SU(2) Chiral Sigma Model and Properties of Neutron Stars

Pradip Kumar SAHU* and Akira OHNISHI

Division of Physics, Graduate School of Science
Hokkaido University, Sapporo 060-0810, Japan

(Received July 12, 2000)

We discuss the SU(2) chiral sigma model in the context of nuclear matter using a mean field approach at high density. In this model we include a dynamically generated isoscalar vector field and higher-order terms in the scalar field. With the inclusion of these, we reproduce the empirical values of the nuclear matter saturation density, binding energy, and nuclear incompressibility. The value of the incompressibility is chosen according to recently obtained heavy-ion collision data. We then apply the same dynamical model to neutron-rich matter in beta equilibrium, related to neutron star structure. The maximum mass and corresponding radius of stable non-rotating neutron stars are found to be in the observational limit.

§1. Introduction

It has been argued that chiral symmetry is a good hadron symmetry\(^1\) which ranks only below isotopic spin symmetry. The spontaneous breaking of this symmetry generates the constituent quark masses and hence various hadron masses, including the nucleon mass. Therefore, the theories of dense nuclear matter, where dynamical modifications of hadron masses are expected should possess this symmetry. In recent decades, three-body forces in the equation of state at high density have been studied by several authors.\(^2,3\) These studies also support the importance of studying the chiral sigma model, because the non-linear terms in the chiral sigma Lagrangian can give rise to three-body forces. Theories of dense nuclear matter should be capable of describing the bulk properties of nuclear matter, such as the binding energy per nucleon, saturation density, compression modulus/incompressibility and symmetry energy. Presently there is no theory of dense nuclear matter that describes all the nuclear matter properties and possesses chiral symmetry.

A chiral Lagrangian using a scalar (sigma) field was originally introduced by Gell-Mann and Levy,\(^4\) and later the importance of chiral symmetry in nuclear matter was emphasized by Lee and Wick.\(^5\) The usual theory of pions does not possess the empirically desirable saturation properties for nuclear matter. For this reason, an isoscalar vector field with a dynamically generated mass was introduced into the theory of nuclear matter and it enabled us to have a saturation density in nuclear matter equation of state.\(^6\) In the standard sigma model, the value of the incompressibility parameter of nuclear matter turns out to be quite large — several times the desirable value — and can be reduced only by introducing the scalar field self-interactions with adjustable coefficients. Several papers\(^7,8\) have attempted to

---

\(^*\) JSPS fellow.
Pradip Kumar Sahu and Akira Ohnishi derive the chiral sigma model equation of state at high matter density with normal nuclear matter saturation as well as a desirable incompressibility value. However, in these theories, the mass of the isoscalar vector field is not generated dynamically. This fact can be considered a shortcoming of the chiral symmetry model.

A few years ago, we proposed the SU(2) chiral sigma model to describe the properties of nuclear matter. In that work we adopted an approach in which the mass of the isoscalar vector field is generated dynamically. To ensure the saturation properties of nuclear matter, inclusion of such a vector field is necessary. The nucleon effective mass thus acquires a self-consistent density dependence both on the scalar and the vector meson fields, and these meson fields are treated in the mean-field theory. To describe the nuclear matter properties, we have two parameters in the theory: the ratio of the coupling constants to the scalar and to the isoscalar vector fields. In this approach the value of incompressibility at saturation density is relatively large, which is an undesirable feature, as far as nuclear matter at saturation and higher densities is concerned. In the present calculation, we rectify the above mentioned shortcoming at the nuclear matter saturation density by including higher-order terms of the scalar field potential in our proposed chiral sigma Lagrangian.

In this way, we get two extra parameters in the mean-field approach. These are fitted with the nuclear matter properties at the saturation density.

The paper is organized as follows: In §2 we briefly describe the proposed SU(2) chiral sigma model and the derivation of the equation of state for the nuclear matter density. Section 3 contains the neutron star matter equation of state in beta equilibrium with the neutron star results. A conclusion and summary are presented in §4.

