Calculation of Detailed Component Maps Combining SCHA and Digital Filtering

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The aim of this study is to analyse the integration of sparse three component magnetic observations from observatories and repeat stations with dense total field values from aeromagnetic data, into a homogeneous description of the magnetic field.

The methodology followed implies the independent computation of main and crustal field contributions, each one incorporating the constraints of harmonicity. The crustal anomalous field is calculated for 1980 by digital filtering of the total field values from aeromagnetic surveys; the convolution is made in the spatial domain with space varying coefficient sets to account for the change in direction of the main field. The main field is estimated by updating the DGRF80, whose agreement with the Iberian data has already been demonstrated, with a SCHA model for the secular variation deduced from 360 three component observations over the Iberian Peninsula for the period 1980–1991.

The combination of these two contributions permits one to compute magnetic component maps for the 1991.0 epoch that agree, in the least-squares sense, with the observational data while respecting the mathematical characteristics of a potential field. This methodology seems to be an effective way to produce regional estimations of the internal field.

1. Introduction

The combination of three component magnetic data obtained from ground stations with high resolution aeromagnetic surveys to produce good quality three component maps was used by Le Mouël (1969) for the French territory in 1969 and by Galdeano et al. (1980), to update the three component magnetic maps of France for the 1980.0 epoch.

The approach followed by Le Mouël (1969) was to consider that the magnetic field as observed either in the three components at ground stations or in the total field surveys could be considered as the sum of two different contributions, one corresponding to the main field and the other to the influence of the magnetic crust. A good local approximation to the main field was achieved by the sum of a ‘world field’ (at that time the IGRF65) plus a correction term that should be added in order to fit, as well as possible, the observations made by the French magnetic repeat network. This term was thought to be described by a low-order polynomial of latitude and longitude subjected to two constraints: the amplitude of the vector field recovered should fit the total field aeromagnetic survey data (at its reference epoch) and the vector field model should behave like the gradient of a scalar harmonic potential. The crustal field was calculated from the aeromagnetic survey of France, using the known relationships between the total field and each magnetic component spectrum (Le Mouël, 1969).

The approach we take here is slightly different. The DGRF80 model has been seen to be a good estimation of the main field for the 1980.0 epoch over the Iberian Peninsula (Miranda et al., 1989, Socias, 1994) and so, the problem of calculating a good regional estimation of the main field is reduced to the calculation of a secular-variation (SV) model, valid from 1980 onward.
A derivative-fit method was followed, by numerically differentiating the main field data (first differences of annual values divided by the respective time intervals), and then fitting a Spherical Cap Harmonic Analysis (SCHA) model directly to the resulting SV experimental data. This has the advantage of removing crustal contamination. The SV model thus obtained is a continuous function of time, instead of a series of step functions as occurs with the IGRF SV models for the same periods. The integration of the estimated SV model between 1980.0 and a later epoch, gives the variation in the main-field between those two epochs. The input information we use is given by the ground magnetic Portuguese and Spanish stations and the existing magnetic observatories. Finally, the results obtained can be compared with the global estimates (i.e. DGRF and similar models) and with the observatory data.

The methodology followed in the calculation of the anomalous crustal field is based on digital filtering techniques in the space domain (Galdeano et al., 1980) and we take account of the dependence of the operators on the direction of the main field. This methodology can easily be implemented to process large data sets, where the variation of the main field direction is very significant and must be considered in order to have a good estimation of the filter coefficients.

2. Determination of the Crustal Field

The possibility of calculating the components of the field anomaly ($F_A$) from the total field anomaly is a direct consequence of the fact that $F_A$ is a harmonic function and, so, since the total field magnetic anomaly is given approximately by the projection of $F_A$ in the direction of the main or normal field ($F_N$) we can recover the corresponding magnetic potential by integrating the total field anomaly in the direction of $F_N$; the three components of the magnetic field are then simply the derivatives of the scalar potential in the direction of each coordinate.

