Gravitational Lensing by Dark Matter Halos with Non-universal Density Profiles

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ABSTRACT

The statistics of gravitational lensing can provide us with a very powerful probe of the mass distribution of matter in the universe. By comparing predicted strong lensing probabilities with observations, we can test the mass distribution of dark matter halos, in particular, the inner density slope. In this letter, unlike previous work that directly models the density profiles of dark matter halos semi-analytically, we generalize the density profiles of dark matter halos from high-resolution N-body simulations by means of generalized Navarro-Frenk-White (GNFW) models of three populations with slopes, $\alpha$, of about -1.5, -1.3 and -1.1 for galaxies, groups and clusters, respectively. This approach is an alternative and independent way to examine the slopes of mass density profiles of halos. We present calculations of lensing probabilities using these GNFW profiles for three populations in various spatially flat cosmological models with a cosmological constant $\Lambda$. It is shown that the compound model of density profiles does not match well with the observed lensing probabilities derived from the Jodrell-Bank VLA Astrometric Survey data in combination with the Cosmic Lens All-Sky Survey data. Together with the previous work on lensing probability, our results suggest that a singular isothermal sphere mass model of less than about $10^{13} h^{-1} M_\odot$ can predict strong lensing probabilities that are consistent with observations of small splitting angles.

Subject headings: cosmology:observations—cosmology:theory—gravitational lensing—dark matter—galaxies:clusters:general—galaxies:halos

1. Introduction

Mapping the mass distribution of matter in the universe has been a major challenge for modern observational cosmology. The only direct procedure to weigh matter in the universe is measuring its deflection of light by gravity. The statistics of gravitational lensing
can provide us with a very powerful probe of the mass distribution of the Universe. By comparing predicted lensing probabilities with observations, we can examine the mass distributions of dark matter halos, in particular, their inner density slopes. It is well known that the Jodrell-Bank VLA Astrometric Survey (JVAS) and the Cosmic Lens All-Sky Survey (CLASS) (Browne & Myers 2000; Myers et al. 2003; Browne et al. 2003) have provided us with observations of strong lensing probabilities for small image separations ranging from 0.3″ to 3″. Based on the Cold Dark Matter (CDM) model, which has become the standard theory of cosmic structure formation, the lensing probabilities strongly depend on the density profiles of CDM halos. The lensing model is usually described by a singular isothermal sphere (SIS), the Navarro-Frenk-White (NFW) model (Navarro et al. 1996, 1997), or generalized NFW (GNFW) density profiles of dark halos (Zhao 1996). Li & Ostriker (2002) employed a semi-analytical approach to analyze the gravitational lensing of remote quasars by foreground dark halos and checked the plausibility of various lensing models. They found that no model can completely explain the current observations: the SIS models predict too many lenses with large splitting angles, while the NFW models predict too few small splitting angles. They therefore further developed a two-population halo model for lensing: small mass halos with a steep inner density slope and large mass halos with a shallow inner density slope, concluding that a combination of SIS and NFW halo models can reproduce the current observations reasonably well.

Motivated by the sensitivity of the dependence of the image separation distribution of lenses below 1″ on both the inner mass profile of galactic halos and the faint end slope of the mass and luminosity functions, Ma (2003) compared the traditional approach that models lenses as SISs and the Schechter luminosity function, with another method that invokes a certain halo mass profile and the Press-Schechter mass function. She found that dark matter halos cannot all be SISs, otherwise, the mass function becomes steeper than the luminosity function and leads to a relatively high lensing rate on smaller angular scales.

