ARE THERE PRESSURE WAVES IN THE VACUUM?

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The Higgs vacuum is a kind of medium. In any medium one generally expects sound waves for sufficiently long wavelengths ($\lambda \gg$ mean free path). I briefly describe how the broken-symmetry vacuum can be viewed as a Bose-Einstein condensate of 'phion' particles. This picture yields a natural notion of the 'mean free path.' I speculate that this is at the millimeter-centimeter scale.

1 Introduction

Are there sound waves in the vacuum? The question might seem silly, but here’s my point: (i) the Higgs vacuum of the Standard Model, with its non-zero background field $\langle \Phi \rangle \neq 0$, is not ‘empty,’ but rather is a kind of medium. (ii) For very basic reasons, one expects any medium to propagate pressure waves for sufficiently long wavelengths. At length scales much, much longer than the mean free path ($\text{mfp}$) — the ‘hydrodynamical’ regime — sound waves arise directly from energy-momentum conservation equations linearized about equilibrium. Could this reasoning apply even to the ‘aether’ (the Higgs vacuum) — and, if so, what is the ‘mfp’ scale?

In this talk I briefly outline a description of the Higgs vacuum as, literally, a Bose-Einstein (BE) condensate of particles that I’ll call ‘phions.’ I’ll consider single-component $\lambda \Phi^4$ theory since Goldstone bosons are not directly relevant here. The intrinsic phion size $r_0$ will serve as the inverse of the ultraviolet cutoff. The key variables will be $n$, the number density of phions, and $a$, their scattering length.

In the quantum-field-theory limit, where $r_0 \to 0$, it will turn out that $n \to \infty$ and $a \to 0$, such that $na$ remains finite and determines the Higgs mass squared. The fact that the scattering length $a$ goes to zero reflects the ‘triviality’ of $\lambda \Phi^4$ theory. In this field-theory limit sound waves would not exist; they would be banished to infinitely long wavelengths because the mfp scale is of order $1/(na^2)$ which goes to infinity.

However, if we do not take the cutoff all the way to infinity then the mfp $1/(na^2)$ would be a large, but finite, length scale. If we are also willing to contemplate Lorentz-invariance violation at the cutoff scale, we could naturally expect sound waves to exist at ultra-low momenta. They would be an
example of what Volovik has called “re-entrant violation of Lorentz invariance,” in which Lorentz invariance arises as a low-energy effective symmetry from a non-symmetric fundamental theory, but deviations from Lorentz symmetry occur at ultra-low momenta as well as at very high momenta. I’ll return to these speculations later.

2 Phion condensation

Consider single-component $\lambda \Phi^4$ theory in a region of parameters where the effective potential has both a minimum at $\phi = 0$ and a deeper minimum at $\phi = \pm v$. (Such a situation is possible, as we’ll see.) The symmetric vacuum is then locally, but not globally, stable. The particle excitations above this metastable, ‘empty’ vacuum I will call ‘phions.’ It must be possible to describe the physics in terms of the phion degrees of freedom. The broken-symmetry vacuum must correspond to a BE condensate of phions, and the Higgs bosons must correspond to the quasiparticle excitations of this condensate, in condensed-matter terminology. The issue, though, is; why do phions want to condense?

The answer lies, of course, in the phions’ interactions. The fundamental interaction is the $4$-point vertex. Expressed as an interparticle potential this is a repulsive $\delta^3(r)$ potential with strength $a/m$, where $m$ is the phion mass and $a$ is the scattering length ($a \sim \lambda/m$ in terms of the coupling constant $\lambda$). The $\delta^3(r)$ potential may be regularized by spreading it out over a small size $r_0$, with $1/r_0$ acting as an ultraviolet cutoff. In addition, there is an induced long-range, attractive interaction due to the “fish” diagram involving exchange of two virtual phions. This corresponds to a $-a^2/r^3$ interaction, if we neglect the mass of the exchanged phions. (Including the phion mass basically cuts off this potential at distances greater than $\sim 1/m$.)

[For our purposes this form of the phions’ interparticle potential is effectively exact, provided that $a$ incorporates short-range interactions to all orders. The point is that, just as in the non-relativistic theory of BE condensation, only low-energy ($ka \ll 1$) scattering is involved, and this can be characterized by a single parameter, the $s$-wave scattering length $a$.]

Consider a large box, volume $V$, with periodic boundary conditions that contains $N$ phions. Provided the system is dilute ($na^3 \ll 1$), the ground state corresponds to almost all the phions being Bose-condensed in the zero-momentum mode; hence the energy is

$$E = Nm + \frac{4}{3}N^2a,$$

(1)

$^a$ Throughout this talk I’ll ignore numerical factors of 2, 4$\pi$, etc.
where the first term counts the rest energies of the $N$ phions and the second is the number of pairs times the average energy of a pair, $\bar{u}$. The diluteness assumption means that three-body interactions, etc., can be neglected. Since almost all the phions are in the zero-momentum mode, whose wavefunction is uniform across the box, the average energy of a pair is just

$$\bar{u} = \frac{1}{V} \int d^3r V(r).$$  \hspace{1cm} (2)

Substituting this into (1) and dividing by volume gives the energy density as

$$\mathcal{E} = nm + n^2 \int d^3r V(r).$$  \hspace{1cm} (3)

(I’ve dropped the $\frac{1}{2}$.) The interparticle potential $V(r)$ contains the $(a/m)\delta^{(3)}(r)$ term and the $-a^2/r^3$ term. Integration of the latter yields $\int dr/r$, which is cut off at short distances by the core size $r_0$, but will also need to be cut off at long distances by some $r_{\text{max}}$:

$$\mathcal{E} = nm + n^2 \frac{a^2}{m} - n^2 a^2 \ln \left( \frac{r_{\text{max}}}{r_0} \right).$$  \hspace{1cm} (4)

The crucial question now is; what determines $r_{\text{max}}$? As mentioned earlier, the phion mass will ultimately cut off the $-a^2/r^3$ interaction at distances $\geq 1/m$. However, when $m$ is very small a more important consideration is the ‘screening’ by the background density of phions. Thus, the $\ln(r_{\text{max}})$ will naturally turn into a logarithm of $n$.

