Anisotropy driven reversal of magnetisation in Blume-Capel ferromagnet: A Monte Carlo study

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Abstract: The two dimensional Spin-1 Blume-Capel ferromagnet is studied by Monte Carlo simulation with Metropolis algorithm. Starting from initial ordered spin configuration the reversal of magnetisation is investigated in presence of a magnetic field ($h$) applied in the opposite direction. The variations of the reversal time with the strength of single site anisotropy are investigated in details. The exponential dependence was observed. The systematic variations of the mean reversal time with positive and negative anisotropy was found. The mean macroscopic reversal time was observed to be linearly dependent on a suitably defined microscopic reversal time. The saturated magnetisation $M_f$ after the reversal was noticed to be dependent of the strength of anisotropy $D$. An interesting scaling relation was obtained, $|M_f| \sim |h|^{3/2} f(D|h|^{-\alpha})$ with the scaling function of the form $f(x) = \frac{1}{1+e(x-a)/\beta}$. The temporal evolution of density of $S_i^z = 0$ (surrounded by all $S_i^z = +1$) is observed to be exponentially decaying. The growth of mean density of $S_i^z = -1$ has been fitted in a function $\rho_{-1}(t) \sim \frac{1}{a+e(tc-t)/\sigma}$. The characteristic time shows $t_c \sim e^{-rD}$ and a crossover in the rate of exponential falling is observed at $D = 1.5$. The metastable volume fraction has been found to obey the Avrami’s law.

Keywords: Blume-Capel model, Monte Carlo simulation, Metropolis algorithm, Magnetic anisotropy, Magnetisation reversal
I. Introduction

The field driven reversal of magnetisation in the magnetic materials is an interesting field of modern research. In the technology\cite{1} of magnetic recording\cite{2}, this scale of time is responsible for the speed of recording. The longevity of magnetic storage device depends on the reversal time. The dependence of this reversal time on the temperature, disorder and various physical parameters is a serious matter of investigation to the theoretician as well as the experimentalists. These results will be useful to the technologists to prepare the magnetic sample in such way so that the reversal time can be tuned to the demand of technological world.

To study the reversal of magnetisation in ferromagnetic sample, the simplest choice would be Ising ferromagnet. A big pool of literatures of such study has been developed in last few decades. In this context, the phenomenological Becker-Döring theory\cite{3} is much appealing. Due to its simplicity, the reversal time (or so called nucleation time) can be obtained as function of the temperature of the system and the magnitude of applied magnetic field. However, the prediction of this phenomenological theory needed simulational verification to identify the reversal mechanism via the coalescence of multiple droplets or by the growth of single supercritical droplet. The only method to verify it is the simple Monte Carlo simulation where the growth of droplets can be studied as phase ordering kinetics. The rate of nucleation of crystalline solids in solid-melt system demands to be an important historical study\cite{4} in this context. The idea of the decay of metastable state of such a system opened a field of research. The longevity of metastable states in kinetic Ising ferromagnet is studied\cite{5} to find its dependence on the applied field and system size. A large scale Monte Carlo study is done\cite{6} using multispin coding algorithm to investigate the behaviors of metastable lifetimes in different spatial dimensions. A dynamical magnetisation reversal transition with a pulsed magnetic field was studied\cite{7}. The relaxation of Ising ferromagnet after a sudden reversal of applied magnetic field is also studied\cite{8}. The thermally activated magnetisation switching of small ferromagnetic particles driven by an external magnetic field has been investigated and interestingly a crossover from coherent rotation to nucleation for a classical anisotropic Heisenberg model, has been reported\cite{9}. The rates of growth and decay of the clusters of different sizes, have been studied\cite{10} as functions of external field and temperature. The rate of nucleation of critical nuclei, its speed of expansion and the corresponding changes of free energy is related and is described by famous Kolmogorov–Johnson–Mehl–Avrami law\cite{11–13}. The reversal of magnetisation assisted by heat, in ultra thin films for ultra-high-density information recording has been investigated\cite{14}. The nucleation time was observed\cite{15} to increase in the presence of magnetic field which is spreading over the space in time as compared to that in static field. The behaviours of metastable states in Ising ferromagnet in the presence of a gradient of field\cite{16} and in the simultaneous presence of gradients of fields and the temperature was investigated\cite{17} recently. The dependence of metastable lifetime on the width of quench disorder was investigated\cite{18} recently in random field Ising model.

