Universal Unitarity Triangle and Physics Beyond the Standard Model

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Abstract

We make the simple observation that there exists a universal unitarity triangle for all models, like the SM, the Two Higgs Doublet Models I and II and the MSSM with minimal flavour violation, that do not have any new operators beyond those present in the SM and in which all flavour changing transitions are governed by the CKM matrix with no new phases beyond the CKM phase. This universal triangle can be determined in the near future from the ratio $\Delta M_d/(\Delta M)_s$ and $\sin 2\beta$ measured first through the CP asymmetry in $B_d^0 \to \psi K_S$ and later in $K \to \pi \nu \bar{\nu}$ decays. Also suitable ratios of the branching ratios for $B \to X_{d,s}\nu \bar{\nu}$ and $B_{d,s} \to \mu^+ \mu^-$ and the angle $\gamma$ measured by means of CP asymmetries in B decays can be used for this determination. Comparison of this universal triangle with the non-universal triangles extracted in each model using $\epsilon$, $(\Delta M)_d$ and various branching ratios for rare decays will allow to find out in a transparent manner which of these models, if any, is singled out by experiment. A virtue of the universal triangle is that it allows to separate the determination of the CKM parameters from the determination of new parameters present in the extensions of the SM considered here.
1 Introduction

One of the important goals of particle physics is the determination of the Cabibbo–Kobayashi–Maskawa (CKM) matrix \([1, 2]\). In addition to the leading tree level K and B decays, flavour changing neutral current processes generated at the one loop level in the Standard Model (SM) and sensitive to the top quark couplings \(V_{td(s)}\) play a crucial role in this determination. This program is not only complicated by the presence of hadronic uncertainties but also by the possible existence of new physics that contributes to various quantities through diagrams involving new particles. These new contributions depend on unknown parameters, like the masses and couplings of new particles, that pollute the extraction of the CKM parameters.

We would like to point out that in a certain class of extensions of the SM it is possible to construct measurable quantities that depend on the CKM parameters but are not polluted by new physics contributions. This means that these quantities allow a direct determination of the “true” values of the CKM parameters which are common to the SM and this particular class of its extensions. Correspondingly there exists a universal unitarity triangle common to all these models. Interestingly the quantities required to construct the universal unitarity triangle are essentially free from hadronic uncertainties.

In order to explain our point we use the Wolfenstein parameterization \([3]\) of the CKM matrix and its generalization to include higher order terms in \(\lambda\) \([4]\).

Let us recall first that the four Wolfenstein parameters \(\lambda, A, \varrho\) and \(\eta\) can be determined in the standard manner as follows:

Step 1:

The parameters \(\lambda\) and \(A\) are determined from semileptonic K and B decays sensitive to the elements \(|V_{us}|\) and \(|V_{cb}|\) respectively:

\[
\lambda = |V_{us}| = 0.22, \quad A = \frac{|V_{cb}|}{\lambda^2} = 0.826 \pm 0.041.
\]

As the decays in question are tree level decays with large branching ratios this determination is to an excellent approximation independent of any possible physics beyond the SM.

Step 2:

The parameters \(\varrho\) and \(\eta\) are determined by constructing with the help of various decays the unitarity triangle of fig. 1, where \([4]\)

\[
\bar{\varrho} = \varrho(1 - \frac{\lambda^2}{2}), \quad \bar{\eta} = \eta(1 - \frac{\lambda^2}{2})
\]

(1)
Figure 1: Unitarity Triangle.

describe the apex of this triangle. The lengths CB, CA and BA are equal respectively to

\[ R_b \equiv \sqrt{\bar{\rho}^2 + \bar{\eta}^2} = (1 - \lambda^2/2) \frac{1}{\lambda} |V_{ub}|, \quad R_t \equiv \sqrt{(1 - \bar{\rho})^2 + \bar{\eta}^2} = \frac{1}{\lambda} |V_{td}|. \]  

(3)

