Capacity Analysis for Tunnel Diode Amplifier Assisted Ambient Backscatter Communications

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ABSTRACT

Ambient backscatter communication (AmBC) addresses connectivity, cost, and congestion bottlenecks for the Internet of Things (IoT) deployment. AmBC, by avoiding a dedicated power infrastructure and carrier emitter, allows tags to communicate by simply reflecting the ambient radio frequency (RF) signals to the reader. Thus, the tag can operate in the low-maintenance, battery-free mode and with energy harvesting. However, a fundamental bottleneck is the limited communication range. A novel solution is that the tag can use a tunnel diode amplifier to enhance its backscattered signal power and thus extend the communication range. This paper studies the tunnel diode amplifier solution and investigates the resulting capacity performance. Specifically, we first develop the system’s mathematical model, elaborate on the tunnel diode’s amplification mechanism, and derive its reflection gain. Subsequently, we derive the closed-form channel capacity and propose a reader-tag distance adjustment scheme to improve the system’s performance. Finally, simulation results corroborate our theoretical results. They show that the tunnel diode amplifier can significantly increase the system capacity by an order of magnitude when the tag receives less than -30 dBm of incident RF signal power. They also demonstrate a significant increase in the communication range.

INDEX TERMS

Ambient backscatter communication (AmBC), channel capacity, Internet of Things (IoT), reflection amplifier, tunnel diode (TD).

I. INTRODUCTION

Internet of things (IoT), aiming at realizing ubiquitous connections of massive numbers of devices, is a crucial component of the fourth industrial revolution (industry 4.0) and the sixth-generation communications (6G). McKinsey Global Institute forecasts that the global IoT market value would reach 5.5 ∼ 12.6 trillion dollars by 2030 [1].

A significant challenge that hinders the further development and commercialization of IoT is that many small devices, e.g., sensors and tags, are powered by batteries, have a restricted lifetime, and require frequent replacements/recharging. Although the battery life can sometimes be up to ten years, the production cost and size of batteries still limit the application scenarios of IoT devices [2].

The ambient backscatter communication (AmBC) technology allows IoT devices to achieve a battery-free, self-sustaining, autonomous operational mode. Without using a dedicated power infrastructure and a carrier emitter, AmBC enables the backscatter device (BD) (also known as a tag) to harvest energy from ambient radio frequency (RF) signals [3] or light [4]. Moreover, the BD communicates by...
backscattering (reflecting) the ambient RF signals from sources such as TV, WiFi, cellular, and others. Thus, the tag can eliminate power-hungry RF circuits and thus has only sub-µW power consumption [5]. AmBC thus has the potential to support future IoT applications, including smart logistics, smart agriculture, and smart city [6]. Research efforts on channel estimation, data detection, performance analysis, system design, and other aspects of AmBC have massively surged recently [7], [8], [9], [10], [11], [12], [13], [14].

However, the power level of the backscattered signal received by the reader is tiny compared to that of the direct ambient signal, mainly because of the path loss, reflection loss at the BD, and other losses during the backscattering process. Consequently, the communication distance between the BD and the reader is limited [15].

Integrating a tunnel diode into the BD can amplify the otherwise weak backscattered signals [16], [17], [18], [19]. In [16], a tunnel diode amplifier achieves a reflection gain up to 34.4 dB with an incident RF power level as low as -70 dBm. In [17], a tag with a tunnel diode amplifier, also known as the tunneling tag, is designed, which only consumes 0.2 mW DC power and can realize 17 dB gain at -30 dBm an incident power. In addition, [18] and [19] develop several tunneling tags, which can achieve up to a 40 dB reflection gain while consuming only tens of µW power. Besides, reference [19] shows that the tunneling tags can increase the reader-tag distance by a factor of ten compared to the traditional backscatter system.

As far as we know, the literature has not previously studied the channel capacity gains from the tunneling tag. To address this gap, we derive the AmBC system channel capacity where the tunnel diode amplifier assists the tag in this paper. The major contributions of this paper are as follows:

- We first formulate the mathematical model of the system when the tag is equipped with a tunnel diode as the reflection amplifier, which can be applied to the AmBC system assisted by different tunnel diodes.
- We then introduce the amplification mechanism of the tunnel diodes and suggest their reflection gain function for further analysis.
- We also derive the channel capacity of the AmBC system with a tunnel diode as the reflection amplifier by maximizing the mutual information between the tag symbols and the received symbols. We develop a reader-tag distance adjustment scheme based on the derived expression to improve the system’s capacity.
- We evaluate the channel capacity improvement of this AmBC system over the traditional AmBC system and draw broad conclusions that the tunnel diode amplifier can effectively improve the system capacity and extend the communication range.

The rest of the paper is organized as follows. Section II formulates the system model of the tunnel diode amplifier-assisted AmBC system. Section III introduces the amplification mechanism of the tunnel diode and models the function of the reflection gain. Section IV develops the system’s capacity analytically and discusses the optimal RF source-to-tag distance of the system. Numerical results are provided in Section V. Finally, Section VI concludes this paper.

