There are two competitive opinions about a magnitude of the momentum of light in matter. The Balazs thought experiment gives unambiguous theoretical evidence based only on laws of mechanics that the momentum of light in matter is smaller by \( n \) times than that of the same light in free space where \( n \) is the refractive index of the matter. On the other hand, there is other theoretical evidence confirmed by experimental investigation that the momentum of light in matter is greater by \( n \) times than that in free space. This ancient dilemma cannot be resolved till now. We show that the momentum density flux of light in matter is greater by \( n \) times than that in free space. In the same time we present an alternative interpretation of the Balazs thought experiment where two optical pressures arising in the regions of the matter where leading and trailing edges of the pulse are propagating are taken into account. This enables us to match contradictory results of mentioned experiments. Since the contradiction is overcome, the dilemma disappears.

1. Introduction

The momentum of light in a conventional transparent optical medium is one of known unsolved classic surprises in theoretical physics. There have been extensive debates about the correct expression of the momentum density of electromagnetic radiation in linear media for more than 100 years since the original papers of Minkowski [1] and Abraham [2], and even so there is still some confusion or at least disagreement among authors. In accordance with Minkowski, momentum of light in an optical medium increases by \( n \) times as compared with the momentum of the same light in free space where \( n \) is the refractive index of the medium. In accordance with Abraham, momentum of light in an optical medium decreases by \( n \) times. Various arguments in favor of the Abraham and Minkowski forms are presented in reviews [3–6].

There are several approaches in attempts to resolve the dilemma. The oldest and most widespread one is based on the energy-momentum tensor formalism [1, 2, 6]. In accordance with Griffith [6]

“Both the Minkowski momentum and the Abraham momentum are “correct,” but they speak to different issues”.

“Only the total stress-energy tensor carries unambiguous physical significance, and how one apportions it between an “electromagnetic” part and a “matter” part depends on context and convenience. Minkowski did it one way and Abraham another; they simply regard different portions of the total as “electromagnetic.” Except in vacuum, “electromagnetic momentum” by itself is an intrinsically ambiguous notion”.

“For example, when light passes through matter it exerts forces on the charges, setting them in motion and delivering momentum to the medium. Since this is associated with the wave, it is not unreasonable to include some or all of it in the electromagnetic momentum, even though it is purely mechanical in nature. But figuring out
This position prevents an analysis of processes connected with transformation of the electromagnetic momentum into mechanical one and vice versa at propagation of light in an inhomogeneous optical medium and prevents using a powerful method of a calculation a magnitude of optically induced forces (OIF) by means of an analysis of a change of the momentums. Having known a change of the momentum density fluxes in various points of the matter, the density of OIF can be calculated without knowledge of a distribution of electrical and magnetic fields.

Last time several papers have been published [7–9] devoted to this problem. It is declared that the problem has been resolved at last. The Barnett resolution [7] recognizes that “both forms are correct” and “the Abraham and Minkowski momenta are, respectively, the kinetic and canonical optical momenta.” However, this approach cannot solve the problem that is formulated by Barnett as follows: “why is [sic] it that the experiments supporting one or other of these momenta give the results that they do?” Thus, this resolution gives no possibility to use the approach based on an analysis of a change of the momentum of light in practical applications.

Mansuripur believes also that he has solved the controversy [8]. His solution is based on the generalized expression of the Lorentz force. However this expression is incorrect because it gives incorrect result in simplest cases [10].

Various attempts to resolve the Abraham–Minkowski dilemma are reviewed in the recent paper by Kemp [9] where references to the Barnett and Mansuripur solutions are presented. The following conclusion is derived: “a complete picture of electrodynamics has still yet to be fully interpreted.”

There are the following real experiments performed by Jones et al. [11, 12]. When a mirror is immersed in a dielectric liquid, the radiation pressure exerted on the mirror is proportional to the refractive index of the liquid. An accuracy of this effect was 0.05%. On assumption that the light has the momentum that changes its sign at reflection from the reflector, one can conclude that the momentum of the light in the liquid is greater by n times than the momentum of the same light in free space.

However this conclusion contradicts results obtained in the same time from the Balazs thought experiment [13] based on the generally accepted law of the momentum conservation. It is shown theoretically that a transparent block through which a light pulse is propagating without reflection should be displaced in a direction of the propagation of the light pulse. As a result, the block is moving along the light pulse when the pulse is propagating inside the block. In this case a part of the momentum of the pulse is transferred to the block. Therefore, the momentum of light inside the block is smaller than that of the same pulse propagating in free space. Thus, the momentum of light inside the block corresponds to the Abraham form. As is pointed in review [3], “If argument advanced in favor of the Abraham momentum were to be incorrect, than that would bring into question uniform motion of an isolated body as expressed in the Newton’s first law of motion [sic].”

