ON THE UNBOUNDED OF A CLASS OF FOURIER INTEGRAL OPERATOR ON $L^2(\mathbb{R}^n)$

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Abstract. In this paper, we give an example of Fourier integral operator with a symbol belongs to $\bigcap_{0<\rho<1} S^0_{\rho,1}$ that cannot be extended as a bounded operator on $L^2(\mathbb{R}^n)$.

1. Introduction

A Fourier integral operator is a singular integral operator of the form

$$I(a, \phi) u(x) = \int \int e^{i\phi(x,y,\theta)} a(x, y, \theta) u(y) dyd\theta$$

defined under certain assumptions on the regularity and asymptotic properties of the phase function $\phi$ and the amplitude function $a$. Here $\theta$ plays the role of the covariable.

Fourier integral operators are more general than pseudodifferential operators, where the phase function is of the form $\langle x - y, \theta \rangle$.

Let us denote by $S^m_{\rho,\delta}(\mathbb{R}^{n_1} \times \mathbb{R}^{n_2} \times \mathbb{R}^N)$ the space of $a(x, y, \theta) \in C^\infty(\mathbb{R}^{n_1} \times \mathbb{R}^{n_2} \times \mathbb{R}^N)$, satisfying

$$|\partial_x^\alpha \partial_y^\beta \partial_\theta^\gamma a(x, y, \theta)| \leq C_{\alpha,\beta,\gamma} \lambda^{m-\rho|\gamma|+\delta(|\alpha|+|\beta|)}(\theta), \ \forall (\alpha, \beta, \gamma) \in \mathbb{N}^{n_1} \times \mathbb{N}^{n_2} \times \mathbb{N}^N,$$

where $\lambda(\theta) = (1 + |\theta|)$.

The phase function $\phi(x, y, \theta)$ is assumed to be $a-C^\infty(\mathbb{R}^{n_1} \times \mathbb{R}^{n_2} \times \mathbb{R}^N, \mathbb{R})$ real function, homogeneous in $\theta$ of degree 1.

Since 1970, many efforts have been made by several authors in order to study this type of operators (see, e.g., [1, 3, 6, 7, 8]).

For the Fourier integral operators, an interesting question is under which conditions on $a$ and $\phi$ these operators are bounded on $L^2$ or on the Sobolev spaces $H^s$.

It was proved in [10] that all pseudodifferential operators with symbol in $S^0_{\rho,\delta}$ are bounded on $L^2$ if $\delta < \rho$. When $0 < \delta = \rho < 1$, Calderon and Vaillancourt [2] have proved that all pseudodifferential operators with symbol in $S^0_{\rho,\rho}$ are bounded on $L^2$. On the other hand, Kumano-Go [11] has given a pseudodifferential operator with symbol belonging to $\bigcap_{0<\rho<1} S^0_{\rho,1}$ which is not bounded on $L^2(\mathbb{R})$.

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For Fourier integral operators, it has been proved in [1] that the operator
$I(a, \phi) : L^2 \to L^2$ is bounded if $\delta = m = \rho = 0$. Recently, M. Hasanov [6] constructed a class of unbounded Fourier integral operators on $L^2(\mathbb{R})$ with an amplitude in $S^{0}_{1,1}$.

For $u \in C_{0}^{\infty}(\mathbb{R}^{n})$, the integral operators

$$I(a, S) \varphi(x) = \int \exp(iS(x, \theta)) a(x, y, \theta) \mathcal{F}\varphi(\theta) \, d\theta$$

appear naturally in the expression of the solutions of hyperbolic partial differential equations (see [3, 5, 12]).

If we write formally the expression of the Fourier transformation $\mathcal{F}u(\theta)$ in (1.1), we obtain the following Fourier integral operators

$$I(a, S) u(x) = \int \exp(iS(x, \theta) - y\theta) a(x, y, \theta) u(y) \, dy \, d\theta$$

in which the phase function has the form $\phi(x, y, \theta) = S(x, \theta) - y\theta$. We note that in [13], we have studied the $L^2$-boundedness and $L^2$-compactness of a class of Fourier integral operator of the form (1.2).

