TASI 2011: lectures on Higgs-Boson Physics

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Abstract

In these lectures I briefly review the Higgs mechanism of electroweak symmetry breaking and focus on the most relevant aspects of the phenomenology of the Standard Model Higgs boson at hadron colliders, namely the Tevatron and the Large Hadron Collider. Emphasis is put in particular on the Higgs physics program of both LHC experiments and on the theoretical activity that has entailed from the need of providing accurate predictions for both signal and background in Higgs searches.
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1
1 Introduction

The mechanism through which the electroweak gauge bosons, the $W^\pm$ and the $Z^0$, as well as all elementary fermions, leptons and quarks, develop the mass properties of which we have experimental evidence is unknown at the moment. Generically dubbed as the mechanism of electroweak symmetry breaking (EWSB), this problem has been the core question that all theories proposed as extensions of the Standard Model try to answer.

The so called Higgs mechanism provides a very simple and economical solution to the problem of EWSB, and since it was first proposed in 1964 by Higgs, Kibble, Guralnik, Hagen, Englert and Brout \[1, 2, 3\], it has become de facto part of the Standard Model (SM). By introducing one complex pair of scalar fields with a non trivial potential and a suitable interaction to all matter particles, it achieves the goal of providing mass to both the weak force carriers and the elementary matter particles, at the expense of introducing just one new particle, the by now famous Higgs particle or Higgs boson. Extensions of the SM often generalize the same mechanism to adapt it to more involved symmetry patterns. This is for instance the case of the Minimal Supersymmetric Standard Model (MSSM), where two pairs of complex fields are introduced instead of one, resulting in a final set of several Higgs bosons. While the single Higgs boson of the SM is a neutral scalar (i.e. spinless) particle (which we will denote by $H$), the MSSM has four Higgs bosons, two neutral scalars ($h^0$ and $H^0$), one neutral pseudoscalar ($A^0$), and one charged scalar ($H^\pm$). Extensions of the SM besides the MSSM can have even richer spectra of scalar and pseudoscalar particles originating in the process of EWSB.

Precision studies of the SM and of the MSSM have assumed the corresponding Higgs particle(s) as integral part of the theory and have been able to constrain its (their) masses and couplings. Results from the Tevatron collider have been able to exclude regions of the parameter space of both the SM and the MSSM. Since his inception the LHC has validated and extended the Tevatron bounds and has recently found strong evidence of the existence of a Higgs boson with SM-like properties at about 125-127 GeV \[4, 5\]. Indeed, the discovery of a spin-0 particle compatible with the predictions for a SM Higgs boson has been announced on July, 4th 2012. This comes as one of the most exciting result that we could have ever expected at such an early stage of the LHC and represents a milestone in the history of particle physics.

The success of the LHC Higgs physics program hinges however on the
crucial assumption that experimental data can be compared with very accurate theoretical predictions capable of discriminating between signal and background at a statistically significant level. Theorists have been meeting this challenge by modeling the complexity of hadronic interactions in the context of Quantum Chromodynamics (QCD). Since at high energies QCD is a perturbative quantum field theory (pQCD), QCD effects at colliders can be calculated order by order in the strong coupling constant.

Lower order predictions typically have larger uncertainties associated with them and cannot be meaningfully used to compare to experimental measurements. In preparation for the LHC, a huge theoretical effort has been devoted to provide accurate QCD predictions for all the most important processes that are and will be the core of the LHC physics program. In particular, the Higgs physics program of the LHC has received an incredible amount of attention and most SM Higgs production processes have been calculated to a high level of precision, including a few orders of perturbative QCD corrections. In the context of the LHC Higgs Cross Sections Working Group (LHC-HXSWG) the most up to date theoretical results have been collected for both inclusive and exclusive production cross sections for both SM and MSSM Higgs bosons [6, 7], and a new phase of activity has started that will mainly focus on the identification of the newly discovered particle.

In these lectures I would like to present a self contained introduction to the physics of the Higgs boson(s). In Section 2, after a brief glance at the essence of the Higgs mechanism, I will review how it is embedded in the Standard Model and what constraints are directly and indirectly imposed on the mass of the single Higgs boson that is predicted in this context. Among the extensions of the SM, I will only consider the case of the Minimal Supersymmetric Model (MSSM), and in this context I will mainly focus on those aspects that could be more relevant in distinguishing the MSSM Higgs bosons. Section 3 will review the phenomenology of both the SM and the MSSM Higgs bosons, at the Tevatron and the LHC. Sections 4 and 5 deal specifically with the SM Higgs-boson recent results from the Tevatron and the LHC. Finally, in Section 6 I will summarize the state of the art of existing theoretical calculations for both decay rates and production cross sections of a Higgs boson, and discuss the impact of QCD corrections in the prototype case of the \( gg \to H \) production mode.

Let me conclude by pointing the reader to some selected references available in the literature. The theoretical bases of the Higgs mechanism are nowadays a matter for textbooks in Quantum Field Theory. They are pre-
sented in depth in both Refs. [8] and [9]. An excellent review of both SM
and MSSM Higgs physics, containing a very comprehensive discussion of both
theoretical and phenomenological aspects as well as a thorough bibliography,
can be found in Refs. [10, 11]. The phenomenology of Higgs physics has also
been thoroughly covered in a review paper [12]. Finally, series of lectures
given at previous summer schools [13, 14, 15] can provide further references.

2 Theoretical framework: the Higgs mechanism and its consequences.

In Yang-Mills theories gauge invariance forbids to have an explicit mass term
for the gauge vector bosons in the Lagrangian. If this is acceptable for theo-
ries like QED (Quantum Electrodynamics) and QCD (Quantum Chromody-
namics), where both photons and gluons are massless, it is unacceptable
for the gauge theory of weak interactions, since both the charged ($W^\pm$)
and neutral ($Z^0$) gauge bosons have very heavy masses ($M_W \simeq 80$ GeV,
$M_Z \simeq 91$ GeV). A possible solution to this problem, inspired by similar phe-
nomena happening in the study of spin systems, was proposed by several
physicists in 1964 [1, 2, 3], and it is known today simply as the 

Higgs mechanism. We will review the basic idea behind it in Section 2.1. In Section 2.2
we will recall how the Higgs mechanism is implemented in the Standard
Model and we will discuss which kind of theoretical constraints are imposed
on the Higgs boson, the only physical scalar particle predicted by the model.
Finally, in Section 2.4 we will generalize our discussion to the case of the
MSSM, and use its extended Higgs sector to illustrate how differently the
Higgs mechanism can be implemented in extensions of the SM.

2.1 A brief introduction to the Higgs mechanism

The essence of the Higgs mechanism can be very easily illustrated considering
the case of a classical abelian Yang-Mills theory. In this case, it is realized
by adding to the Yang-Mills Lagrangian

$$\mathcal{L}_A = -\frac{1}{4} F_{\mu\nu}^\alpha F_{\mu\nu}^\alpha \quad \text{with} \quad F_{\mu\nu}^\alpha = (\partial^\mu A^\nu - \partial^\nu A^\mu)^\alpha , \quad (1)$$

a complex scalar field with Lagrangian

$$\mathcal{L}_\phi = (D^\mu \phi)^* D_\mu \phi - \mu^2 \phi^* \phi - \lambda (\phi^* \phi)^2 , \quad (2)$$

4
Figure 1: The potential $V(\phi)$ ($\phi = \phi_1 + i\phi_2$) plotted for an arbitrary positive value of $\lambda$ and for an arbitrary positive (right) or negative (left) value of $\mu^2$.

where $D^\mu = \partial^\mu + igA^\mu$, and $\lambda > 0$ for the scalar potential to be bounded from below. The full Lagrangian

$$\mathcal{L} = \mathcal{L}_A + \mathcal{L}_\phi$$

is invariant under a $U(1)$ gauge transformation acting on the fields as:

$$\phi(x) \rightarrow e^{i\alpha(x)}\phi(x), \quad A^\mu(x) \rightarrow A^\mu(x) + \frac{1}{g}\partial^\mu\alpha(x),$$

while a gauge field mass term (i.e., a term quadratic in the fields $A^\mu$) would not be gauge invariant and cannot be added to $\mathcal{L}$ if the $U(1)$ gauge symmetry has to be preserved. Indeed, the Lagrangian in Eq. (3) can still describe the physics of a massive gauge boson, provided the potential $V(\phi)$ develops a non trivial minimum ($\phi^*\phi \neq 0$). The occurrence of a non trivial minimum, or, better, of a non trivial degeneracy of minima only depends on the sign of the $\mu^2$ parameter in $V(\phi)$. For $\mu^2 > 0$ there is a unique minimum at $\phi^*\phi = 0$, while for $\mu^2 < 0$ the potential develops a degeneracy of minima satisfying the equation $\phi^*\phi = -\mu^2/(2\lambda)$. This is illustrated in Fig. 1 where the potential $V(\phi)$ is plotted as a function of the real and imaginary parts of the field $\phi = \phi_1 + i\phi_2$. In the case of a unique minimum at $\phi^*\phi = 0$ the Lagrangian in Eq. (3) describes the physics of a massless vector boson (e.g. the photon, in electrodynamics, with $g = -e$) interacting with a massive charged scalar particle. On the other hand, something completely different takes place when $\mu^2 < 0$. Choosing the ground state of the theory to be a
particular $\phi$ among the many satisfying the equation of the minimum, and expanding the potential in the vicinity of the chosen minimum, transforms the Lagrangian in such a way that the original gauge symmetry is now hidden or spontaneously broken, and new interesting features emerge. To be more specific, let’s pick the following $\phi_0$ minimum (along the direction of the real part of $\phi$, as traditional) and shift the $\phi$ field accordingly:

$$
\phi_0 = \left( -\frac{\mu^2}{2\lambda} \right)^{1/2} = \frac{v}{\sqrt{2}} \rightarrow \phi(x) = \phi_0 + \frac{1}{\sqrt{2}} (\phi_1(x) + i\phi_2(x)) \, .
$$

(5)

The Lagrangian in Eq. (3) can then be rearranged as follows:

$$
\mathcal{L} = -\frac{1}{4} F^{\mu\nu} F_{\mu\nu} + \frac{1}{2} g^2 v^2 A^\mu A_\mu + \frac{1}{2} (\partial^\mu \phi_1)^2 + \mu^2 \phi_1^2 + \frac{1}{2} (\partial^\mu \phi_2)^2 + gv A_\mu \partial^\mu \phi_2 + \ldots
$$

and now contains the correct terms to describe a massive vector field $A^\mu$ with mass $m_A^2 = g^2 v^2$ (originating from the kinetic term of $\mathcal{L}_\phi$), a massive real scalar field $\phi_1$ with mass $m_{\phi_1} = -2\mu^2$, that will become a Higgs boson, and a massless scalar field $\phi_2$, a so called Goldstone boson which couples to the gauge vector boson $A^\mu$. The terms omitted contain couplings between the $\phi_1$ and $\phi_2$ fields irrelevant to this discussion. The gauge symmetry of the theory allows us to make the particle content more transparent. Indeed, if we parameterize the complex scalar field $\phi$ as:

$$
\phi(x) = e^{i\chi(x)/\sqrt{2}} (v + H(x)) \xrightarrow{U(1)} \frac{1}{\sqrt{2}} (v + H(x)) \, ,
$$

(7)

the $\chi$ degree of freedom can be rotated away, as indicated in Eq. (7), by enforcing the $U(1)$ gauge invariance of the original Lagrangian. With this gauge choice, known as unitary gauge or unitarity gauge, the Lagrangian becomes:

$$
\mathcal{L} = \mathcal{L}_A + \frac{g^2 v^2}{2} A^\mu A_\mu + \frac{1}{2} \left( \partial^\mu H \partial_\mu H + 2\mu^2 H^2 \right) + \ldots
$$

(8)

which unambiguously describes the dynamics of a massive vector boson $A^\mu$ of mass $m_A^2 = g^2 v^2$, and a massive real scalar field of mass $m_H^2 = -2\mu^2 = 2\lambda v^2$, the Higgs field. It is interesting to note that the total counting of degrees of freedom (d.o.f.) before the original $U(1)$ symmetry is spontaneously broken
and after the breaking has occurred is the same. Indeed, one goes from a
theory with one massless vector field (two d.o.f.) and one complex scalar
field (two d.o.f.) to a theory with one massive vector field (three d.o.f.) and
one real scalar field (one d.o.f.), for a total of four d.o.f. in both cases. This
is what is colorfully described by saying that each gauge boson has *eaten up*
one scalar degree of freedom, becoming massive.

We can now easily generalize the previous discussion to the case of a
non-abelian Yang-Mills theory. $L_A$ in Eq. (3) now becomes:

$$L_A = \frac{1}{4} F^{a, \mu\nu} F^a_{\mu\nu} \quad \text{with} \quad F^a_{\mu\nu} = \partial_\mu A^a_\nu - \partial_\nu A^a_\mu + gf^{abc} A^b_\mu A^c_\nu ,$$  \hspace{1cm} (9)

where the latin indices are group indices and $f^{abc}$ are the structure con-
stants of the Lie Algebra associated to the non abelian gauge symmetry Lie

When $\mu^2 < 0$ the potential develops a degeneracy of minima described by
the minimum condition: $\phi^2 = \phi_0^2 = -\mu^2/\lambda$, which only fixes the magnitude of
the vector $\phi_0$. By arbitrarily choosing the direction of $\phi_0$, the degeneracy is
removed. The Lagrangian can be expanded in a neighborhood of the chosen
minimum and mass terms for the gauge vector bosons can be introduced as
in the abelian case, i.e.:

$$\frac{1}{2} (D_\mu \phi_i)^2 \rightarrow \ldots + \frac{1}{2} g^2 (t^a \phi_i) (t^b \phi_i) A^a_\mu A^b_\mu + \ldots$$

$$\phi_{\text{min}} \rightarrow \phi_{\text{min}} + \frac{1}{2} g^2 (t^a \phi_0) (t^b \phi_0) A^a_\mu A^b_\mu + \ldots$$

$$m_{ab}^2$$

7
Upon diagonalization of the mass matrix $m_{\alpha\beta}^2$ in Eq. (12), all gauge vector bosons $A_\mu^a$ for which $t^a \phi_0 \neq 0$ become massive, and to each of them corresponds a Goldstone particle, i.e., an unphysical massless particle like the $\chi$ field of the abelian example. The remaining scalar degrees of freedom become massive, and correspond to the Higgs field $H$ of the abelian example.

The Higgs mechanism can be very elegantly generalized to the case of a quantum field theory when the theory is quantized via the path integral method. In this context, the quantum analog of the potential $V(\phi)$ is the effective potential $V_{\text{eff}}(\varphi_{\text{cl}})$, defined in term of the effective action $\Gamma[\varphi_{\text{cl}}]$ (the generating functional of the 1PI connected correlation functions) as:

$$V_{\text{eff}}(\varphi_{\text{cl}}) = -\frac{1}{VT} \Gamma[\varphi_{\text{cl}}] \quad \text{for} \quad \varphi_{\text{cl}}(x) = \text{constant} = \varphi_{\text{cl}},$$  \hspace{1cm} (13)

where $VT$ is the space-time extent of the functional integration and $\varphi_{\text{cl}}(x)$ is the vacuum expectation value of the field configuration $\phi(x)$:

$$\varphi_{\text{cl}}(x) = \langle \Omega | \phi(x) | \Omega \rangle .$$  \hspace{1cm} (14)

The stable quantum states of the theory are defined by the variational condition:

$$\frac{\delta}{\delta \varphi_{\text{cl}}} \Gamma[\varphi_{\text{cl}}] \bigg|_{\varphi_{\text{cl}} = \varphi_{\text{cl}}} = 0 \quad \longrightarrow \quad \frac{\partial}{\partial \varphi_{\text{cl}}} V_{\text{eff}}(\varphi_{\text{cl}}) = 0 ,$$  \hspace{1cm} (15)

which identifies in particular the states of minimum energy of the theory, i.e., the stable vacuum states. A system with spontaneous symmetry breaking has several minima, all with the same energy. Specifying one of them, as in the classical case, breaks the original symmetry on the vacuum. The relation between the classical and quantum case is made even more transparent by the perturbative form of the effective potential. Indeed, $V_{\text{eff}}(\varphi_{\text{cl}})$ can be organized as a loop expansion and calculated systematically order by order in $\hbar$:

$$V_{\text{eff}}(\varphi_{\text{cl}}) = V(\varphi_{\text{cl}}) + \text{loop effects} ,$$  \hspace{1cm} (16)

with the lowest order being the classical potential in Eq. (2). Quantum corrections to $V_{\text{eff}}(\varphi_{\text{cl}})$ affect some of the properties of the potential and

---

1Here I assume some familiarity with path integral quantization and the properties of various generating functionals introduced in that context, as I did while giving these lectures. The detailed explanation of the formalism used would take us too far away from our main track.
therefore have to be taken into account in more sophisticated studies of the Higgs mechanism for a spontaneously broken quantum gauge theory. We will see how this can be important in Section 2.3 when we discuss how the mass of the SM Higgs boson is related to the energy scale at which we expect new physics effect to become relevant in the SM.

