Total Energy of Cycle and Some Cycle Related Graphs

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Abstract: In this article we write algorithms and MATLAB programs to find the total energy of Cycle and some Cycle related graphs. The concept of total matrix and total energy of a graph G is introduced by K.Palani&M.Lalithakumari in [9]. Let G=(V,E) be a (p,q) simple graph. Let (G) = {v_i/i = 1,2, ... p} and E(G) = {e_i/i = 1,2, ... q}.The total matrix T = (G) of G is a square matrix of order p+q whose (i,j)-entry is defined as:

\[
T = (t_{ij}) = \begin{cases} 
1 & \text{if } v_i \text{ adjacent to } v_j \text{ i ≠ j} \\
1 & \text{if } e_i \text{ adjacent to } e_j \text{ i ≠ j} \\
1 & \text{e_i incident with } v_j \\
0 & \text{otherwise}
\end{cases}
\]

The Total Energy of a graph is the sum of absolute value of the eigen values of its Total matrix (G). For any (p,q) graph G, the total number of eigen value is p+q.

Let \(\lambda_1, \lambda_2, \lambda_3, ..., \lambda_{p+q}\) be the eigen values of T. Then total energy of G is \(TE = \sum_{i=1}^{p+q} |\lambda_i|\).

Key words: Total matrix, Total Energy, Cycle, Dumbbell, Pan, \(<C_1C_{n-1}>\).

AMS Subject Classification: 05C50

I. Introduction

Throughout this article we deal with finite, simple and undirected graphs. The concept of energy of a graph was proposed by Gutman[7] in 1978 as the sum of absolute values of the eigen value of a graph G and is denoted by \(E(G)\). The eigen values of the total matrix T is known as the total eigen values of G. We find the total energy for Path, Star, \(Y_{n+1}\) & Bull Graph.

1.2 Definition

Let \(\lambda_1, \lambda_2, \lambda_3, ..., \lambda_{p+q}\) be the total eigen values of T. Then the spectrum of \(G\) is \(Specr(G) = \{\lambda_1^m,\lambda_2^m,\lambda_3^m, ..., \lambda_{p+q}^m\}\) where \(m\) is the algebraic multiplicity of the total eigenvalues \(\lambda_i\), for \(1 ≤ i ≤ p + q\).
1.3 Definition
The total graph $T(G)$ of a graph $G$ is a graph such that (i) the vertex set of $T(G)$ corresponds to the vertices and edges of $G$ and (ii) two vertices are adjacent in $T(G)$ if and only if their corresponding elements are either adjacent or incident in $G$.

1.4 Definition
Let $G = (V, E)$ be a $(p,q)$ graph. The energy of total matrix of $G$ is called the total energy. It is denoted as $TE(G)$.

That is the total energy of $G = \text{Energy of Total matrix of } G$

$$= \sum_{i=1}^{p+q} |\lambda_i|$$

II. Total Energy of cycle related graphs

2.1 Algorithm to generate the total energy of cycle graph $C_n$

Step I: Assume $G=(V,E)$ to be a $(p,q)$ graph.

Step II: Assume that in the total matrix representation, vertices and edges appear alternatively along both rows and columns.

Step III: Let $T_G = T_e + T_v$.

Step IV: For $i=1$ to $r-1$ assign $t_{i,i+1} = t_{i+1,i} = 1$.

Step V: For $i=1$ to $r-2$ assign $t_{i,i+2} = t_{i+2,i} = 1$.

Step VI: For $i=1 & 2$ assign $t_{i,r} = t_{r,i} = 1$.

Step VII: Assign $t_{1r-1} = t_{r-11} = 1$.

Step VIII: Assume the other entries as zero.

Step IX: Find eigen values of $T$.

Step X: Find Total Energy $TE$.

