Seismic Sources in Stress-Induced Anisotropic Media

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Abstract  Rocks in the upper mantle are extensively subjected to initial stress. The rock exhibits elastic anisotropy under the deviatoric stress, which affects the seismic response of underground structures. This paper aims to study the effect of stress-induced anisotropy on the seismic source. The seismic moment tensor generated by a shear faulting in a homogeneous isotropic medium with initial stress is derived by using the motion description in the intermediate configuration, the constitutive relation of third-order elasticity, and the boundary condition considering the effect of initial stress. Then the seismic moment tensor is decomposed to investigate seismic source characters quantitatively. Our results show that shear faulting in a stress-induced anisotropic medium can produce significant non-double-couple (non-DC) mechanisms, including compensated linear vector dipole (CLVD) and isotropic (ISO) components. ISO and CLVD components vary linearly with the initial shear stress on faults. In addition, stress-induced anisotropy causes the double-couple (DC) to deviate from the plane defined by the fault normal and slip direction. The initial stress not only affects the source intensity but also affects the propagation of seismic waves. The radiation patterns of longitudinal waves have non-uniform lobes, which shows the characteristic of anisotropy. Stress-induced anisotropic parameters are smaller than intrinsic anisotropy at high-stress levels and a nearly linear function of the increased initial stress. These results underscore the importance of considering stress-induced anisotropy in the inversion of focal mechanisms and provide a new perspective for monitoring underground stress.

Plain Language Summary  Because of the Earth’s gravity and crustal movement, underground rocks are subjected to three different principal stresses. The accumulation of tectonic stress is often associated with the occurrence of earthquakes. Getting a state of underground stress may help us determine whether an earthquake is likely to occur in a region. We use the third-order elastic theory, in which the strain energy contains the cubic term of strain, and use Lagrangian boundary conditions to describe the seismic source modeled by a tangential sliding fault. The intensity of seismic sources caused by slipping faults can be characterized by seismic moment tensors. We decompose the tensors into isotropic, double-couple, and compensated linear vector dipole components, and these components are closely related to the stress on the fault plane. Our results show that a shear faulting in stressed media may exhibit a tensile effect. The initial stress affects the source intensity and shape of radiation and affects the propagation of seismic waves. In addition, the symmetry of the stressed medium is also closely related to the initial stress. Future studies may use our work to obtain stress states in the focal region from the source information.

1. Introduction

Due to tectonic stress and gravity, crustal rocks are subjected extensively to initial stress. The knowledge of in situ stress states is of great significance for evaluating the stability of regional crust, reservoir characterization, and improving drilling and production safety. Moreover, the regional stress state can provide the foundation for geological structure interpretation and seismic monitoring and forecast. The seismicity-stress inversion method has been used to estimate the Coulomb stress (Jia et al., 2020). With the development of non-invasive techniques, elastic waves are widely used to investigate the elastic properties of rock. In laboratory measurements, the coda wave interferometry method has been used to monitor velocity changes in medium (Singh et al., 2019; Snieder et al., 2002) and study earthquake focal mechanisms (Robinson et al., 2007). The ambient noise cross-correlation method in field measurements is widely used to detect seismic velocity changes in the seismic source zone (Zaccarelli et al., 2011). We may get the elastic properties of the medium in the focal region by measuring the seismic wavefields. It is necessary to study the effect of stress on the generation and propagation of seismic waves so that we can use the information related to seismic waves to monitor underground stress.

Earth's crustal rocks usually contain cracks or fractures that induce elastic anisotropy under non-hydrostatic stress. Biot (1940, 1965) showed that an isotropic material might possess an induced anisotropy under the initial
anisotropic stress. The initial stress affects seismic wave velocity (Dahlen, 1972a; Korneev & Glubokovskikh, 2013; Nur & Simmons, 1969; Sharma, 2005), anisotropy (Becker et al., 2007; Sayers & Kachanov, 1995; Zheng, 2000), and frequency-dependence (Goodfellow et al., 2015; Oda et al., 1990; Okamoto et al., 2013; Toksoz et al., 1979).

In order to investigate the effect of stress-induced anisotropy on seismic waves, three crucial aspects need to be considered. First, the initial stress will affect the traditional elastic wave motion equation. In seismic exploration, the influence of stress on the wave equation itself is usually ignored, which is valid when the initial stress is minor. The form of the equation of motion depends on the configuration in which the physical quantities are considered. However, a complete form of the motion equation that considers the effect of initial stress in ultrasonic nondestructive testing (Toupin & Bernstein, 1961) and global seismology (Dahlen & Tromp, 1998) has been proposed. In particular, the equation of motion in the intermediate configuration intuitively has a concise form.

Second, the initial stress changes the effective elastic properties of underground rocks. In general, three different approaches, namely, micromechanical theories, adiabatic pressure derivative method, and third-order elasticity (TOE) or acoustoelasticity, have been proposed by geophysicists to quantitatively investigate the effect of stress on elastic moduli of underground media (Sripanich et al., 2021). The micromechanical theories focus on the stress-dependent responses of micromechanical structures such as pores and microcracks (Gurevich et al., 2011; Sayers & Kachanov, 1995; Shapiro & Kaselow, 2005). In this approach, microcracks are commonly treated as compliant parts of pore space, which are more sensitive to stress than stiff pores. In global seismology, Tromp and Trampert (2018) recently proposed the adiabatic pressure derivative method that stems from the non-uniqueness of elastic tensors in continuum mechanics. This method can be conveniently used to estimate approximately the elastic moduli under non-hydrostatic stress by adiabatic pressure derivative under hydrostatic stress (Sripanich et al., 2021; Tromp et al., 2019).

Acoustoelasticity is based on continuum mechanics and higher-order elastic constants (Brugger, 1964; Hughes & Kelly, 1953; Murnaghan, 1951; Pao & Gamer, 1985), which forms the fundamental basis for using ultrasonic waves to estimate stress state in materials. Brugger (1964) and Thurston and Brugger (1964) have given a complete description of the theory of third-order elasticity. Some variants of the TOE theory have been proposed in the seismic exploration based on different application scenarios and assumptions (Johnson & Rosolofosaon, 1996; Norris et al., 1994; Sinha & Plona, 2001). Similar to the theory of acoustoelasticity, Maitra and Al-Attar (2021) investigated the stress dependence of the elastic tensor and discussed the importance of undetermined rotations. In addition, the polynomial approach of Eringen is used to analyze prestressed problems in anisotropic media (Feng et al., 2009, 2011). Among the above theories, TOE theory generally does not rely on micromechanical models and does not assume the source of nonlinearity, making it more general for the characterization of various rocks. Third, boundary conditions may be affected by initial stress. Norris et al. (1994) investigated the acoustoelastic effects in the medium with solid/liquid interfaces. In the medium with a sliding interface, the tractions described before deformation are not necessarily continuous functions of undeformed coordinates because the particles are not adjacent across the interface under the influence of the initial static stress. In particular, this effect also needs to be considered in the description of seismic faults. Dahlen (1972b) considered the influence of initial stress when describing the continuity condition of the fault. Lagrangian description of fault boundary conditions can be found in the book of Dahlen and Tromp (1998). These conditions will influence the description of seismic sources.

Generally, the observed seismic records result from both source and propagation effects. The effect of prestress on the propagation of elastic waves is widely investigated in seismology (Harkrider et al., 1994; Tromp & Trampert, 2018; Walton, 1973; Zatsepin & Crampin, 1997) and borehole geophysics (Hsu et al., 2011; Liu et al., 2019; Sayers, 2007; Schmitt et al., 2012; Sinha & Kostek, 1996; Wang & Tang, 2005). Moreover, the literature describing seismic sources in unstressed media can refer to the groundbreaking work of Burridge and Knopoff (1964). The source effects are also studied in complex conditions, such as anisotropic media (Grecha, 2020; Vavryčuk, 2005; Yao & Wang, 2021) and porous media (Wang et al., 2015). Menke and Russell (2020) investigate the behavior of non-DC components of the moment tensor in a transversely isotropic medium and prove that ISO and CLVD components can be applied to determine the orientation and strength of anisotropy. However, few studies on seismic sources consider the influence of initial stress. Dahlen (1972b) deduced the motion equation and continuity condition of a uniformly rotating, self-gravitating, perfectly elastic Earth model with an arbitrary initial static stress field and gave the equivalent body force of a seismic dislocation. However, the corresponding reference state of elastic moduli is not specified. He concluded that the influence of deviatoric stress on the radiation patterns of P and S waves could be almost ignored in the case of first-order approximation (Dahlen, 1972c). Based on the description of the current configuration, Walton (1973) obtained...
the equivalent force resulting from shear discontinuity in a pre-strained medium, but the results still contain the discontinuity of traction. Boschi (1973) derived expressions of the equivalent body force generated by seismic dislocations by the variational method. However, the effect of initial stress on the elastic properties of the medium in the source area is not considered. Bonafede et al. (1980) derived the first-order corrections of radiation patterns from a dislocation source in prestressed media, but the effect of prestress on fault boundary conditions was neglected in their studies. Norris et al. (1994) showed that the initial stress has a significant influence on boundary conditions of the sliding interface. This paper will consider the effects of initial stress on the fault boundary condition.

In order to describe the seismic source in a stressed medium analytically, we must make some assumptions. Since the ground stress field is relatively stable for a short period, we first assume that the source region is in static equilibrium and undergoes elastic deformation under tectonic stress before an earthquake. Then we assume that the static deformation of the source region is uniform. In other words, initial stress fields do not change with the spatial position of the focal area. The wavelengths of seismic waves in the seismic record are usually much larger than the scale of the source region, so the process of stress release by fault slip can be described in terms of infinitesimal motions produced by the point source (Aki & Richards, 2002). In addition, we use an isotropic propagation medium to calculate the seismic radiation.

In this paper, we utilize the complete form of the motion equation described in the intermediate configuration and the boundary conditions involving the initial stress to derive the seismic moment tensor generated by a shear dislocation in a homogeneous and stress-induced anisotropic medium. Then we quantitatively investigate the effect of stress-induced anisotropy on seismic sources and seismic wave propagation.

2. Basic Equations and Boundary Conditions Describing Seismic Sources

Based on the continuum mechanics theory, we can divide the deformation process of the Earth under in situ stress into two stages. First, before the earthquake, imagine that the Earth moves from a stress-free natural configuration (corresponding to an unloaded state in the laboratory) to an intermediate configuration through quasi-static deformation under the accumulation of long-term tectonic stress. The intermediate configuration is associated with a representative body in static equilibrium observed before the earthquake. Therefore, the physical parameters of intermediate configurations may be associated with the precursors of earthquakes. After the earthquake, the dynamic deformation caused by the earthquake movement deforms the Earth into the current configuration. The definitions of deformation and stresses are shown in Appendix A. We model the seismic response in a perfectly elastic medium with initial stress as a small motion superimposed on a finite elastic deformation to describe the above two motion processes.

In order to distinguish the physical quantity in different states, we specify the symbols related to deformation motion in Table 1. Physical quantities with tildes in the table refer to those quantities in the intermediate state, superscript “0” refers to the initial stress, superscript “t” represents the total stress, and the quantity without superscript represents the incremental stress. Uppercase Latin, lowercase Greek, and lowercase Latin letters are associated with the natural, intermediate, and current states, respectively. The summation convention on repeated subscripts is implied in this paper.

