Energetic Ion Acceleration by Small-scale Solar Wind Flux Ropes

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Abstract. We consider different limits of our recently developed kinetic transport theory to investigate the potential of supersonic solar wind regions containing several small-scale flux ropes to explain the acceleration of suprathermal ions to power-law spectra as observations show. Particle acceleration is modeled in response to flux-rope activity involving contraction, merging (reconnection), and collisions in the limit where the particle gyroradius is smaller than the characteristic flux-rope scale length. The emphasis is mainly on the statistical variance in the electric fields induced by flux-rope dynamics rather than on the mean electric field induced by multiple flux ropes whose acceleration effects are discussed elsewhere. Our steady-state analytical solutions suggest that ion drift acceleration by flux ropes, irrespective of whether displaying incompressible or compressible behavior, can yield power laws asymptotically at higher energies whereas an exponential spectral rollover results asymptotically when field-aligned guiding center motion acceleration occur by reconnection electric fields from merging flux ropes. This implies that at sufficiently high particle energies, drift acceleration might dominate. We also expect compressive flux ropes to yield harder power-law spectra than incompressible flux ropes.

1. Introduction

Spacecraft observations, magnetohydromagnetic (MHD) turbulence theory and simulations suggest that low-frequency turbulence in the solar wind near Earth consists primarily of a quasi-two-dimensional (quasi-2D) component \cite{1,2,3,4}, at least during quiet solar wind conditions. Three-dimensional (3D) compressible MHD turbulence simulations with a strong guide magnetic field support the idea that MHD turbulence is mainly quasi-2D with a strong presence of closely spaced, quasi-2D (helical) small-scale flux ropes (combination of an axial (guide) field and a magnetic island or twisted magnetic field in the plane perpendicular to the guide field) on turbulence inertial range scales that are dynamic (see Dmitruk et al. \cite{4}). Flux-rope dynamics include merging (reconnection) at the current sheets separating neighboring flux ropes, and flux-rope contraction or expansion as well. In the presence of a guide field reconnection can be understood in terms of component reconnection \cite{5}, where it is the component of the magnetic field transverse to the guide field that is involved in the reconnection process of two neighboring flux ropes.

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Dmitruk et al. [4] also use test particle simulations of particles traversing the electric fields induced by multiple dynamic small-scale flux ropes from the 3D MHD turbulence simulation (with a strong guide field) to show that efficient acceleration and hard power-law distributions result for both ions and electrons. This extended and confirmed earlier work where hard power-law spectra using a 2D MHD turbulence simulation with a guide field [6]. Likewise, MHD simulations of turbulent reconnection at large-scale primary current sheets produce multiple small-scale flux ropes often separated by small-scale current sheets that, through contraction and merging dynamics, can efficiently accelerate test particles to produce power-law spectra [6]. Recent simulations of particle acceleration in merging and contracting small-scale flux ropes of particle-in-the-cell (PIC) codes confirm the formation of power-law spectra [7,8]. This is in contrast to previous PIC modeling that produced exponential spectra [9,10,11], possibly due to computational limitations.

Recent observations show that energetic particle fluxes peak at magnetic field discontinuities including coherent structures separated by current sheets and that the highest energetic particle fluxes occur at the strongest discontinuities most often found behind interplanetary shocks [12]. Observational evidence also exists for small-scale flux rope formation and merging activity at the primary current sheets at leading and trailing edges of interplanetary coronal mass ejection (ICME) structures [10,11], and for energetic particle acceleration in regions of multiple small-scale flux ropes formed near the heliospheric current sheet (HCS) [15]. In summary, a growing body of observational, theoretical and computational evidence suggest that coherent small-scale flux-rope dynamics and dissipation could play an important role in the observed formation of suprathermal ion power-law spectra in the supersonic solar wind. This aspect also needs further investigation in addition to the traditional focus on ion acceleration by interplanetary shocks and random compression and rarefaction regions [16,17,18,19].

Building on previous work [9,10,20], Zank et al. [21] developed a comprehensive kinetic theory framework to model energetic charged particle acceleration by multiple quasi-2D small-scale flux ropes on large spatial scales in the presence of a nonuniform large-scale magnetic field and solar wind flow. By focusing on ion acceleration by the mean electric field induced by multiple flux-rope dynamics of contraction and merging, Zank et al. found power-law spectra with indexes depending on the Alfvénic Mach number and on the ratio of the diffusion over the island contraction time scales. This development was further extended by le Roux et al. [22] using a perturbation analysis augmented with nonlinear effects. This approach enables modeling of the acceleration effects of both the mean and the variance in the electric fields induced by multiple contracting and merging flux ropes in the presence of a strong guide field. In this paper we explore different limits of the extended theory [22] by concentrating mainly on the variance in the flux-rope electric fields to investigate their potential in forming energetic ion power-law spectra in the supersonic solar wind.