The determination of the integral and the derivatives is particularly easy in the Fourier domain in the case where the survey area completely contains the crustal anomalies under consideration. In this case, we can consider that the component filters in the spatial domain are the inverse Fourier transforms of the following operators (Galdeano et al., 1980):

$$
G(r, s)_x = \frac{u(\alpha u + \beta v) + i(u\gamma\sqrt{u^2 + v^2})}{\gamma^2(u^2 + v^2) + (\alpha u + \beta v)^2}
$$

$$
G(r, s)_y = \frac{v(\alpha u + \beta v) + i(v\gamma\sqrt{u^2 + v^2})}{\gamma^2(u^2 + v^2) + (\alpha u + \beta v)^2}
$$

$$
G(r, s)_z = \frac{-\gamma(\alpha u + \beta v) + i(\alpha\gamma\sqrt{u^2 + v^2})}{\gamma^2(u^2 + v^2) + (\alpha u + \beta v)^2}
$$

$$
G(0, 0)_x = \frac{\alpha}{1 + \gamma}, \quad G(0, 0)_y = \frac{\beta}{1 + \gamma}, \quad G(0, 0)_z = 1,
$$

for the $X$, $Y$ and $Z$ components, respectively, and where ($\alpha, \beta, \gamma$) are the direction cosines of the main field, $u = 2\pi r/M$ and $v = 2\pi s/N$ are the coordinates in the Fourier domain, and $(M \times N)$ are the dimensions of the filter in the space domain.

As this expression changes with the direction of the main field, the filter coefficients change through the study area. In this situation we are unable to determine the magnetic components in the Fourier domain. However, if we work in the spatial domain, a set of coefficients for each location can be obtained if we divide the area into small size windows, calculate the set of coefficients as in Eq. (1) and the corresponding $\alpha$, $\beta$ and $\gamma$ derivatives for its central point, and reconstruct for each individual point the component filter using a first order Taylor approximation.

There are only five different independent derivative filters, with the following expressions in
the Fourier domain:

\[ G(r, s)^x = \frac{u^2(\gamma^2(u^2 + v^2) - (\alpha u + \beta v)^2)}{(\gamma^2(u^2 + v^2) + (\alpha u + \beta v)^2)^2} - i \frac{2u\gamma \sqrt{u^2 + v^2} (\alpha u + \beta v)}{(\gamma^2(u^2 + v^2) + (\alpha u + \beta v)^2)^2} \]

\[ G(r, s)^y = \frac{u^2(\gamma^2(u^2 + v^2) - (\alpha u + \beta v)^2)}{(\gamma^2(u^2 + v^2) + (\alpha u + \beta v)^2)^2} - i \frac{2u\gamma \sqrt{u^2 + v^2} (\alpha u + \beta v)}{(\gamma^2(u^2 + v^2) + (\alpha u + \beta v)^2)^2} \]

\[ G(r, s)^\alpha = G(r, s)^x, \quad G(r, s)^\beta = G(r, s)^y \]

The error that is made when using this approach can be estimated in the following way: consider different window sizes for the calculation of the Taylor development described above; when the size of the window is of the same order of magnitude as the distance between data points there is really no Taylor expansion to be made; on the contrary, if the window size increases, the error in the use of a linear approximation grows.

To quantify the relationship between the window size and the errors due to the first order approximation used in the calculation of the filters, we present in Fig. 1 a plot of the maximum and minimum errors (in each component) versus the window size, in degrees, for the Portuguese territory. We consider as “true” the result that is obtained when 0.1 degree windows are used, which corresponds to the mean distance between data points.

To get an idea of the relative importance of the misfit, we must consider that the X component varies between -225.0 nT and 169.8 nT, the Y component between -108.4 nT and 130.4 nT, and the Z component between -306.1 nT and 254.5 nT. The analysis of the results shows that if

![Fig. 1. Plot of the maximum and minimum errors in the computation of the crustal field as a function of the window size.](image-url)
we use windows smaller than 2 degrees all the errors are within the accuracy of the measurements.

3. Determination of the Main Field

According to the analysis made of the total field aeromagnetic data for the Portuguese mainland (Miranda et al., 1989) the DGRF80 is a good description for the 1980.0 epoch. Over Spain (Socias, 1994) it was also used as the reference field of the national 1987 aeromagnetic survey. However, after 1980 it is doubtful whether the available global reference fields can be used as a good description of the main field for the Iberian Peninsula. The recent compilations of magnetic data around Iberia (Silva et al., 1995) once again emphasized the misfits obtained when data sets with 20 to 30 years of observations were merged using the SV models taken from the DGRF’s.

