The Piezoaxionic Effect

Amalia Madden - Perimeter Institute for Theoretical Physics

Based on work w/ Asimina Arvanitaki (PI) and Ken Van Tilburg (NYU)
Outline

• **Motivation**
  
  The QCD axion

• **The Piezoaxionic Effect**
  
  P and T violation in nuclei, atoms and crystals
  
  Proposed experimental setup and sensitivity

• **The Ferroaxionic Effect**
  
  Axion-mediated forces
Strong CP Problem

\[ \mathcal{L}_{SM} \supset \frac{\theta_0}{32\pi^2} \text{tr } G\tilde{G} \]

- Physical angle \( \bar{\theta} = \theta_0 + \arg \det[M_q] \)

- Neutron EDM of size \( d_n \sim \bar{\theta} \cdot 10^{-16} \cdot e \cdot \text{cm} \)

- Experimentally, \( d_n \lesssim 10^{-26} \cdot e \cdot \text{cm} \implies |\bar{\theta}| < 10^{-10} \)

\[ \mathcal{L} \supset \frac{a}{32\pi^2 f_a} \text{tr } G\tilde{G} \text{ dynamically solves strong CP problem} \]
Photon vs Gluon Couplings

https://cajohare.github.io/AxionLimits/
Wavy Dark Matter

Bosonic DM has wave-like properties when $n_{DM} > \frac{1}{\lambda_{DM}^3}$. In our galaxy: $m_{DM} < 1eV$.

- Locally, $a(t) \approx a_0 \cos \frac{m_a c^2}{\hbar} t$
- Amplitude $a_0 \propto \sqrt{\rho_{DM}} \frac{1}{m_a}$
- Small frequency spread (coherence) $\delta \omega_a \approx \frac{v^2}{\hbar} \omega_a \approx 10^{-6} \omega_a$
Dark matter production

\[ \ddot{a} + 3H(T)\dot{a} + m^2 a = 0 \]
(H = Hubble parameter)

- \( m < 3H \): frozen
- \( m > 3H \): oscillates around minimum

\[ \frac{\rho_a}{\rho_{total}} = 0.25 < \theta_{initial}^2 > \left( \frac{f_a}{5 \times 10^{12} \text{GeV}} \right)^{7/6} \]
and scales as \( a^{-3} \)
Dark Matter

$m_a \sim 6 \times 10^{-11} eV \left( \frac{10^{17} GeV}{f_a} \right)$
The Piezoaxionic Effect

\[ \mathcal{L} \supset \frac{a}{f_a} G \tilde{G} \]
Piezoelectric Crystals

- Crystal structure breaks parity symmetry
  \((x, y, z) \neq (-x, -y, -z)\)

- Deformation causes electric dipole moment across unit cell (and vice versa).

![Diagram showing piezoelectric effect](image)
Constitutive Equations for Piezoelectricity

\[ \theta_a(t) \equiv \frac{a(t)}{f_a} \]

Stress = \( + \frac{1}{\epsilon} \cdot \text{Strain} \)

Electric Field = \( - \frac{h}{\epsilon} \cdot \text{Strain} \)

\( \theta_a(t) \) is parity even
\( \gamma_a(t) \) is parity odd
\( \theta_a(t) \) is time-reversal odd

Nuclear Spin Direction
The piezoaxionic tensor $\xi$ is **ODD** under parity, and can only be present in piezoelectric materials.

The electroaxionic tensor $\zeta$ is **EVEN** under parity, and can be present in all dielectrics.

**We will focus on $\xi$ in this talk!**
How big is the piezoaxionic effect?
QCD axion dark matter induces an oscillating nuclear electric dipole moment (EDM):

\[ d_n \sim 10^{-16} \frac{\sqrt{\rho_{DM}}}{m_a f_a} \cos m_a t \cdot e \cdot \text{cm} \]

EDM generates an oscillating stress on unit cell:
Schiff Suppression

If we treat an atom as a system of static, point-like particles, nuclear EDM is perfectly shielded by electron cloud [Schiff 1963].

Resolution: Schiff’s theorem violated by finite size effects:

\[ V_e = 4\pi e \mathcal{S} \cdot \nabla(\delta_e(r)) \]

\[ \mathcal{S} \sim e \frac{\tilde{\theta}_a R_0^2}{m_N} \propto A^{2/3} \quad \text{non-deformed nuclei} \]

\[ \mathcal{S} \sim eZ \frac{\tilde{\theta}_a R_0^2}{m_N} \propto Z A^{2/3} \quad \text{pear shaped nuclei} \]
In a piezoelectric crystal, the ground state electron wave function is a mixture of opposite parity orbitals $\epsilon_s$ and $\epsilon_p$:

$$|\psi\rangle_e = \epsilon_s |s\rangle + \epsilon_p |p\rangle$$

The piezoaxionic tensor can be estimated as:

$$\xi \sim \partial_{\text{Strain}} \frac{\langle H_{\text{Schiff}} \rangle}{V_{\text{cell}}} \approx \frac{Z^2}{a_0^4} \frac{dS}{d\theta_a} \times \frac{N_S}{V_{\text{cell}}} \frac{\partial(e_s \epsilon_p^*)}{\partial_{\text{Strain}}} \sim \mathcal{O}(1) \text{ factor}$$

Bigger in strongly piezoelectric materials
strain = \frac{\Delta L}{L}

S = |\xi c^{-1} \hat{I} \vec{\theta}_a| \sim 10^{-26}

\xi = \text{Piezoaxionic tensor}

\hat{I} = \text{nuclear spin direction}

Axion theta angle \propto \frac{\sqrt{\rho_a}}{m_a f_a}

\text{elastic stiffness tensor}

\text{Piezoelectric factor}

\text{Schiff potential}
Resonant Mass Detectors

In the 1960’s:
Weber Bar, $S \sim 10^{-17}$

AURIGA, NAUTILUS,
MiniGrail, $S \sim 10^{-25}$

Goryachev et al. 2014
$S \sim 10^{-22}$

$0.1 - 1 \text{kHz}$

$\text{MHz} - \text{GHz}$
Experimental Setup

1. Find a piezoelectric material with low mechanical noise and big Schiff moments

2. Cool to $\sim mK$

3. Align nuclear spins using a magnetic field

4. Measure tiny oscillating voltage using a SQUID
Materials

Piezoelectric make up a large class of materials - 20 out of 32 symmetry groups!

