I. INTRODUCTION

It is no exaggeration to say that the physical properties of most theoretical models of particle physics are in large part determined by the structure of their vacua. This vacuum structure often determines, for example, whether the apparent symmetries of a model remain manifest or are spontaneously broken, and this in turn governs much of the resulting phenomenology. However, it is also of considerable importance to understand whether the vacuum structure of a model consists of a single, unique ground state, or whether there might also exist one or more metastable states. Even when the true ground state preserves the apparent symmetries of a model, the physical properties associated with the metastable vacua can often differ markedly from those of the ground state. In such situations, the resulting phenomenology of the model might be determined by the properties of a metastable vacuum rather than by those of the true ground state.

This last question takes on a particular urgency within the context of models exhibiting spontaneous (and indeed dynamical [1]) supersymmetry-breaking, for it is well-known (see, e.g., Refs. [2–8]) that the scalar potentials of supersymmetric field theories often contain not only a global minimum corresponding to the true ground state, but also one or more additional local minima which correspond to metastable vacua. Indeed, a number of recent papers [9,10] have sparked a renewed interest in this phenomenon. Since the decay rates of the metastable vacua are often highly suppressed, such metastable configurations are frequently exploited to construct field-theoretic models of dynamical supersymmetry-breaking [11–14] in situations in which the true ground state of the theory is known to preserve supersymmetry by virtue of Witten-index arguments [15]. Such metastable vacuum solutions can also be realized in a string context through explicit D-brane constructions [16].

It has even been suggested [10] that metastable supersymmetry-breaking in such models is inevitable. Even though minor modifications (such as the introduction of explicit $R$-symmetry-breaking operators to give mass to the otherwise massless $R$-axion [17,18]) can modify the Witten index in such theories and thereby reintroduce a supersymmetric vacuum elsewhere in field space, such modifications tend not to spoil the properties of the false vacuum. The phenomenon of metastable supersymmetry-breaking then persists intact.

Despite their ubiquity, dealing with such metastable vacua in supersymmetry-breaking scenarios is generally not particularly easy. There are two primary reasons why this is the case. First, metastable vacua are often difficult to locate, enumerate, or analyze in an arbitrary model — especially one whose ultraviolet completion involves non-perturbative dynamics or duality arguments. While a wealth of tools (including the Witten index [15], the ADS criterion [19], etc.) are available for probing the ground states of supersymmetric field theories, far fewer tools exist for analyzing metastable vacua. In fact, merely establishing whether such vacua even exist in a given model is often difficult.

Second, however, even if one or more metastable minima are identifiable in a given model, it often turns out that the low-energy effective description of such a model will typically have the property that the metastable vacua and the true vacuum are separated by infinite distances in field space. As a result, any low-energy effective description of the theory which is valid in the vicinity of a metastable vacuum generally tends to break down for the true ground state (and vice versa) due to the presence of strong dynamics or unspecified high-scale physics. This implies that precise calculations of quantities which
depend on the global features of the scalar potential — such as the lifetime of the metastable vacuum — are often difficult to perform.

In this paper, we shall present a simple theoretical construction which evades these difficulties. Specifically, we shall present a relatively straightforward supersymmetric model whose vacuum structure exhibits

- a supersymmetric ground state in which \( R \)-symmetry is preserved;
- a metastable state in which both supersymmetry and \( R \)-symmetry are broken, and whose gauge symmetry also differs from that of the true vacuum; and
- a vacuum energy barrier between the two of a sort that results in a long lifetime for the metastable vacuum.

Most importantly, all three of these features will be realized classically, i.e., at tree level, and no non-perturbative physics will be required in order to create either of these vacua or guarantee their stability. Moreover, due to the perturbative nature of our theory, these features are expected to be robust against quantum corrections. Finally, as we shall demonstrate, the true and metastable vacua in our model are separated by only a finite distance in field space. As a result, we will be able to perform explicit calculations of physical quantities such as the vacuum energy, lifetime, and particle mass spectrum of the metastable vacuum.

What makes our construction unique relative to most previous discussions of metastable supersymmetry-breaking in the literature is that the supersymmetry-breaking in our model is sourced by the presence of Fayet-Iliopoulos \( D \)-terms [20]. While \( D \)-term breaking normally poses phenomenological difficulties, we shall see that in our model, this \( D \)-term breaking in turn sources \( F \)-term breaking. This then makes possible the breaking of a global \( R \)-symmetry.

We stress that it is not our aim in this paper to present a complete phenomenological model of metastable supersymmetry-breaking. While this is clearly an important endeavor, our goal here is merely to develop a simple, field-theoretic "kernel" which might ultimately serve as the supersymmetry-breaking sector in a fully developed model. As such, we envision that this kernel might eventually be connected to the Standard-Model sector through a suitable messenger sector, and likewise that this kernel might have a suitable ultraviolet completion that allows it to emerge at sufficiently low mass scales from a fully-developed high-scale theory.

For reasons to be discussed, however, we do believe that a sensible ultraviolet completion of our kernel should not be difficult to construct, and that this kernel could therefore easily be used as a platform for constructing models where supersymmetry is broken at scales parametrically lower than the Planck or string scale. Moreover, as we shall discuss, we believe that our kernel has another important property required of all models of metastable supersymmetry-breaking, namely that the properties of the metastable vacuum in our model will not be spoiled by the introduction of additional operators with small coefficients, even if these operators tend to benignly produce additional supersymmetric vacua elsewhere in field space. Consequently, we believe that a variety of phenomenological model-building requirements — such as successfully coupling the model to a messenger sector, and including an explicit component to \( R \)-symmetry breaking to avoid running afoul of experimental constraints on a massless \( R \)-axion [17] — should be easy to satisfy.

