Hole-hole superconducting pairing in the $t$-$J$ model induced by spin-wave exchange

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We study numerically the hole pairing induced by spin-wave exchange. The contact hole-hole interaction is taken into account as well. It is assumed that antiferromagnetic order is preserved at all scales relevant to pairing. The strongest pairing is obtained for the $d$-wave symmetry of the gap. Dependence of the value of the gap on hole concentration and temperature is presented. For the critical temperature we obtain $T_c \sim 100$ K at the hole concentration $\delta \sim 0.2$–0.3.

74.20.Mn, 74.20.Fg, 71.27.+a, 74.25.Jb

I. INTRODUCTION

Magnetic fluctuations are believed to be a very likely mechanism of pairing in cuprate superconductors. There have been many studies of predominantly phenomenological nature supporting this idea. In the present work we study spin-wave-mediated hole pairing using results obtained from first principles for the undoped $t$-$J$ model.

We base our study on the results of previous papers. It was shown in Ref. [10] that because of spin-wave exchange there is an effective long-range attraction between two holes with opposite spins,

$$U_{\text{eff}}(r) = \frac{\lambda}{r^2}, \quad \lambda < 0. \quad (1)$$

In this potential there is an infinite series of two-hole bound states. However, they have very large sizes and very small binding energies and thus are not directly responsible for high-$T_c$ superconductivity. Very strong pairing in the many-hole problem due to the same potential was demonstrated in Ref. [11], where an infinite set of solutions for the superconducting gap was found. The strongest pairing was either in the $d$-wave or $g$-wave sector. The pairing induced by spin-wave exchange is a long-range phenomenon. However, the attractive potential (1) is too singular and the wave function is known to collapse to the origin. On the one hand, this “collapse” effect substantially enhances pairing. On the other hand, it leads to a dependence of the superconducting gap on short-range dynamics which cannot be studied analytically. For this reason analytical calculations can only estimate the numerical value of the gap and cannot distinguish between $d$- and $g$-wave pairing (which have the same long-range behavior but a different short-range one). In the present work, we calculate the gap numerically, taking into account both spin-wave exchange and contact hole-hole interaction. The $d$-wave pairing is shown to be the strongest.

Recently, $d$-wave pairing was studied in Ref. [12]. Although many results are similar, we believe the spin-wave exchange interaction which we use is more realistic than the atomic limit interaction employed in Ref. [11].

Our paper has the following structure. In Sec. II we present an effective Hamiltonian of the $t$-$J$ model. In Sec. III we calculate the BCS-type pairing of holes at zero temperature. Section IV presents the results of the calculation of the critical temperature. Finally, our conclusions are given in Sec. V.

II. EFFECTIVE HAMILTONIAN FOR DRESSED HOLES

The underlying microscopic physics is described by the $t$-$J$ model defined by the Hamiltonian

$$H = H_t + H_J = -t \sum_{\langle nm \rangle \sigma} (d_{n \sigma}^d d_{m \sigma} + \text{H.c.}) + J \sum_{\langle nm \rangle} \mathbf{S}_n \cdot \mathbf{S}_m, \quad (2)$$

where $d_{n \sigma}^\dagger$ is the creation operator of a hole with spin $\sigma$ ($\sigma = \uparrow, \downarrow$) at site $n$ on a two-dimensional square lattice. The $d_{n \sigma}^\dagger$ operators act in Hilbert space with no double electron occupancy. The spin operator is $\mathbf{S}_n = \frac{1}{2} d_{n \uparrow}^\dagger \sigma_{\alpha \beta} d_{n \beta}$. $\langle nm \rangle$ are the nearest-neighbor sites on the lattice. Below we set $J = 1$ and give all energy values in units of $J$.

