Suppressing cluster cooling flows by self-regulated heating from spatially distributed population of AGNs

Adi Nusser¹, Joseph Silk² and Arif Babul³

¹Physics Department- Technion, Haifa 32000, Israel
²Astrophysics, Oxford University, Keble Road, Oxford OX1 3HR, UK
³Department of Physics and Astronomy, University of Victoria, Victoria, BC V8P 5C2, Canada

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ABSTRACT
Existing models invoking AGN activity to resolve the cooling flow conundrum in galaxy clusters focus exclusively on the role of the central galaxy. Such models require fine-tuning of highly uncertain microscopic transport properties to distribute the thermal flow over the entire cluster cooling core. We propose that the ICM is instead heated by multiple, spatially distributed AGNs. The central regions of galaxy clusters are rich in spheroidal systems, all of which are thought to host black holes and could participate in the heating of the ICM via AGN activity of varying strenghts.

Active AGNs drive bubbles into the ICM. We identify three distinct interactions between the bubble and the ICM: (1) Upon injection, the bubbles expand rapidly in situ to reach pressure equilibrium with their surroundings, generating shocks and waves whose dissipation is the principal source of ICM heating. (2) Once inflated, the bubbles rise buoyantly at rate determined by a balance with the viscous drag force, which results in some additional heating. (3) Rising bubbles expand and compress their surroundings. This process is adiabatic and does not contribute to any additional heating; rather, the increased ICM density due to compression enhances cooling.

Our model sidesteps the “transport” issue by relying on the spatially distributed galaxies to heat the cluster core. We include self-regulation in our model by linking AGN activity to cooling characteristics of the surrounding ICM. We use a spherically symmetric one-dimensional hydrodynamical code to carry out a preliminary study illustrating the efficacy of the model. Our self-regulating scenario predicts that there should be enhanced AGN activity of galaxies inside the cooling regions compared to galaxies in the outer parts of the cluster. This prediction remains to be confirmed or refuted by observations.

Key words: cosmology: theory, observation, dark matter, large-scale structure of the Universe — gravitation

1 INTRODUCTION
The observed properties of the x-ray emitting gas in clusters of galaxies present several challenging puzzles for which satisfactory explanations are yet to be found. The absence of cold gas (with temperatures below \( \sim 1 \) keV) is one of those puzzles (e.g. Peterson et. al. 2001). Because of radiative losses, one would expect the temperature profiles to fall below a keV or so in the central regions where cooling is most efficient. Yet in none of the observed clusters does the temperature drop to the level dictated by cooling alone. Hence efficient heating mechanisms must operate without fail at the cores of all cooling clusters.

The most popular mechanism for suppressing cooling is energy released by an AGN harboured by the central cluster galaxy (cf. Quilis et. al. 2001, Babul et. al. 2002, Kaiser & Binney 2003, Dalla Vecchia et. al. 2004, Omma et. al. 200, Roychowdhury et. al. 2004, Ruszkowski et. al. 2004, Voit & Donahue 2005, Brüggen et. al. 2005, Nipoti & Binney 2005, Roychowdhury et. al. 2005, Sijacki & Springel 2006, and references therein). Relativistic ejecta from the AGN
transform into hot bubbles that eventually reach pressure equilibrium with the ICM and proceed to rise buoyantly away from the center. These bubbles are believed to be one of the means with which the AGN can heat the ICM. For example, mechanical activity associated with the inflation of the bubbles near the center can generate internal gravity waves (ie. disturbances that are subject to a restoring force of gravity or buoyancy) (Omna et al. 2004) and/or sound waves which are believed to eventually dissipate their energy in the ICM. To balance cooling in a cluster of x-ray luminosity of $L_x \sim 10^{44}$erg s$^{-1}$, a central AGN must produce $\sim 10^{46}$erg over the entire life-time of the cluster. This is at the upper limit of the observed range of AGN energy output in galaxy clusters, based on the $pV$ content of x-ray cavities (e.g. Birzan et al. 2004). Since the $pV$ energy content must be a lower estimate of the AGN energy output (see below), it is reasonable to assume that cooling is quenched by AGN feedback. The challenge, however, is to arrange for efficient energy transport from the AGN over the entire cooling core, or out to distances of up to $\sim 100$ kpc.

Typically, studies investigating the impact of AGNs have tended to focus on AGN activity in only the central galaxy, and invoke relatively high values of thermal conduction to redistribute the energy across the cooling core (see, for example, Roychowdhury et. al. 2005). Here, we propose instead that the cluster ICM is heated by the AGN activity in more than just the one central galaxy. The central region of the clusters is rich in giant elliptical galaxies, most of which are presumed to harbour supermassive black holes that are thought to inject radio bubbles into the ICM with a duty cycle of $\sim 10^8$ yr per Hubble time (Best et. al. 2006).

Upon injection, the overpressurized bubbles rapidly expand (virtually in situ) to reach pressure equilibrium with their surroundings, generating waves and shocks, the dissipation of which is the primary mechanism by which the ICM is heated. Once in pressure equilibrium, the bubbles will rise buoyantly. The dissipation of the work done by viscous drag acting to retard the bubbles’ motion will further heat the ICM, though to a much lesser extent. In response to the decreasing pressure, the rising bubbles will expand. This expansion causes a corresponding compression of the ICM. However, while the initial inflation of the cavity may trigger internal gravity waves, compression waves and shocks that heat the ICM, this subsequent expansion-compression is adiabatic in character and no heating of the ICM ensues, even though the ICM’s internal energy will increase due to compression. In fact, the compression actually leads to the erosion of the thermal content of the ICM because the resulting increase in density results in enhanced cooling. This can be seen as follows. The compression of the ICM can be approximated as an adiabatic process so that the “entropy” $S = T/n^{2/3}$ is conserved. The bremsstrahlung cooling time is $t_c \propto T/(nT^{0.6}) \sim n^{0.4}n^{-1.06}$. Therefore, cooling becomes more efficient as $n$ is increased during adiabatic compression.

