A phonon laser utilizing quantum-dot spin states

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We propose a nano-scale realization of a phonon laser utilizing phonon-assisted spin flips in quantum dots to amplify sound. Owing to a long spin relaxation time, the device can be operated in a strong pumping regime, in which the population inversion is close to its maximal value allowed under Fermi statistics. In this regime, the threshold for stimulated emission is unaffected by spontaneous spin flips. Considering a nanowire with quantum dots defined along its length, we show that a further improvement arises from confining the phonons to one dimension, and thus reducing the number of phonon modes available for spontaneous emission. Our work calls for the development of nanowire-based, high-finesse phonon resonators.

Realizing acoustic analogues of active optical devices has been a long-standing challenge. Phonon lasers could provide versatile sources of coherent acoustic waves used for three-dimensional (3D) imaging of nanostructures or creating periodic strain of a material to rapidly modulate its optical or electronic properties. Recent experimental candidates include doped semiconductor superlattices and micro-cavity systems coupled to a radio-frequency mechanical mode1 [1–3], while many other possibilities have been considered theoretically4–10. Despite the obvious analogy between photons and (acoustic) phonons — both being bosonic excitations with a linear dispersion, a realization of the phonon laser is considerably more demanding. The key difficulty stems from the small value of the speed of sound8, s, or, equivalently, from the high value of the phonon density of states (DOS), which makes the threshold for stimulated emission hard to overcome.

A class of highly-controllable quantum systems emerging from the ideas of spintronics and spin-based quantum computing11–14 may offer new regimes of physical parameters in which phonon lasering is feasible, despite the smallness of s. In particular, Zeeman sub-levels of quantum dots (QDs)15 have several desirable properties. They constitute reliable two-level systems with the spin relaxation rate 1/T116–23 low compared to the electron tunneling rates, while the spin-selective tunneling24–27 allows to separately manipulate the populations of the spin-up and spin-down states.

The main requirements for the occurrence of stimulated emission are the following: 1) a population inversion for two levels, 2) phonon emission must dominate over other relaxation channels, and, 3) to overcome the threshold, the emission into the amplified phonon mode should exceed the loss due to a finite phonon lifetime, τQ, in the resonator. Usually, the latter condition is difficult to fulfill. Due to the high DOS for phonons, spontaneous emission competes effectively with stimulated emission into the designated mode, making the population inversion small. Indeed, Chen and Khurgin8 derive for the threshold pump rate (per unit volume),

\[ R_{th} = \frac{\pi T g(\omega_Q)}{\tau_Q} \sim \frac{\omega_Q^2 \Gamma}{s^3 \tau_Q}, \]

where \( \Gamma \) is the width of the electronic level, \( g(\omega) \) is the phonon DOS in 3D, and \( \omega_Q \) is the frequency of the lasing mode (throughout \( \hbar = 1 \) and \( \Gamma_Q = 1 \)). Remarkably, \( R_{th} \) does not depend on the interaction strength between the phonon field and the two-level system, assuming that phonon emission is the sole relaxation channel. According to Eq. (1), the only realistic way of overcoming the threshold consists in using a small frequency \( \omega_Q \). In the context of QDs discussed below, Eq. (1) describes the regime of weak pumping, which corresponds to a small value of the population inversion, and arises when the sequential-tunneling rate is small compared to 1/T1.

In this Letter we show that Zeeman sub-levels in semiconductor QDs are ideal two-level systems for using in phonon lasers. To create population inversion, spin-selective tunneling from the leads is used [Fig. 1(a)]. Spin flip is mediated by the spin-orbit interaction in the QD and accompanied by phonon emission15, 21–22. We find a regime of strong pumping, when the upper, spin-up, level is occupied and the lower, spin-down, level is empty. This regime is accessible with QDs for realistic values of physical parameters. Indeed, the characteristic tunneling rates into and out of the QD that correspond to the onset of this regime are determined by small 1/T1 value, and can be easily adjusted. In this regime, the stronger the phonon field couples to the spin the lower is the threshold for stimulated emission. Furthermore, the ability to tune the Zeeman splitting independently of the size of the QD allows one to control the strength of the spin-phonon coupling and use an optimal phonon mode for lasing. The prescription for the angular frequency of this mode, \( \omega_Q \), is then \( \omega_Q \sim s/a \), where a is the QD size along the phonon propagation.

