Vector hidden-bottom tetraquark candidate: $Y(10750)$

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Abstract

In this article, we take the scalar diquark and antidiquark operators as the basic constituents, construct the $C\gamma_5 \otimes \partial_\mu \otimes \gamma_5 C$ type tetraquark current to study the $Y(10750)$ with the QCD sum rules. The predicted mass $M_Y = 10.75 \pm 0.10$ GeV and width $\Gamma_Y = 33.60^{+16.61}_{-9.45}$ MeV support assigning the $Y(10750)$ to be the diquark-antidiquark type vector hidden-bottom tetraquark state, which has a relative P-wave between the diquark and antidiquark constituents.

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1 Introduction

Recently, the Belle collaboration observed a resonance structure $Y(10750)$ with the global significance of 6.7$\sigma$ in the $e^+e^- \to \Upsilon(nS)\pi^+\pi^-$ ($n = 1, 2, 3$) cross sections at energies from 10.52 to 11.02 GeV using the data collected with the Belle detector at the KEKB asymmetric-energy $e^+e^-$ collider [1]. The Breit-Wigner mass and width are $M_Y = 10752.7 \pm 5.9^{+17.4}_{-3.3}$ MeV and $\Gamma_Y = 35.5^{+17.6}_{-11.3} - 3.3$ MeV, respectively. The $Y(10750)$ is observed in the processes $Y(10750) \to \Upsilon(nS)\pi^+\pi^-$ ($n = 1, 2, 3$), its quantum numbers may be $J^{PC} = 1^{--}$. In the famous Godfrey-Isgur model, the nearby bottomonium states are the $\Upsilon(4S), \Upsilon(5S)$ and $\Upsilon(3D)$ with the masses 10.635 GeV, 10.878 GeV and 10.698 GeV, respectively [2], while in the QCD-motivated relativistic quark model based on the quasipotential approach (the screened potential model), the corresponding masses are 10.586 GeV, 10.869 GeV and 10.704 GeV (10.611 GeV, 10.831 GeV and 10.670 GeV [3], respectively [4]. Without introducing mixing effects, the experimental data $M_Y = 10752.7^{\pm 5.9}_{\pm 3.3}$ MeV cannot be reproduced, if we assign the $Y(10750)$ to be a conventional bottomonium state [5].

The $Y(10750)$ may be a hidden-bottom tetraquark candidate. In Refs. [6, 7], we take the scalar and axialvector diquark operators as the basic constituents, as they are favored quark configurations, introduce a relative P-wave between the scalar (or axialvector) diquark and scalar (or axialvector) antidiquark operators explicitly in constructing the vector tetraquark current operators, and calculate the masses and pole residues of the vector hidden-bottom tetraquark states using the QCD sum rules in a systematic way, and obtain the lowest masses of the vector hidden-bottom tetraquark states up to now, the predictions support assigning the exotic states $Y(4220/4260)$, $Y(4320/4360)$, $Y(4390)$ and $Z(4250)$ to be the vector tetraquarks with the quantum numbers $J^{PC} = 1^{--}$, which originate from the relative P-wave between the diquark and antidiquark constituents. On the other hand, if we take the scalar ($C\gamma_5$-type), pseudoscalar (C-type), vector ($C\gamma_5\gamma_5$-type) and axialvector ($C\gamma_5\gamma_3$-type) diquark operators as the basic constituents, and construct the vector tetraquark current operators having the quantum numbers $J^{PC} = 1^{--}$ without introducing the relative P-wave between the diquark and antidiquark constituents, we can obtain the masses of the lowest vector tetraquark states, which are about 4.34 GeV or 4.59 GeV [8], and are larger or much larger than the measured mass of the $Y(4220/4260)$ from the BESIII collaboration [9], because the pseudoscalar and vector diquarks are not favored quark configurations [8]. In Ref. [10], we take the scalar and axialvector diquark (and antidiquark) operators as the basic constituents to construct the current operators, calculate the masses and pole residues of the hidden-bottom tetraquark states with the quantum numbers $J^{PC} = 0^{++}, 1^{++}, 1^{--}$ and $2^{++}$ systematically using the QCD sum rules, and observe that the masses of the ground state

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hidden-bottom tetraquark states are about 10.61 – 10.65 GeV. The \( Y(10750) \) may be a vector hidden-bottom tetraquark state.

In the present work, we tentatively assign the \( Y(10750) \) as a diquark-antidiquark type vector hidden-bottom tetraquark state with the quantum numbers \( J^{PC} = 1^{--} \), and construct the \( C\gamma_5 \otimes \bar{\gamma}_\mu \otimes \gamma_5 C \) type tetraquark current operator to calculate its mass and pole residue using the QCD sum rules. In calculations, we take into account the vacuum condensates up to dimension 10 in the operator product expansion as in our previous works. Furthermore, we study the two-body strong decays of the vector hidden-bottom tetraquark candidate \( Y(10750) \) with the three-point correlation functions by carrying out the operator product expansion up to the vacuum condensates of dimension 5. In calculations, we take into account both the connected and disconnected Feynman diagrams.

The rest of the paper is organized as follows. In section 2, we obtain the QCD sum rules for the mass and pole residue of the \( Y(10750) \); In section 3, we obtain the QCD sum rules for the hadronic coupling constants in the strong decays of the \( Y(10750) \), then obtain the partial decay widths; Section 4 is given for a short conclusion.

2 The mass and pole residue of the vector tetraquark candidate \( Y(10750) \)

Firstly, we write down the two-point correlation function \( \Pi_{\mu\nu}(p) \) in the QCD sum rules,

\[
\Pi_{\mu\nu}(p) = i \int d^4x e^{ip\cdot x} \langle 0 | T \{ J_\mu(x) J_\nu^\dagger(0) \} | 0 \rangle ,
\]

where \( J_\mu(x) = J_\mu^{(1,\pm)}(x), J_\mu^{(1,0)}(x) \) and \( J_\mu^{(0,0)}(x) \),

\[
J_\mu^{(1,1)}(x) = \frac{\varepsilon^{ijk}\varepsilon^{lmn}}{\sqrt{2}} u^{Tj}(x) C\gamma_5 b^k(x) \bar{\gamma}_\mu \bar{d}^m(x) \gamma_5 C \bar{b} T^n(x) ,
\]

\[
J_\mu^{(1,0)}(x) = \frac{\varepsilon^{ijk}\varepsilon^{lmn}}{2} [u^{Tj}(x) C\gamma_5 b^k(x) \bar{\gamma}_\mu \bar{u}^m(x) \gamma_5 C \bar{b} T^n(x) - d^{Tj}(x) C\gamma_5 b^k(x) \bar{\gamma}_\mu \bar{d}^m(x) \gamma_5 C \bar{b} T^n(x)] ,
\]

\[
J_\mu^{(0,0)}(x) = \frac{\varepsilon^{ijk}\varepsilon^{lmn}}{\sqrt{2}} d^{Tj}(x) C\gamma_5 b^k(x) \bar{\gamma}_\mu \bar{d}^m(x) \gamma_5 C \bar{b} T^n(x) ,
\]

\[
J_\mu^{(1,-1)}(x) = \frac{\varepsilon^{ijk}\varepsilon^{lmn}}{\sqrt{2}} u^{Tj}(x) C\gamma_5 b^k(x) \bar{\gamma}_\mu \bar{u}^m(x) \gamma_5 C \bar{b} T^n(x) + d^{Tj}(x) C\gamma_5 b^k(x) \bar{\gamma}_\mu \bar{d}^m(x) \gamma_5 C \bar{b} T^n(x) ,
\]

where the \( i, j, k, n \) are color indexes, the superscripts \( (1, \pm) \), \( (1, 0) \), \( (0, 0) \) denote the isospin indexes \( (I, I_3) \), \( \bar{\gamma}_\mu = \bar{\gamma}_\mu - \gamma_5 \bar{\gamma}_\mu \). In the isospin limit, i.e. \( m_u = m_d \), the current operators \( J_\mu(x) \) couple potentially to the diquark-antidiquark type vector hidden-bottom tetraquark states which have degenerate masses. In the present work, we choose \( J_\mu(x) = J_\mu^{(1,1)}(x) \) for simplicity.

