Enforced stripping of negative muons from $\mu\text{He}^+$ ions to stimulate muon-catalyzed fusion by cyclotron resonance acceleration

Yosiharu Mori

Institute for Integrated Radiation and Nuclear Science, Kyoto University, Kumatori, Osaka, 590-0494, Japan

E-mail: mori.yoshiharu.4w@kyoto-u.ac.jp

Received May 26, 2021; Revised August 17, 2021; Accepted August 18, 2021; Published August 24, 2021

1. Introduction

Since the first observation of muon catalysis of nuclear fusion [1], muon-catalyzed fusion (MuCF) has been discussed as a candidate for future energy production systems [2,3], as well as an intense neutron source for various applications such as the mitigation of long-lived fission products with nuclear transmutation [4,5]. MuCF can be initiated from muonic molecular ($\mu\text{dt}$) generation by absorbing negative muons into a deuterium and tritium mixture. As the muon mass is approximately 200 times heavier than the electron mass, the two nuclei (deuteron and triton) of a $\mu\text{dt}$ molecule move approximately 200 times more than an ordinary $\text{dt}$ molecule, leading to an immediate $d-t$ nuclear fusion reaction. The energy released in the reaction is carried by a neutron (14.1 MeV) and an $\alpha$ particle (He ion, 3.5 MeV). The fusion rate of this reaction is much faster ($\sim$100 times) than the $\mu\text{dt}$ molecular formation rate. Thus, the fusion cycles could continue during the negative muon lifetime if negative muons were freely released during the fusion reactions.

The formation rate of the $\mu\text{dt}$ molecule, $\lambda_f$, was evaluated experimentally in Ref. [6]. A typical value of $\lambda_f$ for a liquid deuterium/tritium (D/T) mixture (D fraction $C_d = 0.7$, T fraction $C_t = 0.3$) is approximately $(467 \pm 41) \times 10^6 \text{s}^{-1}$ estimated from the values presented in Table 1 of Ref. [6]. For a gas D/T mixture pressure of 10 atm ($C_d = 0.7$, $C_t = 0.3$), $\lambda_f$ is $(347 \pm 93) \times 10^6 \text{s}^{-1}$ at a temperature of 400–500 K. The D/T gas mixture density dependence of $\lambda_f$ is relatively small even at about 10 atm, which has also been clarified experimentally [7].
Thus, the possible number of fusion cycles for a single muon, $\lambda_f \tau_\mu$, is approximately 700–1000, where $\tau_\mu$ is the lifetime of a negative muon (2.2 $\mu$s). However, previous experiments showed that only approximately 100–150 MuCF fusion cycles [8] were initiated for a single negative muon, which was almost an order of magnitude smaller than expected from the $\mu$dt molecular formation rate. This is because of muons sticking to $\alpha$ particles.

The sticking probability of a single MuCF reaction has been evaluated theoretically and experimentally to be approximately 0.6%–0.7% by several groups [9–12]. This means that the net MuCF energy production catalyzed by a single negative muon is about 2.5 GeV. The beam energy cost for a single negative muon production in high-energy hadron accelerators is approximately 5 GeV [13]. The efficiency of MuCF for energy production is less than half that required to achieve scientific break-even. Thus, it is thought that MuCF is not useful for energy production because of the muon sticking process. Efficiently detaching the negative muon from the $\mu$He$^+$ ion is essential to increasing the number of induced fusion reactions.

Colliding $\mu$He$^+$ ions whose energy is more than 200 keV with D/T gas molecules effectively strips the negative muon from the $\mu$He$^+$ ion. The energy loss of the $\mu$He$^+$ ion can be recovered by reacceleration by an external electric field. At a D/T mixture gas pressure of approximately 100 atm, the energy loss of the $\mu$He$^+$ ion is approximately 100 MeV m$^{-1}$, which is difficult to compensate with a static electric field. References [14,15] showed that the efficiency of muon stripping could be enhanced by resonance acceleration of $\mu$He$^+$ ions by a radio-frequency (RF) electric field. They claimed that in this case the muon sticking probability could become less than 20% and the fusion energy catalyzed by a single negative muon could exceed 10 GeV, which can be large enough to surmount the break-even limit in MuCF energy production. A liquid D/T mixture target may be difficult to use in this scheme, as pointed out in Ref. [15], because the He ions heat the target. Only a gas D/T mixture target can be used.

However, some phenomena may severely restrict the efficiency of the scheme: electron capture and detachment reactions of $\mu$He$^+$ ions in the gas D/T mixture. As the neutralization of the $\mu$He$^+$ ion caused by electron capture reduces the energy recovery efficiency of the electric field, the muon stripping efficiency can be largely influenced by the balance between electron capture and detachment. The cross sections of both reactions are approximately $(m_\mu/m_e)^2 \sim 40 000$ times larger than for muon stripping from the $\mu$He$^+$ ion, and they depend sensitively on the energy of the $\mu$He$^+$ ion. At relatively low energies (less than 0.5 MeV), the electron capture process becomes dominant and muon reactivation by energy recovery with an external electric field suffers greatly. These processes have not yet been examined. A detailed analysis of these effects is necessary to understand how the resonance RF acceleration scheme can be useful for muon reactivation from $\mu$He$^+$ ions and increase the number of MuCF chain cycles.

This paper presents studies on these processes based on simulation calculations using particle tracking. In addition, the manner of small-diameter and conical diverged D/T mixture gas flows in which the resonance RF acceleration scheme can stimulate muon reactivations and improve the number of MuCF chain cycles is shown.

