ExoMol line lists – XV. A new hot line list for hydrogen peroxide

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ABSTRACT

A computed line list for hydrogen peroxide, H$_2^{16}$O$_2$, applicable to temperatures up to $T = 1250$ K is presented. A semi-empirical high-accuracy potential energy surface is constructed and used with an $ab$ initio dipole moment surface as input TROVE to compute 7.5 million rotational-vibrational states and around 20 billion transitions with associated Einstein-$A$ coefficients for rotational excitations up to $J = 85$. The resulting APTY line list is complete for wavenumbers below 6000 cm$^{-1}$ ($\lambda < 1.67$ $\mu$m) and temperatures up to 1250 K. Room-temperature spectra are compared with laboratory measurements and data currently available in the HITRAN data base and literature. Our rms with line positions from the literature is 0.152 cm$^{-1}$ and our absolute intensities agree better than 10 per cent. The full line list is available from the CDS data base as well as at www.exomol.com.

Key words: molecular data – opacity – astronomical data bases: miscellaneous – planets and satellites: atmospheres.

1 INTRODUCTION

Terrestrial hydrogen peroxide exists as a trace molecule in the Earth’s atmosphere and contributes to the atmospheres oxidizing budget as well as ozone production and water chemistry (Davis 1974; Chance et al. 1991; Allen et al. 2013; Zins & Krim 2014) and its concentration is now being routinely observed (Allen et al. 2013). Astrophysically there have been multiple detections of H$_2$O$_2$ in the atmosphere of Mars (Clancy, Sandor & Moriarty-Schieven 2004; Encrenaz et al. 2004, 2012; Aoki et al. 2015) with seasonal variation, possibly formed by triboelectricity in dust devils and dust storms (Encrenaz et al. 2012) and may well act as an agent in the oxidation of the Martian surface. Hydrogen peroxide has also been detected in the atmosphere of Europa (Hand & Brown 2013) in the 3.5 $\mu$m region. The first detection of interstellar H$_2$O$_2$ was made by Bergman et al. (2011) and is believed to play an important role in astrophysical water chemistry similar to that on Earth. Du, Parise & Bergman (2012) suggest that H$_2$O$_2$ is produced on dust-grains via the hydrogenation of grain HO$_2$ and released into the gas-phase through surface reactions. On the dust-grain, H$_2$O$_2$ acts as an intermediate in the formation of water and aids in the production of other species such as H$_2$CO, CH$_3$OH, and O$_2$.

Hydrogen peroxide belongs to the peroxide group of molecules with an HO-OH bond dissociation enthalpy of 17050 cm$^{-1}$ (Bach, Ayala & Schlegel 1996) at 0 K. H$_2$O$_2$ is an asymmetric prolate rotor molecule and is the simplest molecule that exhibits internal rotation. This torsional motion gives rise to a double minimum potential curve with respect to its internal rotation coordinates as well as two alignments of the O-H bonds: cis and trans. The consequence of this motion means that there are four sub-levels for each torsional excitation which are characterized by their symmetry. This necessitates the use of an additional quantum number, $\tau$, to unambiguously describe its motion. The molecular states can be classified using the $C_{2h}^{+}(M)$ symmetry group which best describes the torsional splitting caused by the cis and trans tunnelling (Hougen 1984). H$_2$O$_2$ has six vibrational modes: $v_1$ and $v_5$ represent the symmetric and asymmetric O-H stretching, respectively; $v_3$ and $v_6$ represent the O-H bending modes; $v_2$ represents the O-O stretch and the $v_4$ mode represents the torsional excitation with the more common notation of $n$.

Experimental studies of ro-vibrational H$_2$O$_2$ spectra have mostly probed the torsional motion in the ground (Olson et al. 1988), the $v_3$ (Camy-Peyret et al. 1992) and $v_6$ (Perrin et al. 1990, 1995) vibrational modes. Conversely, the higher lying O-H stretching modes, $v_1$ and $v_5$, are poorly studied using high-resolution techniques. The difference between the two stretching bands is about 8–10 cm$^{-1}$ and torsional splitting from the double minimum of the potential gives rise to doubling (Giguere & Srinivasan 1974) in the form of ‘quasi’-degenerate states (Rauhut, Knizia & Werner 2009) that are difficult to resolve with a degree of accuracy. Olson et al. (1988) give an estimate of 3610–3618 cm$^{-1}$ for $v_3$ and 3601–3617 cm$^{-1}$
for \( v_1 \), whilst a Raman study gives a lower value of 3607 cm\(^{-1}\) for the \( v_1 \) band-centre (Giguere & Srinivasan 1974) but determining the accuracy to better than 0.1 cm\(^{-1}\) is difficult.

