MODEL-FREE LQR BASED PID CONTROLLER FOR TRAJECTORY TRACKING OF 2-DOF HELICOPTER: COMPARISON AND EXPERIMENTAL RESULTS

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ABSTRACT

This paper studies the performance of a model-free LQR based PID (i-LQR-PID) controller designed for tracking control problem of a 2-DoF laboratory helicopter. The control problem addressed in 2-DoF helicopter system aims to track the desired pitch and yaw axes trajectories despite disturbed operating conditions. In addition to the unpredictable variations, the 2-DoF helicopter dynamic is highly nonlinear with having strong cross-couplings in their models as well as being open loop unstable system. Thus, we propose a model-free LQR based PID control strategy in order to achieve better trajectory tracking control objectives. Robustness tests are performed experimentally to show the effectiveness of the model-free control.

Keywords 2-DoF Helicopter system · Process control · Model-free control · Linear quadratic regulator (LQR) · LQR based PID control (LQR-PID) · Intelligent LQR based PID (i-LQR-PID) control · Robustness analysis

1 Introduction

Design of control strategies for helicopters has attracted the research community due to its wide range of civil and military applications [Zheng and Zhong (2011)] and it is still an active research topic with several challenges including the presence of high nonlinearities and model uncertainties [Hernandez-Gonzalez et al. (2012)]. Both stabilization and trajectory tracking problems for helicopters systems have been studied in the literature. For instance, a combined feed forward action and saturation feedback was proposed in [Marconi and Naldi (2007)]. A robust Linear Quadratic Regulator (LQR) was introduced for attitude control of 3-DoF helicopter in [Liu et al. (2013)]. Moreover, a backstepping based approach [Raptis et al. (2011)] and an adaptive LQR using Model Reference Adaptive Control (MRAC) scheme [Subramanian and Elumalai (2016)] have been proposed to solve tracking problems in unmanned helicopters. However, there is still a need for robustness enhancement especially under aggressive turbulence effects.

In this study, a model-free LQR-proportional-integral-derivative controller called intelligent LQR-proportional-integral-derivative (i-LQR-PID) controller is introduced as an alternative robust control strategy to the LQR-PID control. The conventional PID controller is one of the most used controller in industry for closed control-loops thanks to its simplicity

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in real time implementation. The design and tuning of such control algorithm has been widely covered and is still an active field of research especially for industrial plants subject to external disturbances [Astrom and Hagglund (2001); Ho et al. (1999); Astrom et al. (1993)]. Indeed, due to the significant variations in the amplitude vibration affected by the external disturbances, satisfactory performance covering the total range of disturbances is difficult to reach with a conventional PID without an external compensation. Thereafter, it is desirable to design robust control strategies without additional computational effort.

To consider modeling errors, system uncertainties, disturbances and actuator faults when designing a controller, a model-free control (MFC) algorithm has been proposed in Flies and Join (2009), Flies and Join (2013). The main feature of this approach consists in updating continuously the input-output behavior using an ultra-local-model. To improve the performance and robustness of conventional controllers with less time and effort expenditure, MFC has been successfully combined to some controllers such as PID controller and more recently the LQR providing the so-called intelligent PID controller (i-PID) and intelligent LQR (i-LQR) [Astrom and Hagglund (2006)]

MFC in general, i-PID and i-LQR controllers in particular have been considered in several applications and their performance have been studied in both simulation and experiments. Examples of such applications include shape memory alloys Gedouin et al. (2009), DC/DC converters Michel et al. (2010), active magnetic bearing Miras et al. (2013), two-dimensional planar manipulator Madonski and Herman (2013), agricultural greenhouse Lafont et al. (2015), quadrotor vehicle and aerospace Choi et al. (2013), Younes et al. (2016), Flies and Join (2017), Menhour et al. (2017), automotive engine Choi et al. (2009), mechanical system Villagra et al. (2012) and Qball-X4 quadrotor vehicle Younes et al. (2014), Younes et al. (2016).

In this paper, an intelligent LQR based PID (i-LQR-PID) controller is designed for reference trajectory tracking of the pitch and yaw angles in a helicopter system. The performance of the i-LQR-PID is evaluated by comparison to LQR-PID controller through experiments. Moreover, robustness analysis is performed and validated experimentally with respect to nominal tracking, exogenous disturbance, parameter uncertainty and wind disturbances through experiments.