In our model, the heating occurs near where the bubbles are produced but since the galaxies that produce these bubbles are distributed throughout the cluster core region, so too is the heating. Modest heat conduction should suffice to distribute further the energy over the ICM between the galaxies. Since the energy deposition is a local phenomenon, it is relatively straightforward to construct a self-regulating model.

To model the interaction of the bubbles with the ICM (which we treat as all material not belonging to the hot and dilute bubbles), we use a spherical one-dimensional (1D) hydrodynamical code. The code treats the bubbles and the ICM as a two-fluid system in a fixed dark matter halo. The code treats the interaction of the bubbles and ICM in a consistent semi-analytical way and assumes that the distribution of bubbles is spherically symmetric. The code integrates the hydrodynamical equations of an ambient medium with thermodynamic properties which depend on the local filling factor of bubbles.

The outline of the paper is as follows. In §2 we derive the equations of motion of the ambient medium and present details of the the interaction of the bubble fluid with the ICM. In §3 we present our scheme for self-regulating feedback and discuss its general properties. We describe the numerical implementation in §4 and show results in §5. We conclude with a summary and general discussion of alternative heating schemes in §6.

2 THE EQUATIONS OF MOTION FOR THE AMBIENT MEDIUM

A 1D hydrodynamical code cannot tackle the intrinsically three-dimensional (3D) problem of the evolution of individual bubbles in the ICM, in particular if bubbles are injected from sources away from the center as in our proposed scenario. Therefore, a formalism for following the evolution of bubbles statistically must be developed. We represent bubbles as a fluid specified by the following physical quantities: the radius of bubbles, their number density, and velocity, all as a function of distance from the center. The bubbles and the ICM are a two-fluid system that is best described as a single fluid which we term the ambient medium. The representation in terms of a single medium is applicable only if local pressure equilibrium between the bubbles and the ICM is established (Appendix A). Hereafter, quantities related to the ambient medium, the bubbles, and the ICM (gas outside bubbles) are denoted by the subscripts, $a$, $b$, and $i$, respectively. Suppose that the volume filling factor of bubbles at $r$ is $F(r)$. The ambient density, $\rho_a$, at position $r$, is defined as

$$\rho_a = (1 - F)\rho_i + F\rho_b,$$  

(1)

in terms of the density inside bubbles, $\rho_b$, and the density of the ICM, $\rho_i$, also at $r$. The local ambient energy, $u_a$, per unit mass as

$$u_a = (1 - F)u_i + F\rho_b u_b, \quad (2)$$

where $u_i$ and $u_b$ are the energy per unit mass of the material inside the bubbles and of the ICM. If the pressure, $p$, is $p = (\gamma - 1)u\rho$ for both the ICM and the material inside the bubbles, Following an analysis similar to that presented in Appendix A we find that the equation of state of the ambient medium is,
\[ p = (\gamma_a - 1)\rho_a u_a \quad (3) \]

where

\[ \gamma_a = \frac{(\gamma_l - 1)(\gamma_b - 1)}{(1 - F)(\gamma_b - 1) + F(\gamma_l - 1)}, \quad (4) \]

and we have assumed local pressure equilibrium between the bubbles and the ICM. These thermodynamical relations supplement the following equations of motion of the ambient medium in a gravitational field \( g(r) \),

\[ \frac{d\nu_a}{dt} = g - \frac{1}{\rho_a} \frac{dp}{dr}. \quad (5) \]

In the absence of dissipative heating and cooling, the adiabatic energy equation is,

\[ \frac{d\nu_a}{dt} = \frac{p}{\rho_a^2} \frac{dp_a}{dr}. \quad (6) \]

Dissipative heating and cooling will affect the energy equation as described below. In the above equations the change of the filling factor as a result of dissipative heating and cooling, production of new bubbles, and bubble motion will have to be taken into account self-consistently.

### 2.1 Motion of bubbles and their interaction with the ICM

In our model, cluster galaxies affect the ICM by the production of over-pressurised bubbles made of hot relativistic plasma with an adiabatic index \( \gamma_b = 4/3 \). Bubbles are produced in response to local cooling in the vicinity of galaxies according to the self-regulating mechanism described in §3.

Let \( n_b(r), v_b(r), \) and \( R_b(r) \), be, respectively, the number density of bubbles, the velocity of bubbles relative to the ambient medium, and the radius of bubbles, all at distance \( r \) from the center. The motion of the bubbles is modeled as that of a pressure-free (i.e., collisionless) fluid with a velocity that is determined by a balance between buoyancy and drag forces with the ICM. Bubbles are assumed to be injected with the same energy per bubble and the same initial pressure so that at distance \( r \) all bubbles have the same size (see equation 7) and the same velocity. Therefore, a hydrodynamical description for the flow of bubbles is self-consistent. The pressure-free assumption is also self-consistent since \( v_b(r) \) is a single-valued function of \( r \) so that no "shocks" can form.

The various phases of the evolution of the bubbles are as follows:

- **Phase I**: A bubble is injected at distance \( r \) from the center. This over-pressurised bubble undergoes a super-sonic expansion in the ICM until its internal pressure drops close to that of the ICM. During this phase of rapid expansion, the bubbles heat the ICM by the production of weak shocks. This expansion is assumed to be quite rapid and such that the bubble does not move significantly away from the distance at which it has been injected.

- **Phase II**: The dilute hot bubble then rises by buoyancy towards more distant regions of lower pressure. The moving bubbles heat the ICM via viscous drag forces. As noted, the velocity of bubbles relative to the ambient medium is determined by a balance between buoyancy and drag forces with the ICM. Rising bubbles also expand adiabatically, compressing the surrounding ICM. This process does not heat the ICM\(^*\). Increased ICM density does, however, elevate the efficiency of cooling.

- **Phase III**: Finally, bubbles are destroyed by hydrodynamical (Raleigh-Taylor and Kelvin-Helmholtz) instabilities (e.g. Soker, Blanton & Sarazin 2002).

