The Effects of Cosmic-Ray Diffusion and Radiative Cooling on the Galactic Wind of the Milky Way

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Abstract

The effects of cosmic-ray diffusion and radiative cooling on the structure of the Galactic wind are studied using a steady-state approximation. It is known that realistic cooling processes suppress the wind from launching. The effects of cosmic-ray diffusion are also supposed to be unfavorable for launching the wind. Both of these effects have not been studied simultaneously in a steady-state approximation of the wind. We find 327,254 solutions of the steady-state Galactic wind and confirm that: the effect of the cosmic-ray pressure depends on the Alfvén Mach number, the mass flux carried by the wind does not depend on the cosmic-ray pressure directly (but depends on the thermal pressure), and the typical conditions found in the Galaxy may correspond to the wind solution that provides metal-polluted matter at a height of ~300 kpc from the disk.

Unified Astronomy Thesaurus concepts: Galactic winds (572); Galactic cosmic rays (567); Milky Way evolution (1052); Star formation (1569)

1. Introduction

Supernovae inject momenta, energies, and cosmic rays (CRs) into the interstellar medium (ISM). These drive the dynamics of the ISM (McKee & Ostriker 1977), and eventually result in cloud formation (Inutsuka et al. 2015). The dynamics of ISM are controlled by the pressures of thermal gas, turbulence, CRs, and magnetic fields, which are comparable to each other (Boulares & Cox 1990; Ferrière 2001). The Galactic archaeological study shows that the overall star formation rate in the Galaxy did not deviate much in the past ~8 Gyr (e.g., Haywood et al. 2016). In this paper, we study this nonobvious star formation history in terms of the mass budget of gaseous matter in the Galaxy.

The star formation rate of the Milky Way (MW), ~1 M⊙ yr−1, indicates a depletion of all gaseous matter in the Galactic disk with a mass of ~10^10 M⊙ within a time of ~1 Gyr (e.g., Kennicutt & Evans 2012). Therefore, to understand the star formation history of the MW, which has been maintained during ~8 Gyr with almost constant rate, we must study the replenishment mechanisms of gaseous matter. Recent observations of metal absorption lines (e.g., Mg II, O VI, etc.) around external galaxies suggest that the circumgalactic medium (CGM) is a huge mass reservoir with a mass of 10^7–10^12 M⊙ (e.g., Tumlinson et al. 2017, and references therein). Since these absorption lines are ubiquitously observed around the host galaxy with a distance more than ~100 kpc, we may naturally consider the galactic wind as a metal transfer mechanism. Once the wind is really driven, the metal-polluted CGM cools significantly by radiative cooling and eventually falls to the host galaxy. Thus, we have to study the possible conditions to launch such an outflow that is a part of the mass cycle, like a galactic fountain flow (Shapiro & Field 1976) but with a scale height of 100 kpc.

Shapiro & Field (1976) suggested the galactic fountain flow based on observations of O VI absorption lines in our Galaxy. Note that for the case of our galaxy, locations of gas responsible for the absorption lines are still not constrained observationally (e.g., Bregman & Lloyd-Davies 2007). Comparing the radiative cooling rate to the reheating rate of gas by supernovae, the galactic fountain flow is considered to have a scale height of ~1 kpc. Breitschwerdt et al. (1991) considered that hot, tenuous gas coexisting with the cool, condensed fountain flow is pushed by the CR pressure and eventually escapes from the Galaxy as the Galactic wind. They showed steady-state solutions without radiative cooling, diffusion of CRs, and other possible heating processes. Radiative cooling was introduced by Breitschwerdt & Schmutzler (1999), but the diffusion of CRs was neglected. Recchia et al. (2016) introduced CR diffusion and heating due to the dissipation of Alfvén waves, but they did not consider radiative cooling. Note that they solved the CR spectrum differing from other studies and showed that the net CR pressure is almost the same as that calculated by fluid approximations. Thus, we study steady-state outflow solutions including radiative cooling, CR diffusion, and heating due to wave dissipation with fluid approximations. Recent numerical simulations study the dynamical role of CRs in launching the Galactic wind (e.g., Girichidis et al. 2018; Hopkins et al. 2018). Girichidis et al. (2018) investigated outflows launched from the midplane of the Galactic disk with solar neighborhood conditions using a local box approximation with a size of 0.5 kpc × 0.5 kpc × ±10 kpc. They found that the CR pressure can efficiently support the launching outflows and strongly affects their phase structure. Their analysis was concentrated at a height of ~3 kpc from the midplane. Hopkins et al. (2018) performed a global simulation in the context of galaxy formation and showed that the CR pressure can drive the Galactic wind (outflow with a height of ~100 kpc) for the

1 However, they studied the effects of Alfvén wave damping using the model of Ipavich (1975), which treats the Galactic system with a spherical geometry acting as a point source of mass and energy at the center. Their main analysis was done by using a model in cylindrical coordinates (the Galactic disk model) without wave damping.
conditions of their simulated galaxy. In their simulation, the inflow is also seen. These results may not be so surprising qualitatively because CRs do not lose their energy compared with the thermal gas; the additional pressure can affect the thermal gas. The aim of this paper is to show the effect of the CR pressure explicitly by analyzing steady-state solutions of the outflow.

This paper is organized as follows. In Section 2, we provide a physical model of the Galactic wind. In Section 3, the wind equation is analyzed, and the role of CRs is discussed. The solutions of the Galactic wind are shown in Section 4. Finally, we summarize our results and discuss future prospects in Section 5.

2. Physical Model

We study a steady outflow taking into account the effects of CRs. Figure 1 shows the model geometry. We approximate the Galactic disk to be axially symmetric and work in cylindrical coordinates. The radial distance of the disk is $R$, and the distance perpendicular to the disk is $z$. There is a hot gaseous layer above the Galactic disk with a thickness of several kpc that may be created due to supernova explosions (the so-called Lockman layer; Lockman 1984; Girichidis et al. 2018). The outflow is solved from $z = 2$ kpc for a range of $1$ kpc $\leq R \leq 10$ kpc in this paper.

Plasma escaping from the galaxy may have a temperature comparable with the virial temperature. Therefore, we presume that the gas within the Lockman layer has a temperature comparable to the virial temperature of the MW for a radius of $\sim 100$ kpc, $T_{\text{vir}} \sim 3 \times 10^6 K(M/M_{\odot})^{1/2}$, and has a number density of $\sim 10^{-3}$ cm$^{-3}$. The existence of gas with such temperature and density at $z = 2$ kpc is implied by X-ray observations (e.g., Nakashima et al. 2018) and numerical simulations (e.g., Girichidis et al. 2018). Supposing a magnetic field with a strength of $B \sim 1 \mu G$, we estimate plasma-$\beta$ as $\beta \sim 60(\mu/0.6)(n/10^{-3} \text{ cm}^{-3})(T/T_{\text{vir}})(B/1 \mu G)^{-2}$, where $\mu$, $n$, and $T$ are the mean molecular weight, number density, and temperature, respectively. Thus, the dynamics of the outflow may not be affected by the magnetic field that may be along the direction of travel of the outflow. We assume that the magnetic field is always along the flow in this paper.

Since the geometry of the MW is that of a thin disk with axial symmetry, the outflow “feels” the gravitational acceleration approximately along the vertical direction $z$ at a relatively small height. Thus, we treat a one-dimensional outflow traveling along the $z$-direction so that the required energy for launching the outflow becomes approximately minimum. When the outflow reaches a height comparable to the size of the Galactic disk, it feels a more multidimensional (i.e., spherical) gravitational acceleration. At such height, the outflow should cross a surface element like that of the spherical coordinate. Hence, following Breitschwerdt et al. (1991), we assume that the outflow travels along a cylindrical flux tube given by

$$A(z) = A_0 \left[1 + \left(\frac{z}{Z_0}\right)^2\right],$$  

(1)

where $A_0$ is the area cross section of the flux tube at $z = 0$, whose actual value is not important in this paper. The transition scale from vertical to spherical is represented by $Z_0 = 15$ kpc.

Along this flux tube, the divergence and gradient operations are represented by

$$\text{div} = \frac{1}{A(z)} \frac{d}{dz} A(z), \quad \text{grad} = \frac{d}{dz}.$$

Thus, the mass and magnetic fluxes are given by

$$\frac{1}{A} \frac{d}{dz} (A\rho v) = 0,$$

(3)

$$\frac{1}{A} \frac{d}{dz} (AB) = 0,$$

(4)

so that $\rho v A = \text{const.}$ and $BA = \text{const.}$, where $\rho$ and $v$ are the mass density and velocity of the fluid, respectively.