The scattering amplitude for one-gluon exchange is proportional to

\[
\left( \frac{\lambda^a}{2} \right)_{ij} \left( \frac{\lambda^a}{2} \right)_{kl} = \frac{1}{3} (\delta_{ij}\delta_{kl} - \delta_{il}\delta_{kj}) + \frac{1}{6} (\delta_{ij}\delta_{kl} + \delta_{il}\delta_{kj}) ,
\]

where

\[
\varepsilon^{mij} \varepsilon^{mjl} = \delta_{ij}\delta_{kl} - \delta_{il}\delta_{kj} ,
\]

\[
2
\]
the $\lambda^a$ is the Gell-Mann matrix. The negative (positive) sign in front of the antisymmetric antitriplet $\bar{3}_c$ (symmetric sextet $6_c$) indicates the interaction is attractive (repulsive), which favors (disfavors) formation of the diquarks in color antitriplet $\bar{3}_c$ (color sextet $6_c$). We prefer the diquark operators in color antitriplet $\bar{3}_c$ to the diquark operators in color sextet $6_c$ in constructing the tetraquark current operators to interpolate the lowest tetraquark states.

At the phenomenological side, we take into account the non-vanishing current-hadron couplings considering the same quantum numbers, and separate the contribution of the ground state vector hidden-bottom tetraquark state in correlation function $\Pi_{\mu\nu}(p)$ [11, 12], which is supposed to be the $Y(10750)$,

$$\Pi_{\mu\nu}(p) = \frac{\lambda^2}{m^2 - p^2} \left( -g_{\mu\nu} + \frac{p_\mu p_\nu}{p^2} \right) + \cdots ,$$  \hspace{1cm} (5)

where the pole residue $\lambda_Y$ is defined by $\langle 0 | J_\mu(0) | Y(p) \rangle = \lambda_Y \varepsilon_\mu$, the $\varepsilon_\mu$ is the polarization vector.

At the QCD side, we carry out the operator product expansion up to the vacuum condensates of dimension 10 in a consistent way, and take into account the vacuum condensates $\langle \bar{q}q \rangle$, $(\frac{\alpha_sGG}{\pi})$, $\langle \bar{q}g_s\sigma Gq \rangle$, $\langle \bar{q}q \rangle^2$, $\langle \bar{q}q \rangle (\frac{\alpha_sGG}{\pi})$, $\langle \bar{q}g_s\sigma Gq \rangle$, $\langle \bar{q}g_s\sigma Gq \rangle^2$ and $\langle \bar{q}q \rangle^2 (\frac{\alpha_sGG}{\pi})$, then obtain the QCD spectral density through dispersion relation, take the quark-hadron duality below the continuum $\langle qg \rangle$, $\langle \bar{q}q \rangle$, $\langle \bar{q}q \rangle$ through a fraction, and perform the Borel transform to obtain the QCD sum rules:

$$\lambda_Y^2 \exp \left( -\frac{M^2}{T^2} \right) = \int_{4m_b^2}^{s_0} ds \rho(s) \exp \left( -\frac{s}{T^2} \right).$$  \hspace{1cm} (6)

For the explicit expression of the QCD spectral density $\rho(s)$ and the technical details in calculating the Feynman diagrams, one can consult Refs. [6, 13].

Then we can obtain the QCD sum rules for the mass of the vector hidden-bottom tetraquark candidate $Y(10750)$ through a fraction,

$$M_Y^2 = \frac{\int_{4m_b^2}^{s_0} ds \frac{d\tau}{d\tau} \rho(s) \exp(-\tau s)}{\int_{4m_b^2}^{s_0} ds \rho(s) \exp(-\tau s)} \left|_{\tau = \frac{1}{T^2}} \right. .$$  \hspace{1cm} (7)

We choose the conventional values (in other words, the popular values) of the vacuum condensates $\langle \bar{q}q \rangle = -(0.24 \pm 0.01 \text{GeV})^3$, $\langle \bar{q}g_s\sigma Gq \rangle = m_0^2 \langle \bar{q}q \rangle$, $m_0^2 = (0.8 \pm 0.1) \text{GeV}^2$, $\langle \frac{\alpha_sGG}{\pi} \rangle = (0.33 \text{GeV})^4$ at the energy scale $\mu = 1 \text{GeV}$ [11, 12, 14], and take the $\overline{MS}$ mass $m_b(m_b) = (4.18 \pm 0.03) \text{GeV}$ listed in "The Review of Particle Physics" [15], and set the $u$ and $d$ quark masses to be zero. Furthermore, we take into account the energy-scale dependence of the parameters at the QCD side from the renormalization group equation [16],

$$\langle \bar{q}q \rangle(\mu) = \langle \bar{q}q \rangle(1 \text{GeV}) \left[ \frac{\alpha_s(1 \text{GeV})}{\alpha_s(\mu)} \right]^{\frac{12}{33 - 2\eta} \mu^2},$$

$$\langle \bar{q}g_s\sigma Gq \rangle(\mu) = \langle \bar{q}g_s\sigma Gq \rangle(1 \text{GeV}) \left[ \frac{\alpha_s(1 \text{GeV})}{\alpha_s(\mu)} \right]^{\frac{2}{33 - 2\eta} \mu^2},$$

$$m_b(\mu) = m_b(m_b) \left[ \frac{\alpha_s(\mu)}{\alpha_s(m_b)} \right]^{\frac{12}{33 - 2\eta} \mu^2},$$

$$\alpha_s(\mu) = \frac{1}{b_0 t} \left[ 1 - \frac{b_1 \log t}{b_0^2} + \frac{b_2^2 (\log^2 t - \log t - 1) + b_0 b_2}{b_0^4 t^2} \right],$$  \hspace{1cm} (8)

where $t = \log \frac{\mu^2}{\Lambda^2}$, $b_0 = \frac{33 - 2\eta}{12 \pi}$, $b_1 = \frac{153 - 19 \eta}{24 \pi}$, $b_2 = \frac{2857 - 3033 \eta + 325 \pi^2}{24 \pi}$, $\Lambda = 210 \text{MeV}$, $292 \text{MeV}$ and $332 \text{MeV}$ for the flavors $n_f = 5$, 4 and 3, respectively [15]. It makes the present analysis, as we study the vector hidden-bottom tetraquark state, it is better to choose the flavor $n_f = 5$, then evolve all the input parameters to the ideal energy scale $\mu$. 

3
The Borel parameter $T^2$ is a free parameter, the continuum threshold parameter $s_0$ is also a free parameter, but we can borrow some ideas from the mass spectrum of the conventional mesons and the established exotic mesons and put additional constraints on the $s_0$ so as to avoid contaminations from the excited states and continuum states. In the conventional QCD sum rules, there are two basic criteria (i.e. "pole dominance at the hadron side" and "convergence of the operator product expansion") to obey. In the QCD sum rules for the multiquark states, we add two additional criteria, (i.e. "appearance of the flat Borel platforms" and "satisfying the modified energy scale formula”), as in the QCD sum rules for the conventional mesons and baryons, we cannot obtain very flat Borel platforms due to lacking higher dimensional vacuum condensates to stabilize the QCD sum rules. Now we search for the optimal values of the two parameters to satisfy the four criteria via try and error.

In Refs.\[13, 17, 18\], we study the hidden-charm and hidden-bottom tetraquark states (which consist of a diquark-antidiquark pair in relative S-wave) with the QCD sum rules, and explore the energy scale dependence of the extracted masses and pole residues for the first time. In the heavy quark limit $m_Q \rightarrow \infty$, the heavy quark $Q$ serves as a static well potential and attracts the light quark $q$ to form a diquark in the color antitriplet $\bar{3}_c$, while the heavy antiquark $\bar{Q}$ serves as another static well potential and attracts the light antiquark $\bar{q}$ to form a antidiquark in the color triplet $3_c$. Then the diquark and antidiquark attract each other to form a compact tetraquark state.

