Charge-dipole and dipole-dipole interactions in two-dimensional materials

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We derive the explicit analytical form for the charge-dipole and dipole-dipole interactions in 2D configuration space. We demonstrate that the reduction of dimensionality can alter the charge-dipole and dipole-dipole interactions in 2D case. The asymptotics of these interactions at large distances coincide to the charge-dipole and dipole-dipole interactions in 3D configuration space.

In classical electrodynamics for description of the field produced by a system of electric charges at large distances the concepts of dipoles and multipole moments are very important and well developed \cite{1,2}. This approach is based on the potential of a single charge in three-dimensional (3D) configuration space. Ordinary matter is more or less uncharged, but it is reach in pair of charges called dipoles. Dipoles are building blocks of bulk dielectric and magnetic materials. Not surprisingly, it turns out to be efficient mathematically to deal with the dipole not as just a pair of individual positive and negative charges.

The last two decades discoveries and studies of two-dimensional (2D) materials attract a considerable interest. Atomically thin materials such as graphene and monolayer transition metal dichalcogenides (TMDC), phosphorene, Xenes (silicene, germanine, stanene), exhibit remarkable physical properties resulting from their reduced dimensionality and crystal symmetry. The family of semiconducting transition metal dichalcogenides is an especially promising platform for fundamental studies of two-dimensional systems, with potential applications in optoelectronics and valleytronics due to their direct band gap in the monolayer. Exciton is the simplest bound complex formed by an electron in a conduction band and hole in a valence band. The description of excitons, trions, biexcitons in 2D material requires knowledge of electrostatic interaction in reduced dimensionality.

The interaction of two charge particles in two-dimensional space are studied in detail and the analytical expression for two charged particle interaction is well known \cite{3,4} and widely used for description of excitonic complexes in 2D materials (see reviews: \cite{5–7}). In 3D configuration space when charged particles interact via the Coulomb potential the corresponding charge-dipole and dipole-dipole potentials are well known. In contrast, the influence of the reduction of dimensionality on the charge-dipole and dipole-dipole interactions in 2D configuration space has not yet been investigated. We still lack of the analytical expression for the charge-dipole and dipole-dipole 2D interactions. Below we derive the explicit analytical form for the charge-dipole and dipole-dipole interactions in 2D space.

\textbf{Charge-charge interaction in 2D configuration space.} An interaction of two charged particles in the context of thin semiconductor films, was derived analytically by Rytova \cite{3} and, decade later, by Keldysh \cite{4}. Due to the lack of screening by the environment above the material layer it was shown that the electron-hole interaction potential in a thin semiconductor layer is not Coulombic. Over the course of decade the celebrated Rytova-Keldysh (RK) potential \cite{3,4} has been widely used to describe the electrostatic interaction of few-body complexes such as excitons, trions, and biexcitons \cite{7} in monolayer transition-metal dichalcogenides, phosphorene and Xenes. This potential describes the non-hydrogenic Rydberg series of neutral excitons. The effective electron-hole Rytova-Keldysh potential, which takes into account screening due to the reduction of dimensionality is given by \cite{3,4}:

\begin{equation}
V_{\text{RK}}(R) = \frac{\pi ke^2}{2\kappa \rho_0} \left[ H_0 \left( \frac{R}{\rho_0} \right) - Y_0 \left( \frac{R}{\rho_0} \right) \right].
\end{equation}

In Eq. (1), \( k = 9 \times 10^9 \text{ N}\cdot\text{m}^2/\text{C}^2 \), \( R = |\mathbf{R}| \) is the magnitude of the relative electron-hole separation, \( \kappa = (\epsilon_1 + \epsilon_2)/2 \) describes the surrounding dielectric environment, where \( \epsilon_1 \) and \( \epsilon_2 \) correspond to the dielectric constants of the materials above and below the monolayer, \( H_0 \) and \( Y_0 \) are the Struve and Bessel functions of the second kind, respectively, and \( \rho_0 \) is the screening length. In the case of a thin semiconductor layer of a finite thickness \( t \) and an isotropic dielectric constant \( \epsilon \), the screening length is evaluated as \( \rho_0 = t\epsilon/2\kappa \) \cite{4}. In the case of atomically thin 2D materials the...
screening length is given by \[8, 9\]:

