Non-adiabatic topological spin pumping

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Abstract
Based on the Floquet scattering theory, we analytically investigate the topological spin pumping for an exactly solvable model. Floquet spin Chern numbers are introduced to characterize the periodically time-dependent system. The topological spin pumping remains robust both in the presence and in the absence of the time-reversal symmetry, as long as the pumping frequency is smaller than the band gap, where the electron transport involves only the Floquet evanescent modes in the pump. For the pumping frequency greater than the band gap, where the propagating modes in the pump participate in the electron transport, the spin pumping rate decays rapidly, marking the end of the topological pumping regime.

1. Introduction

Generally speaking, quantum pumping is a dynamic transport mechanism that dc charge currents can flow under zero bias via a quantum system, in which some parameters are periodically modulated in time. It was originally proposed by Thouless and co-workers in the 1980s, who found that a quantized charge can be pumped during a period of slow variation of potential in the Schrödinger equation [1, 2]. The amount of charge pumped per cycle is directly related to a topological invariant of the system, namely, the Chern number [1, 2]. The same topological invariant was also used to classify the integer quantum Hall effect [3]. There have been continuous interest in the topological charge pumping [4–10].

For weak pumping, where the time-dependent parameters vary so slowly that the system can be treated adiabatically, the pumped current can be obtained by the Berry phase theorem associated with the instant scattering matrix [11, 12]. As has been clearly shown by Graf and coworkers [13, 14], the topological approach and the scattering matrix one provide equivalent descriptions of the adiabatic quantum pumping. However, when the charge pumping was observed in an open quantum dot experimentally, Switkes et al found that the adiabatic theory is inadequate to explain the observation in the strong pumping regime [15]. Then Zhu and Wang developed the Floquet scattering method to study the pumping with a series of time-periodic potentials, their main results in both the weak pumping and strong pumping regimes being consistent with experiment results [16]. Moskalets and Büttiker generalized the Floquet scattering theory to the quantum pumping in mesoscopic conductors [17]. Kim found that the Floquet scattering approach and the adiabatic scattering approach give the exactly equivalent result under the weak pumping condition for a double delta-barriers model [18]. The essential difference of the two methods is that the adiabatic condition is necessary for the adiabatic pumping, while the Floquet pumping only relies on the periodicity of the time-dependent parameters. Moreover, in the Floquet theory, the pumped current may be non-zero when the phase difference between the driven potentials is zero [16] or only a single parameter is time-dependent in the system [19], although the adiabatic theory predicts vanishing pumped current under these conditions.

The topological spin pump is a spin analogue of the Thouless charge pump, which was proposed after the discovery of the topological insulators [20–22]. Topological insulators have non-trivial bulk band topology characterized by unconventional topological invariants [23, 24]. Two-dimensional topological insulators are also called the quantum spin Hall (QSH) systems, whose topological properties are usually described by the $\mathbb{Z}_2$
invariant \([25]\) or spin Chern numbers \([26, 27]\). An important consequence of the non-trivial bulk band topology of the QSH systems is that a pair of helical edge states emerge in the band gap, which make them conductive at the sample boundary and essentially different from ordinary insulators. The \(Z_2\) invariant is well-defined only when the time-reversal (TR) symmetry is present \([25]\). This feature is consistent with the fact that the edge states in the QSH systems are gapless in the presence of the TR symmetry, and usually gapped otherwise. While the spin Chern numbers yield an equivalent description for TR-invariant systems, their robustness does not rely on any symmetries \([27–29]\). Non-zero spin Chern numbers guarantee that edge states emerge in the bulk band gap, which could be gapless or gapped, depending on the symmetry and local microscopic structures of the sample edges \([30]\).

