Spectroscopic factor strengths in $^{27}$Al($d$, $^{3}$He)$^{26}$Mg and $^{27}$Al($d$, $t$)$^{26}$Al reactions

P.C. Srivastava and Vikas Kumar

Department of Physics, Indian Institute of Technology, Roorkee 247 667, India

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Abstract

In the present work we calculated spectroscopic factor strengths for recently measured experimental data for $^{1}$p and $^{1}$n pick-up reactions $^{27}$Al($d$, $^{3}$He)$^{26}$Mg [Phys. Rev. C 93, 044601 (2016)] and $^{27}$Al($d$, $t$)$^{26}$Al [Phys. Rev. C 91, 054611 (2015)] within the framework of the shell-model. We performed calculations with USDA and USDB effective interactions, results are in a good agreement with the experimental data.

1 Introduction

The nature and occupancy of the single-particle orbits for a nucleus can be determine from the spectroscopic factors (SF). Experimentally the SF can be measured with single-particle transfer reactions. These reactions are of two types, first one is stripping in which one nucleon is stripped from the incoming projectile, while the second one is pick-up reaction in which one nucleon is picked up by the projectile. The examples of neutron transfer pick-up reactions are like ($p$, $d$), ($d$, $t$) and ($^{3}$He, $\alpha$), while stripping reactions are like ($d$, $p$), ($t$, $d$) and ($\alpha$, $^{3}$He) [1]. The SF is defined by a matrix element between initial state and final state corresponding to entrance channel and exit channels, respectively. It is possible to describe the capture or emission of single-nucleons in stellar burning processes by calculating the nuclear matrix elements for a single-nucleon spectroscopic factors in the nuclear structure calculations.

The study of SF in different region of nuclear chart are reported in refs. [2, 3, 4, 5, 6, 7, 8]. Survey of excited state neutron spectroscopic factors for $Z = 8 - 28$ nuclei are reported by Tsang et al [9]. In this work they extracted 565 neutron spectroscopic factors for $sd$ and $fp$ shell nuclei by analyzing ($d$, $p$) angular distributions, they compared the experimental results with shell-model.

To study different excited states and their spectroscopic factors for $^{26}$Mg, earlier many experimental results were reported in refs. [10, 11, 12, 13, 14, 15, 16]. Recently, the spectroscopic factors for $^{26}$Mg is reported in ref. [17] using $^{27}$Al($d$, $^{3}$He)$^{26}$Mg reaction. The structure of $^{27}$Al using different reaction channels were reported in refs. [18, 19, 20, 21, 22, 23, 24]. The experimental results for SF of 14 excited states for $^{27}$Al using $^{27}$Al($d$, $t$)$^{26}$Al reaction is reported in ref. [26]. The study of $^{26}$Al and $^{26}$Mg are important for astrophysics point of view. The massive stars throughout the Galaxy dominate in the production of $^{26}$Al [25], and it decays by $\beta^+$ to $^{26}$Mg.

In the present work our aim is to study recently available experimental spectroscopic factors data for $^{27}$Al($d$, $^{3}$He)$^{26}$Mg [17] and $^{27}$Al($d$, $t$)$^{26}$Al [26] in the framework of nuclear shell-model.
Table 1: Extracted values of $C^2S$ for different excited states of $^{26}$Mg from the reaction $^{27}$Al(d, $^3$He) at 25 MeV. We have taken experimental data from ref. [17]. Here $l=0$, $l=2$ are $s_{1/2}$ and $d_{5/2}$ orbitals, respectively.

| Expt. J* | USDA keV | USDB keV | $C^2S$ [Expt.] | $C^2S$ [USDA] | $C^2S$ [USDB] |
|----------|----------|----------|----------------|--------------|--------------|
|          |          |          | $l=0$         | $l=2$        | $l=0$        | $l=2$        |
| 0 0+     | 0 0      | 0 0      | 0.17±0.05     | 0.013 0.846 0.014 0.876 |
| 1806 2+  | 1938 1897| 3109 3007| 0.002 0.57±0.14| 0.021 0.888 0.021 0.909 |
| 2935 2+  | 4377 4317| 4407 4365| 0.072 0.013 0.846 0.014 0.876 |
| 3 3+     | 4450 4449| 4457 4482| 0.002 0.17±0.03| 0.021 0.888 0.021 0.909 |
| 2 2+     | 4616 4586| 5363 5286| 0.002 0.13±0.03| 0.021 0.888 0.021 0.909 |
| 4 4+     | 5897 5892| 5920 5882| 0.002 0.13±0.03| 0.021 0.888 0.021 0.909 |
| 3 3+     | 6165 6179| 6167 6182| 0.036 0.006 0.028 0.006 |

