The generalized second law for the interacting generalized Chaplygin gas model

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Abstract

We investigate the validity of the generalized second law (GSL) of gravitational thermodynamics in a non-flat FRW universe containing the interacting generalized Chaplygin gas with the baryonic matter. The dynamical apparent horizon is assumed to be the boundary of the universe. We show that for the interacting generalized Chaplygin gas as a unified candidate for dark matter (DM) and dark energy (DE), the equation of state parameter can cross the phantom divide. We also present that for the selected model under thermal equilibrium with the Hawking radiation, the GSL is always satisfied throughout the history of the universe for any spatial curvature, independently of the equation of state of the interacting generalized Chaplygin gas model.

Keywords: dark energy theory, generalized Chaplygin gas

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1 Introduction

One of interesting DE models is the Chaplygin gas (Kamenshchik et al. 2000, 2001; Bilic et al. 2002) and its generalization (Bento et al. 2002, 2004) which has been widely studied for interpreting the accelerating universe. It is remarkable that the generalized Chaplygin gas (GCG) equation of state has a well defined connection with string theory and can be obtained from the light cone parameterization of the Nambu-Goto action, associated with a D-brane (Bilic et al. 2007). The GCG is the only gas known to admit a supersymmetric generalization (Zhang et al. 2006). In this model, a single self-interacting scalar field is responsible for both DE and DM, giving also the observed accelerated expansion (García-Compeán et al. 2008). The striking feature of the GCG is that it allows for a unification of DE and DM. This point can be easily seen from the fact that the GCG behaves as a dust-like matter at early times and behaves like a cosmological constant at late stage. This dual role is at the heart of the surprising properties of the GCG model (Wu and Yu 2007; Lu et al. 2009). Moreover, the GCG model has been successfully confronted with various phenomenological tests involving SNe Ia data, CMB peak locations, gravitational lensing and other observational data (Alcaniz et al. 2003; Bento et al. 2003a,b; Bertolami et al. 2004; Bento et al. 2005; Wu and Yu 2007; Lu et al. 2008; Lu et al. 2009).

Here our aim is to investigate the GSL of thermodynamics for the GCG model as a candidate for the unified DM-DE which is in interaction with the baryonic matter in the non-flat universe enclosed by the apparent horizon.

2 Interacting GCG and BM

The GCG model is based on the equation of state

\[ P_{\text{Ch}} = -\frac{A}{\rho_{\text{Ch}}^\alpha}, \]

where A and \( \alpha \) are the GCG constant parameters (Bento et al. 2002, 2004). The case \( \alpha = 1 \) corresponds to the standard Chaplygin gas model (Kamenshchik et al. 2000, 2001; Bilic et al. 2002).

Using Eq. (1), the continuity equation can be integrated to give

\[ \rho_{\text{Ch}} = \rho_{\text{Ch}0} \left[ A_s + \frac{(1 - A_s)}{a^{3(1+\alpha)}} \right]^{1/\alpha}, \]

where \( A_s \equiv \frac{A}{\rho_{\text{Ch}0}^\alpha} \), \( \rho_{\text{Ch}0} \) is the present energy density of the GCG, and a is the cosmic scale factor. Note that the GCG model smoothly interpolates between a non-relativistic matter phase (\( \rho_{\text{Ch}} \propto a^{-3} \)) in the past and a negative-pressure DE regime (\( \rho_{\text{Ch}} = -P_{\text{Ch}} \)) at late times. This interesting feature leads to the GCG model being proposed as a candidate for the unified DM-DE (UDME) scenario (Wu and Yu 2007; Lu et al. 2009).

