Tachyons in de Sitter Space and Analytical Continuation from dS/CFT to AdS/CFT

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Abstract: We discuss analytic continuation from $d$ dimensional Lorentzian de Sitter ($dS_d$) to $d$-dimensional Lorentzian anti-de Sitter ($AdS_d$) spacetime. We show that $AdS_d$, with opposite signature of the metric, can be obtained as analytic continuation of a portion of $dS_d$. This implies that the dynamics of (positive square-mass) scalar particles in $AdS_d$ can be obtained from the dynamics of tachyons in $dS_d$. We discuss this correspondence both at the level of the solution of the field equations and of the Green functions. The $AdS/CFT$ duality is obtained as analytic continuation of the $dS/CFT$ duality.

Keywords: $AdS$-$CFT$ Correspondence.
1. Introduction

One of the most important realizations of the holographic principle [1, 2] is the correspondence between bulk anti-de Sitter (AdS) gravity and boundary conformal field theories (CFT) [3, 4, 5]. The success of the AdS/CFT correspondence has motivated further proposal of holographic dualities between gravitational systems and conformal field theories. In particular, a similar correspondence has been proposed for gravity in de Sitter (dS) spacetime (dS/CFT) [6] (see also for instance [7, 8, 9, 10]).

$d$-dimensional de Sitter spacetime (dS$_d$) looks, at least locally, very similar to $d$-dimensional anti-de Sitter spacetime (AdS$_d$). They are both spacetimes of constant curvature and dS$_d$ can be obtained from AdS$_d$ by flipping the sign (from negative to positive) of the cosmological constant $\Lambda$. Naively, one could therefore expect the dS$_d$/CFT$_{d-1}$ duality to be simply related to AdS$_d$/CFT$_{d-1}$, for instance by means of a analytical continuation. However, local and global properties of de Sitter spacetime lead to unexpected obstructions. AdS$_d$ has a simply connected boundary, which is (a conformal compactification of) $d−1$ dimensional Minkowski space. Conversely, dS$_d$ has two disconnected boundaries conformally related to $d−1$ dimensional Euclidean space. Moreover, the causal structure of dS$_d$ is completely different from that of AdS$_d$. A single de Sitter observer has not access to the whole of dS$_d$. This has
a strong impact on the features of quantum field theories on dS\(_d\). It is essentially responsible for the existence of a family of vacua \([11, 12]\) for quantum scalar fields in dS\(_d\), whereas the corresponding AdS\(_d\) vacuum is essentially unique. Last but not least, dS\(_d\) is a time-dependent gravitational background, which is very poorly understood in the context of string theory.

Because of these difficulties, the status of the dS/CFT correspondence remains unclear and plugged by unresolved problems, which raised criticisms about the very existence of a dS/CFT duality \([13, 14]\). A possible strategy to tackle the problems is to explore in detail the similarities between dS/CFT and AdS/CFT. After all dS\(_d\) is formally very similar to AdS\(_d\), so that one can hope to find some analytical continuation relating dS/CFT to AdS/CFT. This approach has been used in the Euclidean context. It has been shown that AdS\(_d\) can be considered as “negative” Euclideanization of dS\(_d\) \([15]\). However, the Euclidean approach met only partial success, essentially because to achieve real progress one needs not only to analytically continue the spacetime metric but also the Green functions. In the Euclidean context, no analytic continuation relating the AdS vacuum with some of the dS vacua could be found \([12]\).

In our opinion the dS/CFT duality can be understood as an analytical continuation of AdS/CFT in the Lorentzian context. An essential ingredient is represented by a correspondence between tachyons in dS\(_d\) and particles with positive square-mass in AdS\(_d\). In Ref. \([13]\) it was argued that a scalar particle with positive square-mass in dS space is like a tachyon in AdS space. Moreover, investigating the dS\(_2\)/CFT\(_1\) correspondence \([16, 17]\) along the lines of the AdS\(_2\)/CFT\(_1\) duality \([18, 19]\), it was shown that a tachyonic perturbation in the two-dimensional bulk corresponds to a boundary conformal operator of positive dimension. The key idea we use in this paper is very simple. The overall minus sign coming from the analytical continuation of the metric is compensated by the minus sign in front of the tachyon square-mass.

More in detail, in this paper we review the arguments of Ref. \([13]\), based on unitarity bounds, about the correspondence between tachyons in dS\(_d\) and (positive square-mass) particles in AdS\(_d\) (Sect. 2). Using a particular coordinatization of dS\(_d\), we show that a region of Lorentzian dS\(_d\) can be mapped, by means of an analytic continuation, into whole “negative” Lorentzian AdS\(_d\). As a consequence, the scalar field equation for a tachyon in dS\(_d\) becomes that for a scalar with positive square-mass in AdS\(_d\) (Sect. 3). The particular chart used to cover dS\(_d\) and the related causal structure of the region of dS\(_d\) mapped into AdS\(_d\) allow for a consistent mapping of the two spacetimes (Section 4). We also show explicitly that the solutions of the field equation and the Green functions for a tachyon in dS\(_d\) can be analytically continued into the solutions and Green functions of a scalar field in AdS\(_d\). In particular, we argue that the AdS vacuum can be obtained as analytical continuation of a tachyonic dS vacuum (Sect. 5, 6). Finally, the AdS/CFT duality is obtained as analytic continuation of the dS/CFT duality (Sect. 7).
2. Conformal weights and unitarity bounds

The dS\(d\)/CFT\(d-1\) duality puts in correspondence a scalar field of mass \(m\) propagating in \(d\)-dimensional de Sitter space with a \((d-1)\)-dimensional CFT living on the boundary with conformal weights \([6]\)

\[ h_\pm = \frac{1}{2} \left( d - 1 \pm \sqrt{(d - 1) - \frac{4m^2}{\lambda^2}} \right), \tag{2.1} \]

where \(\lambda^2\) is related to the cosmological constant \(\Lambda\) of the de Sitter space by \(\Lambda = (d-2)(d-1)\lambda^2/2\) (In this paper we take \(d > 2\)).

