Applications of Monte-Carlo method for risk assessment of buildings at service stage

Zhang Hui1, a and Song Xiaobing2, *

1Professor, Shanghai Jianke Engineering Consulting Co, Ltd, Shanghai 200240, China
2Asst. Professor, Shanghai Jiao Tong University, Shanghai 200240, China

*Corresponding author e-mail: xbsong@sjtu.edu.cn, zhanghui@jkec.com.cn

Abstract. Monte-Carlo method is considered to be an efficient risk analysis method, but the method is seldom used in the field of buildings at service stage. This paper recognizes major risks of buildings at the stage and provides a risk assessment Monte-Carlo model. The model can be used for regional evaluation of buildings and assessment of important risks, serving for formulation of pertinent measures.

1. Introduction

The service stage of buildings refers to the age after a structure is completed and put into use. In this stage, many key resources need to be integrated such as personnel and equipment to achieve the purpose of reducing its operating costs, increasing investment income and extending the life cycle of buildings. In the whole life cycle of buildings, the phase in service far exceeds the sum of the time of other phases, with the cost accounting for 55%-75% of the total cost. In the recent years, a large number of buildings are competed and put into use in China, as a result, building accidents occur more frequently because of various complicated causes. In order to make management of these buildings more effective, some government departments attempt to manage these buildings in a grid-based management model. Risk assessment is the foundation of grid-based management and mainly includes two aspects: regional evaluation and ranking of important risks. The objects of the former are all buildings in an area, serving for reallocating management resources of different areas, while the objects of the latter are various primary risks, serving for formulation of appropriate measures.

At present, the research on risk assessment of buildings mainly focusses on the phase of construction, such as fault tree analysis (FTA), Fuzzy Comprehensive Evaluation, etc. As an example, based on the fire accidents occurred from 1998 to 2007, Huang Ying [1] established a fire fault tree, analyzed it and got the importance order of basic risks. Compared to the qualitative analysis of FTA, Fuzzy Comprehensive Evaluation divides the whole system into different elements, and get more accurate results by considering both risk size and risk importance, therefore, this method has been used more widely. Tian Yumin and others [2] introduced fire probability into the Fuzzy Comprehensive Evaluation model to study high-rise building fire. Lian Fengmei and others [3] considered high falling and electric shock accidents, built a safety evaluation model of buildings in construction and suggested pertinence measures to prevent relative possible risks. Based on engineering examples, Song Yanhong [4] developed an evaluation system with 16 main indexes that affected the safety of buildings in construction. The system can quantitatively analyze the safety of the construction site, and give suggestions for safe production.
Fault tree analysis method is used for qualitative analysis, while Fuzzy Comprehensive Evaluation method can be applied to semi-quantitative analysis, thus the method has been applied more widely. However, there are two flaws in this method: one is that both risk classification and risk size are determined by experts, which is a subjective process; the other is that the evaluation results are obtained by maximum membership degree method, which is sometimes one-sided and may even get unreasonable conclusions.

This paper collects and analyzes a lot of building accidents that occurred in the service stage and builds a risk database, based on which the paper revises the Fuzzy Comprehensive Evaluation method, and builds the Monte-Carlo model: The model determines distribution of basic risks based on the database, develops a risk assessment system, and adopts risk coefficients as description of building risks instead of maximum membership degree, which makes the results more accurate.

2. The establishment of Monta-Carlo model

2.1. Risk database

Risk assessment is based on data, and research on database has attracted the attention of many scholars. Barry [5] and Ward [6] proposed the construction of a risk library. However, there is little relative study on building risks currently; this paper collects a lot of building accidents that occurred in the service stage from Shanghai Key Laboratory of Engineering Structure Safety, and analyzes them from aspects of building function, structure form, accident type, accident causes, etc.

There are 703 cases of main structure and 831 cases of maintenance structure in the database. Statistic analysis shows that the accidents occurring in the main structure at service stage mainly contain leakage, structural collapse, fire, member failure, elevator fault, failure of power supply system, etc. Figure 1 shows the statistic analysis of accident.

2.2. Risk assessment system

This paper attempts to establish a risk assessment system in the case of structural collapse. Based on the risk database, it can be inferred that the possible situations that lead directly to structural collapse mainly include: insufficient strength of components/nodes, insufficient overall performance and foundation failure. Therefore, the three factors are classified as the first-level risks corresponding to structural collapse.

