Analysis of the Harmonic Transmission Characteristics of HVDC Transmission Based on a Unified Port Theory Model

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ABSTRACT The application of power electronics can introduce flexibility into power systems, but it can also cause harmonic problems. As a typical application of power electronics, high-voltage direct current (HVDC) transmission exhibits very prominent harmonic problems. In recent years, many HVDC protection maloperation events caused by harmonic problems have occurred in the field. The phenomena of harmonic transfer and amplification in HVDC transmission have an undeniable impact on the overall operation of a power system. Previous studies of these phenomena have relied mainly on detailed electromagnetic transient simulations. However, these simulations normally suffer from a large computational burden and low efficiency due to the complexity of the models and the small simulation time steps used. Therefore, an efficient analysis model of harmonics for various HVDC operating modes is presented in this paper. This model is based on port theory; every part of the HVDC transmission system is represented by an equivalent two-port component to construct a unified equivalent circuit. The proposed model is simple to calculate, and only the connections of the two-port components need to be changed to model different operation modes. The proposed two-port model is applied for the harmonic analysis of an HVDC transmission system, and the validity and accuracy of the model are confirmed by detailed electromagnetic simulations. Moreover, the model can be easily applied to HVDC systems for harmonic transfer and amplification, and the effects of the HVDC transmission power and smoothing reactor on harmonic transfer and amplification are analyzed.

INDEX TERMS HVDC transmission, harmonic modeling, two-port, operation model, harmonic transfer and amplification.

I. INTRODUCTION
The proportion of power electronics apparatuses in power systems has grown in recent years, and harmonic problems are becoming increasingly prominent in power systems [1]–[3]. For example, the harmonic problem of line-commutated converter high-voltage direct current (HVDC) transmission is particularly severe [4].

Recently, as an increasing number of HVDC transmission projects have been put into operation, the harmonic interaction between AC and DC systems and the transmission process of harmonics between AC and DC systems have become extremely complicated [5], [6]. In this case, the harmonics generated by AC-side faults or other behaviors of the AC/DC hybrid power grid may be transmitted to the DC side, causing maloperation of the harmonic protection system [7]–[10]. The phenomena of harmonic transfer and amplification in HVDC transmission have an undeniable impact on the operation of the overall power system [11], [12]. Many studies have been conducted on HVDC harmonics in academic and engineering circles. However, these studies have mainly focused on establishing an equivalent model that can accurately reflect the harmonics of the converter and determining how the interaction of AC-side harmonics and DC-side harmonics causes instability [13], [14]. The research on the harmonic calculation, transfer, and amplification of a detailed HVDC transmission system is insufficient [15], [16]. The stability of HVDC was analyzed in [17]–[19]; however, these
analyses rely on small signal models and do not pay much attention to harmonic problems. Based on a dynamic phasor model, the harmonics of a DC system under asymmetric faults were calculated in [20]–[22]; however, the study did not analyze harmonic transfer and amplification in detail. A switching function model was used to calculate the harmonics and inter-harmonics of DC systems for weak AC systems in [23], but the model was a back-to-back model and did not consider DC lines. A new approach for modeling line-commutated AC/DC/AC conversion systems based on modulation function theory was presented and discussed in [24]. This new approach considers the DC-side harmonics caused by the impedance time variance. However, a solution of the time-varying impedance was not given in the paper, and the influence of the line was not considered. A method for directly calculating the harmonic current flowing through a DC conductor and a grounding conductor and the harmonic current induced in the DC side and grounding conductors was proposed in [25]; however, the paper did not consider the different operating modes of the HVDC system. A DC system model and transmission line model were established to analyze the DC harmonics in [26]; this calculation method can effectively calculate the harmonics on the DC side, but the model of the DC line is too simple, and the harmonic calculation method is only for a certain operation mode of the HVDC system. An HVDC harmonic domain model was presented in [27]. The HVDC link was also modeled with an extended control system for a realistic specification of the steady-state operating point. The Newton method, which requires a high initial value and may not converge, was used to solve the problem. In the abovementioned research, a model of the converter was established, and the harmonics on the DC side were calculated. However, the DC line model is not considered adequately, and the operation of the HVDC system is not considered in the analysis of the harmonics.

In this paper, a detailed two-port analysis model of an HVDC transmission system is presented. To cover the different HVDC operation modes, the HVDC system is modeled in detail, including DC smoothing reactors and filters. These complex situations were not modeled in the work presented above. In the two-port model proposed in this paper, the transformer, converter, smoothing reactor and filter, and DC line in the HVDC transmission system are equivalent to one two-port network. The connection of the two ports of each part forms a whole two-port network. According to the different operation modes of HVDC transmission, only the connection mode of the two-port part needs to be modified, and the other parts remain unchanged [28], [29]. In this way, HVDC transmission forms a unified two-port network under different operation modes. After verifying the accuracy of the established model, the phenomena of harmonic transfer and amplification in HVDC transmission are analyzed. This paper also presents an analysis to show the influence of the HVDC transmission power and smoothing reactor on harmonic transmission and amplification.