§2. The SU(2) chiral sigma model and equation of state

The Lagrangian for the SU(2) chiral sigma model can be written as (we choose units in which $\hbar = c = 1$)

$$
\mathcal{L} = \frac{1}{2} \left( \partial_\mu \pi \cdot \partial^\mu \pi + \partial_\mu \sigma \partial^\mu \sigma \right) - \frac{1}{4} F_{\mu\nu} F^{\mu\nu} - \frac{\lambda}{4} \left( x^2 - x^2_0 \right)^2 - \frac{\lambda B}{6m^2} \left( x^2 - x^2_0 \right)^3 - \frac{\lambda C}{8m^4} \left( x^2 - x^2_0 \right)^4 - g_\sigma \bar{\psi} \left( \sigma + i \gamma_5 \tau \cdot \pi \right) \psi + \bar{\psi} \left( i \gamma_\mu \partial^\mu - g_\omega \gamma_\mu \omega^\mu \right) \psi + \frac{1}{2} g_\omega x^2 \omega_\mu \omega^\mu + \frac{1}{24} \xi g_\omega^4 \left( \omega_\mu \omega^\mu \right)^2 - D\sigma,
$$

where $F_{\mu\nu} \equiv \partial_\mu \omega_\nu - \partial_\nu \omega_\mu$ and $x^2 = \pi^2 + \sigma^2$, $\psi$ is the nucleon isospin doublet, $\pi$ is the pseudoscalar-isovector pion field, $\sigma$ is the scalar field, and $D$ is a constant. The Lagrangian includes a dynamically generated isoscalar vector field, $\omega_\mu$, that couples to the conserved baryonic current $j_\mu = \bar{\psi} \gamma_\mu \psi$. $B$ and $C$ are constant parameters included the higher-order self-interaction of the scalar field in the potential. In the fourth order term in the omega fields, the quantity $\xi$ is a constant parameter. For simplicity, we set $\xi$ to zero in our calculation.
The interactions of the scalar and the pseudoscalar mesons with the vector meson generate a mass of the latter through the spontaneous breaking of the chiral symmetry. The masses of the nucleon, the scalar meson and the vector meson are respectively given by

\[ M = g_\sigma x_\omega, \quad m_\sigma = \sqrt{2\lambda x_\omega}, \quad m_\omega = g_\omega x_\omega, \] (2)

where \( x_\omega \) is the vacuum expectation value of the \( \sigma \) field, \( \lambda = (m_\sigma^2 - m_\pi^2)/(2f_\pi^2) \), with \( m_\pi \) the pion mass and \( f_\pi \) the pion decay coupling constant, and \( g_\omega \) and \( g_\sigma \) are the coupling constants for the vector and scalar fields, respectively. Throughout this paper, we limit ourselves to a mean field treatment and ignore the explicit role of \( \pi \) mesons.

The equation of motion of fields is obtained by adopting the mean-field approximation. This approach has been used extensively to obtain field theoretical equation of state models for high density matter. Using the mean-field ansatz, the equation of motion for the isoscalar vector field is

\[ \omega_0 = \frac{n_B}{g_\omega x_\omega^2}, \] (3)

and the equation of motion for the scalar field in terms of \( y \equiv x/x_\omega \) is

\[
(1 - y^2) - \frac{B}{m^2 c_\omega}(1 - y^2)^2 + \frac{C}{m^4 c_\omega^2}(1 - y^2)^3 \\
+ \frac{2c_\sigma c_\omega n_B^2}{M^2 y^4} - \frac{c_\sigma \gamma}{\pi^2} \int_0^{k_F} \frac{k^2 dk}{\sqrt{k^2 + M^*}} = 0, \tag{4}
\]

where \( M^* \equiv yM \) is the effective mass of the nucleon and

\[ c_\sigma \equiv g_\sigma^2/m_\sigma^2, \quad c_\omega \equiv g_\omega^2/m_\omega^2. \] (5)

The quantity \( n_B \) is the baryon density, or equivalently, \( n_B = \frac{\gamma}{(2\pi)^3} \int_0^{k_F} d^3k \), where \( k_F \) is the Fermi momentum and \( \gamma \) is the nucleon spin degeneracy factor.