2 System description

The Quanser 2-DoF laboratory helicopter has been studied in this paper. As illustrated in Fig. 1 the system consists of a helicopter body on a fixed base with two propellers. DC motors drive these propellers which control both the pitch and yaw angles of the helicopter.

Fig. 1 shows the free body diagram of 2-DoF laboratory helicopter. There are two degrees of freedom that are given by the motion around the z axis (yaw), represented by angle Ψ, and the rotation around the y axis (pitch), represented by the angle θ. The control inputs for the system are given by the voltages to the DC motors.

Using the kinematic diagram of the 2-DoF helicopter testbed and Euler Lagrangian energy based approach, the state space representation model governing the dynamics of the 2-DoF helicopter system is given in [1] (see Quanser (2010)).

The nominal plant parameters of the 2-DoF helicopter system are given in Table 1.
The control objective of this study is to track a desired reference trajectory for both pitch and yaw angles using LQR based PID and intelligent LQR based PID controllers.

3 LQR based PID tracking control

Linear-quadratic-regulator (LQR) is an optimal control strategy [Naidu (2002), Choudhary (2014)] which has been widely used in various applications. LQR design is based on the selection of a state feedback gain $K$ such that the cost function or performance index $J$ is minimized. This ensures that the gain selection is optimal for the cost function specified. The inherent robustness, stability and optimality between the control input and speed of response make LQR as the most preferred optimal controller in aerospace applications.

Consider the following state space representation of the augmented 2-DoF helicopter system

$$
\begin{bmatrix}
\dot{\theta} \\
\dot{\Psi} \\
\ddot{\theta} \\
\ddot{\Psi} \\
\dot{I}_\theta \\
\dot{I}_\Psi
\end{bmatrix} =
\begin{bmatrix}
0 & 0 & 1 & 0 & 0 & 0 \\
0 & 0 & 0 & 1 & 0 & 0 \\
0 & 0 & -\frac{K_{pp}}{J_{eq,p} + m_h \ell^2} & 0 & 0 & 0 \\
0 & 0 & 0 & -\frac{K_{pp}}{J_{eq,y} + m_h \ell^2} & 0 & 0 \\
1 & 0 & 0 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 & 0 & 0
\end{bmatrix}
\begin{bmatrix}
\theta \\
\Psi \\
\dot{\theta} \\
\dot{\Psi} \\
I_\theta \\
I_\Psi
\end{bmatrix}
+ \begin{bmatrix}
0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & \frac{K_{pp}}{J_{eq,p} + m_h \ell^2} & 0 & 0 & 0 \\
0 & 0 & \frac{K_{pp}}{J_{eq,y} + m_h \ell^2} & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0
\end{bmatrix}
\begin{bmatrix}
u_p \\
u_y
\end{bmatrix},
$$

$$
Y = \begin{bmatrix}
1 & 0 & 0 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 & 0 & 0
\end{bmatrix}
\begin{bmatrix}
\dot{\theta} \\
\dot{\Psi} \\
\ddot{\theta} \\
\ddot{\Psi} \\
I_\theta \\
I_\Psi
\end{bmatrix},
\quad I_\theta = \int (\theta - \theta_d) \, dt,
\quad I_\Psi = \int (\Psi - \Psi_d) \, dt. \quad (1)
$$

The control objective of this study is to track a desired reference trajectory for both pitch and yaw angles using LQR based PID and intelligent LQR based PID controllers.

The PID control gains are computed using the LQR scheme where $x^T = [\theta \quad \Psi \quad \dot{\theta} \quad \dot{\Psi} \quad I_\theta \quad I_\Psi]$. Using the state feedback control law,

$$
u(t) = -K x(t), \quad (3)$$

The above three transformation matrices can be represented as,

$$
T_{cm} = \begin{bmatrix}
0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 \\
1 & 0 & 0 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0
\end{bmatrix},
\quad K_{pp} = \frac{J_{eq,p}}{J_{eq,p} + m_h \ell^2}.
$$

Thus, the translation and rotation are defined as,

$$
T_{cm} = \begin{bmatrix}
0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 \\
1 & 0 & 0 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0
\end{bmatrix}.
$$

The above three transformation matrices can be represented as,

$$
T_{cm} = \begin{bmatrix}
0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 \\
1 & 0 & 0 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0
\end{bmatrix}.
$$

