THE QUANTUM CORRELATION $R_b$-$R_c$ IN THE MSSM: MORE HINTS OF SUPERSYMMETRY?

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ABSTRACT

We study the correlation of quantum effects on the ratios $R_b$ and $R_c$ within the framework of the MSSM. While in the SM the quantity $R_b$ is in discrepancy with experiment at the $2\sigma$ level from below, and $R_c$ differs from the experimental result at the $1.5\sigma$ level from above, the theoretical prediction for both observables could simultaneously improve in the MSSM, provided that $\tan \beta$ is large enough ($\tan \beta \sim m_t/m_b$) and there exists a light supersymmetric pseudoscalar Higgs, and also a light stop and a light chargino, all of them in the $50\, \text{GeV}$ ballpark. In view of the masses predicted for these SUSY particles, persistence of the “$R_b$-$R_c$ crisis” in the next run of experiments would not only suggest indirect evidence of SUSY, but should also encourage direct finding of SUSY at LEP 200. We also point out the consistency of this picture with other observables and the intriguing possibility that this $Z$-physics scenario might allow getting a hint of SUSY at Tevatron through the simple observation of $t \rightarrow H^+b$.

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The experimentally measured value of the ratio

\[ R_b = \frac{\Gamma_b}{\Gamma_h} \equiv \frac{\Gamma(Z \to b\bar{b})}{\Gamma(Z \to \text{hadrons})} \]

has been a source of conflict and of puzzle within the Standard Model (SM) in recent times; and the situation has steadily worsened ever since the first claimed CDF measurements of the top quark mass, which point towards a rather high value for this parameter: \( m_t = 174 \pm 16 \text{ GeV} \)[1]. Indeed the present experimental value of the ratio (1), which is accurate to a precision better than 1%, is \[ R_b^{\text{exp}} = 0.2202 \pm 0.0020, \]

whereas the theoretical prediction of the SM is found to be (using the CDF result on \( m_t \))[4]

\[ R_b^{\text{SM}} = 0.2160 \pm 0.0006. \]

With an accuracy better than 3 parts per mil, the SM prediction is nonetheless more than 2 standard deviations below the experimental result (2), and it decreases quadratically with the top quark mass due to a large, negative, vertex contribution to the \( b\bar{b} \) mode [5]. In contrast, \( R_b \) is particularly insensitive to the SM Higgs mass and it is also fairly independent of all sorts of oblique corrections.

In parallel with the conflicting ratio \( R_b \), we have another offending ratio:

\[ R_c = \frac{\Gamma_c}{\Gamma_h} \equiv \frac{\Gamma(Z \to c\bar{c})}{\Gamma(Z \to \text{hadrons})}. \]

Its present experimental value carries a relatively large error (~6%) [2, 3],

\[ R_c^{\text{exp}} = 0.1583 \pm 0.0098, \]

but it also defies the prediction of the SM. In fact, in this case the theoretical result [4],

\[ R_c^{\text{SM}} = 0.1713 \pm 0.0002, \]

is off by about one and a half standard deviations above the experimental value and it is extremely precise, for it is practically insensitive to the top quark mass and to the Higgs mass. Both ratios (1) and (4) are independent of \( \alpha_s \).

