On the scalability and convergence of simultaneous parameter identification and synchronization of dynamical systems

Bruno Nery  Rodrigo Ventura
Institute for Systems and Robotics
Instituto Superior Técnico, Lisbon, Portugal

Abstract

The synchronization of dynamical systems is a method that allows two systems to have identical state trajectories, apparr from an error converging to zero. This method consists in an appropriate unidirectional coupling from one system (drive) to the other (response). This requires that the response system shares the same dynamical model with the drive. For the cases where the drive is unknown, Chen proposed in 2002 a method to adapt the response system such that synchronization is achieved, provided that (1) the response dynamical model is linear with a vector of parameters, and (2) there is a parameter vector that makes both system dynamics identical. However, this method has two limitations: first, it does not scale well for complex parametric models (e.g., if the number of parameters is greater than the state dimension), and second, the model parameters are not guaranteed to converge, namely as the synchronization error approaches zero. This paper presents an adaptation law addressing these two limitations. Stability and convergence proofs, using Lyapunov’s second method, support the proposed adaptation law. Finally, numerical simulations illustrate the advantages of the proposed method, namely showing cases where the Chen’s method fail, while the proposed one does not.

1 Introduction

Consider two identical continuous time dynamical systems, designated drive (D) and response (R). It is well known that the state evolution of each system, when taken separately, may differ radically if the initial condition for each system differ, namely in the case of chaotic dynamical systems [6, 4]. However, in the presence of a unidirectional coupling from the drive to the response system, synchronization of their state trajectories is known to occur [10, 8, 5]. In this paper we limit the discussion to the simplest coupling scheme, in which the response system receives the full state vector from the
drive. In this situation it is easy to design a controller that synchronizes both systems, using feedback linearization (Section 2).

Such synchronization assumes that both drive and response have the same dynamical model. This paper addresses the problem of achieving synchronization of a response system, when the dynamical model of the drive is unknown. In particular, we target the problem of simultaneous adaptation and synchronization of a response system, given an unknown drive. Two assumptions are made: (1) the response dynamical model depends linearly on a parameter vector, and (2) there is a value for this vector that makes both systems identical. In 2002, Chen and Lü proposed a method to simultaneously adapt this parameter vector and to make both systems synchronized [3]. Lyapunov second method was used to prove the feasibility of this method, however, due to the construction of the Lyapunov function employed, convergence of the response parameters is not guaranteed. This has two consequences that prevent the general usage of this method. Firstly, it does not scale in complexity: if the dimension of the parameter vector is greater than the dimension of the state vector, convergence is not guaranteed. And secondly, even with a small number of parameters, Chen’s proof does not guarantee effective convergence of the parameters.

In this paper we address both of these problems, presenting a convergence proof for the simultaneous synchronization and adaptation of the response to an arbitrary drive system. Moreover, numerical simulations comparing the proposed approach with Chen’s method illustrate the benefits of the approach.

Chaotic synchronization was first introduced by Pecora and Carrol in 1990 [7]. Since then, many publications have deepend our knowledge about this concept [1, 5, 8, 2]. A method for synchronizing the Rössler and the Chen chaotic systems using active control was proposed by Agiza and Yassen [2], however the approach is specific to these particular systems. Chen and Lü proposed a method to perform simultaneous identification and synchronization of chaotic systems [3], but the results show some limitations, which are discussed in length and addressed in this paper.

The paper is structured as follows: section 2 states formally the problem, followed by the proposed solution in section 3, experimental results are presented in section 4 and section 5 concludes the paper.

