Locus model for space-time fabric and quantum indeterminacies

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A simple locus model for the space-time fabric is presented and is compared with quantum foam and random walk models. The induced indeterminacies in momentum are calculated and it is shown that these space-time fabric indeterminacies are, in most cases, negligible compared with the quantum mechanical indeterminacies. This result restricts the possibilities of an experimental observation of the space-time fabric.

Keywords: atomic space-time, Planck scale, indeterminacies.

I. INTRODUCTION

In this work a very simple atomic or discrete space-time model is presented. The atoms of space-time, that we name loci (locus in singular), have sizes comparable with Planck scale and are located in a mathematical continuous space. All points within a given locus are physically equivalent and can not be differentiated or taken apart.

We can consider these loci as probability distributions for physical coordinates and therefore distances, time intervals and momenta become random variables that can be calculated from the loci distributions. We will find the probability uncertainty for these quantities, without any reference to quantum mechanics, and we will afterwards compare them with the corresponding quantum mechanic indeterminacies.

II. THE LOCUS MODEL

Let us use the continuous real variables \((x, t)\) to denote the mathematical coordinates of a space-time point. We will differentiate these mathematical coordinates

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from the physical coordinates because two points \( x_1 \) and \( x_2 \) separated by a distance comparable with the Planck length \( \ell_p \) are physically indistinguishable and two instants of time separated by an interval comparable with Planck time \( t_p \) can not be considered as physically different. In order to formalize this concept we propose that a physical coordinate is described by space-time region, a locus, centered at a mathematical coordinate and having a width given by the Planck scale. A space-time localization of a particle means that a certain locus is occupied and a physical space-time interval will be determined by the set of loci between two space-time points. The precise shape and boundaries of these loci will not be relevant and we can also imagine soft boundaries that could be described by a probability density (gaussian for instance). In this way, physical coordinates become random variables distributed with probability densities \( L_{x,\ell_p}(\xi) \) and \( T_{t,t_p}(\theta) \) with center at \((x,t)\) and widths \((\ell_p,t_p)\). The physically relevant interval, or distance between two coordinates \( x_1 \) and \( x_2 \), is then a random variable \( x_2 - x_1 \) distributed according to the convolution

\[
\int_{-\infty}^{\infty} d\eta L_{x_2,\ell_p}(\xi - \eta)L_{x_1,\ell_p}(\eta) .
\]

(1)

Let us consider now the physical space between two distant coordinate points, say \( x = 0 \) and \( x = \ell \). We can define a partition of the interval \( 0 < x_1 < x_2, \ldots, < x_N < \ell \) and we have \( \ell = \ell - x_N + x_N - x_{N-1} + x_{N-1} - \cdots + x_2 - x_1 + x_1 - 0 \). So we have decomposed the interval \( \ell \) in a sum of \( N + 1 \) subintervals and therefore this physical length is a random variable distributed as an \( N + 1 \) fold convolution. If \( N \) is large, the distribution will approach a gaussian distribution, regardless of the shape of the locus distribution, with a width given by \( \sqrt{2N - 2\ell_p} \). Accordingly, the distribution of a physical length \( \ell \) will depend on the number of points in the partitions. We can fix the number of subintervals to be approximatively equal to the number of loci fitting in the length \( \ell \). Let us define then \( \delta_L \) to be the space density of loci and then we have \( N = \delta_L \ell \). We can expect that the space density of loci is close to \( 1/\ell_p \) because if the density would be much larger, then we could have physical locations separated by a distance less then \( \ell_p \) and if it were much smaller the transition from one location to the next would not be possible. A physical length \( \ell \), much longer than Planck length \( \ell_p \), is then a random variable with a gaussian distribution

\[
\Xi(\xi) = \frac{1}{\sqrt{2\pi}\sigma_x} \exp\left( -\frac{(\xi - \ell)^2}{2\sigma_x^2} \right) \quad (2)
\]

peaked at \( \xi = \ell \) and with a width \( \sigma_x \) (we reserve \( \Delta_x \) for quantum indeterminacies)

\[
\sigma_x = \sqrt{2\ell \delta_L \ell_p} , \quad (3)
\]
and if we take $\delta_L \approx 1/\ell_p$ we have

$$\sigma_x = \left( \frac{2\ell}{\ell_p} \right)^{1/2} \ell_p .$$  \hspace{1cm} (4)

In a similar way we conclude that a time interval $t$, much longer than Planck time $t_p$, is a random variable with a gaussian distribution

$$\Theta(\theta) = \frac{1}{\sqrt{2\pi}\sigma_t} \exp \left( -\frac{(\theta - t)^2}{2\sigma_t^2} \right)$$  \hspace{1cm} (5)

with a width $\sigma_t$

$$\sigma_t = \sqrt{2t\delta_T t_p} ,$$  \hspace{1cm} (6)

where $\delta_T$ is the loci time density and if we take it $\delta_T \approx 1/t_p$ we have

$$\sigma_t = \left( \frac{2t}{t_p} \right)^{1/2} t_p .$$  \hspace{1cm} (7)