\[
\rho_0 = \frac{2\pi\chi_{2D}}{\kappa},
\]

where \(\chi_{2D}\) is the 2D polarizability, which can be calculated via ab-initio methods or considered as a phenomenological parameter. The screening length typically ranges from roughly 30 to 80 Å \[6\]. The effective interaction potential \[1\] has an asymptotic behavior \(\sim 1/R\) only at large distances between the particles, that follows from \[10, 11\]. This limiting case corresponds to the Coulomb interaction unaffected by the dielectric polarization of a 2D layer, as most of the electric-field lines between two distant charges go outside of the 2D semiconductor. Interestingly enough, in this limiting case two charges are interacting the same way as in vacuum. At smaller distances the potential deviates strongly from the usual \(1/R\) form and the dependence has a logarithmic behavior \(\sim \ln(2/r) - \gamma\) \[8, 10, 11\], where \(\gamma = 0.5772\ldots\) is Euler's constant.

**Charge-dipole interaction in 2D configuration space.** Consider two opposite closely spaced charges in a monolayer that form a dipole with the dipole moment \(d = e\mathbf{r}\) and a single charge placed at a distance \(R\) as shown in Fig. 1. The point charge interacts with the dipole via the RK potential. In this case following notations in Fig. 1 for the charge-dipole interaction we have

\[
V_{cd}(\mathbf{r}) = V_{eh}(R) + V_{bh}(|\mathbf{r} + \mathbf{r}|) = -\frac{\pi ke^2}{2\kappa \rho_0} \left[ H_0 \left( \frac{R}{\rho_0} \right) - Y_0 \left( \frac{R}{\rho_0} \right) \right] + \frac{\pi ke^2}{2\kappa \rho_0} \left[ H_0 \left( \frac{\mathbf{r} + \mathbf{r}}{\rho_0} \right) - Y_0 \left( \frac{\mathbf{r} + \mathbf{r}}{\rho_0} \right) \right],
\]

where

\[
|\mathbf{r} + \mathbf{r}| = R\sqrt{1 + \frac{2\mathbf{r} \cdot \mathbf{r}}{R^2} + \frac{r^2}{R^2}}.
\]

For \(R \gg r \left(1 + \frac{2\mathbf{r} \cdot \mathbf{r}}{R^2} + \frac{r^2}{R^2}\right)^{1/2}\) \(\simeq 1 + \frac{1}{2} \left(\frac{2\mathbf{r} \cdot \mathbf{r}}{R^2} + \frac{r^2}{R^2}\right)\). Considering only linear terms with respect to \(r\), Eq. (3) can be written as

\[
V_{cd}(\mathbf{R}) = -\frac{\pi ke^2}{2\kappa \rho_0} \left[ H_0 \left( \frac{R}{\rho_0} \right) - Y_0 \left( \frac{R}{\rho_0} \right) \right] + \frac{\pi ke^2}{2\kappa \rho_0} \left[ H_0 \left( \frac{1 + \mathbf{r} \cdot \mathbf{r}}{R^2} \right) \right] - Y_0 \left( \frac{R}{\rho_0} \left[1 + \frac{\mathbf{r} \cdot \mathbf{r}}{R^2}\right]\right). \tag{5}
\]

