Model Predictive Control Design of a 3-DOF Robot Arm Based on Recognition of Spatial Coordinates

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Abstract—This paper uses Model Predictive Control (MPC) to optimise the input torques of a Three-Degrees-of-Freedom (DOF) robotic arm, enabling it to operate to the target position and grasp the object accurately. A monocular camera is firstly used to recognise the colour and depth of the object. Then, the inverse kinematics calculation and the spatial coordinates of the object through coordinate transformation are combined to get the required rotating angle of each servo. Finally, the dynamic model of the robotic arm structure is derived and the model predictive control is applied to simulate the optimal input torques of servos to minimise the cost function.

Keywords—colour and depth recognition, inverse kinematics, dynamic model, model predictive control, robotic arm.

I. INTRODUCTION

Robotic arms can be widely used in various areas, such as industry, aerospace and medical care. During the last few decades, the control and design of robotic arms have become major fields of interest within the field of robotics. The technologies of robotic arms consist of computer vision, trajectory planning, mechanical design, electronic control, and control theories.

In the 21st century, researchers began to focus on algorithms of robotic arms, especially the optimisation of the robotic system. In 2004, Babazadeh and Sadati [1] proposed an optimisation algorithm for controlling the multiple-arm robotic system using the gradient method, which is suitable for complicated multi-arm systems. As a consequence of the research and development of robotic arms, classical control theory has been developed and applied to robotic arms. In 2015, Akyürek [2] applied PID control and force feedback to accomplish ambidextrous hand grasping, improving the accuracy and decreasing the iterations. Nowadays, Model Predictive Control (MPC) has been developed to be applied in robotics. As one of the most advanced control theories, MPC can be used to optimise the performance of robotic arms. In [3], the MPC approach was used to control a three-degrees-of-freedom (DOF) manipulator robot under the consideration of a second-order closed loop system. [4] applied MPC algorithm and non-industrial robotic arms to develop the high-precision trajectory tracking. In [5] and [6], MPC was used to demonstrate and test the simulation of a 5 DOF robot arm and an autonomous mobile grasping system.

The initial purpose of this project is to use the robotic arm to grasp the object based on the recognised coordinates and calculated joint angle. Fig. 1 shows the robotic arm designed for this project. Firstly, a monocular camera is used for recognising the spatial coordinates of colour-recognised objects. Then, the spatial coordinates are transformed into joint angle output by inverse kinematics calculation. However, in the real-time implementation, due to the disturbance of external forces and the influence of gravity and other factors, the manipulator cannot accurately run to the target position to grasp. In order to improve the stability of the manipulator and make the servo mechanism accurately work to the specified position, the dynamic model of the manipulator is established in this paper. The optimal input torque of the servo rotation is obtained by model predictive control, which makes the grasping process more accurate and stable. Also, the minimisation of the cost function reduces energy consumption.

The article is organised as follows: Section II introduces the principle of colour and depth recognition using a monocular camera. In Section III, the inverse kinematics model of a 3-DOF robot is presented. Then, combining the above technologies, the scene of automatic object recognition and grasping by a fixed camera is demonstrated. To obtain the state-space model, Section IV provides the dynamic model. In Section V, MPC is applied to minimise the cost function by using the obtained dynamic model. Finally, the results of the MPC simulation along with the operating process are presented in Section VI.

Fig. 1. The overall view of the designing robotic arm.
II. COMPUTER VISION

A. Principle of Colour Recognition

Fig. 2 presents the procedure of colour recognition. First, turn on the camera and blur the image initially to facilitate subsequent processing. Colours are then identified and extracted based on HSV values. Then thresholding and dilation are performed to make the image clear. Since the lines are extracted and located, erosion may cause more spaced breakpoints to cut the lines. Therefore, only dilation operation is performed without erosion. Finally, the contour frame is made according to the colour.

B. Principle of Depth Recognition

Fig. 3 shows the principle of depth recognition by a monocular camera. In order to recognise the depth of the object based on colour recognition, the actual width of the object, measured distance and measured pixel width should be known before recognition to calculate the camera focal length first. Camera focal length can be written as:

\[ F = \frac{PD}{W} \]  

(1)

where \( D \) is the measured distance away from the camera, \( W \) is the actual width of the object and \( P \) is the measured pixel width.

