Neutrino phenomenology in a left-right $D_4$ symmetric model

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Abstract

We present a minimal left-right symmetric flavor model and analyze the predictions for the neutrino sector. In this scenario, the Yukawa sector is shaped by the dihedral $D_4$ symmetry which leads to correlations for the neutrino mixing parameters. We end up with four possible solutions within this model. We further analysed the impact of the upcoming long-baseline neutrino oscillation experiment, DUNE. Due to its high sensitivity, DUNE will be able to rule out two of the solutions. Finally, the prediction for the neutrinoless double beta decay for the model has also been examined.
I. INTRODUCTION

A number of phenomenal experimental evidences over the past two decades have established the fact that neutrinos oscillate through their propagation path \[1\]–[4], which implies non-zero neutrino masses and mixings. This fact provides an undoubtedly motivation for the existence of physics beyond the Standard Model (SM), as neutrinos are massless in the SM. Furthermore, the experimental efforts in understanding the neutrino properties have determined the two mass-squared differences and large lepton mixing angles. From global fits of neutrino oscillation data [5] (other global analysis can be found in [6, 7]), the best fit values and the 1σ intervals for a normal neutrino mass ordering (NO) are given by

\[
|\Delta m^2_{\text{sol}}| = 7.55^{+0.20}_{-0.16} \times 10^{-5} \text{ eV}^2, \quad |\Delta m^2_{\text{atm}}| = 2.50 \pm 0.03 \times 10^{-3} \text{ eV}^2, \\
\theta_{12}^\circ = 34.5^{+1.2}_{-1.6}, \quad \theta_{13}^\circ = 8.45^{+0.16}_{-0.14}, \quad \theta_{23}^\circ = 47.7^{+1.2}_{-1.7}, \quad \text{and} \quad \delta_{\text{CP}}^\circ = 218^{+38}_{-27}. \tag{1}
\]

Moreover, the theory behind the dynamical origin of neutrino mass and their flavor mixing pattern and whether they are Majorana or Dirac fermions, are yet unanswered. The simplest idea behind these shortcomings relies on the assumption that neutrinos are Majorana particles and their tiny masses are generated through a seesaw mechanism [8]–[13]. Interesting extensions of the SM featured by the inherent new physics signatures are those that consider a left-right (LR) symmetric nature [14]–[17]. For instance, LR symmetric models have the virtue of accounting for the small neutrino masses from the contribution of two mechanisms, the type-I and type-II seesaw, which implies the existence of new particles.

The simplest LR symmetric model is dictated by the gauge symmetry group \( \text{SU}(3)_C \otimes \text{SU}(2)_L \otimes \text{SU}(2)_R \otimes U(1)_{B-L} \). In this case, the fermion fields have the following charge assignments [18],

\[
l_L \simeq (1, 2, 1, -1), \quad l_R \simeq (1, 1, 2, -1), \quad Q_L \simeq (3, 2, 1, 1/3) \quad \text{and} \quad Q_R \simeq (3, 1, 2, 1/3), \tag{2}
\]

whereas the scalar potential is formed by two triplets and one bi-doublet whose LR charges are,

\[
\Delta_L \simeq (1, 3, 1, 2), \quad \Delta_R \simeq (1, 1, 3, 2) \quad \text{and} \quad \Phi \simeq (1, 2, 2, 0). \tag{3}
\]

\[\text{Note here that the neutrino oscillation experiments are sensitive to (mass)}^2\text{- differences and hence, the possibility of a massless neutrino is not excluded.}\]
If the LR breaking and the masses of the new scalar fields are of $\mathcal{O}(\text{TeV})$, this minimal setup produces sizeable contributions to lepton flavor violating (LFV) decays, lepton number violation as well as CP violating processes [19–25]. Therefore, this scenario turns out to be appealing for experimental searches among the low-energy LFV processes [26]. Further constraints apply to this model from LHC searches of new physics [27–31]. On the other hand, the LR symmetry is also possible to be broken at higher energies, such as the grand unification theory (GUT) scale, leading to gauge coupling unification [32, 33]. This makes LR models interesting frameworks from the perspective of GUTs like $SO(10)$ [34, 35].

