Flavor Democracy and Type-II Seesaw Realization of Bilarge Neutrino Mixing

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Abstract

We generalize the democratic neutrino mixing Ansatz by incorporating the type-II seesaw mechanism with S(3) flavor symmetry. For only the triplet mass term or only the conventional seesaw term large neutrino mixing can be achieved only by assuming an unnatural suppression of the flavor democracy contribution. We show that bilarge neutrino mixing can naturally appear if the flavor democracy term is strongly suppressed due to significant cancellation between the conventional seesaw and triplet mass terms. Explicit S(3) symmetry breaking yields successful neutrino phenomenology and various testable correlations between the neutrino mass and mixing parameters. Among the results are a normal neutrino mass ordering, 0.005 \leq |U_{e3}| \leq 0.057, 1 - \sin^22\theta_{23} \geq 0.005, positive \text{\textsc{J}}_{\text{CP}} and moderate cancellation in the effective mass of the neutrinoless double beta decay.

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The elegant Super-Kamiokande [1], SNO [2], K2K [3] and KamLAND [4] experiments have provided us with very convincing evidence that the long-standing solar neutrino deficit and the atmospheric neutrino anomaly are both due to neutrino oscillations, which can naturally occur if neutrinos are massive and lepton flavors are mixed. A big puzzle is that the mass scale of three active neutrinos (i.e., $\nu_e$, $\nu_\mu$ and $\nu_\tau$) is extremely low, at most of $\mathcal{O}(0.1)$ eV. In addition, lepton flavor mixing involves two remarkably large angles, $\theta_{12} \sim 33^\circ$ and $\theta_{23} \sim 45^\circ$ in the standard parametrization. To understand the smallness of neutrino masses, a number of theoretical and phenomenological ideas have been proposed in the literature [5]. Among them, the most natural idea is the seesaw mechanism [6]. While the seesaw mechanism itself can qualitatively explain why neutrino masses are so small, it is unable to make any concrete predictions unless a specific lepton flavor structure is assumed. Hence an appropriate combination of the seesaw mechanism and possible flavor symmetries [7] or texture zeros [8] is practically needed, in order to quantitatively account for the neutrino mass spectrum and the bilarge lepton mixing pattern. Some interesting attempts in this direction [9] have been made recently.

In this letter we aim to interpret current experimental data on neutrino masses and lepton flavor mixing angles by incorporating the type-II seesaw mechanism [10] with S(3) flavor symmetry and its explicit breaking. Our physical motivation is rather simple. The charged lepton mass matrix with S(3)$_L \times$ S(3)$_R$ symmetry (i.e., flavor democracy) and the effective Majorana neutrino mass matrix with S(3) permutation symmetry may in general be written as

$$M_l^{(0)} = \frac{c_l}{3} \begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix},$$

$$M_\nu^{(0)} = c_\nu \left[ \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} + r_\nu \begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix} \right],$$

in which $c_l$ and $c_\nu$ measure the corresponding mass scales of charged leptons and light neutrinos, and $r_\nu$ is in principle an arbitrary parameter. A soft breakdown of the above permutation symmetry can lead to realistic lepton mass matrices $M_l = M_l^{(0)} + \Delta M_l$ and $M_\nu = M_\nu^{(0)} + \Delta M_\nu$ with proper mass eigenvalues. Then the lepton flavor mixing matrix $U$ arises from the mismatch between the diagonalization of $M_l$ and that of $M_\nu$. It has been noticed in Refs. [11, 12, 13, 14, 15] that $r_\nu$ must be vanishing or strongly suppressed such that a bilarge neutrino mixing pattern can be generated. In the spirit of ’t Hooft’s naturalness principle [16], however, $|r_\nu| = \mathcal{O}(1)$ seems more likely than $r_\nu = 0$ or $|r_\nu| \ll 1$. The point will become clear when the smallness of $c_\nu$ is attributed to the seesaw mechanism. We find that the conventional (type-I) seesaw mechanism cannot help out (see also [14]), but the type-II seesaw scenario may provide a natural interpretation of small neutrino masses and bilarge lepton mixing angles even in the case of $|r_\nu| = \mathcal{O}(0.1)$ to $\mathcal{O}(1)$.