As the camera continues to move closer or farther away from the target, the real distance of the object from the camera can be calculated by using the similarity transformation:

\[ D' = \frac{WF}{P} \]  

(2)

III. INVERSE KINEMATICS

A. Denavit-Hartenberg Model

The purpose of conducting inverse kinematics calculation is to transform the spatial coordinates input to the joint angle output, so that the robotic arm could move to the given spatial coordinate to grasp the objects. The procedure can be described as follows. Firstly, the Denavit-Hartenberg (DH) model is used to establish the structure of the robotic arm. The structure of the robotic arm refers to [7], as shown in Fig. 4. The joints are labelled as 1, 2, 3, 4. The servos at joints 1, 2 and 3 are responsible for moving while the servo at joint 4 is used for grasping. Table 1 shows the parameters of each joint, where \( a_i \) and \( \alpha_i \) represent the distance and angle of rotation from \( \hat{z}_i \) to \( \hat{z}_{i+1} \) along the \( \hat{x}_i \)-axis, and \( d_i \) and \( \theta_i \) (\( q_i \)) represent the distance and angle of rotation from \( \hat{x}_{i-1} \) to \( \hat{x}_i \) along the \( \hat{z}_i \)-axis, respectively.

As [8] proposed, the transformation matrix of coordinate system \([i]|i-1\) relative to \([i-1]\) is

\[
[i-1]^T = \begin{bmatrix}
c_\theta_i & -s_\theta_i & 0 & a_{i-1} \\
s_\theta_i c_\alpha_{i-1} & c_\theta_i c_\alpha_{i-1} & -s_\alpha_{i-1} & -d_i s_\alpha_{i-1} \\
s_\theta_i s_\alpha_{i-1} & c_\theta_i s_\alpha_{i-1} & c_\alpha_{i-1} & d_i c_\alpha_{i-1} \\
0 & 0 & 0 & 1
\end{bmatrix}
\]

(3)

TABLE I. THE PARAMETERS OF EACH JOINTS

| Joints Number | Parameters | \( a_i \) | \( \alpha_i \) | \( d_i \) | \( \theta_i \) (\( q_i \)) |
|---------------|------------|-----------|---------------|-----------|------------------|
| 1             | \( a_1 \)  | 0         | \( \pi/2 \)    | 0         | \( \theta_1 \) (\( q_1 \)) |
| 2             | \( a_2 \)  | 0         | 0             | 0         | \( \theta_2 \) (\( q_2 \)) |
| 3             | \( a_3 \)  | 0         | 0             | 0         | \( \theta_3 \) (\( q_3 \)) |
| 4             | 0          | 0         | 0             | 0         |                  |

In the following section, \( \cos(\theta_i) \) and \( \sin(\theta_i) \) are represented as \( c\theta_i \) and \( s\theta_i \) (or \( c_i \) and \( s_i \)) for simplification. Similarly, \( c_{ij} \) and
The elements of \( i \) hold the physical meaning: \( r_{ij} \) (i, j = 1, 2, 3) represent the rotation matrix, and \( p_x \), \( p_y \) and \( p_z \) represent the spatial coordinates. The purpose of this calculation is to use \( p_x \), \( p_y \) and \( p_z \) to represent \( \theta_1 \), \( \theta_2 \) and \( \theta_3 \). The rotation matrix \( r_{ij} \) (i, j = 1, 2, 3) is:

\[
\begin{bmatrix}
0 & -s_{01} & c_{01} \\
0 & -s_{02} & c_{02} \\
0 & -s_{03} & c_{03}
\end{bmatrix}
\]

The spatial coordinates are:

\[
p_x = a_3(c_1c_2c_3 - c_1s_2s_3) + c_1c_2a_2 \tag{7a}
\]

\[
p_y = a_3(c_2s_1c_3 - s_1s_2s_3) + c_2s_1a_2 \tag{7b}
\]

\[
p_z = a_3s_2s_3 + s_2a_2 \tag{7c}
\]