This paper is organized as follows. Section II describes the HVDC transmission system and the whole equivalent two-port network. The specific equivalent model of the converter and the DC line are described in detail in Section III. Section IV describes the equivalent method of the different connections of the two ports and the port processing of the HVDC in different operating modes. Section V presents a case study and discussion comparing the processing results of the proposed model with those of the PSCAD/EMTDC and then analyzes the harmonic transmission characteristics. The conclusions are given in section VI.

II. HVDC TRANSMISSION SYSTEM

The typical schematic diagram of an HVDC transmission system is shown in FIGURE 1. In this diagram, $U$ stands for the voltage, $I$ stands for the current, and $Q$ represents the connection switch. The subscript $r$ stands for the rectifier side, the subscript $i$ stands for the inverter side, the subscript 1 stands for pole 1, and the subscript 2 stands for pole 2. $R_{gr}$ and $R_{gi}$ represent the grounding electrode resistances of the rectifier side and the inverter side, respectively. The representation in the following figure is similar.

According to the components, the HVDC transmission is divided into different parts. The dashed lines in the schematic are represented as a two-port network. The specific equivalent circuit inside each two-port network will be described in the following sections. In this way, the operation of HVDC transmission is associated with the different connections of the two-port network. The two-port equivalent schematic is shown in FIGURE 2.

Based on the principle of HVDC transmission, the AC system is connected to the end of the port. With the two-port equivalent circuit, the node admittance matrix $Y$ of the circuit can be obtained. The Norton equivalent principle is used to
equate the harmonic voltage source to the harmonic injection current $I$; the solution of $YU = I$ can provide the voltage of the corresponding node so that the corresponding current value can be obtained.

III. COMPONENT EQUIVALENT MODEL
To obtain the equivalent circuit of the HVDC transmission, it is necessary to model each part to obtain its corresponding two-port network.

A. SWITCHING FUNCTION MODEL
Modulation theory is applied to study the converters of an HVDC transmission system because of its clear physical concept, simple calculation process, and high precision. The basic principle is to regard the converter as a modulation switch circuit for connecting the DC and AC systems [2], [30]. The voltage and current in the AC/DC system can be converted accordingly by the modulated switching function.

According to the working principle of the switching function, the relationship between the DC voltage and current and the AC voltage and current in different states of the converter can be described. For the 6-pulse converter shown in FIGURE 3, the expression is given by equations (1) and (2):

$$U_d = u_a S_{ua} + u_b S_{ub} + u_c S_{uc}$$

where $U_d$ and $I_d$ are the DC-side voltage and current, respectively; $u_a$, $u_b$, $u_c$ and $i_a$, $i_b$, $i_c$ represent the three-phase voltage and current on the AC side, respectively; $S_{ua}$, $S_{ub}$, $S_{uc}$ represent the ABC three-phase voltage switch functions; and $S_{ia}$, $S_{ib}$, $S_{ic}$ represent the ABC three-phase current switch functions.

In general, trigger pulses of HVDC systems are generated at equal intervals. In harmonic analysis, the switching function of the voltage and current modulation of the converter can be decomposed into a Fourier series based on theory. For a 6-pulse converter, the switching function can be expressed by equation (3):

$$
\begin{align*}
S_a &= \sum_{n=1}^{\infty} A_n \cos n \omega t \\
S_b &= \sum_{n=1}^{\infty} A_n \cos\left(n \omega t - \frac{2\pi n}{3}\right) \\
S_c &= \sum_{n=1}^{\infty} A_n \cos\left(n \omega t + \frac{2\pi n}{3}\right)
\end{align*}
$$

In the above expressions, $A_n$ is a coefficient and $\omega$ is the angular frequency.

The phase voltage switching function and current switching function of the 6-pulse converter are shown in FIGURE 4. $S_{ua}$ is the switching function without considering the commutation process, $S_{ia}$ is the voltage switching function considering the commutation process, and $S_{ia}$ is the current switching function considering the commutation process [15], [31].

FIGURE 2. The two-port equivalent schematic diagram.

FIGURE 3. A schematic diagram of the 6-pulse converter.

FIGURE 4. A diagram of the switching function.

When the commutation angle $\mu$ is neglected, the expression of the coefficient is:

$$
A_n = \frac{2}{\pi} \int_{0}^{\pi} f(x) \cos nx dx \\
= \frac{2}{\pi} \left( \int_{0}^{\pi/3} \cos nx dx - \int_{\pi/3}^{\pi} \cos nx dx \right) \\
= \frac{4}{n\pi} \sin \frac{n\pi}{2} \cos \frac{n\pi}{6}
$$

