A Submillimeter Level Relative Navigation Technology for Spacecraft Formation Flying in Highly Elliptical Orbit

Xiaoliang Wang 1,†, Deren Gong 1,*, Yifei Jiang 1, Qiankun Mo 1, Zeyu Kang 1, Qiang Shen 1, Shufan Wu 1 and Dengfeng Wang 2,†

1 School of Aeronautics and Astronautics, Shanghai Jiao Tong University, Shanghai 200240, China; xlwang123@sjtu.edu.cn (X.W.); jamesjiang@sjtu.edu.cn (Y.J.); moqk@163.com (Q.M.);
kangzeyu@sjtu.edu.cn (Z.K.); qiangshen@sjtu.edu.cn (Q.S.); shufan.wu@sjtu.edu.cn (S.W.)
2 Institute of Space Radio Technology, Xi’an 710100, China; dfwang_aero@163.com
* Correspondence: drgong@sjtu.edu.cn; Tel.: +86-159-2156-7606
† These authors contributed equally to this work.

Received: 24 September 2020; Accepted: 13 November 2020; Published: 15 November 2020

Abstract: Spacecraft formation flying (SFF) in highly elliptical orbit (HEO) has attracted much attention since many applications in space explore, while precise guidance navigation and control (GNC) technology, especially precise ranging, conducted the basis of success for such SFF missions. In this paper, we introduced a novel K band microwave ranging (MWR) equipment that aimed for the on-orbit verification of submillimeter level precise ranging technology in future HEO SFF missions. The ranging technique is a synchronous dual one-way ranging (DOWR) microwave phase accumulation system, which achieved tens of microns of ranging accuracy in laboratory environment. Detailed design and development process of MWR equipment are provided, with ranging error sources analyzed, and relative orbit dynamic models for HEO formation scenes are given with real perturbations considered. Moreover, an adaptive Kalman filter algorithm is introduced for SFF relative navigation design, incorporating with process noise uncertainty. The performance of SFF relative navigation while using MWR are tested in a hardware in loop (HIL) simulation system within a high precision six degree of freedom (6-DOF) moving platform. The final range estimation errors from MWR using adaptive filter are less than 35 µm m and 8.5 µm/s for range rate, which demonstrated the promising accuracy for future HEO formation mission applications.

Keywords: spacecraft formation flying; highly elliptical orbit; microwave ranging; relative navigation

1. Introduction

Spacecraft formation flying (SFF) has attracted much attention, since it can performing space missions with more reliability, adaptability, and low life-cycle cost, compared with traditional monolithic spacecraft [1]. Several SFF missions have successfully deployed in low Earth orbit (LEO), such as Gravity Recovery and Climate Experiment (GRACE) mission for precise Earth gravity field measurement [2], EO-1/LandSat 7 for Earth observation [3], TanDEM-X mission for generate high-precision digital elevation models by using a high-resolution interferometric synthetic-aperture radar [4,5], and PRISMA, for millimeter level SFF technology on orbit demonstration [6].

With the development of space technologies in recent decades, scientists have attempted explore outer space far beyond Earth, which turn their eyes on highly elliptical orbit (HEO) SFF technologies. The most famous ones including Magnetospheric Multiscale Mission (MMS) launched by the National Aeronautics and Space Administration (NASA), a Solar Terrestrial Probes mission that study the
Earth’s magnetosphere [7]. The Cluster II mission directed by the European Space Agency (ESA) aimed towards space physics observation [8]. Those two HEO SFF missions comprising of four instrumented spacecraft operated in an adjustable pyramid-like constellation, and verified guidance navigation and control (GNC) technology on orbit.

The realization of precise SFF GNC technologies including development of actuators, metrologies and algorithms, while the metrologies system conducting the baseline of GNC, since it provide the direct relative states measurement between the formation spacecrafts. Typically, the metrologies system will cover both radio frequency (RF) and optical metrologies. The RF terminal receive signals from other spacecraft, and process time-of-flight spread-spectrum code and carrier phase that allow for retrieving distance and line-of-sight (LOS) measurement, with an accuracy of 1 cm (distance) and 1° (LOS). The optical metrologies can be conducted using geometric and interferometric concepts, achieving different ranges and accuracies. Optical position metrology for XEUS is reported as a coarse and fine sensor, which achieves accuracy of tens of microns, with relative distance of maximum 120 m [9,10]. The dual wavelength interferometer (DWI) is chosen as a fine longitudinal sensor for DARWIN mission with ranging accuracy of about 10 µm within 250 m distance [11,12].

China’s space agency has recently approved a HEO SFF mission, which we called highly elliptical formation flying demonstration (HEFFD) mission, consisting of two spacecraft for science observation, quite similar as Proba-3 mission [13,14]. The two formation spacecrafts are preliminary designed in different formation configurations with relative distance of several meters to hundreds of kilometers, according to the real task on orbit. Besides the original observe data obtained, the most expecting reward of this mission is the design and development of technologies that enable the future SFF missions in HEO scenario, especially for on orbit demonstration of high precision GNC technology. The primary ranging payload is designed based on optical metrology, which deliver microns level relative lateral/longitudinal ranging between spacecraft during science observation period in apogee.

Moreover, a K band microwave ranging (MWR) payload is developed particularly to this HEO formation missions for the reasons below:

- In addition to optical metrology, (a) MWR can provide precise ranging results, as a backup equipment, during science observation period around apogee and (b) avoid sunlight disturbance, that happened in optical, for specific space science missions as solar corona observation.
- Microwave ranging relies on carefully designed transceiver antenna with relative wider main lobe angle of a few degrees, which greatly reduced the burden of high precision pointing mechanism used for optical. At the same time, microwave ranging can operate in pseudo code mode or carrier mode for coarse/precise ranging if necessary, as required by mission control, providing flexible solution to GNC system.
- Good inheritance of microwave ranging technology from existing space experience. MWR has been developed and tested previously, and successfully verified in one LEO formation mission [15].
- As an essential supplement to MWR system, BeiDou III B1C/B2a dual frequency new generation navigation signal receive and process technology can be fully tested and verified in HEO formation missions. The B1C/B2a dual frequency receiver on board can provide high precise time synchronization solutions with the precision of nanosecond, which served as a time scale benchmark between formation spacecrafts. Moreover, the receiver can provide precise stand alone navigation solutions while using precise orbit determination algorithm.
- Microwave can also be used for real-time data transmission between spacecraft for original science data exchange, differential GPS measurement data transmission for relative navigation, etc.

