Cosmic Strings and Cosmic Superstrings

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In these lectures, I review the current status of cosmic strings and cosmic superstrings. I first discuss topological defects in the context of Grand Unified Theories, focusing in particular in cosmic strings arising as gauge theory solitons. I discuss the reconciliation between cosmic strings and cosmological inflation, I review cosmic strings dynamics, cosmic string thermodynamics and cosmic string gravity, which leads to a number of interesting observational signatures. I then proceed with the notion of cosmic superstrings arising at the end of brane inflation, within the context of brane-world cosmological models inspired from string theory. I discuss the differences between cosmic strings and their solitonic analogues, I review our current understanding about the evolution of cosmic superstring networks, and I then briefly describe the variety of observational consequences, which may help us to get an insight into the stringy description of our Universe.

1. Introduction

Provided our understanding about unification of forces and big bang cosmology are correct, it is natural to expect that topological defects, appearing as solutions to many particle physics models of matter, could have formed naturally during phase transitions followed by spontaneously broken symmetries, in the early stages of the evolution of the Universe. Certain types of topological defects (local monopoles and local domain walls) may lead to disastrous consequences for cosmology, hence being undesired, while others (cosmic strings) may play a useful rôle.

Cosmic strings [1] are linear topological defects, analogous to flux tubes in type-II superconductors, or to vortex filaments in superfluid helium. These objects gained a lot of interest in the 1980’s and early 1990’s, since they offered a potential alternative to the cosmological inflation for the origin of initial density fluctuations leading to the Cosmic Microwave Background (CMB) temperature anisotropies and the observed structure in the Universe. They however lost their appeal, when it was found that they lead to inconsistencies in the power spectrum of the CMB. It was later shown [2] that cosmic strings are generically formed at the end of an inflationary era, within the framework of Supersymmetric Grand Unified Theories (SUSY GUTs). Hence cosmic strings have to be included as a sub-dominant partner of inflation. This theoretical support gave a new boost to the field of cosmic strings, a boost which has been more recently enhanced when it was shown that cosmic superstrings [3] (fundamental or one-dimensional Dirichlet branes) can play the rôle of cosmic strings, in the framework of brane-world cosmologies.

A realistic cosmological scenario necessitates the input of high energy physics; any models describing the early stages of the evolution of the Universe have their foundations in general relativity and high energy physics. Comparing the theoretical predictions of such models against current astrophysical and cosmological data, results to either their acceptance or their rejection, while in the first case it also fixes the free parameters of the models (see e.g., Ref. [4]). In particular, by studying the properties of cosmic superstring networks and comparing their phenomenological consequences against observational data, we expect to pin down the successful and natural inflationary model and get some insight into the stringy description of the Universe. Cosmic strings/superstrings represent a beautiful example of the strong and fruitful link between cosmology and high energy physics.

In what follows, I will summarise the material I
had presented in my lectures during the summer school at Cargèse (June 2008). I will highlight only certain aspects of the subject, which I consider either more important due to their observational consequences, or more recently obtained results.

2. Topological Defects in GUTs

In the framework of the hot big bang cosmological model, the Universe was originally at a very high temperature, hence the initial equilibrium value of the Higgs field $\phi$, which plays the role of the order parameter, was at $\phi = 0$. Since the Planck time, the Universe has, through its expansion, steadily cooled down and a series of phase transitions followed by Spontaneously Symmetry Breaking (SSBs) took place in the framework of GUTs. Such SSBs may have left behind topological defects as false vacuum remnants, via the Kibble mechanism.

The formation or not of topological defects and the determination of their type, depend on the topology of the vacuum manifold $\mathcal{M}_n$. The properties of $\mathcal{M}_n$ are described by the $k$th homotopy group $\pi_k(\mathcal{M}_n)$, which classifies distinct mappings from the $k$-dimensional sphere $S^k$ into the manifold $\mathcal{M}_n$. Consider the symmetry breaking of a group $G$ down to a subgroup $H$ of $G$. If $\mathcal{M}_n = G/H$ has disconnected components — equivalently, if the order $k$ of the non-trivial homotopy group is $k = 0$ — two-dimensional defects, called domain walls, form. The space-time dimension $d$ of the defects is given in terms of the order of the non-trivial homotopy group by $d = 4 - 1 - k$. If $\mathcal{M}_n$ is not simply connected — equivalently, if $\mathcal{M}_n$ contains loops which cannot be continuously shrunk into a point — cosmic strings form. A necessary, but not sufficient, condition for the existence of stable strings is that the fundamental group $\pi_1(\mathcal{M}_n)$ is non-trivial, or multiply connected. Cosmic strings are linear-like defects, $d = 2$. If $\mathcal{M}_n$ contains unshrinkable surfaces, then monopoles form: $k = 1, d = 1$. If $\mathcal{M}_n$ contains non-contractible three-spheres, then event-like defects, textures, form: $k = 3, d = 0$.

Depending on whether the symmetry is local (gauged) or global (rigid), topological defects are respectively, local or global. The energy of local defects is strongly confined, while the gradient energy of global defects is spread out over the causal horizon at defect formation. Global defects having long range density fields and forces, can decay through long-range interactions, hence they do not contradict observations, while local defects may be undesirable for cosmology. In what follows, I will discuss local defects, since we are interested in gauge theories, being the more physical ones. Patterns of symmetry breaking which lead to the formation of local monopoles or local domain walls are ruled out, since they should soon dominate the energy density of the Universe and close it, unless an inflationary era took place after their formation. This is one of the reasons for which cosmological inflation — a period in the earliest stages of the evolution of the Universe, during which the Universe could be in an unstable vacuum-like state having high energy density, which remained almost constant — was proposed. Local textures are insignificant in cosmology since their relative contribution to the energy density of the Universe decreases rapidly with time.

Even in the absence of a non-trivial topology in a field theory, it may still be possible to have defect-like solutions, since defects may be embedded in such topologically trivial field theories. However, while stability of topological defects is guaranteed by topology, embedded defects are in general unstable under small perturbations.

Let me discuss the genericity of cosmic string formation in the context of SUSY GUTs, which contain a large number of SSB patterns leading from a large gauge group $G_{\text{GUT}}$ to the Standard Model (SM) gauge group $G_{\text{SM}} \equiv \text{SU}(3)_C \times \text{SU}(2)_L \times \text{U}(1)_Y$. The minimum rank of $G_{\text{GUT}}$ has to be at least equal to 4, to contain the $G_{\text{SM}}$ as a subgroup; we set the upper bound on the rank $r$ of the group to be $r \leq 8$. The embeddings of $G_{\text{SM}}$ in $G_{\text{GUT}}$ must be such that there is an

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1. http://www.lpthe.jussieu.fr/cargese/

2. The concept of spontaneous symmetry breaking has its origin in condensed matter physics.

3. Note that when we say cosmic strings we refer to local one-dimensional topological defects.
agreement with the SM phenomenology and especially with the hypercharges of the known particles. The large gauge group \( G_{\text{GUT}} \) must include a complex representation, needed to describe the SM fermions, and it must be anomaly free. A detailed investigation [2] has concluded that \( G_{\text{GUT}} \) could be either one of \( \text{SO}(10), \text{E}_6, \text{SO}(14), \text{SU}(8), \text{SU}(9) \); flipped \( \text{SU}(5) \) and \( [\text{SU}(3)]^3 \) are included within this list as subgroups of \( \text{SO}(10) \) and \( \text{E}_6 \), respectively. The formation of domain walls or monopoles, necessitates an era of supersymmetric hybrid inflation to dilute them. Considering GUTs based on simple gauge groups, the type of supersymmetric hybrid inflation will be of the F-type. The baryogenesis mechanism will be obtained via leptogenesis, either thermal or non-thermal. More precisely, for non-thermal leptogenesis, cosmic string formation is unavoidable, while for thermal leptogenesis, cosmic string formation arises in 98% of the acceptable SSB schemes. If the requirement of having \( \mathbb{Z}_2 \) unbroken down to low energies; the successful SSB schemes should end at \( G_{\text{SM}} \times \mathbb{Z}_2 \). Taking all these considerations into account, a detailed study of all SSB schemes leading from a \( G_{\text{GUT}} \) down to the \( G_{\text{SM}} \), by one or more intermediate steps, shows that cosmic strings are generically formed at the end of hybrid inflation.

The results [2] can be summarised as follows: If the large gauge group \( G_{\text{GUT}} \) is the \( \text{SO}(10) \), then cosmic strings formation is unavoidable. The genericity of string formation in the case that the large gauge group is the \( \text{E}_6 \), depends upon whether one considers thermal or non-thermal leptogenesis. More precisely, for non-thermal leptogenesis, cosmic string formation is unavoidable, while for thermal leptogenesis, cosmic string formation arises in 98% of the acceptable SSB schemes. If the requirement of having \( \mathbb{Z}_2 \) unbroken down to low energies is relaxed and thermal leptogenesis is considered as being the mechanism for baryogenesis, then cosmic string formation accompanies hybrid inflation in 80% of the SSB schemes. The SSB schemes of either \( \text{SU}(6) \) or \( \text{SU}(7) \), as the large gauge group, down to the \( G_{\text{SM}} \), which could accommodate an inflationary era with no defect (of any kind) at later times are inconsistent with proton lifetime measurements, while minimal \( \text{SU}(6) \) and \( \text{SU}(7) \) do not predict neutrino masses, implying that these models are incompatible with high energy physics phenomenology. Higher rank groups, namely \( \text{SO}(14), \text{SU}(8) \) and \( \text{SU}(9) \), should in general lead to cosmic string formation at the end of hybrid inflation. In all these schemes, cosmic string formation is sometimes accompanied by the formation of embedded strings. The strings which form at the end of hybrid inflation have a mass which is proportional to the inflationary scale.