2 Problem statement

Consider two dynamical systems, called drive and response, with a unidirectional coupling between them. Throughout this paper we will assume that both drive and response systems are identical, apart from a parameter vector, which is unknown. The goal of the adaptation law is to determine
this parameter vector. Consider the drive system modeled by
\[ \dot{x} = f(x) + F(x) \theta, \]
where \( x(t) \in \mathbb{R}^n \) is the state vector, and \( \theta \in \mathbb{R}^m \) is a parameter vector. The nonlinear functions that support the model are \( f : \mathbb{R}^n \to \mathbb{R}^n \) and \( F : \mathbb{R}^n \to \mathbb{R}^{(n \times m)} \). The coupling between the drive and the response systems consists in a bias term, called synchronization input, from the drive to the response. The response system is identical to the drive, except for the parameter vector \( \alpha \in \mathbb{R}^m \) and the synchronization input \( U \),
\[ \dot{y} = f(y) + F(y) \alpha + U(y, x, \alpha), \]
where \( y(t) \in \mathbb{R}^n \) is the response state vector, and \( U : \mathbb{R}^n \times \mathbb{R}^n \times \mathbb{R}^m \to \mathbb{R}^n \) is the synchronization control function. This function \( U \) realizes the controller that, given the state input from the drive, synchronizes the response system.

Define the state error \( e = y - x \) and the parameter error \( \Delta = \alpha - \theta \); the simultaneous adaptation and synchronization problem consists in the design of a controller \( U \) and of a parameter adaptation law for \( \alpha \) such that both \( \lim_{t \to \infty} e(t) = 0 \) and \( \lim_{t \to \infty} \Delta(t) = 0 \).

Chen proposes in [3] a solution to this problem in the form of an adaptation law for \( \alpha \).

**Assumption 1.** There is a controller \( U \) and a scalar function \( V(e) \) that, for \( \alpha = \theta \), satisfies both (i) \( c_1 ||e||^2 \leq V(e) \leq c_2 ||e||^2 \) and (ii) \( \dot{V}(e) \leq -W(e) \), for \( c_1, c_2 \) positive constants, \( W(e) \) a positive definite function, and \( U(x, x, u, \theta) = 0 \).

For example, the controller
\[ U(y, x, \theta) = -e + f(x) - f(y) + [F(x) - F(y)] \theta, \]
and the function \( V(e) = \frac{1}{2} e^T e \) satisfy this assumption.

**Theorem 1.** Under Assumption 1, the adaptation law
\[ \dot{\alpha} = -F^T(x) [\text{grad} V(e)]^T \]
stabilizes the system at the equilibrium point \( e = 0, \alpha = \theta \).

**Proof.** See [3].

In the proof of this theorem, Chen employs the Lyapunov function
\[ V_1(e, \alpha) = \frac{1}{2} e^T e + \frac{1}{2} \Delta^T \Delta. \]
There is an hidden assumption in the proof: it only holds if \( U(y, x, \alpha) - U(y, x, \theta) = [F(x) - F(y)] \Delta \) (which is true if controller (3) is used).
Still, two problems remain that compromise the applicability of this result. The first one is that (4) does not guarantee strict definite positiveness of $-\dot{V}_1$; in particular, $V_1(0, \alpha) = 0$ for all values of $\alpha$. This means that, as the synchronization error $e$ approaches zero, the magnitude of the parameter error $\Delta$ is not guaranteed to decrease. The second problem concerns the null space of $F^T(x)$: according to (4), the parameter vector $\alpha$ remains changed, as long as grad $V(e)$ lies in the null space of $F^T(x)$. Taking for instance $V(e) = \frac{1}{2} e^T e$, while the state error $e$ lies in this null space, the parameter vector $\alpha$ remains unchanged, even if $\alpha \neq \theta$.

3 Proposed solution

Let us first obtain a controller function $U$ that achieves synchronization, assuming that the true value of the parameter vector is known, $\alpha = \theta$. In this situation, the error state $e$ has the following dynamics

$$\dot{e} = f(y) - f(x) + [F(y) - F(x)] \theta + U(y, x, \theta).$$

(6)

Considering now the positive definite Lyapunov function

$$V(e) = \frac{1}{2} e^T e,$$

(7)

its time derivative is $\dot{V} = e^T \dot{e}$. Taking the controller

$$U(y, x, \theta) = -K e - f(y) + f(x) - [F(y) - F(x)] \theta,$$

(8)

where $K$ is a $(n \times n)$ positive definite matrix, we have that $\dot{e} = -K e$. Matrix $K$ is thus related with the synchronization rate. Since $-\dot{V} = e^T K e$ is a positive definite function, for a positive definite $K$, system (6) is globally uniformly asymptotically stable \[ \text{II} \] at the equilibrium point $e = 0$. Note that this controller satisfies Chen’s Assumption referred in the previous section.

