An Orbifold Compactification with Three Families from Twisted Sectors

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ABSTRACT

We obtain a three generational $SU(3)_c \times SU(3)_w \times U(1)^4 \times [SO(12) \times U(1)^2]'$ model from an orbifold construction with the requirement that three generations arise from twisted sectors. There exist supersymmetric vacua realizing the standard model. In one example the anomalous $U(1)$ breaks the gauge symmetry down to $SU(3)_c \times SU(2)_w \times U(1)_Y \times SO(12)'$. 
The orbifold construction of four dimensional string models [1] has attracted a
great deal of attention due to its possibility of obtaining a standard-like model [2]
and the relative simplicity of model building compared to the other constructions
[3,4,5]. In order to achieve a string derived standard model, however, several
phenomenologically desirable features have to be realized: (i) three families, (ii) a
successful hypercharge assignment, (iii) a good prediction of $\sin^2 \theta_W$ at electroweak
scale, (iv) a good prediction of mixing angles in the quark sector, and (v) a strong
CP solution. In this paper, we focus on (i) and (ii) with an eye on a possible
solution of (iv).

An important ingredient toward a superstring standard model is the role of
the hidden sector for supersymmetry breaking [6]. A hidden sector confining group
is better to be present. A desirable hidden sector is two hidden sector confining
groups with a comparable group size for a realistic determination of gauge coupling
constant at the string scale [7]. In Ref. [7], a hidden sector with $SU(10) \times SU(9)$
has been considered. This large hidden sector cannot be obtained in symmetric
orbifold

constructions. The reason for considering the large hidden sector gauge group
has been to obtain almost the same but different $\beta$ functions. This condition
for $\beta$ function can be achieved by two same size groups but different numbers
of hidden matter fields. The hidden sector $E_8'$ can have $SU(5)' \times SU(5)'$ as a
candidate subgroup in this scenario; but this cannot be obtained by our application
of shift vectors. Therefore, the largest hidden sector factors with comparable sizes
is $SU(4)' \times SU(4)'$. Our model, however, does not realize this factor group but one
hidden confining group $SO(12)'$. Nevertheless, we may anticipate the comparable
but slightly different $\beta$ functions if one $SO(12)'$ is broken to $SU(4)' \times SU(3)'$ at a
somewhat lower scale than the string scale by vacuum expectation values of Higgs
fields.

Another motivation for building a 4-dimensional superstring model is to un-
derstand the flavor problem. The three generation superstring standard models
proposed so far derive the $SU(2)$ doublets of quarks from the untwisted sector [2]; thus realistic fermion mass spectrum cannot be obtained. To understand the flavor problem, the quark doublets must arise from twisted sectors. This condition is very restrictive in the orbifold construction of 4-dimensional superstring models. With this condition one cannot obtain $SU(3) \times SU(2) \times U(1)$ models or $SU(5) \times U(1)$ models. The smallest group constructed in this way is $SU(3) \times SU(3) \times U(1)$'s.

We will consider a model with two Wilson lines to obtain multiplicity of three automatically. The shift vector and Wilson lines are

$$v = \begin{pmatrix} \frac{1}{3} & \frac{1}{3} & \frac{2}{3} & \frac{1}{3} & \frac{2}{3} & 0 & 0 \end{pmatrix}(0, 0, 0, 0, 0, 0, 0),$$
$$a_1 = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ \frac{1}{3} & \frac{1}{3} & \frac{2}{3} & \frac{1}{3} & \frac{1}{3} & 0 & 0 \end{pmatrix},$$
$$a_3 = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ \frac{2}{3} & \frac{1}{3} & \frac{1}{3} & \frac{1}{3} & \frac{1}{3} & \frac{1}{3} & \frac{1}{3} \end{pmatrix}.\tag{1}$$

from which we obtain the desired gauge group

$$SU(3)_c \otimes SU(3)_w \otimes [U(1)]^4 \otimes SO(12)' \otimes [U(1)']^2.\tag{2}$$

