Study of the Curie temperature in a transverse Ising film with the next nearest-neighbor interactions

Z P Li, Z X Lu
School of Mechanical & Vehicle Engineering, Linyi University, Linyi 276005, China
E-mail: lizhipeng@lyu.edu.cn

Abstract. The mean-field approximation is used to study the Curie temperature in a transverse Ising film with the next nearest-neighbor interactions. The main emphasis of our research is focused on the dependence of Curie temperature on the decaying exponent $\delta$. In the meanwhile, how the effect of film thickness $N$ on the character of phase transition is also calculated.

1. Introduction
The transverse Ising film has been concerned for long time in theory and experiment [1-10]. Theoretically, the transverse Ising model, from the microcosmic point of view, has been used to study the phase transition properties of ferroelectric thin films or ferroelectric superlattices [2-10]. By taking into account the nearest-neighbor interactions, Wang et al. [2] and Sy [4], respectively, investigated the surface modification effect on the Curie temperature $T_c$ of a ferroelectric thin film with the mean-field approximation. In fact, the long-range interaction dominates in a ferroelectric thin film. As a consequence, the next nearest-neighbor interactions could not be neglected. In some literatures [5-8], by considering different ranges of exchange interactions, the mean-field approximation and effective-field theory with correlations are used to calculate the dependence of Curie temperature $T_c$ on various exchange interaction parameters. Recently, we studied the phase transitions in a transverse Ising film with the next nearest-neighbor interactions by using the mean-field approximation [9]. Very recently, the nearest-neighbor interactions and the next nearest-neighbor interactions as well as the third nearest-neighbor interactions have been included when we calculated the phase diagram of a ferroelectric thin film by the use of mean-field approximation [10].

In the present paper, the Curie temperature in a transverse Ising film with the next nearest-neighbor interactions has been studied by using the mean-field approximation. The dependence of Curie temperature $T_c$ on the film thickness $N$ is shown. Meanwhile, the influence of decaying exponent $\delta$ on the phase transition properties is also obtained.

2. Model and formulas
Figure 1 is the three-dimension structure diagram for the transverse Ising film with a thickness $N$. The Hamiltonian of this system is [5-10]

$$H = -\frac{1}{2} \sum_{ij} J_{ij} S_i^x S_j^z - \Omega_i \sum_{i \in b} S_i^x - \Omega_i \sum_{i \in s} S_i^x,$$

(1)

where $S_i^x$ and $S_i^z$, respectively, is the spin-1/2 component along the $x$ and $z$ direction at the site $i$. $J_{ij}$ denotes the interchange interaction between the two pseudo-spins. $J_{ij}$ is while the two pseudo-spins at the surface layer, otherwise $J_{ij}$ is while the transverse field on the surface layer, however $\Omega_i = \Omega_s$ while the transverse field in the bulk layer. $\delta$...
represents the decaying exponent.

With the mean-field approximation [5, 6, 9, 10], the polarization in the i-th layer of the transverse Ising film is

$$R_i = \langle S_{iz} \rangle = \frac{\langle H_{iz} \rangle}{2} \frac{|H_i|}{\tanh \left( |H_i|/2k_B T \right)} , \quad (2)$$

where

$$\langle H_{iz} \rangle = \sum_j \frac{J_{ij}}{\delta} S_j^z , \quad (3)$$

$$|H_i| = \sqrt{\Omega_i^2 + \langle H_{iz} \rangle^2} . \quad (4)$$

**Figure 1.** The 3-D structure diagram for a transverse Ising film.

For an N-layer transverse Ising film, the equation (2) will be expanded to N linear equations [9, 10]. For the layer 1,

$$\tau_s R_1 = 4 J_s R_1 + J_b R_2 + 4 \frac{J_s}{\sqrt{2}} R_1 + 4 \frac{J_b}{\sqrt{2}} R_2 , \quad (5)$$

for the layer N,

$$\tau_s R_N = 4 J_s R_N + J_b R_{N-1} + 4 \frac{J_s}{\sqrt{2}} R_N + 4 \frac{J_b}{\sqrt{2}} R_{N-1} , \quad (6)$$

for the other layers,

$$\tau_s R_i = 4 J_s R_i + J_b R_{i+1} + 4 \frac{J_s}{\sqrt{2}} R_i + 4 \frac{J_b}{\sqrt{2}} R_{i+1} + 4 \frac{J_b}{\sqrt{2}} R_{i-1} + 4 \frac{J_b}{\sqrt{2}} R_{i+1} . \quad (7)$$

