The relevance of the study is substantiated by the fact that when managing the processes of oil transportation through main pipelines, it becomes necessary to determine and select the optimal operating modes of the oil pipeline units, taking into account the fuzziness of some part of the initial information. In this regard, solving the problem of multi-criteria selection of effective operating modes for an oil heating station for a hot oil pipeline system, which is often described in a fuzzy environment, based on the apparatus of fuzzy set theories, is an urgent scientific and practical problem. A method for the synthesis of models in the conditions of fuzzy output parameters of the objects has been developed, with the help of which fuzzy models of the investigated oil heating station of the main oil pipeline have been built. Based on the modification and combination of various optimality principles, mathematical formulations of the problem of multi-criteria selection of effective modes for an oil heating station in a fuzzy environment are obtained. By modifying and adapting the principles of guaranteed results and equality in a fuzzy environment, a heuristic method has been developed for solving the formulated problem of selecting object's operation modes using the initial fuzzy information. The proposed heuristic method for multi-criteria selection in a fuzzy environment is based on the use of the experience and knowledge of the decision-maker. The proposed approach is implemented in the formulation and solution of the problem of multi-criteria selection of operating modes of the oil heating station in Atyrau of the Uzen-Atyrau-Samara main oil pipeline. As a result of the application of the proposed method, an improvement in the degree of fulfillment of a fuzzy restriction on environmental impact was achieved by 2%, as well as the optimal values of the operating parameters of the object were improved: the temperature was reduced by 1.85 % (3.67 K), pressure – by 0.84 % (kPa) and fuel consumption – by 2.9 % (0.0002 kg/s). The obtained results have confirmed the effectiveness of the proposed approach to solving the assigned tasks.

Keywords: mathematical models, optimization of operation modes of the oil heating station of main oil pipelines under conditions of fuzzy initial information.

1. Introduction

Main oil pipelines are parts of a complex, extensive technological system with a considerable length. The system of main oil pipelines includes oil pumping stations, oil heating stations (when pumping viscous oils), and linear sections (the pipeline itself), as well as telecommunications, automation, telemechanics, and fire-fighting devices [1, 2]. Compared to other types of oil transportation, main oil pipelines are a more efficient and environmentally safe mode of transportation of oil, oil products, and gases [3]. The schematic flow diagram for pumping oil through hot main pipelines, studied in this work, is shown in Fig. 1.

Oil from the field through pipeline 1 is supplied to tank farm 2 of the main pumping station. The tanks are equipped with heating devices that maintain a temperature that allows oil to be pumped out by booster pumps 3.

Oil, pumped by booster pumps through additional heating and adapting the principles of guaranteeing results and equality in a fuzzy environment, is supplied to the intake of mainline pumps 5. Mainline pumps get the oil to the main pipeline.
The object of this work is the Atyrau oil heating station at the Uzen-Atyrau-Samara “hot” oil pipeline. The oil heating station is designed to ensure the emergency-free and uninterrupted operation of tubular heating furnaces and structures at them, as well as to ensure the optimal technological mode of the oil pipeline operation. When controlling the operating modes of the oil heating station, the following optimization problems are solved:

- minimization of the cost of heating oil and pumping it through the pipeline;
- minimization of operating costs and the amount of fuel used;
- increased productivity;
- maximizing the degree of reliability of mechanisms and devices, etc.

All oil heating stations of “hot” main oil pipelines, incl. Atyrau point at the Uzen-Atyrau-Samara oil pipeline, are equipped with the pipe still heaters of “G9P02B” type. The number of heaters is designed on the condition that 1 heater can heat up 300–600 m³ of oil per hour from 35–40 °C to temperatures not exceeding 70 °C.

Thus, when formalizing and solving problems of mathematical modeling of oil pipeline process units, incl. oil heating station, on the basis of modern mathematical methods, little attention is paid to the methods of developing a system of mathematical models of technological objects of the oil pipeline system in conditions of uncertainty associated with the fuzziness of the initial information. Since in practice, process units of main oil pipelines often function precisely with large measurement errors, approaches to solving the problems discussed above in these conditions are very relevant.

 Attempts to extend traditional modeling methods to quantitatively difficult-to-describe objects, such as process units of main oil pipelines, have not yet yielded good results in practice, despite the significant development of mathematical methods and computer technology. In practice, such units and processes are managed quite well by an experienced human operator (production personnel), a decision-maker. (DM). A human operator in such cases copes fairly successfully with uncertainty and complexity. Unlike PC, a person uses fuzzy qualitative concepts and is quite successful in navigating a complex environment. In this regard, a problem arises of how to use the knowledge and experience of a person, which are formalized in plain or professional language, i.e., fuzzy information.

The problem of energy saving is one of the most important tasks of the oil and gas sector in the field of pipeline transport. The automated control system of main oil pipelines ensures their reliable and uninterrupted operation but does not determine the optimal modes of oil transportation. The energy efficiency of oil pipeline transport largely depends on the organization and management of the technological regime of the main oil pipeline and is achieved by modeling the optimal conditions of its operation.

A very important and urgent task is to increase the efficiency of pipeline transportation of high-setting, high-viscosity oils, which make up a significant share of the total volume of oil produced to the Republic of Kazakhstan.

One of the ways to reduce the cost of transporting oil through pipelines is to reduce energy costs by choosing rational pumping modes. The main oil pipelines are operated for a significant part of time under transitional conditions when the volumes of pumped oil differ from the optimal flow rates calculated at the design stage. In such situations, it is naturally necessary to solve the problem of the most efficient load distribution in the network of main oil pipelines.

2. Literature review and problem statement

The work [5] presents the results of a study of pipe still heaters with radiating heater walls, which can be used to heat oil in oil main pipelines for the transportation of high-viscosity oils. The principle of operation of such heaters is described, and the methodology for estimating the main indicators of such heaters is given. The advantage of this work is that a convenient method for estimating the parameters of industrial heaters is proposed, and as a disadvantage, it can be mentioned that it does not disclose the issues of calculation and modeling using computers.

The author of the work [6] proposes an approach to the use of computers for carrying out the process design of pipe still heaters. The advantage of this work is that it investigates and discloses the main issues of using computers to simulate the thermal mode of operation of pipe still heaters. However, this work does not address the issues of designing heaters with a lack and fuzziness of initial information, which often arises in practice, which can be considered as a disadvantage. The paper [7] studies the structure of pipe still heaters and increasing the efficiency of their operation in oil refining and transportation of high-viscosity oils by technological methods. This is its advantage since this paper addresses specifically the issues of modeling and optimization of an oil pipeline facility pumping high-viscosity oil. But as its disadvantage, it can be noted that it does not investigate methods for increasing the efficiency of pipe still heaters based on mathematical methods, methods of mathematical modeling, and optimization.

In the works [8–11], the issues of mathematical modeling of pipe still heaters of the oil heating station and the issues of optimization of their operation modes are investigated. Namely, in [8] the approaches to solving some problems of optimization of modes of transport of high-viscosity oils with heating and the use of hydrocarbon diluents were investigated and proposed. In the work [9], mathematical models of pipe still heaters based on analytical and statistical methods are proposed. Mathematical modeling and design of industrial, including pipe still, heaters are considered in the work [10]. And in [11], the results of mathematical modeling...
of the thermal operation of industrial heaters under deterministic conditions are presented on the basis of the heat balance equation. The fact that these works comprehensively investigated the issues of modeling and optimization of the operating modes of pipe still heaters can be attributed to their advantages. But these works mainly consider the physicochemical basis of the heating process, propose deterministic and statistical models of pipe still heaters, i.e., the problems of lack and fuzziness of initial information have not been considered, which is their disadvantage. In these and in other analyzed works, no attention is paid to mathematical modeling and optimization of the operating modes of pipe still heaters of the oil heating station in conditions of fuzziness of the initial information. Thus, the results of the literature review show the relevance and expediency of research on the development of mathematical models and optimization of the operating modes of the oil heating station of the main oil pipelines in conditions of fuzziness of the initial information.

3. The aim and objectives of the study

The aim of the study is to develop a method for synthesizing models in conditions of fuzziness of some part of the initial information, on the basis of which fuzzy models of oil heating stations are built, which are used to optimize the process of heating oil in them.