Thus, the translation and rotation are defined as,

$$
T_{cm} = \begin{bmatrix}
0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 \\
1 & 0 & 0 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0
\end{bmatrix}.
$$

Thus, the translation and rotation are defined as,

$$
T_{cm} = \begin{bmatrix}
0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 \\
1 & 0 & 0 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0
\end{bmatrix}.
$$

Thus, the translation and rotation are defined as,
Table 1: Nominal parameters of 2-DoF Helicopter model.

| Symbol  | Unit  | Value  | Description                                |
|---------|-------|--------|--------------------------------------------|
| $B_p$   | N/V   | 0.8    | Equivalent viscous damping about pitch axis |
| $B_y$   | N/V   | 0.318  | Equivalent viscous damping about yaw axis. |
| $J_{eq,p}$ | kg.m$^2$ | 0.0384 | Total moment of inertia about pitch axis.   |
| $J_{eq,y}$ | kg.m$^2$ | 0.0432 | Total moment of inertia about yaw axis.     |
| $m_h$   | kg    | 1.3872 | Total moving mass of the helicopter         |
| $\ell$  | m     | 0.186  | Length along helicopter body from pitch axis |
| $K_{pp}$ | N.m/V | 0.204  | Thrust force constant of pitch motor/propeller. |
| $K_{py}$ | N.m/V | 0.0068 | Thrust torque constant acting on pitch axis from yaw motor/propeller. |
| $K_{yp}$ | N.m/V | 0.0219 | Thrust torque constant acting on yaw axis from pitch motor/propeller. |
| $K_{yy}$ | N.m/V | 0.072  | Thrust torque constant acting on yaw axis from yaw motor/propeller. |
| $u_p$   | V     | ±24    | Pitch motor voltage.                        |
| $u_y$   | V     | ±15    | Yaw motor voltage.                          |

The LQR controller proposes an optimal control strategy that minimizes the following cost function.

$$J(u^*) = \int_0^\infty [x^T(t)Qx(t) + u^T(t)Ru(t)] \, dt,$$

where $Q$ and $R$ are some weighting matrices that penalize the state variables and the inputs. The control input using the LQR approach is given by

$$K = R^{-1}B^TP.$$

The transformation matrix $P$ is the solution of the following Algebraic Riccati Equation

$$A^TP + PA + Q - PB R^{-1}B^TP = 0.$$

The LQR usually offers good robustness properties but its performance deteriorates under severe uncertainties which are for instance due to actuator failure or structural damage [Choudhary (2009)]. To enhance the trajectory tracking performance of the LQR, an intelligent LQR based PID controller framework is proposed in the next section.

4 Intelligent LQR based PID tracking control

The main idea behind a model-free technique is to update continuously the behavior input/output using an ultra-local model. Therefore, the resulting control strategy is robust with respect to un-modeled dynamics and uncertainties of the system. Model-free control has been successfully applied in several applications (Fließ and Join (2013, 2014, 2017); Choi et al. (2009, 2013); Villagra et al. (2009, 2012); Younes et al. (2016); Menhour et al. (2017)).
The following ultra-local model is usually used in MFC

\[ y^{(\nu)}(t) = F(t) + \alpha u(t) \]  

(7)

where \( u \) is the control input, \( y \) is the output, \( F \) refers to all unknown model of the system, disturbances and uncertainties which is estimated from available input/output measurements, \( \nu \) is the differentiation order, \( \alpha \) is a constant which is chosen such that \( \alpha u \) and \( y^{(\nu)} \) are of the same order of magnitude.

**Remark 1** The proper value of \( \nu \) has to be chosen. This value usually depends on prior knowledge on the system and its relative degree. It may also depend on the nature of the control input. Indeed, studies have shown that with an intelligent Proportional-Integral-Derivative (iPID) and iPD controllers the value \( \nu = 2 \) is suitable while for iPI and iP a value \( \nu = 1 \) is more suitable [Younes et al. (2016)].

In this paper, the proper value of \( \nu \) is setting to 2, due to the fact that the relative degree of system (1) is 2 and i-PID controller is also used in the 2-DoF helicopter system (1).

