Horizon Entropy

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Abstract

Although the laws of thermodynamics are well established for black hole horizons, much less has been said in the literature to support the extension of these laws to more general settings such as an asymptotic de Sitter horizon or a Rindler horizon (the event horizon of an asymptotic uniformly accelerated observer). In the present paper we review the results that have been previously established and argue that the laws of black hole thermodynamics, as well as their underlying statistical mechanical content, extend quite generally to what we call here “causal horizons”. The root of this generalization is the local notion of horizon entropy density.

1 INTRODUCTION

Black hole thermodynamics was born a little over three decades ago from two simple questions: “What is the most efficient way to extract the rotational energy of a black hole?” and “Can the second law of thermodynamics be violated by dumping entropy across a black hole horizon?” The answer to the first question is that the most efficient energy extraction occurs when the horizon area remains constant, and that moreover if the area increases the process is classically irreversible\cite{1,2}. More generally, Hawking’s area theorem\cite{3} showed that the horizon area cannot decrease provided that only positive energy can flow across the horizon.

Jacob Bekenstein’s proposed answer\cite{4} to the second question was that it is indeed not possible to violate the second law, provided one attributes to the horizon itself an entropy proportional to its area. The analogy between horizon area and entropy was already motivated by the energy extraction results and the area theorem. Bekenstein added information theoretic arguments, interpreting the horizon area as a measure of the missing information for an outside observer, and deducing that the entropy must be a constant of order unity times the area in Planck units. He thus introduced the generalized second law (GSL) of thermodynamics, which states that the sum of the black hole entropy plus the matter entropy outside the black hole “never decreases”\cite{5}.

The GSL took on a more precise form after Hawking’s discovery\cite{6} that a black hole of surface gravity $\kappa$ radiates at a temperature $T_H = \kappa/2\pi$.\footnote{We use in this paper Planck units, with $\hbar = c = G = k = 1$.} When taken together with the zeroth law\cite{7}, which states that the surface gravity is constant over the horizon, and the first law\cite{7} for isolated black holes,

$$\Delta M = \frac{\kappa}{8\pi} \Delta A + \Omega \Delta J$$

(1)

(where $M$ and $J$ are the mass and angular momentum of the black hole and $\Omega$ is the angular velocity), this yielded the coefficient $(1/4)$ of proportionality between horizon
area and entropy (as well as attributing a truly thermal character to a black hole). The GSL is then the statement that no physical process can lead to a decrease of the sum of the entropy outside a horizon $S_{\text{outside}}$ and the Bekenstein-Hawking entropy $S_{\text{BH}} = A/4$ of the event horizon,

$$\Delta(S_{\text{outside}} + A/4) \geq 0.$$ (2)

The GSL was initially postulated with a positive energy condition in mind, in which case the horizon area of a black hole can only increase. However, it turns out that the GSL holds even during quantum evaporation of the black hole via Hawking radiation\[5, 9], when a negative energy flux across the horizon produces a decrease of area. In that case, the reason for the validity of Eq. (2) is that Hawking radiation carries enough entropy so that the increase of $S_{\text{outside}}$ outweighs the decrease of the black hole entropy. Convincing arguments have been given which establish more generally that, in any quasi-stationary process, the GSL will hold provided the ordinary second law holds for matter\[10, 11, 12]. Taken together, these results imply that black holes are thermodynamic systems.

The generalization of thermodynamics from black hole horizons to de Sitter horizons was initiated by Gibbons and Hawking\[13, 14] shortly after the discovery of the Hawking effect, and is by now quite well accepted, despite the observer-dependence of de Sitter horizons. Nevertheless the corresponding generalization to acceleration horizons is less well accepted, especially with regard to the entropy they carry, although they bear a close resemblance to de Sitter horizons. Bekenstein’s original information theoretic concept of the nature of black hole entropy certainly applies to all horizons, even highly non-stationary ones, as does the area increase theorem (see below). The ultimate significance of black hole thermodynamics hangs on this issue of generality, and we thus consider it an ideal topic for our contribution to this festschrift for Jacob.

In this article we take a broad look, both historical and fresh, at the question how general are the laws of horizon thermodynamics. One of the key questions is whether the local notion of horizon entropy density is a valid concept, or whether something essentially global is involved. At the classical level this question concerns the interplay of horizon area and the gravitational field equations. At the quantum level it concerns the role of horizon area in determining the density of states factor in partition functions, in irreversible processes, and in transition rates. The latter statistical role is the more fundamental, since it is related to the underlying quantum states, and it gives rise to the thermodynamic laws.

Using both the classical and quantum viewpoints, we shall argue that local entropy density of a horizon is indeed a valid concept, and that consequently the laws of black hole thermodynamics extend quite generally to any causal horizon. At the statistical level a causal horizon should thus be conceived as an ensemble whose mean properties are the well-known geometrical ones. This conception was pioneered by Bekenstein, who emphasized from the outset the physical relevance of random exchanges which might some times lead to a decrease of the area\[15]. From this vantage point, the GSL emerges as “a statistical law which becomes overwhelmingly probable in the limit of a macroscopic system”\[5].

After all this, do we understand horizon entropy?\[16] We close with a discussion of the question what does horizon entropy count, a question whose answer we believe, despite some glimpses, remains shrouded in mystery.
2 CAUSAL HORIZONS

By a causal horizon we mean here the boundary of the past of any timelike curve \( \lambda \) of infinite proper length in the future direction. This definition was introduced by Gibbons and Hawking\cite{13} under the name “event horizon”, however we shall call it a “causal horizon” to emphasize its generality, since today “event horizon” refers almost exclusively to the case of a black hole event horizon. More generally, the boundary of the past of any set of events is a kind of causal horizon. This more local notion has not been used as much in the context of horizon thermodynamics as that associated with an infinite future. Nevertheless it does appear to have some thermodynamic or statistical role to play, as evidenced for example in \cite{17, 18}. Four examples are given in Fig. 1.

![Carter-Penrose diagrams of causal horizons in (conformal compactifications of) various spherically symmetric spacetimes. Each generic point in one of the diagrams represents a 2-sphere of symmetry, and the radial null lines make 45 degree angles with the vertical. The points on the left hand edge of each diagram lie at the origin of spherical symmetry, hence they represent just a single point. The points on the right hand edge of the de Sitter diagram lie at the antipode of the 3-sphere, hence they also represent a single point. Future null infinity is denoted by \( I^+ \).

