Explanation of small $I_cR_n$ values observed in inhomogeneous superconductors

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For an inhomogeneous high-$T_c$ superconductor, band-filling dependence of Josephson $I_cR_n$ product is deduced at $T=0$ K by means of measurement; this is an extension of the Ambegaokar - Baratoff (AB) theory based on the $s$-wave theory. The product is given by $J_{obs}R_n \equiv \rho \Delta_i = \rho I_cR_n$, where $0 < \rho < 1$ is band filling (or local density), and $\Delta_i$ is the intrinsic superconducting true gap and small. When $\rho = 1$, $J_{obs}R_n = I_cR_n$ is the intrinsic Josephson true product (or the AB product), where $J_i$ is the intrinsic Josephson true current occurring by Cooper pair. When $0 < \rho < 1$, $J_{obs}R_n$ is an average of $I_cR_n$ over the measurement region and is the effect of measurement. The $J_{obs}R_n$ explains small $I_cR_n$ values observed by experiments. Furthermore, the intrinsic gap, $8.5 < \Delta_i < 17$ meV, is analyzed from $I_cR_n$ data of Bi$_2$Sr$_2$CaCu$_2$O$_{8+x}$. $d$-wave superconductive components do not exist in Bi-2212 crystals.

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From the discovery of a high-$T_c$ superconductor until recently, pairing symmetry for the mechanism of high-$T_c$ superconductivity has been controversial because intrinsic physical information of superconductors is not obtained for intrinsically inhomogeneous superconductors with a metal phase and an insulator phase with the $d_{x^2-y^2}$-wave symmetry. The intrinsic inhomogeneity in which a homogeneous metal region is about $14\AA$ was revealed by scanning tunnelling microscopy. The inhomogeneity is due to the metal-insulator instability. In inhomogeneous superconductors, the fact that the energy gap decreases with an increasing local density was also revealed by experiments and a theoretical consideration. Recently, for an inhomogeneous superconductor, an analysis method for the intrinsic density of states and the intrinsic superconducting gap was developed by means of measurement. The method disclosed the identity of gap anisotropy and revealed that pairing symmetry of a high-$T_c$ superconductor is an $s$-wave.

However, there are still two unresolved problems to be clarified on the Josephson $I_cR_n$ product. One is that the product decreases with an increasing superconducting gap (Fig. 1). The other is that $I_cR_n$ values in $c$-axis Josephson pair (or intrinsic Josephson) tunneling experiments are much smaller than the all-$s$-wave Ambegaokar-Baratoff limit. On the basis of this experimental result, it has been interpreted that the ratio of the $s$-wave component to the full $d$-wave one in the $s + d$ mixed states is very small. In addition, the smaller $I_cR_n$ value was also observed in Pb/I/NbSe$_2$ junctions.

In this paper, we deduce band-filling (or doping) dependence of the Josephson $I_cR_n$ product, by using the means of measurement suggested in previous papers; this is an extension of the Ambegaokar - Baratoff theory. The intrinsic superconducting gap is analyzed from early published $I_cR_n$ data.

Fractional charge has been demonstrated in previous papers. These will be reviewed briefly. In an inhomogeneous superconductor with two phases of a metal region and an insulating region, when it is measured such as photoemission spectroscopy, a spectral-weight value in $k$-space is observed, but the inhomogeneous phases are not able to be deduced from the observed spectral weight. In other words, a reverse transformation from $k$-space into real-space is not defined, (Fig. 2 (a)). This indicates that two real- and $k$- spaces are not mathematically equivalent. The inhomogeneous superconductor is different from the metal with both the electronic structure of one electron per atom and mathematically equivalence between two spaces. In order to overcome this problem, we think out that an measured data is an averaged data. When the inhomogeneous superconductor is measured, carriers in the metal region should be
averaged over lattices (or atoms) in the entire measurement region. Then, the inhomogeneous superconductor is changed into a homogeneous one with the electronic structure of one effective charge per atom, (Fig. 2 (b)). The observed effective charge becomes $e' = \rho e$, where $0 < \rho = n/L \leq 1$ is band filling (or local density), $n$ is the number of carriers in the metal region, and $L$ is the number of total lattices in the measurement region. The fractional effective charge is justified only when the inhomogeneous system is measured. Otherwise, it becomes true charge in the metal region.

