Relativistic transformations of quasi-monochromatic optical beams

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A monochromatic plane wave recorded by an observer moving with respect to the source undergoes a Doppler shift and spatial aberration. We investigate here the transformation undergone by a generic, paraxial, spectrally coherent quasi-monochromatic optical beam (of finite transverse width) when recorded by a moving detector. Because of the space-time coupling engendered by the Lorentz transformation, the monochromatic beam is converted into a propagation-invariant pulsed beam traveling at a group velocity equal to that of the relative motion, and which belongs to the recently studied family of ‘space-time wave packets’. We show that the predicted transformation from a quasi-monochromatic beam to a pulsed wave packet can be observed even at terrestrial speeds.

An observer moving with respect to an optical source emitting a monochromatic plane wave (MPW) records a Doppler-shifted MPW [1–4]. What are the changes observed by a detector moving with respect to a source emitting instead a generic monochromatic optical beam (i.e., a transversely localized field)? Previously tackled questions regarding relativistic transformations of optical fields have sometimes revealed surprising answers. For example, Terrell [5] and Penrose [6] showed that the length of an object in an image captured by an instantaneous shutter does not depend on the observer’s velocity, thus disabusing the physics community of the notion of a ‘visible’ Lorentz contraction [7]. Recently, it has been shown that angular-momentum carrying optical fields exhibit exotic effects in a frame moving orthogonally to the optical axis, including an optical analog of the relativistic spin Hall effect [8, 9], transverse orbital angular momentum and spatio-temporal vortices [10, 11], and relativistic spin-orbit interactions [12].

We analyze here the transformation of a generic quasi-monochromatic beam when recorded by an observer moving with respect to the source along the beam axis. Previous studies of such a transformation have revealed several mathematical results, the central one of which is that a strictly monochromatic beam is recorded as a finite-bandwidth pulsed beam [13–16]. The space-time coupling engendered by the Lorentz transformation yields a propagation-invariant wave packet whose group velocity is the relative velocity between source and detector [15]. Remarkably, the observed wave packet is a realization of so-called ‘space-time wave packets’ (STWPs) [17–22], which have been recently synthesized via spatio-temporal spectral-phase modulations [23–26].

Here we examine Lorentz transformations of spectrally coherent quasi-monochromatic optical beams, and provide an physically intuitive picture that underpins their conversion into STWPs. In this picture, an angularly induced Doppler broadening leads to the formation of an STWP, and we emphasize the impact of the beam’s spatial width on the induced spatio-temporal field structure. Crucially, by relaxing the ideal monochromaticity assumption, we find that the spectral linewidth of the source determines a minimum observer velocity for these effects to be detectable. We examine the potential for observing such effects at terrestrial speeds with currently available narrow-linewidth lasers.

To set the stage for analyzing the Lorentz transformation of optical beams, we first examine the case of MPWs in one transverse dimension $x$ (without loss of generality); see Fig. 1. An MPW at frequency $\omega$ emitted by a source $S$ at rest in the inertial frame $O(x, z, t)$ is Doppler-shifted to $\omega' = \sqrt{\frac{1 - \beta}{1 + \beta}} \omega$ in the frame $O'(x', z', t')$ moving at a velocity $v = \beta c$ along the common $z$-axis [Fig. 1(a)]. An MPW travelling in $O$ at an angle $\varphi$ with the $z$-axis is transformed in $O'$ to a frequency $\omega' = \gamma(1 - \beta \cos \varphi) \omega$ travelling at an angle $\varphi' = \cos^{-1} \left[ \frac{\cos \varphi - \beta}{1 - \beta \cos \varphi} \right]$ (the Doppler spatial aberration [8]), where $\gamma = 1/\sqrt{1 - \beta^2}$ [Fig. 1(b)].

These changes can be visualized on the surface of the spectral light-cone [22, 27]. The wave vector $\vec{k} = (k_x, k_z)$ for an MPW in $O$ is represented by a point on the surface $k_x^2 + k_z^2 = (\omega/c)^2$, where $k_x = \omega \sin \varphi$ and $k_z = \omega \cos \varphi$.

