How to identify a Strange Star

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Contrary to young neutron stars, young strange stars are not subject to the r-mode instability which slows rapidly rotating, hot neutron stars to rotation periods near 10 ms via gravitational wave emission. Young millisecond pulsars are therefore likely to be strange stars rather than neutron stars, or at least to contain significant quantities of quark matter in the interior.

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It has recently been shown, that the emission of gravitational radiation due to r-mode instabilities in hot, young neutron stars severely limits the rotation period of these stars [1–6]. Within the first year, where the star cools to 10^8 K, the star will spin down to a few percent of the Kepler-limit [7] (the ultimate rotation frequency where the stellar equator “falls off”), corresponding to a rotation period of 10–20 milliseconds. This scenario has several interesting ramifications [1–6]. First of all, the “slow” rotation predicted for young pulsars seems to agree with the few observed periods of pulsars known to be young, such as Crab (the rapid millisecond pulsars with periods as small as 1.56 ms are expected to be old, cold pulsars spun-up by angular momentum accretion in binary systems, and are therefore not subject to the r-mode instability). Second, the spin-down involves emission of up to 10^{52} ergs of gravitational waves, making the radiation potentially observable. And third, the instability rules out a scenario where millisecond pulsars with periods as small as 1.56 ms are expected to be young, such as Crab (the rapid millisecond pulsars in even as the rotation frequency goes to zero, at least negative angular momentum from a mode with negative angular momentum in the corotating frame, thereby making the angular momentum and energy increasingly negative. In contrast to other known modes, the r-mode instability does not require a large rotation rate, but sets in even as the rotation frequency goes to zero, at least in a non-viscous system (for a review of instabilities in rotating relativistic stars the reader is referred to [8]).

Important for the r-mode instability scenario is the effect of internal fluid dissipation in the star that tends to suppress the instabilities. Lindblom, Owen, and Morsink [9], and Andersson, Kokkotas, and Schutz [4] have shown that for typical neutron star equations of state, the shear viscosity limits the r-mode instability at temperatures below 10^8 K, whereas bulk viscosity is the limiting factor above this temperature. And they find that the rotation rate for standard assumptions about neutron star cooling is reduced to 7–8% of the Keplerian limit.

Whereas the shear viscosity of quark matter [9] is roughly comparable to that of neutron star matter for the parameter range of interest here, the bulk viscosity of quark matter is larger by many orders of magnitude [10]. Therefore, one should expect the r-mode instability to be significantly suppressed if the stars are made up of u-, d-, and s-quarks (so-called strange stars [4]), rather than ordinary neutron star matter. A similar result would apply if quark matter is metastable rather than absolutely stable, so that a neutron star contains quark phase in its central regions.

In the following I will show that this is indeed the case. The r-mode instability does not play any role in a young strange star. Therefore, finding a young pulsar with a rotation period below 5–10 milliseconds would be a strong indication of the existence of strange stars, or at least a significant quark content in neutron stars, and therefore of the stability or metastability of bulk strange quark matter. In addition it would re-introduce the possibility of forming some millisecond pulsars by accretion-induced collapse of white dwarf stars. Unfortunately it would also remove r-mode instability braking of young neutron stars as a strong source of gravitational radiation.

The shear viscosity of strange quark matter due to quark scattering was calculated in [9]. The results for $T \ll \mu$, where $T$ is the temperature, and $\mu \approx 300$ MeV the quark chemical potential, can be written as

\[
\eta \approx 1.7 \times 10^{18} \left( \frac{0.1}{\alpha_S} \right)^{5/3} \rho_{15}^{14/9} T_9^{-5/3} \text{g cm}^{-1} \text{s}^{-1},
\]

where $\alpha_S$ is the fine structure constant of the strong interactions, $T_9 \equiv T/10^9$ K, and $\rho_{15} \equiv \rho/10^{15}$ g cm^{-3}.

The bulk viscosity of strange quark matter [10] depends mainly on the rate of the non-leptonic weak interaction [12]

\[
\eta + \delta \leftrightarrow s + u.
\]
Because the strange quark is much more massive than up and down quarks, this reaction changes the concentrations of down and strange quarks in response to the density changes involved in vibration or rotational instabilities, thereby causing dissipation. This dissipation is most efficient if the rate of reaction \( \delta \mu = \mu_s - \mu_d \), where \( \mu_i \approx 300 \text{ MeV} \) are the quark chemical potentials. This assumption is not correct at low temperatures (\( 2\pi T \ll \delta \mu \)), where the dominating term in the rate is proportional to \( \delta \mu^3 \). Furthermore, the rate in \( \delta \mu \) is too small by an overall factor of 3, and another discrepancy of 2–3 orders of magnitude, perhaps due to unit conversions, appears as well. Taken together, these effects led to an upward correction of the bulk viscosity by several orders of magnitude, and thereby increases the importance for the astrophysical applications. The non-linearity of the rate also means, that the bulk viscosity is no longer independent of the amplitude of the density variations. The resulting bulk viscosity is (in cgs-units, with strange quark mass \( m_s \), \( T \), and \( \mu_d \approx 359 \text{ MeV} \rho_{15}^{1/3} \) in MeV, and the frequency of the perturbation \( \omega \) in s\(^{-1}\))

