Shear wave velocity and shear modulus of silty sand

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ABSTRACT

The determination of shear wave velocity ($V_s$) in soil and the associated small-strain shear modulus ($G_0$) plays an important role in many geotechnical applications. In the last decades extensive laboratory experiments have been carried out to study these two properties for clean quartz sands. Often natural sands are not clean but contain some amount of fines (referred to as silty sand in practice). The effect of fines on the shear wave velocity and shear modulus is therefore a matter of concern. This paper presents an experimental study on mixtures of clean quartz sand and crushed silica fines at different percentages of fines. A series of resonant column (RC) and bender element (BE) tests were carried out for a range of confining stresses and void ratios. It is found that $G_0$ values obtained from the BE tests are significantly greater than the ones obtained from the RC tests, indicating the importance of testing method for silty sand. For either method, however, the $G_0$ value of the mixture always decreases with an increase of fines content. An empirical model is proposed for estimating $G_0$ which allows for both the effect of fines and the stress and density dependence, and it is shown to possess a reasonably good performance.

Keywords: shear wave velocity, shear modulus, fines; sands; laboratory tests

1 INTRODUCTION

Soil is often assumed to be elastic at strain levels less than 0.001%. Characterization of the soil behavior at such small strains requires accurate determination of elastic stiffness (i.e., small-strain shear modulus, $G_0$). According to the theory of elasticity, $G_0$ can be estimated from the shear wave velocity as follows:

$$G_0 = \rho V_s^2$$

where $\rho$ and $V_s$ are the soil density and shear wave velocity, respectively. Numerous experimental studies have been conducted on clean sands, yielding a fundamental understanding of the small-strain shear modulus (see Clayton, 2011). It is widely recognized that the confining stress and the packing density play an important role in controlling $G_0$. Compared with clean sands used in research, natural sands often contain some amount of fines. Given the important role of fines in altering the liquefaction behavior of sand (Yang and Wei, 2012), the effect of fines on $G_0$ and $V_s$ becomes a matter of great concern. While there are some studies in the literature (e.g., Carraro et al., 2009; Salgado et al., 2000), the effect of fines is not yet well understood and it remains a difficult task to establish a relationship that can be used to estimate $G_0$ values for sands with fines. The effect of testing method in the determination of $G_0$ values for silty sand is also not clear.

This paper presents several interesting results from a specifically designed experimental study that was aimed to address the above concerns. A state-of-the-art resonant column (RC) apparatus incorporating the bender element (BE) function for measurement of $G_0$ and $V_s$ was used in the study. The laboratory measurements by using the two methods are compared in this paper, and based on these measurements an empirical relationship is proposed for estimating $G_0$ values of silty sand under a range of confining stresses and void ratios.

2 TEST MATERIALS AND METHOD

In this study, the silt-sized crushed silica fines of different percentages were mixed with a base material, clean Toyoura sand. The fines content covers a wide range, from 5% to 30%. The particle size distribution curves of the mixtures are shown in Figure 1 and their physical properties are summarized in Table 1.

Table 1. Properties of test materials.

|        | Toyoura sand | Crushed silica |
|--------|--------------|----------------|
| $G_s$  | 2.65         | 2.68           |
| $e_{\max}$ | 0.967       | -              |
| $e_{\min}$ | 0.633       | -              |
| $C_u$  | 1.392        | 2.182          |
| $d_{50}$ | 216 $\mu$m  | 54 $\mu$m     |
The schematic illustration of the set-up of the testing system is shown in Figure 2. The apparatus can accommodate a soil specimen of 50mm in diameter and 10mm in height, with an air-filled cell pressure up to 1MPa. The resonant column is of bottom-fixed and top-free configuration with a pair of bender elements installed on the top cap and bottom pedestal as the transmitter and receiver, respectively. Each piezoelectric element is 11mm in width and 1.2mm in thickness, with the penetration depth of 2.0mm. Specimens were prepared using the under-compaction moist tamping method to overcome the problem of segregation between the quartz sand and the silica fines. For each specimen, void ratio was used as a target for the control of the packing state. Upon the completeness of sample preparation, a stand-up suction of 25kPa was applied. The sample was then flushed in a sequence with CO₂ and de-aired water. The cell and the back pressure were increased accordingly to ensure saturation of the specimen. B-check was performed afterwards and B-value greater than 0.95 was obtained for all specimens. In each test, the stepwise isotropic confining pressure was applied as 50kPa, 100kPa, 200kPa, 400kPa, and 500kPa. In bringing the specimen to a specific confining stress level, the specimen was first consolidated for 15min at each stress level, and the corresponding deformation was measured by the internal high resolution LVDT so as to update the void ratio. This was then followed by the BE and RC testing.

### 3 RESULTS AND DISCUSSION

#### 3.1 Measurements of \( G_0 \) from BE and RC tests

In the BE tests, sinusoid input signals of various frequencies (\( f_{\text{input}} = 1\text{kHz} \) to 80kHz) were adopted. To identify the first arrival of shear wave, the output signals were compared in a whole view. For uniform glass beads of different sizes, Yang and Gu (2013) examined various methods for interpretation of BE signals for the first arrival, and found that the start-start method using the characteristic point S2 in the time domain to be most appropriate. This point was also identified in the shear wave signals in the mixtures tested, as shown in Figure 3. Here the shear wave signals in clean Toyoura sand and in a mixture of the sand and the silica fines at a percentage of 30% are compared for a representative frequency of 10kHz. To take into account the coupling effect between the pore water and the soil skeleton during shear wave propagation, the effective soil density is adopted to estimate \( G_0 \) (Yang and Gu, 2011; Youn et al., 2008). In accordance with Biot’s theory, the effective soil density is always smaller than the saturated density in the BE test due to its high frequency response, whereas the two values are nearly the same in the RC test which usually involves much lower frequencies.

