Modelling of suspended particles motion in channel

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Abstract. The paper considers the application of the scheme based on linear combination
of the Upwind and Standard Leapfrog difference schemes with weight coefficients obtained
by minimizing the approximation error. The proposed difference scheme has shown its effectiveness
in solving diffusion-convection problems in the range of grid Peclet numbers $2 \leq Pe \leq 20$, which
allows us to apply this class of schemes for the numerical solution of computational Oceanology
problems. The problem of soil dumping was considered as a practical example of application of
the proposed schemes.

1. Introduction

In the numerical solution of problems of transport of suspensions in shallow water bodies on
the basis of Central difference schemes, a problem arises associated with a drop in accuracy for
large values of the grid Peclet numbers [1]. One solution to this problem is to grind the step
on the spatial grid, which entails an increase in labor intensity. Another approach to solving
this class of problems is the use of other difference schemes, for example, the Upwind Leapfrog
scheme [2], which were developed to solve problems of aeroacoustics [3, 4]. Difference schemes
Standard Leapfrog scheme and Upwind Leapfrog scheme have quite similar properties: they are
non-dissipative and have a second order of accuracy with respect to steps in the spatial and
temporal coordinate directions. Despite these properties of the scheme, they have low accuracy.
Greater accuracy compared to the classical Upwind and Standard Leapfrog schemes showed
Upwind Leapfrog scheme with limiters solutions [5, 6]. In the work [7] it is proposed to use a
linear combination of the Upwind and Standard Leapfrog difference schemes.

In a linear combination of the Upwind and Standard Leapfrog difference schemes with similar
properties, there is often mutual compensation for approximation errors, and the resulting
scheme has better properties than the original schemes. The purpose of this work is to use
the developed difference scheme, which is a linear combination of the Upwind and Standard
Leapfrog difference schemes with weight coefficients obtained from the condition of minimizing
the order of approximation error, to solve the problems of suspension transport.
2. Solution of the transfer problem based on the linear combination of the Upwind and Standard Leapfrog difference schemes

Consider the transfer equation [8]

\[ \dot{q}_i + u q'_x = 0, \]  

where \( t \in [0, T], x \in [0, L], q(0, x) = q^0(x), q(t, 0) = 0, u = \text{const}. \)

Let introduce uniform computational grid

\[ \omega = \omega_h \times \omega_T, \]  

where \( \tau = t^{n+1} - t^n = \text{const}, \quad \omega_h = \{ x_i | x_i = i h, \quad i = 0, 1, \ldots, N, \quad Nh = L \}, \quad \omega_T = \{ t^n | n = 0, 1, \ldots, T \}. \)

The following finite difference schemes can be used to solve the problem numerically:

- Upwind Leapfrog scheme [2]:

\[ \frac{q_i^{n+1} - q_i^n}{2 \tau} + \frac{q_{i-1}^{n} - q_{i-1}^{n-1}}{2 \tau} + u \frac{q_i^n - q_i^{n-1}}{h} = 0, \quad u \geq 0; \]  

\[ \frac{q_i^{n+1} - q_i^n}{2 \tau} + \frac{q_{i+1}^{n} - q_{i+1}^{n-1}}{2 \tau} + u \frac{q_{i+1}^{n} - q_{i}^{n}}{h} = 0, \quad u < 0; \]  

- Standard Leapfrog scheme:

\[ \frac{q_i^{n+1} - q_i^n}{2 \tau} + u \frac{q_{i+1}^{n} - q_{i-1}^{n}}{2 h} = 0. \]  