Furthermore, overload, fire and insufficient strength of members, which cause insufficient strength of components/nodes, are classified as corresponding secondary risks. Similarly, overturning and loss of stability are the secondary risks corresponding to insufficient overall performance; Foundation damage and excessive deformation of base are the secondary risks corresponding to insufficient of foundation.

Based on the above thoughts, the framework on the causes of structural collapse can be constructed, which is the basis of the risk assessment system.
2.3. Risk coefficients

After the risk assessment system is established, the risk coefficient of the top event can be calculated by layered calculation method. From the basis events, the risk factor of upper layer events can be calculated according to the lower layer events until the top event is calculated.

Since the importance of each event is not exactly the same, the product of the risk coefficient and weight coefficient of the lower layer event are taken as the risk coefficient of the upper layer event. If it is assumed that \( \mu = \{ \mu_1, \ldots, \mu_m \} \) is the risk coefficient set of the lower layer events in a branch, as well as \( W = \{W_1, \ldots, W_m\} \) is the corresponding weight coefficient, then the risk coefficient of the upper layer event of the branch can be calculated by the following form:

\[
R = [\mu_1, \ldots, \mu_m] \times [W_1 \ldots W_m]
\]

(1)

Where, \( \mu_i \) and \( W_i \) are respectively the risk coefficient and weight coefficient of the i-th event in the lower layer, and \( R \) is the risk coefficient of the upper event.

2.3.1. Definition of risk coefficients of basic events. Since the collection of database is based on Sutra probability and binary state hypotheses, which divides the working state of unit and system into "security" or "failure", the database obeys N-heavy Bernoulli distribution. However, in the practice, binary state hypotheses are not valid, and it is in the most cases that events are in the middle state of normality and failure instead. This paper puts forward the viewpoint about intermediate state, considering that the damage probability of risk events exactly reflects the risk coefficient. According to probability theory, the damage probability obeys Beta distribution.

The Beta distribution, with two positive parameters \( \alpha \) and \( \beta \), is a conjugate distribution of the Bernoulli distribution and the binomial distribution. If it is assumed that the total amount of buildings in an area is \( \alpha + \beta \) and the number of a certain event is \( \beta \), the risk coefficient function of the event can be expressed as Equation 3. The mean value of the function is \( \beta / (\alpha + \beta) \), and the domain is between 0-1.
\[ f(x; \alpha, \beta) = \frac{1}{B(\alpha, \beta)} x^{\alpha-1} (1 - x)^{\beta-1} \]  

(2)

2.3.2. Determination of risk weight of events. Risk weight is calculated by the Analysis Hierarchy Process (AHP), whose main ideas are as follows: If there are \( n \) events, an \( n \)-order pairwise comparison matrix can be obtained by comparing and judging the importance between every two events. Then the risk weight of every event is determined by calculating principal eigenvalue of this matrix and corresponding eigenvector [7].

The principal eigenvalue and eigenvector are obtained as follows——An example of pairwise comparison matrix \( [A_{ij}]_{n \times n} \).

1. Normalize the elements in \( [A_{ij}]_{n \times n} \) by column:
   \[ E_{ij} = \frac{A_{ij}}{\sum_{i=1}^{n} A_{ij}} \quad (i = 1, 2, \ldots n; j = 1, 2, \ldots n) \]  
   (3)

2. Sum the elements of matrix \( E \) by raw, \( F = [F_1, F_1, \ldots F_n]^T \)
   \[ F_i = \sum_{j=1}^{n} e_{ij} \quad (i = 1, 2, \ldots n) \]  
   (4)

3. Normalize the vector \( F \) to get \( W \):
   \[ W = [W_1, W_1, \ldots W_n]^T \]  
   (5)
   \[ W_i = \frac{F_i}{\sum_{j=1}^{n} F_j} \]  
   (6)

4. Calculate principal eigenvalue
   \[ \gamma_{max} = \frac{1}{n} \sum_{i=1}^{n} \frac{(AW)^i}{W_i} \]  
   (7)

5. Consistency check:
   ① Consistency Index (CI)
   \[ CI = (\gamma_{max} - n)/(n - 1) \]  
   (8)

   ② Random Consistency Index (RI) can be found in relative standard

   ③ Consistency Ratio (CR)
   \[ CR = \frac{CI}{RI} \]  
   (9)

When CR is less than the threshold value of 0.1, the consistency check shows a high level of consistency.

2.4. The application of the Monte-Carlo model

2.4.1. Regional evaluation of buildings. Monte-Carlo method, which can be used for risk assessment, is a probability-based simulation method. Provided that the probability distributions of random variables are known or in accordance with the assumed distribution, a group of random numbers are generated by random sampling and put into analysis model, thus the evaluation index of a project can be got; when
the number of samples is enough, the distribution of index can provide a reference for management of the project [8].