This paper focus on the principle, design, development, and test of MWR equipment that will be deployed in HEFFD mission. The following content is organized, as follows: Section 2 introduces the submillimeter level accuracy microwave ranging technique and MWR equipment development. Section 3 describes the relative orbit models and perturbations considered during real formation flight in space. Section 4 provides the relative navigation filter algorithm that deals with process
noise uncertainty, and the simulation results using MWR and proposed methods are finally given in Section 5.

2. Submillimeter Level MicroWave Ranging

2.1. MWR Measurement Modeling

MWR measures the biased distance between the antenna phase centers of two satellites through microwave carrier phase accumulation, doppler drift, and then derives the distance variation and its rate between two satellites. Basically, the whole system is a synchronous dual one-way ranging (DOWR) microwave phase accumulation ranging system [16]. Three techniques were adopted by MWR that ensure ranging accuracy:

• correction of ionospheric influence by dual frequency measurement during space flight around perigee;
• synchronous bidirectional ranging comparison that cancels long-term instability of oscillator source; and,
• carrier phase measurement by frequency difference to improve the accuracy.

Figure 1 shows the principle of DOWR microwave phase measurement.

Substitute the receiving phase to sending phase

\[ \phi_i(t + \Delta t_i) = \phi_i(t + \Delta t_i - \tau^j_i) \]  

where \( \tau^j_i \) is the propagation time from satellite \( j \) to \( i \).

Each phase \( \phi_i(t) \) is the summation of reference phase \( \tilde{\phi}_i \) and phase error due to oscillator \( \delta \phi_i \). Phase rate \( \dot{\phi}_i(t) \) represent as reference frequency \( \dot{f}_i \). Finally, the DOWR can be written as:
\[ \Theta(t) = (f_1^2 + f_2^2) \]
\[ + \{ [\delta\phi_1(t) - \delta\phi_1(t - \tau_1^2)] + [\delta\phi_2(t) - \delta\phi_2(t - \tau_2^2)] \} \]
\[ + (f_1 - f_2)(\Delta t_1 - \Delta t_2) \]
\[ + (\delta\phi_1 - \delta\phi_2)(\Delta t_1 - \Delta t_2) + N + I + d + \epsilon \]  

(4)

where \( f_1^2 + f_2^2 \) denotes the real phase measurement;

\( \{ [\delta\phi_1(t) - \delta\phi_1(t - \tau_1^2)] + [\delta\phi_2(t) - \delta\phi_2(t - \tau_2^2)] \} \) denote the phase error due to oscillator, and the middle-term / long-term oscillator phase error can be eliminate if the propagation time is longer than one millisecond using DOWR;

\( (f_1 - f_2)(\Delta t_1 - \Delta t_2) \): time-tag error;

\( (\delta\phi_1 - \delta\phi_2)(\Delta t_1 - \Delta t_2) \): coupling term of Oscillator and time-tag;

\( N + I + d + \epsilon \): other errors; and,

\( (f_1^2 + f_2^2) \approx (f_1 + f_2)\tau + \Delta\Theta_{\text{TOF}}(t) \) with time of flight (TOF) defined as \( \tau = \rho(t)/c \).

\( \Delta\Theta_{\text{TOF}}(t) \): correction of TOF, which can be calibrated while using precise orbit determination technique.

Dual ranging \( R(t) \equiv \lambda\Theta(t) \), with

\[ \lambda = c/(f_1 + f_2) \]  

(5)

Subsequently, we have

\[ R(t) = \rho(t) + \rho_{\text{TOF}}(t) + \rho_{\text{err}}(t) + N' + I' + d' + \epsilon' \]  

(6)

where \( \rho(t) \): instant distance, \( \rho_{\text{TOF}}(t) \): correction of Time of Flight (TOF), \( \rho_{\text{err}}(t) \): ranging error due to oscillator and time-tag, \( N'_i \): ambiguity, \( I' \): ionosphere error, \( d' \): phase bias due to other reasons, and \( \epsilon' \): ranging error due to measurement noise.

The one-way phase measurement of \( \phi_i(t + \Delta t_i) \) is transferred to instant distance \( \rho(t) \), and DOWR ranging \( R_{K\&R_{Ka}} \) can be derived while using K&Ka band measurement. Finally, DOWR ranging without ionosphere influence can be given as

\[ R = \frac{f_{K_{Ka}}^2 R_{Ka} - f_{K_{K}}^2 R_{K}}{f_{K_{Ka}}^2 - f_{K_{K}}^2} \]  

(7)

The principle of differential frequency phase measurement is to change the reference signal and measured signal into intermediate frequency (IF), or lower frequency, by mixing the radio frequency (RF) and local frequency, then followed by phase compare. Because the IF signal entering the phase measurement system is much lower than the RF, the phase period expanded, which greatly improves the phase measurement accuracy.

2.2. Ranging Error Sources Analysis

Carrier phase measurement errors are mainly caused by the following reasons: local oscillator phase noise, system noise, dynamic stress error, etc.

