Comparison of Mixed and Multiplicative Models when Trend Cycle Component is Linear

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Authors’ contributions

This paper was carried out in collaboration between both authors. Author KCND designed the study, performed the statistical analysis, wrote the protocol and wrote the first draft of the manuscript. Author JCN managed the analyses and literature searches of the study. Both authors read and approved the final manuscript.

Article Information

DOI: 10.9734/AJARR/2020/v12i430295

Editor(s):
(1) Dr. Him Lal Shrestha, Kathmandu Forestry College, Nepal.

Reviewers:
(1) Feriaa Ahmed Elbashir Mohamed, University of Jeddah, Saudi Arabia.
(2) Hamid H. Hussien, King Abdulaziz University (KAU), Saudi Arabia.
Complete Peer review History: http://www.sdiarticle4.com/review-history/59211

Received 17 May 2020
Accepted 23 July 2020
Published 04 August 2020

ABSTRACT

The purpose of this study is to present the linear trend cycle component with the emphasis on the choice between mixed and multiplicative models in time series analysis. Most of the existing studies have adequately dwelt more on choice of model between additive and multiplicative, with little or no regards to the mixed model. The main aim of this study is to compare the row, column and overall means and variances for mixed and multiplicative models using Buys-Ballot table for seasonal time series. Specific objectives are 1) to obtain and compare the expected values of means for mixed and multiplicative models 2) to estimate and compare trend parameters and seasonal indices (when there is no trend, that is (b = 0)). The study indicate that column variances \( \hat{\sigma}^2_j \) of the Buys-Ballot table depends on the season \( j \) only through the square of the seasonal effect \( S_j^2 \) for mixed model and it is for multiplicative model, a quadratic function of the column \( j \) and square of the seasonal effect \( S_j^2 \).

Keywords: Time series decomposition; trend cycle component; mixed model; multiplicative model; expected value; buys-ballot table.

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1. INTRODUCTION

Time series decomposition method involve the separation of an observed time series into components representing trend (long term direction), seasonal (calendar related movements), cyclical (long term oscillations) and irregular (short term fluctuations) components. For short period of time series data, the cyclical component is superimposed into the trend and the observed time series can be decomposed into the trend-cyclical component \( (M_t) \), seasonal component \( (S_t) \) and the irregular/residual component \( (e_t) \).

Additive Model:

\[ X_t = M_t + S_t + e_t \]  \hspace{1cm} (1)

Multiplicative Model:

\[ X_t = M_t \times S_t \times e_t \]  \hspace{1cm} (2)

and Mixed Model

\[ X_t = M_t \times S_t + e_t \]  \hspace{1cm} (3)

It is always assumed that the seasonal effect, when it exists, has period \( s \), that is, it repeats after \( s \) time periods.

\[ S_{t+s} = S_t, \text{ for all } t \]  \hspace{1cm} (4)

Additive model is assumed that, the sum of the seasonal components over a complete period/year is zero, ie ,

\[ \sum_{j=1}^{s} S_{t+j} = 0 \]  \hspace{1cm} (5)

Similarly, the assumption for both multiplicative and mixed models is that, the sum of the seasonal components over a complete period is \( s \).

\[ \sum_{j=1}^{s} S_{t+j} = s \]  \hspace{1cm} (6)

The additive model, which is the simpler to use arithmetically, assumes that the actual time series data is the sum of the four basic separate effects. It assumes that the effect of the trend, the season, the cycles and the residuals are equal in absolute terms throughout the period of time. This assumption is usually true when short periods are involved or where the rate of growth or decline in the trend is small and transformation is not needed. However, this study will consider mixed and multiplicative models.

Oladugba, et al, [2] presented brief description between additive and multiplicative models in time series decomposition. In their opinion, the seasonal fluctuation exhibits constant amplitude with respect to the trend in additive model. While amplitude of the seasonal fluctuation depends on trend in multiplicative model. Nwogu, et al [3] and Dozie, et al [4] proposed Chi-Square test based on the seasonal variances of the Buys-Ballot table. The test has been theoretically verified to be quite successful and efficient for choice between mixed and multiplicative models in time series analysis.