Setting \( \nu = 2 \) in equation (7), we have

\[ \ddot{y}(t) = F(t) + \alpha u(t) \]  

(8)

Hence, the corresponding i-PID can be written as

\[ u(t) = - \left( \ddot{\hat{F}}(t) - \dot{\hat{y}}^2(t) + K_P e(t) + K_D \dot{e}(t) + K_I \int_0^t e(\tau) \, d\tau \right) / \alpha \]  

(9)

where

- \( y^d \) is the desired reference trajectory,
- \( \ddot{\hat{F}}(t) \) is the estimate of \( F(t) \) which is described as follows,
  \[ \ddot{\hat{F}}(t) = \ddot{\hat{y}}(t) - \alpha u(t), \]  

(10)

- \( \ddot{\hat{y}} \) is the estimate of \( \ddot{y} \),
- \( e(t) = y(t) - y^d(t) \) is the trajectory tracking error,
- \( K_P, K_I \) and \( K_D \) are the PID gains.

Once \( \ddot{\hat{y}} \) is obtained, the loop is closed by (9) and the modified expression of (8) is given as (10).

Substituting equation (10) in (9), adding and subtracting the derivare of the controlled output \( \ddot{y} \) yields

\[ \ddot{e}(t) + K_P e(t) + K_D \dot{e}(t) + K_I \int_0^t e(\tau) \, d\tau = e_F(t), \]  

(11)

where \( e_F(t) = \ddot{y}(t) - \ddot{\hat{y}}(t) = F(t) - \ddot{\hat{F}}(t) \). Subsequently, with a good estimate \( \ddot{\hat{F}}(t) \) of \( F(t) \) \( i.e \ e_F(t) = F(t) - \ddot{\hat{F}}(t) \approx 0 \), equation (11) yields

\[ \ddot{e}(t) + K_P e(t) + K_D \dot{e}(t) + K_I \int_0^t e(\tau) \, d\tau \approx 0, \]  

(12)

which ensures an excellent tracking of the reference trajectory. This tracking is moreover quite robust with respect to uncertainties and disturbances which can be important in the 2-DoF helicopter stabilization setting such as considered here. This robustness feature is explained by the fact that \( F \) includes all the effects of unmodeled dynamics and disturbances, without trying to distinguish between its different components. Furthermore, the approximation of PID design parameters becomes therefore quite straightforward. This is a major benefit when compared to “classic” PIDs.

![Figure 3: Schematic diagram of LQR-PID model-free control of 2-DoF helicopter.](image-url)
5 Simulation and experimental results: Comparison and discussions

Fig. 4 illustrates the hardware in the loop experimental system of a 2-DoF helicopter. Its consists of helicopter plant, data acquisition board, computer and power amplifier. This system is a real-time control experimental system with the support of real-time software QUARC. The blue arrows represent the acquisition of current states, while the gray arrows are actions of control solution. Integrated encoders measure the pitch and yaw angles. The controller is supervised by commands from a Matlab/Simulink based human machine interface.

To evaluate the effectiveness of the intelligent LQR based PID (i-LQR-PID) over the LQR based PID (LQR-PID) controller in tracking the desired trajectories despite disturbed operating conditions, the pitch and yaw motors are driven simultaneously by model-free control laws. LQR-PID controller is mainly suitable for linear systems or linearized systems around a specific operating point. However, the tuning of the LQR-PID control law proves to be challenging and highly dependent on the model of the 2-DoF helicopter system. In this work, the 2-DoF helicopter system is decomposed into two-SISO subsystems that are linked to each other and two model-free controls are designed to control simultaneously the pitch and yaw angles. The gain \( \alpha \) for each controller is determined empirically. Based on the state representation given in (1), \( Q \) and \( R \) matrices that achieve an optimal control are given as follows:

\[
Q = \text{diag} \begin{bmatrix} 200 & 150 & 100 & 200 & 50 & 50 \end{bmatrix},
\]

\[
R = \text{diag} \begin{bmatrix} 1 & 1 \end{bmatrix} \text{ with } \alpha_{\text{pitch}} = 1.3 \text{ and } \alpha_{\text{yaw}} = 0.43.
\]

Using the above weighting matrices, the LQR state feedback controller gain \( K \) for the system is given as

\[
K = \begin{bmatrix} 18.9 & 1.98 & 7.48 & 1.53 & 0.45 & 7.03 & 0.77 & 7.03 \\ -2.22 & 19.4 & -0.45 & 11.9 & 19.4 & -0.77 & 21.7 & 19.4 \\ K_P & K_D & K_I \end{bmatrix}.
\]