From the point of view of \( \Gamma_b (\Gamma_c) \), there is an excess (deficit) of \( \sim 8 \text{ MeV} (\sim 25 \text{ MeV}) \) of beauty (charm) produced in \( Z \) decays as compared to SM expectations. It is thus a challenge to any theory proposing an extension of the SM to ameliorate the prediction of these observables. In particular, Supersymmetry (SUSY) and more specifically the Minimal Supersymmetric Standard Model [6], which is supposed to be the most predictive
framework for physics beyond the SM, must be carefully contrasted with experiment \[4\] in all phenomenological fronts \[5\]. Here we shall take the point of view that there is a “\(R_b - R_c\) crisis” in the SM and shall explore its consequences in the MSSM. Failure of the MSSM to improve the theoretical prediction of the SM, or evidence that it manifestly worsens it, could be interpreted in the negative sense for SUSY. However, it should be well borne in mind that the present status of the experimental information on \(R_b\) and \(R_c\) is not robust enough to be brandished as a lethal weapon to kill the MSSM, nor to confirm it. Moreover, consistency of the MSSM with additional observables will, of course, be necessary before jumping into conclusions. As a matter of fact, the door is still open to the possibility that the SM itself will be perfectly consistent with both \(R_b^{\text{exp}}\) and \(R_c^{\text{exp}}\) without resorting at all to any form of new physics. We are referring to the experimental conundrum associated to the process of b-tagging and its anticorrelation to c-tagging whose resolution might simultaneously render the \(R_b\) and \(R_c\) crises non-existent in the SM \[3, 10\]. Be as it may, while this technical problem remains unsettled, we had better prepare the ground to confront the MSSM with the present and future experimental data on these observables.

Ever since the appearance of the first experimental measurements of \(R_b\), several analyses of supersymmetric radiative corrections to that ratio have been published in the literature \[11, 12, 13\]. More recently, \(R_b\) has been considered in constrained minimal SUSY and in specific supergravity models, and in general to test model building in the framework of supersymmetric Grand Unified Theories \[14, 15, 16\]. Running in parallel with this, a numerical analysis of complete electroweak radiative corrections to the full \(Z\)-width, \(\Gamma_Z\), in the general MSSM has been presented by the authors in Ref.\[17\]; and it was immediately particularized to the ratio \(R_b\) in Ref.\[19\] whose notation and definitions we shall adopt hereafter. The aim of the present letter is to extend the latter analysis of \(R_b\) to situations not explicitly addressed in that reference and present the simultaneous prediction of \(R_c\) within the MSSM. For the numerical evaluation we shall borrow Models I and II as defined in Refs.\[17\]-\[20\], the first model being general enough from the phenomenological point of view and the second one containing the supergravity-based canonical ingredients for gauge coupling unification. Nonetheless, in the ignorance of the ultimate unification theory (if any), and as a means to constrain the profiles of the truly fundamental physical theory, we shall not commit ourselves to any particular Yukawa coupling unification model, which, if taken seriously enough, should eventually fit in with

\[3\]Within errors the MSSM is at least as successful as the SM in the prediction of observables from global fits to all precision data \[8\], which is certainly not the case for the rival composite and technicolour approaches \[9\].

\[4\]That study is based on exact calculations of all supersymmetric, oblique and non-oblique, one-loop effects on \(\Gamma_Z\). The cumbersome analytical details are presented in Ref.\[18\].
the general conditions derived in this study.

In actual practice, we subordinate our combined analysis of \( R_b - R_c \) to the MSSM prediction of \( \Gamma_Z \) [17], which is experimentally bound to lie within the interval [3]

\[
\Gamma_Z^{\text{exp}} = 2.4974 \pm 0.0038 \, \text{GeV} .
\] (7)

The \( \Gamma_Z \)-constraint may severely restrict the freedom that we have to optimize the ratios (1) and (4). For any of these ratios we may decompose the MSSM theoretical prediction as follows:

\[
R_{b,c}^{\text{MSSM}} = R_{b,c}^{\text{RSM}} + \delta R_{b,c}^{\text{MSSM}},
\] (8)

where

\[
\delta R_{b,c}^{\text{MSSM}} = \delta R_{b,c}^{\text{SUSY}} + \delta R_{b,c}^H = R_{b,c}^{\text{RSM}} \left( \frac{\delta \Gamma_{b,c}^{\text{MSSM}}}{\Gamma_{b,c}^{\text{RSM}}} - \frac{\delta \Gamma_h^{\text{MSSM}}}{\Gamma_h^{\text{RSM}}} \right)
\] (9)

