Determination of the theoretical trajectory of a tractor at a turn

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Abstract. The most complete idea of the stability of the curvilinear motion of a tractor is given by the nature of the change in the trajectory of motion of its kinematic center and the coordinates of the turn: the abscissa and ordinate of the headland, showing both the transverse and longitudinal deviations of the machine-tractor unit, including those determining its productivity and the required quality of agricultural operations. According to the operating conditions, the methods of motion of the unit are chosen, both on the main array of the field and on the headland. The paper presents explicit equations for the analytical calculation of the theoretical coordinates of the points of the kinematic center trajectory of the tractor when it performs a circular loopless rotation, which allowed determining the theoretical trajectory of a complete cycle of circular loopless rotation of a tractor depending on the following parameters. These parameters are as follows: design - base, distances between the axes of the pivots, maximum angles of rotation of the internal wheels; operational - forward speed of a tractor, the angular speed of the steered wheels in the transverse plane. The author made the attempt to analyze and compare the most universal ways of turn: the front steered wheels and the front and rear steered wheels by turning them in different directions relative to the core.

1. Introduction

In order to reduce economic costs, to achieve high productivity of the machine and tractor unit (MTU) and the required quality of agricultural operations, it is necessary to ensure such a mode of operation that would most fully comply with the technical characteristics and agrotechnical requirements and operating conditions, for which the following aspects are taken into account:

1. The types of agricultural machines, which are determined by agrotechnical requirements
2. The number of machines in MTU and the speed mode of its operation, which satisfies the permissible limits of agricultural requirements, depending on the technical and operational properties of the machines.
3. The modes of motion of the unit, both on the main array of the field and on the headland based on the operating conditions.
Thus, the technology for the production of MTU should provide the compliance with the kinematic characteristics of its motion and agrotechnical conditions of the work.

2. Problem statement
MTU in the field conditions during technological operations for sowing agricultural crops, processing soil, plants, etc. undergoes a path of considerable length, alternating productive work (working strokes) with unproductive work (idle runs and turns). The constantly and almost cyclically repeating nature of changes in working strokes, idle runs and turns exactly determines the way the unit moves across the field at each of its sections. At the same time, for example, when working on medium-sized fields in the aisles of row crops, an MTU spends up to a third of the shift time on turns.

It is necessary to take into account that the deterioration in the quality of work, the increase in dynamic loads on both a tractor and an agricultural machine, the increase in the level of psychomotor costs of a driver, the decrease in the technical and economic indicators of the performance of agrotechnical operations, partial and in some cases complete, destruction of cultivated plants directly depends on the wrong way of driving the machine on the headland. In addition, on the headland, the MTU propellers compact and spray the soil more significantly than in the main section of the field thereby reducing its crop yields.

3. Materials and methods
It is recommended to study MTU curvilinear motion by the nature of its behavior during a circular loopless turn, since most of the used turning methods are based on its combinations.

The curvilinear motion of MTU, carried out mainly on the headlands, is the most complex element of its kinematics, due to the fact that individual points of the unit move at different speeds and describe trajectories that are significantly different from each other. As a rule, in practice, a turn is a gradual transition from an infinitely large radius to a minimum one (during the transition from motion along a straight section to motion along a curve of constant radius – “entering a turn”) [1]; then motion along a section of constant curvature (“steady turn”) [2, 3] and, finally, the transition from the minimum - to an infinitely large radius (when passing from a section of constant radius to straight motion – “the exit from the turn”). However, in most studies, it is accepted that turns are made with the minimum permissible constant radius.

The most difficult ones are the sections both in terms of dynamics and kinematics – “corner entry” and “steady turn” [1-6], on which the operational properties of the unit are noticeably deteriorated: due to large abruptly varying loads and intense lateral slip, significant deviation from the required trajectory of motion, the dynamic loading of the tractor, agricultural machine and soil increases.

The studies showed that the necessary and fairly complete assessment of the kinematic characteristics of the curvilinear motion of MTU can be carried out by the nature of the change in the trajectory outlined on the surface of one of its characteristic points (center of gravity, middle of the bridge, etc.), taken as the kinematic center. Since the experimental studies to assess the influence of the design and operational parameters of the tractor and MTU based on it on the change in kinematic characteristics during curvilinear motion are a very laborious and rather expensive process, they require a large amount of experiments and the determination of many parameters, including in the field conditions of real operation, then, it is obvious that a calculated assessment is more acceptable - the analytical description of the process of cornering by methods of mathematical modeling [2, 3, 7-13].

