Abstract

We characterize the structure and origins of missingness for 159 cross-sectional return predictors and study missing value handling for portfolios constructed using machine learning. Simply imputing with cross-sectional means performs well compared to rigorous expectation-maximization methods. This stems from three facts about predictor data: (1) missingness occurs in large blocks organized by time, (2) cross-sectional correlations are small, and (3) missingness tends to occur in blocks organized by the underlying data source. As a result, observed data provide little information about missing data. Sophisticated imputations introduce estimation noise that can lead to underperformance if machine learning is not carefully applied.

JEL Classification: G0, G1

Keywords: stock market predictability, stock market anomalies, missing values, machine learning
1 Introduction

A growing literature applies methods from machine learning (ML) to asset pricing. These studies combine the information in dozens, or even hundreds, of cross-sectional stock return predictors. Freyberger et al. (2020) combine 62; Kelly et al. (2023) combine 138; and Han et al. (2022) combine up to 193 predictors. Each of these papers finds that there are economic gains to expanding the set of predictors beyond the five used in Fama and French (2015).\textsuperscript{1}

Buried in this literature is the problem of missing values. When learning from many predictors, the standard practice of dropping stocks with missing values is often untenable. For example, applying the standard practice to the 125 most-observed predictors in the Chen and Zimmermann (2022) dataset drops 99% of stocks. So even though imputing missing values may seem dangerous, ML researchers often have no choice but to impute.

We examine missing data handling for 159 predictors using a broad array of imputation and return forecasting methods. We focus on cross-sectional expectation-maximization (EM) and cross-sectional mean imputation, but also examine several other methods, including some that incorporate time-series information. Our baseline forecasts use principal component regressions (PCR), but we also examine neural networks and gradient boosting. These methods are chosen to provide an overview and intuition behind missing data issues, rather than to advocate for a particular imputation algorithm.

We find that imputing with cross-sectional means (as in Kozak, Nagel and Santosh (2020); Gu, Kelly and Xiu (2020)) does a surprisingly good job of capturing the potential expected returns. Imputing with means and then sorting stocks on return forecasts from a three-layer neural network leads to long-short decile returns of 66% per year equal-weighted or 37% per year value-weighted. Imputing with cross-sectional EM leads to nearly identical returns: 67% equal-weighted and 39% value-weighted.\textsuperscript{2} This invariance is seen across most fore...

\textsuperscript{1}Lewellen (2015) finds that regressions on 15 predictors lead to an annualized Sharpe Ratio of around 0.8 (excluding micro-caps). Freyberger et al. (2020); Kelly et al. (2023); and Han et al. (2022) find Sharpe ratios in excess of 1.8, and as high as 2.4 (excluding micro-caps or value-weighting). Other papers that combine many predictors include Haugen and Baker (1996); Green et al. (2017); DeMiguel et al. (2020); Gu et al. (2020); Lopez-Lira and Roussanov (2020); Kozak et al. (2020); Chen and Velikov (2022); Simon et al. (2022); Chen et al. (2023).

\textsuperscript{2}The returns from PCR in mean-imputed data are 51% and 32% (equal- and value-weighted, respectively), compared to 54% and 36%, in EM-imputed data. All results use the usual assumption of no transaction costs, including no shorting fees.
casting and imputation methods, despite the fact that EM has strong theoretical properties and is recommended in textbooks on missing data handling and machine learning (Little and Rubin (2019); Efron and Hastie (2016)).

This invariance is intuitive given three properties of cross-sectional predictor data. The first is that missingness occurs in large blocks organized by time. For common stocks on major exchanges that are missing book-to-market this month, 81% are also missing book-to-market in every previous month. As a result, there is little time-series information about missing values.

The second key property is that predictors are largely uncorrelated cross-sectionally (Green, Hand and Zhang (2013); Chen and Zimmermann (2022)). Almost all correlations lie between $-0.25$ and $+0.25$, and the first 10 principal components span only 40% of total variance. Thus, most observed predictors contain little cross-sectional information about the missing predictors.

The third key property is that missingness tends to occur in blocks organized by the underlying data. For stocks that are missing book-to-market, about 90% are also missing earnings-to-price. So even though book-to-market and earnings-to-price have a nontrivial cross-sectional correlation of around 0.3, this correlation can rarely be used for imputing book-to-market.

Overall, we find the observed predictors provide little information about the missing predictors, so one might as well impute with cross-sectional means.

In some settings, EM imputation can even lead to underperformance. Our baseline results form forecasts separately for micro-cap, small-cap, and big stocks, motivated by the fact that predictability can differ by market cap and market liquidity (Fama and French (2008); Chen and Velikov (2022)). If we instead form forecasts using all stocks simultaneously, EM often leads to smaller value-weighted returns compared to simple mean imputation, though equal-weighted returns are generally similar.

This underperformance is consistent with the idea that sophisticated imputations introduce estimation noise. This noise can lead to underperformance, particularly if the forecasts are not carefully structured. Consistent with this interpretation, we find that EM imputation errors are larger among small stocks in random masking exercises.

All together, we recommend using simple cross-sectional mean imputation for handling missing values when applying machine learning to cross-sectional return predictors. In theory, EM should be less biased, more efficient, and thus
lead to more accurate forecasts. In practice, the missingness and covariance structures of cross-sectional predictors imply that the improvements are small, and estimation noise can even lead to underperformance.

The high dimensionality of cross-sectional predictors naturally raises the question of how many of these dimensions contribute to expected returns. Our principal components (PC) regression tests find that the answer is subtle and depends on multiple measurement choices. Computing PCs the standard way, we find that about 75 PCs contribute to the potential equal-weighted returns of 50% per year, while 40 PCs contribute to the potential value-weighted returns of about 20%. However, standard PC analysis is an unsupervised method that ignores return information. Using Huang et al. (2022)’s scaled PCA to incorporate return information leads to a much smaller dimensionality: about 30 PCs are required when equal-weighting while 15 are required when value-weighting.

We also describe the origins of missing values in the Chen and Zimmermann (2022) dataset. We find there are three main drivers: (1) missing underlying data, (2) predictors requiring a long history of underlying data, and (3) using missingness as a substitute for interaction effects. Requiring accounting variables leads to an observed share of around 70% in 1985 and requiring analyst forecasts drops the observed share to about 50%. Variables that require a long history of data include 5-year sales growth, which is observed for about 40% of stocks. Interaction effects are implicit in predictors like asset tangibility and payout yield, which are replaced with missing values for non-manufacturing firms and stocks with non-positive payout, respectively. These drivers of missingness can themselves interact and lead to extremely low rates of observability. For example, Asquith et al. (2005)’s institutional ownership among high short interest stocks predictor requires both specialized underlying data and uses missingness to represent interaction effects, leading to a 0.2% of stocks having observations in 1985.

We post all of our code at https://github.com/jack-mccoy/missing_data. The EM imputed data can be found at https://sites.google.com/site/chenandrewy/.

1.1 Related Literature

In contemporaneous work, Freyberger, Höppner, Neuhierl and Weber (2023) (FHNW), Bryzgalova, Lerner, Lettau and Pelger (2023) (BLLP), and Beckmeyer and Wiedemann (2023) also study missing values in cross-sectional predictor data. Each paper advocates a different imputation algorithm, though all avoid
modeling the missingness process. FHNW use moment conditions designed for cross-sectional predictors, BLLP use Xiong and Pelger (2023)'s latent factor method, and Beckmeyer and Wiedemann use a masked language model. Our paper takes a more neutral approach and compares textbook imputation methods (Little and Rubin (2019)) with the cross-sectional mean imputation used in asset pricing. Our approach leads us to focus on practical issues, like whether imputation affects inferences made from standard ML algorithms like neural networks and gradient boosting (Efron and Hastie (2016)). We also are distinct in providing a detailed discussion of the origins of missingness for 159 predictors and exploring the intuition for why simple mean imputation seems to perform well in ML applications, including using neural network models and FHNW’s regularized non-linear strategies.

The contemporaneous papers also differ in their predictor and stock return datasets. In this respect, FHNW is the most similar to ours. We both use 80+ predictors from the CZ open-source dataset and cover all common stocks in CRSP. Beckmeyer and Wiedemann (2023) examine 143 characteristics from the Jensen et al. (Forthcoming) open source dataset and restrict stock-months to those with data on at least 20% of the predictors. BLLP examine 45 characteristics from Kozak et al. (2020) and restrict stocks to those with at least one Compustat characteristic. Given the size and breadth of these data, and the diversity of statistical methods, the empirical results in these studies are complementary.

Our results provide a rigorous justification for the ubiquitous use of mean imputation in machine learning papers. Almost every paper that attempts to combine large sets of predictors imputes with cross-sectional means or medians, including Gu, Kelly and Xiu (2020); Freyberger, Neuhierl and Weber (2020); Kozak, Nagel and Santosh (2020). This result is important because the algorithms in these papers are often complex, so a simple imputation algorithm is helpful for transparency.