On top of gauge symmetries, one can impose additional global symmetries that relate the flavor structure of the SM. In the last decade, there have been a tremendous amount of works in that direction, for reviews see [36, 37]. Nevertheless, It is particularly interesting the interplay between the LR symmetry and a discrete flavor symmetry. This combination shapes and correlates the Yukawa sector, giving predictions for the flavor observables, i.e. masses and mixings [38–42]. In this work, we study the effects of combining a non-Abelian discrete flavor symmetry $D_4$ with LR symmetry. The $D_4$ flavor symmetry group has been explored in [43–54], not in combination with a LR symmetric model to the best of our knowledge. Among many of the consequences of this model, is the appearance of two-texture zeros in the neutrino mass matrix, in a similar way to other discrete flavor symmetry models. Under the Glashow-Frampton- Marfatia classification [55] for the two-zero texture Majorana neutrino mass matrices, we get an $A_2$ texture zero matrix. This model also predicts a non trivial mass matrix for the charged leptons.

The outline of the paper is as follows: in Sec. II we present the model and charge assignments. There, we describe the lepton sector, that is, we give the invariant Lagrangian of the theory. We explain the procedure of our analysis in Sec. III as well as show our results for the neutrino predictions within the model. Our final comments and summary are given in Sec. IV.

II. LEFT-RIGHT $D_4$ SYMMETRIC MODEL

We consider an extension of the minimal left-right symmetric model by adding a $D_4$ flavor symmetry. Besides postulating a symmetry that shapes the Yukawa sector, we add
two flavon fields, $\xi$ and $\eta$ transforming as a singlet and doublet under $D_4$, respectively. In Table I we provide the matter content and charge assignments of the model. In this framework, the symmetry breaking goes like

$$\text{LRSM} \otimes \text{GF} \xrightarrow{\eta, \xi} \text{LRSM} \xrightarrow{\Phi} \text{SM} \xrightarrow{\Phi} \text{SU}(3)_C \otimes \text{U}(1)_{em},$$

where $\text{GF} = D_4 \otimes \mathbb{Z}_2$ and its breaking is associated to the non-zero vevs of the flavon fields $\langle \xi \rangle$ and $\langle \eta \rangle$.

|            | $\psi_{L_i}$ | $\psi_{R_i}$ | $\Delta_L$ | $\Delta_R$ | $\Phi$ | $\eta$ | $\xi$ |
|------------|--------------|--------------|------------|------------|--------|--------|--------|
| SU(2)$_L$  | 2            | 1            | 3          | 1          | 2      | 1      | 1      |
| SU(2)$_R$  | 1            | 2            | 1          | 3          | 2      | 1      | 1      |
| U(1)$_{B-L}$ | -1          | -1           | 2          | 2          | 0      | 0      | 0      |
| $D_4$      | $2^0\oplus 1$| $2^0\oplus 1$| 1          | 1          | 1      | 2      | 1      |
| $\mathbb{Z}_2$ | 1           | 1            | 1          | 1          | -1    | -1    | -1    |

**TABLE I:** Matter content and charge assignments of the left-right $D_4$ model.

We assume the following sequential symmetry breaking $\Lambda_F >> \Lambda_{LR} >> \Lambda_{EW}$, where $\Lambda_F$ is the flavour breaking scale and $\Lambda_{LR}$ is the left-right symmetry breaking scale$^2$.

