COSMIC-RAY PROPAGATION: NONLINEAR DIFFUSION PARALLEL AND PERPENDICULAR TO MEAN MAGNETIC FIELD

HUIRONG YAN\textsuperscript{1} and A. LAZARIAN\textsuperscript{2}

Received 2007 September 21; accepted 2007 October 16

ABSTRACT

We consider the propagation of cosmic rays in turbulent magnetic fields. We use the models of magnetohydrodynamic turbulence that were tested in numerical simulations, in which the turbulence is injected on large scales and cascades to small scales. Our attention is focused on the models of the strong turbulence, but we also briefly discuss the effects that the weak turbulence and the slab Alfvénic perturbations can have. The latter are likely to emerge as a result of instabilities within the cosmic-ray fluid itself, e.g., beaming and gyroresonance instabilities of cosmic rays. To describe the interaction of cosmic rays with magnetic perturbations we develop a nonlinear formalism that extends the ordinary quasi-linear theory that is routinely used for the purpose. This allows us to avoid the usual problem of 90° scattering and enables our computation of the mean free path of cosmic rays. We apply the formalism to the cosmic-ray propagation in the Galactic halo and in the warm ionized medium. In addition, we address the issue of the transport of cosmic rays perpendicular to the mean magnetic field and show that the issue of cosmic-ray subdiffusion (i.e., propagation with retracing the trajectories backward, which slows down the diffusion) is only important for restricted cases when the ambient turbulence is far from what numerical simulations suggest to us. As a result, this work provides formalism that can be applied for calculating cosmic-ray propagation in a wide variety of circumstances.

Subject headings: acceleration of particles — cosmic rays — ISM: magnetic fields — MHD — scattering — turbulence

Online material: color figures

1. INTRODUCTION

The propagation and acceleration of cosmic rays (CRs) is governed by their interactions with magnetic fields. Astrophysical magnetic fields are turbulent, and therefore, the resonant and nonresonant (e.g., transient time damping [TTD]) interaction of CRs with MHD turbulence is the accepted principal mechanism to scatter and isotropize CRs (see Schlickeiser 2002). In addition, efficient scattering is essential for the acceleration of CRs. For instance, scattering of CRs back into the shock is a vital component of the first-order Fermi acceleration (see Longair 1997). At the same time, stochastic acceleration by turbulence is entirely based on scattering.

It is generally accepted that properties of turbulence are vital for the correct description of CR propagation. Historically, the most widely used model is the model composed of slab perturbations and two-dimensional (2D) MHD perturbations (see Bieber et al. 1988). The advantage of this empirical model is its simplicity and the ability to account for the propagation of CRs in the magnetosphere given a proper partition of the energy between the two types of modes.

Numerical simulations (see Cho & Vishniac 2001; Maron & Goldreich 2001; Müller & Biskamp 2000; Cho et al. 2002; Cho & Lazarian 2002, 2003; Biskamp 2003; see Cho et al. 2003 and Elmegreen & Scalo 2004 for reviews), however, do not show 2D modes, but instead show Alfvénic modes that exhibit scale-dependent anisotropy consistent with predictions in Goldreich & Sridhar (1995, hereafter GS95). The approach in the latter work makes productive use of the earlier advances in understanding of MHD turbulence that can be traced back to Iroshnikov (1963) and Kraichnan (1965) and the classical work that followed (see Montgomery & Turner 1981; Higdon 1984; Montgomery et al. 1987).

A careful analysis shows that there is no big gap between the reduced MHD and the GS95 model. In fact, it was shown in Lazarian & Vishniac (1999) that the numerical results in Matthaeus et al. (1998) are consistent with GS95 predictions. While particular aspects of the GS95 model, e.g., the particular value of the spectral index, are the subject of controversies (see Müller & Biskamp 2000; Boldyrev 2006, 2006; Beresnyak & Lazarian 2006; Gogoberidze 2007; Mason et al. 2007), we think that, at present, the GS95 model provides a good starting point for developing models of CR scattering as was done in Chandran (2000), Yan & Lazarian (2002, 2004, hereafter YL02 and Paper I, respectively), Brunetti & Lazarian (2007), etc. In particular, the latter three papers used the decomposition of MHD turbulence over Alfvén, slow, and fast modes as in Cho & Lazarian (2003) and identify the fast modes as the major source of CR scattering in the interstellar and intracluster media.

However, while the turbulence injected on large scales may correspond to the GS95 model and its extensions to the compressible medium (Lithwick & Goldreich 2001; Cho & Lazarian 2002, 2003), one should not disregard the possibilities of generation of additional perturbations by CRs themselves. Indeed, the

\footnote{To address quantitatively these controversies, we need much better numerical resolution. For instance, hydrodynamic turbulence simulations in Kritsuk et al. (2007) showed that, only starting with the 1028 cubic, the bottleneck effects stop dominating the measured spectral slope. While the simulations in Kowal & Lazarian (2007) show that the bottleneck is less important for their MHD code, the result of the spectral slope is still uncertain. At the same time, the particular theoretically predicted features of turbulence, for instance, the existence of the scale-dependent anisotropy, can be reliably established, while the exact scaling of this dependence, e.g., like $k \sim k^{-\alpha}$ as in GS95 model or $k \sim k^{-\alpha}$, where $\alpha^* < 2/3$ (see Beresnyak & Lazarian 2006), also requires higher resolution simulations.}

\textsuperscript{1} Canadian Institute of Theoretical Astrophysics, 60 St. George Street, Toronto, ON M5S 3H8, Canada; yanhr@cita.utoronto.ca.

\textsuperscript{2} Astronomy Department, University of Wisconsin, Madison, WI 53706; alazarian@wisc.edu.
slab Alfvenic perturbation can be created, e.g., via streaming instability (see Wentzel 1974; Cesarsky 1980) or kinetic gyroresonance instability (see its application for CR transport in Lazarian & Beresnyak 2006). These perturbations, which are present for a range of CR energies (e.g., \( \lesssim 100 \text{ GeV} \) for the instabilities above in the interstellar medium [ISM]) owing to nonlinear damping arising from ambient turbulence (YL02; Paper I; Farmer & Goldreich 2004; Lazarian & Beresnyak 2006), should also be incorporated into the comprehensive models of CR propagation and acceleration.

At present, the propagation of the CRs is an advanced theory, which makes use of both analytical studies and numerical simulations. However, these advances have been done within the turbulence paradigm which is being changed by the current research in the field. As we discussed above, instead of the empirical 2D + slab model of turbulence, numerical simulations suggest anisotropic Alfvenic modes (an analog of 2D, but not an exact one, as the anisotropy changes with the scale involved) + fast modes and/or slab modes. This calls for important revisions of the CR propagation, which is the subject of the current paper.

The perturbations of turbulent magnetic field are usually accounted for by direct numerical scattering simulations (Giacalone & Jokipii 1999; Mace et al. 2000; Qin at al. 2002) or by quasi-linear theory (QLT; see Jokipii 1966; Schluekeiser 2002). The problem with direct numerical simulations of scattering is that the present-day MHD simulations have rather limited inertial range. At the same time, creating synthetic turbulence data which would correspond to scale-dependent anisotropy with respect to the local magnetic field (which corresponds, e.g., to the GS95 model) is challenging and has not been practically realized, as far as we know.

While QLT allows one to easily treat the CR dynamics in a local magnetic field system of reference, a key assumption in QLT, that the particle’s orbit is unperturbed, makes one wonder about the limitations of the approximation. Indeed, while QLT provides simple physical insights into scattering, it is known to have problems. For instance, it fails in treating 90° scattering (see Jones et al. 1973, 1978; Volk 1973, 1975; Owens 1974; Goldstein 1976; Felice & Kulsrud 2001) and perpendicular transport (see Kota & Jokipii 2000; Matthaeus et al. 2003).

