The neutron electric dipole form factor in the perturbative chiral quark model

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Abstract. We calculate the electric dipole form factor of the neutron in a perturbative chiral quark model, parameterizing CP-violation of generic origin by means of effective electric dipole moments of the constituent quarks and their CP-violating couplings to the chiral fields. We discuss the relation of these effective parameters to more fundamental ones such as the intrinsic electric and chromoelectric dipole moments of quarks and the Weinberg parameter. From the existing experimental upper limits on the neutron EDM we derive constraints on these CP-violating parameters.

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1. Introduction

The origin and manifestations of CP violation have been important issues in particle physics since its discovery in neutral kaon decays in 1964 [1]. So far CP violation has been observed only in $K$ and $B$ hadron decays [2], results that are consistent with the Kobayashi-Maskawa formulation of a complex quark mixing matrix [3], within the Standard Model (SM) of electroweak interactions. However, CP violation is not only important in hadron decays but it is also an inevitable ingredient for the generation of the observed baryon asymmetry of the Universe [4]. Yet, the observed CP violation within the SM is by far insufficient to explain it. This evidence is pointing to physics beyond the SM, whose contribution to CP violation in the K and B systems may be irrelevant but should be dominant for the Universe’s baryon asymmetry. Thus, there are powerful motivations to study all possible CP odd observables and distinguish its underlying sources. Among those observables, a great deal of effort has been directed to the study of the electric dipole moment (EDM) of leptons, neutrons and neutral atoms. Both SM and various non-SM sources of CP violation have been considered (for recent reviews see e.g. Ref. [5, 6]). These studies have been particularly stimulated by the expectation of great improvements (2 to 4 orders of magnitude) in the experimental sensitivities to EDMs in the next decade (for review see Ref. [6]).

As it is well known, there are two sources of CP-violation within the SM: the complex phase of the Cabibbo-Kobayashi-Maskawa (CKM) matrix in the weak interaction sector, and the $\theta$-term in the strong interaction sector [7]-[10]. On the one hand, the complex phase of the CKM matrix is well established and provides a consistent explanation of the observed CP odd effects in hadron decays, but predicts an imperceptible contribution to the EDMs, far below the sensitivity of present or foreseeable experiments. Indeed, the CKM prediction for the neutron EDM, $d_n$, ranges from $10^{-31}$ to $10^{-33}$ $e\cdot cm$ [11] while its present experimental upper limit [12] is

$$|d_n| < 0.63 \times 10^{-25} \, e\cdot cm.$$  \hspace{1cm} (1)

On the other hand, the presence of a $\theta$-term is still an open issue; it leads to a sizable neutron electric dipole moment [13, 14] and may significantly contribute to atomic EDMs, while it is insignificant for CP violation in hadron decays. Indeed, the non-observation of EDM for the neutron imposes a very strict upper bound on the value of $\theta$, of the order of $10^{-10}$. This unnaturally small value of $\theta$, which is otherwise not restricted by theory, is called the strong CP problem. One elegant solution was proposed by Peccei and Quinn (PQ) [15], which makes the $\theta$ parameter to vanish dynamically. Other important contributions to the atomic and neutron EDMs may arise from physics beyond the SM.

In the calculations of atomic and neutron EDMs one faces the problem of translation of the CP violation mechanism formulated at the quark-gluon level, to the processes at the hadronic or atomic level. This translation must resort to hadronic and nuclear models and perform a careful treatment of the effects from the cloud of atomic electrons. The validity of the estimates then depends on the validity of these models.
The purpose of the present work is to use the PCQM model to calculate the neutron EDM and electric dipole form factor (EDFF) induced by the standard CP-odd QCD $\theta$-term as well as by generic CP-violating effective interactions arising from physics beyond the SM. The latter correspond to effective quark-photon and quark-gluon interactions induced by intrinsic electric and chromoelectric dipole moments of the constituent quarks and the Weinberg gluon operator at a more fundamental level. In the present paper we do not discuss concrete mechanisms for the generation of these CP-odd interactions, but concentrate on the hadronic structure aspects of the derivation of the neutron EDM in terms of these generic CP-violating parameters at low energies.

The problem of the neutron EDM has been studied within different theoretical approaches: current algebra and chiral perturbation theory [14, 16, 17, 18], chiral quark models [19]-[21], lattice QCD [22, 23], QCD sum rules [24, 25], an approach based on solutions of Schwinger-Dyson and Bethe-Salpeter equations [26], etc. We use the PCQM which describes baryons as bound states of three relativistic valence quarks confined in a static potential and supplemented by a cloud of pseudoscalar Goldstone bosons, as required by chiral symmetry. This model has already been successfully applied to the electromagnetic $N-\Delta$ transition, meson-nucleon sigma-terms and other baryon properties [27].

This paper is organized as follows. Sect. 2 contains a brief review of the PCQM. In Sect. 3 we apply the PCQM calculation of the EDFF and the EDM of the neutron induced by the individual EDMs of the quarks and their CP-violating couplings to the chiral fields. In Sections 4 we extract constraints for the corresponding quark EDMs and CEDMs from the existing experimental limits on the neutron EDM and discuss the phenomenological implications of our results.

2. The Perturbative Chiral Quark Model

The basis of the perturbative chiral quark model (PCQM) [27] is an effective chiral Lagrangian describing the valence quarks of baryons as relativistic fermions moving in an external field (static potential)

$$V_{\text{eff}}(r) = S(r) + \gamma^0V(r), \quad \text{with } r = |\vec{x}|,$$

which in the SU(3)-flavor version are supplemented by a cloud of Goldstone bosons ($\pi, K, \eta$). Treating Goldstone fields as small fluctuations around the three-quark core, the linearized effective Lagrangian is written as:

$$\mathcal{L}_{\text{eff}}(x) = \bar{q}(x)\gamma^5\Phi(x)F S(r)q(x) + \sum_{i=1}^{8} \frac{1}{2} \left[\partial_{\mu}\Phi_i(x)\right]^2 - \bar{q}(x)i\gamma^5\hat{\Phi}(x)S(r)q(x),$$

where we explicitly defined the linearized coupling of Goldstone bosons to quarks as:

$$\mathcal{L}^{\text{str}(1)}(x) = -\bar{q}(x)i\gamma^5\hat{\Phi}(x)F S(r)q(x).$$
The additional term $\mathcal{L}_{\chi_{SB}}$ in Eq. (3) contains the mass contributions both for quarks and mesons, which explicitly break chiral symmetry:

$$L_{\chi_{SB}}(x) = -\bar{q}(x)FMq(x) - \frac{B}{2}Tr[\hat{\Phi}^2(x)M].$$

(5)

Here, $\hat{\Phi} = \sum_{i=1}^{8} \Phi_i \lambda_i$ is the octet matrix of pseudoscalar mesons, $F = 88$ MeV is the pion decay constant in the chiral limit, $M = \text{diag}\{\hat{m}, \hat{m}, m_s\}$ is the mass matrix of current quarks (we restrict to the isospin symmetry limit $m_u = m_d = \hat{m}$) and $B = -<0|\bar{u}u|0>/F^2$ is the quark condensate constant. We rely on the standard picture of chiral symmetry breaking and for the masses of pseudoscalar mesons we use the leading term in their chiral expansion (i.e. linear in the current quark mass):

$$M^2_\pi = 2\hat{m}B, \quad M^2_K = (\hat{m} + m_s)B, \quad M^2_\eta = \frac{2}{3}(\hat{m} + 2m_s)B.$$  