By summing squares of (7a), (7b) and (7c):

\[
p_x^2 + p_y^2 + p_z^2 = a_x^2 + a_y^2 + 2a_2a_3c_3 \tag{8}
\]

\[
\Rightarrow c_3 = \frac{p_x^2 + p_y^2 + p_z^2 - a_x^2 - a_y^2}{2a_2a_3}, \quad s_3 = \sqrt{1 - c_3^2} \tag{9}
\]

Therefore,

\[
\theta_3 = \arctan \left( \frac{\sqrt{1 - c_3^2}}{c_3} \right) \tag{10}
\]

Since different signs of \( \theta \) lead to different solutions, one combination of solutions is chosen here. According to the projection of the end effector of the robot arm on the X-Y plane, the \( \theta_1 \) could be calculated,

\[
\theta_1 = \arctan \left( \frac{p_x}{p_y} \right) \tag{11}
\]

\( \theta_2 \) can be calculated via the sum of the squares of \( p_x \) and \( p_y \):

\[
p_x^2 + p_y^2 = (c_1a_2 + a_3c_2c_3)^2 \tag{12}
\]

\[
\Rightarrow c_2 = \frac{\sqrt{p_x^2 + p_y^2 - a_x^2 - a_y^2}}{a_2 + a_3c_3} \tag{13}
\]

Then substitute \( s_2 = (p_z - a_3s_2s_3)/(a_2 + a_3c_3) \) obtained from (7c) into (2):

\[
c_2 = \frac{\sqrt{p_x^2 + p_y^2} (a_2 + a_3c_3) + p_xa_3s_2s_3}{a_2 + a_3^2 + 2a_2a_3c_3} \tag{14}
\]

\( \theta_2 \) could then be calculated:

\[
\theta_2 = \arctan \left( \frac{\sqrt{p_x^2 + p_y^2} (a_3s_2s_3) + p_x(a_2 + a_3c_3)}{\sqrt{p_x^2 + p_y^2} (a_2 + a_3c_3) + p_xa_3s_2s_3} \right) \tag{15}
\]

Therefore, the set of solutions chosen here is shown in (11), (15) and (10). Parameters \( c_2 \) and \( s_2 \) can be referred to (9). Part of the derivation process refers to [7].

B. Combination of Computer Vision and Inverse Kinematics

Fig. 5 shows the procedure of automatically identifying objects and grasping them with the fixed camera. Firstly, the spatial coordinates of objects are obtained by colour and depth recognition, and then the camera coordinate system is transformed into the world coordinate system by matrix transformation. Through inverse kinematics calculation, the robotic arm can operate the specific angle to move to the position of the object to grasp. Also, considering the robustness, it is supposed to determine whether the input has a solution and is within the operating range of servos. Fig. 6 presents the location plan of the camera and robotic arm.
The height of the camera is 8 cm, which means the fixed coordinate of the camera is (10, 12.5, 8) cm. The position offset of the camera relative to the robotic arm will be used in the matrix transformation.

In the upper left corner of Fig. 7, a GUI interface that includes six quantities presents the HSV values, which can be adjusted to identify different colours. In the test shown in Fig. 7, green was chosen as the colour to be identified. The code for colour recognition refers to Heywood [9].

The blue frame in Fig. 7 represents the result of colour identification and the red frame represents the depth identification. It can be seen that the red and blue frames nearly overlap. This is because the recognition of depth is based on the result of colour identification. The coordinates in Fig. 7 represent the x-axis, y-axis coordinates and depth respectively. Fig. 8 shows the scene of grasping the target. Within the range of inverse kinematics solutions, the robotic arm can accurately grasp objects.

To obtain recognition accuracy, we use manual measurement to measure the spatial coordinates of the objects located in the center of the camera's field of view and at the upper, lower, left, and right margins as shown in Figure 6. Then compare the measured coordinates with the coordinates recognized by the camera. The observation results show that the spatial coordinate error of the object closer to the center is smaller (the error of the x, y-coordinate, and the depth is approximately ± 1cm), compared to the error of the x and y-coordinate of the object close to the margin, which is approximately ± 2cm.

Some conditions should be guaranteed to ensure the recognition accuracy of the proposed system. Firstly, the length and width of the object should be known in advance to do the similarity transformation, and the orientation of the object should be the same as when measuring the length and width. Secondly, the object needs to be put within the camera's field of view (120 degrees). The variation of lighting intensity should also be within a certain range to ensure the accuracy of colour recognition.

