Hydrologic cycle of moist air in the lower atmosphere

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Abstract. A hydrological cycle in the atmosphere plays an important role in the Earth system. The hydrological cycle in the atmosphere is a source of fresh water for life on earth. Starting from the density of heavy water, it is converted into a mild vapor density which allows an increase in water vapor from the sea surface to the dew point where the process of changing water vapor into liquid phase can take place. An atmospheric machine is represented by a modified Carnot cycle below water dome containing mixed water vapor and liquid in an atmospheric layer. This study aims to learn changes in thermodynamic parameters, such as heat and work during the moist air experiencing thermodynamic processes and the correlation of temperature and humidity to rainfall. Basic information recorded by radiosonde is used as mixed water vapor and liquid parameters representing an initial atmospheric condition in producing fresh water. The results will provide information required to figure out the process of evaporation and condensation in the troposphere and the relationship between temperature and humidity to rainfall.

1. Introduction
A hydrological cycle in the atmosphere plays an important role in the Earth system. The hydrological cycle in the atmosphere is a source of fresh water for life on earth. Starting from the density of heavy water, it is converted into a mild vapor density which allows an increase in water vapor from the sea surface to the dew point where the process of changing water vapor into liquid phase can take place. Water in the atmosphere can be in three phases, liquid, vapor, and ice. A change in a liquid phase into a vapor phase is called evaporation and its reverse is called condensation. While a change in a liquid phase into a solid phase is called freezing and its reverse is called liquefaction. A change in a solid phase into a vapor phase is called sublimation and its reverse is called a deposition. When water vapor which is part of natural air turns into liquid or solid particles, these particles become foreign matter in the atmosphere and cause clouds, fog, rain, snow, moisture or hailstone. Formation of water in a vapor phase in the air is very important in determining weather conditions.

Atmospheric stability has to do with the weather conditions of an area, one of which can be seen by understanding the characteristics of the atmospheric boundary layer. In a study stated, the comparison between day and night tends to show the atmosphere during the day is not stable compared to the night. So it is quite important to conduct a study related to the hydrological cycle during the day. In the atmospheric hydrological cycle, the evaporation process or the occurrence of condensation can be described in the thermodynamic framework. Several other studies also discuss a thermodynamic approach to the hydrological cycle in the atmosphere. The thermodynamic framework referred to this study relates to a Carnot cycle in terms of each process to obtain efficiency values and the relation of temperature and humidity to rainfall.

Our study will be focused on the atmosphere above the Indonesian Maritime Continent (IMC). In a study stated, one of the classifications of climate areas in Indonesia can be determined by looking at rainfall patterns. In general, the presentation of rainfall depends on the evaporation process of sea
levels but some global factors such as the presence of ENSO or IOD events, resulting in drought
despite evaporation. \[8\]

However, the focus of this study is to understand the hydrological cycle in
the atmosphere locally in terms of the thermodynamic framework. Consequently, it is interesting to study
the hydrological cycle above the sea surface of the equatorial region
discussing thermodynamic parameters on the evaporation and condensation processes. This study aims to learn
changes in thermodynamic parameters, such as heat and work during the moist air experiencing thermodynamic processes
and the correlation of temperature and humidity to rainfall.

2. Modified Carnot Cycle

Carnot cycle consists of taking a working substance in a system through the four processes that
together constitute a reversible and cyclic transformation. \[9\] In a case of an ideal Carnot cycle, process
1 to 2 is isotherm, process 2 to 3 is an adiabatic expansion, process 3 to 4 is isotherm and process 4 to
1 is adiabatic compression. In this study, the Carnot cycle is modified to slightly different processes
underwater dome, namely process 1 to 2 is isotherm and isobaric (evaporation), process 2 to 3 is
adiabatic expansion, process 3 to 4 is isotherm and isobaric (condensation) and process 4 to 1 is
adiabatic compression which is depicted in Figure 1. The equation of state of water vapor as an ideal
gas can be written using subscripts \(v\) as follows

\[
P V = m R_v T; \quad R_v = 461.5 \text{ Jkg}^{-1}\text{K}^{-1}
\]

\[P = \frac{m}{V} R_v T = \rho R_v T \alpha = R_v T; \quad \alpha = \frac{1}{\rho}
\]

Where \(P, V, m, T, R_v, \rho\) and \(\alpha\) stand for respective pressure, volume, mass, temperature, gas constant
for water vapor, density, and specific density. \[10\]

The relative humidity is the ratio of the actual amount of water vapor to the saturation amount at a
certain temperature, and expressed by the following equation: \[11\] At the ocean surface where the
process of evaporation takes place, relative humidity will be: \[12\]

\[
\mathcal{H}_H = \frac{e_H}{e_s(T_H)}
\]