Indeed, many attempts have been made to improve the QLT, and various nonlinear theories have been attempted (see Dupree 1966; Volk 1973, 1975; Jones et al. 1973; Goldstein 1976). Currently, we observe a surge of interest in finding a way to go beyond QLT. Those include recently developed nonlinear guiding center theory (see Matthaeus et al. 2003), weakly nonlinear theory (Shalchi et al. 2004), and second-order QLT (Shalchi 2005a, 2006; Webb et al. 2006; Qin 2007; Le Roux & Webb 2007). At the same time, most of the analysis so far has been confined to traditional 2D + slab models of MHD turbulence. Following the reasoning above, we think that it is important to extend the work of the nonlinear treatment of CR scattering to models of MHD turbulence that are supported by numerical simulations.

Propagation of CRs perpendicular to the mean magnetic field is another important problem in which QLT encounters serious difficulties. Compound diffusion, resulting from the convolution of diffusion along the magnetic field line and diffusion of field line perpendicular to the mean field direction, has been invoked to discuss transport of CRs in the Milky Way (Getmansev 1963; Lingenfelter et al. 1971; Allan 1972). The role of compound diffusion in the acceleration of CRs at quasi-perpendicular shocks was investigated by Duffy et al. (1995) and Kirk et al. (1996).

Indeed, the idea of CR transport in the direction perpendicular to the mean magnetic field being dominated by the field line random walk (FLRW; Jokipii 1966; Jokipii & Parker 1969; Forman et al. 1974) can be easily justified only in a restricted situation where the turbulence perturbations are small and CRs do not scatter backward to retrace their trajectories. If the latter is not true, the particle motions are subdiffusive, i.e., the squared distance diffused grows not as \( r^n \) but as \( r^{1/n} \), where \( \alpha < 1 \), e.g., \( \alpha = 1/2 \) (Kota & Jokipii 2000; Mace et al. 2000; Qin et al. 2002; Shalchi 2005b). If true, this could indicate a substantial shift in the paradigm of CR transport, a shift that surely dwarfs a modification of magnetic turbulence models from the 2D + slab to a more simulation-motivated model that we deal with here.

It was also proposed that with substantial transverse structure, i.e., transverse displacement of field lines, perpendicular diffusion is recovered (Qin et al. 2002). Is it the case in the MHD turbulence models we deal with?

How realistic is the subdiffusion in the presence of turbulence? The answer for this question apparently depends on the models of turbulence chosen. In this paper we again seek the answer for this question within the domain of numerically tested models of MHD turbulence.

There are three major thrusts of the paper:

1. Extend QLT by taking into account the magnetic mirroring effect on large scales;
2. Describe CR propagation in the Milky Way (e.g., calculate CR mean free path for different phases of the ISM);
3. Address the problem of perpendicular transport of CRs.

In what follows, we discuss the CR transport in incompressible turbulence in § 2. We describe in § 2.1 the dispersion of guiding center of CRs and introduce the broadened resonance function to replace the \( \delta \)-function in QLT, following which we discuss the scattering in strong and weak incompressible turbulence in §§ 2.2 and 2.3, respectively. Then we consider the scattering by fast modes in § 3 and apply the analysis to the ISM and get the mean free path for different phases of the ISM (§ 4). In § 5 we study the perpendicular transport of CRs on both large and small scales. We also discuss the applicability of subdiffusion. Discussion and summary are provided in §§ 6 and 7, respectively.

2. CR TRANSPORT IN INCOMPRESSIBLE TURBULENCE

2.1. General Formalism

It was demonstrated that scattering by the Alfvenic turbulence is substantially suppressed due to its anisotropy (Chandran 2000; YL02). On the other hand, resonant mirror interaction (so-called transit time damping [TTD]) can arise from the slow modes (also known as the pseudo-Alfven modes in the incompressible limit), and it is not subjected to the suppression from anisotropy. One may speculate that TTD is the alternative in this case.

The only requirement for TTD is that the projected particle speed is comparable to the phase speed of the magnetic field compression\(^4\) \( v \simeq \omega / k_z \) for particles to have enough collisions with the moving magnetic mirrors before they leak out of them. In QLT, this means that only particles with a specific pitch angle can be scattered. For the rest of the pitch angles, the interaction is still dominated by gyroresonance, the efficiency of which is negligibly small for the Alfvenic anisotropic turbulence. The quantitative treatment of this interaction is in YL02, which used the empirical magnetic field fluctuation tensor from Cho et al. (2002), provided the reduction factor is of 20 orders of magnitude for CRs of 100 GeV energies. This leads to a very large mean free

\(^4\) The compressions of the magnetic field in incompressible turbulence are related to the pseudo-Alfven mode, which is the limiting case of the slow mode (see Alfven & Falthammar 1963).
path for CRs. With the resonance broadening, however, we expect that a wider range of pitch angles can be scattered through TTD, including 90°.

The basic assumption of the QLT is that particles follow unperturbed orbits. In reality, the particle’s pitch angle varies gradually with the variation of the magnetic field due to conservation of the adiabatic invariant $v_i^2/B$, where $B$ is the total strength of the magnetic field (see Landau & Lifshitz 1975). Since $B$ is varying in the turbulent field, so is the projection of the particle speed $v_\perp$ and $v_\parallel$. This results in broadening of the resonance. Indeed, the average uncertainty of the parallel speed $\Delta v_\parallel$ is given by (see Völk 1975)

$$\frac{\Delta v_\parallel}{v_\parallel} = \left[ \frac{\langle (B - B_0)^2 \rangle^2}{B_0^2} \right]^{1/4} \approx \left[ \frac{\langle \delta B^2 \rangle}{B_0^2} + O \left( \frac{\langle \delta B^2 \rangle^2}{B_0^4} \right) \right]^{1/4}. \quad (1)$$

The variation of the velocity is mainly caused by the magnetic perturbation $\delta B_\parallel$ in the parallel direction. This is true even for the incompressible turbulence we discuss in this section. For the incompressible turbulence, the parallel perturbation arises from the pseudo-Alfven modes. The perpendicular perturbation $\delta B_\perp$ is a higher order effect, which we neglect in this paper.

The propagation of a CR can be described as a combination of a motion of its guiding center and the CR’s motion about its guiding center. Because of the dispersion of the pitch angle $\Delta \mu$ and therefore of the parallel speed $\Delta v_\parallel$, the guiding center is perturbed about the mean position $z_0 = v_\parallel t$ as they move along the field lines. As a result, the perturbation $\delta B(x, t)$ as the CRs view when moving along the field gets a different time dependence.