The topological spin pump has an intimate connection to the topological invariants underlying the QSH effect. As an observable effect, it provides a possible route to investigate the topological invariants experimentally. Based on a spin-conserved model of an antiferromagnetic chain, Shindou proposed the prototype of a topological spin pump \([31]\). Fu and Kane established the more general concept of a \(Z_2\) pump without limitation of spin conservation \([32]\). In the \(Z_2\) pump, while the amount of spin pumped per cycle is not integer-quantized in the absence of spin conservation, the pumping process is protected by a \(Z_2\) topological invariant, provided that the TR symmetry is present. Meidan, Micklitz, and Brouwer classified topological spin pumps based on general properties of the scattering matrix \([33]\). They showed that in the weak coupling limit, topological spin pumps are characterized by the appearance of symmetry-protected gapless end states during the pumping cycle, similarly to the \(Z_2\) pump. Several different methods have been put forward to realize the \(Z_2\) pump experimentally, including a Luttinger liquid \([34]\), a double-corner junction in a topological insulator \([35]\), and quantum wires proximity coupled to a superconductor \([36]\).

Recently, Zhou et al \([37]\) investigated the effect of TR symmetry breaking on the topological spin pumping by introducing randomly distributed magnetic impurities with classical spins into the one-dimensional model used by Fu and Kane \([32]\). In contradiction to the previous belief, the magnetic impurities only affect the amount of spin pumped per cycle in a perturbative manner rather than destroy the spin pumping effect immediately. While the \(Z_2\) invariant can no longer be defined in this situation as the TR symmetry is explicitly broken, the spin pumping effect can be attributed to the spectral flow of the spin-polarized Wannier functions driven by non-zero spin Chern numbers. Chen et al proposed that such spin-Chern pumping effect might be observed in a two-dimensional topological insulator with oscillating dual gate voltages subject to an in-plane ac electric field \([38]\).

In the absence of disorder, they showed that the effective one-dimensional system for each transverse momentum between two critical values \(-k_c^+\) and \(+k_c^+\) acts as a spin-Chern pump, and all the individual transverse momenta between \(-k_c^+\) and \(+k_c^+\) join together to contribute to a bulk spin pumping current in proportion to the width of the pump.

So far, existing theoretical works on the topological spin pump are limited to the adiabatic regime, which is difficult to reach in experiments. It is unclear how the topological properties of the spin pump evolve and whether the topological spin pumping remains to be robust in the more general non-adiabatic regime. Therefore, theoretical study of the topological spin pumping beyond the adiabatic approximation is highly desirable. In this work, we study the non-adiabatic topological spin pumping based on an exactly solvable model. We introduce the Floquet spin Chern numbers to describe the topological properties of the spin pump, which extend the adiabatic spin Chern numbers to more general non-adiabatic regime. Based on the Floquet scattering matrix theory, the spin pumped per cycle is calculated from the Floquet scattering matrix theory in the absence and in the presence of the TR symmetry. We show that spin pumping through the Floquet evanescent modes remains robust and insensitive to the parameters of the system, as long as the pumping frequency is smaller than the band gap. For the pumping frequency greater than the band gap, the spin pumping involves the propagating modes in the pump, and decreases rapidly, which marks the boundary of the topological spin pumping regime.

In the next section, we first introduce the model Hamiltonian. Then we define the Floquet spin Chern numbers, which are applicable to the more general non-adiabatic regime. In section 3, the spin pumped per cycle in the presence of the TR symmetry is calculated by using the Floquet scattering matrix method. In section 4, the spin pumped in the absence of the TR symmetry is calculated, and the result is compared with that obtained in the adiabatic approximation. The final section contains a summary.

### 2. Floquet spin Chern numbers

Let us start from the effective continuum electron model of a one-dimensional zigzag atomic chain proposed in \([37]\), which was used to describe the body of a topological spin pump.
The Hamiltonian $H_0$ is given by:

$$H_0 = v_F p_x \sigma_y + \alpha(t) \sigma_x + g(t) s_z \sigma_z. \quad (1)$$

Here, $s_z$ is the Pauli matrix describing electron spin, and $\sigma_{x,y,z}$ are the Pauli matrices associated with sublattices. The first term originates from the nearest neighbor hopping with $v_F$ as the Fermi velocity. The second term can be induced by agitating oscillatory shear deformation of the substrate, on which the one-dimensional atomic chain is deposited, with $\alpha(t) = \alpha_0 \cos \omega t$ and $\alpha_0$ as the strength of the deformation potential.