To achieve this aim, the following research objectives shall be implemented:

– to develop a method for the synthesis of a fuzzy model of process units with clear input and fuzzy output parameters;
– to develop fuzzy and statistical models of the oil heating station, describing the dependence of the station performance, temperature, and pressure at the outlet of the furnaces on the input parameters;
– to formalize the task of multi-criteria selection in a fuzzy environment of effective operating modes of the oil heating station of the main oil pipeline;
– to formulate a mathematical shape of the problem of multi-criteria selection of effective operating modes of the oil heating station in a fuzzy environment and to develop a heuristic method to solve it;
– to apply the research results when solving the problem of multi-criteria selection of the effective mode of the investigated oil heating station.

4. Materials and methods

How can human abilities be transferred to a computer for modeling, optimization, and control of complex industrial facilities? To solve such a problem, special methods of fuzziness formalization and processing of fuzzy, high-quality information are required, for example, expert evaluation methods [4, 12], the mathematical apparatus of fuzzy set theories [13–15].

In this work, to solve the problems of uncertainty arising from the random nature of the initial information, the known probabilistic methods are used to determine more accurate values of the measured parameters by processing an array of statistical data. Many complex production facilities operating under conditions of uncertainty are related to quantitatively difficult-to-describe objects since quantitative data on some of their parameters and conditions are absent or lacking. However, the missing part of the initial information about the parameters and states of such objects can be supplemented by fuzzy information from experts. Therefore, for the collection, formalization, and use of fuzzy information for the development of models of quantitatively difficult-to-describe objects, this paper proposes a method for the synthesis of fuzzy models based on the methods of expert evaluations and theories of fuzzy sets, and for the multi-criteria selection of operating modes in a fuzzy environment, a heuristic method based on the modification of various give-and-take schemes and application of the apparatus of fuzzy sets is proposed.

To study the process and operating modes of the oil heating station of main oil pipelines, the methodology and methods of system analysis are used [16–18], which makes it possible to use the available initial information of a different nature in the development of models and optimization of the object in conditions of uncertainty and fuzziness of the initial information. During the collection and processing of initial information, the development of mathematical models, in the formulation and solution of the problem of decision-making on the multi-criteria selection of operating modes for the oil heating station, experimental-statistical methods [19–21], methods of multi-criteria optimization and decision-making theories are used [22–27]. In the work, with the purpose of collecting, formalizing, and using fuzzy information, methods of expert evaluations and theories of fuzzy sets are used [4, 12–15].

The study was conducted on the basis of theoretical and experimental methods and methods of expert evaluation. To study the technology and operating modes of the oil heating station of the main oil pipelines, the methods of system analysis were used, which allow using the available source information of various types for the development of models and optimization of the research object. The statistical data collected by experimental methods on the operation of the object and expert information collected from experts and processed by methods of fuzzy set theories are used as the initial information [16].

To construct a fuzzy model for evaluating the performance of an oil heating station, the method of synthesis of a fuzzy model of technological objects proposed in this paper is used. Statistical models for determining the measured output parameters of an oil heating station are built on the basis of an experimental statistical method. For the structural identification of models, the method of sequential inclusion of regressors and its modifications were used, and their parameters were identified on the basis of the modified least-squares method. The numerical values of the model parameters were determined using the “Regress” software package.

When setting and solving problems of optimizing the operating modes of an oil heating station, modified methods of multi-criteria selection are used, adapted in this work for working in a fuzzy environment. At the same time, to modify the methods of multi-criteria selection, the ideas of compromise schemes of the main criterion and the ideal point are used, which allow solving the problems of multi-criteria selection. To check the adequacy of the developed models, a function that minimizes deviations of the model (calculated) and real (experimental) values of the output parameters of the object with the same values of the input parameters is used as a criterion for evaluating the adequacy [17].
5. Research results: development of models and optimization of operating modes of an oil heating station (OHS) in a fuzzy environment

5.1. Development of a method for synthesizing a fuzzy model of objects with fuzzy output parameters

As you know, models in a fuzzy environment are built on the basis of the linguistic values of the input and output parameters of the modeled object. At the same time, on the basis of logical rules of conditional inference and methods of expert assessments, a rule base is built, which is used for fuzzy modeling of the object under study. In this case, the built models are called linguistic models since they have the structure of logical rules for conditional inference:

\[ \text{IF} \text{ fuzzy condition } \text{THEN} \text{ fuzzy output.} \]

At present, various software packages are known, for example, the Fuzzy Logic Toolbox applications of the MatLab system, which are designed for building linguistic models and modeling with visualization of modeling results [28]. However, in practice, situations are often encountered when, for a process unit, part of the initial information required to build its mathematical model is available in a quantitative, clear form, and a part of information in a fuzzy form. Especially often cases occur where the input operating parameters, for example, temperature, pressure, and others, are measured and available in a quantitative form, and some output parameters of an object that assess the quality of its work are characterized by fuzziness. Such output parameters of process units include many quality indicators of made products (smell, color, flavor, etc.), object performance, and others. In these conditions, problems of partial fuzziness of the initial information appear. Such poorly or difficult to measure or completely unmeasurable output parameters are estimated with the participation of a person, DM in natural language, that is, they are expressed fuzzily [13, 15].

In this paper, we propose a modified method for synthesizing a fuzzy model under the conditions of the fuzzy output parameter of the object, i.e., partial fuzziness of the initial information in order to concretize and improve the method proposed in [29]. The essence of the modification lies in a more rigorous use of the methods of expert assessments and the apparatus of fuzzy set theories for the formation, processing and the use of the initial fuzzy information for the synthesis of a fuzzy model. In addition, the proposed method provides for the use of the Fuzzy Logic Toolbox application features during fuzzification and defuzzification of fuzzy parameters.

Let us consider the main content of the proposed modified method for the synthesis of a fuzzy model using the example of the object of study, i.e., the oil heating station (OHS) in Atyrau of the Uzen-Atyrau-Samara main oil pipeline.

The input parameters of the investigated oil heating station, which affect the heating of the pumped oil and the quality of the OHS operation, are \( x_i, i=1,2,3,4 \), namely:

- \( x_1 \) – temperature at the OHS inlet, the value of which may fluctuate within the range of 30–38 °C;
- \( x_2 \) – pressure at the OHS inlet. The value of this parameter at the studied facility according to the regulations can be from 8 to 12 kg/cm²;
- \( x_3 \) is the fuel consumption at the inlet of the OHS heaters, which can take a value from 20 to 30 kg/h;
- \( x_4 \) – flow rate, i.e., the volume of oil supplied to the OHS inlet, the value of which may fluctuate within the range of 700–720 t/h.

In practice, the values of all described input parameters are measured by various measuring instruments and means (thermometers, manometers, flow meters) and are clear. Some problems of inaccuracy associated with the precision of measuring instruments and external random factors are solved by the appropriate methods of probability theory and mathematical statistics [19–21].

The output parameters of the OHS, allowing to assess its performance, include the productivity \( (\tilde{y}_j) \) of the OHS, as well as the temperature \( (y_2) \) and pressure \( (y_3) \) at its outlet. Here the first parameter, that is, the performance of the OHS is indicated with the uppercase symbol \( \tilde{y} \) (tilde), which means this parameter is not directly measured, is characterized by fuzziness and is estimated with the participation of a human operator, the DM of the OHS. The rest of the output parameters – temperature \( y_2 \) and pressure \( y_3 \) at the OHS outlet, which are monitored, are measured using appropriate means. Therefore, for the development of mathematical models for determining the temperature and pressure at the outlet of the OHS: \( y_j=f(x), j=2,3 \), it is possible to apply well-known experimental and statistical methods for building the models, which allows obtaining statistical data of the models of the object. In the general structure of the above models, the vector \( x \) denotes the above input parameters of the OHS, i.e., \( x=(x_1, x_2, x_3, x_4) \). And to build a model for determining the performance of the OHS, which is characterized by fuzziness, it is necessary to synthesize a fuzzy model that allows evaluating the performance of the OHS, based on the knowledge and experience of the DM, expressed in the form of fuzzy information.

In this paper, based on the example of the investigated OHS, the following method for synthesizing a fuzzy model of a process unit with clear input and fuzzy output parameters is proposed. The block diagram of the implementation algorithm of the proposed method for the synthesis of fuzzy models using fuzzy values of the output parameters is shown in Fig. 2.

Description of the main blocks of the block diagram of the method for the synthesis of fuzzy process units in the conditions of clear values of the input parameters and fuzzy output parameters.