\( \text{H}_2\text{O}_2 \) has been a benchmark system for developing methods aiming to treat large amplitude motion (Luckhaus 2000; Mladenovic 2002; Yu & Muckerman 2002; Carter, Handy & Bowman 2009). Recent calculations on the ro-vibrational states for \( \text{H}_2\text{O}_2 \) include the \textit{ab initio} computation using CCSD(T)-F12 electronic structure calculations of band frequencies accurate to about 4.0 cm\(^{-1}\) by Rauhut et al. (2009), models of the peroxide stretches by Bacelo & Binning (2005), a discrete variable representation calculation for levels up to 6000 cm\(^{-1}\) by Chen, Ma & Guo (2001), Lin & Guo (2003) and finally, a potential energy surface (PES) calculations by Koput, Carter & Handly (1998) and Kuhn et al. (1999). Calculation which also consider transition intensities are rather rarer and a recent example is provided by Carter, Sharma & Bowman (2011). The peroxide system was used to benchmark the large amplitude calculations of MULTIMODE (Bowman, Carter, & Huang 2003) up to \( J = 20 \) and showed good agreement against HITRAN line intensities but the PES used had a root mean square (rms) of \( \approx 20 \) cm\(^{-1}\) against experimental band centres. However, this PES has been superseded by the higher accuracy \textit{ab initio} PES of Małyszek & Koput (2013) which was further modified by Polyansky et al. (2013). This modified PES was used for our room-temperature line list (Al-Refaie et al. 2015a) and provides the starting point for the refinements performed here.

Experimental transitions frequencies and intensities for \( \text{H}_2\text{O}_2 \) are available in the HITRAN 2012 data base (Rothman et al. 2013) but only for room-temperature modelling up to 1800 cm\(^{-1}\). This region covers the torsional, O-H bending modes and O-O stretch but misses the O-H stretches in the 3750 cm\(^{-1}\) region. Only a few studies deal with absolute intensities of \( \text{H}_2\text{O}_2 \) in the far-infrared (Zumwalt & Giguere 1941; Perrin et al. 1995, 1996) with only PNINL-IR (Sharpe et al. 2004) data providing integrated intensities in the mid-infrared region (Johnson et al. 2009). The thermal decomposition of hydrogen peroxide at 423 K makes it difficult and dangerous to study at higher temperatures.

Theoretical line lists can be used to fill in gaps in the experimental data both in terms of wavelength and temperature coverage. The ExoMol project (Tennyson & Yurchenko 2012) aims to produce comprehensive theoretical molecular line lists to aid in studies of the atmospheres of exoplanets, cool stars and other (hot) bodies. A room-temperature line list for \( \text{H}_2\text{O}_2 \) was previously computed by us (Al-Refaie et al. 2015a) using the PES of Polyansky et al. (2013) and a new \textit{ab initio} dipole moment surface (DMS). It provides about 1 billion transitions at up to 8000 cm\(^{-1}\). However it is limited as the rotational excitation of \( J = 40 \) makes it inadequate for high temperature modelling and the lower energy cutoff means that coverage above 4000 cm\(^{-1}\) rapidly becomes incomplete. This work aims to build upon this line list by refining the PES towards spectroscopic accuracy and extending the temperature and frequency range for which the resulting line list is applicable.

## 2 Method

### 2.1 PES refinement

Our previous room-temperature \( \text{H}_2\text{O}_2 \) line list (Al-Refaie et al. 2015a) was computed using the \textit{ab initio} PES of Malyszec & Koput (2013) with the small adjustment of the \textit{ab initio} equilibrium geometry and height of the torsional barrier proposed by Polyansky et al. (2013). This PES reproduces the known empirical energy levels with an rms of about 1–2 cm\(^{-1}\). However, by utilizing empirical band-centre shifting (Yurchenko et al. 2009) during the computation of the Hamiltonian, this was significantly reduced to 0.001–0.1 cm\(^{-1}\) in the room-temperature line list. The empirical shifting can be thought of as an addition to the \textit{ab initio} PES producing an ‘ad hoc’ PES which we will refer to as ‘\( \text{H}_2\text{O}_2 \)-2015’ in comparisons below. Whilst band-centre shifting can reproduce experimental energies simply, its accuracy and predictive ability is limited to vibrational bands whose band-centre positions are already experimentally characterized. A more robust method of correcting the PES is through fitting or refining to experimental energies. The Theoretical ROVibrational Energies (\textit{TRove}) nuclear motion program used by us here, see below, provides PES refinement capabilities and has been successful in producing accurate PES for molecules such as \( \text{H}_2\text{CO} \) (Yachmenev et al. 2011), \( \text{NH}_3 \) (Yurchenko, Barber & Tennyson 2011b), \( \text{SO}_3 \) (Underwood et al. 2014), \( \text{PH}_3 \) (Sousa-Silva et al. 2015) and \( \text{CH}_4 \) (Yurchenko & Tennyson 2014).