For a black hole in asymptotically flat spacetime, the event horizon can be defined as the boundary of the past of all of \( I^+ \), however an equivalent definition is the boundary of the past of any timelike curve that goes to future timelike infinity \( i^+ \). Equivalently, the event horizon is the boundary of the past of \( i^+ \) itself. Therefore, the black hole horizon is defined with reference to the intrinsic asymptotic structure of the spacetime,
without referring to a particular class of observers.

An asymptotic de Sitter horizon is defined in a spacetime that is asymptotically de Sitter in the future. It is defined by the boundary of the past of a timelike worldline reaching a point $p$ at $I^+$ (which is spacelike in asymptotically de Sitter spacetime), or equivalently the boundary of the past of $p$. Since there are many different points at $I^+$ there are many inequivalent asymptotic de Sitter horizons. An asymptotic de Sitter horizon is thus said to be “observer-dependent”, in contrast to a black hole event horizon.

Our other two examples are observer-dependent in the same sense. An asymptotic anti-de Sitter horizon, defined in a spacetime that is asymptotically anti-de Sitter at spatial infinity, is the boundary of the past of a timelike worldline reaching a point $p$ at $I^+$ (which is here timelike,) or equivalently the boundary of the past of $p$. An asymptotic Rindler horizon (ARH), defined in a spacetime that is asymptotically flat, is the boundary of the past of an accelerated worldline that goes to a point $p$ on $I^+$ (which is here null,) or equivalently the boundary of the past of $p$.

Let us bring out the nature of an ARH in more detail. The boundary of the past of $p$ is a cone in compactified Minkowski spacetime, however it is actually a plane in Minkowski space. To show this we use coordinates $(u, v, x, y)$ for which the line element takes the form $ds^2 = du dv - dx^2 - dy^2$. The point $p$ could be the endpoint $v \to \infty$ of the light ray $(u_0, v, x_0, y_0)$. The points $(u, v, x, y)$ to the past of $(u_0, v_0, x_0, y_0)$ are those with $u < u_0$ and $(x_0 - x)^2 + (y_0 - y)^2 < (u_0 - u)(v_0 - v)$. In the limit $v_0 \to \infty$ this includes all points with $u < u_0$. The ARH is the boundary of this region, which is the null plane $u = u_0$. Thus in Minkowski spacetime an ARH is just what is usually meant by a “Rindler horizon”. Note that the location of this horizon is independent of the transverse coordinates $x_0$ and $y_0$ of the light ray. It is nevertheless observer dependent since it depends on $u_0$. Note also that the horizon has infinite cross sectional area. An asymptotically flat spacetime approaches Minkowski space near $I^+$, so an ARH in general will asymptotically approach a Rindler horizon.

A causal horizon whose generators coincide with some Killing flow is a Killing horizon, i.e. a null hypersurface generated by a Killing flow. A bifurcate Killing horizon (in four dimensions) is a pair of Killing horizons which intersect in a particular two dimensional spacelike cross section—called the bifurcation surface—on which the Killing vector vanishes. Examples of these occur in Minkowski, de Sitter, Anti de Sitter, and Schwarzschild spacetimes. In the Minkowski case the pair of planes $u = 0$ and $v = 0$ (in the above mentioned coordinates) comprise a bifurcate Killing horizon for which the Killing field is the hyperbolic rotation generator $-u \partial_u + v \partial_v$, and the bifurcation surface is the $x$-$y$ plane at $u = v = 0$. This is the bifurcate Rindler or acceleration horizon, and the Killing field generating it is called the boost Killing field. The “other sheet” of the Schwarzschild horizon is called the past Killing field. The “other sheet” of the Schwarzschild horizon is called the past horizon since it is defined by the boundary of the future of past null infinity (in the same asymptotic region whose future defines the future horizon). Similarly in Minkowski spacetime for example the other sheet is the past acceleration horizon. It should be emphasized that the various examples of Killing horizons are all indistinguishable in a neighborhood of the bifurcation surface smaller than the curvature scale, hence the acceleration horizon provides a universal template for all of them.

While the notion of bifurcate Killing horizon is a global one, it can be localized in two ways. Obviously, one can focus on a neighborhood of the bifurcation surface or even just part of it. Somewhat less obviously, a neighborhood of a piece of a single Killing horizon can be extended to a neighborhood of a bifurcate Killing horizon including the bifurcation surface, provided the surface gravity is constant and nonvanishing on the horizon[19]. The constancy of the surface gravity—the zeroth law of horizon
thermodynamics—can be derived either using the Einstein equation and the dominant energy condition[7] or from the assumption that a neighborhood of the horizon is static or stationary-axisymmetric with a \(t-\phi\) reflection isometry[20, 21].

In [13] Gibbons and Hawking argue that in empty spacetimes satisfying the Einstein equations with a cosmological constant (or with electromagnetic fields) stationary causal horizons are necessarily stationary axisymmetric Killing horizons. Their argument follows the same lines as those that had been given previously for the special case of asymptotically flat black hole horizons. (Although not stated explicitly in [13], it seems the horizon cross sections are assumed to be compact in these arguments.) They also point out that the proof in Ref. [7] of the zeroth law for such Killing horizons extends immediately to the case of a cosmological constant. Alternatively, the zeroth law follows without the use of field equations or energy conditions as discussed above.

3 THE SECOND LAW OF CAUSAL HORIZONS

Gibbons and Hawking[13] quote a “second law of event horizons” (whose proof is referred to a forthcoming publication that seems never to have appeared) which states that the area of causal horizons cannot decrease provided that (i) what the observer can see at late times can be predicted from a spacelike surface (a cosmic censorship assumption), and (ii) the energy-momentum tensor satisfies the strong energy condition. Similar theorems were quoted by Penrose[22] under both the strong and null energy conditions, and with several different formulations of the cosmic censorship assumption (though we have not been able to find their actual proofs in print.) It should be noticed that all these theorems concern the classical version of the second law in which only positive energy fluxes are considered.

A simple but not entirely satisfactory proof that causal horizons satisfy the classical area theorem can be given under the assumptions that (i) the null energy condition holds, and (ii) the null geodesics generating the horizon are future complete, i.e. have infinite affine length in the future. This proof, which is just Hawking’s original proof of the black hole area theorem[3], works by contradiction. Suppose the congruence of horizon generators were converging at some point \(p\). Then the null energy condition implies that this focusing cannot be reversed, and will therefore produce a caustic that would be reached at finite affine parameter along the generator to the future of \(p\). Such a caustic is in essence an intersection of infinitesimally separated null horizon generators, which is incompatible with the defining property of the horizon as the boundary of the past of an infinite timelike curve. Hence the expansion of the null congruence of horizon generators must be greater than or equal to zero everywhere. That is, the area of any bundle of horizon generators is non-decreasing.

