LHC Signatures for Cascade Seesaw Mechanism

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Abstract

Cascade seesaw mechanism generates neutrino mass at higher dimension (5+4$n$) operators through tree level diagram which bring the seesaw scale down to TeV and provide collider signatures within LHC reach. In particular, both Type-II scalar and Type-III heavy fermion seesaw signatures exist in such a scenario. Doubly charged scalar decays into diboson is dominant. We perform a thorough study on the LHC signals and the Standard Model background. We draw the conclusion that multilepton final state from interplay of doubly charged scalar and heavy fermion can provide distinguishable signatures from conventional seesaw mechanisms.

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I. INTRODUCTION

The effective dimension-five Weinberg operator [1] violates lepton number by two units and provides an explanation for the smallness of neutrino masses successfully if neutrinos are Majorana fermions. Three tree-level realizations of this operator correspond to three types of seesaw mechanism through exchange of singlet fermion [2], triplet scalar [3], and triplet fermion [4] respectively. These theories are often contemplated in the context of the grand unified theories which live in scales much higher than the electroweak theories, therefore, are beyond the reach of the present collider experiments. Even though, the phenomenological studies aimed at the TeV scale were investigated in the literature. In particular, the like-sign dilepton production at the hadron colliders for Type-I and Type-II seesaw mechanisms were addressed in [5] and [6] respectively. On the other hand, leptons plus jets final states from fermion triplet decays in Type-III seesaw were also studied in [7]. It was suggested that the discrimination of these underlying theories is possible by utilizing various multi-lepton signals [8].

Theoretically, several model building methods provide opportunities to investigate the neutrino mass generation directly by lowering the scale to the reach of colliders. For example, one may introduce extra particles and/or discrete symmetries to generate neutrino masses radiatively [9]. Alternatively, the so-called ”inverse seesaw mechanism” extends the conventional seesaw mass matrix with one additional vector-like singlet, $N = N_R + N_L$, and it turns out the light neutrino masses receive double seesaw suppression factor $\epsilon_L m_D^2/m_N^2$ ($m_D$, $m_N$ and $\epsilon_L$ are Dirac mass terms between $\nu_L - N_R$, $N_L - N_R$ and new $U(1)$ symmetry breaking scale associated to $N_L$ respectively) [10]. For a small $\epsilon_L$ and a TeV scale $m_N$, one would give the sub-eV neutrino masses without severely tuning Yukawa couplings [11]. A general extension of the vector-like singlet $N$ to higher multiplets was also proposed in Ref. [12].

Another method to lower the seesaw scale is that the neutrino mass originates from higher dimension operators [13]. In particular, there is a class of models called ”cascade seesaw mechanism” [14] and a similar idea can be found in [15]. In this case the neutrino masses are generated at tree level from a diagram as those of Type-I and Type-III seesaw mechanisms by introducing both new scalar and fermion multiplets, carrying quantum numbers
\((I_\Phi, Y_\Phi) = (n + \frac{1}{2}, 1)\) and \((I_\Sigma, Y_\Sigma) = (n + 1, 0)\) for \(n \geq 1\). In this type of model, the lepton number is violated by the mass insertion of the neutral component of the new fermion multiplet and the light neutrino masses are generated via a dimension-(5 + 4n) operator. The higher dimensionality of neutrino mass operator is due to the development of the vacuum expectation value (VEV) of higher isospin scalar field \(\Phi^{n+\frac{1}{2}}\) (scalar isospin \(n + \frac{1}{2}\) is denoted in this way hereafter). In order to generate a naturally small VEV for \(\Phi^{n+\frac{1}{2}}\) one needs to engineer the scalar potential in a way similar to the Type-II seesaw mechanism. In the context of Type-II seesaw mechanism the sign of the scalar triplet quadratic term is positive and the lepton number breaking term (trilinear term of scalar triplet and Higgs doublet) will trigger the VEV development of scalar triplet. As a result, a seesaw structure between the VEV and the mass of the triplet scalar can be obtained. The development of \(\langle \Phi^{n+\frac{1}{2}} \rangle\) is realized by generating VEVs of a series of isospin multiplets step by step so that the neutrino mass is suppressed by intermediated \(\Phi\) and \(\Sigma\) with masses located within the reach of LHC.

