Understanding $K/\pi$ ratio distribution in the mixed events

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The event mixing method is analyzed for the study of the event-by-event $K/\pi$ ratio distribution. It is shown that there exists some correlation between the kaon and pion multiplicities in the mixed events. The $K/\pi$ ratio distributions in the mixed events for different sets of real events are shown. The dependence of the distributions on the mean $K/\pi$ ratio, mean and variance of multiplicity distribution in the real events is investigated systematically. The effect of imperfect particle identification on the $K/\pi$ ratio distribution in the mixed event is also considered.

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I. INTRODUCTION

The strangeness production in high energy heavy ion collisions is an important topic. Because there is no strangeness content in the initial states of the collisions, the strangeness in the final states is a result of the complex interactions in the collisions. It has been proved that the strangeness in the final states is sensitive to the dynamics of the process. The enhanced strangeness production in ultra-relativistic heavy ion collisions may be caused by: (i) equilibrated gluon rich plasma phase; (ii) baryon-junctions; (iii) diquark breaking and sea-diquark; (iv) strong color fields etc. In particular, the enhanced production may be related to the formation of the long searching deconfined quark-gluon plasma (QGP) and can shed light on the time scale of chemical freeze-out and therefore carry information about the early stage of a heavy ion collision.

In recent years, with the advent of large acceptance detectors, event-by-event (EBE) analysis has been performed in many investigations. The philosophy of EBE physics is based on following speculation: Although the conditions for QGP production may be reached in every event, the fact that a phase transition from QGP state to hadronic final state is a critical phenomenon implies that deconfined QGP state may occur only in a very small sub-sample of events. So the fluctuations accompanying the phase transition will, in effect, be averaged out in the conventional ensemble analyses. The EBE analysis searches for fluctuations of observables at the event level, so it can be used to select interesting or anomalous event candidates with specific dynamical properties. So the EBE analysis can provide dynamical information which cannot be obtained from the traditional inclusive spectra.

To estimate the amount of dynamical contributions to the distribution of EBE observables, the method of event mixing has been used to create reference events which may contain only statistical fluctuations and have the same inclusive (momentum and etc.) spectra as the original ones. To create mixed events one first sets up a track pool (“event-mixer”) and inserts many (a few hundred or more) real events into the mixer. Then, by selecting particles from different original events in the pool one can get a non-real (mixed) event. In the process of constructing a mixed event only one track in any original event is allowed to be used. The number of particles in a mixed event is a random integer satisfying the multiplicity distribution determined from the original real events in the filled pool. Thus, the mixed events are random samples from the same inclusive track population as the real events, but there is no internal momentum correlations in them. So, the mixed events may be useful in the study of correlations due to some conservation laws, such as energy-momentum conservation law, since the constraint of such conservation laws plays no role in the mixed events.

Recently the EBE analysis is used in experiments to analyze flavor (strangeness) fluctuations in high energy heavy ion collisions. In one of our earlier papers, we tried to calculate the trivial $K/\pi$ ratio distribution for events having statistically independent multiplicity distributions for kaons and pions. For that pure statistical case, the $K/\pi$ ratio distribution was shown to be approximately Gaussian, and its width depends on the shape of the distributions of kaon and pion multiplicities. That result is the starting point for investigating any dynamical effect in the production of strange kaons, though it is not for the real situation in physical problems. In real experimental case, the multiplicity distributions for kaons and pions may be not independent, and the mixed events were used in the study of transverse momentum fluctuations, where the inclusive transverse momentum distributions for real and mixed events are the same, the inclusive distributions of kaon and pion multiplicities are not necessarily the same for real and mixed events. One can see this point clearly in Sect. To the best of our knowledge, it is not known yet what will be the $K/\pi$ ratio distribution in the mixed events before event mixing is actually performed when the ratio distribution...
in real events is given. Such knowledge is important for us to understand the true effect of event mixing. It will also speed up the real data analysis and save a lot of computing resources. We should investigate theoretically on which quantities in the real events the $K/\pi$ ratio distribution in the mixed events depends. The first question one should ask is: What is the dependence of the ratio distribution in the mixed events on that in the original events? If such dependence is strong, the mixed events can hardly be used to estimate the dynamical contribution in the real data.