In the framework of the Friedmann-Robertson-Walker (FRW) metric

\[ ds^2 = -dt^2 + a^2(t) \left( \frac{dr^2}{1 - kr^2} + r^2d\Omega^2 \right), \]

the first Friedmann equation has the following form

\[ H^2 + \frac{k}{a^2} = \frac{8\pi}{3} (\rho_{\text{Ch}} + \rho_{\text{b}}), \]
where we take $G = 1$ and $k = 0, 1, -1$ represent a flat, closed and open FRW universe, respectively. Also $\rho_{\text{Ch}}$ and $\rho_b$ are the energy density of GCG and BM, respectively. In terms of the dimensionless energy densities as

\[
\Omega_b = \frac{8\pi\rho_b}{3H^2}, \quad \Omega_{\text{Ch}} = \frac{8\pi\rho_{\text{Ch}}}{3H^2}, \quad \Omega_k = \frac{k}{a^2H^2},
\]

the first Friedmann equation yields

\[
\Omega_b + \Omega_{\text{Ch}} = 1 + \Omega_k.
\]

The energy equations for the GCG and BM with $\omega_b = 0$ are

\[
\dot{\rho}_{\text{Ch}} + 3H(1 + \omega_{\text{Ch}})\rho_{\text{Ch}} = -Q, \quad (7)
\]

\[
\dot{\rho}_b + 3H\rho_b = Q, \quad (8)
\]

where following Kim et al. (2006), we choose $Q = 3b^2H(\rho_{\text{Ch}} + \rho_b)$. The interaction between the GCG and BM can be detected by local gravity measurements (Hagiwara et al. 2002; Peebles and Ratra 2003). Following Guendelman and Kaganovich (2008), the interaction between the BM and the DE will cause the appearance of the fifth force. However, according to the results of the fifth force experiments, a coupling of the DE (modeled by a light scalar field) to visible (baryonic) matter is strongly suppressed. According to Guendelman and Kaganovich (2008), a new aspect introduced by modern cosmology to this problem is the question of why the coupling of the light scalar (DE) to visible matter is strongly suppressed while similar coupling to DM is energetic. Discovery of DE and cosmic coincidence interpreted as evidence of the existence of an unsuppressed DE to DM coupling, turns the fifth force problem into an actual and even burning fundamental puzzle.

Taking time derivative of Eq. (2) yields

\[
\dot{\rho}_{\text{Ch}} = -3(1 - A_s)H \frac{\rho_{\text{Ch}0}}{a^3(1+\alpha)} \left[ A_s + \frac{(1 - A_s)}{a^3(1+\alpha)} \right]^{\frac{\alpha}{1+\alpha}}. \quad (9)
\]

Substituting Eq. (9) in (7) gives the equation of state (EoS) parameter of the interacting GCG model as

\[
\omega_{\text{Ch}} = -\frac{A_s a^{3(1+\alpha)}}{1 - A_s + A_s a^{3(1+\alpha)}} - b^2 \left( \frac{1 + \Omega_k}{\Omega_{\text{Ch}}} \right). \quad (10)
\]

Observational constraints on the GCG model as the UDME from the joint analysis of the latest astronomical data give the best-fit values of the GCG model parameters. In this respect, Wu and Yu (2007) obtained $(A_s = 0.76, \alpha = -0.005)$ for non-flat universe with $(\Omega_k = 0.04, \Omega_{\text{Ch}} = 0.995)$. Also Lu et al. (2009) derived $(A_s = 0.73, \alpha = -0.09)$ for flat universe with $(\Omega_{\text{Ch}} = 0.956)$. Equation (10) shows that in the absence of interaction between GCG and BM, $b^2 = 0$, and for the present time, $a = 1$, $\omega_{\text{Ch}} = -A_s > -1$ and cannot cross the phantom divide. However, in the presence of interaction, $b^2 \neq 0$, taking $(A_s = 0.76, \alpha = -0.005, \Omega_k = 0.04, \Omega_{\text{Ch}} = 0.995)$ given by Wu and Yu (2007), $(A_s = 0.73, \alpha = -0.09, \Omega_k = 0.0, \Omega_{\text{Ch}} = 0.956)$ given by Lu et al. (2009) and $a = 1$ for the present time, Eq. (10) clears that the phantom EoS, i.e. $\omega_{\text{Ch}} < -1$, can be achieved when $b^2 > 0.23$ and $b^2 > 0.26$ for non-flat and flat universe, respectively.