In this paper we investigate the LHC collider signatures for such a general scenario of tree level seesaw mechanism with a heavy fermion exchange. We find one of the general features of the cascade seesaw mechanism is that both the Type-II and Type-III seesaw particles exist. This paper is organized as follows: we introduce the generic idea of cascade seesaw mechanism in Section-II. In Section-III we briefly describe the minimal version of the cascade seesaw mechanism, and then we further investigate its collider phenomenology in detail in Section-IV. Then we draw the conclusion in Section-V.

II. CASCADE SEESAW MECHANISM

The requirement of high scale canonical seesaw mechanism is because the neutrino Dirac mass term \(m_D\) is usually considered at the electroweak scale. Naively, if one takes \(m_D \simeq m_e\) the seesaw scale is down to \(O(100)\) GeV, which can be realized by generalizing the tree level seesaw diagram as shown in Fig. 1. Yukawa interactions involving the Standard Model (SM) left-handed lepton doublet \(l_L\) are written as

\[
\mathcal{L}_{\text{Yukawa}} \supset y_{\alpha\beta} \bar{l}_{L\alpha} H l_{R\beta} + g_{\alpha i} \bar{l}_{L\alpha} \Phi^{n+\frac{1}{2}} \Sigma_i + h.c.,
\]

1 We use the convention \(Q = I_3 + \frac{Y}{2}\) in this paper.
where \( y, g \) are Yukawa couplings, \( c \) is the charged conjugation, \( H \) denotes the SM Higgs doublet, \( \alpha \) and \( \beta \) refer to flavor indices \( e, \mu, \tau \), and \( i \) is the new fermion (\( \Sigma \)) generation index. \( \Sigma \) field is self-conjugated and can form a Majorana mass term, \( \frac{1}{2} \Sigma^c M \Sigma + \text{h.c.} \), expanded as

\[
\frac{1}{2} \Sigma^{(n+1)} c M \Sigma^{-(n+1)} - \frac{1}{2} \Sigma^{(n)} c M \Sigma^{-n} - \ldots + \frac{1}{2} \Sigma^{0} c M \Sigma^{0} + \ldots + \frac{1}{2} \Sigma^{-(n+1)} c M \Sigma^{+(n+1)}. 
\]

(2)

We choose a real and diagonal basis here without the loss of generality. The charged Dirac fermions can be defined as \( \Sigma^{+(n+1)} + (\Sigma^{-(n+1)})^c, \Sigma^{+n} - (\Sigma^{-n})^c, \ldots \), and the neutral Majorana fermion is \( \Sigma^0 + (\Sigma^0)^c \). In such a way the chiral anomaly is cancelled and the lepton number is violated by the Majorana mass term of \( \Sigma^0 \). The diagram can be divided into two parts, \( i.e. \) Dirac mass from the second term in Eq. (1) and a heavy fermion intermediary. When \( \Phi^{n+\frac{1}{2}} \) is taken to be the SM Higgs doublet, one can retain the Type-I and Type-III seesaw mechanisms with iso-singlet and iso-triplet fermion as the intermediary respectively.

Different from the canonical seesaws, the black dot vertices in Fig. 1 represent how \( \Phi^{n+\frac{1}{2}} \) develops its VEV via the cascade chain effect involving multi-Higgs external lines. On the other hand, the constraints from electroweak precision measurements put an upper limit on the scalar multiplet VEV. Since \( \Phi^{n+\frac{1}{2}} \) carries higher isospin, the tree level \( \rho \) parameter is given by

\[
\rho = \frac{n(n+2)}{1 \sqrt{2}} \langle \Phi^{n+\frac{1}{2}} \rangle^2 + \frac{1}{2} \langle v^2 + \langle \Phi^{n+\frac{1}{2}} \rangle^2 \rangle \left( \frac{v^2}{\langle \Phi^{n+\frac{1}{2}} \rangle^2} \right),
\]

