Behaviour of ultrasonic waves in porous rigid materials: an anisotropic Biot-Attenborough model

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Abstract. The anisotropic pore structure and elasticity of cancellous bone cause wave speeds and attenuation in cancellous bone to vary with angle. Anisotropy has been introduced into Biot theory by using an empirical expression for the angle-and porosity-dependence of tortuosity. Predictions of a modified anisotropic Biot–Attenborough theory are compared with measurements of pulses centred on 100 kHz and 1 MHz transmitted through water-saturated porous samples. The samples are 13 times larger than the original bone samples. Despite the expected effects of scattering, which is neglected in the theory, at 100 kHz the predicted and measured transmitted waveforms are similar.

1. Introduction
Understanding the propagation of acoustic waves through cancellous bone is an important prerequisite to improving the diagnosis of osteoporosis by ultrasound. Bone essentially has two types of structure, both having the same mineralised collagen composition. Cortical bone has porosity less than 30% and may generally be considered to be solid; cancellous bone has porosity greater than 30% and consists of a complex open-celled porous network of rod- and plate-shaped elements termed trabeculae. The porosity of human cancellous bone ranges between 70% and 95%, the remaining volume being perfused with bone marrow. In the adult human vertebral body for example, both horizontal and vertical trabeculae range from 50–120 μm in thickness, and spaced at intervals of between 1200 – 5000 μm and 700 and 2000 μm respectively [1]. During childhood, more bone is added than is being taken away.

During early adulthood, the amounts removed and added are the same. If however, more bone is removed than is being added, we have a condition called osteoporosis which literally means ‘porous bone’ and describes a period of largely asymptomatic bone loss leading to skeletal fragility and increased risk of fracture. It is caused by hormonal imbalance (oestrogen & testosterone) and long-term cortico-steroid use. It is also caused by low bone mass, as well as a weakened structure [2] and [3]. One in three women and one in five men over the age of 50 will break a bone attributed to osteoporosis. It is second only to cardiovascular disease as a global healthcare problem [4]. Osteoporosis leads to nearly 9 million fractures annually worldwide [5], and over 300,000 patients present with fragility fractures to hospitals in the UK each year [6]. Direct medical costs from fragility fractures to the UK healthcare economy were estimated at £1.8 billion in 2000, with the potential to increase to £2.2 billion by 2025, and with most of these costs relating to hip fracture care [7]. To improve the prediction of fracture risk by ultrasound it is important to understand the propagation of acoustic waves through cancellous bone. The theory that is mostly used for investigation of acoustic
wave propagation in cancellous bone is Biot theory which was specifically developed to describe acoustic wave propagation in fluid-saturated porous elastic media [8] and [9]. The Biot theory allows for an arbitrary microstructure, with separate motions considered for the solid elastic framework (bone) and the interspersed fluid (marrow), induced by the ultrasonic wave, and also includes energy loss due to viscous friction between solid (bone) and fluid (marrow). The Biot theory predicts a shear wave. The Biot model has since been used extensively to describe the wave motion in trabecular (cancellous) bone [10], [11], [12], [13], [14], [15] and [16]. Attenborough et al. [17] presented tortuosity deduced from audio-frequency measurements in air-filled cancellous bone replicas and showed that there was strong anisotropy.

Laugier and Haiat [18] have presented an extensive view of the mathematical and numerical models to understand Quantitative ultrasound (QUS) potential and the types of variables that can be determined by QUS in order to characterize bone strength. A reasonable protocol has been presented for case-finding purposes, which relies on a combined assessment of clinical risk factors (CR.F) and heel QUS by Hans and Krieg [19]. The clinical applications of QUS in the detection and management of osteoporosis and osteoporosis-related fractures have been reviewed. Foiyet et al. [20] have used ultrasound waves to characterize and determine the structural and material properties of cortical bone. The frequency spectrum of Lamb type of waves is the input data of the method of parameters identification, based on a least mean square algorithm. Resonant ultrasound spectroscopy (RUS) has been used to measure the anisotropic elasticity of the cortical bone despite the strong viscoelastic damping. RUS does not require a minimum size for the tested specimens, it may be appropriate for the characterization of small bone specimens. Normal and pathological bone material properties can be investigated by using RUS.

