Coupling model of jointed rock mass and rock bolt in offshore LPG underground storage

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Abstract
Most of the surrounding rocks of offshore liquefied petroleum gas (LPG) reservoirs occur in jointed rock mass. This paper is aimed at the reinforcement of the layered composite rock masses. First, the structural mechanical model of the elastic deformation and completely plastic deformation for the bolt was constructed. Then, the equations of axial force, shear force, and bending moment in the cross section of the bolt were obtained by the force-method canonical equation. The shear-slip resistance effect of the bolt was decomposed into the friction effect and the dowel effect. In addition, the quantitative relationship between shear resistance and the characteristic angles (anchoring angle, friction angle, dilatancy angle, and deflection angle) and the geometrical and physical parameters of the bolt were derived. Finally, the influence of each characteristic angle on the mechanical properties of the bolt was analyzed in details and the model was verified. The analysis results showed that the laterally constrained load of the rock stratum on the bolt can be characterized by a power function, but the optimal power should be no less than 10. The dominant position of either the friction effect or the dowel effect was changed with the matching of different characteristic angles. The increase of the friction angle and dilatancy angle of the joint surface of the rock is beneficial to improve the lateral shear resistance of the bolt. The effect of the deflection angle on the lateral shear resistance of the bolt was affected by the anchoring angle. The optimal anchoring angle of the bolt should be in the range of 50°-60°. The results in this study have important theoretical significance to guide the offshore anchoring engineering of layered composite rock masses.

Keywords
anchor shear resistance, anchoring effect, characteristic angle, failure mode, layered composite rock mass
1 | INTRODUCTION

With the development of the world’s industry, countries have stepped up their development of various new energy sources and reserves of traditional energy. Compared with coal, petroleum, and other fuels, liquefied petroleum gas (LPG) has the characteristics of rich resources, clean combustion, and high thermal efficiency. Therefore, petrochemical energy storage as a national strategy has attracted wider attention. Because the underground storage tank has lower cost, safe operation, and less environmental pollution than the ground tank with the same capacity, the way of LPG storage has gradually changed from aboveground storage to underground storage.\(^1,2\) Due to the particularity of the stored material, the overall stability of the underground cavern is highly demanded. The surrounding rocks of the underground reservoirs in the eastern part of China are mainly hard rocks such as granite, granite porphyry, and syenite porphyry. And the surrounding rock stress is medium, the main joint intersects the axis of the chamber at a large angle, and the inclination angle is larger. It is more favorable for the stability of the cavern, and the self-stability of the surrounding rock is better.\(^3\) However, there are layered composite rock masses with weak structural plane in the upper LPG underground storage caverns, which is not conducive to the stability of the caverns. This brings huge security risks to the construction and later operation of the reservoir.

The layered composite rock mass is a natural geological body composed of rock with different lithology and structural plane (joint surface), which is common in offshore underground engineering, mines, and slope engineering. The discontinuous rock leads to significant changes in the mechanical properties.\(^4\) Due to the existence of the joint surface in the rock masses, under the external loads, the layers of the rock have the risk of shear slips along the direction of the joint surface or separation in the direction perpendicular to the joint surface, which can cause large-scale collapse and spalling of the rock masses. This risk results in great challenges to the support of surrounding rock in offshore reservoir, the roof in mine roadway, and the reinforcement of slope.\(^5-8\) Fully grouted bolts are commonly used in the reinforcement of the layered rock masses. In the anchoring engineering of a single rock mass, the fully grouted bolt has always been regarded as a purely tensioned rod. But for the composite rock mass with joint surface, the fully grouted bolt also provides a lateral shearing effect, and its reinforcement mechanism and failure mechanism are very complicated. Therefore, understanding the anchoring mechanics of the layered composite rock mass has a guiding significance for the anchorage engineering of offshore storage.