At half-filling (one hole per site) the $t$-$J$ model is equivalent to the Heisenberg antiferromagnet model which
has long-range Néel order in the ground state. Under

doping the long-range antiferromagnetic order is de-
stroyed. However, local antiferromagnetic order is pre-
served. We assume that the magnetic correlation length
\( \xi_{magn} \) is not smaller than the typical wavelength of holes,
\( \xi_{magn} \geq \frac{1}{p_F} \sim \frac{1}{\sqrt{\delta}} \) (\( \delta \ll 1 \) is the concentration of
holes). Thus we have antiferromagnetic order at all scales relevant to
the problem. This assumption does not contradict experi-
timental data.\[1\]

We treat the \( J \) term of the Hamiltonian (2) using the
linear spin-wave approximation (see Ref. \[2\] for a review).
Define the Fourier transformations
\[
\begin{align*}
\alpha^\dagger_q &= \sqrt{\frac{2}{N}} \sum_{n \in \uparrow} S_n^\dagger e^{i q \cdot \mathbf{r}_n}, \\
\beta^\dagger_q &= \sqrt{\frac{2}{N}} \sum_{n \in \downarrow} S_n^\dagger e^{i q \cdot \mathbf{r}_n},
\end{align*}
\]
where the notation \( n \in \uparrow (n \in \downarrow) \) means that site \( n \) is on
the spin-up (down) sublattice. Introducing the Bogoli-
ubov canonical transformation
\[
\begin{align*}
\alpha^\dagger_q &= U_q \alpha_q - V_q b_{-q}^\dagger, \\
\beta^\dagger_q &= U_q \alpha_q^\dagger - V_q b_{-q},
\end{align*}
\]
we write the Heisenberg Hamiltonian \( H_J \) as
\[
H_J = E_0 + \sum_q \omega_q (\alpha^\dagger_q \alpha_q + \beta^\dagger_q \beta_q),
\]
where \( E_0 \) is the antiferromagnetic background energy.
The summation over \( q \) is restricted to the Brillouin zone
of one sublattice \( \gamma_q = \frac{1}{2} (\cos q_x + \cos q_y) \geq 0 \).
The spin-wave dispersion and the transformation coefficients are given by
\[
\begin{align*}
\omega_q &= 2 \sqrt{1 - \gamma_q^2}, \\
\omega_q \approx \sqrt{2} |q| &\text{ at } |q| \ll 1, \\
U_q &= \sqrt{1 + \frac{\omega_q}{2}}, \\
V_q &= -\text{sgn}(\gamma_q) \sqrt{1 - \frac{\omega_q}{2}}.
\end{align*}
\]
The spin waves created by \( \alpha^\dagger_q \) and \( \beta^\dagger_q \) have definite
values of spin projection. Due to Eqs. (3) and (4), \( \alpha^\dagger_q |0\rangle \) has
\( S_z = -1 \) and \( \beta^\dagger_q |0\rangle \) has \( S_z = +1 \). Here \( |0\rangle \) is the wave
function of the quantum Néel state.