Therefore, we set the task of the determination of the theoretical trajectories of the kinematic center of the unit for various methods of wheel control, their comparison and evaluation when making a circular loopless turn, based on the condition of wheel rolling without slipping and the coincidence of the instantaneous centers of rotation of all tractor wheels, that is, correct turn. In order to solve this problem, we chose two of the most universal kinematic methods of turn as the object of research. In these methods the direction of motion was changed by turning either only by the front (turning method I), or only front and rear, asynchronously - in different directions (turning method II) by controlled
wheels relative to tractor core [1]. The design diagram (Figure 1) shows the method of turning II. Turning method I was obtained when the rear axle wheels were in neutral position.

The point on the longitudinal axis of a tractor corresponding to the intersection of the perpendicular to it, drawn from the instantaneous center of rotation \( O_T \) (point E in Figure 1), is taken as the kinematic center. In case of turning by the front wheels, the kinematic center of a tractor corresponds to the middle of its rear axle (coincides with point D).

![Design diagram of a wheeled tractor with all steering wheels](image)

**Figure 1.** Design diagram of a wheeled tractor with all steering wheels

In the work [1], we obtained in a parametric form the equations to determine the current coordinates \( x(t) \) and \( y(t) \) of the theoretical curve of the trajectory of the kinematic center of a vehicle at the entrance to the turn, corresponding to points \( D \) - for the method of turning I or \( E \) - for the method of turning II, according to the scheme in Figure 1 [14-17].

\[
x(t) = v \int_{0}^{t} \sin \left[ \int_{0}^{t} \frac{B}{2} \left( t \omega_1 \tau + t \omega_2 \tau \right) + L \right] \cos \omega_1 \tau \cos \omega_2 \tau \, d\tau \]

\[
y(t) = v \int_{0}^{t} \cos \left[ \int_{0}^{t} \frac{B}{2} \left( t \omega_1 \tau + t \omega_2 \tau \right) + L \right] \cos \omega_1 \tau \cos \omega_2 \tau \, d\tau
\]

where \( L \) – tractor base, \( m \); \( B \) – distance between the axles pivots of the steering linkage of the tractor, \( m \); \( \nu \) – forward speed of the tractor kinematic center, \( m \cdot s^{-1} \); \( \omega_1 \) and \( \omega_2 \) – the angular speeds of rotation of the front and rear steerable wheels of a tractor in the transverse plane relative to its core, \( \text{s}^{-1} \); \( t, \tau \) – time of motion of a tractor at the turn, \( s \).

The authors also performed the assessment of the influence of some design and operational indicators of a tractor on the parameters of the curve of this trajectory.

However, it is known [14, 17] that during the construction and analysis of the theoretical dependence of a curvilinear trajectory, it is more convenient to use its explicit analytical specification.
After a nonlinear approximation of the data obtained using the parametric equations of coordinates (1) and (2) of the curve of the trajectory of the entrance to the turn [1, 14], we obtained an explicit form of the function described in general form by the formula [18-21].

$$y(x) = px^q,$$

where $p$ and $q$ – quite definite constants, $0 < q < 1$.

Thus, at the turn section, the trajectory of motion with a high degree of accuracy is approximated by an explicit function (3), since the comparison of the curves of the turn entry trajectories calculated from the parametric setting of functions (1, 2) and by formula (3) showed that they practically coincide, since the maximum error in the approximation was $2 \times 10^{-3}$ m [1].

At the end of the “turn entry” section, when the steered wheels reach the maximum angles at a certain time $t = t_{\text{max}}$, the tractor switches to a section of steady motion along a trajectory with a constant radius of curvature $R_T$.

