PLANET FORMATION IN STELLAR BINARIES. II. OVERCOMING THE FRAGMENTATION BARRIER IN α CENTAURI AND γ CEPHEI-LIKE SYSTEMS

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Received 2014 August 21; accepted 2014 October 24; published 2014 December 23

ABSTRACT

Planet formation in small-separation (∼20 AU) eccentric binaries such as γ Cephei or α Centauri is believed to be adversely affected by the presence of the stellar companion. Strong dynamical excitation of planetesimals by the eccentric companion can result in collisional destruction (rather than growth) of 1–100 km objects, giving rise to the “fragmentation barrier” for planet formation. We revise this issue using a novel description of secular dynamics of planetesimals in binaries, which accounts for the gravity of the eccentric, coplanar protoplanetary disk, as well as gas drag. By studying planetesimal collision outcomes, we show, in contrast to many previous studies, that planetesimal growth and subsequent formation of planets (including gas giants) in AU-scale orbits within ∼20 AU separation binaries may be possible, provided that the protoplanetary disks are massive (∼10−2 M⊙) and only weakly eccentric (disk eccentricity ≲ 0.01). These requirements are compatible with both the existence of massive (several MJ) planets in γ Cep-like systems and the results of recent simulations of gaseous disks in eccentric binaries. Terrestrial and Neptune-like planets can also form in lower-mass disks at small (sub-AU) radii. We find that the fragmentation barrier is less of a problem in eccentric disks that are apsidally aligned with the binary orbit. Alignment gives rise to special locations, where (1) relative planetesimal velocities are low and (2) the timescale of their drag-induced radial drift is long. This causes planetesimal pileup at such locations in the disk and promotes their growth locally, helping to alleviate the timescale problem for core formation.

Key words: binaries: close – planetary systems – planets and satellites: formation – protoplanetary disks

1. INTRODUCTION

Planets are known to be able to form in a variety of environments, some of which are believed to be hostile to their genesis. A good illustration of this statement is provided by planets detected in close binary systems, such as γ Cephei (Hatzes et al. 2003). This eccentric (eγ = 0.41), relatively small semi-major axis (aγ = 19 AU) system consists of two stars of masses Mγ = 1.6 M⊙ and M* = 0.41 M⊙. It harbors a giant planet with the projected mass Mpl sin i = 1.6 MJ in an orbit with a semi-major axis of apl ≈ 2 AU and an eccentricity of ep = 0.12 around the primary.

Several more such systems that fall into the S-type according to classification of Dvorak (1982) are known at present (Chauvin et al. 2011; Dumusque et al. 2012). Two of them—HD 196885 (Correia et al. 2008) and HD 41004 (Zucker et al. 2004)—harbor giant planets in orbits with apl = 1.6–2.6 AU. Two more—α Cent (Dumusque et al. 2012) and Gl 86 (Queloz et al. 2000)—host planets at smaller separations, apl ≈ 0.04 AU and α pl ≈ 0.11 AU, respectively. These systems exhibit a diversity of planetary masses, with an Earth-like planet (Mpl sin i = 1.1 M⊕) orbiting our neighbor α Cent (compare Hatzes 2013) and other binaries hosting gas giants with Mpl sin i = (1.6–4.0)MJ (Chauvin et al. 2011).

The existence of planets in these tight binaries has posed a serious challenge for planet formation theories. The expectation of inward rather than outward planet migration due to disk–planet interaction (Ward 1986) suggests that such planets form in situ, at 1–2 AU (as we show in this work, it is very difficult to form them even further out). At these separations, gravitational instability is a very unlikely avenue of planet formation (Rafikov 2005, 2007). An alternative model of core accretion (Harris 1978; Mizuno 1980) relies on formation of a massive core by collisional agglomeration of a large number of planetesimals, possibly starting at small, <1 km, sizes. It is this stage of planetesimal growth in tight binaries that presents significant problems to existing planet formation theories.

Indeed, it has been known since the work of Heppenheimer (1978) that an eccentric stellar companion can drive very large planetesimal eccentricities, ∼0.1 at AU-scale separations. This would cause planetesimals to collide at high relative speeds of a few km s−1. As this is much higher than the escape speed from the surface of even a 100 km object (about 100 m s−1), collisions between planetesimals should lead to their destruction rather than growth, introducing a fragmentation barrier for planet formation (see Section 4). The theoretical expectation of suppressed planet formation in α < 20 AU binaries has been largely corroborated by recent observations (Wang et al. 2014).

The premise of our present work is that the key to solving the fragmentation barrier puzzle lies in achieving a better understanding of planetesimal dynamics. However, some alternative suggestions have also been considered over the years. For example, Thebault et al. (2008, 2009) proposed that tight planet-hosting binaries could have started on more extended orbits, which were subsequently shrunk by interactions with other stars in their birth cluster. Paardekooper & Leinhardt (2010) propose a solution involving a non-standard mode of planetesimal accretion. It may also be possible that planetesimals are born big (Johansen et al. 2007), with sizes exceeding 102 km, in which case they are safe from collisional destruction from the start. These possibilities would need to be invoked if we were not able to resolve the fragmentation barrier puzzle by the better treatment of planetesimal dynamics alone, underscoring the importance of this aspect of the problem.

The dynamics of planetesimals in binaries are complicated by a plethora of agents affecting their motion. It has been recognized for some time that not only the companion gravity but also gas drag affects planetesimal motion (Marzari & Scholl...
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(Xie & Zhou 2009; Xie et al. 2010). More recently, it was shown that the gravity of the protoplanetary disk in which planetesimals reside has a dominant effect on their dynamics (Rafikov 2013; hereafter R13). The tendency of protoplanetary disks in binaries to become eccentric further complicates this issue; see Silsbee & Rafikov (2015; hereafter SR15).

Generalizing these efforts, Rafikov & Silsbee (2015; hereafter Paper I) combined different physical ingredients—gravity of an eccentric disk, perturbations due to the companion star, and gas drag—to present a unified picture of planetesimal dynamics in binaries in secular approximation. They came up with analytical solutions for planetesimal eccentricity and explored the behavior of relative velocities between planetesimals of different sizes.

Our present goal is to use these dynamic results to understand planetesimal growth in tight binaries with a particular focus on the fragmentation barrier issue. We couple them with a recent approximation, i.e., neglecting short-term gravitational perturbations in a model with a planet in a non-precessing disk with an eccentric orbit and a small inclination between the disk and the binary (Regály et al. 2009; Statler 2001; Ogilvie 2001; SR15). Here we assume a power-law dependence of the disk surface density at the periastron of the fluid trajectory with semi-major axis \(a_d\). Surface density at an arbitrary point in the disk can be uniquely specified once \(e_p(a_d)\) and \(\Sigma_p(a_d)\) are known. We assume that disk mass \(M_d\) is contained near \(a_{out}\), the surface density distribution is given by

\[
\Sigma_p(a_d) = \frac{2 - p}{2 \pi} \frac{M_d}{a_{out}} \left(\frac{a_{out}}{a_d}\right)^p \approx 3 \times 10^3 \text{g cm}^{-2} \text{M}_d^{-2} a_{out}^{-5} a_d^{-1},
\]

where \(p\) is the power-law index (\(p = 1\) in the numerical estimate), \(M_d \approx M_d/(10^{-2} \text{M}_\odot)\), \(a_{out} \approx a_{out}/(5 \text{AU})\), and \(a_d \equiv a_d/\text{AU}\). Equation (2) neglects disk ellipticity and assumes \(p < 2\), so that most of the disk mass is concentrated near \(a_d\). Unless stated otherwise (see Section 5) we will be using a disk model with \(p = 1\) and \(q = -1\) in our calculations, i.e., \(\Sigma_p(a_d) \propto a_d^{-1}\) and \(e_p(a_d) \propto a_d^{-1}\); see R13 and SR15 for motivation. We assume a disk with \(a_{out} = 5 \text{AU}\).

Planetesimals of radius \(d_p\) orbit the primary within the disk and are coplanar with it and the binary. Their orbits are described by the semi-major axis \(a_p\), the eccentricity \(e_p\), and the apsidal angle (with regard to the binary apsidal line) \(\varpi_p\). The latter two are often combined for convenience into the planetesimal eccentricity vector \(e_p = (k_p, h_p) = e_p \cos \varpi_p, \sin \varpi_p\). Everywhere in this work, we assume \(e_p \ll 1\) as well as \(e_s \ll 1\).

3. SUMMARY OF THE RESULTS ON PLANETESIMAL DYNAMICS

In Paper I we obtained a number of important results on the dynamics of planetesimals in binaries in a secular approximation, i.e., neglecting short-term gravitational perturbations (Murray & Dermott 1999). Our calculations simultaneously accounted for the gravity of the massive eccentric protoplanetary disk, binary companion, and gas drag.

Gravitational perturbations due to the binary companion and the eccentric disk excite planetesimal eccentricity at the rates determined by the eccentricity excitation terms \(B_{\text{h}}\) for the binary and \(B_d\) for the disk, given by Equations (7, PI) and (8, PI), respectively ("PI" means that the referenced equation can be found in Paper I). At the same time, the axiymmetric component of the gravity of these perturbers drives apsidal precession of planetesimal orbits at rates \(\dot{\varpi}_p\), (binary, Equation (5, PI)) and \(\dot{\varpi}_d\) (disk, Equation (6, PI)). We invariably find that in disks massive enough to form Jupiter mass planets, \(M_d \gtrsim 10^{-2} \text{M}_\odot\), planetesimal precession, and often eccentricity excitation, are dominated out to a few AU by the gravity of the disk. This finding is a novel result of R13, SR15, and Paper I.