The characteristic phase function $e^{ik_0 z}$ of the perturbation $\delta B(x, t)$ deviates from that for plane waves. Assuming the guiding center has a Gaussian distribution along the field line, one obtains

$$f(z) = \frac{1}{\sqrt{2\pi} \sigma_z} e^{-\frac{(z-z_0)^2}{2\sigma_z^2}}. \quad (2)$$

Integrating over $z$, one gets

$$\int_{-\infty}^{\infty} \frac{dk_0 |e^{ik_0 z}|^2}{e^{k_0^2 \sigma_z^2}} = 1.$$ \quad (3)

From equation (1), we obtain

$$\sigma_z^2 = \left[ \frac{\Delta v_\parallel^2}{v_\parallel^2} \right] = \left[ \frac{\delta B_\perp^2}{B_0^2} \right]^{1/4}.$$ \quad (4)

Perturbation $\delta B_\parallel$ exists owing to the pseudo-Alfven modes in the incompressible turbulence. Inserting equation (3) into the expression for $D_{\mu\mu}$ (see Völk 1975; Paper I), we obtain

$$D_{\mu\mu} = \frac{\Omega^2 (1 - \mu^2)}{B_0^2} \int d^3 k \sum_{n=0}^{\infty} R_n(k_1 v_\parallel - \omega \pm n\Omega) \times \left[ I^A(k) \frac{n^2 f_2(w)}{w^2} + \frac{k_1^2}{k^2} f_2'(\omega) I^M(k) \right]. \quad (5)$$

where $\Omega$ and $\mu$ are the Larmor frequency and pitch-angle cosine of the CRs, respectively, $J_n$ represents the Bessel function, $w = k_1 v_\parallel / \Omega = k_1 L R (1 - \mu^2)^{1/2}$, where $R = \omega / (\Omega \Omega)$ is the dimensionless rigidity of the CRs and $L$ is the injection scale of the turbulence, $k_1$ and $k_2$ are the components of the wavevector $k$ perpendicular and parallel to the mean magnetic field, respectively, $\omega$ is the wave frequency, $I^A(k)$ is the energy spectrum of the Alfven modes, and $I^M(k)$ represents the energy spectrum of magnetosonic modes, which in our case at hand are the pseudo-Alfven modes. In QLT, the resonance function $R_0 = \pi \delta(k_1 v_\parallel - \omega \pm n\Omega)$. Now due to the perturbation of the orbit, it should be

$$R_n(k_1 v_\parallel - \omega \pm n\Omega) = \text{Re} \left( \int_0^\infty dt e^{i(k_1 v_\parallel + n\Omega t - (1/2)k_1^2 v_\parallel^2 t^2)} \right) \approx \frac{\sqrt{\pi}}{|k_1| \Delta v_\parallel} \exp \left[ - \frac{(k_1 v_\parallel - \omega + n\Omega)^2}{k_1^2 v_\parallel^2 (1 - \mu^2)} \right]. \quad (6)$$

where $M_\alpha = \delta V/v_A = \delta B/B_0$ is the Alfvenic Mach number and $v_A$ is the Alfven speed. We stress that equations (5) and (6) are generic and applicable to both the incompressible and compressible mediums.

For gyroresonance ($n = \pm 1, 2, \ldots$), apparently the result is similar to that from QLT for $\mu \gg \Delta \mu / \nu$. In this limit, equation (6) represents a sharp resonance and becomes equivalent to a $\delta$-function when put into equation (5). In general, the result is different from that of QLT, especially at $\alpha \to 90^\circ$, the resonance peak happens at $k_{\parallel, res} \sim \Omega / \Delta v$ in contrast to the QLT result $k_{\parallel, res} \sim \Omega / \Delta v \sim \infty$.\footnote{We show below that, due to the anisotropy, the scattering coefficient $D_{\mu\mu}$ is still very small if the Alfven and the pseudo-Alfven modes are concerned.}

On the other hand, the dispersion of $v_\parallel$ means that CRs with a wider range of pitch angles can be scattered by the pseudo-Alfven modes through TTD ($n = 0$), which is marginally affected by the anisotropy and much more efficient than the gyroresonance. Below we consider both the cases for scattering in strong and weak turbulence.

The nonlinear approach we use here is based on particle trapping by large-scale magnetic perturbations (Völk 1973, 1975). The difference is that we have a Gaussian profile (eq. [6]) resonance and he adopted a Heaviside step function. Formally, our approach also has a similarity to the second-order QLT that Shalchi (2005a) proposed for slab modes, although his approach is based on a different set of approximations.

### 2.2. Strong MHD Turbulence

In strong MHD turbulence, we assume that the Alfven and the pseudo-Alfven modes follow the scaling obtained in Cho et al. (2002), which is consistent with the GS95 model,

$$I^A(k) = I^M(k) = \frac{L^{-1/3} M_\alpha^{4/3}}{6\pi} \exp \left( - \frac{L^{1/3} k_{\parallel}}{M_\alpha^{1/3} k_{\perp}^{2/3}} \right). \quad (7)$$
The pitch-angle scattering arising from TTD with pseudo-Alfvén modes is given by equation (5) with \( n = 0 \),

\[
D_{\mu\nu}^T = \frac{v_0 \sqrt{\pi} (1 - \mu^2)}{6 \pi L R^2} \int_1^{k_{\text{max}}} d^3 x \frac{x_\perp^{-10/3} M_A^{4/3}}{(x_\perp^2 + x_\parallel^2) \Delta \mu} J_1^2(w) \times \exp \left[ -\frac{x_\perp^{2/3} M_A^{4/3}}{\Delta \mu^2} \left( \frac{\mu - v_A}{v_{\perp}} \right)^2 \right],
\]

where \( x = kL, x_\parallel = k_1 L \) are the normalized wavenumbers and \( k_{\text{max}} \) is the maximum wavevector of the turbulence, corresponding to the dissipation scale \( k^{-1}_{\text{diss}} \). Since all scales contribute to TTD and the magnetic perturbation increases with scale, the interaction is dominated by the large-scale moving mirrors \( k \sim 1/L \). Accordingly, we can make an estimate of the above equation. For CRs with small rigidities \( R = v/(L \Omega) \ll 1 \), \( w = x_\perp R(1 - \mu^2)^{1/2} \ll 1 \), and \( J_1(w) \sim w/2 \). On the large scale, the anisotropy of the turbulence is small, and we approximate \( (x_\perp^2 + x_\parallel^2) \) in the above expression by \( 2 x_\perp^2 \). Then we get from equation (8)

\[
D_{\mu\nu}^T \approx \frac{\sqrt{\pi} M_A^{7/2} v_0}{16L} \left( 1 - \mu^2 \right)^{3/2} \left[ -E_1(q x_\parallel) - e^{-q x_\parallel} \right]_{x_\parallel = \text{max}} \times \exp \left[ -\frac{(\mu - v_A/v_\perp)^2}{\Delta \mu^2} \right],
\]

where \( E_1 \) represents the function of the exponential integral \( E_1(x) = \int_0^\infty \exp(-xt)/t \) and \( q = (x_{\parallel \text{max}} M_A^2)^{-2/3} \). As shown in Figure 1, the above expression provides a good approximation for equation (8).

For gyroresonance, the interaction is dominated by the first-order harmonics, because higher order harmonics operate on smaller scales where turbulence energy is decreasingly small accord-
Fig. 3.—Scattering coefficients in weak incompressible turbulence. Dashed line represents TTD (i.e., $n = 0$) in the weak turbulence on large scales. Solid line and line of crosses (analytical approximation from eq. [13]) refer to TTD with the weak pseudo-Alfvén modes on large scales, and dashed line represents TTD with the strong turbulence on small scales. We see that gyroresonance (i.e., $n$ not equal to zero) which is denoted by the dash-dotted line is negligible because of the strong anisotropy. [See the electronic edition of the Journal for a color version of this figure.]

