Combined fit to BaBar and Belle data on $e^+e^- \rightarrow \pi^+\pi^-\psi(2S)$

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Abstract

A combined fit is performed to the BaBar and Belle measurements of the $e^+e^- \rightarrow \pi^+\pi^-\psi(2S)$ cross sections for center-of-mass energy between threshold and 5.5 GeV. The resonant parameters of the $Y(4360)$ and $Y(4660)$ are determined. The mass is $4355^{+9}_{-10} \pm 9$ MeV/$c^2$ and the width is $103^{+17}_{-15} \pm 11$ MeV/$c^2$ for the $Y(4360)$, and the mass is $4661^{+9}_{-8} \pm 6$ MeV/$c^2$ and the width is $42^{+17}_{-12} \pm 6$ MeV/$c^2$ for the $Y(4660)$. The production of the $Y(4260)$ in $\pi^+\pi^-\psi(2S)$ mode is found to be at 2$\sigma$ level, and $B(Y(4260) \rightarrow \pi^+\pi^-\psi(2S)) \cdot \Gamma_{e^+e^-}$ is found to be less than 4.3 eV/$c^2$ at the 90% confidence level, or equal to $7.4^{+2.1}_{-1.7}$ eV/$c^2$ depending on it interferes with the $Y(4360)$ constructively or destructively. These information will shed light on the understanding of the nature of the $Y$ states observed in initial state radiation processes.

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I. INTRODUCTION

Charmonium spectroscopy is of great interest to both the experimentalists and the theorists since its first discovery more than 30 years ago. With the successful running of the two $B$-factories at SLAC and KEK, charmonium physics was also revitalized as more and more charmonium and charmonium-like states were observed in $B$ decays, in initial state radiation (ISR) processes, in double-charmonium productions, and in two-photon processes. The observation of the $X(3872)$ \cite{1}, $Y(4260)$ \cite{2,3}, $Z(4430)$+ \cite{4} and so on may suggest the existence of new type of hadronic states besides the conventional mesons ($q\bar{q}$) and baryons ($qqq$) in quark model. Among these many newly observed states, those with $J^{PC} = 1^{--}$ are becoming very puzzling since there are too many of them between 4-5 GeV/$c^2$ than expected from the potential models \cite{5,6}. In addition to the excited $\psi$ states observed in the inclusive hadronic cross section \cite{7,8} (the $\psi(4040)$, $\psi(4160)$, $\psi(4415)$), there are four new structures observed in ISR processes (the $Y(4008)$ \cite{3}, $Y(4260)$ \cite{2,3}, $Y(4360)$ \cite{9,10}, $Y(4660)$ \cite{9}). The overpopulation of the vector states in this mass range may suggest at least one of them are non-conventional charmonium state \cite{11}.

In the measurement of the $\pi^+\pi^-(2S)$ cross section via ISR at BaBar \cite{10}, a broad resonance like structure was observed with mass $4324 \pm 24$ MeV/$c^2$ and width $172 \pm 33$ MeV/$c^2$; while with more luminosity, the Belle experiment found it is indeed due to two narrow resonances with masses of $4361$ MeV/$c^2$ and $4664$ MeV/$c^2$ \cite{9}. A close examination of the BaBar observation shows that there is one bin with high cross section at around the second resonance observed in Belle experiment although it is not very significant.

In this paper, we try to perform a combined fit to $e^+e^- \rightarrow \pi^+\pi^-\psi(2S)$ cross sections measured by the BaBar and Belle experiments with two resonances, to obtain a better estimation of the resonant parameters of the $Y(4360)$ and $Y(4660)$. We also study the possible production of the $Y(4260)$ in $\pi^+\pi^-\psi(2S)$ mode.

