Chiral-invariant CP-violating Effective Interactions in Z Decays to three Jets\footnote{Research supported by BMBF, contract No. 05 6HD 93P(6)}

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Abstract:
Tests of CP violation by appropriate momentum correlations in $Z \rightarrow 3$ jets and in particular in $Z \rightarrow b\bar{b}X$ probe CP-violating effective couplings – that manifest themselves as form factors – which conserve the quark chirality and quark flavour. By giving two examples we show that such couplings can be induced at one-loop order in extensions of the Standard Model with CP violation beyond the Kobayashi-Maskawa phase. In one of the models we compute the chirality-conserving part of the CP-violating $Zb\bar{b}$-gluon amplitude for massless $b$ quarks, determine the resulting effective dimension $d = 6$ couplings in the local limit, and discuss the possible size of the effects. Finally we show that in models with excited quarks the chiral-invariant CP-violating effective interactions could be quite large if appropriate couplings are of a size characteristic of a strong interaction.
1 Introduction

The large number of Z boson events at LEP1 and at the SLAC linear collider have provided precision tests of the Standard Model (SM), and allow for searches of new physics effects. In [1, 2, 3] it was proposed to use the reactions $Z \to n \geq 3$ jets and in particular $Z \to b \bar{b}X$ for tests of CP nonconservation beyond the one induced by the phase of the Kobayashi-Maskawa (KM) matrix [4]. Related proposals were made in [5, 6, 7, 8].

CP-violating interactions in $Z \to b \bar{b}X$ would affect at the parton level correlations among parton energies/momenta and parton spins. While the partonic momentum directions are reconstructed from the jet directions of flight the spin-polarization of the $b$ quark (and of lighter quarks) cannot, in general, be determined with reliable precision due to fragmentation. As far as CP tests in the above reactions are concerned this implies that useful observables are primarily those which originate from CP-odd partonic momentum correlations. With these correlations only chirality-conserving effective couplings (which do not flip the quark helicity) can be probed with reasonable sensitivity [1, 2, 3]. This situation is in contrast to $\tau^+\tau^-$ and $t\bar{t}$ production where the fermion polarizations can be traced in the decays. With regard to CP tests [9] this circumstance allows to search in $Z \to \tau^+\tau^-$ for a CP-violating dipole form factor of the $\tau$ [10, 11] which is chirality-flipping.

In the framework of a manifestly $SU(2)_L \times U(1)_Y$ gauge-invariant effective Lagrangian approach it was shown in [12] that CP-violating interactions of dimension $d = 6$ (after symmetry breaking) that are chirality-conserving and flavour-diagonal can exist with couplings which are a priori not related to the couplings of the $d = 5$ dipole interactions. In renormalizable theories these couplings can only be due to quantum corrections. In this letter we show that there are extensions of the SM with CP violation beyond the KM phase where such “couplings” – that is to say, form factors – are induced at one-loop order in the $Zb\bar{b}$-gluon amplitude even for vanishing $b$ quark mass. In one of the models discussed below we determine the resulting effective couplings in the “local limit” where the masses of the particles in the loop are much larger than the external energies and discuss the possible strength of these couplings.

Finally we make an estimate for models with excited quarks and show that one can obtain rather large CP-violating and chirality-conserving effective interactions relevant for $Z \to b\bar{b}X$. The price one has to pay is the introduction of a new type of strong interactions for quarks.

2 Parameterization of CP violation in $Z \to b + \bar{b} + G$

We consider the amplitude for the following partonic reaction at the $Z$ resonance:

$$e^+ + e^- \to Z(p) \to b(p_1) + b(p_2) + G(k) ,$$

(1)
where \( G \) denotes a gluon and the letters in brackets denote the four-momenta of the particles. The amplitude (1) may be affected by CP-violating interactions. In the effective Lagrangian approach these interactions are parameterized by the couplings of local operators composed of the \( Z, b, \bar{b}, \) and gluon fields. Including terms of operator dimension \( d \leq 6 \) (counted after spontaneous breaking of the electroweak symmetry) the effective CP-violating Lagrangian relevant for (1) contains helicity-flipping \( d = 5 \) electric, chromoelectric, and weak dipole moment operators and helicity-conserving \( d = 6 \) operators. Here we are interested only in the latter terms which read \([1, 12]\)