2. Solutions of Ion Acceleration by Multiple Contracting and Merging Small-scale Flux Ropes

Our kinetic transport theory was developed using standard guiding center kinetic theory as a basis, resulting in a focused transport equation with Fokker-Planck coefficients for particle diffusion in momentum space due to interaction with several flux ropes. This limits the theory to particle gyroradii \( r_p \ll L_I \), where \( L_I \) is the characteristic scale size of the small-scale flux ropes [22]. Small-scale flux ropes observed near the heliospheric current sheet at Earth, for example, have a characteristic scale size of \( L_I \approx 0.01 \) AU [15]. Given that the gyroradius of a 1 keV proton is \( \sim 5 \times 10^{-6} \) AU at 1 AU, our theory applies to a wide range of suprathermal particle energies. In our theory we can choose between the limit where the particle parallel scattering mean path \( \lambda_\parallel < L_I \) or the opposite limit where \( \lambda_\parallel > L_I \). Here we pursue solutions where energetic ions are diffusing along the flux-rope magnetic field (\( \lambda_\parallel < L_I \)). In this limit the
simplifying assumption of a near-isotropic particle distribution is most plausible, enabling us to reduce the basic focused transport equation we derived to a Parker-like transport equation that can be solved analytically [22]. Thus, the solutions discussed below are perhaps best applicable in the enhanced turbulence conditions that one typically find behind interplanetary shocks.

2.1. Drift Acceleration by Incompressible Contracting and Merging Flux Ropes

In this section we discuss briefly ion drift acceleration by the variance in the electric fields induced by multiple contracting and merging flux ropes exhibiting incompressible behavior [9,10]. The basic idea is that ions are energized during flux-rope contraction or merging because they mainly experience curvature drift in the direction of the induced electric fields generated at the strongly curved magnetic field at endpoints of contracting flux ropes, or in the outflow regions of reconnection sites at the center of merging neighboring flux ropes [10,11]. Energy loss is associated with negative betatron acceleration [23] arising from particles conserving their magnetic moment while the magnetic field strength decreases with time during incompressible contraction or merging of flux ropes [10]. Thus, when particles scatter efficiently, as assumed here, they will undergo net stochastic acceleration from experiencing in a random fashion both the mean induced electric field and weakening magnetic field when traversing several contracting and merging flux ropes. If the mean induced electric field and magnetic field of rate of change is zero, or the particles have a purely isotropic distribution, the net energy gain from incompressible flux rope behavior is zero [9,10,22]. However, even during such conditions, the variance in random statistical fluctuations in the induced electric fields and magnetic field time variations of multiple contracting and merging flux ropes results in stochastic acceleration [20,22]. This is the focus of this section.

We assume a spherically-symmetric heliosphere with a radial solar wind outflow from the Sun where spatially parallel diffusing energetic ions feel the effects of large-scale solar wind advection and adiabatic cooling in the supersonic solar wind, and experience momentum diffusion from drift acceleration by incompressible contracting and merging flux ropes. Simplifying our theory accordingly, we solve analytically a steady-state Parker transport equation for the direction-averaged distribution $f_0(r, p')$ as a function of heliocentric distance $r$ in the fixed frame and particle momentum $p'$ in the solar wind flow frame. The equation is

$$U_0 \frac{\partial f_0}{\partial r} - \frac{1}{r^2} \frac{\partial}{\partial r} \left( r^2 \kappa_{rr} \frac{\partial f_0}{\partial r} \right) - \frac{2}{3} \frac{U_0}{r} p' \frac{\partial f_0}{\partial p'} - \frac{1}{p'^2} \frac{\partial}{\partial p'} \left( p'^2 D_{pp}^{INC} \frac{\partial f_0}{\partial p'} \right) = \frac{\dot{N} \delta(r - r_0) \delta(p - p_0)}{16\pi r_0^2 p_0^2}. \quad (1)$$