The equation of state is calculated from the diagonal components of the conserved total stress tensor corresponding to the Lagrangian together with the mean-field equation of motion for the fermion field and a mean-field approximation for the meson fields. The total energy density, \( \varepsilon \), and pressure, \( P \), of the many-nucleon system are the following:

\[
\varepsilon = \frac{M^2(1 - y^2)^2}{8c_\sigma} - \frac{B}{12c_\sigma c_\omega}(1 - y^2)^3 + \frac{C}{16m^2 c_\omega^2 c_\sigma}(1 - y^2)^4 \\
+ \frac{c_\omega n_B^2}{2y^2} + \frac{\gamma}{\pi^2} \int_0^{k_F} \frac{k^2 dk}{\sqrt{k^2 + M^*}},
\]

\[
P = -\frac{M^2(1 - y^2)^2}{8c_\sigma} + \frac{B}{12c_\sigma c_\omega}(1 - y^2)^3 - \frac{C}{16m^2 c_\omega^2 c_\sigma}(1 - y^2)^4 \\
+ \frac{c_\omega n_B^2}{2y^2} + \frac{\gamma}{6\pi^2} \int_0^{k_F} \frac{k^2 dk}{\sqrt{k^2 + M^*}}. \tag{6}
\]
The energy per nucleon is \( E/A = \epsilon/n_B \), where \( \gamma = 4 \) for symmetric nuclear matter.

In the above equations, we have four parameters: the nucleon coupling to the scalar and the vector fields, \( c_\sigma \) and \( c_\omega \), and the coefficients in the scalar potential terms, \( B \) and \( C \). These are obtained by fitting the saturation values of binding energy/nucleon (~16.3 MeV), baryon density (0.153 fm\(^{-3}\)), and effective (Landau) mass (0.85 MeV).\(^{10}\) The nuclear incompressibility is somewhat uncertain at saturation and therefore we choose it between 250 MeV and 350 MeV, i.e. \( \sim 300 \) MeV, in accordance with recent heavy-ion collision data.\(^{11,12}\) These values are \( c_\omega = 1.9989 \) fm\(^2\), \( c_\sigma = 6.8158 \) fm\(^2\), \( B = -100 \) and \( C = -133.6 \). In Fig. 1, we present the energy/nucleon as a function of the baryon density. The solid curve (MCH) corresponds to the modified chiral sigma model presented above, where the nuclear incompressibility is around 300 MeV. This equation of state is much softer than that of the original chiral sigma model (CH), which is represented by the dotted curve. This is due to the additional terms \( B \) and \( C \) in the modified chiral sigma model in the potential term in equation (1). For comparison, we display the dashed curve (NL3), which was derived from recent heavy-ion collision data\(^{13}\) with incompressibility around 340 MeV, and the dotted curve corresponds to the original chiral sigma model,\(^9\) with very high incompressibility.

§3. Equation of state in neutron star matter

In the interior of neutron stars, i.e. at high density, the neutron chemical potential exceeds the combined mass of the proton and electron. Therefore, asymmetric matter, with an admixture of electrons, rather than pure neutron matter, is a more likely composition of matter in neutron star interiors. The concentrations of neutrons, protons and electrons can be determined from the condition of beta equilibrium \((n \leftrightarrow p + e + \bar{\nu})\) and from charge neutrality, assuming that neutrinos are not degenerate. We have

\[
\mu_n = \mu_p + \mu_e, \quad n_p = n_e, \tag{7}
\]

where \( \mu_i \) is the chemical potential of particle species \( i \). For the purpose of describing neutron-rich matter, we include the interaction due to the isospin triplet \( \rho \) meson in the Lagrangian (1). The following terms are included in the Lagrangian:
\(-\frac{1}{4} G_{\mu\nu} \cdot G^{\mu\nu} + \frac{1}{2} m_{\rho}^2 \rho_{\mu} \cdot \rho^{\mu} - \frac{1}{2} g_{\rho} \bar{\psi}(\rho_{\mu} \cdot \tau^{\mu}) \psi \). \hspace{1cm} (8)

Here $G_{\mu\nu} \equiv \partial_{\mu} \rho_{\nu} - \partial_{\nu} \rho_{\mu}$. Using the mean-field approximation in the equation of motion for $\rho$, the following density dependence equation is obtained:

$$\rho_{\rho}^3 = \frac{g_{\rho}}{2m_{\rho}^2}(n_p - n_n). \hspace{1cm} (9)$$

From the semi-empirical nuclear mass formula, the symmetric energy coefficient is

$$a_{\text{sym}} = \frac{c_{\rho}k_F^3}{12\pi^2} + \frac{k_F^2}{6\sqrt{k_F^2 + M^*}}. \hspace{1cm} (10)$$