Massless chiral fermions arise from untwisted and twisted sectors. The fermions in the twisted sectors have opposite chirality from the fermions in the untwisted sector. The massless chiral superfields satisfy

for untwisted sector

$$p_I \cdot v = -\frac{1}{3} \mod 1,$$
$$p_I \cdot a_i = 0 \mod 1, \quad (i = 1, 3)\tag{3}$$

for twisted sectors

$$p_I \cdot p_I = \begin{cases} \frac{2}{3} \quad \text{(multiplicity 9)} \\ \frac{4}{3} \quad \text{(multiplicity 3)} \end{cases}.\tag{4}$$

If we add one more Wilson line, the multiplicities in the twisted sectors become 3 and 1 for $p_I^2 = \frac{2}{3}$, and $\frac{4}{3}$, respectively. In this sense, two Wilson line models are most attractive because all chiral fields appear as multiples of 3.
There are nine twisted sectors distinguished by the shift and Wilson lines,

\[
T_0 : \; v, \quad T_1 : \; v + a_1, \quad T_2 : \; v - a_1,
\]
\[
T_3 : \; v + a_3, \quad T_4 : \; v - a_3, \quad T_5 : \; v + a_1 + a_3,
\]
\[
T_6 : \; v + a_1 - a_3 \quad T_7 : \; v - a_1 + a_3 \quad T_8 : \; v - a_1 - a_3
\]

For the twisted sector \(T_0\), we present all massless chiral fields. We drop the multiplicity 3 throughout the paper. The momenta satisfying \(p_I^2 = \frac{2}{3}\) are

\[
p_I = (0 0 \bar{1} 0 0 \bar{1} 0 0)(\cdots) \quad 3 \cdot 1
\]
\[
p_I = (- - - - - - + +)(\cdots) \quad 3 \cdot 1
\]
\[
p_I = (- - - - - - -)(\cdots) \quad 3 \cdot 1
\]

where \(\bar{1}, +, -, \) and \(\bar{-}\) represent \(-1, 1/2, -1/2\) and \(-3/2\), respectively. The momenta satisfying \(p_I^2 = \frac{4}{3}\) are

\[
\begin{pmatrix}
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & \bar{1} & \bar{1} & \bar{0} & 0 & 0 & 0 & 0 \\
\bar{1} & 0 & \bar{1} & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & \bar{1} & \bar{1} & 0 & 0 & 0 \\
0 & 1 & \bar{1} & 0 & \bar{1} & 0 & 0 & 0 \\
0 & 1 & \bar{1} & 0 & \bar{1} & 0 & 0 & 0 \\
\bar{1} & 0 & \bar{1} & 0 & \bar{1} & 0 & 0 & 0 \\
\bar{1} & 0 & \bar{1} & 0 & \bar{1} & 0 & 0 & 0 \\
\end{pmatrix} = (3_c^*, 3_w^*)
\]

\[
\begin{pmatrix}
\bar{1} & \bar{1} & \bar{1} & 0 & 0 & \bar{1} & 0 & 0 \\
\bar{1} & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\
\end{pmatrix} = (3_c, 1), \quad \begin{pmatrix}
0 & 0 & \bar{1} & \bar{1} & \bar{1} & \bar{1} & 0 & 0 \\
0 & 0 & \bar{1} & \bar{1} & \bar{1} & \bar{1} & 0 & 0 \\
0 & 0 & \bar{1} & 0 & \bar{1} & 0 & 0 & 0 \\
\end{pmatrix} = (1, 3_w)
\]

