Two-dimensional black holes in accelerated frames: quantum aspects

R Balbinot†§ and A Fabbri‡∥

† Dipartimento di Fisica dell’Università di Bologna and INFN sezione di Bologna, Via Irnerio 46, 40126 Bologna, Italy
‡ SISSA-ISAS and INFN sezione di Trieste, Via Beirut 2–4, 34014 Trieste, Italy

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Abstract. By considering charged black-hole solutions of a one-parameter family of two-dimensional dilaton gravity theories, one finds the existence of quantum mechanically stable gravitational kinks with a simple mass-to-charge relation. Unlike their Einsteinian counterpart (i.e. extreme Reissner–Nordström), these have nonvanishing horizon surface gravity.

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1. Introduction

Hawking’s discovery [1] that semiclassically black holes radiate has triggered a widespread debate concerning the ultimate fate of an evaporating black hole. According to one school of thought, black holes evaporate completely in a finite time into the vacuum ([1], for an explicit two-dimensional model see, for instance, the RST model [2]). Others advocate the existence of stable (zero-temperature) remnants of finite (probably Planckian) mass which should represent the endpoint of the evaporation process (see, for example, the review articles [3]). Connected with these two alternative issues is the question concerning unitarity violations in the quantum mechanics of the black-hole formation–evaporation process.

Zero-temperature remnants are easily found in Einstein gravity coupled to electromagnetism (or to some other Abelian long-range field). This theory contains charged black-hole solutions (Reissner–Nordström) provided the mass $m_0$ of the hole equals or exceeds, in Planck units, the conserved Abelian charge $Q$. According to the conventional picture a black hole of mass $m_0 > |Q|$ will evaporate quantum mechanically until it reaches the extremal value $m_0 = |Q|$, at which point the Hawking temperature vanishes and the evaporation ceases. Thus the extremal Reissner–Nordström solutions are expected to be the endpoint of Hawking evaporation and correspond to stable quantum ground states¶. Similar results are also found in dilaton gravity theories coupled to Abelian fields (see, for example, [4, 5]).

The analysis of quantum fields propagating on these extreme black-hole spacetimes and their backreaction on the geometry has been performed extensively over the past few
years using two-dimensional approximation schemes [6, 7]. They revealed that, despite
the stability against the evaporation process, the expectation values \( \langle T_{ab} \rangle \) of the energy–
momentum tensor of a massless scalar field, as measured by a free-falling observer, is
divergent on the horizon\(^†\). Their backreaction on the geometry is, however, not problematic,
since in the ‘quantum-corrected geometry’ the metric and the scalar curvature \( R \) appear to
be well behaved there.

Quite different is the situation when no Abelian field is present: zero-temperature
remnants of finite mass with regular horizon (gravitational kinks) simply do not exist. The
‘Boulware vacuum’ construction of these kinds of objects produces unbounded \( \langle T_{ab} \rangle \) at the
black-hole horizon. This divergence is stronger than the one encountered in the previous
(extreme) case and as a result of the backreaction now a diverging scalar curvature \( R \) is
encountered as one approaches the horizon [9, 10].

The purpose of this paper is an investigation of the existence of kink-like solutions in
a two-dimensional dilaton gravity theory proposed recently which generalizes the original
CGHS theory. In [11] the existence of one-parameter theories all leading to the RST action
[2] at the semiclassical level has been shown; they are exactly solvable both classically and
semiclassically. They are described by the classical action

\[
S_n = \frac{1}{2\pi} \int d^2x \sqrt{-g} \left[ e^{-2\phi/n} \left( R + \frac{4}{n} (\nabla \phi)^2 \right) + 4\lambda^2 e^{-2\phi} \right],
\]

where \( R \) is the scalar curvature associated with the two-dimensional metric tensor \( g_{ab} \), \( \phi \)
is the dilaton field. \( n \) is the parameter labelling the different theories; \( n = 1 \) reproduces the
usual CGHS action [12].

Static black-hole solutions of these theories for \( n \neq 1 \) have the striking feature that
the natural frame in which the metric is static is not ‘asymptotically’ Minkowskian but is
Rindler-like. This, as we shall see, is the key feature to understanding the existence of our
gravitational kinks.

The introduction, in these theories, of an electromagnetic term coupled to the dilaton
and to the gravitational field breaks the conformal invariance of the theory, which is no
longer exactly solvable at the semiclassical level. However, as we shall see, classical
exact solutions describing charged black holes can be found and also the corresponding
semiclassical corrections to these geometries can be worked out perturbatively. We shall
show the conditions these solutions have to fulfil in order to describe gravitational kinks,
the properties of which will then be compared to their extreme Reissner–Nordström-like
counterpart.