Finally, we point out that our kernel is remarkably simple, consisting of only two \( U(1) \) gauge groups and a superpotential of the Wilson-line type. This gives our kernel a relevance beyond the narrow issue of supersymmetry-breaking. For example, structures of this sort appear naturally — and in fact are almost unavoidable — in heterotic and Type I orientifold string models, and as such we expect our discussion of the vacuum structure of our models to have a relevance in string theory that transcends their potential use as kernels in supersymmetry-breaking model-building. One possible implication is that the presence of long-lived metastable vacua in such models should have a significant effect on overall landscape statistics [21], which could in turn affect the methodologies that have been employed for statistical samplings of explicit heterotic [22] and Type I [23] string vacua. More phenomenologically, the non-trivial vacuum structure of the multiple-\( U(1) \) theories we discuss has the possibility to lead to new observable signatures for \( Z' \) physics.

II. THE MODEL

Our model is surprisingly simple. Working in terms of \( \mathcal{N}=1 \) supersymmetry, our model consists of two \( U(1) \) gauge groups denoted \( U(1)_a \) and \( U(1)_b \), with respective gauge couplings \( g_{a,b} \) and Fayet-Iliopoulos terms \( \xi_{a,b} \), and five chiral superfields with charges as shown in Table I. Given this configuration, the most general renormalizable superpotential is given by

\[
W = \lambda \Phi_1 \Phi_2 \Phi_3 + m \Phi_4 \Phi_5 .
\]  

(1)

Indeed, the (\( \Phi_1, \Phi_2, \Phi_3 \)) “core” of this model is nothing but a two-site linear moose of the sort that has been discussed intensely in the literature in connection with deconstruction [24], flux compactifications of string theory [25], and even toy field-theoretic models of the string landscape [26]. To this core, we have then simply added
and $F$ gauge theory coupled to matter includes both dimensionful parameters with appropriate powers of $g$. Our model is then defined in terms of the four remaining choices ($\lambda, m, \xi, \xi_b$). However, for the purpose of analyzing the metastability inherent in this model, we shall consider the gauge couplings to be equal, $g_a = g_b \equiv g$, and we shall set $g = 1$ for convenience. Our model is then defined in terms of the four remaining parameters ($\lambda, m, \xi, \xi_b$). However, for the purpose of analyzing the metastability inherent in this model, we will begin by considering the simple parameter choice ($\lambda, m, \xi, \xi_b$) = (1, 1, 5, 0), where we have rescaled all dimensionful parameters with appropriate powers of an overall, unspecified mass scale. Other choices for the parameters shall be discussed below.

In general, the scalar potential for a supersymmetric gauge theory coupled to matter includes both $D$-term and $F$-term contributions and can be written in the form

$$V = \frac{1}{2} \sum_a g_a^2 D_a^2 + \sum_i |F_i|^2 ,$$

(2)

where

$$D_a = \xi_a + \sum_i q_i^{(a)} |\phi_i|^2 , 
\quad F_i = - \frac{\partial W^*}{\partial \phi_i^*} .$$

(3)

In the model under consideration here, the scalar potential, including both $D$-term and $F$-term contributions, is given by

$$V = \lambda^2 (|\phi_1|^2 |\phi_2|^2 + |\phi_1|^2 |\phi_3|^2 + |\phi_2|^2 |\phi_3|^2 ) + m^2 (|\phi_4|^2 + |\phi_5|^2 ) + \frac{1}{2} g_a (\xi_a - |\phi_1|^2 + |\phi_2|^2 + |\phi_4|^2 - |\phi_5|^2 )^2 + \frac{1}{2} g_b (\xi_b - |\phi_2|^2 + |\phi_3|^2 + |\phi_4|^2 - |\phi_5|^2 )^2 .$$

(4)

Note that $V$ is a function only of absolute squares of fields; we can therefore take the vacuum expectation values of all the $\phi_i$ to be real and positive-semidefinite without loss of generality. For a given choice of the model parameters ($\lambda, m, \xi, \xi_b$), the extrema of the scalar potential can then be obtained by solving the coupled simultaneous equations

$$\frac{\partial V}{\partial \phi_i} = 0 \quad (i = 1, \ldots, 5) .$$

(5)

However, a solution is a local minimum only if the eigenvalues of the $10 \times 10$ mass matrix

$$M^2 \equiv \begin{pmatrix}
\frac{\partial^2 V}{\partial \phi_1^2} & \frac{\partial^2 V}{\partial \phi_1 \phi_2} & \cdots & \frac{\partial^2 V}{\partial \phi_1 \phi_5} \\
\frac{\partial^2 V}{\partial \phi_2 \phi_1} & \frac{\partial^2 V}{\partial \phi_2^2} & \cdots & \frac{\partial^2 V}{\partial \phi_2 \phi_5} \\
\vdots & \vdots & \ddots & \vdots \\
\frac{\partial^2 V}{\partial \phi_5 \phi_1} & \frac{\partial^2 V}{\partial \phi_5 \phi_2} & \cdots & \frac{\partial^2 V}{\partial \phi_5^2}
\end{pmatrix}$$

(6)

are all non-negative and the number of zero eigenvalues is precisely equal to the number of Goldstone bosons eaten by the massive gauge fields. (Indeed, additional zeroes would indicate the presence of classical flat directions.) In what follows, however, we will use the term “vacuum” loosely to refer to any extremum of the potential and employ adjectives such as “stable” and “unstable” to distinguish the eigenvalues of the mass matrix. Of course, a “metastable” vacuum exists only when two or more vacua exist and are stable according to the above definitions; all but the vacuum with lowest energy are considered metastable. Finally, we shall refer to a given vacuum as potentially breaking $R$-symmetry if its associated field configuration implies that any of the $F$-terms receive a non-zero VEV.