The wavefunctions of the time-dependent Hamiltonian equation (1) is difficult to obtain for arbitrary parameter sets. However, we find that in the special case $\alpha_0 = g_0$, the problem is exactly solvable. We will confine ourselves to this case, but the conclusion is expected to be applicable to more general cases. The Hamiltonian $H_0$ conserves spin $s_z$, so that the two spin species can be treated separately. The time-dependent Schrödinger equation for the spin-up electrons is

$$i \hbar \frac{\partial \Psi_{\uparrow}(x, t)}{\partial t} = H_{\uparrow}(x, t), \quad (2)$$

$$H_{\uparrow}(x, t) = v_F p_x \sigma_y + \alpha(t) \sigma_x + g(t) \sigma_z. \quad (3)$$

Since the Hamiltonian is periodic in time, the Floquet theorem is applicable to this equation. By setting $\Psi_{\uparrow}(x, t) = e^{-i E_\uparrow t / \hbar} \psi_{\uparrow}(x, t)$, where $E_\uparrow \in [0, \hbar \omega)$ is the Floquet energy and $\psi_{\uparrow}(x, t)$ is a periodic function in time $\psi_{\uparrow}(x, t) = \psi_{\uparrow}(x, t + T)$ with period $T = 2\pi / \omega$, equations (2) and (3) are derived to be

$$i \hbar \frac{\partial \psi_{\uparrow}(x, t)}{\partial t} = H^f_{\uparrow}(x, t), \quad (4)$$

$$H^f_{\uparrow}(x, t) = v_F p_x \sigma_y + \alpha(t) \sigma_x + g(t) \sigma_z - E_{\uparrow}. \quad (5)$$

Using a unitary transformation $\psi_{\uparrow}(x, t) = U \psi_{\uparrow}(x, t)$ with $U = e^{-i \omega t / \hbar}$, which means transforming the system from the laboratory reference frame to a rotating reference frame, we obtain

$$i \hbar \frac{\partial \psi_{\uparrow}(x, t)}{\partial t} = H_{\uparrow}(x, t), \quad (6)$$

$$H_{\uparrow} = i \hbar \frac{\partial U}{\partial t} U^\dagger + U H^f_{\uparrow}(t) U^\dagger,$$

$$= v_F p_x \sigma_y + g_0 \sigma_x + \frac{\hbar \omega}{2} \sigma_y - E_{\uparrow}. \quad (7)$$

Since the Hamiltonian $H_{\uparrow}$ is time-independent, we can easily obtain the eigenfunctions of $H_{\uparrow}$ as

$$\psi_{\uparrow}^{\pm}(x, t) = \frac{1}{\sqrt{2}} \left( \begin{array}{c} 1 \\ \pm e^{i \varepsilon_\pm t / \hbar} \end{array} \right) e^{ik_{\uparrow}x - i(\varepsilon_\pm t / \hbar) t / \hbar}, \quad (8)$$

where $e^{i \varepsilon_\pm} = [g_0 + i(v_F / \hbar k_x + \hbar \omega/2)]/|\varepsilon_\pm|$. The corresponding eigenenergies are

$$\varepsilon_\pm = \pm \sqrt{g_0^2 + (v_F / \hbar k_x + \hbar \omega/2)^2}. \quad (9)$$

The system has a band gap between $-g_0$ and $g_0$ with $g_0 > 0$. To ensure $\psi_\uparrow(x, t) = U^{-1} \psi_\uparrow(x, t)$ to be periodic in time, the eigenenergies need to take quantized values $\varepsilon_\pm = E_{\uparrow} = (n + 1/2) \hbar \omega$, from which the allowable values of $k_x$, namely, $k_0$ for any given $E_{\uparrow}$ can be obtained.

