The effects of couple stress lubricants and surface roughness on squeeze EHL motion between porous medium layer and elastic ball

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Abstract
This paper investigated the effects of porous media layer (PML) and couple stress (CS) lubricants at pure squeeze action in an isothermal elastohydrodynamic lubrication (EHL) point contact with surface roughness (SRN) under constant load condition. The modified transient stochastic Reynolds equation was derived in polar coordinates by means of the Christensen's stochastic theory and the Stokes's CS fluid theory. The Gauss-Seidel method (GSM) and the finite difference method (FDM) were both used to solve simultaneously modified transient stochastic Reynolds, force equilibrium, elasticity deformation, and rheology equations. The simulation results revealed that the greater permeability ($K$) and porous layer thickness ($F$) are, the smaller central pressures ($P_c$) are, and the smaller film thicknesses ($H$) are. The times required for achieving maximum central pressures ($P_{c_{\text{max}}}$) decrease with increasing $K$ and $F$. Due to squeeze and elastic deformation effects, the slope values of central velocity ($V_c$) change from negative to positive. The magnitude of $V_c$ and $V_{\text{min}}$ decreased with decreasing $K$ and $F$. The $H_{\text{cmin}}$ of circular type RN are greater than that of radial type RN. The $P_{c_{\text{max}}}$ of radial type RN are slightly greater than that of circular type RN.

Keywords
Surface roughness, porous medium layer, couple stress lubricants, EHL, transient Squeeze

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Introduction
Due to self-contained oil reservoir and good low friction features, porous bearings have been extensively used in industry for a long time. They were widely used in clutches, brakes, and so on. The surface roughness (SRN) effect played a very important role in tribology domain, especially when SRN and film thickness have similar order of magnitude. In order to reduce the friction between the contact surfaces and improve the durability of the friction parts, different kinds of lubricating additives are joined to the base lubricant. Their rheological properties belong to non-Newtonian lubricants types. Many micro continuum theories are usually used to investigate rheological characteristics of non-Newtonian lubricants. The high pressure is accompanied by elastic deformation between contact pairs during squeeze process. Therefore, the effects of porous media layer (PML) and couple stress lubricants...
(CS) on squeeze elastohydrodynamic lubrication (EHL) motion with SRN are worth discussing.

Many tribological simulation methods\textsuperscript{1–6} have been presented to explore the effects of SRN. These papers used finite difference method (FDM) and finite element method (FEM) to analyze simultaneously the EHL characteristics of different surface roughness patterns using measured 3D or given stochastic gauss distribution surface roughness. They thought that the rough peak contacts of two surfaces and transient process during start-up are important topics in the future. Over the past years, the generation of the dimple on pure squeeze motion with Newtonian viscous fluid has attracted the attention of many scholars.\textsuperscript{7–9} But non-Newtonian lubricants, surface roughness, and porous media layer did not be considered. These effects are very important influence parameters in modern industry. Adding lubricant additives in lubricants can improve the load-carrying capacity and reduce the friction parameter.\textsuperscript{10,11} Many micro-continuum theories have been developed to describe the effects of additives. Stokes theory\textsuperscript{12} considered the effects of couple stress, body couple, and asymmetric tensor. This model aims to investigate the effects of particle sizes. It can be applied to additives in lubricants. In recent years, many researchers\textsuperscript{13–15} have dedicated their studies on the effect of journal bearing using couple stress lubricant because of the extensive use of additives in lubricants for heavy load applications in industries. Many studies\textsuperscript{16–18} investigated the lubrication characteristics of porous journal bearings with different surface roughness patterns using non-Newtonian lubricants, but they did not consider elastic deformation. They believed that the permeability of porous media layer has significant effects for the characteristics of the porous journal bearings. However, the study on the effects of porous media and non-Newtonian lubricant with surface roughness on EHL circular contact at squeeze motion is insufficient. Therefore, these effects on squeeze EHL motion are necessary for further consideration.

This paper explores the effects of the CS lubricant between PML and elastic ball with SRN on squeeze EHL motion for fixed load condition. The finite difference method (FDM), Gaussian elimination method, the explicit method, and Gauss-Seidel iteration method (GSM) were used to calculate simultaneously the effects of the SRN, PML, and CS lubricants for pressure and film thickness distributions at each time step in the EHL region during pure squeeze process, but without asperities contact. To study the SRN behavior, Christensen stochastic model is adopted as it is a randomly varying quantity.