The outflow mainly consists of nonrelativistic, thermal particles that are forced by the pressures of CRs and Alfvén waves. The basic equations (equation of motion and energy fluxes) of this system can be given by (see Breitschwerdt et al. 1991),

$$\frac{\rho v}{dz} = -\frac{d}{dz} (P_g + P_t + P_h) - \frac{\rho}{\mu G},$$

(5)

$$\frac{1}{A} \frac{d}{dz} \left[ A \left( \frac{1}{2} \nu + \frac{\rho}{\gamma - 1} \frac{P_h}{\nu} \right) \right] = -\nu \frac{d}{dz} (P_g + P_h) - \rho \frac{d\Phi}{dz} - n^2 A + Q_v,$$

(6)

$$\frac{1}{A} \frac{d}{dz} \left[ A \left( \frac{\gamma c^2 (v + V_\Lambda) P_t}{\gamma c^2 - 1} - \frac{\kappa}{\gamma c^2 - 1} \frac{dP_t}{dz} \right) \right] = (v + V_\Lambda) \frac{dP_t}{dz},$$

(7)
where $P_g$, $V_A$, and $\gamma_g = 5/3$ are the pressure, Alfvén speed, and adiabatic index of the thermal particles, respectively. The gravitational acceleration due to the stars in the galaxy and dark matter halo is $d\Phi/dz$, as given in Section 2.1. The pressures of the CRs and Alfvén waves, $P_r$, and $P_w$, are taken into account for the gas dynamics so that they appear in the equation of motion (5). Equations (7) and (8) describe the transport of CRs and energy density of Alfvén waves, respectively, where $\kappa$ is the spatial diffusion coefficient of CRs, and $\gamma_e = 4/3$ is the adiabatic index for relativistic particles. These sets of equations were derived previously, e.g., by Achterberg (1981a). The dissipation of Alfvén waves is represented by $Q_w > 0$, which results in the heating of the thermal particles (discussed in Section 2.2). $A$ is the radiative cooling function for the thermal particles and is given in Section 2.3. Here, we omit the hadronic losses of CRs that are inefficient in the tenuous medium.\(^5\)

2.1. Gravitational Acceleration

For the gravitational potential formed by the stars in the Galactic bulge and disk, Miyamoto & Nagai (1975) gave a convenient pair of functions in cylindrical coordinates as

$$\Phi_{BD}(R, z) = -\sum_{i=1}^{2} \frac{GM_i}{R^2 + (a_i + \sqrt{z^2 + b_i^2})^2},$$

where the gravitational constant is $G$, and the fit parameters are $a_1 = (0; 7.258)$ kpc, $b_1 = (0.495; 0.520)$ kpc, and $M_i = (2.05 \times 10^{10}; 2.547 \times 10^{11}) M_\odot$ for the bulge and disk, respectively.

For the dark matter halo, we assume a Navarro–Frenk–White (NFW)-like density profile (Navarro et al. 1996) as

$$\rho(x) = \frac{\rho_0}{x(1 + x)} - \frac{\rho_0}{x_e(1 + x_e)},$$

where the dark matter density is $\rho$, the normalized galactocentric radius is $x = r/r_c$, and the parameters $\rho_0$ and $x_e = r_e/r_c$ characterize the total mass and extent of the dark matter halo, respectively. The gravitational acceleration of the dark matter halo is obtained from the Poisson equation,

$$\frac{1}{r^2} \frac{d}{dr} \left( r^2 \frac{d\Phi_{\text{DM}}}{dr} \right) = 4\pi G \rho_\text{DM},$$

as

$$d\Phi_{\text{DM}} = \frac{4\pi G m_\odot}{\rho_\text{DM} \rho}, \quad (x < x_e)$$

where the total mass of the dark matter halo is

$$M_{\text{DM}} = 4\pi r_e^3 \left[ \ln(1 + x_e) - \frac{x_e}{1 + x_e} - \frac{x_e^2}{3(1 + x_e^2)} \right].$$

(13)

Setting the parameters as $\rho_0 = 1.06 \times 10^7 M_\odot \text{kpc}^{-3}$, $M_{\text{DM}} = 10^{12} M_\odot$, and $r_e = 300 \text{kpc}$ (e.g., Sofue 2012), we obtain the core radius as $r_c = 15.408 \text{kpc}$.

The total gravitational acceleration is written as

$$\frac{d\Phi}{dz} = \frac{d\Phi_{BDM}}{dz} + \frac{z}{\sqrt{R^2 + z^2}} \frac{d\Phi_{\text{DM}}}{dr},$$

(14)

and is represented by Figure 2 for $R = 1 \text{kpc}$ (left panel) and $R = 8 \text{kpc}$ (right panel).

2.2. Interplay between Cosmic Rays and Alfvén Waves

CRs can excite Alfvén waves in the background plasma (Lerche 1966, 1967; Wentzel 1968; Kulsrud & Pearce 1969). In the fluid approximation of CRs, the generation rate of the wave can be estimated as $-V_A dP_c/rdz$ (e.g., Kulsrud 2005) and appears on the right-hand side of Equations (7) and (8) as the energy sink and source term, respectively. Note that $dP_c/rdz < 0$.

On the other hand, Alfvén waves in a high $\beta$-plasma can be dissipated via (at least) nonlinear Landau damping (Lee & Volk 1973; Achterberg 1981b). We assume a local equilibrium between the wave generation and dissipation so that the energy sink term of the wave energy in Equation (8), $-Q_w$, is equal to $V_A dP_c/rdz$ (Volk & McKenzie 1981; Zirakashvili et al. 1996).

The dissipated energy is converted to the thermal energy of gas; therefore $Q_w$ appears on the right-hand side of Equation (6) as the heating term.

CRs are scattered by Alfvén waves, which results in the diffusion of CRs. The actual diffusion coefficient of CRs currently remains uncertain. We assume one of the most frequently invoked coefficients that is estimated from the observations of CR compositions (see Hayakawa et al. 1958; Ginzburg & Syrovatskii 1964; Gabici et al. 2019, for a recent review),

$$\kappa = 3.3 \times 10^{28} \text{cm}^2 \text{s}^{-1} \left( \frac{P_M/P_w}{10^6} \right) \left( \frac{B}{1 \mu \text{G}} \right)^{-1},$$

(15)

where $P_M = B^2/8\pi$. For the dependence of $\kappa$, we adopt the case of pitch-angle scattering due to small-amplitude Alfvén waves (Jokipii 1966). The energy density of Galactic CRs may be mostly deposited by CRs with energies of $1$–$10$ GeV (e.g., Acero et al. 2016). Since an Alfvén wave generated by a CR has a wavelength comparable to the associated CR Larmor radius, the wave pressure $P_w$ is almost given by the field disturbance with a wavelength of $\sim 10^{13} \text{cm} (E_{10}/10 \text{GeV}) (B/1 \mu \text{G})^{-1}$. Pitch-angle scattering becomes strong when the CR interacts with the wave that has a comparable wavelength with the CR’s Larmor radius. Thus, we regard that the momentum averaged diffusion coefficient $\kappa$ is represented by the coefficient of $\sim \text{GeV}$ CRs.

2.3. Radiative Cooling

Radiative cooling rates generally depend on the temperature via the ionization balance of gas. In our case, the gas is tenuous, and its temperature may be around the virial temperature of the MW.
as \( T_{\text{ad}} \sim 3 \times 10^6 \, \text{K} (M/10^{12} \, M_\odot) (r/100 \, \text{kpc})^{-1} \). In such situation, atomic line emissions are the most important processes.

For calculations of the radiative cooling function, we approximate that the gas is in the optically thin limit for simplicity. Hence, the atomic ionization state (level population) is given by the ratio of the collisional ionization rate (excitation rate) to the recombination rate (spontaneous transition rate). In this paper, we omit photoionization and the charge–exchange reaction.\(^3\) We calculate the bound–bound, free–bound, free–free, and two-photon decays.

The radiation power of line emission due to the transition from the upper level \( u \) to the lower level \( l \) (bound–bound) is estimated as (e.g., Osterbrock & Ferland 2006)

\[
P_{ul} = n_e n_{\text{ion}} E_{ul} q_{ul},
\]

where \( n_e \) and \( n_{\text{ion}} \) are the number densities of the electron and atom, respectively. The emitted photon energy is the subtraction of the upper energy level \( E_u \) and the lower energy level \( E_l \), \( E_{ul} = E_u - E_l \). The collisional excitation rate \([\text{cm}^3 \, \text{s}^{-1}]\) is given by

\[
q_{ul} = 8.629 \times 10^{-7} \frac{\Omega_{ul}}{g_l} \frac{e^{-\frac{E_{ul}}{kT}}}{\sqrt{T}} ,
\]

where \( g_l \) is the statistical weight of the lower level, \( k \) is Boltzmann constant, and \( T \) is the temperature of gas. The collision strength is

\[
\Omega_{ul} = \frac{8\pi}{\sqrt{3}} \frac{f_{ul}}{E_{ul,\text{Ryd}}} \tilde{g}(T),
\]

where \( f_{ul} \) is the oscillator strength, and \( E_{ul,\text{Ryd}} \) is the photon energy given in the Rydberg unit. The averaged Gaunt factor is \( \tilde{g} \). The value of the averaged Gaunt factor is around unity, and determines the detailed temperature dependence of the excitation rate. The precise data of the excitation rate (or \( g \)) are, however, still not available. In this paper, we use the following fitting function (Mewe 1972)

\[
\tilde{g}(T) = 0.15 + 0.28 \left[ \log \left( \frac{\chi + 1}{\chi} \right) - 0.4 \left( \frac{1 + \chi}{(1 + \chi)^2} \right) \right],
\]

where \( \chi = E_{ul}/kT \), for neutral atoms, while we assume \( \tilde{g} = 1 \) for ionized atoms. Note that the cooling function mainly depends on the ionization balance of atoms rather than \( \tilde{g} \). For the oscillator strength and energy levels, we use the data table given by the National Institute of Standards and Technology. For the calculation of the cooling function, it is sufficient to consider only the allowed transitions from the ground state.