Previous work on the influence of anisotropic pore structure and elasticity in cancellous bone has been extended by developing an anisotropic Biot–Allard model allowing for angle dependent tortuosity and elasticity by Aygün et al. [21]. The extreme angle dependence of tortuosity corresponding to the parallel plate microstructure used by Hughes et al. [15] has been replaced by angle dependent tortuosity values based on data for slow wave transmission through air-filled bone replicas. Audio-frequency data obtained at audio-frequencies in air-filled bone replicas are used to derive an empirical expression for the angle-and porosity-dependence of tortuosity. Most recently, Aygün et al. [22, 23] have transmitted ultrasonic signals through water saturated stereolithograpical bone replicas. It is found that the expected occurrence of scattering does not cause significant discrepancies between predictions and data at 100 kHz (which would be equivalent to 1.3 MHz in real bone), perhaps as a consequence of the fact that the samples behave as low pass filters. Scattering should be even more important at 1 MHz (equivalent to 13 MHz in real bone) where the fast and slow wavelengths are 3 mm and 1.5 mm respectively.

Aim of this work is to investigate the dependence of ultrasonic wave propagation upon the material and structural properties of porous rigid materials. Stereolithograpical (see Figure 1) bone replicas and ceramic have been used for the investigation of the influences of perforation and thinning in cancellous bone on the acoustical and mechanical properties of the bone structure.

![Figure 1. Views of three stereolithograpical (STL) bone replicas and a porous ceramic.](image-url)
2. Theory
A porous rigid sample of length \(L\) is subjected to an incident ultrasonic wave in fluid (water), \(P'\) (see Figure 2). Part of incident wave is reflected back into the fluid, \(P_r\), while other part is transmitted through the sample, \(P_t\). Fellah et al. have presented an analytical model in order to describe the viscous interaction between fluid and a porous elastic structure. The Fourier transform of the transmitted field is given by Fellah et al. as:

\[
P_i(x, \omega) = \tilde{T}(\omega) \exp \left( -j \omega \frac{(x-L_{TP})}{c_0} \right) \varphi(\omega), \quad x \geq L_{TP}
\]

where \(\varphi(\omega)\) is the Fourier transform of the incident field \((P'(t))\), \(\tilde{T}(\omega)\) is the Fourier transform of the transmission kernel, \(\omega\) is the angular frequency of motion, \(c_0\) is the speed of sound in fluid, and \(L_{TP}\) is the transmission path. A more detailed consideration of the transformed field can be found in the paper by Fellah et al. The transmission coefficient \(T(\omega)\), which is the Fourier transform of \(\tilde{T}\).

\[
\text{Figure 2. Geometry of a porous rigid material}
\]

Aygün et al. [21] have introduced a transverse anisotropy into Biot-Allard model by allowing angle-and-porosity dependent tortuosity, and angle-dependent elasticity. Tortuosity is known to have an important influence on high frequency sound propagation in fluid saturated porous media. A heuristic form for porosity- and angle-dependent tortuosity is proposed by Aygün et al. [21] as:

\[
\alpha_\phi = 1 - r \left( 1 - \frac{1}{\phi} \right) + k \cos^2(\theta)
\]

where \(\phi\) is the porosity, \(\theta\) is the variable between 0\(^\circ\) and 90\(^\circ\), \(r\) and \(k\) can be considered adjustable. A range of possible values of \(r\) and \(k\) have been found by comparing predictions of equation (2) for \(\theta = 0^\circ\) and 90\(^\circ\) respectively with values deduced from air-filled replica [17] of known porosity. Values of \(r\) and \(k\) are found by solving the resulting simultaneous equations.

To predict transmission through an anisotropic poroelastic sample it is necessary to allow for elastic anisotropy also. The dependence of skeletal frame modulus (Young’s modulus, \(E_b\), Bulk Modulus, \(K_b\), and rigidity modulus, \(\mu_b\)) in terms of bone volume fraction \((1 - \phi)\) and the Young’s modulus of the solid material of the frame \((E_s)\) are given by Williams [24]:

\[
E_b = E_s (1 - \phi)^n, \\
K_b = E_s (1 - 2\nu_b), \\
\mu_b = E_s / (1 + 2\nu_b),
\]

where \(\nu_b\) is the Poisson’s ratio of frame, and the exponent \(n\) varies from 1 to 3 according to Gibson [25], depending on the angle \((\theta)\) with respect to the dominant structural orientation according to \(n = n_1\)
\[ \sin^2(\theta) + n_2 \cos^2(\theta). \]

Values of \( n_1 = 1.23 \) and \( n_2 = 2.35 \) are chosen by Lee et al.\[26\] to be consistent with the work of Williams \[24\].

