A general method for determining the masses of semi invisibly decaying particles at hadron colliders

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We present a general solution to the long standing problem of determining the masses of pair-produced, semi invisibly decaying particles at hadron colliders. We define two new transverse kinematic variables, $M_{CT_1}$ and $M_{CT_2}$, which are suitable one-dimensional projections of the transverse mass $M_{CT}$. We derive analytical formulas for the boundaries of the kinematically allowed regions in the ($M_{CT_1}, M_{CT_2}$) and ($M_{CT_1}, M_{CT_2}$) parameter planes, and introduce suitable variables $D_{CT j}$ and $D_{CT}$ to measure the distance to those boundaries on an event per event basis. We show that the masses can be reliably extracted from the endpoint measurements of $M_{CT_1}^{max}$ and $D_{CT j}^{min}$ (or $D_{CT j}^{min}$). We illustrate our method with dilepton $t\bar{t}$ events at the LHC.

The ongoing run of the Large Hadron Collider (LHC) at CERN will finally provide the first glimpse of physics at the TeV scale. In large part, the excitement surrounding the LHC is fueled by the anticipation of the unknown: no one knows for sure where or how the first signal of new physics beyond the standard model (BSM) will show up. Yet, complementary and independent arguments from particle physics and astrophysics suggest that the best place to look for new physics is a channel with missing transverse energy $E_T$, caused by unseen new particles contributing to the dark matter of the Universe.

Unfortunately, the study of missing energy signatures poses a tremendous challenge at hadron colliders like the LHC. The first fundamental difficulty is related to the very nature of hadron collisions, where in each event the partonic center-of-mass energy $\sqrt{s}$ and longitudinal momentum $p_z$ of the initial state are unknown. To make matters worse, the lifetime of the dark matter particle is typically protected by a new parity symmetry, which guarantees that in every event the missing particles come in pairs, thus proliferating the number of unknown parameters describing the final state event kinematics.

The generic topology of a “new physics” $E_T$ event is sketched in Fig. 1. Consider the inclusive production of an identical pair of new “parent” particles $P$. Each parent $P$ decays semi invisibly to a set $V_i$ ($i = 1, 2$) of standard model (SM) particles, which are visible in the detector, and a dark matter particle $C$ (from now on referred to as the “child”) which escapes detection. In general, the parent pair is accompanied by a number of additional “upstream” objects $U$ (typically jets) with total transverse momentum $U_T$. They may originate from various sources such as initial state radiation or decays of even heavier particles. We shall not be interested in the exact details of the physics responsible for $U$, adopting a fully inclusive approach to the production of the parents $P$. Given this general setup, the goal is to determine independently the mass $M_p$ of the parent and the mass $M_c$ of the child in terms of $U$, $V_1$ and $V_2$.

In the past, several approaches to this problem have been proposed, but each has its own limitations. For example, the classic method of invariant mass endpoints [1,2] only applies when the visible SM particles in $V_i$ arise from a sufficiently long decay chain. Attempts at direct reconstruction [3] of the children momenta are again limited to long decay chains only. In this letter, we shall consider the extreme, most challenging example where each visible set $V_i$ consists of a single SM particle of fixed mass $m_i$. A perfect testing ground for this scenario is provided by dilepton $t\bar{t}$ events (already observed at the LHC [4]) and we shall use that example in our numerical illustrations below. The role of the parent $P$ (child $C$) will be played by the SM $W$-boson (SM neutrino), each $V_i$ is a SM lepton ($e$ or $\mu$), while $U$ is composed of the two $b$-jets from the top quark decays, plus any additional QCD jets from initial state radiation (ISR).

For such extremely short decay chains, the only viable alternative at the moment is provided by the methods based on the $M_{T2}$ variable [5]. There, at least in principle, the individual masses $M_p$ and $M_c$ can be determined by observing a “kink” feature in the $M_{T2}$ endpoint as a function of a hypothesized trial mass $M_c$ for $C$ [6], or by

![FIG. 1: The generic event topology under consideration. All particles visible in the detector are clustered into three groups: upstream objects $U$ with total transverse momentum $U_T$, and two composite visible particles $V_i$ ($i = 1, 2$), each with invariant mass $m_i$ and total transverse momentum $p_{iT}$.](image-url)
exploring the \( U_T \) dependence of the \( M_{T2} \) endpoint. Compared to those \( M_{T2} \) approaches, our method here has two advantages. First, it is simpler – it uses only the observed objects \( U, V_1 \) and \( V_2 \) in the event and makes no reference to the missing particle kinematics (or mass). Second, it is more precise, since it utilizes the whole kinematic boundary of the relevant two-dimensional distribution and not just the kinematic endpoint of its one-dimensional projection. We proceed in three easy steps.