2. Quantum fields in two-dimensional Reissner–Nordström spacetime

In this section we collect the most important results (for our purposes) concerning quantum
effects in two-dimensional Reissner–Nordström spacetime. This spacetime is the unique
solution of Einstein–Maxwell theory which is spherically symmetric and asymptotically
flat. The two-dimensional section (\( \theta = \text{constant}, \phi = \text{constant} \)) of this solution is described
by the line element

\[
ds^2 \equiv -e^{2\rho} \, du \, dv = -f \, du \, dv,
\]

\(^†\) Numerical evaluations of \( \langle T_{\mu\nu} \rangle \) in four spacetime dimensions seem, however, not to reproduce this divergence
[8], which therefore should be considered as just an artefact of the two-dimensional approximation scheme.
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where the coordinates $u$ and $v$ are defined by

$$
du = dt - \frac{dr}{f}, \quad dv = dt + \frac{dr}{f},
$$

and

$$
f = 1 - \frac{2m_0}{r} + \frac{Q^2}{r^2},
$$

(2.3)

(we take $m_0 > |Q|$). Equation (2.1) describes a black hole of mass $m_0$ and Abelian charge $Q$. The coordinates $(u, v)$ become, as $r \to \infty$, the usual retarded and advanced time of special relativity. Associated with the static nature of these solutions is the existence of a Killing vector $\partial_t$. The zeros of $f$ represent the Killing horizons where the norm of $\partial_t$ vanishes. These null surfaces are the event horizon at $r = r_+$ and the inner (Cauchy) horizon at $r = r_-$, where

$$
r_{\pm} = m_0 \pm \sqrt{m_0^2 - Q^2}.
$$

(2.4)

We also define, for later use, the surface gravity $k$ at both horizons

$$
k_{\pm} \equiv \frac{1}{2} |\partial_r f|_{r_\pm} = \frac{\sqrt{m_0^2 - Q^2}}{r_{\pm}^2}.
$$

(2.5)

The scalar curvature is simply

$$
R = - f''
$$

(2.6)

and from equation (2.3) we see that $r = 0$ is the location of the singularity and $r \to \infty$ defines the flat asymptotic region, where the spacetime metric becomes Minkowski-like.

The Penrose diagram of this spacetime is represented (for $m_0 > |Q|$) in figure 1.

When $m_0 = |Q|$ the black hole is called extremal and in this case

$$
f = \left(1 - \frac{m_0}{r}\right)^2.
$$

(2.7)

**Figure 1.** Penrose diagram of the Reissner–Nordström spacetime for $m_0 > |Q|$. Double lines represent the singularity, broken lines the curves $r = \text{constant}$, regular lines the horizons and thick lines the asymptotic region.
This implies \( r_+ = r_- = m_0 \) and \( k_\pm = 0 \). The causal structure of this solution is depicted in figure 2.

The expectation values of the energy–momentum tensor of \( N \) massless scalar fields \( f_i \) existing on a two-dimensional spacetime can be derived, knowing the conformal anomaly, simply by integrating the conservation equations in the gauge of equation (2.1). The result is

\[
\langle T_{uu}\rangle = -\frac{N}{12\pi} \left( \partial_u \rho \partial_u \rho - \partial_u^2 \rho - \frac{1}{8} t_u(u) \right), \\
\langle T_{vv}\rangle = -\frac{N}{12\pi} \left( \partial_v \rho \partial_v \rho - \partial_v^2 \rho - \frac{1}{8} t_v(v) \right), \\
\langle T^{a\ a}\rangle = \frac{NR}{24\pi}.
\] (2.8)

(2.9)

(2.10)

Here units are chosen such that \( \hbar = c = G = 1 \). \( t_u(u) \) and \( t_v(v) \) are functions of their arguments and depend on the choice of the quantum state in which the expectation values have to be taken. For our static spacetime these are just constants to be fixed later and

\[
\langle T_{uu}\rangle = -\frac{N}{96\pi} \left( ff'' - \frac{1}{2} f'^2 + t_u \right), \\
\langle T_{vv}\rangle = -\frac{N}{96\pi} \left( ff'' - \frac{1}{2} f'^2 + t_v \right), \\
\langle T^{a\ a}\rangle = -\frac{Nf''}{24\pi}.
\] (2.11)

(2.12)

(2.13)

where \( f \) is given in (2.3) or (2.7).

Because of the asymptotic Minkowskian behaviour of the metric, it seems reasonable to require that, as \( r \to \infty \), \( \langle T_{ab}\rangle \) vanishes. This simply fixes the constants

\[
t_u = t_v = 0.
\] (2.14)

This choice of state corresponds to the ‘Boulware vacuum’, i.e. the state that corresponds asymptotically to the usual vacuum state of Minkowski quantum field theory. However,
when \( m_0 > |Q| \), this ‘natural choice’ is not appropriate to correctly describe the vacuum polarization induced by the fields \( f_i \) near the event horizon. In fact, in terms of a system of coordinates \((U, V)\) regular on the event horizon \( r_+ \) (Kruskal coordinates)

\[
U = -\frac{e^{-k_+ u}}{k_+}, \quad V = \frac{e^{k_+ v}}{k_+},
\]

and vanishing on the future and past sheet of this surface, respectively, one has

\[
\langle T_{UU} \rangle \sim -\frac{1}{U^2}, \quad \langle T_{VV} \rangle \sim -\frac{1}{V^2},
\]

which are clearly divergent for \( U \to 0, V \to 0 \). The backreaction of this source on the geometry [9, 10] produces a diverging curvature on the event horizon (however, the ADM mass stays finite due to the asymptotic behaviour of \( \langle T_{ab} \rangle \) in the Boulware state). The presence of this singularity indicates that the Boulware state cannot be the relevant quantum state for describing vacuum polarization in a black-hole spacetime.

