A 2 DOF vibration harvester for broadband and multi-frequency harvesting using a single electro-magnetic transducer

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Abstract. The narrow bandwidth of resonant vibration energy harvesters has long been seen as a drawback to exploitation. The narrow bandwidth is necessitated by the requirement to sufficiently amplify small source vibrations, but results in devices vulnerable to changes in excitation frequency, de-tuning due to ageing of components, and also makes efficient harvesting from sources with multiple frequency components difficult. In this paper a harvester based on a 2 degree of freedom oscillator is presented that not only enjoys the wider bandwidth of the higher order system, but configures the electromagnetic transducer in such a way that it requires no more components than the transducer of a typical single degree of freedom harvester. Theoretical models of the harvester system predict a range of possible frequency response functions dependent on easily-adjusted electrical parameters. These predictions are validated with experimental results.

1. Introduction

In attempts to broaden the frequency response of vibration energy harvesters, authors have proposed various ingenious modifications to the standard mass-spring-damper resonant mechanical amplifier, including devices with non-linear characteristics, electrical tuning and devices featuring multiple degrees of freedom (DOF). Non-linear devices have been well documented in the literature (e.g. [1]) over recent years and undoubtedly have some positive characteristics; however, since fundamentally superposition does not apply they are unable to display broad-band response in respect to simultaneous excitation at multiple frequencies. Electrically tuned harvesters can in theory replicate any frequency response, but in practice have limits imposed by non-ideal electrical transduction [2].

Multiple DOF (linear) devices do not have the limitations of single frequency excitation, nor the complexity of electrically tuned systems. A growing number have been reported in the literature, from the simple combinations of many 1 DOF systems combined in one package, e.g. the multiple beams described by Ferrari et al. [3], to coupled devices with higher-order behavior, such as the device described by Tang and Zuo [4], or the 3 DOF device of Wong et al. [5]. Many of the devices based on higher order responses can be derived from the classic vibration absorber described by Den Hartog [5].

In this paper a novel 2 DOF vibration harvester is described which configures a typical voice coil transducer in a non-typical way such that both coil and magnet assembly are individually sprung with
respect to the base and energy is extracted by damping their relative motion. Arranging the voice coil in this manner results in the ability to harvest power over an extended frequency range, typical of higher order systems, but without the penalty of additional electromechanical transducers.

2. Theoretical 2 DOF harvester

The schematic for the 2 DOF harvester can be seen in figure 1.

Figure 1. Schematic of the 2 DOF system, where \( m_1 \) is the coil mass with \( k_1 \) and \( C_{m1} \) as the associated compliance and parasitic loss, and where \( m_2 \) is the magnet assembly with \( k_2 \) and \( C_{m2} \) as its associated compliance and parasitic loss. \( C_e \) is the electrical damping.

The motion of the 2 DOF harvester depicted in figure 1 is described by equations 1 and 2.

\[
\begin{align*}
\ddot{x}_1 &= \frac{k_1}{m_1} (y - x_1) - \frac{C_{m1}}{m_1} (\ddot{x}_2 - \dot{y}) - \frac{C_e}{m_1} (\dot{x}_1 - \dot{x}_2) + \ddot{y} \\
\ddot{x}_2 &= \frac{k_2}{m_2} (y - x_2) - \frac{C_{m2}}{m_2} (\ddot{x}_2 - \dot{y}) - \frac{C_e}{m_2} (\dot{x}_2 - \dot{x}_1) + \ddot{y}
\end{align*}
\]

Figure 2 shows the power frequency response of the device, from numerical simulation of equations 1 and 2, when the harvester is subject to excitation with constant displacement and output power is that developed in the electrical damping \( C_e \). Three differing values of \( C_e \) are simulated and are classified as lightly, moderately and heavily damped conditions due to their effect on the frequency response of the system.

The device has the characteristic ‘dual peak’ frequency response of a third order system when the relative damping applied by the transducer is light, however when the damping is greater, the system behavior tends to that of a second order single DOF system, illustrated in figure 2. Because the system is linear, it is capable of simultaneously responding usefully to excitation from multiple frequencies (unlike non-linear systems).