Researchers in China and other countries have performed many studies on the load transfer mechanism of bolts under the single tension condition. There has been great progress in theoretical models, experiments, and numerical simulations.\(^9-11\) In recent years, anchoring failures caused by the shearing effect have frequently been reported in various types of anchoring projects. The issue of anchoring failure has aroused concerns of engineers and researchers. Indoor shear testing is the most direct means to investigate the performance of such type of anchoring systems. Bjurstrom\(^12\) first conducted the shear test for anchored jointed granite. In the test, the transfer pattern of the shear stress at the joint surface of the bolt was analyzed. Based on the results, the bolt can effectively inhibit the mutual sliding between the jointed rock masses and improve the stability of the rock. The researchers conducted a large number of indoor single-shear tests or double-shear tests and revealed the influence of anchoring parameters such as bolt diameter, anchoring angle, dilatancy angle, friction angle, and pretension load on the anchoring effect of composite rock masses.\(^13-19\) The related results showed that the shear stiffness of the bolt was positively correlated with the diameter, dilatancy angle, friction angle, and pretension load. Comparing to the bolt vertical to the joint surface, the inclined bolt was more effective in reducing the lateral deformation of the bolt and strengthening the shear resistance of the jointed rock mass. As a result, the shear displacement was reduced. The anchoring effect is closely related to the anchoring angle. In addition, some studies revealed that the anchoring effect was also related to the strength of the rock\(^20\) and the roughness of the bedding plane.\(^21\) Comparing to the soft rock, the hard rock had faster yielding speed but smaller shear displacement. When the bedding plane had rough surface, the rock tended to break and the shear resistance of the composite rock mass was reduced. Indoor experiments require a lot of manpower, material resources, and financial resources and are vulnerable to the constraints of space and equipment. On contrast, numerical simulation is an alternative research method, in which various complicated factors can be considered. Three types of numerical models have been established to investigate the anchorage of composite rock masses, that is, finite difference model\(^22,23\), finite element model,\(^18,24-27\) and particle flow model.\(^28\) These models and calculation results further revealed various influencing factors and their interaction for the anchoring effects of composite rock mass from macroscopic and mesoscopic perspectives. However, the elastic deformation behavior of materials was investigated in most models, while the plastic mechanical behavior of bolts was rarely studied. Li et al\(^29\) used FLAC3D to select the appropriate constitutive relationship and analyzed the effects of concrete strength, anchoring angle, and bolt diameter on the shear behavior of the double-shear system. Saadat et al\(^30-32\) implemented new cohesive DEM framework, simulated the shear fracture behaviors of rock-like materials, grout-like materials, joint interface, and polycrystalline rock from elasticity to softening; and analyzed the influences of pretension stress, rib angle, roughness coefficient of structural plane, CNL, and CNS boundary conditions on shear strength of structural plane in detail. The simulation results agreed well with the experimental results. Comparing to indoor experiments and numerical simulations, theoretical
analysis revealed the intrinsic mechanism of anchoring effect more clearly. At present, the following two methods are mainly used in the theoretical analysis on the anchoring effect of composite rock mass: (a) the model based on the empirical formula summarized in the indoor test, which was suitable for a specific anchoring structure but had poor versatility and cannot be popularized, (b) anchoring mechanics model based on the theory of elastic-plastic mechanics, the bolt was considered as an elastic foundation beam. In this model, the interaction between the bolt and the rock is systematically revealed. The results showed that the bolt was bent and deformed under the action of lateral load. The intersection point between the bolt and the joint surface was defined as the reverse-bend point, at which the bending moment was zero and the shear force was the maximum. There were two special points on both sides of the reverse-bend point, which can be considered as the plastic hinge. At these two special points, the bending moment was the largest and the shear force was zero. The equations of the axial force, shear force, and displacement of the reverse-bend point were derived under the yielding condition of the bolt. Although the elastic foundation beam theory can be used to describe the deformation behavior of the bolt to some extent, it requires complicated calculation and is not applicable for the plastic behavior of the bolt. Therefore, in some studies, the bolt was considered as a statically indeterminate beam and a structural mechanics model was established for the bolt. This model can be used to describe elastic behavior and plastic behavior of the bolt. However, the current structural mechanics model is still immature. Only a few impact parameters were included in the model, and the applicable conditions were constrained. Therefore, the structural mechanics model still needs further improvement and development.

Based on the previous studies, although the indoor test is real and intuitive, it is almost impossible to be used to analyze the impact of each factor on the anchoring system in detail. In addition, it is challenging to obtain patterned results using the indoor test. The numerical simulation is affected by various factors such as the constitutive material, boundary conditions, and mesh quality of the material; thus, the results are not easy to converge and the reliability is low. Consequently, using indoor tests and numerical simulation methods, it is challenging to obtain general results that can be generalized. Theoretical analysis can be used to establish a universal mechanical model, which can clearly reveal the influence mechanism of each parameter on the anchoring effect. However, the current theoretical analysis has the following problems: First, the laterally constrained load of rock on the bolt is assumed to be a specific function, which is inconsistent with the real scenario. Second, the anchoring angle is considered to be acute, while the obtuse angle has not been considered. Third, interface characteristics of anchored rock mass have not been fully considered. The innovations of this paper are as follows:

a. Assume the transverse constraint load in the elastic stage to distribute as a power function
b. Analyze anchoring angle as acute, right, and obtuse
c. Take the anchoring angle, dilatancy angle, deflection angle, and friction angle into account comprehensively

Therefore, in this paper, by addressing the above problems and deficiencies, the structural mechanical model theory of the layered rock mass was further developed. The results can provide theoretical guidance for the reinforcement of the composite rock mass in offshore storage.

2 ANCHORING MECHANICAL MODEL OF LAYERED COMPOSITE ROCK STRATUM

Figure 1 shows the bending deformation of a bolt in the layered composite rock stratum. Under the combined effect of the bending sinking effect of the upper rock and the lateral shear resistance including the friction between layers and the horizontal pressing force, the horizontal shearing slip is generated along the joint surface. Under the joint action of tensile force and shear force, the bolt produces the combined deformation of tension and bending. The fully grouted bolt has a large shear stiffness, which can restrain the mutual sliding or the tendency of mutual sliding for the composite rock. This restraining effect is called lateral shear resistance effect. This shear resistance effect is manifested in two aspects: the dowel effect, that is, the stiffness of the bolt is used to enhance the shear resistance of the joint surface, and the friction effect, that is, the friction between the bolt and the bonding
layer provides the sliding resistance. The related studies have shown that the lateral deformation zone of the bolt was mainly concentrated near the joint surface within the range of 4 to 6 times as long as the diameter of the bolt.38

As shown in Figure 2, in the vicinity of the joint surface, a debonding failure zone and an extrusion failure zone are observed between the bolt and the bonding layer. Both failure zones are in an antisymmetric distribution. In addition, outside the lateral deformation region of the bolt, the bolt and the bonding layer are still tightly bonded. Therefore, the bolt in the lateral deformation region can be considered as a beam which is fixed at both ends. Under the action of lateral load, the bolt undergoes three phases of deformation, that is, elastic deformation, elastoplastic deformation, and completely plastic deformation. In the elastoplastic deformation phase, there is a gradual transition from the elastic deformation of the bolt at the plastic hinge position to the plastic deformation near the joint surface. The load is complicated in the elastoplastic deformation phase due to the dynamic changes from elastic deformation phase to plastic deformation phase. On contrast, in the elastic phase and the fully plastic phase, the load is relatively stable and simple. In fact, the bolt achieves yielding at the end of the elastic phase, and the damage occurs at the beginning of the completely plastic phase. The stress state during the yielding and damage of the bolt should be used to evaluate the anchoring performance of the bolt. Therefore, the following analyses are focused on the elastic phase and the completely plastic phase of the bolt, while the elastic-plastic transition phase of the rock bolt with complex and variable restraint load is not considered.

