Primordial black holes in interferometry

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Abstract

If there is a population of black holes distributed randomly in space, light rays passing in their vicinity will acquire random phases. In the “two-slit” model of an interferometer this can, for a high density of black holes, lead to a diffusion in the phase difference between the two arms of the interferometer and thus to a loss of coherence or “visibility” in interferometric observations. Hence the existence of “fringe contrast” or “visibility” in interferometric observations can be used to put a limit on the possible presence of black holes along the flight path.

We give a formula for this effect and consider its application, particularly for observations in cosmology. Under the assumption that the dark matter consists of primordial black holes, we consider sources at high $z$, up to the CMB. While the strongest results are for the CMB as the most remote source, more nearby sources at high $z$ lead to similar effects.

The effect increases with the baseline, and in the limiting case of the CMB we find that with earth-size baselines a non-zero “visibility” would limit the mass of possible primordial black holes, to approximately $M/M_\odot \leq 10^{-1}$. Although such limits would not appear to be as strong as those obtained, say from microlensing, they involve a much different methodology and are dominated by very early times (see Table 1). Longer baselines lead to more stringent limits and in principle with extreme lengths, the method could possibly find positive evidence for primordial black holes. In this case, however, all other kinds of phase averaging would have to be constrained or eliminated.

1 Introduction

When there are scattering centers between a source of radiation and an observer, the associated deflections lead to a spread of the incoming directions. The usual concern with this effect is the intrinsic limitation on the angular resolution it presents for the observer. But if the nature of the intervening medium or scattering centers is unknown, it can also be used to obtain information on these.
In a recent paper [1] we discussed how this could be used to limit, or perhaps to discover primordial black hole dark matter. With the CMB as the source [2] and assuming the dark matter composed of black holes, we found that there is a limit on the angular resolution \( \delta \alpha_{\text{lim}} \approx 3 \times 10^{-7}\sqrt{M/M_\odot} \text{ rad} \). Combining this with the milliradian resolution demonstrated [3] in the observations of “acoustic peaks” one concludes that the (average) mass \( M \) of possible black holes is constrained to about \( M/M_\odot \leq 10^7 \).

The scattering process in question is the gravitational deflection of photons, as in the bending of light by the sun,

\[
\delta \alpha = \frac{4GM}{b} = \frac{2r_s}{b} = 6.3 \times 10^{-13} (ly/b) (M/M_\odot),
\]

with \( \delta \alpha \) the deflection angle in radians, \( b \) the impact parameter of the ray or photon, and \( r_s = 2GM \) the schwarzschild radius associated with \( M \), the mass of the black hole or gravitating object (\( c=1 \) units, \( ly=\text{light year}, M_\odot=\text{solar mass} \)).

At present the most likely explanation for the dark matter seems to be a new kind of elementary particle, which might be detected in the laboratory, especially by the cryogenic technique [4]. But black holes are an interesting alternative [5]. It would evidently be interesting to be able to extend our argument to smaller angles, and so to eliminate, or conceivably to discover, smaller mass primordial black holes.

However, there appears to be a difficulty. There are very likely no very fine, prominent features on the CMB, analogous to the “acoustic peaks”, but subtending very small angles. How then, could one exhibit a high resolution, or the lack of it, when there are no fine features to observe?

2 Principle

Here we would like to propose an solution to this question, and to apply it to the possible primordial black holes at the CMB. As briefly alluded to in [1], the answer lies in examining the coherence of the wavefront at large separations. If instead of a perfect plane wave from some source, one has an incoherent sum of slightly differing incoming directions, the phases at widely separated points will become uncorrelated or incoherent. That is, the coherence over the wavefront at large separations is closely connected to the angular spread, the limiting resolution, of the incoming waves.

In long baseline interferometry, which is used in astronomy to achieve high angular resolution, one has a form of the two-slit interference arrangement [6]. The high resolution arises from a long baseline \( B \) between two “slits” or receivers so that \( \delta \alpha_{\text{min}} \approx \frac{1}{2\pi B} \), with \( \lambda \) the wavelength of the radiation. For long baseline interferometry to function it is evidently necessary that the two paths from the source to the “slits” be coherent. If the two rays which should ultimately interfere encounter different perturbations or scatterings along the way to the receivers, there will be a loss of interference and a breakdown of the method, giving an intrinsic limiting resolution.
When $B$ is small and the two paths traverse essentially the same environments they will be coherent. As $B$ increases, at some point this will no longer be true and interference breaks down. In the following we examine how this would happen due to the presence of black holes along the flight path from the CMB, or possible more nearby sources. The resulting loss of “visibility” or “fringes”, or the absence of such loss, can then be used as a limit, or an indication for the black holes.