We can now compare these results with other models with an essential indeterminacy in space-time points. An early proposal was made by Karolyhazy\[1\] that combined Heisenberg’s uncertainty principle with Schwarzschild horizon in order to estimate a minimal uncertainty in the measurement of a distance $\ell$ and a time $t$ given by

$$\sigma_x \sim \left( \frac{\ell}{\ell_p} \right)^{1/3} \ell_p , \text{ and } \sigma_t \sim \left( \frac{t}{t_p} \right)^{1/3} t_p .$$  \hspace{1cm} (8)

Due to the $1/3$ exponent, these indeterminacies are much smaller than the ones resulting from the loci model. These results, obtained from a heuristic argument, were rediscovered in the context of a quantum foam model for the space-time fabric\[2\] and they gain support from other apparently independent arguments\[3\]. Indeed it was shown that the same result can be obtained as a consequence of the Holographic Principle and also from Black Holes physics and even from Information and Computer Theory. The loci model result, with an $1/2$ exponent, was also obtained from a random walk\[4, 5\] model. An interesting feature of these random walk models is that they can be modified in order to obtain also the $1/3$ exponent by the introduction of some memory in the random walk that increases the probability of returning to the previous position like a repentant walker. One could motivate such a memory by an attractive self interaction between the actual and the previous position of a particle.
III. INDUCED INDETERMINACIES IN MOMENTUM

In this section we will deduce the momentum indeterminacy induced by the space-time indeterminacies. Although we will concentrate on the indeterminacies of the locus model, the conclusions are also valid for the other models with a 1/3 exponent. Let us consider a free particle of mass \( m \) moving a distance \( \ell \) during a time interval \( t \). Since these quantities are random variables with distributions given in Eqs.(2, 5) the momentum of the particle, given by 

\[ p = \frac{m \ell}{t} \]

will also be a random variable with the distribution corresponding to the quotient of random variables. Therefore the momentum \( p \) will be distributed according to the probability density function \( \Pi(\varpi) \) given by

\[ \Pi(\varpi) = \int_{-\infty}^{\infty} d\xi \int_{-\infty}^{\infty} d\theta \; \Xi(\xi) \; \Theta(\theta) \; \delta \left( \varpi - \frac{m \xi}{\theta} \right) = \int_{-\infty}^{\infty} d\theta \left| \frac{\theta}{m} \right| \Xi(\varpi \theta) \; \Theta(\theta) . \tag{9} \]

If we insert the gaussian densities given in Eqs.(2,5) we can obtain the momentum density distribution in terms of the Error Function and considering that \( t \gg t_p \) we get the approximation

\[ \Pi(\varpi) \approx \frac{\sqrt{pmc}}{2\pi} \left( \frac{\ell}{\varpi^2 + pmc} \right)^{1/2} \exp \left( -\frac{(\varpi - p)^2}{4 \frac{m \ell}{mc} \left( \varpi^2 + pmc \right)} \right) . \tag{10} \]

This distribution is peaked at \( \varpi = p = \frac{m \ell}{t} \) and its width can be estimated from the denominator of the exponent. However we can obtain the momentum indeterminacy more rigorously from the definition

\[ \sigma_p^2 = \int_{-\infty}^{\infty} d\varpi \; (\varpi - p)^2 \; \Pi(\varpi) \]

\[ = \int_{-\infty}^{\infty} d\varpi \int_{-\infty}^{\infty} d\xi \int_{-\infty}^{\infty} d\theta \; (\varpi - p)^2 \; \Xi(\xi) \; \Theta(\theta) \; \delta \left( \varpi - \frac{m \xi}{\theta} \right) \]

\[ = \int_{-\infty}^{\infty} d\xi \int_{-\infty}^{\infty} d\theta \; (m \frac{\xi}{\theta} - p)^2 \; \Xi(\xi) \; \Theta(\theta) \]

\[ = m^2 \int_{-\infty}^{\infty} d\xi \; \xi^2 \; \Xi(\xi) \int_{-\infty}^{\infty} d\theta \; \frac{1}{\theta^2} \; \Theta(\theta) - 2mp \int_{-\infty}^{\infty} d\xi \; \Xi(\xi) \int_{-\infty}^{\infty} d\theta \; \frac{1}{\theta} \; \Theta(\theta) + p^2 \]

\[ = m^2 \sigma_x^2 + t^2 \left( \frac{1}{\theta^2} \right) - 2mpt \left( \frac{1}{\theta} \right) + p^2 \]

\[ = m^2 \sigma_x^2 + t^2 \left( \frac{1}{\theta^2} \right) + m^2 t^2 \left( \frac{1}{\theta^2} \right) - 2t \left( \frac{1}{\theta} \right) + \frac{1}{t^2} \] . \tag{11} \]

