Experimental study on Statistical Damage Detection of RC Structures based on Wavelet Packet Analysis

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Abstract. A novel damage indicator based on wavelet packet transform is developed in this study for structural health monitoring. The response signal of a structure under an impact load is normalized and then decomposed into wavelet packet components. Energies of these wavelet packet components are then calculated to obtain the energy distribution. A statistical indicator is developed to describe the damage extent of the structure. This approach is applied to the test results from simply supported reinforced concrete beams in the laboratory. Cases with single damage are created from static loading, and accelerations of the structure from under impact loads are analyzed. Results show that the method can be used for the damage monitoring and assessment of the structure.

1. Introduction
In recent years, damage assessment of structures has drawn great attention from various engineering practitioners. Damage identification techniques can be classified into either local or global methods. Most existing techniques, such as visual, acoustics, magnetic field, eddy current etc., are effective but local in nature. They require that the vicinity of the damage is known a priori and the position of the structure being inspected is readily assessable. The global methods quantify the healthiness of a structure by examining changes in its vibrational characteristics or the static behaviour under load. The core of this group of methods is to seek some damage indices that are sensitive to structural damage. Doebling et al. (1998) and Brownjohn (2007) presented a literature review on the damage assessment methodologies based on parameters such as the natural frequencies, mode shapes, mode shape curvature, flexibility matrix and stiffness matrix. However, the modal properties, such as natural frequencies and mode shapes are poor indicators of damage, and more sophisticated methods have been derived based on the second derivative of mode shapes. Also a large number of measurement locations are needed to provide sufficient resolution on the mode shapes.

Most of the vibration-based damage assessment methods require modal properties that are obtained from traditional Fourier transform (FT). However, when the damage is very small, the damage-induced changes of physical structural properties are always too insignificant to reveal the damage.
using the FT-based method. In addition, the measured vibration signals are often contaminated with noise. The Wavelet Transform (WT) based method for vibration signal analysis is gradually adopted in many areas due to its good time-frequency localization. Hou et al. (2000) used a simple structural model with multiple breakable springs subjected to harmonic excitation to show that the wavelet transform can successfully be used to identify both abrupt and cumulative damages. The wavelet packet transform (WPT) is an extension of the WT that provides complete level-by-level decomposition. The WPT enables the extraction of features from signals with combined stationary and non-stationary characteristics and arbitrary time-frequency resolution. Sun and Chang (2002) concluded that the WPT-based component energy is a sensitive condition index for structural damage assessment. This index is sensitive to changes of structural rigidity and insensitive to measurement noise. The WPT component energy combined with well-trained neural network models was used to identify the location and the severity of damage. Yam et al. (2003) also extracted the structural damage feature based on energy variation of structural vibration responses decomposed using wavelet packet, and neural network is used to establish the mapping between the structural damage feature and damage status. This method needs accurate model information for both the healthy and damaged conditions to train the neural network model. However, it is difficult and challenging in practice, especially for complex structures. Zabel (2004) used the wavelet packet component energies as the damage index for damage assessment of reinforced concrete structures. Law et al. (2005) developed a method to identify damage in structures using wavelet packet sensitivity. The sensitivity of wavelet packet transform component energy with respect to local change in the system parameters is derived analytically basing on the dynamic response sensitivity. The sensitivity-based method is then used for damage detection of structures.

Because all vibration-based damage detection processes rely on experimental data with inherent uncertainties, statistical analysis procedures are necessary if one is to state in a quantifiable manner that changes in the vibration response of a structure are indicative of damage as opposed to operational and/or environmental variability. Farrar et al. (2001) and Sohn et al. (2000) cast the structural health monitoring problem in the context of a statistical pattern recognition paradigm. This paradigm can be described as a four-part process: 1) Operational evaluation; 2) data acquisition and cleansing; 3) feature extraction and data reduction; and 4) statistical model development. Most references focus on methods for extracting damage-sensitive features from vibration response measurements. Few of them take a statistical approach to quantify the observed changes in these features. Worden et al. (2000) developed a statistical method for damage detection using the outlier analysis. The damage sensitive features are assumed to have a Gaussian distribution with an estimated mean value and covariance matrix. The problem is one of novelty detection. Features are first extracted from a baseline system to be monitored and subsequent data are then compared to see if the new features are outliers which significantly depart from the rest of the population. In fact, many statistical procedures can be used for the problem of novelty detection and a literature review was presented by Markou and Singh (2003). Sun and Chang (2004) developed a statistical pattern classification method based on the WPT for structural health monitoring. The dominant component energies are defined as a novel condition index. Two damage indicators based on the sum of absolute difference and square sum of difference are proposed. These two indicators basically quantify the deviations of the wavelet packet signatures from the baseline reference. The statistical process control method is used to determine the threshold value for the damage indicators. Any indicator that exceeds the threshold would cause a damage alarm.