We also show that stimulated phonon emission can be envisioned even in the weak pumping regime for suffi-
ciently small values of the Zeeman splitting \( \Delta_Z \). However, in 3D, the threshold value in Eq. (11) is too demanding because the rate of spontaneous emission is too high. By proceeding to a 1D situation in which the phonons are emitted only along a nanowire, we show that \( \tau_{th} \) can be strongly reduced. In this case all the “wrong” phonon modes which could have been emitted in the direction perpendicular to the propagation of the lasing mode are excluded.

We first consider an idealized situation depicted in Fig. (a), where a QD is tunnel coupled to two half-metal ferromagnets \( \mu_L \) and \( \mu_R \). The electrons from the left lead can only tunnel into the higher-energy spin-up state. In order to proceed to the right lead, the electron should flip spin and transit to the lower, spin-down, state by emitting a phonon. The case of ferromagnetic leads containing both spin species, and the role of the leakage current due to the spin-orbit interaction are discussed later. The QD Hamiltonian is

\[
H_{\text{QD}} = \sum_{s = \uparrow, \downarrow} \epsilon_s d_s ^\dagger d_s + U n_{\uparrow} n_{\downarrow},
\]

where \( \epsilon_{\uparrow} = \epsilon + \Delta_Z/2 \) and \( \epsilon_{\downarrow} = \epsilon - \Delta_Z/2 \) are the energies of the two Zeeman sub-levels (\( \Delta_Z > 0 \)), \( d_s ^\dagger \) are the fermionic creation operators, and \( n_s = d_s ^\dagger d_s \). The on-site Coulomb energy \( U \) is assumed to be larger than the source-drain bias \( eV = \mu_L - \mu_R \). In the sequential-tunneling regime, as shown in Fig. (a), we neglect the doubly-occupied state \( d_{\uparrow} ^\dagger d_{\uparrow} ^\dagger |0\rangle \), keeping only the empty-dot \( |0\rangle \), spin-up \( |\uparrow\rangle = d_{\uparrow} ^\dagger |0\rangle \), and spin-down \( |\downarrow\rangle = d_{\downarrow} ^\dagger |0\rangle \) states. The sequential-tunneling condition implies \( \epsilon = 0 \) \( 2\gamma \) and \( eV > \Delta_Z \).

Coupling between the QD and the leads is described by the tunneling Hamiltonian

\[
H_T = \sum_{l \sigma \sigma_s} t_{l \sigma \sigma_s} ^\dagger c_{l \sigma_s} d_{\sigma} + \text{h.c.},
\]

where \( t_{l \sigma \sigma_s} ^\dagger \) is the matrix of tunneling amplitudes and \( c_{l \sigma_s} ^\dagger \) creates an electron with momentum \( k \) and spin \( \sigma = \uparrow, \downarrow \) lead \( l = L, R \). The relevant physical quantity here is the matrix of rates \( \Gamma_{l \sigma \sigma_s} = \pi \sum_{\sigma'} \left( t_{l \sigma \sigma_s} ^\dagger \right)^2 \nu_{\sigma' \sigma_s} \), where \( \nu_{\sigma'} \) is the DOS of spin species \( \sigma \) in lead \( l \). In our idealized situation, only \( \Gamma_{L \uparrow \downarrow} \) and \( \Gamma_{R \downarrow \uparrow} \) are different from zero. The associated sequential-tunneling rates, marked by arrows in Fig. (a), read

\[
W^L \equiv W^L_{\uparrow \downarrow} = 2 \Gamma_{L \uparrow \downarrow} \mu_L, \quad W^R \equiv W^R_{\downarrow \uparrow} = 2 \Gamma_{R \downarrow \uparrow} \left[ 1 - f(\epsilon_{\uparrow} - \mu_R) \right],
\]