II. SYSTEM MODEL

Consider an AmBC system comprising an RF source (S), a backscatter tag with a tunnel diode amplifier (T), and a reader (R) (Fig. 1 (a)). Fig. 1 (b) shows the block diagram of the tag. Let $d_{sr}$, $d_{st}$, and $d_{tr}$ be the $S - R$, $S - T$, and $T - R$ distances, respectively. Let $h_{sr}$, $h_{st}$, and $h_{tr}$ represent the $S - R$, $S - T$, and $T - R$ channel coefficients, respectively. Following [7], [20], we assume the existence of line-of-sight signals, and hence the channel coefficients follow the normalized Rician distribution.

\[
    h_m \sim CN \left( \frac{\kappa}{\kappa + 1} \sigma_m, \frac{\sigma_m^2}{\kappa + 1} \right), m \in \{sr, st, tr\} \tag{1}
\]

We assume channel state information (CSI) is available at the reader via pilot-assist channel estimation.
techniques [7], [21], [22], [23]. Besides, if the CSI is not available, the non-coherent detection and the cloud-based methods can be applied to realize backscatter communication [12], [24], [25], [26].

The tag performs backscatter modulation by reflecting the incident RF signals from the RF source. Precisely, the tag will dynamically adjust its internal impedance. The resulting mismatch induces the reflected RF signal with a specific phase and amplitude. They will make up the signal constellations. This process is also known as load modulation [27]. Finally, the reader can receive both the RF ambient signal and the tag-reflected RF signal. The former acts as an interference in the detection process of the reader.

We denote the ambient RF source signal by random \( s(n) \) with zero mean and unit variance. The received signal at the tag can be written as

\[
y_t(n) = \frac{\sqrt{P_s} h_{tr}^*}{\sqrt{\sigma_n^2}} s(n),
\]

where \( P_s \) is the transmit power of the RF source, and \( \alpha \) is the large-scale path loss factor.

The tag will adjust its reflection coefficient \( \Gamma \) by dynamically changing the load impedance \( Z_L \) to modulate its information. It can be expressed as

\[
\Gamma = \frac{Z_L - Z_A^*}{Z_L + Z_A} = |\Gamma| e^{i \theta},
\]

where \( Z_A \) and \( Z_L \) are antenna impedance and load impedance, respectively, and \( |\Gamma| \) and \( \theta \) are the amplitude and phase of the reflection coefficient, respectively.

Consequently, the backscattered signal can be expressed as

\[
x_t(n) = |\Gamma| e^{i \theta} x(n)y_t(n).
\]

The noise contribution from the tag is negligible and can be omitted [7], [20].

Denote the antenna impedance by

\[
Z_A = R_A + jX_A, \quad (R_A > 0)
\]

where \( R_A \) and \( X_A \) are the real and imaginary parts of \( Z_A \).

Typically, the tag applies on-off key (OOK) modulation. To achieve it, the tag switches the load impedance \( Z_L \) between two values, which generates binary OOK modulated signals. Specifically, when the tag chooses \( Z_L = Z_A^* \) so that \( |\Gamma| = 0 \), which is the absorption state; when the tag sets \( Z_L \neq Z_A^* \) and hence \( |\Gamma| \neq 0 \), which is the backscattering state. The absorption and backscattering state can represent binary data ‘0’ and ‘1’, respectively.

With conventional tags, the resistance of load impedance \( \Re\{Z_L\} \) is always positive, where \( \Re\{z\} \) denotes the real part of the complex number \( z \). The positive impedance ensures that the magnitude of the reflection coefficient cannot exceed unity. Whereas for the tag with a tunnel diode amplifier, the real part \( \Re\{Z_L\} \) can be negative. Thus, the amplitude of the reflection coefficient, \( |\Gamma| \), can be greater than unity. This condition results in the amplification of the backscatter signals. By exploiting these two options, the tag may or may not amplify the signals depending on practical operating scenarios. We will discuss these next.

Without loss of generality, the signal received at the reader can be expressed as

\[
y(n) = \frac{\sqrt{P_s} h_{tr}^*}{\sqrt{\sigma_n^2}} s(n) + \frac{h_{tr}}{\sqrt{\sigma_n^2}} x_t(n) + \omega(n)
= \begin{cases} 
\sqrt{P_s} h_0 s(n) + \omega(n); & x(n) = 0, \\
\sqrt{P_s} h_1 s(n) + \omega(n); & x(n) = 1,
\end{cases}
\]

where \( h_0 = \frac{h_{tr}}{\sqrt{\sigma_n^2}} \), \( h_1 = \frac{h_{tr}}{\sqrt{\sigma_n^2}} + |\Gamma| e^{i \theta} \), and \( \omega(n) \) is the additive white Gaussian noise (AWGN) term with zero mean and \( \sigma_n^2 \) variance.

### III. TUNNELING AMPLIFIER

This section briefly describes the amplification mechanism of the tunnel diode and derives the reflection gain in dB as a function of the input power.

A tunnel diode is a two-port device with a heavily doped positive-to-negative (P-N) junction about 10 nm wide. The heavy doping causes a broken band gap, where conduction band electron states on the N-side are roughly aligned with valence band hole states on the P-side. The depletion layer of the tunnel diode is small. So the electrons can directly tunnel across it from the n-side conduction band into the p-side valence band. This quantum tunneling effect can result in a reverse current and thus a negative impedance [18], [28], [29].

