Actuarial Study of Career Pensions - A Case Study of Universities

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Keywords: Pension insurance, population model, pension income and expenditure model

Abstract: Since the reform of pension insurance for domestic institutions and institutions has been officially implemented on October 1, 2014, how to ensure the sustainability of pension insurance for institutions and institutions is an issue worth exploring. This paper takes the pension insurance fund of a university institution in Chengdu as the research object and predicts the population life span through the existing population prediction model. The income and expenditure data of the university pension insurance fund are calculated, an actuarial model of the income and expenditure of the university pension insurance fund is established, and finally, based on the results of data analysis, relevant countermeasures are proposed.

1. Introduction

A pension is a social insurance system in which the state raises funds through multiple channels in the form of legislation to give retired workers a certain amount of financial compensation so that they can enjoy basic livelihood protection after retirement. The core issue of the operation of pension insurance is the problem of maintaining a balance between fundraising and application[1].

With the continuous development of society, especially the aging of the population, many countries around the world are facing pension reform, and the issue of pension sustainability has become a global pain point. Specifically, the pension issue involves the survival cycle of the population, the unit contribution rate of pension, the individual contribution rate of pension, the payment rate of pension, the financial subsidy rate of pension, and the policy guarantee of pension. [2]Especially in 2020, the sudden COVID-19 and the global economic downturn will have a huge impact on the pension problem.

The issue of Chinese pensions has also received attention from foreign scholars, such as Feldstein and Liberman (2006) in western developed countries, who conducted a study on the potential of realizing the income and expenditure of Chinese pension funds in the context of China's unified pension model. Jean-Claude Cosset, Narjess Boubakri,(2016) argue that when the market environment is favorable, the rate of return on investment will far exceed the rate of inflation, and pensions can be invested in the stock market at this time to generate returns.

Domestic studies related to the pension issue include: Peng Du, Zhenwu Zhai, and Wei Chen (2005) made a prediction analysis of China's population aging development trend in the next 10
years based on the total population and the age-specific population structure obtained from the 2000 census, and made corresponding adjustments based on the statistics of the number of people in school by age and sex of the population. Li Yu, and Lintao Yang (2014) considered urban-rural development differences and predicted urban-rural population development trends separately by establishing a bilinear population prediction marquee. The prediction results show that the total population of China will reach its peak around 2025, and then the total population will experience a slow decline in the following 10 years. With the help of a time series model, Minglong Yu (2016) evaluates the operation of employee pension insurance in Jiaxing City between 2015 and 2020, and the results show that the ability to pay pension insurance in Jiaxing City is gradually decreasing, i.e., the future development trend is not optimistic, and puts forward policy recommendations such as appropriately increasing the contribution rate to increase revenue, improving government financial support and increasing the level of coordination.

Some scholars use econometric methods to build a multiple regression model of the pension insurance fund gap for analysis; Lechuan Zhang (2012) integrates factors such as demographic changes, economic development level, and urbanization rate to study the pension insurance fund gap problem in China based on a macro perspective; Peng Zhang (2016) uses the GM(1,1) model and the pension insurance fund income and expenditure gap forecasting model to forecast the current balance of the pension insurance fund in Hebei Province. Lu Liu (2017) takes urban enterprise workers' basic pension insurance co-ordinated account in Henan Province as the research object, classifies its participants by policy, and models fund income and expenditure separately for different categories; according to the measurement results, the size of the pension gap will expand significantly and increase at a significant rate after it reappears in 2050 with the gradual decrease of population and pension gap.

2. Population Life expectancy projections

We first use the Lee-Carter model to project population longevity, an important indicator in determining the pension balance. The Lee-Carter model was proposed by Lee Ronald. And Carter Lawrence R. in 1992[3]. The two demographers used U.S. mortality data, blend, and time series analysis to calculate an age-specific mortality formula. The model reduces the influence of subjective speculation on the predicted outcomes, and the most significant feature of their approach is the use of stochastic processes to model future uncertainty[4].

Mortality and its indicators.