Since it is impossible in a frame of the energy–momentum tensor formalism to resolve the controversy, we will consider a controversy between unambiguous results of real and thought experiments and terms connected with names of Abraham and Minkowski we will not use. In this case momentum that is smaller by n times than that in free space we will be denoted as B-momentum meaning that a magnitude of the momentum is derived from the Balazs thought experiment. Accordingly, the momentum which magnitude is greater by n times in matter than that in free space will be denoted by J-momentum bearing in mind that its magnitude is derived from the Jones et al. experiments [11, 12]. By the way, in accordance with our thought experiment [14] a conclusion was derived theoretically that the momentum of a continuous light wave propagating in matter corresponds to the J-momentum. No assumption about kinds and physical origin of OIF responsible for an increase of the momentum is made. Thus, there are two unambiguous rival thought experiments [13, 14] where no assumption about kinds of optically induced force and their physical origin is made. These experiments are a safe ground for analysis of a magnitude of momentums and OIF in matter.

Hitherto all attempts to match rival results of the experiments were failed because a reason of this discrepancy was not found out. Recognition that both results are correct because they correspond to experiment is insufficient. An explanation is required about why the momentums are different. We present our explanation and show that it can be presented many years ago because the explanation is based on classical laws of mechanics.

2. Notation and Basic Definitions

Above all things, it is worthwhile to note that our notion about the momentum of light is taken from mechanics where this notion was introduced several centuries ago for a body of mass M moving at speed v as a product of M v. In accordance with Newton the momentum characterizes the “quantity of motion.” A reason of a change of the momentum p is a force f and in accordance with the second Newton law dp/dt = f. In a closed system (one that does not exchange any matter with the outside and is not acted on by outside forces) the total momentum is constant. This fact, known as the law of conservation of momentum, is implied by Newton’s laws of motion [15, 16]. A magnitude of the momentum of a light pulse propagating in vacuum is generally accepted and is given by $\mathcal{E}_\text{pulse}/c$, where $\mathcal{E}_\text{pulse}$ is the energy of the light pulse.

If a continuous plane light wave is considered, its momentum is equal to infinity. In this case the momentum of the wave propagating through a cross-section of unit area per unit time is considered. This momentum is equal to the momentum density flux (MDF). MDF of a continuous plane
light wave propagating in vacuum at speed $c$ is given by $(W_0 c)/c = W_0$ [J/m$^3$] where $W_0 = \varepsilon_0 E_0^2/2$ is the energy density of the light wave and $E$ is the amplitude of the strength of the alternate electrical field of the light wave. We will call this MDF by the electromagnetic MDF. A mechanical pressure $P$ applied to a body transmits to the body the mechanical MDF equal to $P$ [N/m$^2$ = 1/m$^3$].

There are optically induced forces (OIF) produced by the light propagating in an optical medium. As a result, the light interacts with matter (an exchange of the momentums between the light and matter takes place). The law of the conservation of the momentums and the third Newton law are valid at this interaction. As a result, each OIF changes the mechanical MDF of matter. In turn, a counterpart of the OIF (COIF) that arises in accordance with the third Newton law changes the electromagnetic MDF of light. Thus, each interaction is accompanied by a redistribution between mechanical and electromagnetic MDFs. A sum of these MDFs is not changed. Thus, OIF is responsible for a change of the electromagnetic momentum and COIF is responsible for a change of the electromagnetic momentum. Usually, relations between electromagnetic and mechanical MDFs before interaction are known. The mechanical momentum of any light wave in free space is equal to zero. Having known a distribution of OIF in space and time, a behavior of the mechanical and electromagnetic MDFs in space and time can be calculated.

3. Momentum of Light Derived from the Balazs Thought Experiment

A main argument in favor of the B-momentum in matter is the Balazs thought experiment [13] that is described in last time many times [3–6, 8]. A behavior of a transparent block of an optical medium through which a light pulse is propagating is considered. The behavior is explained correctly on assumption that the B-momentum takes place within the block. The J-momentum would predict a motion of the block in the opposite direction to the incident pulse.

The following 1D structure is considered. A light pulse of a plane light wave is propagating along the $z$-axis in free space at speed of light $c$. The pulse enters a block of thickness $D$ and refractive index $n$, is propagating within the block, and leaves the block preserving its initial energy and momentum. The optical medium of the block is as simple as possible. The medium is linear, dispersionless, lossless, homogeneous, and nonmagnetic. It is supposed that measures are undertaken to exclude reflections when the light pulse enters and leaves the block. For example, the block is confined by slabs of thickness $d$, where $D \gg d \gg \lambda$ and the refractive index $n$ is changed gradually from 1 to $n$ in the slab before the front face of the block and from $n$ to 1 in the slab after the back face. Here $\lambda$ is the wavelength of light. As is known, reflections of light wave entering the block through such slab can be neglected.

