CORRELATION BETWEEN PLASMA AND TEMPERATURE CORRECTIONS TO THE CASIMIR FORCE

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When comparing experimental results with theoretical predictions of the Casimir force, the accuracy of the theory is as important as the precision of experiments. Here we evaluate the Casimir force when finite conductivity of the reflectors and finite temperature are simultaneously taken into account. We show that these two corrections are correlated, i.e. that they can not, in principle, be evaluated separately and simply multiplied. We estimate the correlation factor which measures the deviation from this common approximation. We focus our attention on the case of smooth and plane plates with a metallic optical response modeled by a plasma model.

1. Motivations

After its prediction in 1948, the Casimir force has been observed in a number of ‘historic’ experiments. It has recently been measured with an improved experimental precision. The recent experiments should allow for an accurate comparison between the measured force and the theoretical prediction and this is important for at least two reasons.

First, accurate experiments are devoted to searches for hypothetical new forces predicted by theoretical unification models with nanometric to millimetric ranges or by tests of Newtonian gravity at millimetric to centimetric distances. At submillimetric distances, the Casimir effect dominates the hypothetical new force so that the latter would appear as a difference between experimental measurements and theoretical expectations of the Casimir force.

Then, the Casimir force is the most accessible effect of vacuum fluctuations in the macroscopic world. As the existence of vacuum energy raises difficulties at the interface between the theories of quantum and gravitational phenomena, it is worth...
testing this effect with the greatest care and highest accuracy. Now, as far as a theory-experiment comparison is concerned, the accuracy of theory is as crucial as the precision of experiments. If a given accuracy, say at the 1% level, is aimed at in the comparison, then the theory as well as the experiment must be mastered at this level independently from each other.

The differences between the real experimental conditions and the ideal situation considered by Casimir play a key role in this discussion. Casimir calculated the force between perfectly plane, flat and parallel plates in the limit of zero temperature and perfect reflection. This is the reason why the Casimir formula \( F_{\text{Cas}} \) only depends on the distance \( L \), the area \( A \) (supposed to be much larger than \( L^2 \)) and the two fundamental constants \( c \) and \( \hbar \)

\[
F_{\text{Cas}} = \frac{\hbar c A \pi^2}{240L^4}
\]

This is a remarkably universal feature, especially since the force is independent of the fine structure constant in contrast to the Van der Waals forces. This indicates that the response of perfect mirrors to the fields is saturated, since they reflect 100% of the incoming light.

But experiments are performed with mirrors which do not reflect perfectly the field at all frequencies. For example, conduction electrons have an optical response described by a plasma model so that metallic mirrors show perfect reflection only at frequencies smaller than the plasma frequency \( \omega_p \). Hence the Casimir force between metal plates does fit the Casimir formula \( F_{\text{Cas}} \) only at distances \( L \) larger than the plasma wavelength \( \lambda_p = \frac{2\pi}{\omega_p} \). For metals used in the recent experiments, this wavelength lies in the \( 0.1 \mu m \) range (\( \sim 107 \) nm for Al and 136 nm for Cu and Au). At distances smaller than or of the order of \( \lambda_p \), the finite conductivity of the metal produces a reduction of the force which can be described by a plasma correction factor \( \eta_P \)

\[
F^P = \eta_P^P F_{\text{Cas}} \quad \eta_P^P < 1
\]

At the same time, experiments are performed at room temperature and the radiation pressure of thermal field fluctuations is superimposed to that of vacuum field fluctuations. This effect can be described by a temperature correction factor which increases the Casimir force at distances larger than or of the order of a thermal wavelength \( \lambda_T = \frac{\hbar c}{k_B T} \) (\( \sim 7 \mu m \) at room temperature)

\[
F^T = \eta_T^T F_{\text{Cas}} \quad \eta_T^T > 1
\]

Now, the plasma correction \( \eta_P^P \) has been defined at zero temperature while the thermal correction \( \eta_T^T \) is usually computed for perfect reflection. Since the plasma wavelength is much smaller than the thermal wavelength, the whole correction \( \eta_F \) giving the force \( F \) when both effects are simultaneously accounted for is commonly
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calculated by multiplying the plasma and thermal correction factors. This is however an approximation and the discussion of its accuracy is the main motivation of the present paper. To this aim, we write the whole correction factor as

\[ F = \eta_F F_{\text{Cas}} \eta_T = \eta_F^P \eta_T^T (1 + \delta_F) \] (4)

A null value for \( \delta_F \) would justify the common approximation where the plasma and thermal corrections are computed independently from each other and then multiplied. In contrast, a non null value represents a correlation of the two corrections which must be taken into account in an accurate evaluation of the Casimir force.

In the present paper, we consider the initial Casimir geometry with perfectly plane, flat and parallel plates and thus restrict our attention on conductivity and thermal corrections. Since the correlation between these two corrections is appreciable only at distances where the plasma model is a good description of metals, we focus our attention on this model (see a more precise argument below).

2. The plasma and thermal corrections to the Casimir force

We now present the evaluation of the correction factors in the Casimir geometry.

A cavity built on partly transmitting mirrors can be dealt with using the Fabry-Prot theory. Field fluctuations impinging the cavity have their energy either enhanced or decreased inside the cavity, depending on whether their frequency is resonant or not with a cavity mode. The radiation pressure associated with these fluctuations then exerts a force on the mirrors which is directed either outwards or inwards respectively. It is the balance between these outward and inward contributions, integrated over the wavevectors associated with the field modes, which gives the net Casimir force.