We decided to use the experimental data obtained at several stations in the Iberian Peninsula during the period 1980–1991, to update the DGRF80, and, since our data are limited to a restricted area, we adopted the SCHA method proposed by Haines (1985a, 1985b, 1988). In this way we make sure that the main-field model corresponds to the gradient of a differentiable potential field, and that the basis functions are orthogonal in the domain of the data. This has the advantage of providing better accuracy in the process of least-squares inversion.

3.1 The SCHA method

The analytical truncated solution for internal sources of the Laplace equation in spherical coordinates, in a spherical cap of half angle \( \theta_0 \), subject to the following boundary conditions on \( \theta \)

\[
V(r, \theta_0, \phi) = f(r, \phi),
\]

\[
\frac{\partial V(r, \theta_0, \phi)}{\partial \theta} \bigg|_{\theta=\theta_0} = g(r, \phi),
\]

(where \( f(r, \phi) \) and \( g(r, \phi) \) are arbitrary functions), is given by

\[
V(r, \theta, \phi, t) = \sum_{k=0}^{K_{\text{MAX}}} \sum_{m=0}^{k} \left( \frac{a}{r} \right)^{n_k(m)+1} P_{n_k(m)}^m(\cos \theta) \{ g_k^m(t) \cos m\phi + h_k^m(t) \sin m\phi \}.
\] (2)

In these equations, \( (r, \theta, \phi) \) are the usual spherical coordinates (with the z-axis along the spherical cap axis), and \( a \) is the Earth’s surface radius. \( P_{n_k(m)}^m(\cos \theta) \) are the associated Legendre functions for which (Haines, 1985a)

\[
\frac{dP_n^m(\cos \theta_0)}{d\theta} = 0
\] (3a)

or

\[
P_n^m(\cos \theta_0) = 0.\] (3b)

The \( n \) roots of these equations are real but not necessarily integer, and are never simultaneous solutions of both equations. \( k, \) with \( k \geq m, \) is an integer subscript that was introduced to distinguish between the roots of Eqs. (3a) and (3b) at a given \( m, \) and to order them. The values of \( n_k(m) \) for which \( k - m \) is even, are the roots of Eq. (3a) and are ordered as the 0, 2nd, 4th, ..., roots, and those for which \( k - m \) is odd, are the roots of Eq. (3b) and are ordered as the 1st, 3rd, 5th, ... roots.

Finally, \( g_k^m(t) \) and \( h_k^m(t) \) are the harmonic coefficients of the expansion, sought as polynomial functions of time:

\[
g_k^m(t) = \sum_{q=0}^{Q_{\text{MAX}}} g_{k,q}^m t^q,
\]
This choice of basis functions makes it possible to represent the gradient components of the potential as the sum of term-wise derivatives of Eq. (2). We then adopted the SCHA method, as implemented by Haines (1988), to model simultaneously the $X$ (north), $Y$ (east) and $Z$ (vertical) components of the SV field (Haines, 1985b).

3.2 The data used and the parameters chosen

4 observatories and 39 repeat stations in the Iberian Peninsula were considered, with their locations indicated in Fig. 2. The input data consisted of the first differences of the annual mean values of the magnetic components, divided by their respective time intervals and then assigned to an intermediate epoch, for the period 1980–1991. A total number of 360 ($X, Y, Z$) data points were used. The data were equally weighted in the least-squares fitting procedure.

We placed the cap’s pole in the middle of the Peninsula, at a point with coordinates (40°N, 5°W). A fourth order spatial expansion was made ($K_{\text{MAX}} = 4$). This is considered to be a good choice when modelling the SV field, which has a relatively long characteristic wavelength (Haines, 1985b; García et al., 1991). The time variation assumed for the SV model coefficients was linear.

Fig. 2. Location of observatories (stars) and repeat stations (solid circles) whose data were used in determining the SCHA SV model. Also shown is the boundary of the 5° spherical cap.
(\(Q_{\text{MAX}} = 1\)), as there are no ‘jerks’ of the main field identified in this period (Alexandrescu et al., 1995), and the chosen reference epoch was 1986.0 (so \(t = \text{year} - 1986.0\) in Eq. (4)).

