Fast dynamic aperture optimization with reversal integration

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Outline

❖ Principle
❖ Proof-of-principle with Hènon map
❖ Applications of dynamic aperture optimization
  ❖ Dedicated light source rings (NSLS-II, or MBA)
  ❖ Collider ring (BNL-EIC e-ring)
❖ Summary
Principle

- Old idea: no new physics, empirical method, but effective
- Sensitivity to initial condition: nonlinearity, deterministic, but unpredictable
- Lyapunov exponent: separation growth rate describes the chaos

\[ |\delta Z(t)| = e^{\lambda t} |\delta Z(0)| \]
Launch two particles, which have a "small" difference in their initial conditions.

Two trajectories will separate from each other due to (1) initial difference (2) round-off error (3) chaos.

How to specify a "small" value in computer? (see next slide)

Alternative: Forward-Reversal Integration (FRI)
Round-off errors in computers

- Round-off error of floating point value as defined by IEEE-754 standards depend on:
  - number of bit (computer)
  - absolute value (number)
  - round-off method, ie. floor, ceiling and nearest (software)
- Manually specifying a “small” value at the machine precision is doable, however, complicated
Laslett (1957): a linear map (no chaos) is numerically irreversible due to round-off error, a “consistent error which grows in direct proportion to the number of iterations executed” by computer.

For a chaotic trajectory, the difference grows exponentially (LE) by scaling the accumulated round-off errors.

Motivation: could we optimize a nonlinear lattice by control the growth rate of irreversibility? (ie. sensitivity to initial condition)
Proof-of-principle with Hènon map

\[
\begin{pmatrix} x \\ p \end{pmatrix}_n = \begin{pmatrix} \cos \mu & \sin \mu \\ -\sin \mu & \cos \mu \end{pmatrix} \begin{pmatrix} x \\ p - x^2 \end{pmatrix}_{n-1}
\]

❖ Is chaos visible with F-R integration?
❖ How much iterations (turns) is needed to observe chaos?
❖ Color-bar \( \Delta = \log_{10} |z_0 - z'| \) (courtesy Y. Hao)
Applications to DA optimization at NSLS-II

- Method: by tuning nonlinear knobs (sextupole, octupole) to reduce the chaos within the desired DA
- Implementation: slicing and dividing the volume of DA
- Using multi-objective optimizer
“Early” chaos indicator: “fast”

- NSLS-II lattice: **1 turn (15 supercells)** can provide visible difference
- We don’t attempt to extract the profile of DA with 1-turn F-R integration.
- Rule out the uncompetitive candidates (smaller DA ↔ larger chaos) during optimization
- Color-bar: \[ \Delta = \log_{10} \sqrt{\Delta x^2 + \Delta p_x^2 + \Delta y^2 + \Delta p_y^2} \]
  
  Linear action: J
MOGA optimization results

- Many solutions can meet the requirement
- An elite solution has been confirmed by a multi-turn tracking, frequency map analysis (FMA), and a beam test at the NSLS-II ring
- A multi-bend achromat lattice for 4th-gen. LS has been tested
FMA vs FRI for APS-U 7BA lattice

(courtesy M. Borland)
BNL-EIC e-ring (10GeV 60Deg)

- Challenges:
  - Large ring: impractical if the optimizer driven by multi-turns (N>100?) tracking
  - High dimension parameter space: 36 families sextupoles
  - Low periodicity: 1
Different slicing method

- A large momentum acceptance is needed, which is difficult for FODO lattice
- 16 turns 5D tracking is used for optimization
- 1024 turns 6D tracking is used to confirm the DA at with synchro-betatron coupling

6D tracking

5D tracking
Implementation at ELEGANT

7.11 chaos_map

- type: major action command.
- function: compute chaos map from tracking. Note that the number of turns tracked is set by the run_control command.
- can use parallel resources (Pelegant)
- Command syntax, including use of equations and subcommands, is discussed in 7.2.
- NB: this feature is new in 2019.4 and somewhat experimental. Please report problems on the forum.

```c
&chaos_map
  STRING output = NULL;
double xmin = -0.1;
double xmax = 0.1;
double ymin = 1e-6;
double ymax = 0.1;
double delta_min = 0;
double delta_max = 0;
long nx = 20;
long ny = 21;
long ndelta = 1;
long forward_backward = 0;
double change_x = 1e-6;
double change_y = 1e-6;
long verbosity = 1;
&end
```

Thanks to M. Borland
Since Version 2019.4

Mode switch between the forward-only and F-R integrations
Manually input a “small” difference. Not recommended)
Summary

❖ A early chaos indicator using forward reversal integration has been demonstrated to optimize DA
❖ It is fast because only a very few turns is needed to rule out uncompetitive candidates
❖ It has been integrated into the ELEGANT code (different magnets, fringe field, undulator - kickmap, RFC, etc. have been implemented)