The Upwind Leapfrog scheme and Standard Leapfrog scheme showed low accuracy in solving model problems [5, 7]. Consider the difference scheme, which is a linear combination of the Upwind and Standard Leapfrog difference schemes with weights 2/3 and 1/3, respectively [1, 7]

\[ \frac{q_i^{n+1} - q_i^n}{\tau} + \frac{4}{3} \left( \frac{q_{i-1}^{n} - q_{i-1}^{n-1}}{2 \tau} + u \frac{q_i^n - q_i^{n-1}}{h} \right) + \frac{1}{3} \left( \frac{q_i^{n} - q_i^{n-1}}{2 \tau} + u \frac{q_{i+1}^{n} - q_{i+1}^{n-1}}{3h} - u \frac{q_{i+1}^{n} - q_{i}^{n}}{3h} \right) = 0, \quad u \geq 0; \]  

\[ \frac{q_i^{n+1} - q_i^n}{\tau} + \frac{4}{3} \left( \frac{q_{i+1}^{n} - q_{i+1}^{n-1}}{2 \tau} + u \frac{q_{i+1}^{n} - q_{i}^{n}}{h} \right) + \frac{1}{3} \left( \frac{q_i^{n} - q_i^{n-1}}{2 \tau} + u \frac{q_{i+1}^{n} - q_{i}^{n}}{3h} - u \frac{q_{i+1}^{n} - q_{i}^{n-1}}{3h} \right) = 0, \quad u < 0. \]

Scheme (5) has a local approximation error with respect to the dummy node \((i-1/3, n)\) equal to \( \frac{(1-\epsilon)}{6} u h^2 q_{xx} + O(h^3) \). Thus, for small Courant numbers, a scheme (5) having an approximation error of \( O(\epsilon h^2) \) is preferable to the original schemes (3) and (4) whose approximation errors are \( O(h^3) \).

Model problem I. It is required to find the solution of the transfer equation

\[ \frac{\partial q}{\partial t} + u \frac{\partial q}{\partial x} = 0, \]

where \( u = 0.5 \ m/s, \ 0 \leq t \leq T, \ 0 \leq x \leq L, \ q(t, 0) = 0 \) and with initial conditions \( q^0(x) = \theta(20 - x) - \theta(10 - x) \), where \( \theta(x) \) is the Heaviside function.

We compare the calculations based on the proposed scheme (5) with the results obtained using the Upwind Leapfrog and Standard Leapfrog difference schemes with the constraints of the solution [5, 6].

Figure 1 shows the values of the errors of the numerical solution of the Model problem I based on schemes (5) the Upwind Leapfrog and Standard Leapfrog difference schemes with limiters depending on the values of the Courant numbers. The length of the time interval \( T \) is equal to 100 s. The time step \( \tau \) is from 0.02 s to 2 s. The Courant numbers range from 0.01 to 1.
Figure 1. The error values of the numerical solution of the Example 1 depending on the values of the Courant numbers (1 – scheme (5), 2 – Upwind Leapfrog difference scheme with limiters, 3 – Standard Leapfrog difference schemes with limiters).

3. Solution of the diffusion problem
Consider the case of the heat equation with constant coefficients:

\[ q_t = \mu q_{xx} + f, \quad t > 0, \quad 0 < x < l, \]  
\[ q(x, t) \big|_{t=0} = q^0(x), \quad 0 \leq x \leq l, \]  
\[ q(0, t) = q_0(t), \quad q(l, t) = q_l(t), \quad t \geq 0. \]

We assume the necessary smoothness of the functions included in the relations (6)–(8), and the consistency of the initial and boundary conditions.

For the numerical solution of the problem (6) cover the calculated area with a uniform grid:

\[ w_h = \left\{ t^n = n\tau, \quad x_i = ih; \quad n = 0..N_t, \quad i = 0..N_x; \quad N_t\tau = T, \quad N_xh = l \right\}, \]

where \( \tau \) is the time step, \( h \) is the space step, \( M \) is the upper bound in time, \( N \) is the number of nodes in space.

The difference scheme for the diffusion equation

\[ \frac{q_i^{n+1} - q_i^n}{\tau} = \mu \frac{q_i^{n+\sigma} - 2q_i^{n+\sigma} + q_i^{n-\sigma}}{h^2} + f_i, \]

where \( q_i^{n+\sigma} = \sigma q_i^{n+1} + (1 - \sigma) q_i^n, \quad \sigma \in [0, 1] \) is the weight of the scheme.

The necessary stability condition obtained on the basis of the harmonic method leads to the following inequality [9]:

\[ \gamma = \frac{\tau\mu}{h^2} \leq \frac{1}{2}. \]

Although this estimate is a strict constraint for explicit difference schemes, in practice the time step needs to be taken even less.

