On the spectrum and polarization of magnetar flare emission

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Abstract. Bursts and flares are among the distinctive observational manifestations of magnetars, isolated neutron stars endowed with an ultra-strong magnetic field ($B \approx 10^{14} - 10^{15}$ G). It is believed that these events arise in a hot electron-positron plasma, injected in the magnetosphere, due to a magnetic field instability, which remains trapped within the closed magnetic field lines (the “trapped-fireball” model). We have developed a simple radiative transfer model to simulate magnetar flare emission in the case of a steady trapped fireball. We assume that magnetic Thomson scattering is the dominant source of opacity in the fireball medium, and neglect contributions from second-order radiative processes. The spectra we obtained in the 1–100 keV energy range are in broad agreement with those of available observations. The large degree of polarization ($\gtrsim 80\%$) predicted by our model should be easily measured by new-generation X-ray polarimeters, like IXPE, XIPE and eXTP, allowing one to confirm the model predictions.

1. Introduction
The most distinctive manifestation of magnetars, a particular kind of isolated neutron stars (NSs) observationally identified with Soft Gamma Repeaters (SGRs) and Anomalous X-ray pulsars (AXPs), is the emission of short ($\approx 10^{-2} - 1$ s) energetic ($\approx 10^{36} - 10^{41}$ erg) X-ray bursts and longer ($\approx 1 - 50$ s), even more energetic ($\approx 10^{41} - 10^{43}$) intermediate flares. Furthermore, three SGRs have been observed to also emit giant flares, the most powerful events ever observed from compact objects [1, 2, for reviews]. According to the magnetar model, first developed by [3], the strong internal magnetic field of magnetars is believed to develop a large toroidal component, able to exert a strong magnetic stress on the conductive star crust. In contrast to what happens in ‘normal’ NSs, where this force can be balanced by the rigidity of the crust, in the case of magnetars the internal stresses are strong enough to displace single surface elements. As a result, the external magnetic field acquires in turn a non-zero toroidal component, becoming twisted, and this makes possible for charged particles to fill the magnetosphere, streaming along the closed field lines [4, 5]. Although the mechanisms that trigger magnetar bursting activity are still not completely clear, [6, 7] suggested a model according to which magnetar flares originate in the sudden re-arrangements of the external field lines that follow the crustal displacements triggered by the strong internal field. This injects in the magnetosphere an Alfvén pulse that
dissipates into an electron-positron pair plasma, which remains magnetically confined within the closed field lines (the “trapped fireball”).

In this work we study the spectrum and polarization of the radiation emitted from a steady trapped fireball, providing simulations directly comparable with observations. Although our model relies on a series of simplifying assumptions, the simulated spectra we obtained in the 1–100 keV energy range, that can be suitably described in terms of the superposition of two blackbody components, turn out to be in broad agreement with the observations available so far [8, 9, 10]. Furthermore, the large degree of polarization expected for magnetar burst radiation can be easily measured by new-generation X-ray polarimeters, such as IXPE, XIPE and εXTP, allowing to confirm the model predictions. In section 2 we discuss the radiative processes which take place in the fireball medium, while in section 3 and 4 we deal with the problem of radiative transfer and describe the structure of our numerical codes. The results of our simulations are presented in section 5, while discussion and conclusions are reported in section 6.

2. Radiative processes in strong magnetic fields

We assume that magnetic scattering is the main source of opacity in the fireball medium. For the sake of simplicity, we consider scattering in the Thomson limit; the full case of Compton scattering will be addressed in a sequel work. In the presence of strong magnetic fields, photons emitted from the surface of a neutron star are expected to be linearly polarized in two normal modes, the ordinary (O) and the extraordinary (X), with the photon electric field oscillating either parallel or perpendicular to the \((\mathbf{k}, \mathbf{B})\) plane, with \(\mathbf{k}\) the photon propagation direction and \(\mathbf{B}\) the star magnetic field [11, 12, 13]. In the hypothesis (well verified for magnetar bursts) that the photon energy \(\varepsilon\) is much smaller than the electron cyclotron energy \(\varepsilon_B = m_e c^2 B / B_Q\), with \(m_e\) the electron mass, scattering of photons onto electrons/positrons is non-resonant. In the electron rest frame, and neglecting the electron recoil, the scattering cross-sections depend on the polarization state of both the ingoing and the outgoing photons [14, 15, 16]:

\[
\begin{align*}
\left[ \frac{d^2\sigma}{d\varepsilon'd\Omega} \right]_{\text{OO}} &= \frac{3}{8\pi} \sigma_T \left( 1 - \mu_B^2 \right) \left( 1 - \mu_{Bk}^2 \right) \delta, \\
\left[ \frac{d^2\sigma}{d\varepsilon'd\Omega} \right]_{\text{OX}} &= \frac{3}{8\pi} \sigma_T \left( \frac{\varepsilon}{\varepsilon_B} \right)^2 \mu_{Bk}^2 \nu_{Bk}^2 \delta, \\
\left[ \frac{d^2\sigma}{d\varepsilon'd\Omega} \right]_{\text{XO}} &= \frac{3}{8\pi} \sigma_T \left( \frac{\varepsilon}{\varepsilon_B} \right)^2 \mu_{Bk}^2 \nu_{Bk}^2 \delta, \\
\left[ \frac{d^2\sigma}{d\varepsilon'd\Omega} \right]_{\text{XX}} &= \frac{3}{8\pi} \sigma_T \left( \frac{\varepsilon}{\varepsilon_B} \right)^2 (1 - \nu_{Bk}^2) \delta,
\end{align*}
\]

(1)

where a prime labels the quantities after scattering, \(\sigma_T\) is the Thomson cross section, \(\mu_{Bk}\) is the cosine of the angle \(\theta_{Bk}\) between the photon direction and the local magnetic field, \(\nu_{Bk}\) is the cosine of the associated azimuth \(\phi_{Bk}\) and \(\delta = \delta(\varepsilon - \varepsilon')\) is the Dirac \(\delta\)-function. The previous expressions hold as far as the vacuum contributions in the dielectric tensor dominate over the plasma ones [17]. From equations (1) it appears clearly that all the cross-sections that involve X-mode photons are suppressed by a factor of \((\varepsilon / \varepsilon_B)^2 \propto (\varepsilon B_Q / B)^2\) with respect to the O–O cross-section, which is essentially of the order of \(\sigma_T\). This implies that the medium becomes optically thin for extraordinary photons at much larger Thomson depths with respect to the ordinary ones.

Among the additional second-order processes that can take place in magnetized plasma there are thermal bremsstrahlung, photon splitting and double-Compton scattering. The bremsstrahlung between particles with the same charge becomes important only for high particle energies [18, above 300 keV, outside the range we considered our work], so that the only relevant bremsstrahlung process turns out to be the electron-positron branch, slightly enhanced with respect to the electron-ion one [19, 20]. Also in this case, however, the amplitude turns out
to be negligible for magnetar burst radiation, allowing to neglect the contributions of thermal bremsstrahlung in our work. Moreover, although photon splitting can be important in a wider range of photon energies [21, 22], also in this case, the cross section of the process is much smaller than the Thomson cross section in the weak-field limit (and for photon energies $\lesssim$ 100 keV), while one should consider an additional suppression factor in the strong-field limit [23, 24]. For these reasons we can safely neglect also photon splitting effects in the model.

Therefore, double-Compton scattering turns out to be the unique process responsible for photon production in the fireball in our model. According to [25], for large scattering depths photons follow a Planck distribution at energies low enough to make double-Compton scattering dominant, while at higher energies scattering tends to establish a Bose-Einstein distribution.

However, as shown [26], the photon chemical potential remains rather small, allowing to solve the photon transport assuming local thermal equilibrium (LTE) at large depth for both O- and X-mode photons [27].

3. Radiative transfer in the fireball atmosphere
We solved the radiative transfer equations for both the ordinary and the extraordinary photons in the geometrically thin surface layers of the fireball, which we term the atmosphere. The latter is divided into a number of patches, each labelled by the intensity of the magnetic field at the patch centre and by the angle $\theta_B$ between $B$ and the local normal $z$. Once the viewing geometry is set, i.e. the angles $\chi$ and $\xi$ that the star rotation axis $\Omega$ makes with the observer line-of-sight (LOS) and the star magnetic axis, respectively, the contributions of all the patches in view are summed together to derive the spectral and polarization properties of the emitted radiation.