If, similar to the case of strong MHD turbulence, the pseudo-Alfvén modes in the weak turbulence follow the same scaling as the shear Alfvén modes, then equation (5) provides

$$D_{\mu\mu}^G = \frac{v \sqrt{\pi} (1 - \mu^2)}{2L^2} \int \frac{1}{(x_1^2 + 1)\Delta \mu} J_1^2(w)x_1^{-2} \times \exp \left[ -\left( \frac{\mu - \nu_A}{\Delta \mu} \right)^2 \right].$$

(12)

In the case where the rigidity $R$ is much less than 1, the Bessel function $J_1^2(w)$ can be replaced by the first-order approximation $w^2/4$. The above integration can then be evaluated analytically,

$$D_{\mu\mu}^G = \frac{v \sqrt{\pi}}{8L} (1 - \mu^2)^{3/2} M_A^{3/2} \tan^{-1} \left( M_A^{-2} - \frac{1}{2} \right) \times \exp \left[ -\left( \frac{\mu - \nu_A}{\Delta \mu} \right)^2 \right].$$

(13)

The result is compared with the numerical evaluation of equation (12) in Figure 3. Figure 3 presents the scattering coefficients owing to various interactions in an incompressible medium with $M_A = 0.1$. We account for TTD interactions with both the weak turbulence on large scales ($k \leq k^{-1} \geq \ell_0$) and the strong turbulence on small scales ($k^{-1} < \ell_0$). Here, TTD is present for a smaller range of $\mu$ compared to the case of $M_A \simeq 1$ (Fig. 1), as $\Delta \mu \sim M_A^{-2} \ll \mu$. For the rest of the pitch angles, the turbulence can only scatter CRs through gyroresonance, which is inefficient because of the turbulence anisotropy.

3. CR SCATTERING IN COMPRESSIBLE MHD TURBULENCE

In compressible turbulence, the pseudo-Alfvén modes become the slow modes, and an additional type of perturbation, fast modes, are present. The latter were identified as the major scattering agent for the MHD turbulence that is injected at large scales (YL02; Paper I). The papers were using QLT, however. Therefore, it is necessary to check the validity of this conclusion using the modified resonance function given by equation (6).

Therefore, below we provide calculations for scattering induced by the fast modes. The scatterings by the Alfvén and the slow waves are similar to those by the Alfvén and the pseudo-Alfvén modes discussed in §2. The latter has similarities to the former claimed on theoretical grounds (GS95) and confirmed using numerical simulations (Cho & Lazarian 2003).

For gyroresonance, according to equations (5) and (6), the pitch-angle diffusion coefficient is given by

$$D_{\mu\mu}^G = \frac{v \sqrt{\pi} (1 - \mu^2)}{2L^2} \int_1^{k_{\max}} d\xi \int_0^{1/\Delta \mu} \frac{x^{-5/2} \xi}{\Delta \mu} J_1(w)^2 \times \exp \left[ -\left( \frac{\mu - 1/(\xi R)}{\Delta \mu} \right)^2 \right],$$

(14)

where $\xi = \cos \theta$ is the cosine of wave pitch angle $\theta$. For TTD,

$$D_{\mu\mu}^T = \frac{v \sqrt{\pi} (1 - \mu^2)}{2L^2} \int_1^{k_{\max}} d\xi \int_0^{1/\Delta \mu} \frac{x^{-5/2} \xi}{\Delta \mu} J_1(w)^2 \times \exp \left[ -\left( \frac{\mu - \nu_A/(\xi R)}{\Delta \mu} \right)^2 \right].$$

(15)

For CRs with sufficiently small rigidities $R < 1/(k_{\max}L)$, the Bessel function can be approximated by the first-order asymptotics, and we obtain

$$D_{\mu\mu}^T = \frac{\sqrt{\pi}}{4} \frac{v (1 - \mu^2)^{3/2}}{\Delta \mu} \exp \left[ -\left( \frac{\mu - \nu_A/\xi}{\Delta \mu} \right)^2 \right] \times \int_0^{1/\Delta \mu} d\xi \sqrt{k_{\max}(\xi)} \frac{\xi (1 - \xi^2)}{L},$$

(16)

where $k_{\max}(\xi)$ is the cutoff wavenumber at $\xi$.

The scattering by the fast modes is exhibited in Figure 4, where we provided the numerical evaluations of equations (14) and (15) as well as the analytical approximation given by equation (16). We adopt viscous damping for the illustrative calculations (with the physical parameters in the warm ionized medium [WIM], see Table 1). Realistic calculation for the interstellar medium is given in §4. As we see, the TTD interaction dominates for large pitch angles until 90°. Gyroresonance is important for small pitch angles. While the TTD interaction is expanded for a much wider range (including 90°) compared to the QLT result (see, e.g., Fig. 3 in Paper I), the gyroresonance in QLT and non-linear theory (NLT) are comparable. This is because gyroresonance happens on a local scale $k_{\ell_1}^{-1} \sim r_1$ unlike TTD. The influence of large-scale trapping is thus limited. Especially for the range $\mu > \Delta \mu$, we see marginal difference. For large pitch angles, indeed there is a discrepancy between NLT and QLT. In fact, we see a similar trend in the result of Shalchi (2005a), which has close relations with the NLT by Völk (1973, 1975). However, because of the dominance of TTD in this range, this difference does not count. All in all, nonlinear effect is important for CR scattering, particularly for the contribution from TTD interaction; for gyroresonance, nevertheless, one can use the quasi-linear approximation to calculate the scattering of CRs at small pitch angles.

Note that although the overall contribution from gyroresonance is smaller than that from TTD, gyroresonance plays an important role in confining the CRs at small pitch angles. Without sufficient
4. CR PROPAGATION IN GALAXY

The scattering by fast modes is influenced by the medium properties, as the fast modes are subject to linear damping, e.g., Landau damping (see Ginzburg 1961). In Paper I we showed that the CR scattering is different in different ISM phases, but could not calculate the mean free path as we faced the scattering at 90° problem. Using the approach above we revisit the problem of the CR propagation in the selected phases of the ISM. In particular, we shall make quantitative predictions for the parallel mean free path of CRs in the Galactic halo and WIM (see Table 1 for a list of fiducial parameters appropriate for the idealized phases) assuming that the turbulence is injected at large scales.

4.1. Halo

In the Galactic halo (see Table 1), the Coulomb collisional mean free path is \( \sim 10 \, \text{pc} \); the plasma is thus in a collisionless regime. The cascading rate of the fast modes is (Cho & Lazarian 2002)

\[
\tau_k^{-1} = (k/L)^{1/2} \delta V^2 / V_{\text{ph}}.
\]

By equating it with the collisionless damping rate

\[
\Gamma_c = \frac{\sqrt{\pi} \beta \sin^2 \theta}{2 \cos \theta k v_A} \left[ \sqrt{\frac{m_e}{m_i}} \exp \left( - \frac{m_e}{\beta m_i \cos^2 \theta} \right) + 5 \exp \left( - \frac{1}{\beta \cos^2 \theta} \right) \right],
\]

we obtain the turbulence truncation scale \( k_c = k_{\text{max}} \).

\[
k_c L = 4 M_A^2 m_i \cos^2 \theta \exp \left( \frac{2 m_e}{\beta m_i \cos^2 \theta} \right),
\]

where \( \beta = P_{\text{gas}} / P_{\text{mag}} \).

The scale \( k_c \) depends on the wave pitch angle \( \theta \), which makes the damping anisotropic. As the turbulence undergoes turbulent cascade and/or the waves propagate in a turbulent medium, the angle \( \theta \) is changing. As discussed in Paper I, the field wandering defines the spread of angles. During one cascading time, the fast modes propagate a distance \( \pi r_{\text{cas}} \) and see an angular deviation \( \tan \delta \theta \simeq (\tan^2 \delta \theta_{||} + \tan^2 \delta \theta_{\perp})^{1/2} \), which is

\[
\tan \delta \theta \simeq \sqrt{\frac{M_A^2 \cos \theta}{27 (kL)^{1/2}} + \left( \frac{M_A^2 \sin^2 \theta}{kL} \right)^{1/3}}.
\]

The parameters of idealized interstellar phases are a subject of debate. Recently, even the entire concept of the phase being stable entities has been challenged (see Gazol et al. 2007 and references therein). Similar to in Paper I, we were guided in choosing the numbers by our communications with D. Cox (2006, private communication). However, we accept that different parts of the interstellar medium can exhibit variations of these parameters (see Wolfire et al. 2003 and references therein).