Model problem II. Consider the problem arising in the simulation of slurry transport in shallow waters [10]. It is required to find the solution of the two-dimensional diffusion equation for the region extended in one direction

\[ q_t = \mu_x q_{xx} + \mu_y q_{yy}, \quad t > 0, \quad 0 < x < l_x, \quad 0 < y < l_y, \]
\[ \mu_x = 100 \, m^2/s, \mu_y = 0.5 \, m^2/s, \, l_x = 2000 \, m, \, l_y = 5 \, m \]

with initial conditions:
\[ q(x, y, t)|_{t=0} = (\theta(1100 - x) - \theta(900 - x)) (\theta(3 - y) - \theta(2 - y)), \quad 0 \leq x \leq l_x, \quad 0 \leq y \leq l_y \]

and boundary conditions in the Dirichlet form.

Parameters of the calculated grid are: space steps \( h_x = 100 \, m \) and \( h_y = 0.5 \, m \), the length of the time interval \( T = 600 \, s \). Figure 2 shows the solutions of Model problem II on the basis of: 1 – scheme with weights, 2 – explicit scheme.

**Figure 2.** Function of dependence of approximation error on time step: 1 – for an explicit scheme, 2 – for scheme with weights.

Figure 2 shows that the error of achievement for the explicit scheme of the time step restriction is significantly less than for the scheme with weights. The explicit scheme has a stable solution under the constraint \( \tau \leq O(h^2) \) [11], and the scheme with weights at \( \sigma \geq 0.5 \) has no time step restrictions. In practice, in order that the error of calculations on the basis of the difference scheme (9) with weight \( \sigma \) was in the acceptable range you must use the following restriction on the time step: \( \tau \leq \Delta \cdot (\sum_{i=1}^{r} \mu_i/h_i^2)^{-1} \), where \( r \) is the dimension of the space, \( \Delta \) is a parameter describing the ratio of the step size \( \tau \) needed to the accuracy of the calculations was within the acceptable range to step \( \tau_{\text{max}} \) obtained from limits on the stability of the explicit scheme. For an explicit scheme, \( \Delta = 0.01 \) is recommended, and for a scheme with a weight of \( \sigma = 0.5 \) – \( \Delta = 0.3 \).

**Comment.** From the results of the calculation of the Model problem I (Figure 1) it can be seen that the proposed scheme (5) is more accurate than the Upwind Leapfrog scheme with limiters solves the problem of convection at small Courant numbers (\( c = |u| \tau/h \leq 0.1 \)). It follows from the results of the calculation of the diffusion problem (Model problem II and Figure 2) that for explicit schemes there is a restriction \( \tau \leq \Delta \cdot \tau_{\text{max}}, \tau_{\text{max}} = h^2/2\mu, \Delta = 0.01 \). From these estimates it follows, \( \Delta |u| h/2\mu \leq 0.1 \), where \( Pe = |u| h/\mu \leq 0.2/\Delta = 20 \), where \( Pe \) is the grid number of Peclet [1]. The proposed approximation of the convective transport operator will be effective in this range of Peclet numbers (the case of absence of monotonicity of schemes constructed on the basis of Central difference approximations \( Pe > 2 \) is considered).

4. Approximation of the two-dimensional convection-diffusion equation
Consider the two-dimensional convection-diffusion equation:
\[ c'_t + uc'_x + vc'_y = (\mu c'_x)_x + (\mu c'_y)_y + f \quad (10) \]

with boundary conditions:
\[ c'_n(x, y, t) = \alpha_n c + \beta_n, \quad (11) \]

where \( u, v \) are the components of the velocity vector, \( \mu \) is the coefficient of turbulent exchange, \( f \) is the function describing the intensity and distribution of sources.