We showed that in the case of a non-precessing disk with a fixed orientation with respect to the binary apsidal line, the planetesimal eccentricity \(e_p\) is an analytic function of the planetesimal size \(d_p\) and system parameters given by the expressions (22, PI)-(28, PI), (32, PI), and (33, PI). The latter enter equations through the two key variables—characteristic eccentricity \(e_s\) and size \(d_e\)—defined by Equations (29, PI) and
Figure 1. Maps of the relative eccentricity $e_{12}$ (left color bar) and velocity $v_{12} = e_{12} v_e$ (right color bar) for planetesimals of different sizes $d_1$ and $d_2$ (see Paper I for similar maps). Calculation is done at $a_p = 1$ AU for the $\gamma$ Cephei system for our standard ($p = 1$, $q = -1$), aligned ($\varpi_d = 0$) disk with $M_d/M_p = 0.01$ and disk eccentricity at its outer edge ($a_p = 1$ AU) $e_0 = 0.03$ (resulting in $e_c \approx 2.45 \times 10^{-3}$) and ($b$) $e_0 = 0.01$ ($e_c \approx 3.15 \times 10^{-4}$). Contours illustrate collisional outcomes using different fragmentation criteria: catastrophic destruction (3) in panel (a), white and erosion (6) in panel (b), black. Planetesimals are destroyed in collisions of pairs of objects within corresponding contours. Solid and dashed contours are for strong and weak planetesimals. The extent of the destruction zone (arrow) and the smallest and largest ($\delta s$ and $\delta l$) sizes of planetesimals that get destroyed are illustrated in panel (a). In panel (b), the parameter $\chi$ measures the extent of the erosion zone: it represents a lower limit on the size ratio of objects that lead to erosive collisions.

(31, PI), respectively. The dependence of $e_c$ and $d_c$ on the system parameters was explored in great detail.

Our analytic solutions allow us to produce maps of the relative eccentricity $e_{12} = |e(d_1) - e(d_2)|$ for pairs of planetesimals of different sizes $d_1$ and $d_2$: an example is shown in Figure 1. We also derived a distribution of approach velocities for colliding planetesimals (Section 8 of Paper I) and shown it to be rather narrow, with the approach velocity $v_{12}$ constrained to lie within the range $(1/2) v_K e_{12} < v_{12} < v_K e_{12}$, where $v_K$ is the local Keplerian speed. Thus, maps such as those shown in Figure 1 directly characterize the typical velocity at which planetesimals collide, $v_{12} \sim v_K e_{12}$, and allow us to understand their collision outcomes; see Section 4.

The $e_{12}$ and $v_{12}$ maps in Figure 1 are made for the $\gamma$ Cephei system at $a_p = 1$ AU for the standard ($p = 1$, $q = -1$), aligned ($\varpi_d = 0$) disk with $M_d/M_p = 10^{-2}$ and $e_0 = 0.03$, 0.01 (resulting in $e_c = 2.45 \times 10^{-3}$, $3.15 \times 10^{-4}$, respectively). One can clearly see that planetesimals exhibit small relative eccentricity in a blue region around the diagonal line $d_1 = d_2$. This low $e_{12}$ “valley” appears because planetesimals with similar sizes follow similar orbits, and collide with low relative speed. The valley is narrowest at $d_1, d_2 \sim 0.1$–1 km (depending on $e_c$), which corresponds to the characteristic size $d_c$ given by Equation (31, PI). For $d_1, d_2 \ll d_c$, planetesimals experience apsidal alignment and their relative eccentricities are lowered by gas drag. For $d_1, d_2 \gg d_c$, apsidal alignment is accomplished...
by the disk and companion gravity, again resulting in small \( e_{12} \). On the contrary, planetesimals of very different sizes (upper left and lower right regions) are not aligned and exhibit high relative eccentricity, with \( e_{12} \approx e_c \) given by Equation (29, PI).

We also obtained some analytical results on planetesimal eccentricity behavior in precessing disks; see Section 6 of Paper I. We did this in two limiting cases: when binary gravity dominates over that of the disk and vice versa. These asymptotic results are used to understand planetesimal growth in precessing disks in Section 7.

4. PLANESETIMAL COLLISION OUTCOMES

The description of the dynamical behavior of planetesimals provided in Paper I is used in this work to understand the outcomes of their collisions.

There are different ways in which planetesimal collisional evolution can be characterized. A high-velocity collision is usually considered catastrophic when the mass of the largest surviving remnant is less than half of the combined mass of objects \( M_{\text{tot}} = m_1 + m_2 \) involved. In this work, we use a fragmentation prescription developed by Stewart & Leinhardt (2009), which suggests that a collision is catastrophically disruptive if

\[
\frac{Q_R}{Q_{R,D}} > 1, \quad \left(3\right)
\]

\[
Q_R = \frac{M_t v_{\text{coll}}^2}{2M_{\text{tot}}}, \quad \left(4\right)
\]

\[
Q_{R,D}^* = q_s R_C^{\mu_c(3-2\phi)} v_{\text{coll}}^2 q_g R_C^{3\mu_c/2} v_{\text{coll}}^2, \quad \left(5\right)
\]

where \( Q_R \) is the appropriately scaled kinetic energy of the collision, \( M_t = m_1 m_2 / (m_1 + m_2) \) is the reduced mass of the colliding objects, and \( v_{\text{coll}} \) is the collision speed at the moment of contact. The energy threshold for catastrophic disruption \( Q_{R,D}^* \) depends on constants \( q_s, \mu_c, \phi, \) and \( q_g \) related to the material properties of the planetesimals; \( R_C \) is the radius of a sphere with mass \( M_{\text{tot}} \) and a density of 1 g cm\(^{-3}\). Following Stewart & Leinhardt (2009), we use \( \mu_c = 0.4, \phi = 7, q_c = 500, \) and \( q_g = 10^{-4} \) (in proper CGS units) for our weak planetesimals and \( \mu_c = 0.5, \phi = 8, q_c = 7 \times 10^4, \) and \( q_g = 10^{-4} \) for strong ones.

On the other hand, even if condition (3) is not satisfied and catastrophic disruption is avoided, collisional growth is not guaranteed—it requires that the largest object (e.g., \( m_1 \)) is not eroded in a collision. Erosion occurs when the largest remnant is less massive than the more massive body involved in a collision. According to Stewart & Leinhardt (2009), erosion happens whenever

\[
\frac{Q_R}{Q_{R,D}} > 2 \frac{m_2}{M_{\text{tot}}}, \quad m_2 < m_1. \quad \left(6\right)
\]

This condition is far more prohibitive for growth than (3) since \( m_2 \) can be much less than \( m_1 \). Growth in a given collision occurs only when the condition (6) is violated.

Figure 2 illustrates the two collisional criteria (3) and (6) by showing the critical (minimum) relative planetesimal velocity \( v_{\text{coll}} \) that leads to either catastrophic destruction (panel a) or erosion (panel b) of the bodies of different sizes. Various curves correspond to different size ratios and internal strengths of the objects involved in a collision.

In the case of catastrophic disruption, the critical \( v_{\text{coll}} \) is a sensitive function of the size ratio of objects involved. Collisions of objects of similar size are clearly more destructive than those of planetesimals with very different sizes as the former are characterized by lower critical \( v_{\text{coll}} \). For collisionally strong objects (solid curves), we find that most destructive collisions (requiring the lowest relative speed \( \sim 10 \text{ m s}^{-1} \) for destruction of equal-mass objects) involve \( \sim 300 \text{ m planetesimals, almost independent of the mass ratio. For collisionally weak objects, this size is } \sim 100 \text{ m and } v_{\text{coll}} \sim 1 \text{ m s}^{-1} \) for \( m_1 = m_2 \).

In the case of erosion, the critical \( v_{\text{coll}} \) attains minimum values roughly at the same sizes. However, the dependence on mass ratio is very weak and vanishes in the limit of \( m_2 \ll m_1 \). This follows from Equation (4) which demonstrates that in this limit, \( Q_R \propto m_2 \), canceling the dependence on \( m_2 \) on the right-hand side of the condition (6). This difference in behaviors between the two collisional criteria has important implications as we show next.

We note at this point that the critical velocity curves shown in Figure 2(b) are not likely to be applicable for the case of erosion by very small objects. In this limit, one would expect cratering and mass loss from the target to be determined by its local material properties (Housen & Holsapple 2011), rather than global ones as suggested by the Stewart & Leinhardt

![Figure 2](image_url)
Figure 3. Variation of the destruction (white contours) and erosion (black contours) zones with the disk eccentricity and $e_c$. Calculations are done for the same parameters as in Figure 1, except that now we use (a) $e_0 = 0.1$ (resulting in $e_c \approx 10^{-2}$) and (b) $e_0 = 8.8 \times 10^{-3}$ (resulting in $e_c \approx 1.9 \times 10^{-3}$). Note that in panel (b) catastrophic disruption never presents a problem for planetesimal growth (no white contours).

The procedure used for calculating the relative eccentricity of colliding planetesimals $\epsilon_{12}$ in both the non-precessing and precessing disks is outlined in the Appendix.

Maps of $\epsilon_{12}$, $v_{12} = \epsilon_{12}v_K$ such as the one presented in Figure 1 show that $\epsilon_{12}$ is a function of $d_1$, $d_2$, meaning that the same is true for $v_{\text{coll}}$ in our approach. We can then use these maps to directly illustrate collision criteria for both strong and weak planetesimals. In Figure 1(a) the two regions inside the white boundaries stretching along the $d_1 = d_2$ line represent the “zone of destruction”: planetesimals with sizes falling into this region get catastrophically destroyed in mutual collisions. The extent of such a zone in $d_p$ is indicated with a white arrow, and the largest and smallest planetesimal sizes that get destroyed in collisions are denoted $d_L$ and $d_S$.