2. METHODOLOGY

The method adopted in this study is Buys-Ballot procedure for time series decomposition. For further details of Buys-Ballot table/procedure, see Wei [5], Iwueze and Nwogu [6,7,8], Iwueze and Ohakwe [9], Dozie [10], Dozie, et al [4], Dozie and Ijomah [11]. Nwogu, et al, [3] and Dozie, et al, [4] derived the row, column and grand means and variances of the Buys-Ballot table for mixed model given in Table 1, while comparing them with those of the multiplicative model. As Table 1 shows, the rows, columns and overall means and variances are not the same for both mixed and multiplicative models. However, the expected values of rows, columns and overall means are the same for both multiplicative and mixed models (see Table 2).

2.1 Expected Values of Means for Mixed and Multiplicative Models

Using the expression in Table 1, expected values of the means for mixed and multiplicative models are obtained. The row mean for mixed model is;

\[ \bar{X}_i = a - bs(i-1) + \frac{b}{s} \sum_{j=1}^{s} jS_j + \bar{e}_i \]

\[ E(\bar{X}_i) = E\left[ a - bs(i-1) + \frac{b}{s} \sum_{j=1}^{s} jS_j \right] + E(\bar{e}_i) \]  \hspace{1cm} (7)
Hence, the expected value of row mean is

\[ E(X_i) = a - bs + bsi + \frac{b}{s} \sum_{j=1}^{s} jS_j \]

\[ E(\bar{e}_i) = 0 \]

where

For multiplicative model, the row mean is

\[ \bar{X}_i = a - bs(i - 1) + \frac{b}{s} \sum_{j=1}^{s} jS_j + bsi \bar{e}_i \]

\[ E(\bar{X}_i) = E[a - bs(i - 1) + \frac{b}{s} \sum_{j=1}^{s} jS_j + bsi \bar{e}_i] \]

Thus, the expected value of row mean is

\[ E(\bar{X}_i) = a - bs + bsi + \frac{b}{2} \sum_{j=1}^{s} jS_j \]

\[ E(\bar{e}_i) = 1 \]

Where,

The column mean for mixed model is

\[ \bar{X}_j = a + b\left(\frac{n-s}{2}\right) + bj \]

\[ E(\bar{X}_j) = E[a + b\left(\frac{n-s}{2}\right) + bj]S_j + E(\bar{e}_j) \]

Therefore, the expected value of the column mean is

\[ E(\bar{X}_j) = a + b\left(\frac{n-s}{2}\right) + bj \]

\[ E(\bar{e}_j) = 0 \]

where

for multiplicative model, the column mean is

\[ \bar{X}_j = a \bar{e}_j + \frac{bs}{m} \sum_{i=1}^{m} i \bar{e}_j - bs \bar{e}_j + bj \bar{e}_j \]

\[ S_j \]

\[ E(\bar{X}_j) = E\left[a \bar{e}_j + \frac{bs}{m} \sum_{i=1}^{m} (i - 1) - bs + bj \right]E(S_j) \]

\[ = \left[a + \frac{bs(m - 1)}{2} - bs + bj \right]S_j \]

Hence, the expected value of column mean is

\[ E(\bar{X}_j) = \left[a + b\left(\frac{n-s}{2}\right) + bj \right]S_j \]

Overall mean for mixed model is

\[ \bar{X}_m = a + b\left(\frac{n-s}{2}\right) + bc_i + \bar{e}_m \]

\[ E(\bar{X}_m) = E[a + b\left(\frac{n-s}{2}\right) + bc_i] + E(\bar{e}_m) \]

Hence, the expected value for the overall mean is

\[ E(\bar{X}_m) = a + b\left(\frac{n-s}{2}\right) + bc_i \]

\[ E(\bar{e}_m) = 0 \]

where

2.2 Estimation of Trend Parameters and Seasonal Indices

The periodic and grand means are used to estimate parameters of trend line. The length of periodic interval is taken to be s. Using the expression in (7) and (9), we obtain both mixed and multiplicative models as;

\[ \hat{X}_i = a - b(s - c_i) + (bs)i \]

\[ \equiv \alpha + \beta_i \]

Hence,

\[ a = \alpha + b(s - c_i) \]

\[ \hat{b} = \frac{\beta}{s} \]

For mixed model, when \( b = 0 \), that is when there is no trend,
\[ X_i = a + e_i \]  

