An Augmented Automatic Choosing Control of a Formal Linearization Filter Type for Nonlinear Systems

Tomohiro Hachino¹, Hitoshi Takata¹, Soichiro Osako¹, Kazutomo Yunokuchi¹, Hiromi Miyajima¹ and Kazuo Komatsu²

¹ Kagoshima University, 1-21-40 Korimoto, Kagoshima 890-0065, Japan
² Kumamoto National College of Technology, 2659-2 Suya, Koshi, Kumamoto 861-1102, Japan
E-mail: hachino@eee.kagoshima-u.ac.jp

Abstract This paper is concerned with single nonlinear feedback control for nonlinear systems with noisy measurement. A given nonlinear system is linearized piecewise to design the linear optimal controllers, which are then smoothly united into a single nonlinear feedback controller by an automatic choosing function. The state estimation is carried out using a nonlinear filter on the basis of the following formal linearization. This filter is synthesized by applying the extended Kalman filter technique to the augmented system that adds some polynomials of nonlinear measurement terms to the given nonlinear system. This is called an augmented automatic choosing control of a formal linearization filter type (AACCFLF). Simulation results are shown to demonstrate the effectiveness of the proposed method.

Keywords: nonlinear control, augmented automatic choosing control, nonlinear filter, formal linearization

1. Introduction

There are many estimation and control approaches for nonlinear systems [1]-[8]. Most controllers are synthesized by linearizing a given nonlinear system to which the linear estimation and control theories are applied. One such control theory is based on a truncation at the first order of the Taylor expansion [1],[3]. This control law, which is called linear optimal control (LOC), is easy to implement in many practical nonlinear systems, such as electric power systems, but is only useful in a small region or in essentially linear ones. Controllers based on a change of coordinates in differential geometry [4] are effective in a wider region but are not easy to implement in many practical systems. Controllers based on an automatic choosing control law [5],[7], which is an extension of the LOC, are more practical but they have not yet been studied sufficiently.

Therefore, this paper is concerned with a nonlinear feedback controller with the automatic choosing function [5],[7], a formal linearization filter [8], and the linear optimal control theory [3],[6] for nonlinear systems with noisy measurement. This filter is designed by adding some polynomials of nonlinear measurement terms to the given nonlinear system, and then by applying the extended Kalman filter technique to the resulting augmented system [8]. Considering the nonlinearity, the given region of the system is divided into some subdomains. For each subdomain, the system equation is linearized by the Taylor expansion to enable the application of the LQ control theory [3],[6]. Constant terms in this linearization are treated as coefficients of stable zero dynamics [7]. The resulting linear controls are smoothly united by the automatic choosing function to yield a single nonlinear feedback control. This controller is called an augmented automatic choosing control of formal linearization filter type (AACCFLF).

Numerical simulations are carried out to illustrate the following. The estimation accuracy of systems is improved as the order of the formal linearization filter used in the AACCFLF increases. The stability of electric power systems with the AACCFLF is considerably better than that of systems with the LOC.

2. Statement of the Problem

The plant is assumed to be described by the nonlinear dynamic and noisy measurement equations

\[ \dot{x} = f(x) + Bu, \quad x \in D \subset R^n \]  (1)
where \( \cdot = d/dt, \ x = [x_1, \ldots, x_n]^T \) is an \( n \)-dimensional state vector, \( u = [u_1, \ldots, u_r]^T \) is an \( r \)-dimensional control vector, \( y = [y_1, \ldots, y_m]^T \) is an \( m \)-dimensional measurement vector, \( f \) and \( h \) are nonlinear vector-valued functions with \( f(0) = 0 \) and are continuously differentiable, \( B \) is an \( n \times r \) driving matrix, \( v \) is white Gaussian noise of \( N(v : 0, V) \), \( V \) is an \( m \times m \) covariance matrix, and \( T \) denotes transpose.

