Quantum spin transistors in superconducting circuits

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Transistors play a vital role in classical computers, and their quantum mechanical counterparts could potentially be as important in quantum computers. Where a classical transistor is operated as a switch that either blocks or allows an electric current, the quantum transistor should operate on quantum information. In terms of a spin model the in-going quantum information is an arbitrary qubit state (spin-1/2 state). In this paper, we derive a model of four qubits with Heisenberg interactions that works as a quantum spin transistor, i.e. a system with perfect state transfer or perfect blockade depending on the state of two gate qubits. When the system is initialized the dynamics complete the gate operation, hence our protocol requires minimal external control. We propose a concrete implementation of the model using state-of-the-art superconducting circuits. Finally, we demonstrate that our proposal operates with high-fidelity under realistic decoherence.

I. INTRODUCTION

When you look inside your personal computer, you will find integrated circuits filled with billions of minuscule transistors. Each transistor has a very simple job: it is a switch for opening or closing an electronic gate. Despite the limited functionality of each transistor, they achieve great things together, such as running your entire computer system. This approach to computing – connecting many simple devices into larger powerful structures – is called modular computing.

Modular computing allows highly scalable and computationally capable classical computers. The same strategy can be employed in the quantum case[1], where various hybrid technologies[2,3] such as cold atoms and photons[4,5], superconducting circuits[6–9] and optomechanical systems[10] have been proposed. Essentially, we require few-qubit modules that may readily enter into a larger network. Depending on the structure and operation of the network, one can achieve conditional dynamics[11] and ultimately build quantum computers[12], quantum simulators[13,14], or quantum neural networks[15–17] for quantum machine learning[18–21].

Inspired by the crucial role the transistor plays in classical computers, we turn our attention towards its quantum analogue. Thought of as a module in a larger network[22], the quantum transistor is a link in a quantum information channel and works as a switch for quantum state transfer. Implementations of such a gate has been studied as the atomtronic transistor in ultra-cold atoms[23,24], spintronic transistors[25,26], and photonic transistors based on light-matter interactions[27,28]. However, the usefulness of a quantum transistor is all the more clear when implemented with technologies that also allow for long-lived general-purpose qubits.

As the quantum transistor must enter as a component of a larger network, it must be capable of being mass-produced as a standard off-the-shelf unit. Thus, we propose in the work a design in superconducting circuits, a technology which shows great potential for producing commercial chips for quantum computing[6,20,37–40]. Specifically we show that an implementation using four interacting transmon qubits[41] is capable of realizing a high-quality quantum transistor. However, the transmon qubits can readily be exchanged for other types of qubits, such as Xmon qubits[42], flux qubits[43,44], fluxonium qubits[45], phase qubits[47,48], C-shunted flux qubits[49], and potentially other types of superconducting qubits. In Figure 1 we show a sketch of our proposed circuit and the resulting spin model of four interacting spins (qubits).

![FIG. 1. Sketch of a superconducting circuit of four transmon qubits (red) coupled via resonators (green). Hovering above the circuit is the effective model of four spins/qubits (purple) interacting via Heisenberg XX couplings (blue) or Heisenberg XXZ couplings (blue and orange). Operated as a transistor, the two middle qubits comprise the gate, which is coupled to a left and a right qubit.](image-url)

The four qubits in our implementation comprise of two gate qubits in the center connected to a left and a right qubit. Considered as a module in a larger network, the left and right qubits may connect the transistor to other parts of the network. In that respect, the transistor acts
FIG. 2. Illustration of the transistor’s two functions with the gate as a traffic light, and the left and right qubit states loaded onto trucks. The graphs show simulation results for an ideal model ('ideal model'), our proposed circuit model with realistic parameters ('circuit model') and the circuit model with realistic decoherence noise included ('circuit model with noise'), with $|\psi\rangle = (|\uparrow\rangle + re^{i\theta}|\downarrow\rangle)/\sqrt{1 + r^2}$ for $0 \leq r \leq 1$ and $0 \leq \theta < 2\pi$. The cartoons a)-d) illustrate the situations at various points on the graphs. Closed gate: The upper graph show the fidelity for remaining in the initial four-qubit state as a function of time. Ideally, the initial truck configuration a) will never change, i.e. a constant remain fidelity. However, in a realistic scenario, due to cross-talk couplings and noise, a small probability of running the red light emerges, indicated as ghostly trucks crossing the junction in b). Open gate: The lower graph show the state transfer fidelity as a function of time. The trucks initially on each side of the junction in c) will ideally cross the junction and transfer their load of quantum states to the opposite side, as seen in d). This state transfer happens with almost unit state fidelity also in the realistic scenario with noise.

as a traffic junction in a road for quantum information, as depicted in Figure 2. Initially, an arbitrary qubit state $|\psi\rangle$ is loaded onto the left truck, and a down-state $|\downarrow\rangle$ is loaded onto the right truck. The two gate qubits constitute a traffic light, which may be red (closed gate) or green (open gate) depending on the their two-qubit state. For a perfect transistor with law-abiding truck drivers, the trucks would never run a red light and would always cross with unit fidelity for a green light, thereby ensuring perfect blockade or transfer of the left and right qubit states. Thus the mechanism behind our transistor is blocking and allowing quantum state transfer. Typically this type of dynamics is studied in closed quantum systems [50–55] where the dynamics is symmetrical, but directional state transfer has also been studied recently [56, 57], which is interesting from a network point of view.

The first main result of this paper is providing a theoretical model for a four-qubit transistor. A significant finding is that our model does not require fine-tuning of the parameters in order to operate as a perfect or near-perfect transistor. We assess the performance of the transistor by the state-fidelity of the blocked or transferred state, shown as the solid lines on the plots on Figure 2. When the gate is closed, the fidelity to remain in the initial (four-qubit) state is constant unity: The red light is on forever, and the trucks never move. On the other hand, when the gate is open, the state transfer fidelity grows smoothly to unity: The green light is on, and after some time the two trucks have crossed the junction. The initial state is here taken as the range $|\psi\rangle = (|\uparrow\rangle + re^{i\theta}|\downarrow\rangle)/\sqrt{1 + r^2}$ for $0 \leq r \leq 1$ and $0 \leq \theta < 2\pi$.

Our second main result is proposing a physical realizable model in superconducting circuits. This includes realizing – for the first time to our knowledge – a Heisenberg XXZ coupling between two superconducting qubits. In addition to the desired model, the superconducting circuit also give rise to a small ‘cross-talk’ coupling between the left and right qubits which causes the closed gate to leak slightly over time, however, this problematic coupling can be suppressed. Simulation results are shown as dashed lines on Figure 2.

Finally, we add some experimentally realistic dephasing to the model and simulate its behavior. The dynamics of the noisy system is seen as the dotted lines on Figure 2. The most notable difference from the noiseless circuit simulations (dashed lines) are a faster leakage of the
closed transistor. However, within the relevant time-scale of operation, i.e. the transfer time, the state fidelity for the ideal state stays above 0.95, demonstrating very robust functionality.