In type-II seesaw models with three right-handed neutrinos, the neutrino mass term
reads
\[ -\mathcal{L}_{\text{mass}} = \frac{1}{2} (\nu, \nu^c) \begin{pmatrix} M_L & M_D \\ M_D^T & M_R \end{pmatrix} \begin{pmatrix} \nu^c \\ \nu \end{pmatrix} R, \]  
(2)
where \( \nu \) denotes the column vector of three neutrino fields, \( M_D \) stands for the \( 3 \times 3 \) Dirac neutrino mass matrix, \( M_L \) and \( M_R \) represent the symmetric \( 3 \times 3 \) mass matrices of left-handed and right-handed Majorana neutrinos respectively. As \( M_L \) results from a \( SU(2)_L \) triplet term of the Yukawa interactions, its scale might be considerably lower than the gauge symmetry breaking scale \( v \approx 174 \text{ GeV} \). On the other hand, the scale of \( M_R \) can naturally be much higher than \( v \), because right-handed neutrinos are \( SU(2)_L \) singlets and their corresponding mass term is not subject to gauge symmetry breaking.

The strong hierarchy between the scales of \( M_R \) and \( M_L \) or \( M_D \) allow us to make some safe approximations in diagonalizing the \( 6 \times 6 \) neutrino mass matrix in Eq. (2) and arrive at an effective mass matrix for three light (essentially left-handed) neutrinos [10]:

\[ M_\nu \approx M_L - M_D M_R^{-1} M_D^T. \]  
(3)

For a phenomenological study of neutrino masses and lepton flavor mixing, we assume a discrete left-right symmetry between \( M_L \) and \( M_R \), whose mass scales are characterized respectively by the vacuum expectation values (vevs) of two triplet fields, \( v_L \) and \( v_R \). Consequently, the usual left-right symmetric relation \( v_L v_R = \gamma v^2 \) holds, where \( \gamma \) is a model-dependent factor of \( \mathcal{O}(1) \). As investigated recently, the interplay of the two terms in the type-II seesaw formula can result in several interesting effects. One can, e.g., upgrade a hierarchical neutrino mass spectrum to a quasi-degenerate one [17] or create deviations from the bimaximal neutrino mixing pattern [18]. In this letter we shall take advantage of possible cancellations hidden in the type-II seesaw mechanism, which is an intriguing feature when the two mass terms on the right-hand side of Eq. (3) contribute to \( M_\nu \) with comparable magnitudes.

Imposing \( S(3) \) flavor symmetry on \( M_L \) and \( M_R \) and allowing for soft symmetry breaking, we write down

\[ M_L = v_L \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} + r_\nu \begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix} + \Delta M_L, \]

\[ M_R = v_R \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} + r_\nu \begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix} + \Delta M_R. \]  
(4)

On the other hand, the flavor democracy or \( S(3)_L \times S(3)_R \) symmetry can be imposed on the Dirac neutrino mass matrix \( M_D \) and the charged lepton mass matrix \( M_l \), whose eigenvalues appear to be hierarchical as those of up- or down-type quarks [19]. Once soft symmetry
breaking is taken into account, \( M_D \) and \( M_l \) read

\[
M_D = \frac{c_D}{3} \begin{pmatrix}
1 & 1 & 1 \\
1 & 1 & 1 \\
1 & 1 & 1 \\
\end{pmatrix} + \Delta M_D, \\
M_l = \frac{c_l}{3} \begin{pmatrix}
1 & 1 & 1 \\
1 & 1 & 1 \\
1 & 1 & 1 \\
\end{pmatrix} + \Delta M_l. \\
\tag{5}
\]

To make concrete predictions, one has to specify the patterns of \( \Delta M_L, \Delta M_R, \Delta M_D \) and \( \Delta M_l \). For the sake of simplicity, we follow Refs. [11] and [15] to take

\[
\Delta M_L = v_L \begin{pmatrix}
-\delta_M & 0 & 0 \\
0 & +\delta_M & 0 \\
0 & 0 & \varepsilon_M \\
\end{pmatrix}, \\
\Delta M_R = v_R \begin{pmatrix}
-\delta_M & 0 & 0 \\
0 & +\delta_M & 0 \\
0 & 0 & \varepsilon_M \\
\end{pmatrix}, \\
\tag{6}
\]

where left-right symmetry has been implemented. For the Dirac fermion sector, we choose

\[
\Delta M_D = \frac{c_D}{3} \begin{pmatrix}
-i\delta_D & 0 & 0 \\
0 & +i\delta_D & 0 \\
0 & 0 & \varepsilon_D \\
\end{pmatrix}, \\
\Delta M_l = \frac{c_l}{3} \begin{pmatrix}
-i\delta_l & 0 & 0 \\
0 & +i\delta_l & 0 \\
0 & 0 & \varepsilon_l \\
\end{pmatrix}. \\
\tag{7}
\]