The procedure implemented in \textit{TRove} describes a correction surface \( \Delta V \) to the initial (\textit{ab initio}) surface \( V \). The new refined surface \( V \) can therefore be written as \( V = V + \Delta V \) and the new Hamiltonian as \( \mathcal{H} = H + \Delta \mathcal{V} \), where \( H \) is the Hamiltonian of the starting point for the refinement. The wavefunctions from \( H \) are used as basisfunctions for \( \mathcal{H} \). \( \Delta V \) is expanded in Taylor series and the expansion coefficients, \( \Delta \mathcal{V}_{f,i} \), are obtained in a variational least-squares fit to spectroscopic data via the objective function F:

\[
F = \sum_i^{N} w_i (E_{i}^{\text{obs}} - E_{i})^2 = 0,
\]

where \( N \) is the number of observed energies \( E_{i}^{\text{obs}} \), \( E_{i} \) are the calculated energies and \( w_i \) are the weights.

Here the original 282 expansion coefficients of the \textit{ab initio} PES are reduced to 163 by removing the symmetry-related O-H stretching (\( ij \)) and bending (\( kl \)) terms from the input PES and simply linking them in the computation of potential energy terms in the Hamiltonian. This ensures that the symmetry of these terms is preserved during the fitting process. The quality of the fit is determined by the quality and vibrational diversity of the input data set. Two sources of experimental data come from line-positions provided in the literature and transitions from HITRAN. The HITRAN data set sources come from observations by Perrin et al. (1995), Perrin et al. (1996), Perrin et al. (1990) and Klee et al. (1999) with literature line-positions from Fland et al. (1989), Olson et al. (1988) Giguere (1950), Zumwalt & Giguere (1941) and Camy-Peyret et al. (1992). This empirical data set provides the \( \nu_4, \nu_3 + \nu_4, \nu_4 + \nu_5 \) and \( \nu_2 \) vibrational terms. Unfortunately there is little to no reliable data on the \( \nu_1 \) and \( \nu_3 \) energy levels, their reported band-centre values vary significantly in literature making them unsuitable for the refinement. This hampers the vibrational diversity that would aid in construction of an extensive fitted PES. However, these terms can be indirectly improved by including higher \( J \) values from other vibrational states.

Our input data set includes all energies for \( J \leq 4 \) up to 4000 cm\(^{-1}\). The weights \( w_i \) used have an arbitrary range of values that are normalized in the fit. The energies given in literature are the simplest to include in the refinement process and are given the highest weighting. Here the pure torsional band at \( J > 6 \) from Camy-Peyret et al. (1992) and Olson et al. (1988) are given the highest weighting of \( w_i = 100 \). The \( \nu_2, \nu_3, \nu_5, \nu_3 + \nu_4, \nu_4 + \nu_5 \) energies and \( \text{H}_2\text{O}_2 \) band centres (except for \( \nu_1 \) and \( \nu_3 \)) from Camy-Peyret et al. (1992),
Table 1. Comparison of $N$ weighted experimental data points in the fit and non-weighted root mean squared deviation of both H2O2-2016 (this work) and H2O2-2015 (Al-Refai et al. 2015a) for each data set.

| Weight | $N$ | H2O2-2016 rms (cm$^{-1}$) | H2O2-2015 rms (cm$^{-1}$) | Comment
|--------|-----|--------------------------|--------------------------|---------|
| 100    | 43  | 0.001                    | 0.000                    |         |
| 10–20  | 144 | 0.004                    | 0.007                    |         |
| 1–9    | 186 | 0.539                    | 1.369                    |         |

Table 2. Comparison of $N$ experimental data points in the fit and weighted root mean squared deviation of both H2O2-2016 (this work) and H2O2-2015 (Al-Refai et al. 2015a).

| $J$ | $N$ | H2O2-2016 wrms (cm$^{-1}$) | H2O2-2015 wrms (cm$^{-1}$) |
|-----|-----|---------------------------|---------------------------|
| 0   | 34  | 0.238                     | 0.254                     |
| 1   | 47  | 0.079                     | 0.320                     |
| 2   | 81  | 0.096                     | 0.345                     |
| 3   | 116 | 0.183                     | 0.404                     |
| 4   | 132 | 0.154                     | 0.287                     |
|     | 18  | 0.150                     | 0.321                     |

Giguere (1950), Perrin et al. (1990), Flaud et al. (1989) and Zummault & Giguere (1941) are given weights $10 < w_i < 20$.