The above mentioned studies are basically done on Ising (or anisotropic Heisenberg) model. In the case of spin-$\frac{1}{2}$ Ising model, the reversal is governed microscopically by the spin flip. The spin would flip from +1 to -1 due to the application of external magnetic field. The metastability and its lifetime is function of the temperature and the magnitude of external magnetic field. What will happen if the spin component has become perpendicular to the applied magnetic field? Precisely, how does the anisotropy take part in reversal mechanism in the spin-1 Blume-Capel\cite{19,20} model? Metastability and nucleation in the Spin-1 Blume-Capel (BC) ferromagnet was studied and found the different mechanism of transition\cite{21}. The first order phase transition was studied\cite{22} in the spin-1 BC model by effective field theory. The critical point temperature was determined\cite{23} in the
BC model by using Wang-Landau Monte Carlo simulation. The dynamical phase transition was studied [24] in a randomly diluted single site anisotropic BC model in presence of time varying oscillating magnetic field. The dynamical phase transition in BC model driven by propagating and standing magnetic field wave is studied [25] recently. The metastability in the BC model with distributed anisotropy was studied [26] using different dynamics. The first order phase transition and the tricritical behaviour in the BC model was also studied [27] recently.

In the present article, we have extensively investigated the role of single site anisotropy to the reversal mechanism in BC model. We employed the Monte Carlo simulation using Metropolis single spin flip algorithm. The time dependence of the metastable volume fraction is investigated. The saturated magnetisation, after the reversal, was found to follow an interesting scaling behaviour. We have organised the manuscript as follows: in the section -II we have defined the BC model and the simulation method, the results are reported briefly in section-III and the paper ends with concluding remarks in section-IV.

II. The Model and Simulation method:

The spin-1 Blume-Capel ferromagnet is modelled by the following Hamiltonian,

\[ H = -J \sum_{<i,j>} S_i^z S_j^z + D \sum_i (S_i^z)^2 - h \sum_i S_i^z, \]

where \( S_i^z \) is the z-component of the spin \( (S = 1) \) at i-th lattice site. \( S_i^z \) can assume three values, +1, 0 and -1. The first term signifies the contribution to the energy due to the nearest neighbours ferromagnetic \( (J > 0) \) exchange interaction. Second term is considered to model the effect of single site anisotropy (or magnetocrystalline anisotropy arising from the crystal field generated by crystal structure) with strength \( D \). For simplicity, we have considered the uniform \( D \). The Zeeman energy involving the interaction of applied magnetic field \( (h) \) with each spin, is being represented by the third term. We have considered a two dimensional ferromagnetic square lattice of size \( L \times L \) with periodic boundary conditions applied to both directions.

Let us briefly describe the method of Monte Carlo simulation employed here. Initially, the system is considered to be in perfectly ordered state where all the spins are pointing up \( S_i^z = +1 \) \( \forall i \). A site (i-th say) has been chosen randomly. The present value of \( S_i^z \) at that chosen site is \( S_i^z(\text{initial}) \). The updated value may be any of the three (+1, 0 and -1) possible values. The final test value of \( S_i^z \) is chosen randomly from any of these three values with equal probability. Let this test value is labelled as \( S_i^z(\text{final}) \). The probability of \( S_i^z \), to assume the final value \( S_i^z(\text{final}) \) from its initial value \( S_i^z(\text{initial}) \) is given by Metropolis transition probability: [28, 29],

\[ P(S_i^z(\text{initial}) \rightarrow S_i^z(\text{final})) = \text{Min}[1, \exp(-\frac{\Delta H}{kT})], \]

where \( \Delta H \) is the change in energy (calculated from equation-1) due to the change in the value of \( S_i^z \), from \( S_i^z(\text{initial}) \) to \( S_i^z(\text{final}) \). \( k \) is the Boltzmann constant and \( T \) is the temperature of the system. The temperature of the system is measured in the unit of \( J/k \). For simplicity, we set \( J = 1 \) and \( k = 1 \) throughout the simulational study. The acceptance of the final value \( S_i^z(\text{final}) \) is determined just by comparing a random number with the Metropolis transition probability. The test move is accepted only when the random number (uniformly distributed in the range [0,1]) is less than or equal to \( P(S_i^z(\text{initial}) \rightarrow S_i^z(\text{final})) \). In this way total \( L^2 \) number of randomly chosen spins are updated. This is usually called random updating scheme and \( L^2 \) number of random
updates constitutes one Monte Carlo Step per Spin (MCSS) and acts as the unit of time in the problem.