For the calculations of continuum components, we follow Mewe et al. (1986; free–bound) and Gronenschild & Mewe (1978; free–free and two-photon decays). Then, integrating for the photon frequency, we obtain the net radiation power and thus the cooling function.

We solve for the 10 most abundant elements H, He, C, N, O, Ne, Mg, Si, and Fe (Asplund et al. 2009). The ionization cross sections are given by Janev & Smith (1993) for H and Lennon et al. (1988) for the others. The fitting functions for those data are given by the International Atomic Energy Agency. We summarize the literature for the recombination rates in Table 3. We fit those data with Chebyshev polynomials with 20 terms. For hydrogen-like atoms, we follow the fitting function given by Kotelnikov & Milstein (2019).

Figure 3 shows the calculated cooling function \( \Lambda \) in the collisional ionization equilibrium (black line) that is consistent...
with the cooling function given by Cloudy (green; Ferland et al. 2017). Since we omit the charge–exchange reaction, which works at $T \sim 10^8$ K, the function is overestimated due to the survived H and lower ionized ions. Interestingly, our $\Lambda$ shows a depression at $T \sim 3 \times 10^8$ K compared with the function given by Cloudy, around which the gas can be thermally stable ($d\ln \Lambda/d\ln T \gtrsim 2$). This may be due to the updated recombinations rates of Fe. In this paper, we concentrate to study the nature of Galactic wind, deferring further analysis on the depression for future work. In this article, we assume the collisional ionization equilibrium and use this cooling function.

Compton heating and photoionization heating are competitive processes against radiative cooling in general. We neglect them for simplicity. In the case of the MW (irradiated by the metagalactic radiation field), they do not dominate over cooling around the virial temperature unless $n \ll 10^{-5}$ cm$^{-3}$ (e.g., Gnat 2017). The outflow with a mass transfer rate of $\sim \rho v^2 R^2 \sim 0.2 M_{\odot}$ yr$^{-1}$ ($n/10^{-3}$ cm$^{-3}$) ($v/100$ km s$^{-1}$) (R/10 kpc)$^2$ that is comparable to the star formation rate of the MW has a number density of $n \sim 10^{-3}$ cm$^{-3}$ $\gg 10^{-5}$ cm$^{-3}$. When the number density reaches $10^{-5}$ cm$^{-3}$, the heating rate dominates over the cooling rate at $T \gtrsim 2 \times 10^4$ K with the solar metallicity and metagalactic radiation field at the current time (Gnat 2017). The cooling rate becomes comparable to that in the case of collisional ionization equilibrium (CIE) at $T \gtrsim 5 \times 10^4$ K. At a higher temperature, the cooling rate is smaller than the CIE case by a factor of 2 due to the photoionization yielding highly ionized ions. Note that, at a given electron temperature, an increment in highly ionized ions reduces the line intensity because a potential energy of the bound electrons becomes large. Thus, radiative heating is not expected to be important, and we neglect it for simplicity.

Heating due to the dissipation of Alfvén waves can be comparable to radiative cooling. Defining a scale height of the CR pressure as $H_{\text{cr}} \equiv P_{\text{cr}} |dP_{\text{cr}}/dz|^{-1}$, we estimate the ratio of the radiative cooling rate to the heating rate due to the wave dissipation as

$$\frac{n^2 \Lambda}{Q_{\text{w}}} \approx 0.91 \left(\frac{n}{10^{-3} \text{ cm}^{-3}}\right)^{5/2} \left(\frac{B}{1 \mu G}\right)^{-1} \times \left(\frac{P_{\text{cr}}}{0.3 \text{ eV cm}^{-3}}\right)^{-1} \left(\frac{H_{\text{cr}}}{10 \text{ kpc}}\right) \left(\frac{\Lambda}{10^{-22} \text{ erg cm}^{-3} \text{s}^{-1}}\right).$$

(20)

Since the transport of the CRs is determined by a combination of the advection and diffusion as described by Equation (7), the characteristic length scale can be estimated as $H_{\text{cr}} \sim n/\kappa /v \sim 10$ kpc ($\kappa/3 \times 10^{28}$ cm$^2$ s$^{-1}$)($v/10$ km s$^{-1}$)$^{-1}$. Thus, the e gas heating process may be mainly determined by the dissipation of Alfvén waves rather than the radiative process.

3. Wind Equation and Transonic Point Analysis

From the basic Equations (5)–(8), we can derive the wind equation as

$$\frac{v'}{v} = \left(\frac{A'}{A}\right) \left(\frac{C_g^2 + C_w^2}{v^2} - \frac{V_g^2}{v^2} \right),$$

(21)

where the prime indicates the derivative with respect to $z$ (e.g., $v' = dv/dz$), and we define

$$C_g^2 \equiv \frac{\gamma g P_g}{\rho},$$

(22)

$$C_w^2 \equiv \frac{3M_A + 1}{2(M_A + 1)} \frac{P_w}{\rho},$$

(23)

$$V_g^2 \equiv \frac{A}{A'} \left[ \frac{d\Phi}{dz} + \frac{\gamma g - 1}{\rho v} \mathcal{H} + \frac{1}{\rho dz} \right],$$

(24)

$$\mathcal{H} \equiv -n^2 \Lambda - V_{\text{cr}} \frac{dP_{\text{cr}}}{dz}.$$  

(25)

We assume $v' > 0$ for the smooth outflow solution. We will see that the role of CRs is important for this condition.

First, we review the primal properties of a steady flow in a gravitational potential. Considering the simplest case $C_{\text{cr}} = 0$, $P_{\text{cr}} = 0$, $P_w = 0$, and $\mathcal{H} = 0$ (i.e., analogous to a simple adiabatic solar wind solution) as an example, the sign of the wind equation depends on whether the flow is subsonic ($v < C_g$) or supersonic ($v > C_g$). The subsonic flow with a negligible gravitational acceleration ($d\Phi/dz < C_g A')$ shows always $v' < 0$. This reflects simply the adiabatic expansion in vacuum. In contrast to this, a sufficiently large gravitational pull ($d\Phi/dz > C_g A'$) corresponds to a negative numerator on the right-hand side of Equation (21). Since the denominator is negative in the subsonic region, this results in $v' > 0$. The accelerated flow passes the transonic point at which the condition of $v' = C_g^2 = V_g^2$ should be satisfied so that $v'$ is finite and positive. For a supersonic flow, the internal energy is no longer important for the fluid dynamics, and the condition of $v' > 0$ is $d\Phi/dz < C_g^2 A'/A$. This means that the mean kinetic energy of particles in the fluid element $\sim mC_g^2$ is larger than the gravitational potential energy $m\Phi$, i.e., the particles should be gravitationally unbound.

In our case, the CRs can accelerate the fluid by a term of $vdP_{\text{cr}}/dz$ and can heat the fluid via wave generation and dissipation ($V_g dP_{\text{cr}}/dz$). The former makes the rate of adiabatic losses large, that is, too much CR pressure results in $v' < 0$ for a subsonic flow. On the other hand, for a supersonic flow, this additional pressure support helps to make $v' > 0$. The latter acts in the opposite sense. The heating acts against the radiative or adiabatic cooling; thus a sufficient amount of CRs tends to make $v' > 0$ for a subsonic flow. For a supersonic flow, too much heating results in too high thermal pressure that dominates over the ram pressure of the fluid. In such a case, the condition of the smooth wind solution $v' > 0$ everywhere is not satisfied due to the existence of a pressure bump. These effects can be seen by rewriting the wind equation. Considering a subsonic flow as an example, from the condition of $v' > 0$, we obtain

$$\left(\frac{\gamma g - 1}{M_A} - 1\right) \frac{dP_{\text{cr}}}{dz} < \rho \frac{d\Phi}{dz} - \frac{\gamma g - 1}{v} n^2 \Lambda - A' \frac{C_g^2 + C_w^2}{A}.$$

(26)

Thus, when $M_A \gg \gamma g - 1$ ($M_A \ll \gamma g - 1$), the CR pressure acts as the term of $vdP_{\text{cr}}/dz$ (the heating), and $dP_{\text{cr}}/dz$ has an upper (lower) limit. For a supersonic flow, the inequality reverses. As we discussed in Section 2, plasma-$\beta$ has $\beta \sim 60$ at the bottom.
region of the outflow. Thus, the Alfvén Mach number is about \( M_A \sim \sqrt{\beta} \sim 8 \gg 2/\beta \). Hence, the CRs contribute to realize the steady-state outflow via the term \( vdP_{cr}/dz \), which acts as an additional pressure support for a supersonic flow. Note that the above argument appears because we adopt the steady-solution with \( v' > 0 \) everywhere.

