A new 2-D DOA estimation method based on coprime array MIMO radar

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ABSTRACT
Aiming at the problem that traditional Direction of Arrival (DOA) estimation methods cannot handle multi-source with high accuracy while increasing the degree of freedom, proposing a new two-dimensional DOA estimation method based on coprime array MIMO radar (SM-MIMO-CA). Firstly, to ensure the accuracy of multi-source estimation under the finite number of array elements, proposing a new array model based on coprime array MIMO radar (MIMO-CA). Array model uses a special combination of irregular transmitting array and uniform linear array receiving array. Besides, to decrease the complexity and raise the accuracy of two-dimensional DOA estimation, proposing a new DOA estimation method based on coprime array MIMO radar. This method uses the sparse array topology of virtual array elements to analyze a larger number of sources, and combines the compressed sensing method to process the sparse array. This method obtains a larger array aperture with a smaller number of elements, and improves the resolution of the azimuth angle, so the DOA estimation accuracy is improved and the complexity is reduced. Finally, experiments are verified the effectiveness and reliability of the SM-MIMO-CA in improving the degree of freedom of the array, decreasing the complexity, and enhancing the accuracy of DOA.

CCS CONCEPTS
• Computing methodologies; • Modeling and simulation; • Model development and analysis; • Model verification and validation; • Simulation theory; • Systems theory; • Theory of computation; • Design and analysis of algorithms; • Data structures design and analysis; • Data compression;

1 INTRODUCTION
In the uniform linear array of the traditional direction of arrival estimation (DOA) method, the estimable target sources’ number is less than the array elements’ number. Classical methods such as MUSIC method [1-2] or ESPRIT method [3-4] use $N$ array elements to estimate at most $N-1$ target signals, and the array’s degree of freedom (DOF) is limited. Therefore, in the case of a certain number of array elements, how to optimize the array structure to obtain a bigger array caliber to enhance DOA estimation accuracy and multi-target resolution has always been a hot issue for scholars [1-6].

In recent years, with the continuous in-depth study of the array element structure, scholars have proposed many non-uniform array structures [7-8]. For example, the coprime Array [8-11]. To enhance the ability of two-dimensional DOA estimation, a multi-input multiple-output (MIMO) radar is proposed [12-13]. Although, the references [14-15] combine MIMO and a coprime array to estimate DOA, the coprime array still uses a uniform linear array, which does not have very high accuracy for two-dimensional DOA estimation. Reference [16] proposed the use of compressed sensing for sparse matrix processing, however, it’s not used in the MIMO coprime array structure. Reference [17] proposes a fast DOA estimation method for Nand $N + 1$ parallel uniform linear arrays, but when there are many sources, additional matching is required and the sensors are not fully utilized. References [18-19] proposed a new array, but in the case that array has the same number of elements, it is impossible to detect more sources.

Therefore, this paper proposes a new MIMO array-based coprime array combination (MIMO-CA). The transmitting array of the array combination is a special irregular array, and the receiving
array is a uniform linear array. Then, using the new array (MIMO-CA) proposed in this paper, this paper combines the methods of compressed sensing [20], and proposes a new two-dimensional DOA method based on sparse matrix. First of all, the method can be realized by constructing an equivalent array of sparse array, that is, using the sparse array topology of virtual array elements to analyze a larger number of two-dimensional DOA sources, and can be able to match the corresponding angle. Besides, by converting the two-dimensional DOA estimation problem into two 1-D DOA estimation problems, only one variable can be estimated, thus reducing the computational complexity. Then, when information sources’ number is greater or equal to the number of array elements, a virtual differential array is established, and least squares operations and sparse reconstruction are performed. The sparse matrix is processed through compressed sensing, so that M + 4 array elements can identify 2M² sources.

2 ARRAY CONFIGURATION AND SIGNAL MODEL

2.1 Array model

Following the guidelines throughout this template will also improve the accessibility of your manuscript and increase the audience for your work. Ensure that heading styles are applied as instructed, tables are created using Word’s table feature (rather than an image), figures have a text equivalent, and list styles are applied as instructed.

The number of transmit arrays targeted by MIMO is 4, and the receive arrays are M. Because of the nature of the MIMO array model, the array could be virtualized so that the number of virtualized arrays is 4M. In Figure 1, the array arrangement of the transmitting array, and receiving array is Muniform linear arrays with 2Md-spacing along the x-axis direction. Where, d = λ/2.