The calculated area is inscribed in a rectangle. For the numerical implementation of a discrete mathematical model of the problem a uniform grid is introduced:
\[ w_h = \{ t^n = n\tau, x_i = ih_x, y_j = jh_y; \ n = 0, \ldots, N_t, \ i = 0, \ldots, N_x, j = 0, \ldots, N_y ; N_t \tau = T, N_x h_x = l_x, N_y h_y = l_y \}, \]

where \( \tau \) is the time step, \( h_x, h_y \) are space steps, \( N_t \) is the upper time bound, \( N_x, N_y \) are space bounds.

Cells are rectangles, they can be filled, partially filled, or empty. Cell centers and nodes are spaced \( h_x/2 \) and \( h_y/2 \) at coordinates \( x \) and \( y \), respectively. Denote by \( o_{i,j} \) the fullness of the cell \((i, j)\). In the neighborhood of the node \((i, j)\) are cells \((i, j)\), \((i+1, j)\), \((i, j+1)\), \((i+1, j+1)\).

The coefficients \( q_0 \), \( q_1 \), \( q_2 \), \( q_3 \), \( q_4 \) describing the volume of fluid (VOF) of the corresponding control areas \( D_0 \), \( D_1 \), \( D_2 \), \( D_3 \), \( D_4 \) located in the vicinity of the cell are introduced. The value of coefficient \( q_0 \) characterizes the filling area \( D_0 \), where
\[ D_0 : x \in \left( x_{i-1/2}, x_{i+1/2} \right), \ y \in \left( y_{j-1/2}, y_{j+1/2} \right), \]
\[ q_1 - D_1 : x \in \left( x_i, x_{i+1/2} \right), \ y \in \left( y_{j-1/2}, y_{j+1/2} \right), \]
\[ q_2 - D_2 : x \in \left( x_{i-1/2}, x_i \right), \ y \in \left( y_{j-1/2}, y_{j+1/2} \right), \]
\[ q_3 - D_3 : x \in \left( x_{i-1/2}, x_{i+1/2} \right), \ y \in \left( y_{j}, y_{j+1/2} \right), \]
\[ q_4 - D_4 : x \in \left( x_{i-1/2}, x_{i+1/2} \right), \ y \in \left( y_{j-1/2}, y_j \right). \]

The filled parts of the regions \( D_m \) will be called \( \Omega_m \), where \( m = 0, \ldots, 4 \). In accordance with this, the coefficients \( q_m \) can be calculated by the equations:
\[ (q_m)_{i,j} = \frac{S_{\Omega_m}}{S_{D_m}}, (q_0)_{i,j} = \frac{o_{i,j} + o_{i+1,j} + o_{i+1,j+1} + o_{i,j+1}}{4}, \]
\[ (q_1)_{i,j} = \frac{o_{i+1,j} + o_{i+1,j+1}}{2}, \]
\[ (q_2)_{i,j} = \frac{o_{i,j} + o_{i,j+1}}{2}, \]
\[ (q_3)_{i,j} = \frac{o_{i+1,j+1} + o_{i,j+1}}{2}, \]
\[ (q_4)_{i,j} = \frac{o_{i,j} + o_{i+1,j}}{2}. \]

To approximate the homogeneous equation (10) for large grid Peclet numbers, use space splitting schemes:
\[ \frac{c^{n+1/2} - c^n}{\tau} + u \left( c^n \right)_x' = \left( \mu \left( c^n \right)_x' \right)_x', \quad (12) \]
\[ \frac{c^{n+1} - c^{n+1/2}}{\tau} + v \left( c^{n+1/2} \right)_y' = \left( \mu \left( c^{n+1/2} \right)_y' \right)_y'. \quad (13) \]

To approximate the system of equations (12)–(13) we will use the scheme (5), obtained as a result of linear combination of the Upwind and Standard Leapfrog difference schemes, while taking into account the VOF of cell:
– difference scheme for the equation (12) describing the transfer along the Ox direction:

\[
\frac{2q_{1,i,j} + q_{0,i,j} c_{i,j}^{n+1/2} - c_{i,j}^{n}}{3} \frac{n}{\tau} + \frac{5u_{i-1/2,j} q_{2,i,j}}{3h_x} c_{i,j}^{n} - c_{i,j}^{n-1/2} + u_{i+1/2,j} \min (q_{1,i,j}, q_{2,i,j}) c_{i,j}^{n} - c_{i,j}^{n-1/2} + \frac{2\Delta_x c_{i-1,j}^{n} q_{2,i,j} + \Delta_x c_{i,j}^{n} q_{0,i,j}}{3} - 2\mu_{i-1/2,j} q_{2,i,j} c_{i,j}^{n} - c_{i,j}^{n-1/2} - \frac{|q_{1,i,j} - q_{2,i,j}| \mu_{i,j} c_{i,j}^{n} + \beta_x}{h_x}, \quad u_{i,j} \geq 0; (14)
\]

