Shape Coefficient Study of The Extended End-plate Connections Base on Three Parameters Power Model

Wei Liu 1*

1 Department of Architectural Engineering, Hefei University, Anhui 230601, China
*Corresponding author’s e-mail: lwdd001@163.com

Abstract. Based on the three parameter power model, the finite element models of the extended end-plate semi-rigid connections are established by using FEA software ANSYS10.0, and the moment-rotation curve of the semi-rigid connections are obtained. By fitting the moment-rotation curve with numerical analysis method, two approximate formulas for calculating the shape coefficient of semi-rigid connections are obtained, and compared with the existing experimental data. The results shows that semi-rigid connections $M-\theta$ curves obtained by the fitted formula of shape coefficient are in consistent with the $M-\theta$ curve of FEA model and experimental data, which is based on the three parameter power model. The correctness and applicability of the approximate formula of shape coefficient are confirmed, and provides the theoretical basis for the design method of semi-rigid connections in practical engineering.

1. Introduction

In the process of model analysis of practical engineering, although it is convenient to analyze the model by considering beam-column connection as rigid connection or hinge connection, it is not in line with the actual situation. Experiments have shown that any beam-column connection is not simply considered as a rigid connection or hinge connection, but a semi-rigid connection between a rigid connection and a hinge connection[1].

Article 5.1.4 of China’s newly promulgated Standard for Steel Structure Design(GB50017-2017) stipulates that when semi-rigid connections are adopted between beams and columns, the influence of the change of the intersection angle of beams and columns shall be taken into account. In process of internal force analysis, the moment-rotation curve of connections shall be assumed, and the design of connections shall be in conformity with the assumed moment-rotation curve in joint design[2]. Although the Standard for Design of Steel Structures (GB50017-2017) requires that the influence of moment-rotation curves of semi-rigid connections on the mechanical properties of joints should be considered in the model analysis stage, no specific method for obtaining moment-rotation curves is given.

The moment-rotation curve of semi-rigid connections has obvious non-linear characteristics. In reference[1] and reference[3], the three-parameters power model is proposed to simulate the moment-rotation curve of semi-rigid connections.

$$M = \frac{R_n \theta}{[1+(\theta/\theta_n)^n]^\gamma}$$  

(1)
Where, \( R_0 \) is the initial stiffness of the connection, \( \theta_0 \) is the reference plastic angle (\( \theta_0 = M_u / R_u \)), \( M_u \) is the ultimate bearing capacity of the connection, \( n \) is the shape coefficient of three parameters power model. Initial stiffness \( R_0 \) and ultimate bearing capacity \( M_u \) can be obtained by component method and minimum value method\(^4\)[\(^5\)], and can also be obtained by numerical analysis. However, shape coefficient \( n \) has no specific physical meaning. At present, scholars at home and abroad have little research on shape coefficient \( n \), and have not given a formula that can be applied to practical engineering.

In this paper, the extended end-plate semi-rigid connections are selected as the research objects. Based on the three-parameter model and the numerical analysis method, the moment-rotation curve is obtained, the initial stiffness and ultimate bending moment are obtained, and the formula for calculating the shape factor suitable for practical engineering is fitted.

2. Calibration of finite element model

In order to calibrate the finite element model, this paper chooses Dr. Shi Gang's experimental model of Tsinghua University\(^6\), as shown in Figure 1. The section size of the specimens is shown in Table 1. There are four specimens in the original test. Specimens SC1-SC4. The specimen is a symmetrical structure. The beam end is a cantilever end. Fixed boundary conditions are applied at the symmetrical section of the column. By applying load at the cantilever end of the beam, the load gradually increases until the specimen is destroyed.

![Figure 1. Dimensional drawing of the FEA model (Unit:mm).](image1)

![Figure 2. The FEA model.](image2)
Table 1. Section size table of the test model (Unit:mm).

| Specimen number | End-plate thickness | Bolt diameter | End plate stiffener | Stiffening rib of column web | Thickness of stiffeners | $l$ | $h$ | $l_p$ | $b_p$ | $p_1$ | $p_2$ | $p_3$ | $p_4$ | $p_5$ | $p_6$ |
|-----------------|---------------------|---------------|--------------------|-----------------------------|------------------------|-----|-----|-------|-------|------|------|------|------|------|------|
| SC1             | 25                  | 20            | yes                | yes                         | 10                     | 1200| 2000| 500   | 200   | 46   | 54   | 50   | 50   | 62   | 176 |
| SC2             | 20                  | 24            | yes                | yes                         | 10                     | 1200| 2000| 500   | 200   | 46   | 54   | 50   | 50   | 62   | 176 |
| SC3             | 25                  | 24            | yes                | yes                         | 10                     | 1200| 2000| 500   | 200   | 46   | 54   | 50   | 50   | 62   | 176 |
| SC4             | 16                  | 20            | yes                | yes                         | 10                     | 1200| 2000| 500   | 200   | 46   | 54   | 50   | 50   | 62   | 176 |