\[
\zeta \approx 1.10 \times 10^{29} m_s^2 \omega^{-2} \rho_{15}^{1/3} \times \left[ 3 \left( \frac{m_s^2}{4 \mu_d v_0} \right)^2 + 4 \pi^2 T^2 \right] \text{ g cm}^{-1}\text{s}^{-1}.
\]  

For typical values \( (m_s = 100 \text{ MeV}, \mu_d = 300 \text{ MeV}, \omega = \omega_r = 2\pi\Omega/(l(l+1)) = 2 \times 10^3 \text{ s}^{-1}) \) this is \( \zeta \approx 1.6 \times 10^{30} \left[ 93 (\Delta v/v_0)^2 + 0.29 T_s^2 \right] \text{ g cm}^{-1}\text{s}^{-1} \), where \( \Delta v/v_0 \) is the perturbation amplitude.

Kokkotas and Stergioulas\(^ {13} \) give an analytic description of the \( r \)-mode instability in uniform density neutron stars, finding results in good agreement with more sophisticated numerical calculations. The approximation of constant density is even better for a strange star, except very close to the gravitational instability limit, so for the estimates in the present investigation it is safe to use their approximate formulae for damping times etc. In particular, the damping time due to shear viscosity is given by

\[
\tau_{sv} = \frac{3}{4\pi(l-1)(2l+3)} \frac{M}{\eta R},
\]

and the corresponding timescale for damping due to bulk viscosity

\[
\tau_{bv} = \frac{3(2l+5)}{2\pi(l+1)^3} \frac{\Gamma^4 M}{\zeta R},
\]

with \( \Gamma \approx 5 \) to simulate an almost uniform density.

The growth time due to emission of gravitational waves \( (\tau_{gw} = -2E/(dE/dt)|_{gw}) \) is lowest in order in the angular velocity, \( \Omega \), given by

\[
\tau_{gw} = \frac{c^{2l+3}}{24G} \frac{[(2l + 3)!]^2}{(2l + 3)(l-1)!^2} \frac{(l+1)^{2l+2}}{(l+2)} \frac{\Omega^{-2l-2}}{M R^{2l+3}}.
\]

If the timescale for gravitational radiation emission is short compared to the damping timescales \( \tau_{sv} \) and \( \tau_{bv} \), the star will spin down.

To find the critical angular velocity for a given stellar model as a function of temperature one solves the equation

\[
\frac{1}{\tau_{gw}} + \frac{1}{\tau_{sv}} + \frac{1}{\tau_{bv}} = 0.
\]

In the case of hot neutron stars, the \( l = 2, m = 2 \) r-mode instability was found to be decisive in the sense that it corresponds to the longest viscous timescales.

Introducing \( R_{10} \equiv R/10\text{km}, M_{1.5} \equiv M/1.5M_\odot \), where \( M_\odot \) is the solar mass, and \( \Omega_3 = \Omega/10^3\text{s}^{-1} \), one gets

\[
\tau_{gw} = -1.29 \times 10^8 s \Omega_3^{-6} M_{1.5}^{-3} R_{10}^{-4}.
\]

For strange quark matter the viscous timescales are

\[
\tau_{sv}(\text{SQM}) = 1.01 \times 10^8 s (\alpha S/0.1)^{5/3} M_{1.5}^{-5/9} R_{10}^{11/3} T_9^{5/3},
\]

\[
\tau_{bv}(\text{SQM}) = 5.75 \times 10^{-2} s m_{100}^{-4} \Omega_3^2 R_{10}^{2} T_9^{-2}.
\]

Here \( m_{100} \equiv m_s/100\text{MeV} \), and for simplicity only the \( T \)-dependent term, not the \( \Delta v/v_0 \)-term in Eq. (3) has been included.