In this paper, we try to study systematically the effect of event mixing on the EBE $K/\pi$ ratio distribution. By $K/\pi$ we denote the ratio of positively charged kaons to pions in the events. For the sake of simplicity, we assume that there are only pions and kaons in the phase space under study. This paper is organized as following. After some general discussion in Sect. II, we will start in Sect. III from the simplest case by assuming that the multiplicity and $K/\pi$ ratio are both fixed in the original events. For this case, the effect of event mixing on the $K/\pi$ ratio distribution can be treated with rigorously and analytically. Then we will study the influence of $K/\pi$ ratio distribution in real events on the event mixing in Sect. IV. There it is assumed that all the real events have common fixed multiplicity but different EBE $K/\pi$ ratios. Sect. V will devote to the event mixing for original events having a non-trivial multiplicity distribution but with a common fixed EBE $K/\pi$ ratio. Sect. VI is for the most general case for mixing of events with distributed multiplicity and distributed $K/\pi$ ratio. In Sect. VII we will consider the influence of imperfect particle identification on the ratio distributions in the real and mixed events. Sect. VIII is for discussion and conclusion.

II. GENERAL DISCUSSIONS

We first give some general formula for the calculation of some quantities characterizing the $K/\pi$ ratio distribution. Suppose the probability for an event (real or mixed) to have $N - k$ pions and $k$ kaons is $P(N, k)$. Define a generating function

$$G(x, y) = \sum_k \sum_\pi x^k y^n P(\pi + k, k) \quad ,$$

then we have a global $K/\pi$ ratio $R_g$ as

$$R_g \equiv \frac{\langle k \rangle}{\langle \pi \rangle} = G_x'(1, 1)/G'_\pi(1, 1) \quad ,$$

where the subscript together with a prime represents derivative of the generating function with respect to the variables. In the expression, $\langle \cdot \cdot \cdot \rangle$ represents average over all possible configuration.

In an EBE analysis, one measures EBE ratio $r_e \equiv k/\pi$ for every event and observes the distribution of the EBE ratio. For such an analysis, one can get

$$R_e \equiv \langle k/\pi \rangle = \frac{d}{dx} \bigg|_{x=1} \int_0^1 dy \frac{G(x, y) - G(x, 0)}{y} \quad ,$$

$$\langle k^2/\pi^2 \rangle = \left( \frac{d^2}{dx^2} + \frac{d}{dx} \right) \bigg|_{x=1} \int_0^1 dy \frac{G(x, y) - G(x, 0)}{y} \ln \frac{1}{y} \quad ,$$

then one can get the width of the EBE $K/\pi$ ratio distribution as $\sigma = \sqrt{\langle k^2/\pi^2 \rangle - \langle k/\pi \rangle^2}$. In deriving last equation condition $1 - P(N, N) \approx 1$ is used, which is quite a good approximation for current experimental analysis.

III. MIXING OF EVENTS WITH FIXED MULTIPLICITY AND FIXED $K/\pi$ RATIO

We begin to study the effect of event mixing from the simplest case. We assume that every real original event has the same fixed numbers of kaons and pions. Then the $K/\pi$ ratio in the original events is fixed, so does the total multiplicity $N$. We denote in this paper such a case by FF (fixed multiplicity and fixed $K/\pi$ ratio). No chemical fluctuations in the real events are assumed.