In this equation $U_0$ is the solar wind flow speed, $\kappa_{rr} = \kappa_{||}^{eff} \cos^2 \psi$ is the radial diffusion coefficient as a result of projecting the the effective parallel diffusion coefficient $\kappa_{||}^{eff}$ in the radial direction ($\psi$ is the solar wind magnetic field spiral angle), $D_{pp}^{INC}$ is the momentum diffusion coefficient for modeling energetic ion acceleration in response to the variance in the electric fields induced by multiple incompressible contracting and merging flux ropes, and $\dot{N}$ is the constant rate of source particle production at a fixed heliocentric distance $r_0$ and momentum $p_0$. In the assumed limit of efficient particle scattering and a strong guide field, parallel diffusion was modeled by extending the usual quasi-linear theory perturbation analysis for undisturbed guiding center motion to include diffusive distortion of particle guiding center trajectories [24]. Effectively, diffusion was modeled as a combination of particle gyro-resonant scattering by parallel propagating Alfvén waves and of scattering in response to the variance encountered in the multi-island end-point magnetic fields that reflect/transmit particles because of the magnetic mirroring force. This generates a nonlinear expression for effective parallel diffusion that needs to be solved. A simplified solution for parallel diffusion was found upon realizing that on large spatial scales scattering by flux ropes can be more efficient than scattering by Alfvén waves in solar wind.
conditions near 1 AU. The expression for parallel diffusion is

$$\kappa_{\parallel}^{\text{eff}} \approx V_A \left( \frac{25}{9\pi} \frac{L_{\text{INC}}}{\langle (\delta B_{\text{INC}}^2)/B_0^2 \rangle} \right),$$

(2)

where $V_A$ is the Alfvén speed, $L_{\text{INC}}$ is the characteristic length of incompressible flux ropes, and $\langle (\delta B_{\text{INC}}^2)/B_0^2 \rangle$ is the mean fluctuation energy density of the incompressible multi-flux-rope magnetic field normalized to the energy density of the mean magnetic field. Note that parallel diffusion due to random magnetic mirroring and transmission by flux ropes is independent of particle speed. The momentum diffusion coefficient is

$$D_{pp}^{\text{INC}} = p^2 \frac{3\pi}{50} \frac{V_A}{L_{\text{INC}}} \left( \frac{\langle (\delta B_{\text{INC}}^2)/B_0^2 \rangle}{B_0^2} \right) \left( \frac{\langle B_{\text{INC}} \rangle}{B_0} \right)^4,$$

(3)

where $\langle B_{\text{INC}} \rangle$ is the mean multi-flux-rope magnetic field strength. Upon conveniently introducing the radial dependencies $\kappa_{\parallel} = V_{\kappa_0}r$ and $D_{pp}^{\text{INC}} = p^2D_0r^{-1}$, whereby both $V_{\kappa_0}$ (a speed determining the rate of diffusive transport) and $D_0$ (a speed determining the rate of second order Fermi acceleration) are constants proportional to the Alfvén speed, we find a solution to equation (1) given by

$$f_0(r,p') = \frac{N}{32\pi^3 r_0^2 p_0^2 V_{\kappa_0}} \left( \frac{V_{\kappa_0}}{D_0} \right) \left( \frac{r}{r_0} \right)^{(1-U_0/2V_{\kappa_0})} \left( \frac{p'}{p_0} \right)^{-(3+2U_0/3D_0)/2} \times$$

$$\times K_0 \left( b \sqrt{\ln^2(r/r_0) + (V_{\kappa_0}/D_0) \ln^2(p'/p_0)} \right),$$

(4)

where $b$ is determined by the expression

$$b^2 = \left( 1 - \frac{U_0}{2V_{\kappa_0}} \right)^2 + \frac{1}{4} \frac{D_0}{V_{\kappa_0}} \left( 3 + \frac{2U_0}{3D_0} \right)^2.$$

(5)

and $K_0$ is the modified Bessel function of the second kind (a decaying function with increasing argument size).

The accelerated part of the particle spectra ($p'/p_0 > 1$) converge to power laws with similar power-law indexes for $p' >> p_0$ over a wide range of radial distances at and beyond the particle source position. The power law index can be estimated by assuming $p' \gg p_0$ in the argument of the $K_0$ Bessel function in equation (4), which leads asymptotically to a power-law particle distribution given by

$$f_0(p') \propto \left( \frac{p'}{p_0} \right)^{-c(1+\sqrt{1+(V_{\kappa_0}/D_0)(d/c)^2})},$$

(6)

$$c = \frac{1}{2} \left( 3 + \frac{2U_0}{3D_0} \right),$$

$$d = 1 - \frac{U_0}{2V_{\kappa_0}}.$$
stochastic acceleration, and when diffusive particle escape from flux ropes reduces the particle trapping (acceleration) time in individual flux ropes. Advection, on the other hand, tends to harden the spectrum because it effectively reduces diffusion, and particle escape from flux ropes is less efficient. Consequently, the trapping and acceleration time in individual flux ropes increase and the spectrum hardens.