Li & Ostriker (2003) further proposed a lensing model based on three populations of halos distinguished by halos mass: Population 1 corresponds to normal galaxies of mass $10^{10}h^{-1}M_\odot < M < 10^{13}h^{-1}M_\odot$ with centers dominated by baryonic matter and inner density profile slopes of $\alpha = 2$ (SISs); Population 2 corresponds to groups or clusters of galaxies of mass $M > 10^{13}h^{-1}M_\odot$ with centers of dark matter and $\alpha = 1.3$ (GNFW models); Population 3 corresponds to dwarf galaxies or sub-galactic objects with mass $M < 10^{10}h^{-1}M_\odot$, whose centers lack baryons and are also dominated by dark matter, and also described by GNFW models of slope $\alpha = 1.3$. Their results showed that both LCDM and OCDM cosmological models are marginally consistent with current lensing observations, while the SCDM model can be ruled out.
Besides these semi-analytical approaches, we can directly employ the results of high-resolution simulations for the density profiles of dark matter halos in the calculation of lensing probabilities. Jing & Suto (2000) performed a series of high-resolution N-body simulations to examine the density profiles of dark matter halos. They found a clear systematic correlation between halo mass and the slope of the profile at 1% of the virial radius, the slope of which is \( \sim -1.5, -1.3 \) and \(-1.1\) for profiles of galaxies, groups and clusters, respectively. They concluded that dark matter density profiles, especially in their inner regions, do not follow universal forms such as the NFW model. Therefore, consideration of the dependence of the slopes of density profiles on the masses of halos in calculating lensing probabilities seems to be necessary and reasonable. In this letter, we generalize Jing & Suto (2000)'s cosmological simulation results for density profiles of dark matter halos using the three-population GNFW models with slopes of \( \alpha \propto -1.5, -1.3 \) and \(-1.1\) for galaxies, groups and clusters, respectively. This is an alternative and independent way to examine the slopes of density profiles of CDM halos.

2. General Formalism for Lensing

2.1. Lensing Equation for GNFW models

The GNFW density profile can be expressed in the form \( \rho(r) = \frac{\rho_s r_s^3}{r^\alpha (r + r_s)^{3-\alpha}} \) (Zhao 1996) where \( 0 < \alpha < 3 \). We generalize the inner slope from the simulation of dark matter halos using the GNFW model with a varying \( \alpha \) which is dependent of the masses of halos. We take \( \alpha = 1.5 \) for \( M < M_{c1} \), \( \alpha = 1.3 \) for \( M_{c1} < M < M_{c2} \) and \( \alpha = 1.1 \) for \( M > M_{c2} \), respectively, where \( M_{c1} \sim 10^{13} M_\odot \) corresponds to the cooling mass scale (Porciani & Madau 2000; Kochanek & White 2001) and \( M_{c2} \sim 10^{14} M_\odot \). The mass of a dark halo within \( r_{200} \) can be defined as \( M = 4\pi \int_0^{r_{200}} \rho r^2 dr = 4\pi \rho_s r_s^3 f(c_1) \), and \( r_{200} \) is the radius of a sphere around a dark halo within which the average mass density is 200 times the critical mass density of the universe. The function \( f(c_1) = \int_0^{c_1} x^2 dx / x^\alpha (1 + x)^{3-\alpha} \) and \( c_1 = r_{200} / r_s \) is the concentration parameter

\[
c_1(M_{15}, z) = c_{\text{norm}} \frac{2 - \alpha}{1 + z} [10M_{15}]^{-0.13},
\]

where \( z \) is halo redshift, \( c_{\text{norm}} = 8 \) (Bullock et al. 2001) and \( M_{15} = M/(10^{15} h^{-1} M_\odot) \) is dimensionless halo mass. So \( \rho_s \) and \( r_s \) can be related to mass \( M_{15} \) and redshift \( z \) by

\[
\rho_s = \rho_c E^2(z) \frac{c_1^3}{3 f(c_1(M_{15}, z))}, \quad r_s = \frac{1.626}{c_1} \frac{M_{15}^{1/3}}{E^2/3(z)} h^{-1} \text{Mpc},
\]