Since the curve of the turn entry trajectory (3), must smoothly and continuously pass into a circle of radius $R_T$ at the end of the section - in its upper part, then in order to determine the theoretical trajectory in the section of steady motion, using the conditions of its smoothness and continuity, we carried out gluing of functions of the initial section of the turn (turn entry) and the circle (steady turn), having previously determined the coordinates of its center $O_T$ and received a piecewise-smooth function of the form $y = f(x)$ [18-21]:

$$f(x) = \begin{cases} (p(v)x^q, & x \leq x_0, \\ R_T^2 - \left( x - x_0 - \frac{R_T p(v) q \cdot x_0^{q-1}}{1 + \left( p(v) q \cdot x_0^{q-1} \right)^{1/2}} \right)^2 \right)^{1/2} + p(v) \cdot x_0^q - \frac{R_T}{\left(1 + \left( p(v) q \cdot x_0^{q-1} \right)^{1/2}\right)^{3/2}}, & x > x_0. \end{cases}$$

Thus, matching the circle of radius $R_T$ with the curve of the turn entry section at the top point, we obtained the current coordinates of the theoretical MTU trajectory, corresponding to the selected kinematic center of a tractor, for two stages of turn – “turn entry” and “steady turn” [1-3].

For the section “exit from the turn” we will consider the trajectory of the “mirror” trajectory of the entrance into the turn, since when leaving the turn, a transition occurs from a more dynamically loaded state to a less loaded one.

The curves of motion trajectories according to (4) by turning method I $\omega_1 = 0.157 \, \text{c}^{-1}$, $\omega_2 = 0 \, \text{c}^{-1}$, $t=4$ c, $a_{1v} = 28.84^\circ$ (the maximum average angle of rotation of the front axle wheels relative to the core in the transverse plane - corresponds to the maximum internal turn with respect to the center of rotation of the wheel of the front axle $\alpha_{1}^{\prime}\prime = 36^\circ$), $= 0^\circ$ (the maximum average angle of rotation of the wheels of the rear axle relative to the core in the transverse plane - corresponds to the maximum angle of rotation of the internal turn with respect to the center of rotation of the wheel of the rear axle $\alpha_{2}^{\prime}\prime = 0^\circ$), $B = 2.2$ m, $L = 2.5$ m, $R_{T1} = 4.54$ m [11] and turning method II $(\omega_1 = 0.157 \, \text{c}^{-1}$, $\omega_2 = 0.157 \, \text{c}^{-1}$, $t=4$ c, $a_{1v} = 23.9^\circ$ corresponds to $a_{1v} = 36^\circ$), $a_{2v} = 23.9^\circ$ (corresponds to $a_{2}^{\prime}\prime = 36^\circ$), $B = 2.2$ m, $L = 2.5$ m, $R_{T2} = 2.82$ [1]) at speeds $v = 1.5 \, \text{m/s}$ are presented in Figure 2.
Figure 2. Trajectories of turn at $v=1.5 \text{ m} \cdot \text{s}^{-1}$: 1 – method I: $\omega_1 = 0.157 \text{ s}^{-1}$, $\omega_2 = 0 \text{ s}^{-1}$; 2 – method II: $\omega_1 = 0.157 \text{ s}^{-1}$, $\omega_2 = 0.157 \text{ s}^{-1}$

Figure 3. Trajectories of turn by method I at $v=1.25 \text{ m} \cdot \text{s}^{-1}$: 1 – $L=2.0 \text{ m}$, $B=1.8 \text{ m}$; 2 – $L=2.5 \text{ m}$, $B=1.8 \text{ m}$; 3 – $L=2.5 \text{ m}$, $B=2.2 \text{ m}$

Figure 4. Trajectories of turn at $v=1.5 \text{ m} \cdot \text{s}^{-1}$: 1 – method I: $\omega_1 = 0.21 \text{ s}^{-1}$, $\omega_2 = 0 \text{ s}^{-1}$; 2 – method II: $\omega_1 = 0.21 \text{ s}^{-1}$, $\omega_2 = 0.21 \text{ s}^{-1}$

At the same time, it was revealed that with the turning method I, the maximum abscissa (width) of the headland was $x_{\text{max}1} = 9.81 \text{ m}$, the maximum ordinate (height) of the headland $y_{\text{max}1} = 7.67 \text{ m}$, the turning time $t_{\text{max}1} = 13.74 \text{ s}$ and the length of the turning trajectory $S_{\text{max}1} = 20.62 \text{ m}$, and the method turn II – $x_{\text{max}2} = 6.78 \text{ m}$, $y_{\text{max}2} = 5.59 \text{ m}$, $t_{\text{max}2} = 9.86 \text{ s}$ and $S_{\text{max}2} = 14.8 \text{ m}$ (Figure 2). The application of method I lead to the increase in the above mentioned parameters compared to method II by 3.03 m (30.9%), 2.08 m (27.12%), 3.9 s (28.24%) and 5.82 m (28.24%) respectively.

