Delayed Choice Quantum Coherent Control

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A Bell test for a particular implementation of coherent control is shown to certify the quantumness of that scenario. Specifically, a multi-photon ionization experiment with alkali atoms is devised to demonstrate the analogy between coherent control and single-photon interference in a Young double-slit. We describe two complementary experimental situations, which are characterized by a random photoelectron spin polarization with particle-like behavior on the one hand, and by spin controllability and wave-like nature on the other. The simultaneous realization of both situations naturally leads to an implementation of quantum delayed choice.

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In certain implementations of “coherent control” [1], an approach to coherently manipulating atomic and molecular processes, two or more laser-induced dynamical path alternatives interfere. Here, two fundamentally different lines of description compete: First, by analogy with Young’s double-slit experiment, one can interpret the probability of the process’ outcomes—the control objective or yield—as a phase-dependent pattern resulting from the quantum-coherent interference of mutually exclusive path alternatives embodied in the excitation pathways [1] [2]. Or second, one views the phase-controllability as the manifestation of the non-linear response of the system to the superposition of phase-coherent incident laser fields which can therefore be perceived as an inherently classical phenomenon [3].

In general, a phenomenon can be reproduced by a classical model even if the underlying physics is quantum, especially when the observables examined cannot reveal true quantum attributes. For example, the pattern of a Young double-slit is the same whether it builds up photon by photon or whether it arises from classical coherent light. It is only when the complementarity between the photon’s wave and particle property is observed that the very essence of the double-slit experiment can be understood: Whereas the wave property is manifest in the fringes of the interference pattern, the particle aspect corresponds to the knowledge about the path the photon has taken through the slits. Quantum mechanics allows a display of either one of these features, or even a blend of the two, depending on the experimental situation. Hence, if the analogy of coherent control to the double-slit is justified, then quantum interference in coherent control can be exposed in a similar way. In this Letter, we propose an experiment in which the quantum features within a specific coherent control implementation can be identified.

The technique of coherent control is highly versatile [1]. Here, we exclusively examine the interaction between a single heavy alkali atom and ionizing bichromatic radiation [4]. The atom is assumed to be tightly trapped in order to prevent recoil motion that might otherwise provide which-way information. Neglecting hyperfine structure, we specify the hydrogen-like electronic bound states in the standard spectroscopic notation as |nljm⟩ with n the principle quantum number and E(nlj) the energy. The continuum states are denoted by |Eljm+⟩ [5]. In the basis of these states, the atomic Hamiltonian H_A is diagonal. The bound and continuum solutions Rnlj, Re lj of the radial Schrödinger equation depend on both the orbital angular momentum ℓ and the total angular momentum j; mj denotes its projection onto the z-axis in the laboratory frame. Using Clebsch-Gordan coefficients, we write ⟨x|nljm⟩ =...
\[ \sum_{m_1, m_2} \langle lm_l \frac{1}{2} m_s | jm_j \rangle R_{nlj}(x) Y_{lm_l}(\hat{x}) | m_s \rangle, \] where \( m_l \) is the orbital angular momentum quantum number,  the projection on the \( z \)-axis and \( | m_s \rangle \) describes the electron's spin projected onto the same axis. An analogous expansion exists for the continuum wave functions \( | x | E jm_j^+ \rangle \). The quantized radiation field, with canonical Hamiltonian \( \mathcal{H}_R \), constitutes an auxiliary degree of freedom \( \mathcal{R} \). We treat the light-matter interaction \( V \) within the electric dipole approximation, \( V = e \sum_{kk} \hat{a}_{kk} (\hat{a}_{kk}^\dagger x \cdot \hat{e} + \text{h.c.}) \), where \( \hat{g}_k \) is the vacuum field strength and \( \hat{a}_{kk} \) the photon annihilation operator with wave vector \( k = \omega_k k/c \) and polarization \( \hat{e} \perp k \).

Consider the case where the continuum at energy \( E' \) can be reached from the ground state \( nS_{1/2} m_j \) mainly by two dynamical pathways \[ (3) \]: (i) Absorption of a single photon of energy \( h \omega_1 = E' - E(nS_{1/2}) \) or (ii) absorption of two photons, each with energy \( h \omega_2 = h \omega_1/2 \). This setup implements a coherent control interferometer (CCI), a variant of a Mach-Zehnder interferometer (MZI): The ground state is spin-1/2 and constitutes the interferometer's two input ports, whereas the final spin projection \( m_s' = \pm 1/2 \) of the ejected photoelectron \( \Pi \) is identified with the two output ports.