(6)

In our analysis we use the following set of parameters:

$$\hat{m} = 7 \text{ MeV}, \quad m_s = 25\hat{m}, \quad B = M^2_\pi/2\hat{m} = 1.4 \text{ GeV}.$$  

The meson masses satisfy the Gell-Mann-Oakes-Renner and Gell-Mann-Okubo relations. In addition, the linearized effective Lagrangian fulfills PCAC. The properties of baryons, which are modeled as bound states of valence quarks surrounded by a meson cloud, are then derived using perturbation theory. At zeroth order, the unperturbed Lagrangian simply describes a nucleon as three relativistic valence quarks which are confined by an effective one-body static potential $V_{\text{eff}}(r)$ in the Dirac equation. We denote the unperturbed three-quark ground-state as $|\phi_0\rangle$, with the normalization $\langle \phi_0 | \phi_0 \rangle = 1$. We expand the quark field $q$ in the basis of eigenstates generated by this potential as

$$q(x) = \sum_{\alpha} b_{\alpha} u_{\alpha}(\vec{x}) \exp(-iE_{\alpha}t)$$

(7)

where the quark wave functions $\{u_{\alpha}\}$ in orbits $\alpha$ are the solutions of the Dirac equation including the potential $V_{\text{eff}}(r)$. The expansion coefficients $b_{\alpha}$ are the corresponding single quark annihilation operators. All calculations are performed at an order of accuracy $O(1/F^2, \hat{m}, m_s)$. In the calculation of matrix elements, we project the quark diagrams on the respective baryon states. The baryon states are conventionally set up by the product of SU(6) spin-flavor and SU(3)$_c$ color wave functions, where the nonrelativistic single quark spin wave function is replaced by the relativistic solution $u_{\alpha}(\vec{x})$ of the Dirac equation.

In our description of baryons we use the effective potential of Eq. (2), which is given by a sum of a scalar potential $S(r)$ providing confinement and the time component of a vector potential $\gamma^0V(r)$. Obviously, other possible Lorenz structures (e.g., pseudoscalar or axial) are excluded by symmetry principles. It is known from lattice simulations that a scalar potential should be a linearly rising one and the vector potential is thought to be responsible for short-range fluctuations of the gluon field configurations [28]. In our study we approximate $V_{\text{eff}}(r)$ by a relativistic harmonic oscillator potential with a quadratic radial dependence [27]

$$S(r) = M_1 + c_1r^2, \quad V(r) = M_2 + c_2r^2.$$  

(8)
The model potential defines unperturbed wave functions for the quarks, which are subsequently used to calculate baryon properties. This potential has no direct connection to the underlying physical picture and is thought to serve as an approximation of a realistic potential. Notice that this type of potential was extensively used in chiral potential models [29]-[31]. A positive feature of this potential is that most of the calculations can be done analytically. As was shown in Refs. [29]-[31] and later on also checked in the PCQM [27], this effective potential gives a reasonable description of baryon properties and can be treated as a phenomenological approximation of the more fundamental forces dictated by QCD.

The use of a variational Gaussian ansatz for the effective potential (8) gives the following solution for the ground state (for the excited quark states we proceed by analogy):

\[ u_0(\vec{x}) = N \exp\left(-\frac{\vec{x}^2}{2R^2}\right) \left(1 + i\rho \frac{\vec{x}}{R}\right) \chi_s \chi_f \chi_c, \]  

where \( N = \left(\frac{\pi^{3/2}R^3(1 + 3\rho^2/2)}{2}\right)^{-1/2} \) is a normalization constant, and \( \chi_s, \chi_f, \chi_c \) are the spin, flavor and color quark wave functions, respectively. The parameter \( \rho \), setting the strength of the ”small component”, can be related to the axial charge \( g_A \) of the nucleon. In the leading order (3-quark-core) approximation, this relation is [27]

\[ g_A = \frac{5}{3}\left(1 - \frac{2\rho^2}{1 + \frac{3}{2}\rho^2}\right). \]  

The parameters of the effective potential \( V_{\text{eff}} \) can also be expressed in terms of \( \rho \) and \( R \):

\[ M_1 = \frac{1 - 3\rho^2}{2\rho R}, \quad M_2 = \mathcal{E}_0 - \frac{1 + 3\rho^2}{2\rho R}, \quad c_1 = c_2 = \frac{\rho}{2R^3}. \]  

Here, \( \mathcal{E}_0 \) is the single-quark ground-state energy. In our calculations we use the value \( g_A=1.25 \). Therefore, we have only one free parameter in the model, \( R \). In our numerical study, \( R \) is varied in the region from 0.55 fm to 0.65 fm, which is set and constrained by nucleon phenomenology [27]. Such a variation of the parameter \( R \) slightly changes the physical quantities up to 5% [27]. In this paper we also test the sensitivity of the neutron EDM to a variation of \( R \).

The expectation value of an operator \( \hat{A} \) is defined as

\[ \langle \hat{A} \rangle = \langle 0 | \sum_{n=1}^{\infty} \frac{i^n}{n!} \int d^4x_1 \ldots \int d^4x_n T[\mathcal{L}_I(x_1) \ldots \mathcal{L}_I(x_n)\hat{A}] | 0 \rangle^B \]  

where \( \mathcal{L}_I \) is the full interaction Lagrangian which may contain both CP-even and CP-odd terms, as discussed below. The superscript “\( B \)” in Eq. (12) indicates that the matrix elements are projected on the respective baryon states and the subscript “\( c \)” refers to contributions from connected graphs only.

For the evaluation of Eq. (12) we apply Wick’s theorem with the appropriate propagators for quarks and mesons. For the quark field we use a vacuum Feynman propagator for a fermion in a binding potential. In the calculation of meson-quark loops
we include only the ground state in the quark propagator, which leads to the following truncated form:

\[ iG_q(x, y) = \langle 0| T\{q(x)\bar{q}(y)\}|0\rangle \rightarrow \theta(x_0 - y_0)u_0(\vec{x})\bar{u}_0(\vec{y})e^{-iE_0(x_0 - y_0)}. \] (13)

Notice that in our previous papers [27] we estimated explicitly the contribution of the low-lying excited quark states in the quark propagator to the physical quantities. Their contribution is about 10 to 15% with respect to the ground state contribution. Therefore, a restriction of the quark propagator to the ground states is a reasonable approximation. For completeness we also calculate the corrections to the neutron EDM due to the inclusion of excited quark states: the first \( p \)-states (\( 1p_{1/2} \) and \( 1p_{3/2} \) in the non-relativistic notation) and the second excited states (\( 1d_{3/2}, 1d_{5/2} \) and \( 2s_{1/2} \)), i.e. we restrict to the low-lying excited states with energies smaller than the typical scale of \( \Lambda = 1 \text{ GeV} \) of low-energy approaches.