Transitions from HITRAN require additional work. In order to determine the upper state of a transition requires the assignment of the lower state energy. Fortunately, HITRAN provides the lower state energy for all transitions in the data base. However, lower state energies require corroborations from literature data and/or the $ab$ initio energies for the upper state energy to be included in the fit with $1 < w_i < 9$ based on confidence of the datum. Each input datum must be correlated with a theoretically computed energy level which, in this work, was straightforward due to the good agreement given by the initial $ab$ initio PES.

Special measures must be taken in order to ensure that the refinement process does not lead to unphysical shapes for the new PES due to a limited sampling of the experimental data not covering all the complexity of the PES of HOOH. For example, the high stretching or bending overtones are poorly represented in the experimental and therefore it is important to retain the $ab$ initio quality of the original PES by Polyansky et al. (2013). To this end we constrain the PES around $ab$ initio energies at each geometry (Yurchenko et al. 2003, 2011a; Yachmenev et al. 2011; Sousa-Silva et al. 2015).

The new PES is called H2O2-2016. Table 1 describes the rms for states of a particular weight. The high quality of the H2O2-2016 energies computed without any empirical band shifts shows that this new semi-empirical PES performs better overall than the $ab$ initio band-shifted PES especially for the lower weighted states. Weights $10 > 10$ relate to vibrational states that were involved in the band-shifting which gives H2O2-2015 its low rms values. Comparing weights lower than 10 suggests that the predictive ability of the H2O2-2016 PES is greatly enhanced. The overall comparison as a function of the rotational quantum number $J$ with a weighted rms is given in Table 2.

Overall H2O2-2016 improves the rms deviations of H2O2-2015 by more than a factor of 2. Table 3 highlights residuals for $J \geq 30$ for the $v_3$ and $v_4$ line positions from Camy-Peyret et al. (1992) and Perrin et al. (1990) and shows excellent agreement with an overall rms of 0.064 cm$^{-1}$. The rms deviation for all 2734 states in HITRAN up to $J = 49$ and energy up to 3461.02 cm$^{-1}$ is 0.834 cm$^{-1}$. Vibrational terms that correspond to the highest weighted states have an rms of 0.192 cm$^{-1}$. Around 12 states related to higher excited torsional modes $n > 3$ have an rms of 5.2 cm$^{-1}$ and may well be due to misassignments. This PES is therefore of there of improved accuracy and is the one used below. The coefficients defining this PES are given in the Supplementary Information.

2.2 Variational computation

TROVE (Yurchenko, Thiel & Jensen 2007) was employed to compute the ro-vibrational energies of H2O2. TROVE is a variational nuclear motion solver and was used to successfully produce the room-temperature H2O2 line list as well as hot line lists for NH3 (Yurchenko et al. 2013), CH4 (Yurchenko & Tennyson 2014), PH3 (Sousa-Silva et al. 2015), H2CO (Al-Refai et al. 2015b) and SO3 (Underwood et al. 2016).

TROVE can operate with any coordinate system of our choosing by utilizing an approximate kinetic energy operator (KEO). For H2O2, this approximate KEO was shown by Polyansky et al. (2013) to produce results that are in very good agreement with the exact KEO code WAVE4 (Kozin et al. 2004). However, the computational cost is greatly reduced for rotationally excited states by using TROVE. Convergence of the KEO usually requires an expansion of the order of 6 or 8 (Yurchenko et al. 2007); 6 being chosen for this work and 8 for the potential energy expansion as suggested from the previous H2O2 calculation by Polyansky et al. (2013).

Basis sets and wave functions are symmetrized to the $D_{2h}(M)$ molecular symmetry group (Bunker & Jensen 1998) which is isomorphic to the $C_{\infty v}(M)$ symmetry group. This has the benefit of factorizing the Hamiltonian into smaller independent blocks for diagonalization. The irreducible representations of this group are $A_g, A_u, B_{1g}, B_{1u}, B_{2g}, B_{2u}, B_{3g}$ and $B_{3u}$. However, the states corresponding to $B_{2g}, B_{2u}, B_{1u}$ and $B_{3u}$ have zero nuclear statistical weight and therefore these matrix blocks are not needed for $J > 0$. The corresponding $J = 0$ energies from these representations are also unphysical, but computed and kept as vibrational band centres for reference purposes.