The deceleration parameter is given by

\[
q = -\left( 1 + \frac{\dot{H}}{H^2} \right). \quad (11)
\]
Taking time derivative of both sides of Eq. (4), and using Eqs. (5), (6), (7) and (8), we get
\[ q = \frac{1}{2} \left( 1 + \Omega_k + 3\Omega_{\text{Ch}}\omega_{\text{Ch}} \right), \tag{12} \]
Substituting Eq. (10) in (12) yields
\[ q = \frac{1}{2} \left[ -3A_s a^{3(1+\alpha)} \Omega_{\text{Ch}} + (1 - 3b^2)(1 + \Omega_k) \right]. \tag{13} \]
Now taking \( A_s = 0.76, \alpha = -0.005, \Omega_k = 0.04, \Omega_{\text{Ch}} = 0.995 \) given by Wu and Yu (2007), \( (A_s = 0.73, \alpha = -0.09, \Omega_k = 0.0, \Omega_{\text{Ch}} = 0.956) \) given by Lu et al. (2009) and \( a = 1 \) for the present time we get \( q = -0.61 - 1.56b^2 \) and \( q = -0.55 - 1.5b^2 \) for non-flat and flat universe, respectively. These results show that the deceleration parameter is always negative even in the absence of interaction between GCG and BM. Therefore the GCG model in the present time can drive the universe to accelerated expansion.

3 GSL of thermodynamics

Here, we study the validity of the GSL in which the entropy of the GCG and BM inside the horizon plus the entropy of the horizon do not decrease with time (Wang et al. 2006).

The location of the apparent horizon \( \tilde{r}_A \) in the FRW universe according to Cai et al. (2009) is obtained as
\[ \tilde{r}_A = H^{-1}(1 + \Omega_k)^{-1/2}. \tag{14} \]
For \( k = 0 \), the apparent horizon is same as the Hubble horizon, i.e. \( \tilde{r}_A = H^{-1} \).

Following Cai and Kim (2005), the associated Hawking temperature on the apparent horizon is given by
\[ T_A = \frac{1}{2\pi \tilde{r}_A} \left( 1 - \frac{\dot{\tilde{r}}_A}{2H\tilde{r}_A} \right), \tag{15} \]
where \( \frac{\dot{\tilde{r}}_A}{2H\tilde{r}_A} < 1 \) ensure that the temperature is positive.

Taking time derivative of both sides of (14) and using Eqs. (4), (5), (6), (7), (8), (11) and (12), one can rewrite Eq. (15) in real dimension form as
\[ T_A = \frac{\hbar c}{k_B 8\pi (1 + \Omega_k)^{1/2}} (1 + \Omega_k - 3\Omega_{\text{Ch}}\omega_{\text{Ch}}), \]
\[ = \frac{\hbar c}{k_B 4\pi (1 + \Omega_k)^{1/2}} (1 + \Omega_k - q), \tag{16} \]
where \( \hbar, c \) and \( k_B \) are the reduced Planck constant, speed of light and Boltzmann constant, respectively. Now taking \( H_0 = 72 \text{ Km s}^{-1}\text{Mpc}^{-1} \) for the present time, one can estimate the Hawking temperature on the apparent horizon as \( T_{A_0} \sim \frac{\hbar cH_0}{k_B} \sim 10^{-22} \text{ K} \).

Following Maartens (1996), the BM (non-relativistic matter) temperature in the universe scales as \( T \propto a^{-2} \), so like Gong et al. (2007b) we assume here that the GCG temperature has a similar behavior \( T \propto a^{-n} \) to avoid the negative entropy problem, where \( n \) is an arbitrary constant. It is not necessary to take \( n = 2 \) to ensure that the GCG is in equilibrium with the BM, since their dispersion relations could be completely different (see Lima and Alcaniz 2004; Gong et al. 2007a). Since the usual BM temperature in the universe decreases as the universe expands, we expect that the GCG temperature also preserves this property. Santos et al. (2006) showed that the GCG temperature in the adiabatic evolution from a dust-like to a de Sitter
cosmological model remains in the range \(0 < T < T_+\), where \(T_+ \sim 10^{32}\) K, the temperature of the Planck era, is the maximum temperature of the GCG when it fills small volumes. Therefore the temperature of the universe filled with the GCG cools down as expected. This shows that the temperature behaviour of the GCG is different from that of the DE with constant EoS parameter \(\omega\). The temperature of the GCG decreases instead of increasing. This tells us that the increasing or decreasing behaviour of the temperature of the universe dominated by DE is model dependent; it is not a general property associated with the DE (Gong et al. 2007a).

