Simulation of fiber optic sensors of electrical quantities based on Bragg gratings with instrumental errors correction

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Abstract. The principle and an example of simulating a network high-sensitivity sensor of electrical quantities based on fiber Bragg gratings interlinked with a sensor-actuator structure, and a method for instrumental errors correction are described.

1. Introduction
Creation and improvement of sensors based on fiber Bragg gratings (FBG) is one of the most promising areas for development of informational measuring equipment. Among the features that set them apart from other types of passive fiber-optic sensors are compactness, light weight, low cost, immunity to unwanted electromagnetic interference, high reliability, durability, speed, possibility of remote multi-parameter distributed sensing using network technologies (Fiber Optics Sensing System - FOSS), etc. [1-9]. In particular, FOSS is capable of processing information at speeds of up to 5000 measurements per second which is 1000 times better than traditional fiber-optic technologies; moreover, each 40-foot optical fiber provides up to 2000 readout points with adjustable spatial resolution [7]. Pressure, deformation, vibration, micro displacements, temperature, acceleration, high voltage are measured with FBG sensors, and using sensor-actuator structures (SAS) interfaced with FBG can improve metrological abilities of sensors and expand the list of measured values: acoustic fields, presence and concentration chemicals in gas and liquid media, corrosion of metal structures, small electrical and magnetic quantities, etc. [10-15].

At the same time, analysis of publications shows the need to systematize knowledge about the features of operation of FBG sensors with SAS in severe operating conditions in order to develop recommendations for maintaining and increasing their sensitivity, accuracy, and stability when exposed to a wide range of external disturbing factors (EDF).

These difficulties increase significantly when designing such advanced sensors as electric field FBG sensors which, in fact, are not considered in the current literature. Development of theoretical foundations and principles of structural construction ensuring their long-term uninterrupted functioning is required for sensors of this type [16-18]. As part of their vision of this problem, the authors proposed a model of an electric field FBG sensor; to increase sensitivity, a SAS of two ring electrets and an electrostrictive element and a methodology for choosing an instrumental error correction algorithm when exposed to a limited number of dominant EDF was introduced into this
sensor. A simple mathematical model is formulated in this paper; the model can be considered as the basic one for the type of electric field sensor presented in the paper.

2. Principle of electric field sensor operation

Block diagram (Figure 1a) and algorithm for operation of sensor and its components were developed to build the model. The sensor contains optical fiber 1 with FBG 2, electrostrictive element 3, and two electret plates 4 and 5. The first plate is connected to one edge of the electrostrictive element, and the second end of the electrostrictive element freely moves in the opening in the second plate. Optical fiber is rigidly fixed on the electrostrictive element along the axis of its strain. Electret plates are located in the electric field affected zone relative to each other at a distance $l_{EE}$ which is equal to the distance between the electrodes. The electrostrictive element is exposed to the electric field generated by the electrodes and electret plates as well as to external disturbing factors.

Due to pre-stretching of the optical fiber by an amount $\Delta l_0$ in the absence of an external electric field, the zero point of the quadratic characteristic is shifted to a region of high steepness (Figure 1, b).

The resulting electric field intensity between electret plates is

$$E_{EE} = E_0 + E$$

where $E_0$ and $E$ – intensity of the electric field generated by electret plates and electrodes, respectively.

Change in the field intensity leads to increase or decrease of strain of electrostrictive element with an optical fiber attached to it and, as a result, to FBG period change and shift of the central wavelength $\lambda_{BG}$ of the signal reflected from the grating. Geometrical dimensions of the sensor components are determined by the required sensitivity and range of measured values of the electric field intensity $E$, geometrical and physical parameters of the electret plates, and resolving limit provided by the method of reading and analysis.

![Figure 1. Fiber optic electrical field sensor: a – block diagram, b – positional characteristic.](image)

3. Calculation of potential energy of dipole in the field of two oppositely charged rings

In the context of designing sensors that react to magnetic and electrostatic fields, the promising ones are designs where highly nonhomogeneous fields are created. Configuration of the fields should be chosen in such a way that potential energy of interaction of the sensor sensitive element with external electric or magnetic field had spatially separated pronounced minima. Then, sensitive elements placed in the vicinity of two such minima will experience forces directed to the minima points which will lead to fiber optic tension.

We will consider the system shown in Figure 2 for illustrative purposes. Two oppositely charged rings of radius $R$ are located in planes perpendicular to $z$ axis the distance between which is $L$. The origin of coordinates is shifted to the middle of the segment connecting the centers of the rings. The sensing element is simulated by a point electric dipole of $p$ value, which is directed along $z$ axis. Let the upper ring be negatively charged, and the lower ring is positively charged with linear charge...
densities $-\sigma$ and $\sigma$, respectively. In other words, a certain potential difference is applied to the rings. It is obvious that the system has axial symmetry and all points located on any circle with centers on $z$ axis and in planes perpendicular to $z$ axis will be in the same physical conditions. Let us calculate the potential of the electrostatic field of this system in a point with coordinates $(\rho, z)$.