For any fixed value of the temperature \( T \) of the system and the applied external magnetic field \( h \), the magnetisation of the system is determined by

\[
M(t) = \frac{1}{L^2} \sum_{i} S_i^z
\]

To study the reversal of the magnetisation, precisely one has to calculate the minimum time required (in MCSS), to have the negative magnetisation, starting from a perfectly ordered configuration.

### III. Simulational results:

We have considered that the values of all the spins are \( S_i^z = 1 \), as the initial configuration. This may be imagined that all the spins are in the positive z-directions of a square lattice in xy-plane. Now a magnetic field \( h \) in opposite direction (along negative direction of z-axis) is applied to the system. That means, \( h \) assumes negative value. We have studied the time evolution of the magnetisation \( M(t) \) of the system for different strength of anisotropy \( D \) at any fixed temperature \( T \). The 'reversal time' \( \tau \) is defined as the time by which the magnetisation changes its sign (from initially chosen positive value to the negative value). It is observed that reversal time \( \tau \) of the magnetisation decreases with the increase in the strength of anisotropy \( D > 0 \) (fig.1). Additionally we noticed that the saturation magnetisation \( M_f \), after complete reversal, also varies with the strength \( D \) of the anisotropy. \( M_f \) is determined by taking time average of the magnetisation after reaching saturation (flatness of the plots in negative magnetisation region in Fig.1). In the case of negative anisotropy \( M_f \) reaches a negative value (close to -1) i.e. a considerably large number of the spins are flipped to \( S_i^z = -1 \) state (along the direction parallel to the applied magnetic field). In the case of positive anisotropy \( D > 0 \), as the magnitude of the anisotropy becomes stronger, \( |M_f| \) decreases and finally reaches zero. Actually for negative \( D \) the system behaves as a spin-1/2 model where the system tries to be settled down in either of two states (either the spins are 1 or the spins are -1) by minimizing its energy. As \( D \) is increased beyond \( D = -0.5 \), the reversal time will be large enough. But for positive anisotropy, spins tend to assume another value \( (S_i^z = 0) \) favourable for minimizing the energy. As a result, the mean density of \( S_i^z = 0 \) starts to grow as the positive anisotropy is increased. It is clear that, due to large positive \( D > 0 \) anisotropy, the value of the magnetisation of the system is mostly determined by \( D \) (unlike the situation of negative \( D \) where it was prefereably determined by the applied magnetic field \( h \)).

Fig.1 depicted the reduction in reversal time with the increase in anisotropy \( D \) of a single system. Now we have dedicated 10000 number of different random samples and calculated the reversal time for each. Fig.2 shows normalised probability distribution of those reversal times for different strength \( D \) of anisotropy. The most probable reversal time and the spreading (standard deviation) of the distribution, decreases with increasing anisotropy (positive \( D \)) (fig.2b). Similar study has been carried out and depicted in fig.2d for both negative and positive anisotropy having same absolute value \( |D| = 0.5 \). Distribution for \( D = -0.5 \) has a huge spread compared to that
for $D = +0.5$. Due to the presence of statistical distribution of the reversal time ($\tau$), we have considered the mean reversal time ($\tau_{av}$) to investigate the metastable behaviours of this system.

We have studied the nature of the variation of the mean reversal time ($\tau_{av}$) with anisotropy (for both positive $D$ fig.4a and negative $D$ fig.4b). Mean reversal time is calculated by simply averaging the reversal times obtained for 10000 numbers of different random samples. The mean $\tau_{av}$ of the reversal times and its standard deviation $\sigma_r$ (fig.4h, 4b) were found to decrease exponentially ($\tau_{av} \sim e^{-gD}$) with the increase in positive anisotropy. In the presence of negative anisotropy, both of them increase exponentially ($\tau_{av} \sim e^{-g'D}$) with the increase in the absolute value of the strength of anisotropy. Positive stronger anisotropy compel the system to lower the absolute value of $\tau_{av}$ of magnetisation, due to the production of large number of $S_i = 0$ (which contributes nothing to the magnetisation). Stronger value of negative $D$ will map the system onto an equivalent spin-$\frac{1}{2}$ Ising ferromagnet, where the single spin flip would require more cost of energy than that of a Blume-Capel ferromagnet with positive $D$, which has a possibility of transition from $S_i = +1$ to $S_i = 0$. This is a possible reason of getting smaller reversal time in the case of larger positive $D$, in the BC model. In both cases (i.e., $D > 0$ and $D < 0$), a crossover is observed in the rate ($r'$) of change of the exponential function of $\sigma_r \sim e^{-r'D}$.