We also assume that the patch dimensions are small enough with respect to the radial scale to neglect the curvature of the closed field lines that contain the fireball, so that we can treat the atmosphere in the plane-parallel approximation. Under these assumptions the radiative transfer equations assume a relatively simple form,

$$\mu_z \frac{dn_i}{d\tau} = C_i n_i - S_i ,$$  

(2)

where $n_i$ are the photon number intensities ($i = O, X$), $d\tau$ is the infinitesimal Thomson depth, $\mu_z$ is the cosine of the angle between the photon direction and the local normal, $C_i$ are terms that do not depend on $\tau$ and $S_i$ represents the source terms. However, due to the suppression factor $B^{-2}$ that affects the X-mode cross sections (see equations 1), integrating the equations for the Thomson depth could require long computational times. In particular, the X-mode photosphere will lie at different heights in the fireball atmosphere for different photon energies, while at the same time the medium remains optically thick for ordinary photons at all the photon energies.

For these reasons, we adopted the approach used by [26], solving the radiative transfer equations in terms of the Rosseland mean optical depth $\tau_R$, which is related to the Rosseland mean cross section $\sigma_X = \sigma_{XX} + \sigma_{XO}$ of the X-mode photons.

4. Numerical implementation
We developed a specific FORTRAN code to solve the radiative transfer in the fireball atmosphere. We assumed a maximum optical depth $\tau_R^{\text{max}} = 1000$ at the base of the atmosphere, since this guarantees that the medium is optically thick for X-mode photons over the entire energy range 1–100 keV, up to $\tau_R^{\text{min}} = 0$ at the top. A good approximation for the temperature distribution in a scattering-dominated medium in the diffusion regime turns out to be $T = T_b(1+0.75\tau_R)^{1/2}$ [26], where $T_b$ is the bolometric temperature (we assumed $T_b = 10$ keV). We checked a posteriori this provides a good approximation to the numerical solution one obtains solving the energy balance in the fireball also at small optical depths. Using this temperature distribution, one can simply relate the Thomson scattering depths $\tau_O$ and $\tau_X$ for O- and X-mode photons, respectively, to
the Rosseland mean optical depth. In order to avoid too long runs, we decide to integrate the two radiative transfer equations for $\tau_{O,X} \leq 10$ only, taking $n_{O,X}$ as Planckian distributions at the local temperature $T$ otherwise. Since, for the chosen values of the parameters, the X-mode photosphere lies always at larger $\tau_R$ with respect to the O-mode one, we solve the transfer equation for $n_X$ only in the case of $\tau_X \leq 10$ and $\tau_O > 10$, taking $n_O$ as Planckian [26, 27]. For the sake of simplicity, we assume the external magnetic field as dipolar, with polar intensity $B_p = 10^{14}$ G; the fireball is then limited by the close field lines that reach a maximum distance $R_{\text{max}}$ from the star, taken as twice the stellar radius $R_{\text{NS}} = 10$ km. For this reason, all the patches in which the fireball atmosphere is divided are characterized by an angle $\theta_R = 90^\circ$.

In order to sum together the contributions coming from all the patches in view, we used an IDL ray-tracing code [27, 28, 29]. Once the viewing angles $\chi$ and $\xi$ have been chosen, the code solves the terminator condition $z \cdot \mathbf{\ell} = 0$, where $\mathbf{\ell}$ is the unit vector of the observer LOS, to determine the point of the fireball surface which enter into view at different rotational phases. Since in general the fireball terminator shape turns out to be quite complicate, we decide to neglect the general relativistic corrections in our code. Nevertheless, as we checked in previous works [28], general relativity marginally influences the behavior of the polarization observables at infinity, and the qualitative conclusions we discuss in the following hold in general. Finally, we considered two different emission geometries, the model a, in which photons come from the entire torus of the field lines that limit the trapped fireball, and the model b, in which the emitting region is only a portion of this torus. In this latter case, radiation emitted from the planar slices that limit the emitting part of the torus is also considered. QED effects on the polarization modes are treated as discussed in [28, 30]; in particular, since we assumed the vacuum contributions to the dielectric tensor as dominant with respect to the plasma ones, vacuum resonance effects turn out to be negligible in our model.