**Theoretical analysis**

The squeeze film mechanism as shown in Figure 1, a rough elastic sphere is approaching an infinite plate with rough porous medium layer (PML) for constant load condition, and neglect temperature effect. The compressible couple stress lubricant is filled between the ball and the plate. The local film thickness can be considered to include smooth part ($\bar{h}$) and random part ($\delta$):

$$\bar{h} = h(t) + \delta(r, \theta, \xi)$$  \hspace{1cm} (1)

According to the Stokes microcontinuum theory\textsuperscript{12} and the usual assumption of EHL applicable to a thin film, the reduced momentum equations and the continuity equation governing the motion of the lubricant given in polar coordinates can be obtained as:

$$\frac{\partial p}{\partial r} = \frac{\mu}{\rho} \frac{\partial^2 v_r}{\partial z^2} - \eta \frac{\partial^4 v_r}{\partial z^4}$$  \hspace{1cm} (2)

$$\frac{\partial p}{\partial z} = 0$$  \hspace{1cm} (3)

$$\frac{1}{r} \frac{\partial}{\partial r} \left( r \frac{\partial v_r}{\partial r} \right) + \frac{\partial v_z}{\partial z} = 0$$  \hspace{1cm} (4)

The velocity boundary conditions (BC) at the surfaces of the porous medium layer and elastic ball are:

$$v_r(r, 0) = 0, \quad \frac{\partial^2 v_r(r, 0)}{\partial z^2} = 0, \quad v_z(r, 0) = -v_z *$$  \hspace{1cm} (5)

$$v_r(r, \bar{h}) = 0, \quad \frac{\partial^2 v_r(r, \bar{h})}{\partial z^2} = 0, \quad v_z(r, \bar{h}) = \frac{\partial \bar{h}}{\partial t}$$  \hspace{1cm} (6)
Where \( v_2* \) is the modified Darcy velocity component in the porous region. The couple stress lubricant is governed by Darcy’s law, which accounts for the polar effects in the porous region\(^6\) given by

\[
v_2* = -\frac{k}{\mu(1 - \beta)} \frac{\partial p*}{\partial z} \quad (7)
\]

where

\( k \) is permeability parameter, \( \varphi \) is porous film thickness, \( \beta \) is ratio of microstructure size to pore size, \( \beta = (\eta/\mu)/k \).

Due to continuity of the fluid motion in the porous region, the pressure \( p* \) satisfies the Laplace equation.

\[
\nabla^2 p* = 0 \quad (8)
\]

Integrating equation (2) using BC, the velocity component \( v_r \) is:

\[
v_r = \frac{1}{\mu} \frac{\partial p}{\partial r} \left[ \varphi(\varphi + \bar{h})/2 + \bar{f} \left( 1 - \cosh(\frac{2\varphi - \bar{h}}{2l})/\cosh(\frac{\bar{h}}{2l}) \right) \right] \quad (9)
\]

where

\( l \) is characteristic length of the couple stress fluids, \( l = (\eta/\mu)^{1/2} \).

Suppose the porous layer thickness is small. The pressure is continuous (\( p = p* \)) at \( z = 0 \). Integrating equation (8) across the porous layer thickness using \( \partial p* / \partial z = 0 \) at \( z = -\varphi \), the equation can be derived as:

\[
\frac{\partial p*}{\partial z} \bigg|_{z=0} = -\varphi \frac{1}{r} \frac{\partial}{\partial r} \left( r \frac{\partial p}{\partial r} \right) \quad (10)
\]

Substituting equation (10) into equation (7), the \( v_2* \) at \( z = 0 \) can be derived as:

\[
v_2* = \frac{k\varphi}{\mu(1 - \beta)} \frac{1}{r} \frac{\partial}{\partial r} \left( r \frac{\partial p}{\partial r} \right) \quad (11)
\]