Considering the nonlinearity of system (1), we introduce a vector-valued function \( C : D \rightarrow R^L \) that defines the separative variables \( \{C_j(x)\} \), where \( C = [C_1 \cdots C_i \cdots C_L]^T \) is continuously differentiable. Let \( D \) be a domain of \( C^{-1} \). For example, if \( x_{[2]} \) is the element that has the highest nonlinearity in system (1) (see Eq. (43)), then

\[
C(x) = x_{[2]} \in D \subset R \ (L = 1)
\]

The domain \( D \) is divided into some subdomains:

\[
D = \bigcup_{i=0}^{M} D_i \quad \text{where} \quad D_{M+1} = D - \bigcup_{i=0}^{M} D_i \quad \text{and} \quad C^{-1}(D_0) \supset 0 \quad \text{for} \quad D_i \ (0 \leq i \leq M)
\]

Endowed with a lexicographic order is the Cartesian product \( D_i = \Pi_{j=1}^L [a_{ij}, b_{ij}] \), where \( a_{ij} < b_{ij} \).

We here introduce an automatic choosing function of the sigmoid type:

\[
I_i(x) = \prod_{j=1}^L \left\{ 1 - \frac{1}{1 + \exp (2N (C_j(x) - a_{ij}))} - \frac{1}{1 + \exp (-2N (C_j(x) - b_{ij}))} \right\}
\]

where \( N \) is a positive real value and \(-\infty \leq a_{ij} < b_{ij} \leq \infty \). \( I_i(x) \) is analytic and almost unity on \( C^{-1}(D_i) \); otherwise, it is almost zero (see Fig. 1).

The aim of this study is to design a nonlinear feedback control AACCFLF by smoothly uniting the sectionwise controls and by using a formal linearization filter.

3. Design of Control

The nonlinear function \( f \) of system (1) is linearized by the Taylor expansion truncated at the first order about the point \( \hat{x}_i \) in \( C^{-1}(D_i) \) and \( \hat{x}_0 = 0 \) on each subdomain \( D_i \) (see Fig. 2):

\[
f(x) \simeq f(\hat{x}_i) + A_i(x - \hat{x}_i) = A_i x + w_i
\]

where

\[
A_i = \left. \frac{\partial f(x)}{\partial x} \right|_{x = \hat{x}_i}, \ w_i = f(\hat{x}_i) - A_i \hat{x}_i
\]

We introduce stable zero dynamics:

\[
\dot{x}_{[n+1]} = -\sigma \dot{x}_{[n+1]}
\]

(\( \dot{x}_{[n+1]}(0) \simeq 1, \quad 0 < \sigma < 1 \))

where the value of \( \sigma \) is selected such that \( \sigma = -\hat{x}_{[n+1]} / \dot{x}_{[n+1]} \leq -\hat{x}_{[k]} / \dot{x}_{[k]} \) holds for all \( k \) \((k = 1, \ldots, n)\). This is to keep \( \dot{x}_{[n+1]} \simeq 1 \) for a reasonable period of time when system (1) is not on \( C^{-1}(D_0) \).

We approximate \( f \) as

\[
f(x) \simeq A_i x + w_i \simeq A_0 x + w_{[0]} \quad \text{for} \quad f(0) = 0 \quad \text{in Eq. (3)}.
\]

We assume that the control is designed, using Eq. (3), as

\[
u = \sum_{i=0}^{M} u_i I_i(\dot{x})
\]

where \( \dot{x} \) is an estimate of \( x \).

Note that \( \sum_{i=0}^M I_i(\dot{x}) = 1 \) for Eq. (3). Substituting Eqs. (5) and (6) into system (1), the dynamic equation becomes

\[
\dot{x} = f(x) + Bu
\]

\[
= \sum_{i=0}^{M} f(x) I_i(\dot{x}) + \sum_{i=0}^{M} Bu_i I_i(\dot{x})
\]

\[
\simeq \sum_{i=0}^{M} (A_i x + w_i \dot{x}_{[n+1]} + Bu_i) I_i(\dot{x})
\]

In order to attain a control gain of \( u_i \), we here set \( I_i(\dot{x}) = 1 \), in which \( N = -a_{ij} = b_{ij} \rightarrow \infty \) in Eq. (3).