\* For two Wilson lines, the original 27 degenerate states of the twisted sector are distinguished by 9 different but triply degenerate states.
where we neglected the eight zero entries of \( E_8 \). We also showed the representation content in the gauge group \( SU(3)_c \times SU(3)_w \). Similarly, we find the other chiral fields which are, taking the opposite chiralities of the twisted and untwisted sectors and ignoring the multiplicity 3,

\[
\begin{align*}
UT : & \quad (3^*_c, 1), (1, 3^*_w) \\
T0 : & \quad (3_c, 3_w), 3(3^*_c, 1), 3(1, 3^*_w), 9 \cdot 1 \\
T1 : & \quad (3^*_c, 1), (1, 3_w), 3 \cdot 1 \\
T2 : & \quad (3_c, 1), (1, 3^*_w), 3 \cdot 1 \\
T3 : & \quad (3_c, 1), (3^*_c, 1), (1, 3_w), (1, 3^*_w), 6 \cdot 1 \\
T4 : & \quad (3_c, 1), (3^*_c, 1), (1, 3_w), (1, 3^*_w), 6 \cdot 1 \\
T5 : & \quad (3_c, 1), (3^*_c, 1), (1, 3_w), (1, 3^*_w), 6 \cdot 1 \\
T6 : & \quad (12)', 6 \cdot 1 \\
T7 : & \quad (3_c, 1), (1, 3_w), 3 \cdot 1 \\
T8 : & \quad (3_c, 1), (3^*_c, 1), (1, 3_w), (1, 3^*_w), 6 \cdot 1.
\end{align*}
\]

Striking out vectorlike combinations under \( SU(3)_c \times SU(3)_w \times SO(12)' \), we obtain a three generation model

\[
(3_c, 3_w) + 3(3^*_c, 1) + 3(1, 3^*_w) + \text{singlets}
\]

where the multiplicity 3 is not written. Of course, the charged lepton singlets are hidden in the singlets.
The six $U(1)$ charges are defined as

\begin{align}
Q_1 &= (1 1 -1 0 0 0 0 0)(0 \cdots 0) \\
Q_2 &= (0 0 0 1 1 -1 0 0)(0 \cdots 0) \\
Q_3 &= (0 0 0 0 0 -1 0)(0 \cdots 0) \\
Q_4 &= (0 0 0 0 0 0 -1)(0 \cdots 0) \\
Q_5 &= (0 \cdots 0)(-1 -1 -1 -1 0 0 0) \\
Q_6 &= (0 \cdots 0)(0 0 0 0 -1 -1 -1 -1)
\end{align}

(10)

It is tedious but straightforward to calculate the $Q$ charges of the matter superfields. We define new $U(1)$ charges as $P_1, P_2, P_3, P_4, P_5,$ and $X$,

\begin{align}
P_1 &= \frac{1}{6}(Q_1 + Q_2) \\
P_2 &= \frac{1}{2}(Q_3 + Q_4) \\
P_3 &= \frac{1}{12}(Q_1 - Q_2) + \frac{1}{4}(Q_3 - Q_4) \\
P_4 &= \frac{1}{12}(Q_1 - Q_2) - \frac{1}{12}(Q_3 - Q_4) - \frac{1}{6}Q_6 \\
P_5 &= \frac{1}{4}Q_5 \\
X &= \frac{1}{12}(Q_1 - Q_2) - \frac{1}{12}(Q_3 - Q_4) + \frac{1}{12}Q_6
\end{align}

(11)

The trace of $X$ charge is

\begin{align}
\sum_{i=3,3^c} X(i) &= -2, \\
\sum_{i=3,3^c} X(i) &= -2, \\
\sum_{i=12} X(i) &= -1,
\end{align}

\begin{align}
\sum_{i=\text{all including singlets}} X(i) &= -24
\end{align}

(12)

This compactification exhibits the anomalous $U(1)_X$ whose gauge boson becomes massive by absorbing the model independent axion $a_{MI}$. $X$ is the charge of
this $U(1)_X$. Note that the divergence of the corresponding current is

$$\partial^\mu J^X_\mu = +\{F_c \tilde{F}_c\} + \{F_w \tilde{F}_w\} + \{F' \tilde{F}'\} + \cdots \quad (13)$$

where $\{F_c \tilde{F}_c\} \equiv (1/32\pi^2)F^a_{\mu\nu}\tilde{F}^{a\mu\nu}$ ($a = SU(3)_c$ adjoint index), etc., and $\cdots$ denote $U(1)_F \tilde{F}'s$ with the same coefficient +1. Even though the sum of $X$ charges for color triplets and antitriplets differs from the $X$ charge of 12' by a factor of 2, the anomaly coupling given above is the same because the indices of the representations differ by a factor of $1/2$, viz. $l(3_c \text{ or } 3_c^*) = 1/2$ and $l(12') = 1$ where $\text{Tr } T_i T_j = l\delta_{ij}$. Thus the anomaly coefficient of $U(1)_X$ match those of the model-independent axion. The model-independent axion becomes the longitudinal degree of the $U(1)_X$ gauge boson.