where \( f(\epsilon) = [1 + \exp(\epsilon/T)]^{-1} \) is the Fermi distribution function. The reverse rates, \( W^L_{\downarrow \uparrow} \) and \( W^R_{\uparrow \downarrow} \), are obtained from Eq. (1) by replacing \( f(\epsilon) \to 1 - f(\epsilon) \). However, these rates are suppressed at low temperatures, when \( T \ll eV - \Delta_Z \).

For the lasing phonon mode, we write \( H_Q = \omega_Q (N_Q + 1/2) \), where \( N_Q = a^\dagger a \) is the phonon number operator, with \( a^\dagger \) creating a phonon in the lasing mode. Well above the threshold \( N_Q \) is large (\( N_Q \gg 1 \)) and the lasing mode acts as a classical field, capable of driving Rabi oscillations in the QD.

The term describing the coupling between the QD and the lasing mode reads

\[
H_a = \sum_{\sigma \sigma_s} M_{\sigma \sigma_s} a^\dagger a + \text{h.c.}, \tag{5}
\]

where \( M_{\sigma \sigma_s} \) are matrix elements of the spin-phonon interaction, obtained by taking into account a combined effect of spin-orbit interaction and magnetic field \( 21 \ 22 \ 30 \). In nanowires, such as InAs or InSb, the spin-orbit interaction is rather strong \( 31 \ 32 \), facilitating an efficient spin-phonon coupling.

The coupling of the spin to the phonon continuum, \( \text{i.e.} \) to all modes except the lasing mode, is identical in nature to Eq. (5) and is obtained from Eq. (3) by summing over the phonon modes. This coupling leads to spin relaxation \( 21 \ 22 \ 30 \) with the rate \( 1/T_1 = w_{\uparrow \downarrow} + w_{\downarrow \uparrow} \), where \( w_{\sigma \sigma'} \) are rates for phonon-assisted transitions. One can estimate \( 21 \ 22 \ 30 \)

\[
w_{\sigma \sigma'} \simeq 2 \pi |M_{\sigma \sigma'}|^2 V g (\Delta_Z) [1 + N(\Delta_Z)], \tag{6}
\]

where \( V \) is the sample volume in 3D (or length of nanowire in 1D) and \( N(\epsilon) = [\exp(\epsilon/T) - 1]^{-1} \) is the Bose-Einstein distribution function. Equation (6) gives the rate for phonon emission. The rate for phonon absorption, \( w_{\sigma' \sigma} \), is obtained from Eq. (6) by replacing

\[
\exp(\epsilon/T) \rightarrow 1 - \exp(\epsilon/T), \tag{7}
\]
1 + N(Δ_2) by N(Δ_2). For low temperature, when \( T \ll Δ_2 \), we set \( \frac{u_{\bar{Q}}}{u_Q} = 0 \).

We describe the QD by a density matrix \( \hat{\rho} \), which include diagonal and off-diagonal elements. The master equations can be derived in the standard way [34]. The key point is that we treat the laser mode as a classical field, assuming that its population is large \( N_Q \gg 1 \). Similar treatment for an electron coupled to an oscillating magnetic (ESR) field was used in Ref. [35]. After applying the rotating wave approximation, we obtain [36]

\[
\begin{align*}
\frac{d\rho_\uparrow}{dt} &= W^L_\uparrow \rho_0 - W^L_\uparrow \rho_\uparrow + w_\uparrow \rho_\uparrow - \frac{\gamma N_Q (\rho_\uparrow - \rho_\downarrow)}{\tau_q}, \\
\frac{d\rho_\downarrow}{dt} &= W^R_\downarrow \rho_0 - W^R_\downarrow \rho_\downarrow + w_\downarrow \rho_\downarrow - \frac{\gamma N_Q (\rho_\uparrow - \rho_\downarrow)}{\tau_q}, \\
\frac{d\rho_0}{dt} &= W^L_0 \rho_\uparrow + W^R_0 \rho_\downarrow - (W^L_0 + W^R_0) \rho_0, \\
\frac{dN_Q}{dt} &= \gamma N_Q (\rho_\uparrow - \rho_\downarrow) - \frac{N_Q}{\tau_q}.
\end{align*}
\]  
\( (7) \)