2.2 MATLAB program to generate the total energy of cycle graph $C_n$

```matlab
% "T" is the Total matrix of a graph
% "K" is the eigen values of the matrix
% "TE" is the Total Energy of the graph
% r=p+q
% p,q refers the number of vertices and edges of $C_n$
for i=1:r-1
    T(i,i+1)=1;
    T(i+1,i)=1;
end
for i=1:r-2
    T(i,i+2)=1;
```
\begin{verbatim}
T(i+2,i)=1;
end
for i=1:2
T(i,r)=1;
T(r,i)=1;
end
T(1,r-1)=1;
T(r-1,1)=1;
T
K=eig(T);
TE=sum(abs(K))

2.3 Illustration

When the above program is executed for \( G_6 \), the output will be \( TE=18.93 \)

2.4 Algorithm to generate the total energy of Dumbbell graph \( D_n \).

**Step I:**
Assume \( G=(V,E) \) to be a \((p,q)\) graph.

**Step II:**
Assume that in the total matrix representation, vertices and edges appear alternatively along both rows and columns.

**Step III:**
Let \( r = p + q \).

**Step IV:**
For \( i=1 \) to \( p-1 \) assign \( t_{i,i+1} = t_{i+1,i} = 1 \).

**Step V:**
For \( i=p+2 \) to \( r-1 \) assign \( t_{i,i+1} = t_{i+1,i} = 1 \).

**Step VI:**
For \( i=1 \) to \( p-2 \) assign \( t_{i,i+2} = t_{i+2,i} = 1 \).

**Step VII:**
For \( i=p+2 \) to \( r-2 \) assign \( t_{i,i+2} = t_{i+2,i} = 1 \).

**Step VIII:**
For \( i=1 \) to \( 2 \) assign \( t_{ip} = t_{pi} = 1 \).

**Step IX:**
For \( i=q+1 \) to \( q+2 \) assign \( t_{iq} = t_{qi} = 1 \).

**Step X:**
For \( i=p+2 \) to \( p \) assign \( t_{iq} = t_{qi} = 1 \).

**Step XI:**
Define \( t_{ip}\) = \( t_{ip-1} = t_{q+1} r-1 = t_{r-1-q} +1 = 1, t_{q+1} q+1 = t_{q+1} q+1 = 1, t_{q+2} q+2 = t_{q+2} q+2 = 1, t_{q+1} q+1 = 1 \).

**Step XII:**
Assign \( t_{1p-1} = t_{r-1} 1 = 1 \).

**Step XIII:**
Assume the other entries as zero.

**Step XIV:**
Find eigen values of \( T \).

**Step XV:**
Find Total Energy \( TE \).
2.5 MATLAB program to generate the total energy of Dumbbell graph $D_n$

% "T" is the Total matrix of a graph
% "K" is the eigen values of the matrix
% "TE" is the Total Energy of the graph
% $r=p+q$
% $p,q$ refers the number of vertices and edges of $D_n$
for i=1:p-1
    T(i,i+1)=1;
    T(i+1,i)=1;
end
for i=p+2:r-1
    T(i,i+1)=1;
    T(i+1,i)=1;
end
for i=1:p-2
    T(i,i+2)=1;
    T(i+2,i)=1;
end
for i=p+2:r-2
    T(i,i+2)=1;
    T(i+2,i)=1;
end
for i=1:2
    T(i,p)=1;
    T(p,i)=1;
end
for i=q+1:q+2
    T(i,r)=1;
    T(r,i)=1;
end
for i=p-2:p
    T(i,q)=1;
    T(q,i)=1;
end
T(1,p-1)=1;
T(p-1,1)=1;
T(q+1,r-1)=1;
T(r-1,q+1)=1;
T(q+1,q)=1;
T(q,q+1)=1;
T(q+2,q)=1;
T(q,r)=1;
T(r,q)=1;
K=eig(T);
TE=sum(abs(K))

2.6 Illustration
When the above program is executed for $D_5$, the output will be $TE=36.7617$
2.7 Algorithm to generate the total energy of Pan graph $P_{a_n}$.

Step I:
Assume $G=(V, E)$ to be a $(p,q)$ graph.

Step II:
Assume that in the total matrix representation, vertices and edges appear alternatively along both rows and columns.

Step III:
Let $r = p + q$.