FIG. 1. A monochromatic plane wave (MPW) emitted in the rest frame $O$ is Doppler-shifted in the frame $O'$ moving along the $+z$-axis. (b) An off-axis MPW in $O$ is Doppler-shifted and undergoes an angular rotation in $O'$. (c) The on-axis MPW is Doppler-shifted along the light line $k_z = \omega/c$ ($k_x = 0$) in the Fourier domain, whereas (d) an off-axis MPW is shifted along a fixed-$k_z$ hyperbola on the light-cone surface.
The Lorentz-transformed wave-vector components are: $k'_x = k_x; k'_z = \gamma (k_z - \beta \omega/c)$ and $\omega' = \gamma(\omega - c\beta k_z)$. Because $k'^2 + k'^2_z = (\omega')^2$, the structure of the light-cone itself is Lorentz-invariant, so that the points corresponding to MPWs in $O$ and $O'$ can be represented on the same surface. The MPW in Fig. 1(a) corresponds to a point on the light-line $k_x = 0$, along which its Doppler-shifted counterpart in $O'$ is displaced [Fig. 1(c)]. In contrast, the point representing the off-axis MPW in $O$ [Fig. 1(b)] is displaced in $O'$ along a constant-$k_x$ hyperbola [Fig. 1(d)].

Now consider a generic monochromatic beam emitted by the source $S$ in $O$ [Fig. 2(a)], which is a superposition of plane waves (spatial bandwidth $\Delta k_z$, inverse of the beam width $\Delta x$) at all the same frequency $\omega_0$ but traveling at different angles $\varphi$ with the $z$-axis $[28, 29]$. The spectral support for such a beam is the circle $k^2_z + k_z^2 = k_0^2$ at the intersection of the light-cone with a horizontal iso-frequency plane $\omega = \omega_0$ [Fig. 2(b)]; here $k_0 = \omega_0/c$. Because the Doppler shift depends on the relative velocity $v$ and angle $\varphi$ between source and detector, the MPWs in $O$ undergo different Doppler shifts in $O'$ [Fig. 2(c)], and the associated points along the circle on the light-cone in $O$ are displaced in $O'$ differently along the constant-$k_x$ hyperbolas [Fig. 2(d)]. Consequently, a finite spectral bandwidth $\Delta \omega'$ is Doppler-induced in the initially monochromatic beam, whose coherence guarantees that the space-time-coupled field in $O'$ is pulsed [Fig. 2(c)]. The spectral support is Lorentz-transformed from a horizontal circle in $O$ into a tilted ellipse in $O'$ $[15]$ at the intersection of the light-cone with the plane $k'_z = (\omega' - \omega'_0)c \tan \theta$, which is parallel to the $k_z$-axis but makes an angle $\theta$ with the $k'_z$-axis, where $\tan \theta = -\beta$ [Fig. 2(d)]. The linear relationship between $k'_z$ and $\omega'$ indicates the absence of dispersion in the observed wave packet, which travels in $O'$ rigidly without diffraction at a group velocity $\tilde{v} = c \tan \theta = -v$ $[22, 24]$. Such a field corresponds to a so-called subluminal ‘baseband’ STWP $[22, 23, 29]$, which have been recently synthesized with group velocities in the range $0.07c < \tilde{v} < c$ $[24, 30, 31]$. It will of course be challenging to produce such STWPs via relative motion between the source and detector.

In the paraxial regime $\Delta k_z \ll k_0$, the ellipse in $O'$ can be approximated $[23]$ by a parabola $\Omega'(k'_z) = \frac{c k'_z^2}{2k_0^2(1 - \cos \theta)}$ [Fig. 3(a)], where $\Omega' = \omega' - \omega'_0$. The initially monochromatic beam acquires a bandwidth $\Delta \omega' = \frac{1}{2} \gamma |\beta| \omega_0(\Delta k_z)^2$ via space-time coupling. Although $\Delta \omega'$ is independent of the sign of $\beta$ (i.e., it is symmetric with respect to approaching or receding observers), the carrier frequency $\omega'_0$ in contrast is highly asymmetric around $\beta = 0$ [Fig. 3(b)]. The resulting on-axis ($x = 0$) pulsewidth is $\Delta T' \sim \frac{\omega_0}{c}$, where $\Delta T'$ is the Rayleigh range of the initial monochromatic beam. At a wavelength $\lambda_o = 800$ nm and beam width $\Delta x = 40 \mu m$ ($\Delta k_z \sim 1.6$ mm), relative motion at $v = 0.8c$ results in $\Delta T' \sim 4$ ps (a bandwidth $\Delta \omega' \sim 0.25$ nm). The pulsewidth is reduced to $\Delta T' \sim 250$ fs when the beam

FIG. 2. Lorentz transformation of a monochromatic beam. (a) A monochromatic beam in $O$ is a superposition of plane waves of the same frequency $\omega_0$, travelling in different directions, and (b) its spectral support on the light-cone is an iso-frequency circle. (c) In the moving frame $O'$, each plane wave undergoes an angle-dependent Doppler shift. (d) The spectral support for the field in (c) is the intersection of the light-cone with a plane that makes an angle $\theta$ with the $k'_z$-axis.