For comparison the shear viscosity in ordinary neutron star matter is dominated by electron-electron scattering for \( T_9 < 1 \). The corresponding timescale is 31.
\[ \tau_{sv}(\text{NS}) = 3.4 \times 10^7 \text{s} M_1^{-1} R_1^6 T_9^{-2}. \quad (11) \]

If the proton fraction in the neutron star is below \( \frac{1}{3} \), the modified URCA-process is the dominant weak interaction, giving a bulk viscosity damping time \([1]\):
\[ \tau_{sv}(\text{NSmU}) = 4.33 \times 10^9 \text{s} \Omega_3^2 M_1^{-1} R_5^3 T_9^{-6}. \quad (12) \]

If a larger proton fraction is available, as happens in some equations of state, the faster direct URCA-process is active \([1]\), and the bulk viscosity timescale changes to
\[ \tau_{sv}(\text{NSdU}) = 227 s \left( q x_{01} \right)^{-1/3} \Omega_3^2 M_1^{1/3} T_9^{-4}, \quad (13) \]

where \( q x_{01} \approx 1 \) (c.f. \([1]\)).

Inserting these timescales in Eq. (6), one can solve for a critical angular velocity, \( \Omega_c \), for the onset of the \( r \)-mode instability. Figure 1 shows the results as functions of angular velocity within a matter of days to months.

In contrast, the transition between shear- and bulk viscosity damping is seen to take place at \( T \approx 10^9 K \), whereas direct URCA-processes reduces the transition temperature to \( 10^6 K \). The time required to cool the interior of a neutron star to \( 10^6 K \) by the modified URCA mechanism is less than a year, and via the direct URCA mechanism, the cooling timescale is roughly \( t_{cool} \approx 20s T_{9}^{-4} \) (see \([1]\) for a review of neutron star cooling), so in any case, the neutron star will be subject to significant \( r \)-mode damping of the rotation rate within a matter of days to months.

For neutron stars with proton fraction below \( \frac{1}{3} \), the transition between shear- and bulk viscosity damping is seen to take place at \( T \approx 10^9 K \), whereas direct URCA-processes reduces the transition temperature to \( 10^6 K \). The time required to cool the interior of a neutron star to \( 10^6 K \) by the modified URCA mechanism is less than a year, and via the direct URCA mechanism, the cooling timescale is roughly \( t_{cool} \approx 20s T_{9}^{-4} \) (see \([1]\) for a review of neutron star cooling), so in any case, the neutron star will be subject to significant \( r \)-mode damping of the rotation rate within a matter of days to months.

If the initial rotation rate of the strange star is very rapid, say half the Kepler frequency, the \( r \)-mode instability will set in after a few years for low strange quark mass, but increasing the rotation period even above 2 ms will take more than \( 10^3 \) years (and to reach the ultimate limit of 3 ms takes at least a hundred times longer).

These results were derived keeping only the temperature dependent term in the bulk viscosity of strange quark matter, Eq. (1) \([2]\). The actual viscosity is even higher due to the presence of the nonlinear term depending on the amplitude of the density perturbation. Thus the bulk viscosity will dominate at even lower temperatures than found here, but a detailed numerical study of the eigenmodes in a stellar model is required to include the nonlinear term properly.

It therefore seems safe to conclude, that strange stars differ significantly from ordinary neutron stars in terms of their (lack of) sensitivity to the spin-down due to the \( r \)-mode instability. A young, rapidly rotating pulsar with period below 5–10 milliseconds could therefore very well be a strange star, if strange quark matter is absolutely stable, or a neutron star with a significant core of strange quark matter, if strange matter is only metastable (a so-called hybrid star) \([22]\). At present no such young millisecond pulsars (i.e. young pulsars with periods below 10 milliseconds) are known, which may either indicate, that pulsars are neutron stars rather than quark stars, or perhaps that the stars are all formed with low angular momenta for some other reason. Old neutron stars being spun up by angular momentum transfer in binaries are not subject to the \( r \)-mode instability, so the presently known millisecond pulsars could be either neutron or strange stars, whereas an old submillisecond pulsar because of the extremely high bulk viscosity of strange quark matter might be a strange star but probably not a neutron star \([4,6,10]\).

More detailed calculations will be necessary to set the exact limits for the rotation frequency of neutron stars, hybrid stars and quark stars as a function of age and cooling history, but the discovery of even a single young pulsar in the millisecond regime would be most exciting for the strange quark matter hypothesis.

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[22] Another interesting possibility would be that some other exotic phase in the neutron star led to a dramatic increase in the viscosity. The direct URCA process is an example of a process that does increase the bulk viscosity, but in a less dramatic way. Nuclear matter with high strangeness content or even more exotic phases might experience something similar. However, since such phases are typically concentrated near the center of the star, nothing as dramatic as seen for a strange star with high viscosity quark phase almost to the surface should be expected.