When \( X = [x^T, \dot{x}_{[n+1]}]^T \), then Eqs. (4) and (7) yield an approximated linear equation:

\[
\dot{X} = \bar{A}_i X + \bar{B} u_i
\]
where

\[
\bar{A}_i = \begin{bmatrix} A_i & w_i \\ 0 & -\sigma \end{bmatrix}, \quad \bar{B} = \begin{bmatrix} B \\ 0 \end{bmatrix}
\]

Therefore, we apply the LQ control theory to obtain the control formula as follows. Consider that the system and cost function

\[
\Sigma : \left\{ \begin{array}{l}
\dot{X} = \bar{A}_i X + \bar{B} u_i \\
J_i = \frac{1}{2} \int_0^\infty (X^T Q X + u_i^T R u_i) dt
\end{array} \right.
\]

are given. Then an application of the linear optimal control theory [3] yields

\[
u_i(X) = -F_i X
\]

\[
F_i = R^{-1} \bar{B}^T P_i
\]

where the \((n+1) \times (n+1)\) matrix \(P_i\) satisfies the Riccati equation

\[
P_i \bar{A}_i + \bar{A}_i^T P_i + Q - P_i \bar{B} R^{-1} \bar{B}^T P_i = 0
\]

Here, \(Q = Q^T > 0\) and \(R = R^T > 0\) are positive symmetric matrices. Values of \(Q\) and \(R\) are appropriately determined on the basis of engineering experience [6].

4. Design of Filter

4.1 Simple measurement equation

We here design a nonlinear filter for state estimation by making use of the methods of formal linearization [8] and the extended Kalman filter [3].

Here, we consider a simple case of the measurement equation (2), which is assumed to be divided into a linear part

\[
y_L = H_L x + w
\]

and a scalar nonlinear part

\[
\eta = h(x) + v_1
\]

Consequently, Eq. (2) is rewritten as

\[
y = \begin{bmatrix} y_L \\ \eta \end{bmatrix} = \begin{bmatrix} H_L x \\ h(x) \end{bmatrix} + \begin{bmatrix} w \\ v_1 \end{bmatrix}
\]

where \(y_L \in \mathbb{R}^{m-1}\), \(\eta \in \mathbb{R}, \ h \in \mathbb{R}, \ w \in \mathbb{R}^{m-1}, \ v_1 \in \mathbb{R}\), \(H_L\) is an \((m-1) \times n\) constant matrix, and \(v = [w^T, v_1]^T\). The covariance matrix of noise is

\[
V = Evv^T = E \begin{bmatrix} w \\ v_1 \end{bmatrix} \begin{bmatrix} w^T & v_1 \end{bmatrix} = \begin{bmatrix} W & 0 \\ 0 & V_1 \end{bmatrix}
\]

where \(w\) and \(v_1\) are independent of each other (\(w \perp v_1\)), and \(Ev = 0\).

4.2 Augmented measurement equation

We consider the polynomial \(\eta\) of Eq. (13) as follows. For simplicity, noises are assumed to be produced from noise generators,

\[
N g_j = \{v^0_j : v^0_j \in \mathcal{N}(0, \sigma^2_0)\} \quad (j = 1, 2, \cdots, \mu)
\]

where \(v^0_j\) is white Gaussian noise of variance

\[
\sigma^2_0 = \frac{1}{\| h \|^2} V_1
\]

and is independent of the state \(x\) and other noises, \(v^0_j \perp v^0_k \perp x_i (j \neq k)\). Here, \(\| \cdot \|\) is a norm such that

\[
\| z \| = \sup_{x \in \mathcal{D}} \sqrt{z^T(x)z(x)}
\]

for any vector \(z\), and

\[
\| z \| = \sup_{x \in \mathcal{D}} \| z(x) \|
\]

for a scalar \(z\).

We define the functions

\[
g_j(x) = \frac{h^j(x)}{\| h^j \|} \quad (j = 1, 2, \cdots, \mu)
\]

that satisfy

\[
| g_j(x) | \leq 1 \quad \text{for all } x \in \mathcal{D}
\]

and \(g_j \in C^\infty\) if \(h \in C^\infty\). Similarly, we also define the \(j\)th polynomial function of \(\eta\) as

\[
p_j(\eta) = \frac{\eta^j}{\| \eta \|^j} \quad (j = 1, 2, \cdots, \mu)
\]

by considering \(\eta \approx h(x)\) in the case of \(v_1 \approx 0\), because \(\eta = h(x) + v_1\). Let us assume that the polynomial function \(p_j(\eta)\) can be described by the sum of the measurement function \(g_j(x)\) and noise \(v^0_j\). Therefore, we have the following polynomial equations of \(\eta\) by setting

\[
v^0_j = h^j \| v^0_j \| \quad (j = 1, 2, \cdots, \mu)
\]

