Bimaximal Neutrino Mixings from Lopsided Mass Matrices

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Abstract

Current solar and atmospheric neutrino oscillation data seem to favor a bimaximal pattern for neutrino mixings where the matrix elements $U_{e2}$ and $U_{\mu 3}$ are of order one, while $U_{e3}$ is much smaller. We show that such a pattern can be obtained quite easily in theories with “lopsided” mass matrices for the charged leptons and the down type quarks. A relation connecting the solar and atmospheric neutrino mixing angles is derived, $\tan^2 \theta_{atm} \simeq 1 + \tan^2 \theta_{sol}$, which predicts $\sin^2 2\theta_{atm} \simeq 0.97$ corresponding to the best fit LMA solution for solar neutrinos. Predictive schemes in $SO(10)$ realizing these ideas are presented. A new class of $SO(10)$ models with lopsided mass matrices is found which makes use of an adjoint VEV along the $I_{3R}$ direction, rather than the traditional $B - L$ direction.
§1. Introduction

Recent data seems somewhat to favor either the LMA (large mixing angle MSW) or the LOW solution to the solar neutrino problem over the small mixing angle solution \[1, 2\]. Taken together with the atmospheric neutrino results \[3\] and the CHOOZ reactor experiment \[4\], this would imply a so-called “bimaximal” pattern of mixing, with \(U_{e2}\) and \(U_{\mu 3}\) large and \(U_{e3}\) small (\(U\) is the leptonic mixing matrix) \[5\]. Another possibly significant feature of the data is that the atmospheric neutrino mixing angle is not merely large, but seems to be nearly maximal. The best-fit value at present is \(\sin^2 2\theta_{atm} \simeq 1.0\) \[3\].

In this paper we make several points relevant to these observations.

1. The so-called “lopsided” models \[6, 7, 8, 9, 10, 11\] provide a very simple way of accounting for the bimaximal pattern of mixing, and in particular have no difficulty in obtaining the LMA solar solution, unlike certain other kinds of bimaximal schemes.

2. By combining the idea of lopsided mass matrices with a nonabelian flavor symmetry one can explain in a simple way the near maximality of \(\theta_{atm}\). In the simplest case one obtains the relation \(\tan^2 \theta_{atm} = 1 + \tan^2 \theta_{sol}\). In the limit of small solar angle this gives maximal atmospheric angle; while for the best-fit LMA value \[12\] of \(\tan^2 \theta_{sol} \simeq 0.4\), it gives \(\sin^2 2\theta_{atm} = 0.97\).

3. The lopsided bimaximal idea can be straightforwardly implemented in the context of \(SO(10)\), and in that case a prediction relating quark masses and mixings to the atmospheric neutrino mixing angle arises.

4. A new class of predictive \(SO(10)\) models for quark and lepton masses is found which makes use of an adjoint VEV along the \(I_{3R}\) direction. Bimaximal mixing pattern for neutrinos can be obtained easily in this class of models, along with several predictions relating the charged fermion masses and mixings. This provides a new way of looking at quark and lepton masses in \(SO(10)\), different from the traditional way where the VEV of the adjoint points along the \(B - L\) direction.

§2. Bimaximal mixing

Imagine that the leptonic mixing angles come primarily from the diagonalization of the charged lepton mass matrix \(L\), which has the following “lopsided” form:

\[
L = \begin{pmatrix}
- & - & - \\
- & - & \epsilon \\
\rho' & \rho & 1
\end{pmatrix} m_D. \tag{1}
\]