\[
\mathcal{L}_{\text{CP}}^{d=6}(x) = \bar{b}(x) T^a \gamma^\mu [h_{Vb} + h_{Ab} \gamma_5] b(x) Z^\mu (x) G^a_{\mu \nu}(x),
\]

where \( T^a \) are the generators of \( SU(3)_c \) in the fundamental representation, \( G^a_{\mu \nu} \) denotes the gluonic field strength tensor, and \( h_{Vb,Ab} \) are real coupling constants of mass dimension \(-2\). The interactions (2) induce a CP-odd term in the amplitude of (1). Interference with the CP-even SM terms leads to CP-odd correlations among the momenta and momentum directions of the \( b, \bar{b}, \) and gluon. The momenta and in particular the momentum directions are given to good accuracy by the corresponding jet variables. Detailed studies were made in \([1, 2, 3]\). These CP-odd partonic and jet correlations, respectively, are not suppressed by (powers of) \( m_b/\sqrt{s} \). (Here \( m_b \) denotes the \( b \) quark mass and \( \sqrt{s} \) is the c.m. energy.) It is convenient to work with dimensionless coupling constants \( \hat{h}_{Vb,Ab} \) which are defined by

\[
\hat{h}_{Vb,Ab} = \frac{e g_s}{\sin \theta_W \cos \theta_W m_Z^2} h_{Vb,Ab},
\]

Here \( e > 0 \) denotes the positron charge, \( g_s \) is the QCD coupling constant, and \( m_Z \) is the \( Z \) boson mass.

Alternatively one may parameterize CP-violating effects in (1) by performing a form factor decomposition of the amplitude (cf. \([8]\) for an analysis of \( Z \rightarrow bb\gamma \) which also applies to (1)). For some remarks concerning the relation and the respective advantages of the effective Lagrangian and the form factor approaches we refer to \([13]\). Restriction to on-shell \( Z \) decays, massless \( b \) quarks, and imposition of \( SU(3)_c \) gauge invariance implies that the \( Zb\bar{b}G \) vertex function contains six pairs of chirality-conserving form factors \( (h^{(i)}_V + h^{(i)}_A \gamma_5), \) \( i = 1 \ldots 6 \). One obtains for the CP-violating part of the \( T \)-matrix element:

\[
 i \mathcal{T}_{CP} = e_Z^\mu e_G^\nu \bar{u}(p_2) T^a \Lambda_{\mu \nu} u(p_1),
\]

where

\[
\Lambda_{\mu \nu} = (k g_{\mu \nu} - k_{\mu} \gamma_{\nu} + h^{(1)}_V + h^{(1)}_A \gamma_5 + r_{\mu \nu})
\]

and \( e_Z^\mu \) \((e_G^\mu)\) is the polarization vector of the \( Z \) boson (gluon) in ordinary (and colour) space. The pair of form factors exhibited in (5) has mass dimension \(-2\) and its accompanying Lorentz structure is identical to the structure induced by (2). The remainder \( r_{\mu \nu} \) contains other CP-odd form factor pairs which have mass dimension \(-4\), i.e., their Lorentz structure contains two more powers of external momenta \([8]\). In the local limit (cf. below) these form factors correspond to dimension \( d \geq 8 \) interactions, whereas \( h^{(1)}_V, h^{(1)}_A \) become identical to the couplings \( h_{Vb}, h_{Ab} \), respectively, which are defined in (2).
### 3 Extended Higgs models

We now discuss extensions of the standard electroweak theory in which chirality-conserving and CP-violating terms – in particular terms with a structure as described by (2), (3) – are generated in the $Zb\bar{b}G$ amplitude at one-loop order. The simplest possibility is the extension of the Standard Model by an additional Higgs doublet, which allows for CP violation in the neutral Higgs sector [14]. For 2-Higgs doublet models with natural flavour conservation at the tree level [15] CP-violating neutral Higgs boson exchange at one-loop order leads to chirality-conserving structures in the above amplitude [16]. However, these terms are proportional to $m_b^2$. (For instance one factor of $m_b$ comes from the chirality-flipping Higgs boson coupling, the other one from the mass term necessary to flip the chirality back.) Thus the resulting effects at the $Z$ peak are uninterestingly small.