When \(j = 1\), we have \(p_1(\eta) = g_1(x) + v^0_1\) or \(\| \eta \| = \| h(x) \| + v^0_1\), thus

\[
\eta = h(x) + v \| v^0_1 \| = h(x) + v_1
\]

When \(j = 2\), we have \(p_2(\eta) = g_2(x) + v^0_2\) or \(\frac{\eta^2}{\| \eta \|^2} = \frac{h^2(x)}{\| h \|^2} + v^0_2\), thus

\[
\eta^2 = h^2(x) + v \| v^0_2 \| = h^2(x) + v_2
\]
When \( j = j \), we have \( p_j(\eta) = g_j(x) + v_j \) or \( \eta_j = \frac{h_j(x) + v_j}{\| h_j \|} \), thus

\[
\eta_j = h_j(x) + v_j \quad (26)
\]

When \( j = \mu \), we have \( \eta_\mu = g_\mu(x) + v_\mu \) or \( \eta_\mu = \frac{h_\mu(x) + v_\mu}{\| h_\mu \|} \), thus

\[
\eta_\mu = h_\mu(x) + v_\mu \quad (27)
\]

Using these polynomials \( \{ \eta_j \} \), an augmented measurement vector \( Y \) and noise vector \( v \) are introduced as

\[
Y = \begin{bmatrix}
y_L
\eta
\eta^2
\vdots
\eta^{\mu}
y_1
v_1
\vdots
v_\mu
\end{bmatrix}, \quad v = \begin{bmatrix} w \\tilde{\nu} \end{bmatrix} \quad (28)
\]

where

\[
\tilde{\nu} = \begin{bmatrix} v_1 
v_2 
\vdots 
v_\mu \end{bmatrix} \quad (29)
\]

Thus, Eq. (14) yields

\[
Y = \begin{bmatrix}
y_L
\eta
\eta^2
\vdots
\eta^{\mu}
y_1
h(x) + v_1
\vdots 
h_\mu(x) + v_\mu
\end{bmatrix} + v = h(x) + v \quad (30)
\]

where its noise covariance is

\[
V = \begin{bmatrix} W & 0 
0 & V_1 \Psi \end{bmatrix} \quad (31)
\]

from Eq. (48) of the Appendix.

Note that each polynomial of \( \{ \eta_j : j = 1, \ldots, \mu \} \) is treated as a new measurement in the formal linearization approach [8]. The scalar case of nonlinear measurement \( \eta \) has been considered in this paper, but it can be expanded to a multidimensional case of \( \eta \in R^m \) (1 < \( m \leq \mu \)) straightforwardly.

4.3 Application of extended Kalman filter approach

Equations (1) and (30) are expanded by the Taylor expansion about an assumed known optimal estimate \( x(t) = \hat{x}(t) \) and are truncated at the first order:

\[
\dot{x}(t) \simeq f(\hat{x}(t)) + F(t)e(t) + Bu \quad (32)
\]

\[
Y(t) \simeq h(\hat{x}(t)) + H(t)e(t) + v(t) \quad (33)
\]

where \( e(t) = x(t) - \hat{x}(t) \), \( F(t) = \partial f(x)/\partial x^T \bigg|_{x=\hat{x}(t)} \), and \( H(t) = \partial h(x)/\partial x^T \bigg|_{x=\hat{x}(t)} \).

The extended Kalman filter approach is applied to the linearized system of Eqs. (32) and (33), so that the filter becomes

\[
\dot{x}(t) = f(\hat{x}(t)) + K(t) \{ Y(t) - h(\hat{x}(t)) \} + Bu \quad (34)
\]

where \( K(t) \) is the filter gain of

\[
K(t) = S(t)H(t)^T V^{-1} \quad (35)
\]

\[
\dot{S}(t) = F(t)S(t) + S(t)F^T(t) - S(t)H(t)^T V^{-1} H(t)S(t) \quad (36)
\]

with initial values of \( \dot{x}(0) = \bar{x}_0 \) and \( S(0) = S_0 \).

