A new combination approach based on improved evidence distance

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Abstract

Dempster-Shafer evidence theory is a powerful tool in information fusion. When the evidence are highly conflicting, the counter-intuitive results will be presented. To adress this open issue, a new method based on evidence distance of Jousselme and Hausdorff distance is proposed. Weight of each evidence can be computed, preprocess the original evidence to generate a new evidence. The Dempster’s combination rule is used to combine the new evidence. Comparing with the existing methods, the new proposed method is efficient.

Keywords: Evidence theory, Conflict evidence, Evidence distance, Combination rule, Target recognition

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1. Introduction

Dempster-Shafer evidence theory\cite{1, 2} has attracted more and more attentions recently years. It can handle with uncertain and incomplete information in many fields, such as target recognition, information fusion and decision making\cite{3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21}. While the evidence are highly conflicting, the Dempster’s combination rule will generate counter-intuitive results, such as the typical conflictive example proposed by Zadeh\cite{22}. In the last decade researchers have proposed many approaches to cope with this open issue and certain effort have been obtained. The existing methods can be mainly classified into two categories. The first strategy regards that Dempster’s combination rule is incomplete and modifying the combination rule as alternative, such as Yager’s method\cite{23}, Smet’s method\cite{24, 25} and Lefevre’s method\cite{26}, etc. The second strategy believes that Dempster’s rule has perfect theoretical foundation and preprocessing the original evidence before combination, such as Haenni’s method\cite{27}, Murphy’s method\cite{28} and Deng’s method\cite{29}, etc. We believe that Dempster’s rule is excellent and has been widely applied in recent years. In this paper, preprocessing the original evidence for highly conflicting is adopted. The method of Deng proposed\cite{29} in 2004 based on the evidence distance can deal with the conflicting evidence and that the correct sensor can be quickly recognized. The evidence distance of Deng’s method reflects the difference between evidences distance roughly, but can not reflect the degree of difference. In this paper, we propose a new method weighted averaging the evidence, improving Deng’s method\cite{29}. The new method takes both Jousselme\cite{30} and Hausdorff\cite{31} evidence distance into account. Thus, the weights of evidence are more appropriate.

The remainder of this paper is organized as follows. Section 2 presents some preliminaries. The proposed method is presented in section 3. Numerical examples and applications are used to demonstrate the validity of the proposed method in section 4. A short conclusion is drawn in the last section.

2. Preliminaries

In this section, some concepts of Dempster-Shafer evidence theory\cite{1, 2} are briefly recalled. For more information please consult Ref.\cite{32}. The Dempster-Shafer evidence theory is introduced by Dempster and then developed by Shafer.
In Dempster-Shafer evidence theory, let \( \Theta = \{\theta_1, \theta_2, \ldots, \theta_n\} \) be the finite set of mutually exclusive and exhaustive elements. It is concerned with the set of all subsets of \( \Theta \), which is a powerset of \( 2^{\Theta} \), known as the frame of discernment, denotes as

\[
\Omega = \{\emptyset, \{\theta_1\}, \{\theta_2\}, \ldots, \{\theta_n\}, \{\theta_1, \theta_2\}, \ldots, \{\theta_1, \theta_1, \ldots, \theta_n\}\}
\]

The mass function of evidence assigns probability to the subset of \( \Omega \), also called basic probability assignment (BPA), which satisfies the following conditions

\[
m(\emptyset) = 0, 0 \leq m(A) \leq 1, \sum_{A \subseteq \Theta} m(A) = 1.
\]

\( \emptyset \) is an empty set and \( A \) is any subsets of \( \Theta \).

Dempster’s combination rule\[^1, 2\] is the first one within the framework of evidence theory which can combine two BPAs \( m_1 \) and \( m_2 \) to yield a new BPA \( m \). The rule of Dempster’s combination is presented as follows

\[
m(A) = \frac{1}{1 - k} \sum_{B \cap C = A} m_1(B)m_2(C) \quad (1)
\]

with

\[
k = \sum_{B \cap C = \emptyset} m_1(B)m_2(C) \quad (2)
\]

Where \( k \) is a normalization constant, namely the conflict coefficient of BPAs.

3. New combination approach

The method of Murphy\[^28\] purposed regards each BPA as the same role, little relevant to the relationship among the BPAs. In Deng’s weighted method\[^29\], each BPAs play different roles, that depended on the extent to which they are accredited in system. The similarity of Deng’s method between two BPAs is ascertained by Jousselme distance function\[^30\].