At the top in the process of condensation is:

\[
\mathcal{H}_C = \frac{e_C}{e_s(T_C)}
\]

Where \(\mathcal{H}_H, \mathcal{H}_C, e_H, e_{dry}, e_s(T_H)\) and \(e_s(T_C)\) stand for respective humidity at the ocean surface and at
the top, hot and cold vapor pressure and saturated vapor pressure at the given hot and cold temperature.

\[\text{Figure 1. A modified Carnot cycle (Zemansky and Dittman, 1997).}\]
3. Heterogeneous system

Heterogeneous systems are systems of two phases, the liquid phase and the vapor phase. Heterogeneous systems have temperatures and pressures that vary with height. Gibbs $G$ free energy, expressed in equations involving $U, T, S, V, p$ which stands for internal energy, temperature, entropy, volume and pressure. Gibbs free energy function:

$$G = U + pV - TS$$

(05)

$$dG = dU + pdV + Vdp - SdT - TdS$$

(06)

$$dG = -SdT + Vdp + (dU + pdV - TdS);$$

(07)

Because of $Tds = dU + pdV$, hence:

$$dG = -SdT + Vdp$$

(08)

The system is containing liquid water mass ($m_l$) and water vapor mass ($m_g$). Gibbs free energy function has additional terms of two phases of water.

$$dG = -SdT + Vdp + \mu_l dm_l + \mu_g dm_g$$

$$dU + pdV - Tds = \mu_l dm_l + \mu_g dm_g$$

$$dU + pdV - \mu_l dm_l - \mu_g dm_g = Tds$$

$$dU + dW - d(m_l \mu_l + m_g \mu_g) = Tds$$

(09)

Expression in specific terms:

$$du + dw - d\mu = dq$$

(10)

The transition from state-1 to state-2 occurs at sea surface with temperature $T_H$ as the process of isotherm-isobaric evaporation. A parcel of liquid water mass $m_l$ is transformed into water vapor mass by keeping that $m_l = m_g$. This water vapor will joint with $m'_g$ as a residual water vapor mass come down from the upper layer. The quality of mixing is expressed as $x = m'_g/m_t$. Where $m_t$ is a total of liquid water and water vapor masses which is expressed by $m_t = m_l + m'_g$.

Accordingly, $m_l/m_t = m_g/m_t = 1 - x$. Thermodynamic potential (specific Gibbs free energy) during the evaporation is zero implied that $\mu_g = \mu_l$. This is supported by the fact that during a change of phase the Gibbs free energy remains the same value, nevertheless the first differential of Gibbs free energy to the temperature having different values.

For the change of volume and specific volume due to the isotherm isobaric process (evaporation):

$$\Delta V_{12} = (V_{g2} - V_{11}) + (V'_{g2} - V'_{g1}) = \Delta v_{12} = (1 - x)(v_{g2} - v_{11}) + x(v'_{g2} - v'_{g1})$$

Because of: $V_{11} \ll V_{g2}$ and $v_{11} \ll v_{g2}$, the change of volume:

$$\Delta V_{12} = V_{g2} + (V'_{g2} - V'_{g1}) = \Delta v_{12} = (1 - x)v_{g2} + x(v'_{g2} - v'_{g1})$$

(11)

This transition is isotherm-isobaric; consequently, the second term in the right-hand side will be isochoric which is having a zero value. Simply the change of specific volume is $\Delta v_{12} = (1 - x)v_{g2}$.

During an isotherm process, there will be no change in internal energy, $\Delta U_{12} = 0$. Heat and specific heat required to evaporate are:

$$\Delta q_{12} = (1 - x)R(T_H)$$

(12)
Since the change of phase form liquid to vapor involves expansive volume, the system will exert the work to the environment, an important note  \( \frac{RT_H}{p(T_H)} \gg v_1 \)

\[
\Delta W_{12} = m_t p(T_H) \left( \frac{RT_H}{p(T_H)} - v_1 \right) = (1 - x)R(T_H)
\]  

(13)

The transition from state-2 to state-3 is in the process of adiabatic expansion. A parcel of water vapor mass  \( m_g \) is rising and joining with a residual vapor  \( m'_g \) come downing from the upper layer at the sea surface just after evaporation. Total water vapor mass is  \( m_t = m_g + m'_g \) to the level where a process of condensation will be possible. During this transition, there will be a change in thermodynamic potential at the sea surface to its value at the possible level of condensation.

\[
\Delta m_t \mu = m_t \mu_C - m_t \mu_H \\
\Delta \mu = R(T_C \ln \mathcal{H}_C - T_H \ln \mathcal{H}_H)
\]

(14)

Because there will be a change in thermodynamic potential at the sea surface, this process called pseudo-adiabatic. So,  \( \Delta q_{23} \neq 0 \). Heat specific for the process:

\[
\Delta q_{23} = R(T_H \ln \mathcal{H}_H - T_C \ln \mathcal{H}_C)
\]