What is not entirely satisfactory about the above proof is the assumption that the horizon generators are future complete. In the black hole case this assumption can be replaced by the cosmic censorship assumption that no singularities (to the future of a partial Cauchy surface) are visible from \(I^+\) [23] [24]. (To exploit this assumption it is necessary to use the device of deforming the horizon slightly outward to another surface containing points that are causally connected to \(I^+\).) For a general causal horizon similar proofs were long ago reported [13] [22] (but not published) under different formulations of the cosmic censorship assumption. Recently, the area theorem for general “future horizons” has been reexamined and proved by Chruściel et al. [25], without assuming piecewise smoothness of the horizon.

Besides such general results, area theorems for various special cases of horizons other than standard black hole horizons have been established by several authors. A theorem was proved by Davies[26] for the case of asymptotically de Sitter causal horizons.
in Friedmann cosmologies with a cosmological constant, under the assumption that the fluid satisfies the dominant energy condition. An area increase theorem for black holes in asymptotically de Sitter spacetime was proved by Hayward et al. and Shiromizu et al. An explicit proof of the area increase theorem for the cosmological event horizon (defined as the past Cauchy horizon of $I^+$) in asymptotically de Sitter spacetimes containing a black hole was given by Maeda et al.

4 THE FIRST LAW OF CAUSAL HORIZONS

In this section we consider several versions of the first law of black hole thermodynamics and discuss how they extend to all causal horizons. We close the section with the argument that the ultra-local version of this first law implies, in a precise sense, the Einstein equation as a corresponding equation of state.

Stationary comparison version

Having shown that causal horizons in stationary spacetimes satisfy the zeroth and second laws, Gibbons and Hawking went on to derive a version of the first law. In particular, for a pair of infinitesimally close spacetimes with both cosmological and black hole horizons, minus the variation of the integral of Killing energy density on a spatial surface bounded by the horizons is given by

$$-\delta E_{\text{Killing}} = \left(\kappa_C/8\pi\right)\delta A_C + \left(\kappa_H/8\pi\right)\delta A_H + \Omega_H \delta J_H.$$  

(3)

The Killing vector $\xi^a$ employed here is the one that generates the cosmological horizon. The Killing energy density is defined by $T_{ab}\xi^a n^b$, where $n^b$ is the unit normal to the spatial surface, $\kappa_C$, $\kappa_H$, $A_C$ and $A_H$ are the surface gravities and areas of the cosmological and black hole horizons, $J_H$ is the angular momentum of the black hole (defined by the integral of the curl of the rotational Killing field over the horizon), and $\Omega_H$ is the angular velocity of the black hole relative to the cosmological horizon. No angular momentum term appears for the cosmological horizon since the Killing vector $\xi^a$ generates that horizon. Note that although this Killing field has no natural normalization, that presents no difficulty since each term in Eq. scales with the undetermined normalization in the same manner.

This version of the first law is a generalization of Eq. 1, which arises when taking the asymptotically flat space-time limit $\kappa_C \to 0$. The generalization involves not the variation of the ADM mass—which is not defined in a de Sitter background—but instead the variation of the matter Killing energy. It can thus be used to describe variations in the asymptotically flat case between two stationary configurations at fixed ADM energy, in which the matter energy is redistributed, with some going into the black hole. Formulated this way, the first law is more local, since no reference to what is happening at infinity is required.

Physical process version

Both of the previously mentioned versions of the first law Eq. 1 and Eq. 3 involve comparisons of two stationary solutions to the field equations. A yet more local version of the first law emerges when one enquires into the dynamical process by which the horizon area adjusts to a small flow of energy across the horizon. This relation, which Wald called the “physical process version” of the first law in the case of black hole
horizons, was first established in 31 (see also 32 for a comprehensive review.) The physical process version of the first law states that

$$\delta S = \delta E_H / T_H,$$

where $$\delta S = \delta A / 4$$ is one fourth the horizon area change, $$\delta E_H = \int_H T_{ab} \xi^a d\Sigma^b$$ is the flux of “energy” across the horizon, defined with respect to the horizon generating Killing field $$\xi^a$$ of the stationary background spacetime which is being perturbed, and $$T_H$$ is $$1/2\pi$$ times the surface gravity $$\kappa = |\nabla_a \xi|$$ of the horizon generating Killing field.

To exhibit its local nature, let us review the derivation of the physical process first law. We first discuss the approach using affine parametrization and then discuss the approach using Killing parametrization. The expansion $$\theta$$ of the null congruence of horizon generators measures the fractional rate of increase of the cross-sectional area element with respect to an affine parameter $$\lambda$$. Hence the change in the area of a bundle of generators over a finite range of affine parameter is

$$\Delta A = \int_B \theta d^2A d\lambda,$$

where the integral is over a patch $$B$$ of the horizon consisting of the finite range of the chosen generators. This integral can be related to the energy flux using the identity $$\theta = d(\lambda \theta)/d\lambda - \lambda d\theta/d\lambda$$, together with the Raychaudhuri equation for $$d\theta/d\lambda$$ and the Einstein equation. The Raychaudhuri equation is

$$\frac{d\theta}{d\lambda} = -\frac{1}{2} \theta^2 - \frac{1}{2} \sigma^2 - R_{ab} k^a k^b,$$

where $$\sigma^2 = \sigma_{ab} \sigma^a \sigma^b$$ is the squared shear of the congruence and $$k^a$$ is the affine tangent vector to the congruence, and the Einstein equation reads

$$R_{ab} - \frac{1}{2} R g_{ab} = 8\pi T_{ab}.$$

Eq. (6) thus yields

$$\Delta A = [\lambda \theta A]_1^2 + \int_B \frac{1}{2} (\theta^2 + \sigma^2) d^2A d\lambda, + \int_B 8\pi T_{ab} \lambda k^a d\Sigma^b.$$

Eq. (8) is an identity satisfied by any null congruence in a spacetime satisfying the Einstein equation. It becomes the first law when we specialize to a situation involving a small change of a stationary background spacetime, with the congruence corresponding to a small deformation of the Killing horizon. We now see how this happens by examining each term in the equation.