Figure 3 is a simplified diagram to calculate the lateral deformation zone of the bolt in the elastic phase. The lateral deformation zone is denoted as $B_1B_2$. For the convenience of analysis, it is assumed that the rock layers on both sides of the joint surface have the same lithology, the lengths of the bolt in both rock layers are both $L$, and $L = 3d$, where $d$ is the diameter of the bolt. The forces exerted by grout on bolt can be divided into the bond force of grout and the load transmitted by grout from rock. It is considered that the effect of the load transmitted by grout from rock is more significant. In order to simplify the analysis, it is approximately considered that the rock directly acts on the bolt and the grout is ignored. Considering the generality, the transversely constrained load $q$ on the bolt by the rock layer is distributed in the form of a power function. The function expression is $q(x) = \frac{q_0}{L^x}x^n$, in which the maximum value $q_0$ appears in the joint surface, that is, the point of $O$. The shear force at two points $B_1$ and $B_2$ is zero. $n$ is the power of the assumed power function of lateral load, and $x$ is the direction of bolt axis.

The simplified force diagram of the bolt in the completely plastic stage is similar to that in the elastic stage, except that the changes of the laterally constrained load, as shown in Figure 4. In this case, $q(x) = p_u$, where $p_u$ denotes the load when the bolt enters complete plastic deformation.

3 | ANALYSIS OF ANCHORING EFFECT OF LAYERED COMPOSITE ROCK STRATUM

3.1 | Analysis of elastic anchoring effect considering lateral shear effect

From Figure 3, the constraint is released at the point of $O$, and the basic static system of the lateral deformation section of the bolt can be obtained, as shown in Figure 5. $X_1$, $X_2$, and $X_3$ are the axial force, the shear force, and the bending moment
in the cross section of the bolt at the position of the joint surface, respectively.

The canonical equation of the force method is as follows:

\[ \begin{cases} 
\delta_{11}X_1 + \delta_{12}X_2 + \delta_{13}X_3 + \Delta_{1q} = \Delta_1 \\
\delta_{21}X_1 + \delta_{22}X_2 + \delta_{23}X_3 + \Delta_{2q} = \Delta_2 \\
\delta_{31}X_1 + \delta_{32}X_2 + \delta_{33}X_3 + \Delta_{3q} = \Delta_3 
\end{cases} \]  

(1)

where \(\delta_{ij}\) refers to the generalized displacement caused by the \(j\)th unit force in the \(i\)th direction, \(\Delta_{ij}\) refers to the generalized displacement of the external load in the \(i\)th direction, and \(\Delta_i\) refers to the generalized displacement in the \(i\)th direction.

The rock layer exerts a lateral load on the bolt through the bonding layer. The shear force and the bending moment in the cross section of the bolt at the position of the joint surface, respectively.

The approximate differential equation of the deflection curve of the beam is as follows:

\[ EL\theta'' = M \]  

(4)

Substitute Eq. (3) into the above equation; then, the angle and deflection in any cross section of the bolt can be obtained.

\[ \theta(x) = \omega'(x) = \frac{1}{EI} \frac{q_0}{(n+1)(n+2)(n+3)} L^3 x^{n+3} \]  

(5)

\[ \omega(x) = \frac{1}{EI} \frac{q_0}{(n+1)(n+2)(n+3)(n+4)} L^4 x^{n+4} \]  

(6)

Since the length of the lateral deformation section of the bolt is shorter than 10 times of the diameter of the bolt, the bolt cannot be considered as the Euler-Bernoulli beam. Considering the transverse shear deformation caused by the shear force, the bolt should be considered as the Timoshenko beam.\(^{44,45}\) In this condition, the generalized displacement matrix generated by the excess constraining force and the external load should be expressed as follows\(^{46}\):

\[ \delta_y = \begin{bmatrix} \frac{L}{EI} & 0 & 0 \\
0 & \frac{L^3}{3EI} + \frac{KL}{GA} & \frac{L^2}{2EI} \\
0 & \frac{L^2}{2EI} & \frac{L}{EI} \end{bmatrix} \]  

(7)

\[ \Delta_q = \begin{bmatrix} 0 \\
-\left( \frac{1}{EI} \frac{q_0 L^4}{(n+1)(n+2)(n+3)(n+4)} + \frac{k}{GA} \right) \\
\frac{1}{EI} \frac{q_0 L^4}{(n+1)(n+2)(n+3)} \end{bmatrix} \]  

(8)

where \(E\) and \(G\) represent the elastic modulus and shear modulus of the bolt, respectively, \(A\) and \(I\) refer to the area and the inertia moment of the cross section of the bolt, \(k\) is the shear deflection factor, which is 10/9 for the circular cross section.

Since the point of \(O\) is the inflection point, at which the bending moment is 0, let \(X_3 = 0\) and substitute Eqs. (7) and (8) into Eq. (1); then, \(X_2\) and \(q_0\) can be solved.

\[ X_2 = \frac{[6EIGAL^2 + 6(1+3)(n+4)kE^2L^2]}{3(n+1)(n+4)kEIL^2 - (2n+5)GAL^3} \Delta_2 - \frac{6(n+4)EIGAL^2}{3(n+1)(n+4)kEIL^2 - (2n+5)GAL^3} \Delta_3 \]  

(9)

\[ q_0 = \frac{(n+1)(n+2)(n+3)(n+4)}{3(n+1)(n+4)kEIL^2 - (2n+5)GAL^3} \left( \frac{2EIGAL^2 + 6kE^2L^2}{3(n+1)(n+4)kEIL^2 - (2n+5)GAL^3} \right) \Delta_2 - \frac{3EIGA}{3(n+1)(n+4)kEIL^2 - (2n+5)GAL^3} \Delta_3 \]  