Effects which do not vanish for $m_b \to 0$ are possible for instance in models with $n > 2$ charged Higgs bosons and intrinsic CP violation in the charged Higgs sector (resulting from CP violation in the charged mixing matrix; i.e., the scalar potential). Moreover non-diagonal $ZH^+ H^-_j$ couplings are required. They can appear if there are Higgs representations other than doublets. For definiteness we consider an extension of the standard $SU(2)_L \times U(1)_Y$ electroweak theory with three $SU(2)$ Higgs doublets and one singlet with $Y = 1$. The physical particle spectrum of the model contains three charged Higgs bosons $H_{1,2,3}^\pm$ with an off-diagonal interaction with $Z$ bosons being of the generic form

$$L_{ZH_{1,2}H_2} = -i \frac{e}{\sin \theta_W \cos \theta_W} \kappa_{12} Z^\mu \partial_\mu H_2^- - (\partial_\mu H_1^+) H_2^- \right] + \text{h.c.} \quad (6)$$

and similar terms with $H_1, H_3$ and $H_2, H_3$. Here $\kappa_{12}$ is a real parameter and $H_{1,2,3}^+$ denotes a physical charged Higgs boson in the mass basis. The interaction of one of these bosons with $t$ and $b$ quarks reads

$$L_{H_{1,2}tb} = -(2\sqrt{2}G_F)^{1/2}(\alpha_i m_b \bar{t}L b_R + \beta_i m_t \bar{t}R b_L) H_i^+ + \text{h.c.} \quad (7)$$

Here $G_F$ denotes the Fermi constant and $q_{R,L} = \frac{1}{2}(1 \pm \gamma_5)q$. Due to CP-violating charged Higgs boson mixing the phases of the complex numbers $\alpha_i, \beta_i$ differ from the KM phase, and in general $\text{Im}(\alpha_i \beta_i^*) \neq 0$. For generating non-zero form factors (3) in the limit of massless $b$ quarks $\text{Im}(\beta_j \beta_k^*) \neq 0 \ (j \neq k)$ is required. This is possible if $t_R$ has complex Yukawa couplings to more than two Higgs doublets. Such couplings are not excluded for the third generation fermions. A detailed discussion of this model – which we shall call model I for short – will be given elsewhere.

The interactions (3), (4) generate the matrix element (4) at one-loop order if $\text{Im}(\beta_j \beta_k^*) \neq 0$ for $j \neq k$ and if the Higgs masses differ from each other. The diagrams of Fig. 1 depict the left-handed contribution to the vertex function which survives the limit $m_b \to 0$. The Yukawa interaction (7) leads also to a right-handed vertex function which is, however, proportional to $m_b^2$ and therefore uninteresting. Putting the intermediate $Z$ boson on-shell and the mass of the $b$ quark to zero,

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$^3$Since terms proportional to $p_\mu$ are irrelevant for the reaction (1), the result given below applies also to off-shell $Z$ bosons.
we find that the left-handed vertex function represented by Figs. 1a, b contains all
the CP-violating form factors of mass dimension $-2$ and $-4$ mentioned above. We
are interested here only in the dimension $-2$ form factors for which we obtain the
following contribution from the $ZH_1H_2$ coupling in (6):
\[
h^{(1)}_V = -h_A^{(1)} = \text{Im}(\beta_1\beta_2^*)\sqrt{2}G_F\frac{eg_sK_{12}}{16\pi^2\sin\theta_W\cos\theta_W}f_{12}. \tag{8}
\]

The amplitude $f_{12}$ is given as a function of the rescaled $b$ and $\bar{b}$ energies (in the $e^+e^-$
c.m. frame) $x = 2E_b/\sqrt{s}$, $\bar{x} = 2E_{\bar{b}}/\sqrt{s}$ by
\[
f_{12}(x, \bar{x}) = \frac{m_t^2}{s} \int_0^1 dx_1 \int_0^{1-x_1} dx_2 [I_a(x, \bar{x}, x_1, x_2) + I_b(x, \bar{x}, x_1, x_2)] . \tag{9}
\]