The standard construction of this triangle involves the ratio \(|V_{ub}/V_{cb}|\) extracted from inclusive and exclusive tree level B decays and flavour changing neutral current processes such as \(B_d^0 - \bar{B}_d^0\) mixing (the mass difference \((\Delta M)_d\)) and indirect CP violation in \(K_L\) decays (the parameter \(\varepsilon\)), both sensitive to the CKM element \(V_{td}\). There is also a constraint coming from the lower bound on the mass difference \((\Delta M)_s\) describing \(B_s^0 - \bar{B}_s^0\) mixing. In particular in the case of \(B_{d,s}^0 - \bar{B}_{d,s}^0\) mixings the following formulae for \((\Delta M)_{d,s}\) resulting from box diagrams are used:

\[
(\Delta M)_{d,s} = \frac{G_F^2}{6\pi^2} \eta_B m_{B_{d,s}} (\hat{B}_{B_{d,s}} F_{B_{d,s}}^2) M_W^2 F_{tt} |V_{td}|^2 \]

(4)

Here \(F_{tt}\) is a function of \(m_t\) and \(M_W\) resulting from box diagrams with top quark exchanges, \(\hat{B}_B\) is a non-perturbative parameter, \(F_B\) is the B meson decay constant and \(\eta_B\) the short distance QCD factor [5, 6] common to \((\Delta M)_d\) and \((\Delta M)_s\).

Similarly, the experimental value for \(\varepsilon\) combined with the theoretical calculation of box diagrams describing \(K^0 - \bar{K}^0\) mixing gives the constraint for \((\bar{\rho}, \bar{\eta})\) in the form of the following hyperbola [7]:

\[
\bar{\eta} \left[ (1 - \bar{\rho}) A^2 \eta_2 F_{tt} + P_c(\varepsilon) \right] A^2 \hat{B}_K = 0.226.
\]

(5)

Here \(\hat{B}_K\) is a non-perturbative parameter analogous to \(\hat{B}_{B_{d,s}}\), \(\eta_2\) is a short distance QCD correction [8], \(F_{tt}\) is the function present also in (4) and \(P_c(\varepsilon) = 0.31 \pm 0.05\) [8] summarizes charm–charm and charm–top contributions.
Combining the two steps above one can determine the range of values of $(\bar{\rho}, \bar{\eta})$ consistent with all present data. Analyses of this type can be found in [7, 9, 10, 11]. In the future this procedure can be generalized to include CP asymmetries in B decays sensitive to the angles of the unitarity triangle and various branching ratios for K and B decays sensitive to the sides and the height of this triangle [7]. If the SM is the correct theory all these measurements should result in a unique value of $(\bar{\rho}, \bar{\eta})$.

This procedure of testing the SM can be applied to its extensions as well. Step 1 remains unchanged as this determination, based on tree level decays, is insensitive to physics beyond the SM. On the other hand Step 2 can be affected by new physics due to:

- New contributions to box diagrams modifying the function $F_{tt}$ and to the analogous functions describing various penguin diagrams contributing to rare K and B decays. This introduces new parameters into the box function $F_{tt}$ and the penguin functions that in the SM depend only on $m_t$ and $M_W$.

- New contributions to box and penguin diagrams that are not proportional to the same combination of CKM matrix elements as the SM top contribution (for example, new contributions to $P_t$ in eq. (3) or new contributions to $(\Delta M)_{d,s}$ proportional to $|V^*_{cs}V_{cb}|^2$).

- New complex phases beyond the one present in the CKM matrix.

- New local operators contributing to the relevant amplitudes beyond those present in the SM. This would introduce additional non-perturbative factors $B_i$ and new box and penguin functions.