The favored heavy diquark configurations are the scalar and axialvector diquark operators $\varepsilon^{ijk}q^T_3C_5\gamma_5Q^k$ and $\varepsilon^{ijk}q^T_3C_5\gamma_5Q^k$ in the color antitriplet $\bar{3}_c$ \[19\]. If there exists an additional P-wave between the light quark and heavy quark, we can obtain the pseudoscalar and vector diquark operators $\varepsilon^{ijk}q^T_3C_5\gamma_5\gamma_5Q^k$ and $\varepsilon^{ijk}q^T_3C_5\gamma_5\gamma_5Q^k$ in the color antitriplet without introducing the additional P-wave explicitly, as multiplying a $\gamma_5$ can change the parity, the P-wave effect is embodied in the underlined $\gamma_5$. On the other hand, we can introduce the P-wave explicitly, and obtain the vector and tensor diquark operators $\varepsilon^{ijk}q^T_3C_5\gamma_5\bar{\gamma}_\alpha \gamma_\alpha Q^k$ and $\varepsilon^{ijk}q^T_3C_5\gamma_5\bar{\gamma}_\alpha \gamma_\alpha Q^k$ in the color antitriplet.

We can take the $C$, $C_{\gamma_5}$, $C_{\gamma_5\alpha}$, $C_{\gamma_5\alpha\gamma_5}$, $C_{\gamma_5\bar{\gamma}_\alpha}$ and $C_{\gamma_5\bar{\gamma}_\alpha\gamma_5}$ type diquark and antidiquark operators (also the $C_{\sigma_{\alpha\beta}}$ and $C_{\sigma_{\alpha\beta}\gamma_5}$ type diquark operators, which have both $J^P = 1^+$ and $1^-$ components) as the basic constituents to construct the tetraquark current operators with the $J^{PC} = 0^{++}$, $1^{++}$, $1^{+-}$, $1^{--}$ and $2^{++}$ to interpolate the hidden-charm or hidden-bottom tetraquark states, the P-wave lies between the light quark and heavy quark (or between the light antiquark and heavy antiquark) if any, in other words, the P-wave lies inside the diquark or antidiquark, while the diquark and antidiquark are in relative S-wave \[8, 10, 13, 17, 18\]. In this case, we introduce the effective heavy quark mass $M_Q$ and virtuality $V = \sqrt{M_X^2/Y/Z} \rightarrow (2M_Q)^2$ to characterize the tetraquark states, and suggest an energy scale formula $\mu = \sqrt{M_X^2/Y/Z} - (2M_Q)^2$ to choose the optimal energy scales of the QCD spectral densities \[13, 17, 18\].

On the other hand, if there exists a relative P-wave, which lies between the diquark and antidiquark constituents, we have to consider the effect of the P-wave and modify the energy scale formula,

$$\mu = \sqrt{M_X^2/Y/Z} - (2M_Q + P_E)^2,$$

(9)

where the $P_E$ denotes the energy costed by the relative P-wave \[9, 7\]. The $Y(10750)$ lies near the $\Upsilon(4S)$ and $\Upsilon(5S)$, the energy gap between the masses of the $\chi_b(4P)$ and $\Upsilon(4S) (\chi_b(5P)$ and $\Upsilon(5S))$ is about $0.14 \sim 0.15$ GeV ($0.12 \sim 0.14$ GeV) in the potential models \[2, 3\]. In this article, we study the vector hidden-bottom tetraquark state, there exists a relative P-wave between the bottom diquark and bottom antidiquark constituents. In the present case, the relative P-wave is estimated to cost about $0.12$ GeV, we can modify the energy scale formula to be,

$$\mu = \sqrt{M_Y^2} - (2M_b + 0.12 \text{ GeV})^2 = \sqrt{M_Y^2} - (10.46 \text{ GeV})^2,$$

(10)
where we choose the updated value $M_b = 5.17\text{ GeV}$\cite{21}. The value $P_E = 0.12\text{ GeV}$ is reasonable, as the QCD sum rules indicate that the ground state hidden-bottom tetraquark mass is about $10.61 - 10.65\text{ GeV}$\cite{10}, the vector hidden-bottom tetraquark mass is estimated to be $10.73 - 10.77\text{ GeV}$, which is in excellent agreement with (at least is compatible with) the experimental data $M_Y = 10752.7 \pm 5.9_{-1.1}^{+0.7}\text{ MeV}$ from the Belle collaboration\cite{11}.

In Ref.\cite{10}, we study the scalar, axialvector and tensor diquark-antidiquark type hidden-bottom tetraquark states $Z_b$ (where the bottom diquark and bottom antidiquark are in relative S-wave) with the QCD sum rules systematically, and choose the continuum threshold parameters as $\sqrt{s_0} = M_{Z_b} + 0.55 \pm 0.10\text{ GeV}$, which works well and is consistent with the assumption $M_{Z'_b} - M_{Z_b} = M_Y - M_T = 0.55\text{ GeV}$\cite{15}. In this article, we assume $M_{T'} - M_Y = M_{T'} - M_T = 0.55\text{ GeV}$ and choose the continuum threshold parameter as $\sqrt{s_0} = M_Y + 0.55 \pm 0.10\text{ GeV}$.

In numerical calculations, we observe that the continuum threshold parameter $\sqrt{s_0} = 11.3 \pm 0.1\text{ GeV}$, Borel parameter $T^2 = (6.3 - 7.3)\text{ GeV}^2$ and energy scale $\mu = 2.5\text{ GeV}$ work well. The pole contribution from the ground state tetraquark candidate $Y(10750)$ is about $47 - 70\%$, the pole dominance is well satisfied. The predicted mass is about $M_Y = 10.75\text{ GeV}$, which certainly obeys the modified energy scale formula.

In numerical calculations, we observe that the contributions of the vacuum condensates $\langle \bar{q}q \rangle$, $\langle \bar{q}q, \sigma Gq \rangle$, $\langle \bar{q}q \rangle^2$ and $\langle \bar{q}q \rangle \langle \bar{q}q, \sigma Gq \rangle$ are large, the values change quickly with variation of the Borel parameter $T^2$ in the region $T^2 < 6.3\text{ GeV}^2$, the convergent behavior is bad, we have to choose $T^2 \geq 6.3\text{ GeV}^2$. At the Borel window, $T^2 = (6.3 - 7.3)\text{ GeV}^2$, the contributions of the vacuum condensates $\langle \bar{q}q \rangle$, $\langle \bar{q}q, \sigma Gq \rangle$, $\langle \bar{q}q \rangle^2$ and $\langle \bar{q}q \rangle \langle \bar{q}q, \sigma Gq \rangle$ have the hierarchy $D_3 \gg |D_B| \sim D_B \gg |D_8|$, where we use the symbol $D_n$ to denote the contributions of the vacuum condensates of dimension $n$. The contributions of the vacuum condensates $\langle \bar{q}q \rangle^2 \langle \bar{q}q, \sigma Gq \rangle$ and $\langle \bar{q}q \rangle \langle \bar{q}q, \sigma Gq \rangle$ are very small and cannot affect the convergent behavior of the operator product expansion, the contribution of the vacuum condensates of dimension 10 is $(2 - 6)\%$. We can obtain the conclusion that the operator product expansion is well convergent.

Now we obtain the numerical values of the mass and pole residue of the tetraquark candidate $Y(10750)$ from the QCD sum rules in Eqs.(6)-(7), and take into account all uncertainties of the input parameters, and plot the predicted mass and pole residue with variations of the Borel parameter $T^2$ explicitly in Figs.1-2,

$$M_Y = 10.75 \pm 0.10\text{ GeV},$$
$$\lambda_Y = (1.89 \pm 0.31) \times 10^{-1}\text{ GeV}^6. \quad (11)$$

It is obvious that there appear platforms in the lines of both the mass and pole residue in the Borel window, see Figs.1-2. Now the four criteria of the QCD sum rules for the vector tetraquark states are all satisfied\cite{13 17 18}, and we expect to make reliable or reasonable predictions.

