Controlling tax evasion fluctuations

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Abstract

We incorporate the behaviour of tax evasion into the standard two-dimensional
Ising model and augment it by providing policy-makers with the opportunity
to curb tax evasion via an appropriate enforcement mechanism. We discuss
different network structures in which tax evasion may vary greatly over time
if no measures of control are taken. Furthermore, we show that even minimal
enforcement levels may help to alleviate this problem substantially.

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1 Introduction

In economics, the problem of tax evasion from a multi-agent-based perspective has received little attention so far (see Bloomquist (2006) for a recent overview). Realistic models on tax evasion appear to be necessary, because tax evasion remains to be a major predicament facing governments (see Andreoni et al., 1998; Lederman, 2003; Slemrod, 2007).

Experimental evidence (see Gächter, 2006) suggests that tax payers usually condition their decision regarding whether to pay taxes or not on the tax evasion decision of the members of their group (“conditional cooperation”). Conditional cooperators are more likely to evade taxes if they have the impression that many others evade. On the other hand if most others behave honestly, an individual is less likely to cheat on her taxes. Gächter presents the findings of public goods experiments and argues that conditional cooperation primarily motivates people to either contribute to the provision of a public good or to free-ride. Frey and Torgler (2006) provide empirical evidence on the relevance of conditional cooperation for tax morale. They find a positive correlation between people’s tax morale, which is measured by asking whether tax evasion is justified if the chance arises, and their perception regarding how many others evade paying tax. Conditional cooperation from the viewpoint of the standard economic theory may be explained by changes in risk aversion due to changes in equity (Falkinger, 1995).

We decide to use the Ising model, because it allows to model conditional cooperation in a multi-agent-based fashion. It allows to consider a large number of agents who interact locally with each other and base their decision whether to evade taxes or not on the behaviour of the other agents in their group.

We incorporate the behaviour of tax evasion and add an enforcement mechanism into the standard two-dimensional square lattice Ising spin model. We aim to extend the study of Zaklan et al. (2008), which illustrates how, in a world where agents are conditionally cooperative, different levels of enforcement affect aggregate tax evasion over time. We define enforcement to consist of two components: a probability of an audit \( p \) each person is subject to in every period and a length of time detected tax
evaders need to remain honest for \( k \) periods. We embed our tax evasion model into different network structures and find for these networks that fluctuation in aggregate tax evasion behaviour may arise if no enforcement is used. For our simulations we make use of the “tunnelling” process at temperatures slightly below the critical temperature. This process has been used for two decades in the field of physics, and also by Hohnisch et al. (2005) for the IFO-Index. We use it to illustrate a second important and maybe less obvious effect of enforcement, as we define it: we provide evidence, by simulation, that even minimal levels of enforcement may help to reduce the presence of fluctuations in tax evasion. Such fluctuations can be completely prevented in the considered networks by setting the enforcement measures to sufficiently high, but realistic, levels. Everybody then remains compliant for most of the time.

The remainder of our manuscript is organised as follows: in section 2 we present our model, which is based on the standard two-dimensional Ising model on a square lattice. In section 3 we describe the evolution of the aggregate tax evasion behaviour that our model generates under different enforcement regimes. In section 4 we additionally embed our model into the Barabási-Albert network and the Voronoi-Delaunay random lattice and discuss the resulting tax evasion dynamics.