5. Synthesis of AACCFLF

On the basis of the formulations in Sects. 3 and 4, we have the following AACCFLF formula.

[AACCFLF formula]

\[
\dot{x}(t) = f(\hat{x}(t)) + K(t) \{ Y(t) - h(\hat{x}(t)) \} + Bu(t) \quad (37)
\]

\[
\dot{x}_{[n+1]}(t) = -\sigma \hat{x}_{[n+1]}(t) \quad (\hat{x}_{[n+1]}(0) \simeq 1) \quad (38)
\]

\[
u(t) = \sum_{i=0}^{M} u_i(t) I_i(\hat{x}(t)) \quad (39)
\]

\[
K(t) = S(t)H(t)^T V^{-1} \quad (40)
\]

where

\[
u_i(t) = -R^{-1} B^T P_i \hat{X}(t) \quad (41)
\]

\[
\hat{X}(t) = [\hat{x}^T(t), \hat{x}_{[n+1]}(t)]^T \quad (42)
\]

\[
P_i A_i + A_i^T P_i Q - P_i B R^{-1} B^T P_i = 0 \quad (43)
\]

\[
\dot{S}(t) = F(t)S(t) + S(t)F^T(t) - S(t)H(t)^T V^{-1} H(t)S(t) \quad (44)
\]

Since this formula is of a structure-specified type, each parameter included in the above equations must be suitably selected so that the feedback control system (1) with AACCFLF can be stabilized globally.

6. Numerical Simulations

Numerical simulations are carried out to illustrate the estimation accuracy and the excellent control of the AACCFLF.
6.1 Estimation

Consider a pendulum in which the bob is connected to a rod of zero mass. Let \( \theta \) denote the angle subtended by the rod and the vertical axis through the pivot point. Such a system is written as

\[
\ddot{\theta} + a_1 \dot{\theta} + a_2 \sin \theta = 0
\]

(37)

We assume that the position of the bob is measured from above and the measurement equation is

\[
y = a_3 \sin \theta + v
\]

(38)

Setting the state variables as \( x_1 = \theta \) and \( x_2 = \dot{\theta} \), the given system is rewritten as

\[
\dot{x} = \left( -a_2 \sin x_1 - a_1 x_2 \right) \equiv f(x)
\]

(39)

\[
y = a_3 \sin x_1 + v \equiv h(x) + v
\]

(40)

where \( n = 2 \), \( r = 0 \), \( m = 1 \), \( B = 0 \), \( H_L = 0 \), and \( \mu \)

is varied from 1 to 4. Parameters are set as \( a_1 = 0.5 \), \( a_2 = 980.7 / 400 \), \( a_3 = 1 \), \( x(0) = [1.8, 0.8]^T \), \( \dot{x}(0) = 0 \), \( V = 0.05 \cdot \text{diag}(1, \ldots, 1) \), and \( S(0) = 1.5 \cdot \text{diag}(1, 1) \).

Figure 3 shows the logarithm of \( J(t) \), which is the integral square error of estimation

\[
J(t) = \int_0^t (x(t) - \hat{x}(t))^T (x(t) - \hat{x}(t)) \, dt
\]

(41)

for the various orders \( \{\mu\} \) of Eq. (30).

The simulation result indicates that the estimation accuracy can be improved as the order \( \mu \) increases.

6.2 Control

Consider a field excitation control problem of a single-machine power system, the Ozeki Power Plant of Kyushu Electric Power Company in Japan. This system is assumed to be described [7] by

\[
\begin{align*}
\frac{d\delta}{dt} + \hat{D}(\delta)\dot{\delta} + P_e(\delta) &= P_m \\
\dot{P}_e(\delta) &= E_1^2 Y_{11} \cos \theta_{11} + E_1 \tilde{V} Y_{12} \cos(\theta_{12} - \delta) \\
\tilde{E}_1 + T_{do} \dot{E}_q' &= \tilde{E}_d \\
\dot{E}_1 &= E_q' + (X_d - X_d') I_d(\delta) \\
I_d(\delta) &= -E_1 Y_{11} \sin \theta_{11} - \tilde{V} Y_{12} \sin(\theta_{12} - \delta) \\
\dot{D}(\delta) &= \tilde{V}^2 \left\{ T_{do}' (X_d' - X_d'') \sin^2 \delta \\
&\quad + \frac{T_{do}' (X_d - X_d'')}{(X_d + X_e)^2} \cos^2 \delta \right\}
\end{align*}
\]