Here \(\rho' \sim \rho \sim 1\), whereas \(\epsilon \ll 1\). The dashes represent elements that are small compared to the ones shown. The convention being used is that the left-handed lepton fields multiply the mass matrix from the right. The diagonalization of this matrix can be
done in stages, the first stage being to rotate in the space of \( \ell_2^- \) and \( \ell_1^- \) by an angle which we will call \( \theta_s \), satisfying \( \tan \theta_s = \rho'/\rho \). This brings the matrix to the form

\[
L' = \begin{pmatrix} - & - & - \\ - & \epsilon & - \\ 0 & \sigma & 1 \end{pmatrix} m_D, \quad (2)
\]

where \( \sigma \equiv \sqrt{\rho'^2 + \rho^2} \). (Note that all parameters shown in \( L \) can be made real by field redefinitions.) The next stage is to rotate in the space of \( \ell_3^- \) and the new \( \ell_2^- \) by an angle which we will call \( \theta_a \), satisfying \( \tan \theta_a = \sigma \). This brings the matrix to the form

\[
L'' = \begin{pmatrix} - & - & - \\ - & \frac{\sigma}{\sqrt{\sigma^2+1}} & \frac{1}{\sqrt{\sigma^2+1}} \epsilon \\ 0 & 0 & 1 \end{pmatrix} m_D. \quad (3)
\]

The rotations needed to complete the diagonalization involve only small rotations of the left-handed leptons, and we will therefore neglect them. (An important point is that the (2,1) and (2,2) elements of \( L \) were assumed small compared to \( \epsilon \). Otherwise, there would still be required a large rotation in the 1-2 plane to diagonalize \( L'' \), and that would induce a large \( U_{e3} \).) The unitary matrix \( U_L \) required to diagonalize \( L^\dagger L \) is thus approximately

\[
U^\dagger_L \cong \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos \theta_a & -\sin \theta_a \\ 0 & \sin \theta_a & \cos \theta_a \end{pmatrix} \begin{pmatrix} \cos \theta_s & -\sin \theta_s & 0 \\ \sin \theta_s & \cos \theta_s & 0 \\ 0 & 0 & 1 \end{pmatrix} = \begin{pmatrix} c_s & s_a & 0 \\ c_a s_s & c_a s_s & -s_a \\ s_a s_s & s_a c_s & c_a \end{pmatrix}. \quad (4)
\]

The full leptonic mixing matrix is given by \( U_{MNS} = U^\dagger_L U_\nu \), where \( U_\nu \) is the unitary matrix required to diagonalize the neutrino mass matrix. However, since we are assuming \( U_\nu \cong I \), \( U_{MNS} \) is given approximately by the matrix in Eq. (4), which has the bimaximal mixing pattern, with \( U_{e2} \) and \( U_{\mu 3} \) both of order unity and \( U_{e3} \) small. A very important point is that no constraint whatsoever has had to be placed on the neutrino masses. The questions of neutrino mixing and neutrino mass are completely decoupled in this scenario. This means, in particular, that there is no difficulty in obtaining the neutrino mass ratios appropriate to any of the large angle solar solutions, LMA, LOW, and VAC.

In many published models the bimaximal mixing comes from the diagonalization of the neutrino mass matrix \([13]\). It is instructive to compare the present idea to some of these other approaches. Consider the following three forms of \( M_\nu \), the light neutrino mass matrix obtained after seesaw diagonalization.

\[
M^A_\nu \sim \begin{pmatrix} - & - & - \\ - & \sigma^2 & \sigma \\ - & \sigma & 1 \end{pmatrix} m_\nu, \quad M^B_\nu \sim \begin{pmatrix} \rho^2 & \rho \rho' & \rho' \\ \rho' & \rho^2 & \rho \\ \rho & \rho' & 1 \end{pmatrix} m_\nu, \quad M^C_\nu \sim \begin{pmatrix} - & 1 & \sigma \\ 1 & - & - \\ \sigma & - & - \end{pmatrix} m_\nu. \quad (5)
\]
As before, the dashes indicate elements smaller than the ones explicitly shown, and $\rho$, $\rho'$ and $\sigma$ are assumed to be of order unity.