#### 2.1.1 Initial parameters of an injected bubble

Let a bubble be injected with an initial energy \( E_{bi} \) and an initial internal pressure \( p_{bi} \). Since \( E_{bi} = u_{bi} p_{bi} 4\pi R_{bi}^3 / 3 \), where \( p_{bi} \) and \( u_{bi} \) are the initial mass density and energy per unit mass in the bubble, the equation of state \( p_b = (\gamma_b - 1)p_{bi} u_{bi} \), yields,

\[ R_{bi} = \left[ \frac{3}{4\pi} (\gamma_b - 1) \frac{E_{bi}}{p_{bi}} \right]^{1/3}. \quad (7) \]

#### 2.1.2 Bubble radius as a function of distance

The bubble radius, \( R_b(r) \), at any distance \( r \) from the center can be estimated from the adiabatic condition \( p_b \rho_b^{\gamma_b} = \text{const} \). This condition must hold in Phase I and Phase II because the speed of sound of the relativistic plasma in the bubble is much larger than its expansion velocity. The adiabatic condition yields

\[ R_b(r) = R_{bi} \left[ \frac{p_{bi}}{p(r)} \right]^{\frac{3}{\gamma_b}}, \quad (8) \]

which for \( \gamma_b = 4/3 \) gives the weak dependence, \( R_b \propto (p_{bi} / p)^{1/4} \), on the pressure ratio.

#### 2.1.3 Heating by shocks

We estimate the energy transferred from bubbles to the ICM by means of weak shocks during Phase I. The equation of state of the material inside the bubble gives \( E_b = (4\pi/3)R_b^3 p_b / (\gamma_b - 1) \) for the thermal energy of a bubble of radius \( R_b \) with internal pressure \( p_b \). At the end of the rapid expansion in Phase I we assume that \( p_b \approx p(r) \) where \( p(r) \) is the ICM pressure at the distance at which the bubble has been injected. Substituting \( p = p(r) \) in (8) to get \( R_b \), we find that the thermal energy at the end of Phase I is

\[ E_{b, 0} = E_{bi} \left( \frac{p}{p_{bi}} \right)^{1-1/\gamma_b}. \quad (9) \]

This equation implies that \( E_{b, 0} \propto p R_b^3 \) can be substantially smaller than the initial energy of the bubble. The heat transferred to the ICM by the generation of shocks is

* Deviation from adiabatic compression may result in bulk motions in the ICM. These motions can eventually dissipate into heat via turbulence or ordinary viscosity. The maximum heating rate that is obtained this way is slightly less that the drag heating rate discussed in §2.1.4

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ble coalescence, changing bubble shapes and orientations, bubble-bubble interactions mediated by bubble wakes, bubble and dissipation of shocks, wakes and waves in this medium, lem, involving a host of complex phenomena including the radiative cooling is, in general, a highly non-trivial prob-

-ation field (per unit mass) as measured in a frame of reference attached to the ambient medium at distance

where 0 \( \leq C_{\text{eff}} \leq 1 \) is an efficiency parameter. In the present treatment, we assume that the energy transferred to the ICM is deposited locally. The expression (10) does not take into account the energy needed to place the bubble with initial radius \( R_{\text{bi}} \) and energy \( E_{\text{bi}} \) into the ambient medium. For a given \( E_{\text{bi}} \), this energy is negligible if \( R_{\text{bi}} \) is sufficiently small.

2.1.4 Buoyancy and heating of the ICM by drag with rising bubbles

The dynamical evolution of the system comprised of an expanding, rising bubble in a bubbly medium subject to radiative cooling is, in general, a highly non-trivial problem, involving a host of complex phenomena including the compressible nature of the bubbly medium, the evolution and dissipation of shocks, wakes and waves in this medium, bubble-bubble interactions mediated by bubble wakes, bubble coalescence, changing bubble shapes and orientations, etc. We write the equations of motion of a bubble in Phase II. We restrict ourselves to a simple situation of a system involving a single spherical bubble rising along the radial direction in the ICM. The equation of motion governing the bubble’s rise is

\[
\frac{d}{dt} (M_a v_b) = F_b - F_{\text{drag}}
\]

where the two terms on the right represent the buoyancy force acting on the bubble of radius \( R_b \) and the drag force on the bubble as it moves through the ICM. The left-hand term is the rate of change of the Kelvin impulse (also known as the added-mass force) corresponding to the displacement of the ambient medium due to the bubble’s rising motion. Analytic expressions for the “virtual added-mass” of the bubble can be obtained for simplified motions. For a laminar flow generated by a spherical bubble moving in an incompressible fluid the expression for the virtual added mass is (e.g. Kendoush 2003),

\[
M_a = C_{AM} \frac{4 \pi}{3} R_b^3 \rho_a \quad C_{AM} = \left( \frac{1}{2} + \frac{3}{2} \frac{\dot{R}^2}{v_b^2} \right)
\]

where \( \dot{R} \) is the rate of expansion of the bubble. We have neglected the inertia of the bubble as it is much smaller than the added-mass term.

A bubble of radius \( R_b \) at distance \( r \) experiences a buoyant force of

\[
F_{\text{bu}} = \frac{4 \pi}{3} R_b^3 \rho_a g_{\text{eff}}
\]

where we have assumed that \( \rho_b \ll \rho_a \). Here, \( g_{\text{eff}} \) is the gravitational force field (per unit mass) as measured in a frame of reference attached to the ambient medium at distance \( r \). The actual gravitational field, \( g \), differs from \( g_{\text{eff}} \) only when the system deviates from hydrostatic equilibrium, such as in a cooling runaway where \( g_{\text{eff}} \approx 0 \). We write the drag term on the bubble as it moves through the ICM with velocity \( v_b \) as

\[
F_{\text{drag}} = \frac{\pi}{2} C_d R_b^2 \rho_s v_b^2
\]

where \( C_d \) is the drag coefficient and we have assumed that \( v_b \ll c_s \). In our model all bubbles at the same distance share the same velocity. But in a realistic situation, bubbles are generated with different initial energies and at random directions. So a bubble may collide with others. Hence, to model this effect we write \( \rho_s \) instead of \( \rho_b \) in the drag equation.

