Arctic Amplification of Anthropogenic Forcing: A Vector Autoregressive Analysis

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Abstract

Arctic sea ice extent (SIE) in September 2019 ranked second-to-lowest in history and is trending downward. The understanding of how internal variability amplifies the effects of external CO\textsubscript{2} forcing is still limited. We propose the VARCTIC, which is a Vector Autoregression (VAR) designed to capture and extrapolate Arctic feedback loops. VARs are dynamic simultaneous systems of equations, routinely estimated to predict and understand the interactions of multiple macroeconomic time series. Hence, the VARCTIC is a parsimonious compromise between full-blown climate models and purely statistical approaches that usually offer little explanation of the underlying mechanism. Our "business as usual" completely unconditional forecast has SIE hitting 0 in September by the 2060’s. Impulse response functions reveal that anthropogenic CO\textsubscript{2} emission shocks have a permanent effect on SIE – a property shared by no other shock. Further, we find Albedo- and Thickness-based feedbacks to be the main amplification channels through which CO\textsubscript{2} anomalies impact SIE in the short/medium run. Conditional forecast analyses reveal that the future path of SIE crucially depends on the evolution of CO\textsubscript{2} emissions, with outcomes ranging from recovering SIE to it reaching 0 in the 2050’s. Finally, Albedo and Thickness feedbacks are shown to play an important role in accelerating the speed at which predicted SIE is heading towards 0.

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1 Introduction

In 2019, the minimum extent of Arctic sea ice ranked second-to-lowest in history, with the lowest occurring in September 2012. A persistent retreat of SIE may further accelerate global warming and threaten the composition of the Arctic’s ecosystem (Screen and Simmonds (2010)). While depriving many people from their traditional livelihood, a retreating ice cover has already offered new shipping routes and oil exploration projects over recent years (Meier et al., 2014), increasing business activity in the region.

Motivation. The Coupled Model Intercomparison Project (CMIP), assembles estimates of long-run projections of Arctic sea ice from a large number of climate models. These models try to reproduce the geophysical dynamics and interrelations among various variables, influencing the evolution of global climate. With CMIP now being in its 6th phase (CMIP6), climate models provide more realistic forecasts of the Arctic’s sea ice cover compared to its predecessor CMIP5 (see Stroeve et al. (2012), Notz et al. (2020)). The majority of contributors to CMIP6 see the Arctic’s sea ice to retreat below the $1 \times 10^6 \text{ km}^2$ mark before the year 2050. Despite following the hitherto accepted physical laws of our climate, the chaotic nature of the latter, i.e. the still obscure interplay of various climate variables, imposes a major burden on climate models. In trying to replicate the observed behavior of our climate, each model is re-run several times with differing initial conditions resulting in a wide range of projections of key climate variables (Notz et al., 2020). In addition to this tuning, these simulations require large amounts of input data and a coupling of various sub-models (Taylor et al., 2012).

The above raises the question whether an approach that is statistical and yet multivariate can paint a more conciliating picture. This means estimating a statistical system that depicts the interaction of key variables describing the state of the Arctic. In such a setup, the downward SIE path will be an implication of a complete dynamic system based on the observed climate record. We can provide a formal statistical assessment of different hypotheses about the historical path of SIE and their implications for the future. We can quantify a specific effect – and the uncertainty surrounding its estimation – without resorting to use a climate model.

Feedback Loops. Our analysis focuses on the interaction between anthropogenic carbon dioxide ($\text{CO}_2$) forcing and internal variability. The former is already widely suspected to be the main driver behind long-run SIE evolution (see Meier et al. (2014), Notz (2017)). Internal variability consists of feedback loops that are well documented in the literature (see Parkinson and Comiso (2013), Winton (2013), Stuecker et al. (2018), McGraw and Barnes (2020)) and are crucial for further enhancing the predictability of the Arctic’s sea ice cover (Wang et al. (2016), Notz et al. (2020)). It is clear that only an approach that considers the interaction of many variables in a flexible way – and thus numerous potential sources for feedback loops – has a chance to depict a reliable statistical portrait of the Arctic. The still high variation in CMIP6 Arctic sea ice projections (see Notz et al. (2020)) suggests that there is widespread un-
certainty around the question to what extent internal variability can amplify external forcing. In order to quantify this, we present a methodology from economics to achieve just that.

**The VARctic.** Our analysis focuses on the evolution of the long-term trajectory of SIE and the interconnected forces behind it. The modeling approach we propose achieves a desirable balance between purely statistical and theoretical/structural approaches. In many fields statistical approaches have a much better forecasting record than theory-based models. An obvious drawback is that the successful model provides little to no explanation of the underlying mechanism.\(^1\)

A Vector Autoregression (VAR) lives in a useful middle ground. It is a statistical model, but yet generates forecasts by iterating a complete system of difference equations in multiple endogenous variables. These interactions can be analyzed and provide an explanation for the resulting forecasts. When estimated with Bayesian techniques, VARs are known to provide competitive forecasts — very often as good as black-box models (Giannone et al. (2015)).\(^2\) In the light of all these considerations, we propose the VAR for the Arctic (VARCTIC) statistical approach that can (i) generate long-run forecasts, (ii) explain them as the result of feedback loops and external forcing (iii) allows us to analyze how the Arctic responds to exogenous impulses/anomalies.