1. Carrier phase measurement error due to system noise

Carrier phase measurement error due to system noise can be given as \( (1\sigma) \)

\[ \sigma_{\text{noise}} = \frac{\lambda}{2\pi} \sqrt{\frac{B_n}{\epsilon/n_0}} (1 + \frac{1}{2Tc/n_0}) \]  

(8)
where $B_n = 2$ Hz: noise bandwidth of phase lock loop (PLL); $\lambda$: length of carrier wave (K band $12.3 \times 10^3 \mu \text{m}$, Ka band $9.2 \times 10^3 \mu \text{m}$); $T = 0.02$ s: PLL pre detection integration time; $c/n_0 = 10^{65/10}$: carrier to noise ratio($C/N_0 = 65$ dB-Hz).

With calculation, the measurement error due to system noise are $\sigma_{\text{nois-K}} = 15.4 \mu$m ($1\sigma$), $\sigma_{\text{nois-Ka}} = 11.5 \mu$m ($1\sigma$).

2. Carrier phase measurement error caused by phase noise of local oscillator

The carrier phase measurement error (for third-order tracking loop) introduced by the local oscillator Allen variance can be given by the following empirical formula:

$$\sigma_{\text{Allen3}} = 160 \frac{\lambda}{360} \frac{\sigma_A(\tau) f_c}{B_n} \text{ (} \mu\text{m)}$$

where $B_n = 2$ Hz: the band width of PLL, $\tau = 1/B_n = 1$ s: the short-term stability length of Allen variance, $\sigma_A(\tau) = 3 \times 10^{-13}$: Allen variance, $f_c$: carrier frequency, K = 24.5 GHz, Ka = 32.7 GHz.

With calculation, the phase measurement error due to local oscillator Allen variance are $\sigma_{\text{Allen3-K}} = 20.1 \mu$m ($1\sigma$) for K band and $\sigma_{\text{Allen3-Ka}} = 15 \mu$m ($1\sigma$) for Ka band.

3. Phase measurement error duo to dynamic stress

Dynamic stress error is closely related to tracking loop order. For three-order PLL, dynamic stress is given as

$$\sigma_{\text{PLL3}} = 0.4828 \frac{d^3 R}{dt^3} B_n^3 \text{ (} \mu\text{m)}$$

where $d^3 R/dt^3$ is line of sight (LOS) accelerate rate (\(\mu\text{m}/s^3\)). For PLL bandwidth $B_n = 2$ Hz, the LOS accelerate rate $10 \mu$m/s^3, and dynamic error $\sigma_{\text{PLL3}} = 0.60 \mu$m ($1\sigma$).

4. Summary of phase measurement error

The total measurement error while using three-order PLL can be modeled as:

- **K band**: $\sigma_{\Sigma-K} = \sqrt{\sigma_{\text{nois-K}}^2 + \sigma_{\text{Allen3-K}}^2 + \sigma_{\text{PLL3}}^2} = \sqrt{15.4^2 + 20.1^2 + 0.6^2} = 25.3$ (\(\mu\text{m}\))
- **Ka band**: $\sigma_{\Sigma-Ka} = \sqrt{\sigma_{\text{nois-Ka}}^2 + \sigma_{\text{Allen3-Ka}}^2 + \sigma_{\text{PLL3}}^2} = \sqrt{11.5^2 + 15^2 + 0.6^2} = 19$ (\(\mu\text{m}\))

2.3. MWR Payload Development

The MWR system payload has been carefully designed and developed, with the diagram of one spacecraft shown in Figure 2. Several things have to be determined during design phase, including detailed system composition, frequency planning, link budget analysis, system measurement error analysis, antenna polarization mode and optimization, high precision time synchronization, on-ground test and verification, etc.

Currently, MWR system payload prototypes have been developed that consist of two sets of identical equipment mainly include (a) K/Ka band transceiving antenna: single corrugated horn antenna that transmit-receive K/Ka band dual frequency microwave signals; (b) frequency reference unit: ultra-stable-oscillator (USO) is used as the frequency reference source of the whole system; (c) K/Ka band microwave channel: up-convert the reference frequency to transmission carrier frequency (TCF) and down convert the TCF, receiving from another satellite, to single carrier signal that complete amplification process and mixing reception; and, (d) digital signal processing unit: sampling and digital processing of down converted K/Ka band carrier phase signal and BeiDou signal to complete high-precision carrier phase extraction. Here, we provide a brief introduction of those four components, as:
Figure 2. Block schematic of microwave ranging (MWR) system, mainly including K/Ka horn transceiving antenna with quadrature coupler, frequency reference unit, K/Ka band microwave channel, and digital signals process platform.

2.3.1. Antenna Scheme Design

The MWR antenna is composed of K/Ka dual frequency corrugated horn and dual polarization quadrature coupler, as shown in Figure 3 below. The antenna was specially designed while using horn configuration, which take advantages of small volume, compact connection with feed source, high precision installation accuracy, and small thermal deformation (smaller thermal deformation of horn antenna made of invar steel). The disadvantage is that it is difficult to achieve high gain performance. Fortunately, it was finally solved by choosing the appropriate size of the corrugated horn, with a huge number of simulation and calculation.

Figure 3. Illustrations of MWR transceiving antenna. (a) three dimensional antenna structure design including corrugated horn and K/Ka dual polarization quadrature coupler. (b) tested results of antenna pattern (radiation angle deviated from line-of-sight (LOS) direction vs. antenna gain) for both K/Ka band.
2.3.2. Frequency Reference Unit

Frequency reference units include USO and frequency multiplier, which provides frequency reference for microwave channel, signals process unit of MWR, and BeiDou receiver.

USO is achieved by high stability/low phase noise constant temperature oscillator design, which includes (1) oscillator circuit and (2) temperature control circuit. The Oscillator circuit is used in order to realize low phase noise function, including pierce main oscillator circuit, amplification, spectrum purity filter and power supply circuit. Continuous temperature control circuit is adopted, since it guarantees the best short-term oscillator stability (less than $1 \times 10^{-10}/s$) and temperature frequency characteristics ($-40^\circ C \sim +80^\circ C$).