where $c_{\rho} \equiv \frac{g_{\rho}^2}{m_{\rho}^2}$ and $k_F = (6\pi^2 n_B/\gamma)^{1/3}$. We fix the coupling constant $c_{\rho}$ by requiring that $a_{\text{sym}}$ correspond to the empirical value, 32 MeV. This gives $c_{\rho} = 4.66$ fm\(^2\). The inclusion of the $\rho$ meson in the Lagrangian will contribute the term $m_{\rho}^2 \rho_0^2/2$ to the energy density and pressure. Then the equation of state for neutron rich nuclear matter in beta equilibrium is calculated using the expression for the energy density $\varepsilon$ and the pressure $P$ as follows:

$$\varepsilon = \frac{M^2(1 - y^2)^2}{8c_{\sigma}} - \frac{B}{12c_{\omega}c_{\sigma}}(1 - y^2)^3 + \frac{c_{\omega}n_B^2}{2y^2} + \frac{C}{16m_{\omega}^2c_{\omega}c_{\sigma}}(1 - y^2)^4 + \sum_i \varepsilon_{FG} + \frac{1}{2} m_{\rho}^2 \rho_0^2,$$

$$P = -\frac{M^2(1 - y^2)^2}{8c_{\sigma}} + \frac{B}{12c_{\omega}c_{\sigma}}(1 - y^2)^3 + \frac{c_{\omega}n_B^2}{2y^2} - \frac{C}{16m_{\omega}^2c_{\omega}c_{\sigma}}(1 - y^2)^4 + \sum_i P_{FG} + \frac{1}{2} m_{\rho}^2 \rho_0^2. \hspace{1cm} (11)$$

In these equations $\varepsilon_{FG}$ and $P_{FG}$ are the relativistic non-interacting energy density and pressure of the baryons (with effective masses) and electrons ($i$), respectively.

Figure 2, displays the pressure versus the total mass-energy density for neutron-rich matter in beta equilibrium. The solid curve corresponds to the modified chiral sigma model MCH considered in the present calculation. Inclusion of the two parameters, $B$ and $C$ in the potential term, gives a reasonable value for the incompressibility at the saturation density of nuclear matter. Hence, the MCH model is much softer than the original CH model with regard to the neutron star matter equation of state. The physics behind this is that the pressure generated by the self-interaction of scalar fields at high density is less; i.e., the pressure with density decreases more than that for the CH model. The dashed curve (NL3) represents the neutron rich equation of state, which was derived from the heavy-ion collision data. From Fig. 2, we note, in comparison with the NL3 model, that the MCH model is stiffer in the low density range and becomes softer above three times the normal nuclear matter density.
The mass and radius of a neutron star are characterized by its structure. These are determined from the equations that describe the hydrostatic equilibrium of degenerate stars without rotation in general relativity, the Tolman-Oppenheimer-Volkoff (TOV) equations:

\[
\frac{dp}{dr} = -\frac{G(\epsilon + p/c^2)(m + 4\pi r^3 p/c^2)}{r^2(1 - 2Gm/rc^2)}, \quad \frac{dm}{dr} = 4\pi r^2 \epsilon.
\] (12)

Here \(p\) and \(\epsilon\) are the pressure and total mass-energy density, and \(m(r)\) is the mass contained in a volume of radius \(r\). The quantity \(G\) is the gravitational constant, and \(c\) is the velocity of light. To integrate the TOV equations, one needs to know the equation of state for the entire expected density range of the neutron star, starting from the high density at the center to the surface densities. Therefore, we construct a composite equation of state for the entire neutron star density span by joining our equation of state for high density neutron rich matter to those with (i) \(10^{14} \text{ to } 5 \times 10^{10} \text{ g cm}^{-3}\), (ii) \(5 \times 10^{10} \text{ g cm}^{-3} \text{ to } 10^{3} \text{ g cm}^{-3}\), and (iii) less than \(10^{3} \text{ g cm}^{-3}\). Thus we integrate the TOV equations for the newly constructed equation of state and given central density \(\epsilon(r = 0) = \epsilon_c\) with the boundary condition \(m(r = 0) = 0\) to give \(R\) and \(M\). The radius \(R\) is defined by the point where \(P \sim 0\), or, equivalently, \(\epsilon = \epsilon_s\), where \(\epsilon_s\) (7.8 g cm\(^{-3}\)) is the density expected at the star surface. The total mass is then given by \(M = m(R)\).

