B-spline Function Modeling of Electric Heating Flow Regulating Valve

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Abstract. In this paper we propose a modeling approach of electric heating flow regulating valve with B-spline function and recursive least squares method. And by means of the analysis of the structure and working principle of electric heating flow regulating valve, its correctness is validated with MATLAB simulation. The results indicate that this modeling method can realize the minimum error approximation of real working state of electric heating flow regulating valve.

1. Introduction

As a flow regulating device, the flow regulating valve is widely used in petrochemical, metallurgy, electric power and other industrial department to control fluid flow in the production process. And it is also applied in the heating, air conditioning and refrigeration systems for the temperature control.

Now most of flow regulating valve is the electric control device in the industry and fluid medium flow is mainly controlled by changing the size of valve opening with motor. However, due to the wear of motor reciprocating rotary and mechanical parts when the valve is working, this type of flow regulating valve shows the characteristics of a large loss, high energy consumption, the failure rate and high production cost. In this paper, a new kind of flow regulating valve is researched which directly uses the expansion force of temperature-sensing paraffin instead of motor to move the valve core. It is mainly composed of intelligent control device, paraffin driver and the plunger valve stroke model [1-3]. Obviously this flow regulating valve without motor has the same function as the flow regulating valve with motor and can reduce the mechanical wear.

For flow valve, the flow characteristic curve is a technical indicator describing flow control ability and an important basis of precise flow control [4-5]. Due to internal mechanical structure limitation of flow regulating valve, the precision measurement of valve opening is very difficult in the process of modeling. In addition, many research results have shown that the correlation between valve opening and the flow also be affected by other factors and cause many uncertainties. Therefore, in order to obtain current research results in practical application of flow regulating valve characteristic expression and precise flow control, there are many problems still need to be solved. In this paper, the proposed method uses the flow variation to replace valve opening variation, and the characteristic expression between a given temperature of paraffin actuator and flow in electric heating flow regulating valve is established. Secondly, corresponding to the regulating characteristics of electric heating flow regulating valve, based on the measurement data of two physical quantities, the function relationship between the given temperature and flow rate of paraffin actuator was established by system identification method. Obviously this method can avoid the influence of the internal
mechanical structure or the differential pressure $\Delta P_0$ of regulating valve in modeling and it can also effectively reduce the system complexity and increase the modeling speed.

In view of this, a feature modeling with correlation between paraffin temperature of electric heating flow regulating valve and flow is proposed by combining the B-spline function with recursive least squares method in this paper. This method can make curve to pass through each data point with the lower order of function, and it can overcome the existing problems of traditional method as the poor stability and complex calculation. And it has high precision and can be convenient online operations.

2. Structure and working principle of electric heating regulating valve

In order to establish the mathematical model described by correlation between paraffin temperature of electric heating regulating valve and flow, the structure and working principle of the flow regulating valve should need to be analyzed. The mechanical structure of the paraffin driver is shown in Figure 1 and it is mainly composed of an electric heating actuator and the control device. The electric heating actuator includes paraffin actuator, the mandrill, the hold-down nut, and the power line, the fixed position of the spring, the transmission spring, the movement plate, plastic shell and the device shell.

![Figure 1. The internal structure diagram of paraffin driver.](image)

The working principle of electric heating regulating valve is based on thermal expansion and non-compression of paraffin. The working process can be expressed as follows: firstly, electric heating regulating valve at room temperature is set as normally open state (or closed state); when the flow need to be changed, electric heating actuator is energized to change the paraffin temperature, the expansion or contraction of paraffin under different temperature will make the cone valve core moving to increase or decrease the valve opening, then flow of fluid through valve is changed (if the valve opening is increased, the fluid flow through valve is increased, on the contrary, the fluid flow through valve is decreased).

3. The mathematical model described by correlation between paraffin temperature of electric heating regulating valve and flow

Based on the analysis of the structure and operating principle of electric heating regulating valve and the mathematical model described by correlation between paraffin temperature of electric heating regulating valve and flow, the flow variation $\Delta L$ of fluid medium can be controlled by the temperature variation $\Delta T$ of paraffin actuator, The specific method is as follows.

Through synthesis of B-spline function and recursive least squares method, the mathematical model described by correlation between given paraffin temperature of electric heating regulating valve and flow can be established and its calculation process stated below:

First, suppose that real-time temperature of the paraffin actuator in electric heating flow regulating valve is $T(t)$ and real-time flow of fluid medium is $L(t)$ at time $t$. In view of each time points (such as $t = 1, 2, \cdots$), the corresponding real-time temperature $T(t)$ and real-time flow $L(t)$ can be measured to draw out correlation curve of function $L = f(T)$ relating to flow $L$ and...
temperature $T$. Second, B-spline functions is used to approximate the curve $L = f(T)$ relating to flow $L$ and temperature $T$ and its control coefficient can be obtained by the recursive least squares algorithm.