Our PC analyses relate to the ongoing debate about the factor structure of the cross-section of returns. Our findings are consistent with the idea that there are many dimensions of predictability if one is interested in all stocks (Green, Hand and Zhang (2014); Bessembinder, Burt and Hrdlicka (2021)) and that a moderately strong factor structure is possible if one considers only large stocks (Green, Hand

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3 Other machine learning style papers that examine imputing with cross-sectional averages include Green et al. (2017); DeMiguel et al. (2020); Fulop et al. (2022); Han et al. (2021); Azevedo et al. (2022); Da et al. (2022); Simon et al. (2022); Zhang and Aaraba (2022); Jensen et al. (2022).
Hand and Zhang (2017); Lettau and Pelger (2020); Kozak et al. (2020)). Our findings complement Lopez-Lira and Roussanov (2020), who document large Sharpe ratios in strategies that hedge the principal components of individual stock returns.

2 The Structure and Origins of Missing Predictor Data

We describe the structure and origins of missingness among published cross-sectional predictors. We also explain why imputation is required for large scale machine learning studies.

2.1 Cross-Sectional Predictor Data

Our predictors data begins with the August 2023 release of the CZ dataset. This release contains 212 cross-sectional predictors published in academic journals. We merge this data with CRSP and keep only common stocks (SHRCD 10, 11, or 12) listed on the NYSE, NYSE MKT, or NASDAQ (EXCHCD 1, 2, 3, or 31, 32, 33). This ensures that the missing data problems we study apply to widely-studied common stocks, and do not occur because of the rise of ETFs or other more exotic securities. The screens we use follow Banz (1981), Fama and French (1992), and Bali et al. (2016) among others.

We drop the 33 discrete predictors because the missing values in this subset are closely related to the interpretation of these predictors by CZ. Many of these predictors draw on event studies (e.g. exchange switch), which CZ interpret as a binary firm characteristic (following McLean and Pontiff (2016)). Others are based on double-sorts (e.g. analyst recommendation and short interest). In both cases, CZ use missing values for any stocks that belong in neither the long or short portfolio, though there are other ways to interpret these studies.

We drop an additional 20 predictors because they have months with fewer than 2 stock observations in the 1985-2021 sample. 18 of these predictors use data that does not begin until after 1985. The remaining predictors are junk stock momentum (Mom6mJunk) and firm age - momentum (FirmAgeMom), which

4Previous versions of this paper did not apply these screens. We thank Jun Kyung Auh for pointing out this issue.
have small gaps in recent years. This screen ensures the predictor data is relatively well-behaved and avoids the use of special techniques to handle predictor-months with no data.

The final 159 predictors includes not only those based on CRSP and Compustat, but also predictors that use IBES analyst forecast data. This inclusion is important, as analyst forecasts may offer a distinct dimension of predictability. Unfortunately, it excludes option-based predictors, as option prices are not observed until the 1990s.

2.2 Where Does Missingness Come From?

Table 1 presents a list of selected predictors that illustrates the origins of missingness. The predictors are sorted by the share of stocks with observations in 1985, a year that is fairly representative of other years. The observed share ranges from 99.8% for size all the way down to 0.2% for institutional ownership among high short interest stocks. We curate this list from the full list of 159 predictors, which can be found in the Internet Appendix (Table IA.1).

[Table 1 about here]

The most-observed predictors use only recent CRSP data. These predictors include short-term reversal, idiosyncratic volatility, and size, and are observed for nearly 100% of stocks. Requiring a longer history of CRSP data leads to a small but non-trivial share of missing values. 12-month momentum is observed for 92% of stocks. A similar amount of CRSP data is required for coskewness, beta, and two of the zero trade predictors.

Requiring accounting variables drops the observed share into the 60% to 75% range. Predictors that fall in this range include B/M and asset growth. Many other accounting-based predictors fall in this range, including employee growth, leverage, taxes, external financing, and accruals.

Rates of observability below 50% are driven by a wide variety of issues. Most of these predictors require a long history of accounting or CRSP data, such as return seasonality in years 11-15 or revenue growth rank, which uses the past 5 years of sales growth. Long histories of data are also required for intangible value, Titman, Wei and Xie (2004)’s investment predictor, and composite debt issuance.

But many of the less-observable predictors are missing values simply because the data items they require are often missing. Predictors that use analyst forecast
data are observed for at most 50% of stocks, such as EPS forecast revision. Earnings surprise streak requires a long history of EPS forecasts, leading to an even smaller share. Data items are also often missing for less-common accounting variables, like advertising expense.

Still other predictors have missing values due to a diverse range of specialized filters. Asset tangibility applies only to manufacturing firms, while IPO and age only applies to recent IPOs. Payout yield drops stocks with non-positive payout. In CZ’s interpretation, turnover volatility drops stocks in the top two size quintiles, and operating profitability drops stocks in the bottom size tercile. The original Piotroski F-score (PS) study only examines stocks in the highest BM quintile.

At the bottom of Table 1 are selected predictors that are observed for less than 10% of stocks in 1985. These very low rates of observability are sometimes due to highly specialized data. Conglomerate return uses Compustat segment data and customer momentum uses the BEA input-output tables. Yet others combine specialized data with the use missingness to substitute for interaction effects, such as institutional ownership for high short interest.

As seen in these details, missingness is sometimes used as a substitute for modeling non-monotonicity and interaction effects. Incorporating these interactions is non-trivial. It requires going into the code behind the individual predictors, removing the missingness assignments, and carefully introducing new interaction variables. We leave this work for future research.

### 2.3 Summarizing Missingness for Individual Predictors

Table 2 describes missingness at the individual predictor level. For each predictor, we compute the share of stocks that have observed predictors in selected months between 1970 and 2000. Panel (a) shows order statistics across predictors.

[Table 2 about here]

Though many cross-sectional predictability studies begin in 1963, at least 5% of our 159 predictors have no observations until about 1980. Several of these predictors draw on analyst forecast data, which is not generally observed until 1985 (using IBES).

Missingness reaches a steady state in 1985, as the order statistics are largely unchanged after this date. In this steady state, the typical predictor is observed
for 70% of stocks, and 75% of predictors are observed for roughly 50% of stocks. While this distribution suggests that missing values are a relatively modest problem, we will see that combining large sets of predictors compounds the issue.

This steady state suggests that missingness in the time series exhibits structural breaks: predictor data is continually missing until the data begins being published, after which the data is continually observed.

2.4 Why Imputation is Often Required

When combining predictors, one typically needs a numerical value for all predictors for each stock-month in consideration. This requirement compounds the magnitudes in Panel (a) of Table 2, leading to a dramatically larger missingness problem.

This problem is illustrated in Figure 1, which shows the “missingness map” for the 159 predictors among common stocks in June 1990. Roughly half of the map is non-shaded, indicating that roughly half of the data is observed. This does not mean, however, that roughly half of stocks can be used in a machine learning algorithm without imputation. In general, only stocks with no missing values can be used, corresponding to columns in the missingness map that are entirely non-shaded. Zero columns satisfy this requirement.

[Figure 1 about here]

Panel (b) of Table 2 quantifies this problem. This panel examines the prevalence of missing values when combining \( J \) predictors, where \( J \) ranges from 25 to 150. The \( J = 25 \) row examines the 25 predictors with the most observations in selected months. It shows that one can combine 25 predictors with minimal missing data issues. In June 1985, 96% of predictor-stocks have observations and 86% of stocks have predictor data for all of these 25 predictors.

But when combining for 75 or more predictors, one needs to take a stand on imputation. As seen in the \( J = 75 \) row, simply dropping stocks with missing values would result in dropping all but 16% of the data. For \( J = 125 \), essentially the entire dataset is gone if one follows the traditional missing value handling used in individual predictor studies.

Returning to Figure 1, the complex missingness patterns show that a general purpose missing data imputation is required. There are dozens of distinct
shapes in Figure 1, suggesting that dozens of equations are required to model the specific missingness mechanisms. It is unlikely that an analyst could convincingly present such a complicated model. The only alternative is to assume Rubin (1976) ignorability, which leads to our focus on EM methods (Section 3.2).

2.5 The Structure of Missingness

Figure 1 sorts the predictors by the type of data they focus on (according to the CZ documentation). This sort shows that missingness occurs in blocks, organized by data type. Stocks that are missing some accounting predictors are often missing every single accounting predictor. The same pattern arises with analyst forecast predictors, as seen in the dark vertical lines that span the “Analyst” section of the plot.

Block structures also occur within data categories, consistent with the fact that missingness often occurs due to the requirement of having a certain amount of historical data. This block structure suggests that extracting information about missing values from cross-sectional observations may be difficult.