One of the problems in the SCHA calculation deals with the choice of the cap half angle, \(\theta_0\). Though the data we have is confined to a region that corresponds approximately to a spherical cap of half angle 5° around the defined pole (Fig. 2), we had to consider a greater cap half angle in the modelling. This was done to avoid boundary instabilities observed when considering small values of \(\theta_0\), due in part to the bad coverage of the low degree spatial harmonic content, as pointed out by Torta et al. (1992). We analyzed the following three aspects for the choice of \(\theta_0\): the standard error of the estimate of the spherical cap harmonic fit; the spatial power spectra obtained; and the similarity between the shapes of the charts generated by the SCHA and by the IGRF models.

In what concerns the standard error of the estimate (square root of the residual sum of squares divided by the number of degrees of freedom) of the SCHA models, we can see that for values of \(\theta_0\) greater than 12° the differences are not physically significant, with small oscillations around 17.05 nT/year that persist to higher values of \(\theta_0\) (Fig. 3).

Referring to the second point, we computed the sub-periodic spatial power spectra of the potential field (Haines, 1991), with the total spatial power for degree \(n\) given by:

\[
S_n = \frac{a^2}{(2n + 1) \sin^2(\theta_0/2)} \sum_{m=0}^{n} [(\hat{g}_{m}^n)^2 + (\hat{h}_{m}^n)^2].
\]  

(5)

We denote by \(\hat{g}_m^n\) and \(\hat{h}_m^n\) the coefficients of a global potential that is given by Eq. (2) when \(\theta \leq \theta_0\), and that is zero when \(\theta > \theta_0\). These coefficients are computed from the spherical cap

![Graph showing standard error of estimate and number of significant coefficients obtained when fitting a SCHA model to the SV data, for different values of the parameter \(\theta_0\). For all models, a fourth-order spatial expansion and a first-order time expansion were considered.](image-url)
harmonic coefficients $g^m_k$ and $h^m_k$ in Eq. (2) following Haines (1991):

$$\begin{align*}
\left\{ g^m_k \right\} &= \frac{\delta_m (2n + 1) \sin \theta_0}{4} \\
\left\{ h^m_k \right\} &= \times \left[ dP^m_n (\cos \theta_0) \frac{\delta_m}{\delta \theta} \sum_{k \equiv m \text{ even}}^\infty \frac{g^m_k}{h^m_k} \frac{P^m_{nk}(m)}{(nk(m) - n)(nk(m) + n + 1)} \\
&\quad - dP^m_n (\cos \theta_0) \frac{\delta_m}{\delta \theta} \sum_{k \equiv m + 1 \text{ odd}}^\infty \frac{g^m_k}{h^m_k} \frac{P^m_{nk}(m)}{(nk(m) - n)(nk(m) + n + 1)} \right]
\end{align*}$$

where

$$\delta_m = \begin{cases} 
2 & m = 0 \\
1 & m > 0
\end{cases}.$$

The sample power spectra (divided by $a^2$) for both the secular variation potential (SVP) for the reference epoch 1986.0 (using the zero-order terms of the time polynomial Eq. (4)), and for the secular acceleration potential (SAP) (using the first-order terms in Eq. (4)), were obtained in this way. They are plotted together as a function of the parameter $\theta_0$ in Fig. 4.

Starting with $\theta_0 = 12^\circ$, we can see that there is a sudden jump in the lower degree spatial power content of the SVP around $\theta_0 = 18^\circ$. This spectrum shows a maximum in $n = 4$. The relative distribution of the power content is however different for the SAP spectrum, with a maximum around $n = 20$. From $\theta_0 = 18^\circ$ to $\theta_0 = 28^\circ$ the shapes of the two spectra become similar and the maximum values approach each other up to the point where they practically coincide. The charts for these models still show some important irregularities and spurious features in some epochs, mainly in the $X$ and $Y$ components. For higher values of $\theta_0$, the two spectra maintain the same shape and relative position, although the intensity of the power content increases progressively. We conclude that for $\theta_0 < 28^\circ$ the shape of the SVP spectrum will be distorted when moving away from 1986.0, with a power increase in the higher degrees. For $\theta_0 \geq 28^\circ$, however, the shape of the SVP spatial power spectrum will be virtually the same during all the time interval of the model, and the required low degree representativeness condition is fulfilled. This is coincident with an observed stability in the charts of the model from $\theta_0 = 28^\circ$ onward, probably related to the improvement in the rate of convergence, which depends on the spectral decay of the field.