(42)

The output is assumed to be given by \( \dot{\delta} \) and \( P_e(\delta) \), which are easily measurable. \( \tilde{E}_d \) is a control variable. Here, \( \delta \) is the phase angle, \( \dot{\delta} \) the rotor speed, \( \bar{M} \) the inertia coefficient, \( \hat{D}(\delta) \) the damping coefficient, \( P_m \)

the mechanical input power, \( P_e(\delta) \) the generator output power, \( \tilde{V} \) the reference bus voltage, \( E_1 \) the open circuit voltage, \( E_{fd} \) the field excitation voltage, \( X_d \) the direct axis synchronous reactance, \( X_d' \) the direct axis transient reactance, \( X_e \) the external impedance, \( Y_{11} \) \( Y_{12} \) the direct admittance of the network, \( Y_{12} \) \( \theta_{11} \) the mutual admittance of the network, and \( I_d(\delta) \) the direct axis current of the machine.

Set \( x=[x_1, x_2, x_3]^T=[E_1 - \tilde{E}_1, \delta - \hat{\delta}_0, \dot{\delta}]^T \) and \( u = E_{fd} - \tilde{E}_{fd} \), so that

\[
\begin{align*}
\dot{x} &= \begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{bmatrix} = \begin{bmatrix} f_1(x) \\ f_2(x) \\ f_3(x) \end{bmatrix} + \begin{bmatrix} b_1 \\ 0 \\ 0 \end{bmatrix} u \\
y &= \begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix} = \begin{bmatrix} H_{Lx} \\ h(x) \end{bmatrix} + v
\end{align*}
\]

(43)

(44)

where

\[
\begin{align*}
f_1(x) &= \frac{1}{k T_{do}} \left( x_1 + \tilde{E}_1 - \tilde{E}_{fd} \right) \\
&\quad + (X_d - X_d') \tilde{V} Y_{12} \cos \left( \theta_{12} - x_2 - \hat{\delta}_0 \right) \\
f_2(x) &= x_3 \\
f_3(x) &= \frac{\tilde{V} Y_{12}}{M} \left( x_1 + \tilde{E}_1 \right) \cos \left( \theta_{12} - x_2 - \hat{\delta}_0 \right) \\
&\quad - Y_{11} \cos \theta_{11} \left( x_1 + \tilde{E}_1 \right)^2 - \frac{\tilde{D}(x)}{M} x_3 \frac{P_m}{M} \\
H_{Lx} &= \begin{bmatrix} 0 & 0 & 1 \end{bmatrix} x_3 \\
h(x) &= Y_{11} \cos \theta_{11} (x_1 + \tilde{E}_1)^2 \\
&\quad + \tilde{V} Y_{12} (x_1 + \tilde{E}_1) \cos \left( \theta_{12} - x_2 - \hat{\delta}_0 \right) \\
\tilde{D}(x) &= \tilde{V}^2 \left( T_{do}' (X_d' - X_d'') \sin^2 \left( x_2 + \hat{\delta}_0 \right) \\
&\quad + \frac{T_{do}' (X_d - X_d'')}{(X_d + X_e)^2} \cos^2 \left( x_2 + \hat{\delta}_0 \right) \right) \\
b_1 &= \frac{1}{k T_{do}}
\end{align*}
\]
Steady-state values are as follows.