where \( \rho_\uparrow + \rho_\downarrow + \rho_0 = 1 \) is due to Coulomb blockade and \( \gamma = 2 |M_{\uparrow\downarrow}|^2 / \Gamma \). The quantity \( \gamma N_Q \) is the rate of Rabi flips. The quantity \( \Gamma = (W^L_0 + W^R_0) / 2 + 1 / T_2 \) is the decay rate of the off-diagonal component \( \rho_{\uparrow\downarrow} \) of the density matrix. It includes the component due to tunneling from the up and down spin states to the left and right leads, see Eq. (1), and intrinsic decoherence rate \( 1 / T_2 \). The last equation of system (7) describes the occupation of the lasing mode. The decay rate \( 1 / \tau_Q \) represents the loss of phonons due to scattering processes, including escape through the mirrors.

Equation (7) is written for a single QD in the system. When there are \( N_D \) identical QDs, and distance between them is larger than the phonon wave length, then in the system of equations for quantities \( \hat{\rho} \) and \( N_Q / N_D \), \( \gamma \) is replaced everywhere by \( \gamma_D = \gamma N_D \). Since \( \gamma \) is proportional to the coupling constant \( |M_{\uparrow\downarrow}|^2 \), it means that the normalization volume (for the phonon wave function) which enters the problem is equal to \( n^{-1} = V / N_D \), i.e. the volume per one QD.

Next, we seek a stationary solution of Eq. (7) and conditions for the onset of stimulated emission. We do not present the explicit expressions for \( \hat{\rho} \) and give only the equation for the population inversion. For non-trivial solutions, for which \( N_Q \) does not vanish identically, from the last line in Eq. (7) we obtain

\[
\rho_\uparrow - \rho_\downarrow = 1 / (\gamma_D \tau_Q).
\]  
\( (8) \)

This equation has a simple physical meaning, namely, in the stationary regime the incoming rate to the lasing mode should be equal to the decay rate \( 1 / \tau_Q \). The number of phonons per QD reads

\[
N_Q / N_D = \frac{(W^R - 1 / T_1) (\tau_Q - 1 / \gamma_D)}{2 + W^R / W_L} - \frac{1}{\gamma_D T_1}.
\]  
\( (9) \)

From the condition \( N_Q > 0 \) one gets

\[
\frac{1}{\gamma_D \tau_Q} < \frac{T_1 W^R - 1}{T_1 W^R + 1 + W^R / W_L}.
\]  
\( (10) \)
determining the threshold value of \( \tau_Q \) that corresponds to the onset of the stimulated phonon emission [see Fig. 1 (b)]. The quantity \( T_1 W^R \) should be larger than unity. Further we assume the inequality \( T_1 W^R \gg 1 \).

Two pumping regimes can be distinguished [see Fig. 1 (b)]. The weak pumping \( T_1 W^L \ll 1 \) corresponds to almost empty dot, \( \rho_0 \approx 1 \), and Coulomb blockade does not play any role. From Eq. (10) the threshold value of the pump rate \( W^L_{\text{th}} \) is

\[
W^L_{\text{th}} = \frac{1}{T_1} \frac{1}{\gamma_D \tau_Q} \approx \frac{g \Gamma}{\tau_Q n}.
\]  
\( (11) \)

where we used \( 1 / T_1 \sim |M_{\uparrow\downarrow}|^2 g V \) and \( g \) is the phonon DOS calculated at the Zeeman energy. Eq. (11) is valid for the case of any dimensionality, \( n \) is the corresponding concentration of the QDs. Note that Eq. (11) is similar to Eq. (1), and the coupling constant drops out.