Consider now the positive definite Lyapunov function

$$V(e, \Delta) = \frac{1}{2} e^T e + \frac{1}{2} \Delta^T \Delta.$$

(9)

This function is zero if and only if both the response is synchronized with the drive, and its parameters equal the drive ones. The dynamics of the error $e$, while using the controller (8), is then

$$\dot{e} = -K e + F(x) \Delta.$$

(10)

By left multiplying this equation by $F^T(x) L$, where $L$ is a $(n \times n)$ positive definite matrix (which is related with the adaptation rate; see below), and transposing the result, one gets the relation

$$\Delta^T F^T(x) L^T F(x) = \dot{e}^T L^T F(x) + e^T K^T L^T F(x).$$

(11)

We are now in condition to prove the main result of this paper:
Theorem 2. Assuming that there is a constant matrix $L$ such that $G(x) = F^T(x) L^T F(x)$ is positive definite for all $x$, the adaptation law

$$\dot{\alpha} = -F^T(x) \left[ (L K + I) e + L \dot{e} \right],$$

(12)

together with the controller (8), globally uniformly stabilizes both the error system (10) at $e = 0$, and the parameter error at $\Delta = 0$.

Proof. Considering the Lyapunov function (9), we have $\dot{V} = e^T \dot{e} + \Delta^T \dot{\Delta}$. Taking the adaptation law (12) together with (11), while noting that $\dot{\Delta} = \dot{\alpha}$, one obtains

$$\dot{V} = -e^T K e - \Delta^T G(x) \Delta.$$  

(13)

Since $G(x)$ is assumed positive definite, $-\dot{V}$ is also positive definite, from which we can conclude that $(e, \Delta) = (0, 0)$ is a globally uniformly asymptotically stable equilibrium point of (10).

This theorem implies both synchronization ($y = x$) and correct identification of the parameters ($\alpha = \theta$). Note that the practical use of the proposed adaptation law (12) requires knowledge of the error time derivative $\dot{e}$, which in principle can be obtained (or estimated) from the error evolution.

The choice of the constant matrices $K$ and $L$ have impact on the convergence rate. If $\alpha = \theta$, the error system is $\dot{e} = -K e$, meaning that the error decreases asymptotically to zero according to a first-order linear dynamics with a time constant determined by $K$. If $e = 0$, the parameter error has the dynamics $\dot{\Delta} = -F^T(x) L F(x) \Delta$, and thus the magnitude of $L$ impacts on the convergence rate of the parameters. Simple choices for $K$ and $L$ are diagonal matrices with constant values, $K = k I$ and $L = l I$, for $k$ and $l$ two positive scalars. Thus, the state and parameter error dynamics become $\dot{e} = -k e$ and $\dot{\Delta} = -l F^T(x) F(x) \Delta$.

Since $F(x)$ is a $(n \times m)$ matrix, its rank is lower or equal to $\min(n, m)$, and thus the rank of $G(x)$ is also lower and equal to $\min(n, m)$. However, in order for $G(x)$ to be positive definite, its rank has to be equal to $m$ (the dimension of the parameter vector $\theta$), and thus $n \geq m$ is a necessary condition for $G(x)$ to be full rank. This means that there is an upper bound to the amount or parameters $m$, in order for convergence to be guaranteed. This largely limits the flexibility of the response system to adapt to arbitrary drive systems, in particular with a large amount of parameters.