Finally, we analyze the conditions of the transonic point following Breitschwerdt et al. (1991).\(^4\) Defining \( C_s^* = C_s^2 + C_w^2 \), we rewrite the wind equation as

\[
\frac{dv}{dz} = \frac{v}{z - v + C_s}(v - C_w),
\]

where

\[
N = zA'(C_s^2 - V_g^2).
\]

At the transonic point \( z = z_{tr} \), the conditions \( v^2 = C_s^2 \) and \( C_s^2 = V_g^2 \) should be satisfied simultaneously so that the velocity gradient is finite and positive. We obtain the gradient at \( z_{tr} \) by the linearization of the equation around the transonic point as

\[
\frac{dv}{dz} \approx \frac{v}{2z_{tr}} \left( \frac{1}{v - C_s} \right) \left( z - z_{tr} \right) N_z
+ \left( \frac{v}{v - v_{tr}} \right) N_{\eta}.
\]

(29)

where \( N_z = (\partial N/\partial z)_z \) and \( N_{\eta} = (\partial N/\partial v)_z \). Then, introducing the transformations

\[
\eta = \frac{v - C_s}{C_s} \quad \text{and} \quad \zeta = \frac{z - z_{tr}}{z_{tr}},
\]

we obtain

\[
\frac{d\eta}{d\zeta} = \alpha \zeta + \beta \eta,
\]

(30)

\[
\alpha = \frac{2z_{tr}}{C_s} N_z,
\]

(31)

\[
\beta = \frac{1}{2C_s} N_{\eta}.
\]

(32)

Substituting \( \eta = w\zeta \), we find the solution of this differential equation as

\[
\left( \frac{dv}{dz} \right)_w = \frac{C_s}{z_{tr}} v,
\]

(33)

\[
w = \frac{\beta \pm \sqrt{\beta^2 + 4\alpha}}{2}.
\]

(34)

Since we consider the outflow solution \( (dv/dz > 0) \), we take the positive sign in front of the square root of \( w \). We show the expressions of \( N_z \) and \( N_{\eta} \) in the Appendix.

To find the wind solution, we set the values \( (v, \rho, P_s, P_{cr}, dP_{cr}/dz, P_{cr}, \text{ and } B) \) at the transonic point as a boundary condition. The location of the transonic point is given by the condition \( v^2 = C_s^2 = V_g^2 \) because \( V_g^2 \) contains the term of \( dB/dz \), which is a given function of \( z \). The velocity gradient at the transonic point is calculated by using Equation (34), and then

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4 The original definition of \( C_s \) by Breitschwerdt et al. (1991) includes the CR pressure by neglecting the CR diffusion. Recchia et al. (2016) considered the CR diffusion and also defined \( C_s \) with the CR pressure. In such definition, however, the expression of \( C_s \) shows an apparent diverging point. To avoid this, we treat the CR pressure separately.

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Table 1

| Parameter Range of the Boundary Conditions, Where \( n_u, T_u, P_{cr,u}, \text{ and } H_{cr,u} \) are the Number Density, Temperature, Pressure of CRs, and Scale Height of CRs \( (H_{cr,u} = f_{cr,ud}/dT_{cr,u}/dz) \) at the Transonic Point |
|------------------|------------------|------------------|------------------|
| \( n_u \) | \( 10^{-3} \) cm\(^{-3} \) | \( 10^8 \) K | 0.1 eV cm\(^{-3} \) |
| \( T_u \) | \( 10^{-2} \) cm\(^{-3} \) | \( 10^7 \) K | 10 eV cm\(^{-3} \) |
| \( P_{cr,u} \) | \( 10^4 \) K | 0.1 eV cm\(^{-3} \) | 100 kpc |
| \( H_{cr,u} \) | 16 | 16 | 16 |

Note. Each parameter ranges from \( n_u \) to \( P_{cr,u} \) divided by \( N \) in the logarithmic space (see Equation (35)).

The gradients of other values \( (dP_{cr}/dz, dP_{cr}/dz, dP_{cr}/dz) \) are obtained. Integrating these differential equations from the transonic point toward the top and bottom boundaries, we can find a smooth solution with \( v' > 0 \) at arbitrary \( z \). Note that this method ensures the flow passing through the transonic point; however, it is not guaranteed whether the solution continues at both boundaries with a positive and finite \( v' \). For example, some boundary conditions lead to a flow showing \( v^2 \rightarrow C_s^2 \rightarrow V_g^2 \) at the subsonic region between the transonic point and bottom boundary that results in a divergence of the velocity gradient. Such behavior results from the effects of radiative cooling, and the topology of the flow is different from the case of no radiative cooling (e.g., Breitschwerdt et al. 1991; Recchia et al. 2016). We can efficiently exclude such “failed” solutions by starting from the transonic point.

4. Results

To derive the outflow solutions, we set boundary conditions at the transonic point \( z = z_{tr} \). We fix two parameters as \( B_0 = 1 \) \( \mu \)G and \( P_{cr,u} = 1.56 \times 10^{-8} \) eV cm\(^{-3} \) (i.e., \( k_T = 5.3 \times 10^{38} \) cm\(^{-2} \) s\(^{-1} \)). The subscript “\( u \)” indicates the values at the transonic point. We have chosen logarithmically spaced \( N \) different values of other parameters for \( n, T, P_{cr}, \text{ and } H_{cr} \), that can be expressed by the following formula:

\[
f = \log f_m + \frac{i - 1}{N} \log f_{m1} (i = 1 - N),
\]

(35)

where \( f \) symbolically indicates \( n, T, P_{cr}, \text{ and } H_{cr} \), and \( f_m, f_{m1}, \text{ and } N \) are summarized in Table 1. Note that \( dP_{cr,u}/dz = -P_{cr,u}/H_{cr,u} \). Then, we integrate Equations (5)–(8) from the transonic point toward the bottom boundary \( z_{bt} = 2 \) kpc and toward the top boundary \( z_{tp} = 350 \) kpc by the forth Runge–Kutta method, respectively. For those boundary conditions, the successful outflow solutions we consider satisfy \( v' > 0 \), \( P_{cr'} < 0 \), and \( T > 10^4 \) K at \( z_{bt} < z < z_{tp} \), and \( v < C_s \) at \( z_{bt} < z < z_{tp} \), and \( v > C_s \) at \( z_{bt} < z < z_{tp} \), where the subscripts “\( bt \)” and “\( tp \)” denote the values at the bottom boundary and top boundary, respectively. We apply this procedure for 10 horizontal positions, \( R = (1; 2; 3; 4; 5; 6; 7; 8; 9; 10) \) kpc, and find 327,254 solutions in total as a result.

We discuss the nature of the outflow using the solution at \( R = 8 \) kpc for \( n_u \approx 2.74 \times 10^{-3} \) cm\(^{-3} \), \( T_u \approx 2.74 \times 10^6 \) K, \( P_{cr,u} \approx 0.316 \) eV cm\(^{-3} \), and \( H_{cr,u} \approx 32.2 \) kpc as a representative result. Note that \( P_{cr,u} = n_u k_T T_u \approx 0.614 \) eV cm\(^{-3} \). The values at the bottom boundary of this solution are \( n_u \approx 5.23 \times 10^{-3} \) cm\(^{-3} \), \( T_u \approx 4.70 \times 10^6 \) K, \( P_{cr,bt} \approx 0.456 \) eV cm\(^{-3} \), \( H_{cr,bt} \approx 33.9 \) kpc, and \( B_{bt} \approx 1.44 \mu \)G, respectively. The left panel of Figure 4 shows the velocities of \( v \) (solid line), \( C_s \) (dots), and \( V_g \) (dashed line). Note
that \( C_\infty \simeq C_g = \sqrt{\frac{\gamma}{\gamma - 1}} \frac{P_g}{\rho_g} \). The transonic point is located at \( z_{tr} \simeq 9.54 \text{ kpc} \). \( V_g \) becomes negative at \( z \sim 20 \text{ kpc} \) because in Equation (24), the term of \( \nu dP_c / dz \) dominates over the gravitational acceleration (i.e., \( \nu' \approx P_c / \rho v \)). \( C_g \) rapidly decreases from \( z \sim 20 \text{ kpc} \) due to radiative cooling. The minimum temperature is about \( 1.043 \times 10^4 \text{ K} \), at which \( \beta \simeq 1.070 \). Then, the cooling is balanced by the heating due to the wave dissipation. Since the cooling rate \( n^2 \Lambda \) decreases with increasing velocity and area cross section of the flux tube (\( \rho \propto 1 / \sqrt{\Lambda} \)), the outflow is slightly heated due to wave dissipation. The outflow begins to be adiabatically cooled again from \( z \sim 100 \text{ kpc} \) because the CR pressure drops exponentially. This exponential decay of \( P_{cr} \) results from the effects of CR diffusion. Note that, if we consider a case of no diffusion of CRs (\( \kappa = 0 \)), the CR transport Equation (7) could be rewritten as \( P_{cr} \propto [(\nu + V_\Lambda) \lambda]^{-1} \) (e.g., Breitschwerdt et al. 1991). The right panel of Figure 4 shows the pressures of \( P_{cr} \) (solid line), \( P_g \) (dots), and \( P_w \times 10^5 \) (dashed line). The CR pressure is well represented by \( \exp(-z//H_{cr, br}) \) for this solution.