With
\[
\rho(m_1, m_2) \equiv \frac{x_1 m_2^2 + x_2 m_1^2 + (1 - x_1 - x_2)m_t^2}{s}, \tag{10}
\]
where $m_{1,2}$ denote the charged Higgs masses, we have
\[
I_a = \frac{2x_1x_2(1 - x - \bar{x}) - x_2(1 - x) - x_1(1 - \bar{x}) + 2\rho(m_1, m_2)}{[x_2(1 - x) - x_1(1 - \bar{x})]^2}
\times \ln \left[ \frac{\rho(m_1, m_2) - x_2(1 - x_2)(1 - x) - x_1 x_2 x}{\rho(m_1, m_2) - x_1(1 - x_1)(1 - \bar{x}) - x_1 x_2 \bar{x}} \right] \tag{11}
\]
- $(m_1 \leftrightarrow m_2)$

and

\[
I_b = \frac{\ln [\rho(m_1, m_2) - x_1 x_2 - x_1(1 - x_1 - x_2)(1 - \bar{x})]}{1 - \bar{x}}
- \frac{\ln [\rho(m_1, m_2) - x_1 x_2 + (1 - x_2)(1 - x_1 - x_2)(1 - x)]}{1 - x} \tag{12}
- $(m_1 \leftrightarrow m_2)$.

In (8) $I_a$ ($I_b$) is the contribution from the one particle irreducible (reducible) dia-
gram(s) in Fig. 1. Correspondingly, $I_b$ could in principle have poles at $x = 1$ and
\(\bar{x} = 1\). However, in (12) these pole terms cancel when the contribution $(m_1 \leftrightarrow m_2)$
is subtracted. Thus, $h_V^{(1)}$, $h_A^{(1)}$ in (8) are regular for $x \to 1$, $\bar{x} \to 1$ and in the local
limit discussed below they get equal contributions from the diagrams (a) and (b) of
Fig. 1.

The form factor $h_V^{(1)}$ does not show any pronounced variation as a function of $x,$
$\bar{x}$ in the kinematically allowed range. In order to make contact with the effective
Lagrangian approach we assume that the particle masses in the loops of Fig 1a, b are
much larger than the external momenta involved. This local limit can formally
be realized by assuming $s \ll m_t^2, m_1^2, m_2^2$ in (3). With $r_{1,2} \equiv m_{1,2}^2/m_t^2$ we obtain then
\[ f_{12}(x, \bar{x}) \rightarrow \hat{f}(r_1, r_2) = \frac{2}{3(r_1 - r_2)} \left\{ -\frac{2[(r_1 - r_2)^2 + r_1r_2(1 - r_1)(1 - r_2)]}{(1 - r_1)^2(1 - r_2)^2} \right. \\
- \frac{r_1^2 \ln(r_1)(r_1 - 3r_2 + r_1r_2 + r_2^2)}{(1 - r_1)^3(r_1 - r_2)} + \frac{r_2^2 \ln(r_2)(r_2 - 3r_1 + r_1r_2 + r_1^2)}{(1 - r_2)^3(r_1 - r_2)} \right\} . \quad (13) \]

The function \( \hat{f}(r_1, r_2) \) is an antisymmetric and regular function of the mass ratios \( r_{1,2} \). The maximal value of its modulus is \(|\hat{f}(1, 0)| = |\hat{f}(0, 1)| = 1/9\). From Fig. 2 we see that \(|\hat{f}(r_1, r_2)| \simeq 0.07\) if one Higgs mass is around 90 GeV and the other one is around 300 GeV or larger. Furthermore a numerical study shows that replacing, for a given set of particle masses, the form factor function \( f_{12}(x, \bar{x}) \) by its local limit \( \hat{f}(r_1, r_2) \) is a very good approximation in the whole kinematic range of \( x, \bar{x} \).

We can now estimate the strength of the effective interaction (2). It is characterized by the dimensionless couplings (3) for which we get from (8)–(13) after summing the contributions of all pairs \( H_j, H_k \ (j \neq k) \)

\[ \hat{h}_{Vb} = -\hat{h}_{Ab} = \sum_{1 \leq j < k \leq 3} \sqrt{2}G_Fm_Z^2 \frac{\kappa_{jk} \text{Im}(\beta_j^* \beta_k)}{16\pi^2} \hat{f}(r_j, r_k) . \quad (14) \]

How large could \( \hat{h}_{Vb,Ab} \) possibly be? The parameters \( \kappa_{jk} \) will in general not exceed values of order one. If the couplings of the charged Higgs bosons to the right-handed top quark are substantially enhanced, \(|\beta_j| = \mathcal{O}(1)\), then \( \hat{h}_{Vb,Ab} \) can reach the percent level.