In Figure 1(b), black contours delineate “zones of erosion”: collisions of objects falling within the corresponding contour result in mass loss by the larger planetesimal, hindering

(2009) prescription. Then the critical velocity (in the strength-dominated regime, in the absence of ejecta re-accumulation) should become independent of the target size as the projectile-to-target size ratio tends toward zero; this is not what Figure 2(b) shows. To avoid this issue in the following, we do not explore erosion in the limit of very large size ratios of colliding bodies; see Section 5.2.

4.1. Relative Velocities and Collision Outcomes

We now couple this understanding of different collisional outcomes with the dynamical results of Paper I and proceed as follows. We compute the relative collision velocity of the two objects $v_{\text{coll}}$ as $v^2_{\text{coll}} = \epsilon_{12}^2v_K^2 + 2G(m_1 + m_2)/(d_1 + d_2)$, where $d_1$, $d_2$ are the sizes of planetesimals with masses $m_1$, $m_2$. Note that by using the maximum possible approach velocity $\epsilon_{12}v_K$ for calculating $v_{\text{coll}}$, we are being conservative, since the actual approach speed may be as small as $(1/2)\epsilon_{12}v_K$, see Section 3.
growth. The extent of the erosion zone is characterized by the dimensionless parameter $\chi$, which is the smallest target-to-projectile size ratio of objects that can get eroded in a collision for a given set of system parameters; see Figure 1(b) for illustration of this definition. The overall morphology of the erosion zone is similar to “erosion regions” found by Thébault et al. (2008) in $d_1 - d_2$ space using numerical integration of planetesimal orbits and fragmentation criteria different from ours; see their Figures 2, 6, and 7. Note, however, that our Figure 1(b) shows the erosion zone over a much broader range of planetesimal sizes.

Both the “islands of destruction” in Figure 1(a) and the “islands of erosion” in Figure 1(b) exhibit a narrow “channel” between them at $d_1 = d_2$ where growth is possible. This common feature is due to the fact that $e_{12} \to 0$ when $d_1$ and $d_2$ are exactly the same because $e_\perp$ is a function of planetesimal size only. At the same time, the general morphologies of the destruction and erosion regions are different—the former does not extend too far from the $d_1 = d_2$ line because catastrophic destruction of a target planetesimal in collisions of very different objects (either $d_1/d_2 \ll 1$ or $d_1/d_2 \gg 1$) would require very high relative velocity; see Figure 2(a). On the other hand, erosion is possible even for collisions of highly unequal objects; see Figure 1(b), simply because the critical $v_{\text{crit}}$ becomes independent of $d_1/d_2$ as $d_1/d_2 \to 0$. As expected, for collisionally weak objects, both the destruction and the erosion zones are more extended in $d_1 - d_2$ space, as shown by the dashed contours in Figure 1.

The extent of these zones sensitively depends on the value of the eccentricity scale $e_\perp$. This is illustrated in Figure 3 where the variation of these zones with the characteristic planetesimal eccentricity $e_\perp$ is shown for strong planetesimals; the rest of the parameters are as in Figure 1. For high values of the disk eccentricity (at its outer edge), $e_\perp = 0.1$ (panel (a)), one obtains high $e_\perp \approx 10^{-2}$, which results in very extended destruction and erosion zones. The former zone has $d_1/d_2 \approx 300$, while for the latter $\chi \approx 1$. In other words, the growth-friendly channel between the two lobes of the erosion zones is extremely narrow, making planetesimal agglomeration highly unlikely in this case.

Lowering $e_\perp$ to 0.03 ($e_\perp \approx 2.45 \times 10^{-3}$) as in Figure 1(a) shrinks the size of the destruction zone so that it presents danger for planetesimals within a size range of only about an order of magnitude, $d_1/d_2 \sim 0$. Reducing disk eccentricity even further as in Figure 3(b) ($e_\perp = 8.8 \times 10^{-3}$, $e_\perp \approx 1.9 \times 10^{-3}$), we find that the catastrophic destruction zone fully disappears.

At the same time, erosion zones tend to persist even in disks with very small eccentricity. For example, one finds $\chi \approx 3$ for $e_\perp = 0.01$ ($e_\perp \approx 3.15 \times 10^{-4}$); see Figure 1(b). This means that planetesimals in such a disk cannot erode a larger object if its mass is $\lesssim 30$ times higher. In Figure 3(b), where the destruction zone vanishes completely, the erosion zone with $\chi \sim 10$ is still present and may affect growth of planetesimals with radii $\sim 0.05$–1 km.

### 5. IMPLICATIONS FOR PLANETESIMAL GROWTH IN BINARIES

We now use our understanding of the collisional outcomes described in Section 4 to explore the possibility of planetesimal growth in binaries as a function of the two key protoplanetary disk characteristics—disk mass $M_d$ and its eccentricity at the outer edge $e_\perp$ (defined by Equation (1)); we fix the disk model to have $p = 1$, $q = -1$.

In Figure 4 we present maps of collisional outcomes for strong planetesimals in the $M_d - e_\perp$ space. Each map uses parameters of a particular planet-hosting binary—HD 196885, γ Cep, and HD 41004 (Chauvin et al. 2011)—selected because they host Jupiter-mass planets in AU-scale orbits. These maps are computed at the distance from the primary $a_p$ equal to the present-day semi-major axis of the planet (shown in the panels); planet mass is indicated by the vertical red dashed line in each panel. Calculations used to produce this figure assume that the disk is aligned with the binary, i.e., $\sigma_d = 0$. The effect of non-zero $\sigma_d$ is explored further in Section 6.1.

#### 5.1. Accounting for Catastrophic Disruption

For each point in the two-dimensional space $M_d - e_\perp$, we construct the relative velocity distribution for planetesimals of different sizes as shown in Figure 1. Using this map of $e_{12}$ parameters for which the catastrophic destruction of planetesimals of any size never happens. The black region is a part of the phase space where growth with some erosion, limited by the condition $\chi > \chi_{\text{min}} = 10^{2/3} \approx 4.6$ (see Figure 1(b)) can take place. The purple and cyan lines are the $|A_d| = |A_d|$ and $|B_d| = |B_d|$ conditions, i.e., $M_d = M_d, \Delta = 0$ and $M_d = M_d, |B_d| = |B_d|$ curves defined by Equations (49, P1) and (52, P1).

![Figure 4. Map of the conditions favorable for planetesimal growth in the $M_d - e_\perp$ space for three binaries (labeled on panels) harboring Jupiter mass planets in AU-scale orbits with $a_p \sim$ AU. Planetary semi-major axes are indicated and their $M_d, \sin i$ are shown with red dashed lines in each panel. Gray areas correspond to disk parameters for which the catastrophic destruction of planetesimals of any size never happens. The black region is a part of the phase space where growth with some erosion, limited by the condition $\chi > \chi_{\text{min}} = 10^{2/3} \approx 4.6$ (see Figure 1(b)) can take place. The purple and cyan lines are the $|A_d| = |A_d|$ and $|B_d| = |B_d|$ conditions, i.e., $M_d = M_d, \Delta = 0$ and $M_d = M_d, |B_d| = |B_d|$ curves defined by Equations (49, P1) and (52, P1).](image-url)
and the recipe provided in Section 4, we determine whether the catastrophic destruction zone (white contours in Figures 1 and 3) defined by the condition (3) appears in it. If it does not, then the corresponding points in $M_d - e_0$ space in Figure 4 are colored gray. The resultant gray region in this figure covers part of the parameter space in which catastrophic collisions do not present a danger to planetesimal growth.

In the opposite case, when the white contours appear in the $\epsilon_{12}$ maps, catastrophic disruption gets in the way of planetesimal growth. Parts of $M_d - e_0$ phase space, in which planetesimal growth is interrupted by catastrophic collisions, are not colored and lie outside the gray regions in Figure 4.

5.2. Accounting for Erosion

Even if catastrophic fragmentation is avoided (i.e., outside of white region in Figure 4), planetesimal growth may still be complicated by the erosion of growing objects in numerous collisions with smaller planetesimals. To address this issue, we check whether for given values of $M_d$ and $e_0$, the erosion condition (6) gets satisfied for any $d_1$, $d_2$ in a corresponding map of $\epsilon_{12}$ (i.e., whether black contours such as in Figures 1 and 3 appear in the $\epsilon_{12}$ map). If it does, we need to decide how dangerous it can be for growth, which is a non-trivial issue.

First, demanding erosion to be completely absent as a necessary condition for planetesimal growth is likely too conservative. Even if some collisions are erosive, planetesimals should still be able to grow provided that the mass gain in non-erosive collisions exceeds the mass loss in erosive impacts. Examination of the erosion zone shape in Figures 1(b) and 3 shows that a collisional evolution is complicated by the erosion of growing objects in numerous collisions with more massive objects of comparable size result in mergers, adding a substantial amount of mass and easily resulting in net mass gain and an overall growth of planetesimals. This can naturally be the case if the planetesimal size distribution is such that at all times most mass is concentrated in the largest objects.

The exact balance of mass loss and gain depends on the velocity and mass spectrum of colliding planetesimals. The results of Paper I allow us to predict the former. However, the latter can be known only after a self-consistent calculation of planetesimal coagulation and evolution of the mass spectrum is performed. Such a calculation needs the improved dynamic input from Paper I and requires an understanding of the inclination distribution of planetesimals in binaries, which is a key input for calculation of their collision rate. These developments are beyond the scope of the present work, as our current main goal is simply to understand the general implications of the improved description of planetesimal dynamics (Paper I) on their collisional evolution.

