Study on rising flow characteristics of triple bubbles

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Abstract: In order to study the influence of bubble size and distance on the interaction between bubbles, this paper discussed the rising movement characteristics of triple bubbles in static water based on Fluent software. By analyzing the rising track, flow fields and velocity of bubbles, the results show that, when bubble diameters are 4mm, 4mm, 4mm, the distance between the left bubble and the middle bubble is smaller than the minimum coalescence distance, and they coalesce. When bubble diameters are 6mm, 4mm, 4mm, the velocity and track of the small bubble at the bottom will be affected by the wake of the big bubble as the big bubble rises, but they do not coalesce. When bubble diameters are 2mm, 6mm, 2mm, the smaller bubbles on both sides are attracted by the middle bubble and they coalesce; there is almost no vertical rise or deviation process.

1. Introduction
The phenomenon of bubble movement is common in the fields of agriculture, navigation, petroleum, water conservancy, etc. At present, there are many studies on the movement characteristics and influencing factors of single bubble, however in actual production and life, bubbles exist in groups. It is of great scientific significance to conduct in-depth research on the movement characteristics of bubbles and the interaction among bubbles. Many scholars have done a lot of research on it. Bai Lina et al. [1-4] studied velocity, trajectory of single bubble, and the effects of liquid surface tension and buoyancy on bubble deformation in still water. Du Yuhao [5] used the interface tracing method to simulate the rising of 1-4 equal diameter bubbles arranged horizontally. Liao et al. [6,7] simulated the interaction between bubbles in different positions, proposing a new bubble coalescence and fragmentation model. Aladjem et al. [8] studied the interaction between two continuous bubbles in a vertical tube experimentally. Combining the PLIC algorithm with the gas-liquid two-phase flow control equation, Wang Tai [9] studied the coalescence of coaxial equal diameter double bubbles. Yang [10] simulated the coalescence and interaction of two vertical coaxial bubbles in viscous liquid. Robinson P. B. [11] used boundary integral method to study the interaction among bubbles. Sun Tao et al. [12,13] simulated the rising process of bubble group and the interaction among bubbles in stagnant liquids by Boltzmann method. Huang Ying [14] studied the motion characteristics of bubbles from the twin-orifice by the visual experimental method. Li Xin [15] summarized the influence factors of single bubble rising, the coalescence and rupture of multiple bubbles rising through experiment and numerical calculation. Li Zhangrui [16] simulated the dynamic characteristics of three-dimensional bubble by the boundary integral method. All in all, the present study on multiple bubbles contribute to about equal diameter bubbles mostly, but less about unequal diameter bubbles. Therefore, this paper...
mainly used Fluent software to study the rising behavior of three horizontal bubbles with equal and unequal diameters. Through a comparison of the movement characteristics in different cases, the influence of bubble size and distance on the interaction among bubbles was discussed and analyzed.

2. Mathematical physical model [24]

In this paper, bubble rising motion is simulated using the Volume of Fluid method (VOF). In the VOF model, tracking the phase interface is completed by solving the continuous equation to the volume fraction of single-phase or multi-phase. For the phase \( q \), the equation is in the following:

\[
\frac{1}{\rho_q} \frac{\partial (\alpha_q \rho_q)}{\partial t} + \sum_{p=q}^{n} \frac{1}{\rho_p} \nabla (\alpha_p \rho_p \mathbf{v}_p) = \frac{S_{\alpha_q}}{\rho_q} + \sum_{p=1}^{n} (m_{pq} - m_{qp})
\]  

(1)

where, \( m_{pq} \) is the mass transfer from phase \( p \) to phase \( q \), \( m_{qp} \) is the mass transfer from phase \( q \) to phase \( p \), \( \alpha_q \) is the fluid volume fraction of phase \( q \), \( \rho_q \) is the fluid density of phase \( q \), and \( \mathbf{v}_q \) is the flow velocity of phase \( q \). The constraints on the volume fraction of each phase are as follows:

\[
\sum_{q=1}^{n} \alpha_q = 1
\]  

(2)

where, if \( \alpha_q = 0 \), phase \( q \) is water, \( \alpha_q = 1 \), phase \( q \) is air, and \( 0 < \alpha_q < 1 \), the unit contains the interface between fluid \( q \) and other multi-phase fluids.

Momentum equation

\[
\frac{\partial}{\partial t} (\rho \mathbf{v}) + \nabla \cdot (\rho \mathbf{v} \mathbf{v}) = -\nabla p + \nabla \cdot \left[ \mu \left( \nabla \mathbf{v} + \nabla \mathbf{v}^T \right) \right] + \rho g + F
\]  

(3)

where, \( \mu \) is the viscosity coefficient of fluid motion, \( p \) the pressure, \( F \) is the source term, \( \rho \) the density, \( \mathbf{v} \) the fluid speed, and \( g \) the acceleration of gravity.

3. Model Validation

By virtue of ANSYS ICEM, this paper established grid division and a model with a size of 150*500mm and a calculation area of 150*300mm. In comparison with the movement parameters of a 2.01mm diameter bubble in the static water at the temperature of 8 °C in the experiment by Liu Liu [25], the experimental model was shown in Figure 1. In order to rule out the influence of grid number on bubble simulation, different grid numbers were adopted. For the bubble with a diameter of 2.01mm, the volume fraction distribution of dispersed phases was described in Figure 3. It could be seen that the difference in grid number only affected the definition of a bubble boundary. The larger the grid number was, the clearer the bubble boundary was, and the higher the resolution was. In this paper, the grid number was fixed at 442,316.