Missingness is extremely persistent. For predictor-stock combinations that are missing in June 1990 (everything except for the lightest shade), the vast majority were never observed before June 1990 (darkest shade). Only a tiny share has observations in the past 12 months. This structure implies that there is very little time-series information about missing values.

This missingness structure leads us to focus on linear cross-sectional imputations in our baseline methods. These methods focus on the part of the data which contains more information about the missing values (the cross-section), and they avoid over-parameterizing the model when the amount of information in the cross-section is likely limited.

3 Baseline Imputation Methods

The missingness patterns imply that one must impute data when combining more than 100 predictors. This section describes our baseline imputation methods: EM and simple mean.

These imputations are important benchmarks. EM is recommended in the influential Little and Rubin (2019) textbook on missing data. Simple mean im-
putation is the most common method in ML-style asset pricing.

We examine several additional imputations in Section 5.

3.1 Preprocessing

Before applying any imputation, we use the following preprocessing. First, we winsorize across stocks symmetrically at the 1% level for each predictor-month. Second, we apply Hawkins and Weisberg (2017)'s extension of Box and Cox (1964) to make each predictor approximately normal. Finally, we standardize predictors to have zero mean and unit variance.

The Box-Cox transform is not commonly found in asset pricing, but it is a natural extension of the log transformation that is typically applied to size and B/M (Fama and French (1992)). More broadly, cross-sectional predictability studies rarely impose a functional form on the relationship between the predictor predictors and expected returns, so one might as well standardize the predictors to be approximately normal.

3.2 Cross-Sectional EM Imputation

There are $N$ stocks, and stock $i$ in month $t$ has a vector of predictor values $X_{i,t}$. Missing values are represented by $X_{\text{miss},i,t}$ and $X_{\text{obs},i,t}$, which are the missing and observed subvectors of $X_{i,t}$.

Suppose $\hat{\Sigma}_t$ is a reasonable estimate for $\Sigma_t$, the cross-stock covariance matrix of $X_{i,t}$. For example, one could let $\hat{\Sigma}_t$ be the sample covariance across pairs of stocks that have predictor data, as in BLLP. Then, an intuitive way to impute $X_{\text{miss},i,t}$ is to use:

$$\hat{X}_{\text{miss},i,t} = \hat{\beta}_{i,t}' X_{\text{obs},i,t},$$  \hspace{1cm} (1)$$

where

$$\hat{\beta}_{i,t} = \hat{\Sigma}_{\text{obs,obs},i,t}^{-1} \hat{\Sigma}_{\text{obs,miss},i,t},$$  \hspace{1cm} (2)$$

and $\hat{\Sigma}_{\text{miss,obs},i,t}$ and $\hat{\Sigma}_{\text{obs,obs},i,t}$ are the submatrices of $\hat{\Sigma}_t$ corresponding to the

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5For example, if $X_{i,t} = [1 \ 2 \ 4 \ 5]'$, and the first and third elements are missing, then $X_{\text{miss},i,t} = [1 \ 3]'$ and $X_{\text{obs},i,t} = [2 \ 4]$. $X_{i+1,t}$ may have a different set of missing predictors, and thus the length of $X_{\text{miss},i+1,t}$ and $X_{\text{obs},i+1,t}$ may be different compared to $X_{\text{miss},i,t}$ and $X_{\text{obs},i,t}$. 

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missing and observed predictors for stock $i$. Analogous to the classic OLS formula $\hat{\beta} = [X'X]^{-1}X'y$, Equations (1)-(2) use the covariance between the LHS and RHS variables to forecast the LHS. Equation (1) omits the intercept since we standardize variables to have mean zero (Section 3.1).

The problem with this approach is that it is not self-consistent. That is, the imputed data would generally imply

$$\hat{\Sigma}_t \neq N^{-1} \sum_{i=1}^{N} \hat{X}_{i,t}' \hat{X}_{i,t},$$

(3)

where $\hat{X}_{i,t}$ combines $\hat{X}_{\text{miss},i,t}$ and $\hat{X}_{\text{obs},i,t}$. In this case, it is unclear whether you should be using $\hat{\Sigma}_t$ or $N^{-1} \sum_{i=1}^{N} \hat{X}_{i,t}' \hat{X}_{i,t}$ to be imputing missing values.

A secondary issue is that Equations (1)-(2) do not tell you how to deal with higher order missing terms. That is, not only is $X_{\text{miss},i,t}$ unknown, but so is $X_{\text{miss},i,t}'X_{\text{miss},i,t}$. The expected value of $X_{\text{miss},i,t}'X_{\text{miss},i,t}$ is different than the outer product of Equation (1), and is potentially important for a rigorous estimate of $\Sigma_t$. An intuitive way to estimate this higher order term is to use

$$\left[\bar{X}_{t,i}'X_{t,i}\right]_{\text{miss},i} = \Sigma_{\text{miss,miss},i,t} - \hat{\beta}_{i,t}' \Sigma_{\text{obs,miss},i,t} + \hat{X}_{\text{miss},i,t}' \hat{X}_{\text{miss},i,t},$$

(4)

which can be derived from Gaussian updating formulas.

The EM algorithm solves the self-consistency problem by iterating between Equations (1)-(2) and Equation (3):

1. E-step: impute missing data with the observed data and the current guess $\hat{\Sigma}_t$, using Equations (1) and (2).

2. M-step: estimate a new guess $\hat{\Sigma}_t^\text{new}$ by plugging the imputed data from step 1 into the RHS of Equation (3). Add to $\hat{\Sigma}_t^\text{new}$ the higher order correction implied by Equation (4) for submatrices corresponding to $\hat{X}_{\text{miss},i,t}' \hat{X}_{\text{miss},i,t}$. \footnote{Some descriptions of EM omit the higher order correction (e.g. Roweis (1997); Efron and Hastie (2016)).}

These two steps are repeated until $\|\hat{\Sigma}_t^\text{new} - \hat{\Sigma}_t\|_\infty \leq \delta$. We use $\delta = 10^{-4}$.

The E- and M-steps can be applied to almost any imputation and estimation formulas. Our main results focus on the simple cross-sectional regression formula (Equations (1)-(2)). This method avoids look-ahead bias and is intuitive
given the missingness structure described in Section 2.5. We examine variations that assume a factor structure and take on time-series information in Section 5.

More formally, this EM algorithm can be derived from maximum likelihood (Dempster et al. (1977)), assuming that the process generating the missing data is “ignorable” in the sense of (Rubin (1976)). A sufficient condition for ignorability is that data is “missing at random,” which is a technical condition that is best described as the weakest general condition that leads to ignorability (Little (2021)). This notion of “missing at random” is different than the one used in Xiong and Pelger (2023), whose notion of “missing at random” is often described as “missing completely at random” (Little and Rubin (2019)). We provide a formal derivation of our EM algorithm and discuss these technical issues in the Internet Appendix.

This EM algorithm is sometimes described as “requiring” the assumption of normality (e.g. Freyberger et al. (2023), BLLP). But while normality along with ignorability are sufficient conditions for the validity of EM, they are not necessary assumptions. Indeed, simulation studies find this algorithm works well even if the data are not normal (Little (1988); Azen et al. (1989); Graham and Schafer (1999); King et al. (2001)). This robustness is consistent with the intuition that EM is an iterative form of OLS, and like OLS, is maximum likelihood under ideal conditions but works well in real world settings. Little (1988) shows that allowing for fat tails leads to the same algorithm but with smaller weights on the extreme observations. As we winsorize our data before imputation, such a fat tail extension should have little effect on our results.

### 3.3 Simple Mean Imputation

Simple mean imputation replaces missing values with the mean observed value conditional on the predictor-month. Given our pre-processing (Section 3.1), simple mean imputation amounts to replacing missing predictor values with zeros, and median imputation is equivalent.

Mean imputation could lead to significantly biased inferences (BLLP). But Equations (1) and (2) show this may not be the case. Mean imputation is the special case of EM (and thus maximum likelihood) in which the off-diagonal terms of \( \hat{\Sigma}_t \) are zero. In this case, \( \hat{\Sigma}_{\text{miss,obs}|i,t} \) is a matrix of zeros, the imputation slopes \( \hat{\beta}_{i,t} \) is a vector of zeros, and \( \hat{X}_{\text{miss}|i,t} \) is also a vector of zeros.
More generally, if the off-diagonal terms of $\Sigma_t$ are close to zero for predictor pairs that can be used for imputation, then mean imputation should be similar to EM and other algorithms focused on $\Sigma_t$.

