Acoustic properties of overheated liquid with gas nuclei during temperature increasing

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Abstract. The dynamics of weak harmonic disturbances in a superheated water-air bubble medium is studied when in addition to water vapor there is an inert gas (e.g. air) that does not participate in phase transitions. The maps of the stability zones of the considered systems are analyzed depending on the value of the liquid overheating on the “volume content - bubble radius” plane with an increase in the equilibrium pressure from 0.1 to 10 MPa. The effect of the initial pressure amplitude and the overheating (from hundredths to one degree) on the dispersion of harmonic waves as well as the dependence of the increment on the bubble radius for unstable systems are investigated.

1. Introduction

It is known that a bubble liquid is a system with unique wave properties [1, 2]. Problems of the wave propagation in such media have been of great interest to researchers for almost half a century due to the wide spread of this systems in nature and their intensive use in modern technology.

Numerical investigations on the developed in [1] models of a gas-liquid mixture for dynamics of acoustic pressure waves in bubble media are conducted in papers [3–7].

The oblique incidence of an acoustic signal to the interface between a vapor–gas–drop medium and air was considered in [8]. The authors found that, if a wave is incident to the interface of the vapor–gas–drop mixture, there is a critical incidence angle at which the wave is totally reflected. In addition, the same authors considered the propagation of sound in a fog [9]. The authors of [10] studied the problem of wave reflection and transmission to the interface between bubbly and pure liquids, when a liquid is “cold” i.e. there is only gas in the bubbles. The propagation of small perturbations in a superheated liquid containing gas nuclei is studied in [11] without taking diffusion into account.

Analysis of the previous works shows that the intensity of attenuation of sound disturbances in the considering systems is mainly determined by the thermophysical characteristics of the gas in the bubbles. The aim of this work is to analyze the effect of fluid overheating on the value of the phase velocity and attenuation coefficient when directly falling on the interface between bubbly and pure liquids.
2. Governing equations and dispersive analysis

Let us consider one-dimensional acoustic waves. Let us turn the vertical axis $0x$ perpendicular to the interface plane between single- and two-phase media and set the origin of coordinates $x = 0$ on the interface. The horizontal axis $0y$ is directed along the interface (figure 1).

Let us assume that a liquid in region $x > 0$ has temperature $T_0$ and pressure $p_0$ and a liquid in region $x < 0$ at the same temperature and pressure contains spherical bubbles with radii $a_0$ which in turn contain vapor and a gas insoluble in a liquid phase. Let a plane harmonic wave be incident at some angle to a flat interface between a pure and gas-saturated liquid (figure 1). This system is described by equations (1 - 12) from [12]. It should be noted, that, if a liquid state is far enough from critical, conditions $p_\nu = p_S(T_0)$ are fulfilled [1].

The main calculation procedure is presented in [12]. Let us note its key points.

The solution of the above system will be searched in the form of a damped running wave:

\[ p_l, p_g, \nu, a \sim \exp \left( i (Kx - \omega t) \right), \quad T_i = T_i(r) \exp \left( i (Kx - \omega t) \right), \quad (i = g, l), \]

\[ k' = k(r) \exp \left[ i (Kx - \omega t) \right], \quad K = k + i\delta, \quad C_p = \omega/k, \quad i = \sqrt{-1}, \]

where $K$ is the wave vector, $\delta$ and $C_p$ are the damping coefficient and the phase velocity of the wave, respectively. From the condition for the existence of a solution of this type, taking into account the acoustic discharge effects [13] of bubbles, we obtain the dispersion equation

\[ \frac{K^2}{\omega^2} = \left( 1 - \alpha_{g0} \right)^2 + 3\rho_{l0}^0 \omega^2 a_0^2 \frac{(1 - \alpha_{g0})}{\psi}, \quad (1) \]

\[ \psi = \frac{3\gamma p_{g0}}{Q} - \frac{\rho_{l0}^0 \omega^2 a_0^2}{\xi} - 4i\rho_{l0}^0 \nu_{(\mu)} \omega - \frac{2\sigma}{a_0}, \]

\[ p_{g0} = p_0 + \frac{2\sigma}{a_0}, \quad \xi = 1 - i\omega t_A, \quad t_A = \frac{a_0}{\sqrt{\nu_{0\alpha_{g0}}}}, \]