### 3 Calculation

The basis for the calculation will be the formula for the phase acquired by a photon or light ray traversing a weak gravitational field. In addition to the plane wave term in the phase, there is an extra gravitational phase $\phi_G$ which according to general arguments [7] is

$$\phi_G = \frac{1}{2} \int h_{\mu\nu} k^\mu dx^\nu$$  \hspace{1cm} (2)

where $h_{\mu\nu}$ is a small deviation from an overall metric tensor, $k$ the four-momentum or wave vector of the photon, and the integration is along the path of the ray.

We need the difference in phases $\delta \phi_G$ for two parallel rays 1,2 in the vicinity of a black hole, with impact parameters $b_1, b_2$. Integrating the difference according to Eq 2 with the schwarzschild metric and taking the difference $\delta b = b_1 - b_2$ to be small compared to that for the pair as a whole, $b$, one finds,

$$\delta \phi_G = \omega r_s \delta b \int_{-\infty}^{\infty} dz \frac{d}{db} \frac{1}{\sqrt{z^2 + b^2}} = 2 \omega r_s \frac{\delta b}{b},$$  \hspace{1cm} (3)

where $r_s$ is the schwarzschild radius of the body, and $\omega = k$ the frequency of the radiation. The phase difference increases linearly with the ray separation $\delta b$. The assumption $\delta b/b << 1$ seems justified since while $\delta b$ will be given by the baseline, and so be on the of order of earth or perhaps solar system size, we anticipate $b$ on the order of light years or more.

There is a close relation between the classic bending formula Eq 1 and Eq 3. When $b_1$ and $b_2$ lie along a radius vector, this calculation for $\delta \phi_G$ is the same as needed for the bending of a wavefront, where the scattering angle is given by the transverse derivative of the phase. That is, for a scattering process with long range forces and smoothly varying phase $\phi(b)$, one has for the bending of a plane wave $\delta \alpha = k_T/k = \frac{1}{\omega} \frac{d\phi(b)}{db}$, where $k_T$ is the transverse momentum to the photon. In Eq 3 this amounts to dividing by $\omega \delta b$ which yields, in fact, Eq 1.

### 4 Random $\delta \phi_G$

A pair of rays starting at a point on the source and arriving at the two ‘slits’ or receivers will experience different fields and so acquire different phases along their paths. If the dark matter is made of black holes, presumed random in
their locations, this will lead to a random $\delta \phi_G$, which, if it is large enough, will lead to loss of coherence or “visibility” in the interference pattern.

In view of the small values of the factors in Eq 3 $\delta \phi_G$ from a single passage near a black hole will be small. However with many repeated passages, which would occur if the dark matter is made of black holes, it is possible that a substantial $\delta \phi_G$ will accumulate.

The sign of $\delta \phi_G$ in a single passage depends on the orientation of the ray pair with respect to the radius vector from the black hole, sometimes ray 1 will be closer to the black hole and sometimes ray 2, so that the average value of $\delta \phi_G$ will be zero. However the accumulated $\delta \phi_G$ will fluctuate, with the character of a random walk. In such problems the expected magnitude of the quantity, is characterized by the variance, here $(\delta \phi_G)^2$ and it is this quantity that we wish to study.

When the two rays are combined in the detector, the $\delta \phi_G$ will lead to fluctuating factors such as $\cos(\delta \phi_G)$ multiplying the interference term. The significance of $(\delta \phi_G)^2$ may be seen with a gaussian probability distribution $P_{\text{gauss}}$ for $\delta \phi_G$:

$$\cos(\delta \phi_G) = \int d(\delta \phi_G) \cos(\delta \phi_G) P_{\text{gauss}} = e^{-\frac{1}{2}(\delta \phi_G)^2}$$

(4)

When $(\delta \phi_G)^2$ becomes larger than one, interferences are “washed out”. We shall use this condition,

$$(\delta \phi_G)^2 > 1$$

(5)

to estimate when coherence is lost and the effects of the possible black holes would become significant.