A symmetry-adapted basis-set is constructed by a multistep contraction scheme by solving the 1D Schrödinger equation via the Numerov–Cooley method (Noumerov 1924; Cooley 1961) for the basis-functions $\phi_n(\zeta_j)$ ($i = 1, 2, \ldots, 6$), where $\zeta_1$ represents the O-O stretching coordinate, $\zeta_2$ and $\zeta_3$ represent the O-H stretching coordinates, $\zeta_4$ and $\zeta_5$ represents the O-H bending modes and $\zeta_6$ represents the torsional mode. $n_i$ is the local mode quantum number assigned by TROVE. The basis functions $\phi_n$ are then used to form a product-type basis set, which is truncated by the polyad number $P$:

$$P = 4n_1 + 8(n_2 + n_4 + n_5 + n_6) + n_6 \leq P_{\text{max}}.$$  

We use $P_{\text{max}} = 42$ as it was found to give good convergence (Polyansky et al. 2013). The second step requires computing a contracted basis set by reducing the six-dimensional coordinate system into four subspaces: $(\zeta_1, \zeta_4, \zeta_6), (\zeta_2, \zeta_5), (\zeta_3, \zeta_5)$ and $(\zeta_6)$ based on their permutation properties. The reduced Hamiltonian is solved using the
primitives $\phi_n$ as basis functions to obtain the symmetrized vibrational eigenfunctions $\Phi_{iJ}(\zeta_1), \Phi_{iK}(\zeta_2, \zeta_3), \Phi_{iL}(\zeta_1, \zeta_2, \zeta_3)$ and $\Phi_{iM}(\zeta_2, \zeta_3)$. The vibrational contracted basis set is then formed from symmetrized products of these eigenfunctions which is also truncated via equation (2). A final contraction step is performed by solving the $J = 0$ problem and replacing the bulky primitive and contracted vibrational basis set with the more compact $J = 0$ wavefunctions $\Psi_{J=0}^{iL}$ truncated at 12 000 cm$^{-1}$. The benefit of this is that the computation of the Hamiltonian matrix elements for $J > 0$ is more efficient and reduces the size of the matrix. Whilst this form means we can replace the diagonal vibrational terms with experimental band centres, the low rms error of H2O2-2016 PES band centres makes this procedure unnecessary. Our final ro-vibrational wavefunction has the form:

$$
\Psi_{J=0}^{iL} = \sum_{iJ,K,p} c_{iJ,K,p} \chi_{J,K,p}^{iL}(J, K, p),
$$

(3)

where $|J, K, p\rangle$ are rigid rotor functions with parity $p$ defined by Yurchenko et al. (2005b) and $c_{iJ,K,p}$ are the eigenvector coefficients obtained by diagonalization of the Hamiltonian matrix. The
linear algebra libraries LAPACK (Anderson et al. 1999) and SCALAPACK (Blackford et al. 1997) were employed to solve for the eigenvalues and eigenvectors.

TROVE assigns six local mode \( n_i \) quantum numbers to each state by finding the largest contributing \( v_{i,k,ho} \). These can be reassigned to match the more standard normal modes \( v_i \) generally used in literature. However, TROVE only provides six quantum numbers for reassignment, when seven are required by the inclusion of \( \tau \). The \( \tau \) quantum number can be preserved in the reassignment in TROVE by utilizing the following form:

\[
v_4 = 4n + i,
\]

where \( n \) is the excitation and \( i \) is the symmetry where \( i = 0, 1, 2, 3 \) is \( A_g, B_{2u}, B_{2g} \) and \( A_u \), respectively. To retrieve \( n \) and \( \tau \) simply requires

\[
\tau = (v_4 \text{ mod } 4) + 1, \quad n = \left\lfloor \frac{v_4}{4} \right\rfloor.
\]

\[\text{(5)}\]

### 2.3 DMS and intensities

An electric DMS is required in order to compute transition intensities. The \textit{ab initio} DMS of Al-Refaie et al. (2015a) was utilized; this was computed at the CCSD(T)-f12b (Bartlett & Musiał 2007) level of theory in the frozen-core approximation and is applicable to energies up to \( 6 \times 10^3 \) cm\(^{-1}\).

The eigenvectors, obtained by diagonalization, are used in conjunction with the DMS to compute the required linestrengths and Einstein-A coefficients of transitions that satisfy the rotational selection rules

\[
J' - J'' = 0, \pm 1, J' + J'' \neq 0
\]

and the symmetry selection rules:

\[
A_g \leftrightarrow A_u, \quad B_{1g} \leftrightarrow B_{1u}
\]

applied to para and ortho states, respectively. States with \( B_{2g}, B_{2u}, B_{3g} \) \( \text{and } B_{3u} \) symmetry are forbidden due their zero nuclear statistical weights.

