Manipulation of Gaussian derivative pulses and vector solitons in an anomalous-dispersion fiber laser

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Both positive- and negative-polarity Gaussian monocycle and doublet pulses, for which the pulse shapes are the first and second derivatives of Gaussian functions respectively, are generated in a ring-cavity erbium-doped fiber laser from polarization-locked vector solitons by using passive optical technology. The pulse states are found to be the superposition of bright and dark solitons with different widths, amplitudes, time delays, polarizations, and wavelengths. Qualitative analysis of the properties of vector solitons are performed by solving coupled complex Ginzburg-Landau equations. By theoretical representing the envelopes of bright soliton by sech and dark soliton by tanh functions, the incoherent superposition of these two components have simulated the experimental observations, and the underlying mechanisms on the formation of monocycle and doublet pulses are attributed to the polarization locking of bright and dark solitons. The results of tunable vector soliton generation are compared with atomic solitons in the system of Bose-Einstein condensation. Since the soliton generation of fiber lasers and Bose-Einstein condensation have many properties in common and thus the simulation and propagation of pulsating waves may open a new route to explore the classical solitary dynamics in nonlinear optics and its quantum analogy in ultracold fields.

I. INTRODUCTION

Fiber lasers have been found many applications in laser science, optical communication, frequency metrology, and biomedical technology, etc. Many rare-earth-doped fibers have been used as the active media of fiber lasers. Among these active media, ytterbium-doped fibers (YDFs) and erbium-doped fibers (EDFs) have attracted much attention for either continuous-wave (CW) or pulsed fiber lasers [1–6]. Semiconductor optical amplifiers (SOA) have also been utilized as the gain media in fiber lasers to provide additional lasing wavelengths or functionalities [7, 8]. Various fiber lasers with different wavelengths and properties have become practical and useful test beds for laser physics and nonlinear dynamics.

The interests to study nonlinear dynamics of solitary waves may date back to 1960s since Zabusky and Kruskal studied the numerical solutions of the Korteveg-de Vries equation [9]. Subsequently, more soliton solutions were found in the systems of different fields described by, for example, the Burger equation [10], the sine-Gordon equation [11], and the nonlinear Schrödinger equation. The common feature of these equations is that they are integrable, which makes them to be analytically solved. The well-known methodology in solving the nonlinear equations relies on the basis of the change of independent variables or functions. The calculations, however, may also deliver the shock wave or kink solutions. In contrast to the extended-wave solutions, an ideal soliton or solitary wave will show the characteristic of the nonlinear eigenmodes, which connects the propagation speed of the solitary wave with its amplitude. A solitary wave also shows the characteristic of neither dispersive nor dissipative propagation in the medium. The strictness establishes the physical correspondence showing that, at any time, as the matter-wave soliton passing through an interface between two materials, the mass and the linear momentum of the flow have to be conserved, whereas it is energy that has to be conserved in the scenario to an optical soliton. To date, researches on families of solitons have flourished on a large scale over fluid mechanics, atomic physics [12–15], optics, and also biology. Regarded as a spatially or temporally coherent structure, the soliton solutions can exist provided that nonlinearity can be counterbalanced by the dispersive property of the medium.

Depending on the sign of fiber dispersion, either bright or dark solitons can be generated in single-mode fibers (SMFs), which are governed by the nonlinear Shrödinger equation (NLSE) [16]. It was theoretically shown that the bright solitons can be formed in anomalous-dispersion fibers, while the dark solitons can be formed in normal-dispersion fibers. In birefringent fibers, a new type of optical solitons involving two orthogonal polarization components were found, which are termed as vector solitons [17]. For fiber lasers, the propagation of intracavity light field should be described by the complex Ginzburg-Laudau Equation (CGLE) [1]. Owing to the strong optical intensity and large interaction length, several soliton families have been observed in fiber lasers. As compared with birefringent optical fibers, the formation of vector solitons in a fiber laser cavity is not only decided by the fiber birefringence, group velocity dispersion, and nonlinear Kerr effect, but also by the gain, loss, nonlinear absorption, optical filtering, and the cavity boundary condition. When operated in pulse states, most of the lasers produce bright pulses. However, it was shown that dark pulses or bright-dark pulses could be formed in fiber lasers [18–21]. Shao et al. reported the formation of induced dark solitary pulses in a net anomalous dispersion cavity fiber laser, where the induced solitons were formed by the cross-phase modulation (XPM) between the two orthogonal polarization components of the fiber birefringence in the laser cavity [22]. Zhang et al. observe a new type of vector dark soliton in a fiber ring laser, which consists of stable localized structures (polarization domain walls) separating the two orthogonal linear polarization eigenstates of the laser
A series of vector solitons have been observed in weakly birefringent fiber laser cavities [19]. When polarization maintaining fibers are used in the cavity, a fiber laser is considered to be highly birefringent. Different from a weakly birefringent cavity, the group velocity mismatch between the two orthogonal polarization components cannot be ignored in a highly birefringent fiber laser [20].