The simulation results of Fluent and experimental results of bubble velocity were shown in Figure 2. Specifically speaking, the numerical simulation result of bubble velocity was highly close to the experimental result in the rising acceleration period, while the simulation value of bubble terminal velocity was slightly less stable than the experimental value, with an error within ± 10%.
Figure 1. Sketch of the experimental apparatus
Figure 2. Bubble velocity as a function of distance
Figure 3. Contours of the dispersed phase volume fractions at different mesh numbers

4. Result Analysis

In this paper, unstructured grids were used to simulate the rising movement of three bubbles under three different working conditions. The initial calculation state was shown in Figure 4, with $d_0$ representing the bubble distance. In addition, the liquid phase was in red while the gas phase was in blue.

In working condition (a), the rising track of bubbles was shown in Figure 5, and the flow field change was shown in Figure 7. At the beginning of bubble movement, all three bubbles rose in a nearly vertical direction. When $t = 0.045s$, the left and right bubbles deviated to both sides, while the middle bubble continued to rise nearly vertically. This was because the flow field vortices generated by the bubbles on both sides aimed at the middle bubble started to interact in opposite directions. Specifically, the vortex on the right side of the left bubble was counter clockwise, while the vortex on the left side of the right bubble was clockwise. Therefore, their effect on the middle bubble just neutralized each other, making the middle bubble rise in a nearly vertical direction. Subsequently, since the deviation of the bubbles on both sides steadily increased, their influence on the middle bubble became increasingly small, and the movement of three bubbles was similar to that of a single bubble. When $t = 0.81s$, the left bubble and the middle bubble coalesced. Then the fused bubble and the right bubble rose to the liquid level. Figure 6 described the variation of bubble rising velocity with time. When $t = 0.38s$, as the middle bubble deviated to the left, the velocity flow fields of the three bubbles changed step by step. When $t = 0.5s$, the right bubble moved as a single bubble, while the left
bubble moved under the influence of the wake of the middle bubble. Accordingly, the velocity of the three bubbles all increased in varying degrees.

![Figure 5 Track of bubbles rising in (a)](image)

![Figure 6 Bubbles velocity as a function of time in (a)](image)

![Figure 7 Flow field diagram of rising bubbles in (a)](image)

![Figure 8 Track of bubbles rising in (b)](image)

![Figure 9 Bubbles velocity as a function of time in (b)](image)

In working condition (b), the bubble rising track was shown in Figure 8, and the flow field change was shown in Figure 10. When the bubbles started to rise, the track of the three bubbles was almost the same as that in working condition (a). The difference lay in that the offset distance of the right bubble was larger than that of the left bubble. This was because the size of the right bubble was small, and the force generated by the clockwise flow field vortex made the right bubble offset on its right side to a large extent. By contrast, the force produced by the counter clockwise flow field vortex of the left bubble made the left bubble offset on its left side to a small extent. In the meantime, due to the larger intensity and range of the vortex flow field of the left bubble, it was more attractive to the middle bubble, causing the middle bubble to shift to the left. Afterwards, the three bubbles caught up each other when rising. As the left bubble rose, the influence of its wake on the other two bubbles became small. As a result, the middle bubble moved like a single bubble, and the right bubble still moved under the influence of its wake. Figure 9 described the variation of bubble rising velocity with time. The influence of the left bubble wake on the middle bubble was larger than that on the right bubble, so
the velocity oscillation amplitude of the middle bubble was larger than that of the right bubble. As the force of the left bubble wake became small, the velocity of the middle bubble gradually stabilized.

![Flow field diagram of rising bubbles in (b)](image)

**Figure 10** Flow field diagram of rising bubbles in (b)

![Track of bubbles rising in (c)](image)

**Figure 11** Track of bubbles rising in (c)

![Bubbles velocity as a function of time in (c)](image)

**Figure 12** Bubbles velocity as a function of time in (c)

![Flow field diagram of rising bubbles in (c)](image)

**Figure 13** Flow field diagram of rising bubbles in (c)

In working condition (c), the bubble rising track was shown in Figure 11, and the flow field change was shown in Figure 13. Because the bubbles on both sides were small in size, they can be easily attracted to the middle by the middle bubble. Therefore, there was almost no vertical rise or outward deviation on both sides. When $t = 0.07s$, the left bubble and the middle bubble first moved close to each other and then coalesced; when $t = 0.09s$, the right bubble and the fused bubble coalesced. Finally, a large bubble was formed after the coalescence when $t = 0.1s$. At this time, the rising height reached 16.5mm. Then the large bubble was similar to a single bubble rising to the liquid level by floating left and right. Figure 12 described the variation of bubble rising velocity with time. In the process of coalescence, the velocity of bubbles on both sides changed greatly, while the velocity of the middle bubble was relatively stable due to its large size.

5. **Conclusion**

Based on Fluent software, this paper has numerically simulated the rising movement of triple bubbles in static water, studied the influence of size and distance on the interaction between bubbles, analyzed the rising track and velocity of three bubbles. The following conclusions have been obtained:

1. The track of a single bubble shows an irregular s-shaped curve, and the velocity increases rapidly to the maximal and then tends to be stable after fluctuation. (2) When isodiametric three bubbles (4mm, 4mm, 4mm) rising, they go up vertically at first, and then the bubbles on both sides deviate. When the three bubbles rise in oscillation, the distance between the left bubble and the middle bubble is smaller than the minimum coalescence distance, and they coalesce. When three bubbles with different diameters (6mm, 4mm, 4mm) rise, the velocity and track of the small bubble at the bottom
will be affected by the wake of the big bubble as the big bubble rises, but they do not coalesce. Because of the small size of the right bubble, its offset distance is larger than that of the left bubble. When three bubbles with different diameters (d=2mm, 6mm, 2mm) rise, the smaller bubbles on both sides are attracted by the middle bubble and they coalesce, there is almost no vertical rise or deviation process.

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