Consider the limit when spatial advection of ions by the solar wind is approximately in balance with spatial diffusion according to the limit $U_0/V_0 \rightarrow 2$ (similar to the diffusion-advection limit in standard cosmic-ray modulation theory). Then, diffusive escape of energetic ions from flux ropes is relatively inefficient which limits the steepening of the spectrum. Accordingly,

$$f_0(p') \propto \left(\frac{p'}{p_0}\right)^{-\left(3+\frac{2}{3}U_0D_0\right)},$$

so that the power-law index of energetic ions in the supersonic solar wind is determined by a competition between stochastic acceleration by flux ropes and adiabatic cooling in the expanding solar wind. If in addition, stochastic acceleration of energetic ions by flux ropes is weakly dominated by adiabatic cooling, $(2U_0/3D_0 \rightarrow 2)$, the ion spectrum becomes a $f_0(p') \propto p'^{-5}$ as is often observed in the solar wind [24]. Such an outcome could be a consequence of a quasi-steady state that forms if one extend our test particle solution to take into account self-consistent interaction between energetic ions and flux ropes [18,25].

In le Roux et al. [22] we also showed that when realistic parameters are incorporated in the full solution of equation (4), acceleration in quiet solar wind conditions near 1 AU is inefficient. Efficient acceleration to hard power-law spectra was found in strong flux-rope conditions expected behind interplanetary shocks. This raises the interesting question of how flux rope acceleration downstream near interplanetary shocks might modify diffusive shock acceleration. Our preliminary results suggest that flux rope acceleration behind shocks can easily result in hardening of the diffusive shock acceleration spectrum. For a discussion of this issue, see Zank et al. [26].

2.2. Field-Aligned Guiding Center Motion Acceleration by Merging Flux Rope Reconnection Electric Fields

In the case of merging neighboring flux ropes with a strong guide field, field-aligned guiding center motion through the reconnection zone occurs mostly parallel or anti-parallel to the direction of the reconnection electric field pointing perpendicular to the magnetic island (twist) part of the flux rope. Therefore, energy gain or energy loss can occur. In the case of multiple merging flux ropes with a finite mean reconnection electric field, energetic ions undergoing efficient scattering will experience 2nd order Fermi acceleration with a net energy gain as long as the particle distribution is not purely isotropic. This case was discussed by Zank et al. [21]. Instead, we focus on how energetic ions undergo stochastic acceleration in response to the statistical variance in the reconnection electric field they encounter when traversing multiple merging flux ropes in the supersonic solar wind, an effect that produces a net energy gain irrespective of whether the particle distribution is isotropic or not [22].

In this limit of our theory, we solve the following steady-state Parker-type transport equation for energetic ions given by

$$U_0 \frac{\partial f_0}{\partial x} - \kappa_x \frac{\partial^2 f_0}{\partial x^2} - \frac{1}{p'^2} \frac{\partial}{\partial p'} \left( p'^2 D_{REC} \frac{\partial f_0}{\partial p'} \right) = \frac{n_0 U_0 \delta(x) \delta(p - p_0)}{4\pi^2 p_0^3}.$$  

(8)

In equation (8) it is assumed for simplicity that energetic ions propagate in a one-dimensional heliosphere (Cartesian geometry) where the solar wind flow occurs in the $x$-direction, and $x$
is distance relative to the source particles with a momentum \( p_0 \) which are injected into the heliosphere at an arbitrary distance labeled \( x = 0 \) at a fixed rate determined by the flux \( n_0 U_0 \) (\( n_0 \) is the number density of the source particles advected with the solar wind flow at a velocity \( U_0 \)). A consequence of assuming planar geometry is the neglect of adiabatic cooling of energetic ions. This is reasonable for energetic ions far from the Sun where adiabatic cooling is weak, or in places where stochastic acceleration is relatively efficient, such as in regions of enhanced turbulence behind heliospheric shocks. The spatial diffusion coefficient in the \( x \)-direction is \( \kappa_{xx} = \kappa_{xx}^{REC} \cos^2 \psi \) where the expression for \( \kappa_{xx}^{REC} \) is the same as in equation (2). Therefore, \( \kappa_{xx} \) is independent of particle momentum. The expression for the momentum diffusion coefficient \( D_{pp}^{REC} \) models the effect of the statistical variance in the reconnection electric fields associated with multiple merging flux rope pairs on ion acceleration. Its expression is

\[
D_{pp}^{REC} = p'^2 \frac{\pi V_A}{10 L_M} \left( \frac{L_M}{r_g} \right)^2 \left( \frac{\langle \delta B_{M}^2 \rangle}{B_0^2} \right)^2 ,
\]

