Class2Simi: A Noise Reduction Perspective on Learning with Noisy Labels

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Abstract
Learning with noisy labels has attracted a lot of attention in recent years, where the mainstream approaches are in pointwise manners. Meanwhile, pairwise manners have shown great potential in supervised metric learning and unsupervised contrastive learning. Thus, a natural question is raised: does learning in a pairwise manner mitigate label noise? To give an affirmative answer, in this paper, we propose a framework called Class2Simi: it transforms data points with noisy class labels to data pairs with noisy similarity labels, where a similarity label denotes whether a pair shares the class label or not. Through this transformation, the reduction of the noise rate is theoretically guaranteed, and hence it is in principle easier to handle noisy similarity labels. Amazingly, DNNs that predict the clean class labels can be trained from noisy data pairs if they are first pretrained from noisy data points. Class2Simi is computationally efficient because not only this transformation is on-the-fly in mini-batches, but also it just changes loss computation on top of contrastive learning. Thus, a natural question is raised: does learning in a pairwise manner mitigate label noise? This motivates us to introduce a pairwise manner to deal with label noise.

On the other hand, methods employing pairwise manners are very prevailing and have made a great success in machine learning, e.g., supervised metric learning and unsupervised contrastive learning (Qi et al., 2019; Boudiaf et al., 2020; Chen et al., 2020; He et al., 2020), where relationships between data points are exploited. Intuitively, the pairwise manners require less pointwise supervision information, i.e., class labels, and might be robust to label noise. In this paper, we naturally ask a question: does learning in a pairwise manner mitigate label noise? This motivates us to introduce a pairwise manner to deal with label noise.

Here we propose a noise reduction perspective on handling label noise: Class2Simi, i.e., transforming training data with noisy class labels into data pairs with noisy similarity labels. A class label shows the class that an instance belongs to, while a similarity label indicates whether or not two instances belong to the same class. We theoretically prove that through this transformation, the noise rate becomes lower (see Theorem 2). This is because, given a data pair, of which if one point has an incorrect class label or even if both points have incorrect class labels, the similarity label could be correct. Moreover, this transformation also reduces a multi-class classification problem into a binary classification problem. In label noise learning, the binary problem is easier to handle and a lower noise rate usually results in higher classification performance (Patrini et al., 2017).

Specificaly, we illustrate the transformation and the robustness of similarity labels in Figure 1. In the middle column, we can see the noisy similarity labels of example-pairs \((x_2, x_3)\) and \((x_2, x_4)\) are correct, although there is one mislabeled point in \((x_2, x_5)\), and two mislabeled points in \((x_2, x_4)\). Moreover, if we assume that the noisy class labels in Figure 1 are generated according to the latent clean class labels and the class transition matrix (the \(ij\)-th entry of this matrix denotes the probability that the clean class label \(i\) flips into the noisy class label \(j\)), the noise rate of...
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Figure 1. An illustration of the transformation from class labels to similarity labels. Note that \( y \) stands for the noisy class label and \( \bar{y} \) for the latent clean class label. The labels marked in red are incorrect. If we assume the class label noise is generated according to the transition matrix presented in the upper part of the right column, it can be calculated that the noise rate for the noisy class labels is 0.5 while the noise rate for the noisy similarity labels is 0.25. Note that the transition matrix for similarity labels can be calculated by exploiting the class transition matrix as in Theorem 1.

class labels is 0.5. Meanwhile, the corresponding similarity transition matrix can be derived from the class transition matrix with the class-priors (see Theorem 1). The noise rate of similarity labels is 0.25, which is the proportion of the number of incorrect similarity labels to the number of total similarity labels.

To handle the transformed data pairs with noisy similarity labels, the connection between noisy similarity posterior and clean class posterior should be established. Intuitively, noisy similarity posterior can be linked to clean similarity posterior, and then clean class posterior can be inferred from clean similarity posterior. For the first part, we can draw on the philosophy of dealing with noisy class labels, e.g., selecting reliable data pairs for training, and correcting the similarity loss to learn a robust similarity classifier. For the second part, plenty of similarity metrics can be adopted. As an example, we could adapt the Forward (Patrini et al., 2017) to learn clean similarity posterior from data with noisy similarity labels. Then, by using the inner product of the clean class posterior (Hsu et al., 2019) to approximate clean similarity posterior, the clean class posterior (and thus the robust classifier) can thereby be learned.