In this article we give an example of a Fourier integral operator, in higher dimension, of the form (1.1) with symbol $a(x, \theta) \in \bigcap_{0 < \rho < 1} S^{0}_{\rho,1}$ independent on $y$, that cannot be extended to a bounded operator in $L^2(\mathbb{R}^{n})$, $n \geq 1$. Here we take the phase function in the form of separate variable $S(x, \theta) = \varphi(x) \psi(\theta)$.

2. The boundedness on $C_{0}^{\infty}(\mathbb{R}^{n})$ and on $D'(\mathbb{R}^{n})$

If $\varphi \in C_{0}^{\infty}(\mathbb{R}^{n})$, we consider the following integral transformations

$$(I(a, S) \varphi)(x) = \int_{\mathbb{R}^{n}} \exp(iS(x, \theta)) a(x, \theta) \mathcal{F}\varphi(\theta) \, d\theta$$

$$= \int_{\mathbb{R}^{n} \times \mathbb{R}^{n}} \exp(i(S(x, \theta) - y\theta)) a(x, \theta) \varphi(y) \, dy \, d\theta$$

for $x \in \mathbb{R}^{n}$ and $N \in \mathbb{N}$.

In general the integral (2.3) is not absolutely convergent, so we use the technique of the oscillatory integral developed by L. Hörmander in [3]. The phase function $S$ and the amplitude $a$ are assumed to satisfy the hypothesis

(H1) $S \in C_{\infty}(\mathbb{R}^{n} \times \mathbb{R}^{n}_{0})$ (S real function)

(H2) $\forall \beta \in \mathbb{N}^{N}$, $\exists C_{\beta} > 0$;

$$|\partial_{\beta}^{\beta} S(x, \theta)| \leq C_{\beta}(x) \lambda^{(1 - |\beta|)}(\theta), \forall (x, \theta) \in \mathbb{R}_{x}^{n} \times \mathbb{R}_{\theta}^{n}$$

where $\lambda(\theta) = (1 + |\theta|)$ and $(1 - |\beta|)_{+} = \max(1 - |\beta|, 0)$.

(H3) $S$ satisfies

$$\left(\frac{\partial S}{\partial x} - \frac{\partial S}{\partial \theta} - y\right) \neq 0, \forall (x, \theta) \in \mathbb{R}_{x}^{n} \times (\mathbb{R}_{\theta}^{n} \setminus \{0\})$$

Remark 2.1. If the phase function $S(x, \theta)$ is homogeneous in $\theta$ of degree 1, then it satisfies (H2).

For any open $\Omega$ of $\mathbb{R}_{x}^{n} \times \mathbb{R}_{\theta}^{n}$, $m \in \mathbb{R}$, $\rho > 0$ and $\delta \geq 0$ we set
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\[ S_{\rho, \beta}^m (\Omega) = \left\{ a \in C^\infty (\Omega) : \begin{array}{c} \forall (\alpha, \beta) \in \mathbb{N}^n \times \mathbb{N}^N, \exists C_{\alpha, \beta} > 0; \\ |\partial^\alpha \partial^\beta_\theta a(x, \theta)| \leq C_{\alpha, \beta} \lambda^{\alpha - \rho|\beta| + \delta|\alpha|}(\theta). \end{array} \right\} \]

**Theorem 2.2.** If $S$ satisfies (H1), (H2), (H3) and if $a \in S_{\rho, \beta}^m (\mathbb{R}_x^n \times \mathbb{R}_\theta^n)$, then $I(a, \phi)$ is a continuous operator from $C_0^\infty (\mathbb{R}^n)$ to $C^\infty (\mathbb{R}^n)$ and from $\mathcal{E}' (\mathbb{R}^n)$ to $D' (\mathbb{R}^n)$, where $\rho > 0$ and $\delta < 1$.