Models of another class in which a non-zero matrix element (4) can be induced at one-loop order are \( SU(2)_L \times U(1)_Y \) gauge theories with exotic Higgs representations. It is known [17,18] that models with Higgs boson multiplets other than doublets or singlets can have tree level \( H^\pm W^\mp Z \) couplings (\( H^+ \) denotes again a physical charged Higgs boson in the mass basis) which may be parameterized as follows [18]:

\[ \mathcal{L}_{HWZ} = -\frac{e}{\sin \theta_W \cos \theta_W} m_Z \xi \mathcal{W}^\mu W^- + h.c. \quad (15) \]

Here \( \xi \) is a real parameter which depends on the vacuum expectation values and on the quantum numbers of the scalar fields. It should be noted that the experimental constraint of the \( \rho \) parameter being close to 1 does not imply that \( \xi \) must be very small. In these models there is usually more than one singly charged physical Higgs state. Depending on the scalar potential there can be CP-violating mixing of these states. Then the couplings of the charged Higgs bosons to quarks can have a CP-violating phase being different from the KM phase. The interaction of one of these bosons \( H^+ \) is of the generic form (7) with complex couplings \( \alpha, \beta \).

If \( \text{Im}(\beta V_{tb}^*) \neq 0 \), where \( V_{tb} \) is the KM mixing matrix element, these interactions \( \hat{h}_{Vb,Ab} \) which we call model II generate the left-handed amplitude depicted in Fig. 3a, b with CP-violating form factors which remain non-zero for \( m_t = 0 \). In the local limit \( s/m_t^2 \rightarrow 0 \ (i = H, W, t) \) we obtain for the dimensionless couplings (3)

\[ \hat{h}_{Vb} = -\hat{h}_{Ab} = \sqrt{2}G_Fm_Z^2 \frac{\xi \text{Im}(\beta V_{tb}^*)}{16\pi^2} \hat{f} , \quad (16) \]
where \( \tilde{f} \) is the local limit of the form factor function associated with \( h^{(1)} \). It is of the same order of magnitude as \( f \) in (13). We may expect the absolute value of \( \tilde{h}_{Vb} \) evaluated in model II to be smaller than the one of \( h_{Vb} \) in model I because for enhanced Yukawa coupling \( \beta \) the coupling \( (13) \) grows only linearly with \( \beta \).

In this section we have discussed in detail only the form factors \( h^{(1)}_V, h^{(1)}_A \) and their local limits. When comparing the predictions of these models with experimental data, the other CP-violating form factors of mass dimension \(-4\) should also be taken into account. In this note our intention was to show that in the context of spontaneously broken \( SU(2)_L \times U(1)_Y \) gauge theories with CP violation beyond the KM phase there can be CP-violating contributions to \( Z \rightarrow b + \bar{b} + G \) at one-loop approximation which conserve the quark chirality and quark flavour. In the (possibly hypothetical) limit where the particle masses in the loop are much larger than the external momenta these contributions lead in particular to dimension \( d = 6 \) chiral-invariant effective quark gauge-boson interactions. The corresponding couplings are not suppressed by small quark masses.

## 4 Models with excited quarks

Let us assume that there exist excited quarks (cf. e.g. [19]). This would be natural in a scenario where quarks have substructure and participate in a new type of strong interaction. In particular, we assume that \( b \) quarks have excited partners \( b' \), which could have spin \( \frac{1}{2} \) or \( \frac{3}{2} \). For simplicity we consider a \( b' \) of spin \( \frac{1}{2} \) and mass \( m_{b'} \). Due to colour gauge invariance we expect the \( b'b \) Gluon couplings to be chirality-flipping dipole couplings. Because weak \( SU(2) \) gauge invariance is broken at the scale of LEP energies chirality-conserving \( Zb'b \) couplings are a priori possible. For the sake of demonstrating that couplings of the form given in (2) can be generated we consider the following effective interactions of \( b' \) to \( b \) quarks, \( Z \) bosons and gluons:

\[
\mathcal{L}' = -\frac{e}{2 \sin \theta_W \cos \theta_W} Z \mu b' \gamma^\mu (g'_V - g'_A \gamma_5) b - i \frac{g_s}{2m_{b'}} \hat{d}_c g'_c \sigma^{\mu \nu} \gamma_5 T^a b C_{\mu \nu}^a + \text{h.c.} \quad (17)
\]

Here \( g'_V, g'_A \) and \( \hat{d}_c \) are complex parameters, which can be expected to be of order one if the underlying dynamics is strongly interacting. In addition to \( \hat{d}_c \), the chromoelectric dipole transition form factor \( b \rightarrow b' \), there will be also a chromomagnetic transition form factor \( \hat{d}_m \) which we omit for brevity.