**Preview of Results.** Our "business as usual" forecast has SIE reaching 0 in September of the 2060s and predicts SIE to be below one million \(km^2\) by the mid 2050s. By studying the impulse response functions of a Bayesian VAR, we report that \(CO_2\) shocks have the unique property of an everlasting impact on SIE. Additionally, we document that the corresponding responses of Albedo, Air Temperature and Thickness largely amplify the middle-run impact of \(CO_2\) anomalies on SIE. Further, we use conditional forecast analysis to evaluate the long-run effect of the systematic \(CO_2\) increase. We consider three different \(CO_2\) emission scenarios and show that abiding by the Paris Accord could eventually bring back SIE to 2010s levels. The two other standard \(CO_2\) paths lead SIE to 0 in the 2050s or the 2070s. We find that internal variability as characterized by Albedo and Thickness feedbacks, while not the original source of the decay, amplify external forcing greatly. In the worst-case scenario for the \(CO_2\) path, canceling these forces starting from 2020 would postpone reaching \(1 \times 10^6 \ km^2\) by a decade.

**Roadmap.** We first discuss the data and its transformation in section 2. Secondly, we discuss the VAR model, its identification and Bayesian estimation in section 3. Section 4 contains the empirical results which comprise of (i) a long-run forecast of SIE, (ii) impulse response functions of the VAR, (iii) exploring the transmission mechanism (feedback loops) and (iv) conditional forecasting analysis.

\(^1\)This is also a well-known concern in Machine Learning, which generated an ongoing interpretability literature that aims at opening the so-called black box.

\(^2\)Via informative priors, Bayesian shrinkage can help in reducing the variance of the densely parameterized VAR estimates – the same way LASSO or Ridge regularization helps in Machine Learning.
2 Data

Our data set comprises eighteen time series, proxying the Arctic’s climate system, and accounting for potential feedback loops among different components. The sample covers monthly observations from 1980 through 2018. Even if the accuracy of estimates of various variables has improved over the last decades, uncertainties and measurement errors still remain. In particular, providing data for long-term analysis often requires the merging of data sources of different quality and reliability (Meier et al., 2014). Nevertheless, our data is derived from well accepted sources (see Stroeve and Notz (2018)), such as the National Snow & Ice Data Center’s (NSIDC) Sea Ice Index series (Fetterer et al. (2017)), which proxies SIE, NASA’s Modern Era Retrospective analysis for Research and Applications (MERRA-2) for atmospheric variables (Gelaro et al., 2017), or the Pan-Arctic Ice Ocean Modeling and Assimilation System (PIOMAS) (Zhang and Rothrock, 2003).

We combine 8 variables, which importance has been highlighted by the existing literature (Meier et al. (2014)), into VARCTIC 8, our benchmark specification. Fortunately, variables can easily be added/removed from a VAR and Bayesian shrinkage ensures that a larger model will not overfit – the latter aspect is further explained in section 3.5. Therefore, we consider in the appendix a VARCTIC 18 which includes an additional 10 series from the reanalysis product MERRA2 (Gelaro et al. (2017)) as a robustness check. To summarize compactly, the two specifications considered in this paper are:

I. **VARCTIC 8**: CO₂, TotCloudCover, PrecipCMAP, AirTemp, SeaSurfTemp, SIE, Thickness, Albedo;

II. **VARCTIC 18**: SWGNT, SWTNT, CO₂, LWGNT, TotCloudCover, TAUTOT, PrecipCMAP, TS, AirTemp, SeaSurfTemp, LGAB, LWTUP, LGEM, SIE, Age, Thickness, EMIS, Albedo.

A comprehensive overview of all variables (including those of VARCTIC 18), their acronyms and links to data providers can be found in the appendix in Table 1. The justification for the subset of these variables included in VARCTIC 8 and the extensions of VARCTIC 18 will be discussed extensively in section 3.3.

The raw data is highly seasonal as displayed in Figure 15. However, the feedback loops we wish to estimate and extrapolate reside in the (stochastic) trend components and short-run anomalies, which is a limited part of the SIE’s variance. That is, we wish our VAR to explain long-run fluctuations and short-run variability rather than seasonality. Hence, we proceed to transform the data so that the resulting VARCTIC is fitted on deviations from seasonal means. For our benchmark analysis, we use a simple and transparent transformation: we de-seasonalized our data by regressing a particular variable \( y^{raw} \) on a set of monthly dummies.
That is, for each variable we run the regression

\[ y_{t}^{raw} = \sum_{m=1}^{12} \alpha_{m} D_{m} + \text{residual}, \]

and \( y_{t} \) is defined as \( y_{t} \equiv y_{t}^{raw} - \sum_{m=1}^{12} \hat{\alpha}_{m} D_{m} \). \( D_{m} \) is an indicator that is 1 if the date \( t \) is in month \( m \) and 0 otherwise. The estimates of \( \alpha_{m} \)'s, \( \hat{\alpha}_{m} \)'s, are obtained by ordinary least squares. This is exactly equivalent to de-meaning each data series month by month and is a more flexible approach to modeling seasonality than using Fourier series.\(^3\) Finally, we keep our filtered data \( y \) in levels. We do not want to employ first differences or growth rate transformations to make the data stationary. Such an action would suppress long-run relationships which are an important object of interest. Figure 1 shows the data after being filtered with monthly dummies.\(^4\)

Figure 1: Deseasonalized Series: 8 Variables

Pre-processing the data can influence results. Moreover, Diebold and Rudebusch (2019) and Meier et al. (2014) document seasonal variability in SIE trends. As a natural robustness check, we also consider a very different approach to eliminate seasonality. In appendix A.6, we reproduce our results with a data set of stochastically de-seasonalized variables obtained from the approach of structural time series (Harvey (1990) and Harvey and Koopman (2014)). In short, this extension allows for seasonality to evolve (slowly) over time, which could be a feature of some Arctic time series.