is the total MSSM departure of these ratios from the corresponding Reference Standard Model (RSM) values, \( R_{b,c}^{\text{RSM}} \), which are identified with (3) and (4), respectively [19]. Our calculation of \( \delta R_{b,c}^{\text{MSSM}} \) includes full treatment of the genuine supersymmetric part, \( \delta R_{b,c}^{\text{SUSY}} \), induced by squarks, charginos and neutralinos; and also includes full treatment of the additional Higgs part, \( \delta R_{b,c}^H \), induced by the Higgs sector (charged and neutral) of the MSSM. In contradistinction to the SM, these extra Higgs-quark interactions are potentially significant due to the presence of enhanced Yukawa couplings involving top and bottom quarks. At the level of the superpotential they have strengths

\[
h_t = \frac{g m_t}{\sqrt{2} M_W \sin \beta}, \quad h_b = \frac{g m_b}{\sqrt{2} M_W \cos \beta},
\] (10)

shared by the higgsino couplings with the corresponding quarks and squarks. In practice, the supersymmetric Yukawa couplings in the mass-eigenstate basis are interwoven with the gauge couplings in the complete bottom-stop (sbottom)-chargino (neutralino) interaction Lagrangian and therefore result in a rather complicated structure [4]. Numerically we will see that these couplings are a rather efficient source of non-oblique non-standard one-loop contributions. In particular, both the SUSY and the additional Higgs vertex corrections could be responsible for relatively important quantum effects on the \( Z \rightarrow b\bar{b} \) partial width, especially if the sparticles are not too heavy.

The signs of all the extra MSSM quantum effects are not coincident. Thus, on one hand, the SUSY vertex corrections to \( Z \rightarrow b\bar{b} \) are positive whereas those to \( Z \rightarrow c\bar{c} \) are negative. These signs correspond to the natural regions of parameter space explored in Refs.[17, 19]. For contrived values of the parameters, they could be different, but we consider it to be unlikely. On the other hand, the supersymmetric neutral Higgs

\footnote{For detailed formulae, see e.g. eqs.(18)-(19) of Ref.[21].}
corrections to $Z \rightarrow b\bar{b}$ can be either positive or negative whereas the charged Higgs effects are always negative. (Of course, all Higgs contributions to the $Z \rightarrow c\bar{c}$ partial width are negligible.) Therefore, in principle we have more freedom in the MSSM to juggle with the various contributions in such a way to compensate for the SM “deficit” on the $b\bar{b}$ mode while at the same time to cancel out the SM “surplus” on the $c\bar{c}$ mode. This extra freedom notwithstanding, the success of the MSSM should not be viewed as a trivial adjustment of the parameters; for if the signs described above would have been just the opposite, then the supersymmetric contributions would aggravate both the $R_b$ and $R_c$ crises within the MSSM. Fortunately, the dominant supersymmetric quantum effects just happen to go in the right direction.

It should be emphasized that we are including the MSSM corrections not only on the numerators $\Gamma_b$ and $\Gamma_c$ of the ratios (1) and (4) but also on all partial widths involved in the denominator $\Gamma_h$. As a consequence, the SUSY virtual effects on the $b\bar{b}$ mode (which are positive and constitute the largest among the SUSY corrections to the total $Z$-width [17]) do increase the theoretical value of both $\Gamma_b$ and $\Gamma_h$. Thus, as a side effect on $R_c$, the positive SUSY corrections to $\Gamma_b$ effectively reinforce the negative SUSY contributions to $\Gamma_c$. All in all, one hopes that the potential magnitude of the Yukawa couplings [10] allows for a noticeable shift of $R_b$ up while at the same time for a shift of $R_c$ a bit down. Thereby both $R_b^{MSSM}$ and $R_c^{MSSM}$ are expected to be in better agreement with (2) and (3), respectively, than $R_b^{SM}$ and $R_c^{SM}$.