We consider the effects of the amount of CRs. Figure 5 shows three solutions at \( R = 8 \text{ kpc} \) for \( P_{cr, br} \simeq 0.316 \text{ eV cm}^{-3} \) (blue), \( P_{cr, br} \simeq 0.750 \text{ eV cm}^{-3} \) (green), and \( P_{cr, br} \simeq 2.37 \text{ eV cm}^{-3} \) (red). The other parameters are the same as in Figure 4. Table 2 summarizes the values at the bottom boundary for these three solutions. As shown in Table 2, the thermal pressure \( P_{g,bt} \) is anticorrelated to the CR pressure \( P_{cr, br} \). Since the acceleration by large pressures overwhelming the gravitational acceleration at the subsonic region makes \( v' < 0 \) (see Section 3), the total pressure may be regulated to a certain degree to satisfy the given boundary conditions at the transonic point. Note that the thermal pressures at the transonic points are the same as each of the three solutions (i.e., the total pressures are different); therefore, the regulation of the total pressure at the bottom boundary is not so obvious.

Figure 6 shows the total pressure \( P_g + P_{cr} \) (left panel) and ratio of the pressures \( P_g / P_{cr} \) (right panel) for these three solutions. In the case of the relatively small CR pressure (blue line), a relatively large thermal pressure \( P_g = nkT \) is required. Since the temperature should be around the virial temperature, this requirement results in a relatively large density. Then, the outflow suffers radiative cooling at a rate of \( n^2 \Lambda \), and the CR pressure becomes the dominant component to accelerate the outflow. Contrary to this, when the CR pressure is relatively large (red line), the required thermal pressure becomes low, which results in a relatively small density. Since the cooling rate depends on \( n^2 \) and the heating rate depends on \( P_{cr} \), the temperature (and thermal pressure) remains high. Thus, a smaller (larger) CR pressure results in a slower and colder (faster and hotter) outflow. Note that the amount of CRs at a local point is also determined by the diffusion coefficient. Figure 7 shows the spatial profile of the coefficient \( \kappa(z) / \kappa_w \). The hotter wind results from a larger \( \kappa \) because the CRs can reach at a higher \( z \).

Here, we confirm the above discussion from the overall trend of the solutions. Figure 8 shows the values at the bottom and top boundaries for the derived solutions at \( R = 8 \text{ kpc} \). The left panel shows the relation between \( P_{g,bt} + P_{cr, br} \rho_{bt} v_{bt} \) and \( T_{sp} \).
The total pressure range is $1 - 10 \text{ eV cm}^{-3}$. The mass flux shows a roughly linear dependence on the total pressure. As shown in Figure 9, this mainly results from the dependence of $v_{\text{bx}}$ on $P_{\text{tot}}$, while $v_{\text{bx}}$ itself does not depend on the total pressure (a scattering of $v_{\text{bx}}$ is about a factor of 2). Since the temperature should be above the virial temperature (indeed our solutions show $T_{\text{vir}} \approx 10^{6} - 10^{7} \text{ K}$), this dependence simply reflects the equation of state $P_\text{g} = nkT$. Then, a faster outflow at the subsonic region can reach a larger height, at which the gravitational acceleration is negligible compared with the rate of the adiabatic cooling, so it tends to make $v' < 0$. Thus, the larger pressure results in a more massive outflow with a velocity rather than accelerating the outflow with fixed mass. The right panel of Figure 8 shows the relation between $P_{\text{g,bt}}/P_{\text{cr, bt}}$, $\rho_{\text{ht}}$, and $T_{\text{g}}$. As we discussed above, a relatively small (large) CR pressure results in a cold (hot) outflow. Thus, the mass flux is determined by the total pressure, while the properties of the outflow (especially the temperature) are controlled by the amount of CRs.

We discuss the fate of the outflow in the three cases summarized in Table 2. Figure 10 shows the ratio of the cooling rate $\dot{\Lambda}_{\text{cool}}$ to the heating rate $n^2 \Lambda / Q_\text{d}$ (left panel) and the cooling time $t_{\text{cool}} = kT / n \Lambda$ (right panel). Since the heating due to wave dissipation decreases with the exponential decrement in the CR pressure, the gas eventually cools. Considering the solution with $P_{\text{cr, at}} \approx 0.316 \text{ eV cm}^{-3}$ (blue) as an example, the cooling time is shorter than 1 Gyr. Such cooled gas eventually decelerates due to the gravitational pull, that is, we cannot regard that the solution continues to infinity. It may be difficult to think that such solution represents a stable and steady-state solution, rather the solution may be related to phenomena for which the gas falls back to the Galactic disk. In the following, we denote the solutions with $t_{\text{cool}} < 1 \text{ Gyr}$ and $n^2 \Lambda / Q_\text{d} > 1$ at the top boundary as “fall back” solutions. Note that, since the number density becomes smaller than $10^{-2} \text{ cm}^{-3}$ at $z \gtrsim 100 \text{ kpc}$, the radiative heating rate can dominate over the cooling rate at $T \gtrsim 2 \times 10^{4} \text{ K}$ (see Section 2.3). The wind could be isothermal with $T \approx 2 \times 10^{4}$, and the isothermal temperature increases (decreases) with decreasing (increasing) number density. Thus, if the “fall back” phenomena are related to a condensation of gas, the radiative heating may not affect the expectation of the “fall back” phenomena. Condensation and precipitation are indeed likely processes of a thermally unstable gas.

Finally, we estimate the total mass carried by the outflow per unit time as

$$\Delta M_i \equiv (\rho v) x_{i} \times 2 \pi R_{i} \Delta R_i \times 2, \quad (36)$$

$$M_{\text{tot}} = \sum_{i=1}^{10} \Delta M_i \quad (37)$$

$$R_{i} = i \text{ kpc}, \quad \Delta R_i = 1 \text{ kpc}, \quad (38)$$

where $i$ denotes the horizontal location $R_{i} = i \text{ kpc}$. The carried mass per unit time at a radius of $R = R_i$ is $\Delta M_i$, which is estimated from the mass flux $\rho v$ integrated along the axial symmetric ring at $R = R_i$ with a width of $\Delta R_i = 1 \text{ kpc}$. The factor of 2 indicates the two directions of the outflow, $+z$ and $-z$. The total carried mass per unit time is $M_{\text{tot}}$. Since we suppose axial symmetry, this estimation may give an upper limit of the mass carried by the outflow. Note that $\Delta M_i$ and $M_{\text{tot}}$ were defined in a way similar to previous studies (e.g., Breitschwerdt et al. 1991). Since we have many possible sets of solutions, we derive a statistical average. Figure 11 shows the average of $M_{\text{tot}}$ as a function of the CR pressure at the bottom boundary. The green boxes indicate the average for all $M_{\text{tot}}$.
Figure 12 shows the averaged number density and temperature at the bottom boundary for the CR pressure of \( P_{\mathrm{cr, bt}} \) larger than the heating rate at the top boundary have a short cooling time of \( \tau_{\text{cool}} \) at the vertical height of \( z \). Figure 8 shows the averaged number density and temperature of\( \rho_n \) and \( T_n \) (left panel) and mass flux \( \rho_v \) (right panel): The relation between the ratio of pressures \( P_{\mathrm{g, bt}}/P_{\mathrm{cr, bt}} \) and mass flux \( \rho_v \) The color shows the temperature \( T_n \) at the top boundary \( (z = z_{\text{top}}) \).

The temperature is around the virial temperature in both cases. The boundary conditions are consistent with X-ray observations (e.g., Nakashima et al. 2018) and numerical simulation (Girichidis et al. 2018); therefore the “fall back” solutions are expected to be realized.

5. Summary and Discussion

We have solved the steady-state Galactic wind including the effects of radiative cooling, CR diffusion, and heating due to the dissipation of Alfvén waves that are excited by CRs. We have found 327,254 solutions in total. For the dynamics of the thermal gas, the CR pressure acts as the term \( \nu dP_{\mathrm{cr}}/dt \) or heating depending on the Alfvén Mach number. The former is dominant for \( M_A \gg \gamma \sim 1 \), while the latter is dominant for \( M_A \ll \gamma \sim 1 \). The mass flux of the outflow mainly depends on the thermal pressure. The CR pressure determines the cooling and heating balance. When the CR pressure is smaller than the thermal pressure, the outflow tends to have a lower temperature at the vertical height of \( z \sim 100 \text{ kpc} \). A fraction of the solutions have a short cooling time of \( < \tau_{\text{cool}} \) with the cooling rate larger than the heating rate at the top boundary \( z = z_{\text{top}} = 350 \text{ kpc} \). For these solutions, \( P_{\mathrm{g, bt}}/P_{\mathrm{cr, bt}} \gtrsim 1 \) is required. We have regarded that such an outflow finally falls back to the Galactic disk. The total mass carried by the outflow per unit time has been estimated as \(~10 M_{\odot} \text{ yr}^{-1} \). Thus, in terms of the mass budget, the Galactic star formation history is controlled by whether the outflow falls back to the Galactic disk or not, which is determined by the ratio of the thermal pressure to the CR pressure.