FIG. 3. (a) Spatio-temporal spectrum of paraxial STWPs in $O'$ for different observer velocities $\beta$. (b) Dependence of the STWP central frequency $\omega'_0$ (black solid curve, left axis) and bandwidth $|\Delta \omega'|$ (red dashed curve, right axis) on $\beta$, normalized to the frequency $\omega_0$ of the monochromatic beam in $O$ having $\Delta k_z = 0.1k_0$. 
The time-averaged intensity axial plane

To investigate this possibility, we must first drop the assumption of a strictly monochromatic field, for which any non-zero relative velocity can in principle lead to the formation of a detectable STWP via idealized space-time coupling $\delta(\Omega' - \Omega'(k_z'))$. Rather, a realistic finite-energy field is inevitably quasi-monochromatic, and hence possesses a finite linewidth $\delta \Omega$ in $\Omega$. When transformed in $\Omega'$ into an STWP, the precise delta-function correlation $\delta(\Omega' - \Omega'(k_z'))$ is relaxed to $g(\Omega' - \Omega'(k_z'))$ [30, 35], where $g(\cdot)$ is a narrow spectral function whose width corresponds to a finite spectral uncertainty $\delta \Omega'$. The spectral uncertainty $\delta \Omega$ sets a minimum relative velocity $v_{\text{min}}$ between source and detector that is required for a detectable STWP:

$$v_{\text{min}} \sim 2c \left( \frac{\delta \Omega}{\omega_0} \right) \left( \frac{\delta k_x}{k_0} \right)^2 \sim 2c \frac{\delta \Omega}{\delta \Omega'} \sim \frac{c}{\Delta T'},$$

where $\Delta T' \sim 1/\delta \Omega$ is the pulselength of the field in $\Omega$.

This minimal requirement on the relative velocity can be understood from several perspectives. The spectral uncertainty $\delta \Omega$ is the finite bandwidth of the spectral support for the quasi-monochromatic field on the light-cone surface [Fig. 2(b)]. The Doppler-induced bandwidth $\Delta \Omega'$ results in an on-axis pulselength $\Delta T' \sim \frac{\delta \Omega}{\omega_0}$ that is independent of the initial linewidth $\delta \Omega'$. For the relative motion to produce a detectable STWP, the spectral tilt angle $\theta$ must be sufficient for the new spectral support on the light-cone to be distinguishable from the initial spectrum. This requires that $\Delta \Omega'$ exceed the spectral uncertainty, $\Delta \Omega > \delta \Omega'$, which sets a minimal spectral tilt angle, and hence a minimal relative velocity. A different perspective is gleaned from considering the maximum propagation distance of an STWP $L_{\text{max}} \sim \frac{\omega_0}{\delta \Omega'}$ [30, 35]. Observing the STWP in $\Omega'$ requires that $L_{\text{max}}$ be larger than the axial pulse length $c \Delta T' \sim \frac{c}{\Delta T'}$, thereby leading to the result in Eq. 1.

We illustrate in Fig. 5 the consequences of Eq. 1 starting with a quasi-monochromatic beam of $\frac{\delta \Omega}{\omega_0} = 100 \mu m$ (Rayleigh range $z_R \approx 5 \text{ mm}$) and spectral linewidth $\frac{\delta \Omega}{\omega_0} = 300 \text{ Hz} \left( \delta \Omega' = 1 \text{ ms} \right)$ centered at $\omega_0 = 200 \text{ THz}$ ($\lambda_0 \approx 1.55 \mu m$) [Fig. 5(a)], which is observed by a moving detector [Fig. 5(b)]. From Eq. 1, $v_{\text{min}} \approx 1.3 \times 10^7$ or $v_{\text{min}} \approx 140 \text{ km/h}$, so that an observer at $v = 5 \text{ km/h}$ ($|v| < v_{\text{min}}$) records a conventionally diffracting quasi-monochromatic beam [Fig. 5(b)]. However, an observer at $v = 320 \text{ km/h}$ ($|v| > v_{\text{min}}$) records an STWP with $\Delta T' \approx 0.5 \text{ ms}$, $L_{\text{max}} \approx 10 \text{ mm}$, and $v = 320 \text{ km/h}$ [Fig. 5(c)]. An even faster observer moving at $v = -1600 \text{ km/h}$ detects an STWP of shorter pulselength $\Delta T' \approx 100 \mu s$ and longer propagation distance of $L_{\text{max}} \approx 50 \text{ mm}$ [Fig. 5(d)].