We define a Floquet spin Chern number for the spin-up electrons

$$C^f_\uparrow = \frac{1}{\pi} \int_0^T dt \int_{-\infty}^{\infty} dk_x \text{Im} \left\langle \partial_{k_x} \psi^\dagger_\uparrow \right| \partial_{k_x} \psi^\dagger_\uparrow \right\rangle. \quad (10)$$

Equation (10) is an extension of the adiabatic spin Chern number to the more general non-adiabatic regime. In the adiabatic limit, $\hbar \omega \to 0$ and $E_{\uparrow}$ becomes negligible, $\psi_{\uparrow}$ recovers the adiabatic wavefunction, and one can easily see that the Floquet spin Chern numbers just become the adiabatic spin Chern numbers. By substituting $\psi_\uparrow(x, t) = U^{-1} \psi_\uparrow(x, t)$ and equation (8) into equation (9), it is straightforward to derive the Floquet spin Chern number to be $C^f_\uparrow = 1$. Similarly, for the spin-down electrons, through using the inverse unitary transformation $U = e^{-i \omega t / \hbar}$, one can obtain $C^f_\downarrow = -1$. Therefore, the resulting Floquet spin Chern numbers are the same to the adiabatic ones. The system is topological, and can pump pure spin.

### 3. Spin pumping with TR symmetry

Now we consider a pumping system, where a pump body described by Hamiltonian equation (1) lies at $x < 0$, and a lead is at $x > 0$ with Hamiltonian

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The system is topological, and can pump pure spin.
Here we consider the scattering process for a spin-up electron with energy $E_0 = E_F = 0$ incident from the lead. The reflection waves contain different energy modes $E_{m} = m \hbar \omega$ with $m$ as an integer, due to absorption or emission of photons. There is also probability for the electron to transmit into the pump body. The Floquet eigenenergy can be chosen to be the incident energy $E_{fl} = E_F = 0$ for convenience. We first consider the case where the spin of the incident electron is parallel to the $z$ axis. The electron spin is conserved in the scattering process, and will be omitted for a while. On the basis of $(|1\rangle, |−1\rangle)$ with the kets as the eigenstates of $\sigma_z$, the wave function in the lead is given by
\[
\Psi_{fl}^\dagger(x, t) = \sum_{m = \infty}^{\infty} \left[ \frac{\delta_{m,0}}{\sqrt{2}} \right] e^{-i k_m x} + \frac{t_{m}^\dagger}{\sqrt{2}} \left[ \begin{array}{c} 1 \\ i \end{array} \right] e^{i k_m x} e^{-i E_m t / \hbar},
\]
where $k_m = m \omega / \nu_F$. In the pump body, depending on the electron energy is in the gap or in the band, the wave function can be evanescent modes or propagating modes. The wave function is generally written as
\[
\Psi_{p}^\dagger(x, t) = U^\dagger \sum_{n = \infty}^{\infty} \frac{t_{n}^\dagger}{\sqrt{|a_n|^2 + |b_n|^2}} \left[ \begin{array}{c} a_n \\ b_n \end{array} \right] e^{i k_n x - i \sigma_n t / \hbar},
\]
where $a_n = 1 - i \nu (\nu k_n^2 + \omega / 2) / g_0$, $b_n = \varepsilon_n / g_0$, and $\varepsilon_n = (n + 1/2) \lambda \omega$ are the eigenenergies in the pump in the rotating reference frame, as has been obtained in section 2. The wave vectors $k_n$ are given by $v_F / \hbar k_n^2 = - \hbar \omega / 2 \pm \sqrt{\varepsilon_n^2 - g_0^2}$, where the sign is for $\varepsilon_n < -g_0^2$ and otherwise. In equation (12), the sum function represents the wave function in the pump in the rotating reference frame. After a unitary transformation $U^\dagger = e^{i \theta \sigma_z / 2}$, the wave function goes back into the laboratory reference frame, so that the wave functions in the pump and lead can be connected directly. Here, it is worth pointing out that while the eigenenergies $E_n = m \lambda \omega$ in the lead are taken to be integer values in units of $\lambda \omega$, the eigenenergies $\varepsilon_n = (n + 1/2) \lambda \omega$ in the pump are half-integers in the rotating reference frame. In equation (12), the wave function in the rotating reference frame can be generally considered as a superposition of two eigenstates of $\sigma_y$, namely, $\chi_+ = (1, i)^T$ and $\chi_- = (1, -i)^T$. When acting on $\chi_+$ or $\chi_−$, $U^\dagger$ becomes $e^{i \theta \sigma_z / 2}$ or $e^{-i \theta \sigma_z / 2}$, which effectively shifts the energy levels by $\hbar \omega / 2$ or $- \hbar \omega / 2$. This fact ensures that the energy levels in the laboratory reference frame take integer values, so that the energy conservation law for the electron-photon system can be satisfied during the scattering process, as expected. We recall that the energy spectrum in the pump has a band gap between $-g_0$ and $g_0$. Indeed, if $|\varepsilon_n| < g_0$, the wave vector $k_n$ is a complex number, corresponding to an evanescent mode. Otherwise, the wave vector is real, corresponding to a propagating mode.