This paper is organized as follows: In Section II beginning from a very general four-qubit Heisenberg model, we determine which conditions the interaction constants must fulfill in order to realize a transistor and thus end up with an ideal transistor model. Using the idealized model as a stepping stone, we introduce in Section III a slightly more sophisticated model that can realistically be implemented with superconducting qubits. Section IV provides simulations of the proposed implementation.

Unless stated otherwise, we use units where \( \hbar = 2e = 1 \).

## II. A SIMPLE FOUR-QUBIT TRANSISTOR

The concept of the quantum transistor is illustrated in Fig. 3a and comprises a left (L) qubit, a right (R) qubit, and a gate. The gate is operated as one logical qubit whose state controls the operation on two qubits. Initially, the left qubit (target qubit) may be in an arbitrary state, \( |L\rangle_i = a|\uparrow\rangle + b|\downarrow\rangle \), but the right qubit does not necessarily enjoy the same degree of freedom. Concretely, in this paper we consider \( |R\rangle_i = |\downarrow\rangle \). The gate (the control qubit) can be configured in an “open” and “closed” state. If the gate is open, the state of the left and right qubits are interchanged, \( |L\rangle_f = |R\rangle_i \), and \( |R\rangle_f = |L\rangle_i \). In the spin system picture, Fig. 3b, we think of this operation as a state transfer from the target L qubit to the R qubit, although the transfer is completely symmetrical.

On the other hand, if the gate is closed, nothing happens, \( |L\rangle_f = |L\rangle_i \), and \( |R\rangle_f = |R\rangle_i \). In the spin system picture, we say that the closed gate blocks the state transfer. Notice that the transistor’s operation is very similar to that of the CSWAP (Fredkin gate\(^{25,59}\)) that exchanges two target qubit states conditional to the state of a control qubit. In fact, we may consider the quantum transistor as a restricted CSWAP where the right qubit must be initialized in the \( |\downarrow\rangle \) state.

In Ref. \(^{31}\), Marchukov et al. showed that a linear Heisenberg spin chain of four qubits can operate as a quantum transistor. In their design two strongly coupled qubits constitute the gate, while the left and right qubits were each coupled weakly to one of the gate qubits. The strong coupling between the gate qubits was used to detune the closed gate state from the right and left qubit states, hence suppressing state transfer through the gate. However, since the chain was linear, there was a very small oscillating probability for transfer even when the gate was closed. Furthermore, they also found that they needed a Heisenberg XXZ chain in order to operate the transistor, i.e. different X- and Z-couplings was a key ingredient in their design.

We now seek to improve the four-qubit transistor by asking the very general question: Which conditions does transistor functionality put on a general four-qubit system? We require the transistor to be capable of being perfectly open and closed. By ‘perfect’ we mean that the open transistor shall transfer the left/right qubit states across the gate with unit fidelity (without altering the gate state) and that the closed transistor shall exhibit no dynamics at all. In this regard the model of Ref. \(^{31}\) is not strictly closed. However, going beyond a simple linear chain allows for a much greater class of models.

While the initial left qubit state is arbitrary, the right qubit state is initially assumed to be spin-polarized, say, in the down-direction:

\[
|L\rangle_i = a|\uparrow\rangle + b|\downarrow\rangle, \quad |a|^2 + |b|^2 = 1 \quad (1) \\
|R\rangle_i = |\downarrow\rangle \quad (2)
\]

Let \( |\text{open}\rangle \) and \( |\text{closed}\rangle \) be general states in the gate subspaces of total spin projection \(-1\) and \(0\), respectively (counting up/down-spin as plus/minus one half):

\[
|\text{open}\rangle = |\downarrow\rangle \quad (3) \\
|\text{closed}\rangle = \cos \theta |\uparrow\rangle + \sin \theta |\downarrow\rangle \quad (4)
\]

With \( \theta = \pm \pi/4 \) the above definitions \((1)-(3)\) coincide with the ones in Ref. \(^{31}\). For the dynamics, we will here consider a Hamiltonian of the following type:

\[
H_0 = J_Z^{(2)} \sigma_z^{(2)} \sigma_z^{(3)} + \sum_{i>j} J_{X} \sigma_x^{(i)} \sigma_x^{(j)} + J_{Y} \sigma_y^{(i)} \sigma_y^{(j)} \sigma_+^{(i)} \sigma_+^{(j)} \quad (5)
\]

where \( J_{X,Y}^{i,j} \) are XZ-coupling strengths between qubit \( i \) and \( j \), \( \sigma_{x,y}^{(i)} = (\sigma_{x,y}^{(i)} \pm i \sigma_y^{(i)})/2 \) and \( \sigma_z^{(i)} \) are the Pauli operators for qubit \( i \). For now we assume that the coupling strengths are real and time-independent. This model describe a Heisenberg XXZ-interaction between qubit 2 and 3, which will constitute the gate, and Heisenberg XX-interactions between all other pairs of qubits. The left and right qubit will be labeled 1 and 4, respectively. We nickname the model the diamond model due to the

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**FIG. 3.** The concept of a quantum transistor. (a) Schematic using gate notation. The gate state controls the operation performed on the two qubits. (b) As a spin system, one may think of two qubits (L and R) coupled through the remainder of the spin system, called the gate.
visualization in Fig. 4. Most notably, this model deviates from the one in Ref. [31] by going beyond the typically studied linear chain. This allows quantum interference between several paths between the left and right qubit to either enhance or reduce state transfer across the gate. Contrary to the linear chain in Ref. [31], interference will allow us to close the transistor perfectly. Network Hamiltonians similar to the diamond model has been studied recently in Ref. [21] for a $\sqrt{\text{SWAP}}$ gate, which will turn out to be related to the open state of our transistor.


gate

FIG. 4. Illustration of the diamond model presented in Eq. 5. The purple dots with strike-through arrows represent qubits labeled 1-4, where qubit 1 and 4 are also labeled $L$ and $R$, respectively, and qubit 2 and 3 comprise the gate. The solid (dotted) lines denote $X$-couplings ($Z$-couplings) and are labeled with their strengths $J_{ij}^{X,Z}$.

