An extended study on the supersymmetric SO(10) models with natural doublet-triplet splitting

Qian Wan\textsuperscript{1} and Da-Xin Zhang\textsuperscript{2}

School of Physics, Peking University, Beijing 100871, China

Abstract

In the supersymmetric SO(10) models, the doublet-triplet splitting problem can be solved through the Dimopoulos-Wilczek mechanism. This mechanism is extended in the non-renormalizable version. Improvement on the realistic model is also made.
1 Introduction

The Supersymmetric (SUSY) Grand Unified Theories (GUTs) are very important to search for physics beyond the Standard Model (SM). Among these models, the SUSY SO(10) models are very predictive. In SUSY SO(10) models, the fermions of the three generation are contained in three spinor representations $16 (\psi)$, including right-handed neutrinos which are responsible for explaining the neutrino oscillation data through the seesaw mechanism.

The renormalizable SUSY SO(10) models can be constructed successfully to achieve many important features in the same models, such as gauge coupling unification, fermion masses, proton decays, and doublet-triplet splitting (DTS). In contrast, the non-renormalizable (NR) SUSY SO(10) models are more difficult to be constructed successfully, since in these models we need to take into account all allowed interactions which are consistent with the symmetry of the models, even these interactions come from higher order operators. These difficulties are very transparent in the realization of the DTS through the Dimopoulos-Wilczek (DW) mechanism. The only known DW mechanism in the NR SUSY SO(10) models was realized in [12], in contrast to those in the renormalizable models [11, 13, 14, 15]. The DW mechanism in the NR SUSY SO(10) models was further studied in [16] in an effort to construct a realistic model. However, to construct a successful fermion mass sector, the model in [16] is very complicated and even incomplete.

In the present work, we will extend the DW mechanism in the NR models and improve the model in [16]. We will first give an extended study on the NR version of the DW mechanism. We find that the DW mechanism can be applied in the presence of new high dimension couplings. Then by modifying the model in [16], we solve the difficulties in the fermion mass sector. We will also discuss the proton decay problem in the new model.

2 Extended study on the DW mechanism

In SO(10) models, symmetry breaking requires the existence of two sectors in general. The first sector, which breaks SO(10) into its subgroup $SU(3)_c \times SU(2)_L \times SU(2)_R \times U(1)_{B-L}$ or $SU(3)_c \times SU(2)_L \times U(1)_{I_{3R}} \times U(1)_{B-L}$, needs Higgs fields in the SO(10) representation 45 or 210. The second sector is rank breaking which breaks $SU(2)_R \times U(1)_{B-L}$ or $U(1)_{I_{3R}} \times U(1)_{B-L}$ into $U(1)_{Y}$ of the SM. This rank breaking sector needs fields carrying $B-L$ numbers to develop Vacuum Expectation Values (VEVs). The required fields are usually 126 + $\overline{126}$ in the renormalizable models or 16 + $\overline{16}$ in the NR models. These two sectors need to be coupled together to avoid extra massless Goldstones which are not permitted at low energy.

The problem of DTS in GUT theories is to answer the question why the weak doublets of the minimal SUSY Model (MSSM) are so light compared to the color triplets in the same Higgs multiplets. In realizing DTS through the DW mechanism, there are several versions can be used. In the simplest NR models, a 45-plet ($A$) is needed with the superpotential

$$W_{DW} = \frac{m_A}{2} A^2 + \frac{1}{M^*} A^4 + \cdots, \quad (1)$$
where $M_*$ stands for some large mass scale, and terms of higher dimensions are not displayed explicitly. The symbol $A^4$ are SO(10) invariants of the form $(A^2)_X(A^2)_X$, where $X = 1, 54, 210, 770$ are symmetric representations of SO(10), so the second term in (1) stands for four terms with different couplings which are suppressed for simplicity. In the following, analogue couplings will also be suppressed.