Given the matter content shown in Table I the Yukawa Lagrangian (up to dimension-5) for the leptons can be expressed as

$$\mathcal{L}_Y \supset \bar{\ell}_{LD} \left( y_1 \frac{\xi}{\Lambda_F} \Phi + \bar{\eta}_1 \frac{\xi}{\Lambda_F} \bar{\Phi} \right) \ell_{RD} + \bar{\ell}_{LD} \left( y_2 \frac{\eta}{\Lambda_F} \Phi + \bar{\eta}_2 \frac{\eta}{\Lambda_F} \bar{\Phi} \right) \ell_{Rs}$$

$$+ \bar{\ell}_{Ls} \left( y_3 \frac{\eta}{\Lambda_F} \Phi + \bar{\eta}_3 \frac{\eta}{\Lambda_F} \bar{\Phi} \right) \ell_{RD} + \bar{\ell}_{Ls} \left( y_4 \frac{\xi}{\Lambda_F} \Phi + \bar{\eta}_4 \frac{\xi}{\Lambda_F} \bar{\Phi} \right) \ell_{Rs}$$

$$+ \frac{Y_{L_1}}{2} \bar{\ell}_{LD}^T C (i \sigma_2) \Delta_L \ell_{LD} + \frac{Y_{L_2}}{2} \bar{\ell}_{Ls}^T C (i \sigma_2) \Delta_L \ell_{Ls}$$

$$+ \frac{Y_{R_1}}{2} \bar{\ell}_{RD}^T C (i \sigma_2) \Delta_R \ell_{RD} + \frac{Y_{R_2}}{2} \bar{\ell}_{Rs}^T C (i \sigma_2) \Delta_R \ell_{Rs} + \text{h.c.}$$

where the bi-doublet $\Phi$ can be read as

$$\Phi = \begin{pmatrix} \phi_1^0 & \phi_1^+ \\ \phi_2^- & \phi_2^0 \end{pmatrix}.$$  

$^2$ With this assumption the flavon fields decouple from the theory having only an impact on the Yukawa couplings. Then, in this energy regime the scalar potential is approximate to the minimal LRSM one [18]. Additionally, since $\Lambda_{LR} >> \Lambda_{EW}$, the new scalars do not have an important contribution to LFV processes [26].
Note that the Dirac neutrino mass matrices stem from the dimension-5 operators. Hence, from Eq. (4) after spontaneous symmetry breaking (SSB), one gets that the mass matrix for charged leptons as

\[ M_\ell = \frac{1}{\sqrt{2}} (Y'_L v_2 + \bar{Y}'_L v_1) , \]  

(6)

where

\[ Y'_L = \frac{1}{\Lambda_F} \begin{pmatrix} 0 & y_1 v_\chi & y_2 v_\eta_2 \\ y_1 v_\chi & 0 & y_2 v_\eta_1 \\ y_3 v_\eta_2 & y_3 v_\eta_1 & y_4 v_\chi \end{pmatrix} \quad \text{and} \quad \bar{Y}'_L = \frac{1}{\Lambda_F} \begin{pmatrix} 0 & \bar{y}_1 v_\chi & \bar{y}_2 v_\eta_2 \\ \bar{y}_1 v_\chi & 0 & \bar{y}_2 v_\eta_1 \\ \bar{y}_3 v_\eta_2 & \bar{y}_3 v_\eta_1 & \bar{y}_4 v_\chi \end{pmatrix} , \]  

(7)

with \( \langle \chi \rangle \equiv v_\chi, \langle \eta \rangle \equiv (v_{\eta_1}, v_{\eta_2})^T \) and \( \Phi \)

\[ \langle \Phi \rangle = \begin{pmatrix} v_1 \\ 0 \\ v_2 \end{pmatrix} . \]  

(8)

Assuming a vev alignment \( \langle \eta \rangle \sim (1, 0)^T \), the mass matrix for the charged leptons becomes

\[ M_\ell = \frac{1}{\sqrt{2} \Lambda_F} \begin{pmatrix} 0 & (y_1 v_2 + \bar{y}_1 v_1) v_\chi & 0 \\ (y_1 v_2 + \bar{y}_1 v_1) v_\chi & 0 & (y_2 v_2 + \bar{y}_2 v_1) v_\eta_1 \\ 0 & (y_3 v_2 + \bar{y}_3 v_1) v_\eta_1 & (y_4 v_2 + \bar{y}_4 v_1) v_\chi \end{pmatrix} . \]  

(9)

