On the Consistence Conditions to Braneworlds Sum Rules within Scalar-Tensor Gravity for Arbitrary Dimensions

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We derive an one-parameter family of consistence conditions to braneworlds in the Brans-Dicke gravity. The sum rules are constructed in a completely general frame and they reproduce the conditions already obtained in General Relativity theory just by using a right limit of the Brans-Dicke parameter.

PACS numbers: 04.50.+h; 98.80Cq

I. INTRODUCTION

In the recent years braneworld models are consolidating a new branch of high energy physics. Among many interesting physical insights \cite{1}, they provide an elegant solution to the hierarchy problem \cite{2, 3, 4}. The development of new braneworld models is increasing in direct proportion to the possibilities raised in the scope of a extra dimensional world.

On the other hand, formal advances of string theory point into a scalar tensorial theory as an effective theory which gives the right approach to the gravitational phenomena \cite{5}. Unification models such as supergravity, superstrings and M-theory \cite{6} effectively predict the existence of a scalar gravitational field acting as a mediator of the gravitational interaction together with the usual rank-2 tensorial field. In this context, the analysis of braneworld consistence conditions are indeed necessary. Models with large extra dimensions, in which our universe is usually performed as a brane, have typical size scales much larger than the scales involved in quantum gravity. Therefore, the sum rules arise manly in a gravitational theory scenario. These sum rules were obtained in ref. \cite{7} for five dimensions in General Relativity\textsuperscript{1} (GR) and extended to an arbitrary number of

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\textsuperscript{1} It is directly applied in the Randall-Sundrum model \cite{3, 4}. 

dimensions in ref. [8], also in GR.

The main purpose of this work is to generalize the braneworld sum rules, already obtained in the GR set up, to the case of the scalar tensorial gravity. In particular, we work with the Brans-Dicke (BD) theory [9], since it is the simplest scalar-tensor theory we have in the literature. The paper is structured as follows: in the next Section we establish the notation and then, generalize to the case of the BD gravity framework. After arriving to the main general results we particularize the analysis to braneworld models obtained in ref. [10]. In the final Section we conclude with some remarks.

II. GENERALIZED SUM RULES

In this section we obtain the generalized braneworld sum rules within the BD gravity framework. First we fix the notation and, since the geometrical part of the Einstein’s equation is the same as in the BD case, we derive the basic setup to obtain the consistence conditions following the prescription of ref. [8]. After some general considerations, we extend the analysis in order to incorporate the scalar field terms (generically called as dilaton here). Basically, we obtain an one-parameter family of consistence conditions to the BD case by a circular integration in the internal compact space.

A. Notation and Conventions

Following the standard notation used in refs. [7, 8], we analyze a D-dimensional bulk spacetime endowed with a non-factorable geometry, which metric is given by

\[ ds^2 = G_{AB}dX^A dX^B = W^2(r)g_{\alpha\beta}dx^\alpha dx^\beta + g_{ab}(r)dr^a dr^b , \]

where \( W^2(r) \) is the warp factor, assumed to be a smooth integrable function, \( X^A \) denotes the coordinates of the full D-dimensional spacetime, \( x^\alpha \) stands for the \( (p + 1) \) non-compact coordinates of the spacetime and \( r^a \) labels the \( (D - p - 1) \) directions in the internal compact space\(^2\). Note that this type of metric encodes the possibility of existing \( q \)-branes \( (q > p) \) [8]. In this case, the \( (q - p) \) extra dimensions are compactified on the brane and constitute part of the internal space. This possibility is important in the hybrid compactification models context.

The D-dimensional spacetime Ricci tensor can be related with the brane Ricci tensor as well as

\(^2\) As an example, if \( D = 5, \ p = 3 \) and \( W(r) = e^{-2k|r|} \) one arrives to the Randall-Sundrum model.
with the internal space partner by the equations

\[ R_{\mu\nu} = \tilde{R}_{\mu\nu} - \frac{g_{\mu\nu}}{(p + 1)W^{p-1}}\nabla^2 W^{p+1}, \tag{2} \]

and

\[ R_{ab} = \tilde{R}_{ab} - \frac{p + 1}{W} \nabla_a \nabla_b W, \tag{3} \]