2 Shell Model Analysis

We performed the shell-model calculation for spectroscopic factor using the two most suitable effective interactions USDA and USDB for sd shell nuclei using shell-model code NuShellX [27]. The Hamiltonians, USDA and USDB are based on a renormalized $G$ matrix by fitting two-body matrix elements with experimental data for binding energies and excitation energies for the $sd$ shell nuclei [28, 29]. The USDA and USDB interactions are fitted by varying 30 and 56 linear combinations of one- and two-body matrix elements, respectively. The rms deviations of 170 keV (USDA) and 130 keV (USDB) were obtained between experimental and theoretical energies. In the present work before calculating the SF, we first examine the wavefunctions of concerned nuclei. The comparison between the calculated and experimental levels for $^{26}$Mg and $^{26}$Al reported in Figs. 1-2. The results of USDB interaction is much better than USDA interaction. In the case of $^{26}$Al we calculated only the g.s.(0$^+$). With the comparison of theoretical and experimental spectroscopic factors it is possible to check the predictive power of effective interaction used in the shell-model calculations.

We can define the spectroscopic factor in terms of the reduced matrix elements of $a^+$ by:

$$S = \frac{1}{2J+1} |\langle \psi^A \omega_J | a^+_k | \psi^{A-1} \omega_{J'} \rangle|^2.$$  (1)

where the $(2J+1)$ factor is by convention associated with heavier mass $A$. Here, $\omega$ indices distinguish the various basis states with the same $J$ value.

2.1 Calculation of $C^2S$ for 1p pick-up reaction $^{27}$Al(d, $^3$He)$^{26}$Mg

Experimentally the states of $^{26}$Mg [17] were studied by assuming pick-up from $d_{5/2}$ orbital only, and also few states by assuming configuration mixing of two lower orbital of $sd$ shell - $d_{5/2}$ and $s_{1/2}$ single particle orbitals. In the Table I, we compared the experimental $C^2S$ values with shell-model results for USDA and USDB effective interactions. As in the experiment the $C^2S$ values were very
Figure 1: (Color Online) Comparison between calculated and experimental [?] data for $^{26}\text{Mg}$.

Table 2: Extracted values of $C^2S$ for different excited states of $^{26}\text{Al}$ from the reaction $^{27}\text{Al}(d, t)$. We have taken experimental data from ref. [26]. Here $l = 0$, $l = 2$ are $s_{1/2}$ and $d_{5/2}$ orbitals, respectively.

| Expt. keV | J$^+$ | $C^2S$ [USDA] keV | $C^2S$ [USDB] keV | $C^2S$ [Expt.] | $C^2S$ [USDA] l=0 | $C^2S$ [USDA] l=2 | $C^2S$ [USDB] l=0 | $C^2S$ [USDB] l=2 |
|-----------|-------|-------------------|-------------------|----------------|----------------|----------------|----------------|----------------|
| 0         | 5+    | 0                 | 0                 | ...           | 0.73±0.21       | ...             | 1.066          | ...             |
| 228.3     | 0+    | 234               | 229               | ...           | 0.09±0.03       | ...             | 0.137          | ...             |
| 416.8     | 3+    | 541               | 520               | ...           | 0.32±0.07       | 0.180           | 0.002          | 0.201          |
| 1057.7    | 1+    | 813               | 1034              | ...           | 0.17±0.05       | 0.345           | 0.031          | 0.004          |
| 1759.0    | 2+    | 1675              | 1583              | ...           | 0.038±0.006     | 0.028           | 0.004          | 0.031          |
| 1850.6    | 1+    | 1800              | 1818              | ...           | 0.019±0.004     | ...             | 0.005          | 0.007          |
| 2068.8    | 2+    | 2172              | 2126              | ...           | 0.26±0.06       | 0.007           | 0.423          | 0.007          |
| 2365.1    | 3+    | 2369              | 2155              | ...           | 0.13±0.02       | 0.0004          | 0.146          | 0.008          |
| 2545.3    | 3+    | 2964              | 2402              | ...           | 0.16±0.03       | 0.0000          | 0.006          | 0.002          |
| 3159.8    | 2+    | 3343              | 3236              | ...           | 0.06±0.01       | 0.010           | 0.044          | 0.010          |
| 3402.6    | 5+    | 3366              | 3398              | ...           | 0.06±0.01       | ...             | 0.080          | ...             |
| 4705.3    | 4+    | 4736              | 4857              | ...           | 0.27±0.08       | ...             | 0.001          | ...             |

26$\text{Mg}$
Table 3: Wavefunctions of different states for $^{26}\text{Mg}$. The spectroscopic factors corresponding to these states are shown in the table I.