Then, we consider the mixing of the original events. By construction, the mixed events have the same total multiplicity $N$ as the original ones. When a mixed event is constructed, $N$ particles are selected from the mixer, and one picks up one and only one particle randomly from tracks of a randomly chosen original event. Since the numbers of kaons and pions are assumed fixed in the original events, the selected particle from an original event can be a pion or a kaon with definite probability. Assume the multiplicity ratio of kaons to pions is $R$, the probability for the chosen
FIG. 1: Distribution of EBE $K/\pi$ ratios in the mixed events from a set of original events with fixed multiplicity and fixed kaon contents.

particle to be a kaon is $R/(1 + R)$, and that for a chosen particle to be a pion is $1/(1 + R)$. Then, the probability for a mixed event to have $k$ kaons and $\pi = N - k$ pions is prescribed by binomial distribution

$$P_{FF}(N, k) = \binom{N}{k} \left( \frac{R}{1 + R} \right)^k \left( \frac{1}{1 + R} \right)^{N-k}.$$  \hspace{1cm} (5)

It is obvious that the multiplicity distributions for kaons and pions in the mixed events are determined by the total (kaons plus pions) multiplicity $N$ together and the $K/\pi$ ratio $R$ in real events. One sees that the inclusive kaon (or pion) multiplicity distribution in the mixed events is very different from that in original events.

The EBE $K/\pi$ ratio distribution in the mixed events together with its dependence on the multiplicity $N$ and the original ratio $R$ can be obtained from Monte Carlo simulation. To get a mixed event one can generate $N$ random numbers. If a random number is less than $R/(1 + R)$, we say that a kaon is produced. Otherwise, a pion is thought to be produced. If the total kaon number in a mixed event is $k$, one can get a $K/\pi$ ratio for the mixed event $r_e = k/(N - k)$. In Fig. 1, the normalized $r_e$ distributions $P(r_e)$ are shown for $R=0.15$ and 0.2 as examples. For each case 5 million mixed events are generated. For each $R$, four values of $N$, 200, 250, 300 and 350, are taken. Gaussian fits to all the distributions are shown in the same figure. For fixed $R$, one can see that the width of the distribution decreases with the increase of $N$. For the same $N$, the width increases with the increase of $R$.

The mean values and widths of the distributions can be obtained from a Gaussian fit to the histograms in Fig. 1 and the fitted results are shown with error bars in Fig. 2 as functions of inverse root of multiplicity, $1/\sqrt{N}$. It seems that finite particle number effect plays the only role in the mixing process for both $R = 0.2$ and 0.15 since $\sigma/R \propto 1/\sqrt{N}$ is quite a good approximation for the points shown in the figure. The slope for $R = 0.15$ is slightly larger than that for $R = 0.20$. The mean and the width of the distributions can also be calculated from Eq. (5). To use our general equations Eqs. (3) and (4), one needs to have the generating function for distribution Eq. (5). The generating function for such a distribution is

$$G_{FF}(x, y) = \left( \frac{Rx + y}{1 + R} \right)^N.$$  \hspace{1cm} (6)

The calculated results for six multiplicities $N$ are also shown in Fig. 2 as solid circles linked by curves. One can find that the mean values from two different ways agree with each other quite well. But there is slight difference between the widths from the two ways. The slight difference indicates that the EBE $K/\pi$ ratio distribution in the mixed events is not a perfect Gaussian. For smaller $R$, the difference between the results from two ways is smaller.
FIG. 2: Mean values $R_e$ and relative widths $\sigma/R_e$ of the EBE $K/\pi$ ratio distributions in the mixed events as functions of the inverse root of the event multiplicity. The points with error bars are fitted results of Fig. 1 to Gaussian distributions, and points linked by curves are from theoretical calculation.

IV. MIXING OF EVENTS WITH FIXED MULTIPlicity AND DISTRIBUTED $K/\pi$ RATIOS

As the second simple case for investigating effect of event mixing on $K/\pi$ ratio distribution, we assume that the real events have fixed common multiplicity $N$ but different strange contents (FD). Thus the real events have a non-trivial $K/\pi$ ratio distribution.