A basic difficulty, however, is that the present-day DE temperature has not been measured. Lima and Alcaniz (2004) using a very naive estimate obtained the present value of the DE temperature as \(T_{\text{DE}}^0 \sim 10^{-6}\) K in a non-flat FRW universe and in the absence of interaction with BM. Zhou et al. (2009) showed that the DM temperature in the absence of interaction with the DE behaves same as the BM as \(T_{\text{DM}} \propto a^{-2}\). They estimated \(T_{\text{DM}}^0 \sim 10^{-7}\) K for the present time. Since the temperatures of the DE and the BM at the present time differs from that of the horizon \(T_{\Lambda_0} \sim 10^{-22}\) K, the systems must interact for some length of time before they can attain thermal equilibrium. Although in this case the local equilibrium hypothesis may no longer hold (Das et al. 2002, Wang et al. 2008; Pavón and Wang 2009; Zhou et al. 2009), Karami and Ghaffari (2010) showed that the contribution of the heat flow between the horizon and the fluid in the GSL in non-equilibrium thermodynamics is very small, \(O(10^{-7})\). Therefore the equilibrium thermodynamics is still preserved.

The entropy of the universe containing the GCG and BM is given by Gibb’s equation (Izquierdo and Pavón 2006a)

\[ T dS = dE + P dV, \]  
(17)

where \(V = 4\pi\tilde{r}_A^3/3\) is the volume of the universe and \(T = T_{\Lambda}\). Also

\[ E = \frac{4\pi\tilde{r}_A^3}{3}(\rho_{\text{Ch}} + \rho_b), \]  
(18)

\[ P = P_{\text{Ch}} + P_b = P_{\text{Ch}} = \frac{\omega_{\text{Ch}} \rho_{\text{Ch}}}{8\pi} \Omega_{\text{Ch}} \omega_{\text{Ch}}. \]  
(19)

Taking time derivative of both sides of (17) and using Eqs. (4), (5), (6), (7), (8), (18) and (19), we obtain

\[ T_{\Lambda} \dot{S} = -4\pi H \tilde{r}_A^3 \rho_{\text{Ch}} (1 + u + \omega_{\text{Ch}}) \left(1 - \frac{\dot{\tilde{r}}_A}{H \tilde{r}_A}\right), \]  
(20)

where \(u = \rho_b/\rho_{\text{Ch}}\).

Also the evolution of the geometric entropy on the apparent horizon \(S_{\Lambda} = \pi\tilde{r}_A^2\) (Izquierdo and Pavón 2006a) is obtained as

\[ T_{\Lambda} \dot{S}_{\Lambda} = 4\pi H \tilde{r}_A^3 \rho_{\text{Ch}} (1 + u + \omega_{\text{Ch}}) \left(1 - \frac{\dot{\tilde{r}}_A}{2H \tilde{r}_A}\right). \]  
(21)

Finally, adding Eqs. (20) and (21) yields the GSL as

\[ T_{\Lambda} \dot{S}_{\text{tot}} = 2\pi\tilde{r}_A^2 \rho_{\text{Ch}} (1 + u + \omega_{\text{Ch}}) \dot{\tilde{r}}_A, \]  
(22)

where \(S_{\text{tot}} = S + S_{\Lambda}\) is the total entropy.

Taking time derivative of the apparent horizon (14) and using Eq. (4), one can obtain

\[ \dot{\tilde{r}}_A = 4\pi H \tilde{r}_A^3 \rho_{\text{Ch}} (1 + u + \omega_{\text{Ch}}). \]  
(23)
Substituting Eq. (23) into Eq. (22) gives

\[ T_A \dot{S}_{\text{tot}} = 8\pi^2 H r_A^5 \rho_{\text{Ch}}^2 (1 + u + \omega_{\text{Ch}})^2 \geq 0, \]

which shows that the GSL for the universe containing the interacting GCG with BM enclosed by the dynamical apparent horizon is always satisfied throughout the history of the universe for any spatial curvature, independently of the EoS parameter of the interacting GCG model.

