmetry breaking. In the standard approach, the GMOR-relation (Eq. 1) and the value of the quark condensate also allow a determination of the absolute values of the light quark masses \( u \) and \( d \) independently of other sum rules approaches, such as the pseudoscalar [4, 5, 6] for \( (m_u + m_d) \) and the scalar sum rules for \( (m_d - m_u) \) [7], and the present lattice extrapolations [3].

2 The chiral condensate from QCD sum rules.

The chiral condensate plays an important role in the QCD sum rule [10] analysis of many channels and the standard value [11, 12] of \( \langle \bar{\psi}\psi \rangle (1 \text{ GeV}) = -(229 \pm 9 \text{ MeV})^3 \) has led to interesting results [12], many of which have been checked experimentally. However, if one wants to extract the value of \( \langle \bar{\psi}\psi \rangle \) directly from phenomenological data, correlations with other (non) perturbative parameters limit the accuracy severely. In the light meson sum rules, the chiral quark condensate effects are relatively negligible compared with the ones of the gluon condensate \( \langle \alpha_s G^2 \rangle \), as it is often multiplied by the small light quark mass. In the nucleon sum rules [13, 14, 15, 12], which seem, at first sight, a very good place for determining \( \langle \bar{\psi}\psi \rangle \), we have two form factors for which spectral sum rules can be constructed, namely the form factor \( F_1 \) which is proportional to the Dirac matrix \( \gamma \cdot p \) and \( F_2 \) which is proportional to the unit matrix. In \( F_1 \) the four quark condensates play an important role, but these are not chiral symmetry breaking and are related to the condensate \( \langle \bar{\psi}\psi \rangle \) only by the factorization hypothesis [10] which is known to be violated by a factor 2-3 [13, 16, 12]. The form factor \( F_2 \) is dominated by the condensate \( \langle \bar{\psi}\psi \rangle \) and the mixed condensate \( \langle \bar{\psi}\gamma \sigma G\psi \rangle \), such that the baryon mass is essentially determined by the ratio \( M_0^2 \) of the two condensates:

\[
M_0^2 = \langle \bar{\psi}\gamma \sigma G\psi \rangle / \langle \bar{\psi}\psi \rangle
\]  \( (3) \)

Therefore from nucleon sum rules one gets a rather reliable determination of \( M_0^2 \) [14, 15]:

\[
M_0^2 = (.8 \pm .1) \text{ GeV}^2.
\]  \( (4) \)

A sum rule based on the ratio \( F_2/F_1 \) would in principle be ideally suited for a determination of \( \langle \bar{\psi}\psi \rangle \) but this sum rule is completely unstable [15] due to fact that odd parity baryonic excitations contribute with different signs to the spectral functions of \( F_1 \) and \( F_2 \). In the correlators of heavy mesons (\( B, B^* \) and \( D, D^* \)) the chiral condensate gives a significant direct contribution in contrast to the light meson sum rules [12], since, here, it is multiplied by the heavy quark mass. However, the dominant contribution to the meson mass comes from the heavy quark mass and therefore a change of a factor two in the value of \( \langle \bar{\psi}\psi \rangle \) leads only to a negligible shift of the mass. However, from the \( B-B^* \) splitting one gets a precise determination of the mixed condensate \( \langle \bar{\psi}\gamma \sigma G\psi \rangle \) with the value [17]

\[
\langle \bar{\psi}\gamma \sigma G\psi \rangle = -(9 \pm 1) \times 10^{-3} \text{ GeV}^5,
\]  \( (5) \)

which combined with the value of \( M_0^2 \) given in Eq. [10] gives our first result for the value of \( \langle \bar{\psi}\psi \rangle \) at the nucleon scale:

\[
\langle \bar{\psi}\psi \rangle (M_N) = -[(225 \pm 9) \text{ MeV}]^3
\]  \( (6) \)

\footnote{The errors quoted here and in the following come from the variations of the different input parameters and of the sum rule variables at given order of perturbation theory.}
in good agreement with the standard value in Eq. (2).