It is evident from (4) and (5) that any modification of the function $F_{tt}$ will change the values of the extracted $(\bar{\rho}, \bar{\eta})$. A recent analysis of this type within the MSSM can be found in [9]. Similar comments apply to the extraction of $(\bar{\rho}, \bar{\eta})$ from various branching ratios for rare K and B decays. Moreover if new phases are present in the extensions of the SM, CP violating asymmetries will generally measure different quantities than $\alpha$, $\beta$ and $\gamma$ in fig. 1. For instance the CP asymmetry in $B \rightarrow \psi K_S$ will no longer measure $\beta$ but $\beta + \theta_{NP}$ where $\theta_{NP}$ is a new phase. Strategies for dealing with such situations have been developed. See for instance [12, 13] and references therein.

The presence of new physics and of new phases will be signaled by inconsistencies in the $(\bar{\rho}, \bar{\eta})$ plane. In order to sort out which type of new physics is responsible for deviations from the SM expectations one has to study many loop induced decays and many CP asymmetries. Some ideas in this direction can be found in [12, 13].
While in principle a global fit of all experimental data can be used to test the SM and its extensions it is desirable to develop strategies which allow to make these tests in a transparent manner.

Here we will concentrate on models like the SM, the Two Higgs Doublet Models (TDHM) I and II and the MSSM with minimal flavour violation, that do not have any new operators beyond those present in the SM [14] and in which all flavour changing transitions are governed by the CKM matrix with no new phases beyond the CKM phase. Furthermore, in these models the only sizable new contributions are proportional to the same CKM parameters as the SM top contributions. That is, only the values of the functions describing top-mediated contributions to box and penguin diagrams are modified.

We would like to point out that the models in this class share a useful property. Namely, the CKM parameters in these models extracted from a particular set of data are independent of the contributing loop functions like $F_{tt}$, they are universal in this class of models. Correspondingly there exists a universal unitarity triangle. The determination of this universal unitarity triangle and of the corresponding CKM parameters has four virtues:

- The CKM matrix can be determined without the knowledge of new unknown parameters present in these particular extensions of the SM.

- Because the extracted CKM matrix is also valid in these models, the dependence of various quantities on the new parameters becomes more transparent. In short: the determination of the CKM matrix and of the new parameters can be separated from each other, as opposed to the present strategies discussed in step 2 above.

- The comparison of the predictions for a given observable in the SM and in this kind of extensions can then be done keeping the CKM parameters fixed.

- The extraction of the universal CKM parameters is essentially free from hadronic uncertainties.

In what follows we will list the set of quantities which allow a determination of the universal unitarity triangle. Subsequently we will indicate how the models in this class can be distinguished from each other and from more complicated models which bring in new complex phases and new operators.
2 Determination of $R_t$

In order to illustrate our point let us consider (4). Using this formula one finds

$$\frac{|V_{td}|}{|V_{ts}|} = \xi \frac{m_{B_s}}{m_{B_d}} \sqrt{\frac{(\Delta M)_d}{(\Delta M)_s}} \equiv \kappa, \quad \xi = \frac{F_{B_s}\sqrt{\bar{B}_{B_s}}}{F_{B_d}\sqrt{\bar{B}_{B_d}}}.$$  \hspace{1cm} (6)

This ratio depends only on measurable quantities $(\Delta M)_{d,s}$, $m_{B_{d,s}}$ and the non-perturbative parameter $\xi$. Now to an excellent accuracy \cite{4}

$$|V_{td}| = |V_{cb}|\lambda R_t, \quad |V_{ts}| = |V_{cb}|(1 - \frac{1}{2}\lambda^2 + \bar{\eta}\lambda^2)$$  \hspace{1cm} (7)

with $\bar{\eta}$ defined in (2). We note next that through the unitarity of the CKM matrix, the present experimental upper bound on $(\Delta M)_d/(\Delta M)_s$ and the value of $|V_{ub}/V_{cb}|$ one has in all these models $0 \leq \bar{\eta} \lesssim 0.5$, where $\xi = 1.16 \pm 0.07$ \cite{17} has been used. Consequently $|V_{ts}|$ deviates from $|V_{cb}|$ by at most 2.5%. This means that to a very good accuracy $R_t$ is given by

$$R_t = \frac{\kappa}{\lambda}$$  \hspace{1cm} (8)

independently of new parameters characteristic for a given model and of $m_t$. If necessary the $O(\lambda^2)$ corrections in (7) can be incorporated in (8). This will be only required when the error on $\xi$ will be decreased below 2%, which is clearly a very difficult task.