According to the nature of the MIMO radar, the virtual array is obtained, and then when constructing the co-prime array, just discard the last element of sub-array 2 to form the co-prime array. The result is shown in Figure 2

The coprime array has three sparse linear uniform arrays. Sub-array 1 has 2Marray elements, and the array element spacing is Mm, and sub-arrays 2 and 3 have M−1and Marray elements, and the array element spacing is 2Md. Array element spacing is λ/2. By choosing M ∈ N⁺ and 2M ∈ N⁺ to be relatively prime (where N⁺ is represented as a set of positive integers). For simple distinction, let 2M = N. Then the array sensor is located at:

\[
\{ (x, y) \mid (0, 2Mmd) \cup (d, Nm_1d) \cup (d + Ld, MNd + Nm_2d) \}
\] (1)

Where 2m ∈ [0, 2M − 1], m₁ ∈ [1, M − 1], m₂ ∈ [0, M − 1], n, m₁, m₂ ∈ N⁺.

2.2 Signal model

These sub-arrays are not collinear, and are set in parallel at distances and Ld, (L ∈ N⁺) respectively, that is, the minimum unit spacing along the x-axis. Then the signal output as follow:

\[
x(l) = [a_1(\theta_1, \varphi_1) \otimes a_i(\theta_1, \varphi_1), a_1(\theta_2, \varphi_2) \otimes a_i(\theta_2, \varphi_2), \ldots, a_1(\theta_k, \varphi_k) \otimes a_i(\theta_k, \varphi_k)] S(l) + n(l)
\] (2)

Where \( \theta_k, \varphi_k \) are the angles of the k-th source respectively. \( \otimes \) is expressed as the Kronecker product. \( n(l) \) is another noise vector.\( a_i(\theta_k, \varphi_k) = a_i(\theta_k, \varphi_k) \otimes a_i(\theta_k, \varphi_k) \) and \( a_i(\theta_k, \varphi_k) = a_i(\theta_k, \varphi_k) \otimes a_i(\theta_k, \varphi_k) \). Let \( a_i(\theta_k, \varphi_k) \otimes a_i(\theta_k, \varphi_k) = a_i(\theta_k, \varphi_k), \) assume that the relationship after virtual is:

\[
x_i(t) = \sum_{q=1}^{Q} a_i(\theta_q, \varphi_q) e^{j2\pi \frac{\lambda}{M} \sin(\theta_q)\cos(\phi_q) s_q(t) + n_i(t)
\] (3)

Where,

\[
a_i(\theta_q, \varphi_q) = [e^{j2\pi \frac{y_{q1}^1}{M} \sin(\theta_q)\cos(\phi_q)}, \ldots, e^{j2\pi \frac{y_{qJ}^1}{M} \sin(\theta_q)\cos(\phi_q)}]^T
\] (4)

Formula (4) represents \( (\theta_q, \varphi_q) \) corresponding to the steering vector of the i-th sub-array, where \( q = 1, \ldots, Q, i = 1, 2, 3, y_{qj}^1, 1 \leq j \leq N_i^q \) is the y coordinate of the i-th sensor. \( N_i^q \) is the total number of sensors in the i-th sub-array. \( x_i(t) \) is the i-th sub-array’s position along the x-axis. The noise vector element is in the i-th sub-array \( n_i(t) \).

In order to avoid the need for spectral peak search like MUSIC algorithm and Capon algorithm, the problem of 2-D DOA estimation is converted into two 1-D problems. \( \alpha_q, \beta_q \in [0°, 180°] \) are respectively expressed as the angle between the incident direction and the y direction and the x direction. The relationship between \( \alpha_q, \beta_q \) and \( \theta_q, \varphi_q \) is:

\[
\cos(\alpha_q) = \sin(\theta_q) \sin(\varphi_q)
\] (5)

\[
\cos(\beta_q) = \sin(\theta_q) \cos(\varphi_q)
\] (6)

Therefore, received data vector in the formula (3) is:

\[
x_i(t) = \sum_{q=1}^{Q} a_i(\alpha_q) e^{j2\pi \frac{\lambda}{M} \cos(\beta_q) s_q(t) + n_i(t)
\] (7)