2. MuCF chain cycle

The negative muon chain cycle in MuCF in a D/T system is shown schematically in Fig. 1. The process can be described by a series of rate equations as shown in the Appendix, which are specified with the following parameters: fusion rate $\lambda_f$, sticking rate $\omega$ to form a $\mu$He$^+$ ion (sticking), muon
Fig. 1. Schematic diagram of the MuCF chain cycle. $\lambda_f$ is the fusion rate, $\omega$ is the muon sticking rate to form a $\mu$He$^+$ ion, and $\alpha$ is the muon stripping rate from the $\mu$He$^+$ ion.

stripping rate $\alpha$ from $\mu$He$^+$ ion (stripping), muon lifetime $\tau_\mu$, and $\mu$He$^+$ ion lifetime $\tau^*$. The fusion rate $\lambda_f$ is mostly determined by the $\mu$dt molecule formation process [8].

2.1. Cycling rate

The number of chain reactions, $N^\infty_f / N^0_\mu (t = +\infty)$, induced by a single negative muon is given by Eq. (A.13), including the muon sticking and stripping processes as shown in the Appendix. This leads to

$$
\frac{N^\infty_f}{N^0_\mu} = \left[ \frac{1}{\lambda_f \tau_\mu} + \frac{\omega}{\lambda_f} \left( \frac{1}{\tilde{\alpha} \tau^* + 1} \right) \right]^{-1},
$$

where $\tilde{\alpha}$ is the time-averaged stripping rate, and $\tau^*$ is the effective lifetime of the $\mu$He$^+$ ion given by Eq. (A5).

The first term in the bracket on the right-hand side of Eq. (1) implies the number of MuCF reactions. The second term presents the muon sticking probability including the muon stripping process from the $\mu$He$^+$ ion. The second term can be written as

$$
W = W_0 \left( \frac{1}{\tilde{\alpha} \tau^* + 1} \right), \quad W_0 = \frac{\omega}{\lambda_f}.
$$

Here, $W_0$ represents the initial muon sticking probability just after the fusion reaction when the muon stripping is ignored, $\tilde{\alpha} = 0$. To evaluate the sticking probability, which is reduced by muon stripping, we define a parameter, $\epsilon$, as

$$
\epsilon = \frac{W}{W_0}.
$$

The MuCF fusion reaction rate, $\lambda_f$, has been estimated theoretically [10,16,17], predicting that $\lambda_f$ is approximately $0.5-1 \times 10^9$ s$^{-1}$ for $\mu$dt fusion. These estimates were experimentally confirmed in Refs. [8,11,18]. Thus, $\lambda_f \tau_\mu \sim 1000$. 

3/18
The initial muon sticking probability, \( W_0 \), has been calculated theoretically \([9,10]\) and also measured by several experiments \([11,12]\), and the results are almost in agreement with

\[
W_0 = \frac{\omega}{\lambda_f} \sim 10^{-2}.
\]  

(4)

According to Eq. (1), the total number of fusion chain cycles induced by a single negative muon, \( N_f \infty / N_\mu^0 \), is mostly determined by the muon sticking probability if the muon stripping process is ignored:

\[
\frac{N_f \infty}{N_\mu^0} = \left[ \frac{1}{\lambda_f \tau_\mu} + W_0 \right]^{-1} \sim 100.
\]  

(5)

Thus, the energy production in MuCF from a single negative muon is estimated to be approximately 3 GeV, which is too small to achieve scientific break-even in MuCF for energy production because energy of approximately 5–6 GeV is necessary for the production of a single negative muon. If the sticking probability \( W \) is reduced fivefold or more by enhancing negative muon stripping, both scientific and technical break-even can be achieved. Thus, energy production with MuCF could become a reality.

2.2. Muon stripping

The reactivation of a negative muon by stripping from the \( \mu^+ \)He \(^+\) ion helps to increase the total number of MuCF cycles. The muon stripping cross section of the \( \mu^+ \)He \(^+\) ion in hydrogen gas can be estimated from the electron detachment cross section of the He \(^+\) ion. The muon orbital radius of the \( \mu^+ \)He \(^+\) ion is as small as the electron–muon mass ratio compared with that of the electron of the He \(^+\) ion. Thus, the stripping cross section can be evaluated with the electron stripping process of He \(^+\) ions, such as

\[
\sigma_s(E) \sim \kappa^2 \sigma_{es}(E),
\]  

(6)

where \( \kappa \) is the ratio of the electron mass to negative muon mass (\( \kappa = m_e/m_\mu \)). The electron stripping cross sections of the He \(^+\) ion, \( \sigma_{es}(E) \), have been obtained experimentally \([19–21]\).

The muon stripping cross section has been examined and evaluated by several groups \([22,23]\) empirically and theoretically. On the other hand, in this study it is assumed that electron stripping from He \(^+\) ions has undergone a similar process and the negative muon stripping cross-section can be scaled from the electron stripping cross section of the He \(^+\) ion. The negative muon stripping cross sections estimated in this study, including the energy dependence, are close to agreement with the values obtained by the previous works. The maximum cross section evaluated in this study, for example, is about \( 2.5 \times 10^{-22} \) cm\(^{-2}\) at 0.6 MeV, which is about 0.7 times smaller than that obtained in Ref. \([22]\).

The efficiency of the negative muon reactivation depends on the averaged stripping rate \( \bar{\alpha} \) given by Eq. (A7), which is derived from rate equations based on the linearized reaction model. The time elapsed for \( \mu^+ \)He \(^+\) ions slowing down, \( \tau_1 \), is given by Eq. (A4) using the stopping power \( S(E) \), and \( \tau_1 \) decreases in proportion to the increase in gas density.