where \( \tilde{R}_{ab} \), \( \nabla_a \) and \( \nabla^2 \) are respectively the Ricci tensor, the covariant derivative and the Laplacian operator constructed by the internal space metric \( g_{ab} \). \( \tilde{R}_{\mu\nu} \) is the Ricci tensor derived from \( g_{\mu\nu} \). Let us denote the three curvature scalars by \( R = G_{AB} R^{AB}, \tilde{R} = g^{\mu\nu} \tilde{R}_{\mu\nu} \) and \( \tilde{R} = g^{ab} \tilde{R}_{ab} \). Therefore, the traces of equations (2) and (3) give

\[ \frac{1}{p + 1} \left( W^{-2} \tilde{R} - R_{\mu}^{\mu} \right) = pW^{-2} \nabla W \cdot \nabla W + W^{-1} \nabla^2 W \tag{4} \]

and

\[ \frac{1}{p + 1} \left( \tilde{R} - R_a^{a} \right) = W^{-1} \nabla^2 W, \tag{5} \]

where \( R_{\mu}^{\mu} \equiv W^{-2} g^{\mu\nu} R_{\mu\nu} \) and \( R_a^{a} \equiv g^{ab} R_{ab} \) (in such a way that \( R = R_{\mu}^{\mu} + R_a^{a} \)). It is not difficult to see that, if \( \xi \) is an arbitrary constant,

\[ \nabla \cdot (W^{\xi} \nabla W) = W^{\xi+1} (W^{-2} \nabla W \cdot \nabla W + W^{-1} \nabla^2 W). \tag{6} \]

The combination of the equations (4), (5) and (6) leads to

\[ \nabla \cdot (W^{\xi} \nabla W) = \frac{W^{\xi+1}}{p(p + 1)} \left[ \xi (W^{-2} \tilde{R} - R_{\mu}^{\mu}) + (p - \xi) (\tilde{R} - R_a^{a}) \right]. \tag{7} \]

This is a very important relation. In particular, it will provide the consistence conditions since the left-hand side must vanish under a closed integration over the internal compact space.

**B. The Scalar-Tensor Gravity Case**

Once established the usual notation and conventions, it is time to look at the dilaton field \( (\phi) \) contributions. The Einstein-Brans-Dicke (EBD) equation is given by

\[
R_{MN} - \frac{1}{2} G_{MN} R = \frac{8\pi}{\phi} T_{MN} + \frac{w}{\phi^2} \left( \nabla_M \phi \nabla_N \phi - \frac{1}{2} \nabla_A \phi \nabla^A \phi G_{MN} \right) \\
+ \frac{1}{\phi} \left( \nabla_M \nabla_N \phi - \frac{8\pi}{3 + 2w} T G_{MN} \right), \tag{8}
\]
where $T_{MN}$ is the matter stress-tensor (everything except $\phi$), $T$ is the trace and $w$ the BD parameter. We remark that the scalar part of the BD set of equations was already taken into account in the last term of the right-hand side of eq. (8). From eq. (8) it is easy to note that

$$R_{MN} = \frac{8\pi}{\phi} \left( T_{MN} + \frac{2(1+w)T}{(2-D)(3+2w)}G_{MN} \right) + \frac{w}{\phi^2} \nabla_M \phi \nabla_N \phi + \frac{1}{\phi} \nabla_M \nabla_N \phi. \quad (9)$$

Now, if we call $T^\mu_{\mu} \equiv W^{-2}g^{\mu\nu}T_{\mu\nu} \ (T = T^\mu_{\mu} + T_m^m)$, it is possible to express $R^\mu_{\mu}$ and $R^m_m$ by

$$R^\mu_{\mu} = \frac{8\pi}{\phi(D-2)(3+2w)} \left( (3D+2w(D-p-3)-2(p+1))T^\mu_{\mu} - 2(1+w)(p+1)T_m^m \right) + \frac{wW^{-2}}{\phi^2} \nabla^\nu \phi \nabla_\nu \phi + \frac{W^{-2}}{\phi} \nabla^\nu \nabla_\nu \phi, \quad (10)$$

and

$$R^m_m = \frac{8\pi}{\phi(D-2)(3+2w)} \left( (D+2w(p-1)-2(p-2))T_m^m - 2(1+w)(D-p-1)T^\mu_{\mu} \right) + \frac{w}{\phi^2} \nabla^m \phi \nabla_m \phi + \frac{1}{\phi} \nabla^m \nabla_m \phi. \quad (11)$$