2 The model

We use the standard Glauber kinetics of the Ising model on a \( 20 \times 20 \) square lattice (in section 4 we will analyse our model for other lattice types). In every time period each lattice site is occupied by an individual who can either be an honest taxpayer \( (S_i = +1) \) or a tax evader \( (S_i = -1) \). The small number of agents may be imagined to represent the elite of a country, whose tax evasion behaviour it may be interesting to look at, given the different enforcement regimes of the tax authority. For our analysis we assume that everybody is honest initially. Each period individuals have the opportunity to become the opposite type of agent as they were in the previous period. Each agent’s social network, which is made up of four next neighbours, may either prefer tax evasion, reject it or be indifferent.
Tax evaders have the greatest influence to turn honest citizens into tax evaders if they constitute a majority in the given neighbourhood. If the majority evades, one is likely to also evade. On the other hand, if most people in the vicinity are honest, the respective individual is likely to become a decent citizen if she was a tax evader before. How strong the influence from the neighbourhood is can be controlled by adjusting the temperature, $T$. Total energy is given by the Hamiltonian $H = -\sum_{<i,j>} J_{ij} S_i S_j - B \sum_i S_i$. We choose $J = 1$ and $B = 0$. For very low temperatures, the autonomous part of decision-making almost completely disappears. Individuals then base their decisions solely on what most of their neighbours do. A rising temperature has the opposite effect. Individuals then decide more autonomously. It is well known that for $T > T_c$ ($\approx 2.269$), half of the people are honest and the other half cheat, while for $T < T_c$ states coordinated on cheating or compliance prevail for most of the time.

As an enforcement measure, we introduce a probability of an efficient audit ($p$). If tax evasion is detected, the individual must remain honest for a certain number of periods. We denote the period of time for which detected tax evaders are punished by the variable $k$. One time unit is one sweep through the entire lattice. Audits are stochastically independent from other agents and from the history any agent has.

3 Dynamics of the model

The top-left panel of Figure 1 illustrates the baseline setting, i.e. no use of enforcement, for the square lattice. We depict the dynamics of tax evasion over 50,000 time steps. Although everybody is honest initially, it is not possible to predict which level of tax compliance will be reached at some time step in the future. Agents are usually either mostly compliant or mostly non-compliant, whereas the system typically remains in either state for a while. Switching from a mostly compliant to a mostly

\footnote{The autonomous part of individual decision-making is responsible for the emergence of the tax evasion problem, because some initially honest tax payers decide to evade taxes and then exert influence on others to do so as well.}
non-compliant society, or vice versa, is favoured by both the small number of agents
and the temperature, which needs to be somewhere close to the critical level (we use
$T \approx T_c$). If, by chance, more than 50\% of agents start to prefer the opposite action
of the currently dominating one, this strategy will then start to prevail for a while.
As soon as there is a majority for the previously dominating strategy regarding tax
evasion, aggregate behaviour is then likely to reverse again. If more agents or a
temperature further below the critical level are picked, it would take longer for a
switch in aggregate evasion behaviour to occur. Apparently, a suitable measure of
control is needed to prevent agents from repeatedly falling into non-compliance.

– Figure 2 goes about here –

Figure 2 illustrates different simulation settings for the square lattice, where for each
considered combination of degree of punishment ($k = 1$, 10 and 50) and audit rate
($p = 0.5, 10$ and 90\%) the corresponding dynamics of tax evasion is depicted over
50,000 time steps. Surprisingly, even very small levels of enforcement (e.g. $p = 0.5\%$
and $k = 1$) suffice to almost completely prevent fluctuation in aggregate tax evasion
behaviour and to establish mainly compliance. Only seldomly tax evasion then
becomes the predominant aggregate choice of action. Both, a rise in audit probability
(greater $p$) and a higher penalty (greater $k$), work to flatten the time series of
tax evasion and to shift the band of possible non-compliance values towards more
compliance. If the audit rate is increased to the level of 1\%, even for very small
penalties ($k = 1$) then an upsurge in tax evasion will not occur any longer (not
displayed). Since high income earners are audited more often ($p \approx 10\%$) than
average income tax payers ($p \approx 1\%$), we look at how results change if higher levels
of enforcement are used ($p = 10\%$). Interestingly, higher audit rates only reduce
the level of tax evasion marginally. The simulations illustrate that even extreme
enforcement measures (e.g. $p = 90\%$ and $k = 50$) cannot fully resolve the problem
of tax evasion.
4 Modifications