(3)

where we take the SM Higgs VEV \( \langle H \rangle = \frac{v}{\sqrt{2}} \). From \( \rho = 1.0004^{+0.0003}_{-0.0004} \), the first term in the numerator denotes the deviation from 1 and constrains the \( \langle \Phi^{n+\frac{1}{2}} \rangle \lesssim O(1) \text{ GeV} \). The key point to obtain a small VEV of \( \Phi^{n+\frac{1}{2}} \) in the cascade seesaw mechanism is the relation between lepton number violation and the development of \( \langle \Phi^{n+\frac{1}{2}} \rangle \). To make it clear, there
exist one renormalizable coupling of scalar multiplet to Higgs fields in the potential, which is linear to the lowest isospin state of $\Phi^{n+\frac{1}{2}}$, $\Phi^{3/2} \tilde{H} \tilde{H}$. Here $\Phi^{3/2}$ is the quadruplet by taking $n = 1$ and this term together with the Yukawa interactions shown in Eq. (1) violates lepton number explicitly. The terms relevant to the spontaneous symmetry breaking are

$$V^{\frac{3}{2}} \supset -\mu_H^2 H^\dagger H + \lambda_H (H^\dagger H)^2 + \mu^2_{3/2} \Phi^{3/2} \Phi^{3/2} - [\kappa \Phi^{3/2} \tilde{H} \tilde{H} + \text{h.c.}],$$

where the first two terms are the Higgs potential. The quadratic term of $\Phi^{3/2}$ is set to be positive, instead the $\kappa$ term will induce the VEV development. As a result, the VEV of $\Phi^{3/2}$ receives a suppression factor, $\langle \Phi^{3/2} \rangle \propto v^3 / \mu^{2}_{3/2} \Phi^{3/2}$ which is similar to Type-II seesaw mechanism.

To generalize this case to higher isospin $(k+\frac{1}{2})$ multiplets, one has to utilize the next-to-higher isospin $(k-\frac{1}{2})$ field as a bridge to develop the VEV. For example, the quartic term $\Phi^{5/2} \Phi^{3/2} \tilde{H} \tilde{H}$ will induce $\langle \Phi^{5/2} \rangle$ after $H$ and $\Phi^{3/2}$ develop VEVs. The procedure can be applied to a sequence of scalar fields $\Phi^{k+\frac{1}{2}}$.

$$V^{n+\frac{1}{2}} \supset -\mu^2_H H^\dagger H + \lambda_H (H^\dagger H)^2 + \sum_{k=1}^{n} \mu^2_k \Phi^{k+\frac{1}{2}} \Phi^{k+\frac{1}{2}} - \sum_{k=1}^{n} [\lambda_k (\Phi^{k+\frac{1}{2}} \Phi^{k-\frac{1}{2}} H \tilde{H}) + \text{h.c.}].$$

Notice that another reason for the positivity of $\mu_k^2$ terms is to prevent the tachyonic fields for higher isospin multiplets. The general VEV of $\Phi^{n+\frac{1}{2}}$ can be derived as

$$\langle \Phi^{n+\frac{1}{2}} \rangle = \frac{v^{2n+1}}{2^{n+1}} \prod_{k=1}^{n} \frac{1}{2 \sqrt{2k+1} \mu^2_k} \lambda^*_k,$$

here the factor $v^{2n+1}$ is corresponding to $2n+1$ Higgs external lines illustrated in Fig. 1. It can be read that the VEV of $\Phi^{n+\frac{1}{2}}$ is suppressed by integrating out the intermediate scalar multiplets via the cascade chain. Therefore, the general seesaw formula for the neutrino masses can be expressed as

$$m_{\nu_{\alpha\beta}} = \sum_{i} \frac{(-1)^n g_{\alpha i} g_{i \beta}}{(2n+3)M_{\Sigma_i}} \frac{v^{4n+2}}{2} \prod_{k=1}^{n} \frac{1}{2 \sqrt{2k+1} \mu^2_k} \lambda^*_k.$$