For comparison, the combined maximum power frequency response of two independent 1 DOF harvesters with the same input conditions and a combined proof mass the same as the total mass of the 2 DOF system is shown in figure 3. The ‘maximum power’ frequency response refers to the fact that the resistive load at each particular frequency is the optimum for power generation. The wider power bandwidth of the 2 DOF system can be seen, although it will be noted that the improved response for frequencies between the two outer peaks is at the expense of frequencies outside of this range, in comparison with the dual 1 DOF devices.

The values used in these simulations were taken from the experimental device described in the next section of this paper.
3. Prototype device

The device has been prototyped at a meso scale and is shown in figure 4. The assembled device is shown in figure 4a, and partially dismantled to reveal the separate springing of the magnet and coil in figure 4b. All parts were custom manufactured for the prototype including planar springs water-jet cut from beryllium copper. The voice coil and magnet assembly were numerically optimized for maximum harvested power for a fixed volume.

![a) Device assembled][1] ![b) device partially dismantled][2]  
**Figure 4. Prototype 2 DOF harvesting device**

The measured parameters of the device are given in table 1. The mass of each of the seismic elements was altered during testing to produce the desired resonant frequency for each individual mass/spring pair. Two conditions were tested: one with resonances set at 24.7 / 27.5 Hz to reproduce a closely spaced resonant pair, and 24.7 / 40.8 Hz producing two distinct resonances.
Table 1. Properties of prototype device

| Property                        | Value  |
|---------------------------------|--------|
| Mechanical Damping Coefficient $C_{m1}C_{m2}$ (Ns/m) | 0.03   |
| Length Coil (m)                 | 14.8   |
| Internal Resistance (Ω)         | 14.3   |
| BL (Tm)                         | 6.70   |
| $(BL)^2/R_i$                    | 3.14   |
| Spring Stiffness $K_1=K_2$ (N/m) | 1715   |

4. Experimental Results

The experimental harvester was tested on a rig consisting of a feedback controlled shaker driven by a Data Physics Quattro dynamic signal processor which also recorded the voltage output across the load resistance. Figure 5 gives experimental frequency response derived from swept-sine measurements. The two separate graphs correspond to the case of light and high damping which produce the differing responses in the 2 DOF system. Each graph contains 2 traces which show the frequency response of the 2 DOF system with closely spaced resonances (green, 24.7/27.5 Hz) and wide spaced resonances (red, 24.7/40.8 Hz). A third trace show the response of the system with the voice coil clamped such that the harvester behaves as a 1 DOF system with a single resonance at the lower frequency (black 24.7 Hz). An interesting phenomenon can be seen in the heavily damped case where the low value of load resistance tends to force the oscillators to behave as one. In this situation the overall damping of the system is low and the response of the coupled oscillators can be very large.

![Experimental frequency response curves for a 1 DOF device (tuned to 24.7Hz, black) and two configurations of the 2 DOF device (tuned to 24.7 & 27.5, green, and tuned to 24.7Hz & 40.8 Hz, red). Three different electrical loads are applied corresponding to damping which illustrates the three principle behaviours of the 2 DOF system.](image-url)
Figure 6 reveals the locus of the power maxima for the 24.7 / 27.5 Hz tuned condition as electrical damping is changed. This plot illustrates how the electrical damping modifies the system response at low load resistances.

In several tests the harvester was excited by two distinct frequencies and the total harvested power was found to be close to the theoretical combination of two individual oscillators. A limitation on the system used in this manner was found to be the differential displacement since the transducer must accommodate the maximum excursion of both mass/spring systems.

![Figure 6. Locus of the peak power frequency as electrical damping is changed.](image)

5. Conclusion

The prototype device displayed the predicted extended frequency response of a 2 DOF system and the realised design was able to efficiently couple the additional degree of freedom (over a conventional resonant harvester) by making both parts of the transducer into individual mass/spring systems.

Depending on the spacing of the resonant frequencies of the each mass-spring system, differing characteristics could be achieved, making a single broader peak or two distinct peaks. The relative motion of the two mass/spring systems is damped by the electrical load, hence the overall damping seen by the system is not a monotonic function of applied electrical load. The clearest manifestation of this is in the case of zero load resistance which results in the system approximating a lightly damped oscillator with compliance and mass equivalent to the combination of that of the individual oscillators.

In the extent of testing to-date the 2 DOF system has proved capable of harvesting from two frequencies of excitation simultaneously, demonstrating the significant advantage of higher order devices.

References

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