The central mortality rate (or probability of death) is the number of deaths on the age interval \((x, x+1)\) divided by the number of survivors at \((x, x+1)\) for a person aged \(x\) in year \(t\). The ratio of

\[ m_{x,t} = \frac{d_{x,t}}{l_{x,t}} \quad (1) \]

The central mortality rate or initial mortality rate was obtained from the calculation. Of these, \(d_{x,t}\) and \(l_{x,t}\) denote the number of dead population, exposed population between \([x, x+1]\) years for a person aged \(x\) in year \(t\), respectively[5].

The model splits the mortality change into two factors, the first of which is a time factor that varies over time, and the other is an age factor that is age-related and independent of the time factor. The equation of the model is.

\[ \ln(m_{x,t}) = \alpha_x + \beta_x k_t + \epsilon_{x,t} \quad (2) \]

where \(m_{x,t}\) denotes the central mortality rate at age \(x\) in year \(t\), and the parameter \(\alpha_x\) denotes the logarithmic mean of the mortality rate of the population at age \(x\), which is used to describe the age factor that does not vary over time; the parameter \(\beta_x\) denotes the time-varying coefficient of
mortality at age \( x \), which is used to describe the age factor for \( k_t \), the sensitivity of the age factor; the parameter \( k_t \) denotes the mortality level at year \( t \), which is used to describe the time-varying year factor \( \epsilon_{x,t} \) is the central mortality random error term of the model[6].

This paper utilizes age- and sex-specific population mortality data for cities nationwide. The original data were obtained from the China Demographic Statistical Yearbook and the China Population and Employment Statistical Yearbook, respectively, to obtain a total of the number of exposed population, a number of deaths, and mortality data by gender for the urban population of China from 1996 to 2019, and to give specific data for each age for each year. Using the annual average population by sex, age \( d_{x,t} \) and death population \( l_{x,t} \) data, and then obtain the central mortality rate for the combined male and female population aged 60-90+ years from 1995-2019 \( m_{x,t} \) [7].

In this paper, the weighted least squares (WLS) method is used to estimate several parameters of the model.

Using R software for \( \beta_x \), and \( k_t \). Weighted least squares (WLS) were applied to estimate and calculate \( \alpha_x \), and \( \beta_x \) of the fitted values from the weighted least squares (WLS) method to find the parameters in the Lee-Carter model for men and women \( k_t \) of the estimated values. For the time factor \( k_t \), a stochastic process with a drift term is used for modeling, and from \( d = \frac{k_t - k_1}{T} \) The estimates of the drift parameters by gender can be obtained. The predicted values are as follows.

The resulting graphs of mortality by sex for the 60-90 age group over time to 2020-2040 are shown in Figure1 and Figure2 below.

![Figure 1: Predicted male mortality over time](image1)

![Figure 2: Predicted female mortality over time](image2)

The above graph shows a gradual decrease in mortality and an increase in life expectancy by age and sex over time.

Once the population mortality rate is projected using the Lee-Carter model, it is possible to project the average future life expectancy based on the sex-specific population mortality rate. As shown in Table 1.

| Year | Average remaining life expectancy for men | Average remaining life expectancy for women |
|------|------------------------------------------|--------------------------------------------|
| 2020 | 22.84                                     | 26.12                                      |
| 2025 | 23.16                                     | 26.28                                      |
| 2030 | 23.55                                     | 27.37                                      |
| 2035 | 24.62                                     | 27.58                                      |
| 2040 | 24.75                                     | 27.82                                      |

The resulting average remaining life expectancy for men and women at age 60 over the years is...
shown in Figure 3 and Figure 4.

![Figure 3: Expected remaining life over time for men](image1.png)  
![Figure 4: Expected remaining life over time for women](image2.png)

3. Actuarial model construction for university retirement

The actuarial model construction of pensions in universities and other institutions is divided into three steps: the first step actuarial model construction of pension income in universities, the second step actuarial model construction of pension expenditure in universities, and the third step actuarial model construction of pension income and expenditure in universities[8].

3.1 Calculation of pension income in universities

According to the policy of "Decision on the Reform of the Pension Insurance System for Institutional Staff" (Guo Fa [2015] No. 2) issued by the State Council in January 2015, put the personnel of institutions of higher education who are eligible to participate in the insurance are divided into "old people", "middle people" and "new people". The three categories are "old man", "middle man" and "new man". So the university pension income is calculated and aggregated according to the different contribution policies of these three categories of personnel.