To assess it quantitatively, we produce in Figs.1-5 the combined plots of $R_b^{MSSM}$ and of $R_c^{MSSM}$ relevant to our analysis. We have numerically surveyed the general MSSM parameter space using the 8-tuple procedure devised in Ref.[19]:

$$(\tan \beta, m_{A^0}, M, \mu, m_{\tilde{\nu}}, m_{\tilde{\nu}^c}, m_\tilde{b}, M_{LR})$$

from which the whole sparticle spectrum is determined in Model I. Throughout our numerical analysis, it is understood that the SUSY parameters in (11) will be scanned in the wide intervals given in eq.(14) of Ref.[19] under certain conditions to be specified in each case. Notice that because of the 8-dimensional nature of the parameter space, all our numerical searches and optimizations are highly CPU-time demanding. Just to get an idea, the working out of our figures took a few hundred hours of net CPU-time in an IBM (RS 6000, 390/3BT) and in an “α” (DEC 3000, 300/AXP).

Our search for admissible points is optimized by fixing the pseudoscalar mass, $m_{A^0}$, to a few light values bordering the phenomenological lower bounds [22, 23]. In a regime of high $\tan \beta$ these choices insure that the positive contribution to $R_b^{MSSM}$ from the neutral Higgs sector of the MSSM is large enough to override the negative contribution from the charged Higgses (Cf. Fig.3 of Ref.[19]). For heavy pseudoscalar masses this
situation is not possible; and, as we have shown in the latter reference, the position of $R_b$ becomes far less comfortable in the MSSM. Further optimization of $R_b^{\text{MSSM}}$ (i.e. additional positive contributions to that quantity) is achieved by searching over regions of the SUSY parameter space where at least one chargino and one stop have a mass as close as possible to their present lower phenomenological bounds \cite{22, 23, 24}. Thus, to start with, we restrict the search of points (11) within the subspace

$$45 \text{ GeV} < m_{\tilde{t}_1} < 60 \text{ GeV}, \quad 48 \text{ GeV} < M_{\tilde{\chi}_1^\pm} < 60 \text{ GeV}, \quad M_{\tilde{\chi}_1^0} > 20 \text { GeV}, \quad (12)$$

where $m_{\tilde{t}_1}, M_{\tilde{\chi}_1^\pm}, M_{\tilde{\chi}_1^0}$ are the masses of the lightest stop, chargino and neutralino, respectively. (The other stop-chargino-neutralino mass eigenvalues can, of course, be heavier.) This will insure at the same time a sizeable negative contribution to $R_c^{\text{MSSM}}$.

From Fig.1a we learn that to restore the ratio $R_b^{\text{MSSM}}$ within 1 $\sigma$ of $R_b^{\exp}$ we have to require $\tan \beta \geq 22$. Furthermore, since $R_b^{\text{MSSM}}$ is more yielding than $R_c^{\text{MSSM}}$ (Fig.1b), we have focused our work on optimizing $R_c^{\text{MSSM}}$; thus our figures actually show the simultaneous solution curves for $R_b^{\text{MSSM}}$ corresponding to the best solution curves for $R_c^{\text{MSSM}}$ obtained in the aforementioned intervals of parameter space.

Our optimum curves concentrate around the lightest values of the stop and chargino masses in the range (12). Although this could be expected, we did not try to fix the masses $m_{\tilde{t}_1}$ and $M_{\tilde{\chi}_1^\pm}$ beforehand, but rather we let our code to search for the optimum values automatically within the parameter subspace under consideration. (Decoupling of SUSY in the asymptotic regime should not preclude the possibility of interesting local behaviours.)

In contrast to $R_b^{\text{MSSM}}$, it turns out that it is not possible to drag $R_c^{\text{MSSM}}$ into the experimental range at the strict 1 $\sigma$ level. However, simple inspection of Figs.1a-1b shows that for any value of $\tan \beta$ that makes $R_b^{\text{MSSM}}$ compatible with $R_b^{\exp}$ at 1 $\sigma$, makes $R_c^{\text{MSSM}}$ compatible with $R_c^{\exp}$ at 1.25 $\sigma$. Therefore, on condition that

$$\tan \beta \gtrsim 20, \quad (13)$$

it is possible to simultaneously solve the “$R_b$ crisis” at 1 $\sigma$ and the “$R_c$ crisis” at 1.25 $\sigma$ within the MSSM. To appreciate the sensitivity of the curves in Fig.1 to the variation of the parameters, we fix e.g. $m_{A^0} = 40 \text{ GeV}$ and sample our best solution curve for $R_c^{\text{MSSM}}$ over the range (12). The result is represented in the form of a narrow band in Fig.2b, whose darkened part is compatible with $R_c^{\exp}$ to within 1.25 $\sigma$. The one-to-one map of this darkened region onto the ($R_b^{\text{MSSM}}, \tan \beta$)-plane is the other darkened band shown in Fig.2a, part of which is excluded by the upper and lower 1 $\sigma$ cuts on $R_b^{\exp}$.