Since the \( \Theta(\theta) \) distribution is sharply peaked at \( \theta = t \) we can take as good approximation \( \langle 1/\theta^2 \rangle = 1/t^2 \) and \( \langle 1/\theta \rangle = 1/t \) and with this, the parenthesis in last equation
vanishes. With this we become then the simple expression

\[ \frac{\sigma_p}{p} = \frac{\sigma_x}{\ell}. \]  

(12)

This is a general result but if we specialize it for the locus model, using Eqs. (4 and 7), we obtain

\[ \frac{\sigma_p}{p} = \frac{\sigma_x}{\ell} = \frac{\sigma_t}{t} \left( \frac{mc}{p} \right)^{1/2} = \left( \frac{2\ell p}{\ell} \right)^{1/2}. \]  

(13)

IV. COMPARISON WITH QUANTUM INDETERMINACIES

In the estimation of the indeterminacies due to the space-time fabric with a 1/3 exponent, some quantum mechanical arguments have been used. However, since these arguments were heuristic, there is no guaranty that the indeterminacies obtained are compatible with rigourous quantum mechanical indeterminacies and their correlations manifest in the uncertainty principle. Quantum mechanical indeterminacies are ubiquitous but nonintuitive and it is therefore dangerous to identify them with indeterminacies arising in idealized measurement procedures. Indeed, it is well known that some heuristic arguments using Heisenberg’s principle may lead to erroneous results and as D. Griffith warns “when you hear a physicist invoke the uncertainty principle, keep a hand on your wallet”. Furthermore the 1/2 exponent indeterminacies are larger than the 1/3 ones and were derived without reference to quantum mechanics. It is therefore necessary to compare the space-time fabric indeterminacies \( \sigma_x \) and \( \sigma_p \) with the quantum mechanical indeterminacies \( \Delta_x \) and \( \Delta_p \), in particular with their correlations \( \Delta_x \Delta_p \geq \hbar/2 \). From Eq. (13) above we immediately obtain for the product of the space-time fabric indeterminacies

\[ \sigma_x \sigma_p = 2p\ell, \]  

(14)

and therefore for a momentum smaller than some value we have a product of indeterminacies below the quantum mechanical bound \( \hbar/2 \). This value of momentum turns out to be enormous, 5 kg m/s or \( 10^{28} \) ev/c, many orders of magnitude bigger than the highest energy cosmic rays observed. We must conclude that the indeterminacies due to the space-time fabric lie deep below the quantum mechanical indeterminacies. This places severe limits on the observability of the space-time indeterminacies in the kinematics of a particle because any observation will encounter the quantum mechanical limits long before the space-time indeterminacies are approached. There are proposals to observe space-time indeterminacies in extragalactic light by interferometric techniques taking \( \ell \) large enough, close to the observable universe length
\( \ell \sim 10^{20} \text{m} \). In such a long travel, the photons in the locus or random walk models would develop an indeterminacy \( \sigma_x \sim 1.7 \times 10^{-3} \text{m} \), much longer than the wave length, making the light incoherent and therefore no interference should be observed. In the case of the other models, the indeterminacy accumulated is \( \sigma_x \sim 3 \times 10^{-15} \text{m} \), not sufficient to destroy coherence. The observation of interference fringes seem to exclude the 1/2 exponent models. The observation of the space-time indeterminacies for these 1/3 exponent models faces the difficulties imposed by quantum mechanics: in order to test an indeterminacy of \( \sigma_x \sim 3 \times 10^{-15} \text{m} \) the quantum mechanical indeterminacy must be even smaller \( \Delta_x < \sigma_x \) but this implies a momentum indeterminacy \( \Delta_p > \hbar/(2\sigma_x) \) and this turns out to be 30 Mev/c requiring very high energy gamma rays.

V. CONCLUSIONS

The locus model presented is very simple and intuitive, however some modifications and refinements could be necessary when the observations of space-time indeterminacies becomes feasible. Several possible changes in the model can be implemented. One of them could to modify the space and time loci densities \( \delta_L \) and \( \delta_T \) considering the possibility of very high or very low loci densities or overlap. Another modification could be to consider physical space and time coordinates not as independent random variables but instead described by a joint probability distribution function. There are also more speculative possibilities where we could describe the propagation in terms of a deformation of the loci in the direction of propagation by an amount related to the energy enclosed. As is also the case for the other quantum foam models, the locus model violates Lorentz covariance and we could assume the loci to be at rest in the reference frame where the 2.7K background radiation is isotropic, or to assume that they move with a velocity distribution with a large spread in order to recover approximatively Lorentz covariance.

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