3. Measurements

The experimental procedure used by Fellah et al. \[13\] has been followed to perform measurements in a water tank (see Figure 1). Two 100 kHz and two 1 MHz plane piezoelectric transducers have been used for experiments. 400 V pulses are provided by a 5058PR Panametrics pulser/receiver. Electronic interference is removed by 1000 acquisition averages. Once we start revolving the bone replica sample around itself, its thickness that ultrasonic wave transmitted through became equal to \( L \cdot \cos(\theta) \) where \( \theta \) is the angle of propagation.

![Experimental setup for ultrasonic measurements.](image)

When a wave impinges on a STL bone replica, part of the wave is reflected back. The part of the wave penetrating into the sample undergoes mode conversion into fast and slow components which are transmitted through the STL bone replica. The measurements have been made parallel to the trabeculae direction. The stereolithographical bone replicas used in the measurements are in the form of 57 mm cubes. The incident (reference) signals generated by 100 kHz and 1 MHz transducers and transmitted through fluid (water) are shown in Figure 4a and Figure 5a, and their spectra are shown in Figure 4b and Figure 5b, respectively. The waveform at 100 kHz looks like a complex waveform consisting of more than one signal while the waveform at 1 MHz looks like a sine wave with its amplitude reducing by time.
4. Comparisons between predictions and data
Wave transmission through replicas has been predicted at normal angles and results have been compared with measurements performed in a water filled tank. The parameters used in the predictions
are listed in Table 1. The elastic moduli of the bone replicas made of resin have been taken to be equal to the elastic modulus of resin which is 6.04 GPa (DSM Somos) and is smaller than the elastic modulus of real bone which is 20 GPa [24]. Assuming that the permeability of the bone is $5 \times 10^{-9}$ m$^3$ [10], the permeability of bone replicas has been taken to be $13^2$ times higher because of the magnification of the actual size of the bone microstructure by 13 times in each direction. The assumed characteristics of the saturating fluid (water) are: density $\rho_f = 1000$ kg/m$^3$, viscosity $\eta = 10^{-3}$ kg ms$^{-1}$, speed of sound in water $c_0 = 1490$ m/s. Measured and predicted transmitted signals are travelling through the bone replicas in the same direction as the trabecular alignment ($x$ direction). The incident signal shown in Figure 4a has been used for predictions. Predicted transmitted waves of STL Iliac Crest (ICF), and Femoral Head (FRA) replicas versus time are compared with measured transmitted waves in water at 100 kHz in Figures 6 and 7, respectively. The agreement between predicted and measured transmitted waveforms is reasonable. The initial parts of the measured and predicted transmitted waveforms can be identified as the fast wave arrival while the second and major parts of the transmitted waveforms can be identified as the slow wave contribution. Arrival times for measured and predicted fast waves of ICF bone replica at 100 kHz are $3.4 \times 10^{-5}$ s and $3.8 \times 10^{-5}$ s respectively. Arrival times for measured and predicted fast waves of FRA bone replica at 100 kHz are $3 \times 10^{-5}$ s and $3.5 \times 10^{-5}$ s respectively. A possible reason of late arrival of fast wave might be signal leakage caused by refraction from the CAB replica at higher angles and transmission paths that fall outside the receiving transducers. Fifteen times of enlargement of the original sample in 3 directions might be another reason for late arrival. It is not possible to determine the exact arrival time for slow waves because of overlapping between fast and slow waves.

![Figure 6](image-url)

**Figure 6.** Comparison of predicted and measured transmitted waveforms through a water-saturated ICF bone replica at 100 kHz at normal incidence.
Figure 7. Comparison of predicted and measured transmitted waveforms through a water-saturated FRA bone replica at 100 kHz at normal incidence.