**Proof.** See [6], [4, pages 50-51]. \hfill \Box

**Corollary 2.3.** Let $\varphi (x), \psi (\theta) \in C^\infty (\mathbb{R}^n, \mathbb{R})$ two functions, $\psi (\theta)$ is homogeneous of degree 1 ($\psi (\theta) \neq 0$) and $\varphi (x)$ satisfies

\[ (\varphi') (x) \neq 0, \forall x \in \mathbb{R}^n. \]  

Then the operator

\[ (Fu) (x) = \int_{\mathbb{R}^n} e^{i\varphi (x) \psi (\theta)} a(x, \theta) \mathcal{F} u(\theta) \, d\theta, \quad u \in S(\mathbb{R}^n) \]

maps continuously $C_0^\infty (\mathbb{R}^n)$ to $C^\infty (\mathbb{R}^n)$ and $\mathcal{E}' (\mathbb{R}^n)$ to $D' (\mathbb{R}^n)$ for every $a \in S_{\rho, \beta}^m (\mathbb{R}_x \times \mathbb{R}_\theta)$, where $\rho > 0$ and $\delta < 1$.

**Proof.** For the phase function $S(x, \theta) = \varphi (x) \psi (\theta)$, (H1), (H2) and (H3) are satisfied. \hfill \Box

3. The unboundedness of the operator $F$ on $L^2(\mathbb{R}^n)$

In this section we shall construct a symbol $a(x, \theta)$ in the Hörmander space $\bigcap_{0 < \rho < 1} S_{\rho, 1}^m (\mathbb{R}_x^n \times \mathbb{R}_\theta^n)$, such that the Fourier integral operator $F$ can not be extended as a bounded operator in $L^2(\mathbb{R}^n)$.

**Lemma 3.1.** (Kumano-Go [11]). Let $f_0 (t)$ be a continuous function on $[0, 1]$ such that

\[ f_0 (0) = 0, \quad f_0 (t) > 0 \text{ in } [0, 1]. \]  

Then, there exists a continuous function $b(t)$ on $[0, 1]$ such that $b(t)$ satisfies the conditions

\[ \begin{cases} f_0 (t) \leq b(t) \text{ on } [0, 1], \\ b \in C^\infty ([0, 1]), \ b(0) = 0, \ b' (t) > 0 \text{ in } [0, 1], \\ |b''(t)| \leq C_n t^{-n} \text{ in } [0, 1], \quad n \in \mathbb{N}^*, \ C_n > 0. \end{cases} \]

**Definition 3.2.** It is obvious that an operator $A$ is extended as a bounded operator in $L^2(\mathbb{R}^n)$ if and only if there exists a constant $C > 0$ such that

\[ \|Au\|_{L^2} \leq C \|u\|_{L^2} \text{ for any } u \in S(\mathbb{R}^n). \]

**Theorem 3.3.** Let $A$ be an operator, given at least for $x \in [0, \beta^n]$ ($\beta < 1$), by

\[ (Au) (x) = \int_{\mathbb{R}^n} u (g (x) z) \mathcal{F} \Psi (z) \, dz, \quad u \in S(\mathbb{R}^n), \]

we denote here by $[0, \beta^n] = \prod_{j=1}^n [0, \beta[j.}
If \( \Psi \in \mathcal{S}(\mathbb{R}^n) \), \( \Psi(0) = 1 \) and the function \( g \in C^0([0, \beta[^n, \mathbb{R}_+] \) satisfies

\[
\lim_{|x| \to 0^+} \frac{g(x)}{|x|^\delta} = 0,
\]

(3.4) \( \forall i \in \{1, \ldots, n\} \); \( x_i \rightarrow g(x_1, \ldots, x_i, \ldots, x_n) \) is increasing on \( [0, \beta[^n \).