Direct integration can show that the potential of the field created by the lower (positively charged) ring is equal to the following expression:

$$\varphi_2(\rho, z) = -\frac{4R\sigma}{\sqrt{(\rho + R)^2 + \left(\frac{L}{2} + z\right)^2}} K(k_2),$$

(1)

Where $K(x)$ – complete elliptic integral of the first kind:

$$K(x) = \int_0^{\frac{\pi}{2}} \frac{d\varphi}{\sqrt{1 - x^2 \sin^2 \varphi}},$$

(2)

function argument $K(x)$ in the formula (1) is determined by expression:

$$k_2 = \sqrt{\frac{4\rho R}{(\rho + R)^2 + \left(\frac{L}{2} + z\right)^2}}.$$  

(3)

To calculate the potential of the entire system in a given point, let us use the principle of superposition, i.e. the potential of the field created by two rings will be equal to the sum of the potentials created by each ring:

Similarly, the potential created by the upper (negatively charged) ring is calculated:

$$\varphi_1(\rho, z) = -\frac{4R\sigma}{\sqrt{(\rho + R)^2 + \left(\frac{L}{2} - z\right)^2}} K(k_1),$$

(4)

where argument of complete elliptic integral of the 1st kind is equal to the following expression:

$$k_1 = \sqrt{\frac{4\rho R}{(\rho + R)^2 + \left(\frac{L}{2} - z\right)^2}}.$$  

(5)

To calculate the potential of the entire system in a given point, let us use the principle of superposition, i.e. the potential of the field created by two rings will be equal to the sum of the potentials created by each ring:
Intensity of electrostatic field in the system under consideration will be determined by the standard formula:
\[
\vec{E}(\rho, z) = -\nabla \varphi(\rho, z) = -\frac{\partial \varphi}{\partial \rho} \hat{n}_\rho - \frac{\partial \varphi}{\partial z} \hat{n}_z,
\]
where \(\hat{n}_\rho\) – the unit vector perpendicular to \(z\) axis, \(\hat{n}_z\) – the unit vector directed along \(z\) axis. When calculating the gradient in a cylindrical coordinate system, the fact that field has axial symmetry, i.e. it is independent of azimuth angle is taken into account.

Let us place a point electric dipole \(p\) in the point under consideration. Potential energy of the dipole interaction with the electric field of the rings is calculated by the standard formula as a scalar product of dipole moment vector and electrostatic field intensity vector with opposite in sign:
\[
U(\rho, z) = -\langle \vec{p}, \vec{E} \rangle.
\]
We assume that the dipole moment is directed along \(z\) axis:
\[
\vec{p} = p\hat{n}_z.
\]
Inserting formulae (7) and (9) into formula (8), we obtain the following expression for the potential energy of the point dipole interaction with the electrostatic field of a two-ring system:
\[
U(\rho, z) = p \frac{\partial \varphi}{\partial z} = -\frac{4R\sigma}{\sqrt{(\rho + R)^2 + \left(\frac{L}{2} - z\right)^2}} \left(\rho + \frac{L}{2} + z\right) E(k_1) - \frac{4R\sigma}{\sqrt{(\rho - R)^2 + \left(\frac{L}{2} - z\right)^2}} \left(\rho - \frac{L}{2} + z\right) E(k_2),
\]
where \(E(x)\) – complete elliptic integral of the 2\(^{nd}\) kind:
\[
E(x) = \int_0^\frac{x}{x^2} \sqrt{1 - x^2} \sin^2 \varphi \, d\varphi.
\]
Figure 3 shows a typical form of potential energy (10) as a function of two variables.
Radial distance (the distance from the point where dipole is located to $z$ axis) is measured in fractions of $R$, the radius of the rings. Distance along $z$ axis is measured in fractions of $L$, the distance between the rings.

Figure 3 shows that the potential energy of the dipole has two pronounced minima in the vicinity of the rings at a distance $R$ from the system axis. A dipole placed in the vicinity of the ring will be drawn to it, and in case of fixing the opposite end of the optic fiber it will experience tension.

The considered physical system and the mathematical model formulated for it with a highly inhomogeneous potential energy of sensing element can be considered as the reference one for a wide range of electric field sensors designs. An example of the proposed model application is a realistic calculation of the electric field sensor which design is described in section 2.