For the meson fields we use their free Feynman propagators:

\[ i\Delta_{ij}(x - y) = \langle 0| T\{\Phi_i(x)\Phi_j(y)\}|0\rangle = \delta_{ij} \int \frac{d^4k}{(2\pi)^4 i} \exp\left[-ik(x - y)\right] \frac{1}{M^2 \Phi - k^2 - i\epsilon}. \] (14)

3. The neutron electric dipole Form Factor and Moment in the PCQM

Here we apply the above described PCQM model to the calculation of the electric dipole form factor (EDFF) and moment (EDM) of the neutron. In this model, having the constituent quarks and pseudoscalar mesons as basic degrees of freedom, the effective chiral CP-odd Lagrangian at low energies can be written in the following generic form

\[ \mathcal{L}^{(1)}_{CPV}(x) = -\frac{i}{2} \bar{q}(x) d_q \sigma^{\mu\nu} \gamma_5 q(x) F_{\mu\nu}(x) + i \bar{q}(x) e^{i\gamma_5 \Phi(x)/(2F)} \gamma_5 h_q e^{i\gamma_5 \Phi(x)/(2F)} q(x) \]

\[ = -\frac{i}{2} \bar{q}(x) d_q \sigma^{\mu\nu} \gamma_5 q(x) F_{\mu\nu}(x) + i \bar{q}(x) \gamma_5 h_q q(x) \]

\[ - \frac{1}{2F} \bar{q}(x) \{ h_q, \Phi(x) \} q(x) + O(\Phi^2) \], \hspace{1cm} (15)

where \( F_{\mu\nu} \) is the electromagnetic field strength tensor, the symbol \( \{ , \} \) denotes anticommutator, \( d_q = \text{diag}(d_u, d_d, d_s) \) and \( h_q = \text{diag}(h_u, h_d, h_s) \) are diagonal matrices of the effective EDMs of the constituent quarks and their CP-violating couplings to the chiral fields, respectively. In Eq. (15) we expanded the quark-meson term in powers of \( \Phi/F \) and kept the leading non-trivial term linear in \( \Phi \). It turns out that the first term in this expansion does not contribute to the EDFF at one loop because the diagrams involving this vertex do not contain the required spin-flip structure \( \bar{\sigma}_N \cdot \vec{q} \), the product of the neutron spin operator \( \bar{\sigma}_N \) and of the 3-momentum of the photon \( \vec{q} \). Therefore, in the following we keep only the linear term in \( \Phi \) in Eq. (15).

We assume that the quark-meson Lagrangian (15) originates from a more fundamental level of quark and gluon CP-odd interactions, so that the effective parameters \( d_q \) and \( h_q \) can be related to those more fundamental CP-odd parameters. We discuss these questions in section [11].
Now, we calculate the neutron EDFF, \(D_n(Q^2)\), starting from the Lagrangian (15). The EDFF is defined in the standard way through the neutron matrix element of the electromagnetic current as
\[
\langle n(p')|J^\mu(0)|n(p)\rangle = \bar{u}_n(p') \left[ \gamma^\mu F_n^{(1)}(Q^2) + \frac{i}{2m_N} \sigma^{\mu\nu} q_\nu F_n^{(2)}(Q^2) \right. \\
\left. - \sigma^{\mu\nu} \gamma_5 q_\nu D_n(Q^2) + (\gamma^\mu q^2 - 2m_N q^\mu) \gamma_5 A_n(Q^2) \right] u_n(p),
\]
where, in addition to \(D_n(Q^2)\), \(F_n^{(1)}(Q^2)\) and \(F_n^{(2)}(Q^2)\) are the well-known \(CP\)-even neutron electromagnetic form factors and \(A_n(Q^2)\) is the neutron anapole moment form factor. The neutron EDM, \(d_n\), is defined as the value of the neutron EDFF at zero recoil, namely \(d_n = D_n(0)\).

To guarantee electromagnetic gauge invariance in non-covariant approaches (see discussion in Refs. [32, 27]) we have to work in the Breit frame, which defines the momenta of the transferred photon and the initial and final neutron states respectively as
\[
q = (0, \vec{q}), \quad p = (E, -\vec{q}/2), \quad p' = (E, \vec{q}/2),
\]
where \(E = \sqrt{m_N^2 + \vec{q}^2/4}\) is the nucleon energy, \(m_N\) the nucleon mass, and \(q^2 \equiv -Q^2 = -\vec{q}^2\) the momentum transfer squared. We can now use Eq. (16) in the Breit frame and our model w.f. of Eq. (9) to extract the form factor \(D_n(Q^2)\) as the term in \(\langle J^0(0) \rangle\) proportional to \(\bar{\sigma}_N \cdot \vec{q}\), namely
\[
\langle J^0(0) \rangle = \frac{E}{m_N} \chi_{N_s}^\dagger \left[ E \chi_s \cdot \vec{q} \right] D_n(Q^2) + \ldots .
\]
Here \(\chi_{N_s}\) and \(\chi_{N_s}^\dagger\) are the nucleon spin w.f. in the initial and final state, and
\[
\langle J^0(0) \rangle = \langle n(p')| \sum_{n=1}^{\infty} \frac{i^n}{n!} \int d^4x_1 \ldots \int d^4x_n T[\mathcal{L}_I(x_1) \ldots \mathcal{L}_I(x_n)J^0(0)]|n(p)\rangle_c,
\]
where \(\mathcal{L}_I\) is the full interaction Lagrangian and the subscript “c” refers to connected graphs only. In PCQM, the electromagnetic current operator
\[
J^\mu(x) = J^\mu_\Phi(x) + J^\mu_{CPV}(x)
\]
contains a CP-conserving electromagnetic part, given in terms of the charged pseudoscalar meson fields:
\[
J^\mu_\Phi(x) = e \left[ \pi^-(x) i\partial^\mu \pi^+(x) + K^-(x) i\partial^\mu K^+(x) \right] + \text{h.c.}
\]
and a CP-violating part involving the quarks:
\[
J^\mu_{CPV}(x) = i \partial_\nu \left[ \bar{q}(x) d_q^\nu \sigma^{\mu\nu} \gamma_5 q(x) \right],
\]
which is derived from the Lagrangian (15). The interaction Lagrangian \(\mathcal{L}_I\) in Eq. (18) up to one loop order is the linearized strong interaction Lagrangian between quark and chiral fields, which consists of the CP-conserving interaction given in Eq. (4) and the CP-violating Lagrangian of Eq. (15):
\[
\mathcal{L}_I(x) = -\bar{q}(x) i\gamma^5 \frac{\Phi(x)}{F} S(r) q(x) - \frac{1}{2F} \bar{q}(x) \{ h_q, \Phi(x) \} q(x).
\]
Up to one loop, the neutron EDFF receives contributions from three types of effective diagrams, shown in Fig. 1: the “$h$-terms” (Fig. 1a), which are proportional to the CP-violating couplings $h_q$ of Eq. (22), the “tree-level terms” (Fig. 1b) and the “one-loop corrections” (Fig. 1c), both due to the CP-violating current of Eq. (21) and proportional to the couplings $d_q$. The diagrams in Figs. 1a and 1c encode the chiral corrections to the neutron EDM. The diagram in Fig. 1a has contributions from charged pion and kaon clouds, while the diagram in Fig. 1c contains contributions from $\pi$, $K$ and $\eta$-meson clouds.