After symmetry breaking, the field $A$ can develop two VEVs along the SM singlet directions. They are labeled by the representations under the SO(10) subgroup $SU(4)_c \times SU(2)_L \times SU(2)_R$ as $A_1(1, 1, 3)$ and $A_2(15, 1, 1)$. The VEV in (1) is in the form

$$\langle W_{DW} \rangle = \frac{m_A}{2} (A_1^2 + A_2^2) + \frac{1}{M_*} (A_1^4 + A_1^2 A_2^2 + A_2^4).$$  (2)

In the bracket of the second term, we have also suppressed the CG coefficients. In (2) the dependence on $A_1$ is at least quadratic and thus the F-term corresponding to $A_1$ is proportional to $A_1$. Then, the F-flatness conditions give several solutions, among them we pick up the DW solutions

$$A_1 = 0, \quad A_2 \sim \sqrt{m_A M_*}$$  (3)

to realize DTS when coupled with two different Higgs in the 10[10]. In additional to (2), the higher dimensional terms with even numbers of $A$s are also of the forms $A_1^{2m} A_2^{2n}$ $(m, n$ are integers). There are also higher dimensional terms with odd numbers of $A$s in the form

$$\epsilon \cdot A^5 \equiv \epsilon^{ij_1 j_2 \cdots i_{10}} A_{i_1 j_2} \cdots A_{i_9 j_{10}},$$  (4)

whose contribution to the VEV is $A_1^2 A_2^3$. All these higher terms do not spoil the DW solutions (3).

The difficulty in the NR models is that the fields $A(45)$ need to be coupled with the rank breaking Higgs fields 16 + 16. These couplings include those which are linear in $A_1$. When the linearly coupled terms are added in addition to (2), the DW solutions disappear. In [12] the difficulty is overcome by introducing two pairs of 16 + 16 ($C, \bar{C}, C', \bar{C}'$) through

$$W_{Rank} = \left( \bar{C}C' + \bar{C}'C \right) \left( S + \frac{1}{M_*} \bar{C}C + \frac{1}{M_*} ZA \right) + \frac{1}{M_*^2} \bar{C}'C' Z^2 S,$$  (5)

and pick up the solutions $\langle C' \rangle = \langle \bar{C}' \rangle = 0$. Here $S$ and $Z$ are SO(10) singlets. Later on, in [16] a realistic model has been attempted to construct.

Together with (1) and (5), the SO(10) symmetry is broken down into the SM gauge symmetry. The F- and D-flatness conditions give

$$s \sim \frac{c^2}{M_*}, \quad z \sim \frac{c^2}{A_2}, \quad A_2 \sim \sqrt{m_A M_*},$$  (6)

where $s = \langle S \rangle$, $z = \langle Z \rangle$ and $c = \langle C \rangle \equiv \langle \bar{C} \rangle$. As will be introduced in the next section, an anomalous gauge $U(1)$ symmetry is usually used to forbid the unwanted couplings of a model. The D-term of this symmetry has a Fayet-Iliopoulos term of the order $-\sqrt{\xi} \sim (0.1 - 1) M_* \sim 10^{17-18}$ GeV [18, 19], which is to be canceled by the VEVs of those U(1) charged fields $\phi_i$ as $\xi + \sum Q_i \phi_i^2 = 0$. Typically we have $c^2 + s^2 + A_2^2 \sim -\frac{1}{4} \xi$ [16], then

$$c, z \sim (1 - 10) \times M_{GUT}; \quad s \sim (0.01 - 0.1) \times M_{GUT}; \quad A_2 \approx M_{GUT}.$$  (7)
These VEVs will be used in the following study on the fermion sector of the MSSM.

To generate the massless doublets of the MSSM while keeping the color triplets heavy, two 10-plets $H$ and $H'$ are introduced to couple with $A(45)$ with the DW solutions through

$$H A H' + \frac{m_{H'}}{2} H'H'.$$

Here $H = H(1,2,2) + H(6,1,1)$ (and similar for $H'$). This is the standard form of superpotential in realizing DTS through the DW mechanism\[10\]. In the NR models, there are also higher dimensional terms in additional to (8) in the form

$$\frac{1}{M^2_m} H A^{2m+1} H' \ (m = \text{integer})$$

which does not give mass to the doublets $H(1,2,2)$ since $A^{2m+1}_{\pm}$ contain no VEV in either $(1,1,1)$ or $(1,1,3)$ direction. Terms $H A^{2m} H'$ will spoil the DW mechanism which need to be forbidden by imposing extra symmetries on the models.