The condition Eq. (11) has a simple physical meaning, and can be derived in the following way. At the threshold, the incoming rate to the lasing mode should exceed the decay rate \( \rho_\uparrow \gamma_D > 1 / \tau_Q \). On the other hand, the pump rate at the threshold is equal to the spontaneous emission rate, \( W^L_{\text{th}} = \rho_\uparrow / T_1 \). Excluding \( \rho_\uparrow \) from these equations, one obtains Eq. (11).

For the ratio of the threshold pump rates in 3D and 1D we obtain

\[
\frac{W^L_{\text{th,3D}}}{W^L_{\text{th,1D}}} \sim \frac{g_1 n_1}{g_1 n_3} \sim \frac{A}{\lambda_{ph}} \gg 1,
\]  
\( (12) \)

where \( g_1 = 1 / \pi s \) is the 1D phonon DOS, \( \lambda_{ph} \) is the phonon wave length, and \( A \) is the area of the sample in the transverse direction (perpendicular to the direction of
rewritten in terms of the phonon mean free path \( \tau \) versus the Zeeman splitting \( \Delta_z \); maximal coupling is achieved at \( \Delta_z \approx \hbar s/a \).

In the opposite regime of strong pumping \( T_1 W^L \gg 1 \), the effective threshold pump rate \( W^L_\rho \) of the upper level saturates at the \( 1/T_1 \) value. In this regime \( \rho_\parallel \approx 1 \), and because of the Pauli principle an electron cannot tunnel from the left lead into the QD until the electron inside the QD flips its spin and gets to the lower level, leaving the QD. Thus, spin-flip transition, which happens with the rate \( 1/T_1 \), is the bottle-neck process in this regime. From the condition that the incoming rate to the lasing mode exceeds the decay rate, we obtain for the threshold value of \( \tau_Q \) the following inequality: \( 1/\gamma_D\tau_Q^{th} < 1 \). This inequality follows also from Eq. (11). Note that the phonon DOS drops out, and the condition for \( \tau_Q^{th} \) is determined only by the coupling constant \( |M_{\parallel \parallel}|^2 \), as function of \( \Delta_z \) in Fig. 2(b). The dependence of threshold value of \( \tau_Q \) on the tunneling rate \( W^L \) is shown in Fig. 1(b). The plateau value of \( \tau_Q^{th} \propto 1/\gamma_D \) can be reduced by choosing the material with a strong spin-orbit interaction like InAs. The curve corresponding to a much higher value of \( 1/T_1 \) and a small value of population inversion is described by Eq. (11), and goes above the plateau in the strong tunneling regime in Fig. 1(b).

For strong pumping the threshold condition can be rewritten in terms of the phonon mean free path (\( \tau_Q^{th} \)). Specifically, for 1D,

\[
N_D l_{\text{ph}}^{th} / L_z = \Gamma T_1,
\]

where \( L_z \) is the distance between the mirrors. Assuming also that \( \gamma_D \tau_Q^{th} \gg 1 \), i.e. well above the threshold, we obtain for the number of phonons in strong tunneling regime

\[
N_Q \approx \frac{W^L W^R}{2W^L + W^R \theta Q N_D}. \tag{14}
\]

Our consideration so far did not take into account the leakage current, i.e. when the spin-up electron can tunnel directly to the right lead without flipping its spin inside the QD, and an electron from the left lead can directly tunnel into the spin down state of the QD. Such processes are possible for minority carriers in ferromagnets and because with the spin-orbit interaction the spin-up/down directions are not exactly collinear. Adding the corresponding terms into system Eq. (7), with the tunneling rates \( W_{\theta 0}^L \equiv W^L \) and \( W_{\theta 0}^R \equiv W^R \), we derive a new threshold equation instead of Eq. (10). In the case of relatively strong leakage when \( W^R \gg 1/T_1 \), the threshold equation takes the form

\[
\frac{1}{\gamma_D \tau_Q} < \frac{W^L W^R - \tilde{W}^L \tilde{W}^R}{W^L W^R + W^L \tilde{W}^R + W^R \tilde{W}^R}. \tag{15}
\]

We see that even in the case of strong spin-orbit coupling, when \( W^R \) and \( W^R \) are of the same order of magnitude, but not very close to each other \( (\tilde{W}^R < W^R, \ W^L < W^L) \), the threshold condition is similar to that we had before in the strong tunneling case without leakage, when the right hand side of Eq. (15) was unity.