The negative impedance of a tunnel diode is evident from its \( I - V \) curve. Fig. 2 depicts the ideal characteristic \( I - V \) curve of a tunnel diode [18]. This curve shows that when the tunnel diode is properly biased, current \( I \) decreases with bias voltage \( V_{bias} \) (regions II and IV).

Therefore, the resistance of a tunnel diode can be set to a negative value. For instance, consider the negative impedance design of backscatter devices. The tag in [15] uses an MSP430 microcontroller to output a voltage and leverages a matching network configured for a specific impedance value of the tunnel diode to bias the output voltage between 65 mV and 150 mV, yielding an impedance approximately to \(-287 \Omega\).
Accordingly, the most critical parameter that controls the V or less fixed. Therefore, it is reasonable to assume that f is constant, and the environment’s temperature remains more

\[ \text{Ideal characteristic curve of a tunnel diode.} \]

The reflection coefficient is described as a function of parameters of a tunneling tag over the binary input and binary output (BIBO) channel. The input is the ideal OOK state of the tag.

Remark 2: A tunneling tag may not be activated if the input power \( P_{in} \) is less than its activation threshold. To ensure this does not happen, the tag cannot limit itself to only reflection locking. In such a case, the tag cannot work correctly.

3) When \( P_{in} \) is greater than \( P_u \), no amplification is achieved because the mismatch of \( Z_A \) and \( Z_L \) results in a reflection coefficient whose amplitude is less than one. The tag may then perform even worse than the traditional tag. To avoid this, the authors of [15] propose a switchover mechanism so that the tag can sense the received signal power \( P_{in} \) and switch its operating mode from the amplification mode to the standard backscatter mode when \( P_{in} \geq P_u \). Using this mechanism, we can fix the reflection gain \( G \) as a constant \( \eta \) less than or equal to 0 dB.

Remark 1: The parameters \( a, b, c, f, P_l, \) and \( P_u \) in (11) are determined by the specific circuit designs of a tag. The values of these parameters can be determined according to the measured data of the tag.

In this section, we analyze the capacity of the AmBC system with a tunneling tag over the binary input and binary output (BIBO) channel. The input is the ideal OOK state of the tag. The output is the detected symbols from the energy detector of the reader.

A. CAPACITY OF BIBO CHANNEL
We assume \( s(n) \sim \mathcal{CN}(0, 1) \). The received signal at the reader, (6), is distributed as

\[
\gamma(n) \sim \begin{cases} 
\mathcal{CN}(0, \sigma_0^2); & x(n) = 0, \\
\mathcal{CN}(0, \sigma_1^2); & x(n) = 1,
\end{cases}
\]
where \( \sigma_0^2 \triangleq |h_0|^2 P_s + \sigma_n^2 \) and
\[
\sigma_1^2 \triangleq |h_1|^2 P_s + \sigma_n^2 = \frac{h_{tr}}{\sqrt{d_{tr}^2 + 10^{10} G\left(\frac{d_{tr}}{d_{sr}^2}\right)^2}} h_{sr} \frac{h_{sr}}{\sqrt{d_{sr}^2 + 10^{10} G\left(\frac{d_{sr}}{d_{tr}^2}\right)^2}} P_s + \sigma_n^2.
\]

The detector needs to know the parameters \( \sigma_0^2 \) and \( \sigma_1^2 \) for computing its decisions. However, these parameters can be obtained after estimating the channel coefficients through coordination among nodes [32], or the blind estimation with short training of the tag [7]. Such estimation can be well conducted, especially for a sparse channel, and pioneering work in [33] provides an essential and effective tool to measure channel sparsity.

We define the binary input distribution as \( \text{Prob}(x(n) = 0) = p, \text{Prob}(x(n) = 1) = 1 - p \), the binary output distribution as \( \text{Prob}(\hat{x}(n) = 0) = q, \text{Prob}(\hat{x}(n) = 1) = 1 - q \). Let \( P_{0|i} \) denote the conditional probability of output \( j \) given input \( i \). The conditional probability \( P_{0|i} \) of the output 0 given the input \( i \) can be computed as [7]:
\[
P_{0|i} = \Gamma \left(N, \frac{T_h}{\sigma_i^2}\right) / \Gamma(N),
\]
where \( N \) is the ratio of the data rates of the RF source to the tag, \( T_h \) is the detection threshold defined as
\[
T_h = \frac{N \sigma_0^2 \sigma_1^2}{\sigma_0^2 - \sigma_1^2} \ln \frac{\sigma_0^2}{\sigma_1^2},
\]
and \( \Gamma(N, x) \) denotes the upper incomplete Gamma function
\[
\Gamma(N, x) = \int_x^\infty t^{N-1} e^{-t} dt.
\]

Theorem 1: The channel capacity of the BIBO channel can be expressed as
\[
C_{\text{BIBO}} = -h(P_{0|0}) + (P_{0|0} - 1) D(P_{0|0}, P_{0|1}) + \log \left(1 + 2D(P_{0|0}, P_{0|1})\right),
\]
where the binary entropy function is defined as
\[
h(x) \triangleq -x \log x - (1 - x) \log(1 - x), \quad 0 \leq x \leq 1,
\]
and the function \( D(\cdot, \cdot) \) is
\[
D(x, y) \triangleq \frac{h(x) - h(y)}{x - y}.
\]

See Appendix A for the proof.