### A. Closed transistor

When the transistor is initialized in its closed state, no dynamics in the system is allowed. This condition imposes constraints on the couplings strengths in Eq. 5. Formally, for every time $t > 0$, unitary time-evolution must not change the initial state:

$$\forall t > 0: \langle L|\text{closed}\rangle|R_i\rangle \xrightarrow{t} \langle L|\text{closed}\rangle|R_i\rangle. \tag{6}$$

The full four-qubit state is denoted as a product state of those of the left qubit, the two-qubit gate, and the right qubit. The condition in Eq. 6 states that the initial four-qubit state must be an eigenstate of the Hamiltonian. The Hamiltonian of Eq. 5 is spin-preserving, so we consider the problem in each subspace $B_k$ of total spin projection $k = 0, \pm 1, \pm 2$. The initial state for the closed transistor is a linear combination of states in $B_{-1}$ and $B_0$, namely

$$\cos \theta \left| \uparrow \uparrow \downarrow \downarrow \right\rangle + \sin \theta \left| \downarrow \downarrow \uparrow \uparrow \right\rangle \in B_{-1} \tag{7}$$

$$\cos \theta \left| \uparrow \downarrow \uparrow \downarrow \right\rangle + \sin \theta \left| \uparrow \downarrow \downarrow \uparrow \right\rangle \in B_0. \tag{8}$$

Expressing the Hamiltonian of Eq. 5 as a six by six matrix in $B_0$, we get six equations that must be fulfilled if the state in Eq. 5 is to be an eigenstate. We quickly realize that $J_{12}^X$ must vanish, which is a reasonable requirement as this coupling connects the left and right qubits directly, allowing state transfer to bypass the closed gate. Requiring the state of Eq. 7 to be an eigenstate of the four by four Hamiltonian in $B_{-1}$ with the same energy as the one of Eq. 5, we get four additional equations. Comparing all the obtained equations, we find that

$$\begin{pmatrix} J_{12}^X & J_{13}^X \\ J_{12}^X & J_{13}^X \end{pmatrix} \begin{pmatrix} \cos \theta \\ \sin \theta \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}. \tag{9}$$

This equation is only satisfied if $J_{12}^X = J_{13}^X$ and $\sin \theta = \pm \cos \theta$, i.e. $\theta = \pm \pi/4$ as in Ref. [31]. Thus the closed gate state is what is sometimes referred to as a dark state. The remaining equations imply $J_{24}^X = J_{23}^X$. The energy of the initial state, $\langle L|\text{closed}\rangle|R_i\rangle$, is $E_c = -J_{23}^X \pm J_{23}^X$. In total, we have reduced the number of free parameters to four coupling strengths and one sign choice.

The model derived here remains closed for more general states than first anticipated. In fact, even when the right qubit state is arbitrary, $|R_i\rangle = \epsilon |\uparrow\rangle + d |\downarrow\rangle$ with $|\epsilon|^2 + |d|^2 = 1$, the transistor is kept closed. This means that noise on the right qubit state does not result in leakage through the gate.

### B. Open transistor

In the case of an open transistor, we wish to exchange the left and right qubit states after some time $t_f$ of unitary time-evolution:

$$|L_i|\text{open}\rangle|R_i\rangle \xrightarrow{t_f} |R_i|\text{open}\rangle|L_i\rangle. \tag{10}$$

In the subspaces $B_{-2}$ and $B_{-1}$, this amounts to

$$\begin{align*}
\text{In } B_{-2}: & \quad |\uparrow\downarrow\downarrow\downarrow\rangle \xrightarrow{t_f} |\downarrow\downarrow\downarrow\uparrow\rangle \tag{11} \\
\text{In } B_{-1}: & \quad |\uparrow\uparrow\downarrow\downarrow\rangle \xrightarrow{t_f} |\downarrow\uparrow\downarrow\uparrow\rangle. \tag{12}
\end{align*}$$

Since $|\downarrow\uparrow\downarrow\uparrow\rangle$ is the only state in $B_{-2}$, it is an eigenstate of any total-spin conserving Hamiltonian such as the one we consider. Hence, the requirement of Eq. 11 is trivially fulfilled (up to a global phase). On the other hand, $B_{-1}$ consists of four states, and the Hamiltonian in this basis is thus a four by four matrix. Comparing the time-evolved state with $|\downarrow\uparrow\downarrow\uparrow\rangle$, we can derive criteria the Hamiltonian need to fulfill for the transition in Eq. 12 to happen. To simplify the analytic solution, we constrain our parameters as follows. First, we pick the sign $\theta = \pi/4$, and hence

$$|\text{closed}\rangle = \frac{1}{\sqrt{2}} (|\uparrow\downarrow\rangle + |\downarrow\uparrow\rangle). \tag{13}$$

Next, we impose left/right symmetry in the parameters: $J_{12}^X = J_{24}^X = -J_{13}^X = -J_{23}^X$. Finally, simulations show that $J_{23}^X$ cause unwanted interference in the state transfer, so we set $J_{23}^X = 0$. A sketch of this reduced model with

\[ Of course, the four-qubit state will acquire an overall phase factor, but this does not change the state.
merely two parameters $J_{23}^X$ and $J_{12}^X$, both assumed non-zero, is shown in Fig. 5.

With these simplifications, we can find simple expressions for the eigenstates and eigenergies of the four by four Hamiltonian in the subspace $B_{-1}$. The non-normalized eigenstates are

\[ |E_1 \rangle = |\uparrow \uparrow \downarrow \downarrow \rangle + |\downarrow \downarrow \uparrow \uparrow \rangle, \]

\[ |E_2 \rangle = |\uparrow \downarrow \downarrow \uparrow \rangle - |\downarrow \uparrow \uparrow \downarrow \rangle, \]

\[ |E_3 \rangle = |\uparrow \uparrow \downarrow \uparrow \rangle - \zeta |\downarrow \downarrow \uparrow \uparrow \rangle + \zeta |\uparrow \uparrow \downarrow \downarrow \rangle + |\downarrow \downarrow \uparrow \uparrow \rangle, \quad (14) \]

\[ |E_4 \rangle = |\uparrow \downarrow \downarrow \uparrow \rangle + \zeta^{-1} |\downarrow \uparrow \uparrow \downarrow \rangle - \zeta^{-1} |\downarrow \downarrow \uparrow \uparrow \rangle + |\uparrow \uparrow \downarrow \downarrow \rangle, \]

where $\zeta = (J_{23}^X + L)/2J_{12}^X$ with $L = 4(J_{12}^Z)^2 + (J_{23}^Z)^2$, and the energies are $E_1 = -J_{23}^X$, $E_2 = +J_{23}^X$, $E_3 = -L$ and $E_4 = +L$. In the basis of eigenstates, we may express the states

\[ |\uparrow \downarrow \downarrow \rangle = \frac{|E_2 \rangle}{\langle E_2 |E_2 \rangle} + \frac{|E_3 \rangle}{\langle E_3 |E_3 \rangle} + \frac{|E_4 \rangle}{\langle E_4 |E_4 \rangle} \]

\[ |\downarrow \uparrow \uparrow \rangle = \frac{|E_2 \rangle}{\langle E_2 |E_2 \rangle} + \frac{|E_3 \rangle}{\langle E_3 |E_3 \rangle} + \frac{|E_4 \rangle}{\langle E_4 |E_4 \rangle} \quad (15) \]

and note here that the only difference is the sign on the first term. Thus the $|\uparrow \rangle$ state at the left qubit position is transferred to the right qubit port with unity fidelity in time $t_f$ if and only if the time-evolution flips the relative sign between $|E_2 \rangle$ and the states $|E_3 \rangle$ and $|E_4 \rangle$, i.e. $e^{-i(E_2 - E_3)t_f} = e^{-i(E_2 - E_4)t_f} = -1$. Solving these equations, we find the transfer time as $t_f = \pi/|J_{23}^X|$ and a criterion on the ratio of coupling strengths, similar to the one found in e.g. Ref. [99].

\[ \frac{|J_{12}^X|}{|J_{23}^X|} = \sqrt{m^2 - \frac{1}{4}}, \quad m = 1, 2, 3, \ldots, \quad (16) \]

the simplest case ($m = 1$) yielding $J_{12}^X = \pm \sqrt{3/4}J_{23}^Z$.