To understand the reason, of the above mentioned observations, clearly, we have investigated the dynamics of the mean density (fig.5) of the three values of $S_i$, i.e., $+1, 0$ and $-1$ for different values of $D$. For negative ($D < 0$) anisotropy, the mean density of $S_i = +1$ is almost fixed and maximum (approximately equal to unity) in the metastable region and suddenly starts to decrease near reversal time and vanishes after some time (as a result of complete reversal dominated by large population of $S_i = -1$). As a result, the density of $S_i = -1$ remains zero in the metastable region the suddenly accuires a large value (due to reversal) and eventually aquires maximum value when complete reversal is achieved. The mean density of $S_i = 0$ remains almost zero through out all the time in presence of negative anisotropy ($D < 0$). This fact arises due to the behaviour of the BC model which is similar to that of Ising model in the case of large negative anisotropy. As the positive anisotropy rises, the active role of $S_i = 0$ comes into the picture. This shows a maximum near the reversal time for some moderate values of anisotropy ($D > 0$). Some snapshots (fig.6) of instantaneous spin configurations, are captured near the time of reversal in the presence of different anisotropy ($D > 0$) to have a clear idea. From fig.6 it is observed that, close to the time of reversal of the magnetisation, the density of $S_i = +1$ and $S_i = -1$ are equal and the density of $S_i = 0$ gets peaked (for moderate value of positive $D$) or a larger value (for higher $D$). This is obviously true because for vanishingly small net magnetisation, the densities of $S_i = +1$ and $S_i = -1$ must be approximately equal, in the thermodynamic limit. The values of this equal density (of $S_i = +1$ and $S_i = -1$) and the density of the $S_i = 0$ are legitimately noticed to be varied with anisotropy ($D > 0$).

If we plot (fig.7a) the densities($\rho_r$) of these three values ($+1, 0$ and $-1$) of $S_i$ in the vicinity of the time of reversal ($\tau$) with anisotropy ($D > 0$), it is observed that the densities of $S_i = +1$ and $S_i = -1$ reduce with increasing anisotropy whereas the density of $S_i = 0$ increases. Interestingly, a strength of anisotropy has been noticed, in presence of which the three values ($+1, 0$ and $-1$) of $S_i$ have almost equal density near the time of reversal. Since, the stronger field ($h$) accelerates the reversal process for a particular value of anisotropy ($D$), the $\rho_r$ of $S_i = +1$ and $S_i = -1$ are reduced and that for $S_i = 0$ is increased. So obviously that particular value of $D(> 0)$, for which $\rho_r$ of all $S_i = +1$, $S_i = 0$ and $S_i = -1$ are equal, is reduced in presence of a stronger applied field (fig.7b). For clarity, in fig.7d we have excluded the density of $S_i = -1$ as it is same as that of $S_i = +1$.

How does the macroscopic mean reversal time is connected to any microscopic scale of time? To
address this interesting question, we have studied the evolution of (in fig.8) density ($\rho_i^0$) of $S_i^z = 0$, surrounded by all (four nearest neighbours) $S_i^z = +1$, with time in presence of the anisotropy ($D > 0$ here). That density $\rho_i^0$ is found to decay exponentially with time i.e. $\rho_i^0 = ae^{-bt}$. The microscopic scale of time is defined from the exponential decay. So, $\frac{1}{b} = \tau_a$ is the microscopic scale of time in the present issue. Some snapshots (fig.9) are captured at different time steps in presence of a particular anisotropy to have an idea about the evolution of the density $\rho_i^0$. Now the $\tau_a$ is determined for several values of anisotropy and plotted them with the reversal time ($\tau$) that we have defined earlier. It follows a straight line (fig.10, $\tau_a \sim c\tau$, where $c$ is a constant). This interesting observation prompted us to have the idea of getting microscopic scale of time ($\tau_a$) which is related to the macroscopic reversal time ($\tau$). It may be noted here, that both time scales are measured in the case of single sample only (no averaging is carried out over different random sample).