In the long-time limit, the change of the electron's spin can be described by a completely positive map that is constructed from matrix elements \( \langle K'E^m_k | m_k \rangle | nS_{1/2} m_j \rangle \) of the scattering operator \( S = \mathbb{1} - 2\pi i \int dE \delta(E - E_0) \mathcal{T}(E + i0) \delta(E - E_0) \), where \( | m_k \rangle \) is the \( m \)-photon number state of the mode \( k \) and \( E_0 = H_A + H_R \) the free Hamiltonian of atom and radiation. The transition operator describes a perturbative expansion \( T(z) = V + V G_0(z) T(z) \) in powers of \( V \) and the unperturbed resolvent \( G_0(z) = (z - H_0)^{-1} \).

Evaluating the scattering matrix element to first order gives the probability amplitude for absorbing a single photon and ejecting an electron with (asymptotic) wavevector \( K' \): \( \langle \delta(\omega - \omega_1) \delta_{m,m'+1} \sqrt{m} \rangle | l'(m', m_j) / \hbar \) and

\[
 t_1(m', m_j) = \frac{2\pi h^2 g_k}{\sqrt{m c K}} \sum_{m_l} Y_{lm_l}(K') \sum_{j'} \langle lm_l \frac{1}{2} m_s | jm_j \rangle \langle j' m'_j \rangle \langle m'_j | A_1 (P j' S_{1/2}, \hat{e}) | m_j \rangle D_1(E' j' nS_{1/2}), \tag{1a} \]

\[
 \langle m'_j | A_1(l' j' l j, \hat{e}) | m_j \rangle = \sqrt{2 \pi h^2} \int (010|010 \rangle \sum_{m_{l'}} \sum_{q} \varepsilon_q \langle lm_{l'} 1q | l' m_{l'} \rangle \sum_{m_s} \langle l'm'_j \frac{1}{2} m_{s} | jm_j \rangle \langle lm_{l'} \frac{1}{2} m_s | m_j \rangle, \tag{1b} \]

The dipole selection rules dictate that \( t_1 \) only connects to \( P \) continuum states, \( l' = 1 \), since the \( A_1 \) matrix element vanishes unless \( l \) and \( l' \) differ by one. Moreover, the projections \( \varepsilon_q = \hat{e} \cdot \hat{e}_q \) of the incident-field polarization onto the spherical basis vectors \( \hat{e}_q = \hat{z} \pm i \hat{y} / \sqrt{2} \), \( \hat{e}_0 = \hat{z} \) determine the respective shares of \( \sigma^- (|\Delta m_j = \pm 1\rangle \) and \( \pi \)-transitions \( (|\Delta m_j = 0\rangle \) in the absorption amplitude. The radial integrals \[ [8]. \] \[ D_1(E' l' j' n l) = \frac{e^{\int_0^\infty dx x^3 R_{E' l' j'}^\Pi(x) R_{nlj}(x), \text{etc.}}}{c} \text{are not explicitly calculated here and serve as empirical parameters.} \]

In the same manner, we derive the two-photon transition amplitude \( t_2(m'_s, m_j) \) in second order perturbation theory. It can be written as \( \delta(\omega - \omega_2) \delta_{m,m'+2} \sqrt{m(m-1)} \) \( t_2(m'_s, m_j) / 2 \hbar \), where we defined

\[
 t_2(m'_s, m_j) = \frac{2\pi h^3 g_k^2}{\sqrt{m c K}} \sum_{l' m_j} Y_{l'm_j}(K') \sum_{j'} \langle l'm'_j \frac{1}{2} m_{s} | jm_j \rangle \sum_{j''} \langle m'_j | A_2 (l' j' P j'' S_{1/2}, \hat{e}) | m_j \rangle D_2(E' l' j' P j'' nS_{1/2}), \tag{2a} \]

\[
 \langle m'_j | A_2(l' j' l'' j'' l j, \hat{e}) | m_j \rangle = \sum_{m_{j''}} \langle m'_j | A_1(l' j' l'' j'' \hat{e}) | m_{j''} \rangle \langle m''_j | A_1(l'' j'' l j, \hat{e}) | m_j \rangle, \tag{2b} \]

\[
 D_2(E' l' j' l'' j'' n l j) = \sum_{n''} \frac{D_1(E' l' j' n'' l'' j'' j') D_1(n'' l'' j'' n l j)}{E' - E(n'' l'' j'')} + \frac{dE''}{E'' - E'} \frac{D_1(E' l' j' E'' l'' j'') D_1(E'' l'' j'' n l j)}{E'' - h \omega + i0}. \tag{2c} \]

