$b \to dd\bar{s}$ transition and constraints on new physics in $B^-$ decays

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The $b \to dd\bar{s}$ transition gives extremely small branching ratios within the standard model, thus providing an appropriate ground for testing new physics. Using renormalization group technique we determine the Wilson coefficients and the mixing of the operators which contribute to the $b \to dd\bar{s}$ transition. We consider contributions to this decay mode from the supersymmetric standard model with and without $R$-parity, as well as from a model with an additional neutral $Z'$ gauge boson. Using Belle and BaBar upper bounds for the $B^- \to \pi^-\pi^-K^+$ branching ratio we constrain contributions of these new physics scenarios. Then we calculate branching ratios for two- and three-body nonleptonic $B^-$ meson decays driven by the $b \to dd\bar{s}$ transition, which might be experimentally accessible.

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I. INTRODUCTION

Among many ongoing searches for physics beyond the standard model (SM), rare $B$ meson decays seem to offer good opportunities for discovering new physics. In particular the experimental results on decay rates and the parameters describing CP violation in the $B$ meson nonleptonic two-body weak decays such as $B \to \pi K$ and $B \to \phi K_S$ have attracted a lot of attention during the last few years (see e.g. [1] and references therein). In the theoretical explanation of these decay rates and CP violating parameters it is usually assumed that an interplay of the SM contributions and new physics occurs. On the other hand, there are processes of the type $b \to ss\bar{d}$ and $b \to dd\bar{s}$ which are extremely rare within the SM. A careful study of the $b \to ss\bar{d}$ transition has been done [2,3,4,5,6,7,8] and the decay $B^- \to \pi^-K^-K^-$ has been suggested as the most appropriate mode among possible candidates for experimental searches. The upper limit was first determined in [7] and subsequently constrained by both $B$ factories [8,9]. These upper bounds gave an unique opportunity to determine constraints on a variety of scenarios of new physics such as the minimal supersymmetric standard model (MSSM) with and without $R$-parity violation (RPV), variations of the two Higgs doublets model (THDM), and models with additional neutral gauge bosons. Using constraints from this decay rate the $\Delta S = 2$ two body decays of $B^-$ were considered [4,10] as well as $\Delta S = 2$ decays of $B_s, \Pi^{0^+}$.

The $b \to dd\bar{s}$ transition has not been subject of such intensive theoretical studies although experimental information on the upper bound for the $B^- \to \pi^-\pi^-K^+$ decay rate already exists. Namely, the BaBar collaboration has reported that $\text{BR}(B^- \to \pi^-\pi^-K^+) < 1.8 \times 10^{-6}$ [4], while the Belle collaboration found $\text{BR}(B^- \to \pi^-\pi^-K^+) < 4.5 \times 10^{-6}$ [11]. Hopefully soon the LHC-b would give even better constraints.

Some time ago Grossman et al. [12] have investigated the decay mechanisms of $B \to K\pi$ decays and found that new physics might give important contributions to the relevant observables. Within their study of penguin operators which could receive contributions due to new physics, these authors also included the effects of the $\Delta S = 1$ transition. In their search for the explanation of the $B \to K\pi$ puzzle, the authors of [13] have investigated the $B \to K\pi$ decay mode within a model with an extra flavor changing $Z'$ boson, making predictions for the CP violating asymmetries in these decays. $Z'$ mediated penguin operators have also been considered in many other scenarios. Contributions of supersymmetric models with and without RPV in the same decay channel were discussed in Ref. [14]. The difficulty with this decay mode is that the SM contribution is the dominant one. The use of quantum chromodynamics in the treatment of the weak hadronic $B$ meson decays is not a straightforward procedure. Numerous theoretical studies have been attempted to obtain the most appropriate framework to describe nonleptonic $B$ meson decays to two light meson states. But even the most sophisticated approaches such as QCD factorization (BBNS and SCET) [15,16,17,18,19,20,21,22,23] still have parameters which are difficult to obtain from “first principles”. Consequently, searches for new physics in decay modes dominated by SM contributions suffer from large uncertainties.

In this paper we suggest to search for the effects of new physics in rare decays for which the SM gives negligible contributions. We only consider $B^-$ meson decays driven by the $b \to dd\bar{s}$ transition, since the $\bar{u}$ anti-quark is a spectator in this process and one should not worry about possible contributions of the SM penguins. The measurement of decay rates for the modes in which the “exotic” $b \to dd\bar{s}$ transition occurs might give an unique opportunity to constrain parameters describing new physics. These constraints may then be compared with those obtained from other processes such as $K^0 - \bar{K^0}$ and $B^0 - \bar{B^0}$.

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transitions.
In the second section, we describe the $b \rightarrow dd\bar{s}$ decay and consider contributions of various new physics models. First we determine the Wilson coefficients of the hadronic operators contributing to the effective Hamiltonian in an extended operator basis, which is applicable for variety of the new physics scenarios. Namely, we investigate inclusive $b \rightarrow dd\bar{s}$ within the MSSM with and without RPV, and within an extension of the SM where an additional flavor changing $Z'$ neutral boson appears. In Sec. III we write explicit expressions for the transition matrix elements entering in exclusive nonleptonic decay rates. Then we consider contributions of various new physics models to the experimental searches. First we study the three-body decay $B^- \rightarrow \pi^- \pi^- K^+$, which has been already investigated by both $B$ meson factories. Then we derive decay rates for two-body decays $B^- \rightarrow \pi^- K^0$, $B^- \rightarrow \rho^- K^0$, $B^- \rightarrow \pi^- K^{*0}$, $B^- \rightarrow \rho^- K^{*0}$, and three-body decay $B^- \rightarrow \pi^- D^- D^+$. In Sec. IV we comment on possibilities to observe effects of new physics in considered decays and summarize our results.

II. INCLUSIVE PROCESSES

The effective weak Hamiltonian encompassing the $b \rightarrow dd\bar{s}$ process has been introduced by the authors of \[12\] in the case of $B \rightarrow K\pi$ decays. Following their notation we write it as

$$\mathcal{H}_{\text{eff}} = \sum_{n=1}^{5} \left[ C_n \mathcal{O}_n + \tilde{C}_n \tilde{\mathcal{O}}_n \right],$$