In the present study we need to extend the coupling $H A H'$ to the case in the presence of spinors $C(16), \bar{C}(1\bar{6})$. These couplings are of the forms

$$\frac{1}{M^2_s} H A \left( C^2 + \bar{C}^2 \right),$$

which have escaped attentions in the literature. Note that $16 \times 16 = 10_S + 120_A + 126_S$, $1\bar{6} \times 1\bar{6} = 10_S + 120_A + 1\bar{2}6_S$, where the labels $A, S$ stand for anti-symmetric and symmetric, respectively, $CC$ and $\bar{C}\bar{C}$ thus contain no 120. Also, because 10, 126 + 1\bar{2}6, 45 are tensors of ranks 1,5,2, respectively, they do not couple together. Consequently, in (10) $CC$ and $\bar{C}\bar{C}$ can be only in the representation 10 so that the DW mechanism can be applied in this case.

The operators of the form $H A^{2m}(C^2 + \bar{C}^2)$ also destroy the DW mechanism which need to be forbidden by introducing an extra symmetry, as in the case of the operators $H A^{2m} H'$. In the presence of the higher dimensional couplings

$$\frac{1}{M^2_{s+m}} H A^{2m+1} \left( C^2 + \bar{C}^2 \right),$$

$CC$ ($\bar{C}\bar{C}$) can be either 10 or 126 ($1\bar{2}6$). The case $(CC)_{126}$ (and similarly ($\bar{C}\bar{C})_{\bar{1}26}$) needs to be discussed as following. Note that $10 \times 126 = 210 + 1050$\[17\], neither 210 nor 1050 has a SM singlet direction which couples with odd number of $A_2$ fields. More generally, two fields (or two products of fields with each product in a irreducible representation) can only couple to VEVs with either even or odd numbers of $A_2$ fields. The exceptions come from the couplings of the form (4) which are zero in the DW cases with $A_1 = 0$. This completes the feasibility of the DW mechanism in the presence of the higher dimensional couplings (11).

### 3 Toward a realistic model

The model in \[16\] has difficulties in the SM fermion mass sector, where a complete set of Yukawa couplings were not given. In the present work, we will improve the model in \[16\] to solve the
difficulties. As in [16], the Higgs sector contains an adjoint \( A(45) \), two 10-plets \( H + H' \), two pairs of spinor-anti-spinor superfields \( C/\bar{C} + C'/\bar{C}' \) and two singlets \( S \) and \( Z \). A \( Z_2 \) assisted anomalous \( U(1) \) symmetry is needed to discard harmful couplings. The charges of the Higgs fields and those of the three matter families \( f_i \) under \( U(1)_A \times Z_2 \) are listed in Table 3.

| \( Q \) | \( \bar{C} \) | \( \bar{C}' \) | \( C \) | \( C' \) | \( S \) | \( Z \) | \( A \) | \( H \) | \( H' \) | \( f_i \) |
|------|-------|-------|------|------|------|------|------|------|------|------|
| \( Z_2 \) | +     | +     | +    | +    | -    | -    | +    | -    | -    | +    |

Table 1: \( U(1)_A \times Z_2 \) charges for all the superfields.

The main difference of the charges in Table 3 from those in [16] are introduced to replace the Higgs coupling \( H\bar{C}\bar{C} \) by \( H\bar{C}\bar{C}' \). The superpotential for the Higgs sector is

\[
W_H = HH'A + H\bar{C}\bar{C}' + \frac{1}{M_*}(H'H^2Z + H'AC^2 + HC'^2S) + \frac{S}{M_*^2}(H'CC'Z + H'^2Z^2 + H'AC^2) + \frac{Z^2}{M_*^3}(H'\bar{C}\bar{C}'Z + H'^2C\bar{C} + H'^2AZ + H'ACC' + H'AC^3C) + \cdots ,
\]

following which the electro-weak doublets have the mass matrix

\[
M_D = \begin{pmatrix}
0 & 0 & c & 0 \\
0 & 0 & s + \frac{c^2}{M_*} + \frac{zA_2}{M_*^2} & 0 \\
0 & s + \frac{c^2}{M_*} + \frac{zA_2}{M_*^2} & c\frac{s^2}{M_*^2} + \frac{czA_2}{M_*^2} & 0 \\
0 & \frac{c^2}{M_*} + \frac{csA_2}{M_*^2} & \frac{cs^2}{M_*^2} + \frac{cs^2A_2}{M_*^2} & M_{H'}
\end{pmatrix}
\]

where the columns are \( H_u, \bar{C}_u, \bar{C}_u', H'_u \) and the rows are \( H_d, C_d, C'_d, H'_d \), respectively, and

\[
M_{H'} = \frac{c^2 + zA_2}{M_*} + \frac{s^2 + z^2}{M_*^2} + \cdots .
\]

In the 4th column, the entry (1,4) comes from the coupling \( HAH' \) so that only the VEV \( A_1 = 0 \) can be taken. The entry (2,4) comes from \( H'ACC \) where the product \( CC \) can only take the representation 10 of SO(10), and again, only the VEV \( A_1 = 0 \) can be taken. As was discussed after (11), higher dimensional operators do not break this result.