The matrix \( M_\ell \) can be diagonalised by a bi-unitary transformation as

\[ \text{diag}(m_e, m_\mu, m_\tau) = U_\ell M_\ell V_\ell^\dagger , \]  

(10)

and the neutrino mass matrix is given by

\[ m_\nu = \begin{pmatrix} m_L & m_D & m_D \\ m_D & m_L & m_D \\ m_D & m_D & m_R \end{pmatrix} , \]  

(11)

where \( m_L = \sqrt{2} Y_L v_L \) and \( m_R = \sqrt{2} Y_R v_R \), with

\[ Y_{L(R)} = \begin{pmatrix} 0 & Y_{L_1(R_1)} & 0 \\ Y_{L_1(R_1)} & 0 & 0 \\ 0 & 0 & Y_{L_2(R_2)} \end{pmatrix} . \]  

(12)

In this scenario, the Dirac neutrino mass matrix turns out to be

\[ m_D = \frac{1}{\sqrt{2}} (Y'_L v_1 + \bar{Y}'_L v_2) . \]  

(13)
After the SSB, light neutrino eigenstates acquire their masses through the type-I and type-II seesaw mechanism. Hence, since $v_R \gg v_L, v_1, v_2$, light-neutrino masses are given by,

$$M^\text{light}_\nu = m_L - m_D m_R^{-1} m_D^T.$$  \hfill (14)

The left-right symmetric nature of the theory demands a relation between the Yukawa couplings mediating the interaction of leptons with the scalar triplets, i.e. $Y_R = Y_L$.

The left-right exchange symmetry can be realized through either C or P transformations. Here we choose to use P-transformations, which demand the hermiticity of Dirac type fermion mass matrices, that is,

$$M_\ell = M_\ell^\dagger, \quad m_D = m_D^\dagger.$$  \hfill (15)

### III. NEUTRINO PHENOMENOLOGY

From Eq. (9) one can notice that the mass matrix for charged leptons is non-diagonal. The left-right symmetry gives further relations for leptonic Yukawas, as mentioned in the previous section. Using this fact, the mass matrix for charged leptons, as given by Eq. (9), can be recasted as

$$M_\ell = \begin{pmatrix} 0 & a_\ell & 0 \\ a_\ell^* & 0 & b_\ell \\ 0 & b_\ell^* & c_\ell \end{pmatrix}.$$  \hfill (16)

The phases of this matrix can be absorbed in a pair of diagonal phase matrices ($P$ and $P'$), which will define a real charged lepton matrix basis

$$\tilde{M}_\ell = P M_\ell P'.$$  \hfill (17)

In this basis the neutrino mass matrix becomes

$$\tilde{M}_\nu = P^T M_\nu P,$$  \hfill (18)

where $M_\nu$ is the neutrino mass matrix in the interaction basis, Eq. (14). Since $\tilde{M}_\ell$ is symmetric, it is diagonalised as

$$\text{diag}(m_e, m_\mu, m_\tau) = O_\ell \tilde{M}_\ell O_\ell^T,$$
where $O_\ell$ is an orthogonal matrix and one can easily get the expressions for $a_\ell$, $b_\ell$ and $c_\ell$ in terms of the charged fermion masses. This is done by computing the invariants of the charged leptons mass matrix, namely $\text{Tr}M_\ell$, $\text{Tr}M_\ell^2$ and $\text{det}M_\ell$. Then, the matrix elements in Eq. (16) as functions of the masses read as

$$a_\ell = \pm \frac{\sqrt{m_\ell m_\mu m_\tau}}{\sqrt{m_\ell - m_\mu + m_\tau}},$$

$$b_\ell = \pm \frac{\sqrt{-m_\mu + m_\tau} \sqrt{-m_\ell^2 + m_\mu m_\mu - m_\ell m_\mu + m_\mu m_\tau}}{\sqrt{m_\ell - m_\mu + m_\tau}},$$

$$c_\ell = m_\ell - m_\mu + m_\tau. \quad (19)$$

With this information one can compute the rotation matrix $O_\ell$ and is given by

$$O_\ell = \begin{pmatrix}
0.998 & -\text{sgn}(a_\ell)0.070 & \text{sgn}(a_\ell b_\ell)0.001 \\
-\text{sgn}(a_\ell)0.068 & 0.969 & \text{sgn}(b_\ell)0.236 \\
-\text{sgn}(a_\ell b_\ell)0.017 & -\text{sgn}(b_\ell)0.235 & 0.972
\end{pmatrix}. \quad (20)$$