IV. DYNAMIC MODEL

A. State-Space Model

According to [8], the Lagrangian function of the manipulator can be written as:

\[ L = T(q, \dot{q}) - V(q) \] (16)

where \( T \) is the kinetic energy and \( V \) is the potential energy. Consider the Lagrange equation:

\[ \frac{d}{dt} \left( \frac{\partial L}{\partial \dot{q}} \right) - \frac{\partial L}{\partial q} = \tau \] (17)

The standard manipulator dynamic equation is derived as:

\[ M(q, \dot{q}) \ddot{q} + N(q, \dot{q}) \dot{q} + g(q) = \tau \] (18)

where \( M(q, \dot{q}) \) is the inertia matrix, \( N(q, \dot{q}) \) is the Centrifugal and Coriolis matrix and \( g(q) \) is the gravitational vector:

\[
M(q, \dot{q}) = \begin{bmatrix}
d_{11} & d_{12} & d_{13} \\
d_{21} & d_{22} & d_{23} \\
d_{31} & d_{32} & d_{33}
\end{bmatrix}, \\
N(q, \dot{q}) = \begin{bmatrix} c_1 \\
c_2 \\
c_3 \end{bmatrix}^T
\]

(19)

where \( g_0 = 9.8 \text{ m/s}^2 \) and \((x_d, y_d, z_d)\) represents the centroid coordinates of link \( i \). Due to the space limitations, \( M(q, \dot{q}) \) and \( N(q, \dot{q}) \) cannot be shown here. They are related to the inertia tensor of three links:

\[
I = \begin{bmatrix} I_{xx} & I_{xy} & I_{xz} \\
I_{yx} & I_{yy} & I_{yz} \\
I_{zx} & I_{zy} & I_{zz} \end{bmatrix}
\]

(20)

Rewrite the (19) into the following form:

\[
\ddot{q} = -M^{-1}N\dot{q} - M^{-1}g + M^{-1}\tau
\]

(21)

According to [5], the controller can be chosen as follows to compose the linear-like state-space model:

\[
u = -M^{-1}(\tau - g)
\]

(22)

Then, the state-space model can be derived as:

\[
\begin{bmatrix} \ddot{q} \\ \dot{q} \end{bmatrix} = \begin{bmatrix} 0_{3x3} & I_{3x3} \\ 0_{3x3} & -M^{-1}N \end{bmatrix} \begin{bmatrix} \dot{q} \\ u \end{bmatrix}
\]

(23)

where

\[
\begin{bmatrix} u \\ \dot{q} \end{bmatrix} = \begin{bmatrix} q_1 \\ q_2 \\ q_3 \\ \dot{q}_1 \\ \dot{q}_2 \\ \dot{q}_3 \end{bmatrix}^T
\]

(24)
During the operation, the desired joint angle $q_d$ is set as a constant. The error between desired joint angle and real joint angle is set as:

$$e = q_d - q$$  \hspace{1cm} (25)$$

Then,

$$\dot{e} = \dot{q}_d - \dot{q} = -\ddot{q}$$  \hspace{1cm} (26)$$

The state-space model can be rewritten as

$$\begin{bmatrix} \dot{e} \\ \dot{q} \end{bmatrix} = \begin{bmatrix} 0 & -1 \\ 0 & -M^{-1}N \end{bmatrix} \begin{bmatrix} e \\ q \end{bmatrix} + \begin{bmatrix} 0 \\ I \end{bmatrix} u$$  \hspace{1cm} (27)$$

Also, $u = -M^{-1}(\tau - g)$

Follows the form of:

$$\dot{x} = Ax + Bu$$  \hspace{1cm} (28)$$

Considering the fixed point:

$$\begin{cases} \dot{e} = 0 \Rightarrow \dot{q}_d = 0 \\ \dot{q} = 0 \Rightarrow u = M^{-1}N \Rightarrow \tau = g \end{cases}$$  \hspace{1cm} (29)$$

It indicates that when the joint angular velocity is zero and the input torque is the gravity of the joint, the system will be stable, which corresponds to the actual situation.

