Confinement and the Short Type I' Flux Tube

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Abstract

We show that the recent world-sheet analysis of the quantum fluctuations of a short flux tube in type II string theory leads to a simple and precise description of a pair of “stuck” D0branes in an orientifold compactification of the type I' string theory. The existence of a stable type I' flux tube of sub-string-scale length is a consequence of the confinement of quantized flux associated with the scalar dualized ten-form background field strength $*F_{10}$, evidence for a $-2$-brane in the BPS spectrum of M theory. Using heterotic-type I duality, we infer the existence of an M2brane of finite width $O(\sqrt{\alpha'})$ in M-theory, the strong coupling resolution of a spacetime singularity in the D=9 twisted and toroidally compactified $E_8 \times E_8$ heterotic string. This phenomenon has a bosonic string analog in the existence of a stable short electric flux tube arising from the confinement of photons due to tachyon field dynamics. The appendix clarifies the appearance of nonperturbative states and enhanced gauge symmetry in toroidal compactifications of the type I' string. We account for all of the known disconnected components of the moduli space of theories with sixteen supercharges, in striking confirmation of heterotic-type I duality.
1 Introduction

Recently, a precise world-sheet description of the quantum fluctuations of a short flux tube of characteristic length $O(\sqrt{\alpha'})$ has been given, both in the bosonic and the type II string theories [1, 2]. We will show in this paper that this analysis leads to a simple and elegant description of a pair of “stuck” D0branes in an orientifold compactification of the type I’ string theory [5, 8, 7]. Away from the orientifold planes, the type I’ theory coincides with the massive IIA string theory [3, 4, 6] compactified on the interval $S^1/\mathbb{Z}_2$. This theory has $O(16)\times O(16)$ gauge fields living on the worldvolume of eight pairs of D8branes located symmetrically on orientifold planes at $X^9=0, \pi R_9$. The I’ theory is $T_9$-duality equivalent to the type IB string with 32 space-filling D9branes carrying Chan-Paton labels of the group $O(16)\times O(16)$. Upon inclusion of a Wilson line, the IB theory can be mapped by an S-duality to the $SO(32)$ heterotic string compactified on $S^1$, to the eleven-dimensional M theory compactified on $S^1\times S^1/\mathbb{Z}_2$. Following a relabeling of the coordinates $X^9, X^{10}$, this theory coincides with the strong coupling limit of the $E_8\times E_8$ heterotic string compactified on $S^1$. In summary, we have [5, 8]:

\begin{align}
\text{Heterotic} & \quad S & \quad \text{type I} & \quad T_9 & \quad \text{type I’} & \quad S & \quad \text{M theory} \\
\text{on } S_1 & \quad \mapsto & \quad \text{on } S^1 & \quad \mapsto & \quad \text{on } S^1 & \quad \mapsto & \quad \text{on } S^1\times S^1/\mathbb{Z}_2 .
\end{align}

Consider a type IB background with a pair of D1branes wrapped on the compact $X^9$ direction, in addition to 32 D9branes with $O(16)\times O(16)$ Chan-Paton labels. Let us briefly review the appearance of nonperturbative states in the spinor representations of $O(16)$ [5, 7]. A more extensive discussion of enhanced gauge symmetry in toroidal compactifications of the type I’ string theory, and the appearance of disconnected components of the moduli space of theories with sixteen supercharges, is given in the appendix. Although there are by now several algebraic and/or geometric descriptions of the $E_8\times E_8$ enhanced symmetry point in nine dimensions, including bound state analyses of D0branes, we find the following explanation of the enhanced gauge symmetry by far the simplest: a $T_9$ duality transformation maps the IB background with wrapped Dstrings into a I’ background with a pair of D0branes threading a stack of eight pairs of symmetrically placed D8branes at each of two orientifold planes. The total “width” of each stack is within a distance of $O(\sqrt{\alpha'})$. Note that this configuration preserves the full spacetime supersymmetry of the $O(16)\times O(16)$ theory. As is well-known [4, 8, 9, 10], there is a jump of $\pm \mu_8$ in the background 9-form gauge potential each time a D0brane crosses a D8brane, associated with the creation of a fundamental string connecting the D0brane to the D8brane. From the requirement of vanishing dilaton gradients at both orientifold planes, the net number of positive and negative jumps must exactly cancel. A sequence of jumps may be labeled $(\pm, \pm, \pm, \pm, \pm, \pm, \pm, \pm)$, with a corresponding gradient cancelling sequence at the other orientifold plane. Then the total number of distinct sequences gives the familiar counting, $128=2+56+70$, for an even number of plus signs. We obtain, of course, the identical counting for sequences with an odd number of plus signs. In the limit of coincidence, the fundamental strings threading the stack of D8branes at each orientifold plane go to zero length giving massless states. The additional 128 massless states at each orientifold plane thus enhances the gauge symmetry to $E_8\times E_8$.

In the absence of an external electric field, the stuck D0brane pairs threading the pairs of D8brane stacks are absorbed into the orientifold planes [3]. The dynamics of the “stuck” pair of D0branes
at sub-string length distance scales in the presence of a constant external electric field, $F_{09}$, will be the main subject of this paper. We will show that the stuck D0brane and its mirror image are connected by a type I' flux tube of characteristic length, $R \leq \sqrt{\alpha'}$, a $T_9$ duality transformation converting this to a configuration of Dstrings wrapping the $X^9$ coordinate with spatial separation $R$ in an orthogonal direction. The existence of quantized constant background $C_0$ and $B_2$ gauge potentials on the worldvolume of the D8branes, necessitated by stability, will become evident in our analysis. Consider the moduli space of the nine-dimensional $E_8 \times E_8$ theory. Dbrane charge conservation has been invoked to argue that while a single D0brane cannot move into the bulk spacetime away from the orientifold planes, a D0brane pair can do so. We will show that the analysis in [2] gives a simple and elegant solution to the problem of quantizing the fluctuations of the “stuck” D0branes in a constant external electric field, at weak type I’ coupling. We find a short distance $r^{-9}$ force between the “dressed” D8brane stack and its mirror at each orientifold plane, giving evidence for the existence of $-2$-branes coupling to the scalar $*F_{10}$ field strength. Using the heterotic-type I duality chain given above, we are led to infer also the existence of an M2brane of finite width stretched between the orientifold planes of M theory compactified on $S^1 \times S^1 / \mathbb{Z}_2$ [5].

The stability of a type I’ flux tube of sub-string scale length will be linked to the confinement of quantized flux associated with a background scalar dualized ten-form field strength, $*F_{10}$. Consider the $T_9$-dualized IIB theory with a pair of Dstrings wrapping the $X^9$ direction. The necessity of quantized constant background $B_2$ and $C_0$ fields in this IIB background can be inferred from the identification of a corresponding soliton solution of the IIB theory. The IB background has the classical geometry of a pair of massive solitonic strings, wrapped about the compact $X^9$ direction. A solitonic solution of the massive N=2A supergravity [3, 9] with precisely these characteristics was found in [10]; it requires background $C_0$ and $B_2$ gauge potentials. Application of the SL(2,$\mathbb{Z}$) U-duality symmetry of the twisted $T_9$-duality equivalent IIB string theory implies a quantization of the mass parameter of the massive supergravity in inverse units of the radius $R_{09}$ [9]. We also infer the existence of an entire $(p, q)$ multiplet of massive solitonic type II strings [9]. The geometry of the solution in [10] describes a solitonic string perpendicular to a D8brane, but which breaks the $SO(1, 1)$ symmetry unlike the macroscopic fundamental strings of [11]. We consider IIB solitonic strings wrapped about the compact $X^9$ direction. The type IIB theory is self-dual, and an S-duality transformation maps this object into a Dstring wrapped about the $X^9$ direction. Consider lifting a IIB background with wrapped Dstrings into a corresponding solution of the IB string theory. For constant background $C_0$ and $B_2$ fields, this is implemented by simply modding out by orientation reversal. A $T_9$-duality transformation maps the IB background to a I’ background with a D0brane intersecting a D8brane in a point: a “stuck” D0brane, pinned to lie on an orientifold plane of the I’ theory. Embedding this soliton in a compactification of the anomaly-free I’ theory with 8 D8brane pairs at each orientifold plane, completes the picture described above.