In block 2, the processed values of the input, mode parameters are entered \( x_i, i=1, n \), which are obtained through measurement. Those basic parameters that affect the course of the process in the facility and its output parameters are selected as such parameters.

In block 3, an expert assessment is arranged and carried out to describe the hard-to-measure, but indistinctly estimated output parameters of the object \( \tilde{y}_j, j=1, m \). Here, the Delphi method can be recommended as the most convenient and effective method of expert assessment, which eliminates the problem of conformism and allows one to consistently increase the coefficient of concordance, i.e., the consistency of expert opinions. In addition, by using a computer network, the speed and performance of this method can be greatly increased.

In block 4, on the basis of an expert assessment, fuzzy values of linguistic variables are determined that describe the output parameters of the object \( \tilde{y}_j, j=1, m \). Thus, in this block, a term-set is defined, the elements of which are the values of fuzzy variables that estimate the output fuzzy parameters of the object.
Fuzzification of the output parameters for each term $\hat{y}^q_j$, $j=1,m$, of fuzzy parameters, i.e., the procedure for constructing the membership function is carried out in block 5. At the same time, as practice shows, it is convenient to select membership functions of the Gaussian type, different options that are available in the Fuzzy Logic Toolbox application of the MatLab system. Here one can also use a similar structure of exponential dependence with tuning coefficients $Q^q_i$ and $N^q_i$ proposed by us in [29], which also have a Gaussian (bell-shaped) type:

$$\mu_i(\hat{y}_j) = \exp\left(\frac{Q^q_i(\hat{y}_j - y^{0q})}{N^q_i}\right).$$  \hspace{1cm} (1)

In the above expression (1), $y^{0q}$ means a fuzzy variable that is more closely corresponding to the selected term. This parameter is determined from the condition $\mu_i^{0q}(\hat{y}_j) = \max \mu_i(\hat{y}_j)$, i.e., its membership function as the maximum value of the membership function of the fuzzy output parameters of the object.

Block 6 solves the problem of structural identification of the developed model. In this block, to identify the structure of the developed model, it is proposed to use the idea of the method of sequential regressors (first the linear, then the nonlinear part of the regression models) until the condition of adequacy of the developed regression models is met.
The use of the structure of regression models is justified by the fact that the input parameters of the object are quantitatively measured and are clear, that is, there are statistical data related to them available. Arguably, there are data on the output parameters of the object, which are first obtained in the form of fuzzy information from experts, then on different given sets of level $\alpha$, based on the corresponding formulas of the theory of fuzzy sets, they are converted into clear statistical data.

Thus, the structure of a fuzzy model can be identified in the form of fuzzy regression equations in the following form:

$$
\hat{y}_j = \tilde{a}_0 + \sum_{i=1}^{n} \tilde{a}_{yi} x_i + \sum_{j=1}^{m} \tilde{a}_{ij} x_j + ..., \quad j = 1, m, \quad (2)
$$

where $\hat{y}_j$, $j = 1, m$ – fuzzy output parameters of the object, assessed by experts; $\tilde{a}_{yi}$, $\tilde{a}_{ij}$ – identifiable fuzzy parameters (coefficients) of the model.

In block 7, a set of level $\alpha$ is selected, i.e., $\alpha_s$ sections for the output parameters $y^{(i)}_j$, $j = 1, m$, $q = \frac{1}{\alpha}$ and fuzzy regression coefficients $\tilde{a}^{(i)}_{yi}$, $\tilde{a}^{(i)}_{ij}$, $i = 1, n$, $k = i$, $j = 1, m$, $q = \frac{1}{\alpha}$, i.e., the procedure for defuzzification of fuzzy parameters is implemented. The results of this block allow further application of well-known parametric identification methods, for example, the least-squares method for parametric identification.

In the next block 8, the fuzzy models are converted, the structure of which is identified in block 6 using the sections selected in block $\alpha_s$, $q = \frac{1}{\alpha}$, 7 into a system of equivalent clear tasks at the $\alpha$ levels:

$$
y^{(i)}_j = a^{(i)}_0 + \sum_{i=1}^{n} a^{(i)}_{yi} x_i + \sum_{j=1}^{m} a^{(i)}_{ij} x_j + ..., \quad j = 1, m, \quad (3)
$$

where $a^{(i)}_{yi}$, $a^{(i)}_{ij}$ are the values of the regression coefficients at a set of $\alpha$ levels, $q$ is the number of levels, $\alpha$-sections. It is recommended to select $a_{yi}$, $q = \frac{1}{\alpha}$ sections larger than 0.5 to 1 (a reliable part of the original fuzzy information).

To determine the specific values of the regression coefficients, the obtained values $a^{(i)}_{yi}$, $a^{(i)}_{ij}$ are combined according to the formula:

$$
\tilde{a}_{yi} = \frac{1}{\alpha} \sum_{\alpha=0.1}^{0.5} a^{(i)}_{yi} \quad \text{or} \quad \mu_{a_{yi}}(a_{yi}) = \sup \min \left\{ a_{yi}, \mu^{(i)}_{a_{yi}}(a_{yi}) \right\}, \quad (4)
$$

Parametric identification of models, which allows one to determine the values of the regression coefficients of the models (3) obtained in block 8, is performed in block 9 based on the least-squares method. As a result, the values of the regression coefficients on the selected $\alpha$-sections are obtained.

Then, for the purpose of computer modeling, the obtained values of the regression coefficients on $\alpha$-sections based on expression (4) are combined into a single coefficient for each regressor.

In block 10, the condition for the adequacy of the constructed models is checked. The condition for the adequacy of the developed models is matching of the calculated $y^{(i)}_j$ (obtained on the basis of the model) and real (obtained experimentally) $y^{(i)}_j$ values of the output parameters of the simulated object, with the same values of the input parameters in the permissible range of their variation. The condition of adequacy in a formalized form can be written, for example, as follows:

$$
\left| y^{(i)}_j - y^{(i)}_j \right| \leq 0.05,
$$

where 0.05 is the permissible deviation between the model and real values of the output parameters, that is, the metric (distance) between $y^{(i)}_j$ and $y^{(i)}_j$ must be equal to or less than the specified accuracy 0.05. It is proposed to use the Euclidean metric.

If this condition is not met, then it is necessary to conduct an analysis in block 11 and find out the reason for the inadequacy of the constructed models and go to the corresponding blocks to eliminate the identified causes of inadequacy. The main reasons for the inadequacy of the developed models include possible errors in the structural identification of the model or in the parametric identification of unknown coefficients. This cycle is repeated until the specified accuracy of adequacy is reached.

As soon as the adequacy of the model is ensured, one can move to block 12, where the obtained results are displayed in the form of a structure of synthesized models with identified parameters, which are recommended for modeling the operating modes of the object under study.

5.2. Development of fuzzy and statistical models of an oil heating station

In this subsection, a fuzzy model for assessing the performance of the investigated oil heating station is developed using the proposed above method for synthesizing a fuzzy model of process units with fuzzy output parameters and its statistical models based on an experimental-statistical method.

In order to assess the influence of the input, operating parameters of the OHS on its performance with the distinctness of the initial information, using the Delphi method, an expert assessment was carried out and, on the basis of the described method of synthesis of a fuzzy model, the following structure of the fuzzy model was determined:

$$
\tilde{y}_i = \tilde{a}_{i1} + \tilde{a}_{i2} x_{i1} + \tilde{a}_{i3} x_{i1} - \tilde{a}_{i4} x_{i1} + \tilde{a}_{i5} x_{i1} + \\
+ \tilde{a}_{i6} x_{i1} + \tilde{a}_{i7} x_{i1} - \tilde{a}_{i8} x_{i1} + \tilde{a}_{i9} x_{i1} + \tilde{a}_{i10} x_{i1}, \quad (5)
$$

where $\tilde{y}_i$ – performance of the OHS; $\tilde{a}_{i0}, \tilde{a}_{i1}, ..., \tilde{a}_{i10}$ – fuzzy regression coefficients identified on the basis of the modified least-squares method; $x_{i1}, x_{i2}, x_{i3}, x_{i4}$ – respectively, temperature, pressure, fuel and oil consumption at the inlet of the OHS.

Here, to solve the problem of parametric identification, that is, to identify the fuzzy coefficients $\tilde{a}_{i0}, \tilde{a}_{i1}, ..., \tilde{a}_{i10}$ of the obtained fuzzy model of the OHS of the oil pipeline system, it is recommended to use the method of identifying fuzzy parameters of the models proposed in [29]. This technique is based on the combination of a modified least-squares method, experiment planning, expert judgment, and fuzzy set theories.

