A Squint Multi-aperture Chirp Scaling Algorithm for the Multi-aperture SAS with Small Squint Angle

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Abstract. Squint multi-aperture synthetic aperture sonar (SAS) data is more challenging to process than data about its common side-looking counterpart because the geometry is complicated. While traditional multi-aperture algorithms mainly handle general side-looking cases, they do not work well for squint cases. In this paper, a squint multi-aperture chirp scaling algorithm (SMACSA) is proposed that is suited to process multi-aperture SAS data with small squint angle. In the derivation of the imaging algorithm, the squint multi-aperture signal is converted into the squint monostatic signal by the phase compensation factor of range dependent and the delay compensation factor on reference range, then the multi-aperture chirp scaling algorithm (SMACSA) is first proposed by drawing on the classic chirp scaling algorithm (CSA). The greatest improvement between SMACSA and CSA is the azimuth walk introduced by non-stop-hop-stop mode. Finally, the validity of SMACSA is proved by computer simulation.

1. Introduction

Synthetic aperture sonar (SAS) is an important equipment for offshore exploration and environmental surveillance [1]. Synthetic aperture sonar (SAS) achieves high azimuth resolution by using small antenna of uniform linear motion to synthesize a large aperture[2]. But the factors of ocean currents and asymmetric platforms may lead to squint phenomenon appearing. Because the squint angle of multi-aperture SAS can lead to delay errors and the Doppler effect of sonar is more significant than that of radar, the multi-aperture SAS image will be defocused, even if the squint angle is small.

In 2017, Wu proposes first a squint multi-aperture range Doppler algorithm for the narrow beam and small squint angle [3]. The range Doppler algorithm [4] is the most widely used algorithm in the synthetic aperture imaging systems. However, under certain conditions, two disadvantages can become apparent. First, a high computing load is experienced when a long kernel is used to obtained high accuracy in the range cell migration correction (RCMC) operation. Second, it is not easy to incorporate the azimuth frequency dependence of second range compression (SRC) [5], which can limit its accuracy in certain wide-aperture cases.

The chirp scaling algorithm (CSA) was developed specifically to eliminate the interpolator used for RCMC[6]. It is based on a scaling principle described by Papoulis[7], whereby a frequency modulation is applied to a chirp encoded signal to achieve a shift or scaling of the signal. Using this “chirp scaling” principle, the required range-variant RCMC shift can be implemented, using phase multiples instead of a time domain interpolator. The algorithm has the additional benefit that SRC can
be made azimuth frequency dependent. This benefit arises because the data are available in the two dimensional frequency domain at a convenient stage in the processing. However, in order to make SAS use scaling principle to achieve RCMC in the squint case RCMC, this paper proposes first a squint multi-aperture chirp scaling algorithm (SMACSA).

2. Derivation of the Squint Imaging Algorithm

2.1. Multi-aperture Data Processing

After demodulation to baseband, the received signal of the $i$ th aperture comes from the point target $P$, which is expressed as

$$ss_i(\tau, t; r) = p\left(\tau - \frac{R(t; r)}{c}\right) \cdot \alpha(t) \cdot \exp\left\{i \pi k \left(\tau - \frac{R(t; r)}{c}\right)^2\right\} \cdot \exp\left\{-j \frac{2\pi f_0}{c} R(t; r)\right\}$$

(1)

where $\tau$ is the fast time, $t$ is the slow time, $p(\cdot)$ is the pulse envelope, $\alpha(t)$ is the antenna weighting, $k$ is the FM rate, $c$ is the speed of sound, $f_0$ is the carrier frequency, $R(t; r)$ is the instantaneous slant range, $r$ is the closest range between SAS and the target. According to the reference[8], $R(t; r)$ is formulated as follows:

$$R(t; r) = \sqrt{r^2 - 2vdr \sin \theta_{sq} + v^2t^2} + \sqrt{r^2 + d_i^2 - 2v\left(t + \frac{2r}{c}\right)(r \sin \theta_{sq} - d_i \cos \theta_{yaw}) + v^2 \left(t + \frac{2r}{c}\right)^2}$$

(2)

where $v$ is the platform velocity and $d_i$ is the baseline between the transmitter and the $i$ th aperture, $\theta_{sq}$ is the squint angle, $\theta_{yaw}$ is the yaw angle.