5 Calculation of $(\delta \phi_G)^2$

To calculate the total variance we take the individual black holes to be uncorrelated and use the principle of the addition of variances. We do this by calculating the variance from one encounter as given by Eq 3 and then multiply by the number of encounters along the flight path.

These encounters will take place in the general background spacetime, which we take to be that of the FRW metric given by $ds^2 = dt^2 - a^2 dv^2$ with expansion factor $a = (t/t_{\text{now}})^2/3$, $t_{\text{now}} = 2.9 \times 10^{17}s \ [8]$. In this connection one notes that Eq 2 is a scalar in general relativity, being the contraction of two tensors. Thus we may use the evaluation Eq 3 carried out for an ambient flat space, in the local frames of black holes along the flight path and simply add the various $(\delta \phi_G)^2$.

Of course the parameters needed, such as the frequency, and the separation of the two rays, will depend on the location in the general background spacetime.

There is a difficulty, however, associated with the long ranged nature of the gravitational field. In order to define the probability of an “encounter” one must assign some range or cross section to the field around each constituent, here the black holes. But for the gravitational field there is no such parameter, the force has an infinite range. We shall handle this problem by assuming
that the picture of a single, independent constituents holds up to some distance \( b_{\text{max}} \), with a corresponding cross section \( \sigma = \pi b_{\text{max}}^2 \). This seems a reasonable procedure since the contributions to \((\delta \phi_G)^2\) are largely at small \( b \). The \( b_{\text{max}} \) parameter can be plausibly be taken to be the typical distance between black holes. We shall see below that the choice of \( b_{\text{max}} \) enters into the final result only logarithmically.

## 5.1 Number of Encounters

With this understanding, we first consider the number of black holes encountered in an interval of cosmic time \( dt \). Along the flight path, the number of black holes per unit area in a time interval \( dt \) is \( d\rho = \rho(t) dt \), where \( \rho(t) \) is the number density. Associating with each black hole a “cross section” \( \sigma \), one has for the number of encounters \( dN \) in a time \( dt \)

\[
dN = \sigma d\rho = \rho(t) \sigma dt
\]

## 5.2 Single Passage

For the \((\delta \phi_G^2)_{\text{single}}\) associated with a single encounter, we assume the \( b \) for the ray pair to be uniformly distributed across the area of \( \sigma \), integrate the square of Eq 3 over this area and divide by the area.

**Angular Factor:** First we consider the angular factor involved in \( \delta b \), which is the origin of the fluctuating sign. Let \( b_1 \) and \( b_2 \) be the respective vectors from the black hole to the points of closest approach, and \( d = (b_1 - b_2) \) the vector connecting them. Then for the \( \delta b \) needed in Eq 3 one finds

\[
\delta b = b_1 - b_2 = 2d \cdot \frac{b}{2b} = d \cos \theta,
\]

with \( \theta \) the angle between \( d \) and the radius vector \( b = \frac{1}{2}(b_1 + b_2) \). This relation follows from writing \( b_1, b_2 = b \pm \frac{1}{2}d \), squaring, taking the difference, and using \( b_1 + b_2 \approx 2b \). For the nearby universe and the line of sight perpendicular to the baseline, \( d \) is simply the baseline \( B \). (In the following we take the direction of observation to be a right angles to the baseline, otherwise a trigonometric factor can be included.) For the angular integration one thus obtains

\[
\int_0^{2\pi} d\theta (\delta b)^2 = \pi d^2
\]

**Integration Over \( b \):** For the remaining \( b \) integration one now has

\[
(\delta \phi_G^2)_{\text{single}} = \frac{1}{\sigma} 4\pi(\omega r_s d)^2 \int_{b_{\text{min}}}^{b_{\text{max}}} \frac{db}{b} = \frac{1}{\sigma} 4\pi(\omega r_s d)^2 \ln\left(\frac{b_{\text{max}}}{b_{\text{min}}}\right),
\]

with \( b_{\text{min}}, b_{\text{max}} \) some smallest, largest, impact parameters.
5.3 General Formula

Eq. 9 must now be multiplied by the number of encounters in an interval of cosmic time.