A structural damage can cause shifts in the vibration energy across the frequency spectrum of interest. Therefore, the energy of structural vibration response at different frequency bandwidth contains information of the structural damage, and the energy variation in one or several frequency components can indicate the damage status of the structure.
This above-mentioned property of vibration energy shifts across the frequency spectrum with damage is employed in a novel damage classification method based on wavelet packet transform and statistical analysis as shown below for structural health monitoring. The response signal of a reinforced concrete structure under an impact load is normalized and then decomposed into wavelet packet components. Energies of these wavelet packet components are then calculated, and statistical similarity test based on an F-test is used to identify damage in the reinforced concrete structures with these wavelet packet component energy distributions. A statistical indicator is developed to describe the damage extent of the structure. Experimental study is carried out on simply supported reinforced concrete beams. Different damage cases are created using static loading. Accelerations of the structure under impact loads are analysed. Results show that the method can be used with no reference baseline measurement and model for damage monitoring and assessment of the structure.

2. Wavelet packet component energy analysis
The continuous wavelet transform of a square-integrable signal $f(t)$ is defined as (Mallat, 1999)

$$W_f(u,s) = f(t) \otimes \psi_s, (t) = \frac{1}{\sqrt{s}} \int_{-\infty}^{\infty} f(t) \psi_s\left(\frac{t-u}{s}\right) dt$$  

(1)

where $t$ is time, and $\otimes$ denotes the convolution of two functions. $\psi_s, (t)$ is the dilatation of $\psi(t)$ by the scale factor $s$. $u$ is the translation indicating the locality. $\psi^*(t)$ is the complex conjugate of $\psi(t)$ which is a mother wavelet satisfying the following admissibility condition:

$$\int_{-\infty}^{\infty} \left| \frac{\Psi(\omega)}{\omega} \right|^2 d\omega < +\infty$$  

(2)

where $\Psi(\omega)$ is the Fourier transform of $\psi(t)$. The existence of the integral in (2) requires that

$$\Psi(0) = 0 \ \text{i.e.,} \ \int_{-\infty}^{\infty} \psi(t) dt = 0$$  

(3)

The reconstruction of the original signal can be expressed as

$$f(t) = \frac{1}{C_\psi} \int_{-\infty}^{\infty} \int_{\frac{d}{s}}^{\infty} W_f(u,s) \Psi\left(\frac{t-u}{s}\right) ds du$$  

(4)

with

$$C_\psi = 2\pi \int_{0}^{\infty} \left| \Psi(r) \right|^2 \frac{dr}{r} < \infty.$$  

Various forms of wavelet basis function $\psi(t)$ have been developed. One of the most useful practical methods for signal decomposition is the wavelet packet analysis. A wavelet orthogonal basis decomposes the frequency axis in dyadic intervals whose sizes have an exponential growth. Coifman et al (1992) generalized this fixed dyadic construction by decomposing the frequency in intervals whose bandwidths may vary. Each frequency interval is covered by the time-frequency boxes of wavelet packet functions that are uniformly translated in time in order to cover the whole plane.

In the present study, the vibration responses are standardized prior to wavelet transform as follow (Sohn and Farrar, 2001)
\[ x(t) = \frac{\hat{x}(t) - u_\tilde{x}}{\sigma_\tilde{x}} \]  \hspace{1cm} (5)

where \( \hat{x}(t) \) is the original signal and \( u_\tilde{x} \) and \( \sigma_\tilde{x} \) are the mean and standard deviation of \( \hat{x}(t) \), respectively. The standardization procedure is applied to all measured responses. The measured response is represented by Haar wavelet basis function through the dyadic wavelet transformation. The bandwidths of each level of the dyadic wavelet transform are octaves, and this enables a direct comparison of the energy content of the wavelet packets as shown below. The WPT component function of the standardized measured response \( x(t) \), i.e. \( x'_j(t) \), can be reconstructed as,

\[ x'_j(t) = \sum_{k=-\infty}^{\infty} c_{j,k}^i \phi_{j,k}^i(t) = R_j^i c_j^i = R_j^i D_j^i x(t) \]  \hspace{1cm} (6)

where \( i \) denotes the \( i \)th WPT and \( j \) denotes the \( j \)th level of decomposition, and \( R_j^i = [\psi_{j,0}^i \quad \psi_{j,2}^i \quad \cdots \quad \psi_{j,i}^i] \), \( (l = 0,1,\ldots,N/2^j - 1) \)