Single-particle properties in the \( t-J \) model are by now
well established (see Ref. [2]). A single hole is a mag-
netic polaron of a small radius, i.e., a “bare” hole that is
“dressed” by virtual spin excitations. A single hole has
a ground state with a momentum of \( k = (\pm \pi/2, \pm \pi/2) \).
The energy is almost degenerate along the line \( \cos k_x + \\
\cos k_y = 0 \) which is the edge of the magnetic Brillouin
zone (see, e.g., Refs. [14, 23]). The hole dispersion may be
well approximated by the analytical expression [2]
\[
\begin{align*}
\epsilon_k &= \sqrt{\frac{\Delta_0^2}{4} + 4 \gamma_k^2 (1 + y)} \\
&\quad - \sqrt{\frac{\Delta_0^2}{4} + 4 \gamma_k^2 (1 + y) - 4 \gamma_k^2 (1 + y) \gamma_k^2} \\
&\quad + \frac{1}{4} \beta_2 \gamma_k^2; \\
\Delta_0 &\approx 1.33, \quad x \approx 0.56, \quad y \approx 0.14,
\end{align*}
\]
where the parameters \( \Delta_0, x, y \) are some combinations
of the ground state spin correlators.\[2\] Near the band bot-
tom \( k_0 = (\pm \pi/2, \pm \pi/2) \) the dispersion (9) can be pre-
sent in the usual quadratic form
\[
\epsilon_p \approx \frac{1}{2} \beta_1 p_1^2 + \frac{1}{2} \beta_2 p_2^2, \quad \beta_2 \ll \beta_1,
\]
where \( p_1 (p_2) \) is the projection of \( k - k_0 \) on the direction orthogon-
al (parallel) to the face of the magnetic Brillouin zone (Fig. 1).
From Eq. (6) for \( t \gg \Delta_0/4 \) we have
\[
\beta_1 = \frac{x + y}{\sqrt{1 + y}} \approx 0.65t.
\]
According to Refs, \[3, 22\] and \[24\], \( \beta_2 \approx 0.1t \) at \( t \geq \Delta_0/4 \).
The wave function of a single hole may be written in the
form \( \psi_{k,\sigma} = h_{k,\sigma}^\dagger |0\rangle \). At large \( t \) the composite hole
operator \( h_{k,\sigma}^\dagger \) has complex structure. For example, at
\( t/J = 3 \), very roughly, the weight of a bare hole in \( \psi_{k,\sigma} \)
is about 25%; the weight of configurations “bare hole +
1 magnon” is ~50%, and of configurations “bare hole +
2 or more magnons” ~25%. These estimations are based
on the approach with a minimal string ansatz [13] and furt-
her renormalization due to additional magnons.\[24\] How-
ever, other approaches like finite cluster diagonaliza-
tion [17] or numerical solution of Dyson’s equation [18, 19]
give very close results. We have to stress that the dressed
hole is a normal fermion.

The interaction of a composite hole with spin waves is
of the form (see, e.g., Refs. [18, 19, 23, 24])
\[
\begin{align*}
H_{h,sw} &= \sum_{k, q} g_{k, q} \left( h_{k+q, \uparrow}^\dagger h_{k, \uparrow} \alpha_q + h_{k+q, \downarrow}^\dagger h_{k, \downarrow} \beta_q + H.c. \right), \\
g_{k, q} &= 2f \sqrt{\frac{1}{N} \gamma_k U_q + \gamma_k + V_q V_q}.
\end{align*}
\]
For arbitrary \( t \) the coupling constant \( f \) was calculated in
Ref. [23, 24]. The plot of \( f \) as a function of \( t \) is presented in
Fig. 2. For large \( t \) the coupling constant is \( t \)-independent
\( f \approx 2 \).

Let us stress that even for \( t > J \) the interaction (10)
between quasiholes and spin waves has the form as for
\( t \ll J \) (i. e., as for bare hole operators) with an added
renormalization factor (of the order of \( J/t \) for \( t \gg J \)).
This is a remarkable property of the \( t-J \) model which is
due to the absence of a single-loop correction to the
vertex. This property was first found perhaps in Ref. [2].
In Refs. [18, 19, 23, 24] it was demonstrated explicitly that vertex
corrections with different kinematic structure are of the
order of few percent at \( t/J \approx 3 \). There is also a weak
dependence of the coupling constant \( f \). The plot in Fig. 2
corresponds to the long-wavelength limit \( q = 0 \) because,
as we will see later, the small \( q \)'s are most important for
pairing. At \( q \approx \pi \), the factor \( f \) is \((10-17\%) \) bigger than at
\( q = 0 \) [see the discussion between Eqs. (13) and (14) of
Ref. [13]. The influence of this correction on the pairing is
negligible.
Note that the hole scattering between different pockets makes a large contribution to the pairing. However, in the two-sublattice formalism which we use, there are no spin waves with \( q = g = (\pm \pi, \pm \pi) \) and such scattering takes place via umklapp processes with \( q \sim p_F \ll 1 \). One could use another description: Expand the Brillouin zone for spin waves and include \( q \approx g \) into consideration explicitly. Then, due to antiferromagnetic order the points \( q = 0 \) and \( q = g \) are equivalent, and the coupling constants in the effective Hamiltonian (10) are exactly equal, \( f_{g=0} = f_{g=g} \). Certainly the kinematic structure of the vertex \( \cdot \cdot \cdot \) reflects this symmetry: \( g_{q,q} = g_{q,q+g} \).