1) Universities and other institutions employees "old people" pension income

Before the pension reform, universities and colleges implemented the pension insurance system for public institutions, and working employees did not need to pay pension insurance personally, and their pensions depended on social coordination and financial protection. Therefore, the pension income of "old people" who work in universities and other institutions is zero.

2) The "middle" and "new" employees of universities and other institutions pension income

The "middle-aged" and "newcomers" are not required to pay insurance premiums before the start of the pension "integration" reform, but after the start of the reform, the basic pension insurance premiums mainly include unit After the reform is launched, the basic pension insurance premiums mainly include the social integration contribution of the unit and the individual contribution, the proportion of which is 12% of the corresponding contribution amount salary for the unit and 8% of the individual contribution salary for the individual.

Individual contribution in year \( t \), year \( I_{t,i}^{\text{per}} \) is:

\[
I_{t,i}^{\text{per}} = w_{c,t,i} \times \alpha \quad (3)
\]

The total amount of pension paid by the unit in year \( I_t^{\text{soc}} \) is:

\[
I_t^{\text{soc}} = w_{c,t,i} \times \beta \quad (4)
\]
where \( t \geq 2014 \).

So the total basic pension income of the "middleman" and "newcomer" in year \( t \) is:

\[
I_t = \sum_{i=1}^{C_t^d} I_{t,i} \times 12 + I_t^{soc} \quad (5)
\]

(Where \( C_t^d \) represents the number of persons in category \( f \) who paid pension in year \( t \) (\( fm \) and \( fn \) represent "middle person" and "new person" respectively).)

In which when the personal salary is greater than 300% of the local average salary of the on-the-job employees in the previous year, it is not included in the personal contribution salary base. When the personal salary is less than 60% of the average salary of local employees on duty in the previous year, the personal contribution salary base is calculated according to 60% of the average salary of local employees on duty in the previous year.

### 3.2 Calculation of pension expenditure in universities

The main purpose of the pension expenditure in universities is to take the pension "integration" reform as the node, and divide the personnel of university institutions and institutions who are eligible to participate in the insurance into "old people", "middle people" and "new people". The three types of personnel receive pensions according to the corresponding policies and methods. The pension expenditure of the university is the sum of the pension expenditure of the three categories of personnel.

1) "Old people" pension expenses for employees of universities and other institutions

The calculation of pension "old man" is based on a certain percentage of one's basic salary, plus a certain amount of retirement allowance, and the percentage is determined according to one's working years, for example, if one has worked for 15 years or less than 20 years, the percentage is 70%, if one has worked for 20 years or less than 30 years, if one has worked for 30 years or less, if one has worked for 30 years or less, if one has worked for 35 years or more, if one has worked for 35 years or more, the percentage is 80%, 85% and 90% respectively. Less than 35 years, full 35 years and above, the accrual ratio is 80%, 85% and 90% respectively. The retirement allowance is generally the hardship and remote area allowance, special teacher allowance, teaching age allowance, nursing age allowance, etc.

The basic monthly pension of individual \( i \) in year \( tE_t^p \) is:

\[
E_{t,i}^p = w_{1,t,i} \times \theta_i + w_{2,t,i} \quad (6)
\]

So the total basic pension expense for the "old man" in year \( tE_t^{pas} \) is

\[
E_{t}^{pas} = \sum_{i=1}^{C_t^p} E_{t,i}^p \times 12 \quad (7)
\]

2) Universities and other institutions employees "middle-aged" pension expenditure

According to the national policy, only after 15 years of contributions and deemed contributions have been accumulated, the insured will be issued a basic pension after retirement. The monthly standard of basic pension is based on the average salary of employees on duty in the previous year and the indexed average contribution salary index of the employee; the monthly standard of personal account pension is the amount of personal account savings divided by the number of months of accrual, and the number of months of accrual is determined by the average life...
expectancy of the urban population at the time of retirement and the retirement age of the employee. The transitional pension is jointly determined by the average salary of employees on duty in the previous year at the time of retirement and the deemed contribution index, as well as the number of years of deemed contribution and the transitional coefficient. The principle of the transitional pension is "reasonable convergence of treatment levels and smooth transition between old and new policies". According to the principle of transition, the principle of granting transitional pensions to the "middle-aged" is to make up for the less accumulated pensions in terms of the number of years of deemed contributions. In this paper, we assume that the standard pension for the "middle-aged" is a certain replacement rate of the previous year's average salary. The amount of pension expenditure can be calculated.