- High density of nuclei with large Schiff moments and low radioactivity
- Good acoustic properties (high Q-factor)
- Strong piezoelectric properties
- Structural similarity to well-known resonator crystals.

| Class | Candidates | Similar Crystals |
|-------|------------|------------------|
| 32    | NaDyH₂S₂O₉ | SiO₂ (quartz)    |
|       | BiPO₄      | Ga₅La₃SiO₁₄ (langasite) |
|       | UOF₄       | tourmaline        |
| 3m    | UCd        | LiNbO₃ (lithium niobate) |
| 4mm   | DySi₃Ir    | Li₂B₄O₇ (lithium tetraborate) |
|       | DyAgSe₂    |                 |
| 42m   | DyAgTe₂    | NH₆PO₄ (ADP)     |
|       | Dy₂Be₂GeO₇ | KH₂PO₄ (KDP)     |
| mm₂   | UCO₅       | Ba₂NaNb₅O₁₅ (barium sodium niobate) |

Candidate materials collected from the database at https://materialsproject.org/
Scanning

- Grow a series of crystals of different thicknesses
- Vary *electrical* resonance frequency using capacitor and inductor
Backgrounds:

Fluctuating nuclear spins
Small effect

Fluctuating magnetic impurities in material
$\lesssim$ ppm

Magnetization noise $\rightarrow$ fictitious EMF

Vibrational noise
Systematic, demonstrated at AURIGA

Noise:

Thermal noise limited, main sources: crystal mechanical noise and SQUID noise
Idealized Forecast

*parameter space above QCD axion line tuned in mass and vacuum alignment

BBN: K. Blum, R. T. D’Agnolo, M. Lisanti, B. R. Safdi (2014)

GWs: J. Zhang, Z. Lyu, J. Huang, M. C. Johnson, L. Sagunskii, M. Sakellariadou, H. Yang (2021).

Sun: A. Hook, J. Huang (2018)

WDs: R Balkin, J Serra, K Springmann, S Stelzl, A Weiler (2022)

Superradiance: A. Arvanitaki, S. Dubovsky (2011)
Axion-Electron Coupling

\[ H_{aee} \simeq -\frac{G_{aee}}{2} \sigma_e \left( \nabla a + \frac{\dot{a}}{m_e} \right) \]

P EVEN
T ODD

P ODD
T EVEN
Future Directions

• Precise Schiff moment calculations for stable, octupole deformed nuclei

• Density functional theory (DFT) calculations for $\xi$ and $\zeta$

• Experimental investigation of suitable materials
1880: Curie brothers discover “direct” piezoelectric effect
Stress -> Charge

1881: Gabriel Lippman predicts “converse” effect from thermodynamics
Charge->Stress

Curie brothers experimentally verify
Ferroaxionic effect

\( \mathcal{L} \supset \frac{a}{f_a} G\tilde{G} \)

\( \mathcal{L} \supset G_{aNN} \nabla a \cdot \sigma_N \)

(As seen yesterday in Andy Geraci's talk!)
Monopole-dipole potential:

$$U_{s,p}(r) = \frac{g_s g_p}{m_N} \left( \frac{m_a}{r} + \frac{1}{r^2} \right) e^{-m_ar} \left( \hat{r} \cdot \hat{\sigma} \right)$$

$$= \gamma \vec{B}_{eff} \cdot \hat{\sigma}$$
$$(\Box + m^2) a = g_s n_N$$

$$g_s \approx \frac{4\pi e}{A f_a} \frac{\partial \mathcal{S}}{\partial \theta_a} \mathcal{M}_e \cdot \mathbf{I}$$

$\mathcal{M}_e$, the electronic matrix element, inherits its direction from the electric polarization vector of the ferroelectric.
Spin-polarized Source mass

\[ \sigma_1 \]

Quartz sample block

\[ \sigma_2 \]

Polarized 3He sample

SQUID magnetometer

SC shield

\[ d \]
| Summary |
|-----------------|-----------------|-----------------|-----------------|
| ![Diagram](image) | \[
\frac{a}{f_a} G\tilde{G} \\
G_{aee} \sigma_e \cdot \frac{\dot{p}_e}{m_e}
\] | \(10^{-11} \text{eV to } 10^{-7} \text{eV}\) | Must be DM |
| ![Diagram](image) | \[
\frac{a}{f_a} G\tilde{G} \\
G_{aNN} \nabla a \cdot \sigma_N
\] | \(10^{-5} \text{eV to } 10^{-2} \text{eV}\) | Doesn’t need to be DM |
• QCD axion DM can excite vibrational modes in piezoelectric crystals via its model-independent coupling to gluons.

• Ferroelectric crystals can source QCD axion mediated forces, that could be detected using an NMR sample.

• Complimentary to cavity experiments