**Step I. Orthogonal decomposition of the observed transverse momenta with respect to the \( \vec{U}_T \) direction.** The Tevatron and LHC collaborations currently use fixed axes coordinate systems to describe their data. Instead, we propose to rotate the coordinate system from one event to another, so that the transverse axes are always aligned with the direction \( T \) selected by the measured upstream transverse momentum vector \( \vec{U}_T \) and the direction \( T \) orthogonal to it (see Fig. 2). The visible transverse momentum vectors from Fig. 1 are then decomposed as

\[
\vec{p}_{T1} \equiv \frac{1}{\vec{U}_T} (\vec{p}_T \cdot \vec{U}_T) \vec{U}_T, \\
\vec{p}_{T0} \equiv \vec{p}_T - \vec{p}_{T1} = \frac{1}{\vec{U}_T} \vec{U}_T \times (\vec{p}_T \times \vec{U}_T).
\]

**Step II. Constructing the transverse and longitudinal contrasverse masses \( M_{CT1} \) and \( M_{CT2} \).** Our starting point is the original contrasverse mass variable \( M \)

\[
M_{CT} = \sqrt{m_1^2 + m_2^2 + 2(e_{1T} e_{2T} + \vec{p}_{1T} \cdot \vec{p}_{2T})},
\]

where \( e_{IT} \) is the “transverse energy” of \( V_i \)

\[
e_{IT} = \sqrt{m_i^2 + |\vec{p}_{IT}|^2}.
\]

For events with \( U_T = 0 \), \( M_{CT} \) has an upper endpoint which is insensitive to the unknown \( \sqrt{s} \), providing one relation among \( M_p \) and \( e \)

\[
M_{CT}^{max}(U_T = 0) = \sqrt{m_1^2 + m_2^2 + 2m_1m_2 \cos(\zeta_1 + \zeta_2)},
\]

where

\[
sinh \zeta_i = \frac{\sqrt{(M_p^2, M_e^2, m_i^2)}}{2M_pm_i},
\]

\[
\lambda(x, y, z) = x^2 + y^2 + z^2 - 2xy - 2xz - 2yz.
\]

Unfortunately, the \( U_T = 0 \) limit is not particularly interesting at hadron colliders (especially for inclusive studies), since a significant amount of upstream \( U_T \) is typically generated by ISR (and other) jets. One possible fix is to use all events, but modify the definition \( \zeta \) to approximately compensate for the transverse \( \vec{U}_T \) boost. One then recovers a distribution whose endpoint is still given by \( \zeta \). Alternatively, one could stick to the original \( M_{CT} \) variable, and simply account for the \( U_T \) dependence of its endpoint as

\[
M_{CT}^{max}(U_T) = \sqrt{m_1^2 + m_2^2 + 2m_1m_2 \cosh(2\eta + \zeta_1 + \zeta_2)}
\]

where \( \zeta_i \) were already defined in \( \zeta \), and

\[
sinh \eta = \frac{U_T}{2M_p}, \quad \cosh \eta = \sqrt{1 + \frac{U_T^2}{4M_p^2}}.
\]

Our approach here is to utilize the one-dimensional projections from eqs. (12) and construct one-dimensional analogues of the \( M_{CT} \) variable

\[
M_{CT1} \equiv \sqrt{m_1^2 + m_2^2 + 2(e_{1T} e_{2T} + \vec{p}_{1T} \cdot \vec{p}_{2T})},
\]

\[
M_{CT2} \equiv \sqrt{m_1^2 + m_2^2 + 2(e_{1T} e_{2T} + \vec{p}_{1T} \cdot \vec{p}_{2T})},
\]

where the corresponding “transverse energies” are

\[
e_{CT1} \equiv \sqrt{m_1^2 + |\vec{p}_{CT1}|^2}, \quad e_{CT2} \equiv \sqrt{m_2^2 + |\vec{p}_{CT2}|^2}.
\]

The benefit of the decomposition \( \zeta \) is that one gets “two for the price of one”, i.e. two independent and complementary variables instead of the single variable \( \zeta \).