Finally, let us observe that at the quantum level the choice of gauge becomes a delicate issue. For example, in the unitarity gauge of Eq. (7) the particle content of the theory becomes transparent but the propagator of a massive vector field $A^\mu$ turns out to be:

$$\Pi^{\mu\nu}(k) = -\frac{i}{k^2 - m_A^2} \left( g^{\mu\nu} - \frac{k^\mu k^\nu}{m_A^2} \right), \quad (17)$$

and has a problematic ultra-violet behavior, which makes more difficult to consistently define and calculate ultraviolet-stable scattering amplitudes and cross sections. Indeed, for the very purpose of studying the renormalizability of quantum field theories with spontaneous symmetry breaking, the so called renormalizable or renormalizability gauges ($R_\xi$ gauges) are introduced. If we consider the abelian Yang-Mills theory of Eqs. (1)-(3), the renormalizable gauge choice is implemented by quantizing with a gauge condition $G$ of the form:

$$G = \frac{1}{\sqrt{\xi}} (\partial_\mu A^\mu + \xi g v \phi_2), \quad (18)$$

in the generating functional

$$Z[J] = C \int DA D\phi_1 D\phi_2 \exp \left[ i \int (\mathcal{L} - \frac{1}{2} G^2) \right] \det \left( \frac{\delta G}{\delta \alpha} \right), \quad (19)$$

where $C$ is an overall factor independent of the fields, $\xi$ is an arbitrary parameter, and $\alpha$ is the gauge transformation parameter in Eq. (4). After having reduced the determinant in Eq. (19) to an integration over ghost fields ($c$ and $\bar{c}$), the gauge plus scalar fields Lagrangian looks like:

$$\mathcal{L} - \frac{1}{2} G^2 + \mathcal{L}_{\text{ghost}} = -\frac{1}{2} A_\mu \left( -g^{\mu\nu} \partial^2 + \left( 1 - \frac{1}{\xi} \right) \partial^\mu \partial^\nu - (gv)^2 g^{\mu\nu} \right) A_\nu$$

$$+ \frac{1}{2} (\partial_\mu \phi_1)^2 - \frac{1}{2} m_{\phi_1}^2 \phi_1^2 + \frac{1}{2} (\partial_\mu \phi_2)^2 - \frac{\xi}{2} (gv)^2 \phi_2^2 + \cdots$$

$$+ \bar{c} \left[ -\partial^2 - \xi (gv)^2 \left( 1 + \frac{\phi_1}{v} \right) \right] c, \quad (20)$$

9
such that:

\[
\langle A^\mu(k)A^\nu(-k) \rangle = -i \frac{g^{\mu\nu} - \frac{k^\mu k^\nu}{k^2}}{k^2 - m_A^2} + \frac{-i\xi}{k^2 - \xi m_A^2} \left( \frac{k^\mu k^\nu}{k^2} \right),
\]

\[
\langle \phi_1(k)\phi_1(-k) \rangle = -i \frac{1}{k^2 - m_{\phi_1}^2},
\]

\[
\langle \phi_2(k)\phi_2(-k) \rangle = \langle c(k)c(-k) \rangle = -i \frac{1}{k^2 - \xi m_A^2},
\]

where the vector field propagator has now a safe ultraviolet behavior. Moreover we notice that the \(\phi_2\) propagator has the same denominator of the longitudinal component of the gauge vector boson propagator. This shows in a more formal way the relation between the \(\phi_2\) degree of freedom and the longitudinal component of the massive vector field \(A^\mu\), upon spontaneous symmetry breaking.

### 2.2 The Higgs sector of the Standard Model

The Standard Model is a spontaneously broken Yang-Mills theory based on the \(SU(2)_L \times U(1)_Y\) non-abelian symmetry group\[^8,9\]. The Higgs mechanism is implemented in the Standard Model by introducing a complex scalar field \(\phi\), doublet of \(SU(2)\) with hypercharge \(Y_\phi = 1/2\),

\[
\phi = \begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix},
\]

with Lagrangian

\[
\mathcal{L}_\phi = (D^\mu \phi)^\dagger D_\mu \phi - \mu^2 \phi^\dagger \phi - \lambda(\phi^\dagger \phi)^2,
\]

where \(D_\mu \phi = (\partial_\mu - igA_\mu^a \tau^a - ig'Y_\phi B_\mu)\), and \(\tau^a = \sigma^a/2\) (for \(a = 1, 2, 3\)) are the \(SU(2)\) Lie Algebra generators, proportional to the Pauli matrix \(\sigma^a\). The gauge symmetry of the Lagrangian is broken to \(U(1)_{em}\) when a particular vacuum expectation value is chosen, e.g.:

\[
\langle \phi \rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v \end{pmatrix} \text{ with } v = \left( \frac{-\mu^2}{\lambda} \right)^{1/2} \text{ (\(\mu^2 < 0, \lambda > 0\))},
\]

Upon spontaneous symmetry breaking the kinetic term in Eq. \([23]\) gives origin to the SM gauge boson mass terms. Indeed, specializing Eq. \([12]\) to
the present case, and using Eq. (24), one gets:

\[
(D^\mu \phi)^\dagger D_\mu \phi \rightarrow \cdots + \frac{1}{8} (0 \ v) \left( g A^a_\mu \sigma^a + g' B_\mu \right) \left( g A^b_\mu \sigma^b + g' B_\mu \right) \left( \begin{array}{c} 0 \\ v \end{array} \right) + \cdots \\
\rightarrow \cdots + \frac{1}{2} v^2 \left[ g^2 (A^1_\mu)^2 + g^2 (A^2_\mu)^2 + (-g A^3_\mu + g' B_\mu)^2 \right] + \cdots 
\]

(25)

One recognizes in Eq. (25) the mass terms for the charged gauge bosons \( W^{\pm}_\mu \):

\[
W^{\pm}_\mu = \frac{1}{\sqrt{2}} (A^{1}_\mu \pm A^{2}_\mu) \rightarrow M_W = g v, 
\]

(26)

and for the neutral gauge boson \( Z^0_\mu \):

\[
Z^0_\mu = \frac{1}{\sqrt{g^2 + g'^2}} (g A^3_\mu - g' B_\mu) \rightarrow M_Z = \sqrt{g^2 + g'^2} \frac{v}{2}, 
\]

(27)

while the orthogonal linear combination of \( A^3_\mu \) and \( B_\mu \) remains massless and corresponds to the photon field \( (A_\mu) \):

\[
A_\mu = \frac{1}{\sqrt{g^2 + g'^2}} (g' A^3_\mu + g B_\mu) \rightarrow M_A = 0, 
\]

(28)

the gauge boson of the residual \( U(1)_{em} \) gauge symmetry.

The content of the scalar sector of the theory becomes more transparent if one works in the unitary gauge and eliminate the unphysical degrees of freedom using gauge invariance. In analogy to what we wrote for the abelian case in Eq. (7), this amounts to parameterize and rotate the \( \phi(x) \) complex scalar field as follows:

\[
\phi(x) = \frac{e^{\frac{i}{\sqrt{2}} (x(x) - x) \Phi}}{\sqrt{2}} \left( \begin{array}{c} 0 \\ v + H(x) \end{array} \right) \xrightarrow{SU(2)} \phi(x) = \frac{1}{\sqrt{2}} \left( \begin{array}{c} 0 \\ v + H(x) \end{array} \right), 
\]

(29)

after which the scalar potential in Eq. (23) becomes:

\[
\mathcal{L}_\phi = \mu^2 H^2 - \lambda v H^3 - \frac{1}{4} H^4 = -\frac{1}{2} M_H^2 H^2 - \sqrt{\frac{\lambda}{2}} M_H H^3 - \frac{1}{4} \lambda H^4. 
\]

(30)

Three degrees of freedom, the \( \chi^a(x) \) Goldstone bosons, have been reabsorbed into the longitudinal components of the \( W^{\pm}_\mu \) and \( Z^0_\mu \) weak gauge bosons. One
real scalar field remains, the Higgs boson $H$, with mass $M^2_H = -2\mu^2 = 2\lambda v^2$ and self-couplings:

$$H \rightarrow H = -3i \frac{M^2_H}{v}$$

Furthermore, some of the terms that we omitted in Eq. (25), the terms linear in the gauge bosons $W^\pm_\mu$ and $Z^0_\mu$, define the coupling of the SM Higgs boson to the weak gauge fields:

$$V^\mu_\nu \rightarrow H = 2i \frac{M^2_H}{v} g^{\mu\nu}$$

We notice that the couplings of the Higgs boson to the gauge fields are proportional to their mass. Therefore $H$ does not couple to the photon at tree level. It is important, however, to observe that couplings that are absent at tree level may be induced at higher order in the gauge couplings by loop corrections. Particularly relevant to the SM Higgs-boson phenomenology that will be discussed in Section 3 are the couplings of the SM Higgs boson to pairs of photons, and to a photon and a $Z^0_\mu$ weak boson:

as well as the coupling to pairs of gluons, when the SM Lagrangian is extended through the QCD Lagrangian to include also the strong interactions:
The analytical expressions for the $H\gamma\gamma$, $H\gamma Z$, and $Hgg$ one-loop vertices are more involved and will be given in Section 3.1. As far as the Higgs boson tree level couplings go, we observe that they are all expressed in terms of just two parameters, either $\lambda$ and $\mu$ appearing in the scalar potential of $L_\phi$ (see Eq. 23) or, equivalently, $M_H$ and $v$, the Higgs-boson mass and the scalar-field vacuum expectation value. Since $v$ is measured in muon decay to be $v = (\sqrt{2}G_F)^{-1/2} = 246$ GeV, the physics of the SM Higgs boson is actually just function of its mass $M_H$.

The Standard Model gauge symmetry also forbids explicit mass terms for the fermionic degrees of freedom of the Lagrangian. The fermion mass terms are then generated via gauge invariant renormalizable Yukawa couplings to the scalar field $\phi$:

$$L_{\text{Yukawa}} = -\Gamma^i_{\bar{u}Q_L} \phi^c u^i_R - \Gamma^i_{\bar{d}Q_L} \phi d^i_R - \Gamma^i_{\bar{e}L} \phi l^i_R + h.c.$$  

(31)

where $\phi^c = -i\sigma^2 \phi^*$, and $\Gamma_f$ ($f = u, d, l$) are matrices of couplings arbitrarily introduced to realize the Yukawa coupling between the field $\phi$ and the fermionic fields of the SM. $Q^i_L$ and $L^i_L$ (where $i = 1, 2, 3$ is a generation index) represent quark and lepton left handed doublets of $SU(2)_L$, while $u^i_R$, $d^i_R$ and $l^i_R$ are the corresponding right handed singlets. When the scalar fields $\phi$ acquires a non zero vacuum expectation value through spontaneous symmetry breaking, each fermionic degree of freedom coupled to $\phi$ develops a mass term with mass parameter

$$m_f = \Gamma_f \frac{v}{\sqrt{2}} .$$  

(32)

where the process of diagonalization from the current eigenstates in Eq. (31) to the corresponding mass eigenstates is understood, and $\Gamma_f$ are therefore the elements of the diagonalized Yukawa matrices corresponding to a given fermion $f$. The Yukawa couplings of the $f$ fermion to the Higgs boson ($y_f$) is proportional to $\Gamma_f$: 

![Diagram of one-loop vertex](image)
As long as the origin of fermion masses is not better understood in some more general context beyond the Standard Model, the Yukawa couplings $y_f$ represent free parameter of the SM Lagrangian. The mechanism through which fermion masses are generated in the Standard Model, although related to the mechanism of spontaneous symmetry breaking, requires therefore further assumptions and involves a larger degree of arbitrariness as compared to the gauge boson sector of the theory.

2.3 Theoretical constraints on the SM Higgs boson mass

Several issues arising in the scalar sector of the Standard Model link the mass of the Higgs boson to the energy scale where the validity of the Standard Model is expected to fail. Below that scale, the Standard Model is the extremely successful effective field theory that emerges from the electroweak precision tests of the last decades. Above that scale, the Standard Model has to be embedded into some more general theory that gives origin to a wealth of new physics phenomena. From this point of view, the Higgs sector of the Standard Model contains actually two parameters, the Higgs mass ($M_H$) and the scale of new physics ($\Lambda$).

In this Section we will review the most important theoretical constraints that are imposed on the mass of the Standard Model Higgs boson by the consistency of the theory up to a given energy scale $\Lambda$. In particular we will touch on issues of unitarity, triviality, vacuum stability, fine tuning and, finally, electroweak precision measurements.

2.3.1 Unitarity

The scattering amplitudes for longitudinal gauge bosons ($V_L V_L \to V_L V_L$, where $V = W^\pm, Z^0$) grow as the square of the Higgs-boson mass. This is easy to calculate using the electroweak equivalence theorem [8, 9], valid in the high energy limit (i.e. for energies $s = Q^2 \gg M_V^2$), according to which
the scattering amplitudes for longitudinal gauge bosons can be expressed in terms of the scattering amplitudes for the corresponding Goldstone bosons, i.e.:

\[ \mathcal{A}(V_L^1 \ldots V_L^n \rightarrow V_L^1 \ldots V_L^m) = (i)^n(-i)^m \mathcal{A}(\omega^1 \ldots \omega^n \rightarrow \omega^1 \ldots \omega^m) + O \left( \frac{M_V^2}{s} \right), \]

(33)

where we have indicated by \( \omega^i \) the Goldstone boson associated to the longitudinal component of the gauge boson \( V^i \). For instance, in the high energy limit, the scattering amplitude for \( W_L^+ W_L^- \rightarrow W_L^+ W_L^- \) satisfies:

\[ \mathcal{A}(W_L^+ W_L^- \rightarrow W_L^+ W_L^-) = \mathcal{A}(\omega^+ \omega^- \rightarrow \omega^+ \omega^-) + O \left( \frac{M_W^2}{s} \right), \]

(34)

where

\[ \mathcal{A}(\omega^+ \omega^- \rightarrow \omega^+ \omega^-) = -\frac{M_H^2}{v^2} \left( \frac{s}{s-M_H^2} + \frac{t}{t-M_H^2} \right). \]

(35)

Using a partial wave decomposition, we can also write \( \mathcal{A} \) as:

\[ \mathcal{A} = 16\pi \sum_{l=0}^{\infty} (2l+1) P_l(\cos \theta) a_l, \]

(36)

where \( a_l \) is the spin \( l \) partial wave and \( P_l(\cos \theta) \) are the Legendre polynomials. In terms of partial wave amplitudes \( a_l \), the scattering cross section corresponding to \( \mathcal{A} \) can be calculated to be:

\[ \sigma = \frac{16\pi}{s} \sum_{l=0}^{\infty} (2l+1) |a_l|^2, \]

(37)

where we have used the orthogonality of the Legendre polynomials. Using the optical theorem, we can impose the unitarity constraint by writing that:

\[ \sigma = \frac{16\pi}{s} \sum_{l=0}^{\infty} (2l+1) |a_l|^2 = \frac{1}{s} \text{Im} [\mathcal{A}(\theta = 0)] , \]

(38)

where \( \mathcal{A}(\theta = 0) \) indicates the scattering amplitude in the forward direction. This implies that:

\[ |a_l|^2 = \text{Re}(a_l)^2 + \text{Im}(a_l)^2 = \text{Im}(a_l) \rightarrow |\text{Re}(a_l)| \leq \frac{1}{2}. \]

(39)
Via Eq. (39), different amplitudes can then provide constraints on $M_H$. As an example, let us consider the $J = 0$ partial wave amplitude $a_0$ for the $W_L^+W_L^- \to W_L^+W_L^-$ scattering we introduced above:

$$a_0 = \frac{1}{16\pi s} \int_{-s}^0 A dt = -\frac{M_H^2}{16\pi v^2} \left[ 2 + \frac{M_H^2}{s - M_H^2} - \frac{M_H^2}{s \log \left( 1 + \frac{s}{M_H^2} \right)} \right]. \quad (40)$$

In the high energy limit ($M_H^2 \ll s$), $a_0$ reduces to:

$$a_0 \xrightarrow{M_H^2 \ll s} \frac{M_H^2}{8\pi v^2}, \quad (41)$$

from which, using Eq. (39), one gets:

$$M_H < 870 \text{ GeV}. \quad (42)$$

Other more constraining relations can be obtained from different longitudinal gauge boson scattering amplitudes. For instance, considering the coupled channels like $W_L^+W_L^- \to Z_LZ_L$, one can lower the bound to:

$$M_H < 710 \text{ GeV}. \quad (43)$$

Taking a different point of view, we can observe that if there is no Higgs boson, or equivalently if $M_H^2 \gg s$, Eq. (39) gives indications on the critical scale $\sqrt{s_c}$ above which new physics should be expected. Indeed, considering again $W_L^+W_L^- \to W_L^+W_L^-$ scattering, we see that:

$$a_0(\omega^+\omega^- \to \omega^+\omega^-) \xrightarrow{M_H^2 \gg s} -\frac{s}{32\pi v^2}, \quad (44)$$

from which, using Eq. (39), we get:

$$\sqrt{s_c} < 1.8 \text{ TeV}. \quad (45)$$

Using more constraining channels the bound can be reduced to:

$$\sqrt{s_c} < 1.2 \text{ TeV}. \quad (46)$$

This is very suggestive: it tells us that new physics ought to be found around 1-2 TeV, i.e. exactly in the range of energies that will be explored by the Tevatron and the Large Hadron Collider.
2.3.2 Triviality and vacuum stability

The argument of triviality in a $\lambda \phi^4$ theory goes as follows. The dependence of the quartic coupling $\lambda$ on the energy scale ($Q$) is regulated by the renormalization group equation

$$\frac{d\lambda(Q)}{dQ^2} = \frac{3}{4\pi^2} \lambda^2(Q). \quad (47)$$

This equation states that the quartic coupling $\lambda$ decreases for small energies and increases for large energies. Therefore, in the low energy regime the coupling vanishes and the theory becomes trivial, i.e. non-interactive. In the large energy regime, on the other hand, the theory becomes non-perturbative, since $\lambda$ grows, and it can remain perturbative only if $\lambda$ is set to zero, i.e. only if the theory is made trivial.

