ABSTRACT. A new class of nano-structure devices is suggested, based on interference from the order parameter phase gradient of a single superconductor (S) in contact with a single normal metallic lead (N). By solving the Bogoliubov - de Gennes equation in two dimensions, it is demonstrated that the electrical conductance of a normal conductor of width $M$ in contact with a superconductor will oscillate as the phase gradient $\nu$ at $90^\circ$ to the interface is increased. This effect is enhanced by the presence of a Schottky barrier at the interface and is also present in $N - S - N$ structures.

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Following the pioneering work of Spivak and Khmel’nit’skii[1], it has long been recognised that the conductance of a phase coherent normal structure with two superconducting inclusions should oscillate with the phase difference $\phi$ between the order parameters of the inclusions[2-4]. While the details of this effect is still a subject of discussion[5-9], the underlying principle behind such Andreev interferometers is clear; the wavefunction of a quasi-particle which Andreev reflects at a normal-superconducting (N-S) interface acquires the local phase of the order parameter. Therefore if two such interfaces are present, interference between partial waves reflecting from the separate interfaces yields an Andreev reflection coefficient $R_a$ of the form $R_a = A + B \cos \phi + \ldots$, where the dots indicate the presence of higher harmonics, details of which depend on the geometry and underlying disorder of the sample. If $\mu$ is the common condensate chemical potential of the inclusions and $\mu_1 = \mu + eV$ the chemical potential of an external normal reservoir of electrons, the current $I$ flowing from the reservoir into the superconductor is $I = (2e^2/h)G_{NS}V$, where $G_{NS} = 2R_a$ is the BTK boundary conductance[10]. Consequently through measurements of the electrical conductance, it has been possible to confirm the existence of Andreev interferometers in nano-structure devices formed from tunnel junctions[11] and metallic interfaces[12,13].

The aim of this Letter is to suggest a new class of devices based on interference from the order parameter phase gradient of a single superconductor. Such Andreev phase gradiometers should be easier to construct than a structure containing two interfaces and may open up the possibility of measuring unambiguous Andreev interference effects in phase coherent normal-semiconductor structures. Four examples of phase gradiometers are shown in figure 1. In examples A,B,C, which are studied in detail below, the measured current $I$ flows vertically from a normal, crystalline external lead at the bottom of the figure to another at the top. The leads are connected to normal reservoirs at potentials $V_1$ and $V_2$ respectively and we shall compute the total electrical conductance $G = (h/2e^2)I/(V_1 - V_2)$ in the presence of a superconducting order parameter of the form
\[ \Delta(r) = \Delta_0 \exp(i vx), \] with the \( \bar{x} \)-axis chosen to be horizontal. In practice one can envisage producing a phase gradient \( v \) of this kind, at 90° to the measured current \( I \) by applying a control current from left to right. Example A of figure 1 shows a clean superconductor in contact with metallic leads. Example B shows a (N-I-S-N) structure with a single insulating (I) barrier at one interface. Example C shows a N-I-N-S-N structure, with a metallic region between the insulating barrier and the superconductor.

The key principle we wish to establish is that the conductance of these devices is an oscillatory function of the phase gradient \( v \). One expects this behaviour, because the order parameter acts as a complex, off-diagonal scattering potential and therefore for a two-dimensional N-S interface of finite width \( M \), scattering matrix elements will be sensitive to the total phase change \( Mv \). Consequently transport coefficients should be oscillatory functions of \( v \), with period \[ \bar{\tau} = 2\pi/M. \] A further aim of this Letter is to determine which if any of the examples shown in figure 1 yield an oscillation which is a finite fraction of the overall conductance.

The central quantity needed to compute transport properties of a phase coherent sample possessing a Hamiltonian \( H \) and connected to external current carrying leads, is the quantum mechanical scattering matrix \( s(E, H) \), with sub-matrices \( s_{L,L'}^{\alpha,\beta}(E, H) \), which describe the scattering of excitations of energy \( E \) from all incoming \( \beta \) channels of lead \( L' \) to all outgoing \( \alpha \) channels of lead \( L \) (where \( \alpha, \beta = +1 \) for particles and -1 for holes).