Second, recently Windmark et al. (2012) and Garaud et al. (2013) have shown that planetesimal growth can proceed even in the presence of collisional barriers. This possibility arises when the coagulation–fragmentation process is treated in a statistical sense, allowing for a distribution of collisional outcomes. Unlike the deterministic approach that is usually employed, this way of treating planetesimal growth allows low probability events—formation of massive objects immune to collisional destruction—to occur, given a large total number of bodies in the system through a series of “lucky” collisions. As a result, some planetesimals can grow even though the majority get destroyed. In our case, this may allow growth if some degree of erosion and even a chance of catastrophic fragmentation (i.e., $d_1/d_2 > 1$; see Figure 1(a)) are present.

To account for these arguments, we assume planetesimal growth to be possible in the presence of some erosion as long as it is not too significant. More specifically, we will assume that planetesimals can grow (in a statistical sense) if the extent of the erosion zone is limited by some minimum value of the parameter $\chi$ defined in Section 4 and Figure 1(b). In this work, we use a fiducial value

$$\chi_{\text{min}} = 10^{2/3} \approx 4.6,$$

which means that a growing planetesimal cannot be eroded in collisions with projectiles more massive than $10^{-2}$ of its own mass. We choose this particular value of $\chi_{\text{min}}$ simply for illustrative purposes, while in practice it should be determined based on planetesimal coagulation models (Windmark et al. 2012; Garaud et al. 2013). It is also low enough that we do not need to worry about the applicability of the critical velocity curves in Figure 2(b) in the $\chi \to \infty$ limit; see the discussion in Section 4.

Black regions in Figure 4 cover the part of the $M_d - e_0$ parameter space where the condition $\chi > \chi_{\text{min}} = 10^{2/3}$ is fulfilled. We assume planetesimal growth to be possible there despite some degree of erosion in collisions with small objects.

5.3. Specific Systems

A general conclusion that can be drawn from Figure 4 is that, given our growth criteria, planetesimal accretion may be possible in tight binaries at the semi-major axes of the present day planets, as long as the disk mass is high and the disk eccentricity is low. Growth is also possible along a narrow extension of the colored region toward higher $e_0$ and lower $M_d$, roughly along the cyan line $|B_{d2}| = |B_{e0}|$ describing the equality of planetesimal excitation by the binary and the disk. The origin of this growth-friendly region is connected to the existence of the valley of stability (see Section 3) in aligned disks, which is further discussed in Section 6.1.

Focusing on specific systems, Figure 4(c) shows that in situ planetesimal growth (i.e., at the observed semi-major axis of the planet) is easiest in the HD 41004 system (Zucker et al. 2008); see Figure 4(a). Previously, Thébault (2011) realized that HD 196885 presents the most serious challenge for in situ planetesimal growth. This is mainly because of the large $a_{pl} \approx 2.6$ AU, making planetesimal accretion with some erosion possible only in very massive disks with $M_d \gtrsim 1.5 \, M_\odot$ and for $e_0 \lesssim 0.08$. Note that at very high $M_d$ an ejection resonance corresponding to commensurability $A = n_b$ between the planetesimal apsidal precession and the binary mean motion (Touma & Wisdom 1998), can appear in the disk. This would additionally disturb dynamics of planetesimals and complicate their growth (see Paper I).

Not too different is $\gamma$ Cephei (Figure 4(b)) with its high $a_{pl} \approx 2$ AU and $M_d \approx 1.6 \, M_\odot$: here planetesimal growth with $\chi > 10^{2/3}$ requires $M_d \gtrsim 0.1 \, M_\odot$ and $e_0 \lesssim 0.007$. Alternatively,
growth should also be possible if disk parameters fall within the valley of stability (see Section 6.1), which can be quite wide at its lower right end.

Figure 4 reveals some additional important details. First, the purple vertical lines in Figure 4 mark the location of the secular resonance, where the planetesimal precession rate \( A = A_d + A_b \) becomes zero; see Section 7.1 of Paper I. At a disk mass of \( M_{d,A=0} \), corresponding to this resonance (Equation (49, PI)), the value of \( e_c \) diverges in secular approximation, meaning that planetesimals collide at very high speeds, resulting in their destruction.

Second, planet masses \( (M_d \sin i) \) indicated by the red dashed lines in Figure 4 never fall below \( M_{d,A=0} \) at the corresponding semi-major axis. Under the natural assumption of \( M_d > M_{pl} \), we can conclude that the protoplanetary disk mass \( M_d \) must have exceeded \( M_{d,A=0} \) by at least a factor of several. Based on the results of SR15 and Paper I, this inevitably implies that the in situ growth of planetesimals toward forming cores of gas giants should always occur in either DD or DB dynamic regimes in the classification of SR15, i.e., when \( |A_d| \gtrsim |A_b| \) (to the right of the purple line in Figure 4) and disk gravity dominates the planetesimal precession rate. This important fact was completely overlooked prior to the work of R13 and SR15.

6. SENSITIVITY TO MODEL PARAMETERS

Next we explore the sensitivity of our results to the different parameters of the calculation, such as the disk orientation (Section 6.1), radial distribution of the gas surface density and eccentricity (Section 6.2), and distance from the primary (Section 6.3). We focus on the \( \gamma \) Cephei system and vary our inputs one by one. The results are then compared with Figure 4(b), allowing us to isolate the most important factors affecting planetesimal growth.

6.1. Role of the Disk Orientation

We start by analyzing how planetesimal growth is affected as we vary the disk orientation with respect to the binary apsidal line quantified via the angle \( \sigma_d \).

An important feature of the perfectly aligned disk visible in Figure 4(b) is the “safe zone” favorable for growth, which extends toward the upper left corner of the \( M_d - e_0 \) map. Its origin lies in the presence of the dynamic “valley of stability” in the \( M_d - e_0 \) phase space for aligned disks. This feature is easily visible in Figure 4(a) of Paper I as a narrow region, within which the characteristic eccentricity \( e_c \) is low. Compared with Figure 4(b) we see that the shape of the growth-friendly region in \( M_d - e_0 \) space mirrors the overall morphology of the dynamical valley of stability.

An in depth discussion of the “valley of stability” properties is provided in Section 7.2 of Paper I where it is shown, in particular, that for \( M_d \gtrsim M_{d,A=0} \), this valley stretches close to the \( |B_d| = |B_b| \) curve (cyan line in Figures 4 and 5) defined by Equation (52, PI), which corresponds to the equality of the planetesimal eccentricity excitation contributions provided by the disk \( (B_d) \) and the binary companion \( (B_b) \). This dynamical feature makes planetesimal growth possible even in low-mass disks with \( M_d \gtrsim 3 \times 10^{-3} M_\odot \), as long as the disk eccentricity \( e_0 \) takes on a particular value of the order of several percent. The valley of stability vanishes for \( M_d \sim M_{d,A=0} (\approx 1.6 \times 10^{-3} M_\odot \) for \( \gamma \) Cep) because a secular resonance appears at this disk mass driving \( e_c \) to very high values and making growth impossible. However, for even lower disk masses the valley of stability re-emerges, making planetesimal growth possible even in low-mass disks \( (M_d \lesssim 10^{-3} M_\odot) \) but only at a certain (narrow) range of the disk eccentricity \( e_0 \approx 0.1 \) given by Equation (54, PI).

As the disk orientation changes away from perfect alignment, the valley of stability starts to shrink. Figure 5(a) shows that even a relatively small misalignment of \( \sigma_d = 10^\circ \) is enough to eliminate the growth-friendly zone for \( M_d \lesssim M_{d,A=0} \). Planetesimal growth without catastrophic disruption is then possible only for \( M_d \gtrsim 10^{-2} M_\odot \), but it may still proceed at disk eccentricity \( e_0 \approx 0.01-0.04 \) (upper left of the grey region). Growth allowing for some erosion with \( \chi < 10^{2/3} \) requires \( M_d \gtrsim 0.03 M_\odot \) and \( e_0 \lesssim 0.015 \) (upper left of the black region). At the same time, the overall morphology of the growth-friendly zone remains roughly the same as in the aligned case—a relatively narrow region extending toward the upper left corner of the \( M_d - e_0 \) parameter space.

At \( \sigma_d = 25^\circ \) growth, avoiding catastrophic fragmentation is possible if \( M_d \gtrsim 0.02 M_\odot \) and \( e_0 \lesssim 0.02 \). Erosion with
\( \chi < 10^{2/3} \) is not an obstacle for growth only for \( M_d \gtrsim 0.08 M_\odot \)
and \( e_0 \lesssim 0.006 \).

Finally, for an anti-aligned disk (\( \sigma_d = \pi \)), the growth avoiding catastrophic destruction is still possible for \( M_d > 0.04 M_\odot \), \( e_0 < 0.01 \). Planetesimal growth with even modest erosion (\( \chi < 10^{2/3} \)) is certainly not possible in such a disk if its mass is below \( \sim 0.2 M_\odot \).

These results demonstrate that both the valley of stability and the extended region favorable to planetesimal growth in Figure 5 are endemic to relatively well-aligned disks. We conclude that the maximum disk misalignment at which the valley of stability can still facilitate planetesimal growth is \( \sigma_d \approx 10^0\text{--}15^\circ \).

Simulation results regarding the value of \( \sigma_d \) for non-precessing disks are rather mixed. Most of the simulations of Müller & Kley (2012) are consistent with relatively well-aligned disks and \( \sigma_d < 10^0 \). This would greatly facilitate planetesimal growth in binaries. At the same time, Paardekooper et al. (2008) and Marzari et al. (2012) find \( \sigma_d \approx \pi \), i.e., anti-alignment. Part of the reason for the discrepancy between the different studies may lie in the method used to determine disk eccentricity (Marzari et al. 2009)—whether it is based on osculating orbital elements of fluid elements or on fitting the isodensity contours of the disk. Thus, the numerical evidence regarding the actual value of \( \sigma_d \) is inconclusive at the moment.