Inserting these last two equations\(^3\) in (7) one has

$$\nabla \cdot (W^2 W) = \frac{W^{\xi+1}}{p(p+1)} \left( \xi W^{-2} \tilde{R} + (p-\xi)\tilde{R} \right) - \frac{8\pi}{\phi(D-2)(3+2w)} \left( T^\mu_{\mu} \left[ \xi(5D+4w(D-p-2)-2(2p+5))-2p(1+w)(D-p-1) \right] + T_m^m \left[ \xi(-4wp - D+2(1-2p)) + p(D-p-2) + 2w(p-1) \right] \right) - \frac{w}{\phi^2} \left[ \xi W^{-2} \nabla^\mu \phi \nabla_\mu \phi + (p-\xi)\nabla^m \phi \nabla_m \phi \right] - \frac{1}{\phi} \left[ \xi W^{-2} \nabla^\mu \nabla_\mu \phi \right] + (p-\xi) \nabla^m \nabla_m \phi \right). \quad (12)$$

An important characteristic about braneworld models in the BD theory is that, generally, the scalar field depends only on the large extra dimensions \(^{10}\). This type of dependence is indeed useful since the projected EBD equation on the brane leads to important subtle modifications but still resembles the Einstein’s equation \(^{11}\). Keeping it in mind, let us considerer hereon the $\nabla_\mu \phi = 0$ case. In an internal compact space the follow identity is respected

$$\oint \nabla \cdot (W^\xi W) = 0, \quad (13)$$

\(^3\) It is important to remark that when one takes the limit $w \to \infty \ (\phi \to 1/G_N)$ the expressions $R^\mu_{\mu}$ and $R^m_m$ recover the case analyzed in General Relativity.
therefore from eq. (12) we have

\[ \oint W^{\xi+1} \left( T_{\mu}^{\nu} \left[ \xi((5D + 4w(D - p - 2)) - 2(2p + 5)) - 2p(1 + w)(D - p - 1) \right] + T_{\mu}^{m} \left[ \xi(-4wp - D + 2(1 - 2p)) + p(D - 2(p - 2) + 2w(p - 1))] \right) - \left( \frac{8\pi}{\phi} \right)^{-1}(D - 2)(3 + 2w) \times \left[ \xi W^{-2} \hat{R} + (p - \xi) \hat{R} \right] + \left( \frac{8\pi}{\phi} \right)^{-1}(D - 2)(3 + 2w) \left( \frac{u}{\phi^2} \nabla^m \phi \nabla_m \phi + \frac{1}{\phi} \nabla^m \nabla_m \phi \right) \right) = 0. \]  

The above equation provides an one-parameter family of consistence conditions in arbitrary dimensions in the scope of the BD gravity. It is important to stress that if one reintroduces the \((3+2w)^{-1}\) factored term and takes \(w \to \infty\) \((\phi \to 1/G_N)\), by the usual L’Hôpital limit, the case analyzed in General Relativity is recovered as expected\(^4\).

Equation (14) is quite general and self-consistent. However, in that form it is not very useful. Going forward, we rewrite eq. (14) in terms of an adequate energy-momentum tensor form, after what the generalized sum rules may be applied to any particular model. We shall use the same stress-tensor \textit{Ansatz} of ref. [8], since it is very complete. So, we write the stress-tensor in the form

\[ T_{MN} = -\Lambda G_{MN} - \sum_i T_q^{(i)} P[G_{MN}]^{(i)} \Delta^{(D-q-1)}(r - r_i) + \tau_{MN}, \]  

where \(\Lambda\) is the cosmological constant, \(T_q^{(i)}\) is the \(i\)-th q-brane tension, \(\Delta^{(D-q-1)}(r - r_i)\) is the covariant combination of delta functions which positions the brane\(^5\), \(P[G_{MN}]^{(i)}\) is the pull-back of the bulk metric and any other matter contribution is due \(\tau_{MN}\). From this \textit{Ansatz} one obtains

\[ T_{\mu}^{\nu} = -(p + 1)\Lambda + \tau_{\mu}^{\nu} - \sum_i T_q^{(i)} \Delta^{(D-q-1)}(r - r_i)(p + 1), \]  

and

\[ T_{\mu}^{\nu} = -(D - p - 1)\Lambda + \tau_{\mu}^{m} - \sum_i T_q^{(i)} \Delta^{(D-q-1)}(r - r_i)(q - p). \]  

Now, substituting these expressions in equation (14) we get, after some algebra, the following form to the consistence conditions

\[ \oint \frac{W^{\xi+1}}{\phi} \left( -\Lambda(c(p + 1) + bD) - \sum_i (cp + a + bq)T_q^{(i)} \Delta^{(D-q-1)}(r - r_i) + aT_{\mu}^{\nu} + bT_{\mu}^{m} \right) \]

\[ - \left( \frac{8\pi}{\phi} \right)^{-1}(D - 2)(3 + 2w) \left[ \xi W^{-2} \hat{R} + (p - \xi) \hat{R} \right] + \left( \frac{8\pi}{\phi} \right)^{-1}(D - 2)(3 + 2w) \times \left( \frac{u}{\phi^2} \nabla^m \phi \nabla_m \phi + \frac{1}{\phi} \nabla^m \nabla_m \phi \right) = 0, \]  

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\(^4\) See, for instance, eq. (13) of reference [8].