\(^3\)Of course, if we were using higher-frequency data – like daily observations, then the Fourier approach would be much more parsimonious and potentially preferable (Hyndman, 2010). The dummies approach to taking out seasonality only requires 12 coefficients with monthly but 365 with hypothetical daily data.

\(^4\)CO\(_2\) was not de-seasonalized.
3 The VARCTIC

Reenforcing feedback loops have been the subject of countless climate studies. However, recently proposed statistical approaches to model them – as advocated in McGraw and Barnes (2020) and McGraw and Barnes (2018) – are yet incomplete in their quest to fully unlock the potential of macroeconometrics. In this section, we review the VAR: the model; its identification; its Bayesian estimation. Furthermore, we discuss the construction of the long-forecasts and impulse response functions as tools to understand the VARCTICs’ results.

3.1 Vector Autoregressions and Climate

Vector Autoregressions are dynamic simultaneous systems of equations. They can characterize a linear dynamic system in discrete time. The methodology was introduced to macroeconomics by Sims (1980) and is now so widely used that it almost became a field of its own (see Kilian and Lütkepohl (2017)). It is a multivariate model in the sense that $\mathbf{y}_t$ in

\[ A \mathbf{y}_t = \Psi_0 + \sum_{p=1}^{P} \Psi_p \mathbf{y}_{t-p} + \mathbf{\epsilon}_t, \]

is an $M$ by 1 vector. This means the dynamic system incorporates $M$ variables. Each of these variables is predicted by its own lags and lags of the $M - 1$ remaining variables. The matrix $A$ characterizes how the $M$ different variables interact contemporaneously. Finally, the disturbances are mutually uncorrelated disturbances with mean zero:

\[ \mathbf{\epsilon}_t = [\epsilon_{1,t}, \ldots, \epsilon_{M,t}] \sim N(0, \mathbf{I}_M). \]

Equation (1) is the so-called structural form of the VAR which cannot be estimated because $A$ is not identified by the data. An estimable version is the reduced-form VAR

\[ \mathbf{y}_t = \mathbf{c} + \sum_{p=1}^{P} \Phi_p \mathbf{y}_{t-p} + \mathbf{u}_t, \]

where $\mathbf{c} = A^{-1}\Psi_0$, $\Phi_p = A^{-1}\Psi_p$ and $\mathbf{u}_t$ are now regression residuals

\[ \mathbf{u}_t = [u_{1,t}, \ldots, u_{M,t}] \sim N(0, \Sigma_u) \]

with $\Sigma_u = A^{-1}'A^{-1}$ by construction. While standard, obtaining an estimate of $A$ by decomposing $\Sigma_u$ is more complicated than running a simple regression is addressed on its own in section 3.3.

The methodology has many advantages over simple autoregressive distributed lags (ARDL) regression that have gained some popularity in the econometric and climate literature. In test-
ing for the importance of blocks of lags, McGraw and Barnes (2018) argue in favor of doing Granger causality tests rather than running ARDL regressions that exclude lags of the dependent variable. In essence, this is a well-known omitted variable bias that led eventually to the adoption of VARs by the macroeconometric profession. For instance, if the true data generating process is a VAR for each of the M equations, excluding lags of \( y_{m,t} \) in equation \( m \) will lead to spurious results. The advantages the authors described for Granger causality tests are thus de facto included in a VAR analysis.\(^5\) Their argument for inclusion of lags of the dependent variable can be interpreted as one for completeness of the modeled dynamic system, as guaranteed by an adequately specified VAR.

### 3.2 Obtaining Long-Run Forecasts from a VAR

The symmetry of the VAR allows for it to generate forecasts by simply iterating the model.\(^6\) Assuming the chosen variables to characterize the system completely, we can forecast its future state by iterating a particular mapping. To do so, we use a representation that exploits the fact that any VAR\((P)\) (that is, with \( P \) lags) can be rewritten as a VAR\((1)\) using the so-called companion matrix.\(^7\) Thus, obtaining forecasts amounts to iterate

\[
\hat{Y}_{t+1} = F(\hat{Y}_t) = \kappa + \Phi \hat{Y}_t, \quad \text{to obtain} \quad \hat{Y}_{t+h} = f^h(Y_t). \tag{3}
\]

This equation provides forecasts of all variables, \( h \) periods from time \( t \). An obvious \( t \) to consider is \( T \), the end of the sample. The fact that we can obtain predictions by simply iterating the system, is of particular interest for generating scenarios for the Arctic. First, the prediction will rely on an explainable mechanism – potentially mixing external forcing and internal feedback loops – rather than a purely statistical relationship. Second, our forecast does not rely in any way on external data or forecasts made exogenously by some other entity which would rely on assumptions implicitly incompatible with ours. Nevertheless, in some cases, it may be desirable to mix some external forecasts/scenarios of certain variables (like \( \text{CO}_2 \)) that may be less successfully characterized by the VAR and we do just that in section 4.4.

The canonical macroeconomic VAR analysis seeks to explain business cycle fluctuations (expansions, recessions). Thus, many applications focus on modeling growth rates or deviations from trend rather than levels. In such setups, information about the deterministic long-run component has been suppressed, which implies that the VAR prediction (3) usually converges to a constant for each variable as \( h \to \infty \). This behavior is in line with standard macroeconomic theory: an equilibrium path implies market forces balancing each other until

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\(^5\)This connection will be further discussed in sections 3.4 and 4.3.