To determine the influence of the input parameters on the temperature and pressure at the outlet of the OHS on the basis of an experimental-statistical approach using the ideas of the method of sequential inclusion of regressors, the following structure of statistical models was identified:

$$
y_j = a_{j0} + a_{j1} x_{j1} + a_{j2} x_{j2} + a_{j3} x_{j3} + a_{j4} x_{j4} + \\
+ a_{j5} x_{j5} + a_{j6} x_{j6} + a_{j7} x_{j7} + a_{j8} x_{j8}, \quad j = 2, 3, \quad (6)
$$

where $y_j$, $j = 2, 3$ – respectively, temperature and pressure at the outlet of the OHS; $a_{j0}, a_{j1}, ..., a_{j8}$ – the OHS; regression coefficients identified by parametric identification methods; $x_{j1}, x_{j2}, x_{j3}, x_{j4}, x_{j5}, j = 2, 3$ – respectively, temperature, pressure, fuel and oil consumption at the inlet of the OHS.
To identify unknown fuzzy coefficients $\tilde{a}_q$ in the fuzzy model of the OHS (1), the fuzzy sets describing the quality indicators of production are divided into the following sets of the level $\alpha=0.5, 0.80; 1$. In accordance with the selected level, the values of the input $x_{ij}$, $i=1,4; j=1,2$ and output $y$, parameters are observed at each level $\alpha_q$, $q=\frac{1}{3}$.

For each level of $\alpha_q$, $q=\frac{1}{3}$, fuzzy multiple regression equations (3), i.e., the fuzzy model for the OHS, we rewrite the following system of equations:

$$y_{i}^{*} = a_{i}^{*} + a_{i}^{*}x_{1i} + a_{i}^{*}x_{2i} + a_{i}^{*}x_{3i} + a_{i}^{*}x_{4i},$$

$$q=\frac{1}{3}$$

Thus, the fuzzy model describing the fuzzy dependence of the OHS performance on $y$, the input parameters $x_{i}$, $i=1,4$ is brought into a clear model on the sets of levels $\alpha$ (not affecting or very slightly affecting the $y$, regressors are zeroed).

Then the obtained values of the coefficients $\tilde{a}_q^n$, $i=0.7, j=\frac{1}{3}, q=\frac{1}{3}$ for the model (8) are combined using the formula (4) from the theory of fuzzy sets. As a result of this procedure, the following model for assessing the performance of the OHS was obtained, suitable for computer modeling:

$$y_{i} = 2.0722557x_{1i} + 7.0735457x_{2i} - 5.0683345x_{3i} + 0.0982337x_{4i} + 0.0883025x_{5i} + 0.7017238x_{6i} - 0.3734534x_{7i} + 0.0778345x_{8i} - 0.3217358x_{9i},$$

As a result of the determination by the least-squares procedure, the following model for assessing the performance of the OHS was obtained, suitable for computer modeling:

$$y_{i} = 0.0000001 + 0.5882353x_{1i} - 0.5000000x_{2i} + 0.4000000x_{3i} - 0.0070423x_{4i} + 0.0216263x_{5i} - 0.0500000x_{6i} + 0.0294118x_{7i}x_{8i},$$

In the parametric identification of the unknown regression coefficients of the models (7), which determine the temperatures ($y_2$) and pressures ($y_3$) at the output of the OHS from the input parameters $x_{i}$, $i=1,4$ based on the least-squares method and after neglecting regressors with zero (or almost zero) coefficients, the following is obtained:

$$y_{i} = 0.000000001 - 0.0235294x_{1i} + 0.32000000x_{2i} + 0.03200000x_{3i} + 0.00225352x_{4i} - 0.00138408x_{5i} + 0.04000000x_{6i} + 0.00705882x_{7i}x_{8i}.$$
modes of the OHS is an urgent task for the transportation of high-viscosity oil. At the same time, the selected operating modes of the oil pipeline facilities must ensure the extreme values of the control criteria, for example, maximum productivity, maximum pumping volume, minimum environmental impact, etc.

Making optimal decisions on the selection of effective operating modes of the facility is a process that involves a decision-maker (DM). In the process of choosing a solution, alternatives are evaluated, i.e. possible solutions, and the selection of the best one is made according to the given criteria [22, 31]. Evaluation and selection of the best solution in a production environment, as well as in our problem, are performed under multi-criteria conditions, in line with the criteria of economic, environmental, technological nature. The challenge is complicated by the fact that these economic and environmental criteria can be conflicting and also not clearly described. Under these conditions, the problem of multi-criteria selection, i.e. when the initial set of alternatives \( A \) is known, but the optimality principle is unknown \( opt \), is reduced to the task of investigating the preferences of the DM and building on this basis an adequate model for choosing the best alternative according to the given criteria [32–35].

The importance of solving such problems that are relevant for science and practice and the interest in the concept of analysis and comparison of DMs of multi-criteria alternatives performed when choosing a solution led to the emergence of many works on multi-criteria selection and decision making [22–32, 35, 36]. But it should be noted that in these and other works, the problems of multi-criteria selection of operating modes of complex technological systems in conditions of contradictory, poorly formalized and vaguely described criteria and alternatives have not been sufficiently investigated and solved. In this regard, this work investigates and solves the main problems of multi-criteria selection problems with the fuzziness of the initial information to determine the optimal operating modes of process units on the example of an oil heating station of the main hot oil pipeline.

On the basis of the approach to the formalization of the problem of multi-criteria selection described in [20], we formalize a specific problem of choosing the operating modes of an OHS hot oil pipeline.

For a formalized description of the problem of multi-criteria selection of the optimal operating mode of the OHS, we introduce the following notation:

\[
f(x) = f_1(x), f_2(x), f_3(x) - \text{the vector describing the output parameters of the OHS, i.e., productivity } f_1(x), \text{ temperature } f_2(x) \text{ and pressure } f_3(x) \text{ at the outlet of the OHS.}
\]

As known, these parameters in practice are considered as criteria that assess the performance of an OHS inlet; pressure; temperature; the components of this vector are possible minimum and maximum values, which are determined by the technological regulations of the facility [37–39].

Under these conditions, the problem of multi-criteria selection of the optimal operating modes of the OHS can be represented as a problem of finding such a value of the vector of input parameters \( x = [x_1, x_2, x_3, x_4] \) that provides the optimal vector of criteria for assessing the quality of the object’s operation \( f(x) = f_1(x), f_2(x), f_3(x) \). In addition, the vector \( x = [x_1, x_2, x_3, x_4] \) must also ensure that the conditions of all existing constraints are met, which are associated with environmental limits or resources and are usually fuzzy and \( \phi_\gamma(x) \geq b_\gamma, q \geq 1.2 \).

Then, on the basis of the above formalization, it is possible to formulate a mathematical formulation of the problem of choosing the optimal operating modes of the oil heating station of the main oil pipelines, taking into account the fuzziness of the constraints.

To outline the formulation of the problem of the multi-criteria selection of operating modes of the OHS, the problem should be reduced to a convenient form for using the mathematical apparatus of the methods of the fuzzy sets theory. For this purpose, the following designations are introduced:

\[
- \mu_\gamma(x) = \mu_\gamma^1(x), \mu_\gamma^2(x), \mu_\gamma^3(x) - \text{the normalized vector of local criteria evaluating the efficiency of the OHS operation modes, i.e., } f_1(x), f_2(x), f_3(x);
- \mu_\gamma(x) = \mu_\gamma^1(x), \mu_\gamma^2(x) - \text{the values of the membership function, which estimate the degree of fulfillment of the constraints } \phi_\gamma(x) \geq b_\gamma, q \geq 1.2.
\]

These membership functions are based on information from DMs, experts, including environmental specialists [40].

In addition, let \( \gamma = [\gamma_1, \gamma_2, \gamma_3] \) and \( \beta = [\beta_1, \beta_2] \), respectively, be the importance vectors of local criteria and constraints, the components of which are weighting coefficients. It is assumed that the weighting coefficients of the mutual importance of local criteria and constraints can be determined and assigned by the DM in the process of solving the problem.