(1)

where $C_i$ and $\tilde{C}_i$ denote effective Wilson coefficients multiplying the complete operator basis of all the four-quark operators which can contribute to the process $b \rightarrow dd\bar{s}$. We choose

$$\mathcal{O}_1 = \bar{d}_L^i \gamma^\mu b_R^i \bar{d}_R^j \gamma^\mu s_L^j,$$

$$\mathcal{O}_2 = \bar{d}_L^i \gamma^\mu b_R^i \bar{d}_R^j \gamma^\mu t_L^j,$$

$$\mathcal{O}_3 = \bar{d}_L^i \gamma^\mu b_R^i \bar{d}_R^j \gamma^\mu s_L^j,$$

$$\mathcal{O}_4 = \bar{d}_L^i \gamma^\mu b_R^i \bar{d}_R^j \gamma^\mu t_L^j,$$

$$\mathcal{O}_5 = \bar{d}_L^i \gamma^\mu b_R^i \bar{d}_R^j \gamma^\mu s_L^j,$$

(2)

plus additional operators $\tilde{\mathcal{O}}_{1,2,3,4,5}$, with the chirality exchanges $L \leftrightarrow R$. In these expressions, the superscripts $i, j$ are SU(3) color indices. All other operators with the correct Lorentz and color structure can be related to these by operator identities and Fierz rearrangements. We perform our calculations of inclusive and exclusive decays at the scale of the $b$ quark mass ($\mu = m_b$), therefore we have to take into account the renormalization group running of these operators from the interaction scale $\Lambda$. At leading log order in the strong coupling, the operators $\mathcal{O}_{1,2}$ mix with the anomalous dimension matrix

$$\gamma(\mathcal{O}_1\mathcal{O}_2) = \frac{\alpha_s}{2\pi} \begin{pmatrix} -8 & 0 \\ -3 & 1 \end{pmatrix}.$$  

(3)

The same holds for operators $\mathcal{O}_{3,4,5}$ ($\gamma(\mathcal{O}_1\mathcal{O}_3) = \gamma(\mathcal{O}_1\mathcal{O}_5)$) due to Fierz identities, while the operator $\mathcal{O}_3$ has anomalous dimension $\gamma(\mathcal{O}_3) = \alpha_s/\pi$. Anomalous matrices for chirally flipped operators $\tilde{\mathcal{O}}_{1,2,3,4,5}$ are identical to these.

Within the SM only the operator $\mathcal{O}_3$ contributes to the $b \rightarrow dd\bar{s}$ transition at one loop with the Wilson coefficient

$$C_{3}^{\text{SM}} = \frac{G_F^2}{4\pi^2} m_W^2 V_{tb} V_{td}^* \left[ V_{ts} V_{td}^* f \left( \frac{m_W^2}{m_t^2} \right) \right. + \left. V_{cs} V_{cd}^* g \left( \frac{m_W^2}{m_c^2} \frac{m_s^2}{m_W^2} \right) \right],$$

(4)

where the functions $f(x)$ and $g(x, y)$ were given explicitly in \[2\]. Using numerical values of the relevant Cabibbo-Kobayashi-Maskawa (CKM) matrix elements from PDG \[27\] and including the $V_{td}$ phase one finds $|C_{3}^{\text{SM}}| \leq 2.5 \times 10^{-13}$ GeV$^{-2}$. Renormalization group running from the weak interaction scale to the bottom quark mass scale, due to anomalous dimension of the operator $\mathcal{O}_3$, induces only a small correction factor which can be safely neglected. The inclusive $b \rightarrow dd\bar{s}$ decay width within the SM is then \[14\]

$$\Gamma_{\text{inc}}^{\text{SM}} = \frac{C_{3}^{\text{SM}} \frac{m_b^5}{48(2\pi)^3}}{10^{14}},$$

(5)

which leads to the branching ratio of the order $10^{-14}$.

Next we discuss contributions of several models containing physics beyond the SM: the MSSM with and without RPV and a model with an extra $Z'$ boson. For the THDM on the other hand, the contributions to the $b \rightarrow dd\bar{s}$ transition coming from charged Higgs box diagrams were found to be negligible. Namely, due to the CKM matrix elements suppression they would be even smaller than those found in \[3\] for the analogue case of $b \rightarrow ss\bar{d}$. Consequently we choose to neglect them. In addition, the tree level neutral Higgs exchange amplitude is proportional to $|\xi_b \xi_{dL}|/m_H^2$, where $\xi_b$ and $\xi_{dL}$ are flavor changing Yukawa couplings and $m_H$ is a common Higgs mass scale. This ratio is constrained from the neutral meson mixing $\mathcal{R}$. Using presently known values of $\Delta m_K$ and $\Delta m_B$ \[27\] one can obtain an upper bound of $|\xi_b \xi_{dL}|/m_H^2 < 10^{-13}$ GeV$^{-2}$ rendering also this contribution negligible \[43\].

In the MSSM, like in the SM, the main contribution comes from the $\mathcal{O}_3$ operator, while the corresponding Wilson coefficient is here

$$C_{3}^{\text{MSSM}} = \frac{\alpha_s^2}{216 m_d^2} \left( \frac{\delta_{3L}^{dL}}{\delta_{3L}^{dL}} \right) \left( \delta_{3L}^{dL} \right)_{LL} [24 x f_6(x) + 66 \tilde{f}_6(x)],$$

(6)

as found in analyses \[29\] taking into account only contributions from the left-handed squarks in the loop. The recent limits on $\delta_{3L}^{dL}$ \[29, 30, 31\] disallow significant contributions from the mixed and the right-handed squark mass insertion terms. Therefore, we only include the dominant contributions given in the above expression.
We follow Ref. [30] and take \( x = m_{\tilde{q}}^2/m_{\tilde{q}^2} = 1 \) and the corresponding values of \([\delta^d_{13}]_{LL}(x = 1) \leq 0.14\) and \([\delta^d_{21}]_{LL}(x = 1) \leq 0.042\) [28]. We take for the average mass of squarks \( m_{\tilde{q}} = 500 \) GeV and for the strong coupling constant \( \alpha_S = 0.12 \), and find \( |C^MSSM| \leq 1.6 \times 10^{-12} \) GeV\(^{-2}\). Using Eq. [30] and substituting for the correct Wilson coefficient one finds the MSSM prediction for the inclusive \( b \to d \bar{s} s \) decay branching ratio of the order of \( 10^{-12} \).

If RPV interactions are included in the MSSM, the part of the superpotential which becomes relevant here is \( W = \lambda'_{ijk} L_i Q_j d_k \), where \( i, j, \) and \( k \) are family indices, and \( L, Q \) and \( d \) are superfields for the lepton doublet, the quark doublet, and the down-type quark singlet, respectively. The tree level effective Hamiltonian receives contributions from the operators \( \mathcal{O}_4 \) and \( \mathcal{O}_6 \) with the Wilson coefficients defined at the interaction scale \( \Lambda \sim m_{\tilde{q}} \)

\[
C_4^{\text{RPV}} = -\frac{3}{2} \frac{\lambda_{31} \lambda_{n12}}{m_{\tilde{q}}^2} \\
\tilde{C}_4^{\text{RPV}} = -\frac{3}{2} \frac{\lambda_{31} \lambda_{n13}}{m_{\tilde{q}}^2}.
\]

which is of the order \( \tilde{f}_{\text{QCD}}(m_{\tilde{q}}) \lesssim 0.4 \) for the chosen sneutrino mass range. The relevant part of the effective Hamiltonian we use in this scenario is then

\[
\mathcal{H}^{\text{eff,RPV}} = f_{\text{QCD}}(\mu) \left[ C_4^{\text{RPV}} \mathcal{O}_4(\mu) + \tilde{C}_4^{\text{RPV}} \tilde{\mathcal{O}}_4(\mu) \right] \\
+ \tilde{f}_{\text{QCD}}(\mu) \left[ C_4^{\text{RPV}} \mathcal{O}_5(\mu) + \tilde{C}_4^{\text{RPV}} \tilde{\mathcal{O}}_5(\mu) \right].
\]