\(^6\)Further, it does not rely on the matrix \( A \).

\(^7\)In short, any VAR\((P)\) in \( M \) variables can be rewritten as a VAR\((1)\) in \( M \times P \) variables so the theoretical analysis can be carried out with the less burdensome VAR\((1)\) (Kilian and Lütkepohl (2017)). \( Y_t \) are stacked \( y_{t-p} \)'s for \( p = 1, \ldots, P \). Complete lay out of \( \Phi \) in equation (3) can be found in the appendix in equation A.3.
a steady state is reached. This paper’s application has quite different properties: it is clear that
the Arctic ecosystem is on a diverging path. Hence, for the VARCTIC to capture that salient
feature of the data, we expect the VAR to have an explosive root because of the joint action of
external forcing and internal variability. Fortunately, we can still iterate forward a diverging
system to obtain a prediction. However, that prediction will diverge rather than converge, and
will do so until a physical limit of 0 is reached.

3.3 Identification

While conditional and unconditional forecasting are important byproducts of our VARCTIC,
another important objective of our analysis is to understand the underlying mechanism from
a statistical standpoint. For instance, we will later show that anthropogenic CO\textsubscript{2} is a key
driver of the long-run forecast (cutting emissions dramatically would prevent SIE from going
to 0). This important result rests solely on the reduced-form VAR. However, to uncover and
interpret the mechanism that amplifies CO\textsubscript{2}’s effect on SIE, we need an identification scheme
for instantaneous relationships.

Fundamentally, the typical time series identification problem originates from simultaneity
in the data. That is, multivariate time series data can tell us whether

\[ X_{t-1} \rightarrow Y_t \quad \text{or} \quad Y_{t-1} \rightarrow X_t \]

is more plausible. This is predictive causality in the sense of Granger (1969). However, the
data by itself cannot distinguish

\[ X_t \rightarrow Y_t \quad \text{from} \quad X_t \leftarrow Y_t. \]

This is a simple example of the simultaneity problem: one correlation between \( X_t \) and \( Y_t \) can be
generated by two different causal structures. Fortunately, the vast VAR literature has provided
many tools to address the identification problem.

If one seeks to do structural analysis with the VAR, the above problem boils down to the
need for \( A \) in equation (1)). Yet, the data only procures us with the variance-covariance matrix
of the residuals \( \hat{\Sigma} \). The identification problem emerges from the fact that \( A \) is not the only ma-
trix that can satisfy \( \hat{\Sigma}_u = A^{-1} A^{-1} \). For instance, for any orthogonal rotation matrix \( H \) (which
has the property of \( H = H^{-1} \)) we can have \( \bar{A} = HA \) and thus \( \hat{\Sigma}_u = \bar{A}^{-1} \bar{A}^{-1} \) is also satisfied by
construction. Mercifully, there exist many ways to pin down a single \( A \) matrix without having
to delve into too much theory, which is partially responsible for the popularity of VARs among
applied economists.\footnote{Much more on this can be found in Kilian and Lütkepohl (2017)}

Furthermore, if the cross-equation correlation of residuals (off-diagonal
elements of \( \hat{\Sigma}_u \)) is small, then the impact on results of the chosen scheme will be rather limited.
The strategy we opt for is the traditional Cholesky decomposition of \( \hat{\Sigma}_{ii'} \), which implies a causal ordering of the data. It implies that residuals of a variable at position \( i \) are only constituted of structural shocks \( \epsilon_i \) (the unpredictable and uncorrelated disturbances of equation (1)) of variables situated before it. Numerous more sophisticated alternatives have been proposed over the years (Kilian and Lütkepohl (2017)). The prime motivation for this is the occurrence of "puzzles" (IRFs with counter-intuitive signs) especially when VARs are used to estimate the effect of monetary policy on inflation. As figure 3 will show later, there are no such puzzles in our application. Even better, we will find that by shuffling the ordering around (within reasonable bounds), our results do not change in any significant way (see section A.4).

**THE ORDERING.** When using Cholesky decomposition to identify a VAR, the ordering of the variables may influence the effect and transmission of shocks. Only if variable \( i \) is ordered below variable \( j \), will a shock to \( j \) affect variable \( i \) contemporaneously. Otherwise, variable \( i \) will experience the effect of that very shock only with a lag.

It is a relatively well established fact that the melting SIE and the responsible Arctic environment are both results of exogenous (to other Arctic variable) human action (Dai et al. (2019), Notz and Stroeve (2016)). We view the Arctic system as being subject to feedback loops that may amplify the effect of exogenous shocks way beyond their original impact. However, the original stimulus is very likely to be anthropogenic, given that without the unprecedented increase in \( \text{CO}_2 \) emissions and subsequent rise in global temperature, none of these mechanisms would have been so evident in effect (Amstrup et al. (2010), Melillo et al. (2014)). In 2018, carbon dioxide accounted for 81.3% of greenhouse gases emitted in the United States. These greenhouse gases absorb infrared radiation, preventing the inherent heat content from being transmitted into outer space, triggering a response of global temperature (EPA, 2020) and initiating a cycle of knock-on effects on other factors of internal climate variability. In our benchmark VARCTIC, \( \text{CO}_2 \) is the only representative of the group of external forcing variables. Therefore, we order \( \text{CO}_2 \) first, meaning that shocks to any of the other variables may only impact \( \text{CO}_2 \) with a one-period lag.\(^9\)

