Experimental and numerical evaluation of natural frequencies and mode shapes in a satellite model-plate connecting interface without vibration isolators

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Abstract This paper numerically and experimentally examines the natural frequencies and mode shapes in a satellite model-plate connecting interface installed on flexible gas-inflated jacks in order to assess the effect of external impacts on the natural frequencies of the satellite model and to thus generate a reference for appropriate vibration isolators for transportation and launch as required. The experimental determination of the frequencies and mode shapes of vibration in the system was carried out using modern software in conjunction with the hardware loading model satellite (LMS). Numerical calculation of the natural frequencies and mode shapes of the research object was performed in the finite element package ANSYS Workbench, with results suggesting that the suspension system (gas inflatable jacks) was sufficiently flexible and did not affect the natural frequencies at which the system under investigation, the satellite model-connecting interface plate, began to deform. Experimental determination of the natural frequencies and mode shapes of vibration for the satellite model-connecting interface plate was then conducted in the range from 44 Hz to 200 Hz; the differences in natural frequencies between the calculations and experiments averaged 10 to 15%.

Keywords: Transport satellite, Satellite natural frequency, Satellite mode shapes.

1. Introduction
In any system, dynamic concerns can cause serious trouble where the effect of free vibrations is significant with respect to the inherent properties of any system. Undamped systems have thus been the basis of many different research investigations into forced motion in damped systems in order to conserve the total energy in these systems, in contrast to the use of forced motion in which the system is driven by outside forces.
Modern satellite technologies are now used globally for a wide range of functions, both social and military. Communication applications are totally dependent on such systems, and they are also used to observe weather patterns to predict extreme weather conditions and for other observational purposes. It
has therefore become important to protect the sensitive equipment in satellites and related electronic units from undesired vibrations that may cause damage during transport or launching operations. One of the most serious issues to consider is structural resonance due to the natural frequency, which may affect the satellite body and cause it to become unsettled so that sensitive equipment becomes separated. In addition, vibrations could make mechanical problems such as fatigue and cracks occur. This makes it essential to determine, and thus plan to counteract, the natural frequencies at which the body may vibrate during transportation or launch [1].

Determining and understanding the natural frequencies and mode shapes offers a basis for conserving any system, and specifying the main effects, based on accurate measurement of system parameters, is necessary to allow the development of techniques to address such problems [2] by protecting satellites during transportation from overloads, vibration, or external influences, important problems that have arisen repeatedly in engineering practice. Moreover, determination of the fundamental natural frequencies of a satellite and its mode shapes represents a step towards the more important goal of determining the appropriate isolator required to keep a satellite within permissible limits of vibration and specifying its properties.

The article in [3] provided a description protection system designed to preserve a of satellite from vibration, electrical, or climatic influences during transportation by land and air. The values of pressure during transportation by plane were calculated as in Figure 1.

![Figure 1. Pressure differences during flight](image)

A programme for testing the satellite vibration isolation system was presented, though the work did not contain experimental data and did not mention the mass and frequency characteristics of the transported satellite, despite these being a necessary basis for evaluating isolator efficiency.

As part of this study of the stiffness of vibration isolators and the assessment of their location, other articles describing vibration isolation systems were considered. The article in [4] offered a diagram of an LVIS system to provide vibration and shock protection based on mechanical and pneumatic vibration isolators designed based on calculations of natural frequency and vibration due to external effects (Figure 2).
In addition to the presentation of vibration protection systems, many previous studies have offered methods for calculating natural frequencies and shapes using finite element packages. One of these studies [5] offered a nonlinear analysis of natural frequencies in a simple vibration isolator; this analysis was performed without supporting experimental research, however. The calculation results and the design diagram for the vibration isolator in that study are shown in Figure 3.

Figure 3 shows the zones of amplification and attenuation of oscillations, as well as the resonant frequency $f_0$. Further calculations for a more complex vibration isolation system with and without load were also performed.