II. THE DATA

Both BaBar and Belle experiments reported cross sections of $e^+e^- \rightarrow \pi^+\pi^-\psi(2S)$ for center-of-mass energy ranges from threshold to 5.5 GeV. The integrated luminosity of the BaBar data sample is 298 fb$^{-1}$ while that of the Belle data sample is 673 fb$^{-1}$, with $\sim 90\%$ of the data were collected at the $\Upsilon(4S)$ resonance ($\sqrt{s} = 10.58$ GeV), while the rest were taken off the $\Upsilon(4S)$ peak.

Figure \[1\] shows the data, good agreement between BaBar and Belle results is observed, and the two structures are evident.

III. THE FORMULAE AND THE LIKELIHOOD FIT

The Breit-Wigner form of a single resonance used in this analysis is the same as in Refs. \cite{9,10}, i.e.,

$$BW(\sqrt{s}) = \frac{\sqrt{12\pi\Gamma_{ee}B(R \rightarrow f)\Gamma_{tot}}}{s - M^2 + iM\Gamma_{tot}} \sqrt{\frac{PS(\sqrt{s})}{PS(M)}},$$

(1)
FIG. 1: The cross sections of $e^+e^- \to \pi^+\pi^-\psi(2S)$ measured at BaBar (stars with error bars) and Belle (dots with error bars). (a) shows all the data, while (b) shows the region with the $Y$ resonances.

where $M$ is the mass of the resonance, $\Gamma_{\text{tot}}$ and $\Gamma_{e^+e^-}$ are the total width and partial width to $e^+e^-$ respectively, $B(R \to f)$ is the branching fraction of $R$ decays into final state $f$, and $PS(\sqrt{s})$ is the three-body decay phase space factor.

In fitting to the data, we assume all the cross sections are due to the $Y(4360)$ and $Y(4660)$ resonances, and they are added coherently, that is,

$$
\sigma(\sqrt{s}) = |BW_1(\sqrt{s}) + BW_2(\sqrt{s}) \cdot e^{i\phi}|^2,
$$

where $BW_1$ and $BW_2$ represent the two resonances and $\phi$ is the relative phase between them.

We fit the data using a binned maximum likelihood method with MINUIT in the CERN Program Library [12]. To take the Poisson distribution of the small number of events in each $\pi^+\pi^-\psi(2S)$ mass bin into consideration, we start from the observed number of events in each mass bin instead of from the measured cross section. For each $\pi^+\pi^-\psi(2S)$ mass bin, the probability of observing $n_{i}^{\text{obs}}$ events when the expected number of events is $\lambda_i$ is

$$
f_i = \frac{\lambda_i^{n_{i}^{\text{obs}}}}{n_{i}^{\text{obs}}!} e^{-\lambda_i}.
$$

Here

$$
\lambda_i = \sigma_i \varepsilon_i \mathcal{L}_i \mathcal{B}(\psi(2S) \to \pi^+\pi^-J/\psi) \mathcal{B}(J/\psi \to \ell^+\ell^-) + n_{i}^{\text{bkg}},
$$

where $\sigma_i$ is the mean cross section in the $i$-th bin (with bin width $\Delta m$), which is calculated as

$$
\sigma_i = \frac{1}{\Delta m} \int_{m-\Delta m/2}^{m+\Delta m/2} \sigma(x) dx;
$$
FIG. 2: The results of the binned maximum likelihood fit to $e^+e^- \rightarrow \pi^+\pi^-\psi(2S)$ data from Belle and BaBar. The curves show the best fit with two coherent Breit-Wigners and the contribution from each component. The interference between the two amplitudes is not shown. The two dashed curves at each peak show the two solutions (see text).

the number of background events in each bin, $n_{\text{bkg}}^{\text{Belle}} = 0.115$ for a 25 MeV/$c^2$ mass bin for Belle, and $n_{\text{bkg}}^{\text{BaBar}}$ is neglected for BaBar. $\varepsilon_i L_i B(\psi(2S) \rightarrow \pi^+\pi^- J/\psi) B(J/\psi \rightarrow \ell^+\ell^-)$ for each bin is obtained through $(n_{\text{obs}}^{i} - n_{\text{bkg}}^{i})/\sigma_{\text{exp}}^{i}$ from the $\pi^+\pi^-\psi(2S)$ invariant mass distributions and cross sections presented in Refs. [9, 10].

