Application Analysis of LS Channel Estimation for Transform Domain Communication System

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Abstract. To reduce the influence of wireless transmission channel on transform domain communication system (TDCS) and improve the channel estimation accuracy, a system model of Least Square (LS) channel estimation on TDCS is established. In this paper, the influence of the unavailable frequency existing in basic function is analysed for the LS channel estimation, then the expression of mean square error (MSE) is derived. After that, the frequency pilot insertion pattern and the interpolation algorithm suitable for TDCS is proposed. The simulation results verify the paper’s theoretical analysis, and the effectiveness of the proposed algorithm is proved by the comparison of the MSE and BER performance.

1. Introduction

In order to meet the rapidly growing demand for wireless broadband communications and provide faster and more reliable QoS (quality of service), future communication systems require more intelligent waveforms to tackle frequency band scarcity and various interferences problems. TDCS as an implementation of cognitive radio (CR), has been widely concerned in the communication field. By sensing the surrounding radio environment and dynamically distributing the available spectrum resources, TDCS can actively avoid interferences[1-2]. However, wireless communication channel has a great impact on TDCS system performance[3], it’s necessary to accurately estimate the wireless communication channel impulse to perform equalization correction on the distorted signal. The accuracy of TDCS channel estimation is the basic for demodulation and equalization at the receiving end.

The Least Square (LS) algorithm is a typical pilot assisted channel estimation method, it has been widely used in OFDM[4]. However the TDCS basic function has spectrum holes that doesn’t carry signal energy, the traditional LS algorithm cannot be directly applied to TDCS channel estimation. Most of the current research on TDCS channel estimation considers setting ideal conditions, lacks the influence analysis of spectrum holes on the accuracy of channel estimation and the design of TDCS pilot pattern. In paper [5], the LS channel estimation algorithm is applied in TDCS, the improved channel estimation algorithm combined with time-sliding average and time-forgetting average are proposed to improve channel estimation performance. In paper [6], a least square iterative channel estimation algorithm is proposed to improve the channel estimation performance of TDCS at very low
SNR. But both of them lack analysis of the influence of spectrum holes on channel estimation performance. In paper [7], a fast Fourier transform TDCS channel estimation method based on cyclic code shift keying (CCSK) modulation characteristics is proposed which can avoid complex RAKE receivers and achieve fast data demodulation. But it isn’t suitable for use in high SNR and complex channel environments. In paper [8], Two TDCS comb pilot pattern designs are given which improves the channel estimation performance in time-varying channels. However it uses traditional methods to recover non-pilot frequency point channel information which will lead to a larger channel estimation error of subcarrier at the edge of spectrum holes.

In this paper, we present the LS channel estimation model for both TDCS in time domain and frequency domain, and analyze the influence of the number of unavailable frequency points on channel estimation accuracy. The frequency domain pilot insertion pattern and the recovery scheme for the channel response estimation at the non-pilot position is given. The rest of this paper is organized as follows: In Section 2, we simply introduce the TDCS channel estimation system and the two pilot insert patterns. Then, the LS method and the application analysis in TDCS is introduced in Section 3, comprehensive simulations and analysis are presented in Section 4 and conclusions are given in Section 5.

2. TDCS Channel Estimation Model

Based on the traditional TDCS transceiver characteristic, this paper designs a TDCS Least Square channel estimation model based on cyclic prefix (CP), as shown in figure 1.

![TDCS channel estimation model](image1)

Figure 1. TDCS channel estimation model

TDCS first senses the electromagnetic spectrum of the environment, the whole spectrum band is divided into $N$ spectral points. A spectrum mask vector $A = \{A_0, A_1, ..., A_{N-1}\}$ is used to tag the availability of such spectral points. The value of $A_k$ is set to 1 if the magnitude is smaller than the given threshold or set to 0 if the magnitude is bigger, as shown in figure 2.