The wave functions in the pump and lead are connected through the continuity equation at $x = 0$
\[
\Psi_{fl}^\dagger(x = 0^+, t) = \Psi_{p}^\dagger(x = 0^−, t).
\]
Substitute equations (11) and (12) into equation (13), and notice that the wave functions on both sides of equation (13) are Fourier series in time. When two Fourier series are equal, the components of the same order in two series must be equal. Consequently, we derive the following equations
\[
\sqrt{2} \delta_{m,0} \left[ \begin{array}{c} 1 \\ i \end{array} \right] + \sqrt{2} t_{m}^\dagger \left[ \begin{array}{c} 1 \\ i \end{array} \right] = \frac{t_{m}^\dagger}{\sqrt{|a_m|^2 + |b_m|^2}} \left[ \begin{array}{c} a_m - i b_m \\ b_m \end{array} \right] e^\theta \left[ \begin{array}{c} 1 \\ i \end{array} \right] + \frac{t_{m-1}^\dagger}{\sqrt{|a_{m-1}|^2 + |b_{m-1}|^2}} \left[ \begin{array}{c} a_{m-1} + i b_{m-1} \\ b_{m-1} \end{array} \right] e^{-i \theta \sigma_z / 2} \left[ \begin{array}{c} 1 \\ i \end{array} \right].
\]
It follows from this equation that the $m$th order reflection amplitudes are coupled to the $m$th order and $(m − 1)$th order transmission amplitudes. We notice that because $\left[ \begin{array}{c} a_m \sigma_z \end{array} \right] = 0$, the electron helicity $\sigma_z$ is conserved. Therefore, when the incident wave in the lead is $\chi_− = (1, -i)^T$, which is an eigenstate of $\sigma_y$, the reflection wave in the lead should be the other eigenstate of $\sigma_y$, i.e., $\chi_+ = (1, i)^T$. The transmission wave functions are linear combinations of $\chi_+$ and $\chi_-$. It is easy to find $t_{m}^\dagger = t_{m-1}^\dagger = 0$ for $m = 0$. The only non-zero coefficients are
\[
r_{1}^\dagger = \frac{a_1 - i b_1}{a_1 + i b_1},
\]
\[
t_{1}^\dagger = \frac{\sqrt{2} \left( |a_1|^2 + |b_1|^2 \right)}{a_1 + i b_1}.
\]
The above result indicates that only the single-photon assisted transport happens in the topological Floquet scattering process, as illustrated in figure 1(a). If $2g_0 > \hbar \omega$, $\varepsilon_1 = - \hbar \omega / 2$ is in the band gap, and the wave function in the pump is evanescent mode with exponential decay. $a_{-1} = 1 - i \sqrt{\left( \hbar \omega / 2g_0 \right)^2 - 1}$ and $b_{-1} = - \hbar \omega / 2g_0$ are real, so $|r_{-1}^\dagger|^2 = 1$. If $2g_0 \leq \hbar \omega$, there is actual transport of probability into the pump.
through the propagating mode. By defining the electron velocity \( v_{\text{rel}} = \frac{\partial H}{\partial k_x} |_{\psi_{\text{in}}} \) and denoting the component of the Floquet wave functions correspondent to \( \psi_{\text{out}} \) \( | \psi_{\text{out}} \rangle \), it is easy to check \( | r_1^\uparrow \rangle^2 + | r_1^\downarrow \rangle^2 \psi_{\text{in}} = 1 \), and the conservation law of the probability current is fulfilled. Similarly, by considering the case, where the spin of the incident electron is antiparallel to the \( z \)-axis, we can obtain \( r_1^\downarrow = (r_1^\downarrow)^s \) and \( t_1^\uparrow = (t_1^\uparrow)^s \), and all other transmission and reflection amplitudes are zero. The scattering process for a spin-down incident electron is illustrated in figure 1(b). We notice that while the incident electrons in the two spin channels have the same energy, the reflection electrons have different energies, which will give rise to the pumping effect in this Floquet transport picture.