– difference scheme for the equation (13) describing the transfer along the Oy direction:

\[
\frac{2q_{4,i,j} + q_{0,i,j} c_{i,j}^{n+1/2} - c_{i,j}^{n}}{3} \frac{n}{\tau} + \frac{5v_{i,j-1/2} q_{4,i,j}}{3h_y} c_{i,j}^{n} - c_{i,j}^{n-1/2} + v_{i,j+1/2} \min (q_{3,i,j}, q_{4,i,j}) c_{i,j}^{n} - c_{i,j}^{n-1/2} + \frac{2\Delta_y c_{i,j+1}^{n} q_{4,i,j} + \Delta_y c_{i,j}^{n} q_{0,i,j}}{3} - 2\mu_{i,j-1/2} q_{4,i,j} c_{i,j}^{n} - c_{i,j}^{n-1/2} - \frac{|q_{3,i,j} - q_{4,i,j}| \mu_{i,j} c_{i,j}^{n} + \beta_y}{h_y}, \quad v_{i,j} \geq 0; (15)
\]
5. Solution of hydrodynamics test problems

Consider a stationary fluid flow between two infinitely long coaxial circular cylinders:

\[ \begin{align*}
    u u'_x + v v'_y &= -\rho^{-1} P'_x + \mu \Delta u, \\
    u v'_x + v u'_y &= -\rho^{-1} P'_y + \mu \Delta v, \\
    \frac{r_1^2}{r_2} &= \sqrt{x^2 + y^2}. 
\end{align*} \tag{16} \]

**Model problem III.** The problem of finding the numerical flow of a viscous fluid between two coaxial semi-cylinders \((x \geq 0)\) is considered. The radius of the inner cylinder \(r_1 = 5 \text{ m}\) the Radius of the outer cylinder \(r_2 = 10 \text{ m}\) the Calculated area is inscribed in a rectangle with dimensions 10 by 20 \(\text{m}\) \((0 \leq x \leq 10 \text{ and } -10 \leq y \leq 10)\). In the section of the cylinder, the plane \(x = 0\) sets the velocity field \(u'(0, y) = -5/y \text{ m/s}, \ v(0, y) = 0 \text{ m/s}\). In all other grid nodes, the velocity field is calculated. On the inner and outer walls of the cylinder, sliding and non-flowing conditions are set.

The analytical solution of Model problem III in the Cartesian coordinate system will take the form:

\[ \begin{align*}
    u(x, y) &= -\frac{5y}{x^2 + y^2}, \ v(x, y) = \frac{5x}{x^2 + y^2}, \\
    P(x, y) &= P(r_1) - \frac{12.5\rho}{x^2 + y^2} + \rho/2. 
\end{align*} \tag{17} \]

The errors of numerical solutions are most clearly visible on coarse grids. We describe the parameters of the coarse mesh. Steps in spatial directions are 1 \(\text{m}\), time step 0.1 \(\text{s}\), grid sizes \(21 \times 11\) nodes, the length of the counting interval 10 \(\text{s}\), the density of the medium \(\rho = 1000 \text{ kg/m}^3\), the coefficient of turbulent exchange \(\mu = 1 \text{ m}^2/\text{s}\).

Figure 3 shows the numerical solution of the problem (16) in case (a) the system of diffusion-convection equations was solved on the basis of the Central difference scheme, in case (b) the system of diffusion-convection equations was solved on the basis of difference schemes (14), (15). Figure 3 (c) shows the difference of the velocity vector fields calculated on the basis of the Central difference scheme and the difference schemes (14), (15).