### III. THE MINIMAL MODEL

It is obvious that the minimal scenario in this class of models appears when $n = 1$. In addition to the SM model particles, we have two extra fields, a scalar quadruplet and a fermion quintuplet,

$$\Phi^{3/2} = (\Phi^+, \bar{\Phi}^+ \Phi^0, \Phi^-)^T \quad \text{and} \quad \Sigma = (\Sigma^{++}, \Sigma^+, \Sigma^0, \Sigma^-, \Sigma^{-})^T.$$
As we can see the scalar triplet in Type-II seesaw and the fermion triplet in Type-III seesaw mechanisms are embedded in the particle content. Thus the general features of cascade seesaw mechanism are the coexistence of Type-II and Type-III seesaw mechanisms. The scalar potential is given by

$$V(H, \Phi^{3/2}) = -\mu^2 H \dagger H + \lambda(H \dagger H)^2 + \mu_{\Phi^{3/2}}^2 \Phi^{3/2} \Phi^{3/2} + \lambda_1 (\Phi^{3/2} \Phi^{3/2})^2$$

$$+ \lambda_2 \bar{\Phi}^{3/2} \Phi^{3/2} \bar{\Phi}^{3/2} \Phi^{3/2} + \lambda_3 H \dagger H \Phi^{3/2} \Phi^{3/2} + \lambda_4 H \dagger H \bar{\Phi}^{3/2} \Phi^{3/2}$$

$$+ (\lambda_5 \bar{\Phi}^{3/2} \bar{H} H \bar{H} + \lambda_6 H H \bar{\Phi}^{3/2} \bar{\Phi}^{3/2} + \lambda_7 H \bar{\Phi}^{3/2} \Phi^{3/2} \Phi^{3/2} + \text{h.c.}),$$

(9)

and it leads the VEV of $\Phi^{3/2}$ to be $\lambda_5^* v^3 / \sqrt{3} \mu_{\Phi^{3/2}}^2$. The tree level contribution to the neutrino mass is obtained,

$$m_{\nu_{\alpha \beta}} = -\frac{1}{6} \lambda_5^* v^6 \sum_i \frac{g_{\alpha i} g_{i \beta}}{M_{\Sigma_i}},$$

(10)

with $i$ stands for the number of the fermion quintuplet. For $v = 174$ GeV, $\lambda_5 = 10^{-3}$, $M_{\Sigma} \sim 10^2$ GeV and $m_{\nu} \sim 0.1$ eV, we have Yukawa couplings around $10^{-2} - 10^{-1}$.

The relatively large Yukawa couplings would significantly enhance the search probability at the LHC. The model would also give flavor changing interactions due to the mismatch between the gauge eigenstates and mass eigenstates of neutrinos. The two eigenstates can be related by an unitary matrix,

$$\begin{pmatrix} \nu_L \\ \Sigma^0 \end{pmatrix} = U \begin{pmatrix} \nu_{mL} \\ \Sigma^0 \end{pmatrix} \quad \text{with} \quad U = \begin{pmatrix} U_{PMNS} & V_{\nu \Sigma} \\ V_{\Sigma \nu} & 1 \end{pmatrix}.$$

(11)

Here we assume $M_{\Sigma} \gg m_D$. The gauge neutral current can be written as

$$\mathcal{L}_{NC} = \frac{g}{c_W} \left[ \frac{1}{4 \sqrt{2}} \bar{\nu} \left( U_{PMNS}^\dagger V_{\nu \gamma} \gamma^\mu (1 - \gamma_5) - V_{PMNS}^T V_{\nu \gamma}^* \gamma^\mu (1 + \gamma_5) \right) \right] \Sigma^0$$

$$+ \frac{\sqrt{3}}{8} \bar{\nu} V_{\nu \gamma}^* \gamma^\mu (1 + \gamma_5) \Sigma^+ \right] Z_\mu + \text{h.c.},$$