where \( E(z) = \frac{\Omega_m}{3} \left( \frac{1}{1+z} \right)^{3/2} \).
where \( \rho_{\alpha} \) is the critical mass density of the universe today and the expansion rate \( E(z) = \sqrt{\Omega_m(1+z)^3 + \Omega_\Lambda} \). The lensing equation for the GNFW profile is given by \( y = x - \mu_s g(x)/x; \) \( \xi = \vec{x} r_s \) and \( \vec{n} = \vec{y} r_s d^3_s/dL^4 \) are the position vectors in the lens plane and the source plane respectively, \( g(x) = \int_0^y u du \int_0^\infty (u^2 + z^2)^{-\alpha/2} \left[(u^2 + z^2)^{1/2} + 1\right]^{-3+\alpha} dz, \) and

\[
\mu_s \equiv \frac{4\rho_s r_s}{\Sigma_{cr}} = 0.002(\frac{\rho_s}{\rho_{\alpha}})(\frac{r_s}{1h^{-1}Mpc})(\frac{d^4_R}{c/H_0}),
\]

where \( \mu_s \) is a parameter on which the efficiency of producing multiple images is strongly dependent. Here \( \Sigma_{cr} = \frac{c^2}{4\pi G} \frac{d^3_s}{dL^4 dS^2} \) is the critical surface mass density, and \( d^4_R = d^4_L d^4_{LS} / d^4_S \). \( d^4_s \) and \( d^4_L \) are the angular diameter distances from the observer to the source and to the lens object respectively, while \( d^4_{LS} \) is the same quantity but from the lens to the source object. The lensing equation for a GNFW density profile with different \( \alpha \) and setting \( \mu_s = 1 \) is plotted in Fig.1. The curves are symmetrical with respect to the origin. Multiple images can be formed when \( |y| \leq y_{cr} \), where \( y_{cr} \) is the maximum value of \( y \) when \( x < 0 \) or the minimum value for \( x > 0 \). Generally speaking, there exist three images for \( |y| < y_{cr} \). We will just consider the outermost two images stretched by the splitting angle \( \Delta \theta \) when more than two images are formed.

### 2.2. Lensing Probability for GNFW models

We can write the cross-section as \( \sigma(M, z) \approx \pi y^2_s r^2_s \vartheta (\Delta \theta - \Delta \theta_0) \) with \( \Delta \theta > \Delta \theta_0 \) in the lens plane for multiple images produced by a GNFW lens at \( z \). \( \vartheta \) is a step function and the splitting angle \( \Delta \theta \) is given by \( \Delta \theta = r_s \Delta x / d^4_L \approx 2 x_0 r_s / d^4_L \) where \( x_0 \) is the positive root of the lensing equation \( y(x) = 0 \). The lensing probability with image separations larger than \( \Delta \theta \) is given by Schneider et al. (1992)

\[
P(\Delta \theta) = \int_0^{z_s} \frac{dP_p(z)}{dz} dz \int_0^\infty \bar{n}(M, z) \sigma(M, z) dM,
\]