Final, correlation model of electric heating flow regulating valve relating to temperature $T$ and flow $L$ is eventually established as following.

$$L = \sum_{\rho=-\xi}^{\xi} c_{\rho} \Omega_{\xi}(T - \rho) + e_j$$  \hspace{1cm} (1)

$$\Omega_{\xi}(T - \rho) = \frac{1}{\xi^2} \sum_{j=0}^{\xi^2-1} j^T c_{j+1} \left( T - \rho + \frac{\xi+1}{2} - j \right)^\xi$$  \hspace{1cm} (2)

Where $\xi$ is non-negative integer. $\hat{C}_i$ is determined by $l$ sample data points, and $\hat{C}_{l+1}$ can be determined by $l+1$ sample data points and $\hat{C}_l$ can be identified by recursive method. And if we omit the specific derivation process and apply synthesis of B-spline function and recursive least squares method, the recursive estimation formula of control coefficient is given as follows.

$$\begin{align*}
\hat{C}_{l+1} &= \hat{C}_l + Q_{l+1} M_{l+1}^T \left( L_{l+1} - M_{l+1} \hat{C}_l \right) \\
Q_{l+1} &= Q_l - Q_l M_{l+1}^T (I + M_{l+1} Q_l M_{l+1}^T)^{-1} M_{l+1} Q_l \\
M_{l+1} &= [p_{l+1,-\xi}, p_{l+1,-\xi+1}, \ldots, p_{l+1,l-1}] 
\end{align*}$$  \hspace{1cm} (3)

Where $Q_l = [P_l^T P_l]^{-1}$. In recursive process, let $\hat{C}_0 = 0$, $Q_0 = \varepsilon \times I_{l+\xi+1}$ be initial value, and $\varepsilon$ is sufficiently large real number and the unknown vector dimension of control coefficient is constant. It can be proved that the estimated values of control coefficient are convergent and the only. And it ensures the correctness of $\hat{C}_l$ in the whole process.

Then, substituting obtained control coefficient into (1) yields correlation model relating to flow $L$ and temperature $T$. And if this correlation model is expressed by incremental form, we can establish the model of electric heating flow regulating valve relating to temperature variation $\Delta T$ of paraffin actuator and flow variation $\Delta L$.

$$\begin{align*}
\Delta L &= \sum_{\rho=-\xi}^{\xi} c_{\rho} \Omega_{\xi} (\Delta T - \rho) + e_j \\
\hat{C}_{l+1} &= \hat{C}_l + Q_{l+1} M_{l+1}^T (\Delta L_{l+1} - M_{l+1} \hat{C}_l) \\
Q_{l+1} &= Q_l - Q_l M_{l+1}^T (1 + M_{l+1} Q_l M_{l+1}^T)^{-1} M_{l+1} Q_l \\
M_{l+1} &= [p_{l+1,-\xi}, p_{l+1,-\xi+1}, \ldots, p_{l+1,l-1}] 
\end{align*}$$  \hspace{1cm} (4)

4. The experimental simulation and results analysis

In order to get more real experimental results, we use a real laboratory heating system as the experiment platform to obtain the experimental data needed in simulation process. The correctness of the theory research can be verified by simulation experiment and experiment results analysis. The correctness of the theory research can be verified by simulation experiment and experiment results analysis.

With real experimental data, the different order polynomial functions and different order B-spline function are respectively selected to establish their recursive least squares mathematical model. Then a MATLAB program of recursive method is used to obtain system control coefficient [6] and draw the
fitting curve and error curve of these models. These results are compared and analyzed, and the curve on flow change of electric heating regulating valve with paraffin temperature change is drawn, which is used as an ideal simulation object model. In this experiment, the paraffin temperature is the increasing sequence in [28, 58] and flow of regulating valve is the measured value associated with it. The data samples are shown in table 1.