It is obvious that Class2Simi suffers information loss because we can not recover the class labels from similarity labels, which implies that learning only from similarity labels can only cluster data points but can not identify the semantic classes of clusters. In Hsu et al. (2019), a pointwise cluster can be learned from similarity labels. However, in our case, the pairs with similarity labels are constructed from points with class labels, and we could acquire the semantic class information of clusters by pretraining the model from points with class labels without any additional information. Note that when class labels of points are corrupted, leading to noisy similarity labels, the proposed pretraining still works because the noisy class is assumed to be dominated by its clean class in label noise learning. Thus we do not suffer the major information loss in noisy similarity learning.

It is worthwhile to mention Class2Simi increases the computation cost very slightly, compared with the standard pointwise training. As shown in Figure 2, most computation is still pointwise. Only the computation of the pairwise enumeration layer (Hsu et al., 2018) and the loss are pairwise, while both the forward and backward propagation are pointwise. The pairwise enumeration layer was verified to only introduce a negligible overhead to the training time (Hsu et al., 2019). Moreover, the transformation is on-the-fly in mini-batches, which means the pairs are quadratic on the batch size other than the whole sample size.

The contributions of this paper are summarized as follows:

- We propose a noise reduction perspective on learning with noisy labels, which transforms class labels into similarity labels, reducing the noise rate.
- We provide a way to estimate the similarity transition matrix \( T_s \) by theoretically establishing its relation to the class transition matrix \( T_c \). We show even if the \( T_c \) is roughly estimated, the induced \( T_s \) still works well.
2. Problem Setup and Related Work

Let \((X, Y) \in \mathcal{X} \times \{1, \ldots, c\}\) be the random variables for instances and clean labels, where \(\mathcal{X}\) represents the instance space and \(c\) is the number of classes. However, in many real-world applications (Zhang et al., 2017; Zhong et al., 2019; Li et al., 2019; Tanno et al., 2019; Zhang et al., 2018b; Xia et al., 2021; Feng et al., 2020; Chou et al., 2020; Wu et al., 2020b; Zhu et al., 2021; Yu et al., 2020; Berthon et al., 2021), the clean labels cannot be observed. The observed labels are noisy. Let \(\hat{Y}\) be the random variable for the noisy labels. What we have is a sample \(\{(x_1, \hat{y}_1), \ldots, (x_n, \hat{y}_n)\}\) drawn from the noisy distribution \(P_{\hat{Y}}\) of the random variables \((X, Y)\). We aim to learn a robust classifier that could assign clean labels to test data by exploiting the sample with noisy labels.

Existing methods for learning with noisy labels can be divided into two categories: algorithms that result in statistically inconsistent or consistent classifiers. Methods in the first category usually employ heuristics to reduce the side-effect of noisy labels, e.g., selecting reliable samples (Han et al., 2018b; Yu et al., 2019; Wei et al., 2020; Wu et al., 2020a; Xia et al., 2020b), reweighting samples (Ren et al., 2018; Jiang et al., 2018; Ma et al., 2018; Kremer et al., 2018; Reed et al., 2015), correcting labels (Tanaka et al., 2018; Zheng et al., 2020), designing robust loss functions (Zhang & Sabuncu, 2018; Xu et al., 2019; Liu & Guo, 2020; Ma et al., 2020), employing side information (Vahdat, 2017; Li et al., 2017), and (implicitly) adding regularization (Li et al., 2021; Vait et al., 2017; Vahdat, 2017; Han et al., 2018a; Zhang et al., 2018a; Guo et al., 2018; Hu et al., 2020; Zhang et al., 2021; Han et al., 2020a). Those methods empirically work well in many settings. Methods in the second category aim to learn robust classifiers that could converge to the optimal ones defined by using clean data. They utilize the transition matrix, which denotes the probabilities that the clean labels flip into noisy labels, to build consistent algorithms (Natarajan et al., 2013; Scott, 2015; Liu & Tao, 2016; Patrini et al., 2017; Northcutt et al., 2017; Yu et al., 2018; Kremer et al., 2018; Hendrycks et al., 2018; Liu & Guo, 2020; Yao et al., 2020b; Xia et al., 2020c). The idea is that given the noisy class posterior probability and the transition matrix, the clean class posterior probability can be inferred.

Note that the noisy class posterior and the transition matrix can be estimated by exploiting the noisy data, where the transition matrix additionally needs anchor points (Liu & Tao, 2016; Patrini et al., 2017). Some methods assume anchor points have already been given (Yu et al., 2018). There are also methods showing how to identify anchor points from the noisy training data (Liu & Tao, 2016).