2.1. Equations of Motion in the Presence of Initial Stress

Based on the description of stress tensors in different configurations, the equations of motion can be written in terms of total stress in the form

| Table 1 |
| --- |
| **Notations in the Three Configurations** |
| Configuration | Position | Volume/Surface | Density | PK stress | SK stress |
| Natural \( B_0 \) | \( X \) | \( V_0, S_0, N \) | \( \rho_0 \) | \( T_A^0, T_B^0, T_C^0 \) | \( S_{KL}^0, S_{KL}^0, S_{KL}^0 \) |
| Intermediate \( \hat{B} \) | \( \xi \) | \( \hat{V}, \hat{S}, \hat{N} \) | \( \hat{\rho} \) | \( \hat{T}_{ij}^0, \hat{T}_{ij}^0, \hat{T}_{ij}^0 \) | \( \hat{S}_{ij}^0, \hat{S}_{ij}^0, \hat{S}_{ij}^0 \) |
| Current \( B(t) \) | \( x \) | \( V, S, n \) | \( \rho \) | | |
\[
\begin{align*}
T_{ij}^\alpha + \rho_0 f_{ij}^\alpha &= \rho_0 \omega_{ij,\alpha}, \quad X \in V_0 \\
\tilde{T}_{ij}^\alpha + \tilde{\rho} f_{ij}^\alpha &= \tilde{\rho} \tilde{\omega}_{ij,\alpha}, \quad \xi \in \tilde{V} \\
t_{ij}^\alpha + \rho f_{ij}^\alpha &= \rho t_{ij,\alpha}, \quad x \in V
\end{align*}
\]

(1)

where \( f_{ij}^\alpha = f_{ij}^\alpha + f_{ij} \) denote the mass density of the total body force, \( f_{ij}^\alpha \) is the mass density of body force associated with static deformation. In view of Equations A5, A10, and A16, stresses among the three configurations are related by

\[
T_{ij}^\alpha = \hat{\rho} / \rho_0 F_{sA} T_{ij}^\alpha = (J_0)^{-1} F_{sA} T_{ij}^\alpha, \quad t_{ij}^\alpha = \rho / \rho_0 F_{iA} T_{ij}^\alpha = (J)^{-1} F_{iA} T_{ij}^\alpha
\]

(2)

where \( J_0 = \text{det}(F_{sA}) \) and \( J = \text{det}(F_{iA}) \) are the Jacobians of the transformation associated with static and total deformation, respectively.

The small-on-large theory assumes that dynamic deformation is much smaller than static deformation, that is

\[
\mathbf{u} \ll \mathbf{w}, \quad \frac{\partial \mathbf{u}}{\partial X} \ll \frac{\partial \mathbf{w}}{\partial X}
\]

(3)

where \( \mathbf{u} \) and \( \mathbf{w} \) denote dynamic and static displacements, respectively. A detailed description of static and dynamic deformation is given in Appendix A. The assumption (Equation 3) means that the rotation and strain generated by the dynamic disturbance are very small. Since the coordinate systems are chosen to be consistent in the initial and the current configurations, we will no longer strictly distinguish the displacements \( \mathbf{u}(\xi) \) and \( \mathbf{u}(x) \) in the following sections. The reader can easily discern the meaning by context.

In the ‘small-on-large’ theory, total stresses are approximately expanded into two parts by a linear approximation (Norris et al., 1994):

\[
\begin{align*}
T_{ij}^\alpha(X) &= T_{ij}^\alpha_0(X) + T_{ij}(X) + \ldots, \quad X \in V_0 \\
\tilde{T}_{ij}^\alpha(\xi) &= \tilde{T}_{ij}^\alpha_0(\xi) + \tilde{T}_{ij}(\xi) + \ldots, \quad \xi \in \tilde{V} \\
t_{ij}^\alpha(x) &= t_{ij}^\alpha_0(x) + t_{ij}(x) + \ldots, \quad x \in V
\end{align*}
\]

(4)

where \( T_{ij}^\alpha_0, \tilde{T}_{ij}^\alpha_0 \), and \( t_{ij}^\alpha_0 \) are the initial PK stresses described in the natural, intermediate, and current states, respectively. The second terms of the right hand in Equation 4 are incremental stresses associated with dynamic deformation. In the deformation process of Earth, the fault boundary in the initial configuration is usually known. Therefore, it is convenient to use the PK stress tensor to describe the seismic source with initial stress. The initial stress in the intermediate configuration is in static equilibrium, that is

\[
\tilde{T}_{ij,\alpha} + \tilde{\rho} f_{ij}^\alpha = 0
\]

(5)

Upon inserting Equation 4 into Equation 1 and considering Equation 5, the equation of motion the intermediate configuration can be written in the form

\[
\tilde{T}_{ij,\alpha} + \tilde{\rho} f_{ij} = \tilde{\rho} \frac{\partial^2 \mathbf{u}_{ij}}{\partial \xi^2}, \quad \forall \xi \in \tilde{V}
\]

(6)

2.2. Constitutive Relation Expressed by the Incremental Stress

In order to consider the effect of initial stress on the elastic properties of the medium, we use the third-order elasticity to establish the constitutive relation between PK stress and dynamic displacement gradient. Assume that the stored energy is wholly transformed into internal energy by an isotropic elastic deformation process. Then, the PK stress tensor that meets the definition of thermodynamics can be expressed as (Eringen & Suhubi, 1974)

\[
T_{ij}^\alpha(X) = \rho_0 \frac{\partial W}{\partial F_{ij}}
\]

(7)
where $W$ denotes the internal energy per unit mass stored during the static and dynamic deformation. $F_{kk}$ is the deformation gradient tensor between the current and natural configurations, defined in Equation A5 in Appendix A. The PK stress can usually be expressed as a function of the deformation gradient. Constitutive relation related to incremental PK stress can be obtained by expanding the total PK stress tensor in terms of the deformation gradient in the intermediate state. Accordingly, the total PK stress can be expressed by

$$\tilde{T}_k^0(X) = T_{kk}^0(X) + \frac{\partial T_{kk}^0}{\partial F_{kk} \mid_1} (F_{IL} - F_{uL} \delta_{uL}) + \ldots = T_{kk}^0(X) + A_{kkL}^0 u_L + \ldots$$  \hspace{1cm} (8)

where $A_{kkL}^0$ called elasticities used by Eringen and Suhubi (1974), and the subscript “1” denotes quantities in the intermediate state, that is, $\mathbf{u} = 0$. Using the relation (Equation 7), we obtain an explicit formula for the elasticities $A_{kkL}^0$ and initial PK stress in terms of the energy density $W$:

$$T_{kk}^0 = \rho_0 \frac{\partial W}{\partial F_{kk} \mid_1}$$  \hspace{1cm} (9)

$$A_{kkL}^0 = \frac{\partial T_{kk}^0}{\partial F_{kk} \mid_1} = S_{kkL} \delta_{uL} + \rho_0 \frac{\partial^2 W}{\partial E_{KM} \partial E_{MN} \mid_1} \tilde{F}_{IL} \tilde{F}_{LM}$$  \hspace{1cm} (10)

where we have used the SK stress defined by $S_{kkL} = \rho_0 \frac{\partial W}{\partial E_{KL} \mid_1}$.

To obtain the relation between the incremental PK stress and energy density, we substitute Equations 9 and 10 into Equation 8 and use the relation (Equation 2). Neglecting terms of second-order in $\|\mathbf{u}\|$ and comparing with Equation 4, after some manipulation, we obtain

$$\tilde{T}_{ij} = B_{ij\beta} u_{\beta}$$  \hspace{1cm} (11)

where $B_{ij\beta}$ are the components of the equivalent elastic tensor associated with the initial Cauchy stress $\tilde{T}_{ij}^0$ in the intermediate configuration. The elastic tensor $\mathbf{B}$ can be written in the form

$$B_{ij\beta} = \tilde{T}_{ij}^0 \delta_{\beta j} + C_{ij\beta}$$  \hspace{1cm} (12)

where $C_{ij\beta}$ is defined by

$$C_{ij\beta} = \rho_0 (j_0)^{-1} \frac{\partial^2 W}{\partial E_{KL} \partial E_{MN} \mid_1} \tilde{F}_{IL} \tilde{F}_{LM} \tilde{F}_{MN}$$  \hspace{1cm} (13)

The relation (Equation 11) is the general constitutive relation of PK stress in the initial configuration. We see that the equivalent elastic tensors $\mathbf{B}$ and $\mathbf{C}$ satisfy the symmetry relations, respectively:

$$B_{ij\beta} = B_{\beta j i}$$  \hspace{1cm} (14)

$$C_{ij\beta} = C_{\beta j i} = C_{ij\beta}$$  \hspace{1cm} (15)

Assume that the strain energy possesses a Taylor expansion of the form associated with the Lagrangian strain (Brugger, 1964; Toupin & Rivlin, 1960), that is,

$$\rho_0 W = c_0 + c_{AB} E_{AB} + \frac{1}{2} c_{ABCD} E_{AB} E_{CD} + \frac{1}{6} c_{ABCD EF} E_{AB} E_{CD} E_{EF} + \ldots$$  \hspace{1cm} (16)

where $c_{ABCD}$ and $c_{ABCD EF}$ are the second-order and third-order elastic moduli in the natural state. For an elastic medium, strain energy density and stress are zero in the absence of strain, so $c_0, c_{AB} = 0$. Inserting representations (Equations 16 and A4) in Appendix A into Equation 13 and ignoring quadratic and higher-order terms in the static displacement gradient, the effective elastic tensor related to static deformation is given by

$$B_{ij\beta} = c_{ij\beta} (1 - w_{h\beta} + \tilde{T}_{ij}^0 \delta_{\beta j} + c_{k\beta j} u_{\beta K} + c_{k\beta j} w_{i K} + c_{k\beta j} w_{j K} + c_{k\beta j} w_{k K})$$  \hspace{1cm} (17)

Upon using Equation 11 we rewrite Equation 6 in the form
(B_{ij}; \bar{u}_{i,j})_{,\mu} + \bar{p} \bar{j}_{j} = \bar{\rho} \bar{u}_{i,j} \tag{18}

The results (Equations 17 and 18) are the basic equations describing the motion in stressed media, which have been obtained in different ways in previous studies (Norris et al., 1994; Pao & Gamer, 1985; Sinha, 1982). Next, these equations will be used to establish reciprocal relations of incremental wavefields and investigate the effect of initial stress on seismic sources and the resulting seismic waves.

2.3. Boundary Conditions Across the Surface of a Shear Fault

Generally, the interface conditions include kinematical boundary conditions and dynamic boundary conditions. Kinematical boundary conditions on the welded or solid-solid boundaries require that particles maintain contact across a deformed interface. The Lagrangian description of this condition is precisely expressed as

$$[u]^+ = 0 \tag{19}$$

where the notation $[q]^+$ denotes the jump in quantity $q$ on the surface approached from the positive and negative sides of its positive normal. In order to ensure that material particles do not separate at the slipping boundary allowing tangential slip, Equation 19 must be replaced by

$$[\mathbf{N} \cdot u]^+ = 0 \tag{20}$$

Dynamic boundary conditions require that traction vectors must be continuous across both a welded and slipping deformed boundary:

$$[u \cdot \mathbf{t}]^+ = 0 \tag{21}$$

This condition on a welded boundary can be conveniently written in terms of the PK stress in the form

$$[\mathbf{N} \cdot \mathbf{T}^{PK}]^+ = 0 \tag{22}$$

Since two adjacent points need not be adjacent after deformation, Equation 22 is not valid in the presence of tangential slip. Note that the relation (Equation 21) is exact at adjacent positions in the current configuration. However, the sliding of particles at the interface makes pointwise conditions difficult to apply in intermediate coordinates directly.

Now assume that the fault that produces earthquakes is pre-existing and not caused by the initial stress. The depiction of boundary conditions across the fault surface is shown in Figure 1. The initial fault surface $\Sigma_0$ with a boundary curve $\partial \Sigma_0$ is embedded in Earth model $V$ with the surface $\partial V$. The fault surface may or may not intersect Earth's surface. The initial surface elements $d\hat{S}^+$ with the unit normal $\hat{\mathbf{N}}^+$ and $d\hat{S}^-$ with the unit normal $\hat{\mathbf{N}}^-$ are two surface elements centered on position $\xi^+$ and $\xi^-$ on the front and back sides of the initial fault surface $\Sigma_0$, respectively. The displacements of the particle $\xi^+$ and $\xi^-$ are denoted by $u^+$ and $u^-$, respectively.