\footnote{This is still well below the approximate perturbative limit $\tan \beta \sim 70$. Incidentally we note that the large $\tan \beta$ region is favoured by recent $t - b - \tau$ Yukawa coupling $SO(10)$ unification models \cite{25}.}
Let us remark that the $\Gamma_Z$-constraint mentioned above is innocuous in Figs.1-2, due to the small SUSY correction to the $Z$-width within the parameter subspace ([12]). The smallness of the correction stems from the large, negative, vacuum polarization effects on $\Gamma_Z$ from chargino-neutralinos in that region of parameter space (Cf. Fig.1 of Ref.[17]) which nevertheless cancel out to a large extent in $R_{b,c}$.

In Figs.3a-3b we display the best (candidate) solution curves for $R_{c^{MSSM}}$ in correspondence with the simultaneous solution curves for $R_{b^{MSSM}}$ when the chargino-stop spectrum is scanned in the intermediate mass region up to the LEP 200 discovery range:

$$60 GeV < m_{\tilde{t}_1}, M_{\Psi^{\pm}_1} < 90 GeV .$$

Again, our code projects the best solution curves for the lightest values of the stop and chargino masses in this range. However, since this time all the supersymmetric vacuum polarization corrections are positive, and therefore add up to the leading (positive) vertex contributions, the effect of the $\Gamma_Z$-constraint becomes patent: It cuts-off the (candidate) solution curves and prevents them from exiting the experimentally allowed region. In the same conditions as before, a simultaneous solution of the “$R_b - R_c$ crisis”, however, does exist for

$$\tan \beta \gtrsim 25 .$$

We have checked that in running through the values of the interval ([14]) from lower to higher masses $m_{\tilde{t}_1}$ and $M_{\Psi^{\pm}_1}$, the cut-off effect from the $\Gamma_Z$-constraint trims away a larger and larger portion of the optimum solution curves.

The critical cut-off situation shown in Figs.4a-4b corresponds to a numerical search in the vicinity of the LEP 200 unaccessible range

$$90 GeV < m_{\tilde{t}_1}, M_{\Psi^{\pm}_1} < 110 GeV .$$

Here both $R_{b^{MSSM}}$ and $R_{c^{MSSM}}$ find themselves in deep water. Indeed, these curves are severely cut-off; and whereas $R_{b^{MSSM}}$ scarcely penetrates into the experimental domain of $R_{b^{exp}}$ at 1 $\sigma$, $R_{c^{MSSM}}$ is unable to reach $R_{c^{exp}}$ at all, not even at 1.25 $\sigma$. Hence, in the mass interval under consideration a simultaneous solution to the “$R_b - R_c$ crisis” within the MSSM does not exist for any value of $\tan \beta$, unless the error on $R_{c^{exp}}$ is extended up to 1.5 $\sigma$, i.e. up to the compatibility range of the SM itself. Notice that even in this case the MSSM is in better shape than the SM, for the MSSM could still be marginally consistent with $R_b$ at 1 $\sigma$ (for $\tan \beta \gtrsim 30$) whereas the SM would be at variance with experiment by more than 2 $\sigma$.