![Figure 3](image-url)

**Model problem IV.** It is required to find the solution of the problem of transport of substances (10) between two coaxial cylinders on the basis of the Central difference scheme and schemes (14), (15) thus the flow field was set by the function (17) with the initial distribution:

\[ c^0(x, y) = (\theta(1 - x) - \theta(0.5 - x)) (\theta(-8.5 - y) - \theta(-9 - y)). \]

Figure 4 shows the numerical solution of substances transfer problem (Model problem IV) at small grid Peclet numbers. The case of fulfillment of the stability condition of the Central
difference scheme is considered. The coefficient of turbulent exchange is taken to be $0.5 \text{ m}^2/\text{s}$. The calculated interval is $20 \text{ s}$. Figure 5 shows the calculation of substances transfer problem (Example 4) at large grid Peclet numbers. The coefficient of turbulent exchange is taken to be $0.01 \text{ m}^2/\text{s}$ in this case, diffusion exchange is practically absent. Figure 4, 5 denoted: (a) – the initial distribution of the concentration field, (b) – the result of calculating the concentration field based on the Central difference scheme, (c) – the result of calculating the concentration field based on the difference schemes (14), (15).

**Figure 4.** Numerical solution of the problem of transfer of substances between two coaxial cylinders at small grid Peclet numbers.

**Figure 5.** Numerical solution of the problem of transfer of substances between two coaxial cylinders at large grid Peclet numbers.

Figure 4, 5 demonstrate that the proposed difference schemes, constructed on the basis of a linear combination of the Upwind and Standard Leapfrog difference schemes with weight coefficients $2/3$ and $1/3$, respectively, obtained as a result of minimizing the order of approximation error, for the diffusion-convection problem have a lower grid viscosity and, as a consequence, more accurately describe the behavior of the solution in the case of large grid Peclet numbers.
6. Mathematical model of transport of suspended particles

To describe the transport of suspended particles, the diffusion-convection equation is used, which can be written in the following form [14]:

\[ \frac{\partial c}{\partial t} + (uc)_x + (vc)_y + ((w + w_g)c)_z = \mu \left( \frac{\partial^2 c}{\partial x^2} + \frac{\partial^2 c}{\partial y^2} \right) + \nu \frac{\partial c}{\partial z} + f, \]  

(18)

where \( c \) is the impurity concentration; \( \vec{V} = \{u, v, w \} \) are the components of the velocity vector field; \( w_g \) is the rate of suspension deposition in the vertical direction; \( \mu, \nu \) is the horizontal and vertical coefficients of turbulent diffusion; \( f \) is the function describing the intensity of the distribution of pollutant sources.

Add boundary conditions to the equation (18):

\[ C'_n = 0, \quad (\vec{V}, \vec{n}) \leq 0, \quad \mu C'_n + w_g V_n = 0, \quad (\vec{V}, \vec{n}) > 0, \]

where \( \vec{n} \) is the external normal to the boundary of the computational domain; \( V_n \) is the normal component of the velocity vector.

A three-dimensional model of shallow water hydrodynamics [15, 16] was used to calculate the components of the velocity vector of the water medium.

The calculation of fish productivity of the area of work is carried out. A model of suspended particle transport has been developed and implemented programmatically. The developed numerical algorithms and the complex of programs implementing them were used for the study of hydrobiological processes during repair dredging. Dredging on the approach channel will be performed by a self-draining dredger of the type ZS-TR 1300/2-2162. Productivity of the equipment on soil makes: self-draining dredger – 778 \( m^3/h \). Technical characteristics of the equipment used in the disposal (the amount of the hold, the frequency of discharge, duration of disposal time): the amount of the hold in the development of the Sands – 1000 \( m^3 \); the rate of loading of a soil of the hold – 741 \( m^3 \); the duration of disposal operations at the underwater dump – 0.1 \( h \).