In the current study the natural frequencies and mode shapes for a satellite model-plate connecting interface installed on flexible gas-inflated jacks was investigated both numerically, using ANSYS Finite Element Software and assuming the satellite as a rigid body, and experimentally, using a satellite simulator with pre-calculated dimensions and weight, to assess the effect of external impacts on the natural frequencies of the satellite model.

2. Governing Equations
For a mass-spring system, the alternating transfer between potential energy (spring) and kinetic energy (mass or inertia) is defined as the free vibration; by equating the value of these energy quantities, the natural frequency can thus be defined, being the point where the system will oscillate normally without the need for external forces [6].
The interchange of energy between potential and kinetic forms in any system is known as vibration, and for each cycle of vibration in a damped system, some energy is dissipated [7]. In a spring, the force acting along its length is

\[ F = K(x - u) \]  

(1)

The free vibration of an undamped system is illustrated in Figure 4:

\[ F = m \ddot{x} \]  

(2)

In a viscous damper, however, the applied force is proportional to the relative velocity of the connection points:

\[ F = c (\dot{x} - \dot{u}) \]  

(3)

The force \( m \ddot{x} \) applied by the mass on the spring is thus equal in magnitude and opposite in direction to the force \( kx \) applied by the spring on the mass:

\[ m \ddot{x} + kx = 0 \]  

(4)

where \( x = 0 \) determines the equilibrium position of the mass.

The solution to Equation (4) can thus be found by

\[ x = A \sin \sqrt{\frac{k}{m}} t + B \cos \sqrt{\frac{k}{m}} t \]  

(5)

where \( \sqrt{\frac{k}{m}} \) represents the angular natural frequency and

\[ \omega_n = \sqrt{\frac{k}{m}} \text{ Rad/sec} \]  

(6)

The sinusoidal oscillation of the mass is continuous, and the time to complete each cycle is known as the period:

\[ \tau = \frac{2\pi}{\omega_n} \]  

(7)

The natural frequency is the reciprocal of the period:
\[ f_n = \frac{1}{\tau} = \frac{\omega_n}{2\pi} = \frac{1}{2\pi} \sqrt{\frac{k}{m}} = \frac{1}{2\pi} \sqrt{\frac{kg}{W}} \]  

(8)

where \( W = mg \) is the weight of the rigid body forming the mass of the system.

Equation (5) can be written in a form that includes the frequency relation in Equation (6) for displacement in oscillatory motion:

\[ x = A \sin \omega_n t + B \cos \omega_n t = C (\omega_n t + \theta) \]  

(9)

where \( C = (A^2 + B^2)^{1/2} \) and \( \theta = \tan^{-1} B/A \).

The angle \( \theta \) is called the phase angle.

The deflection of spring \( k \) due to the gravity force of the associated mass is the static deflection of a simple mass-spring system, \( \delta_{st} = mg/k \).

Substituting this into Equation (8) thus gives

\[ f_n = \frac{1}{2\pi} \sqrt{\frac{g}{\delta_{st}}} \]  

(10)

3. Experimental determination of the natural frequencies and vibration modes of the system

The experimental determination of the frequencies and modes of vibration of the loading model satellite-connecting interface plate system was carried out using a complex loading model satellite (LMS). To create this, a satellite model with a plate was installed on flexible gas-inflatable jacks to remove contact effects (Figure 5).

![Loading model satellite](image1)

![Connecting interface plate](image2)

**Figure 5.** Loading model satellite-connecting interface plate.

Vibration accelerations on the object were measured at 12 points in three mutually perpendicular directions using one- and three-component PCB accelerometers with sensitivities of 100 mV/g.
Excitation was performed using a PCB 086D50 modal hammer with a piezoelectric sensitivity of 0.23 mV / N, and the response from the sensors was recorded using a SCADAS Mobile data collection system. LMS Test.Lab v13 package, Impact Testing module, software was also implemented.