The azimuth spectrum of the signal corresponding to the double root range history shown in (2) can not be directly obtained by the stationary phase principle (POSP) [9]. To solve this problem, the double root range history is approximated by the single root range history, as shown in Fig.1.

According to the sampling point position shown in Fig.1, we can get the range history of a single root's equivalent sampling points and obtain the approximate range history expression by compensating the error between the single root range history and the range history in (2).

**Figure 1.** Displaced phase centre antenna.
In (3), the first term is the range history of a single root, and the second term is the correction. Before the azimuth sampling rearrangement, it is necessary to compensate the phase and time delay component produced by the correction in (3) [8]. Phase compensation is performed in the two-dimensional time domain, the compensation factor is

$$
\psi_i (r_i; d_i) = e^{j 2 \pi \frac{f_0}{c} \left[ \frac{r^2}{4r} + \frac{d_i}{\cos(\theta_{sv})} \right] + \frac{d_i \sin(\theta_{av})}{\cos(\theta_{sv})}}
$$

To save the amount of computation, the delay error of the whole scene is compensated by the delay error of the reference range $r_{ref}$ (the swath center), the expression of the phase compensation factor is given by

$$
\phi_i (f_i; d_i) = e^{j 2 \pi \frac{f_0}{c} \left[ \frac{r^2}{4r} + \frac{d_i}{\cos(\theta_{sv})} \right] + \frac{d_i \sin(\theta_{av})}{\cos(\theta_{sv})}}
$$

Where $f_0$ is the range frequency. Then the signal can be arranged in accordance with the position of the sampling points and receiver apertures and a monostatic signal is given by

$$
S_s(t; r) = P(f_i) \cdot \omega_o(t) \cdot \exp \left\{ -j \frac{\pi f_i^2}{k} \right\} \cdot \exp \left\{ -j \frac{2 \pi R(t; r)}{c} \left( f_0 + f_i \right) \right\}
$$

$$
R(t; r) = 2 \sqrt{r^2 - 2vr \sin \theta_{av} \left( t + \frac{r}{c} \right) + v^2 \left( t + \frac{r}{c} \right)^2}
$$

Where $P(f_i)$ is the range spectrum envelope.

2.2. Derivation of CSA

By POSP [9], the azimuthal Fourier transform of (6) is carried out, and the two-dimensional frequency domain expression of the point target signal is given by

$$
SS(f_r, f_s; r) = P(f_s) W_s(f_s - f_{dc}) \exp \left\{ j \theta_{sd}(f_r, f_s; r) \right\}
$$

where $f_r$ is the azimuth frequency, $W_s(f_s)$ is the azimuth spectrum envelope, $f_{dc}$ is the Doppler center frequency, $\theta_{sd}(f_r, f_s; r)$ is the phase of the SAS transfer function. To facilitate the development of the processing algorithm, we expand $\theta_{sd}(f_r, f_s; r)$ around $f_r$. The expanded series are truncated at the second-order term and expressed as follows:

$$
\theta_{sd}(f_r, f_s; r) = -\frac{4 \pi r \cos \theta_{av} f_0}{c} \gamma(f_s) + 2 \pi \left( \frac{r \sin \theta_{av}}{v} \right) f_s - 2 \pi \frac{2r \cos \theta_{av}}{c} \gamma(f_0) f_r - \frac{\pi k_s f_r^2}{2}
$$
The system parameters are listed in Table 1. In order to verify the validity of the SMACSA in the paper, simulations are carried out in this section.

3. Simulation Results

In order to verify the validity of the SMACSA in the paper, simulations are carried out in this section. The system parameters are listed in Table 1.

where \( \gamma(f_s) = \sqrt{1 - \frac{c^2 f_s^2}{4 v^2 f_0^2}} \) is the factor of the range migration, \( \frac{1}{k_s} = \frac{c r \cos \theta_n f_s^2}{2 v^2 f_0^2 \gamma^2(f_s)} + \frac{1}{k} \) is the range dependence of the range frequency rate. The range-Doppler form of the point scatter response:

\[
sS(r, f_s; r) = \exp \left\{ j \pi k_s \left( \frac{\cos \theta_n}{\gamma(f_s)} - 1 \right) \left( r - \frac{2 r_n \cos \theta_n}{v} f_s \right)^2 \right\}
\]

(10)

Provided chirp scaling function is

\[
S_c(r, f_s) = \exp \left\{ j \pi k_s \left( \frac{\cos \theta_n}{\gamma(f_s)} - 1 \right) \left( r - \frac{2 r_n \cos \theta_n}{v} f_s \right)^2 \right\}
\]