(9)

where \( L_M \) is the characteristic scale size of merging flux ropes, \( r_g \) is the maximum particle gyroradius (pitch angle is 90°), and \( \langle \delta B_{M}^2 \rangle / B_0^2 \) is the mean fluctuation energy density of multiple merging flux-rope magnetic fields normalized to the energy density of the mean magnetic field. Assuming non-relativistic particles, inspection of \( D_{pp}^{REC} \) reveals that it is approximately independent of \( p' \). This forms a strong contrast with the previous case discussed in Section 2.1 where \( D_{pp}^{INC} \propto p'^2 \). For simplicity, we assume that both \( \kappa_{xx} \) and \( D_{pp}^{REC} \) are independent of distance \( x \). Thus, we specify \( D_{pp}^{REC} = D_0 \), where \( D_0 \) is a constant in units of momentum\(^2\)/time instead of speed as in the previous solution. We also set \( \kappa_{xx} = \kappa_0 \) where \( \kappa_0 \) is a constant in units of speed times length instead of speed as before. We find an analytic solution for the particle distribution which is

\[
f_0(x, p') = \frac{\dot{N}}{8\pi^2 p_0^2 p' D_0} \left( \frac{D_0}{\kappa_0} \right)^{1/2} \left( \frac{p_0}{p} \right) \times 
\left[ K_0 \left( \alpha \sqrt{(p' - p_0)^2 + (D_0/\kappa_0)^2} \right) - K_0 \left( \alpha \sqrt{(p' + p_0)^2 + (D_0/\kappa_0)^2} \right) \right].
\]

(10)

where \( \alpha = (U_0/2\kappa_0)\sqrt{\kappa_0/D_0} \). For \( p' \gg p_0 \), the accelerated ion spectrum asymptotically becomes

\[
f_0(p') \propto \frac{\epsilon(p')}{\sqrt{\alpha p'(1 - \epsilon(p'))}} \left[ 1 - \sqrt{\frac{1 - \epsilon(p')}{{\epsilon(p')}^2 + 1 + \epsilon(p') e^{-2\alpha p'}}} \right] e^{-\alpha p'(1 - \epsilon(p'))},
\]

(11)

where \( \epsilon(p') = p_0/p' \ll 1 \). Note that when \( \epsilon(p') \rightarrow 0 \) (\( p'/p_0 \rightarrow \infty \)), \( f_0(p') \rightarrow 0 \). Thus, the accelerated ion spectrum rolls over exponentially with increasing momentum and a power law is not found. This can be understood by comparing the transport terms in equation (8) using dimensional analysis. The left-hand-side of the equation then becomes \( f_0(p')[1/T_C + 1/T_D + D_0/p'^2] \), where \( T_C \) is the spatial advection time scale, \( T_D \) is the spatial diffusion time scale, and \( D_0 \) is the momentum diffusion coefficient, all independent of particle momentum. This implies that at higher energies the efficiency of spatial transport of ions across flux ropes is maintained. In comparison, transport in momentum space associated with the variance in the reconnection electric fields becomes less efficient at higher energies. In contrast, dimensional analysis of drift acceleration of energetic ions by multiple incompressible flux ropes in Section 2.1 shows that momentum diffusion maintains its efficiency relative to spatial transport at all energies, thus allowing a power-law spectrum to form.
2.3. Drift Acceleration by Compressible Contracting and Colliding Flux Ropes

Here we investigate the possibility that flux ropes can at times also exhibit compressive behavior, thus affecting energetic ion acceleration in a different way. When new elongated magnetic islands form in the plane perpendicular to the guide field and grow initially due to magnetic reconnection as a result of neighboring x-points forming on a current sheet, this process is accompanied by a rapid increase in the density and magnetic field strength as the x-point outflows advect magnetic field lines, causing them to pile-up toward the center of the evolving magnetic island. This suggests that the initial island formation and growth phase is compressible [27,28]. Accordingly, the area surrounded by an individual island magnetic field-line loop decreases while at the same time the path length between the loop endpoints shortens as the loop end points contract [21]. Similar compressible island behavior can be expected when neighboring magnetic islands collide after being attracted to each other. After collision the magnetic islands can either rebound or merge. In addition, Particle-in-cell simulations show that after neighboring islands merge, the newly formed island structures oscillate with alternating expansion and contraction motions [8]. During such pulsations, it is possible magnetic islands are compressive for at least part of the time.