Conversely, the AdS\(d\)/CFT\(d-1\) duality tells us that a scalar field of mass \(m\) propagating in \(d\)-dimensional anti-de Sitter space is in correspondence with a \(d\)-dimensional boundary CFT with conformal weight \([5]\)

\[ \Delta = \frac{1}{2} \left( d - 1 + \sqrt{(d - 1) + \frac{4m^2}{\lambda^2}} \right), \tag{2.2} \]

now the spacetime has negative cosmological constant \(-\Lambda\). The similarity between Eq. (2.1) and Eq. (2.2) is striking. In the case of the dS\(d\)/CFT\(d-1\) correspondence the CFT is unitary only if the Strominger bound, \(m^2 \leq (d - 1)(\lambda^2/4)\), for the mass of the scalar is satisfied \([6]\). For AdS\(d\)/CFT\(d-1\) the unitarity bound is given by the Breitenlohner-Freedman bound, \(m^2 \geq -(d - 1)(\lambda^2/4)\). The two bounds are related by the transformation \(m^2 \rightarrow -m^2\). Moreover, by changing the sign of \(m^2\) in Eq. (2.2) we reproduce one of the solutions in Eq. (2.1), \(\Delta(-m^2) = h_+(m^2)\). Thus, scalar fields with positive \(m^2\) in AdS\(d\) are like tachyons in dS\(d\) and vice versa. This fact was first observed in Ref. \([15]\).

The previous statement represents just a conjecture, unless one can map explicitly the dynamics of tachyons in dS\(d\) into that of scalar fields in AdS\(d\). Moreover, one should also be able to show explicitly that the transformation \(m^2 \rightarrow -m^2\) maps the dS\(d\)/CFT\(d-1\) correspondence into the AdS\(d\)/CFT\(d-1\) one. This is not so easy, for at least two reasons, already pointed out in the literature: (a) the topology of dS\(d\) is completely different from that of AdS\(d\). In particular, the boundary \(\mathcal{B}\) of AdS\(d\) is simply connected and has the topology of \(S^1 \times S^{d-2}\). dS\(d\) has two disconnected boundaries, \(\mathcal{I}^\pm\), each with topology \(S^{d-1}\). (b) until now no analytic continuation between Lorentzian dS\(d\) and Lorentzian AdS\(d\) could be found.

The previous features represent a strong obstruction that has to be overcome if one wants to find a relationship between propagation of scalars in dS\(d\) and AdS\(d\) or, more in general, between dS\(d\)/CFT\(d-1\) and AdS\(d\)/CFT\(d-1\). For instance, the existence of the two boundaries \(\mathcal{I}^\pm\) for dS\(d\) implies that there are two independent boundary conditions (corresponding to two independent sets \(\phi_\pm\) of boundary modes), which can be imposed on the asymptotic behavior of the scalar field. Once the dS\(d\)/CFT\(d-1\)
correspondence is implemented, this implies the existence of two roots \( h_\pm \) in Eq. (2.1) for the conformal weights of the boundary CFT, whereas in the AdS\(_d\)/CFT\(_{d-1}\) only the single root \( \Delta \) of Eq. (2.2) is present. Also, the analytic continuation from AdS\(_d\) to dS\(_d\) is a rather involved problem. Owing to the different topologies, one cannot switch from AdS\(_d\) to dS\(_d\) just by using the analytic continuation \( \lambda \rightarrow i\lambda \). A possibility to circumvent the problems is to work in the Euclidean rather then in the Lorentzian. Euclidean AdS\(_d\) can be considered as “negative” Euclideanization of dS\(_d\) \[15\]. However, for obvious reasons the Euclidean formulation cannot be used if one wants to show the existence of a correspondence between tachyons on dS\(_d\) and particles with positive square-mass on AdS\(_d\).

In the next section we will show that Lorentzian dS\(_d\) can be analytically continued into “negative” Lorentzian AdS\(_d\), i.e. usual anti-de Sitter spacetime with a metric tensor of opposite signature. In the following we will only consider tachyons on dS\(_d\) and the corresponding scalar particles in AdS\(_d\). In this case one has \( h_+ > 0 \) and \( h_- < 0 \). However, our discussion could be easily generalized to scalar fields in dS\(_d\) satisfying the Strominger bound and corresponding tachyons in AdS\(_d\) satisfying the Breitenlohner-Freedman bound.

3. Analytic continuation from dS\(_d\) to negative AdS\(_d\)

d\(_d\)-dimensional de Sitter spacetime can be defined as the hyperboloid

\[
\eta_{AB} X^A X^B = \frac{1}{\lambda^2}, \quad \eta_{AB} = (-1, \ldots 1), \quad A, B = 0, 1 \ldots d, \tag{3.1}
\]

embedded in the \( d + 1 \)-dimensional Minkowski spacetime, \( ds^2 = \eta_{AB} dX^A dX^B \). The parametrization of the hyperboloid suitable for our analytic continuation has been given in Ref. \[8\]

\[
\lambda X^d = \sqrt{\lambda^2 r^2 + 1} \sin \lambda t, \quad \lambda X^{d-1} = \sqrt{\lambda^2 r^2 + 1} \cos \lambda t \\
X^0 = r \cosh \theta, \quad X^j = r \omega^j \sinh \theta, \quad j = (1 \ldots d - 2), \tag{3.2}
\]

with \( r > 0, \quad 0 \leq t \leq 2\pi/\lambda, \quad \theta > 0 \) and \( \omega^j \) parametrizing the \( S^{d-3} \) sphere, \( \omega^1 = \cos \theta_1, \omega^2 = \sin \theta_1 \cos \theta_2, \ldots, \omega^{d-2} = \sin \theta_1 \ldots \sin \theta_{d-3} \). The induced metric on dS\(_d\) is

\[
dS^2 = -(1 + \lambda^2 r^2)^{-1} dt^2 + (1 + \lambda^2 r^2) dt^2 + r^2 (d\theta^2 + \sin^2 \theta d\Omega^2_{d-3}) \tag{3.3}
\]

where \( d\Omega^2_{d-3} \) is the metric over the \( S^{d-3} \) sphere. Notice that the coordinates \((\theta, \theta_k)\) parametrize a hyperbolic space \( H^{d-2} \) with metric

\[
d\Sigma^2 = d\theta^2 + \sin^2 \theta d\Omega^2_{d-3}. \tag{3.4}
\]

The \( r = \text{const} \) sections of the metric have \( S^1 \times H^{d-2} \) topology. The coordinatization \( (3.2) \) gives an hyperbolic slicing of dS\(_d\) and do not cover the whole de Sitter hyperboloid, but just a region of it. In particular, only one boundary (for instance \( \mathcal{I}^+\),
which can be reached letting $r \to \infty$ of $\text{dS}_d$ is visible in these coordinates. Moreover, using this coordinate system $\mathcal{I}^+$ has $S^1 \times H^{d-2}$ topology. Later on this paper, when discussing the causal structure of the spacetime, we will come back to this point.