If the small deformation is caused by some matter stress tensor at order $$\epsilon$$, then the metric perturbation and therefore $$\theta$$ and $$\sigma$$ will be of order $$\epsilon$$. Hence the term $$\theta^2$$ can be neglected. 2

If the upper limit of integration is chosen in a region where the horizon is again stationary, then the net area change of a given bundle of generators must be finite, hence $$\theta$$ must vanish faster than $$1/\lambda$$, so the term $$\lambda \theta A$$ vanishes. To understand the behavior of the lower limit it is helpful to relate the affine parameter $$\lambda$$ to the Killing

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2 Actually, neglect of this term is not justified if the generators develop caustics or if the integration region extends to infinity, as in the case of an asymptotic Rindler horizon, as will be discussed below.
parameter $v$ of the background stationary spacetime. In general the relation between affine and Killing parameters on a Killing horizon is

$$\lambda = ae^{\kappa v} + b,$$

where $a$ and $b$ are arbitrary constants. If we exploit the shift freedom of the affine parameter to set $b = 0$, then as the Killing parameter $v$ goes into the past, the affine parameter $\lambda$ goes to zero exponentially fast. The term $(\lambda \theta A)_1$ is already first order in $\epsilon$ from $\theta$, hence to first order we can use this relation between the background values of $\lambda$ and $v$ to conclude that this term is exponentially suppressed provided the stretch of (background) Killing time over which the first law is being applied is long compared with $\kappa^{-1}$. This temporal restriction of the applicability of the first law corresponds to an “equilibration time” (albeit backwards in time). Indeed, the expansion defined with respect to Killing time also vanishes exponentially.

The last step is to relate $\lambda k^a$ in the energy flux term to the Killing vector $\xi^a$. Since the energy is a first order perturbation, this relation can be evaluated in the strictly stationary background on the Killing horizon. We thus have $\lambda k^a = \lambda (dv/d\lambda) \xi^a$, where $v$ is the Killing parameter. From the relation (9) with $b = 0$ we have $\lambda (dv/d\lambda) = \kappa^{-1}$, hence

$$\lambda k^a = \kappa^{-1} \xi^a.$$  

(10)

When substituted into Eq. (8) this yields the physical process form of the first law Eq. (4) when the patch $B$ is taken to be a piece of the horizon bounded in the past and the future by two complete cross-sections.

**Adiabatic version**

In the preceding formulation of the first law for physical processes it was assumed only that the neighborhood of the horizon is nearly stationary, so that the perturbation is small. If one assumes further that it is adiabatic, then an even more local form of the first law applies. As pointed out above, the horizon equilibration rate is given by $\kappa$, hence adiabatic means slow with respect to that scale. To extract the adiabatic limit, it is useful to re-express the Raychaudhuri equation (6) in terms of derivatives with respect to the Killing parameter $v$ rather than the affine parameter. This produces the equation

$$\hat{\theta} = \kappa \hat{\theta} - \frac{1}{7} \hat{\sigma}^2 - \frac{1}{2} \hat{\sigma}^2 - R_{ab} \dot{k}^a \dot{k}^b,$$

(11)

where $\hat{\theta} = (d\lambda/dv)\theta$, $\sigma = (d\lambda/dv)\sigma$, $\dot{k}^a = (d\lambda/dv)k^a$ and $\kappa = (d^2\lambda/dv^2)/(d\lambda/dv)$ (cf. Eq. (9)). For a slow perturbation we have $\hat{\theta} \ll \kappa$, $\hat{\sigma} \ll \kappa$, and $d\hat{\theta}/dv \ll \kappa \hat{\theta}$, so both the left hand side and the second order terms in the expansion and shear can be dropped, which yields

$$\hat{\theta} \approx \kappa^{-1} R_{ab} \dot{k}^a \dot{k}^b.$$  

(12)

When this is substituted in Eq. (5) and the Einstein equation is used, we see that, when the evolution is adiabatic, the first law Eq. (4) holds over any patch and over any stretch of time of the slowly evolving horizon.

**Extension to all causal horizons**

While the preceding derivations apply for black hole horizons, they are localized on the horizon and make no reference to spatial infinity. Therefore it is tempting to apply them to all causal horizons. Possible obstructions concern the normalization of the Killing
vector, the possibly infinite horizon area, and convergence of the upper limit. Let us consider these in turn.

First, the energy flux $\delta E_H$ and the temperature $T_H$ scale in the same way under a change of normalization of the Killing field. Hence, as in Eq. (3), this normalization cancels out so no choice is required. (In the stationary black hole case the horizon generating Killing field is a linear combination of a time translation and a rotation, $\xi^a = (\partial/\partial t)^a + \Omega_H (\partial/\partial \phi)^a$. The time translation Killing field $(\partial/\partial t)^a$ is normalized at infinity, so the change of “energy” in Eq. (4) is expressed as $\delta M - \Omega_H \delta J$.)

Second, the derivation of the first law can be applied to a bundle of generators forming a finite subset of the horizon, so no infinite area need be confronted. Moreover, the change of total area $\delta A$ can be finite, even if the total area itself is infinite. This change can be defined by considering first a finite bundle of horizon generators whose cross-sectional area is then taken to infinity.

Third, convergence of the upper limit $(\lambda \theta A)^2$ of Eq. (8) can be analyzed in each bundle of generators. As explained above this upper limit vanishes if and only if the cross sectional area of any finite bundle is finite at infinity. This corresponds to the condition that the horizon is in equilibrium in the asymptotic future. An ARH satisfies this condition, since it is just a Rindler horizon at infinity. (We have not checked whether the horizon of a point at $I^+$ in AdS is asymptotically stationary.)

The physical process first law thus applies to all situations in which a causal horizon goes through an evolution that is nearly stationary and settles down in the future.

Asymptotic Rindler horizons

In the limit that the cosmological constant goes to zero, the stationary comparison version of the first law for cosmological horizons becomes the first law for Rindler horizons. To illustrate this, we suppose a rest mass $\delta m$ follows a Killing orbit on which the norm of the Killing field is $\xi$. The Killing energy of this mass is $\xi \delta m$, so the first law implies that the disappearance of the mass increases the horizon area by $\delta A_C = (8\pi \xi / \kappa_C) \delta m$. In de Sitter space we have $\kappa_C / \xi = \sqrt{a^2 + \Lambda}$, where $a$ is the proper acceleration of the Killing orbit. If the mass sits at the center where $a = 0$, then in the limit as $\Lambda$ goes to zero the area change diverges. If instead $a \neq 0$ then the ratio $\kappa_C / \xi$ approaches $a$ and the area change is finite. It is interesting to note that, when $\Lambda$ is nonzero, $\kappa_C / 2\pi \xi$ is equal to the local Unruh/de Sitter temperature $T_{local}$ that would be experienced by an accelerated mass in the Euclidean vacuum state $[^{34}]$. (As $\Lambda$ goes to zero this becomes just the Unruh temperature for the accelerated mass and as $a$ goes to zero it becomes just the de Sitter temperature.) The disappearance of the mass thus increases the horizon entropy by an amount equal to $\delta m / T_{local}$ for all values of $\Lambda$ and $a$.