Our analysis was motivated by analogous considerations of short electric flux tubes in the bosonic string [1, 12]. A world-sheet description of a short electric flux tube in the bosonic string was given in [1], accomplishing in part the long-desired goal of a consistent theoretical framework for the short distance behavior of the Wilson loop expectation value [13]. The bosonic string does not have RR gauge potentials. The analysis in [1] did not address the underlying bosonic string dynamics that can give rise to a stable electric flux tube of short length. Remarkably, precisely this question has been addressed in recent studies of tachyon condensation in open string field theory [14, 2], giving evidence for the formation of an electric flux tube with confinement scale of order the fundamental string length. The open strings are confined to a spatial volume of order
$\alpha'$. Precisely this behavior was found in [1]. The heavy quark-antiquark pair interact via a scale invariant $-1/r$ potential through single photon exchange, for spatial separations restricted to the distance regime: $2\pi\alpha' \tanh^{-1} v \leq r \leq \alpha'$, where $v$ is the relative velocity of quark and antiquark. The lower bound is the minimum distance that can be probed by a semi-classical heavy quark in this background [13]. The coefficient of the potential is dimensionless, a measure of the effective number of degrees of freedom in the critical bosonic string: $V(r) = -(d-2)/r$. Corrections to the static term are parameterized by the following background field dependent variables, $z = |r_{\text{min}}/r|^2$, $u^2/\pi$, and $|u|^2$, where $u^i = \tanh^{-1}(\alpha' F^{0i})$.

In [2], this world-sheet analysis was extended to the type II superstring theory, but with important differences due to the imposition of consistency conditions that both eliminate the tachyon and require the absence of a static force between heavy quark and antiquark. The chosen configuration of quark loops therefore preserves only one-quarter of the supersymmetries of the type II theory, giving rise to a short distance potential that is qualitatively different from the result quoted above:

$$V_{\text{super}}(r, u) = -\frac{|u|^4}{r^{d-2}} 2^{4\pi^2/\alpha'^4} \Gamma\left(\frac{9}{2}\right) + O(|u|^6). \quad (2)$$

The leading term in the potential is now \textit{dimensionful}, indicating that something more than the single photon exchange described above is at work here. Note the absence in the potential of $O(|u|^4, |u|^2)$ terms, and any subleading corrections of $O(|u|z)$. Since the tachyon has been explicitly eliminated, the question arises as to what plays the role of the confining tachyon field dynamics of the bosonic string? We began by noting that the general reasoning in [2] can nevertheless be applied here, but with confinement originating in the dynamics of background RR and NS gauge fields. This observation led us to consider the potential for the background gauge fields of the massive IIA supergravity theory, dimensionally reducing to nine dimensions where it is T-duality equivalent to a twisted dimensional reduction of the massless IIB supergravity [3]. This theory is very rich and should offer an interesting arena to study the dynamics of massive background gauge fields. Note that the configuration of massive solitonic strings breaks spacetime supersymmetry, suggesting that a full analysis of the moduli space dynamics, including possible tachyon instabilities along the lines of [4, 12], could be of interest. However, as explained at the outset, the answer to our question regarding the short type I' flux tube turns out to be much simpler. It relies solely on the quantization of scalar $*F_{10}$ flux.

Nevertheless, consideration of this theory and its solitonic solutions yields some important insights as explained above. Following a brief introduction to the massive IIA supergravity theory in section 2, we identify a particular classical soliton, originally found in [10], which takes the form of a solitonic string perpendicular to the D8brane of the massive IIA supergravity. We compactify on a circle the coordinate $X^9$, giving a wrapped soliton string. The soliton is $T_9$-duality equivalent to a IIB soliton string carrying quantized $C_0$ charge. Upon S dualizing the wrapped soliton string in the self-dual IIB theory, we obtain a D1brane wrapped about the $X^9$ direction carrying quantized 2

\footnotetext{We follow the conventions in [3], where the $B$ field in the world-sheet action is dimensionless while the Maxwell field strength $F$ is dimensionful. This also holds for the Dirac-Born-Enfield worldvolume action. The precise form of the $\alpha'F$ corrections, inclusive of numerical factors, appears in Eq. (48) of [3]. Note that no restrictions are placed on the value of the background fields other than the upper bound $F^{0i} \leq F_{\text{crit}}^{0i}$ in the electric case. We should also note that, strictly speaking, the analyses in [3,1] are carried out for magnetic background $F^{0i}$ fields, since the coordinate $X^9$ is obtained by an analytic continuation, $X^j \rightarrow iX^9 \pm \mathbb{R}$. The result can be appropriately interpreted in either case.}
C_0 charge. The flux of the RR scalar is necessarily confined: there is no corresponding propagating field. Thus, a configuration of a pair of closely separated Dstrings with unit winding number in an orthogonal compact direction is inherently stable. The quantized C_0 charge will give rise to a quantized theta angle for the D9brane worldvolume gauge fields in the IB theory as described below.

A significant simplification in the analysis of the short flux tube comes about by lifting the configuration of wrapped soliton strings into the IB string theory, which we do in section 3, restricting at the same time to constant background fields. Using simple world-sheet techniques of perturbative open and closed string theory [16], and the well-known type I-heterotic string duality [5], we give a solution to the problem of quantizing the fluctuations of the short type I' flux tube. We show that the result in [2] leads to a precise computation of the potential between a pair of “dressed” D8branes at short spatial separations of order r^2<\alpha' in the I' theory. Note that, as in [15], the presence of a background electric field, F_{09}, is crucial in enabling a probe of sub-string length distance scales, effectively resolving the stuck D0brane pair. We quote the systematic expansion in powers of \alpha^4 F_{09}^4 of the potential derived in [2], arising in the presence of a constant external electric field.

Finally, we come to the interesting question of the strong coupling dual of the short type I' flux tube. An S duality transformation maps the short type I' flux tube into a M2brane configuration in M theory [4, 8]: a finite width membrane of characteristic width O(\sqrt{\alpha'}), stretched between the orientifold planes of the eleven-dimensional theory. The S-duality leaves the Yang-Mills gauge fields unchanged. We infer that the membrane configuration carries two species of quantized instanton charge. The appendix contains auxiliary material on the appearance of nonperturbative states and enhanced gauge symmetry in toroidal compactifications of the IB theory with wrapped Dstrings in addition to D9branes. As an illustration of the usefulness of the extended heterotic-IB-IIB duality chain, exploited extensively in our analysis above, we provide some missing details of the heterotic-type I duality map, accounting for all of the disconnected components of the moduli space of the theory with sixteen supercharges. This resolves some of the puzzles raised in [20].

2 Massive String Solitons and the Type II Flux Tube

We begin by adapting the arguments for the tachyon potential in [12] to that of the massive background gauge fields in the type II theory. The D=10 massive IIA supergravity [3] has bosonic field content \((G, \Phi, C_3, C_1, B_2)\), where the NS-NS two-form, \(B_2\), is massive due to the generalized Higgs mechanism, and the \(C_n\) are RR n-form potentials. Using the field redefinitions given in section 2 of [3], and with the conventions of [8], the action takes the form:

\[
S_{IIA} = \frac{1}{2\kappa_{10}^2} \int d^{10}X \sqrt{-G} \left\{ e^{-2\Phi} [R + 4\partial^\mu \Phi \partial_\mu \Phi - \frac{1}{2} |H_3|^2] - \frac{1}{2} |F_2|^2 - \frac{1}{2} |\tilde{F}_4|^2 - \frac{1}{2} M^2 \right\} - \frac{1}{4\kappa_{10}^2} \int d^{10}X \left[ B_2 \wedge dC_3 \wedge dC_3 + M dC_3 \wedge B_2 \wedge B_2 + M^2 B_2 \wedge B_2 \wedge B_2 \wedge B_2 \wedge B_2 \wedge B_2 \right],
\]

(3)

We assume a mostly positive signature spacetime metric and normalize the kinetic term as follows: \(|F_\mu|^2 = \frac{1}{p!} G^\mu_1 \cdots \mu_p \cdots G^\nu_1 \cdots \nu_p F_{\mu_1 \cdots \mu_p} F_{\nu_1 \cdots \nu_p} \). The wedge products simply denote contractions with the appropriate epsilon tensor; there is no metric dependence. Finally, the gravitational couplings satisfy the relation \(\kappa_{d}^2 = 2\pi R_d \kappa_{d-1}^2\).
where the gauge invariant field strengths are defined:

\[ F = dA + MB_2, \quad H_3 = dB_2, \quad \bar{F}_4 = dC_3 - C_1 \wedge F_3 + \frac{1}{2} MB_2 \wedge B_2 \quad . \quad (4) \]

By introducing a 9-form potential, \( C_9 \), acting as a Lagrange multiplier for the mass the action may be rewritten in terms of the RR field strength \( F_{10} \). Note that the ten-form field strength in the RR sector of the IIA theory can be Hodge-dualized to a scalar field strength \( *F_{10} \).