### Table 1

| ISM     | \( T \) (K) | \( C_S \) (km s\(^{-1}\)) | \( n \) (cm\(^{-3}\)) | \( L_{\text{mp}} \) | \( L \) (pc) | \( B \) (\( \mu G \)) | \( \beta \) | Damping                  |
|---------|-------------|----------------|----------------|-----------------|-------------|----------------|---------|---------------------|
| Halo    | \( 10^9 \)  | 91             | \( 10^{-3} \)  | \( 4 \times 10^{19} \) | 100         | 5             | 0.14    | Collisionless             |
| WIM     | 8000        | 8.1            | 0.1            | \( 6 \times 10^{12} \) | 50          | 6             | 0.077   | Collisionless and viscous |

Note.—The dominant damping mechanism for the fast mode turbulence is given in the last column.
As is evident, the damping scale given by equation (19) varies considerably especially when \( \theta \to 0^\circ \) and \( \theta \to 90^\circ \). For the quasi-parallel modes, the randomization \([\times(kL)^{-1/4}]\) is negligible since the turbulence cascade continues to very small scales. On small scales, most energy of the fast modes is contained in these quasi-parallel modes (Paper I; Petrosian et al. 2006).

For the quasi-perpendicular modes, the damping rate (eq. [18]) should be averaged over the range \( 90^\circ - \theta \approx 90^\circ \). Equating equation (17) and equation (18) averaged over \( \Delta \theta \), we get the averaged damping wavenumber (see Fig. 5). The field line wandering has a marginal effect on the gyroresonance, whose interaction with the quasi-perpendicular modes is negligible (Paper I). However, TTD scattering rates of moderate-energy CRs (<10 TeV) will be decreased owing to the increase of the damping around 90\(^\circ\) (see Fig. 6). For higher energy CRs, the influence of damping is marginal and so is that of field line wandering.

We adopt QLT for calculating the gyroresonance (see Paper I). As we see from Figure 4, the QLT result in the range \( \mu > \Delta \mu \) provides a good approximation for our calculations using the nonlinear approximation given by equation (14). For CRs with sufficiently small rigidities, the resonant fast modes \([k_{\text{res}} \approx 1/(R \mu)]\) are on small scales with a quasi-slab structure (see Fig. 5). For the scattering by these quasi-parallel modes, the analytical result that follows from the QLT approximation (see Paper I) for the gyroresonance is

\[
\frac{D_{\nu \mu}^G}{D_{pp}^G} = \frac{\pi \mu_0^2 (1 - \mu^2)}{4L R^0.5} \left[ \frac{1}{7} \left[ 1 + (R \mu)^2 \right]^{-7/4} - \left( \tan^2 \theta + 1 \right)^{-7/4} \right. \\
\left. \times \frac{m^2 v^2}{3} \left\{ 1 + (R \mu)^2 \right\}^{-3/4} - \left( \tan^2 \theta + 1 \right)^{-3/4} \right],
\]

where \( \tan \theta = k_{\perp,\text{res}}/k_{\perp,\text{res}} \). This justifies our use of the analytical approximation above.

Once we know the functional form of \( D_{\nu \mu} \), we can obtain the corresponding mean free path (Earl 1974),

\[
\frac{\lambda_{\perp}}{L} = \frac{3}{4} \int_0^1 d\mu \frac{\nu(1 - \mu^2)^2}{(D_{\nu \mu}^T + D_{\nu \mu}^T) L},
\]

where \( D_{\perp}^T \) can be obtained by equations (15)–(16) and (19). Inserting \( D_{\perp}^T \) and the QLT result for \( D_{\perp}^T \) into the above expression, we get the mean free path of CRs in the halo.

The mean free path is sensitive to the scattering by gyroresonance at small pitch angles, due to the influence of damping on the fast modes on small scales. Figure 7 shows the pitch-angle
The weak dependence of the mean free path (see Fig. 9) for the moderate-energy (e.g., \(< 1 \text{ TeV}\)) CRs in the halo results from the fact that gyroresonance changes marginally with the CR energy (see Fig. 7). Gyroresonance happens on small scales where the fast modes develop a quasi-slab structure because of the damping (see Fig. 5). In the case of the halo, the critical \(\theta_c\) changes more slowly compared to the case in the WIM (see Fig. 9); the scattering by gyroresonance is thus marginally changing with the energy (see eq. [21] and Fig. 7). For higher energy CRs with larger gyroscales, the influence of damping is small, and thus, the CR mean free path begins increasing with energy.

4.2. WIM

In the WIM, the Coulomb collisional mean free path is \(l_{\text{mfp}} = 6 \times 10^{12} \text{ cm}\) and the plasma \(\beta \approx 0.11\). Suppose that the turbulence energy is injected from large scales, then the compressible turbulence is subjected to the viscous damping besides the collisionless damping. By equating the viscous damping rate with the cascading rate (eq. [17]), we obtain the truncation scale

\[
 k_cL = \begin{cases} 
 x_c(1 - \xi^2)^{-2/3}, & \beta \ll 1, \\
 x_c(1 - 3\xi^2)^{-4/3}, & \beta \gg 1,
\end{cases}
\]

where \(x_c = (6\rho_0 V^2 L/(\eta_0 V_A)^2)^{1/3}\) and \(\eta_0\) is the longitudinal viscosity. In the low-\(\beta\) regime, the motions are primarily perpendicular to the magnetic field so that \(\partial v_x/\partial x = \bar{n}/n \sim B/B\). The longitudinal viscosity enters here as the result of distortion of the Maxwellian distribution (see Braginskii 1965). The transverse energy of the ions increases during compression because of the conservation of the adiabatic invariant \(v_z^2/B\). If the rate of compression is faster than that of collisions, the ion distribution in the momentum space is bound to be distorted from the Maxwellian isotropic sphere to an oblate spheroid with the long axis perpendicular to the magnetic field. As a result, the transverse pressure becomes greater than the longitudinal pressure, resulting in a stress \(\sim n_0 \partial v_y/\partial x\). The restoration of the equilibrium increases the entropy and causes the dissipation of energy.

The viscous damping scale is compared to the collisionless cutoff scale (eq. [19]) in Figure 5. As shown there, both viscous damping and collisionless damping are important in the WIM. Viscous damping is dominant for small \(\theta\), and collisionless damping takes over for large \(\theta\) except for \(\theta = 90^\circ\). This is because collisionless damping increases with \(\theta\) much faster than the viscous damping. For sufficiently small wave pitch angles, the viscous damping is too small to prevent the fast modes from cascading down to scales smaller than the mean free path \(l_{\text{mfp}}\). Because of the similar quasi-slab structure on small scales, equation (21) can also be applied in the WIM. The results are illustrated in Figure 8. Compared to the case in the halo, we see that the qualitative difference stands in the gyroresonance, because gyroresonance is sensitive to the quasi-slab modes for which dampings differ in the halo and WIM.

The mean free paths of CRs are given in Figure 9. We see the mean free path first decreases with the energy until 100 GeV. This is because of the influence of viscous damping on gyroresonance (see Fig. 5). The lower the energy of the CRs is, the less efficient is the gyroresonance according to equation (21) (see Fig. 8). For higher energy CRs, for which rigidities are larger than \(R \gtrsim (3 \times 10^6)^{-1}\), the maximum wavenumber of the resonant wave modes are determined by collisionless damping (see Fig. 5). As a result, the mean free path grows with energy similar to the case in the halo where collisionless damping is dominant.