(10)

According to Eq. (2), the shear force at point \(O\) can be obtained as follows:

\[ X_2 = \frac{L}{n+1} q_0 \]  

(11)

Combining Eqs. (9)-(11), the relationship between \(\Delta_3\) and \(\Delta_2\) can be obtained as follows:

\[ \Delta_3 = \frac{[6(n+4) - 3(n+2)(n+3)(n+4)] GAL}{[6 - 2(n+2)(n+3)(n+4)] GAL^2 - 6(n+1)(n+3)(n+4)kEIL} \Delta_2 \]  

(12)
Substituting Eq. (12) into Eqs. (9) and (10), \(X_2\) and \(q_0\) can be expressed as a function of \(\Delta_2\):

\[
X_2 = \frac{3(n+2)(n+3)(n+4)EIGA}{3(n+1)(n+3)(n+4)kEIL + [(n+2)(n+3)(n+4)-3]GAL} \Delta_2 \quad (13)
\]

\[
q_0 = \frac{3(n+2)(n+3)(n+4)EIGA}{3(n+1)(n+3)(n+4)kEIL + [(n+2)(n+3)(n+4)-3]GAL} \Delta_2 \quad (14)
\]

Therefore, the axial force and shear force acting at the point of \(O\) in the cross section can be expressed as follows:

\[
N_O = X_1 = \frac{EA}{L} \Delta_1 \quad (15)
\]

\[
Q_0 = X_2 = \frac{3(n+2)(n+3)(n+4)EIGA}{3(n+1)(n+3)(n+4)kEIL + [(n+2)(n+3)(n+4)-3]GAL} \Delta_2 \quad (16)
\]

Figure 6(A,B) show the positional relationship between the anchoring angle \(\alpha\), the dilatancy angle \(\beta\), the friction angle of the structural plane \(\varphi\), and the deflection angle \(\theta\), \(\Delta_1\), and \(\Delta_2\) in the transverse deformation section of the bolt within different anchoring angle ranges.

From the positional relationship shown in the figure, the following equation can be obtained:

\[
\frac{\Delta_2}{\Delta_1} = \tan(\alpha - \beta) \quad (17)
\]

Substituting Eqs. (15) and (16) into Eq. (17), the relationship between \(Q_0\) and \(N_O\) can be obtained as follows:

\[
\frac{Q_0}{N_O} = W \tan(\alpha - \beta) \quad (18)
\]

where \(W = \frac{3(n+2)(n+3)(n+4)G}{3(n+1)(n+3)(n+4)kEIL + [(n+2)(n+3)(n+4)-3]GAL}\) is a physical quantity related to the geometric parameters, the physical parameters, and the constrained load distribution of the bolt.

M. Holmberg\(^{47}\) considered the deflection angle of the bolt from the perspective of static balance and proposed to use the following resistance of the bolt to characterize the anchoring effect.

\[
R = N_O \cos(\alpha - \theta) + Q_0 \sin(\alpha - \theta) \pm |N_O \sin(\alpha - \theta) - Q_0 \cos(\alpha - \theta)| \tan \varphi \quad (19)
\]

According to the reports by Pellet,\(^{39}\) the dowel effect in the above equation is as follows:

\[
R_d = N_O \cos(\alpha - \theta) + Q_0 \sin(\alpha - \theta) \quad (20)
\]

The friction effect is as follows:

\[
R_f = [N_O \sin(\alpha - \theta) - Q_0 \cos(\alpha - \theta)] \tan \varphi \quad (21)
\]

Combining Eqs. (18), (20), and (21), the following equation can be obtained:

\[
\frac{R_f}{R_d} = \frac{\sin(\alpha - \theta) - W \cos(\alpha - \theta) \tan(\alpha - \beta)}{\cos(\alpha - \theta) + W \sin(\alpha - \theta) \tan(\alpha - \beta)} \tan \varphi \quad (22)
\]
by the combination of axial force and bending moment, so the yield criterion can be expressed as follows:

$$\frac{N_A}{N_{\text{yield}}} + \frac{M_A}{M_{\text{yield}}} = 1$$ (23)

where $N_{\text{yield}}$ is the limit of the tensile yield with the value of $A\sigma_{\text{yield}}$, and $M_{\text{yield}}$ is the elastic limit bending moment with the value of $\frac{M_{\text{yield}}}{2\pi}$. The tangential shear stress at the interface of the bonding layer and bolt is negligible, that is, $N_A = N_0$. The shear force at point $B_1$ is 0, $M_A = \frac{Q_O}{n+2}$. Thus, Eq. (23) can be represented by $N_O$ and $Q_O$, as shown in the following equation:

$$\frac{N_O}{A_1} + \frac{Q_O}{A_2} = \sigma_{\text{yield}}$$ (24)

where $A_1 = \frac{\pi d^4}{4}$, $A_2 = \frac{(\alpha+2)\pi d^4}{32L}$.

Combining Eqs. (18) and (24), the axial force and shear force in the cross section of $O$ can be obtained when the section of $B_1$ is yielding:

$$N_O = \frac{A_1A_2}{A_2 + A_1W \tan (\alpha - \beta)} \sigma_{\text{yield}}$$ (25)

$$Q_O = \frac{A_1A_2W \tan (\alpha - \beta)}{A_2 + A_1W \tan (\alpha - \beta)} \sigma_{\text{yield}}$$ (26)

Therefore, based on Eqs. (19), (25), and (26), the resistance of the bolt to the initial plastic hinge can be determined as follows:

$$R_y = \cos (\alpha - \theta) + \frac{W \tan (\alpha - \beta) \sin (\alpha - \theta) + [\sin (\alpha - \theta) - W \tan (\alpha - \beta) \cos (\alpha - \theta)] \tan \phi}{A_2 + A_1W \tan (\alpha - \beta)} \frac{A_1A_2 \sigma_{\text{yield}}}{x}$$ (27)

### 3.2 Analysis of plastic anchoring effect considering the lateral shear effect

From Figure 4, the constraint is removed at the point of $O$ and the basic static system can be obtained, as shown in Figure 7. $X_1$, $X_2$, and $X_3$ refer to the axial force, the shear force, and the bending moment in the cross section of the bolt in the position of the joint surface in the completely plastic phase.