(12)

and the gauge charged current as

$$\mathcal{L}_{CC} = g \left[ -\frac{\sqrt{3}}{2 \sqrt{2}} \bar{\nu} \left( V_{PMNS}^\dagger V_{\nu \gamma} \gamma^\mu (1 - \gamma_5) + V_{PMNS}^T V_{\nu \gamma}^* \gamma^\mu (1 + \gamma_5) \right) \Sigma^+ \right. \right.$$  

$$\left. - i V_{\nu \gamma} \gamma^\mu (1 - \gamma_5) \Sigma^0 + \frac{3}{2} F V_{\nu \gamma}^* \gamma^\mu (1 + \gamma_5) \Sigma^{++} \right] W^-_\mu + \text{h.c..}$$

(13)

Just like the canonical seesaw mechanism, in the minimal realizations the mixing between the light and heavy neutrinos is predicted as $V_{\nu \Sigma} \approx \sqrt{m_{\nu} / M_{\Sigma}}$ thus is suppressed. This makes
some processes with mixing parameters involved difficult to produce at the LHC. However, it is known that a significant mixing between light-heavy neutrinos can be obtained as large as $10^{-2}$ which is around the experimental upper limit. Hence the mixings decouple from the mass ratio, if $V_{i\Sigma}$ is of rank 1 or $\text{Tr}(m_D^T M_{\Sigma}^{-1} m_D) = 0$ for three generations of $\Sigma$ field [18].

In the following section we will study the processes which are insensitive to the mixings.

IV. COLLIDER PHENOMENOLOGIES

Both Type-II and Type-III seesaw particles exist in the minimal cascade seesaw model. The doubly charged scalar in Type-II seesaw model and the exotic heavy fermion in Type-III seesaw model have been searched at LEP [19], Tevatron [20] and the LHC [21–23]. The up-to-date lower limits for the mass of the doubly charged scalar are obtained to be 409 GeV, 398 GeV and 375 GeV if the 100% branching ratio of the doubly charged scalar decays into $e^\pm e^\pm$, $\mu^\pm \mu^\pm$ and $e^\pm \mu^\pm$ is assumed respectively [22, 23]. However, we shall emphasize here that the above constraints do not apply to the doubly charged scalar in cascade seesaw models in which the tree level $\Phi^{\pm\pm}$ coupling to dilepton is absent. The branching ratios of like-sign dilepton channels are always negligible, which is independent of the multi-scalar VEV. The decay channels of $\Phi^{\pm\pm}$ are $W^{\pm(*)} W^{\pm(*)}$ and $\Sigma^{\pm l\pm}$ if kinematically allowed. We plot the decay widths and the branching ratios of these two decay channels in Fig. 2 by taking $M_{\Sigma^{\pm}} = 420$ GeV as the benchmark point [24]. We find the diboson channel is dominant in the case of Yukawa coupling $g = 0.01$ and scalar VEV $V_{\Phi} = 1$ GeV. A recent study sets the limits of doubly charged scalar mass to be below 43 GeV and 60 GeV by using the data at LEP and at the LHC with 7 TeV collision energy and the $4.7 \text{ fb}^{-1}$ integrated luminosity. A lower limit is evaluated to be 85 GeV if the extrapolation of the data to 20 $\text{ fb}^{-1}$ is used [25]. These are the constraints we should adopt in our discussion. For the searches of the fermionic triplet lepton in Type-III seesaw model, the CMS collaboration has reported the lower limits ranging from 180 to 210 GeV in events selected with 3 isolated leptons (the range depends on the selected lepton flavors) at $\sqrt{s} = 7$ TeV and an integrated luminosity of $4.9 \text{ fb}^{-1}$ [26]. While the ATLAS collaboration excludes the heavy lepton mass below 245 GeV in the event selection of at least 4 charged lepton in the final states at $\sqrt{s} = 8$ TeV and $5.8 \text{ fb}^{-1}$ of luminosity with the mixing parameter at $O(10^{-2})$. Interestingly, the probability to have equal to or more than the observed number of events with a background
FIG. 2: Decay width and branching ratio of doubly charged scalar decaying into diboson and heavy fermion with scalar VEV equals to 1 GeV, heavy fermion to be 420 GeV and Yukawa coupling equals to 0.5, 0.1 and 0.01, respectively.

only hypothesis, $p_0$, is found to be 0.20 at heavy fermion mass around 420 GeV \cite{24}. More data accumulation will be helpful to demonstrate this signal event.