![Spectrum mask vector A schematic diagram](image2)

Figure 2. Spectrum mask vector $A$ schematic diagram
A pseudorandom (PR) poly-phase vector \( \mathbf{E} = \{ e^{j\theta_0}, e^{j\theta_1}, ..., e^{j\theta_{N-1}} \} \) is generated in transmitter side to make the waveform noise-like, which is multiplied one-by-one with the spectrum mask vector \( \mathbf{A} \) to get a spectral vector \( \mathbf{B} = \mathbf{A} \cdot \mathbf{E} \), the basic function \( \mathbf{b} = \{ b_0, b_1, ..., b_{N-1} \} \) is obtained by performing an IFFT operation on the spectral vector \( \mathbf{B} \), and it can be expressed:

\[
 b_n = \lambda \frac{1}{\sqrt{N}} \sum_{k=0}^{N-1} A_k e^{j \frac{2\pi k n}{N}}, n = 0, 1, ..., N - 1
\]

(1)

Where \( \lambda = \sqrt{N / N_A} \) ensures the normalized transmission power. TDCS transmitted signal \( \mathbf{x} = \{ x_0, x_1, ..., x_{N-1} \} \) is generated by M-ary CCSK modulation of the basic function \( \mathbf{b} \):

\[
 x_n = \lambda \frac{1}{\sqrt{N}} \sum_{k=0}^{N-1} A_k e^{j \frac{2\pi k m}{M}} e^{-j \frac{2\pi k n}{N}}
\]

(2)

Where \( m \) represents the \( \log_2 M \) bits of data\(^9\).

The two classical patterns of pilots are block pilots and comb pilots. Figure 3 shows the two different pilots insertion method of TDCS. The idea of block pilots is to insert pilot symbols once every certain time in the available signal, and the pilot symbols occupy all available subcarriers of the TDCS signal. Channel estimation based on block pilots only requires time domain interpolation, and the calculation is simple. However, Comb pilots insert a pilot symbol at a certain frequency, and the pilot symbols occupy each TDCS signal. Therefore, channel estimation based on comb pilots only requires frequency domain interpolation. Since the actual radio channel parameters may change during a TDCS symbol period, comb pilots are more widely used in wireless channel estimation\(^10\).

\[
 h(n) = \sum_{l=0}^{L-1} h_l \delta(n-l)
\]

(3)

Where \( L \) represents the number of channel taps, \( h_l \) is the channel gain of the \( l \) path. After removing CP, the received signal can be expressed as:

\[
 \mathbf{y} = \mathbf{x} \otimes \mathbf{h} + \mathbf{w}
\]

(4)

Where \( \mathbf{y} = [y_0, y_1, ..., y_N]^T \) represents the received signal, \( \mathbf{x} = [x_0, x_1, ..., x_N]^T \) represents the transmitted signal, \( \mathbf{a} \otimes \mathbf{b} \) stands for linear convolution operation, \( \mathbf{w} = [w_0, w_1, ..., w_N]^T \) is a Gaussian
white noise vector with a mean value of 0 and a variance of $\sigma_w^2$. The formula (4) can also be written in matrix form:

$$ y = Gx + w $$

(5)

$G$ is the cyclic matrix composed of the channel impulse response:

$$ G = \begin{bmatrix}
  h_0 & 0 & \ldots & 0 & 0 & \ldots & h_2 & h_1 \\
  h_1 & h_0 & \vdots & 0 & \ldots & \vdots & \vdots & \vdots \\
  \vdots & h_1 & \vdots & 0 & \ldots & \vdots & \vdots & \vdots \\
  h_{L-1} & \vdots & h_0 & 0 & \ldots & \vdots & \vdots & \vdots \\
  0 & h_{L-1} & \ldots & h_l & h_0 & \ldots & 0 & 0 \\
  0 & 0 & \ldots & h_1 & \ldots & \vdots & \vdots & \vdots \\
  \vdots & \vdots & h_{L-1} & \vdots & h_0 & 0 & \ldots & 0 \\
  0 & 0 & \ldots & 0 & h_{L-1} & \ldots & h_l & h_0
\end{bmatrix} $$