The situation in the Standard Model is more complicated, since the running of $\lambda$ is governed by more interactions. Including the lowest orders in all the relevant couplings, we can write the equation for the running of $\lambda(Q)$ with the energy scale as follows:

$$32\pi^2 \frac{d\lambda}{dt} = 24\lambda^2 - (3g'^2 + 9g^2 - 24y_t^2)\lambda + \frac{3}{8}g'^4 + \frac{3}{4}g'^2g^2 + \frac{9}{8}g^4 - 24y_t^4 + \cdots \quad (48)$$

where $t = \ln(Q^2/Q_0^2)$ is the logarithm of the ratio of the energy scale and some reference scale $Q_0$ square, $y_t = m_t/v$ is the top-quark Yukawa coupling, and the dots indicate the presence of higher order terms that have been omitted. We see that when $M_H$ becomes large, $\lambda$ also increases (since $M_H^2 = 2\lambda v^2$) and the first term in Eq. (48) dominates. The evolution equation for $\lambda$ can then be easily solved and gives:

$$\lambda(Q) = \frac{\lambda(Q_0)}{1 - \frac{3}{4\pi^2} \lambda(Q_0) \ln \left( \frac{Q^2}{Q_0^2} \right)} . \quad (49)$$

When the energy scale $Q$ grows, the denominator in Eq. (49) may vanish, in which case $\lambda(Q)$ hits a pole, becomes infinite, and a triviality condition needs to be imposed. This is avoided imposing that the denominator in Eq. (49) never vanishes, i.e. that $\lambda(Q)$ is always finite and $1/\lambda(Q) > 0$. This condition gives an explicit upper bound on $M_H$:

$$M_H^2 < \frac{8\pi^2 v^2}{3 \log \left( \frac{Q^2}{Q_0^2} \right)} , \quad (50)$$
obtained from Eq. (49) by setting $Q = \Lambda$, the scale of new physics, and $Q_0 = v$, the electroweak scale.

On the other hand, for small $M_H$, i.e. for small $\lambda$, the last term in Eq. (48) dominates and the evolution of $\lambda(Q)$ looks like:

$$\lambda(\Lambda) = \lambda(v) - \frac{3}{4\pi^2} \frac{y_t^4}{v^2} \log \left( \frac{\Lambda^2}{v^2} \right).$$

(51)

To assure the stability of the vacuum state of the theory we need to require that $\lambda(\Lambda) > 0$ and this gives a lower bound for $M_H$:

$$\lambda(\Lambda) > 0 \quad \rightarrow \quad M_H^2 > \frac{3v^2}{2\pi^2} y_t^4 \log \left( \frac{\Lambda^2}{v^2} \right).$$

(52)

More accurate analyses include higher order quantum correction in the scalar potential and use a 2-loop renormalization group improved effective potential, $V_{\text{eff}}$, whose nature and meaning has been briefly sketched in Section 2.1.

2.3.3 Indirect bounds from electroweak precision measurements

Once a Higgs field is introduced in the Standard Model, its virtual excitations contribute to several physical observables, from the mass of the $W$ boson, to various leptonic and hadronic asymmetries, to many other electroweak observables that are usually considered in precision tests of the Standard Model. Since the Higgs-boson mass is the only parameter in the Standard Model that is not directly determined either theoretically or experimentally (previous to discovery), it can be extracted indirectly from precision fits of all the measured electroweak observables, within the fit uncertainty. This is actually one of the most important results that can be obtained from precision tests of the Standard Model and greatly illustrates the predictivity of the Standard Model itself. All available studies can be found on the LEP Electroweak Working Group and on the LEP Higgs Working Group Web pages [16, 17] as well as in their main publications. An excellent series of lectures on the subject of Precision Electroweak Physics is also available from a previous TASI school [19]. The correlation between the Higgs-boson mass $M_H$, the $W$ boson mass $M_W$, the top-quark mass $m_t$, and the precision data is illustrated in Figs. 2 and 3. Apart from the impressive agreement existing between the indirect determination of $M_W$ and $m_t$ and their experimental measurements we see in Fig. 2 that the 68% CL contours from LEP, SLD,
Figure 2: Comparison of the indirect measurements of $M_W$ and $m_t$ (LEP I+SLD data) (solid contour) and the direct measurement ($p\bar{p}$ colliders and LEP II data) (dashed contour). In both cases the 68% CL contours are plotted. Also shown is the SM relationship for these masses as a function of the Higgs-boson mass, $m_H$. The arrow labeled $\Delta\alpha$ shows the variation of this relation if $\alpha(M_Z^2)$ is varied by one standard deviation. From Ref. [16].

and Tevatron measurements select a SM Higgs-boson mass region roughly below 200 GeV. Therefore, assuming no physics beyond the Standard Model at the weak scale, all available electroweak precision data are consistent with a light Higgs boson.

The actual value of $M_H$ emerging from the electroweak precision fits strongly depends on theoretical predictions of physical observables that include different orders of strong and electroweak corrections. As an example, in Fig. 2 the magenta arrow shows how the yellow band would move for one standard deviation variation in the QED fine-structure constant $\alpha(m_Z^2)$. It also depends on the fit input parameters. As we see in Fig. 3, $M_H$ grows for larger $m_t$ and smaller $M_W$. The limits deduced from Fig. 2 and 4 is
Figure 3: The 68% confidence level contour in \( m_t \) and \( M_H \) for the fit to all data except the direct measurement of \( m_t \), indicated by the shaded horizontal band of \( \pm 1\sigma \) width. The vertical band shows the 95% CL exclusion limit on \( M_H \) from direct searches. From Ref. [16].

summarized as

\[
\begin{align*}
M_H &= 94^{+29}_{-24} \text{ GeV} & \text{for } m_t = 173.2 \pm 0.9 \text{ GeV }, \\
M_H &< 152(171) \text{ GeV} \text{ (95% CL)} & \text{and } M_W = 80.385 \pm 0.015 \text{ GeV } .
\end{align*}
\]

(53)

A large region of the \( \Delta \chi^2 \) band in Fig. 4 in particular the region about the minimum, is already excluded, and values of \( M_H \) very close to the experimental lower bound seem to be favored. It is fair to conclude that the issue of constraining \( M_H \) from electroweak precision fits is open to controversies and, at a closer look, emerges as a not clear cut statement. With this respect, Fig. 5 illustrates the sensitivity of a few selected electroweak observables to the Higgs-boson mass as well as the preferred range for the SM Higgs-boson mass as determined from all electroweak observables. One can observe that \( M_W \) and the leptonic asymmetries prefer a lighter Higgs boson, while \( A_{FB}^{bc} \) and the NuTeV determination of \( \sin^2 \theta_W \) prefer a heavier Higgs boson. A certain tension is still present in the data. We could just think that things
Figure 4: $\Delta \chi^2 = \chi^2 - \chi^2_{\text{min}}$ vs. $M_H$ curve. The line is the result of the fit using all electroweak data; the band represents an estimate of the theoretical error due to missing higher order corrections. The vertical band shows the 95% CL exclusion limit on $M_H$ from direct searches. The solid and dashed curves are derived using different evaluations of $\Delta \alpha_{\text{had}}^{(5)}(M_Z^2)$. The dotted curve includes low $Q^2$ data. From Ref. [16].
Figure 5: Preferred range for the SM Higgs-boson mass $M_H$ as determined from various electroweak observables. The shaded band shows the overall constraint on the mass of the Higgs boson as derived from the full data set. From Ref. [16].
will progressively adjust and, after the discovery of a light Higgs boson at the LHC, this will result in yet another amazing success of the Standard Model. Or, one can interpret the situation depicted in Fig. 5 as an unavoidable indication of the presence of new physics beyond the Standard Model and only more accurate studies of the newly discovered spin-0 particle at the LHC will help shed some light on the puzzle.

### 2.3.4 Fine-tuning

One aspect of the Higgs sector of the Standard Model that is traditionally perceived as problematic is that higher order corrections to the Higgs-boson mass parameter square contain quadratic ultraviolet divergences. This is expected in a $\lambda \phi^4$ theory and it does not pose a renormalizability problem, since a $\lambda \phi^4$ theory is renormalizable. However, although per se renormalizable, these quadratic divergences leave the *inelegant* feature that the Higgs-boson renormalized mass square has to result from the *adjusted* or *fine-tuned* balance between a bare Higgs boson mass square and a counterterm that is proportional to the ultraviolet cutoff square. If the physical Higgs mass has to live at the electroweak scale, this can cause a fine-tuning of several orders of magnitude when the scale of new physics $\Lambda$ (the ultraviolet cutoff of the Standard Model interpreted as an effective low energy theory) is well above the electroweak scale. Ultimately this is related to a symmetry principle, or better to the absence of a symmetry principle. Indeed, setting to zero the mass of the scalar fields in the Lagrangian of the Standard Model does not restore any symmetry to the model. Hence, the mass of the scalar fields are not protected against large corrections.

Models of new physics beyond the Standard Model should address this fine-tuning problem and propose a more satisfactory mechanism to obtain the mass of the Higgs particle(s) around the electroweak scale. Supersymmetric models, for instance, have the remarkable feature that fermionic and bosonic degrees of freedom conspire to cancel the Higgs mass quadratic loop divergence, when the symmetry is exact. Other non-supersymmetric models, like little Higgs models, address the problem differently, by interpreting the Higgs boson as a Goldstone boson of some global approximate symmetry. In both cases the Higgs mass turns out to be proportional to some small deviation from an exact symmetry principle, and therefore intrinsically small.

As suggested in Ref. [20], the *no fine-tuning* condition in the Standard Model can be softened and translated into a *maximum amount of allowed*
Figure 6: The SM Higgs-boson mass $M_H$ as a function of the scale of new physics $\Lambda$, with all the constraints derived from unitarity, triviality, vacuum stability, electroweak precision fits, and the requirement of a limited fine-tuning. The empty region is consistent with all the constraints and less than 1 part in 10 fine-tuning. From Ref. [20].
fine-tuning, that can be directly related to the scale of new physics. As derived in Section 2.1 upon spontaneous breaking of the electroweak symmetry, the SM Higgs-boson mass at tree level is given by \( M_H^2 = -2\mu^2 \), where \( \mu^2 \) is the coefficient of the quadratic term in the scalar potential. Higher order corrections to \( M_H^2 \) can therefore be calculated as loop corrections to \( \mu^2 \), i.e. by studying how the effective potential in Eq. (16) and its minimum condition are modified by loop corrections. If we interpret the Standard Model as the electroweak scale effective limit of a more general theory living at a high scale \( \Lambda \), then the most general form of \( \mu^2 \) including all loop corrections is:

\[
\bar{\mu}^2 = \mu^2 + \Lambda^2 \sum_{\lambda_i} c_n(\lambda_i) \log^n(\Lambda/Q) ,
\]

where \( Q \) is the renormalization scale, \( \lambda_i \) are a set of input parameters (couplings) and the \( c_n \) coefficients can be deduced from the calculation of the effective potential at each loop order. As noted originally by Veltman, there would be no fine-tuning problem if the coefficient of \( \Lambda^2 \) in Eq. (54) were zero, i.e. if the loop corrections to \( \mu^2 \) had to vanish. This condition, known as Veltman condition, is usually over constraining, since the number of independent \( c_n \) (set to zero by the Veltman condition) can be larger than the number of inputs \( \lambda_i \). However the Veltman condition can be relaxed, by requiring that only the sum of a finite number of terms in the coefficient of \( \Lambda^2 \) is zero, i.e. requiring that:

\[
\sum_{0}^{n_{max}} c_n(\lambda_i) \log^n(\Lambda/M_H) = 0 ,
\]

where the renormalization scale \( \mu \) has been arbitrarily set to \( M_H \) and the order \( n \) has been set to \( n_{max} \), fixed by the required order of loop in the calculation of \( V_{eff} \). This is based on the fact that higher orders in \( n \) come from higher loop effects and are therefore suppressed by powers of \((16\pi^2)^{-1}\). Limiting \( n \) to \( n_{max} \), Eq. (55) can now have a solution. Indeed, if the scale of new physics \( \Lambda \) is not too far from the electroweak scale, then the Veltman condition in Eq. (55) can be softened even more by requiring that:

\[
\sum_{0}^{n_{max}} c_n(\lambda_i) \log^n(\Lambda/M_H) < \frac{v^2}{\Lambda^2} .
\]

This condition determines a value of \( \Lambda_{max} \) such that for \( \Lambda \leq \Lambda_{max} \) the stability of the electroweak scale does not require any dramatic cancellation in
In other words, for $\Lambda \leq \Lambda_{\text{max}}$ the renormalization of the SM Higgs-boson mass does not require any fine-tuning. As an example, for $n_{\text{max}} = 0$, $c_0 = (32\pi^2 v^2)^{-1}3(2M_W^2 + M_Z^2 + M_H^2 - 4m_t^2)$, and the stability of the electroweak scale is assured up to $\Lambda$ of the order of $4\pi v \simeq 2$ TeV. For $n_{\text{max}} = 1$ the maximum $\Lambda$ is pushed up to $\Lambda \simeq 15$ TeV and for $n_{\text{max}} = 2$ up to $\Lambda \simeq 50$ TeV. So, just going up to 2-loops assures us that we can consider the SM Higgs sector free of fine-tuning up to scales that are well beyond where we would hope to soon discover new physics.

For each value of $n_{\text{max}}$, and for each corresponding $\Lambda_{\text{max}}$, $M_H$ becomes a function of the cutoff $\Lambda$, and the amount of fine-tuning allowed in the theory limits the region in the ($\Lambda, M_H$) plane allowed to $M_H(\Lambda)$. This is well represented in Fig. 6 where also the constraint from the conditions of unitarity (see Section 2.3.1), triviality (see Section 2.3.2), vacuum stability (see Section 2.3.2) and electroweak precision fits (see Section 2.3.3) are summarized. Finally, the main lesson we take away from this plot is that if a Higgs boson is discovered new physics is just around the corner and should manifest itself at the LHC.

### 2.4 The Higgs sector of the Minimal Supersymmetric Standard Model

In the supersymmetric extension of the Standard Model, the electroweak symmetry is spontaneously broken via the Higgs mechanism introducing two complex scalar $SU(2)_{L}$ doublets. The dynamics of the Higgs mechanism goes pretty much unchanged with respect to the Standard Model case, although the form of the scalar potential is more complex and its minimization more involved. As a result, the $W^\pm$ and $Z^0$ weak gauge bosons acquire masses that depend on the parameterization of the supersymmetric model at hand. At the same time, fermion masses are generated by coupling the two scalar doublets to the fermions via Yukawa interactions. A supersymmetric model is therefore a natural reference to compare the Standard Model to, since it is a theoretically sound extension of the Standard Model, still fundamentally based on the same electroweak symmetry breaking mechanism.

Far from being a simple generalization of the SM Higgs sector, the scalar sector of a supersymmetric model can be theoretically more satisfactory because: (i) spontaneous symmetry breaking is radiatively induced (i.e. the sign of the quadratic term in the Higgs potential is driven from positive to
negative) mainly by the evolution of the top-quark Yukawa coupling from the scale of supersymmetry-breaking to the electroweak scale, and (ii) higher order corrections to the Higgs mass do not contain quadratic divergences, since they cancel when the contribution of both scalars and their super-partners is considered (see Section 2.3.4).

At the same time, the fact of having a supersymmetric theory and two scalar doublets modifies the phenomenological properties of the supersymmetric physical scalar fields dramatically. In this Section we will review only the most important properties of the Higgs sector of the MSSM, so that in Section 3 we can compare the physics of the SM Higgs boson to that of the MSSM Higgs bosons.

I will start by recalling some general properties of a Two Higgs Doublet Model in Section 2.4.1 and I will then specify the discussion to the case of the MSSM in Section 2.4.2. In Sections 2.4.3 and 2.4.4 I will review the form of the couplings of the MSSM Higgs bosons to the SM gauge bosons and fermions, including the impact of the most important supersymmetric higher order corrections. A thorough introduction to Supersymmetry and the Minimal Supersymmetric Standard Model has been given during this school by Prof. H. Haber to whose lectures I refer [21].

2.4.1 About Two Higgs Doublet Models

The most popular and simplest extension of the Standard Model is obtained by considering a scalar sector made of two instead of one complex scalar doublets. These models, dubbed Two Higgs Doublet Models (2HDM), have a richer spectrum of physical scalar fields. Indeed, after spontaneous symmetry breaking, only three of the eight original scalar degrees of freedom (corresponding to two complex doublet) are reabsorbed in transforming the originally massless vector bosons into massive ones. The remaining five degrees of freedom correspond to physical degrees of freedom in the form of: two neutral scalar, one neutral pseudoscalar, and two charged scalar fields.