During the state transfer in Eq. (12), the state $|\downarrow \uparrow \uparrow \rangle$, being an eigenstate with energy $J_{23}^X$, accumulates a phase factor $e^{i\pi t_f} = -1$. Thus, the initial state evolves,

\[ |L_i \rangle_{\text{open}} |R_i \rangle = (a|\uparrow \rangle + b|\downarrow \rangle) |\downarrow \rangle |\downarrow \rangle |\downarrow \rangle |\downarrow \rangle. \quad (17) \]

So in order to achieve the total state transfer suggested in Eq. (10), we must apply a single-qubit phase gate on the right qubit to fix the sign, an operation which can be done in zero time [61]. This is a simple task, and we conclude that the diamond model is capable of functioning as a quantum transistor.

One may wonder whether the open transistor works for an arbitrary right qubit state, as in the case of the closed transistor. This would indeed be the case if $|\uparrow \uparrow \downarrow \rangle$ was an eigenstate, but it is not, and such a term in the initial state becomes a messy state in $B_0$ as time passes. Therefore, we cannot relax the requirement $|R_i \rangle = |\downarrow \rangle$. If the transistor was able to operate with an arbitrary initial right qubit state, it would constitute a conditional swap operation on two arbitrary left and right qubit states [55, 59]. Such a gate, called CSWAP or Fredkin gate, is universal for quantum computing, and a simple realization of this gate is much-coveted. However, we speculate that the CSWAP could be possible if we promote the two gate qubits to qutrits (three-level systems), thereby extending the Hilbert space, giving more degrees of freedom for a complete CSWAP.

### III. TOWARDS AN IMPLEMENTATION WITH SUPERCONDUCTING CIRCUITS

In the previous section, we saw that a four-qubit system with Heisenberg interactions could function as a quantum transistor. We kept the model simple in order to gain analytic insight. Now, we use the simple model as a stepping stone towards a more realistic case, and consider how such a diamond transistor may be realized in a real physical system, specifically in superconducting circuits.

We consider the spin Hamiltonian to be

\[ H_0 = \frac{1}{2}(\Omega + \Delta)(\sigma_z(1) + \sigma_z(4)) + \frac{1}{2}\Omega(\sigma_z(2) + \sigma_z(3)), \quad (18) \]

where $\Delta$ denote a detuning of qubit 1 (left qubit) and qubit 4 (right qubit) from the frequency $\Omega$ of qubits 2 and 3 (the gate qubits). For the interaction part, we consider:

\[ H_{\text{int}} = J_1\sigma_z(2)\sigma_z(3) + J_2\sigma_\mu(2)\sigma_\mu(3) + J_2(\sigma_\mu(1) + \sigma_\mu(4))(\sigma_z(2) - \sigma_z(3)). \quad (19) \]

These type of interaction terms are naturally realized with superconducting circuits. Assuming $|2\Omega + \Delta| \gg |\Delta|$, we employ the rotating wave approximation and ignore the fastest oscillating terms, such that the system Hamiltonian in the frame rotating with $H_0$ becomes:

\[ H = J_1\sigma_z(2)\sigma_z(3) + J_2(\sigma_z(1) + \sigma_z(3)) + J_2(\sigma_z(2) - \sigma_z(3))e^{i\Delta t} \]

\[ + J_2(\sigma_\mu(1) + \sigma_\mu(4))(\sigma_z(2) - \sigma_z(3))e^{-i\Delta t}. \quad (20) \]
Notice that this model, sketched in Fig. 6, is a special case of the diamond model defined in Eq. (5). In fact, if we set $\Delta = J_x = 0$ and $J_2 = \pm \sqrt{3/4}J_z$, the model reduces to the final model of the previous section.

From the previous section, we already know that the model functions as an open transistor. So, what we need to resolve is the question about the open transistor. The detuning $\Delta$ between the gate qubits and the left/right qubits results in a time-oscillating coupling. We solve the time-evolution analytically using the Floquet formalism\[62\], and the transfer time can be determined in a way similar to what was done in Section II B. Under the assumption that the detuning is much larger than the other parameters, the Z-coupling $J_z$ is a small quantity of Eq. (16). When strong driving is applied through $\Delta$, we claim that the superconducting circuit in Fig. 7 will do the job. In addition to the couplings in Eq. 20, we also get an unwanted coupling between the left and right qubit so that the Hamiltonian implemented by the circuit is:

$$ H = J_z \sigma_z^{(2)} \sigma_z^{(3)} + J_x \sigma_+^{(2)} \sigma_-^{(3)} + \sigma_z^{(2)} \sigma_z^{(3)} $$

$$ + J_y (\sigma_+^{(1)} \sigma_-^{(4)} + \sigma_-^{(1)} \sigma_+^{(4)}) e^{i \Delta t} $$

$$ + J_y (\sigma_+^{(1)} \sigma_-^{(4)} + \sigma_-^{(1)} \sigma_+^{(4)}) e^{-i \Delta t} $$

$$ + J_4 (\sigma_+^{(1)} \sigma_-^{(1)} + \sigma_-^{(1)} \sigma_+^{(1)}) . $$

As we remarked in Section II A, the cross-talk term of strength $J_4$ allows state transfer to bypass the gate, resulting in a leaking closed transistor. Fortunately, we can suppress this coupling by making the capacitance $C_R$ large. Details on the analysis of the circuit and expressions for the parameters in the Hamiltonian in terms of circuit parameters are provided in Appendix B.

In this section we wish to study the performance of the diamond transistor in a realistic setting. Our simulations shall therefore be based on the Hamiltonian of Eq. (22) where the parameters are found from realistic circuit parameters using the relationships in Appendix B. An experimental realization of the system will necessarily

FIG. 6. Illustration of the diamond model we wish to implement using superconducting qubits, given as $H$ in Eq. (20). Notation on the figure is explained in the caption of Fig. 4.

FIG. 7. Circuit diagram for the circuit that implements the Hamiltonian of Eq. (22). Each circuit element has been labeled with its properties, so that the capacitors are labeled with their capacitances $C, C', J, J', C_R$, the inductors with their inductances $L, L', L_R$ and the Josephson junctions by their Josephson energies $E_J, E_J', E_R, E_R$. Each node in the circuit is labeled by a letter $A-G$. The transfer time is given entirely by the Z-coupling in the gate, it being the only energy scale of the system.