Studies of large sets of cross-sectional predictors find that the off-diagonal terms of $\Sigma_t$ are mostly close to zero (Green et al. (2017); Chen and Zimmermann (2022)). We verify this result in Section 4.3. Moreover, the predictor pairs that can be used for imputation often cross data categories (Figure 1), implying the relevant covariances are even smaller than those that have been documented. Lastly, assuming the off-diagonal terms are small can be thought of as a structural restriction, implied by the idea that predictors need to be novel to be worthy of publishing (Chen and Zimmermann (2020)).

Mean imputation also has the advantages of tractability and transparency. These advantages are notable in our setting due to the number and complexity of the missingness patterns in Figure 1. For each missingness pattern, EM imputation implies a different matrix inversion, and every matrix inversion must be done in every iteration of the algorithm. There are other general-purpose algorithms that are more tractable, but it can be difficult to formally derive their statistical properties under general assumptions (e.g. Roweis (1997)’s pPCA). The transparency of mean imputation is also important, given the complexity of the ML procedures that are applied to imputed data.

4 EM vs Simple Mean Imputation

This section provides the main results. We compare the performance of EM vs mean imputation for predicting long-short returns in single sorts (Section 4.1) or PCR portfolios (Section 4.2). We explain why this performance is largely similar (Section 4.3) and why EM can sometimes underperform (Section 4.4).

We also show how Huang et al. (2022)’s scaled PCR leads to a smaller dimensionality of expected returns, though imputations still have little effect on inferences about expected returns (Section 4.5).

4.1 Single-Predictor Strategies

We start with single-predictor strategies because they are so familiar to asset pricing researchers. It is unlikely that missing data handling has a large effect
here, but these results are helpful “descriptive” statistics before the main results.

Our single-predictor strategies go long the 500 stocks with the strongest predictor and short the 500 stocks with the weakest predictor each month. We do this for two missing data handling methods: (1) EM imputation and (2) dropping stocks with missing values.

This design allows us to examine the most common method for handling missing values (dropping missing) while simultaneously removing “predictor dilution” effects that can occur when using imputed data. Intuitively, the imputed data covers more stocks, and if these additional stocks have more neutral predictors, the extreme deciles using imputed data will display less predictability simply because of this additional coverage. Fixing the number of stocks in each leg controls for this issue.

Table 3 shows the resulting distribution of mean returns and Sharpe ratios. EM generally improves the distribution of performance, but the typical effect is small, and it comes at the cost of worse performance for many predictors. The distribution of mean returns shifts to the right overall, but the left tail shifts to the left, particularly in equal-weighted portfolios. Using Sharpe ratios, the typical improvement is effectively zero, while both tails shifts outward.

[Table 3 about here]

### 4.2 159-Predictor Strategies Using PCR

We now examine the more compelling setting for studying missing values: strategies that combine information from many predictors. We combine information using principal components regression (PCR), a simple method for handling potentially correlated predictors and potential overfitting problems.

Our PCR analysis forms “real-time” long-short strategies as follows: For each month $t$ between January 1995 and December 2021, we apply the following steps:

1. Separate stocks into three size groups (micro, small, and big) according to market cap by month. Micro is below the 20th percentile NYSE market cap and big is above the 50th percentile NYSE market cap, following Fama and French (2008).

2. For each size group, find the principal components (PCs) of the predictors using data from the past 120 months. Then use OLS to predict stock returns
using the first $K$ PCs.

3. Form a portfolio that goes long the top decile of predicted returns and short the bottom decile. Hold this position for one month.

We make inferences on each size group separately because predictability differs by market cap (Fama and French (2008)) and market liquidity (Chen and Velikov (2022)). Within each size group, OLS and PCA are run pooling observations.\footnote{Pooling can be justified if $\{N^{-1}\sum_{i=1}^{N} X_{i,t}X_{i,t}^t\}_{t=1,2,3,...}$ is weakly dependent, so that the pooled estimator converges to $E\left(X_{i,t}X_{i,t}^t\right)$.}

Figure 2 shows the result. It plots the mean returns (Panel (a)) and Sharpe ratios (Panel (b)) as a function of the number of PCs used to forecast returns. The various lines show equal- or value-weighting, and EM or simple mean imputation.

Figure 2 also suggests an extremely high dimensionality of expected returns. Mean returns and Sharpe ratios increase in the number of PCs, up to at least 40 PCs. This dimensionality may be overstated, however, as PCA does not incorporate return information, and can be thought of as an “unsupervised learning” model. We revisit this issue in Section 4.5, where we examine a “supervised” version of PCA.

### 4.3 Imputation Information and Predictor Correlations

Why do EM and simple mean imputation lead to similar inferences about expected returns?
The answer is that the observed predictors contain little information about the missing predictors. This fact is illustrated in Figure 3, which shows the distribution of correlations between pairs of predictors. The “Observed” lines use all pairs of stocks that have predictor data in the given month (“available case” or “pairwise complete” correlations). The “EM Algo” lines use the correlations estimated from EM.

The correlations cluster near zero, consistent with Chen and Zimmermann (2022). As a result, mean imputation should be a reasonable approximation of EM (Section 3.3). Intuitively, the low correlations imply that the observed value of one predictor tells you relatively little about a missing predictor. Similarly, principal components are relatively uninformative. 10 PCs capture only 40% of total variance. These results are stable over time and do not depend on whether you use available case or EM correlations.

Consistent with the interpretation that the observed data is uninformative, we find that the imputation slopes (Equation (2)) cluster close to zero (Internet Appendix Figure IA.4).

Figure 3 actually overstates the potential information gains. As discussed in Section 2.4, missingness tends to occur in blocks organized around the data source. If a stock is missing an accounting-based predictor, it is likely to be missing all accounting-based predictors. So while earnings-to-price and book-to-market may be correlated, this correlation typically cannot be used to impute book-to-market. Instead, one would need to impute with a more distant predictor like coskewness or trading volume, which are unlikely to tell us much about book-to-market.

Similarly, while Figure 3 focuses on cross-sectional information, one might argue that there is time-series information from, say, the persistence in book-to-market. However, missingness also tends to occur in blocks organized around time (Figure 1). If book-to-market is missing now, it is extremely likely to be missing last year, so this time-series correlation is also not informative.

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8 The relationship between PCA and imputation slopes is difficult to formalize but we show high dimensionality is associated with small imputation slopes in simulations in the Internet Appendix.

9 Though the observed and EM distributions look identical, we show in the Internet Appendix that they differ slightly.
4.4 Imputation Noise and Pitfalls

EM imputation not only adds little information, it also introduces noise. This noise can lead to poor forecasting performance if the data is not carefully handled.

Figure 4 illustrates the noise introduced by imputation. It shows imputation errors from a masking exercise: we randomly mask 10% of observed stock-predictor values, impute with EM, and calculate the error as the difference between the masked value and the imputed value.

For stocks with below-median market cap, the RMSE is at least 0.70. For comparison, imputing with simple means would lead to an RMSE of 1.00, since the data is standardized (Section 3.1). Thus, the imputation errors are quite large, especially among small stocks.

Indeed, the RMSEs in Figure 4 are surely understated, as our random masking does not account for the fact that accounting predictors are typically missing together. To mask, we draw all observed predictor-stock combinations with equal probability, so we will often mask book-to-market without masking the other accounting-based predictors. Thus, the imputation exercise in Figure 4 can “cheat,” by imputing book-to-market with earnings-to-price, even though in the real world these variables are almost always missing together.

Imputation noise can lead to poor forecasting performance, as seen in Figure 5. This figure repeats the long-short portfolio exercise from Section 4.2, but instead of estimating predicted returns by size group, it runs pooled estimations, estimating a single model for all size groups simultaneously. Thus, it ignores the fact that predictability is stronger in small stocks, as well as the fact that imputation errors are larger for small stocks.

In value-weighted portfolios, EM leads to much worse performance than simple mean imputation if more than 75 PCs are used. Importantly, simple mean imputation leads to value-weighted returns as high as 25% per year, while EM imputation tops out at 20%. This problem is much smaller in equal-weighted portfolios, in which returns top out at around 53% per year, regardless of the imputation method.
This underperformance is consistent with the pattern in imputation errors. EM underperformance happens when many PCs are used, forecasts are pooled, and performance is evaluated using value-weighting. In this setting, the large imputation errors in small stocks affect the forecasts of large stocks, and these large-stock forecasts are the focus of the performance evaluation.

These results illustrate a pitfall of sophisticated imputations. The noise introduced by sophisticated imputation may outweigh the information gained. This risk is higher if forecasts are not aligned with performance evaluation, as in Figure 5's value-weighted results. Section 5 shows that this pitfall is not just a property of EM imputation and PCR, but is also seen using other methods.

This pitfall leads us to recommend simple mean imputation for cross-sectional asset pricing research. While sophisticated imputations more fully incorporate information, they also introduce noise and can lead to underperformance. Given the complexity of most imputation and ML forecasting methods, our judgment is that the benefits of simple mean imputation outweigh the costs.