In the spirit of many medium to large BVAR applications to macroeconomic data (Bernanke et al. (2005), Christiano et al. (1999), Stock and Watson (2005) and Bańbura et al. (2010)), we classify the variables, describing the internal climate variability, into fast-moving and slow-moving ones. Cloud Cover, Precipitation and Air Temperature are classified as fast-moving. Absorbing short- and longwave radiation, clouds have a significant impact on the earth's energy balance and thus its overall heat content (Carslaw et al., 2002). But clouds, or the accumulation of condensed water and dust particles, eventually carry precipitation with not unambiguously determined effects on SIE (Parkinson and Comiso (2013), Meier et al. (2014)). We order both variables, Cloud Cover and Precipitation, before the temperature variables, i.e. Air Tempera-

\(^9\) Meier et al. (2014) give an in-depth description of the various internal factors, their mutual interaction and their response to carbon dioxide.
ture (AT) and Sea Surface Temperature (SST). Besides AT, also SST, especially warmer water from the Atlantic ocean, contributed to shaping the historically unprecedented decline of SIE over the last four decades (Meier et al., 2014). Here we follow Parkinson and Comiso (2013) who state that besides the cooling effects of a melting ice cover, SST is highly influenced by currents and winds transferring warmer energy from lower to higher latitudes. We therefore place SST at the boundary of fast and slow moving variables.

The last block of variables comprises, SIE, Thickness, and Albedo. The measure of extent is not the only feature of Arctic sea ice. Thickness is an underestimated determinant of how SIE reacts to both external forcing and internal variability (Meier et al. (2014), Parkinson and Comiso (2013)). Thicker layers make the ice more resilient and increase Albedo, while thin ice is more easily advected by winds, making SIE more sensitive to extreme events (Meier et al., 2014). We order Thickness – and Albedo – after SIE because we hypothesize that the effect of shocks of the former can only influence the latter with a certain delay. For instance, shocks to Thickness via increased water precipitation or strong winds will immediately reduce Thickness but SIE only with a certain lag. Last but not least, we regard Albedo, i.e. the proportion of incoming shortwave radiation that is not absorbed by the surface, as being driven contemporaneously by all other factors.

**ON EXCLUDED POTENTIAL MECHANISMS.** We consider VARCTIC 18 in part as a way to confirm that key phenomena are already accounted for in VARCTIC 8. For example, studies have emphasized the role of incoming long- and shortwave radiation and their interactions with SIE and Thickness (see Burt et al. (2016), Dai et al. (2019)). The impact of downwelling longwave radiation (DLW) on SIE is not direct, but transmitted via DLW’s influence on AT. Here, the thickness of sea ice is crucial, as thinner sea ice is more susceptible to DLW than thicker layers (Park et al., 2015). As we will show later (like in figure 12), accounting for both short- and longwave radiation in VARCTIC 18, the forecast of an ice-free Arctic deviates only marginally from the ice-free date projected by the VARCTIC 8. This result suggests that short- and longwave radiation do not have a direct impact on SIE, but rather affect the evolution of the Arctic’s sea ice cover via other variables (e.g. AT and Thickness), which VARCTIC 8 already accounts for.

In a similar line of thought, upper-ocean-heat content may also contribute to the evolution of SIE. Examining the Barents Sea, Årthun et al. (2012) attribute the increased sea ice loss to an elevated influx of ocean heat. Studies have, however, shown that anomalies in the temperature of the upper-ocean layers and anomalies in SST do coincide during winter and spring especially over the Barents Sea (Park et al., 2015). We regard this as evidence for making an extension of both VARCTIC models dispensable.
3.4 Impulse Response Functions

Since Sims (1980), the dominant approach for studying the properties of the VAR around its deterministic path has been impulse response functions (IRFs) to structural shocks. Thanks to the orthogonalization strategy discussed in 3.3, we converted plain regression residuals into orthogonal shocks. The dynamic effect of these specific disturbances (the impulse) can be analyzed as that of a randomly assigned treatment. IRFs take this structural meaning as a response to a fundamental shock when we rather look at uncorrelated impulses from \( \epsilon_{m,t} \). Uncorrelatedness of the latter implies the "keeping everything else constant" interpretation – hence, a causal meaning for IRFs – is guaranteed by construction.

As we just argued, a natural way of understanding the VAR is to look at its response to plausibly exogenous impulses called shocks. It is natural to wonder what is the meaning of such shocks in a physical system. Mechanically, these shocks are the difference between the realized state of a variable and its predicted value as per the previous state of the dynamic system. These unpredictable anomalies, which emerge from outside a well-specified VARCTIC, are the key to understand the dynamic properties of the model. A now obvious example of a shock will be that of CO\(_2\) emissions reduction in 2020: it is inevitable that the observed emissions will be lower than what was predicted by the endogenous system since the latter excludes "pandemics". Any model that is partially incomplete will be subject to external shocks. The study of such exogenous impulses may be alien-sounding, especially when contrasted with the deterministic environment of a climate model. Nevertheless, understanding the properties of a climate model by conditioning on a particular RCP scenario is equivalent to conditioning on a series of shocks. Hence, one can understand the VARCTIC and its IRFs as expanding the number of potentially exogenous sources of forcing. Of course, our later focus on CO\(_2\) shocks is expressively motivated by the fact that the latter is a well-accepted source of exogenous forcing in climate systems.