The massless eigenstates of (13) are

\[
H^0_u = H_u, \quad H^0_d = \frac{c}{\sqrt{c^2 + M_*^2}}H_d - \frac{M_*}{\sqrt{c^2 + M_*^2}}C_d,
\]

they are the Higgs doublets of the MSSM. In contrast, in [16] the \( H^0_d \) has components from all the four down-type Higgs doublets, which differ from the present model in the fermion sector of the MSSM.

In the spectrum, there is no major difference between this work and [16]. As in [16], threshold effects can be adjusted to be small in realizing gauge coupling unification. We refer [16] for more details.
4 Fermion masses and proton decays

With Table 3, the most general Yukawa couplings are

\[ W_Y = f_i f_j \left( H + \frac{1}{M_*} C^2 + \frac{1}{M_*^2} (\bar{C}^2 S + H A^2) + \frac{Z^2}{M_*^3} (C C' + H' Z) + \frac{S Z^2}{M_*^4} \bar{C} \bar{C}' \right. \]
\[ \left. + \frac{Z^4}{M_*^5} C'^2 + \frac{S Z^4}{M_*^6} \bar{C}^2 \right), \] (16)

in which the fermion masses are given by

\[ f_i f_j \left( H + \frac{C^2}{M_*} + \frac{C^2 S}{M_*^2} + \frac{H A^2}{M_*^2} \right) \supset f_i f_j \left( H_u + H_d \right) \]
\[ + \frac{c}{M_*} C_d + \frac{s}{M_*^2} \bar{C}^2 + \frac{A^2_2}{M_*^2} (H_u + H_d), \] (17)

where the third term contributes to the Majorana masses for the right-handed neutrinos at \( s c^2 / M_*^2 \sim 10^{14} \text{GeV} \) when \( s \sim 0.1 M_{\text{GUT}} \) is taken. This term contributes neither to Dirac masses for the neutrinos nor to the charged fermion masses, so data on large neutrino mixing can be fitted without further constraints. In the fourth term, \( H A^2 \) can be 10 or \( 126 \) but not 120 when \( A_1 = 0 \) is taken, and 126 gives Georgi-Jarlskog type corrections to fermion masses[20]. (17) also contribute to proton decays through dimensional-five operators when replacing the doublets by the color triplets of the same representations.

In (16),

\[ f_i f_j \left( \frac{C C' Z^2}{M_*^4} + \frac{H' Z^3}{M_*^3} + \frac{\bar{C} C' S Z^2}{M_*^4} \right) \] (18)

contributes to proton decays but not to fermion masses, but these contributions are power suppressed so the couplings can be set to zeros safely. The remaining terms in (16),

\[ f_i f_j \left( \frac{C'^2 Z^4}{M_*^5} + \frac{\bar{C}'^2 S Z^4}{M_*^6} \right) \] (19)

contribute to neither, thus are irrelevant to the present study.

In summary, the Yukawa couplings (16) can be simplified as

\[ f_i f_j \left( Y_H H + Y_C \frac{1}{M_*} (C^2)_{10} + Y_C \frac{s}{M_*^2} (\bar{C}^2)_{126} + Y_{HAA} \frac{1}{M_*^2} (H A^2)_{126} \right) \]
\[ \left. _{ij} \right), \] (20)

where the Yukawa couplings \( Y \)’s are added explicitly. We have been omitted the contributions from \( (H A^2)_{10} \) which is a small correction to the \( Y_H \) term. Together with (15), the Dirac mass matrices of fermions are given by