The numerical value $M_Y = 10.75 \pm 0.10\text{ GeV}$ from the present QCD sum rules is in excellent agreement with (at least is compatible with) the experimental data $M_Y = 10752.7 \pm 5.9_{-1.1}^{+0.7}\text{ MeV}$ from the Belle collaboration\cite{11} (see Fig.1), which favors assigning the $Y(10750)$ as the diquark-antidiquark type vector hidden-bottom-tetraquark state, which has a relative P-wave between the diquark and antidiquark constituents. The relative P-wave between the diquark and antidiquark constituents hampers the rearrangements of the quarks and antiquarks in the color and Dirac-spinor spaces to form the quark-antiquark type meson pairs, which can interpret (is compatible with) the small experimental value of the width $\Gamma_Y = 35.5_{-11.3}^{+17.6} + 3.9\text{ MeV}$\cite{14}.

At the charm sector, the calculations based on the QCD sum rules favors assigning the $Y(4220/4260)$, $Y(4320/4360)$ and $Y(4390)$ to be the vector tetraquark states with a relative P-wave between the scalar (or axialvector) diquark and scalar (or axialvector) antidiquark pair\cite{6 7}. Furthermore, the QCD sum rules favors assigning the $X'(3860)$ to be the scalar-diquark-scalar-antidiquark type scalar tetraquark state, where the diquark and antidiquark constituents are in relative S-wave\cite{20}. Analogous arguments survive both in the bottom and charm sectors, however, unambiguous assignments call for more experimental data and more theoretical works.
Figure 1: The mass of the vector hidden-bottom tetraquark candidate \(Y(10750)\) with variation of the Borel parameter \(T^2\).

Figure 2: The pole residue of the vector hidden-bottom tetraquark candidate \(Y(10750)\) with variation of the Borel parameter \(T^2\).

In Fig.3 we plot the predicted mass of the vector hidden-bottom tetraquark candidate \(Y(10750)\) with variation of the energy scale \(\mu\) for central values of the input parameters. From the figure, we can see that the predicted mass decreases monotonically and quickly with the increase of the energy scale \(\mu\). If we abandon the modified energy scale formula

\[
\mu = \sqrt{M^2_{X/Y/Z} - (2M_Q + P_E)^2},
\]

we are puzzled about which energy scale should be chosen. If we choose the typical energy scale \(\mu = 2 \text{ GeV}\), analogous pole contribution (47 − 70)%, analogous \(D_{10}\) contribution (2 − 6)%, we have to postpone the continuum threshold parameter to much larger value \(\sqrt{s_0} = 11.75 \pm 0.10 \text{ GeV}\), then we obtain the Borel window \(T^2 = (6.6 − 7.6) \text{ GeV}^2\), and the central values of the predicted mass and pole residue \(M_Y = 11.20 \text{ GeV}\) and \(\lambda_Y = 2.13 \times 10^{-7} \text{ GeV}^6\). The predicted mass \(M_Y = 11.20 \text{ GeV}\) is much larger than the experimental data \(M_Y = 10752.7 \pm 5.9_{-1.1}^{+0.7} \text{ MeV}\) from the Belle collaboration \cite{1}. The modified energy scale formula can enhance the pole contribution remarkably and improve the convergent behavior of the operator product expansion remarkably. On the other hand, if we choose the typical energy scale \(\mu = 3 \text{ GeV}\), analogous calculations lead to a mass about 10.42 GeV, which is smaller than the S-wave hidden-bottom tetraquark masses 10.61 − 10.65 GeV \cite{10} and should be abandoned.
Figure 3: The predicted mass of the vector hidden-bottom tetraquark candidate $Y(10750)$ with variation of the energy scale $\mu$ for central values of the input parameters.

3 The decay width of the vector tetraquark candidate $Y(10750)$

Now we study the partial decay widths of the $Y(10750)$ as a vector hidden-bottom tetraquark candidate with the three-point QCD sum rules, and write down the three-point correlation functions firstly,

$$
\Pi_\nu(p,q) = i^2 \int d^4x d^4y e^{ipx} e^{iqy} \langle 0 | T \{ J_B(x) J_B(y) J_\nu^\dagger(0) \} | 0 \rangle,
$$

$$
\Pi^{1,\alpha\beta\nu}(p,q) = i^2 \int d^4x d^4y e^{ipx} e^{iqy} \langle 0 | T \{ J_{B^*,\alpha}(x) J_{B^*,\beta}(y) J_\nu^\dagger(0) \} | 0 \rangle,
$$

$$
\Pi^{1,\mu\nu}(p,q) = i^2 \int d^4x d^4y e^{ipx} e^{iqy} \langle 0 | T \{ J_{B^*}(x) J_{B^*}(y) J_\nu^\dagger(0) \} | 0 \rangle,
$$

$$
\Pi^{2,\mu\nu}(p,q) = i^2 \int d^4x d^4y e^{ipx} e^{iqy} \langle 0 | T \{ J_\eta_b(x) J_{\omega,\mu}(y) J_\nu^\dagger(0) \} | 0 \rangle,
$$

$$
\Pi^{3,\mu\nu}(p,q) = i^2 \int d^4x d^4y e^{ipx} e^{iqy} \langle 0 | T \{ J_\Upsilon,\mu(x) J_f^0(y) J_\nu^\dagger(0) \} | 0 \rangle,
$$

where

- $J_B(x) = \bar{b}(x)i\gamma_5 u(x)$,
- $J_{B^*,\alpha}(x) = \bar{b}(x)\gamma_\alpha u(x)$,
- $J_{B^*}(x) = \bar{b}(x)i\gamma_5 b(x)$,
- $J_\eta_b(y) = \frac{\bar{u}(y)\gamma_\mu u(y) + \bar{d}(y)\gamma_\mu d(y)}{\sqrt{2}}$,
- $J_{\omega,\mu}(y) = \bar{b}(x)\gamma_\mu b(x)$,
- $J_{\Upsilon,\mu}(x) = \frac{\bar{u}(y)u(y) + \bar{d}(y)d(y)}{\sqrt{2}}$,
- $J_f^0(y) = J_\nu^\dagger(0,0)$.

At the phenomenological side, we insert a complete set of intermediate hadronic states with the same quantum numbers as the current operators into the three-point correlation functions and
isolate the ground state contributions \[11, 12\].

\[
\Pi_\nu(p, q) = \frac{f_{Bm}^2}{m_B^2} \frac{\lambda_Y G_{YYB^*}}{(p^2 - m_Y^2)(p^2 - m_B^2)(q^2 - m_B^2)} i(p - q)^\alpha \left(-g_{\alpha\nu} + \frac{p_{\nu}p_{\alpha}}{p^2}\right) + \ldots \\
\Pi_\nu(p^2, p^2, q^2) (-ip_\nu) + \ldots ,
\]

\[
\Pi^3_{\alpha\beta\nu}(p, q) = \frac{f_{B^*m}^2}{m_B^2} \frac{\lambda_Y G_{YYB^*} \varepsilon_{\alpha\beta\rho\sigma} p^\rho}{(p^2 - m_Y^2)(p^2 - m_B^2)(q^2 - m_B^2)} i(p - q)^\sigma \left(-g_{\nu\sigma} + \frac{p_{\sigma}p_{\nu}}{p^2}\right) \left(-g_{\alpha\rho} + \frac{p_{\rho}p_{\alpha}}{p^2}\right) + \ldots \\
\Pi(p^2, p^2, q^2) (-i\varepsilon_{\mu\alpha\beta} p^\alpha q^\beta) + \ldots ,
\]

\[
\Pi^2_{\mu\nu}(p, q) = \frac{f_{m\omega}^2}{2m_\omega} \frac{\lambda_Y G_{Ym\omega} \varepsilon_{\alpha\beta\rho\sigma} q^\rho}{(p^2 - m_\omega^2)(p^2 - m_\omega^2)(q^2 - m_\omega^2)} i(p - q)^\sigma \left(-g_{\mu\rho} + \frac{p_{\rho}p_{\mu}}{p^2}\right) \left(-g_{\nu\sigma} + \frac{p_{\sigma}p_{\nu}}{p^2}\right) + \ldots \\
\Pi(p^2, p^2, q^2) (-i\varepsilon_{\mu\alpha\beta} p^\alpha q^\beta) + \ldots ,
\]