The obtained expression is a generalization of Lifshitz’s formula which is valid for any couple of mirrors described by arbitrary frequency dependent reflection amplitudes obeying the general properties of scattering theory. Since any real mirror is transparent at the high frequency limit, a regular expression is naturally obtained, which is free from the divergencies usually associated with the infiniteness of vacuum energy.

At non zero temperature, the Casimir force may be written as the Poisson formula given by equation (7) in . This formula is used as the starting point of the following calculations after specialization to the case of metallic mirrors with an optical response described by the plasma model .

\[ \text{Lifshitz’s results were not originally written in terms of reflection coefficients. U. Mohideen and V.M. Mostepanenko have recently drawn our attention to the reference where Kats wrote Lifshitz’s results in this manner.} \]

\[ \text{Thermal corrections to the Casimir force have also been evaluated with a Drude model used to describe absorption in the metal and they have led to controversial results. As far as this controversy is concerned, note that equation (17) of coincides with the Poisson formula used in the present paper (equation (7) of) and leads to results at variance with the ones obtained in . For a detailed discussion of the interplay between metallic and temperature corrections in the general case, see the contribution of G.L. Klimchitskaya to this volume.} \]
We have numerically evaluated the Casimir force with the plasma wavelength corresponding to Aluminium and at room temperature. The global correction factor obtained in this manner is shown on figure 1 for the experimentally relevant distance range $0.1 - 10 \mu m$ and it is compared with the plasma correction factor (evaluated at zero temperature) and the thermal correction factor (evaluated for perfect mirrors).

The thermal correction is negligible at short distances and enhances the force at large distances whereas the conductivity correction may be ignored at large distances and decreases the force at small distances. As a consequence, the global correction factor $\eta_F$ behaves roughly as the product $\eta_P^F \eta_T^F$ of the two correction factors evaluated separately. But both corrections are appreciable in the intermediate distance range and it is therefore necessary to discuss more precisely the accuracy of the decorrelation approximation.

3. The correlation between correction factors

In order to assess the quality of the decorrelation approximation, we now evaluate the correlation factor $\delta_F$ introduced in (4).

This factor is plotted on figure 2 as a function of the distance $L$ for the plasma wavelengths corresponding respectively to Al, Cu-Au, and two additional plasma wavelengths chosen to emphasize the correlation effect. It turns out that the correlation factor lies in the % range for Al, Cu-Au, that is precisely the accuracy which is usually aimed at. The positive sign obtained for $\delta_F$ means that the correlation increases the theoretical value of the force.
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This is the main result of the present contribution: approximating the global correction factor as the product of the plasma and thermal correction factors evaluated independently is sufficient for rough estimates, with a precision worse than 1%. But the correlation $\delta_F$ between these two corrections should be taken into account when an accuracy beyond the 1% level is needed. At this point, let us emphasize that the correlation is appreciable at distances larger than 1$\mu$m where the plasma model is known to be a good effective description of the metallic optical response \cite{9}. This justifies the use of this model in the present paper which is devoted to the study of the correlation effect \cite{c}.

Figure 2 also shows that the correlation factor is proportional to the value of the plasma wavelength while keeping the same functional dependence on the distance. It has been possible to prove that $\delta_F$ obeys a simple scaling law \cite{13}

$$\delta_F = \frac{\lambda_P}{\lambda_T} \Delta_F$$ (5)

It is proportional on one hand to the ratio $\frac{\lambda_P}{\lambda_T}$ of the two wavelengths which characterize respectively the plasma and thermal effects and, on the other hand, to the universal function $\Delta_F$ which does only depend on $\frac{1}{\lambda_T}$. This scaling law is valid for $\lambda_P \ll \lambda_T$, which is the situation of interest for experiments with ordinary metals at room temperature.

\*At shorter distances, say around 0.1-0.5$\mu$m, a more complete description of the metallic optical response must be used in order to obtain accurate estimates of the Casimir force \cite{11}. Since the temperature correction is negligible in this distance range, the estimation can be simplified by considering only the contribution of vacuum fluctuations.
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An analytical derivation of (5) has been given in 13 through a perturbative development of the force to first order in $\frac{\lambda}{\lambda^2}$. The resulting expression is found to fit well the results of the complete numerical integration presented above, with an accuracy now much better that the 1% level. It provides one with a simple method for getting an accurate theoretical expectation of the Casimir force in presence of plasma and thermal corrections: the force is indeed given by equations (4, 5) and the analytical expressions of $\eta_\lambda^P$, $\eta_\lambda^T$ and $\Delta F$ available from 13. This solves the problem of the accurate evaluation of the Casimir force between two metallic planes at room temperature at distances where the plasma model can be used.

When addressing the problem of accuracy of theoretical predictions, we must keep in mind other corrections involved in recent measurements of the Casimir force 3, in particular the geometry correction - experiments are not performed with two plane plates but with a sphere and a plane - and the roughness correction. This entails that not only the accuracy of the approximations used to treat these effects should be carefully studied for perfect mirrors in vacuum but also that the correlations between geometry, roughness, conductivity and temperature corrections have to be evaluated.

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