Conformal Invariance, Accelerating Universe and The Cosmological Constant Problem

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Abstract

We investigate a conformal invariant gravitational model which is taken to hold at pre-inflationary era. The conformal invariance allows to make a dynamical distinction between the two unit systems (or conformal frames) usually used in cosmology and elementary particle physics. In this model we argue that when the universe suffers phase transitions, the resulting mass scales introduced by particle physics should have variable contributions to vacuum energy density. These variations are controlled by the conformal factor that appears as a dynamical field. We then deal with the cosmological consequences of this model. In particular, we shall show that there is an inflationary phase at early times. At late times, on the other hand, it provides a mechanism which makes a large effective cosmological constant relax to a sufficiently small value consistent with observations. Moreover, we shall show that the conformal factor acts as a quintessence field that leads the universe to accelerate at late times.

1 Introduction

There is a fundamental conflict between observations and theoretical estimates on the value of the cosmological constant. In view of the cosmological observations we have an upper limit equivalent to $\frac{\Lambda}{G} \sim (0.003 \text{ ev})^4$. On the other hand, the standard model of particle physics implies that the universe has undergone a series of phase transitions at early epoch of its evolution contributing to the vacuum energy density 120 order of magnitude larger than this observational bound. Understanding of such a large discrepancy remains as one of the main problems of theoretical physics. There have been many attempts trying to resolve the problem

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Most of them are based on the belief that $\Lambda$ may not have such an extremely small value at all the time and there should exist a dynamical mechanism working during evolution of the universe which provides a cancellation of the vacuum energy density at late times [2, 3].

Among different kinds of such models of a decaying cosmological constant, more promising ones may be those which consist of a scalar field nonminimally coupled to gravity which itself is described by the usual Einstein-Hilbert action with a cosmological term [3]. The scalar field evolves with cosmic expansion in such a way that its energy density which is unstable due to the gravitational interaction compensates the vacuum energy density. Generic feature of these models is that they results in an asymptotic behavior $\Lambda \sim t^{-2}$ which in the present epoch roughly gives the observed upper bound. Nevertheless, as an immediate consequence of a nonminimal coupling these models entail an effective gravitational coupling which also behaves asymptotically as $G_{\text{eff}} \propto t^{-2}$, namely that gravity turns off at the cost of having a small cosmological constant.

This dramatic behavior encourages one to think about theories in which the gravitational coupling itself appears as a dynamical field, namely the scalar-tensor theories of gravitation. Along this line of thought we have concerned here with a particular form of these theories which is conformally invariant. The conformal invariance implies that the theory is invariant under local changes of units of length and time or local unit transformations [4]. In such transformations different unit systems or conformal frames are related via spacetime dependent conversion (or conformal) factors. Thus there exists a dynamical distinction between any two different unit systems in a conformal invariant model. The reason for introducing of such a model to study the cosmological constant problem is the important fact that the observational estimates and the theoretical predictions are actually carried out in two different unit systems, the unit systems usually used in cosmology and elementary particle physics. It is generally assumed that these two unit systems are related by a constant conversion factor. In other terms they are transformed by a global unit transformation.

In a local unit transformation, on the other hand, changes of unit systems find a dynamical meaning. We have already shown [5] that this dynamical changes of unit systems can be taken as a basis for constructing a cancellation mechanism which reduces a large effective cosmological constant to a sufficiently small value. In the present work, we intend to study the effects of the model introduced in [5] both on the early and the late times asymptotic behaviour of the scale factor in the standard cosmological model. In particular, we wish to answer the questions that how this model affect the inflation at early times and whether it leads to an accelerating universe at late times.