In matrix $M^A_\nu$, a large rotation angle, satisfying $\tan \theta_a \cong \sigma$, is required to diagonalize the 2-3 block. This produces a large atmospheric neutrino mixing angle. The magnitude of the solar neutrino mixing angle depends on the magnitude of the small elements in $M^A_\nu$, and may also be large. Because of the approximately “factorized” or rank-1 form of the 2-3 block of this matrix, there is only one large mass eigenvalue, so that the desired hierarchy $m_1, m_2 \ll m_3$ results. Matrix $M^A_\nu$ can thus give a satisfactory bimaximal mixing. However, there is a price to be paid for this: in order for both the mixing angles and the neutrino masses to come out right a certain precise relationship had to be assumed to exist among the elements of $M_\nu$ — namely, the approximately factorized or rank-1 structure of the 2-3 block. Moreover, for the solar neutrino mixing angle to be large, further assumptions have to be made about the small elements of $M^A_\nu$.

Matrix $M^B_\nu$ has the apparent advantage over matrix $M^A_\nu$ that both the atmospheric and the solar neutrino mixing angles automatically come out to be order one. However, it does not give a realistic bimaximal scheme, since $U_{e3}$ is of order one rather than small. The reason is the following. To diagonalize $M^B_\nu$, requires first rotating in the 1-2 plane by an angle with $\tan \theta_s = \rho'/\rho$, and then in the 2-3 plane by an angle with $\tan \theta_a = \sigma \equiv \sqrt{\rho'^2 + \rho^2}$. This is the same as what was required to diagonalize the charged lepton mass matrix in Eq. (1). However, because one is diagonalizing the neutrino mass matrix in this case, the resulting MNS matrix is the adjoint of what was obtained in Eq. (4). (Recall that $U_{MNS} = U_{L}^\dagger U_{\nu}$.) Thus, here $U_{e3} = \sin \theta_a \sin \theta_s$. Moreover, as in the previous example, matrix $M^B_\nu$ requires a form in which the elements are in a special precise relationship to each other.

Matrix $M^C_\nu$ is the typical “inverted hierarchy” form [14], with an approximate $L_e - L_\mu - L_\tau$ symmetry and automatically gives bimaximal mixing, with $U_{e3}$ very small. This can be seen as follows. The matrix $M^C_\nu$ can be diagonalized in stages, as in the other examples. In this case, however, the first stage is to rotate in the 2-3 plane by an angle $\theta_a$ such that $\tan \theta_a = \sigma$. This brings the matrix to a “pseudo-Dirac” form with large and equal (1,2) and (2,1) elements and all other elements small. The next stage is a rotation by $\pi/4$ in the 1-2 plane. Thus, $U_\nu$ has the form given in Eq. (4) with $\theta_s \cong \pi/4$. The fact that the solar neutrino angle typically comes out very close to maximal is certainly acceptable for the LOW and VAC solutions, but may not be acceptable for the LMA solution if it turns out that the LMA fits require $\tan^2 \theta_{sol}$ to be significantly smaller than one. At the moment, the best-fit LMA value is $\tan^2 \theta_{sol} \sim 0.4$, but maximal mixing is within the 99% confidence level contours given in [12].

One sees from these comparisons that obtaining bimaximal mixing from the diagonalization of the charged lepton mass matrix simplifies the problem by neatly separating the questions of neutrino mass and neutrino mixing.

§3. Nearly maximal atmospheric neutrino mixing

At present the best fit to the atmospheric neutrino angle is $\sin^2 2\theta_{atm} \sim 1.0$. This is
difficult to obtain as a prediction from models. Several types of model, indeed, predict 
that the solar angle should be very close to maximal — for instance, the “inverted 
hierarchy” models \[14\] just described and “flavor democracy” models \[15\]. But maximal 
*atmospheric* neutrino mixing is much harder to achieve. The reason is simple. The 
most obvious way to get nearly maximal mixing of two neutrino flavors is by a pseudo-
Dirac form of the mass matrix: \[
\begin{pmatrix}
\delta & 1 \\
1 & \delta'
\end{pmatrix} ,
\] with \(\delta, \delta' \ll 1\). This form also gives nearly 
degenerate neutrinos. Therefore, if such a form is assumed for the 1-2 block of \(M_\nu\), 
to give maximal solar neutrino mixing angle, it also typically gives \(\Delta m^2_{\text{sol}} \ll \Delta m^2_{\text{atm}}\), 
as desired. However, if the 2-3 block of the neutrino mass matrix is assumed to have 
a pseudo-Dirac form, to give maximal atmospheric neutrino mixing angle, it typically 
gives \(\Delta m^2_{\text{atm}} \ll \Delta m^2_{\text{sol}}\), which is wrong.