Interaction between two holes can be caused by exchange of one spin wave. Alongside that there is a contact hole-hole interaction. One can say that it is due to exchange of several hard spin-wave excitations. The Hamiltonian of the contact hole-hole interaction was derived in Refs. 26,10 using a variational approach:

\[
H_{h,h} = \frac{8}{N} \sum_{k_1,k_2,k_3,k_4} \left[ A_1 \gamma_{k_1-k_3} + \frac{1}{2} \gamma_{\gamma_{k_1+k_3}+\gamma_{k_2+k_4}} \right] h_{k_1}^\dagger h_{k_2}^\dagger h_{k_3} h_{k_4}^\dagger \delta_{k_1+k_2,k_3+k_4},
\]

where

\[
A = 16 t \nu \mu^3 (1 - 7 \mu^2) - \frac{1}{4} - 2 \mu^2 - 18.5 \mu^4 + 84 \mu^6 + 10 \alpha t \nu \mu^3, \quad C = \frac{2}{3} \alpha t \nu \mu^3,
\]

and the Fermi momentum \( \nu = \frac{1}{2} \left[ \frac{3}{2} + \frac{2 S_1}{S_1} \right]^{1/2}, \quad \mu = \frac{t}{\left( S_1 (3/2 + 2 S_1) \right)^{1/2}}, \quad S_1 = [9/16 + 4 t^2]^{1/2}. \]

The coefficients \( A \) and \( C \) in Eq. (11) were derived in first order in \( \alpha \), where \( \alpha \) is the coefficient in front of the transverse part to the Heisenberg interaction: \( S_n S_m \rightarrow S_n^z S_m^z + \frac{1}{2} (S_n^+ S_m^+ - S_n^- S_m^-) \). Since the physical value is \( \alpha = 1 \), contributions of higher orders are important. In order to estimate them, we will set \( \alpha = 0.6 \). This choice is made so that results for the binding energy of short-range two-hole bound states obtained by finite lattice diagonalization [31,32] would agree with results obtained [33,34] by using the effective interaction (11). Actually at \( \alpha = 0.6 \) the contact interaction \( H_{h,h} \) is very small and practically does not influence the pairing.

To summarize, we conclude that the dynamics of holes on the antiferromagnetic background is described by the effective Hamiltonian

\[
H_{\text{eff}} = \sum_{k \sigma} \epsilon_k h_{k \sigma}^\dagger h_{k \sigma} + \sum_{q} \omega_q (\alpha_q^\dagger \alpha_q + \beta_q^\dagger \beta_q) + H_{h,sw} + H_{h,h},
\]

which is expressed in terms of the composite hole \( h_{k \sigma} \) and spin-wave \( \alpha_q, \beta_q \) operators. It includes free holes and spin waves and their interactions \( H_{h,sw} \) and \( H_{h,h} \) [given by Eqs. (10) and (11)].

### III. SUPERCONDUCTING STATE

For the small concentrations \( \delta \ll 1 \) under consideration, holes are localized in momentum space in the vicinity of the minima of the band, \( k_0 = (\pm \pi/2, \pm \pi/2) \), and the Fermi surface consists of ellipses (see Fig. 1). The Fermi energy and Fermi momentum of noninteracting holes are

\[
\epsilon_F = \frac{1}{2} \pi (\beta_1 \beta_2)^{1/2} \delta, \quad p_F \sim (\pi \delta)^{1/2}.
\]

The Fermi momentum \( p_F \) is measured from the center of the corresponding ellipse. Let us stress that the numerical value of \( \epsilon_F \) is very small. For realistic superconductors \( t/J \approx 3 \) (see, e.g., Refs. 22,23). Therefore at \( \delta = 0.1 \) and \( J = 0.15 \text{ eV} \) one gets \( \epsilon_F \approx 15 \text{ meV} \approx 175 \text{ K} \). In pairing, the exchange of spin waves with typical momentum \( q \sim p_F \ll 1 \) is the most important. The energy of such spin waves is much higher than the typical energy of a pair,

\[
\omega_q \sim p_F \sim (\pi \delta)^{1/2} \gg \epsilon_F \sim (\beta_1 \beta_2)^{1/2} \delta.
\]

The situation is quite similar to that for the two-hole bound state problem [31] and much different from the situation with the usual phonon-induced pairing where Debye’s frequency is much lower than the Fermi energy.