5. Results

The intensities calculated in the radiative transfer code and processed in the ray-tracer are then used to obtain the simulated spectra and the polarization observables (both phase-resolved and phase-averaged) as observed at infinity. Since observations of the ‘burst forest’ emitted by SGR 1900+14 in 2006 [8] suggest that the spectrum of the intermediate flares (and of normal bursts too) is thermal and well reproduced by the superposition of two blackbodies, we attempted to fit the phase-averaged spectra with two Planckian distributions (with temperatures $T_1$ and $T_2$ and normalizations $A_1$ and $A_2$), although comparing results from our simplified model with real observations could be premature. Figure 1 shows the phase-averaged total spectrum of the radiation emitted from the fireball in the case of a limited emitting region (model b) with angular opening $\Delta \phi = 90^\circ$, for $\chi = 60^\circ$ and $\xi = 30^\circ$, together with the best fit and the single blackbody components [27, for further details]. Under our assumptions, the values of $T_1$ and $T_2$ as well as of the ratio $A_2/A_1$ turn out to be in broad agreement with the observations, although the temperature of the softer component ($\sim 2$ keV) turns out to be somehow lower than the observed one ($\sim 5$ keV). No significant difference is expected switching from model a to model b or changing the viewing geometry. Moreover, we found that our model is able to reproduce the typical pulsating profile of the observed magnetar flare decay tails [27].

The observed polarization signal, simulated taking into account both vacuum polarization and the geometrical effects due to the magnetic field topology [28], confirms that, according to our model, radiation collected from a magnetar flare is expected to be highly polarized in the extraordinary mode. In particular, the phase-resolved linear polarization fraction $\Pi_L$ attains a value generally higher than 80 per cent for photon energies between 1–50 keV, while a decrease up to $\sim 40$ per cent is expected only at higher energies. This reflects the behavior of the scattering cross-sections (see equations 1), according to which X-mode photons dominate the spectrum at almost all the photon energies, where $\tau_X$ becomes comparable with $\tau_O$, above $\sim 50$ keV. Such
Figure 1. Phase-averaged photon spectrum (black solid line) of the radiation emitted in the case of model b (with $\Delta \phi = 90^\circ$), fitted by the superposition of two blackbody components for $\chi = 60^\circ$ and $\xi = 30^\circ$. The best fit is marked by the light-blue dashed line, and the single blackbody components at the temperatures $T_1$ (blue dotted line) and $T_2$ (red dotted line) are also shown.

A high polarization degree will be readily measurable by new-generation X-ray polarimeters like IXPE, XIPE and eXTP, as confirmed by the simulations reported in Figure 2. Here, we refer to an event characterized by an X-ray flux $F_X = 4.68 \times 10^{-7}$ erg cm$^{-2}$ s$^{-1}$ in the 1–10 keV energy range and a duration $t_{\text{exp}} = 1.737$ s, for different viewing geometries. It can be noted that both polarization fraction and angle measurements recover the values expected from the theoretical model with an acceptable degree of accuracy (within $1\sigma$), although the errors on the polarization angle increase by decreasing the corresponding polarization degree.

Figure 2. Simulation of the phase-averaged linear polarization fraction (triangles with error bars) of the XIPE (blue), IXPE (orange) and eXTP (green) polarimeters to the signal predicted by our theoretical model (in the case of model b) for a duration $t_{\text{exp}} = 1.737$ s, X-ray flux $F_X = 4.68 \times 10^{-7}$ erg cm$^{-2}$ s$^{-1}$ in the 1–10 keV energy range and different viewing geometries (red crosses).

6. Conclusions
We have revisited the problem of modelling the spectral and polarization properties of the radiation emitted during magnetar flares in the context of the trapped-fireball model [6, 7]. Our code integrates the radiative transfer equations for both ordinary and extraordinary photons in the fireball atmospheric layer, divided into a number of different patches. This model generalizes the approach presented in [26], where only the case of $B$ parallel to the patch normal was treated. The outputs of the radiative transfer code have then been reprocessed through a ray-tracing code, in order to obtain the spectra and the polarization observable distributions as measured by a distant observer. The results we obtained from the spectral analysis are compatible with observations, also in reproducing the typical pulse profile of magnetar flare decay tails. Finally,
the model predicts a high degree of polarization, that promises to be readily measured by the recent polarimetric missions under development, allowing to verify the predictions of our theoretical model.

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