Unlike the stationary comparison version, the physical process version of the first law does not hold for an asymptotic Rindler horizon since the ‘nearly stationary’ condition cannot be satisfied. We now explain the reason for this in two ways, first from the perspective of the ARH itself, and then by describing the ARH as the infinite mass limit of a Schwarzschild horizon.

Suppose a planet in free fall wanders across the ARH. Although the boost energy flux of the planet is finite, it can be shown that the net area change caused by the passage of the planet is infinite. This is essentially because every null geodesic on the
ARH—no matter how far from the planet in the transverse direction—is eventually focused backwards in time to a focal point on the line behind the planet, opposite to the direction of the endpoint of the ARH. That is, all the horizon generators originate on the line behind the planet. Hence an infinite area is added to the ARH so evidently the first law (4) is not satisfied.

The reason the physical process first law is not satisfied is that the process is not nearly stationary. The Raychaudhuri equation (6) of course applies, but the terms neglected to arrive at the first law cannot be neglected. For example this is clear from the fact that there are focal points, where the expansion goes to infinity and hence $\theta^2$ cannot be neglected. Moreover, since all generators originate on the line of caustics, the ultimate area of the annulus of generators between asymptotic cylindrical radii $\rho$ and $\rho + \Delta\rho$ must grow from zero without any energy having crossed the congruence. According to (5) all of this area must come from integrating the squared expansion and shear. The shear will dominate (since it is the shear that generates the expansion in this case[31]). For any bundle of generators this produces a finite final area, however when integrated over an infinite cross section and infinite affine parameter range it produces an infinite total area change. Thus, even though the squared shear is $O(\epsilon^2)$ in the perturbation, it has an infinite net effect.

Further insight is offered by viewing the ARH as the infinite mass limit of a Schwarzschild horizon of mass $M$. The planet crossing the ARH can be modeled by a planet that falls in freely from infinity and crosses the Schwarzschild horizon. This situation has been analyzed in [35], where it was shown that the process is nearly stationary for the purposes of the first law only if $R \gg \sqrt{mM}$, where $R$ is the radius of the planet and $m$ is the mass. In the limit of $M$ going to infinity, this requires that the planet have an infinite radius. Hence, for any finite sized planet crossing, an ARH will not satisfy the physical process version of the first law. The fact that the area increase is infinite can also be understood using the infinite mass limit, since the planet comes in from infinity with finite Killing energy $m$ and the surface gravity $\kappa = 1/4M$ goes to zero. Hence, if the physical process form of the first law were to apply, the area change would be given by $\delta A \sim (8\pi/\kappa)m = 32\pi mM$, which diverges as $M$ goes to infinity. Failure of the nearly stationary condition would only add to this divergence.

It should be emphasized that although the nearly stationary physical process first law (4) does not hold, the expression (5) for the horizon area change remains valid. This should be considered an extension of the first law to non-equilibrium thermodynamics, as discussed at the classical level in [31, 32] and at the quantum statistical level in [36, 37]. It would be interesting to pursue this line of thought further.

**Local Rindler horizons and the Einstein equation of state**

We have just seen how the Einstein equation enforces the first law for near-stationary transitions of causal horizons. To conclude this section let us show how this logic can be turned around to derive the Einstein equation as an “equation of state” of spacetime[17]. To render the argument entirely local we make use of the notion of a

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4A similar but simpler example of a non-nearly stationary process is the formation of a black hole by an infalling thin spherical shell of mass $M$. The horizon grows in area from the focal point in the center where it originates, and this area growth is not properly accounted for by the first law. However, if a second such shell with mass $\Delta M \ll M$ follows, the transition does obey the first law.

5This condition can also be expressed by saying that the surface area of the planet is much larger than the increase of the horizon area according to the first law.

6As the planet falls, a small fraction ($\sim 0.01m/M$) of the rest mass is radiated as gravitational waves, hence the Killing energy crossing the horizon approaches $m$ as $m/M$ goes to zero.
“local Rindler horizon” (LRH). An LRH is defined as the boundary of the past (on one side) of a spacelike two-surface element \( S \) through a point \( p \), with \( S \) adjusted so that the generators of the LRH have vanishing expansion and shear at \( p \). Thus the LRH reaches equilibrium at \( p \). The area change of a small bundle of generators approaching the “equilibrium point” \( p \) is given by Eq. (5), with
\[
\theta \approx -\lambda R_{ab}k^a k^b,
\]
where the affine parameter \( \lambda \) has been set to zero at \( p \), and \( k^a \) is the affinely parametrized horizon generator. If we restrict attention to a sufficiently small neighborhood, then the effect of any classical energy momentum tensor makes only a small perturbation of the local Minkowski space, hence the near-stationary analysis holds. According to Eq. (10) we can replace \(-\lambda k^a\) by \(\kappa^{-1}\xi^a\), where \(\xi^a\) is the boost Killing field on the LRH. The first law Eq. (4) will hold on the small bundle of generators, with \(T_H\) equal to the Unruh temperature associated with \(\xi^a\), if and only if at \( p \) we have
\[
R_{ab}k^a k^b = 8\pi T_{ab}k^a k^b.
\]
If this is to hold for all LRH’s through \( p \) then it must hold for all \( k^a \), which implies that \(G_{ab} = 8\pi T_{ab} + f_{g_{ab}}\), where \(f\) is an arbitrary function. The Bianchi identity and the conservation of \(T_{ab}\) then imply that \(f\) is a constant, which can be identified with an undetermined cosmological constant.

It is difficult to resist concluding from this argument that the horizon entropy density proportional to area is a more primitive concept than the classical Einstein equation, which appears as a thermodynamic consequence of the interplay of entropy and causality. From the point of view of the quantum action principle this conclusion is not so surprising. The action gives rise to the entropy at the quantum statistical level, and to the field equations at the level of the classical mean field. It is to the former, deeper level of description that we now turn.