At a gas density equivalent to a pressure of 1 atm, \( \tau_1 \) becomes approximately \( 0.5 \times 10^{-7} \) s. The muon sticking probability \( W \), which depends on \( \bar{\alpha} \tau^* \) shown in Eq. (2), becomes constant at a large gas pressure because \( \tau^* \) reaches \( \tau_1 \). The solid line in Fig. 2 shows the normalized muon sticking probability \( W/W_0 \) obtained from Eq. (2) as a function of the target medium density. The
sticking probability decreases slightly as the gas pressure increases, and becomes constant at large gas pressure.

As mentioned above, the sticking probability is largely affected by the energy loss of the $\mu$He$^+$ ion. If this energy loss can be effectively reduced by energy recovery with an external electric field, the sticking probability will decrease and the number of cycles will inevitably increase. In the gas pressure range of 10–100 atm relevant to the problem, the required strength of the electric field to compensate for the energy loss effectively is approximately 10–100 MV m$^{-1}$. Such high electric field strengths cannot be achieved with electrostatic fields, but may be achieved with RF electric fields. In addition, electrostatic fields accelerate muons as well as ions simultaneously, interfering with the formation of muon molecules and ultimately reducing the efficiency of MuCF.

Thus, it is necessary to accelerate only the $\mu$He$^+$ ions resonantly with the RF electric field (cyclotron resonance acceleration). The number of MuCF cycles increases owing to the reactivation of negative muons using the RF resonant acceleration scheme. However, a process that might substantially reduce the reactivation efficiency in the resonant acceleration scheme is the electron capture process of the $\mu$He$^+$ ion to form the $\mu$He$^0$ atom.

2.3. Electron capture and detachment

The possible processes that affect the negative muon sticking probability under RF acceleration are the electron capture and detachment reactions of the $\mu$He$^+$ ion and the $\mu$He atom:

Capture: $\mu$He$^+ + D_2(T_2) \rightarrow \mu$He$^0 + D_2^+(T_2^+)$;
Detachment: $\mu$He$^0 + D_2(T_2) \rightarrow \mu$He$^+ + D_2(T_2) + e^-$. 

Until now, no direct experiment exists to measure these cross sections. However, since the $\mu$He$^+$ ion is a hydrogen-like ($H^+$) ion, the reaction cross sections of electron capture and detachment of the $\mu$He$^+$ ion can be scaled from those of the $H^+$ ion and $H^0$ atom processes.
Fig. 3. Equilibrium fractions of $\mu^+\text{He}$ ions ($F^+$) and $\mu\text{He}$ atoms ($F^0$) as functions of the $\mu\text{He}$ ion/atom energy in a thick hydrogen gas target of more than $10^{18}$ atom cm$^{-2}$.

The experimental electron capture cross section ($\sigma_{10}$) of the $H^+$ ion and detachment cross section ($\sigma_{01}$) of the $H^0$ atom as a function of energy were summarized in Ref. [24]. The energy dependence of the cross sections for the $\mu\text{He}^+$ ion and atom is scaled to those for the $H^+$ ion and $H^0$ atom by considering the mass difference between the electron and muon, similar to the stripping cross section shown in Eq. (6). The cross sections for both interactions in this energy range are around $10^{-16}$ cm$^2$, which is more than $10^4$ times bigger than the negative muon stripping cross section. Also, both cross sections are very energy dependent.

At a relatively large gas thickness ($nl > 10^{17}$ cm$^2$), the equilibrium fractions of $\mu\text{He}$ atoms ($F^0$) and $\mu\text{He}^+$ ions ($F^+$) can be approximated by the following equations using $\eta = \sigma_{0+}/\sigma_{+0}$. Here, $\sigma_{0+}$ and $\sigma_{+0}$ are the electron detachment and capture cross sections respectively, which are estimated from those of the hydrogen atom and ion.

$$F^0 \sim \frac{1}{1 + \eta}, \quad F^+ \sim \frac{\eta}{1 + \eta}.$$  \hspace{1cm} (7)

Figure 3 shows the equilibrium fractions of $\mu\text{He}^+$ ions ($F^+$) and $\mu\text{He}$ atoms ($F^0$) as functions of energy. As can be seen from the figure, $F^+$ decreases gradually with a reduction in the ion energy. Approximately 50% of $\mu\text{He}^+$ ions are neutralized at 200 keV and 90% at less than 50 keV. This process should be considered to evaluate the stripping efficiency of the negative muon from the $\mu\text{He}^+$ ion stimulated by reacceleration with an external electric field. The previous works, however, have not taken this process into account.

3. Muon stripping from the $\mu\text{He}^+$ ion with cyclotron resonance acceleration

The number of cycles of MuCF can be enhanced by increasing the stripping rate $\tilde{\alpha}$ of $\mu^-$ from the $\mu\text{He}^+$ ion. If the energy loss of the $\mu\text{He}^+$ ion is compensated by accelerating with an external electric field, $\tilde{\alpha}$ becomes large as shown in Eq. (A7). A static electric field cannot be used for this purpose because it accelerates not only the $\mu\text{He}^+$ ion but also the negative muon, which disturbs the $\mu dt$ molecule formation and decreases the fusion rate. Radio-frequency acceleration with cyclotron
resonance has mass selectivity, which is useful for accelerating only $\mu$He$^+$ ions and recovering their energy losses.