Step IV:
For $i=1$ to $r-1$ assign $t_{i+1} = t_{i+1i} = 1$.

Step V:
For $i=1$ to $r-3$ assign $t_{i+2} = t_{i+2i} = 1$.

Step VI:
For $i=1$ and $2$ assign $t_{i-2} = t_{i-2i} = 1$.

Step VII:
For $i=r-4$ to $r-3$ assign $t_{i+3} = t_{i+3i} = 1$.

Step VIII:
Assign $t_{r-3} = t_{r-3r} = 1$.

Step IX:
Assume the other entries as zero.

Step X:
Find eigen values of $T$.

Step XI:
Find Total Energy $T_E$.

2.8 MATLAB program to generate the total energy of Pan graph $P_{a_n}$

```matlab
% "T" is the Total matrix of a graph
% "K" is the eigen values of the matrix
% "TE" is the Total Energy of the graph
% r=p+q
% p,q refers the number of vertices and edges of $P_{a_n}$
for i=1:r-1
    T(i,i+1)=1;
    T(i+1,i)=1;
end
for i=1:r-3
    T(i,i+2)=1;
    T(i+2,i)=1;
end
for i=1:2
    T(i,r-2)=1;
    T(r-2,i)=1;
end
for i=r-4:r-3
    T(i,i+3)=1;
    T(i+3,i)=1;
end
T(1,r-3)=1;
```
T(r-3,1)=1;
T
K=eig(T);
TE=sum(abs(K))

2.9 Illustration

When the above program is executed for $Pa_5$, the output will be $TE=20.4252$

2.10 Algorithm to generate the total energy of graph $<C_n, C_{n-1}>$.

Step I:
Assume $G= (V, E)$ to be a $(p,q)$ graph.

Step II:
Assume that in the total matrix representation, vertices and edges appear alternatively along both rows and columns.

Step III:
Let $r = p + q$.

Step IV:
For $i=1$ to $r-1$ assign $t_{i,i+1} = t_{i+1,i} = 1$.

Step V:
For $i=1$ to $q$ assign $t_{i,i+2} = t_{i+2,i} = 1$.

Step VI:
For $i=q+2$ to $r-2$ assign $t_{i,i+2} = t_{i+2,i} = 1$.

Step VII:
For $i=1\&2$ assign $t_{i,q+1} = t_{q+1,i} = 1$.

Step VIII:
For $i=p$ to $q+1$ assign $t_{i,r} = t_{r,i} = 1$.

Step IX:
For $i=p\&q$ assign $t_{i,i+3} = t_{i+3,i} = 1$.

Step X:
Assign $t_{q+2,r} = t_{r,q+2} = 1$, $t_{q,r-1} = t_{r-1,q} = 1$, $t_{1,q} = t_{q,1} = 1$

Step XI:
Assume the other entries as zero.

Step XII:
Find eigen values of $T$.

Step XIII:
Find Total Energy $TE$.

2.11 MATLAB program to generate the total energy of graph $<C_n, C_{n-1}>$.

```matlab
% "T" is the Total matrix of a graph
% "K" is the eigen values of the matrix
% "TE" is the Total Energy of the graph
% r=p+q
% p,q refers the number of vertices and edges of $<C_n, C_{n-1}>$.
for i=1:r-1
    T(i,i+1)=1;
    T(i+1,i)=1;
end
```
for i=1:q
T(i,i+2)=1;
T(i+2,i)=1;
end
for i=q+2:r-2
T(i,i+2)=1;
T(i+2,i)=1;
end
for i=1:2
T(i,q+1)=1;
T(q+1,i)=1;
end
for i=p:q+1
T(i,r)=1;
T(r,i)=1;
end
for i=p:q
T(i,i+3)=1;
T(i+3,i)=1;
end
T(q+2,r)=1;
T(r,q+2)=1;
T(q,r-1)=1;
T(r-1,q)=1;
T(q,1)=1;
K=eig(T);
TE=sum(abs(K))

2.12 Illustration

When the above program is executed for $G_5, G_4$, the output will be $TE=30.5443$

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