The impulse response function of a variable \( m \) to a one standard deviation shock of \( \epsilon_{m,t} \) is defined as

\[
IRF(\tilde{m} \rightarrow m, h) = E(y_{m,t} | \tilde{y}_t, \tilde{m} = \sigma_{\epsilon_m}) - E(y_{m,t} | \tilde{y}_t, \epsilon_{t, \tilde{m}} = 0).
\]

10Mathematically, we took a linear combination of the VAR residuals (an unpredictable change in a variable of interest, \( u_t \)) such that \( u_t = A\epsilon_t \).

11Of course, one could look at how the system respond to an impulse from a residual \( u_{m,t} \), but the interpretation will be rather weak because those are cross-correlated across equations.

12For instance, air temperature (AT) is largely determined by the previous values of variables in the system, but not completely.

13For macroeconomists, the idea of a single shock driving an otherwise completely endogenous system echoes back to the foundational work of Kydland and Prescott (1982) in which productivity shocks are considered as the sole driver of all aggregate macroeconomic fluctuations. That is, productivity is not predictable by the cyclical system and driven by exogenous forces. Hence, in that setup, one would need an external productivity growth scenario to construct long-run forecasts the same way RCPs are needed in climate science. Lastly, it is worth mentioning that the VAR paradigm moves away from the original Kydland and Prescott (1982) setup by (among many other things) allowing for more than one shock – a necessary feature for economic models to be reconciled with reality.
Thus, it is the expected difference between an Arctic system that responded to and propagated an unexpected CO$_2$ increase and the same system where no such increase occurred. In a linear VAR with one lag ($P = 1$), the IRF of all variables can easily be computed from the original estimates using the formula

$$\text{IRF}(\tilde{m} \to m, h) = \Psi^h A^{-1} e_{\tilde{m}}$$

where $e_{\tilde{m}}$ is vector with $\sigma_{\epsilon_{\tilde{m}}}$ in position $\tilde{m}$ and zero elsewhere. This just means that we are looking at the individual effect of $\epsilon_{\tilde{m}}$ while all other structural disturbances are shut down.$^{14}$

The latter discussion focused on analyzing how our dynamic system responds to an external/unpredictable impulse, which is a standard way of interpreting VAR systems. Of course, we are also interested in the "systematic" part of the VAR that is responsible for the propagation of shocks when they do occur – the IRF transmission mechanism. In section 4.3, we focus our attention on CO$_2$ and Air Temperature shocks and quantify the amplification effect of different channels.

### 3.5 Bayesian Estimation

Over the years, many extensions to Sims (1980) original work have been proposed and some have specific advantages that make them more adequate for our application. Precisely, we use a Bayesian VAR in the tradition of Litterman (1980). There are two crucial advantages of doing so. First, Bayesian inference does not change whether the VAR system is stationary or not (Fanchon and Wendel 1992). We are effectively modeling variables in levels and expecting at least one explosive root. Frequentist inference is notoriously complicated in such setups (Choi 2015) and even standard approaches for non-stationary data have well-known robustness problems (Elliott 1998). From a practical point of view, using non-stationary data (as we think is necessary here) means that standard test statistics (like popular Granger Causality tests) will be undermined by faulty standard errors, potentially leading to erroneous conclusions.

Second, for us to consider a system of many variables estimated with a relatively small number of observations, Bayesian shrinkage can be beneficial to out-of-sample forecasting performance and help in reducing estimation uncertainty (like those of IRFs). In fact, VARs are known to suffer from the curse of dimensionality as the number of parameters scales up very fast with the number of endogenous variables.$^{15}$ Via informative priors, Bayesian inference provides a natural way to impose soft/stochastic constraints (that is, constraints are not imposed to bind) and yet keep inference (Bàribura et al. 2010).$^{16}$ Furthermore, we are interested in transformations (forecasting paths, impulse response functions) of the parameters rather than the parameters themselves. Inference for such objects can easily and naturally be

$^{14}$In the case of a linear VAR with $P > 1$ lags, we must use the companion matrix form. The relevant formula (equation (A.4)) can be found in the discussion of appendix A.2.

$^{15}$Such a situation motivates McGraw and Barnes (2020) use of the LASSO.

$^{16}$For instance, doing a VAR with LASSO would induce some form of shrinkage but inference is far from easy.
obtained by transforming draws from the posterior distribution. All these procedures are well established in the macroeconometrics community and packages are available in most statistical programming software (Dieppe et al. (2016)).

Bayesian inference implies the use of priors which degree of informativeness is usually determined by the user. To be as agnostic as possible, we use the technique of Giannone et al. (2015) to choose the tightness of priors as to optimally balance bias and variance in a data-driven way. The prior structure, however, must be chosen. We estimate our benchmark Bayesian VARCTIC with a standard Minnesota prior. In this simple framework, $\Sigma$, the variance-covariance matrix of the VAR residuals, is treated as known. Thus, the remaining parameters of the model reduce to the vectorized matrix $\beta = \text{vec} \left( [\Phi_1 \cdots \Phi_p \mathbf{c}]^\top \right)$ of dimension $(M^2p + M) \times 1$. The posterior distribution of $\beta$, $\pi(\beta | y)$, is obtained by the product of the likelihood function of the data $f(y | \beta)$, and the prior distribution of $\beta$, $\pi(\beta)$. Hence, by sampling from the posterior distribution

$$\pi(\beta | y) \propto f(y | \beta) \pi(\beta)$$

we can quantify both the uncertainty around $\beta$, but also more interesting transformations of it, such as IRFs and forecasts. The prior distribution for $\beta$ is the multivariate normal distribution $\pi(\beta) \sim N(\beta_0, \Sigma_0)$. The Minnesota prior is a specific structure for values of both $\beta_0$ and $\Sigma_0$. An extended discussion the prior, its motivation for time series data and details on the exact values of hyperparameters used can all be found in section A.3. Finally, we fix the number of lags in the VARCTIC 8 to $P = 12$ and to $P = 3$ in VARCTIC 18 respectively. The total number of posterior draws is 2000.