Regularity of the stress tensor on both the past and future event horizon is obtained by the (Hartle–Hawking) state given by the choice

\[
t_u = t_v = 2k_+^2, \quad (2.17)
\]

which ensures a sufficiently rapid vanishing of \( \langle T_{uu} \rangle = \langle T_{vv} \rangle \) on \( r = r_+ \). In fact, as \( r \to r_+ \)

\[
\langle T_{uu} \rangle = \langle T_{vv} \rangle \sim \text{constant} \times (r - r_+)^2
\]

which implies regularity of \( \langle T_{ab} \rangle \) in the Kruskal frame (2.15)†. However, as a result of this choice of state, asymptotically

\[
\langle T_{uu} \rangle = \langle T_{vv} \rangle \to \frac{Nk_+^2}{48\pi},
\]

i.e. the stress tensor is no longer vanishing. Physically the Hartle–Hawking state is a thermal state, describing thermal equilibrium of a black hole and a heat bath at the Hawking temperature \( T_H = k_+/2\pi \)‡.

Summarizing the basic result, we have that as long as \( m_0 > |Q| \) the two conditions of vanishing of \( \langle T_{ab} \rangle \) at infinity and regularity on the event horizon cannot be fulfilled simultaneously.

The situation is significantly different in the extremal case \( m_0 = |Q| \). Now with \( r_+ = r_- \) and \( k_+ = 0 \) the Boulware and the Hartle–Hawking state become, in some sense, the same state. Nevertheless, one can show (see, for example, [6, 13]) that at the horizon a free-falling observer will measure an energy density

\[
\rho_{\text{obs}} \sim f'''
\]

which is divergent because \( f \) now has a double zero at \( r_+ \) (see equation (2.7)). However, the divergence in expression (2.20) is in some sense weak, since the geometry obtained by including the backreaction of this \( \langle T_{ab} \rangle \) (quantum-corrected geometry) has regular curvature on the horizon§.

We should stress the fact that here we focus our attention on static black-hole configurations, quantum-mechanically stable, of finite mass and regular at the event horizon. These are natural candidates for the final state of a black-hole formed dynamically by the

† This construction does not, however, prevent the stress tensor from diverging on the inner (Cauchy) horizon.
‡ Due to the behaviour in (2.19), the ADM mass of the geometry obtained by including the backreaction of \( \langle T_{ab} \rangle \) is infinite. Usually one eliminates this problem by enclosing the system in a box.
§ However, a mild singularity appears in the second derivative of \( R \) as seen by a free-falling observer.
collapse of matter which then evaporates. The only candidate found is therefore the quantum version of the extremal Reissner–Nordström black hole.

Other examples with the same features are found in two-dimensional dilaton gravity theories like the charged extension of the CGHS model (see, for example, [4, 5]).

3. Charged black holes in accelerated frames

In [11] a one-parameter class of simple models of two-dimensional dilaton gravity theory was considered. The model is described by the action (1.1)

\[ S_n = \frac{1}{2\pi} \int d^2x \sqrt{-g} \left[ e^{-2\phi/n} \left( R + \frac{4}{n} \nabla^2 \phi \right) + 4\lambda^2 e^{-2\phi} \right]. \] (3.1)

The equations of motion derived from this action are

\[ g_{\mu\nu} \left[ \frac{4}{n} \left( -\frac{1}{2} + \frac{1}{n} \right) (\nabla \phi)^2 - \frac{2}{n} (\nabla^2 \phi) - 2\lambda^2 e^{(2-2n)\phi/n} \right] + \frac{4}{n} \left( 1 - \frac{1}{n} \right) \partial_\mu \phi \partial_\nu \phi + \frac{2}{n} \nabla_\mu \partial_\nu \phi = 0, \] (3.2)

\[ \frac{R}{n} - \frac{4}{n^2} (\nabla \phi)^2 + \frac{4}{n} \nabla^2 \phi + 4\lambda^2 e^{(2-2n)\phi/n} = 0. \] (3.3)

As shown in appendix A, static solutions of these equations describing black holes can be found.