We claim that the model specified in Eqs. (1) and (4) has all the desired features outlined in Sect. I. In particular, the vacuum structure of the model can be explicitly described as follows:

- First, there exists a stable, supersymmetric ground state, henceforth denoted ‘A’, with $V = 0$ and unbroken $R$-symmetry in which only one field ($\phi_1$) receives a non-zero VEV. As a result, the gauge group $U(1)_b$ is preserved in this vacuum state.

- Second, there exists a metastable vacuum with $V = 9/2$, henceforth denoted ‘B’, in which both supersymmetry and $R$-symmetry are broken and in which two fields ($\phi_3$ and $\phi_5$) receive non-zero VEVs. The gauge group is entirely broken in this metastable vacuum.

- Finally, there exist three additional unstable solutions to Eq. (5), henceforth denoted ‘C’, ‘D’, and ‘E’, with even higher vacuum energies. One preserves both $U(1)_a$’s, one preserves a linear combination of $U(1)_a$ and $U(1)_b$, and one breaks the $R$-symmetry altogether, leaving behind a heavy pseudoscalar in the process. All three break supersymmetry, but only two break $R$-symmetry.

A complete listing of the vacuum structure of this model is given in Table II. Fig. 1 shows the behavior of the scalar potential $V$ evaluated between the metastable vacuum solution B (our tree-level “nest”), the true vacuum solution A (the “ground” state), and the saddle-point solution C which connects them.
TABLE II. The complete classical vacuum structure of the model in Eq. (1) with $(\lambda, m, \xi_a, \xi_b) = (1, 1, 5, 0)$ and $g = 1$. For each of the five extrema (labelled A through E) of the scalar potential in Eq. (4), we have listed the corresponding VEVs $(v_1, v_3, v_5)$, the value of the scalar potential, and the stability properties of that extremum. We have also indicated whether supersymmetry and $R$-symmetry are broken or unbroken at that extremum, along with the surviving (unbroken) gauge group. Each of these solutions has $v_2^2 = v_4^2 = 0$; likewise, all dimensionful quantities are quoted in terms of the overall arbitrary mass scale associated with our model. Note that the unstable extrema $D$ and $E$ are local maxima in the restricted three-dimensional VEV subspace $(v_1, v_3, v_5)$, but are actually saddle points in the full five-dimensional VEV space $(v_1, v_2, v_3, v_4, v_5)$.

| Label | $(v_1, v_3, v_5)$ | $V$ | Stability | SUSY | $R$-symmetry | Gauge Group |
|-------|------------------|-----|-----------|------|--------------|-------------|
| A     | $(\sqrt{5}, 0, 0)$ | 0   | Stable    | Yes  | Yes          | $U(1)_{h}$ |
| B     | $(0, 2, 2)$       | 9/2 | Metastable| No   | No           | None        |
| C     | $(\sqrt{3}/2, \sqrt{7}/2, \sqrt{5}/2)$ | 45/8 | Unstable  | No   | No           | None        |
| D     | $(0, 0, \sqrt{2})$ | 17/2 | Unstable  | No   | No           | $U(1)_{(a-b)}$ |
| E     | $(0, 0, 0)$       | 25/2| Unstable  | No   | Yes          | $U(1)_a \times U(1)_b$ |

FIG. 1. The supersymmetric ground state of our model and the metastable “nest” above it. *Left figure:* A surface plot of the scalar potential $V$ evaluated on the unique two-dimensional plane within the three-dimensional $(v_1, v_3, v_5)$ field space which simultaneously contains the true vacuum solution A, the metastable vacuum solution B, and the saddle-point solution C between them. Projected below the surface plot is a contour plot for $V$, showing the shortest path (blue) in field space connecting these three solutions. *Right figure:* The scalar potential $V$ evaluated along this shortest path. Field-space distances are quoted relative to the metastable vacuum B along this path, and all units are in terms of the overall unspecified mass scale associated with our model. Note that the field-space distance between the metastable and true ground states is finite and $\sim O(1)$; nevertheless, we shall see in Sect. IV that the lifetime of this metastable vacuum exceeds the age of the universe even when this unspecified mass scale is taken to be as high as the Planck scale.

III. EXPLORING THE PARAMETER SPACE

Thus far, we have presented a model in which there exists both a supersymmetric ground state and a non-supersymmetric metastable state at tree level. This is the model with the particular choices $(\lambda, m, \xi_a, \xi_b) = (1, 1, 5, 0)$ and $g = 1$. In this section, we seek to understand the behavior of this model as all of these parameters are varied, with an eye towards determining the conditions under which a non-supersymmetric metastable vacuum emerges relative to a supersymmetric ground state.