\[
\frac{d(\delta \phi^2)}{G} = dN \times \left(\frac{\delta \phi^2}{G}\right)_{\text{single}}
\]

\[
= dt \rho \sigma \frac{1}{\sigma} 4\pi (\omega r_s d)^2 \ln \left(\frac{b_{\text{max}}}{b_{\text{min}}}\right)
\]

\[
= dt 4\pi \rho (\omega r_s d)^2 \ln \left(\frac{b_{\text{max}}}{b_{\text{min}}}\right).
\]

(11)

As anticipated, \(\sigma\) has cancelled and the \(b\) limits are only in the logarithm. The appearance of the logarithm is an interesting feature of the present problem, in contrast to the more familiar situation of particles or photons passing through ordinary neutral matter. There one deals with the shielded coulomb field or the short ranged nuclear force, while here one has the unshielded, long range, gravitational force.

It remains to insert values for \(b_{\text{max}}, b_{\text{min}}\). For \(b_{\text{max}}\) we take the distance between the black holes, namely \(\rho - 1/3\). Beyond this distance one must consider the summed fields from many objects, leading to smooth and relatively weak fields. For the lower limit we take \(b_{\text{min}} = r_s\).

The final result is thus

\[
\left(\frac{\delta \phi^2}{G}\right) = 4\pi r_s^2 \int dt \rho(t) \left(\omega(t) d(t)\right)^2 \ln \left(\frac{\rho^{-1/3}(t)}{r_s}\right)
\]

(12)

writing \(\omega d = 2\pi d/\lambda\), and where we have explicitly indicated those quantities which might depend on \(t\).

**Parameters:** For quantitative estimates, we shall use light years (ly) for lengths and solar masses (\(M_\odot\)) for mass. Thus the number density is \(\rho(t) = \rho_0(t)(M_\odot/M)/\text{ly}^3\) for a dark matter mass density \(\rho_0(t)M_\odot/\text{ly}^3\), where \(\rho_0(t)\) is a dimensionless density parameter. We will be assuming that a well defined mass or average mass can be assigned to the black holes, if this is not true the arguments may have to be reconsidered. Furthermore, anticipating the use of earth-size baselines and microwave wavelengths, as in ref.[9], we introduce a length parameter \(d_0 = 1.0 \times 10^4\ km\) and a wavelength parameter \(\lambda_0 = 1.0 \times 10^{-1}\ cm\). Collecting all factors and inserting \(r_s = 3.0\ km(M/M_\odot) = 3.2 \times 10^{-13}(M/M_\odot)\text{ly}\) as in Eq.1 Eq.12 becomes

\[
\left(\frac{\delta \phi^2}{G}\right) = (5.1 \times 10^{-3})(M/M_\odot) \int dt/y \rho_0(t) \left(\frac{d/d_0}{\lambda/\lambda_0}\right)^2 \ln \left(\frac{\rho^{-1/3}(t)}{r_s}\right)
\]

(13)

\[
= (5.1 \times 10^{-3})(M/M_\odot) \int dt/y \rho_0(t) \left(\frac{d/d_0}{\lambda/\lambda_0}\right)^2 \left((1/3) \ln \rho_0(t) - (2/3) \ln (M/M_\odot) + 29\right)
\]

\[
= (1.5 \times 10^{-1})(M/M_\odot) \int dt/y \rho_0(t) \left(\frac{d/d_0}{\lambda/\lambda_0}\right)^2 \left(1 + (0.011) \ln \rho_0(t) - (0.023) \ln (M/M_\odot)\right)
\]
(Since we have $c = 1$ units and are dealing with light rays, one may use a distance label $r$ instead of $t$. In that case one writes $\int dt/y \rightarrow \int dr/ly$). We also note that with this notation the dimensionless quantity $\int dt/y \rho_o$ is simply the coefficient in the mass column density; that is, the mass column density is $\int dt/y \times \rho_o \times M_\odot/ly^2$.

Because of the small coefficient of the $ln \rho_o(t)$ term, it may be possible to replace it by some typical value and move it outside the integral. In that case Eq (13) becomes

$$\langle \delta \phi_G^2 \rangle \approx C \int dt/y \rho_o(t) \left( \frac{d(t)/d_o}{\lambda(t)/\lambda_o} \right)^2$$

$$C = (1.5 \times 10^{-1})(M/M_\odot) \left( 1 + (0.011)ln \rho_o^{typ} - (0.023)ln(M/M_\odot) \right)$$

As should be expected, the effect vanishes with $M \rightarrow 0$, as the medium becomes smooth and less “lumpy”.