When the commutation angle $\mu$ is considered, the expression of the coefficient is:

$$
A_{nu} = \frac{2}{\pi} \left( \int_{0}^{\pi/3-\mu/2} \cos nx dx + \int_{\pi/3+\mu/2}^{\pi} \cos nx dx \right. \\
- \left. \int_{2\pi/3-\mu/2}^{\pi} \cos nx dx - \int_{2\pi/3+\mu/2}^{\pi} \cos nx dx \right) \\
= A_n \cos \frac{\mu}{2}
$$
On the basis of the switching function model of the converter, the harmonics caused by the harmonic voltage from the AC side to the DC side will be analyzed. Take the 6-pulse AC converter as an example. Consider the general case where the AC bus of the converter has a voltage with a frequency of $f_1$; its expression is:

$$\begin{align*}
A_{ni} &= 2 \left\{ \int_0^{\pi/3 - \mu/2} \cos nx \, dx - \int_{\pi/3 + \mu/2}^{\pi} \cos nx \, dx \right\} \\
&= A_n \sin \frac{n \mu}{2} \left( \frac{n \mu}{2} \right)
\end{align*}$$

where $A_n$ is the amplitude of the AC-side voltage and $\theta_f$ is the corresponding initial phase angle.

As explained in [32] and [33], research on the harmonics of the DC system shows that the non-characteristic harmonic voltage of $n > 1$ is an order of magnitude lower than the non-characteristic harmonic voltage of $n = 1$ for 12-pulse converters. Therefore, considering only the lower order non-characteristic harmonic voltages would be adequate for industry practice.

By substituting equation (7) into equation (1), a dominant harmonic voltage with a frequency of $(f_1 - f_1)$ is produced on the DC side after the converter modulation:

$$E_{dc(f-f_1)}^+ = \frac{3}{2} A_{ni} U_f \cos \left[ (\omega_f - \omega_1)t + \theta_f \right]$$

$$= \frac{3\sqrt{3}}{\pi} \cos \frac{\mu}{2} \times U_f \cos \left[ (\omega_f - \omega_1)t + \theta_f \right]$$

The analysis of equation (8) indicates that if there is a positive second-order harmonic voltage disturbance in the AC system, the number of harmonic components generated on the DC side will be reduced by 1 after the corresponding modulation effect of the converter. That is, a 50Hz voltage component with an amplitude of $\frac{3\sqrt{3}}{\pi} \times U_f$ can be obtained.

Similarly, if the DC current of the converter is:

$$I_{dch} = I_{dc} \cos(\omega t + \phi)$$

where $I_{dc}$ is the amplitude of the small signal on the DC-side and $\phi$ is the corresponding initial phase angle, then the AC-side three-phase current expression can be obtained by applying equation (2) for the corresponding calculation.

It can be seen from equation (10) that the DC-side small signal $I_{dch}$ will generate two sets of dominant uncharacteristic harmonics on the AC side, one of which has a positive sequence and an angular frequency of $\omega + \omega_1$. $\varphi_a, \varphi_b, \varphi_c$ are the angles of other components.

$$\begin{align*}
I_a &= I_{dch} S_{ia} = \frac{1}{2} A_{ni} I_{dc} \left\{ \cos \left[ (\omega + \omega_1)t + \varphi \right] + \cos \varphi \right\} \\
I_b &= I_{dch} S_{ib} = \frac{1}{2} A_{ni} I_{dc} \times \left\{ \cos \left[ (\omega + \omega_1)t + \varphi - \frac{2\pi}{3} \right] + \cos \varphi \right\} \\
I_c &= I_{dch} S_{ic} = \frac{1}{2} A_{ni} I_{dc} \times \left\{ \cos \left[ (\omega + \omega_1)t + \varphi + \frac{2\pi}{3} \right] + \cos \varphi \right\}
\end{align*}$$

Therefore, if there is a current of 50Hz on the DC side of the converter, and a 100Hz current with an amplitude of $\frac{2\sqrt{3}}{\pi \mu} \times I_{dc}$ will be generated in the AC system.

### B. DC LINE MODEL

An actual HVDC transmission line generally consists of multiple parallel wires. The frequency-dependent model [34], [35] is used in this paper. There is electromagnetic coupling between the wires, and the electromagnetic process is complex. Therefore, the wires should be decoupled.

In the frequency-dependent model, modal decoupling is usually carried out by means of Clarke’s matrix or a modal representation using a propagation matrix. However, the question of how to deal with the coupling relationship between two lines after those lines are connected to external components has not been studied. To adapt to the different operating modes of the HVDC transmission system, a unified model of the DC lines should be established. The key point is how to solve for the coupling between the lines. Therefore, a method based on the elimination of mutual inductance is used in this paper to construct a unified two-port matrix of the HVDC transmission system lines in different operating modes. Two mutual inductance branches are taken as examples to illustrate the principle of removing the mutual inductance between the phases, as shown in FIGURE 5.