4.3. Other Phases

In the hot ionized medium (HIM), the plasma is also in a collisionless regime, but the density is higher and the plasma-\(\beta\) is larger than 1. The damping by protons thus becomes substantial especially at small pitch angles. The damping truncates the turbulence at much larger scales than the gyroscales of the CRs of the energy range we consider. No gyroresonance can happen and some other mechanisms are necessary to prevent CRs streaming freely along the field. The turbulence injected from small scales might play an important role (see § 5).
In partially ionized gas one should take into account an additional damping that arises from ion-neutral collisions (see Kulsrud & Pearce 1969; Lithwick & Goldreich 2001; Lazarian et al. 2004). In the latter work, a viscosity-damped regime of turbulence was predicted at scales less than the scale $k_{\text{amb}}^{-1}$ at which the ordinary magnetic turbulence is damped by ionic viscosity. The corresponding numerical work, e.g., Cho et al. (2003), testifies that for the viscosity-damped regime the parallel scale stays equal to the scale of the ambipolar damping, i.e., $k_{\parallel} = k_{\text{amb}}$, while $k_{\perp}$ increases. In that respect, the scattering by such magnetic fluctuations is analogous to the scattering induced by the weak turbulence (see § 2.3). The difference stems from the spectrum of $k_{\perp}$ being shallower than the spectrum of the weak turbulence. The predicted values of the spectrum for the viscosity-damped turbulence $\tilde{E}(k_{\perp}) \sim k_{\perp}^{-3}$ (Lazarian et al. 2004) are in rough agreement with simulations. More detailed studies of scattering in partially ionized gas will be provided elsewhere.

5. PERPENDICULAR TRANSPORT

While in the earlier sections we dealt entirely with the diffusion parallel to the magnetic field, in this section we deal with the diffusion perpendicular to the mean magnetic field. The assumption that CRs follow the magnetic field averaged over their Larmor radius is pretty accurate in most situations, e.g., for Galactic CRs.

Compound diffusion happens when particles are restricted to the magnetic field lines and perpendicular transport is solely due to the random walk of field line wandering (see Kota & Jokipii 2000). In the three-dimensional turbulence, field lines are diverging away due to shearing by the Alfvén modes (see Lazarian & Vishniac 1999; Narayan & Medvedev 2001; Lazarian 2006, 2007). Since the Larmor radii of CRs are much larger than the minimum scale of eddies $l_{\text{min}}$, field lines within the CR Larmor orbit are effectively diverging away owing to shear by the Alfvénic turbulence. The cross-field transport thus results from the deviations of field lines at small scales, as well as FLRW at large scales [$\gtrsim \text{min}(L/M_\Lambda^2, L)$].

Most recently, the diffusion in magnetic fields was considered for thermal particles in Lazarian (2006, 2007). In what follows we modify the results of these studies for the case of CRs.

For perpendicular diffusion, the important issue is the frame of reference. We emphasize that we consider the diffusion perpendicular to the mean field direction in the global frame of reference.

5.1. Perpendicular Diffusion on Large Scales

High-$M_\Lambda$ turbulence.—High-$M_\Lambda$ turbulence corresponds to the field that is easily bent by the hydrodynamic motions at the injection scale as well as the hydro energy at the injection scale is much larger than the magnetic energy, i.e., $\rho v^2 \gg B^2$. The turbulence in clusters of galaxies is the high-$M_\Lambda$ turbulence. In this case the magnetic field becomes dynamically important on a much smaller scale, i.e., the scale $l_\Lambda = L/M_\Lambda^2$ (see Lazarian 2006). If $l_\parallel > l_\Lambda$, the CR diffusion is controlled by the straightness of the field lines, and

$$D_\perp = D_\parallel \approx 1/3l_\Lambda v, \quad M_\Lambda > 1, \quad l_\parallel > l_\Lambda.$$  

(24)

The diffusion is isotropic if scales larger than $l_\Lambda$ are concerned. In the opposite limit $l_\parallel < l_\Lambda$, the stiffness of the $B$ field is negligible. The CR diffusion is insensitive to the topology of the magnetic field. In the global frame of reference, there is no distinction between the perpendicular and parallel direction. Naturally, a result for isotropic turbulence, namely,

$$D_\perp = D_\parallel \approx 1/3l_\parallel v,$$  

(25)

holds.

Low-$M_\Lambda$ turbulence.—For strong magnetic field, i.e., the field that cannot be easily bent at the turbulence injection scale, individual magnetic field lines are aligned with the mean magnetic field. The diffusion in this case is anisotropic. As we mentioned earlier, if the turbulence is injected at scale $L$ it stays weak for the scales larger than $l_\text{inj}$ given by equation (11) and it is strong at smaller scales.

Consider first the case of the CR parallel mean free path larger than the injection scale of the turbulence, i.e., $\lambda_\parallel > L$. The perturbations of the field are uncorrelated over scales larger than $LM_\Lambda^{-1}$ in the direction perpendicular to the mean magnetic field. Indeed, this perpendicular distance corresponds to the particle moving a parallel distance of the order $L$, which is the scale of the energy injection, which in a simplified picture of turbulence is the maximal scale over which the magnetic perturbations are correlated.

In this situation the random walk steps in the perpendicular direction are of $l_\text{inj}$ length. Thus, to diffuse over a distance $R$ with random walk of $l_\text{inj}$ one requires $(R/l_\text{inj})^2$ steps. If the time of the individual step is $L/v_\text{inj}$, then

$$D_\perp = \frac{R^2}{\delta t} = \frac{R^2}{(R/l_\text{inj})^2L/v_\text{inj}} \approx \frac{1}{3Lv_\text{inj}M_\Lambda^4}, \quad M_\Lambda < 1, \quad \lambda_\parallel > L.$$  

(26)

This is similar to the case discussed in the FLRW model (Jokipii 1966). However, we obtain the dependence of $M_\Lambda^2$ instead of their $M_\Lambda^2$ scaling. This difference is not critical for environments like the Milky Way or solar wind, for which $M_\Lambda \sim 1$, but may be important for other environments where strong slightly perturbed magnetic field is present, e.g., the solar corona.

What would be the CR diffusion perpendicular to the mean magnetic field in the opposite case of $\lambda_\parallel < L$? The time of the individual step is $L^2/D_\parallel$. Therefore, the perpendicular diffusion coefficient is

$$D_\perp = \frac{R^2}{\delta t} \approx \frac{R^2}{(R/l_\text{inj})^2L^2/D_\parallel} = D_\parallel M_\Lambda^4, \quad M_\Lambda < 1, \quad \lambda_\parallel < L,$$  

(27)

which coincides with the result obtained for the diffusion of electrons in magnetized plasma (Lazarian 2006). The turbulence in the interplanetary medium is, in this regime, with $M_\Lambda \lesssim 1$. From equation (27), we obtain a constant ratio of $\lambda_\perp/\lambda_\parallel = D_\perp/D_\parallel = M_\Lambda^4$, consistent with the Palmer consensus (Palmer 1982).

We mention parenthetically that our arguments above can be repeated for any random walk process in the perpendicular direction with a step $\delta x$. If we can write $D_\perp = (\delta x/\delta z)^2/D_\parallel$, we, naturally, recover the result in equation (27).

5.2. Perpendicular Diffusion on Small Scales

The diffusion of CR on the scales $\ll L$ may be different. We consider particular examples below.