We shall assume that gravity is described by a conformal invariant gravitational model at early universe, specifically before breaking the gauge symmetry of fundamental interactions at GUT energy scale. When the gauge symmetry is spontaneously broken the resulting vacuum energy density as a dimensional parameter may be considered in different conformal frames. To deal with the cosmological implications of such a parameter it is essential to note that the cosmological and the quantum frames are dynamically distinct and therefore the vacuum energy density, as a mass scale introduced by particle physics, should take a variable configuration in the cosmological frame. This variation is then controlled by the conformal factor that appears

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1From now on these are referred to as the cosmological and the quantum unit systems, respectively.
as a dynamical field in our model. We shall show that this dynamical field plays a key role in a cancellation mechanism that works essentially due to expansion of the universe. It also appears as a quintessence field which causes the universe to accelerate at late times.

We organize this paper as follows: In section 2, we introduce a conformal invariant gravitational theory consisting of a real scalar field which is conformally coupled to gravity. It is the classical analogue of the model which we have previously investigated in [6]. In section 3, it is argued that introduction of an effective cosmological constant to the model requires that one considers a dynamical distinction between the cosmological and the quantum unit systems. We then study the consequences of this distinction in two parts. Firstly, consideration of the model at early times reveals that it can bring the universe into an inflationary phase. Secondly, we shall show that while the model leads to a damping behavior for the effective value of the cosmological constant at late times, it avoids the aforementioned problem on the gravitational coupling. Moreover, evolution of the scale factor indicates that the model gives rise to an accelerating expansion for the universe at late times. In section 4, we summarize and discuss our results.

Throughout this paper we work in units in which $\hbar = c = 1$ and the sign conventions are those of MTW [7].

## 2 The model

We shall consider a gravitational system which consists of a scalar field $\phi$ and the gravitational field, described by the action\footnote{This action has been investigated in different contexts. See, for example, [6, 8].}

$$
S = -\frac{1}{2} \int d^4 x \sqrt{-g} \left( g^{\mu\nu} \nabla_\mu \phi \nabla_\nu \phi + \frac{1}{6} R \phi^2 \right) \quad (1)
$$

where $\nabla_\mu$ denotes a covariant differentiation and $R$ is the Ricci scalar. Note that the action is not involved the free gravitational field contribution. The gravitational coupling is here dynamic and is given by $\sim \phi^{-2}$.

The remarkable feature of (1) is that it is invariant under conformal transformations

$$
g_{\mu\nu} = e^{2\sigma} \bar{g}_{\mu\nu} \quad (2)
$$

$$
\phi(x) = e^{-\sigma} \bar{\phi}(x) \quad (3)
$$

where $\sigma$ is a smooth dimensionless spacetime function. This means that the theory described by the action (1) can be described in many different conformal frames which are dynamically equivalent. They correspond to various configurations one assigns to the scalar field $\phi$ or various choices of local standards of units. Therefore different conformal frames may be distinguished by local values of some dimensional parameters which enter the theory.

Variation of (1) with respect to $g^{\mu\nu}$ and $\phi$ yields, respectively,

$$
G_{\mu\nu} = -6\phi^{-2} (\nabla_\mu \phi \nabla_\nu \phi - \frac{1}{2} g_{\mu\nu} \nabla_\gamma \phi \nabla^\gamma \phi) + \phi^{-2} (\nabla_\mu \nabla_\nu g_{\mu\nu} - g_{\mu\nu} \Box) \phi^2 \quad (4)
$$
\[ \Box \phi - \frac{1}{6} R \phi = 0 \]  
\text{(5)}

Here \( \Box \equiv g^{\mu \nu} \nabla_\mu \nabla_\nu \) and \( G_{\mu \nu} \) is the Einstein tensor. One should recognize that the equations (4) and (5) are not independent. Indeed, the trace of (4) gives

\[ \phi (\Box - \frac{1}{6} R) \phi = 0 \]  
\text{(6)}

which contains the equation (5). This is a direct consequence of the absence of a dimensional parameter in the model. In the next section we shall introduce a cosmological constant which leads the field equations to be independent. Note that addition of a dimensional parameter in a conformal invariant model would not necessarily mean the breakdown of the symmetry since in a local unit transformation all the dimensional parameters are expected to be transformed according to their dimensions. The conformal invariance is broken when one singles out a particular conformal frame in which the dimensional parameter or the gravitational coupling \( \phi^{-2} \) take on preferred constant values.