So the monthly basic pension for individual $i$ in year $t$ is:

$$E_{t, i}^{\text{Basic}} = \bar{w}_{t-1} \times \frac{1 + f_i}{2} \times (s_i + n_i)\%$$  
(8)

So the monthly personal account pension paid out to individual $i$ in year $t$ is:

$$E_{t, i}^{\text{Personal}} = f_i \times \bar{w}_{t-1} \times \alpha \times 12 \times n_i \times (1 + r_t)^{n_i}$$  
(9)

So the monthly expense of the transition pension for individual $i$ in year $t$ is:

$$E_{t, i}^{\text{Transition}} = \bar{w}_{t-1} \times \varphi_i \times s_i \times \varepsilon$$  
(10)

So the monthly basic pension expended in year $t$ for individual $i$ is:

$$E_{t, i}^m = E_{t, i}^{\text{Basic}} + E_{t, i}^{\text{Personal}} + E_{t, i}^{\text{Transition}}$$  
(11)

So the total basic pension expense of the "middle man" in year $t$ is:

$$E_t^{\text{mid}} = \sum_{i=1}^{f_m} E_{t, i}^m \times 12$$  
(12)

In the above equation $w_{1, t, i}$ represents the monthly base salary of individual $i$ in year $t$. $w_{2, t, i}$ represents the monthly retirement allowance subsidy for individual $i$ in year $t$.  

3) Pension expenses for "newcomers" in universities and other institutions

The "newcomers" are only paid the basic household pension and the individual account pension; since the contribution period is at least 15 years, the "newcomers" will appear after many years, and those who retire before then are all "middle-aged". The "newcomers" will not appear until many years later because the contribution period is at least 15 years. In this paper, it is assumed that the pensions for "middle-aged" and "newcomers" are basically the same, with a certain replacement rate of the previous year's average wage[9].

The monthly basic pension for individual $i$ in year $t$ is:

$$E_{t, i}^{\text{Basic}} = \bar{w}_{t-1} \times \frac{1 + f_i}{2} \times n_i\%$$  
(13)

The monthly personal account pension paid out to individual $i$ in year $t$ is:

$$E_{t, i}^{\text{Personal}} = f_i \times \bar{w}_{t-1} \times \alpha \times 12 \times n_i \times (1 + r_t)^{n_i}$$  
(14)

The basic pension of the monthly expenditure individual $i$ in year $t$ is:

$$E_{t, i}^{\text{new}} = E_{t, i}^{\text{Basic}} + E_{t, i}^{\text{Personal}}$$  
(15)

So the total basic pension expense for the "newcomer" in year $t$ is:
\[ E_t^{new} = \sum_{i=1}^{c_t^f} E_{t,i}^n \times 12 \quad (16) \]

In the above equation, \( z_t^f \) represents the number of pensioners in category \( f \) in year \( t \) (\( fp, fm \) and \( fn \) represent "old people", "middle people" and "new people" respectively). \( w_{t-1} \) represents the average monthly salary of employees on duty in the province in the previous year. \( n_i \) represents the actual number of years of contribution for individual \( i \). \( s_i \) represents the number of years of deemed contribution for individual \( i \). \( \varphi_i \) represents the deemed contribution index for individual \( i \). \( f_i \) represents the actual average contributory wage index for individual \( i \). \( r_t \) represents the interest rate credited to the basic pension insurance individual account in year \( t \); \( M \) number of months of accrual.

### 3.3 Actuarial model construction of the pension gap in universities

Adding up the pension income and expenses of universities and other institutions in year \( t \), respectively, yields.

\[ I_t = \sum_{i=1}^{c_t^f} I_{t,i}^{per} \times 12 + I_{t}^{soc} \quad (17) \]

\[ E_t = E_t^{pas} + E_t^{mid} + E_t^{new} \quad (18) \]

The total income minus the total expenditure for each year is then discounted and finally the difference between income and expenditure for each year is discounted and summed to obtain the pension shortfall projection model (where \( c \) is the discount rate).

\[ P_{gap} = \sum_{t=2020}^{2040} \left( \frac{I_t-E_t}{(1+c)^{t-2020}} \right) \quad (19) \]

In the above equation, \( E_t^{pas} \) represents the total basic pension expenditure of the "elderly" in year \( t \), \( E_t^{mid} \) represents the total basic pension expenditure of the "middle-aged" in year \( t \), \( E_t^{new} \) represents the total basic pension expenditure of "newcomers" in year \( t \).