3. Cosmic Strings and Inflation

An appealing solution to the drawbacks of the standard hot big bang model is to introduce, during the very early stages of the evolution of the Universe, a period of accelerated expansion, known as cosmological inflation [7]. The inflationary era took place when the Universe was in an unstable vacuum-like state at a high energy density, leading to a quasi-exponential expansion. The combination of the hot big bang model and the inflationary scenario provides at present the most comprehensive picture of the Universe at our disposal. Inflation ends when the Hubble parameter

\[
H = \sqrt{8 \pi \rho / (3 M_{\text{Pl}}^2)}
\]

(where \( \rho \) denotes the energy density and \( M_{\text{Pl}} \) stands for the Planck mass) starts decreasing rapidly. The energy stored in the vacuum-like state gets transformed into thermal energy, heating up the Universe and leading to the beginning of the standard hot big bang radiation-dominated era.

Inflation is based on the basic principles of general relativity and field theory, while when the principles of quantum mechanics are also considered, it provides a successful explanation for the origin of the large scale structure, associated with the measured temperature anisotropies in the CMB spectrum. Despite its remarkable success, inflation still remains a paradigm in search of model. An inflationary model should be inspired from a fundamental theory, while its predictions should be tested against current data. In addition, releasing the present Universe from its acute dependence on the initial data, inflation is faced with the challenging task of proving itself generic [5], in the sense that inflation would take place without fine-tuning of the initial conditions.

Theoretically motivated inflationary models
can be built in the context of supersymmetry or Supergravity (SUGRA). $N=1$ supersymmetry models contain complex scalar fields which often have flat directions in their potential, thus offering natural candidates for inflationary models. In this framework, hybrid inflation driven by F-terms or D-terms is the standard inflationary model, leading generically to Abrikosov-Nielsen-Olesen strings, called F-term strings.

The scalar potential, as a function of the scalar complex component of the respective chiral superfields $\Phi_\pm, S$, is

$$V(\phi_+ , \phi_-, S) = |F_{\Phi_+}|^2 + |F_{\Phi_-}|^2 + |F_S|^2$$

$$+ \frac{1}{2} \sum_a g_a^2 D_a^2 . \quad (2)$$

The F-term is such that $F_{\Phi_i} \equiv |\partial W/\partial \Phi_i|_{\theta=0}$, where we take the scalar component of the superfields once we differentiate with respect to $\Phi_i = \Phi_\pm, S$. The D-terms are $D_a = \partial_i (T_a)^i_j \phi^j + \xi_a$, with $a$ the label of the gauge group generators $T_a$, $g_a$ the gauge coupling, and $\xi_a$ the Fayet-Iliopoulos term. By definition, in the F-term inflation the real constant $\xi_a$ is zero; it can only be nonzero if $T_a$ generates an extra U(1) group. In the context of F-term hybrid inflation the F-terms give rise to the inflationary potential energy density while the D-terms are flat along the inflationary trajectory, thus one may neglect them during inflation.

The potential, has one valley of local minima, $V = \kappa^2 M^4$, for $S > M$ with $\phi_+ = \phi_- = 0$, and one global supersymmetric minimum, $V = 0$, at $S = 0$ and $\phi_+ = \phi_- = M$. Imposing initially $S \gg M$, the fields quickly settle down the valley of local minima. Since in the slow-roll inflationary valley the ground state of the scalar potential is non-zero, supersymmetry is broken. In the tree level, along the inflationary valley the potential is constant, therefore perfectly flat. A slope along the potential can be generated by including one-loop radiative corrections. Hence, the scalar potential gets a little tilt which helps the inflaton field $S$ to slowly roll down the valley of minima.

The one-loop radiative corrections to the scalar potential along the inflationary valley lead to the effective potential $V_{\text{eff}}^F (|S|)$

$$V_{\text{eff}}^F (|S|) = \kappa^2 M^4 \left\{ 1 + \frac{\kappa^2 \mathcal{N}}{32 \pi^2} \left[ 2 \ln |S|^2 \kappa^2 \right] \frac{A^2}{A^2} + (z + 1)^2 \ln (1 + z^{-1}) \right. $$

$$\left. + (z - 1)^2 \ln (1 - z^{-1}) \right\} . \quad (3)$$

with $z = |S|^2 / M^2$, and $\mathcal{N}$ stands for the dimensionality of the representation to which the complex scalar components $\phi_+, \phi_-$ of the chiral superfields $\Phi_+, \Phi_-$ belong. This implies that the
effective potential, Eq. (3), depends on the particular symmetry breaking scheme considered.

D-term inflation can be easily implemented within high energy physics (e.g., SUSY GUTs, SUGRA, or string theories) and it avoids the $\eta$-problem. Within D-term inflation, the gauge symmetry is spontaneously broken by introducing Fayet-Iliopoulos (FI) D-terms. In standard D-term inflation, the constant FI term gets compensated by a single complex scalar field at the end of the inflationary era, which implies that standard D-term inflation ends always with the formation of cosmic strings, called D-term strings. A supersymmetric description of the standard D-term inflation is insufficient, since the inflaton field reaches values of the order of the Planck mass, or above it, even if one concentrates only around the last 60 e-folds of inflation. Thus, D-term inflation has to be studied in the context of supergravity [4,5].

Standard D-term inflation requires a scheme

$$G_{\text{GUT}} \times U(1) \frac{M_{\text{GUT}}}{\Phi_+ \Phi_-} H \to G_{\text{SM}}.$$  

It is based on the superpotential

$$W = \lambda S \Phi_+ \Phi_-, \quad (4)$$

where $S, \Phi_+, \Phi_-$ are three chiral superfields and $\lambda$ is the superpotential coupling. It assumes an invariance under an Abelian gauge group $U(1)_{\xi}$, under which the superfields $S, \Phi_+, \Phi_-$ have charges 0, +1 and −1, respectively. It also assumes the existence of a constant FI term $\xi$. In D-term inflation the superpotential vanishes at the unstable de Sitter vacuum (anywhere else the superpotential is non-zero), implying that when the superpotential vanishes, D-term inflation must be studied within a non-singular formulation of supergravity. Various formulations of effective supergravity can be constructed from the superconformal field theory. To construct a formulation of supergravity with constant FI terms from superconformal theory, one finds [9] that under U(1) gauge transformations in the directions in which there are constant FI terms $\xi_\alpha$, the superpotential $W$ must transform as $\delta_\alpha W = \eta_\alpha \partial^\alpha W = -i(g\xi_\alpha/M_{Pl}^2)W$; one cannot keep any longer the same charge assignments as in standard supergravity.

D-term inflationary models can be built with different choices of the Kähler geometry. Various cases have been explored in the literature. The simplest case is that of D-term inflation within minimal supergravity [4]. It is based on

$$K_{\text{min}} = \sum_i |\Phi_i|^2 = |\Phi_-|^2 + |\Phi_+|^2 + |S|^2, \quad (5)$$

with $f_{ab}(\Phi_i) = \delta_{ab}$.

Another example is that of D-term inflation based on Kähler geometry with shift symmetry,

$$K_{\text{shift}} = \frac{1}{2}(S + \bar{S})^2 + |\phi_+|^2 + |\phi_-|^2, \quad (6)$$

and minimal structure for the kinetic function [5].

One can also consider consider [5] a Kähler potential with non-renormalisable terms:

$$K_{\text{non-renorm}} = |S|^2 + |\Phi_+|^2 + |\Phi_-|^2 + \frac{f_+}{M_{Pl}^2} |\Phi_+|^2 + \frac{f_-}{M_{Pl}^2} |\Phi_-|^2 + b |S|^4/M_{Pl}^4, \quad (7)$$

where $f_{\pm}$ are arbitrary functions of $(|S|^2/M_{Pl}^2)$ and the superpotential is given in Eq. (4).

Having the superpotential, one must proceed in the same way as in F-term inflation and write down the three level scalar potential and then include the one-loop radiative corrections.

Let me finally note that different approaches [10] have been proposed in order to avoid cosmic string formation in the context of D-term inflation. For example, one can add a non-renormalisable term in the potential, or add an additional discrete symmetry, or consider GUT models based on non-simple groups, or finally introduce a new pair of charged superfields so that cosmic string formation is avoided within D-term inflation.

4. Cosmic String Dynamics

The world history of a cosmic string can be expressed by a two-dimensional surface in the four-dimensional string world-sheet:

$$x^\mu = x^\mu(\zeta^a), \quad a = 0, 1; \quad (8)$$
the world-sheet coordinates \( \zeta^0, \zeta^1 \) are arbitrary parameters, \( \zeta^0 \) is time-like and \( \zeta^1 (\equiv \sigma) \) is space-like.