The observations and studies on dark and bright-dark solitons have enriched the research fields of nonlinear optics and fiber lasers. Milián et al. present experimental and numerical data on the supercontinuum generation in an optical fiber pumped in the normal dispersion range where the seeded dark and the spontaneously generated bright solitons contribute to the spectral broadening [24]. Vector solitons can also be used in the alignment and rotation of individual plasmonic nanoparticles [24], vectorial control of matter magnetization [25], and polarization division multiplexing in optical communication [26]. Due to the potential applications in microwave photonics and pulse shaping, many optical methods have been proposed in past few years to generate Gaussian monocycle or doublet pulses, which have the pulse shapes of the first and second derivatives of Gaussian functions respectively [27]. However, most of these methods incorporated active technologies and multiple light sources, which are inevitably complicated and very expensive. The generation and manipulation of bright and dark solitons in fiber lasers may provide an economic solution to the optical pulse synthesizing.

In previous studies, we have achieved passive optical pulse generation in fiber lasers using a semiconductor optical amplifier as the gain medium [28]. Various waveforms, including square wave, staircase wave, triangular wave, bright pulse, and dark pulse have been observed. With proper cavity tuning, it is likely to obtain more kinds of waveforms in fiber lasers. In this paper, we first demonstrate the generation of Gaussian monocycle and doublet pulses in a ring-cavity erbium-doped fiber laser (EDFL) using passive optical technology. By adjusting intracavity polarization controllers, the fiber laser can be switched among continuous-wave, monochromatic, and doublet states. The polarization of laser output is measured to study the origin of pulse formation. As only very few NLSEs can be solved analytically [29], the pioneering work of Zakharov and Shabat [30] stimulated great enthusiasm in finding solutions of integrable NLSEs. Recently, Agalarov et. al. successfully modified the coupled NLSEs to Manakov and Makhankov U(n,m)-vector models and found vector solitons with unconventional dynamics for the passive fiber considering four-wave mixing terms [31]. However, in the low birefringent fiber with gain, loss, and optical filtering, the soliton dynamics can no longer be traced by solving NLSE with inverse scattering transform method. Alternatively, in this work, the interactions of system parameters among group velocity dispersion (GVD), self-phase modulation (SPM), XPM, gain, loss, and optical filtering, and their combined effects on the shaping of the bright and dark solitons are further qualitatively analyzed by solving coupled CGLEs. While the systems of fiber lasers and Bose-Einstein condensation (BEC) have many things in common, the results of tunable vector soliton generation are compared with atomic solitons in the system of BEC in the final part of this work. As the domain wall solitons can be create in the two-component condensate with all-repulsive nonlinear interactions [32,33], we find that the optical domain wall between orthogonal polarizations can be manipulated by the adjustment of intracavity polarization in the fiber laser.

The paper is organized as follows. In section II, we introduce the experimental setup. In section III, we discuss the details for the generation and manipulation of fiber solitons. Then, we construct the theoretical model to configure the formation of vector solitons. We also compare the optical solitons with atomic solitons and decisively determine the mechanism in the formation of vector solitons in the anomalous-dispersion fibers. Finally, in section IV we give a brief conclusion of the results.