We neglect the \( \tilde{f}_{\text{QCD}} \) suppressed contributions of \( \mathcal{O}_5, \tilde{\mathcal{O}}_5 \) to the amplitudes in the cases where the operators \( \mathcal{O}_4, \tilde{\mathcal{O}}_4 \) give non-zero contribution. The inclusive \( b \to d \bar{s} s \) decay rate induced by the RPV model becomes

\[
\Gamma_{\text{inc,RPV}} = \frac{m_{\tilde{q}}^5 f_{\text{QCD}}^2(m_{\tilde{q}})}{256(2\pi)^3} \left( |C_4^{\text{RPV}}|^2 + |	ilde{C}_4^{\text{RPV}}|^2 \right).
\]

Present experimental bounds on the individual RPV couplings contributing to the effective Wilson coefficients \( C_4^{\text{RPV}} \) and \( \tilde{C}_4^{\text{RPV}} \) do not constrain this mode, and we extract the bounds on the relevant combination from exclusive decays in Sec. [14].

In many extensions of the SM [32], an additional neutral gauge boson appears. Heavy neutral bosons are also present in grand unified theories, superstring theories and theories with large extra dimensions [33]. This induces contributions to the effective tree level Hamiltonian from the operators \( \mathcal{O}_{1,3} \) as well as \( \mathcal{O}_{1,3} \). Following [32, 33], the Wilson coefficients for the corresponding operators read at the interaction scale \( \Lambda \sim m_{Z'} \)

\[
f_{\text{QCD}}(\mu) = \begin{cases} 
\frac{\alpha(\mu)}{\alpha(\Lambda)}^{24/23}, & \Lambda < m_t \\
\frac{\alpha(\mu)}{\alpha(\Lambda)}^{24/23} - \frac{\alpha(\mu)}{\alpha(m_t)}^{24/21}, & \Lambda > m_t 
\end{cases}
\]

(8)

The renormalization group running of the operators induces a common correction factor for \( C_4^{\text{RPV}}(\mu) = f_{\text{QCD}}(\mu) C_4^{\text{RPV}} \) and \( \tilde{C}_4^{\text{RPV}}(\mu) = f_{\text{QCD}}(\mu) \tilde{C}_4^{\text{RPV}} \):

\[
C_4^{\text{RPV}} = \frac{-4G_F y B_{d_{12}}^{d_{13}}}{\sqrt{2}} C_4^{\text{RPV}}, \quad \tilde{C}_4^{\text{RPV}} = \frac{-4G_F y B_{d_{12}}^{d_{13}}}{\sqrt{2}} \tilde{C}_4^{\text{RPV}},
\]

\[
C_3^{\text{RPV}} = \frac{-4G_F y B_{d_{12}}^{d_{13}}}{\sqrt{2}} C_3^{\text{RPV}}, \quad \tilde{C}_3^{\text{RPV}} = \frac{-4G_F y B_{d_{12}}^{d_{13}}}{\sqrt{2}} \tilde{C}_3^{\text{RPV}},
\]

(12)

where \( y = \frac{g_3^2}{g_2^2} (\rho_1 \sin^2 \theta + \rho_2 \cos^2 \theta) \) and \( \rho_i = m_{W}/m_{Z'}^2 \sin^2 \theta W \). In this expression \( g_1, g_2, m_1 \) and \( m_2 \) stand for the gauge couplings and masses of the Z and Z' bosons, respectively, while \( \theta \) is their mixing angle. Again renormalization group running induces corrections and mixing between the operators. As already mentioned, the mixing of operators \( \mathcal{O}_{1,2} \) and their chirally flipped counterparts is identical to that of operators \( \mathcal{O}_{1,5} \) since these operators are connected via Fierz rearrangement. Thus the same scaling and mixing factors \( f_{\text{QCD}} \) and \( f_{\text{QCD}} \) apply. For the operator \( \mathcal{O}_3 \) on the other hand the renormalization can be written as \( C_3^{\text{RPV}}(\mu) = f_{\text{QCD}}(\mu) C_3^{\text{RPV}} \) with
\[ f_{QCD}(\mu) = \begin{cases} 
\left[ \frac{\alpha_s(\mu)}{\alpha_s(\Lambda)} \right]^{-6/23}, & \Lambda < m_t \\
\left[ \frac{\alpha_s(\mu)}{\alpha_s(\Lambda)} \right]^{-6/21}, & \Lambda > m_t \end{cases} \] (13)

In particular for a common Z' boson scale of \( m_{Z'} \approx 500 \text{ GeV} \), one gets numerically \( f_{QCD}(m_b) \approx 2 \), \( \tilde{f}_{QCD}(m_b) \approx 0.4 \) and \( f'_{QCD}(m_b) \approx 0.8 \). The full contributing part of the effective Hamiltonian in this case is

\[ \mathcal{H}_{\text{eff}}^{Z'} = f_{QCD}(\mu) \left[ C_1^{Z'} \mathcal{O}_1(\mu) + \check{C}_1^{Z'} \mathcal{\check{O}}_1(\mu) \right] + f_{QCD}(\mu) \left[ C_1^{Z'} \mathcal{O}_2(\mu) + \check{C}_1^{Z'} \mathcal{\check{O}}_2(\mu) \right] + f_{QCD}(\mu) \left[ C_3^{Z'} \mathcal{O}_3(\mu) + \check{C}_3^{Z'} \mathcal{\check{O}}_3(\mu) \right]. \] (14)

For the inclusive \( b \to d\bar{s}s \) decay rate the \( \mathcal{O}_2 \) and \( \check{\mathcal{O}}_2 \) are numerically suppressed due to the \( f_{QCD} \) factor and we write

\[ \Gamma^{Z'}_{\text{inc}} = \frac{m_5^2}{192(2\pi)^3} \left[ 3f_{QCD}^2(m_b) \left( |C_{1}^{Z'}|^2 + |\check{C}_{1}^{Z'}|^2 \right) + 4f_{QCD}^2(m_b) \left( |C_{3}^{Z'}|^2 + |\check{C}_{3}^{Z'}|^2 \right) \right]. \] (15)

In Sec. IV we discuss bounds on Wilson coefficients \( C_{1,3}^{Z'} \) and \( \tilde{C}_{1,3}^{Z'} \) which might be estimated from the \( B^{-} \to \pi^{-}\pi^{-}K^{+} \) decay rate.