Where the parameters \( \tau_s \) and \( \tau_b \) is respectively defined as follows:

$$\tau_s = 2\Omega_s \coth \left( \Omega_s / 2k_B T \right) , \quad (8)$$

$$\tau_b = 2\Omega_b \coth \left( \Omega_b / 2k_B T \right) . \quad (9)$$

According to [9], the Curie temperature \( T_c \) for an N-layer transverse Ising film is calculated by

$$X_0 \hspace{1cm} -1$$

$$-1 \hspace{1cm} X_0 \hspace{1cm} -1$$

$$-1 \hspace{1cm} X_0 \hspace{1cm} \ddots$$

$$X_0 \hspace{1cm} -1$$

$$-1 \hspace{1cm} X_0 \hspace{1cm} -1$$

$$-1 \hspace{1cm} X_0 \hspace{1cm} X_{1 \times N}$$

$$= 0$$


where

\[
X_s = \frac{\tau_s - 4J_s - 4\frac{J_b}{\sqrt{2}\sigma}}{J_b + 4\frac{J_b}{\sqrt{2}\sigma}}, \quad X_b = \frac{\tau_b - 4J_b - 4\frac{J_b}{\sqrt{2}\sigma}}{J_b + 4\frac{J_b}{\sqrt{2}\sigma}}.
\]  

(11)

3. Numerical results

Within the framework of the mean-field approximation [9, 10], the Curie temperature of a transverse Ising film with the next nearest-neighbor interaction can be calculated from equations (5-11). In the following numerical calculations, the transverse field \(\Omega_s=\Omega_b=\Omega\) is supposed for convenience.

![Figure 2](image2.png)

**Figure 2.** The dependence of Curie temperature \(T_c\) on the film thickness \(N\) for \(J_s=J_b\).

![Figure 3](image3.png)

**Figure 3.** The dependence of Curie temperature \(T_c\) on the film thickness \(N\) for \(J_s>J_b\). All curves are for \(J_b=1.0, \Omega=1.0\). (a) \(J_s=1.35\), (b) \(J_s=1.80\).

Figure 2 shows the dependence of Curie temperature \(T_c\) on the film thickness \(N\) for different values of decaying exponent \(\delta\) when \(J_s=J_b\). In Figure 2, \(J_s=J_b=1.0\) and \(\Omega=1.0\) are selected. For a certain decaying exponent \(\delta\), it is obvious that the Curie temperature \(T_c\) increases with the film thickness \(N\) increases. On the other hand, for the transverse Ising thin film with a certain thickness \(N\), with the increase of decaying exponent \(\delta\), however, the Curie temperature \(T_c\) decreases. In addition, the larger
the value of decaying exponent $\delta$, the closer to each other the value of Curie temperature $T_c$. These characters are similar to those of in [9]. However, Figure 2 is more intuitionistic and explicit.

Figure 3 shows the dependence of Curie temperature $T_c$ on the film thickness $N$ for different values of decaying exponent $\delta$ when $J_s>J_b$. In Figure 3(a), $J_s=1.35$, $J_b=1.0$ and $\Omega=1.0$. It is obvious that the surface interchange interaction $J_s$ has an important impact on the Curie temperature $T_c$. For a different surface interchange interaction $J_s$, all the curves of $T_c$~$N$ have some significant changes. The curvature and direction of every curve is obvious differently. However, the larger the decaying exponent $\delta$, the closer the Curie temperature $T_c$. These features still remain.

Figure 4. The dependence of Curie temperature $T_c$ on the film thickness $N$ for different $J_s$.

All curves are for $J_b=1.0$, $\Omega=1.0$, $\delta=5.0$.

Figure 4 shows the dependence of Curie temperature $T_c$ on the film thickness $N$ for different values of surface interchange interaction $J_s$ when the decaying exponent $\delta$ is fixed. Obviously, for a certain $J_s$ which is smaller than $J_s=1.363$, the Curie temperature $T_c$ increases with the film thickness $N$ increases. Eventually, the curves of $T_c$~$N$ for different values of $J_s$ would be close to the bulk transition temperature $T_c=1.989$ with film thickness $N$ increases. These features are similar to those of in [10].

4. Conclusions

In conclusion, the mean-field approximation has been used to calculate the Curie temperature for a transverse Ising film by taking into account the next nearest-neighbor interaction. The dependence of the Curie temperature $T_c$ on the film thickness $N$ for different values of decaying exponent $\delta$ is discussed. Meanwhile, the effect of surface interchange interaction $J_s$ on the Curie temperature $T_c$ is also executed. Our results are also compared with those of the third nearest-neighbor interaction [10].

References

[1] Scott J F 2007 Science 315 954
[2] Wang C L, Zhong W L and Zhang P L 1992 J. Phys.: Condens. Matter 4 4743
[3] Wang C L, Smith S R P and Tilley D R 1994 J. Phys.: Condens. Matter 6 9633
[4] Sy H K 1993 J. Phys.: Condens. Matter 5 1213
[5] Wang Y G, Zhong W L and Zhang P L 1997 Solid State Commun. 101 807
[6] Xin Y, Wang C L, Zhong W L and Zhang P L 1999 Phys. Lett. A 260 411
[7] Yao D L, Wu Y Z and Li Z Y 2002 Phys. Status Solidi B 231 3
[8] Tabyaoui A, Madani M, Ainane A and Saber M 2005 Physica A 358 150
[9] Li Z P 2015 IFEEA 364
[10] Li Z P 2016 Phase Transitions 89 1119