\(^5\) For a complete discussion about the expression of \(\Delta^{(D-q-1)}(r - r_i)\) see the Appendix of ref. [8].
where \( a \equiv \xi[5D + 4w(D - p - 2) - 2(2p + 5)] - 2p(1 + w)(D - p - 1), \ b \equiv \xi[-4wp - D + 2(1 - 2p)] + p[D - 2(p - 2) + 2w(p - 1)] \) and \( c \equiv a - b \).

The above considerations can be considerably simplified if the scalar field has a power-law form. Note that if \( \varphi = [(w + 1)(C_1 r + C_2)]^{w+1} \), (19) disappears. Therefore, there exist a particular solution of the dilaton field in which the “kinetic” term does not intervene in any consistence condition. In other words, in this solution the derivative part of the scalar field does not impose an additional constraint to the model. It is a quite interesting characteristic. However, it is not an usual solution to the scalar field [10]. Moreover, in models within BD gravity the explicit form of the \( \varphi \) field composes the warp factor. So, solutions as [19] are not potentially interesting to the solution of the hierarchic problem. Hereon, we shall still consider the last term of eq. (18).

C. A Particular Case Example

In order to study a specific example in BD theory, let us particularize our analysis to braneworld models inspired in one of those previously found in this framework [10]. In the models proposed in [10], all the bulk-brane structure were generated by using local and global cosmic string as sources. The final scenario is composed by 4-branes embedded in a bulk of six-dimensions. Therefore, they are models of hybrid compactification. The on-brane dimension is compactified into a \( S^1 \) cycle and the dilaton field depends only on the large transverse dimension. With these specifications, we have \( D = 6, p = 3 \) and \( q = 4 \), and consequently, \( a = 2\xi(15 + 2w) - 2(17 + 6w), \ b = -4\xi(4 + 3w) + 12(1 + w) \) and \( c = 2\xi(23 + 8w) - 2(23 + 12w) \). Then, the equation (18) gives

\[
\oint W^{\xi+1} \left( -4\Lambda[35\xi - w(\xi + 3) - 14] - 2\sum_i [26\xi + w(\xi - 9) - 31] T_4^{(i)} \Delta^{(1)}(r - r_i) \\
+ \tau_\mu^\mu [\xi(15 + 2w) - (17 + 6w)] + \tau_m^m [-2\xi(4 + 3w) + 6(1 + w)] - 2(3 + 2w) \left( \frac{8\pi}{\phi} \right)^{-1} \\
\times [\xi W^{-2} \tilde{R} + (3 - \xi) \tilde{R}] + 2(3 + 2w)(3 - \xi) \left( \frac{8\pi}{\phi} \right)^{-1} \left( \frac{w}{\phi^2} \nabla^m \phi \nabla_m \phi + \frac{1}{\phi} \nabla^m \nabla_m \phi \right) \right) = 0. \quad (20)
\]

The analysis can be simplified if we assume empty bulk models and do not take into account contributions of any matter fields on the brane (\( \tau_\mu^\mu = 0 = \tau_m^m \)). Besides, in braneworld models in the BD theory, the cosmological constant is no longer constant and generally it depends on the
large extra dimension. Therefore, let us take ($\Lambda = 0$) for simplicity. Finally, note that different choices of $\xi$ lead to different contributions, so we begin with $\xi = -1$ since it eliminates the overall warp factor on the left-hand side of equation (20). With these several simplifications, the eq. (20) reads

$$
\oint \left( \frac{2w}{\phi^2} \nabla^m \phi \nabla_m \phi + \frac{2}{\phi} \nabla^m \nabla_m \phi - 4\bar{R} + W^{-2}\bar{R} \right) = -\frac{8\pi(57 + 10w)}{3 + 2w} \sum_i T_4^{(i)} \oint \frac{\Delta^{(1)}(r - r_i)}{\phi}. \tag{21}
$$