The branch voltage equation of the two mutual inductance branches shown in FIGURE 5 (a) can be expressed by a matrix as follows:

$$\begin{align*}
\begin{bmatrix}
\dot{U}_p - \dot{U}_q \\
\dot{U}_r - \dot{U}_s
\end{bmatrix} &= \begin{bmatrix}
z_{pq} & z_{pm} & z_{pr} \\
z_{mr} & z_{ms} & z_{rs}
\end{bmatrix} \begin{bmatrix}
I_{pq} \\
I_{mr}
\end{bmatrix}
\end{align*}$$

(11)
By a transformation, the following can be obtained:

\[
\begin{bmatrix}
I_{pq} \\
I_{rs} \\
I_{qp} \\
I_{sr}
\end{bmatrix} = \begin{bmatrix}
y'_{pq} & y'_m & -y'_{pq} & -y'_m \\
y'_m & y'_{rs} & -y'_m & -y'_{rs} \\
-y'_{pq} & -y'_m & y'_{pq} & y'_m \\
-y'_m & -y'_{rs} & y'_m & y'_{rs}
\end{bmatrix} \begin{bmatrix}
\dot{U}_p - \dot{U}_q \\
\dot{U}_r - \dot{U}_s
\end{bmatrix}
\]

(12)

where

\[
\begin{align*}
y'_{pq} &= \frac{z_{rs}}{z_{rs} z_{pq} - z_{m}^2} \\
y'_m &= \frac{z_{pq}}{z_{rs} z_{pq} - z_{m}^2} \\
y'_m &= \frac{z_{m}}{z_{rs} z_{pq} - z_{m}^2} \end{align*}
\]

According to the above equation, a nonmutual inductance equivalent circuit with the mutual inductance eliminated can be made, as shown in FIGURE 5(b). The equivalent circuit is a noninductive circuit consisting of four nodes and six branches. The admittance values of each branch are \(y'_{pq}, y'_m\), \(y'_m\), \(y'_{pq}\), \(-y'_m\), and \(-y'_m\).

Through the equivalent circuit without mutual inductance, it is convenient to use the node voltage method to obtain the corresponding node voltage equations as:

\[
\begin{bmatrix}
I_{pq} \\
I_{rs} \\
I_{qp} \\
I_{sr}
\end{bmatrix} = \begin{bmatrix}
y'_{pq} & y'_m & -y'_{pq} & -y'_m \\
y'_m & y'_{rs} & -y'_m & -y'_{rs} \\
-y'_{pq} & -y'_m & y'_{pq} & y'_m \\
-y'_m & -y'_{rs} & y'_m & y'_{rs}
\end{bmatrix} \begin{bmatrix}
\dot{U}_p \\
\dot{U}_r \\
\dot{U}_q \\
\dot{U}_s
\end{bmatrix}
\]

(13)

### C. HVDC CONTROL SYSTEM

The ability to achieve multiple fast modes of regulation is one of the main advantages of HVDC systems. The operating point of the HVDC system can be changed by adjusting the firing angle of the converter valve or the ratio of the converter transformer according to the operating conditions of the AC/DC system.

Normally, the HVDC rectifier side achieves constant current control by reducing the firing angle. Different from the rectifier side, the inverter side usually adopts both constant extinction angle control and constant current control [36]-[38]. The overall control scheme is shown in FIGURE 6. In the figure, \(I_{d,rec}\) and \(I_{d,inv}\) represent the measured value and reference value, respectively, of the rectifier side current; \(U_{d,inv}\) and \(U_{d,inv}\) are the measured values of the voltage and current, respectively, on the inverter side; \(\gamma_s\) and \(\gamma_d\) are the measured values of the arc extinction angles of the Y-bridge and D-bridge, respectively, on the inverter side; \(\gamma_{ord}\) is the inverter side arc extinction angle reference value; and \(\alpha_{rec}\) and \(\alpha_{inv}\) are the rectifier side and inverter side firing angles, respectively, of the control output.

The control system shown in FIGURE 6 is used in the CIGRE HVDC benchmark system [36], [37] and is also used in the case studies in this paper.

Modern HVDC systems generally adopt the equal-interval triggering mode, which is followed by the phase locked loop (PLL) to generate the synchronous sawtooth signal. According to the PLL following phase, six equally spaced pulse signals will be generated when the sawtooth signal is reset and compared with the trigger angle pulse signal of the pole control layer to realize the trigger command to the valve [19], [39].

The control block diagram of the PLL is shown in FIGURE 7. In the figure, \(u_a\), \(u_b\), and \(u_c\) are the input AC voltages; the output is the following phase \(\theta\). \(K_p\) and \(K_i\) are the parameters of the control part [19]. The control system parameters are shown in APPENDIX A.

### IV. TWO-PORT CONNECTION

To obtain the overall equivalent circuit, it is necessary to determine the connection relationship of each two-port network.

#### A. EQUIVALENT TWO-PORT NETWORK FOR THE TRANSFORMER AND CONVERTER UNION

Taking Y-Y as an example, if the converter transformer is equivalent to a two-port network, then the equivalent diagram is shown in FIGURE 8. In the figure, \(U_A\) and \(I_A\) are the primary-side voltage and current, respectively; \(U_a\) and \(I_a\) are the secondary-side voltage and current, respectively; \(n\) is the transformer ratio; and \(X_T\) is the leakage inductance of the transformer [40]. When other connections are adopted for the transformer, the derivation is similar.