5 STATISTICAL MECHANICS OF CAUSAL HORIZONS

In this section we discuss, at the quantum statistical level, the extension of the notion of black hole Boltzmann entropy to stationary states of all causal horizons, i.e. to Killing horizons. The starting point for this generalization is the work of Gibbons and Hawking[14] which showed that semiclassical evaluation of the partition function for the gravitational field yields \(A/4\) for the entropy of both black holes and de Sitter space. Here we examine the role of horizon entropy in determining the rates for dynamic processes, and we argue that this role extends in particular to acceleration horizons. We also briefly consider a few approaches to explicit “state counting” of horizon degrees of freedom.

Pair creation

A first indication in favor of the extension of the statistical notion of black hole entropy to all causal horizons arises from the pair creation probability for magnetically charged black holes in a magnetic field[38]. This probability is given as a product of two exponentials:
\[
P(q, M; B) \propto e^{A/4} \times e^{\Delta A_{\text{accel}}/4}.
\]
The second factor contains \(\Delta A_{\text{accel}}(q, M, B)\): the change of area of the acceleration horizon[35] which is associated with the creation of the black hole pair. This term
persists even when no black holes but only monopoles are created, and it reproduces the well known instanton contribution $\exp(-\pi M^2/qB)$ in the test field approximation $G \to 0$ where $G$ is Newton’s constant. The first factor tells us that the probability is enhanced over that of monopoles by $\exp(A/4)$, where $A(q,M)$ is the horizon area of a black hole of mass $M$ and magnetic charge $q$.7

This is consistent with the usual statistical interpretation of $A/4$ as the logarithm of the number of black hole states. From Eq. (15) we see that the two area changes determine the probability in the same manner. It is therefore compelling to apply the same statistical interpretation to both of them. Perhaps the strongest indication in favour of treating both areas on the same footing arises from the fact that an accelerated black hole will be in a statistical ensemble since it will experience Unruh radiation. The random exchanges of thermal photons between the acceleration horizon and the hole will bring the whole system into equilibrium. It is to be noticed that the equilibrium situation is determined by Eq. (15) and corresponds to the maximum probability where $\partial_M(A + \Delta A_{acc}) = 0$, i.e. when Hawking and Unruh temperatures agree. In this $S_{system} = (A + \Delta A_{acc})/4$ does act as the entropy of the whole system. The random energy exchanges will also spread both the black hole mass and the area of the acceleration horizon around their mean values. The spread in the mass of the hole is related to the heat capacity and it is given, as usual, by $(\partial^2 M/S_{system})_{equal}^{-1/2}$. We emphasize that the conclusion that the acceleration horizon is in an ensemble requires no assumption other than that the creation probability Eq. (15) also delivers the equilibrium distribution. The considerations of the following subsection support this natural assumption.

Consideration of pair creation of black holes in de Sitter space yields another probability given as the exponential of two area changes. In this case, people would likely agree that both terms possess a statistical interpretation. Despite the similarity between the two cases, Hawking and Horowitz nevertheless assert that the acceleration horizon area should not be interpreted as counting an entropy. They give two reasons. The first is that the usual evaluation of the entropy from the partition function cannot be performed since the period of the Euclidean section is fixed and cannot be varied. The second is that the acceleration horizon is observer dependent and hence the information behind the horizon “can be recovered by observers who simply stop accelerating”. Restated, they assert the unitarity of the evolution on a foliation cutting across the acceleration horizon. The last two statements can be made for a black hole horizon (or a de Sitter horizon) however, so they do not seem to preclude the association of area with entropy for the acceleration horizon.

**Transition rates for systems in contact with a Killing horizon**

Further evidence for the statistical entropy of causal horizons is obtained from where it is explained why the transition rates for systems in contact with a Killing horizon are governed by changes in the horizon area. The analysis applies to black hole horizons with the evaporation process itself, to acceleration horizons with the Unruh effect experienced by accelerated systems, and to cosmological horizons with the processes related to the de Sitter temperature. In all cases, when taking into account the gravitational back-reaction, the excitation rates are governed by $\exp(\delta A/4)$ in place of the thermal Boltzmann factor $\exp(-\delta E_K/T_H)$ which is known to govern Hawking radiation for

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7One might have thought that it should be the total area of the two black holes appearing here, however that is not the case. The reason is that the pair carries the quantum numbers of the vacuum, so their quantum degrees of freedom are strictly correlated. Thus the degeneracy of states is that of a single black hole.
example. In these expressions, $\delta E_K$ is the change in Killing energy, $T_H$ the horizon temperature measured in the same units as $\delta E_K$, and $\delta A$ is the corresponding area change. In the test field approximation, $\delta A$ is related to the matter change by Eq. (3). Hence we recover the transition rate in a thermal state, as previously obtained in the case of Hawking radiation or transitions of an accelerated detector [43]. The novelty consists of taking into account the back-reaction of the spacetime geometry in the evaluation of individual transition amplitudes (like one does it with recoil effects [45]), and no longer in the mean as one does when using the semi-classical Einstein equations. As a byproduct, one notices that canonical distributions (governed by Hawking or Unruh temperature) are replaced by microcanonical distributions in which the heat capacity of the horizon is properly accounted for.

To understand how this replacement occurs, recall that the action must sometimes be supplemented by boundary terms in order to have a well-defined action principle, since the action should be stationary when varied within the class of metrics under consideration. Consider $S_h$, the action for matter+gravity in its hamiltonian form and applied to the class of metrics which belong to the “one black hole sector” [46], namely metrics which are asymptotically flat and which also possess an inner horizon. The total variation of $S_h$ reads

$$\delta S_h = \int d\Sigma_3 (\pi \delta g + p \delta q)_{11}^2 + \text{bulk terms} + \delta M (t_2 - t_1) - \frac{\delta A}{8\pi} (\Theta_2 - \Theta_1).$$  \hspace{1cm} (16)

The first terms give the initial and final momenta of gravity and matter respectively. The bulk terms vanish on-shell. The third term arises from the outer boundary. It is given by the variation of the ADM mass times the lapse of asymptotic proper time. The last term arises from the inner boundary and it is given by the change of the horizon area times the lapse of the hyperbolic angle $\Theta$. (On-shell and for stationary metrics, $d\Theta/dt = \kappa$ gives the surface gravity.) For the class of processes involving exchanges between a black hole and surrounding matter at fixed ADM energy $M$, the outer boundary term vanishes. Hence one should not add to $S_h$ the energy term $-\int dt M$. On the other hand, since the area then adjusts itself dynamically, it should not be fixed. Thus, one must get rid of the inner boundary term in Eq. (16) by working with the (Legendre transformed) exchange action

$$S_{\text{exchange}} = S_h + \frac{A}{8\pi} \Theta. \hspace{1cm} (17)$$

As noticed by Carlip and Teitelboim [46], quantization of this action yields for the matter + gravity wave function the unusual Schrödinger equation

$$i \frac{d}{d\Theta} \Psi = -\frac{\hat{A}}{8\pi} \Psi, \hspace{1cm} (18)$$

where $\hat{A}$ is the operator valued horizon area which acts on both the matter and gravity sectors. The system at fixed ADM energy thus evolves according to $\Theta$ and not the time at infinity. Eq. (18) is remarkable. It shows that the area itself is the operator-valued geometrical quantity that replaces the Hamiltonian in governing exchanges.