3. Class2Simi meets noisy supervision

In this section, we propose a new perspective for learning from noisy data. Our core idea is to transform class labels to similarity labels first, and then handle the noise manifested on similarity labels.

3.1. Transformation on labels and the transition matrix

As in Figure 1, we combine every 2 instances in pairs, and if the two instances have the same class label, we assign this pair a similarity label 1, otherwise 0. If the class labels are corrupted, the generated similarity labels also contain noise.

The definition of the similarity transition matrix is similar to the class one. The elements in a similarity transition matrix denote probabilities that clean similarity labels \(\tilde{H}\) flip into noisy similarity labels \(\hat{H}\), i.e., \(T_{s,mn} := P(\hat{H} = n | H = m)\). The dimension of the similarity transition matrix is always \(2 \times 2\). Since the similarity labels are generated from class labels, the similarity noise is determined and, thus can be calculated, by the class transition matrix.

**Theorem 1** Assume that the dataset is balanced (each class has the same amount of instances, and \(c\) classes in total), and the noise is class-dependent. Given a class transition matrix \(T_{c}\), such that \(T_{c,ij} = P(\hat{Y} = j | Y = i)\). The elements of the corresponding similarity transition matrix \(T_{s}\) can be calculated as

\[
\begin{align*}
T_{s,00} &= \frac{c^2 - c - (\sum_j (\sum_i T_{c,ij})^2 - ||T_c||^2_{\text{Fro}})}{c^2 - c}, \\
T_{s,01} &= \frac{\sum_j (\sum_i T_{c,ij})^2 - ||T_c||^2_{\text{Fro}}}{c^2 - c}, \\
T_{s,10} &= \frac{c - ||T_c||^2_{\text{Fro}}}{c}, \\
T_{s,11} &= \frac{||T_c||^2_{\text{Fro}}}{c}.
\end{align*}
\]

A detailed proof is provided in Appendix A.

**Remark 1** Theorem 1 can easily extend to the setting where the dataset is unbalanced in classes by multiplying each \(T_{c,ij}\) by a coefficient \(n_i\). \(n_i\) is the number of instances from the \(i\)-th class.
Note that the similarity labels are only dependent on class labels. If the class noise is class-dependent, the similarity noise is also ‘class-dependent’ (class means similar and dissimilar). Under class-dependent label noise, a binary classification is learnable as long as \( T_{00} + T_{11} > 1 \) (Menon et al., 2015), where \( T \) is the corresponding binary transition matrix; a multi-class classification is learnable if the corresponding transition matrix \( T_c \) is invertible. For Class2Simi, in the most general sense, i.e., \( T_c \) is invertible, \( T_{s,00} + T_{s,11} > 1 \) holds. Namely, the learnability of the pointwise classification implies the learnability of the reduced pairwise classification. A proof is provided in Appendix B. However, the latter cannot imply the former: As shown in Figure 1, the class transition matrix is not invertible, and thus the pointwise classification is not learnable while the reduced pairwise classification is learnable. Note that this ‘learnable’ is only for the binary pairwise classification in this case. Technically, two conditions must be met to learn a pointwise classifier from pairwise data: (1) The reduced pairwise classification is learnable; (2) The semantic class information is learnable. Generally, the second condition is equivalent to the learnability of the pointwise classification. Thus the learnability for a pointwise classifier of the two learning manners is consistent.

**Theorem 2** Assume that the dataset is balanced (each class has the same amount of samples), and the noise is class-dependent. When the number of classes \( c \geq 8 \), the noise rate of noisy similarity labels is lower than that of the noisy class labels.

A detailed proof is provided in Appendix C.

In multi-class classification problems, the number of classes is usually larger than 8. As \( c \) becomes larger, the range of ‘dissimilarity’ of data pairs becomes larger, which is conducive to the reduction of the noise rate. Through Class2Simi, the number of \( d \)-pairs (with similarity label 0) is \((c-1)\) times as much as that of \( s \)-pairs (with similarity label 1). Meanwhile, compared with the original noise rate of noisy class labels, the noise rate of noisy similarity labels of \( s \)-pairs is higher and that of \( d \)-pairs is lower, while the overall noise rate of data pairs is lower, which partially反映 that the impact of label noise is less bad. Notably, the flip from ‘dissimilar’ to ‘similar’ should be more adversarial and thus more important. In practice, it is common that one class has more than one clusters, while it is rare that two or more classes are in the same cluster. If there is a flip from ‘similar’ to ‘dissimilar’ and based on it we split a (latent) cluster into two (latent) clusters, we still have a high chance to label these two clusters correctly later. If there is a flip from ‘dissimilar’ to ‘similar’ and based on it we join two clusters belonging to two classes into a single cluster, we nearly have zero chance to label this cluster correctly later. As a consequence, the flip from ‘dissimilar’ to ‘similar’ is more adversarial and important, thus deserving a larger weight when calculating the noise rate. Here we assign all data pairs the same weight, otherwise, there would be a more reduction of the noise rate. On balance, considering the reduction of the overall noise rate is meaningful.