It is a simple matter to calculate \( \hat{h}_{Vb,Ab} \) in this type of model from the diagrams of Fig. 4. For \( m_{b'} \gg m_Z \) we get

\[
\hat{h}_{Vb} = \frac{m_Z^2}{m_{b'}^2} \text{Re}(\hat{d}_c g'_c),
\]

\[
\hat{h}_{Ab} = -\frac{m_Z^2}{m_{b'}^2} \text{Re}(\hat{d}_c g'_c). \quad (18)
\]

If \( m_{b'} \) is not too far away from \( m_Z \) the couplings \( \hat{h}_{Vb,Ab} \) get somewhat enhanced due to the propagator of the virtual \( b' \) quark in Fig. 4. In addition the contribution of
the chromomagnetic transition coupling $\hat{d}_m$ to $\hat{h}_{Vb}$, $\hat{h}_{Ab}$ is proportional to $\text{Im}(\hat{d}_m g'_{V})$ and $\text{Im}(\hat{d}_m g'_{A})$, respectively. Thus we conclude that in this model $\hat{h}_{Vb,Ab} = \mathcal{O}(1)$ may be possible if the mass of the $b'$ quark is not too far away from the $Z$ mass. In [20] the lower limit $m_{q'} > 540$ GeV on the masses of excited quarks was published, but this applies to excited $u$ and $d$ quarks only and does not exclude a lighter $b'$ quark.

In [3] it was pointed out that the CP-odd couplings (2), which enhance the $Z \rightarrow b\bar{b}G$ decay rate, would explain the discrepancy of the experimental value of $R_b = \Gamma(Z \rightarrow b\bar{b}X)/\Gamma(Z \rightarrow \text{hadrons})$ with the corresponding theoretical SM value (cf. [21]) if they were of order one. Our present study indicates that couplings of this order of magnitude might be generated if there are sufficiently light excited $b$ quarks. Of course, $R_b$ receives not only incoherent contributions from $\hat{h}_{Vb,Ab}$ but also coherent ones from the interference of CP-invariant contributions induced by (17) with the SM contributions to the amplitude of $Z \rightarrow b\bar{b}G$. These coherent contributions to $R_b$ could be more important than the incoherent ones and they may also be positive.

We add a remark concerning $b'$ production at the Tevatron collider. Depending on the $b'$ mass, the couplings (17) may lead to anomalous $b\bar{b}'$ production, that is, anomalous $bbG$ production with subsequent high $p_T$ dileptons from semileptonic $b$ and $\bar{b}$ decay.

5 Conclusions

We have discussed in this letter some ways to generate chiral-invariant CP-violating couplings relevant for the decay $Z \rightarrow b\bar{b}G$. We found that models with extra charged Higgs particles can induce such couplings which survive the $m_b \rightarrow 0$ limit. The resulting dimensionless coupling constants $\hat{h}_{Vb,Ab}$ were estimated to be at most at the per cent level for realistic Higgs masses and Yukawa couplings. Larger couplings $\hat{h}_{Vb,Ab}$ can be obtained in models with an excited $b$ quark, $b'$, with a mass not too far away from the $Z$ boson mass.

We think that further experimental investigations of $Z \rightarrow b\bar{b}G$ (and also $Z \rightarrow b\bar{b}\gamma$) should be a very worthwhile undertaking. First studies in this direction have already been presented for $Z \rightarrow b\bar{b}G$ in [22, 23, 24].

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References

[1] W. Bernreuther, U. Löw, J. P. Ma and O. Nachtmann, Z. Phys. C43 (1989) 117.

[2] J. Körner, J. P. Ma, R. Münch, O. Nachtmann and R. Schöpf, Z. Phys. C49 (1991) 447.

[3] W. Bernreuther, G. W. Botz, D. Bruß, P. Haberl and O. Nachtmann, Z. Phys. C68 (1995) 73.

[4] M. Kobayashi and T. Maskawa, Prog. Theor. Phys. 49 (1973) 652.