The total contribution of Fig. 1a to the neutron EDFF is then composed by charged pion and Kaon clouds as:

$$D_n^{[h]}(Q^2) = \sum_{\Phi=\pi,K} D_n^{[h;\Phi]}(Q^2), \quad D_n^{[h;\Phi]}(Q^2) \equiv d_n^{[h;\Phi]} F^{[h;\Phi]}(Q^2) \quad (23)$$

where the normalization $F^{[h;\Phi]}(0) = 1$ defines $d_n^{[h;\Phi]}$ as the contributions of the charged $\pi$ and $K$ clouds of Fig.1a to the neutron EDM:

$$d_n^{[h;\Phi]} \equiv D_n^{[h;\Phi]}(0) = e c_n^{[h;\Phi]} \frac{g_{\pi NN} \bar{g}_{\Phi NN}}{2m_N} \int \frac{d^3k}{(2\pi)^3} \frac{F_{\pi NN}(\vec{k}^2)^2 F_{\pi NN}(\vec{k}^2)}{w_\Phi(\vec{k}) w_\Phi(\vec{k}) + 2} \quad (24)$$

and $F^{[h;\Phi]}(Q^2)$, the form factor normalized to unity at zero recoil is given by:

$$F^{[h;\Phi]}(Q^2) = \frac{I^{[h;\Phi]}(Q^2)}{I^{[h;\Phi]}(0)}, \quad (25)$$

$$I^{[h;\Phi]}(Q^2) = \frac{m_N}{E} \int \frac{d^3k}{(2\pi)^3} \frac{F_{\pi NN}[(\vec{k} + \vec{q})^2] F_{\pi NN}(\vec{k}^2)}{w_\Phi(\vec{k} + \vec{q}) w_\Phi(\vec{k})} \quad (26)$$

Here $\Phi = \pi$ or $K$, $w_\Phi(\vec{q}) = \sqrt{M_\Phi^2 + \vec{q}^2}$ is the meson energy, and $c_n^{[h;\Phi]}$ is the SU(6) spin-flavor factor, which is $c_n^{[h;\pi]} = 1/2$ for the pion-loop and $c_n^{[h;K]} = 1/5$ for the kaon-loop diagram.

The expression for $d_n^{[h;\Phi]}$ in Eq. (24) is written in terms of the known $\pi NN$ coupling which satisfies the Goldberger-Treiman relation

$$g_{\pi NN} = g_A \frac{m_N}{F}, \quad (27)$$

where $g_A = 1.25$ is the axial nucleon charge and the CP-violating couplings $\bar{g}_{\Phi NN}$, which are:

$$\bar{g}_{\pi NN} = \frac{h_u + h_d}{2F} \gamma, \quad \bar{g}_{K NN} = \frac{h_u + h_s}{2F} \gamma. \quad (28)$$

Here $\gamma$ is the isovector-scalar two-quark condensate in the nucleon, given by:

$$\langle N|\bar{q} \gamma_3 q |N \rangle = \gamma \bar{u}_{N} \gamma_3 u_{N}. \quad (29)$$

This factor $\gamma$ coincides with the so-called relativistic reduction factor [27]. In the PCQM, $\gamma = 5/8$ [27]. Finally, $F_{\pi NN}$ and $\bar{F}_{\pi NN}$ are the normalized CP-conserving and CP-violating form factors that appear in PCQM [27], and which regularize the divergent loop integral:

$$F_{\pi NN}(\vec{k}^2) = \exp(-\vec{k}^2 R^2/4) \left[ 1 + \frac{\vec{k}^2 R^2}{8} \left( 1 - \frac{5}{3g_A} \right) \right], \quad (30)$$
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\[ \bar{d}^{[h;\pi]}_n = e \frac{g_{\pi NN} \bar{g}_{\pi NN}}{4 \pi^2 m_N} \log \frac{m_N}{M_\pi}. \]  

Now, it is worth checking if our result for the pion-cloud contribution to \( d^{[h;\pi]}_n \) in Eq. (24) is consistent with the model-independent prediction derived in Ref. [14] for the leading term in the chiral expansion:

\[ \bar{d}^{[h;\pi]}_n = \frac{g_{\pi NN} \bar{g}_{\pi NN}}{4 \pi^2 m_N} \log \frac{m_N}{M_\pi}. \]  

To this end, we drop the normalized form factors \( F_{\pi NN} \) and \( \bar{F}_{\pi NN} \) in Eq. (24) by substituting \( F_{\pi NN} = \bar{F}_{\pi NN} = 1 \), and analyze this equation using alternatively cutoff or dimensional regularizations. Both methods of regularization give the same result, which also coincide with Eq. (31). Therefore, our approach is consistent with QCD in the local limit, when \( F_{\pi NN} = \bar{F}_{\pi NN} = 1 \). The nontrivial form factors \( F_{\pi NN} \) and \( \bar{F}_{\pi NN} \) provide an ultraviolet convergence for the EDM. Notice that Eq. (31) has been derived in ChPT when the strong CP-violating \( \pi NN \) coupling constant \( \bar{g}_{\pi NN} \) is defined by the \( \theta \)-term. This result is also valid for general CP-violating couplings. The difference is encoded in the redefinition of

\[ \bar{g}_{\pi NN}(\bar{\theta}) = \bar{\theta} \frac{m}{F} \gamma \to \bar{g}_{\pi NN}(h_u, h_d) = \frac{h_u + h_d}{2F} \gamma. \]  

The leading contribution of the kaon-cloud diagram is also proportional to the chiral logarithm but contains a model-dependent coefficient \( c^{[h;K]} = 1/5 \):

\[ \bar{d}^{[h;K]}_n = c^{[h;K]} e \frac{g_{\pi NN} \bar{g}_{\pi NN}}{4 \pi^2 m_N} \log \frac{m_N}{M_K}. \]  

We remark that the coefficient \( c^{[h;K]} \) was calculated previously in Heavy Baryon Chiral Perturbation Theory (HBChPT) [17], where it was expressed through the parameters of the chiral Lagrangian as:

\[ c^{[h;K]} = \frac{D - F}{D + F} \frac{b_F - b_D}{b_F + b_D}. \]  

Here \( D \) and \( F \) are the axial-vector couplings, and \( b_D \) and \( b_F \) are low-energy constants. Using the actual values [17] \( D = 0.80 \) and \( F = 0.46 \) fixed from a fit of semileptonic hyperon decays, and \( b_D = 0.079 \) GeV\(^{-1} \), \( b_F = -0.316 \) GeV\(^{-1} \) determined from the calculation of baryon masses and the \( \pi-N \) sigma-term up to fourth order in the chiral expansion, we deduce the prediction of HBChPT for \( c^{[h;K]} = 0.45 \). This is more than a factor two larger than the prediction of our model.