\[
\Pi^3_{\mu\nu}(p, q) = \frac{f_{Tm\omega}^2}{m_\omega} \frac{\lambda_Y G_{Y\omega\omega} \varepsilon_{\alpha\beta\rho\sigma} q^\rho}{(p^2 - m_\omega^2)(p^2 - m_\omega^2)(q^2 - m_\omega^2)} i(p - q)^\sigma \left(-g_{\mu\sigma} + \frac{p_{\sigma}p_{\mu}}{p^2}\right) \left(-g_{\nu\sigma} + \frac{p_{\sigma}p_{\nu}}{p^2}\right) + \ldots \\
\Pi(p^2, p^2, q^2) (-i\varepsilon_{\mu\alpha\beta} p^\alpha q^\beta) + \ldots ,
\]

where we have used the definitions for the decay constants and hadronic coupling constants,

\[
\langle 0|J_B(0)|B(p)\rangle = \frac{f_{Bm}^2}{m_B}, \\
\langle 0|J_{B^*\mu}(0)|B^*(p)\rangle = f_{B^*mB^*\varepsilon_\mu}^B, \\
\langle 0|J_{b\mu}(0)|b(p)\rangle = \frac{f_{m^2}}{2m_B}, \\
\langle 0|J_T(0)|\omega(p)\rangle = f_{Tm\omega^\omega}, \\
\langle 0|J_\mu(0)|\omega(p)\rangle = f_{m_\omega^\mu}, \\
\langle 0|J_0(0)|f_0(p)\rangle = f_{f_0m_0},
\]

(20)
\[
\langle B(p)B(q)|X(p') \rangle = -(p-q)^a \xi_\alpha^Y G_{YBB},
\]
\[
\langle B^*(p)B^*(q)|X(p') \rangle = (p-q)^a \xi_\alpha^Y \xi_\beta^{B^*} \xi^{B^*+\beta} G_{YBB^*},
\]
\[
\langle B^*(p)B(q)|X(p') \rangle = -\varepsilon^{\alpha\beta\rho\sigma} p_\alpha^\rho \xi_\beta^{B^*} \xi^Y G_{YBB^*},
\]
\[
\langle \eta_b(p)\omega(q)|X(p') \rangle = -\xi_\alpha^\omega \xi_\beta^\eta G_{Y\eta\omega},
\]
\[
\langle \Upsilon(p)f_0(q)|X(p') \rangle = (p-q)^a \xi_\alpha^Y \xi^\Upsilon^{T} \xi^{\omega+\bar{\beta}} G_{Y\Upsilon\omega},
\]
\[
\langle \Upsilon(p)\omega(q)|X(p') \rangle = (p-q)^a \xi_\alpha^Y \xi_\beta^\Upsilon \xi^{\omega+\bar{\beta}} G_{Y\Upsilon\omega},
\]
(21)

the \( \xi_\mu^{B^*}, \xi_\mu^Y, \xi_\mu^\omega \) and \( \xi_\mu^Y \) are the polarization vectors of the conventional mesons and tetraquark candidate \( Y(10750) \), respectively. The \( G_{YBB}, G_{YBB'}, G_{YBB^*}, G_{Y\eta\omega}, G_{Y\Upsilon f_0} \) and \( G_{Y\Upsilon\omega} \) are the hadronic coupling constants. In calculations, we observe that the hadronic coupling constant \( G_{Y\Upsilon\omega} \) is zero at the leading order approximation, and we will neglect the process \( Y(10750) \rightarrow \Upsilon \omega \rightarrow \Upsilon\pi^+\pi^-\pi^0 \).

We usually assign the lowest scalar nonet mesons \( \{ f_0/\sigma(500), a_0(980), \kappa_0(800), f_0(980) \} \) as the tetraquark states, and assign the higher scalar nonet mesons \( \{ f_0(1370), a_0(1450), \kappa_0(1430), f_0(1500) \} \) as the conventional \( 3P_0 \) quark-antiquark states \[22, 23, 24\]. In this article, we assume \( f_0 = f_0(1370) \) with the symbolic quark structure \( f_0^{(1370)} = \bar{u}u + \bar{d}d \).

We study the components \( \Pi(p'^2, p^2, q^2) \) of the correlation functions in Eq. (14)-(19), and carry out the operator product expansion up to the vacuum condensates of dimension 5. We calculate both the connected and disconnected Feynman diagrams, take into account the perturbative terms, quark condensate and mixed condensate, and neglect the tiny contributions of the gluon condensate. Then we obtain the QCD spectral densities through dispersion relation, match the hadron side with the QCD side of the components \( \Pi(p'^2, p^2, q^2) \), perform double Borel transform with respect to \( P^2 = -p^2 \) and \( Q^2 = -q^2 \) by setting \( p^2 = p'^2 \) in the hidden-bottom channels and \( p^2 = 4p'^2 \) in the open-bottom channels to obtain the QCD sum rules for the hadronic coupling
constants,
\[
\frac{f_{\bar{b}}^2 m_{\bar{b}}^2}{m_b^2} \frac{\lambda_Y G_{YYB}}{4 (m_Y^2 - m_B^2)} \left[ \exp \left( -\frac{m_{\bar{b}}^2}{T_1^2} \right) - \exp \left( -\frac{m_{\bar{b}}^2}{T_2^2} \right) \right] \exp \left( -\frac{m_B^2}{T_2^2} \right) \\
+ (C_{Y'B^+} + C_{Y'B^-}) \exp \left( -\frac{m_{\bar{b}}^2}{T_1^2} \right) \\
- \frac{1}{512 \pi^4} \int_{m_b^2}^{s_{\bar{b}}^0} ds \int_{m_b^2}^{s_{\bar{b}}^0} du \left( 1 - \frac{m_{\bar{b}}^2}{s} \right)^2 \left( 1 - \frac{m_{\bar{b}}^2}{u} \right)^2 \frac{m_b^2}{s} (3s^2 - 5su - 14sm_b^2 + 4um_b^2) \\
\exp \left( -\frac{s}{T_1^2} - \frac{u}{T_2^2} \right) \\
- \frac{m_b(q\bar{q})}{192 \pi^2} \int_{m_b^2}^{s_{\bar{b}}^0} du \left( 1 - \frac{m_{\bar{b}}^2}{u} \right)^2 (u + 11m_b^2) \exp \left( -\frac{m_{\bar{b}}^2}{T_1^2} - \frac{u}{T_2^2} \right) \\
+ \frac{m_b(q\bar{q})}{192 \pi^2} \int_{m_b^2}^{s_{\bar{b}}^0} ds \left( 1 - \frac{m_{\bar{b}}^2}{s} \right)^2 \left( 3s - 19m_b^2 + \frac{4m_b^4}{s} \right) \exp \left( -\frac{s}{T_1^2} - \frac{m_{\bar{b}}^2}{T_2^2} \right) \\
- \frac{m_b(q\bar{q}, \sigma Gq)}{768 \pi^2} \int_{m_b^2}^{s_{\bar{b}}^0} du \left( 1 - \frac{m_{\bar{b}}^2}{u} \right)^2 \left( 27 - u + 29m_b^2 \right) \exp \left( -\frac{m_{\bar{b}}^2}{T_1^2} - \frac{u}{T_2^2} \right) \\
- \frac{m_b(q\bar{q}, \sigma Gq)}{384 \pi^2 T_1^2} \int_{m_b^2}^{s_{\bar{b}}^0} ds \left( 1 - \frac{m_{\bar{b}}^2}{s} \right)^2 \left( 9 - u + 11m_b^2 \right) \exp \left( -\frac{s}{T_1^2} - \frac{m_{\bar{b}}^2}{T_2^2} \right) \\
- \frac{m_b(q\bar{q}, \sigma Gq)}{768 \pi^2} \int_{m_b^2}^{s_{\bar{b}}^0} ds \left( 1 - \frac{m_{\bar{b}}^2}{s} \right)^2 \left( 1 - 4m_b^2 \right) \left( 3s - m_{\bar{b}}^2 \right) \exp \left( -\frac{s}{T_1^2} - \frac{m_{\bar{b}}^2}{T_2^2} \right) \\
\exp \left( -\frac{s}{T_1^2} - \frac{2m_{\bar{b}}^2}{T_2^2} \right) \\
- \frac{m_b(q\bar{q}, \sigma Gq)}{768 \pi^2} \int_{m_b^2}^{s_{\bar{b}}^0} du \left( 1 - \frac{m_{\bar{b}}^2}{u} \right)^2 \left( -9 + 7u + 11m_b^2 \right) \exp \left( -\frac{m_{\bar{b}}^2}{T_1^2} - \frac{u}{T_2^2} \right) \\
- \frac{m_b(q\bar{q}, \sigma Gq)}{768 \pi^2} \int_{m_b^2}^{s_{\bar{b}}^0} ds \left( 1 - \frac{m_{\bar{b}}^2}{s} \right)^2 \left( \frac{4m_b^2}{s} - 1 + \frac{1}{T_2^2} \right) \left[ 24m_b^2 - 3s + \left( 1 - \frac{4m_b^2}{s} \right) m_{\bar{b}}^2 \right] \\
\exp \left( -\frac{s}{T_1^2} - \frac{2m_{\bar{b}}^2}{T_2^2} \right) \\
+ \frac{m_b(qq, \sigma Gq)}{256 \pi^2} \int_{m_b^2}^{s_{\bar{b}}^0} du \left( 1 - \frac{m_{\bar{b}}^2}{u} \right)^2 \left( 5 - \frac{4m_b^2}{u} \right) \exp \left( -\frac{m_{\bar{b}}^2}{T_1^2} - \frac{u}{T_2^2} \right) \\
+ \frac{m_b(qq, \sigma Gq)}{128 \pi^2} \int_{m_b^2}^{s_{\bar{b}}^0} ds \left( 1 - \frac{m_{\bar{b}}^2}{s} \right)^2 \left( 2 - \frac{5m_b^2}{s} + \frac{2m_b^4}{s^2} \right) \exp \left( -\frac{s}{T_1^2} - \frac{m_{\bar{b}}^2}{T_2^2} \right) \\
- \frac{m_b(qq, \sigma Gq)}{256 \pi^2} \int_{m_b^2}^{s_{\bar{b}}^0} ds \left( 1 - \frac{3m_b^2}{s} + \frac{8m_b^4}{s^2} - \frac{2m_b^6}{s^3} \right) \exp \left( -\frac{s}{T_1^2} - \frac{m_{\bar{b}}^2}{T_2^2} \right), \tag{22}
\]
\[
\frac{f_B m_B^2 \lambda Y B^* B^*}{4 (m_Y^2 - m_B^2)} \left[ \exp \left( \frac{-m_B^2}{T_1^2} \right) - \exp \left( \frac{-m_Y^2}{T_1^2} \right) \right] \exp \left( \frac{-m_B^2}{T_2^2} \right) \\
+ C_{V' B^+ V' B^-} \exp \left( \frac{-m_B^2}{T_1^2} \right) \exp \left( \frac{-m_B^2}{T_2^2} \right)
\]