The Einstein-A coefficient for a particular transition from the initial state \( i \) to the final state \( f \) is given by

\[
A_{if} = \frac{8\pi^4\nu_{lf}^3}{3h}(2J_f + 1) \sum_{A=X,Y,Z} |\langle \Psi_f | \hat{\mu}_A | \Psi_i \rangle|^2,
\]

\[\text{(8)}\]

where \( J_i \) is the rotational quantum number for the initial state, \( h \) is Planck constant, \( \nu_{lf} \) is the transition frequency (hc \( \nu_{lf} = E_f - E_i \)), \( \Psi' \) and \( \Psi \) represent the eigenfunctions of the final and initial states, respectively, \( \hat{\mu}_A \) is the electronically averaged component of the dipole moment along the space-fixed axis \( A = X, Y, Z \) (see also Yurchenko et al. 2005a). From this the absolute absorption intensity is determined by

\[
I(f \leftrightarrow i) = \frac{A_{if}}{8\pi c g_m(2J_f + 1)} \exp \left( -\frac{c_2 E_i}{kT} \right) \frac{Q}{\nu_{lf}^2} \times \left[ 1 - \exp \left( -\frac{c_2 E_i}{kT} \right) \right],
\]

where \( c_2 = hc/k \) is the second radiation constant, \( E_i \) is the energy term value, \( T \) the absolute temperature and \( g_m \) is the nuclear spin statistical weight factor. \( Q \), the partition function, is given by

\[
Q = \sum_i g_i \exp \left( -\frac{c_2 E_i}{kT} \right),
\]

\[\text{(9)}\]

where \( g_i \) is the degeneracy of a particular state \( i \) with the energy term value \( E_i \). For H\(_2\)O\(_2\), \( g_i \) is \( g_{nm}(2J_i + 1) \) with \( g_{nm} = 1 \) for \( A_g \) and \( A_u \) symmetries and \( g_{nm} = 3 \) for \( B_{1g} \) and \( B_{1u} \) symmetries. The transitions were computed using the energy limits \( h \times 6000 \) and \( h \times 12000 \) cm\(^{-1}\) for the lower and upper states, respectively, giving a complete coverage of the region \( 0 \sim 6000 \) cm\(^{-1}\) (\( \lambda > 1.67 \) μm).

The intensities were computed using an enhanced version of TROVE that utilizes the NVIDIA graphics processing units (GPU) allowing for the computation of 5000–30 000 transitions per second on a single GPU. The GPUs utilized were the nVidia M2090, K20 and the K40 models. A paper discussing this
will be published elsewhere (Al-Refaie, Tennyson & Yurchenko 2016).

3 RESULTS

The final hot line list named APTY contains 7560 352 states and almost 20 billion transitions that completely cover the 0–6 000 cm$^{-1}$ region. An extended line list is provided which contains an additional 8 billion transitions in the 6000–8000 cm$^{-1}$ region with reduced completeness for higher temperature. Radiative lifetimes are also computed via the methodology presented by Tennyson et al. (2016a). Fig. 2 presents the lifetimes computed for states up to 6000 cm$^{-1}$. Here the complicated rotational structure of H$_2$O$_2$ gives rise to long lived lower state energies with lifetimes that can reach up to 30 million years. Extract from the ExoMol-form (Tennyson, Hill & Yurchenko 2013; Tennyson et al. 2016b) states and transition files are given in Tables 4 and 5. Spectra at arbitrary temperatures can be computed using the Einstein-A coefficients from the transition files.

An estimate of the temperature applicability of the line list can be performed by checking the partition function convergence of equation (9), which is computed via explicit summation (Vidler & Tennyson 2000). The convergence can be measured by computing $(Q_{\text{tot}} - Q_{\text{tot} - 1})/Q_{\text{tot}}$ where $Q_{\text{tot}}$ is the partition function for all energy levels up to rotational excitation $J$. For 296 K, the partition function converges to within 0.001 per cent at $J = 37$ which matches the room-temperature line list’s $J$ limit of $J = 40$. At higher temperatures, it is well converged up to at least 1500 K where the estimated error is only 0.2 per cent at $J = 85$. This can be attributed to the good coverage of $J$ states computed that contribute to the overall population. A second partition $Q_{\text{lim}}$ can be evaluated by only including states that fall below the $hc$ 6 000 cm$^{-1}$ lower state energy limit of the line list and compared against $Q$ by computing the ratio $Q_{\text{lim}}/Q$ to assess the completeness of the full line list. Fig. 2 shows that at up to 800 K, the partition functions are essentially the same. At

| $f$ | $i$ | $A_f$ |
|-----|-----|------|
| 4893324 | 4865894 | 2.5558e-33 |
| 6064947 | 6004042 | 5.0185e-24 |
| 978337 | 813317 | 2.3963e-29 |
| 5920140 | 5983274 | 9.1885e-25 |
| 4882627 | 4920666 | 1.5740e-25 |
| 2328674 | 2173645 | 7.2614e-33 |

Note: $f$: upper state counting number; $i$: lower state counting number; $A_f$: Einstein-A coefficient in s$^{-1}$.  