(21)

For multiplicative model, when \( b = 0 \), that is when there is no trend,

\[ X_i = a \]  

(22)

Estimation of \( S_j, \ j = 1, 2, \ldots, s \)

The seasonal and grand means are used to estimate the seasonal indices. The length of periodic interval is also taken to be \( s \). Using the expression in (11) and (13), we obtain both mixed and multiplicative models

\[ \tilde{X}_j = \left[ a + b \left( \frac{n-s}{2} \right) + bj \right] S_j \]

\[ = [\alpha + \beta j] S_j \]  

Where,

\[ \alpha = a + b \left( \frac{n-s}{2} \right) \]

\[ \beta = b \]  

(24)

\[ \therefore \ S_j = \frac{\tilde{X}_j}{a + b \left( \frac{n-s}{2} \right) + b} \]  

(27)

For mixed model, where there is no trend \((b = 0)\), we obtain from (11)

\[ \hat{S}_j = \frac{\tilde{X}_j}{a + \hat{e}_j} \]  

(28)

For multiplicative model, when \( b = 0 \), that is when there is no trend, we obtain from (13)

\[ \hat{S}_j = \frac{\tilde{X}_j}{a \hat{e}_j} \]

Linear trend-cycle component:

\[ M_t = a + b_t, \quad t = 1, 2, \ldots, n = ms \]

\[ c_i = \frac{1}{s} \sum_{j=1}^{s} j S_j \]

Where,

3. REAL LIFE DATA

The purpose of this section is to discuss real life example, based on monthly time series data on number of baptism collected from Assumpta Cathedral Owerri, Imo State, Nigeria for a period of 2009 to 2018 given in Appendix A. The time series plots of actual and transformed data sets are given in Figs. 1 and 2. The expression of linear trend and seasonal indices for both mixed and multiplicative models given as

\[ \tilde{X}_j = 2.584 + 0.0201j \]  

(29)

Using (25),(26) and (27)

\[ \hat{b} = 0.0201 \]

\[ \hat{a} = 2.584 - 0.0201 \left( \frac{120 - 12}{2} \right) \]

As shown in appendix A and Fig. 1, the series clearly exhibits a declining trend. The slope of the trend line represents decline on number registered baptism in Assumpta Cathedral Parish Owerri annually.

\[ \hat{a} = 1.4986 \]

\[ \hat{S}_j = \frac{\tilde{X}_j}{2.584 + 0.0201j} \]

Note: mixed satisfies

\[ \sum_{j=1}^{s} S_j = s \]  

as in (6)

Also, multiplicative model satisfies

\[ \sum_{j=1}^{s} S_j = s \]  

as in (6)
Table 1. Estimates of means and variances for mixed and multiplicative models