The selection rules for the two photon process are such that only partial waves with \( l' = 0 \) or 2 contribute to the final amplitude. The states accessed on each route are thus of different parity and angular momentum and, clearly, there exist measurement schemes for which the one- and two-photon ionization pathways become abso-
the laboratory’s $\hat{m}_j = m_j - m'_j = 1$, whereas two-photon ionization leaves the electronic spin unchanged. The processes are thus distinguishable by their outcomes. (b) This is not the case in the closed setup c, for which the transition amplitudes between all spin combinations are finite. For $m_j = \frac{1}{2}$ (and $d_2 = 0$), these amplitudes are balanced. For $m_j = \frac{3}{2}$, they are not, illustrated here by the width of the arrows.

In the current setup, the choice of the measurement does not originate from the CCI, but is hosted externally. Therefore, observation of any interference in the spin degree of freedom necessitates a partial or full erasure of the path knowledge stored in the outgoing matter wave \cite{9}. Here, this is accomplished by projecting onto a specific direction of emission, $\hat{K}'$.

Path information might also become available due to entanglement between the spin and the radiation. To ensure the spin not on the $\hat{m}_j = -\frac{1}{2}$ states, $|\alpha\rangle = |\alpha\rangle \otimes |\phi_\alpha\rangle$ and $|\alpha\rangle = |\phi_\alpha\rangle \otimes |\phi_\beta\rangle$ are eigenstates of the annihilation operator. If used, any remaining which-way knowledge must stem from unbalanced a priori probabilities of the ionization pathways. Aiming for maximum interference contrast, we realize an unbiased interferometer by appropriately adjusting the coherent state amplitude moduli:

$$|\alpha_{\omega_0}\rangle = \frac{6\sqrt{\pi} \lambda e^{i\kappa_1}}{\hbar g_k} \left[ -D_1(E'P_1^2 S_\frac{1}{2}) + D_1(E'P_1^2 S_\frac{1}{2}) \right]$$

$$|\alpha_{\omega_0}\rangle^2 = \frac{90\sqrt{\pi} \lambda e^{i\kappa_2}}{\hbar^2 g_k^2} \left[ -5D_2(E'S_\frac{1}{2} P_2^2 S_\frac{1}{2}) - 10D_2(E'S_\frac{1}{2} P_2^2 S_\frac{1}{2}) + 5D_2(E'D_2^2 P_2^2 S_\frac{1}{2}) + 9D_2(E'D_2^2 P_2^2 S_\frac{1}{2}) \right]^{-1}.$$  (4b)

It is evident that this adjustment necessitates spin-orbit coupling and knowledge of the $D_1$ and $D_2$ parameters. The field’s phases, $\phi_\alpha = \arg \alpha_{\omega_0}$ and $\phi_\beta = \arg \alpha_{\omega_0}$, can still be chosen freely. $\kappa_1$ and $\kappa_2$ are both material phase factors and $\lambda$ is a positive, constant scaling factor. The value of $\lambda$ is irrelevant for now, since the probability amplitudes are eventually conditioned on successful detection of the photoelectron in the channel $\hat{K}'$.

For the choices above, $|t_{\omega_0}|^2 = |t_{\omega_0}|^2$, $\nu_0 = 1$, and $\theta_0 = \phi_{\omega_0} - 2\phi_{\omega_0} + \kappa_1 - \kappa_2$. Hence, the interference pattern can be used to infer the phase difference $\kappa_1 - \kappa_2$ between even and odd parity continuum wave functions, as originally illustrated in Ref. \cite{6}.