Figure 1. $h_b$ is the height of beam section; $b_c$ is the width of column section; $h_c$ is the height of column section; $l$ is the length of beam; $h$ is the length of column; $l_p$ is the length of end plate; $b_p$ is the width of end plate; $p_1$ is the transverse distance of bolt; $p_2$ is the transverse distance of bolt; $p_3$ is the longitudinal distance of bolt; $p_4$, $p_5$ is the distance of bolt from the upper and lower flanges of beam; $p_6$ is the distance of web bolt; $t_p$ is the thickness of end plate; $t_s$ is the thickness of stiffening rib.

The section of beam and column is welded H-beam, the section of beam is H300×200×8×12, and the section of column is H300×250×8×12. The bolt grade is 10.9 friction type high strength bolts, there are M20 and M24. Except for friction type high strength bolts, the materials of other specimens and parts are all Q345 steel. The main parameters of the specimens are shown in Table 1 and the size of the specimens is shown in Figure 1. The thickness of stiffening ribs of column web and end plate is 10 mm, and the stiffening ribs of end plate are isosceles right triangle, and the side length is 100 mm.

In the factory, the friction surface of the connected component is shot blasted, and the coefficient of friction surface is 0.44. The pre-tightening forces of 10.9 grade M20 and M24 friction type high strength bolts are 170 kN and 250 kN respectively.

The schematic diagram of the finite element model is shown in Figure 2. The end of the column is fixed and the concentrated load is applied at the end of the beam. In order to accurately simulate the mechanical properties of semi-rigid connections, the meshes at the core of joints, such as column flange, end plate, bolt and beam end connected with bolt, are denser, while the meshes of other parts are sparse.
All materials are isotropic, poisson's ratio is 0.3, and von Mises yield criterion is used for yield criterion. The stress-strain curve of friction type high strength bolt adopts trilinear follow-up strengthening model. The data of three key points are shown in Figure 3. According to the material property test data, the ideal elastic-plastic model is adopted for the stress-strain model of structural steel except for high strength bolts. The yield strength is 363.3 MPa and the elastic modulus is 204227 MPa for steel plate with thickness greater than 16 mm. For steel plate with thickness less than or equal to 16 mm, the yield strength is 391.1 MPa and the elastic modulus is 190707 MPa. The stress-strain curve is shown in Figure 4.

Table 2 gives the comparison and analysis of the initial stiffness and ultimate bending moment obtained by finite element simulation with the experimental values. It can be seen that the errors of the initial stiffness and ultimate bending moment obtained by ANSYS are smaller.

| Specimen number | Initial stiffness $R_{ki}$/(kN·m/rad) | Ultimate bending moment $M_u$/(kN·m) |
|-----------------|---------------------------------------|-------------------------------------|
|                 | Finite Element Value | Test value | Error (%) | Finite Element Value | Test value | Error (%) |
| SC1             | 45265                  | 46096     | 1.81      | 330                  | 322        | 2.48      |
| SC2             | 45096                  | 46066     | 2.1       | 395                  | 390        | 1.28      |
| SC3             | 46957                  | 47469     | 1.08      | 409                  | 410        | 0.24      |
| SC4             | 41059                  | 41634     | 1.38      | 291                  | 296        | 1.69      |

The moment-rotation curves of specimens SC1-SC4 obtained by finite element analysis and the bending moment-rotation curves obtained by experiments are given in Figure 5 – Figure 8. Through comparison, it can be seen that the bending moment-rotation curves obtained by ANSYS are in good agreement with the bending moment-rotation curves obtained by experiments.

Through the above analysis, it can be seen that the results obtained by ANSYS are in good agreement with those obtained by experiments, regardless of the initial stiffness, ultimate bending moment or bending moment-rotation curve of semi-rigid connections with extended end plates, which provides a solid foundation and feasibility for further parametric analysis.

![Figure 5. Moment-rotation curve of model SC1](image1)

![Figure 6. Moment-rotation curve of model SC2](image2)
3. FEA model

In order to obtain the main influencing factors of initial stiffness and ultimate bending moment of semi-rigid joints with extended end plates, parametric analysis is carried out. These parameters include column flange thickness $t_{cf}$, column web thickness $t_{cw}$, beam section height $h_b$, nominal diameter $D$ of bolt (including pretension $P_{re}$), end plate thickness $t_p$, end plate length $l_p$, end plate width $b_p$, stiffener rib thickness $t_s$, bolt transverse spacing $p_2$, bolt longitudinal spacing $p_4$, $p_5$.