The interaction between two holes with opposite spins and opposite momenta is

\[
V_{k,k'} = -\frac{g_{k,q} g_{k',-q}}{-\omega_q - E_k - E_{k'}} + \frac{8}{N} (A \gamma_{k-k'} + C \gamma_{k+k'}). \quad (16)
\]
The first term here is due to the spin-wave exchange diagrams shown in Fig. 3. The minus sign before this term takes into account the fact that spin-wave exchange makes the spin flip for both holes. For the same reason, the momentum transfer is the sum (not the difference) of the hole momenta \( q = k + k' \). The energy denominator in Eq. (16) takes into account the energy of the spin-wave \( \omega_q \), and the energies \( E_k \) and \( E_{k'} \) of the two holes in intermediate unpaired state. In fact, the account of \( E_k \) and \( E_{k'} \) is the contact interaction (1). We believe that numerically the wave function (17) satisfies the conventional BCS equation just to justify it. We discuss this question below. The second term in Eq. (16) is the contact interaction (1).

We use the usual BCS wave function for the ground state of the many-hole system

\[
|\Psi\rangle = \prod_k (u_k + v_k h_{k\uparrow}^+ h_{-k\downarrow}^+)|0\rangle.
\]

Thus we suppose that all quasiparticles are in the condensate. For strong interactions the validity of this assumption is under question because there is no parameter to justify it. We believe that numerically the wave function (17) is good. Anyway one may consider the wave function (17) as a trial one in the variational method. In this case the large gain in energy which we get is a justification of the wave function.

The gap \( \Delta_k \) corresponding to the wave function (17) satisfies the conventional BCS equation

\[
\Delta_k = -\frac{1}{2} \sum_{k'} V_{kk'} \frac{\Delta_{k'}}{\sqrt{\xi_k + \Delta_{k'}}},
\]

where \( \xi_k = \epsilon_k - \mu \), \( \mu \) being the the chemical potential fixed by the hole density

\[
\delta = 2 \sum_k v_k^2.
\]

It is well known that the excitation energy of fermions in BCS theory is \( E_k = \sqrt{\xi_k + \Delta_k^2} \). Just this energy enters Eq. (14) for the effective hole-hole interaction. Equation (18) is obtained by variation of the average value of the Hamiltonian with respect to the parameters \( u_k \) and \( v_k \),

\[
\frac{\delta}{\delta u_k} \langle \Psi | H - \mathcal{E} | \Psi \rangle = \frac{\delta}{\delta v_k} \langle \Psi | H - \mathcal{E} | \Psi \rangle = 0.
\]

Here \( \mathcal{E} \) is the energy of the ground state. The effective interaction (18) itself depends on the parameters \( u_k \) and \( v_k \) via the dependence of \( E_k \) on the gap \( \Delta_k \). Nevertheless, in the variational equations (20) we have to set

\[
\frac{\delta}{\delta u_p} V_{kk'} = \frac{\delta}{\delta v_p} V_{kk'} = 0,
\]

and therefore we get the usual BCS equation (18). Explanation of the condition (21) is as follows. The spin-wave exchange part of the interaction (16) is due to the second order of perturbation theory. Therefore, the actual denominator in the spin-wave contribution is \( \mathcal{E} - \mathcal{E}_{\text{excited}} \), and it does not depend explicitly on \( u_k \) and \( v_k \). The self-consistency condition \( \mathcal{E} - \mathcal{E}_{\text{excited}} = -\omega_q - E_k - E_{k'} \) appears after solving Eqs. (20) and (21). From the practical point of view this question is not important because due to the condition (13) the dependence of \( V_{kk'} \) on the gap is very weak.