4.5 Huang et al. (2022)'s Scaled PCR and the Dimensionality of Expected Returns

The PCR results (Figures 2 and 5) imply that there are dozens of PCs that contribute to the cross-section of expected returns. This section shows that the dimensionality of expected returns is actually subtle and depends on multiple measurement decisions. To show this result, we repeat the portfolio formation exercise in Section 4.2, but use Huang et al. (2022)'s scaled PCA instead of standard PCA.

We apply Huang et al. (2022)'s scaled PCA as follows: For each predictor \( j \), we estimate \( r_{i,t} = \alpha_j + \gamma_j X_{i,j,t-1} + \epsilon_{i,j,t} \) using OLS, where \( X_{i,j,t-1} \) is the value of predictor \( j \) for stock \( i \) in month \( t-1 \). Then we define the scaled predictor \( \hat{X}_{i,j,t} = \gamma_j X_{i,j,t} \) and apply standard PCA to \( \hat{X}_{i,j,t} \) to obtained scaled PCs. This rescaling provides a simple and intuitive method for incorporating return information into the PC estimation. Huang et al. (2022) show that scaled PCA outperforms standard PCA using theory, simulation, and a macroeconomic forecasting exercise.

The performance of scaled-PCR portfolios is shown in Figure 6. Compared to Figure 2, the implied dimensionality is much smaller, especially if it is ap-
plied to EM-imputed data. In this specification, only 30 PCs are required to capture the potential equal-weighted returns, compared to about 70 PCs using standard PCR. Focusing on very large stocks implies a much smaller dimensionality. Scaled PCR requires only about 15 PCs to capture the potential value-weighted returns (Panel (b)), compared to about 30 PCs using standard PCR.

Still, mean returns depend relatively little on how missing data is handled. Through most of Figure 6, the lines for EM and simple mean imputation largely overlap. And the potential “real-time” mean returns top out at around 50% per year equal-weighted or 20% value-weighted. Whether the underlying data is imputed with EM or just simple means does not matter. We find broadly similar results using CAPM and FF6 alphas in the Appendix (Figure A.2).

An important exception to this invariance is in value-weighted portfolios that use more than 125 PCs. Here, EM imputation underperforms, consistent with the idea that the estimation noise using EM can exceed the information gained (Section 4.4).

5 Comparing Six Imputations using Six Return Forecasting Methods

Our baseline results examine just two imputation methods and two return forecasting methods. This section expands the analysis in both directions.

We examine four additional imputations, including ones that incorporate time-series information. We examine four additional return forecasting methods, including two based on neural networks.

The main results continue to hold: ad-hoc imputations and more sophisticated imputations lead to largely similar inferences about expected returns.

5.1 Four Additional Imputation Methods

We examine the following additional imputations:

1. **Cross-sectional EM on AR1 Residuals:** We estimate an AR1 model on Box-Cox transformed predictors using the past 60 months of data and apply
cross-sectional EM to the residuals. To impute missing predictors, we add
the AR1 prediction to the cross-sectional EM-imputed residuals. This al-
gorithm takes on time-series information, which BLLP find is important in
their imputation error tests.

2. **Practical EM for Probabilistic PCA**: We assume Box-Cox transformed pre-
dictors follow a 10-dimensional factor structure and impute using Roweis
(1997)’s “practical” EM algorithm for missing data. This practical EM algo-
rithm simply imputes missing predictors using the factor model prediction
in the E-step, and as such is not a maximum likelihood estimate (it ignores
higher order terms like Equation (4)). However, it is easily implemented
using the pcaMethods R library and provides a regularized alternative to
our cross-sectional EM imputation.

3. **Industry - Size Decile Means**: Within each 3-digit SIC code, we sort stocks
into size deciles. We then replace missing Box-Cox transformed predictors
with their means within each industry-size group. This imputation applies
the intuition that industry and size are important determinants of a firm’s
economics.

4. **Last Observed**: Replace missing values with the last observed value within
the last 12 months. If there is no observed value within the last 12 months,
we replace missing values with cross-sectional means. This imputation
takes on time-series information while avoiding using stale information.
It is also easy to implement

For additional details see Appendix A.1.

### 5.2 Four Additional Return Forecasting Methods

We examine the following additional return forecasting methods:

1. **Simple OLS**: We simply regress future stock returns on predictors using
OLS without any regularization. Simple OLS provides a helpful bench-
mark.

2. **Gradient-Boosted Regression Trees (GBRT)**: We model future stock re-
turns using as the average of $B$ shallow regression trees. The model is fitted
by gradient boosting: starting from an existing tree, the tree is “boosted” by
adding a new tree, where the new tree is chosen to step along the gradient of a squared-error loss function, and this process is continued $B$ times. $B$ and other hyperparameters are tuned following Gu et al. (2020) and Li et al. (2022), and the algorithm is implemented with Microsoft’s open source lightGBM library.

3. **3-Layer Neural Network (NN3):** We model the relationship between future stock returns and predictors as the average of an ensemble of feed-forward neural networks with three hidden layers. We fit the model using an Adam optimizer with batch normalization and early stopping. Tuning and other specification details follow Gu et al. (2020), who find that neural network performance peaks at three hidden layers. We implement the algorithm with Google’s open source Tensorflow and Keras libraries.

4. **1-Layer Neural Network (NN1):** This forecast uses the same model as NN3, but with only one hidden layer. This model highlights the robustness of NN3, which we find is by far the best-performing forecasting method.

We apply these methods either to all stocks at the same time or by size group. For additional details on the neural network specifications see Appendix A.2.

### 5.3 Out-of-Sample Forecasting and Hyperparameter Tuning

We impute missing values using one of the six methods and then construct “out-of-sample” portfolios using one of the six forecasting methods.

Portfolio construction uses the following procedure. For each June $t$ between 1995-2021, we

1. Separate the data between January 1985 and $t$ into a training sample and a validation sample. The validation sample is either the latter half of the data or the past 12 years, whichever sample is shorter. The remainder of the data is the training sample.

2. For each size group (micro, small, big), tune hyperparameters and fit the return model.

   (a) For all hyperparameter sets in a pre-selected set-of-sets, fit returns to lagged imputed data using the training sample and measure the RMSE in the validation sample.
(b) Using the hyperparameter set with the smallest RMSE, fit returns to lagged imputed data from January 1985 to $t$.

3. For each month $t, t+1, \ldots, t+11$, sort stocks on the predicted return and form long-short decile portfolios.

This procedure follows Gu et al. (2020) closely, though we add the size group separation, which we find generally improves performance (see Section 4.4). We also differ in requiring that at least half of the January 1985 to $t$ subsamples are used for training. Gu et al. do not need this requirement because their data goes back to 1957.

With the exception of OLS, the forecasting methods require hyperparameter tuning. We tune hyperparameters following Gu et al. (2020) as closely as possible (see their Appendix Table A.5). For PCR and sPCR, we select the number of PCs from $\{10, 30, 50, 70, 90\}$. For GBRT, the depth is 1 or 2, the learning rate is 0.01 or 0.10, and the number of trees is in $\{1, 250, 500, 750, 1000\}$. For NN1 and NN3, the L1 penalty is either $10^{-5}$ or $10^{-3}$, the learning rate is 0.001 or 0.010, and other hyperparameters are fixed: batch size is 10,000, epochs is 100, early stopping patience is 5, the ensemble number is 10, and we use the Adam optimizer with default parameters in the Keras interface to TensorFlow.

5.4 Results Across a Broad Array of Methods

Table 4 shows the result across all combinations of the 6 imputations (rows) and 6 forecasting methods (columns). For each forecasting method, we examine estimating separately for micro, small, and big stocks (“By Size”) or estimating all stocks together (“Pool”). We examine both equal-weighted (Panel (a)) and value-weighted (Panel (b)) returns.

[Table 4 about here]

5.4.1 Performance Across Imputations

The central feature of Table 4 is that returns depend little on the imputation method. Neural networks outperform other forecasting methods (columns) and

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10We differ from Gu et al. (2020) by using the standard L2 objective in GBRT rather than a Huber loss. Our implementation of GBRT uses lightGBM, which currently has poor documentation of the objective function, and we find that objective = 'huber' works poorly on simulated and empirical data. Li et al. (2022) does not mention using Huber loss.
returns are much higher in equal-weighted portfolios (Panel (a)). But moving across imputations (rows), there is little change in mean returns. For example, applying a one-layer neural network (NN1) to mean imputation leads to equal-weighted returns of 65% per year, compared to 64% imputing with EM on AR1 residuals. Similarly, applying a one-layer neural network to mean imputation leads to value-weighted returns of 40% per year, compared to 39% per year using EM on AR1 residuals. This strong performance for mean imputation is consistent with the idea that the observed data provide little information about the missing data (4.3).