Then the operator \( A \) cannot be extended to a bounded operator in \( L^2(\mathbb{R}^n) \).

**Proof.** Using the Fourier inversion formula in \( \mathcal{S}(\mathbb{R}^n) \), we have

\[
(Au)(x) = \int_{\mathbb{R}^n} u(g(x)z) \mathcal{F}\Psi(z) \, dz, \quad u \in \mathcal{S}(\mathbb{R}^n).
\]

Then there exists a constant \( N_0 > 0 \) such that

\[
(3.5) \quad \left| (2\pi)^{-\frac{n}{2}} \int_{[-N,n]^n} \mathcal{F}\Psi(z) \, dz \right| \geq \beta \quad \text{for any } N \geq N_0.
\]

Setting for \( \varepsilon > 0 \)

\[
u_{\varepsilon}(z) = \begin{cases} (2\pi)^{-\frac{n}{2}}, & \text{for } z \in [-\varepsilon, \varepsilon]^n \\ 0, & \text{for } z \notin [-\varepsilon, \varepsilon]^n \end{cases}
\]

Then, using the density of \( \mathcal{S}(\mathbb{R}^n) \) in \( L^2(\mathbb{R}^n) \), we see that \( (Au_{\varepsilon})(x) \) must be

\[
(3.6) \quad (Au_{\varepsilon})(x) = (2\pi)^{-\frac{n}{2}} \int_{[-\frac{\varepsilon}{\varepsilon}, \frac{\varepsilon}{\varepsilon}]^n} \mathcal{F}\Psi(z) \, dz \quad \text{for } x \in [0, \beta[^n.
\]

By (3.4) for any \( p \in \mathbb{N}^n \) there exists a small \( \varepsilon_p \geq 0 \) such that

\[
\frac{\varepsilon_p}{g(p\varepsilon_p, \ldots, p\varepsilon_p)} \geq N_0 \quad \text{and } p\varepsilon_p \leq \beta.
\]

It follows from the condition (3.4) that

\[
\frac{\varepsilon_p}{g(x)} \geq \frac{\varepsilon_p}{g(p\varepsilon_p, \ldots, p\varepsilon_p)} \geq N_0, \quad \text{holds for } x \in [0, p\varepsilon_p]^n,
\]

so that, using (3.5) and (3.6), we have

\[
(3.7) \quad \|Au_{\varepsilon_p}\|_{L^2}^2 = \int_{\mathbb{R}^n} |Au_{\varepsilon_p}(x)|^2 \, dx \geq \int_{[0, p\varepsilon_p]^n} |Au_{\varepsilon_p}(x)|^2 \, dx \geq \beta^2 (p\varepsilon_p)^n.
\]

Assume that \( A \) is bounded on \( L^2(\mathbb{R}^n) \). According to (3.3), there exists \( C > 0 \) such that:

\[
\beta^2 (p\varepsilon_p)^n \leq \|Au_{\varepsilon_p}\|_{L^2}^2 \leq C^2 (2\varepsilon_p)^n \quad \text{for any } p.
\]

Which is a contradiction. \( \square \)

Let \( K(t) \) be a function from \( \mathcal{S}(\mathbb{R}) \) such that \( K(t) = 1 \) on \([-\delta, \delta] \) \( (\delta < 1) \), \( b(t) \in C^0(0, 1] \) be a continuous function satisfying conditions (3.2) and \( \varphi(x), \psi(\theta) \in C^\infty(\mathbb{R}^n, \mathbb{R}) \) with \( \psi(\theta) \) homogeneous of degree 1 \( (\psi(\theta) \neq 0). \) We assume that \( \varphi(x) \) satisfies

\[
(3.8) \quad |\varphi(x)| \leq C |x| \quad \text{for } |x| \leq 1.
\]

We remark that if the function \( \varphi(x) \) is homogeneous of degree 1, then it satisfies (3.8).