| Measures | Linear trend-cycle component: $M_t = a + b t$, $t = 1, 2, \ldots, n = m s$ | Mixed model |
|----------|--------------------------------------------------------------------------------|-------------|
| $X_i$    | $a - bs + \frac{b}{s} \sum_{j=1}^{s} jS_j + bsi$ * $\bar{c}_i$ | $a - bs + bsi + \frac{b}{s} \sum_{j=1}^{s} jS_j + \bar{c}_i$ |
| $X_j$    | $a \bar{c}_j + \frac{bs}{m} \sum_{i=1}^{m} i e_{ij} - bs \bar{c}_j + bj \bar{c}_j$ * $S_j$ | $a + b \left( \frac{n-s}{2} \right) + bj$ * $S_j + \bar{c}_j$ |
| $X$      | $a + b \left( \frac{n-s}{2} \right) + bC_1$ | $a + b \left( \frac{n-s}{2} \right) + bC_1 + \bar{c}_j$ |
| $\sigma_i^2$ | $\left\{ \left[ a + bs(i-1) \right] + bC_1 \right\}^2 + \text{var}\left[ a + bs(i-1) \right] S_j \right\} \sigma_i^2$ | $\left\{ \left[ a + bs(i-1) \right] + bC_1 \right\}^2 + \text{var}\left[ a + bs(i-1) \right] S_j + bjS_j \right\} + \sigma_i^2$ |
| $\sigma_j^2$ | $\frac{b^2 (n^2 - s^2)}{12} + \left[ a + b \left( \frac{n-s}{2} \right) + bj \right]^2 S_j \right\} \sigma_j^2$ | $\frac{b^2 n(n+s)}{12} S_j^2 + \sigma_j^2$ |
| $\sigma_i^2$ | $\frac{b^2 (n^2 - s^2)}{12} + \left[ a + b \left( \frac{n-s}{2} \right) + C_1 \right]^2 \right\}$ | $\frac{b^2 (n^2 - s^2)}{12} + \left[ a^2 + 2ab \left( \frac{n-s}{2} \right) + \frac{b^2(n-s)(2n-s)}{6} \right] \text{Var}(S_j) \right\} + \sigma_i^2$ |
| $\sigma_j^2$ | $\frac{b^2 (n^2 - s^2)}{12} + \left[ a + b \left( \frac{n-s}{2} \right) + C_1 \right]^2 \right\}$ | $\frac{b^2 (n^2 - s^2)}{12} + \left[ a^2 + 2ab \left( \frac{n-s}{2} \right) + \frac{b^2(n-s)(2n-s)}{6} \right] \text{Var}(S_j) \right\} + \sigma$ |
| $\sigma_i^2$ | $\frac{b^2 (n^2 - s^2)}{12} + \left[ a + b \left( \frac{n-s}{2} \right) + C_1 \right]^2 \right\}$ | $\frac{b^2 (n^2 - s^2)}{12} + \left[ a^2 + 2ab \left( \frac{n-s}{2} \right) + \frac{b^2(n-s)(2n-s)}{6} \right] \text{Var}(S_j) \right\} + \sigma$ |
### Table 2. Expected values of means for multiplicative and mixed models

| Measures | Multiplicative model | Mixed model |
|----------|----------------------|-------------|
| \( \bar{X}_i \) | \( [a - bs + bsi] + \frac{b}{s} \sum_{j=1}^{s} jS_j \) | \( a - bs + bsi + \frac{b}{s} \sum_{j=1}^{s} jS_j \) |
| \( \bar{X}_j \) | \( a + b\left(\frac{n-s}{2}\right) + bj \) \( S_j \) | \( a + b\left(\frac{n-s}{2}\right) + bj \) \( S_j \) |
| \( \bar{X}_- \) | \( a + b\left(\frac{n-s}{2}\right) + bc_1 \) | \( a + b\left(\frac{n-s}{2}\right) + bc_1 \) |

### Table 3. Estimates of trend and seasonal indices

| Parameter | Multiplicative model | Mixed model |
|-----------|----------------------|-------------|
| \( a \) | \( a + b(s - c_i) \) | \( a + b(s - c_i) \) |
| \( b \) | \( \beta \) \( s \) | \( \beta \) \( s \) |
| \( S_j \) | \( \frac{\bar{X}_{-j}}{a + b\left(\frac{n-s}{2}\right) + bj} \) | \( \frac{\bar{X}_{-j}}{a + b\left(\frac{n-s}{2}\right) + bj} \) |

**Fig. 1. Time plot of the actual series**

Dozie and Nwanya; AJARR, 12(4): 32-42, 2020; Article no.AJARR.59211
Table 4. Estimates of trend and seasonal indices (when there is no trend \((b = 0)\))

| Parameter | Multiplicative model | Mixed model |
|-----------|----------------------|-------------|
| \(\bar{X}_t\) | \(a\) | \(a\) + \(e_{i}\) |
| \(\bar{X}_j\) | \(a\) | \(a\) + \(e_{i}\) |
| \(\bar{X}_t\) | \(a\) | \(a\) + \(e_{i}\) |
| \(S_j\) | \(\bar{X}_j\) | \(\bar{X}_j\) |

Table 5. Estimates of trend parameters

| Parameter | Mixed model values | Multiplicative model values |
|-----------|--------------------|-----------------------------|
| \(a\)    | 1.4986             | 1.4986                      |
| \(b\)    | 0.0201             | 0.0201                      |