5.1 Trajectory tracking

The performance analysis of 2-DoF helicopter system has been carried out for pitch and yaw angles through simulations and experiments. The initial position of the helicopter is set to \( \theta = -40.5^\circ \) and \( \Psi = 0^\circ \). To assess the control tracking performance of the intelligent LQR based PID (i-LQR-PID) controller under nominal conditions, a square trajectory with an amplitude of \( \pm 10^\circ \) is given as a reference signal. Figs. 5 and 6 which show the simulated pitch and yaw tracking responses of the pitch reference step scheme for both i-LQR-PID and LQR-PID provide satisfactory results and highlight that the pitch and yaw angles from the i-LQR-PID settles faster at the reference value with a short convergence time of 4 seconds.

The experimental pitch and yaw tracking responses under pitch reference step are shown in Figs. 7 and 8 respectively. In Fig. 7 the zoomed view response of i-LQR-PID controller for the pitch angle over the time interval [26 – 29] seconds gives a better trajectory tracking response. The yaw angle for both controllers shows an offset from the desired trajectory. However, the i-LQR-PID controller reduces the yaw offset angle and presents the smallest tracking error than the LQR-PID controller. The tracking trajectory performance is also evaluated by Root Mean Square (RMS), Standard
Deviation (STD) and Mean for both pitch and yaw angles. The performances indices are given in Table 2 and Table 3 for both pitch and yaw angles respectively. The RMS represents the tracking error between the desired trajectory and the output, STD estimates the central tendency of the distribution of the output and the mean is used to indicate the spread of control results and evaluate the precision of the system. From Tables 2 and 3, it can be clearly seen that i-LQR-PID controller achieves better trajectory tracking performances than LQR-PID.

Table 2: Pitch tracking errors

| Controller          | RMS   | STD   | Mean   | Interval     |
|---------------------|-------|-------|--------|--------------|
| LQR\_PID            | 1.5123| 1.2983| 0.7757 | 26 – 30sec.  |
| Intelligent LQR\_PID| 0.9428| 0.9193| 0.2096 | 26 – 30sec.  |

5.2 Parameter uncertainty

The robustness of the control scheme against model variation is assessed in this subsection. Model uncertainty is introduced at the yaw angle. Two test cases are considered where +5% and −5% of $K_{pp}$ pitch force constant parameter variations have been introduced respectively. Figs. 9 and 10 show both pitch and yaw responses of i-LQR-PID and Figs. 11 and 12 illustrate both pitch and yaw responses of LQR-PID controller respectively for both levels of uncertainties. The level of the oscillations in case of 5% of variations is higher when using i-LQR-PID controller. Furthermore, the convergence time of yaw angles for i-LQR-PID controller is faster and the maximum offset is smaller and much better than LQR-PID controller response.

5.3 Disturbance rejection

The robustness of i-LQR-PID is assessed in two test cases where short term disturbance and continuous disturbance are respectively introduced into the 2-DoF helicopter system.

- Short term disturbance
Table 3: **Yaw** tracking errors

| Controller                | RMS (pitch) | STD (pitch) | Mean (pitch) | Interval   |
|---------------------------|-------------|-------------|--------------|------------|
| LQR                       | 6.9951      | 0.2825      | 6.9893       | 26 – 30sec.|  
| Intelligent LQR-PID      | 2.3304      | 0.1981      | 2.3220       | 26 – 30sec.|  

The short term bounded pulse external disturbance function which 10° disturbance magnitude, 35 seconds period, 25 seconds phase delay and 10% pulse width is introduced into the yaw control angle from the time interval [25 – 30] seconds. Figs. [13] and [14] which illustrate the pitch and yaw responses during a short term disturbance experiment for both i-LQR-PID and LQR-PID controllers, demonstrate the ability of both controllers to reduce the deviation and brings back the response to set-point in around t = 5 seconds. Moreover, i-LQR-PID provides better satisfactory results in reducing the pitch and yaw amplitudes in shorter time.