(6)

The received signal is transformed into the frequency domain and enters the equalization filter for frequency domain equalization to eliminate the influence of the multipath fading. We uses the Minimum Mean Square Error (MMSE) equalization method. The Frequency signal can be expressed:

$$ 21()HH w D I D D  −=  +H H HYY $$

(7)

Where $D$ is the diagonal matrix formed by the channel frequency response $H$. The time domain signal $y'=\text{IFFT}(Y')$, which is correlated with the conjugate form of the receiving end basic function $b'$ to obtain the final data $d(t)$.

3. TDCS LS Channel Estimation

To transform formula (4) into the frequency domain, we can get the expression:

$$ Y = D_X H + W = D_X Q h + W $$

(8)

Where $X = [X_0, X_1, \ldots, X_{N-1}]^T$ and $Y = [Y_0, Y_1, \ldots, Y_{N-1}]^T$ represents the transmitted signal and the received signal in frequency domain. $D_X$ is the diagonal matrix composed of vector $X$. $Q$ is the normalized Fourier transform matrix in $N \times L$. According to formula (8), the received pilot signal can be expressed as:

$$ Y_p = D_p Q_p h + W_p $$

(9)

Where $Y_p$ and $W_p$ represents the vector consisting of $Y$ and $W$ corresponding to $P$ rows. $D_p$ and $Q_p$ represents the matrix consisting of $D_X$ and $Q$, corresponding to $P$ rows. $P = \{k_0, k_1, \ldots, k_{P-1}\}$ is the set of pilots positions. The LS estimation of $h$ can be expressed as:

$$ \hat{h} = (D_p Q_p)^{-1} Y_p - h + (D_p Q_p)^{-1} W_p $$

(10)

3.1. Application Analysis of Block Pilot in TDCS

Block pilot insertion is to add pilot symbols (0 or 1) evenly to the transmitted data in time domain, and then through CCSK modulation the transmitted signal is formed and enters the wireless channel through the transmitter. The pilot block is the same length as the data block. The pilot set can be expressed as $P = \{k_0, k_1, \ldots, k_{N-1}\}$, the pilot location LS channel estimation in frequency domain can be expressed as:

$$ \hat{H} = Q_p (D_p Q_p)^{-1} Y_p = D_p^{-1} Y_p = H + D_p^{-1} W_p $$

(11)

In wireless communication system, mean square error is usually used to evaluate the accuracy of channel estimation. The mean square error of each subcarriers of block pilot can be expressed as:
\[
MSE = E \left( \| \hat{\mathbf{H}} - \mathbf{H} \|^2 \right) = \text{tr} \left\{ (\hat{\mathbf{H}} - \mathbf{H})(\hat{\mathbf{H}} - \mathbf{H})^H \right\} \\
= \text{tr} \left\{ (D_p^{-1}\mathbf{W}_p)(D_p^{-1}\mathbf{W}_p)^H \right\} = \text{tr} (\sigma^2 \mathbf{I}) \\
= \sigma^2 \sum_{i=0}^{N_p - 1} \frac{1}{|X_i|} \tag{12}
\]

In the formula (12), $\| \mathbf{A} \|$ is Frobenius norm of matrix $\mathbf{A}$, $E$ is mathematical expectation, and $\text{tr} \left\{ \mathbf{A} \right\}$ represents the trace of $\mathbf{A}$. It can be seen from the above formula that the MSE of LS channel estimation is related to noise and pilot power. When the power of pilot subcarrier is limited, the performance of LS channel estimation is greatly affected by noise power, so noise reduction algorithm is needed, such as time domain average noise reduction and IDFT transform domain noise reduction algorithms. Time domain average noise reduction algorithm is obtained by averaging the channel estimates value of $N$ pilot symbols, and the noise variance is reduced to $1/N$ of the original LS channel estimates variance. IDFT transform domain noise reduction is to transform the LS channel estimation value into the time domain by IDFT. Since the multipath delay is concentrated in the front part of the time slot, the latter segment can be set to zero to achieve the purpose of noise reduction. The channel estimation results of several algorithms are compared as shown in figure 4. The simulation channel is cost207 typical urban Channel (cost207TUx6), and the number of basic function subcarriers is 256.