II. EXPERIMENTAL SETUP

The schematic setup of ring-cavity EDFL for Gaussian monocycle- and doublet-pulse generation is shown in Fig. 1. The cavity consists of a 980/1550 WDM coupler, a 90-cm erbium-doped fiber (LIEKKI Er80-4/125), two fiber polarization controllers, an output coupler, and a fiber isolator. The erbium-doped fiber is pumped by a laser diode (LD) with center wavelength of 974 nm. The EDFL output signals are detected and characterized by a high-speed InGaAs detector, a digital oscilloscope, a sampling oscilloscope with O/E module, an optical power meter, an RF spectrum analyzer, and an optical spectrum analyzer. The polarization states of output laser pulses are analyzed by using a polarization controller and a polarization beam splitter.

III. RESULTS AND DISCUSSION

A. Generation and manipulation of fiber solitons

For the ring-cavity EDFL of Fig. 1, the LD current is set to 210 mA. After properly tuning of the intracavity polariza-
tion controllers (PCs), we obtain the monocycle pulses of dark-bright structure shown in Fig. 2(a), which has a repetition rate of 13.2 MHz and average output power of 6.5 mW. By further adjustments of the PCs, the relative delay and widths of the peak and valley can be tuned to obtain the doublet pulses of Fig. 2(c). The pulse repetition rate of doublets is the same as the monocycle pulses, but the output power is slightly decreased to 5.8 mW. Another feature of different central hump intensities for two pulses can also be seen from the optical spectra of Figs. 2(b) and 2(d). For this ring-cavity EDFL, monocycle and doublet pulses of reversed shape (i.e., bright-dark for monocycle pulses and valley-centered for doublet pulses) have also been obtained by tuning the polarization controllers.

Figs. 3(a) and 3(b) show respectively the curve-fittings of monocycle and doublet pulses. The monocycle pulse can be fitted by the first derivative of Gaussian function as

$$y = y_0 + \left( \frac{A}{w^2} \right) (x - x_c) \exp \left[ -\frac{(x - x_c)^2}{2w^2} \right],$$  \hspace{1cm} (1)

where $y_0 = 2.14641 \times 10^{-4}$, $A = 5.98484 \times 10^{-4}$, $w = 4.17598$ ns, and $x_c = 54.75168$ ns. The doublet pulse can be fitted by the second derivative of Gaussian function as

$$y = y_0 + \left( \frac{A}{w^2} \right) \left[ 1 - \frac{(x - x_c)^2}{w^2} \right] \exp \left[ -\frac{(x - x_c)^2}{2w^2} \right],$$  \hspace{1cm} (2)

where $y_0 = 2.07505 \times 10^{-4}$, $A = 0.00184 \times 10^{-4}$, $w = 2.69183$ ns, and $x_c = 48.62335$ ns.

After analyzing the polarization states, we found that both the monocycle and doublet pulses are composed of two orthogonal polarization components. As shown in Fig. 4(a), the monocycle pulses are synthesized by bright (black curve) and dark solitons (red curve), which have the characteristics of polarization domain wall solitons as in [18]. Owing to the incoherent superposition of the two polarization components with fixed time delay and different structures, the bright and dark solitons are locked to form the anti-symmetric monocycle pulse of Fig. 2(a). The optical spectra of these two constituent polarization components are shown in Fig. 4(b), where they are found to have different spectral distributions.

