ELRADGEN 2.0: Monte Carlo generator for simulation of radiative events in polarized elastic electron-proton scattering

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Abstract. A new version of Monte Carlo generator ELRADGEN for simulation of real photon emission in elastic electron-proton scattering is presented. The extensions in the new version include opportunity to deal with polarized particles: longitudinally polarized electron and arbitrary polarized proton. Simulation strategy, specifications of used kinematics, structure of the contributions to the observed cross section, cross-checks, and numerical results for BLAST experimental setup are presented and briefly discussed.

1 Introduction

Exclusive real photon production in lepton-nucleon scattering plays a rather important role in the investigation of the nucleon structure. The measurement of this process in different kinematic regions allows researchers to obtain the information about the generalized parton distributions \[1,2\] and the generalized polarizabilities \[3,4\]. In some cases this process appears as background to lepton-nucleon scattering in elastic \[5\] and inelastic \[6\] channels.

Here we present a new version of the Monte Carlo generator ELRADGEN. The previous one \[7\] was developed for simulation of real hard photon emission from the lepton legs as background to unpolarized elastic electron-proton scattering. In the new version 2.0 we extend the generator to deal with initial polarized particles: longitudinally polarized lepton and arbitrary polarized proton. Both new and previous versions of this generator are based on the results of ref. \[8\] which were obtained using the Bardin-Shumeiko covariant approach for the extraction and cancellation of the infrared divergence \[9\].

2 Kinematics and Method of Generation

For the simulation of exclusive radiative events in polarized electron-proton scattering

\begin{equation}
 e(k_1, \xi_L) + p(p_1, \eta) \longrightarrow e'(k_2) + p'(p_2) + \gamma(k)
\end{equation}

\((k^2 = 0, k_1^2 = k_2^2 = m^2, p_1^2 = p_2^2 = M^2)\) we choose three kinematic variables: a transfer momentum squared \(t = -(k_1 - k_2 - k)^2\), the inelasticity \(v = (p_2 + k)^2 - M^2\), and the azimuthal angle \(\phi_k\) between the planes \((q, k)\) and \((k_1, k_2)\) depicted in Fig. 1 (a). Together with the kinematic variables characterizing Born contribution to elastic scattering

\begin{equation}
 Q^2 = -q^2 = -(k_1 - k_2)^2, \quad S = 2k_1 p_1, \quad \phi,
\end{equation}

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they represent a full set variables for reconstruction of the four–vectors of all final particles in any frame.

The four–vectors $\xi_L$ and $\eta$ in (1) describe the longitudinally polarized electron and the arbitrary polarized proton, respectively. The proton polarization is characterized by two angles $\theta_\eta$ and $\phi_\eta$ as presented in Fig. 1(b). The explicit expressions for polarized vectors can be found in [6].

Simulation of exclusive radiative events requires a separation of the total (or the radiatively corrected) cross section $\sigma_{\text{obs}}$ into the contribution of real hard photon emission $\sigma_{\text{rad}}(v_{\text{min}})$ and remaining part containing Born, soft photon, and additional virtual particle contributions $\sigma_{\text{BSV}}(v_{\text{min}})$. It can be performed by introducing a separation parameter, namely, the minimum inelasticity value $v_{\text{min}}$ that can be associated with missing mass square resolution of the detector [10] when the final proton are not detected. The sum of these two positive parts

$$\sigma_{\text{obs}} = \sigma_{\text{rad}}(v_{\text{min}}) + \sigma_{\text{BSV}}(v_{\text{min}})$$

(3)

does not depend on $v_{\text{min}}$ while $\sigma_{\text{BSV}}(v_{\text{min}})$ and $\sigma_{\text{rad}}(v_{\text{min}})$ do. The numerical details of these dependencies are presented in the next section.

The explicit expressions for these two contributions are similar to those for $\sigma_{\text{rad}}(v_{\text{min}})$ and $\sigma_{\text{non–rad}}(v_{\text{min}})$ from [7] with two exceptions: i) abbreviation $\text{BSV}$ is used instead of $\text{non–rad}$ for the remaining part of the cross section and ii) two structure functions representing the contributions of the polarized parts of the cross sections are additionally used.