Figure 1(a) shows positions of the light pulse of duration $\tau$ and length $L = c\tau$ in a form of hollow arrows before and after penetrating through the block. Figure 1(b) shows positions of the same pulse propagating in free space outside the block. A time of propagation of the pulse through the block in Figure 1(a) at distance $L + D$ is equal to $\tau + T$ where $T = D/(c/n)$ is time of propagation of the leading edge of the pulse through the block. The pulse in Figure 1(b) propagates at distance $c(Dn/c+\tau) = Dn+L$ during this time. The distance is greater than the distance $L + D$ in Figure 1(a) by $D(n-1)$. Positions of the centers of mass-energy in Figure 1(a) and Figure 1(b) are identical if the following condition is satisfied, $Mc^2 \Delta z = W_0LSD(n-1)$ where $M$ is the mass of the block, $\Delta z$ is a displacement of the block when the pulse has left the block, $W_0$ is the energy density of the pulse in free space, $S$ is the area of the cross-section of the block, and $W_0LS$ is the energy of the light pulse. In this case

$$\Delta z = \frac{W_0LSD(n-1)}{Mc^2}. \quad (1)$$

Since the system consisting of the pulse and block is closed, the momentum conservation law for momenta before and after entrance of the pulse into the block gives

$$\frac{W_0LS}{c} = Mv_{\text{block}} + P_{\text{pulse}}, \quad (2)$$

where $P_{\text{pulse}}$ is the momentum of the pulse inside the block and $v$ is the velocity of the block when the pulse is propagating inside the block. Since the block is shifted by $\Delta z$ during time interval $\tau + D/(c/n)$, the velocity of the block is $v_{\text{block}} = \Delta z/[(\tau + D)/(c/n)]$. Then from (1), (2) we have

$$P_{\text{pulse}} = W_0\tau \left[1 - \frac{1 - 1/n}{1 + L/(Dn)}\right]. \quad (3)$$

As is seen, the momentum of the pulse inside the block decreases by $n$ times in a limiting case only when length of the pulse $L$ tends to zero. It is that this case is considered in the abovementioned publications. In this case the pressure on the front face of the block is given by

$$P = W_0 \left(1 - \frac{1}{n}\right). \quad (4)$$
A generally accepted interpretation of this result is the following. The MDF of the light in free space is equal to $W_0$. The mechanical MDF that enters the block is given by (4). Then MDF of the light inside the block is equal to $W_0/n$. As is noted by authors of [3], “only the conservation of momentum and the uniform motion of the center of mass-energy are used, and it is difficult to see how any components of our derivation could seriously be open to question.” Indeed, (4) derived for the pulse where $\tau \ll T$ is not open to question. However, a conclusion that the MDF of the light inside the block is equal to $W_0/n$ is open to question when forces arising in regions of the block where leading and trailing edges of the pulse are propagating are taken into account. These forces are not taken into account because it was supposed that the pulse enters the block instantly. A propagation of one photon is considered usually.

There is another thought experiment [14]. A continuous light wave is reflecting in serial from two parallel reflectors of an optical resonator located in free space. Block in Figure 1 is imbedded in the plane optical resonator. It is shown that a net force applied to the block is equal to zero. The pressure on the front face of the block produce by a travelling light wave is given by

$$P_M = W_0 (1 - n).$$  \hspace{1cm} (5)

The pressure on the back face is equal to $-P_M$. In this case the block is expanded by the pressures only. An increase of the MDF at the entrance of a continuous light wave into the block is explained as follows. There should be a counterpart of the negative pressure $P_M$ applied to the front face of the block. This counterpart is positive and is applied to the object that is responsible for appearance of pressure $P_M$. Only the continuous light wave can be this object. The positive counterpart increases the MDF of the continuous light wave from $W_0$ to $nW_0$.

### 4. Resolution of a Contradiction between Balazs’ and Jones’s Experiments

We have to admit that the MDF of a light pulse decreases in matter by $n$ times but the MDF of a continuous light wave increases in matter by $n$ times. A difference is connected with edges of the light pulse that are absent in a continuous light wave [17]. Let us assume that there are additional pressures in the regions where the leading and trailing edges of the light pulse are propagating. A joint action of the additional pressure produced by the leading edge along with the negative pressure given by (5) provides, as a result, the positive pressure given by (4). In this case a magnitude of this additional pressure is given by

$$P_A = W_0 \left( n - \frac{1}{n} \right).$$  \hspace{1cm} (6)

Analogously, the additional pressure produced by the trailing edge of the pulse should be equal to $-P_A$. In this case a process of propagation of a light pulse through the block in the Balazs thought experiment at $\tau < T$ looks like as follows. When only the leading edge of the pulse is propagating inside the block, there are two pressures applied to the block. Negative pressure given by (5) is applied to the front face of the block. Positive pressure given by (6) is applied to the region where the leading edge is propagating. Time instants when these pressures are terminated are identical and are equal to $\tau = \tau$. As a result, a total pressure on the block is given by $W_0(1 - n) + W_0(n - 1/n) = W_0(1 - 1/n)$. This is in accordance with (4). Thus, the pressure applied to the block obtained from the Balazs thought experiment can be obtained on an assumption that additional pressure in accordance with (6) takes place in the region where the leading edge of the pulse is propagating. Unlike the interpretation of the Balazs thought experiment that the pressure on the front face of the block is positive and is given by (4), there is the negative pressure given by (5). Additional pressure $P_A$ given by (6) should be taken into account to obtain the pressure in accordance with (4).