B. CAPACITY OPTIMAL S – T DISTANCE

The \( S - T \) distance determines the amount of large-scale path loss. Thus, the input power \( P_{in} \) of the tag and the reflection gain function \( G \) are functions of it. Consequently, the capacity of the BIBO channel is a function of the distance \( d_{st} \).

As the channel coefficients remain static during the channel coherence time, we can improve the channel capacity by optimizing the distance \( d_{st} \).

Therefore, we aim to derive the optimal \( d_{st}^* \) that maximizes \( C_{\text{BIBO}} \).
\[
d_{st}^* = \arg \max_{d_{st}} C_{\text{BIBO}}(d_{st}),
\]
where the \( T - R \) distance \( d_{st} \) must satisfy the condition \( |d_{tr} - d_{sr}| < d_{st}^* < d_{tr} + d_{sr} \) to form a triangle. At the same time, it also need meets the condition \( d_{st} < \sqrt[3]{\gamma P_s |h_{sr}|^2 / P_I} \) to ensure the power \( P_{in} \) is greater than the threshold \( P_I \) thus the tag can backscatter signals. Therefore, we define \( d_{min} \triangleq |d_{tr} - d_{sr}|, \) and \( d_{max} \triangleq \min \{d_{tr} + d_{sr}, \sqrt[3]{\gamma P_s |h_{sr}|^2 / P_I}\} \).

Clearly, as the distance \( d_{st} \) varies, the input power \( P_{in} \) may fall into one of the following two conditions: (1) \( P_{in} > P_u \); (2) \( P_t \leq P_{in} \leq P_u \). Therefore, we transform the original optimization problem (20) into two optimization problems for different distance intervals, i.e., the interval \( I_1 \) without reflection gain, and the interval \( I_2 \) with reflection gain. These two intervals can be represented as
\[
I_1 = [d_{min}, \min\{d_{max}^\text{amp}, d_{max}\}],
\]
\[
I_2 = [\max\{d_{min}^\text{amp}, d_{min}\}, d_{max}].
\]

where \( d_{min}^\text{amp} \triangleq \sqrt[3]{\gamma P_s |h_{sr}|^2 / P_u} \) is the minimum distance that the tag has a reflection gain. We find the maximum values in the two intervals, respectively, and then select the distance which maximizes the channel capacity.

In the interval \( I_1 \), the input power \( P_{in} \) is strong and there is no amplification gain. Obviously, the optimal distance in this range is \( d_{min} \).

Whereas in the interval \( I_2 \), the capacity varies according to the reflection gain. The derivative of (17) is
\[
\frac{\partial C_{\text{BIBO}}(d_{st})}{\partial d_{st}} = \left[D(P_{0|0}, P_{0|1}) - \log \left(\frac{1 - P_{0|0}}{P_{0|0}}\right)\right] \frac{\partial P_{0|0}}{\partial d_{st}}
\]
\[
+ \left[P_{0|0} \frac{1}{1 + 2D(P_{0|0}, P_{0|1})} \right] \frac{\partial D(P_{0|0}, P_{0|1})}{\partial d_{st}},
\]
where
\[
\frac{\partial D(P_{0|0}, P_{0|1})}{\partial d_{st}} = \frac{1}{P_{0|0} - P_{0|1}} \times \left[\log \left(\frac{1 - P_{0|0}}{P_{0|0}}\right) \frac{\partial P_{0|0}}{\partial d_{st}}
\right]
\]
\[
- \log \left(\frac{1 - P_{0|1}}{P_{0|1}}\right) \frac{\partial P_{0|1}}{\partial d_{st}} - D(P_{0|0}, P_{0|1})
\]
\[
\times \left[\frac{\partial P_{0|0}}{\partial d_{st}} - \frac{\partial P_{0|1}}{\partial d_{st}}\right].
\]

According to (14), the derivative of \( P_{0|i} \) with respect to \( d_{st} \) is computed as
\[
\frac{\partial P_{0|i}}{\partial d_{st}} = \frac{\partial P_{0|i}}{\partial \sigma_i^2} \frac{\partial \sigma_i^2}{\partial d_{st}},
\]
where
\[
\frac{\partial P_{0|i}}{\partial \sigma_i^2} = \frac{e^{-\frac{T_h}{\tilde{\sigma}_i}} \left(\frac{\tilde{\sigma}_i}{\sigma_i^2}\right)^{N-1}}{\Gamma(N)} \left[\frac{N \sigma_0^2 \ln \left(\frac{\tilde{\sigma}_0^2}{\sigma_0^2}\right)}{\sigma_i^2 (\sigma_0^2 - \sigma_1^2)^2} - \frac{N \sigma_0^2 \ln \left(\frac{\tilde{\sigma}_0^2}{\sigma_0^2}\right)}{(\sigma_0^2 - \sigma_1^2)^2}\right].
\]
\[
\frac{\partial P_0(s)}{\partial \sigma_1} = e^{-\frac{T_o}{\sigma_1}} \frac{I_o}{\Gamma(N)} \left\{ \frac{N}{\alpha_0 - \sigma_1^2} - \frac{N \sigma_0^2 \ln \left( \frac{\alpha_0}{\sigma_1} \right)}{\left( \sigma_0 - \sigma_1^2 \right)^2} \right\}.
\]