The transition from the “free regime” of Figs.1-2 into the “cut-off regime” of Figs.3-5 turns on for $m_{\tilde{t}_1}, M_{\Psi^{\pm}_1} > 55 GeV$, reaching a maximum somewhere beyond the LEP
200 unaccessible range \([16]\) and then becoming again less and less severe as long as the
sparticle masses become effectively decoupled. As stated, this to-and-fro behaviour is due
to a balance between the oblique and non-oblique corrections and to the fact that the
formers (latters) are leading effects on \(\Gamma_Z (R_{b,c}^{MSSM})\) but virtually cancel in \(R_{b,c}^{MSSM}\).

Finally, we display in Figs.5a-5b the case corresponding to very heavy sparticles, where
the ratios \(R_{b,c}^{MSSM}\) recover from the critical cut-off behaviour. This set-up corresponds in
good approximation to what we have termed Model II. Here the effect of the \(\Gamma_Z\)-constraint
is in fact not too harmful, for the supersymmetric contributions (oblique and non-oblique)
are very small and care is needed only to control the Higgs effects. The situation depicted
in Fig.5 actually corresponds to the asymptotic cut-off regime where the various sparticles
are infinitely heavy \([7]\). We see that in this asymptotic regime a simultaneous MSSM
solution exists for \(R_b\) at 1 \(\sigma\) and for \(R_c\) at 1.25 \(\sigma\) provided

\[
\tan \beta \gtrsim 45,
\]

i.e. for significantly larger values of \(\tan \beta\) than in the light and intermediate SUSY cases,
eqs.(13), (15). Needless to say, for sparticle masses well beyond the LEP 200 discovery
limit and at the same time a pseudoscalar mass \(m_A > 70\, GeV\), the position of \(R_{b,c}\) in
the MSSM would be as untenable as in the SM. It is nevertheless quite remarkable that
the sole presence of a light pseudoscalar may greatly alleviate the “\(R_b - R_c\) crisis” at the
modest expense of a large value of \(\tan \beta\). This feature, which is automatic in the MSSM
Higgs sector, could also be achieved in general two-Higgs-doblet-models, but only after a
suitable tuning of the parameters.

In Ref.[19] we showed that \(R^{\exp}_b\) could tolerate (at 1 \(\sigma\)) a SUSY spectrum in the vicinity
of the LEP 200 unaccessible range \([10]\), so long as one keeps a light pseudoscalar Higgs
and a large value of \(\tan \beta\). From the present study we realize that this is no longer possible
if we want at the same time to match up \(R_c^{MSSM}\) with \(R_c^{\exp}\) to an accuracy better than
1.25 \(\sigma\). Admittedly, the MSSM achievement on this ratio may not be too spectacular;
after all the experimental error on \(R_c\) is much larger than that on \(R_b\) and therefore the
experimental situation of \(R_c\) is still loose enough to undergo potentially important changes
in the near future. All the same, the improvement of the prediction of \(R_b\) in the MSSM
is clear-cut and we can build on the fact that the corresponding effect on \(R_c\) has at least
the right trend and it can be quantitatively acceptable in situations like the ones depicted
in Figs.1-3 and 5.

The nature of the solutions in these figures is however rather different. In fact, although
the comfortable solution in Figs.1-2 is free from the \(\Gamma_Z\)-constraint and prefers the lightest
possible values for some sparticle masses, as a drawback it is confined to the narrow

\(^7\)Slight differences with respect to Fig.5 of Ref.[13] are due to updating of \(R^{\exp}_b\) in the present study.
interval \( (12) \). In contrast, the solution in Figs.3 and 5, in spite of being a cut-off solution, it is perfectly compatible with a relatively light or a very heavy (decoupled) sparticle spectrum. Thus, in the very end, what is needed from the MSSM is either a light or a heavy SUSY spectrum (not an intermediate one!), together with a light pseudoscalar and a large value of \( \tan \beta \). With all these ingredients, the MSSM is able to cook a fairly satisfactory resolution of the “\( R_b - R_c \) crisis” which encourages LEP 200 to find at least a CP-odd (and a CP-even) supersymmetric Higgs, and in favourable circumstances (Figs.1-2) even a stop and a chargino.