While the ratio $(\Delta M)_d/(\Delta M)_s$ will be the first one to serve our purposes, there are at least two other quantities which allow a clean measurement of $R_t$ within the class of extensions of the SM considered. These are the ratios

$$\frac{Br(B \to X_{d}\nu\bar{\nu})}{Br(B \to X_{s}\nu\bar{\nu})} = \left|\frac{V_{td}}{V_{ts}}\right|^2$$  \hspace{1cm} (9)

$$\frac{Br(B_d \to \mu^+\mu^-)}{Br(B_s \to \mu^+\mu^-)} = \frac{\tau_{B_d}m_{B_d}F_{B_d}^2}{\tau_{B_s}m_{B_s}F_{B_s}^2} \left|\frac{V_{td}}{V_{ts}}\right|^2$$  \hspace{1cm} (10)

which similarly to $(\Delta M)_d/(\Delta M)_s$ measure

$$\left|\frac{V_{td}}{V_{ts}}\right|^2 = \lambda^2 \left(1 - \bar{\eta}\right)^2 + \bar{\eta}^2 \approx \lambda^2 R_t^2.$$  \hspace{1cm} (11)

Out of these three ratios the cleanest is (9), which is essentially free of hadronic uncertainties \cite{16}. Next comes (10), involving $SU(3)$ breaking effects in the ratio of $B$ meson decay constants. Finally, $SU(3)$ breaking in the ratio of bag parameters $\bar{B}_{B_d}/\bar{B}_{B_s}$ enters in addition in (9). These $SU(3)$ breaking effects should eventually be calculable with reasonable precision from lattice QCD.
It should be remarked that the branching ratio for the rare decay $K^+ \rightarrow \pi^+ \nu \bar{\nu}$ is known to provide a clean measurement of $V_{td}$ and consequently of $R_t$ [7]. However, this branching ratio alone cannot serve our purposes because it is sensitive to new physics contributions.

3 Determination of $\beta$ and $\gamma$

In order to complete the determination of $\bar{\rho}$ and $\bar{\eta}$ in the universal unitarity triangle one can use $\sin 2\beta$ extracted either from the CP asymmetry in $B_d \rightarrow \psi K_S$ [7] or from $K \rightarrow \pi \nu \bar{\nu}$ decays [18]. In the first case one has to measure the time dependent asymmetry

$$a_{CP}(t, \psi K_S) = -\sin(2\beta) \sin((\Delta M)d_t)$$

(12)

that allows a measurement of the angle $\beta$ without any hadronic uncertainties. In the second case the measurements of $Br(K^+ \rightarrow \pi^+ \nu \bar{\nu})$ and $Br(K_L \rightarrow \pi^0 \nu \bar{\nu})$ are required. Then [18]:

$$\sin 2\beta = \frac{2r_s}{1 + r_s^2}$$

(13)

with

$$r_s(B_1, B_2) = \sqrt{\frac{\sigma(B_1 - B_2) - P_c(\nu \bar{\nu})}{\sqrt{B_2}}}.$$  

(14)

Here $\sigma = 1/(1 - \lambda^2/2)^2$ and $B_{1,2}$ stand for the “reduced” branching ratios

$$B_1 = \frac{Br(K^+ \rightarrow \pi^+ \nu \bar{\nu})}{4.11 \cdot 10^{-11}} \quad B_2 = \frac{Br(K_L \rightarrow \pi^0 \nu \bar{\nu})}{1.80 \cdot 10^{-10}}.$$  

(15)

It should be stressed that $\sin 2\beta$ determined in this manner depends only on two measurable branching ratios and on $P_c(\nu \bar{\nu}) = 0.42 \pm 0.06$ which is completely calculable in perturbation theory [18]. Moreover, hadronic uncertainties in these decays have been found to be negligibly small [20, 16]. As analyzed in [18], a measurement of both branching ratios within $\pm 10\%$ will allow the determination of $\sin 2\beta$ within $\pm 0.05$.