and \( D_j^i = H^{i+1}D_j^i, D_j^i = H^j, D_j^i = G^j \). \( H^{i+1} \) and \( G^{i+1} \) are matrices formed by the low-pass filter function and high-pass function respectively (Mallat, 1999). \( c_{j,k}^i \) is the wavelet packet coefficients for the response with \( c_j^i = D_j^i x(t) \). \( \phi_{j,k}^i(t) \) is the wavelet packet function. The \( i \)th wavelet packet transform component energy of the response \( x(t) \) at the \( j \)th level of decomposition, \( E_{ij}^j \) is defined as,

\[ E_{ij}^j = (x_j^i)^T (x_j^i) = x^T (R_j^i D_j^i)^T (R_j^i D_j^i) x = x^T T_j^i x \]  \hspace{1cm} (7)

The total energy of the primary signal is equal to the sum of energy contents in various frequency bands. A non-dimensional vector at the \( j \)th level of decomposition can be written as follows

\[ P = \left\{ p(1), p(2), \cdots, p(2^j-1) \right\} \]  
\[ = \left\{ E_{1j}^j / E, E_{2j}^j / E, \cdots, E_{2^j-1}^j / E \right\} \]  \hspace{1cm} (8)

where \( E = \sum_{j=1}^{2^j-1} E_{ij}^j \) is the total energy of the vibration response. \( p(i) \) is the non-dimensional energy of the \( i \)th wavelet packet component. This vector is in fact the non-dimensional energy distribution in the different wavelet packets.

3. Statistical damage assessment

If there is damage in RC structures, the energy will fluctuate in various frequency bands. The energy distribution of the response within the frequency bandwidth is represented by the different statistical moments of the distribution, and its mean, variance, skewness and kurtosis, i.e. \( \mu, \sigma', S \) and \( K \), are taken as damage indicators in the damage classification study.
\[ \mu = \sum_{k=1}^{n} n_k p(n_k) \]
\[ \sigma^2 = \sum_{k=1}^{n} (n_k - \mu)^2 p(n_k) \]
\[ S = \frac{\sum_{k=1}^{n} (n_k - \mu)^2 p(n_k)}{\sigma^2} \]
\[ K = \frac{\sum_{k=1}^{n} (n_k - \mu)^4 p(n_k)}{\sigma^4} \]  \hspace{1cm} (9)

where \( n_k \) is the number of the \( k \)th wavelet packet, and \( p(n_k) \) is its probability density function.

4. Experimental setup

One four-metre long uniform rectangular reinforced concrete beam with 3.8m simply supported span as shown in Figure 1, was tested in the laboratory. The beam cross-section is 300mm high and 200mm wide. There are three 20mm diameter mild steel bars at the bottom of the beam, and two 6mm diameter steel bars at the top of the beam section. 6mm diameter mild steel links are provided at 195mm spacing over the whole beam length. The beam end rests on top of 50mm diameter steel bar at each end which in turn rests on top of a solid steel support fixed to a large concrete block on the strong floor of the laboratory. A piece of thin rubber pad is placed between the steel bar and the bottom of the concrete beam for level adjustment. The reinforcement in the beam corresponds to a steel percentage of 1.57%. The compression strength of concrete is 54.4MPa, and the density, tensile strength, Young’s modulus, and Poisson Ratio of concrete are respectively 2351.4kg/m\(^3\), 3.77MPa, 30.2GPa and 0.16. The Young’s Modulus and yield stress of the mild steel bars are respectively 181.53GPa and 300.07MPa.

The beam was incrementally loaded at mid-span to different static load levels creating a single damage zone using three-point loading as shown in Figure 1(a). The load levels were from zero up to the failure load of the beam. The crack locations and lengths were monitored in addition to the displacement measurements. Details of the loading steps and the crack damage conditions are listed in Table 1. At each load level, twenty impact hammer tests were conducted after the beam had been unloaded for twenty minutes. Seven accelerometers evenly distributed at the bottom and along the beam as shown in Figure 1(b) measured the responses. Impact force excitation was applied at 3/8L from the left support using a Dytran Instruments model 5803A 12 lbs instrumented impulse hammer. INV300 data acquisition system was used to collect the data from all the 8 channels.
frequency is 2000Hz, and 8192 data were recorded for each hammer test. The fundamental frequency of the beam at each load level is also shown in Table 1. The fundamental frequency is noted to reduce with increase of damage at each load level.