Note that $O_\ell$ in Eq. (20) is determined up to sign combinations of the parameters $a_\ell$ and $b_\ell$.

Regarding the neutrino mass matrix, this is obtained using eqs. (12-14) and turns out to be,

$$\tilde{M}_\nu = \begin{pmatrix}
0 & a_\nu & 0 \\
a_\nu & d_\nu & b_\nu \\
0 & b_\nu & c_\nu
\end{pmatrix}, \quad (21)$$

where $a_\nu$, $b_\nu$ and $c_\nu$ and $d_\nu$ are complex numbers. Then, in this model the light-neutrino masses are computed through diagonalization of $\tilde{M}_\nu$ in Eq. (21). This mass matrix and the neutrino mass eigenstates are related as follows,

$$\tilde{M}_\nu = U_\nu^*(\theta_{12}^0, \theta_{23}^0, \theta_{13}^0, \delta_0) \text{diag}(m_{\nu_1}, m_{\nu_2}, m_{\nu_3}) U_\nu(\theta_{12}^0, \theta_{23}^0, \theta_{13}^0, \delta_0)^T, \quad (22)$$

where $m_{\nu_i}$ are the light neutrino masses and the unitary matrix $U_\nu$ follows the PDG parameterization \cite{PDG}. Therefore, in this model, the lepton mixing matrix (also known as Pontecorvo-Maki-Nakagawa-Sakata (PMNS) mixing matrix \cite{PMNS1, PMNS2}) is defined by\textsuperscript{3},

$$V_L(\theta_{12}, \theta_{23}, \theta_{13}, \delta) = O_\ell^T U_\nu(\theta_{12}^0, \theta_{23}^0, \theta_{13}^0, \delta_0) K, \quad (23)$$

\textsuperscript{3} Similar structure for charged leptons and neutrinos was obtained in the context of $S_3$ flavor symmetry \cite{S31, S32}.
where the angles $\theta_{ij}$ correspond to the mixing angles determined by neutrino oscillation experiments, $\delta$ represents the Dirac type CP-violating phase and $K$ is the Majorana phase diagonal matrix.

A. Results

Fig. 1 shows the results from our numerical scan in the $(\sin^2 \theta_{23} - \delta)$ plane. Note that depending on the sign of $(a_l, b_l)$, see Eq. (20), there are four possible solutions which correlate the atmospheric angle $\theta_{23}$ and the Dirac type CP-violating phase $\delta$. These are denoted as: in light-red $A = (+, -)$; in light-blue $B = (-, -)$; in light-green $C = (+, +)$; and, in light-pink $D = (-, +)$. In the left-panel, the $(\sin^2 \theta_{23} - \delta)$ plane depicts the allowed regions considering the latest global analysis of neutrino oscillation data [5] at 1, 3, and 5$\sigma$ C. L., respectively. These contours are shown using the red, orange, and yellow colors, respectively. The best-fit value has been marked with a ‘black-dot’. It can be seen from the left-panel that the solution $A$ is ruled out by the present data at 5$\sigma$ C. L., whereas the solution $D$ is marginally allowed at 3$\sigma$ C. L., but only for the CP-conserving values, namely around $\delta = 0, 2\pi$. We also notice that the solutions $B$ and $C$ are allowed at 1$\sigma$ C. L. Furthermore, it can be seen that among the four cases only the solution $C$ is able to explain the latest best-fit value of neutrino oscillation data.