3.1. Two existing evidence distance

The evidence distance proposed by Jousselme\[^30\] is presented as follows
Definition 1. Let $m_1$ and $m_2$ be two BPAs defined on the same frame of discernment $\Theta$, containing $N$ mutually exclusive and exhaustive hypotheses. The metric $d_{BPA}$ can be defined as follows

$$d_{BPA}(m_1, m_2) = \sqrt{\frac{1}{2} (m_1 - m_2)^T D (m_1 - m_2)}$$ (3)

$D$ is a $2^N \times 2^N$ similarity matrix, indicates the conflict of focal element in $m_1$ and $m_2$, where

$$D(A, B) = \frac{|A \cap B|}{|A \cup B|}$$ (4)

$|A \cup B|$ is the cardinality of subset of the union $A$ and $B$, where $A$ and $B$ may belong to the same BPA or come from different BPAs. $|A \cap B|$ indicates the conflict degree between elements $A$ and $B$. When two elements have no common object, they are highly conflicting.

Another evidence distance proposed by Sunberg\cite{33} is presented as follows

Definition 2. Let $m_1$ and $m_2$ be two BPAs defined on the same frame of discernment $\Theta$, containing $N$ mutually exclusive and exhaustive hypotheses. The distance of two BPAs referred to as $d_{Haus}$ is defined as follows

$$d_{Haus}(m_1, m_2) = \sqrt{\frac{1}{2} (m_1 - m_2)^T D_H (m_1 - m_2)}$$ (5)

with

$$D_H(i,j) = S_H(A_i, A_j) = \frac{1}{1 + CH(A_i, A_j)}$$ (6)

Where $H(A_i, A_j)$ is the Hausdorff distance\cite{31} between focal elements $A_i$ and $A_j$. $A_i$ and $A_j$ may belong to the same BPA or come from different BPAs. Positive number $C$ is a user-defined tuning parameter. $C$ is set to be 1, in this paper, for simplicity. It is defined according to

$$H(A_i, A_j) = \max\{\sup_{b \subseteq A_i} \inf_{c \subseteq A_j} d(b, c), \sup_{c \subseteq A_j} \inf_{b \subseteq A_i} d(b, c)\}$$ (7)

Where $d(b, c)$ is the distance between two elements of the sets and can be defined as any valid metric distance on the measurement space\cite{31}. 

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While the elements are real numbers, the Hausdorff distance may be simplify as

\[ H_R(A_i, A_j) = \max\{|\min(A_i) - \min(A_j)|, |\max(A_i) - \max(A_j)|\} \]  \hspace{1cm} (8)

The below example is used to illustrate the difference between Jousselme distance[30] and Hausdorff distance[31].

**Example 1.** There are five orderable mutually exclusive and exhaustive hypotheses elements: 1, 2, 3, 4 and 5 on the same frame of discernment \( \Theta \).

By (4), the Jousselme distance matrix \( D \) between each elements in a BPA can be obtained as follows

\[
D = \begin{bmatrix}
1 & 0 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 & 0 \\
0 & 0 & 1 & 0 & 0 \\
0 & 0 & 0 & 1 & 0 \\
0 & 0 & 0 & 0 & 1 \\
\end{bmatrix}
\]

Utilize Hausdorff distance in (6), the Hausdorff distance matrix \( D_H \) between each elements in a BPA can be obtained as follows

\[
D_H = \begin{bmatrix}
\frac{1}{3} & \frac{1}{3} & \frac{1}{3} & \frac{1}{3} & \frac{1}{3} \\
\frac{1}{3} & \frac{1}{3} & \frac{1}{3} & \frac{1}{3} & \frac{1}{3} \\
\frac{1}{3} & \frac{1}{3} & \frac{1}{3} & \frac{1}{3} & \frac{1}{3} \\
\frac{1}{3} & \frac{1}{3} & \frac{1}{3} & \frac{1}{3} & \frac{1}{3} \\
\frac{1}{3} & \frac{1}{3} & \frac{1}{3} & \frac{1}{3} & \frac{1}{3} \\
\end{bmatrix}
\]

It is clearly that, the five elements have no object in common. The similarity between each elements are the same value zero in Jousselme distance matrix. In case of this, Jousselme distance matrix can not show the detailed distance of each elements in an orderable system. However, Hausdorff distance matrix can calculate the detail similarity between each orderable elements.

### 3.2. New combination approach

In this subsection, we purpose an improved combination approach based on Deng’s method[29]. The new method takes advantage of Hausdorff distance[31] to update Jousselme distance[30].
Definition 3. Let \( m_1 \) and \( m_2 \) be two BPAs defined on the same frame of discernment \( \Theta \), containing \( N \) mutually exclusive and exhaustive hypotheses. The distance between \( m_1 \) and \( m_2 \) can be defined as

\[
d_{Com}(m_1, m_2) = \sqrt{\frac{1}{2} (m_1 - m_2)^T D_{Com} (m_1 - m_2)}
\]

with

\[
D_{Com}(i, j) = D(i, j) \ast D_H(i, j)
\]

\( D_{Com} \) is a \( 2^N \times 2^N \) similarity matrix, indicates the metric of focal elements in \( m_1 \) and \( m_2 \). \( D(i, j) \) is the distance matrix in (4) and \( D_H(i, j) \) is the distance matrix in (6).