The corresponding steering vector is:

\[
a_i(\alpha_q) = [e^{j2\pi \frac{\lambda}{M} \cos(\alpha_q)}, \ldots, e^{j2\pi \frac{\lambda}{M} \cos(\alpha_q)}]^T
\] (8)
On the premise of satisfying the conditions of $Q < N_r Q$ to obtain the noise subspace, an effective method is proposed in this section to achieve the equivalence of the difference array with a larger number of DOF. In addition, enhancing the estimation accuracy of DOA by the group sparse array technology, and the differential covariance formulas of $x_i(t)$ and $x_k(t)$ are constructed:

$$R_{x_k} = E[x_i(t) x_k^H(t)] = \sum_{a_q=1}^{Q} a_q^2 e^{j2\pi (x_q - x_k) \cos(\beta_i)} a_q^H a_q$$

$$+ n_i(t)n_k^H(t) = \begin{pmatrix} A_i R_{x_k} A_i^H & A_i \sigma_n^2 N_q^H \end{pmatrix}$$

\[\text{where } R_{x_k} = E[s(t) H(t)] = \text{diag}([\sigma_1^2, \ldots, \sigma_Q^2]) \text{is the covariance matrix of the } Q \times Q \text{dimensional signal. In addition,}

\[D_i = b_i R_{x_k} = \text{diag}\left\{e^{j2\pi (x_q - x_k) \cos(\beta_i)}, \ldots, e^{j2\pi (x_q - x_k) \cos(\beta_Q)}\right\}\]

When $i = k$, it becomes the identity matrix. Quantify the matrix $R_{x_k}$ to obtain the following measurement vector:

$$z_{ik} = \text{vec}(R_{x_k}) = \begin{pmatrix} \tilde{A}_{ik} b_{ik} \end{pmatrix}, \quad i \neq k$$

Where,

$$\tilde{A}_{ik} = [\tilde{a}_{ik}(\alpha_1), \ldots, \tilde{a}_{ik}(\alpha_Q)]$$

$$b_{ik} = [\sigma_1^2 e^{j2\pi (x_q - x_k) \cos(\beta_1)}, \ldots, \sigma_Q^2 e^{j2\pi (x_q - x_k) \cos(\beta_Q)}]^T$$

Due to the relatively prime nature of $M$ and $N$, the DOF in the common array is greatly increased, so that more information sources $N_r$ can be estimated with fewer array elements.

3 2-D DOA ESTIMATION METHOD

The next subsections provide instructions on how to insert figures, tables, and equations in your document.

3.1 2-D DOA estimation method for sparse array

The signal vector, $Z_{ik}, 1 \leq i, k \leq 3$ in equation (11), can be sparsely expressed on the entire discrete angle grid as:

$$z_{ik} = \begin{pmatrix} \tilde{A}_{ik} b_{ik} \end{pmatrix}, \quad \text{where } \tilde{A}_{ik} = [\tilde{a}_{ik}(\alpha_1), \ldots, \tilde{a}_{ik}(\alpha_Q)]$$

\[b_{ik} = [\sigma_1^2 e^{j2\pi (x_q - x_k) \cos(\beta_1)}, \ldots, \sigma_Q^2 e^{j2\pi (x_q - x_k) \cos(\beta_Q)}]^T\]

The respective steering vector of each vector $z_{ik}$ is:

$$\Phi_{ik} = \begin{pmatrix} \bar{A}_{ik} \end{pmatrix}, \quad \text{where } \bar{A}_{ik} = [\tilde{a}_{ik}(\alpha_1), \ldots, \tilde{a}_{ik}(\alpha_Q)]$$

Where, $\tilde{A}_{ik}$ is the sparse vector, and its non-zero entry position corresponds to the DOA estimated by $a_q$, $q = 1, \ldots, Q$. For not the same sub-arrays, non-zero items usually have different values and share the same position when searching [10]. In other words, $b_{ik}$ exhibits a set of sparsity on all pairs of sub-arrays. Therefore, the estimation of $a_q$, $q = 1, \ldots, Q$ can be solved in the sparse reconstruction framework [10, 16]. In this paper, the complex multi-task Bayesian compressed sensing method is introduced into the SM-MIMO-CA method [10, 20].

