Orbifold GUT model with nine Higgs doublets

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Abstract. We describe a non-supersymmetric orbifold GUT based on SU(5) symmetry. It is a modification of Kawamura’s 5-D orbifold GUT model. The difference lies in the choice of Higgs scalars as we have allowed only 5-plets of SU(5) in the GUT scale. This variant was originally proposed by Brahmachari and Raychoudhuri. Proton decay problem and the doublet triplet splitting problems are solved by extra dimensional mechanism. The unification scale is around $5 \times 10^{13}$ GeVs. In low energy there are nine Higgs doublets. One at the 100 GeV region and eight others degenerate at around 1.4 TeV. It is an attractive non-supersymmetric extension of standard model with very rich collider physics phenomenology.

Keywords: SU(5), orbifold, higgs doublets
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INTRODUCTION

The SU(5) model attempts to unify the strong, weak, and electromagnetic interactions in the smallest simple group. It has many attractive features which are well recognized, however it suffers from a few major difficulties which are actually generic to the idea of grand unification itself.

1. Because quarks and leptons reside in unified multiplets and there are B- and L-violating interactions, gauge boson exchanges can result in proton decay. If these gauge bosons are appropriately heavy, the decay rate will be very small. Their masses, in the usual formulation, are, however, not arbitrary but rather determined by the scale where the different gauge couplings unify. The proton decay lifetime is therefore a robust prediction of the model. No experimental signature of proton decay has been found yet and the model is disfavoured. More complicated unification models involving several intermediate mass-scales can evade this problem.

2. The low energy Higgs doublet, responsible for electroweak breaking, is embedded in a 5 representation of SU(5). The other members of this multiplet are color triplet scalars which must have a mass near the unification scale and no such scalars have been observed at the electroweak scale. This leads to an unnatural mass splitting among the members of the same SU(5) multiplet. This is termed the double-triplet splitting problem.

3. The non-supersymmetric version does not have a natural dark matter candidate. Neutral members of SU(5) particle spectrum decay quickly.

These unwelcome features of the SU(5) model can be tackled in an elegant way if unified SU(5) symmetry exists in a 5-D world. Low energy 4-D SU(3)$_c \times SU(2)_L \times U(1)$ symmetric recovery when the extra dimension is compactified on a $S^1/(Z_2 \times Z'_2)$ orbifold. This situation is realized when space-time is considered to be factorized into a product of 4D Minkowski space-time $M^4$ and the orbifold $S^1/(Z_2 \times Z'_2)$. The coordinate system consists of $x^\mu = (x^0, x^1, x^2, x^3)$ and $y = x^5$. There are two distinct 4-D branes; one at $y=0$ and another at $y = \pi R/2$. On the $S^1$, $y=0$ is identified with $y = \pi R$ while $y = \pm \pi R/2$ are identified with each other.

As in usual SU(5) model, fermions are put into 5 plet and the 10 plet. We assume, as is common, that the fermions are fixed in the 4-D brane at $y=0$ whereas gauge bosons and the scalars penetrate inside the bulk. The discrete $Z_2$ and $Z'_2$ symmetries, permit the expansion of any 5-D field $\phi$ in the following mode expansions according to whether they are even or odd fields.

$$\phi_{++}(y) = \sqrt{\frac{4}{2\delta_0 \pi R}} \sum_{n=0}^{\infty} \phi_{++}^{(2n)} \cos \frac{2ny}{R}; \quad M_n = \frac{2n}{R}$$  \hspace{1cm} (1)
TABLE 1. Parity assignments for different components of 5-D gauge field. These assignments were first given by Kawamura.

| Components       | $Z_2$ | $Z'_2$ |
|------------------|-------|--------|
| $A_{\mu} \to (1,1)+(1,3)+(8,1)$ | +     | +      |
| $A_{\mu} \to (3,2)+(\bar{3},2)$ | +     | -      |
| $A_5 \to (1,1)+(1,3)+(8,1)$       | -     | -      |
| $A_5 \to (3,2)+(\bar{3},2)$       | -     | +      |

FIGURE 1. $X$ and $Y$ gauge bosons cannot couple to fermions as they have negative $Z'_2$ parity. This makes the proton stable.