In these estimates we have assumed a uniform and constant relation between the black holes and dark matter. Variations on this assumption can be taken into account by changes in the density factor $\rho$ in the above. If for example, one would like to assume that only a certain fraction of the dark matter is black holes, the fact that that $\langle \delta \phi_G^2 \rangle$ is linear in the density implies that the results need only to be reduced by this fraction. In addition, our assumption that the black holes are distributed randomly could be violated if there is a tendency for them to “clump”. Such concentrations, however, would amount to a small increase in the effective mass $M$ in our formulas, for the given amount of dark matter, and so would only increase the effect. This originates from the factor $r_s^2 \sim M^2$ in the formulas.

6 CMB

We now consider the application of the formula to rays traveling from the CMB to the Milky Way. For the density we take

$$\rho_0 \approx \frac{1}{a^3(t)}(1.0 \times 10^{-9}),$$

(15)

as follows from taking the present cosmological dark matter energy density at $1/4$ the critical density [8], and applying the cosmological expansion factor $a(t) = (t/t_{now})^{2/3}$.

To simplify the integral we replace the time dependent $ln a(t)$ arising in $ln \rho_o(t)$ by its value at $t_{cmb}$, namely $-7.0$. Since $ln a(t)$ varies from zero to this value in the time integration, this simplification gives a small underestimate of the total integral. This gives $ln \rho^{typ} = 0.3$ and one then has for $C$ in Eq (14)
\[ C = (1.5 \times 10^{-1})(M/M_\odot)(1 - (0.023)ln(M/M_\odot)) \] and so

\[ \overline{\langle \delta \phi_G^2 \rangle} \approx 1.5 \times 10^{-10}(M/M_\odot)(1 - (0.023)ln(M/M_\odot)) \int dt/y 1 \frac{a^3(t)}{\lambda(t)/\lambda_0} \left( \frac{d(t)/d_o}{\lambda(t)/\lambda_o} \right)^2 \] (16)

Since \( \int dt \) will be at least \( 10^{10} \) years, \( \overline{\langle \delta \phi_G^2 \rangle} \) can easily be greater than one, and satisfy the condition Eq 5.

### 6.1 Time Integration

We are thus left with the integral

\[ I = \int_{t_{cmb}}^{t_{now}} \frac{dt}{y} \left( \frac{d(t)/d_o}{\lambda(t)/\lambda_0} \right)^2 \] (17)

to evaluate.

**Frequency Shift:** In the rest frame of a black hole at time \( t \), one has the frequency or wavelength red shifted, so that if \( \omega \) is the frequency of observation, one has \( \omega \to \omega/a(t) \) or \( \lambda \to a(t)\lambda \).

**Ray Separation:** The remaining question is the ray separation \( d(t) \). The rays must originate from a common point to interfere. In euclidean space, without general relativistic considerations, one would have \( d(t) = B t_{now} - t_{cmb} \), where we chose the constants to give \( d = 0 \) at emission and \( B \) at the baseline of the detector.

To transfer this reasoning to the general relativistic situation, we use the “conformal time” \( \eta(t) = 3 t_{now} x^{1/3} \), and introduce the dimensionless time variable \( x \) so that

\[ x = t/t_{now} \quad a = x^{2/3} \quad \eta = 3 t_{now} x^{1/3} \quad x_{cmb} = 2.7 \times 10^{-5} \] (18)

In terms of \( \eta \) one has \( ds^2 = a^2(d\eta^2 - dr^2) \). Light rays travel in “straight lines” in these coordinates, so for the coordinate separation, one has the euclidean result as before, 

\[ d_{coord}(\eta) = \frac{B}{\eta_{now} - \eta_{cmb}} (\eta - \eta_{cmb}) \].