Thirty-seven finite element models are obtained by changing the above parameters. Due to the space limitation, Table 3 gives the main parameters of some finite element models. Model 1 is the reference model, while the other 36 models are obtained by changing the parameters on the basis of model 1. The meanings of the parameters in the table are shown in Figure 1.

### Table 3: Section size table of partial finite element model (Unit:mm)

| Model  | Parametric transformation | Column Section | Beam Section | Bolt diameter | End plate thickness | Stiffening rib of column web | Thickness of stiffeners | $l$ | $h_b$ | $b_p$ | $P_{re}$ |
|--------|---------------------------|----------------|--------------|---------------|---------------------|-----------------------------|--------------------------|-----|------|------|--------|
| 1      | Benchmark model           | H-300x300×10x15 | H-200x200×8x12| 20            | Yes                | Yes                         | Yes                      | 10  | 1200 | 2000 | 50     |
| 2      | Whether to Set Stiffening Ribs | H-300x300×10x15 | H-200x200×8x12| 20            | Yes                | No                          | Yes                      | 10  | 1200 | 2000 | 50     |
| 3      |                           |                |              |               |                     |                             |                          | 10  | 1200 | 2000 | 50     |
| 4      |                           |                |              |               |                     |                             |                          | 10  | 1200 | 2000 | 50     |

4. Analysis of finite element results

The initial stiffness and ultimate bending moment of the semi-rigid end-plate connection finite element model are obtained by the 37 finite element models are analysed. The finite element solutions and failure modes of the initial stiffness and ultimate bending moment of some finite element models are given in Table 4.
### Table 4 Initial stiffness and ultimate bearing moment of partial finite element model

| Model Number | Initial stiffness $R_{ki}$ (kN·m/rad) | Ultimate bearing moment $M_u$ (kN·m) | Failure modes |
|--------------|---------------------------------------|--------------------------------------|---------------|
| 1            | 21269                                 | 140                                  | Bolt break    |
| 2            | 18246                                 | 165                                  | Bolt break    |
| 3            | 15019                                 | 164                                  | Bolt break    |
| 4            | 12156                                 | 165                                  | Bolt break    |

5. Study on shape coefficient

Based on 37 finite element models, the bending moment-rotation curve of the model is obtained by numerical analysis. The three-parameter model is used as the reference model, and the shape coefficient $n$ is fitted by the least square principle.

Through the analysis of a large number of finite element models, it is found that the shape factor $n$ is related to the ratio of the ultimate moment $M_u$ to the initial stiffness $R_{ki}$ of semi-rigid connections with extended end plates, that is $\theta_0 = M_u / R_{ki}$ ($\theta_0$ is the reference plastic rotation angle). In order to facilitate the design of practical engineering, equation 2 and equation 3 are fitted to approximate the value of shape coefficient $n$ by using the least square principle.

When $\log \theta_0 > -3.53$:

$$n = 1.398 \log \theta_0 + 4.631$$  \hspace{1cm} (2)

When $\log \theta_0 \leq -3.53$:

$$n = 0.95$$  \hspace{1cm} (3)

6. Verification of test database

In order to further verify the correctness of the approximate formula for calculating the shape factor of semi-rigid connection of extended end-plate, the database of semi-rigid connection of extended end-plate collected by W.F.Chen is selected as the object of verification [1]. The initial stiffness test value, the ultimate bending moment test value and the approximate shape factor of the three-parameter model are obtained respectively. Finally, the bending moment-rotation curve and the bending moment obtained from the test are drawn.
Because some test models in the database lack key data, such as bolt diameter, bolt end spacing, bolt spacing and material property test data, all data can not be verified. There are also some unsatisfactory test results, for example, the bending moment-rotation curve obtained is very different from the expected curve, and some specimens are semi-rigid connection of one-side outrigger end plate. Therefore, 20 test models of semi-rigid connection of outrigger end plate are selected from the database. As the final research object, they are compared with equation 1 and equation 2 to verify the approximate calculation of shape coefficient \( n \). The correctness and applicability of the formula.

Table 5 gives the experimental values of initial stiffness and ultimate bending moment of 20 test models and the approximate solution of shape factor obtained by equation 2 and equation 3. Based on the three-parameter model, the bending moment-rotation curves of 20 test models are plotted and compared with the bending moment-rotation curves obtained from the test. The approximate solution of the shape coefficient \( n \) of the test model is given in each graph. It can be seen that the three-parameter model obtained by equation 2 and equation 3 agrees well with the experimental curve and can be applied to practical engineering design. For space reasons, the bending moment-rotation curves of model V-95 and model V-96 are compared, as shown in Figure 9 and Figure 10.
7. Conclusion
Aiming at the semi-rigid connection of extended end plate, based on the three-parameter model theory, the approximate calculation formula of shape coefficient $n$ is given through the finite element model analysis, and compared with the experimental data, the correctness of the formula is verified, which provides design basis for future engineering design.

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