### III. EXCLUSIVE \( B^{-} \) DECAY MODES

In calculating decay rates of various \( B \) meson decay modes based on the \( b \to d\bar{s}s \) quark transition, one has to calculate matrix elements of the effective Hamiltonian operators between meson states. As a first approximation, we use the naive factorization of three-body amplitudes, and express the resulting two-body transition amplitudes between mesons in terms of the standard weak transition form factors, as dictated by the Lorentz covariance. For the \( B \to \pi(\mu) \) transitions we use form factors calculated in the relativistic constituent quark model, with numerical input from the lattice QCD at high momentum transfer squared. For the \( D_s \to D \) and \( K \to \pi \) transition form factors we use results of Refs. [10, 35], where heavy meson effective theory and chiral Lagrangian approach were used.

For the decays of the \( B^{-} \) meson to three pseudoscalar mesons \( P, P_1 \) and \( P_2 \) we first derive a general expression for the factorized matrix element of the \( \mathcal{O}_3 \) operator, relevant in the framework of SM (MSSM)

\[ \langle P_2(p_2)P_1(p_1)\bar{d}q_{\mu} s|0\rangle \langle P(p)|\bar{d}\gamma^{\mu} b|B^{-}(p_B)\rangle = (t - u)F_{1}^{P_1P_2}(s)F_{P}^{P}(s) \]
\[ + \frac{(m_{P_1}^2 - m_{P_2}^2)(m_{P_2}^2 - m_{P}^2)}{s} \left[ F_{1}^{P_1P_2}(s)F_{P}^{P}(s) - F_{0}^{P_1P_2}(s)F_{0}^{P}(s) \right]. \] (16)

Because only vector currents contribute in the above expression, it also applies for operators \( \mathcal{O}_3, \mathcal{O}_1 \) and \( \mathcal{O}_1 \). Form factors \( F_1 \) and \( F_0 \) are defined in the Appendix A and the Mandelstam kinematical variables are \( s = (p_B - p)^2, t = (p_B - p_1)^2 \) and \( u = (p_B - p_2)^2 \).

In the context of RPV, the contributions of the operators \( \mathcal{O}_4 \) and \( \mathcal{\check{O}}_4 \) to hadronic amplitudes are dominant. One can use the Dirac equation to express scalar (pseudoscalar) density operators in terms of derivatives of vector (axial-vector) currents

\[ \bar{q}_i q_j = i\frac{\partial_\mu (\bar{q}_i \gamma^{\mu} q_j)}{m_{q_j} - m_{q_i}}, \] (17a)
\[ \bar{q}_i \gamma^{5} q_j = i\frac{\partial_\mu (\bar{q}_i \gamma^{\mu} \gamma^{5} q_j)}{m_{q_j} + m_{q_i}}. \] (17b)

Using these relations we derive an expression for the factorized matrix element of the \( \mathcal{O}_4 \) and \( \mathcal{\check{O}}_4 \) operators, contributing only with their scalar parts

\[ \langle P_2(p_2)P_1(p_1)\bar{d}s|0\rangle \langle P(p)|\bar{d}b|B^{-}(p_B)\rangle = \frac{(m_{P_1}^2 - m_{P_2}^2)(m_{P_2}^2 - m_{P}^2)}{(m_{b} - m_{d})(m_{s} - m_{d})} F_{1}^{P_1P_2}(s)F_{P}^{P}(s). \] (18)

In the case of the \( Z' \) model one encounters contributions of the operators \( \mathcal{O}_{1,2,3} \) and \( \mathcal{\check{O}}_{1,2,3} \). The color nonsinglet operators \( \mathcal{O}_2 \) and \( \mathcal{\check{O}}_2 \) can be Fierz rearranged to \( \mathcal{O}_4 \) and \( \mathcal{\check{O}}_4 \) and then Eq. 18 applies as well. Remaining operators are all of the \( V \pm A \) form and their contribution to the amplitude is already given by Eq. 10.

In two-body decays with a vector meson \( V \) and a pseudoscalar meson \( P \) in the final state we sum over the polarizations of \( V \). The sum in our case reduces to

\[ \sum_{\nu} |\epsilon_{\nu}^\star (p_{V} \cdot p_{B})|^2 \frac{\lambda(m_{V}^2, m_{P}^2, m_{P}^2)}{4m_{V}^2}, \] (19)

where \( \epsilon_{\nu}^\star \) is the polarization vector of \( V \) and \( \lambda \) is defined as \( \lambda(x, y, z) = (x + y + z)^2 - 4(xy + yz + zx) \).
For decay to two vector mesons in the final state we use the helicity amplitudes formalism as described in Ref. [30]. Non-polarized decay rate is expressed as an incoherent sum of helicity amplitudes

\[ \Gamma = \frac{|p_1|}{8\pi m_B} \left( |H_0|^2 + |H_{+1}|^2 + |H_{-1}|^2 \right), \tag{20} \]

where \( p_1 \) is momentum of the vector meson in \( B^- \) meson rest frame and helicity amplitudes are expressed as

\[ H_{\pm 1} = a \pm \sqrt{\lambda(m_B^2, m_1^2, m_2^2)} c, \tag{21a} \]

\[ H_0 = -\frac{m^2 - m_1^2 - m_2^2}{2m_1m_2} a - \frac{\lambda(m_B^2, m_1^2, m_2^2)}{4m_1^2m_2^2} b. \tag{21b} \]

Vector meson masses are denoted by \( m_{1,2} \), while definition of the constants \( a, b \) and \( c \) is given by general Lorentz decomposition of the polarized amplitude

\[ H_\lambda = \epsilon_{\mu \nu}^1(\lambda)\epsilon_{\nu 2}(\lambda) \left( ag^{\mu \nu} + \frac{b}{m_1m_2} p_B^\mu p_B^\nu \right. \]

\[ + \frac{ic}{m_1m_2} \epsilon^{\mu \nu \rho \sigma} p_1^\rho p_2^\sigma \right), \tag{22} \]

where \( \epsilon_{1,2} \) and \( p_{1,2} \) are the vector mesons polarizations and momenta.