| $J^\pi$ | USDA | % | Configuration | USDA | % | Configuration |
|---------|-------|---|---------------|-------|---|---------------|
| 0$^+$   | 60    | $\pi(d_{5/2})^4 \otimes \nu(d_{5/2})^6$ | 62    | $\pi(d_{5/2})^4 \otimes \nu(d_{5/2})^6$ |
| 2$^+$   | 50    | $\pi(d_{5/2})^4 \otimes \nu(d_{5/2})^6$ | 52    | $\pi(d_{5/2})^4 \otimes \nu(d_{5/2})^6$ |
| 2$^+$   | 29    | $\pi(d_{5/2})^3(s_{1/2})^1 \otimes \nu(d_{5/2})^5(s_{1/2})^1$ | 29    | $\pi(d_{5/2})^4 \otimes \nu(d_{5/2})^6(s_{1/2})^1$ |
| 3$^+$   | 34    | $\pi(d_{5/2})^4 \otimes \nu(d_{5/2})^5(s_{1/2})^1$ | 35    | $\pi(d_{5/2})^4 \otimes \nu(d_{5/2})^5(s_{1/2})^1$ |
| 4$^+$   | 52    | $\pi(d_{5/2})^4 \otimes \nu(d_{5/2})^6$ | 52    | $\pi(d_{5/2})^4 \otimes \nu(d_{5/2})^6$ |
| 2$^+$   | 27    | $\pi(d_{5/2})^4 \otimes \nu(d_{5/2})^6$ | 25    | $\pi(d_{5/2})^4 \otimes \nu(d_{5/2})^6$ |
| 2$^+$   | 29    | $\pi(d_{5/2})^3(s_{1/2})^1 \otimes \nu(d_{5/2})^6$ | 33    | $\pi(d_{5/2})^3(s_{1/2})^1 \otimes \nu(d_{5/2})^6$ |
| 2$^+$   | 30    | $\pi(d_{5/2})^3 \otimes \nu(d_{3/2})^3(d_{5/2})^5$ | 33    | $\pi(d_{5/2})^3 \otimes \nu(d_{3/2})^3(d_{5/2})^5$ |
| 4$^+$   | 23    | $\pi(d_{5/2})^4 \otimes \nu(d_{5/2})^5(s_{1/2})^1$ | 26    | $\pi(d_{5/2})^4 \otimes \nu(d_{5/2})^5(s_{1/2})^1$ |
| 4$^+$   | 27    | $\pi(d_{5/2})^4 \otimes \nu(d_{5/2})^5(s_{1/2})^1$ | 31    | $\pi(d_{5/2})^4 \otimes \nu(d_{5/2})^5(s_{1/2})^1$ |
| 3$^+$   | 48    | $\pi(d_{5/2})^4 \otimes \nu(d_{5/2})^5(s_{1/2})^1$ | 50    | $\pi(d_{5/2})^4 \otimes \nu(d_{5/2})^5(s_{1/2})^1$ |

Figure 2: (Color Online) Comparison between calculated and experimental [NNDC] data for $^{26}\text{Al}$. 

26$^{\text{Al}}$
Figure 3: Comparison between calculated and experimental [17] data of spectroscopic factors for $^{26}$Mg.

Table 4: Wavefunctions of different states for $^{26}$Al. The spectroscopic factors corresponding to these states are shown in the table II.