### 3 Cosmological implications

#### 3.1 Vacuum-dominated era

It is generally believed that at GUT energy scale the universe has passed through a certain disordered phase associated with the gauge symmetry of the grand unified theories. When the gauge symmetry is spontaneously broken the structure of the vacuum drastically changes in the sense that it acquires a large amount of energy density which appears as a large effective cosmological constant. The key question which should be answered at this stage is that how this vacuum energy density should be coupled to gravity.

To clarify this point we remark that the use of two different unit systems are conventional for measuring this energy density. On one hand, the upper bound set by observations is obtained in a unit system which is defined in terms of large scale cosmological parameters (the cosmological unit system). On the other hand, the theoretical predictions are based on a natural unit system which is suggested by quantum physics (the quantum unit system). One usually presupposes that these two unit systems should be indistinguishable up to a constant conversion factor in all spacetime points. It means that they should transform to each other by a global unit transformation. Such a global transformation clearly carries no dynamical implications and the use of a particular unit system is actually a matter of convenience.

Here we would like to consider a different approach. We introduce a theoretical scheme in which an explicit recognition is given to the distinguished characteristics of the cosmological and the quantum unit systems. In such a theoretical scheme one should no longer accept the triviality one usually assigns to a unit transformation.

We first assume that gravity is described by the conformal invariant gravitational model (1), before the universe goes through phase transition. In the context of this conformal invariant model we intend to consider local unit transformations. In fact, the conformal invariance of (1)
implies that there exists a dynamical distinction between any two different conformal frames (or unit systems) because they are generally related via a spacetime dependent conversion factor. When the universe goes through phase transition, one should incorporate the resulting vacuum energy density to the model (1) by noting this dynamical distinction. We should therefore write the action (1) as

\[ S = -\frac{1}{2} \int d^4 x \sqrt{-g} \left\{ g^{\mu \nu} \nabla_\mu \phi \nabla_\nu \phi + \frac{1}{6} (R - 2 \Lambda e^{-2\sigma}) \phi^2 \right\} \quad (7) \]

where \( g^{\mu \nu} \) and \( \phi \) are the metric tensor and the scalar field in the cosmological frame. These are related to \( \bar{g}^{\mu \nu} \) and \( \bar{\phi} \) by the relations (2) and (3). Here \( \bar{\Lambda} \) is a typical mass scale introduced by elementary particle physics measured in the quantum frame. Note that since \( \bar{\Lambda} \) carries the dimension of squared mass it appears in the action (7) with an exponential factor \( e^{-2\sigma} \). The corresponding value of \( \bar{\Lambda} \) in the cosmological frame is \( \Lambda \) which is given by

\[ \Lambda = \bar{\Lambda} e^{-2\sigma} \quad (8) \]

As a consequence various mass scales introduced by elementary particle physics should have variable contributions to vacuum energy density in the cosmological frame. To study the evolution of such mass scales we let the above action involve a kinetic term for \( \sigma \). In this way we consider \( \sigma \) as a dynamical field. This seems to be necessary to account for the dynamical distinction between any two different unit systems. The action (7) takes then the form

\[ S = -\frac{1}{2} \int d^4 x \sqrt{-g} \left\{ g^{\mu \nu} \nabla_\mu \phi \nabla_\nu \phi + \left( \frac{1}{6} (R - 2 \Lambda e^{-2\sigma}) + \alpha g^{\mu \nu} \nabla_\mu \sigma \nabla_\nu \sigma \right) \phi^2 \right\} \quad (9) \]

where \( \alpha \) is a dimensionless constant parameter. This action can now be used to describe a vacuum-dominated universe since it excludes any matter contribution.

