Diffraction pattern by nanometric thin films under illumination of an orbital angular momentum beam with integer topological charge

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Abstract. The orbital angular momentum of light has a big contribution in many engineering applications like optical communications, because this physical property allows eigenstates characteristic of the wavefront rotation when the beam is propagated. The nature of these eigenstates allows that information can be encoded and gives immunity to electromagnetic interference, allowing an increase of bandwidth, cadence and capacity of the communication channel. This work shown the methodology using nanometric thin films like Titanium based (TiO₂) grown over strontium titanate (SrTiO₃) support, to distinguish and discriminate a well-defined integer value of the topological charge of an OAM beam.

1. Introduction
Optical beams with orbital angular momentum OAM, have a spatial phase structure \( e^{im\theta} \) where \( m \) is the topological charge TC, which can take integer values [1]. These beams have great potential for generating and improving existing engineering applications such as optical communications [2], optical tweezers and particle manipulation [3], encoding and multiplexing [4]; since no need to interrupt the light beam for the transfer of data, provides immunity to electromagnetic interference, increases bandwidth, the rate of transmission and the ability of information transmitted by the channel, making the communication process more efficient [5]. Because of these properties, it is necessary to deepen the study of the TC detection. The current used techniques are interferometry [6] and digital image processing [7]; however, it can be found in a simple way from the associated pattern when the AOM beam is diffracted by a triangular opening [8]. Numerical and experimental far-field diffraction pattern by a border for OAM beams with integer topological charge, are analyzed by a triangular equilateral aperture. Results are presented below and we show that thanks to use of those patterns, we can discriminate the spatial distribution information on the front-wave of OAM beams.

2. Results and discussion
For a Bessel-Gauss beam with topological charge \( m \), we use the description of the propagating electrical field amplitude denoted by [9]:

\[
U(\rho, \theta, z_0) = c_0 e^{-\frac{\rho^2}{2w^2(z_0)}} e^{im\theta} \rho \left[ I_{m-1} \left( \frac{\rho^2}{2w^2(z_0)} \right) - I_{m+1} \left( \frac{\rho^2}{2w^2(z_0)} \right) \right]
\] (1)
Where \( c_0 = t \frac{\sqrt{\pi}}{2w_0} e^{\frac{i z}{2z_r}} e^{\frac{i m}{2} m e^{i k z_0}} \) is a complex number, \( w(z_0) \) the radius of the beam on the opening plane, \( R(z) = z \left[ 1 + \left( \frac{z_r}{z} \right)^2 \right] \) the wavefront radius, \( z_r = \frac{k w_0^2}{2} \) the Rayleigh range, \( w_0 \) the beam-waist, \( k \) the wave number of the monochromatic beam and, \( I \left( \frac{m \pm 1}{2} \right) \) the modified Bessel function. The factor \( e^{i m \theta} \) expresses OAM, so that the value of \( m \) indicates the number of wavefront rotations around the propagation axis, is the topological charge (See Figure 1).

![Figure 1](image1.png)

**Figure 1.** Magnitude and phase of a Bessel-Gaussian beam with topological charge \( m = 3 \).

This equation describes the field distribution at \( z = z_0 \), where is located the nanometric thick edge, centered on the optical axis of the system. After, an equilateral triangular aperture, AT, is used. This equilateral triangular aperture provides a simple and easy method for determining the magnitude and sign of the topological charge of the optical vortex, by counting the number of light spots along one side of the diffraction pattern [6, 7]. When an OAM beam is diffracted by a nanometric thick edge and after far field analyzed through AT, a characteristic pattern of lines that offers an alternative way to find the magnitude and sign of TC occurs (see Figure 2). Experimentally, we used nanometric thin films with transmittance very close to zero (\( TiO_2 @ 532nm \)) and thickness comparable to the wavelength.

![Figure 2](image2.png)

**Figure 2.** Diffraction pattern of an OAM beam with \( m = 3 \), after analysis by a triangular equilateral aperture AT.

For openings, generally the far field diffraction is mainly due to the interference of the diffracted beam by each edge, where each one has finite length with infinitely steepening. By combining the edge with the equilateral triangular aperture (see Figure 4), used here for analysis, we obtain the results shown in Figure 3. Linking already intensity distributions from the Equation (2).

\[
I_{Bor\;des\;ABC} = I_{nA} + I_{nB} + I_{nC} - I_{nAB} - I_{nBC} - I_{nCA} \tag{2}
\]
Where $I_{n_1}$ is the intensity diffraction pattern due to the opening $i$ (The addition is not the Born rule, it's just a simple way to determine the topological charge).

Suppose the edge thickness $d$, with nanoscale dimensions higher than wavelength ($d \gg \lambda$), and transmittance $T_1 = T_F \cdot T_g$ (See Figure 5). The film transmission, in a fixed position relative to the axis of the donut modifies its far field diffraction pattern.

A simple model which describes through their transmittance parameters, like thickness and refractive index, the influence produced by the nanometer thin film edge over a vortex, is proposed. As shown in Figure 5, the geometry of the proposed model, allows to introduce a complex term in mathematical Equation 1. The refractive index of a material ($n_1$), generally is associated with a propagation delay of the electromagnetic wave, we relates directly to a phase term. The phase must keep a proportionality between the total revolution ($2\pi \text{rad}$) and wavelength ($\lambda$) according to the thickness ($d$) of the film. From the standpoint of strength, it is considered a term of proportionality ($T_1$). When scanning transmittance with the model of Figure 5, is obtained the results of Figure 6.

![Figure 5](image5.png)

**Figure 5.** Geometry for interaction between a OAM diffracted beam by a straight edge of comparable thickness to the incident wavelength.

Numerically, diffracted beams are calculated with TC for $m = \pm 1, \pm 5$, diffraction patterns obtained are shown in Figure 7. The experimental implementation was performed with the setup shown schematically in Figure 8. Patterns obtained for far-field diffraction as shown in Figure 9.
Figure 7. Far-field diffraction of an OAM beam with integer TC, $m=\pm 1 \rightarrow \pm 5$, by the edge of the thin film, seen through AT (Numerical simulation results).

Figure 8. Schematic experimental setup: Laser (La), spatial light modulator (SLM) Polarizer (Po), Analyzer (An), Straight Border (SB), Triangular Equilateral Aperture (TEA), Fourier lens (FL) and CCD camera.

Figure 9. Experimental far-field diffraction, for $m=\pm 1 \rightarrow \pm 5$. Straight edge thin film: Ti=0.2%, e≈660nm.

3. Conclusions
The Fraunhofer diffraction patterns obtained by simulation and experimentally for light beams with integer orbital angular momentum, by the edge of different thin film thickness and transmittance are reported for first time, to our knowledge. The proposed mathematical model is in accord with numerical simulations and experimental results. The results obtained, experimentally and numerically, are in agreement. The proposed method is useful especially for high topological charges, bigger than $m=20$. 
From far-field diffraction pattern of an OAM beam by the edge of a thin film and seen through an equilateral triangular aperture, we can discern the value and sign of its topological charge, by a quick inspection. It is also possible to infer the value and sign of the topological charge, adding the diffraction patterns of rotated borders, individually and by couples, without using the Born rule.

We propose a new technique for determining the transmittance in thin films, using a beam with very high topological charge and analysis of their far-field diffraction pattern.

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