Without loss of generality, assume that the initial surface elements $d\hat{S}^+$ and $d\hat{S}^-$ on the fault surface are combined into the continuous surface element $dS$ centered on the current position $x$ after undergoing the dynamic deformation so that

$$\xi^+ + u^+ = \xi^- + u^- = x \text{ and } [u]^+ = - (\xi^+ - \xi^-) \tag{23}$$

The slip $[u]^+$ denotes tangential displacement occurring on the undeformed fault surface in this paper. To convert the boundary conditions of the current configuration to the intermediate coordinate system, we rewrite the precise Eulerian boundary condition across the fault surface in the form (Dahlen & Tromp, 1998)

$$[\mathbf{n} dS \cdot \mathbf{t}]^+ = 0 \tag{24}$$

Upon using definitions (Equations A11 and A13), the condition (Equation 24) can be written in the intermediate configuration in terms of PK stress in the form
\[
\hat{\mathbf{N}}^+ d\hat{S}^+ \cdot (\mathbf{T}^0 + \mathbf{T}^{PK+}) = \hat{\mathbf{N}}^- d\hat{S}^- \cdot (\mathbf{T}^0 + \mathbf{T}^{PK-})
\]  \tag{25}

Using the representation (Equation A9) and some proper manipulations, infinitesimal fault surface areas \(d\hat{S}_0^+\) and \(d\hat{S}_0^-\), and the corresponding unit normal vectors \(\hat{\mathbf{N}}^+\) and \(\hat{\mathbf{N}}^-\) at the points \(\xi^+\) and \(\xi^-\) are related by

\[
d\hat{S}^+ - d\hat{S}^- = -(\nabla_{\Sigma_0} \cdot [\mathbf{u}]^+^+) d\hat{S}^\pm
\]
\[
\hat{\mathbf{N}}^+ - \hat{\mathbf{N}}^- = \left(\nabla_{\Sigma_0} [\mathbf{u}]^+^\pm\right) \cdot \hat{\mathbf{N}}^\pm
\]  \tag{26, 27}

where \(\nabla_{\Sigma_0} = \nabla - \hat{\mathbf{N}} \cdot \nabla\) is the surface gradient operator on the surface \(\Sigma_0\). Since the Eulerian initial traction \(\hat{\mathbf{N}} \cdot \mathbf{T}^0\) is continuous at two neighboring points on either side of the fault, the initial tractions at the point \(\xi^+\) and \(\xi^-\) are related by

\[
\hat{\mathbf{N}}^+ \cdot \mathbf{T}^{0\pm} - \hat{\mathbf{N}}^- \cdot \mathbf{T}^{0\pm} = -[\mathbf{u}]^\pm \cdot \nabla_{\Sigma_0} \left(\hat{\mathbf{N}} \cdot \mathbf{T}^0\right)
\]  \tag{28}

Upon inserting Equations 26–28 into Equation 25, we obtain the dynamic boundary condition in terms of the incremental PK stress in the form

\[
\left[\hat{\mathbf{N}} \cdot \mathbf{T}^{PK} - \nabla_{\Sigma_0} \cdot (\mathbf{u} \hat{\mathbf{N}} \cdot \mathbf{T}^0)\right]^+ = 0
\]  \tag{29}

Dahlen (1972b) used this condition to describe the continuity of the fault plane in the self-gravitating Earth with initial stress, but initial stress does not produce static deformation. Norris et al. (1994) also considered a similar boundary condition in studying the acoustoelasticity of fluid-solid composite systems. Correct to first order in \(\|\mathbf{u}\|\), the fault boundary conditions are summarized as follows:

\[
\begin{cases}
\hat{\mathbf{N}}(\xi) \cdot [\mathbf{u}(\xi, t)]^\pm = 0, & \forall \xi \in \Sigma_t \\
[u(\xi, t)]^\pm = 0, & \forall \xi \in \partial\Sigma_0 = \partial\Sigma_t \cap \partial V
\end{cases}
\]  \tag{30}

The condition (Equation 30) shows that normal displacements are continuous across the fault plane, while displacements remain continuous at the point where the fault boundary does not intersect with the free boundary.

\[
\left[\hat{\mathbf{N}}(\xi) \cdot \mathbf{T}^{PK}(\xi, t)\right]^\pm = \nabla_{\Sigma_0} \cdot \left\{[\mathbf{u}(\xi, t) \cdot \hat{\mathbf{N}}(\xi) \cdot \mathbf{T}(\xi)]^\pm\right\}, \quad \forall \xi \in \Sigma_0
\]  \tag{31}
3. Seismic Moment Tensor

3.1. Seismic Moment Tensor Generated by a Shear Faulting in a Stressed Medium

We will use the above basic equation to depict the seismic source in a stress-induced medium. As shown in Figure 1, the surfaces of the Earth model \( V \) include an outer surface \( \partial V \) and two adjacent fault surfaces \( \Sigma_0^+ \) and \( \Sigma_0^- \). Strictly, the representation theorem is satisfied on the surface \( \partial V + \Sigma_0^+ + \Sigma_0^- \). We take the surface \( \partial V \) as a free surface. The physical quantities below are described using the coordinate system consistent with the intermediate configuration. So for the convenience of description, we no longer distinguish the subscripts of a physical quantity and use the lowercase Latin letter subscript. In order to maintain the consistency in form with traditional wavefield reciprocity relation, the volume density of the body force is adopted in the following derivation. Based on the work of Boschi (1973), the reciprocity relation in terms of incremental wavefields can be written in the following form

\[
\begin{align*}
\mathbf{u}_m(\xi, t) &= \int_{-\infty}^{\infty} d\tau \int_{V} \mathbf{f}_V^\prime (\eta, \tau) G_{m\ell}(\xi, t - \tau; \eta, 0) dV(\eta) \\
&- \int_{-\infty}^{\infty} d\tau \int_{\Sigma_0^+} \left\{ G_{m\ell}(\xi, t - \tau; \eta, 0) \left[ \tilde{N}_i(\xi) \tilde{N}_j(\xi, \tau) \right]_+ \right. \\
&\left. - \tilde{N}_i(\xi) [u_j(\xi, \tau)]_+^0 B_{ij\ell} G_{\ell k\ell}(\xi, t - \tau; \eta, 0) \right\} dS(\xi)
\end{align*}
\]

where \( \mathbf{G} \) is the Green's function tensor related to the Equation 18 described in the intermediate configuration. \( \mathbf{f}_V^\prime \) is the volume density of the incremental body force in the intermediate state. Using the boundary conditions (Equations 30 and 31), after some simplified manipulation, the response of the Earth model with faults system can be written in the form

\[
\begin{align*}
\mathbf{u}_m(\xi, t) &= \int_{-\infty}^{\infty} d\tau \int_{V} \mathbf{f}_V^\prime (\eta, \tau) G_{m\ell}(\xi, t - \tau; \eta, 0) dV(\eta) \\
&+ \int_{-\infty}^{\infty} d\tau \int_{\Sigma_0^+} \left\{ \tilde{N}_i(\xi) [u_j(\xi, \tau)]_+^0 \left( B_{ij\ell} + \delta_{ij} \right) G_{\ell k\ell}(\xi, t - \tau; \eta, 0) \right\} dS(\xi)
\end{align*}
\]

where \( \eta \) and \( \zeta \) denote the arbitrary position in volume \( V \) and on the fault plane, respectively.

We are mainly interested in the response of the shear faulting, then take the incremental force \( \mathbf{f} \) in Equation 33 to be zero. By introducing the surface moment-density tensor \( \mathbf{m} \), we can rewrite the Equation 33 in the form

\[
\mathbf{u}_m(\xi, t) = \int_{-\infty}^{\infty} d\tau \int_{\Sigma_0^+} \{ m_{ij\ell}(\xi, \tau) G_{ij\ell k}(\xi, t - \tau; \eta, 0) \} dS(\xi)
\]

The surface moment-density tensor is defined by

\[
m_{ij\ell} = H_{ij\ell} \tilde{N}_i(\xi) [u_j(\xi, \tau)]_+^0
\]

where

\[
H_{ij\ell} = B_{ij\ell} + \delta_{ij} \delta_{\ell k}
\]

It is easily verified that the equivalent elastic tensor \( H_{ij\ell} \) is only symmetric with respect to subscripts \( k \) and \( l \), so the moment-density tensor \( \mathbf{m} \) is symmetric. Equation 34 gives the seismic response of a shear dislocation source
in stressed media. It is worth noting that these results are applicable for a finite fault embedded in anisotropic media subjected to initial stress.

Since the wavelengths of seismic waves are much larger than the source size and the high-frequency components of seismic waves are weak and decay fast, the point-source approximation is reasonably employed to simplify the result (34). The response of an ideal fault is written in the form (Aki & Richards, 2002)

\[ u_n(\xi, t) = M_{kl} \ast G_{ml,k} \]  

where

\[ M_{kl} = \int m_{kl} dS = \int H_{ijkl} \frac{\eta}{\nu} [u_j]^+ dS \]  

The quantity \( M \) is known as the moment tensor, which plays an important role in the focal mechanism. The slip \( \mathbf{u} \) in the point-source approximation is considered as an average displacement \( \overline{\mathbf{u}} \) that occurs on the center of the fault plane \( \Sigma_0 \) by Dirac distribution in space

\[ \mathbf{u}(\xi, t) = \overline{\mathbf{u}}(t) S_0 \delta(\xi - \zeta) \]  

where \( S_0 \) is the total area of the fault plane. We consider a planar fault with the unit normal \( \mathbf{n}^0 \) and slip direction \( v^0 \) in the intermediate configuration. The moment tensor (Equation 38) can be simplified as

\[ M_{kl} = \overline{u}(t) S_0 n^0_i v^0_j H_{ijkl} \]  

where \( \overline{u}(t) \) denotes the amplitude of the average displacement \( \overline{u} \). This result is virtually identical to that in an unstressed medium, and the only difference is that effective elastic tensor \( \mathbf{H} \) replaces the usual tensor of elastic moduli.

Note that the displacement gradient tensor in Equation 17 can be decomposed into strain and rotation, regardless of geometric nonlinearity that is

\[ w_{ij} = E_{ij} + \tilde{\omega}_{ij} \]  

where \( \tilde{\omega}_{ij} = 1/2 (w_{ij} - w_{ji}) \) are the components of rotation associated with the initial static deformation. So we can rewrite the equivalent elastic tensor in the form

\[ B_{ijkl} = c_{ijkl} \left( 1 - E_{mn} \right) + e_{ijkl} E_{lm} + c_{ijkl} E_{mn} + c_{ijkl} E_{km} + c_{ijkl} E_{km} + c_{ijkl} E_{km} + c_{ijkl} E_{km} + c_{ijkl} E_{km} + c_{ijkl} E_{km} + c_{ijkl} E_{km} \]  

The result (Equation 42) shows that the equivalent elastic tensor \( \mathbf{B} \) associated with the seismic moment tensor includes the contributions of strain, rotation, and individual stress terms associated with the initial static deformation. The equivalent elastic tensor can not generally be uniquely expressed as a function of the initial stress due to the influence of undetermined rotation (Maitra & Al-Attar, 2021). However, for non-trivial material symmetry groups, such as isotropic materials, the effect of undetermined rotation can be ignored so that the equivalent elastic tensor can be expressed as a function of the initial stress. The rotation caused by tectonic movement is usually less than the strain level in strata. And laboratory measurements show that the second-order elastic constant is usually several orders of magnitude smaller than the third-order elastic constants. Therefore, even for anisotropic materials, if the rotation is smaller than the strain component, the effect of uncertain rotation on the equivalent elastic tensor can be ignored in geophysical applications (Fuck & Tsvankin, 2009; Prioul et al., 2004; Sarkar et al., 2003; Sinha & Kostek, 1996). Nevertheless, the rotation effect may need to be considered at the epicenter of a large earthquake.