where \( D_p(z) = c/H_0 \int_0^z dz / (1 + z) E(z) \) is the proper distance from the observer to the lens at redshift \( z \). The physical number density \( \bar{n}(M, z) \) of virialized dark halos of masses between \( M \) and \( M + dM \) is expressed as \( \bar{n}(M, z) = n(M, z)(1 + z)^3 \), where \( n(M, z) = \rho_0 f(M, z) / M \) is the comoving number density (Press & Schechter 1974). \( \rho_0 \) is the mean mass density of the universe today, and \( f(M, z) = -\sqrt{2 \frac{d_\Delta(z)}{dM}} \frac{dln \Delta}{dM} \exp[-\frac{\Delta^2(M)}{2\Delta^2}] \) where \( \Delta^2(M) = \frac{1}{2\pi^2} \int_0^\infty P(k) W^2(k r_M) k^2 dk \) is the present variance of the fluctuations within a sphere containing a mass \( M \). The power spectrum of CDM density fluctuations \( P(k) = A k T^2(k) \) is given by Eisenstein & Hu (1999), where \( A \) is the amplitude normalized to \( \sigma_8 = \Delta(r_M = 8h^{-1}Mpc) \).
Fig. 1.— The lensing equation for a GNFW model with $\alpha=1.5$, 1.3 and 1.1 respectively.
Since the redshift distribution of quasars in the JVAS/CLASS survey is still poorly known, we adopt the mean value of $z_s = 1.27$ estimated by Marlow et al. (2000). In this letter, we will use spatially flat $\Lambda$CDM models characterized by the matter density parameter $\Omega_m$, vacuum energy density parameter $\Omega_\Lambda$ and Hubble constant $h = 0.75$, and will calculate lensing probabilities with image separations greater than $\Delta \theta$ according to GNFW halo profile models with varying $\alpha$. In the definition of cross-section or lensing probability, we will just consider the criterion $\Delta \theta$, and neglect another one $q_r$ (the brightness ratio of the outermost two images) which is the ratio of the corresponding absolute values of the magnifications (Schneider et al. 1992). In order to investigate the effect of central black holes or bulges on lensing probability, Chen (2003a,b) introduced $q_r$ into the calculation for lensing cross-section. Due to the existence of central black holes or galactic bulges, $y_{cr}$ becomes extremely large when $|x|$ approaches zero. Thus $y_{cr}$ can be determined by the consideration of $q_r$ together with $\Delta \theta$. However for GNFW halo models in the absence of central black holes or galactic bulges, we can see in Fig.1 that the lensing equation curves are so smooth that we do not need to define cross-section by $q_r$.

Our numerical results are shown in Fig.2 together with the observational ones, which for $6'' \leq \Delta \theta \leq 15''$ from JVAS/CLASS takes the form of an upper limit. Based on the high-resolution simulation results (Jing & Suto 2000), we adopt a compound GNFW model for dark halos with three populations as described above. For the cosmological models, we first choose the new result from the Wilkinson Microwave Anisotropy Probe (WMAP): $\Omega_m = 0.27$, $\sigma_8 = 0.84$ (Bennett et al. 2003; Spergel et al. 2003) and $M_{c1} = 10^{13}M_\odot$, the numerical result of which is depicted by a thin solid line in Fig.2. It is clear that the predicted lensing probabilities are much lower than the observed values. Previous works (Chen 2003a,b) showed that lensing probability is sensitive to $\Omega_m$, $\sigma_8$ and $M_{c1}$. More specifically, it increases with $\Omega_m$, $\sigma_8$ and $M_{c1}$. In order to improve the prediction, we take $\Omega_m$, $\sigma_8$ and $M_{c1}$ to be the higher values of 0.4, 1.2 and $5 \times 10^{13}M_\odot$ respectively. As shown by the dashed line in Fig.2, the derived lensing probabilities are still inconsistent with the observations although they have increased greatly compared to the former model. Thus we take the upper limit of 1.6 for $\sigma_8$, as used in literature to date without changing the other parameters. The results are shown with a dotted line in Fig.2. There is almost no improvement in the small angle separations compared with the case of $\sigma_8 = 1.2$, apart from the increase of the predicted probabilities on large angles. On the other hand, the lensing probabilities for both of cases $\sigma_8 = 1.2$ and $\sigma_8 = 1.6$ lie above the observed upper limit for large angle separations.
Fig. 2.— The lensing probabilities for image separations greater than $\Delta \theta$ using the compound GNFW dark halo profile models with varying $\alpha$. The solid-line histogram represents observed lensing probabilities from JVAS/CLASS (Chen 2003a; Chae et al. 2002). The thick solid horizontal line denotes the upper limit of the null result for lenses with $6'' \leq \Delta \theta \leq 15''$ from JVAS/CLASS. The other three curves are the predicted lensing probabilities in $\Lambda$CDM cosmology.
3. Conclusions and Discussion