To tackle this problem we propose augmenting the $F(x)$ matrix with extra rows, as many as needed, in order for $G(x)$ to become full rank. First, let us designate by $x^*(t)$ a new state vector consisting in the concatenation of time delayed versions of the original state vector $x(t)$,

$$x^* = \left[ x_0 \ x_1 \ \cdots \ x_r \right]^T,$$

(14)

5
where \( x_i(t) = x(t - i \delta) \), for \( i = 1 \ldots r \) and a \( \delta > 0 \). Using this state vector, the drive system becomes
\[
\dot{x}^* = f^*(x^*) + F^*(x^*) \theta,
\] (15)
where
\[
f^*(x^*) = \begin{bmatrix} f(x_0) \\ \vdots \\ f(x_r) \end{bmatrix} \quad \text{and} \quad F^*(x^*) = \begin{bmatrix} F(x_0) \\ \vdots \\ F(x_r) \end{bmatrix}.
\] (16)
This augmented system is equivalent to (1), as the additional state dimension corresponds to time delayed versions of the original system. The response system, with state vector \( y^* \in \mathbb{R}^{(r+1)n} \) takes the form
\[
\dot{y}^* = f^*(y^*) + F^*(y^*) \alpha + U^*(y^*, x^*, \alpha).
\] (17)
These two coupled systems with state vectors \( x^* \) and \( y^* \) can be viewed as a new pair of drive and response systems by themselves, with error vector \( e^* = y^* - x^* \). Thus, the results obtained above can be directly applied here: the synchronization controller becomes
\[
U^*(y^*, x^*, \alpha) = -K^* e^* - f^*(y^*) + f^*(x^*) - [F^*(y^*) - F^*(x^*)] \alpha,
\] (18)
where the matrix \( K^* \) can be a \(((r+1)n \times (r+1)n)\) block diagonal formed by \( K \) matrices,
\[
K^* = \begin{bmatrix} K & 0 & 0 \\ 0 & K & 0 \\ \vdots & \vdots & \ddots \end{bmatrix}.
\] (19)
The adaptation law becomes then
\[
\dot{\alpha} = -F^T(x^*) [(L^* K^* + I) e^* + L^* \dot{e}^*],
\] (20)
where \( L^* \) is a \(((r+1)n \times (r+1)n)\) matrix, which can also take the form of a block diagonal in the same fashion as \( K^* \) above,
\[
L^* = \begin{bmatrix} L & 0 & 0 \\ 0 & L & 0 \\ \vdots & \vdots & \ddots \end{bmatrix}.
\] (21)
If both \( K^* \) and \( L^* \) have the block diagonal structure as in (19) and (21), the adaptation law (20) can be simplified into
\[
\dot{\alpha} = -\sum_{i=0}^{r} F^T(x_i) [(L K + I) e_i + \dot{e}_i],
\] (22)
where \( e_i = y_i - x_i \) and \( \dot{e}_i = \dot{y}_i - \dot{x}_i \).

With the above augmented system, we can prove convergence when \( n < m \) with the following Corollary:
Corollary 1. If matrix $G^*(x) = (F^*)^T (L^*)^T F^*(x)$ is full rank for all $x$, then the response system (17), together with the adaptation law (20), globally uniformly stabilizes both the error system (10) at $e = 0$, and the parameter error at $\Delta = 0$.

Proof. The equivalent drive (15) and the response (17) systems satisfy the conditions of Theorem 2, as long as $G^*(x)$ is full rank.

The rank of $G^*(x)$ cannot be guaranteed \textit{a priori}, but a necessary condition Corollary can still be stated:

Corollary 2. If $F$ has rank $n < m$, then $r \geq \lceil \frac{m}{n} - 1 \rceil$ is a necessary condition for $G^*$ to be full rank.

Proof. The rank of $G^* = (F^*)^T (L^*)^T F^*$ is at most $\min[(r+1)n, m]$. Since $G^*$ is a $m \times m$ matrix, in order to be full rank, $(r+1)n \geq m$ has to hold. Therefore, $r \geq \frac{m}{n} - 1$, but since $r$ is an integer, its lower bound is $\lceil \frac{m}{n} - 1 \rceil$. \hfill $\square$

In general, as $r$ is arbitrary, one can expect that there is a value of $r$ large enough that makes $G^*$ full rank.

Comparing the obtained adaptation law (22) with (12) above, one can observe that the gradient of the parameters depends on several time delayed samples of the error $e$ (as well as on their derivatives $\dot{e}$). A possible intuition to this result comes from the observation that, if $m > n$, the degrees of freedom of $e$ are not enough to produce a meaningful gradient for $\alpha$, if the law (12) is employed. However, with (22), which depends on $e^*$ with $(r+1)n$ degrees of freedom, the gradient of $\alpha$ can have the full dimensionality of $m$.