Expand the Struve \(H_0 \left( \frac{R}{\rho_0} \left[1 + \frac{\mathbf{r} \cdot \mathbf{r}}{R^2}\right]\right)\) and Bessel \(Y_0 \left( \frac{R}{\rho_0} \left[1 + \frac{\mathbf{r} \cdot \mathbf{r}}{R^2}\right]\right)\) functions in terms of power of \(\frac{r}{R}\) when \(R \gg r\) and consider linear terms with respect to \(r\):

\[
H_0 \left( \frac{1 + \frac{\mathbf{r} \cdot \mathbf{r}}{R^2}}{\rho_0} \right) \simeq H_0 \left( \frac{R}{\rho_0} \right) + H_0' \left( \frac{R}{\rho_0} \right) \frac{\mathbf{r} \cdot \mathbf{r}}{R^2} = H_0 \left( \frac{R}{\rho_0} \right) + H_{-1} \left( \frac{R}{\rho_0} \right) \frac{\mathbf{r} \cdot \mathbf{r}}{R^2}, \tag{6}
\]

\[
Y_0 \left( \frac{1 + \frac{\mathbf{r} \cdot \mathbf{r}}{R^2}}{\rho_0} \right) \simeq Y_0 \left( \frac{R}{\rho_0} \right) + Y_0' \left( \frac{R}{\rho_0} \right) \frac{\mathbf{r} \cdot \mathbf{r}}{R^2} = Y_0 \left( \frac{R}{\rho_0} \right) + Y_{-1} \left( \frac{R}{\rho_0} \right) \frac{\mathbf{r} \cdot \mathbf{r}}{R^2}. \tag{7}
\]

Here we use that \(H_0' = H_{-1}\) and \(Y_0' = -Y_1 = -Y_{-1}\) \[10, 11, 13\]. Using (6) and (7) Eq. (5) can be written as

\[
V_{cd}(\mathbf{R}) = \frac{\pi ke^2}{2\kappa \rho_0} \left[ H_{-1} \left( \frac{R}{\rho_0} \right) - Y_{-1} \left( \frac{R}{\rho_0} \right) \right] \frac{\mathbf{r} \cdot \mathbf{r}}{R}, \tag{8}
\]

or

\[
V_{cd}(\mathbf{R}) = \frac{\pi ke^2}{2\kappa \rho_0} \left[ H_{-1} \left( \frac{R}{\rho_0} \right) - Y_{-1} \left( \frac{R}{\rho_0} \right) \right] \frac{\mathbf{r} \cdot \mathbf{d}}{\rho_0 R}. \tag{9}
\]
Consider the asymptotic of \( V_{ed} (\mathbf{R}) \) interaction when \( R \to \infty \). For the difference of \( H_{-1} (x) - Y_{-1} (x) \) when \( x \to \infty \) we have \([10][11][14]\)

\[
H_{\nu} (x) - Y_{\nu} (x) \xrightarrow{x \to \infty} \frac{(\frac{x}{2})^{\nu-1}}{\sqrt{\pi} \Gamma (\nu + 1/2)}. \tag{10}
\]

For \( \nu = -1 \) \( H_{-1} \left( \frac{R}{\rho_0} \right) - Y_{-1} \left( \frac{R}{\rho_0} \right) = \frac{4\rho_0^2}{\pi R^2} \frac{1}{\sqrt{\pi} (-1/2)} \). Therefore,

\[
V_{ed} (\mathbf{R}) \xrightarrow{R \to \infty} -\frac{ke}{\kappa} \frac{\mathbf{R} \cdot \mathbf{d}}{R^3}, \tag{11}
\]

where we use \( \Gamma (-1/2) = -2\sqrt{\pi} \). Thus, one can conclude that in 2D configuration space the charge-dipole interaction has the form \([10]\) that has the asymptotic \([11]\).

Evidently, the charge-dipole potential goes like \( \frac{1}{R^2} \) at large \( R \) and it falls off more rapidly than potential \([1]\). For the charge-dipole interaction in 3D configuration space we have well known expression

\[
V_{ed}^C (\mathbf{R}) = -\frac{ke}{\kappa} \frac{\mathbf{R} \cdot \mathbf{d}}{R^3}. \tag{12}
\]

We can conclude that \( V_{ed} (\mathbf{R}) \) in 2D and \( V_{ed}^C (\mathbf{R}) \) in 3D configuration spaces, respectively, vary as \( \frac{1}{R^2} \) at large separation of the charge and the dipole.