Given the difficulty of determining rotation, we consider an isotropic Earth model in a stress-free natural state. The isotropic model is defined by two independent second-order Lame constants \( \lambda \) and \( \mu \), and three independent third-order Lame constants \( v_1, v_2, \) and \( v_3 \) used by Toupin and Bernstein (1961). Then, the second-order and third-order elastic tensors can be expressed as
\[ c_{ijkl} = \lambda \delta_{ij} \delta_{kl} + 2\mu I_{ijkl} \]  
\[ c_{ijkl\mu
u} = \nu_1 \delta_{ij} \delta_{kl} \delta_{\mu\nu} + 2\nu_2 (\delta_{ij} I_{\mu\nu\lambda} + \delta_{kl} I_{\mu\lambda\nu}) + \delta_{\mu\nu} I_{ijkl} \]

where \( I_{ijkl} = 1/2 (\delta_{ik} \delta_{jl} + \delta_{il} \delta_{jk}) \) is the fourth-order unit tensor. Using Voigt notation of the sixth-order elastic tensor, it is easy to obtain that \( \nu_1 = c_{123}, \nu_2 = c_{144} \) and \( \nu_3 = c_{456} \).

For initial isotropic materials, rotation-dependent terms in the equivalent elastic tensor can be ignored (Maitra & Al-Attar, 2021). The elastic tensor \( \mathbf{H} \) in Equation 36 for an initially isotropic material can be written in the form

\[ H_{ijkl} = \left[ \lambda + (\nu_1 - \lambda) E_{nm} \right] \delta_{ij} \delta_{kl} + 2 \left[ \mu + (\nu_2 - \mu) E_{nm} \right] I_{ijkl} + 2 (\lambda + \nu_2) \left( \delta_{ij} \tilde{E}_{ik} + \delta_{ij} \tilde{E}_{jk} \right) + 2 (\mu + \nu_3) \left( \delta_{kl} \tilde{E}_{ik} + \delta_{kl} \tilde{E}_{jk} \right) + \tilde{\tau}_{ik} \delta_{ij} + \tilde{\tau}_{ij} \delta_{kl} \]  

Using the linear constitutive relation to describe the strain produced by the initial stress, we obtain

\[ \tilde{E}_{ij} = -\frac{\lambda}{6\mu K} \tilde{t}_{ik} \delta_{ij} + \frac{1}{2\mu} \tilde{t}_{ij}^0 \]  

where \( K = \lambda + 2\mu/3 \) is the bulk modulus. Since initial hydrostatic stress does not cause anisotropy in the isotropic reference medium, we decompose initial stress into the isotropic and deviatoric parts:

\[ \tilde{t}_{ij}^0 = \frac{1}{3} \tilde{t}_{ik} \delta_{ij} + \tilde{t}_{ij}^0 \]  

Upon combining the result (Equations 40–47) and defining a normalized parameter \( \tilde{\tau} = \tilde{\tau}_0/\mu \), we obtain the moment tensor in the form

\[ \mathbf{M}(t) = M_0 \left\{ \left[ 1 + P_1 \right] \left( \mathbf{n}^0 \cdot \mathbf{\tilde{\tau}} + \mathbf{o}^0 \mathbf{n}^0 \right) + P_3 \left( \mathbf{n}^0 \cdot \mathbf{\tilde{\tau}} \cdot \mathbf{n}^0 + \mathbf{o}^0 \mathbf{n}^0 \cdot \mathbf{\tilde{\tau}} \right) \right\} 
+ P_1 \left( \mathbf{n}^0 \cdot \mathbf{\tilde{\tau}} + \mathbf{o}^0 \cdot \mathbf{\tau}^0 \mathbf{n}^0 + \mathbf{o}^0 \cdot \mathbf{\tilde{\tau}} \mathbf{n}^0 + \mathbf{o}^0 \mathbf{n}^0 \cdot \mathbf{\tilde{\tau}} \right) \]  

where

\[ P_1 = \frac{(3\lambda + 3\mu + 3\nu_2 + 4\nu_3)}{9K}, \quad P_2 = \frac{(\lambda + \nu_2)}{\mu}, \quad P_3 = \frac{(\mu + \nu_3)}{\mu} \]  

The quantity \( M_0 = \mu S_0 \mathbf{\tilde{\sigma}}(t) \) is the scalar moment tensor in the unstrained isotropic medium. The first term in Equation 49 is associated with the isotropic part of initial stress, while the second and third terms represent the effect of deviatoric stress. We see that the moment tensor generated by a shear fault is only affected by the third-order constant \( \nu_3 \), while \( \nu_1 \) does not contribute to the seismic moment tensor in a stress-induced medium. This result is mainly based on the following two reasons: on the one hand, in Equation 45, we can see that the product of \( \nu_1 \) and the bulk strain produced by initial stress is similar to the action of the conventional second-order Lamé constant \( \lambda \); on the other hand, the slip direction and the fault normal are perpendicular to each other for shear faults. Because of the nature of the Kronecker symbol, \( \nu_1 \) disappears in seismic moment tensors. However, for tensile earthquakes in stress-induced media, there is no doubt that the contribution of \( \nu_1 \) to the seismic moment tensor should be considered.

### 3.2. Decompositions of Seismic Moment Tensor

The symmetric moment tensor can be decomposed into isotropic and deviatoric parts.

\[ \mathbf{M} = \mathbf{M}_{ISO} + \mathbf{M}_D \]  

where

\[ \mathbf{M}_{ISO} = \frac{1}{3} \text{tr}(\mathbf{M}) \mathbf{I} = M_0 \left( \frac{3P_2 + 4P_3 + 2}{3} \right) \mathbf{n}^0 \cdot \mathbf{\tilde{\tau}} \mathbf{o}^0 \mathbf{I} \]
We define two tensors related to the fault normal and slip direction:
\[
S = \frac{1}{2} \left( n^o v^o + v^o n^o \right), \quad A = \frac{1}{2} \left( n^o v^o - v^o n^o \right)
\] (52)

Considering deviatoric moment tensor as the perturbation of the moment tensor in an unstressed medium and normalizing \( \mathbf{M}_D \) with respect to \( M_0 \), we obtain
\[
\overline{\mathbf{M}}_D = \mathbf{M}_D/M_0 = 2S + \mathbf{M}_{INC}
\] (53)

where \( \mathbf{M}_{INC} \) is the incremental part of the deviatoric seismic moment tensor due to initial stress,
\[
\mathbf{M}_{INC} = 2P_3 S + (2P_3 + 1) \left( S \cdot \overline{\tau} + \overline{\tau} \cdot S \right) - \left( \frac{4P_3 + 2}{3} \right) \left( S : \overline{\tau} \right) \mathbf{I} + \left( \overline{\tau} \cdot \mathbf{A} - \mathbf{A} \cdot \overline{\tau} \right)
\] (54)

For an isotropic medium in the absence of initial stress, \( \mathbf{M}_{INC} = 0 \), so Equation 53 degenerates to the result of a classical shear fault. In this case, the eigenvalues and eigenvectors of the tensor \( \overline{\mathbf{M}}_D \) are of the form (Stein & Wyssession, 2003)
\[
\begin{align*}
\lambda_1 &= -1, \quad \lambda_2 = 0, \quad \lambda_3 = 1 \\
\mathbf{d}_1 &= \frac{1}{\sqrt{2}} \left( v^o - n^o \right), \quad \mathbf{d}_2 = v^o \times n^o, \quad \mathbf{d}_3 = \frac{1}{\sqrt{2}} \left( v^o + n^o \right)
\end{align*}
\] (55)

When \( \| \mathbf{M}_{INC} \| \ll \| 2S \| \), we obtain the approximate eigenvalues and principal axis of the moment tensor in a stressed medium by perturbation theory (Wilkinson, 1988):
\[
\lambda'_i = \lambda_i + \Delta \lambda_i, \quad \mathbf{d}'_i = \mathbf{d}_i + \Delta \mathbf{d}_i, \quad (i = 1, 2, 3)
\] (56)

We derive explicit expressions for the first-order perturbations of eigenvalues and eigenvectors:
\[
\begin{align*}
\Delta \lambda_1 &= -P_3 - P_3 v^o \cdot \overline{\tau} \cdot v^o - \left( P_3 + 1 \right) n^o \cdot \overline{\tau} \cdot n^o + \frac{2P_3 + 1}{3} n^o \cdot v^o \\
\Delta \lambda_2 &= \frac{4P_3 + 2}{3} n^o \cdot \overline{\tau} \cdot v^o \\
\Delta \lambda_3 &= P_3 + P_3 v^o \cdot \overline{\tau} \cdot v^o + \left( P_3 + 1 \right) n^o \cdot \overline{\tau} \cdot n^o + \frac{2P_3 + 1}{3} n^o \cdot \overline{\tau} \cdot v^o
\end{align*}
\] (57)

\[
\Delta \mathbf{d}_1 = D_{12} \mathbf{d}_2 + D_{13} \mathbf{d}_3, \quad \Delta \mathbf{d}_2 = -D_{12} \mathbf{d}_1 - D_{23} \mathbf{d}_3, \quad \Delta \mathbf{d}_3 = -D_{13} \mathbf{d}_1 + D_{23} \mathbf{d}_2
\] (58)

where
\[
\begin{align*}
D_{12} &= \frac{1}{2} \left[ \left( 1 + 2P_3 \right) \left( \mathbf{d}_1 \cdot \overline{\tau} \cdot \mathbf{d}_2 \right) - \left( \mathbf{d}_2 \cdot \overline{\tau} \cdot \mathbf{d}_1 \right) \right] \\
D_{13} &= \frac{1}{4} \left( \mathbf{d}_1 \cdot \overline{\tau} \cdot \mathbf{d}_1 - \mathbf{d}_3 \cdot \overline{\tau} \cdot \mathbf{d}_3 \right) \\
D_{23} &= \frac{1}{2} \left[ \left( 2P_3 + 1 \right) \left( \mathbf{d}_2 \cdot \overline{\tau} \cdot \mathbf{d}_1 \right) - \mathbf{d}_1 \cdot \overline{\tau} \cdot \mathbf{d}_2 \right]
\end{align*}
\] (59)

The eigenvectors of moment tensor are generally denoted by \( \mathbf{p}, \mathbf{t} \) and \( \mathbf{b} \), which define the principal axes system of moment tensor represented by \( P, T, \) and \( B \) axes. These axes correspond to the minimum, maximum and intermediate eigenvalues, respectively. Physically, three axes represent the directions of the maximum compressive, maximum tensional, and intermediate stresses generated at the source, respectively.