### 6.2. Sensitivity to the Disk Model

In Figure 6 we test the sensitivity of our results on collisional outcomes to other details of the adopted disk model. Namely, we vary power-law indices \( p \) and \( q \) characterizing \( \Sigma_d(r) \) and \( e_d(r) \). Comparison with the middle panel of Figure 4 shows that variations of the \( \Sigma_d \) profile (i.e., of \( p \)) do not induce noticeable changes. However, results are sensitive to the eccentricity profile—the model with \( q = -1/2 \) (\( e_d \propto a_d^{1/2} \)) in Figure 6(b) yields higher disk eccentricity \( e_\delta \) at the same semi-major axis and for the same \( e_0 \) than the \( q = -1 \) model (\( e_d \propto a_d \)); see Equation (1). This has a detrimental effect on planetesimal growth and shrinks the size of the growth-friendly zone in the \( M_d - e_0 \) space.

### 6.3. Variation with the Location in the Disk

Calculations shown in Figure 4 are performed at a single location—present day semi-major axis of the planet in each system. In Figures 7 and 8 we illustrate how the conditions favorable for planetesimal growth change as the distance to the star is varied.

Our discussion of collisional outcomes in Section 4 shows that the detrimental effect of catastrophic collisions for planetesimal growth can be characterized by the sizes \( d_l \) and \( d_s \) of the largest and smallest objects that get destroyed; see Figure 1 for illustration. We can describe the effect of catastrophic collisions via the ratio \( d_l/d_s \), which exceeds unity whenever such collisions are possible for some planetesimal sizes. The higher \( d_l/d_s \) is, the more extended the catastrophic disruption zone is and the more difficult it is for growing planetesimals to avoid being destroyed in such collisions. The white regions in the maps in Figure 4 correspond to \( d_l/d_s > 1 \), while in the gray regions catastrophic collisions are absent for any planetesimal sizes.

In Figure 7 we illustrate the sensitivity of planetesimal growth to catastrophic disruption by showing the maps of \( d_l/d_s \) as a function of both the disk mass \( M_d \) and the semi-major axis \( a_p \), for several values of the disk eccentricity at its outer edge \( e_0 \). Calculation is done for an aligned disk in the \( \gamma \) Cep system.

For a high \( e_0 = 0.1 \) we see two regions favorable to growth (i.e., the ones where \( d_l/d_s \) is unity or at least less than \( \sim 10 \)). First, there is a thin dark blue band along the \( |B_b| = |B_d| \) (cyan) curve, corresponding to the “valley of stability”; see Equation (52, PI). Second, close to the star, planetesimal dynamics are completely dominated by disk gravity (the DD dynamic regime in the classification of SR15), so that \( e_\delta \sim e_\varpi \), which is small in the inner disk (for our \( q = -1 \)). For the lower eccentricity models shown in panels (b) and (c), most of the DD regime (high \( M_d \), small \( a_d \); see SR15) is favorable for planet formation. It may seem surprising that the \( e_0 = 0.01 \) case appears to be slightly more favorable than the \( e_0 = 0 \) case. This is because of the existence of the valley of stability for \( e_0 \neq 0 \) (panel b), which slightly widens the growth-friendly zone in the DD regime; see Section 7.2 of Paper I.

It is also interesting that the upper left corner of the high \( e_0 \) map shown in panel (a) is more favorable for planetesimal growth than in maps corresponding to lower \( e_0 \). This is caused by the degeneracy of the particular choice \( e_0 = 0.1 \) mentioned in Section 7.2 of Paper I (see Equation (54, PI)), which causes \( e_\delta \) to be low in the corresponding region (the BB regime in the classification of SR15) of Figure 4(c) of Paper I.

Next, in Figure 8 we illustrate the sensitivity of planetesimal growth to erosion by showing the maps of \( \log \chi \), where \( \chi \) is
the lowest target-to-projectile size ratio for which erosion is possible for some planetesimal size; see Section 4 and Figure 1 for details. Large values of $\chi$ (red) correspond to the situation when erosion occurs only in collisions with very small objects, which do not result in appreciable mass removal from the target. Such collisions are unlikely to prevent planetesimal growth as long as such small objects do not account for the dominant fraction of the disk mass.

One can see that the behavior of $\chi$ in $M_d-e_0$ space largely replicates that of $d_t/d_s$ in Figure 7—safe zones near the valley of stability, as well as at high $M_d$ and small $a_p$. Growth-unfriendly regions (blue) lie toward higher $a_p$ and at small disk masses. Thus, planetesimal growth is easiest in massive disks and closer to the star.

7. PLANETESIMAL GROWTH IN PRECESSING DISKS

In this section, we analyze planetesimal growth in disks that do not have a fixed orientation with respect to the binary orbit but precess at some rate $\dot{\bar{\sigma}}_d$. We do this by following the same procedure as in Section 4, but calculating the relative planetesimal velocity using the results of Section 6 of Paper I; see the Appendix. Results are shown in Figure 9 where we display regions in the $M_d-e_0$ space favorable for planet formation at 2 AU in $\gamma$ Cephei for two different values of the disk precession rate $\dot{\bar{\sigma}}_d$, expressed here in units of the local value of the planetesimal precession rate $A$. Note that the value of $A$ varies within each panel since it is a function of $M_d$.

Calculations described in the Appendix for the case of precessing disk do not provide an analytical solution for $|A - \dot{\bar{\sigma}}_d| e_x+B_d - |B_b|$ (here, again, $A = A_d + A_b$), which excludes certain parts of the $M_d-e_0$ phase space (blue bands) from Figure 9. In the rest of the figure, we use the results for strong (Section 6.1 of Paper I) and weak (Section 6.2 of Paper I) binary perturbation cases, depending on the circumstances. This makes our treatment of collision outcomes in the precessing disk somewhat approximate. Nevertheless, we can understand the main effects of disk precession on planetesimal collisional outcomes by comparing these results with Figure 4(b).

First of all, the valley of stability ceases to exist because disk-secondary apsidal alignment is not possible in a precessing disk. This tends to dramatically reduce the size of the growth-friendly zone in precessing disks, even far from the center of the valley of stability.

Second, in the strong binary perturbation regime, below the blue band, planetesimal growth conditions are independent of $\dot{\bar{\sigma}}_d$. This is because the $e_p$ solutions obtained in Section 6.1 of Paper I for this regime are independent of $\dot{\bar{\sigma}}_d$, since eccentricity excitation by the disk is weak. The size of the low-$e_0$
Given that for high $M_d$, planetesimal dynamics are in the DD regime, for non-pathological disk models (i.e., for surface density slope $0 < q < 3$, SR15) $A < 0$, i.e., planetesimal apsidal precession is retrograde relative to its mean motion. Then we conclude from the condition (9) that slow ($|\dot{\sigma}_d| < 2|A|$) prograde precession of the disk is favorable for planetesimal growth. This is indeed seen at the high $M_d$ end of Figure 9(a), although the magnitude of the effect is small because of the small adopted value of $|\dot{\sigma}_d/A| = 0.1$ (see below for the characteristic value of $|\dot{\sigma}_d/A|$).

On the contrary, retrograde or fast prograde ($|\dot{\sigma}_d| > 2|A|$) disk precession shrinks the size of the growth friendly zone, as demonstrated by Figure 9(b) and (c) for $\dot{\sigma}_d = 0.3A$. This is a bit counter-intuitive as one may naively expect fast precession to result in effective azimuthal averaging of the disk potential, suppressing planetesimal eccentricity excitation by the non-axisymmetric component of the disk gravity, and lowering $|\epsilon_p|$ in agreement with R13 and SR15. However, this argument loses its validity in presence of gas drag, which provides an important contribution to the value of $\epsilon_p$. For that reason planetesimal growth is facilitated by disk precession only when the somewhat non-trivial condition (9) is fulfilled.

For our fiducial disk with $p = 1$ one finds (R13)

$$|A_d| = n_p \frac{M_d}{M_p} \frac{a_p}{a_{\text{out}}} = n_b \left( \frac{(1 - \mu) a^2}{a_p a_{\text{out}}^2} \right)^{1/2} \frac{M_d}{M_p} \approx 0.1 n_b \frac{a_{\text{out}}^{3/2}}{a_p} \left( \frac{M_d}{M_p} \right) 10^{-5},$$

where the numerical estimate is for the $\gamma$ Cep parameters and $n_b = [G(M_p + M_\star)/a_{\text{out}}^3]^{1/2}$ is the mean rate of the binary.

At the same time, simulations of disks in eccentric binaries tend to find a variety of outcomes depending on the detailed physics that goes into the calculations, with both prograde (Okazaki et al. 2002; Marzari et al. 2009) and retrograde (Kley & Nelson 2008; Müller & Kley 2012) precession possible. Numerical results suggest that typically $|\dot{\sigma}_d| \sim (1-2) \times 10^{-5} n_b$ (Marzari et al. 2009; Müller & Kley 2012), which is considerably smaller than $|A_d|$ evaluated at the semi-major axis of the planet, $|\dot{\sigma}_d| \sim 0.1 |A_d|$. In this case, according to Figure 9(a), even if precession is prograde its effect on planetesimal growth in high-mass disks is going to be small (or slightly negative mainly through the elimination of the valley of stability in precessing disks).