4 Experimental results

This section presents numerical results illustrating the theoretical results derived above. Two classical chaotic systems were used: the Lorenz oscillator [12], commonly used in the chaotic synchronization literature for numerical simulations [5, 8, 3], and the Rössler attractor, designed to behave similarly to the Lorenz system while being easier to understand [9]. Simultaneous identification and synchronization is simulated, while comparing the performance of Chen’s method [10] with the one proposed here. For the Chen’s method we used controller (3) with the adaptation law (4), and for our method we used controller (18) with the adaptation law (22).

The Lorenz oscillator is a three-dimensional dynamical system that behaves chaotically for a certain set of parameters [12]. In the form of (1), it can be written as

$$
\begin{bmatrix}
\dot{x} \\
\dot{y} \\
\dot{z}
\end{bmatrix} =
\begin{bmatrix}
0 & -y -xz \\
y - x & 0 & 0 \\
x y & 0 & x
\end{bmatrix}
+ \begin{bmatrix}
y - x & 0 & 0 \\
x & 0 & 0 \\
0 & 0 & -z
\end{bmatrix}
\begin{bmatrix}
\theta_1 \\
\theta_2 \\
\theta_3
\end{bmatrix}
$$

(23)
where $x$, $y$ and $z$ are state variables and $\theta_1$, $\theta_2$ and $\theta_3$ are system parameters. The Lorenz oscillator was synchronized with a response system, which is specified by four parameters. In the form of (2), it can be written as

$$
\begin{bmatrix}
\dot{u} \\
\dot{v} \\
\dot{w}
\end{bmatrix} =
\begin{bmatrix}
0 & -v & 0 & 0 \\
-v - uw & 0 & u & 0 \\
uw & 0 & 0 & -w & 1
\end{bmatrix}
\begin{bmatrix}
\alpha_1 \\
\alpha_2 \\
\alpha_3 \\
\alpha_4
\end{bmatrix}
+ 
\begin{bmatrix}
u_1 \\
u_2 \\
u_3
\end{bmatrix}
$$

where $u$, $v$ and $w$ are state variables and $\alpha_1$, $\alpha_2$, $\alpha_3$ and $\alpha_4$ are the parameters. Note that the rank of the $F(x)$ matrix in (24) is at most 3, while the response systems uses 4 parameters: $\alpha_4$ is an unnecessary parameter that is not present in the drive (23), being artificially introduced to comparing the two approaches when $m > n$. As it was shown before, under these conditions Chen’s method is not guaranteed to converge, while Corollary 2 requires $r \geq 1$ for $G^*$ to be full rank, and thus a necessary condition for convergence (as Corollary 1).

For this simulation, the classical parameter values for the Lorenz system were used: $[\theta_1, \theta_2, \theta_3, \theta_4]^T = [10, 28, 8/3, 0]^T$. The initial states of the drive system and the controlled system were arbitrarily set to $[8, 9, 10]^T$ and $[3, 4, 5]^T$, respectively. The parameters of the response system had zero initial condition. The $L$ and $K$ parameters were set to $10I$ and $0.1I$. 

Figure 1: Lorenz system: graph of parameter identification results for $\alpha_4$. Solid line: Chen’s method (4), dotted and dash-dotted lines: proposed method (22) for $r = 3$ and for $r = 5$. 
Figure 2: Lorenz system: graph of parameter identification results for $\alpha_4$ using Chen’s method [4].