Fig. 4(c) shows the two polarization components of doublet pulses that are also synthesized by bright (black curve) and dark (red curve) solitons. We observed that the the extremities of bright and dark components are almost coincident, and they become miscible without apparent domain walls. For doublets, the dark solitons have two spectral peaks, while the bright pulses have larger intensity toward longer wavelengths. Therefore, the underlying mechanism in the formation of monocycle and doublet pulses is similar, i.e., the polarization-locking of bright and dark solitons with different temporal widths, amplitudes, and time delays. When the centers of bright and dark pulses coincide, Gaussian doublet pulses are generated. On the other hand, when the centers of bright and dark pulses are detuned, Gaussian monocycle pulses will be formed.
For this ring-cavity EDFA, Gaussian monocycle and doublet pulses with different polarities can also be generated. At LD pump current of 250 mA, we properly adjusting the intracavity polarization controllers and have obtained the monocycle pulse of positive polarity shown in Fig. 5(a), which has a reduced average output power of 4.3 mW. The variation of output power can be attributed to the weak polarization-dependent loss induced by the bent intracavity fibers [34]. The experimental data (black curve) can be fitted by the first derivative of Gaussian function plus a constant term (red curve). The monocycle pulses are found to be composed of bright and dark pulses, and the pulse train is shown in the inset of Fig. 5(a). The separation between the peak and valley in the monocycle pulse is 9.3 ns. We find that the positions, widths, and amplitudes of the bright and dark components can be tuned by adjusting the polarization controllers, and their polarization states are orthogonal. Fig. 5(b) is the optical spectrum of the bright-dark pulses, which has center wavelength of 1567.6 nm and 3-dB bandwidth of 0.2 nm. The RF spectrum is shown in the inset of Fig. 5(b). For this EDFA, monocycle pulse of negative polarity can also be obtained by carefully adjusting the polarization controllers, and the results are shown in Figs. 5(c) and 5(d). The separation between the valley and peak is 9.1 ns, with optical spectrum centered at 1567.6 nm and 3-dB bandwidth of 0.1 nm. The output power is slightly increased to 4.4 mW.

By further adjustment of the intracavity polarization controllers, Gaussian doublet pulses of negative [Fig. 6(a)] and positive polarities [Fig. 6(c)] have been observed at the LD pump current of 250 mA, and the output powers become 4.4 mW and 4.9 mW, respectively. Fig. 6(b) shows the optical spectrum of the negative-polarity doublet pulses, which has center wavelength of 1567.6 nm and 3-dB bandwidth of 0.1 nm. The center wavelength of positive-polarity doublet pulses in Fig. 6(d) is slightly shifted to 1568.4 nm, with 3-dB bandwidth of 0.2 nm.

B. Theoretical modelling of vector solitons

From the polarization analysis we claim that the two electric-field components of both monocycle and doublet pulses are mutually orthogonal. Furthermore, we also claim that the monocycle and doublet pulses can be synthesized from the incoherent superposition of bright and dark solitary waves. The propagation of vector solitons can be described by the coupled CGLEs

\[
\frac{\partial u}{\partial z} = i\beta u - \frac{\partial u}{\partial t} - \frac{i}{2} k'' \frac{\partial^2 u}{\partial t^2} + \frac{k'''}{6} \frac{\partial^3 u}{\partial t^3} + i\gamma \left( |u|^2 + \epsilon_1 v^2 \right) u + i\epsilon_2 v^2 u e^{-2i\rho z} + \frac{g}{2} u + \frac{g}{2\Omega_z^2} \frac{\partial^2 u}{\partial t^2} \tag{3}
\]

\[
\frac{\partial v}{\partial z} = -i\beta v - \frac{\partial v}{\partial t} - \frac{i}{2} k'' v^2 \frac{\partial^2 v}{\partial t^2} + \frac{k'''}{6} v^3 \frac{\partial^3 v}{\partial t^3} + i\gamma \left( |v|^2 + \epsilon_1 u^2 \right) v + i\epsilon_2 u^2 v e^{2i\rho z} + \frac{g}{2} v + \frac{g}{2\Omega_z^2} \frac{\partial^2 v}{\partial t^2}, \tag{4}
\]

in which \( u \) and \( v \) represent the envelopes of the two phase-locked polarization modes propagating along the erbium-doped fiber laser with net anomalous dispersion and birefringence, \( u^* \) and \( v^* \) are the complex conjugates of \( u \) and \( v \). \( 2\beta \) is the wave number difference between two modes, \( k'' \) and \( k''' \) are the 2nd and 3rd order dispersion coefficients, respectively, \( \gamma = n_2 \alpha_{0i} / (c A_{eff}) \), represents the nonlinearities in the fibers, where \( n_2 \) is the optical Kerr coefficient, \( \alpha_{0i} \) is the central angular frequency and \( A_{eff} \) is the effective mode area, \( \rho = 2\beta \) counts for the linear birefringence, \( g \) is the saturable gain coefficient, and \( \Omega_z \) is the bandwidth of the laser gain or