According to the corresponding relationship of the transformer ratio, the T-parameter of the equivalent port of the
transformer can be obtained.

\[
\begin{bmatrix}
\dot{U}_A \\
\dot{I}_A
\end{bmatrix} =
\begin{bmatrix}
T_{T11} & T_{T12} \\
T_{T21} & T_{T22}
\end{bmatrix}
\begin{bmatrix}
\dot{U}_a \\
\dot{I}_a
\end{bmatrix}
\]

(14)

In the above conversion,

\[
\begin{cases}
T_{T11} = \frac{1}{n} \\
T_{T12} = -n \ast Z_T \\
T_{T21} = 0 \\
T_{T22} = -n
\end{cases}
\]

Here, \(Z_T\) is the transformer leakage inductance at the corresponding frequency.

Similarly, the 6-pulse converter is treated as a port, as shown in FIGURE 9. In the figure, \(U_{acp}\) and \(I_{acp}\) are the AC-side voltage and current, and \(U_{dch}\) and \(I_{dch}\) are the DC-side voltage and current, respectively.

\[\begin{bmatrix}
\dot{U}_{acp} \\
\dot{I}_{acp}
\end{bmatrix} =
\begin{bmatrix}
T_{C11} & T_{C12} \\
T_{C21} & T_{C22}
\end{bmatrix}
\begin{bmatrix}
\dot{U}_{ach} \\
\dot{I}_{ach}
\end{bmatrix}
\]

(15)

In the above formula,

\[
\begin{cases}
T_{C11} = 1/(\frac{3\sqrt{3}}{\pi} \cos \frac{\mu}{2}) \\
T_{C12} = 0 \\
T_{C21} = 0 \\
T_{C22} = -\frac{2\sqrt{3}}{\pi \mu} \sin \frac{\mu}{2}
\end{cases}
\]

The two two-port cascades of the transformer and inverter are shown in FIGURE 10.

\[T_1\] is the T-parameter after the two-port networks are cascaded. In this way, for 6-pulse HVDC transmission, the equivalent two-port parameters of the transformer and the converter can be obtained.

In addition, 12-pulse HVDC transmission is equivalent to a combination of two 6-pulse systems, as shown in FIGURE 11.

\[G_0 = \frac{1}{T_A} \begin{bmatrix}
T_C & -\Delta T \\
1 & T_B
\end{bmatrix}
\]

(18)

In the equation, \(\Delta T = T_A T_D - T_B T_C\).

Using the change of formulas (17) and (18), the \(G\)-parameters of the two two-port networks corresponding to the 12-pulse system can be obtained.

Two two-port networks are connected in parallel on the left side and in series on the right side. The connection parameters can be obtained using equation (19).

\[G = G_1 + G_2\]

(19)

The inverter side adopts a similar method; the only difference is that the port connection is different.

**B. DIFFERENT OPERATION MODES**

There are several typical modes of operation for HVDC transmission [28]. By changing the different combinations in Part 2 of FIGURE 2, it is possible to construct different operation modes.

The wiring diagram of Part 2 for bipolar ground operation mode is shown in FIGURE 12(a). It can be seen from the figure that the two poles are symmetrical when operating in bipolar ground mode, and the voltage on the ground electrode is zero. The monopolar ground operation mode requires a modification of the partial wiring, as shown in FIGURE 12(b). In the monopolar ground operation mode, the pole 2 DC line is disconnected from the connected smoothing reactor. Although only one DC line is in operation, the circuit structure of the rectifier side and the inverter side are still symmetrical. When the HVDC system is in the monopolar metallic operation mode, the wiring of Part 2 is shown in FIGURE 12(c). It can be seen from the figure that the pole 2 DC line is directly connected to the smoothing reactor of pole 1. In addition, the rectifier side and the inverter side are still symmetrical.
side are asymmetrical at this time, mainly because there is a grounding point on the inverter side.

**C. PORT CONSOLIDATION AND PORT GROUNDING PROCESSING IN DIFFERENT OPERATION MODES**

Although HVDC transmission has different modes of operation, when forming the equivalent matrix, it is only necessary to modify the corresponding part of the matrix according to different port connections.

1) **PORT CONSOLIDATION**

The two ports of the transformer and converter are described in the previous sections, and the port of the DC line is also explained in detail. The admittance matrix is used when the port is connected by a node [42], [43]. The node combination is shown in FIGURE 13.

When two nodes are combined into one node, the injection current of the new node is equal to the sum of the injection currents of the original two nodes. For example, for the node $a$, $b$ combination, the merged node becomes $a$, and so there is $\dot{U}_a + \dot{U}_b = \dot{U}'_a$ and $\dot{I}_a + \dot{I}_b = \dot{I}'_a$; the superscript ‘‘ indicates a new value. At this time, the network equation will be reduced by one order. If $\dot{U}_a$ is replaced by $\dot{U}'_a$, then $\dot{U}_a$ is eliminated; $\dot{I}_a$ is replaced by $\dot{I}'_a$, and $\dot{I}_a$ is eliminated. Correspondingly, the $b$th row of the admittance matrix is added to the $a$ row, and the $b$th column is added to the $a$ column. In this way, the connected port admittance matrix can be obtained.