Typically, the magnitude of added-mass term is much smaller than the buoyancy and the drag terms and therefore, to first order, the rise velocity of the bubbles can be estimated by equating the drag and buoyancy forces (e.g. Churazov et. al. 2001), yielding:

\[
v_b^2 = \frac{8 g_{\text{eff}} R_b}{3 C_d} \left( 1 + \frac{8 g_{\text{eff}} R_b}{3 C_d c_s^2} \right)^{-1}
\]

As a bubble rises, the gravitational potential energy of the system (the bubble and the surrounding medium) decreases. A Rising bubble also expands and its internal energy decreases. The total energy available as the bubble rises from position 1 (point of injection) to position 2 is

\[
\Delta E_2 = (E_{b1} - E_{b2}) + \int_{r_1}^{r_2} 4 \pi \frac{4 \pi}{3} R_b^3 \rho_a g_{\text{eff}} \, dr
\]

where the first difference pair, corresponding to the change in the bubble’s internal energy, it can be evaluated using equation the condition \( p/\rho_b^\gamma = \text{const} \), for adiabatic changes of the bubble fluid as

\[
(E_{b1} - E_{b2}) = E_{b1} \left( \frac{p_1}{p_{\text{bi}}} \right)^{1-1/\gamma_b} \left[ 1 - \left( \frac{p_2}{p_1} \right)^{1-1/\gamma_b} \right]
\]

Assuming that the ambient medium is in hydrostatic equilibrium, the second term too can be straightforwardly evaluated, yielding:

\[
\int_{r_1}^{r_2} 4 \pi \frac{4 \pi}{3} R_b^3 \rho_a \frac{\partial \phi}{\partial r} \, dr = \gamma_b (E_{b1} - E_{b2})
\]

where \( \phi \) is the gravitational potential. Consequently, \( \Delta E_2 = (1 + \gamma_b) (E_{b1} - E_{b2}) \). This is the energy available to lift the bubble against the drag force, overcome the inertia of displacing the surrounding fluid, as well as compress the ambient medium.

In general, it is not possible to determine how much of this energy that will go towards compressing the ambient medium and how much will towards heating the ICM, be it due to drag or through the eventual dissipation of the wakes and motions associated with the displacement of the ICM,
unless one knows the details of the rise-expansion process. In the limit that the bubble rises slowly, maintaining pressure equilibrium with its surroundings at all times, the bubble expansion/ICM compression is adiabatic and the change in the internal energy of the bubble goes entirely towards raising the internal energy of the ambient medium. In this case, the total energy available for heating the ICM is simply that due to the change in the potential energy and the corresponding heating rate (in units of energy per unit time) is

\[ \dot{H}_{\text{heat}} = \frac{4\pi}{3} R_{\text{b}}^3 \rho_b g_{\text{eff}} \nu_b. \]  

(20)

It is instructive to relate this heating term to the physical parameters of the bubbles and their injection rate. Assume that bubbles are injected within the sphere of radius \( R_{\text{inj}} \) around the center, at a rate \( \dot{N}_{\text{inj}} \) per unit volume. Neglecting bubble destruction, the total number of bubbles within a distance \( r \gg R_{\text{inj}} \) is \( \dot{N}_{\text{inj}} (4\pi/3) R_{\text{inj}}^3 / \nu_b \), so that the number density within \( r < R_{\text{inj}} \) is \( \dot{n}_b (r) \approx \dot{N}_{\text{inj}} R_{\text{inj}}^3 / (\nu_b r^2) \). Approximating \( \dot{n}_b (r) \approx \dot{n}_b \) in (20) and using (15) gives

\[ \dot{H}_{\text{heat}} \approx \frac{4\pi}{3} \dot{N}_{\text{inj}} R_{\text{inj}}^3 g_{\text{eff}} \frac{R_{\text{inj}}^3}{r^2}. \]  

(21)

It is interesting that this does not depend on the drag coefficient, \( C_{\|} \). We can further simplify this expression by noting the bubble energy at \( r \) is \( E_b \sim 4\pi R_{\text{b}}^3 \rho_b / (\gamma_b - 1) \). The final result is

\[ \dot{H}_{\text{heat}} \sim (\gamma_b - 1) \dot{N}_{\text{inj}} g_{\text{eff}} \frac{R_{\text{inj}}^3}{r^2} \frac{E_b}{p}. \]  

(22)

For \( r < R_{\text{inj}} \) the heating rate is obtained by replacing \( R_{\text{inj}} \) with \( r \) in the last expression.

In the numerical code, the heating is computed explicitly using (20). We do so because apart from the initial acceleration phase, the bubble’s motion should be well described by the “slow rise” approximation. In the worst case scenario, we are underestimating heating rate in Phase IIby a factor of \( 1 + 1/\gamma_b \). This uncertainty is not of concern because in our adopted scenario where the bubbles are initially injected in a highly overpressurized state \( (p_i \ll p_b) \), the heating of the ICM during Phase 0dominates over the heating by the rising, expanding bubble.

3 SELF-REGULATING FEEDBACK

We write the energy equation in terms of the ICM entropy \( S_1 = u_1 / \rho_1^{\gamma - 1} \) as

\[ \rho_1^{\gamma - 1} \dot{S}_1 = \dot{H} - \frac{u_1}{t_{\text{cool}}} \]  

(23)

where \( t_{\text{cool}} \) is given in Appendix B. As explained before, two sources contribute to \( \dot{H} \): a) heating by weak shocks generated during the initial rapid expansion of the overpressurized bubbles, and b) heating by drag with the ICM as bubbles rise up by buoyancy. Adiabatic work done by the bubbles as they move upward does not change \( S_1 \) and therefore does not contribute to \( \dot{H} \). We introduce \( R_{\text{inj}} \), the distance from the center within which bubbles are injected. At \( r > R_{\text{inj}} \), bubbles can be present only as a result of flow of bubbles from regions with \( r < R_{\text{inj}} \). Given the injection rate (number per unit volume per unit time), \( \dot{N}_{\text{inj}} (t, r) \) of bubbles, we write the local heating rate (energy per unit mass of the ICM) via weak shocks as

\[ \dot{H}_{\text{wsh}} = \dot{N}_{\text{inj}} \frac{\Delta E_1}{\rho_1}, \]  