Further, the seismic moment tensor in an anisotropic medium can be decomposed into the isotropic (ISO), double-couple (DC), and compensated linear vector dipole (CLVD) parts (Jost & Herrmann, 1989; Knopoff & Randall, 1970; Vavryčuk, 2001):
\[
\mathbf{M} = \mathbf{M}_{ISO} + \mathbf{M}_{DC} + \mathbf{M}_{CLVD}
\] (60)

In the case of first-order approximation, the principal components of the moment tensor can be decomposed into
\[
\mathbf{M}_{\text{ISO}} = \left( \lambda_{0,\text{ISO}}^0 + \lambda_{\text{ISO}} \right) \mathbf{M}_0 \mathbf{E}_{\text{ISO}} \\
\mathbf{M}_{\text{DC}} = \left( \lambda_{0,\text{DC}}^0 + \lambda_{\text{DC}} \right) \mathbf{M}_0 \mathbf{E}_{\text{DC}} \\
\mathbf{M}_{\text{CLVD}} = \left( \lambda_{0,\text{CLVD}}^0 + \lambda_{\text{CLVD}} \right) \mathbf{M}_0 \mathbf{E}_{\text{CLVD}}
\]

(61)

where \( \lambda_{0,\text{ISO}}^0 = \lambda_{\text{CLVD}}^0 = 0 \) and \( \lambda_{0,\text{DC}}^0 = 1 \) are the parameters related to the unstressed state, respectively. The basis tensors for ISO, DC, and CLVD components are in the form

\[
\mathbf{E}_{\text{ISO}} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}, \quad \mathbf{E}_{\text{DC}} = \begin{bmatrix} -1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}, \quad \mathbf{E}_{\text{CLVD}} = \begin{bmatrix} 0 & -1 & 0 \\ 0 & 0 & 2 \end{bmatrix}
\]

(62)

The additional components of the moment tensor \( \lambda_{\text{ISO}}, \lambda_{\text{CLVD}} \) and \( \lambda_{\text{DC}} \) can be expressed as

\[
\begin{align*}
\lambda_{\text{ISO}} &= \frac{3P_3 + 4P_5 + 2}{3} n^0 \cdot \mathbf{n} \cdot \mathbf{n}^0 \\
\lambda_{\text{CLVD}} &= \frac{4P_3 + 2}{3} n^0 \cdot \mathbf{n} \cdot \mathbf{n}^0 \\
\lambda_{\text{DC}} &= P_3 + P_5 n^0 \cdot \mathbf{n} \cdot \mathbf{n}^0 + (P_3 + 1) n^0 \cdot \mathbf{n} \cdot \mathbf{n}^0 - (2P_3 + 1) n^0 \cdot \mathbf{n} \cdot \mathbf{n}^0
\end{align*}
\]

(63)

The percentage of the ISO component is determined by

\[
c_{\text{ISO}} = \frac{1}{3} \frac{\text{Trace}(\mathbf{M})}{|M_{\text{MAX}}|} \times 100\%
\]

(64)

where \( M_{\text{MAX}} \) denotes the eigenvalue of the seismic moment tensor with the maximum absolute value. The percentages of the CLVD component and the DC component are given, respectively

\[
c_{\text{CLVD}} = 2\varepsilon \left( 100\% - |c_{\text{ISO}}| \right)
\]

(65)

\[
c_{\text{DC}} = 100\% - |c_{\text{ISO}}| - |c_{\text{CLVD}}|
\]

(66)

The parameter \( \varepsilon \) measures the size of CLVD relative to DC (Julian et al., 1998; Sipkin, 1986)

\[
\varepsilon = \frac{M_{\text{MIN}}}{M_{\text{MAX}}}
\]

(67)

where \( M_{\text{MIN}} \) and \( M_{\text{MAX}} \) denote eigenvalues of the deviatoric moment with the maximum and minimum absolute values, respectively.

Generally, the sum of the ISO and CLVD components is called non-DC components of the moment tensor, and the sum of the absolute values of the DC and non-DC components equals 100%. We see that the non-DC components are non-zero for the shear faulting in a stressed medium, which results from the anisotropy induced by non-hydrostatic stress. The CLVD component is mainly related to initial stress through the third-order elastic constant \( \nu_3 \). Similar to the moment tensor in an intrinsic anisotropic medium, stress-induced anisotropy will significantly affect the behavior of the non-DC components of the seismic moment tensor.

4. Results

4.1. The Rock Parameters for Calculations

In order to understand the effect of initial stress on the seismic source, the properties of focal mechanisms and the calculation of fault plane solutions in stress-induced media will be illustrated numerically. We consider 15 rocks investigated in laboratory experiments. These rocks mainly include sandstone, limestone, granite, marble, and thermally cracked rocks. The parameters of these rocks are presented in Table 2. For convenience, the principal
The initial static stresses represented by the principal stress are set to \( \sigma_{11} = -30 \text{ MPa} \), \( \sigma_{22} = -15 \text{ MPa} \), and \( \sigma_{33} = -20 \text{ MPa} \).

4.2. Non-DC Components Generated by the Shear Faulting

The effects of initial stress on seismic moment tensors are manifested in two aspects: the magnitude of initial stress affects the amplitude of the effective elastic constant in the moment tensor, and the elastic anisotropy induced by deviatoric stress makes the shear dislocation source produce non-DC components. To estimate the levels of the non-DC components in various rock models presented in Table 2, we generate 10,000 seismic events produced by shear faults with a uniform random distribution of spatial orientations, which are defined by angles of the strike, dip, and slip. The definitions of parameters related to the spatial orientations of the fault are presented in Figure 2. Then, we use Equation 48 to calculate seismic moment tensors and investigate the DC, ISO, and CLVD components quantitatively by Equations 64–66.

We investigate the characteristics of non-DC components generated by shear faults in the model of Berea sandstone, Barre granite, and Dry D82 marble with the initial stress. For the Berea SS (A) model, the percentages of ISO and CLVD are in the ranges of −20.0–20.0 and −1.8–1.8, respectively (see Figure 3). We can see that the ISO is more significant than CLVD at the initial stress above. The percentages of ISO and CLVD are in the intervals (−8.0, 8.0) and (−6.0, 6.0) in Barre granite, respectively. While in Dry D82 marble, they are in the intervals (−18.0, 18.0) and (−11.0, 11.0), respectively (see Figures 4 and 5). The CLVD component increases significantly and is comparable to ISO in the model of the latter two rocks. This is because the third-order elastic constants \( A_{32} \) and \( A_{33} \) are comparable in the two types of rocks. Overall, shear faulting produces remarkable non-DC components in all rock models in the presence of initial stress.

The extreme values of the DC, CLVD, and ISO in all rock models are summarized in Table 3. The table shows that the percentage of ISO is larger than that of CLVD, except for three rock models (Buff SS, Massilon SS, and Dry Castlegate sandstone). At the stress levels given above, the maximum absolute value of ISO generally remains in the range of 5–25 and CLVD in the range of 5–20. The most significant difference between ISO and CLVD components is found in Berea SS(A) because the third-order elastic constant \( v_3 \) of Berea SS(A) is significantly smaller than the constant \( v_2 \). The highest value of CLVD is almost 23% in Dry Castlegate sandstone, and

| Rock samples          | \( \rho \) (kg/m\(^3\)) | \( K \) (GPa) | \( \mu \) (GPa) | \( v_1 \) (GPa) | \( v_2 \) (GPa) | \( v_3 \) (GPa) |
|-----------------------|--------------------------|---------------|---------------|---------------|---------------|---------------|
| Berea SS(A)           | 2.120                    | 4.1           | 4.5           | 3,160         | −3,216        | −112.8        |
| Berea SS(B)           | 2.080                    | 3.7           | 3.7           | −2,090        | −2,425        | −1,558.3      |
| Buff SS               | 2.270                    | 9.9           | 9.5           | 356           | −1,382        | −1,053.8      |
| Hanson SS             | 2.270                    | 10.4          | 10.0          | −2,608        | −2,121        | −842.8        |
| Limestone1078         | 2.170                    | 21.0          | 12.2          | −5,152        | −746          | −301.8        |
| Limestone1083         | 2.490                    | 29.6          | 20.6          | −3,736        | −6,434        | −2,432        |
| Massilon SS           | 2.090                    | 6.1           | 6.3           | −4,460        | −5,670        | −4,382        |
| Portland SS           | 2.140                    | 9.7           | 7.3           | −680          | −419          | −280          |
| Westerly granite      | 2.650                    | 29.9          | 23.6          | −2,300        | −20227        | −3,517.8      |
| Barre granite         | 2.650                    | 13.8          | 18.2          | 142           | −3,442        | −1,650        |
| Dry F32 Fontainebleau sandstone | 2.414     | 15.3          | 11.7          | −81700        | −56950        | −21225        |
| Dry F32 thermally cracked | 2.414    | 7.8           | 5.7           | −53900        | −47050        | −8,725        |
| Dry D82 marble        | 2.870                    | 30.0          | 21.3          | −50100        | −15250        | −5,075        |
| Dry Castlegate sandstone | 2.000      | 3.67          | 3.25          | −95.9         | −162.5        | −340.8        |
| Dry Indiana Limestone | 2.210                    | 16.28         | 10.85         | −10550        | −5,643        | −873.3        |

Note: The parameters in this table were calculated from data presented or collected in the following publications: samples 1–9 are from Winkler and Liu (1996), samples 10–13 are from Johnson and Rasolofosaon (1996), sample 14 is from Sinha and Plona (2001), sample 15 is from Winkler and McGowan (2004).
the lowest value is almost 2% in Berea SS(A). The highest value of ISO is 30% in the Dry F32 thermally cracked because many microcracks are produced by cyclic thermal loading. The above results are consistent with the Equation 63, which shows that the CLVD component is dominated by the third-order elastic constant $\varepsilon_3$, while the ISO component is determined by the third-order elastic constants $\varepsilon_2$ and $\varepsilon_3$ together.

In seismic practice, fault plane solutions are almost always obtained under the assumption of the isotropic seismic source region. The approximate values of the fault normal and slip direction can be calculated from the eigenvectors of the moment tensor using the following relations:

$$\mathbf{n}_{\text{app}}^0 = (\mathbf{p} + \mathbf{t}) / \sqrt{2}$$

$$\mathbf{v}_{\text{app}}^0 = (\mathbf{p} - \mathbf{t}) / \sqrt{2}$$

where $\mathbf{p}$ and $\mathbf{t}$ denote the unit vectors corresponding to the P and T axes. We use the pair of $\mathbf{n}_{\text{app}}^0$ and $\mathbf{v}_{\text{app}}^0$ to calculate the error between the true and approximate values of the fault normal and slip directions. The maximum possible deviation between the actual fault orientations and the approximate values calculated by Equations 68 and 69 is listed in the last column of Table 3. The table indicates the maximum deviation occurs in the Dry Castlegate sandstone rock model, at around 6.5°, which is the rock with the most significant percentage of CLVD component. It seems that the deviation is positively correlated with the percentage of the CLVD component.

Figure 2. Schematic of a slipping planar fault. $\phi_s$ is measured clockwise from north.

Figure 3. Histograms of the isotropic (a) and compensated linear vector dipole (b) components produced by the shear faulting in Berea SS (A).
The non-DC component is closely related to the initial stress level. We investigate the changes of ISO, DC, and CLVD components as well as shear stress on faults with various oriented fault planes. For different azimuth angles, the components of moment tensor and initial shear stress on fault are shown as functions of the dip angle and rake angle for Barre granite in Figures 6 and 7, respectively. We see that the absolute value of the percentage of the ISO component is more significant than that of the CLVD component. The percentage of ISO and CLVD is negatively correlated with the shear stress on the fault plane, while the decrease of the DC component is consistent with the absolute value of shear stress on the fault plane.

In order to evaluate the validity of the results obtained by perturbation approximation, we use a numerical method to decompose the seismic moment tensor and compare it with the result of Equation 63 for the stress-induced model of Dry D82 marble. We change the three principal components of the initial stress from zero to the state set above in equal proportion to obtain various fault stress states. Figure 8 shows that the results obtained using the perturbation approximation almost coincide with the exact value, proving that the perturbation approximation is valid. The result (Equation 63) may help to obtain shear stress on the fault plane from the ISO and CLVD components.