IV. NUMERICAL SIMULATIONS

As already mentioned, we wish to implement the diamond model of Eq. (20) using superconducting qubits. We claim that the superconducting circuit in Fig. 7 will do the job. In addition to the couplings in Eq. 20, we also get an unwanted coupling between the left and right qubit so that the Hamiltonian implemented by the circuit is:

$$ H = J_z \sigma_z^{(2)} \sigma_z^{(3)} + J_x \sigma_+^{(2)} \sigma_-^{(3)} + \sigma_z^{(2)} \sigma_z^{(3)} $$

$$ + J_y (\sigma_+^{(1)} \sigma_-^{(4)} + \sigma_-^{(1)} \sigma_+^{(4)}) e^{i \Delta t} $$

$$ + J_y (\sigma_+^{(1)} \sigma_-^{(4)} + \sigma_-^{(1)} \sigma_+^{(4)}) e^{-i \Delta t} $$

$$ + J_4 (\sigma_+^{(1)} \sigma_-^{(1)} + \sigma_-^{(1)} \sigma_+^{(1)}) . $$

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\[3\] A patent application pertaining to the circuit, and the XXZ gate in particular, has been filed with the European Patent Office (application number 17185721.2 - 1879).
introduce noise, and so we include realistic dephasing noise in the simulations, too. We do not consider spin flip noise, which could potentially flip between the open and closed gate, since the closed flux loop inherent to the gate will likely make flux noise dominant[41].

Specifically, we use the spin model parameters in Panel B of Table I. This set of parameters is found from the circuit parameters in Panel A. We stress that we did not fine-tune any of the parameters in Table I and that the transistor properties reported in this section are inherent to the diamond model.

Note that the energy ratios $\frac{E_{J,i}}{E_{C,i}}$ suitable for transmon qubits ($i = 1, 2$), and $E_{J,2}/E_{J,1} \sim \frac{E_{J,1}}{E_{J,2}}$. Also note that $C_R$ has been chosen large to suppress the unwanted cross-talk of strength $J_4$ such that the gate can be closed effectively on the time-scale of operation. Though a bit higher than in typical designs, we note that inductances of the order $L$ and $L_R$ can be realized with today’s technology[63].

### A. Time-evolution and transition fidelities

To model decoherence in the system from dephasing noise, we consider the Lindblad master equation,

$$\dot{\rho} = -i[H, \rho] + \sum_{i=1}^{4} \gamma_i \left[ \sigma_z^{(i)} \rho \sigma_z^{(i)} - \frac{1}{2} (\rho \sigma_z^{(i)} \sigma_z^{(i)} \rho) \right],$$

with $\rho$ the density matrix, $H$ the Hamiltonian of Eq. (22) and $\sqrt{\gamma} \sigma_z^{(i)}$ the collapse operator causing phase flip of qubit $i$, and $\gamma_i$ being the corresponding rate. We set $\gamma_1 = \gamma_2 = \gamma_3 / 2 = \gamma_4 / 2 = \gamma$, modeling shorter decoherence time on gate than the left/right qubits. State-of-the-art values for the decoherence rate are $\gamma / 2 \pi \sim 0.01 \text{MHz} / 2 \pi = 0.0016 \text{MHz}$, corresponding to the time-scale $1 / \gamma \sim 100 \mu s$. As we will see, decoherence plays a role on a time-scale two order of magnitude magnitudes larger than the operation time of the transistor, and is not considered an eminent threat to our protocol.

In order to determine the time-evolution of a pure state $|\psi\rangle = (|\uparrow\rangle + e^{i\theta}|\downarrow\rangle) / \sqrt{T + r^2}$ for $0 \leq r \leq 1$ and $0 \leq \theta < 2\pi$, we set where the $|\downarrow\rangle$ dominates, because transfer dynamics in this case is trivial with both transfer and remain fidelity $\sim 1$. The right qubit is initialized according to Eq. (2), and the gate is either open or closed, as defined in Eqs. (5) and (13).

Simulation results for scenarios including and excluding cross-talk and noise are shown in Figure 8 (closed gate) and Figure 9 (open gate).

When the gate is closed, we compute the fidelity with the final state set equal to the initial one. As is seen
Transfer fidelity, $F(t)$, shows in Fig. 10 the fidelity for the open gate with initial and final states as given in the main text. Simulation data is obtained including cross-talk and without noise ($\gamma/2\pi = 0$) and varying detuning frequencies $\Delta$. The remaining parameters are taken from Table I.

from the simulations in Figure 8, a non-zero cross-talk term, $J_4 \neq 0$, causes the gate to leak slightly over time, and, not surprisingly, decoherence speeds up the process. However, during the operation of the open gate explained below ($t \sim 0.7 \mu s$), the transistor remains well-closed, $F_{i \to f} > 0.95$.

For the open gate we will define the fidelity using the transition in Eq. (17), due to the accumulated sign difference of $|\uparrow \downarrow \downarrow \downarrow \rangle$ and $|\downarrow \uparrow \downarrow \downarrow \rangle$ observed in Section III. We see in Figure 9 that the state transfer fidelity almost reaches unity around $t = 0.7 \mu s$. The transfer fidelity is only barely reduced by dephasing noise. In fact, the transfer time is a little faster than the analytic prediction of $0.84 \mu s$ computed from Eq. (22) and the result from the ideal model (with $J_4 = 0$), which is consistent with the effect of the additional small cross-talk term.

Another analytic prediction for the transfer time is its scaling with $|\Delta|$ and $J_2^{-2}$. Simulations verify this behavior very accurately. In reality $\Delta$ and $J_2$ cannot be changed independently since they are connected through the circuit parameters. However, in order to single-out the role of $\Delta$ and illustrate its role in the transfer time, we show in Fig. 10 the fidelity for the open gate with various values of $\Delta$ and the remaining spin model parameters fixed. Besides illustrating the tunability of the transfer time, this figure clearly demonstrates that the state transfer is perfect or nearly perfect no matter the exact parameter values, rendering the transistor properties very robust.

B. State preparation

In order to operate the transistor successively, we need a scheme for preparing the state of the input qubit and the gate. It is advantageous if we can address the gate qubits exclusively so that the gate may be switched on or off independently of the left and right qubits. This is the case if the left/right qubits are far detuned from the gate qubits, i.e. when $\Delta$ is sufficiently large. Picking $\Delta \sim 2\pi \mathrm{GHz}$, we may therefore address the gate qubits with microwave radiation without affecting the input and output qubits. In experiments, the input and output frequencies may be tuned in situ using flux control lines.

Next we need to ensure that the gate can be opened and closed in a controlled way. Suppose we start in the open gate state, and we wish to close the gate, or the opposite. This can be achieved by driving the nodes $D$ and $E$ on the circuit in Fig. 7, thereby introducing the following additional term in the interaction-picture Hamiltonian:

$$H_d(t) = iA \cos(\omega_d t) \times \left[ (\sigma_+^{(2)} + \sigma_-^{(3)}) e^{i\Omega t} + (\sigma_-^{(2)} + \sigma_+^{(3)}) e^{-i\Omega t} \right].$$

This driving term, like the remaining Hamiltonian, preserves the total spin. When starting from any of the triplet states shown in Fig. 11, we can therefore ignore the singlet state $(|\uparrow\rangle - |\downarrow\rangle)/\sqrt{2}$. The driving introduces Rabi oscillations between the closed and open states provided the driving frequency matches the energy difference, $\omega_d = (\Omega - 3J_2)$, and $A \ll J_2$. Thus starting from $|\text{open}\rangle$ (or $|\text{closed}\rangle$) a $\pi$-pulse, $\omega_d t = \pi$, would kick the gate state to $|\text{closed}\rangle$ (or $|\text{open}\rangle$) in about $0.05 \mu s$. With $J_2 \sim 30 \cdot 2\pi \mathrm{MHz}$ the energy difference between the open or closed state and $|\uparrow\rangle$ are far enough from $\omega_d$ as
to not accidentally populate $|↑↑\rangle$. Using the mechanism described here, we can thus switch between the open and closed transistor using a simple external microwave drive.