Frequency multipliers provide double and eight multiplier reference frequency with a power amplification, distribution, multiplier, and isolation amplification process.

2.3.3. Microwave Channel

The differential frequency phase measurement method is adopted in order to achieve high-precision phase measurement in MWR system. The phase measurement frequency of Ka band is designed as 600 kHz. By making the reference frequency source of USO from two spacecraft differ by 66 Hz, the carrier phase measurement can be completed with identical MWR equipment onboard two spacecraft. The method is to use the local transmitting carrier as the local oscillator for mixing, down convert the receiving carrier, directly generate a phase measuring frequency point of about 600 kHz (i.e., the direct mixing technology of the transmitting carrier), and then send it to the digital signal processing unit for carrier phase measurement.

The MWR microwave channel consists of a RF module, local oscillator, and secondary power supply. (a) The RF module uses a local oscillator signal to convert 32.7 GHz RF signal to 600.098 kHz intermediate frequency (IF) signal with certain gain. The RF module gain can be adjusted by tele control command, with gain state parameters telemetry output. (b) The local oscillator uses the input reference signal in order to generate the local oscillator signal for RF module frequency down convert. (c) The secondary power provides the required secondary voltage for the RF module, and it controls the ON/OFF switch of the equipment through the tele control command, and it provides telemetry information of switch state.

Fundamental wave mixing technical is adopted for RF module down convert, which has the advantages of low frequency conversion loss and low noise coefficient; usually, the frequency conversion loss is less than 10 dB. Moreover, it can better suppress high-order mixing by products, when single balance or double balance method is used. The central frequency of fundamental mixing local oscillator signal is chosen as 32.7 GHz, according to the frequency relationship between input and output signals.

2.3.4. Signals Process Hardware Platform

The digital signal process unit is the most important one among the whole MWR system, since it processes both K/Ka dual frequency signals and BeiDou dual frequency navigation signals in high quality. New generation BeiDou III B1C/B2a navigation signals are chosen and technique of dual frequency receiver development is adopted, which aim to provide high precision time synchronization between satellites to less than 0.3 ns that is required by MWR DOWR measurement.

In order to ensure the accuracy of 0.3 ns time synchronization that is required by MWR, the dual frequency navigation receiver is integrated designed with K/Ka signals process hardware platform that shares a unique USO frequency source. The hardware platform includes the MWR measurement processing module, BeiDou measurement processing module, and external interface module, as shown in Figure 4.

ADC1 of hardware platform, as shown in Figure 2, is composed of two dual channel ADC chips, which conduct the analog-to-digital conversion of K/Ka-band IF signals. The digitized K/Ka-band
signals are sent to FPGA for digital signal correlation processing to obtain the original measurement. DSP completes the onboard relative ranging calculation, channel control, and data packaging to the GNC computer and TT&C external interface. Similar to the MWR process, ADC2 completes the analog-to-digital conversion of L band BeiDou B1C/B2a signals, sending to BeiDou FPGA for digital signal correlation and original measurement processing. BeiDou DSP completes the function of high precision time synchronization, channel control, and navigation positioning solution.

Figure 4. Signals process hardware platform, including K/Ka band dual frequency MWR process module, BeiDou III B1C/B2a dual frequency navigation process module, and external interface module.

3. Relative Dynamic for Spacecraft Formation

Unlike the traditional relative orbit motion equation that was expressed in the directions of $x$: Radial, $y$: In-Track and $z$: Cross-Track(RIC), with origin located in master spacecraft [17,18]; here, we use a radar system that was suggested by Eggleston and Dunning [19] by variables transformation as:

$$x = \varrho \cos \phi \cos \theta$$

$$y = \varrho \cos \phi \sin \theta$$

$$z = \varrho \sin \phi$$

where $\varrho$ denotes the relative distance between the formation spacecrafts, $\theta$ and $\phi$ are the azimuth and relative elevation angle. The relative motion equations can be finally transformed to [20]:

$$\ddot{\eta} = f(\eta, \dot{\eta}, t) + G(\eta)w$$

where $\eta = [\varrho \, \theta \, \phi]^T$, $w = [w_\varrho \, w_\theta \, w_\phi]^T$, $f = \{f_\varrho \, f_\theta \, f_\phi\}$ with

$$f_\varrho(\eta, \dot{\eta}, t) = (\varrho^2 + 2\varrho \dot{\theta} + \dot{\theta}^2)\varrho \cos^2 \phi + \varrho \dot{\varrho}^2$$

$$- \frac{\mu(r \cos \theta \cos \phi + \varrho)}{[r^2 + \varrho^2 + 2r \varrho \cos \theta \cos \phi]^2} + \frac{\mu}{r^2} \cos \theta \cos \phi$$

$$f_\theta(\eta, \dot{\eta}, t) = 2(\omega + \dot{\theta})\varrho \tan \phi - \omega - 2(\omega + \dot{\theta})\frac{\dot{\varrho}}{\varrho}$$

$$+ \frac{\mu(r \sin \theta \sec \phi)}{\varrho[r^2 + \varrho^2 + 2r \varrho \cos \theta \cos \phi]^2} - \frac{\mu}{r^2 \varrho} \sin \theta \sec \phi$$

$$f_\phi(\eta, \dot{\eta}, t) = -\frac{1}{2}(\omega + \dot{\theta})^2 - \frac{2\dot{\theta}}{\varrho} - \frac{\mu}{r^2 \varrho} \cos \theta \sin \phi$$

$$+ \frac{\mu \varrho \cos \theta \sin \phi}{\varrho[r^2 + \varrho^2 + 2r \varrho \cos \theta \cos \phi]^2}$$