(27)

The derivative of \(G(d_{st})\) can be calculated as
\[
\frac{\partial G(d_{st})}{\partial d_{st}} = 2a \frac{\partial [h_{st}^2 P_s]}{\partial d_{st}^a} \left( \frac{h_{st}^2 P_s}{d_{st}^a} - b \right)
\]
\[\times e^{-\left( \frac{\partial [h_{st}^2 P_s]}{\partial d_{st}^a} a \right)^2.}\]

(28)

Substituting (24)-(29) into (23), we can have the partial derivative of \(C_{BIBO}\) regarding \(d_{st}\). The optimal distance in the interval \(I_2\) lies on the point that satisfies \(\frac{\partial C_{BIBO}}{\partial d_{st}} = 0\). Unfortunately, a closed-form solution appears intractable.

Nevertheless, since \(C_{BIBO}(d_{st})\) is a function of a single variable, we can simply obtain the solution by one-dimensional searching. Here we use the modified golden section searching (MGSC) method since the function may be unimodal or non-unimodal because of the different parameters of the amplification function \(G\). The main idea of the MGSC method is discarding the searching intervals whose value is greater than the borders of the first search interval [34].

The overall process to obtain the optimal distance \(d_{st}\) is shown in Algorithm 1. Line 1 initializes the parameters of the algorithm. Line 2 to line 11 determine whether the TD has amplification gain in the range \([d_{min}, d_{max}]\). If there is no gain, the algorithm outputs \(d_{min}\) as the optimal distance. If there is a gain, it further finds the interval where the gain exists and initializes the border parameter of the searching method. Line 12 to line 39 are the basic steps of MGSC. As space is limited, we omit detailed descriptions.

A use-case scenario for AmBC with tunneling-diode tags is an intelligent agriculture farm, where the animals roam around the farm, and each animal wears a tag that can record the state information of things such as plants, temperature, and others. The tag caches the data in its memory and sends it to the channel when the condition is good enough. When the animals move to a place that fits for transmitting data, the tag will send data to the reader. A central controller might help the data transmission by adjusting the position of the animals. We should note that the tag does not consistently achieve the optimal distance. A suboptimal distance is acceptable [35], [36].

Algorithm 1 Distance Searching Algorithm

**Input:** Parameters: \(d_{sf}, d_{tr}, d_{st}, h_{sf}, h_{tr}, h_{st}, \alpha, b, c, f, P_1, P_{st}, G, N, \alpha\), the stop length \(\epsilon\).

**Output:** The optimal distance \(d_{st}^*\)

1. Initialize: \(k = 1, f(z) = -C_{BIBO}(z), d_{min} = |d_{sf} - d_{tr}|, d_{max} = \min \{d_{sf} + d_{st}, \sqrt{P_s|h_{st}|^2/P_f} \}, d_{imp} = \sqrt{\frac{G}{P_s|h_{st}|^2/P_f}}\).
2. if \(d_{max} \leq d_{imp}\) then
   3. Return \(d_{st}^* = d_{min}\).
   4. else
      5. \(d_1 = d_{min}, z_1^* = d_{max}\).
      6. if \(d_{imp} \leq d_{min}\) then
         7. \(I_1 = \emptyset, I_2 = [d_{min}, d_{max}], z_1^* = d_{min}\).
      8. else
         9. \(I_1 = [d_{min}, d_{imp}], I_2 = [d_{imp}, d_{max}], z_1^* = d_{imp}\).
      10. end if
      11. if \(k > 2\) then
          12. \(z_1^* = z_{1,k - 1}^* + 0.382(z_{1,k}^* - z_{1,k - 1}^*), z_2^* = z_{1,k}^* + 0.618(z_{1,k}^* - z_{1,k - 1}^*).\)
          13. \(t^k = \min(f(z_{1,k}^*)), t \in \{1, 2, 3, 4\}.\)
          14. if \(t^k > 2\) then
              15. \(z_{1,k}^* - z_{1,k - 1}^* < \epsilon,\) then
                  16. \(d_2 = \frac{z_{1,k}^* + z_{1,k - 1}^*}{2},\) Return \(d_{st}^* = \arg \min f(x).\) \(\{d_1, d_2\}\).
              17. else
                  18. \(z_{1,k + 1}^* = z_{1,k}^* + 0.382(z_{1,k}^* - z_{1,k - 1}^*),\)
                  19. \(z_{2,k + 1}^* = z_{1,k}^* + 0.618(z_{1,k}^* - z_{1,k - 1}^*).\)
                  20. if \((-1)^k f(z_{1,k}^*) < (-1)^k f(z_{1,k + 1})\) then
                      21. \(t^k = t^k + 1, k = k + 1,\) go to 14.
                  22. else
                      23. \(k = k + 1,\) go to 14.
                  24. end if
                  25. end if
                  26. else
                      27. \(z_{1,k}^* - z_{1,k - 1}^* < \epsilon,\) then
                          28. \(d_2 = \frac{z_{1,k}^* + z_{1,k - 1}^*}{2},\) Return \(d_{st}^* = \arg \min f(x).\) \(\{d_1, d_2\}\).
                      29. else
                          30. \(z_{1,k + 1}^* = z_{1,k}^* + 0.382(z_{1,k}^* - z_{1,k - 1}^*),\)
                          31. \(z_{2,k + 1}^* = z_{1,k}^* + 0.618(z_{1,k}^* - z_{1,k - 1}^*).\)
                          32. if \((-1)^k f(z_{1,k}^*) \leq (-1)^k f(z_{1,k + 1})\) then
                              33. \(k = k + 1,\) go to 14.
                          34. else
                              35. \(t^k = t^k + 1, k = k + 1,\) go to 14.
                          36. end if
                          37. end if
                  38. end if