Now, let us examine the diagram Fig. 1b. In this calculation we use the well-known quark model formula for the projection of the spin-flavor part of the Lagrangian [15] between neutron states:

\[ \langle n \uparrow | \sum_{i=1}^{3} (d_q)_{ii} (\hat{\sigma} \cdot \hat{q}) | n \uparrow \rangle = \frac{4}{3} d_d - \frac{1}{3} d_u, \]  

where the sum is over the constituents, \( d_q \) is a diagonal \( 2 \times 2 \) flavor matrix of the quark EDMs, \( \hat{q} = \hat{q}/|\hat{q}| \) is the unit vector along the momentum \( \hat{q} \), and \( \hat{\sigma} \) are the Pauli matrices in spin space.
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Then the contribution of the diagram in Fig. 1b (a tree-level contribution) to the neutron EDFF is given by
\[
D_n^{[d:\text{tree}]}(Q^2) = \left( \frac{4}{3} d_d - \frac{1}{3} d_u \right) \left( \frac{1}{2} + \frac{3}{10} g_A \right) F^{[d]}(Q^2),
\]
(36)
\[
F^{[d]}(Q^2) = \frac{m_N}{E} \exp(-Q^2 R^2/4) \left[ 1 + \frac{Q^2 R^2}{1 + 3 g_A/5} \right],
\]
where \( F^{[d]}(0) = 1. \)

Finally, we present the loop correction diagram of Fig. 1c. This one has three contributions: from \( \pi, K \) and \( \eta \) loops, which we denote as:
\[
D_n^{[d:\text{loop}]}(Q^2) = D_n^{[d:\pi]}(Q^2) + D_n^{[d:K]}(Q^2) + D_n^{[d:\eta]}(Q^2),
\]
(37)
and where each meson loop contribution is given by:
\[
D_n^{[d:\Phi]}(Q^2) = -c^{[d:\Phi]} \frac{g_{\pi NN}^2}{200 m_N^2} F^{[d]}(Q^2) \int \frac{d^4 k}{(2\pi)^4} \frac{k^2 F_{\pi NN}(k^2)}{w_\Phi^2(k)}, \quad \Phi = \pi, K, \eta.
\]
Here \( c^{[d:\Phi]} \) are SU(6) spin-flavor factors: \( c^{[d:\pi]} = 7d_u + 2d_d, \ c^{[d:K]} = 6d_s \) and \( c^{[d:\eta]} = 4d_u/3 - d_d/3. \)

The total neutron EDFF, up to one loop in the chiral expansion, is then given by the tree level contributions of Fig. 1b and the loop contributions of Figs. 1.a and 1.c:
\[
D_n(Q^2) = D_n^{[\text{tree}]}(Q^2) + D_n^{[\text{loop}]}(Q^2),
\]
\[
D_n^{[\text{tree}]}(Q^2) \equiv D_n^{[d:\text{tree}]}(Q^2),
\]
\[
D_n^{[\text{loop}]}(Q^2) \equiv \sum_{\Phi=\pi,K} D_n^{[h:\Phi]}(Q^2) + \sum_{\Phi=\pi,K,\eta} D_n^{[d:\Phi]}(Q^2),
\]
where
\[
D_n^{[d:\text{tree}]}(Q^2) = a^{[\text{tree}]} \left( \frac{4}{3} d_d - \frac{1}{3} d_u \right) F^{[d]}(Q^2),
\]
\[
D_n^{[d:\pi]}(Q^2) = a^{[\pi]} \left( \frac{7}{9} d_u + \frac{2}{9} d_d \right) F^{[d]}(Q^2),
\]
\[
D_n^{[d:K]}(Q^2) = a^{[K]} d_s F^{[d]}(Q^2),
\]
\[
D_n^{[d:\eta]}(Q^2) = a^{[\eta]} \left( \frac{4}{3} d_u - \frac{1}{3} d_d \right) F^{[d]}(Q^2),
\]
(39)
\[
D_n^{[h:\pi]}(Q^2) = \frac{e}{\Lambda^2} b^{[\pi]} \left( \frac{1}{2} h_u + \frac{1}{2} h_d \right) F^{[h:\pi]}(Q^2),
\]
\[
D_n^{[h:K]}(Q^2) = \frac{e}{\Lambda^2} b^{[K]} \left( \frac{1}{2} h_u + \frac{1}{2} h_s \right) F^{[h:K]}(Q^2),
\]
where in the contributions of the diagram in Fig. 1a, for convenience, we use the dimensional parameter, \( \Lambda = 4\pi F_\pi \simeq 1.2 \text{ GeV} \), the scale of spontaneously broken chiral symmetry, where \( F_\pi = 92.4 \text{ MeV}. \) The corresponding numerical coefficients \( a^{[\text{tree}]} = 7/8, \ a^{[\pi]} = 3 \times 10^{-2}, \ a^{[K]} = 6.3 \times 10^{-3}, \ a^{[\eta]} = 8.5 \times 10^{-4}, \ b^{[\pi]} = 2.92 \) and \( b^{[K]} = 0.10 \) are obtained from the evaluation of Eqs. (24), (36) and (37).
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It is convenient to separate the contributions from the individual quarks as:

\[ D_n(Q^2) = \sum_{q=u,d,s} D_{n,q}(Q^2) = \sum_{q=u,d,s} \left[ \Delta_q^{[d]} d_q F_{q}^{[d]}(Q^2) + \Delta_q^{[h]} \tilde{h}_q F_{q}^{[h]}(Q^2) \right], \] (40)

where we define \( \tilde{h}_q = (e/\Lambda^2_q) h_q \) and

\[ F_{q}^{[h]}(Q^2) = \frac{b^{[\pi]} F_{q}^{[h;\pi]}(Q^2) + b^{[K]} F_{q}^{[h;K]}(Q^2)}{b^{[\pi]} + b^{[K]}}, \]

\[ F_{d}^{[h]}(Q^2) = F_{q}^{[h]}(Q^2), \] (41)

\[ F_{s}^{[h]}(Q^2) = F_{q}^{[h;K]}(Q^2). \]

The coefficients \( \Delta_q^{[d]} \) and \( \Delta_q^{[h]} \) are:

\[ \Delta_u^{[d]} = -\frac{1}{3} a_{[\text{tree}]} - \frac{7}{9} a^{[\pi]} - \frac{4}{3} a^{[n]} = -0.316, \]

\[ \Delta_d^{[d]} = \frac{4}{3} a_{[\text{tree}]} - \frac{2}{9} a^{[\pi]} + \frac{1}{3} a^{[n]} = 1.16, \]

\[ \Delta_s^{[d]} = -a^{[K]} = 0.006, \] (42)

\[ \Delta_u^{[h]} = \frac{1}{2} b^{[\pi]} + \frac{1}{2} b^{[K]} = 1.51, \]

\[ \Delta_d^{[h]} = \frac{1}{2} b^{[\pi]} = 1.46, \]

\[ \Delta_s^{[h]} = \frac{1}{2} b^{[K]} = 0.05. \]

The numerical results for the \( \theta \)-term contribution to the neutron EDFF (including the pion and kaon loops) are shown in Fig. 2. In Fig. 3 we plot the \( Q^2 \)-dependence of the \( d \) quark contributions to the neutron EDFF arising from its individual EDM, \( d_d \), as well as its CP-violating coupling to mesons, \( h_d \), contained in the first and second terms of Eq. (41), respectively. Since the values of \( d_d \) and \( \tilde{h}_q \) and their relative signs are not known, we display these two types of contributions separately, in terms of normalized form factors: \( F_{n.d}^{[d]}(Q^2) = \Delta_d^{[d]} F_d^{[d]}(Q^2) \) for the individual \( d \) quark EDM contribution, and \( F_{n.d}^{[h]}(Q^2) = \Delta_d^{[h]} F_d^{[h]}(Q^2) \) for the \( d \) quark contribution via its CP-violating coupling to mesons.