The denotations of variables/matrix $\mu$, $r$, $\omega$, $G(\eta)$ and $w$ in (14)–(17) can be found in [20], which are not explained here for short. For the purpose of real formation simulation, the master and slaver
spacecrafts are separately propagated in inertial frame, and the relative position and velocity are computed with differences and transformed into the master RIC frame and radar system in (11)–(17), considered to be the real orbit values. Accelerations of gravitational and non-gravitational are considered, with the models shown in Table 1, as:

Table 1. Accelerations of gravitational and non-gravitational model.

| Items                                      | Model                                      |
|--------------------------------------------|--------------------------------------------|
| GA—the geopotential effect of the Earth   | 20th order and degree                      |
| GA—Sun, and Moon gravities                | high-precision DE405/LE405 planetary ephemerides model |
| GA—solid Earth tides                      | IERS Conventions 1996                     |
| GA—ocean tides                             | Center for Space Research 3.0 model        |
| NGA—the atmospheric drag                  | NRLMSISE-00 empirical model                |
| NGA—the solar radiation pressure          | IERS Standards 1992                       |

Note: GA (gravitational accelerations), NGA (non-gravitational accelerations).

4. Mission Orbit and Relative Navigation Filter

4.1. Real Formation Mission Orbit

The formation flying orbit analyzed here, quite similar as PROBA-3, is a virtual structure SFF mission that will perform in HEO, which we called HEFFD mission before. HEFFD mission is initially designed as an on-orbit development, verification and validation platforms for multiple task forces. The main object is the test of Guidance, Navigation, and Control (GNC) technology in space flight, especially for accuracy ranging metrology units and high performance propulsion system that allow fine formation maneuvers.

The HEFFD mission orbit design is a trade-off analysis process that considered the requirements of (a) high altitude during formation flying stage since low gravity gradient environment; (b) simple Sun-Earth geometry when SFF, during Solar observation phase; (c) launch capability; and, (d) ground station location.

Here, we provide the baseline orbit for HEFFD mission in Table 2 as:

Table 2. Highly elliptical formation flying demonstration (HEFFD) mission orbit parameters (Master spacecraft).

| Parameter             | Value | Unit |
|-----------------------|-------|------|
| Apogee altitude       | 60,530| km   |
| Perigee altitude      | 600   | km   |
| Inclination           | 59    | deg  |
| Argument of Perigee   | 42    | deg  |
| RAAN                  | 25    | deg  |
| True anomaly          | 0     | deg  |

and the slaver spacecraft is supposed to be performing follow on flight relative to master spacecraft, with a minimal distance of 10 km around apogee.

Some preliminary designed parameters of master/slaver spacecrafts include: mass: 200 kg/400 kg; reference area: 1.81 m²/3.34 m²; dimensional drag coefficient: 1.29/1.5; dimensional solar radiation pressure coefficient: 1.2/3.27 [21,22].

Figures 5 and 6 provided the relative range and elevation angle values for three orbit periods. Clearly, the orbit calculation start from perigee with a relative formation distance of about 120 km, and gradually approached to about a minimum of 10 km that is suitable to mission operation. The bolded red lines in Figure 5 demonstrated the relative orbit values of range and elevation angles, while using dynamic equations of radar model (11)–(17). The azimuth angle values are not shown here, since the co-orbit formation configuration in this simulation. Moreover, the green lines in Figure 5
shown the relative range and elevation values by using propagated perturbation orbit dynamic using model of Table 1. The results illustrated that the real perturbed relative formation orbit drifted quickly from radar model during simulation, and Figure 6 illustrated the drift bias within three orbit periods.

According to Figures 5 and 6, the relative orbit dramatically drifted. Formation ranging distance drifted gradually near each perigee arc, almost reached to $-40\ km$ at the end of third orbit time, for example, and converged quickly during apogee arc. The formation mission configuration can be carefully designed initially, when considering detailed orbit perturbations. However, sophisticated relative model is not suitable to navigation filter calculation since huge of computation burden on-board for real space mission. Finally, by careful consideration, apogee arcs are chosen as the mission operation orbit, which is used to high precision formation GNC technology verification. The mission apogee arcs, occupying about half of orbit period time, are clearly shown in Figures 5 and 6, by using bolded blue star dots. The relative navigation algorithm has to be carefully designed, which consider real orbit drift and model uncertainty.

Figure 5. Formation range and elevation angles (design and real values). The red lines denote the designed spacecraft formation flying (SFF) relative range (top) and relative elevation angles (bottom) during three orbit periods, and the green lines denote the same values of SFF in real orbit perturbations. The blues lines denote the mission operation orbit during apogee, for SFF precision guidance navigation and control (GNC) technology verification.
4.2. Relative Navigation Using Adaptive Kalman Filter

According to the relative orbit model (11)–(17), the class of systems considered here is given as

\[
\begin{align*}
\mathbf{x}_k &= f(\mathbf{x}_{k-1}, \mathbf{u}_{k-1}) + G\mathbf{w}_{k-1} \\
\mathbf{y}_k &= h(\mathbf{x}_k) + \mathbf{v}_k
\end{align*}
\]  

(18)

where \( \mathbf{x}_k \in \mathbb{R}^n \) denotes the state vector, \( \mathbf{u}_k \in \mathbb{R}^m \) denotes the control vector. The output vector is \( \mathbf{y}_k \in \mathbb{R}^p \) and the non-linear model is \( f(\mathbf{x}_{k-1}, \mathbf{u}_{k-1}) \). The process/measurement noise \( \mathbf{w}_k \) and \( \mathbf{v}_k \) are assumed to be independent, identically distributed Gaussian random variables with

\[
\mathbf{w}_k \sim \mathcal{N}(0, Q_k), \quad \mathbf{v}_k \sim \mathcal{N}(0, R_k)
\]  

(19)

The traditional Kalman filter provides the optimal estimation of states by using a recursive scheme of propagating states and measurement update. The filter optimally blends the prior state and information update through gain matrix, which balances uncertainty in both measurements and dynamics model.