High-$M_\Lambda$ turbulence.—Consider the diffusion on scales that are $\lambda_\parallel < |k^{-1}_\parallel| < l_\Lambda$, i.e., on scales at which CRs are in the diffusive

\footnote{For the sake of simplicity we disregard the effects of the inverse cascade that can increase the correlation scale of magnetic perturbations.}
regime, but the magnetic fields are strong enough to influence turbulent motions, the mean deviation of a field in a distance \( k^{-1} = \delta z \) is given by (Lazarian & Vishniac 1999)

\[
\langle (\delta x)^2 \rangle^{1/2} = \left( \frac{[\delta z | M_\perp|^3]}{3^3 L^{11/2}} \right), \quad M_\perp > 1.
\]  

(28)

Thus, for scales much less than \( L \)

\[
D_\perp \approx \left( \frac{\delta x}{\delta z} \right)^2 D_\parallel \sim \frac{[\delta z | M_\perp|^3]}{3^3 L^{11/2}} \quad D_\parallel \sim (k_l)_{l_A}^{-1}, \quad M_\perp > 1,
\]  

(29)

which for a limiting case \( |k_l| \sim l_A^{-1} \) gets the result consistent with equation (24).

Low-M_\perp turbulence.---On scales larger than \( l_R \), the turbulence is weak (see § 2.3). The mean deviation of a field in a distance \( \delta z \) is given by Lazarian (2006) as

\[
\langle (\delta x)^2 \rangle^{1/2} = \left( \frac{[\delta z | M_\perp|^3]}{3^3 L^{11/2}} \right) M_\perp^2, \quad M_\perp < 1.
\]  

(30)

For the scales \( L > |\delta z| > l_R \), we combine equation (30) with

\[
|\delta z| = \sqrt{D_\parallel \delta t}
\]  

(31)

and get for scales much less than \( L \)

\[
D_\perp \approx \left( \frac{\delta x}{\delta z} \right)^2 D_\parallel \sim \frac{D_\parallel |\delta z|}{3^3 L^{11/2}} M_\perp^2 \sim D_\parallel |k_l|^4 L^{11/2},
\]  

(32)

which for a limiting case \( |k_l| \sim L^{-1} \) coincides up to a factor with equation (27).

Equations (29) and (32) certify that the perpendicular diffusion at scales much less than the injection scale accelerates as \( z \) grows. The reason is that there is no random walk on small scales up to the injection scale of the strong MHD turbulence \( l_R \) for \( M_\perp < 1 \) and \( l_R \) for \( M_\perp > 1 \). The diffusion on scales less than the turbulent injection scale is important for describing propagation and acceleration of CRs in supernovae shells, clusters of galaxies, etc.

5.3. Subdiffusion

The diffusion coefficient in equation (27), i.e., \( D_\parallel M_\perp^2 \), means that the transport perpendicular to the dynamically strong magnetic field is a diffusion, rather than subdiffusion, as it was stated in a number of recent papers. Let us clarify this point by obtaining the necessary conditions for the subdiffusion to take place.

In the papers discussing compound diffusion (see Kótá & Jokipii 2000; Webb et al. 2006), \( (\delta x)^2/|\delta z| = D_{\text{pat}} \) is a spatial diffusion constant coefficient. If we adopt this, we shall indeed get from equation (31) the perpendicular diffusion coefficient

\[
D_\perp = \left( \frac{\delta x}{\delta z} \right)^2 D_\parallel = \frac{D_{\text{pat}} |D_\parallel|}{|\delta z|} = D_{\text{pat}} D_\parallel^{1/2} (\delta t)^{-1/2}.
\]  

(33)

Therefore, the perpendicular transposition will be \( \chi^2 = D_\perp \delta t = D_{\text{pat}} D_\parallel^{1/2} (\delta t)^{1/2} \) in accordance with the findings in the aforementioned papers.

The major implicit assumption in the reasoning above is that the particles trace back their trajectories in the \( x \)-direction on the scale \( \delta z \). If this is not true, the introduction of the diffusion coefficient \( D_{\text{pat}} \) does not make sense.

When is it possible to talk about tracing particle trajectories back? In the case of random motions at a single scale only, the distance over which the particle trajectories get uncorrelated is given by the Rechester & Rosenbluth (1978) model. On scales larger than the Rechester & Rosenbluth scale \( k^{-1} > L_{RR} \), the separation between field lines \( \delta x \) grows monochromatically with the distance \( \delta z \) along the magnetic field, no retracing can happen in this case. Assuming that the damping scale of the turbulence is larger than the CR Larmor radius, this model, when generalized to the anisotropic turbulence, provides (Narayan & Medvedev 2001; Lazarian 2006)

\[
L_{RR} = l_{R|\perp|} \ln \left( l_{R|\perp|}/r_L \right).
\]  

(34)

where \( l_{R|\perp|} \) is the parallel scale of the cutoff of turbulent motions, \( l_{R|\perp|} \) is the corresponding perpendicular scale, and \( r_L \) is the CR Larmor radius. The assumption of \( r_L < l_{R|\perp|} \) can be valid, for instance, for the Alfvenic motions in partially ionized gas. However, it is easy to see that, even in this case, the corresponding scale is rather small, and therefore, subdiffusion is not applicable for the transport of particles in the Alfvenic turbulence over scales \( \gg l_{R|\perp|} \).

If \( r_L > l_{R|\perp|} \), as it is a usual case for the Alfven motions in the phase of the ISM with the ionization larger than \( \approx 93\% \), where the Alfvenic motions go to the thermal particle gyroradius (see the estimates in Lithwick & Goldreich 2001; Lazarian et al. 2004), the subdiffusion of CRs is not an applicable concept for the Alfvenic turbulence. This does not preclude subdiffusion from taking place in particular models of magnetic perturbations, e.g., in the slab model considered in Shalchi (2005b), but we believe in the omnipresence of the Alfvenic turbulence in interstellar gas (see Armstrong et al. 1995).

6. DISCUSSIONS

The present paper extends our study in Paper I. As in Paper I we mostly deal with the magnetic perturbations that are part of the large-scale turbulent cascade, which is consistent with the big power law in the sky observed via radio-scattering and scintillation techniques (Armstrong et al. 1995). In both papers we use the description of the MHD turbulence that follows from numerical simulations.

In Paper I we have the CR scattering calculated in the selected interstellar environments making use of quasi-linear theory (QLT). Because of the limitations of the QLT, we could not provide calculations of the mean free path in Paper I, which limited the utility of the study. In this paper we extended the nonlinear approach suggested in Voll (1975) to treat the scattering, which allows us to calculate the mean free paths that arise from CR interactions with the fast modes. In doing so, similar to Paper I, we take into account damping of the fast modes in the presence of the field wandering induced by the Alfvenic modes.

Our results show that in the WIM and halo of our Galaxy, confinement of bulk CRs are mostly due to the compressible modes. We obtain CR mean free paths of about a few parsec, consistent with what observations indicate. The major difference with the earlier picture is the dependencies of CR transport parameters on the medium properties. The dependence appears as a result of damping of the fast modes. For low-energy CRs (\( \leq 100 \) GeV), if dominated by viscous damping, the mean free path of CRs would decrease with energy; with collisionless damping, however, CRs’ mean free path stays almost a constant. Field line wandering in general increases the damping of the fast modes and reduces the scattering efficiency of CRs. For higher energy CRs, the influence
of damping is limited, and their mean free path increases with energy. The dependencies on the turbulence damping and therefore the phase properties should have various implications for the ratio of secondary to primary elements, diffuse Galactic γ-ray emission, and the cosmic microwave background synchrotron foreground. With precise measurements, the understanding of cosmic microwave background is now constrained by our understanding of the foreground. The variation of CR index over the Galaxy may paralyze the synchrotron templates. Such variations can be addressed on the basis of the more elaborate CR propagation theory.