Based on the above information, the mathematical formulation of the problem of multi-criteria selection of the optimal operating modes of the oil heating station under fuzzy constraints can be written as the following fuzzy mathematical programming problem:

\[
\max_{x \in \Omega} \mu_\gamma^i(x), \ i = 1, 3, \quad (12)
\]

\[
X = \{x: \arg \max_{x \in \Omega} \mu_\gamma^i(x), q \geq 1.2\}. \quad (13)
\]

To obtain the correct mathematical formulation of the above problem (12), (13), it is necessary to modify various principles of optimality (compromise schemes) for working in a fuzzy environment. Thus, it is possible to obtain various mathematical formulations of the problems of multi-criteria selection of modes of the oil heating station. An effective approach to solving the multi-criteria selection problems obtained in this way with fuzzy constraints is the development and application of appropriate heuristic methods, which are based on the involvement of DMs, experts to use their experience, knowledge and intuition. The advantage of using heuristic methods for solving the considered multi-criteria selection problems in a fuzzy environment is that they take into account human intuition, intelligence and creativity, and also provide flexibility when taking into account the
DM’s preferences in the process of choosing the best solution [41, 42].

It should be noted that this formulation of the problem of multi-criteria selection of operating modes of the OHS (12), (13) with fuzzy constraints can be written in general form for any process unit in similar situations with an arbitrary number of criteria and constraints, i.e., there may be \( i = \overline{1,m} \) and \( q = \overline{1,L} \).

5.4. Mathematical formulation of the problem of choosing effective modes of OHS operation and the method of its solution

Let us present the results of the general mathematical formulation, formalized above, of the problem of multi-criteria selection of effective modes of operation of process units on the example of OHS. To do this, we use a modification of various compromise schemes, that is, we adapt them to work in a fuzzy environment and develop heuristic methods for solving the obtained problem of multi-criteria selection of operating modes of a process unit.

First, we consider a situation when one of the criteria vectors is the most important and it can be considered as the main criterion that needs to be optimized. In addition, in this case, the DM, who solves the problem of selection and the experts involved, consultants can single out this main criterion and determine the boundary values for the remaining local criterion, which are taken into account as constraints. This allows you to take into account the local criteria based on the main criterion (MC) method. In addition, in the situation under consideration, it is assumed that the DM can determine the weighting coefficients of the importance of these constraints and the idea of the Pareto optimality (PO) principle can be used for them.

In these situations, for the mathematical formulation of the problem of multi-criteria selection of operating modes of a process unit according to the selected criteria, we use a combination of the main criterion methods (for local criteria) and the Pareto optimality principle (for fuzzy constraints) [23, 43]. As already noted, the DM, experts identify the main criterion \( \mu_{1,i}(x) \), which is optimized and sets the boundary values for the remaining local criteria \( \mu_{i',i} \), \( i = \overline{2,m} \) and also assigns weighting coefficients for fuzzy constraints \( \beta_{1}, ..., \beta_{L} \) that take into account their mutual importance. According to the requirement of the Pareto optimality principle, the number of evaluated objects, in this case, the restrictions should not be too many, that is, no more than \( 7\pm2 \).

In this case, the formulation of the problem of multi-criteria selection of optimal operating conditions of an object with a vector of fuzzy constraints can be written as follows [44]:

\[
\max_{x,\Omega} \mu_{1,i}(x), \tag{14}
\]

\[
X = \left\{ x: x \in \Omega : \arg \max_{x,\Omega} \beta_{i} \mu_{i}(x) \wedge \beta_{i} > 0 \right\}, \tag{15}
\]

In the above formulation of the problem (14), (15), the following new notation is introduced:

\( \wedge \) – the sign of logical “and”, which requires the truth of all statements that are associated with them;

\( \mu_{1,i} \) – boundary values of local criteria, transferred to the composition of constraints and are considered as constraints \( \mu_{i',i}(x) \), \( i = \overline{2,m} \). The values of these constraints are assigned with the help of DMs and experts.

The solution to the posed problem of multi-criteria selection (14), (15) depends on the value of \( \mu_{1,i} \), \( i = \overline{2,m} \) and \( \beta_{1}, ..., \beta_{L} \). Therefore, by changing the values of the boundary values and/or weighting coefficients of the constraints, it is possible to obtain a set of solutions \( x \in \Omega \), \( i = \overline{2,m} \), \( q = \overline{1,L} \) then the DM selects the best solution from the obtained set of solutions.

To solve the problem of selecting the operating modes of a process unit when setting (14), (15) on the basis of a modification of the method of the main criterion and the Pareto principle of optimality and their combined use, a heuristic method can be developed.

Such a heuristic method for solving the problem of multi-criteria selection, which is based on the combined application of the modified principles of the main criterion and Pareto optimality, makes it possible to effectively solve the posed problem (14), (15) in a fuzzy environment under the conditions of applicability of the indicated principles of optimality. However, in practice, when solving problems of multi-criteria selection of operating modes, it is not always possible to single out the main criterion and determine the coefficients of mutual importance of objects. Then the application of the main criterion method and the idea of the Pareto principle of optimality does not give the desired results or is impossible. In these situations, when solving problems of selection in a fuzzy environment, one has to apply the idea of other principles, modifying them in advance to work in a fuzzy environment.

In practice, various production situations may arise, for example, the characteristics of the pumped raw materials change, production plans are adjusted, the characteristics of units and devices change, etc. Accordingly, the criteria for choosing solutions, the availability of one or another source information can also change. Such situations require the formulations of the problems of selecting the optimal operating modes of the object and the development of effective algorithms for their solution adapted to them.

Next, we present the results of the formulation of the problem of multi-criteria selection in a fuzzy environment of operating modes of process units and the development of a heuristic based on a modification and combination of other optimality principles.

Suppose it is necessary to obtain solutions with a multi-criteria selection that provide results, and fuzzy constraints are of approximately equal importance. Then, based on the modification of the principles of maximin (guaranteed result) and equality, it is possible to obtain a mathematical formulation of the multi-criteria selection problem with fuzzy constraints and develop a heuristic method for its solution.

Suppose \( \mu_{1,i}(x) \), \( \mu_{2,i}(x) \), ... \( \mu_{n,i}(x) \) are the normalized values of the criteria that assess the efficiency of the operating modes of process units, \( \mu_{1}(x) \), ... \( \mu_{n}(x) \) – the membership functions, assessing the degree of fulfillment of fuzzy constraints, where \( m \) is the number of criteria, \( L \) is the number of fuzzy constraints. The values of the criteria and constraints depend on the vector of input, mode parameters \( x = (x_{1}, ..., x_{k}) \), consisting of \( n \) components.
Statement of the problem of multi-criteria selection based on the modification and combination of the principle of guaranteed results (GR) and the principle of equality (PE):

\[
\max_{x \in X} \mu'_i(x), \quad \gamma = (\gamma_1, ..., \gamma_m), \quad \beta = (\beta_1, ..., \beta_n), \quad \mu = (\mu_1, ..., \mu_n).
\]

\[
X = \left\{ x : (x_1, ..., x_n) \in \Omega \wedge \mu(x) = \mu(x) \wedge \arg \max_{\mu_i(x)} \beta_i(x) \wedge \right. \left. \arg \min_{\mu_i(x)} \beta_i(x) \right\},
\]

\[
\gamma = (\gamma_1, ..., \gamma_m) \quad \text{and} \quad \beta = (\beta_1, ..., \beta_n).
\]

To solve the obtained formulation of the multi-criteria selection problem (16), (17), a heuristic method is proposed, based on modified principles of guaranteed result and equality (for fuzzy constraints). The structure of the proposed heuristic method GR+PE includes the following main points:

1. The DM, experts assign a number of priorities for local criteria \( I = \{1, ..., m\} \), while the most important criterion should have priority 1, the next most important criterion should have priority 2, etc.

2. With the involvement of DMs, experts, a vector of weighting coefficients is set for all local criteria except for the highest priority criterion \( \gamma = (\gamma_1, ..., \gamma_m) \), which determines the mutual importance of local criteria \( \mu_i(x), i \in I \), \( I = \{2, ..., m\} \) and ensures their equality.

3. On the basis of an expert assessment, the values of the vector of weighting coefficients are determined \( \beta = (\beta_1, ..., \beta_n) \).

4. A term-set is determined that describes fuzzy constraints and membership functions are constructed that estimate the degree of fulfillment of fuzzy constraints \( \mu_i(x) \),

\[
q = \prod_{i=1}^{n} \beta_i \geq 0, \quad q = \prod_{i=1}^{n} \beta_i \geq 0.
\]

5. To search for an effective decision, the DM determines a number \( p_q \), \( q = \prod_{i=1}^{n} \beta_i \) of steps at each \( q \) coordinate.