It is quite difficult to find a form of \(M_\nu\) that both gives maximal atmospheric neutrino 
mixing angle and \(\Delta m^2_{\text{sol}} \ll \Delta m^2_{\text{atm}}\). The ingenious model of Ref. \[16\] shows what is 
required to obtain this result.

It is much easier to obtain maximal atmospheric neutrino mixing through the charged 
lepton mass matrix \[17\], precisely because that decouples the neutrino mixing pattern 
from the neutrino mass pattern. Consequently, the requirement that \(\Delta m^2_{\text{sol}} \ll \Delta m^2_{\text{atm}}\) 
presents no difficulty. All that is needed is that there be a nonabelian symmetry relating 
\(\mu^-\) and \(\tau^-\) so that the parameter \(\rho\) in Eq. (1) comes out to have magnitude 1. This can 
be done in various ways. One possibility is that \((\mu^-_L, \tau^-_L) \equiv \psi^-\) form a doublet of the 
permutation group \(S_3\), while the \(e^-\) is a singlet. If the \(S_3\) is broken by a doublet “flavon” 
field \(\chi\), with its VEV given by \(\langle \chi_i \rangle = (1, i)\) (this form of the VEV can emerge from certain 
simple forms of the Higgs potential as shown below), then the desired \((3,2)\) and \((3,3)\) 
elements of the matrix \(L\) given in Eq. (1) can arise from the term \(\tau^+_L \psi^- \langle \chi_i \rangle \langle H_d \rangle\). This 
will make \(|\rho| = 1\) in Eq. (1).

Simple Higgs potentials can be constructed with flavon fields that have the desired 
VEV structure. As an example, consider the potential corresponding to an \(S_3\) doublet 
flavons \(\chi \equiv (\chi_1, \chi_2)\). The renormalizable potential for \(\chi\) that is invariant under \(S_3\) 
as well as a \(U(1)\) symmetry is \[18\]

\[
V(\chi) = \mu^2(\chi_1^* \chi_1 + \chi_2^* \chi_2) + \lambda(\chi_1^* \chi_1 + \chi_2^* \chi_2)^2 + \lambda_2(\chi_1^* \chi_2 - \chi_2^* \chi_1)^2 \\
+ \lambda_3[(\chi_1^* \chi_2 + \chi_2^* \chi_1)^2 + (\chi_1^* \chi_1 - \chi_2^* \chi_2)^2].
\]  

(6)

The VEVs can be parametrized as \(\langle \chi_1 \rangle \equiv r \cos \theta\), \(\langle \chi_2 \rangle \equiv r \sin \theta e^{i\phi}\). Minimization of \(V\) 
with respect to \(\phi\) and \(\theta\) leads to \(\phi = \pm \pi/2\) and \(\theta = \pm \pi/4\), corresponding to \((\lambda_2 + \lambda_3)\) 
having positive sign. This is the desired VEV, written as \(\langle \chi \rangle \equiv r(1, i)\). Realistic charged 
fermion masses can be induced by making use of flavon fields which get VEVs of the form 
\((1,0)\) and \((0,1)\) in the space of the second and the third families. For an analysis of 
alternative ways of inducing this VEV structure in the context of supersymmetric models 
see Ref. \[17\].

If \(|\rho| = 1\) in Eq. (1), the following relations obtain: \(\tan \theta_{\text{sol}} = \rho'/\rho = \rho'\), and 
\(\tan \theta_{\text{atm}} = \sigma = \sqrt{\rho^2 + \rho'^2} = \sqrt{1 + \rho'^2}\). Together these imply that
\[
\tan^2 \theta_{atm} = 1 + \tan^2 \theta_{sol},
\]

or, for the more usually quoted quantity,

\[
\sin^2 2\theta_{atm} = \frac{1 + \tan^2 \theta_{sol}}{(1 + \frac{1}{2} \tan^2 \theta_{sol})^2}.
\]

One sees that as \(\tan^2 \theta_{sol}\) varies between 0 and 1, \(\sin^2 2\theta_{atm}\) varies between 1 and \(8/9\). (The point \(\tan^2 \theta_{sol} = 1, \sin^2 2\theta_{atm} = 8/9\), is the same as the prediction of flavor democracy models in the pure flavor democracy limit.) For the currently favored “best fit” value of \(\tan^2 \theta_{sol} \simeq 0.4\), we have \(\sin^2 2\theta_{atm} \simeq 0.97\), in excellent agreement with data.