The spin-dependent electrical current pumped into the lead can be expressed in terms of the Floquet scattering matrix \( S \) of the lead as [17, 40]

\[
I^s = \frac{e}{\hbar} \int dE \left\{ f_s^{\text{out}}(E) - f_s^{\text{in}}(E) \right\},
\]

(17)

Here, \( f_s^{\text{out}}(E) = \sum_{n=-\infty}^{\infty} \sum_{\nu=\uparrow,\downarrow} \left| S_{\nu}^s(E_{\text{in}}, E) \right|^2 f(E) \) with \( s = \uparrow, \downarrow \) is spin-dependent distribution function for outgoing electrons, and \( f_s^{\text{in}}(E) \) for incoming ones, where \( f(E) \) is the Fermi distribution function.

At zero temperature, substituting the scattering coefficients into equation (17), we can obtain

\[
I^\uparrow = \frac{e}{\hbar} \int_0^{\hbar/2} | r_1^\uparrow |^2 dE = \frac{e\hbar}{2\pi} | r_1^\uparrow |^2,
\]

(18)

\[
I^\downarrow = \frac{e}{\hbar} \int_{-\hbar/2}^0 | r_1^\downarrow |^2 dE = -\frac{e\hbar}{2\pi} | r_1^\downarrow |^2.
\]

(19)

Since \( r_1^\uparrow = (r_1^\uparrow)^s \), the pumped currents in the two spin channels are just opposite, i.e., \( I^\uparrow = -I^\downarrow \). The total charge current \( I^c = I^\uparrow + I^\downarrow \) vanishes, and the spin current is given by \( I^s = \hbar (I^\uparrow - I^\downarrow) / 2e \). It is convenient to consider the spin pumped per cycle \( \Delta s = 2\pi I^s / \hbar \), which is derived to be

\[
\Delta s = \hbar | r_1^\uparrow |^2 = \begin{cases} \frac{\hbar}{1 - \lambda} & \hbar \omega < 2g_0, \\ \frac{\hbar}{1 + \lambda} & \hbar \omega \geq 2g_0, \end{cases}
\]

(20)

with \( \lambda = \sqrt{1 - (2g_0 / \hbar \omega)^2} \). In figure 2, the spin pumped per cycle is plotted as a function of the single-photon energy \( \hbar \omega \). When the single-photon energy \( \hbar \omega \) is less than the band gap \( 2g_0 \), the pumped spin is quantized and independent of the parameters. The spin pumping remains to be topological, which can be attributed to the non-zero Floquet spin Chern number. While this result is consistent with the the adiabatic approximation, we

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**Figure 1.** The Floquet scattering processes. (a) For an electron wave with energy \( E_0 = E_g = 0 \) incoming from the spin-up channel of the lead, the reflection wave mode has an energy \( E_{-1} = -\hbar \omega / 2 \), with \( \hbar \omega \) as the energy of a single photon. In our model, only the single-photon processes can occur, and the multi-photon assisted processes are prohibited. In the pump part, the transmission wave mode has an energy \( E_{-1} = -\hbar \omega / 2 \) in the rotating reference frame. The case for an electron wave incoming from the spin-down channel is illustrated in (b). The fact that the eigenenergies in the rotating reference frame take half-integer values in units of \( \hbar \omega \) is because of the unitary transformation \( U = e^{-i\theta_0/2} \), which shifts the energy levels by \( \pm \hbar \omega / 2 \) with respect to those in the laboratory reference frame, as explained below equation (12). The evanescent modes in the band gap between the energy levels of the transmission waves for both the spin-up and spin-down cases, i.e., \( \varepsilon_1 \) and \( \varepsilon_{-1} \), are in the band gap, and the spin pumping is topological.
need to point out that the adiabatic approximation is valid only in the limit $\hbar \omega \to 0$, and the present calculation is strict for any value of $\hbar \omega$. When the single-photon energy is greater than the band gap, the pumped spin is not quantized, and dependent on the parameters. The spin pumping is no longer related to the topology of the Floquet energy band, because the incident electrons can now be transmitted to the Floquet propagating modes in the pump. In addition, by similar analysis, when the Fermi energy is non-zero, either above or below the middle of the band gap, the region for the topological spin pumping will shrink, and the condition for the topological spin pumping becomes $\hbar \omega/2\epsilon_0 < 1 - |E_0|/\epsilon_0$.