When the concept of measurement is applied to an inhomogeneous superconductor, the observed energy gap, $\Delta_{\text{obs}}$, was given by

$$\Delta_{\text{obs}} = \Delta_i / \rho,$$

where $\Delta_i$ is the intrinsic superconducting true gap determined by the minimum bias voltage. For understanding of Eq. (1), Fig. 3 is given. The $0 < \rho \leq 1$ is band filling (local density or homogeneous factor), which indicates the extent of the metal region. The validity of Eq. (1) was given by many tunneling experiments. Ambeegaokar and Baratoff generalized the Josephson tunnel theory and calculated the coherent tunneling supercurrent on the basis of the BCS theory for a s-wave homogeneous superconductor. The supercurrent at $T = 0$ K was given by

$$J = \frac{\pi}{2} R_n^{-1} \Delta,$$

where $R_n = (2\pi\hbar/e^2T)$, $T$ is the tunneling matrix, and $\Delta$ is a superconducting energy gap.

![FIG. 2. (a) In an inhomogeneous superconductor, a reverse transformation from k-space to real-space is not defined, which is a problem. $n$ is the number of electrons in metal region. $L$ is the number of lattices in the measurement region. $0 < \rho = n/L \leq 1$ is defined. The spectral weight in k-space increases with an increasing $\rho$. (b) When the metal region is averaged over lattices in the measurement region, the inhomogeneous superconductor become homogeneous when measured. The two spaces are mathematically equivalent. $e' = \rho e$ is a fractional effective charge. When $\rho = 1$, it is metal.]

![FIG. 3. (a) When an inhomogeneous superconductor is measured, the homogeneous metal region in Fig. 2 (b) is averaged over lattices in the measurement region. Then, $e'V' = \rho eV' = \rho \Delta_{\text{obs}}$ is given. (b) In an inhomogeneous superconductor, if only the homogeneous metal region is measured, $eV = \Delta_i$ is given.]

In an inhomogeneous superconductor, the averaged metallic system has the electronic structure of one effective charge per atom, as shown in Fig. 2 (b), which is mathematically equivalent to the electronic structure of the metal used in the BCS theory. The metal for k-space used in the BCS theory has the electronic structure of one electron per atom, as shown when $\rho = 1$ in Fig. 2 (b). The Josephson current and the product derived in the Ambegaokar and Baratoff theory can be used without formula’s change even in the inhomogeneous superconductor by replacing true charge by the effective charge because the averaged effective charge is invariant under transformation. Particular calculations are not necessary because it had already been given by Ambegaokar and Baratoff. In addition, similar calculations have been given when the Brinkman-Rice picture was extended. Thus, the observed supercurrent, $J_{\text{obs}}$, is given by substituting $\rho$ in $R_n$ and $\Delta$ with $e' = \rho e$ and $\Delta_{\text{obs}} = \Delta_i / \rho$ by

$$J_{\text{obs}} = (4\pi^2T)\rho \Delta_i = \rho J_i,$$

where $J_i$ is the intrinsic true supercurrent. The observed Josephson product is also given by Eq. (3) by

$$J_{\text{obs}} R_n = \frac{\pi}{2} \rho^2 \Delta_{\text{obs}} = \frac{\pi}{2} \rho \Delta_i = \rho J_i R_n,$$

where $\Delta_i$ is constant.