When the trailing edge enters the block, the negative pressure in accordance with (5) disappears. In the same time the negative additional pressure $-P_M$ in accordance with (6) in the region where the trailing edge is propagating arises. A sum of pressures produced by the leading and trailing edges of the pulse is equal to zero and the center of mass of the block moves uniformly.

When the leading edge leaves the block, positive pressure $P_M$ in accordance with (5) arises on the back face of the block. In the same time positive pressure in the region where the leading edge is located disappears but the negative additional pressure produced by the trailing edge of the pulse remains. As a result, a sum of pressures is negative and is equal to $-W_0(1 - 1/n)$. This pressure provides a negative acceleration to the center of mass of the block. The center of mass stops when the trailing edge leaves the back face of the block. This picture is in a full compliance with results of the Balazs thought experiment. As is seen no notion about the Abraham or Minkowski momentums has been used.

Thus, two possible interpretations of the Balazs thought experiment are possible. The first one is generally accepted. The momentum of the light pulse is decreased inside the block by $n$ times. The second one is the following. The momentum of the light pulse is increased by $n$ times but additional pressures in accordance with (6) arise in the regions where leading and trailing edges of the light pulse are propagating.

To choose a correct interpretation, let us consider one more thought experiment where experimental evidence is available. Let a light wave of the energy density $W_0$ be propagating in free space and enter the block at $t = 0$ where it is propagating in an optical medium and be reflecting from an ideal reflector located on the back face of the block (Figure 2). This block is different from the block in Figure 1. The slab with gradually decreasing refractive index on the back face of the block is absent.

Let us first consider a propagation of continuous light wave in the block. In accordance with the first alternative, the momentum flux density of the continuous light wave inside the block is equal to $W_0/n$, the pressures applied to the front face, to the reflector, and total pressure applied to the block are given by $2W_0(1 - 1/n)$, $2W_0/n$, and $2W_0$, respectively.
In accordance with the second alternative these pressures are given by $2W_0(1-n)$, $2W_0n$, and $2W_0$. A difference between these alternatives is the following. In accordance with the first alternative all pressures are positive. The pressure applied to the reflector is equal to double MDF that is reflected from the reflector. Then the pressure is equal to $2W_0/n$ for the first alternative and $2W_0n$ for the second one. In accordance with the Jones experiment the pressure on the reflector produced by the light propagating in matter is greater by $n$ times than the pressure on the reflector produced by the same light propagating in free space. Thus, the first alternative contradicts results of the Jones et al. experiment.

Let us next consider a propagation of a light pulse in the block in Figure 2 to show that the second alternative remains valid, unlike the first one. Let $\tau \gg T$ and let the leading and trailing edges be propagating in free space. The leading edge is propagating after reflection in backward direction. The trailing edge is propagating in the forward direction and has not entered the block yet. Since the additional force in free space in accordance with (6) is equal to zero, this situation is similar to that considered above for a continuous light wave. The total pressure $P$ applied to the system consisting of the block and reflector is equal to a change of the momentum flux density of light in free space per unit time. This change is equal to $P = 2W_0$. The pressure applied to the front face of the block in accordance with (5) is equal to $P_M = 2W_0(1-n)$. Multiplier 2 arises because the pressure $P_M$ is produced by forward and backward waves. From relation $P = P_M + P_R$, where $P_R$ is the pressure applied to the reflector, we have $P_R = 2W_0n$. It is not surprising because there are no edges of the pulse within the block and the light pulse can be considered as a continuous light wave in this case.

Let us next assume that $\tau < T$ and $\tau_\tau \ll \tau$ where $\tau_\tau$ is duration of edges of the pulse. When the leading edge is entering the block, two kinds of pressures arise in accordance with (5) and (6). Their sum given by (4) is consistent with the Balazs thought experiment. When the trailing edge of the pulse enters the block, the net pressure applied to the block becomes equal to zero because the negative pressure $P_M$ in accordance with (5) disappears and the positive additional pressure $P_A$ produced by the leading edge in accordance with (6) is compensated by the negative pressure $-P_A$ produced by the trailing edge.