The general solution describing a static uncharged black hole can be given in the ‘Schwarzschild–Rindler’ gauge \((\sigma, t)\) as

\[ ds^2 = e^{2(1-n)\lambda \sigma} \left( -f \, dt^2 + \frac{1}{f} \, d\sigma^2 \right), \] (3.4)

\[ \phi = -n\lambda \sigma, \] (3.5)

where \( f \) is defined as

\[ f = 1 - \frac{2m_0}{\lambda} e^{2\phi/n}. \] (3.6)

Proceeding to the usual ADM construction, one finds that \(2m_0\) represents the mass of the black hole. It is easy to realize that ‘asymptotically’, i.e. as \( \sigma \to \infty \), the metric in (3.4) becomes flat but it is expressed in Rindler coordinates instead of the usual Minkowski ones.

Adding to the action (3.1) an electromagnetic term of the form

\[ S_{EM} = \frac{1}{2\pi} \int d^2x \sqrt{-g} e^{-2a\phi} \left( -2F_{\mu\nu} F^{\mu\nu} \right), \] (3.7)

where \( a = (2-n)/n \) and \( F_{\mu\nu} \) is the electromagnetic tensor, one can find solutions describing black holes which carry an Abelian charge \( Q \). The equations of motion for \( F_{\mu\nu} \) derived from (3.7) are

\[ \nabla_{\nu} (e^{-2a\phi} F^{\mu\nu}) = 0, \] (3.8)

which are easily integrated leading to

\[ F_{\mu\nu} = Q e^{2a\phi} e_{\mu\nu}, \] (3.9)
where $e_{\mu\nu} = e_{(\mu\nu)}$ and $e_{01} = \sqrt{-g}$. The solution of the field equations for the metric and the dilaton are again given by (3.4) and (3.5) but where $f$ is now

$$f = 1 - \frac{2m_0}{\lambda} e^{2\phi/n} + \frac{Q^2}{\lambda^2} e^{4\phi/n}. \quad (3.10)$$

This solution describes static black holes of mass $m_0$ and Abelian charge $Q$.

The causal structure of these solutions can be studied following the scheme of [14].

The line element is transformed to the chiral form

$$ds^2 = -h(v, x) dv^2 + 2 dv dx \quad (3.11)$$

by the coordinate transformation

$$x = \frac{e^{2(1-n)\sigma}}{2(1-n)\lambda}, \quad v = t + \int \frac{d\sigma}{f}, \quad (3.12)$$

which yields

$$h(v, x) = 2(1-n)\lambda x - \frac{2m_0}{\lambda} [2(1-n)\lambda x]^{n/(n-1)} + \frac{Q^2}{\lambda^2} [2(1-n)\lambda x]^{(n+1)/(n-1)}. \quad (3.13)$$

With this choice of coordinates $v$ labels ingoing null lines and $x$ is an appropriate normalized affine parameter on them. Starting from the form (3.11) of the metric one can construct the fundamental building blocks and the resulting Penrose diagrams. For the purposes of the present paper we need just to outline some basic characteristics of the spacetimes. For $n < 1$ and $m_0 > |Q|$ the solution presents two horizons located at $\sigma = \sigma_{\pm}$, where

$$e^{2\lambda\sigma_{\pm}} = e^{-2\phi_{\pm}/n} = \frac{(m_0 \pm \sqrt{m_0^2 - Q^2})}{\lambda}. \quad (3.14)$$

$\sigma_+$ is the location of the outer (event) horizon, and $\sigma_-$ is the inner (Cauchy) horizon. For $m_0 = |Q|$ the two horizons coalesce. The essential singularity is at $\sigma = -\infty$, whereas the asymptotically flat region is at $\sigma = +\infty$.

Our solution, apart a conformal factor, is the same as the charged (Reissner–Nordström-like) solution of the CGHS ($n = 1$) model. The corresponding Penrose (conformal) diagrams are therefore the same as those shown in figures 1 and 2. The presence of the conformal factor, however, makes the metric approach a Rindler-like form asymptotically instead of the usual Minkowski one. One can introduce Minkowskian null coordinates $y^\pm$ defined by

$$\lambda y^+ = e^{\lambda(1-n)u} \frac{1}{(1-n)}, \quad (3.15)$$

$$-\lambda y^- = e^{-\lambda(1-n)u} \frac{1}{(1-n)}, \quad (3.16)$$

where $u = t - \int d\sigma/f$ and $v = t + \int d\sigma/f$, but the metric is no longer static in this frame. It takes the form (for $Q = 0$)

$$ds^2 = \frac{dy^+ dy^-}{[1 + (2m_0/\lambda)(-\lambda^2(1-n)^2y^+y^-)^{1/(n-1)}]^n}. \quad (3.17)$$

For $n > 1$ one has the same features described above except for the fact that $\sigma = +\infty$ no longer corresponds to true infinity, since inertial observers can reach this region in a

\[\text{Note that for } n = 2 \text{ the electromagnetic field is completely decoupled from the dilaton and the insertion of } S_{EM}, \text{ once equations (3.9) have been taken into account, is then equivalent to the addition of a two-dimensional cosmological constant to the action.}\]
finite proper time. One can show that the null surface $\sigma = +\infty$ is an acceleration horizon for $\sigma = \text{constant observers}$. A useful quantity in the description of horizons is the local surface gravity $k$, defined by (see, for example, [15])

$$\kappa = \frac{1}{2} \frac{\partial_\sigma g_{tt}}{\sqrt{-g_{\sigma\sigma} g_{tt}}} = |\lambda(1-n) f + \frac{1}{2} f,\sigma|.$$  