\[ Q = 1 + \left( \frac{\gamma - 1}{k_0} H\nu_{kh} + \frac{\gamma}{1 - k_0} H\nu_{kh}(z) \right) \left( \frac{H\nu}{k_0} + \frac{\gamma kh(z)}{(1 - k_0)\beta \sinh \nu(y)} \right)^{-1}, \]
\[ \sinh \nu(x) = 3(1+x(A_0x \tanh x(A_0 - 1))(A_0x - \tanh x(A_0 - 1))^{-1})x^{-2} \]
\[ \sinh \nu(x) = 3(1+x)x^{-2}, \]
\[ \nu(x) = 3(1+x(x(A_0x \tanh x(A_0 - 1))(A_0x - \tanh x(A_0 - 1))^{-1})x^{-2} \]
\[ A_0 = \alpha^{-1/\beta}_0, \quad \nu = \sqrt{-i\omega a_0^2/\nu_0(T)}, \quad z = \sqrt{-i\omega a_0^2/D}, \quad \beta = (\gamma - 1)\eta\mu\chi^2, \]
\[ \eta = \rho_0^0 c_1, \quad \chi = \frac{c_0 T_0}{l}, \quad H_s = \frac{B_v}{B_0}, \quad H_a = \frac{B_a}{B_0}, \quad H = H_v - H_a. \]

From the dispersion equation written above (1) in assumption \( \omega \to 0 \) the formula below follows for the equilibrium velocity of sound

\[ C_e = \sqrt{\frac{\rho_0^0}{\rho_0^0 \alpha^0_0}} \left( 1 - k_0 \right) H_a + k_0 \alpha^0_0 \frac{\gamma}{\beta} - \frac{2}{3} \frac{\sigma}{a_0 \rho_0^0 \alpha^0_0}, \]

and it generalizes the well-known Malloch and Landau formulas [14]. In particular, at the boiling point \( T_0 = T_s(p_0) \) taking into account the expressions from the dispersion equation, we obtain

\[ C_e = \sqrt{k_0 \left( \frac{\rho_0^0}{\rho_0^0} \right)^2 \left( \frac{B_0}{B_c} \right)^2 \frac{l^2}{C_1 T_0} + \frac{4}{3} \frac{\sigma}{a_0 \rho_0^0 \alpha^0_0}}. \]

3. Numerical results

Numerical calculations for water with vapor-gas bubbles were carried out on the basis of above-mentioned dispersion equation. The data from [15] were used as the values of the physical and thermophysical parameters. Lines 1, 2, and 3 in the figures correspond to the following static pressure values \( p_0 = 0.1, 1, \) and 10 MPa. Indexes \( a, b \) and \( c \) correspond to the values of the liquid overheating \( \Delta T_0 = 0.01, 0.1 \) and 1 K respectively.

Figure 2 shows the lines defining on the plane \((\alpha^0_0, a_0)\) the boundary between stable and unstable states of superheated liquid with vapor-air void fraction at the above mentioned pressure values \( p_0 = 0.1, 1 \) and 10 MPa with corresponding saturation temperature \( T_s(p_0) = 373, 453, 584 \) K. It is shown that these lines consist of two sections, ascending one and descending one. On the ascending part of line which is almost vertical, corresponding mass concentration \( k_0 \) of steam increases. Moreover, for pressures \( p_0 = 1 \) and 10 MPa for all three values of overheating mass concentration reaches the value \( k_0 = 1 \) (bold points on the graphs). For the case of \( p_0 = 1 \) MPa with \( \Delta T_0 = 1 \) K for volume contents of bubbles not more than ten percent \( (\alpha^0_0 \leq 0.1) \) this value for concentration \( (k_0 = 1) \) is not reached ever. On the descending part of line the mass concentration of steam keeps its constant value \( (k_0 = 1) \), that is, these line sections correspond to the steam – water bubble system. Zones located outside the curves correspond to stable states of media and inside the curves to unstable states. In the case of pure vapor bubbles the lines completely define the boundary between stable and unstable states. Besides the zones located to the right of these lines correspond to stable stats and to the left for the unstable one. This case was studied in [16]. It is easy to see from the figure that, firstly, the volume content of vapor-gas bubbles has little effect on the pattern of the curves defining the stability boundary. Secondly, the presence of gas in the bubbles expands the stability zone of the superheated liquid. In this case, the smaller the overheating, the more to the right the boundary between stable and unstable states is shifted. Therefore, the less overheating, the more the stability zone becomes.