Now we can turn to study the collider signatures of the cascade seesaw mechanism. Throughout the following analysis we ignore the mass splittings between the components in the multiplet for simplicity. The pair production of both doubly charged scalars $\Phi^{\pm\pm}\Phi^{\mp\mp}$ and exotic heavy leptons $\Sigma^+\Sigma^0$ are shown in Fig. 3 for 8 TeV (left panel) and 14 TeV (right panel) LHC. Our result is consistent with previous studies \cite{6–8,27}. Comparing the pair production cross section between the doubly charged scalars and the heavy fermions, fermion pair production has a larger cross section over scalar about $\mathcal{O}(10^2)$ which is naively because $\Phi^{\pm\pm}\Phi^{\mp\mp}$ is produced via quark-antiquark and is much less than quark-quark in the proton-proton collision. At 8 TeV with integrated luminosity assumed to be 20 fb$^{-1}$, there is still more than 1 number of event expected with the scalar mass up to 600 GeV. For future 14 TeV run of the LHC, more optimistic number of events can be obtained with 300 fb$^{-1}$ luminosity for both scalar and fermion mass up to 1 TeV as shown in the right panel of Fig. 3.

Since the doubly charged scalar decays into leptons is highly suppressed, we will consider the specific process of $pp \to Z^*/\gamma^* \to \Phi^{\pm\pm}\Phi^{\mp\mp} \to 2W^+2W^-$ and its signatures at LHC with $M_\Phi = 300$ GeV. The total cross section of this process can be obtained through the general formula $\sigma = \int f_a(x_1, Q^2)f_b(x_2, Q^2)\hat{\sigma}_{q\bar{q} \to 4l + E_T}(x_1x_2s)dx_1dx_2$, with $\hat{\sigma}_1 = \int \frac{1}{2s} |M|^2 d\text{lips}_8$, and $\hat{\sigma}_2 = \hat{\sigma}_1(p_1 \leftrightarrow p_2)$. $\text{lips}_8$ represents the 8-body final state Lorentz
FIG. 3: The cross section for doubly charged scalar and fermion pair production process $pp \rightarrow \Phi^{++}\Phi^{--}/\Sigma^{+}\Sigma^{0}$ with centre of mass energy at 8 (14) TeV luminosity assumed to be 20 (300) fb$^{-1}$.

invariant phase space, $f_a(x_1)(f_b(x_2))$ is the parton distribution function (PDF) of initial state quarks, $\sqrt{s}$ is the center of mass energy (c.m.) of parton-parton collision, and $\hat{\sigma}$ is the partonic level cross section for $q\bar{q}$ process. The eight body final state cross section and contributions from the SM background is also listed in Table I. at 8 TeV and 14 TeV LHC. For the process $\Phi^{++}\Phi^{--} \rightarrow 2W^+2W^- \rightarrow 4l + E_T$, basic cuts including transverse momentum $p_T$, missing energy $E_T$, pseudorapidity $|\eta|$ and minimal separation $\Delta R_{\text{min}}$ cuts are chosen to be

$$p_T > 30\text{GeV}, E_T > 30\text{GeV}, |\eta| < 2.5 \text{ and } \Delta R_{\text{min}} = \min(\sqrt{\Delta \eta^2 + \Delta \phi^2}) > 0.4,$$

(14)

respectively according to different distributions of signal and the SM background. To be more realistic, we also perform simple detector simulation by smearing the leptons and jets energies according to the assumption of the Gaussian resolution parametrization

$$\frac{\delta(E)}{E} = \frac{a}{\sqrt{E}} \oplus b,$$

(15)