To examine whether the results generated by the square lattice are robust, we extend our analysis to other frequently used network structures. Specifically, we make use of the Voronoi-Delaunay random lattice and the Barabási-Albert network model. The construction of the Voronoi-Delaunay lattice (i.e. tessellation of the plane for a given set of points) is defined as follows (Lima et al., 2000). For each point, one first needs to determine the polygonal cell, consisting of the region of space nearer to that point than to any other point. Whenever two such cells share an edge, they are considered to be neighbours. From the Voronoi tessellation, one can obtain the dual lattice by the following procedure: when two cells are neighbours, a link is placed between the two points located in the cells. From the links, one obtains the triangulation of space. The network constructed in this manner, which we use for simulation, is called the Voronoi-Delaunay lattice.

The Barabási-Albert network (Barabási and Albert, 1999) is grown such that the probability of a new site to be connected to one of the already existing sites is proportional to the number of connections the existing site has already accumulated over time: individuals with many friends are more likely to gain new friends than loners.

In these variations of our simple square lattice model we also choose 400 agents and depict the resulting tax evasion dynamics over 50,000 time steps.

The remaining pictures in Figure 1 illustrate the dynamics in the baseline setting in these additional network structures of our model. Both networks, the Voronoi-Delaunay lattice and the Barabási-Albert network, support our findings in the case of the square lattice, namely that fluctuations in tax evasion behaviour may occur if no enforcement mechanism is implemented.

— Figure 3 goes about here —

Figure 3 illustrates the tax evasion dynamics for the Voroi-Delaunay lattice (first column) and the Barabási-Albert (other two columns) network, if different degrees of enforcement are used.
For the Voronoi-Delaunay random lattice we also find that fluctuations in tax evasion can be reduced substantially by implementing very low probabilities of an audit. For an audit rate of $p = 1\%$ no fluctuations occur any longer and society remains mainly honest over time. Obviously higher punishments, i.e. higher levels of $k$, also lower the amount of non-compliance.

The next two columns illustrate that enforcement in the Barabási-Albert network is less efficient than in the Voronoi-Delaunay network or in the simple square lattice. As can be seen, fluctuations still occur for $p = 1\%$, even for high levels of punishment (e.g. $k = 10$). The last column illustrates the tax evasion dynamics, holding the audit rate constant at $p = 4.5\%$. This is the minimal audit rate which (almost always) prevents tax evasion from fluctuating, even for the lowest considered level of punishment (i.e. $k = 1$), and is much higher than in the other two network models, yet still at a realistic level.

## 5 Conclusion

Tax evasion can vary widely across nations, reaching extremely high values in some developing countries. Wintrobe and Gërshani (2004) explain the observed higher level of tax evasion in generally less developed countries with a lesser amount of trust that people accord to governmental institutions. Empirical evidence for the importance of trust for tax compliance may, for example, be found in Hyun (2005) and in Torgler (2004). So far we have neglected the effects of public opinion, which may vary greatly across countries, on tax evasion. It therefore seems worthwhile to extend the setting of our simple model in a further study, to specifically analyse the aspect of varying tax compliance across countries.
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Figure 1: The baseline setting is given if no enforcement measure is implemented to control tax evasion. The figures above illustrate the baseline settings for the different network structures we use. For the square lattice we take $T = 2.265$, for the Voronoi-Delaunay lattice $T_c = 3.802$ and for the Barabási-Albert network $0.8 \cdot T_c = 0.8 \cdot m \cdot \log(NSITES)/2$ (where $m = 4$ and $NSITES = 400$). All simulations are performed over 50,000 time steps.
Figure 2: The square lattice model of tax evasion with various degrees of enforcement. \((T = 2.265 \text{ and } 50,000 \text{ time steps})\)
Figure 3: The first column illustrates the resulting tax evasion dynamics for different enforcement regimes if the Voronoi-Delaunay network is used. The next two columns depict the tax evasion dynamics in the case of the Barabási-Albert network. Again, we use 50,000 time steps.