We assume that the \( X^9 \) Neumann direction is compact, a circle of radius \( R^9 \). Compactifying the massive type IIA supergravity on \( S^1 \) yields the massive type II supergravity in nine dimensions with the field content:

\[ (G_{\mu\nu}, \Phi, C_{\mu\nu\lambda}, C_\mu, B_{\mu\nu}) \oplus \left( C_{\mu\nu9}, B_{\mu9}, \frac{G_{\mu9}}{G_{99}}, \sqrt{-G_{99}}, C_9 \right) , \quad (5) \]

where the indices \( \mu, \nu \) now run from 0, \( \cdots \), 8. Note that the subscripts denote spacetime indices here. Under a target space \( T \)-duality transformation, the D=9 massive IIA theory is mapped to the twisted dimensional reduction of the massless IIB theory described in \( \llbracket 9 \rrbracket \). The field content above is therefore isomorphic to the set:

\[ \{ G_{\mu\nu}, \Phi, C_{\mu\nu\lambda}, C_\mu, B_{\mu\nu}, B_\mu, \tilde{A}_\mu, e^\chi, l_c \} , \quad (6) \]

where the last four entries coincide with those in Eq. \( \llbracket 3 \rrbracket \) above; the remaining notation is self-evident. Owing to the \( T_9 \) equivalence with the IIB theory which has an inherent SL(2,R) symmetry, the p-form potentials are found to fit naturally into \( SL(2, R) \) doublets: \( \tilde{H}_{\mu\nu}^{(i)}, \tilde{F}_{\mu
u}^{(i)}, \tilde{B}_{\mu
u}^{(i)}, \) and \( \tilde{A}_{\mu}^{(i)} \), where the index \( i=1, 2 \) distinguishes gauge fields originating in the NS-NS and RR sectors. We will interchangeably use this notation when convenient. The action takes the form \( \llbracket 9 \rrbracket \):

\[
S_{\Pi} = \frac{1}{2\kappa_9^2} \int d^9x \sqrt{-G} e^{-2\Phi} [ R + 4\partial^\mu\Phi \partial_\mu\Phi - |d\chi|^2 - \frac{1}{2} |\tilde{H}_3^{(1)}|^2 - \frac{1}{2} e^{2\chi} |\tilde{F}_2^{(1)}|^2 - \frac{1}{2} e^{-2\chi} |dB_1|^2 ] \\
- \frac{1}{4\kappa_9^2} \int d^9x \sqrt{-G} \left\{ e^\chi [M^2 + |\tilde{H}_3^{(2)}|^2 + |\tilde{F}_2^{(2)}|^2 + |F_4|^2 + e^{-\chi} |dl_c - MB_1|^2 ] \\
- \frac{1}{4\kappa_9^2} \int d^9x \left[ \tilde{F}_4 \wedge \tilde{F}_4 \wedge dB_1 + \tilde{F}_4 \wedge \tilde{H}_3^{(1)} \wedge \tilde{H}_3^{(2)} \right] \right. . \quad (7)
\]

We have made the field redefinitions \( \llbracket 9 \rrbracket \):

\[
\begin{align*}
\tilde{A}_1^{(1)} &= C_1 - e^\chi \tilde{A}_1, \quad \tilde{A}_1^{(2)} = \tilde{A}_1 \\
\tilde{B}_2^{(2)} &= C_2 - C_1 \wedge B_1 + e^\chi l_c \tilde{A}_1 \wedge B_1, \quad \tilde{B}_2^{(1)} = B_2, \\
\tilde{F}_2^{(1)} &= d\tilde{A}_1^{(1)} + l_c d\tilde{A}_1^{(2)} + M(B_2 - \tilde{A}_1^{(2)} \wedge B_1), \quad \tilde{F}_2^{(2)} = d\tilde{A}_1^{(2)} \\
\tilde{H}_3^{(1)} &= dB_2 + \tilde{A}_1^{(2)} \wedge dB_1, \quad \tilde{H}_3^{(2)} = dC_2 - \tilde{A}_1^{(1)} \wedge dB_1 - l_c dB_2 - MB_1 \wedge B_2 \\
\tilde{F}_4 &= F_4 + \tilde{A}_1^{(1)} \wedge dB_2 - e^\chi B_1 \wedge \tilde{A}_1^{(1)} \wedge d\tilde{A}_1^{(2)} + \frac{1}{2} MB_2 \wedge B_2 - MB_2 \wedge \tilde{A}_1 \wedge B_1 ,
\end{align*}
\]

The field content and action can be mapped to that of the twisted dimensionally reduced massless IIB theory, \( C_{\mu\nu\lambda} \rightarrow C_{\nu\mu\lambda}, C_\mu \rightarrow C_{9\mu}, l_c \rightarrow C_0 \), with the remaining NS potentials arising from the particular dimensional reduction and field redefinitions given in section 5 of the first of the references in \( \llbracket 4 \rrbracket \). The D=9 type II theory has an \( SL(2, Z) \) U-duality symmetry. Consider the \( SL(2, Z) \) transformation parameterized, \( a=d=1, b=n, \) and \( c=0 \). Since,

\[
C_0 \rightarrow C_0 + n, \quad (p, q) \rightarrow (p - nq, q), \quad n \in Z , \quad (9)
\]
where \((p, q)\) denotes the bound state of \(p\) fundamental and \(q\) Dstrings, we find that a fundamental string, \(q=0\), is mapped to another fundamental string, but one that carries \(n\) units of RR scalar charge. The significance of this periodicity will become apparent later on. Note that as a consequence of the relation between \(l_c\) and \(C_0\), the mass parameter of the massive IIA supergravity is likewise quantized in units of \(R_9\):\
\[
M = \frac{n\alpha'}{R_{9B}}, \quad n \in \mathbb{Z}.
\] (10)

The nontrivial potential for the massive \(B\) field in the massive IIA supergravity theory is reminiscent of that for the tachyon field in the bosonic string. An analysis of both kinetic and potential terms in this action would be complicated. However, it is always possible to work in a scaling limit in which the potential dominates over the kinetic terms in the action. This is the essential observation used in \([12]\), permitting arguments in favor of the formation of localized macroscopic closed string solitons.

Accepting for the moment the ansatz that a stable and localized flux tube can form in the type II theory, what are its consequences? We begin by identifying it with a particular soliton solution of the \(D=9\) massive IIA supergravity. Soliton solutions of the massive IIA supergravity theory have been studied in \([10]\). Among those with vanishing 3-form background, and preserving also an \(SO(8)\) symmetry, is the “massive” string:

\[
B_{09} = -\frac{1}{2}f(r, z)dz \wedge dt, \quad f(r, z) = 1 + \frac{kr^6}{r^6} + \frac{M(z - M r^2/16)}{r^6},
\]

\[
e^{-2\Phi} = f^{-1}, \quad l_c = e^{\chi} = f^{3/8},
\]

\[
ds^2 = f^{3/4}\left((dz)^2 + (dt)^2\right) + f^{-1/4}\left((dr)^2 + r^2(d\Omega_7)^2\right),
\] (11)

where we have defined, \(t=X_0,\ z=X_9,\) and \(r\) is the radial coordinate in the 8-dimensional space perpendicular to the massive string. It was noticed in \([10]\) that the spacetime metric of the massive string soliton reduces to that of the macroscopic fundamental strings found in \([11]\), upon setting the mass parameter, \(M,\) to zero. Note, however, that even for \(M=0,\) there is a nonvanishing background for the RR scalar. Compactifying \(X^9\) on a circle, \(S^1,\) of radius \(R_9,\) gives a wrapped massive string soliton.