Assume that the $N$ particles in a mixed event are picked up from real events labelled $i_1, i_2, \cdots, i_N$ and that the corresponding $K/\pi$ ratio in those events are $r_{i_1}, r_{i_2}, \cdots, r_{i_N}$, respectively. Not losing any generality, we assume that the particles picked from events $i_1, i_2, \cdots, i_k$ are kaons and others are pions. Since we have no interest in the order of the original events from which the kaons are picked up, the probability for such a configuration is then

$$
\left( \begin{array}{c} N \\ k \end{array} \right) \frac{r_1}{1 + r_1} \frac{r_2}{1 + r_2} \cdots \frac{r_k}{1 + r_k} \frac{1}{1 + r_{k+1}} \cdots \frac{1}{1 + r_N}.
$$

(7)

Let the mean $K/\pi$ ratio in the original real events be $R$, and rewrite $r_i$ as $R + \delta_i$. Since the mixed events are constructed by randomly picking up particles from randomly chosen original events, the final probability for a mixed event to have $k$ kaons and $N - k$ pions can be obtained from above expression by averaging it over all possible configurations. Since $\langle \delta_i^{2m+1} \rangle = 0$ for $m = 0, 1, 2, \cdots$, one can get

$$
P_{FD}(N,k) = P_{FF}(N,k) - \frac{\sigma_0^2 N}{(1 + R)^2} [P_{FF}(N - 1, k - 1) - P_{FF}(N, k)] + O(\sigma_0^4),
$$

(8)

where $\sigma_0^2 = \langle \delta_i^2 \rangle$ is the width squared for the ratio distribution in the real events. As stated above, the value of $\sigma_0^2$ is determined by the involved statistical and dynamical contributions in the strangeness production. Normally, $\sigma_0^2$ is quite small, therefore the contribution from higher order terms can be safely neglected. From last equation one can see that the width of the EBE $K/\pi$ distribution will be slightly larger than in the case with fixed multiplicity and fixed $K/\pi$ ratio. From this equation the mean and width of $K/\pi$ ratio distribution can be calculated. To compare with the calculation, we can also perform a Monte Carlo simulation for the case. We assume that the distribution of $K/\pi$ ratios in the original events is a Gaussian with a relative width $\sigma_0/R = 0.2$. With the same choices of $R$ and $N$ as in Fig. 1, eight distributions of the EBE $K/\pi$ ratios in 5 million mixed events are shown in Fig. 3. One can compare this figure with Fig. 1 and find that the corresponding distributions are almost the same. This is not surprising since the width $\sigma_0$ used in the simulations shown in Fig. 3 is only 0.03 for $R = 0.15$ or 0.04 for $R = 0.2$. Thus the $\sigma_0^2$ term
FIG. 3: Distributions of the EBE $K/\pi$ ratios in the mixed events from original ones with fixed multiplicity and distributed ratios. The $K/\pi$ ratio distributions in original events are assumed to have a relative width $\sigma_0/R = 0.2$ for illustration.

In Eq. (8) has little effect on the final results. The mean values and relative widths of the distributions are also very close to those shown in Fig. 2 and will not be discussed any more. The insensitivity of the relative width $\sigma/R_e$ (or the ratio distribution) in the mixed events to the detail of the ratio distribution of original real events is crucial for detecting dynamical contributions in experimental data analysis from the event mixing method. Because the original distribution is a mixture result from dynamical and statistical contributions and the dynamical correlations in the original events are moved in the mixed events, the difference between the two distributions from real and mixed events can be used to quantify the dynamical contribution to the distribution in the real events. But it should be pointed out that the dynamical contribution can make the width of the ratio distribution larger or smaller, as have been found in [17]. So, we think it better to use $\sigma_{\text{dyn}}^2 = (\sigma_{\text{mixed}} - \sigma_0)^2$ to measure the dynamical contribution instead of $\sigma_{\text{mixed}}^2 - \sigma_0^2$ in [17].

V. MIXING OF EVENTS WITH DISTRIBUTED MULTIPLICITY AND FIXED $K/\pi$ RATIO

In real experiments the event multiplicity is normally not fixed but has a non-trivial distribution. The multiplicity distribution will certainly affect the $K/\pi$ ratio distribution in the mixed events, because in Sect. IV we have seen that the ratio distributions for mixed events with different multiplicities are different. To understand the event mixing for real experimental events, it is an important step to investigate the mixing of events with some multiplicity distribution.