Over distances that are large compared to the width of the string, but small compared to the horizon size, solitonic cosmic strings can be considered as one-dimensional objects and their motion can be well-described by the Nambu-Goto action. Thus, the string equation of motion, in the limit of a zero thickness string, is derived from the Goto-Nambu effective action

\[
S_0[x^\mu] = -\mu \int \sqrt{-\gamma} d^2 \zeta , \tag{9}
\]

where \( \gamma = \det(\gamma_{ab}) \) with \( \gamma_{ab} = g_{\mu\nu} x^\mu_a x^\nu_b \) and \( \mu \) stands for the linear mass density, with \( \mu \sim T_c^2 \), where \( T_c \) is the critical temperature of the phase transition followed by SSB leading to cosmic string formation. By varying the action, Eq. (9), with respect to \( x^\mu(\zeta^a) \), and using \( d\gamma = \gamma_{\nu\sigma} d\gamma^{\nu\sigma} \), we get the string equation of motion:

\[
x^\mu_{;\alpha} + \Gamma^\mu_{\nu\sigma} \gamma^{\alpha\nu} x^\sigma_{;b} = 0 ; \tag{10}
\]

\( \Gamma^\mu_{\nu\sigma} \) is the four-dimensional Christoffel symbol.

We have neglected the friction \( \mu \), due to the scattering of thermal particles off the string. For strings formed at the grand unification scale, friction is important for a very short period of time. For strings formed at a later phase transition (e.g., closer to the electroweak scale), friction would dominate their dynamics through most of the thermal history of the Universe.

By varying the action with respect to the metric, the string energy-momentum tensor reads

\[
T^{\mu\nu} \sqrt{-g} = \mu \int d^2 \zeta \sqrt{-\gamma} a^2(x^\mu_{;\nu} x^\nu_{;\mu} - \delta^{\nu\sigma}(x^\sigma_{;\nu} x^\nu_{;\sigma} (\zeta^a))) . \tag{11}
\]

In an expanding Universe, the cosmic string equation of motion is most conveniently written in comoving coordinates, where the Friedmann-Lemaître-Robertson-Walker (FLRW) metric takes the form \( ds^2 = a^2(\tau)[d\tau^2 - dr^2] ; a(\tau) \) is the cosmic scale factor in terms of conformal time \( \tau \) (related to cosmological time \( t \), by \( dt = ad\tau \)).

Under the gauge condition \( \zeta^0 = \tau \), the comoving spatial string coordinates, \( x(\tau, \sigma) \), are written as a function of \( \tau \), and the length parameter \( \sigma \).

For a string moving in a FLRW Universe, the equation of motion, Eq. (10), can be simplified in the gauge for which the unphysical parallel components of the velocity vanish,

\[
\dot{x} \cdot x' = 0 ; \tag{12}
\]

overdots and primes denote derivatives with respect to \( \tau \) and \( \sigma \), respectively. In these coordinates, the Goto-Nambu action yields the following string equation of motion in a FLRW metric:

\[
\ddot{x} + 2(\frac{\dot{a}}{a}) \dot{x}(1 - \dot{x}^2) = \left( \frac{1}{\epsilon} \right) \left( \frac{x'}{\epsilon} \right)' . \tag{13}
\]

The string energy per unit \( \sigma \), in comoving units, is \( \epsilon \equiv \sqrt{x'^2/(1 - x^2)} \), implying that the string energy is \( \mu a \int d\sigma \). One usually fixes entirely the gauge by choosing \( \epsilon \) so that \( \epsilon = 1 \) initially.

The string equation of motion is much simpler in Minkowski space-time. Equation (10) for flat space-time simplifies to

\[
\partial_a(\sqrt{-\gamma} a_{ab} x'^b) = 0 . \tag{14}
\]

We impose the conformal gauge

\[
\dot{x} \cdot x' = 0 , \quad \dot{x}^2 + x'^2 = 0 ; \tag{15}
\]

overdots and primes denote derivatives with respect to \( \zeta^0 \) and \( \zeta^1 \), respectively. In this gauge the string equation of motion is just a two-dimensional wave equation:

\[
\ddot{x} - x'' = 0 . \tag{16}
\]

To fix entirely the gauge, we set \( t \equiv x^0 = \zeta^0 \), which allows us to write the string trajectory as the three-dimensional vector \( x(\sigma, t) \), where \( \zeta^1 = \sigma \), the space-like parameter along the string. Hence, the constraint equations, Eq. (14), and the string equation of motion, Eq. (15), become

\[
\dot{x} \cdot x' = 0 , \quad \dot{x}^2 + x'^2 = 1 , \quad \ddot{x} - x'' = 0 . \tag{17}
\]

The above equations imply that the string moves perpendicularly to itself with velocity \( \dot{x} \), that \( \sigma \) is proportional to the string energy, and that the string acceleration in the string rest frame is inversely proportional to the local string curvature radius. A curved string segment tends to straighten itself, resulting to string oscillations.
The general solution to the string equation of motion in flat space-time, Eq. (16), is

$$\mathbf{x} = \frac{1}{2}[\mathbf{a}(\sigma - t) + \mathbf{b}(\sigma + t)] ,$$

where \( \mathbf{a}(\sigma - t) \) and \( \mathbf{b}(\sigma + t) \) are two continuous arbitrary functions which satisfy

$$\mathbf{a}^2 = \mathbf{b}^2 = 1 .$$

Hence, \( \sigma \) is the length parameter along the three-dimensional curves \( \mathbf{a}(\sigma) \), \( \mathbf{b}(\sigma) \).

The Goto-Nambu action describes to a good approximation cosmic string segments which are separated. However, it leaves unanswered the issue of what happens when strings cross; a study which necessitates full field theory. When two strings of the same type collide, they may either pass simply through one another, or they may reconnect (intercommute). A necessary, but not sufficient, condition for string reconnection is that the initial and final configurations be kinematically allowed in the infinitely thin string approximation. Numerical simulations (and analytical estimates) of type-II (and weakly type-I) strings in the Abelian Higgs model suggest that the probability that a pair of strings will reconnect, after they intersect, is close to unity. The results are based on lattice simulations of the corresponding classical field configurations in the Abelian Higgs model; the internal structure of strings is highly non-linear, and thus difficult to treat via analytical means. String-string and self-string intersections lead to the formation of new long strings and loops. String intercommutations produce discontinuities, kinks, in \( \mathbf{x} \) and \( \mathbf{x}' \) on the new string segments at the intersection point, composed of right- and left-moving pieces travelling along the string at the speed of light.

The first analytical studies of the evolution of a cosmic string network have shown the existence of scaling, in the sense that the string network can be characterised by a single length scale, roughly the persistence length or the inter-string distance \( \xi \) which grows with the horizon. This important property of cosmic strings renders them cosmologically acceptable, in contrast to local monopoles or domain walls. Early numerical simulations have shown that the typical curvature radius of long strings and the characteristic distance between the strings are both comparable to the evolution time \( t \). The energy density of super-horizon strings in the scaling regime is given (in the radiation-dominated era) by \( \rho_{\text{ong}} = \kappa \mu t^{-2} \), where \( \kappa \) is a numerical coefficient \( (\kappa = 20 \pm 10) \). Assuming that the super-horizon strings are characterised by a single length scale \( \xi(t) \), implies

$$\xi(t) = \kappa^{-1/2} t .$$

The typical distance between the nearest string segments and the typical curvature radius of the strings are both of the order of \( \xi \). These results agree with the picture of the scale-invariant evolution of the string network and with the one-scale hypothesis. Further numerical investigations however revealed dynamical processes, such as the production of small sub-horizon loops, at scales much smaller than \( \xi \) \[14\]. In response to these findings, a three-scale model was developed \[15\] which describes the network in terms of three scales: the energy density scale \( \xi \), a correlation length \( \xi \) along the string, and a scale \( \zeta \) related to local structure on the string. The small-scale structure (wiggliness), which offers an explanation for the formation of the small sub-horizon sized loops, is basically developed through intersections of long string segments. Aspects of the three-scale model have been checked \[16\] evolving a cosmic string network in Minkowski space-time.

The sub-horizon strings (loops), their size distribution, and the mechanism of their formation remained for years the least understood parts of the string evolution. Recently, numerical simulations of cosmic string evolution in a FRW Universe (see, Fig. 1), found evidence of a scaling regime for the cosmic string loops in the radiation- and matter-dominated eras down to the hundredth of the horizon time. The scaling was found without considering any gravitational back reaction effect; it was just the result...
of string intercommutations. The scaling regime of string loops appears after a transient relaxation era, driven by a transient overproduction of string loops with length close to the initial correlation length of the string network. Subsequently, numerical [18] and analytical [19] studies supported the results of Ref. [17].

Let me note that there are two approaches of developing numerical simulations of cosmic string evolution. Either cosmic strings are modelled as idealised one-dimensional objects, or field theoretic calculations have been considered. In particular, for the field theoretic approach, the simplest example of an underlying field theory containing local U(1) strings, namely the Abelian Higgs model, has been recently employed [20].