Note that \( \delta_{M,D,l} \) and \( \varepsilon_{M,D,l} \) are small perturbative parameters and their magnitudes are at most of \( \mathcal{O}(0.1) \). Note also that we have introduced imaginary perturbations in \( \Delta M_D \) and \( \Delta M_l \), in order to accommodate leptonic CP violation. Calculating the effective neutrino mass matrix \( M_\nu \) by using Eqs. (3)–(7), we obtain

\[
M_\nu \approx v_L \begin{pmatrix}
1 & 0 & 0 \\
0 & 1 & 0 \\
0 & 0 & 1 \\
\end{pmatrix} + r_\nu \begin{pmatrix}
1 & 1 & 1 \\
1 & 1 & 1 \\
1 & 1 & 1 \\
\end{pmatrix} + \begin{pmatrix}
-\delta_M & 0 & 0 \\
0 & +\delta_M & 0 \\
0 & 0 & \varepsilon_M \\
\end{pmatrix} + \frac{c_D^2}{v_R} \left[ (1 - \bar{\varepsilon}_M) \begin{pmatrix}
1 & 1 & 1 \\
1 & 1 & 1 \\
1 & 1 & 1 \\
\end{pmatrix} + \frac{1}{3} \begin{pmatrix}
-2i\delta_D & 0 & \varepsilon_D - i\delta_D \\
0 & +2i\delta_D & \varepsilon_D + i\delta_D \\
\varepsilon_D - i\delta_D & \varepsilon_D + i\delta_D & 2\varepsilon_D \\
\end{pmatrix} \right], \\
\tag{8}
\]

where \( \bar{c}_D \equiv c_D / \sqrt{3(1 + 3r_\nu)} \), \( \bar{\varepsilon}_M \equiv \varepsilon_M / [3(1 + 3r_\nu)] \), and terms of \( \mathcal{O}(\delta_{M,D}^2) \) and \( \mathcal{O}(\varepsilon_{M,D}^2) \) have been neglected. It is quite obvious that the matrices proportional to \( v_L \) and \( \bar{c}_D^2 / v_R \) in Eq. (8) arise respectively from \( M_L \) and \( M_D M_R^{-1} M_D^T \). Their relative contributions to \( M_\nu \) can be classified into three typical cases:
• In the limit of $v_L \to 0$, we are left with the conventional (type-I) seesaw result of $M_\nu$, whose leading term displays flavor democracy. Because both $M_t$ and $M_\nu$ come from the explicit (soft) breaking of flavor democracy in this special case (similar to the case of democratic quark mass matrices [5]), no large lepton flavor mixing can appear. To suppress or avoid such a flavor democracy term in the type-I seesaw expression of $M_\nu$ (and thereby to open the possibility of generating large neutrino mixing angles), other possible flavor symmetries (such as $Z_3$ symmetry [14]) have to be taken into account.

• In the limit of $\tilde{c}_D^2/v_R \to 0$, we obtain $M_\nu \approx M_L$. This pure triplet case can accommodate current experimental data of solar and atmospheric neutrino oscillations, if $\varepsilon_M \gg \delta_M \sim r_\nu$ is satisfied [15]. To be more specific, $r_\nu/\varepsilon_M \sim 6.1 \times 10^{-3}$ has been obtained in Ref. [15] without any fine-tuning. As $\varepsilon_M = \mathcal{O}(0.1)$ is most plausible, the magnitude of $r_\nu$ must be of $\mathcal{O}(10^{-3})$ or $\mathcal{O}(10^{-4})$. Such a small result implies that the two $S(3)$ symmetry terms in $M_L$ are not balanced — one of them (i.e., the flavor democracy term) is strongly suppressed. This seems unnatural in some sense, since $|r_\nu| = \mathcal{O}(1)$ is more or less expected from the point of view of ’t Hooft’s naturalness principle.