In the current setup, the choice of the measurement basis selects either the spin’s particle or wave property. We deem this unsatisfactory, since the duality does not originate from the CCI, but is hosted externally. Therefore, we would like to integrate the choice between the properties directly into the CCI. To this end, we have to determine a set of parameters $(k\hat{e})_{0c}, (m - 2k\hat{e})_{0o}, (m - 2k\hat{e})_{2o}$, which-way information can be erased and visibility restored. The maximally observable fringe visibility in the subensemble selected by any such projection is restricted by the bias of the interference: Full visibility, $\nu_o = 1$, is only attainable if $|t_{\omega_0}|^2 m_{1o} = |t_{\omega_0}|^2 m_{2o}(m_{2o} - 1) > 0$.

The reappearance of interference as described above cannot serve as a device-independent proof of nonclassicality. Without considering and understanding the details of the light-matter interaction, we cannot decide whether actual which-way labels have been erased (quantum erasure) or whether there were none to begin with (non-erasing quantum erasure) \cite{9}. In the former case the spin and the radiation field would have shared an entangled state, whereas in the latter case they may have been in a classical mixture.
that realizes a balanced closed interferometer \(c\), complementing the open configuration \(o\) above. We find that, on the one hand, the light for the one-photon ionization should be elliptically polarized, \(\sqrt{14} \hat{e}_{1c} = \sqrt{5/3 + \sqrt{2}} (\hat{x} + \sqrt{2} \hat{z}) - i \sqrt{9 - 3\sqrt{2}} \hat{y}\), and be emitted in the same direction in which we detect the photoelectron, \(\hat{k}_{1c} = \hat{K}'\). It then follows that \(t_{1c}(\frac{1}{2}, \frac{1}{2}) = t_{1c}(\frac{1}{2}, -\frac{1}{2})\). Two-photon absorption, on the other hand, will be actuated by a circularly polarized field, \(\sqrt{2} \hat{e}_{2c} = (2\sqrt{2} \hat{x} + \hat{z})/3 + i \hat{y}\), that shines in from the direction \(\hat{k}_{2c} = (\hat{x} - 2\sqrt{2} \hat{z})/3\) making an angle of \(\pi - \arcsin 3\) with the \(\hat{z}\)-axis, see Fig. 1. The selection rules prohibit two-photon \(\sigma\)-transitions that end in S states. Only \(t_2(\frac{1}{2}, \frac{1}{2})\) and \(t_2(-\frac{1}{2}, -\frac{1}{2})\) contain S-wave \(D_2\) factors and the above settings are tailored to eliminate them. If we can select an \(E'\) for which \(D_2(E'D_2^* P_2^* n S_2^*) = 0\), then \(t_2(\frac{1}{2}, \frac{1}{2}) = -t_2(\frac{1}{2}, -\frac{1}{2})\), which is the most desirable situation. In general, however, the amplitudes are somewhat biased and have a mutual phase shift different from \(\pi\), see below.

Simultaneous unbiased interference for both input ports of the CCI, \(m_j = \pm \frac{1}{2}\), cannot be achieved. Hence, in the following, we restrict ourselves to the port \(m_j = -\frac{1}{2}\). To prevent the emergence of which-path knowledge in the radiation field, we, as before, utilize a direct product \(|c\rangle\) of coherent states \(|(\alpha k\hat{e})_{1c}\rangle, |(\alpha k\hat{e})_{2c}\rangle\) with amplitudes

\[
|\alpha_{1c}| = \sqrt{\frac{7}{2(\pi - 2\sqrt{2})}} |\alpha_{1o}|, \tag{5a}
\]

\[
|\alpha_{2c}| = 180 \sqrt{\pi} \lambda e^{i \kappa_{2c}} \left(\frac{D_2(E'D_2^* P_2^* n S_2^*)}{i(\sqrt{2} + 2) h^2 \beta^2 d}\right) \left[D_2(E'D_2^* P_2^* n S_2^*) + D_2(E'D_2^* P_2^* n S_2^*)^{-1}\right]^{-1}, \tag{5b}
\]

and phases \(\phi_{1c}, \phi_{2c}\). In Eq. (5b), the material phase \(\kappa_{2c}\) assures positivity of the right hand side. Eventually, with \(D_2(E'D_2^* P_2^* n S_2^*) = 0\), we have \(|t_1(m', \frac{1}{2})\alpha_{1c}| = |t_2(m', -\frac{1}{2})\alpha_{2c}|\), see Fig. 2(b).