(11)

After the multiplication of (10) by the chirp scaling phase function (11), By the a range Fourier transform squint SAS transfer function of chirp-scaled signal is obtained:

\[
SS(f_s, f_s; r) = P(f_s) W_s(f_s - f_a) \exp \left\{ -j \frac{4 \pi r \cos \theta_n f_0}{c} \gamma(f_s) \right\} \exp \left\{ j 2 \pi \left( \frac{r}{c} + \frac{r \sin \theta_n}{v} \right) f_s \right\}
\]

(12)

\[
\times \exp \left\{ -j \frac{\pi r}{k_c \cos \theta_n f_s^2} \right\} \exp \left\{ -j \frac{4 \pi r_n}{c} \left( \frac{\cos \theta_n}{\gamma(f_s)} - 1 \right) f_s \right\}
\]

\[
\times \exp \left\{ \frac{4 \pi k_s}{c^2} \left( \frac{r}{c} + \frac{r \sin \theta_n}{v} \right)^2 \left( 1 - \frac{D}{2 \cos(\theta_n)} \right)^2 \exp \left\{ -j \frac{4 \pi r f_s}{c} \right\}
\]

where the first exponential term is the azimuthal modulation item, the second exponential term is the azimuth walk item introduced by the non-stop-hop-stop mode and the squint mode, the third exponential term is the range modulation item, the fourth exponential term is the range position item of the target, the fifth exponential term is the the bulk range cell migration, the sixth exponential term is the residual phase.

The range compression, SRC, and the bulk range cell migration can then be performed via conjugate multiplication. According to (12), the phase function at this step is given by

\[
\phi_{\text{ol}}(f_s, f_s) = \exp \left\{ j \frac{\pi r}{k_c \cos \theta_n f_s^2} \right\} \exp \left\{ j \frac{4 \pi r_n}{c} \left( \frac{\cos \theta_n}{\gamma(f_s)} - 1 \right) f_s \right\}
\]

(13)

After the range inverse Fourier transform, the azimuth compression, the residual phase correction and the azimuth move correction are applied, via conjugate multiplication, to the range Doppler domain. According to (12), this phase function is given by

\[
\phi_d(f_s, f_s) = \exp \left\{ j \frac{4 \pi r \cos \theta_n f_0}{c} \gamma(f_s) \right\} \exp \left\{ -j 2 \pi \left( \frac{r}{c} + \frac{r \sin \theta_n}{v} \right) f_s \right\}
\]

(14)
Table 1. SAS System Parameters.

| Carrier frequency | Bandwidth | Pulse width | PRI | Antenna length(transmitter) | Antenna length(aperture) | Velocity | Aperture number |
|-------------------|-----------|-------------|-----|-----------------------------|--------------------------|----------|----------------|
| 150kHz            | 20kHz     | 10ms        | 200ms| 0.08m                       | 0.04m                    | 2.5m/s   | 25             |

A simulation based on a flat earth model is presented. Five point targets are used in the simulation. These point targets are illuminated at the same time. The separation between adjacent point targets is 2m in azimuth direction and is 2m in range direction.

Fig. 2 shows the result of processing five simulated point targets. Table 2 summarizes the point target analysis for the cases that the squint angle is 2°. The simulation results in Fig.2 and Table 2 show that the range resolution, the azimuth resolution, the sidelobe ratio, the integral sidelobe ratio and the target location all reach the theoretical value.

| Table 2. Image Quality Parameters. |
|------------------------------------|
| Squint angle | Azimuth resolution | Range resolution | Azimuth PSLR | Range PSLR | Azimuth ISLR | Range ISLR |
|------------|-------------------|------------------|--------------|------------|--------------|------------|
| 2°         | 4.15 cm           | 3.70 cm          | -15.95dB     | -13.23dB   | -14.45dB     | -11.57dB   |

4. Conclusion
To solve the imaging problem of squint multi–apertures SAS, a squint multi-aperture chirp scaling algorithm (SMACSA) is proposed that is suited to process multi-aperture SAS data with small squint angle. This paper draws from the classical CSA and the side-looking imaging algorithm for multi-aperture SAS and provides the small squint angle imaging algorithm for the first time. Finally, this paper proves the validity of the proposed algorithm via a computer simulation experiment.

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