In cyclotron resonance acceleration of a charged particle, a circular polarized (rotating) RF electric field perpendicular to the magnetic field works most effectively, as pointed out in Ref. [15]. A left circularly polarized field can accelerate positively charged particles. A linear polarized field also accelerates the particles in the cyclotron resonance. The linearly polarized field is a composition of left and right circularly polarized fields, and positively charged particles are accelerated only by the left circularly polarized field. In a linearly polarized field, the acceleration efficiency is half that of a circularly polarized field. In the RF cavities with high electric field strength that have been demonstrated so far [25,26] the electric field is linearly polarized, so this study assumes a linearly polarized electric field.

Multi-particle tracking simulations are useful for understanding how RF acceleration effectively induces stripping. In particular, the electron capture effects that may disturb the energy recovery in the resonance RF acceleration can be analyzed in detail.

3.1. Simulation procedure

To evaluate the muon stripping efficiency by recovering the $\mu$He$^+$ ion energy with cyclotron resonance RF reacceleration, multi-particle tracking simulation was carried out. The three-dimensional particle position and momentum were determined for each small time step in the simulation, including the following processes that affect the muon stripping: particle energy loss, energy straggling, multiple Rutherford scattering, and neutralization caused by electron capture and detachment.

The cross section of the $\mu^-$ stripping from the $\mu$He$^+$ ion was estimated from the electron stripping cross section of the He$^+$ ion with mass ratio $\kappa = m_e/m_\mu$ in Eq. (6). The energy losses (stopping power) at a gas pressure of 1 atm employed in the simulation are also shown in Fig. A.1.

The effects of energy straggling and multiple Rutherford scattering were evaluated as random processes at each time step with the energy-dependent Gaussian distributions given in Refs. [27,28]. The neutralization probability of the $\mu$He$^+$ ion in the electron capture and detachment processes was also determined at each time step using the equilibrium values $F_+(\mu$He$^+)$ and $F_0 (\mu$He$^0)$, as shown in Fig. 3.

In the simulation, three-dimensional particle motions, including the processes described above, were tracked by numerically solving the equations of motion with a leap-frog method. A typical number of events was 10,000, and the time-step size for the leap-frog integration method was 5 ps in the simulation. The processes which affect the particle velocities such as energy loss ($-\Delta \vec{v}_i$), energy straggling ($\delta \vec{v}_s$), and multiple Rutherford scattering ($\delta \vec{v}_r$) were treated as instantaneous kicks at each time step as given by

$$
\Delta \vec{v} = -\Delta \vec{v}_i + \delta \vec{v}_s + \delta \vec{v}_r. \tag{8}
$$

The thermal motions of the gas molecules were ignored in the simulation.

The coordinates used in the simulations are shown in Fig. 4. The external magnetic field and RF electric field directions are along the $z$ and $y$ axes, respectively. The $\mu$He$^+$ ions were assumed to be isotropically emitted from the origin ($x = y = z = 0$). Typical particle trajectories with and without energy recovery are also shown in Fig. 4. In this case, a magnetic field of 5 T and an electric field of 50 MV m$^{-1}$ were assumed. The RF frequency is 18.5 MHz, which is equivalent to the cyclotron resonance frequency of the $\mu$He$^+$ ion at 5 T. The D/T mixture pressure is 5 atm. In addition, no effects of the electron capture and detachment processes were included in this case.
Fig. 4. Examples of the $\mu$He$^+$ ion trajectories obtained by simulations at a medium target density of 5 atm with a magnetic field of 5 T. The purple line shows the trajectory when the RF electric field strength in energy recovery by cyclotron resonance acceleration is zero, and the red line shows the trajectory when the RF electric field strength is 50 MV m$^{-1}$.

As shown in Fig. 2, the $\mu^-$ sticking probability ($W/W_0$) evaluated from Eq. (2) decreases slightly and becomes almost constant at large gas pressure, if the energy losses of the $\mu$He$^+$ ion are not recovered. The simulation results are also plotted with crosses in this figure, which show good agreement with the analytical values evaluated in Eq. (2).

3.2. Stripping of negative muons
3.2.1. Effect of the electron capture process
The reacceleration of $\mu$He$^+$ ions to compensate for the energy losses with the RF electric field helps to improve the $\mu^-$ stripping efficiency from $\mu$He$^+$ ions. Simulations were carried out to examine the effects of the RF electric field on decreasing the sticking probability. Figure 5 shows the simulation results for the muon sticking probabilities as a function of the strength of the RF electric field for various D/T gas pressures in a magnetic field of 5 T. In this case, the electron capture effects described in Sect. 2.3 are not included.

The muon sticking probability decreases gradually as a function of RF electric field strength and becomes almost zero when the strength exceeds a certain value for each gas pressure. At a gas pressure of 7.5 atm, for example, the muon sticking probability becomes zero above an RF electric field strength of approximately 50 MV m$^{-1}$.

The RF electric field can be obtained using an ordinary vacuum RF cavity with Be windows [25], and also a gas-filled RF cavity [26]. The frequency of these RF cavities is 805 MHz, and the electric field strength obtained is 50 MV m$^{-1}$. According to Kilpatrick’Ts rule of thumb [29], the discharge limit of a vacuum RF cavity is almost proportional to the square root of the RF frequency. The cyclotron resonance frequency of $\mu$He$^+$ ions is 18.5 MHz to 74 MHz when the external magnetic field is 5 T to 20 T. Assuming that the maximum electric field strength is proportional to the square root of the frequency, the electric field strength ranges from 7.6 MV m$^{-1}$ to 15.2 MV m$^{-1}$. On the other hand, in an RF cavity filled with high-pressure hydrogen gas at 40 atm, an electric field strength of 50 MV m$^{-1}$ has been obtained [26]. The frequency dependence of the maximum electric field
Fig. 5. Sticking probabilities ($W/W_0$) as a function of the electric field strength for various gas pressures. No electron capture process is included.