**V. SIMULATION RESULTS**

This section numerically evaluates the capacity of the proposed AmBC system. The path loss factor is set as \(\alpha = 2\); the reflection gain of the traditional backscatter tag is set as \(\eta = 0 \text{ dB}\); the noise floor is set as \(\sigma_0^2 = -90 \text{ dBm}\) [37], [38].

We consider three tunnel diode amplifiers from [18] and [19]. Their original data is shown as Table 1. We fit the
TABLE 1. The original data of tunnel diode amplifiers.

| Tunnel diode 1 | Tunnel diode 2 | Tunnel diode 3 |
|---------------|---------------|---------------|
| $P_{in}$ Gain | $P_{in}$ Gain | $P_{in}$ Gain |
| -81.16 32.63 | -83.87 27.91 | -75.82 36.20 |
| -76.71 39.34 | -76.90 25.08 | -72.61 31.07 |
| -74.97 39.03 | -71.97 24.04 | -69.78 19.97 |
| -71.87 37.03 | -69.45 24.56 | -67.64 18.70 |
| -66.84 32.84 | -64.32 21.83 | -66.57 13.15 |
| -61.61 28.96 | -60.55 20.05 | -65.18 9.35 |
| -56.97 24.35 | -55.52 17.22 | -63.74 8.87 |
| -51.74 19.84 | -50.97 15.96 | / / |
| -44.68 12.82 | -46.03 13.03 | / / |
| -41.77 9.57 | -40.61 10.31 | / / |
| -37.32 4.22 | -35.68 7.11 | / / |

TABLE 2. Fitted parameters of tunnel diode amplifiers.

| Index | $a$ | $b$ | $c$ | $f$ | $P_{in}$ | $P_{out}$ |
|------|-----|-----|-----|-----|---------|----------|
| 1    | 36.69 | -76.23 | 27.23 | 2 | -84dBm | -32dBm |
| 2    | 22.93 | -84.95 | 37.11 | 5 | -84dBm | -32dBm |
| 3    | 34.63 | -78.24 | 9.67 | 4 | -79dBm | -55dBm |

reflection gains according to [19]. Table 2 shows the fitted parameters, and Fig. 3 shows the fitted curves.

Fig. 4 illustrates the channel capacity versus the transmit signal-to-noise ratio (SNR) $\rho$ with different data rate ratios, $N$. We set the Rician factors as $\kappa = 10$, and the scale parameters $\sigma_{st} = \sigma_{sr} = \sigma_{tr} = 1$, and the distance $d_{st} = d_{sr} = d_{tr} = 10$ m. For the sake of brevity, System $n$ denotes the system with tunnel diode (TD) amplifier $n$ from Table 2. The system without a TD amplifier is named the Traditional system. We can see that tunneling-tag AmBC almost consistently outperforms the baseline (i.e., AmBC with traditional, passive tags). Interestingly, the channel capacities of TD-based systems are not monotonically increasing with $P_{in}$. The input RF signal power affects the TD amplifier’s reflection gain. Therefore, the TD-assisted system capacity curve tracks that of reflection gain as a function of $P_{in}$, a valuable insight into the adjustment and optimization of AmBC. The ratio between the data of the RF source and that of the tag, $N$, is a critical parameter. Fig. 4 also shows that a larger $N$ leads to higher capacity. Intriguingly, however, the gain because of increasing $N$ has a marginal value effect, i.e., increasing the data rate of the RF source (or slowing the tag rate) will face diminishing capacity gain returns.

Fig. 5 depicts the channel capacity versus distance $d_{st}$ and $d_{tr}$ when transmit signal-to-noise ratio $\rho = 50$ dB and the channel coefficient $h_{sr} = 0.86$, $h_{tr} = 0.52$, $h_{tr} = 0.69$. From Fig. 5 (a), we can find that each system achieves different performance levels at various distances. TD 1 offers the best performance overall, and all the TD-assisted systems outperform the baseline. We can see from Fig. 5 (b) that the channel capacity of all the systems decreases with $d_{tr}$. This trend is easy to explain because, as transmit power, $P_{in}$, remains fixed, the received signal power at the tag’s input gets attenuated because of the path loss, which increases with the distance. This explains the downtrend in all curves with the
channel capacity is to optimize the distance between the RF source to the reader. Fig. 8 shows the searched distance $d_{st}$ with Fig. 7 (a) shows the optimal distance $d_{st}^*$ versus the RF power $P_s$. It shows that in the traditional system, the optimal placement of the tag is close to the RF source, while that of the TD-assisted system depends on the RF power level. Of course, the optimal distance of the tunneling tag is much larger than that of the traditional system. Fig. 7 (b) shows the optimal capacities versus the transmit SNR $\rho$. This figure shows that the curves gradually reach 0.05 (bps/Hz) as the transmit SNR $\rho$ increases. The capacities of all three TD-assisted AmBC systems show a precipitous drop beyond a certain distance. This drop occurs because of interference from the RF source to the reader. Fig. 8 shows the searched distance $d_{st}$ and the corresponding capacity for given $d_{st}$. The stop length $\epsilon$ is set as $= 0.01$. Although the searched distance may differ from the optimal distance in Fig. 7 (a), the capacity performance is very close, demonstrating the effectiveness of the search method.