4 Results

A VAR contains many coefficients – there are $8 \times (8 \times 12 + 1) = 776$ in the baseline VARCTIC 8. Staring at them directly is unproductive and a single coefficient (or even a specific block) carries little meaning by itself. As it is common with VARs in macroeconomics, we rather study the properties of the VARCTIC by looking at its implied forecasts and its impulse response functions.

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17Setting priors’ tightness in such a way can be understood as analogous (at a philosophical level) to setting tuning parameters using cross-validation in Machine Learning.

18This choice is motivated by the fact that it facilitates the optimization of hyperparameters. As it turns out, optimizing tuning parameters has more impact on resulting IRFs and their respective credible regions than treating $\Sigma$ as unknown, when using for instance an Independent Normal Wishart (with Gibbs sampling).

19In the latter case, the credible region will naturally comprehend the uncertainty from the act of recursive forecasting itself, but also the fact that it relies on unknown parameters that must be estimated.

20As a reference, a Ridge regression would imply $\beta_0 = 0$ and $\Sigma_0$ being a diagonal matrix with identical diagonal elements.

21The same arithmetic gives a total of 990 parameters in the VARCTIC 18.
4.1 The "Business as Usual" Forecast

We report here the unconditional forecast of our main VAR. The VARCTIC 8 suggests SIE to hit the zero lower bound around 2060 (see Figure 2), whereas in the VARCTIC 18 specification, the Arctic would be ice-free about the same time (see Figure 12). The shaded area shows 90% of all the potential paths of the respective VARCTIC. That is, the VARCTIC 8 dates the Arctic to be totally ice-free for the first time somewhere between 2052 and 2073 with a probability of 90%. The VARCTIC 18 slightly extends that timeframe to the year 2079. For the two models, the median scenario has SIE being less than one $10^6$ km$^2$ by 2054 and 2060 respectively. The one $10^6$ km$^2$ is more likely an interesting quantity since the "regions north of Greenland/Canada will retain some sea ice in the future even though the Arctic can be considered as 'nearly sea ice free' at the end of summer." (Wang and Overland (2009)). The corresponding credible regions mark the period 2047-2065 for the VARCTIC 8 and 2047-2069 for the VARCTIC 18 respectively. These dates and time spans range in the close neighborhood of previous climate model simulations (see Jahn et al. (2016)). For both VARCTICs, less than 5% of the simulated paths hit 0 before 2050, making it an unlikely scenario according to our calculations. In essence, the two models suggest SIE melting at a rate that is slower than Diebold and Rudebusch (2019)'s results, but much faster than most CMIP5 models (Stroeve et al. (2012)) and in line with the latest CMIP6 calculations (Notz et al. (2020)).

4.2 Impulse Response Functions

Figure 3 shows the impulse response functions with the 90% credible region. Thus, the blue band reflects parameters’ uncertainty and contains 90% of the posterior draws from VARCTIC 8. We display the response of SIE to a positive shock of one standard deviation to any of the model’s $M$ variables. Hence, when the credible region contains the 0 line, it means that more than 5% of the posterior draws produce an IRF which sign is opposite to that of the posterior mean (the dark blue line). This implies that such an IRF does not describe a significant phenomenon, implying that the posterior probability of observing the opposite-signed effect is non-negligible.

The resulting impact of CO$_2$ anomalies on SIE is sizable and most importantly, durable. While the sign of the response is highly uncertain and weak for more than a year, CO$_2$ shocks emerge to have a permanent downward effect on SIE. The relevance of the CO$_2$/SIE relation is

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22 We include in the graph the in-sample deterministic component of the VAR (as discussed in Giannone et al. (2019), which is essentially a long-run forecast starting from 1980 (the same sort of which we are doing right now for the next decades) using the VAR estimates of 12 lags.

23 Diebold and Rudebusch (2019) stress the point that quadratic trends are likely to differ across months (especially summer vs non-summer months). We accommodate for that using a refinement of the stochastically de-seasonalized series in section A.6. An interesting but more sophisticated extension – that would however be beyond the scope of this paper – is to estimate a smooth-transition VARCTIC where dynamics are allowed to vary across seasons. In fact, many other such non-linearities/state-dependencies could be investigated and tested against our benchmark linear VARCTIC.
not a surprise (Notz and Stroeve (2016)). Moreover, this behavior is distinct from other shocks that rather have a significant short-run effect but no significant effect after more than a year. More precisely, the effect of CO₂ impulses takes almost a year to settle in (not significant for approximately 10 periods) but ends up having a permanent downward effect on trend SIE of approximately $-0.005 \times 10^6$ km² month after month. This mechanically implies that a one-off CO₂ deviation from its predicted value/trend leads to a cumulative impact that is ever increasing in absolute terms (as displayed later in Figure 4b). It is important to remember that this is the effect of an unexpected increase in CO₂ which is to be contrasted with the systematic effect that will be studied later. However, in the framework of this section – where CO₂ is allowed to endogenously respond to Arctic variables, this is as close as one can get to obtain an experimental/exogenous variation needed to evaluate a dynamic causal effect. $-0.005 \times 10^6$ is roughly 0.1% of the last deterministic trend value of SIE, which is about the size of the Great Salt Lake. CO₂ shocks, by construction of our linear VAR, have mean 0 and there are approximately as many positive and negative shocks in-sample. The linearity and symmetry of the VAR imply that these permanent effects are present for both upward and downward deviations from the deterministic trend.