(3.18)

One finds that at the black-hole horizons $\sigma_{\pm}$

$$\kappa_{\pm} = \frac{1}{2} |f,\sigma_{\pm}| = \lambda e^{2\lambda \sigma_{\pm}} (e^{2\lambda \sigma_{\pm}^2} - e^{2\lambda \sigma_{\pm}^2}).$$  

(3.19)

Note that, provided $m_0 > |Q|$, always $k_- > k_+$ and $k_+ = k_- = 0$ for the extreme $m_0 = |Q|$ hole. In the case where $n > 1$ one can also define the surface gravity at the acceleration horizon

$$k_{ah} \equiv k(\sigma = +\infty) = \lambda(n - 1).$$  

(3.20)

Since in what follows we are interested in solutions exhibiting an asymptotic region, only the case $n < 1$ will be considered.

4. Quantum kinks

The behaviour of a conformally invariant quantum field propagating on the charged black-hole solution of our two-dimensional dilaton gravity action equations (3.1) and (3.7) is analysed within the framework developed in section 2. We first write the metric in a conformal flat form

$$ds^2 = -e^{2\rho} du dv = -F f du dv,$$

(4.1)

where $F = e^{2(1-n)\lambda \sigma}$ and $f$ is defined in (3.10). $u$ and $v$ are the usual advanced and retarded null coordinates $du = dt + d\sigma/f$, $dv = dt - d\sigma/f$. The stress tensor $\langle T_{ab} \rangle$ is given as before by (2.8)–(2.10). Again for static equilibrium configurations we require that $t_u = t_v = C = \text{constant}$. The value of this constant reflects the choice of quantum state in which to evaluate the expectation values.

Our choice is to consider as ‘physical’ only those configurations which have a regular event horizon, i.e. Hartle–Hawking state. The quantum field operator should therefore be expanded in normal modes

$$e^{-iwx^+}, \quad e^{-iwx^-}$$  

(4.2)

where $x^\pm$ are Kruskal-like coordinates, regular on the event horizon. These are related to our ‘Schwarzschild–Rindler’ frame by the following coordinate transformations:

$$x^+ = \frac{e^{k_u} \nu}{k_+}, \quad x^- = -\frac{e^{-k_+ u}}{k_+}.$$  

(4.3)

The value of $C$ is therefore simply the Schwarzian derivative between the Kruskal and the Schwarzschild–Rindler sets. The end result is that the stress tensor $\langle T_{ab} \rangle$ evaluated in the Hartle–Hawking state $|H\rangle$ is

$$\langle H|T_{uu}|H\rangle = \langle H|T_{vv}|H\rangle = \frac{N}{96\pi} \left[ ff,\sigma\sigma - \frac{1}{2} f^2,\sigma - 2(1-n)^2 \lambda^2 f^2 + 2k_+^2 \right].$$  

(4.4)

$$\langle H|T_{av}|H\rangle = \frac{NR}{24\pi} = -\frac{N}{24\pi} e^{-2(1-n)\lambda \sigma} \left[ ff,\sigma\sigma + 2(1-n)\lambda f,\sigma \right].$$  

(4.5)

These expressions can be checked to be regular on both the future and the past sheet of the event horizon.
Note that, in general, the Kruskal coordinates \( x^\pm \) are not inertial coordinates at infinity and therefore \( \langle H | T_{\mu\nu} | H \rangle \) does not vanish asymptotically, namely as \( \sigma \to +\infty \)
\[
\langle H | T_{\mu\nu} | H \rangle \simeq \frac{N}{48\pi}[k_+^2 - \lambda^2(1-n)^2]
\] (4.6)
and in terms of the Minkowski coordinates at infinity \( y^\pm \), defined in (3.15) and (3.16), one has
\[
\langle H | T_{\mu\nu} | H \rangle \simeq \frac{N}{48\pi(1-n)^2\lambda^2 y^\pm}[k_+^2 - \lambda^2(1-n)^2],
\] (4.7)
\[
x^\pm = \pm \frac{1}{k_+}[(\pm(1-n)\lambda y^\pm)^{1/(1-n))}].
\] (4.8)
However, when the surface gravity \( k_+ \) of the hole satisfies
\[
k_+ = \lambda(1-n)
\] (4.9)
we see that (4.7) vanishes and the relation (4.8) becomes trivial, i.e. \( x^\pm = y^\pm \). This is the basic result of our paper. From the definition of \( k_+ \) (equation (3.19)) one can rewrite the condition (4.9) as
\[
Q^2 = \frac{4n}{(1+n)^2}m_0^2,
\] (4.10)
which implies \( m_0 > |Q| \) for \( n \neq 1 \) (\( n \) positive).