\[
= \frac{1}{1536 \pi^4} \int_{m_b^2}^{s^b_0} \int_{m_b^2}^{s^b_0} ds \, du \left( 1 - \frac{m_b^2}{s} \right)^2 \left( 1 - \frac{m_b^2}{u} \right)^2 \frac{m_b^2}{s} \left( 2m_b^4 + 46sm_b^2 - 8um_b^2 - 9s^2 + 5su \right) \\
\exp \left( \frac{-s}{T_1^2} - \frac{u}{T_2^2} \right) \\
+ \frac{m_b \langle \bar{q} q \rangle}{192 \pi^2} \int_{m_b^2}^{s^b_0} du \left( 1 - \frac{m_b^2}{u} \right)^2 \left( u - 13m_b^2 \right) \exp \left( \frac{-m_b^2}{T_1^2} - \frac{u}{T_2^2} \right) \\
+ \frac{m_b \langle \bar{q} q \rangle}{192 \pi^2} \int_{m_b^2}^{s^b_0} ds \left( 1 - \frac{m_b^2}{s} \right)^2 \left( 3s - 17m_b^2 + \frac{2m_b^4}{s} \right) \exp \left( - \frac{s}{T_1^2} - \frac{m_b^2}{T_2^2} \right) \\
- \frac{m_b \langle \bar{q} g \sigma G q \rangle}{1152 \pi^2} \int_{m_b^2}^{s^b_0} du \left( 1 - \frac{m_b^2}{u} \right)^2 \left( 45 + \frac{7u - 55m_b^2}{T_1^2} \right) \exp \left( - \frac{m_b^2}{T_1^2} - \frac{u}{T_2^2} \right) \\
+ \frac{m_b \langle \bar{g} g \sigma G g \rangle}{384 \pi^2 T_1^4} \int_{m_b^2}^{s^b_0} ds \left( 1 - \frac{m_b^2}{s} \right)^2 \left( 1 + \frac{3s - m_b^2}{T_2^2} \right) \exp \left( - \frac{s}{T_1^2} - \frac{m_b^2}{T_2^2} \right) \\
- \frac{m_b \langle \bar{q} g \sigma G q \rangle}{576 \pi^2} \int_{m_b^2}^{s^b_0} ds \left( 1 - \frac{m_b^2}{s} \right)^2 \left( 1 - \frac{4m_b^2}{s} \right) \exp \left( - \frac{s}{T_1^2} - \frac{m_b^2}{T_2^2} \right) \\
+ \frac{m_b \langle \bar{g} g \sigma G g \rangle}{384 \pi^2 T_1^4} \int_{m_b^2}^{s^b_0} ds \left( 1 - \frac{m_b^2}{s} \right)^2 \left[ -3s + 16m_b^2 + \frac{2m_b^4}{s} + \left( 1 - \frac{4m_b^2}{s} \right) m_b^2 \right] \exp \left( - \frac{s}{T_1^2} - \frac{m_b^2}{T_2^2} \right) \\
+ \frac{m_b \langle \bar{q} g \sigma G q \rangle}{768 \pi^2} \int_{m_b^2}^{s^b_0} ds \left( 1 - \frac{m_b^2}{s} \right)^2 \left( 13 - \frac{35m_b^2}{s} + \frac{16m_b^4}{s^2} \right) \exp \left( - \frac{s}{T_1^2} - \frac{m_b^2}{T_2^2} \right) \\
+ \frac{m_b \langle \bar{g} g \sigma G g \rangle}{768 \pi^2} \int_{m_b^2}^{s^b_0} du \left( 1 - \frac{m_b^2}{u} \right) \left( 5 - \frac{32m_b^2}{u} \right) \exp \left( - \frac{m_b^2}{T_1^2} - \frac{u}{T_2^2} \right) \\
+ \frac{m_b \langle \bar{q} g \sigma G q \rangle}{2304 \pi^2} \int_{m_b^2}^{s^b_0} ds \left( 4 - \frac{9m_b^2}{s} + \frac{27m_b^4}{s^2} - \frac{10m_b^6}{s^3} \right) \exp \left( - \frac{s}{T_1^2} - \frac{m_b^2}{T_2^2} \right),
\]
\[
\frac{f_{BB'MB'\lambda YGYB'B}}{4m_b(m^2_Y - m^2_{B'})} \left[ \exp \left( -\frac{m^2_{B'}}{T_1^2} \right) - \exp \left( -\frac{m^2_Y}{T_1^2} \right) \right] \exp \left( -\frac{m^2_B}{T_2^2} \right) + C_{Y'B'+Y'B-} \exp \left( -\frac{m^2_{B'}}{T_1^2} \right) \exp \left( -\frac{m^2_B}{T_2^2} \right)
\]
\[
\frac{m_b}{256\pi^4} \int_{m_b^2}^{s_0} ds \int_{m_b^2}^{s_0} du \left( 1 - \frac{m_b^2}{s} \right)^2 \left( 1 - \frac{m_b^2}{u} \right)^2 (s + u - 2m_b^2) \exp \left( -\frac{s}{T_1^2} - \frac{u}{T_2^2} \right)
\]
\[
- \frac{\langle \bar{q}q \rangle}{96\pi^2} \int_{m_b^2}^{s_0} du \left( 1 - \frac{m_b^2}{u} \right)^2 (u - m_b^2) \exp \left( -\frac{m_b^2}{T_1^2} - \frac{u}{T_2^2} \right)
\]
\[
- \frac{\langle \bar{q}q \rangle}{96\pi^2} \int_{m_b^2}^{s_0} ds \left( 1 - \frac{m_b^2}{s} \right)^2 (s - m_b^2) \exp \left( -\frac{s}{T_1^2} - \frac{m_b^2}{T_2^2} \right)
\]
\[
+ \frac{\langle \bar{q}q, \sigma Gq \rangle}{288\pi^2 T_1^4} \left( 1 + \frac{3m_b^2}{4T_1^2} \right) \int_{m_b^2}^{s_0} du \left( 1 - \frac{m_b^2}{u} \right)^2 (u - m_b^2) \exp \left( -\frac{m_b^2}{T_1^2} - \frac{u}{T_2^2} \right)
\]
\[
+ \frac{m_b^2 \langle \bar{q}q, \sigma Gq \rangle}{384\pi^2 T_1^2} \int_{m_b^2}^{s_0} ds \left( 1 - \frac{m_b^2}{s} \right) (s - m_b^2) \exp \left( -\frac{s}{T_1^2} - \frac{m_b^2}{T_2^2} \right)
\]
\[
+ \frac{\langle \bar{q}q, \sigma Gq \rangle}{384\pi^2} \int_{m_b^2}^{s_0} ds \left( 1 - \frac{m_b^2}{s} \right)^2 \left( 1 + \frac{3m_b^2}{4T_1^2} \right) \int_{m_b^2}^{s_0} du \left( 1 - \frac{m_b^2}{u} \right)^2 (u - m_b^2) \exp \left( -\frac{m_b^2}{T_1^2} - \frac{u}{T_2^2} \right)
\]
\[
+ \frac{\langle \bar{q}q, \sigma Gq \rangle}{384\pi^2} \int_{m_b^2}^{s_0} ds \left( 1 - \frac{m_b^2}{s} \right)^2 \left( 1 + \frac{3m_b^2}{4T_1^2} \right) \int_{m_b^2}^{s_0} du \left( 1 - \frac{m_b^2}{u} \right)^2 (u - m_b^2) \exp \left( -\frac{m_b^2}{T_1^2} - \frac{u}{T_2^2} \right),
\]
\[
(24)
\]
\[
\frac{f_{m_b, m_m^2, m_\omega, \lambda YGYm_\omega}}{2m_b(m^2_Y - m^2_{m_\omega})} \left[ \exp \left( -\frac{m^2_m}{T_1^2} \right) - \exp \left( -\frac{m^2_Y}{T_1^2} \right) \right] \exp \left( -\frac{m^2_\omega}{T_2^2} \right) + C_{Y'\omega + Y'\omega} \exp \left( -\frac{m^2_m}{T_1^2} \right) \exp \left( -\frac{m^2_\omega}{T_2^2} \right)
\]
\[
= \frac{m_b}{64\sqrt{2}\pi^2} \int_{4m_b^2}^{s_0} ds \int_0^{s_0} du \sqrt{1 - \frac{4m_b^2}{s}} u \exp \left( -\frac{s}{T_1^2} - \frac{u}{T_2^2} \right) - \frac{\langle \bar{q}q \rangle}{24\sqrt{2}\pi^2} \int_{4m_b^2}^{s_0} ds \sqrt{1 - \frac{4m_b^2}{s}} (s - 4m_b^2) \exp \left( -\frac{s}{T_1^2} \right)
\]
\[
+ \frac{\langle \bar{q}q, \sigma Gq \rangle}{72\sqrt{2}\pi^2 T_2^2} \int_{4m_b^2}^{s_0} ds \sqrt{1 - \frac{4m_b^2}{s}} (s - 4m_b^2) \exp \left( -\frac{s}{T_1^2} \right) - \frac{\langle \bar{q}q, \sigma Gq \rangle}{96\sqrt{2}\pi^2} \int_{4m_b^2}^{s_0} ds \sqrt{1 - \frac{4m_b^2}{s}} \exp \left( -\frac{s}{T_1^2} \right),
\]
\[
(25)
\]
Table 1: The Borel windows, hadronic coupling constants and partial decay widths of the $Y(10750)$ as the vector hidden-bottom tetraquark state.