At the same time, having multiple scalar doublets in the Yukawa Lagrangian (see Eq. (31)) allows for scalar flavor changing neutral current. Indeed, when generalized to the case of two scalar doublet $\phi_1$ and $\phi_2$, Eq. (31) becomes (quark case only):

$$ L_{Yukawa} = - \sum_{k=1,2} \Gamma_{ijk}^u \bar{Q}_L^i \Phi_k^c u_R^j - \sum_{k=1,2} \Gamma_{ijk}^d \bar{Q}_L^i \Phi_k^d d_R^j + h.c. , $$

(57)
where each pair of fermions \((i, j)\) couple to a linear combination of the scalar fields \(\phi^1\) and \(\phi^2\). When, upon spontaneous symmetry breaking, the fields \(\phi^1\) and \(\phi^2\) acquire vacuum expectation values

\[
\langle \Phi^k \rangle = \frac{v^k}{\sqrt{2}} \quad \text{for} \quad k = 1, 2 ,
\]

(58)

the parameterization of \(\mathcal{L}_{Yukawa}\) of Eq. (57) in the vicinity of the minimum of the scalar potential, with \(\Phi^k = \Phi^{\tau k} + \mu^k\) (for \(k = 1, 2\)), gives:

\[
\mathcal{L}_{Yukawa} = -\bar{u}_L \sum_k \Gamma_{ij,k}^u \frac{v^k}{\sqrt{2}} u_R^j - \bar{d}_L \sum_k \Gamma_{ij,k}^d \frac{v^k}{\sqrt{2}} d_R^j + \text{h.c.} + \text{FC couplings} ,
\]

(59)

where the fermion mass matrices \(M_{ij}^u\) and \(M_{ij}^d\) are now proportional to a linear combination of the vacuum expectation values of \(\phi^1\) and \(\phi^2\). The diagonalization of \(M_{ij}^u\) and \(M_{ij}^d\) does not imply the diagonalization of the couplings of the \(\phi^k\) fields to the fermions, and Flavor Changing (FC) couplings arise. This is perceived as a problem in view of the absence of experimental evidence to support neutral flavor changing effects. If present, these effects have to be tiny in most processes involving in particular the first two generations of quarks, and a safer way to build a 2HDM is to forbid them all together at the Lagrangian level. This is traditionally done by requiring either that \(u\)-type and \(d\)-type quarks couple to the same doublet (Model I) or that \(u\)-type quarks couple to one scalar doublet while \(d\)-type quarks to the other (Model II). Indeed, these two different realization of a 2HDM can be justified by enforcing on \(\mathcal{L}_{Yukawa}\) the following \textit{ad hoc} discrete symmetry:

\[
\begin{align*}
\Phi^1 &\rightarrow -\Phi^1 \quad \text{and} \quad \Phi^2 \rightarrow \Phi^2 \\
d^i &\rightarrow -d^i \quad \text{and} \quad u^j \rightarrow \pm u^j
\end{align*}
\]

(60)

The case in which FC scalar neutral current are not forbidden (Model III) has also been studied in detail. In this case both up and down-type quarks can couple to both scalar doublets, and strict constraints have to be imposed on the FC scalar couplings in particular between the first two generations of quarks.

2HDMs have indeed a very rich phenomenology that has been extensively studied. In these lectures, however, we will only compare the SM Higgs boson phenomenology to the phenomenology of the Higgs bosons of the MSSM, a particular kind of 2HDM that we will illustrate in the following Sections.
2.4.2 The MSSM Higgs sector: introduction

The Higgs sector of the MSSM is actually a Model II 2HDM. It contains two complex $SU(2)_L$ scalar doublets:

$$\Phi_1 = \left( \frac{\phi^+_1}{\phi^0_1} \right), \quad \Phi_2 = \left( \frac{\phi^0_2}{\phi^-_2} \right),$$

with opposite hypercharge ($Y = \pm 1$), as needed to make the theory anomaly-free. $\Phi_1$ couples to the up-type and $\Phi_2$ to the down-type quarks respectively. Correspondingly, the Higgs part of the superpotential can be written as:

$$V_H = (|\mu|^2 + m_1^2)|\Phi_1|^2 + (|\mu|^2 + m_2^2)|\Phi_2|^2 - \mu B\epsilon_{ij}(\Phi^*_i\Phi^*_j + h.c.)$$

$$+ \frac{g^2 + g'^2}{8} (|\Phi_1|^2 - |\Phi_2|^2)^2 + \frac{g^2}{2} |\Phi_1^*\Phi_2|^2,$$

in which we can identify three different contributions:

(i) the so called $D$ terms, containing the quartic scalar interactions, which for the Higgs fields $\Phi_1$ and $\Phi_2$ correspond to:

$$\frac{g^2 + g'^2}{8} (|\Phi_1|^2 - |\Phi_2|^2)^2 + \frac{g^2}{2} |\Phi_1^*\Phi_2|^2,$$

(ii) the so called $F$ terms, corresponding to:

$$|\mu|^2(|\Phi_1|^2 + |\Phi_2|^2);$$

(iii) the soft SUSY-breaking scalar Higgs mass and bilinear terms, corresponding to:

$$m_1^2|\Phi_1|^2 + m_2^2|\Phi_2|^2 - \mu B\epsilon_{ij}(\Phi^*_i\Phi^*_j + h.c.).$$

Overall, the scalar potential in Eq. (62) depends on three independent combinations of parameters, $|\mu|^2 + m_1^2$, $|\mu|^2 + m_2^2$, and $\mu B$. One basic difference

Another reason for the choice of a 2HDM is that in a supersymmetric model the superpotential should be expressed just in terms of superfields, not their conjugates. So, one needs to introduce two doublets to give mass to fermion fields of opposite weak isospin. The second doublet plays the role of $\phi^c$ in the Standard Model (see Eq. (31)), where $\phi^c$ has opposite hypercharge and weak isospin with respect to $\phi$. 

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with respect to the SM case is that the quartic coupling has been replaced by gauge couplings. This reduced arbitrariness will play an important role in the following.

Upon spontaneous symmetry breaking, the neutral components of $\Phi_1$ and $\Phi_2$ acquire vacuum expectation values

$$
\langle \Phi_1 \rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v_1 \end{pmatrix}, \quad \langle \Phi_2 \rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} v_2 \\ 0 \end{pmatrix},
$$

and the Higgs mechanism proceed as in the Standard Model except that now one starts with eight degrees of freedom, corresponding to the two complex doublets $\Phi_1$ and $\Phi_2$. Three degrees of freedom are absorbed in making the $W^\pm$ and the $Z^0$ massive. The $W$ mass is chosen to be: $M^2_W = g^2(v_1^2 + v_2^2)/4 = g^2v^2/4$, and this fixes the normalization of $v_1$ and $v_2$, leaving only two independent parameters to describe the entire MSSM Higgs sector. The remaining five degrees of freedom are physical and correspond to two neutral scalar fields

$$
h^0 = -(\sqrt{2}\text{Re}\phi_2^0 - v_2) \sin \alpha + (\sqrt{2}\text{Re}\phi_1^0 - v_1) \cos \alpha, \quad H^0 = (\sqrt{2}\text{Re}\phi_2^0 - v_2) \cos \alpha + (\sqrt{2}\text{Re}\phi_1^0 - v_1) \sin \alpha,
$$

one neutral pseudoscalar field

$$
A^0 = \sqrt{2}(\text{Im}\phi_2^0 \sin \beta + \text{Im}\phi_1^0 \cos \beta),
$$

and two charged scalar fields

$$
H^\pm = \phi_2^\pm \sin \beta + \phi_1^\pm \cos \beta,
$$

where $\alpha$ and $\beta$ are mixing angles, and $\tan \beta = v_1/v_2$. At tree level, the masses of the scalar and pseudoscalar degrees of freedom satisfy the following relations:

$$
M^2_{H^\pm} = M_A^2 + M_W^2, \quad M^2_{H^0, h} = \frac{1}{2} \left( M_A^2 + M_Z^2 \pm ((M_A^2 + M_Z^2)^2 - 4M_Z^2M_A^2\cos^2 2\beta)^{1/2} \right),
$$

making it natural to pick $M_A$ and $\tan \beta$ as the two independent parameters of the Higgs sector.
Eq. (70) provides the famous tree level upper bound on the mass of one of the neutral scalar Higgs bosons, $h^0$:

$$M_h^2 \leq M_Z^2 \cos 2\beta \leq M_Z^2,$$  \hspace{1cm} (71)

which already contradicts the current experimental lower bound set by LEP II: $M_h > 93.0$ GeV [22]. The contradiction is lifted by including higher order radiative corrections to the Higgs spectrum, in particular by calculating higher order corrections to the neutral scalar mass matrix. Over the past few years a huge effort has been dedicated to the calculation of the full one-loop corrections and of several leading and sub-leading sets of two-loop corrections, including resummation of leading and sub-leading logarithms via appropriate renormalization group equation (RGE) methods. A detailed discussion of this topic can be found in some recent reviews [12, 23, 24] and in the original literature referenced therein. For the purpose of these lectures, let us just observe that, qualitatively, the impact of radiative corrections on $M_{h\text{max}}^2$ can be seen by just including the leading two-loop corrections proportional to $y_t^2$, the square of the top-quark Yukawa coupling, and applying RGE techniques to resum the leading orders of logarithms. In this case, the upper bound on the light neutral scalar in Eq. (71) is modified as follows:

$$M_h^2 \leq M_Z^2 + \frac{3g^2m_t^2}{8\pi^2M^2_H} \left[ \log \left( \frac{M^2_S}{m_t^2} \right) + \frac{X_t^2}{M^2_S} \left( 1 - \frac{X_t^2}{12M^2_S} \right) \right],$$  \hspace{1cm} (72)

where $M^2_S = (M^2_{t_1} + M^2_{t_2})/2$ is the average of the two top-squark masses, $m_t$ is the running top-quark mass (to account for the leading two-loop QCD corrections), and $X_t$ is the top-squark mixing parameter defined by the top-squark mass matrix:

$$\begin{pmatrix} M^2_{Q_t} + m_t^2 + D^t_L & m_tX_t \\ m_tX_t & M^2_{R_t} + m_t^2 + D^t_R \end{pmatrix},$$  \hspace{1cm} (73)

with $X_t \equiv A_t - \mu \cot \beta$ ($A_t$ being one of the top-squark soft SUSY breaking trilinear coupling), $D^t_L = (1/2 - 2/3 \sin \theta_W)M_Z^2 \cos 2\beta$, and $D^t_R = 2/3 \sin^2 \theta_W M_Z^2 \cos 2\beta$. Fig. 7 illustrates the behavior of $M_h$ as a function of $\tan \beta$, in the case of minimal and maximal mixing. For large $\tan \beta$ a plateau (i.e. an upper bound) is clearly reached. The green bands represent the variation of $M_h$ as a function of $m_t$ when $m_t = 175 \pm 5$ GeV. If top-squark mixing is maximal, the
Figure 7: The mass of the light neutral scalar Higgs boson, $h^0$, as a function of \( \tan \beta \), in the \textit{minimal mixing} and \textit{maximal mixing} scenario. The green bands are obtained by varying the top-quark mass in the \( m_t = 175 \pm 5 \text{ GeV} \) range. The plot is built by fixing $M_A = 1 \text{ TeV}$ and $M_{SUSY} \equiv M_Q = M_U = M_D = 1 \text{ TeV}$. From Ref. [12].

The upper bound on $M_h$ is approximately $M_h^{\text{max}} \simeq 135 \text{ GeV}$. The behavior of both $M_{h,H}$ and $M_{H^\pm}$ as a function of $M_A$ and $\tan \beta$ is summarized in Fig. 8 always for the case of maximal mixing. It is interesting to notice that for all values of $M_A$ and $\tan \beta$ the $M_H > M_h^{\text{max}}$. Also we observe that, in the limit of large $\tan \beta$, i) for $M_A < M_h^{\text{max}}$: $M_h \simeq M_A$ and $M_H \simeq M_h^{\text{max}}$, while ii) for $M_A > M_h^{\text{max}}$: $M_H \simeq M_A$ and $M_h \simeq M_h^{\text{max}}$.

2.4.3 MSSM Higgs-boson couplings to electroweak gauge bosons

The Higgs-boson couplings to the electroweak gauge bosons are obtained from the kinetic term of the scalar Lagrangian, in strict analogy to what we have explicitly seen in the case of the SM Higgs boson. Here, we would like to recall the form of the $H_iVV$ and $H_iH_jV$ couplings (for $H_i = h^0, H^0, A^0, H^\pm$, and $V = W^\pm, Z^0$) that are most important in order to understand the main features of the MSSM plots that will be shown in Section 3.

\footnote{This limit is obtained for $m_t = 175 \text{ GeV}$, and it can go up to $M_h^{\text{max}} \simeq 144 \text{ GeV}$ for $m_t = 178 \text{ GeV}$.}
Figure 8: The mass of the light ($h^0$) and heavy ($H^0$) neutral scalar Higgs bosons, and of the charged scalar Higgs boson ($H^\pm$) as a function of the neutral pseudoscalar mass $M_A$, for two different values of $\tan \beta$ ($\tan \beta=3, 30$). The top-quark mass is fixed to $m_t = 174.3$ GeV and $M_{SUSY} = M_Q = M_U = M_D = 1$ TeV. The maximal mixing scenario is chosen. From Ref. [12].

First of all, the couplings of the neutral scalar Higgs bosons to both $W^\pm$ and $Z^0$ can be written as:

$$g_{hVV} = g_V M_V \sin(\beta - \alpha) g^{\mu\nu}, \quad g_{HVV} = g_V M_V \cos(\beta - \alpha) g^{\mu\nu},$$

where $g_V = 2 M_V / v$, while the $A^0VV$ and $H^\pmVV$ couplings vanish because of CP-invariance. As in the SM case, since the photon is massless, there are no tree level $\gamma\gamma H_i$ and $\gamma Z^0 H_i$ couplings.

Moreover, in the neutral Higgs sector, only the $h^0 A^0 Z^0$ and $H^0 A^0 Z^0$ couplings are allowed and given by:

$$g_{hAZ} = \frac{g \cos(\beta - \alpha)}{2 \cos \theta_W} (p_h - p_A)^\mu, \quad g_{HAZ} = -\frac{g \sin(\beta - \alpha)}{2 \cos \theta_W} (p_H - p_A)^\mu,$$

where all momenta are incoming. We also have several $H_i H_j V$ couplings involving the charge Higgs boson, namely:

$$g_{H^+H^-Z} = -\frac{g}{2 \cos \theta_W} \cos 2\theta_W (p_{H^+} - p_{H^-})^\mu,$$

$$g_{H^+H^-\gamma} = -ie (p_{H^+} - p_{H^-})^\mu,$$
\[ g_{H^\pm H^\pm} = \pm i \frac{g}{2} \cos(\beta - \alpha)(p_h - p_{H^\pm})^\mu, \]
\[ g_{H^\pm H^0} = \pm i \frac{g}{2} \sin(\beta - \alpha)(p_h - p_{H^\pm})^\mu, \]
\[ g_{H^\pm A^\pm} = \frac{g}{2} (p_A - p_{H^\pm})^\mu. \]

At this stage it is interesting to introduce the so called decoupling limit, i.e. the limit of \( M_A \gg M_Z \), and to analyze how masses and couplings behave in this particular limit. \( M_{H^\pm} \) in Eq. (70) is unchanged, while \( M_{h,H} \) become:

\[ M_h \simeq M_{h}^{\text{max}} \quad \text{and} \quad M_H \simeq M_A^2 + M_Z^2 \sin^2 2\beta. \quad (77) \]

Moreover, as one can derive from the diagonalization of the neutral scalar Higgs-boson mass matrix:

\[ \cos^2(\beta - \alpha) = \frac{M_h^2 (M_Z^2 - M_h^2)}{M_A^2 (M_H^2 - M_h^2)} \overset{M_A \gg M_Z}{\longrightarrow} \frac{M_h^2 \sin^2 4\beta}{4 M_A^4}. \quad (78) \]

From the previous equations we then deduce that, in the decoupling limit, the only light Higgs boson is \( h^0 \) with mass \( M_h \simeq M_{h}^{\text{max}} \), while \( M_H \simeq M_{H^\pm} \simeq M_A \gg M_Z \), and because \( \cos(\beta - \alpha) \rightarrow 0 \) (\( \sin(\beta - \alpha) \rightarrow 1 \)), the couplings of \( h^0 \) to the gauge bosons tend to the SM Higgs-boson limit. This is to say that, in the decoupling limit, the light MSSM Higgs boson will be hardly distinguishable from the SM Higgs boson.

Finally, we need to remember that the tree level couplings may be modified by radiative corrections involving both loops of SM and MSSM particles, among which loops of third generation quarks and squarks dominate. The very same radiative corrections that modify the Higgs boson mass matrix, thereby changing the definition of the mass eigenstates, also affect the couplings of the corrected mass eigenstates to the gauge bosons. This can be reabsorbed into the definition of a renormalized mixing angle \( \alpha \) or a radiatively corrected value for \( \cos(\beta - \alpha) \) (\( \sin(\beta - \alpha) \)). Using the notation of Ref. [12], the radiatively corrected \( \cos(\beta - \alpha) \) can be written as:

\[ \cos(\beta - \alpha) = K \left[ \frac{M_Z^2 \sin 4\beta}{2 M_A^2} + O \left( \frac{M_A^4}{M_A^4} \right) \right], \quad (79) \]

where

\[ K \equiv 1 + \frac{\delta M_{11}^2 - \delta M_{22}^2}{2 M_Z^2 \cos 2\beta} - \frac{\delta M_{12}^2}{M_Z^2 \sin 2\beta}, \quad (80) \]
δM_{ij} are the radiative corrections to the corresponding elements of the CP-even Higgs squared-mass matrix (see Ref. [12]). It is interesting to notice that on top of the traditional decoupling limit introduced above ($M_A \gg M_Z$), there is now also the possibility that $\cos(\beta - \alpha) \to 0$ if $K \to 0$, and this happens independently of the value of $M_A$.

