Application of FFT Interpolation Correction Algorithm Based
on Window Function in Power Harmonic Analysis

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Abstract. In the power harmonic detection algorithm, the disadvantages of Fast Fourier Transform (FFT) are difficult to accurately detect in non-synchronous sampling and non-integer periodic truncation to need be solved. Therefore, this paper considers various advantages and disadvantages of traditional window functions such as Hanning window and new window function, for example: Blackman-Harris window and Nuttall window. Four three-order Nuttall window functions was adopted, which are more suitable for power harmonic analysis. Meanwhile, the double-line interpolation analysis and algorithm correction of the Nuttall window function are carried out. Finally, the algorithm is compared and simulated. By using this algorithm the phase error is stable at 0.00001°, and the amplitude error is almost 0, which fully meets the requirements of harmonic measurement standards.

1. Introduction
With the advancement of power electronization in power system, harmonic detection and control in power grid has attracted wide attention due to the large number of power electronic devices connected to the grid. In order to maintain the safe and stable operation of power grid, high-precision harmonic component detection is of great significance [1-8]. Fast Fourier Transform is the most commonly used method for harmonic analysis. However, the disadvantage of FFT is that it is prone to "spectrum leakage" and "fence effect" when non-synchronous sampling and non-integer periodic truncation occur, and cannot accurately measure the harmonic data of power grid. Therefore, it is necessary to adopt a window function with good performance to process FFT algorithm to inhibit the spectrum effect. The traditional window functions are rectangular window, Hanning window, Hamming window and Blackman window, which have different advantages and disadvantages. In fact, the large attenuation of the first side lobe and the fast attenuation of the peak value of the side lobe are the most effective ways to inhibit the spectral effect, but the larger width of the main lobe often brings adverse effects on signal analysis, so for different kinds of signals, this is a process of comprehensive evaluation and compromise selection. There are two general principles, including: 1) The main lobe of window spectrum should be as narrow as possible to inhibit spectrum leakage. 2) The maximum amplitude of the side lobe relative
to the main lobe is minimized so that the energy accumulates in the main lobe and the attenuation deceleration of the side lobe is increased [9-12]. The power harmonic measurement standard requires that the error of amplitude and the phase angle should not exceed 5% [13]. In order to improve the accuracy and calculation speed of harmonic analysis, a kind of four-third-order Nuttall window function is studied importantly, and the Nuttall window function is double-line interpolated, then the fifth-order polynomial fitting is performed. Through numerical simulation, the results show that this algorithm can achieve more accurate results while meeting the harmonic measurement standards in the same kind of algorithms.

2. Analysis and Comparison of Window Functions

Traditional window functions such as rectangular windows, Hanning windows, Hamming windows and Blackman windows. The DFT spectrum are shown in Figs. 1. The normalized frequency is used as abscissa and the maximum decibel value is 0 dB as ordinate in spectrum diagram.

![Window Function Spectrum Diagram](image)

**Figure 1.** Window function spectrum diagram

Through numerical analysis, four related indexes of window function are obtained as follows: Table 1.

| Window function | First Side lobe Attenuation(dB) | Main Lobe bandwidth | Sidelobe Peak Attenuation(dB/oct) |
|-----------------|----------------------------------|---------------------|-----------------------------------|
| Rectangular     | -13                              | 4 $\pi /N$          | -6                                |
| Hamming         | -41                              | 8 $\pi /N$          | -6                                |
| Hanning         | -32                              | 8 $\pi /N$          | -18                               |
| Blackman        | -58                              | 12 $\pi /N$         | -18                               |
Rectangular window is suitable for extracting main lobe frequency, but not for extracting amplitude; Hanning window has widened main lobe and reduced side lobe, which is suitable for extracting frequency and amplitude of multiple frequency component signals; Hanning window is similar to Hanning window, but the attenuation speed of side lobe is slower; as for Blackman window, because the equivalent noise bandwidth is larger than Hanning window, the accuracy of frequency identification is lower.

For power harmonic analysis, we often pay more attention to the amplitude and phase angle, it is not difficult to conclude that Hanning window is more suitable for harmonic analysis and extraction through comprehensive analysis. However, due to the mutual restriction of the main lobe bandwidth and the attenuation of the first side lobe, some sub-harmonic analysis often loses its reference value. At this time, window functions with better performance, such as Blackman-Harris window function and Nuttall window function, are proposed.

1) The Blackman-Harris window function is expressed as:

\[ w(n) = 0.35875 - 0.48829\cos\left(\frac{2\pi n}{N}\right) + 0.14128\cos\left(\frac{4\pi n}{N}\right) - 0.01168\cos\left(\frac{6\pi n}{N}\right) \]

Its spectrum analysis is shown in Fig 2. From the Fig 3, we can see that the main lobe width of Blackman-Harris window function is \(16\pi / N\); First side lobe attenuation is -92dB; The peak attenuation velocity of side lobe is 6dB/oct.