However, for SFF missions in the HEO scene, the real relative orbit drifted gradually away from the designed model, as in Figure 6. The reason why the orbit drift is inevitable is due to the unknown process noise from perturbations. Some adaptive approaches have to be used for the on-line estimation of process noise, which will increase the accuracy of predicted state error covariance matrix and increase the stability of filter.

Here, we use a recursive adaptive algorithm dealing with process noise uncertainty by minimizing a function in real time, which is determined by the difference of residual covariance computed by the filter and residual sequence generated by the filter. The adaptive approach can be summarized, as follows:
Suppose that we have the predicted state error covariance matrix $\bar{P}(k)$, then $\bar{P}_y(k) = H\bar{P}(k)H^T + R(k)$ is the predicted output covariance matrix. The adaptive Kalman filter for unknown process noise is to minimize the following criterion function

$$S_k = \text{tr}(\Delta \bar{P}_y^2(k)) = \text{tr}[(\bar{P}_y(k) - \bar{P}_y^0(k))^2]$$

where $\bar{P}_y^0(k)$ is the unbiased estimation of $\bar{P}_y(k)$. The traditional Massachusetts Institute of Technology (MIT) rule is used here to derive the adaptive law, which is a popular method for on-line estimation of unknown parameters in a control system [23,24]. The solution of criterion function 20, including the process of negative gradient, discretization, and recursive scheme for process noise update. The whole adaptive processes have been successfully used in the HEO SFF scene, which are not shown here for conciseness [25].

5. Test and Simulation

Extensive tests of the MWR payload have been done in the laboratory environment, which conducted in a precise moving platform, as shown in Figure 7. The moving platform can perform six degree of freedom (6-DOF) movement with an accuracy of microns in position and milliarcseconds in attitude orientation. The test work is divided into two stages: first, the assessment of MWR ranging accuracy by simply K band microwave signals process through transmitting and receiving. The output of this stage would be the knowledge of best accuracy that MWR can provide, which is considered as a fundamental specification for the next move. The second stage is hardware in loop (HIL) simulation. For the purpose of indepth analysis of relative navigation filter performance using real MWR measurement equipment, a HIL simulation is proposed with all of the payloads fixed in the 6-DOF platform that will be deployed onboard in future SFF missions. Relative navigation errors that are provided by different measurement payloads and different filter algorithms can be obtained at this stage, and the trade-off analysis for SFF mission reward and measurement capability could be determined.

Figure 7. Six degree of freedom moving platform that achieved accuracy of microns in position and milliarcseconds in attitude orientation.
5.1. Assessment of MWR Ranging Accuracy

With extensive tests of MWR that were conducted in the 6-DOF platform, the results verified the microns level high accuracy specifications that could be applied to real space formation missions in the future. Table 3 provided the primary designed parameters of a whole ranging system, with a relative distance of 100 km as an example. Figures 8 and 9 provided the real ranging test results from the 6-DOF moving platform under stable longitudinal velocity of 5 \( \mu \text{m/s} \). The ranging error (the differential between real measurement data from MWR and optical moving platform sensor) can be achieved to be less than 40 \( \mu \text{m} \) during the test process, and 1.6 \( \mu \text{m/s} \) range rate error can be obtained.

Table 3. Parameters of MWR ranging payloads.

| Items                               | Symbols | Unit         | K Band       | Ka Band       |
|-------------------------------------|---------|--------------|--------------|--------------|
| Frequency                           | \( f \) | GHz          | 24.517002    | 32.692746    |
| Transmitting power                  | \( Ptx \) | W            | 10           | 10           |
| Transmitting power                  | \( Ptx \) | dBW          | 10           | 10           |
| Transmitting feed loss              | \( Lfx \) | dB           | 0.5          | 0.5          |
| Transmit antenna gain               | \( Gtx \) | dB           | 16           | 16           |
| Effective Isotropic Radiated Power  | EIRP    | dBW          | 25.5         | 25.5         |
| Space distance                      | \( d \) | km           | 100          | 100          |
| Flux density                        | \( Fd \) | dBW/m\(^2\)  | -85.49209857 | -85.49209857 |
| Space propagate loss                | \( Ld \) | dB           | -160.2311193 | -162.7308    |
| Power level of receiving antenna    | \( Prxa \) | dBW         | -134.7311193 | -137.2308    |
| Receiving antenna gain             | \( Grx \) | dB           | 15           | 15           |
| Polarization mismatch loss          | \( Lpol \) | dB          | 1            | 1            |
| Antenna gain of receiving system port | \( Grxs \) | dB           | 14           | 14           |
| Receiving feed loss                 | \( Lfrx \) | dB           | 0.5          | 0.5          |
| Power level of receiver input port  | \( Prxi \) | dBm         | -91.23111929 | -93.7308004 |
| Noise temperature of receiving antenna | \( Ta \) | K            | 150          | 150          |
| Temperature of receiving feeder     | \( Tf \) | K            | 290          | 290          |
| Receiver noise factor               | \( Rf \) | dB           | 10           | 10           |
| Receiver noise temperature          | \( Terx \) | K            | 228          | 228          |
| Receiver noise temperature          | \( Terx \) | dBK         | 23.57934847  | 23.57934847  |
| ENT of Receiver port                | \( Td \) | K            | 393.2248687  | 393.2248687  |
| ENT of Receiver port                | \( Td \) | dBK          | 25.94640976  | 25.94640976  |
| G/T value of Receiver port          | \( G/T \) | dB/K         | -12.44640976 | -12.44640976 |
| ENT at the receive system port      | \( Terxs \) | K           | 441.2055593  | 441.2055593  |
| ENT at the receive system port      | \( Terxs \) | dBK         | 26.44640976  | 26.44640976  |
| G/T value of Receive system port    | \( G/T \) | dB/K         | -12.44640976 | -12.44640976 |
| Carrier to noise ratio              | \( C/N_0 \) | dB-Hz      | 80.82247095  | 78.3227902   |
Figure 8. MWR ranging errors of 2400 samples that data collected from 6-DOF moving platform under condition of relative longitudinal velocity 5 µm/s. The ranging errors are less than 40 µm during test.