When the pulse is reflecting from the reflector, the MDF of the pulse changes its direction and pressure $P_R = 2W_0n$ is applied to the reflector in time interval $\tau$. The additional pressures in the regions where the leading and trailing edges of the pulse are propagating are both negative at reflection and their sum is equal to $-2W_0(n - 1/n)$. As a result, the total momentum density transferred by the light pulse to the block when the pulse enters the block propagates inside the block, reflects from the reflector, propagates in backward direction, and leaves the block that is equal to $\tau W_0[(1 - n) + (n - 1/n) + 2n - 2(n - 1/n) + (n - 1/n) + (1 - n)] = 2\tau W_0$. As might be expected, the total momentum density transferred to the system comprising of the block and reflector is independent on $n$ and is equal to a change of the momentum density of light pulse propagating in free space $2\tau W_0$. The momentum density transmitted to the reflector is equal to $2\tau nW_0$. The momentum density transferred to the block is equal to $2\tau W_0(1 - n)$. A sum of momentum densities transmitted to reflector and block is equal to $2\tau W_0$. If a binding between the reflector and block is absent, the block will move to the left and the reflector will move to the right.

Thus, a light pulse can be imagined as a segment of a continuous light wave where additional interaction between light and matter takes place in the regions where edges of the light pulse are propagating. The total MDFs in the light wave and pulse are identical and are equal to $W_0n$. The positive pressure in the region where the leading edge is propagating produces positive mechanical MDF $W_0(n - 1/n)$ in the matter. The negative counterpart of this pressure decreases the electromagnetic MDF of the pulse from $nW_0$ to $nW_0 - W_0(n - 1/n) = W_0n$. The negative pressure in the region where the trailing edge is propagating produces negative mechanical MDF $-W_0(n - 1/n)$ in the matter. The negative and positive MDFs are deleted mutually in the regions located behind the light pulse. Thus, the propagation of the light pulse in matter is accompanied by propagation of two kinds of the momentum. These are the electromagnetic momentum of density $\tau W_0/n$ and the mechanical momentum density $\tau W_0(n - 1/n)$. Their sum is equal to $\tau nW_0$ and corresponds to $J$-momentum. As was shown, the pressure produced by the light pulse on the reflector is equal to $2\tau nW_0$. Thus, the mechanical component of the total momentum takes part in producing the pressure on the reflector.

The mechanical component of the total momentum produces the displacement of the block that can be calculated from the following proportion. Pressure in accordance with (4) causes a displacement of the block $\Delta z$ given by (1). Then the pressure given by (6) causes the following displacement: $\Delta z_1 = \Delta z(n - 1/n)/(1 - 1/n) = \Delta z(n + 1)$. The pressure produces the displacement not only the center of mass of the block but also the whole block because all parts of the block are displaced at identical distance. The pressure applied to the front face of the block causes the negative displacement given by $\Delta z_2 = \Delta z(1 - n)/(1 - 1/n) = -\Delta zn$. As is seen, $\Delta z_1 + \Delta z_2 = \Delta z$. Although the negative displacement of the center mass of the block is equal to $\Delta z_2$ due to the pressure given by (5), the displacement of the regions near the front face of the block is significantly greater. The displacement of the regions near the rear face of the block is equal to zero because the displacement arising near the front face is propagating at sound speed that is smaller by five orders of magnitude than the light speed. The transient processes will be terminated when the light pulse has propagated at great distance above $10^5D$.

The additional pressures in accordance with (6) are not taken into account in the generally accepted interpretation of the Balazs thought experiment. Because of this an erroneous
conclusion that the momentum of pulse decreases in matter was derived.

As is seen, a transmission of the momentum to the block differs essentially from the simplest view accepted in the generally accepted interpretation of the Balazs thought experiment where it is supposed that the momentum of the pulse simply decreases by \( n \) times at entering the block and recovers its value at exiting the block. In reality, it is a complex procedure where the propagation is accompanied by various forces arising in various regions of space in various time instants. These forces produce various mechanical momentums of different signs in different regions of the block. The light pulse transmits to the block mechanical momentums of different signs and leaves the block at light speed. In this time mechanical transient processes initiated by OIF are not terminated although the displacement of the center of mass of the block is equal to \( \Delta z \) when the trailing edge of the light pulse has left the block. The inner forces that take part in the transient processes change positions of various parts of the block but do not change the displacement of the center of mass.

Thus, a joint consideration of all OIFs enables us to match rival results of the Balazs and Jones experiments. No notion about the Abraham or Minkowski momentum of light is used to interpret a behavior of the block derived from the Balazs thought experiment as well as the pressure produced on a reflector in the Jones et al. experiments.