What we have just shown is a remarkable property of our charged black-hole solution: provided that the mass and the charge of the hole satisfy equation (4.10), the quantum state \( |H \rangle \) yields a stress tensor regular on the event horizon and vanishing asymptotically. These black holes are therefore natural candidates for gravitational kinks. With \( k_+ \neq 0 \), the event and inner horizons of the black hole are distinct and located at, respectively,
\[
e^{2i\sigma_+} = \frac{2}{(1+n)} \frac{m_0}{\lambda}, \quad e^{2i\sigma_-} = \frac{2n}{(1+n)} \frac{m_0}{\lambda}.
\] (4.11)
The backreaction of the stress tensor on the geometry can be evaluated perturbatively (see appendix B for the details) yielding for the quantum-corrected geometry of the kink (we reinsert \( \hbar \) in the formulae)
\[
f(\sigma) \sim 1 - 2m_0 \frac{\hbar}{\lambda} e^{-2i\sigma} + \left[ \frac{4n}{(1+n)^2\lambda^2} \frac{m_0^2}{\hbar N m_0} \left( n(n^2 - 2n + 3) \right) \right] e^{-4i\sigma} + O(e^{-6i\sigma}),
\] (4.12)
which shows that the ADM mass of the quantum kink is still \( 2m_0 \). For the dilaton field we find
\[
e^{-\phi/n} \sim e^{\phi \sigma} \left( 1 + \hbar \frac{N m_0}{18\pi \lambda} (2n - n^2)e^{-4i\sigma} \right).
\] (4.13)
In appendix B we also give the corrected value of the event horizon \( \bar{\sigma}_+ \). Both metric and dilaton field are regular there.

A very different behaviour is expected near the Cauchy horizon since \( \langle T_{\mu\nu} \rangle \) diverges badly there, namely
\[
\langle T_{\mu\nu} \rangle \simeq -\frac{\alpha}{x^{7/2}},
\] (4.14)
where
\[
\alpha = \frac{k_+^2 - k_-^2}{k_+^2}
\] (4.15)
and $x^+$ is now the Kruskal advanced null coordinate on the Cauchy horizon vanishing there. The quantum-corrected geometry must therefore be very different from the classical one. This problem has been addressed in [16], to which we refer the reader for the details, where it has been shown that near the Cauchy horizon the semiclassical equations can be solved yielding

$$e^{-2\phi/n} \simeq -\alpha \ln(-k_- x^+) + \text{finite terms.} \quad (4.16)$$

Furthermore, the Ricci scalar in the same limit behaves as

$$R \sim \frac{1}{x^+ (-\ln(-k_- x^+))^{2-n}}, \quad (4.17)$$

which clearly diverges for every value of $n$.

This singularity of the geometry indicates that, once quantum effects are taken into account, the black-hole spacetime has a singular boundary at the Cauchy horizon and any continuation of the spacetime beyond this surface is therefore meaningless.

5. Conclusion

In this paper we have found the existence of charged black-hole solutions of a one-parameter ($n$) two-dimensional dilaton gravity action which generalizes the CGHS action. This latter corresponds to the value $n = 1$.

At the classical level the static black holes of charge $Q$ do not seem to differ drastically from the usual Reissner–Nordström-like solutions of general relativity. They have two horizons (outer and inner) provided that $m_0 > |Q|$. For $m_0 = |Q|$ the two horizons coalesce and we have an extreme black hole with zero surface gravity on the horizon. However the presence of the conformal factor (which is non-trivial for $n \neq 1$) in the expression of the metric equation (3.4) makes the physics of these objects more subtle. These black holes are, in fact, static only when viewed by a Rindler-like frame. Asymptotically to this frame one can associate an acceleration, with respect to an inertial (Minkowsky) frame,

$$a_M = \lambda (1-n). \quad (5.1)$$

So one can think of our black holes as being accelerated with this acceleration.

Focusing our attention on semiclassical effects, we looked for configurations which might represent stable ground states for black-hole evaporation. To this end we searched for static, finite-mass black-hole solutions with regular event horizon and vanishing quantum radiation at infinity. These configurations (gravitational kinks) do indeed exist provided the surface gravity $k_+$ of the event horizon equals the above acceleration $a_M$,

$$k_+ = a_M. \quad (5.2)$$

This relation constraints the mass and charge of the kink to fulfil the following equation:

$$Q^2 = \frac{4n}{(1+n)^2} m_0^2. \quad (5.3)$$

From equation (5.2) we see that the surface gravity of the event horizon of these black holes is, in general, nonvanishing (except for $n = 1$), i.e. the event horizon is not degenerate. Furthermore, from equation (5.3) we see that the mass of these states can be made arbitrarily large compared to $Q$ by lowering $n$ towards zero. Note that equation (5.3) requires $m_0 > |Q|$ and therefore our extremal solution $m_0 = |Q|$ is not a kink.