The results for the star structure parameters are listed in Table I and displayed in Fig. 3. This figure plots the mass as a function of the central density. The models are the same as in Fig. 2.

In Table I, we also list additional parameters of interest. These are the moment of inertia \(I\), and the surface redshift \(z = \frac{1}{\sqrt{1 - 2GM/Rc^2}} - 1\) as a function of the central density of the star. (For details, see Ref. 9.) These are important for the dynamics and transport properties of pulsars. From Fig. 3 and Table I, we see that the maximum masses of the stable neutron stars are \(2.1M_\odot\), \(2.2M_\odot\) and \(2.5M_\odot\) and corresponding radii are 12.1 km, 11.6 km and 13.8 km for MCH, NL3 and CH equation.
Table I. Neutron star structure parameters.

| $\varepsilon_c$ \((g \text{ cm}^{-3})\) | $R$ \((\text{km})\) | $M/M_\odot$ | $z$ | $I$ \((g \text{ cm}^2)\) |
|---|---|---|---|---|
| $2.0 \times 10^{14}$ | 18.33 | 0.19 | 0.015 | $0.11 \times 10^{45}$ |
| $6.0 \times 10^{14}$ | 13.77 | 1.38 | 0.19 | $1.77 \times 10^{45}$ |
| $8.0 \times 10^{14}$ | 13.57 | 1.74 | 0.27 | $2.38 \times 10^{45}$ |
| $1.0 \times 10^{15}$ | 13.28 | 1.93 | 0.32 | $2.66 \times 10^{45}$ |
| $1.5 \times 10^{15}$ | 12.62 | 2.10 | 0.40 | $2.73 \times 10^{45}$ |
| $1.8 \times 10^{15}$ | 12.30 | 2.12 | 0.43 | $2.64 \times 10^{45}$ |
| $2.0 \times 10^{15}$ | 12.11 | 2.12 | 0.44 | $2.57 \times 10^{45}$ |
| $2.5 \times 10^{15}$ | 11.74 | 2.10 | 0.46 | $2.39 \times 10^{45}$ |
| $3.0 \times 10^{15}$ | 11.44 | 2.07 | 0.47 | $2.24 \times 10^{45}$ |

of states, respectively. The corresponding central densities are $2.0 \times 10^{15} \text{ g cm}^{-3}$ (> 7 times nuclear matter density, where the nuclear matter density is $2.8 \times 10^{14} \text{ g cm}^{-3}$), $2.0 \times 10^{15} \text{ g cm}^{-3}$ (> 7 times nuclear matter density) and $1.5 \times 10^{15} \text{ g cm}^{-3}$ (> 5 times nuclear matter density) for MCH, NL3 and CH, respectively, at the maximum neutron star masses. From the neutron star structure point of view, our present model MCH is comparable to that of the NL3 model, which was constructed using recent heavy-ion collision data. The maximum masses calculated using our models are in the range of recent observations, where the observational consequences are given below.

Very recently, it has been observed that the best determined neutron star masses are found in binary pulsars, and they all lie in the range $1.35 \pm 0.04 M_\odot$, except for those of the non-relativistic pulsars PSR J1012+5307, for which $M = (2.1 \pm 0.8) M_\odot$. There are several measured X-ray binary masses, and the heaviest among them are Vela X-1 with $M = (1.9 \pm 0.2) M_\odot$ and Cygnus X-2 with $M = (1.8 \pm 0.4) M_\odot$. From the recent discovery of high-frequency brightness oscillations in low-mass X-ray binaries, the large masses of the neutron stars are around $M = 2.3 M_\odot$, $M = 2.1 M_\odot$ and $M = 1.9 M_\odot$ in QPO 4U 1820-30, QPO 4U 1608-52 and QPO 4U 1636-536,
This provides a new method to determine the masses and radii of the neutron stars. Our results lie in the range of those predicted by the observational limits.\(^{19}-22\)

\section*{Summary and conclusion}

We have discussed the \(SU(2)\) chiral sigma model, in which the isoscalar vector field is generated dynamically. In the present calculation, we have modified the \(SU(2)\) chiral sigma model by including two extra terms in the Lagrangian to ensure an appropriate value of the incompressibility at the saturation density of symmetric nuclear matter. By employing these and using a mean field approximation, we have obtained the nuclear equation of state at high densities. This is compatible with the equation of state that was derived using recent heavy-ion collision flow data.\(^{11}-13\)