To see this, consider two, well-separated phions that are trying to interact by exchanging a pair of virtual phions. The two virtual phions have to travel through the condensate and thus will experience collisions with background phions. These collisions with zero-momentum particles, proportional to $n$ times $a$, behave as mass insertions in the propagator. They convert each phion propagator into the propagator for a quasiparticle (a Higgs boson) whose mass squared is thus

$$M^2 = m^2 + 8\pi na.$$  \hspace{1cm} (5)

It turns out that the second term completely dominates, so the Higgs mass is much, much greater than the phion mass. The scale for $r_{\text{max}}$ is then set by $1/M \sim 1/\sqrt{n a}$ (a much shorter length scale than $1/m$).

The energy density, from (3), thus has the form

$$\mathcal{E} = \text{sum of } n, \ n^2, \ n^2 \ln n \ \text{terms.}$$  \hspace{1cm} (6)
The three terms represent (i) one-particle rest energies, (ii) the energy cost of repulsive, short-range, two-particle interactions, and (iii) the energy gain due to attractive, long-range, two-particle interactions (with the \( \ln n \) arising because the incipient infrared divergence is cut off by the background density effect). For sufficiently small phion mass (iii) wins, so that an empty box \((n = 0)\) is energetically disfavoured, compared to a condensate of some optimal density \(n_v\), where \(n = n_v\) is the non-trivial minimum of \(\mathcal{E}(n)\).

This physics can be translated back into familiar field-theory terms by recognizing that \(n\) translates to \(\frac{1}{2m\phi^2}\), where \(\phi\) is the background field value \(\langle \Phi \rangle\). The energy density as a function of \(n\) then translates into the field-theoretic effective potential as a function of \(\phi\).

### 3 Hierarchy of Length Scales

It is straightforward to analyze where the phase transition occurs in terms of the parameters. In units where the Higgs mass \(M_h \sim \sqrt{n_va}\) is finite, we need \(m\), \(1/n_v\), and \(a\) to tend to zero like \(\epsilon^{1/2}\), where \(\epsilon = 1/\ln(\text{cutoff}/M_h)\) and the cutoff is \(1/r_0\). The fact that the scattering length \(a\) must vanish reflects the ‘triviality’ of \(\lambda \Phi^4\) theory. ‘Triviality’ is usually viewed as an embarrassment, which is odd because it naturally does something very desirable; it generates a hierarchy. In fact, ‘TRIVIALITY’ means HIERARCHY.

‘Triviality’ means that there are two, vastly different, physical length scales; the Compton wavelength \(M_h^{-1}\) (finite) and the scattering length \(a\) (infinitesimal). By keeping the cutoff finite, but as large as we like, we can have a hierarchy of physical length scales. In fact, the hierarchy is quite rich, as illustrated below:

|       | \(r_0\) | \(a\) | \(n_v^{-1/3}\) | \(M_h^{-1}\) | \(\xi\) |
|-------|--------|------|----------------|--------------|--------|
| \(e^{-1/2}\) | \(\epsilon^{1/2}\) | \(\epsilon^{1/6}\) | 1 | \(\epsilon^{-1/2}\) |

The fact that the average phion spacing, \(n_v^{-1/3}\), is much greater than \(a\) corresponds to diluteness. Both these scales are small compared to the physical length scale set by \(M_h^{-1}\). There is also a very long length scale, denoted by \(\xi\), which corresponds to the phion mfp, \(1/(na^2)\), which happens to be the same order as \(m^{-1}\).

A natural speculation is to identify \(a\) with the Planck length. In this case \(\epsilon\), instead of going to zero, is the tiny number \(10^{-34}\). In that case the
$mfp$ is at the millimeter/centimeter scale. Sound waves with wavelengths much longer than this scale would correspond to the quasiparticle spectrum having a ‘phonon branch,’ \( E \sim v_s p \), at ultra-low momenta, \( p \ll m \). This ‘phonon branch’ would have to join on to the normal spectrum \( E \sim \sqrt{p^2 + M_h^2} \) at \( p \sim m \), implying that the speed of sound \( v_s \) is of order \( M_h/m \), which is much greater than the speed of light (by a factor of \( 10^{17} \) in this scenario).

Two points should be made about this vastly superluminal sound velocity:

(i) Sound waves exist only for long wavelengths, so there is no way to produce a sharp wavefront; thus, superluminal signalling is still impossible. (ii) The velocity \( v_s \) is relative to the condensate rest frame, but in a moving frame the apparent sound velocity could be very different. Indeed, in principle, one could use this effect to determine the aether rest frame.\(^b\)

There is no space here to discuss some independent reasons to suspect that the Higgs spectrum has a ‘phonon branch’ (which would shrink into a discontinuity in the propagator at \( p^\mu p_\mu = 0 \) in the infinite-cutoff limit). I refer the reader to the following references.\(^6\), \(^7\), \(^8\) In particular, there are longstanding arguments that, as \( p \to 0 \), the radial Higgs propagator should behave as \( |p|^{d-4} \) in \( d \) dimensions.\(^7\) There is also direct evidence for peculiar infrared behaviour from lattice simulations.\(^8\)

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\(^b\) In the infinite-cutoff limit, where exact Lorentz invariance is recovered, the condensate rest frame still exists, but it cannot be determined experimentally.
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