Table 6. Estimates of Seasonal indices

| \(j\) | \(\bar{X}_j\) | \(\hat{S}_j\) |
|-------|---------------|---------------|
| 1     | 2.2380        | 0.8594        |
| 2     | 2.8480        | 1.0853        |
| 3     | 2.6060        | 0.9855        |
| 4     | 2.7018        | 1.0140        |
| 5     | 2.8670        | 1.0680        |
| 6     | 2.7110        | 1.0024        |
| 7     | 2.7860        | 1.0225        |
| 8     | 2.9392        | 1.0708        |
| 9     | 2.7190        | 0.9834        |
| 10    | 2.5305        | 0.9086        |
| 11    | 2.8780        | 1.0263        |
| 12    | 2.7500        | 0.9734        |

\[ \sum_{j=1}^{s} \hat{S}_j = 12.0000 \]

Fig. 2. Time plot of the transformed series
### Table 7. Estimates of parameters of trend and seasonal indices

| Parameter | Multiplicative model values | Mixed model values |
|-----------|-----------------------------|--------------------|
| $\hat{a}$ | 1.4986 | 1.4986 |
| $\hat{b}$ | 0.0201 | 0.0201 |
| $\hat{S}_1$ | 0.8594 | 0.8594 |
| $\hat{S}_2$ | 1.0853 | 1.0853 |
| $\hat{S}_3$ | 0.9855 | 0.9855 |
| $\hat{S}_4$ | 1.0140 | 1.0140 |
| $\hat{S}_5$ | 1.0680 | 1.0680 |
| $\hat{S}_6$ | 1.0024 | 1.0024 |
| $\hat{S}_7$ | 1.0225 | 1.0225 |
| $\hat{S}_8$ | 1.0708 | 1.0708 |
| $\hat{S}_9$ | 0.9834 | 0.9834 |
| $\hat{S}_{10}$ | 0.9086 | 0.9086 |
| $\hat{S}_{11}$ | 1.0263 | 1.0263 |
| $\hat{S}_{12}$ | 0.9734 | 0.9734 |

$$\sum_{j=1}^{12} \hat{S}_j = 12.0000$$

As given in appendix B and Fig. 2, the slope of the trend line is negative. The transformed series clearly exhibits a decreasing trend.

### 4. CONCLUSION

Results of the study show that 1) the means and variances of the Buys-Ballot table for mixed and multiplicative models are not the same. 2) the expected values of the means are the same both for mixed and multiplicative models 3) the computed values of estimated trend parameters and seasonal indices are the same for the two models, but different when there is no trend. The column variances ($\hat{\sigma}_j^2$) of the Buys-Ballot table depends on the season $j$ only through the square of the seasonal effect $S_j^2$ for mixed model. It is for multiplicative model, a quadratic function of the column $j$ and square of the seasonal effect $S_j^2$.

### COMPETING INTERESTS

Authors have declared that no competing interests exist.

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Appendix A. Buys-Ballot table for the actual data on number of Registered Baptism in Assumpta Cathedral Parish Owerri (2009-2018)

| Year | Jan. | Feb. | Mar. | Apr. | May | Jun. | Jul. | Aug. | Sept. | Oct. | Nov. | Dec. | \( \bar{X}_i \) | \( \sigma_i^2 \) |
|------|------|------|------|------|-----|------|------|------|-------|------|------|------|----------|----------|
| 2009 | 16   | 34   | 21   | 12   | 23  | 24   | 14   | 27   | 13    | 11   | 39   | 27   | 21.75    | 80.93    |
| 2010 | 16   | 24   | 20   | 15   | 11  | 26   | 23   | 18   | 22    | 13   | 28   | 23   | 20.08    | 25.54    |
| 2011 | 8    | 18   | 14   | 14   | 26  | 14   | 24   | 18   | 21    | 19   | 25   | 10   | 17.33    | 36.24    |
| 2012 | 14   | 14   | 18   | 14   | 15  | 26   | 14   | 19   | 16    | 24   | 10   | 20   | 18.0     | 19.8     |
| 2013 | 17   | 21   | 20   | 23   | 16  | 11   | 27   | 22   | 14    | 12   | 5    | 21   | 17.42    | 37.72    |
| 2014 | 8    | 11   | 14   | 23   | 32  | 22   | 13   | 20   | 25    | 13   | 19   | 15   | 17.92    | 46.81    |
| 2015 | 5    | 21   | 9    | 10   | 18  | 15   | 9    | 20   | 18    | 10   | 15   | 16   | 13.83    | 25.95    |
| 2016 | 9    | 18   | 11   | 12   | 12  | 13   | 10   | 27   | 9     | 18   | 18   | 14   | 14.25    | 27.30    |
| 2017 | 6    | 10   | 11   | 18   | 12  | 11   | 10   | 11   | 8     | 11   | 10   | 10   | 10.67    | 7.88     |
| 2018 | 5    | 13   | 9    | 14   | 13  | 8    | 18   | 20   | 9     | 10   | 17   | 10   | 12.17    | 20.15    |