We can now obtain negative $\text{AdS}_d$ just by using the analytic continuation

$$ \theta \to i\theta, $$  \hspace{1cm} (3.5)

in the de Sitter metric (3.3). We get

$$ ds^2 = - \left[ -(1 + \lambda^2 r^2) dt^2 + (1 + \lambda^2 r^2)^{-1} dr^2 + r^2 d\Omega_{d-2}^2 \right], \hspace{1cm} (3.6)$$

where now $d\Omega_{d-2}^2$ is the metric over the $S^{d-2}$ sphere, and the hyperbolic coordinate $\theta$ has become after the analytic continuation an angular coordinate of the sphere.

The metric (3.6) is (minus) the metric that can be used over the whole $\text{AdS}$ hyperboloid. The $\text{AdS}$ spacetime with metric given by Eq. (3.6) can be defined as the hyperboloid

$$(X^d)^2 + (X^{d-1})^2 - \sum_{k=0}^{d-2}(X^k)^2 = \frac{1}{\lambda^2} \hspace{1cm} (3.7)$$

embedded in $d + 1$-dimensional flat space with metric

$$ ds^2 = - \left( -(dX^d)^2 - (dX^{d-1})^2 + \sum_{k=0}^{d-2}(dX^k)^2 \right). \hspace{1cm} (3.8) $$

Notice that the signature of the embedding space is opposite to the usual one used to define the $\text{AdS}$ spacetime. The $\text{AdS}$ hyperboloid (3.7) and $\text{AdS}$ embedding space (3.8) can be obtained as the analytic continuation $X^j \to iX^j, j = 1 \ldots (d-2)$ (which is the transformation (3.3) written in terms of the coordinates $X$) of the $\text{dS}$ hyperboloid (3.1) and the $\text{dS}$ embedding space.

The analytic continuation (3.5) changes the topology of the $r = \text{const}$ sections of the metric. In the $\text{AdS}$ case (3.6) the topology of the $r = \text{const}$ sections is $S^1 \times S^{d-2}$. The analytic continuation $\theta \to i\theta$ maps $H^{d-2} \to S^{d-2}$. It transforms the $\mathcal{I}^+$ boundary of $\text{dS}_d$ with topology $S^1 \times H^{d-2}$ into the boundary $\mathcal{B}$ of $\text{AdS}_d$ with topology $S^1 \times S^{d-2}$.

The metric (3.7) corresponds to a parametrization of AdS space that covers the whole hyperboloid. In the following we will also make use of (negative) AdS metric written in terms of the Poincaré coordinates (see e.g. [20])

$$ ds^2 = - \left[ \frac{du^2}{\lambda^2 u^2} + u^2 \left( -d\tau^2 + dx_j dx^j \right) \right], \hspace{1cm} (3.9)$$

where $0 \leq u < \infty$ and $j = 1 \ldots (d-2)$. These coordinates cover only half of the $\text{AdS}$ hyperboloid. The metric (3.9) can be obtained as analytic continuation of the de Sitter metric in planar coordinates

$$ ds^2 = -dt^2 + e^{2\lambda t} dx_a dx^a, \hspace{0.5cm} a = 1 \ldots (d-1), \hspace{1cm} (3.10) $$


with $-\infty < \hat{t} < \infty$. Performing in Eq. (3.10) the analytic continuation

$$x_j \to ix_j, \quad j = 1, \ldots, (d-2),$$

(3.11)

and setting $e^{\lambda t} = u, \quad x_{d-1} = \tau$, we get immediately the metric (3.9).

Let us now consider a tachyonic scalar field $\phi$ with square-mass $-m^2$ propagating in the de Sitter spacetime (3.3), the field equation reads

$$(\nabla^2_{dS} + m^2)\phi = 0,$$

(3.12)

where the the operator $\nabla^2_{dS}$ is calculated with respect to the dS metric (3.3). Performing in this equation the analytic continuation (3.5), we get the field equation for scalar field with a positive square-mass propagating in the AdS spacetime,

$$(\nabla^2_{AdS} - m^2)\phi = 0,$$

(3.13)

where the the operator $\nabla^2_{AdS}$ is now calculated with respect to the usual AdS metric, which is given by Eq. (3.4) with opposite sign. The minus sign coming from the analytic continuation is compensated by the change of sign of the square-mass of the scalar.

Of course the same holds true if we consider a tachyon propagating in dS endowed with the metric (3.10) and perform the analytic continuation leading to the AdS metric in the form (3.9).

We therefore conclude that, at least at the classical level, tachyonic propagation in dS is simply related by the analytic continuation (3.5) to propagation of particles with positive square-mass in AdS.

In our discussion, we did not take into consideration the global structure of both dS and AdS. We have already pointed out in Sect. 2 that topologically AdS and dS are very different. Therefore, our previous conclusion is strictly true only locally. In the next section we will discuss the impact that global features of the spacetime have on the correspondence between tachyons on dS and particles with positive square-mass on AdS.

4. Causal structure of the spacetime

The coordinates $(r, t, \theta, \theta_k)$ appearing in the hyperbolic slicing (3.3) of dS do not cover the full de Sitter hyperboloid but just a region of it. On the other hand, as it is evident from the metric (3.6), the same coordinates cover the whole AdS spacetime. This means that the analytic continuation (3.5) does not map the whole dS into entire AdS but just a region of the dS hyperboloid into the whole of the AdS hyperboloid. In particular, only the $\mathcal{I}^+$ (i.e $r \to \infty$) boundary of dS is visible in the particular coordinatization chosen, and it is mapped by means of the transformation (3.5) into the single boundary $\mathcal{B}$ of AdS.

Although this fact does
not have any influence on the local behavior of the solution for the field equations (3.12), (3.13), the boundary condition we have to use when we solve these equations depend crucially on the global features defined by the chart used to cover dS.

The region of dS covered by the coordinates \((r, t, \theta, \theta_k)\) can be easily described by deriving the coordinate transformation connecting them with the global coordinates \((\hat{\tau}, \hat{\theta}_a)\), \(a = 1, \ldots, (d - 1)\) that cover the whole of de Sitter space. In terms of these coordinates the de Sitter metric reads (see for instance Ref. [9])

\[
ds^2 = -d\hat{\tau}^2 + \frac{1}{\lambda^2} \cosh^2(\lambda \hat{\tau})d\Omega^2_{d-1},
\]

where \(-\infty < \hat{\tau} < \infty\) and \(\hat{\theta}_a\) parametrize the \(S^{d-1}\) sphere whose metric is \(d\Omega^2_{d-1}\).