### A. \( B^- \rightarrow \pi^- \pi^- K^+ \)

Hadronic matrix element entering in the amplitude for \( B^- \rightarrow \pi^- \pi^- K^+ \) in SM (MSSM) is readily given by Eq. [10], after identifying \( P = \pi^- \), \( P_1 = K^+ \), \( P_2 = \pi^- \) and using appropriate form factors given in the Appendix A. Eq. [18] is used instead for RPV, while the \( Z' \) amplitude incorporates both Eqs. [10] and [18]. There are two contributions in each model to this mode, with an additional term with the \( u \leftrightarrow s \) replacement in Eqs. [10] and [18], representing an interchange of the two pions in the final state. After phase space integration, the decay rates can be written very compactly with only Wilson coefficients left in symbolic form:

\[ \Gamma^{(MS)SM}_{\pi K} = |C^{(MS)SM}_3|^2 \times 2.0 \times 10^{-3} \text{ GeV}^5, \tag{23} \]

\[ \Gamma^{\text{RPV}}_{\pi K} = |C_4^{\text{RPV}} + \tilde{C}_4^{\text{RPV}}|^2 \times 9.2 \times 10^{-3} \text{ GeV}^5, \tag{24} \]

\[ \Gamma^{Z'}_{\pi K} = |C_1^{Z'} + \tilde{C}_1^{Z'}|^2 \times 1.0 \times 10^{-2} \text{ GeV}^5 \]

\[ + |C_2^{Z'} + \tilde{C}_2^{Z'}|^2 \times 1.3 \times 10^{-3} \text{ GeV}^5 \]

\[ + \text{Re} \left[ \left( C_1^{Z'} + \tilde{C}_1^{Z'} \right) \left( C_2^{Z'} + \tilde{C}_2^{Z'} \right)^* \right] \times 6.7 \times 10^{-3} \text{ GeV}^5. \tag{25} \]

### B. \( B^- \rightarrow \pi^- D^- D_s^+ \)

In calculation of the \( B^- \rightarrow \pi^- D^- D_s^+ \) decay rate again we use Eqs. [10] and [18] now with substitutions \( P = \pi^- \), \( P_1 = D_s^+ \) and \( P_2 = D^- \). Numerically this yields

\[ \Gamma^{(MS)SM}_{\pi DD_s} = |C_3^{(MS)SM}|^2 \times 8.7 \times 10^{-9} \text{ GeV}^5, \tag{26} \]

\[ \Gamma^{\text{RPV}}_{\pi DD_s} = |C_4^{\text{RPV}} + \tilde{C}_4^{\text{RPV}}|^2 \times 8.4 \times 10^{-5} \text{ GeV}^5, \tag{27} \]

\[ \Gamma^{Z'}_{\pi DD_s} = |C_1^{Z'} + \tilde{C}_1^{Z'}|^2 \times 1.5 \times 10^{-5} \text{ GeV}^5 \]

\[ + |C_2^{Z'} + \tilde{C}_2^{Z'}|^2 \times 5.5 \times 10^{-9} \text{ GeV}^5 \]

\[ + \text{Re} \left[ \left( C_1^{Z'} + \tilde{C}_1^{Z'} \right) \left( C_2^{Z'} + \tilde{C}_2^{Z'} \right)^* \right] \times 5.7 \times 10^{-7} \text{ GeV}^5. \tag{28} \]

These decay rates are suppressed due to the small phase space in comparison to the rates of the \( B^- \rightarrow \pi^- \pi^- K^+ \) decay.

### C. \( B^- \rightarrow \pi^- K^0 \)

The authors of Ref. [32] addressed this decay mode as the wrong kaon mode, being highly suppressed in the SM compared to the decay with \( K^0 \) in the final state. The operators \( O_{1,3} \) and \( \tilde{O}_{1,3} \) that are present in SM (MSSM) and \( Z' \) model have the following contribution:

\[ \left\langle K^0(p_K)\left| \bar{d}\gamma^5 s \right| 0 \right\rangle \left\langle \pi^-(p_\pi)\left| \bar{d}\gamma^\mu b \right| B^-(p_B) \right\rangle \]

\[ = i(m_B^2 - m_\pi^2)f_{K^0B}(m_K^2). \tag{29} \]

Operators \( \tilde{O}_4 \) and \( \tilde{O}_4 \), relevant for the RPV and \( Z' \) models result in

\[ \left\langle K^0(p_K)\left| \bar{d}\gamma^5 s \right| 0 \right\rangle \left\langle \pi^-(p_\pi)\left| \bar{d}\gamma^\mu b \right| B^-(p_B) \right\rangle \]

\[ = \frac{im_B^2(m_B^2 - m_\pi^2)}{(m_b - m_d)(m_\pi + m_d)} f_{K^0B}(m_K^2). \tag{30} \]

However, in the latter two models, the two chirally flipped contributions to the amplitude have opposite signs, resulting in a slightly different combination of Wilson coefficients in comparison with the \( B^- \rightarrow \pi^- \pi^- K^+ \) decay rate

\[ \Gamma^{(MS)SM}_{\pi K} = |C_3^{(MS)SM}|^2 \times 3.9 \times 10^{-4} \text{ GeV}^5, \tag{31} \]

\[ \Gamma^{\text{RPV}}_{\pi K} = |C_4^{\text{RPV}} - \tilde{C}_4^{\text{RPV}}|^2 \times 4.9 \times 10^{-4} \text{ GeV}^5, \tag{32} \]

\[ \Gamma^{Z'}_{\pi K} = |C_1^{Z'} - \tilde{C}_1^{Z'}|^2 \times 9.5 \times 10^{-4} \text{ GeV}^5 \]

\[ + |C_3^{Z'} - \tilde{C}_3^{Z'}|^2 \times 2.5 \times 10^{-4} \text{ GeV}^5 \]

\[ - \text{Re} \left[ \left( C_1^{Z'} - \tilde{C}_1^{Z'} \right) \left( C_3^{Z'} - \tilde{C}_3^{Z'} \right)^* \right] \times 9.8 \times 10^{-4} \text{ GeV}^5. \tag{33} \]
Using form factors parameterizing $^{A3}_{x}$ of the pseudoscalar to vector meson transition we derive the following two factorized matrix elements of axial-vector and pseudoscalar operators:

$$
\langle K^0(p_K) | d \gamma^\mu \gamma^5 s | 0 \rangle \langle p^-(\epsilon_{p}, p_{p}) | d \gamma^\mu \gamma^5 b | B^-(p_B) \rangle
$$

$$
= 2m_{p}f_{K}A^{AB}_0(m^2_\rho)\epsilon^*_p \cdot p_B,
$$

$$
\langle K^0(p_K) | d \gamma^5 s | 0 \rangle \langle p^- (\epsilon_{p}, p_{p}) | d \gamma^5 b | B^-(p_B) \rangle
$$

$$
= \frac{2m_{p}m^2_\rho}{(m_b + m_d)(m_s + m_d)}f_{K}A^{PB}_0(m^2_\rho)\epsilon^*_p \cdot p_B.
$$