where $\delta(E)/E$ is the energy resolution, $a$ ($b$) is a sampling (constant) term, and $\oplus$ denotes a sum in quadrature. We take $a = 5\%$, $b = 0.55\%$ for leptons and $a = 100\%$, $b = 5\%$ for jets respectively [28, 29] and use Madgraph to perform background analysis [30]. We find that in the case of 4-lepton final state at 14 TeV, there is 5 number of events and 1.6 $\sigma$ significance, which makes this process observable. With one $W$ decays hadronally, we choose additional basic cuts for jets as $p_T^j > 20\text{GeV}$, and $|\eta_j| < 2.5$. After basic cuts, the significance of $l^+2l^-2j$ process becomes small. However, the lower limit on the doubly charged scalar mass is rather
\begin{table}[h]
\centering
\begin{tabular}{|c|c|c|c|c|c|}
\hline
[TeV] & Signal & Background & S/B & S/\sqrt{S+B} \\
\hline
\hline
$\Phi^+\Phi^- \rightarrow 2W^+2W^- \rightarrow$ & & & & \\
14 & $2l^+2l^- + \not{E}_T$ & $W^+W^-W^-W^-$ & 0.50 & 2.24 \\
+basic cuts & & & & \\
8 & 0.29 & 0.004 & 3.4 & 0.09 & 0.034 \\
\hline
$t^+l^-W^+ \rightarrow 2l^- 2j + \not{E}_T$ & $t\not{E}_T W^- W^-W^+2j W^-Z2j$ & & & \\
$P_T^{l,j} > 20(30)\text{GeV}$ & 45 & 304.5 & 48.24 & 16722 & 0.003 & 0.344 \\
& 23 & 146 & 23.8 & 7958 & 0.002 & 0.25 \\
$\not{E}_T > 30\text{GeV}$ & 22.4 & 140 & 18.8 & 3392 & 0.006 & 0.37 \\
& 19.1 & 125.3 & 13.9 & 2531 & 0.007 & 0.37 \\
$|\eta_{l,j}| < 2.5$ & 10.2 & 122 & 11.8 & 2165 & 0.004 & 0.21 \\
$\Delta R > 0.4$ & 5.8 & 7.09 & 0.814 & 355.4 & 0.016 & 0.302 \\
8 & & & & \\
\hline
$\Sigma^+\Sigma^0 \rightarrow$ & & & & \\
14 & $l^+l^-Z^0 \rightarrow 3l^+l^- + 2j$ & $t\not{E}_T W^- W^-Z W^+ZZ$ & & \\
& 177 & 381.9 & 0.1854 & 6.249 & 0.46 & 7.4 \\
& 4.2 & 6.1 & 0.0043 & 0.1994 & 30 & 2.4 \\
8 & & & & \\
\hline
$t^+l^-l^+W^+ \rightarrow 2l^- 2j + \not{E}_T$ & $W^+W^-W^+Z$ & & & \\
14 & 177 & 0.4137 & & 427.8 & 13.1 \\
8 & 4.2 & 0.01 & & 420 & 2.04 \\
\hline
$t^+l^-l^-W^+ \rightarrow 3l^+l^- + \not{E}_T$ & $W^+W^-Z2j$ & & & \\
14 & 29.5 & 0.0417 & 2.191 & 13.2 & 5.24 \\
8 & 0.7 & 0.001 & 0.076 & 9.1 & 0.79 \\
\hline
\end{tabular}
\caption{Number of events of scalar $M_\Phi = 300$ GeV and fermion $M_\Sigma = 300$ GeV pair production and decay as well as corresponding SM backgrounds at centre-of-mass energy 8 TeV (20fb$^{-1}$) and 14 TeV (300fb$^{-1}$). In both signal and backgrounds, charged lepton of $e^-$ and $\mu^-$ are included. In heavy fermion decay process, $|V_{l\Sigma}| = g = 0.01$ is adopted as mixing parameter and Yukawa coupling.}
\end{table}

weak, and can be as low as 85 GeV in cascade seesaw. According to our estimation, the number of events from $l^+2l^-2j + \not{E}_T$ signal can increase from $\mathcal{O}(10)$ to $\mathcal{O}(10^4)$ for doubly charged scalar mass from 300 GeV to 165 GeV and predict large enough significance for observation. It will significantly increase the testability of the model at the LHC.