The canonical equation of the force method is the same as Eq. (1). When the bolt is in the completely plastic deformation phase, the shear force and bending moment in any cross section can be expressed as follows:

$$Q(x) = p_vx$$ (28)

$$M(x) = \frac{1}{2}p_vx^2$$ (29)

The deflection angle and deflection are as follows:

$$\theta (x) = \frac{1}{EI} \frac{1}{6} p_v x^3$$ (30)

$$\omega (x) = \frac{1}{EI} \frac{1}{24} p_v x^4$$ (31)

The generalized displacement matrix generated by the excessive constraining force is the same as that in Eq. (7). The generalized displacement matrix generated by the external load is as follows:

$$\Delta q = \begin{bmatrix} 0 \\ -\left(\frac{p_1L^4}{24EI} + \frac{k_{33}L^2}{2GA}\right) \\ -\frac{k_{33}L^2}{6EI} \end{bmatrix}$$ (32)

Let $X_3 = 0$, the relationship between $N_O$ and $Q_O$ can be obtained using the similar method in the elastic phase.

$$\frac{Q_O}{N_O} = W_p \tan (\alpha - \beta)$$ (33)

where $W_p = \frac{24 AG}{12EI + 27GA L^2}$.

When the $N_O$ and $Q_O$ meet the following failure criterion, the damaging occurs at point $O$ of the bolt.

$$\left(\frac{N_O}{N_b}\right)^2 + \left(\frac{Q_O}{Q_b}\right)^2 = 1$$ (34)

According to the Mises criterion, $\sigma = \sqrt{3\tau}$, the above equation can be written as follows:

$$\frac{N_O^2}{D^2} + \frac{3Q_O^2}{D^2} = \sigma_p^2$$ (35)
where \( D = \frac{n \sigma_b}{4} \), \( \sigma_b \) is the tensile strength of the bolt. Combining Eqs. (33) and (35), the axial force and shear force in the cross section near the joint surface can be obtained when the bolt is damaged.

\[
N_\alpha = \frac{1}{\sqrt{1 + 3W^2_{p} \tan^2(\alpha - \beta)}} D\sigma_b
\]  

(36)

\[
Q_\alpha = \frac{W_p \tan(\alpha - \beta)}{\sqrt{1 + 3W^2_{p} \tan^2(\alpha - \beta)}} D\sigma_b
\]  

(37)

Thus, based on Eqs. (19), (36), and (37), when the damage occurs, the resistance of the bolt is as follows:

\[
R_p = \frac{\cos(\alpha - \theta) + W_p \tan(\alpha - \beta) \sin(\alpha - \theta) + \sin(\alpha - \theta) - W_p \tan(\alpha - \beta) \cos(\alpha - \theta)}{\sqrt{1 + 3W^2_{p} \tan^2(\alpha - \beta)}} \tan \phi \sigma_b
\]  

(38)

4 | ANALYSIS AND DISCUSSION ON ANCHORING EFFECT OF LAYERED COMPOSITE ROCK STRATUM

From Eqs. (27) and (38), the influencing factors for the anchoring resistance include anchoring angle, friction angle, deflection angle, dilatancy angle, the constraint load distribution of rock stratum on the bolt, and the geometric and physical parameters of the bolt. The influence of the geometric parameters of the bolt has been investigated in many studies; thus, it is not discussed in this paper. In the following section, the anchoring angle, the friction angle, the deflection angle, and the dilatancy angle are referred to as characteristic angles that affect the lateral shearing effect of the bolt. The following discussions focus on the effects of lateral load distribution and characteristic angles on the resistance of the bolt.

The physical and geometric parameters of the bolt are shown in Table 1.

| TABLE 1 Mechanical parameters of the bolt |
|------------------------------------------|
| E/GPa | \( \mu \) | \( k \) | \( d/mm \) | \( \sigma_{yield}/MPa \) | \( \sigma_{s}/MPa \) |
|-------|--------|-------|--------|-----------------|----------------|
| 206   | 0.28   | 10/9  | 25     | 345             | 490             |

In order to discuss the resistance of the bolt, the lateral mutual restraint between the rock stratum and the bolt has been simplified into three distributions, that is, uniform distribution (\( n = 0 \)), triangular distribution (\( n = 1 \)), and parabolic distribution (\( n = 2 \)).\(^{37,41,42}\) In comparison, the power function distribution proposed in this paper is more general and universal for the elastic phase. Under the condition of \( \alpha = 30^\circ, \beta = 6^\circ, \theta = 9^\circ, \) and \( \varphi = 45^\circ, \) the power variation pattern of the resistance force with the constraint power function when the bolt is yielding can be shown in Figure 8. With the increase in the power \( n, \) the resistance of the yielding bolt first increases sharply, then tends to be gentle, and finally approaches a stable value. Therefore, the theoretical values calculated by the uniform distribution, the triangular distribution, and the parabolic distribution are all smaller than the actual value. As power number \( n \) is smaller, the deviation is greater. This deviation is consistent with the results by Liu et al.\(^{42}\) that the theoretical value is slightly smaller than the experimental value. In the following analysis and discussions, \( n \) is set to be 10.