The presence of the anomalous $U(1)_X$ has desirable Fayet–Iliopoulos $D$-terms, reducing the rank of the gauge group. For supersymmetry, we must satisfy [8,9]

$$\langle D^{(X)} \rangle = \langle \frac{2g}{192\pi^2} \text{Tr } X + \sum_i X(i)\phi^*(i)\phi(i) \rangle = 0 \quad (14)$$

One can find vacua satisfying the above supersymmetry condition. One may reduce the rank of the gauge group by the condition of vanishing $D$-terms, and breaking $SU(3)_w$ down to $SU(2)_w$. One must check also that the resulting three generation model has the correct hypercharge. We find several models satisfying this criteria.

For example, giving a vacuum expectation values to the following $3_w, 3_w^*$ and singlets,
\[
\begin{array}{cccccc}
P_1 & P_2 & P_3 & P_4 & P_5 & X \\
3 (T_0) : & 0 & 0 & 1 & 1 & 0 & 1 \\
3^* (T_7) : & 0 & 0 & -1 & 1 & 0 & -1 \\
1 (T_0) : & -1 & 3 & 0 & 0 & 0 & 0 \\
1 (T_0) : & -1 & -3 & 0 & 0 & 0 & 0 \\
1 (T_6) : & 0 & 0 & 0 & -2 & 0 & 0 \\
1 (T_6) : & 0 & 0 & 0 & 0 & 0 & 2 \\
1 (T_6) : & 0 & 0 & 0 & 0 & 3 & -1 \\
1 (T_6) : & 0 & 0 & 0 & 0 & -3 & -1 \\
1 (T_7) : & 1 & 3 & 0 & 2 & 0 & 0 \\
1 (T_7) : & 1 & -3 & 0 & 2 & 0 & 0 \\
\end{array}
\]

we obtain the desired electroweak hypercharge

\[
Y = Y_3 + \frac{1}{3} P_3,
\]

where \(Y_3\) is proportional to the 8th generator of \(SU(3)_w\), \(Y_3 = \text{diag.(1/6,1/6,-1/3)}\). Thus we obtain the supersymmetric standard model \(SU(3) \times SU(2) \times U(1)_Y \times SO(12)\). All the other \(U(1)\)'s are broken.\(^\star\) Removing the vectorlike representations, we obtain three families at low energy. Among vectorlike representations, there appear \(Q_{em} = \pm 1/3, \pm 2/3\) leptons and \(Q_{em} = \pm 1/3\) quarks.

The compactification realized above hints a few interesting directions toward string derived supersymmetric standard models. Firstly, the standard model gauge group can arise through the Fayet–Iliopoulos mechanism even though the 4-D string model possess a much larger gauge symmetry. This Fayet–Iliopoulos mechanism is like the Higgs mechanism, and the available Higgs fields are restricted. A grand unification such as \(SU(5) \times U(1)\) can be broken down to the standard model gauge

\(^\star\) If we want an extra \(U(1)\), we can remove \((-1-3 0 0 0 0) (T_0)\) and \((1 3 0 2 0 0) (T_7)\) fields in Eq. (16). Then the additional \(U(1)\) gauge charge is \(Y' = 3P_1 + P_2\) which can be used to guarantee a long proton lifetime.
group by the Fayet–Iliopoulos mechanism, but a model with a much larger gauge symmetry may run into the difficulty due to the lack of needed Higgs fields. In the present case, the gauge group is small enough, $SU(3) \times SU(3) \times U(1)$’s; there exist a number of needed Higgs fields realizing supersymmetric standard model. Second, the requirement of quark doublets from twisted sectors is very restrictive. For example, it requires that there should exists a $\tilde{v}^\dagger$ taking the form,‡