4. Instrumental error correction
One of the issue of importance in simulating the FBG sensors is the models development with introduction of elements and algorithms to correct the instrumental errors (IE) caused by influence of EDF. In some cases, it allows to improve significantly the basic sensors accuracy and stability without significant changes in design of components and units.

| Methods of FOS/FBG instrumental errors correction | Auxiliary measurement method | Method of reference standards | Algorithms of functional processing | Inverse transformation method |
|-----------------------------------------------|-----------------------------|-----------------------------|-----------------------------------|-------------------------------|
| EDF influencing the IE parameters             | IE                          | referenc devices             | arithmetic processing of digital signal | Digital signal or analog signal into the measurable value |
|                                              |                             | referenc devices             | logic processing of digital signal | Deformation s of FBG into the measurable value |

**Figure 4.** Classification of methods of instrumental errors correction of FBG sensors.

Figure 3 shows classification of IE correction methods that have already been implemented in various types of sensor devices and can be considered in existing and prospective models of FBG sensors [19-21]. These methods include: method of reference standards, inverse transformation method, auxiliary measurement method, test method, and filtering algorithms. Having certain advantages, the listed methods consider and ensure, to different extents, the requirements imposed on the measuring equipment in terms of dimensions, response time, simplicity and uniformity of design,
resistance to extreme EDF, and other parameters. E.g., the method of reference standards requires highly stable sources of measurable units in the form of reference values; it is hardly implementable for a number of FBG sensors, but can be widely used in the electronic unit (interrogator) maintaining one to several hundred sensors in the form of reference signals. The inverse transformation method requires significant complication of the sensor design because it is necessary to introduce accurate and stable inverse converters of a digital code into the measurable value (in case of the active method of inverse transformation). On the other hand, this method is widely and successfully used directly in EU when the incoming signal is interpolated.

Implementation of test methods is limited by difficulties of test formation with regard to structural construction and hardware-time costs, therefore, as a rule, their use in FBG sensors is limited and requires additional research. Algorithms for functional processing provide either duplication of the single measuring channels and introduction of additional measuring channels or introduction of additional measurement cycles in one basic channel. Sometimes the method of auxiliary measurements is referred to the separate group. This method is based on identification and correction of the IE dominant components. This method distinctive feature is in possibility of direct or indirect measurement of EDF parameters functionally related to the transformation error, auxiliary FBG sensors, which contributes to decrease in the number of correction channels. While selecting the optimal algorithms, it is possible to carry out an ordinary statistical processing (averaging), construction of approximating polynomials, logic and logic-digital processing of signals using fixed or flexible program. Statistical processing, as the simplest filtering algorithm, does not always provide the required accuracy even in case of many measuring channels. Logic signal processing allowing to correct IE against generalized criterion without separation of individual components can be considered more effective for FBG sensors.

For measuring the electrical values, the method of auxiliary measurements is best for multisensory measurement systems where one common group of sensors sensitive to specific EDFs allows to correct data from many EQ sensors. Figure 4a shows a structural-algorithmic scheme of IE correction model, each of which is a function of one EDF. Graphically, EDF effect is represented by nonlinear components of the transformation function (Figure 4b). Correction within the whole range of the measurable value $E$ consists in determining and entering the total correction into the measurement result with reduction of each IE component to the measurable parameter.

![Diagram](image)

**Figure 5.** IE correction by auxiliary measurement method: a) structural-algorithmic scheme; b) positional characteristics.

### 5. Conclusion
The most significant scientific results of the article.

1. The principle of simulating FBG sensors with sensory-actuator structures (SAS) is presented; this principle allows to expand the list of measured values and significantly increase the sensitivity of sensors to measured electrical quantities.
2. It is demonstrated that introduction of an electric friction element and electret plates into the FBG sensor of electric field shifts zero point to the area of high steepness and stability, positional characteristics of the sensor.

3. A mathematical model of the sensor with a highly inhomogeneous potential energy of the sensing element in an external electric field is formulated. The model is based on a simple physical system consisting of a point dipole interacting with the electrostatic field of a two-ring system. In particular, the model allows to determine the distance between the rings of electret ensuring SAS maximum sensitivity to an external electric field for the sensor designed in the paper.

3. A method is proposed for choosing an IE correction method for FBG sensors of electrical quantities; it is focused on the features of their functioning. It is shown that implementation of hardware and software tools allows most effective correction of IE dominant components by their direct and indirect measurement at the point of electrical quantities readout.

4. Combined use of correction methods allows to ensure fulfillment of the necessary and sufficient conditions for maximum accuracy of FBG sensors and multisensor networks based on them.

5. Solution of problems concerning increasing the sensitivity and accuracy of existing and new types of FBG sensors with SAS of different operation principles remains relevant.