\[
\phi_{-+}(y) = \sqrt{\frac{4}{\pi R}} \sum_{n=0}^{\infty} \phi_{-+}^{(2n+1)} \sin \left(\frac{(2n+1)y}{R}\right); \quad M_n = \frac{2n+1}{R}
\]

\[
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\]

\[
\phi_{--}(y) = \sqrt{\frac{4}{\pi R}} \sum_{n=0}^{\infty} \phi_{--}^{(2n+2)} \sin \left(\frac{(2n+2)y}{R}\right); \quad M_n = \frac{2n+2}{R}
\]

GUT SYMMETRY BREAKING

In this scheme 24 Higgs has been excluded. Instead of using Higgs mechanism, we use orbifold properties for breaking $SU(5)$ symmetry. $A_{\mu}$ has + parity and $A_5$ has - parity under $Z_2$. Further more $(3,2)$ and $(\bar{3},2)$ components of $A_{\mu}$ has negative $Z'_2$ parity. In other words $Z_2$ can distinguish between usual 4-D and the extra fifth component of $A_{\mu}$ (5-D gauge field), whereas $Z'_2$ can distinguish between SM gauge bosons and the extra $SU(5)$ gauge bosons. Thus $Z'_2$ assignments break SU(5) symmetry, because a mass splitting among 24 gauge bosons are introduced. Out of 24, 12 have mass-less modes but remaining 12 do not. We choose $M_{GUT} = \frac{1}{R}$, where $R$ is the radius of fifth dimension. Therefore GUT scale is same as compactification scale.

Proton decay problem

It was noted by Dienes, Dudas and Gherghetta[8], that if fermions are restricted to orbifold fixed points, all $Z'_2$ odd type wave functions, such as $X$ and $Y$ gauge bosons will vanish at orbifold fixed points. Thus there is no coupling of $X$ and $Y$ gauge bosons to low energy quarks and leptons, forbidding proton decay. Later on, Altarelli and Feruglio[10] noted that the absence of tree level amplitudes can provide an explanation of present negative experimental results. They also noted that even-though the idea of forbidding proton decay by a suitable discrete symmetry is not new, its physical origin is clear in the present context.
There are three equations and three unknowns. Solving for the unknowns we get four dimensional doublets containing only.

There is proton decay non-observation problem in conventional GUTs. There is also doublet triplet splitting problem in conventional GUTs. These two problems can be solved if SU(5) symmetry exists in a 5-D world. The simplest model there are 6 quarks and 3 leptons. Therefore the total number of fermions are nine per generation. Perhaps we have one doublet for each of them. It is an interesting puzzle. We have compactified the fifth dimension on a $S^1/Z_2 \times Z'_2$

### TABLE 2. Decomposition of SU(5) representations up-to dimension 24, where $Z_2$ and $Z'_2$ are assigned in $\mathbf{5}$ and $\mathbf{\bar{5}}$ only. Higher representations are obtained by group multiplication.

| $SU(5)$ ⊃ | $SU(3)_c \times SU(2)_L \times U(1)_Y$ |
|------------|------------------------------------------|
| $\mathbf{5}$ ⊃ | (1,2,1/2)$_+ + (3,1,-1/3)$_+ - |
| $\mathbf{\bar{5}}$ ⊃ | (1,2,-1/2)$_+ + (3,1,1/3)$_+ - |
| $\mathbf{10}$ ⊃ | (1,1,1)$_+ + (3,1,-2/3)$_+ + (3,2,1/6)$_+ - |
| $\mathbf{15}$ ⊃ | (1,3,1)$_+ + (3,2,1/6)$_+ + (6,1,-2/3)$_+ + |
| $\mathbf{24}$ ⊃ | (1,1,0)$_+ + (1,3,0)$_+ + |
| | | $+(3,2,-5/6)$_+ - $+(3,2,5/6)$_+ - + (8,1,0)$_+ + $ |

**Doublet triplet splitting**

Our model has only $\mathbf{5}$ plets of Higgs fields. From Eqn. [1], Eqn. [4] we see that only $\phi_{++}$ has a mass-less mode. Therefore, once we include one 5-dimensional $\mathbf{5}$ plet, one four dimensional doublet remains mass-less whereas the triplet has a mass of the order of the GUT scale. Similarly for ‘$n_5$’ numbers of 5-dimensional $\mathbf{5}$ plets, ‘$n_5$’ numbers of four dimensional doublets remains mass-less at low energy.