Even though our model contains five parameters $(g, \lambda, m, \xi_a, \xi_b)$, we can always rescale our $(\lambda, m, \xi_a, \xi_b)$ parameters so as to absorb the gauge coupling $g$ completely and without loss of generality. To do this, we
can simply rewrite our expressions in terms of the new variables \((\tilde{\lambda}, \tilde{m}, \tilde{\xi}_{a,b}, \tilde{v}_i) \equiv (\lambda/g, m/\sqrt{g}, g\xi_{a,b}, \sqrt{g}v_i)\), and then drop the tildes. We shall therefore use these rescaled variables in what follows.

It is straightforward to generalize our Solutions A through E for arbitrary \((\lambda, m, \xi_a, \xi_b)\), obtaining

\[
\begin{align*}
A : & \quad v_1^2 = \xi_a, \quad v_2^2 = 0, \quad v_3^2 = 0 \\
B : & \quad v_1^2 = 0, \quad v_2^3 = \xi_a - \xi_b - m^2, \quad v_3^2 = \xi_a - m^2 \\
C : & \quad \begin{cases} 
  v_1^2, 3 = \frac{\xi_a - \xi_b}{2(\lambda^2 + 1)} + \frac{m^2}{2\lambda^2}, \\
  v_2^2 = \frac{1}{2} \left[ \xi_a + \xi_b + \frac{m^2}{\lambda^2} (1 - \lambda^2) \right].
\end{cases} \\
D : & \quad v_1^2 = v_2^3 = 0, \quad v_3^2 = \frac{1}{2} (\xi_a + \xi_b - m^2) \\
E : & \quad v_1^2 = v_3^2 = v_3^2 = 0
\end{align*}
\]

where we continue to have \(v_2^2 = v_4^2 = 0\) for each solution. Substituting these results into Eq. (4), we then obtain the vacuum energies for each of these field configurations:

\[
\begin{align*}
V_A &= \frac{1}{2} \xi_a^2, \quad V_B = m^2 \xi_a - \frac{1}{2} m^4 \\
V_C &= \frac{1}{4\lambda^2(1 + \lambda^2)} [\lambda^4 (\xi_a - \xi_b)^2 + 2m^2 \lambda^2 (1 + \lambda^2) (\xi_a + \xi_b) + m^4 (1 - \lambda^2)] \\
V_D &= \frac{1}{4} (\xi_a - \xi_b)^2 + \frac{1}{2} m^2 (\xi_a + \xi_b) - \frac{1}{4} m^4 \\
V_E &= \frac{1}{2} (\xi_a^2 + \xi_b^2)
\end{align*}
\]

Clearly, these solutions exist only for those combinations of parameters \((\lambda, m, \xi_a, \xi_b)\) for which these expressions for the \(v_i^2\) are non-negative. For example, we see from Eq. (7) that Solution A does not exist as a solution to the equations in Eq. (5) unless \(\xi_2 \geq 0\).

However, even when these solutions exist, we cannot easily determine from Eqs. (7) and (8) whether they correspond to stable or unstable extrema of the potential. For this, we must examine the eigenvalues of the mass matrix in Eq. (6) and demand that they satisfy the conditions discussed below Eq. (6). It is this which is the subtle part of the analysis, and the results are shown in Fig. 2. For any arbitrary values of \(\lambda\) and \(m\), we have illustrated in Fig. 2 those regions in the two-dimensional \((\xi_a, \xi_b)\) plane for which each of our solutions not only exists but is also stable. Such \((\xi_a, \xi_b)\) plots are similar to those originally drawn in Ref. [26], and from these plots it is relatively straightforward to see how these regions deform as \(\lambda\) and \(m\) are varied.

FIG. 2. Within the two-dimensional \((\xi_a, \xi_b)\) plane, we have indicated the regions in which Solutions A through E exist (light shading) or both exist and are stable (darker shading). In drawing these regions, we have implicitly assumed \(\lambda^2 > 1\) when laying out the coordinates on each axis, but the results shown are general and apply even when \(\lambda^2 \leq 1\), with the shaded regions deforming accordingly. Note that the solid horizontal line (red) within Solution A indicates a line of vacua with unbroken supersymmetry.
Given these results, we can proceed to determine the general conditions on our model parameters \((\lambda, m, \xi_a, \xi_b)\) for which various types of metastability emerge. First, we observe from Fig. 2 that only for \(\xi_b = 0\) does our Solution A correspond to a supersymmetric vacuum; for all other \(|\xi_b| < m^2\) this vacuum state is non-supersymmetric. Thus \(\xi_b = 0\) is a necessary condition for having a supersymmetric ground state. We also observe from Fig. 2 that our theory will also have a non-supersymmetric metastable vacuum as long as \(\xi_a > (1 + 1/\lambda^2)m^2\). We thus conclude that both vacua are non-supersymmetric. Thus, we observe from Fig. 2 that only for \(\xi_b = 0\) does our Solution A correspond to the metastable vacuum. This is exactly what would be required for both gravity- and gauge-mediated scenarios of metastability.

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supersymmetry-breaking.

We can also calculate the particle spectra corresponding to each of the two vacuum states in our model. The analysis is straightforward, but may have important phenomenological implications if our model is to be used as a potential supersymmetry-breaking sector (or more generally as a potential hidden sector coupled to the Standard Model at an intermediate scale). Of course, as part of the spectrum analysis, we must take into account the effects of both gauge-symmetry breaking and supersymmetry-breaking, when relevant.