Estimation the flow rate in the channel shows, that at a tidal flow rate of 0.5 \( m/s \), the duration of the tide of 6 hours and the ebb rate of 0.25 \( m/s \), the average speed of the actual transfer of suspended particles with a diameter of 0.05 \( mm \) will be 0.12 \( m/s \). The average speed of the actual transfer of suspended particles with a diameter of 0.05 \( mm \), which cause the greatest harm to aquatic biological resources, on the dumps will be 0.2 \( m/s \). Table 1 shows the initial data and parameters of the calculated area.

**Table 1.** The initial data for model of suspended particles motion and calculated area.

| The initial data                        | Parameters of the calculated area |
|----------------------------------------|----------------------------------|
| The depth of the reservoir             | 10 \( m \)                       |
| The volume of loading                  | 741 \( m^3 \)                    |
| Flow rate                              | 0.2 \( m/s \)                    |
| Deposition rate (Stokes)               | 2.042 \( mm/s \)                 |
| Soil density                           | 1600 \( kg/m^3 \)                |
| Percentage of dusty particles \((d < 0.05 \( mm \)) in sandy soils\) | 26.83 %                          |
|                                        | Calculated interval               |
|                                        | 2 \( h \)                        |

Figure 6.a shows the results of calculation of the vertical profile of the horizontal component of the flow velocity vector. Figure 6.b shows the dependence on time (hour) of the volume of
Figure 6. Vertical profile of the horizontal component of the flow velocity vector (a), depending on the time of the volume of water with the contents of suspended particles: 1 – more than 100 mg/l, 2 – more than 20 mg/l, 3 – more than 0.25 mg/l (b).

Figure 7. Suspended particle concentration field values after: 15 min; 30 min; 1 h; 2 h after discharge of hold.

Based on the obtained materials, calculate the total amount of contaminated water at soil dumps (table 2).

The actual assessment of the impact on the fish food supply is impossible without the use of the most modern and optimized mathematical models that allow to predict both the spread of
Table 2. The volume of contaminated water at the dumping ground.

| Plot number | Total volume of polluted water at a single discharge, million m$^3$ | Number of discharges | Total volume of water with concentrations of pollutants, million m$^3$ |
|-------------|-------------------------------------------------|----------------------|---------------------------------------------------------------|
|             | including water with concentrations of pollutants, million m$^3$ |                      | >0.25 mg/l >20 mg/l >100 mg/l                                 |
| 1           | 1,285                                           | 0.89                 | 0.245 0.15 124                                               |
| 2           | 1,12                                            | 0.813                | 0.202 0.105 50                                               |
| 3           | 1,279                                           | 0.889                | 0.24 0.15 45                                                 |

suspension plumes in the aquatic environment and the change in the bottom relief due to the deposition of suspended soil particles in the sediment. The quantity of polluted waters at soil dumps and at work of dredging equipment for calculation of damage to fish stocks is calculated, and values of areas at which death of bottom vegetation on dumps and in areas of dredging are calculated. On the basis of the developed software package, it was found that reducing the size of the areas of soil dumps allows to minimize the damage caused to biotopes.

7. Conclusions
The paper presents a comparison of the calculations of the transfer problem based on the proposed scheme with the results obtained using the scheme, which is a linear combination of the Upwind and Standard Leapfrog difference schemes [6], as well as the two-parameter difference scheme [6] of the third order of accuracy. In the norm of the grid space $L_1$ the dependences of the errors of the numerical solution of model problems on the basis of the above schemes depending on the values of the Courant numbers are given. Of the presented examples, it follows that for small Courant numbers (0.1 or less) of the proposed scheme and scheme, the resulting linear combination of the Upwind and Standard Leapfrog difference schemes, much more other, considered in the work schemes. It should be noted that the proposed scheme has a stable solution in the range of Courant numbers from 0 to 1.

Also presented are the results of modeling the problem of suspension transport on the basis of the proposed scheme, which is a linear combination of the Upwind and Standard Leapfrog difference schemes. Mathematical models of transport of suspended particles allow to predict the spread of plumes of suspension in the aquatic environment and changes in the bottom relief due to the deposition of suspended soil particles in the sediment. The calculations made allowed to consider the possibility of optimizing the areas of existing soil dumps. Optimization of the size of the areas of soil dumps allows to minimize the damage caused to biotopes.

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