Finally, we sum over polarizations of the $\rho$ meson using Eq. (19), and the unpolarized decay rates read

$$
\Gamma^{(MS)SM}_{\rho K} = |C^3_{(MS)SM}|^2 \times 3.9 \times 10^{-4} \text{ GeV}^5,
$$

$$
\Gamma^{RPV}_{\rho K} = |C^4_{RPV} + \tilde{C}^{RPV}|^2 \times 4.9 \times 10^{-4} \text{ GeV}^5,
$$

$$
\Gamma^\prime_{\rho K} = |C^3_{1} + \tilde{C}^{3}_{1}|^2 \times 2.3 \times 10^{-3} \text{ GeV}^5
$$

$$
+ |C^3_{2} + \tilde{C}^{3}_{2}|^2 \times 2.5 \times 10^{-4} \text{ GeV}^5
$$

$$
- \text{Re} \left( \left| \left( C^3_{1} + \tilde{C}^{3}_{1} \right) \left( C^3_{2} + \tilde{C}^{3}_{2} \right)^* \right| \right)
$$

$$
\times 1.5 \times 10^{-3} \text{ GeV}^5.
$$

E. $B^- \rightarrow \pi^- K^{0}$

Factorized matrix element is here a product of vector meson $K^{0}$ creation amplitude $^{A2b}_{x}$ and $B^- \rightarrow \pi^-$ transition amplitude. Operators which involve vector currents result in

$$
\langle K^0(\epsilon_K, p_K) | d \gamma^\mu s | 0 \rangle \langle p^- (\epsilon_p, p_p) | d \gamma^\mu b | B^-(p_B) \rangle
$$

$$
= 2g_{K}F_1^{AB}(m^2_{K^{*}})\epsilon^*_K \cdot p_B,
$$

while the density operators $\bar{Q}_3$ and $\bar{Q}_2$ do not contribute, as a result of Eqs. (17) and (18). Consequently, in the RPV model this mode is dominated by the operators $\bar{Q}_5$ and $\bar{Q}_6$ which are, as mentioned in Sec. II suppressed by the renormalization group running. Using Fierz rearrangements, we write them down as $\bar{Q}_1$, $\bar{Q}_2$, and yield an additional 1/2 suppression factor.

$$
\Gamma^{(MS)SM}_{\pi K^*} = |C^3_{3(0)SM}|^2 \times 7.4 \times 10^{-4} \text{ GeV}^5,
$$

$$
\Gamma^{RPV}_{\pi K^*} = |C^4_{RPV} + \tilde{C}^{RPV}|^2 \times 2.9 \times 10^{-5} \text{ GeV}^5
$$

$$
\Gamma^\prime_{\pi K^*} = |C^3_{1} + \tilde{C}^{3}_{1}|^2 \times 2.9 \times 10^{-3} \text{ GeV}^5
$$

$$
+ |C^3_{2} + \tilde{C}^{3}_{2}|^2 \times 4.7 \times 10^{-4} \text{ GeV}^5
$$

$$
+ \text{Re} \left( \left| \left( C^3_{1} + \tilde{C}^{3}_{1} \right) \left( C^3_{2} + \tilde{C}^{3}_{2} \right)^* \right| \right)
$$

$$
\times 2.3 \times 10^{-3} \text{ GeV}^5.
$$

IV. DISCUSSION AND RESULTS

We have investigated the $b \rightarrow d\bar{s}d$ transition within the SM, MSSM without and with RPV terms and within a model with an extra $Z'$ gauge boson. The SM contribution leads to extremely small branching ratio for this transition.

First we have calculated the effects of QCD on the Wilson coefficients caused by the renormalization group run-
ning. The moderate increase of the MSSM compared to the SM predictions is still too insignificant for any experimental search. The MSSM with RPV terms, however, might give significant contributions and a possibility to shrink down the parameter space even further. The $Z'$ model exhibits its structure through interplay of different interaction scale couplings and might also give opportunity to constrain its relevant parameters. In the case of the two Higgs doublet model we do not expect any sizable effect as already noticed in the case of $b \to ssd$ decays [3].

In the $b \to d\bar{s}s$ decay a particular combination of the model parameters appear which can be constrained using the $B^- \to \pi^-\pi^-K^+$ decay mode. In our calculation we have relied on the naive factorization approximation, which is as a first approximation sufficient to obtain correct gross features of new physics effects. One might think that the nonfactorizable contributions might induce large additional uncertainties, but we do not expect them to change the order of magnitude of our predictions. Additional uncertainties might originate in the poor knowledge of the input parameters such as form factors. However, we do not expect these to exceed more than 30%.

Using the stringest experimental bound for the $\text{BR}(B^- \to \pi^-\pi^-K^+) < 1.8 \times 10^{-6}$ and normalizing the masses of sneutrinos to a common mass scale of 100 GeV we derive bounds on the RPV terms given in Eq. (47)

$$\left| \sum_{n=1}^{3} \left( \frac{100 \text{ GeV}}{m_{\tilde{\nu}_n}} \right)^2 (\lambda'_{n31}\lambda'^*_{n12} + \lambda'_{n21}\lambda'^*_{n13}) \right| < 8.9 \times 10^{-5}. \quad (47)$$

Complementary bounds coming from measurements of $K^0 \to \bar{K}^0$ and $B^0 - \bar{B}^0$ mixings have been established in Refs. [37, 38].

$$\begin{align*}
\text{Re} \left[ \sum_{n=1}^{3} \left( \frac{100 \text{ GeV}}{m_{\tilde{\nu}_n}} \right)^2 \lambda'_{n31}\lambda'^*_{n12} \right] &< 2.6 \times 10^{-5}, \quad (48a) \\
\text{Im} \left[ \sum_{n=1}^{3} \left( \frac{100 \text{ GeV}}{m_{\tilde{\nu}_n}} \right)^2 \lambda'_{n31}\lambda'^*_{n12} \right] &< 2.9 \times 10^{-5}, \quad (48b) \\
\text{Re} \left[ \sum_{n=1}^{3} \left( \frac{100 \text{ GeV}}{m_{\tilde{\nu}_n}} \right)^2 \lambda'_{n21}\lambda'^*_{n13} \right] &< 2.9 \times 10^{-4}. \quad (48c)
\end{align*}$$

From Eqs. (48a) and (48b) it becomes apparent that the $\lambda'_{n31}\lambda'^*_{n12}$ term is negligible in Eq. (47), and the bound becomes simpler

$$\left| \sum_{n=1}^{3} \left( \frac{100 \text{ GeV}}{m_{\tilde{\nu}_n}} \right)^2 \lambda'_{n21}\lambda'^*_{n13} \right| < 8.9 \times 10^{-5}, \quad (49)$$

now being more restrictive than Eq. (48c), obtained from $B^0 - \bar{B}^0$ mixing.