Similarly, in the right-panel of Fig. 1 we show the compatibility of the model by considering the simulated results of the next generation long baseline oscillation experiment, DUNE [61]. The allowed parameter space of DUNE in the $(\sin^2 \theta_{23} - \delta)$ plane is found using the latest best-fit value of neutrino oscillation data. For the numerical simulation of DUNE, the GLoBES package was used [62, 63] along with the auxiliary files in Ref. [64]. It was assumed a running time of 3.5 years in both neutrino and antineutrino modes for DUNE, i.e. DUNE[3.5 + 3.5]. The detailed numerical procedure that have been followed to simulate data coincides with the one performed in [65, 66]. Notice from the right-panel that DUNE results would significantly improve the precision of both the parameters. It is observed that $\sin^2 \theta_{23}$ is constrained to values between $(0.45, 0.58)$, whereas $\delta$ is restricted to the range $(0.95, 1.88)\pi$ at 5$\sigma$ C. L. after DUNE[3.5 + 3.5] running time. Therefore, one can infer that the precise measurement of both parameters ($\theta_{23}$ and $\delta$) by DUNE, the solution $D$ will be ruled out at 5$\sigma$ C. L., still allowed by the latest global-fit data.
FIG. 1: Allowed parameter space in \((\sin^2 \theta_{23} - \delta)\) plane for the four-solutions. Various colors viz, light-red, light-blue, light-green, and light-pink show correlation for A = (+, -), B = (-, -), C = (+, +), and D = (-, +), respectively. The solid contours for the left (right)-panel depicts the allowed region for ‘global-fit data’ ('simulated results of DUNE') corresponding the latest best-fit value as shown by black-dot.

In this model we also have a prediction for lepton number violating processes such as the neutrinoless double beta decay \((0\nu\beta\beta)\). Ongoing experiments that are looking for the signatures of \(0\nu\beta\beta\) decays are namely, GERDA Phase-II \([67]\), CUORE \([68]\), SuperNEMO \([69]\), KamLAND-Zen \([70]\) and EXO \([71]\). The half-life of these processes can be expressed as \([72, 73]\),

\[
(T_{0\nu}^{1/2})^{-1} = G_{0\nu}|M_{0\nu}(A, Z)|^2|\langle m \rangle_{ee}|^2,
\]  

where \(G_{0\nu}\) represents the two-body phase-space factor, \(M_{0\nu}\) is the nuclear matrix element and \(|\langle m \rangle_{ee}|\) is the effective Majorana neutrino mass. The expression of \(|\langle m \rangle_{ee}|\) is given by,

\[
|\langle m \rangle_{ee}| = \left| \sum_{i=1}^{3} m_i V_{ei}^2 \right|
\]

where \(V_{L}\) stands for lepton mixing matrix as mentioned in Eq. \((22)\). Fig. 2 shows the prediction for \(0\nu\beta\beta\) decay. For comparison, we first show the allowed 3\(\sigma\) parameter space in \((m_{light} - |\langle m \rangle_{ee}|)\)-plane using the latest global analysis of neutrinos oscillation data \([5]\), as shown by the gray color. We proceed to compute the effective Majorana neutrino mass in Eq. \((25)\) for the allowed solutions, namely for B, C and D. The color code of the prediction is the same as the one used in Fig. 1. Current upper bound on \(|\langle m \rangle_{ee}|\) comes from KamLAND-Zen collaboration and is delimited by the dark-yellow horizontal band. The two black lines on this band corresponds to the uncertainty of the nuclear matrix element, \(|M_{0\nu}|\) in eq. \((24)\).
In addition, the light green-vertical band represents the bound on $m_{\text{light}}$ coming from the cosmological limit on the sum of neutrino masses provided by the Planck Collaboration, namely $\sum m_\nu < 0.12$ eV at the 95% C.L. \cite{74,75}. Furthermore, as pointed out before, from the left-panel Fig. 1 on can observe that DUNE can rule out solution D. This also has an impact for the prediction of $0\nu\beta\beta$. As a final remark, notice that the allowed solutions are compatible only with normal neutrino mass ordering.