• The two mass terms of $M_\nu$ in Eq. (8) are comparable in magnitude and lead to significant cancellation. A particularly interesting possibility is that the two flavor democracy terms, which are proportional to $r_\nu$ and $(1 - \tilde{\varepsilon}_M)$ respectively, may essentially cancel each other. In this case,

$$r_\nu \approx \frac{\tilde{c}_D^2}{v_Lv_R}(1 - \tilde{\varepsilon}_M) = \frac{\tilde{c}_D^2}{\gamma v^2}(1 - \tilde{\varepsilon}_M)$$

(9)

is likely to be of $\mathcal{O}(0.1)$ to $\mathcal{O}(1)$ (e.g., $c_D \sim m_t \approx v$ might hold in a specific GUT framework with lepton-quark symmetry, such as some SO(10) models). We carry out a careful numerical analysis of this typical type-II seesaw scenario and find that the bilarge neutrino mixing pattern can actually be reproduced without fine-tuning. Before presenting our numerical results, we would like to give some more comments on the consequences of Eq. (9).

Note that the possibility of $r_\nu \sim -1/3$, which may significantly enhance the magnitude of $\tilde{\varepsilon}_M$, is found to be disfavored in fitting current neutrino oscillation data. In the following we will constrain ourselves to positive and small perturbative parameters. With the definition $\zeta_\nu \equiv c_D^2/(\gamma v^2)$, from which $\tilde{c}_D^2/(\gamma v^2) = \zeta_\nu/[3(1 + 3r_\nu)]$ can be expressed, we then obtain

$$r_\nu \approx \frac{1}{6} \left(-1 \pm \sqrt{1 + 4\zeta_\nu}\right)$$

(10)

by solving Eq. (9) in the leading-order approximation (i.e., in the neglect of $\tilde{\varepsilon}_M$). This rough result clearly shows that $|r_\nu|$ is most likely to be of $\mathcal{O}(0.1)$ to $\mathcal{O}(1)$, provided $\zeta_\nu = \mathcal{O}(1)$ holds. Typically, taking for instance $\zeta_\nu = 2$, we arrive at $r_\nu \approx 1/3$ or $r_\nu \approx -2/3$. Now the
question is whether in the outlined framework bilarge neutrino mixing can be achieved. Inserting Eq. (10) into (8) gives

\[
M_\nu \approx v_L \begin{pmatrix}
1 - \delta_M - 2 i \hat{\delta}_D & 0 & -\hat{\epsilon}_D + i \hat{\delta}_D \\
0 & 1 + \delta_M + 2 i \hat{\delta}_D & -\hat{\epsilon}_D - i \hat{\delta}_D \\
-\hat{\epsilon}_D + i \hat{\delta}_D & -\hat{\epsilon}_D - i \hat{\delta}_D & 1 + \varepsilon_M - 2 \hat{\epsilon}_D
\end{pmatrix},
\]

(11)

where \( \hat{\epsilon}_D \equiv \varepsilon_D \zeta_\nu/[9(1 + 3 r_\nu)] \) and \( \hat{\delta}_D \equiv \delta_D \zeta_\nu/[9(1 + 3 r_\nu)] \). One may diagonalize this symmetric mass matrix by the transformation \( U_\nu M_\nu U_\nu^T = \text{Diag}\{m_1, m_2, m_3\} \), where \( U_\nu \) is a unitary matrix and \( m_i \) (for \( i = 1, 2, 3 \)) denote the physical masses of three light neutrinos. It is obvious that \( m_1 \approx m_2 \approx m_3 \) must hold to leading order. The observed solar and atmospheric neutrino mass-squared differences \( \Delta m^2_\odot \equiv \Delta m^2_{21} \approx 10^{-5} \text{eV}^2 \) and \( \Delta m^2_\text{A} \equiv \Delta m^2_{32} \approx 10^{-3} \text{eV}^2 \) are proportional to \( v_L^2 \), and their different magnitudes are governed by the relevant perturbative parameters \( (\delta_M, \varepsilon_M, \text{etc}) \). The presence of \( \hat{\delta}_D \) and \( \hat{\epsilon}_D \) makes it possible to generate suitable rotation angles in \( U_\nu \). The mismatch between \( U_\nu \) and the unitary matrix \( U_t \), which is defined to diagonalize \( M_t \) (i.e., \( U_t M_t U_t^T = \text{Diag}\{m_\mu, m_\tau, m_\nu\} \)) and given by \[15\]

\[
U_t \approx \begin{pmatrix}
\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 \\
\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{6}} & \frac{1}{\sqrt{6}} \\
\frac{1}{\sqrt{3}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{3}}
\end{pmatrix} + i \sqrt{\frac{m_\nu}{m_\mu}} \begin{pmatrix}
\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{6}} & -\frac{2}{\sqrt{6}} \\
\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 \\
0 & 0 & 0
\end{pmatrix} + \frac{m_\mu}{m_\tau} \begin{pmatrix}
\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{6}} & \frac{1}{\sqrt{6}} \\
-\frac{1}{\sqrt{12}} & \frac{1}{\sqrt{12}} & \frac{1}{\sqrt{12}} \\
\frac{1}{\sqrt{3}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{3}}
\end{pmatrix},
\]