From the U-duality of the equivalent IIB theory, we can infer both the quantization conditions in Eqs. (9), (10), and also the existence of an entire \((p, q)\) multiplet of massive string solitons. The type IIB string theory is self-dual, and an S-duality transformation exchanges RR and NS-NS backgrounds, mapping the singly wound massive fundamental string to a multiplet of massive \((p, q)\) strings wrapped on the circle \(S^1: \)

\[
(1, 0) \quad S \quad (0, 1) \quad C_0 \quad \rightarrow \quad C_0 + n \quad (-n, 1).
\] (12)

The quantum number \(n\) corresponds to instanton number. This leads to a theta term in the action of the Yang-Mills gauge fields lying in the worldvolume of the D1branes, obtained in the last step of the duality chain. The symmetry \(C_0 \rightarrow C_0 + n\) corresponds to the periodicity of the theta angle, \(\theta \rightarrow \theta + 2n\pi.\) Fixing this symmetry nonperturbatively restricts the soliton to carrying unit instanton winding number, \(n=1.\)
Finally, consider a pair of massive solitonic strings wrapped about the $X^9$ direction, with possible spatial separation $R$ in a direction orthogonal to $X^9$. Under S-duality, we obtain a pair of D1branes coupled to a background NS two-form field. Following [12], let us “tune” the potential to the point in the moduli space corresponding to the IIB vacuum with vanishing D1brane charge.

This finally brings us to the setup in [2]: the Wilson loops are loops of closed string with Dirichlet boundary conditions for all embedding spacetime directions, but in a IIB vacuum with vanishing energy density for the RR $F_3$ field. In the “$n$” theta vacuum the Wilson loops carry $n$ units of instanton charge. What is the significance of this charge? The answer is a form of flux confinement, although for reasons quite different from those at work in the tachyon field dynamics discussed in [12]. This question can be addressed much more clearly upon lifting the IIB solitons into a background of the type I theory, as we now demonstrate.

3 Flux Quantization and the Type I Flux Tube

To address the question posed above, we begin by noting that the mass parameter of the massive IIA theory can be expressed as the expectation value of the IIA scalar field strength, $\ast F_{10}$, with trivial equation of motion:

$$d \ast F_{10} = 0, \quad \ast F_{10} = \text{constant} \ .$$

($13$)

$\ast F_{10}$ does not give rise to a propagating field, but can nevertheless contribute to the vacuum energy density. Under T$_9$-duality, we have mapped the IIA background to a background with quantized expectation value for the IIB RR scalar, $C_0$. Open strings terminating on the Wilson loops and carrying $\ast F_{10}$ charge would be confined by the resulting linear potential. Why is the confinement scale as small as $\alpha'$? We don’t have a good answer, except to point out that this is consistent with the assumption that the end-points of open strings carry $\ast F_{10}$ charge: they would have to be confined. Observations similar to this have appeared in many places in the Dbrane literature [3, 8, 23]. To the best of our knowledge, this is the first time the notion of flux quantization as an avenue for confinement has been applied to macroscopic soliton strings.

A much cleaner analysis emerges if we restrict ourselves to constant background fields, simultaneously lifting the soliton configurations described above into type I string theory. We will find that the computations become remarkably simple, giving concrete evidence for the validity of our conjecture. We will use only perturbative techniques of open and closed string theory in constant background fields [16], as well as the well-known type I-heterotic string duality [4]. The action for the type IIB string theory, dimensionally reduced to D=9, is obtained by setting to zero the IIB fields, $C_0$, $B_2$, $C_4$, and also their dimensional reductions, $B_1$ and $\bar{A}_1$, in Eqs. (7) and (8). This leaves us with the simplified action:

$$S_1 = \frac{1}{2\kappa_9^2} \int d^9X \sqrt{-G} e^{-2\Phi} [R + 4\partial^\mu \Phi \partial_\mu \Phi - |d\chi|^2]$$

$$- \frac{1}{4\kappa_9^2} \int d^9X \sqrt{-G} e^\chi [|\tilde{F}_3|^2 + |\tilde{F}_2|^2] - \frac{1}{2g_9^2} \int d^9X \sqrt{-G} e^{-2\Phi} \text{Tr} |F_2|^2 \ ,$$

($14$)

Such a tuning is hypothetical in the framework of [4], given the absence of the tachyon field, but it helps clarify the physical interpretation of these results.

We should note here the early observation in [4], and comments following Eq. (12.1.23) in [8].
where $F_2$ is the generalized DBI two-form field strength on the world-volume of D9-branes wrapped on the circle $S^1$, originating in the open string sector. We have defined:

$$\tilde{F}_3 = dC_2 - \frac{k_2^{10}}{g_{10}^2} \text{Tr}_V \left[ A_1 \wedge dA_1 - i \frac{2}{3} A_1 \wedge A_1 \wedge A_1 \right], \quad \tilde{F}_2 = dC_{10}, \quad F_2 = dA_1, \quad e^\chi = \sqrt{G_{99}}.$$  

(15)

where the trace denotes the sum over the vector representation of the nonabelian gauge group, and $A_1$ is the DBI vector potential. The $Z_2$ moded orientifold projection leaves open the possibility of half-integer shifts in the constant value of the background $B$ field, and integer shifts in the constant value of the RR scalar.

The presence of quantized constant $C_0$ and $B_2$ background fields on the worldvolume of the D9-branes shows up in Chern-Simons couplings to the DBI field strength:

$$I_1 = \int d^{10} X \, C_0 \, \text{Tr}_V (B_2 \wedge F_2 \wedge F_2 \wedge F_2 \wedge F_2)$$

$$I_2 = \int d^{10} X \, \text{Tr}_V (B_2 \wedge F_2 \wedge F_2 \wedge F_2 \wedge F_2).$$  

(16)

There has been extensive analysis of type I orientifolds with half-integer background $B_{\mu\nu}$ fields [20]. Under heterotic-type I duality, such orientifolds are known to map into variants of the CHL string with $B_2$ mapped to a quantized theta angle in the heterotic string theory [22]. Consider the massive string soliton given in Eq. (11). The solution has an asymptotically flat limit when $M \to 0$ and the background fields take the constant values, $B_{09} = \frac{1}{2}$, $l_c = 1$, and $e^\chi = 1$ (this is with the radius of the $X^9$ coordinate rescaled to unity). At this particular point in the moduli space of constant backgrounds, the orientifold projection can be carried out giving a corresponding soliton string in the type I theory. Recall that the mass parameter is quantized in units of the radius $R_9$. The IB soliton is stable precisely because it supports RR gauge fields, including the quantized $C_0$ flux. A much simpler description of the type I soliton with unit winding number at weak type I string coupling is obtained from the world-sheet analysis of [2], to which we now turn.

### 3.1 Quantum Fluctuations of the Short Type I' Flux Tube

Consider quantizing the fluctuations of a short flux tube linking the “stuck” D0brane with its mirror image in the D=9 I’ background with 8 D8brane stacks at each of two orientifold planes in the presence of a constant external electric field, $F_{09}$. As explained above, the worldvolume of the D8branes supports, in addition, quantized constant background fields, $B_2$ and $C_0$. Consider the duality relations [8]:

$$g_{1'} \propto g_I R_{9I}^{-1}, \quad R_{9I'} \propto R_{9I}^{-1}.$$  

(17)

Thus, for small type I string coupling, the type I’ theory is always weakly coupled even at infinite $R_{9I}$ radius. The effect of the background electric field is to induce a background field dependent force between the solitons, thus resolving the “stuck” D0brane pair at sub-string length distance scales [15]. T-orbifold maps this to a configuration of wrapped Dstrings with small spatial separation $R$ in the $X^8$ direction, in a constant external electric field, $F_{09}$, lying in the worldvolume of 32 space-filling D9branes carrying $O(16) \times O(16)$ Chan-Paton labels, in addition to the quantized constant $B_2$.

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It would be interesting to investigate type I backgrounds in $D<10$ with quantized constant background 5-form field strength. We are not aware of such an analysis.