From a knowledge of \( s(E, H) \), a matrix of reflection and transmission coefficients can be constructed \( P_{L,L'}^{\alpha,\beta}(E) = \text{Trace}\{s_{L,L'}^{\alpha,\beta}(s_{L,L'}^{\alpha,\beta})^\dagger\} \), in terms of which the zero temperature, two probe electrical conductance, in units of \( 2e^2/h \), can be written\cite{14,15},

\[
G = T_0 + T_a + \frac{2(R_aR'_a - T_aT'_a)}{R_a + R'_a + T_a + T'_a} \tag{1}
\]

The coefficients \( R_0 = P_{L,L}^{++}(0), \ T_0 = P_{L',L}^{++}(0) \ (R_a = P_{L,L}^{-+}(0), \ T_a = P_{L',L}^{-+}(0)) \) are probabilities for normal (Andreev) reflection and transmission of quasi-particles from the lower reservoir \( L \), while \( R'_0, T'_0 \ (R'_a, T'_a) \) are corresponding probabilities for quasi-particles
from the upper reservoir $L'$. In the presence of $N$ open channels per lead, these satisfy

$$R_0 + T_0 + R_a + T_a = R'_0 + T'_0 + R'_a + T'_a = N$$

and

$$T_0 + T_a = T'_0 + T'_a.$$ 

In the limit of negligible quasi-particle transmission, the resistance reduces to a simple sum of two BTK boundary resistances,

$$G^{-1} = 2/R_a + 2/R'_a$$

associated with Andreev reflection into the separate reservoirs.

Given the spatial form of the superconducting order parameter $\Delta(x)$ the scattering matrix can be computed by solving the Bogoliubov - de Gennes equation, as outlined in [15]. In what follows, we present the results of detailed numerical simulations of a two dimensional tight binding system, described by a Bogoliubov - de Gennes operator of the form

$$H = \begin{pmatrix} H_0 & \Delta \\ \Delta^* & -H_0^* \end{pmatrix}.$$ 

In this equation $H_0$ is a nearest neighbour tight binding model on a square lattice, with off-diagonal hopping elements of value $-\gamma$ and $\Delta$ a diagonal order parameter matrix. The scattering region is chosen to be $M$ sites wide and is connected to external leads of width $M$, as shown in figures 1A to 1C. For convenience we make the choice $\gamma = 1$ and throughout the whole structure, except in a barrier region, the diagonal elements of $H_0$ are set to $10^{-3}$. By choosing a value which is close to, but not identically zero one obtains a normal host material close to half-filling, while avoiding a discontinuity in the number of open channels at $E = 0$. For structures B and C, within the region occupied by the barrier, diagonal elements of $H_0$ are set to $3\gamma$. Finally within the superconductor, the magnitude $\Delta_0$ of the order parameter is chosen to be $\Delta_0 = 0.5\gamma$ and since $\mu \approx 4\gamma$, $\Delta_0/\mu \approx 10^{-1}$, which is typical of a cuprate superconductor. In what follows, for each structure and a given choice of $v$, the scattering matrix at $E = 0$ is obtained numerically, using a transfer matrix technique outlined in appendix 2 of reference[15]. For a lattice constant $a$, the Landau critical velocity of a such a homogeneous superconductor is $v^* = \Delta_0/(a\gamma)$. In what follows we choose $a = 1$ and therefore $v^* = 0.5$. 