### 2.4.4 MSSM Higgs-boson couplings to fermions

As anticipated, $\Phi_1$ and $\Phi_2$ have Yukawa-type couplings to the up-type and down-type components of all $SU(2)_L$ fermion doublets. For example, the Yukawa Lagrangian corresponding to the third generation of quarks reads:

$$L_{Yukawa} = -h_t \left[ \bar{t}_R \phi_1^0 t_L - \bar{t}_R \phi_1^+ b_L \right] - h_b \left[ \bar{b}_R \phi_2^0 b_L - \bar{b}_R \phi_2^- t_L \right] + \text{h.c.}$$  \hspace{1cm} (81)

Upon spontaneous symmetry breaking $L_{Yukawa}$ provides both the corresponding quark masses:

$$m_t = h_t \frac{v_1}{\sqrt{2}} = h_t \frac{v \sin \beta}{\sqrt{2}} \quad \text{and} \quad m_b = h_b \frac{v_2}{\sqrt{2}} = h_b \frac{v \cos \beta}{\sqrt{2}},$$  \hspace{1cm} (82)

and the corresponding Higgs-quark couplings:

\begin{align*}
g_{htt} &= \frac{\cos \alpha}{\sin \beta} y_t = \left[ \sin(\beta - \alpha) + \cot \beta \cos(\beta - \alpha) \right] y_t, \hspace{1cm} (83) \\
g_{hb\bar{b}} &= -\frac{\sin \alpha}{\cos \beta} y_b = \left[ \sin(\beta - \alpha) - \tan \beta \cos(\beta - \alpha) \right] y_b, \\
g_{Htt} &= \frac{\sin \alpha}{\sin \beta} y_t = \left[ \cos(\beta - \alpha) - \cot \beta \sin(\beta - \alpha) \right] y_t, \\
g_{Hbb} &= \frac{\cos \alpha}{\cos \beta} y_b = \left[ \cos(\beta - \alpha) + \tan \beta \sin(\beta - \alpha) \right] y_b, \\
g_{Att} &= \cot \beta y_t, \quad g_{Abb} = \tan \beta y_b, \\
g_{H^{\pm} t\bar{b}} &= \frac{g}{2\sqrt{2}M_W} \left[ m_t \cot \beta (1 - \gamma_5) + m_b \tan \beta (1 + \gamma_5) \right],
\end{align*}

where $y_q = m_q/v$ (for $q = t, b$) are the SM couplings. It is interesting to notice that in the $M_A \gg M_Z$ decoupling limit, as expected, all the couplings in Eq. (83) reduce to the SM limit, i.e. all $H^0$, $A^0$, and $H^{\pm}$ couplings vanish, while the couplings of the light neutral Higgs boson, $h^0$, reduce to the corresponding SM Higgs-boson couplings.
The Higgs boson-fermion couplings are also modified directly by one-loop radiative corrections (squarks-gluino loops for quarks couplings and slepton-neutralino loops for lepton couplings). A detailed discussion can be found in Ref. [12, 11] and in the literature referenced therein. Of particular relevance are the corrections to the couplings of the third quark generation. These can be parameterized at the Lagrangian level by writing the radiatively corrected effective Yukawa Lagrangian as:

\[
\mathcal{L}_{\text{Yukawa}}^{\text{eff}} = -\varepsilon_{ij} \left[ (h_b + \delta h_b)\bar{b}_R Q^j_L \Phi_2^i + (h_t + \delta h_t)\bar{t}_R Q^i_L \Phi_1^j \right] + \Delta h_b \bar{b}_R Q^k_L \Phi^{k*} - \Delta h_t \bar{t}_R Q^k_L \Phi^{k*} + \text{h.c.},
\]

where we notice that radiative corrections induce a small coupling between \(\Phi_1\) and down-type fields and between \(\Phi_2\) and up-type fields. Moreover the tree level relation between \(h_b, h_t, m_b\) and \(m_t\) are modified as follows:

\[
m_b = \frac{h_b v}{\sqrt{2}} \cos \beta \left(1 + \frac{\delta h_b}{h_b} + \frac{\Delta h_b \cot \beta}{h_b} \right) \equiv \frac{h_b v}{\sqrt{2}} \cos \beta (1 + \Delta_b),
\]

\[
m_t = \frac{h_t v}{\sqrt{2}} \sin \beta \left(1 + \frac{\delta h_t}{h_t} + \frac{\Delta h_t \tan \beta}{h_t} \right) \equiv \frac{h_t v}{\sqrt{2}} \sin \beta (1 + \Delta_t),
\]

where the leading corrections are proportional to \(\Delta h_b\) and turn out to also be \(\tan \beta\) enhanced. On the other hand, the couplings between Higgs mass eigenstates and third generation quarks given in Eq. (83) are corrected as follows:

\[
g_{ht\bar{t}} = \frac{\cos \alpha}{\sin \beta} y_t \left[ 1 - \frac{1}{1 + \Delta_t} \frac{\Delta h_t}{h_t} (\cot \beta + \tan \alpha) \right],
\]

\[
g_{hb\bar{b}} = -\frac{\sin \alpha}{\cos \beta} y_b \left[ 1 + \frac{1}{1 + \Delta_b} \left( \frac{\delta h_b}{h_b} - \Delta_b \right) (1 + \cot \alpha \cot \beta) \right],
\]

\[
g_{Ht\bar{t}} = \frac{\sin \alpha}{\sin \beta} y_t \left[ 1 - \frac{1}{1 + \Delta_t} \frac{\Delta h_t}{h_t} (\cot \beta - \cot \alpha) \right],
\]

\[
g_{Hb\bar{b}} = \frac{\cos \alpha}{\cos \beta} y_b \left[ 1 + \frac{1}{1 + \Delta_b} \left( \frac{\delta h_b}{h_b} - \Delta_b \right) (1 - \tan \alpha \cot \beta) \right],
\]

\[
g_{A\bar{t}} = \cot \beta y_t \left[ 1 - \frac{1}{1 + \Delta_t} \frac{\Delta h_t}{h_t} (\cot \beta + \tan \beta) \right],
\]

\[
g_{A\bar{b}} = \tan \beta y_b \left[ 1 + \frac{1}{(1 + \Delta_b) \sin^2 \beta} \left( \frac{\delta h_b}{h_b} - \Delta_b \right) \right].
\]
\[
g_{H^0} \simeq \frac{g}{2\sqrt{2}M_W} \left\{ m_t \cot \beta \left[ 1 - \frac{1}{1 + \Delta_t} \frac{\Delta h_t}{h_t} (\cot \beta + \tan \beta) \right] (1 + \gamma_5) \right. \\
+ \left. m_b \tan \beta \left[ 1 + \frac{1}{(1 + \Delta_b) \sin^2 \beta} \left( \frac{\delta h_b}{h_b} - \Delta_b \right) \right] (1 - \gamma_5) \right\},
\]

where the last coupling is given in the approximation of small isospin breaking effects, since interactions of this kind have been neglected in the Lagrangian of Eq. [84].

### 3 Higgs searches

The search for a SM-like Higgs boson and for more exotic Higgs bosons of the kind predicted by multi-Higgs models is one of the most important goals of the physics programme of both the Tevatron and the LHC. After LEP ended its lifetime by setting a lower bound on the mass of a SM-like Higgs at 114.4 GeV, the Tevatron has excluded larger windows of the SM-Higgs (mass) parameter space and is still actively analyzing data collected during Run II, while the LHC is breaking new ground with an unprecedented amount of high energy data and has confirmed and extended the Tevatron exclusion bounds, promising to confirm or exclude the existence of a SM Higgs boson by the end of 2012, i.e. before the 2013 shutdown. Indeed on July 4th 2012 both the ATLAS and CMS experiments at CERN announced the discovery of a spin-0 particle with SM-Higgs-like properties and mass around 125-127 GeV. Two days earlier, on July 2nd 2012, the Tevatron experiments, CDF and D0, presented their Summer 2012 results and showed how their data would confirm or at least not contradict a discovery of a Higgs boson at about 126 GeV. Searches for more exotic Higgs bosons are meanwhile providing more and more stringent bounds on supersymmetric as well as non-supersymmetric extensions of the SM.

Having discussed the nature and implications of EWSB via the Higgs mechanism in Section 2, we can now turn to investigate the more phenomenological aspects of Higgs searches. In this Section I would like to build some background to understand the main properties of Higgs searches at hadron colliders and then specialize the discussion to Higgs searches at both the Tevatron and the LHC. For the sake of clarity, I will focus on the case of a SM Higgs boson and take the opportunity to go in detail on some important aspects. The possibility of interpreting the ATLAS and CMS signals in the
context of various extensions of the SM is being thoroughly investigated by the theory community. Since the discussion was not part of these lectures when they were originally delivered, and since including it would go far beyond the scope of this lectures, I would rather not touch on it and refer the interested reader to the rapidly growing literature on the subject.

### 3.1 SM Higgs-boson decay branching ratios

Different search channels are at the moment distinguished by the corresponding Higgs-boson decay channels. A precise calculation of both production cross sections and decay widths with their respective uncertainties is therefore essential to a correct interpretation of the data. In this section we will review the main decay properties of a SM Higgs boson and the major sources of uncertainties in the theoretical calculation of the corresponding decay widths.

In Section 2.2 we have derived the SM Higgs couplings to gauge bosons and fermions. In particular we have seen that, at the tree level the SM Higgs boson can decay into pairs of electroweak gauge bosons ($H \to W^+W^-$, $ZZ$), and into pairs of quarks and leptons ($H \to QQ, l^+l^-$), while at one-loop it can also decay into two photons ($H \to \gamma\gamma$), two gluons ($H \to gg$), or a $\gamma Z$.
pair \((H \to \gamma Z)\). Fig. 9 represents all the decay branching ratios of the SM Higgs boson as functions of its mass \(M_H\). The SM Higgs-boson total width, sum of all the partial widths \(\Gamma(H \to XX)\), is represented in Fig. 9.

In particular, Fig. 9 shows that a light Higgs boson \((M_H \leq 130 - 140 \text{ GeV})\) behaves very differently from a heavy Higgs boson \((M_H \geq 130 - 140 \text{ GeV})\). Indeed, a light SM Higgs boson mainly decays into a \(b\bar{b}\) pair, followed hierarchically by all other pairs of lighter fermions. Loop-induced decays also play a role in this region. \(H \to gg\) is dominant among them, and it is actually larger than many tree level decays. Unfortunately, this decay mode is almost useless, in particular at hadron colliders, because of background limitations.

Among radiative decays, \(H \to \gamma\gamma\) is tiny, but it is actually phenomenologically very important because the two photon signal can be seen over large hadronic backgrounds. On the other hand, for larger Higgs masses, the decays to \(W^+W^-\) and \(ZZ\) dominates. All decays into fermions or loop-induced decays are suppressed, except \(H \to \bar{t}t\) for Higgs masses above the \(\bar{t}t\) production threshold. There is an intermediate region, around \(M_H \simeq 160 \text{ GeV}\), i.e. below the \(W^+W^-\) and \(ZZ\) threshold, where the decays into \(WW^*\) and \(ZZ^*\) (when one of the two gauge bosons is off-shell) become important. These are indeed three-body decays of the Higgs boson that start to dominate over the \(H \to b\bar{b}\) two-body decay mode when the largeness of the \(HWW\) or \(HZZ\) couplings compensate for their phase space suppression.\(^4\) The different decay pattern of a light vs a heavy Higgs boson influences the role played, in each mass region, by different Higgs production processes at hadron and lepton colliders.

The curves in Fig. 9 are obtained by including all available QCD and electroweak (EW) radiative corrections. Indeed, the problem of computing the relevant orders of QCD and EW corrections for Higgs decays has been thoroughly explored and the results are nowadays available in public codes like HDECAY\(^{25}\), which has been used to produce Fig. 9. Indeed it would be more accurate to represent each curve as a band, including both parametric (due to the variation of the input parameters \(\alpha_s, m_c, m_b,\) and \(m_t\)) and theoretical uncertainties (resulting from approximations in the theoretical calculations, the dominant effects being due to missing higher orders). The effect of including both kinds of uncertainties have been studied in detail in Ref. [6] and [7] and is illustrated in Fig. 10 where the right-hand-side plot

\(^4\)Actually, even four-body decays, corresponding to \(H \to W^*W^*, Z^*Z^*\) may become important in the intermediate mass region and are indeed accounted for in Fig. 9.
Figure 10: SM Higgs decay branching ratios (left) and width (right) as a function of $M_H$. From Ref. [6].

gives an expanded view of the low mass region. Furthermore, for $H \to WW$ and $H \to ZZ$ the full decay chains into all possible 4-fermion final states have been calculated including NLO QCD and EW corrections, and have been included in the PROPHECY4f Monte Carlo event generator [26, 27], which also takes into account all possible interferences between common final states as well as leading two-loop heavy-Higgs corrections. This has been used in estimating the overall uncertainties of Fig. 10.

The theoretical uncertainties are most relevant for the $H \to gg$, $H \to Z\gamma$, and $H \to t\bar{t}$ branching ratios, reaching $O(10\%)$. For the $H \to b\bar{b}$, $H \to c\bar{c}$, and $H \to \tau\tau$ branching ratios they remain below a few per cent. Parametric uncertainties are relevant mostly for the $H \to c\bar{c}$ and $H \to gg$ branching ratios, reaching up to $O(10\%)$ and $O(5\%)$, respectively. They are mainly induced by the parametric uncertainties in $\alpha_s$ and $m_c$. The parametric uncertainties resulting from $m_b$ affect the Br($H \to b\bar{b}$) at the level of 3%, and the parametric uncertainty from $m_t$ influences in particular the Br($H \to t\bar{t}$) near the $t\bar{t}$ threshold. For the $H \to \gamma\gamma$ channel the total uncertainty can reach up to about 5% in the relevant mass range. Both theoretical and parametric uncertainties on the $H \to ZZ$ and $H \to WW$ channels remain at the level of 1% over the full mass range, giving rise to a total uncertainty below 3% for $m_H > 135$ GeV.
3.1.1 General properties of radiative corrections to Higgs decays

All Higgs-boson decay rates are modified by both EW and QCD radiative corrections. QCD corrections are particularly important for $H \rightarrow Q\bar{Q}$ decays, where they mainly amount to a redefinition of the Yukawa coupling by shifting the mass parameter in it from the pole mass value to the running mass value, and for $H \rightarrow gg$. EW corrections can be further separated into: i) corrections due to fermion loops, ii) corrections due to the Higgs-boson self-interaction, and iii) other EW corrections. Both corrections of type (ii) and (iii) are in general very small if not for large Higgs-boson masses, i.e. for $M_H \gg M_W$. On the other hand, corrections of type (i) are very important over the entire Higgs mass range, and are particularly relevant for $M_H \ll 2m_t$, where the top-quark loop corrections play a leading role. Indeed, for $M_H \ll 2m_t$, the dominant corrections for both Higgs decays into fermion and gauge bosons come from the top-quark contribution to the renormalization of the Higgs wave function and vacuum expectation value.

Several higher order radiative corrections to Higgs decays have been calculated in the large $m_t$ limit, specifically in the limit when $M_H \ll 2m_t$. Results can then be derived applying some very powerful low energy theorems. The idea is that, for an on-shell Higgs field ($p_H^2 = M_H^2$), the limit of small masses ($M_H \ll 2m_t$) is equivalent to a $p_H \rightarrow 0$ limit, in which case the Higgs couplings to the fermion fields can be simply obtained by substituting

$$m_i^0 \rightarrow m_i^0 \left(1 + \frac{H^0}{v^0}\right),$$

in the (bare) Yukawa Lagrangian, for each massive particle $i$. In Eq. (87) $H^0$ is a constant field and the upper zero indices indicate that all formal manipulations are done on bare quantities. This induces a simple relation between the bare matrix element for a process with ($X \rightarrow Y + H$) and without ($X \rightarrow Y$) a Higgs field, namely

$$\lim_{p_H \rightarrow 0} A(X \rightarrow Y + H) = \frac{1}{v^0} \sum_i m_i^0 \frac{\partial}{\partial m_i^0} A(X \rightarrow Y).$$

When the theory is renormalized, the only actual difference is that the derivative operation in Eq. (88) needs to be modified as follows

$$m_i^0 \frac{\partial}{\partial m_i^0} \rightarrow \frac{m_i}{1 + \gamma_m \frac{\partial}{\partial m_i}}.$$
where $\gamma_{m_i}$ is the mass anomalous dimension of fermion $f_i$. This accounts for the fact that the renormalized Higgs-fermion Yukawa coupling is determined through the $Z_2$ and $Z_m$ counterterms, and not via the $H f \bar{f}$ vertex function at zero momentum transfer (as used in the $p_H \rightarrow 0$ limit above).