6. The steps are calculated to change the coordinates of the weighting vector \( \beta = (\beta_1, ..., \beta_n) \) by the formula \( h_q = \frac{1}{p_q} \),

\[
q = \prod_{i=1}^{n} \beta_i \geq 0.
\]

7. The vector of weighting vectors \( \beta^1, \beta^2, ..., \beta^n \) is constructed \( N = (p_1 + 1)(p_2 + 1) ... (p_n + 1) \) by changing the coordinates at the sections \( [0, 1] \) with the step calculated at step 6 \( h_q \).

8. The problem of maximizing the first criterion is solved, i.e., \( \max_{x \in X} \mu'_i(x) \) (16) with a set of feasible solutions \( X \), determined by expression (17) for all \( \gamma = (\gamma_1, ..., \gamma_m) \) and \( \beta = (\beta_1, ..., \beta_n) \).

The obtained current solutions are displayed: \( x(\gamma, \beta) \) – the desired vector of input parameters that provide the corresponding values of the first criterion having the highest priority \( \mu'_i(x) \), the values of other local criteria taken into account in the constraints \( \mu_i(x) \), \( \mu_i(x) \), \( \mu_i(x) \), \( \mu_i(x) \), \( \mu_i(x) \) and the values of the membership function, which are equal and describe the degree of fulfillment of fuzzy constraints \( \mu_i(x) \), \( \mu_i(x) \), \( \mu_i(x) \).

9. The obtained solutions are presented to the DM for analysis and selection of the best solution. If the current solutions obtained satisfy the DM, then based on his preference, he selects the final, i.e., the best solution and goes to step 10. If the current decisions do not satisfy the DM, then the values of the vector of weighting coefficients of constraints \( \beta = (\beta_1, ..., \beta_n) \) and/or local criteria are changed

\[
\gamma = (\gamma_1, ..., \gamma_m). \quad \text{Then the return to point 4 is carried out and the next cycle to improve the current solutions begins.}
\]
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main oil pipeline, we use the modified principles of guaranteed result and equality discussed above.

To formalize the problem to be solved, we introduce the following concretization: $\mu_1(x) = (\mu_1^1(x), \mu_1^2(x), \mu_1^3(x))$ – a normalized vector of local criteria that describe the quality of the object’s operation, that is, evaluate the efficiency of the OHS operation modes; $I_2 = \{1, 2, 3\}$ – priorities of local criteria, determined on the basis of information from DMs and experts. Suppose $\mu_1^1(x)$ is the volume of oil at the outlet of the OHS, heated to a temperature of 70 °C, which has the highest priority, i.e., 1; $\mu_1^2(x)$ – temperature at the outlet of the OHS with priority $2$; $\mu_1^3(x)$ – pressure at the outlet of the OHS, which has priority $3$.

The guaranteed results of criteria with priorities $2$ and $3$ are determined on the basis of the maximal principle, taking into account their importance coefficients $\gamma_2$ and $\gamma_3$. The degree of fulfillment of fuzzy constraints that assess the environmental condition is estimated by membership functions $\mu_2(x), \mu_2(x)$, which are built on the basis of expert assessment methods. As it can be seen, the criteria and fuzzy constraints depend on the vector of input parameters of the OHS $x = (x_1, x_2, x_3, x_4)$, the components of which are, respectively: temperature; pressure; fuel consumption and oil consumption at the OHS inlet. These dependences of the criteria are determined on the basis of mathematical models of the OHS, for example (9)–(11), which are developed in Section 3.2.

The weighting coefficients that allow using the principle of equality for fuzzy constraints will be denoted by $\beta_1, \beta_2$, which are assigned by the DM and/or experts.

On the basis of the above formalization using the modified above principles of GR and PE, the general formulation of the multi-criteria selection problem (16), (17) with fuzzy constraints can be specified and noted as follows:

$$\max_{x \in \mathbb{X}} \mu_1^i(x),$$

$$X = \left\{ x : (x_1, x_2, x_3, x_4) \in \Omega \wedge \arg \min_{x \in \mathbb{X}} \gamma \mu_2^i(x) \wedge \arg \left( \beta \mu_1(x) = \beta \mu_1(x) \right), I_2 = \{2, 3\} \right\}.$$  \hspace{1cm} (19)

To solve the obtained task, we apply the heuristic method proposed in Section 5.4, based on the modified principles of guaranteed result and equality.

The DM in the process of solving the multi-criteria selection problem (18), (19) on the basis of the GR+PE method iteratively selects the best solution that satisfies him and provides an effective mode of operation of the OHS. In this case, such values of the vector of input parameters are determined $x_1, x_2, x_3, x_4$ that provide the maximum value of the criterion with the highest priority $\mu_1^1(x)$ – guaranteed values of other local criteria $\mu_1^2(x), \mu_1^3(x)$ and the maximum degrees of fulfillment of fuzzy constraints $\mu_1(x), \mu_1(x)$. Thus, the choice of the final decision is made taking into account the preference of the DM.

Let us present the results of solving the problem posed for the multi-criteria selection of effective operating modes of the OHS (18), (19) using the developed heuristic method based on the modification of the principles of guaranteed result and equality (GR+PE):

1. The DMs, experts set a number of priorities for local criteria $I_2 = \{1, 2, 3\}$. In this case, the highest priority 1 was assigned to the most important criterion $\mu_1^1(x)$ evaluating the performance of the OHS, the next priority was assigned to the criterion $\mu_1^2(x)$ that determines the temperature at the outlet of the OHS, and priority 3 was assigned to the criterion $\mu_1^3(x)$ that evaluates the pressure at the outlet of the OHS.

2. With the involvement of DMs, experts, a vector of weighting coefficients for all local criteria, except for the criterion with the highest priority $\gamma = (\gamma_2, \gamma_3) = (0.7, 0.3)$, which determines the mutual importance of local criteria, is set $\mu_1^1(x), \mu_1^2(x)$.

3. On the basis of an expert assessment, the values of the vector of weighting coefficients were determined $\beta = (0.06, 0.4)$, taking into account the requirements $\sum_1^2 \beta_1 = 1 \wedge \beta_2 = 0$, $q = 1.2$, which, taking into account the importance of fuzzy constraints, make it possible to ensure the equality $\beta \mu_1(x) = \beta \mu_1(x)$.

4. To describe fuzzy constraints, the following term-set is defined: “below the norm”; “norm”; “above the norm” and for them with the use of exponential dependence (1), membership functions were constructed that estimate the degree of fulfillment of fuzzy constraints $\mu_1(x), \mu_1(x)$.

$$\mu_1(x) = \exp(0.15 |a_1 - 0.01|^{0.5});$$

$$\mu_1(x) = \exp(0.10 |a_2 - 0.05|^{0.5}),$$

where $a_1, a_2$ are the average numerical values of fuzzy parameters, respectively; maximum permissible concentrations for soil and air around the OHS, and other parameters are 0.15; 0.3 and 0.10; 0.5 fast and slow tuning coefficients identified to fit the membership function curve for analytical expressions.

5. To search for a solution, the DM determines a number of steps at each $q$ coordinate $p_\alpha, q = 1.2, p_\alpha=5, p_\beta=5$.

6. The steps for changing the coordinates of the weighting vector are calculated on the computer $\beta = (0.6, 0.4)$ by the formula $h_{\alpha} = \frac{1}{p_\alpha}, q = 1.2: h_{\alpha} = \frac{1}{p_\alpha} = \frac{1}{5} = 0.2, h_\beta = \frac{1}{p_\beta} = \frac{1}{5} = 0.2$.

7. A set of vectors $\beta^{\alpha1, \beta^{\alpha2}, \ldots, \beta^{\alpha5}}$ has been constructed, $N = (5 + 1)(5 + 1) = 36$, the coordinates change on the segments [0, 1] with a step $h_{\alpha} = h_\beta = 0.2$.

8. The problem of maximizing the first criterion, which has the highest priority, is solved on the computer, i.e., maximizing the performance of the OHS $\max \mu_1^1(x)$ on the set of feasible solutions $X$, determined by expression (17) for all $\gamma = (0.7, 0.3)$ and $\beta = (0.6, 0.4)$. The problem of maximization, taking into account the imposed constraints, was solved on the basis of mathematical programming methods using the FinPlus software package [47].