5. String Thermodynamics

It is well-known in string theory, that the degeneracy of string states increases exponentially with energy, namely
\[ d(E) \sim e^{\beta_H E} \, . \] (20)

Hence, there is a maximum temperature \( T_{\text{max}} = 1/\beta_H \), the Hagedorn temperature [21]. Let us consider, in the microcanonical ensemble, a system of closed string loops in a three-dimensional box. Intersecting strings intercommute, but otherwise they do not interact and are described by the Goto-Nambu equation of motion. The statistical properties of a system of strings in equilibrium are characterised by only one parameter, the energy density of strings, \( \rho \), defined as \( \rho = E/L^3 \), with \( L \) the size of the cubical box. The behaviour of the system depends on whether it is at low or high energy densities, and it undergoes a phase transition at a critical energy density, the Hagedorn energy density \( \rho_H \). Quantisation implies a lower cutoff for the size of the string loops, determined by the string tension \( \mu \). The lower cutoff on the loop size is roughly \( \mu^{-1/2} \), implying that the mass of the smallest string loops is \( m_0 \sim \mu^{1/2} \).

For a system of strings at the low energy density regime (\( \rho \ll \rho_H \)), all strings are chopped down to the loops of the smallest size, while larger loops are exponentially suppressed. Thus, for small enough energy densities, the string equilibrium configuration is dominated by the massless modes in the quantum description. The energy distribution of loops, given by the number \( dn \) of loops with energies between \( E \) and \( E + dE \) per unit volume, is [21][22]
\[ dn \propto e^{-\alpha E} E^{-5/2} dE \quad (\rho \ll \rho_H) \, , \] (21)
where \( \alpha = (5/2m_0) \ln(\rho_H/\rho) \).

However, as the energy density increases, more and more oscillatory modes of strings get excited. In particular, once a critical energy density, \( \rho_H \), is reached, long oscillatory string states begin to appear in the equilibrium state. The density at which this happens corresponds to the Hagedorn temperature. The Hagedorn energy density — achieved when the separation between the smallest string loops is of the order of their sizes — is approximately \( \mu^2 \), and then the system undergoes a phase transition characterised by the appearance of super-horizon (infinitely long) strings.

At the high energy density regime, the energy distribution of string loops is [21][22]
\[ dn = A m_0^{9/2} E^{-5/2} dE \quad (\rho \gg \rho_H) \, , \] (22)
where \( A \) is a numerical coefficient independent of \( m_0 \) and of \( \rho \). Equation (22) implies that the
mean-square radius $R$ of the sub-horizon loops is
\[ R \sim m_0^{-3/2} E^{1/2} . \]  
(23)
Hence the large string loops are random walks of step approximately $m_0^{-1}$. Equations (22) and (23) imply
\[ dn = A'R^{-4}dR \quad (\rho \gg \rho_H) , \]  
where $A'$ is a numerical constant. Thus, at the high energy density regime, the distribution of closed string loops is scale invariant, since it does not depend on the cutoff parameter. The total energy density in sub-horizon string loops is independent of $\rho$. Increasing the energy density $\rho$ of the system of strings, the extra energy $E - E_H$, where $E_H = \rho_H L^3$, goes into the formation of super-horizon long strings, implying
\[ \rho - \rho_1 = \text{const} \quad (\rho \gg \rho_H) , \]  
where $\rho_1$ denotes the energy density in super-horizon loops (often called in the literature as infinite strings).

Clearly, the above analysis describes the behaviour of a system of strings of low or high energy densities, while there is no analytic description of the phase transition and of the intermediate densities around the critical one, $\rho \sim \rho_H$. An experimental approach to the problem has been proposed in Ref. [23] and later extended in Ref. [24].

The equilibrium properties of a system of cosmic strings have been studied numerically in Ref. [24]. The strings are moving in a three-dimensional flat space and the initial string states are chosen to be a loop gas consisting of the smallest two-point loops with randomly assigned positions and velocities. This choice is made just because it offers an easily adjustable string energy density. Clearly, the equilibrium state is independent of the initial state. The simulations revealed a distinct change of behaviour at a critical energy density $\rho_H = 0.0172 \pm 0.002$. For $\rho < \rho_H$, there are no super-horizon strings, their energy density, $\rho_1$, vanishes. For $\rho > \rho_H$, the energy density in sub-horizon string loops is constant, equal to $\rho_H$, while the extra energy goes to the super-horizon string loops with energy density $\rho_1 = \rho - \rho_H$. Thus, Eqs. (24) and (25) are valid for all $\rho > \rho_H$, although they were derived only in the limit $\rho \gg \rho_H$. At the critical energy density, $\rho = \rho_H$, the system of strings is scale-invariant. At higher energy densities, $\rho > \rho_H$, the energy distribution of sub-horizon string loops at different values of $\rho$ were found to be identical within statistical errors, and well-defined by a line $dn/dE \propto E^{-5/2}$. Thus, for $\rho > \rho_H$, the distribution of sub-horizon string loops is still scale-invariant, but in addition the system includes super-horizon string loops, which do not exhibit a scale-invariant distribution. The number distribution for super-horizon loops goes as $dn/dE \propto 1/E$, which means that the total number of super-horizon string loops is roughly $\log(E - E_H)$. So, typically the number of long strings grows very slowly with energy; for $\rho > \rho_H$ there are just a few super-horizon strings, which take up most of the energy of the system.

The above numerical experiment has been extended [24] for strings moving in a higher dimensional box. The Hagedorn energy density was found for strings moving in boxes of dimensionality $d_B = 3, 4, 5$ [24]:

\[ \rho_H = \begin{cases} 
0.172 \pm 0.002 & \text{for } d_B = 3 \\
0.062 \pm 0.001 & \text{for } d_B = 4 \\
0.031 \pm 0.001 & \text{for } d_B = 5 
\end{cases} \]  
(26)

Moreover, the size distribution of sub-horizon string loops at the high energy density regime was found to be independent of the particular value of $\rho$ for a given dimensionality of the box $d_B$. The size distribution of sub-horizon string loops was found [24] to be well defined by a line

\[ \frac{dn}{dE} \sim E^{-(1 + d_B/2)} , \]  
(27)
where the space dimensionality $d_B$ was taken equal to 3, 4, or 5. The statistical errors indicated a slope equal to $-(1 + d_B/2) \pm 0.2$. Above the Hagedorn energy density the system is again characterised by a scale-invariant distribution of sub-horizon string loops and a number of super-horizon string loops with a distribution which is not scale invariant.
6. Cosmic String Gravity

The gravitational properties of cosmic strings are very different than those of non-relativistic linear mass distributions. Straight cosmic strings produce no gravitational force in the surrounding matter: $\nabla^2 \Phi = 0$, where $\Phi$ stands for the Newtonian potential. Cosmic strings have relativistic motion, implying that oscillating string loops can be strong emitters of gravitational radiation. Since super-horizon cosmic strings have wiggles and small-scale structure due to string intercommutations, they are also sources of gravitational radiation [25]. A gravitating string is described by a coupled system of Einstein, Higgs and gauge field equations, for which no exact solution is known. We thus usually make two simplifications: we consider the cosmic string thickness to be much smaller than any other relevant dimension and the cosmic string gravitational field to be sufficiently weak, so that linearised Einstein equations can be used (for $G\mu \ll 1$).

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Hence, the effect of a cosmic string is to introduce an azimuthal deficit angle $\Delta$, whose magnitude is determined by the symmetry breaking scale $T_c$ leading to the cosmic string formation, namely $\Delta = 8\pi G\mu$. Thus, the string metric $ds^2 = dt^2 - dz^2 - dr^2 + (1 - 8\pi G\mu) r^2 d\theta^2$ describes a conical space leading to interesting observational effects on the propagation of light (i.e., double images of light sources located behind cosmic strings) and of particles (i.e., discontinuous Doppler shift effects). The centre-of-mass velocity $u$ of two particles, moving towards a string at the same velocity $v$ reads

$$u = v \sin(\Delta/2) [1 - v^2\cos^2(\Delta/2)]^{-1/2}. \quad (28)$$

Considering that one of the particles carries a light source, while the other one is an observer, one realises that the observer will detect a discontinuous change in the frequency $\omega$ of light, given by [26]

$$\frac{\delta \omega}{\omega} = \frac{v}{\sqrt{1 - v^2}} \Delta. \quad (29)$$

This discontinuous change in the frequency has its origin in the Doppler shift: particles start moving towards each other, decreasing their distance, once the line connecting the particles crosses the string.

In the framework of gravitational instability, topological defects in general and cosmic strings in particular, offered an alternative to the inflationary paradigm for the origin of the initial fluctuations leading to the observed large-scale structure and the measured anisotropies of the CMB temperature anisotropies. The angular power spectrum of CMB is expressed in terms of the dimensionless coefficients $C_\ell$, in the expansion of the angular correlation function in terms of the Legendre polynomials $P_\ell$ reads [6]

$$\langle 0 \left| \frac{\delta T}{T}(n) \frac{\delta T}{T}(n') \right| 0 \rangle \big| \theta = \cos \vartheta = \cos \vartheta \rangle = \frac{1}{4\pi} \sum_\ell (2\ell + 1) C_\ell P_\ell(\cos \vartheta) W_\ell^2, \quad (30)$$

where $W_\ell$ stands for the $\ell$-dependent window function of the particular experiment. Equation (30) compares points in the sky separated by an angle $\vartheta$. The value of $C_\ell$ is determined by fluctuations on angular scales of the order of $\pi/\ell$. The angular power spectrum of anisotropies observed today is usually given by the power per logarithmic interval in $\ell$, plotting $\ell(\ell+1)C_\ell$ versus $\ell$.