4.3. Radiation Patterns Generated by a Shear Faulting

The seismic motions result from both propagation effects and source effects. This paper focuses on the study of seismic sources in stress-induced anisotropic media. Due to the relatively large stress level in the source region, the medium in the focal area can be considered stress-induced anisotropic, while the medium outside the fault region is isotropic. So we first use isotropic Green's function to investigate the effect of initial stress on source

![Figure 4](attachment:image1.png)

**Figure 4.** Histograms of the isotropic (a) and compensated linear vector dipole (b) components produced by the shear faulting in Barre granite.

![Figure 5](attachment:image2.png)

**Figure 5.** Histograms of the isotropic (a) and compensated linear vector dipole (b) components produced by the shear faulting in Dry D82 marble.
By using seismic moment tensors in stress-induced media expressed by Equation (71), we calculate the far-field radiation patterns for the four different faults listed in Table 4. The far-field radiation patterns of P and S waves calculated according to Equation 72 and Equation 73 using the four different faults are illustrated in Figures 9 and 10. The illustrations are shown from a perspective perpendicular to the fault plane. Since there are two polarizations for S waves, the resultant radiation patterns of S waves are investigated. The amplitude of radiation patterns with the initial stress is normalized to that in the absence of initial stress. The amplitude of radiation patterns of both P and S waves increases significantly in the presence of initial stress because the closure of micro-cracks in the rock caused by initial compressive stress increases rock moduli. For shear faults in the homogenous rock model of Dry D82 Marble, radiation patterns of P waves exhibit four uniform lobes in the absence of initial stress.
The inhomogeneity of the radiation pattern results from the non-DC components generated by the shear faulting due to the stress-induced anisotropy. The relationship between inhomogeneity and anisotropy caused by initial stress is worth further study. However, the synthetic radiation patterns of shear waves exhibit no prominent characteristics related to stress-induced anisotropy.

4.4. Synthetic Seismograms of the Shear Faulting in Stress-Induced Anisotropic Media

To investigate changes in the seismic record due to the effect of initial stress on the seismic source, we first calculate synthetic seismograms using an isotropic analytical Green's functions. As shown in Figure 11, we consider a full-space model and choose fault IV as the seismic source to compute the synthetic seismograms. The source time function is a Ricker wavelet with a peak frequency of 1.0 Hz (thus, the maximum frequency is about 3 Hz) and a time shift of 1.0 s. The source depth is fixed at 10 km.

Figure 12 shows the synthetic seismograms from a dislocation source in the model of Dry D82 Marble. It can be seen that the amplitudes of both P and S waves in the stress-induced model are more significant than those in the unstressed model, which is consistent with the analysis in the previous section. P waves dominate the radial component, while S waves dominate transverse components. The P-wave packets in transverse components...
Figure 7. The initial shear stress on faults and the percentage of the isotropic, DC, and compensated linear vector dipole components as a function of the rake in the model of Dry D82 marble.

Figure 8. The exact values and perturbation approximations of isotropic, compensated linear vector dipole (a), and DC (b) components as a function of the fault shear stress. All the results are normalized to the scalar seismic moment tensor $M_0$. Solid lines represent exact results derived from numerical calculations, and dashed lines represent results derived from perturbation approximation.
are caused by the inconsistency of the fault orientation with the coordinate system. In addition, the initial stress has a stronger influence on the amplitude of the P wave than that of the S wave. When the epicentral distance is not considerable, a visible shear wave packet can be observed in the radial component, mainly from the contribution of the near-field and middle-field S wave.

The stress-induced anisotropy affects not only the seismic source but also the Green's function related to seismic wave propagation. Green's functions in intrinsically anisotropic media have been extensively investigated in the previous literature (Pan, 2019; Tonon et al., 2001; Wang & Achenbach, 1995).

Based on the similarity between stress-induced anisotropy and intrinsic anisotropy, the Green's functions in the frequency domain in stress-induced anisotropic media can be written in the following form

\[ G_{\mu \nu}(\mathbf{x} - \mathbf{x}', \omega) = G_{\mu \nu}^{S}(\mathbf{x} - \mathbf{x}') + G_{\mu \nu}^{R}(\mathbf{x} - \mathbf{x}') \]

where the superscripts S and R denote the static (singular) and dynamic (regular) parts, respectively. \( \xi \) and \( \zeta \) denote the field point and source point, respectively. The specific form of Green's function in stress-induced anisotropic media is given in Appendix B. Using Equations 37 and 74, the synthetic seismograms from seismic moment tensor sources of shear faults in stress-induced anisotropic media can be calculated by the following form

\[ u_m(\mathbf{x}, t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} M_{\mu \nu}(\omega) G_{\mu \nu}(\mathbf{x}, \omega) e^{-i\omega t} d\omega \]

Based on Equation 75, some numerical examples are given to investigate the effect of stress-induced anisotropy on Green's functions and synthetic seismograms. Similarly, we use the full-space model shown in Figure 11. Next, we investigate the effect of stress-induced anisotropic Green's functions on synthetic seismograms generated by the shear faulting in stressed media. Figure 13 shows the synthetic seismograms generated by the shear fault in the unstressed and stressed Dry D82 marble model. The initial stress is selected as \( \bar{\sigma}_{11} = -30\text{MPa}, \bar{\sigma}_{22} = -20\text{MPa}, \bar{\sigma}_{33} = -15\text{MPa} \). The product of fault area and average slip displacement is assumed to be 10,000 m². We not only consider the effect of initial stress on the seismic source but also investigate the effect of initial stress on the propagation effect of seismic waves. The influence of stress on the seismic source is mainly reflected in the amplitude of the seismic wave. The initial compressive stress can enhance the intensity of the seismic source (as shown in Figure 12). We can see that the seismic waves calculated from stressed Green's

| Fault styles                      | Strike \( \phi_s \) (°) | Dip \( \delta \) (°) | Rake \( \lambda \) (°) |
|----------------------------------|--------------------------|---------------------|----------------------|
| Fault I: Dip-slip normal fault   | 60                       | 60                  | −90                  |
| Fault II: Dip-slip reverse fault | 60                       | 30                  | 90                   |
| Fault III: Left-lateral strike-slip fault | 60   | 60                  | 0                    |
| Fault IV: Oblique slip fault     | 60                       | 60                  | 45                   |

| Table 4 Fault Parameters Used in Calculations |
|-----------------------------------------------|
| Fault styles | Strike \( \phi_s \) (°) | Dip \( \delta \) (°) | Rake \( \lambda \) (°) |
|------------------------------------------------|
| Fault I: Dip-slip normal fault | 60                       | 60                  | −90                  |
| Fault II: Dip-slip reverse fault | 60                       | 30                  | 90                   |
| Fault III: Left-lateral strike-slip fault | 60   | 60                  | 0                    |
| Fault IV: Oblique slip fault | 60                       | 60                  | 45                   |

Figure 9. The far-field radiation patterns of P waves in the unstressed (top plots) and stressed (bottom plots) rock model of Dry D82 Marble by using different fault styles of (a) Fault I, (b) Fault II, (c) Fault III, and (d) Fault IV. The red lines and green lines denote fault normal and slip direction.
function arrive faster, which is attributed to the increase in the velocity of seismic waves caused by the initial stress. The effect of initial compressive stress on Green's function also decreases the amplitude of seismic waves. However, the combined effect of these two reasons makes the amplitude of seismic waves slightly smaller than that in the unstressed medium.

Moreover, the phenomenon of shear wave splitting can be observed in the seismograms calculated by stressed Green's function, which mainly results from the anisotropy caused by the deviatoric initial stress. The magnitude of the fast S wave is larger in the horizontal component, while the magnitude of the slow S wave is larger in the vertical component. However, the anisotropy induced by the deviatoric stress is not very pronounced relative to the time shift of the seismic wave caused by the initial stress.

4.5. Comparison of Stress-Induced Anisotropy and Intrinsic Anisotropy

For the initial isotropic reference material, the undetermined rotation in Equation 42 can be ignored. Even so, because of the asymmetry of the last term in the Equation 42 under non-hydrostatic stress, the equivalent elastic tensor $A_B$ lacks classical elastic symmetry, which is not invariant under the interchange of either $i\leftrightarrow j$ or $k\leftrightarrow l$.

Therefore, stress-induced anisotropy cannot be strictly equivalent to intrinsic anisotropy for analysis. The effect of the asymmetric term on the stiffness coefficient (ratio of asymmetric terms to the overall value of effective elastic constant) is less than 0.1%, or the effect on the velocity is less than 0.05% (Prioul et al., 2004). Since the asymmetry is below the accuracy required by seismic methods, we reasonably ignore the asymmetric term for analyzing stress-induced anisotropy. Therefore, the equivalent elastic tensor can be written in the form

$$A_{ijkl} = C_{ijkl}$$

Then, the elastic tensor in Equation 76 can be expressed in Vo

$$C_{ijkl} = C_{(ij)(kl)}$$

The Voigt notation is obtained by $ij$ or $kl$: $11 \rightarrow 1$, $22 \rightarrow 2$, $33 \rightarrow 3$, $23 \rightarrow 4$, $13 \rightarrow 5$, $12 \rightarrow 6$. 

Figure 10. The resultant far-field radiation patterns of S waves in the unstressed (top plots) and stressed (bottom plots) rock. Other marking instructions are the same as in Figure 9.

Figure 11. Schematic of source-receiver configuration used for synthetic seismogram. The source is placed at 10 km from the origin on the $\xi_3$-axis.
For an initial reference material with at least orthotropic symmetry and isotropic third-order elastic tensor, it can be proved that the equivalent elastic tensor under initial stress always has orthotropic anisotropy or higher symmetry (Fucking & Tsvankin, 2009). Therefore, we adopt the parameter presented in Tsvankin (1997) to evaluate the degree of anisotropy of the equivalent elastic tensor. The anisotropy parameters are summarized below:

\[
\varepsilon^{(1)} = \frac{C_{22} - C_{33}}{2C_{33}}, \quad \delta^{(1)} = \frac{(C_{23} + C_{44})^2 - (C_{33} - C_{44})^2}{2C_{33}(C_{33} - C_{44})}, \quad \gamma^{(1)} = \frac{C_{66} - C_{55}}{2C_{55}}
\]

\[
\varepsilon^{(2)} = \frac{C_{11} - C_{33}}{2C_{33}}, \quad \delta^{(2)} = \frac{(C_{13} + C_{55})^2 - (C_{33} - C_{55})^2}{2C_{33}(C_{33} - C_{55})}, \quad \gamma^{(2)} = \frac{C_{66} - C_{44}}{2C_{44}}
\]

\[
\delta^{(3)} = \frac{(C_{12} + C_{66})^2 - (C_{11} - C_{66})^2}{2C_{11}(C_{11} - C_{66})}
\]

Figure 12. The synthetic seismograms from the shear faulting (fault VI) in the rock model of Dry D82 Marble. The radial component (a) and transverse components in θ direction (b) and φ direction (c) are received at \(\xi_1 = 20\) km in the unstressed model (solid black line) and stressed model (red shot dashed line), respectively. (d) The radial component is received at \(\xi_1 = 100\) km. \(u_0\) is the maximum displacement in the unstressed model.
For VTI media,

\[ \varepsilon^{(2)} = \varepsilon^{(1)} = \epsilon, \quad \delta^{(2)} = \delta^{(1)} = \delta, \quad \gamma^{(2)} = \gamma^{(1)} = \gamma, \quad \delta^{(3)} = 0 \]

(81)

which correspond to the anisotropic parameters in Thomsen (1986).