Restricting to the $(\lambda, m, \xi_a, \xi_b)$ region indicated in Eq. (9), our results are as follows. Recall that our model consists of five complex scalars $\phi_i$ ($i = 1, \ldots, 5$), five Weyl fermions $\psi_i$, two $U(1)$ gauge fields $A_a, b$, and their superpartner gauginos $\lambda_{a, b}$. For the true vacuum solution, we then find that $U(1)_b$ remains unbroken, while $U(1)_a$ is broken spontaneously by the VEV of $\phi_1$. The corresponding gauge boson then acquires a squared mass $M^2_a = 2g^2_{\lambda}$. As a result of the unbroken supersymmetry, the gaugino associated with the $U(1)_b$ gauge group remains massless; thus Solution A always gives rise to a massless field with odd $R$-parity unless there are other sources of supersymmetry-breaking beyond our model. The lightest remaining physical states in the spectrum are the $\phi_4, 5$ fields, which obtain squared mass $m^2$, and their superpartners.

In the metastable vacuum, the situation is of course quite different. Here, we find that the non-zero expectation values for $\phi_3$ and $\phi_5$ spontaneously break both $U(1)_a$ and $U(1)_b$, resulting in a pair of heavy $Z'$ gauge bosons with squared masses

$$M^2_a = (3 \pm \sqrt{5})(\xi_a - m^2).$$

A massless Goldstone fermion also appears in the spectrum (as one would expect, since supersymmetry is spontaneously broken in this vacuum configuration), and its identity is an admixture of $\psi_4$ and $\lambda_a$. All of the remaining physical particles are massive, with the identity of the lightest scalar depending on $(\lambda, m, \xi_a)$ and consisting of either $\phi_4$ with squared mass $2m^2$; $\phi_1$ with squared mass $\lambda^2(\xi_a - m^2) - m^2$; or an admixture of $\phi_3$ and $\phi_5$ with squared mass $(3 - \sqrt{5})(\xi_a - m^2)$. The situation for the corresponding fermions is similar, but more complicated as the mixing between the $\psi_i$ fields and the gauginos is far less trivial. However, we already see from these results that unlike the true vacuum, our metastable vacuum does not give rise to dangerous massless matter with odd $R$-parity. This once again provides an explicit illustration of the phenomenological impact of being in a metastable vacuum rather than the true ground state.

Up to this point, we have focused on those conditions under which our model contains both a supersymmetric ground state and a metastable state at tree level. We have not yet tackled the question of perturbativity, which is necessary not only for calculability but also for ensuring that our tree-level solutions remain intact against radiative corrections. However, it is easy to restore the factors of gauge coupling that were missing from Eq. (9), obtaining

\[
\begin{align*}
\xi_a > \left(1 + \frac{g^2}{\lambda^2}\right) \frac{m^2}{g^2} \\
\xi_b = 0.
\end{align*}
\]

Likewise, the condition for perturbativity is given by

$$g^2 \ll 16\pi^2.$$  \hspace{1cm} (14)

Thus, combining these results, we find that our model will not only function as advertised but also be perturbative provided $\xi_b = 0$ and

$$\frac{1}{16\pi^2} \ll \frac{g^2}{m^2} < \frac{\xi_a}{m^2 - \frac{1}{\lambda^2}}.$$  \hspace{1cm} (15)

Note that our original parameter choices from Sect. II amply satisfy these requirements. While the general result in Eq. (15) technically places a lower bound on the gauge coupling, this lower bound can be as small as we wish provided $\xi_a$ is taken sufficiently high or $m$ is taken sufficiently small. Thus, loop corrections can be arbitrarily suppressed in our model, ensuring that our tree-level solutions survive radiative corrections.

### IV. THE LIFETIME OF THE METASTABLE VACUUM

In order to be of phenomenological interest for model-building, the lifetime of any given metastable vacuum must be at least on the order of the present age of the universe. Such a state decays to the true vacuum via instanton transitions, the rate (per unit volume) for which may be parametrized as $[28]

$$\frac{\Gamma_{\text{inst}}}{\text{Vol}} = A e^{-B}.$$  \hspace{1cm} (16)

We will not be particularly concerned with the form of the coefficient $A$, and will focus our attention on the exponent $B \equiv S_E(\phi) - S_E(\phi_+)$, usually referred to as the bounce action, which represents the difference between the Euclidian action $S_E(\phi)$ at some point in field space and the action $S_E(\phi_+)$ at the metastable minimum. In general, bounce actions must be either computed numerically or evaluated analytically in some limiting regime, since only a handful of potentials are known which admit exact, closed-form expressions for $B$.

In this paper, we shall proceed as in Ref. [29] by approximating the potential along the classical path between the metastable and stable vacua as a triangle. In this approximation, the bounce action depends on four
parameters: $\Delta \phi_s$, which is the distance in field space between the top of the potential barrier (in this case, Solution C) and the metastable (+) or truly stable (−) vacuum (in this case, Solutions B and A respectively); and $\Delta V$, which is the potential difference between the top of the barrier and each respective vacuum state. All four of these quantities are directly calculable in terms of $(\lambda, m, \xi, \xi_b)$ from the expressions in Eqs. (7) and (8).