Lower mass disks ($M_d \sim 10^{-3} M_\odot$), containing enough mass to form only terrestrial or Neptune-like planets have lower $|A|$. If they precess at the slow rates found in simulations they may have $|\dot{\sigma}_d| \sim |A|$ satisfied. However, as shown in Figure 9, planetesimal growth is strongly suppressed in such low-mass disks. Thus, planetesimal growth in low-mass precessing disks must be rather difficult, at least at separations $> 1$ AU. This is contrary to the non-precessing aligned disk case, in which the existence of the valley of stability permits collisional growth even for $M_d \lesssim 10^{-2} M_\odot$, see Section 5 and Figure 4.

It is also worth noting that simulations with improved treatment of the gas thermodynamics (Marzari et al. 2012; Müller & Kley 2012) and including self-gravity (Marzari et al. 2009) tend to produce non-precessing disks, properties of which we explored in previous section. Thus, disk precession is unlikely to strongly affect our conclusions regarding planetesimal growth in S-type binaries.

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**Figure 9.** Same as Figure 4(b) but for a precessing disk with $p = 1$, $q = -1$ around $\gamma$ Cephei at 2 AU. The disk precession rate is indicated on each panel in units of the local planetesimal precession rate $A$ (which itself depends on $M_d$ within each panel). The analytical description of planetesimal dynamics fails within blue regions, which are excluded from the panels. See the text for a discussion.

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growth-friendly region varies only because the extent of the excluded region (blue band) depends on $\dot{\sigma}_d$.

Third, in the weak binary perturbation regime, above the blue band, the extent of the growth-friendly zone does depend on $\dot{\sigma}_d$. To understand this, we recall (see Section 6.2 of Paper I) that the overall planetesimal eccentricity scale in a precessing disk is given by $\epsilon_p$, defined by Equation (45, PI). Above the blue band $A \approx A_d$, and we can use expressions (6, PI) and (8, PI) for $A$ and $B \approx B_d$. Recalling that for our disk model with $p = 1$, $q = -1$ the coefficients in these expressions are $\psi_1 = -0.5, \psi_2 = 1.5$ we can write

$$\frac{\epsilon_p}{\epsilon_e} = 1.5 \left( 1 - \frac{\dot{\sigma}_d}{A} \right)^{-1} - 1. \quad \text{(8)}$$

This ratio is equal to 0.5 in a non-precessing disk, when $\epsilon_p \rightarrow \epsilon_e$. A simple analysis of Equation (8) then shows that $|\epsilon_p| < \epsilon_e$ and non-zero precession suppresses planetesimal eccentricity and relative velocity compared to the case of a non-precessing disk if

$$-2 < \frac{\dot{\sigma}_d}{A} < 0. \quad \text{(9)}$$
8. RADIAL MIGRATION OF PLANETESIMALS

Apart from the eccentricity evolution, the non-conservative gas drag causes inspiral of planetesimal orbits—an effect that was not accounted for in SR15. We now turn our attention to this important issue.

Calculation of the radial drift $\dot{a}_p$ is a more delicate procedure than that of the eccentricity damping. As shown by Adachi et al. (1976), even in the case of a circular disk one has to account for the radial variation of the gas density $\rho_g$ when computing $\dot{a}_p$. Calculation becomes even more complicated in the case of an eccentric disk with its non-axisymmetric surface density profile. Accounting for the difference in azimuthal velocities of gas and particles that results from the radial pressure gradient can be highly non-trivial in the case of an eccentric disk.

For that reason, we have chosen to describe radial planetesimal drift $\dot{a}_p$ using an empirical generalization of the appropriate results of Adachi et al. (1976) for the case of an eccentric disk. This generalization is physically motivated and reduces to the known results in the case of the circular disk with $e_r = 0$. Namely, we use the Equation (4.21) of Adachi et al. (1976), in which we simply set $i = 0$ and replace $e$ with the relative particle-gas eccentricity $e_r$. As a result, we find

$$\dot{a}_p = -\frac{\pi}{\tau_d} \left[ \frac{5}{8} e_r^2 + \eta^2 \right]^{1/2} \left[ \frac{\alpha}{4} + \frac{5}{16} e_r^2 + \eta \right], \quad (11)$$

where

$$\tau_d = e_r \tau_d = \frac{4\pi}{3C_0} \frac{D^{-1}}{\sqrt{2}} D^{-1}$$

$$\approx 6\tau_c \frac{1}{C_0} \frac{a_{\text{out}}}{M_{p}^{1/2} M_{p}^{-2}} \frac{h}{r} \frac{d}{d_p}$$

is the characteristic timescale, $\tau_d$ is the eccentricity damping time defined by Equation (18, PI), and

$$\eta = \frac{1}{2} (\alpha + s) \left( \frac{c_s}{n_p} \right)^2 = \frac{1}{2} \left( \frac{p + s + 3}{2} \right) \left( \frac{h}{r} \right)^2 \quad (13)$$

is the measure of the azimuthal particle-gas drift caused by the pressure support in the gas disk. The different parameters entering these expressions are the logarithmic slopes of the gas density and temperature $\alpha = -\partial \ln \rho_g / \partial \ln r$ and $s = -\partial \ln \rho_g / \partial \ln T$, related via $\alpha = p + (3 - s)/2$, see Equation (2).

In this work we will use power-law temperature profile $T(r) = T_1 (r/\text{AU})^{-2}$, with $T_1$ being the gas temperature at 1 AU, so that

$$\frac{h}{r} \approx 4 \times 10^{-2} \left( \frac{M_\odot}{M_p} \frac{T_1}{100 \text{K}} \right)^{1/2} \left( \frac{\text{AU}}{r} \right)^{1-s/2} \quad (14)$$

In our calculations we normally take $s = 1/2$ and $T_1 = 400$ K (the central stars of compact planet-hosting binaries are usually somewhat more massive than the Sun).

Note that the characteristic timescale of the radial drift in the case $e_r \gg \eta^{1/2}$ is $|d \ln a_p/dt|^{-1} \approx \tau_d e_r^{-3} = \tau_d e_r^{-2}$, which is much longer than the eccentricity damping time $\tau_d$. For smaller $e_r$, migration time lengths even further. The slowness of the radial drift allows us to treat $a_p$ as a constant while following the evolution of planetesimal eccentricities.

Radial drift depends steeply on $e_r$, and can be rather fast for strongly dynamically excited planetesimals. Because of the radial pressure support in the gaseous disk resulting in the non-zero value of $\eta$, $\dot{a}_p$ does not completely vanish even as $e_r \rightarrow 0$. This is not the case for eccentricity evolution—eccentricity damping naturally vanishes for $e_r = 0$.

In Figure 10 we map the migration time $\tau_m \equiv |d\ln a_p/dt|^{-1}$ in $M_d - a_p$ space for two different planetesimal sizes, $a_p = 0.3 \text{ km}$ (left) and $a_p = 3 \text{ km}$ (right), and two values of the disk eccentricity at $a_{\text{out}}$, $e_0 = 0.01$ (top) and $e_0 = 0.1$ (bottom). The calculation is done for an aligned disk in the $\gamma$ Cephei system.

These maps clearly show many non-trivial features and significant variation as we change $e_0$ and $a_p$. To better understand them we overplot the lines of $A = 0$ (purple) and $|B_0| = |B_2|$ (cyan) conditions. Interestingly, no significant feature is seen in the $\tau_m$ maps at the location of the $A = 0$ secular resonance. This is in contrast to the characteristic eccentricity maps in Figure 4 of Paper I, which show the divergence of $e_r$ at this resonance caused by $e_r \sim |A|^{-1} \text{ scaling}$, see Equation (29, PI). This difference is easily explained by looking at the Equation (28, PI), which shows that the relative planetesimal–gas velocity $e_r \propto |A| e_r$ thus removing singularity at $A = 0$. Upon closer inspection, one can see only a mild reduction of $\tau_m$ in a broad region surrounding the $A = 0$ curve. It is caused by the local $e_r \propto \left[1 + A^2 \sigma_2^2\right]^{-1/2}$ dependence on $A$, increasing $e_r$, and decreasing $\tau_m$ where $A \rightarrow 0$ according to Equation (11).

At the same time, in all panels one can easily see a band of increased $\tau_m$, which runs close to the $|B_0| = |B_2|$ (blue) curve. Its location is independent of $d$, but is sensitive to $e_0$, with higher disk eccentricity pushing this valley of high $\tau_m$ further from the star. Comparing this with Figure 4 of Paper I, we conclude that this feature is caused by $e_r \rightarrow 0$ within this band. Since this is possible only in the aligned disk (see Section 6.1 and Section 7.2 of Paper I), such a feature would not be present in a misaligned or precessing disk.

In a disk with $\sigma_2 \approx 0$, however, migration time can become very long in this region of parameter space: $\tau_m \sim \text{Myr}$ is
quite typical within the valley of high $\tau_m$, stretching along the $|B_p| = |B_e|$ curve, especially for large $d_p$ and higher $e_0$. In this region, $e_c$ can be so small that $\tau_m$ becomes determined solely by the non-zero value of $\eta$ in Equation (11), which is due to the radial pressure support in a gas disk:

$$\tau_m \rightarrow \frac{\tau_a}{\pi \eta^2} \propto \alpha_p^{7/4} M_d^{-1}.$$  

To arrive at the last scaling, we used Equations (2), (12)–(14) and adopted $p = 1, q = -1$.