The importance of this study goes beyond the ISM. For instance, Brunetti & Lazarian (2007) treated acceleration of CRs for plasma in clusters of galaxies by appealing to the fast modes, which is an approach to CRs similar to that in Paper I. We believe that the nonlinear treatment may be useful for such cases as well. In addition, stochastic acceleration by the MHD turbulence is a promising mechanism for generating high-energy particles during solar flares (see, e.g., Petrosian & Liu 2004 and references therein). An application to the acceleration of CRs in solar flares will be given in Yan et al. (2008).

In our treatment we attempted to use the scalings (1) that are consistent with numerical calculations and (2) whose amplitudes we can estimate with a sufficient degree of precision. Therefore, our present study does not deal with scattering of CRs by the fast modes on the scales $l > l_{M_A}^2 M_A < 1$, i.e., on the scales where the Alfvénic turbulence is in the weak regime. It was suggested by Chandran (2005) that the weak fast modes at small pitch angles tend to steepen due to the coupling with the Alfvén modes. When the resulting scaling of the fast modes becomes clearer, our approach will be applicable to them.

We have not quantitatively dealt in the present paper with the case of the slab Alfvén modes created by instabilities.9 The CR scattering by the perturbations created by those modes may dominate over the gyroresonance with the fast modes, especially for CRs of low energies, i.e., whose gyroresonance with the fast modes is inefficient due to the fast mode damping (see estimates in Lazarian & Beresnyak 2006). Progress in quantitative description of the nonlinear stages of the instabilities that can create slab modes should enable comprehensive models that include both the fast modes and the slab modes.

In addition, we addressed the issue of perpendicular diffusion, the issue that we did deal with in Paper I. We found that, similar to the case of thermal diffusion discussed in Lazarian (2006), the diffusion of CRs depends on the Alfvénic Mach number $M_A$. We found that the suppression of the perpendicular diffusion compared to the parallel one scales as $M_A^2$ for $M_A < 1$. Approaching the issue of subdiffusion, we found that it is negligible for CRs in the Alfvénic turbulence.

7. SUMMARY

Our result can be briefly summarized as follows.

1. Treatment of the scattering in both strong and weak MHD turbulence has been generalized to account for perturbations of the particle orbits. We found that the nonlinear treatment is essential for calculating mean free paths of CRs.

2. Our calculations of scattering rates performed for different modes of MHD turbulence, assuming that the turbulence is injected at large scales, confirm the dominance of the fast modes for scattering of the bulk CRs in the WIM and Galactic halo.

3. We obtained the relation between the CR diffusion coefficient parallel to the magnetic field and the CR diffusion coefficient perpendicular to the magnetic field. We show that CR transport perpendicular to the magnetic field depends strongly on the Alfvén Mach number of the turbulence.

We are grateful to the anonymous referee for his/her valuable comments and suggestions. We thank Chris Thompson for fruitful discussions. H. Y. is supported by CITA and the National Science and Engineering Research Council of Canada. A. L. acknowledges the NASA grant X5166204101, the NSF grant ATM 06-48699, as well as the NSF Center for Magnetic Self-Organization in Laboratory and Astrophysical Plasmas.

9 Streaming instability (see Cesarsky 1980) is an example of such instability. However, the instability is suppressed by both ion-neutral damping (Kulsrud & Pearce 1969) and the ambient turbulence (YL02; Farmer & Goldreich 2004; Paper I; Lazarian & Beresnyak 2006). Another example is the gyroresonance instability discussed in the context of CRs in Lazarian & Beresnyak (2006).

REFERENCES

Alfvén, H., & Fälthammar, C. G. 1963, Cosmical Electrodynamics (Oxford: Clarendon)
Allan, H. R. 1972, Astrophys. Lett., 12, 237
Armstrong, J. W., Rickett, B. J., & Spangler, S. R. 1995, ApJ, 443, 209
Beresnyak, A., & Lazarian, A. 2006, ApJ, 640, L175
Bieber, J. W., Smith, C. W., & Matthaeus, W. H. 1988, ApJ, 334, 470
Biskamp, D. 2003, Magnetohydrodynamic Turbulence (Cambridge: Cambridge Univ. Press)
Boldyrev, S. 2005, ApJ, 626, L37
———. 2006, Phys. Rev. Lett., 96, 115002
Braginskii, S. I. 1965, Rev. Plasma Phys., 1, 265
Brunetti, G., & Lazarian, A. 2007, MNRAS, 378, 245
Cesarsky, C. 1980, ARA&A, 18, 289
Chandran, B., & Lazarian, A. 2002, Phys. Rev. Lett., 88, 245001
Cho, J., & Lazarian, A. 2000, Phys. Rev. Lett., 85, 4566
———. 2005, Phys. Rev. Lett., 95, 265004
Cho, J., & Lazarian, A. 2002, Phys. Rev. Lett., 88, 245001
Cho, J., Lazarian, A., & Vishniac, E. T. 2002, ApJ, 564, 291
———. 2003, in Turbulence and Magnetic Fields in Astrophysics, ed. E. Falgarone & T. Passot (Berlin: Springer), 56
Cho, J., & Vishniac, E. 2000, ApJ, 539, 273
Duffy, F., Kirk, J. G., Gallant, Y. A., & Dendy, R. O. 1995, A&A, 302, L21
Dupree, T. H. 1966, Phys. Fluids, 9, 1773
Earl, J. A. 1974, ApJ, 193, 231
Elmegreen, B., & Scalo, J. 2004, ARA&A, 42, 211

Farmer, A., & Goldreich, P. 2004, ApJ, 604, 671
Felice, G. M., & Kulsrud, R. M. 2001, ApJ, 553, 198
Forman, M. A., Jokipii, J. R., & Owens, A. J. 1974, ApJ, 192, 535
Gazol, A., Kim, J., Vázquez-Semadeni, E., & Luis, L. 2007, in ASP Conf. Ser. 365, SINS – Small Ionized and Neutral Structures in the Diffuse Interstellar Medium, ed. M. Havercorn & W. M. Goss (San Francisco: ASP), 154
Getman, A. V. 1963, Soviet Astron., 6, 477
Giacalone, J., & Jokipii, J. R. 1999, ApJ, 520, 204
Ginzburg, V. L. 1961, Propagation of Electromagnetic Waves in Plasma (New York: Gordon & Breach)
Gogoberidze, G. 2007, Phys. Plasmas, 14, 022304
Goldreich, P., & Sridhar, H. 1995, ApJ, 438, 763 (GS95)
Goldstein, M. 1976, ApJ, 204, 900
Higdon, J. C. 1984, ApJ, 285, 109
Iroshnikov, P. 1963, Astron. Zh., 40, 742
Jokipii, J. R. 1966, ApJ, 146, 480
Jokipii, J. R., & Parker, E. N. 1969, ApJ, 155, 777
Jones, F. C., Birmingham, T. J., & Kaiser, T. B. 1973, ApJ, 180, L139
———. 1978, Phys. Fluids, 21, 347
Kirk, J. G., Duffy, P., & Gallant, Y. A. 1996, A&A, 314, 1010
Koita, J. & Jokipii, J. R. 2000, ApJ, 531, 1067
Kowal, G., & Lazarian, A. 2007, ApJ, 666, L69
Kraichnan, R. 1963, Phys. Fluids, 8, 1385
Kritsuk, A. G., Norman, M. L., Padoan, P., & Wagner, R. 2007, ApJ, 665, 416
Kulsrud, R., & Pearce, W. P. 1969, ApJ, 156, 445