Figure 9. MWR ranging rate errors of 2400 samples that data collected from 6-DOF moving platform under condition of relative longitudinal velocity 5 µm/s. The ranging errors are less than 1.6 µm/s during test.
5.2. Hardware in Loop Simulation for SFF Relative Navigation

A Hardware in loop (HIL) simulation is conducted here, with all of the deployed payloads fixed in the 6-DOF platform, aiming for indepth analysis of relative navigation filter performance by using real MWR equipment.

When considering the sharp orbit drift during perigee time, mission designers proposed a mature S band ranging (SBR) equipment, besides BeiDou dual frequency receiver, as a backup payload for both relative measurement data transfer and TT&C payloads. SBR is a reliable medium accuracy measurement payload that operates in an amateur frequency of 2450 MHz ± 50 Hz. The ranging accuracy of SBR is 1 m(1σ) in pseudo noise (PN) code measurement mode, 0.01 m(1σ) in carrier phase measurement mode, and the data rate achieved a maximum of 1 Mbit/s. The whole ranging process during the HEFFD mission is scheduled like this: (1) The BeiDou B1C/B2a dual frequency receiver is operated in the whole flight, which provide relative carrier phase differential measurement of centimeter level accuracy, and provide synchronized time-tag of MWR. (2) The SBR payload is used for reliable medium accuracy level distance measurement and data transfer, which is arranged to operate during whole flight time. (3) The relative ranging values from BeiDou receiver and SBR are simply optimally fused at the output level. It will be regarded as baseline formation distance data in HEFFD mission. (4) The MWR payload is especially developed for HEFFD mission, and it will only be used during the apogee orbit period for submillimeter level ranging and precise GNC technology validation. (5) Dual frequency interference optical ranging payload is also used with a ranging accuracy of 1 micron(1σ), which provides the reference distance data for the validation process.

The platform is precisely installed with transmitting/receiving payloads and antennas of SBR, MWR, and optical equipment, with a guaranteed accuracy of a fixed position deviation within two microns. Figure 10 demonstrates the block diagram of whole simulation platform used in this paper. The whole simulation system is coordinated by a central control computer, which conducts the HEFFD mission formation scenario design, high precision moving platform control, payload operation, data collection, and analysis. Note that a state of art high fidelity SPIRENT GSS9000 simulator is used for the generation of BeiDou B1C/B2a signals and real HEFFD mission scene. Dual frequency BeiDou receivers from both formation spacecrafts are connected directly to the simulator, for the purpose of differential carrier phase measurement and precise time synchronization with an accuracy of 0.3 ns(1σ).

Some of the parameters used in this simulation include: initial date of simulation: 8 April 2021, 14:15:00 (GMT+08:00), sample time interval: 1 s. The spectral densities of the process noise components \( w_x, w_y, w_z \) in Equation (19), which are each given by \( \sqrt{5} \times 10^{-11} \text{ m/s}^{3/2} \) [20]. The individual standard deviation for initial states \( \eta \) and \( \dot{\eta} \) is given by 0.01 km for \( q \), 0.1 deg for \( \theta \), 0.1 deg for \( \phi \), \( 1 \times 10^{-4} \text{ km/s} \) for \( \dot{\phi}(0) \), 0.01 deg/s for \( \dot{\theta}(0) \), and 0.01 deg/s for \( \phi(0) \). 0.05 and 0.05 deg/s² for leading orbit angular velocity and its rate, and 50 m and 0.01 m/s for the leading orbit radius and its rate.
Figure 10. Block diagram of simulation platform, mainly including high precision 6-DOF moving platform, optical/SBR/MWR/BeiDou payloads, SPIRENT GNSS simulator, central control computer, and standard Can bus.

Figure 11 illustrated the estimation error (the differential of estimated values and real values from model of Table 1) of HEFFD relative range and range rate, by using BeiDou/SBR measurement and traditional Kalman filter algorithm. The performance of this situation is not so good since the estimation error divergent gradually over time, especially during perigee periods (bolded red lines in Figure 11), and converge a little bit near apogee (blue lines in Figure 11). The range estimation error can be up to 80 cm around perigee, and reduced to about 10 cm during apogee flight. The range rate error achieves maximum of 1 mm/s during perigee, and 0.03 mm/s during apogee. Similar estimation performance of elevation, azimuth angle and their rates can be obtained, which will be provided later in detail.

Clearly, this is not an optimal result for engineering application. The reason for the filter divergence is this: the formation dynamic model used in the filter, radar model, is not accurate enough for the states prediction through recursive calculation at each sampling time. The process noise values of $w_x$, $w_y$, $w_z$ are changing consistently according to the orbit perturbations, which need to be adjusted through adaptive approach from measurement update. For the purpose of comparison, the data that are analyzed in Figure 11 are redrawn in Figures 12 and 13 with new results. The green line in Figure 12 demonstrates the relative ranging errors while using the adaptive filter introduced in Section 4, from BeiDou/SBR equipment. It can be seen that the estimation accuracy notably improved during both perigee and apogee orbit time, by using the process noise adaptive filter algorithm. The range estimation error is less than 10 cm around perigee, and reduced to 1 cm during apogee. The range rate estimation errors were also significantly reduced, as in Figure 13, which achieved less than 15 µm/s around apogee.
Figure 11. Range and range rate estimation errors from BeiDou/SBR equipment. The red lines denote the estimation errors accordingly, and the short blue lines denote the same data during apogee time.