| Decays | $T^2(\text{GeV}^2)$ | $G$ | $\Gamma(\text{MeV})$ |
|--------|-----------------|------|---------------------|
| $Y(10750) \rightarrow B^+B^-$ | 5.1 - 5.9 | $3.70^{+1.34}_{-1.31}$ | $6.61^{+6.39}_{-3.85}$ |
| $Y(10750) \rightarrow B^+B^+$ | 5.5 - 6.3 | $3.89^{+1.08}_{-1.45}$ | $8.79^{+2.23}_{-5.33}$ |
| $Y(10750) \rightarrow B^+B^-$ | 4.0 - 4.8 | $0.01 \text{GeV}^{-1}$ | $0.02$ |
| $Y(10750) \rightarrow \eta_b\omega$ | 2.6 - 3.6 | $0.30^{+0.12}_{-0.10} \text{GeV}^{-1}$ | $2.64^{+1.70}_{-1.69}$ |
| $Y(10750) \rightarrow \Upsilon_{f_0(1370)} \rightarrow \Upsilon \pi^+\pi^-$ | 2.5 - 3.5 | $1.32^{+0.10}_{-0.09} \text{GeV}$ | $0.08^{+0.20}_{-0.06}$ |
| $Y(10750) \rightarrow \Upsilon_\omega$ | 2.6 - 3.6 | 0 | 0 |

The input parameters at the hadron side are chosen as $m_Y = 9.4603 \text{GeV}$, $m_{\eta_b} = 9.3987 \text{GeV}$, $m_\omega = 0.78265 \text{GeV}$, $m_{B^+} = 5.27925 \text{GeV}$, $m_{B^{*+}} = 5.3247 \text{GeV}$, $m_{B^{0*}} = 0.13957 \text{GeV}$, $\sqrt{s_{B^0}} = 5.8 \text{GeV}$, $\sqrt{s_{Y}} = 9.9 \text{GeV}$, $\sqrt{s_{\Upsilon}} = 9.9 \text{GeV}$ \cite{15}, $\sqrt{s_{\Upsilon}} = 1.3 \text{GeV}$, $f_{\omega} = 215 \text{MeV}$ \cite{26}, $m_{f_0} = 1.35 \text{GeV}$, $f_{f_0} = 546 \text{MeV}$, $\sqrt{s_{f_0}} = 1.8 \text{GeV}$ (This work), $f_{B^+} = 194 \text{MeV}$, $f_{B^{*+}} = 213 \text{MeV}$ \cite{27} \cite{28}, $f_X = 649 \text{MeV}$ \cite{29}, $f_{\eta_b} = 667 \text{MeV}$ \cite{30}.

We set the Borel parameters to be $T_1^2 = T_2^2 = T^2$ for simplicity in the QCD sum rules for the hadronic coupling constants $G_{YBB}$, $G_{YB^{*+}B^*}$, $G_{YB^0B^0}$ and $G_{Y\eta_b\omega}$, while in the QCD sum rules for the hadronic coupling constant $G_{Y\Upsilon f_0}$, the contribution in the $u$ channel can be factorized out explicitly, we take the local limit, i.e. $T_2^2 \rightarrow \infty$ and $T_1^2 = T^2$. The unknown parameters are chosen as $C_{Y^*B^+}^* + C_{Y^*B^-}^* = 0.0441 \text{GeV}^8$, $C_{Y^*B^{*+}B^*}^* + C_{Y^*B^{*+}B^*}^* = 0.0454 \text{GeV}^8$, $C_{Y^*B^{*+}B^*}^* + C_{Y^*B^{*+}B^*}^* = 0.00145 \text{GeV}^7$, $C_{Y^*\eta_b}^* + C_{Y^*\omega}^* = 0.000125 \text{GeV}^7$ and $C_{Y^*\Upsilon f_0}^* = 0.00238 \text{GeV}^9$ to obtain platforms in the Borel windows, which are shown in Table 1. In Fig.1 we plot the hadronic coupling constants $G$ with variations of the Borel parameters $T^2$ in the Borel windows. The Borel windows $T_{\text{max}}^2 - T_{\text{min}}^2 = 1.0 \text{GeV}^2$ for the hidden-bottom decays and $T_{\text{max}}^2 - T_{\text{min}}^2 = 0.8 \text{GeV}^2$ for the open-bottom decays, where the $T_{\text{max}}^2$ and $T_{\text{min}}^2$ denote the maximum and minimum of the Borel parameters. We choose the same intervals $T_{\text{max}}^2 - T_{\text{min}}^2$ in all the QCD sum rules for the hadronic coupling constants in the two-body strong decays \cite{23}, which work well for the decays of the $Z_c(3900)$, $X(4140)$, $Z_c(4000)$, $Y(4660)$, etc.