§4. Embedding in grand unified models

One of the main virtues of lopsided mass matrices, which has been emphasized in the literature [6, 8, 9, 10, 11], is that in the context of grand unified theories they very elegantly account for the disparity between the observed 2-3 mixings in the quark and lepton sectors, i.e. the fact that \(U_{\mu 3} \simeq 0.7\) whereas \(V_{cb} \simeq 0.04\). The explanation lies in the fact that \(SU(5)\) relates the charged lepton mass matrix \(L\) to the transpose of the down quark mass matrix \(D\). In fact, in the “minimal” \(SU(5)\) model \(L = D^T\) exactly. In the form of \(L\) shown in Eq. (1), it is the \(O(1)\) elements \(\rho\) and \(\rho’\) that control the mixing of the left-handed fields and give large \(U_{\mu 3}\). However, if \(D\) is similar to the transpose of this form, then it is the small entry \(\epsilon\) (cf. Eq. (1)) that controls the mixing of the left-handed down quarks of the second and third family, namely \(V_{cb}\). It should be noted that \(SU(5)\) relates \(L\) only to \(D\), and not to the up quark mass matrix \(U\) or the neutrino Dirac mass matrix \(N\). Therefore, one expects that \(D\) should be lopsided if \(L\) is, but there is no reason to suppose that \(U\) and \(N\) are. In fact, in lopsided models that give a good account of quark and lepton masses and mixings, only \(D\) and \(L\) are assumed to have lopsided forms. This is true also of the realistic \(SO(10)\) lopsided models that have been constructed [8, 10].

Where the form in Eq. (1) differs from most published lopsided models is in the large element \(\rho’\). (However, the model of Ref. [3], had a form for \(L\) much like Eq. (1), with an entire row of large elements.) The presence of the large element \(\rho’\) puts significant constraints on the building of realistic models of quark and lepton masses. The point has to do with the so-called Georgi-Jarlskog factors of 3: \(m_s \approx m_{\mu}/3\) and \(m_d \approx 3m_e\) (at the GUT scale) [14]. If the charged lepton mass matrix has the form shown in Eq. (1), then the simplest way to get the first Georgi-Jarlskog factor is by assuming that \(D\) has the form:

\[
D = \begin{pmatrix}
- & - & \rho’ \\
- & - & \rho \\
- & -\epsilon/3 & 1
\end{pmatrix} m_D.
\]

The factor of \(-1/3\) in the \(\epsilon\) term relative to the corresponding term in \(L\) is easily
explained as being due to the $SO(10)$ generator $B - L$. Exactly this factor appears in the $SO(10)$ lopsided models of Refs. [8, 10].

To get the second Georgi-Jarlskog factor of three requires, as is well-known, that $\det D \cong \det L$. Barring some accidental cancellations, this forces the (1,1) and (1,2) elements of $L$ (and correspondingly the (1,1) and (2,1) elements of $D$) to vanish, or at least to be negligibly small. One can also, without loss of generality rotate to make the (2,1) element of $L$ and the (1,2) element of $D$ vanish. This leads to the virtually unique forms for $D$ and $L$. It is straightforward to generalize the $SO(10)$ model of Refs. [8, 10] to obtain the following realistic mass matrices:

$$L = \begin{pmatrix} 0 & 0 & \delta' \\ 0 & \delta & \epsilon \\ \rho' & \rho - \epsilon & 1 + \kappa \end{pmatrix} m_D, \quad D = \begin{pmatrix} 0 & 0 & \rho' \\ 0 & \delta & \rho + \epsilon/3 \\ \delta' & -\epsilon/3 & 1 + \kappa \end{pmatrix} m_D,$$

$$N = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & \epsilon \\ 0 & -\epsilon & 1 \end{pmatrix} m_U, \quad U = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & -\epsilon/3 \\ 0 & \epsilon/3 & 1 \end{pmatrix} m_U. \tag{10}$$