[5] J. F. Donoghue and G. Valencia, Phys. Rev. Lett. 58 (1987) 451; ibid. 60 (1988) 243 (E).

[6] J. Bernabéu and M. B. Gavela, in: CP Violation, ed. by C. Jarlskog (World Scientific, Singapore 1989);
M. B. Gavela, F. Iddir, A. Le Yaouanc, L. Oliver, O. Pène and J. C. Raynal, Phys. Rev. D39 (1989) 1870.

[7] G. Valencia and A. Soni, Phys. Lett. B263 (1991) 517.

[8] K. J. Abraham and B. Lampe, Phys. Lett. B326 (1994) 175.

[9] W. Bernreuther and O. Nachtmann, Phys. Rev. Lett. 63 (1989) 2787;
W. Bernreuther, G. W. Botz, O. Nachtmann and P. Overmann, Z. Phys. C52 (1991) 567.

[10] P. D. Acton et al. (OPAL collab.), Phys. Lett. B281 (1992) 405;
R. Akers et al. (OPAL collab.), Z. Phys. C66 (1995) 31.

[11] D. Buskulic et al. (ALEPH collab.), Phys. Lett. B297 (1992) 459;
D. Buskulic et al. (ALEPH collab.), Phys. Lett. B346 (1995) 371.

[12] W. Bernreuther and O. Nachtmann, Phys. Lett. B268 (1991) 424.

[13] W. Bernreuther and O. Nachtmann, “Some remarks on the search for CP violation in Z decays”, hep-ph/9603331, to be published in Z. Phys. C.

[14] T. D. Lee, Phys. Rev. D8 (1973) 1226.

[15] S. Weinberg, Phys. Rev. D42 (1990) 860.

[16] T. Schröder, diploma thesis, Universität Heidelberg (1990) unpublished;
S. Bar-Shalom et al., preprint SLAC-PUB-95-6765 (1995).

[17] J. A. Grifols and A. Méndez, Phys. Rev. D22 (1980) 1725.

[18] R. S. Chivukula and H. Georgi, Phys. Lett. 182B (1986) 181;
M. S. Chanowitz and M. Golden, Phys. Lett. 165B (1985) 105;
J. F. Gunion, R. Vega and J. Wudka, Phys. Rev. D42 (1990) 1673.

[19] U. Baur, I. Hinchcliffe and D. Zeppenfeld, Int. J. Mod. Phys. A2 (1987) 1285;
U. Baur, M. Spira and P. Zerwas, Phys. Rev. D42 (1990) 815.

[20] F. Abe et al. (CDF collab.), Phys. Rev. Lett 72 (1994) 3004.
[21] The LEP Collaborations ALEPH, DELPHI, L3, OPAL and the LEP Electroweak Working Group, “A Combination of Preliminary LEP Electroweak Measurements and Constraints on the Standard Model”, CERN-PPE/95-172 (1995).

[22] M. Steiert, “Suche nach CP-verletzenden Effekten in hadronischen 3-Jet Erignissen mit bottom Flavour, $Z^0 \rightarrow b\bar{b}G$”, diploma thesis, Univ. of Heidelberg (unpublished); M. Steiert, J. von Krogh, talk presented at the spring meeting of the German Physical Society, Karlsruhe 1995.

[23] S. Dhamotharan, “Suche nach CP-Verletzung in $Z \rightarrow b\bar{b}g$ Zerfällen mit dem Detektor ALEPH am LEP”, diploma thesis, Univ. of Heidelberg (unpublished).

[24] D. Buskulic et al. (ALEPH collab.), “Search for CP Violation in the Decay $Z \rightarrow b\bar{b}g$”, CERN-PPE/96-71 (1996)
Figure Captions

Fig. 1: CP-violating contributions to the $ZbbG$ amplitude in model I for massless $b$ quarks. Permutated diagrams and diagrams where the gluon is emitted from the $b$ quark are not drawn.

Fig. 2: Dependence of $\hat{f}_{12}$ defined in eq. (13) as a function of the charged Higgs masses $m_1$ and $m_2$. The top mass is chosen to be $m_t = 175$ GeV.

Fig. 3: CP-violating contributions to the $ZbbG$ amplitude in model II for massless $b$ quarks.

Fig. 4: Contribution to $ZbbG$ from an excited quark $b'$. The permuted diagram is not shown.
Fig. 2
Fig. 3

Fig. 4