The obtained current solutions: $x(\gamma, \beta)$ – the required vector of input parameters that provide the corresponding values of the performance of the OHS $\mu_1(x(\gamma, \beta))$, the values of local criteria that estimate the temperatures $\mu_1^1(x(\gamma, \beta))$ and pressures $\mu_1^2(x(\gamma, \beta))$ at the outlet of the OHS, as well as the values of the membership function, which are equal and describe the degree of fulfillment of fuzzy constraints $\mu_1(x(\gamma, \beta)) = \mu_1(x(\gamma, \beta))$. To optimize and estimate the values of the criteria that depend on the vector of input parameters, in this step $x = (x_1, x_2, x_3, x_4)$ we used models (9), (10) and (11) constructed in Section 5.2, i.e., the values of the criteria are determined as follows:

$$\mu_1(x) = \exp(0.15 |a_1 - 0.01|^{0.5});$$

$$\mu_1(x) = \exp(0.10 |a_2 - 0.05|^{0.5}),$$

where $a_1, a_2$ are the average numerical values of fuzzy parameters, respectively; maximum permissible concentrations for soil and air around the OHS, and other parameters are 0.15; 0.3 and 0.10; 0.5 fast and slow tuning coefficients identified to fit the membership function curve for analytical expressions.

5. To search for a solution, the DM determines a number of steps at each $q$ coordinate $p_\alpha, q = 1.2, p_\alpha=5, p_\beta=5$.

6. The steps for changing the coordinates of the weighting vector are calculated on the computer $\beta = (0.6, 0.4)$ by the formula $h_{\alpha} = \frac{1}{p_\alpha}, q = 1.2: h_{\alpha} = \frac{1}{p_\alpha} = \frac{1}{5} = 0.2, h_\beta = \frac{1}{p_\beta} = \frac{1}{5} = 0.2$.

7. A set of vectors $\beta^{\alpha1, \beta^{\alpha2}, \ldots, \beta^{\alpha5}}$ has been constructed, $N = (5 + 1)(5 + 1) = 36$, the coordinates change on the segments [0, 1] with a step $h_{\alpha} = h_\beta = 0.2$.

8. The problem of maximizing the first criterion, which has the highest priority, is solved on the computer, i.e., maximizing the performance of the OHS $\max \mu_1^1(x)$ on the set of feasible solutions $X$, determined by expression (17) for all $\gamma = (0.7, 0.3)$ and $\beta = (0.6, 0.4)$. The problem of maximization, taking into account the imposed constraints, was solved on the basis of mathematical programming methods using the FinPlus software package [47].

The obtained current solutions: $x(\gamma, \beta)$ – the required vector of input parameters that provide the corresponding values of the performance of the OHS $\mu_1(x(\gamma, \beta))$, the values of local criteria that estimate the temperatures $\mu_1^1(x(\gamma, \beta))$ and pressures $\mu_1^2(x(\gamma, \beta))$ at the outlet of the OHS, as well as the values of the membership function, which are equal and describe the degree of fulfillment of fuzzy constraints $\mu_1(x(\gamma, \beta)) = \mu_1(x(\gamma, \beta))$. To optimize and estimate the values of the criteria that depend on the vector of input parameters, in this step $x = (x_1, x_2, x_3, x_4)$ we used models (9), (10) and (11) constructed in Section 5.2, i.e., the values of the criteria are determined as follows:
9. The obtained current solutions of each iteration are presented to the DM (the operator who controls the oil heating process) for analysis and selection of the best solution. In iterations 1 through 4, the DM was not satisfied with the results obtained and made adjustments to the values of the weights that had not been previously used. Thus, the values of the weights of the criteria were varied; the remaining criteria, the values of temperature and pressure at the OHS outlet, were left unchanged. Proceeding to step 4 to improve the solution. At iteration 5, the DM was satisfied with the obtained solutions for the implementation of the final solutions selected by him and moved on to step 10.

10. The best solutions, chosen by the DM, which satisfy him, have been implemented:

- \(\gamma\) – the optimal values of the vector of input parameters, values of temperature, pressure, fuel consumption and oil consumption at the OHS inlet;
- \(\mu_1(x, y, \beta)\) – the maximum value of productivity, that is, the criterion with the highest priority, which is provided \(x'(y, \beta)\);
- \(\mu_2(x, y, \beta)\) – the remaining criteria, the values of temperature and pressure at the outlet of the OHS at the optimal value of the vector of input parameters;
- \(\mu_3(x, y, \beta)\) – the maximum degree of fulfillment of fuzzy constraints, achieved with the optimal value of the vector \(x'(y, \beta)\).

The obtained results of the final choice of the DM at iteration 5 are entered in Table 1 below.

Thus, to substantiate the advantages of the heuristic methods for synthesizing fuzzy models and solving problems of multi-criteria selection based on the MC+IP method, comparisons are made with similar results obtained by other well-known methods for solving the problem under consideration. To improve the results in the practical application of the proposed approaches to solving the practical problem, a DM who participates in solving the problem was trained. As a result, an improved solution has been achieved.

6. Discussion of the results of the study of the developed models and methods

The main advantages of the proposed methods include:

- the proposed method for the synthesis of fuzzy models based on the apparatus of the theories of fuzzy sets makes it possible to develop effective models of process units in conditions of fuzzy output parameters;
- the developed heuristic method for solving the problem of multi-criteria optimization in a fuzzy environment through the use of knowledge, experience, and intuition of experienced DMs, expert specialists allows one to effectively solve the problems of multi-criteria selection of optimal modes of their work based on the maximum use of the initial fuzzy information.

In the developed method for the synthesis of a fuzzy model of process units with fuzzy output parameters, the structure of fuzzy models (block 3) in the form of fuzzy regression equations is identified based on the idea of the method of sequential inclusion of regressors. And the identification of the fuzzy coefficients of the fuzzy regression equations is identified based on the set of level \(\alpha\) and the least-squares method (block 9). The identification of the structure of fuzzy models is carried out on the basis of the system analysis of the object and the method of sequential inclusion of regressors, the essence of which is the sequential

| No. | Criteria and constraints | Deterministic method | Statistical approach | MC+IP method | Proposed GR+PE method | Real values obtained experimentally |
|-----|--------------------------|----------------------|---------------------|--------------|-----------------------|-----------------------------------|
| 1   | OHS productivity, kg/s, \(y_1\) | 0.1960               | 0.1962              | =0.1970      | =0.1970               | 0.1967                            |
| 2   | Temperature at the heater outlet, K, \(y_2\) | 321.43               | 324.50              | 323.15       | 323.10                | 323.20                            |
| 3   | Pressure at the heater outlet, kPa, \(y_3\) | 101.62               | 96.570              | 96.54        | 96.53                 | 96.55                            |
| 4   | Degree of fulfillment of a fuzzy constraint \(1 - \mu_1(x', y, \beta)\) | –                  | –                   | 1.0          | 1.0                   | –                                |
| 5   | Degree of fulfillment of a fuzzy constraint \(2 - \mu_2(x', y, \beta)\) | –                  | –                   | 0.98         | 1.0                   | –                                |
| 6   | Optimal value of the vector \(x' = (x_{1*}, x_{2*}, x_{3*})\): \(x_{1*}\) – optimal temperature at the heater inlet, K | 308.15               | 307.77              | 306.15       | 301.10                | 307.15                            |
| 7   | \(x_{2*}\) – optimal pressure at the heater inlet, kPa | 120.25               | 119.95              | 117.04       | 117.00                | 119.40                            |
| 8   | \(x_{3*}\) – optimal fuel consumption, kg/s | 0.0075               | 0.0071              | 0.0069       | 0.0067                | 0.0072                            |
| 9   | \(x_{4*}\) – optimal volume of raw materials (oil) at the heater inlet, kg/s | 0.1970               | 0.1970              | 0.1970       | 0.1970                | 0.1970                            |

Note: (–) parameters are not defined or measured. The time it takes to solve problems in methods is almost the same.
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inclusion of sequential regressors until the conditions for the adequacy of the model to real data are met. In this case, the linear part is first added to the model, then the nonlinear part with an increase in the degree and factors of pair interaction. For parametric identification, one can use a fuzzy equivalent of the least-squares method [47].