These mass matrices arise from the following Yukawa terms: The entries denoted ‘1’ come from $(16_3 16_3) 10_H$. The $O(1)$ elements $\kappa, \rho,$ and $\rho'$ come from $(16_3 16_H)(16, 16'_H)$, where $i = 1, 2, 3$, the multiplets in the parentheses are contracted into $SO(10)$ vectors; $16_H$ breaks $SO(10)$ down to $SU(5)$, and the $16'_H$ breaks the electroweak interactions. The elements $\epsilon$, which appear antisymmetrically, come from $16_2 16_3 10_H 45_H$, where the VEV of the adjoint Higgs lies in the $B - L$ direction. Such an adjoint VEV is what would be desired to achieve doublet–triplet splitting without fine-tuning via the Dimopoulos-Wilczek mechanism in $SO(10)$ [20]. The elements $\delta$ and $\delta'$ come from terms of the same form as the $\kappa, \rho$ and $\rho'$ terms, but with of course different family indices. These are exactly the same kinds of operators that appear in the models of Refs. [3, 8, 10].

These mass matrices give a quite satisfactory fit to all the quark and lepton masses and mixings. In the approximation $1 \sim \rho \sim \rho' \sim \kappa \gg \epsilon \gg \delta \sim \delta'$, so that the observed mass hierarchy is correctly reproduced, the following mass relations are obtained at the GUT scale:

$$m_b \cong m_\tau, \quad m_s \cong m_\mu/3, \quad m_d \cong 3m_e, \quad m_u/m_t \cong 0 . \tag{11}$$

The first three are the Georgi-Jarlskog relations, all of which work quite well when compared with experimental values of the masses. $m_u/m_t$ is predicted to be zero by these forms. Experimentally, it is about $10^{-5}$, which is about two orders of magnitude less than the corresponding ratio for the down quarks, $m_d/m_b$. A tiny non-zero value of $m_u$ can easily arise from some higher-dimension operator.

The parameter $\kappa$ in Eq. (10) is necessary in order to have adequate CP violation in the CKM matrix. We may redefine $1 + \kappa$ to be simply 1 with an appropriate redefinition of $m_D$ in Eq. (10). The parameter $\epsilon$ in $L$ and $D$ of Eq. (10) will then be different from $\epsilon$ in $N$ and $U$. Let us then rename $\epsilon$ appearing in $L$ and $D$ as $\epsilon z$. In this redefined notation (we denote the redefined $\delta, \delta', \rho, \rho'$ by the same symbols) we have $m_s/m_b \cong$
\[
\frac{\sigma}{(1 + \sigma^2)}(z\epsilon/3), V_{us} \cong \delta/(z\epsilon/3), V_{ub} \cong \delta/(1 + \sigma^2), V_{cb} \cong (\epsilon/3)(z/(1 + \sigma^2) - 1). \]

From these relations, we obtain the following prediction:

\[
\tan \theta_{atm} = \frac{\tan 2\theta_C}{2|V_{ub}|} \left(\frac{m_s}{m_b}\right),
\]

(12)

where \( \theta_C \) is the Cabibbo angle, and \( \theta_{atm} \) is the atmospheric neutrino mixing angle that comes from the charged lepton matrix. (The contribution from the neutrino sector is assumed to be small.) Note that \( \theta_{atm} \) is of order unity, as needed for atmospheric neutrino oscillations.

The mixing parameter \( U_{e3} \) is predicted to be

\[
|U_{e3}| \simeq \frac{\sin \theta_C}{3} |U_{\mu 3}| \simeq (0.04 - 0.05)
\]

(13)

where the factor \( \sin \theta_C/3 \) arises from the small rotation needed to complete the diagonalization of \( L'' \) of Eq. (3). (The factor 3 arises because \( m_\mu \cong 3m_s \).) This prediction will provide a test of this class of models.