In order to see some new features of the topological Floquet pumping, in comparison to the topologically trivial pumping, we consider the effect of varying the strength of the oscillating Zeeman field on the pumped spin. As can be seen from figure 2, when the strength $\epsilon_0$ is large enough such that $\hbar \omega/2\epsilon_0 < 1$, the pumped spin is quantized to 2 in units of $\hbar$, which is the same as the adiabatic pumping [37]. However, in the topologically trivial pumping, such as the pumping in the one-dimensional system with time-dependent double delta barriers [18], the Floquet pumping and the adiabatic pumping yield an equivalent result only for small strength of the oscillating Zeeman field. The difference may be understood from the fact that the strength $\epsilon_0$ determines the magnitude of the band gap in the topological pumping of this system. As long as the condition $\hbar \omega < 2\epsilon_0$ is fulfilled, the pumping process is governed by the topological property of the system.

4. Spin pumping without TR symmetry

To study the spin pumping in the absence of the TR symmetry, we include into the model a magnetic impurity with potential $H_M = V(x)s_x$. $V(x)$ is taken to be a square potential centered at $x = 0$ with height $V_0$ and width $d$. For simplicity, we can take the limit $d \to 0$ and keep $V_0 = V_0 d$ finite. It can be shown that the scattering effect of the impurity potential is equivalent to imposing a unitary boundary condition for the electron wavefunctions [37]

$$\Psi_{ii}^L(x = 0^+, i) = S\Psi_{ii}^R(x = 0^-, i),$$

(21)

where $S = e^{-i\phi}e^{-i\phi k}$, with $\phi = V_0/\hbar\nu$. The magnetic impurity explicitly breaks the TR symmetry of the system.

We consider the scattering problem for an spin-up electron incident from the lead. The presence of the magnetic impurity destroys the spin conservation, and we now need to explicitly include both spin degrees of freedom in the electron wave functions. On the basis $\{\uparrow, \downarrow, |1\rangle, |\downarrow, -1\rangle, \downarrow, \downarrow, \downarrow, \downarrow, \downarrow\}$ with the kets as the eigenstates of $\sigma_z$ and $\sigma_x$, the wavefunction in the lead is given by

$$\Psi_{ii}^L = \sum_{m=-\infty}^{\infty} \left[ \frac{\epsilon_{m,0}}{\sqrt{2}} \begin{pmatrix} -1 \\ 0 \\ 0 \end{pmatrix} e^{-i\delta_{m,0}x} + \frac{r_{m,0}^{\uparrow\downarrow}}{\sqrt{2}} \begin{pmatrix} 1 \\ i \\ 0 \end{pmatrix} e^{i\delta_{m,0}x} + \frac{r_{m,0}^{\downarrow\uparrow}}{\sqrt{2}} \begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix} e^{i\delta_{m,0}x} \right] e^{-imx}.$$

(22)
In the pump, the wave function can be written as

$$\Psi_{\Pi}^{t} = \sum_{n=-\infty}^{\infty} \left[ \frac{t_{M,n}^{+} e^{i\omega t \phi}}{\sqrt{a_{n}^2 + b_{n}^2}} e^{ik_{n}x} + \frac{t_{M,n}^{-} e^{-i\omega t \phi}}{\sqrt{c_{n}^2 + d_{n}^2}} e^{ik_{n}x} \right] e^{-i(n+1/2)\omega t}.$$  \hspace{1cm} (23)