The strategy for simulation of an event can be outlined as follows. Two contributions $\sigma_{\text{rad}}(v_{\text{min}})$ and $\sigma_{\text{BSV}}(v_{\text{min}})$ to $\sigma_{\text{obs}}$ are calculated using a predetermined value of the $v_{\text{min}}$. Then the channel of scattering (i.e., the process with or without real hard photon emission) is simulated according to the partial contributions of these two parts to the radiatively corrected cross section. If the channel with the real hard photon emission is chosen, three photonic variables $t$, $v$ and $\phi_k$ are simulated according to their calculated distributions as they contribute to $\sigma_{\text{rad}}(v_{\text{min}})$ (see Fig.2 and ref. [7] for details). Finally using these variables together with Born ones (2) which can be simulated according to the Born cross section or be externally predetermined, the four-vectors of all final particles in any frame are reconstructed.

The cross section of the process (1) when the real hard photon emitted from the lepton legs is expressed through the nucleon form factors which depend only on one integration variable, namely, on $t$. Therefore to have an convenient opportunity to apply this generator to different

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1 Here and later we define $\sigma \equiv d\sigma/dQ^2d\phi$
fits or models of nucleon form factors, the integration over \( t \) has to be used numerical only, while the analytical integration over the other photonic variables \( v \) and \( \phi_k \) is possible and allows to speed up the process of event generation. The analytical integration over these two variables was used in previous version of this generator for unpolarized scattering \([7] \). However when we deal with the arbitrary polarized proton the analytical integration over \( v \) is a rather difficult due to non-trivial dependence of transverse component of the proton polarized vector on this variable. Therefore, in present version of the generator the analytical integration over \( v \) is used for unpolarized particle scattering only.

### 3 Numerical test

One cross-check which has to be done first is to investigate how well the simulated distributions of photonic variables \( t \), \( v \) and \( \phi_k \) reproduce those theoretically calculated. Fig. 2 provides such an illustration for the polarized electron proton scattering at BLAST kinematic conditions \([11] \) \((E_{\text{beam}} = 850\text{ MeV}, Q^2 = 0.2 \text{ GeV}^2, \theta = 48^0)\), with \( \phi = \phi_N \), \( P_LP_N = -1 \) and \( v_{\text{min}} = 10^{-4} \text{ GeV}^2 \).

![Fig. 2. Histograms (points) and corresponding probability densities (solid lines) for variables describing the exclusive real hard photon production in polarized electron proton scattering at BLAST kinematic conditions \([11] \) \((E_{\text{beam}} = 850\text{ MeV}, Q^2 = 0.2 \text{ GeV}^2, \theta = 48^0)\), with \( \phi = \phi_N \), \( P_LP_N = -1 \) and \( v_{\text{min}} = 10^{-4} \text{ GeV}^2 \).](image)

Two studies were illustrated in Tab. \([11] \): i) the investigation of the \( v_{\text{min}} \)-dependence of \( \sigma_{\text{rad}}(v_{\text{min}}) \), \( \sigma_{\text{BSV}}(v_{\text{min}}) \) and their sum; and ii) the comparison of the ratio of the radiatively corrected cross section \( \sigma_{\text{obs}} \) to the Born contribution \( \sigma_0 \) obtained by our generator and Fortran code MASCARAD \([8] \) with the same input parameters as a simplest comparison of these two codes. Specifically, Tab. \([11] \) demonstrates that the observable cross section does not almost change with decreasing \( v_{\text{min}} \) from 1 to \( 10^{-4} \text{ GeV}^2 \) while its components \( \sigma_{\text{rad}}(v_{\text{min}}) \) and \( \sigma_{\text{BSV}}(v_{\text{min}}) \) change essentially: \( \sigma_{\text{rad}}(v_{\text{min}}) \) increases and \( \sigma_{\text{BSV}}(v_{\text{min}}) \) decreases. The comparison with MASCARAD results in a good agreement as well.
Table 1. The $v_{min}$-dependence of the radiative, BSV and observable contributions to electron-proton scattering with polarized target for different spin orientation in the Born units and results of comparison with MASCARAD [8] at BLAST kinematic conditions [11] ($E_{beam} = 850$ MeV, $Q^2 = 0.2$ GeV$^2$, $\theta_{\eta} = 48^\circ$), with $\phi = \phi_{\eta}$.