An advantage of the SCHA method over a SHA model, is that it enables a smaller scale homogeneous definition of the harmonic field in regions where we have a greater density of data, keeping a small number of coefficients for the model. Choosing too high a $\theta_0$ value does not take full advantage of the method, since we approach the SHA conditions ($\theta_0 = 90^\circ$), losing the definition of smaller scale features. For all these reasons we opted for the value $\theta_0 = 28^\circ$.

There are 50 $((K_{\text{MAX}} + 1)^2(Q_{\text{MAX}} + 1))$ coefficients present, in theory, in the expansion of the $X$, $Y$ and $Z$ components of the SV field. From this number, a selection of the statistically significant coefficients was made according to the stepwise regression procedure adopted by Haines (1985b), at an $F$-level of 4. The final number of coefficients of our model is 9, corresponding to a standard error of estimate of 17.052 nT/year.

### 3.3 Results for the SV field

Figure 5 shows the charts obtained from the SCHA SV model, for 1982.5 and 1987.5. These are the intermediate epochs of the five year intervals 1980–1985 and 1985–1990, for which the IGRF-DGRF (Langel, 1992) secular variation values are closest to the real annual variation. The DGRF80 and DGRF85 isopors in $X$, $Y$ and $Z$ (obtained respectively from (DGRF85 –}
Fig. 4. Sub-periodic spatial power spectra divided by $a^2$ for SCHA models, obtained for different values of $\theta_0$. Solid circles correspond to $S_n$ values determined using the constant coefficients; open circles correspond to $S_n$ values determined using the first-order coefficients in time. Units are $\text{nT}^2\text{yr}^{-2}$ for the secular variation spectra and $\text{nT}^2\text{yr}^{-4}$ for the secular acceleration spectra.

DGRF80)/5 and (IGRF90 - DGRF85)/5) are thus also plotted, and the charts can be directly compared with the corresponding SCHA ones.

Except for the $Y$-component in 1982.5, the SCHA and DGRF isopors display a very similar behavior. There is however a translational shift of corresponding isopors in the two models, especially in the $X$ and $Y$ components for the 1987.5 epoch.
Fig. 5. Comparison between SCHA (with $\theta_0 = 28^\circ$) and DGRF SV models, for 1982.5 and 1987.5 epochs. Units are nT/year.
3.4 Results for the main or normal field

The SCHA SV model was then integrated from 1980.0 onward, taking as the integration constant the DGRF80 main field model. This was accomplished with the same computer program (Haines, 1988), and a quadratic time-dependent SCHA main field model was obtained for the Iberian Peninsula. Figure 6 shows charts of the total variation of each component between the reference epoch 1980.0 and 1991.0, for the SCHA and the DGRF models.

We next proceeded to find out how these differences reflected on the prediction of the time evolution of the field at a given station. In Fig. 7 the two integrated models are displayed, as well as the observed annual mean values, for two observatories in the Peninsula: Coimbra (Portugal) and San Pablo de los Montes (Spain). The two curves and the set of data points are all subtracted from the DGRF80 main field value of the corresponding component for the specific station. For the X-component, the time variation of the SCHA model corresponds better to the time variation of the data. For the Y-component, the two models display a similar time variation, which is in accordance with the data variation. The SCHA model shows however a slower growth. With respect to the Z-component, the SCHA model time variation seems to be closer to the observed time variation.

For all plots, we calculated the shift $\Delta$ that should be added to each model curve $f(x)$, in order to minimize the sum of the squares of the deviations of the $N$ data points $y_i$, relative to the shifted curve. $\Delta$ is such that

$$\frac{d}{d\Delta} \sum_{i=1}^{N} [(f(x_i) + \Delta) - y_i]^2$$

Fig. 6. Comparison between the total variation of the main-field components for the period 1980.0–1991.0, as given by the SCHA ($\theta_0 = 28^\circ$) and the IGRF/DGRF models. Units are nT.
is a zero, which leads to

$$\Delta = \frac{\sum_{i=1}^{N} (y_i - f(x_i))}{N}.$$  

The calculated shift values in the X, Y and Z components can be accounted for by local crustal anomalies and regional fields which are not considered in main-field models.