Variation of (9) with respect to \( g^{\mu \nu} \), \( \phi \) and \( \sigma \) yields, respectively,

\[ G_{\mu \nu} + \bar{\Lambda} e^{-2\sigma} g_{\mu \nu} = 6 \phi^{-2} \tau_{\mu \nu} \quad (10) \]

\[ \square \phi - \frac{1}{6} R \phi + \frac{1}{3} \bar{\Lambda} \phi e^{-2\sigma} - \alpha \phi \nabla_\gamma \sigma \nabla^\gamma \sigma = 0 \quad (11) \]

\[ \frac{1}{\sqrt{-g}} \nabla_\mu (\sqrt{-g} \phi^2 g^{\mu \nu} \nabla_\nu \sigma) = \frac{1}{3\alpha} \bar{\Lambda} \phi^2 e^{-2\sigma} \quad (12) \]

where

\[ \tau_{\mu \nu} = -\left( \nabla_\mu \phi \nabla_\nu \phi - \frac{1}{2} g_{\mu \nu} \nabla_\gamma \phi \nabla^\gamma \phi \right) + \frac{1}{6} \left( \nabla_\mu \nabla_\nu - g_{\mu \nu} \square \right) \phi^2 - \alpha \phi^2 \left( \nabla_\mu \sigma \nabla_\nu \sigma - \frac{1}{2} g_{\mu \nu} \nabla_\gamma \sigma \nabla^\gamma \sigma \right) \quad (13) \]

The exponential coefficient of \( \bar{\Lambda} \) emphasizes that this mass scale belongs to a unit system which is different from that used in cosmology. Intuitively, one expects that there should be no distinction between the cosmological and the quantum unit systems at sufficiently early times so that
\( e^{-2\sigma} \to 1 \)
\[
\Lambda = \bar{\Lambda} e^{-2\sigma} \to \bar{\Lambda} \text{ as } t \to 0 \tag{14}
\]
This can be taken as an early-time boundary condition for the dynamical field \( \sigma \). In an expanding universe the distinction between these two unit systems is expected to increase with time since all cosmological scales enlarge as the universe expands. Thus the conformal factor \( e^{2\sigma} \) must be an increasing function of time. According to (8), this automatically provides us with a dynamical mechanism for reducing the mass scale \( \Lambda \) in the cosmological frame. It is important to note that this mechanism essentially works due to cosmic expansion.

Before studying this mechanism we would like to focus on the behaviour of the field equations at early times. To do this, we apply the field equations to a homogeneous and isotropic universe. In particular, we specialize to a spatially flat Friedman-Robertson-Walker metric
\[
ds^2 = -dt^2 + a^2(t)(dx^2 + dy^2 + dz^2) \tag{15}
\]
where \( a(t) \) is the scale factor. The homogeneity and isotropy require that the fields \( \phi \) and \( \sigma \) be only functions of time. The equations (10), (11) and (12) become
\[
3\left(\frac{\dot{a}}{a}\right)^2 - \bar{\Lambda}e^{-2\sigma} + 3\frac{\dot{\phi}^2}{\phi^2} + 6\frac{\ddot{a}}{a} + 3\alpha \dot{\sigma}^2 = 0 \tag{16}
\]
\[
\frac{\ddot{\phi}}{\phi} + 3\frac{\dot{a}}{a} + \left(\frac{3}{a} + \frac{\dot{a}^2}{a^2}\right) - \alpha \dot{\sigma}^2 - \frac{1}{3}\bar{\Lambda}e^{-2\sigma} = 0 \tag{17}
\]
\[
\ddot{\sigma} + \left(3\frac{\dot{a}}{a} + 2\frac{\dot{\phi}}{\phi}\right)\dot{\sigma} + \frac{\bar{\Lambda}}{3\alpha} e^{-2\sigma} = 0 \tag{18}
\]
where the overdot indicates differentiation with respect to \( t \). We take\(^5\)
\[
a \sim e^{nt} \tag{19}
\]
\[
\phi \sim e^{mt} \tag{20}
\]
and
\[
\sigma = \xi t \tag{21}
\]
We substitute these and the condition (14) into (16), (17) and (18) to obtain
\[
n = \xi = \frac{1}{\sqrt{\alpha(4\alpha + 1)}} \sqrt{\frac{1}{3}\bar{\Lambda}} \tag{22}
\]
\[
m = (2\alpha - 1)n \tag{23}
\]
\(^5\)The relations (19) and (20) indicate the behaviour of \( a(t) \) and \( \phi(t) \) up to some proportionality constants. This is due to the fact that the field equations do not change under scaling of \( a(t) \) and \( \phi(t) \). These proportionality constants, of course, do not have any role in our arguments.
These results indicate that the scale factor grows exponentially and the spacetime geometry is described by the de Sitter metric. This inflation continues to be a solution until $e^{-2\sigma} \approx 1$ holds. After a sufficiently long time this approximation is no longer valid and the inflation ends. The point which we wish to make here is that our model does not provide any contradiction with existence of an inflationary phase at early times. This is important since the basic idea of inflation seems to be the only reasonable programme, suggested so far, to resolve the cosmological puzzles such as the flatness and the horizon problems [9].