![FIG. 5. (color online) (a) Temporal profile and (b) optical spectrum of the bright-dark pulses generated in the EDFA. (c) Temporal profile and (d) optical spectrum of the dark-bright pulses. The insets in (a) and (c) show the pulse trains, while the insets in (b) and (d) show the RF spectra.](image1)

![FIG. 6. (color online) (a) Temporal profile and (b) optical spectrum of the positive-polarity Gaussian doublet pulses generated in the EDFA. (c) Temporal profile and (d) optical spectrum of the negative-polarity pulses. The insets in (a) and (c) show the pulse trains, while the insets in (b) and (d) show the RF spectra.](image2)
optical filtering effect. Since the two orthogonal polarizations are locked together to form a vector soliton, they should have the same group velocity, and thus $\delta = 0$. Meanwhile, since we have pulses with durations much longer than tens of femtoseconds, the higher-order dispersion involving $k''^m$ term can be reasonably ignored in the following calculations. Taking the rotational invariance in an isotropic material, the contribution of the 3rd order nonlinear polarization implies that $\epsilon_1 = 2/3$ and $\epsilon_2 = 1/3$, and the term involving the birefringence would lead to the degenerate four-wave mixing. It’s well known that in the highly birefringent fiber without net gain and $\epsilon_1 = 1$, Manakov showed that the coupled CGLEs can be solved by the inverse scattering method [35]. In this circumstance, the coupled CGLEs support the solutions for stable vector solitons of both fundamental mode and high-order modes.

On the other hand, the gain saturation and bandwidth filtering effects have to be considered in a real fiber laser system. We assume that the fiber laser is composed of $N$-segments of fibers and each has length $L_n$ for $n = 1, 2, 3, \ldots, N$, with a specific set of parameters. Instead of applying the inverse scattering method, a trial solution of the bright soliton in one segment is assumed to be in the form of

\[
u_n(z,t) = u_{0,n} \text{sech}(p_1 z) \exp\{-i q_n \ln[\cosh(p_1 z)]\} \times \exp[i(\sigma_n - a_1 \rho_n) z],
\]

(5)

and a dark solution has the form of

\[
u_n(z,t) = v_{0,n} [\tanh(p_2 z) - i \sqrt{1 - B_n^2}] \times \exp\{-i q_n \ln[\cosh(p_2 z)]\} \exp[i(\sigma_n + a_2 \rho_n) z].
\]

(6)

For a specific segment (for simplicity, we neglect the label $n$ in below), $u_0$ and $v_0$ denote the soliton amplitudes, the inverse of $p_1$ and $p_2$ denote the widths of the pulses, $q$ sets the value of chirping, and the parameter $B$ determines the intensity dip of the dark soliton. Only for $B = 1$ the intensity of the dip center would decrease to zero, otherwise for $|B| < 1$ the intensity of the dip center approaches a finite value and the dark soliton may be termed a grey soliton.

For weak birefringent fibers without considering gain, loss, optical filtering, and for solitons with long temporal duration traveling therein, the substitutions of Eqs. (5) and (6) into coupled CGLEs give $a_1 = a_2 = \frac{1}{2}$, $q = 0$, $p_1^2 = p_2^2 = 2g/k''$, $u_0^2 = (\sigma - 2\beta)/\gamma$, and $v_0^2 = (\sigma + 2\beta)/\gamma$, where the introduction of a tunable parameter $\sigma$ sets the allowance for the formation of bright and dark solitons. These formulas have exactly confirmed Christodoulides’s predictions [15]. By separately equating the real part and imaginary part of Eqs. (5) and (6) without dropping gain, loss, and optical filtering effects, we obtain

\[q = \frac{3k''\Omega^2_{\text{g}}}{2g} \pm \sqrt{\frac{9gk''^2\Omega^2_{\text{g}}}{4g^2} + 2},
\]

(7)

\[p_1^2 = \frac{g}{2} \left[ \frac{g}{2\Omega^2_{\text{g}}} (q^2 - 1) - \frac{3}{2} k'' q \right]^{-1},
\]

(8)

\[p_2^2 = \frac{g}{2} \left[ \frac{g}{2\Omega^2_{\text{g}}} (q^2 - 1) - k'' q \right]^{-1},
\]