The likelihood is defined as the product of $f_i$ over all the $\pi^+\pi^-\psi(2S)$ mass bins, that is, $L = \prod_i f_i$. In reality, $-2\ln L$ is minimized to get the best estimation of the parameters.

IV. THE $Y(4360)$ AND $Y(4660)$

We fit the Belle data on $\pi^+\pi^-\psi(2S)$ between threshold and 5.5 GeV (in 60 bins) and the BaBar data in the same energy range (in 30 bins) simultaneously. Figure 2 shows the fit results. Two equally good solutions are found with the two amplitudes interfere with each other differently. The masses and the widths of the two resonances are identical but the partial widths to $e^+e^-$ and relative phases are different in these two solutions, as shown in Table I. The statistical significance of the $Y(4660)$ is calculated by comparing the likelihood of the fit with and without it, and we found it is 6.1$\sigma$ in the combined fit, while that quoted in Belle experiment is 5.8$\sigma$ [9].

To validate our fitting method, we fit the Belle data only with two coherent Breit-Wigners, and the results are shown in Table I together with the Belle results [9] from an unbinned maximum likelihood fit method. From the table, we found that while the two fits agree with each other reasonably well, the mass of the second resonance shifts by about 9 MeV/$c^2$, 

\begin{equation}
\sigma(\pi^+\pi^-\psi(2S))(pb)
\end{equation}
TABLE I: Fit results to the combined BaBar and Belle data on $e^+e^- \to \pi^+\pi^-\psi(2S)$. The errors are statistical only. $M$, $\Gamma_{\text{tot}}$, and $\mathcal{B}\Gamma_{e^+e^-}$ are the mass (in MeV/$c^2$), total width (in MeV/$c^2$), and product of the branching fraction to $\pi^+\pi^-\psi(2S)$ and the $e^+e^-$ partial width (in eV/$c^2$), respectively. $\phi$ is the relative phase (in degrees).

| Parameters | Solution I | Solution II |
|------------|------------|-------------|
| $M(Y(4360))$ | 4355$^{+9}_{-10}$ | 4355$^{+9}_{-10}$ |
| $\Gamma_{\text{tot}}(Y(4360))$ | 103$^{+17}_{-15}$ | 103$^{+17}_{-15}$ |
| $\mathcal{B}\Gamma_{e^+e^-}(Y(4360))$ | 11.1$^{+1.3}_{-1.2}$ | 12.3$^{+1.2}_{-1.2}$ |
| $M(Y(4660))$ | 4661$^{+9}_{-8}$ | 4661$^{+9}_{-8}$ |
| $\Gamma_{\text{tot}}(Y(4660))$ | 42$^{+17}_{-12}$ | 42$^{+17}_{-12}$ |
| $\mathcal{B}\Gamma_{e^+e^-}(Y(4660))$ | 2.2$^{+0.7}_{-0.6}$ | 5.9$^{+1.6}_{-1.6}$ |
| $\phi$ | 18$^{+23}_{-24}$ | $-74^{+16}_{-12}$ |

TABLE II: The binned fit to Belle data. The errors are statistical only. The definitions of the parameters and the units are the same as in Table I. Numbers in parentheses are taken from Ref. [9] from an unbinned fit.