- **Continuous disturbance**

The continuous disturbance function $10\sin(25t + 10)$ which 10° disturbance magnitude is introduced during the step command tracking of pitch control angle 2-DoF helicopter. Figs. [15] and [16] show the response of both i-LQR-PID and LQR-PID controllers framework during the continuous disturbance. It can be noted that the deviation in magnitude is restricted within ±0.3° for both i-LQR-PID and LQR-PID controllers. The ability of both controllers to reject the disturbance and track the reference signal is highlighted in the zoomed view of the pitch response shown in Fig. [15].

### 5.4 Robustness under wind gusts disturbances

In real time scenarios, in addition to varying magnitudes the helicopter needs often to adapt to directional changes. Hence, position tracking control problem under aggressive wind turbulence effects is investigated. The wind gusts is generated by an electrical fan and with fixed velocity. The wind velocity parallels the pitch axis when the pitch and yaw angles are set to $\theta = -40.5^\circ$ and $\Psi = 0^\circ$ respectively. Figs. [18] and [19] show the maneuvering performance of both i-LQR-PID and LQR-PID controllers for pitch and yaw angles. The i-LQR-PID controller provides better trajectory tracking results.

Table 4: RMS tracking errors of **Pitch** and **Yaw** angles under wind disturbances

| Controller          | RMS (pitch) | RMS (yaw) | Interval  |
|---------------------|-------------|-----------|-----------|
| LQR-PID             | 5.8598      | 7.7559    | 0 – 45sec.|  
| Intelligent LQR-PID | 5.8673      | 4.0847    | 0 – 45sec.|  

The performance indices are given in Table 4 for both pitch and yaw angles respectively. From Table 4 it can be clearly seen that i-LQR-PID controller achieves better trajectory tracking performances than LQR-PID.
6 Conclusion

In this paper, i-LQR-PID controller has been proposed to control the pitch and yaw angles so as to make the system to track the reference trajectory. The performance of the presented i-LQR-PID has been evaluated and compared to the LQR-PID controller in closed-loop to accommodate the disturbances present in the 2-DoF helicopter system. Simulation and experimental results of 2-DoF helicopter for different level of magnitudes and direction have shown that i-LQR-PID controller is more effective and robust than LQR-PID controller for tracking references under aggressive turbulence effects, while it preserves its simplicity of implementation.

References

B. Zheng and Y. Zhong. Robust attitude regulation of a 3-DOF helicopter benchmark: Theory and experiments. IEEE Trans. Ind. Electron., 58:660–670, 2011.
M. Hernandez-Gonzalez, A. Alanis, and E. Hernandez-Vargas. Decentralized discrete-time neural control for a quanser 2-DOF helicopter. Appl. Soft Comput., 12:2462–2469, 2012.
L. Marconi and R. Naldi. Robust full degree-of-freedom tracking control of a helicopter. Automatica, 43(11):1909–1920, 2007.
H. Liu, G. Lu, and Y. Zhong. Robust LQR attitude control of a 3-DOF laboratory helicopter for aggressive maneuvers. IEEE Trans. Ind. Electron., 60(10):4627–4636, 2013.
I. A. Raptis, K. P. Valavanis, and W. A. Moreno. A novel nonlinear backstepping controller design for helicopters using the rotation matrix. IEEE Trans. Control Syst. Technol., 19(2):465–473, 2011.
R. G. Subramanian and V. K. Elumalai. Robust MRAC augmented baseline LQR for tracking control of 2-DoF helicopter. Robotics and Autonomous Systems, 86:70–77, 2016.
K. Astrom and T. Hagglund. The future of PID control. Control Engineering Practice, 9(11):1163–1175, 2001.
W.K. Ho, K.W. Lim, C.C. Hang, and L.Y. Ni. Getting more phase margin and performance out of PID controllers. Automatica, 35:1579–1585, 1999.
Figure 15: Pitch tracking responses under pitch reference step during continuous disturbance.

Figure 16: Yaw tracking responses under pitch reference step during continuous disturbance.

Figure 17: Experimental setup under wind gust.

K. Astrom, T. Hagglund, C. C. Hang, and W. K. Ho. Automatic tuning and adaptation for PID controllers—a survey. *Control Engineering Practice*, 9(4):699–714, 1993.

M. Fliess and C. Join. Model-free control and intelligent PID controllers: towards a possible trivialization of nonlinear control. In *European Symposium on Martensitic Transformations*, pages 1531–1550, Saint Malo, France, 2009.