VI. CONCLUSION

This paper investigated the channel capacity of tunnel diode amplifier-assisted AmBC systems. Specifically, we formulated the system model and developed the mathematical model for the reflection gain of the tag. We then derived the closed-form channel-capacity expression. Finally, we provided simulation results to corroborate the theoretical derivations. They show that using the tunnel diode amplifier in AmBC can significantly improve the channel capacity and increase the coverage range. Another way to maximize the channel capacity is to optimize the distance between the RF source and the tag. Our study sheds light on this matter too. We also find that the system performance depends on the amplification range and the peak reflection gain of the tunnel diode amplifier to a significant degree. Consequently, to maximize the system gain from its use, the performance of the tunnel diode amplifier should be optimized in terms of its operating point and other factors.
The necessary and sufficient condition on the input distribution \( p^* = [p^*_1, 1 - p^*_1] \) to achieve capacity is

\[
I(x = 0; \hat{x})_{p=p^*} = I(x = 1; \hat{x})_{p=p^*} = E,
\]

where \( I(x = 0; \hat{x}) \) is the mutual information for \( x = 0 \) averaged over the output, and \( E \) is a scalar that is greater than 0.

Utilizing the Bayes Rule, the mutual information can be express as

\[
I(x = i; \hat{x}) = \sum_{j=0}^{1} P_{ji} \log \frac{P_{ji}}{\sum_{k=0}^{1} P(x = k)P_{jk}}
\]

\[
= \frac{P_{0|j}}{q} \left( 1 - P_{0|i} \right) \log \frac{1 - P_{0|i}}{1 - q} + \frac{P_{0|j}}{q} \left( 1 - P_{0|i} \right) \log \left( 1 - q \right)
\]

Substituting (A.3) into (A.2), we have

\[
p^* = \frac{q^* - P_{0|1}}{P_{0|0} - P_{0|1}}, \quad q^* = \frac{1}{1 + 2\sqrt{P_{0|1}P_{0|0}}}.
\]

By using (A.4), we can obtain (17).