| Parameters | Solution I | Solution II |
|------------|------------|-------------|
| $M(Y(4360))$ | 4359$^{+9}_{-10}$ (4361$^{+9}_{-10}$) | 4361$^{+9}_{-10}$ |
| $\Gamma_{\text{tot}}(Y(4360))$ | 85$^{+18}_{-15}$ (74$^{+18}_{-15}$) | 74$^{+18}_{-15}$ |
| $\mathcal{B}\Gamma_{e^+e^-}(Y(4360))$ | 11.9$^{+2.5}_{-1.7}$ (10.4$^{+2.5}_{-1.7}$) | 12.8$^{+2.5}_{-1.7}$ (11.8$^{+2.5}_{-1.7}$) |
| $M(Y(4660))$ | 4655$^{+11}_{-8}$ (4664$^{+11}_{-8}$) | 4664$^{+11}_{-8}$ |
| $\Gamma_{\text{tot}}(Y(4660))$ | 40$^{+17}_{-15}$ (48$^{+17}_{-15}$) | 48$^{+17}_{-15}$ |
| $\mathcal{B}\Gamma_{e^+e^-}(Y(4660))$ | 3.4$^{+5.0}_{-0.9}$ (3.0$^{+5.0}_{-0.9}$) | 6.0$^{+2.4}_{-3.5}$ (7.6$^{+2.4}_{-3.5}$) |
| $\phi$ | 1$^{+34}_{-30}$ (39$^{+34}_{-30}$) | $-56^{+91}_{-22}$ ($-79^{+91}_{-22}$) |

which is comparable to the error quoted by Belle. One of the reasons of the shift is the fitting method, that is, due to the binning of the experimental data.

In principle, the difference between a binned and an unbinned likelihood fit should be very small when the data sample is large enough. However, in our special case, the data sample is rather small (62 events in BaBar experiment and 110 events in Belle), and thus the fluctuation due to binning is not small. In other words, the lost of information in the binning process can not be neglected. We test this by doing toy experiments with Monte Carlo (MC) simulation. We generate 100 MC samples each with the same number of observed events as in Belle experiment (110 events) [9]. For each sample, we do binned and unbinned likelihood fits and check the difference between the fit values of the parameters. Figure 3 shows the distributions of the differences in masses and widths from the two kinds of fit, and Table III shows the fit results to the distributions with Gaussian functions.

It can be seen that the mean values of the differences are consistent with zero as expected, while the standard deviations are significantly different from zero, indicating that the expected uncertainties introduced by using a binned fit are at a few MeV/$c^2$ level from the unbinned fit. The observed difference in the mass of the $Y(4660)$ between binned and unbinned fits can be explained as the uncertainty introduced by the binning procedure. We take this kind of differences as one extra source of the systematic error. It should be noted
FIG. 3: The distributions of the parameter difference between binned and unbinned likelihood fits. The curves show the fits with Gaussian functions. (a) $M_{\text{bin}}(Y(4360)) - M_{\text{unbin}}(Y(4360))$, (b) $\Gamma_{\text{tot}}^{\text{bin}}(Y(4360)) - \Gamma_{\text{tot}}^{\text{unbin}}(Y(4360))$, (c) $M_{\text{bin}}(Y(4660)) - M_{\text{unbin}}(Y(4660))$, and (d) $\Gamma_{\text{tot}}^{\text{bin}}(Y(4660)) - \Gamma_{\text{tot}}^{\text{unbin}}(Y(4660))$.

TABLE III: Expected differences between binned and unbinned fits. $\Delta M$ and $\Delta \Gamma_{\text{tot}}$ are the differences in mass (in MeV/$c^2$) and in total width (in MeV/$c^2$). $\mu$ is the mean value and $\sigma$ is the standard deviation of a Gaussian distribution, the errors are statistical.

| Parameters          | $\mu$  | $\sigma$ |
|---------------------|--------|----------|
| $\Delta M(Y(4360))$ | $0.3 \pm 0.3$ | $2.6 \pm 0.3$ |
| $\Delta \Gamma_{\text{tot}}(Y(4360))$ | $0.2 \pm 0.6$ | $4.4 \pm 0.5$ |
| $\Delta M(Y(4660))$ | $-0.8 \pm 0.4$ | $3.0 \pm 0.5$ |
| $\Delta \Gamma_{\text{tot}}(Y(4660))$ | $-1.0 \pm 0.7$ | $5.2 \pm 0.6$ |

that the correlations between the parameters are neglected and we do not try to quantize the uncertainties of the $B \cdot \Gamma_{e^+e^-}$ measurements here to simplify the error estimation procedure.