When dealing with label noise, a low noise rate has many benefits. The most important one is that the noise-robust algorithms will consistently achieve higher performance when the noise rate is lower (Han et al., 2018b; Xia et al., 2019; Patrini et al., 2017). Another benefit is that, when the noise rate is low, the complex instance-dependent label noise can be well approximated by class-dependent label noise (Cheng et al., 2020), which is easier to handle.

### 3.2. Learning with noisy similarity labels

In order to learn a multi-class classifier from similarity labeled data, we should establish relationships between class posterior probability and similarity posterior probability. Here we employ the relationship established in (Hsu et al., 2019), which is derived from a likelihood model. As in Figure 2, we denote the predicted clean similarity posterior by the inner product between two categorical distributions: \( \hat{S}_{ij} = f(X_i)^\top f(X_j) \). Intuitively, \( f(X) \) outputs the predicted categorical distribution of input data \( X \) and \( f(X_i)^\top f(X_j) \) can measure how similar the two distributions are. For clarity, we visualize the predicted similarity posterior in Figure 3. If \( X_i \) and \( X_j \) are predicted belonging to the same class, i.e., \( \arg\max_{m \in c} f_m(X_i) = \arg\max_{n \in c} f_n(X_j) \), the predicted
The pipeline of the proposed Class2Simi is summarized in Figure 2. The softmax function outputs an estimation for the clean class posterior, i.e., \( f(X) = \hat{P}(Y|X) \), where \( \hat{P}(Y|X) \) denotes the estimated class posterior. Then a pairwise enumeration layer is added to calculate the predicted clean similarity posterior \( \hat{S}_{ij} \). According to Equation (1), by pre-multiplying the transpose of the noise similarity transition matrix, we can obtain the predicted noisy similarity posterior \( \hat{S}_{ij} \).

Therefore, by minimizing \( L_{\text{c2s}} \), we can learn a classifier for predicting noisy similarity labels. Meanwhile, before the transition matrix layer, the pairwise enumeration layer will output a prediction for the clean similarity posterior, which guides \( f(X) \) to predict clean class labels.

**Remark 2** For a better understanding, we formulate Class2Simi in the form combined with Forward as an illustration. However, Class2Simi is a meta method that can be applied on top of sample selection, loss correction, label correction, and many other label noise learning methods. We provide another implementation with Reweight in Appendix D.

### 3.3. Implementation

The proposed algorithm is summarized in Algorithm 1. Since learning only from similarity labels will lose the semantic class information, we load the model trained on the data with noisy class labels to provide the semantic class information for similarity learning in Stage 2.

### 3.4. Generalization error

We formulate the above problem in the traditional risk minimization framework (Mohri et al., 2018). The expected and empirical risks of employing estimator \( f \) can be defined as

\[
R(f) = E_{(X_i, X_j, y_i, y_j, H_{ij}, T_s)} \sim D^s_\nu [\ell(f(X_i), f(X_j), T_s, H_{ij})],
\]

and

\[
R_n(f) = \frac{1}{n^2} \sum_{i=1}^n \sum_{j=1}^n \ell(f(X_i), f(X_j), T_s, \hat{H}_{ij}),
\]

where \( n \) is the training sample size of the noisy data. Assume that the neural network has \( d \) layers with parameter matrices \( W_1, \ldots, W_d \), and the activation functions \( \sigma_1, \ldots, \sigma_{d-1} \) are Lipschitz continuous, satisfying \( \sigma_j(0) = 0 \). We denote by \( H : X \rightarrow W_d \sigma_{d-1}(W_{d-1} \sigma_{d-2}(\ldots \sigma_1(W_1 X))) \in \mathbb{R} \) the standard form of the neural network. \( H = \text{argmax}_{\hat{h}_i} \in \{1, \ldots, c\} \hat{h}_i \). Then the output of the softmax function is defined as
Algorithm 1 Class2Simi

**Input:** training data with noisy class labels; validation data with noisy class labels.

**Stage 1: Learn \( T \):**
1. Learn \( g(X) = \hat{P}(Y|X) \) by training data with noisy class labels, and save the model for Stage 2;
2. Estimate \( \hat{T} \), following the optimization method in (Patrini et al., 2017);
3. Transform \( \hat{T} \) to \( \hat{T} \).