Parameter assumptions.

1) Assuming that the age of participation in the workforce is all 28 years old, the retirement age for men is 60 years old, the retirement age for women is 50 years old, and the retirement age for female cadres is 55 years old.

2) According to the Chengdu City Statistical Yearbook, the average monthly wages of on-the-job workers in Chengdu for 2018-2020 are 7334, 8127 and 8705, respectively. It is assumed that the wage growth rate is equal to the GDP growth rate. When the economy develops to a higher level, the GDP growth rate remains stable.

3) In projecting future trends such as pension shortfalls, it is assumed that no policy effects or other unforeseen events occur during the projected time interval, and that uncertainties such as lump-sum catch-up contributions and early retirement are ignored.

4) Deemed contribution index and transition coefficient. With reference to the basic pension insurance for enterprise employees, it is assumed that the deemed contribution index is 1 and the transition coefficient is 1.3% adopted by most provinces.

5) The number of employees in institutions such as universities: mainly including serving teachers, party and government work cadres, administrative and logistical staff, etc.

Measurement results and their analysis. The above parameter settings are set into Equation 1-17, and the results can be obtained. The specific results are shown in Table 2.

73
### Table 2: Current Shortfall of Pension Funds

| Year | millions | Year | millions |
|------|----------|------|----------|
| 2021 | 15.3     | 2031 | 78.2     |
| 2022 | 20.1     | 2032 | 86.7     |
| 2023 | 25.1     | 2033 | 92.5     |
| 2024 | 31.5     | 2034 | 100.2    |
| 2025 | 37.1     | 2035 | 112.1    |
| 2026 | 42.4     | 2036 | 123.8    |
| 2027 | 49.6     | 2037 | 134.2    |
| 2028 | 55.2     | 2038 | 148.7    |
| 2029 | 64.2     | 2039 | 162.8    |
| 2030 | 70.6     | 2040 | 188.1    |

As the above table shows, the overall gap of Chengdu college pension fund does exist. The actuarial projection results show that the pension insurance fund is decreasing in the current period as time goes by, and the college pension gap will not be able to cover its expenses, and it has been maintaining a growing gap with an upward trend of values, after which the gap will deepen with policy changes.

### 4. Conclusions

This paper uses actuarial theory and combines the provisions of contributions and benefits in the Decision on the Reform of the Pension Insurance System for Institutional Staff (Guo Fa [2015] No. 2) issued by the State Council in January 2015 to measure the income and expenditure of the pension fund of a university in Chengdu using the pension data of a university in recent years. It is found that the pension problem in colleges and universities is expanding along with the deepening of aging[12]. Meanwhile, there are still many problems in the pension insurance system, especially the multi-level pension insurance system has not been really developed, and it is still a single-pillar pension insurance model in essence. The reform should start from the following aspects.(i) Relaxing the investment restrictions on pensions and implementing diversified investment of pensions[13]. Rationalize and optimize, and revitalize the stock. Give full play to the benefits of pensions. At present, we have not calculated the investment income of pensions in our calculation, the policy of this piece is still to be improved and it is difficult to obtain the data, it will be an effective supplement if we can fully utilize the investment channels and investment income of pensions.(ii) Raising the retirement age. According to our previous model's calculations, as society develops, human life expectancy will increase and raising the retirement age can effectively alleviate the pension funding shortage. It is recommended that the retirement age be extended in stages, by one month for each year from 2023 to 2040, to alleviate the pension shortfall.(iii) Accelerate the "three pillars" pension insurance system and optimize the structure of the pension system. The current pension system is overly dependent on the first pillar of basic social pension insurance, while the second pillar of occupational pension is developing very slowly, although universities have initially established an occupational pension system, which is an effective supplement to pension insurance, but on the other hand, the third pillar of personal savings insurance has just started, and commercial pension insurance has not yet been developed. It is recommended to accelerate the development.

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