2) **PORT GROUNDING**

The port needs to eliminate these nodes when the grounding point occurs [43]. Assuming that node $p$ is the node to be erased and node $p$ is given later, the network equation represented by the admittance matrix is used. The expression after matrix partitioning is given by equation (20).

\[
\begin{bmatrix}
Y_I & Y_{II} & Y_{III} & Y_{IV} \\
\dot{U}_n & \dot{U}_p & \end{bmatrix}
\begin{bmatrix}
\dot{I}_n \\
\dot{I}_p 
\end{bmatrix}
= \begin{bmatrix}
\dot{I}_n \\
\dot{I}_p 
\end{bmatrix}
\]

After eliminating node $p$, the following formula can be obtained:

\[
(Y_I - Y_{II}Y_{IV}^{-1}Y_{III})\dot{U}_n = \dot{I}_n - Y_{II}Y_{IV}^{-1}\dot{I}_p 
\]

Equation (21) is the node admittance matrix after node $p$ is eliminated, and its corresponding diagram is shown in FIGURE 14.

**V. CASE STUDY AND DISCUSSION**

In this section, two case studies with an analysis and a discussion are presented. Section A is mainly used to prove the accuracy of the model. Section B uses the verified model to analyze harmonic transfer and amplification, and this part focuses on the analysis of the HVDC transmission power and the effect of the DC reactor on harmonic transmission and amplification.

A. **HARMONIC QUANTITATIVE CALCULATION ANALYSIS**

This section is divided into two parts to verify the accuracy and validity of the two-port model. The first part uses the CIGRE HVDC benchmark system, and the second part uses the actual engineering model.

1) **THE CIGRE HVDC BENCHMARK SYSTEM MODEL**

To verify the effectiveness of the proposed method, the line of the CIGRE HVDC benchmark system was changed to the double coupled line, and the line length was 1254 km. The tower structure of DC line is shown in APPENDIX B. The model of the modified CIGRE HVDC benchmark system based on the two-port model was developed in MATLAB.

Taking the positive sequence 2nd harmonic of the rectifier-side AC system as an example, an 8% (24.24 kV) harmonic
source was added to the AC system. The simulation and calculation were performed on a computer with a 3.40 GHz i7-6700 processor, 8 GB of RAM memory, and the 64-bit version of Windows 10. The computation (CPU) time required by the methods was recorded. The calculated value of the equivalent circuit was compared with the simulated value of PSCAD/EMTDC, as shown in TABLE 1.

| Method | Voltage, kV | Current, A | Voltage, kV | Current, A | Total time/s |
|--------|-------------|------------|-------------|------------|--------------|
| Simulated value | 41.5361 | 41.8945 | 0.8556 |
| Calculated value | 18.2046 | 19.8143 | 8.1224 |
| Operation mode | 19.7580 | 21.0356 | 6.0732 |
| Bipolar | 144.0716 | 146.5659 | 1.7018 |
| Monopolar | 42.0106 | 40.3642 | 4.0789 |
| Ground | 44.6025 | 42.6359 | 4.6173 |
| Operation mode | 18.2185 | 19.1937 | 5.0820 |
| Monopolar | 111.0193 | 108.3051 | 2.5061 |
| Metallic | 30.3392 | 31.1147 | 2.4922 |
| Operation mode | 14.1777 | 14.8119 | 4.2815 |
| Bipolar | 12.3170 | 13.3455 | 7.7061 |
| Monopolar | 95.6147 | 99.1634 | 3.5786 |

TABLE 1 shows that the CPU time required by the proposed method is shorter than that required by the simulation method, and the proposed method can offer simulation-level accuracy with a marked reduction in computing burden. However, the CIGRE HVDC benchmark system has certain limitations in terms of the AC filters. Therefore, the following simulation in this manuscript uses the actual engineering model.

2) THE ACTUAL ENGINEERING MODEL

Based on the PSCAD/EMTDC platform, combined with a ±500kV HVDC transmission engineering model, a simulation is carried out. The specific parameters of the model are shown in APPENDIX C.

Taking the positive sequence 2nd harmonic of the rectifier-side AC system as an example, a 9% (27.27 kV) harmonic source is added to the AC system. When the HVDC system operates in bipolar ground mode, the simulation value of the 50Hz harmonic voltage and current can be obtained by measuring the corresponding node. Similarly, the same harmonic is added to the rectifier-side AC system, and the calculated value of the 50Hz harmonic can be obtained by the corresponding calculation. The calculated value of the equivalent circuit is compared with the simulated value of PSCAD/EMTDC in TABLE 2.

Comparing the data, it can be found that the calculated values of the voltage and current of the DC line are close to the simulated values when the HVDC system is in three different operation modes; the relative error is small, which indicates the harmonic calculation method is accurate.