Substituting \( v_r \) into equation (4) and integrating across the film thickness with equations (5) and (6), the modified transient Reynolds equation can be derived as:

\[
\frac{\partial}{\partial t} \left( \frac{\rho r}{\mu} \xi(\bar{h}, k, \varphi, \beta, l) \frac{\partial p}{\partial r} \right) = 12r \frac{\partial p\bar{h}}{\partial t} \quad (12)
\]

where

\[
\xi(\bar{h}, k, \varphi, \beta, l) = \bar{h}^3 - 12\bar{h}^2 + 24\bar{h}^3 \tanh\left( \frac{\bar{h}}{2l} \right) + \frac{12k\varphi}{(1 - \beta)} \quad (13)
\]

Taking the expected values of both sides of (12) by Christensen\(^1\) theory, the modified transient stochastic Reynolds equation can be obtained as:

\[
\frac{\partial}{\partial t} \left( \frac{\rho r}{\mu} E[\xi(\bar{h})] \frac{\partial E(p)}{\partial r} \right) = 12r \frac{\partial}{\partial t} [\rho E(\bar{h})] \quad (14)
\]

The expectancy operator is defined by

\[
E(\Theta) = \int_{-\infty}^{+\infty} (\Theta)\lambda(\delta) d\delta \quad (15)
\]

Most engineering rough surfaces are Gaussian in nature.\(^7\) Choose a polynomial function close to the Gaussian distribution:

\[
\lambda(\delta) = \left\{ \begin{array}{ll} \frac{35}{32c} (c^2 - \delta^2)^3, & -c \leq \delta \leq +c \\ 0, & \text{else} \end{array} \right. \quad (16)
\]

The function \( \lambda \) terminates at \( c = \pm 3\sigma \). In the current analysis, the radial type RN (RTRN) and circular type RN (CTRN) structures\(^19\) are worthy of attention. The structures have the long narrow ridges and valleys forms extending in \( r \) and \( \theta \) directions, respectively. For RTRN, the oil film can be expressed as:

\[
\bar{h} = h(t) + \delta(r, \xi) \quad (17)
\]

By means of equations (15) and (16), the dimensionless form of the equation (14) can be derived for RTRN as:

\[
\frac{\partial}{\partial \bar{x}} \left( \frac{\overline{\rho X}}{\mu} E[\xi(\bar{H}, \bar{k}, \overline{\varphi}, \overline{\beta}, L)] \frac{\partial \bar{P}}{\partial \bar{x}} \right) = \frac{8\pi \overline{W} \overline{X} \partial(\bar{p}\bar{H})}{\partial \bar{T}} \quad (18)
\]

where

\[
E[\xi(\bar{H}, \bar{k}, \overline{\varphi}, \overline{\beta}, L)] = \frac{35}{32C^7} \int_{-c}^{c} \left[ \bar{H}^3 - 12\bar{H}L^2 + 24L^3 \tanh\left( \frac{\bar{H}}{2L} \right) + \frac{12\bar{K}\overline{\varphi}}{(1 - \overline{\beta})} \right] (C^2 - \delta^2)^3 d\delta \quad (19)
\]

For CTRN, the oil film can be expressed as:

\[
\bar{h} = h(t) + \delta(\theta, \xi) \quad (20)
\]

The dimensionless form of the equation (14) can be derived for CTRN as:

\[
\frac{\partial}{\partial \bar{x}} \left( \frac{\overline{\rho X}}{\mu} E[1/\xi(\bar{H}, \bar{k}, \overline{\varphi}, \overline{\beta}, L)] \frac{\partial \bar{P}}{\partial \bar{x}} \right) = \frac{8\pi \overline{W} \overline{X} \partial(\bar{p}\bar{H})}{\partial \bar{T}} \quad (21)
\]

where

\[
E[1/\xi(\bar{H}, \bar{k}, \overline{\varphi}, \overline{\beta}, L)] = \frac{35}{32C^7} \int_{-c}^{c} \frac{1}{(C^2 - \delta^2)^3} \left[ \bar{H}^3 - 12\bar{H}L^2 + 24L^3 \tanh\left( \frac{\bar{H}}{2L} \right) + \frac{12\bar{K}\overline{\varphi}}{(1 - \overline{\beta})} \right] d\delta \quad (22)
\]