Type I soliton strings with similar solution for the metric and dilaton fields were obtained in [17]. An important difference here is the presence of the RR scalar charge.
We have included the antisymmetric tensor field, $B_{90}$ of the underlying type II theory. In the presence of a constant background electric field, consistency conditions were chosen so as to preserve one quarter of the spacetime supersymmetries. This defines the supersymmetric Wilson loop boundary value problem solved in [2], where the conditions on the world-sheet are Dirichlet in all ten embedding coordinates of the type I string. To each other. We begin with the open string boundary conditions in the presence of a constant endpoint of the stretched string acquire a constant velocity component in the $X^9$ direction. To the Dirichlet boundary conditions. Gauge fixing world-sheet reparameterizations of the cylinder $X^{0} \equiv \text{Minkowskian time}$, the spatial separation of the endpoints of the stretched string takes the form: $r^2 = R^2 + v^2 \tau^2$, where $\tau$ is the zero mode of Minkowskian time [8].

This leads naturally to the formulation of the string path integral with boundaries on fixed curves in an embedding spacetime first considered in [24]. Let us briefly summarize the computation of $Z$ as given in [4]. The classical action for the stretched open string of length $r$ is simply $S = r^2 l/4\alpha'$, where we have assumed the fiducial world-sheet metric $ds^2 = l^2(d\sigma^1)^2 + (d\sigma^2)^2$, with $l$ the intrinsic length of the world-sheet boundary. The one-loop vacuum amplitude with boundaries on fixed curves, $C_i$, $C_f$, can be rewritten in the form of a potential [13, 8, 2]:

$$A = -i \int_{-\infty}^\infty d\tau \ V_{\text{loop}}[r(\tau)]$$

$$= -i \int_{-\infty}^\infty d\tau \int_0^l dl \ (2\pi^2 \alpha'/l)^{-1/2} \tanh(u) \ e^{-r^2 l/4\alpha'} Z[l, u] ,$$

where $Z$ is the gauge fixed one loop string amplitude including fluctuations of all world-sheet degrees of freedom, and the factor in square brackets is the normalization for the integral over the zero mode, $X^0$. Note that the zero modes for coordinates $X^1$ through $X^9$ are absent in the path integral due to the Dirichlet boundary conditions. Gauge fixing world-sheet reparameterizations of the cylinder metric, including the Jacobian from boundary diffeomorphisms originally computed in [24], gives the measure for the open string modulus, $l$, obtained in [1]. Supersymmetrizing that result, and upon including quantum fluctuations of both bosonic and fermionic world-sheet fields, we obtain the one-loop amplitude derived in [3]:

$$Z[l, u] = \frac{1}{2} N^2 \int_0^\infty dl \ \left\{ \frac{[g(\frac{l}{2})^{-3}]}{i \Theta_1(ul/2\pi, \frac{l}{2})} \sum_{(\beta, \alpha)} C^\beta_\alpha \Theta^3_{(\beta, \alpha)}(0, il/2) \Theta(ul/2\pi, il/2) \right\} ,$$

where we include a factor of $N^2$ for $N$ allowed values for the Chan-Paton index labeling a Wilson
loop wrapped about the $X^9$ coordinate. The labels $\beta, \alpha\in\mathbb{N}$ sum over spin structure and the open string boundary conditions on world-sheet fermions, the four possible choices denoting R-R, R-NS, NS-R, and NS-NS sectors of the closed string, from the viewpoint of the closed string tree channel. Requiring the absence of both the tachyon and a zero field (static) vacuum energy density implies the phases: $C^0_0=\ldots=C^i_0=-C^j_1=-C^k_1=1^\mathcal{I}$.  

The short distance behavior of the one-loop amplitude is dominated by the term with the massless open string states circulating around the loop. We will perform the integration over $l$ for this term, giving the vacuum energy density inclusive of the quantum fluctuations of the massless open string modes. The result has a systematic expansion in powers of $\tanh^{-1}(2\pi\alpha^4 F_9)=u$, for weak fields. It takes the precise form $^3$:

$$V(r, u) = -N^2(8\pi^2\alpha')^{-1/2} \int_0^\infty dl \ e^{-\pi^2 l/2\pi\alpha'} \frac{1}{2} \frac{\sinh(t)}{u} \left[12 + 4\cos(2ul) - 16\cos(ul)\right]$$

$$= -N^2(8\pi^2\alpha')^{-1/2} \int_0^{\pi/u} \frac{d\ell}{u} \ e^{-\pi^2 l/2\pi\alpha'} \frac{l-1/2}{u} \frac{\tanh(u)}{u} \left[\sum_{k=1}^{\infty} C_k(u) 2^k + \sum_{k=1}^{\infty} \sum_{m=1}^{\infty} C_{k,m}(ul)^{2(k+m)}\right].$$

(21)

where the numerical coefficients in the sum are given by:

$$C_k = \frac{4(-1)^k(2^{2k} - 1)}{(2^n)!}$$

$$C_{k,m} = \frac{8(-1)^k(2^{2m}-1 - 1)}{(2^n)!|B_{2m}|} \left(\frac{2^{2k} - 4}{2^m} \right).$$

(22)

The $B_{2m}$ are the Bernoulli numbers. Note that the $k=1$ term vanishes in both sums and the leading background field dependence of the amplitude is $O(\alpha'^4 F^4_9)$.

Integrating over $l$ gives a systematic expansion for the potential in powers of $u^2/r^4$. As in $^4$, we identify a dimensionless scaling variable, $z=\gamma_{\text{min}}^2/r^2$, where $r_{\text{min}}^2=2\pi\alpha'u$. We find that the background field dependent corrections to the leading term in the potential are succinctly expressed as a convergent power series in the single variable $z$:

$$V(r, u) = -N^2(8\pi^2\alpha')^{-1/2} \frac{\tanh(u)}{u} \frac{r}{\sqrt{2\pi\alpha'}} \left[\sum_{k=1}^{\infty} C_k z^{2k} \gamma(2k+1/2, \pi/z) \right.$$

$$+ \sum_{k=1}^{\infty} \sum_{m=1}^{\infty} C_{k,m} z^{2(k+m)} \gamma(2(k+m)+1/2, \pi/z)\biggr].$$

(23)

The existence of a critical limiting value for the external electric field sets an upper bound on the regime of validity for this result. With the restriction $z<1$, and for weak fields, $u<<1$, the expression in Eq. (23) becomes increasingly accurate.

Now consider lifting this result into the unoriented type IB theory. In the $T_9$-dualized $I'$ theory, the effect of the constant external electric field, $F_9=\partial_a A_9$, is to shift the locations of the stacks of

\footnote{We are careful to distinguish $V_{\text{loop}}$ from the potential between soliton strings wrapped on the worldvolume of $n$ coincident, and space-filling, D9branes. The distinction is the contribution to the vacuum energy density: the expression $V_{\text{loop}}$ holds in the IIB vacuum without Dbranes, except for the (confined) flux of the RR scalar probed at short distances, $r<\alpha'$. The corresponding interpretation in the IB theory is explained below.}
D8branes off of the orientifold planes:

\[ W = (e^{i\theta}, e^{-i\theta}, \ldots, e^{i(\pi-\theta)}, e^{-i(\pi-\theta)}), \quad X^9(0) = \theta R_{9I}, \quad X^9(1) = (\pi - \theta) R_{9I}, \quad \theta_I = i 2\pi R_{9I} A_9^I, \quad (24) \]

the index \( I \) labeling the D8branes. We assume the constant values: \( \theta, \pi - \theta \), for D8brane stacks at each of two orientifold planes. Each stack is paired with its mirror image in an orientifold plane, and the spatial separation of a D8brane and its image is assumed to be small, a distance of \( O(\sqrt{\alpha'}) \).