In fig.1 we observed that the anisotropy ($D$) of the system affects the saturation magnetisation after complete reversal ($|M_f|$). Now in fig.11a, the dependence of ($|M_f|$) on the anisotropy of the system is checked in presence of different values of applied field at a fixed temperature. Now all the plots are scaled (Fig.11b) by rescaling the anisotropy from $D$ to $D_s = D|h|^{-\alpha}$ and the saturation magnetisation from $|M_f|$ to $|M_f|^s = |M_f||h|^{-\beta}$. All the data points are observed to be collapsed to a single plot which is now fitted (Fig.11) to a scaling function, $f(x) = \frac{1}{1+e^{(x-a)/b}}$ where $f(x) = |M_f||h|^{-\beta}$ and $x = D|h|^{-\alpha}$. From fig.11, it is observed that scaling exponent $\alpha$ plays the crucial role ($\beta$ is quite small) in collapsing the data. The $|M_f|$ changes very slowly in the region of negative anisotropy ($D < 0$) and also in the region of strong positive anisotropy ($D > 0$) of the system. Because, in the presence of negative anisotropy ($D < 0$), all the spins try to align along (parallel or antiparallel) the direction of the applied field. In contrast, the strong positive anisotropy ($D > 0$) forbids the spins to be aligned along (parallel or antiparallel) the direction of the applied magnetic field. In between these two regions $|M_f|$ changes rapidly because, as the positive anisotropy becomes stronger, probability of microscopic transition from $S_i^z = +1$ to $S_i^z = 0$ increases. So the density of $S_i^z = 0$ increases with the increase in anisotropy causing significant reduction in $|M_f|$.

Above scaling analysis has been studied for different temperatures (fig.12). Below the temperature $T = 1.0$ and above $T = 1.5$ this scaling behaviour was not found to show significantly good data collapse. So, for a certain range (approximately from $T = 1.0$ to $T = 1.5$) of temperature, the system shows a fair scaling behaviour (as verified by good data collapse). Within this range, for six different temperatures, we have obtained the values of the scaling exponent $\alpha$ by simple trial and error method. And also fitted (fig.12) to the function $f(x) = \frac{1}{1+e^{(x-a)/b}}$ similarly as fig.12. The value of $D_s$ at which $((D_s)_c$ is actually the value of the fitting parameter $a$) $|M_f|^s$ becomes 0.5, is determined for each case. It $((D_s)_c$ was found to decrease linearly (fig.13a) as the temperature ($T$) is increased. On the other hand, the scaling exponent $\alpha$ is observed to increase exponentially with the increase in temperature (fig.13b). Since the value of other scaling exponent $\beta$ is too small to study any systematic variation with temperature.

In the present study, we have also investigated whether the system obeys Avrami’s law regarding the decay of metastable volume fraction in presence of anisotropy. According to the KJMA theory, metastable volume fraction decays exponentially with a power of time. Avrami’s law depicts that metastable volume fraction (relative abundance of $S_i^z = 1$) decays exponentially with $t^{d+1}$ in a $d$ dimensional system (closer to the critical temperature $T = 0.8T_c$ here). So for a two dimensional Blume-Capel ferromagnet, the logarithm of the metastable volume fraction is plotted (fig.14) against the third power of time i.e., $(\frac{t}{\tau})^3$ (nondimensionalised by reversal time $\tau$) which fits fairly
to a straight line. This confirms that the Avrami’s law holds good in the case of $S = 1$ Blume-Capel ferromagnetic system with anisotropy. We have checked that law in the presence of three different strength of anisotropy ($D = 0.5$, $D = 1.0$ and $D = 1.5$) (shown in Fig.[14]) keeping the temperature of the system fixed at $T = 0.8T_c$ for each case [30]. Here also, the system is found to follow the Avrami’s law for different values of $D$.

The cooperatively interacting many body system, with large degrees of freedom, is found to suffer from the finite size effect. To study such effect, we have investigated the mean reversal time (obtained from 1000 samples) and also its standard deviation against strength of anisotropy for three different size of lattice ($L = 100, 200$ and $300$). Fig.[15] depicts that the mean reversal time $\tau_{av}$ is beyond any finite size effect in the system size considered in our present study. On the other hand, the standard deviation $\sigma_\tau$ of reversal times, decreases for larger system sizes. This is obviously in conformity with general statistical mechanical study.