2) Four third-order Nuttall window functions is expressed as:

\[ w(n) = 0.338946 - 0.481973\cos\left(\frac{2\pi n}{N}\right) + 0.161054\cos\left(\frac{4\pi n}{N}\right) - 0.018027\cos\left(\frac{6\pi n}{N}\right) \]

Its spectrum analysis diagram is shown in Fig.3:

![Figure 2. Blackman-Harris window function spectrum diagram (right)](image-url)
Figure 3. Four third-order Nuttall window function spectrum (left)

From the Fig 4, we can see that the main lobe width of four third-order Nuttall window function is $16\pi / N$; First side lobe attenuation is -83dB; The peak attenuation velocity of side lobe is 30dB/oct. This window function with large attenuation of the first side lobe and fast attenuation of the peak value of the side lobe has the characteristics of small leakage of the spectrum. It is very suitable for the harmonic amplitude and phase angle values. Therefore, the basic algorithm of harmonic analysis is FFT algorithm based on four third-order Nuttall window functions [14-15].

3. Study on Double Line Interpolation Algorithm

All manuscripts must be in English, also the table and figure texts, otherwise we cannot publish your It is difficult to accurately detect harmonic parameters in asynchronous sampling and non-integer period truncation. But double interpolation algorithm can solve this problem very well.

Let a signal $x(t)$ be sampled discretely at $f_s$ sampling frequency, and the formula 3 is obtained.

$$x(n) = A \cos(2\pi n \frac{f_0}{f_s} + \varphi_0)$$ (3)

Where: $f_0$ : the frequency of signal $x(n)$; $A$ : the amplitude of signal $x(t)$; $\varphi_0$ : the initial phase angle of signal $x(t)$.

Formula 4 is obtained by Euler transformation after adding windows to signal $x(n)$ and FFT:

$$X(f) = \frac{A}{2} e^{i\varphi_0} \{ W[2\pi \left(\frac{f-f_0}{f_s}\right)] + W[2\pi \left(\frac{f+f_0}{f_s}\right)] \}$$ (4)

For DTFT discrete sampling of Formula 4, the sampling interval is: $\Delta f = \frac{f_0}{N}$, In this paper, only positive frequency signals are considered, and then the spectrum of the windowed signal is obtained:

$$X(k\Delta f) = \frac{A}{2} e^{i\varphi_0} W[2\pi \left(\frac{k\Delta f - f_0}{f_s}\right)]$$ (5)
Where: $W(\cdot)$: FFT transform of window function.

Let $k_1$, $k_2$ be the first and second largest spectral lines at the peak point $k_0$ or so, respectively, $y_1 = |X(k_1\mathcal{V}f)|$ and $y_2 = |X(k_2\mathcal{V}f)|$, meeting $k_1 \leq k_0 \leq k_2 = k_1 + 1$. At the same time, $\alpha$ parameters are introduced $\alpha = k_0 - k_1 - 0.5$ and set as [-0.5, 0.5]. Another $\beta$ parameter is defined $\beta = (y_2 - y_1)/(y_2 + y_1)$, substitution:

$$\beta = \frac{|W[2\pi(-\alpha + 0.5)]|}{|W[2\pi(-\alpha - 0.5)]|}$$

Formula 6 defines the functional relationship between $\alpha$ and $\beta$ as $\beta = g(\alpha)$ or $\alpha = g^{-1}(\beta)$. The frequency correction formula $f_0 = k_0\mathcal{V}f = (\alpha + k_1 + 0.5)\mathcal{V}f$ is obtained, that is formula 7:

$$A = \frac{|W[2\pi(k_1 - k_0)]|}{|W[2\pi(k_2 - k_0)]|} = \frac{2(y_1 + y_2)}{2(2\pi(-\alpha - 0.5))}$$

When the length of data stage $N$ is large, formula 7 can be further simplified, that is formula 8:

$$A = N^{-1}(y_1 + y_2)v(\alpha)$$

At the same time, from formula 8, the initial phase correction formula can be obtained as follows:

$$\varphi_0 = \arg\{X(k\mathcal{V}f)\} + \arg\{W[2\pi(k - k_0)]\}$$

4. Bispectral Line Interpolation Correction Formula Based on Four Third Order Nuttall Window

Based on the above analysis, it is not difficult to see that the calculation of bispectrum interpolation algorithm is still somewhat cumbersome, especially for the application in embedded real-time monitoring system, if Nuttall window function is added, the amount of calculation is relatively large. Accordingly, a polynomial approximation numerical analysis algorithm is used to obtain $\alpha = g^{-1}(\beta)$. The general idea is to take a series of values of $\alpha$, whose range is [-0.5, 0.5], and substitute formula 6 to get corresponding $\beta$ values. Through data fitting, formula 13 can be obtained:

$$\alpha = g^{-1}(\beta) = \alpha_0 + \alpha_1\beta + \alpha_2\beta^2 + \ldots + \alpha_{2m+1}\beta^{2m+1}$$