**Renormalization group equations**

Let us define new quantities $m_{k,l} = \ln(M_k/M_l)$ and $b'_{k,l}$ coefficients range $M_k \leftrightarrow M_l$

$$2\pi \alpha^{-1}_i(M_Z) = 2\pi \alpha^{-1}_X + b'_{X,l}M_{X,l} + b'_{l,Z}M_{l,Z}$$

(5)

Where $b$ coefficients are defined as,

$$b'_{X,l} = \left(\begin{array}{c} 41/10 \\ -19/6 \\ -7 \end{array}\right) + \frac{n_5}{3} \left(\begin{array}{c} 3/10 \\ 1/2 \\ 0 \end{array}\right)$$

(6)

There are three equations and three unknowns. Solving for the unknowns we get,

$$\alpha^{-1}_X = 38.53$$

(7)

$$M_{l,Z} = 26.98 - 194.75/n_5$$

(8)

$$M_{X,l} = 194.75/n_5$$

(9)

Because $M_{l,Z} \geq 0$ we obtain $n_5 \geq 8$. For the case of $n_5 = 8$ we get,

$$M_l = 1.39 \text{ TeV} \quad M_X = 5.0 \times 10^{13} \text{ GeV}$$

(10)

Therefore we see that this simple model predicts one doublet at the $m_Z$ scale plus eight more doublets at around 1.4 TeV. Therefore this is a nine Higgs doublet model. Obviously, this model is very interesting from the point of view of collider physics phenomenology. In Figure [2] we have plotted the unification scenario our case has the label (8,0,0,0). More general cases can be found in the reference [11] which also discusses the present case in detail.

**CONCLUSIONS SPECULATIONS AND OUTLOOK**

There is proton decay non-observation problem in conventional GUTs. There is also doublet triplet splitting problem in conventional GUTs. These two problems can be solved if SU(5) symmetry exists in a 5-D world. The simplest model containing only $\mathbf{5}$ plets has nine doublets $(1+8)$ at low energy. If we count the number of fermions in standard model, there are 6 quarks and 3 leptons. Therefore the total number of fermions are nine per generation. Perhaps we have one doublet for each of them. It is an interesting puzzle. We have compactified the fifth dimension on a $S^1/Z_2 \times Z'_2$. 

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FIGURE 2. Gauge unification in various models. Labels of cases are \((n_5, n_{10}, n_{15}, n_{24})\). As a first approximation we have used one intermediate scale which is given by the mass scale of extra scalars allowed by \(S^1/Z_2 \times Z'_2\) compactifications.

orbifold. These extra symmetries can make the proton quite stable. This type of model being non-supersymmetric, there is no R-parity violating proton decay. There is no neutrino mass in this model. However we can add heavy right handed neutrinos in the GUT scale. This is a heavy right handed singlet fermion of mass \(10^{13}\) GeV or so. Then via see-saw mechanism one can produce a small majorana mass of the order of

\[
m = \frac{m_D^2}{M_{GUT}}
\]

Here, we can see that, in our case \(M_{GUT} = 5 \times 10^{13}\). Therefore, we can make an order of magnitude estimate of the neutrino mass,

\[
m = \frac{(10^{12})^2}{5 \times 10^{13}} = 0.2 \times 10^{-9} \text{ GeV} = 0.2 \text{ eV}.
\]

We should study Kaluza-Klein type dark matter in this model. The lightest Kaluza-Klein state with (at-least one) negative parity could be stable and its lifetime can be comparable to the age of the universe. It can be seen that \(Z_2\) parity is the same for all multiplets of scalars. Only the \(Z'_2\) changes. Therefore is \(Z_2\) redundant? We have kept it here even though it is of no use for scalars. This is because if we want to construct the supersymmetric version of the theory, we may have to use it. One can study \(b - \tau\) unification in this model. The simplest way is to couple only one doublet to Fermions. However all nine doublets contribute to gauge coupling unification. At last let us comment on the 5-D Planck scale where quantum gravity becomes non-negligible,

\[
M_{\text{planck}}^{5d} = M_P^{2/3} \times M_c^{1/3}
\]

For our case, \(M_P = 10^{19}\) GeV and \(M_c = M_{GUT} = 10^{13}\) GeV, Therefore the 5-D Planck scale is at

\[
M_{\text{planck}}^{5d} = 10^{16.87} \text{ GeV}
\]

We see that the Planck scale is higher than unification scale as expected. This result follows from our requirement that GUT scale and 5-D compactification scale should be one and the same.

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