Two applications of Eq. (18) were considered in [42]. First is the thermalisation of a two level atom held at some fixed distance from a black hole. This type of situation also applies to the thermalisation of inertial systems in de Sitter space and leads to expressions that can be put in parallel to the static version of the first law Eq. (3). The other case concerns self-gravitating effects within Hawking radiation as studied in [47, 48]. The results found there emerge simply and transparently when the system
is analyzed using the conjugate variables $\Theta$ and $A$ rather than the pair of asymptotic variables $t$ and $M$.

The essential new point is that, via the gravitational back reaction, each matter energy eigenstate $j$ is correlated to an area eigenstate with eigenvalue $A_j$. Thus it evolves according to (18) with a $\Theta$-time dependence given by $\exp(iA_j\Theta/8\pi)$. (The entanglement of the gravitational subspace of eigenvalue $A_j$ to the matter state $j$ is not specific to the present case: because of the constraints, this gravity-matter entanglement always occurs when working with the solutions of the Wheeler-DeWitt equation[49].)

As an illustration the system can be taken to be a two-level atom held fixed at some distance from the hole (for a more complete discussion see [42]). To mediate the interactions we introduce a radiation field which allows for transitions from ground state $j$ to excited state $j'$ by absorption or emission of a photon. To first order in the coupling between the atom and the radiation field, the transition amplitude from $j$ to $j'$ accompanied by the emission of a photon of frequency $\Omega$ is given by

$$
T_{j \rightarrow j'+\Omega} = C \int d\Theta \Psi_j (\Psi_{j'} \phi_{\Omega})^* \\
= \tilde{C} \int d\Theta e^{i(A_j - A_{j'})\Theta/8\pi} e^{-i\Omega e^{-\Theta}}.
$$

In the first line, the wave function $\Psi_j$ characterizes the time evolution of the initial state of the system: a black hole plus the detector in its ground state with the radiation field in its vacuum state. The product in parenthesis characterizes the final state: the black hole and the excited detector plus the photon of frequency $\Omega$. Their wave functions factorize because we neglect the gravitational back-reaction of the radiation. (This is a legitimate assumption when computing the rates to leading order in the back-reaction.) The overall constants $C$ and $\tilde{C}$ will play no role since we shall be interested in the ratio $R = |T_{j \rightarrow j'+\Omega}/T_{j' \rightarrow j+\Omega}|^2$ which determines the equilibrium distribution. In the second line, we have grouped together the two `unperturbed' stationary states with their corresponding area-eigenvalues. The last factor is the wave function of a photon of (Kruskal) frequency $\Omega$. Its properties are fixed by the following considerations. Since we are interested in near equilibrium transitions, the initial state of the radiation field must be stationary and regular on the horizon. It must therefore be either the Hartle-Hawking vacuum[13] or the Unruh vacuum, depending on the condition one imposes on infalling configurations. It is most economical (though physically less direct) to characterize one-particle states by making use of modes which individually incorporate the requirement of regularity at the expense of being non-stationary. (The other option would be to work with stationary modes (like Schwarzschild modes $e^{-i\lambda \Theta}$) and to express the Unruh or Hartle-Hawking state as a thermal state in terms of these modes.) When working with the first option, the modes are of the form $\exp(-i\Omega U)$, where $U$ is a null outgoing coordinate which is regular on the future horizon (see e.g. [50] for a discussion of the various relevant modes). When evaluated at the (stationary) location of the two level atom, one has $U(\Theta) = -c e^{-\Theta}$, where the constant $c$ can be put to unity by a shift in $\Theta$.

Before proceeding, it is of interest to display the corresponding expression one would obtain by working in a fixed background. Instead of Eq. (19) one would have

$$
T_{j \rightarrow j',\Omega} = C' \int dt \psi_j (\psi_{j'} \phi_{\Omega})^* \\
= \tilde{C}' \int dt e^{-i(E_j - E_{j'})t} e^{-i\Omega e^{-\Theta}}.
$$
where $E_j$ and $E_{j'}$ are the eigen-energies of the two atom states measured with the asymptotic time which defines the surface gravity $\kappa$.

When working with the non-stationary modes, the ratio $R$ of transition rates can be simply evaluated, at fixed $\Omega$, by recognizing from (19) that $T_{j'\rightarrow j+\Omega}$ is related to $T_{j\rightarrow j'+\Omega}$ by a shift $\Theta \rightarrow \Theta - i\pi$ of the $\Theta$ contour. This yields $R = \exp(\Delta_j - \Delta_{j'})/4$ for the case when the gravity-matter entanglement has been taken into account in the amplitudes. On the other hand when using the background fixed transitions the ratio of the rates is

$$R = \exp \left(-\left(E_{j'} - E_j\right)(2\pi/\kappa)\right),$$

which gives rise to a Boltzmann distribution with the horizon temperature $\kappa/2\pi$. As mentioned above, the first law Eq. (3) guarantees that these results agree to first order in $G$. The only difference is that, in the first case, the specific heat of the horizon has been taken into account through the gravity-matter entanglement of the wave functions. Let us emphasize that in both cases, the ratio of the transition rates has been computed unconventionally by making use of the (non-stationary) Kruskal modes. The stationarity of the processes follows from the fact that the above ratios are independent of $\Omega$.

The principle of detailed balance implies that the ratio $p_{j'}/p_j$ of equilibrium occupation probabilities for the detector + horizon system, with the field in the Unruh or Hartle-Hawking state, is given by the ratio $R = \exp(\Delta_j - \Delta_{j'})/4$ of transition rates. The equilibrium distribution is also the microcanonical distribution, hence we infer that the number of states in a configuration with area $A$ is proportional to $\exp A/4$. Since the above analysis is couched in terms of $A$ and $\Theta$, which are defined at the horizon, it is entirely local to the neighborhood of the Killing horizon and therefore takes the same form for black hole, de Sitter, and acceleration horizons. It was therefore concluded in [12] that a quantum statistical interpretation of the entropy $A/4$ should apply to all “event horizons”.