The theorem summarized by Eq. (88) is valid also when higher order radiative corrections are included. Therefore, outstanding applications of Eq. (88) include the determination of the one-loop $Hgg$ and $H\gamma\gamma$ vertices from the gluon or photon self-energies, as well as the calculation of several orders of their QCD and EW radiative corrections. Indeed, in the $m_t \rightarrow \infty$ limit, the loop-induced $H\gamma\gamma$ and $Hgg$ interactions can be seen as effective vertices derived from an effective Lagrangian of the form:

$$\mathcal{L}_{\text{eff}} = \frac{\alpha_s}{12\pi} F^{(a)\mu\nu} F^{(a)\mu\nu} \frac{H_v}{v} (1 + O(\alpha_s)) \ ,$$

(90)

where $F^{(a)\mu\nu}$ is the field strength tensor of QED (for the $H\gamma\gamma$ vertex) or QCD (for the $Hgg$ vertex). The calculation of higher order corrections to the $H \rightarrow \gamma\gamma$ and $H \rightarrow gg$ decays is then reduced by one order of loops! Since these vertices start as one-loop effects, the calculation of the first order of corrections would already be a strenuous task, and any higher order effect would be a formidable challenge. Thanks to the low energy theorem results sketched above, QCD NNLO corrections have indeed been calculated.

### 3.1.2 Higgs-boson decays to gauge bosons: $H \rightarrow W^+W^-, ZZ$

The tree level decay rate for $H \rightarrow VV$ ($V = W^\pm, Z$) can be written as:

$$\Gamma(H \rightarrow VV) = \frac{G_F M_H^3}{16 \sqrt{2} \pi} \delta_V \left(1 - \tau_V + \frac{3}{4} \tau_V^2 \right) \beta_V \ ,$$

(91)

where $\beta_V = \sqrt{1 - \tau_V}$, $\tau_V = 4M_V^2/M_H^2$, and $\delta_{W,Z} = 2, 1$.

Below the $W^+W^-$ and $ZZ$ threshold, the SM Higgs-boson can still decay via three (or four) body decays mediated by $WW^*$ ($W^*W^*$) or $ZZ^*$ ($Z^*Z^*$) intermediate states. As we can see from Fig. 4, the off-shell decays $H \rightarrow WW^*$ and $H \rightarrow ZZ^*$ are relevant in the intermediate mass region around $M_H \simeq 160$ GeV, where they compete and overcome the $H \rightarrow b\bar{b}$ decay mode. The decay rates for $H \rightarrow VV^* \rightarrow V f_i \bar{f}_j$ ($V = W^\pm, Z$) are given by:

$$\Gamma(H \rightarrow WW^*) = \frac{3g^4M_H}{512\pi^3} F \left(\frac{M_W}{M_H} \right) \ ,$$

(92)
\[ \Gamma(H \rightarrow ZZ^*) = \frac{g^4 M_H}{2048(1 - s_W^2)^2 \pi^3} \left( 7 - \frac{40}{3} s_W^2 + \frac{160}{9} s_W^4 \right) F\left( \frac{M_Z}{M_H} \right) , \]

where \( s_W = \sin \theta_W \) is the sine of the Weinberg angle and the function \( F(x) \) is given by

\[ F(x) = -(1 - x^2) \left( \frac{47}{2} x^2 - \frac{13}{2} + \frac{1}{x^2} \right) - 3 \left( 1 - 6x^2 + 4x^4 \right) \ln(x) + 3 \frac{1 - 8x^2 + 20x^4}{\sqrt{4x^2 - 1}} \arccos \left( \frac{3x^2 - 1}{2x^3} \right) . \]  

(93)

### 3.1.3 Higgs-boson decays to fermions: \( H \rightarrow QQ, l^+l^- \)

The tree level decay rate for \( H \rightarrow f \bar{f} \) \((f = Q, l, Q = \text{quark}, l = \text{lepton})\) can be written as:

\[ \Gamma(H \rightarrow f \bar{f}) = \frac{G_F M_H}{4\sqrt{2} \pi} N_c m_f \beta_f^3 , \]  

(94)

where \( \beta_f = \sqrt{1 - \tau_f} \), \( \tau_f = 4 m_f^2 / M_H^2 \), and \( (N_c)^Q = 1, 3 \). QCD corrections dominate over other radiative corrections and they modify the rate as follows:

\[ \Gamma(H \rightarrow Q\bar{Q})_{QCD} = \frac{3G_F M_H}{4\sqrt{2} \pi} \bar{m}_Q(M_H) \beta_q^3 \left[ \Delta_{QCD} + \Delta_t \right] , \]  

(95)

where \( \Delta_t \) represents specifically QCD corrections involving a top-quark loop. \( \Delta_{QCD} \) and \( \Delta_t \) have been calculated up to three loops and are given by:

\[ \Delta_{QCD} = 1 + 5.67 \frac{\alpha_s(M_H)}{\pi} + (35.94 - 1.36 N_F) \left( \frac{\alpha_s(M_H)}{\pi} \right)^2 + (164.14 - 25.77 N_F + 0.26 N_F^2) \left( \frac{\alpha_s(M_H)}{\pi} \right)^3 , \]  

(96)

\[ \Delta_t = \left( \frac{\alpha_s(M_H)}{\pi} \right)^2 \left[ 1.57 - \frac{2}{3} \ln \frac{M_H^2}{m_t^2} + \frac{1}{9} \ln^2 \frac{\bar{m}_Q(M_H)}{M_H^2} \right] , \]

where \( \alpha_s(M_H) \) and \( \bar{m}_Q(M_H) \) are the renormalized running QCD coupling and quark mass in the \( \overline{MS} \) scheme. It is important to notice that using the \( \overline{MS} \) running mass in the overall Yukawa coupling square of Eq. (93) is very important in Higgs decays, since it reabsorbs most of the QCD corrections,
including large logarithms of the form $\ln(M_H^2/m_Q^2)$. Indeed, for a generic scale $\mu$, $\bar{m}_Q(\mu)$ is given at leading order by:

$$
\bar{m}_Q(\mu)_{LO} = \bar{m}_Q(m_Q) \left( \frac{\alpha_s(\mu)}{\alpha_s(m_Q)} \right)^{2b_0/\gamma_0} (97)
$$

$$
= \bar{m}_Q(m_Q) \left( 1 - \frac{\alpha_s(\mu)}{4\pi} \ln \left( \frac{\mu^2}{m_Q^2} \right) + \cdots \right),
$$

where $b_0$ and $\gamma_0$ are the first coefficients of the $\beta$ and $\gamma$ functions of QCD, while at higher orders it reads:

$$
\bar{m}_Q(\mu) = \bar{m}_Q(m_Q) f(\frac{\alpha_s(\mu)}{\pi}) \frac{f(\alpha_s(m_Q)/\pi)}{f(\alpha_s(m_Q)/\pi)} ,
$$

(98)

where, from renormalization group techniques, the function $f(x)$ is of the form:

$$
f(x) = \left( \frac{25}{6} x \right)^{\frac{11}{6}} [1 + 1.014x + \ldots] \text{ for } m_c < \mu < m_b ,
$$

(99)

$$
f(x) = \left( \frac{23}{6} x \right)^{\frac{11}{2}} [1 + 1.175x + \ldots] \text{ for } m_b < \mu < m_t ,
$$

$$
f(x) = \left( \frac{7}{2} x \right)^{\frac{4}{7}} [1 + 1.398x + \ldots] \text{ for } \mu > m_t .
$$

As we can see from Eqs. (98) and (99), by using the $\overline{MS}$ running mass, leading and subleading logarithms up to the order of the calculation are actually resummed at all orders in $\alpha_s$.

The overall mass factor coming from the quark Yukawa coupling square is actually the only place where we want to employ a running mass. For quarks like the $b$ quark this could indeed have a large impact, since, in going from $\mu \simeq M_H$ to $\mu \simeq m_b$, $\bar{m}_n(\mu)$ varies by almost a factor of two, making therefore almost a factor of four at the rate level. All other mass corrections, in the matrix element and phase space entering the calculation of the $H \rightarrow Q\bar{Q}$ decay rate, can in first approximation be safely neglected.

### 3.1.4 Loop induced Higgs-boson decays: $H \rightarrow \gamma\gamma, \gamma Z, gg$

As seen in Section 2.2, the $H\gamma\gamma$ and $H\gamma Z$ couplings are induced at one loop via both a fermion loop and a W-loop. At the lowest order the decay rate
for $H \to \gamma\gamma$ can be written as:

$$
\Gamma(H \to \gamma\gamma) = \frac{G_F \alpha^2 M_H^3}{128\sqrt{2}\pi^3} \left| \sum_f N^f_c Q_f^2 A^H_f(\tau_f) + A^H_W(\tau_W) \right|^2 ,
$$

(100)

where $N^f_c = 1, 3$ (for $f = l, q$ respectively), $Q_f$ is the charge of the $f$ fermion species, $\tau_f = 4m_f^2/M_H^2$, the function $f(\tau)$ is defined as:

$$
f(\tau) = \begin{cases} 
\arcsin^2 \frac{1}{\sqrt{\tau}} & \tau \geq 1 \\
-\frac{1}{4} \left[ \ln \frac{1 + \sqrt{1-\tau}}{1-\sqrt{1-\tau}} - i\pi \right]^2 & \tau < 1
\end{cases}
$$

(101)

and the form factors $A^H_f$ and $A^H_W$ are given by:

$$
\begin{align*}
A^H_f &= 2\tau \left[ 1 + (1-\tau)f(\tau) \right] , \\
A^H_W(\tau) &= -[2 + 3\tau + 3\tau(2-\tau)f(\tau)] .
\end{align*}
$$

(102)

On the other hand, the decay rate for $H \to \gamma Z$ is given by:

$$
\Gamma(H \to \gamma Z) = \frac{G_F^2 M_W^2 \alpha M_H^3}{64\pi^4} \left( 1 - \frac{M_Z^2}{M_H^2} \right)^3 \left| \sum_f A^H_f(\tau_f, \lambda_f) + A^H_W(\tau_W, \lambda_W) \right|^2 ,
$$

(103)

where $\tau_i = 4M_i^2/M_H^2$ and $\lambda_i = 4M_i^2/M_Z^2$ ($i = f, W$), and the form factors $A^H_f(\tau, \lambda)$ and $A^H_W(\tau, \lambda)$ are given by:

$$
\begin{align*}
A^H_f(\tau, \lambda) &= 2N^f_c \frac{Q_f(I_{3f} - 2Q_f \sin^2 \theta_W)}{\cos \theta_W} \left[ I_1(\tau, \lambda) - I_2(\tau, \lambda) \right] , \\
A^H_W(\tau, \lambda) &= \cos \theta_W \left\{ \left[ (1 + \frac{2}{\tau}) \tan^2 \theta_W - \left( 5 + \frac{2}{\tau} \right) \right] I_1(\tau, \lambda) \\
&\quad + 4 \left( 3 - \tan^2 \theta_W \right) I_2(\tau, \lambda) \right\} ,
\end{align*}
$$

(104, 105)

where $N^f_c$ and $Q_f$ are defined after Eq. (100), and $I^f_f$ is the weak isospin of the $f$ fermion species. Moreover:

$$
\begin{align*}
I_1(\tau, \lambda) &= \frac{\tau \lambda}{2(\tau - \lambda)} + \frac{\tau^2 \lambda^2}{2(\tau - \lambda)^2} [f(\tau) - f(\lambda)] + \frac{\tau^2 \lambda}{(\tau - \lambda)^2} [g(\tau) - g(\lambda)] , \\
I_2(\tau, \lambda) &= -\frac{\tau \lambda}{2(\tau - \lambda)} [f(\tau) - f(\lambda)] ,
\end{align*}
$$

(106)
and
\[ g(\tau) = \begin{cases} \sqrt{\tau - 1} \arcsin \frac{1}{\sqrt{\tau}} & \tau \geq 1 \\ \sqrt{\frac{1}{\tau} - 1} \ln \left[ \frac{1 + \sqrt{1 - \tau}}{1 - \sqrt{1 - \tau}} + i\pi \right] & \tau < 1 \end{cases} \] (107)

while \( f(\tau) \) is defined in Eq. (101). QCD and EW corrections to both \( \Gamma(H \rightarrow \gamma\gamma) \) and \( \Gamma(H \rightarrow \gammaZ) \) are pretty small and for their explicit expression we refer the interested reader to the literature [28, 10].

As far as \( H \rightarrow gg \) is concerned, this decay can only be induced by a fermion loop, and therefore its rate, at the lowest order, can be written as:
\[
\Gamma(H \rightarrow gg) = \frac{G_F^2 \alpha_s^2 M_H^3}{36 \sqrt{2} \pi^3} \left| \frac{3}{4} \sum_q A_H^q(\tau_q) \right|, \tag{108}
\]
where \( \tau_q = 4m_q^2/M_H^2 \), \( f(\tau) \) is defined in Eq. (101) and the form factor \( A_H^q(\tau) \) is given in Eq. (104). QCD corrections to \( H \rightarrow gg \) have been calculated up to NNLO in the \( m_t \rightarrow \infty \) limit, as explained in Section 3.1.1. At NLO the expression of the corrected rate is remarkably simple
\[
\Gamma(H \rightarrow gg(g), q\bar{q}g) = \Gamma_{LO}(H \rightarrow gg) \left[ 1 + E(\tau_Q) \frac{\alpha_s^{(NL)}}{\pi} \right], \tag{109}
\]
where
\[
E(\tau_Q) \xrightarrow{M_H^2 \ll 4m_q^2} \frac{95}{4} - \frac{7}{6} N_L + \frac{33 - 2N_F}{6} \log \left( \frac{\mu^2}{M_H^2} \right). \tag{110}
\]
When compared with the fully massive NLO calculation (available in this case), the two calculations display an impressive 10% agreement, as illustrated in Fig. 11, even in regions where the light Higgs approximation is not justified. This is actually due to the presence of large constant factors in the first order of QCD corrections. We also observe that the first order of QCD corrections has quite a large impact on the lowest order cross section, amounting to more than 50% of \( \Gamma_{LO} \) on average. This has been indeed the main reason to prompt for a NNLO QCD calculation of \( \Gamma(H \rightarrow gg) \). The result, obtained in the heavy-top approximation, has shown that NNLO QCD corrections amount to only 20% of the NLO cross section, therefore pointing to a convergence of the \( \Gamma(H \rightarrow gg) \) perturbative series. We will refer to this discussion when dealing with the \( gg \rightarrow H \) production mode, since its cross section can be easily related to \( \Gamma(H \rightarrow gg) \).
Figure 11: The QCD correction factor for the partial width $\Gamma(H \rightarrow gg)$ as a function of the Higgs-boson mass, in the full massive case with $m_t = 178$ GeV (dotted line) and in the heavy-top-quark limit (solid line). The strong coupling constant is normalized at $\alpha_s(M_Z) = 0.118$. From Ref. [10].

### 3.2 MSSM Higgs-boson branching ratios

The decay patterns of the MSSM Higgs bosons are many and diverse, depending on the specific choice of supersymmetric parameters. In particular they depend on the choice of $M_A$ and $\tan \beta$, which parameterize the MSSM Higgs sector, and they are clearly sensitive to the choice of other supersymmetric masses (gluino masses, squark masses, etc.) since this determines the possibility for the MSSM Higgs bosons to decay into pairs of supersymmetric particles and for the radiative induced decay channels ($h^0, H^0 \rightarrow gg, \gamma\gamma, \gamma Z$) to receive supersymmetric loop contributions.

In order to be more specific, let us assume that all supersymmetric masses are large enough to prevent the decay of the MSSM Higgs bosons into pairs of supersymmetric particles (a good choice could be $M_\tilde{g} = M_Q = M_U = M_D = 1$ TeV). Then, we only need to examine the decays into SM particles and compare with the decay patterns of a SM Higgs boson to identify any interesting difference. From the study of the MSSM Higgs-boson couplings in Sections 2.4.3 and 2.4.4 we expect that: i) in the decoupling regime, when $M_A \gg M_Z$, the properties of the $h^0$ neutral Higgs boson are very much the
Figure 12: Branching ratios for the $h^0$ and $H^0$ MSSM Higgs bosons, for $\tan \beta = 3, 30$. The range of $M_H$ corresponds to $M_A = 90 \text{ GeV} - 1 \text{ TeV}$, in the MSSM scenario discussed in the text, with maximal top-squark mixing. The vertical line in the left hand side plots indicates the upper bound on $M_h$, which, for the given scenario is $M_{h}^{\text{max}} = 115 \text{ GeV}$ ($\tan \beta = 3$) or $M_{h}^{\text{max}} = 125.9 \text{ GeV}$ ($\tan \beta = 30$). From Ref. [12].
Figure 13: Branching ratios for the $A^0$ and $H^+$ MSSM Higgs bosons, for $\tan\beta = 3, 30$. The range of $M_{H^\pm}$ corresponds to $M_{A^0} = 90$ GeV − 1 TeV, in the MSSM scenario discussed in the text, with maximal top-squark mixing. From Ref. [12].

same as the SM Higgs boson; while away from the decoupling limit $ii)$ the decay rates of $h^0$ and $H^0$ to electroweak gauge bosons are suppressed with respect to the SM case, in particular for large Higgs masses ($H^0$), $iii)$ the $A^0 \rightarrow VV$ ($V = W^\pm, Z^0$) decays are absent, $iv)$ the decay rates of $h^0$ and $H^0$ to $\tau^+\tau^-$ and $b\bar{b}$ are enhanced for large $\tan\beta$, $v)$ even for not too large values of $\tan\beta$, due to $ii)$ above, the $h^0, H^0 \rightarrow \tau^+\tau^-$ and $h^0, H^0 \rightarrow b\bar{b}$ decay are large up to the $t\bar{t}$ threshold, when the decay $H^0 \rightarrow t\bar{t}$ becomes dominant, $vi)$ for the charged Higgs boson, the decay $H^+ \rightarrow \tau^+\nu_\tau$ dominates over $H^+ \rightarrow t\bar{b}$ below the $t\bar{t}$ threshold, and vice versa above it.