Figure 3: Lorenz system: plot of parameter identification results for $\alpha_3$. Top plot: Chen’s method [1], middle and bottom plots: proposed method [22] for $r = 3$ and for $r = 5$. 
Figure 1 shows the numerical results of parameter identification for parameter $\alpha_4$. Note the trend for the parameter convergence to be faster for higher values of $r$. Figure 2 shows the results of parameter identification for the parameter $\alpha_4$ for Chen’s method over a longer time horizon. While Chen’s method is not able to identify this parameter even after 1000 seconds, our method allows for a significantly faster convergence (under 200 seconds). Figure 3 shows the results of parameter identification for $\alpha_3$. Table 4 shows the time it takes for the parameter identification error to fall below a percentage of the real parameter value. Note again that the convergence is faster for higher values of $r$. It is interesting to note that, for instance, during the last 20 seconds of the simulation, the coefficient of variation$^2$ of the root mean square error is of $4.42 \times 10^{-3}$ for Chen’s method, while for our method it is of $1.21 \times 10^{-4}$ ($r = 3$) and $1.36 \times 10^{-6}$ ($r = 5$). Chen’s method is not able to correctly identify this parameter, with its value oscillating around the true value of $\theta_3$. Our method, however, allows for a lower variance in the parameter identification. Figure 4 shows the synchronization error, as measured by the Lyapunov function (7). Both Chen’s method and ours are able to drive the synchronization error to zero. Our method, however, shows near-instantaneous convergence. Also, the magnitude of the

---

$^1$All simulations were performed using Python together with SciPy and PyDDE libraries.

$^2$The coefficient of variation is defined as the ratio $\sigma/|\mu|$, where $\sigma$ is the standard deviation and $\mu$ the sample mean.
Table 1: Time to reach identification error ranges for parameter $\alpha_3$ (in simulation seconds)

| Method         | Identification error |
|----------------|----------------------|
|                | 1%       | 0.1%     | 0.01%    | 0.001%   |
| Classical Chen |          |          |          |          |
| $r = 1$        | 0.4      | 261.5    | 612.1    | 942.7    |
| $r = 2$        | 10.3     | 104.8    | 226.7    | 308.5    |
| Extended Chen  |          |          |          |          |
| $r = 3$        | 2.5      | 58.4     | 106.5    | 163.8    |
| $r = 4$        | 2.5      | 31.1     | 70.9     | 94.8     |
| $r = 5$        | 2.5      | 21.6     | 43.5     | 67.3     |

error is reduced by comparison to Chen’s method.

Similar results were obtained using the Rössler attractor. In the form of (1), it can be written as

$$
\begin{bmatrix}
\dot{x} \\
\dot{y} \\
\dot{z}
\end{bmatrix} =
\begin{bmatrix}
-\theta_1 & -\theta_2 & -\theta_3 \\
\theta_1 & \theta_2 & \theta_3 \\
\theta_2 & \theta_3 & \theta_4
\end{bmatrix}
\begin{bmatrix}
\theta_1 \\
\theta_2 \\
\theta_3
\end{bmatrix}
$$

where $x$, $y$ and $z$ are state variables and $\theta_1$, $\theta_2$ and $\theta_3$ are system parameters. The Rössler system was synchronized with a response system specified by four parameters. In the form of (2), it can be written as

$$
\begin{bmatrix}
\dot{u} \\
\dot{v} \\
\dot{w}
\end{bmatrix} =
\begin{bmatrix}
-\theta_1 & -\theta_2 & -\theta_3 & -\theta_4 \\
\theta_1 & \theta_2 & \theta_3 & \theta_4 \\
\theta_2 & \theta_3 & \theta_4 & \theta_4
\end{bmatrix}
\begin{bmatrix}
\alpha_1 \\
\alpha_2 \\
\alpha_3 \\
\alpha_4
\end{bmatrix}
$$

where $u$, $v$ and $w$ are state variables and $\alpha_1$, $\alpha_2$, $\alpha_3$ and $\alpha_4$ are the parameters. Again, the rank of the $F(x)$ matrix is 3, while the number of parameters is 4.

For this simulation, the commonly used parameter values for the Rössler system were used: $[\theta_1, \theta_2, \theta_3, \theta_4]^T = [0.1, 0.1, 14, 0]^T$. The initial states of the drive system and the controlled system were arbitrarily set to $[8, 9, 10]^T$ and $[3, 4, 5]^T$, respectively. The parameters of the response system had zero initial condition. The $L$ and $K$ parameters were set to $10I$ and $0.1I$ (for $I$ being the identity matrix with appropriate dimensions).

