Application and verification of Sarrau formula in order to calculate pressure acting on the projectile’s base in the 120 mm Leopard 2 A5 tank’s barrel with piezoelectric pressure sensors and Doppler radar usage

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Abstract: Semi-empiric Sarrau formula relates to a pressure acting on a breech with a pressure acting on the base of a projectile during a shot. Pressure acting on a breech is relatively easy to measure in comparison with pressure acting on projectile’s base. Exact value of projectile’s base pressure can be used in mechanical calculations for projectile, such as strength of a projectile’s body and its acceleration which is essential for example in designing of fuzes. In the paper authors make an attempt for definition coefficient of Sarrau formula to calculate a force acting on projectile’s base. In the paper results of 120 mm Leopard 2 A5 barrel’s shooting tests were used. The results contain pressure in the chamber during a shot measured with a piezoelectric pressure sensor and muzzle velocity measured with a Doppler radar. Piezoelectric sensor registered pressure waveforms. On that basis force acting on the breech was calculated, subsequently force acting on the projectile’s base was calculated with Sarrau formula, kinetic energy and work of the projectile in the barrel. Combining those, muzzle velocity was calculated and compared with muzzle velocity measured with Doppler radar. On that basis the coefficient was determined. The coefficient was later verified with the following shooting tests on 120 mm barrel. Conclusions describe issues during gathering data and results of Sarrau formula verification.

1. Introduction
Sarrau formula is the formula relating to pressure acting on a breech with pressure acting on a projectile’s base. Pressure acting on the breech is easily measurable, but pressure acting on a projectile’s base during a shot is a complex issue. Originally it was used for black powder but it is also applicable to modern gun powder. This is empiric formula and contains coefficients for various type of guns and ammunitions. To determine the coefficient, pressure chart during a shot was analyzed. Data in pressure charts were provided thanks to piezoelectric pressure sensors. Piezoelectric pressure sensors are widely used in defense technology. Pressure generated by gun propellant at a moment of a shot change dimension of crystal inside of sensor. That generates an electric charge. It is converted by an amplifier to voltage signal, then converted to pressure values. In the following experiment Doppler radar was used also. It is very reliable method for muzzle velocity calculations. Radar’s work is based on a well-known Doppler effect – change frequency of a wave in dependence on moving source in relation to an observer. The goal of the experiment is determining force and velocity of projectile in
the bore of the gun. These parameters have significant influence for e.g. projectile’s strength calculation.

2. Process of the shot
Process of the shot can be divided for few stages:

- Initial stage – primer ignites a charge, pressure in the gun chamber starts to increase, but projectile isn’t moving yet.
- Primary stage – projectile starts to move in the gun bore, pressure is still increasing. Pressure reaches its maximum, propellant charge is still burning, pressure starts to decrease but projectile is still accelerating.
- Secondary stage – propellant charge is burned, projectile is still accelerated but its acceleration is decreasing. Projectile leaves a muzzle of the gun barrel.

Pressure-time chart can look different depends on the type of the gun propellant. Rapid burning propellant will achieve maximum pressure sooner than leisure burning propellant. On the contrary the leisure burning propellant will assure greater pressure while projectile leaves a muzzle.

During a shot in the gun bore very complicated process has an occurrence. The gas – effect of a burning propellant moves a projectile, and following decompression phenomena in the chamber and the bore occur. The process of that is very turbulent. Particles of gas bounce from each other, from a bore’s wall, and a breech. Additionally gas heats a barrel changing its dimensions. Gas particles travels around in the space between breech and a moving projectile. Their density is in one place high and in a different place is low. All these factors make that the determination of the pressure acting directly on a projectile is very hard to be obtained precisely.

To determine projectile’s pressure very precisely, the wireless piezoelectric sensors could be used. The sensors should be attached to the projectile’s base then. Those sensors are very expensive. Damaging or losing one of them is probably in that solution of measurement system.

The second solution relies on wired piezoelectric sensors and specially prepared barrel. The barrel should have openings in the wall for sensors deployed along it, i.e. from breech and to the muzzle. Each sensor measures pressure when projectile passes the openings. That measurement solution is expensive and time consuming.

There are few calculation solutions (formulas) relating to pressure acting on the projectile base and breech, like the Pidduck-Kent [1] or Sarrau formula. In the paper, the Sarrau formula is verified on the basis of firings from the 120 mm gun of Leopard 2 tank.