The nuclear equation of state is reasonably softer than the original \(SU(2)\) chiral sigma model we considered previously.\(^9\) We then employed the same nuclear equation of state in the neutron star matter calculation, where the composition of the star matter consists of neutrons, protons and electrons. The mass and radius of the neutron star were calculated from the TOV equations, and were found to be \(2.1M_\odot\) and 12.1 km, respectively. The corresponding central density is \(2.0 \times 10^{15}\) g cm\(^{-3}\) (\(>7\) times the nuclear matter density, \(2.8 \times 10^{14}\) g cm\(^{-3}\)) for the modified chiral sigma model. These values are comparable to the values calculated using the recent NL3 model, which was recently constructed using heavy-ion collision data. The overall structural values are in good agreement with those given by the quasi-periodic oscillation method,\(^{22}\) a new method to determine the masses and radii of neutron stars.

In the future we plan to investigate more systematically the properties of neutron stars in the presence of other baryons, i.e., hyperons and mesons with (hidden-)strangeness in the \(SU(3)\) chiral sigma model with the proper interactions between them derived from recent experiments. Moreover, we are interested in searching for a phase transition to quark matter (a new state of matter) by implementing this equation of state in heavy-ion collision simulation calculations.

\section*{Acknowledgements}

PKS would like to acknowledge the support of the Japan Society for the Promotion of Science (ID No. P 98357), Japan.

\section*{References}

1) N. K. Glendenning, Ann. of Phys. 168 (1986), 246.
2) A. D. Jackson, M. Rho and E. Krotscheck, Nucl. Phys. A407 (1985), 495.
3) T. L. Ainsworth, E. Baron, G. E. Brown, J. Cooperstein and M. Prakash, Nucl. Phys. A464 (1987), 740.
4) M. Gell-Mann and M. Levy, Nuovo Cim. 16 (1960), 705.
5) T. D. Lee and G. C. Wick, Phys. Rev. D9 (1974), 2291.
6) J. Boguta, Phys. Lett. B120 (1983), 34; B128 (1983), 19.
7) M. Prakash and T. L. Ainsworth, Phys. Rev. C36 (1987), 346.
8) N. K. Glendenning, Nucl. Phys. A480 (1988), 597.
9) P. K. Sahu, R. Basu and B. Datta, Astrophys. J. 416 (1993), 267.
10) P. Möller, W. D. Myers, W. J. Swiatecki and J. Treiner, Atomic Data Nucl. Data Tables 39 (1988), 225.
11) P. K. Sahu, A. Hombach, W. Cassing, M. Effenberger and U. Mosel, Nucl. Phys. A640 (1998), 493.
12) P. K. Sahu, W. Cassing, U. Mosel and A. Ohnishi, Nucl. Phys. A672 (2000), 376; P. K. Sahu, Phys. Rev. C62 (2000), 045801.
13) P. K. Sahu and A. Ohnishi, Science of Hadrons under Extreme Conditions, JAERI-Conf 2000-011 (2000), 19.
14) C. W. Misner, K. S. Thorne and J. A. Wheeler, *Gravitation* (San Francisco, Freeman) (1970).
15) J. B. Hartle, Astrophys. J. 150 (1967), 1005.
16) J. W. Negle and D. Vautherin, Nucl. Phys. A207 (1973), 298.
17) G. Baym, C. J. Pethick and P. G. Sutherland, Astrophys. J. 170 (1971), 299.
18) R. P. Feynmann, N. Metropolis and E. Teller, Phys. Rev. 75 (1949), 308.
19) J. van Paradijs, *The many faces of neutron stars*, ed. R. Buccheri, J. van Paradijs and M. A. Alpar, (Kluwer Academic Publishers, 1998).
20) O. Barziv, in preparation; M. H. van Kerkwijk, J. van Paradijs and E. J. Zuiderwijk, Astron. Astrophys. 303 (1995), 497.
21) J. A. Orosz and E. Kuulkers, Mon. Not. Ro. Astron. Soc. 305 (1999), 132.
22) M. C. Miller, F. K. Lamb and P. Psaltis, Astrophys. J. 508 (1998), 791; H. Heiselberg, Proc. of the 10th Int. Conf. on Recent Progress in ManyBody Theories, World Scientific, (1999).
23) S. E. Thorsett and D. Chakrabarty, Astrophys. J. 512 (1999), 288.