In figure 3 the dependence of the increment \( \omega' \) on the radius \( a_0 \) at different values of overheating \( \Delta T_0 \) and for \( p_0 = 1 \) and 10 MPa (lines 2 and 3, respectively) is shown. \( \omega \) is calculated from the dispersion equation from the condition \( \omega = \omega' \cdot i \) [16]. There is \( a_0 = 10^{-4} \) volume content of bubbles. It can be seen that with the radius increase, the increment increases from zero to the maximum value, then decreases monotonically to zero. Maximum value of
Figure 2. Lines defining the boundary between stable and unstable states of superheated water with vapor-air bubbles on the plane \((a_0, a_0)\).

Figure 3. The dependence of the increment \(\omega'\) on the radius \(a_0\) at different values of overheating \(\Delta T_0\). Designations on the lines correspond to figure 2.
Figure 4. The dependence of the phase velocity (a) and attenuation coefficient (b) on the disturbance frequency at various values of the superheat value $\Delta T_0$ for pressure in the liquid $p_0 = 1$ MPa.

The increment is achieved at the maximum possible radius of vapor-gas bubbles (bold points on figure 3) obtaining under the condition $p_{g0} = 0$ ($k_0 = 1$). In addition, all of the dependences of the increment on the radius for different overheatings lay on one curve.

Figures 4 and 5 show the phase velocity and attenuation coefficient versus perturbation frequency for stable systems, displaying the effect of overheating $\Delta T_0$ for pressure in the liquid $p_0 = 1$ MPa (figure 4) and 10 MPa (figure 5). Equilibrium bubble radius is $a_0 = 10^{-7}$ m (figure 4) and $10^{-8}$ m (figure 5). Lines 1, 2, 3, 4, and 5 in the figures correspond to the values of the overheating $\Delta T_0 = 0.05, 0.5, 1, 5$ and 10 K. Initial volume content is $\alpha_0 = 10^{-3}$. For
Figure 5. The dependence of the phase velocity (a) and attenuation coefficient (b) on the disturbance frequency at various values of the superheat value $\Delta T_0$ for pressure in the liquid $p_0 = 10$ MPa.

The low-frequency range ($\omega \leq \omega_R$, $\omega_R = a_0^{-1} \sqrt{3\gamma/\rho_l^0}$ – Minnaert frequency of natural bubble oscillations) overheating has some influence on the phase velocity and attenuation coefficient. For the higher frequencies overheating has not important value. In addition, it can be seen from the figure that for all values of overheating the same dependence of the phase velocity on frequency is observed. This feature is associated with a high enough mass concentration of inert gas in the bubbles and as a result the elasticity of the bubbles is mainly determined by the mass content of the gas.
4. Conclusion
On the basis of analytical and numerical calculations, maps of zones of stability in the area \((\alpha_0, a_0)\) of a bubble vapor–liquid mixture are plotted depending on the magnitude of the superheated liquid \(\Delta_0\). It has been established that the presence of insoluble gas in bubbles substantially expands the range of values of the volume content \(\alpha_0\) and the radius of bubbles \(a_0\) in the plane \((\alpha_0, a_0)\). It turns out that these effects are greatly enhanced with an increase in the vapor concentration due to an increase in the temperature of the system. With increasing static pressure \(p_0\), the stability zone, where the system is purely vapor-liquid expands, and the stability zone, where gas is present in the bubbles, narrows with the same overheating \(\Delta T_0\).

The effect of fluid overheating \(\Delta T_0\) on the magnitude of the phase velocity and attenuation coefficient when the system is in a steady state is considered. It was established that for the considered equilibrium radii in the stability zone, overheating does not significantly affect the change in phase velocity and attenuation coefficient, which is associated with a rather high concentration of inert gas in the bubbles. The damping decrement in the stable region does not change more than twofold.

This article is a continuation of the work [17–19].

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