Figure 10 shows that $\tau_m$ is higher for higher $e_0$ in high $\tau_m$ regions. This is somewhat counterintuitive as one naively expects higher disk eccentricity to result in larger planetesimal velocities, driving faster, rather than slower, migration. This contradiction is resolved by understanding that even for the same $d_p$ we are comparing the values of $\tau_m$ at special locations, where $e_c \to 0$. Their position is roughly described by Equation (52, PI) for $|B_p| = |B_e|$, from which one infers their $a_p \propto (e_0 M_d)^{1/2}$. Plugging this into Equation (15), one finds that $\tau_m \propto \sqrt[4]{e_0} M_d^{-1/8}$ i.e., maximum $\tau_m$ is indeed longer for higher disk eccentricity. This is simply a reflection of the fact that for higher $e_0$ the valley of small $e_c$ moves out to larger $a_p$. The same reasoning also explains why $\tau_m$ increases along the high $\tau_m$ valley as both $a_p$ and $M_d$ get smaller.

Note the long values of $\tau_m$ in the upper left corner of Figure 10(c) and (d). They are caused by a particular choice of $e_0 = 0.1$ for which $e_c$ becomes very small globally in the BB regime when the gravity of the binary companion dominates over that of the disk (SR15). This coincidence has been previously discussed in Section 7.2 of Paper I; see Equation (54, PI).

The existence of a localized peak of $\tau_m$ has important implications for planetesimal growth. In a disk with fixed values of $e_0$ and $M_d$, planetesimals in the outer parts of the disk rapidly migrate inward until they reach the high $\tau_m$ valley. In a narrow range of semi-major axes corresponding to this valley, their radial drift significantly slows down, resulting in a local increase of the surface density of solids of different sizes. Given the dramatic local increase of $\tau_m$, one can expect the planetesimal density there to exceed its initial local value by orders of magnitude. Concentrating a large amount of solid material at a particular semi-major axis is expected to substantially increase the local density of embryos, and hasten the process of core formation. Moreover, according to Figure 4 of Paper I, the high $\tau_m$ valley is also the location where $e_c$ becomes very small, providing favorable conditions for planetesimal growth. These points are further discussed in Section 9.

9. PLANET FORMATION IN BINARIES

Now we apply our understanding of planetesimal growth and migration described in previous sections to clarify the circumstances under which planets of different masses can form in disks within binaries.

We note that overcoming the fragmentation barrier does not fully guarantee that planet formation is possible. It is still subject to a number of other threats. In particular, even if the relative collisional velocities of planetesimals are reduced to the point of avoiding fragmentation, they may nevertheless remain high enough to make planetesimal coagulation very slow. This might, for instance, be the case if the collision velocities still exceed the escape speeds for planetesimals involved in a collision, implying that gravitational focusing is inefficient. Given the inferred shorter protoplanetary disk lifetimes within binaries (Cieza et al. 2009; Kraus et al. 2012), this may present a problem for growing solid cores that are massive enough to trigger the core accretion within the nebula lifetime (Rafikov 2006). Radial migration of solids reducing their surface density can also be an issue, although likely not a very serious one as we have shown in Section 8.

In the following, we will address the likelihood of planet formation focusing predominantly on the fragmentation barrier issue.

9.1. Conditions for Giant Planet Formation

The presence of planets with $M_{pl} \sin i$ of order several $M_J$ inevitably implies that their parent protoplanetary disks must have been massive, $M_d \gtrsim 10^{-2} M_\odot$. Indeed, the disk mass cannot be much lower than at least several $M_J$, otherwise the disk simply would not contain enough gas to form these massive objects. This argument must hold even despite the observational evidence against massive disks in small separation binaries coming from sub-millimeter observations (Harris et al. 2012).

Planet masses indicated by the vertical red lines in Figure 4 are no more than an order of magnitude lower than $M_d$ at the edge of the (gray) growth-friendly zone for $e_0 \sim 10^{-2}$. Then, under the natural constraint $M_d \gtrsim 10^{-2} M_\odot$, Figure 4 clearly implies that unimpeded planetesimal growth leading to giant planet formation at AU-scale separations in binaries is possible provided that disk eccentricity is low, $e_0 \lesssim 10^{-2}$. This is an important requirement for giant planet formation in small separation ($a_b \approx 20$ AU) binaries, which is inspired by planetesimal dynamics alone. It represents one of the key results of this work.

Unfortunately, we do not have direct measurements of circumstellar disk eccentricities in young stellar binaries and cannot address the $e_d$ constraint directly. Simulations of disks in eccentric binaries with $e_b = 0.4$ tend to find rather low values of $e_d \lesssim 0.05$ (Marzari et al. 2009, 2012; Müller & Kley 2012; Picogna & Marzari 2013). In fact, Regály et al. (2011) claim that for $e_b > 0.2$, the protoplanetary disk does not develop permanent eccentricity in their simulations, and deviations from axisymmetry are minimal. This is in contrast to simulations of disks in circular (or low-$e_b$) binaries, which often demonstrate high $e_d \approx 0.5$–0.5 (Kley et al. 2008; Regály et al. 2011). Such a dichotomy is likely caused by the smaller truncation radii of the disks in high $e_b$ binaries (Regály et al. 2011), reducing the companion’s perturbation on them. Disks in circular binaries can extend further out, potentially creating conditions for disk eccentricity excitation via the Lubow (1991) mechanism.

Based on this, we conclude that the existing numerical results are roughly compatible with the conditions needed to overcome the fragmentation barrier and form giant planets within massive disks ($M_d \gtrsim 10^{-2} M_\odot$) in AU-scale orbits, namely, low $e_0$ of the order of several percent; see Figure 4. Note that in very massive disks ($M_d \gtrsim 0.1 M_\odot$), this conclusion holds for arbitrary disk orientation as well as for precessing disks; see Figures 5 and 9.

Even in high $M_d$ disks, the presence of the valley of stability facilitates planet formation. Indeed, as we already mentioned, it is necessary, but not sufficient for giant planet formation to be able to overcome the fragmentation barrier. One must also be able to form cores of several Earth masses before the disk has time to disperse. According to Kraus et al. (2012), the majority of gas disks in tight binaries ($a_b < 40$ AU) disperse within one Myr. This creates tension with the core accretion scenario as accumulation of a core large enough to accrete gas
is expected to take at least several Myr (Hubickyj et al. 2005). However, in our aligned disk model the core formation would occur much more rapidly at certain locations. Figures 10(a) and (b) show that in low $e_0$, high $M_d$ systems, migration time $\tau_m$ becomes very long at semi-major axes of 2-3 AU. Planetesimals from the outer disk migrating inward due to gas drag would accumulate at these locations. A corresponding increase of the surface density of solids, combined with lowered relative velocities of planetesimals at the same locations (see Figure 4), has the potential to locally speed up growth of planetary cores and resolve the timescale issue.

Interestingly, three out of five presently known planet-hosting tight binaries have planets at $a_{pl} = 1.6$–2.6 AU, and all three are massive giants with $M_{pl} \sin i > 1.6M_J$ (Chauvin et al. 2011). We suggest that this may be not a coincidence but, possibly, the evidence for in situ formation of these giants, facilitated by the local pileup of solids, in low-eccentricity ($e_0 \lesssim 0.01$), high-mass ($M_d \gtrsim 10^{-2} M_\odot$) disks, which were aligned ($\sigma_d \approx 0$) with the orbits of their binary companions.

We also speculate that the observed clustering of the binary eccentricity in $\gamma$ Cep-like systems (with $a_b \approx 20$ AU) around $e_b \sim 0.4$–0.5 (Chauvin et al. 2011; Dumusque et al. 2012) is directly linked to lower disk eccentricities $e_\alpha$, in such binaries, as suggested by simulations (Regály et al. 2011). This makes such eccentric binaries more favorable for overcoming the fragmentation barrier and forming planets than their circular counterparts. On the other hand, in highly eccentric systems, $e_b \to 1$, disks would be truncated at radii that are too small to contain enough mass for planet formation. Thus, the apparent clustering of $e_b$ of compact ($a_b \approx 20$ AU) planet-hosting binaries around 0.4–0.5 may be not coincidental.

### 9.2. Earth- and Neptune-like Planet Formation

Formation of terrestrial (as in the $\alpha$ Cen system; Dumusque et al. 2012) or Neptune-size planets may also proceed in massive disks, in which case, the conclusions of Section 9.1 would apply directly. At the same time, just based on the mass budget, low-mass planets might also be expected to form in lower mass ($M_d \sim 10^{-3} M_\odot$) disks. Sub-millimeter observations suggest that such disks are more abundant than their more massive counterparts in binaries with separations of the order of several tens of AU (Harris et al. 2012). However, satisfying the planetesimal growth constraints formulated in Section 9.1 for low $M_d$ becomes problematic, as can be inferred from the presence of extended growth-unfriendly (white) zones at small $M_d$ in Figure 4. According to Figures 5 and 9, planetesimal growth is essentially impossible in low-$M_d$ disks which are misaligned with the binary orbit or precess.

However, in aligned disks, low-mass planet formation may still be possible even for $M_d \lesssim 10^{-6} M_\odot$. In such disks the valley of stability (see Figure 4) provides conditions favorable for planet formation even for $M_d \lesssim 10^{-2} M_\odot$ and for relatively high $e_0 \sim 0.1$. Moreover, disk evolution may naturally drive even high $M_d$ systems toward the valley of stability at a given semi-major axis. Indeed, even if the disk starts at relatively high $e_0 \sim 0.1$ and high $M_d \gtrsim 5 \times 10^{-3} M_\odot$, above the black region in Figure 4, over time, its viscous evolutions will reduce $M_d$ and ultimately bring the disk into the valley of stability, making low-mass planet formation quite natural at this point.

Within the localized regions corresponding to the valley of stability, one would again have a combination of both the increased density of solids due to planetesimal accumulation induced by the non-uniform planetesimal drift and the suppression of relative planetesimal velocities. Both factors promote planetesimal growth. Figures 7 and 10 clearly show that in low-mass disks $M_d \gtrsim 10^{-5} M_\odot$ with relatively high eccentricities $e_0 \sim 0.1$, such low $e_\alpha$ and high $\tau_m$ regions lie at semi-major axes of 1–2 AU. Earth- or Neptune-like planets may form there.