Suppose that the multiplicity distribution in the original events is $M(N)$ and that all the real events have the same fixed $K/\pi$ ratio $R$. We denote this set of original events by DF. Since the mixed events have the same total multiplicity distribution as the original ones, the probability for a mixed event to have $k$ kaon particles and $N - k$ pions is then

$$P_{\text{DF}}(N, k) = M(N)P_{\text{FF}}(N, k).$$  (9)

From this probability one can get the inclusive kaon (pion) number distribution by summing over pion (kaon) number. It is clear that the joint probability $P_{\text{DF}}(k + \pi, k)$ cannot in general be equal to the product of the inclusive pion and kaon number distributions. One can see this point from a simple fact that

$$\langle k\pi \rangle = \sum_k \sum_\pi k\pi P_{\text{DF}}(k + \pi, k) \neq \sum_k \sum_\pi kP_{\text{DF}}(k + \pi, k) \times \sum_k \sum_\pi \pi P_{\text{DF}}(k + \pi, k) = \langle k \rangle \langle \pi \rangle.$$
This point can be directly checked even for the case FF with Eq. (5). From that equation one can get
\[ \langle k\pi \rangle = N\langle k \rangle - \langle k^2 \rangle, \quad \langle k \rangle\langle \pi \rangle = N\langle k \rangle - \langle k \rangle^2. \]

They cannot be equal to one another for mixed events. That means there is some (non-dynamical) correlation between multiplicities of kaons and pions in the mixed events though the mixed events are constructed by randomly choosing one particle from each randomly chosen real event. The correlation is caused by the demand that the mixed events have the same total multiplicity distribution as the real events.

Despite such an intrinsic correlation between multiplicities of kaons and pions in the mixed events, we can go further to analyze the \( K/\pi \) ratio distribution in the those events.

Let
\[ \sum_N z^N M(N) = f(z), \tag{10} \]
and using Eqs. (5) and (6), the generating function for \( P_{DF}(N, k) \) can be written as
\[ \sum_k \sum_{\pi} x^k y^\pi P_{DF}(k + \pi, k) = f \left( \frac{Rx + y}{1 + R} \right). \tag{11} \]

It is clear that the ratio distribution in the mixed events depends on the shape of total multiplicity distribution \( f(z) \) and the mean ratio \( R \). In many experiments the multiplicity distribution can be parameterized as a negative binomial (NB) or modified negative binomial (MNB) distributions, whose generating functions are, respectively,
\[ G_{NB}(z) = (1 - r(z - 1))^{-\kappa}, \]
\[ G_{MNB}(z) = \left( \frac{1 - \Delta(z - 1)}{1 - r(z - 1)} \right)^\kappa. \]

The NB distribution can be regarded as a special case of the MNB distribution with parameter \( \Delta = 0 \). To see the dependence of the ratio distribution in the mixed events on the shape of total multiplicity distribution or on the parameters in the generating functions, we fix \( R = 0.15 \) and the mean multiplicity \( \langle N \rangle = 270.13 \), the same as in [15, 16]. For the NB distribution, \( \langle N \rangle = r\kappa \), therefore after \( \langle N \rangle \) is fixed only one parameter is left which can be
used to specify the variance of total multiplicity distribution. In our calculation we choose \( r \) to change from 0.05 to 0.89. The calculated mean value and relative width of the ratio distribution in the mixed events are shown in Fig. 4. The mean value seems independent of the value of parameter \( r \), while the width decreases with \( r \) slightly. Since the relative variance of the multiplicity distribution has a linear dependence on the parameter \( r \) in the NB distribution, the dependence of the ratio \( \sigma/R_e \) on the relative variance of the multiplicity distribution is similar to that shown in 4.