Using the triangle approximation, calculating $B$ is relatively straightforward [29]. When

$$\frac{\Delta \phi}{\Delta \phi_+} \geq \frac{\sqrt{1 + c + 1}}{\sqrt{1 + c - 1}}, \quad (17)$$

with $c \equiv (\Delta V_-/\Delta V_+)(\Delta \phi_+ / \Delta \phi_-)$, the bounce action is given by [29]

$$B = \frac{32\pi^2}{3} \frac{1 + c}{(1 + c - 1)^5} \left( \frac{\Delta \phi_+}{\Delta V_+} \right). \quad (18)$$

By contrast, when the inequality in Eq. (17) is not satisfied, the appropriate expression is instead given by [29]

$$B = \frac{\pi^2}{96} \left( \frac{\Delta V_+}{\Delta \phi_+} \right)^2 R_T^3 
\times \left( -\beta_+^2 + 3c\beta_+^2 \beta_- + 3c\beta_-^2 \beta_+ - c^2 \beta_-^2 \right), \quad (19)$$

where $\beta_+$ and $R_T$ are given by

$$\beta_+ \equiv \left( \frac{8\Delta \phi_+^2}{\Delta V_+} \right)^{1/2}, \quad R_T \equiv \frac{1}{2} \left( \frac{\beta_+^2 + c\beta_-^2}{c\beta_- - \beta_+} \right). \quad (20)$$

It can be verified that these solutions match smoothly at the point where Eq. (17) is saturated.

In Fig. 3, we plot the bounce action $B$ as a function of $\xi_a$ for several different values of $\lambda$. For these plots, we have taken $m = 1$ and $\xi_b = 0$ (implying that the true ground state preserves supersymmetry), using the rescaled parameters introduced at the beginning of Sect. III. Note that the bounce action $B$ — and hence the lifetime of the false vacuum — increases with increasing $\xi_a$. Thus if $\xi_a$ is taken to be sufficiently large, the lifetime of the metastable vacuum in our model will exceed the present age of the universe, rendering such a vacuum phenomenologically viable. For smaller values of $\xi_a$, the lifetime of the metastable vacuum decreases until $\xi_a$ reaches its minimum allowed value in Eq. (9). At this point, the bounce action $B$ drops to zero: the metastable vacuum B merges with the saddle-point Solution C and becomes unstable.

Note that with our original choice of model parameters $(\lambda, m, \xi_a, \xi_b) = (1, 1, 5, 0)$ and $g = 1$ as in Sect. II, we find that $B \approx 1300$. This is more than sufficient to guarantee a metastable lifetime exceeding the age of the universe even if the overall unspecified mass scale of our model is taken to be as high as the Planck scale.

![Fig. 3. The bounce action (i.e., the exponential suppression factor for the decay rate of our metastable vacuum), plotted as a function of $\xi_a$ for several different values of $\lambda$. For this plot, we have taken $m = 1$ and $\xi_b = 0$ (where the true ground state is supersymmetric). For sufficiently large $\xi_a$, we see that $B$ can easily exceed $\sim O(10^3)$, implying that the lifetime of our metastable vacuum can easily exceed the age of the universe.]

V. DEGENERATE GROUND STATES AND A "$\theta$-VACUUM"

As we have seen, our model in Eq. (1) gives rise to metastable vacua. However, as we shall now demonstrate, our model can also give rise to something potentially even more interesting: a doubly-degenerate classical ground state. In a quantum-mechanical setting, this implies that our true quantum-mechanical ground state is an effective "$\theta$-vacuum" of the theory. This is yet another phenomenon illustrating the highly non-trivial vacuum structure inherent in our model.

To illustrate this phenomenon, it is important to realize that in general, Solutions A through E are not the only solutions to the equations in Eq. (5). While Solutions A through E represent the complete set of solutions which exist along the line $\xi_b = 0$, other solutions may also exist for other values of $(\xi_a, \xi_b)$. For example, one other solution giving rise to a stable vacuum is

$$F: \quad v_3^2 = -\xi_b, \quad v_1^2 = v_2^2 = v_4^2 = v_5^2 = 0; \quad (21)$$

this solution exists in the $\xi_b < 0$ region, and is stable within the subregion

$$|\xi_a| < m^2, \quad |\xi_a| < -\lambda^2 \xi_b. \quad (22)$$

This vacuum even has unbroken supersymmetry when $\xi_a = 0$. Note that the regions of existence, stability, and unbroken supersymmetry for Solution F in the $(\xi_a, \xi_b)$
plane are the same as those for Solution A, only rotated by $\pi/2$ in the clockwise direction.

For $\lambda \leq 1$, the stable vacuum associated with Solution F does not simultaneously coexist with any other stable vacua. However, for $\lambda > 1$, the stable vacuum associated with Solution F coexists with the stable vacuum associated with Solution A within the region

$$\xi_a > -\lambda^2 \xi_a, \quad \xi_b < \xi_a / \lambda^2, \quad \xi_a, -\xi_b < m^2. \quad (23)$$

This then provides yet another region of parameter space in which our model simultaneously contains both a true ground state and a metastable ground state, both non-supersymmetric. For $\xi_a > -\xi_b$, the true ground state corresponds to Solution A, while for $\xi_a < -\xi_b$, the true ground state corresponds to Solution F.

An extremely interesting situation occurs when we further demand that $\xi_a = -\xi_b$: along this line, our two stable solutions A and F are exactly degenerate. This then provides an example of a situation in which our model exhibits two degenerate classical ground states. Ultimately, the double degeneracy of the classical ground state which emerges for $\xi_a = -\xi_b$ is the manifestation of a deeper $\mathbb{Z}_2$ reflection symmetry which involves the simultaneous exchanges of the two chiral superfields $\Phi_1 \leftrightarrow \Phi_3$ and the two gauge groups $U(1)_a \leftrightarrow U(1)_b$. In other words, this symmetry is a manifestation of the underlying reflection/parity symmetry of the original two-site moose upon which our model is built.