Fig. 6. Sticking probabilities as a function of the electric field strength with and without straggling and multiple Rutherford scattering effects. The purple line shows the case without straggling and multiple Rutherford scattering effects, and the blue line the case with both effects. The gas pressure is 10 atm, and no electron capture process is included in the simulations.

strength in a gas-filled RF cavity is of interest. Based on the above, in this study we mainly evaluate the negative muon sticking probability in the range of RF electric field strength up to 50 MV m$^{-1}$.

The energy straggling ($\delta \vec{v}_s$) and multiple Rutherford scattering ($\delta \vec{v}_r$) have a slight effect on the behavior of the sticking probability near zero, as shown in Fig. 6. Thus, at a D/T gas pressure of less than approximately 7–8 atm, the muon sticking probability can be decreased to less than 0.2 with a technically available RF electric field. This results in at least five times or more increase in the number of chain reactions of MuCF cycles, albeit at a relatively low D/T gas pressure, clearing the scientific and even technical break-even criteria of MuCF.
Fig. 7. Sticking probabilities \((W/W_0)\) as a function of the electric field strength for various gas pressures. The electron capture process is included.

If the \(\mu\)He\(^+\) ion captures an electron and becomes a neutral \(\mu\)He atom, the energy recovery efficiency by the RF acceleration becomes small and the muon stripping probability decreases. Because the electron capture and detachments are much faster processes (>10,000 times) than the muon stripping process, the simulations treat them as probability processes based on the equilibrium states of the \(\mu\)He\(^+\) ion and \(\mu\)He atom.

Figure 7 shows the sticking probabilities including the electron capture and detachment effects in the stripping process. As can be seen from this figure, the electron capture process largely affects the stripping efficiency. Even at a D/T gas pressure of 5 atm, the sticking probability is only reduced to 0.5 in an RF electric field of approximately 50 MV m\(^{-1}\). The sticking probability becomes less than 0.1 for a technically achievable strength of RF electric field only when the D/T gas pressure is below 1 atm. Thus, it is almost impossible to reach the break-even criteria of MuCF by decreasing the muon sticking probability with RF acceleration at a practical D/T gas pressure.

3.2.2. Muon sticking probability in localized gas streams

Because of the neutralization of \(\mu\)He\(^+\) ions by electron capture, a considerable RF electric field strength of more than 100 MV m\(^{-1}\) is required to significantly reduce the muon sticking probability at a D/T gas pressure of more than 5 atm. If the energy loss rate of \(\mu\)He\(^+\) ions moving in the D/T gas decreases effectively, the required strength of the RF electric field can be reduced.

In order to reduce the energy loss rate, a scheme using a locally filamented D/T target has been proposed [15]. In his idea, many fine D/T target rods with a small filling factor are placed in the cyclotron resonance acceleration region. Thus, \(\mu\)He\(^+\) ions from the D/T solid or liquid target rod are extracted and accelerated in the D/T free region to compensate for the energy loss at the D/T target rod. This allows the balancing of energy loss and acceleration, and the stripping of negative muons from \(\mu\)He\(^+\) ions. However, this scheme has the serious problem of removing the He\(^{++}\) ion fusion energy at very low temperature. This issue does not arise in the D/T mixture gas scheme that is discussed in this paper.
To evaluate the effects of the localized D/T mixture gas streams on the muon sticking probability, a spatial arrangement of the gas streams is assumed as shown in Fig. 8. From a technical point of view, a conically diverged shape of the stream may be easier for a Laval nozzle to make a supersonic flow compared with a cylindrical stream shape. The entrance and exit radii of each gas stream are defined as $r_i$ and $r_o$, respectively. The radius of each gas stream is increased proportionally from the entrance to the exit. The spatial occupation of the gas stream is given by Eq. (9). The length of each stream is set to be twice the cyclotron radius of a 3.4 MeV $\mu$He$^+$ ion, which is approximately 0.2 m in a magnetic field of 5 T.

The pressure at the exit, $p_o$, is assumed to be one-tenth that at the entrance, $p_i$, and the gas pressure during the flow is inversely proportional to the cross section of the stream ($p/p_i = (r/r_i)^2$).

The $\mu$He$^+$ ions are assumed to be generated at the origin ($x = y = z = 0$) shown in Fig. 8 and emitted isotropically. The spatial occupation of the gas stream is defined as

$$\Phi = \frac{\pi r^2}{d^2}. \tag{9}$$

The directions of the external magnetic field and RF electric field are also shown in this figure.

Figure 9 shows the results of a tracking simulation for various stream radii with a gas pressure of 100 atm as a function of the occupation factor. Here, the strength of the RF electric field is 50 MV m$^{-1}$, the magnetic field is 5 T, and the electron capture processes of $\mu$He$^+$ ions are included. Compared with the homogeneous D/T gas mixture shown in Fig. 7, the sticking probability is greatly reduced even at a high gas stream pressure. For example, the sticking probability decreases to 0.2 when the occupation factor is approximately 0.02 at a gas pressure of 100 atm. The sticking probability depends only on the occupation factor and not on the size of the gas stream.