The task of the final stage of the proposed method (block 10) is to check the adequacy of the model to the object according to the minimum criterion \( \| y_i^{\text{d}} - y_i \| \leq 0.05 \). The model is considered adequate to the object if the parameters of the object are calculated with its help with a given accuracy, for example, 0.05, match the real data obtained experimentally. To determine a specific value of a fuzzy coefficient \( \alpha_{ij} \), \( i = 1, \ldots, n \), \( j = 1, \ldots, m \), \( q = 1, \ldots, q \), they are combined according to the rules of the theory of fuzzy sets with the expression (4).

The application of the approach of sequential inclusion of regressors does not guarantee obtaining an adequate model for a certain number of cycles, but it allows obtaining a sufficiently adequate model, and its efficiency depends on a priori information, as well as the experience and knowledge of the researcher, i.e., on heuristics. Thus, the proposed method for synthesizing a fuzzy model of process units with fuzzy output parameters is heuristic.

When developing fuzzy and statistical models of the OHS, describing the dependence of the productivity, temperature, and pressure at the outlet on the input parameters, fuzzy information from experts on the assessment of performance, and statistical data on the values of the temperature and pressure of the OHS were used. The structure of the fuzzy model for assessing the performance of the OHS is determined on the basis of the method of sequential inclusion of regressors in the form of fuzzy regression equations (2).

The set of values of the fuzzy coefficients of the model for assessing the efficiency of the OHS is first identified on the set of level \( \alpha \), then they are combined according to formula (4) and a specific numerical value is determined by defuzzification using the center of gravity method. Regression models describing the dependence of the temperature and pressure at the outlet of the OHS on the input parameters are determined using the well-known methods of regression analysis and the least-squares method [48].

Based on the results of the analysis and discussion of the solution to the problem of choosing effective operating modes of the investigated oil heating station using the proposed heuristic method, the following can be noted:

- when solving the problem of multi-criteria selection of OHS modes, the developed heuristic method based on a combination of the principle of guaranteed result and the principle of equality makes it possible to improve the results in comparison with the results obtained by other methods. At the same time, the results obtained based on the GR and PE method are more consistent with real data, in comparison with the results of the deterministic, statistical approaches and the fuzzy MC+IP method (based on the methods of the main criterion and the ideal point);

- high adequacy and efficiency of the selected operating mode of the OHS are ensured by taking into account the complex, not formalized connections between the input and output parameters of the OHS when synthesizing a fuzzy model of an object, taking into account the knowledge, experience, and intuition of DMs, experts;

- the advantages of the proposed heuristic method for solving the multi-criteria selection problem based on the modified result method and the principle of equality are also ensured by the fact that this method allows obtaining guaranteed values of the criteria and ensuring the maximum fulfillment of fuzzy constraints;

- the reliability of the results of solving the problem of the multi-criteria selection of operating modes of the OHS in a fuzzy environment is ensured by the correct use of expert assessment methods and the mathematical apparatus of the theory of fuzzy sets, as well as decision-making methods. As can be seen from Table 1 that compares the solution results obtained on the basis of the proposed method based on the modification of the principles of maximin and equality, they are in good alignment with real data. In addition, local criteria 2 and 3 achieve their guaranteed results, and the degree of fulfillment of fuzzy constraints reaches maximum values (1.0; 1.0);

- this study has time constraints necessary for organizing and conducting expert assessments of fuzzy parameters, as well as for formalizing the obtained fuzzy information. In addition, for recently commissioned facilities, the number of experts to assess and collect the missing baseline information may be limited;

- as disadvantages of this research, we can point out complexity of program-based realization of the proposed heuristic methods for solving the task of multi-criteria selection; and some points of the method are presented in an enlarged format. Besides, the need for preliminary training of decision-makers may arise with the purpose of efficient solution of the task in a dialogue with a computer. Further, the work shall be developed in the area of detailed explicitation of the proposed methods when representing them in the form of more detailed algorithms. Also, in the long run, mathematical tools should be developed for different management systems on the basis of the proposed modeling and decision-making methods, for example, for intelligent systems to support decision-making concerning control of operation modes of process facilities.

7. Conclusions

1. A method has been developed for the synthesis of a fuzzy model of the process units under the conditions of clear input and fuzzy output parameters using the capabilities of the Fuzzy Logic Toolbox application for the fuzzification of fuzzy parameters. A block diagram of the algorithm for implementing the proposed method for synthesizing a fuzzy model of the process units is built. The developed method is based on the use of expert assessment methods and theories of fuzzy sets, as well as on the modification of the method of sequential inclusion of regressors for working in a fuzzy environment. The essence of the idea used is that next regressors with fuzzy coefficients are added to the structure of a fuzzy regression model until the required adequacy of the model is achieved. Thus, the proposed method uses the idea of the method of sequential inclusion of regressors – when identifying the structure of a fuzzy regression model, and the least-squares method, modified on the basis of the \( \alpha \)-level – when parametric identification.

2. Fuzzy and statistical models of the oil heating station have been developed, which, respectively, describe the dependence of the flow rate, temperature, and pressure at the outlet of the heaters on the input operating parameters. The fuzzy model for assessing the performance of the OHS is
based on fuzzy information from DMs, experts and has the structure of fuzzy multiple regression equations, which are determined based on the method of sequential inclusion of regressors. Statistical models that determine the temperature and pressure at the outlet of the OHS, depending on the values of the input parameters are built on the basis of experimental statistical methods and have the structure of nonlinear regression equations. The parameters of the developed models are identified by the modified least-squares method using the REGRESS software.

3. The problem of multicriteria selection of effective operation modes of the OHS of the main oil pipeline in a fuzzy environment has been formalized. The formalization and general formulation of the problem of multi-criteria selection are carried out on the basis of the methods of expert assessments, decision-making, and the mathematical apparatus of the theory of fuzzy sets.

4. The mathematical formulation of the problem of multi-criteria selection of effective operating modes of the OHS in a fuzzy environment was formulated and obtained on the basis of a modification of the MC method and the PO principle, as well as the GR and PE principles. On the basis of the combined use of the modified principles of MC and PE, a heuristic method for solving the formulated problem using the initial fuzzy information has been developed. The proposed heuristic method is based on the adaptation of the maximin principle (for criteria) and the principle of equality (for fuzzy constraints) to work in a fuzzy environment, on the involvement of a DM in the process of solving the problem of choice and using his experience, knowledge, experience and intuition.

5. The obtained research results are applied to solve the problem of multi-criteria selection of the effective mode of the OHS in a fuzzy environment has been formalized. The formalization and general formulation of the problem of multi-criteria selection on the environmental conditions of production. The problem of multi-criteria selection based on the heuristic method is solved in a fuzzy environment. In this case, unlike the known methods, the original fuzzy problem is not replaced with equivalent deterministic problems. This makes it possible to maximize the use of the initial fuzzy information and obtain an adequate solution to the production problem under fuzzy conditions. Thus, the effective application of multi-criteria selection methods in a fuzzy environment is ensured. The practical advantage of the proposed approach to solving multi-criteria selection problems in a fuzzy environment is that, depending on the production situation and the availability of initial information of a different nature, the DM is given the opportunity to select a more suitable, acceptable way to solve the problem from the proposed set of algorithms.

Comparing the results obtained using the models and methods proposed in this paper for selecting the effective operating mode of the oil heating station in comparison with the known similar results (Table 1), the following conclusions can be made:

- fuzzy approaches in comparison with the deterministic and statistical approaches allow increasing the performance of the OHS by 0.001 kg/s or 216 kg/h, i.e. by 0.5 %. At the same time, the heuristic method proposed in this work provides an effective operation mode of the object with less energy consumption, for example, at a temperature lower by 0.05 K and pressure lower by 0.01 kPa;
- the heuristic selection method proposed in this paper based on the principles of guaranteed result and equality provides the maximum values of the accessory function for fulfilling fuzzy constraints equal to 1, i.e., the requirements of fuzzy constraints are met 100 %;
- the developed method for selecting the operating mode of the OHS in a fuzzy environment by optimizing the values of the operating parameters in comparison with the best-known method allows improving the decisions. At the same time, the temperature of the object was reduced by 1.85 %, pressure by 0.04 %, and fuel consumption by almost 3 %, which ensures saving energy and resources in production.

Acknowledgments

We are grateful to Zhasulan Tuleuev, Yerbol Tulegenov and Bakytzhan Bultekov, who provided very important information for this study. The research data was sponsored by the Science Committee of the Ministry of Education and Science of the Republic of Kazakhstan (Grant No.of the research fund AP08853680–Intelligent decision support system for managing the operating modes of a catalytic reforming unit).

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