$|\text{open}\rangle = |↓↓\rangle$, $|Ω - 3J_z| \sim 10 \cdot 2\pi \text{GHz}$

$|\text{closed}\rangle = \frac{1}{\sqrt{2}}(|↑↓\rangle + |↓↑\rangle)$, $|Ω + 3J_z| \sim 10 \cdot 2\pi \text{GHz}$

$|↑↑\rangle$

FIG. 11. Sketch of the triplet gate states, typical parameters are $Ω \sim -10 \cdot 2\pi \text{GHz}$ and $J_z \sim -30 \cdot 2\pi \text{MHz}$.

V. CONCLUSIONS AND OUTLOOK

We have discussed the notion of a quantum transistor, and introduced a model for such a device comprised of four qubits interacting via Heisenberg XX and Heisenberg XXZ couplings. Using basic quantum mechanics and Floquet theory, we showed that our "diamond model" is capable of operating as a transistor without fine-tuning the spin model parameters. Then we proposed a concrete implementation of the model as a superconducting circuit, and demonstrated its capability of operating with high-fidelity in a realistic noisy setting. Seeking a compromise between fast state transfer and a well-closed gate, we simulated the transistor with one example circuit parameter choice.

Our proposed transistor model is readily implementable in state-of-the-art experiments with superconducting qubits, and may serve as a vital ingredient in larger networks for quantum computation. In fact, the transistor is very closely related to the CSWAP (Fredkin gate), which is a universal gate for quantum computing, i.e., a network of CSWAP gates could perform any quantum computation. The CSWAP exchanges two arbitrary qubit states conditioned by the value of a control qubit, and consequently we may regard our transistor as a CSWAP where one of these swapped qubits is not arbitrary but is restricted to the $|↓\rangle$ state.

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Appendix A: Driven system

Consider the periodically driven time-dependent Hamiltonian of Eq. \[20\], which can be cast as

$$H = H_0 + H_1 e^{i\Delta t} + H_1^* e^{-i\Delta t}, \quad (A1)$$

with

$$H_0 = J_2 (\sigma^{(2)}_z \sigma^{(3)}_z + \sigma^{(2)}_+ \sigma^{(3)}_- + \sigma^{(2)}_- \sigma^{(3)}_+)$$

$$H_1 = J_2 (\sigma^{(1)}_+ + \sigma^{(4)}_+)(\sigma^{(2)}_- - \sigma^{(3)}_-). \quad (A2)$$

We now wish to analyze the open gate: see whether perfect or near-perfect state transfer is possible and, if so, find an expression for the transfer time. Floquet theory is developed to treat such periodically driven systems, and the time-evolution operator can be expressed as an exponential of a Floquet Hamiltonian. For typical parameter values in superconducting qubits, the driving frequency $\Delta \sim 2\pi \text{GHz}$ is about a thousand times larger than the energy scale set by $J_1$, $J_2$, and $J_3$. In other words: While the open gate permits one cycle of state transfer, the driving terms in the Hamiltonian has undergone a thousand oscillations. So it is appropriate to compute the Floquet Hamiltonian using an inverse-frequency expansion known as the Magnus expansion. To first order in $\Delta^{-1}$, the Magnus expansion states that the (stroboscopic) Floquet Hamiltonian can be expressed as

$$H_F = H_0 + \frac{1}{\Delta} \left( [H_1, H_0^+] - [H_1, H_0] + [H_1^*, H_0] \right). \quad (A3)$$

After some Pauli operator gymnastics we find:

$$[H_1, H_0^+] = +J_2^2 (\sigma^{(2)}_z \sigma^{(3)}_+ + \sigma^{(2)}_- \sigma^{(3)}_+)(\sigma^{(1)}_+ + \sigma^{(4)}_+) - J_2^2 (\sigma^{(2)}_- \sigma^{(3)}_+ + \sigma^{(2)}_- \sigma^{(3)}_-)(\sigma^{(1)}_+ + \sigma^{(4)}_-) - J_2^2 (\sigma^{(1)}_+ \sigma^{(4)}_+ + \sigma^{(4)}_+ \sigma^{(1)}_-)(\sigma^{(2)}_- + \sigma^{(3)}_+) - J_2^2 (\sigma^{(1)}_+ \sigma^{(4)}_+ + \sigma^{(4)}_+ \sigma^{(1)}_-)(\sigma^{(2)}_- + \sigma^{(3)}_+) \quad (A4)$$

$$[H_1, H_0] = J_2 J (\sigma^{(2)}_z \sigma^{(3)}_+ - \sigma^{(3)}_z \sigma^{(2)}_+)(\sigma^{(1)}_+ + \sigma^{(4)}_+)$$

$$[H_1^*, H_0] = - [H_1, H_0^+] \quad (A4)$$

where we have defined $\tilde{J} = 2J_2 + J_3$ for later convenience.

The operator $U(T, 0) = e^{-iH_F T}$ takes the system from time zero through one driving cycle of period $T = 2\pi \Delta^{-1}$. Therefore, successive application of this operator $n$ times, $U(nT, 0) = e^{-iH_F nT}$, \(A5\) will take the system from time zero to time $nT$. Since the driving period $T$ is very small compared to the state transfer time, we will consider $t = nT$ a continuous time-variable.
The Floquet Hamiltonian $H_F$, like $H$, conserves the total spin projection. So, as we are interested in the time-evolution of $|\uparrow\downarrow\downarrow\downarrow\rangle$, it suffices to diagonalize $H_F$ in the subspace $B_1$. The (non-normalized) eigenstates have the same form as in the non-driven case studied in the main text:

$$|E_1\rangle = |\uparrow\uparrow\uparrow\uparrow\rangle + |\downarrow\downarrow\uparrow\uparrow\rangle,$$

$$|E_2\rangle = |\downarrow\downarrow\downarrow\uparrow\rangle - |\uparrow\downarrow\uparrow\uparrow\rangle,$$

$$|E_3\rangle = |\uparrow\uparrow\downarrow\downarrow\rangle - \tilde{\zeta}|\uparrow\uparrow\uparrow\downarrow\rangle + \tilde{\zeta}|\downarrow\downarrow\uparrow\uparrow\rangle + |\downarrow\downarrow\downarrow\uparrow\rangle,$$

$$|E_4\rangle = |\uparrow\uparrow\downarrow\downarrow\rangle + \tilde{\zeta}^{-1}|\downarrow\downarrow\uparrow\uparrow\rangle - \tilde{\zeta}^{-1}|\uparrow\downarrow\uparrow\uparrow\rangle + |\downarrow\downarrow\downarrow\uparrow\rangle,$$

where $\tilde{\zeta}$ is a very complicated function of $J_2$, $J_x$, $J_z$ and $\Delta$ whose exact form is irrelevant for our analysis. The energies are $E_1 = J_x - J_z$, $E_2 = J_z$, $E_3 = -(J_x + \kappa)/2$ and $E_4 = -(J_x - \kappa)/2$, with

$$\kappa = \sqrt{\frac{64J_x^2}{\Delta^2} + \frac{16J_x^2}{\Delta^2}} + J^2. \quad (A7)$$