For the MNB distribution, there are three parameters \( \kappa, r, \) and \( \Delta \). In our calculation, we fix \( R = 0.15 \) and the mean multiplicity \( \langle N \rangle = \kappa(r - \Delta) = 270.13 \), the same as in the case for NB distribution, and we choose the relative variance of the multiplicity distribution \( w = (\langle N^2 \rangle - \langle N \rangle^2)/\langle N \rangle = r + \Delta + 1 \) to be 1.0, 2.0, 2.2, and 2.5. Then the left parameter \( \kappa \) is changed from 100 to 300. The results for the mean values and the relative widths of the ratio distribution are shown in Fig. 5. The mean \( R_e \) is the same constant for all the choice of parameters, while the relative width \( \sigma/R_e \) is independent of \( \kappa \) and increases a little with the variance \( w \) of the multiplicity distribution. The \( w \) dependence of the relative width \( \sigma/R_e \) is a direct consequence of the multiplicity \( N \) dependence shown in Fig. 2.

An important observation is that the calculated width \( \sigma/R_e \) for the mixed events is much smaller than 23% given in 15, 16, 17.

VI. MIXING OF EVENTS WITH DISTRIBUTED MULTIPLICITY AND DISTRIBUTED \( K/\pi \) RATIO

The last and the most general case for our investigation of the mixing of real events is that the real events have both the total multiplicity and \( K/\pi \) ratio distributions (DD). For given total multiplicity distribution \( M(N) \) the corresponding joint probability for a mixed event to have \( k \) kaons and \( N - k \) pions is

\[
P_{DD}(N, k) = M(N)P_{FD}(N, k).
\]

Because of the result in Sect. 4 that the ratio distributions in the mixed events for cases FF and FD are almost the same, one has reason to expect that the results from Eq. (12) will be almost the same as those from Eq. (11). So, this case will not be discussed any more.
VII. EFFECT OF IMPERFECT IDENTIFICATION ON EVENT MIXING

In high energy heavy ion collisions, the identity of a particle is obtained from its charge and \( dE/dx \) information with the help of the Bethe-Bloch function. The rate of energy loss \( dE/dx \) depends on the charge, mass and momentum of the particle in a complex way, and different particles may have very close \( dE/dx \) values in some momentum range. Therefore, normally one cannot make definite conclusion about a particle's identity from such information. Instead one can only say, for example, that a track is a pion or a kaon with a probability less than 1. This means that a track which is thought to be a kaon may in fact be a pion, and vise versa. If one tries to get \( K/\pi \) ratio with the Maximum-Likelihood Method from the \( dE/dx \) information, as done in [17], such uncertainty will play a role in the \( K/\pi \) ratio distribution. Since the efficiency of identification depends on the momenta of particles, it is extremely difficult to model the identification process. In the following we only show that the identification efficiency has important effect on the observed \( K/\pi \) ratio distribution. To do that, we assume that the identification efficiency is the same for all kaons (or pions).

For simplicity, we assume that only kaons and pions exist in the final states. Considering the identification uncertainty of a track from its \( dE/dx \) information, we assume that the probability for a thought-to-be kaon to be indeed a kaon is \( p_1 \), and that for a thought-to-be pion to be a real pion is \( p_2 \). We assume that the values of \( p_1 \) (\( p_2 \)) is the same for all thought-to-be kaons (pions). For an event with \( K \) thought-to-be kaons and \( N - K \) thought-to-be pions, the apparent \( K/\pi \) ratio is \( R = K/(N - K) \). But the real numbers of kaons and pions may change from event to event even if \( N \) and \( K \) are the same for all events, and the probability \( P(N,k) \) of having \( k \) real kaons and \( N - k \) real pions can be calculated. In such a case, the \( K/\pi \) ratio will have an intrinsic non-trivial distribution, although it is assumed that every original event has the same numbers of thought-to-be kaons and pions. It is not difficult to prove that the generating function of the intrinsic distribution \( P(N,k) \) is

\[
G_{\text{intrinsic}}^{N,K}(x, y) = (p_1 x + (1 - p_1) y)^K ((1 - p_2)x + p_2 y)^{N-K}.
\]