Transmitted waveforms at the different oblique angles versus time have been measured in order to understand the dependence of the wave transmission on the propagation angle. Measured transmitted waveforms of CAB bone replica subjected ultrasonic propagation at different angle of propagation are quite different from each other as seen in Figure 8. Generally, measured and predicted transmitted waveforms through CAB bone replica are very similar. The arrival time of the ultrasonic waves changes when the angle of propagation was varied. Since the tortuosity equation used in this paper depends on the angle of propagation, varying the angle changes the tortuosity. The arrival time of the ultrasonic wave depends on the quantity of the bone in the path of the wave transmission. Only two parameters, Poisson’s ratio of frame and viscous characteristic length, were adjusted in order to obtain the ‘best-fit’. In particular, the predictions are very sensitive to the assumed values of viscous characteristic length. The ‘best-fit’ characteristic length values (Table 1) for the replicas are about 13 times those found for real bone in the literature i.e. between 5 and 10 µm [13, 14].
Table 1: Input parameters for Stereolithograpical (STL) bone replicas.

| Parameters                  | Iliac Crest, ICF | Femoral Head, FRA | Calcaneus, CAB | Porous Ceramic |
|-----------------------------|------------------|-------------------|---------------|---------------|
| Density, $\rho_s$, kg/m$^3$ | 1233.4           | 1227              | 1171          | 950           |
| Young’s modulus, $E_s$, GPa | 6.04             | 6.04              | 6.04          | 2.6           |
| Poisson’s ratio of solid, $v_s$ | 0.30          | 0.30              | 0.30          | 0.29          |
| Poisson’s ratio of frame, $v_b$ | 0.36           | 0.40              | 0.34          | 0.35          |
| Porosity, $\phi$           | 0.8386           | 0.7426            | 0.8822        | 0.62          |
| Permeability, $k_0$, m$^3$ | $845 \times 10^{-9}$ | $845 \times 10^{-9}$ | $845 \times 10^{-9}$ | $5 \times 10^{-10}$ |
| Form factor, $c$            | 1.2              | 1.2               | 1.2           | 1.2           |
| Form factor, $c'$           | $c/2$            | $c/2$             | $c/2$         | $c/2$         |
| Viscous characteristic length, $L$, $\mu$m | 100             | 60                | 150           | 38            |
| $r$                         | 0.888            | 0.591             | 0.816         | 0.56          |
| $k$                         | 0.468            | 0.684             | 0.574         | 0.44          |

Figure 8. Measured and predicted transmitted waveforms through CAB bone replica versus time with 1MHz transducers at 10°, 12°, 20°, and 29°.
Predicted transmission coefficient as a function of frequency for ceramic is shown in Figure 9. Porous ceramic is obtained by mixing clay and plastic then burning the plastic in a kiln at Laboratory of Acoustics and Thermal Physics at K. U. Leuven. Porous ceramic used for measurements is in the form of 65 mm squares with 30 mm thickness. Predicted and measured transmitted waveform in the porous ceramic at oblique incidence (25°) is presented as a function of propagation time in Figure 9. There is reasonable agreement between predictions and data. It seems that less than 3% of the incident wave amplitude is transmitted through porous ceramic at 1 MHz. Most of the ultrasonic wave is reflected back into the water. Generally measured and predicted transmitted waveforms are similar except for the initial parts of the transmitted waveforms.

**Figure 9.** Measured and predicted transmitted waveforms through porous ceramic versus time at 25°.

**5. Conclusion and further work**

Predictions of a modified anisotropic Biot-Allard theory which neglecting scattering, have been compared with measurements of pulses centered on 100 kHz and 1 MHz transmitted through water saturated porous rigid samples at normal and oblique angles. The predictions and data are in reasonable agreement.

Further works need to be done on porous solid materials (ceramics) with different porosity values to investigate the Osteoporosis by using structural borne vibration. A sinusoidal vibration should be applied to porous samples. This will cause a structural borne sound wave to propagate through the samples. The resulting vibration should be measured using either an ultrasound probe or an accelerometer. Based on these results, a hand held, low cost and portable device should be designed and developed to detect the Osteoporosis in the bone with the aim of improving early detection rates.

The chief debilitating consequence of osteoporosis is fracture, and early detection of the condition can allow interventions reducing the likelihood of occurrence [25]. As hip fracture invariably leads to hospital admission, early detection of the condition could significantly reduce the overall long term treatment costs by reducing admission to hospital [26]. Due to anticipated low costs of the product, it is anticipated that access to early screening would improve with increased overall take up by clinics compared to other technologies.
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