From the coordinate transformation linking global and hyperbolic coordinates, one easily finds that the coordinates \((r, t, \theta, \theta_k)\) cover only the region

\[
\cosh^2(\lambda \hat{\tau}) \sin^2 \hat{\theta}_1 \ldots \sin^2 \hat{\theta}_{d-2} \geq 1
\]

of dS. The south and north pole, \(\hat{\theta}_1, \ldots, \hat{\theta}_{d-2} = 0, \pi\) are not covered by the hyperbolic coordinate system \((r, t, \theta, \theta_k)\). This feature explains why the topology of the \(r = \text{const}\) (and in particular of the boundary \(r = \infty\)) sections of the metric (3.3) is \(S^1 \times H^{d-2}\) whereas that of the \(\hat{\tau} = \text{const}\) (and in particular of the boundaries \(I^\pm\)) sections of the the metric (4.1) is \(S^{d-1}\).

It is evident from Eq. (4.2) that given a point of the the \(S^{d-1}\) sphere with coordinates \((\hat{\theta}_1 \ldots \hat{\theta}_{d-1})\) its future, \(O^+\), (or past \(O^-\)) light-cone, is never completely covered by the hyperbolic coordinate system unless Eq. (4.2) is identically satisfied. This is true only for points with \(\hat{\theta}_1 \ldots \hat{\theta}_{d-2} = \pi/2, 0 \leq \hat{\theta}_{d-1} \leq 2\pi\), i.e for the equator of the \(S^{d-1}\) sphere. We conclude that the hyperbolic coordinates cover the region corresponding to the causal future (if the coordinate \(r > 0\)) or the causal past (if the coordinate \(r < 0\)) of an observer sitting at the equator of the \(S^{d-1}\) sphere. The Penrose diagram of the space-time can be easily constructed writing the metric (4.1) in conformal coordinates, \((T, \hat{\theta}_a)\), with \(\cos(\lambda T) = [\cos(\lambda \hat{\tau})]^{-1}\),

\[
ds^2 = \frac{1}{\cos^2(\lambda T)} (-dT^2 + \lambda^{-2}d\Omega^2_{d-1}),
\]

where \(-\pi/2 \leq \lambda T \leq \pi/2\).

\[\text{Figure 1: Penrose diagram for dS}_{d}\]
Taking the angular coordinates of the sphere all constant but one, \( \hat{\theta}_1 = \hat{\theta} \), we get the Penrose diagram shown in figure 1. The vertical lines \( \hat{\theta} = 0, \pi/2, \pi \) represent the north pole, the equator and the south pole, respectively. The horizontal lines \( \lambda T = -\pi/2, \pi/2 \) the two boundaries \( \mathcal{I}^\pm \) of \( dS_d \). The region enclosed between the two lines \( \lambda T = \pm(\hat{\theta} + \pi/2) \) is the region covered by the hyperbolic coordinates. \( \mathcal{O}^\pm \) are respectively the future and past light-cone for an observer sitting at \( T = 0 \) at the equator. From our previous discussion emerges clearly that if we restrict, as we did in Eq. (3.3), the coordinate \( r \) to range only over \([0, \infty]\) we cover only the region \( \mathcal{O}^+ \) of figure 1. This means that \( \mathcal{O}^+ \) is the region of \( dS_d \) which is mapped by the analytic continuation (3.5) into entire \( AdS_d \).

Because \( \mathcal{O}^+ \) is the future light-cone of an observer at the equator, we have reached the important result: the portion of \( dS_d \) corresponding to the causal future of a single observer can be mapped by the simple analytic continuation (3.5) into the whole \( AdS_d \). Translated in terms of the dynamics of the scalar field \( \phi \) considered in the previous section: the dynamics of a tachyon defined in the region \( \mathcal{O}^+ \) of \( dS_d \), is analytically continued by Eq. (3.3) into the dynamics of a scalar field (with positive square-mass) defined over the whole \( AdS_d \).

Let us conclude this section by observing that the correspondence between \( \mathcal{O}^+ \subset dS_d \) and \( AdS_d \) is very natural from the physical point of view. It is well-known that a single observer cannot have access to the entire de Sitter spacetime. Developing a field theory over \( dS_d \) we should limit ourself to consider only regions over which a single observer has a causal control (or from which can be causally controlled). This point of view has been already advocated in Ref. [8, 12].

5. Analytic continuation of the solutions for the scalar field

In this section we will show explicitly that a solution of the field equation (3.12) for a tachyon in \( dS_d \) can be analytically continued into a solution of the field equation (3.13) for a scalar field in \( AdS_d \).

Let us first consider the wave equation (3.12) with the operator \( \nabla^2 \) evaluated on the metric (3.3). The differential equation (3.12) is separable. The solution takes the form

\[
\phi(r, t, \Omega) = f(r) h(t) \hat{Y}_{d-2}(\Omega),
\]

where \( \Omega = (\theta, \theta_1 \ldots \theta_{d-3}) \). \( h(t) \) is solution of the equation

\[
\frac{d^2 h}{dt^2} = -\lambda^2 n^2 h,
\]

The choice of \( \mathcal{O}^+ \) instead of \( \mathcal{O}^- \) is a matter of conventions. We could have as well mapped \( \mathcal{O}^- \) into \( AdS_d \).
with \( n \) integer since \( t \) is periodic. \( \hat{Y}_{d-2}(\Omega) \) are the eigenfunctions of the \( \nabla^2_{d-2} \) operator evaluated on the hyperbolic space \( H^{d-2} \) with metric (3.4),

\[
\nabla^2_{d-2} \hat{Y}_{d-2} = l(l + d - 3) \hat{Y}_{d-2}. \tag{5.3}
\]

\( f(r) \) is solution of the radial equation \((t = d/dr)

\[
f'' + \left( \frac{d - 2}{r} + \frac{2r\lambda^2}{(1 + \lambda^2 r^2)} \right) f' + \left[ \frac{n^2 \lambda^2}{(1 + \lambda^2 r^2)^2} - \frac{l(l + d - 3)}{r^2(1 + \lambda^2 r^2)} - \frac{m^2}{(1 + \lambda^2 r^2)} \right] f = 0.
\tag{5.4}
\]

One can easily check that the wave equation (3.13) for a scalar in \( \text{AdS}_d \), once the solution is separated as \( \phi(r, t, \Omega) = f(r)h(t)Y_{d-2}(\Omega) \) with \( \Omega = (\theta_1, \ldots, \theta_{d-2}) \), exactly reproduces both Eq. (5.2) and (5.4) for \( h(t) \) and \( f(r) \), whereas Eq. (5.3) is replaced by the equation

\[
\nabla^2_{d-2} Y_{d-2} = -l(l + d - 3) Y_{d-2}.
\tag{5.5}
\]

where now the operator \( \nabla^2_{d-2} \) is evaluated on the \( S^{d-2} \) sphere. \( Y_{d-2} \) are the \((d-2)\)-dimensional spherical harmonics.