Variation of the density of $S^z_i = -1$ ($\rho_{(-1)}$) is studied as function of time and shown in fig.[16a] for three different values of anisotropy $D$. The density $\rho_{(-1)}$, is found to fit with a function like $f(x) = \frac{1}{a + e^{-bx/c}}$. Stronger anisotropy raises the parameter $a$, whereas reduces the the value of $b$. From the function $f(x)$, a characteristic time ($t_c$) (i.e. the value of the parameter $b$ in a sense) is defined at which $f(x) = \rho_{(-1)} = \frac{1}{a+1}$. Whereas $\frac{1}{a+1}$ is the saturated density of $S^z_i = -1$, achieved after a very long of time ($t \rightarrow \infty$). The $t_c$ can give an approximate idea about the influence of anisotropy on the rate of growth of density of $S^z_i = -1$. The variation of $t_c$ with $D$ (positive) is also studied here (fig.[16a]). The data are exponentially fitted separately in two regimes of $D$. It is observed that $t_c \sim e^{-rD}$. Interestingly, the value of $r = 1.85$ in the weak anisotropy regime and $r = 0.84$ in the strong anisotropy regime. A crossover is observed around $D = 1.5$. In the presence of stronger magnetic anisotropy $D$, the rate of achieving the density $\frac{1}{a+1}$ is faster than that in the regime of weaker $D$. Because, in stronger $D$ regime, density of $S^z_i = 0$, plays a dominating role. The change from $S^z_i = +1$ to $S^z_i = -1$ in the low density of $S^z_i = 0$, is less probable than that for high density of $S^z_i = 0$. As a result, the density of $S^z_i = -1$ takes shorter time to saturate in the limit of high $D$.

IV. Summary

In this article, the reversal of magnetisation is studied in a two dimensional anisotropic ($S = 1$) Blume-Capel ferromagnet by Monte Carlo simulation using Metropolis single spin flip algorithm. The reversal time was studied in details as function of the strength of the anisotropy $D$. The statistical distribution of the reversal times (for different random samples) was also studied. The most probable value of this distribution was found to be strongly dependent of $D$. The most probable reversal time decreases as the strength of anisotropy $D(>0)$ increases. The mean (or average) reversal time is found to be exponentially decreasing with $D$. On the other hand, the mean reversal time was found to increase exponentially with the magnitude of $D$, for negative anisotropy. The standard deviation of reversal times shows an interesting behaviour. For positive $D$, it falls exponentially with the magnitude of $D$. However, the rate of falling is higher in the low anisotropy regime. In the case of negative anisotropy, a qualitatively contrast behaviour is observed. We have also studied the evolution of densities of the different values of $S^z_i$ i.e. $+1, 0$ and $-1$ in a microscopic level for better understanding of the reversal phenomena in the Blume-Capel model. The mean density $\rho_{0}^z$, i.e. density of $S^z_i = 0$ (surrounded by all $S^z_i = +1$) is studied as
function of time. It is observed to decay exponentially with time. Here, the microscopic time is found to depend linearly on the macroscopic reversal time.

After the reversal, the magnetisation was observed to reach a saturated value (with some fluctuation of course). This saturated magnetisation $M_f$, after the reversal, is found to be strongly dependent on the strength of anisotropy $D$. An interesting scaling relation was found $|M_f| \sim |h|^\beta f(D|h|^{-\alpha})$ with the scaling function of the form $f(x) = \frac{1}{1+e^{\beta x}}$. The scaling relation was obtained in a certain range of temperature ($T$) by employing the method of simple data collapse. The scaling exponent $\alpha$ has a dependence on the temperature ($T$) of the system.

The growth of mean density of $S_i^z = -1$ is observed to be fitted in a function like $\rho_{-1}(t) \sim \frac{1}{a+e^{(t-t_c)/c}}$. The characteristic time ($t_c$) behaves like $t_c \sim e^{-rD}$. A crossover of the values of $r$ is detected at $D = 1.5$.

We have also studied the functional form of the metastable volume fraction (i.e., relative abundance of $S_i^z = +1$). It is observed the the metastable volume fraction decays exponentially with the third power of time ($t$) indicating the Avrami’s law.

The size dependences of the reversal time and its fluctuations are also checked for three different system sizes. The mean reversal time does not show any remarkable change however the standard deviation of reversal time decreases with the increase of the system size which is quite expected in general statistical analysis.