Using self-lag and cross-lag, this paper re-arranged the vector $z_{ik}$ so that the algorithm effectively reduces the grid error.

$$z_{ik} = \Phi_{ik} b_{ik} + \epsilon_{ik}, \quad 1 \leq i, k \leq 3$$

Where $\epsilon_{ik}$ is the noise subspace, an effective method is proposed in this section to achieve the equivalence of the difference array with a larger number of DOF. In addition, enhancing the estimation accuracy of DOA by the group sparse array technology, and the differential covariance formulas of $x_i(t)$ and $x_k(t)$ are constructed:
\[ b_{ik} = (\hat{A}^H \hat{A})^{-1} \hat{A}^H \hat{b}_k, i \neq k \]  

(27)

Where,

\[ \hat{A}_{ik} = [a_{ik}(\hat{a}_1), ..., a_{ik}(\hat{a}_q)] \]  

(28)

Therefore: \( \hat{\beta}_q, q = 1, ..., Q \) estimates:

\[ \hat{\beta}_q = \cos^{-1}(-\text{phase}(\hat{b}_q)/\pi) \]  

(29)

Where, \( \hat{b}_q \) is the \( q \)-th element of vector \( \hat{b}_{ik} \), so \( \hat{\beta}_q \) automatically matches \( \hat{a}_q \), and \( \hat{a}_q \) can be obtained in the same way, so by formulas (6) and (7)

\[ \hat{\theta}_q = \sin^{-1}[\sqrt{\cos^2(\hat{a}_q) + \cos^2(\hat{\beta}_q)}] \]  

(30)

\[ \hat{\phi}_q = \tan^{-1}\frac{\cos(\hat{a}_q)}{\cos(\hat{\beta}_q)} \]  

(31)

4 EXPERIMENTAL RESULTS AND ANALYSIS

4.1 Degree of Freedom Analysis

In the case of one-dimensional, the obtained co-array is equivalent to the traditional coprime array, that is, the number of estimated signals can reach: \( Q_{av} = MN \), that is, \( Q_{av} = M^2 \). For a given number of physical antennas \( N_t = 2M + N - 1 = 4M - 1, Q_{av} \) can be obtained in the following form:

The maximum number of sources can be estimated:

\[ Q_{av} = MN = M^2 \]  

(32)

Subject to:

\[ N_t = 2M + N - 1 = 4M - 1, M < N, M, N \in \mathbb{N}^+ \]  

(33)

Making them as equal as possible. Under these circumstances, estimated signals \( Q_{av} \) ’s maximum number is:

\[ Q_{\text{max}} = \left\lfloor \frac{N_t(N_t + 2)}{8} \right\rfloor \]  

(34)

As shown in Figure 3 above, we compare the \( Q_{\text{max}} \) value of this method with the TDUL-PM method, TPAUL method, PUL-RARE method, TDSR-CS in the references [21], [19], [23], [24] The methods are compared in the Figure 3. Although the value of \( Q_{\text{max}} \) of all methods increases with the increase of \( N_t \), and SM-MIMO-CA and TPAUL are obviously better than other methods.

4.2 Two-dimensional DOA estimation performance comparison

4.2.1 The relationship between signal-to-noise ratio and mean square error

To verify the ability of the two-dimensional DOA estimation of SM-MIMO-CA, the SM-MIMO-CA is compared with the TPAUL [21], the TDUL-PM [19], the TDSR-CS [23] and the PUL-RARE [24]. Perform 100 Monte Carlo simulations for each method.

Where, the number of Monte Carlo experiments is \( I \), and sources’ number is \( Q \). Let \( M = 4 \), that is, the array configuration of the \( N_t = 4M - 1 = 15 \) antenna. In addition, let \( L = 20 \). Assume that \( \theta \)-field sources with the same power are on the elevation plane \( (\theta_q, \phi_q) \), where \( \theta_q \in [0^\circ, 90^\circ], \phi_q \in [-90^\circ, 90^\circ], q = 1, ..., Q \). The grid interval in the angular space is set to 0.1\(^\circ\), and \( a = b = c = d = 0 \).

Figure 4 and Figure 5 show the use of SM-MIMO-CA method and TPAUL method, TDUL-PM method, TDSR-CS method, PUL-RARE method when \( Q = 3, T = 500 \). The estimated performance is compared, and the RMSE changes of the method under not the same signal-to-noise ratios (SNR) are investigated. By comparing RMSE under different SNR, it is concluded that SM-MIMO-CA has
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5 CONCLUSION

Aiming at the problem that traditional array signal processing methods cannot handle multiple sources with high accuracy while increasing the DOF, this paper proposes SM-MIMO-CA method. First of all, a new MIMO coprime array model is proposed, which can effectively estimate multiple sources with a smaller number of arrays, and improve the DOF and accuracy of DOA estimation. Besides, based on the array model, SM-MIMO-CA method is proposed. This method combines compressed sensing theory to process the MIMO coprime array that has been sparsely processed, reducing the

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