For \( x = (x_1, \ldots, x_n), \theta = (\theta_1, \ldots, \theta_n) \in \mathbb{R}^n, \) set

\[
q(x, \theta) = e^{-i\varphi(x)\psi(\theta)} \prod_{j=1}^n K(b(||x||) |x| \theta_j)
\]
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Lemma 3.4. The function $q \in C^\infty([-1,1]^n \times \mathbb{R}^n_0)$ and the following estimate holds:

$$\forall (\alpha, \beta) \in \mathbb{N}^n \times \mathbb{N}^n, \exists C_{\alpha, \beta} > 0;$$

$$(3.9) \quad \left| \partial_x^\alpha \partial_\theta^\beta q(x, \theta) \right| \leq C_{\alpha, \beta} \lambda^{\alpha - |\beta|} (\theta) \left\{ b \left( \lambda^{-1} (\theta) \right) \right\}^{-|\beta|} \text{ on } [-1,1]^n \times \mathbb{R}^n_0.$$

Proof. We adopt here the same strategy of Kumano-Go [11] lemma 2.

Since $\psi (\theta)$ is homogeneous of degree 1 ($\psi (\theta) \neq 0$), it will be sufficient to check the estimate for $n = 1$ on $[-1,1] \times \mathbb{R}_\theta$, i.e.

$$\forall (j, k) \in \mathbb{N} \times \mathbb{N}, \exists C_{j,k} > 0;$$

$$(3.10) \quad \left| \partial_x^j \partial_\theta^k \left[ e^{-i\varphi(x)\theta} K \left( b \left( |x| \right) x \theta \right) \right] \right| \leq C_{j,k} \lambda^{-j-k} (\theta) \left\{ b \left( \lambda^{-1} (\theta) \right) \right\}^{-k}.$$

Since $K (t) \in S(\mathbb{R})$ and $K^{(n)} (t) = 0$ on $[-\delta, \delta]$, $n \in \mathbb{N}^*$, then

$$(3.11) \quad \left| \partial^l K (t) \right| \leq C_l, \forall l \in \mathbb{N}$$

$$(3.12) \quad \left| \partial^l K^{(n)} (t) \right| \leq C_l n, \forall n \in \mathbb{N}^*, \forall l \in \mathbb{Z}.$$

By Leibnitz’s formula we have

$$\partial_x^j \partial_\theta^k \left[ e^{-i\varphi(x)\theta} K \left( b \left( |x| \right) x \theta \right) \right] = \sum_{j_1+j_2=j, k_1+k_2=k} C^{j,k}_{j_1,j_2,k_1,k_2} \partial_x^{j_1} \partial_\theta^{k_1} \left[ K^{(k_1)} \left( b \left( |x| \right) x \theta \right) \right] \partial_x^{j_2} \left[ e^{-i\varphi(x)\theta} \right],$$

where $C^{j,k}_{j_1,j_2,k_1,k_2} = \frac{\partial^{j_1}}{j_1!} \frac{\partial^{k_1}}{k_1!}$.

Then, by means of (3.9) and (3.2), we have for constants $C^{j,k}_{j_1,j_2,k_1,k_2}$

$$(3.13) \quad \max_{\sum j_1+j_2=j, k_1+k_2=k} \left\{ \left| \partial_x^{j_1} \partial_\theta^{k_1} \left[ e^{-i\varphi(x)\theta} K \left( b \left( |x| \right) x \theta \right) \right] \right| \right\} \leq C_{j,k} K_{j+k} (b \left( |x| \right) x \theta)$$

$$(3.14) \quad K_0 (t) = |K (t)|, \quad K_p (t) = \max_{1 \leq p' \leq p} \left| K^{(p')} (t) \right|, \quad p \in \mathbb{N}^*.$$