### 3.2 Matter-dominated era

To apply the model to a matter-dominated universe we should first add a matter system in the action (9). We write

$$S = -\frac{1}{2} \int d^4x \sqrt{-g} \left\{ g^{\mu\nu} \nabla_\mu \phi \nabla_\nu \phi + \phi^2 \left( \frac{1}{6} (R - 2\Lambda e^{-2\sigma}) + \alpha g^{\mu\nu} \nabla_\mu \sigma \nabla_\nu \sigma \right) \right\} + S_m[g_{\mu\nu}]$$

where $S_m[g_{\mu\nu}]$ is the matter field action. The gravitational equations for the action (24) will be

$$G_{\mu\nu} + \Lambda e^{-2\sigma} g_{\mu\nu} = 6\phi^{-2} (T_{\mu\nu} + \tau_{\mu\nu})$$

where

$$T_{\mu\nu} = \frac{2}{\sqrt{-g}} \frac{\delta}{\delta g^{\mu\nu}} S_m[g_{\mu\nu}]$$

The field equations of $\phi$ and $\sigma$ remain unchanged in the presence of matter. We may put the equation (11) into the trace of (25) to obtain

$$T_{\gamma} = \frac{1}{3} \Lambda \phi^2 e^{-2\sigma}$$

which is a relation between the trace of the matter stress-tensor and the vacuum energy density. This relation have important implications that we describe in the following:

Let us first take $T_{\mu\nu}$ to be the stress-tensor of a perfect fluid with energy density $\rho$ and pressure $p$

$$T_{\mu\nu} = \rho u_\mu u_\nu + p(g_{\mu\nu} + u_\mu u_\nu)$$

where $u_\mu$ is the four velocity of the fluid. For a pressureless fluid, (27) takes the form

$$\rho \phi^{-2} \sim \Lambda e^{-2\sigma}$$

In an expanding universe $\rho \phi^{-2}$ decreases. The relation (29) then predicts that the same thing happens for $\Lambda$. Two important results arise from this statement. Firstly, the expansion of the universe induces the reduction of $\Lambda$. This requires that the conformal factor $e^{2\sigma}$, or the $\sigma$ field, be an increasing function of time. In the previous subsection we discussed that this is indeed the case in a vacuum-dominated universe. In this section we shall show that this is also true in a matter-dominated universe.

\*Note that $\xi > 0$ in the relation (21).
Secondly, the conformal invariance of our model does not allow to incorporate naively a constant mass scale such as $\bar{\Lambda}$. Due to (27), this would not be dynamically consistent with the field equations. This emphasizes the role of the dynamical distinction between the cosmological and the quantum unit systems in our model.