Four measurement points were selected at the edges of the connecting interface plate, one in the middle of the plate, with another at the top of the satellite model, and three points each on the upper rods (ends and middle). The installation points of the accelerometers are shown in Figure 6.

![Figure 6. Installation points of accelerometers (wire model).](image)

In order to experimentally determine the vibration modes in the longitudinal, transverse and vertical directions, vibrations were excited by concussion with a modal hammer sequentially at two points: along the + Y and –Z axis at point 7 and along the + Z axis at point 2. The places of impact on which the modal hammer were used are shown in Figure 7.

The natural vibration frequencies of the system obtained during the experiment are presented in Table 1.

![Figure 7. Locations of vibration excitation](image)
Table 1. Experimental natural vibration frequencies of the loading model-connecting interface plate.

| No | Natural frequency, Hz |
|----|-----------------------|
| 1  | 44,2251               |
| 2  | 58,4257               |
| 3  | 66,6538               |
| 4  | 69,7465               |
| 5  | 73,9891               |
| 6  | 83,3589               |
| 7  | 92,5037               |
| 8  | 111,7864              |
| 9  | 143,7629              |
| 10 | 146,6559              |
| 11 | 167,5326              |
| 12 | 182,2841              |
| 13 | 211,9937              |

4. **Theoretical study of natural frequencies and vibration modes**

A three-dimensional model was built in the SolidWorks engineering package based on the approved drawings of the satellite loading model and connecting interface plate.

Numerical calculation of the natural frequencies and mode shapes of the research object were performed in ANSYS Workbench, and the imported solid geometric model of the object was broken into finite elements of 20 mm diameter. The resulting model is shown in Figure 8.

![Figure 8. Satellite design model.](image)
We assume that the platform is absolutely rigid, its flexibility will be taken into account in the subsequent calculation with a finite element. Now, setting the permissible minimum $f_{1\text{min}}$ and maximum $f_{1\text{max}}$ natural frequencies, we find that the corresponding boundaries of the suspension stiffness range in the direction of the $y$-axis from the condition that the natural frequency of vertical translational vibrations falls into the permissible range of 10-13 Hz.

**Figure 9.** Determination of mass and inertial characteristics of a really made simulator.

Based on this calculation, the natural oscillation frequencies of the system were obtained, as presented in Table 2.

**Table 2.** Calculated natural vibration frequencies of the satellite model-connecting interface plate system.

| No | Natural frequency (Hz) |
|----|------------------------|
| 1  | 54,239                 |
| 2  | 71,172                 |
| 3  | 76,619                 |
| 4  | 81,771                 |
| 5  | 82,272                 |
| 6  | 92,662                 |
| 7  | 92,766                 |
| 8  | 101,06                 |
| 9  | 130,52                 |
| 10 | 170,47                 |
| 11 | 178,24                 |
| 12 | 196,69                 |
| 13 | 209,69                 |
| 14 | 239,23                 |

5. **Comparison of calculated and experiment results**

Table 3 shows the corresponding calculated and experimental natural forms. The difference in frequencies can be mainly explained by the idealised geometry of the design model, the conditions for joining the slab and the load-model together and attaching the rods of the load-model to the upper plate, and the effect of the necessary welds.
A frequency range of up to 200 Hz was considered, as above this frequency, the vibration modes of the stiffeners and the more complex vibration modes of the plate and rods appear; the ability to detect these effectively is limited by the number of measuring channels.