**Figure 13.** Synthetic seismograms generated by the shear fault (Fault IV) in the Dry D82 marble model calculated from the stressed source and unstressed (solid magenta black line), stressed (solid red line) Green's functions, and the unstressed source and Green's function (black dashed line). (a) and (b) denote horizontal displacements, (c) donotes the vertical displacement. The source depth is fixed at 10 km. The receiver is located at \( A_1 = 200 \) km, \( A_2 = 200 \) km.
The rock model of Jurassic shale with intrinsic anisotropy in a stress-free reference state is used to study the difference between stress-induced anisotropy and intrinsic anisotropy. Figure 14 shows the variation of anisotropic parameters of Jurassic shale with hydrostatic stress. In the absence of initial stress, Jurassic shale is transversely isotropic with a vertical axis of symmetry, and the elastic constants of the rock are shown in Table 5. It is noteworthy that hydrostatic stress does not change the symmetry of anisotropy but affects the magnitude of the anisotropy parameters. The variation of anisotropy parameters is less than 0.1 at low initial stress levels and less than 0.07 at high initial stress levels. At low-stress levels, the predicted values of the anisotropy parameters by the acoustoelastic theory are slightly different from the experimental data. In contrast, there is a good quantitative agreement with the experimental data at high-stress levels. The difference at low initial stress levels is mainly due to the strong nonlinearity caused by the closure of the compliant cracks.

Since cracks in underground rock may close and produce viscoelastic deformation under long-term stress, seismic observations related to stress changes are mainly based on a reference state with a high-stress level. Therefore, we assume that the medium is subjected to hydrostatic stress of 40 MPa in the reference state to investigate the anisotropy parameters at the high-stress level. The bulk density of Jurassic shale in the reference state is calculated to be about 2545 kg/m$^3$. Figure 15 shows the anisotropy parameters predicted by acoustoelastic theory for low-stress and high-stress levels. Anisotropy parameters have relatively significant changes or even several times at low-stress levels, especially for the parameters $\delta^{(2)}$, $\gamma^{(2)}$, $\delta^{(2)}$ that control the anisotropy in the $\xi_1 - \xi_3$ plane. According to the results in Figure 14, the anisotropy predicted by acoustoelastic theory at low-stress levels may be greater than in the actual rock medium. This is due to the strong nonlinearity of the closure of rock cracks at low-stress levels. For the reference state with hydrostatic stress of 40 MPa, the variation of the anisotropy parameter is less than 0.06. Under the action of in situ stress in deep Earth, the compliant cracks are usually closed, which reduces the nonlinear effect of rocks. The fact that anisotropy parameters vary relatively little for the reference state of higher hydrostatic pressure may be more consistent with the actual situation. It is noteworthy that stress-induced anisotropy in seismic records may be significantly observed due to the accumulation of initial stress over time.

5. Conclusions

Using the motion description in the intermediate state and the third-order elasticity and considering the effect of initial stress on the boundary conditions of the fault plane, we obtain the seismic moment tensor of shear dislocation sources in stress-induced anisotropic media. Then the seismic moment tensor is decomposed to study the effect of stress-induced anisotropy on the seismic source. We also study the effect of initial stress on seismic waves generated by the stressed seismic source. The main conclusions are as follows:

Shear faulting on planar faults in stress-induced media can produce significant non-DC mechanisms, including CLVD and ISO components. The amount of the non-DC components depends on the initial stress, symmetry of stress-induced media, and faulting orientation. The CLVD component is dominated by the third-order elastic constant $\nu_3$, while the ISO component depends on third-order elastic constants $\nu_2$ and $\nu_3$. The results show that ISO and CLVD components vary linearly with the initial shear stress on faults. The relation provides a new perspective for monitoring induced stress by continuous inversion of moment tensor.

Table 5

| Reference state | $c_{11}$ (MPa) | $c_{13}$ (MPa) | $c_{13}$ (MPa) | $c_{44}$ (MPa) | $c_{66}$ (MPa) | Hydrostatic pressure (MPa) | $\nu_1$ (GPa) | $\nu_2$ (GPa) | $\nu_3$ (GPa) |
|-----------------|----------------|----------------|----------------|----------------|----------------|---------------------------|---------------|---------------|---------------|
| 0 MPa           | 31.3           | 21.0           | 14.3           | 4.3            | 8.6            | 0–30                      | 5,800         | −5,300        | 1,837.5       |
| 40 MPa          | 44.0           | 30.1           | 17.4           | 13.5           | 7.9            | 30–100                    | 40            | −420          | −77.5         |
Because of anisotropy induced by deviatoric initial stress, the P and T axes of the seismic moment tensor deviate from the plane defined by the normal and slip directions of the fault. If the stress-induced anisotropy is ignored, the fault normal and slip direction in stressed media obtained based on the isotropy assumption would slightly deviate from the actual solutions. Therefore, stress-induced anisotropy may be taken into account in the inversion of the fault plane solution.

Even if the medium outside the focal area is isotropic, induced stress also affects the radiation of the seismic source. The amplitude of the radiation patterns of P and S waves in the isotropic reference medium increases significantly under compressed stress. Moreover, the radiation pattern of the P wave shows the significant characteristic of anisotropy with non-uniform lobes. The initial stress has a greater effect on the amplitude of the P wave than that of the S wave. Regardless of the initial stress, the radial component has a significant shear wave package due to the contribution of near-field and midfield waves. In general, the initial stress affects the source intensity and the propagation of seismic waves. On the one hand, the increase of equivalent elastic tensor caused by the initial compressive stress increases the source intensity; on the other hand, the effect of the initial compressive stress on Green's function leads to the decrease of seismic wave amplitude. For the Dry D82 marble model, the combined effect results in the amplitude of the seismic waves in stressed media being slightly smaller than that in unstressed media.

Stress also affects the anisotropy of the medium. Hydrostatic stress does not change the symmetry of anisotropy but affects the anisotropy parameters. The results show that non-hydrostatic stress can lead to lower symmetry of the medium. The initial stress has a more significant effect on the anisotropy parameters at low-stress levels than high-stress levels due to the nonlinear effect of the crack closure. We note that the stress-induced anisotropic parameters are smaller than intrinsic anisotropy at high-stress levels and a nearly linear function of the increased initial stress. Even at the reference state of high-stress levels, the accumulation of initial stress over time makes stress-induced anisotropy potentially observable in seismic records. This variation of anisotropy with stress may help us obtain information on in situ stress from seismic observation records.

**Appendix A: Deformations and Definitions of Stress**

We introduce three states to describe the deformation process corresponding to infinitesimal motions superimposed on a large static deformation. The motion of a particle among different states is shown in Figure A1. We call the original state of the unstressed medium the natural state or undeformed state, in which the particle occupies the space domain $V_0$ and surface $S_0$, corresponding to the natural configuration $B_0$. Consider the state occupying a spatial region $\tilde{V}$ and surface $\tilde{S}$ in the initial configuration $\tilde{B}$, henceforth calling the intermediate state or initial state. The intermediate state is obtained from the natural state undergoing a finite static displacement $\mathbf{w}$. The dynamic displacement $\mathbf{u}$ superimposed on the intermediate state deforms the solid into another time-dependent configuration $B(t)$ that occupies a region $V(t)$, usually called the current state or final state.
In order to avoid the complexity of coordinate system transformation, we use a consistent coordinate system to describe the motion in the three configurations. We use upper Latin, lower Greek, and lower Latin subscripts to represent the components of the physical field in the natural, intermediate, and current state, respectively. For example, the coordinates of a material particle in the natural configuration are denoted by $\mathbf{A} \mathbf{A} \mathbf{A} \mathbf{K} (\mathbf{K} = 1, 2, 3)$, in the intermediate configuration by $\mathbf{A} \mathbf{A} \mathbf{A} \alpha (\alpha = 1, 2, 3)$, and in the current configuration by $\mathbf{A} \mathbf{A} \mathbf{A} \mathbf{k} (\mathbf{k} = 1, 2, 3)$. Functional relations specify the deformations between three different configurations:

$$\mathbf{x} = \mathbf{x} (\mathbf{\xi} \mathbf{,} t), \ \mathbf{\xi} = \mathbf{\xi} (\mathbf{X}) \quad (A1)$$

We adopt $\mathbf{w} = \mathbf{w} (\mathbf{X})$ and $\mathbf{u} = \mathbf{u} (\mathbf{X}, t)$ to denote the static displacement produced by the initial stress and dynamic displacement by seismic disturbance, respectively. Using the Lagrange motion description, it is possible to relate the particle in the current configuration to that in the intermediate configuration in the form

$$\mathbf{x} (\mathbf{X}, t) = \mathbf{u} (\mathbf{X}, t) + \mathbf{\xi} (\mathbf{X}), \ \mathbf{\xi} (\mathbf{X}) = \mathbf{X} + \mathbf{w} (\mathbf{X}) \quad (A2)$$

The total displacement from the natural state to the current state is

$$\mathbf{u} = \mathbf{x} - \mathbf{X} = \mathbf{u} + \mathbf{w} \quad (A3)$$

The static deformation gradient tensor is denoted by

$$\tilde{\mathbf{F}}_{\alpha \beta} = \mathbf{\xi}_{\alpha \beta} = \delta_{\alpha \beta} + \mathbf{w}_{\alpha \beta} \quad (A4)$$

where the subscript comma represents partial derivative with respect to $\mathbf{X} \mathbf{A}$. We will continue this convention hereafter. The deformation gradient tensors of the current configuration to the natural configuration and the intermediate configuration can be expressed as

$$\mathbf{F}_{i A} = \mathbf{x}_{i A} = \mathbf{x}_{i A} \mathbf{\xi}_{A, A} = \tilde{\mathbf{F}}_{i A} (\delta_{i A} + \mathbf{u}_{i A}) \quad (A5)$$

$$\mathbf{x}_{i A} = \delta_{i A} + \mathbf{u}_{i A} \quad (A6)$$

where

$$\delta_{k A} = \delta_{k A} = \mathbf{e}_k \cdot \mathbf{e}_A \quad (A7)$$

---

**Figure A1.** Schematics of the deformation among the natural (reference), intermediate, and current configurations.
The quantities defined by equation (Equation A7) are called shifters, which correspond to a Kronecker symbol when \( \mathbf{e}_k \) and \( \tilde{\mathbf{e}}_s \) are coincident (Eringen & Suhubi, 1974). The Cartesian base vectors \( \mathbf{e}_k \) and \( \tilde{\mathbf{e}}_s \) are unit vectors along with the coordinates \( x_k \) and \( \tilde{x}_s \). The total Lagrangian strain in the current state \( \mathbf{x} \), measured with respect to the natural state \( \mathbf{X} \), is

\[
E_{AB} = \frac{1}{2} (F_{IA}F_{IB} - \delta_{AB}) = \tilde{E}_{AB} + \tilde{F}_{IA}F_{IB}\varepsilon_{ij}^{d},
\]

(A8)

where \( \tilde{E}_{AB} \) represents the static strain in the intermediate state, \( \varepsilon_{ij}^{d} \) represents the incremental strain in the intermediate state due to the dynamic disturbance.

At present, there are three different ways to define or measure the stress associated with the different configurations in ultrasonic and geophysical research. Forces in continuum mechanics are usually expressed by a stress tensor acting on a surface element. The geometrical relationships of the surface element between undeformed and deformed configuration and the corresponding surface traction forces are shown in Figure A2. Let \( d\mathbf{f}^E \) and \( d\mathbf{f}^L \) are the surface forces that act upon the undeformed patch \( \mathbf{N}dS_0 \) at the position \( \mathbf{X} \) and the deformed patch \( ndS \) at the position \( \mathbf{x} \), respectively. The surface elements between the two configurations can be expressed as

\[
dS = JNdS_0 \cdot \mathbf{F}^{-1}
\]

(A9)

where \( J \) is the Jacobian of the transformation \( \mathbf{F} \). More importantly, according to the Lagrangian conservation of mass law, the density change between two configurations is related by

\[
\rho_0 = \rho J
\]

(A10)

Cauchy stress is the most common stress associated with the Eulerian momentum conservation law. The Eulerian Cauchy stress \( \mathbf{t}^E \) defines the surface force \( d\mathbf{f}^E \) acting on the deformed patch by

\[
d\mathbf{f}^E = ndS \cdot \mathbf{t}^E
\]

(A11)

Naturally, the corresponding Lagrangian Cauchy stress is defined as

\[
\mathbf{t}^L(\mathbf{X}, t) = \mathbf{t}^E(\mathbf{x}(\mathbf{X}, t), t)
\]

(A12)

The second type of stress tensor often found in the finite deformation theory is the so-called first Piola-Kirchhoff stress, or PK stress for short. This quantity is convenient to represent the Lagrangian momentum conservation law and is defined by

\[
d\mathbf{f}^E = NdS_0 \cdot \mathbf{T}^PK
\]

(A13)

The PK stress is a measure of the force per unit undeformed area. The surface force \( d\mathbf{f}^E \) in equation (Equation A11) acts upon the point \( \mathbf{x} \) in the deformed configuration, whereas the surface element \( \mathbf{N}dS_0 \) is attached to the point in the undeformed configuration. Therefore, the PK stress is a two-point tensor.