To find the power spectrum induced by topological defects, one has to solve in Fourier space, for each given wave vector $k$, a system of linear perturbation equations with random sources:

$$\mathcal{D}X = \mathcal{S}, \quad (31)$$

where $\mathcal{D}$ denotes a time dependent linear differential operator, $X$ is a vector which contains the various matter perturbation variables, and $\mathcal{S}$ is the random source term, consisting of linear combinations of the energy momentum tensor of the defect. For given initial conditions, Eq. (31) can be solved by means of a Green’s function, $G(\tau, \tau')$. To compute power spectra or, more generally, quadratic expectation values of the form

\[\langle \delta T(\theta_1) \delta T(\theta_2) \rangle.\]
\[ \langle X_j(\tau_0, \mathbf{k})X_m^*(\tau_0, \mathbf{k}') \rangle, \] one has to calculate
\[ \langle X_j(\tau_0, \mathbf{k})X_m^*(\tau_0, \mathbf{k}') \rangle = \int_{\tau_{in}}^{\tau_{out}} d\tau \mathcal{G}_{jm}(\tau, \mathbf{k}) \]
\[ \times \int_{\tau_{in}}^{\tau_{out}} d\tau' \mathcal{G}_{lm}^*(\tau', \mathbf{k}') \langle S_m(\tau, \mathbf{k})S_n^*(\tau', \mathbf{k}') \rangle. \] (32)

To compute power spectra, one should know the unequal time two-point correlators \( \langle S_m(\tau, \mathbf{k})S_n^*(\tau', \mathbf{k}') \rangle \) in Fourier space, calculated by means of heavy numerical simulations.

The first determinations of the CMB power spectrum from cosmic strings were based on the assumption that cosmic strings are of infinitesimal width. They were thus realized by either employing Nambu-Goto simulations of connected string segments, or a model involving a stochastic ensemble of unconnected segments. However, a number of questions there have been later raised regarding the accuracy of these approaches. More recently, the CMB power spectrum contribution from cosmic strings has been addressed using field-theoretic simulations of the Abelian Higgs model, the simplest example of an underlying field theory with local U(1) strings. All approaches agree on the basic form of the cosmic string power spectrum, namely a power spectrum with a roughly constant slope at low multipoles, rising up to a single peak, and consequently decaying at small scales. This result is common of all models, in which fluctuations are generated continuously and evolve according to inhomogeneous linear perturbation equations.

In topological defects models, fluctuations are constantly generated by the non-linear defect evolution. This characteristic, combined with the fact that the random initial conditions of the source term of a given scale leak into other scales, destroy perfect coherence. The incoherent aspect of active perturbations affects the structure of secondary oscillations, namely secondary oscillations may get washed out. Thus, in topological defects models, incoherent fluctuations lead to a single bump at smaller angular scales (larger \( \ell \)), than those predicted within any inflationary scenario.

The cosmic string CMB power spectrum was found to have a broad peak at \( \ell \approx 150 - 400 \). Decomposing the power spectrum into scalar, vector and tensor modes, it was shown that the origin of this broad peak lies in both the vector and scalar modes, which peak at \( \ell \approx 180 \) and \( \ell \approx 400 \), respectively. This analysis concluded that the cosmic string power spectrum is dominated by vector modes for all but the smallest scales.

The position and amplitude of the acoustic peaks, as found by the CMB measurements, are in disagreement with the predictions of topological defect models. As a consequence, CMB measurements rule out pure topological defect models in general, and cosmic strings in particular, as the origin of initial density perturbations leading to the observed structure formation.

Since cosmic strings are expected to be generally formed in the context of SUSY GUTs, one should consider mixed perturbation models where the dominant rôle is played by the inflaton field but cosmic strings have also a contribution, small but not negligible. Restricting ourselves to the angular power spectrum, we can remain in the linear regime. In this case,
\[ C_\ell = \alpha C_\ell^I + (1 - \alpha)C_\ell^S, \] (33)
where \( C_\ell^I \) and \( C_\ell^S \) denote the (COBE normalised) Legendre coefficients due to adiabatic inflaton fluctuations and those stemming from the cosmic string network, respectively. The coefficient \( \alpha \) in Eq. (33) is a free parameter giving the relative amplitude for the two contributions. Comparing the \( C_\ell \), calculated using Eq. (33) – where \( C_\ell^I \) is taken from a generic inflationary model and \( C_\ell^S \) from numerical simulations of cosmic string networks – with data obtained from the most recent CMB measurements, one gets that a cosmic string contribution to the primordial fluctuations higher than 14% is excluded up to 95% confidence level.

Let us now return to F- and D-term hybrid inflation and investigate the constraints on the free parameters of the model (namely masses and couplings) so that the cosmic string contribution to the CMB data is within the allowed limits imposed from recent CMB measurements. Considering only large angular scales one can get the contributions to the CMB temperature anisotropies analytically. The quadrupole
anisotropy has one contribution coming from the inflaton field, and one contribution coming from the cosmic string network. Fixing the number of e-foldings to 60, the inflaton and cosmic string contribution to the CMB depend on the parameters of the model. For F-term inflation the cosmic string contribution to the CMB data is consistent with CMB measurements provided

\begin{equation}
M \lesssim 2 \times 10^{15}\text{GeV} \iff \kappa \lesssim 7 \times 10^{-7}.
\end{equation}

(34)

The superpotential coupling \(\kappa\) is also subject to the gravitino constraint which imposes an upper limit to the reheating temperature, to avoid gravitino overproduction. Within the framework of SUSY GUTs and assuming a see-saw mechanism to give rise to massive neutrinos, the inflaton field decays during reheating into pairs of right-handed neutrinos. This constraint on the reheating temperature can be converted to a constraint on the parameter \(\kappa\). The gravitino constraint on \(\kappa\) reads [4] \(\kappa \lesssim 8 \times 10^{-3}\), which is rather weaker.

The tuning of \(\kappa\) can be softened if one allows for the curvaton mechanism. The curvaton is a scalar field that is sub-dominant during the inflationary era as well as at the beginning of the radiation dominated era following inflation. In the context of supersymmetric theories such scalar fields are expected to exist, and in addition, if embedded strings accompany the formation of cosmic strings, they may offer a natural curvaton candidate, provided the decay product of embedded strings gives rise to a scalar field before the onset of inflation. Assuming the existence of a curvaton field there is an additional contribution to the temperature anisotropies and the CMB measurements impose [4] the following limit on the initial value of the curvaton field

\[
\psi_{\text{init}} \lesssim 5 \times 10^{13} \left(\frac{\kappa}{10^{-2}}\right)\text{GeV} \quad \text{for} \quad \kappa \in [10^{-6}, 1].
\]

Figure 2. For D-term inflation in minimal SUGRA, cosmic string contribution to CMB quadrupole anisotropies as a function of the superpotential coupling constant \(\lambda\), for various values of the gauge coupling \(g\). Figure taken from Ref. [4].

D-term inflation can also be compatible with CMB measurements, provide we tune its free parameters. In the case of minimal SUGRA, consistency between CMB measurements and theoretical predictions impose [4] that \(g \lesssim 2 \times 10^{-2}\) and \(\lambda \lesssim 3 \times 10^{-5}\), which can be expressed as a single constraint on the Fayet-Iliopoulos term \(\xi\), namely \(\sqrt{\xi} \lesssim 2 \times 10^{15}\) GeV. The results are illustrated in Fig.2. The fine tuning on the couplings can be softened if one invokes the curvaton mechanism and constrains the initial value of the curvaton field to be [5]

\[
\psi_{\text{init}} \lesssim 3 \times 10^{14} \left(\frac{g}{10^{-2}}\right)\text{GeV} \quad \text{for} \quad \lambda \in [10^{-1}, 10^{-4}].
\]

For D-term inflation based on Kähler geometry with shift symmetry, the cosmic string contribution to the CMB anisotropies is dominant, in contradiction with the CMB measurements, unless the superpotential coupling is [5] \(\lambda \lesssim 3 \times 10^{-5}\). Finally, in the case of D-term inflation based on a Kähler potential with non-renormalisable terms, the contribution of cosmic strings dominates if the superpotential coupling \(\lambda\) is close to unity. The constraints on \(\lambda\) read [5]

\[
(0.1 - 5) \times 10^{-8} \leq \lambda \leq (2 - 5) \times 10^{-5}
\]

or, equivalently

\[
\sqrt{\xi} \leq 2 \times 10^{15}\text{GeV},
\]

implying \(G\mu \leq 8.4 \times 10^{-7}\). Thus, higher order Kähler potentials do not suppress cosmic string contribution.
Apart the temperature power spectrum, important constraints on cosmic string scenarios might also arise in the future from measurements of the polarisation of the CMB photons. More precisely, the B-polarisation spectrum offers an interesting window on cosmic strings since inflation has only a weak contribution. Scalar modes may contribute to the B-mode only via the gravitational lensing of the E-mode signal, with a second inflationary contribution coming from the subdominant tensor modes.

Cosmic strings can also become apparent through their contribution in the small-angle CMB temperature anisotropies. More precisely, at high multipoles \( \ell \) (small angular resolution), the mean angular power spectrum of string-induced CMB temperature anisotropies can be described \([31]\) by \( \ell^{-\alpha} \), with \( \alpha \sim 0.889 \). Thus, a non-vanishing cosmic string contribution to the overall CMB temperature anisotropies may dominate at high multipoles \( \ell \) (small angular scales). In an arc-minute resolution experiment, strings may be observable \([31]\) for \( G\mu \) down to \( 2 \times 10^{-7} \).