During the early compressive phase of flux-rope contraction, one can think of particle energization occurring due to curvature drift in the direction of the induced electric fields generated at endpoints of contracting flux ropes, as was discussed above for later phase incompressible contraction. However, in the compressive phase particle, positive betatron acceleration also contributes to energization as a consequence of the increase in the flux-rope field strength with time [21]. This is in contrast to later phase incompressible contraction where negative betatron acceleration produce energy loss because the field strength decreases with time [9]. The compressive phase is captured in our Parker-like transport equation in terms of the compression of the flux-rope plasma flow ($\nabla \cdot U_{\text{COM}} < 0$). Therefore, when energetic ions traverse multiple compressive flux ropes with a net compression (rarefactions can also occur as discussed above), energetic ions will experience a first order Fermi acceleration effect.

A key difference between incompressible and compressible flux ropes is that the mean contraction effect of multiple incompressible flux ropes only results in a net energy gain for efficiently scattering energetic ions if the particle distribution is not completely isotropic. Thus, a successful 2nd order Fermi acceleration process involving a net mean incompressible contraction effect from several flux ropes is tied to the anisotropic part of the energetic ion distribution. In the case of compressible flux ropes, the mean contraction (compression) from multiple flux ropes is associated with 1st order Fermi acceleration for efficiently scattering ions forming a completely isotropic distribution. When efficiently scattering ions form a nearly, but not completely isotropic distribution, both 1st order Fermi acceleration (associated with the isotropic part of the distribution) and 2nd order Fermi acceleration (associated with the anisotropic part of the distribution) occur in reponse to the net mean compressive contraction of several flux ropes. In addition, 2nd order Fermi acceleration also occurs when energetic ions experience the variance from statistical variations in the divergence of the plasma flow in multiple compressive flux ropes (random compressions and decompressions relative to the assumed mean compression of many flux ropes). This effect occurs irrespective of the anisotropy of the particle distribution.

Below, we investigate the effect of both 1st order Fermi acceleration associated with the mean compression, and 2nd order Fermi acceleration by the variance in the compression of multiple compressible flux ropes on energetic ion acceleration. The 2nd order Fermi acceleration effect from the mean compression associated with the anisotropic part of the ion distribution is ignored because it decays much more strongly with increasing momentum compared to 2nd order Fermi acceleration from the variance in the compression of multiple flux ropes. We solve analytically the steady-state Parker transport equation in spherical geometry for a radial solar wind similar...
to equation (1). The equation is as follows:

\[ U_0 \frac{\partial f_0}{\partial r} - \frac{1}{r^2} \frac{\partial}{\partial r} \left( r^2 \kappa_{rr} \frac{\partial f_0}{\partial r} \right) + \frac{1}{3} U_{eff}^2 \frac{\partial}{\partial p} \left( p^2 D_{COM} \frac{\partial f_0}{\partial p} \right) - \frac{1}{p^2} \frac{\partial}{\partial p} \left( p^2 D_{pp} \frac{\partial f_0}{\partial p} \right) = \frac{\dot{N} \delta (r - r_0) \delta (p - p_0)}{16\pi^2 \gamma_{COM}^4 G p_0^6}, \]

where \( \kappa_{rr} = \kappa_{eff} \cos^2 \psi \), \( U_{eff} = U_{COM} - 2U_0 \), and \( U_{COM} \approx V_A/L_{COM,0} \). The term containing \( U_{eff} \) in equation (12) models the competition between particle energization when particles experience a mean compression effect from multiple flux ropes and energy loss when particles undergo adiabatic cooling in the radially expanding solar wind flow. The expression for \( U_{COM} \) indicates the mean plasma flow velocity associated with several flux ropes undergoing compression, and \( L_{COM,0} \) is the characteristic scale size of compressible flux ropes normalized at the energetic ion source distance \( r = r_0 \) to which it is normalized. In equation (12) it has been assumed that \( L_{COM} \propto r \) to model approximately the observed increase in the size of small-scale flux ropes in the solar wind [15,29]. In equation (12), \( D_{pp}^{COM} \) is the momentum diffusion coefficient of energetic ions experiencing the effect of the variance in the divergence of the plasma flow when encountering multiple compressible flux ropes. The expression for \( D_{pp}^{COM} \) is

\[ D_{pp}^{COM} = p^2 \frac{\pi}{25} \frac{V_A}{L_{COM}} \left( \frac{\delta B_{COM}^2}{B_0^2} \right)^2. \]

The expression for \( \kappa_{eff} \) is the same as in equation (2), except that here we focus on the magnetic field statistics of compressible flux ropes. As in equation (1), for convenience we introduce the radial dependencies \( \kappa_{rr} = V_{n0} r \) and \( D_{pp}^{INC} = p^2 D_0 r^{-1} \), where both \( V_{n0} \) (a speed determining the rate of diffusive transport) and \( D_0 \) (a speed determining the rate of second order Fermi acceleration) are constants proportional to the Alfvén speed.