Figure A2. Schematics of the surface element and the corresponding surface traction forces in two different configurations.
The third type of stress tensor, known as the second Piola-Kirchhoff stress, hereinafter referred to as SK stress, is the most convenient for expressing the constitutive relation of a perfectly elastic material. The SK stress denoted by $\mathbf{T}^{SK}$ is given by

$$ d\mathbf{t}^E = \mathbf{F}^{-1} \cdot d\mathbf{t}^E = N dS_0 \cdot \mathbf{T}^{SK} \quad (A14) $$

where the force $d\mathbf{t}^E$ is obtained by mapping the actual force $d\mathbf{t}^E$ acting upon the displaced point $\mathbf{x}$ to the undeformed material particle $\mathbf{X}$ in terms of the deformation gradient $\mathbf{F}$; that is,

$$ d\mathbf{t}^E = \mathbf{F}^{-1} \cdot d\mathbf{t}^E \quad (A15) $$

Using Equations A9–A14, we can obtain the relationship between three different forms of stress:

$$ \mathbf{T}^{SK} = \mathbf{T}^{PK} \cdot \mathbf{F}^{-T} = \mathbf{J} \mathbf{F}^{-1} \cdot \mathbf{t}^E \cdot \mathbf{F}^{-T} \quad (A16) $$

It is easy to prove that the SK stress is a second-order symmetric tensor similar to the Cauchy stress.

### Appendix B: Green's Function in Stress-Induced Anisotropic Media

Green's functions in intrinsically anisotropic media have been extensively studied. Wang and Achenbach (1995) derived the displacement Green's function in general anisotropic media using the three-dimensional Radon transform. By using a similar approach, Green's function in a stress-induced anisotropic medium can also be written in the following form

$$ G_{\mu\nu}(\zeta - \zeta, \alpha) = G_{\mu\nu}^{S}(\zeta - \zeta) + G_{\mu\nu}^{R}(\zeta - \zeta, \alpha) \quad (B1) $$

where the superscripts $S$ and $R$ denote the singular (static) and regular (dynamic) parts, respectively. Note that the Fourier transform pairs used here are in the form

$$ Q(\omega) = \int_{-\infty}^{\infty} q(t)e^{i\omega t} dt \quad (B2) $$

The static and dynamic parts of Green's function are given in the form (Dravinski & Niu, 2002; Tonon et al., 2001; Wang & Achenbach, 1995)

$$ G_{\mu\nu}^{S}(\zeta - \zeta) = \frac{1}{8\pi^2} \int_{d|d|=1} \Gamma_{\mu\nu}^{(d)}(d) d\Omega(d) \quad (B3) $$

where $d\Omega(d) \in D^S = \{0 \leq \varphi \leq 2\pi\}$ is the domain of integration for the static part, which takes place over a unit circle, $|d| = 1$.

$$ G_{\mu\nu}^{R}(\zeta - \zeta, \alpha) = \frac{1}{4\pi^2} \int_{\mathbf{n} \cdot (\zeta - \zeta) - 1 < 0} \sum_{n=1}^{3} \frac{k_{\mu}E_{\mu n}E_{\nu m}}{2\rho c_n^2} e^{i\alpha |d|} dS(\mathbf{n}) \quad (B4) $$

where $dS(\mathbf{n}) \in D^R = \{0 \leq b \leq 1; 0 \leq \varphi \leq 2\pi\}$ is the domain of integration for the regular part, which takes place over a half of a unit sphere, $|\mathbf{n}| = 1$. The geometric descriptions of the two integral domains are presented in Figure B1. $E_{\mu n}$ are the components of the eigenvector of $\Gamma(\mathbf{n})$

$$ \Gamma_{\mu\nu}(\mathbf{n})E_{\mu m} = k_{\mu}E_{\nu m}, \quad m = 1, 2, 3. \quad (B5) $$

Underlined indices indicate that the summation convention is being suppressed. $\Gamma_{\mu\nu}(\mathbf{n})$ denotes the Christoffel matrix. For stressed media, it can be written in the form

$$ \Gamma_{\mu\nu}(\mathbf{n}) = B_{\mu\nu\rho\eta}n_{\rho} \quad (B6) $$

$c_\alpha$ and $k_\alpha$ are the phase velocities and wave numbers defined by

$$ c_\alpha = \sqrt{k_\alpha/\bar{\rho}}, \quad k_\alpha = \alpha/c_\alpha \quad (B7) $$
The unit vector $\mathbf{n}$ denotes the direction of wave propagation. As shown in Figure B1, let $\mathbf{e}$ be a unit vector in the direction of the position vector $\mathbf{x} = \xi - \zeta$, so that $\mathbf{e} = \frac{\mathbf{x}}{r}$. Let $\mathbf{d}$ be a unit vector in the plane normal to the position vector $\mathbf{x}$, thus $\mathbf{d} \cdot \mathbf{e} = 0$. Then unit vectors $\mathbf{n}$ and $\mathbf{d}$ are related through

$$\mathbf{n} = \sqrt{1 - b^2} \mathbf{d} + be$$

(B8)

where $\mathbf{d}$ can be expressed as a function of $\mathbf{e}$ and $\varphi$ in the form

$$\mathbf{d} = (1 - e_3^2)^{-1/2} \left( e_2 \cos \varphi + e_1 e_3 \sin \varphi, -e_1 \cos \varphi + e_2 e_3 \sin \varphi, (1 - e_3^2) \sin \varphi \right)$$

(B9)

Notice that the integral for the static parts of Geen’s function has been simplified by

$$\mathbf{n} \cdot \xi = \left( \sqrt{1 - b^2} \mathbf{d} + be \right) \cdot \mathbf{e} = rb$$

(B10)

By converting the integral of the singular part into the integral of $\mathbf{e}$ and $\varphi$ in the form $\mathbf{d}$, and converting the integral of the regular part over the unit sphere into the integral of $\mathbf{d}$ and $\varphi$, we can easily obtain the numerical Green’s function of the displacement field.

Accordingly, we can obtain the derivative of Green’s function with respect to the coordinates of the sound source

$$G_{pkq}(\xi - \zeta, \omega) = \frac{\partial}{\partial \xi_q} G^S_{pkq}(\xi - \zeta) + \frac{\partial}{\partial \xi_q} G^R_{pkq}(\xi - \zeta, \omega)$$

(B11)

where

$$\frac{\partial}{\partial \xi_q} G^R_{pkq}(\xi - \zeta, \omega) = \frac{1}{4\pi^2} \int_{n(\xi - \zeta) > 0} \sum_{m=1}^{M} \frac{k_m^2 P_{pkq}}{2\rho c_m} e^{i k_m n(\xi - \zeta)} d\mathbf{n}$$

(B12)

$$\frac{\partial}{\partial \xi_q} G^S_{pkq}(\xi - \zeta) = \frac{1}{8\pi^2} \int_0^{2\pi} G^b_{pkq}(\varphi) d\varphi$$

(B13)

where the integrand $G^b_{pkq}(\varphi)$ is denoted by

---

**Figure B1.** Geometric relationship of integral domains fixed at the source point $\zeta$. The domain of integration $D^S = \{0 \leq \varphi \leq 2\pi, 0 \leq b \leq 1\}$ for the regular parts of Geen’s function and the domain of integration $D^R = \{0 \leq \varphi \leq 2\pi\}$ for the singular part.
\[ G_{pqk}(\varphi) = \left\{ \frac{\partial}{\partial b} g_{pqk}[n(b, d(\varphi))] \right\}_{b=0} \cdot g_{pqk}(n) = n_b \Gamma^{-1}_p(n) \quad (B14) \]

**List of symbols**

- \( A_{KLJ}^0 \): Eringen elasticity
- \( B_{ijkl} \): Equivalent elastic tensor associated with the PK stress
- \( C_{ijkl} \): Equivalent elastic tensor satisfying elastic symmetries
- \( c_{ijkl} \): Second-order elastic constants.
- \( C_{ijklm} \): Third-order elastic constants.
- \( c^{ISO} \): Percentages of ISO component.
- \( c^{DC} \): Percentages of DC component.
- \( c^{CLVD} \): Percentages of CLVD component.
- \( d' \): Eigenvectors of the seismic moment tensor.
- \( E_{ij} \): Total Lagrangian strain.
- \( \tilde{E}_{ij} \): Initial Lagrangian strain.
- \( F \): Deformation gradient tensor.
- \( F_{ab} \): Total deformation gradients.
- \( \tilde{F}_{ab} \): Static deformation gradients.
- \( f, f_0 \): Total and initial body forces.
- \( \tilde{f} \): Incremental body force.
- \( f^V \): Volume density of the incremental body force.
- \( G_{ij} \): Green’s function.
- \( G^S_{ij} \): Static Green’s function.
- \( G^K_{ij} \): Dynamic Green’s function.
- \( H_{ijkl} \): Equivalent elastic tensor in seismic moment tensor.
- \( j_0, J \): Jacobians of static and total deformation gradients.
- \( K \):Bulk modulus.
- \( k_m \): Wavenumber.
- \( m_{kl} \): Surface moment-density tensor.
- \( M_{kl} \): Seismic moment tensor.
- \( M_{ISO} \): Isotropic component of the seismic moment tensor.
- \( M_{DC} \): Double-couple component of the moment tensor.
- \( M_{CLVD} \): Compensated linear vector dipole component of the moment tensor.
- \( M_0 \): Scalar seismic moment tensor.
- \( N, \tilde{N}, n \): Unit normal of the surface in natural, initial, and current states.
- \( n^0 \): Fault normal unit vector.
- \( \nu_0 \): Slip direction unit vector.
- \( p, t, b \): Principal axes of the seismic moment tensor.
- \( \tilde{S}_0 \): Area of fault plane in the intermediate state.
- \( S_{KL}^0 \): Second Piola-Kirchhoff stress in the natural state.
- \( T_{KJ}^0, T_{nj}^0 \): Total first Piola-Kirchhoff stresses.
- \( T_{KJ}, T_{nj} \): Incremental first Piola-Kirchhoff stresses.
- \( t_{ij}^0, t_{ij}^f, t_{ij} \): Total, initial, and incremental Cauchy stresses.
- \( T_{PK} \): First Piola-Kirchhoff stress tensor.
- \( T^{SK} \): Second Piola-Kirchhoff stress tensor.
- \( t^E, t^L \): Eulerian and Lagrangian Cauchy stress tensor.
- \( V_0, \tilde{V}, V \): Volumes in natural, initial, and current states.
- \( W \): Internal energy per unit mass.
- \( w, u, u' \): Static, dynamic, and total displacements.
- \( X_K \): Lagrangian coordinates in the natural state.
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Data Availability Statement

This is a theoretical paper and does not include any data other than those explicitly presented in the text and tables.
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