Table 1. Scheme of Static load test, fundamental frequency and crack conditions

| Damage States | Load P(kN) | Frequency (Hz) | Crack range (mm) | Max. Height (mm) |
|---------------|------------|----------------|------------------|------------------|
| 01            | 0          | 30.43          | 0.00             | 0.00             |
| 02            | 2          | 30.69          | 0.00             | 0.00             |
| 1             | 10         | 30.02          | -                | -                |
| 2             | 17         | 29.66          | -                | 92(1)            |
| 3             | 25         | 29.34          | 1.21             | 132(7)           |
| 4             | 35         | 28.76          | 1.80             | 166(11)          |
| 5             | 45         | 29.20          | 1.80             | 181(11)          |
| 6             | 50         | 28.91          | -                | -                |
| 7             | 55         | 28.73          | -                | -                |
| 8             | 60         | 28.30          | -                | -                |
| 9             | 67         | 28.10          | 2.40             | 192(14)          |
| 10            | 75         | 26.96          | 2.40             | 300(14)          |

Note: 1) (*) denotes number of cracks; 2) – denotes no measurement is recorded.

5. Experimental results
The first 1000 data of the response from 5/8L were decomposed into six levels of wavelet packets. The frequency bandwidth of each decomposed wavelet packet is 15.625Hz. Figure 2 shows the measured responses, their Fourier spectrum and wavelet packet components at 5/8L for different damage states. Only the first 32 WP are shown in the figure because the remaining WP contains very small vibration energy. Table 2 shows the average values of the four damage indicators from Equation (9) for all states. Figure 3 shows the range and average of the damage indicator (mean) for all states. Figure 4 shows the average damage indicator (mean) for all states from different measuring locations. The following observations are obtained from these results:

![Impact responses and their spectrum at 5/8L](image)
1) The wavelet packet component energy changes with the different damage states as shown in Figure 2. The energy ratio of the second wavelet packet component increases with the damage state number while that in the fourth component reduces. This shows that the energy change of the wavelet packet components is clearly related with damage in the reinforced concrete beam.

2) Observations in Figure 3 and Table 2 show that all the states can be classified into six groups according to their damage indicator (mean): States 01 and 02 form the first group. States 1, 2 and 3 are in the second group. State 4 is the third group. States 5, 6 and 7 form the fourth group. States 8 and 9 are in the fifth group and State 10 is the sixth group. The average values of the damage indicators are close to each other in each group. The fundamental frequency in Table 1 also show similar pattern. Therefore the damage process can be represented by these groups, which are renamed as six configurations with distinctly different behaviour and damage pattern. The average variance, skewness and kurtosis in Table 2 also exhibit similar pattern as the average mean.

3) Figure 4 shows that the average damage indicator (mean) is different at different measuring locations. Since the damage in beam is symmetrical about mid-span, the indicators at 1/8L and 7/8L, 1/4L and 3/4L, 3/8L and 5/8L are close to each other and are in pairs. It is noted that this damage indicator is a measure of the central frequency of the energy distribution in the frequency spectrum. Different measured location corresponds to a different combination of the modal responses. Figure 4 shows that the first five configurations exhibit similar behaviour throughout the damage process, while the final damage configuration which corresponds to a failure state, has
significantly different composition of response components as measured at mid-span. This means that any sensor could give an indication of the damage process as seen in Figure 4 but sensor when located at the damage could detect drastic change in the indicator associated with failure of the cross-section.

Table 2. Average damage indicators for Beam 1 from responses at 5/8L

| Damage States | Mean | Variance | Skewness | Kurtosis |
|---------------|------|----------|----------|----------|
| 01            | 7.65 | 9.44     | 3.09     | 14.01    |
| 02            | 7.43 | 9.23     | 3.15     | 14.28    |
| 1             | 5.86 | 6.54     | 3.93     | 23.07    |
| 2             | 6.16 | 7.28     | 3.82     | 21.54    |
| 3             | 5.76 | 6.80     | 4.08     | 24.80    |
| 4             | 4.50 | 4.72     | 5.33     | 44.25    |
| 5             | 5.32 | 6.00     | 4.55     | 31.27    |
| 6             | 5.38 | 5.99     | 4.15     | 27.18    |
| 7             | 5.47 | 6.08     | 4.09     | 26.25    |
| 8             | 4.56 | 4.94     | 4.67     | 34.64    |
| 9             | 4.25 | 4.34     | 4.87     | 38.71    |
| 10            | 5.11 | 6.08     | 4.58     | 32.37    |

6. Conclusions
A novel methodology for structural health monitoring has been developed based on statistical damage indicator. Experimental results in the laboratory show that it is an effective indicator of the damage extent of the structure. From the energy distribution of the response obtained at mid-span of the reinforced concrete beam in the laboratory, the value (mean) is found robust to detect both local and global damage close to mid-span, while those of the higher order statistics are more robust to detect local damage at different locations of the structure. It is believed that the combine use of all four indicators from energy distribution at selected frequency range could monitor the extent and location of local damages in a structure.

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