Classically, a doubly degenerate vacuum state is an exciting phenomenon. Since the system must ultimately settle into one or the other ground state, what results is a spontaneous breaking of the reflection/parity symmetry of our moose. One could even imagine the universe having chosen different ground states in different spacetime locations, leading to domain-wall formation and other topological defects.

However, in a quantum-mechanical setting, the situation becomes even more intriguing. Because the scalar potential barrier between the degenerate $|A\rangle$ and $|F\rangle$ vacua is finite, and because these two vacua are separated by only a finite distance in field space, there will generally be tunneling between these two vacua. The existence of our moose reflection symmetry then implies that the true energy eigenstates of our system are not $|A\rangle$ or $|F\rangle$ independently, but rather the reflection-symmetry eigenstates

$$|\theta\rangle \equiv \frac{1}{\sqrt{2}} (|A\rangle + e^{i\theta} |F\rangle) \quad \text{for} \quad \theta \in \{0, \pi\}. \quad (24)$$

This is exactly what we would expect from an ordinary quantum-mechanical double-well analysis. As a result of the mixing between these states, the energy of the symmetric $|\theta=0\rangle$ state is lower than that of the antisymmetric $|\theta=\pi\rangle$ state. As a result, the true quantum-mechanical vacuum of our theory is the $|\theta=0\rangle$ state, while the $|\theta=\pi\rangle$ state becomes a metastable vacuum. This, then, provides yet another way of achieving metastability in our model. Note that the energy of the true ground state in such a setup is lower than the classical energy associated with either Solution A or Solution F individually. Of course, despite our use of the $\theta$ variable, we emphasize that this sort of “$\theta$-vacuum” should not be confused with the traditional QCD $\theta$-vacuum; indeed, the only gauge symmetries in our model are abelian.

It is interesting to speculate that in a more complicated model, even more degenerate ground states might emerge. The true vacua of such a model would then correspond to Bloch waves across these degenerate ground states, with the corresponding energy eigenvalues populating nearly continuous energy “bands”. This could then provide one possible way of realizing the proposal in Ref. [30] for addressing the cosmological-constant problem.

VI. DISCUSSION

In this paper, we have presented a remarkably simple construction which simultaneously gives rise to a supersymmetric ground state and a non-supersymmetric metastable state, with both arising classically in a theory with no flat directions or infinite distances in field space. The key feature of our construction is that the supersymmetry-breaking in our model arises not only through $F$-term breaking, but also through $D$-term breaking. Since all of the relevant physics is perturbative, we were able to perform explicit calculations of the lifetimes and particle spectra associated with such vacua and demonstrate that these lifetimes can easily exceed the present age of the universe.

As we have shown, the supersymmetry and R-symmetry in our construction are broken at tree level in a perturbative theory where no flat directions appear in the classical potential and where all minima appear at finite locations in field space. This gives our construction a distinct advantage relative to constructions which have appeared in much of the prior literature on metastable supersymmetry-breaking. For example, the effective O’Raifeartaigh-inspired models discussed in Refs. [10,14] contain runaway directions along which supersymmetric global minima occur at infinite distances in field space. While it has been demonstrated [9] that runaway directions of this sort can be successfully stabilized by non-perturbative dynamics at large but finite field values, the presence of such non-perturbative dynamics renders the potential barrier between stable and metastable vacua difficult to describe. As a result, explicit lifetime calculations are challenging to perform.

The prior literature also contains models in which metastable supersymmetry-breaking occurs at finite field...
values. However, these scenarios generally contain classical flat directions which may or may not be lifted at the quantum level. For example, the model described in Ref. [2] leads to both supersymmetric and non-supersymmetric flat directions at tree level, separated by a flat-topped "ridge" of degenerate points in field space. Although the degeneracy along the non-supersymmetric flat direction (as well as that along the ridge) is lifted at the quantum level, the supersymmetric flat direction remains flat to all orders in perturbation theory. Thus, one would require non-perturbative physics or some modification of the model itself in order to achieve a truly stable ground state. Likewise, the model outlined in Ref. [13] also contains a metastable flat direction at tree level. Since this flat direction is destabilized by quantum corrections, some additional dynamics (such as a supergravity contribution to the scalar potential, as discussed by the authors of Ref. [13]) is required for true stability.

By contrast, the "kernel" presented in this work utilizes only finite field values and perturbative dynamics, yet possesses neither classical flat directions nor runaway behavior. As such, we believe that this represents an interesting alternative to previous supersymmetry-breaking scenarios. Indeed, our experience suggests that within the context of traditional dynamical supersymmetry-breaking scenarios, such models are relatively rare, and we are not aware of any models with these characteristics in the prior metastability literature. Moreover, while the supersymmetry-breaking in our model is sourced by $D$-terms, this in turn triggers $F$-term breaking as well. Thus our kernel avoids the phenomenological problems normally associated with pure $D$-term-breaking models, and indeed a potential method of mediating supersymmetry-breaking to a visible sector was outlined below Eq. (11).