(24)

where \( \Delta E_1 \) is given by (10). As a recipe for self-regulated feedback, we assume that bubbles are generated only if \( \dot{S}_1 \) is negative. We write the flux of injected bubbles at a point \( r \ll R_{\text{inj}} \) as

\[ \dot{N}_{\text{inj}} = \frac{\eta}{E_{\text{hi}}} \frac{\dot{S}_1}{S_1} c^2 \quad \text{for} \quad \dot{S}_1 < 0, \]  

(25)

and zero, otherwise. The free parameter \( \eta \) represents the product of the mass fraction of cold gas that accretes onto the AGNs and the efficiency of AGNs at transforming the accreted mass into bubbles. The cold gas is assumed to be generated at a rate (in units of mass per unit volume per unit time) of \( \rho_i \dot{S}_1 / S_1 \). This cold gas is assumed to form by condensation via the development of thermal instabilities (e.g. Field 1965). Finally, \( c \) is the speed of light and, as before, \( E_{\text{hi}} \) is the energy with which a bubble is injected.

We can use the above formalism to investigate the role of heating by weak shocks, the dominant source of heating in our model. Substitute \( \Delta E_1 \) and \( \dot{N}_{\text{inj}} \) from (10) and (25), respectively, into (24) yields

\[ \dot{H} = -u_\alpha \frac{\dot{S}_1}{S_1}, \quad \text{where} \quad u_\alpha = C_{\text{wsh}} \eta c^2 \left[ 1 - \left( \frac{p}{p_b} \right)^{1 - \gamma_b / 3} \right]. \]  

(26)

where \( p = p(r) \) is the ambient pressure at \( r \) and \( p_b \) is the internal bubble pressure with which the bubbles are injected. This expression holds for \( \dot{S}_1 < 0 \). For \( \dot{S}_1 > 0 \), we have \( \dot{H} = 0 \). Substituting (26) in (23) and solving for \( \dot{S}_1 \) we get

\[ \dot{S}_1 = \frac{\dot{S}_i}{\tau_{\text{cool}}}, \]  

(27)

where

\[ \tau_{\text{cool}} = \left( 1 + \frac{u_\alpha}{u} \right) t_{\text{cool}}. \]  

(28)

The above equation implies that cooling is substantially suppressed at low temperatures, \( u \ll u_\alpha \), while at higher temperatures, shocks are unable to suppress cooling. For an ICM at temperature corresponding to velocity \( V \) an efficient suppression of cooling at distance \( r \) occurs if

\[ C_{\text{wsh}} \eta \sim \left( \frac{V}{c} \right)^2 \approx 10^{-5} \left( \frac{V}{10^2 \text{km} \text{s}^{-1}} \right)^2, \]  

(29)

where we have assumed \( p_b \gg p \). Note that since \( \dot{N}_{\text{inj}} \propto 1 / E_{\text{hi}} \), neither the shock nor the drag heating depend on \( E_{\text{hi}} \).
4 NUMERICAL MODEL

We develop a numerical hydrodynamical model to describe the evolution of the ICM and the bubbles in a fixed dark matter halo. The density profile in the halo is approximated by the NFW profile (e.g. Hayashi et al. 2004) of the form

$$\rho(s, t) = \frac{\delta_c \rho(t)}{s(1 + cs)^2},$$

where $$s = r/R_v$$ and $$R_v$$ is the virial radius of the halo. The parameter $$\delta_c$$ is determined by the condition that the mean density within $$r = R_v$$ is 178 times the background density, $$\bar{\rho}(t)$$. We adopt the value $$c = 5$$ for the concentration parameter (e.g. Balogh et al. 2005).

The ambient medium is represented by lagrangian shells of fixed mass. In each time-step, the equations of motion of the bubble and the ambient fluids are solved in the following order:

(i) advance the ambient shells according to equations (5) and (6) without radiative cooling or dissipative heating,

(ii) update bubble properties in each shell as a result of the change in incurred in the ambient medium (see Appendix A) after the hydrodynamic step (i),

(iii) estimate dissipative heating (by weak shocks and drag) and radiative cooling of ICM and update the corresponding ambient pressure and bubble properties without moving the ambient shells,

(iv) inject the ambient medium with new bubbles according to the recipe outlined in (§3).

The integration of the hydrodynamic equations (5) and (6) includes artificial viscosity (e.g. Richtmyer & Morton 1967; Thoul & Weinberg 1995; Nusser & Pointecouteau 2005) in order to deal with any abrupt changes in the velocity which may arise if a cooling runaway develops. The bubble fluid is moved through the shells of the ambient medium using a first order flux conserving scheme. The velocity of this fluid relative to the shells is given by (16). Finally, a leapfrog time integration scheme is used.

5 RESULTS

We adopt a flat cosmology with baryonic and dark matter density parameters respectively $$\Omega_b = 0.044$$ and $$\Omega_m = 0.27$$, a cosmological constant corresponding to a density parameter of $$\Omega_k = 0.686$$, and a Hubble constant of $$H_0 = 70 \text{km s}^{-1} \text{Mpc}^{-1}$$ (e.g. Spergel et al. 2003).

The number of shells describing the ambient medium is 50 in all of the runs. The initial shells are placed at logarithmically spaced distances from the center with the nearest shell $$r = 15\text{kpc}$$ and the farthest lying at the virial radius of the halo. The initial temperature profile is assumed to be isothermal with a temperature, $$T$$ (in keV), given by

$$1.16 \times 10^7 k_B T/\mu m_p = (2/3)GM/R_v,$$

where $$k_B$$ is the Boltzmann constant, $$\mu$$ is the mean molecular weight and $$m_p$$ is the mass of the proton. The equation for hydrostatic equilibrium implies a slope of $$-9/4$$ for the gas density at $$r = R_v$$. The gas density at any $$r$$ ($$r < R_v$$) is solved numerically using the equation of hydrostatic equilibrium including gas gravity. In solving this equation, the gas to dark matter mass ratio at $$r = R_v$$ is taken to be the background ratio $$\Omega_b/\Omega_m$$. The mass in each ambient shell is then estimated from the gas density profile. Because of the limited number of shells, a constant temperature does not necessarily guarantee that the shells are actually in strict hydrostatic equilibrium. Therefore, given the mass in each shell, we use the discrete form of the equations of motion as they appear in the numerical hydrodynamical code (e.g. Weinberg & Thoul (1995) to obtain a temperature (thermal energy per baryon) in each shell such that strict equilibrium is achieved at the initial time. This yields nearly isothermal temperature profiles with deviations that increase with radius.