| $J^+$ | USDA | %     | Configuration                       | %     | Configuration                       |
|------|------|-------|-------------------------------------|-------|-------------------------------------|
|      |      |       |                                     |       |                                     |
| 5+   | 59   | π(d$_{5/2}$)$^0$ ⊗ ν(d$_{5/2}$)$^0$ | 62    | π(d$_{5/2}$)$^0$ ⊗ ν(d$_{5/2}$)$^0$ |
| 0+   | 63   | π(d$_{5/2}$)$^5$ ⊗ ν(d$_{5/2}$)$^0$ | 65    | π(d$_{5/2}$)$^5$ ⊗ ν(d$_{5/2}$)$^5$ |
| 3+   | 16   | π(d$_{5/2}$)$^4$ω(d$_{1/2}$)$^1$ ⊗ ν(d$_{5/2}$)$^5$ | 18    | π(d$_{5/2}$)$^4$ω(d$_{1/2}$)$^1$ ⊗ ν(d$_{5/2}$)$^5$ |
| 1+   | 53   | π(d$_{5/2}$)$^5$ ⊗ ν(d$_{5/2}$)$^5$ | 57    | π(d$_{5/2}$)$^5$ ⊗ ν(d$_{5/2}$)$^5$ |
| 2+   | 15   | π(d$_{3/2}$)$^1$ω(d$_{1/2}$)$^1$ ⊗ ν(d$_{5/2}$)$^5$ | 16    | π(d$_{3/2}$)$^1$ω(d$_{1/2}$)$^1$ ⊗ ν(d$_{5/2}$)$^5$ |
| 1+   | 16   | π(d$_{5/2}$)$^5$ ⊗ ν(d$_{5/2}$)$^1$ω(d$_{1/2}$)$^1$ | 19    | π(d$_{5/2}$)$^4$ω(d$_{1/2}$)$^1$ ⊗ ν(d$_{5/2}$)$^4$ω(d$_{1/2}$)$^1$ |
| 2+   | 39   | π(d$_{5/2}$)$^5$ ⊗ ν(d$_{5/2}$)$^5$ | 42    | π(d$_{5/2}$)$^5$ ⊗ ν(d$_{5/2}$)$^5$ |
| 3+   | 32   | π(d$_{5/2}$)$^5$ ⊗ ν(d$_{5/2}$)$^5$ | 30    | π(d$_{5/2}$)$^5$ ⊗ ν(d$_{5/2}$)$^5$ |
| 3+   | 17   | π(d$_{5/2}$)$^5$ ⊗ ν(d$_{5/2}$)$^5$ | 21    | π(d$_{5/2}$)$^5$ ⊗ ν(d$_{5/2}$)$^5$ |
| 2+   | 12   | π(d$_{5/2}$)$^4$ω(d$_{1/2}$)$^1$ ⊗ ν(d$_{5/2}$)$^4$ω(d$_{1/2}$)$^1$ | 13    | π(d$_{5/2}$)$^4$ω(d$_{1/2}$)$^1$ ⊗ ν(d$_{5/2}$)$^4$ω(d$_{1/2}$)$^1$ |
| 5+   | 16   | π(d$_{5/2}$)$^5$ ⊗ ν(d$_{5/2}$)$^5$ | 17    | π(d$_{5/2}$)$^5$ ⊗ ν(d$_{5/2}$)$^5$ |
| 4+   | 16   | π(d$_{5/2}$)$^4$ω(d$_{1/2}$)$^1$ ⊗ ν(d$_{5/2}$)$^4$ω(d$_{1/2}$)$^1$ | 17    | π(d$_{5/2}$)$^5$ ⊗ ν(d$_{5/2}$)$^5$ |
Figure 4: Comparison between calculated and experimental [26] data of spectroscopic factors for $^{26}$Al.
large for \( l = 2 \) \((d_{5/2})\) transfer as compared with \( l = 0 \) \((s_{1/2})\) transfer for the first three states. The shell-model results are larger for first two states, thus assigning larger single particle characteristics to these states. In the present work we have also predicted \( C^2 S \) values for states up to \( \sim 6 \) MeV, although experimental data is not available. In the Fig. 3 we have also shown variation of \( C^2 S \) for experiment and theory. In this we have also plotted \( \sum C^2 S \) values for theory and experiment.

### 2.2 Calculation of \( C^2 S \) for \( 1n \) pick-up reaction \( ^{27}\text{Al}(d,t)^{26}\text{Al} \)

The states of \(^{26}\text{Al}\) \([26]\) were experimentally studied by assuming pick-up from \( d_{5/2} \) and \( s_{1/2} \) single particle orbitals, while \( 6^+ \) state at 3507 keV by pick-up from \( g_{9/2} \) orbital. In the Table II, we compared experimental \( C^2 S \) values with shell-model results for USDA and USDB effective interactions. The experimentally observed values for spectroscopic factors up to \( \sim 4.7 \) MeV reported in ref. \([26]\). In the present work we interpreted these measured SFs in term of shell-model calculations. The experimental \( C^2 S \) values for \( d_{5/2} \) states given in the table II. The SF results with shell-model is slightly higher than experimental value for \( 5^+ \) state. For some states the SFs are very small this is because in these cases the wavefunctions are very fragmented. There are two states \( 2^+ \) at 1759 keV and \( 4^+ \) at 4705 keV, with very small calculated spectroscopic factors. This is because large cancellations of contributions from different components of the wave functions. In the Fig. 4 we plotted \( C^2 S \) for experiment and theory. In this we have also plotted \( \sum C^2 S \) values for theory and experiment. The theory and experiment showing same trends.

### 3 Summary

Motivated with recent experimental results of spectroscopic factors for \(^{27}\text{Al}(d,{}^{3}\text{He})^{26}\text{Mg}\) and \(^{27}\text{Al}(d,t)^{26}\text{Al}\) reactions. In the present work we performed shell-model calculations for spectroscopic factors with new Hamiltonians for \( sd \) shell USDA and USDB. The predictions from shell-model are found to be in a good agreement with the experimental data. The SFs provide an independent test for the USDA and USDB interactions used in the present shell-model calculations. The present spectroscopic factor results for \(^{26}\text{Mg}\) and \(^{26}\text{Al}\) are important for nucleosynthesis.

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