Vibration control of loosely supported cross-flow heat exchanger tube undergoing fluid elastic instability

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Abstract: In this theoretical work, the effect of support stiffness in controlling the nonlinear vibration of loosely supported cross-flow heat exchanger tube and support-impact forces is studied. The bending of slender heat exchanger tube is modelled using Euler-Bernoulli beam theory. The cross-flow induced forces on tube are modelled using quasi-steady flow model whereas the loose support is modelled with trilinear spring model. The Galerkin discretisation and time integration are used to solve the nonlinear governing differential equation of motion. From the results, it is observed that with the reduction of support stiffness, the chaotic motion of the tube gradually changes to periodic motion, the drastic changes in the acceleration of the tube during its motion reduces, and also the magnitude of the contact forces greatly reduces. Thus, if the support is made of low stiffness material or placing a low stiffer buffer layer over stiff support, the vibration and impact forces can be significantly reduced.

1. INTRODUCTION

From the several decades, the flow-induced vibration (FIV) had been a major problem in the heat exchangers, which are the crucial components used for cooling or heating processes in many engineering systems like power plants, chemical plants, petroleum refineries, nuclear reactors, etc. Especially, in case of cross-flow heat exchangers, the high velocity of cross-flow cause different kinds of instabilities in slender flexible heat exchanger tubes leading to their severe vibrations [1–5]. For limiting the amplitude of oscillations of the tubes, the supports are provided through rigid baffles at different locations along the span. However, a small clearance is provided between baffles and tubes for the sake of thermal expansion and tube/support alignment. When the amplitude of vibration exceeds this clearance, the tube starts impact against the rigid support and causes the fretting failure of tubes. Such a failure leads to different problems like the shutdown of an entire engineering system, production loss, threat to the human safety and the expose of radioactive fluid due to the mixing of primary and secondary fluids (in nuclear power plants). Thus, the FIV of heat exchanger tubes attracted the interest of many researchers and led to the vast amount of literature on the study of flow-induced instabilities/vibrations of the tubes subjected to external [4,6] and internal flows [7,8].

In the heat exchanger, the external cross-flow mainly causes the fluid elastic instability (FEI) in the tube through Hopf bifurcation when the flow velocity exceeds a particular value. This instability causes the flutter of tubes, while corresponding amplitude of vibration gradually increases with the flow velocity [9]. A good number of theoretical and experimental works have been conducted [10–16] in the literature to understand the mechanism of FEI and to develop an accurate model to analyze the same. The studies on the mechanism of FEI revealed that at low mass damping parameter, it primarily arises through negative fluid damping [13], which is mainly produced by the effect of motion of the
tube and its surrounding fluid. Whereas for the high mass damping parameter, FEI appears though the fluid elastic-stiffness-type mechanism, which is due to the coupled effect of motion of the neighbouring tubes and fluid. Thus, it is showed that at low mass damping parameters, one flexible tube in an array of rigid tubes undergoes FEI at an approximately same flow velocity as an array of flexible tubes [13]. This is valid mainly for certain array geometries like rotated triangular and in-line square arrays [17, 18]. Hence, many works have been conducted on the dynamics of a single flexible tube in an array of rigid tubes [9, 19–33]. Most of the studies have considered the effect of loose supports, where the tube impacts continuously against the support, making the dynamics complex due to the appearance of chaotic motion. This dynamics was highly dependent on the kind of support, such as anti-vibration bar (AVB) [9, 19, 26–28], tube support plate (TSP) [29–32], square flat bar support (SFB) [20, 21, 33], Rhomboid flat bar support (RFB) [33].

The AVB support generally limits the tube motion to the cross-stream plane. Hence, a significant number of works have analysed the planar dynamics of the tube in the cross-stream plane. Paidoussis and Li [9] have shown for the first time that the clamped-clamped flexible tube undergoes chaos with increasing flow velocity just after the starting of impact with AVB, where the support is modeled as a trilinear spring at its (tube) mid location. Assuming the fretting wear at one of the clamped ends, Wang and Ni [19] analysed the dynamics of a cantilever tube with a loose support at one end. The chaotic and quasi-periodic motion appears when the loose support is at the end of the tube, but such motion gradually disappear as the location of loose support is gradually changed towards the other end. By applying the axial tension on the simply supported tube with an intermediate loose support (modeled by cubic spring), Sadath et al. [26], have shown that the flow velocity corresponding to the beginning of chaotic motion can be increased. Hassan and Mohany [28] investigated the nonlinear dynamics of multi-span U-tubes supported by AVBs along the span with an axial offset. They reported that the axial offset is beneficial to reduce work rate and impact forces.

