Negative group velocity waves in rectilinear thin wires array

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Abstract. A well-known metamaterial consisting of rectilinear thin wires array forming a 2D-square lattice within the non-conductive matrix was theoretically investigated in this paper. The frequency dependencies of wavelength of smoothed electromagnetic field distributions inside the wired structure, show that this material has a wide band in which there are negative group velocity waves. This effect is clearly observed in the case when the wavelength of the smoothed distribution of the electromagnetic field is much longer than the distance between the wires. For this case, also figures of field distribution from point source were modeled. They show that negative refraction and focusing of electromagnetic radiation observed, ie. the studied material is left-handed. Unlike most of the left-handed metamaterials that are composed of two separate inter-elements (typically, a split ring and a rod), in this paper it is proposed to use only one type of inclusion, namely the conductive rod.

1. Introduction

When electrical permittivity $\epsilon(\omega)$ and magnetic permeability $\mu(\omega)$ are both simultaneously negative, the direction of the phase velocity is opposite to the direction of energy current. Thus, the directions of the field vectors $\mathbf{E}$ and $\mathbf{H}$, and the direction of the propagation wave vector $\mathbf{k}$ form a left-handed coordinate system. Therefore, these media are left-handed media (LHM) [1]. Furthermore, the index of refraction $n(\omega)$ used in the interpretation of Maxwell’s equations should be taken as the negative square root as following $n = -\sqrt{\epsilon(\omega)\mu(\omega)}$. In [2, 3, 4] to create a LHM it was proposed to use the structures consisting of miniature split ring resonators, acting as magnetic dipoles, and straight segments of wire. Structures proposed in [5] were made of only one S-shaped element which yields an overlapping negative permittivity and negative permeability response over a frequency band of about 2.6 GHz. In this paper, we assume that a metamaterial consisting of an array of only conductive wires forming a square lattice within a non-conducting matrix can be the LHM. Similar structures were considered in [6, 7, 8]. In [6], effective permittivity at low frequencies was analytically determined, and, in [7], bandgaps for wire-medium were analytically obtained. However, currently, there are no papers containing such data as frequency dependencies of wavelength of smoothed electromagnetic field distributions (SEFD) $-\lambda_{se,fd}$ inside the wired structure at $0.65 < a/\lambda_m < 1$ ($\lambda_m$ - a wavelength of an electromagnetic field in the non-conducting matrix, $a$ - the distance between the wires).
Figure 1. The geometry of a model and electric field distribution in two 2D unit cells. The color change corresponds to a change in the electric field intensity along the $OY$ axis for an arbitrary frequency ($E_y$).

and a statement that array of wires can be a left-handed material on its own, where wavelength of SEFD much more distance between the wires.

2. Model of composite material based on a wire-structure
Let us consider a composite structure consisting of an array of rectilinear conductive wires (Fig. 1) forming a square lattice in a non-conducting matrix. The distance between the wires ($a$) and their length is much greater than the radius ($r$). Such a structure was previously studied in [6, 7, 8]. A response of the structure under investigation (Fig. 1) to external electromagnetic radiation depends on the direction and phase of this radiation. In the present paper the case when electric field $E$ is parallel to the axis of a wire (Fig. 1) is considered. Let an incident electromagnetic wave propagates along axis $z$, and the vector of electric field be parallel to axis $y$. Thus, the model shown in Fig. 1 corresponds to a composite layer which is infinite in $xy$ plane and which is formed by periodically arranged rectilinear conducting wires.

3. Wavelength of smoothed electromagnetic field distributions in a slab of wire metamaterial. Focusing of electromagnetic waves through the wired structure
In the case when reflection at the slab boundary is small for frequencies of 2nd photonic transparency range at $0.65 < a/\lambda_m < 1$ [9], the electric field distribution is obtained in the form of a ”comb” (Fig. 2). In the figure there are long-wave oscillations of the effective field, on the background of which small interference oscillation can be seen.

For the comprehension of nature of field oscillations, the interference oscillations can be smoothed with various filters, for example, the Fourier filter. Fig. 2 shows that the wavelength obtained after filtering ($\lambda_{sefd}$) coincides with the wavelength outside the structure and with the values obtained in [8, 9].

From Fig. 2, 3 it is seen that where wavelength of SEFD much more distance between the wires the wave vector and the phase velocity of the oscillations of the effective field are opposite to the direction of energy current. That is, the smoothed electromagnetic field distributions is the same as the field distribution in the left-hand metamaterial. The ability of such kind of structures to be left-handed metamaterial was also shown in the research of the EMW negative refraction of EMW [10, 11]. For additional verification of this result, figures of field distribution for the case of point source were modeled. They show that in this frequency band negative refraction and focusing of electromagnetic radiation from a point source are observed. That cannot be done with the help of Bragg scattering mechanism.