An iterative numerical solution of Eq. (18) is straightforward. We present results for \( E_\mu \) corresponding to realistic superconducting systems (13). Since the inverse mass \( \beta_2 \) [see Eqs. (7), (8)] is known with rather poor accuracy, we use several values of the mass ratio \( a = \beta_1/\beta_2 \). We take \( \beta_1 \) from Eq. (1) and then set \( \beta_2 = \beta_1/a \). The constant of the hole-magnon interaction (10) is \( f = 1.80 \) at \( t = 3 \).

The symmetry group of the square lattice is \( C_{4v} \). The solutions of Eq. (18) belong to certain representations of this group. In agreement with Ref. [1] the strongest pairing is in the \( B_1 \) representation \([d\ \text{wave}, \text{Fig. 4(a)}]\) and in the \( A_2 \) representation \([g\ \text{wave}, \text{Fig. 4(b)}]\). Consider first the \( d\)-wave pairing. The map of the gap for the hole concentration \( \delta = 0.1 \) and the mass ratio \( a = \beta_1/\beta_2 = 7 \) is presented in Fig. 5. In Fig. 6 we give the map of \( v_\mu^2 \) which is the mean occupation number of a single-hole quantum state. We observe that despite a big value of gap the hole density distribution changes quite sharply at crossing the Fermi surface. For other mass ratios and hole concentrations Fig. 5 is also approximately valid because it gives the gap in units of \( \Delta_{\text{max}} \) and we found that with changing \( \delta \) and \( a \), the whole gap function is multiplied by some factor but the \( k \) dependence is not much changed,

\[
\Delta_k(\delta, a) \approx \frac{\Delta_{\text{max}}(\delta', a')}{\Delta_{\text{max}}(\delta', a')} \Delta_k(\delta', a').
\]

Due to interaction between holes, the ideal gas relation (14) between the chemical potential \( \mu \) and the hole concentration is given in Fig. 7. Comparing the plots of the gap \( \Delta_1 \) and the chemical potential \( \mu \) (Fig. 7), we conclude that \( \Delta_1 \approx 0.7\mu \). This is really a very strong coupling limit and virtually all holes are involved in pairing. This is to be contrasted with the usual situation when only a small portion of electrons \( n \) take part in pairing and the gap is proportional to the Debye frequency.

The \( g\)-wave pairing is weaker and we will not present complete results for this case. Due to the above mentioned similarity of the long-range (small \( q \)) behavior of
the $d$ and $g$ waves which arises from having the same number of zeros at the Fermi surface, the value of the gap for the $g$ wave for the above parameters is of the same order as for the $d$ wave. It is also interesting that the $g$ wave does not depend on details of the contact part of the interaction \cite{14} while the $d$-wave gap is substantially suppressed by adding repulsion to the short-range interaction. Thus, under certain conditions the $g$-wave solution may be relevant to the problem. Table I gives more information about solutions at several parameters including the difference of the free energy $F = \langle \Psi | H - \mu N_h | \Psi \rangle$ ($N_h$ is the number of holes) between the superconducting and normal states,

$$F_S - F_N = 2 \sum_{k} \xi_k v_k^2 + \frac{1}{2} \sum_{k,k'} V_{k,k'} u_k v_k u_{k'} v_{k'}. \quad (24)$$

It is convenient to calculate free energy per hole and use the difference $f_S - f_N = \frac{1}{N_h}(F_S - F_N)$.

IV. CRITICAL TEMPERATURE

Due to the condition \cite{13} the spin-wave frequency in the hole-hole interaction \cite{10} is large in comparison with the hole excitation energy: $\omega_q \gg E_k$. It means that retardation is small and the interaction is almost instantaneous. It is well known that in this case the equation for the gap at $T \neq 0$ is

$$\Delta_k = -\frac{1}{2} \sum_{k'} V_{k,k'} \Delta_{k'} \tanh \frac{E_{k'}}{2T}. \quad (25)$$

In Fig. 9 we present the calculated dependence of $\Delta_1$ on temperature at hole concentration $\delta = 0.1$. Figure 10 gives the dependence of the critical temperature $T_c$ on hole concentration. The approximate relation \cite{23} derived analytically in Ref. \cite{11} is qualitatively fulfilled. In real units ($J = 0.15$ eV), Fig. 10 gives (taking $a = 7$) $T_c = 51$ K at $\delta = 0.1$ and $T_c = 86$ K at $\delta = 0.3$. Let us stress that in our calculation we do not use any fit. The only input is the values of $t$ and $J$.