The theory has \( U(8)\times U(8) \) gauge symmetry on the worldvolume of the D8branes.\[ \text{Note that this configuration gives vanishing dilaton and metric gradients in the space in between the two stacks.} \]

The result of the electric field has been to induce a separation in the \( X^9 \) direction of a given stack of D8branes and its mirror branes, symmetrically about each of the orientifold planes. Since we are assuming weak fields, the separations are small, \( \Delta X^9 = \frac{2\pi\alpha'F_{09}}{\Delta X^0} \). Parameterizing the constant velocity component as above, we see that the spatial separation of the D0brane pair is given by the expression obtained above, \( r^2 = R^2 + v^2 \tau^2 \), with \( \tau \) the zero mode of Minkowskian time. The proper time evolution of the short flux tube connecting the D0brane and its mirror, of spatial length \( r \), gives a cylindrical world-sheet with boundaries, respectively, on the worldvolumes of a stack of D8branes and its mirror stack. The orientifold plane slices the fiducial cylindrical world-sheet in half along the \( \sigma^2 = \frac{1}{2} \) coordinate axis, where \( \sigma^2 \) is closed string “proper” time. The resulting world-sheet can be understood as follows. A Mobius strip is a disc with boundary \( B_1 \), with a crosscap sewn into its interior. Call the circle in the surface bounding the crosscap, \( C \). Now consider cutting open an additional hole, \( B_2 \), in the surface, along a circle circumscribed \textit{inside} the crosscap region. The crosscap is effectively removed, and the resulting surface is orientable, with two boundaries. Thus, the world-sheet joining the closed world-lines of the D0brane pair has the topology of a cylinder. As a consequence, the result for the potential given in Eq. (21), with \( N=8 \), is unchanged for the type I' flux tube, and its physical interpretation is now clear. The potential corresponds to the short distance interaction of a pair of “dressed” D8brane stacks: spatially extended objects with 8 noncompact spatial dimensions, carrying both D8brane and D0brane charge, in a generalized background electric field, \( F_{09} \). A quick check shows that the \( r \) dependence is as expected, an \( r^{-9} \) potential corresponding to objects with \( \ast C_9 \) charge in ten dimensions.

We close by noting that the “dressed” D8brane has the correct properties to be identified with a \(-2\)-brane, a conjectured BPS object in the massive D=9 supergravity (see comments in [18] and [19]). Notice that, although we have eliminated the tachyon in our framework, the identification of a D\(-2\)-brane as a D-instanton sphaleron in the full configuration space of the massive IIA theory [19] is not inconsistent with our analysis. Also, modulo numerical factors, which are anyhow suppressed in reference [18], the coefficient of the potential obtained above fits the form of the nonlinear brane-tension relations derived in [18] as a consequence of generalized RR flux quantization: \( 2^4 \pi^{7/2} \alpha'^4 = \tau_0 \tau^{-2} \kappa^2 \). Notice that the \( u^4 r^{-9} \) dependence of the short distance potential also fits the expected behavior [13] for a BPS state, identical at short and long distances. This is a bit mysterious to us at the moment.

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9If the \( X^9 \) coordinate is interpreted as resulting from an analytic continuation, \( X^m \rightarrow iX^9 \), the quantized background \( B_{m9} \) field can imply a further reduction of the D9brane gauge symmetry to a single \( U(8) \) (see appendix) [2].
3.2 Strong Coupling Dual of the Short Type I' Flux Tube

Finally, we come to the interesting question of the strong coupling dual of the short type I' flux tube. The type I' theory compactified on $S^1$ has as its strong coupling dual the eleven dimensional M theory compactified on $S^1 \times S^1 / \mathbb{Z}_2$:

$$R_{10M} \propto g_{I'}^{2/3}, \quad R_{9M} \propto g_{I'}^{-1/3} R_9,$$

where we relabel the directions: $(9, 10)_{I'} \rightarrow (10, 9)_M$. Thus, the $X^{10M}$ direction is a segment with boundaries on orientifold planes, each carrying 8 pairs of coincident D8branes. M theory on $S^1 \times S^1 / \mathbb{Z}_2$ is the strongly coupled $E_8 \times E_8$ heterotic string theory. Using the basic heterotic-type I duality relations [5, 8]:

$$g_{I'} \propto g_h^{-1/2} R_{9h}^{3/2}, \quad R_{9I'} \propto g_h^{1/2} R_{9h}^{1/2},$$

where the subscript $h$ refers to the $E_8 \times E_8$ heterotic string. Under heterotic-IB duality, the short type I' flux tube joining a “stuck” D0brane with its mirror D0brane running in the $X^8$ direction is mapped to a heterotic soliton “string” of length $R \ll O(\sqrt{\alpha'})$ also in the $X^8$ direction. Note that lengths in the transverse directions are unchanged under the weak-strong coupling duality. The usual duality chain [8] then implies:

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“F_8” in Heterotic \rightarrow “F_8” in Heterotic \rightarrow “D_8” in type IB \rightarrow “D_{89}” in type I’ \rightarrow “M_{8,10}” in M theory.
```

The short I’ flux tube has been mapped to a finite width M2brane stretched between the orientifold planes in eleven dimensions. Note that a heterotic “string” of sub-string scale length is no string at all and, from the viewpoint of the nine-dimensional heterotic theory, is a singularity in the classical spacetime geometry. This singularity is resolved at strong coupling. From the duality chain given above, we infer that the singularity carries two species of quantized instanton charge. One of these, associated with the half-integer quantized background $B_{09}$ field, is a quantized theta angle analogous to that characterizing toroidal compactifications without vector structure [22]. The other theta angle corresponds to the quantized $C_0$ background, and can be identified with the familiar Yang-Mills instanton winding number. It would be extremely interesting to characterize this geometric singularity more precisely.

4 Conclusions

We have shown that the recent computation [2] of the quantum fluctuations of a short flux tube in type II string theory leads to a simple description of the short distance potential between the “dressed” D8branes of a D=9 type I' orientifold in a constant background electric field $\mathcal{F}_{09}$. Dressed D8branes carry both D0brane and D8brane charge, and their worldvolumes support quantized constant background $B_2$ and $C_0$ fields in addition to the usual nonabelian gauge fields. The nonabelian gauge symmetry is $U(8) \times U(8)$. In the absence of an external electric field, this configuration describes a nonperturbative background of the I' theory with a pair of “stuck” D0branes threading a stack of 8 D8branes at each of two orientifold planes. Confinement is absolute in this case, and the D8brane stacks with stuck D0branes are absorbed into the orientifold planes as a consequence of the confining $*F_{10}$ flux [3, 4]. We have shown that the constant external electric field resolves the
“stuck” D0brane pair, enabling a probe of sub-string-length distance scales. The characteristic $r^{-9}$ fall-off of the short distance potential fits naturally with the interpretation of dressed D8branes as objects coupling to the dualized $\ast C_9$ potential. They can possibly be identified as $-2$-branes. The existence of $-2$-branes in the BPS spectrum of M theory has been conjectured previously [13], but to the best of our knowledge there is no known construction of such objects.

Using heterotic-type I' duality, we are led to infer also the existence of an M2brane of finite string-scale width, stretched between the orientifold planes in eleven dimensions. The finite width M2brane allows interpretation as the strong coupling resolution of a spacetime singularity in the twisted and toroidally compactified $E_8 \times E_8$ heterotic string. From our knowledge of the constant background fields of the IB soliton, we infer that the heterotic singularity carries two species of instanton charge, one of them corresponding to the familiar Yang-Mills instanton winding number. The short type I' flux tube is a beautiful illustration of confinement as a consequence of flux quantization. An analogous phenomenon is believed to occur in the bosonic string theory, but confinement arises instead in the dynamics of the tachyon field [12, 14]. We have shown that the results in [1] give a precise world-sheet computation, inclusive of numerical factors, of the quantized fluctuations of the short electric flux tube in the bosonic string. This includes the systematic corrections to the leading $r^{-1}$ static potential arising from single photon exchange. The corrections are expressed in the form of an expansion in powers of the dimensionless variable $\alpha'^4 F_0^{\frac{1}{9}}$.

A T9 duality maps the I' background into a nonperturbative background of the IB theory with wrapped Dstrings lying in the worldvolume of 32 space-filling D9branes carrying $O(16) \times O(16)$ Chan-Paton labels and supporting, in addition, quantized constant background $B_2$ and $C_0$ fields. In the absence of the external electric field, the nonabelian gauge symmetry is $E_8 \times E_8$. The extended heterotic-IB-IIB duality map leads to an identification with a particular soliton solution of the massive D=9 type II supergravity [3, 4]. The solution takes the form of a “massive” string perpendicular to the worldvolume of a D8brane [11]. We use the SL(2,Z) U-duality symmetry of the nine-dimensional type II theory to infer both the quantization of the constant background $C_0$, and the existence of an entire $(p, q)$ multiplet of massive type II solitonic strings. The mass parameter of the IIA supergravity is quantized in inverse units of the IIB radius. An S-duality in the self-dual IIB theory, followed by an orientation projection at the point in the moduli space with quantized $C_0$ and $B_2$, gives a IB background with Dstrings wrapping the $X^9$ direction. Consequently, the I' soliton described above has no free parameters other than the constant external electric field introduced in order to resolve the dressed D8brane stack and its mirror at each orientifold plane.

Our analysis is based on the nonperturbative D=9 IB theory with massless gauge bosons in the spinor representation of $O(16)$ [3, 4]. Consequently, we give a brief discussion in the appendix of nonperturbative states and the appearance of enhanced gauge symmetry in the toroidally compactified type IB string theory, in backgrounds with wrapped D1branes in addition to the D9branes. Such backgrounds preserve all of the spacetime supersymmetries of the ten-dimensional type I string, and are in one-to-one correspondence with the heterotic CHL orbifolds in spacetime dimensions $D \leq 9$ [24, 25]. We clarify some missing details in the heterotic-type I duality map, addressing the puzzles raised in [21]. Using the extended heterotic-IB-IIB duality chain, we are able to account for all of the known disconnected components of the moduli space of theories with sixteen supercharges [24, 26].
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A Enhanced Gauge Symmetry in the Type I’ theory

In this appendix we clarify the appearance of enhanced gauge symmetry in toroidal compactifications of the Type I’ string theory, filling in some of the missing details in the heterotic-type I duality map. As a consequence, we will account for all of the disconnected components of the moduli space of the theory with sixteen supercharges \[22\], in striking confirmation of heterotic-type I duality \[5\]. An essential ingredient is played by toroidal compactifications of the IB theory with a Dstring pair wrapping any of the compact dimensions, in the presence of a half-integer quantized background \(B\) field of even rank. The backgrounds support, in addition, 32 D9branes as necessitated by anomaly cancellation of the ten-dimensional theory. Such Dbrane configurations preserve all of the spacetime supersymmetries. They are in one-to-one correspondence with the CHL orbifolds \[22, 26\], in precise accord with heterotic-type I duality. Our discussion will clarify some of the puzzles raised in the second of the references in \[20\].

We begin with the type IB theory compactified on \(S^1\), with 32 space-filling D9branes carrying Chan-Paton indices of the group \(O(16) \times O(16)\). Consider, in addition, a pair of Dstrings wrapped about the compact \(X^9\) direction. This can equivalently be described in the \(T_9\)-duality equivalent Type I’ theory, where it corresponds to a “stuck” D0brane pair threading a stack of 8 D8brane pairs at either of two orientifold planes at \(X^9=0, \pi R_{9}\). Massless gauge bosons in the spinor representations of \(O(16)\) appear as a consequence of fundamental string creation arising from the crossing of D8brane stacks by stuck D0branes \[7\] or, equivalently, of space-filling D9branes with \(O(16) \times O(16)\) Chan-Paton labels by wrapped D1branes, enhancing the gauge symmetry to \(E_8 \times E_8\) as described in the introduction. Thus, in \(nine\) spacetime dimensions and below, where the moduli spaces of the heterotic \(E_8 \times E_8\) and \(SO(32)\) theories are known to be connected upon inclusion of Wilson lines, there exists a perfectly good type IB description of points in the moduli space with \(E_8 \times E_8\) enhanced gauge symmetry. This is in accordance with heterotic-type I duality. Note that the radius of the compact dimension, \(X^9\), is arbitrary.

Now consider lowering the compactification radius, \(R_{9}\) \[5\]. The \(E_8 \times E_8\) heterotic string theory compactified on \(S^1\) displays, in addition, an \(SU(2)\) enhanced gauge symmetry at the self-dual radius, \(R_{9}=\sqrt{\alpha'}\) \[8\]. This point in the moduli space cannot be directly accessed in the perturbative type IB description since the type I string coupling is growing strong (see Eq. \(26\)) \[3\]. However, we can always use the duality chain to first map the IB background into a corresponding IIB background by simply undoing orientation reversal. An S-duality transformation in the self-dual IIB theory is then used to map the IIB background with wrapped Dstrings into a IIB background with wrapped fundamental strings:

\[
F_{[m]} \text{ in IIB on } S^1 \quad \leftrightarrow \quad S \quad \text{D1}_{[m]} \text{ in IIB on } S^1 \quad \leftrightarrow \quad \Omega \quad \text{D1}_{[m]} \text{ in IB on } S^1 \quad \leftrightarrow \quad T_m \quad \text{“D0” in I’ on } S^1/\mathbb{Z}_2, \quad (28)
\]
where the notation $F_{[m]}$ denotes a fundamental string wrapped about the compact $X^m$ direction, and likewise for the Dstrings. “D0” denotes a “stuck” D0brane lying in the worldvolume of D8branes on an orientifold plane. The IIB theory with wrapped fundamental strings displays an enhanced gauge symmetry at the self-dual radius of the compact dimension, $X^m$, analogous to the appearance of enhanced gauge symmetry in the heterotic string. Thus, the precise counting of additional massless gauge bosons and their interactions is most easily performed in the IIB background with wrapped $F$ strings, using an S-duality in the self-dual IIB theory, followed by the action of $\Omega$, to recover the IB spectrum with wrapped Dstrings. The precise match of the counting of massless gauge bosons associated with enhanced gauge symmetry at the self-dual radii is striking confirmation of the internal consistency of the extended heterotic-IB-IIB duality chain. The extension of this analysis to compactifications on $T^d$, with $d \leq 9$, is evident: the massless spectrum of a wrapped IIB fundamental string, followed by an S-duality in a self-dual theory plus orientation projection, coincides with that of a wrapped heterotic fundamental string [8]. We are using here the fact that the orientation projection, $\Omega$, is a freely acting $Z_2$ symmetry on the massless spectrum of the IIB theory in the presence of unbroken spacetime supersymmetries. In particular, it is freely reversible. Secondly, we used the self-duality of the IIB theory in order to identify the massless spectrum at weak and strong coupling. It would be extremely interesting to extend these considerations both for backgrounds with fewer spacetime supersymmetries, and for states in the massive spectrum of the nonperturbative IIB theory.