The BC for equations (18) and (21) are:
The lubricant viscosity-pressure relation put forward by Roelands et al. can be presented as:

\[ \mu = \exp\{(\ln \mu_0 + 9.67)(1 + 5.1p/10^9)^{-1}\} \] (24)

The lubricant density-pressure relation put forward by Dowson and Higginson were displayed as:

\[ \tilde{\rho} = 1 + \frac{0.6p/10^9}{1 + 1.7p/10^9} \] (25)

In EHD point contact, the film thickness can be presented as:

\[ h = h_0 + \frac{r^2}{2R} + \zeta \] (26)

The \( h_0 \) is rigid division, the \( r^2/2R \) is geometric profile, the \( \zeta \) is elastic distortion. The dimensionless form of the equation (26) can be transformed into:

\[ \tilde{H}_i = \tilde{H}_0 + \frac{X_i^2}{2} + \tilde{\zeta} \] (27)

The deformation can be calculated as the sum of the deformation contributions from all pressure points \( j \) at the discrete points \( i \):

\[ \tilde{\zeta}_i = \sum_{j=1}^{n} D_{ij}P_j \] (28)

The \( D_{ij} \) are influence coefficients.

Due to the constant load must be maintained all the time during the squeeze process, the force balance equation can be presented as:

\[ 3 \int_{0}^{\infty} PXdX = 1 \] (29)

Results and discussion

To discuss the effects of the CS lubricant between PML and elastic ball with SRN on squeeze EHL motion for fixed load condition. The modified transient stochastic Reynolds equation (18) and (21) with their boundary condition (23a)-(23c), film thickness equation (27), viscosity-pressure relation (24), density-pressure relation (25), and force equilibrium equation (29) must be solved simultaneously. The flow chart for the solution procedure is shown in Figure 2. The finite difference method (FDM), Gaussian elimination method, the explicit method, and Gauss-Seidel iteration method were used to calculate the numerical solutions of film thickness profiles \( (H) \) and pressure distributions \( (P) \) at each time step on pure squeeze motion, but without asperities contact. The computational parameters used in this research are listed in Table 1. The maximum
The analysis area initially selected was $X_{max} = 16.0$. Since more than half of the analysis area was cavitation, the $X_{max}$ was reduced to half of its initial area, and so on, until $X_{max} = 2.0$. The grid consists of 401 nodes uniformly distributed in the computational domain. The typical problem with $W = 1.048 \times 10^{-8}$, $G = 3500$, $h_{00} = 1000\text{nm}$, $L = 0.08$, $\phi = 0.08$, $K = 0.0127$, and $C = 0.08$ was solved.

Figures 3 and 4 show the relative change in the $P$ and $H$ for an elastic sphere approaching a rough porous medium layer with CS lubricant under fixed load condition for different $K$ (0.0127, 0.0191) and $\phi$ (0.08, 0.16) values, respectively. The $P$ is very flat with a relatively thick film thickness, but as the $H$ decreases, the $P$ becomes steeper. The peak pressure is always at the center. Figure 3 shows the central pressure ($P_c$) gradually increases as the central thickness ($H_c$) decreases (from $t = 0.60\text{ms}$ to $t = 5.78\text{ms}$, $K = 0.0191$) until $H_c$ decreases to 0.19 at $t = 5.78\text{ms}$. After $t = 5.78\text{ms}$, the peak pressure decreases as the $H$ decreases (from $t = 5.78\text{ms}$ to $t = 7.20\text{ms}$, $K = 0.0191$) until the minimum film thickness ($H_{min}$) decreases to 0.002 at $t = 7.20\text{ms}$ without asperities contact. Due to the constant load must be maintained (equation (29)), the pressure distribution increases at central region, but the pressure distribution decreases outside central region. The position of the $H_{min}$ is at the center ($X = 0$). When elastic deformation occurs, the dimple produces at central region, the position of the $H_{min}$ is moved to near the periphery of Hertz contact ($X = 1$). Due to lubricant leaks from $H_{min}$ position, the dimple becomes smaller at the final period. The lubricants are squeezed into porous layer due to squeeze motion. The greater the $K$ is, the smaller the $H$ is, and the smaller the $P$ is at central region, but the greater the $P$ is outside central region due to the constant load must be maintained. Figure 4 shows the $P_c$ gradually increases as the $H_c$ decreases (from $t = 0.6\text{ms}$ to $t = 3.82\text{ms}$, $\phi = 0.16$) until $H_c$ decreases to 0.22 at $t = 3.82\text{ms}$. After $t = 3.82\text{ms}$, the peak pressure decreases as the $H$ decreases (from $t = 3.82\text{ms}$ to $t = 5.38\text{ms}$, $\phi = 0.16$) until the minimum film thickness ($H_{min}$) decreases to 0.002 at $t = 5.38\text{ms}$ without asperities contact. The position of the $H_{min}$ is moved out of the center ($X = 0$). The greater the $\phi$ is, the smaller the $H$ is, and the smaller the $P$ is at central region, but the greater the $P$ is outside central region. The pressure and film thickness decrease with increasing permeability parameter ($k$) and porous layer thickness ($\phi$). The pressure distributions are less concentrated on the central region with PML. Therefore, PML has damper and oil storage effects.