To verify the applicability of the model, the accuracy of the model for different degrees of harmonics is considered below. After the model is stably operated, 1%–19% of the phase voltage is superimposed on the rectifier side, and the interval is 2%. The voltage and current on both sides are recorded. The rectifier-side calculated value (RCV), the rectifier-side simulated value (RSV), the inverter-side calculated value (ICV) and the inverter-side simulated value (ISV) are compared.

The voltage and current of the RCV, RSV, ICV, and ISV of the bipolar ground operation mode are shown in FIGURE 15. The details of the other two operating modes are shown in APPENDIX D.

It can be seen from the comparison in the figure that the calculated value is very similar to the simulated value. Thus, the model still has high accuracy for different degrees of harmonics.

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TABLE 3. Current transfer for different transmission powers.

| Power/ p.u. | CV/SV | Rectifier-side I/A | Inverter-side I/A | Amplification factor |
|-------------|-------|---------------------|-------------------|----------------------|
| 0.1         | CV    | 41.0698             | 102.9082          | 2.5056               |
|             | SV    | 40.1871             | 102.8329          | 2.5588               |
| 0.3         | CV    | 42.3111             | 105.1921          | 2.4862               |
|             | SV    | 40.3107             | 105.5928          | 2.6194               |
| 0.5         | CV    | 43.1159             | 107.3360          | 2.4895               |
|             | SV    | 40.0962             | 106.5814          | 2.6581               |
| 0.7         | CV    | 44.1337             | 109.6798          | 2.4851               |
|             | SV    | 41.0962             | 106.6094          | 2.5941               |
| 0.9         | CV    | 44.0382             | 109.6264          | 2.4893               |
|             | SV    | 41.3988             | 107.7290          | 2.6022               |

TABLE 4. Current transfer for different smoothing reactors.

| L/H | CV/SV | Rectifier-side I/A | Inverter-side I/A | Amplification factor |
|-----|-------|---------------------|-------------------|----------------------|
| 0.25| CV    | 10.3706             | 95.6181           | 9.2201               |
|     | SV    | 11.0119             | 99.3335           | 8.9935               |
| 0.30| CV    | 14.1777             | 95.6147           | 6.7440               |
|     | SV    | 14.8119             | 99.1634           | 6.6948               |
| 0.35| CV    | 18.3562             | 96.2338           | 5.2426               |
|     | SV    | 19.8986             | 101.6241          | 5.1070               |
| 0.40| CV    | 22.785              | 97.1247           | 4.2629               |
|     | SV    | 23.3067             | 103.1825          | 4.4271               |

In addition, it can be seen intuitively from the figure that, as the harmonic source increases, both the voltage component and the current component increase in the rectifier side or the inverter side. The current component is amplified by the transmission of the line; the voltage component is reduced by the line transmission. Therefore, it is necessary to pay special attention to the amplified component of the DC current transmitted to the opposite side. The size of this current component largely determines whether the protection will malfunction.

B. HARMONIC TRANSFER AND AMPLIFICATION

In this section, the above-described verified model is used to analyze the effects of different quantities on harmonic transmission, with a focus on the amplification of the current transfer to the opposite side. The amplification factor is defined as the ratio of the terminal harmonic amplitude to the terminal harmonic amplitude.

The proposed model can be used to analyze the relationship between the DC current amplification factor and the operating and system parameters. The HVDC transmission power and the DC smoothing reactor will be analyzed as an example.

TABLE 5. Control system parameters.

| Controller | Parameter | Rectifier | Inverter |
|------------|-----------|-----------|----------|
| Current controller | Proportional gain | 1.0989 | 0.63 |
| Arc extinction angle control | Proportional gain | / | 0.7506 |
| Current measurement | Integral gain | / | 0.0544 |
| Voltage measurement | Integral gain | 0.0012 | 0.0012 |
| Phase locked loop | Proportional gain | 10 | 10 |
| | Integral gain | 50 | 50 |

The results will not only show the characteristics of the harmonic transfer and amplification but also illustrate the accuracy of the two-port model.

1) HVDC TRANSMISSION POWER

In actual operation, HVDC engineering often does not operate at the rated power due to seasonal problems with loads and water sources. Therefore, by changing the power of HVDC
transmission, it is important to investigate whether the HVDC transmission power has an influence on the transmission of harmonics in the DC line.

The harmonic amplitude and position are the same as in the previous section. The voltage and current components on both sides are recorded when the HVDC power is between 0.1 p.u. and 0.9 p.u. By comparing the calculated value (CV) with the simulated value (SV), the effects of the HVDC transmission power on the harmonic transfer amplification are analyzed while also further verifying the accuracy of the model.

The current CV and SV of the monopolar ground operation mode are shown in TABLE 3. The details of the other two operating modes are shown in APPENDIX E.

As the power changes, the current component changes, but the change is not large. The current amplification factor also changes little, and the value fluctuates around a fixed value. Thus, the magnitude of the current component is basically independent of the HVDC transmission power.