4.2 | Influence of characteristic angle on friction effect and dowel effect

The change of four characteristic angles, that is, anchoring angle, friction angle, dilatancy angle, and deflection angle, can cause the change in the contributions of the friction effect and the dowel effect of the bolt, thus affecting the anchoring ability of the bolt. The ratio of the friction effect \( R_f \) to the dowel effect \( R_d \) is defined as the superiority ratio of the friction effect, \( f.\)

\[
f = \frac{R_f}{R_d}
\]  

(39)

\( f \) can quantify the advantage of the friction effect over the dowel effect; \( |f| > 1 \), the shear-slip resistance effect of the bolt is more dependent on the friction effect; \( |f| < 1 \), the shear-slip resistance effect of the bolt is more dependent on the dowel effect; and \( |f| = 1 \), the effect of the two effects is the same.

Figure 9 shows the influence of the friction angle and the anchoring angle on the superiority ratio of friction effect, \( f.\) The sign in the figure is caused by the change in the direction of the axial force and the shear force, and does not represent a numerical relationship. In the figure, the area between the two dotted lines, that is, \( f = \pm 1 \), represents the corresponding values of the friction angle and the anchoring angle when the dowel effect is dominant, while
the area outside the two dotted lines is the corresponding variation pattern of the friction angle and the anchoring angle when the friction effect is dominant. As the friction angle increases, the range of anchoring angles that causes the dominant position of the friction effect increases. For example, when $\phi = 35^\circ$, the range of the anchoring angle under the condition of $|f| > 1$ is $70^\circ \leq \alpha \leq 130^\circ$. When the friction angle is increased to $55^\circ$, the range of the anchoring angle under the condition of $|f| > 1$ is extended to $50^\circ \leq \alpha \leq 150^\circ$, which is increased by 66.7%. Under the same anchoring angle, as the friction angle gets larger, the friction effect of the bolt is more obvious. When $\alpha > 90^\circ$, the friction effect of the bolt is more significant.

Figure 10 shows the variation of the superiority ratio of friction effect, $f$, under different dilatancy angles and anchoring angles. Under different dilatancy angles, the anchoring angle has the same range that causes the dominant position of the friction effect, that is, $60^\circ \leq \alpha \leq 140^\circ$. When the anchoring angle is in the range of $80^\circ \leq \alpha \leq 110^\circ$, the dilatancy angle has a significant effect on the friction anchoring effect. On the other hand, when the anchoring angle is in other range of the interval, the dilatancy angle has little effect on the friction effect. In the vicinity of $\frac{\pi}{2}$, the value of the tangent function has a large change rate; thus, a slight angular change will result in a significant change of $f$. When $80^\circ \leq \alpha \leq 90^\circ$, as the dilatancy angle gets larger, the friction anchoring effect is more obvious. On the other hand, when $90^\circ \leq \alpha \leq 110^\circ$, as the dilatancy angle gets smaller, the friction anchoring effect is more obvious.

Figure 11 shows the effect of the deflection angle and the anchoring angle on $f$. The deflection angle is negatively correlated with $f$. Under the same anchoring angle, as the deflection angle gets larger, the $f$ is smaller. The influence of deflection angle is different from that of the friction angle. When $\alpha \leq 90^\circ$, as the deflection angle gets larger, the friction anchoring effect is weaker, and the anchoring angle for the dominant friction effect has a smaller range. On the other hand, when $\alpha > 90^\circ$, as the deflection angle gets larger, the friction anchoring effect is more significant, and range of the anchoring angle for the dominant friction effect is increased. Therefore, when $\alpha \leq 90^\circ$, the presence of the deflection angle reduces the friction effect and improves the dowel effect. As the anchoring angle gradually approaches 90 degrees, the influence of the deflection angle is more and more significant.

From Figures 9-11, the three types of characteristic angles (friction angle, dilatancy angle, and deflection angle) and anchoring angle have similar effects on the bolt friction effect in different matching relations. The evolution curve has an almost symmetric distribution. The symmetric center is in the range of $90^\circ \leq \alpha \leq 100^\circ$. When $\alpha \leq 90^\circ$, the superiority ratio of the friction effect increases with the increase of the anchoring angle. When $80^\circ < \alpha \leq 170^\circ$, with the increase of the anchoring angle, the friction effect first has a slight increase and then decreases gradually. When $|f| > 1$, the friction effect plays a more dominant role than the dowel effect. The friction angle has a positive influence in improving the superiority ratio of the friction effect. On the contrary, the deflection angle has a negative influence in improving $f$. The influence of the dilatancy angle on the friction effect and
the dilatancy effect is mainly concentrated in the vicinity of $\alpha = 90^\circ$, while the influence is negligible in other regions. For bolts that are perpendicular to the joint surface, the effects of the three angles on the lateral shearing effect of the bolt should be considered.

### 4.3 Influence of characteristic angle on the resistance force for initial plastic hinge of bolt

In the process of transverse shear deformation, the bolt is subjected to tensile force and bending moment. Firstly, the bolt enters the elastic phase, in which elastic deformation occurs. As the load further increases, the position of the plastic hinge starts to yield and enter the plastic phase. At this time, both the elastic zone and the plastic zone exist inside the bolt. As the load is continuously increased, the plastic zone continuously extends from the position of the plastic hinge to the vicinity of the joint surface. When the cross section of the joint surface reaches the failure condition, the bolt is damaged and failed. Therefore, the shear strength of the bolt under the initial generation of the plastic hinge and the failure condition is the key to measure the lateral shear resistance.