Figure 12. Relative range estimation errors from different payloads & filters. The red line: relative range estimation errors of BeiDou/SBR payload using traditional Kalman filter algorithm (same data during apogee time in blue lines); green line: estimation errors of BeiDou/SBR payload using adaptive filter; black lines: estimation errors of MWR payload using adaptive filter during apogee time.
However, centimeter level ranging accuracy is not enough for the real HEFFD mission. The black lines in Figure 12 and 13 illustrated the MWR results, by using adaptive approach, during apogee periods in the simulation. With data collection, statistics, and analysis after simulation, the final accuracy of MWR achieves 35 μm in range and 8.5 μm/s in range rate. Table 4 below provided the maximum estimation error and error root mean square (RMS) during three orbiting periods, for the full states, by using different relative filter algorithms and ranging equipment. The results clearly demonstrated the effectiveness of an daptive filter that incorporates process noise uncertainty, and submillimeter level ranging accuracy for formation flight in HEO using MWR technology.

### Table 4. Relative navigation estimation error during simulation.

| Parameter          | Unit   | KF-BeiDou/SBR* | AF-BeiDou/SBR* | AF-MWR |
|--------------------|--------|----------------|----------------|--------|
| range              | μm     | ME*            | Apogee         | Perigee | Apogee |
|                    |        | 7.8 × 10^3    | 8.65 × 10^4   | 9.12 × 10^4 | 1.05 × 10^4 | / | 3.5 × 10^3 |
|                    |        | 3.02 × 10^3   | 5.65 × 10^4   | 3.64 × 10^4 | 8.27 × 10^3 | / | 9.50 |
| range rate         | μm/s   | ME             | 1 × 10^3      | 3 × 10^4   | 9 × 10^4   | 1.5 × 10^4 | / | 8.5 |
|                    |        | RMS            | 5.91 × 10^2   | 2.54 × 10^4 | 4.51 × 10^4 | 9.8    | / | 2.6 |
| elevation angle    | deg    | ME             | 5.10 × 10^-2  | 5.10 × 10^-2 | 4.10 × 10^-3 | 3.10 × 10^-3 | / | 1.40 × 10^-3 | |
|                    |        | RMS            | 4.21 × 10^-2  | 3.40 × 10^-2 | 3.10 × 10^-3 | 2.20 × 10^-3 | / | 9.00 × 10^-4 |
| elevation rate     | deg/s  | ME             | 1.90 × 10^-3  | 1.70 × 10^-3 | 9.20 × 10^-4 | 7.00 × 10^-4 | / | 1.20 × 10^-4 |
|                    |        | RMS            | 1.80 × 10^-3  | 1.54 × 10^-3 | 8.14 × 10^-4 | 5.22 × 10^-4 | / | 5.24 × 10^-5 |
| azimuth angle      | deg    | ME             | 6.10 × 10^-2  | 4.80 × 10^-2 | 4.02 × 10^-3 | 3.02 × 10^-3 | / | 1.50 × 10^-3 |
|                    |        | RMS            | 6.12 × 10^-3  | 1.59 × 10^-3 | 7.03 × 10^-4 | 6.21 × 10^-4 | / | 8.16 × 10^-5 |
| azimuth rate       | deg/s  | ME             | 1.90 × 10^-3  | 1.90 × 10^-3 | 8.60 × 10^-4 | 8.00 × 10^-4 | / | 1.30 × 10^-4 |
|                    |        | RMS            | 1.62 × 10^-3  | 1.59 × 10^-3 | 7.03 × 10^-4 | 6.21 × 10^-4 | / | 8.16 × 10^-5 |

*note: KF (typical Kalman filter), AF (Adaptive filter), ME (Maximum Error).
6. Conclusions

In this study, we presented a novel submillimeter level accuracy K band microwave ranging (MWR) payload that is used for on-orbit demonstration of precise ranging technology for future HEO SFF mission, which we called highly elliptical formation flying demonstration (HEFFD) mission. The MWR uses DOWR microwave phase accumulation technique and detail development of MWR is introduced, including K/Ka dual frequency corrugated horn antenna, frequency reference unit, microwave channel, and signals process platform. Relative orbit models with perturbations and improved adaptive filter algorithm that corporate with process noise uncertainty are presented, which are used for SFF relative navigation system design.

The MWR equipment is tested in two stages: an assessment of ranging accuracy and HIL simulation within the SFF GNC system. First, MWR is fixed in a high precision 6-DOF moving platform and extensively tested under condition of relative velocity 5 µm/s. The ranging errors achieved less than 40 µm and 1.6 µm/s for range rate errors. Second, a complex HIL simulation system is constructed which incorporate with optical sensor, BeiDou dual frequency receiver and SBR equipment, aiming to evaluate real relative navigation filter performance during HEFFD mission. With data statistics and analysis, the final MWR range estimation accuracy while using the adaptive filter algorithm achieved less than 35 µm and 8.5 µm/s for range rate, which clearly demonstrated the promising submillimeter level performance for future HEO formation missions.

Author Contributions: Conceptualization, X.W. and D.W.; methodology, Y.J., Q.M. and D.W.; software, X.W.; validation, X.W. and D.W.; investigation, Z.K. and Q.S.; resources, D.G.; writing—original draft preparation, X.W.; writing—review and editing, X.W. and D.G.; supervision, S.W.; project administration, X.W.; funding acquisition, X.W. All authors have read and agreed to the published version of the manuscript.

Funding: This research was funded by Shanghai Nature Science Fund under contract No. 19ZR1426800, Shanghai Jiao Tong University Global Strategic Partnership Fund (2019 SJTU-UoT), WF610561702, and Shanghai Jiao Tong University Young Teachers Initiation Programme, AF4130045.

Acknowledgments: We are grateful to the anonymous reviewers for their comments that significantly improved the quality of the manuscript.

Conflicts of Interest: The authors declare no conflict of interest, and the funders had no role in the design of the study; in the collection, analyses, or interpretation of data; in the writing of the manuscript, or in the decision to publish the results.

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