Cosmic strings should also induce deviations from Gaussianity. On large angular scales such deviations are washed out due to the low string contribution, however on small angular scales, optimal non-Gaussian string-devoted statistical estimators may impose severe constraints on a possible cosmic string contribution to the CMB temperature anisotropies.

Finally, let me emphasise that one should keep in mind that all string-induced CMB temperature anisotropies were performed for Abelian strings in the zero thickness limit with reconnection probability equal to unity and winding number equal to one. Even though in any model where fluctuations are constantly induced by sources (seeds) having a non-linear evolution, the perfect coherence which characterises the inflationary induced spectrum of perturbations gets destroyed \([32]\), there is still no reason to expect that quantitatively the results found for conventional cosmic string models will hold in more general cases.

### 7. Superconducting Cosmic Strings

Before finishing this brief review on cosmic strings, let me mention the case of superconducting strings \([33]\), in the sense that in a large class of high energy physics theories, strings have similar electromagnetic properties as those of superconducting wires. Such objects carry large electric currents and hence their interaction with the cosmic plasma can lead to a variety of distinct astrophysical effects.

Cosmic strings are characterised as superconductors if electromagnetic gauge invariance is broken inside the strings, a situation which can occur for instance when a charged scalar field develops a non-zero expectation value in the vicinity of the string core. Superconducting strings appear also in models with fermions, which acquire masses through a Yukawa coupling to the Higgs field of the strings. Thus, depending on the considered model we can have bosonic or fermionic string superconductivity. In the first case, bosons can condensate and acquire a non-vanishing phase gradient, while in the second one, fermions may propagate in the form of zero modes along the string.

Applying an electric field on a superconducting string, the string will develop a growing electric current according to

\[
\frac{dJ}{dt} \sim \frac{ce^2}{\hbar} E, \tag{35}
\]

where \( E \) stands for the field component along the string and \( e \) denotes the elementary charge.

In the case of fermionic superconductivity, fermions are massless inside the string, whereas they have a finite mass, \( m \), outside the string. Under the effect of an electric field, a current \( J \) results, growing in time until it reaches a critical value \( J_c \sim emc^2/\hbar \), when the particles inside the string, moving at relativistic speeds, have sufficient energy to leave the string. Thus, when the string current reaches its critical value, \( J_c \), parti-

---

7 Note that also a vector field, whose flux is trapped inside a non-Abelian string can lead to a superconducting string.

8 The fermion mass is model-dependent, but it is bounded from above by the symmetry breaking scale of the string.
cles get produced at a rate
\[ \dot{n} \sim eE/\hbar , \]  
where \( n \) stands for the number of fermions per unit length. Note that \( J_c \), even though model-dependent, does not exceed a maximum value, given by \( J_{\text{max}} \sim e(\mu c^3/\hbar)^{1/2} \). Depending on their energy scale, superconducting strings may carry huge currents. In the case of bosonic superconductivity, the model-dependent critical current \( J_c \) — determined by the energy scale at which scale invariance is broken — is again bounded by \( J_{\text{max}} \), defined as above.

Superconducting strings can also develop growing currents in magnetic fields, according to
\[ \frac{dJ}{dt} \sim \frac{e^2}{\hbar} vB , \]
where \( v \) is the speed of the moving string segment in a magnetic field \( B \). For a string loop carrying sufficiently large currents, electromagnetic radiation can overtake gravitational radiation, becoming the dominant energy loss mechanism.

Superconducting string loops may be problematic in cosmology. For a current-carrying loop, the energy per unit length is not equal to the tension, their difference equals the string current. Such a loop can rotate and the resulting centrifugal force — which can be expected to be very much stronger than the inefficient magnetic spring repulsion effect — may balance the tension. When a rotating string loop reaches an equilibrium state — defined by the balance between the string tension and the centrifugal force — it is called a vorton \[34\]. Vortons can be formed at, or soon after, the phase transition followed by SSB leading to the string formation, and they possess a net charge as well as a current. If vortons are stable (certainly a model-dependent issue \[35\]), they will scale as matter in the Universe, dominating over its energy density. In this sense, vortons may constrain models for superconducting strings.

8. Cosmic Superstrings

The recent interplay between superstring theory and cosmology has led to the notion of cosmic superstrings \[3\], providing the missing link between superstrings and their classical analogues.

The possible astrophysical role of superstrings has been advocated already more than twenty years ago. More precisely, it has been proposed \[36\], that superstrings of the O(32) and \( E_8 \times E_8 \) string theories are likely to generate string-like stable vortex lines and flux tubes. However, in the context of perturbative string theory, the high tension (close to the Planck scale) of fundamental strings ruled them out \[36\] as potential cosmic string candidates. Luckily, this picture has changed in the framework of brane-world cosmology, which offers an elegant realisation of nature within string theory. Within the brane-world picture, all standard model particles are open string modes. Each end of an open string lies on a brane, implying that all standard model particles are stuck on a stack of \( D_p \)-branes, while the remaining \( p - 3 \) of the dimensions are wrapping some cycles in the bulk. Closed string modes (e.g., dilaton, graviton) live in the high-dimensional bulk. Brane interactions lead to unwinding and thus evaporation of higher dimensional \( D_p \)-branes. We are eventually left with \( D_3 \)-branes — one of which could indeed play the role of our Universe \[37\] — embedded in a \((9+1)\)-dimensional bulk and cosmic superstrings (one-dimensional \( D \)-branes, called \( D \)-strings, and fundamental strings, called \( F \)-strings).

Brane annihilations provide a natural mechanism for ending inflation. To illustrate the formation of cosmic superstrings at the end of brane inflation, let us consider a \( D_p \)-\( \bar{D}_p \) brane-anti-brane pair annihilation to form a \( D_1 \) brane. Each parent brane has a \( U(1) \) gauge symmetry and the gauge group of the pair is \( U(1) \times U(1) \). The daughter brane possesses a \( U(1) \) gauge group, which is a linear combination, \( U(1)_- \), of the original two \( U(1) \)'s. The branes move towards each other and as their inter-brane separation decreases below a critical value, the tachyon field, which is an open string mode stretched between the two branes, develops an instability. The tachyon couples to the combination \( U(1)_- \). The rolling of the tachyon field leads to the decay of the parent branes. Tachyon rolling leads to spontaneously symmetry breaking, which supports de-
fects with even co-dimension. So, brane annihilation leads to vortices, D-strings; they are cosmologically produced via the Kibble mechanism. The other linear combination, U(1)$_{+}$, disappears since only one brane remains after the brane collision. The U(1)$_{+}$ combination is thought to disappear by having its fluxes confined by fundamental closed strings. Such strings are of cosmological size and they could play the rôle of cosmic strings [38]; they are referred to in the literature as cosmic superstrings [39].

9. Differences between Cosmic Strings and Cosmic Superstrings

Cosmic superstrings [3], even though cosmologically extended, are quantum objects, in contrast to solitonic cosmic strings which are classical objects. Hence, one expects a number of differences to arise as regarding the properties of the two classes of objects. As I have earlier discussed, the probability that a pair of cosmic strings will reconnect, after having intersect, equals unity. The reconnection probability for cosmic superstrings is however smaller (often much smaller) than unity. The corresponding intercommutation probabilities are calculated in string perturbation theory. The result depends on the type of strings and on the details of compactification. For fundamental strings, reconnection is a quantum process and takes place with a probability of order $g_s^2$ (where $g_s$ denotes the string tension). It can thus be much less than one, leading to an increased density of strings [40], implying an enhancement of various observational signatures. The reconnection probability is a function of the relative angle and velocity during the collision. One may think that strings can miss each other, as a result of their motion in the compact space. Depending on the supersymmetric compactification, strings can wander over the compact dimensions, thus missing each other, effectively decreasing their reconnection probability. However, in realistic compactification schemes, strings are always confined by a potential in the compact dimensions [41]. The value of $g_s$ and the scale of the confining potential will determine the reconnection probability. Even though these are not known, for a large number of models it was found [41] that the reconnection probability for F-F collisions lies in the range between $10^{-3}$ and 1. The case of D-D collisions is more complicated; for the same models the reconnection probability is anything between 0.1 to 1. Finally, the reconnection probability for F-D collisions can vary from 0 to 1.

Brane collisions lead not only to the formation of F- and D-strings, they also produce bound states, $(p,q)$-strings, which are composites of $p$ F-strings and $q$ D-strings [42]. The presence of stable bound states implies the existence of junctions, where two different types of string meet at a point and form a bound state leading away from that point. Thus, when cosmic superstrings of different types collide, they can not intercommute, instead they exchange partners and form a junction at which three string segments meet. This is just a consequence of charge conservation at the junction of colliding $(p,q)$-strings. For $p = np'$ and $q = nq'$, the $(p,q)$ string is neutrally stable to splitting into $n$ bound $(p',q')$ strings. The angles at which strings pointing into a vertex meet, is fixed by the requirement that there be no force on the vertex. In general, a $(p,q)$ and a $(p',q')$ string will form a trilinear vertex with a $(p + p', q + q')$ or a $(p - p', q - q')$ string. This leads certainly to the crucial question of whether a cosmic superstring network will reach scaling, or whether it freezes leading to predictions inconsistent with our observed Universe.