It is obvious that from the point of view of ensuring better maneuverability the turning method II is the most rational one, in which the wheels of the front and rear axles are steered (Figure 1). However, due to the fact that at the same time, because of significant (more than 40%) decrease in the instantaneous turning radius in comparison with method I (from $R_{T2} = 2.82 \text{ m}$ to $R_{T1} = 4.54 \text{ m}$) at the same angles of rotation of the corresponding wheels of the axles and the consequent increase of the centrifugal force of inertia acting on the machine almost 1.6 times of the lateral force, the directional stability decreases and the dynamic loads on MTU and the soil increase. The motion along an arc of trajectory of greater curve and a smaller radius also leads to the increase in the moment turning a
tractor around the vertical axis, thereby increasing the risk, not only of loss of directional stability, but also of the development of the rear axle skidding.

The lengths of the sections of the steady turn for turning methods I and II were \( S_{11} = 8.62 \text{ m} \) and \( S_{32} = 2.79 \text{ m} \) respectively and the time of their passage - \( t_{11} = 5.74 \text{ s} \) and \( t_{32} = 1.86 \text{ s} \). However, with equal length and time of passage of transitional sections of the turn \( S_{x} = 12 \text{ m} \) and \( t_{x} = 8 \text{ s} \) using method II the share of the total length of the trajectories of the stages “turn entry” and “turn exit” in its total length \( S_{\text{max}2} \) is 1.4 times and, consequently, the fraction of the transit time, where both kinematic and dynamic factors change more intensively than in the section of a steady turn, is 1.4 times greater. Moreover, curvilinear motion with a variable radius of curvature of the trajectory makes up a significant part, more than a half of the entire trajectory of the unit at a turn: method I - 58.1%, method II - 81.1%.

Figure 3 shows the trajectories of turning the kinematic center of a tractor using the most common method I, also constructed from the results of calculations using formula (4). It was determined that with a change in the tractor base (Figure 1) from \( L = 2.0 \text{ m} \) (Figure 3, curve 1) to \( L = 2.5 \text{ m} \) (Figure 3, curve 2) with the same following initial parameters: \( B = 1.8 \text{ m} \); \( v = 1.25 \text{ m} \text{s}^{-1} \); \( \alpha_{1} = 0.157 \text{ s}^{-1} \); \( \omega_2 = 0 \text{ s}^{-1} \); \( t = 4 \text{ s} \); \( \alpha_1^* = 36^\circ \); \( \alpha_2^* = 0^\circ \), the width of the headland increases by 14.5% (\( x_{\text{max}1} = 7.79 \text{ m} \) and \( x_{\text{max}2} = 9.11 \text{ m} \)) and its height - by 12.6% (\( y_{\text{max}1} = 5.76 \text{ m} \) and \( y_{\text{max}2} = 6.59 \text{ m} \)), the minimum turning radius increases by 15.8% (\( R_{\text{max}1} = 3.653 \text{ m} \) and \( R_{\text{max}2} = 4.341 \text{ m} \)), the maximum average steering angle of the front axle wheels increases by 4.1% (\( \alpha_{11\text{m}} = 28.7^\circ \) and \( \alpha_{12\text{m}} = 29.94^\circ \)), the total length of the trajectory and the time to complete the entire maneuver increases by 11.1% (\( S_{\text{max}1} = 16.21 \text{ m} \) and \( S_{\text{max}2} = 18.22 \text{ m} \); \( t_{11} = 12.97 \text{ s} \) and \( t_{12} = 14.6 \text{ s} \)), the length of the section of a steady turn and the time of its passage increases by 24.5% (\( S_{51} = 6.21 \text{ m} \) and \( S_{52} = 8.22 \text{ m} \); \( t_{51} = 4.97 \text{ s} \) and \( t_{52} = 6.56 \text{ s} \)).

With the total length of the trajectories of the transitional turning sections \( S_{1} = 10 \text{ m} \) and the time of their passage \( t_{x} = 8 \text{ s} \) at \( L = 2.0 \text{ m} \), the share of \( S_{1} \) and \( t_{x} \) in its total length \( S_{\text{max}2} \) is 54.9%, which is 1.12 times less than at \( L = 2.0 \text{ m} \). when we have 61.7% of \( S_{1} \) share in \( S_{\text{max}1} \).