Both extractions of $\sin 2\beta$ are to an excellent accuracy independent of the new parameters characteristic for a given model. In particular $P_c(\nu \bar{\nu})$ being proportional to $V_{cs}^* V_{cd}$ receives only negligible new contributions in the class of models considered [14].

Concerning the determination of the angle $\gamma$, the two theoretically cleanest methods are: i) the full time dependent analysis of $B_s \rightarrow D_s^+ K^- \text{ and } B_s \rightarrow D_s^- K^+$ [21] and ii) the well known triangle construction due to Gronau and Wyler [22] which uses six decay rates $B^\pm \rightarrow D_{CP}^0 K^\pm$, $B^+ \rightarrow D^0 K^+$, $\bar{D}^0 K^+$ and $B^- \rightarrow D^0 K^-$, $\bar{D}^0 K^-$. Variants of the latter method which could be more promising experimentally have been proposed in [23, 24].
Both methods involve only tree diagrams and are unaffected by new physics contributions in the class of models considered. It appears that these methods will give useful results at later stages of CP-B investigations. In particular the first method will be feasible only at LHC-B. Clearly any other method for the determination of $\gamma$ in which new physics of the type considered here can be eliminated could also be used. For a recent review of $\gamma$ determinations we refer to [25] and references therein.

4 Determination of the Universal Unitarity Triangle

Once $R_t$ and $\sin 2\beta$ have been determined as discussed above, $\bar{\rho}$ and $\bar{\eta}$ can be found through [26]

$$\bar{\eta} = a \frac{R_t}{\sqrt{2}} \sin 2\beta \cdot r_b(\sin 2\beta), \quad \bar{\rho} = 1 - \bar{\eta} r_b(\sin 2\beta)$$

(16)

where

$$r_b(z) = (1 + b \sqrt{1 - z^2})/z, \quad a, b = \pm.$$  

(17)

Thus for given values of $(R_t, \sin 2\beta)$ there are four solutions for $(\bar{\rho}, \bar{\eta})$ corresponding to $(a,b) = (+, +), (+, -), (-, +), (-, -)$. As described in [26] three of these solutions can be eliminated by using further information, for instance coming from $|V_{ub}/V_{cb}|$ and $\varepsilon$, so that eventually the solution corresponding to $(a,b) = (+, +)$ is singled out

$$\bar{\eta} = \frac{R_t}{\sqrt{2}} \sin 2\beta \cdot r_+(\sin 2\beta), \quad \bar{\rho} = 1 - \bar{\eta} r_+(\sin 2\beta).$$

(18)

We will illustrate this with an example below.

On the other hand $\bar{\rho}$ and $\bar{\eta}$ following from $R_t$ and $\gamma$ are simply given by

$$\bar{\eta} = R_b \sin \gamma \quad \bar{\rho} = R_b \cos \gamma$$

(19)

with

$$R_b = \cos \gamma \pm \sqrt{R_t^2 - \sin^2 \gamma}.$$  

(20)

Comparing the resulting $R_b$ with the one extracted from $|V_{ub}/V_{cb}|$ (see (3)) one of the two solutions can be eliminated.

As an alternative to $\sin 2\beta$ or $\gamma$ one could use the measurement of $\sqrt{\bar{\rho}^2 + \bar{\eta}^2}$ by means of $|V_{ub}/V_{cb}|$ but this strategy suffers from hadronic uncertainties in the extraction of $|V_{ub}/V_{cb}|$. Similarly using $|V_{ub}/V_{cb}|$ and $\gamma$ one can construct the the universal unitarity triangle by means of (19).