However, we require the physical separation so \( a(t) d_{coord}(t) \) should be used for \( d \). The \( a \) factors cancel then in the ratio \( d/\lambda \) and the integral becomes

\[ I = (t_{now}/y) \left( \frac{B/d_o}{\lambda/\lambda_o} \right)^2 \int_{x_{cmb}}^{1} dx \frac{1}{x^2} \left( \frac{\eta - \eta_{cmb}}{\eta_{now} - \eta_{cmb}} \right)^2 \] (19)

\[ = (t_{now}/y) \left( \frac{B/d_o}{\lambda/\lambda_o} \right)^2 \int_{x_{cmb}}^{1} dx \frac{1}{x^2} \left( \frac{x^{1/3} - x_{cmb}^{1/3}}{1 - x_{cmb}^{1/3}} \right)^2 \]

In Eq 19 we have a convenient separation into factors depending on the observational arrangement \( (B, \lambda) \) and a “cosmological integral” \( \int_{x_{cmb}}^{1} ... \) independent of the setup.
6.2 CMB Result

Examination of the integrand shows that with \(x_{\text{cmb}}^{1/3} = 3.0 \times 10^{-2}\) from Eq[15], it is strongly peaked around \(x \sim 10^{-4}\). Since \(x_{\text{cmb}} = 2.7 \times 10^{-5}\), this shows that the effect would come predominantly close to the CMB and that a positive observation would support the idea that the black holes are primordial. For the integration one finds the integrated value \(\int_{x_{\text{cmb}}}^{1} \ldots \approx 32\). As in [1], there is an enhancement at early times, but due to the requirement of the convergence of the rays at their origin, to a much lower power. Combining with Eq[16] one has finally,

\[
\overline{\delta \phi_{C}^{2}} = (4.5 \times 10^{-9})(M/M_{\odot})(t_{\text{now}}/y)\left(\frac{B/d_{\odot}}{\lambda/\lambda_{0}}\right)^{2} (1 - (2.3 \times 10^{-2}) \ln(M/M_{\odot}) ) \tag{20}
\]

after inserting \(t_{\text{now}} = 2.9 \times 10^{17}\) sec = 0.92 \times 10^{10}y.

6.3 Sources More Nearby

We have chosen to evaluate the effect with the CMB as source since this is at the greatest distance available. However interferometry on the CMB [10] has, until now, not been with the earth-sized baselines as used by the EHT project [9].

It may thus be of interest to inquire as to the size of the effect for more nearby sources, say for \(z \leq 10\), where there can be localized sources. To do this, we must evaluate the integral \(\int_{x_{\text{cmb}}}^{1} \ldots \) in Eq[19] with \(x_{\text{cmb}}\) replaced by the \(x\) of the source, which we shall call \(x_{o}\).

In the Table we show the results for a few values of \(x_{o}\). The first two columns gives the ‘distance’ of the source expressed as \(x_{o}\) or equivalently the redshift factor \(z_{o}\). The second column gives the location of the peak of the integrand, in terms of its \(x\) or \(z\). Finally, the last column shows the value of the integral \(\int_{x_{o}}^{1} \ldots\). For comparison the last row shows the CMB result from the subsetion above. The integral is given by the expression \(\int_{x_{o}}^{1} \ldots = (1/1 - x_{o}^{1/3})^{2}(x_{0}^{1/3} - 3 + 3x_{0}^{1/3} - x_{o}^{2/3})\).

One sees that the effect for such nearer sources would be reduced by a factor of only about twenty or thirty compared to the CMB, despite the very great differences in “distance”.

7 Galactic Contributions

On their way to the earth, rays from the CMB or other sources will also pass through the galaxy, first presumably through a large, extended, but tenuous
Table 1: Values of the “cosmological integral” \( \int_{x_o}^{x} \) ... in Eq 19 for some sources in the range 1 < \( z < 10 \), and finally in the last row, for the CMB. The first column gives the scaled time variable \( x_o \) for the source, the second column its redshift factor \( z_o \) and the third column where the integral has its peak in terms of these quantities. One notes that while the effect is largest for the CMB, by about a factor 20, the other estimates are not enormously smaller.

| “Distance” \( x_o \) | “Distance” \( z_o \) | Peak Location \( x, z \) | \( \int_{x_o}^{x} \) ...
|----------------------|----------------------|----------------------|----------------------
| 0.032                | 8.9                  | 0.1, 3.6             | 1.4                  |
| 0.08                 | 4.4                  | 0.25, 1.5            | 1.3                  |
| 0.10                 | 3.6                  | 0.35, 1.0            | 1.2                  |
| 2.7 \times 10^{-6}   | 1.1 \times 10^4      | 10^{-4}, 460         | 32                   |