When the initial plastic hinge is generated, the anchor resistance of the bolt can be calculated by Eq. (27). Figure 12 shows the evolution law of the anchor resistance when the initial plastic hinge is generated by the bolt under different friction angles and anchoring angles. As the friction angle gets larger, the lateral shear resistance of the bolt is larger, and the shear resistance is stronger. The illustrated evolution law can be divided into three regions: I peak region ($10^\circ \leq \alpha \leq 90^\circ$), II mutation region ($90^\circ < \alpha \leq 110^\circ$), and III decreasing region ($110^\circ < \alpha \leq 170^\circ$). When the anchoring angle is within Zone I, the shearing capacity of the inclined mounted bolt is better than that of the vertically mounted bolt. In this zone, when the anchoring angle is in the range of $30^\circ < \alpha < 60^\circ$, the shear resistance performance of the bolt is the greatest. The optimal angle is not a fixed value but increasing as the friction angle increases. In Zone II, the shear performance of the bolt is maximized when $\alpha = 100^\circ$. In Zone III, the shear resistance of the bolt is approximately linearly attenuated with the increase of the anchoring angle. As the friction angle gets larger, the attenuation is greater.

Figure 13 shows the variation of the shear resistance of the bolt under different dilatancy angles and anchoring angles. The variation curve can also be divided into three zones.
Except for Zone II, the increase of dilatancy angle causes only a small increase in the lateral shear resistance of the bolt with the initial plastic hinge. The optimum anchoring angle in Zone I is between 40° and 50°. In Zone II, the shear resistance of the bolt is still the strongest when \( \alpha = 100 \), and the maximum resistance increases with the increase of the dilatancy angle. In Zone III, the resistance of the bolt under different dilatancy angles is attenuated almost by the same amplitude.

Figure 14 further shows the influence of the deflection angle and anchoring angle on the shear resistance of the bolt. The influence of the deflection angle on the resistance of the bolt is more complicated at the initial generation of the plastic hinge. Similarly, the evolution curve of resistance is also divided into the above three zones. In Zone I, when 10° ≤ \( \alpha \) ≤ 55°, the deflection angle suppresses the lateral shear resistance of the bolt. When 55° ≤ \( \alpha \) ≤ 90°, the deflection angle enhances the lateral shear resistance of the bolt. In Zones II and III, the increase in the deflection angle can significantly improve the lateral shear resistance of the bolt. Moreover, in Zone III, the attenuation rate of the bolts is approximately the same under the different deflection angles.

Based on Figures 12-14, the following conclusions can be obtained: In Zone I, as the anchoring angle increases, the resistance of the bolt first increases and then decreases as the initial plastic hinge is generated. A maximum value appears in Zone II, which should be affected by the tangent function. In Zone III, the resistance of the bolt almost attenuates linearly with the increase of the anchoring angle. In the three zones, the resistance value of the bolt near the joint surface is negative, indicating that the direction of the resistance is the same as the external load; thus, the bolt does not have the capacity of lateral shear resistance under the anchoring angle. Based on the combined influence of the friction angle, the dilatancy angle, and the deflection angle, when the bolt experiences the transition from the elastic phase to the initial plastic hinge, the lateral shear resistance of the bolt is the strongest when 40° ≤ \( \alpha \) ≤ 60° in Zone I. However, considering the whole area, when \( \alpha = 100° \), the yield condition of the bolt is last reached; thus, the bolt anchor has the strongest lateral shear resistance. The three types of characteristic angles, that is, friction angle, dilatancy angle, and deflection angle, play different roles in influencing the lateral shear resistance of the bolt. The friction angle plays a significant role in promoting the resistance. The dilatancy angle has negligible influence on the improvement of the resistance. The deflection angle can promote or inhibit the resistance of the bolt, depending on the range of the anchoring angle.
4.4 Influence of characteristic angle on shear resistance of bolts

When the bolt is failed, the resistance can be calculated by Eq. (38). Figure 15 shows the effect of the friction angle on the shear resistance of the bolt when it is damaged. Under the same anchoring angle, as the friction angle is greater, the resistance of the bolt is greater. Under different friction angles, the resistance of the bolt has the same evolution law with the change of the anchoring angle. The curve can be divided into the following three regions: Zone I, that is, peak zone (10° ≤ α ≤ 90°); Zone II, that is, trough zone (90° < α ≤ 110°); and Zone III, that is, decrement zone (110° < α ≤ 170°). The curves in Figures 16 and 17 can be divided into the same zones. In Zone I, as the anchoring angle increases, the resistance of the bolt first increases and then decreases. Thus, for composite rock mass with rough joint surfaces, the anchoring effect is better when the anchoring angle is in acute. In Zone II, when α = 100°, the resistance of the bolt reaches the minimum value, which increases as the friction angle increases. When α ≥ 110°, the resistance of the bolt decreases with the increase of the anchoring angle. In addition, the attenuation of the resistance is faster with the increase of the friction angle.

Figure 16 shows the influence of the dilatancy angle on the shear resistance when the bolt is in failure mode. From Figure 16, the increase of the dilatancy angle has little influence on the lateral shear resistance of the bolt when it is damaged. The influence is only significant in Zone II. The optimum anchoring angle in Zone I is in the range between 50° and 60°. In Zone II, the minimum value is reached when α = 100°. In addition, the minimum value decreases as the dilatancy angle increases. In Zone III, at different dilatancy angles, the resistance of the bolt is attenuated by the same amplitude.

Figure 17 shows the influence of the deflection angle on the shear resistance of the bolt under the condition of failure. In Zone I, when 10° ≤ α ≤ 55°, the deflection angle suppresses the lateral shear resistance of the bolt. When 55° ≤ α ≤ 90°, the deflection angle enhances the lateral shear resistance of the bolt. In Zone II, there is also a minimum value when α = 100°, and the minimum value increases as the deflection angle increases. In Zone III, the resistance of the bolt exhibits the same attenuation pattern at different deflection angles.

Based on the results of Figures 15-17, the curves of the resistance over the change of all the three types of characteristic angles all have the shape of roughly “inverted W.” In Zone I, with the gradual increase of the anchoring angle, the lateral shear resistance of the bolt is generally increased in the failure mode, but the increasing rate is gradually slowing down. In Zone II, when α = 100°, the lateral shear resistance...
of the bolt in the failure mode reached a minimum value. In Zone III, the shear resistance of the bolt still decreases with the increase of the anchoring angle.