| $v_{min}$ GeV$^2$ | $P_L/P_N$ | $\sigma_{rad}/\sigma_0$ | $\sigma_{BSV}/\sigma_0$ | $\sigma_{obs}/\sigma_0$ | MASCARAD |
|-------------------|-----------|-------------------------|--------------------------|-------------------------|-----------|
| $10^{-1}$         | 0.01562   | 0.02342                 | 1.018                    | 1.033                   | 1.053     |
| $10^{-2}$         | 0.1299    | 0.1484                  | 0.9029                   | 0.9040                  | 1.033     |
| $10^{-3}$         | 0.2580    | 0.2776                  | 0.7726                   | 0.7727                  | 1.031     |
| $10^{-4}$         | 0.3873    | 0.4070                  | 0.6388                   | 0.6388                  | 1.026     |
| $10^{-5}$         | 0.5192    | 0.5389                  | 0.5046                   | 0.5046                  | 1.024     |

4 Conclusion and Outlook

In the present report the new version of the Monte Carlo generator ELRADGEN for simulation of real photon events within elastic electron-proton scattering generalized for longitudinally polarized lepton and arbitrary polarized target is presented.

Numerical test of new version of this code shows a good agreement with the Fortran code MASCARAD [8] and reveals lack of dependence on minimum inelasticity value $v_{min}$ with accuracy up to 1%. Besides we found that the distributions of generated radiative events are in coincidence with corresponding probability density.

The present approach is rather general and can be extended in many other different ways including i) the development of this generator for transferred polarization from lepton beam to recoil proton [13] for measurement of electromagnetic form-factors of the proton in polarized scattering [14][15]; ii) its further generalization for the investigation of electroweak effects such as axial form factors of the nucleon [16] and parity violation elastic scattering [17]; and iii) its generalization for practical involvement in the experiments with the measurement of generalized parton distribution [12] as well as generalized polarizabilities [14].

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References

1. M. Diehl, Phys. Rept. 388, (2003) 41
2. A. Airapetian et al. [The HERMES Collaboration], arXiv:0911.0095 [hep-ex]
3. P. A. M. Guichon et al., Nucl. Phys. A 591, (1995) 606
4. D. Drechsel et al., Phys. Rev. C 57, (1998) 941
5. L. C. Maximon and J. A. Tjon, Phys. Rev. C 62, (2000) 054320
6. I. V. Akushevich and N. M. Shumeiko, J. Phys. G 20, (1994) 513
7. A. V. Afanasev, I. Akushevich, A. Ilyichev, B. Niczyporuk, Czech. J. Phys. 53, (2003) B449
8. A. V. Afanasev, I. Akushevich, N. P. Merenkov, Phys. Rev. D 64, (2001) 113009
9. D. Yu. Bardin, N. M. Shumeiko, Nucl. Phys. B 127, (1977) 242
10. A. Afanasev, E. Chudakov, A. Ilyichev, and V. Zykinov, Comput. Phys. Commun. 176, (2007) 218
11. D. Hasell et al., Nucl. Instrum. Meth. A 603, (2009) 247
12. S. I. Bilenkaya, S. M. Bilenkii, Yu. M. Kazarinov and L. I. Lapidus, Pisma Zh. Eksp. Teor. Fiz. 19, (1973) 613
13. A. Akhiezer and M. P. Rekalo Sov. J. Part. Nucl. 4, (1974) 277
14. M. K. Jones et al. [Jefferson Lab Hall A Collaboration], Phys. Rev. Lett. 84, (2000) 1398
15. O. Gayou et al. [Jefferson Lab Hall A Collaboration], Phys. Rev. Lett. 88, (2002) 092301
16. T. Gorringe, H. W. Fearing, Rev. Mod. Phys. 76, (2004) 31
17. D. H. Beck, R. D. McKeown, Ann. Rev. Nucl. Part. Sci. 51, (2001) 189