Modified Monte Carlo method for integral

S Iskandar

Mathematic Department, Universitas Negeri Medan, Jl. Williem Iskandar, Medan, Indonesia

* said.iskandar.alidrus@gmail.com

Abstract. The integral value of a function will be determined by the result of the function of the integral function. In fact many integral functions that are difficult to solve sometimes cannot be solved by exact sciences. There are several ways such as by changing its function to the approach to the Monte Carlo method uses the principle of generating random numbers to test a function. The use of randoms is very closely related to the value of the error obtained. This study aims to minimize the error value of integration by using the Monte Carlo method. The initial integration of the Monte Carlo method with 10000 random points has a fairly large average error value of 0.078, so that the follow-up is done using pias in the Monte Carlo integration.

1. Introduction

Determination of the integral value of course initially only solved analytically by using the fundamental theorem of calculus, but because of functions that were difficult to solve analytically, then numerical integrals emerged. Numerical methods are techniques in which mathematical problems are formulated in such a way that they can be solved by mathematical operations, where the use of these methods produces a near solution that is not exactly the same as the real solution. But the level of accuracy can be seen from the smallest possible error.

Determination of the integral value is certainly included in the deterministic problem. Deterministic models have been developed to solve the problem of defining integral values, such as the Romberg method, the trapezoidal method, the rectangular method and other methods. These methods are included in computational algorithms that use a deterministic process because they produce a definite output each time the calculation process is carried out. Determination of the integral value of course which includes deterministic problems can also be solved using a stochastic approach, one of them using the Monte Carlo Method.

In a previous study, the Monte Carlo Method in Calculating Integral of Doubling Certainly said that the Monte Carlo method is one of the classes in computational algorithms that use random sampling (Pseudo-random) to produce integral problem solving. But in a study conducted by Ermawati (2017a) entitled Comparison of the Integral Numerical Solution of Two Fold Algebraic Functions with the Romberg Method and Monte Carlo Simulation, the Monte Carlo method still has a greater error value compared to the Romberg method even though Monte Carlo uses an iteration of 10,000 for algebraic functions [1][2].
2. Integral

Integral is the study of definitions, properties, and applications of two interconnected concepts, indeterminate integrals and definite integrals. The process of finding the value of an integral is called integration. Integration is symbolized by the symbol \( \int \).

The symbol \( \int \) was introduced by Leibniz and called the integral sign. The symbol is the letter S which is bent and selected because the integral is a limit of the amount. The procedure for calculating integrals is called integration (Stewart 2009).

Some integral uses in everyday life, including determining the area of a field, determining the volume of a rotating object, determining the length of the arc and so on. Integral is not only used in mathematics, but also in other fields such as economics, physics, biology, engineering and other fields of science [3][4].

If \( f \) is a continuous function defined for \( a \leq x \leq b \), then the interval \([a, b]\) is divided into \( n \) subintervals of the same width, \( \Delta x=(b-a)/n \). For example, \( x_0(=a), x_1, x_2, ..., x_n(=b) \) are subinterval endpoints and choose the sample points \( x_1^*, x_2^*, ..., x_n^* \) in this subinterval, so \( x_i^* \) is located in the interval the \( i \)-th of \([x_{i-1}, x_i]\) then the integral of course \( f \) from \( a \) to \( b \) is

\[
\int_a^b f(x) \, dx = \lim_{n \to \infty} \sum_{i=1}^{n} f(x_i^*) \Delta x
\]

Provided the limit exists and gives the same results for all possible sample point selections. If the limit exists, then it says \( f \) is integrated in \([a, b]\).

3. Methods

3.1. Monte Carlo Method

According to pratibha (2003) Monte Carlo is a numerical method used to find mathematical solutions (can consist of many variables) that are difficult to solve, for example with integral calculus, or other numerical methods. One important use of the Monte Carlo method is to compute a function. The basic idea is to take a number of random points on the abscissa of the integration boundary, then calculate the value of the function and add it up. Taking the number of sample points can be chosen arbitrarily as needed. The monte carlo method is very important in computational physics and other applied fields, and has applications that range from the calculation of esoteric quantum chromodynamics to aerodynamic design. This method proved to be efficient in solving radians’ field differential equations. So this method is used in the calculation of global illuminations that produce photorealistic images of three-dimensional models, which are applied in video games, architecture, design, films produced by computers, special effects in film, business, economics and other fields[5][6].

Seeing from how the monte carlo method works is a method that gives all the possible values of a variable. The monte carlo method is a method that utilizes the strong law of large number in calculating, meaning that the more random variables used the better the exact value approach. The monte carlo method uses the average as its exact value estimator. Because this algorithm requires repetition (repetition) and calculations that are very complex, the Monte carlo method is generally carried out using a computer, and uses various computer simulation techniques.[7]

4. Result and Discussion

According to Salusu (2008) the Monte Carlo method is based on the middle price proposition, namely: If \( f(x) \) is continuous at intervals \([a, b]\) then there is a number \( c \) with \( a < b < c \), such that

\[
\frac{1}{b-a} \int_a^b f(x) \, dx = f(c)
\]  

Or

\[
\int_a^b f(x) \, dx = (b - a)f(c)
\]
This method is used to find the approach value of an integral. For functions consisting of one variable for example $\int_{a}^{b} f(x)dx$:

a. Choose $n$ point $c_1, c_2, \ldots, c_n$ in interval; $[a,b]$

b. Determine the average value of the function $f(x)$

$$\bar{f} = \frac{1}{n} \sum_{i=1}^{n} f(c_i) \text{ where } c_i = a + \left(i - \frac{1}{2}\right)h \text{ for } i = 1,2,\ldots,n \text{ and } h = \frac{b-a}{n} \quad (4)$$

c. Find $\int_{a}^{b} f(x)dx = (b-a)\bar{f}$

d. Error $=(b-a)\sqrt{\frac{\bar{f}^2-(\bar{f})^2}{n}}$ where $\bar{f}^2 = \frac{1}{n} \sum_{i=1}^{n} f^2(x_i)$

**Figure 1.** Flowchart for Monte Carlo Method

Test Function 1

$$\int_{-1}^{3}(2x^2 - 8)dx = \frac{2}{3}x^3 - 8x = -\frac{40}{3} \quad (5)$$
Figure 2. Result Area Integration for function1, result is -12.7665

Test Function 2

\[ \int_{-3}^{3} x^4 - 2x - (\sin(2x+1))^2 \, dx \]  

Figure 3. Result Area Integration for function2, result is 92.6473
Test Function 3

\[ \int_{-3}^{3} 2e^{-2x} - 3(\cos(3x - 1))^4dx \]  

(7)

Figure 4. Result Area Integration for function3, result is 390.433

5. Conclusion

Based on the results and discussion that has been done, it can be concluded that the error value in Monte Carlo integration can be minimized by dividing the integration limit interval into several parts (pias). The use of pias does not change the basic concept of the Monte Carlo method while still relying on the use of random points, it's just that the shooting area which was originally too large is divided into small parts to increase the accuracy of the integration results.

Pias affects the accuracy of the Monte Carlo method, especially on graphs or curves that go up and down (complicated) in a shooting area. The small shooting area makes the random point more effective without the need to use it in large quantities. The accuracy of the Monte Carlo integration results reaches a precision of up to four decimal places with a random number of 10,000 points and as many as 1,000 pias.

6. References

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