The variable \( M_{CT1} \) in particular is very useful for our purposes. To illustrate the basic idea, it is sufficient to consider the most common case, where \( V_i \) is approximately massless (\( m_i \approx 0 \)), as the leptons in our \( t\bar{t} \) example. A crucial property of \( M_{CT1} \) is that its endpoint is independent of \( U_T \):

\[
M_{CT1}^{max} = \frac{M_p^2 - \lambda^2}{M_p}, \quad \forall U_T.
\]

In fact the whole \( M_{CT1} \) distribution is insensitive to \( U_T \):

\[
\frac{dN}{dM_{CT1}} = N_{0,1} \delta(M_{CT1}) + (N_{tot} - N_{0,1}) \frac{dN}{dM_{CT1}},
\]

where \( N_{0,1} \) is the number of events in the zero bin \( M_{CT1} = 0 \). Using phase space kinematics, we find that the shape of the remaining (unit-normalized) zero-bin-subtracted distribution is simply given by

\[
\frac{dN}{dM_{CT1}} \equiv -4 \hat{M}_{CT1} \ln \hat{M}_{CT1}
\]

in terms of the unit-normalized \( M_{CT1} \) variable

\[
\hat{M}_{CT1} = \frac{M_{CT1}}{M_{CT1}^{max}}.
\]
The observable $M_{CT\perp}$ distribution for our $t\bar{t}$ example is shown in Fig. 3 for 10 fb$^{-1}$ of LHC data at 7 TeV. Events were generated with PYTHIA $^{10}$ and processed with the PGS detector simulator $^{11}$. We apply standard background rejection cuts as follows $^{11}$: we require two isolated, opposite sign leptons with $p_T > 20$ GeV, $m_{l+\bar{l}} > 12$ GeV, and passing a Z-veto $|m_{l+\bar{l}} - M_Z| > 15$ GeV; at least two central jets with $p_T > 30$ GeV and $|\eta| < 2.4$; and a $\not{E}_T$ cut of $|\not{E}_T| > 30$ GeV ($\not{E}_T > 20$ GeV) for events with same flavor (opposite flavor) leptons. We also demand at least two $b$-tagged jets, assuming a flat $b$-tagging efficiency of 60%. With these cuts, the SM background from other processes is negligible $^{11}$.

Fig. 3 demonstrates that the $M_{CT\perp}$ endpoint can be measured quite well. Since the theoretically predicted shape $^{15}$ is distorted by the cuts, we use a linear slope with Gaussian smearing, and fit for the endpoint and the resolution parameter. We find $M_{CT\perp}^{\text{max}} = 80.9$ GeV; at least two central jets with $p_T > 30$ GeV and $|\eta| < 2.4$; and a $\not{E}_T$ cut of $|\not{E}_T| > 30$ GeV ($\not{E}_T > 20$ GeV) for events with same flavor (opposite flavor) leptons. We also demand at least two $b$-tagged jets, assuming a flat $b$-tagging efficiency of 60%. With those cuts, the SM background from other processes is negligible $^{11}$.

![FIG. 3: Zero-bin subtracted $M_{CT\perp}$ distribution after cuts, for $t\bar{t}$ dilepton events. The yellow (lower) portion is our signal, while the blue (upper) portion shows $t\bar{t}$ combinatorial background with isolated leptons arising from $\tau$ or $b$ decays.](image)

For a fixed representative value $U_T = 75$ GeV. The solid lines show the corresponding boundaries defined in $^{[20]}$ and $^{[23]}$, for the correct value of $M_{CT\perp}^{\text{max}}$ and several different values of $M_p$ as shown.

![FIG. 4: Scatter plots of (a) $M_{CT\perp}$ versus $M_{CT\parallel}$ and (b) $M_{CT\perp}$ versus $M_{CT}$, for a fixed representative value $U_T = 75$ GeV.](image)

used 10,000 events at the parton level. For a given value of $M_{CT\perp}$, the allowed values of $M_{CT\parallel}$ are bounded by

$$M_{CT\parallel}^{(lo)}(M_{CT\perp}) \leq M_{CT\parallel} \leq M_{CT\parallel}^{(hi)}(M_{CT\perp}),$$

where $M_{CT\parallel}^{(lo)}(M_{CT\perp}) = 0$ and

$$M_{CT\parallel}^{(hi)}(M_{CT\perp}) = M_{CT\perp}^{\text{max}} \left( \sqrt{1 - M_{CT\perp}^{\text{max}} \cosh \eta + \sinh \eta} \right).$$