3. Process of the experiment and data analysis
The experiment took place in the Military Institute of Armament Technology in Poland. Weapon used in the test, it was 120 mm Rh120L44 barrel. The barrel is a part of special ballistic stand, designed for purpose of ammunition tests. The barrel is adapted to work with pressure sensor. It is threaded into a barrel at a distance of around 150 mm from a breech. The experiment was conducted on two types of ammunition i.e.: HE and APFSDS-T-TP ones. The experiments were performed during last two years. Due to every shot, pressure data chart and muzzle velocity were obtained.

3.1. Pressure data
Pressure data were received from a Kistler 6213B sensor. PC oscilloscope (PicoScope) was connected with charge amplifier and was able to collect pressure data charts. The pressure data chart was very accurate. It was collected with a step of 0,05 µs. One shot lasted around 5 ms. There were caught 10000 values of pressure starting from the time of gun propellant charge ignition to the time when projectile left a muzzle. The sensor was located in the gun chamber, so all pressure data charts present a pressure in the chamber - not pressure near projectile’s base. As the projectile was moving in the bore, the difference between these two pressure values grew.

Piezoelectric pressure sensor has got a few disadvantages. Opening for sensor in the barrel can be covered by dust, and also signal wire between sensor and amplifier is very sensitive for any outsource
noises. After a certain number of measurements pressure sensor will be decalibrated. All these factors may influence on waveform pressure. Determining the exact value of the coefficient, it could be a successful method of verifying a waveform.

Below (figure 1) the pressure curve is presented. This is a pressure in the bore of 120 mm Leopard barrel’s during a shot (19th November 2018 Shot No. 5). Vertical axis represents voltage signal [V]. To get values of pressure, multiplication by 50 [MPa/V] is necessary. In this waveform pressure peak is 414.9 MPa. Horizontal axis represents time [ms]. The “0” value is a start trigger for proper work of measurement system – information to save to the memory given value of time before and after a trigger.

![Pressure waveform of 120 mm barrel’s shot.](image)

In this waveform the initial stage of pressure wave forming can be observed. Pressure is equal zero then starts increasing very fast then increases a little bit slower. It means that projectile has started moving in the bore. The maximum pressure was reached around 2 ms later. There is no significant mark in the waveform when projectile leaves a muzzle of the gun barrel.

3.2 Radar data
The Doppler radar was located 4 meters behind the muzzle and 2 meters offset from a barrel’s axis. This was essential to registering a track of Doppler curve as close to muzzle as possible. The radar used in this experiment was Weibel SL-528 PE.

The radar measured radial velocity. The tangential velocity was calculated by internal software of the radar system.

4. Calculations and results
The basic form of the Sarrau equation is [1]:

\[ P_x = p \left( 1 + \Theta \frac{\omega}{q} \right) \]  

(1)

where:
The pressure acting on the breech, $p_b$, pressure acting on the projectile’s base, $p_p$, mass of the propellant charge of the artillery (projectile) round, $q$, mass of the projectile, $q$, and empiric coefficient, $\Theta$, are known. The pressure acting on the breech is always higher than a pressure acting on a projectile [2]. The above variables are known, except $\Theta$ (theta value). It is an empirical coefficient, which is usually between 0.33 and 0.67. As it is easy to notify that higher theta gives lower pressure on projectile’s base, hence the lower theta gives higher pressure and better (higher) muzzle velocity of the projectile result. Calculations presented in the paper determine more precisely value of the coefficient for 120 mm tank’s barrel.

The first stage, there are muzzle velocity calculations. To do that a pressure waveform has been divided into high number of trapezoids (figure 2).

![Figure 2. Pressure waveform divided into trapezoids for numerical analysis purposes.](image)

The projectile velocity was calculated in the following way:

The pressure during a shot is no constant, but is a function of time – $p(t)$.

$$q \cdot \frac{dv}{dt} = A \cdot p(t)$$  \hspace{1cm} (1)

$A$ – area of cross section of the barrel.

In order to calculate muzzle velocity, the above differential equation must be solved.

$$dv = \frac{A}{q} \cdot p(t) dt$$  \hspace{1cm} (2)
\[ dv = \frac{A}{q} \cdot p(t) \, dt \]  
\[ v = \frac{A}{q} \int_{t_1}^{t_2} p(t) \, dt \]

where:

- \( t_1 \) – time when pressure starts to increase,
- \( t_2 \) – time when projectile leaves the barrel.