**Stage 2: Learn the classifier \( f(X) = \hat{P}(Y|X) \):**
4. Load the model saved in Stage 1, and train the whole pipeline showed in Figure 2.

**Output:** classifier \( f \).

\[
f_i(x) = \frac{\exp(h_i(x))}{\sum_{j=1}^{c} \exp(h_j(x))}, i = 1, \ldots, c.
\]

We can then obtain the following generalization error bound.

**Theorem 3** Assume the parameter matrices \( W_1, \ldots, W_d \) have Frobenius norm at most \( M_1, \ldots, M_d \), and the activation functions are \( L \)-Lipschitz, positive-homogeneous, and applied element-wise (such as the ReLU). Assume the transition matrix is given, and the instances \( X \) are upper bounded by \( B \), i.e., \( \|X\| \leq B \) for all \( X \), and the loss function \( \ell \) is upper bounded by \( M \). Then, for any \( \delta > 0 \), with probability at least \( 1 - \delta \),

\[
R(\hat{f}) - R_n(\hat{f}) \leq M \left( \frac{\log 1/\delta}{2n} + \frac{(T_{c,11} - T_{c,01})2Bc(\sqrt{2d\log 2} + 1)\prod_{i=1}^{d} M_i}{T_{c,11} \sqrt{n}} \right).
\]

A detailed proof are provided in Appendix E.

Theorem 3 implies that if the training error is small and the training sample size is large, the expected risk \( R(\hat{f}) \) of the representations for noisy similarity posterior will be small. If the transition matrix is well estimated, the clean similarity posterior as well as the classifier for the clean class will also have a small risk according to Equation (1) and the Class2Simi relations. This theoretically justifies why the proposed method works well. In the experiment section, we will show that the transition matrices are well estimated and that the proposed method significantly outperforms the baselines.

In Class2Simi, a multi-class classification is reduced to a pairwise binary classification. For data pairs, if a surrogate loss is classification-calibrated, minimizing it leads to minimizing the zero-one loss on the pointwise random variables in the limit case. Otherwise, we cannot guarantee the worst-case learnability of learning pointwise labels from pairwise labels, but it cannot imply the average-case non-learnability either. Theoretically, (Bao et al., 2020) proved that when the pairwise labels are all correct, for the special case \( c = 2 \), a good model for predicting \( s/d \)-pairs must also be a good model for predicting the original classes, under mild assumptions. In practice, it seems fine to use non-classification-calibrated losses. According to (Tewari & Bartlett, 2007), the multi-class margin loss (i.e., one-vs-rest loss) and the pairwise comparison loss (i.e., one-vs-one loss) are proved to be non-calibrated, but they are still the main multi-class losses in (Mohri et al., 2018; Shalev-Shwartz & Ben-David, 2014).

4. Experiments

**Experiment setup.** We employ three widely used image datasets, i.e., MNIST (LeCun, 1998), CIFAR-10, and CIFAR-100 (Krizhevsky et al., 2009), one text dataset News20, and one real-world noisy dataset Clothing1M (Xiao et al., 2015). News20 is a collection of approximately 20,000 newsgroup documents, partitioned nearly evenly across 20 different newsgroups. Clothing1M has 1M images with real-world noisy labels and additional 50k, 14k, 10k images with clean labels for training, validation and test, and we only use noisy training set in the training phase. Note that the similarity learning method of Class2Simi is based on clustering because there is no class information. Intuitively, for a noisy class, if most instances in it belong to another specific class, we can hardly identify it. For example, assume that a class with noisy labels \( i \) contains \( n_i \) instances with ground-truth labels \( i \) and \( n_j \) instances with ground-truth labels \( j \). If \( n_j \) is bigger than \( n_i \), the model will cluster class \( i \) into \( j \). Unfortunately, in Clothing1M, most instances with label ‘5’ belong to class ‘3’ actually. Therefore, we merge the two classes and denote the modified dataset by Clothing1M* which contains 13 classes. For all the datasets, we leave out 10% of the training data as a validation set, which is for model selection.