Writing $|b \left( |x| \right) x| = |b \left( |x| \right) x \theta| |\theta|^{-1}$ and $|x|^{-1} = |b \left( |x| \right) x \theta|^{-1} b \left( |x| \right) |\theta|$, then there exists a constant $C$

$$(3.15) \quad |x| = \max_{1 \leq p' \leq p} \left| K^{(p')} (t) \right|.$$

Finally from (3.11) to (3.15), we obtain (3.10).
Lemma 3.5. For any continuous function \( b_0 (t) \) on \([1, +\infty[\) such that
\[
 b_0 (t) > 0, \quad \lim_{t \to +\infty} b_0 (t) = +\infty,
\]
then there exists a continuous function \( b (t) \) on \([0, 1]\) which satisfies conditions (3.2) such that we have on \([-1, 1]^n \times \mathbb{R}_\theta^n\)
\[
(3.17) \quad |\partial_x^\alpha \partial_\theta^\beta q(x, \theta)| \leq C_{\alpha, \beta} |\lambda^{\alpha-|\beta|} (\theta) \{ b_0 (\lambda (\theta)) \}| |\beta|, \quad \alpha, \beta \in \mathbb{N}^n.
\]
Proof. Setting
\[
\begin{align*}
  f_0 (t) &= \{ b_0 (t^{-1}) \}^{-1} \quad \text{on } [0, 1] \\
  f_0 (0) &= 0,
\end{align*}
\]
then \( f_0 \) is a continuous function on \([0, 1]\) which verifies condition (3.1). Then, by lemma 3.5 there exists a continuous function \( b (t) \) which satisfies (3.2). Noting that
\[
\{ b (\lambda^{-1} (\theta)) \}^{-1} \leq \{ f_0 (\lambda^{-1} (\theta)) \}^{-1} = b_0 (\lambda (\theta))
\]
this gives (3.17). \( \square \)

Lemma 3.6. Let \( \{ b_l (t) \}_{l \in \mathbb{N}} \) be a sequence of continuous functions on \([1, +\infty[\) which satisfy (3.16). Then, there exists a continuous function \( b_0 (t) \) verifying (3.16), such that, for any \( l_0, \)
\[
b_l (t) \geq b_0 (t) \quad \text{on } [t_0, +\infty[ \quad , \quad l = 1, ..., l_0
\]

Finally, our but is to give an unbounded Fourier integral operator of the form (2.5) with symbol in \( \bigcap_{0 < \rho < 1} S^0_{\rho, 1} (\mathbb{R}_x^n \times \mathbb{R}_\theta^n) \).

Theorem 3.7. There exist a Fourier integral operator \( F \) of the form (2.5), with symbol \( a \in \bigcap_{0 < \rho < 1} S^0_{\rho, 1} (\mathbb{R}_x^n \times \mathbb{R}_\theta^n) \), which cannot be extended to be a bounded operator on \( L^2 (\mathbb{R}^n) \).
Proof. Let \( \phi (s) \) be a \( C_0^\infty (\mathbb{R}) \) function such that
\[
\begin{align*}
  \phi (s) &= 1 \quad \text{on } [-\beta, \beta] \quad (\beta < 1) \\
  \text{supp} \phi &\subset [-1, 1].
\end{align*}
\]
Define a \( C^\infty \)-symbol \( a (x, \theta) \) by
\[
a (x, \theta) = e^{-i \varphi (x) \psi (\theta)} \prod_{j=1}^n \phi (x_j) K (b (|x|) |x| \theta_j) \quad \text{in } \mathbb{R}_x^n \times \mathbb{R}_\theta^n
\]
where \( K (t) \) and \( b (t) \) are the functions of lemma 3.3. Let \( b_l (t) = \log \ldots \log (C_l + t) \) defined on \([1, +\infty[\) and \( C_l \) some large constant, then by lemmas 3.5 and 3.6 we have \( , \forall (\alpha, \gamma) \in \mathbb{N}^n \times \mathbb{N}^n, \forall l \in \mathbb{N} \)
\[
(3.18) \quad |\partial_x^\alpha \partial_\theta^\gamma a (x, \theta)| \leq C_{\alpha, \gamma, l} |\lambda^{\alpha-|\gamma|} (\theta) \{ b_l (\lambda (\theta)) \}| |\gamma| ,
\]
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$C_{\alpha,\gamma,l}$ are constants, so that $a(x,\theta) \in \bigcap_{0<\rho<1} S^0_{\rho,1}(\mathbb{R}^n_x \times \mathbb{R}^n_\theta)$. Furthermore the corresponding Fourier integral $F$ is