Other good channels for the determination of $\langle \bar{\psi} \psi \rangle$ are the sum rules for the semileptonic decays of heavy pseudoscalars to light vector mesons \cite{18, 19, 20}, where the chiral condensate plays a dominant role. For the decay $D \to K^* \ell \nu$ there exist now rather precise data for the semileptonic form factors. In a previous analysis \cite{18, 19} the range of the exponential sum rules parameters $\tau_1, \tau_2$ of the three-point function was fixed by the corresponding two point functions and the choice of standard parameters allowed successful predictions for the form factors. In this analysis where we want to extract from experiment the fundamental predictions for the form factors. In this analysis where we want to extract from experiment the fundamental quantity $\langle \bar{\psi} \psi \rangle$, we do not make a preselection and choose the values of $\tau_1$ and $\tau_2$ by requiring stability of the three-point function in both (uncorrelated) parameters. Making use of the standard definition of the semileptonic decay form factors (see e.g. PDG 96 \cite{21}) we find that the above-mentioned stability criterion is only fulfilled for the vector form factor $V$. For that quantity at zero momentum transfer, we obtain the sum rule:

$$V(0) = \frac{m_c(m_D + m_{K^*})}{4f_Df_{K^*}m_Dm_{K^*}} \exp[(m_D^2 - m_{c}^2)\tau_1 + m_{K^*}^2\tau_2]$$

$$\times \langle \bar{\psi} \psi \rangle \left\{ -1 + M_D^2\left(-\frac{\tau_1}{3} + \frac{m_c^2}{4}\tau_1^2 + \frac{2m_c^2 - \delta_{c}m_s}{6}\tau_1\tau_2\right) 
- \frac{16\pi}{9}\alpha_s\rho(\bar{\psi} \psi)\frac{2m_c}{9}\tau_1\tau_2 - \frac{m_c^3}{36}\tau_1^4 
- \frac{2m_c^3 - \delta_{c}m_s}{36}\tau_1^2\tau_2 + \frac{m_c^2}{9}\tau_1^2 + \frac{2\delta_{c}}{9}\tau_1\tau_2 + \frac{4\tau_2}{9}m_c \right\}$$

$$+ \frac{e^{m_c^2\tau_1}}{\langle \bar{\psi} \psi \rangle} \int_0^{s_{20}} ds_2 \int_{s_2 + m_c^2}^{s_{10}} ds_1 \rho_c(s_1, s_2)e^{-s_1\tau_1 - s_2\tau_2}$$

with

$$\rho_c(s_1, s_2) = \frac{3}{4\pi^2(s_1 - s_2)^3} \times$$

$$\left\{ m_c((s_1 + s_2)(s_1 - m_{c}^2) - 2s_1s_2) + m_c((s_1 + s_2)s_2 - 2s_2(s_1 - m_{c}^2)) \right\}$$

The factor $\rho$ expresses the uncertainty in the factorization of the four quark condensate. In our numerical analysis, we start from the following set of standard parameters given in Eqs. (2) and (4) and: $f_{K^*} = 0.15$ GeV ($f_{\pi} = 93.3$ MeV), $m_c$(pole) = (1.42 ± 0.02) GeV, where the value of the charm quark mass comes from \cite{22}. We enlarge the value of the charm pole mass until half of $M_{J/\psi}$ (so-called constituent mass) i.e $m_c = 1.42 \pm 0.13$ GeV, in order to be conservative. We use the recent value of the four-quark condensate correlated to the value of the gluon condensate obtained in \cite{23}, which is consistent with the value $\rho = 2 \sim 3$ for the condensate value in Eq. (2). The value of $f_{D^*} \simeq (1.35 \pm 0.07)f_{\pi}$ is consistently determined by a two-point function sum rule including radiative corrections, where the sum rule expression can, e.g., be found in \cite{22}. The following parameters enter only marginally: $m_c(1$ GeV) = (0.15 0.19) GeV, $s_{10} = (5 \sim 7)$ GeV$^2$, $s_{20} = (1.5 \sim 2)$ GeV$^2$. With the previous inputs, we obtain a stationary point in the two variables $\tau_1$ and $\tau_2$ at the values 0.8 and 0.9 GeV$^{-2}$ respectively and $V(0) = 1.12$ which agrees well with the experimental value $V(0) = (1.1 \pm 0.2)$ \cite{21}. Normalized to $\langle \bar{\psi} \psi \rangle$, the relative contributions of the other terms are respectively: -0.32 for the mixed condensate, +0.01 for the four quark condensate and 0.25 for the perturbative one, thus showing that at these values of the $\tau$-variables the local OPE converges quite well, and appears to be in a good control. Now, we use the experimental value $V(0) = (1.1 \pm 0.2)$. In this way, we obtain the following range of values for the chiral condensate evaluated at the $\tau$-sum rule scale of 0.8-0.9 GeV$^{-2}$:

$$0.6 \leq \langle \bar{\psi} \psi \rangle/(-0.229 \text{GeV})^3 \leq 1.5,$$

where the lower (resp. higher) bound corresponds to the small (resp. big) value of the charm quark mass. Releasing the absolute value of the mixed condensate and keeping the ratio $M_D^2$ fixed would weaken the upper limit in Eq. (9) to 2.2 since the mixed condensate has the opposite sign of the quark condensate. The result in Eq. (9) of the analysis of the semileptonic decay is thus in perfect agreement with the result in Eq. (6) of the combined nucleon and $B-B^*$ splitting analysis. The previous lower bound excludes smaller values of $\langle \bar{\psi} \psi \rangle$ which may appear within the framework of generalized chiral perturbation theory \cite{8} and questions the reliability of the recent estimate based on variational approach within perturbation theory \cite{24}.
3 Bounds and values of the light quark masses

The previous upper limit is specially interesting since it gives, through the GMOR relation in Eq. (9), also a lower limit on the sum of light quark masses:

\[(m_u + m_d)(1 \text{ GeV}) \geq 9.4 \text{ MeV}.\] (10)

The lower bound in Eq. (9) can be exploited by using the positivity of the \(m_q^2\) term [3], which could be dominant in the generalized chiral perturbation theory approach. In this way, one can also derive the upper limit, independently on the way the chiral symmetry is realized:

\[(m_u + m_d)(1 \text{ GeV}) \leq 15.7 \text{ MeV}.\] (11)

Therefore, one can conclude the range of values for the sum of light quark masses:

\[9.4 \text{ MeV} \leq (m_u + m_d)(1 \text{ GeV}) \leq 15.7 \text{ MeV},\] (12)

which is independent on the nature of the realization of chiral symmetry breaking. Combining the lower bound, obtained within a GMOR realization of the chiral symmetry breaking, with the ratio of the light quark masses from chiral perturbation theory, namely [3]:

\[\frac{m_s}{1/2(m_u + m_d)} = 25.7 \pm 2.6,\] (13)

one obtains:

\[m_s(1 \text{ GeV}) \geq (121 \pm 12) \text{ MeV}.\] (14)

The lower bounds in Eqs. (10) and (14) are in agreement with the direct determinations of the quark masses from the pseudoscalar sum rule [5, 6], but disagree with recent extrapolations from Monte Carlo simulations [9]. These lower bounds also agree with the lower bounds from the pseudoscalar [4, 5] and scalar [6] sum rules. However, we expect that, in view of the value of the chiral condensate obtained from the combined nucleon and \(B^*-B\) sum rules, the lower bounds in Eqs. (10) and (14), though interesting, is relatively weak compared to the real estimate. Indeed, the value of the quark condensate in Eq. (6) from these combined sum rules leads to the estimate:

\[(m_u + m_d)(1 \text{ GeV}) \simeq 15 \text{ MeV}, \quad m_s(1 \text{ GeV}) \simeq 182 \text{ MeV},\] (15)

which is in good agreement with the pseudoscalar [4, 5, 6] and earlier scalar [7, 8] sum rule results. The above range of values of the strange quark mass also agrees with the recent estimate of \(m_s(1 \text{ GeV})=(197 \pm 29)\) MeV from the \(e^+e^- \rightarrow \text{hadrons}\) [11] and the preliminary result of about 200 MeV from \(\tau\)-decay data [25]. It should be noted that some of these previous analyses, especially [6] starts from more general principles than our investigation and therefore the limits are somewhat weaker.