5. A Physical Origin of Pressures Arising at a Propagation of a Light in Matter

Let us first analyze an origin of pressure on the front face of the block given by (5). As is shown in Figure 1, there is a slab of thickness \( d \) where the refractive index is changed gradually along the \( z \)-axis from 1 to \( n \). The density force inside the slab is given by

\[
 f = \frac{dP}{dz} = -\frac{W_0 dn}{dz} = -\frac{dn}{dz} \frac{E_0^2}{2}. \tag{7}
\]

It is desirable to express \( f \) in terms of the strength of the electrical field in the plane where the density force is determined rather than in terms of the strength \( E_0 \) in free space. Since there are no reflections inside the slab, the energy density of a continuous light wave in the slab \( W = n^2 \varepsilon_0 E^2 / 2 \) is greater by \( n \) times than energy density \( W_0 = \varepsilon_0 E_0^2 / 2 \) in free space, where \( E \) and \( E_0 \) are the electrical fields in the slab and free space, respectively. In this case \( E_0^2 = nE^2 \). Then (7) can be presented as follows:

\[
 f = -n \frac{dn}{dz} \frac{E^2}{2} = -\frac{dn}{dz} \frac{E^2}{4}. \tag{8}
\]

On the other hand, it is known from electrostatics since time of Maxwell [18, 19] that density force in a dielectric located in a constant electrical field \( E \) is given by

\[
 f_{ES} = -\nabla (\varepsilon) \varepsilon_0 \frac{E^2}{2}. \tag{9}
\]

As is seen, force \( f_{ES} \) is proportional to the square of the electrical field \( E \). In this case force \( f_{ES} \) is different from zero in an alternate electrical field also, in particular, in an electrical field of light wave. Averaging \( f_{ES} \) over period of oscillation, we obtain (8) from (9). Thus, the density force that increases the MDF of the light entering the block through the slab is a counterpart of the force \( f_{ES} \) that is known for a long time in electrostatics. Brevik calls this kind of force the Abraham-Minkowski force [20]. Kemp calls this kind the Helmholtz force [9]. However this name is more suitable for internal OIF arising due to an inhomogeneity of the electrostrictive pressure [21]. As a kind of force \( f_{ES} \) was known much earlier, we will call it the Maxwell-like force because Maxwell was the first who had studied it.

Thus, OIF calculated on the basis of the electrostatic approach in accordance with (9) (ES approach) is identical to that calculated on the basis of the approach where a change of the momentum flux density is analyzed (CM approach) [14].

Let us next analyze an origin of pressures in the regions where the leading and trailing edges of a light pulse are propagating. The pressure in the region where the leading edge is propagating is given by (6). The density force produced the pressure on assumption that the energy density \( W \) is changed from 0 to \( W_0 \) is given by \( f_A = (n - 1/n)dW/dz \). Since \( z = tc/n \), we have \( f_A = ((n^2 - 1)/c)(dW/dt) \). Since there are no reflections at the entrance of the light pulse into the block, we have \( W = S/c \), where \( S \) is the energy flux density \( S = [E \times H] \). Thus, we have

\[
 f_A = \frac{(n^2 - 1) d[E \times H]}{c^2} \tag{10}
\]

Density force in (10) will be called by A-force. It turns out that the A-force is expressed by the same formula as the Abraham force that is considered along with the Abraham momentum in the Abraham form of the energy-momentum tensor formalism [22, 23]. Usually the term “Abraham force” is used along with term “Abraham momentum.” A combination of the Minkowski form of the MDF and the Abraham form of the density force for matching rival experiments causes misunderstanding and objection. That’s why we deliberately do not use the familiar terms of the energy-momentum tensor formalism and use the concepts of mechanics only. We denote the types of forces and momentum terms taken from the names of researchers who have studied these types.

Thus, an attempt to coordinate results of unambiguous thought experiments leads to the need to recognize the existence of the optical pressure arising in the regions of an optical medium where the intensity of light is changed in time.

A sum of forces given by (9) and (10) gives the following expression for OIF produced by the light, in which intensity is changed in time

\[
 f = -\nabla (n^2) \varepsilon_0 \frac{E^2}{2} + \frac{(n^2 - 1)}{c^2} \frac{d[E \times H]}{dt}. \tag{11}
\]

Equation (11) has long been known [22, 23] and used at present time [20].
Thus, our explanation corresponds to the experiments. An inclusion of the A-force into consideration of propagation of light pulses in the Balazs thought experiment enables us to match rival results obtained from the Balazs and Jones experiments. No notions about the Abraham and Minkowski momentums of light are required in this case.

6. Discussion

The A-forces are not taken into account in the generally accepted interpretation of the Balazs thought experiment. Because of this an erroneous conclusion that the momentum of pulse decreases in matter was derived. In reality, the total momentum of the light pulse corresponds to the J-form \( W_0n \). Due to the A-force in the regions where leading and trailing edges are propagating the total MDF is divided into two components: the mechanical component \( W_0(n-1/n) \) and the electromagnetic components \( W_0/n \) that is equal numerically to B-form. The pressure in accordance with (5) causes a negative displacement of the center of mass of the block. The pressure in accordance with (6) causes a positive displacement of the center of mass of the block. A sum of these displacements is positive and corresponds to (1).