A difference between the Rindler horizon and both the black hole and de Sitter ones is that in the former case the temperature of the horizon is perceived as a physical temperature only by observers who follow the accelerated worldlines of the horizon generating Killing field (i.e. it is the Unruh temperature), while in the black hole and de Sitter cases there are also unaccelerated observers who perceive a nonzero temperature. This difference plays no role in the above analysis however.

Other approaches to counting horizon states

Of the many other approaches to calculating black hole entropy from quantum first principles, it seems to us that none present any evidence that the notion of universal horizon entropy density is invalid. For examples we briefly consider here the approaches of Carlip[51] using the structure of the near-horizon component of the diffeomorphism group, the canonical quantum gravity calculations in the loop approach[52], and the calculations of string theory.

Carlip’s approach, being based on representations of the residual local diffeomorphism symmetry modulo horizon boundary conditions, is as applicable to any Killing horizon as it is to black hole horizons. It does not rely on the global properties of the horizon, but instead appears consistent with the notion of horizon entropy density. The loop quantum gravity approach as currently formulated involves an induced Chern-Simons theory on the horizon, and applies to both compact black hole and de Sitter horizons. For technical reasons however it does not apply to non-compact Rindler horizons or even to a Rindler horizon compactified to a 2-torus. It is not yet clear what result would be obtained for the entropy of a Rindler horizon using this approach.

The string calculations are of two types. In the weakly coupled D-brane context, near-extremal configurations at weak coupling are found to have the same entropy as
the black hole with the corresponding charges at strong coupling (for a review see [53]). To apply this reasoning to other causal horizons seems problematic, since it is unclear how to identify an appropriate weakly coupled cousin. The second (and related) string approach is in the context of the AdS/CFT duality, wherein the black hole entropy is identified with ordinary thermal entropy in the CFT (for a review see [54]). Hawking, Maldacena, and Strominger [55] applied these ideas to understand the entropy in a two dimensional de Sitter brane world, embedded in AdS, in terms of entanglement entropy of the conformal field theory vacuum across the de Sitter horizon, and they made arguments for the extension of this result to higher dimensions. Somewhat similarly, Das and Zelnikov [56] have identified the mapping between the Unruh radiation for an accelerated observer in AdS and a thermal appearance of the ground state in the CFT for a corresponding class of observers. Presumably the acceleration horizon entropy can be identified with some definite CFT entropy along these lines, although that has not been shown explicitly.

In summary, it is fair to say that, wherever the question can be addressed, all of these approaches to counting black hole states apply in principle equally well to all Killing horizons. This seems to be consistent with the universal interpretation of horizon entropy we are advocating here.

6 WHAT DOES HORIZON ENTROPY COUNT?

We have argued that the entire framework of black hole thermodynamics and in particular the notion of black hole entropy extends to any causal horizon. What are the implications of this conclusion?

First of all, our attention is deflected away from the black holes and towards the horizons in black hole thermodynamics. It is sometimes considered a mystery how a black hole horizon could be capable of carrying so much entropy when after all it has no local significance in but is rather defined teleologically in terms of the future evolution of the spacetime. For example, it is regarded as puzzling that when a star collapses and forms a black hole, the entropy suddenly rockets up to a value many orders of magnitude greater than it was in the star, “just because” the horizon has formed. This becomes much less mysterious when it is realized that in essence the black hole really has nothing to do with it. Any causal horizon is endowed with a surface entropy density of 1/4.

The realization that horizon entropy is an intrinsically observer dependent notion raises the obvious question of what are the states that the horizon entropy counts? The notion that it counts the number of internal configurations, i.e. configurations behind the horizon, was argued against in [57] on various grounds. It seems only even possibly viable if the holographic conjecture [58] holds, i.e. if the entire description of the world behind any horizon can be fully described on its bounding surface. It was argued in [57] that the holographic conjecture is at best valid when there is no trapped surface behind the horizon, but it may otherwise in some sense be true. Whether or not it is true, the fact remains that, for the observers who remain confined to the “outside” of the horizon, the horizon entropy somehow captures the number of ways that the world beyond the horizon can affect the world outside.

Black hole pair creation rates are sometimes cited for a counter-argument to this viewpoint, since the rates are proportional to $\exp A/4$ which is interpreted as a density of states factor counting degeneracy of all black hole states. However it must be recalled that the instanton method for computing a tunneling rate just yields the probability for the most probable transition and the ones in its immediate vicinity. Hence the
degeneracy factor is not counting all interior states of the black hole since these are beyond the scope of the instanton method.

One clue to the nature of the horizon states counted by $A/4$ comes from an old analysis by Candelas and Sciama [36, 37]. They showed how the relationship between near equilibrium transition rates for a system in contact with a horizon and horizon area is extended to non-equilibrium processes. They interpreted the viscous dissipation rate of a shearing horizon, via the fluctuation-dissipation theorem, in terms of the quantum gravitational spectrum of shear fluctuations. This explains "why" a horizon has a coefficient of viscosity, and suggests that it is, at least in part, the quantum shear states of the fluctuating horizon that the entropy counts. This seems a good beginning, but it is surely not the whole story. In addition to the shear viscosity term in Eq. (6), for the horizon evolution there is also a bulk viscosity ($\theta^2$) term. Moreover, even when both these terms are absent the horizon acts as a "perfect dissipator" [36, 37] via just the area expansion.

Another idea is that the horizon entropy arises from the entanglement of short distance field fluctuations on either side of the horizon [59]. This notion is quite appealing since it traces the entropy directly to the defining character of the horizon as a causal barrier that hides information, and it naturally accounts for the scaling with area. Moreover, it allows the generalized second law to be understood as a consequence of causality [60]. However, it is problematic in the quantitative measure of missing information (which appears to be infinite in ordinary field theory and to depend on the number of species) and in the neglect of the quantum fluctuations of the horizon itself. These issues are tied up with the renormalization of Newton's constant [61], though a completely satisfactory understanding at the statistical level is far from being at hand. The finiteness problem is absent in the loop quantum gravity approach [52], which can be viewed as a counting of intertwiners that characterize the possible ways a spin network inside the black hole could link to the one in the exterior. However, that approach remains to be tied to the semiclassical limit in a quantitatively convincing manner. Interesting perspectives on some of these issues are given in the review article by Bekenstein [16].

While no definitive answer as to the ultimate nature of horizon entropy seems immediately at hand, an abundance of insight has been gleaned from the three decades of work. Perhaps the time is ripe to synthesize this insight and make the leap to a new conception.

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