As far as QCD and EW radiative corrections go, what we have seen in Sections 3.1.2-3.1.4 for the SM case applies to the corresponding MSSM decays too. Moreover, the truly MSSM corrections discussed in Sections 2.4.3 and 2.4.4 need to be taken into account and are included in Figs 12 and 13.
3.3 Direct bounds on both SM and MSSM Higgs bosons from LEP

LEP2 has searched for a SM Higgs at center of mass energies between 189 and 209 GeV. In this regime, a SM Higgs boson is produced mainly through Higgs-boson strahlung from $Z$ gauge bosons, $e^+e^- \rightarrow Z^* \rightarrow HZ$, and to a lesser extent through $WW$ and $ZZ$ gauge boson fusion, $e^+e^- \rightarrow WW, ZZ \rightarrow H\nu\bar{\nu}, H e^+e^-$ (see Fig. 14). Once produced, it decays mainly into $b\bar{b}$ pairs, and more rarely into $\tau^+\tau^-$ pairs. The four LEP2 experiments have been looking for: i) a four jet final state ($H \rightarrow b\bar{b}, Z \rightarrow q\bar{q}$), ii) a missing energy final state ($H \rightarrow b\bar{b}, Z \rightarrow \nu\bar{\nu}$), iii) a leptonic final state ($H \rightarrow b\bar{b}, Z \rightarrow l^+l^-$) and iv) a specific $\tau$-lepton final state ($H \rightarrow b\bar{b}, Z \rightarrow \tau^+\tau^- \text{ plus } H \rightarrow \tau^+\tau^-, Z \rightarrow q\bar{q}$). The absence of any statistical significant signal has set a 95% CL lower bound on the SM Higgs boson at

$$M_{H_{SM}} > 114.4 \text{ GeV}.$$ 

LEP2 has also looked for the light scalar ($h^0$) and pseudoscalar ($A^0$) MSSM neutral Higgs bosons. In the decoupling regime, when $A^0$ is very heavy and $h^0$ behaves like a SM Higgs bosons, only $h^0$ can be observed and the same bounds established for the SM Higgs boson apply. The bound can however be lowered when $m_A$ is lighter. In that case, $h^0$ and $A^0$ can also be pair produced through $e^+e^- \rightarrow Z \rightarrow h^0A^0$ (see Fig. 14). Combining the different production channels one can derive plots like those shown in Fig. 15, where the excluded ($M_h, \tan \beta$) and ($M_A, \tan \beta$) regions of the MSSM parameter space are shown. The LEP2 collaborations [22] have been able to set the following bounds at 95% CL:

$$M_{h,A} > 93.0 \text{ GeV},$$ 

obtained in the limit when $\cos(\beta - \alpha) \simeq 1$ (anti-decoupling regime) and for large $\tan \beta$. The plots in Fig. 15 have been obtained in the maximal mixing...
Figure 15: 95% CL exclusion limits for MSSM Higgs parameters from LEP2: $(M_h, \tan \beta)$ (left) and $(M_A, \tan \beta)$ (right). Both the maximal and no-mixing scenarios are illustrated, for $M_S = 1$ TeV and $m_t = 179.3$ GeV. The dashed lines indicate the boundaries that are excluded on the basis of Monte Carlo simulations in the absence of a signal. From Ref. [11].
scenario (explained in Section 2.4.2). For no-mixing, the corresponding plots would exclude a much larger region of the MSSM parameter space.

Finally, the LEP collaborations have looked for the production of the MSSM charged Higgs boson in the associated production channel: $e^+e^- \rightarrow \gamma, Z^* \rightarrow H^+H^-$. An absolute lower bound of

$$M_{H^\pm} > 79.3 \text{ GeV}$$

has been set by the ALEPH collaboration, and slightly lower values have been obtained by the other LEP collaborations.

Both the Tevatron and the LHC have extended these bounds as we will discuss in Sections 4 and 5.

### 3.4 SM Higgs production at hadron colliders

The parton level processes through which a SM Higgs boson can be produced at hadron colliders are illustrated in Figs. 16 and 17.

Figures 18 and 19 summarize the cross sections for all these production modes as functions of the SM Higgs-boson mass, at the Tevatron with $\sqrt{s} = 1.96$ TeV and at the LHC with $\sqrt{s} = 14$ TeV. These figures have been produced during the TeV4LHC workshop [30], and contain most known orders of QCD corrections as well as the fairly up to date input parameters. They serve the purpose of illustrating the different relevance of different production processes at both the Tevatron and the LHC and allows us to discuss some general phenomenological aspects of hadronic Higgs production. We postpone further details about QCD corrections till Section 6 where we will discuss the uncertainties involved in the prediction of Higgs production cross sections and will give more accurate plots including both EW and QCD updated effects.
The leading production mode is gluon-gluon fusion, $gg \rightarrow H$ (see first diagram in Fig. 16). In spite of being a loop-induced process, it is greatly enhanced by the top-quark loop. For light- and intermediate-mass Higgs bosons, however, the very large cross section of this process has to compete against a very large hadronic background, since the Higgs boson mainly decays to $b\bar{b}$ pairs, and there is no other non-hadronic probe that can help distinguishing this mode from the overall hadronic activity in the detector. To beat the background, one has to employ subleading Higgs decay modes, like $H \rightarrow \gamma\gamma$, and this dilutes the large cross section to some extent. For larger Higgs masses, above the $ZZ$ threshold, on the other hand, gluon-gluon fusion together with $H \rightarrow ZZ$ produces a very distinctive signal, and make this mode a “gold-plated mode” for detection. For this reason, $gg \rightarrow H$ plays a fundamental role at the LHC over the entire Higgs-boson mass range, but is of very limited use at the Tevatron, where it can only be considered for Higgs boson masses very close to the upper reach of the machine ($M_H \simeq 200$ GeV).

Weak boson fusion ($qq' \rightarrow qq'H$, see second diagram in Fig. 16) and the associated production with weak gauge bosons ($q\bar{q} \rightarrow WH, ZH$, see third diagram in Fig. 16) have also fairly large cross sections, of different relative size at the Tevatron and at the LHC. $qq' \rightarrow WH, ZH$ is particularly important at the Tevatron, where only a relatively light Higgs boson ($M_H < 200$ GeV) is accessible. In this mass region, $gg \rightarrow H, H \rightarrow \gamma\gamma$ is too small at the Tevatron, while $qq' \rightarrow qq'H$ is suppressed (because the initial state is $p\bar{p}$). On the other
Figure 18: Cross sections for SM Higgs-boson production processes at the Tevatron, Run II ($\sqrt{s}=1.96$ TeV). From Ref. [30].

Figure 19: Cross sections for SM Higgs-boson production processes at the LHC ($\sqrt{s}=14$ TeV). From Ref. [30].
hand, \(qq' \rightarrow qq'H\) becomes instrumental at the LHC (\(pp\) initial state) for low- and intermediate-mass SM Higgs bosons, where its characteristic final state configuration, with two very forward jets, has been shown to greatly help in disentangling this signal from the hadronic background, using different Higgs decay channels.

Finally, the production of a SM Higgs boson with heavy quarks, in the two channels \(q\bar{q}, gg \rightarrow Q\bar{Q}H\) (with \(Q = t, b\), see Fig. 17), is sub-leading at both the Tevatron and the LHC, but has a great physics potential. The associated production with \(tt\) pairs is too small to be relevant for the Tevatron, but will play an important role at the LHC, where enough statistics will be available to fully exploit the signature of a \(ttH, H \rightarrow b\bar{b}\) final state. Indeed, this channel has not been used for discovery but will certainly become important now that the properties of the discovered spin-0 particle need to be thoroughly investigated, since it offers the unique possibility of directly measuring one of its most important couplings, namely the coupling to top quarks. On the other hand, the production of a SM Higgs boson with \(b\bar{b}\) pairs is tiny, since the SM bottom-quark Yukawa coupling is suppressed by the bottom-quark mass. Therefore, the \(b\bar{b}H, H \rightarrow b\bar{b}\) channel is the ideal candidate to provide evidence of new physics, in particular of extension of the SM, like supersymmetric models, where the bottom-quark Yukawa coupling to one or more Higgs bosons is enhanced (e.g., by large \(\tan \beta\) in the MSSM). \(bbH\) production is kinematically well within the reach of the Tevatron, RUN II. First studies from both CDF [33] and D0 [34] have translated the absence of a \(b\bar{b}h^0, H^0, A^0\) signal into an upper bound on the \(\tan \beta\) parameter of the MSSM. A difficult channel to measure at the LHC, because of the large hadronic background, it could however offer a striking signal of new physics if observed.

4 Higgs searches at the Tevatron

In a recent note [35] the CDF and D0 collaboration presented combined results of direct searches for the SM Higgs boson in \(p\bar{p}\) collisions at 1.96 TeV. They combined the most recent results of all the Tevatron Higgs-boson searches in the mass range \(m_H = 100 - 200\) GeV. These analyses sought signals of a SM Higgs boson produced through associated production with an EW vector boson (\(q\bar{q} \rightarrow HW/Z\)), through gluon-gluon fusion (\(gg \rightarrow H\)), and through vector boson fusion (\(qq' \rightarrow Hqq'\)), corresponding to integrated lumi-
nosities ranging from 5.4 to 10 fb\(^{-1}\). They studied the \(H \rightarrow b\bar{b}, H \rightarrow W^+W^-,\)
\(H \rightarrow ZZ, H \rightarrow \tau^+\tau^-,\) and \(H \rightarrow \gamma\gamma\) decay signatures. The greatest sensitivity was reached using \(H \rightarrow W^+W^-\) (with the \(W\)s decaying leptonically) in the \(m_H > 125\) GeV region and looking for \(q\bar{q} \rightarrow HW/Z\) with \(H \rightarrow b\bar{b}\) (with the \(W\) or \(Z\) decaying leptonically) in the \(m_H < 125\) GeV mass region.

To quantify the expected sensitivity across the whole mass range, CDF and D0 have studied the distribution of the Log-Likelihood Ratios (LLR) for different hypothesis (background only, signal+background) Results have been presented in terms of \(\text{LLR}_b\) and \(\text{LLR}_{s+b}\) defined as,

\[
\text{LLR} = -2 \ln \frac{p(\text{data}|H_1)}{p(\text{data}|H_0)},
\]

where \(H_1\) denotes the test hypothesis, which admits the presence of SM backgrounds and a Higgs-boson signal, while \(H_0\) is the null hypothesis, for only SM background, and \(\text{data}\) is either an ensemble of pseudo-experiment data constructed from the expected signal and backgrounds, or the actual observed data.

As an example, in Fig. [20] we see the LLR distributions for the combined CDF+D0 analyses as functions of the Higgs-boson mass. The solid black line
corresponds to the observed data (LLR_{obs}). The dashed black and red lines represent the median for the background-only hypothesis (LLR_{b}) and the signal-plus-background hypothesis (LLR_{s+b}). The shaded bands represent the one and two standard-deviation departures from the median for LLR_{b}, assuming that no signal is present and only statistical fluctuations and systematic effects are present. We note that the separation between the medians of the LLR_{b} and LLR_{s+b} distributions provides a measure of the discriminating power of the search. Moreover, the value of LLR_{obs} relative to LLR_{s+b} or LLR_{b} indicates if the data distribution resembles more the case in which a signal is present or not. With this in mind, Fig. 20 shows that the data are consistent with a background-only hypothesis for m_{H} > 145 GeV, except above 190 GeV, where the signal-plus-background and background-only hypotheses cannot be separated very well. On the other hand, for m_{H} from 110 to 140 GeV we see an excess in the data consistent with the expectation for a SM Higgs boson in this mass range (red dashed line). We notice that in this region the ability of separating LLR_{s+b} from LLR_{b} is at the two-σ level. It is interesting to compare these results to what one would obtain by artificially injecting a signal for a SM Higgs with m_{H} = 125 GeV. This is shown in Fig. 21 where the solid black line now represents the artificial Higgs signal.

The probability of observing a signal-plus-background-like outcome without the presence of a signal, i.e. the probability that an upward fluctuation of the background provides a signal-plus-background-like response as observed data, is defined as,

\[ 1 - CL_{b} = p(LLR \leq LLR_{obs}|H_{0}) \ , \tag{112} \]

while the probability of a downward fluctuation of the sum of signal and background in the data is defined as,

\[ CL_{s+b} = p(LLR \geq LLR_{obs}|H_{1}) \ , \tag{113} \]

where LLR_{obs} is the value of the test statistic computed for the data. A small value of CL_{s+b} denotes inconsistency with H_{1}.

To facilitate comparison with the Standard Model, CDF and D0 have presented their resulting limit divided by the SM Higgs-boson production cross section, \( \sigma_{SM} \), as a function of the Higgs-boson mass. This is illustrated in Fig. 22. This figure is rich of information and we will discuss it in detail. First of all, Fig. 22 includes all existing limits on the SM Higgs-boson mass,
including previous limits from LEP and limits provided by the LHC till they recently announced discovery of a spin-0 particle with mass around 126 GeV. The line patterns and colors have the same meaning as in Figs. 20 and 21, i.e. the black solid line indicates the observed limit, the black dashed line indicates the expected limit in the background-only hypothesis, while the green and yellow bands represent the variation of the expected limit by one and two standard deviations respectively.

Since the figure shows the 95% CL of the ratio $R = \sigma/\sigma_{SM}$, a value of the limit observed ratio that is less or equal to one excludes that mass at the 95% C.L. The Tevatron combined analyses therefore exclude the regions $100 < m_H < 103$ GeV and $147 < m_H < 180$ GeV, as shown in Fig. 22 by the green vertical bands.

On the other hand, if the solid black line is above 1.0 and also somewhat above the dotted black line (an excess), then there might be a hint that the Higgs exists with a mass at that value. If the solid black line is at the upper edge of the yellow band, then there may be 95% certainty that this is above the expectations. It could be a hint for a Higgs boson of that mass, or it could be a sign of background processes or of systematic errors that are not well understood. Indeed, in Fig. 22 we see that the limit curve
Figure 22: Observed and expected (background-only hypothesis) 95% C.L. upper limits on the ratios to the SM cross section, as function of the Higgs-boson mass, for the combined CDF and D0 analyses. From Ref. [35].

goes much above the upper edge of the yellow band in the region between 115 and 140 GeV, and this could point to the fact that a Higgs boson may indeed be contributing to the data in that mass range. Still, in the same region the calculated (expected) background has not reached yet enough sensitivity since the black dashed line (as well as the one and two standard deviation bands) goes above the SM=1 threshold. Therefore the indication of a Higgs-like fluctuation is in this case statistically weak. In the words of the experiments, this excess only causes the observed limits not to be as stringent as expected.

5 Higgs searches at the LHC

Since it started running in 2010, the LHC has been accumulating an unprecedented amount of data and has past all expectations in providing exclusion limits for the SM Higgs boson. In 2011, the LHC delivered to ATLAS and CMS up to 5.1 fb$^{-1}$ of integrated luminosity of $pp$ collisions at 7 TeV center-of-mass energy fulfilling all the data quality requirements to search
for the SM Higgs boson. In 2012 the center-of-mass energy was increased to 8 TeV, and the accelerator delivered up to extra 5.9 fb$^{-1}$ of quality data by July 2012, when the amazing discovery of a spin-0 particle with mass around 125-127 GeV and SM-like Higgs-boson properties has been delivered to the world [4, 5]. At the same time the 95% C.L. exclusion limits for a SM Higgs boson have been updated to 110 < $m_H$ < 122.5 GeV plus 127 < $m_H$ < 600 GeV by CMS [5] and to 110 < $m_H$ < 122.6 GeV plus 129.7 < $m_H$ < 558 GeV by ATLAS [4]. Results and details of the search have been published by the two collaborations in Refs. [4, 5] which update recent analyses appeared earlier in 2012 [37, 36].

The LHC experiments have looked for a SM Higgs boson in the wide mass range between the experimental LEP bound (114 GeV) and about 600 GeV. The main production mode in this range is gluon fusion ($gg \rightarrow H$), followed by vector-boson fusion ($qq' \rightarrow V qq'$) and the associated productions with weak gauge bosons ($qq'^{(0)} \rightarrow ZH/WH$) and top quarks ($qg, gg \rightarrow t\bar{t}H$). In all combined analyses so far the following channels have been considered: $H \rightarrow \gamma\gamma$, $H \rightarrow ZZ^*$, $H \rightarrow WW^{(*)}$, $H \rightarrow b\bar{b}$, and $H \rightarrow \tau^+\tau^-$. The crucial channels in the discovery have been the $H \rightarrow \gamma\gamma$ channel in the low mass region and the $H \rightarrow ZZ^{(*)} \rightarrow 4l$ over the whole mass range. Both channels provide a high-resolution invariant mass for fully reconstructed candidates in the respective mass regions. The plots in Fig. 23 are from the ATLAS presentation at CERN on July 4th, 2012: they show the excess of data points around 125-127 GeV in both channels.