**Table 3.** Calculated and experimental natural vibration frequencies

| Natural vibration frequency, Hz | Natural mode of vibration                          | Computation | Experiment |
|--------------------------------|----------------------------------------------------|--------------|------------|
| 54,239                         | Oscillations of rods in antiphase in the direction of the axis y |              | Oscillations of rods in antiphase in the direction of the axis y |
| 44,225                         |                                                    |              |            |
| Natural vibration frequency, Hz | Natural mode of vibration |
|--------------------------------|--------------------------|
| 71.172                         | In-phase vibration of the rods in the direction of the axis y |
|                                | ![Diagram](image1.png)   |
|                                | ![Diagram](image2.png)   |

**Experiment**

| 58,426                         | In-phase vibration of the rods in the direction of the axis y |
|                                | ![Diagram](image3.png)   |
| Natural vibration frequency, Hz | Natural mode of vibration |
|-------------------------------|--------------------------|
| **Computation**               | Oscillations of rods in antiphase in the direction of the axis $z$ |
| 76,619                        |                          |
| **Experiment**                | Oscillations of rods in antiphase in the direction of the axis $z$ |
| 69,747                        |                          |
| Natural vibration frequency, Hz | Natural mode of vibration |
|--------------------------------|---------------------------|
| Computation                    | Oscillations of the rods in antiphase in the direction of the axis $y$ with oscillation of the plate around the axis $x$ |
| 81.771                         |                           |
| Experiment                     | Oscillations of the rods in antiphase in the direction of the axis $y$ with oscillation of the plate around the axis $x$ |
| 73.989                         |                           |
| Natural vibration frequency, Hz | Natural mode of vibration |
|--------------------------------|---------------------------|
| 82,272                         | Oscillations of the rods in antiphase in the direction of the axis $z$ with the oscillation of the plate and rods around the axis $z$ |
| 66,654                         | Oscillations of the rods in antiphase in the direction of the axis $z$ with the oscillation of the plate and rods around the axis $z$ |
| Natural vibration frequency, Hz | Natural mode of vibration |
|-------------------------------|---------------------------|
| 92,662                        | Oscillations of the product in the direction of the axis $y$ |
| **Experiment**                |                           |
| 83,359                        | Oscillations of the product in the direction of the axis $y$ |
| Natural vibration frequency, Hz | Natural mode of vibration |
|--------------------------------|--------------------------|
| 92,766                         | Torsional vibrations of the product around the axis y |

In the experiment, the form is not identified.
| Natural vibration frequency, Hz | Natural mode of vibration |
|--------------------------------|---------------------------|
| **Computation**                |                           |
| 101,060                        | In-phase vibrations of the rods in the direction axis $z$ and the oscillation of the plate around the axis $x$ |
| **Experiment**                 |                           |
| 92,504                         | In-phase vibrations of the rods in the direction axis $z$ and the oscillation of the plate around the axis $x$ |
| Natural vibration frequency, Hz | Natural mode of vibration |
|--------------------------------|---------------------------|
| 130,520                        | Plate bending oscillations about the axes $x$ and $z$ |
| 146,656                        | Plate bending oscillations about the axes $x$ and $z$ |
| Natural vibration frequency, Hz | Natural mode of vibration |
|-------------------------------|---------------------------|
|                               | In-phase vibrations of the rods in the direction axis \( z \) and the oscillation of the plate around the axis \( x \) |
| Computation                   |                           |
| 170.470                       |                           |
| Experiment                    |                           |
| 143.763                       |                           |

In-phase vibrations of the rods in the direction axis \( z \) and the oscillation of the plate around the axis \( x \).
| Natural vibration frequency, Hz | Natural mode of vibration |
|--------------------------------|--------------------------|
| 178,240                        | Vibrations of the plate around the axis z |
| 111,786                        | Vibrations of the plate around the axis z |

**Natural forms that failed to clearly compare with experiment**
| Natural vibration frequency, Hz | Natural mode of vibration |
|--------------------------------|---------------------------|
| 196,690                        | Vibrations plates and rods|
| 209,690                        | Plate bending             |
| Natural vibration frequency, Hz | Natural mode of vibration |
|-------------------------------|--------------------------|
| **239,230**                  | Plate bending            |

**Computation**

**Experiment**

167,533
A comparative analysis of the calculated and experimental natural frequencies and forms of the system was also carried out, and a qualitative coincidence in natural forms was revealed such that the differences in natural frequencies between calculation and experiment (in the range up to 200 Hz) averaged 10 to 15%.