Mean imputation outperforms in some settings. This outperformance is concentrated in value-weighted portfolios using pooled return forecasts (Panel (b), “Pool” columns). This result is consistent with the idea that sophisticated imputations can lead to additional noise in small-cap stocks, which can lead to poor forecasting performance if the data is not carefully handled (Section 4.4). This pitfall leads us to recommend simple mean imputation for cross-sectional asset pricing research.

There is one setting in which mean imputation can lead to significant underperformance. Using GBRT, mean imputation can lead to noticeably lower returns compared to EM, perhaps because regression trees deal poorly with the degenerate distributions that result from mean imputation. One should not over-interpret this result, however, since tree-based algorithms often have their own specialized methods for handling missing values, which we do not employ because of our goal of evaluating “general-purpose” imputations.\footnote{lightGBM can handle missing values by adding a second greedy stage to each split assessment.}

Adding time-series information to the imputations has little effect. The “Last Obs” row differs little from the other ad-hoc imputations and similarly the “EM AR1” row differs little from the other EM imputations. This result is consistent with the fact that missingness occurs in large time-series blocks (Figure 1). It also illustrates the robustness of our main results to the incorporation of time-series information, which BLLP find is important for imputation accuracy under simulated missingness patterns. We also find that the BLLP local backward cross-sectional imputation has little effect on estimated expected returns (Appendix Figure A.1).

Similarly, regularizing EM by using pPCA has little effect. The “pPCA10” rows differ little from the other EM rows. We also find that using industry / size means
has little effect.

5.4.2 Improvements due to Forecasting by Size Group

A second theme in Table 4 is that forecasting by size group significantly improves performance in value-weighted portfolios. Using PCR or sPCR, value-weighted mean returns increase from around 20% per year, to around 33% per year when forecasting by size group. Non-trivial gains from forecasting by size group are also seen in OLS- or GBRT-based value-weighted portfolios.

These results are consistent with the fact that predictability differs by market cap and liquidity (Fama and French (2008); Chen and Velikov (2022)). We also find that the dimensionality of expected returns differs significantly across size groups (Section 4.2). Thus, the forecasting model should vary by size group, and assuming that the expected return structure found among big stocks is the same as those found in small stocks will lead to sub-optimal forecasts.

Indeed, we find that our hyperparameter tuning, which is performed by size group (Section 5.2), finds a much lower dimensionality for large stocks. This variable dimensionality allows the PCR and sPCR value-weighted returns in Table 4 to far exceed the value-weighted returns in Figures 2 and 6, which fix the dimensionality for across size groups.

5.4.3 Performance Across Forecasting Methods

A third theme from Table 4 is that ML-style methods can underperform relative to simple OLS. Moving across the columns of Table 4, outperformance relative to OLS is only seen using neural network forecasts.

These results may be surprising, given that many papers tout the outperformance of machine learning methods (Gu et al. (2020); Freyberger et al. (2020); Simon et al. (2022)). However, they are consistent with these papers once one examines the details. Gu et al. (2020) find that applying PCR and GBRT to 94 predictors leads to returns that are not much larger than those obtained from applying linear regression to just three predictors. Freyberger et al. (2020) and Simon et al. (2022) both examine continuous non-linear models of returns, which are differ substantially from linear PCR models and the discontinuous GBRT models.

Moreover, the underperformance of PC-based methods is intuitive given the fact that mean returns are largely monotonic in the number of PCs used (Figures
2, 5, and 6). OLS always uses all PCs in the data. In contrast, PC-based methods need to tune the number of PCs, which can lead to too few PCs being used.

The outperformance of neural network forecasts is large. Mean returns increase from around 50 percent per year to 65 percent using equal-weighting, and from around 30 percent per year to 40 percent using value-weighting. In fact, we find that other neural network architectures work quite well also, and that performance is fairly robust to ensembling and other tuning choices if early stopping is set following Gu et al. (2020). These results support the strong performance of neural networks found in other papers (Gu et al. (2020); Simon et al. (2022)) and shows that this strong performance is robust to imputation methods.

6 Conclusion

In the age of machine learning, the standard practice of dropping stock-months with missing predictors is often untenable. Previous ML studies apply ad-hoc imputations or data adjustments, with little discussion of their motivation or study of alternatives. The goal of our paper is to provide guidance on this problem.

We recommend using simple mean imputation for ML studies. This recommendation may seem surprising, since mean imputation essentially ignores all information about the missing data. However, the missingness and the covariance structures in cross-sectional predictor data imply that there is little information about missing values. As a result, one might as well ignore all information. Consistent with this idea, we find mean imputation leads to similar portfolio returns and Sharpe ratios across a broad array of imputation and return forecasting methods.

Our PC analyses relate to the ongoing debate about the factor structure of the cross-section of returns. Our findings are consistent with a very high dimensionality for all stocks. In contrast, there is the potential for a strong factor structure among very large stocks. We also find that the results depend on the methodology used to compute PCs, with supervised methods like Huang et al. (2022) implying far fewer PCs are required to fully capture the cross-section.
A Appendix

A.1 Additional Imputation Method Details

A.1.1 Cross-Sectional EM with Time-Series Information

Our main EM imputation handles each cross-section independently. This imputation incorporates time-series information.

We impute in three steps for each “forecasting month” $\tau$:

1. Using OLS estimate $\hat{\phi}_j$ for the AR1 model

$$X_{i,j,t} = \phi_j X_{i,j,t-h} + \epsilon_{i,j,t} \quad (5)$$

where $X_{i,j,t}$ is Box-Cox transformed predictor (as in Section 3.3), $h \in \{1, 3, 12\}$ is the periodicity of the underlying data updates (based on Chen and Zimmermann (2022)’s documentation), $\phi_j$ is the persistence of predictor $j$, and $\epsilon_{i,j,t}$ is a zero mean shock. In this step, we use observed data from month $\tau - 59$ to $\tau$.

2. Using EM, estimate $\hat{\Omega}_\tau$ for

$$\hat{\epsilon}_{i,\tau} = \begin{bmatrix} \hat{\epsilon}_{i,1,\tau} \\ \hat{\epsilon}_{i,2,\tau} \\ \vdots \\ \hat{\epsilon}_{i,J,\tau} \end{bmatrix} \sim \text{MVN}(0, \Omega_\tau). \quad (6)$$

where $\hat{\epsilon}_{i,j,\tau} = X_{i,j,\tau} - \hat{\phi}_j X_{i,j,t-h}$. Assuming standard regularity conditions, this two-step estimation is consistent (Wooldridge (1994)).

3. For missing $X_{i,j,\tau}$, impute by combining the prediction of the AR1 model with the residual estimate—that is, use $\hat{\phi}_j X_{i,j,t-h} + \hat{\epsilon}_{i,j,\tau}$ (if $X_{i,j,t-h}$ is observed) or $\hat{\epsilon}_{i,j,\tau}$ (if $X_{i,j,t-h}$ is not observed).

A.1.2 Practical EM for Probabilistic PCA

We did not restrict the covariance matrix in our main EM imputation. This imputation imposes a factor structure and estimates using a variant of EM (Roweis (1997), see also Tipping and Bishop (1999)).
We estimate the following model:

\[
X_{i,t} = \Lambda_t F_{i,t} + \epsilon_{i,t} \tag{7}
\]

\[
F_{i,t} \sim \text{MVN}(0, I)
\]

\[
\epsilon_{i,t} \sim \text{MVN}(0, \sigma^2 I) \tag{8}
\]

where \(X_{i,t}\) is a \(J\)-dimensional vector of predictors for stock \(i\) in month \(t\), \(F_{i,t}\) is a \(K\)-dimensional vector of factor realizations for stock \(i\) in month \(t\), \(\Lambda_t\) is a \(J \times K\) matrix that maps factors into predictors, and \(\sigma^2\) is a scalar.

The estimation alternates between two steps for each forecasting month \(t\):

1. **E-step**: Given a guess for \(\Lambda_t\) and \(F_{i,t}\), replace the missing components of \(X_{i,t}\) with \(\Lambda_t F_{i,t}\). Find the expectations of \(F_{i,t}\) and \(F_{i,t}' F_{i,t}^\prime\) conditional on \(\Lambda_t\), \(\sigma\), and \(X_{i,t}\).

2. **M-step**: Plug the expectations of \(F_{i,t}\) and \(F_{i,t}' F_{i,t}^\prime\) into the log-likelihood assuming that all \(X_{i,t}\) and \(F_{i,t}\) are observed, and maximize to obtain new \(\Lambda_t\) and \(F_{i,t}\).

The algorithm iterates these two steps until the objective in the M-step no longer improves.