Here we discuss the boundary conditions at the \( z = 2 \text{ kpc} \) for the “fall back” solutions using \( T_{\text{bt}} \approx 5 \times 10^6 \text{ K} \), \( n_{\text{bt}} \approx 5 \times 10^{-3} \text{ cm}^{-3} \), \( P_{\mathrm{cr, bt}} \approx 0.5 \text{ eV cm}^{-3} \), and \( H_{\mathrm{cr, bt}} \approx 34 \text{ kpc} \) as an example (see Table 2). The temperature comparable to the virial temperature for a distance of \( ~100 \text{ kpc} \) may be required to drive the wind, and it is consistent with X-ray observations (e.g., Nakashima et al. 2018). A number density of \( \sim 10^{-3} \text{ cm}^{-3} \) may be nontrivial, though it is supported by numerical simulations (e.g., Girichidis et al. 2018). If the hot, tenuous gas above the Galactic disk secularly exists due to

the balance of energy injection by supernovae and radiative loss, \( \eta_{\mathrm{SN}}/(\pi R^2 \gamma z) \sim n^2 \Lambda \), where \( L_{\mathrm{SN}} \) and \( \eta \) are the energy injection rate of supernovae and conversion efficiency, respectively, we obtain

\[
\frac{L_{\mathrm{SN}}}{4 \pi R^2 \gamma z} \sim 3 \times 10^{-3} \text{ cm}^{-3} \left( \frac{n}{10^{-2}} \right)^2 \left( \frac{L_{\mathrm{SN}}}{10^{42} \text{ erg s}^{-1}} \right)^{1/2} \frac{N}{10 \text{kpc}} \left( \frac{10^{22} \text{ erg cm}^{-3} \text{s}^{-1}}{L_{\mathrm{SN}}} \right)^{-1/2}.
\]

(39)

The energy injection rate of the supernovae, \( L_{\mathrm{SN}} \sim 10^{42} \text{ erg s}^{-1} \), corresponds to an event rate of three times per hundred years. It is expected that roughly a tenth of the kinetic energy of a supernova is converted to turbulence in the ISM. Thus, a conversion efficiency of \( \eta \sim 10^{-2} \) means that roughly a tenth of the energy of the ISM turbulence is consumed to form hot, tenuous gas. The wind may have a velocity comparable to the virial velocity, \( v_{\text{vir}} \sim 298 \text{ km s}^{-1} \left( N/10^{12} M_{\odot} \right)^{1/2} (\nu/100 \text{ kpc})^{-1/2} \), which is consistent with our model calculation. The total mass carried by the wind per unit time can be estimated as

\[
\dot{M} \sim \pi R^2 m_p n_{\text{bt}} v_{\text{vir}} \sim 6.4 M_{\odot} \text{ yr}^{-1} \left( \frac{R}{10 \text{kpc}} \right)^2 \left( \frac{n_{\text{bt}}}{3 \times 10^{-3} \text{ cm}^{-3}} \right) \left( \frac{v_{\text{vir}}}{300 \text{ km s}^{-1}} \right).
\]

(40)

Note that the CR pressure is required to transfer the metals at a height of \( ~100 \text{ kpc} \) by the wind. When the CR pressure satisfies \( P_{\mathrm{g, bt}}/P_{\mathrm{cr, bt}} \gtrsim 1 \), the wind suffers significant radiative cooling. In this case, the solution cannot be extended to infinity. This might be related to the possible existence of “fall back” phenomena, which are interesting as gas replenishment mechanisms. Thus, to realize a steady Galactic system, the condition of

\[
P_{\mathrm{cr, bt}} \lesssim \frac{0.78 \text{ eV cm}^{-3}}{3 \times 10^6 \text{ K}} \left( \frac{n_{\text{bt}}}{3 \times 10^{-3} \text{ cm}^{-3}} \right).
\]

(41)
may be required. Since the wind carries CRs, we can estimate an injection power of CRs in the Galactic disk as

\[
L_{\text{cr}} \sim \frac{P_{\text{cr,bt}}}{\gamma_c - 1} v_{\text{vir}} R^2 \lesssim 3.2 \times 10^{41} \text{ erg s}^{-1} \times \left( \frac{P_{\text{cr,bt}}}{0.78 \text{ eV cm}^{-3}} \right) \times \left( \frac{v_{\text{vir}}}{300 \text{ km s}^{-1}} \right) \left( \frac{R}{10 \text{ kpc}} \right)^2.
\]

Thus, if CRs are injected by supernovae, the wind suffers significant radiative cooling. This system can be stable for variations of the star formation rate. Let us suppose that the star formation rate increases from the current average rate of a few \( M/\text{yr} \). Then, the CR pressure becomes larger than the thermal pressure due to an increment in the star formation rate (i.e., increment in the event rate of the supernovae), and the wind can reach a height of \( z > 350 \text{ kpc} \) without significant radiative cooling. Thus, the Galactic disk secularly loses the gaseous matter, leading to a decrement in the star formation rate. In contrast, with decreasing CR pressure, the radiative cooling of wind becomes significant, which may lead to “fall back” phenomena and an increment in the star formation rate. This self-regulation effect by the wind possibly explains the constant star formation rate averaged by a timescale of \( \sim 1 \text{ Gyr} \) (e.g., Haywood et al. 2016). Hence, it is important to study the conversion efficiency \( \eta \) and existence of the “fall back” phenomena, which are not analyzed in this article. We will address them in our future work.

We compare our model with current observations of MW’s CGM. Miller & Bregman (2015) analyzed emission line measurements of O VIII and O VII from X-ray Multi-mirror Mission Newton European Photon Imaging Camera Metal Oxide Semi-conductor spectra and gave some constraints for the hot CGM assuming a one-dimensional density structure,

\[
n(r) \approx \frac{n_o r_\beta^{3\beta}}{r^{3\beta}}, \tag{43}
\]

where \( r \) is the galactocentric radius. From the O VIII observations, the parameters \( n_o, r_\beta, \) and \( \beta \) were derived as \( n_o r_\beta^{3\beta} = (1.35-1.5) \times 10^{-2} \) and \( \beta = 0.50-0.54 \), where we omit to display the 1σ error, and the range of parameter values results from optical depth corrections. These constraints are mainly derived from O VIII measurements. They estimated the total mass of the X-ray emitting gas as \( M_{\text{MB15}} = (2.9-3.8) \times 10^9 M_\odot \) for \( r < 50 \text{ kpc} \) and \( M_{\text{MB15}} = (2.7-4.3) \times 10^9 M_\odot \) for \( r < 250 \text{ kpc} \).

In their analysis, the CGM was assumed to have a constant temperature profile in CIE with fixed \( \log(T) = 6.3 \) and a metallicity of \( Z = 0.3Z_\odot \). Although these constraints are based on a different situation from our wind, the total mass of our wind should be comparable to the estimated mass given by the intensity of the emission lines (almost equivalent to the column density). The effects of the lower metallicity (\( Z = 0.3Z_\odot \)) are discussed later. The order of magnitude estimate of the total mass may be written as

\[
M_w \sim \dot{M} \frac{z}{v_{\text{vir}}} \sim 2.1 \times 10^9 M_\odot \left( \frac{R}{10 \text{ kpc}} \right)^2 \times \left( \frac{n_{\text{bt}}}{3 \times 10^{-3} \text{ cm}^{-3}} \right) \left( \frac{z}{100 \text{ kpc}} \right). \tag{44}
\]

If “fall back” phenomena really occur over the cooling time of \( \sim 1 \text{ Gyr} \), the expected mass inflow rate \( (-\dot{M}_w/1 \text{ Gyr} \sim 1 M_\odot/\text{yr}) \) onto the disk might be sufficiently large in replenishing the gas of the Galactic disk. From our model calculations, we estimate the statistical average of the total mass as

\[
\Delta M_{w,i} = 2\pi \Delta R \rho \int_{z_{\text{bt}}}^{z_0} dz \rho (R_i, z), \tag{45}
\]

\[
\langle M_w \rangle = \sum_{i=0}^{10} \langle \Delta M_{w,i} \rangle \times 2, \tag{46}
\]

\[
R_i = i \text{ kpc,} \quad \Delta R_i = 1 \text{ kpc}. \tag{47}
\]

Then, we obtain \( \langle M_w \rangle \approx 0.51 \times 10^9 M_\odot \) for all solutions and \( \langle M_w \rangle \approx 0.66 \times 10^9 M_\odot \) for the “fall back” solutions. Note that, in our scenario, the observationally constrained mass should include other components, such as metal-polluted IGM heated by the wind termination shock. In addition, the mass of the inflow from the CGM to the Galactic disk (i.e., the total mass of the CGM) should be larger than the mass carried by the wind because comparable mass is consumed for the star formation.

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5 The analyzed hot X-ray emitting medium was referred to as “hot halo” but the derived length scale is comparable with the CGM we supposed. Therefore, we refer to the “hot halo” of Miller & Bregman (2015) as “CGM” in this article.
Thus, the total mass of the wind estimated by our model can still be consistent with the constraints given by Miller & Bregman (2015) in terms of the mass budget that may explain the observed star formation history.