Figure 2. $Q_{\text{lim}}/Q$ against temperature where $Q_{\text{lim}}$ is the partition function computed using only energy levels below our lower state threshold of 6000 cm$^{-1}$ and $Q$ is our estimate of the full partition function.
Table 6. Comparisons of H$_2$O$_2$ partition functions as function of temperature for this work those used in HITRAN (Fischer et al. 2003).

| $T$/K | APTY                         | HITRAN                    |
|-------|-------------------------------|---------------------------|
| 75    | 895.506                       | 894.866                   |
| 150   | 3215.866                      | 2811.187                  |
| 255   | 7360.598                      | 7336.856                  |
| 300   | 10 126.961                    | 10 087.090                |
| 500   | 31 246.17                     | 30 990.11                 |
| 1000  | 232 439.8                    | 226 152.5                 |
| 1500  | 1031 673.6                   | 993 983.8                 |
| 3000  | 21 847 680                   | 15 151 254                |

1250 K about 90 per cent of the population of states is represented by $Q_{\text{lim}}$ but this falls to $\approx$80 per cent at 1500 K giving the upper temperature for which APTY is reasonably complete as 1250 K. Usage of the line list at higher temperatures runs the risk of losing opacity due to missing contributions. The ratio $Q_{\text{lim}}/Q$ can be used to estimate this. However, the decomposition of H$_2$O$_2$ means that it is unlikely to be an important species above 1000 K. The partition function tabulated in steps of 1 K, alongside an associated cooling function, is given as Supplementary Information to this Paper.

Table 6 compares our partition function against HITRAN; we see that at temperatures less than 1000 K we agree better than 1 per cent. For temperatures 1000–1500 K, the APTY partition function is greater by 2–4 per cent suggesting that the explicit summation method gives higher, and probably better values, than the more approximate method used by HITRAN (Fischer et al. 2003). However, at 3000 K APTY’s $Q$ is 30 per cent lower than the HITRAN value which can be attributed to the eigenvalue cutoff of 12 000 cm$^{-1}$ and $J = 85$. Studies on ammonia and phosphine have shown that considerably extended lists of energy levels are required to get converged partition sums at these elevated temperatures (Sousa-Silva et al. 2014).

Fig. 3 is a simulated spectrum of the APTY line list computed at $T = 296$ K. This highlights the coverage and sheer number and

Figure 3. Overview of our synthetic spectrum at $T = 296$ K against HITRAN data (Rothman et al. 2013).

Figure 4. The fundamental bands compared to the HITRAN data base (Rothman et al. 2013) at $T = 296$ K: (a) Torsional and (b) $\nu_6$ bands.
Figure 5. The $\nu_1$ and $\nu_3$ band region with APTY against PNNL-IR data at 323.15 K (Sharpe et al. 2004) with HWHM = 0.300 cm$^{-1}$.

Figure 6. Cross-section comparison of peaks in the $\nu_1$ and $\nu_5$ band region with APTY (this work) and H2O2-2015 (Al-Refaie et al. 2015a) against PNNL-IR data at 323.15 K (Sharpe et al. 2004) with HWHM = 0.300 cm$^{-1}$.

Table 7. Band intensities, in $10^{-17}$ cm$^{-1}$/molecule cm$^{-2}$.

| Band            | Frequency range (cm$^{-1}$) | Ref.         | Obs    | Calc    | (O-C)/O (per cent) |
|-----------------|-----------------------------|--------------|--------|---------|--------------------|
| Torsional       | 0–1427                      | Perrin et al. (1996) | 4.0400 | 3.7450  | 7.3                |
| $\nu_3$         | 750–1100                     | Johnson et al. (2009) | 0.0157 | 0.0165  | -5.43              |
| $\nu_6$         | 1125–1393                    | Johnson et al. (2009) | 1.7458 | 1.7651  | -1.10              |
| $\nu_6$         | 1170–1380                    | Klee et al. (1999) | 1.8500 | 1.7633  | 4.68               |
| $\nu_6$         | 1170–1380                    | Perrin et al. (1995) | 1.0030 | 1.7633  | -75.80             |
| $\nu_2 + \nu_6$ | 2300–2900                    | Johnson et al. (2009) | 0.0830 | 0.055   | 33.78              |
| Multiple bands  | 2300–2900                    | Johnson et al. (2009) | 0.0830 | 0.0797  | 3.97               |
| $\nu_1, \nu_5$ region | 3300–3800                   | Johnson et al. (2009) | 0.8356 | 0.8724  | -4.40              |
density of transitions available compared to the current edition of the HITRAN data base (Rothman et al. 2013). Fig. 4 compares our results with specific regions in the HITRAN data base, the torsional and $v_5$ bands. Comparisons of the two show excellent agreement in replicating both line position and intensities.