As in the case of our baseline model (Equation (2)), Equations (7)-(8) imply that the ignorable quasi-likelihood can be optimized using EM (Tipping and Bishop (1999)) (see Internet Appendix A.2). This algorithm does not follow the full EM method, as it does not include higher moments of missing \(X_{i,t}\) in the E-step. However, the computer science literature finds that less formal variants of EM work well on this model using simulated data (Roweis (1997); Ilin and Raiko (2010)). Our implementation uses the R library pcaMethods, which follows Roweis (1997)’s algorithm.

### A.1.3 Bryzgalova, Lerner, Lettau and Pelger (2023)’s Factor Model with Added Time-Series Information

BLLP’s local backward cross-sectional model is similar to the imputations we study in that it can be used to form forecasts without look-ahead bias. They emphasize the importance of time-series information, so we examine how their method for adding time-series information affects our scaled-PCA results here.
We find similar results in our other forecasting methods but show just the scaled-PCA results for brevity.

Our implementation of BLLP’s local backward cross-sectional imputation proceeds as follows. For each month of predictor data:

1. Compute the available case cross-sectional covariance matrix of predictors. If there are no overlapping observations for a pair of predictors, we use zero as the covariance matrix entry.

2. Form a matrix using the $K$ eigenvectors with the largest eigenvalues. The rows of these matrix are the “loadings” of each predictor.

3. Regress predictor-stock values on the predictor loadings. The slopes are called “characteristic factors” and there should be $K$ for each stock.

4. Regress predictors on characteristic factors and lagged predictors that are observed within the last 24 months. We only used lagged predictors that have signal updates according to the Chen-Zimmermann documentation (only use lags 12 and 24 for annual predictors).

5. Use the slopes from step 4 to impute missing values.

We apply scaled-PCA (by size) and form portfolios. The results are in Figure A.1.

A.2 Additional Forecasting Method Details

GBRT details are fully described in Sections 5.2 and 5.3.

Neural network details follow Gu et al. (2020). NN1 has a single hidden layer of 32 neurons and NN3 has hidden layers with 32, 16, and 8 neurons. All layers are connected with the ReLU activation function. The objective is L2 with a L1 penalty to weight parameters and is minimized using the Adam extension of Stochastic Gradient Descent under early stopping with a patience parameter of 5 in batches of 10,000, for 100 epochs. The L1 penalty and learning rate are tuned as described in the main text. Last, we form final return forecasts as the ensemble average of 10 neural network forecasts.

A.3 Appendix Exhibits
Each month, we forecast returns using scaled principal components regression and form long-short deciles portfolios. 'BLLP loc B-XS' imputes using the local backward cross-sectional model in BLLP, where we use six factors to be consistent with Section 4.1 of BLLP, and lags used in the second stage regression are restricted to relevant predictor update months based on documentation for the Chen and Zimmermann (2022) dataset. **Interpretation:** Taking on time-series information following BLLP results in similar estimates as simple mean imputation.

(a) Mean Returns

![Graph of Annualized Mean Return vs Number of PCs]

(b) Sharpe Ratios

![Graph of Annualized Sharpe Ratio vs Number of PCs]
Figure A.2: Missing Data Effects on Alphas from Scaled PCA

Each month, we forecast returns using scaled principal components regression and form long-short deciles portfolios, as in Figure 6. We then measure CAPM alphas (Panel (a)) or Fama-French 5 factor + momentum alphas (Panel(b)). **Interpretation:** Simple mean imputation also performs well when examining alphas.

(a) CAPM

![Graph showing CAPM Alphas](image)

(b) FF6

![Graph showing FF6 + Mom Alphas](image)
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Table 1: Missingness in 1985 for Selected Predictors

Predictors are selected to illustrate the causes of missingness (see discussion in text). ‘% Obs’ is the mean share of stock-months for which the predictor is observed in 1985, for common stocks listed on the NYSE, NYSE MKT, and NASDAQ. The full list of 159 continuous predictors with at least 2 observations in every month between 1985 and 2021 is in the Internet Appendix (Table IA.1). Interpretation: Missingness is due to the availability of the underlying data (accounting, analyst forecast, etc), the requirement of having a long history of underlying data (Ret Seasonality Yr 11-15), and is also imposed to approximate interaction effects (IPO and Age).

| Predictor                  | Reference                          | Data Focus | % Obs |
|----------------------------|------------------------------------|------------|-------|
| Size                       | Banz 1981                          | Price      | 99.8  |
| 12-Month Momentum          | Jegadeesh and Titman 1993          | Price      | 91.6  |
| Book-to-Market             | Statman 1980                       | Accounting | 74.4  |
| Asset Growth               | Cooper, Gulen and Schill 2008      | Accounting | 67.1  |
| EPS Forecast Revision      | Hawkins, Chamberlin, Daniel 1984   | Analyst    | 46.0  |
| Ret Seasonality Yr 11-15   | Heston and Sadka 2008              | Price      | 45.3  |
| Revenue Growth Rank        | Lakonishok, Shleifer, Vishny 1994  | Accounting | 40.5  |
| Payout Yield               | Boudoukh et al. 2007               | Accounting | 35.9  |
| Asset Tangibility          | Hahn and Lee 2009                  | Accounting | 34.9  |
| Advertising Expense        | Chan et al. 2001                   | Accounting | 31.3  |
| IPO and Age                | Ritter 1991                        | Other      | 14.2  |
| Earnings Surprise Streak   | Loh and Warachka 2012              | Accounting | 14.1  |
| Conglomerate Return        | Cohen and Lou 2012                  | Price      | 8.4   |
| Customer Momentum          | Cohen and Frazzini 2008             | Other      | 4.9   |
| Inst Own for High Short Int| Asquith Pathak and Ritter 2005     | Other      | 0.2   |
Table 2: The Missing Value Problem for Large-Scale Predictability Studies

Data consists of 159 continuous predictors from Chen and Zimmermann (2022) with at least 2 observations for common stocks listed on major exchanges every month between 1985 and 2021. Panel (a) computes the % of stocks with observations for individual predictors and then computes percentiles across predictors. Panel (b) selects the \( J \) most-observed predictors and then computes the % observed among these using two methods: ‘Predictor-Stocks’ sums predictor-stock observations and then divides by \( J \times \) the number of stocks. ‘Stocks w/ all \( J \’ \) counts the number of stocks with all \( J \) predictors and divides by the number of stocks. **Interpretation:** Missing values are a much larger problem when combining many predictors. When combining 75 or more predictors, one must impute missing values or drop most stocks.

### Panel (a): Individual Predictors

| Predictor Percentile | % of Stocks With Predictor Data in June 1970 | 1975 | 1980 | 1985 | 1990 | 1995 | 2000 |
|----------------------|--------------------------------------------|------|------|------|------|------|------|
| 5                    | 0                                          | 0    | 3    | 14   | 14   | 15   | 16   |
| 25                   | 25                                         | 25   | 34   | 42   | 43   | 45   | 45   |
| 50                   | 67                                         | 67   | 53   | 66   | 66   | 69   | 71   |
| 75                   | 82                                         | 82   | 72   | 80   | 78   | 79   | 87   |
| 95                   | 100                                        | 100  | 97   | 97   | 99   | 99   | 99   |

### Panel (b): Combining \( J \) Predictors

| \( J \) Stocks w/ \( J \) Predictors Data (%) | Stocks w/ \( J \) Predictors (%) | Predictor-Stocks Data (%) | Stocks w/ \( J \) Predictors (%) | Predictor-Stocks Data (%) | Stocks w/ \( J \) Predictors (%) | Predictor-Stocks Data (%) |
|-----------------------------------------------|----------------------------------|---------------------------|----------------------------------|---------------------------|----------------------------------|---------------------------|
| 25                                           | 89.2                             | 49.7                      | 96.2                             | 86.4                      | 95.8                             | 88.1                      |
| 50                                           | 82.1                             | 37.4                      | 87.9                             | 42.1                      | 91.0                             | 53.5                      |
| 75                                           | 76.2                             | 9.7                       | 81.3                             | 16.1                      | 85.7                             | 23.8                      |
| 100                                          | 71.3                             | 0.0                       | 76.7                             | 10.1                      | 81.4                             | 11.2                      |
| 125                                          | 63.2                             | 0.0                       | 70.7                             | 0.9                       | 75.0                             | 0.8                       |
| 150                                          | 55.8                             | 0.0                       | 63.5                             | 0.0                       | 67.6                             | 0.0                       |
Table 3: Missing Data Effects on Single Predictor Strategies

Strategies long the 500 stocks with the most positive predictors and short the 500 stocks with the most negative predictors. ‘EM Imputed’ uses cross-sectional EM imputation (Section 3.2) and ‘Missing Dropped’ drops stock-months with missing predictors. For both methods, we drop predictor-months with less than 1000 stock-predictor observations. ‘EM Improvement’ is ‘EM Imputed’ minus ‘Missing Dropped,’ computed at the predictor level. **Interpretation:** Though EM generally improves performance, the typical effect is small, and it comes at the cost of worse performance for many predictors.