![Figure 4. Channel estimation accuracy comparison](image)

In formula (13), $N_c$ represents the available subcarrier frequency point and $N_u$ represents the unavailable subcarrier frequency point with interference. $\xi_i$ is the channel estimation MSE of the i-th subcarrier. The above formula indicates that the MSE of LS channel estimation with unavailable frequency is related to the noise power and the channel frequency response at the unavailable
subcarrier position. It’s hard to quantitatively analyse the trend of MSE by theoretical formula, the trend of MSE along with the number of available subcarriers will be analysed in the simulation part.

3.2. Application Analysis of Comb Pilot in TDCS

Comb pilot pattern is inserted with a pilot symbol at a certain frequency, and pilot symbols exist in each TDCS transmission signal. Comb pilot channel estimation requires frequency domain interpolation to obtain channel frequency response at data position. The pilot position set can be expressed as \( P = \{k_0, k_1, ..., k_{N_p-1}\} \), \( N_p \) is the number of pilots symbols. According to the formula 10, the channel frequency response of data subcarrier position can be obtained as:

\[
\hat{H} = Q_d (D_p Q_p^{-1})^{-1} Y_r = H + Q_d (D_p Q_p^{-1})^{-1} W_p
\]

\( Q_d \) and \( Q_p \) represents the normalized Fourier transform matrix corresponding to the data position and pilot position, \( D_p \) is the diagonal matrix of pilot symbols in frequency domain. The mean square error of each subcarriers of comb pilot can be expressed as:

\[
MSE = E \left( \| \hat{H} - H \|^2 \right) = \text{tr} \left\{ (\hat{H} - H)(\hat{H} - H)^H \right\} = \text{tr} \left\{ \sigma^2 Q_d (Q_p^{-1} D_p^H D_p Q_p^{-1})^{-1} Q_d^H \right\} = \text{tr} \left\{ \sigma^2 (Q_d Q_p^{-1})^{-1} |D_p|^{-1} (Q_d Q_p^{-1})^{-1} \right\}
\]

In TDCS channel estimation, because the location of interference cannot be predicted and the basic function spectrum is discontinuous, the pilot cannot be inserted at equal spacing or randomly. Therefore, the determination of pilot pattern and the channel information recovery at the non-pilot position are the key points of the TDCS pilot estimation in frequency domain. Here we presents two pilots insertion patterns with a certain number of pilots, which is shown in figure 5:

![Two unequal spacing pilot patterns](image)

**Figure 5. Two unequal spacing pilot patterns**

The first pilot pattern is shown in figure a. This method is still evenly placed, and the pilots located at the unavailable frequency points are moved to the adjacent available subcarriers. The second method is shown in figure b, which is designed in equal spacing of all available subcarriers according to the pilot number.

After estimating the channel response of the pilot position by using the LS method, it is needed to obtain the channel response of the data position by interpolation. Common interpolation methods for channel estimation include linear interpolation, Gaussian interpolation, cubic interpolation, and DFT/IDFT transform domain interpolation. Due to the discontinuity of the available spectrum of the TDCS basic function, the conventional function interpolation method will cause the subcarrier channel estimation error at the edge position of the discontinuous spectrum band to be larger. Transform
domain interpolation is often used to in non-uniform pilots’ interpolation. Because of the non-uniformity of pilots, the direct use of IDFT or DFT will lead to channel estimation position offset and deviation from the actual value. Therefore, a modified transform domain interpolation scheme for TDSCS is proposed in this paper, the steps of the algorithm are as follows:

Step 1: Estimating the channel response at the pilot location by LS: \( \hat{H}_{N_p} = [H_1, H_2, ..., H_{N_p-1}] \).