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For $M = 30$ and a superconductor of length $M' = 20$, figure 2 shows the variation of the various scattering coefficients, along with the electrical conductance $G$. Since this structure is symmetric about a horizontal line passing through the centre of the superconductor, all scattering coefficients from lead 1 are identical to those from lead 2 and the conductance formula (1) reduces to $G = T_o + R_a$. Although careful inspection of the curves of figure 2 reveal a periodic modulation with $v$, the effect is clearly negligible. We have carried out simulations for a range of $M$ and $M'$ and have found only a negligible effect for all structures of type A.

Figure 3 shows results for three systems of type B, with widths $M = 15, 30$ and 45, a potential barrier of length 5 and a long superconductor of length $M' = 150$. The latter is chosen to yield negligible transmission through the device, so that equation (2) provides a good approximation to $G$. Furthermore the device is now asymmetric and the overall resistance is dominated by the boundary conductance $2R_a$ of the lower interface. It is clear from this figure that the period of oscillation is inversely proportional to the width of the device and that by introducing a barrier, the relative size of the effect is increased. This enhancement is reminiscent of the increase of zero bias anomalies through the presence of a Schottky barrier[16-21].

We have performed numerical simulations of a variety of structures and in all cases find that the presence of a barrier at the interface enhances this effect. For a simple structure such as that modelled in figure 3, a familiar Fraunhoffer diffraction pattern is obtained, which can be understood by examining the overlap between an outgoing plane wave with a transverse wavevector shifted by the continuous variable $v$ and the discrete number of allowed wavevectors defining open channels in the external leads. As an example of a more complex system, figure 4 shows results for structure C of figure 1. In this case, the system is of width $M = 30$, the superconductor of length $M' = 9$, the insulating barrier of length 5 and the normal, crystalline, metallic region separating the barrier from the superconductor is of length 6. For this structure resonances associated
with multiple scattering within the normal region yield a more complex interference pattern, which is reflected in the non-trivial variation of $G$ with $v$.

The aim of these simulations has been to demonstrate that an oscillatory dependence of $G$ on the phase gradient $v$ is a generic feature of hybrid superconducting structures and to identify systems for which the effect is non-negligible. To avoid relying on approximate analytical solutions, we have chosen to produce results based on exact solutions of the Bogoliubov - de Gennes equation in two dimensions. The simulation leading to figure 3 reveals that this effect is enhanced by the presence of a Schottky barrier at the interface and therefore it should be possible to observe these oscillations in semiconductor-superconductor structures. Figure 3 also shows that the effect is present in the BTK boundary conductance and therefore the superconductor itself can be used as one of the external reservoirs. From the point of view of constructing the simplest possible experiment, this leads us to suggest the structure sketched in example D of figure 1 as a possible candidate. In this example a superconducting loop is connected via a Schottky barrier to a normal lead and the boundary conductance $2R_a = (h/2e)I/(\mu_1 - \mu)$ is measured as a function of the magnetic field through the loop. Since the latter produces a surface screening current and therefore a phase gradient on the N-S interface, the boundary conductance will oscillate on a field scale which depends on the width $M$ of the normal lead and can therefore be chosen to be distinct from that that of a flux quantum through the loop. Finally to observe this effect, one notes that the period $\tau = 2\pi/M$ must be smaller than the critical velocity $v^*$, which implies that the width $M$ must be greater than the zero temperature coherence length of the superconductor.
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Figure Captions.

Figure 1. Sketches A,B and C show the three distinct structures used in the simulations. Sketch D shows the simplest possible candidate for an experimentally realisable Andreev phase gradiometer.

Figure 2. This shows the variation with $v$ of $G$, $R_o$, $R_a$, $T_o$ and $T_a$ with $V$, for a system of type A. For convenience all quantities are divided by the number $N$ of open channels.

Figure 3. This shows the variation with $v$ of $G$ and $R_a$ for a system of type B with 3 different widths, $R_a$ is the Andreev reflection co-efficient for the superconducting interface containing the barrier. All quantities are divided by the number of open channels.

Figure 4. This shows variation with $v$ of $G$ and $R_a$ for a system of type C. Both quantities are divided by the number of open channels.
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