We have still to show that the \((d-2)\)-dimensional spherical harmonics \( Y_{d-2} \) are analytical continuation of the eigenfunctions \( \hat{Y}_{d-2} \) of the Laplace-Beltrami operator in the hyperbolic space \( H^{d-2} \). The Laplace-Beltrami operator has a discrete spectrum in \( S^{d-2} \) (\( l \) in Eq. (5.5) must be a nonnegative integer) but a continuous spectrum in \( H^{d-2} \) (\( l \) in Eq. (5.3) in general is a real number). For the correspondence between \( \hat{Y}_{d-2} \) and \( Y_{d-2} \) to be one-to-one, we need to restrict \( l \) in Eq. (5.3) to be also a nonnegative integer. If this is the case, both equations (5.3) and (5.5) can be solved in terms of ultraspherical (Gegenbauer) polynomials. Using the explicit form of \( \hat{Y}_{d-2} \) and \( Y_{d-2} \) one can then show that \( Y_{d-2} \) is the analytical continuation \( \theta \to i\theta \) of \( \hat{Y}_{d-2} \).

Let us now discuss the solutions of the radial equation (5.4). The solution of this differential equation can be expressed in terms of hypergeometric functions

\[
f(r) = r^l F\left( \frac{1}{2}(l + h_+), \frac{1}{2}(l + h_-), 1 + \frac{n}{2}, 1 + \lambda^2 r^2 \right), \tag{5.6}
\]

where \( h_\pm \) are given by Eq. (2.1).

In the following sections we will need the asymptotical behavior \( r \to \infty \) of the scalar field \( \phi \). This can be easily read from Eq. (5.9), taking into account that \( h_+ > 0 \) and \( h_- < 0 \) we have

\[
\phi \sim r^{-h_+} h(t) \hat{Y}_{d-2}(\Omega) = r^{-h_+} \phi_+(t, \Omega). \tag{5.7}
\]

Notice that although the wave equation allows two possible behaviors at infinity, \( \phi \sim r^{-h_\pm} \), the falloff \( \phi \sim r^{-h_+} \) is always subleading with respect to \( \phi \sim r^{-h_-} \), as long as we use the hyperbolic slicing (3.3) where only the boundary \( I^+ \) is visible. The absence of the subleading behavior becomes a boundary condition, one can
consistently impose on $\phi$. This corresponds to appropriate boundary conditions for the Green functions (see Sect. 6). The previous discussion does not apply if we take $dS_d$ in the spherical slicing \([4.4]\), in which both boundaries $I^\pm$ are visible. For consistency we have now to keep both behaviors for $\phi$. The subleading behavior in $I^+$ becomes leading in $I^-$ and vice versa. The possibility of ruling out consistently, the falloff behavior $\phi \sim r^{-h_+}$ for the scalar field in $dS_d$ is an important consistency check of our analytic continuation from $dS_d$ to $AdS_d$. For a scalar field in $AdS_d$ only one falloff for $\phi$ appears in the Green functions.

So far we have considered a solution for $\phi$ in $AdS_d$ with global coordinates \([3.6]\) as analytic continuation of a solution in $dS_d$ in hyperbolic coordinated \([3.3]\). We can also consider a solution for $\phi$ in $AdS_d$ in Poincaré coordinates \([3.9]\) as analytic continuation of a solution in $dS_d$ in the planar slicing \([3.10]\).

The solution of the wave equation in the background metric \([3.10]\) is

$$\phi_{dS} = A e^{i k_a x_a} u^{(1-d)/2} Z_{\nu}(k u^{-1}), \quad (5.8)$$

where $A$ is an integration constant, $k_a$ are real (we consider for simplicity only oscillatory solutions on $I^+$), $k^2 = \sum_{a=1}^{d-1} k_a^2$, $u = e^{\lambda t}$ and $Z_{\nu}$ are Bessel function with $\nu = (1/2) \sqrt{(d-1)^2 + 4m^2/\lambda^2}$.

Analytically continuing $x_j \to ix_j$, $j = 1\ldots(d-2)$, the solution \([5.8]\), we get a solution of the wave equation in the AdS background metric \([3.9]\)

$$\phi_{AdS} = A e^{-k_j x_j + i k_{d-1} \tau} u^{(1-d)/2} Z_{\nu}(k u^{-1}). \quad (5.9)$$

The asymptotic behavior $\hat{t} \to \infty$ ($u \to \infty$) of solutions \([5.8]\), \([5.9]\) is easily found to be

$$\phi \sim e^{-\lambda h_+} \phi_- (x) = u^{-h_+} \phi_- (x, \tau). \quad (5.10)$$

6. Green functions

Our next task is to consider Green functions. Our goal is to obtain Green functions for a scalar with square-mass $m^2$ propagating in $AdS_d$ as analytical continuation of a tachyon with square-mass $-m^2$ propagating in $dS_d$. There are two main problems that we have to face in order to achieve this goal. (a) we can construct various $SO(1,d)$ invariant vacua for a scalar field in $dS_d$ but the $SO(2, d-1)$ invariant vacuum for a scalar in $AdS_d$ is unique. (b) because we are considering propagation of a tachyon in $dS_d$, infrared divergent terms are expected to appear in the correlation functions.

$SO(1,d)$-invariant Green functions on $dS_d$ have the general form \([1]\)

$$G(x, y) = F_1(P) \Theta(x, y) + F_2(P) \Theta(y, x), \quad (6.1)$$
where $P$ depends on the geodetical distance $D(x, y)$ between the points $(x, y)$, $P = \cos \lambda D(x, y)$ and $\Theta(x, y) = (0, 1/2, 1)$ respectively for $x$ being in the (past of, spacelike separated from, future of) $y$.

In principle one could also consider more general $O(1, d)$-invariant Green functions. The non-connected group contains also the inversion, which in our parametrization (3.3) acts as $r \rightarrow -r$. Because our coordinate system covers only the region $r > 0$ we have to exclude this case. It is also evident that for a field theory defined on $dS_d$ parametrized as in Eq. (3.3) $F_2 = 0$. This follows from the discussion of the causal structure of the spacetime of Sect. 4. Our coordinate system covers only the future light-cone of an observer sitting at the equator of the $S^{d-1}$ sphere. Owing to the spherical symmetry there is nothing particular about the equator, so the previous feature must hold for every point of the sphere.