Step 2: Perform an IDFT transform of \( N_p \) on \( \hat{H}_{N_p} \) to obtain time domain channel estimation \( \hat{h} \).

Step 3: Zero the end of \( \hat{h} \) to make it with \( N \) length: \( \hat{h} = [\hat{h}, 0, 0, ..., 0] \).

Step 4: Perform a DFT transform of \( N \) on \( \hat{H}_N \) to get the frequency domain channel estimation value \( \hat{H}_N \). Replace channel estimation coefficients at pilot positions in \( \hat{H}_N \) with \( \hat{H}_{N_p} \).

Step 5: Perform an IDFT transform of \( N \) on \( \hat{H}_N \) to get the new time domain channel estimation value \( \hat{h} \).

Step 6: Zero the value of \( L+1 \) to \( N \) of \( \hat{h} \) and get the final channel estimation value \( \hat{H} \) after an IDFT transform.

4. Simulation Results

In this part, the number of subcarriers of the basic function is 256, and the 2-ary CCSK modulation is used in the modulation part. Monte Carlo simulation is 10000 times. For the multipath Rayleigh fading channel, a typical cost207TUx6 channel is used, the Doppler shift is 200 Hz.

The LS channel estimation performance comparison under different available subcarrier ratios is shown in figure 6, Where \( \alpha \) is the proportion of available subcarriers number to total subcarriers number.

![Figure 6. LS channel estimation MSE comparison](image)

As shown in the figure 6, when SNR is greater than -1, reducing the number of available subcarriers of the basic function will increase MSE, while when SNR is less than -1, reducing the number of available subcarriers of base function will decrease MSE. According to formula (13), in cost207TUx6 channel environment, noise is a major factor affecting MSE at low SNR. Increasing unavailable subcarriers will reduce the impact of high noise, while at high SNR, the impact of noise on MSE will be weakened, and more available subcarriers will increase the accuracy of LS channel estimation.

The TDACS BER performance comparison of different noise reduction algorithms is shown in figure 7. ‘Know’ is the system BER curve under the accurate estimation of channel response. It can be seen that the TDACS BER without channel estimation is very high, and the communication requirements cannot be met. The joint reduction method has the best BER performance. Compared with the LS channel estimation and time-average noise reduction algorithm, the joint reduction method has a signal-to-noise ratio gain of about 2dB and 4dB.
Figure 7. Different noise reduction BER comparison

Figure 8 is the comparison of channel estimation accuracy between the interpolation method designed in this paper and the existing function interpolation method. The pilots’ number is 64. From the simulation results, the MSE of linear function interpolation is the largest, and the MSE decreases after IDFT noise reduction. The interpolation scheme designed in this paper can accurately estimate the channel coefficients and reduce the impact of noise. Compared with the linear interpolation method after noise reduction, the MSE is reduced by 0.7 orders of magnitude, and it also has better channel estimation performance in low SNR environment.

Figure 8. Different interpolation method MSE comparison

Figure 9 shows the BER performance comparison of several interpolation algorithms in TDCS under low SNR environment. It can be seen that the interpolation scheme designed in this paper has the lowest bit error rate. Compared with linear interpolation and linear interpolation after noise reduction, there are 1 dB and 2 dB SNR gains obtained when the BER is 0.01.

Figure 9. Different interpolation method BER comparison
5. Conclusions
In this work, we design a Pilot-Assisted channel estimation scheme for TDCS, discuss the influence of unavailable frequency points on channel estimation in time domain, and derive the expression of MSE. The pilot pattern of TDCS in frequency domain is given, and the interpolation scheme of non-pilot position is designed. The comparison between the MSE and the BER performance shows that the frequency domain interpolation scheme designed in this paper is more suitable for TDCS channel estimation with unavailable frequency points.

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