The dominant systematic uncertainties are those on the measurement of the integrated luminosity and on the theoretical predictions of the signal production cross sections and decay branching ratios, as well as those related to detector response that impact the reconstruction analyses in various reconstructing procedures. More details on both the uncertainties from the measure of the integrated luminosity and the detector response can be find in the ATLAS and CMS papers [36, 37] as well as in the discovery papers [4, 5]. The degree of accuracy reached in the theoretical predictions of both production cross sections and branching ratios will be discussed in Sec. 6. Fig. 24 shows the 95% CL upper limits on the signal strength. The various curves and bands have the same meaning as in Fig. 20, as reminded in the figure caption. From our discussion of Fig. 20 we can clearly see that the observed cross section exceeds the expected background well beyond the two standard-deviation level, in a region where the expected background is determined with enough sensitivity to test the SM-Higgs boson hypothesis.
Figure 23: Distributions of the reconstructed invariant for the selected candidate events and for the total background and signal expected in the $H \rightarrow \gamma \gamma$ (left) and the $H \rightarrow ZZ^{(*)} \rightarrow 4l$ (right) channels, in the low mass region. From Ref. [39] and Ref. [4].

This amazing result brought us by the LHC experiments opens a new chapter in the study of EWSB. The newly discovered spin-0 particles will have to be studied in all its properties, in particular its couplings to gauge bosons and fermions to disentangle possible hints of new physics through subtle deviations from the SM-Higgs boson pattern. For instance, it will important to test deviations that may discriminate the supersymmetric nature of the discovered spin-0 particle. With this respect, all production and decay modes will have to be used in order to control the largest possible number of couplings and determine them from multiple sources. Cleaver strategies and accurate knowledge of both production cross sections and branching ratios will be crucial to a successful implementation of the Higgs-boson physics program in the future of the LHC experiments.

6 Theoretical predictions for SM Higgs production at hadron colliders

Given the elusive nature of a Higgs-like signal, a precise theoretical prediction of both signal and background total cross-sections and distributions is
Figure 24: ATLAS and CMS results for the 95% confidence upper limit on the signal strength as a function of $m_H$ (full mass range and low mass range only, respectively). The black solid curve indicates the observed limit and the black dashed curve illustrates the median expected limit in the absence of a signal together with the one standard deviation (green) and two standard deviation (yellow) bands. From Refs. [4, 5].
vital to the success of the LHC program. For this reason, all Higgs production channels have received a lot of attention in recent years and they are nowadays calculated including Next-to-Leading-Order (NLO) and sometimes Next-to-Next-to-Lading Order (NNLO) QCD corrections, as well as, in some cases, the first order of electroweak (EW) corrections. More recently, the attention has been shifting on controlling background processes at the same level of accuracy. Thanks to the enormous progress in higher-order perturbative calculations during the last few years, several processes involving multiple jets, multiple gauge bosons, as well as several massive fermions, are now predicted at NLO in QCD. For a thorough introduction to higher-order calculation in quantum field theory and their recent applications I refer to John Campbell’s lectures at this school [40]. In this context, another crucial progress of the last decade has been the development of a consistent interface between NLO parton level calculations and Parton Shower (PS) Monte Carlo generators, the tools commonly used in experimental analyses to model the evolution of high energy hadronic collisions. We nowadays have some main frameworks, namely MC@NLO [41], POWHEG [42, 43], and SHERPA [44], within which a parton level calculation can be consistently matched to the process of radiation emission implemented in PS Monte Carlo programs like PYTHIA [46] and HERWIG [45], including the first order of QCD corrections. This allows for more reliable comparisons with data both in terms of kinematic distributions and overall cross sections. In particular, the impact of QCD corrections in the presence of kinematic cuts and vetos on specific final state particles and/or decay products can be studied more accurately.

The main references for all SM Higgs-boson production channels are collected in Table 1 for the reader’s convenience. They correspond to the original parton-level NLO/NNLO calculations, while we refer to [6, 7] for further developments, including comparison between different calculations as well as results for the NLO interface with parton-shower Monte Carlo programs. These calculations and their developments have been the official reference for Higgs searches at the Tevatron and the LHC. In particular they have been at the core of an extended program of providing consistent state-of-the-art theoretical predictions for Higgs-boson production during the different phases of the LHC (with center-of-mass energies 7 TeV, 8 TeV, and 14 TeV respectively), summarized in the work of the LHC Higgs Cross Section Working Group (LHC-HXSWG) [6, 7]. In this context, common prescriptions to estimate the uncertainties of theoretical predictions deriving from input parameters, parton distribution functions, αs, and residual unknown pertur-


Table 1: Existing QCD corrections for various SM Higgs production processes.

| process | $\sigma_{\text{NLO,NNLO}}$ by |
|---------|-------------------------------|
| $gg \to H$ | S.Dawson, NPB 359 (1991), A.Djouadi, M.Spira, P.Zerwas, PLB 264 (1991)  <br> C.J.Glosser et al., JHEP 0212 (2002); V.Ravindran et al., NPB 634 (2002)  <br> D. de Florian et al., PRL 82 (1999)  <br> R.Harlander, W.Kilgore, PRL 88 (2002) (NNLO)  <br> C.Anastasiou, K.Melnikov, NPB 646 (2002) (NNLO)  <br> V.Ravindran et al., NPB 665 (2003) (NNLO)  <br> S.Catani et al. JHEP 0307 (2003) (NNLL),  <br> G.Bozzi et al., PLB 564 (2003), NPB 737 (2006) (NNLL)  <br> C.Anastasiou, R.Boughezal, F.Petriello, JHEP (2008) (QCD+EW) |
| $q\bar{q} \to (W, Z)H$ | T.Han, S.Willenbrock, PLB 273 (1991)  <br> M.L.Ciccolini, S.Dittmaier, and M.Krämer (2003) (EW)  <br> O.Brien, A.Djouadi, R.Harlander, PLB 579 (2004) (NNLO) |
| $q\bar{q} \to q\bar{q}H$ | T.Han, G.Valencia, S.Willenbrock, PRL 69 (1992)  <br> T.Figy, C.Oleari, D.Zeppenfeld, PRD 68 (2003)  <br> M.L.Ciccolini, A.Denner,S.Dittmaier (2008) (QCD+EW)  <br> P.Bolzoni, F.Maltoni, S.O.Moch, and M.Zaro (2010) (NNLO) |
| $q\bar{q}, gg \to t\bar{t}H$ | W.Beenakker et al., PRL 87 (2001), NPB 653 (2003)  <br> S.Dawson et al., PRL 87 (2001), PRD 65 (2002), PRD 67,68 (2003) |
| $q\bar{q}, gg \to b\bar{b}H$ | S.Dittmaier, M.Krämer, M.Spira, PRD 70 (2004)  <br> S.Dawson et al., PRD 69 (2004), PRL 94 (2005) |
| $gb(\bar{b}) \to b(\bar{b})H$ | J.Campbell et al., PRD 67 (2003) |
| $b\bar{b} \to H$ | D.A.Dicus et al. PRD 59 (1999); C.Balazs et al., PRD 60 (1999).  <br> R.Harlander, W.Kilgore, PRD 68 (2003) (NNLO) |
In the following I would like to discuss the relevance of including different layers of QCD corrections in the calculation of Higgs-boson production cross sections and illustrate it with a prototype example, i.e. the case of gluon-
gluon fusion.

6.1 $gg \to H$ at NNLO: a prototype example

The gluon-fusion process offers a true learning ground to understand the complexity of hadronic cross sections. We can learn about the need of improving the theoretical predictions beyond the LO and even the NLO, the importance of resumming sets of large corrections at all orders, the subtleties of interfacing the NNLO calculation with a PS Monte Carlo.

Most of the basic ideas that motivate the techniques used in the NNLO calculation of the cross section for the $gg \to H$ production process have been already introduced in Section 3.1.4, where we discussed the $H \to gg$ loop-induced decay. In particular we know that in the SM, the main contribution to $gg \to H$ comes from the top-quark loop (see Fig. 26) since:

$$
\sigma_{LO} = \frac{G_F \alpha_s(\mu)^2}{288\sqrt{2}\pi} \left| \sum_q A^H_q(\tau_q) \right|^2 ,
$$

(114)

where $\tau_q = 4m^2_q/M^2_H$ and $A^H_q(\tau_q) \leq 1$ with $A^H_q(\tau_q) \to 1$ for $\tau_q \to \infty$.

As we saw in Section 3.1.4, one can work in the infinite top-quark mass limit and reduce the one-loop $Hgg$ vertex to a tree level effective vertex, derived from an effective Lagrangian of the form:

$$
L_{eff} = \frac{H}{4v} C(\alpha_s) G^{a\mu\nu} G^a_{\mu\nu} ,
$$

(115)
Figure 28: The NLO cross section for $gg \rightarrow H$ as a function of $M_H$. The two curves represent the results of the exact calculation (solid) and of the infinite top-quark mass limit calculation (dashed), where the NLO cross section has been obtained as the product of the $K$-factor ($K = \sigma_{NLO}/\sigma_{LO}$) calculated in the $m_t \rightarrow \infty$ limit times the LO cross section. From Ref. [14].

where the coefficient $C(\alpha_s)$, including NLO and NNLO QCD corrections, can be written as:

$$C(\alpha_s) = \frac{1}{3} \frac{\alpha_s}{\pi} \left[ 1 + c_1 \frac{\alpha_s}{\pi} + c_2 \left( \frac{\alpha_s}{\pi} \right)^2 + \cdots \right]. \quad (116)$$

NLO and NNLO QCD corrections to $gg \rightarrow H$ can then be calculated as corrections to the effective $Hgg$ vertex, and the complexity of the calculation is reduced by one order of loops.

The NLO order of QCD corrections has actually been calculated both with and without taking the infinite top-quark mass limit. The comparison between the exact and approximate calculation shows an impressive agreement at the level of the total cross section, and, in particular, at the level of the $K$-factor, i.e. the ratio between NLO and LO total cross sections ($K = \sigma_{NLO}/\sigma_{LO}$), as illustrated in Fig. 28. It is indeed expected that methods like the infinite top quark mass limit may not reproduce the correct kinematic distributions of a given process at higher order in QCD, but are
very reliable at the level of the total cross section, in particular when the cross section receives large momentum independent contribution at the first order of QCD corrections. As for the $H \to gg$ decay process, the NLO corrections to $gg \to H$ are very large, changing the LO cross section by more than 50%. Since the $gg \to H$ is the leading Higgs-boson production mode at hadron colliders, it has been clear for quite a while that a NNLO calculation was needed in order to understand the behavior of the perturbatively calculated cross section, and if possible, in order to stabilize its theoretical prediction.

The NNLO corrections to the total cross section have been calculated using the infinite top-quark mass limit (see Table 11). The calculation of the NNLO QCD corrections involves then 2-loop diagrams like the ones shown in Fig. 29 instead of the original 3-loop diagrams (a quite formidable task!). Moreover, thanks to the $2 \to 1$ kinematic of the $gg \to H$ process, the cross section has in one case been calculated in the so called soft limit, i.e. as an expansion in the parameter $x = M_H^2 / \hat{s}$ about $x = 1$, where $\hat{s}$ is the partonic center of mass energy (see paper by Harlander and Kilgore in Table 11). The $n$-th term in the expansion of the partonic cross section $\hat{\sigma}_{ij}$,

$$\hat{\sigma}^{(n)}_{ij} = \sum_{n \geq 0} \left( \frac{\alpha_s}{\pi} \right)^n \hat{\sigma}_{ij},$$

(117)

can then be written in the soft limit $(x \to 1)$ as follows:

$$\hat{\sigma}^{(n)}_{ij} = a^{(n)} \delta(1-x) + \sum_{k=0}^{2n-1} b^{(n)}_k \left[ \ln^k(1-x) \right] + \sum_{l=0}^{\infty} \sum_{k=0}^{2n-1} c^{(n)}_{lk} (1-x)^l \ln^k(1-x)$$

purely soft terms

(118)

collinear+hard terms

where we have made explicit the origin of different terms in the expansion. The NNLO cross section is then obtained by calculating the coefficients $a^{(2)}$, $b^{(2)}_k$, and $c^{(2)}_{lk}$, for $l \geq 0$ and $k = 0, \ldots, 3$. In Fig. 30 we see the convergence
Figure 30: $K$-factor for $gg \rightarrow H$ at the LHC ($\sqrt{s} = 14$ TeV), calculated adding progressively more terms in the expansion of Eq. (118). From Harlander and Kilgore as given in Table 1.

Figure 31: Cross section for $gg \rightarrow H$ at the LHC ($\sqrt{s} = 14$ TeV), calculated at LO, NLO and NNLO of QCD corrections, as a function of $M_H$, for $\mu_F = \mu_R = M_H/4$. From Harlander and Kilgore in Table 1.
behavior of the expansion in Eq. (118). Just adding the first few terms provides a remarkably stable $K$-factor. The results shown in Fig. 30 have been indeed confirmed by a full calculation \cite{47}, where no soft approximation has been used.

The results of the NNLO calculation \cite{48, 47} are illustrated in Figs. 31 and 32. In Fig. 31 we can observe the convergence of the perturbative calculation of $\sigma(gg \to H)$, since the difference between NLO and NNLO is much smaller than the original difference between LO and NLO. This is further confirmed in Fig. 32, where we see that the uncertainty band of the NNLO cross section overlaps with the corresponding NLO band. Therefore the NNLO term in the perturbative expansion only modify the NLO cross section within its NLO theoretical uncertainty. This is precisely what one would expect from a good convergence behavior. Moreover, the narrower NNLO bands in Fig. 32 shows that the NNLO result is pretty stable with respect to the variation of both renormalization and factorization scales. This has actually been checked thoroughly in the original papers, by varying both $\mu_R$ and $\mu_F$ independently over a range broader than the one used in Fig. 32. The NNLO calculation has been more recently implemented into the HNNLO code \cite{50} and subsequently extended by including the $H \to \gamma\gamma$, $H \to WW/ZZ \to 4l$, with the possibility to apply arbitrary cuts on the momenta of the partons and of the photons or
leptons that are produced in the final state. Analogous results are available through the FEHiP code [51], used to produce the plots presented in Fig. 34, where we can see how the impact of QCD corrections can vary drastically with different choices of exclusive cuts [52].

The NNLO cross section for $gg \rightarrow H$ has been further improved by Catani et al. [49] by resumming up to the next-to-next-to leading order of soft logarithms. Using the techniques explained in their papers, they have been able to obtain the theoretical results shown in Figs. 33 and 35 for the total and differential cross sections respectively. In particular, we see from Fig. 33 that the NNLO and NNLL results nicely overlap within their uncertainty bands, obtained from the residual renormalization and factorization scale dependence. The residual theoretical uncertainty of the NNLO+NNLL results has been estimated to be 10% from perturbative origin plus 10% from the use of NLO PDF’s instead of NNLO PDF’s. Moreover, in Fig. 35 we see how the resummation of NNL crucially modify the shape of the Higgs-boson transverse momentum distribution at low transverse momentum ($q_T$), where the soft $\ln(M_H^2/q_T^2)$ are large and change the behavior of the perturbative expansion in $\alpha_s$ [53].

Finally, the NLO calculation of $gg \rightarrow H$ has been interfaced with PS Monte Carlo programs (HERWIG and PYTHIA) using both the MC@NLO and POWHEG methods [54]. The NNLO calculation cannot be consistently interfaced with a PS Monte Carlo yet, but the comparison of the implementation of the NLO calculation into POWHEG and MC@NLO with Next-to-Leading-Logarithms (NLL) resummed results and the knowledge of the
Figure 34: Dependence of the LO, NLO, and NNLO cross section from vetoing events with jets in the central region $|\eta| < 2.5$ and $p_T^{\text{jet}} > p_T^{\text{veto}}$. The right plot shows the $K$-factor as a function of $p_T^{\text{veto}}$. The dashed horizontal lines correspond to the NLO and NNLO $K$-factors for the inclusive cross-section. From Ref. [52].

resummed cross section at the Next-to-Next-to-Leading-Logarithm (NNLL) level have allowed to provide an improved interface obtained by rescaling the results of the NLO interface by NNL/NNLL. A sample of results from Ref. [54] is given in Fig. 36.

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Figure 35: The $q_T$ spectrum at the LHC with $M_H = 125$ GeV. The upper plots show: (left) setting $\mu_R = \mu_F = Q = M_H$, the results at NNL+LO accuracy compared with the LO spectrum and the finite component of the LO spectrum; (right) the uncertainty band from the variation of the scales $\mu_R$ and $\mu_F$ at NLL+LO accuracy. The lower plots show the same at NNLL+NLO accuracy. From Ref. [53].
Figure 36: The Higgs-boson $p_T$ spectrum at the LHC with $M_H = 120$ GeV as obtain in the POWHEG and MC@NLO frameworks (left) and compared to the NNL and NNLL results. From Ref. [54].

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