| Percentile Across Predictors | 10  | 25  | 50  | 75  | 90  |
|-----------------------------|-----|-----|-----|-----|-----|
| **Panel (a): Equal-Weighted** |     |     |     |     |     |
| Mean Return (% Ann)         |     |     |     |     |     |
| EM Imputed                  | -1.2| 2.9 | 7.1 | 12.7| 17.5|
| Missing Dropped             | -0.1| 3.1 | 6.2 | 10.2| 15.2|
| EM Improvement              | -2.0| -0.3| 0.5 | 2.1 | 4.4 |
| Sharpe Ratio (Ann)          |     |     |     |     |     |
| EM Imputed                  | -0.05| 0.16| 0.58| 0.86| 1.21|
| Missing Dropped             | -0.01| 0.26| 0.53| 0.89| 1.11|
| EM Improvement              | -0.22| -0.04| 0.01| 0.09| 0.18|
| **Panel (b): Value-Weighted** |     |     |     |     |     |
| Mean Return (% Ann)         |     |     |     |     |     |
| EM Imputed                  | -1.9| 0.5 | 3.1 | 7.0 | 11.2|
| Missing Dropped             | -2.1| 0.8 | 2.6 | 5.5 | 10.4|
| EM Improvement              | -1.5| -0.4| 0.2 | 1.5 | 3.8 |
| Sharpe Ratio (Ann)          |     |     |     |     |     |
| EM Imputed                  | -0.13| 0.04| 0.22| 0.43| 0.54|
| Missing Dropped             | -0.12| 0.04| 0.22| 0.36| 0.51|
| EM Improvement              | -0.10| -0.04| 0.00| 0.06| 0.19|
Table 4: Performance Across Several Imputation and Return Forecast Methods

We impute missing values, forecast returns, and then form long-short decile portfolios recursively each June between 1995-2021. ‘Mean’ imputes with cross-sectional means and ‘EM’ is cross-sectional EM (Section 3.2). Other imputations are defined in Section 5.1. ‘PCR’ and ‘sPCR’ run OLS on PCs or scaled PCs (Section 4.2). Other forecasts are defined in Section 5.2. Tuning is described in Section 5.3. Forecasts are either estimated within market equity groups ('By Size') or using all stocks ('Pool'). **Interpretation:** Across many forecasting tests, mean imputation performs well compared to other imputations, including imputations that incorporate time-series information and ones that regularize using a factor structure (see also Figure A.1). Mean imputation can even outperform EM imputation if the forecasts ignore the size dependence of predictability.

| Panel (a): Equal-Weighted Mean Return (% Annualized) | Forecasting Methods | Linear | Non-Linear |
|-----------------------------------------------------|----------------------|--------|------------|
|                                                     |                      | OLS By Size Pool | PCR By Size Pool | sPCR By Size Pool | GBRT By Size Pool | NN1 By Size Pool | NN3 By Size Pool |
| Ad-hoc Imputations                                   |                      |                  |                |                  |                  |                  |
| Mean                                                |                      | 55               | 51             | 50               | 31               | 40               | 65               | 66               | 66               | 68               |
| Ind / Size                                          |                      | 56               | 53             | 51               | 43               | 38               | 64               | 64               | 65               | 67               |
| Last Obs                                            |                      | 56               | 52             | 49               | 40               | 37               | 65               | 66               | 67               | 68               |
| EM Imputations                                      |                      | 58               | 54             | 52               | 55               | 54               | 52               | 51               | 65               | 65               | 67               | 68               |
| EM AR1                                              |                      | 56               | 51             | 49               | 53               | 50               | 49               | 48               | 64               | 64               | 66               | 67               |
| pPCA10                                              |                      | 57               | 50             | 49               | 54               | 49               | 54               | 53               | 65               | 65               | 67               | 66               |

| Panel (b): Value-Weighted Mean Return (% Annualized) | Forecasting Methods | Linear | Non-Linear |
|-----------------------------------------------------|----------------------|--------|------------|
|                                                     |                      | OLS By Size Pool | PCR By Size Pool | sPCR By Size Pool | GBRT By Size Pool | NN1 By Size Pool | NN3 By Size Pool |
| Ad-hoc Imputations                                   |                      |                  |                |                  |                  |                  |
| Mean                                                |                      | 31               | 32             | 21               | 27               | 23               | 17               | 25               | 40               | 37               | 37               | 40               |
| Ind / Size                                          |                      | 31               | 31             | 21               | 28               | 22               | 22               | 12               | 37               | 37               | 34               | 40               |
| Last Obs                                            |                      | 30               | 38             | 20               | 34               | 21               | 23               | 7                | 38               | 38               | 35               | 45               |
| EM Imputations                                      |                      | 28               | 36             | 16               | 31               | 18               | 32               | 11               | 43               | 34               | 39               | 40               |
| EM AR1                                              |                      | 34               | 35             | 18               | 33               | 18               | 27               | 13               | 39               | 33               | 36               | 40               |
| pPCA10                                              |                      | 30               | 34             | 20               | 31               | 18               | 32               | 17               | 40               | 33               | 38               | 40               |

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Vertical axis represents 159 predictors and horizontal axis represents 6,000 common stocks traded on major exchanges. Shading indicates months since the last observation. Lightest shade is currently observed and the darkest indicates the predictor-stock has never been observed. Predictors are sorted by the type of data they focus on (according to the CZ documentation). **Interpretation:** When combining 159 predictors, a stock can only be used without imputation if it has values for all 159 predictors. No stocks satisfy this requirement. The complexity of missingness suggests assuming Rubin (1976) ignorability is required. Missingness occurs in blocks organized around data type and time. There is very little time series information about missing values.
Figure 2: Missing Data Effects in 159-Predictor Strategies using PCR

Each month, we forecast next month’s returns by running PCR on the past 10 years of data by size group (micro, small, big). Strategies go long-short stocks in the extreme deciles of predicted returns. ‘EM Algo’ imputes missing values with cross-sectional EM (Section 3.2). ‘Simple Mean’ imputes with cross-sectional means (Section 3.3). **Interpretation:** EM and simple mean imputation lead to similar inferences about expected returns and Sharpe ratios.

(a) Mean Returns

(b) Sharpe Ratios
Figure 3: Empirical Distribution of Correlations

All predictors are transformed using an extended Box-Cox algorithm to have approximately normal marginal distributions at the predictor level (Section 3.1). ‘Observed’ uses all available cases at the predictor-pair level (a.k.a. pairwise-complete). ‘EM Algo’ uses the cross-sectional EM-estimated covariance matrix. Interpretation: The correlations cluster near zero, implying that imputation slopes are close to zero (Equation (2)), and thus simple mean imputation should be similar to EM imputation.

(a) June 1990

(b) June 2000

(c) June 2010
We measure out-of-sample imputation errors a la cross-validation as follows: For each cross-section, we randomly divide the stock-predictor observations into 10 groups. We mask one of the groups, impute with cross-sectional EM, and find the imputation errors for the masked stock-predictors. Masking and imputation is repeated for each of the 10 groups. We then calculate RMSE by market equity decile. **Interpretation:** Imputations errors are very large among small stocks, nearly as large as the errors from mean imputation, which is about 1.0 by construction.

![Figure 4: Imputation Errors in a Random Masking Exercise](image-url)
Figure 5: Missing Data Effects in PCR Strategies Using Pooled Forecasts

Like Figure 2, we form portfolios on PCR return forecasts, but now we forecast all stocks together instead of forecasting micro, small, and big stocks separately. ‘EM Algo’ imputes missing values with the cross-sectional EM algorithm (Section 3.2). ‘Simple Mean’ imputes with cross-sectional means (Section 3.3). Interpretation: Large imputation errors in small stocks (Figure 4) lead to poor performance in value-weighted strategies, if micro and big stock returns are assumed to have the same structure. EM introduces estimation noise, which can lead to underperformance relative to mean imputation, particularly if forecasts are not carefully designed.

(a) Mean Returns

(b) Sharpe Ratios
Like Figure 2, we form portfolios on PCR return forecasts, but now we construct PCs using Huang et al. (2022)'s scaled-PCA instead of standard PCA. Panel (b) zooms in on the first 25 PCs. **Interpretation:** PCA significantly overstates the dimensionality of expected returns. Scaled PCA requires far fewer PCs to capture the potential returns, especially if EM is used. The dimensionality of expected returns is subtle and depends on multiple measurement decisions. Mean imputation still does a good job capturing the potential mean returns.

(a) Mean Return: All PCs

(b) Mean Returns: PCs 1-25