Finally, the number of nonradiative (nr) and radiative (ra) transitions per unit time inside the QD are \( I_{nr} = w^\parallel \rho^\parallel - w_\parallel \rho_\parallel; I_{ra} = \gamma N_Q (\rho_\parallel - \rho_\parallel) \equiv N_Q / (N_D \tau_Q) \). A figure of merit of the emitter is the ratio \( \eta = I_{ra} / I_{nr} \). In the most favorable case this ratio reaches \( \eta \approx (2T_1/\tau_Q)(N_Q / N_D) \approx T_1 W^R \). The power output of the phonon laser is written as

\[
P = \hbar \omega Q N_Q / \tau_Q. \tag{16}
\]

We next estimate the relevant parameters. Since the typical length of the QDs is \( a \approx (0.3 \div 1) \times 10^{-5} \text{cm} \), in order to have reasonably strong coupling to the phonons one needs to choose not very large Zeeman gap \( \Delta_z \approx \hbar s/a \), \( 1K < \Delta_z < 5K \) [see Fig. 2(b)]. Therefore, the temperature is also restricted to these values. To have not very high value of the threshold phonon mean free path, we take the tunneling rate \( \Gamma = (10^9 \div 10^{10})s^{-1} \), which corresponds to the current \( I = eI \approx 1 \text{nA} \). Then, assuming for InAs QDs relatively short \( T_1 \approx (10^{-7} \div 10^{-8})s \), and taking \( N_D = 10 \), we obtain from Eq. (13) the ratio \( l_{\text{ph}} / L_z = 10 \). To find how realistic is that, we take \( L_z = 1 \mu m \), and \( \tau_Q \approx 10^{-7} \text{s} \) [37], which corresponds to a phonon mean free path \( (10^{-2} \div 10^{-1}) \text{cm} \). Then we obtain \( l_{\text{ph}} / L_z \sim (10^2 \div 10^3) \). The indicated values for \( l_{\text{ph}} \) were experimentally observed [38] for the THz acoustic phonons in 3D semiconductors. For the number of phonons above the threshold, see Eq. (14), we get \( N_Q \approx N_D \Gamma \tau_Q \approx 10^3 \). For the power, see Eq. (16), one
gets \( P \approx 4.2 \times 10^{-6} \text{erg/sec} \) for \( \Delta Z = 3K \). Taking the diameter of a wire \( 10^{-6} \text{cm} \), we obtain for the power density \( \approx 1 \text{W/cm}^2 \).

In conclusion, we propose to use the Zeeman sub-levels of the ground orbital state of the QD to generate stimulated phonon emission. The frequency of phonons can be easily tuned by changing the external Zeeman field, which allows a reasonably large interaction with phonons for a given QD size [see Fig. 2(b)]. Because of a generally low value of spin-relaxation rate, the strong pumping regime, characterized by a large value of the population inversion, can be easily achieved. We show that a promising practical implementation is a system of elongated QDs embedded into 1D nanowire. The threshold for stimulated emission is greatly reduced in the 1D case, when the phonons propagate only along the wire.

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\[ \rho_{\uparrow \downarrow} = \frac{iM_{\uparrow \downarrow} e^{-i\omega t}}{\Gamma + i(\Delta Z - \omega)} (\rho_{\uparrow \uparrow} - \rho_{\downarrow \downarrow}). \]

This solution is valid under the condition \( \Delta Z \gg \Gamma \gg \frac{\omega}{\omega_0} N_0 \).
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