\[ M_u = \left( Y_H + Y_{HAA} \frac{A^2_2}{M_*^2} \right) \langle H_u^0 \rangle, \]
\[ M_\nu = \left( Y_H - 3 Y_{HAA} \frac{A^2_2}{M_*^2} \right) \langle H_u^0 \rangle, \]
\[ M_d = \frac{c}{M_*} \left( Y_H + 2 Y_C + Y_{HAA} \frac{A^2_2}{M_*^2} \right) \langle H_d^0 \rangle, \]
\[ M_e = \frac{c}{M_*} \left( Y_H + 2 Y_C - 3 Y_{HAA} \frac{A^2_2}{M_*^2} \right) \langle H_d^0 \rangle, \] (21)
following which a small $\tan \beta \equiv \langle H_u^0 \rangle / \langle H_d^0 \rangle \sim O(1)$ is preferred. It can be also noted that in the down-type quark and charged lepton masses, the $Y_{HAA}$ term is suppressed by a factor $A_2^2 / M_2^2$ compared to the $Y_H$ term, implying that the Georgi-Jarlskog mechanism affects the first two generations mainly. Then at GUT scale we have $m_s \sim m_\mu$ instead of $m_s \sim 1/3 m_\mu$ in [19], while the approximate relation $\frac{m_s}{m_\mu} \sim \frac{1}{9} \frac{m_\mu}{m_s}$ can be satisfied by adjusting the $Y_{HAA}$ terms in (21).

Provided that (20) is sufficient in giving fermion masses and thus $Y$’s are determined, we can estimate the proton decay rates. In SUSY GUT models, the dominant contributions for proton decay originate from dimension-5 operators mediated by the color-triplet Higgs(-inos). These operators are then dressed by the wino-sfermion loops. The most typical mode is $p \rightarrow K^+ \nu\bar{\nu}$ with the current limit $\tau > 6.6 \times 10^{33}$ years. With most SUSY parameters $3-100$ TeV, the color-triplet Higgs are required to be $\geq 10^{18-19}$ GeV [21]. In the present case, not all of the 4 pairs of color triplets can mediate proton decay. The amplitudes are then proportional inversely to the effective triplet masses resulting from integrating out those triplets which do not couple to the fermion superfields [22]. The mass matrix for the color triplet Higgs is analogue to (13),

$$M_T = \begin{pmatrix} 0 & 0 & c/s + c^2 M_2^2 + z A_2/M_2^2 & A_2 \cr 0 & 0 & s + c^2 M_2^2 + z A_2/M_2^2 & c A_2/M_2^2 + z A_2/M_2^2 \cr 0 & s + c^2 M_2^2 + z A_2/M_2^2 & c s A_2/M_2^2 + z A_2/M_2^2 \cr A_2 & c A_2 M_2^2 + c s A_2/M_2^2 & c s A_2/M_2^2 + z A_2/M_2^2 \end{pmatrix} \quad \text{(22)}$$

with the columns are $H_T, \tilde{C}_T, \tilde{C}_T', H'_T$ and the rows are $H_T, C_T, C_T', H'_T$, respectively. An effective triplet mass $M_{i,j}^{\text{eff}}$ corresponds to the matrix element (i,j) after integrating out all the other elements in the triplet mass matrix, and is given by [22]

$$M_{i,j}^{\text{eff}} = (-1)^{i+j} \frac{\text{Det}(M)}{\text{Det}(M_{ji}^{\ast})}, \quad \text{(23)}$$

where $\text{Det}(M_{ji}^{\ast})$ is the cofactor corresponding to the determinant of the matrix $M$ with the jth row and the ith column eliminated.

For proton decay through dimension-5 operators, the coefficients are

$$Y_H Y_H \frac{1}{M_{11}^{\text{eff}}} + \frac{c s}{M_2 s} Y_H Y_C \frac{1}{M_{12}^{\text{eff}}} + \frac{c s}{M_2 s} Y_H Y_C \frac{1}{M_{21}^{\text{eff}}} + \frac{c^2 s}{M_3 s} Y_C Y_C \frac{1}{M_{22}^{\text{eff}}}.$$ \quad \text{(24)}$$

Numerically,

$$M_T \sim \begin{pmatrix} 0 & 0 & 10^{16} & 10^{16} \\
0 & 0 & 10^{15} & 10^{15} \\
0 & 10^{15} & 10^{12} & 10^{12} \\
10^{16} & 10^{16} & 10^{13} & 10^{13} \end{pmatrix} \quad \text{(25)}$$

if we take $c, z, A_2 \sim M_{\text{GUT}}$; $s \sim 0.1 M_{\text{GUT}}$,

$$M_{11}^{\text{eff}}, M_{12}^{\text{eff}} \sim 10^{19} \text{GeV}, \quad M_{21}^{\text{eff}}, M_{22}^{\text{eff}} \sim 10^{18} \text{GeV}. \quad \text{(26)}$$

Together with the suppressed Yukawa couplings in (24), the proton decay amplitudes can be suppressed sufficiently.
5 Summary

We have extended the DW mechanism in the NR models to the more general forms. A realistic model is modified into a new version. The fermion masses and the proton decay suppressions are also studied.