**Dipole-dipole interaction in 2D configuration space.** Consider two dipoles \( \mathbf{d}_1 = e \mathbf{r}_1 \) and \( \mathbf{d}_2 = e \mathbf{r}_2 \) interaction in 2D configuration space. One can consider a dipole-dipole interaction as the interactions of a positive and negative charges of one dipole with the second dipole. Following notations in Fig. \([1]\) for a dipole-dipole interaction in 2D space we have

\[
V_{dd} (\mathbf{R}) = V_{dd} \left( \frac{R}{\rho_0} \right) + V_{ed} \left( \frac{\mathbf{R} + \mathbf{r}_2}{\rho_0} \right). \tag{13}
\]

Using \([1]\) for the charge-dipole interaction in Eq. \([13]\), we have

\[
V_{dd} (\mathbf{R}) = \frac{\pi ke^2}{2\kappa \rho_0^3} \left[ H_{-1} \left( \frac{\mathbf{R} + \mathbf{r}_2}{\rho_0} \right) - Y_{-1} \left( \frac{\mathbf{R} + \mathbf{r}_2}{\rho_0} \right) \right] \left( \frac{\mathbf{R} + \mathbf{r}_2}{\rho_0} \right) R \mathbf{R}^1 - \frac{\pi ke^2}{2\kappa \rho_0^3} \left[ H_{-1} \left( \frac{\mathbf{R} + \mathbf{r}_2}{\rho_0} \right) - Y_{-1} \left( \frac{\mathbf{R} + \mathbf{r}_2}{\rho_0} \right) \right] \frac{(\mathbf{R} + \mathbf{r}_2) \cdot \mathbf{r}_1}{|\mathbf{R} + \mathbf{r}_2|}. \tag{14}
\]

We focus on the second term in Eq. \([13]\). For \( R \gg r_2 \) considering the terms linear with respect to \( r_2 \), we have \( |\mathbf{R} + \mathbf{r}_2| = R \left( 1 + \frac{2R r_2}{R^2} + \frac{r_2^2}{R^2} \right)^{1/2} \approx R (1 + \frac{R r_2}{R^2}) \) and \( \frac{1}{|\mathbf{R} + \mathbf{r}_2|} = \frac{1}{R} \left( 1 + \frac{2R r_2}{R^2} + \frac{r_2^2}{R^2} \right)^{-1/2} \approx \frac{1}{R} \left( 1 - \frac{R r_2}{R^2} \right) \) and \([14]\) becomes

\[
V_{ed} \left( \frac{|\mathbf{R} + \mathbf{r}_2|}{\rho_0} \right) = \frac{\pi ke^2}{2\kappa \rho_0^3} \left[ H_{-1} \left( \frac{|\mathbf{R} + \mathbf{r}_2|}{\rho_0} \right) - Y_{-1} \left( \frac{|\mathbf{R} + \mathbf{r}_2|}{\rho_0} \right) \right] \left( \frac{|\mathbf{R} + \mathbf{r}_2|}{\rho_0} \right) R \mathbf{R}^1 - \frac{\pi ke^2}{2\kappa \rho_0^3} \left[ H_{-1} \left( \frac{1 + \frac{R \cdot \mathbf{r}_2}{R^2}}{1 + \frac{R \cdot \mathbf{r}_2}{R^2}} \right) - Y_{-1} \left( \frac{1 + \frac{R \cdot \mathbf{r}_2}{R^2}}{1 + \frac{R \cdot \mathbf{r}_2}{R^2}} \right) \right] \left( \frac{\mathbf{R} + \mathbf{r}_2}{R} \right) \frac{\mathbf{r}_1 \cdot \mathbf{r}_2}{R^3}. \tag{15}
\]