We now can summarize as follows: For the initial state \(|i_o\rangle = |n S_2^* \frac{1}{2} \rangle \otimes |o\rangle\) and in second order in \(V\), we obtain the final state

\[
|f_o\rangle = \frac{e^{i \delta_o}}{\sqrt{2}} \left(|K'_{\frac{1}{2}}\rangle + e^{-i \delta_o} |K'_{-\frac{1}{2}}\rangle\right) \otimes |o\rangle, \tag{6}
\]

where \(\delta_o = \phi_{1o} + \kappa_1\). \(|f_o\rangle\) displays full particle-like statistics. Projected on the \(\hat{z}\) axis, both spin orientations \(m'_i = \pm \frac{1}{2}\) are always equally likely. There is no phase coherent control and the fringe visibility vanishes on both output ports, \(V_o = 0\). If, instead, the CCI is initialized in the state \(|i_c\rangle = |n S_2^* \frac{1}{2} \rangle \otimes |c\rangle\), then ionization leaves the system in the state

\[
|f_c\rangle = \frac{e^{i \delta_c}}{N_{f_c}} \left|\left(\cos \frac{\phi_{1c} + \phi_{2c}}{2} + \frac{1}{2} (7 - 5\sqrt{2}) e^{-i \phi_{1c}/2} d_2\right)|K'_{\frac{1}{2}}\rangle \otimes |c\rangle\right|, \tag{7}
\]

with the global phase \(\delta_c = (\phi_{1c} + 2\phi_{2c} + \kappa_1 + \kappa_{2c})/2\), the interferometric phase difference \(\phi_c = \phi_{1c} - 2\phi_{2c}\), the material phase shift \(\theta_c = \kappa_1 - \kappa_{2c}\), the normalization \(N_{f_c}\), and the empirical factor

\[
d_2 = \frac{-3 e^{-i(\kappa_1 + \kappa_{2c})/2} D_2(E'D_2^* P_2^* n S_2^*)}{5 D_2(E'D_2^* P_2^* n S_2^*) + D_2(E'D_2^* P_2^* n S_2^*)}. \tag{8}
\]

\(|f_c\rangle\) is wave-like: Coherent control of the electron’s spin is possible by virtue of \(\phi_c\). The visibility—and thus the controllability—is finite and reaches the maximum \(N_c = 1\) for \(d_2 = 0\).

One could argue that randomizing between the configurations \(o\) and \(c\), \(\delta_i = (|i_o\rangle\langle i_o| + |i_c\rangle\langle i_c|)/2\), realizes Wheeler’s “delayed choice” gedanken experiment [10]. However, this is not the case, because by using two distinct experimental setups the property of the spin is chosen in advance. In order to render it impossible for the spin to “know” beforehand which property to display, one would have to postpone the choice of setup until after the photoelectron has been emitted. Yet, the nature of the proposed experiments does not literally allow this. Notwithstanding, instead of flipping a coin between \(o\) and \(c\) we can prepare a coherent superposition \(|i\rangle = (\cos \gamma |i_o\rangle + \sin \gamma |i_c\rangle)/N_i\), which gives the final state \(|f\rangle = (\cos \gamma |f_o\rangle + \sin \gamma |N_{f_o} |f_c\rangle)/N_f\) within second order perturbation theory and with \(N_i, N_f\) being normalization factors. Since the states \(|o\rangle\) and \(|c\rangle\) are each direct products of two coherent states in, in total, four mutually orthogonal modes (cf. Fig. 1), the state \(|i\rangle\) features nonclassical correlations of radiation [11]. For \(\gamma = -3\pi/4\) or \(\pi/4\), the superposition is balanced and of the GHZ type.

It can be produced from a single semiclassical coherent light source (a laser) by virtue of a special nonlinear MZI followed by second harmonic generation, controlled absorption, phase manipulation, and polarization. States with variable weighting \(\gamma\) pose an additional challenge [12]. \(|f\rangle\) describes a coherent blend of particle and wave aspects that is conditioned on the state of the radiation field. To simultaneously have and to lack the ability to interfere is unheard-of in classical physics and is clearly quantum.

Since the CCI is conditioned on an auxiliary quantum degree of freedom, the proposed experiment implements “quantum delayed choice” [13]. It is more flexible than Wheeler’s original thought experiment, because the nature of the spin, wave or particle, can be prepared even after its polarization has been detected: Photon counting, say, in the mode 1o, prepares a completely particle-like
state, if the mode is found to be populated with at least one photon, or, if the mode is empty (and the amplitude \( \alpha_k \) sufficiently large), a wave-like state.