The neutron EDM, \( d_n \), is then obtained by definition as the neutron EDFF at zero recoil. Using \( F_{q}^{[d]}(0) = 1 \) and \( F_{q}^{[h]}(0) = 1 \), for \( q = u, d, s \), the resulting EDM can also be expressed in terms of the individual quarks EDMs \( d_q \) and CP-violating couplings \( \tilde{h}_q \), as:

\[ d_n = \sum_{q=u,d,s} \left[ \Delta_q^{[d]} d_q + \Delta_q^{[h]} \tilde{h}_q \right], \] (43)

with the coefficients \( \Delta_q^{[d]} \) and \( \Delta_q^{[h]} \) given in Eq. (42).

Another CP-odd parameter which may have important implications for CP-violation in atoms is the electron-neutron Schiff moment \( S' \), related to the neutron EDM form factor \( D_n(Q^2) \) by

\[ S' = - \left[ \frac{dD_n(Q^2)}{dQ^2} \right]_{Q^2=0}. \] (44)
From the expression (45) we obtain:

$$S' = S'[h] + S'[d],$$

where

$$S'[h] = e \sum_{\Phi=\pi,K} c^{[h;\Phi]} \frac{g_{\pi NN} g_{\Phi NN}}{2 m_N} \int \frac{d^3k}{(2\pi)^3} \frac{F_{\pi NN}(k^2) F_{\pi NN}(k^2)}{w_\Phi^3(k)}$$

$$\times \left[ \frac{3}{4 w_\Phi^2(k)} - \frac{5}{6} \frac{k^2}{w_\Phi^4(k)} \right]$$

is the contribution of the diagram in Fig. 1a and

$$S'[d] = \left[ \left( \frac{4}{3} d_d - \frac{1}{3} d_u \right) \left( \frac{1}{2} + \frac{3}{10} g_A \right) - \sum_{\Phi=\pi,K,\eta} c^{[d;\Phi]} \frac{g_{\pi NN}^2}{200 m_N^2} \int \frac{d^3k}{(2\pi)^3} \frac{k^2}{w_\Phi^3(k)} \right]$$

$$\times \left[ \frac{1}{8 m_N^2} + \frac{3}{2} R^2 \frac{g_A}{1 + 3 g_A/5} \right]$$

$$\equiv \sum_{q=u,d,s} \Delta[d]_q \left[ \frac{1}{8 m_N^2} + \frac{3}{2} R^2 \frac{g_A}{1 + 3 g_A/5} \right]$$

is the contribution of the diagrams in Figs. 1b and 1c. Here $F_{\pi NN}$ and $F_{\pi NN}'$ are the first- and second-order derivatives of the $F_{\pi NN}$ form factor with respect to $k^2$.

Again, as for the case of the EDM, we can show that our Schiff moment is consistent with the ChPT results in leading order of the chiral expansion. As in the case of the neutron EDM, we drop the normalized form factors $F_{\pi NN}$ and $F_{\pi NN}'$ in Eq. (45), substituting $F_{\pi NN} = F_{\pi NN}' = 1$ and, therefore, $F_{\pi NN}' = F_{\pi NN}' = 0$. Then we analyse the leading term in the chiral expansion (which is ultraviolet convergent):

$$S' = e \sum_{\Phi=\pi,K} c^{[h;\Phi]} \frac{g_{\pi NN} g_{\Phi NN}}{2 m_N} \int \frac{d^3k}{(2\pi)^3} \frac{1}{w_\Phi^3(k)} \left[ \frac{3}{4} \frac{k^2}{w_\Phi^3(k)} \right].$$

A straightforward calculation of the integral (46) gives

$$S' = e \sum_{\Phi=\pi,K} c^{[h;\Phi]} \frac{g_{\pi NN} g_{\Phi NN}}{48 \pi^2 m_N M_\Phi^2}.$$ (47)

In the case when we restrict Eq. (47) to the contribution of the pion-cloud, our result coincides with the leading order result of ChPT [33, 34]:

$$S' = e \frac{g_{\pi NN} g_{\Phi NN}}{48 \pi^2 m_N M_\Phi^2}.$$ (48)

Let us now present our results for the electron-neutron Schiff moment $S'$ in terms of the partial contributions:

(a) Diagram in Fig. 1a: partial $\pi$ and $K$ meson loop contributions

$$S'[d;\pi] = 16.22 \text{ GeV}^{-2} \times \left[ \frac{1}{2} h_u + \frac{1}{2} h_d \right], \quad S'[h;K] = 0.21 \text{ GeV}^{-2} \times \left[ \frac{1}{2} h_u + \frac{1}{2} h_s \right].$$ (49)

(b) Diagram in Fig. 1b: tree-level result

$$S'[d;\text{tree}] = 8.79 \text{ GeV}^{-2} \times \left[ \frac{4}{3} d_d - \frac{1}{3} d_u \right].$$ (50)
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(c) Diagram in Fig. 1c: partial π, K and η meson loop contributions

$$S'[^{d\pi}_d] = -0.31 \text{ GeV}^{-2} \times \left[ \frac{7}{9}d_u + \frac{2}{9}d_d \right],$$

$$S'[^{d\pi}_K] = -0.06 \text{ GeV}^{-2} \times d_s,$$

$$S'[^{d\pi}_\eta] = -0.01 \text{ GeV}^{-2} \times \left[ \frac{4}{3}d_u - \frac{1}{3}d_d \right].$$

Next we summarize our result for the Schiff moment in terms of the individual quarks EDMs $d_q$ and CP-violating couplings $\tilde{h}_q$, as:

$$S' = \sum_{q=u,d,s} [\beta[^{d\pi}_d] \Delta[^{d\pi}_q] d_q + \beta[^{h\pi}_q] \Delta[^{h\pi}_q] \tilde{h}_q],$$

with the coefficients $\Delta[^{d\pi}_d]$ and $\Delta[^{h\pi}_q]$ given in Eq. (42). Here, for convenience we introduced the $\beta$-factors: $\beta[^{d\pi}_d] = -F[^{d\pi}_d]'(0)$ and $\beta[^{h\pi}_q] = -F[^{h\pi}_q]'(0)$. Their numerical values are:

$$\beta[^{d\pi}_d] = 10.06 \text{ GeV}^{-2}, \quad \beta[^{h\pi}_u] = 5.44 \text{ GeV}^{-2}, \quad \beta[^{h\pi}_d] = 5.55 \text{ GeV}^{-2}, \quad \beta[^{h\pi}_s] = 2.10 \text{ GeV}^{-2}.$$

4. Limits on the quark EDMs and their phenomenological implications

Here, using Eq. (43) we extract the limits on the quark EDMs, $d_q$, and CP-violating couplings $h_q$, from the existing experimental upper limit on the neutron EDM, shown in Eq. (1). Assuming the absence of accidental cancellations between the different terms in Eq. (43) we obtained the corresponding limits shown in the Table 1.