![Figure 3. Received shear wave signals in clean and silty sands.](image)

Figure 4 compares the measurement of \( G_0 \) value from the BE and RC tests. Interestingly, the \( G_0 \) values from the BE tests are consistently greater than those from the RC tests even under the conditions where the effective soil density has been applied. Evidently this discrepancy is irrespective to the fines content. Refer to the signals shown in Figure 3. The upward arrows represent the arrival times back calculated from the RC test. Under otherwise similar conditions, the shear wave in the BE test arrived earlier in comparison with the time back calculated from the RC test. Similar
observation has also been obtained by Gu and Yang (2012) in testing decomposed granite samples and Toyoura sand prepared using the moist tamping method. However, no apparent discrepancies were observed in testing uniform glass beads and Toyoura sand prepared by the dry tamping method (Yang and Gu, 2013; Gu and Yang, 2012). This finding suggests that the effect of testing method is coupled with the effect of sample preparation method.

### 3.2 Effect of fines content

To further take into account the influence of void ratio, in Figure 5 the void ratio corrected $G_0$ values are plotted against the confining stress that is also normalized by a reference pressure (98kPa). The correction of void ratio was made using the following expression (Hardin and Richart, 1963; Iwasaki and Tatsuoka, 1977):

$$F(e) = \frac{(2.17 - e)^2}{1 + e}$$

(2)

It is found that the above hyperbolic expression is effective for both the clean sand and its mixtures with fines. The trend of each soil can be described using the power function as follows:

$$G_0 = AF(e)\left(\frac{\sigma^*}{P_0}\right)^n$$

(3)

where $A$ and $n$ are two constants reflecting the soil properties and fabric. The best-fit values are summarized in Table 2, giving markedly high coefficients of determination ($R^2$>0.95) for all cases of fines content tested.

The data in Figure 5 clearly indicate that $G_0$ values, either from BE or RC tests, decrease with fines content up to 30%. Accordingly, the value of $A$ successively decreases with fines content, but the value of $n$ remains almost unchanged. This finding suggests that $A$ is sensitive to the presence of fines but the stress exponent is not. Also, it is noted that the value of $n$ is greater than 1/3, a theoretical value predicted for packings of spheres using the Hertz and Mindlin theory.

### Table 2. Fitting parameters using equations 2 & 3.

| FC   | Test method | $A$  | $n$  | $R^2$ |
|------|-------------|------|------|-------|
| 0%   | $G_0$ (BE)  | 114.8| 0.39 | 0.99  |
|      | $G_0$ (RC)  | 94.3 | 0.38 | 0.99  |
| 5%   | $G_0$ (BE)  | 106.0| 0.39 | 0.99  |
|      | $G_0$ (RC)  | 89.4 | 0.38 | 0.99  |
| 10%  | $G_0$ (BE)  | 100.0| 0.39 | 0.99  |
|      | $G_0$ (RC)  | 84.4 | 0.38 | 0.97  |
| 20%  | $G_0$ (BE)  | 86.7 | 0.39 | 0.95  |
|      | $G_0$ (RC)  | 73.0 | 0.38 | 0.95  |
| 30%  | $G_0$ (BE)  | 80.8 | 0.39 | 0.97  |
|      | $G_0$ (RC)  | 68.6 | 0.38 | 0.97  |

### 3.3 Empirical relations

Given the importance of $G_0$ and $V_s$ in geotechnical engineering practice, a number of empirical models for prediction of their values have been proposed in the literature, mainly based on the experimental data on clean quartz sands. Recognizing the significant influence of fines on $G_0$ and $V_s$, an attempt is made here to propose a simple empirical model for estimating $G_0$ values for silty sand. In doing that, the general expression in equation (3) is adopted, but the parameter $A$ (in MPa) is described as an exponential function of fines content (FC) as follows:

$$A(BE) = 114.8e^{-1.3FC}$$

(4)

$$A(RC) = 94.31e^{-1.1FC}$$

(5)

For simplicity, the stress exponent can be taken as 0.39 for BE measurements and 0.38 for RC measurements.
The predictions of $G_0$ values are made using the above equations (4) and (5). In Figure 6, the predicted values are compared with the measurements from the BE and RC tests, respectively. As expected, a good performance is obtained, with the discrepancies of generally less than 10%.

4 CONCLUSIONS

This paper presents an experimental study of the small-strain shear modulus ($G_0$) of sand-fines mixtures. A resonant column apparatus incorporating the bender element function was used and a series of tests were carried out for a wide range of fines contents and for a wide range of confining stresses and void ratios.

For the bender element tests, the start-to-start method was adopted to determine the first arrival of shear wave. It has been found that under otherwise similar conditions, $G_0$ values measured from the bender element tests are always greater than those from the resonant column tests, with the former being about 20% larger, and these discrepancies appear to be irrespective of the fines content. This finding emphasizes the importance of testing method in determination of the small-strain shear modulus of silty sand. On the other hand, a reduction of $G_0$ value due to the presence of fines has always been observed regardless of testing methods.

An empirical model has been proposed which accounts for the influence of fines content as well as the stress and density dependence of $G_0$. In this model the parameter $A$ is described as an exponential function of fines content, whereas the stress exponent $n$ is assumed to be independent of fines content. This simple model exhibits a reasonably good accuracy in comparison with laboratory measurements. Further refinement and improvement of the model can be made when more data become available.

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