$$(F u)(x) = \int_{\mathbb{R}^n_\theta} e^{i\varphi(x)\psi(\theta)} a(x,\theta) \mathcal{F} u(\theta) \, d\theta$$

(3.19)

$$= \prod_{j=1}^n \phi(x_j) \int_{\mathbb{R}^n_\theta} \prod_{j=1}^n K(b(|x|)|x|\theta_j) \mathcal{F} u(\theta) \, d\theta, \quad u \in \mathcal{S}(\mathbb{R}^n).$$

We consider $(F u)(x)$ in $]0,\beta[^n$. Then, using an adequate change of variable in the integral (3.19), we have

$$(F u)(x) = \int_{\mathbb{R}^n_\theta} u(b(|x|)|x|z) \prod_{j=1}^n \mathcal{F} K(z_j) \, dz$$

which has the form of $A$ in theorem 3.3. In addition the function $g(x) = b(|x|)|x|$ satisfies (3.4). Consequently the operator $F$ cannot be extended as a bounded operator on $L^2(\mathbb{R}^n)$.

□

References

[1] Asada, K and Fujiwara, D.: On some oscillatory transformation in $L^2(\mathbb{R}^n)$. Japan. J. Math. vol 4 (2), 1978, p299-361.
[2] Caldéron, A. P. and Vaillancourt, R.: On the boundedness of pseudodifferential operators. J. Math. Soc. Japan 23, 1972, p374-378.
[3] Duistermaat, J.J.: Fourier integral operators. Courant Institute Lecture Notes, New-York 1973.
[4] Egorov, Yu. V. and Shubin, M. A.: Partial differential equations. Vol II, Springer-Verlag, Berlin, 1994.
[5] Egorov, Yu. V. and Shubin, M. A.: Partial differential equations. Vol IV, Springer-Verlag, Berlin, 1994.
[6] Hasanov, M., A class of unbounded Fourier integral operators. J. Math. Analysis and application 225, 1998, p641-651.
[7] Helffer, B.: Théorie spectrale pour des opérateurs globalement elliptiques. Société Mathématiques de France, Astérisque 112, 1984.
[8] Hörmander, L.: Fourier integral operators I, Acta Math. vol 127, 1971, p33-57.
[9] Hörmander, L.: Pseudo-differential operators of type (1.1). Comm. Pure Appl. Math. 33 (1988), p1085-1111.
[10] Hörmander, L.: On the $L^2$ continuity of pseudodifferential operators. Comm. Pure Appl. Math. 24 (1971), p529-555.
[11] Kumano-Go, A.: A problem of Nirenberg on pseudo-differential operators. Comm. Pure Appl. Math. 23 (1970), p115-121.
[12] Messirdi, B. and Senoussaoui, A.: Parametriz du problème de Cauchy $C^\infty$ pour un opérateur différentiel matriciel fortement hyperbolique mani des ordres de Leray-Volević. Journal for Analysis and its Applications. Vol 24, (3), 2005, 581-592.
[13] Messirdi, B. and Senoussaoui, A.: On the $L^2$-boundedness and $L^2$-compactness of a class of Fourier integral Operators. Electronic J. Diff. Equa. Vol 2006, (26), 2006, 1–12.

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