Baryon Spectra and Semileptonic Decay in a Constituent Quark Model

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Baryon mass spectra and semileptonic decay rates of a few heavy baryon decay rates are obtained in a constituent quark model. A modified fit of nonrelativistic Hamiltonian generates configuration-mixed baryon wave functions from the known masses and the measured $\Lambda_c^+ \rightarrow \Lambda e^+ \nu$ rate, with wave functions expanded in both harmonic oscillator and Sturmian bases. The wave functions obtained are used to calculate form factors and decay rates for the semileptonic decays of some of these baryons. The elastic fraction for $\Lambda_c^+ \rightarrow \Lambda^{(*)} \rightarrow \Lambda e^+ \nu$ decay is found to be $0.88 \pm 0.02$, and the $\Lambda_c^+ \rightarrow \Lambda e^+ \nu$ rate is found to be $1.28^{+0.16}_{-0.21}$, close to the experimental value. The elastic fraction of $b \rightarrow c$ ranges from about 50% calculated with the harmonic-oscillator model, to about 67% calculated with the Sturmian model.

I. INTRODUCTION AND MOTIVATION

Semileptonic decay of hadrons are of interest for two basic reasons; they are the primary source of information for the extraction of the Cabbibo-Kobayashi-Maskawa (CKM) matrix elements of the Standard Model from experiment, and the study of the semileptonic decays of baryons provides information about their structure. Here we present results of a calculation of the form factors and rates of the semileptonic decays of heavy baryons ($\Lambda_Q, \Xi_Q$) obtained using a constituent quark model.

Some work has been done in modeling the form factors for the semileptonic decays of heavy baryons, to the best of our knowledge little has been done in treating the decays to excited baryons. In addition, as far as we know, there are no calculations performed outside of the heavy quark effective theory framework, which motivates the present study, more details of which are presented in [1–3].

The semileptonic decay rate for $\Lambda_c \rightarrow \Lambda$ has been measured by the CLEO and Argus collaborations [4, 5], while the Delphi collaboration has only recently published an analysis of the exclusive semileptonic decay of the $\Lambda_b$ [6]. In their analysis of the $\Lambda_c \rightarrow \Lambda$ semileptonic decay, the CLEO collaboration have assumed that the ground state $\Lambda$ saturates the semileptonic decays of the $\Lambda_c$, and cite the absence of any final states of the form $\Lambda \ell \nu$ with additional decay products from the $\Lambda_c$ to support their assumption [5]. The CLEO-c collaboration [7] has published evidence for the observation of the decay $\Omega_b^0 \rightarrow \Omega^- e^+ \nu$, and have measured the product of the branching fraction and cross section but no quantitative value for the branching fraction has yet been published.

II. FORM FACTORS AND THE HAMILTONIAN

We use two bases to expand the wave function and calculate the form factors. One of these is the commonly used harmonic oscillator and the other one is Sturmian basis [8].

We use a non-relativistic quark model similar to that of Isgur and Karl [9]. The phenomenological Hamiltonian we use takes the form

$$H = \sum_i K_i + \sum_{i<j} \left( V_{\text{conf}}^{ij} + H_{\text{hyp}}^{ij} \right),$$

(1)

where $\sum_i K_i$ is the kinetic part of the Hamiltonian, the confining potential includes the spin dependent linear and Coulomb; and the hyperfine potential includes the contact and tensor interactions. The quark masses and the coupling constants of the potential are not fundamental, but are phenomenological parameters obtained from a fit to the spectrum of baryon states.
III. RESULTS

A. Mass Spectra and Size Parameters

Portions of the mass spectra and the size parameters we obtain using our models are shown in table I. In this table, the first two columns identify the state and its experimental mass, while the next two columns show the masses that result from a fit of the Hamiltonian parameters to those states whose experimental masses are known and the last two columns shows the size parameters of each state. ‘HO’ denotes the harmonic oscillator model, while ‘ST’ denotes the sturmian model.

| Baryon state | Masses | Size Parameters |
|--------------|--------|-----------------|
|              | Expt.  | HO             | ST             | (α₁,α₂) | (β₁,β₂) |
| N(1/2⁺)     | 0.94   | 1.00           | 1.07           | 0.35    | -       |
| N(1/2⁻⁺)    | 1.44   | 1.68           | 1.76           | 0.35    | -       |
| N(1/2⁻⁻)    | 1.54   | 1.47           | 1.51           | 0.35    | -       |
| N(3/2⁺)     | 1.72   | 1.72           | 1.77           | 0.34    | -       |
| Δ(3/2⁺)     | 1.23   | 1.24           | 1.29           | 0.34    | -       |
| Λ(1/2⁺)     | 1.12   | 1.11           | 1.09           | (0.42,0.42) | (0.46,0.67) |
| Λ(1/2⁻⁺)    | 1.60   | 1.74           | 1.61           | (0.42,0.42) | (0.46,0.67) |
| Λ(1/2⁻⁻)    | 1.41   | 1.49           | 1.46           | (0.41,0.39) | (0.52,0.61) |
| Λ⁺c(1/2⁺)   | 2.28   | 2.27           | 2.27           | (0.46,0.43) | (0.53,0.61) |
| Λ⁺c(1/2⁻⁻)  | 2.59   | 2.63           | 2.60           | (0.48,0.38) | (0.58,0.56) |
| Ξ(1/2⁺)     | 1.32   | 1.32           | 1.40           | (0.39,0.42) | (0.49,0.65) |
| Ξ(3/2⁺)     | 1.53   | 1.52           | 1.45           | (0.37,0.40) | (0.43,0.71) |
| Ξ(3/2⁻⁻)    | 1.82   | 1.83           | 1.79           | (0.38,0.41) | (0.61,0.53) |
| Ω(3/2⁺)     | 1.67   | 1.66           | 1.60           | 0.40    | -       |
| Ω⁺c(1/2⁺)   | 2.70   | 2.69           | 2.73           | (0.49,0.42) | (0.67,0.47) |