Fig. 4(a) reveals that the endpoint $M_{CT\parallel}^{\text{max}}$ of the one-dimensional $M_{CT\parallel}$ distribution is obtained at $M_{CT\perp} = 0$

$$M_{CT\parallel}^{\text{max}} = M_{CT\parallel}^{(hi)}(0) = M_{CT\parallel}^{\text{max}}(\cosh \eta + \sinh \eta)$$

$$= \frac{1}{2} \left( 1 - \frac{M_p^2}{M_p^2} \right) \left( \sqrt{4M_p^2 + U_T^2} + U_T \right).$$

Notice that events in the zero bins $M_{CT\perp} = 0$ and $M_{CT\parallel} = 0$ fall on one of the axes and cannot be distinguished on the plot.

Now consider the scatter plot of $M_{CT\perp}$ vs $M_{CT}$ shown in Fig. 4(b). $M_{CT}$ is similarly bounded by

$$M_{CT}^{(lo)}(M_{CT\perp}) \leq M_{CT} \leq M_{CT}^{(hi)}(M_{CT\perp}),$$

where this time $M_{CT}^{(lo)}(M_{CT\perp}) = M_{CT\perp}$ and

$$M_{CT}^{(hi)}(M_{CT\perp}) = M_{CT\perp}^{\text{max}} \left( \cosh \eta + \sqrt{1 - M_{CT\perp}^{\text{max}} \sinh \eta} \right).$$

We see that the endpoint $M_{CT\perp}^{\text{max}}$ of the one-dimensional $M_{CT}$ distribution is also obtained for $M_{CT\perp} = 0$:

$$M_{CT\perp}^{\text{max}} = M_{CT}^{(hi)}(0) = M_{CT\perp}^{\text{max}}(\cosh \eta + \sinh \eta) = M_{CT\perp}^{\text{max}}.$$
show linear binned maximum likelihood fits. DFIG. 5: M use only events in the zero bin M[6] A. J. Barr, B. Gripaios and C. G. Lester, JHEP 0802 463 [4] V. Khachatryan et al. [2] K. T. Matchev, F. Moortgat, L. Pape and M. Park, JHEP 0711, 2343 (2003). [1] I. Hinchliffe et al., Phys. Rev. D 55, 5520 (1997); B. C. Allanach et al., JHEP 0009, 004 (2000); B. K. Gjelsten, D. J. Miller and P. Osland, JHEP 0412, 003 (2004). [1] I. Hinchliffe et al., Phys. Rev. D 55, 5520 (1997); B. C. Allanach et al., JHEP 0009, 004 (2000); B. K. Gjelsten, D. J. Miller and P. Osland, JHEP 0412, 003 (2004). [2] K. T. Matchev, F. Moortgat, L. Pape and M. Park, JHEP 0908, 104 (2009). [3] K. Kawagoe, M. M. Nojiri and G. Polesello, Phys. Rev. D 71, 035008 (2005); H. C. Cheng et al., Phys. Rev. Lett. 100, 252001 (2008). [4] V. Khachatryan et al. [CMS Collaboration], arXiv:1010.5994 [hep-ex]. [5] C. G. Lester and D. J. Summers, Phys. Lett. B 463, 99 (1999); A. Barr, C. Lester and P. Stephens, J. Phys. G 29, 2343 (2003). [6] A. J. Barr, B. Gripaios and C. G. Lester, JHEP 0802, 014 (2008); M. Burns, K. Kong, K. T. Matchev and M. Park, JHEP 0903, 143 (2009). [7] K. T. Matchev, F. Moortgat, L. Pape and M. Park, Phys. Rev. D 82, 077701 (2010); P. Konar, K. Kong, K. T. Matchev and M. Park, Phys. Rev. Lett. 105, 051802 (2010). [8] D. R. Tovey, JHEP 0804, 034 (2008). [9] G. Polesello and D. R. Tovey, JHEP 1003, 030 (2010). [10] T. Sjostrand, S. Mrenna and P. Skands, JHEP 0605, 026 (2006). [11] http://www.physics.ucdavis.edu/~conway/research/ software/pgs4-general.htm [12] D. Costanzo and D. R. Tovey, JHEP 0904, 084 (2009); M. Burns, K. T. Matchev and M. Park, JHEP 0905, 094 (2009). [13] I. W. Kim, Phys. Rev. Lett. 104, 081601 (2010).