The literature shows that the loosely supported cross-flow heat exchanger tube undergoes a complex vibration and subjected to excessive impact forces. Although the major amount of works are reported on the mechanics and modelling of FEI, a limited number of works are available on the study of vibration or instability control of heat exchanger tubes [26, 28]. The present work is in this direction, where the effect of support material is explored for the reduction of vibration and the impact forces. In the following sections, first, the derivation of the governing equation of motion of cross-flow heat exchanger tube is presented. In, the numerical results, initially, the validations of the modeling of external cross-flow are presented. Subsequently, the effect of support material on the dynamics of tube as well as impact forces is presented.

2 SYSTEM MODEL AND GOVERNING EQUATION OF MOTION

The schematic diagram of a heat exchanger tube with intermediate loose AVB support and subjected to an external cross-flow is shown in figure 1(a). This system is an idealization of square inline tube array with an intermediate AVB support and subjected to external cross-flow (figure 1(b)) [9,13]. The pinned supports are considered at the ends of the tube. To derive the governing equation of motion of the tube, a reference coordinate system (XYZ) is attached at the one end of the central axis of the tube, while having its X-axis along the central axis. The external cross-flow with \( U_\infty \) velocity is assumed in XY plane along the direction of Y-axis. Hence, the motion of the tube can be considered as planar motion in the XZ plane [14]. The geometrical properties of the tube like outer diameter, inner diameter, length are symbolized as \( D_o \), \( D_i \), \( L \) respectively. Since the slender heat exchanger tube is considered in the present analysis, it is modelled using Euler-Bernoulli beam theory. Considering the principal geometric nonlinearity, stretching of the tube due to its transverse deflection (w), the differential governing equation of motion of a tube can be given as,
\[ m_p \frac{\partial^2 w}{\partial t^2} + E I \frac{\partial^3 w}{\partial x^3} + \int_{0}^{L} \left[ \frac{EA}{2L} \left( \frac{\partial w}{\partial x} \right)^2 \right] \, dx - \frac{E A L}{L} \int_{0}^{L} \left( \frac{\partial^2 w}{\partial x \partial t} \right) \, dx \frac{\partial^2 w}{\partial x^2} + \delta(x-x_b) f(w) = I \]  \tag{1}

where, \( \rho_p \) is density of the tube; \( w(x,t) \) is the transverse displacement of the tube at any location \( x \) along the tube; \( F_{\text{ext}} \) external transverse distributed force per unit length; \( f(w) \) is the force on the tube due to the intermediate support at \( x = x_b \); \( \delta \) is the Dirac delta function. Here, the force due to loose support \( f(w) \) is modeled by using trilinear spring model with impact stiffness \( K_b \) as,

\[
f(w,\dot{w}) = K_b \left[ w - \frac{1}{2} \left( |w+d_c| - |w-d_c| \right) \right] \tag{3}
\]

where, \( d_c \) is the clearance between support and tube’s outer surface (figure 1(a)).

![Figure 1. (a) Schematic diagram of heat exchanger tube with intermediate loose support (AVB) ;(b) square inline tube array](image_url)

To model the fluid dynamic force on the tube due to cross flow, quasi-steady flow model [13] is utilized. This model is most realistically models the cross-flow over tube array based on the concept of time delay (\( \Delta t \)) between tube motion and corresponding generated fluid dynamic forces. This model is popularly used in the literature [9,19,26,27,29,30,32], because it requires less experimental input data and produces results in good agreement with experiments [13,30]. Accordingly, the fluid dynamic force on the tube (per unit length) due to cross flow can be expressed [9] as,

\[
F_{\text{ext}}(w,\dot{w},\ddot{w}) = -m_e \frac{\partial^2 w}{\partial t^2} - B \frac{\partial w}{\partial t} + C w(x,t - \Delta t) \]

\[
m_e = \frac{\pi}{4} \rho_e D_o^2 C_{ma}, \quad B = \frac{1}{2} \rho_e U_e D_o C_D, \quad C = \frac{1}{2} \rho_e U_e^2 D_o \frac{\partial C_L}{\partial w}, \quad \Delta t = \mu \frac{D_o}{U_e} \tag{4}
\]