Integral was calculated with trapezoidal rule commonly used in numerical analysis:

\[ v_{0\text{piezo}} = \frac{A}{q} \cdot \frac{h}{2} \left( p(t_1) + 2 \sum_{i=0}^{n-1} p(t + ih) + p(t_2) \right) \]

where:

- \( h \) – width of every trapezoid which is equal 0.05 ms.

Piezoelectric sensor measures breech pressure, so all values of waveform was converted by means of Sarrau formula. Theta coefficient was picked in that way to make a calculated velocity equal with measured velocity with Doppler radar.

### 4.1. Results of the experiments

On 14th March 2019 the shooting of 120 mm APFSDS-TP ammunition took place. The results of calculations are presented below:

**Table 1. Result of \( \Theta \) calculations.**

| Shot number | \( \Theta \) | \( V_{0\text{piezo}} = V_{0\text{radar}} \) [m/s] | Peak Pressure [MPa] |
|-------------|---------------|---------------------------------|---------------------|
| 1           | 0.4425        | 1734.0                          | 473.4               |
| 2           | 0.4385        | 1749.9                          | 490.5               |
| 3           | 0.4312        | 1745.0                          | 479                 |
| 4           | 0.4384        | 1734.5                          | 475.1               |
| 5           | 0.4454        | 1731.4                          | 465.1               |
| 6           | 0.432         | 1741.5                          | 473.9               |
| 7           | 0.447         | 1727.3                          | 468                 |
| 8           | 0.4445        | 1736.4                          | 468.6               |
| 9           | 0.4483        | 1737.5                          | 468.1               |
| 10          | 0.4495        | 1739.3                          | 474.2               |
| Avg         | 0.44173       | 1737.68                         | 473.7               |

\( \sigma \) – standard deviation,

\( \text{Avg} \) – average value.

The results of shooting APFSDS-TP ammunition on 14 February 2019 are presented below:

**Table 2. Result of \( \Theta \) calculations.**

| Shot number | \( \Theta \) | \( V_{0\text{piezo}} = V_{0\text{radar}} \) [m/s] | Peak Pressure [MPa] |
|-------------|---------------|---------------------------------|---------------------|
| 1           | 0.4365        | 1737.3                          | 475.4               |
| 2           | 0.4378        | 1721.2                          | 481.3               |
| 3           | 0.4396        | 1733.5                          | 470.8               |
| 4           | 0.4250        | 1739.3                          | 475.4               |
Theta coefficient stays similar, i.e. it is between 0.41 and almost 0.45. It is not a small difference but ensures us that the process of every shot is quite similar. Batch of ammunition was different in above days of test. The higher Theta coefficient means that more of total kinetic energy is consumed by a charge, and less of this energy is consumed by a moving projectile.

The results of shooting APFSDS-TP ammunition on 01st March 2018 are presented below:

**Table 3.** Results of $\Theta$ calculations for +15°C seasoned rounds.

| Shot number | $\Theta$ | $V_{0\text{piezo}}=V_{0\text{radar}}$ [m/s] | Peak Pressure [MPa] | Temperature of rounds |
|-------------|----------|-------------------------------------------|---------------------|-----------------------|
| 1           | 0.3980   | 1727.0                                    | 447.4               | +15°C                 |
| 2           | 0.4040   | 1727.0                                    | 452.1               |                       |
| 3           | 0.4040   | 1728.9                                    | 452.7               |                       |
| 4           | 0.4075   | 1722.4                                    | 448.8               |                       |
| 5           | 0.4085   | 1728.6                                    | 456.5               |                       |
| 6           | 0.4145   | 1722.3                                    | 453.8               |                       |
| 7           | 0.4070   | 1731.1                                    | 458.0               |                       |
| 8           | 0.4075   | 1726.4                                    | 453.9               |                       |
| 9           | 0.4165   | 1721.7                                    | 454.1               |                       |
| 10          | 0.4140   | 1730.2                                    | 460.9               |                       |
| Avg         | 0.4082   | 1726.6                                    | 453.8               |                       |
| $\sigma$    | 0.0053294| -                                         | -                   |                       |