As an illustration of the usefulness of the extended heterotic-IB-IIB duality chain, we will work out the IB equivalences of the CHL orbifolds [22, 24]: $Z_n$ orbifolds of the heterotic string compactified on $T^d$, which leave invariant all sixteen supercharges of the D=10 theory. The simplest example, which occurs in all spacetime dimensions, $D \leq 9$, is to mod out by the $Z_2$ outer automorphism symmetry exchanging the two $E_8$ lattices, accompanied by a half-integer shift in $p_m$, with $m \leq d$ [22]. The result is a theory with sixteen supercharges and a moduli space characterized by $8 + d$ abelian vector multiplets, disconnected from the moduli space of the parent theory with $16 + d$ abelian vector multiplets. Its moduli space includes, in particular, an enhanced symmetry point with gauge group $E_8$. Boosts in the $(16 + d, d)$ momentum lattice, more precisely described as the inclusion of Wilson lines accompanied by $Z_2$ order shifts in the $(d, d)$ compactification lattice, can give rise to many distinct enhanced symmetry points. These cover all of the possible simple Lie groups, and including both simply-laced and non-simply-laced examples [22]. More generally, modding out by any finite order element in the known classification of supersymmetry preserving automorphisms of the given $(16 + d, d)$ self-dual lattices, permits a classification of the disconnected components of the moduli space of the theory with sixteen supercharges [26]. To the best of our knowledge, the explicit classification is as yet incomplete, although provided in principle by elements in the Monster group [24]. Nevertheless, many interesting corners of this problem have been elucidated by different authors [27]. Our goal here is simply to identify the equivalent orbifold action on the I’-IB-IIB theory.

10 There exist, also, both type IIA/M theory and F-theory duals of the CHL orbifold [26]. Note that these are low energy dualities. It is not, of course, inconsistent to have more than one low energy dual for a given theory.
11 Although we restrict our discussion to the massless spectrum in what follows as in [26], note that specifying the supersymmetry preserving automorphism of the $(16 + d, d)$ lattice determines the full perturbative heterotic string spectrum. This will also hold for the type I’ dual, upon including nonperturbative states in the spinor representations. The momentum lattice of the CHL $Z_2$ orbifold has been worked out in [24]. A more detailed analysis of the Wilson lines appears in the second of the references in [23].
Let us return to the $I'$ description of the $D=9$ $E_8 \times E_8$ theory by a pair of Dstrings wrapped on $X^9$. The $T_9$-duality equivalent $I'$ description has a pair of stuck D0branes crossing stacks of D8branes at either end-point of the interval. It is evident that this theory has a $\mathbb{Z}_2$ automorphism symmetry, $I$, under exchange of the D8brane stacks. Consider a half-integer shift in the $(1,1)$ compactification lattice of the associated self-dual D=9 IIB theory. In the presence of a half-integer quantized background $B$ field, the contributed shift to the momentum lattice is invariant under a subsequent orientation projection, surviving in the IB theory $[20,8]$:

$$p_{mL} = \frac{n_m}{R_{mB}} + \frac{w^9 R_{mB}}{\alpha'} (G_{9m} + B_{9m})$$

$$p_{mR} = \frac{n_m}{R_{mB}} + \frac{w^9 R_{mB}}{\alpha'} (-G_{9m} + B_{9m})$$

Note that the compactification radius of the circle, $R_{mB}$, coincides with the type I radius, $R_{mI}$, invariant under the orientation projection. The contribution from the winding modes is absent in the IB theory, but a constant $B_{mI} = \frac{1}{2} \alpha'/R_{mB}$ gives a surviving contribution to the $m$ component of the IB momentum, $p_m = \frac{1}{2} (p_{mL} + p_{mR}), m=1, \cdots, d$. In addition, the IB theory has D9brane gauge fields, and the possibility of constant background gauge fields induces a further shift in the IB momentum from Wilson lines $[8]$:

$$p_m \rightarrow (n_m + \frac{1}{2}) \frac{1}{R_{mB}} - q I A_{m}^I - \frac{R_{mB}}{2} A_{m}^I A_{m}^I, \quad w_9 = 1 \quad ,$$

the index $I$ labeling the D9branes. We comment that the expressions in Eqns. (29), (30), are closely analogous to the parameterization of the $(d,d)$ momentum lattice describing a toroidal compactification of the heterotic string with constant background gauge fields $[22,8]$, modulo the transformation of the radii $[7]$, and the above-mentioned quantization of the $\pm G + B$ term. Consider modding out by the symmetry under exchange of the D8brane stacks, accompanied by a translation in the compactified momentum lattice. We can describe this operation using orbifold terminology $[8]$. Denoting a translation vector in the momentum lattice by $v^m$, and the interchange of the D8brane stacks by the symbol $\gamma$, a generic element of the orbifold group, $\mathcal{H}$, is denoted $(v, \gamma(v))$. In this example, both generators are order two, giving sectors, $(1, v, \gamma(v))$. The presence of the translation implies that all of the states in the twisted sectors are heavy. Restricting to the untwisted sector containing states invariant under $\mathcal{H}$, the counting of massless gauge bosons proceeds precisely as in the CHL orbifold $[22,23]$. The gauge bosons of $O(16)$ arise from the symmetric linear combination of fundamental strings stretched between corresponding pairs of D8branes within each of two stacks. Labeling the 16 D8branes, $(1, \cdots, 8, 1', \cdots, 8')$, the untwisted massless gauge bosons correspond to fundamental string states of zero length that are denoted:

$$\frac{1}{\sqrt{2}} \left\{ ([0, \cdots, \pm 1, 0, \cdots, 0, \pm 1, 0, \cdots, 0], 0^8) + (0^8, [0, \cdots, \pm 1, 0, \cdots, 0, \pm 1, 0, \cdots, 0]) \right\} \quad ,$$

where a pair of $\pm$ signs in the $(i,j)$th locations denotes a string joining the $i$th and $j$th D8branes, and the signs denote their orientation. The multiplicity of 4 for strings linking a pair of D8branes located at the orientifold plane accounts for the $SO(8)$ symmetry. In addition, there are massless states in the Cartan subalgebra corresponding to strings with both end-points on the same D8brane denoted: $( [0, \cdots, \pm 1, 0, \cdots, 0], 0^8) + (0^8, [0, \cdots, \pm 1, 0, \cdots, 0])$. Note that the choice of a representation of
the fundamental strings joining D8branes by root vectors was deliberate: the vectors above are already in the standard normalization of length 2. It is well known that there is an integer shift in the value of the quantized constant $\ast F_{10}$ background, $\nu_0$, upon crossing a D8brane [5, 6, 7, 19]. Thus, the vacuum with mass parameter set to zero, $n=0$, corresponds to $\nu_0=\frac{1}{2}$, implying a non-vanishing constant background $C_0$ potential in the $T_9$-duality equivalent IIB vacuum. This has already been noted for the massive string soliton of Section 2 following Eq. (11). The normalizations of nonperturbative type I states in the spinor representation of $O(16)$ are therefore determined as given below. We note that the $(d, d)$ compactification lattice appended to the $(8, 8)$ dimensional gauge lattice constructed above corresponds precisely to the construction of the heterotic string. As a result, the action of an element of the orbifold group on the $(16+d, d)$ Narain lattice [21] determines an isomorphic orbifold action on the type I' background. The enhancement of the gauge symmetry to $E_8$ proceeds upon including the symmetric linear combinations of additional length two vectors filling out the spinors of $O(16)$: $\frac{1}{2}(\pm, \pm, \pm, \pm, \pm, \pm, \pm, 0^8)$ and $\frac{1}{2}(0^8, \pm, \pm, \pm, \pm, \pm, \pm, \pm, \pm)$, with an even number of $+$ signs as described in the introduction, which denote fundamental string creation due to the stuck D0brane crossing a stack of 8 D8branes. Restricting to the symmetric linear combination of fundamental strings created in each of two stacks gives a single $128$.

The extension of this construction to the abelian $Z_n$ orbifolds listed in the first of references in [26] proceeds by finding a point in the moduli space with the required $Z_n$ automorphism symmetry, corresponding to the interchange of $n$ identical stacks of D8branes, accompanied by an order $n$ translation in the momentum lattice. The required $\nu$ is identified by adjusting the background gauge fields appropriately, the Wilson lines parameterizing the locations of the individual D8brane stacks. We will not pursue the analysis further in this paper, although it is evident at this point that the constructions in [26] map isomorphically into the type I' description upon inclusion of nonperturbative type I states [12]. There is an important issue here regarding the imposition of consistency conditions on these theories. In [22], this was done using the fermionic construction, a technique of outstanding reliability. The $(Z_2)^k$ examples of [26] with $k=1, \cdots, 4$ correspond to moduli spaces with $d+2^4/2k$ abelian vector multiplets, and are well-corroborated in the fermionic construction [22]. These theories are easily matched with type I' backgrounds with half-integer quantized $B$ field, examples of which first appeared as the rational type I unorientifolds in [20], as is clear from the discussion above. It should be clear from our discussion that, as in the heterotic string, making an arbitrary distinction between automorphisms of the gauge lattice or automorphisms of the compactification lattice is rather artificial, upon the inclusion of Wilson lines. Thus, there exist well-documented examples with fewer than $d$ vector multiplets in both the fermionic and heterotic orbifold constructions [22, 29], including a four dimensional moduli space which is pure N=4 supergravity with no additional vector multiplets [22]. It would be interesting to study their type I' analogs. We comment that the construction described above can be mapped rather straightforwardly into the fermionic construction of [22]. This would clarify the relationship of the consistency conditions of the fermionic construction to tadpole cancellation, enabling a cleaner analysis of the consistency conditions in type I' theories with less supersymmetry. This introduces many new features into the analysis, and we postpone further discussion to future work.

12This is an important point. In the absence of the $O(2n)$ spinor representations, the equivalence between the heterotic and IIB gauge lattices described above would not hold.
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