Note the appearance of the Euler character $\chi = \frac{1}{4\pi} \oint \bar{R}$. The internal space of the model can be characterized by $\chi$ and, then, for each model this topological invariant contributes in a specific way to the sum rules. For the above case, we have explicitly

$$
\oint \left( \frac{w}{\phi^2} \nabla^m \phi \nabla_m \phi + \frac{1}{\phi} \nabla^m \nabla_m \phi + \frac{W^{-2}}{2} \bar{R} \right) = 8\pi \chi - \frac{4\pi(57 + 10w)}{3 + 2w} \sum_i T_4^{(i)} L_i \phi^{-1}(r_i), \tag{22}
$$

where $L_i$ is the area of the $S^1$ cycle (the extra dimension compactified on the brane) and $\phi(r_i)$ is the dilaton field value on the $i^{th}$-brane at the $r_i$ position. It is time to make a brief comment about this result. In the ref. [11], it was raised the hypothesis of an inconsistence between braneworld models and “pure” BD gravity (where $w$ is expected to be $\sim 1$). The reason was because of the $(w - 1)\lambda$ coupling - where $\lambda$ is the brane tension - which appears in an ubiquitous way in the effective Einstein equation projected on the brane. In the context of this work (as well as in ref. [11]), the BD gravity acts as an intermediate theory between General Relativity and an effective gravity recovered from the low energy limit of the string theory, so the $(w - 1)\lambda$ coupling is not a real problem in this scope. However, for a “pure” BD gravity it seems to be a problem, since one cannot define the contribution of the brane vacuum energy to the projected Einstein equation. As one can see, from eq. (22), it is just an apparent inconsistence, since there is not a definite forbidden value to the BD parameter.

Another interesting choice for the $\xi$ parameter is $\xi = 3$. If we reconsider all the simplifications which lead to (21) but with $\xi = 3$ it results

$$
\frac{3(3 + 2w)}{8\pi} \oint W^2 \bar{R} = (6w - 47) \sum_i T_4^{(i)} W^4(r_i) \phi^{-1}(r_i) L_i. \tag{23}
$$

Note that there is no contribution from the derivatives of the scalar field, as well as, from the Euler number. We remark that, for a negative scalar curvature constant it is possible to have a multiple brane scenario only with negative brane tensions! On the other hand, for a positive scalar curvature constant, it possible, at least, one negative brane tension. We shall comment the results in the next Section.
III. CONCLUDING REMARKS

We generalize the braneworld sum rules to the BD gravity. In the interface between General Relativity and an effective gravity obtained from the low energy limit of the string theory, such a generalization is quite necessary. The results recover the previous case when the limit $w \to \infty$ is taken. The main goal of obtaining consistence conditions in braneworlds is the large application to several models.

On one hand, the constraints imposed by the equation (18), for example, must be respected by any braneworld model in the BD gravity in which the scalar field depends only on the large extra dimension and should be taken into account in more involved models. It is a strong imposition. On the other hand however, as the eq. (18) shows, such a constraint is not too restrictive itself. Of course, the consistence conditions are much more attractive if one assumes a particular model.

It is important to remark the role of the scalar field in the sum rules. It is clear that, except in those cases showed by eq. (23), the shape of the scalar field is strongly related with the consistence of the model. In this context, it is surprising that there is, at least, one solution to the dilaton field (19) which reduces this importance. Note that this result is not obvious from the Einstein-Brans-Dicke equation of motion (8).

To the particular cases analyzed from eq. (21), it is important to stress the possibility of existing only positive or only negative brane tensions. It is in sharp contrast with the Randall-Sundrum model (9), where the tension of the two branes are necessarily equal and opposite. This type of fine tuning between the brane tensions is not a necessity for other models too (see ref. (8)). Besides, as it was shown, the sum rules did not impose any restriction on the BD parameter, solving the apparent inconsistence, raised in ref. (11), between braneworld models and “pure” BD gravity.

To end with, we call the attention to the possibility of the sum rules bringing information about the modulus stabilization problem as well as the stabilization of the scalar field. It can be obtained, in principle, by the analysis of the right magnitude of the last term of the right-hand side of eqs. (22) and (23), for instance. Of course, it does not provide a mechanism of stabilization but it can help to point a concrete research direction. We shall address to these questions in a future work.
Acknowledgments

J. M. Hoff da Silva thanks to CAPES-Brazil for financial support. M. C. B. Abdalla and M. E. X. Guimarães acknowledge CNPq for partial support.

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