Numerical Study of the Gluon Propagator in Lattice Landau Gauge: the Three-Dimensional Case

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We study the infrared behavior of the gluon propagator in lattice Landau gauge, for pure SU(2) lattice gauge theory in a three-dimensional lattice. Simulations are done for nine different values of the coupling $\beta$, from $\beta = 0$ (strong coupling) to $\beta = 6.0$ (in the weak-coupling region). In the limit of large lattice volumes, we observe in all cases a gluon propagator decreasing as the momentum goes to zero.

1. INTRODUCTION

The infrared behavior of the gluon propagator in lattice Landau gauge has been the subject of several numerical studies (see [1,2] and references therein). In fact, although this propagator is a non-gauge-invariant quantity, the study of its infrared behavior provides a powerful tool for increasing our understanding of QCD, and for gaining insight into the physics of confinement in non-abelian gauge theories.

On the lattice, the Landau gauge condition is imposed by finding a gauge transformation which brings the functional $E_U[g]$, defined in eq. (1) below, to a minimum. A lattice configuration which satisfies this minimizing condition belongs to the region $\Omega$ of transverse configurations, for which the Faddeev-Popov operator is nonnegative [3,4]. This region is delimited by the so-called first Gribov horizon, defined as the set of configurations for which the smallest non-trivial eigenvalue of the Faddeev-Popov operator is zero.

The restriction of the path integral, which defines the partition function, to the region $\Omega$ implies a rigorous inequality [5] for the Fourier components of the gluon field $A$. From this inequality, which is a consequence only of the positiveness of the Faddeev-Popov operator, it follows that the region $\Omega$ is bounded by a certain ellipsoid $\Theta$. This bound implies proximity of the first Gribov horizon in infrared directions, and consequent suppression of the low-momentum components of the gauge field, a result already noted by Gribov in Ref. [5]. This bound also causes a strong suppression of the gluon propagator in the infrared limit (i.e. for momentum $p \to 0$). More precisely, Zwanziger proved [6] that, in the infinite-volume limit, the gluon propagator is less singular than $p^{-2}$ in the infrared limit and that, very likely, it vanishes as $p^2$. A gluon propagator vanishing as $p^2$ in the infrared limit was also found (under certain hypotheses) by Gribov [5]. Finally, a gluon propagator vanishing in the infrared limit has been recently obtained as an approximate solution of the gluon Dyson-Schwinger equation [7].

Here we study the infrared behavior of the gluon propagator in the three-dimensional case (preliminary results have been reported in Ref. [8]). Let us recall that nonabelian gauge theories in three dimensions are similar to their four-dimensional counterparts, and results obtained in the three-dimensional case can teach us something about the more realistic four-dimensional theories. Of course, the advantage of using a three-dimensional lattice is the possibility of simulating lattice sizes larger than those used in the four-dimensional case. This is particularly important in the study of the gluon propagator: in fact, Zwanziger’s prediction [6,8] of an infrared-suppressed gluon propagator is valid only in the infinite-volume limit.
2. METHODOLOGY

We consider a standard Wilson action for $SU(2)$ lattice gauge theory in 3 dimensions, with periodic boundary conditions. (Details of notation and numerical simulations will be given in 3.) The gauge field, which belongs to the $\mathfrak{su}(2)$ Lie algebra, is defined as $A_{\mu}(x) \equiv 1/2 \left[ U_{\mu}(x) - U_{\mu}^\dagger(x) \right]$, where $U_{\mu}(x) \in SU(2)$ are link variables. We also define $A_{\mu}^a(x) \equiv \text{Tr} \left[ A_{\mu}(x) \sigma^a \right]/(2i)$, where $\sigma^a$ is a Pauli matrix.

In order to fix the lattice Landau gauge we look for a local minimum of the functional

$$\mathcal{E}_U[g] \equiv 1 - \frac{1}{3V} \sum_{\mu=1}^3 \sum_x \frac{\text{Tr}}{2} U_{\mu}(x),$$

where $V$ is the lattice volume, and $U_{\mu}(g(x)) \equiv g(x) U_{\mu}(x) g(x + \epsilon_{\mu})$. [Here $g(x) \in SU(2)$ are site variables.]

Finally, the lattice gluon propagator in momentum space is defined as

$$D(0) \equiv \frac{1}{9V} \sum_{\mu, a} \left\langle \left[ \sum_x A_{\mu}^a(x) \right]^2 \right\rangle,$$

$$D(k) \equiv \frac{1}{6V} \sum_{\mu, a} \left\langle \left\{ \left[ \sum_x A_{\mu}^a(x) \cos (2\pi k \cdot x) \right]^2 \right. \right.$$  

$$\left. + \left[ \sum_x A_{\mu}^a(x) \sin (2\pi k \cdot x) \right]^2 \right\rangle. \tag{3}$$

3. RESULTS

In Figures 1 and 2 we plot the data for the gluon propagator as a function of the square of the lattice momentum $p^2(k) \equiv 4 \sum_{\mu=1}^3 \sin^2 (\pi k_{\mu})$, for different lattice volumes $V$ and couplings $\beta$. Our data confirm previous results 3 obtained in the strong-coupling regime for the four-dimensional case: the gluon propagator is decreasing as $p$ decreases, provided that $p^2(k)$ is smaller than a turn-over value $p^2_{\text{to}}$. Clearly $p^2_{\text{to}}$ is $\beta$- and volume-dependent. Also, as in four dimensions, the lattice size at which this behavior for the gluon propagator starts to be observed increases with the coupling. In particular, in the strong-coupling regime, this propagator is clearly decreasing as $p^2(k)$ goes to zero, even for relatively small lattice volumes (see the case $\beta = 2.8$ in Figure 1). On the contrary, for $\beta \geq 3.4$, this propagator is increasing (monotonically) in the infrared limit for $V = 16^3$, while it is decreasing (see Figures 2 and 3) for the largest lattice volume considered.

Let us also notice that, at high momenta, there are very small finite-size effects, at all values of $\beta$. The situation is completely different in the small-momenta sector, as already stressed above. In particular, the value $D(0)$ of the gluon propagator at zero momentum decreases monotonically as the lattice volume increases (see for example the case $\beta = 5.0$ in Figure 2). These results suggest a finite value for $D(0)$ in the infinite-volume limit, but it is not clear whether this value would be zero or a strictly positive constant. Therefore, the possibility of a zero value for $D(0)$ in the infinite-volume limit is not ruled out.

4. CONCLUSIONS

As said above, our data in the strong-coupling regime are in qualitative agreement with the results obtained in the four-dimensional case (see Figure 1 in Ref. 3). This strongly suggests to us that a similar analogy will hold — in the limit of large lattice volumes — also for couplings $\beta$ in the scaling region, leading to an infrared-suppressed gluon propagator in four dimensions.

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Figure 1. Plot of the gluon propagator $D(k)$ as a function of the square of the lattice momentum $p^2(k)$ for lattice volumes $V = 16^3$ (□) and $V = 32^3$ (*), with $k = (0, 0, k_t)$, at: (a) $\beta = 2.8$ and (b) $\beta = 3.4$. Error bars are one standard deviation.

Figure 2. Plot of the gluon propagator $D(k)$ as a function of the square of the lattice momentum $p^2(k)$ for lattice volumes $V = 16^3$ (□), $V = 16^2 \times 32$ (+), $V = 32^3$ (*), $V = 32^2 \times 64$ (○), and $V = 64^3$ (✸), with $k = (0, 0, k_t)$, at: (a) $\beta = 4.2$ and (b) $\beta = 5.0$. Error bars are one standard deviation.