The other sources of the systematic errors on the mass and width measurements are not very different from those listed by the Belle experiment \cite{9}, since it is mainly due to the parametrization of the resonance. Taking directly the error from Belle and adding in quadrature with the uncertainty due to the binning, one gets the total systematic errors of 9 MeV/$c^2$, 11 MeV/$c^2$, 6 MeV/$c^2$, and 6 MeV/$c^2$, for $M(Y(4360))$, $\Gamma_{\text{tot}}(Y(4360))$, $M(Y(4660))$, and $\Gamma_{\text{tot}}(Y(4660))$, respectively.
V. THE $Y(4260)$

The $Y(4260)$ was observed in $\pi^+\pi^- J/\psi$ mode \[2, 3\], and its production in $\pi^+\pi^- \psi(2S)$ is not forbidden by any selection rule, except that the phase space is a bit small due to large $\psi(2S)$ mass. We add the $Y(4260)$ amplitude in Eq. 2 coherently with mass $(4247 \text{ MeV}/c^2)$ and width $(108 \text{ MeV}/c^2)$ fixed to the Belle measurement \[3\] to measure its production rate.

Figure 4 shows the fit result with three resonances, the change of $-2 \ln \mathcal{L}$ is 5.2 for two more free parameters, this corresponds to a statistical significance of the $Y(4260)$ of $1.8 \sigma$. That is, the production of the $Y(4260)$ in $\pi^+\pi^- \psi(2S)$ is not significant. There are two pairs of solutions for $B(Y(4260) \rightarrow \pi^+\pi^- \psi(2S)) \cdot \Gamma_{e^+e^-}$, one pair interferes with the $Y(4360)$ constructively, with $B(Y(4260) \rightarrow \pi^+\pi^- \psi(2S)) \cdot \Gamma_{e^+e^-} = 1.4^{+1.6}_{-0.9} \text{ eV}/c^2$, and we set the upper limit as $4.3 \text{ eV}/c^2$ at the 90% confidence level; the other pair interferes with the $Y(4360)$ destructively, and $B(Y(4260) \rightarrow \pi^+\pi^- \psi(2S)) \cdot \Gamma_{e^+e^-} = 7.4^{+2.1}_{-1.7} \text{ eV}/c^2$.

VI. SUMMARY

Based on combined Belle and BaBar data on $e^+e^- \rightarrow \pi^+\pi^- \psi(2S)$, we perform a maximum likelihood fit to get the resonant parameters of the $Y(4360)$ and $Y(4660)$. We obtain $M(Y(4360)) = 4355^{+9}_{-10} \pm 9 \text{ MeV}/c^2$, $\Gamma_{\text{tot}}(Y(4360)) = 103^{+17}_{-15} \pm 11 \text{ MeV}/c^2$, $M(Y(4660)) = 4661^{+8}_{-9} \pm 6 \text{ MeV}/c^2$, and $\Gamma_{\text{tot}}(Y(4660)) = 42^{+12}_{-13} \pm 6 \text{ MeV}/c^2$. These results give the best measurement of the $Y(4360)$ and $Y(4660)$.

There is a faint evidence of $Y(4260) \rightarrow \pi^+\pi^- \psi(2S)$, with $B(Y(4260) \rightarrow \pi^+\pi^- \psi(2S)) \cdot \Gamma_{e^+e^-} < 4.3 \text{ eV}/c^2$ at the 90% confidence level, or $B(Y(4260) \rightarrow \pi^+\pi^- \psi(2S)) \cdot \Gamma_{e^+e^-} = 7.4^{+2.1}_{-1.7} \text{ eV}/c^2$, depending on the interference between the $Y(4260)$ and $Y(4360)$. These numbers should be compared with the couplings of the $Y(4260)$ to $\pi^+\pi^- J/\psi$ in Ref. \[3\] and to $K^+K^- J/\psi$ in Ref. \[13\].
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