Needless to say, metastability in and of itself is not a phenomenological necessity. Indeed, there already exist models in the literature [34] in which supersymmetry and R-symmetry are broken at tree level in the true ground state and in which no metastable vacua appear. However, we believe that metastability offers rich possibilities when it opens up new models to phenomenological viability that would previously have been deemed problematic based on the properties of their ground states. This is certainly true in the model of Ref. [9] as well as other similar models it has inspired, since the ground states of such models are supersymmetric and hence unacceptable on phenomenological grounds. However, this is also true in models with multiple $U(1)$ gauge groups (which form the context of the discussion of this paper) because the ground states of such models frequently involve pure $D$-term breaking — a situation which also yields phenomenological difficulties — as well as potentially unbroken supersymmetry. Thus, metastable supersymmetry-breaking offers a phenomenological advantage within such models as well, and suggests that our model may have broad implications for an even wider class of theories than we have considered here. Moreover, as we pointed out, such models can play the role of self-sufficient supersymmetry-breaking "kernels" which are not overly sensitive to the larger models in which they are embedded. As a result, their supersymmetry-breaking properties are largely unaffected by the additional physics that would be required in order to build a complete phenomenological model of supersymmetry-breaking.

Strong motivation to consider such models and their phenomenology also comes from the fact that structures involving additional $U(1)$ gauge groups arise with great frequency in string theory. Indeed, from this perspective, our work can be interpreted as highlighting an important fact about models involving Fayet-Iliopoulos terms and about the landscape of string-motivated models in general, indicating that substantial regions of the landscape that would not have previously been considered phenomenologically viable are in fact so. This is illustrated by the explicit examinations of the parameter space that we performed in Sect. III.

Before we conclude, a few additional comments are in order. First, the kernel presented in Sect. II includes two dimensionful parameters, $\sqrt{F}$ and $m$. While it is beyond the scope of this paper to present a specific mechanism for dynamically generating these parameters at scales parameterically smaller than the Planck scale $M_{\text{Planck}}$, we will now argue that arranging such a separation of scales may well be feasible. We mentioned in Sect. II that this kernel naturally appears in the low-energy limit of heterotic string models. In such a context, the variant [31] of the Green-Schwarz mechanism [32] that can be used to cancel the mixed gauge anomalies implicit in the charge assignments given in Table I also gives rise to the Fayet-Iliopoulos terms that break supersymmetry. However, it has been argued [27] that the physical scale associated with these terms is not tied to the Planck scale and therefore the supersymmetry-breaking scale can be much smaller than $M_{\text{Planck}}$. Alternatively, one could imagine generating $\xi_a$, $\xi_b$, and $m^2$ dynamically via the retrofitting methods of Ref. [12], or generating an ultraviolet completion of our model [33], perhaps using D-brane configurations along the lines developed in Ref. [34].

As a result of the robustness of the metastable vacuum in our model against small corrections, most of the standard methods of mediating supersymmetry-breaking to the visible sector (gravity mediation [35], gauge mediation [3,4,36], etc.) should be viable options for our model. Indeed, we saw in Sect. III that introducing additional vector-like pairs of chiral superfields with gauge charges identical to those of $\Phi_1$ and $\Phi_2$ and supersymmetric masses greater than $m$ does not destabilize the vacuum structure of the theory. This, therefore, suggests a simple method of coupling our kernel to a messenger sector. In addition, the presence of multiple $U(1)$ groups
makes $Z'$ mediation [37] an intriguing alternative possibility, and perhaps the most natural way of mediating supersymmetry-breaking to the fields of the minimal supersymmetric standard model (MSSM). A scenario of this sort, in which some of the MSSM fields would be charged under one or more hidden-sector $U(1)$ groups, could lead to a rich $Z'$ phenomenology potentially visible at the LHC [33]. It would also be interesting to incorporate kinetic-mixing effects [38] into our analysis [33].

The cosmological implications of this construction are also quite interesting. As discussed in Sect. V, regions of parameter space in this scenario which preserve the $Z_2$ reflection symmetry of the two-site moose give rise to a two-fold vacuum degeneracy and a ground state with vacuum energy lower than that of either of the degenerate classical solutions alone. One could also imagine similar scenarios [33] where a far larger number of degenerate or nearly degenerate vacua exist, resulting in a band structure of the kind explored in Ref. [30]. It may also be possible to construct an extension of this setup in which the cosmological-constant problem is addressed via transitions through a set of non-degenerate vacua, as in Ref. [39], or through other non-traditional tunnelling effects [40] in a vast stringy cosmic landscape. The presence of additional $U(1)$ gauge groups in this setup also has implications for cosmology, as they will inevitably give rise to cosmic strings [41] and other topological defects with rich phenomenologies [42].

Finally, as already mentioned above, the presence of long-lived metastable vacua and non-trivial vacuum structures can have a significant impact on state counting in studies of the string-theory landscape. Because additional $U(1)$ groups with non-trivial Fayet-Iliopoulos terms are a general feature of heterotic and Type I string models, our results suggest that metastable vacua (many of which break supersymmetry, and at potentially high scales) exist on a substantial fraction of the string landscape and must therefore be taken into account when statistical surveys of the landscape are performed. To do this properly, one would have to account for vacuum transitions at finite temperature and ascertain the thermal population of the landscape, taking into account the fact that “holes” in the landscape (i.e., regions where no stable vacuum exists) have been shown to occur in deconstruction-motivated models [26] and can result in isolated “islands” [43] that are not in thermal contact with one another.

We see, then, that the “kernel” we have presented in this paper may be of interest for constructing models of low-scale supersymmetry-breaking, for understanding the phenomenology of $Z'$ physics, and for evaluating various properties of the string landscape. As such, we expect that these sorts of models can serve as fertile arenas from which future investigations into the behavior of supersymmetric field theories might hatch.

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