Given there are \( n \) BPAs in the system, we can calculate the distance between each two BPAs. Thus, the distance matrix is presented as follows

\[
DIM = \begin{bmatrix}
1 & d_{12} & \cdots & d_{1j} & \cdots & d_{1n} \\
\vdots & \vdots & \ddots & \vdots & \cdots & \vdots \\
d_{i1} & d_{i2} & \cdots & d_{ij} & \cdots & d_{in} \\
\vdots & \vdots & \ddots & \vdots & \cdots & \vdots \\
d_{n1} & d_{n2} & \cdots & d_{nj} & \cdots & 1
\end{bmatrix}
\]

Definition 4. Let \( Simi(m_i, m_j) \) be the similarity value between BPA \( m_i \) and \( m_j \), thus the \( Simi(m_i, m_j) \) can be defined as

\[
Simi(m_i, m_j) = 1 - d_{Com}(m_i, m_j)
\]

It is obvious that while the value of distance between two BPAs are bigger, the similarity of two BPAs are smaller, and vice versa. The similarity function can be represented by a matrix as follows

\[
SIM = \begin{bmatrix}
1 & Simi_{12} & \cdots & Simi_{1j} & \cdots & Simi_{1n} \\
\vdots & \vdots & \ddots & \vdots & \cdots & \vdots \\
Simi_{i1} & Simi_{i2} & \cdots & Simi_{ij} & \cdots & Simi_{in} \\
\vdots & \vdots & \ddots & \vdots & \cdots & \vdots \\
Simi_{n1} & Simi_{n2} & \cdots & Simi_{nj} & \cdots & 1
\end{bmatrix}
\]
Definition 5. Let $\text{Supp}(m_i)$ be the support degree of BPA $m_i$ in the system, and the support degree of BPA $m_i$ can be presented as follow

$$\text{Supp}(m_i) = \sum_{j=1}^{n} \text{Simi}(m_i, m_j)$$ (14)

From (13) and (14), we can see that the support degree $\text{Supp}(m_i)$ is the sum of similarity between each BPAs, except itself. The larger the value of $\text{Supp}(m_i)$ is, the more important the evidence will be.

To normalize $\text{Supp}(m_i)$, the $W(m_i)$ of BPA $m_i$ can be obtained as follows

$$W(m_i) = \frac{\text{Supp}(m_i)}{\sum_{i=1}^{n} \text{Supp}(m_i)}$$ (15)

It is obvious that

$$\sum_{i=1}^{n} W(m_i) = 1$$

$W(m_i)$ indicates the important and credible degree of BPA $m_i$ among all BPAs in the system. It can be regard as the weight of BPA $m_i$. After obtained the weight of each BPAs, we take advantage of Dempster’s combination rule\cite{1, 2} to yield a new BPA.

The below example is used to demonstrate the detail processes of the new proposed method.

Example 2. Given there are four BPAs $m_1$, $m_2$, $m_3$ and $m_4$ on the same frame of discernment $\Theta$:

\begin{align*}
  m_1(R) &= 0.3, m_1(S) = 0.5, m_1(T) = 0.2 \\
  m_2(R) &= 0, m_2(S) = 0.5, m_2(T) = 0.5 \\
  m_3(R) &= 0.6, m_3(S) = 0.2, m_3(T) = 0.2 \\
  m_4(R) &= 0.9, m_4(S) = 0, m_4(T) = 0.1
\end{align*}

By (9)-(15), we can obtain the weight of the four BPAs $m_1$, $m_2$, $m_3$ and $m_4$ as follows

$$W(m_1) = 0.2688, W(m_2) = 0.2276, W(m_3) = 0.2752, W(m_4) = 0.2284.$$
Therefore, the new BPA $m_{\text{New}}$ before combination can be obtained as follows

\begin{align*}
  m_{\text{New}}(R) &= 0.3 \times 0.2688 + 0 \times 0.2276 + 0.6 \times 0.2752 + 0.9 \times 0.2284 = 0.4513 \\
  m_{\text{New}}(S) &= 0.5 \times 0.2688 + 0.5 \times 0.2276 + 0.2 \times 0.2752 + 0 \times 0.2284 = 0.3033 \\
  m_{\text{New}}(T) &= 0.2 \times 0.2688 + 0.5 \times 0.2276 + 0.2 \times 0.2752 + 0.1 \times 0.2284 = 0.2454
\end{align*}

There are four BPAs in this example, we apply Dempster’s combination rule to combine the new BPA $m_{\text{New}}$ three times, the results are presented as follows

\begin{align*}
  m(R) &= 0.7744, m(S) = 0.1579, m(T) = 0.0677.
\end{align*}

4. Numerical examples and Applications

It is known that Dempster-Shafer evidence theory\cite{1,2} needs less information than Bayes probability to deal with uncertain information. It is often regarded as the extension of Bayes probability.