8 Conclusions

The examination of interferometric “visibility” at long baselines provides a very high sensitivity method for studying an intervening medium. We have applied the method to the question of primordial black holes in cosmology.
Interferometric observations on the CMB have been considered and carried out over the last decades [10]. If observations with earth-size baselines, as in [9], reveal coherence, a nonzero “visibility”, then according to Eq 20 and Eq 5, primordial black holes down to the \((M/M_\odot) \sim 10^{-1}\) range are ruled out. Longer baselines would of course access smaller \(M\). An earth-moon baseline, \(\sim 4 \times 10^5\) km, for example, leads to \((M/M_\odot)\) in the \(\sim 10^{-4}\) range.

These values are similar but generally weaker than limits found in work of the “macho search” or “microlensing” type [11]. An overview of the limits from different methodologies is provided by a plot in reference [2]. But the interferometric method would provide a different approach to the question, and also refer to quite different epochs in cosmology. The microlensing work uses background stars in the nearby universe, Andromeda or the Milky Way bulge, while the main contribution to Eq 20, as said and as is shown in the Table, is at very high redshift, close to the CMB.

In subsection 6.3 we have discussed applying the method to sources in the range \(1 < z < 10\), leading to somewhat smaller effects. This region could involve different technologies and would cover a different region in cosmology.

9 Remarks

Conceivably, the method could also be used to find positive evidence for primordial black holes. This would be evidently a more difficult matter than the simple setting of limits, since all kinds of other phase averaging, instrumental as well as natural, would have to be eliminated or controlled. In this respect, the simple frequency dependence of Eq 20 which originates in the achromatic behavior of photons in the gravitational field, might be helpful.

Our discussion has been for the ‘two slit’ configuration, which is the paradigm of astronomical interferometry [6], while contemporary long baseline interferometry often uses many baselines together. But our essential effect, due to interfering rays traversing different fields, should be dominated by the largest baseline. For the more complex, multi-baseline arrangements it would be interesting to examine possible improvements in sensitivity and statistics, but the “two slit” estimate for the sensitivity to potential black holes should remain roughly correct.

We stress that our considerations, or equivalently those concerning the resolution, should not be confused with the question of the power spectrum of the CMB. For the present purposes the role of the CMB is simply to provide a remote “source” and we are not concerned with the nature of the CMB itself. This point is discussed in section 5.5 of [1].

References

[1] L. Stodolsky Mod.Phys.Lett.36 A, no.11 (2021), arXiv:1912.01325
Although in ref [1] the origin of the rays is taken at the CMB, one may also consider sources in the nearby universe where a high density of dark matter is expected. In “Limits on primordial black holes from M87”, J. Silk and L. Stodolsky, Phys. Rev. D 105 063506,(2022), arXiv 2201.03591, we apply the argument to the dark matter “spike” expected to surround a Super-Massive-Black Hole, exploiting the high angular resolution of the EHT observations of M87.

Multipoles up to \( l \sim 10^3 \) are discussed in N. Aghanim et al. [Planck Collaboration], arXiv:1807.06207 [astro-ph.CO].

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L. Stodolsky, “Matter and Light Wave Interferometry in Gravitational Fields,” Gen. Rel. Grav. 11, 391 (1979). doi:10.1007/BF00759302. An explanation of effects due to a possible spin of the black holes is found in section 7 of this reference. In place of the \( h_{\mu\nu} \) in the leading effect, Eq 2 one will have a \( g \sim GL/R^2 \) from the angular momentum \( L \) of the black hole. Since \( h_{\mu\nu} \) resembles the newtonian potential \( \sim GM/R \), we see there is an extra power of distance \( R^{-1} \) for the spin effect. With the rays generally passing at large distances compared to the schwarzschild radius, we expect such effects to be subdominant. If the observation of such secondary effects ever becomes feasible, it would also be interesting to consider the consequences for the polarization of the photons.

We take statistical formulas and parameters from the Particle Data Group booklet 2016, extracted from C. Patrignani et al. [Particle Data Group], “Review of Particle Physics,” Chin. Phys. C 40, no. 10, 100001 (2016). doi:10.1088/1674-1137/40/10/100001. Note that our definition of \( a(t) \) introduces a factor 3/2 between \( t_{\text{now}} \) and the hubble constant.
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