### 4.5 Determination of the optimal anchoring angle

Figure 18 shows the relationship between the lateral shear resistance and the anchoring angle at the same characteristic angle near the joint surface when the initial plastic hinge is generated and the bolt is failed. Under the conditions of $\alpha \leq 90^\circ$ and $110^\circ \leq \alpha \leq 140^\circ$, when the initial plastic hinge is generated, the lateral shearing resistance of the bolt near the joint surface is much smaller than that in the failure. This phenomenon indicates that when the bolt reaches the yield condition at the position of the plastic hinge, the joint surface does not meet the conditions of failure yet, which is consistent with the viewpoint of Pellet & Egger. However, under the conditions of $\alpha = 100^\circ$ and $\alpha > 140^\circ$, when the initial plastic hinge is generated, the
lateral shear resistance of the bolt at the joint surface is greater than that in the failure. This phenomenon indicates that when the bolt has not yet yielded at the position of the plastic hinge, it has already been damaged near the joint surface, which is different from the viewpoint of Pellet & Egger. According to the analysis in Sections 4.2-4.4, at $\alpha = 100^\circ$, the final shear resistance of the bolt is not the strongest. When the anchoring angle is in the range of $50^\circ \leq \alpha \leq 60^\circ$, the bolt has the strongest shear resistance and the most significant anchoring effect.

5 | MODEL VERIFICATION

As shown in Figure 19, Chen et al. conducted a shear test on the anchored structural plane, in which a normal stress was applied to the specimen. The load was applied with a shear rate of 0.5 mm/min. The dilatancy coefficient of the structural plane is 0.4, which translates into a dilatancy angle of $21.8^\circ$. The calculation parameters of the composite rock mass are shown in Table 2. Figure 19 shows a shear deformation diagram of the bolt. After deformation, the angle between the axis of the bolt and the horizontal line, that is, the deflection angle, is $\theta = 13^\circ$ (Figure 20). Chen assumed that the failure stress of the bolt is equal to the yield stress in the theoretical calculation. In order to compare the result, the same assumptions were also made in the theoretical calculations in this paper. Table 3 shows the comparison of the results between the model in this paper and the model by Chen and Pellet. The large error between the theoretical results by Pellet and the experimental value is because he assumed that the rock in elastic stage applied linear load to bolt. Using the model established in this paper, when the bolt is failed, the relative error between the theoretical value of the lateral shear resistance and the experimental value is small, which indicates that the theoretical results are close to the experimental results. Thus, the model proposed in this paper is feasible.

6 | CONCLUSIONS

In this paper, the structural mechanics model of bolt with both elastic deformation and complete plastic deformation under the lateral restraint of rock stratum was established for the anchored jointed rock mass of offshore LPG storage. The quantitative relationship between the anchoring performance index, that is, shear resistance and characteristic angle (anchoring angle, friction angle, dilatancy angle, and deflection angle) and geometric and physical parameters of the bolt were derived in both phases. In addition, the model was verified and the optimal anchoring angle of composite rock was proposed. The obtained conclusions are as follows:

1. When the composite rock layer is anchored, the lateral restraint from the rock stratum has a significant influence on the anchoring effect. The relevant experimental results showed that the load can be characterized by the power function, but there was an optimal power value.
When the power value is higher than 10, the results from the power function are in good agreement with the experimental results.

2. The lateral shearing effect of the bolt is the combination of both the friction effect and the dowel effect. The characteristic angle has a greater influence on the dominance of the two types of effects. When the anchoring angle is small, the dowel effect plays a leading role. For the vertically mounted bolt, the friction effect is significantly stronger than the dowel effect. The superiority ratio of the friction effect gets weaker with the increase of the deflection angle and gets higher with the increase of the friction angle and the dilatancy angle. The dilatancy angle only has significant influence on the bolt that is approximately vertically mounted. The lateral shear resistance of the bolt increases with the increase of the friction angle and the dilatancy angle. The friction angle plays a more significantly role in improving the shear resistance, while the influence of the dilatancy angle is not obvious.

3. The mounting angle of the bolt can significantly affect the lateral shear resistance of the bolt. The lateral shear resistance of the bolt is stronger at the acute installation angle than that at the obtuse angle. In particular, compared with other angles, when $\alpha = 100^\circ$, as the load increases, the elastic phase is extended. Thus, at this mounting angle, the bolt is the last to reach the yield limit and enter the plastic phase. However, eventually the bolt is damaged earlier than that with the acute mounting angle. In general, the bolt with acute mounting angles has a better anchoring effect for the rock layer than the vertically mounted bolt, while the bolt with obtuse mounting angles is not as effective as the vertically mounted bolt. Under the action of lateral load, the yielding and breaking positions of the bolt are closely related to the anchoring angle. When $\alpha < 100^\circ$, the bolt first yields at the position of the plastic hinge and then get damaged at the position of the joint surface. When $\alpha > 100^\circ$, the positions of yielding and damage are both located at the joint surface. According to all the results, when the anchoring angle is between $50^\circ$ and $60^\circ$, the performance of the lateral shear resistance of the bolt can be maximized.

4. The anchoring mechanics model established in this paper is consistent with the experimental results. However, there are still some limitations of the model. For example, lateral loads can cause some damage for the weak rock, which results in the uncertainty of the friction angle. Ignore the forces exerted by grout on bolt, and only consider the effect of rock on bolt. In addition, in the completely plastic phase of the model, the deflection angle generated in the elastic phase was neglected, and the deflection angle was still considered to start from zero. As a result, some errors were produced in the final result. This problem will be solved in the next step of our study.

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