where, \( U_e \) is the gap flow velocity of external fluid; \( \rho_e \) is density of external fluid; \( C_D \) and \( C_L \) are the drag and lift coefficients based on gap flow velocity; \( C_{ma} \) is the virtual or added mass coefficient of external fluid. Therefore, the nonlinear governing equation of motion of the loosely supported tube subjected to an external cross-flow can be obtained by combing the equation(1) with the equation (4)[9] as
\[
(m_p + m_c) \frac{\partial^2 w}{\partial t^2} + B \frac{\partial w}{\partial t} - C w(x, t - \Delta t) + E I \frac{\partial^5 w}{\partial x^5 \partial t} + EI \frac{\partial^4 w}{\partial x^4} + \\
- \left[ \frac{E A L}{2 L_0} \left( \frac{\partial^2 w}{\partial x^2} \right)^2 \right] dx + \frac{E A L}{L_0} \left( \frac{\partial w}{\partial x} \frac{\partial^2 w}{\partial x \partial t} \right) dx + \frac{\partial^2 w}{\partial x^2} + \delta(x - x_b) f(w) = 0
\]  

(5)

For expressing the governing equation of motion (equation. (5)) in the dimensionless form, following dimensionless quantities are introduced,

\[
\eta = \frac{w}{D_o}, \quad \xi = \frac{x}{L}, \quad \tau = \Omega_1 t, \quad \Omega_1 = \lambda_1^2 \left( \frac{EI}{m_p L^4} \right)^{1/2}, \quad \hat{m} = \frac{m_o}{\rho I D_o}, \quad u_e = \frac{2\pi U_e}{D_o \Omega_1}, \quad k_e = \frac{AD_o^2}{2L_1}, \quad \beta_e = \frac{m_e}{m_p + m_c}, \quad \alpha = \frac{\lambda_1^2 E^*}{L^2} \left( \frac{I}{Em_p} \right)^{1/2}, \quad d_e = \frac{d_G}{D_o}, \quad \kappa = \frac{K_b}{m_p \Omega_1^2 L}.
\]

(6)

In the equation(6), \( \lambda_1 \) is the dimensionless eigen value corresponding to the first mode of pinned-pinned beam. Introducing these dimensionless quantities of equation. (6) in the equation (5), the dimensionless governing equation of motion can be obtained as,

\[
M \ddot{\eta} + C_1 \dot{\eta} - \left[ k_c \left( \eta^2 \right) d \xi + 2 k_e \alpha \eta \eta' d \xi \right] \eta' + K_1 \eta''' + K_2 \eta'' - K_2 (\eta, \tau - T) + \delta(\xi - \xi_b) \kappa \varphi(\eta) = 0,
\]

\[
M_1 = \frac{1}{1 - \beta_e}, \quad K_1 = \frac{1}{\lambda_1^2}, \quad K_2 = \frac{u_c^2}{8\pi^2 \hat{m}} \frac{\partial C_1}{\partial \eta}, \quad C_1 = \frac{u_c C_D}{4\pi \hat{m}}, \quad C_2 = \frac{m_i}{\lambda_1^2},
\]

\[
T = \mu \frac{2\pi}{u_e}, \quad \varphi(\eta) = \eta - \frac{1}{2} \left( |\eta + d_e| - |\eta - d_e| \right)
\]

(7)

The solution of the above dimensionless governing equation (equation(7)) is evaluated by utilizing the analytical Galerkin method. The normalized eigen functions of the pinned-pinned beam are used as basis functions \( (\psi_i) \). Therefore, the transverse displacement \( \eta(\xi, t) \) of the tube can be expressed in terms of \( N \) numbers of basis functions and corresponding generalized coordinates \( (q_i) \) as,

\[
\eta(\xi, \tau) = \sum_{i=1}^{N} \psi_i(\xi) q_i(\tau) = \psi q
\]

(8)

where, \( \psi \) and \( \psi \) are the column and row vectors of generalized coordinates and the basis functions, respectively. By substituting equation(8) in the equation (7), the discretized form of the governing equation of motion of tube subjected to external cross-flow can be obtained as

\[
M \ddot{q} + G \dot{q} + K q \dot{q}(\tau - T) + k_c \left( q^T C q \right) C q + 2 k_e \alpha \psi(\xi_b) + \kappa \psi(\xi_b)^T \varphi(\eta(\xi_b), \tau) = 0.
\]

\[
M = M_1 I, \quad G = C_1 I + K_1 \alpha A, \quad K = K_1 A, \quad K_z = -K_2 I
\]

\[
A = \int_{0}^{1} \Phi^T d \xi, \quad B = \int_{0}^{1} \Phi^T \Phi' d \xi, \quad C = \int_{0}^{1} \Phi^T \Phi' d \xi, \quad D = \int_{0}^{1} \Phi^T \Phi' d \xi
\]

(9)

where the analytical forms of the matrices \( A, B, C, D \) are given in the literature [34]. The above set of simultaneous delay differential equations (equation(9)) are solved in the MATLAB using an inbuilt
solver ‘dde23’.