This difference is tentatively assigned to the existence of bolted connections in the simulator that were not taken into account in the finite element calculation; in the finite element model, the real contact area of the elements of the satellite model located in the bolted connections is omitted, while changes in
the rigidity of the entire structure actually depend on the tightening force of these bolts. There were also no welds in the finite element model, and residual stresses after welding can also lead to changes in the rigidity of the welded structure and, accordingly, to a change in natural frequencies.

The finite element model can thus be used only for qualitative confirmation of the results of modal tests. Under these conditions, when analysing the results, preference should be given to the natural frequencies and forms of the system as determined by experiment.

6. Conclusions
A numerical-experimental study was conducted to determine the natural frequencies and modes of vibration of a satellite model-connecting interface plate system without damping. According to the results of the calculations and tests, the following points were established:

1. The lowest natural frequency at which the system begins to deform was 44 Hz. In the experiment, the natural frequency was identified as corresponding to the oscillations of the simulator (an absolutely rigid body) on a flexible base (gas inflatable jacks). Without this flexible base, the lowest natural frequency was 4.5 Hz, about 10 times less than the first vibration frequency at which the simulator began to deform on the flexible base. This indicates that the selected suspension (gas inflatable jacks) was sufficiently flexible to not affect the natural frequencies at which the simulator began to deform.
2. The experimentally determined natural frequencies and modes of vibration of the system were available only in the range up to 200 Hz.
3. The first natural frequency of the system found in the experiment was 44 Hz, and the vibrations of the rods corresponded to this frequency. The central part of the model and the plate on the form fluctuate very little, and the lower frequencies thus correspond mainly to the vibrations of the rods.
4. A full set of natural frequencies and forms of the system was experimentally determined to generate initial data for determining the natural frequencies and shapes of the satellite model-connecting interface plate within a proposed vibro-isolation system.

7. Recommendations
Efforts have been made to analyse the natural frequency for the selected satellite system with regard to vibrations due to external effects. In order to reduce the negative effects of vibration, the following suggestions are offered to decrease vibration during the transport of such satellites:

1. A specific finite element model should be constructed and adjusted so that the observed natural frequencies are close to the experimental results.
2. Vibration isolators that depend on dry friction to provide damping are likely to be most suitable to the required container size; transport costs can be reduced where the isolator size is small.
3. Data representing real transport methods should be used to calculate the effect of such vibrations on the product during transportation in different conditions.
Nomenclature

| Symbol | Description                                                                 | Units   |
|--------|-----------------------------------------------------------------------------|---------|
| $g$    | Acceleration Due to Gravity                                                 | m / s²  |
| $m_{pro}$ | Product Weight Together with the Connecting Interface                      | kg      |
| $m_{pl}$ | Platform Weight                                                           | kg      |
| $I_{xc}$, $I_{yc}$, $I_{zc}$ | Moments of Inertia of the "Simulator - Plate" System Relative to the Center of Gravity "Simulator - Plate". | kg.m²   |
| $F$    | Force Acting on System                                                     | N       |
| $k$    | Spring Constant or Stiffness                                               | N/m     |
| $m$    | Rigid Mass.                                                                | kg      |
| $x^\prime$ | Acceleration                                                              | m/s²    |
| $c$    | Damping Coefficient                                                        | N s/m   |
| $t$    | Time                                                                       | s       |
| $A$, $B$, $C$ | Constants                                                                    |        |
| $\omega_n$ | Angular Natural Frequency                                                | rad/sec |
| $\tau$ | Period                                                                    | s       |
| $f_n$  | Natural Frequency                                                         | Hz      |
| $[f_1]_{min}$ | Minimum Natural frequency                                                |        |
| $[f_1]_{max}$ | Maximum Natural Frequency                                                |        |
| $W$    | The Weight of the Rigid Body                                               | N       |
| $\theta$ | The Phase Angle.                                                          | rad     |

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