A comment on the breathing in the corona virus environment

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Abstract
The balance equation more correct than in the previous publication of the author for the number of the corona virus (CV) molecules in the lung is developed, and an important inequality for the needed frequency of the breathing is obtained from it. It is immediate from this inequality that a small-size living creature should better endure the CV-environment.

Keywords
breathing (respiration), corona virus disease

An Analysis of the Respiration Process: Breathing in the Corona Virus Environment

An introduction to the topic of corona virus disease can be found in Gluskin (2020), Mangiarotti et al. (2020), Mangiarotti et al. (2016), Seyed et al. (2020), Cao (2020), Kpae (2020). We focus more on an aspect of human behavior, not the biology.

Below, $N(t)$ is the number of the CV molecules in the lung:

$S$ is the surface area of the lung;
$V$ is the volume of the lung; this volume must oscillate with the breathing, we write it as

$$V(t) = V_0 + \varepsilon(t)$$

where $\varepsilon(t)$ is a small oscillating addition to the average volume $V_0$.

Writing

$$\varepsilon(t) = \varepsilon_0 \sin 2\pi At, \quad \varepsilon_0 << V_0,$$

we introduce the notation $A$ of the frequency (the usual one, in Hertz, not cyclic) of the breathing. $A$ is the parameter that interests us; we shall find for it an important inequality.

The spatial density of the CV molecules in the lung is denoted as $n(t)$. Obviously,

$$n(t) = \frac{N}{V(t)} = \frac{N}{V_0 + \varepsilon(t)} \approx \frac{N}{V_0} \left[ 1 - \frac{\varepsilon(t)}{V_0} \right].$$

The density of the CV molecules in the surrounding air is denoted as $n_0$. In the case of already infected person, it should be
\[ n >> n_0 \]  \hspace{1cm} (4)

and in many cases, \( n_0 \) will be ignored.

With the breathing, the flow of the CV molecules \textbf{out} of the lung can be written as

\[ q_1 = -\beta A n \]  \hspace{1cm} (5)

where \( \beta \) is a constant, and the flow \textit{inside} (into) the lung as

\[ q_0 = \beta A n_0 \]  \hspace{1cm} (6)

Additionally to the flows, the rate of \( \frac{dN}{dt} \) is influenced by the rate of the generation of the new CV molecules by those already present—the essence of the disease. This contribution to \( N \)—the reproduction of the CV molecules on the cells of the lung appears to be like a source. We take this rate to be directly proportional to \( S \) and \( n(t) \), obtaining one more linear by \( n \) term, \( kS n(t) \) (\( k \) is a constant), having the same physical dimension as \( q_0 \) and \( q_1 \).

Finally, the balance equation is

\[ \frac{dN}{dt} = q_1 + q_0 + kSn \]  \hspace{1cm} (7)

which is

\[ \frac{dN}{dt} = -(\beta A - kS)n(t) + \beta A n_0, \]  \hspace{1cm} (8)

in which form (3) for \( n(t) \) will be used.

\section*{THE BASIC INEQUALITY}

The term with \( n_0 \) in (8) can be ignored both because of its relative smallness, and because stability of such a “state-equation” (Kuo 1967)\(^8\) is defined by the (using the state space theory terminology) matrix \([A]\) which here is reduced to the single parameter \(-P\) where

\[ P = \beta A - kS. \]  \hspace{1cm} (9)

That is, it is sufficient to consider the homogenous equation

\[ \frac{dN}{dt} = -Pn(t), \]  \hspace{1cm} (10)

or, using (3)

\[ \frac{dN}{dt} = -\frac{P}{V_0} \left[ 1 - \frac{e(t)}{V_0} \right] N(t), \]  \hspace{1cm} (11)

from which

\[ N(t) = Ke^{-\frac{P}{V_0} \int_0^t \left[ 1 - \frac{e(t)}{V_0} \right] dt} \]

\[ = Ke^{-\frac{\xi(t)}{V_0} + \int_0^t \frac{e(t)}{V_0} dt} \]  \hspace{1cm} (12)
This exponent is composed of two factors, one of which is the strongly influencing

$$Ke^{-\frac{e_p t}{\sigma}}$$

and the other has the degree

$$P \int_{0}^{t} \frac{e(\lambda)}{V_0} d\lambda.$$ 

(14)

Since $e(t) \sim \sin(2\pi At)$

$$\int_{0}^{t} e(\lambda) d\lambda \sim \frac{e(t)}{2\pi A}$$

(15)

and the latter exponential factor is

$$e^{\frac{2}{2\pi A}e(t)} = e^{\left(\beta - \frac{kS}{A}\right) \frac{e(t)}{2\pi A}}.$$ 

(16)

This factor is not very important, because of the smallness of the oscillating $e(t)$.

The essential first factor (13) makes it immediate that in order $N(t)$ not to become infinitely increasing, it is necessary that

$$P = \beta A - kS > 0,$$

(17)

that is,

$$A > \frac{kS}{\beta}.$$ 

(18)

The opposite case just means a catastrophe, even though investigation of the equality $P = 0$ might be of some interest.

It is obvious from (18) that the smaller $S$ is (i.e., of the smaller size is the living breathing creature—for instance, a child, a bird, or a mouse)—the better this surviving inequality can be satisfied.

Even though the balance equation is written for $N(t)$, and not $n(t)$, the specific value $n(t)$ is most interesting, since any measurement of the presence of the CV-molecules in the lungs, made via the respiration—either electrical, optical, or chemical, would directly relate to $n(t)$, and not $N(t)$. Such measurement should be done by different reasons, for instance for defining the moment when the artificial respiration has to be started.

The importance of $n(t)$, is similar to the importance of the density $\sigma$ of the charge on the plate of a capacitor. It is possible to define $\sigma$ as $Q/S$ where $Q$ is the full charge of the plate having area $S$, or define $Q$ as $\sigma S$, but the equality of the electrical field, near the plate, to $\sigma$ gives $\sigma$ especial importance.

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