(12)

measures the strength of lepton flavor mixing — namely, \( U = U_t U_\nu^T \). Although a bilarge neutrino mixing pattern is naturally expected from this democratic type-II seesaw scenario, we find it very difficult to obtain a simple analytical expression of \( U_\nu \) to make the result of \( U \) more transparent. In this case, we shall do a numerical analysis of our phenomenological Ansatz without sticking to the condition given in Eq. (9) or (10).

We first vary \( \zeta_\nu \) between 0.2 and 10 since we expect from the above discussion that in this range \( |r_\nu| \) will be of \( \mathcal{O}(0.1) \) to \( \mathcal{O}(1) \). Larger values of \( \zeta_\nu \) will result in unnaturally large values of \( c_D \) as long as \( \gamma \) is of order one. For the sake of simplicity, here we only take account of \( r_\nu \geq 0 \) but emphasize that a similar analysis for the \( r_\nu \leq 0 \) case is straightforward. Furthermore, all small perturbative parameters appearing in \( M_t \) and \( M_\nu \) are allowed to vary between 0 and 0.2. The relevant neutrino oscillation parameters are required to lie in the following ranges, which are the typical 1σ outcome of recent global analyzes \[20\ [21\ [22:\]

\[
\tan^2 \theta_{12} = 0.34 \ldots 0.44 ,
|U_{e3}|^2 \leq 0.015 ,
\sin^2 2 \theta_{23} \geq 0.95 ,
R_\nu \equiv \frac{\Delta m^2_\odot}{\Delta m^2_\text{A}} = 0.033 \ldots 0.053.
\]

(13)
We plot in Fig. 1 some of the resulting correlations between the model parameters and observables. It is seen that $r_\nu$ indeed is of $O(1)$ for values of $\zeta_\nu$ larger than one. The functional behavior is excellently described by Eq. (10), implying that the flavor democracy contribution to $M_\nu$ is strongly suppressed due to significant cancellation between the conventional seesaw and triplet mass terms. Regardless of the values of $r_\nu$ and $\zeta_\nu$, the neutrino mass ordering is of normal type. Moreover, the rephasing invariant of CP or T violation $J_{CP} = \text{Im}\{U_{e1}U_{\mu2}U_{e2}^*U_{\mu1}^*\}$ is positive\(^1\) and smaller than $\approx 1.2\%$. This quantity measures the strength of leptonic CP and T violation in neutrino oscillations. In addition, the effective mass of the neutrinoless double beta decay $\langle m \rangle = \sum (m_i U_{ei}^2)$ is found to be of order of the common neutrino mass scale $v_L$, which may be at or below the level of $O(0.1)$ eV. The deviation of $\sin^2 2\theta_{23}$ from one and that of $|U_{e3}|$ from zero are always non-vanishing. We see that the atmospheric neutrino mixing parameter $1 - \sin^2 2\theta_{23}$ is larger than $\approx 0.005$. On the other hand, $|U_{e3}|$ is also larger than $\approx 0.005$ but smaller than $\approx 0.06$. This upper limit is given by $|U_{e3}| \approx 2m_e/(\sqrt{6} m_\mu) \approx 0.057$, which is actually the prediction obtained from $\zeta_\nu = 0$ [11]. For this special case, we show the correlation between $\tan^2 \theta_{12}$ and $1 - \sin^2 2\theta_{23}$ in Fig. 2. It is clear that $1 - \sin^2 2\theta_{23}$ varies only slightly. Indeed, $\sin^2 2\theta_{23} \approx 8(1 + m_\mu/m_\tau + R_\nu \cos 2\theta_{12})/9 \approx 0.95$ [15], which has nicely been reproduced by our numerical analysis. For the case of $\zeta_\nu = 1$ we plot the correlations between $1 - \sin^2 2\theta_{23}$ and $|U_{e3}|$ as well as between $\langle m \rangle/v_L$ and $J_{CP}$ in Fig. 3. The result for larger values of $\zeta_\nu$ is found to be essentially the same. Typically, larger values of $|U_{e3}|$ imply larger values of $1 - \sin^2 2\theta_{23}$ and less cancellation [23] in $\langle m \rangle$. On the other hand, $J_{CP}$ becomes larger when $\langle m \rangle$ approaches $v_L$. Note that the numerical analysis only requires to reproduce the ratio of the solar and atmospheric neutrino mass-squared differences. Hence the common neutrino mass scale $v_L$ is basically unspecified and ranges in our Ansatz from $\approx 0.06$ eV to $\approx 0.25$ eV, which is consistent with the limits from laboratory experiments. Taking into account the most stringent cosmological limit on neutrino masses $m_i \leq 0.14$ eV [24] would cut the afore-obtained upper bound of $v_L$ by roughly a factor of two.