The improved convergence performance of the proposed method over Chen’s is clearly visible in Figure 5, while parameter convergence is faster for higher values of $r$. Figure 6 shows the results of parameter identification for $\alpha_1$, which, together with $\alpha_4$, specifies the evolution of the state variable $v$. During the last 20 seconds of the experiment, the coefficient of variation of the root mean square error is of $2.96 \times 10^{-2}$ for Chen’s method, while for our method it is of $6.11 \times 10^{-5}$ ($r = 3$) and $3.32 \times 10^{-7}$ ($r = 5$).
Figure 5: Rössler system: graph of parameter identification results for $\alpha_4$. Solid line: Classical Chen, dotted line: Extended Chen ($r = 3$, dash-dotted line: Extended Chen ($r = 5$)).

Figure 6: Rössler system: plot of parameter identification results for $\alpha_3$. Top plot: Classical Chen, middle plot: Extended Chen ($r = 3$), bottom plot: Extended Chen ($r = 5$).
method cannot identify this parameter correctly, with its value oscillating around the true value of $\theta_3$. On the other hand, our method allows for stable parameter identification. Again, convergence is faster for greater values of $r$. Finally, Figure 7 shows the synchronization error, as measured by the Lyapunov function \( L(t) \). Both methods drive the synchronization error to zero, while our method shows a significantly faster convergence. Also, the magnitude of the error is reduced by comparison to Chen’s method.

5 Conclusions

Building upon previous work in simultaneous parameters identification and synchronization of dynamical systems, this paper proposes an improved method that addresses limitations of the previously published Chen’s method \cite{3}. The proposed method is capable of handling arbitrarily large parameter space dimensions. Convergence proof of the method is provided, using the Lyapunov’s second method. Numerical results illustrate the proposed method, comparing it to Chen’s and showing better performance in terms of both faster and less noisy parameter identification.

References

[1] V. S. Afraimovich, N. N. Verichev, and M. I. Rabinovich. Stochastic synchronization of oscillation in dissipative systems. *Radiophysics and
Quantum Electronics, 29(9):795–803, 1986.

[2] H.N. Agiza and M.T. Yassen. Synchronization of rossler and chen chaotic dynamical systems using active control. *Physics Letters A*, 278(4):191–197, 2001.

[3] Shihua Chen and Jinhu Lü. Parameters identification and synchronization of chaotic systems based upon adaptive control. *Physics Letters A*, 299:353–358, 2002.

[4] C. Grebogi, E. Ott, and J.A. Yorke. Chaos, strange attractors, and fractal basin boundaries in nonlinear dynamics. *Science*, 238(4827):632–638, 1987.

[5] L. Kocarev and U. Parlitz. Generalized synchronization, predictability, and equivalence of unidirectionally coupled dynamical systems. *Physical Review Letters*, 76(11):1816–1819, March 1996.

[6] E.N. Lorenz. Deterministic Nonperiodic Flow. *Atmos. Sci.*, 1963.

[7] Louis M. Pecora and Thomas L. Carroll. Synchronization in chaotic systems. *Physical Review Letters*, 64(8):821–825, 1990.

[8] Louis M. Pecora, Thomas L. Carroll, Gregg A. Johnson, and Douglas J. Mar. Fundamentals of synchronization in chaotic systems, concepts, and applications. *Chaos*, 7(4):520–543, 1997.

[9] O.E. Rössler. An equation for continuous chaos. *Physics Letters A*, 57(5):397–398, 1976.

[10] Nikolai F. Rulkov, Mikhail M. Sushchik, Lev S. Tsimring, and Henry D. I. Abarbanel. Generalized synchronization of chaos in directionally coupled chaotic systems. *Physical Review E*, 51(2):980–994, February 1995.

[11] Shankar Sastry. *Nonlinear Systems: Analysis, Stability, and Control*. Springer, 1999.

[12] W. Tucker. The Lorenz attractor exists. *Comptes Rendus de l’Académie des Sciences-Series I-Mathematics*, 328(12):1197–1202, 1999.