6. References

[1] Campanella C E, Cuccovillo A, Campanella C, Yurt A, Passaro V M N 2018 Fibre Bragg Grating Based Strain Sensors: Review of Technology and Applications Sensors 18 3115

[2] Hisham K H 2019 Fiber Bragg Grating Sensors: Development and Applications (CRC Press) p 120

[3] Qiao X, Shao Z, Bao W and Rong Q 2017 Fiber Bragg Grating Sensors for the Oil Industry Sensors (Basel) 17(3) 429 DOI: 10.3390/s17030429

[4] Allwood G, Wild G and Hinckley S 2017 Fiber Bragg Grating Sensors for Mainstream Industrial Processes Electronics 6 92 DOI: 10.3390/electronics6040092

[5] Chan H M, Parker A R, Piazza A and Richards W L 2017 Fiber-optic sensing system: Overview, development and deployment in flight at NASA (Publisher: IEEE)

[6] Leonovich G I, Danilin A I, Lobakh A E and Zakharov V N 2017 Fiber-Optic Sensor Network of FBG-Sensors with Complex Redundancy Actual Issues of Radio Electronics and Telecommunication. Proceedings of All-Russian Scientific-and-Technical Conference 17-19

[7] Montserrat F V, Lopez-Amo M 2012 Optical Fiber Networks for Remote Fiber Optic Sensors Sensors 12 3929-3951 DOI: 10.3390/s120403929

[8] Leonovich G I, Oleshkevich S V 2016 Hybrid Sensors on Fiber-Optic Bragg Gratings Izvestia of Samara Scientific Center of the Russian Academy of Sciences 18(4-7) 1340-1345

[9] Alwis L, Sun T and Grattan K 2016 Fibre Grating-based Sensor Design for Humidity Measurement in Chemically Harsh Environment Procedia Engineering 168 1317-1320 DOI: 10.1016/j.proeng.2016.11.359

[10] Ambrosino C 2008 Fiber Bragg Grating Sensors and Piezoelectric Actuators in Co-Located Configuration for Active Vibration Control Applications Smart Sensors and Sensing Technology Lecture Notes Electrical Engineering 20

[11] Kurohiji M, Ichiriyama S, Yamasaku N, Okazaki S, Kasai N, Maru Y and Mizutani T 2018 A Robust Fiber Bragg Grating Hydrogen Gas Sensor Using Platinum-Supported Silica Catalyst Film Journal of Sensors 1-8 DOI: 10.1155/2018/5810985

[12] Park C, Han Y, Joo K-I, Lee Y, Kang S-W and Kim H-R 2010 Optical detection of volatile organic compounds using selective tensile effects of a polymer-coated fiber Bragg grating Optics Express 18(24) 24753-24761 DOI: 10.1364/OE.18.024753

[13] Leonovich G I, Zakharov V N and Gorshkov A I 2017 Development of Fiber-Optic Sensor of Electrical Parameters on the Basis of Bragg Gratings and Software Package for Automatic Modeling of its Parameters III International Conference and Youth School Information Technology and Nanotechnology 1507-1511
[14] Leonovich G I, Zakharov V N and Lobakh A E 2018 Fiber-Optic Sensor of the Constant Electric Field Intensity Proceedings of All-Russian Scientific-and-Technical Conference Actual Issues of Radio Electronics and Telecommunication 65-67

[15] Leonovich G I, Zakharov V N and Gorshkov A I 2017 Development of Fiber-Optic Sensor of Electrical Parameters on the Basis of Bragg Gratings and Software Package for Automatic Modeling of its Parameters III International Conference and Youth School Information Technologies and Nanotechnologies (Samara National Research University) 1507-1511

[16] Borisenkov I L, Fedeech A F, Leonovich G I, Kupriyanov S V, Krutov A A and Zakharov V N 2019 A mathematical model of non-uniform micromechanical deformation of optical fiber section under axisymmetric surface loading Nano- and Microsystem Technology 6 331-340

[17] Leonovich G I, Karpeev S V and Paranin V D 2015 Correction of parameters of fiber-optical systems on the basis of the magneto tunable gradient elements CEUR Workshop Proceedings 1490 133-137 DOI: 10.18287/1613-0073-2015-1490-133-137

[18] Leonovich G I 1998 Optoelectronic digital displacement sensors for severe operating conditions: scientific publication (Samara: Samara State Aerospace University) p 256

[19] Grechishnikov V M, Teryaeva O V 2017 Autocorrection of instrumental errors of multisensor information converters Izvestia of Samara Scientific Center of the Russian Academy of Sciences 19(4) 177-182

[20] Butt M A, Khonina S N and Kazanskiy N L 2018 Highly sensitive refractive index sensor based on hybrid plasmonic waveguide microring resonator Waves in Random and Complex Media DOI: 10.1080/17455030.2018.1506191

[21] Karpeev S V, Pavelyev V S, Khonina S N, Kazanskiy N L, Gavrilov A V and Eropolov V A 2007 Fiber sensors based on transverse mode selection Journal of Modern Optics 54(6) 833-844 DOI: 10.1080/09500340601066125