Here, $a_{n}$ and $b_{n}$ are the same as in equation (12), $c_{n} = 1 - i \hbar (\nu_{r} k_{n}^+ - \omega)/g_{0}$, and $d_{n} = b_{n}$. The wave vectors $k_{n}^\pm$ are given by $\nu_{r} k_{n}^+ = \hbar \omega / 2 + \sqrt{\varepsilon_{n}^2 - g_{0}^2}$, where the sign $+$ is for $\varepsilon_{n} < -g_{0}$, and $-$ otherwise. Similarly to solving the Floquet scattering coefficients in section 3, by substituting equations (22) and (23) into equation (21) and by some algebra, we can obtain for the reflection amplitudes $r_{M,1}^{\pm} = r_{M,1}^{\pm} \cos^2 \phi$, $r_{M,1}^{\pm} = - (r_{M,1}^{\pm})^*$, where $r_{M,1}^{\pm}$ is the reflection amplitude in the absence of the magnetic impurity given by equation (15). The transmission amplitudes are obtained as $t_{M,0}^{\pm} = (i t_{M,0}^{\pm})^* \sin \phi$, $t_{M,0}^{\pm} = t_{M,0}^{\pm} \cos \phi$, where $t_{M,0}^{\pm}$ is given by equation (16). It is easy to verify that the reflection and transmission coefficients satisfy the conservation law of probability current. For the case of a spin-down electron incident from the lead, the scattering amplitudes can be obtained as $r_{M,1}^{\pm} = r_{M,1}^{\pm}$, $r_{M,1}^{\pm} = r_{M,1}^{\pm}$, $r_{M,1}^{\pm} = r_{M,1}^{\pm}$, $r_{M,1}^{\pm} = r_{M,1}^{\pm}$, $r_{M,1}^{\pm} = r_{M,1}^{\pm}$, $r_{M,1}^{\pm} = r_{M,1}^{\pm}$, and $r_{M,1}^{\pm} = (r_{M,1}^{\pm})^*$. In both spin channels, only the single-photon assisted scattering process contributes to the electron transport.

From equation (17), the pumped current in the presence of the magnetic impurity can be obtained as

$$I_{M} = e \frac{\omega}{2\pi} \left( |r_{M,1}^{+}|^2 + |r_{M,1}^{-}|^2 - |r_{M,1}^{+}|^2 - |r_{M,1}^{-}|^2 \right),$$ \hspace{1cm} (24)

$$I_{M} = - e \frac{\omega}{2\pi} \left( |r_{M,1}^{+}|^2 + |r_{M,1}^{-}|^2 - |r_{M,1}^{+}|^2 - |r_{M,1}^{-}|^2 \right).$$ \hspace{1cm} (25)

The second and fourth terms in both equations (24) and (25) cancel each other, since $r_{M,1}^{\pm} = r_{M,1}^{\pm}$ and $r_{M,1}^{\pm} = - (r_{M,1}^{\pm})^*$. Using the expressions for the other reflection amplitudes, we find that the charge pumped per cycle is zero, and the spin pumped $\Delta s_{M} = h (I_{M} - I_{M})/2\omega e$ is

$$\Delta s_{M} = \Delta s \cos 2\phi,$$ \hspace{1cm} (26)

where $\Delta s$ is the spin pumped per cycle without the magnetic impurity given by equation (20). For weak impurity potential $\phi \ll 1$, the pumped spin can be expanded to be $\Delta s_{M} \approx \Delta s (1 - 2\phi^2)$. For $\hbar \omega < 2g_{0}$, $\Delta s_{M} = h (1 - 2\phi^2)$, which recovers the result obtained in the adiabatic limit [37]. The impurity potential affects the spin pumping only in a perturbative manner, rather than destroys it immediately. This result proves that beyond the adiabatic regime, the topological spin pumping remains to be robust against TR symmetry breaking.

5. Summary

In summary, we have studied the non-adiabatic topological spin pumping based on an exactly solvable model. A Floquet spin Chern number is defined to describe the non-trivial bulk band topology of the system. It is a generalization of the spin Chern number that was previously introduced in the adiabatic limit to any periodically driven one-dimensional fermionic systems. The spin pumping is topological and robust for pumping frequency smaller than the band gap, where the electron transport involves only the Floquet evanescent modes in the pump. In this regime, the topological spin pumping is stable to TR symmetry breaking. For pumping frequency greater than the band gap, where the electron transport involves the propagating modes in the pump, the pumping spin current decays rapidly and becomes irrelevant to the topological properties of the energy bands.

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