Figure 5 illustrates the $P_c$, $H_c$, and $H_{min}$ versus time for different $K$. The $P_c$ increases rapidly to a maximum value with time at the early period, and then decreases...
slowly toward the 1.0 (Hertzian pressure, \( P_{\text{hertz}} \)) with time at the later period. The \( H \) decreases rapidly with time at the early period and then decreases slowly with time at the later period. The larger the \( K \) is, the smaller the \( P_c \) is, and the smaller the \( H \) is. The times need to achieve the \( P_{c\max} \) and the \( P_{\text{hertz}} \) decrease as the \( K \) increases.

Figure 6 illustrates the \( P_c \), \( H_c \), and \( H_{\min} \) versus time for different \( \phi \). The \( P_c \) increased rapidly to a maximum value with time at the early period, and then decreased slowly toward to the 1.0 (\( P_{\text{hertz}} \)) with time at the later period. The \( H \) decreases rapidly with time at the early period and then decreases slowly with time at the later period. The greater the \( \phi \) is, the smaller the \( P_c \) is, and the smaller the \( H \) is. The times need to achieve the \( P_{c\max} \) and the \( P_{\text{hertz}} \) decrease as the \( \phi \) increases.

Figure 7 illustrates the central normal squeeze velocity (\( V_c \)) and the minimum normal squeeze velocity (\( V_{\min} \)) versus different \( K \). at the early period, the magnitude of the \( V_c \) and \( V_{\min} \) decreased rapidly with time. Due to squeeze and elastic deformation effects, the slope values of the \( V_c \) change from negative to positive. The magnitude of the \( V_c \) and \( V_{\min} \) decreased with decreasing \( K \). The magnitude of \( V_c \) is smaller than the magnitude of \( V_{\min} \) at first half period. The magnitude of \( V_c \) is greater than the magnitude of \( V_{\min} \) at second half period.

Figure 8 illustrates the \( V_c \) and the \( V_{\min} \) for different \( \phi \), at the early period, the magnitude of the \( V_c \) and \( V_{\min} \)
decreased rapidly with time. Due to squeeze and deformation effects, the slope values of the $V_c$ change from negative to positive. The magnitude of the $V_c$ and $V_{min}$ decreased with decreasing $/C_{22}u$. The magnitude of $V_c$ is smaller than the magnitude of $V_{min}$ at first half period. The magnitude of $V_c$ is greater than the magnitude of $V_{min}$ at second half period.

Figure 9 illustrates the $PC_{max}$ and the $HC_{min}$ versus $\phi$ using the CTRN and the RTRN. The larger the $\phi$ is, the smaller the $PC_{max}$ is, and the smaller the $HC_{min}$ is. The $PC_{max}$ of the RTRN are slightly larger than that of the CTRN under the same $\phi$ condition. The $HC_{min}$ of the CTRN are greater than that of the RTRN under the same $\phi$ condition.

Figure 10 illustrates the $PC_{max}$ and the $HC_{min}$ versus $K$ using the CTRN and the RTRN. The larger the $K$ is, the smaller the $PC_{max}$ is, and the smaller the $HC_{min}$ is. The $PC_{max}$ of the RTRN are slightly greater than that of the CTRN under the same $K$ condition. The $HC_{min}$ of the CTRN are greater than that of the RTRN under the same $K$ condition. The CTRN has oil resistance effect. The RTRN has oil guide effect.