With the increase in the distance between the axes of the tractor pivots (Figure 1) from \( B = 1.8 \text{ m} \) (Figure 3, curve 2) to \( B = 2.2 \text{ m} \) (Figure 3, curve 3) and the same other initial data, according to the work (4), we have: the increase in \( x_{\text{max}3} \) by 5.3%, \( y_{\text{max}3} \) - by 7.96%, \( S_{\text{max}3} \) and \( t_{11\text{m}} \) - by 6.83%, \( S_{33} \) and \( t_{33} \) - by 14.0%, \( R_{\text{max}3} \) - by 4.4% and a slight decrease by 2.0%. The share of \( S_{1} \) and \( t_{x} \) in its total length \( S_{\text{max}3} \) at \( B = 2.2 \text{ m} \) is 51.13%, which is only 1.074 times less than at \( B = 1.8 \text{ m} \).

Thus, both with the decrease in the longitudinal base and the track width of a tractor with all other equal conditions the turning radius of a machine, the area of the headland, the length of the trajectory of the circular loopless turn and the time for its implementation decrease. However, at the same time, the most unpleasant tractor vibrations as longitudinal and transverse, transverse-angular and longitudinal-angular ones increase.

The increase in the forward speed of a tractor, for example, with the turning method I (Figure 1), from \( v = 1.25 \text{ m} \text{s}^{-1} \) (Figure 3, curve 3) to \( v = 1.5 \text{ m} \text{s}^{-1} \) (Figure 2, curve 1), with all other equal conditions leads to the increase in the width by 2% and in the height by 5.65% of the headland and the length of the trajectory by 5.2%. However, at the same time, the time for making a turn is reduced by 13.5% and is 15.64 s at \( v = 1.25 \text{ m} \text{s}^{-1} \), and at \( v = 1.5 \text{ m} \text{s}^{-1} \) - 13.75 s.

At the same time, the reduction in the time of entering the turn from \( t = 4 \text{ s} \) (Figure 2) to \( t = 3 \text{ s} \) (Figure 4), which corresponds to the rotation method I \( \omega_{1} = 0.21 \text{ s}^{-1} \), \( \omega_{2} = 0 \text{ s}^{-1} \) and when method II - \( \omega_{1} = 0.21 \text{ s}^{-1} \), \( \omega_{2} = 0.21 \text{ s}^{-1} \), with the remaining parameters unchanged, reduces the width by 3.2% and the height of the headland by 11.7%, the length of the trajectory - by 7.9% in the first case (Figure 2, curve 1 and Figure 4, curve 1), and in the second (Figure 2, curve 2 and Figure 4, curve 2) - by 7.2%, 11.1% and 9.5% respectively.
However, the increase in the speed of a tractor, including during curvilinear motion and, especially, at the transitional stages of turning and with the decrease in the time of their passage, contributes to the increase in the centrifugal force of inertia, uneven resistance to motion, is accompanied by the increase in variable and alternating dynamic loads, which cause the occurrence of increased oscillatory processes in the system “supporting surface-tractor-operator”. Moreover it degrades its smoothness and stability. Therefore, in order to achieve their minimum values, it is necessary to give preference to scientifically grounded rational modes of MTU operation, for example, using the formulas (1), (2) and (4).

4. Conclusion

Therefore, in order to achieve high productivity of MTU and the required quality of agricultural operations, it is necessary:
- to choose one or another rational way of its motion on the headland, corresponding to a specific technological process,
- to operate automatically the control system for turning the rear wheels and identifying the patterns of changing the angles of rotation of the wheels of front and rear axles [9-11], based on the conditions for the increase in the stability of motion, the absence of skidding, reducing the dynamic load and providing, if necessary, the possibility of returning to the entrance to the row spacing with offset by the working width of technological equipment.

The determined analytically valid trajectory of MTU kinematic center for all the stages of circular loopless rotation by nonlinear approximation of the experimental function of an explicit form (4) with a minimum error of no more than 2.2% allows estimating the kinematic characteristics of MTU for various ways of rotation, depending on the main design and operational parameters and also studying it motion along a given trajectory.

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