\begin{align*}
\dot{\bar{M}} &= 0.016095 \text{ [pu]} & T_{d0}' &= 5.09907 \text{ [s]} \\
\dot{\bar{V}} &= 1.0 \text{ [pu]} & T_{d1}' &= 1.2 \text{ [pu]} \\
X_d &= 0.875 \text{ [pu]} & X_d' &= 0.422 \text{ [pu]} \\
Y_{11} &= 1.04276 \text{ [pu]} & Y_{12} &= 1.03084 \text{ [pu]} \\
\theta_{11} &= -1.56495 \text{ [pu]} & \theta_{12} &= 1.56189 \text{ [pu]} \\
X_s &= 1.15 \text{ [pu]} & X_s' &= 0.238 \text{ [pu]} \\
X_q &= 0.6 \text{ [pu]} & X_q' &= 0.3 \text{ [pu]} \\
T_{d0}' &= 0.0299 \text{ [pu]} & T_{d0}' &= 0.02616 \text{ [pu]} 
\end{align*}

Steady-state values are as follows.

\begin{align*}
\dot{E}_I &= 1.52243 \text{ [pu]} & \delta_0 &= 48.57^\circ \\
\dot{\delta}_0 &= 0.0 \text{ [deg/s]} & \dot{E}_{f I} &= 1.52243 \text{ [pu]}
\end{align*}

Set \( X = [x^T, \dot{x}_0^T] = [x_{[1]}, x_{[2]}, x_{[3]}, \dot{x}_{[4]}]^T, C(x) = x_{[2]}, L = 1, M = 1, \mu = 2, N = 15.0, a_1 = 1.05, \chi_0 = 0, \chi_1 = \begin{bmatrix} 0 & 0.90 & 0 - \delta_0 & 0 \end{bmatrix}^T, \sigma = 0.1, R = 1, Q = \text{diag}(1,1,1,1), V = \text{diag}(1,1), X(0) = [0, x_{[2]}(0), x_{[3]}(0), 1]^T, \dot{x}(0) = [0, \dot{x}_{[2]}(0), 0]^T, \bar{X}(0) = \begin{bmatrix} 1 & 1 & 0 \\ 1 & 5 & 10 \\ 0 & 10 & 50 \end{bmatrix}, \) and \( S(0) = \begin{bmatrix} 1 & 1 & 0 \\ 1 & 5 & 10 \\ 0 & 10 & 50 \end{bmatrix}. \)

These values are selected by trial and error. Experiments are carried out for the new control (AACCLF) and the ordinary LOC [1],[3]. Figure 4 depicts the stable regions for AACCLF and LOC. It shows that the stability region of AACCLF includes that of LOC. Figure 5 shows the time responses of \( x_{[1]}, x_{[2]}, x_{[3]}, \) and \( u \) for AACCLF and LOC where \( X(0) = [0, 1, 0, 1]^T \) and \( \bar{X}(0) = [0, 0, 0, 1]^T \). The estimate \( \hat{x}_{[3]} \) of the state \( x_{[3]} \) is obtained from the linear part \( y_{[1]} = H_{L} x + w, \) while \( \hat{x}_{[1]} \) and \( \hat{x}_{[2]} \) are obtained from.
7. Conclusions

We studied an augmented automatic choosing control of the formal linearization filter type (AACCFLF) for nonlinear systems with noisy measurement. The simulation results showed the following. The estimation accuracy could be improved with increasing formal linearization order. The stability of the power system could be considerably expanded by incorporating the AACCFLF. The following is left for future work: the problem of the selection of the optimum parameters \( \{M, N, a_{ij}, b_{ij}, \chi_i, \mu\} \) and the initial values of AACCFLF, which greatly depend on the stable region and trajectory of systems; the global stability problem; and the application to other nonlinear systems with higher nonlinearity.

Appendix: Noise Covariance

The noise \( \nu \) in Eq. (30) has the following property. The expectation of \( \bar{\nu} \) of Eq. (29) becomes

\[
E(\bar{\nu}) = E \begin{bmatrix} v_1 \\ \vdots \\ v_j \\ \vdots \\ v_\mu \end{bmatrix} = E \begin{bmatrix} \| h \| v_1^0 \\ \vdots \\ \| h^j \| v_j^0 \\ \vdots \\ \| h^\mu \| v_\mu^0 \end{bmatrix} = 0
\]