Just like in simple case studied in the main text, we may expand the following states in the eigenbasis:

$$|\uparrow\downarrow\downarrow\downarrow\rangle = \frac{|E_2\rangle}{\langle E_2|E_2\rangle} + \frac{|E_3\rangle}{\langle E_3|E_3\rangle} + \frac{|E_4\rangle}{\langle E_4|E_4\rangle},$$

$$|\downarrow\downarrow\uparrow\uparrow\rangle = \frac{|E_2\rangle}{\langle E_2|E_2\rangle} + \frac{|E_3\rangle}{\langle E_3|E_3\rangle} + \frac{|E_4\rangle}{\langle E_4|E_4\rangle}. \quad (A8)$$

We see that the $|\uparrow\rangle$ state is transferred from the left to the right position when unitary time evolution accounts for the relative sign between $|E_2\rangle$ and the states $|E_3\rangle$ and $|E_4\rangle$, i.e. $e^{-i(E_2 - E_3)t}f = e^{-i(E_2 - E_4)t}f = -1$. This is equivalently expressed as

$$\begin{cases}
(E_2 - E_3)t_f = (2n + 1)\pi \\
(E_2 - E_4)t_f = (2m + 1)\pi
\end{cases} \quad (A9)$$

$$\Leftrightarrow \begin{cases}
(\tilde{J} + \kappa)t = 2(2n + 1)\pi \\
(\tilde{J} - \kappa)t = 2(2m + 1)\pi
\end{cases} \quad (A10)$$

for $n, m \in \mathbb{Z}$.

If $\tilde{J} = 0$, then $\kappa = 8J_x^2/|\Delta|$, and the conditions in Eq. (A10) reduces to a single equation. Seeking the solution for the smallest positive $t_f$ yields the transfer time:

$$t_f = \frac{\pi|\Delta|}{4J_x^2} \quad (\tilde{J} = 0). \quad (A11)$$

If $\tilde{J} \neq 0$, we can simplify $\kappa$ by taking the limit $|\tilde{J}|, |J_2| \ll |\Delta|$. Neglecting term $\sim \Delta^{-2}$ in the square root, Eq. (A7) becomes

$$\kappa \approx \sqrt{\frac{16J_x^2}{\Delta}} + J^2 \approx |\tilde{J}| \left(\frac{8J_x^2}{\Delta J} + 1\right), \quad (A12)$$

where the last approximation is a first order expansion of the square root. With the above approximation for $\kappa$, we see that the two conditions in Eq. (A10) define two time-scales, $(\tilde{J} \pm \kappa)^{-1}$: one short $\sim \tilde{J}^{-1}$ and one much longer $\sim \Delta/J_x^2$. The long time-scale will set the speed limit for the state transfer, and it will fulfill the state transfer condition for the first time when

$$\frac{8J_x^2}{\Delta} t_f = 2\pi \Leftrightarrow t_f = \frac{\pi|\Delta|}{4J_x^2} \quad (\tilde{J} \neq 0). \quad (A13)$$

During this transfer time, the state transfer condition derived from the short time-scale is satisfied $\sim 100$ times, so in practice this condition will also be satisfied at, or very close to, the transfer time of Eq. (A13). Technically, we find the constraint that $\tilde{J}\Delta/8J_x^2$ must be an integer, but since its magnitude is very large, it may be very well approximated by the nearest integer and the constraint can be considered fulfilled. So, for both zero and non-zero $\tilde{J}$, we find that (nearly) perfect state transfer is achieved in time

$$t_f = \frac{\pi|\Delta|}{4J_x^2}. \quad (A14)$$

Appendix B: Implementation with superconducting circuits

In this section the units $2e$ and $\phi_0/2\pi$ are kept explicitly. We see in Fig. 5 a circuit that will implement the diamond Heisenberg model in Eq. (22). The goal of this appendix is to indicate how the implementation works and provide expressions for the parameters in the Hamiltonian in terms of properties of the circuit elements that make up the circuit. To do this, we will model the transmission-line resonators as LC-circuits with capacitances $C_R$ and inductances $L_R$. For each node in the circuit, labeled $A, B, \ldots, G$ in Fig. 5, we have a related flux degree of freedom. Denoting the flux degree of freedom at node $i$ by $\phi_i$, we have the seven degrees of freedom $\phi_A, \phi_B, \ldots, \phi_G$. It is however more advantageous to describe the circuit in terms of the variables

$$\begin{align*}
\phi_1 &= \phi_A \\
\phi_2 &= \phi_D - \phi_E - \phi_C \\
\phi_3 &= \phi_C - \phi_D + \phi_E \\
\phi_4 &= \phi_G \\
\phi_{LR} &= \phi_B \\
\phi_{RR} &= \phi_F \\
\phi_{CM} &= \phi_C + \phi_D + \phi_E.
\end{align*} \quad (B1)$$

Once the system is quantized, each of the four numbered fluxes will correspond to the qubit in Fig. 5 of the same number. Note that the two left/right qubits reside in the two transmon qubits in the far left/right of the circuit in Fig. 7, while the two gate qubits consist of linear combinations of the fluxes $C, D, E$ of the inner part of the circuit. The advantages of this choice of coordinates is threefold. First of all, it implements the $\sigma_z^{(2)}\sigma_z^{(3)}$ coupling
where the coupling between the gate qubits and left/right qubits are anti-symmetric with respect to permutation of qubit 2 and 3 (seen in Fig. 6) as long as the circuit is built symmetrically. Finally, the fact that the left/right qubits are just two transmon qubits at the edge of the circuit should mean the circuit is relatively easy to integrate into larger architectures.

The remaining three coordinates are a center of mass-like coordinate $\phi_{CM}$ and the two resonator-fluxes $\phi_{LR}$ and $\phi_{RR}$, corresponding to the left and right resonator, respectively. These degrees of freedom can be detuned sufficiently through parameter choices that they do not couple significantly to the numbered qubits, and hence can be ignored in the following. Nevertheless, the fact that the coupling between the left/right and gate qubits occurs indirectly through the resonators plays an important role in controlling the strengths of these couplings, as we will later see.