Conclusion

A numerical method was developed to calculate the effects of the CS lubricants between PML and elastic ball with SRN on squeeze EHL motion for fixed load condition. The conclusions include:

1. The larger the $K$ and $/C_{22}u$ are, the smaller the $H$ is, and the smaller the $P$ is at central region, but the greater the $P$ is outside central region.
2. The larger the $K$ and $/C_{22}u$ are, the smaller the $Pc$ is, and the smaller the $H$ is. The times need to achieve the $PC_{max}$ and the $P_{hertz}$ decrease as the $K$ and $\phi$ increase.
3. Due to squeeze and elastic deformation effects, the slope values of the $V_c$ change from negative to positive. The magnitude of the $V_c$ and $V_{min}$ decreased with decreasing $K$ and $/C_{22}u$.
4. The larger the $K$ and $/C_{22}u$ are, the smaller the $PC_{max}$ is, and the smaller the $HC_{min}$ is.
5. For the same $K$ and $/C_{22}u$ conditions, the $HC_{min}$ of the CTRN are larger than that of the RTRN. The $PC_{max}$ of the RTRN is slightly larger than that of the CTRN.

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Appendix

Notations

- $b$: reference Hertzian radius at load $w$ (m)
- $c$: half total range of the random film thickness variable
- $C$: dimensionless half total range of the random film thickness variable, $cR/h^2$
- $D_{ij}$: influence coefficients for deformation calculation
- $E'$: equivalent elastic modulus (Pa)
- $G$: dimensionless material parameter, $\alpha E'$
- $h$: film thickness (m)
- $h_0$: rigid separation (m)
- $h_c$: central film thickness
- $h_{min}$: minimum film thickness
- $H$: dimensionless film thickness, $hR/b^2$
- $k$: permeability parameter
- $K$: dimensionless permeability parameter, $R^2/k/b^4$
- $l$: characteristic length of the couple stress fluids, $l = (\eta/\mu)^{1/2}$
- $L$: dimensionless characteristic length of the couple stress fluids, $lR/b^2$
- $p$: pressure (Pa)
- $p_c$: central pressure (Pa)
- $p_h$: reference Hertzian pressure at load $w$ (Pa)
- $P$: dimensionless pressure, $p/p_h$
- $r$: radial coordinate (m)
- $R$: ball radius (m)
- $t$: time (s)
- $T$: dimensionless time, $tE'/\mu_0$
- $v_r$, $v_z$: velocity of the lubricant in $r$ and $z$ directions, respectively (m/s)
- $v_c$: normal velocity of the ball’s center (m/s)
- $V_c$: dimensionless normal velocity of the ball’s center, $v_cR/E'h^2$
- $w$: load (N)
- $W$: dimensionless load, $w/E'R^2$
- $X$: dimensionless radial coordinate, $r/b$
- $z$: axial coordinate
- $z'$: pressure-viscosity index
| Symbol | Description |
|--------|-------------|
| $\alpha$ | pressure-viscosity coefficient |
| $\eta$ | material constant responsible for couple stress parameter |
| $\mu$ | viscosity of lubricant (Pa s) |
| $\mu_0$ | viscosity at ambient pressure (Pa s) |
| $\mu$ | dimensionless viscosity, $\mu/\mu_0$ |
| $\rho$ | density of lubricant (kg/m$^3$) |
| $\rho_0$ | density of lubricant at ambient pressure (kg/m$^3$) |
| $\bar{\rho}$ | dimensionless density of lubricant, $\rho/\rho_0$ |
| $\delta$ | random part due to the surface asperities |
| $\sigma$ | standard deviation |
| $\Delta$ | elastic deformation (m) |
| $\Delta$ | dimensionless elastic deformation, $R\Delta/b^2$ |
| $\phi$ | porous layer thickness (m) |
| $\bar{\phi}$ | dimensionless porous layer thickness, $R\phi/b^2$ |
| $\beta$ | ratio of microstructure size to pore size, $\beta = (\eta/\mu)/k$ |
| $\xi$ | random variable characterizing the definite roughness arrangement |
| $\lambda$ | probability density distribution for $\delta$ |