One can see that the J-form of the momentum in matter takes place at a steady state where a change of the light intensity in time is absent. The electromagnetic component is equal to the total MFD \( nW_0 \); the mechanical component is equal to zero. This situation takes place in most experiments.

The total MFD of a light pulse in matter is also increasing by \( n \) times. The electromagnetic component of the light pulse in matter is equal to \( M_{EM} = W_0/n \). The mechanical component is equal to \( M_{Mech} = W_0(n-1/n) \). But this is not a single variant.

Let us consider the light pulse in which MDF in free space is equal to \( W_0 \) at \( t < 0 \) and is equal to \( W_0 + \Delta W \) at \( t > 0 \). Since there is a continuous light wave at \( t < 0 \), we have \( M_{total} = W_0n, M_{Mech} = 0, \) and \( M_{EM} = W_0n \). Then at \( t > 0 \) we have \( M_{total} = (W_0 + \Delta W)n, M_{Mech} = \Delta W(n-1/n), \) and, therefore, \( M_{EM} = W_0n + \Delta W/n \). In this case \( M_{total}/M_{EM} = (1+\Delta W/n)/W_0 \). As is seen, \( M_{total}/M_{EM} = 1 \) at \( \Delta W = 0 \) and \( M_{total}/M_{EM} = n^2 \) at \( \Delta W \gg W_0 \). Thus, the electromagnetic component of MDF can be an arbitrary part in interval \( 1 \cdots n^2 \) of the total MDF. However, this part is not a matter of taste [6] but is determined by a specific situation.

Let us compare distributions of OIF in matter in particular cases on the basis of our notion and notions used in reviews and tutorials. Mansuripur describes an exchange between momentums of light pulse \( \vec{E}_{pulse}/c \) in free space that enters the block with the refractive index \( n \) through antireflection \( \lambda/4 \) plate [8]. Here \( \vec{E}_{pulse} \) is the energy of the light pulse. He believes that the mechanic and electromagnetic components of the light pulse are equal, respectively, \((n-1/n)\vec{E}_{pulse}/(2c)\) and \(\vec{E}_{pulse}/(nc)\). The total momentum of the pulse is equal to their sum \((n+1/n)\vec{E}_{pulse}/(2c)\). Besides, the mechanical momentum \(-(n-1)^2\vec{E}_{pulse}/(2nc)\) is transmitted to \(\lambda/4\) plate when the pulse enters the block. A sum of this momentum and the total momentum propagating in the block is equal to the momentum of the pulse in free space \( \vec{E}_{pulse}/c \). As was noted, the law of the conservation of the momentum holds.

However, since MDF of the pulse in accordance with Mansuripur is equal to \( W_0(n+1/n)/2 \), the pressure on the reflector is not proportional to the refractive index \( n \) that contradicts results of the Jones experiment. Our total MDF in matter is equal to \( W_0n \) and, therefore, the pressure on the reflector is proportional to the refractive index \( n \) that agrees with results of the Jones experiment.

Differences between Mansuripur and our analysis are the following. Our total momentum of the pulse, unlike \((n+1/n)\vec{E}_{pulse}/(2c)\), is equal to \( n\vec{E}_{pulse}/c \). Our mechanical momentum, unlike \((n-1/n)\vec{E}_{pulse}/(2c)\) is equal to \((n-1/n)\vec{E}_{pulse}/c \). At last, our mechanical momentum transmitted to \(\lambda/4\) plate, unlike \(-(n-1)^2\vec{E}_{pulse}/(nc)\), is equal to \((1-n)\vec{E}_{pulse}/c \).

As is seen, a magnitude of the mechanical component of MDF in accordance with Mansuripur \( W_0(n+1/n)/2 \) is half of our mechanical MDF given by (6). It is explained by the fact that MDF is produced by the A-force given by (10) that can be presented as follows:

\[
f_1 = \hat{P} \times \vec{B} + \hat{P} \times \vec{B},
\]

where \( E \) and \( B \) are, respectively, the electric field strength and the magnetic induction associated with the light radiation and \( \hat{P} = (e-1)E \) is the polarization of the optical medium.

OF produced by the Lorentz force is presented by the second term in the right-hand part of known expression for the Lorentz density force [4, 6, 8, 9] given by

\[
f_2 = (\vec{P} \cdot \nabla)E + \hat{P} \times \vec{B}.
\]