These characterizing features distinguish substantially our gravitational kinks from the extreme Reissner–Nordström black hole. This latter mass and charge are related by
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$m_0 = |Q|$. Furthermore, the horizon surface gravity vanishes (degenerate horizon), which causes the quantum stress tensor diverge on the degenerate horizon. This divergence, however, has no effect on the spacetime geometry once the backreaction is properly taken into account.

How do we understand the relation (5.2) which leads to the gravitational kink configurations described by (5.3)? We think the situation presented here resembles the discussion of Yi [17] on the quantum stability of accelerated black holes based on the Ernst metric. This metric represents two opposite magnetically charged Reissner–Nordström black holes uniformly accelerated away from each other, where the driving force is an external magnetic field. According to [17] stability under thermal evaporation is obtained by imposing the acceleration temperature (associated with the acceleration horizon of the Ernst metric) to be equal to the Hawking temperature $T_H$ (associated to the event horizon). Consequently, it is shown that the relation between regular coordinates at the event and at the acceleration horizons is of the type $x^± = -1/y^±$, making the Bogolubov transformation between the two bases trivial. This result has, however, been criticized by Massar and Parentani [18], who emphasize the crucial role played by the presence of a second black hole. According to the above authors, the decoherence of the two black holes, the independent spread of their masses around the mean and any breaking of the exact boost invariance of the Ernst metric will lead inevitably to the emission of a steady flux of particles by the holes. Returning to our work, some fundamental differences should be stressed. In our construction there is just one black hole and no acceleration horizon is present; the latter is replaced by null infinity. Furthermore, inertial coordinates there ($y^±$) and regular coordinates on the event horizon ($x^±$) simply coincide: $x^± = y^±$.

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Appendix A. Classical solutions

In this appendix we solve the equations of motion deriving from the action $S = S_a + S_{EM} + S_M$, where $S_a$ is given in (3.1), $S_{EM}$ in (3.7) and $S_M$ is some matter source. We will consider only static frames and a metric of the form

$$ds^2 = e^{2(1-n)\lambda_σ} \left(-f(σ)\, dt^2 + \frac{dσ^2}{f(σ)}\right),$$

where $f(σ)$ is an arbitrary function of its argument to be determined by the field equations. The other unknown function is the dilaton. Following [19], a useful parametrization of $g_{μν}$ and $φ$ is given by the following equations:

$$\frac{e^{-φ/2n}}{λ} = \int dσ e^{2(1-n)\lambda_σ - λ_σ(σ)}, \quad f(σ) = e^{2λ_σ(σ) - 2(1-n)λ_σ} h(σ),$$

where

$$h(σ) = e^{-2φ} \left(1 - \frac{2m(σ)}{λ} e^{2φ/2n} + \frac{Q^2}{λ^2} e^{4φ/2n}\right).$$

† The same relation exists in the case of an extremal Reissner–Nordström black hole between the Kruskal coordinates regular at the horizon and the asymptotic inertial ones.
In terms of the two functions $g(\sigma)$ and $m(\sigma)$ the equations (3.2) can be rewritten in the form

$$\partial_\sigma m = -\frac{T_t^t}{2} e^{(2(1-n)\phi + 2(1-n)\lambda)\sigma - \lambda g(\sigma)}$$  \hspace{1cm} \text{(A.4)}$$

and

$$\partial_\sigma g = (1 - 2n) + \frac{(T^\sigma_{\sigma} - T^t_t)}{2\lambda^2 \hbar(\sigma)} e^{2(1-n)\lambda\sigma - \lambda g(\sigma) + \phi/n},$$  \hspace{1cm} \text{(A.5)}$$

where $T_{ab}$ is the stress tensor associated with the source $S_M$.

In the vacuum, $T_{ab} = 0$, we get simply

$$m = m_0, \quad g(\sigma) = (1 - 2n)\sigma$$  \hspace{1cm} \text{(A.6)}$$

and therefore the solution of the equations of motion is given by

$$f_{cl} = 1 - \frac{2m_0}{\lambda} e^{-2\lambda\sigma} + \frac{Q^2}{\lambda^2} e^{-4\lambda\sigma}, \quad \phi = -n\lambda\sigma.$$  \hspace{1cm} \text{(A.7)}$$

### Appendix B. Quantum-corrected solutions

We now turn to the problem of calculating the quantum corrections to the solution given in (A.7). The difference with respect to the classical case is that in (A.4) and (A.5) the quantum energy–momentum $(T_{ab})$ must be inserted as a source. We write down the following decomposition:

$$m = m_0 + m_q(\sigma), \quad g(\sigma) = (1 - 2n)\sigma + g_q(\sigma)$$  \hspace{1cm} \text{(B.1)}$$

where $m_q$ and $g_q$ are of order $\hbar$. According to (A.4) and (A.5) we get, to the same order,

$$\partial_\sigma m_q(\sigma) = \frac{\langle T_{tt} \rangle}{2 f_{cl}(\sigma)}$$  \hspace{1cm} \text{(B.2)}$$

and

$$\partial_\sigma g_q(\sigma) = \frac{e^{-2\lambda\sigma}}{2\lambda^2 f_{cl}^2(\sigma)} \left( f_{cl}^2(\sigma) \langle T_{\sigma\sigma} \rangle + \langle T_{tt} \rangle \right).$$  \hspace{1cm} \text{(B.3)}$$

where $\langle T_{ab} \rangle$ are calculated on the classical background metric.