From this expression, we know that the real global \( K/\pi \) ratio should be

\[
R_g = \frac{k}{N-k} = \frac{p_1 R + (1 - p_2)}{(1 - p_1)R + p_2},
\]

which may be very different from the apparent ratio \( R = K/(N - K) \). In the limiting case \( p_1 = p_2 = 1/2 \), i.e., when the kaons and pions can't be distinguished, the global ratio \( R_g \) will be 1, no matter how large \( R \) is. Of course, the global ratio is equal to the apparent one if \( p_1 = p_2 = 1 \), as demanded physically.
We are particularly interested in the width $\sigma$ of the intrinsic $K/\pi$ ratio distribution relative to the apparent mean ratio $R$, $\sigma/R$. In the following we discuss the case with $p_1 = p_2 = p$. More general cases can be studied in the same way. For definiteness we take in the following the apparent ratio $R = 0.15$, close to the $K/\pi$ ratios observed in current high energy heavy ion experiments. The mean and relative width of the intrinsic ratio distribution are shown in Fig. 6 as functions of $1/\sqrt{N}$, inverse root of the event multiplicity. This relative width calculated from Eqs. (13), (3), and (4) increases with the $1/\sqrt{N}$. Though linear relation between $\sigma/R$ and $1/\sqrt{N}$ can still be seen, the relation $\sigma \propto 1/\sqrt{N}$ is not longer a good approximation. For poorer identification (smaller $p$) the relative width is larger for the same $N$.

Then we study the effect of event mixing on the $K/\pi$ ratio for the case with imperfect identification. As in Sect. II we first fix the total multiplicity $N$ and the apparent $K/\pi$ ratio $R$. Using Eqs. (3) and (13), one can get the generating function for the joint distribution of real kaon and pion multiplicities in the mixed events as

$$G_{\text{mixed}}^{N,R}(x, y) = \left[ \frac{R_p + (1 - p) x + p + R(1 - p) y}{1 + R} \right]^N,$$

and the resulting mean values and relative width of the $K/\pi$ ratio distribution are shown in Fig. 7 as functions of $1/\sqrt{N}$ for several identification efficiency $p$. The mean values are again constants as in the case without the event mixing, and the relative width decreases with $\sqrt{N}$ but is larger than in Fig. 6 for the same $\sqrt{N}$. As in Fig. 6 the relation $\sigma \propto 1/\sqrt{N}$ is neither a very good approximation though the curves are almost linear. An important observation is that $\sigma/R$ depends now strongly on the identification efficiency $p$ and can be about 23% for $p$ a little less than perfect value 1.0.

Now we can study the mixing of events with a multiplicity distribution and imperfect particle identification. The generating function for the joint kaon and pion multiplicity distribution in the mixed events can now be formulated using Eqs. (13) and (14). When the multiplicity distribution is assumed to be a modified negative binomial distribution, the generating function for the joint kaon and pion multiplicity distribution in the mixed events with imperfect particle identification efficiency $p$ is

$$G_{\text{mixed}}^{\text{MNB}}(x, y) = \left( \frac{1 - \Delta u}{1 - ru} \right)^\kappa \text{ with } u = \frac{[R_p + (1 - p)x + p + R(1 - p)y]}{1 + R}.$$

With this generating function we can perform theoretical calculation. We fix the mean multiplicity $\langle N \rangle = \kappa(r - \Delta) = 270.13$ as before. From Fig. 6 we have seen that the ratio distribution has extremely weak dependence on parameter...
FIG. 8: Relative widths $\sigma/R$ as functions of $w$, which is a measure of the variance of multiplicity distribution, for mixed events with a modified negative binomial multiplicity distribution and without perfect particle identification.