The question arises whether one can implement the scenario under study within a GUT framework. A typical problem will be that, e.g., the triplet term giving rise to $M_L$ is associated with couplings that also contribute to the quark or charged lepton mass terms. Consider a renormalizable SO(10) theory with Higgs fields in the 10-plet and $\overline{126}$ representation. The relevant mass matrices in this case are given by [23]

\[
M_{up} = v_{10}^{up} Y_{10} + v_{126}^{up} Y_{126}, \quad M_{down} = v_{10}^{down} Y_{10} + v_{126}^{down} Y_{126},
\]

\[
M_D = v_{10}^{up} Y_{10} - 3v_{126}^{up} Y_{126}, \quad M_l = v_{10}^{down} Y_{10} - 3v_{126}^{down} Y_{126},
\]

\[
M_L = v_L Y_{126}, \quad M_R = v_R Y_{126},
\]

(14)

with the Yukawa coupling matrices $Y_{10,126}$ and the vevs $v_{10,126}^{up,down}$ for the up- and down-sector, respectively. To link this scenario with ours, $Y_{10}$ will have to correspond to the flavor democracy term. $Y_{126}$ will have to be this term plus a matrix proportional to the

\(^1\)We also find a very fine-tuned region in the parameter space of $(\delta_D, \delta_M, \varepsilon_D, \varepsilon_M)$, in which $|U_{e3}| \approx 0.1$ and $J_{CP} \leq 0$ hold. This possibility seems quite unlikely and can be disregarded.
unit matrix. To assure that the latter term does not significantly contribute to the quark and charged lepton masses, the condition \( v_{\text{up} \text{down}}^{126} \ll v_{\text{up} \text{down}}^{10} \) should be fulfilled. A detailed analysis of this situation is certainly interesting for the sake of model building [26], but it is beyond the scope of the present letter.

To summarize, we have combined the type-II seesaw mechanism with S(3) flavor symmetry and applied this idea to the neutrino phenomenology. Our starting point of view is that a Majorana neutrino mass matrix generally includes two terms allowed by S(3) symmetry, one being a purely democratic matrix and the other proportional to the unit matrix. As a consequence, for a conventional seesaw formula or a pure triplet term no large neutrino mixing can be generated. For both cases the term proportional to the democratic matrix has to be highly suppressed. We have shown here that the suppression of this term can naturally be realized via cancellations in the type-II seesaw scenario, from which the bilarge neutrino mixing pattern is in turn achievable. For the explicit symmetry breaking Ansatz discussed in this letter, we obtain a normal mass ordering, \( 0.005 \leq |U_{e3}| \leq 0.057 \) and \( 1 - \sin^2 2\theta_{23} \geq 0.005 \). Furthermore, we find \( J_{\text{CP}} \geq 0 \) and \( \langle m \rangle / v_L \geq 40\% \). These instructive results can be tested in a variety of forthcoming neutrino experiments.

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Figure 1: Scatter plot of the parameters $\zeta_\nu$ against $r_\nu$ as well as $1 - \sin^2 2\theta_{23}$ against $|U_{e3}|$, $|U_{e3}|$ against $J_{CP}$ and $1 - \sin^2 2\theta_{23}$ against $\langle m \rangle/v_L$ for the case of arbitrary $\zeta_\nu$ and $r_\nu$. 
Figure 2: Scatter plot of $\tan^2 \theta_{12}$ against $1 - \sin^2 2\theta_{23}$ for the case of $\zeta_\nu = 0$.  

Figure 3: Scatter plot of $1 - \sin^2 2\theta_{23}$ against $|U_{e3}|$ and $\langle m \rangle / \nu_L$ against $J_{CP}$ for the case of $\zeta_\nu = 1$. 