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References

[1] M. Gell-Mann, R. Oakes and B. Renner, Phys. Rev. **175** (1968) 2195
[2] H. Leutwyler, QCD 97, Montpellier (1997) and references therein.
[3] J. Stern, QCD 97, Montpellier (1997) and references therein;
   M. Knecht, QCD 94, Montpellier (1994), Nucl. Phys. (Proc. Suppl.) **B,C39** (1995) and references therein.
[4] C. Becchi, S. Narison, E. de Rafael and F.J. Yndurain, Z. Phys. **C8** (1981) 57;
   S. Narison and E. de Rafael, Phys. Lett. **B103** (1981) 57;
   F.J. Yndurain, [hep-ph/9708300](http://arxiv.org/abs/hep-ph/9708300) (1997).
[5] C.A. Dominguez and E. de Rafael, Ann. Phys. **174** (1987) 372;
   S. Narison, Riv. Nuovo Cim. **Vol 10, 2** (1987) 1; Phys. Lett. **B216** (1989) 191;
   J. Bijnens, J. Prades and E. de Rafael, Phys. Lett. **B348** (1995) 226;
   J. Prades, QCD 97, Montpellier (1997).
[6] L. Lellouch, E de Rafael and J. Taron, [hep-ph/9707523](http://arxiv.org/abs/hep-ph/9707523) (1997);
   E. de Rafael, QCD 97, Montpellier (1997) and references therein.
[7] S. Narison, N. Paver, E. de Rafael and D. Treleani, Nucl. Phys. **B197** (1982) 57.
[8] M. Jamin and M. Munz, Z. Phys. **C66** (1995) 633;
   K.G. Chetyrkin, D. Pirjol and K. Schilcher, [hep-ph/9612394](http://arxiv.org/abs/hep-ph/9612394) (1996).
[9] C.R. Allton, QCD 97, Montpellier (1997) and references therein;
   H. Hoeber (sesam collaboration), QCD 97, Montpellier;
   R. Gupta and T. Bhattacharya, Phys. Rev. **D55** (1997) 7203.
[10] M.A. Shifman, A.I. Vainshtein and V.I. Zakharov, Nucl. Phys. B147 (179) 385, 448.
[11] S. Narison, Phys. Lett. **B358** (1995) 113.
[12] S. Narison, *QCD spectral sum rules*, Lecture Notes in Physics, **Vol 26** (1989) ed. World Scientific;
   Acta Phys. Polonica. **26** (1995) 687; *Recent Progress in QCD spectral sum rules*, (to appear).
[13] Y. Chung, H.G. Dosch, M. Kremer and D. Schall, Phys. Lett 102B (1981) 175, Nucl. Phys. **B197** (1982) 57.
[14] B.L. Ioffe, Nucl. Phys. **B188** (1981) 317 E: **B191** (181) 591.
[15] H.G. Dosch, M. Jamin and S. Narison, Phys. Lett. **B220** (1989) 251.
[16] G. Launer, S. Narison and R. Tarrach, Z. Phys. **C26** (1984) 433.
[17] S. Narison, Phys. Lett. **B210** (1988) 238.
[18] P. Ball, V.M.Braun, H.G. Dosch and M. Neubert, Phys. Lett. **B259** (1991) 481
[19] P.Ball, V.M. Braun and H.G. Dosch, Phys. Rev. D44 (1991) 3567.
[20] S. Narison, Phys. Lett. **B283** (1992) 384; **B345** (1995) 166.
[21] R.M. Barnett et al., Phys. Rev. **D54** (1996)1.
[22] S. Narison, Phys. Lett. **B341** (1994) 73.
[23] S. Narison, Phys. Lett. **B361** (1995) 121.
[24] G. Arvanitis, F. Geniet, J.L. Kneur and A. Neveu, [hep-ph/9609247](http://arxiv.org/abs/hep-ph/9609247) (1996);
   J.L.Kneur, QCD 97, Montpellier (1997).
[25] M. Chen (Aleph collaboration), QCD 97, Montpellier (1997); M. Davier, Nucl. Phys. (proc. Suppl.) **B, C55** (1997) 395.