![FIG. 2: The effective Majorana neutrino mass $\langle |m_{ee}| \rangle$ vs the lightest neutrino mass $m_{\text{light}}$. The prediction for the solutions B, C and D are shown by the color codes, which are same as the one used in Fig. 1. Moreover, the latest upper bound on $\langle |m_{ee}| \rangle$ from KamLAND-Zen collaboration are shown by the dark-yellow horizontal band. Also, the current results on the lightest neutrino mass is shown by the light green-vertical band from Planck Collaboration which gives $\sum m_\nu < 0.12$ eV at the 95% C.L.](image)

IV. CONCLUSIONS

We have constructed a minimal left-right symmetric model with the addition of a flavor symmetry, the non-Abelian discrete group $D_4$. We notice that besides the relations in the lepton Yukawas due to the left-right symmetry there are further correlations due to the additional family symmetry behind the theory. For this reason, there are a few free parameters left that can be written in terms of the leptonic observables, namely masses and their mixing angles. This can be observed from the computation of the charged lepton mass matrix as well as the corresponding rotation matrix. The simplicity of the model leads to clear predictions for the neutrino sector. On the one hand, the model turns out to be compatible only with normal neutrino mass ordering and provides a correlation between the
atmospheric angle $\theta_{23}$ and the leptonic CP-violating phase $\delta$. Given the possible solutions of the model there is one, namely the solution A is ruled out by the current neutrino oscillation data. More importantly, due to the high potential of the DUNE experiment to improve the precision of $\theta_{23}$ and to probe $\delta$, it gives further restrictions to the parameter space as shown in the right-panel in Fig. 1. Using this, DUNE will be able to rule out two of the solutions, namely A and D. We also provided the prediction for neutrinoless double beta decay in terms of the lightest neutrino mass for a mass range of $\sim (10^{-3} - 10^{-2})$ eV, which we summarize in Fig. 2.

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**Appendix A: Basics of $D_4$ group**

The dihedral group $D_4$ is a non-Abelian group of order eight and contains five irreducible representations (irreps), denoted as $1, 1', 1'', 1'''$ and $2$. There are two group generators A and B, chosen to be

\[
A = \begin{pmatrix} i & 0 \\ 0 & -i \end{pmatrix} \quad \text{and} \quad B = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}
\]

for $2$ and satisfy the following relations

\[
A^4 = \mathbb{I}, \quad B^2 = \mathbb{I} \quad \text{and} \quad ABA = B
\]

where $\mathbb{I}$ is identity matrix.
The multiplication rules for the 1-dimensional irreps are the following

\[ 1^a \times 1 = 1^a, \quad 1 \times 1^a = 1^a, \quad 1' \times 1'' = 1''', \quad 1' \times 1''' = 1'' \text{ and } 1'' \times 1''' = 1'. \]

For \((s_1, s_2, s_3, s_4) \sim (1, 1', 1'', 1''')\) and \((x_1, x_2)^T \sim 2\) we find

\[
\begin{pmatrix} s_1 x_1 \\ s_1 x_2 \end{pmatrix} \sim 2, \quad \begin{pmatrix} s_2 x_1 \\ -s_2 x_2 \end{pmatrix} \sim 2, \quad \begin{pmatrix} s_3 x_2 \\ s_3 x_1 \end{pmatrix} \sim 2 \quad \text{and} \quad \begin{pmatrix} s_4 x_2 \\ -s_4 x_1 \end{pmatrix} \sim 2.
\]

The product a two-dimensional irreps \(2 \times 2\) decomposes into the four singlets. Taking, for instance, \((x_1, x_2)^T \sim 2\) and \((y_1, y_2)^T \sim 2\) we have that

\[ x_1 y_2 + x_2 y_1 \sim 1, \quad x_1 y_2 - x_2 y_1 \sim 1', \quad x_1 y_1 + x_2 y_2 \sim 1'' \text{ and } x_1 y_1 - x_2 y_2 \sim 1'''. \]

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