Assuming that new physics arises due to an extra $Z'$ gauge boson we derive bounds on the parameters given in Eq. (12). Making the simplest assumption, we neglect interference between Wilson coefficients (third term) in Eq. (24). Experimental bound of this simplified expression now confines $(|C^I_{1'} + \tilde{C}^{Z'}_{1'}|, |C^I_{3'} + \tilde{C}^{Z'}_{3'}|)$ to lie within an ellipse with semiminor and semimajor axes as upper limits

$$\begin{align*}
y \left| B^{d_L}_{12} B^{d_R}_{13} + B^{d_R}_{12} B^{d_L}_{13} \right| &< 2.6 \times 10^{-4}, \quad (50a) \\
y \left| B^{d_L}_{12} B^{d_L}_{13} + B^{d_R}_{12} B^{d_R}_{13} \right| &< 7.1 \times 10^{-4}. \quad (50b)
\end{align*}$$

Complementary bounds, involving the same couplings and $y$, originate from meson mass splittings and CP violation in kaon system and have been derived in Refs. [32, 33].

$$\begin{align*}
y \left| \text{Re}[B^{d_{R,L}}_{12}] \right| &< 10^{-8}, \quad (51a) \\
y \left| \text{Re}[B^{d_{R,L}}_{13}] \right| &< 6 \times 10^{-8}, \quad (51b) \\
y \left| \text{Im}[B^{d_{R,L}}_{12}] \right| &< 8 \times 10^{-11}. \quad (51c)
\end{align*}$$

To combine those bounds with Eqs. (50), one should absorb dimensionless $y$ into the coupling constants by redefinition $\hat{B}^{d_{R,L}}_{12(13)} = \sqrt{y} B^{d_{R,L}}_{12(13)}$. However, when we include the constraints given by Eqs. (50), we obtain no further improvement of the bounds on individual $B$ couplings.

Nevertheless, the bounds (47) and (51) are interesting since they offer an independent way of constraining the particular combination of the parameters, which are not constrained by the $B^0_\rho - B^{\rho}_{0}$, $B^0_\omega - B^{\omega}_{0}$, $K^0 - \bar{K}^0$ oscillations, or by $B^- \to K^-K^-\pi^+$ decay rate (see e.g. [1]).

Using these inputs we predict the branching ratios for the various possible two-body decay modes and the $B^- \to \pi^-D^-D^+\pi^+$ decay. Applying bound (12) to the RPV model decay rates is straightforward, except for the $B^- \to \pi^-K^0$ and $B^- \to \rho^-K^0$ decay modes. In order to make predictions for these two modes, we assume as in Eq. (13), that interference term $C_4^{\text{RPV}}\tilde{C}_4^{\text{RPV}}$ is negligible, which leads to the approximation $|C_4^{\text{RPV}} - \tilde{C}_4^{\text{RPV}}| \simeq |C_4^{\text{RPV}} + \tilde{C}_4^{\text{RPV}}|$. In the case of the $Z'$ model, there are contributions from Wilson coefficients “1” ($C^I_{Z'}$, $\tilde{C}^{Z'}_{2}$) and “3” ($C^I_{3'}$, $\tilde{C}^{Z'}_{3}$). We have already neglected the interference terms between “1” and “3” in Eq. (24) to obtain bounds (11) and we assume that these terms are small for all considered decay modes. Using Eqs. (50) we can now predict branching ratios for decay modes $B^- \to \pi^-D^-D^+$, $B^- \to \pi^-K^0$, and $B^- \to \rho^-K^0$. The remaining two decay rates $B^- \to \pi^-K^0$ and $B^- \to \rho^-K^0$ can be considered after we neglect interference terms $C^I_{2'}\tilde{C}^{Z'}_{1}$ and $C^I_{2'}\tilde{C}^{Z'}_{3}$. The results are summarized in Table IV.

The SM gives negligible contributions. The MSSM is increasing them by two orders of magnitude, which is still insufficient for the current and foreseen experimental searches. Using constraints for the particular combination of the RPV parameters present in the $B^- \to \pi^-\pi^-K^+$ decay we obtain the largest possible branching ratios for the two-body decays of $B^- \to \rho^-K^0$.
Namely, the branching ratio $B$ to observe new physics in the impossible. Therefore, the two-body decay modes with extraction of new physics from this decay mode almost by the Slovenian Research Agency. This work is supported in part by the experimental searches of new physics in the model with an additional $Z'$ boson (V column).

and $B^- \to \pi^- K^0$, while for the $B^- \to \pi^- K^{*0}$ and $B^- \to \rho^- K^{*0}$ the RPV contribution is suppressed by the renormalization group running. Their order of magnitude is $10^{-9}$ and thus still experimentally unreachable. However, these two decay channels are most likely to be observed in the model with an additional $Z'$ boson, if we assume that interference terms are negligible.

Since in the experimental measurements only $K_S$ or $K_L$ are detected and not $K^0$ or $\bar{K}^0$, it might be difficult to observe new physics in the $B^- \to \pi^- K^0$ decay mode. Namely, the branching ratio $\text{BR}(B^- \to \pi^- K_S) = (12.1 \pm 0.7) \times 10^{-6}$ [33] is two orders of magnitude higher than our upper bound for the $\text{BR}(B^- \to \pi^- K^0)$ making the extraction of new physics from this decay mode almost impossible. Therefore, the two-body decay modes with $K^{*0}$ in the final state seem to be better candidates for the experimental searches of new physics in the $b \to d \bar{s}$ transitions.

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### APPENDIX A: FORM FACTORS

We use the standard form factor parameterization of hadronic current matrix elements [40, 41] between two pseudoscalar mesons

$$\langle P_2(p_2) | q_j \gamma^\mu q_i | P_1(p_1) \rangle = F_1(q^2) \left( (p_1 + p_2)^\mu - \frac{m_{P_1}^2 - m_{P_2}^2}{q^2} q^\mu \right) + F_0(q^2) \frac{m_{P_1}^2 - m_{P_2}^2}{q^2} q^\mu, \quad (A1)$$

where $q^\mu = (p_1 - p_2)^\mu$. We also use the standard decay constants of the pseudoscalar $K$ and vector $K^{*0}$ mesons

$$\langle K^0(p) | \bar{q} \gamma^\mu \gamma^5 s | 0 \rangle = i f_K p^\mu, \quad (A2a)$$

$$\langle K^{*0}(q) | \bar{q} \gamma^\mu \gamma^5 s | 0 \rangle = g_{K^*} e_{K^*} q^\mu. \quad (A2b)$$

Matrix element between a pseudoscalar and a vector meson is decomposed as customary

$$\langle V(p_1) | q_j \gamma^\mu q_i | P_1(p_1) \rangle = \frac{2V(q^2)}{m_P + m_V} \epsilon^{\mu\nu\alpha\beta} \epsilon^{*\nu\lambda\rho} p_{1\alpha} p_{2\beta}, \quad (A3a)$$

$$\langle V(p_2) | q_j \gamma^\mu \gamma^5 q_i | P(p_1) \rangle = i \epsilon_V^{\mu\nu\lambda\rho} \frac{2m_V}{m_P + m_V} q^\nu q^\mu A_0(q^2) + i (m_P + m_V) \left[ \epsilon_V^{\mu\nu} - \frac{\epsilon_V^{\mu\lambda}}{q^2} q^\nu \right] A_1(q^2) - \frac{\epsilon_V^{\mu\nu} q^\lambda}{(m_P + m_V)} \left[ (p_1 + p_2)^\mu - \frac{m_{P_1}^2 - m_{P_2}^2}{q^2} q^\mu \right] A_2(q^2), \quad (A3b)$$