We take into account uncertainties of the input parameters, and obtain the hadronic coupling constants, which are shown explicitly in Table 1 and Fig.1. Due to the tiny value of the hadronic
In this article, we take the scalar diquark and scalar antidiquark operators as the basic constituents, and we neglect the uncertainty. Now we calculate the partial decay widths of the two-body strong decays \( Y(10750) \to B^+ B^- , B^{++} B^{--} , B^{*+} B^{*-} \) and \( \eta \omega \) with formula,

\[
\Gamma (Y(10750) \to BC) = \frac{p(m_Y,m_B,m_C)}{24\pi m_Y^2} |T|^2 ,
\]

where \( p(a,b,c) = \sqrt{(a^2-(b+c)^2)(a^2-(b-c)^2)} \), the \( T \) are the scattering amplitudes defined in Eq.\( (24) \), the numerical values of the partial decay widths are shown in Table 1.

We assume the three-body decays \( Y(10750) \to \Upsilon f_0(1370) \to \Upsilon \pi^+ \pi^- \) take place through an intermediate virtual state \( f_0(1370) \), and calculate the partial decay width,

\[
\Gamma(Y \to \Upsilon \pi^+ \pi^-) = \int_{4m^2_\pi}^{(m_Y-m_f)^2} ds |T|^2 \frac{p(m_Y,m_\Upsilon,\sqrt{s})}{192\pi^3 m_Y^2 \sqrt{s}} ,
\]

where

\[
|T|^2 = \frac{(M_Y^2-s)^2 + 2(5M_Y^2-s)M_\Upsilon^2 + M_\Upsilon^4}{4M_Y^2 M_\Upsilon^2} G_f^2 \frac{1}{\Gamma_f (s-m_{f_0}^2)^2 + s\Gamma_0^2 (s) G_{f_0 \pi \pi}^2} ,
\]

\[
\Gamma_f (s) = \Gamma_f (m_{f_0}^2) \frac{m_{f_0}^2}{s} \frac{s-4m_\pi^2}{m_{f_0}^2-4m_\pi^2} ,
\]

\[
\Gamma_f (m_{f_0}^2) = \frac{G_{f_0 \pi \pi}^2}{16\pi m_{f_0}^2} \sqrt{m_{f_0}^2 - 4m_\pi^2} ,
\]

\( \Gamma_f (m_{f_0}^2) = 200 \text{ MeV} \) \cite{15}, the hadronic coupling constant \( G_{f_0 \pi \pi} \) is defined by \( \langle \pi^+(p)\pi^-(q)\rangle f_0(p') = iG_{f_0 \pi \pi} \). If we take the largest width \( \Gamma_f (m_{f_0}^2) = 500 \text{ MeV} \) \cite{15}, we can obtain a slightly larger partial decay width \( \Gamma(Y \to \Upsilon \pi^+ \pi^-) = 0.11^{+0.27}_{-0.08} \text{ MeV} \).

Now it is easy to obtain the total decay width,

\[
\Gamma_Y = \Gamma (Y(10750) \to B^+ B^- , B^{0} \bar{B}^0 , B^{*+} B^{*-} , B^{*0} \bar{B}^{*0} , B^{*+} B^- , B^{+} B^{*-} , B^{*0} \bar{B}^{0} , B^{0} \bar{B}^{*0} , \eta \omega , \Upsilon \pi^+ \pi^-) ,
\]

\[
= 33.60^{+16.64}_{-9.45} \text{ MeV} ,
\]

where we have assume the isospin limit for the \( B \) and \( B^* \) mesons. The predicted width \( \Gamma_Y = 33.60^{+16.64}_{-9.45} \text{ MeV} \) is in excellent agreement with the experimental data 35.5^{+17.6}_{-11.3} +3.9 MeV from the Belle collaboration \cite{11}, which also supports assigning the \( Y(10750) \) to be the diquark-antidiquark type vector hidden-bottom tetraquark state.

In Ref.\( [5] \), Li et al assign the \( Y(10750) \) and \( \Upsilon(10860) \) to be the \( 5^3S_1 - 4^3D_3 \) mixing states, the dominant components of the \( Y(10750) \) and \( \Upsilon(10860) \) are the conventional bottomonium sates \( 4^3D_1 \) and \( 5^3S_1 \), respectively. The decay mode \( 4^3D_1 \to B^* B^* \) is the dominant mode, the decay mode \( 4^3D_1 \to B B^* \) is sizable, while the decay mode \( 4^3D_1 \to B B \) is nearly forbidden. In the present work, we assign the \( Y(10750) \) to be the vector hidden-bottom tetraquark state, its dominant decay modes are \( Y(10750) \to B B \) and \( B^{*+} B^{*-} \), while the partial decay widths for the decays \( Y(10750) \to B B^* \) are tiny. We can search for the \( Y(10750) \) in the processes \( Y(10750) \to B^+ B^- , B^{0} \bar{B}^{0} , B^{*+} B^{*-} , B^{*0} \bar{B}^{*0} , B^{*+} B^- , B^{+} B^{*-} , B^{*0} \bar{B}^{0} , B^{0} \bar{B}^{*0} , \eta \omega , \Upsilon \pi^+ \pi^- \) to diagnose the nature of the \( Y(10750) \).

4 Conclusion

In this article, we take the scalar diquark and scalar antidiquark operators as the basic constituents, construct the \( C_{75} \otimes \gamma_5 \otimes \gamma_5 C \) type tetraquark current by introducing an explicit P-wave between
Figure 4: The hadronic coupling constants with variations of the Borel parameters $T^2$, the $A$, $B$, $C$, $D$ and $E$ denote the hadronic coupling constants $G_{YYB}$, $G_{YBB}$, $G_{YBB}$, $G_{YYY}$ and $G_{YYf_0}$, respectively.
the diquark and antidiquark constituents to study the $Y(10750)$ as a vector tetraquark state with the QCD sum rules. We carry out the operator product expansion up to the vacuum condensates of dimension 10 in a consistent way, and use the modified energy scale formula to choose the ideal energy scale of the QCD spectral density so as to extract the reasonable mass and pole residue. The predicted mass $M_Y = 10.75 \pm 0.10$ GeV is in excellent agreement with (at least is compatible with) the experimental value $M_Y = 10752.7 \pm 5.9^{+0.7}_{-1.1}$ MeV from the Belle collaboration. Furthermore, we study the hadronic coupling constants in the two-body strong decays of the $Y(10750)$ with the three-point correlation functions by carrying out the operator product expansion up to the vacuum condensates of dimension 5. We take into account both the connected and disconnected Feynman diagrams, and obtain the QCD sum rules for the hadronic coupling constants, then obtain the partial decay widths and total width. The predicted width $\Gamma_Y = 33.60^{+16.64}_{-9.45}$ MeV is in excellent agreement with the experimental data $35.5^{+17.6}_{-11.3} -3.3$ MeV from the Belle collaboration. The present calculations favor assigning the $Y(10750)$ as the diquark-antidiquark type vector hidden-bottom tetraquark state with $J^{PC} = 1^{--}$, which has a relative P-wave between the diquark and antidiquark constituents.

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