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Figure 1: Variation of magnetisation ($M(t)$) with time ($t$) at different values of anisotropy ($D$) at fixed temperature $T = 1.2$ in presence of applied field $h = -0.25$. 
Figure 2: (a) Normalised probability ($P_\tau$) distribution of the reversal times ($\tau$) for five different values of positive ($D > 0$) anisotropy at fixed temperature $T = 1.2$ and applied field $h = -0.25$. (b) Normalised probability distribution of the reversal times for the same absolute value of positive and negative anisotropy at same temperature and field as (a).
Figure 3: Semilogarithmic plot of variation of mean reversal time ($\tau_{av}$) with positive anisotropy (a) and negative anisotropy (b) with error or standard deviation $\sigma_\tau$ obtained from the 10000 random samples. Data are fitted to the function $f(x) = ae^{-bx}$ for positive $D$ and $f(x) = ae^{bx}$ for negative $D$ where $f(x) = \tau_{av}$ and $x = |D|$. Temperature is kept $T = 1.2$ and applied field is $h = -0.25$. 
Figure 4: Semilogarithmic plot of the variation of standard deviation ($\sigma_T$) of the mean reversal time with positive anisotropy (a) and negative anisotropy. Data are fitted to the function $f(x) = ae^{-bx}$ for positive $D$ and $f(x) = ae^{bx}$ for negative $D$ where $f(x) = \sigma_T$ and $x = |D|$. (b) Temperature is kept $T = 1.2$ and applied field is $h = -0.25$. 
Figure 5: Variations of the mean densities of $S^z_i = +1$, $S^z_i = 0$ and $S^z_i = -1$ with time for different values of anisotropy ($D$) at fixed values of the temperature ($T = 1.2$) and the applied field ($h = -0.25$). Vertical lines in each figure indicates the time of reversal.
Figure 6: Snapshots of the spin configurations captured at the time of reversal for the different values of the strength of anisotropy $D$. (a) $D = -0.5$ and $\tau = 1516$ MCSS (b) $D = 0.5$ and $\tau = 201$ MCSS (c) $D = 1.0$ and $\tau = 69$ MCSS (d) $D = 1.5$ and $\tau = 28$ MCSS (e) $D = 2.0$ and $\tau = 16$ MCSS (f) $D = 2.5$ and $\tau = 12$ MCSS. Applied field is fixed at $h = -0.25$. 
Figure 7: (a) Variations of the mean densities of $S_i^z = +1$, $S_i^z = 0$ and $S_i^z = -1$ at the time of reversal ($\rho_\tau$) with fixed anisotropy ($D$). Temperature is set to $T = 1.2$ and the applied field is $h = -0.25$ (b) Variation of $\rho_\tau$ with $D$ in the presence of two different values of fields $h = -0.25$ (red) and $h = -0.75$ (blue). Temperature is fixed to $T = 1.2$. 


Figure 8: Temporal evolution of the density of $S_i^z = 0$ surrounded by all neighbouring four $S_i^z = +1$ ($\rho_0^1$) for four different set of values of anisotropy $D$. (Here, $D = 1.5$, $D = 2.0$, $D = 2.5$ and $D = 3.0$). All the four plots are fitted to the function $f(x) = a e^{-bx/10}$ where $f(x)$ stands for $\rho_0^1$ and $x$ stands for $t$. Applied field is $h = -0.25$ and $T = 1.2$. 

$$f(x) = a e^{-bx/10}$$

$b_1 = 0.719 \pm 0.049$

$b_2 = 2.085 \pm 0.108$

$b_3 = 4.283 \pm 0.149$

$b_4 = 6.646 \pm 0.138$
Figure 9: Snapshots of spin configurations captured at different times for the strength of anisotropy $D = 1.5$. (a) $t = 1$ MCSS (b) $t = 10$ MCSS (c) $t = 20$ MCSS (d) $t = 28$ MCSS (reversal time) (e) $t = 50$ MCSS (f) $t = 70$ MCSS. Applied field is $h = -0.25$ and $T = 1.2$. 
Figure 10: Relation between microscopic reversal time ($\tau_a$) and average macroscopic reversal time $\tau$. The value of $\tau_a = \frac{10}{1}$ is calculated from fig. Data are fitted to a straight line $f(x) = ax + b$ where $f(x) = \tau_a$ and $x = \tau$. 