### TABLE 7. Current transfer for different transmission powers.

| Power/ p.u. | CV/SV | Rectifier-side I/A | Inverter-side I/A | Amplification factor |
|-------------|-------|---------------------|-------------------|---------------------|
| 0.1         | CV    | 14.6828             | 123.5156          | 8.4122              |
|             | SV    | 16.2229             | 128.0629          | 7.8939              |
| 0.3         | CV    | 15.1556             | 126.1072          | 8.3208              |
|             | SV    | 16.2475             | 131.0056          | 8.0630              |
| 0.5         | CV    | 15.9798             | 131.9749          | 8.2588              |
|             | SV    | 18.6376             | 132.6986          | 7.1199              |
| 0.7         | CV    | 16.6396             | 136.2520          | 8.1884              |
|             | SV    | 15.8586             | 139.7589          | 8.8127              |
| 0.9         | CV    | 17.2450             | 139.4589          | 8.0869              |
|             | SV    | 19.4852             | 139.9095          | 7.1802              |

The smoothing reactor is used to suppress the ripple in the DC voltage after rectification and is one of the important pieces of equipment of the converter station. Therefore, it is especially

2) DC SMOOTHING REACTOR

The smoothing reactor is used to suppress the ripple in the DC voltage after rectification and is one of the important pieces of equipment of the converter station. Therefore, it is especially
TABLE 8. Current transfer for different smoothing reactors.

(a)(Bipolar operation mode)

| L/H  | CV/SV | Rectifier-side I/A | Inverter-side I/A | Amplification factor |
|------|-------|---------------------|-------------------|----------------------|
| 0.25 | CV    | 9.9410              | 143.9091          | 14.4776              |
|      | SV    | 12.2654             | 144.3162          | 11.7660              |
| 0.30 | CV    | 18.2046             | 144.0716          | 7.9140               |
|      | SV    | 19.8142             | 146.5658          | 7.3969               |
| 0.35 | CV    | 27.7143             | 149.3925          | 5.3904               |
|      | SV    | 30.0384             | 148.9573          | 4.9588               |
| 0.40 | CV    | 37.7900             | 153.8053          | 4.0700               |
|      | SV    | 9.9389              | 143.9107          | 14.4794              |

(b)(Monopolar operation mode)

| L/H  | CV/SV | Rectifier-side I/A | Inverter-side I/A | Amplification factor |
|------|-------|---------------------|-------------------|----------------------|
| 0.25 | CV    | 38.6487             | 106.4351          | 2.7539               |
|      | SV    | 36.4756             | 104.8240          | 2.8738               |
| 0.30 | CV    | 44.6024             | 111.0193          | 2.4891               |
|      | SV    | 42.6339             | 108.3050          | 2.5403               |
| 0.35 | CV    | 54.8730             | 124.0967          | 2.2615               |
|      | SV    | 48.7842             | 111.9172          | 2.2941               |
| 0.40 | CV    | 58.1083             | 120.0628          | 2.0662               |
|      | SV    | 56.9377             | 117.2353          | 2.0590               |

important to analyze the influence of the smoothing reactor on the harmonic transfer amplification.

The harmonic conditions are the same as before. The voltage and current components on both sides are recorded by modifying the value of the smoothing reactor from 0.25 H to 0.4 H. The current CV and SV of the monopolar metallic operation mode are shown in TABLE 4. The details of the other two operating modes are shown in APPENDIX F.

From the data in the table, it can be found that, as the DC smoothing reactor increases, the current component also increases. However, the amplification of the current decreases as the smoothing reactor increases. Thus, the smoothing reactor has an inhibitory effect on the amplification of the harmonics.

VI. CONCLUSION

In this paper, a two-port unified model is proposed. The transformer, converter, smoothing reactor and filter, and DC line in the HVDC transmission system are equivalent to one two-port network. In different operating modes of HVDC transmission, only the connection mode of the two ports needs to be modified. By a comparison with detailed HVDC model simulation results of PSCAD/EMTDC, the conclusions can be drawn as follows:

1) The two-port unified model is a computationally inexpensive option for the analysis of the harmonics of an LCC-HVDC system, due to the small computational burden and improved efficiency. By comparing the results with a simulation, it is found that the CV based on the proposed model are basically consistent with the simulation results for different operation modes, and the accuracy and practicability of the method are illustrated.

2) An analysis of harmonic transfer and amplification are presented by the established two-port unified model. The results show that harmonic transmission and amplification have little to do with the HVDC operating parameters (transmission power) but are closely related to the HVDC system parameters (DC smoothing reactor).

3) All the results show that the two-port unified model can be applied to the assessment of the harmonic current in the initial stage of HVDC project planning. The assessment parameter has a certain reference value for an actual HVDC transmission system and can provide a reference basis for the formulation of a harmonic protection strategy.

However, due to the time-varying nature of the converter, there are certain errors in the quantitative calculation. Therefore, accurately constructing the equivalent circuit of the converter in a quantitative calculation will be the focus of further research and very significant work.

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