Finally, unimpeded planetesimal growth within relatively low mass disks, $M_d \lesssim 10^{-3} M_\odot$, may also be possible close to the star, at sub-AU separations, provided that the disk has a low eccentricity, $e_0 \lesssim 10^{-2}$. This is seen in Figures 7 and 8, which demonstrate small $d_\gamma/dt$ and relatively large $\log \chi$ at small $a_p$. Such a mode of planet formation may have been responsible for the origin of the putative Earth-mass planet in $\alpha$ Cen B (Dumusque et al. 2012).

### 9.3. Comparison with Previous Studies

Our finding that the fragmentation barrier can be overcome, opening a way to planet formation at separations of several AU in tight binaries such as $\gamma$ Cep and $\alpha$ Cen is opposite to the conclusions of many previous studies (Thébault et al. 2008, 2009; Thébault 2011). The main reason for this difference is in the role of (generally non-axisymmetric) protoplanetary disk gravity, which we account for in secular approximation, while other studies included only gas drag and perturbations from the companion. As we showed in Paper I and in this work, this aspect really makes a big difference for the outcome—in disks massive enough to form giant planets, planetesimal precession and eccentricity excitation become dominated by the gravity of the disk rather than that of the companion. Thus, it is very important that future studies of planet formation in binaries, including those that self-consistently evolve the disk using direct hydrodynamical simulations, account for the gravitational effect of the disk on planetesimal motion. This has been previously done in Kley & Nelson (2007) and Fragner et al. (2011) but the complexity of planetesimal dynamics including disk gravity has not been explored in sufficient detail in these studies.

On the other hand, some other previous studies have found planetesimal growth in tight binaries to be possible. Marzari & Scholl (2000) arrived at this conclusion by noticing the apsidal phasing of planetesimal orbits by gas drag. Later, Thébault et al. (2008) showed the associated reduction of the relative speed $v_{12}$ to be a consequence of a single planetesimal size approximation. Thébault et al. (2006) find growth possible for nearly circular binaries with small $e_b$, since in this case, eccentricity forcing by the companion vanishes. However, simulations show that disks tend to develop large eccentricities ($\gtrsim 0.1$) in systems with low $e_b$ (e.g., Marzari et al. 2009, 2012; Regály et al. 2011), which, with disk gravity included, likely would have resulted in severe difficulty for forming planets.

Xie & Zhou (2008) suggested that apsidal alignment between the objects of different sizes, resulting in reduced collisional velocities, should ultimately be possible as the gas disk dissipates. Even though Xie & Zhou (2008) did not account for disk gravity, this effect can still be understood based on our results in Paper I. There we showed that planetesimal eccentricity $e_\gamma$ indeed becomes independent of the planetesimal size when gas dissipates and drag damping time $\tau_d \to \infty$; see Equations (24, PI)–(27, PI). Gas drag is needed in this case to just damp out the free part of the planetesimal eccentricity. In systems that form a Jupiter-sized planet, this effect only works for large planetesimals ($\gtrsim 15$ km) because some of the gas disk must be retained at the dissipation stage to make up the mass of the planet. Proper inclusion of disk gravity does considerably
more to align large planetesimals by reducing the critical size $d_c$, above which such apsidal alignment occurs (see Equations (55) and (56) in Paper I).

Also, Xie & Zhou (2009) and Xie et al. (2010) have shown planetesimal growth to be facilitated if the disk is slightly inclined with respect to the binary orbital plane. In this case, gas drag sets planetesimal nodal lines differently for objects of different size, thus spatially separating orbits of planetesimals with very disparate sizes, which tend to collide with high relative speeds. In this case, growth occurs primarily via collisions of similar-sized objects, which have small relative speeds. Even though we did not consider inclination in this study, its effect on growth of planetesimals of sizes $d_i$ is relevant for this problem—perturbations due to the companion, gas drag, and, most crucially, gravitational effects of an eccentric disk. We used our results to assess the possibility of planetesimal growth in binaries of apexes aligned, low-mass planets can also form in low $M_d$ disks at certain locations (even at $a_p \sim$ AU) where the radially migrating planetesimals (1) accumulate and (2) have low relative velocities, promoting their growth in mutual collisions.

Our results provide a natural way of explaining the existence of planetesimals in small separation binaries, such as $\gamma$ Cep and $\alpha$ Cen, via the improved understanding of planetesimal dynamics. This may eliminate the need to invoke more exotic scenarios for forming such systems.

Our calculations assessed the possibility of planetesimal growth by exploring just the two possible collision outcomes—catastrophic disruption and erosion by objects of certain sizes. The full understanding of planetesimal growth in binaries will require a self-consistent coagulation simulation that would evolve the mass spectrum of objects fully accounting for the complexity of their dynamics in binaries.

Methods developed in this work will be used to understand formation of planets in circumbinary configurations.

10. SUMMARY

We explored planetesimal growth in AU-scale orbits within small-separation ($a_b \approx 20$ AU) binaries using a newly developed secular description of planetesimal dynamics (Paper I), which includes a number of important physical ingredients relevant for this problem—perturbations due to the companion, gas drag, and, most crucially, gravitational effects of an eccentric disk. We used our results to assess the possibility of planetesimal growth in binaries and arrived at the following conclusions:

1. By exploring outcomes of pair-wise planetesimal collisions, we identified ranges of planetesimal sizes for which growth by coagulation is suppressed (Section 4). Inclusion of disk gravity is very important for properly determining the extent of accretion-unfriendly zones.

2. Planetesimal growth uninhibited by fragmentation is possible for a broader range of parameters ($M_d$ and $e_\theta$) in disks, which are apsidally aligned with the binary orbit (Section 6.1).

3. Radial drift of planetesimals caused by gas drag is highly non-uniform in aligned disks, with the drift timescale sharply peaking at AU-scale separations. This causes accumulation of planetesimals at the location where their dynamic excitation is weak and provides favorable conditions for their growth (Section 8). Local concentration of solid material naturally speeds up core formation, which alleviates concerns about the possibility of massive planetesimal formation given the short lifetimes of protoplanetary disks in tight binary systems.

4. Formation of giant planets in observed (AU-scale) configurations in eccentric binaries like $\gamma$ Cep is possible in massive and not very eccentric disks, $M_d \geq 10^{-2} M_\odot$ and $e_\theta \lesssim 0.01$ (Section 9.1). The former condition is consistent with the very existence of massive (several $M_J$) planets in these systems. The latter is in rough agreement with the results of simulations, revealing low disk eccentricity in eccentric ($e_\theta \approx 0.4$) binaries. Planet formation may be inhibited in circular binaries as simulations show disks to develop high eccentricity in such systems. This may naturally explain the clustering of the binary eccentricity around $e_\theta \sim 0.4-0.5$ in currently known compact ($a_b \approx 20$ AU) planet-hosting systems.

5. Terrestrial and Neptune-like planets can form in massive disks just as giant planets can. Their genesis in low-mass ($M_d \lesssim 10^{-2} M_\odot$) disks is possible close to the star ($a_p \lesssim$ AU) but is generally suppressed farther out, at $a_p \gtrsim$ AU. However, if the disk and binary periapses are aligned, low-mass planets can also form in low $M_d$ disks at certain locations (even at $a_p \sim$ AU) where the radially migrating planetesimals (1) accumulate and (2) have low relative velocities, promoting their growth in mutual collisions.

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APPENDIX

RELATIVE ECCENTRICITIES OF PLANETESIMALS

To determine the outcome (destruction or no destruction) of a collision between two bodies of sizes $d_1$ and $d_2$, we need to calculate their relative eccentricity $e_{12} = [(h_1 - h_2)^2 + (k_1 - k_2)^2]^{1/2}$. In the case of a non-precessing disk, we do this by first computing $A \tau_d$ in terms of $d_p$ and $d_i$ for each planetesimal using Equation (32, PI), and then plugging it in to Equation (64, PI) to find $e_{12}$.

For the precessing disk (see Section 7), we do not have analytical expressions for $h_p$ and $k_p$ in general, but we calculate them for two limiting cases (strong and weak binary perturbation cases) using the approach described in Sections 6.1 and 6.2 of Paper I respectively. We start by evaluating Equation (42, PI). If $|(A - \sigma_d)e_\theta + B_d|$ is within a factor of two of $|B_i|$, we exclude this point of the phase space from our calculation as we do not expect analytical limiting behaviors to apply there. If $|B_i| > 2|(A - \sigma_d)e_\theta + B_d|$, then we use Equations (43, PI) to determine $h_p$ and $k_p$. If $|B_i| < 0.5|(A - \sigma_d)e_\theta + B_d|$, then we first compute $(A - \sigma_d)\tau_d$ using Equation (46, PI) and then calculate $e_{i,f,d}$ via Equation (B2, PI) with $B_d = 0$ for each planetesimal. Even though $k_p$ and $h_p$ are not constant for a given object (eccentricity vectors precess together with the disk), their difference is constant and is given by

$$e_{12}^2 = e_1^2 + e_2^2 - 2e_2 e_1 \cos(\phi_1 - \phi_2),$$

$$e_i = \left[\frac{e_i^2 + \tau_d^2(d_i)B_d^2}{1 + \tau_d^2(d_i)(A - \sigma_d)^2}\right]^{1/2},$$

where $e_i (i = 1, 2)$ are the individual forced eccentricities for planetesimals of sizes $d_i$ and $\phi_i$ are their apsidal phase (with respect to the instantaneous direction of the disk periastron) given by Equation (B2, PI) with $\tau_d = \tau_d(d_i)$.
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