In order to discuss possible phenomenological implications of these limits and compare them with the results of other approaches, it would be desirable to express the individual quark EDMs in terms of more fundamental CP violating parameters specifying the origin of CP violation.

The effective CP-odd Lagrangian in terms of the quark and gluon fields up to operators of dimension six has the following standard form [35-39]:

$$L_{\text{eff}}^C(x) = \frac{\bar{q}}{16\pi^2} \text{tr}(\bar{G}_{\mu\nu}G^{\mu\nu}) - \frac{i}{2} \bar{q} d_q^E \sigma^{\mu\nu} \gamma_5 F_{\mu\nu} q - \frac{i}{2} \bar{q} d_q^C \sigma^{\mu\nu} \gamma_5 G^a_{\mu\nu} T^a q - \frac{1}{6} C_W f^{abc} G^a_{\mu\nu} G^b_{\rho\sigma} G^c_{\sigma\rho} \varepsilon^{\mu\nu\rho\sigma},$$

where $G^a_{\mu\nu}$ is the gluon stress tensor, $\bar{G}_{\mu\nu} = \frac{1}{2}\epsilon_{\mu\nu\rho\sigma}G^{\rho\sigma}$ is its dual tensor, and $T^a$ and $f^{abc}$ are the SU(3) generators and structure constants, respectively. In this equation the first term represents the SM QCD $\theta$-term, while the last three terms are the non-renormalizable effective operators induced by physics beyond the SM. The second and third terms are the dimension-five electric and chromoelectric dipole quark operators, respectively, and the last term is the dimension-six Weinberg operator [35]. The quark EDMs, $d_q^E = \text{diag}(d_u^E, d_d^E, d_s^E)$, and quark chromoelectric dipole moments (CEDM), $d_q^C = \text{diag}(d_u^C, d_d^C, d_s^C)$, form diagonal matrices in flavor space. The operators of the above Lagrangian can be induced by physics beyond the SM after integrating out the heavy degrees of freedom.
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The contribution of the QCD $\theta$-term can be related to the parameters of the Lagrangian \((15)\) on the basis of the following standard reasoning. A chiral $U(1)$ transformation in flavor space allows one to remove the gluonic $\theta$-term from the Lagrangian \((54)\) and pass it as a CP-violating complex phase to the quark mass operators. For a small value of $\theta$, the CP-violating term becomes (for details see Refs. \([13, 14, 40]\)):

\[
L^{\text{str}(0)}_{\text{CP V}} = i\bar{\theta}\bar{m}\sum_{q=u,d,s}\bar{q}(x)\gamma_5 q(x),
\]

where $q(x)$ denotes a triplet in flavor space, and so this term is a flavor-$SU(3)$ singlet. The mass coefficient is:

\[
\bar{m} = \frac{m_u m_d m_s}{m_u m_d + m_u m_s + m_d m_s},
\]

which would vanish if any of the quarks were massless. From this term one can construct an effective chiral Lagrangian by introducing the chiral field $e^{i\gamma_5 \hat{\Phi}/F}$ and expanding it in powers of $\hat{\Phi}/F$:

\[
L^{\text{str}}_{\text{CP V}} = i\bar{\theta}\bar{m}\gamma_5 \exp\left(i\frac{\hat{\Phi}}{F}\right) q = i\bar{\theta}\bar{m}\bar{q}\gamma_5 q - \bar{\theta}\bar{m}\bar{q}\frac{\hat{\Phi}}{F}q + O(\hat{\Phi}^2).
\]

For the reasons we gave after Eq. \((15)\), the leading contribution to the EDFF comes from the term linear in $\Phi$ in the expansion of Eq. \((57)\). Comparing the Lagrangians in Eqs. \((15)\) and \((57)\) we obtain the contribution of the $\theta$-term to the parameter $h_q$ which is, naturally, flavor-blind:

\[
h_q = h_q^{\text{BSM}} + \bar{\theta}\bar{m}I_{3\times3},
\]

where $I_{3\times3}$ is the unit matrix in flavor space. Here we have singled out the part $h_q^{\text{BSM}}$ which does not contain the $\theta$ contribution but is due to the other terms in the Lagrangian \((54)\).

In order to relate the parameters $d_q$ and $h_q^{\text{BSM}}$ of the Lagrangian \((15)\) with the parameters $d_q^E$, $d_q^C$ and $C_W$ of the Lagrangian \((54)\) we apply a simplified approach relying on the so-called na"ive dimensional analysis \([35, 41, 42]\). In this way one obtains the following order-of-magnitude relations:

\[
d_q = d_q^E + \frac{e}{4\pi} d_q^C + \frac{e\Lambda}{4\pi} C_W,
\]

\[
h_q^{BSM} = \frac{\Lambda^2}{4\pi} d_q^C + \frac{\Lambda^3}{4\pi} C_W.
\]

We assume all the operators in Eq. \((54)\) to be normalized at an energy scale of about 1 GeV, where the perturbative quark-gluon picture is still reasonable and, on the other hand, occurs an overlap with the chiral symmetry breaking scale $\Lambda_{\chi} = 4\pi F_\pi \simeq 1.2$ GeV, which corresponds to the normalization scale of the Lagrangian \((15)\). Thus, all the parameters in Eqs. \((52)\) and \((50)\) are at the same scale $\Lambda_{\chi} \simeq 1.2$ GeV. In such a case, it is also natural to identify the parameter $\Lambda$ with $\Lambda_{\chi}$. In a specific model beyond-SM, the parameters of the effective Lagrangian \((54)\) could be calculable in terms of the model
parameters at some high energy scale. These parameters then must be QCD-evolved down to the hadronic scale $\Lambda_\chi$ corresponding to the scale of the neutron EDM. The renormalization-group relations of the parameters $d_q^E$, $d_q^C$, $C_W$ in Eqs. (59) and (60) with their values calculated at a high energy scale can be found in Refs. [36, 43].

Now, using Eqs. (58), (59), (60) in Eq. (43), we can express the neutron EDM in terms of the more “fundamental” parameters $\bar{\theta}$, $d_q^E$, $d_q^C$, and $C_W$ of the Lagrangian (54) as

$$d_n = \sum_{q=u,d,s} (z_q^E d_q^E + z_q^C d_q^C) + z_W \Lambda_\chi C_W + (\bar{\theta} z_\theta \times 10^{-16} \, e \cdot cm). \quad (61)$$

The numerical values of the $z$-coefficients are given in Table 2, where we also present for comparison the values of these coefficients derived in some other approaches: QCD sum rules [25, 44], parton quark model [45], SU(6) model [46], MIT bag model [13], current algebra [14], effective chiral approach [16], HBChPT [17], chiral bag model [19], cloudy bag model [20], chiral quark-meson model [21] and extensions of SM [47]-[50].