(15)

The work is exerted by the system to the environment:

\[
\Delta W_{23} = m_t \left[ c_P^g \left( RT_H - RT_C \right) + \left( \frac{C_v^g}{R} \right) (T_H - T_C) \right] > 0
\]

(16)

Due to the work exerted to the environment, the internal energy decreases:

\[
\Delta U_{23} = -\Delta W_{23} = m_t \left( C_v^g (T_C - T_H) - c_P^g \frac{\Delta w_{23}}{m_t} \right) = \Delta U_{23} = -c_P^g (T_C - T_H)
\]

(17)

For the change of Volume and specific volume due to the adiabatic expansion process:

\[
\Delta V_{23} = V_{t3} - V_{t2} = \Delta v_{23} = (1 - x)(v'_{g3} - v_{g2}) + x(v'_{g4} - v'_{g4})
\]

(18)

The transition from state-3 to state-4 occurs at the possible level of condensation with temperature  \( T_C \) as the process of isotherm-isobaric condensation. A parcel of water vapor mass  \( m_t \) is partly transformed into liquid water mass  \( m_l \), and parts remained as a residual water vapor mass  \( m'_g \), by keeping that the total water vapor remains as  \( m_t = m_l + m'_g \).

Thermodynamic potential (specific Gibbs free energy) during the condensation is partly zero implied that  \( m_g \mu_g = m_g \mu_l \). The other part is also zero due to the isotherm-isobaric will be also isochoric, and its thermodynamic potential is also zero.

During an isotherm process, there will be no change in internal energy,  \( \Delta U_{34} = 0 \). Heat is released out of the system:

\[
\Delta q_{34} = -(1 - x)R(T_C)
\]

(19)

The work performed on the system is given by:

\[
\frac{\Delta w_{34}}{m_t} = \Delta w_{34} = -(1 - x)RT_C
\]

(20)

Change of volume and specific volume due to condensation:

\[
\Delta V_{34} = (V_{t4} - V_{g3}) + (V'_{g4} - V'_{g3}) = \Delta v_{34} = (1 - x)(v'_{g4} - v_{g3}) + x(v'_{g4} - v'_{g3})
\]

(21)

Because of:  \( V_{t4} \ll V_{g3} \) and  \( v_{t4} \ll v_{g3} \), the changes of volume are:

\[
\Delta V_{34} = -V_{g3} + (V'_{g4} - V'_{g3}) = \Delta v_{34} = -(1 - x)v_{g3} + x(v'_{g4} - v'_{g3})
\]
By the same reasoning the change in specific volume will be \( \Delta v_{34} = -(1 - x)v_{34} \)

The transition from state-4 to state-1 is in the process of adiabatic compression. Through this transition, some of the remaining vapor and precipitation in liquid form is brought to the surface and reheated to \( T_H \). The specific work performed to the system and related to the change of internal energy:

\[
\Delta w_{41} = -C_p^g(T_H - T_C)
\]  

(22)

There are two parcels water masses come down to the sea surface. The first is liquid water mass \( m_1 \) as precipitation, and the second is water vapor mass \( m_2^g \) experiencing adiabatic compression. During this transition there will be a change in thermodynamic potential from the level of condensation to the sea surface:

\[
\Delta m g \mu = m g \mu_C - m g \mu_H
\]

\( x \Delta \mu = x R(T_C \ln H_C - T_H \ln H_H) \)  

(23)

For the change of volume and specific volume due to adiabatic compression will be:

\[
\Delta V_{41} = (V'_{g1} - V'_{g4}) = \Delta v_{41} = x(v'_{g1} - v'_{g4})
\]  

(24)

| Processes | \( \Delta u \) | \( \Delta w \) | \( \Delta \mu \) | \( \Delta q \) |
|----------------|----------------|----------------|----------------|----------------|
| 1 \( \rightarrow \) 2 | 0 | \((1 - x)RT_H\) | 0 | \((1 - x)RT_H\) |
| 2 \( \rightarrow \) 3 | \(-C_p^g(T_H - T_C)\) | \(C_p^g(T_H - T_C)\) | \(-R(T_C \ln H_C - T_H \ln H_H)\) | \(R(T_H \ln H_H - T_c \ln H_C)\) |
| 3 \( \rightarrow \) 4 | 0 | \(-(1 - x)RT_C\) | 0 | \(-(1 - x)RT_C\) |
| 4 \( \rightarrow \) 1 | \((1 - x)C_p^g(T_H - T_C)\) | \(-C_p^g(T_H - T_C)\) | \(-xR(T_H \ln H_H - T_c \ln H_C)\) | \(-xR(T_H \ln H_H - T_c \ln H_C)\) |
| Total | \(\oint du = (1 - x)R(T_H - T_C)\) | \(\oint dw = +(1 - x)R(T_H - T_C)\) | \(\oint d\mu = \{1 - x\}R(T_H - T_C)\) | \(\oint dq = R(T_H - T_C)\) |

| Where: | \(C_p^g = R = C_p^g\) is used. |

*Recapitulation of all process in reversible Carnot cycle*

Based on Table 1, we can find out the total heat \( dq \) of each process, namely:

\[
\oint dq = \oint dw - \oint d\mu + \oint du = (1 - x)\{R(T_H - T_C) + R\Delta H\}
\]  