In all runs, the code evolves the shells from redshift $$z = 2$$ until $$z = 0$$. In some runs with insufficient feedback to counter cooling, the hydrodynamic time-step becomes exceedingly small. Whenever this happens, the shells colder than 1 keV and denser than 1.5 cm$$^{-3}$$ are held fixed and are not moved by the code at any later time.

Results from simulations for clusters with virial radii of $$R_v = 4 \text{Mpc}$$ and $$R_v = 2 \text{Mpc}$$ are shown in Fig. 1 and Fig. 2, respectively. In the column to the left we show results which include radiative cooling and no heating. For both values of $$R_v$$, effects of cooling become significant already by $$z = 1$$. At $$z < 0.5$$ a cooling runaway develops in the inner regions: cooling becomes increasingly more rapid due to the enhanced density and lower temperature.

The effect of heating by bubbles, according to our feedback recipe described in §3, is explored in the middle and right panels for two values of the efficiency parameter $$\eta$$. To model the effect of multiple AGN activity, bubbles are allowed to be injected anywhere in the ICM (i.e. $$R_{inj} = R_v$$). As noted before the heating rate of the ICM does not depend on the initial bubble energy, $$E_{inj}$$. All results in these two columns are for $$C_i = 1$$ and $$C_{wsh} = 1$$, and an initial bubble pressure of $$10^3$$ times the central ICM pressure. Cooling flows for the two different clusters are completely suppressed for $$\eta$$ as low as 0.01, while a value $$\eta = 10^{-5}$$ gives moderate suppression of cooling as is required by the observations.

6 SUMMARY AND DISCUSSION

We propose that modest radio outbursts from many cluster galaxies may help to quench cluster cooling flows. We appeal to typical massive elliptical galaxies which are thought to have radio outbursts with a duty cycle of $$\sim 10^6$$ yr per Hubble time (Best et al. 2006) and are concentrated in the cluster core (within the central 200 kpc or of order twice the extent of the cooling region). Weak shocks generated during the initial bubble expansion and viscous/turbulent drag on buoyantly rising bubbles are the principal dissipative heating processes. Indirect evidence for multiple heating sources in the form of AGN comes from the frequent presence of x-ray “ghost” cavities that survive long after any associated radio lobes have decayed, and from amorphous radio halos that further suggest an ICM percolated with a distribution of bubbles whose contrast in the X-ray has been diminished due to adiabatic expansion (cf. Heinz & Churazov 2005). Our scenario is in agreement with recent obser-
Figure 1. The ICM electron number density (top), temperature (middle) and entropy (bottom) profiles versus distance from the cluster center (in Mpc). The left column shows profiles obtained with radiative cooling and no energetic feedback. In the middle and right panels feedback is done via bubbles generated in the entire cluster with efficiency parameter $\eta = 10^{-2}$ and $\eta = 10^{-5}$, respectively. The solid, dotted, dashed and dash-dotted curves correspond to redshifts $z = 1.6, 1, 0.5, \text{and } 0$, respectively. The results correspond to an NFW dark halo profile with a virial radius of $R_v = 4 \, \text{Mpc}$ and a concentration parameter of $c = 5$. The number density, temperature, and entropy are given in $\text{cm}^{-3}$, $\text{keV}$, and $\text{keVcm}^2$, respectively.

Figure 2. The same as the previous figure but for a dark halo with a virial radius of $R_v = 2 \, \text{Mpc}$. 
vational evidence for the existence of AGN activity in about 5% of cluster galaxies at low redshift (Martini et al. 2006). Cosmological evolution of the AGN inevitably increases the feedback at earlier epochs (z \lesssim 2). Our scenario alleviates some of the problems that may arise if energy feedback is generated by a single central source (AGN) harboured in the central cD galaxy. The needed energy transport over the entire cooling flow region is not easy to provide if the energy source is associated only with the central AGN (e.g. Mathews, Faltenbacher & Brighenti 2005). One approach is to ensure that mechanical disturbances are able to propagate large distances before they are damped by viscosity (e.g. Ruszkowski, Brüggen & Begelman 2004; Reynolds et al. 2005). This approach has two potential shortcomings. Firstly, the heating rate per baryon of dissipating waves should decay with distance from the central source as r\(^{-3}\). This is too steep a gradient for the heating to affect the ICM over the entire region that is susceptible to cooling. In fact, Roychowdhury et al. (2004) find that the heating leads to a convectively unstable core profile that would potentially destroy any pre-existing metallicity gradient, in conflict with the x-ray observations, and to remedy this, Roychowdhury et al. (2005) invoke relatively high values of thermal conduction to effect rapid transport of the energy out of the center. Secondly, the viscosity has to be finely tuned: too high a value results in dissipation much too close to concentrated near the central source, and too low a value means the heating is inefficient. Since the relevant viscosity coefficient scales as T\(^{5/2}\) (Spitzer 1962), it is difficult to envision that a viscous solution that relies on fine-tuning prevails in all clusters. Moreover, the ICM is weakly magnetized, and the nature of transport processes, like thermal conduction and viscosity, in such media is not well understood.