When $\rho = 1$, Eqs (3) and (4) are the intrinsic supercurrent and the intrinsic product (or Ambegaokar and Baratoff product) caused by true Cooper pairs, respectively. When $0 < \rho < 1$, the equations correspond to the averages of the intrinsic supercurrent and the intrinsic.
product over the measurement region and are the effect of measurement. Eqs (3) and (4) are the extended Ambegaokar - Baratoff (AB) Josephson current and product, respectively.

\[ \rho \text{ dependence in Eq. (4)} \]

\[ \text{ fits well the experimental data measured by break junctions} \]

\[ \text{ (Fig. 4). This reveals that,} \]

\[ \text{ at a fixed doping level, the observed Josephson product decreases with an increasing energy gap} \]

\[ \text{ and that Bi}_2\text{Sr}_2\text{CaCu}_2\text{O}_8 \text{ (Bi-2212) crystals are inhomogeneous, which is a general characteristic of high-} T_c \]

\[ \text{ superconductors} \]

\[ \text{ and the reason why the AB product is not applied directly to experimental data. Moreover,} \]

\[ \text{ because Eq. (4) with} \]

\[ \rho \neq 1 \text{ is an average of the true} \]

\[ I_cR_n \text{ product value (or the AB product value), which is observed in only the metal region with} \]

\[ \rho = 1 \text{ in Fig. 2 (a),} \]

\[ d\text{-wave superconductive components do not exist in the} \]

\[ \text{ homogeneous metal region in Bi-2212 crystals. Note that} \]

\[ \text{ Mourachkine}^{10} \text{ discussed that the decrease of the product in} \]

\[ \text{ Fig. (1) is not intrinsic effect and due to inhomogeneity.} \]

\[ \rho \text{ dependence in Eq. (4) comes from Eq. (3),} \]

\[ \text{ which agrees with a result observed by the intrinsic Josephson junction.} \]

\[ \text{ Note that the magnitude of the intrinsic AB product is basically very small because} \]

\[ \Delta_i \] is small. Considering that the anisotropy of the number of carriers in the c-axis and ab-plane is large \( \rho_{ab\text{-plane}} \ll \rho_{c\text{-axis}} \), the product observed in the c-axis is naturally much less than that in the ab-plane. Thus, the observed small \( I_cR_n \) values \( 10^{-18} \) can be explained by Eq. (4). Additionally, for a Josephson junction by two superconductors with different energy gaps, the \( I_cR_n \) product derived by Anderson \( ^{22} \) is in the context of the above analysis.

We analyze the intrinsic gap of Bi-2212 from experimental data, using Eq. (4). Irie \( et \ al. ^{16} \) suggested that \( I_cR_n \approx 13.3 \text{ meV} \) observed by the intrinsic Josephson junction is \( \frac{1}{3} \) of \( I_cR_n \approx 40 \text{ meV} \) using \( \Delta_{obs} \approx 25 \text{ meV} \) and Eq. (4) with \( \rho = 1 \). The true product value is much less than 40 meV, when both \( \rho \neq 1 \) and \( \Delta_{obs} \neq \Delta_i \) are considered. The intrinsic gap, \( \Delta_i \approx 8.5 \text{ meV} \), is obtained from the observed \( I_cR_n \approx 13.3 \text{ meV} \) by Eq. (4) with \( \rho = 1 \). The intrinsic true gap is slightly larger than 8.5 meV because \( \rho < 1 \) slightly. Mourachkine \( ^{9,10} \) observed the maximum Josephson product of \( I_cR_n \approx 26 \text{ meV} \) for an over-doped crystal, which can be regarded as \( \rho \approx 1 \) without a pseudogap \( ^{21} \), at the minimum energy gap of \( \Delta_{obs} \approx 21 \text{ meV} \). The intrinsic true gap, \( \Delta_i \approx 16.5 \text{ meV} \), is obtained by Eq. (4) with \( \rho = 1 \). The intrinsic true gap is less than that analyzed by Mourachkine. Thus, we conclude that the analyzed intrinsic gap is in \( 8.5 < \Delta_i < 17 \text{ meV} \).

In conclusion, for inhomogeneous high-\( T_c \) superconductors, without the \( d\text{-wave theory}, \) Eq. (4) based on the \( s\text{-wave theory} \) explains the small Josephson-product values observed by experiments.

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