$v(\alpha)$ in Formula 8 is basically the same as Formula 10, so Formula 8 can be rewritten as follows:

$$A = N^{-1}(y_1 + y_2)g + b_2\alpha^2 + \ldots + b_2\alpha^{2j}$$
\[ \phi_0 = \arg[X(k_i Vf)] - \pi(\alpha - (-1)i \times 0.5) \quad i = 1, 2 \]  

(12)

In order to compare analysis, the modified formulas of Hanning window and Blackman-Harris window are also given [15-16].

(1) Four three-order Nuttal window functions and corrected formulas:

\[
w(n) = [0.338946 - 0.481973 \cos(\frac{2\pi n}{N}) + 0.161054 \cos(\frac{4\pi n}{N}) - 0.018027 \cos(\frac{6\pi n}{N})]R_n(n) \\
\alpha = 2.95494514\beta + 0.17671943\beta^3 + 0.09230694\beta^5 \\
A = N^{-1}(g_1 + g_2)(3.20976143 + 0.9187393\alpha^2 + 0.14734229\alpha^4) \]

(13)

(2) Hanning window functions and corrected formulas:

\[
w(n) = \left[0.5 - 0.5\cos(\frac{2\pi n}{N})\right]R_n(n) \\
\alpha = 1.5\beta \\
A = N^{-1}(g_1 + g_2)(2.35619403 + 1.15543682\alpha^2 + 0.32607873\alpha^4 + 0.07891461\alpha^6) \]

(14)

(3) Blackman-Harris window functions and corrected formulas:

\[
w(n) = [0.35875 - 0.48829\cos(\frac{2\pi n}{N}) + 0.14128\cos(\frac{4\pi n}{N}) - 0.01168\cos(\frac{6\pi n}{N})]R_n(n) \\
a = 2.61979085\beta + 0.286567\beta^3 + 0.1283\beta^5 + 0.080241\beta^7 \\
A = N^{-1}(g_1 + g_2)(3.06539676 + 0.965559979\alpha^2 + 0.163556\alpha^4 + 0.01985\alpha^6) \]

(15)

5. Simulation Analysis

In order to verify the theoretical analysis, this section uses MATLAB programming simulation to process a group of harmonic data signals in Table 2 with four three-order Nuttall windows, Hanning windows and Blackman-Harris windows.

Here, the fundamental frequency of the signal is 49.9 Hz, the sampling frequency is 1600 Hz and the data length is 2048.

| Harmonic Number | 1     | 2     | 3     | 4     | 5     | 6     | 7     | 8     | 9     | 10    |
|-----------------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|
| Voltage Amplitude/V | 223.00| 2.00  | 10.00 | 3.00  | 2.70  | 2.00  | 0.10  | 0.40  | 0.00  | 1.00  |
| Frequency/Hz     | 49.90 | 99.90 | 149.50| 199.40| 249.20| 299.00| 349.50| 447.00| 495.00| 555.00|
| Initial phase/o  | 0.00  | 10.00 | 20.00 | 30.00 | 40.00 | 55.00 | 0.00  | 70.00 | 0.00  | 100.00|

Table 2. Harmonic data table

Through the program simulation, the following results are obtained. The specific data are shown in tables 3 to 5, and the phase and amplitude error analysis is shown in tables 6 and 7.
Table 3. Hanning window harmonic analysis data table

| Harmonic Number | 1     | 2     | 3     | 4     | 5     | 6     | 7     | 8     | 9     | 10      |
|-----------------|-------|-------|-------|-------|-------|-------|-------|-------|-------|---------|
| Voltage Amplitude/V | 0.0008 | 0.0007 | 0.0006 | 0.0005 | 0.0004 | 0.0003 | 0.0002 | 0.0001 | 0.0000 | 0.0000 |
| Frequency/Hz | 49.900001 | 99.799431 | 149.69999 | 199.599955 | 249.499984 | 299.39996 | 349.299981 | 399.199993 | 449.100008 | 498.99996 |
| Initial phase/° | -0.000117 | 10.065845 | 20.001105 | 30.00519 | 40.001867 | 50.004551 | 60.003055 | 70.000465 | 80.00001 | 90.00000 |