We intend now to investigate the field equations in a matter-dominated universe. For the metric (15), the equation (25) becomes

$$3\left(\frac{\dot{a}}{a}\right)^2 - \bar{\Lambda}e^{-2\sigma} + 3\frac{\dot{\phi}^2}{\phi^2} + 6\frac{\dot{a}}{a}\frac{\dot{\phi}}{\phi} + 3\alpha\dot{\sigma}^2 = 6\phi^{-2}\rho$$

For late times we take

$$a \sim \left(\frac{t}{t_0}\right)^v$$

$$\phi = \text{const.}$$

$$e^\sigma = \sigma_0 t$$

where $t_0$ is the present age of the universe and $\sigma_0$ is a constant with dimension of mass. Let us first estimate the gravitational coupling. We substitute (32) into the equation (30) to obtain

$$3H^2 - \bar{\Lambda}e^{-2\sigma} + 3\alpha\dot{\sigma}^2 = 6\phi^{-2}\rho$$

where $H = \frac{\dot{a}}{a}$ is the Hubble parameter. Using (29) this equation become

$$3H^2 + 3\alpha\dot{\sigma}^2 \sim \phi^{-2}\rho$$

From the relation (33), one infers that at late times $\dot{\sigma} \rightarrow t^{-1} \sim H$. Thus (35) reduces to

$$H^2 \sim \rho\phi^{-2}$$

Now we may use the observational fact that [10]

$$\rho \sim \rho_c$$

with $\rho_c$ being the critical density

$$\rho_c = \frac{3H_0^2}{8\pi G}$$

Here $G$ is the gravitational constant and $H_0$ is the Hubble constant. From (36) and (37), when $t \rightarrow t_0$ we obtain

$$\phi^{-2} \sim G$$

Thus the constant configuration of $\phi^{-2}$ at late times is given by the gravitational constant. In this case the action (24) reduces to

$$S = -\frac{1}{16\pi G} \int d^4x\sqrt{-g}(R - 2\bar{\Lambda}e^{-2\sigma} + 6\alpha g^{\mu\nu}\nabla_\mu\sigma\nabla_\nu\sigma) + S_m[g_{\mu\nu}]$$
This differs from the usual Einstein-Hilbert action in the sense that it contains a dynamical field \( \sigma \) and a varying cosmological term \( \tilde{\Lambda} e^{-2\sigma} \).

Variation of (40) with respect to \( g^{\mu\nu} \) and \( \sigma \) gives the field equations

\[
G_{\mu\nu} + \tilde{\Lambda} e^{-2\sigma} g_{\mu\nu} + 6\alpha (\nabla_\mu \sigma \nabla_\nu \sigma - \frac{1}{2} g_{\mu\nu} \nabla_\gamma \sigma \nabla_\gamma \sigma) = 8\pi G T_{\mu\nu}
\]  
(41)

\[
\Box \sigma = \frac{\tilde{\Lambda}}{3\alpha} e^{-2\sigma}
\]  
(42)

For the metric (15) and the matter stress-tensor (28), these equations become

\[
3\frac{\dot{a}^2}{a^2} - \tilde{\Lambda} e^{-2\sigma} + 3\alpha \dot{\sigma} = 8\pi G \rho 
\]  
(43)

\[
\frac{\dot{a}^2}{a^2} + 2\frac{\ddot{a}}{a} - \tilde{\Lambda} e^{-2\sigma} - 3\alpha \dot{\sigma} = -8\pi G p 
\]  
(44)

\[
\ddot{\sigma} + 3\frac{\dot{a}}{a} \dot{\sigma} + \frac{\tilde{\Lambda}}{3\alpha} e^{-2\sigma} = 0
\]  
(45)

The equation (43) together with (44) gives

\[
3\frac{\ddot{a}}{a} - \tilde{\Lambda} e^{-2\sigma} - 6\alpha \dot{\sigma}^2 = -4\pi G (\rho + 3p)
\]  
(46)