The tension of solitonic strings is set from the energy scale of the phase transition, followed by a spontaneously broken symmetry, which left behind these defects as false vacuum remnants. Cosmic superstrings however span a whole range of tensions, set from the particular brane inflation model. The tension of F-strings in 10 dimensions is $\mu_F = 1/(2\pi\alpha')$, and the tension of D-strings is $\mu_D = 1/(2\pi\alpha' g_s)$, where $g_s$ stands for the string coupling. In 10 flat dimensions, supersymmetry dictates that the tension of the $(p,q)$ bound states reads

$$\mu_{(p,q)} = \mu_F \sqrt{p^2 + q^2 / g_s^2}. \quad (38)$$

Individually, the F- and D-strings are $\frac{1}{2}$ BPS (Bogomol’nyi-Prasad-Sommerfield) objects, which however break a different half of the su-
persymmetry each. Equation (38) represents the BPS bound for an object carrying the charges of $p$ F-strings and $q$ D-strings. Note that the BPS bound is saturated by the F-strings, $(p, q) = (1, 0)$, and the D-strings, $(p, q) = (0, 1)$. Let me also make the remark that the string tension for strings at the bottom of a throat is different from the (simple) expression given in Eq. (38) and it depends on the choice of flux compactification.

Consider an F- and a D-string, both lying along the same axis. The total tension of this configuration is $(g_n^{-1} + 1)/(2\pi \alpha')$, which exceeds the BPS bound; thus the configuration is not supersymmetric. It can however lower its energy, if the F-string breaks, its ends being attached to the D-string. Since the end points can then move off at infinity, leaving only the D-string behind, a flux will run between the end points of the F-string. Now the tension reads $(g_n^{-1} + O(g_n))/2\pi \alpha'$, and hence the final state represents a D-string with a flux, which is a supersymmetric state.

10. Cosmic Superstring Evolution

The evolution of cosmic superstring networks, is a complicated issue, which has been addressed by numerical [40,43,44,45], as well as analytical [46] approaches.

The first numerical attempt [40], studying independent stochastic networks of D- and F-strings in a flat space-time, has shown that the characteristic length scale $\xi$, giving the typical distance between the nearest string segments and the typical curvature of strings, grows linearly with time

$$\xi(t) \propto \zeta t ;$$

the slope $\zeta$ depends on the reconnection probability $\mathcal{P}$, and on the energy of the smallest allowed loops (i.e., the energy cutoff). For reconnection (or intercommuting) probability in the range $10^{-3} \lesssim \mathcal{P} \lesssim 0.3$, it was shown [40] that

$$\zeta \propto \sqrt{\mathcal{P}} \Rightarrow \xi(t) \propto \sqrt{\mathcal{P}} t .$$

One can find in the literature (e.g., Ref. [38c]) statements claiming that $\xi(t)$ should be instead proportional to $\mathcal{P}t$. If this were correct, then the energy density of cosmic superstrings, of given tension, could be considerably higher than that of their field theory analogues. However, the authors of Ref. [38] have missed out in their analysis that intersections between two long strings is not the most efficient mechanism for energy loss of the string network. The findings of Ref. [40] cleared the misconception about the behaviour of the scale $\xi$, and shown that the cosmic superstring energy density may be higher than in the field theory case, but at most only by one order of magnitude.$^9$

As I have already discussed, in a realistic case $(p, q)$ strings come in a large number of different types, while a $(p, q)$ string can decay to a loop only if it self-intersects or collide with another $(p, q)$ or $(-p, -q)$ string. A collision between $(p, q)$ and $(p', q')$ strings will lead to a new $(p \pm p', q \pm q')$ string, provided the end points of the initial two strings are not attached to other three-string vertices, thus they are not a part of a web. If the collision between two strings can lead to the formation of one new string, on a timescale much shorter than the typical collision timescale, then the creation of a web may be avoided, and the resulting network is composed by strings which are on the average non-intersecting. Then one can imagine the following configuration: A string network, composed by different types of $(p, q)$ strings undergoes collisions and self-intersections. Energy considerations imply the production of lighter daughter strings, leading eventually to one of the following strings: $(\pm 1, 0)$, $(0, \pm 1)$, $\pm (1, 1)$, $\pm (1, -1)$. These ones may then self-intersect, form loops and scale individually. Provided the relative contribution of each of these strings to the energy density of the Universe is small enough, the Universe will not be overclosed.

Let us now study the dynamics of a three-string junction in a simple model. The solutions of the BPS saturated formula

$$\mu_{(p,q)} = \sqrt{[p\mu_{(1,0)}]^2 + [q\mu_{(0,1)}]^2},$$

read

$$\mu_{(p,q)} \sin \alpha = q\mu_{(0,1)} ; \quad \mu_{(p,q)} \cos \alpha = p\mu_{(1,0)},$$

$^9$A discussion and explanation of this misconception can be found in Ref. [46c].
where \( \tan \alpha = q/(pg_b) \). The balance conditions for three strings imply that when an F-string ends on a D-string, it causes it to bend at an angle set by the string coupling; on the other side of the junction there is a (1,1) string. Consider a junction of three strings, with coordinates \( x(\sigma,t) \), tension \( \mu \) and parameter lengths \( L_1(t), L_2(t), L_3(t) \), which are joined at a junction and whose other end terminate on parallel branes. The action for this configuration reads\(^{[14]}\):

\[
S = -\sum_{\alpha=1}^{3} \mu_{\alpha} \int dt \int_{0}^{L_{\alpha}(t)} d\sigma \sqrt{-\gamma^{(\alpha)}} \\
+ \sum_{\alpha=1}^{3} \int dt \dot{l}_{\alpha} \cdot (x(t,L_{\alpha}(t)) - x_{\text{junc}}(t))
\]  

(43)

where the first part stands for the Nambu-Goto terms for the three strings, and \( L_{\alpha} \) denote the Lagrange multipliers to describe the junction, located at position \( x_{\text{junc}} \). From Eq. (43) one can derive the equations of motion as well as the energy conservation. One can easily check that

\[
\frac{\mu_{1}(1-L_{1})}{\mu_{1} + \mu_{2} + \mu_{3}} = \frac{M_{1}(1-c_{23})}{M_{1}(1-c_{23}) + M_{2}(1-c_{13}) + M_{3}(1-c_{12})},
\]

(44)

and cyclic permutations. Note that \( M_{1} = \mu_{1}^{2} - (\mu_{2} - \mu_{3})^{2} \) (and cyclic permutations giving \( M_{2}, M_{3} \)) and \( c_{ij} = a'_{i}(t - L_{i}(t)) \cdot a'_{j}(t - L_{j}(t)) \). Equation (44) implies that the rate of creation of new string must balance the disappearance of old one. Thus, for an F-string with \( \mu_{1} = 1 \), and a D-string with \( \mu_{2} = 1/g_{b} \), the FD-bound state has tension \( \mu_{3} = 1/(1+1/g_{b}^{2} = 1/g_{b} + g_{b}/2 + O(g_{b}^{2}) \). Since the angle \( \alpha \) goes to \( \pi/2 \) in the limit of zero string coupling, we conclude that in the small \( g_{b} \)-limit, the length of the F-string remains constant, while the length of the D-string decreases and the length of the FD-bound state increases. This result has been recently confirmed from numerical experiments\(^{[15]}\).

To shed some light on the evolution of cosmic superstring networks, a number of numerical experiments have been conducted, each of them at a different level of approximation. One should keep in mind that the initial configuration depends on the particular brane inflation scenario, while a realistic network should contain strings with junctions and allow for a spectrum of possible tensions.

I will briefly describe the approach and findings of one of these numerical approaches\(^{[14]}\), which I consider more realistic than others. The aim of that study was to build a simple field theory model of \((p,q)\) bound states, in analogy with the Abelian Higgs model used to investigate the properties of solitonic cosmic string networks, and to study the overall characteristics of the network using lattice simulations. Two models were investigated, one in which both species of string have only short-range interactions and another one in which one species of string features long-range interactions. We thus modelled the network with no long-range interactions using two sets of fields, complex scalars coupled to gauge fields, with a potential chosen such that the two types of strings will form bound states (see, Fig. 3). In this way junctions of 3 strings with different tension were successfully modelled. In order to introduce long-range interactions we considered a network in which one of the scalars forms global strings. This is important if the strings are of a non-BPS species. For example, for cosmic superstrings at the bottom of a Klebanov-Strassler throat the F-string is not BPS while the D-string is.