Still, this projective measurement cannot distinguish between classical and nonclassical correlations in the final state [13]. Experiments to do so often rely on the violation of a Bell inequality [14]. What follows is a proposal of a device-independent Bell test appropriate for the state \( |f⟩ \) with \( \delta_o - \delta_c = (\phi - \pi)/2 \), \( \theta_o = \phi - \pi/2 \), and \( \phi_o + \theta_o = -\phi \) so that \( \phi \) combines the other phase parameters. Adopting the approach of Ref. [13] we define two families of dichotomic measurement operators: On the one hand, \( \Gamma(\zeta) = D_S(\zeta) \beta \sigma_z D_S(\zeta) \) with the rotation \( D_S(\zeta) = \exp(\zeta \sigma_+ - \zeta^\ast \sigma_-) \) measures the spin in the direction of space defined by \( \zeta \), where the Pauli operators \( \sigma_z, \sigma_\pm \) are defined with respect to the basis \( \{ |K^{\frac{1}{2}} \rangle, |K^{\frac{-1}{2}} \rangle \} \). On the other hand,

\[
A(\beta) = \bigotimes_k D_k(\beta_k) \cdot (2|0\rangle \langle 0| - 1) \cdot \bigotimes_k D_{k'}(\beta_{k'})
\]

with the displacements \( D_k(\beta_k) = \exp(\beta_k a_k^\dag - \beta_k^* a_k) \) is a joint photon threshold measurement over all modes \( k = 1o, 1c, 2o, \) and \( 2c \). Assuming unlimited detection efficiency, we have calculated numerically the maximally achievable violation of the Bell-CHSH inequality

\[
|\langle B \rangle| = |(\Gamma(\zeta) \otimes A(\beta)) + (\Gamma(\zeta^\ast) \otimes A(\beta^\ast)) + (\Gamma(\zeta) \otimes A(\beta^\ast)) - (\Gamma(\zeta^\ast) \otimes A(\beta))| \leq 2
\]

by measurements on \( |f⟩ \) for \( \gamma = \pi/4 \) and \( d_2 = 0 \) as a function of \( \phi \). We find that Ineq. [10] can be violated to various degrees for all values of \( \phi \), Fig. 3 refuting any classical explanation of the observed statistics. The maximum value is irrespective of the absolute field amplitudes \( |\alpha_k| \), since the parameters \( \lambda, |\beta_k|, \) and \( |\beta_k'| \) can always be adapted accordingly. The optimized complex phases of \( \beta \) and \( \beta^\ast \) fulfill

\[
\begin{align*}
\beta_{1o} \alpha_{1o}^\ast &= \beta_{2o} \alpha_{2o}^\ast &= \beta_{1c} \alpha_{1c}^\ast &= \beta_{2c} \alpha_{2c}^\ast, \\
\beta_{1c} \alpha_{1c}^\ast &= \beta_{2c} \alpha_{2c}^\ast \\
|\beta_{1o} \alpha_{1o}| &= |\beta_{2o} \alpha_{2o}| &= |\beta_{1c} \alpha_{1c}| &= |\beta_{2c} \alpha_{2c}|
\end{align*}
\]

and are independent of \( |\alpha_k| \). So are the optimal choices for \( \zeta \) and \( \zeta^\ast \). We expect that both a finite detection efficiency and \( d_2 \neq 0 \) have a detrimental impact on the violation.

In conclusion, we have proposed a coherent control experiment within which wave-particle duality, quantum erasure, and quantum delayed choice can be rigorously displayed, thereby solidifying the analogy between coherent control and quantum interference for this specific coherent control implementation. The physics of the proposed experiment does defy any classical explanation, since controllability is conditioned on nonclassical correlations between matter and radiation. This is ascertainable a posteriori by virtue of a Bell test without changes to the coherent control dynamics.

![FIG. 3. Numerically optimized Bell-CHSH inequality test for |f⟩ as a function of the phase φ (solid). The largest violation (|(⟨B⟩)| = 2.260) is achieved both at φ = 2.509 and 5.345, the smallest (2.012) at φ = 0.7846. The dashed line shows |⟨B⟩| for those parameters that maximize the violation at φ = 2.509.](image.png)

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