For \( p = 0 \), we combine this with (43) to obtain

\[
\frac{\dot{a}^2}{a^2} + 2\frac{\ddot{a}}{a} - 3\alpha \dot{\sigma}^2 - \tilde{\Lambda} e^{-2\sigma} = 0
\]  
(47)

Now if we substitute (31) and (33) in the equations (45) and (47) we obtain

\[
v = \frac{2}{3}, \quad -3\alpha
\]  
(48)

\[
\sigma_0 = \sqrt{\frac{\tilde{\Lambda}}{3\alpha (1 - 3v)}}
\]  
(49)

One can take the solution \( v = \frac{2}{3} \) for \( \alpha < 0 \) since only in this case \( \sigma_0^2 > 0 \). This corresponds to the solution of the standard cosmological model for the evolution of the scale factor in a matter-dominated universe.

For \( v = -3\alpha \), we obtain accelerating solutions (with \( v > 1 \)) for \( \alpha < -\frac{1}{3} \). The deceleration parameter

\[
q = -\frac{\ddot{a}a}{\dot{a}^2}
\]  
(50)

is

\[
q = \frac{1}{v} - 1
\]  
(51)
which is negative for $\alpha < -\frac{1}{3}$.

On the other hand, in the cosmological frame we obtain

$$\Lambda = \bar{\Lambda} e^{-2\sigma} \sim t^{-2}$$

where we have used (33) and (49). This result is consistent with the observational bound.

We see that the conformal factor, or equivalently the $\sigma$ field, plays two important role in our model. Firstly, evolution of this dynamical field induced by cosmic expansion damps a large effective cosmological constant. Secondly, it plays the role of a quintessence field that causes the universe to accelerate at late times.

4 Summary and discussion

We have investigated the cosmological consequences of a conformal invariant gravitational model which is assumed to hold during the very early stages of evolution of the universe. The conformal invariance of the model allows us to formalize a theoretical framework in which there exists a dynamical distinction between the two unit systems used in cosmology and elementary particle physics. It is argued that when the universe goes through phase transition the resulting large effective cosmological constant $\bar{\Lambda}$ as a mass scale introduced by particle physics is related to the corresponding mass scale in the cosmological frame by $\Lambda = \bar{\Lambda} e^{-2\sigma}$. Thus all mass scales introduced by particle physics should be considered as variable in the cosmological frame [5]. This automatically suggests a cancellation mechanism caused by expansion of the universe. We emphasize that this feature is also suggested by the relation (27) which is a dynamical consistency relation on the trace of the matter stress-tensor in our gravitational system.

The question which naturally arises is whether such a variable cosmological term alters the standard picture of early history of the universe. To address this question, we have shown that there exists a solution for the field equations at early times exhibiting an exponential growth of the scale factor. It is important to note that in this model there is a natural exit of the universe from this inflationary phase namely when $e^{2\sigma} \approx 1$ does not hold due to growth of $\sigma$.

Our primary interest is to explore the cosmological constant problem. We have shown that the asymptotic solution of the field equations in the matter-dominated era leads to the following consequences:

1) The relation (52) indicates that the cosmological constant in the cosmological frame $\Lambda$ is of the same order of $t^{-2}$ which is consistent with the upper bound set by observations. The smallness of the cosmological constant is therefore related to the fact that the universe is old.

2) The gravitational coupling in the present state of the universe is given by $\phi^{-2} \sim G$.

3) The scale factor exhibits a late-time asymptotic power law expansion $a \propto t^v$ with $v > 1$. This implies that the universe is accelerating and $\sigma$ plays the role of a quintessence field. The acceleration of the universe is generally achieved by negative values of $\alpha$ ($\alpha < -\frac{1}{3}$). This means that $\sigma$ is an ordinary massless scalar field with positive energy density.
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