First, we present the solution of equation (12) when \( D_{pp}^{COM} = 0 \) (1st order Fermi acceleration dominates 2nd Fermi acceleration). The solution is

\[ f_0(x,p') = \frac{\dot{N}}{32\pi^2 \gamma_{COM}^4 V_{n0} \sqrt{\pi} b} \frac{1}{r_0^a} \frac{e^{-\frac{\ln^2(r/r_0)}{4b} + \frac{\ln(p/p_0)}{b}}}{\sqrt{\ln(p/p_0)}^{1/b}} \left( \frac{p}{p_0} \right)^{-a^2/b}, \]

where \( a = 1 - U_0/2V_{n0} \), and \( b = U_{eff}/3V_{n0} = (U_{COM} - 2U_0)/3V_{n0} \). By taking the limit \( p'/p_0 \gg 1 \), we find that the accelerated spectrum asymptotically becomes approximately a power law

\[ f_0(p') \propto \left( \frac{p'}{p_0} \right)^{-1 - U_0/2V_{n0}} \frac{3V_{n0}}{U_{eff}}, \]

weakly modified by a logarithmic function of \( p' \) (not shown). Inspection of the power-law index expression shows qualitatively similar effects of transport processes on the spectral slope of the accelerated ion spectrum as discussed for the case of drift acceleration by incompressible flux ropes: (i) Increasing \( V_{n0} \) steepens the spectrum because particle escape from flux ropes is more efficient, thus reducing the contact time with flux rope compressions. (ii) The presence of solar wind advection reduces the net effect of spatial diffusion, thus reducing the steepening effect from diffusive escape on the spectrum because particles have a longer interaction time with compressive flux ropes. (iii) Increasing the compression speed \( U_{COM} \) hardens the spectrum because this raises the rate of particle energization. (iv) Adiabatic cooling by the expanding solar wind flow reduces efficiency of particle energization by the mean compression of multiple compressive flux ropes, thus causing steepening of the power-law spectrum. Upon again invoking
the diffusion-advection limit \( U_0/V_\kappa \to 2 \) (diffusive escape of energetic ions from flux ropes is finite but inefficient), we find that the accelerated spectrum can be quite hard because then approximately, \( f_0(p') \propto (p'/p_0)^\epsilon \) where \( \epsilon \to 0 \). If alternatively, \( U_{\text{eff}} \approx 0 \), energization by the mean compression from multiple flux ropes is balanced by adiabatic cooling, and the spectrum is very steep, as expected.

The complete solution of equation (12), which is mathematically similar to the solution of equation (1) for incompressible flux ropes, applies best when 2nd order Fermi acceleration by compressive flux ropes is stronger than 1st order Fermi acceleration, because it does not yield the correct solution in the opposite limit (thus the separate solution presented above when the mean compression effect is dominant). Just as in the incompressible flux-rope case, the asymptotic spectrum for \( p'/p_0 \gg 1 \) yields a power-law spectrum, but the power-law index is modified by the mean compression of multiple flux ropes, and 2nd order Fermi acceleration is now determined by compressible flux ropes. The expression for the power-law spectrum for \( p'/p_0 \gg 1 \) is

\[
    f_0(p') \propto \left( \frac{p'}{p_0} \right)^{-c(1 + \sqrt{1 + (V_\kappa D_0/d/c)^2})},
\]

\[
    c = \frac{1}{2} \left( 3 - \frac{1}{3} \frac{U_{\text{eff}}}{D_0} \right),
\]

\[
    d = 1 - \frac{U_0}{2V_\kappa}.
\]

The main differences between the asymptotic energetic ion spectrum for incompressible flux ropes as given by equation (6) and the spectrum compressible flux ropes are that (i) the expression for the stochastic acceleration coefficient \( D_0 \) is different (compare the expressions for \( D_{\text{INC}} \) (equation (3)) with \( D_{\text{COM}} \) (equation (13)), (ii) the constant \( c \) in equation (16) includes \( U_{\text{eff}} = U_{\text{COM}} - 2U_0 \) indicating a three-way competition between particle energization by the mean compression of multiple flux ropes (\( U_{\text{COM}} \)), adiabatic cooling by the radially expanding solar wind flow (\( U_0 \)), and energization by the variance in the compression of multiple flux ropes \( D_0 \), whereas \( c \) in equation (3) includes a two-way competition between adiabatic cooling effect and energization by the variance in the contraction of incompressible flux ropes.