**Table 4.** Results of $\Theta$ calculations for +50°C seasoned rounds.

| Shot number | $\Theta$ | $V_{0\text{piezo}}=V_{0\text{radar}}$ [m/s] | Peak Pressure [MPa] | Temperature of rounds |
|-------------|----------|-------------------------------------------|---------------------|-----------------------|
| 1           | 0.3910   | 1790.5                                    | 513.7               | +50°C                 |
| 2           | 0.3980   | 1781.0                                    | 520.0               |                       |
| 3           | 0.3905   | 1787.0                                    | 519.0               |                       |
| 4           | 0.3910   | 1782.1                                    | 505.0               |                       |
| 5           | 0.3975   | 1753.2                                    | 491.0               |                       |
| 6           | 0.3945   | 1759.9                                    | 495.0               |                       |
| 7           | 0.3885   | 1755.5                                    | 487.0               |                       |
| 8           | 0.3920   | 1758.0                                    | 488.0               |                       |
| Avg         | 0.3929   | 1770.9                                    | 502.3               |                       |
| $\sigma$    | 0.0032186| -                                         | -                   |                       |

Table 3 and table 4 present the best results – i.e. when deviation is the lowest. In dependence on the measurement, change of theta by 0.005 ($\sigma$ value in table 4) changes muzzle velocity about 10 m/s, The
seasoned in 50°C rounds have the lower theta – charge is preheated hence less energy is used for initial warming up a charge, more for the accelerating projectile The piezoelectric sensors were calibrated one day before tests. For all above tests, APFSDS-TP projectiles were used.

On 28th February 2018 the test with HE-TP projectiles took place. HE projectile is different type of projectile than APFSDS one. HE projectile has a different shape, it is a full caliber projectile, much more heavier, and it has different mass of propellant charge.

Results of the test HE TP T ammunition is presented below:

| Shot number | Θ  | V₀ piezo−V₀ radar [m/s] | Peak Pressure [MPa] |
|-------------|----|------------------------|---------------------|
| 2           | 0.3960 | 942.6                  | 437.5               |
| 3           | 0.4025 | 946.2                  | 448.8               |
| 4           | 0.4025 | 946.7                  | 445.4               |
| 5           | 0.4080 | 949.3                  | 455.8               |
| 6           | 0.3900 | 904.3                  | 396.0               |
| 7           | 0.4300 | 950.7                  | 462.5               |
| 8           | 0.4500 | 951.4                  | 465.5               |
| 9           | 0.4250 | 953.3                  | 466.6               |
| 10          | 0.4300 | 952.2                  | 467.6               |
| 11          | 0.4600 | 952.4                  | 473.7               |
| Avg         | 0.4194 | 944.9                  | 451.9               |

Deviation is the highest for all the tests but in these cases theta change by 0.02 will change muzzle velocity only about 5 m/s. In this case, deviations equal 0.022 gives around 5 m/s accuracy. Theta is in similar range of values like for the APFSDS tests despite of different mass of propellant charge and different mass of projectile. That means the theta coefficient value for 120 mm Leopard’s ammunition is around 0.41 ± 8%.

5. Conclusions

In the paper, suitability of Sarrau formula was presented. The Θ coefficient value is between 0.39 and 0.44 and it depends on the type of projectile, initial temperature, date of test and others factors. Theta coefficient was changing a little bit after every shot. The reason of it might be differences of propellant and projectile mass between every shot. The mass tolerances have impact on the value of Θ coefficient. Those tolerances were known but they can’t be published due to protecting producer’s technical data.

However, calculations give good scope for pressure acting on projectile’s base. It can be used for designing projectiles, fuzes, and other elements of the round.

With a defined theta coefficient, it is possible to calculate muzzle velocity. In above calculations, average theta coefficient will provide muzzle velocity with accuracy around ± 10 m/s for APFSDS-TP rounds and ±5 m/s for HE-TP rounds.

The other thing is verification waveforms and/or process of propellant burning. In example, if for one of the shots theta would be very different than 0.4 that means that measurement equipment was faulty, or propellant burning process was other than it should be.

The advantages of the formula, there are relatively simple calculations compared to Pidduck-Kent Solution.
The main disadvantage of Sarrau formula is such that it doesn’t consider a barrel’s length. Obtaining a real projectile’s velocity in the bore or pressure data from the few piezoelectric sensors would give opportunity to make adjustments to the formula regarding a barrel’s length.

Theta coefficient can be defined with a better accuracy if more than one pressure sensor will be used simultaneously. That would eliminate sensor errors and gives a better scope on that phenomena.

6. References
[1] R. N. Jones, H. P. Hitchcock D. R. Villegas, 1964 *Interior Ballistics Of Guns* (Washington D.C., Headquarters United States Army Materiel Command) chapter 5 p 3
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[3] M. Sieriebrakow 1955 *Vnutriennaja Ballistika* (Warsaw, Ministry of Defence Press)