$$\tilde{v} = \left(\begin{array}{cccccccc}
\frac{1}{3} & \frac{1}{3} & \frac{1}{3} & \frac{1}{3} & \frac{1}{3} & 0 & 0 \\
\frac{1}{3} & \frac{1}{3} & \frac{1}{3} & \frac{1}{3} & \frac{1}{3} & 0 & 0 \\
\frac{1}{3} & \frac{1}{3} & \frac{1}{3} & \frac{1}{3} & \frac{1}{3} & 0 & 0 \\
\cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots
\end{array}\right)$$

where $\cdots$ are zeros. Then, it is easy to see that $SU(3) \times SU(3) \times U(1)$’s in the observable sector is the smallest gauge group possible, allowing the quark doublets. Third, the anomalous $U(1)$ allows a low energy global symmetry [10]. However, in the present example the global symmetry is broken by the Fayet–Iliopoulos mechanism (viz. the $(0 \ 0 \ 0 \ 0 \ 0 \ 2)$ field in Eq. (15)), and the axion scale turns out to be too large. However, this phenomenon is not universal, since one may choose a different set of fields for the Fayet–Iliopoulos symmetry breaking. Finally, we comment that models with three Wilson lines do not realize three quark doublets from the twisted sectors.

In this paper, we find a supersymmetric standard model $SU(3)_c \times SU(2)_w \times U(1)_Y \times SO(12)'$. The three quark doublets arise from twisted sectors, which is a desirable feature if the quark mass matrix has to be understood at the string level.

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† $\tilde{v}$ is the vector signifying the twisted sector, e.g. $v + a_1$ for $T1$.
‡ Or even number of $1/3$ entries can be replaced by $2/3$’s. Also, $-$ signs and appropriate number of integers can be added.
References

1. L. Dixon, J. Harvey, C. Vafa, and E. Witten, Nucl. Phys. B261 (1985) 678; B274 (1986) 285; L. E. Ibanez, H. P. Nilles and F. Quevedo, Phys. Lett. B187 (1987) 25.

2. L. E. Ibanez, J. E. Kim, H. P. Nilles, and F. Quevedo, Phys. Lett. B191 (1987) 282; L. E. Ibanez, J. Mas, H. P. Nilles, and F. Quevedo, Nucl. Phys. B301 (1988) 157; J. A. Casas and C. Munoz, Phys. Lett. B214 (1988) 63.

3. B. R. Greene, K. H. Kirklin, P. J. Miron, and G. G. Ross, Phys. Lett. B180 (1986) 69; Nucl. Phys. B278 (1986) 667; B292 (1987) 606.

4. I. Antoniadis, J. Ellis, J. S. Hagelin, and D. V. Nanopoulos, Phys. Lett. B208 (1988) 209.

5. D. Gepner, Nucl. Phys. B296 (1988) 757; Phys. Lett. B199 (1987) 380.

6. J.-P. Derengdinger, L. E. Ibanez and H. P. Nilles, Phys. Lett. B155 (1985) 65; M. Dine, R. Rohm, N. Seiberg, and E. Witten, Phys. Lett. B156 (1985) 55.

7. V Kaplunovsky, L. Dixon, J. Louise, and M. Peskin, SLAC–Pub–5256 (1990).

8. M. Dine, N, Seiberg and E. Witten, Nucl. Phys. B289 (1987) 317; J. J. Atick, L. J. Dixon and A. Sen, Nucl. Phys. B292 (1987) 109; M. Dine, I. Ichinose and N. Seiberg, Nucl. Phys. B293 (1987) 253.

9. A. Font, L. E. Ibanez, H. P. Nilles, and F. Quevedo, Nucl. Phys. B307 (1988) 109; J. A. Casas, E. K. Katehou and C. Muñoz, Nucl. Phys. B317 (1989) 171.

10. J. E. Kim, Phys. Lett. B207 (1988) 434.