(9)

\[u_0^2 = \frac{1}{\gamma} \left[ \sigma - 2\beta + \frac{g}{2} - p_1^2 k'' (q^2 + 1)/2 \right],
\]

(10)

\[v_0^2 = \frac{\sigma + 2\beta - k'' p_2^2 q^2 / 2}{\gamma (2 - B^2)}.
\]

(11)

and unchanged tuning parameters $a_1$ and $a_2$. These expressions reveal that with given system parameters, we are capable of qualitatively depicting the shape of solitons. Since the bright and the dark solitons have orthogonal polarizations, they form the fundamental solutions of the coupled CGLEs for each fiber segment. This implies that the general solution of one segment should be written as the combination of two vectors, i.e., $F_n(z,t) = u_n(z,t)\hat{e}_a + v_n(z,t)\hat{e}_b$. The magnitude of the coefficients $u_{0,n}$ and $v_{0,n}$ should be precisely determined with proper boundary conditions. In a ring-cavity, the boundary conditions require the conservation of pulse energy at the interface of two adjacent fibers and at the interface where the first fiber connects the last one.

The simplest fiber laser with cavity length $L$ can be considered as composed of two segments, i.e., the gain fiber and the passive fiber, with length $L_1$ and $L_2$, respectively. When the solitons propagate inside the fiber of the cross section $A_{\text{eff,n}}$, the energy flowing across the interface of two fibers in a round-trip time $T$ is defined as

\[E_n = A_{\text{eff,n}} \int_0^T \left[ |u_n(z,t)|^2 + (I_b - |v_n(z,t)|^2) \right] dt.
\]

(12)

in which $I_b$ is the CW background of the dark soliton, such as those determined from the polarization-splitting measurements shown in Figs. 4(a) and 4(c). The criteria of energy conservation at the interfaces state that

\[E_1 |_{z=L_1} = E_2 |_{z=L_2}, \quad E_1 |_{z=0} = E_2 |_{z=L}.
\]

(13)

As the boundary conditions of Eq. (13) are employed, the magnitudes of $\sigma_1$ and $\sigma_2$ would no longer be free for tuning, but rather be completely decided. As a consequence, with given sets of parameters, our derivations show that the shapes of the bright and the dark solitons are able to be clearly configured in the whole segments of the fiber laser. Due to the weak polarization-dependent loss in a ring-cavity fiber laser [34], $g$ can be varied for different setting of the intracavity polarization controller. This may explain why the laser output power for 210-mA pumping is lower than that for 250-mA pumping in Sec. IIIA.

The adjustment of polarization controller will set the in- phase time for the constituent frequencies of solitons, thus allows the tuning of peak locations of bright solitons (or the
trough locations of the dark solitons). As a result, the relative delay of bright and dark components can be finely adjusted. The existence of fiber bending in the EDFL, including the three-paddle polarization controller, provides the weak polarization-dependent loss in the cavity. Therefore, the relative width and amplitude of the bright and dark solitons can also be adjusted. By adjusting the relative delay, width, and amplitude of the two polarization components, monocycle or doublet pulses with either positive or negative polarities are obtained.

The generation of Gaussian doublets in a ring-cavity EDFL using passive optical technology have not been reported previously. It is interesting to further study the origin of pulse formation for both the doublet and monocycle pulses. By analyzing the polarization states of the output pulses, we have found that the doublet pulses are composed of bright and dark pulses with different temporal width and orthogonal polarization states (Fig. 4), and the underlying mechanism in the formation of monocycle and doublet pulses is similar—the incoherent superposition of bright and dark pulses with different temporal widths, amplitudes, and time delays. When the centers of bright and dark pulses coincide, doublet pulses are generated. On the other hand, when the centers of bright and dark pulses are separated, monocycle pulses are formed. Fig. 7 demonstrated this observation by theoretically superpose the intensities of a sech\(^2\)-shaped bright pulse and a tanh\(^2\)-shaped dark pulse. In Fig. 7(a), the dark (red dashed curve) and bright (black dashed curve) pulses have approximately equal widths and absolute amplitudes but their extremities are temporally staggered, which results in the dark-bright pulse (blue solid curve) similar to the experimentally observed monocycle pulses in Fig. 3(a). In Fig. 7(b), the extremities of a broad dark pulse with small absolute amplitude (red dashed curve) and a narrow bright pulse with large amplitude (black dashed curve) are temporally aligned, which results in the dark-bright-dark pulse (blue solid curve) similar to the experimentally observed doublet pulses in Fig. 3(b).