Table 4. Blackman-Harris window harmonic analysis data tables

| Harmonic Number | 1     | 2     | 3     | 4     | 5     | 6     | 7     | 8     | 9     | 10      |
|-----------------|-------|-------|-------|-------|-------|-------|-------|-------|-------|---------|
| Voltage Amplitude/V | 0.000173 | 0.000009 | 0.000003 | 0.000002 | 0.000001 | 0.000000 | 0.000000 | 0.000000 | 0.000000 | 0.000000 |
| Frequency/Hz | 49.9 | 99.800022 | 149.700006 | 199.600019 | 249.500027 | 299.400027 | 349.300034 | 399.200041 | 449.100048 | 499.000051 |
| Initial phase/° | -0.000045 | 9.997834 | 19.999312 | 29.99797 | 39.997787 | 49.997076 | 59.996876 | 69.996676 | 79.996476 | 89.996276 |

Table 5. Four third-order Nuttall window harmonic analysis data tables

| Harmonic Number | 1     | 2     | 3     | 4     | 5     | 6     | 7     | 8     | 9     | 10      |
|-----------------|-------|-------|-------|-------|-------|-------|-------|-------|-------|---------|
| Voltage Amplitude/V | 223.000173 | 2.000009 | 10.000015 | 30.000028 | 70.000056 | 1.000004 | 0.100091 | 0.040001 | 0.000000 | 0.000000 |
| Frequency/Hz | 49.98 | 99.800022 | 149.700006 | 199.600019 | 249.500027 | 299.400027 | 349.300034 | 399.200041 | 449.100048 | 499.000051 |
| Initial phase/° | -0.000045 | 9.997834 | 19.999312 | 29.99797 | 39.997787 | 49.997076 | 59.996876 | 69.996676 | 79.996476 | 89.996276 |

Table 6. All kinds of window function harmonic analysis phase error table

| Harmonic Number | 1     | 2     | 3     | 4     | 5     | 6     | 7     | 8     | 9     | 10      |
|-----------------|-------|-------|-------|-------|-------|-------|-------|-------|-------|---------|
| Hanning | 1.17E-04 | 6.57E-02 | 1.01E-03 | 4.19E-03 | 1.87E-03 | 4.55E-03 | 3.87E-02 | 3.06E-03 | 0.00E+00 | 4.65E-04 |
| Blackman-Harris | 4.50E-01 | 2.17E-03 | 6.68E-04 | 2.10E-03 | 2.31E-03 | 2.92E-03 | 4.30E-02 | 1.12E-02 | 0.00E+00 | 3.38E-03 |
| Four Third-order Nuttall | 4.00E-06 | 2.25E-04 | 3.00E-06 | 2.25E-05 | 100E-06 | 1.20E-05 | 7.90E-05 | 2.00E-06 | 0.00E+00 | 1.00E-06 |

Table 7. All kinds of window function harmonic analysis amplitude error table

| Harmonic Number | 1     | 2     | 3     | 4     | 5     | 6     | 7     | 8     | 9     | 10      |
|-----------------|-------|-------|-------|-------|-------|-------|-------|-------|-------|---------|
| Hanning | 8.00E-05 | 2.39E-04 | 1.60E-05 | 1.50E-05 | 2.00E-06 | 4.00E-06 | 0.00E+00 | 0.00E+00 | 0.00E+00 | 1.00E-06 |
| Blackman-Harris | 1.73E-04 | 9.00E-06 | 1.50E-05 | 8.00E-06 | 6.00E-06 | 4.00E-06 | 1.00E-06 | 0.00E+00 | 0.00E+00 | 1.00E-06 |
| Four Third-order Nuttall | 5.00E-06 | 0.00E+00 | 0.00E+00 | 0.00E+00 | 0.00E+00 | 0.00E+00 | 0.00E+00 | 0.00E+00 | 0.00E+00 | 0.00E+00 |

From the above table data, it can be analyzed that compared with the other two window functions, the four three-order Nuttall window functions can better reflect the advantages of small error and high accuracy. Among them, the phase error is stable at 0.00001°, and the amplitude error is almost 0, which fully meets the requirements of harmonic measurement standards. In terms of practical engineering and product development, the calculation is feasible. The method also has a strong detection accuracy for weak amplitude harmonics.

6. Conclusion

In this paper, FFT interpolation modified harmonic analysis algorithm based on four third order Nuttall windows is the core algorithm of the whole paper. The accuracy of FFT algorithm is improved and some spectral effects are inhibit by using this algorithm. The simulation results show that the FFT interpolation correction algorithm based on four third-order Nuttall windows has small detection error, high accuracy, and is suitable for power system harmonic analysis. At the same time, it has certain detection ability for weak amplitude harmonics, which provides a theoretical support for the development of the relevant harmonic analyzer.
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