Expand the Struve \( H_{-1} \left( \frac{R}{\rho_0} \left[ 1 + \frac{R \cdot \mathbf{r}_2}{R^2} \right] \right) \) and Bessel \( Y_{-1} \left( \frac{R}{\rho_0} \left[ 1 + \frac{R \cdot \mathbf{r}_2}{R^2} \right] \right) \) functions in terms of power of \( \frac{R}{R} \) when \( R \gg r_2 \) and consider linear terms with respect to \( r_2 \):

\[
H_{-1} \left( \frac{R}{\rho_0} \left[ 1 + \frac{R \cdot \mathbf{r}_2}{R^2} \right] \right) \approx H_{-1} \left( \frac{R}{\rho_0} \right) + H'_{-1} (x) \bigg|_{x = \frac{R}{\rho_0}} \frac{R \cdot \mathbf{r}_2}{R^2}, \tag{16}
\]

\[
Y_{-1} \left( \frac{R}{\rho_0} \left[ 1 + \frac{R \cdot \mathbf{r}_2}{R^2} \right] \right) \approx Y_{-1} \left( \frac{R}{\rho_0} \right) + Y'_{-1} (x) \bigg|_{x = \frac{R}{\rho_0}} \frac{R \cdot \mathbf{r}_2}{R^2}. \tag{17}
\]

When \( R \gg r_1 \) and \( R \gg r_2 \) by considering only terms linear with respect to \( r_1 \) and \( r_2 \) and using \([16]\) and \([17]\) finally Eq. \([15]\) can be written as

\[
V_{ed} \left( \frac{|\mathbf{R} + \mathbf{r}_2|}{\rho_0} \right) = -\frac{\pi ke^2}{2\kappa \rho_0^3} \left[ H_{-1} \left( \frac{R}{\rho_0} \right) - Y_{-1} \left( \frac{R}{\rho_0} \right) \right] \left( \frac{\mathbf{r}_1 \cdot \mathbf{r}_2}{R} + \frac{\mathbf{r}_1 \cdot \mathbf{r}_2}{R} - \frac{\mathbf{r}_1 \mathbf{R} \cdot \mathbf{r}_2}{\rho_0 R^2} \right) - \frac{\pi ke^2}{2\kappa \rho_0^3} \left[ H'_{-1} \left( \frac{R}{\rho_0} \right) - Y'_{-1} \left( \frac{R}{\rho_0} \right) \right] \frac{\mathbf{R} \mathbf{r}_1 \cdot \mathbf{r}_2}{\rho_0 R^2}. \tag{18}
\]
Let us find $H_{-1}(\frac{R}{\rho_0}) - Y_{-1}(\frac{R}{\rho_0})$ which presents in (18). The recurrence relations for the Struve functions

$H_{\nu-1}(x) + H_{\nu+1}(x) = \frac{2\nu}{x} H_{\nu}(x) + \left(\frac{\pi}{2}\right)^\nu \frac{1}{\sqrt{\pi x}} (\nu + 3/2) \quad \nu \neq 0$

and

$H_{\nu-1}(x) - H_{\nu+1}(x) = 2H'_{\nu}(x) - \left(\frac{\pi}{2}\right)^\nu \frac{1}{\sqrt{\pi x}} (\nu + 3/2) \quad \nu \neq 0$

lead to $H_{\nu}'(x) = H_{\nu-1}(x) + \frac{1}{2} H_{\nu}(x)$. On the other hand, for the second kind Bessel function $Y_{\nu}'(x) = Y_{\nu-1}(x) - \frac{1}{\pi} Y_{\nu}(x)$ [10]. Therefore, for $\nu = -1$ we obtain:

$H_{-1}(\frac{R}{\rho_0}) - Y_{-1}(\frac{R}{\rho_0}) = H_{-2}(\frac{R}{\rho_0}) - Y_{-2}(\frac{R}{\rho_0}) + \frac{\pi}{2\rho_0} \left[ H_{-1}(\frac{R}{\rho_0}) - Y_{-1}(\frac{R}{\rho_0}) \right] \]$