It is worthwhile to note, in passing, a couple of interesting consistency facts of our results with the status of other observables: i) One fact is that the above picture fits pretty well with the theoretical requisites for the branching ratio of \( b \to s \gamma \) to be compatible with experiment within the MSSM. Indeed, a light CP-odd Higgs at large \( \tan \beta \) is perfectly allowed by \( B(b \to s \gamma) \) \[28\]; besides, both a light chargino and a light stop at large \( \tan \beta \) are precisely needed to coexist peacefully with a not too heavy charged Higgs \[29\], i.e. such that to produce a net global radiative correction preserving the CLEO bounds \[30\]; ii) The other consistency fact is that this scenario could also help to cure the bold 3 \( \sigma \) discrepancy between the low energy and high energy determinations of \( \alpha_s(M_Z) \) from global fits to all indirect precision data within the SM. As remarked in Ref.\[31\], any increase of \( \Gamma_h \) coming from physics beyond the SM would be welcome in this respect, for it would diminish the high energy (lineshape) value of \( \alpha_s(M_Z) \) obtained from \( R = \Gamma_h/\Gamma_l \). Now, in our particular MSSM scenario, the balance resulting from the full set of extra electroweak quantum effects on \( \Gamma_h \) ends up with a net positive contribution to that quantity; and, what is more, if we place ourselves in the very same parameter region that we have been exploiting to ameliorate the theoretical predictions of \( R_b \) and \( R_c \), we automatically obtain the necessary 4 per mil enhancement of \( \Gamma_h \) to solve the “\( \alpha_s(M_Z) \) crisis” too\[8\]. Cards are, therefore, laid down on the table just awaiting for the next round of experiments.

A final comment is in order concerning the demand of the present analysis, and of previous analyses \[11\]-\[19\], for light SUSY particles, and more specifically for a light stop, a light chargino–and a fortiori a light neutralino. It obviously suggests that we should see some supersymmetric top quark decay at the next Tevatron run. Thus, even barring the (still open) possibility of light gluinos, we should expect electroweak decays like \( t \to \tilde{t}_1 + \Psi_1^0 \) and \( t \to \tilde{b}_1 + \Psi_1^+ \) if there is a not too heavy sbottom, \( \tilde{b}_1 \). Interesting as they can be, however, all these modes involve genuine SUSY particles and so their actual detection may require some additional effort to tag unconventional final states. Alternatively– and as a distinctive feature of the present analysis–it is amusing to notice that the conditions to solve the “\( R_b - R_c \) crisis” within the MSSM suggest that another,

\[8\] For a detailed study of this issue, see Ref.\[32\]
less exotic, decay mode of the top quark should be $t \rightarrow H^+b$. In fact, we have seen that the correlation $R_b - R_c$ demands not only a light chargino and a light stop but also a light value for the pseudoscalar Higgs mass around 50 GeV. Thus, from the well-known MSSM Higgs mass relations [33], it follows that $m_{H^+} \sim 100$ GeV and therefore $t \rightarrow H^+b$ is expected to be not too much suppressed by phase space with respect to the standard decay mode $t \rightarrow W^+b$. Furthermore, the marked preference that $R_b - R_c$ has for the regime of large values of $\tan \beta$ suggests that the two decay widths [34]

\[
\Gamma(t \rightarrow W^+b) = \frac{G_F m_t^3}{8\pi\sqrt{2}} \left(1 - \frac{M_W^2}{m_t^2}\right)^2 \left(1 + 2\frac{M_W^2}{m_t^2}\right) \tag{18}
\]

and

\[
\Gamma(t \rightarrow H^+b) = \frac{G_F m_t^3}{8\pi\sqrt{2}} \left(1 - \frac{m_{H^+}^2}{m_t^2}\right)^2 \left[\frac{m_b^2}{m_t^2} \tan^2 \beta + \cot^2 \beta\right] \tag{19}
\]

can be comparable. Last but not least, from the fact that $\tan \beta$ could be so large, we expect an additional bonus: namely, that a supersymmetric charged Higgs should most likely decay into $\tau$-lepton and neutrino, rather than into charm-strange quark jets. Indeed this immediately follows from

\[
\Gamma(H^+ \rightarrow \tau^+ \nu_\tau) = \frac{G_F m_{H^+} m_{H^+}}{4\pi\sqrt{2}} \tan^2 \beta \tag{20}
\]

and

\[
\Gamma(H^+ \rightarrow c\bar{s}) = \frac{3G_F m_c^2 m_{H^+}^2}{4\pi\sqrt{2}} \left[\frac{m_c^2}{m_{H^+}^2} \tan^2 \beta + \cot^2 \beta\right] \tag{21}
\]