A quantum field theory on $dS_d$ allows for a family of $SO(1, d)$-invariant vacua [11, 12]. This vacuum degeneracy is essentially due to a peculiarity of the causal structure of $dS_d$, which implies that we can have Green functions with singularities not only when $x$ is in the light-cone of $y$ ($P = 1$) but also when $x$ is in the light cone of the point $\hat{y}$ antipodal to $y$ ($P = -1$) [11]. Of particular relevance is the so-called Euclidean vacuum, which can be defined as the vacuum in which the Green function is singular only if $x$ is in the light cone of $y$, i.e for $P = 1$. These are features of a field theory defined over the whole $dS_d$. From the general theory of field quantization on curved spaces, we know that a particular coordinatization of the space, which may cover only a region of the entire spacetime, singles out a particular vacuum for the field. We can now ask ourselves, which is the vacuum singled out by the coordinatization (3.3). It is not difficult to realize that this is exactly the euclidean vacuum (or a thermalization of it). This follows essentially from the fact that our coordinate system covers only the future light-cone of $x$. We cannot see the singularity for $P = -1$.

Let us now consider analytic continuation of the Green functions (6.1). Because the analytical continuation (3.5) changes the signature of the embedding space from $(-1, 1 \ldots 1)$ to $(1, 1, -1 \ldots -1)$ (see equation (3.8), it switches from $SO(1, d)$-invariant Green functions for a tachyon on $dS_d$ to $SO(2, d-1)$-invariant Green functions for a scalar particle with positive square mass in $AdS_d$. This is evident by considering, for instance, the Wightman function $G(x, y) =< 0 | \{ \phi(x), \phi(y) \} | 0 >$ for a tachyon in $dS_d$,

\[
(\nabla^2_{dS} + m^2) G(x, y) = 0.
\]

The analytical continuation (3.5) maps this equation into that for a scalar particle in $AdS_d$,

\[
(\nabla^2_{AdS} - m^2) G(x, y) = 0.
\]

Summarizing, the Green functions for a scalar field in $AdS_d$ can be obtained as analytical continuation of the Green function for a tachyon in $dS_d$. The unique
AdS-vacuum is obtained as analytical continuation of the tachyonic dS Euclidean vacuum, singled out by the parametrization of dS \(_d (3.3)\).

Let us now solve equation (6.2). By setting \(z = (1 + P)/2\) equation (6.2) takes the form \([21]\)

\[
 z(1 - z) \frac{d^2G}{dz^2} + \left( \frac{d}{2} - dz \right) \frac{dG}{dz} + \frac{m^2}{\lambda^2} G = 0. \tag{6.4}
\]

The solution of this differential equation can be written in terms of the hypergeometric function \(F(h_+, h_-, \frac{d}{2}, z)\), with \(h_\pm\) given by Eq. (2.1). \(F\) has a singularity for \(z = 1\), and, being \(h_- < 0\) also at \(z = \infty\). The singularity at \(z = 1\) is the usual short-distance singularity. Conversely, the presence of the \(z = \infty\) singularity is related to the tachyonic nature of the particle and implies the emergence of correlations that diverge at large distances. This behavior is rather unphysical, for instance implies violation of unitarity for the dual CFT living on the boundary of dS \(_d\). To solve the problem we will look for solutions of Eq. (6.4) that are regular for \(z = \infty\).

For \(h_+ - h_- \neq \text{integer}\) the general solution of the hypergeometric equation (6.4) can be written as

\[
 G(z) = C_1 G_1 + C_2 G_2 = \text{Re} \left\{ C_1 (-z)^{-h_+} F \left( h_+, h_+ + 1 - \frac{d}{2}, h_+ - h_- + 1, \frac{1}{z} \right) \right. \\
 + \left. C_2 (-z)^{-h_-} F \left( h_-, h_- + 1 - \frac{d}{2}, h_- - h_+ + 1, \frac{1}{z} \right) \right\}, \tag{6.5}
\]

where \(C_{1,2}\) are integration constants. Keeping in mind that \(h_+\) is positive but \(h_-\) is negative and that \(F(\alpha, \beta, \gamma, 0) = 1\), regularity of the solution at \(z = \infty\) requires \(C_2 = 0\). The asymptotical \(z \to \infty\) behavior of the solution is,

\[
 G(z) \sim C_1 (-z)^{-h_+}. \tag{6.6}
\]

Let us now consider \(h_+ - h_- = \text{integer}\). This condition implies a discrete tachyon spectrum

\[
 m^2 = \lambda^2 \left[ n^2 + n(d - 1) \right], \tag{6.7}
\]

where \(n\) is a positive integer. The conformal weights are also integer, from Eqs. (2.1),(6.7) it follows, \(h_+ = n + (d - 1)\), \(h_- = -n\). The solution of the differential Eq. (6.4) has again the form \(G(z) = C_1 G_1 + C_2 G_2\), with \(G_1\) given as in Eq. (6.5). \(G_2\) has a complicated expression containing the hypergeometric function \(F\), \(\ln z\) and power series, and diverges for \(z \to \infty\). Again the regularity condition at \(z = \infty\) requires \(C_2 = 0\), so that the solution has the same form as in the previous case.

The integration constant \(C_1\) can be fixed by requiring that the solution has the universal, \(P = 1, (D = 0)\), short-distance behavior \([4]\)

\[
 G \sim \frac{\Gamma(\frac{d}{2})}{2(d - 2)\pi^{d-2}(D^2)^{1-\frac{d}{2}}}. \tag{6.8}
\]
To complete our calculation of the Green functions on dSd and to perform the analytic continuation (3.3) to the Green function in AdSd we need to know explicitly the dependence of $z = (1 + P)/2$ from the coordinates $(r, t, \theta, \theta_b)$ of Eq. (3.3). Using the definition of $P$ in terms of the embedding coordinates $P = \lambda^2 X^A \eta_{AB} X'^B$ and Eq. (3.2), one finds after some algebra

$$P_{dS} = \lambda^2 r r' [- \cosh \theta \cosh \theta' + \sinh \theta \sin \theta' \cos W_{d-3}(\Omega, \Omega')] + \sqrt{\lambda^2 r^2 + 1} (\lambda^2 r'^2 + 1) \cos \lambda (t - t'),$$