Notice that our results for the coefficient $z_\theta$ is dominated by the pion-loop diagram. The kaon-cloud contribution is smaller that the pion contribution by a factor $\sim 1/28$:

$$z_\theta = z_\theta^\pi + z_\theta^K = 1.42,$$

$$z_\theta^\pi = 1.37, \quad z_\theta^K = 0.05. \quad (62)$$

Using Eqs. (58), (59), (60) in Eq. (52), we can write for the electron-neutron Schiff moment the following expression:

$$S' = \sum_{q=u,d,s} (s_q^E d_q^E + s_q^C d_q^C) + s_W \Lambda_\chi C_W + (\bar{\theta} s_\theta \times 10^{-16} \, e \cdot cm). \quad (63)$$

where numerical values of the $s$-coefficients are

$$s_u^E = -4.86 \, \text{GeV}^{-2}, \quad s_d^E = 17.85 \, \text{GeV}^{-2}, \quad s_s^E = -0.09 \, \text{GeV}^{-2},$$

$$s_u^C = 0.41 \, \text{GeV}^{-2}, \quad s_d^C = 1.62 \, \text{GeV}^{-2}, \quad s_s^C = 0.004 \, \text{GeV}^{-2},$$

$$s_W = 2.04 \, \text{GeV}^{-2}, \quad s_\theta = 7.72, \quad (64)$$

and where the separate contributions of pion and kaon meson-loop diagram in Fig. 1a to the coupling $s_\theta$ are $s_\theta^\pi = 7.62$ and $s_\theta^K = 0.1$.

Now, using Eq. (61), we can extract upper limits for the parameters $\bar{\theta}$, $d_q$, $\bar{h}_q$, $d_q^E$, $d_q^C$ and $C_W$ from the current experimental bound on neutron EDM in Eq. (1). These limits are shown in the Table 1. In their derivation we assumed the dominance of each term at a time, thus assuming the absence of significant cancellations between the different terms contributing to the neutron EDM.

We note that, in comparison with the conventional valence quark model which predicts for the neutron EDM

$$d_n = \frac{4}{3} d_d - \frac{1}{3} d_u, \quad (65)$$

the PCQM predicts a small but non-negligible contribution from the strange quark EDM, which appears due to the K-meson loop diagram in Figs. 1a and 1c. In Table 2
we show the predictions for the neutron EDM of some other approaches. With the values of the corresponding $z$-coefficients it is straightforward to derive upper limits on the CP-violating parameters of the Lagrangian within the cited approaches. A general summary is that our limits are less stringent. Notice that all the approaches cited in Table 2, including the PCQM, were successful in describing various properties of baryons, and yet they disagree in the CP-violating sector, as they predict different values of the neutron EDM. This a manifestation of a strong hadronic model dependence in the estimates of the neutron EDM. In view of that, the existing EDM limits on fundamental CP violating parameters could only be treated as a hint for model building rather than as stringent limits.

5. Summary and Conclusions

In this work we applied the perturbative chiral quark model to the calculation of the electric dipole form factor and electric dipole moment of the neutron. We parameterized a generic effect of CP violation in terms of the effective quark electric dipole moments and strong CP-violating quark couplings to the chiral fields, which could have different sources at some more fundamental level. We considered the relations between these effective parameters of the quark-meson Lagrangian with more fundamental parameters of the CP-odd quark-gluon Lagrangian. The latter includes the SM $\theta$-term and the operators which originate from physics beyond the SM, such as the chromoelectric and electric quark dipole moment operators, as well as the Weinberg term. From the existing limits on the neutron EDM we extracted the limits on the individual effective quark EDMs and on the aforementioned more fundamental CP violating parameters, adopting the approach based on the so called naïve dimensional analysis. A general conclusion we made from the comparison of these results with other approaches is that our limits are significantly less stringent. This could be treated as a manifestation of a strong dependence of the predictions for the neutron EDM on the details of the hadronic structure models.

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Table 1. Upper limits on the CP-violating parameters of the Lagrangians in Eqs. (15), (21) and CPVCs \( \tilde{h}_q \): the high-energy constants \( d_q^E \), \( d_q^C \) and from the current experimental bound on the neutron EDM in Eq. (1). All numbers are given in units of \( 10^{-25} \, e \cdot cm \).

| Parameter | \( u \) | \( d \) | \( s \) |
|-----------|-------|-------|-------|
| \( |d_q| \) | 2.0 | 0.5 | 105.0 |
| \( |\tilde{h}_q| \) | 0.4 | 0.4 | 12.6 |
| \( |d_q^E| \) | 1.3 | 0.4 | 68.6 |
| \( |d_q^C| \) | 6.4 | 2.9 | 174.7 |
| \( |\theta| \) | | 0.4 | |
| \( |C_W \Lambda_\chi| \) | | 2.0 | |

Table 2. Theoretical estimates of \( z \)-coefficients in Eq. (61) in various approaches.

| Parameter | Approaches | This work |
|-----------|------------|-----------|
| \( z_u^E \) | - (0.35 ± 0.15) \([44] \); -0.78 \([45] \); -1/3 \([46] \) | -0.48 |
| \( z_d^E \) | (1.4 ± 0.6) \([44] \); 1.14 \([45] \); 4/3 \([46] \) | 1.77 |
| \( z_s^E \) | -0.35 \([45] \) | -0.01 |
| \( z_u^C \) | 0.48 \([18] \); (0.165 ± 0.075) \([44] \); 0.05 \([47] \); 4/9 e ≃ 0.135 \([46] \) | 0.10 |
| \( z_d^C \) | 0.39 \([18] \); (0.33 ± 0.15) \([44] \); 0.05 \([47] \); 8/9 e ≃ 0.270 \([46] \) | 0.21 |
| \( z_s^C \) | 0.08 \([18] \); 0.05 \([47] \); 0.03 \([48] \); 0.008 \([49] \) | 0.004 |
| \( z_W \) | ∼ 0.085 \([35] \); ∼ 0.017 \([44] \); ∼ 0.01 \([50] \) | 0.094 |
| \( z_\theta \) | 2.7 \([13] \); 3.6 \([14] \); 3.3 \([16] \); 6.7 \([17] \); 3.0 \([19] \); 1.4 \([20] \); 1.17 \([21] \); 2.4 \([25] \) | 1.42 |

Figure 1. The contributions to the neutron EDM: “\( h \)-term” diagram (a), “tree-level \( d \)-term” diagram (b) and “one-loop \( d \)-term” diagram (c). The black filled squares correspond to the CP-violating vertices of the Lagrangian \([13] \) or the current \([21] \).
The neutron electric dipole form factor in the perturbative chiral quark model

**Figure 2.** The contributions from pion and kaon loops to the neutron EDM form factor induced by strong CP-violating $\theta$-term, where $D_n^{[\theta]}(Q^2) \equiv D_n^{[h]}(Q^2)$ at the limit $\mathbf{h}_q \equiv \theta \hat{m}_I \mathbf{I}_{3 \times 3}$.

**Figure 3.** The $d$-quark contribution to the neutron EDM form factor via its intrinsic EDM (solid line) and CP-odd coupling to mesons (dashed line). The plotted form factors are: $F_{n,d}^{[d]}(Q^2) = \Delta_d^{[d]} F_d^{[d]}(Q^2)$ (solid line) and $F_{n,d}^{[h]}(Q^2) = \Delta_d^{[h]} F_d^{[h]}(Q^2)$ (dashed line).