The latter expression allows rewrite Eq. (18) as

$$V_{\text{cd}} \left( \frac{R + r'_2}{\rho_0} \right) = -\frac{\pi k e^2}{2\kappa \rho_0^2} \left[ H_{-1} \left( \frac{R}{\rho_0} \right) - Y_{-1} \left( \frac{R}{\rho_0} \right) \right] \frac{R r_1 + r_1 r_2}{R} - \frac{\pi k e^2}{2\kappa \rho_0^2} \left[ H_{-2} \left( \frac{R}{\rho_0} \right) - Y_{-2} \left( \frac{R}{\rho_0} \right) \right] \frac{R r_1 R \cdot r_2}{\rho_0 R^2}. \tag{19}$$

Replacing the second term in Eq. (14) by expression (19) we obtain the dipole-dipole interaction in 2D configuration space

$$V_{\text{dd}} (R) = -\frac{\pi k}{2\kappa \rho_0} \left\{ \left[ H_{-1} \left( \frac{R}{\rho_0} \right) - Y_{-1} \left( \frac{R}{\rho_0} \right) \right] \frac{d_1 \cdot d_2}{\rho_0 R} + \left[ H_{-2} \left( \frac{R}{\rho_0} \right) - Y_{-2} \left( \frac{R}{\rho_0} \right) \right] \frac{R d_1 R \cdot d_2}{\rho_0 R^2} \right\}. \tag{20}$$

Using Eq. (10) for $\nu = -1$ and $\nu = -2$ for the first and second term in Eq. (20), respectively, one can find the asymptotic of $V_{\text{dd}} (R)$ interaction when $R \to \infty$. The first term in Eq. (20) has the following asymptotic behavior

$$\left( \frac{R}{\rho_0} \right)^{-2} - \frac{1}{\sqrt{\pi} (\nu + 3/2)} \frac{d_1 \cdot d_2}{\rho_0 R} = -2 \frac{\rho_0}{\pi} \frac{d_1 \cdot d_2}{R^2}.$$

While the asymptotic of the second term is

$$\left( \frac{R}{\rho_0} \right)^{-3} \frac{1}{\sqrt{\pi} (\nu + 3/2)} \frac{R d_1 R \cdot d_2}{\rho_0^2 R^2} = \frac{6 \rho_0}{\pi} \frac{R d_1 R \cdot d_2}{R^2}.$$

Combining the latter expressions we obtain

$$V_{\text{dd}} (R) \xrightarrow{R \to \infty} \frac{k}{\kappa \rho_0^3} \left[ d_1 \cdot d_2 - 3 \frac{(R \cdot d_1) (R \cdot d_2)}{R^2} \right]. \tag{21}$$

For comparison the dipole-dipole interaction in 3D configuration space has the following form

$$V_{\text{dd}}^C (r) = \frac{k}{\kappa R^3} \left[ d_1 \cdot d_2 - 3 \frac{(R \cdot d_1) (R \cdot d_2)}{R^2} \right], \tag{22}$$

where $\kappa = \varepsilon$ is the dielectric constant of the bulk material. Thus, $V_{\text{dd}} (R)$ asymptotic coincides with the dipole-dipole interaction in 3D configuration space where charges interact via the Coulomb potential. In numerical calculations,

we focus only on freestanding phosphorene and monolayer MoS$_2$. We use for MoS$_2$ polarizability $\chi_{2D} = 6.6$ Å [9] obtained within density functional theory and subsequent the random phase approximation calculations for TMDC monolayers. For the phosphorene polarizability the value $\chi_{2D} = 4.1$ Å [12] is used. The right panel in Fig. 2 presents

![Image](image-url)