For MNIST, CIFAR-10, and CIFAR-100, we use LeNet (LeCun et al., 1998), ResNet-26 with shake-shake regularization (Gastaldi, 2017), and ResNet-56 with pre-activation (He et al., 2016b), respectively. For News20, we first use GloVe (Pennington et al., 2014) to obtain vector representations for the raw text data, and employ a 3-layer MLP with the Softsign active function. For Clothing1M*, we use pre-trained ResNet-50 (He et al., 2016a). Further details for the experiments are provided in Appendix F.1.

**Noisy labels generation.** For clean datasets, we artificially corrupt the class labels of training and validation sets according to the class transition matrix. Specifically, for each instance with clean label \( i \), we replace its label by \( j \) with a probability of \( T_{c,ij} \). In this paper, we consider both symmetric and asymmetric noise settings which are defined in Appendix F.2. Sym-0.2 means symmetric noise type with a 0.2 noise rate and Asym-0.2 means asymmetric noise type with a 0.2 noise rate.
|       | MNIST       |       |       |       |       |       |       |       |
|-------|-------------|-------|-------|-------|-------|-------|-------|-------|
| Co-teaching | 97.34±0.26 | 94.68±0.52 | 93.36±0.47 | 97.37±0.20 | 96.63±0.41 | 91.33±0.38 |       |
| JoCor  | 97.48±0.12  | 96.31±0.20 | 93.18±0.27 | 97.31±0.09 | 95.73±0.29 | 91.43±0.28 |       |
| PHuber-CE | 98.65±0.18 | 98.17±0.15 | 97.63±0.36 | 98.73±0.09 | 98.36±0.25 | 97.37±0.41 |       |
| APL    | 98.77±0.21  | 97.06±0.37 | 97.67±0.35 | 98.72±0.10 | 98.45±0.29 | 97.58±0.25 |       |
| S2E    | 98.96±0.27  | 93.27±2.18 | 89.37±0.70 | 99.19±0.05 | 94.47±1.08 | 92.36±2.40 |       |
| Revision | 98.92±0.09 | 98.42±0.50 | 98.10±0.37 | 98.97±0.06 | 98.58±0.19 | 98.21±0.19 |       |
| Reweight | 98.78±0.16  | 98.26±0.22 | 97.02±0.58 | 98.62±0.19 | 98.12±0.31 | 96.98±0.29 |       |
| Forward | 98.76±0.03  | 98.37±0.25 | 96.89±0.49 | 98.61±0.22 | 98.08±0.33 | 97.43±0.25 |       |
| R-Class2Simi | 99.04±0.06  | 98.87±0.06 | 98.40±0.17 | 99.06±0.05 | 98.75±0.08 | 98.23±0.20 |       |
| F-Class2Simi | 99.26±0.07  | 99.18±0.06 | 98.91±0.09 | 99.26±0.05 | 99.08±0.07 | 98.91±0.07 |       |

|       | CIFAR10     |       |       |       |       |       |       |       |
|-------|-------------|-------|-------|-------|-------|-------|-------|-------|
| Co-teaching | 88.92±0.45 | 85.97±1.02 | 75.97±1.33 | 89.14±0.36 | 84.77±1.08 | 76.07±1.27 |       |
| JoCor  | 88.46±0.25  | 85.19±0.75 | 77.03±0.92 | 88.96±0.70 | 85.19±0.58 | 75.76±1.31 |       |
| PHuber-CE | 90.37±0.26 | 86.05±0.37 | 74.06±0.92 | 90.73±0.22 | 86.06±0.53 | 73.25±1.04 |       |
| APL    | 89.07±0.92  | 85.77±0.84 | 70.06±1.06 | 89.97±0.19 | 85.60±0.91 | 72.33±1.68 |       |
| S2E    | 90.04±1.22  | 82.05±1.95 | 57.96±4.70 | 90.12±0.97 | 83.16±1.58 | 64.77±3.06 |       |
| Revision | 90.02±0.48  | 85.47±0.71 | 73.92±2.02 | 89.77±0.28 | 85.32±1.36 | 75.24±1.87 |       |
| Reweight | 89.05±0.32  | 84.60±0.45 | 74.87±1.18 | 89.28±0.26 | 84.61±0.62 | 72.77±1.91 |       |
| Forward | 89.63±0.20  | 87.08±0.31 | 73.24±1.33 | 90.03±0.41 | 86.64±0.71 | 77.41±0.43 |       |
| R-Class2Simi | 90.91±0.26  | 87.80±0.23 | 79.19±1.65 | 91.07±0.21 | 87.78±0.33 | 78.56±0.63 |       |
| F-Class2Simi | 91.38±0.19  | 88.22±0.19 | 79.45±0.53 | 91.24±0.27 | 87.79±0.36 | 79.05±0.56 |       |