We observe that all these different methods determine the “true” values of $\bar{\eta}$ and $\bar{\rho}$ independently of new physics contributions in the class of models considered. Since $\lambda$
Table 1: Four solutions for $\bar{\eta}$ and $\bar{\rho}$ using $R_t$ and $\sin(2\beta)$.

| $(a, b)$ | $\bar{\eta}$ | $\bar{\rho}$ | $\gamma$ | $R_b$ |
|---------|----------------|----------------|----------|--------|
| $(+, +)$ | 0.35           | 0.15           | 67°      | 0.38   |
| $(+, -)$ | 0.85           | 0.65           | 53°      | 1.07   |
| $(-, +)$ | $-0.35$        | 1.85           | $-11°$   | 1.88   |
| $(-, -)$ | $-0.85$        | 1.35           | $-32°$   | 1.59   |

and $|V_{cb}| = A\lambda^2$ are determined from tree level K and B decays they are insensitive to new physics as well. Thus the full CKM matrix can be determined in this manner. The corresponding universal unitarity triangle common to all the models considered can be found directly from formulae like (16), (18) and (19).

As an example let us take $(\Delta M)_d = 0.471/ps$, $(\Delta M)_s = 16.0/ps$ and $\xi = 1.16$. This gives $R_t = 0.92$. Taking $\sin 2\beta = 0.70$ one finds then by means of (14) the four solutions for the universal unitarity triangle given in table 1. As from the data on $|V_{ub}/V_{cb}|$ we have $R_b \lesssim 0.5$ only the first solution is allowed.

Concentrating on the allowed solution, in table 2 we illustrate with a few examples the accuracy of the determination of the unitarity triangle. The first two rows give the assumed input parameters and their experimental errors. The remaining rows give the results for $\bar{\eta}$, $\bar{\rho}$, $\gamma$ and $R_b$ where errors have been added in quadrature.

The accuracy in the scenario I should be achieved at B-factories, FNAL and HERA-B. Scenarios II and III correspond to B-physics at Fermilab during the Main Injector era, LHC-B and BTeV. It should be stressed that this high accuracy is achieved not only because of our assumptions about future experimental errors in the scenarios considered, but also because of the clean character of the quantities considered.

Having the allowed values of table 1 at hand one can calculate $\varepsilon$, $\varepsilon'/\varepsilon$, $(\Delta M)_d$, $(\Delta M)_s$ and branching ratios for rare decays. As these quantities depend on the parameters characteristic for a given model the results for the SM, the MSSM and other models of this class will generally differ from each other. Consequently by comparing these predictions with the data one will be able to find out which of these models is singled out by experiment. Equivalently, $\varepsilon$, $\varepsilon'/\varepsilon$, $(\Delta M)_d$, $(\Delta M)_s$ and branching ratios for rare decays allow to determine non-universal unitarity triangles that depend on the model considered. Only those unitarity triangles which are the same as the universal triangle survive the test.

It is of course possible that new physics is more complicated than discussed here and
that new complex phases and new operators beyond those present in the SM have to be taken into account. These types of effects would be signaled by:

- Inconsistencies between different constructions of the universal triangle,
- Disagreements of the data with the \((\Delta M)_{d,s}\) and the branching ratios for rare K and B decays predicted on the basis of the universal unitarity triangle for all models of the class considered here.

In our opinion the universal unitarity triangle provides a transparent strategy to distinguish between models belonging to the class considered in this paper and to search for physics beyond the SM. Its other virtues have been listed at the end of the Introduction. Presently we do not know this triangle as all the available measurements used for the construction of the unitarity triangle are sensitive to physics beyond the SM. It is exciting, however, that in the coming years this triangle will be known once \((\Delta M)_{s}\) has been measured and \(\sin 2\beta\) extracted from the CP asymmetry in \(B^0_d \to \psi K_S\). At later stages \(K \to \pi \nu \bar{\nu}\), \(B \to X_{d,s}\nu \bar{\nu}\), \(B_{d,s} \to \mu^+\mu^-\) and future determinations of \(\gamma\) through CP asymmetries in B decays will also be very useful in this respect.

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