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References

[1] Akbar, M., Cai, R.G.: Phys. Rev. D 75, 084003 (2007)
[2] Alcaniz, J.S., Jain, D., Dev, A.: Phys. Rev. D 67, 043514 (2003)
[3] Amendola, L.: Phys. Rev. D 60, 043501 (1999)
[4] Amendola, L.: Phys. Rev. D 62, 043511 (2000)
[5] Bardeen, J.M., Carter, B., Hawking, S.W.: Commun. Math. Phys. 31, 161 (1973)
[6] Bennett, C.L., et al.: Astrophys. J. Suppl. 148, 1 (2003)
[7] Bento, M.C., Bertolami, O., Sen, A.A.: Phys. Rev. D 66, 043507 (2002)
[8] Bento, M.C., Bertolami, O., Sen, A.A.: Phys. Lett. B 575, 172 (2003a)
[9] Bento, M.C., Bertolami, O., Sen, A.A.: Phys. Rev. D 67, 063003 (2003b)
[10] Bento, M.C., Bertolami, O., Sen, A.A.: Phys. Rev. D 70, 083519 (2004)
[11] Bento, M.C., Bertolami, O., Santos, N.M.C., Sen, A.A.: Phys. Rev. D 71, 063501 (2005)
[12] Bertolami, O., Sen, A.A., Sen, S., Silva, P.T.: Mon. Not. Roy. Astron. Soc. 353, 329 (2004)
[13] Bertolami, O., Gil Pedro, F., Le Delliou, M.: Phys. Lett. B 654, 165 (2007)
[14] Bilic, N., Tupper, G.B., Viollier, R.D.: Phys. Lett. B 535, 17 (2002)
[15] Bilic, N., Tupper, G.B., Viollier, R.D.: J. Phys. A: Math. Theor. 40, 6877 (2007)
[16] Cai, R.G., Cao, L.M., Hu, Y.P.: Class. Quantum Grav. 26, 155018 (2009)
[17] Cai, R.G., Kim, S.P.: J. High Energy Phys. 02, 050 (2005)
[18] Caldera-Cabral, G., Maartens, R., Ureña-Lópe, L.A.: Phys. Rev. D 79, 063518 (2009)
[19] Copeland, E.J., Sami, M., Tsujikawa, S.: Int. J. Mod. Phys. D 15, 1753 (2006)
[20] de Bernardis, P., et al.: Nature 404, 955 (2000)
[21] Das, S., Majumdar, P., Bhaduri, R.K.: Class. Quantum Grav. 19, 2355 (2002)
[22] García-Compeán, H., García-Jiménez, G., Obregón, O., Ramírez, C.: J. Cosmol. Astropart. Phys. 07, 016 (2008)
[23] Gong, Y., Wang, B., Wang, A.: J. Cosmol. Astropart. Phys. 01, 024 (2007a)
[24] Gong, Y., Wang, B., Wang, A.: Phys. Rev. D 75, 123516 (2007b)
[25] Guendelman, E.I., Kaganovich, A.B.: J. Phys. A: Math. Theor. 41, 164053 (2008)
[26] Guo, Z.K., Ohta, N., Tsujikawa, S.: Phys. Rev. D 76, 023508 (2007)
[27] Hagiwara, K., et al.: Phys. Rev. D 66, 010001 (2002)
[28] Hawking, S.W.: Commun. Math. Phys. 43, 199 (1975)
[29] Huang, Q.G., Li, M.: J. Cosmol. Astropart. Phys. 08, 013 (2004)
[30] Izquierdo, G., Pavón, D.: Phys. Lett. B 633, 420 (2006a)
[31] Izquierdo, G., Pavón, D.: Phys. Lett. B 639, 1 (2006b)
[32] Jacobson, T.: Phys. Rev. Lett. 75, 1260 (1995)