In the localized D/T mixture gas streams, the sticking probability depends on the product of the gas stream pressure and the occupation factor, $p \times \Phi$, i.e. the mean gas pressure. Figure 10 shows the sticking probabilities as a function of $p \times \Phi$ for gas pressures of 100 and 1000 atm. It was found that the sticking probability was determined mostly by the mean gas pressure, $p \times \Phi$, in the localized D/T mixture gas streams.
Fig. 9. The results of the tracking simulations for negative muon sticking probabilities in the localized D/T mixture gas flow target for various stream radii where the gas target pressure is 100 atm, the magnetic field is 5 T, and the RF electric field strength is 50 MV m$^{-1}$. The closed and open circles in the figure show the sticking probabilities for $r_i = 0.5$ mm and $r_i = 1$ mm, respectively.

Fig. 10. The negative muon sticking probabilities in the localized D/T mixture gas flow target as a function of mean pressure, $p \times \Phi$, for various gas pressures, where the magnetic field and the RF electric field strength are 5 T and 50 MV m$^{-1}$, respectively.

Compared with the sticking probability in the homogeneous D/T mixture gas shown in Fig. 7, the sticking probability using our proposed scheme (the localized D/T mixture gas stream scheme) can be reduced substantially at the mean gas pressure ($p \times \Phi$). For example, at a gas pressure of 5 atm in the homogeneous gas mixture, as shown in Fig. 9, the sticking probability is about 0.6 when the electric field strength is 50 MV m$^{-1}$. On the other hand, in the localized D/T mixture gas stream scheme, the sticking probability is reduced to about 0.25. The localized gas stream scheme helps to reduce the energy loss effectively. In other words, the strength of the RF electric field for energy
Fig. 11. Sticking probabilities in the localized gas stream target as a function of the RF electric field strength at gas pressures of 10 atm and 100 atm.

recovery becomes modest by choosing the optimum radius and occupation factor for various gas pressures.

Figure 11 shows the sticking probabilities as a function of the RF electric field strength when optimizing for a minimum with the radius of the gas streams. As can be seen from this figure, by using a localized conical gas stream scheme, the sticking probability is reduced to approximately 0.2 at an RF electric field strength of 20 MV m$^{-1}$ when the inlet gas pressure of the stream is 10 atm. Even when the gas pressure is 100 atm, the sticking probability decreases to about 0.25 at 30 MV m$^{-1}$.

The reduction of the negative muon sticking probability by the acceleration of $\mu$He$^+$ ions with RF resonant acceleration is very difficult in an ordinary homogeneous D/T gas mixture because of the neutralization by captured electrons. However, by applying the localized gas stream scheme, the sticking probability can be effectively reduced to less than 0.2, and the required strength of the RF electric field becomes modest.

Figure 12 shows the sticking probabilities as a function of the magnetic field strength. The length of the gas stream, $L$ in Fig. 8, needs to be approximately twice the cyclotron resonance radius, which is inversely proportional to the strength of the magnetic field. The sticking probability is almost 0.1 or less when the magnetic field is less than 10 T; however, it increases slightly when the magnetic field is close to 20 T. This is probably because the spatial distance between the streams becomes comparable to or greater than the cyclotron resonance radius.

The dependence of the sticking probability on the background gas pressure at a gas pressure of 100 atm is shown in Fig. 13. In this simulation, it is assumed that the MuCF reactions occur at the center of the D/T target and not in the background region. Thus, the background gas works only for negative muon stripping from $\mu$He$^+$ ions and not for the MuCF reaction. The sticking probability becomes 0.1 when the background pressure is 1 atm or less, as shown in Fig. 13. The effect of background gas pressure on the sticking probability is modest. As described above, in a localized gas scheme with conically diverged streams, the sticking probability can be effectively reduced.
4. Summary

A restriction on achieving energy break-even in muon-catalyzed fusion is the sticking of negative muons to $^\alpha$ particles produced in the fusion reaction. Muon sticking limits the number of MuCF cycles to ten times smaller than that expected from the $\mu d t$ molecule formation rate. The possibility of muon stripping from $\mu He^+$ ions with energy loss recovery using RF acceleration has been proposed by several groups, and some hopes of achieving break-even for MuCF with this scheme were predicted. Three-dimensional multi-particle tracking simulations were performed to evaluate the muon stripping efficiency in this scheme.
It was found that the neutralization process of $\mu\text{He}^+$ ions due to electron capture decreased the energy recovery in RF acceleration and acted against the reduction of the sticking probability. The sticking probability becomes less than 0.1 for a technically achievable strength of the RF electric field of about 50 MV m$^{-1}$ only when the D/T gas pressure is below 1 atm, because of the neutralization process of $\mu\text{He}^+$ ion electron capture. This makes it almost impossible to reach the break-even criteria of MuCF by decreasing the muon sticking probability with the RF acceleration at a practical D/T gas pressure.

To overcome these difficulties, a scheme of using locally placed small-diameter and conical diverged D/T mixture gas flows has been studied. The simulation results showed that at a gas stream pressure of 100 atm, the sticking probability was reduced to approximately 0.1 when the radius of the stream was 1 mm or less. Even in an RF electric field of 30 MV m$^{-1}$, the sticking probability was reduced to approximately 0.2 at a gas stream pressure of 100 atm. Applying the localized gas stream scheme, the sticking probability can be effectively reduced.