(6.9)

where $W_{d-3}(\Omega, \Omega')$ is the geodesic distance between points of coordinates $\Omega = (\theta_1, \ldots \theta_{d-3})$, $\Omega' = (\theta'_1, \ldots \theta'_{d-3})$ on the $S^{d-3}$ sphere. The asymptotic behavior of $P$ on $\mathcal{T}^+$ is

$$\lim_{r, r' \to \infty} P_{dS} = -\lambda^2 r r' [\cosh \theta \cosh \theta' - \sinh \theta \sin \theta' \cos W_{d-3}(\Omega, \Omega') - \cos \lambda (t - t')].$$

(6.10)

We can now perform the analytic continuation $\theta \to i\theta$ in Eq. (6.9) to get the corresponding expressions for (negative) AdSd. We have

$$P_{AdS} = -\lambda^2 r r' [\cosh \theta \cosh \theta' - \sinh \theta \sin \theta' \cos W_{d-3}(\Omega, \Omega') - \cos \lambda (t - t')],$$

(6.11)

where now $W_{d-2}(\Omega, \Omega')$ is the geodesic distance between points on the $S^{d-2}$ sphere. Asymptotically we have,

$$\lim_{r, r' \to \infty} P_{AdS} = -\lambda^2 r r' [\cosh \theta \cosh \theta' - \sinh \theta \sin \theta' \cos W_{d-3}(\Omega, \Omega') - \cos \lambda (t - t')].$$

(6.12)

The minus sign in front of the expression on the right hand of Eq. (6.11) and (6.12) is due to the fact that the analytically continued spacetime is “negative” AdSd. It is a direct consequence of the signature (3.8) of the embedding space.

Analogous calculations enable us to write $P_{ds}$ and $P_{AdS}$ in in the planar, respectively, Poincaré coordinates of Eqs. (3.10), (3.9). We have

$$P_{dS} = \cosh \lambda (\hat{t} - \hat{t}') - \frac{1}{2} e^{\lambda (\hat{t} + \hat{t}')} \lambda^2 |x - x'|^2,$$

$$P_{AdS} = \frac{1}{2} \left[ \frac{u'}{u} + \frac{u}{u'} + \lambda^2 uu' \left(- (\tau - \tau')^2 + |x - x'|^2 \right) \right],$$

(6.13)

where $|x - x'|^2 = \delta_{ab} (x^a - x'^a)(x^b - x'^b)$, $a, b = 1 \ldots (d - 1)$ in the case of $P_{dS}$ and $a, b = 1 \ldots (d - 2)$ in the case of $P_{AdS}$. The corresponding asymptotical expressions are,

$$\lim_{\hat{t}, \hat{t}' \to \infty} P_{dS} = -\frac{1}{2} e^{\lambda (\hat{t} + \hat{t}')} \lambda^2 |x - x'|^2,$$

(6.14)

$$\lim_{u, u' \to \infty} P_{AdS} = \frac{1}{2} uu' \lambda^2 \left(- (\tau - \tau')^2 + |x - x'|^2 \right).$$
7. Analytical continuation from dS/CFT to AdS/CFT

We will now discuss the correlation functions induced on the boundary of dS by the propagation of the scalar field \( \phi \) on the bulk and their analytic continuation. Following Strominger \[6\], the two-point correlator of an operator \( \mathcal{O}_\phi \) on the boundary \( \mathcal{I}^+ \) of dS is derived from the expression

\[
\mathcal{J} = \lim_{r \to \infty} \int_{\mathcal{I}^+} dV_{d-1} dV'_{d-1} \{ (rr')^d \left[ \phi(r, y) \frac{\partial}{\partial r} G(r, y, r', y') \frac{\partial}{\partial r'} \Phi(r', y') \right] \} \bigg|_{r=r'},
\]

(7.1)

where \( G \) is the de Sitter invariant Green function and \( dV_{d-1} \) and \( y = (\theta, t, \Omega) \) denote, respectively, the measure and the coordinates of the boundary \( \mathcal{I}^+ \). For a tachyon in dS, we can find the boundary correlators using Eqs. (5.7), (6.6) and (6.10) in Eq. (7.1). One easily finds the coefficient of the quadratic term in Eq. (7.1), which is identified with the two-point correlator of an operator \( \mathcal{O}_\phi \) dual to \( \phi^- \),

\[
\langle \mathcal{O}_\phi(y) \mathcal{O}_\phi(y') \rangle_{dS} = \kappa_0 \left[ \cosh \Delta \theta - \cos \Delta t \right]^{h_+},
\]

(7.2)

where \( \kappa_0 \) is a constant. We will show later in detail that the short-distance behavior of the correlation functions (7.2) is that pertaining to a Euclidean CFT operator with conformal weight \( h_+ \).

Performing the analytic continuation \( \theta \rightarrow i \theta \) one finds the correlation functions induced on the boundary \( \mathcal{B} \) of AdS by propagation of \( \phi \) on the AdS bulk,

\[
\langle \mathcal{O}_\phi(y) \mathcal{O}_\phi(y') \rangle_{AdS} = \frac{\kappa_0}{\left[ \cos W_{d-2}(\Omega, \Omega') - \cos \lambda(t - t') \right]^{h_+}}.
\]

(7.3)

At short-distance, the correlators describe a \((d-1)\)-dimensional CFT in Minkowski space.

The physical meaning of the correlation functions (7.2), (7.3) and of the analytic continuation relating them, can be better understood considering the simplest, \( d = 3 \), case. For \( d = 3 \), Eq. (7.2) becomes

\[
\langle \mathcal{O}_\phi(\theta, r) \mathcal{O}_\phi(\theta', r') \rangle_{dS} = \frac{\kappa_0}{\left[ \cosh \Delta \theta - \cos \Delta t \right]^{h_+}}.
\]

(7.4)

where \( \Delta \theta = \theta - \theta' \) and \( \Delta t = t - t' \). Correlation functions of this kind have been already discussed in [8]. It was argued that they are thermal correlation functions for a two-dimensional Euclidean CFT, with the compact dimension of the cylindrical geometry being spacelike rather than timelike (remember that in Eq. (7.4) both \( \theta \) and \( t \) are spacelike coordinates). Alternatively, one can interpret (7.4) as a thermal correlator at imaginary temperature \( iT = i \lambda/2\pi \). In fact using complex coordinates, \((w, \bar{w})\) the correlator (7.4) can be written

\[
\langle \mathcal{O}_\phi(w, \bar{w}) \mathcal{O}_\phi(w', \bar{w}') \rangle_{dS} \sim \left[ \sinh(i\pi T \Delta w) \sinh(i\pi T \Delta \bar{w}) \right]^{-h_+}.
\]

(7.5)
For \( d = 3 \) the analytically continued correlator (7.3) is,

\[
\langle \mathcal{O}_\phi(\theta, t)\mathcal{O}_\phi(\theta', t') \rangle_{AdS} = \frac{\kappa_0}{[\cos \Delta \theta - \cos \Delta \lambda t]^{h_+}}
\]

(7.6)

where \( \Delta \theta = \theta - \theta' \) and now \( 0 \leq \theta \leq 2\pi \) is the angular coordinate of the \( S^1 \) sphere.