|       | CIFAR100    |       |       |       |       |       |       |       |
|-------|-------------|-------|-------|-------|-------|-------|-------|-------|
| Co-teaching | 57.14±0.49 | 52.62±1.03 | 37.32±1.67 | 57.82±0.37 | 51.32±0.83 | 35.32±1.68 |       |
| JoCor  | 58.32±0.71  | 51.76±1.07 | 37.02±1.33 | 58.61±0.30 | 49.18±1.05 | 37.09±1.82 |       |
| PHuber-CE | 57.90±0.31 | 52.36±0.77 | 37.93±0.86 | 57.33±0.71 | 51.29±0.96 | 36.03±1.34 |       |
| APL    | 54.03±0.92  | 49.06±0.93 | 36.06±2.02 | 55.62±0.92 | 48.37±0.94 | 35.02±1.72 |       |
| S2E    | 59.37±1.09  | 43.29±1.94 | 30.08±3.91 | 58.92±1.21 | 42.88±2.16 | 29.93±4.05 |       |
| Revision | 56.92±0.97  | 53.26±0.84 | 35.82±2.06 | 58.77±0.93 | 52.72±1.38 | 37.72±1.75 |       |
| Reweight | 49.59±0.74  | 39.72±0.57 | 22.79±1.35 | 48.87±0.96 | 36.65±0.90 | 17.24±1.97 |       |
| Forward | 48.68±0.57  | 39.78±1.23 | 27.01±0.89 | 47.90±0.23 | 37.89±0.57 | 21.71±1.53 |       |
| R-Class2Simi | 55.45±0.55  | 50.38±0.49 | 35.57±0.75 | 54.95±0.65 | 47.56±0.72 | 34.82±0.58 |       |
| F-Class2Simi | 60.26±0.18  | 54.85±0.60 | 40.38±0.58 | 59.10±0.13 | 52.99±0.78 | 38.69±2.84 |       |

**Baselines.** In this paper, we compare our method with the following baselines: **Reweight** (Liu & Tao, 2016), **Forward** (Patrini et al., 2017), and **Revision** (Xia et al., 2019), which utilize a class-dependent transition matrix to model the noise, and learn a robust classifier. Besides, we externally conduct experiments on **Co-teaching** (Han et al., 2018b), which is a representative algorithm of selecting reliable samples for training; **JoCor** (Wei et al., 2020), which employs a joint loss function to select small-loss samples; **PHuber-CE** (Menon et al., 2020), which introduces gra-
and CIFAR-10, indicating Class2Simi is robust against the estimation errors on the transition matrix. To verify this, we add some corresponding pointwise methods.

Results on noisy image datasets. The results in Table 1 and Figure 4 demonstrate that Class2Simi achieves distinguished classification accuracy and is robust against the estimation errors on the transition matrix.

From Table 1, overall, we can see that after the transformation, better performance are achieved due to a lower noise rate and the similarity transition matrix being robust to noise. Even for challenging noise rates of 0.6, Class2Simi achieves good accuracy, uplifting about 5 and 10 points on CIFAR-10 and CIFAR-100 respectively, compared with the corresponding pointwise methods.

In Figure 4, we show that the similarity transition matrix is robust against estimation errors. To verify this, we add some random noise to the ground-truth $T_c$ through multiplying every element in class $T_c$ by a random variable $\alpha_{ij}$. We control the noise rate on the $T_i$ by sampling $\alpha_{ij}$ in different intervals, i.e., 0.1 noise means that $\alpha_{ij}$ is uniformly sampled from $\pm [1.1, 1.2]$. Then we normalize $T_c$ to make its row sums equal to 1. From Figure 4, we can see that the accuracy of Forward drops dramatically with the increase of the noise on $T_c$. By contrast, there is only a slight fluctuation of F-Class2Simi, indicating Class2Simi is robust against the estimation errors on the transition matrix.

Results on noisy text datasets. Results in Table 2 show that the proposed method works well on the text dataset under both symmetric and asymmetric noise settings.