4. Physical explanation for the magnetic response
For are left-handed media (LHM) [1] the index of refraction $n(\omega)$ used in the interpretation of Maxwell's equations should be taken as the negative square root as following $n = -\sqrt{\epsilon(\omega)/\mu(\omega)}$. 


Figure 2. Curve 1 - the dependence of an alternating electric field on coordinate \( z \) along the line passing in between the wires, in sizes of period \( a \). \( a/\lambda_m = 0.9 \). Curve 2: - a signal inside the wired structure, smoothed with Fourier filter (\( \lambda_{sefd} \)).

Figure 3. A signal inside the wired structure, smoothed with Fourier filter at different phases.

In [2, 3, 4] to create a LHM it was proposed to use the structures consisting of miniature split ring resonators, acting as magnetic dipoles, and straight segments of wire, acting as electric dipoles. But difficulty with the definition of physical meaning of permeability at higher frequencies, which is important for theory as well as for the interpretation of experiments, is related in [12] to the fact that it may be impossible to ‘measure’ the permeability by measuring the total induced magnetic moment of a macroscopic body. Indeed, the induced macroscopic current density \( J \) in time-dependent fields arises not only from the magnetization but also from the dielectric polarization \( P \) [13]:

\[
J = c \nabla \times M + \frac{\partial P}{\partial t}.
\]

\( M_{\text{tot}} = M_{\text{tot}}^1 + M_{\text{tot}}^2 \), where \( M_{\text{tot}}^1 = \int M \, dV \) and \( M_{\text{tot}}^2 = (1/2c) \int (r \times \partial P/\partial t) \, dV \).

More often \( M_{\text{tot}}^2 \) can be neglected, and the quantity \( \mu(\omega) \) with reasonable accuracy defines the magnetic moment of a unit volume in the field of the plane electromagnetic wave propagating in the medium. However, for metamaterials created from elements which sizes are much greater than the sizes of molecules, frequencies where effect of \( M_{\text{tot}}^2 \) becomes sufficient are much lower than in natural materials. Using Maxwell equation and the definitions of \( M \) and \( P \), we can immediately evaluate relative contributions to the induced current for a monochromatic electromagnetic wave. For the contribution from the time-dependent dielectric polarization to dominate, i.e., to have \( \partial P/\partial t \gg c \nabla \times M \), which allows neglecting the term \( M_{\text{tot}}^1 \), the
following inequality must be satisfied: $|\epsilon_{\text{eff}} - 1| >> |\mu_{\text{eff}} - 1| \epsilon_{\text{eff}}$. The values from [9], this inequality is satisfied. Also, it is satisfied for the values of effective permittivity of wire medium from works [3, 6, 7] if the permeability is equal to one. Hence, in this case the effective permeability is determined mainly by the polarization current which contribution is probably significant. Effective permeability no longer corresponds to magnetic moment of unit volume and can be unequal to one. According to [12, 13, 14] such permeability is a natural reaction of inhomogeneous fields on magnetic field, i.e. a reaction to the spatial dispersion, which is observed in figures 1, 2. In conclusion, it can be noted that, in general, the permeability determined by us cannot be associated with the total magnetic moment of a macroscopic body. Nevertheless, from a applied point of view (which means that we can estimate the wavelength in the medium, the phase velocity of the wave in the medium and its direction, refraction angles, refractive index, reflection and transmission coefficients), we decided to use this term. In paper [13] it is shown that the appearance of negative group velocity waves (fig. 2) with negative refraction (fig. 3), and, hence, the availability of negative effective permeability is associated with spatial dispersion.

5. Conclusions
Show that well-known metamaterial consisting of rectilinear thin wires array forming a 2D-square lattice within the non-conductive matrix has a wide band with negative group velocity waves, where wavelength of smoothed electromagnetic field distributions much more distance between the wires. Also negative refraction and focusing of electromagnetic radiation from a point source are observed. Unlike most of the left-handed metamaterials that are composed of two separate inter-elements (typically, a split ring and a rod), in this paper it is proposed to use only one type of inclusion, namely the conductive rod.

6. Acknowledgments
This work was supported by the Russian Foundation for Basic Research (Project No. 118-58-53055 GFEN_a, 16-29-14045 ofi_m,17-02-01382), the Foundation for advanced scientific research of the "CSU" (FPNI) and by the Ministry of Education and Science of the Russian Federation (State Contract No. 3.5698.2017/9.10).

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