$$f(x) = ax + b$$

$$a = 0.578 \pm 0.016$$

$$b = -4.319 \pm 0.272$$
Figure 11: (a) Variation of saturated magnetisation ($M_f$) with anisotropy ($D$) at any fixed temperature $T = 1.2$ for different values of applied field ($h$), (b) Scaled saturated magnetisation ($|M_f|h^{-\beta}$) versus scaled anisotropy ($D|h|^{-\alpha}$) at fixed temperature $T = 1.2$. 

\[ f(x) = \frac{1}{1 + e^{(x-a)/b}} \]

\[ a = 2.876 \pm 0.006 \]
\[ b = 0.516 \pm 0.005 \]
Figure 12: Scaling behaviour for another five different values of temperatures (a) $T = 1.0$ (b) $T = 1.1$ (c) $T = 1.3$ (d) $T = 1.4$. 
Figure 13: (a) Variation of the $\langle D_s \rangle_c$ with temperature $T$. The data are fitted with a straight line and (b) the variation of the scaling exponent $\alpha$ with temperature ($T$). The data are fitted to the function $f(x) = ae^{bx}$ where $f(x) = \alpha$ and $x = T$. 
Figure 14: Evolution of metastable volume fraction ($\ln(\frac{N_1}{N})$, where $N_1$ is the number of $S_i^z = +1$ and $N$ is the total number of spins in the lattice) with cube of time ($(\frac{t}{\tau})^3$, where $\tau$ is the reversal time) at three different values of positive anisotropy in presence of applied field $h = -0.25$. Temperature is fixed at $T = 0.8T_c$ for each case ($T = 1.2$ and $\tau = 201$ for $D = 0.5$, $T = 1.1$ and $\tau = 116$ for $D = 1.0$, $T = 0.9$ and $\tau = 75$ for $D = 1.5$). Data are fitted to the straight lines $f(x) = ax + b$ with (i) $a = -0.664 \pm 0.009$, $b = -0.142 \pm 0.005$ for $D = 0.5$ (ii) $a = -0.677 \pm 0.011$, $b = -0.212 \pm 0.006$ for $D = 1.0$ (iii) $a = -0.664 \pm 0.016$, $b = -0.325 \pm 0.007$ for $D = 1.5$. 
Figure 15: (a) Semilogarithmic plot of mean reversal time against positive anisotropy for different size of lattice ($L = 100$, $L = 200$ and $L = 300$) at fixed temperature $T = 1.2$ in presence of applied field $h = -0.25$. Data are fitted to the function $f(x) = ae^{-bx}$ where $x = |D|$ and $f(x) = \tau_{av}$ with (i) $b = 2.23 \pm 0.02$ for $L = 100$ (ii) $b = 2.205 \pm 0.026$ for $L = 200$ (iii) $b = 2.205 \pm 0.028$ for $L = 300$. (b) Semilogarithmic plot of standard deviation of mean reversal time against positive anisotropy for different size of lattice ($L = 100$, $L = 200$ and $L = 300$) at fixed temperature $T = 1.2$ in presence of applied field $h = -0.25$. Data are fitted to the function $f(x) = ae^{-bx}$ where $x = |D|$ and $f(x) = \sigma_\tau$ with (i) $b = 3.68 \pm 0.05$ for $L = 100$ (ii) $b = 3.73 \pm 0.08$ for $L = 200$ (iii) $b = 3.42 \pm 0.08$ for $L = 300$. 
Figure 16: (a) Temporal evolution of density ($\rho_{\langle -1 \rangle}$) of $S_z = -1$ at three different values of magnetic anisotropy ($D = 0.5$, $D = 1.5$, $D = 2.5$) which are fitted to the function $f(x) = \frac{1}{a + e^{\frac{b-x}{c}}}$ where $f(x) = \rho_{\langle -1 \rangle}$ and $x = t$. Temperature is set to $T = 1.2$ and applied field is $h = -0.25$. Values of the parameters are (i) $a = 1.052 \pm 0.001$, $b = 212.793 \pm 0.155$, $c = 48.435 \pm 0.144$ for $D = 0.5$ (ii) $a = 1.236 \pm 0.001$, $b = 36.902 \pm 0.071$, $c = 9.025 \pm 0.062$ for $D = 1.5$ (iii) $a = 3.96 \pm 0.005$, $b = 18.70 \pm 0.255$, $c = 4.80 \pm 0.150$ for $D = 2.5$. (b) Variation of characteristic time ($t_c$) or the values of parameter $b$ (obtained from (a)) with positive $D$. Data are fitted to the exponential function separately in two regimes.