On the basis of the risk analysis model, this paper samples risk coefficients of each basic event, gets a group of inputs, then obtains an output about top event though the model; running for 106 times, the distribution of the risk coefficient of top event can be got. It’s reasonable to determine the safety situation of buildings in an area through the research on this distribution.

2.4.2. Analysis of key basic risks. The occurrence of structure accidents is related to risk probability, as well as the role played by risks in accidents. Both of these two influence should be considered in risk assessment, so this paper calculates the importance of basic events from the two aspects of risk possibility and structural importance.

The probability (p) of occurrence of basic events can be obtained based on their probability distribution from the database. Besides, after a basic event is selected, the path affecting upward can be uniquely determined. So the structural importance of the basic event can be calculated by multiplying the weight coefficients on the path. The magnitude of key risks are function of occurrence probability (p) and structural importance (w).

\[
R_i = \frac{p_i}{p_{\text{max}}} \times 0.5 + \frac{w_i}{w_{\text{max}}} \times 0.5
\]  

Where, \( R_i \) represents the comprehensive importance of the \( i \)-th basic risk; \( p_i \) , \( w_i \) respectively represent the probability and structure importance of the \( i \)-th basic risk. \( p_{\text{max}} \) is the maximum value of probability of all basic events; \( w_{\text{max}} \) is the maximum value of structure importance of all basic events.

3. An example of Monte-Carlo model

3.1. The calculation of risk coefficients of basic events

According to statistics, there are 252 structural collapse cases in the database. Provided that the ratio of the database capacity to sample size is 1/105, it can be estimated that the total number of buildings in the area is 252×105, in which the number of basic risks causing building collapse is \( \beta \). The detailed data is recorded in Table 1. The distributions of risk coefficient of snow disaster, earthquake and wind disaster are shown in Figure 4.
### Table 1. Distribution parameters of basic events

| Basic event                  | $\beta$ | $\alpha+\beta$ | Basic event                  | $\beta$ | $\alpha+\beta$ |
|-----------------------------|---------|-----------------|-----------------------------|---------|-----------------|
| Snow disaster               | 17      | $252 \times 10^5$ | Quality problem of project | 2       | $252 \times 10^5$ |
| Earthquake                  | 13      | $252 \times 10^5$ | Careless use of Electricity | 2       | $252 \times 10^5$ |
| Wind disaster               | 5       | $252 \times 10^5$ | Flammable materials         | 2       | $252 \times 10^5$ |
| Water disaster              | 5       | $252 \times 10^5$ | Incorrect design of foundation | 1     | $252 \times 10^5$ |
| Material corrosion          | 7       | $252 \times 10^5$ | Rainstorm and flood         | 6       | $252 \times 10^5$ |
| Unreasonable Design load    | 17      | $252 \times 10^5$ | Uneven settlement           | 4       | $252 \times 10^5$ |

#### Figure 3. The distribution of risk coefficients of snow disaster, earthquake and wind disaster

3.2. The calculation of risk weight of events

(1) The establishment of pairwise comparison matrix

Based on AHP scale, the PCM of snow disaster, earthquake and wind disaster can be obtained:

$$A = \begin{bmatrix} 1 & 1/3 & 7 & 5 \\ 3 & 1 & 7 & 5 \\ 1/7 & 1/7 & 1 & 1/3 \\ 1/5 & 1/5 & 3 & 1 \end{bmatrix}$$

(2) The calculation of risk weight

Normalize the elements in matrix $A$ by column:

$$A = \begin{bmatrix} 0.23 & 0.20 & 0.39 & 0.44 \\ 0.69 & 0.60 & 0.39 & 0.44 \\ 0.03 & 0.09 & 0.06 & 0.03 \\ 0.05 & 0.12 & 0.17 & 0.09 \end{bmatrix}$$

Sum the elements of matrix $A$ by row and normalize the remained vector.

$$W = [0.1348, 0.5294, 0.0508, 0.1051]^T$$

So the risk weight of improper use, earthquake, wind disaster, snow disaster are respectively 0.1348, 0.5294, 0.0508, 0.1051.

(3) Consistency check:

The consistency check for the thematic layers is $\gamma_{\text{max}} = 4.2319$, $n=4$, $\text{RI}=0.9$, $\text{CI}=0.0773$, and $\text{CR}=0.0859$ is which is less than the threshold value of 0.1, which shows a high level of consistency.