REFERENCES

[1] M. Chui, M. Collins, and M. Patel, “IoT value set to accelerate through 2030: Where and how to capture it,” McKinsey Company, Tech. Rep., 2022.
[2] X. Liu and N. Ansari, “Toward green IoT: Energy solutions and key challenges,” IEEE Commun. Mag., vol. 57, no. 3, pp. 104–110, Mar. 2019.
[3] X. Lu, P. Wang, D. Niyato, D. I. Kim, and Z. Han, “Wireless networks with RF energy harvesting: A contemporary survey,” IEEE Commun. Surveys Tuts., vol. 17, no. 2, pp. 757–789, 2nd Quart., 2015.
[4] A. Varshney, A. Soleiman, I. Mottola, and T. Voigt, “Battery-free visible light sensing,” in Proc. 4th ACM Workshop Visible Light Commun. Syst., Snowbird, UT, USA, Oct. 2017, pp. 3–8.
[5] N. Van Huynh, D. T. Hoang, X. Lu, D. Niyato, P. Wang, and D. I. Kim, “Ambient backscatter communications: A contemporary survey,” IEEE Commun. Surveys Tuts., vol. 20, no. 4, pp. 2889–2922, 4th Quart., 2018.
[6] A. A. Khan, M. H. Rehman, and A. Rachedi, “Cognitive-radio-based Internet of Things: Applications, architectures, spectrum related functionalities, and future research directions,” IEEE Wireless Commun., vol. 24, no. 3, pp. 17–25, Jun. 2017.
[7] J. Qian, F. Gao, G. Wang, S. Jin, and H. Zhu, “Semi-coherent detection and performance analysis for ambient backscatter system,” IEEE Trans. Commun., vol. 65, no. 12, pp. 5266–5279, Dec. 2017.
[8] G. Wang, F. Gao, R. Fan, and C. Tellambura, “Ambient backscatter communication systems: Detection and performance analysis,” IEEE Trans. Commun., vol. 64, no. 11, pp. 4836–4846, Nov. 2016.
[9] X. Kuai, X. Yuan, and Y.-C. Liang, “Message-passing receiver design for multiuser multi-backscatter-device symbiotic radio communications,” IEEE Trans. Wireless Commun., vol. 21, no. 6, pp. 4563–4578, Jun. 2022.
[10] D. Darsena, G. Gelli, and F. Verde, “Modeling and performance analysis of wireless networks with ambient backscatter devices,” IEEE Trans. Commun., vol. 65, no. 4, pp. 1797–1814, Apr. 2017.
[11] I. Qian, Y. Zhu, C. He, F. Gao, and S. Jin, “Achievable rate and capacity analysis for ambient backscatter communications,” IEEE Trans. Commun., vol. 67, no. 9, pp. 6299–6310, Sep. 2019.
[12] D. Darsena, G. Gelli, and F. Verde, “Cloud-aided cognitive ambient backscatter wireless sensor networks,” IEEE Access, vol. 7, pp. 57399–57414, 2019.
[13] H. Guo, Y.-C. Liang, R. Long, and Q. Zhang, “Cooperative ambient backscatter system: A symbiotic radio paradigm for passive IoT,” IEEE Wireless Commun. Lett., vol. 8, no. 4, pp. 1191–1194, Aug. 2019.
[14] W. Zhao, G. Wang, S. Atapattu, T. A. Tsiftsis, and C. Tellambura, “Is backscatter link stronger than direct link in reconfigurable intelligent surface-assisted system?” IEEE Commun. Lett., vol. 24, no. 6, pp. 1342–1346, Jun. 2020.
[15] V. Ambuj, S. Andreas, and V. Thiemo, “Tunnelscatter: Low power communication for sensor tags using tunnel diodes,” in Proc. 25th Annu. Int. Conf. Mobile Comput. Netw. (MobiCom), New York, NY, USA, Aug. 2019, pp. 1–17.
[16] F. Amato, C. W. Peterson, B. P. Degnan, and G. D. Durgin, “A 45 µW bias power, 34 dB gain reflection amplifier exploiting the tunneling effect for RFID applications,” in Proc. IEEE Int. Conf. RFID (RFID), Apr. 2015, pp. 137–144.
[17] F. Farzami, S. Kalhadian, B. Smida, and D. Erricolo, “Ultra-low power reflection amplifier using tunnel diode for RFID applications,” in Proc. IEEE Int. Symp. Antennas Propag. USNC/URSI Nat. Radio Sci. Meeting, San Diego, CA, USA, Jul. 2017, pp. 2511–2512.
[18] F. Amato, C. W. Peterson, B. P. Degnan, and G. D. Durgin, “Tunneling RFID tags for long-range and low-power microwave applications,” IEEE J. Radio Freq. Identif., vol. 2, no. 2, pp. 93–103, Jun. 2018.
[19] F. Amato, H. M. Torun, and G. D. Durgin, “RFID backscattering in long-range scenarios,” IEEE Trans. Wireless Commun., vol. 17, no. 4, pp. 2718–2725, Apr. 2018.
[20] K. Skyvalakis and A. Bletsas, “Asynchronous reception of 2 RFID tags,” IEEE Trans. Commun., vol. 69, no. 8, pp. 5243–5254, Aug. 2021.
[21] Y. Zhao, G. Wang, S. Atapattu, R. He, and Y.-C. Liang, “Channel estimation for ambient backscatter communication systems with massive-antenna reader,” IEEE Trans. Veh. Technol., vol. 68, no. 8, pp. 8254–8258, Aug. 2019.
[22] C. Liu, Z. Wei, D. W. K. Ng, J. Yuan, and Y.-C. Liang, “Deep transfer learning for signal detection in ambient backscatter communications,” IEEE Trans. Wireless Commun., vol. 20, no. 3, pp. 1624–1638, Mar. 2021.
[23] S. Ma, G. Wang, R. Fan, and C. Tellambura, “Blind channel estimation for ambient backscatter communication systems,” IEEE Commun. Lett., vol. 22, no. 6, pp. 1296–1299, Jun. 2018.
[24] D. Darsena, “Noncoherent detection for ambient backscatter communications over OFDM signals,” IEEE Access, vol. 7, pp. 159415–159425, 2019.
[25] M. A. EllMosallamly, M. Pan, R. Jäänti, K. G. Sedddik, G. Y. Li, and Z. Han, “Noncoherent backscatter communications over ambient OFDM signals,” IEEE Trans. Commun., vol. 67, no. 5, pp. 3597–3611, May 2019.
[26] J. Qian, F. Gao, G. Wang, S. Jin, and H. Zhu, “Noncoherent detections for ambient backscatter system,” IEEE Trans. Wireless Commun., vol. 16, no. 3, pp. 1412–1422, Mar. 2017.
[27] F. Rezaei, C. Tellambura, and S. Herath, “Large-scale wireless-powered networks with backscatter communications—A comprehensive survey,” IEEE Open J. Commun. Soc., vol. 1, pp. 1100–1130, 2020.
[28] (2022). Wikipedia Contributors. Tunnel Diode—Wikipedia, the Free Encyclopedia. Accessed: Jul. 19, 2022. [Online]. Available: https://en.wikipedia.org/w/index.php?title=Tunnel_diode&oldid=1085384566.
[29] N. Kluksdahl, A. Kriman, D. Ferry, and C. Ringhofer, “Self-consistent study of the resonant-tunneling diode,” Phys. Rev. B, Condens. Matter, vol. 39, no. 11, p. 7720, Apr. 1989.
[30] B. Razavi, “A study of injection locking and pulling in oscillators,” IEEE J. Solid-State Circuits, vol. 39, no. 9, pp. 1415–1424, Sep. 2004.
[31] A. Varshney and L. Cornejo, “Tunnel emitter: Tunnel diode based low-power carrier emitters for backscatter tags,” in Proc. 26th Annu. Int. Conf. Mobile Comput. Netw. (MobiCom), Sep. 2020, pp. 1–14.
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