Our line list in the $v_1$ and $v_5$ band regions can be validated by simulating absorption cross-sections in the 2.7 μm region and comparing against PNNL-IR data (Sharpe et al. 2004); see Fig. 5. The structure and positions are in good agreement with the overall integrated intensity for APTY in this region being 3 per cent stronger than PNNL. The improvement given by the H2O2-2016 PES can be demonstrated by comparing with our previous room-temperature line list and with the PNNL-IR data; see Fig. 6. The wavelengths of the largest two peaks in this band at 2.738 and 2.736 μm are correctly reproduced by APTY but are shifted by about 0.001 μm for H2O2-2015, showing the improvement in this band due to use of the refined PES. Overall the integrated cross-sections for this band differs only by 3 per cent from PNNL; indeed the entire spectrum up to 6000 cm$^{-1}$ only differs by 3 per cent.
Band intensities can be computed by explicit summation of all transitions within a band and compared against available data. Table 7 shows that for the limit available empirical band intensities we agree with most regions to $\pm 7.3\%$ per cent which is below the estimated experimental uncertainty of $\approx \pm 10\%$ per cent. Two discrepancies are with the $v_2$ band from Perrin et al. (1995) and the $v_2 + v_5$ band from Johnson et al. (2009). The former conflicts with other measurements due to Johnson et al. (2009) and Klee et al. (1999) where integrated absorption intensities were measured directly, whilst the band intensities of Perrin et al. (1995) were obtained by summing a synthetic spectrum of only 27 276 transitions. Our $v_2 + v_5$ band intensity is 33.78 per cent weaker than the experimentally derived value. Johnson et al. (2009) suggests that the assignment of this band is $v_2 + v_5$ compared to the $v_2 + v_3 + v_4$ assignment by Giguere (1950). This is based on a Q-branch peak observed at 2658.62 cm$^{-1}$. The assignments from APTY suggest that the peak observed is actually a convolution of Q-branches of the $(0, 4) \rightarrow v_2 + (0, 4), (0, 4) \rightarrow v_2 + (0, 2) + v_5, (0, 4) \rightarrow v_3 + (4, 4), (0, 4) \rightarrow v_2 + (2, 2), (0, 4) \rightarrow 2v_3 + (2, 3)$ and $(0, 4) \rightarrow v_3 + (2, 2) + v_5$ transitions with an average separation between them at $\approx 0.02$ cm$^{-1}$. Computing the band intensities of all of these bands in this region gives an answer that agrees with value given by Johnson et al. (2009) to 3.97 per cent.

The Kitt Peak Archive provides FTIR spectra of H$_2$O$_2$ covering the wavenumber region up to 6422 cm$^{-1}$, which is only partly assigned. Fig. 7 (lower display) shows an uncalibrated spectrum of H$_2$O$_2$ in the 1.78 $\mu$m region (800628R0.002) recorded by R. H. Hunt in 1980, which covers the $v_1$ and $v_5$ fundamental bands of the hydrogen peroxide. To the best our knowledge these two bands have not been spectroscopically analysed. The upper display of this figure presents our absorption spectrum at 296 K simulated using the Doppler line profile. Our synthetic spectrum resembles all the main features of the experimental data. We would like to encourage a spectroscopic analysis of the Kitt Peak H$_2$O$_2$ spectra in the IR and near-IR regions currently not present in HITRAN. We believe that our theoretical line list with a capability of providing absolute intensities and quantum numbers can assist in the assignment of these spectra.

Fig. 8 presents integrated absorption cross-sections computed using a Doppler profile (Hill, Yurchen & Tennyson 2013) for a range of temperatures. The figure shows how the opacity changes with increasing temperature. We note the particularly dramatic effect raising the temperature has on the absorption by H$_2$O$_2$ in the 13.7 $\mu$m region. This smoothing in the overall spectra can only be modelled if there is adequate coverage and population of rotationally and vibrationally excited states. We also note the strength of the OH stretch feature at about 2.75 $\mu$m; these features are absent from line data bases such as HITRAN because of the absence of assigned spectra in this region. Hopefully APTY can be used to help analyse spectra in this region, as the BYTe NH$_3$ line list is being used to analyse ammonia spectra (Barton et al. 2015, 2016).

4 CONCLUSION

The frequency and Einstein-A coefficients of almost 20 billion transitions of hydrogen peroxide are computed. These transitions cover wavelengths longer than 1.6 $\mu$m and include all rotational excitations up to $J = 85$, making the line list applicable for temperatures up to 1250 K. The line list gives a room-temperature spectrum in excellent agreement with available experimental data and has good predictive ability for bands and line-positions not available experimentally. The new line list may be accessed via www.exomol.com or http://cdsarc.u-strasbg.fr/viz-bin/qcat?J/MNRAS/. The cross-sections of H$_2$O$_2$ can be also generated at www.exomol.com as described by Tennyson et al. (2016b).

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Supporting Information

Additional Supporting Information may be found in the online version of this article:

Table 4. Extract from the H$_2$O$_2$ state file.
Table 5. Extracts from the H$_2$O$_2$ transitions file.

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