**B. Discretisation**

The Forward Euler method is applied to do the discretisation of the state-space equation. The derivative of $x$ can be written as:

$$\dot{x} = \frac{x(k + 1) - x(k)}{T}$$  \hspace{1cm} (30)$$

where $T$ is the sampling time. Substituting (28):

$$\frac{x(k + 1) - x(k)}{T} = Ax(k) + Bu(k)$$  \hspace{1cm} (31)$$

Therefore,

$$x(k + 1) = (TA + I)x(k) + TBU(k)$$  \hspace{1cm} (32)$$

The discrete time state-space can be written as follow:

$$x(k + 1) = A_dx(k) + B_du(k)$$  \hspace{1cm} (33)$$

where $A_d = TA + I, B_d = TB$.

**V. MODEL PREDICTIVE CONTROL**

Model predictive control obtains optimal results by predicting how the system will perform over a certain future period. Set N as the predictive horizon. To minimise a quadratic cost function at time $t = k$, it predicts the state value at time $t = k + 1, t = k + 2, ..., t = k + N$ but only takes the obtained first state. By receding horizon control, optimised results are obtained at each sampling time instant. Set output $y = x$, when at time $t = k$:

$$X_k^T = [x_k, x_{k+1}, x_{k+2}, ..., x_{k+N}]^T$$  \hspace{1cm} (34a)$$

$$U_k^T = [u_k, u_{k+1}, u_{k+2}, ..., u_{k+N-1}]^T$$  \hspace{1cm} (34b)$$

where $X_k$ and $U_k$ represent the sequences of input and predicted state value respectively. The discrete time state-space can be written as:

$$x_k = x_k$$

$$x_{k+1} = A_dx_k + Bdu_k$$

$$x_{k+2} = A_d^2x_k + A_db_u + B_du_{k+1}$$

$$\vdots$$

$$x_{k+N} = A_d^Nx_k + A_d^{N-1}Bu_k + ... + B_du_{k+N-1}$$  \hspace{1cm} (35)$$

This series of equations can be rewritten in the matrix form:

$$X_k = CX_k + FU_k$$  \hspace{1cm} (36)$$

$$C = \begin{bmatrix} I \\ A_d \\ \vdots \\ A_d^N \end{bmatrix}, F = \begin{bmatrix} 0 & 0 & ... & 0 \\ B_d & 0 & ... & 0 \\ \vdots & \vdots & \ddots & \vdots \\ A_d^{N-1}B & A_d^{N-2}B & \ldots & B_d \end{bmatrix}$$

As [10] proposed, the optimisation problem is to minimise the cost function:

$$J = X_k^{N+1}TPX_k^{N+1} + \sum_{s=0}^{N-1} X_s^{s+1}Q X_s^{s+2} + U_s^{s+1}R U_s^{s+1}$$  \hspace{1cm} (37)$$

According to [10], by substituting (36) into (37), the cost function can be rewritten in the quadratic form, which is also the function to be minimised:

$$J = \frac{1}{2}X_k^TYX_k + \min \left(X_k^TM U_k + \frac{1}{2}U_k^TH U_k \right)$$  \hspace{1cm} (38)$$

where $Y$, $M$ and $U$ are obtained from $Q$, $R$, $C$ and $F$.

**VI. RESULTS OF SIMULATION**

**A. Results without External Forces**

This section will combine the previously mentioned techniques to demonstrate how model predictive control can be used to reduce operating errors.

Firstly, as shown in Fig. 7, when the camera sees an object, it first identifies the object according to its colour, and then calculates the depth and three-dimensional coordinates of the object in the camera according to the comparison between the size of the object seen and the actual size. Then, the coordinate transformation and inverse kinematics are applied to get the angle that each joint needs to rotate to reach the target. For example, the recognised coordinates are (10, 15, 10) cm, and the desired joint angle is (28.1, 67.8, 53.8) degrees.