There are many technical issues to be solved to make this scheme a reality. One is how to make such a conically diverged shape of the stream whose size is a few millimeters in diameter. A supersonic flow forming a gas stream with a Laval nozzle could be a possible solution. A small Laval nozzle has been used in various research fields such as gas-phase reaction kinetics [30] and cluster formation [31]. The length of the gas stream required should be at least twice the cyclotron resonance radius and become smaller for a higher magnetic field, which improves the technical feasibility. The sticking probabilities were not significantly influenced by the strength of the magnetic field.

As mentioned in Sect. 3, it is also important to develop a high-frequency cavity that creates a circularly polarized electric field (in this case, a left circularly polarized electric field) along the particle motion instead of a linear polarized electric field. As a candidate, it is interesting to combine two dipole modes with a phase difference of 90° in a transverse electric (TE) mode, e.g. TE$_{111}$, cylindrical RF cavity to generate a circularly polarized electric field [32]. In this scheme, the electric field strength is relatively strong at the center of the cavity, so it may be advantageous for realizing high electric field strength.

The background D/T gas pressure required to keep the sticking probabilities at less than 0.2 is approximately 1 atm or less. This makes it unnecessary to maintain strict boundaries between the gas flows and the background, which could simplify the technical design of the gas flow system. Even a gradually diverging configuration of the gas stream might also be adopted. In conclusion, if high-pressure D/T mixture gas flows of millimeters in diameter are technically realized, the number of MuCF cycles could increase five times or more, making scientific or even technical break-even possible.

Acknowledgements
The author would like to express his sincere appreciation to Drs. C. D. P. Levy, A. Taniguchi, T. Uesugi, K. Tsumori, A. Ando, and Y. Ishi for their valuable comments on the manuscript.

Appendix A. Rate equation
In the linearized model of the MuCF process, the number of fusion reactions, $N_f$, can be expressed by the following rate equations including the muon sticking process:

$$\frac{dN_f}{dt} = \lambda_f N_\mu, \quad (A.1)$$
\[
\frac{dN_\mu}{dt} = -\omega N_\mu + \bar{\alpha} N_I - \frac{1}{\tau_\mu} N_\mu, \quad (A.2)
\]
\[
\frac{dN_I}{dt} = \omega N_\mu - \bar{\alpha} N_I - \left( \frac{1}{\tau_I} + \frac{1}{\tau_\mu} \right) N_I. \quad (A.3)
\]

Here, \( N_\mu \) is the number of negative muons and \( N_I \) is the number of \( \mu \)He\(^+\) ions, respectively. The fusion rate is expressed as \( \lambda_f \), which is assumed to be equivalent to the \( \mu \)dt molecule formation rate, and \( \omega \) is the sticking rate of the negative muon to form the \( \mu \)He\(^+\) ion in the \( \mu \)dt fusion reaction. The negative muon lifetime, \( \tau_\mu \), is \( 2.2 \times 10^{-6} \) s, and \( \tau_I \) is the time duration until the kinetic energy of the \( \mu \)He\(^+\) ion becomes zero:

\[
\tau_I = \int_{E_i}^0 \frac{dE}{(dE/dx) \cdot v_I} = \int_{E_i}^{E_f} \frac{dE}{S(E) \cdot v_I(E)}. \quad (A.4)
\]

Here, \( E_i \) is the initial energy of the \( \mu \)He\(^+\) ion, and \( S(E) = -dE/dx \), which is the stopping power of the \( \mu \)He\(^+\) ion at the D\(_2\)/T\(_2\) target. The effective time of the \( \mu \)He\(^+\) ion concerning the stripping process can be given by

\[
\frac{1}{\tau^*} = \frac{1}{\tau_I} + \frac{1}{\tau_\mu}, \quad (A.5)
\]

where \( \tau^* \) becomes equivalent to \( \tau_\mu \) if the degradation of the \( \mu \)He\(^+\) ion is ignored (\( \tau_I = \infty \)).

The muon stripping (detachment) rate from the \( \mu \)He\(^+\) ion is given by

\[
\alpha = n \sigma_s v_I. \quad (A.6)
\]

Here, \( n \) is the D\(_2\)/T\(_2\) target density, \( \sigma_s \) is the cross section of the stripping process, and \( v_I \) is the \( \mu \)He\(^+\) ion velocity. The stripping cross section \( \sigma_s \) is given by Eq. (5), as shown in Fig. A.1, as a function of the \( \mu \)He\(^+\) ion energy.

Because \( \sigma_s \) and \( v_{He} \) depend on the \( \mu \)He\(^+\) ion energy, \( \alpha \) becomes time dependent. Thus, the time-averaged value of the stripping rate, \( \bar{\alpha} \), in the rate equations, is employed to solve the above rate equations as

\[
\bar{\alpha} = \frac{\int_0^\tau n \sigma_s v_I dt}{\tau_I}. \quad (A.7)
\]

The denominator of this equation, \( R \), can be expressed with the stopping power \( S(E) \) of the medium as

\[
R = \int_0^\tau n \sigma_s v_I dt = \int_{E_i}^{E_f} n \sigma_s dE / S(E). \quad (A.8)
\]

Because \( S(E) \) is proportional to the medium density, \( R \) becomes independent of the medium density.

Figure A.2 shows the integrand of Eq. (A8), \( r = n \sigma_s(E)/S(E) \), as a function of the \( \mu \)He\(^+\) ion energy.