Miller & Bregman (2015) also obtained the subsolar metallicity of the hot CGM by combinations of the emission ([O VIII]) and absorption ([O VII]) analysis and the pulsar’s dispersion measure toward the Large Magellanic Cloud. Since lower metallicities reduce the radiative cooling rate, it might be interesting to investigate how the wind profile may change from our calculations with \( Z = Z_\odot \). The left panel of Figure 13 shows the wind solutions with the same boundary conditions as the case of Figure 5 but the radiative cooling rate is reduced by a factor of 0.1. The reduced cooling rate obviously results in a hotter wind; however, the other profiles of \( v, n, \) and \( P_{\text{cr}} \) are not so changed. The heating rate due to the CRs is still smaller than the reduced cooling rate at a lower \( z \). Once the wind passes the transonic point, the thermal pressure becomes less important and the wind is mainly driven by the CR pressure. Thus, for the typical conditions of the MW we discussed above, the down to 0.1 \( Z_\odot \) does not change our expectations. This is confirmed by the overall trend of the solutions with the reduced cooling function (the right panel of Figure 13). The result is similar to the case of \( Z = Z_\odot \) (Figure 8).

Note that the number of solutions with a reduced cooling rate is 580,014 (127,633 for the “fall back” solutions). Thus, if we compare the same boundary conditions, the reduced cooling rate results in launching the wind “more easily” than the case of the solar metallicity. The average total masses are \( \langle M \rangle \simeq 0.81 \times 10^9 M_\odot \) for all solutions and \( \langle M \rangle \simeq 1.18 \times 10^9 M_\odot \) for the “fall back” solutions. From such insensitive metallicity dependence, we may regard that our picture of the quasi-steady-star formation in the MW has been continued from \( \sim 8 \) Gyr ago when the amount of metals was smaller than the current one.

Comparison of the current CR energy density at the Galactic disk with our estimates may be important. Cerri et al. (2017) studied the CR propagation in the Galactic disk below a height of

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**Figure 10.** Ratio of the cooling rate to the heating rate \( n^2 \Lambda/Q_w \) (left panel) and cooling time \( t_{\text{cool}} = kT/n\Lambda \) (right panel) for the solutions that are the same as those in Figure 5. The color represents the temperature.

**Figure 11.** The statistically averaged total mass carried by the outflow per unit time as a function of the CR pressure (left panel) and total pressure (right panel) at the bottom boundary. The green boxes show the average for all the solutions. The purple boxes show the average for the “fall back” solutions. The vertical thin line indicates \( P_{\text{cr,bt}} = 0.33 \text{ eV cm}^{-3} \), which corresponds to the observed CR energy density around the solar system.

**Figure 12.** The statistically averaged number density at the bottom boundary \( \langle n_{\text{bt}} \rangle \) as a function of the horizontal position \( R \) for a CR pressure of \( P_{\text{cr,bt}} = 0.274-0.422 \text{ eV cm}^{-3} \). The squares and asterisks indicate the averages of all solutions and “fall back” solutions, respectively. The color represents the averaged temperature at the bottom boundary, \( \langle T_{\text{bt}} \rangle \).
Figure 13. (left panel): The solutions at $R = 8$ kpc for the same parameters as in Figure 5 but for a radiative cooling function reduced by a factor of 0.1. The top part shows the velocity, the middle part shows the number density, and the bottom part shows the CR pressure. The color indicates the temperature. (right panel) Same as in Figure 8 but for a radiative cooling rate reduced by a factor of 0.1. The color shows the temperature $T_{\text{up}}$ at the top boundary ($z = z_{\text{up}}$).

Table 3

| Ion    | Literature       | Ion    | Literature       | Ion    | Literature       |
|--------|------------------|--------|------------------|--------|------------------|
| C$^{+1}$ | Nahar & Pradhan (1999) | Mg$^{+5}$ | Arnaud & Rothenflug (1985) | S$^{+12}$ | Mewe et al. (1980a, 1980b) |
| C$^{+2}$ | Nahar & Pradhan (1999) | Mg$^{+6}$ | Zatsarinny et al. (2004) | S$^{+13}$ | Mewe et al. (1980a, 1980b) |
| C$^{+3}$ | Nahar & Pradhan (1997) | Mg$^{+7}$ | Nahar (1995) | S$^{+14}$ | Arnaud & Rothenflug (1985) |
| C$^{+4}$ | Nahar & Pradhan (1997) | Mg$^{+8}$ | Arnaud & Rothenflug (1985) | S$^{+15}$ | Arnaud & Rothenflug (1985) |
| C$^{+5}$ | Nahar & Pradhan (1997) | Mg$^{+9}$ | Arnaud & Rothenflug (1985) | Fe$^{+1}$ | Nahar & Pradhan (1997) |
| N$^{+1}$ | Zatsarinny et al. (2004) | Mg$^{+10}$ | Arnaud & Rothenflug (1985) | Fe$^{+2}$ | Nahar & Pradhan (1997) |
| N$^{+2}$ | Nahar & Pradhan (1997) | Mg$^{+11}$ | Arnaud & Rothenflug (1985) | Fe$^{+3}$ | Nahar & Pradhan (1997) |
| N$^{+3}$ | Nahar & Pradhan (1997) | Si$^{+1}$ | Nahar (2000) | Fe$^{+4}$ | Nahar (1998) |
| N$^{+4}$ | Nahar & Pradhan (1997) | Si$^{+2}$ | Altun et al. (2007) | Fe$^{+5}$ | Nahar & Pradhan (1999) |
| N$^{+5}$ | Nahar (2006) | Si$^{+3}$ | Mewe et al. (1980a, 1980b) | Fe$^{+6}$ | Arnaud & Rothenflug (1985) |
| N$^{+6}$ | Nahar (2006) | Si$^{+4}$ | Zatsarinny et al. (2003) | Fe$^{+7}$ | Nahar (2000) |
| O$^{+1}$ | Nahar (1998) | Si$^{+5}$ | Zatsarinny et al. (2006)* | Fe$^{+8}$ | Arnaud & Rothenflug (1985) |
| O$^{+2}$ | Zatsarinny et al. (2004) | Si$^{+6}$ | Zatsarinny et al. (2003) | Fe$^{+9}$ | Arnaud & Rothenflug (1985) |
| O$^{+3}$ | Nahar (1998) | Si$^{+7}$ | Mitnik & Badnell (2004)* | Fe$^{+10}$ | Lestinsky et al. (2009)* |
| O$^{+4}$ | Nahar (1998) | Si$^{+8}$ | Zatsarinny et al. (2004) | Fe$^{+11}$ | Novotny et al. (2012)* |
| O$^{+5}$ | Nahar (1998) | Si$^{+9}$ | Nahar (1995) | Fe$^{+12}$ | Hahn et al. (2014)* |
| O$^{+6}$ | Nahar (1998) | Si$^{+10}$ | Arnaud & Rothenflug (1985) | Fe$^{+13}$ | Arnaud & Rothenflug (1985) |
| O$^{+7}$ | Nahar (1998) | Si$^{+11}$ | Arnaud & Rothenflug (1985) | Fe$^{+14}$ | Altun et al. (2007)* |
| Ne$^{+1}$ | Arnaud & Rothenflug (1985) | Si$^{+12}$ | Arnaud & Rothenflug (1985) | Fe$^{+15}$ | Murakami et al. (2006)* |
| Ne$^{+2}$ | Zatsarinny et al. (2003) | Si$^{+13}$ | Arnaud & Rothenflug (1985) | Fe$^{+16}$ | Zatsarinny et al. (2004) |
| Ne$^{+3}$ | Mitnik & Badnell (2004)* | S$^{+1}$ | Mewe et al. (1980a, 1980b) | Fe$^{+17}$ | Arnaud & Rothenflug (1985) |
| Ne$^{+4}$ | Zatsarinny et al. (2004) | S$^{+2}$ | Nahar (1995) | Fe$^{+18}$ | Zatsarinny et al. (2003) |
| Ne$^{+5}$ | Nahar (1995) | S$^{+3}$ | Nahar (1995) | Fe$^{+19}$ | Savin et al. (2002)* |
| Ne$^{+6}$ | Arnaud & Rothenflug (1985) | S$^{+4}$ | Altun et al. (2007) | Fe$^{+20}$ | Zatsarinny et al. (2004) |
| Ne$^{+7}$ | Arnaud & Rothenflug (1985) | S$^{+5}$ | Arnaud & Rothenflug (1985) | Fe$^{+21}$ | Arnaud & Rothenflug (1985) |
| Ne$^{+8}$ | Nahar (2006) | S$^{+6}$ | Zatsarinny et al. (2004) | Fe$^{+22}$ | Arnaud & Rothenflug (1985) |
| Ne$^{+9}$ | Nahar (2006) | S$^{+7}$ | Zatsarinny et al. (2006)* | Fe$^{+23}$ | Mewe et al. (1980a, 1980b) |
| Mg$^{+1}$ | Mewe et al. (1980a, 1980b) | S$^{+8}$ | Zatsarinny et al. (2003) | Fe$^{+24}$ | Nahar et al. (2001) |
| Mg$^{+2}$ | Zatsarinny et al. (2004) | S$^{+9}$ | Mitnik & Badnell (2004)* | Fe$^{+25}$ | Nahar et al. (2001) |
| Mg$^{+3}$ | Arnaud & Rothenflug (1985) | S$^{+10}$ | Zatsarinny et al. (2004) | |
| Mg$^{+4}$ | Zatsarinny et al. (2004) | S$^{+11}$ | Nahar (1995) | |

*Note. The superscript * denotes that we use Mewe’s formula for the radiative recombination (Mewe et al. 1980a, 1980b).