B. Semileptonic Decay

The differential decay rates, $dΓ/dq^2$, that we obtain in the two models are shown in Figure 1. In these figures, we show the differential rates for decays to the elastic channel, as well as for two orbital excitations, the states with $J^P = 1/2^-$ and $3/2^-$. 

| $J^P$ | $Γ$(HO) | $Γ$(ST) | Expt. |
|-------|---------|---------|-------|
| 1/2⁺  | 1.80    | 1.32    | 1.05±0.35 |
| 1/2⁻⁻ | 0.15    | 0.14    | -     |
| total | 2.00    | 1.53    | -     |
| $Γ/Γ_{total}$ | 0.90 | 0.86 | 1.0(assumed) |
FIG. 1: Differential decay rates obtained using harmonic oscillator and Sturmian wave functions for $\Lambda_c \to \Lambda(1/2^+)$ decay.

The integrated decay rate for a selection of final states in the two models are shown in Table II. The elastic rates predicted by the ST model is much closer to the experimentally reported rate [5] than that predicted by the HO model. From Table II, it is clear that, while the elastic channel dominates the decay rate of the $\Lambda_c$, it does not saturate the decay. We can compare our predictions for decays to the excited $\Lambda$ states with the assumption made by the CLEO Collaboration [5], that the elastic channel saturates the semileptonic decays of the $\Lambda_c$. In our models, we find that between 10% and 14% of the $\Lambda_c$ semileptonic decays are to excited states. In addition, our branching fraction (of 86% to 90%) to the ground state must represent an upper limit, as we have not included any non-resonant production of multi-particle final states. This elastic fraction is of considerable interest, as a number of decays of the $\Lambda_c$ are normalized to its semileptonic decay [10].

The differential decay rates $d\Gamma/dq^2$ obtained in the two models for a selection of final states in $\Omega_b \to \Omega_c^{(*)} \ell \bar{\nu}_\ell$, are shown in Figure 2. In these figures, we only show the differential rates for the dominant decays to the two elastic channels, with $J^P = 3/2^+$ and $J^P = 1/2^+$, and for two orbital excitations, the states with $J^P = 3/2^-$ and $5/2^-$. 

FIG. 2: Differential decay rates for $\Omega_b \to \Omega_c^{(*)}$ transitions obtained using harmonic oscillator and Sturmian wave functions.
TABLE III: Integrated decay rates for $\Omega_b \to \Omega_c^{(*)}$ in units of $10^{10} s^{-1}$, for a selection of $\Omega_c$ states in the two models we consider.

| Model | $\Omega_b \to \Omega_c^{(*)} e^- \bar{\nu}_e$ | Total width and BR |
|-------|---------------------------------|--------------------|
|       | $3/2^+ \ 1/2^+$ | $3/2^- \ 3/2^+$ | $5/2^- \ 1/2^+$ | $1/2^- \ 1/2^+$ | $\Gamma_{\Omega_c^{(*)}}(1/2^+, 3/2^+)$ | $\Gamma_{\Omega_c^{(*)}}(1/2^+, 3/2^+)/\Gamma_{\Omega_c^{(*)}}$ |
| (HO)  | 1.68 0.71 0.69 0.44 0.70 0.32 0.44 | 4.98 0.48 |
| (ST)  | 2.01 0.87 0.43 0.23 0.46 0.20 0.07 | 4.27 0.67 |

In Table III we show the integrated decay rates obtained for the selected final states in the two models we use. The last two columns of the table present the total decay rate and the ratio of the elastic to the total semileptonic decay rate. The integrated rates for the elastic decay modes ($1/2^+, 3/2^+$) obtained in all models are similar. However, the Sturmian model predicts somewhat smaller rates for decays into the inelastic $\Omega_c$ channels. Both models predict that the elastic decay processes dominate the $\Omega_b$ semileptonic decay but do not saturate it; there significant branching fraction to the inelastic channels.

IV. CONCLUSION

Using a constituent quark model we calculated the form factors and rates for semileptonic decay of heavy baryons. We found that our model form factors of baryon are consistent with HQET predictions for $\Lambda_Q$ and $\Omega_Q$ decays. It is instructive to note that our model prediction for semileptonic decay of $\Lambda_c$ mainly goes to ground state $\Lambda$ with a small but not insignificant (10%-14%) BR to excited state $\Lambda$. We have also calculated rates for semileptonic decays of $\Omega_Q$ and found a significant (33%-51%) BR for $\Omega_b$ decaying to excited states of the $\Omega_c$.

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