The fact that the few Tevatron events collected on the top quark do not make such a distinction, probably means that they correspond to the standard decay mode. Nevertheless, a better statistics and an appropriate experimental search in the future might come across some $\tau$-lepton final states whose parent particle is not just a 80 GeV good-natured $W$-boson but a $\mathcal{O}(100)$ GeV fully-fledged charged Higgs. In principle there is no a priori reason for $\Gamma(t \rightarrow H^+b)$ to be competitive with $\Gamma(t \rightarrow W^+b)$, nor for a non-supersymmetric charged Higgs to decay most likely into the $\tau$-lepton mode rather than into the hadronic mode. Nonetheless for a supersymmetric charged Higgs we do have, in the light of the MSSM quantum effects on $R_b - R_c$, a reasonable motivation to believe in such a scenario. So, at the end of the day, the message for the experimentalists could be something like: find (or manage to find!) $H^+ \rightarrow \tau^+ \nu_\tau$ at Tevatron or at LHC (however difficult as $\tau$-tagging can be in this context) and you might be discovering SUSY!.

9 Potentially important electroweak and strong supersymmetric virtual effects on this decay have recently been recognized in the literature [21, 26, 27].

10 Studies from the LHC collaborations [35] show that for $m_{H^\pm} < 130$ GeV and large $\tan \beta$ it is possible to identify the $H^+ \rightarrow \tau^+ \nu_\tau$ decay on account of the observed excess of events with one isolated $\tau$ as compared to events with an additional lepton [36].
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**Figure Captions**

- **Fig.1** (a) $R_b^{MSSM}$ as a function of $\tan \beta$ for three light pseudoscalar masses $m_{A^0} = 40, 45, 50 GeV$ (curves from top to bottom), in correspondence with (b) the best solution curves for $R_c^{MSSM}$ (from bottom to top). The SUSY spectrum, eq.(11), was scanned in the light chargino-stop mass range (12). The shaded area in (a) corresponds to $R_b^{exp}$ within 1$\sigma$ whereas that in (b) corresponds to the upper part of $R_c^{exp}$ within 1.25$\sigma$. We have taken $m_b = 5 GeV$ and $m_t = 174 GeV$.

- **Fig.2** Sensitivity of the solution curve $m_{A^0} = 40 GeV$ of Fig.1 to a sweep of the parameters across the intervals (12). The narrow darkened bands in (a) and in (b) correspond to $R_b^{MSSM}$ and $R_c^{MSSM}$, respectively, and are in one-to-one correspondence as explained in the text.

- **Fig.3** As in Fig.1, but for a parameter survey in the intermediate chargino-stop region (14), which reaches up to the LEP 200 discovery range.

- **Fig.4** As in Fig.1, but for a parameter survey in the vicinity of the LEP 200 unaccessible region (16).

- **Fig.5** As in Fig.1, but for a sparticle spectrum fully decoupled.
Fig. 2

(a) Graph of $R_b \times 10^{-4}$ vs. $\tan \beta$.

(b) Graph of $R_c \times 10^{-4}$ vs. $\tan \beta$. 

Symbols: $R_b$, $R_c$, $\tan \beta$. 

Note: The graphs illustrate the relationship between $R_b$ and $R_c$ with respect to $\tan \beta$. The shaded areas represent the range of values for each parameter.
Fig. 5

(a) \( R_b \times 10^{-4} \) vs. \( \tan \beta \)

(b) \( R_c \times 10^{-4} \) vs. \( \tan \beta \)