More precisely, the \((p,q)\) string network was modelled\(^{[14]}\) using two sets of Abelian Higgs fields, \( \phi, \chi \). In the case that both species of cosmic strings are BPS, the model is described by the action\(^{[14]}\):

\[
S = \int d^{3}x dt \left[ -\frac{1}{4} F_{\mu \nu}^{2} - \frac{1}{2} (D_{\mu} \phi)(D^{\mu} \phi)^{\ast} - \frac{\lambda_{1}}{4} (\phi \phi^{\ast} - \eta_{1}^{2})^{2} - \frac{1}{4} H^{2} - \frac{1}{2} (D_{\mu} \chi)(D^{\mu} \chi)^{\ast} - \frac{\lambda_{2}}{4} (\phi \phi^{\ast})(\chi \chi^{\ast} - \eta_{2}^{2})^{2} \right],
\]

(45)
where the covariant derivative $D_\mu$ is defined by

\begin{align}
D_\mu \phi &= \partial_\mu \phi - ie_1 A_\mu \phi , \\
D_\mu \chi &= \partial_\mu \chi - ie_2 C_\mu \chi .
\end{align}

(46)

For clarity, we label the $\phi$ field as “Higgs” and the $\chi$ field as “axion”, even though both fields are Higgs-like. The scalars are coupled to the U(1) gauge fields $A_\mu$ and $C_\mu$, with coupling constants $e_1$ and $e_2$ and field strength tensors $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$ and $H_{\mu\nu} = \partial_\mu C_\nu - \partial_\nu C_\mu$, respectively. The scalar potentials are parametrised by the positive constants $\lambda_1, \eta_1$ and $\lambda_2, \eta_2$, respectively. In the case that one species of string is non-BPS, we remove the second gauge field by setting $e_2 = 0$. In this way, this species of string is represented by the topological defect of a complex scalar field with a global U(1) symmetry. Note that such defects are characterised by the existence of long-range interactions — as opposed to local strings in which all energy density is confined within the string, so that local strings have only gravitational interactions — implying different consequences for the evolution of the network.

![Figure 3](image.png)

**Figure 3.** Left: Bound states for local-local ($p, q$) strings. Right: Bound states for local-global ($p, q$) strings. Figure taken from Ref. [44].

Thus, different components of the $(p, q)$ state are expected to exhibit different types of long-range interactions. The evolution of the string networks suggested that the long-range interactions have a much more important rôle in the network evolution than the formation of bound states. In the local-global networks the bound states tend to split as a result of the long-range interactions, resulting in two networks that evolve almost independently. The formation of short-lived bound states and their subsequent splitting only increases the small-scale wiggliness of the local strings. In the case of a local-local network, the absence of long-range interactions allows the bound states to be much longer-lived and significantly influences the evolution of the string network [44]. The most convincing evidence comes from analysing the reverse problem [44], namely that of a bound state splitting as a result of the long-range interactions between strings, presented in Fig. 4. Only in the absence of long-range interactions the strings remain in the $(1, 1)$ state throughout their entire evolution.

![Figure 4](image.png)

**Figure 4.** The total physical volume of the simulation box occupied by Higgs strings (green), axion strings (red), and their bound states (blue). The left panels refer to local-global networks, while the right ones to local-local networks starting from the same initial conditions as the local-global ones. Figure taken from Ref. [44].

Let us now investigate more thoroughly the issue of scaling. The evolution of F-, D-strings and...
their bound states is a rather complicated problem, which necessitates both numerical as well as analytical investigations. As I have already mentioned, junctions may prevent the network from achieving a scaling solution, invalidating the cosmological model leading to their formation. Following the approach of Ref. [44], numerical simulations [45], achieving control over the initial population of bound states, found clear evidence for scaling of all three components — $p$ F-strings, $q$ D-strings and their $(p,q)$ bound states — of the network, independently of the chosen initial configurations, while they concluded that the existence of bound states affects the evolution of the network. In Fig. 5 we show the string correlation length for the Higgs and axion fields, as well as for their bound states, as a function of time. The initial configuration is a local-global network with a large amount of bound states. The corresponding plots for local-local networks are drawn in Fig. 6. Clearly, there is convincing evidence for scaling of the three components of the network for both networks. This scaling is characterised with a distinct change of the correlation length slope during the network evolution. Note that the result holds even in the case of networks with small amounts of bound states.

Moreover, these numerical experiments have shown that for $(p,q)$ strings there is a supplementary energy loss mechanism, in addition to the chopping off of loops; it is this new mechanism that allows the network to scale. More precisely, the additional energy loss mechanism is the formation of bound states, whose length increases, lowering the overall energy of the network.

11. Cosmic Superstrings: A window into String Theory

Cosmic superstrings have gained a lot of interest, the main reason being that they can offer a large (and possibly unique) window into string theory, and in particular shed some light on the appropriate (if any) stringy description of the Universe. Since they interact with Standard Model particles only via gravity, their detection involves their gravitational interactions. Cosmic superstrings, in an analogy to their solitonic analogues, can lead to a variety of astrophysical signatures, like gravitational waves, ultra high en-
ergy cosmic rays, and gamma ray bursts.

At this point, let me however emphasise that given the complexity of the dynamics of a cosmic string network, which we certainly do not fully understand, and the model-dependent initial configuration, any theoretical estimations of the observational signatures of cosmic superstrings have to be taken with caution. Note that even the superstring tension depends on the considered inflationary scenario within a particular brane-world cosmological model.

Gravitational waves is one of the main explored avenues \[47\], in which case three channels of emission have been identified. Radiation can be emitted by cusps, kinks, and/or from the reconnection process itself. Cusps, where momentarily the string moves relativistically, have played a crucial role in discussing the radiation emitted from (ordinary) cosmic strings \[11\]. Kinks, resulting from cosmic string collisions and subsequent reconnection, are basically replaced in the case of cosmic superstrings by junctions. Finally, the radiation emitted from the reconnection process itself, which a sub-dominant process in the case of cosmic strings, may not be negligible in the case of cosmic superstrings because of the small reconnection probability \[P\]. In the scaling regime, the density of long strings goes like \[1/P\] \[40\], implying that the number of reconnection attempts goes like \[1/P^2\], and hence the number of successful reconnections is approximately \[1/P\]. Very recently, the gravitational waveform produced by cosmic superstring reconnections has been calculated \[49\]. Comparing the obtained result to the detection threshold for current and future gravitational wave detectors, it was concluded \[49\] that neither bursts nor the stochastic gravitational background, produced during the cosmic superstring reconnection process, would be detectable by Advanced LIGO. Thus, the most relevant process for gravitational waves emitted from cosmic superstrings turns out to be through their cusps. Hence, one should estimate the abundancy of cusps in cosmic superstrings with junctions.

Following simple geometric arguments, it has been recently shown \[50\] that strings ending on D-branes can indeed lead to cusps, in an analogous way as cusps in ordinary cosmic strings. In particular, cusps would be a generic feature of an F-string ending on two (parallel and stationary) D-strings. Hence, pairs of FD-string junctions, such as those that they would form after intercommutations of F- and D-strings, generically contain cusps. This result opens up a new energy loss mechanism for the network, in addition to the formation and subsequent decay of closed loops and the formation of bound states \[45\]. Phenomenological consequences of cusps from junctions on cosmic superstrings will be most significant at early times, namely towards the end of brane inflation, since then the typical separation of heavy strings is small as compared to the length of F-strings stretched between them \[50\].

12. Cosmic Superstring Thermodynamics

One has to extend previous studies of string thermodynamics in the case of cosmic superstring networks, characterised by the existence of \((p,q)\) bound states and different string tensions. Recently, the Hagedorn transition of strings with junctions has been investigated \[51\], in the context of a simple model with three different types and tensions of string, following an effective field theory approach. More precisely, the authors of Ref. \[51\] translated the thermodynamics of string networks with junctions into the thermodynamics of a set of interacting dual fields. Thus, the Hagedorn transition of the strings becomes a transition of the fields.

In this approach, the equilibrium statistical mechanics of cosmic superstring networks have been studied \[51\], by extending known methods for describing quark deconfinement. It was found \[51\] that as the system is heated, the lightest strings are the first ones to undergo a Hagedorn transition; the existence of junctions does not affect the occurrence of the transition. The system is also characterised by a second, higher, critical temperature above which long string modes of all tensions and junctions, do exist. The existence of multiple tensions indicates

\[\text{\textsuperscript{11}Even though one has to keep in mind that the number of cusps in a realistic cosmic string network has not been estimated, while preliminary numerical studies indicate that it may be rather low \[48\].} \]
the appearance of multiple Hagedorn transitions.

13. Conclusions

In these lectures, I have summarised our current understanding on the physics of cosmic strings and cosmic superstrings. I have discussed their formation, evolution, statistical mechanics and astrophysical/cosmological consequences. This is a topic of active research at present, relating fundamental theoretical ideas with experimental and observational facts.

On the one hand, any successful cosmological scenario, such as the inflationary paradigm, must be inspired from a fundamental theory. On the other hand, any successful high energy physics theory, such as string theory or supersymmetric grand unified theories, must be tested against data; the only available laboratory for the required energy scales, is indeed the early Universe.

Inflation within brane-world cosmological models leads naturally to cosmic superstrings. Inflation within supersymmetric grand unified theories leads generically to cosmic strings, the solitonic analogues of cosmic superstrings. The study of these objects is interesting by itself. In addition, cosmic (super)strings may provide an explanation for the origin of a variety of astrophysical/cosmological observations; they may also offer a test (often a unique one) of fundamental theories of physics, thus shedding some light about the appropriate stringy description of the Universe.

Acknowledgments

It is a pleasure to thank the organisers of the ESF Summer School in High Energy Physics and Astrophysics “Theory and Particle Physics: the LHC perspective and beyond”, which took place at the Cargèse Institute of Scientific Studies in Corsica, for inviting me to present these lectures in such a beautiful and stimulating environment.

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