Let us consider first (B.2). The source is

$$\langle T_{tt} \rangle = 4(\langle T_{vv} \rangle + \langle T_{uu} \rangle + 2\langle T_{uv} \rangle),$$  \hspace{1cm} \text{(B.4)}$$

where (see equations (4.4) and (4.5) and hereafter $f \equiv f_{cl}$)

$$\langle T_{uu} \rangle = \langle T_{vv} \rangle = \frac{N\hbar}{48\pi} \left[ (1 - n)^2 \lambda^2 (1 - f^2) - \frac{1}{4} f^2_\sigma + \frac{1}{2} f f_\sigma \sigma \right]$$  \hspace{1cm} \text{(B.5)}$$

and

$$\langle T_{uv} \rangle = \frac{N\hbar}{48\pi} \left[ \frac{1}{2} f f_\sigma + \lambda (1 - n) f f_\sigma \right].$$  \hspace{1cm} \text{(B.6)}$$

Inserting it we obtain

$$\partial_\sigma m_q(\sigma) = \frac{\hbar}{12\pi} \left[ f_{\sigma\sigma} + \lambda (1 - n) f_{\sigma} + \frac{(1 - n)^2 \lambda^2 (1 - f^2) - \frac{1}{4} f^2_\sigma}{f} \right].$$  \hspace{1cm} \text{(B.7)}$$
and after some algebra we finally find

\[ m(\sigma) = m_0 + \frac{hN}{6\pi} \left\{ m_0 \left[ (1+n) - \frac{(1-n)^2}{2} \right] e^{-2\lambda \sigma} + \frac{m_0^2 n(n-7)}{2\lambda (1+n)} e^{-4\lambda \sigma} \right. \]

\[ + \left. \frac{(1+n)(1-n)^2}{4n} \ln \left[ \frac{e^{-2\lambda \sigma} - (1+n)\lambda/2nm_0}{(1+n)\lambda/2nm_0} \right] \right\}. \quad (B.8) \]

where the kink relation \( Q^2 = (4n/(1+n)^2)m_0^2 \) was used.

For the calculation of \( g_q(\sigma) \) we proceed similarly and from (B.3) we get

\[ \partial_\sigma g_q(\sigma) = \frac{4e^{-2\lambda \sigma} \langle T_{uu} \rangle + \langle T_{vv} \rangle}{\lambda^2 f^2} = \frac{hN e^{-2\lambda \sigma} (1-n)^2\lambda^2(1-f^2) - \frac{1}{2}f_{,\sigma}^2 + \frac{1}{2}ff_{,\sigma\sigma}}{f^2}. \quad (B.9) \]

We obtain

\[ g(\sigma) = (1-2n)\sigma + \frac{hN}{6\pi} \left\{ \frac{(n^2-2n-3)}{2\lambda} e^{-2\lambda \sigma} \right. \]

\[ + \left. \frac{1}{m_0} \frac{(1+n)^2(n-1)}{4n} e^{-2\lambda \sigma} \left( e^{-2\lambda \sigma} - (1+n)\lambda/2nm_0 \right) \right\} \]

\[ - \frac{1}{m_0} \frac{(1+n)^2}{2n} \ln \left[ \frac{e^{-2\lambda \sigma} - (1+n)\lambda/2nm_0}{(1+n)\lambda/2nm_0} \right]. \quad (B.10) \]

We can use these results to determine the quantum-corrected radius of the event horizon, which is located at \( \sigma = \bar{\sigma}^+ \) where

\[ e^{-2\phi(\bar{\sigma}^+)/n} = \frac{2m(\bar{\sigma}^+)}{\lambda(1+n)}, \quad (B.11) \]

that is,

\[ e^{2\lambda \bar{\sigma}^+} = \frac{2m_0}{\lambda(1+n)} + \frac{hN}{6\pi} \left\{ \frac{(1+n)(-5n+14)}{4} + \frac{(n^2-6n-3)}{2n} \ln(1-n) \right. \]

\[ + \left. \frac{(1+n)(5-n)}{2\sqrt{n}} \frac{1 - \sqrt{n}}{1 + \sqrt{n}} \right\}. \quad (B.12) \]

Finally, note that close to the Cauchy horizon \( \sigma \sim \sigma^- \) the perturbative terms \( \sim O(h) \) are huge and diverge at \( \sigma^- \). This signals that in this region our perturbative approach breaks down.

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