C. Optical soliton vs. atomic soliton

The remarkable similarities between matter-wave solitons and optical solitons reveal the close connection between atomic optics and light optics. Their spatio-temporal similarities provide the opportunity to review the underlying properties and mechanisms from the opposite. The analogies and the distinctions between matter-wave solitons and optical solitons are addressed below. We expect this investigation may pave the way for further generation of new-type solitons, as well as property analysis in real fibers of the non-integrable system.

The dynamics of solitons in two-component Bose-Einstein condensates can be described by the coupled Gross-Pitaevskii equations (GPEs),

\[ i\hbar \frac{\partial \psi_1(z,t)}{\partial t} = -\frac{\hbar^2}{2m} \frac{\partial^2 \psi_1(z,t)}{\partial z^2} + V_{ext}(z)\psi_1(z,t) + g_{11}|\psi_1(z,t)|^2\psi_1(z,t) + g_{12}|\psi_2(z,t)|^2\psi_1(z,t) - \mu_1\psi_1(z,t), \]  
\[ i\hbar \frac{\partial \psi_2(z,t)}{\partial t} = -\frac{\hbar^2}{2m} \frac{\partial^2 \psi_2(z,t)}{\partial z^2} + V_{ext}(z)\psi_2(z,t) + g_{22}|\psi_2(z,t)|^2\psi_2(z,t) + g_{21}|\psi_1(z,t)|^2\psi_2(z,t) - \mu_2\psi_2(z,t). \]

In writing Eqs. (14) and (15) we have assumed that the transverse dynamics of cold atoms has been frozen out under extremely tight transverse trapping potentials so that theoretical studies of quasi-1D properties of the condensates and solitons within a shallow \( V_{ext} \) can properly mimic the real experiments. The nonlinear terms represent the hard core collisions between two cold atoms taking places at ultralow temperatures, and the coefficients \( g_{ij} \) count for the collision strength represented in terms of the s-wave scattering length. In the last terms of GPEs, \( \mu \) represents the chemical potential of each component. The conservation of atom number provides the external constraint for Eqs. (14)-(15).

Different from the optical NLSEs that mainly focus on addressing the pulse broadening caused whether by normal dispersion \((k'' > 0)\) or by anomalous dispersion \((k'' < 0)\) effect, and the SPM and XPM due to the Kerr nonlinearity of the material, GPEs describe the Hamiltonian for the condensates. Therefore for the condensates, the role of the kinetic energy terms drives the condensates as if they are moving with normal dispersions. The nonlinear coefficients \( g_{ij} \) can be positive or negative, depending on whether the mutual interaction between two atoms is repulsive \((g_{ij} > 0)\) or attractive \((g_{ij} < 0)\). According to the naive balance condition, one could merely observe dark solitons within the condensate composing of repulsive atoms, and obtain bright solitons from the condensate composing of attractive atoms. In practice, via tuning scattering length near Feshbach resonances, it’s possible to manipulate the interatomic collisions and realize magnetically or optically confined Bose-Einstein condensates as well as solitons in both regimes. It was also reported that the presence of the spatial inhomogeneity strongly affect the motion, stability and interactions of the matter-wave solitons \[16\].

In the system of \(^7\)Li condensate, for example, experiments showed that the balance between the wave packet dispersion and the attractive mean field energy can lead to the formation of bright solitons and bright-dark solitons \[17,40\]. With the help of phase imprinting and Raman transfer, in the system of \(^85\)Rb, for example, experimentalists successfully confirmed the creation of dark-bright solitons by filling the dark
soliton with atoms of different species or in another internal state [41]. Across the soliton, the phase gradient will induce a superfluid velocity for the dark soliton that drives a counterclockwise motion against the condensate. The development of density engineering and phase imprinting techniques brings us the possibility to create bright solitons, dark solitons, dark-bright solitons, and various soliton families in the one-component, multi-component, and spinor condensates [37, 41-44]. As a signature of the formation of solitons from condensates, an observation of undamped collective oscillations or the emission of non-dispersive solitary laser is expected [38, 41].