Defining light-cone coordinates \( \lambda x^+ = \theta + \lambda t, \lambda x^- = \lambda t - \theta \) Eq. (7.4) takes the form of a thermal correlator at temperature \( T = \lambda / 2\pi \) in two-dimensional Minkowski space,

\[
\langle \mathcal{O}_\phi(x^+, x^-)\mathcal{O}_\phi(x'^+, x'^-) \rangle_{AdS} \sim \left[ \sin(\pi T \Delta x^+) \sin(\pi T \Delta x^-) \right]^{-h_+}.
\]

(7.7)

This is what one expects to happen because \( t \) is a timelike, periodic coordinate of the AdS spacetime with metric (3.6).

The analytic continuation (3.3) relating \( dS_3 \) with \( AdS_3 \) maps the thermal Euclidean 2D CFT at imaginary temperature (7.5) living on the boundary of \( dS_3 \) into the thermal Euclidean 2D CFT with same, but real, temperature (7.7) living on the boundary of \( AdS_3 \).

Until now our discussion of the boundary correlation functions was confined to the case in which \( dS_d \) and \( AdS_d \) are described by the metrics (3.3), respectively, (3.6). The emergence of thermal correlation functions is related with the presence a compact direction \( t \) (spacelike for \( dS_d \) and timelike for \( AdS_d \)). This is a peculiarity of the parametrizations (3.3), (3.6), which is in particular relevant when one wants to describe the large-distance behavior of the correlation function. If one wants to describe the short-distance behavior of the correlation functions, the planar (Poincaré) coordinates used in Eq. (3.10) (3.9)) for \( dS_d \) (\( AdS_d \)) are more appropriate. Using the parametrization (3.10) for \( dS_d \), the asymptotic form for the Green function (6.6), for the scalar \( \phi \) (5.10) and for \( P \) (6.14) in the integral (7.1), one finds for the boundary correlators

\[
\langle \mathcal{O}_\phi(x)\mathcal{O}_\phi(x') \rangle_{dS} = \frac{\kappa_0'}{|x - x'|^{2h_+}}.
\]

(7.8)

In planar coordinates, correlators induced by a tachyon on the boundary of \( dS_d \) have the usual short-distance behavior of a \( d - 1 \)-dimensional Euclidean CFT. The operators \( \mathcal{O}_\phi \) have conformal dimension \( h_+ \). The analytic continuation (3.11), which maps \( dS_d \) in planar coordinates into \( AdS_d \) in Poincaré coordinates brings the correlators (7.8) into the form

\[
\langle \mathcal{O}_\phi(t, x)\mathcal{O}_\phi(t', x') \rangle_{AdS} = \frac{\kappa_0'}{[\tau - \tau'|^2 - |x - x'|^2]^{h_+}}
\]

(7.9)

which are the correlation functions for a Lorentzian \((d - 1)\)-dimensional CFT living on the boundary of \( AdS_d \).

The tachyonic nature of the scalar field propagating in de Sitter space is essential to assure that the analytic continuation maps the Euclidean CFT on \( \mathcal{I}^+ \) into the
Minkowskian CFT on $\mathcal{B}$. The tachyonic character of $\phi$ implies that only the weight $h_+$ is positive, whereas $h_-$ is negative. We can therefore use the “unitarity” condition on $G$ discussed in Sect. 6 to rule out operators with conformal dimension $h_-$ from the Euclidean CFT living on the boundary on $dS_d$. This is exactly what we need, since we know that only operators with conformal dimension $h_+$ appear in the Lorentzian CFT living on the boundary of AdS$_d$.

8. Conclusion

In this paper we have investigated the analytical continuation from $d$-dimensional Lorentzian de Sitter to $d$-dimensional Lorentzian anti-de Sitter spacetime. The obstructions to perform this continuation can be removed by choosing a particular coordinatization of $dS_d$ (physically, the coordinate system used by an observer at the equator of the $S^{d-1}$ sphere) and considering tachyons in $dS_d$ in correspondence with particles with positive square-mass in AdS$_d$. An important result of our paper is that the AdS/CFT duality can be obtained as analytical continuation of the dS/CFT duality. The correlations induced by tachyons on the boundary of $dS_d$ are in correspondence with the correlations induced by particles with positive square-mass in the boundary of AdS$_d$.

Because we know much more about AdS/CFT then about dS/CFT, the results of this paper could be very useful to improve our understanding of the dS/CFT duality. In particular, the correspondence we have found between dS/CFT and AdS/CFT could help us to clarify the status of the dS spacetime in the string theory context. Moreover, a feature of our analytic continuation may hold the key for understanding the problems that $dS_d$ inherits from its nature of time-dependent gravitational background. Our analytical continuation exchanges, modulo the overall sign of the metric, the timelike direction of $dS_d$ with the radial spacelike direction of AdS$_d$. At the same time a spacelike direction of the $dS_d$ boundary becomes a timelike direction of the AdS$_d$ boundary. This feature gives a simple explanation of how conserved quantities, associated with spacelike Killing vectors in $dS_d$ are mapped into conserved quantities, associated with timelike Killing vectors in AdS$_d$. The $dS_d \rightarrow$AdS$_d$ analytical continuation can be therefore used to give a natural definition of the mass and entropy of the dS spacetime (see Ref. [22, 23] and Ref. [16] for a discussion of the 2D case.

The exchange of the timelike/spacelike nature of directions, in passing from $dS_d$ to AdS$_d$ seems to be at the core of the relationship between dS/CFT and AdS/CFT. For instance, we have seen in section 7 that boundary thermal correlators, with correlations along spacelike directions, in the case of $dS_d$ become thermal correlators, with correlations along timelike directions, for AdS$_d$. One could also use this feature to try to circumvent the arguments of Ref. [13] against the dS/CFT correspondence.
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