[33] Kamenshchik, A., Moschella, U., Pasquier, V.: Phys. Lett. B 487, 7 (2000)
[34] Kamenshchik, A., Moschella, U., Pasquier, V.: Phys. Lett. B 511, 265 (2001)
[35] Karami, K., Ghaffari, S., Fehri, J.: Eur. Phys. J. C 64, 85 (2009)
[36] Karami, K., Ghaffari, S.: Phys. Lett. B 685, 115 (2010)
[37] Karami, K.: Preprint, arXiv:1002.0431 (2010a)
[38] Karami, K.: J. Cosmol. Astropart. Phys. 01, 015 (2010b)
[39] Kim, H., Lee, H.W., Myung, Y.S.: Phys. Lett. B 632, 605 (2006)
[40] Lima, J.A.S., Alcaniz, J.S.: Phys. Lett. B 600, 191 (2004)
[41] Lu, J., Gui, Y., Xu, L.X.: Eur. Phys. J. C 63, 349 (2009)
[42] Lu, J.B., Xu, L.X., Li, J.C., Liu, H.Y.: Mod. Phys. Lett. A 23, 25 (2008)
[43] Maartens, R.: Preprint, arXiv:astro-ph/9609119 (1996)
[44] Mohseni Sadjadi, H.: Phys. Lett. B 645, 108 (2007)
[45] Padmanabhan, T.: Phys. Rep. 380, 235 (2003)
[46] Pavón, D., Wang, B.: Gen. Relativ. Gravit. 41, 1 (2009)
[47] Pavón, D., Zimdahl, W.: Phys. Lett. B 628, 206 (2005)
[48] Peebles, P.J.E., Ratra, B.: Rev. Mod. Phys. 75, 559 (2003)
[49] Perlmutter, S., et al.: Astrophys. J. 517, 565 (1999)
[50] Perlmutter, S., et al.: Astrophys. J. 598, 102 (2003)
[51] Riess, A.G., et al.: Astron. J. 116, 1009 (1998)
[52] Santos, F.C., Bedran, M.L., Soares, V.: Phys. Lett. B 636, 86 (2006)
[53] Seljak, U., Slosar, A., McDonald, P.: J. Cosmol. Astropart. Phys. 10, 014 (2006)
[54] Sheykhi, A.: J. Cosmol. Astropart. Phys. 05, 019 (2009)
[55] Sheykhi, A., Wang, B.: Phys. Lett. B 678, 434 (2009a)
[56] Sheykhi, A., Wang, B.: Preprint, arXiv:0811.4477 (2009b)
[57] Spergel, D.N.: Astrophys. J. Suppl. 148, 175 (2003)
[58] Spergel, D.N., et al.: Astrophys. J. Suppl. 170, 377 (2007)
[59] Szydlowski, M.: Phys. Lett. B 632, 1 (2006)
[60] Tegmark, M., et al.: Phys. Rev. D 69, 103501 (2004)
[61] Tsujikawa, S., Sami, M.: Phys. Lett. B 603, 113 (2004)
[62] Tsujikawa, S.: Phys. Rev. D 73, 103504 (2006)
[63] Wang, B., Gong, Y., Abdalla, E.: Phys. Lett. B 624, 141 (2005)
[64] Wang, B., Gong, Y., Abdalla, E.: Phys. Rev. D 74, 083520 (2006)
[65] Wang, B., Lin, C.Y., Pavón, D., Abdalla, E.: Phys. Lett. B 662, 1 (2008)
[66] Wu, P., Yu, H.: Astrophys. J. 658, 663 (2007)
[67] Zhang, X., Wu, F.Q., Zhang, J.: J. Cosmol. Astropart. Phys. 01, 003 (2006)
[68] Zhou, J., Wang, B., Gong, Y., Abdalla, E.: Phys. Lett. B 652, 86 (2007)
[69] Zhou, J., Wang, B., Pavón, D., Abdalla, E.: Mod. Phys. Lett. A 24, 1689 (2009)