Having established the important degrees of freedom, the analysis of the circuit is now a relatively straightforward calculation. What is found is that the four degrees of freedom $\phi_1, \phi_2, \phi_3$, and $\phi_4$ each support a qubit, with the energy spacing between the two states of the qubits being identical between the two gate qubits and between the two left/right qubits. Interactions are induced between these qubits through three mechanisms. The most prolific of these mechanisms is capacitative coupling, which occurs as a result of kinetic terms of the form $C_0 \phi_i \dot{\phi}_j$ coupling the $\phi_i$ and $\phi_j$ degrees of freedom. Let us for further reference define the matrix $K$ as the symmetric matrix so that the contributions to the Lagrangian from capacitances take the form

$$L_{kin} = \frac{1}{2} \dot{\phi}^T K \phi ,$$

(B2)

where $\phi = (\phi_1, \phi_2, \phi_3, \phi_4, \phi_{LR}, \phi_{RR}, \phi_{CM})^T$ is the vector of fluxes. Terms of the form $C_0 \phi_i \dot{\phi}_j$ then constitute off-diagonal contributions to $K$, and the strength of the induced interaction between $\phi_i$ and $\phi_j$ will be proportional to the $(i,j)$-entry in the inverse matrix. It is capacitative couplings that are responsible for the couplings between the left/right qubits and the gate qubits, with the coupling strength given by

$$J_2 = -\frac{1}{4} \left( E_{L,1} \left( E_{L,2} + \frac{1}{2} E_{J,2} \right) \right) \frac{(2e)^2}{2E_{C1}E_{C2}} \left( K^{-1} \right)_{(i,j)},$$

(B3)

where $\left( K^{-1} \right)_{(i,j)}$ is the $(i,j)$-entry in the inverse matrix of $K$ and $E_{C1}, E_{C2}, E_{L,i}, E_{J,i}$ are the effective capacitative energies, inductive energies and Josephson energies of the $i$'th qubit. These quantities are related to the circuit parameters as follows:

$$E_{C,i} = \frac{(2e)^2}{8} \left( K^{-1} \right)_{(i,i)},$$

$$E_{L,1} = E_R$$

$$E_{L,2} = \frac{1}{8L} \left( \frac{\Phi_0}{2\pi} \right)^2 + \frac{1}{8L} \left( \frac{\Phi_0}{2\pi} \right)^2 + \frac{3}{32} (E_J + E_J + E_q)$$

$$E_{J,2} = E_J' + \frac{17E_q}{16}$$

$$E_{L,CM} = \frac{E_q}{8}$$

$$E_{L,CM} = \frac{3E_q}{16}$$

and comes from writing the single-qubit parts of the Hamiltonian on the form

$$4E_{C,i} \left( \frac{p_i}{2E} \right)^2 + \left( \frac{\Phi_0}{2\pi} \right)^2 E_{L,i} \phi_i^2 - E_{L,i} \cos \left( \frac{\Phi_0}{2\pi} \phi_i \right),$$

where $p_i$ is the conjugate momentum of the $\phi_i$ degree of freedom and $\Phi_0$ is the magnetic flux quantum. For brevity, the explicit form of the inverse matrix elements of the inverse of the $K$ matrix are omitted since these are sizable expressions.

The second effect inducing coupling in the circuit is the presence of the Josephson junctions. These provide terms of the form $E_J \cos (\phi_i - \phi_j)$. Since we are in the transmon regime ($E_J < E_C$) it is sufficient to expand these cosines to the fourth order in the argument. The result of these terms is therefore, among other things, the presence of the Josephson junctions. These provide sizable expressions.

The second effect inducing coupling in the circuit is the presence of the Josephson junctions. These provide terms of the form $E_J \cos (\phi_i - \phi_j)$.

$$J_z = -\frac{E_{C,2} (E_J' + E_J + 8E_q)}{64 (E_{L,2} + \frac{1}{2} E_{J,2})}.$$  

(B5)

The $J_x$-term contains contributions from both capacitative couplings and Josephson junctions. It also contains contributions from the third and final coupling mechanism: coupling through terms of the form $\frac{1}{2\pi} (\phi_i - \phi_j)^2$ coming from the inductors in the central part of the circuit. The interaction strength is given by

$$J_x = \sqrt{\frac{E_{C,2} E_{L,2} + \frac{1}{2} E_{J,2}}{E_{L,2} + \frac{1}{2} E_{J,2}}} \left( \frac{\Phi_0}{2\pi} \right)^2 \left( \frac{1}{4L} \right)^2 \left( 1 - \frac{1}{4L} \right)$$

$$+ \frac{E_{C,2} E_{L,2} + \frac{1}{2} E_{J,2}}{E_{L,2} + \frac{1}{2} E_{J,2}} \left( \frac{E_J' - E_J}{4} - E_q \right)$$

$$+ \frac{1}{4} \sqrt{E_{L,2} + \frac{1}{2} E_{J,2}} (2e)^2 \left( K^{-1} \right)_{(2,3)}$$

$$- \frac{E_{C,2} (E_J' - E_J - 10E_q)}{32 (E_{L,2} + \frac{1}{2} E_{J,2})}$$

$$+ \frac{E_q}{8} \sqrt{E_{C,2} E_{C,CM}}.$$  

(B6)

In addition to the aforementioned couplings, we get the unwanted cross-talk-coupling between the left and right
qubit with strength

\[ J_4 = \frac{1}{4} \sqrt{\frac{E_{J,1}}{2EC_{1}}} (2e)^2 (K^{-1})_{1,4}. \]  

(B7)

Luckily, this problematic coupling can be eliminated by considering the nature of the interaction. By inspecting the circuit, we see that the capacitative interaction yielding \( J_2 \) occurs through a single resonator, while the \( J_4 \) cross-talk-interaction occurs through both of the resonators as well as a capacitance of the central circuit. The effect of each of these additional capacitances is to scale down the strength of the interaction, and so the relative size of \( J_4 \) compared to \( J_2 \) scales inversely with the capacitances \( C_J \) and \( C_R \):

\[ \frac{J_4}{J_2} \sim \frac{1}{C_J}, \frac{1}{C_R}. \]

By increasing these capacitances, it is therefore possible to scale down the cross-talk to less than 1% of \( J_2 \), which makes the cross-talk sufficiently weak for the transistor to be able to block state transfer with high fidelity (see Section IV).

The frequency of the gate qubits is given by

\[ \Omega = -\sqrt{16E_{C,2}\left(E_{L,2} + \frac{1}{2}E_{J,2}\right)} + \frac{E_{J,2}E_{C,2}}{2\left(E_{L,2} + \frac{1}{2}E_{J,2}\right)} + \frac{E_{C,2}(E'_J + E_J + 8E_q)}{16 \left(E_{L,2} + \frac{1}{2}E_{J,2}\right)} + \frac{5E_q}{32} \left(E_{C,2}\left(E_{L,CM} + \frac{1}{2}E_{J,CM}\right)\right), \]

and the detuning of the left/right qubits from the gate qubits is

\[ \Delta = -\sqrt{8E_{C,1}E_{J,1}} + E_{C,1} - \Omega. \]  

(B8)

(B9)

This exhausts the list of parameters in Eq. 22.

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