$z < 2$–4 kpc to explain the spatial variation of the $\gamma$-ray spectral slope observed by the Fermi Large Area Telescope (LAT) Collaboration (Acero et al. 2016). To derive the CR energy density (or pressure) from the $\gamma$-ray observations, we must know the conditions of the thermal gas including all the components (diffuse hot medium, H I cloud, molecular cloud, etc.) in detail because the $\gamma$-ray photons result from the hadronic interaction between CR protons and thermal protons ($p_{\text{CR}} + p_{\text{ISM}} \rightarrow \pi^0 \rightarrow 2\gamma$). However, the conditions of the thermal gas are not fully understood. Moreover, the propagation of CRs around the molecular cloud is also an unsettled issue in terms of the effects of the magnetic mirror with a local turbulent field (e.g., Lazarian & Xu 2021). Thus, it would be better to discuss both the observational estimate and theoretical prediction rather than the observation only. The propagation model of Cerri et al. (2017) is based on an anisotropic diffusion coefficient tensor given by pitch-angle scattering; the diffusion along the guide magnetic field is stronger than the perpendicular one. The global configuration of
the magnetic field controls the spatial distribution of the CR spectral slope and energy density and is assumed to have a substantial poloidal component. This situation is similar to our model. Their model does not include the various thermal gas conditions, and the calculated energy density and spectral slope of the CR protons are directly compared to the estimated values from the observations. This procedure corresponds to CRs that are approximated to propagate in a diffuse medium with a uniform density structure. The resultant CR energy density of \( \sim 0.3 \text{ eV cm}^{-3} \) (equivalently, \( P_{\text{cr}} \sim 0.1 \text{ eV cm}^{-3} \)) can be consistent with Fermi LAT observations at \( R > 6 \text{ kpc} \). At \( R < 6 \text{ kpc} \), the predicted energy density depleting toward the Galactic center is significantly lower than that estimated from the observations \( \sim 0.3-1 \text{ eV cm}^{-3} \left( P_{\text{cr}} \sim 0.1-0.3 \text{ eV cm}^{-3} \right) \). These estimates satisfy the inequality \( P_{\text{cr,bs}} \lesssim 0.78 \text{ eV cm}^{-3} \) discussed above. Note that the CR pressure of \( \sim 0.1-0.3 \text{ eV cm}^{-3} \) is sufficiently large in launching the wind depending on the thermal gas conditions. Thus, the current state of the MW implies that the cold wind is driven, eventually falling back to the disk in the future.

Observations of external galaxies show that the CGM consists of not only highly ionized species like O VI but also lower-ionized species like H I, C II, Mg II, and so on (Tumlinson et al. 2017, and references therein). The absorption lines of these lower-ionized species are also observed at a distance of \( \sim 100 \text{ kpc} \) from the host galaxy. Our solutions imply that the outflow can transfer such lower-ionized species to a height of \( z \sim 100 \text{ kpc} \) if radiative cooling is efficient. The condition for efficient radiative cooling is \( P_{\text{cr,bs}} \gtrsim 1 \). Note that the ionization state is affected by photoionization, which is omitted in this paper. Thus, we have to pay attention whether the O VI absorption line really indicates the existence of hot gas (see Breitschwerdt & Schmutzler 1999). On the other hand, photonization reduces the number of lower-ionized species. Thus, the existence of lower-ionized species indicates that the temperature of gas is low or the condensation of gas occurs to shield itself from the photoionizing photons. In any case, an efficient cooling process is required. We will study the ionization state and condensation of gas at a height of \( \sim 100 \text{ kpc} \) in future work.

We have assumed that the outflow travels along the vertical direction \( z \) so that the required energy is approximately minimum. This condition, however, is strictly given by \( v \propto \nabla \Phi \). Thus, the outflow may have an at least two-dimensional spatial structure, like a biconical structure, in reality. It is reported by numerical simulation that the CR pressure can affect the spatial structure of wind (Hopkins et al. 2018). The far-ultraviolet observation of NGC 3079 implies an X-shape wind (Hodges-Kluck et al. 2020). Thus, it would be worth investigating the relation between the wind condition and its morphology. The wind morphology may depend on how the diffusion coefficient is assumed. The multidimensional diffusion coefficient (i.e., diffusion tensor) is also an actively discussed issue in the CR transport literature (e.g., Cerri et al. 2017, and references therein). Zirakashvili et al. (1996) studied an axially symmetric wind considering the Galactic disk rotation and introduced an effective CR adiabatic index \( \gamma_{\text{eff}} \) as a possibly useful method,

\[
\frac{\gamma_{\text{eff}}}{\gamma_{\text{eff}} - 1} = \frac{\gamma_{\text{c}}}{\gamma_{\text{c}} - 1} - \left( \frac{\gamma_{\text{c}}}{\gamma_{\text{c}} - 1} \right)^{1 - \frac{1}{\kappa}} \frac{1}{\kappa} \frac{dP_{\text{cr}}}{dz},
\]

so that the CR pressure can be expressed as \( P_{\text{cr}} = \left( A + V_{A} \right)^{-\frac{1}{\kappa}} \). It may simplify the analysis of the transonic “surface” of the multidimensional wind and the systematic parameter study of the diffusion coefficient. We hope to extend our model to a multidimensional one in our future work.

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Software: Cloudy (Ferland et al. 2017).

Appendix

The expressions of \( N_{\zeta} \) and \( N_{\zeta} \) are, respectively,

\[
N_{\zeta} = \frac{A'}{A} \left( C_{g}^{2} + C_{w}^{2} + V_{g}^{2} \right),
\]

\[
\frac{\partial C_{g}^{2}}{\partial z} = - (\gamma_{g} - 1) \left[ \frac{C_{g}^{2} A'}{A} + \gamma_{g} \left( \frac{V_{g} dP_{\text{cr}}}{dz} - n_{\Lambda} \right) \right],
\]

\[
\frac{\partial C_{w}^{2}}{\partial z} = - \frac{3 M_{A}^{2}}{2(3 M_{A} + 1)(M_{A} + 1)} C_{g}^{2} A',
\]

\[
\frac{\partial V_{g}^{2}}{\partial z} = \left( 1 - \frac{Z_{0}^{2} + z^{2}}{2 z^{2}} \right) \times \left( \frac{d\Phi}{dz} - \frac{\gamma_{g} - 1}{M_{A}} \right) + A' \left[ \frac{d^{2}\Phi}{dz^{2}} - \frac{\partial F}{\partial z} \right],
\]

\[
\frac{\partial F}{\partial z} = - \left( \frac{\gamma_{g} - 1}{M_{A}} \right) \frac{n_{\Lambda}}{\rho} \frac{A'}{A} \left( \frac{\partial \Lambda}{\partial T} + \Lambda \frac{A'}{A} \right) - \left( \gamma_{g} - 1 \right) \frac{P_{\text{cr}}}{2 M_{A} \rho} \frac{A'}{A} \rho \frac{A'}{A} \left( \frac{d \ln P_{\text{cr}}}{dz} \right) + \left( \frac{\gamma_{g} - 1}{M_{A}} \right) \left[ \frac{d \ln P_{\text{cr}}}{dz} + \frac{A'}{A} \right]
\]

\[
\times \left( \frac{1}{\rho} \frac{dP_{\text{cr}}}{dz} + \frac{P_{\text{cr}}}{\rho} \frac{\partial}{\partial z} \frac{d \ln P_{\text{cr}}}{dz} \right),
\]

\[
\frac{\partial}{\partial z} \frac{d \ln P_{\text{cr}}}{dz} = - \frac{d \ln P_{\text{cr}}}{dz} \left[ \frac{d \ln P_{\text{cr}}}{dz} \right]^{2} \left( \frac{\gamma_{c} (V + V_{A} / 2) A'}{A} \right),
\]

\[
\frac{\kappa}{\kappa} \frac{d \ln P_{\text{cr}}}{dz} + \frac{\gamma_{c} (V + V_{A} / 2) A'}{A}.
\]
and

\[
\mathcal{N}_v = z \frac{A'}{A} \left[ \frac{C_g^2}{v_A} + \left( \frac{1 + (2M_A + 1)(3M_A + 1)}{2} \right) \right] \times \frac{C_g^2}{v} - A' \left( 2\gamma_g - 1 \right) \left( \frac{n^2 \Lambda}{\rho v^2} \right) - \left( \frac{\gamma_g - 1}{2M_A - 1} \right) \left( \frac{dP_{\perp}}{d\zeta} \right)
\]  

(A2)

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