After model predictive control, the input torque required by the manipulator to reach this coordinate will be obtained. Due to the action of this torque, the error between the space coordinates of the end effector and the coordinates of the object is reduced with MPC. Finally, the robotic arm is able to grasp the target more accurately. Also, the tracking effect and energy consumption of the controller can be changed by adjusting the matrices $Q$, $R$ and $P$, which are more in line with the actual needs.
Set the sampling period as 0.05 seconds. The initial angle set of the three joints is \((0,0,0)\). The desired angle of three joints is \((\frac{28.1\pi}{180}, \frac{67.8\pi}{180}, \frac{53.8\pi}{180})\). According to the design of the state-space model (27), Fig. 9 represents the error between desired joint angle and real joint angle. The results of errors end up at zero indicating that each joint moves to the target position. Fig. 9, 10, 11 and 12 have been obtained by setting proper values of matrices \(Q\), \(R\) and \(P\) to minimise the cost function. From Fig. 9, 10 and 12 we can see that, initially, the input torque of joint 1 has provided a large angular velocity, enabling it to rotate to the target position first. Then, as joint 2 and joint 3 move closer to the target, their torques have increased and finally tend to a constant to support their gravity. The robotic arm tends to be stable in 6 seconds. For this system, the input quantities are the torques obtained in Figure 12. According to the designed controller (22), the torque \(\tau\) is not directly optimized, but calculated from the intermediate quantity \(u\). In Figure 12, the highest torque is close to \(0.06 \text{ kg} \cdot \text{m}\), which is within the torque range of servo (0.2 kg \cdot m).

Fig. 13 and Fig. 14 show the initial and final position of the robotic arm, which is run by the co-simulation of software MATLAB and SolidWorks.

![Fig. 9. Error between desired joint angle and real joint angle.](image9)

![Fig. 10. Angular Velocities of each joint.](image10)

![Fig. 11. Controllers of each joint.](image11)

This gesture of the robotic arm is selected because this is the coordinate where the torque demand of the servo is large. The simulated results illustrate that the torque of servos is capable of supporting the robotic arm and doing the grasping.

### B. Effects of External Forces

In this section, the external forces are considered to test the robustness of MPC in this case. The initial angle set of the three joints is \((0,0,0)\). The desired angle of three joints is \((\frac{28.1\pi}{180}, \frac{67.8\pi}{180}, \frac{53.8\pi}{180})\). As shown in figure 15, from sample 30 to...
35, the torques of joint 2 and 3 are set as the constant 0.07 kg \cdot m to simulate the external forces. In Figure 16, the errors of joint 2 and 3 become larger due to the external force during sample 30 to 35. However, when the external forces are withdrawn, the error decreases again under the action of MPC, and the servos move to the target position and tend to be stable.

In addition, we attempted to use the torque \( \tau \) directly as the controller, to establish the nonlinear state space equation in (39) and linearize it with the Jacobian matrix.

\[
f(x, u) = M^{-1}(-N\dot{q} - g + u) \tag{39}
\]

However, although this allows MPC to directly linearize the input torque, it significantly increases the computational complexity and error, making it difficult to obtain ideal results, especially in hardware operation, which requires high computing power of the microprocessor. The controller can be improved to get a better-optimized input torque to simplify the calculation.

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Fig. 16. Error between desired joint angle and real joint angle with external forces.

VII. CONCLUSION

This project used MPC to control a 3-DOF robot arm by using coordinate transformation and inverse kinematics calculation. The optimal input torques of servos were simulated, making the robotic arm reach the target point accurately and grasp the object.

Compared to other control algorithms such as PID algorithm, MPC has the characteristics of robustness and MIMO (Multiple Input Multiple Output), which can optimize multiple variables simultaneously.

Monocular cameras were used to identify the colour and depth of the object, thus obtaining the spatial coordinates of the object through coordinate transformation. Its major advantage is the lower cost. By adjusting the HSV values, objects of different colours can be conveniently identified for recognition. This project can be applied to industrial scenarios such as sorting different packages in production.

However, compared with binocular cameras, monocular cameras have limitations in accuracy, cannot be used to estimate the depth of objects and should be used under certain conditions to guarantee the recognition results. The coordinate recognition can be improved in the future by using binocular and depth cameras for coordinate recognition.

The current-controlled motor can be used to control the current to control the input torque. By measuring the relationship between the current and torque of the motor, the motor current can be used to get the required torque.