For heavy fermions, specifically we choose the same process as the LHC search \cite{24} and study $pp \rightarrow W^+(k) \rightarrow \Sigma^+\Sigma^0$ decaying into $\Sigma^+ \rightarrow l^+Z^0 \rightarrow l^+l^+l^-$ and $\Sigma^0 \rightarrow l^-W^+ \rightarrow l^-jj'$ i.e., $l^+l^+l^-jj'$ final state. Together with other final states as $3l^+l^-2j$ which violates lepton...
number explicitly, $3l^+2l^-\mathbb{E}_T$ which is purely multi-lepton final state, we find that the SM background is much smaller than the chosen signal processes, providing clean signatures. The pair production of doubly charged fermions is also studied in [17]. For $2l^+2l^-+\mathbb{E}_T$ final state from the doubly charged fermion pair decay, we find that at 8 TeV LHC hundreds of $\Sigma^+\Sigma^-$ can be produced, which also makes the SM background process $W^+W^+W^-W^-$ and $W^+W^-Z$ negligible as shown in the first process of Table. [17] with the same final state. Therefore, heavy fermion production processes provide promising clean channels for LHC study on cascade seesaw mechanism.

In addition, larger multiplets in such a model contains more particles which guarantees us distinctive signatures. We find a novel process as plotted in Fig. 4 that the doubly charged scalar production and then decays into heavy fermions with 8-lepton final state without missing energy. As shown in Fig. 4, $pp \rightarrow \Phi^+\Phi^- \rightarrow \Sigma^+\Sigma^-l^+l^- \rightarrow 4l^+4l^-$ cross section as well as number of events with luminosity 20 fb$^{-1}$ for 8 TeV (left panel) and 300 fb$^{-1}$ for 14 TeV (right panel) are displayed. Such a multi-lepton process is very clean from the SM background. The only possible SM background is from $4Z$ decaying leptonically, but according to our estimation it is at order of $10^{-7}$ fb at 14 TeV LHC thus is negligible. For illustration, we take $M_\Sigma = 420$ GeV Yukawa couplings $g =$0.05, 0.1 and 0.5 respectively. In this process, the mixing parameter $|V_{l\Sigma}|^4$ appeared in the coupling can be cancelled by $|V_{l\Sigma}|^4$ in the propagators using narrow width approximation. As a result, this process is insensitive to the mixing parameter between light and heavy fermions. Obviously, for 8 TeV run of LHC only $g > 0.05$ with scalar mass below 500 GeV we can predict significant signal events for observation. For 14 TeV LHC, with Yukawa coupling approximates to 0.05 and $M_\Phi$ up to 1 TeV, there are still more than 10 number of events expected.
FIG. 5: The cross section for $pp \rightarrow \Phi^{++}\Phi^{--} \rightarrow \Sigma^{+}\Sigma^{-}l^{+}l^{-} \rightarrow 4l^{+}4l^{-}$ at center-of-mass energy equals to 8 TeV (left panel) and 14 TeV (right panel). Different mixing parameters are chosen as Yukawa coupling $g = 0.5$ (black solid line), 0.1 (red dashed line) and 0.05 (green dotted line).

V. CONCLUSION

Higher dimension operators give an explanation to the smallness of the neutrino masses and provide the opportunity to probe the origin of neutrino mass mechanism. Cascade seesaw mechanism is based on the spirit of the canonical seesaw mechanism with the extension of the scalar and fermion sectors to higher dimension representations. The neutrino masses are hence generated at dimension $5+4n$ operators. We review the main consequences of the cascade seesaw mechanism in a general form. A novel signature in cascade seesaw models is that both Type-II and Type-III seesaw particles exist. Then we study the LHC signatures for the minimal model with detailed analysis of signals and the corresponding SM background. For the extra scalar multiplets, we discuss the processes with scalar decaying into diboson which is the dominant process for most of the parameter space. Since such a decay channel of doubly charged scalar receives weaker constraints from experiments, the mass can be smaller enough to produce large number of events for observation. To be consistent with the experimental search on Type III heavy fermion, we study neutral and singly charged heavy fermion production. Clean multi-lepton signatures from SM background are studied. The most non-trivial signature is heavy fermion associated with lepton from doubly charged scalar decay with 8-lepton final state. This process provides a distinctive signal from other seesaw models.
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