If the parameter \( z \) were equal to 1 (which will be the case when the entry \( \kappa \) is absent in Eq. (10)), then there will be not enough CKM type CP violation in this model, as all the mixing angles become approximately real. This is true even when we allow for the parameter \( \delta \) to be complex, since there is a cancellation between the up and the down quark contribution in the phase of the CKM matrix. Allowing for \( z \neq 1 \) (or \( \kappa \neq 0 \)) leads to the desired CP violation, since \( z \) is complex. It is interesting to note that if \( z \) were equal to 1, the charm mass will be predicted to be \( m_c(m_c) \cong (1.1 - 1.2) \) GeV [8]. Furthermore, the relation \( |V_{ub}| \cong (m_s/m_b)^2|V_{us}/V_{cb}| \) will follow, which is in good agreement with experimental values.

\section*{§5. New class of lopsided mass matrices from an \( I_{3R} \) adjoint}

In the preceding example we made essential use of an \( SO(10) \) adjoint VEV along the \( B - L \) direction. Now we show that quite simple and predictive mass matrices can be derived in \( SO(10) \) if the VEV of the single adjoint present in the model points along the \( I_{3R} \) direction (\( I_{3R} \) stands for the third component of the right–handed isospin). There is a simple and elegant realization of the lopsidedness of \( D \) and \( L \) in this scheme. It is worth noting that a single adjoint scalar with its VEV along \( I_{3R} \) direction can lead to a natural doublet-triplet splitting, just as in the case of a single \( B - L \) adjoint [21]. The \( I_{3R} \) adjoint also has some advantages is suppressing Higgsino-mediated proton decay in supersymmetric \( SO(10) \) [22].

Consider the case where the \( B - L \) adjoint that was involved in generating the lopsided mass matrices of Eq. (10) is replaced by an \( I_{3R} \) adjoint. The mass matrices will then have the form
\[
L = \begin{pmatrix}
0 & 0 & \delta' \\
0 & \delta & -\epsilon' \\
\rho' & \rho - \epsilon & 1
\end{pmatrix} m_D, \quad D = \begin{pmatrix}
0 & 0 & \rho' \\
0 & \delta & \rho - \epsilon' \\
\delta' & -\epsilon & 1
\end{pmatrix} m_D, \tag{14}
\]
\[
N = \begin{pmatrix}
0 & 0 & 0 \\
0 & 0 & \epsilon' \\
0 & \epsilon & 1
\end{pmatrix} m_U, \quad U = \begin{pmatrix}
0 & 0 & 0 \\
0 & 0 & \epsilon' \\
0 & \epsilon & 1
\end{pmatrix} m_U.
\]

As in the previous section, the ‘1’ entries arise from \(16_3 \times 16_3 \times 10_H\) coupling. There are two crucial differences compared to Eq. (10). The entry resulting from \(16_3 \times 16_3 \times 10_H \times 45_H\) has now two group contractions. These two are denoted in Eq. (14) as \(\epsilon, \epsilon'\). These entries are proportional to the \(I_{3R}\) charge, so that they are equal in \(D\) and \(L\) (similarly in \(N\) and \(U\)). Lopsided nature arises from the \(\rho\) entry generated through \(16_3 \times 16_3 \times 16_H \times 16_H\) coupling. The parameters \(\rho, \rho'\) are assumed to be much larger than \(\epsilon, \epsilon'\). This model then predicts the following relations:

\[
m_b \cong m_\tau, \quad m_s \neq m_\mu, \quad m_d m_s m_b \cong m_d m_\mu m_\tau. \tag{15}
\]

The inequality for \(m_s\) follows since \(m_s/m_b \cong |\epsilon\sigma/(1 + \sigma^2)|\), while \(m_\mu/m_\tau \cong |\epsilon'\sigma/(1 + \sigma'^2)|\), where \(\sigma \equiv \sqrt{\rho^2 + \rho'^2}\). Thus, although the entries \(\epsilon, \epsilon'\), proportional to \(I_{3R}\) do not distinguish \(L\) from \(D\), and the \(\rho\)-type entries also by themselves do not distinguish \(L\) and \(D\) (these entries do not break \(SU(5)\)), a combination of the two leads to the breaking of \(m_\mu = m_s\) relation, as desired. Unlike in Eq. (10), there is sufficient CP violation in the CKM matrix in this model even without an entry like \(\kappa\) of Eq. (10).