(25)

Where to find the value of \( \Delta H \), that is \( \Delta H = T_H \ln H_H - T_C \ln H_C \)

So, we can calculate efficiency as follows

\[
\eta = \frac{\oint dq}{\oint dw} = \frac{\{1 - x\}R(T_H - T_C) + R\Delta H}{(1 - x)R(T_H - T_C) + R\Delta H} \approx \frac{\{1 - x\}(T_H - T_C) + R\Delta H}{(1 - x)RT_H + R\Delta H}
\]  

(26)

For \( x = 1 \rightarrow \eta = 0 \)

For \( x = 0 \rightarrow \eta = \frac{R(T_H - T_C) + R\Delta H}{RT_H + R\Delta H} = \frac{(T_H - T_C) + T_H \ln H_H - T_C \ln H_C}{T_H + T_H \ln H_H - T_C \ln H_C} = \frac{T_H + T_H \ln H_H - T_C \ln H_C}{T_H + T_H \ln H_H - T_C \ln H_C}
\]  

(27)

Efficiency Carnot:

\[
\eta_C = 1 - \frac{\frac{T_C}{T_H}}
\]  

(28)

For the \( T_H \) value is 300, \( T_C \) is 273, \( R \) is 461.5, \( H_H \) is 0.82, \( H_C \) is 0.78 and \( x \) is from 0 to 1 with interval 0.1.
4. Application
The data to be applied in this study is obtained from Sultan Hasanuddin Airport of Makassar. Software used in this study for processing data in Microsoft Excel 2007. The data collected is in the form of upper air data for 1 year in 2010, such as temperature, relative humidity and rainfall. The focus of this study does not involve phase changes, it does not involve latent heat but discusses about the cycle of humidity, illustrated in Figure 2.

5. Result
Based on the calculated data, we have the curves $q$ value versus $x$ as depicted in Figure 3, and efficiency with $x$ as depicted in Figure 4.

The result provides information concerning the process of evaporation and condensation in the troposphere. From each process it can be explained that when evaporation, between specific heat and mixing quantity ($x$) is inversely proportional to the greater the mixing quantity, the smaller the specific heat. In this process, heat is needed which means heat enters the system. Conversely, when the opposite condensation of the evaporation process with the high value of $x$, the specific heat is also high. In this process, the heat is released which means the heat is released out of the system.

Based on existing data, to see the relationship rainfall to temperature and humidity can be illustrated in Figure 5, by displaying a 1-year graph of data collected every 3 months.
Figure 5. Graph of the relationship of rainfall to temperature and humidity in 1 year for 3 months.
The results from the graph above explain the correlation between temperature and humidity to rainfall. In theory, December, January and February (DJF) the real motion of the sun is in the southern hemisphere so that the wind blowing from North to South is called the West Monsoon, during the third month, the rainy season generally occurs. Six months later, in June, July and August (JJA) precisely, the movement of air masses from south to north is called the East Monsoon, the dry season occurs.

In this study, one-month representative data was presented from DJF, MAM, JJA and SON. In order to be able to compare the differences between the two seasons for one year, the rainy season is represented in January and the dry season is represented in August. January is the month that has high rainfall and almost every day there is rain. As for August, is the month with the lowest rainfall and relatively little amount of rain. It can be seen in the graph above, the higher the humidity, the higher the amount of rainfall applies to each month. Likewise with the temperature affects the quantity of rainfall, it's just different for JJA because at 3 months the rainfall is very minimal.

6. Conclusion
In the troposphere, it is worthwhile to construct a thermodynamic point of view to the hydrologic cycle comprising of a mixture water vapor mass and liquid water mass under the curve of water dome. The simplest cycle is modified Carnot cycle that is enough to figure out the process of evaporation and condensation. From this cycle comes out the efficiency that depends on the difference of temperature and relative humidity between the levels of evaporation and condensation. This result is a preliminary study on the application of thermodynamics in the hydrological cycle. The correlation between temperature and humidity to rainfall is the higher the temperature and humidity, the greater the quantity of rainfall. Needless to say that larger and deeper development is of importance to be conducted further.

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