M. Fliess and C. Join. Model-free control. *Int. J. Contr.*, 86:2228–2252, 2013.

K. J. Astrom and T. Hägglund. *Advanced PID Control*. Instrument Soc. Amer., 2006.

A. P. Gédouin, C. Join, E. Delaleau, J. M. Bourgeot, S. A. Chirani, and S. Calloch. A new control strategy for shape memory alloys actuators. In *15th IFAC Symposium on System Identification (SYSID)*, page 07007, Saint Malo, France, 2009.

L. Michel, C. Join, M. Fliess, P. Sicard, and A. Chériti. Model-free control of DC/DC converters. In *IEEE 12th Workshop on Control and Modeling for Power Electronics (COMPEL)*, pages 1–8, 2010.

J. D. Miras, C. Join, M. Fliess, S. Riachy, and S. Bonnet. Active magnetic bearing: A new step for model-free control. In *Proc. IEEE Conf. Decision & Contr.*, pages 7449–7454, Florence, Italy, 2013.

R. Madonski and P. Herman. Model-free control of a two-dimensional system based on uncertainty reconstruction and attenuation. In *Conference on Control and Fault-Tolerant Systems (SysTol)*, pages 542–547, 2013.

F. Lafont, J. F. Balmat, N. Pessel, and M. Fliess. A model-free control strategy for an experimental greenhouse with an application to fault accommodation. *Comput. Electron. Agricult.*, 110:139–149, 2015.

S. Choi, B. Andréa-Noveli, M. Fliess, H. Mounier, and J. Villagra. Multivariable decoupled longitudinal and lateral vehicle control: A model-free design. In *Proc. IEEE Conf. Decision & Contr.*, pages 2834–2839, Florence, Italy, 2013.

Y. A. Younes, A. Drak, H. Noura, A. Rabhi, and A. E. Hajjaji. Robust model-free control applied to a quadrotor UAV. *J. Intell. Robot Syst.*, 84:37–52, 2016.
Figure 18: Pitch tracking responses under pitch reference step during continuous wind disturbance.

Figure 19: Yaw tracking responses under pitch reference step during continuous wind disturbance.

M. Fliess and C. Join. On ramp metering: Towards a better understanding of ALINEA via model-free control. *Int. J. Contr.*, 90:1018–1026, 2017.

L. Menhour, B. Andréa-Novel, D. Gruyer M. Fliess, and H. Mounier. An efficient model-free setting for longitudinal and lateral vehicle control. validation through the interconnected pro-SiVIC/RTMaps prototyping platform. *IEEE Trans. Intel. Transport. Syst.*, page DOI: 10.1109/TITS.2017.2699283, 2017.

S. Choi, B. Andréa-Novel, M. Fliess, H. Mounier, and J. Villagra. Model-free control of automotive engine and brake for stop-and-go scenarios. In *Proc. European Contr. Conf.*, pages 3622–3627, Budapest, Hungary, 2009.

J. Villagra, , and D. Herrero-Pérez. A comparison of control techniques for robust docking maneuvers for an AVG. *IEEE Trans. Control Syst. Technol.*, 20:1116–1123, 2012.

Y. A. Younes, Y. Drak, A. Noura, H. Rabhi, and A. E. Hajjaji. Model-free control of a quadrotor vehicle. In *International Conference on Unmanned Aircraft Systems*, pages 1126–1131, 2014.

Quanser. *Laser Beam Stabilization Instructor Manual*. Quanser Speciality Experiment Series: LBS Laboratory Workbook, 2010.

D. S. Naidu. *Optimal Control Systems*. CRC. Press, 2002.

S. K. Choudhary. LQR based PID controller design for 3-DOF helicopter system. *International Journal of Computer, Electrical, Automation, Control and Information Engineering*, 8:1371–1376, 2014.

S. K. Choudhary. Robust LQR control for PWM converters: An LMI approach. *IEEE Trans. Ind. Electron.*, 56: 2548–2558, 2009.

M. Fliess and C. Join. Stability margins and model-free control: A first look. In *Proc. European Contr. Conf.*, Strasbourg, France, 2014.

J. Villagra, B. Andréa-Novel, S. Choi, M. Fliess, and H. Mounier. Robust stop-and-go control strategy: An algebraic approach for nonlinear estimation and control. *Int. J. Vehicle Auto. Syst.*, 7:270–291, 2009.