Assuming the diffusion-advection limit \( U_0/V_\kappa \to 2 \),

\[
    f_0(p') \propto \left( \frac{p'}{p_0} \right)^{-3 - \frac{1}{3} \frac{U_{\text{COM}} - 2U_0}{D_0}}.
\]

If 2nd order Fermi acceleration is the dominant mechanism for energy change (\( D_0 \gg U_{\text{COM}} \) and \( D_0 \gg U_0 \)), we get \( f_0(p') \propto p'^{-3} \), just as we found when 2nd order Fermi acceleration dominated in the case of drift acceleration by incompressible flux ropes. When 1st order Fermi acceleration by the mean compression of multiple flux ropes is significant and dominates adiabatic cooling \( U_{\text{COM}} \gg U_0 \), we see that the accelerated spectrum will develop a power-law index with a value \( < 3 \). Thus, compressible flux ropes have the potential to produce a harder power-law spectrum for accelerated ions than incompressible flux ropes because of the additional contribution of 1st order Fermi acceleration by the mean compression. This situation may occur in the region between an ICME and the HCS, because the expanding ICME pushes against the HCS and compresses the confined magnetic islands near the HCS, which often occur in the vicinity of the HCS. An example of the corresponding energetic particle flux enhancement observed between the HCS and the ICME is shown in [12]. Note that these results apply only for energetic test particles. It remains to be investigated whether this difference in the slope of energetic ions for incompressible and compressible flux ropes will survive when acceleration is modeled self-consistently.
3. Summary

There is growing observational evidence that dynamic, small-scale flux rope structures have a significant presence in the supersonic solar wind, especially near primary current sheet structures such as the heliospheric current sheet, and primary current sheets at the leading and trailing edges of the CME structures behind interplanetary shocks, for example [13,14,15]. Observations also increasingly indicate that dissipation of these structures results in the acceleration of suprathermal particles [9,12]. In response we investigated theoretically the possibility that several small-scale flux ropes experiencing contraction, merging, and collisions can accelerate energetic ions to form power-law spectra as simulations suggest [4,11]. This was accomplished by analyzing three distinct limits in the general kinetic transport theory framework that we developed previously [21,22] for energetic ion acceleration by many small-scale flux ropes on inertial scales in the presence of a strong guide magnetic field. The main emphasis was on how the variance in the statistics of flux-rope electric fields induced by flux-rope dynamics affects ion acceleration, because the effect of the mean electric field generated by multiple flux ropes were already discussed before [21]. The three limits considered were (i) energetic ion drift acceleration as a 2nd order Fermi process because of the variance in the induced electric fields from contracting and merging flux-ropes that exhibit incompressible behavior, (ii) energetic ion field-aligned guiding center motion acceleration as a 2nd order Fermi mechanism in response to the variance in the reconnection electric fields generated when neighboring flux ropes merge (reconnect) and, (iii) energetic ion drift acceleration as a 1st order Fermi process due to the mean compression and as a 2nd order Fermi process because of the variance in the compression of several flux ropes exhibiting compressive behavior (e.g., early-time flux-rope contraction or collisions between flux ropes).

Our steady-state analytic solutions, valid for the supersonic solar wind, suggest that both cases of drift acceleration (case (i) for incompressible flux ropes and case (iii) for compressible flux ropes) can result in accelerated ion spectra that form power laws asymptotically for \( p' \gg p_0 \). We found asymptotically an exponential rollover for field-aligned guiding center motion acceleration by reconnection electric fields from merging flux ropes (case (ii)). This implies that whereas field-aligned guiding center motion acceleration by reconnection electric fields might be strong at lower energies, at higher energies drift acceleration by incompressible or compressible flux ropes will dominate. This is to be expected because in our theory \( D_{pp} \propto p^2 \) for the two drift acceleration cases, whereas \( D_{pp} \) is approximately independent of particle momentum for the case of field-aligned guiding center motion acceleration. Assuming that compressible flux ropes are just as probable as incompressible flux ropes, the additional 1st order Fermi acceleration effect from the mean compression in the case of compressible flux ropes is expected to yield a harder power-law spectrum compared to the case of incompressible flux ropes where such a 1st order Fermi acceleration effect is absent.

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