We have generalized the density profiles from high-resolution simulations of dark matter halos according to GNFW models with varying $\alpha$, and developed a compound model incorporating three populations of matter halos. We choose three typical cosmological models: Model A, the new result of WMAP for model parameters: $\Omega_m = 0.27$, $\sigma_8 = 0.84$ and $M_{c1} = 10^{13}M_\odot$; Model B, in which $\Omega_m = 0.4$ is the upper limit adopted for the $\Lambda$CDM model, $\sigma_8 = 1.2$ and $M_{c1} = 5 \times 10^{13}M_\odot$; and Model C, which is the same as Model B except that $\sigma_8$ is taken to be 1.6, the maximum value used so far in the literature. We have compared our predicted results for lensing probabilities with JVAS/CLASS observations. Current observations for lensing probabilities from JVAS/CLASS only cover lensing events on small angle scales, though they also give the upper limit on separation angles over the range $6''$ to $15''$. Our numerical results as shown in Fig.2 show that the predicted lensing probabilities of the halos of three populations in current cosmological models do not agree with the JVAS/CLASS observations. For Model A, the predicted probabilities of lensing events lie below the upper limit observed on large angle scales, but is lower than the observational results by a factor of about one order of magnitude on small separation angles. Even when the upper limit value of $\Lambda$CDM cosmology are used for $\Omega_m$ and $\sigma_8$, such as in Models B and C, the predicted lensing probability on small angle scales still do not match the observations. In the calculation of cross section, we have not considered the uncertainties such as the distribution of source redshifts for quasars, the brightness ratio $q_r$, the scatter effect of the concentration parameter $c_1$ and the variation in $M_{c2}$. As discussed above, consideration of the brightness ratio $q_r$ does not change our conclusion. As for the scatter in the concentration parameter $c_1$, Chen (2003a,b) have investigated its effect on the prediction of lensing probabilities. By averaging lensing probabilities with a log-normal distribution, they found that the scatter in $c_1$ only increases the probabilities on larger image separations but has little effect at smaller separations. As for $M_{c2}$, any variation only contributes to the predicted lensing probabilities on large angle scales. In addition, for the sources in the JVAS/CLASS survey, the redshift distribution is still poorly understood except for the mean redshift $<z_s>=1.27$ as estimated by Marlow et al. (2000). Thus, most of the previous work (Li & Ostriker 2002; Chen 2003a) does not consider the redshift distribution of the sources. The prediction of Dunlop & Peacock (1990) model and the CLASS lensing sub-sample redshift measurements suggest that the redshift distribution for CLASS unlensed sources can be modelled by a Gaussian distribution with mean redshift $<z_s>=1.27$. However this can not affect the conclusion of our letter. Even so, with the ever increasing number of observed lensing events, we look forward to the day when the redshift distribution derived from lensing observations can be taken into account for other aspects of lensing study.

On the other hand, Klypin et al. (2001) have presented a convergence study of the
density profiles of cold dark matter halos simulated with varying mass and force resolutions. They concluded that, on radii larger than the “effective” spatial resolution, the density profiles of dark matter still do not experience any systematical trends even with the further increase of the number of particles or the force resolution. In addition, they also explained the systematic correlation between inner profile slope and halo mass found by Jing & Suto (2000), and pointed out that it is a trend in dark matter halo concentration. Navarro (2003) report recent results of numerical simulations designed to study the inner slopes of density profiles for CDM halos. Their results indicate that the inner slopes of CDM halos are considerably shallower than the asymptotic value of -1.5 proposed by Moore et al. (1999). This point probably indirectly strengthens our argument that the density profiles given by pure numerical simulation for CDM halos are inconsistent with the observational statistics of strong lensing. Therefore, in order to recover the observed strong lensing probabilities from the inner density profiles of mass halos, we can employ the SIS profile with a mass of less than about $10^{13}h^{-1}M_\odot$. This is in agreement with previous work such as Li & Ostriker (2002) and references therein.

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