References

[1] T. E. Clark, T. K. Kuo and N. Nakagawa, An SO(10) supersymmetric grand unified theory, Phys. Lett. B 115 (1982) 26.

[2] C. S. Aulakh and R. N. Mohapatra, Implications of supersymmetric SO(10) grand unification, Phys. Rev. D 28 (1983) 217.

[3] J. Sato, A SUSY SO(10) GUT with an intermediate scale, Phys. Rev. D 53 (1996) 3884.

[4] C. S. Aulakh, B. Bajc, A. Melfo, G. Senjanović, and F. Vissani, The minimal supersymmetric grand unified theory, Phys. Lett. B 588 (2004) 196.

[5] T. Fukuyama, A. Ilakovac, T. Kikuchi, S. Meljanac and N. Okada, General formulation for proton decay rate in minimal supersymmetric SO(10) GUT, Eur. Phys. J. C42 (2005) 191.

[6] B. Bajc, A. Melfo, G. Senjanović, and F. Vissani, Minimal supersymmetric grand unified theory: Symmetry breaking and the particle spectrum, Phys. Rev. D 70 (2004) 035007.

[7] T. Fukuyama, A. Ilakovac, T. Kikuchi, S. Meljanac, and N. Okada, SO(10) Group theory for the unified model building, J. Math. Phys. 46 (2005) 033505.

[8] T. Fukuyama, T. Kikuchi, A. Ilakovac, S. Meljanac, and N. Okada, Detailed analysis of proton decay rate in the minimal supersymmetric SO(10) model, JHEP 0409 (2004) 052.

[9] Z.-Y. Chen, D.-X. Zhang and X.-Z. Bai, Couplings in Renormalizable Supersymmetric SO(10) Models, Int.J.Mod.Phys. A 32 (2017) 1750207.

[10] S. Dimopoulos and F. Wilczek, Incomplete Multiplets in Supersymmetric Unified Models, NSF-ITP-82-07.

[11] M. Srednicki, Supersymmetric Grand Unified Theories and the Early Universe, Nucl.Phys. B 202 (1982) 327.

[12] S. M. Barr and S. Raby, Minimal SO(10) unification, Phys. Rev. Lett. 79 (1997) 4748.

[13] K. S. Babu and S. M. Barr, Natural suppression of Higgsino mediated proton decay in supersymmetric SO(10), Phys. Rev. D 48 (1993) 5354.

[14] K. S. Babu and S. M. Barr, Natural gauge hierarchy in SO(10), Phys. Rev. D 50 (1994) 3529.
[15] Y.-K. Chen and D.-X. Zhang, *A renormalizable supersymmetric SO(10) model with natural doublet-triplet splitting*, JHEP 1501 (2015) 025.

[16] K.S. Babu, J. C. Pati and Z. Tavartkiladze, *Constraining Proton Lifetime in SO(10) with Stabilized Doublet-Triplet Splitting*, JHEP 1006 (2010) 084.

[17] R. Slansky, *Group Theory for Unified Model Building*, Phys. Rept. 79 (1997) 1.

[18] M. B. Green and J. H. Schwarz, *Anomaly cancellations in supersymmetric D = 10 gauge theory and superstring theory*, Phys. Lett. B 149 (1984) 117.

[19] Z. Berezhiani and Z. Tavartkiladze, *More missing VEV mechanism in supersymmetric SO(10) model*, Phys. Lett. B 409 (1997) 220.

[20] H. Georgi and C. Jarlskog, *A New Lepton - Quark Mass Relation in a Unified Theory*, Phys. Lett. B 86 (1979) 297.

[21] J. Ellis, J. L. Evans, N. Nagata, K. A. Olive and L. Velasco-Sevilla, *Supersymmetric proton decay revisited*, Eur.Phys.J. C 80 (2020) 4,332.

[22] X. Li and D.-X. Zhang, *Proton decay suppression in a supersymmetric SO(10) model*, JHEP 1502 (2014) 130.