For the $B^- \to \pi^-$ and $B^- \to \rho^-$ transitions we use form factors calculated in the relativistic constituent quark model with numerical input from lattice QCD at high $q^2$ [34].

$$F_1^{\pi B}(q^2) = \frac{F_1^{\pi B}(0)}{1 - q^2/m_{P_1}^2} [1 - \sigma_1 q^2/m_{P_1}^2], \quad F_1^{\pi B}(0) = 0.29, \quad \sigma_1 = 0.48, \quad \text{(A4a)}$$

$$F_0^{\pi B}(q^2) = \frac{F_0^{\pi B}(0)}{1 - \sigma_1 q^2/m_{P_1}^2 + \sigma_2 q^2/m_{P_1}^2}, \quad F_0^{\pi B}(0) = 0.29, \quad \sigma_1 = 0.76, \quad \sigma_2 = 0.28, \quad \text{(A4b)}$$
The transition form factors between heavy mesons \( D_s^- \to D^- \) have been calculated in the chiral Lagrangian approach by the authors in Ref. \cite{4}. 

\[
F_1^{DD_s}(q^2) = 0, \\
F_0^{DD_s}(q^2) = \frac{q^2}{m_D - m_{D_s}^2} \frac{(g_\pi/4)f_{K(1430)}\sqrt{m_D m_{D_s}}}{q^2 - m_{K(1430)}^2 + i\sqrt{q^2 \Gamma_{K(1430)}}}, 
\]

The same method has been used to obtain the light to light \( K^- \to \pi^- \) meson transition form factors in Ref. \cite{5}.

\[
F_1^{\pi K}(q^2) = \frac{2g_{V\pi K(892)}g_{K\pi}^*}{q^2 - m_{K(892)}^2 + i\sqrt{q^2 \Gamma_{K(892)}(q^2)}}. \\
F_0^{\pi K}(q^2) = \frac{2g_{V\pi K(892)}g_{K\pi}^*}{q^2 - m_{K(892)}^2 + i\sqrt{q^2 \Gamma_{K(892)}(q^2)}} \frac{f_{K(1430)} g_{S\pi K(1430)}}{q^2 - m_{K(1430)}^2 + i\sqrt{q^2 \Gamma_{K(1430)}(q^2)}}. 
\]

Here the decay widths of the resonances \( K^*(892) \) and \( K(1430) \) are taken to be energy dependent.

\[
\Gamma_{K(892)}(q^2) = \left( \frac{m_{K(892)}^2}{q^2} \right)^{5/2} \left( \frac{q^2 - (m_K + m_\pi)^2}{m_{K(892)}^2 - (m_K + m_\pi)^2} \right)^{3/2} \frac{1}{m_{K(892)}^2 - (m_K + m_\pi)^2} \Gamma_{K(892)}, \\
\Gamma_{K(1430)}(q^2) = \left( \frac{m_{K(1430)}^2}{q^2} \right)^{3/2} \left( \frac{q^2 - (m_K + m_\pi)^2}{m_{K(1430)}^2 - (m_K + m_\pi)^2} \right)^{1/2} \frac{1}{m_{K(1430)}^2 - (m_K + m_\pi)^2} \Gamma_{K(1430)}. 
\]

**APPENDIX B: NUMERICAL PARAMETERS**

| \( m_B \) | \( m_{B^*} \) | \( m_{\pi} \) | \( m_{\rho} \) | \( m_{K^+} \) | \( m_{K^0} \) | \( m_{K^0}(892) \) | \( m_{D^+} \) | \( m_{D^*} \) | \( m_{K(1430)} \) | \( m_{b} \) | \( m_{s} \) | \( m_{d} \) |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 5.27 | 5.32 | 0.140 | 0.77 | 0.494 | 0.498 | 0.892 | 1.87 | 1.96 | 1.41 | 4.2 | 0.10 | 0.006 |

**TABLE II:** Meson and quark masses (in GeV) used in our calculations are taken from PDG. \cite{27}

Decay constants of the pseudoscalar \( K^0 \) and vector \( K^{*0} \) mesons are \( f_K = 0.160 \) GeV \cite{34} and \( g_{K^{*0}} = 0.196 \) GeV\(^2\) \cite{10}, respectively. Further numerical parameters relevant for the \( D_s^- \to D^- \) and \( K^- \to \pi^- \) transitions are \( g_{D} = 3.73 \), \( f_{K(1430)} = 0.05 \) GeV, \( \Gamma_{K(1430)} = 0.29 \) GeV, \( g_{S\pi K(1430)} = 3.7 \) GeV, \( g_{V\pi K(892)} = 4.59 \), and \( \Gamma_{K(892)} = 0.051 \) GeV.

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inverse THDM coupling $\xi_{ssd}/m_H$ was not bounded from above due to an unknown upper limit on $\Delta m_{B_s}$. With the recent two-sided bound on the $B_s$ oscillation frequency from the D0 and CDF collaborations it is now possible to constrain this contribution to $|\xi_{ssd}/m_H| < 10^{-12}$. This value is two orders of magnitude smaller than the one used in existing studies. Correspondingly, all the decay rate predictions for the THDM model are diminished by four orders of magnitude and thus rendered negligible.

Before previous work on THDM contributions to the complementary $b \to ssd$ process [26] the relevant effective THDM coupling $|\xi_{ssd}/m_H|$ was not bounded from above due to an unknown upper limit on $\Delta m_{B_s}$. With the recent two-sided bound on the $B_s$ oscillation frequency from the D0 and CDF collaborations it is now possible to constrain this contribution to $|\xi_{ssd}/m_H| < 10^{-12}$. This value is two orders of magnitude smaller than the one used in existing studies. Correspondingly, all the decay rate predictions for the THDM model are diminished by four orders of magnitude and thus rendered negligible.

We find that the contribution of the THDM model to the $b \to ssd$ process is negligible compared to the SM predictions. This conclusion is consistent with the upper limit on $|\xi_{ssd}/m_H|$ from the $B_s$ oscillation frequency, which sets a bound of $|\xi_{ssd}/m_H| < 10^{-12}$. This value is two orders of magnitude smaller than the one used in existing studies. Correspondingly, all the decay rate predictions for the THDM model are diminished by four orders of magnitude and thus rendered negligible.

We find that the contribution of the THDM model to the $b \to ssd$ process is negligible compared to the SM predictions. This conclusion is consistent with the upper limit on $|\xi_{ssd}/m_H|$ from the $B_s$ oscillation frequency, which sets a bound of $|\xi_{ssd}/m_H| < 10^{-12}$. This value is two orders of magnitude smaller than the one used in existing studies. Correspondingly, all the decay rate predictions for the THDM model are diminished by four orders of magnitude and thus rendered negligible.