V. CONCLUSIONS

Using the single spin-wave exchange mechanism suggested in Refs. \cite{11,14} we carried out a numerical \textit{ab initio} calculation of superconducting pairing in the $t$-$J$ model. Both the magnitude of critical temperature and its dependence on hole concentration are in good agreement with experimental data. The calculated critical temperature is still smaller than the highest critical temperature obtained in experiment. However, this may be explained by not knowing the exact parameters. By a relatively small variation of parameters we can get $T_c = 100$–150 K.

The most important remaining problem is the destruction of long-range antiferromagnetic order. Following experimental results \cite{9} we have assumed that antiferromagnetic order is preserved at distances $r \lesssim 1/|p_F|$. The behavior at larger distances is an open question in the present paper.

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TABLE I. Influence of short-range interaction. Changes in the short-range interaction are introduced by increasing the parameter $\alpha$ [see discussion after Eq. (11)] which makes the contact interaction more repulsive. All data are for $T = 0$, $a = 7$, $\mu = 0.058$ (this is the chemical potential at which in the absence of interaction the hole concentration would be $\delta_0 = 0.05$).

| Symmetry | $\alpha$ | $f_s - f_N$ | $\delta$ | $\Delta_{\text{max}}$ | $\Delta_1$ |
|----------|----------|-------------|----------|-----------------|---------|
| $d$-wave | $0.6$    | $-4.42 \times 10^{-3}$ | $0.0606$ | $0.0523$ | $0.0377$ |
| $d$-wave | $0.8$    | $-2.01 \times 10^{-3}$ | $0.0552$ | $0.0317$ | $0.0222$ |
| $g$-wave | $0.6$    | $-1.79 \times 10^{-3}$ | $0.0547$ | $0.0259$ | $0.0210$ |
| $g$-wave | $0.8$    | $-1.79 \times 10^{-3}$ | $0.0547$ | $0.0259$ | $0.0210$ |

FIG. 1. The Brillouin zone of a hole in the $t$-$J$ model.
FIG. 2. The plot of the hole–spin-wave coupling constant $f$.
FIG. 3. Interaction between two holes via a single spin-wave exchange.
FIG. 4. The gap symmetry. (a) $B_1$ type ($d$ wave), (b) $A_2$ type ($g$ wave).
FIG. 5. The contour plot of the $d$-wave gap for $t/J = 3$, the mass ratio $\alpha = \beta_1/\beta_2 = 7$, and hole concentration $\delta = 0.1$. The levels are presented in units of the gap maximal value which at these parameters is equal $\Delta_{\text{max}} = 0.0661$. Dashed curves represent the Fermi surface for the case when one considers the holes like an ideal gas.
FIG. 6. The contour plot of a single hole quantum state mean occupation number $v_k^2$. Parameters are the same as in Fig. 5.
FIG. 7. The chemical potential $\mu$ as a function of hole concentration $\delta$. Dashed curves correspond to an ideal gas of holes. Deviation of dashed curves from linear dependence (1) is due to the deviation of the dispersion relation $E$ from the quadratic expansion $\Delta$. The solid lines. All the curves correspond to $t/J = 3$. The mass ratio is (from top to bottom) $a = \beta_1/\beta_2 = 5, 7, 9$. The maximal value of the gap on the Fermi surface $\Delta_1$ vs hole concentration for $t/J = 3$; the mass ratio is (from top to bottom) $a = \beta_1/\beta_2 = 5, 7, 9$. The maximal value of the gap on the Fermi surface $\Delta_1$ as a function of temperature. The hole concentration is $\delta = 0.05$, $t/J = 3$, the mass ratio $a = \beta_1/\beta_2 = 7$. The critical temperature vs hole concentration. $t/J = 3$, the mass ratio $a = \beta_1/\beta_2 = 7$. 

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