Table 1. Data Samples of flow and paraffin temperature in regulating valve.

| paraffin temperature (°C) | 28 | 30.8 | 33.8 | 36.7 | 39.4 | 41.8 | 43.8 | 45.8 | 47.4 | 48.8 | 50.1 |
|---------------------------|----|------|------|------|------|------|------|------|------|------|------|
| flow (L/min)              | 6.63 | 6.74 | 6.64 | 6.67 | 6.8  | 6.75 | 6.54 | 6.6  | 6.58 | 6.75 | 6.68 |
| paraffin temperature (°C) | 51.1 | 52.3 | 53.1 | 53.9 | 54.6 | 55.2 | 55.8 | 56.3 | 57   | 57.5 | 58   |
| flow (L/min)              | 6.6 | 6.62 | 6.42 | 6.12 | 5.73 | 4.91 | 3.77 | 2.51 | 0    | 0    | 0    |

Based on the data samples of flow and paraffin temperature in Table 1, the curve of ideal model is drawn. By equation (4) the each order of B-spline function model and its corresponding control coefficients is obtained. And the fitting curve of different order polynomial model and different order B-spline function model were drawn as shown in Figure 2 and Figure 3.

Figure 2 shows the fitting effect using polynomial least squares. Among them, the fitting result of quadratic polynomial is the worst and it even cannot achieve full fitting. And the fitting result of fourth order polynomial is the best and it basically can fit the general trend of the experimental data. That means that the higher the order of polynomial, the better the fitting effect. Therefore, if you need a polynomial function accurately to match each data point, a high order of the function is required. However, another problem is that the higher the order of polynomial functions, the worse the function stability. And if the order of function is decreased, the fitting accuracy is also reduced. Obviously if we use the polynomial least squares fitting method, it may make the choice of polynomial order in a dilemma and it cannot achieve the desired result for the research in this paper.
Figure 3. The fitting curve of different order B-spline function mode.

Figure 3 shows the fitting result using B-spline function model to fit the experimental data points. Among them, the fitting result of cubic B-spline function is best, and fitting curves basically can match the experimental data points. And the fitting result of cubic B-spline function is better than quadratic B-spline function and four order B-spline functions. Clearly this one illustrates no linear correlation between the fitting effect and the order of B-spline function. Comparing the Figures 2 and 3, the fitting result of B-spline function recursive least squares method is better than polynomial least squares. And comparing and analyzing the fitting error of different model, the fitting error of different order polynomial and different order B-spline function are shown in Figure 4 and Figure 5, and the fitting error comparisons of fourth order polynomial and different orders B-spline functions are also shown in Figure 6.
Figure 6. The fitting error comparisons of fourth order polynomial and different orders of B-spline function.

Figure 4 shows that the maximum fitting error of quadratic polynomial is 2.56, the maximum fitting error of cubic polynomial is 3.66 and the maximum fitting error of four order polynomial is 2.38. Obviously the fitting error of four order polynomial is the smallest and the fitting effect is the best.

Figure 5 shows that the maximum fitting error of quadratic B-spline function is 1.72, the maximum fitting error of cubic B-spline function is 0.27 and the maximum fitting error of the fourth order B-spline function is 0.83. Obviously the fitting error of cubic B-spline function is the smallest and the fitting effect is the best.

In order to better observe the fitting precision differences between the polynomial and B-spline function, the fitting error of the models is compared in Figure 6. Obviously the fitting error of different order of B-spline function recursive least squares method is smaller than the polynomial least squares. And fitting error of cubic B-spline function is smaller than the quadratic B-spline functions and the fourth order B-spline function. Therefore for fitting method of the mathematical model of B-spline recursive least squares method, the order of B-spline function is not directly proportional to the fitting effect. That is to say that it will lead the fitting error to be increased if the order is too high or too low and it is very important that choosing the appropriate order of B-spline function. Based on the characteristics of electric heating system and above discussion, the paraffin actuator mathematic model using cubic B-spline function recursive least squares method is more suitable.

5. Conclusion
In this paper, the paraffin actuator model of an electric heating flow regulating valve is constructed by B-spline function recursive least squares method and the working principle is analyzed which use thermal expansion and non-compression of liquid medium to adjust the flow of liquid medium. The correlation between a given paraffin temperature and valve opening and the correlation between a given paraffin temperature and flow are explained in detail. And transformation between the given paraffin temperature and flow is also summarized. On this basis, the paraffin actuator mathematical model of B-spline function recursive least squares algorithm is established. In the end, the correctness of the mathematical model is verified by MATLAB simulation and the experimental results are analyzed. The theoretical analysis and experimental results show that the minimum approach error in the modeling process can be realized by B-spline function recursive least square method. This research on mathematical model reveals the correlation between given paraffin temperature and flow, and builds the performance research foundation of electric heating flow regulating valve.
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