Just as a condensate is not equivalent to an atomic soliton, neither is an optical pulse equivalent to an optical soliton. In optical fibers where Kerr effect dominates the nonlinearity and the pulse broadening takes place under the group velocity dispersion, there could only be dark soliton formation in the fibers with normal dispersion and bright soliton formation in the fibers with anomalous dispersion. However, through the XPM between the orthogonal polarization components in a birefringent fiber, simultaneous propagation of bright and dark solitons is indeed possible, leading to the generation of the bright-dark or dark-bright vector soliton pairs. Furthermore, the polarization of optical field within the monocyte pulses switches abruptly between two orthogonal polarizations to form a domain wall, whereas the domain wall disappears for the doublet pulses within the polarization evolves gradually therein. It should be mentioned, the terminology "dark-bright soliton" in the BEC is somewhat different from that in optical solitons. The extremities of dark and bright components in optical dark-bright solitons are detuned, while the extremities for atomic dark-bright solitons are coincident.

In a lossless passive fiber, the formation of stable solitons are attributed to the balancing between SPM and GVD. To generate bright solitons, an intensity threshold should be reached, while the generation of dark solitons is thresholdless. However, in a real fiber laser system, various effects such as saturable gain, XPM, optical filtering, and saturable loss should be considered. As a result, these effects will crucially determine whether a soliton and what kind of a soliton can be stably generated and also dramatically influence the phenomena that can be observed. Relative to an atomic dark-bright or bright-dark soliton that is formed in a superposition manner and displays a spatially localized structure, the relative phase, pulsewidth, and the positions of amplitude extrema for dark and bright solitons in the optical fibers can be easily manipulated via the adjustment of intracavity polarization. As the domain wall solitons can be created in the two-component condensate with repulsive inter- and intra-atomic nonlinear interactions [32, 33], we find that the domain wall between orthogonal polarizations can also be manipulated by the adjustment of intracavity polarization.

IV. CONCLUSION

In this paper, Gaussian monocyte and doublet pulses of positive- and negative-polarities are generated in a ring-cavity erbium-doped fiber laser using passive optical technology. By adjusting polarization controllers, the fiber laser can be switched among continuous-wave, monocyte, and doublet states. After carefully examining the polarization of laser output, we find that both monocyte and doublet pulses are composed of bright and dark solitons having orthogonal polarizations. The temporal widths, amplitudes and time delays of the bright and dark components can be tuned by the intracavity PCs, and the underlying mechanisms on the formation of monocyte and doublet pulses are attributed to the polarization locking of bright and dark solitons. We have observed experimentally that a bright soliton and a dark soliton can be locked to form a monocyte bright-dark or dark-bright vector soliton, depending on the relative time delay between the two polarization components. It is also viable for a bright soliton to be embedded in a broader dark soliton to form a doublet vector soliton, or a dark soliton in a broader bright pulse to form a doublet of reversed polarity. Therefore, the polarization domains and shapes of the vector solitons can be manipulated by the PC adjustments. Without resorting to expensive active components and complicated design, the method used here is simple for applications in optical pulse shaping and laser physics.

The formation of monocyte or doublet pulses are further demonstrated theoretically by the incoherent superposition of the bright soliton with sech and dark soliton with tanh envelope functions, and the results shows good consistency with the experimental observations. The results of tunable vector soliton generation are compared with atomic solitons in the system of BEC. We observed that for optical solitons, the polarization within the monocyte pulses switches abruptly between two orthogonal polarizations to form a domain wall, whereas the domain wall disappears for the doublet pulses since the polarization evolves gradually therein. Future studies can be conducted for the theoretical simulation of domain dynamics under different intracavity gain, dispersion, nonlinearity, and birefringence. Inspired by the experience in studying BEC, we expect that the simulation and propagation of pulse waves may open a novel route to explore the classical solitary dynamics in nonlinear optics and its quantum analogy in ultracold fields.

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