Finally, we calculated the scatter ($\sigma = \sqrt{\sum_{i=1}^{N} [(f(x_i) + \Delta) - y_i]^2/(N - 1)}$) of the data about the shifted curves. The shift and scatter values are shown in Table 1. The first analysis seems to be supported by these results, and we conclude that the SCHA model is clearly a better choice for the X-component at both observatories, and for the Z-component in Coimbra. The DGRF model shows slightly better results when considering the Y-component.
Table 1. $\Delta_{\text{SCHA}}$ and $\Delta_{\text{DGRF}}$ are the values to add to each model curve, in order to minimize the squares of deviations of the data; $\sigma_{\text{SCHA}}$ and $\sigma_{\text{DGRF}}$ are the r.m.s. deviations of the data, relative to the shifted curves.

|          | Coimbra |         |         |          |         | San Pablo |         |         |
|----------|---------|---------|---------|----------|---------|-----------|---------|---------|
|          | $X$     | $Y$     | $Z$     | $X$      | $Y$     | $Z$       | $X$     | $Y$     | $Z$     |
| $\Delta_{\text{SCHA}}$ | 23.67   | -2.12   | 19.91   | 3.00     | 69.51   | -39.53    |         |         |         |
| $\sigma_{\text{SCHA}}$  | 7.97    | 13.66   | 12.04   | 8.15     | 5.99    | 5.35      |         |         |         |
| $\Delta_{\text{DGRF}}$  | 18.73   | -16.39  | 4.97    | 5.93     | 59.59   | -40.27    |         |         |         |
| $\sigma_{\text{DGRF}}$  | 15.96   | 11.87   | 18.21   | 14.91    | 4.00    | 4.56      |         |         |         |

4. Determination of the Component Maps

The component maps can be easily computed by adding the main field and the anomalous field components. An example, concerning the declination over the Iberian Peninsula, is shown in Fig. 8. If we compare the result obtained with the main field models presented before, it is clear that the crustal field is rather important, even for the computation of the declination and inclination maps.

5. Conclusions

The evaluation of the SV of the main field is a main goal of the magnetic repeat station network; however, at least in what concerns the studied data sets, this kind of data is too noisy and does not reflect the expected wavenumber content of the SV field. Only the integration of a significant number of magnetic station data and the use of a strict mathematical model for the representation of the field can assure its usefulness in the computation of regional representations of the magnetic field. This conclusion is emphasized by the comparison between regional SCHA models and “continuous” observatory data, where it shows a good agreement, and by the comparison with individual stations, where the misfits are rather larger.

The misfit between the models and the measurements at individual magnetic stations is generated by the superposition of several effects. First, the observations made at the repeat stations incorporate both the main field, crustal components and local effects. Second, the methods followed in the computation of annual means by comparison between a small number of measurements and a “continuous” magnetic observatory lead to unrealistic evaluations of the secular variation. In what concerns the Portuguese network, this effect is amplified by the perturbations at the Coimbra magnetic observatory (Pais and Miranda, 1995). However, as this last problem can not be very important in the Spanish network, and the observed misfit is clear everywhere over the Iberian Peninsula, the problem of measuring the regional coherency of the SV by the repeat station network remains.

Only a dense network of continuous remote magnetic stations can accurately measure the secular variation of the main field at the Earth Surface. However, the use of SCHA techniques for the derivation of a space-time model for the main field based on both observatory and magnetic repeat stations, can ensure a good approach, as the final representation agrees well where the data are well known (observatories) and displays a regional variability compatible with the expected wavenumber content of the main field.

The main problem always remains of how to fix the cap half-angle that must be used in the SCHA representation. We think that the spectrum stability criterium described above can be a valuable tool in fixing the size of the cap.

Recently, Haines and Newitt (1995) introduced several innovations in the Haines algorithm for SCHA model calculation. These include the variation of the degree of the temporal polynomial...
Fig. 8. Charts of declination of the main-field SCHA model (top), and of the main plus crustal field (bottom) for epoch 1991.0 and altitude 3000 m. Declination is in degrees.
depending on the spherical cap harmonic spatial degree $k$, and the simultaneous use of scalar as well as vector data. It may be interesting in the future to analyse the effect of these improvements on the model that has been calculated for the Iberian Peninsula.

The method described for the computation of the crustal field, by the use of spatially varying digital filters, seems to be very effective, and the errors are very small if windows of lesser than 1 degree are considered for the calculation of the filter coefficients.

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