$\kappa$. So that we fixed it to be 300. Then we can change the relative variance, $w = (\langle N^2 \rangle - \langle N \rangle^2)/\langle N \rangle = 1 + r + \Delta$, of the total multiplicity distribution as a free parameter from 1.9 to 2.9. Since $R_e$ in this case is neither measurable nor meaningful, we focus only on the relative width $\sigma/R$ in the following. The dependence of the relative width of the ratio distribution is shown in Fig. 8. The relative width of the $K/\pi$ ratio distribution increases slowly with $w$ but has a very strong dependence on the identification efficiency $p$. For poorer identification the relative width is larger. The value of the relative width can be close to $23\%$ given in [17] for quite high identification efficiency. For other total multiplicity distributions, the mean and relative width of the EBE $K/\pi$ ration distribution in the mixed events can also be calculated in a similar way.

VIII. DISCUSSION AND CONCLUSION

From above sections one can know how to derive the EBE $K/\pi$ ratio distribution for the mixed events once the total multiplicity distribution and global $K/\pi$ ratio in the real events are known. Then we can understand why the event mixing method can be used to compare with results on $K/\pi$ ratio distribution from real events. Contrary to naive expectation, there may exist correlation between multiplicities of kaons and pions in the mixed events. Although the mean EBE $K/\pi$ ratio in the mixed events is stable for all cases with perfect particle identification, the width shows a quite strong dependence on the total multiplicity distribution and the global $K/\pi$ ratio. But the ratio distribution in the mixed events has an extremely weak dependence on the shape of EBE $K/\pi$ ratio distribution in the real events. This property enables us to estimate the dynamical contributions to the ratio distribution in the real events. For events without perfect particle identification, the intrinsic uncertainty of particle identity makes the relative width of the $K/\pi$ ratio distribution in the mixed events larger and dependent on the multiplicity distribution in a more complicated way. Currently observed relative width of the EBE $K/\pi$ ratio distribution seems to be a combinational result from statistical fluctuations (mainly due to the finite total multiplicity) and the imperfect particle identification. More theoretical and experimental studies are needed.

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[1] E. Shuryak, Phys. Rev. Lett. 68, 3270 (1992).
[2] S.E. Vance and M. Gyulassy, Phys. Rev. Lett. 83, 1735 (1999).
[3] A. Capella and C.A. Salgado, Phys. Rev. C 60, 054906 (1999).
[4] M. Bleicher, W. Greiner, H. Stöcker, and N. Xu, Phys. Rev. C 62, 061901 (2000), and references therein.
[5] P. Koch, B. Müller and J. Rafelski, Phys. Rep. 142, 167 (1986).
[6] L.P. Csernai, Introduction to Relativistic Heavy Ion Collisions, J. Wiley, New York, 1994.
[7] M. Gazdzicki and St. Mrowczynski, Z. Phys. C 54, 127 (1992).
[8] M. Gazdzicki, A. Leonidov and G. Roland, Eur. Phys. J. C 6, 365 (1999).
[9] M.L. Cherry et al. (KLM Collab.), Acta Phys. Pol. B 29, 2129 (1998).
[10] M. Bleicher, M. Bellacem, C. Ernst et al., Phys. Lett. B435, 9 (1998).
[11] St. Mrowczynski, Phys. Lett. B 439, 6 (1998).
[12] St. Mrowczynski, Phys. Lett. B 459, 679 (1999).
[13] F. Liu, A. Tai, M. Gazdzicki et al., Eur. Phys. J. C 8, 649 (1999).
[14] R. Stock, Page 507 in Proceedings of NATO Advanced Study Workshop on Hot Hadronic Matter: Theory and Experiment, Divonne-les-Bains, France, 27 Jun - 1 July 1994.
[15] G. Roland, talk given in Hirschegg '97 Workshop on QCD Phase Transitions, Hirschegg, Germany, 13-18 Jan. 1997, NA49 Note Number 116.
[16] F. Sikler, talk given in Quark Matter ’99, Torino, Italy, 10-15 May, 1999, NA49 Note Number 200.
[17] S.V. Afanasiev et al., NA49 Collaboration, Phys. Rev. Lett 86, 1965 (2001); C. Roland, Ph. D thesis, Frankfurt University, 1999.
[18] C.B. Yang and X. Cai, Int. J. Mod. Phys. A 16, 1227 (2001).