3. RESULTS AND DISCUSSION

In this section, the numerical results are presented to investigate the effect of support stiffness on the nonlinear dynamics of a loosely supported heat exchanger tube subjected to an external flow. The geometrical parameters of the tube \( D_o, D_i, L \) are taken as 27 mm, 24 mm, 2 m, respectively. The material properties of the tube (steel) are taken as \( E = 200 \) GPa, \( \rho_p = 7800 \) kg/m\(^3\), \( E^* / E = 8 \times 10^{-5} \) [35]. The geometrical and material properties are taken such that the mass damping parameter to be less than 300 [13] so that the analysis of single flexible cylindrical tube in an array of rigid tubes is valid. The Reynolds number is also considered to be very high so that the assumption of constant fluid force coefficients is valid [13,31]. Considering the in-line square array with pitch to diameter ratio 1.5, the fluid force coefficients are considered as \( C_D = 0.26, \quad \partial C_L / \partial \eta = -8.1 \) and \( C_{m_a} = 1.2 \) [9,13]. For an inline square array, the time delay parameter (\( \mu \)) can be taken as 1 [9].

In order to verify the present mathematical model, the bifurcation diagram of unsupported cross-flow heat exchanger tube undergoing fluidelastic instability [36] is evaluated. The bifurcation diagram is constructed by plotting the midspan (\( \xi = 0.5 \)) displacement (\( \eta_m \)) corresponding to the zero velocity (\( \eta_m = 0 \)) for each value of cross-flow velocity (\( U_e \)). These results are presented in figure 2 along with the similar results of an identical tube in [36]. The comparison in figure 2 shows the good agreement between present results and the results of [36] that verifies the present mathematical model. Since the Galerkin discretization is used in the present model, a convergence study is conducted and the convergence is observed for \( N = 4 \). Hence, all the following numerical results are evaluated with \( N = 4 \).

3.1. Effect of external flow

The heat exchanger tubes are usually subjected to fluidelastic instability when the cross flow velocity exceeds a certain value known as critical velocity. This instability occurs mainly due to the time lag between the motion of tube and the fluid forces imparted on it. Figure 3(a) illustrates the bifurcation diagram of present cross-flow heat exchanger tube with respect to the external flow velocity. As the flow velocity increases, the tube undergoes Hopf bifurcation at A and B (figure 3(a)) resulting in the flutter phenomena. The corresponding variation of flutter frequencies is illustrated in figure 3(b). It
may be observed from figure 3 that the amplitude and frequency of flutter gradually increases with the flow velocity. In contrast to the figure 2, two Hopf bifurcation points (multiple instability regions) can be seen in figure 3 that may be due to the occurrence of very low mass damping parameter at low flow velocities [13].

3.2. Effect of support stiffness

In order to study the effect of constraint stiffness of loose support, the tube is considered to be supported by an AVB support at its midspan ($\xi = 0.5$). The loose support is modeled using trilinear spring model (equation(7))[9] and the clearance between support and tube is taken as $d_g = 0.16$. The bifurcation diagram is evaluated for different values of dimensionless support stiffness ($\kappa$) 10, 100 and 1000. The stiffness value $\kappa=1000$ approximately denotes the metallic support (steel) [9,37]. Since the present interest lies in studying the effect of support material on the dynamics and impact force, the stiffness value is varied up to the two orders less than that of steel (i.e. $\kappa =1000$ to 10). The low support stiffness can be achieved by using low stiffness materials for AVBs or by using low stiffness materials as buffer layers [38] for AVB’s. Figure 4(a)-(c) shows the bifurcation of loosely supported tube with respect to flow velocity for different support stiffness ($\kappa$) 1000, 100 and 10, respectively. In case of metallic support ($\kappa =1000$), figure 4(a) shows the chaotic motion of tube when the oscillation amplitude exceeds the clearance ($d_g = 0.16$).