On assumption that the leading edge of the light pulse is located in region \( z = 0 \cdots z = l \), we obtain that the pressure produced by the leading edge can be calculated in accordance with (12) as follows:

\[
P = \frac{e\mu - 1}{c^2} \int_0^l \left( \frac{H_y \partial E_x}{\partial t} + E_x \frac{\partial H_y}{\partial t} \right) dz
= \frac{e\mu - 1}{c^2} \int_0^l \left( \frac{H_y \partial E_x}{\partial t} - E_x \frac{\partial E_x}{\partial t} \right) dz
= \frac{e\mu - 1}{2c^2} \int_0^l \left( \frac{\mu_0 \partial H_y^2}{\varepsilon_0 \partial t} + \varepsilon_0 \frac{\partial E_x^2}{\partial t} \right) dz
= \frac{e\mu - 1}{2c^2} \int_0^l \left( \frac{\mu_0 \partial H_y^2}{\varepsilon_0 \partial t} + \varepsilon_0 \frac{\partial E_x^2}{\partial t} \right) dz
= \frac{e\mu - 1}{2e\mu} \left( \mu_0 \varepsilon_0 \partial H_y^2 + \varepsilon_0 \partial E_x^2 \right)
= \frac{e\mu - 1}{e\mu} (W_H + W_E) = \frac{e\mu - 1}{e\mu} W_{TOTAL}
= \left( \sqrt{\frac{e\mu}{\varepsilon_0}} - \frac{1}{\sqrt{e\mu}} \right) W_0,
\]
where $W_{\text{TOTAL}}$ is the total energy density in matter and $W_0$ is the total energy density of the same radiation in free space. Since $\varepsilon = n^2$, (6) is identical to (14) at $\mu = 1$.

As is seen from derivation of (14), the pressure produced by the Lorentz density force in accordance with second term in the right-hand part of (13) is smaller by two times because second terms in brackets of (14) do not take part in calculation of the Lorentz force.

To the best of our knowledge, Jones in 1978 [24] was the first who concluded that the total momentum of light pulse in matter is equal to $pn$, the electromagnetic momentum is equal to $p/n$, and the mechanical momentum is $\left[1 - 1/n^2\right]$ part of the total momentum $pn$, where $p$ is the momentum of the light pulse in free space. However, he cannot explain a physical phenomenon responsible for a rise of the mechanical momentum. He wrote “We are not able to specify the details how this body impulse is created, but merely point out it is demanded by the simple consideration of mechanics [sic].”

Mansuripur in 2010 [8] was the second who showed that the mechanical momentums are created by leading and trailing edges of the light pulse. But he used an approach based on the Lorentz density force and his magnitude of the total momentum of the light pulse in matter contradicts result of Jones et al. experiment.

We are third who have disclosed a physical origin of the Jones “bodily impulse,” corrected calculation of Mansuripur, and showed that the A-force is responsible for a rise of the mechanical momentum in matter rather than the Lorentz force. The main difficulty was to persuade that an approach based on the Lorentz force is inconsistent in spite of the fact that it is used everywhere in last 40 years.

7. Conclusion

The following conclusion can be derived from matching unambiguous results of thought and real experiments known for a long time. A main reason for conclusion that the momentum of light in matter decreases by $n$ times is an erroneous interpretation of results of the Balazs thought experiment because the A-force arising at leading and trailing edges of a light pulse was not taken into account. An inclusion of these forces enables one to match the results of two rival experiments without introducing a notion about rival forms of the momentum. These forces produce the mechanical momentum distributed between leading and trailing edges of the pulse. The mechanical momentum is propagating together with the light pulse at speed $c/n$. The B-momentum in a pure form that is derived from the Balazs thought experiment does not exist. The electromagnetic momentum density flux $W_0/n$ is accompanied by the mechanical momentum density flux $W_0(n - 1/n)$. Their sum $W_0n$ corresponds to the J-form. Besides there is the mechanical momentum density flux $W_0(1 - n)$ arising at an entrance of the light pulse in matter. This momentum is not propagating together with the light pulse.

The momentum of a continuous light wave corresponds to the J-momentum. The mechanical momentum in this case is absent. It is not surprising. When a continuous light wave enters a medium from free space its momentum increases because there is the Maxwell-like force applied to the medium and directed to free space. In accordance with the third Newton law there is a counterpart of this force applied to the light wave that increases its momentum.

No notion about the B-momentum is required for matching unambiguous results of rival experiments. If the A-force is taken into account, a reason for introducing a notion about the B-momentum disappears completely. In reality, there is one form of the momentum in matter consisting of the electromagnetic and mechanical components. A ration between these components depends on a specific case but a sum of these is greater by $n$ times than that of the same light in free space.

Moreover, we can say that we have derived an existence of a kind of optically induced forces called the A-force from two unambiguous thought experiments where a light pulse [13] and continuous light wave [14] are analyzed. Only laws of mechanics were used for this purpose. A-force and J-momentum are sufficient for matching rival experiments. We can conclude that the age-old dilemma about a correct magnitude of the momentum of light in matter is solved. A correct magnitude of the total momentum in matter is presented. It is shown that a distribution of the total momentum between its electromagnetic and material components is not a matter, a taste, as is described in [6] but the distribution can be calculated in each specific case.

Conflict of Interests

The authors declare that there is no conflict of interests regarding the publication of this paper.

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