Cluster structure in light nuclei studied with few-body models

W Horiuchi and Y Suzuki
1 Department of Physics, Hokkaido University, Sapporo 060-0810, Japan
2 Department of Physics, Niigata University, Niigata 950-2181, Japan
3 RIKEN Nishina Center, RIKEN, Wako 351-0198, Japan
E-mail: whoriuchi@nucl.sci.hokudai.ac.jp

Abstract. We present our attempt at describing the low-lying states of \( A = 16 \) nuclei in a single scheme, a \(^{12}\text{C}\) core-plus-few-nucleon model. Promising results are obtained for \(^{16}\text{C}\) and \(^{16}\text{O}\), which suggest the coexistence of single-particle and cluster structure.

1. Introduction
Cluster structure is one of the unique and most interesting phenomena in nuclear many-body systems, and often appears in the excited spectrum of light nuclei. A famous and long-standing problem is the so-called mysterious \( 0^+ \) state in \(^{16}\text{O}\). In contrast to the shell-model-like ground state, the first excited \( 0^+ \) state is known to have a cluster structure of \(^{12}\text{C} + \alpha \) [1], which shows the coexistence of different structures in the spectrum but has not been reproduced microscopically even in modern large-scale calculations. A deep understanding requires a unified description. Since a fully microscopic calculation is not feasible at present, it is interesting to study \(^{16}\text{O}\) in a \(^{12}\text{C}\) core-plus-four-nucleon model, that is, without assuming the existence of an \( \alpha \) cluster.

In this contribution, we present our calculations in a \(^{12}\text{C} + 4\text{N}\) five-body model. In the next section, we briefly explain our approach and in Section 3 we present preliminary results for \(^{16}\text{O}\) as well as \(^{16}\text{C}\). A summary and the future prospects are outlined in Section 4.

2. A core-plus-few-nucleon model
The five-body system we consider here is characterized by the Hamiltonian which contains the nucleon-nucleon (\( N-N \)) potential for valence nucleons and the \(^{12}\text{C}\)-nucleon (\( C-N \)) potential. For the \( N-N \) force the central Minnesota (MN) potential [2] is employed, while for the \(^{12}\text{C}-N\) potential, a Woods-Saxon-plus-derivative form is assumed with parameters determined so as to reproduce the low-lying spectrum of \(^{13}\text{C}\) (\( 1/2^- \), \( 1/2^+ \) and \( 5/2^+ \) states). Our \( C-N \) potential is deep enough to accommodate some redundant or Pauli-forbidden states. To eliminate such states we impose the orthogonality constraint for the motion of the valence nucleons, which is practically achieved by adding a pseudopotential to the Hamiltonian [3]. We assume the \( 0s_{1/2} \) and \( 0p_{3/2} \) harmonic oscillator wave functions for the orbits occupied in \(^{12}\text{C}\).

We express the wave function with total angular momentum \( J \) as a linear combination of many basis states of the following form

\[
\Psi_{JM} = A \left\{ \left[ \Phi_{L\chi S} \right]_J \phi_I(^{12}\text{C}) \right\} \eta_{TM_T},
\]

(1)
where $A$ is an antisymmetrizer which only acts on the valence nucleons. The $\phi_{IM}(^{12}\text{C})$ is the core wave function but the core excitation is ignored here. The $\chi_{SM}(^{3}T_{2}M_{T})$ is the spin (isospin) function of the valence nucleons. For the orbital part, $\Phi_{L}$, we use a correlated Gaussian form reinforced with two global vectors, which enable us to obtain a precise solution of the many-body Schrödinger equation [4]. An advantage of this basis is that its functional form does not change under any coordinate transformation, and thus both cluster- and shell-model-like configurations can be expressed in a single scheme. We have applied this method not only to ground states but also to excited states [5]. In particular, the spectrum of $^{4}\text{He}$ is interesting as an analogue of the $^{16}\text{O}$ spectrum. Similarly to $^{16}\text{O}$, the first excited state of $^{4}\text{He}$ has $J^{\pi} = 0^{+}$ and has been shown to be a cluster state of the $3N + N$ ($^{3}\text{H} + p$ and $^{3}\text{He} + n$) configuration [5, 6].

The variational parameters are optimized following the stochastic variational method [7]. Assuming that we already have some basis states, the parameters of the next one are determined by the following procedure: we generate several candidates with randomly chosen parameters, and select the state which gives the lowest energy among them when included in the basis, and repeat this procedure until the energy is converged.