Narrowing the linewidth to $\frac{\delta \Omega}{\omega_0} = 3 \text{ Hz}$ reduces the threshold to $v_{\text{min}} \approx 1.4 \text{ km/h}$, and recording an STWP becomes accessible to a walking observer, whereas the flying observer records an STWP travelling freely for $L_{\text{max}} = 5 \text{ m}$. Alternatively, $v_{\text{min}}$ can be reduced more effectively by reducing the transverse beam width, due to the quadratic dependence $\beta_{\text{min}} \propto (\Delta x)^2$.

We plot in Fig. 6 the on-axis intensity profiles $I(x', z')$ at $v = 0$ with increasing $\Delta x$ from $\Delta x = 300 \text{ Hz}$, which can be...
FIG. 5. Schematic of a potential test of relativistic transformations of a quasi-monochromatic beam. (a) A beam from a stationary laser (Δx = 100 µm, Δω = 200 THz, Δω = 2 kHz) is recorded by moving observers. (b) A walking observer at v = −1.4 m/s (≈5 km/h) does not detect any change in the beam (|v| < v_min = 40 m/s). (c) A faster observer at v = −90 m/s (≈320 km/h) detects a propagation-invariant STWP of pulsewidth ΔT′ ≈ 0.5 ms traveling at a group velocity v̇ = 90 m/s, having a spatio-temporal spectral structure Ω′ = Ω′(k′_x). (d) An even faster observer at v = −450 m/s (≈1600 km/h) observes an STWP of pulsewidth ΔT′ ≈ 0.1 ms.

viewed as world-lines for the peak of the pulsed field in O’. In O, the long temporal extent of ≈ 1 ms (corresponding to a length of ~ 300 km) combined with the short axial extent Δz ∼ 2R_l = 10 mm renders the peak of the wave packet effectively ‘stationary’, even though the underlying electromagnetic field is travelling at c [Fig. 5(a)]. As the observer moves towards the source, the detected width of the STWP can become significantly shorter than 1 ms when v ≫ v_min, resulting in an STWP peak moving at a group velocity v̇ = −v [Fig. 5(b–d)], and with the STWP propagation invariant within the temporal interval of 1 ms [Fig. 5(d)].

We have considered here a model in which the initial laser spectrum is coherent. However, the narrow-linewidth spectra of realistic laser sources are largely incoherent, corresponding to continuous-wave radiation rather than pulsed [36, 37]. The coherent spectral model utilized here can be obtained by modulating the narrow-linewidth source at a rate higher than its initial linewidth, resulting in a pulse train, each pulse of which is described by the model established here. The Lorentz transformation of a continuous-wave laser source with a spectrally incoherent narrow linewidth requires a different analysis [48, 49], which will be reported elsewhere. Furthermore, our analysis has been restricted to one transverse dimension, which has the advantage of showing a clear structure (the pedestal I_p) emerging as a result of time-space coupling. Incorporating both transverse dimensions in an azimuthally symmetric beam does not change the conclusions except that the pedestal is replaced with a slow ṙ-decay in intensity (r is the radial coordinate) [39, 41].

In contrast to previous tests of special relativity that rely on complex configurations and high-level precision [42–46], the conversion of a generic monochromatic paraxial beam into a propagation-invariant pulsed beam at terrestrial speeds is potentially simpler to realize, especially in light of the current availability of ultranarrow-linewidth lasers (Δω/2π < 300 Hz) and high-speed cameras (>1000 frames/s). Although the Doppler shift is prohibitively difficult to detect at small β, the changes in the spatio-temporal structure of the field can be readily captured. Moreover, the results reported here may lead to new designs and functionalities for so-called spacetime metasurfaces by elucidating what can be achieved at low-speed moving devices [47–50]. Other areas in optical physics have recently explored the ramifications of relativistic transformations of optical fields, including photonic time crystals [51, 52]; realizations of an optical analog of the Mackinnon wave packet [53] via moving dipoles [54, 55]; and reflection and refraction from moving surfaces [56, 61]. Moreover, our findings may open a new perspective on transverse relativistic transformations of monochromatic beams carrying orbital angular momentum [8–12].

In summary, we have analyzed a generic quasi-monochromatic optical beam observed in an axially moving frame, showing that the transformed field is a propagation-invariant wave packet of finite pulsewidth travelling at subluminal group velocities. Moreover, an intuitive physical picture provides the constraint on the relative velocity between source and detector required to observe the predicted phenomena. Our analysis reveals that current technology allows for such a test to be carried out at terrestrial speeds.

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