Working in the approximation \(1 \sim \rho \sim \rho' \gg \epsilon \sim \epsilon' \gg \delta \sim \delta'\), we obtain the following relations for the masses: \(m_b \cong m_\tau \cong \sqrt{1 + \delta^2} m_D, m_s/m_b \cong [(\sigma + \delta^* \rho/\sigma)]/(1 + \sigma^2), m_\mu/m_\tau \cong [(\sigma \epsilon' + \delta^* \rho/\sigma)]/(1 + \sigma^2), m_d m_s m_b \cong m_c m_\mu m_\tau, m_c/m_\mu \cong \epsilon \epsilon' \neq 0, m_u/m_t \neq 0\) all at the unification scale. The CKM mixing angles are given by \(|V_{us}| \cong \delta'/(\epsilon + \delta \rho/\sigma^2), |V_{ub}| \cong \delta/(1 + \sigma^2), |V_{cb}| \cong \epsilon (2 + \sigma^2) - \delta^* \rho/(1 + \sigma^2)\). Here all parameters have been made real by field redefinitions, except \(\delta\). The rephasing invariant CP violation parameter \(\eta\) is given by \(\eta \equiv \text{Im}\{V_{ub}V_{cb}^*/V_{us}V_{cb}\} \cong 2 \epsilon \rho \text{Im}(\delta)/\sigma^2(1 + \sigma^2)|V_{cb}|^2\). From these relations, we obtain the following prediction for the atmospheric neutrino oscillations:

\[
\tan \theta_{\text{atm}} \cong \sigma \cong (m_s/m_b)|V_{us}|/|V_{ub}|, \tag{16}
\]

which is analogous to Eq. (12). We also have a quantitative prediction for \(\tan \theta_{\text{sol}} \equiv \rho'/\rho\). This can be seen by noting that \(\sigma\) is determined from Eq. (16), \(\epsilon'\) from \(m_\mu/m_\tau, \epsilon\) from \(m_c/m_t, \rho \text{Im}(\delta)\) from \(\eta, \rho \text{Re} \delta\) from \(m_s/m_b\), and \(\delta'\) from \(V_{ub}\). The determinant relation \(m_c m_\mu m_\tau \equiv |\rho \rho' \delta|\) then fixes \(\rho'/\rho\).

Consider the input parameters taking the following values. \(|V_{us}| \cong 0.215, |V_{ub}| \cong 0.0036, |V_{cb}| \cong 0.0037, \eta \cong 0.33\), and \(m_c(m_c) \cong 1.35\) GeV, \(m_t = 175\) GeV, and \(U_{e3} = 0.06\), all of which are in reasonably good agreement with current neutrino oscillation data.
In summary, we have presented simple realizations of bimaximal neutrino mixing pattern, making use of lopsided mass matrices for the fermions. This idea has a natural embedding in unified $SO(10)$ models. We have presented two different realizations within $SO(10)$, one making use of the traditional $B-L$ adjoint VEV, and a new class of models making use of an $I_{3R}$ adjoint VEV. We were also able to derive in a simple way the near maximal mixing for atmospheric neutrino oscillation angle, as given in Eq. (8).

Acknowledgments

KSB wishes to thank the Theory Group at Bartol Research Institute for the warm hospitality extended to him during a summer visit when this work started. The work of KSB is supported in part by DOE Grant # DE-FG03-98ER-41076, a grant from the Research Corporation and by DOE Grant # DE-FG02-01ER-45684. The work of SMB is supported in part by DOE Grant # DE-FG02-91ER-40626.

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