3. Results

Figure 1 displays the calculated energy for $^{16}\text{C}$ and the ground and first excited state of $^{16}\text{O}$ as functions of the number of basis functions. The energy curve of $^{16}\text{C}$ converged at 7000 basis states and the energy is very close to experiment. The $^{15}\text{C} + n$ threshold is also well reproduced at $-14.5$ MeV (Exp. $-14.3$ MeV). The energy curves of $^{16}\text{O}$ exhibit slower convergence than that of $^{16}\text{C}$ due to the difficulty of eliminating the forbidden states. The case of $^{16}\text{C}$ is easier than that of $^{16}\text{O}$ because the valence neutrons do not form a strongly bound cluster. When the valence nucleons form a cluster, the wave function would tend to have a large overlap with the forbidden states, and a large basis is required to ensure orthogonality. However, the preliminary results for $^{16}\text{O}$ are promising. The ground state appears below the $^{12}\text{C} + \alpha$ threshold and the first excited state appears just at a few MeV above the ground state, consistently with experiment. The calculation is still being in progress.

It is interesting to see how the cluster structure develops. Figure 2 displays the expectation values of the $\text{C} - \text{N}$ potential and of the $\text{N} - \text{N}$ potential acting between the valence nucleons, as functions of the basis dimension. For $^{16}\text{C}$, the contribution of the $\text{C} - \text{N}$ potential is very large compared to the $\text{N} - \text{N}$ contribution. The state can be regarded as a shell-model-like state, in which the valence nucleons move almost independently around the core.
structure, the two energies of $^{16}$C are almost independent of the basis. In contrast, the curves for the ground state of $^{16}$O change drastically in the region of 4000 to 6000 basis states, where the energy crosses the $^{12}$C+$\alpha$ threshold (See Fig. 1). We expect a phase transition from cluster-to shell-model-like structure at this basis size. Beyond 8000 basis states, where the energies are close to the convergence, the C–N contribution obtained is similar to that of $^{16}$C, whereas the N–N contribution is larger. The N–N energy is approximately half of the N–N contribution of the free alpha particle (−88.03 MeV) obtained with the MN potential [4]. In the ground state of $^{16}$O, the $\alpha$ particle is distorted by the attraction of the core nucleus as well as by the forbidden states. In the first excited state of $^{16}$O, the N–N contribution is dominant and is very close to that of the free alpha particle. This suggests that the first excited state has a well-developed $^{12}$C+$\alpha$ structure as predicted by the cluster model [1]. It should be noted that the expectation value of the C–N potential in the ground state is approximately equal to the N–N contribution in the first excited state of $^{16}$O. Thus two different aspects coexist in the states of $^{16}$O due to the balance of N–N and C–N contributions.

4. Summary and future work
We described the low-lying spectrum of $A=16$ systems with a $^{12}$C+four-nucleon model. Solving the five-body equation is very hard mainly because we face with the difficulty of removing the forbidden states. Our preliminary results for $^{16}$C and $^{16}$O are yet promising: the energies of $^{16}$C (0$^+$) and $^{16}$O (0$^+_1$, 0$^+_2$) are reproduced in a single scheme. The ground state of $^{16}$O seems to be a shell-model-like state, whereas the first excited state has a well-developed $^{12}$C+$\alpha$ structure. To see the clustering more quantitatively, we will calculate the $^{12}$C+$\alpha$ spectroscopic amplitude (factor). The calculation is still being in progress and will be reported elsewhere soon.

References
[1] Suzuki Y 1976 Prog. Theor. Phys. 55 1751 (1976); Prog. Theor. Phys. 56 111
[2] Thompson R, Lemere M and Tang Y C 1977 Nucl. Phys. A 286 53
[3] Kukulin V I and Pomerantsev V 1978 Ann. Phys. (N.Y.) 111 30
[4] Suzuki Y, Horiuchi W, Orabi M and Arai K 2008 Few-Body Syst. 42 33
[5] Horiuchi W and Suzuki Y 2008 Phys. Rev. C 78 034305
[6] Hiyama E, Gibson B F and Kamimura M 2004 Phys. Rev. C 70 031001
[7] Varga K and Suzuki Y 1995 Phys. Rev. C 52 2885