FIG. 2: (Color online) Left panel: The ratios of the Retova-Keldysh and Coulomb potentials, charge-dipole interaction $V_{\text{cd}}$ in 2D configuration space and $V_{\text{cd}}^C$ for bulk materials and the second factors of the dipole-dipole interaction in a monolayer and bulk material. Calculations are performed for the phosphorene and MoS$_2$. Right panel: The universal dependence of the ratio of $V_{\text{cd}} / V_{\text{cd}}^C$ and $V(r) / V(r)^C$ on $R/\rho_0$ for any 2D material.
the ratio of charge-dipole interaction potentials $V_{cd}(R)$ in phosphorene and MoS$_2$ and $V^C_{dd}(R)$ in the same bulk materials. The values of the negative order Struve $H_{-1}\left(\frac{R}{\rho_0}\right)$ and Bessel $Y_{-1}\left(\frac{R}{\rho_0}\right)$ functions were evaluated with the in-built codes in Mathematica. There are five distinguished features: i. the value of $V_{cd}(\mathbf{r})$ is bigger than the value of $V^C_{dd}(\mathbf{R})$; ii. at small distances $V_{cd}(\mathbf{r})$ falls more slowly than the Coulomb potential induced charge-dipole interaction in the same bulk material; iii. at small distances the slope of the ratio fall demonstrate the sensitivity of $V_{cd}(R)$ to the 2D polarizability and dependence on the ratio of dielectric constant of the bulk material and the polarizability of monolayer; v. the asymptotic of the ratio is the value of the dielectric constant of the bulk material. This means that when $R \to \infty$ the charge-dipole interaction in a monolayer is the same as in vacuum.

The both dipole-dipole interactions (20) and (22) have two terms: one is proportional to $\mathbf{d}_1 \cdot \mathbf{d}_2$ and the other one to $(\mathbf{R} \cdot \mathbf{d}_1)(\mathbf{R} \cdot \mathbf{d}_2)$. The comparison of factors in front of $\mathbf{d}_1 \cdot \mathbf{d}_2$ shows that their ratio has the same dependence as the ratio $V_{cd}/V^C_{cd}$. The ratios of factors in front of $(\mathbf{R} \cdot \mathbf{d}_1)(\mathbf{R} \cdot \mathbf{d}_2)$ for phosphorene and MoS$_2$ are shown in Fig. 2. These ratios are smaller than $V_{cd}/V^C_{cd}$ and demonstrate the same features as that are listed above for $V_{cd}/V^C_{cd}$. However, the ratios fall more smoothly than $V_{cd}/V^C_{cd}$. As it is seen from Fig. 2, $V/V^C > V_{cd}/V^C_{cd} > V_{dd}/V^C_{dd}$ and all ratios converging to the dielectric constant of bulk materials. At small distances $V/V^C$ increases more rapidly than $V_{cd}/V^C_{cd}$ and $V_{cd}/V^C_{cd}$ increases more fast than the second term of $V_{dd}/V^C_{dd}$. Interestingly enough, the ratio

$$\frac{[V_{cd}/V^C_{cd}]}{[V/V^C]} = x \frac{H_{-1}(x) - Y_{-1}(x)}{H_0(x) - Y_0(x)}, \quad \text{where} \quad x = \frac{R}{\rho_0}$$

(23)

shows the universality in its dependence on $\frac{R}{\rho_0}$ that is the same for any monolayer material. This ratio we named as a scaled ratio that is the ratio of $V_{cd}(R)$ scaled to the corresponding $V^C_{cd}(R)$ and $V(R)$ scaled to the Coulomb potential. The dependence of this ratio on $R/\rho_0$ is shown on the right panel in Fig. 2.

**Concluding remarks.** In this paper we study the influence of the reduction of dimensionality on the charge-dipole and dipole-dipole interactions in 2D configuration space. We demonstrate that the screened nature of Coulomb interaction imposes peculiarities in the 2D charge-dipole and dipole-dipole interactions behavior. The analytical expression for the charge-dipole and dipole-dipole interactions in 2D configuration space are derived. We hope these charge-dipole and dipole-dipole interactions will find wide application in 2D materials studies. Recently, a new potential form for the electron-hole interaction, which takes into account the three atomic sheets that compose a monolayer of transition-metal dichalcogenides was derived [15]. Without losing any generality our approach can be extended for this form of the potential.

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