We utilize the below example to illustrate the effectiveness of the new proposed method.

**Example 3.** There are five mass functions on the same frame of discernment, the five BPAs are presented as follows\cite{29}

\begin{align*}
  m_1 &: m_1(A) = 0.5, m_1(B) = 0.2, m_1(C) = 0.3 \\
  m_2 &: m_2(A) = 0, m_2(B) = 0.9, m_2(C) = 0.1 \\
  m_3 &: m_3(A) = 0.55, m_3(B) = 0.1, m_3(C) = 0.35 \\
  m_4 &: m_4(A) = 0.55, m_4(B) = 0.1, m_4(C) = 0.35 \\
  m_5 &: m_5(A) = 0.55, m_5(B) = 0.1, m_5(C) = 0.35
\end{align*}

The results of different methods to combine the five BPAs are presented in Table\cite{1}. From Table\cite{1} we can see that Dempster’s combination rule\cite{1,2} can not handle with highly conflicting evidence. Once an element is negativ ed by any BPAs, no matter how strongly it is supported by other BPAs, its probability will always remain zero.

Murphy’s method\cite{28} regards each evidence plays the same role in the system, considered little relations among evidences. Deng\cite{29} improved Murphy’s work and took advantage of an evidence distance as the weight of each evidence. The novel proposed method based on Deng’s method, but utilizes Hausdorff distance to update the distance matrix. Fig\cite{1} indicates that the convergence speed of proposed method is slower than Deng’s method but faster than Murphy’s method, owing to the additional update distance because some sensors may be orderable.
Table 1: Different combination rules to combine highly conflicting evidence.

|                     | $m_1, m_2$ | $m_1, m_2, m_3$ | $m_1, m_2, m_3, m_4$ | $m_1, m_2, m_3, m_4, m_5$ |
|---------------------|------------|-----------------|----------------------|---------------------------|
| Dempster’s combination rule [1, 2] | $m(A) = 0$ | $m(A) = 0$ | $m(A) = 0$ | $m(A) = 0$ |
|                      | $m(B) = 0.8571$ | $m(B) = 0.6316$ | $m(B) = 0.3288$ | $m(B) = 0.1228$ |
|                      | $m(C) = 0.1429$ | $m(C) = 0.3684$ | $m(C) = 0.6712$ | $m(C) = 0.8772$ |
| Murphy’s combination rule [28] | $m(A) = 0.1543$ | $m(A) = 0.3500$ | $m(A) = 0.6027$ | $m(A) = 0.7958$ |
|                      | $m(B) = 0.7469$ | $m(B) = 0.5224$ | $m(B) = 0.2627$ | $m(B) = 0.0932$ |
|                      | $m(C) = 0.0988$ | $m(C) = 0.1276$ | $m(C) = 0.1346$ | $m(C) = 0.1110$ |
| Deng’s combination rule [29] | $m(A) = 0.1543$ | $m(A) = 0.5816$ | $m(A) = 0.8060$ | $m(A) = 0.8909$ |
|                      | $m(B) = 0.7469$ | $m(B) = 0.2439$ | $m(B) = 0.0482$ | $m(B) = 0.0086$ |
|                      | $m(C) = 0.0988$ | $m(C) = 0.1745$ | $m(C) = 0.1458$ | $m(C) = 0.1005$ |
| New proposed combination rule | $m(A) = 0.1543$ | $m(A) = 0.6355$ | $m(A) = 0.7605$ | $m(A) = 0.8761$ |
|                      | $m(B) = 0.7469$ | $m(B) = 0.2229$ | $m(B) = 0.0897$ | $m(B) = 0.0189$ |
|                      | $m(C) = 0.0988$ | $m(C) = 0.1415$ | $m(C) = 0.1468$ | $m(C) = 0.1050$ |
5. Conclusion

Dempster-Shafer evidence theory is a powerful tool to deal with uncertain and imprecise information in widely fields. However the evidence collected may be multifarious, some of them may be highly conflicting, owing to various noise factors, subjective or objective. The original Dempster combination rule can do nothing for these highly conflicting evidence. Modified methods of Dempster’s combination rule are briefly introduced, and all of them have some drawbacks. The new proposed method inherits all the advantages of Deng’s method. It applies Hausdorff distance to update the Jousselme distance and takes more distance information into account. Numerical examples demonstrate that the new proposed method can discern the correct target, effectively.
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