Figure 3. (a) Bifurcation diagram of unsupported tube under external cross-flow; (b) corresponding variation of flutter frequency with flow velocity.
Figure 4. Bifurcation diagrams of loosely supported tube \((d_g = 0.16)\) with respect to external cross-flow velocity for different values of support stiffness, (a) \(\kappa = 1000\), (b) \(\kappa = 100\), (c) \(\kappa = 10\); and (d) corresponding variation of impact forces.

It also induces large impact forces on the tube that may cause local damage to the tube. It may also be observed that the intensity of chaos increases with the flow velocity. Figures 4(b) and 4(c) illustrate the interesting effect of support stiffness that gradually changes the chaotic motion to the periodic motion. The corresponding variation of impact forces is also plotted in figure 4(d) and it shows the significant reduction in the impact force that imparted on the tube with decrease in the support stiffness. This reduction in impact force may significantly help in improving the fatigue life of the heat exchanger tube. It is important to note that figure 4 also shows the support deformation with the reduction in support stiffness. The allowable deformation of support in the heat exchanger array limited by the possibility of impact with the surrounding tubes (pitch to diameter ratio).

Figure 5. (a) Bifurcation diagram of loosely supported tube \((d_g = 0.16)\) with respect to support stiffness \((U_c = 0.85)\); and (b) corresponding variation of impact force.
Figure 5(a) shows the bifurcation diagram of tube with respect to support stiffness at a cross-flow velocity of \( U_e \) 0.85 m/s. The corresponding variation of impact force is presented in figure 5(b). The change of chaotic motion to the periodic motion and the reduction of impact forces can be clearly seen in figure 5. Figure 6 shows the comparison of phase portraits for the two different dimensionless stiffness \( \kappa = 1000 \) and 10. It can be observed that the motion of the tube becomes smooth (velocity changes smoothly with position) and periodic (figure 6) with decrease in support stiffness.

![Figure 6](image)

*Figure 6. Phase portraits corresponding to the motion of tube for different values of support stiffness (\( U_e = 0.85 \)), (a) \( \kappa = 1000 \), (b) \( \kappa = 10 \).*

The drastic changes in the acceleration of the tube during its motion may cause the large fatigue stresses. Generally to improve the fatigue life, high magnitude of impact forces and acceleration as well as drastic changes in the magnitude of acceleration are to be decreased. At a flow velocity of \( U_e = 0.85 \) m/s, figure 7(a)-(c) shows the variation of dimensionless acceleration with time for different support stiffness. The spikes in the acceleration can be seen in its sinusoidal profile (figure 7(a)-(b)) which correspond to the impact of tube with the support. The magnitude of the acceleration spike decreases progressively with the decrease in support stiffness. It can be observed from the figure 7(c) that the acceleration almost becomes sinusoidal for the support stiffness \( \kappa = 10 \). The corresponding variation of dimensionless impact forces can be seen in figure 7(d)-(f). It can be observed that the impact forces reduces almost 10 times with the reduction of support stiffness by 100 times. Along with the reduction in its magnitude, the change of the impact force becomes smooth along with the increased contact time. The contact time is defined as duration of the contact between tube and support during impact. Thus, the reduced support stiffness cushions the impact by increasing the contact time and reducing impact force.
4. CONCLUSIONS

In this work, the effect of support stiffness in controlling the nonlinear vibration of loosely supported cross-flow heat exchanger tube and support-impact forces is studied. Inline square array with pitch to diameter ratio 1.5 is considered. The bending of slender heat exchanger tube is modelled using Euler-Bernoulli beam theory. The cross-flow induced forces on the tube are modelled using quasi-steady flow model whereas the loose support is modelled using trilinear spring model. The Galerkin method is used to discretize the governing equation of motion and resulting delay differential equations are solved in the MATLAB using an inbuilt solver ‘dde23’. The fluid elastic instability induces flutter in tube while its flutter frequency increases linearly with the cross-flow velocity. The impact of tube with support generates high impact forces on the tube and results in its chaotic motion. The drastic changes in the acceleration of the tube and high magnitude impact forces may cause the damage to the tube or its fatigue failure. With the reduction of support stiffness, the chaotic motion gradually changes to periodic motion, the phase portrait becomes smooth (smooth variation of tube velocity during its motion), the acceleration spikes in its sinusoidal variation gradually disappears, the magnitude of the contact forces greatly reduces. But the reduction of the support stiffness is limited by the allowable deformation of support in the heat exchanger array (pitch to diameter ratio) to avoid impact with surrounding tubes. Thus, if the AVB support is made of low stiffness material or by using the low stiffness buffer layer over stiffer AVB, the impact forces and chaotic motion of the tube can be controlled significantly.
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