The model presented here heats the ICM through shocks generated during the initial inflation of the bubbles, and drag forces acting on the rising bubbles. However, the latter relies on some viscosity to dissipate heat but it is not an important source of heating and with respect to the former, we do not need to fine-tune the viscosity since the energy distribution is effected by multiple AGNs hosted in a spatially extended distribution of galaxies. Moreover, it may be possible to avoid the issue altogether. Heinz & Churazov (2005) have recently argued that in a bubble-filled medium of the kind proposed here, the bubbles themselves act as catalysts to convert shock and waves into heat. Additionally, the interaction between the bubbles and the strong shocks will both broaden and distort the shock fronts, causing the features to appear weaker in X-ray observations and resulting in mistaken assumptions that cavity inflation is a gentle process and that the ICM is not subject to strong shocks. Both of these features further contribute to making our model highly viable.

An additional principal feature of our model is the incorporation of self-regulation between heating and cooling. This should inevitably lead to a correlation between the mechanical heating luminosity and the cooling rate. The former is measured by the properties of x-ray cavities and the latter by the cluster x-ray luminosity. Evidence for such a correlation is given by Birzan et al. (2004). In fact, the estimated ratio of mechanical to x-ray luminosity in this latter paper falls short of that needed by Croton et al. (2005) in order to quench cooling flows on group and cluster scales by a factor of 3-10 (Best et al. 2006).

However, the inferred mechanical luminosities are based on cavity pV estimates. In our model, a primary source of energy transfer to the ICM is via weak shocks generated by the expansion of the initially over-pressurised bubble. The initial bubble energy is related to the cavity pV by equation (9), and the resulting injected mechanical energy may exceed pV by a factor of up to 10. Note that this equation neglects the work done by the AGN jet to place the initial seed bubble in the ICM, which can be dissipative and contribute to heating the ICM if the seed bubble is ejected with supersonic speeds. Furthermore, these estimates neglects the energy content in waves detected in a few clusters (Fabian et al. 2003; Nulsen et al. 2005, McNamara et al. 2005). Our model consequently helps in resolving the apparent discrepancy in the estimated feedback required to halt cooling flows even in massive clusters.

An alternative heating source involves cosmic rays in the ICM cooling core, injected by a central AGN (Rephaeli and Silk 1995). The present model provides a distributed source of cosmic rays whose observable signatures could also include a Sunyaev-Zeldovich signal (Pfrommer & Ensslin 2004) as well as an additional hadronic heating source (Colafrancesco, Dar & De Rújula 2004) that might lead to a detectable gamma ray flux. More exotic heating sources invoke collisions between superheavy dark matter particles and protons (Qin & Wu 2001, Chuzhou & Nusser 2004) as well as by the products of dark matter neutralino annihilations (Colafrancesco 2004).

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REFERENCES

Babul, A., Balogh, M.L., Lewis, G.F., Poole, G.B. 2002, MNRAS, 330, 329
Balogh, M.L., Babul, A., Patton, D.R. 1999, MNRAS, 307, 463
Balogh M. L., Babul A., Mark V., McCarthy I. G., Jones L. R., Lewis G. F., Ebeling H. 2005, astro-ph/0511768
Begelman K.G., Broeils A.H., Sanders R.H. 1991, MNRAS, 249, 523
Best P. N., Kaiser C. R., Heckman T.M., Kauffmann G., 2006, astro-ph/0602171
Birzan L., Rafferty D. A., McNamara B.R., Wise M.W., Nulsen P.E.J., 2004, ApJ, 607, 800
Brüggen M., Ruszkowski M., Hallman E. 2005, ApJ, 630, 740
Churazov E., Brüggen M., Kaiser C. R., Böringer H., Forman W., 2001, APJ, 554, 261
Chuzhoy L., Nusser A. astro-ph/0408184
Colafrancesco S. 2004, A&A, 422, 23
Colafrancesco S., Dar A., De Rújula A., 2004, A&A, 413, 441
Croton D., et. al. 2005, MNRAS, 355, 11
Dalla Vecchia, C., Bower, R.G., Theuns, T., Balogh, M.L., Mazzotta, P., Frenk, C.S. 2004, MNRAS, 355, 995
Fabian A.C., Sanders J.S., Allen S.W., Crawford C.S., Iwasawa K., Johnstone R.M., Schmidt R.W., Taylor G.B. 2003, MNRAS, 344, 43
Hayashi E., Navarro J.F., Power C., Jenkins A., Frenk C.S., White S.D.M., Springel V., Stadel J., Quinn T. R., 2004, MNRAS, 355, 794
Heinz S., Churazov E., 2005, astro-ph/0507038
Kaiser C.R., Binney J. 2003, MNRAS, 338, 837
Kendouh A.A., 2003, Phys. Fluids 15, 2782
Martini P., Kelson D. G., Kim E., Mulchaey J. S., Athey A. A., 2006, astro-ph/0602496
Mathews W. G., Faltenbacher A., Brightuti F. 2005, astro-ph/0511151
McNamara B.R., Nulsen P.E.J., Wise M.W., Rafferty D.A., Cartilli C., Sarazin C.L., Blanton E.L. 2005, Nature, 433, 45
Nipoti, C., Binney, J. 2005, MNRAS, 361, 428
Nulsen P.E.J., Humbert D.C., McNamara B.R., Rafferty D., Birzan L., Wise M.W., David L.P. 2005, ApJ, 625, 9
Nusser A., Pointecouteau E., 2006, MNRAS, 366, 969
Omma H., Binney J., Bryan G., Slyz A. 2004, MNRAS, 348, 1105
Peterson J. R., et. al. 2001, A&A, 365, L104
Pfrommer C., Ensslin T. A., 2004, A&A, 413, 17
Qin B., Wu X. 2001, Phys. Rev. Lett., 87, 061301
Quilis, V., Bower, R.G., Balogh, M.L., 2001, MNRAS, 328, 1091
Reynolds, C. S.; McKernan B., Fabian A. C., Stone J. M., Veronico J. C., 2005, MNRAS, 357, 242
Richtmyer R., Morton K.W., 1967, Difference Methods for Initial-Value Problems, New York: Interscience
Roychowdhury, S., Ruszkowski M.; Nath B. B., Begelman, M. C., 2004, ApJ, 615, 681
Roychowdhury, S.; Ruszkowski, M.; Nath, B. B., 2005, ApJ, 634, 90
Ruszkowski M., Brüggen M., Begelman M.C., 2004, ApJ, 611, 158
Sijacki D., Springel V. 2006, MNRAS, 366, 397
Soker N., Blanton E. L., Sarazin C. L., 2002, ApJ, 573, 533
Spergel D.N. et al., 2003, ApJS, 148, 148
Spitzer L., 1962, Physics of Fully Ionized Gases, New York: Interscience
Thoul A.A., Weinberg D.H., 1995, ApJ, 442, 480
Vikhlinin A., et al., 2005, astro-ph/0412306
Voit, G.M., Donahue, M. 2005, ApJ, 634, 955

APPENDIX A: THERMODYNAMICS OF THE BUBBLES AND ICM

A1 Numerical description of bubbles in ambient medium

For simplicity the mass of bubbles is neglected.