From Eqs. (16) and (17), the covariance is

\[
\bar{V} = E[(\bar{\nu} - E(\bar{\nu}))[\bar{\nu} - E(\bar{\nu})]^T] = E(\bar{\nu}\bar{\nu}^T)
\]

\[
= E \begin{bmatrix} v_1^2 & \cdots & v_1 v_\mu \\ \vdots & \ddots & \vdots \\ v_\mu v_1 & \cdots & v_\mu^2 \end{bmatrix} = E \begin{bmatrix} \| h \|^2 (v_1^0)^2 & \cdots & \| h \| \| h^\mu \| v_1^0 v_\mu^0 \\ \vdots & \ddots & \vdots \\ \| h^\mu \| \| h \| v_\mu v_1^0 & \cdots & \| h^\mu \|^2 (v_\mu^0)^2 \end{bmatrix}
\]

\[
= \begin{bmatrix} \| h \|^2 \sigma_0^2 & 0 & \cdots & 0 \\ 0 & \ddots & \vdots & 0 \\ \vdots & \ddots & \| h^j \|^2 \sigma_0^2 & \cdots \\ 0 & \cdots & \cdots & \| h^\mu \|^2 \sigma_0^2 \end{bmatrix}
\]

\[
= (\| h \|^2 \sigma_0^2) \Psi = V_1 \Psi
\]

where

\[
\Psi = \begin{bmatrix} 1 & 0 & \cdots & 0 \\ 0 & (\| h \|^2 \| h \|^2)^2 & 0 & \cdots \\ \vdots & \vdots & \ddots & \vdots \\ 0 & \cdots & \cdots & (\| h^\mu \| \| h^\mu \|^2)^2 \end{bmatrix}
\]

because \( v_j^0 \) and \( v_k^0 \) are assumed to be independent of \( E(v_j^0 v_k^0) = 0 \) \((j \neq k)\). Thus

\[
V = Ev\nu^T = E \begin{bmatrix} w \\ \bar{\nu} \end{bmatrix} [w^T \bar{\nu}^T]
\]

\[
= \begin{bmatrix} W & 0 \\ 0 & V_1 \Psi \end{bmatrix}
\]

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Tomohiro Hachino received his B.S., M.S., and Dr. Eng. degrees in electrical engineering from Kyushu Institute of Technology in 1991, 1993, and 1996, respectively. He is currently an Associate Professor at the Department of Electrical and Electronics Engineering, Kagoshima University. His research interests include the control and identification of nonlinear systems. Dr. Hachino is a member of IEEJ, SICE, and ISCIE.

Hiromi Miyajima received his B.S. degree in electrical engineering from Yamanashi University in 1974 and his M.S. and Dr. Eng. degrees in electrical and communication engineering from Tohoku University in 1976 and 1979, respectively. He is currently a Professor at the Department of Electrical and Electronics Engineering, Tohoku University. His research interests include fuzzy modeling, neural networks, quantum computing, and parallel computing. Dr. Miyajima is a member of IEEJ, IEICE, IEEJ, IPS, and SOFT.

Hitoshi Takata received his B.S. degree in electrical engineering from Kyushu Institute of Technology in 1968 and his M.S. and Dr. Eng. degrees in electrical engineering from Kyushu University in 1970 and 1974, respectively. He is currently a Professor Emeritus and a part-time lecturer at Kagoshima University. His research interests include the control, linearization, and identification of nonlinear systems.

Kazutomo Yunokuchi received his Dr. Eng. degree from the University of Tokyo in 1993. He is currently a Professor at the Department of Information Science and Bioengineering, Kagoshima University. His research interests are concentrated on the information system of the human body. Dr. Yunokuchi is a member of IEEE, IEICE, and IEEJ.

Kazuo Komatsu received his B.S. degree in computer science and his Dr. Eng. degree in electrical engineering from Kyushu Institute of Technology in 1985 and 1995, respectively. He is currently a Professor at the Department of Human-Oriented Information Systems Engineering, Kumamoto National College of Technology. His research interests include formal linearization for nonlinear systems and its applications. Dr. Komatsu is a member of IEEJ.

Soichiro Osako received his B.S. degree in electrical and electronics engineering from Kagoshima University in 2012. He is currently a master’s student at the Department of Electrical and Electronics Engineering, Kagoshima University. His research interests include the control of nonlinear systems.

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