The rate equations above result in \( N_I \) as a function of time using the averaged stripping rate \( \bar{\alpha} \):

\[
N_I = \frac{N_{0\mu} \lambda_f}{(a_+ - a_-)} \left[ \frac{\bar{\alpha} + \frac{1}{\tau^*} + a_+}{a_+} (e^{a_+ t} - 1) - \frac{\bar{\alpha} + \frac{1}{\tau^*} + a_-}{a_-} (e^{a_- t} - 1) \right]. \quad (A.9)
\]

Here, \( N_{0\mu} \) is the initial number of negative muons at \( t = 0 \), and \( a_\pm \) are

\[
a_\pm = -B \pm \sqrt{D}, \quad (A.10)
\]
Fig. A.1. The stripping cross section and the stopping power as a function of $\mu$He$^+$ ion energy. The purple line shows the negative muon stripping cross section obtained from Eq. (7), where $\sigma_s(E)$ is estimated from the experimental values [19–21]. The cross section is normalized by $\pi \left[ a_0 \left( m_e/m_\mu \right) \right]^2$, where $a_0$ is the Bohr radius, and $m_e$ and $m_\mu$ are the electron and muon masses, respectively. The blue line shows the energy loss (MeV cm$^{-1}$) at a medium target density of 1 atm.

Fig. A.2. Integrand of Eq. (A8), $r = n \sigma_s(E)/S(E)$, as a function of the $\mu$He$^+$ ion energy.

where

$$B = \frac{\bar{\alpha} + \omega + \frac{1}{\tau_\mu} + \frac{1}{\tau^*}}{2},$$

(A.11)

$$D = B^2 - \frac{\bar{\alpha} \tau^* + \omega \tau_\mu + 1}{\tau_\mu \tau^*}.$$  

(A.12)
The total number of fusion chain reactions induced by a single negative muon, \( N_f^\infty/N_0^\mu \) at \( t = +\infty \), which includes the muon sticking and stripping processes, can be obtained from Eq. (A10), leading to

\[
\frac{N_f^\infty}{N_0^\mu} = \left[ \frac{1}{\lambda_f \tau_\mu} + \frac{\omega}{\lambda_f} \left( \frac{1}{\alpha \tau^* + 1} \right) \right]^{-1}. \tag{A.13}
\]

References

[1] L. W. Alvarez et al., Phys. Rev. 105, 1127 (1957).
[2] S. Eliezer and Z. Henis, Fusion Tech. 26, 46 (1994).
[3] K. Nagamine, Introductory Muon Science (Cambridge University Press, Cambridge, 2007).
[4] T. Kase et al., Muon Catalyzed Fusion 6, 521 (1991).
[5] H. Okuno, H. Sakurai, Y. Mori, R. Fujita, and M. Kawashima, Proc. Jpn. Acad. Ser. B 95, 430 (2019).
[6] S. E. Jones et al., Phys. Rev. Lett 56, 588 (1986).
[7] W. H. Breunlich et al., Phys. Rev. Lett 58, 329 (1987).
[8] S. E. Jones, A. N. Anderson, A. J. Caffrey, J. B. Walter, K. D. Watts, J. N. Bradbury, P. A. M. Gram, M. Leon, H. R. Maltrud, and M. A. Paciotti, Phys. Rev. Lett 51, 1757 (1983).
[9] J. D. Jackson, Phys. Rev. 106, 330 (1957).
[10] M. Kamimura, AIP Conf. Proc. 181, 330 (1988).
[11] H. Bossy et al., Phys. Rev. Lett. 59, 2864 (1987).
[12] K. Nagamine and M. Kamimura, Adv. Nucl. Phys. 24, 151 (1998).
[13] M. Jandel, M. Danos, and J. Rafelski, CERN-TH-4810-87 (1987).
[14] H. Daniel, Hyperfine Interact. 82, 409 (1993).
[15] R. M. Kulsrud; AIP Conf. Proc. 181, 367 (1988).
[16] S. S. Gerstein and L. P. Ponomarev, Phys. Lett. B 72, 80 (1977).
[17] S. I. Vinitsky et al., Sov. Phys. JETP 47, 444 (1979).
[18] N. Kawamura et al., Phys. Rev. Lett. 90, 043401 (2003).
[19] S. K. Allison, Rev. Mod. Phys. 30, 1137 (1958).
[20] L. I. Pivovar, V. M. Thibaev, and M. T. Novikov, Sov. Phys. JETP 14, 20 (1962).
[21] M. B. Shah and H. B. Gilbody, J. Phys. B: Atom. Mol. Phys. 8, 372 (1975).
[22] L. Bracci and G. Fiorentini, Nature 297, 134 (1982).
[23] C. D. Stodden, H. J. Monkhorst, K. Szalewicz and T. G. Winter, Phys. Rev. A 41, 1281 (1990).
[24] H. Tawara and A. Russek, Rev. Mod. Phys. 45, 178 (1973).
[25] D. Bowring et al., Phys. Rev. Accel. Beams 23, 072001 (2020).
[26] P. Hanlet et al., Proc. 10th European Particle Accelerator Conf., p. 1364 (2006).
[27] S. Kumar and P. K. Diwan, J. Rad. Res. App. Sci. 8, 538 (2015).
[28] G. R. Lynch and O. I. Dahl, Nucl. Instr. Meth. B 58, 6 (1991).
[29] W. D. Kilpatrick, Rev. Sci. Instr. 28, 824 (1957).
[30] A. Canosa, A. J. Ocaña, M. Antifio, B. Ballesteros, E. Jiménez, and J. Albaladejo, Exp. Fluids 57, 152 (2016).
[31] O. F. Hagen and W. Obert, J. Chem. Phys 56, 1793 (1972).
[32] M. A. LaPointe et al., Proc. 12th Int. Particle Accelerator Conf., p. 2744 (2021).