Results on the real-world noisy dataset. Results in Table

| Dataset | MNIST | CIFAR10 | CIFAR100 | News20 |
|---------|-------|---------|----------|--------|
| CE      | 99.30±0.02 | 94.03±0.14 | 58.74±0.51 | 59.86±0.39 |
| C2S     | 99.24±0.05 | 94.05±0.27 | 60.36±0.89 | 59.74±0.20 |

Table 2. Means and Standard Deviations of Classification Accuracy over 5 trials on text datasets.

| Method       | Sym-0.2 | Sym-0.4 | Sym-0.6 | Asym-0.2 | Asym-0.4 | Asym-0.6 |
|--------------|---------|---------|---------|----------|----------|----------|
| Co-teaching  | 55.32±0.28 | 51.09±1.06 | 47.07±0.83 | 55.29±0.41 | 53.08±0.26 | 45.63±0.75 |
| JoCor        | 52.21±0.70 | 49.84±0.92 | 48.83±0.43 | 55.58±0.27 | 49.35±0.62 | 46.21±0.73 |
| PHuber-CE    | 55.73±0.38 | 54.33±0.92 | 45.05±0.49 | 56.76±0.26 | 51.15±0.65 | 41.59±1.05 |
| APL          | 56.91±0.21 | 53.12±1.21 | 43.60±1.28 | 56.11±0.23 | 50.93±1.05 | 43.60±1.28 |
| S2E          | 57.93±0.37 | 47.16±1.32 | 28.53±5.04 | 54.89±1.92 | 50.42±1.71 | 30.67±3.12 |
| Revision     | 58.06±0.19 | 52.30±1.73 | 46.84±1.09 | 56.41±0.77 | 53.44±0.83 | 43.77±1.08 |
| Reweight     | 53.34±1.08 | 50.15±1.33 | 44.73±0.79 | 53.37±0.66 | 49.82±0.44 | 39.46±1.27 |
| Forward      | 57.30±0.32 | 53.94±0.42 | 46.91±1.48 | 53.58±0.54 | 49.90±1.44 | 42.55±3.81 |
| R-Class2Simi | 58.67±0.38 | 56.59±0.74 | 50.48±0.97 | 58.44±0.66 | 55.03±1.55 | 47.75±2.17 |
| F-Class2Simi | 58.27±0.47 | 56.70±1.13 | 50.18±0.89 | 58.46±0.68 | 54.92±1.66 | 46.07±3.54 |

Figure 4. Means and Standard Deviations of Classification Accuracy over 5 trials on MNIST and CIFAR10 with perturbational ground-truth $T_c$.

Table 3. Classification Accuracy on Clothing1M*.

| Method       | Accuracy |
|--------------|----------|
| Co-teaching  | 74.70    |
| JoCoR        | 74.98    |
| PHuber-CE    | 73.16    |
| APL          | 58.93    |
| S2E          | 72.30    |
| Revision     | 74.65    |
| Forward      | 73.88    |
| F-Class2Simi | 75.41    |
| Reweight     | 74.44    |
| R-Class2Simi | 75.76    |

Table 4. Classification Accuracy on clean datasets. CE uses class labels and the cross-entropy loss function. C2S refers to Class2Simi.
show that the proposed method also performs well against agnostic noise.

**Ablation study.** To investigate how the similarity loss function influences the classification accuracy, we conduct experiments with the cross-entropy loss function and the similarity loss function on clean datasets over 3 trials, where the $T_c$ is set to an identity matrix. All other settings are kept the same. As shown in Table 4, on MNIST, CIFAR10, and News20, the similarity loss function does not improve the classification accuracy on clean data, and on CIFAR100, the improvement is marginal. However, in Table 1 and 2, the improvements are significant, which reflects the improvements are mainly benefited from the lower noise rate and the reduced noisy binary paradigm.

### 5. Conclusion

This paper proposes a noise reduction perspective on dealing with class label noise by transforming training data with noisy class labels into data pairs with noisy similarity labels. We establish the connection between noisy similarity posterior and clean class posterior and propose a deep learning framework to learn classifiers from the transformed noisy similarity labels. The core idea is to transform pointwise information into pairwise information, which makes the noise rate lower. We also prove that not only the similarity labels but the similarity transition matrix is robust to noise. Experiments are conducted on benchmark datasets, demonstrating the effectiveness of our method. In future work, investigating different types of noise for diverse real-life scenarios might prove important.

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