As bubbles are transported in the ICM in an infinitesimal time step, the ICM in a shell is either compressed or diluted depending on whether or not number of bubbles in the shell has increased or decreased. This is a purely adiabatic process that conserved the entropies inside a bubble and in the ICM.

Consider a shell with pressure $p_1$ containing bubbles each of radius $R_{b1}$. Assuming that $\delta N$ additional bubbles enter the shell, we would like to estimate the ambient pressure, $p$, the radius, $R_b$, and the filling factor, $F$, after the bubbles enter the shell. For concreteness let $\delta N > 0$, i.e., there is a net increase in the number of bubbles in the shell. Before these bubbles enter the shell, the total material in the shell occupies a volume $V_1 = 4\pi (r_{1}^2 + r_{1}^3)/3$ of which $V_{b1} = F_1 V_1$ is taken by the bubbles and $V_{1b} = (1 - F_1) V_1$ by the ICM.

After $\delta N$ bubbles enter the shell, the same original material occupies a volume $V_1 - \delta V$ where $\delta V = \delta N 4\pi R_b^3/3$ with $R_b$ being the new radius of a bubble in the shell. The entropies in the bubbles and the ICM of the original are conserved so that $p V_1^{\gamma_1} = p_1 V_{b1}^{\gamma_1}$ and $p V_{1b}^{\gamma_b} = p_1 V_{b1}^{\gamma_b}$ where $V_1$ and $V_b$ are the volumes occupied by the ICM and bubbles of the original material after the $\delta N$ bubbles entered the shell. Using these entropy conservation relations in $V_1 - \delta V = V_1 + V_b$ gives

$$1 - \frac{\delta V}{V_1} = F_1 \left( \frac{p_1}{p} \right)^{\frac{1}{\gamma_b}} + (1 - F_1) \left( \frac{p_1}{p} \right)^{\frac{1}{\gamma_1}}.$$  \hspace{1cm} (A1)

Entropy conservation inside a bubble gives $R_b^3 = R_{b1}^3 (p_1/p)^{1/\gamma_b}$ which after substituting in $\delta V = \delta N 4\pi R_b^3/3$ yields

$$1 = (F_1 + \delta N 4\pi R_{b1}^3) \left( \frac{p_1}{p} \right)^{\frac{1}{\gamma_b}} + (1 - F_1) \left( \frac{p_1}{p} \right)^{\frac{1}{\gamma_1}}.$$  \hspace{1cm} (A2)

The bubble radius and their filling factor after the change can easily be computed from the adiabatic conditions once the pressure, $p$, is known.

A1.1 Effect of heating and cooling on the ICM and bubbles

As mentioned previously, the ICM is meant to describe the inter-cluster plasma lying outside the bubbles. As an infinitesimal amount of energy is removed from the ICM by cooling or added, we must describe the reaction of the ICM and bubbles to such infinitesimal change of the energy. Let an energy $\delta E$ be added as heat to the ICM in the shell $i + 1/2$. For concreteness we assume $\delta E > 0$, but the same reasoning applies to cooling $\delta E < 0$. In the calculation we assume that the that the system reacts in the following two step sequence. First the pressure of the ICM is changed to $p_1 + (\gamma_i - 1) \delta E/V_0$ where $V_0 = 4\pi (r_{1i}^2 + r_{1i}^3)/3$ before the bubbles are compressed. Next, the under-pressured bubbles are compressed by the ICM. The expansion of the ICM and the compression of the bubbles in the second step are both assumed to be adiabatic processes. The following adiabatic conditions hold in the second step.

$$p V_i^{\gamma_1} = \left[ p_1 + (\gamma_i - 1) \frac{\delta E}{V_{1i}} \right] V_i^{\gamma_1} \hspace{0.5cm} \text{and} \hspace{0.5cm} p V_b^{\gamma_b} = p_1 V_{b1}^{\gamma_b}.$$  \hspace{1cm} (A3)
where $p$, $V_I$, and $V_b$ are the pressure, the volume of the ICM, and the volume of bubbles at the end of the second step. These conditions give the following equation for $p$

$$1 = F_1 \left( \frac{p_1}{p} \right)^{\frac{1}{\gamma_b}} + (1 - F_1) \left[ \frac{p_1}{p} + (\gamma_1 - 1) \frac{\Delta E}{F_1 V_0 p} \right]^{\frac{1}{\gamma_1}}. \quad (A4)$$

**APPENDIX B: THE COOLING FUNCTION AND THE X-RAY EMISSIVITY**

We approximate the cooling time by (Balogh, Babul & Patton 1999)

$$t_{\text{cool}} = \frac{B_1 \mu m_H T_1(r)^{1/2}}{\rho_1 \left[ 1 + B_2 f_m/T_1(r) \right]}, \quad (B1)$$

where, $\mu$ is the mean molecular weight, $B_1 = 3.88 \times 10^{11} \text{sK}^{-1/2} \text{cm}^{-3}$, $B_2 = 5 \times 10^7 \text{K}$ and $f_m$ is a metallicity-dependent constant that is 1 for solar metallicity and 0.03 for zero metallicity.

The volume X-ray emissivity of the ambient medium is

$$\epsilon = \frac{3}{2} \frac{(1 - F) \rho T_1}{\mu m_H t_{\text{cool}}} \quad (B2)$$