In the similar process that is prescribed to calculate the risk weights of events causing “overload”, the relative weights of the other events are calculated in Figure 3.
3.3. Results

3.3.1. The distribution of risk coefficient of buildings collapse. From the basis events, the risk coefficients of upper layer events are calculated according to the lower layer events until the top event is calculated. After self-compiling program runs for 106 times, the distribution of risk coefficient of structural collapse is shown in Figure 4.

On the basis of graphic analysis, the risk coefficient of regional buildings are mainly distributed between $1 \times 10^{-6}$ and $3 \times 10^{-6}$, and the most possible risk coefficient is $1.95 \times 10^{-6}$. At present, there is no unified standard for the assessment results on regional evaluation of buildings in an area, however, the results can be used to compare security in different areas, providing reference for the management resources invested in these areas.

3.3.2. The comprehensive importance of basic events. The structural importance, which is obtained by multiplying the weight coefficients on the influence path, is substituted into equation 4 along with probability of basic events, then comprehensive importance of the basic events can be calculated (Figure 2).
Table 2. The comprehensive importance of basic events

| Basic events                      | Weight coefficients on influence path | Structure importance | Probability (10^-6) | Importance |
|-----------------------------------|---------------------------------------|----------------------|--------------------|------------|
| Abnormal usage                    | 0.13→0.21→0.33                        | 0.0143               | 6.732              | 0.5215     |
|                                   | 0.13→1.00→0.20→0.14                   |                      |                    |            |
| Earthquake                        | 0.53→0.21→0.33                        | 0.0585               | 5.157              | 0.4757     |
|                                   | 0.53→1.00→0.20→0.14                   |                      |                    |            |
| Snow disaster                     | 0.05→0.21→0.33                        | 0.0055               | 1.986              | 0.1559     |
|                                   | 0.05→1.00→0.20→0.14                   |                      |                    |            |
| Fire                              | 0.11→0.21→0.33                        | 0.0121               | 1.985              | 0.1664     |
|                                   | 0.11→1.00→0.20→0.14                   |                      |                    |            |
| Material corrosion                | 0.11→0.31→0.33                        | 0.0272               | 2.777              | 0.2493     |
| Unreasonable design load          | 0.26→0.31→0.33                        | 0.0643               | 6.751              | 0.6030     |
| Quality problem of project        | 0.63→0.31→0.33                        | 0.1559               | 0.794              | 0.3086     |
| Careless use of Electricity       | 0.14→0.48→0.33                        | 0.0156               | 0.795              | 0.0839     |
|                                   | 0.14→0.48→0.8→0.14                    |                      |                    |            |
| Flammable materials               | 0.86→0.48→0.33                        | 0.0963               | 0.792              | 0.2130     |
|                                   | 0.14→1.00→0.8→0.14                    |                      |                    |            |
| Incorrect design of foundation    | 1.00→0.25→0.52                        | 0.1300               | 0.398              | 0.2378     |
| Rainstorm and Flood               | 0.20→0.75→0.52                        | 0.0780               | 2.377              | 0.3010     |
| Uneven settlement                 | 0.80→0.75→0.52                        | 0.3120               | 1.588              | 0.6176     |

The analysis result shows that the comprehensive importance of uneven settlement, unreasonable design load and improper use are 0.6176, 0.6030 and 0.5215 respectively, which are the most important basic risks. Therefore, pertinent measures can be took to prevent the three risks in the management of buildings in an area.

4. Conclusion and discussion

This paper discusses the method to evaluate risks of buildings at service stage in an area and establishes the Monte-Carlo model to make some new attempts for the regional evaluation. The method has following advantages:

1. In the paper, a database, which contains plenty regional security events, is built. Further more, based on this database, risk coefficient distribution and Monte-Carlo risk assessment system are obtained. The evaluation process is more precise and standardized, which reduces the the subjectivity of the expert assessment.

2. The evaluation result of the Monte Carlo model is not graded, but a distribution image of the risk coefficient by a large number of sampling simulations, so the result is more scientific.

3. The results from the Monte-Carlo model applied to the different areas can be used to compare the security in these areas, which can serve for the scientific and rational measures to prevent building accidents.

In view of the current primary and incomplete regional evaluation of buildings, there is no unified standard for the assessment results. However, with the implementation of grid-based management by government departments, the method of regional evaluation will be continuously improved and play a greater role in risk management.
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Correspondence author: Song Xiaobing.
E-mail: xbsong@sjtu.edu.cn.
Asst. Professor, Shanghai Jiao Tong University, China

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