\( \bar{X}_i \) = 10.4, 18.4, 14.3, 15.6, 18.5, 18.0, 17.5, 19.6, 16.1, 13.0, 20.1, 16.6, 16.34

\( \sigma_i^2 \) = 23.38, 51.38, 24.46, 26.04, 50.78, 35.56, 47.39, 27.16, 32.22, 13.11, 90.1, 35.6, 41.37

Source: Assumpta Cathedral Parish, Owerri 2009-2018
Appendix B. Buys-Ballot table for the transformed data on number of Registered Baptism in Assumpta Cathedral Parish Owerri (2009-2018)

| Year | Jan. | Feb. | Mar. | Apr. | May. | Jun. | Jul. | Aug. | Sept. | Oct. | Nov. | Dec. | $\bar{X}_i$ | $\sigma^2_i$. |
|------|------|------|------|------|------|------|------|------|-------|------|------|------|-----------|-----------|
| 2009 | 2.77 | 3.53 | 3.04 | 2.48 | 3.14 | 3.18 | 2.64 | 3.30 | 2.56   | 2.40 | 3.66 | 3.30 | 3.00      | 0.18      |
| 2010 | 2.77 | 3.18 | 2.30 | 2.71 | 2.56 | 3.26 | 3.14 | 2.89 | 3.09   | 2.56 | 3.33 | 3.14 | 2.91      | 0.11      |
| 2011 | 2.08 | 2.89 | 2.40 | 2.64 | 3.26 | 2.64 | 3.18 | 2.89 | 3.04   | 2.94 | 3.22 | 2.30 | 2.79      | 0.15      |
| 2012 | 2.64 | 2.64 | 2.89 | 3.09 | 2.64 | 2.71 | 3.26 | 2.64 | 2.94   | 2.77 | 3.18 | 2.30 | 2.81      | 0.08      |
| 2013 | 2.83 | 3.04 | 2.30 | 3.14 | 2.77 | 2.40 | 3.30 | 2.64 | 2.48   | 1.61 | 3.04 | 2.72 | 2.22      |           |
| 2014 | 2.08 | 2.40 | 2.64 | 3.14 | 3.47 | 3.09 | 2.56 | 2.30 | 2.22   | 2.56 | 2.94 | 2.71 | 2.76      | 0.17      |
| 2015 | 1.61 | 3.04 | 2.20 | 2.30 | 2.89 | 2.71 | 2.20 | 2.30 | 2.08   | 2.30 | 2.71 | 2.77 | 2.43      | 0.16      |
| 2016 | 2.20 | 2.89 | 2.40 | 2.48 | 2.48 | 2.56 | 2.30 | 3.30 | 2.20   | 2.89 | 2.89 | 2.64 | 2.60      | 0.11      |
| 2017 | 1.79 | 2.30 | 2.30 | 2.40 | 2.89 | 2.48 | 2.40 | 2.30 | 2.08   | 2.40 | 2.40 | 2.30 | 2.34      | 0.06      |
| 2018 | 1.61 | 2.56 | 2.20 | 2.64 | 2.56 | 2.08 | 2.89 | 2.30 | 2.20   | 2.30 | 2.83 | 2.30 | 2.37      | 0.12      |
|      | $\bar{X}_{x,j}$ | 2.24 | 2.85 | 2.47 | 2.70 | 2.87 | 2.71 | 2.79 | 2.73   | 2.64 | 2.53 | 2.88 | 2.68      | 2.67      |

$\sigma^2_{x,j}$ 0.24 0.14 0.09 0.10 0.11 0.14 0.18 0.18 0.17 0.08 0.32 0.15 0.13

Source: Assumpta Cathedral Parish, Owerri 2009-2018

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Peer-review history:
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