Other shocks have important impacts that eventually vanish, which is the traditional IRF shape one would expect to see from a VAR on macroeconomic data. For instance, Air Temperature and Albedo IRFs clearly have the expected sign. However, they do not have the striking lasting impact of CO₂ perturbations. In other words, a one-time Albedo shock will not have a lasting effect on SIE as reported in Figure 2. This does not preclude Albedo to amplify other shocks as we will see in the next section.

By looking at the yearly autocorrelation of SIE, Notz and Marotzke (2012) note that an exceptionally low SIE is usually followed by a higher SIE the following year. Notz (2017) sur-
veys three main sources of negative feedback within a given year. The response of SIE to a SIE anomaly suggest indeed a small negative feedback – usually 6 months later. For instance, as Notz (2017) mentions, this could be the result of thinner ice that forms during winter being able to grow much faster than thicker ice (that itself did not melt in the summer). Nevertheless, this does not preclude Thickness shocks from having an expected and mechanical negative effect in the short run. Furthermore, as we will see later, the response of Thickness to CO$_2$ shocks will itself contribute to the accelerated decay of SIE in the middle run.

For a discussion of VARCTIC 18 results, see section A.5. We now turn to assess the validity of such an hypothesis by opening the black-box of the VAR transmission mechanism. We do so with IRF decompositions.

4.3 Amplification of CO$_2$ and Temperature Shocks by Feedback Loops

The melting of SIE is happening much faster than many other phenomena that are also believed to be set in motion by the steady increase of CO$_2$ emissions. Many recent papers (Notz and Marotzke (2012), Wang and Overland (2012), Serreze and Stroeve (2015), Notz (2017)) argue with theory/climate models or correlations that external CO$_2$ forcing is responsible for the long-run trajectory of SIE. Some of these findings led Notz and Stroeve (2016) to conclude that climate models severely underestimate the impact of CO$_2$ on SIE.
A rather consensual view is that the very nature of the Arctic system leads to the amplification of such external forcing shocks. For instance, the Albedo effect (less ice to reflect heat, more heat, less ice, repeat) has received a lot of attention in the literature. Another hypothesis is that of the thickness channel. As ice melts, older and thicker ice is replaced by newer and thinner ice. The latter is prone to melt faster when the next summer comes around. This thickness channel also has received some recent attention (Meier et al. (2014)). For instance, Parkinson and Comiso (2013) points out that the thinning of Arctic sea ice increases its sensitivity to abnormally adverse weather phenomena. However, things may not be that straightforward as summarized in Notz (2017). The sum of feedbacks at the yearly frequency is in fact likely to be negative. Univariate autocorrelation properties of SIE reveal an annual self-correction mechanism: a low September SIE in year \( t \) is usually followed by higher SIE the same month next year.\(^{24}\) Hence, an understanding – from observational data – on how the Arctic may amplify or not certain external forces is still pending. As Stroeve and Notz (2018) puts it:

\[ \text{It is difficult however, to robustly assess the contribution of internal variability to the observed loss, as this is only possible with climate models, which differ widely in their estimated magnitude of internal variability of the Arctic sea-ice cover.} \]

In general, the VAR can quantify the contribution of different variables in explaining how a dynamic system responds to an external impulse. The VARCTIC encompasses different amplification hypotheses can quantify which channels empirically matter and which ones do not. The only question left is how to convincingly extract that information from our model.

4.3.1 Methodology

A potential approach that has a long history in econometrics is the use of Granger Causality (GC) tests. They have been recently advocated for climate applications by McGraw and Barnes (2018). Nevertheless, those tests only carry very limited information that quite often fall short of answering questions of interest. First, the meaning of the test is not obvious when more than two variables are included and if the researcher is interested in multi-horizon impacts, as discussed in Dufour and Renault (1998). Second, even if for some reason, rejecting the null of a GC test is meaningful somehow, the block of reduced-form coefficients that we know to be of some statistical importance are very hard to interpret. For instance, when there are more than 2 variables, the significant coefficients (by themselves) reveal close to nothing about indirect effects. In sum, in the wake of a GC test rejection, we know some channel matters but we have little to no idea how it matters. In the light of all this, we choose another route that we believe could have wide applicability in empirical climate research, beyond the VARCTIC.

When we observe that SIE is negatively affected by CO\(_2\) shocks, that response can be composed of a direct effect and many complicated indirect effects. Understanding indirect effects

\(^{24}\)The negative autocorrelations are usually quite significant, so it is not a "regression to the mean" illusion.
in the dynamic setup of a VAR is much more intricate than in a static regression setting. This is so because IRFs – for horizons greater than one – are obtained by iterating predictions, which means $X$ can impact $Y$ through $Z$, but also through any of its lags.

We employ a strategy that has been used in macroeconomics to better understand the transmission mechanism in VARs. It consists, rather simply, of shutting down "channels" and plotting what the response to a shock would be, given that channel had been shut. It is the VAR equivalent of the partial (rather than total) derivative interpretation of ordinary least squares regression coefficients. To further clarify, we proceed with examples from the literature that followed Sims and Zha (2006)’s contribution. Bernanke et al. (1997) study the effect of oil price shocks on the US economy, assuming the monetary policy authority had not responded in the usual way it has. This helps understanding whether oil prices themselves or subsequent (and systematic) interest rate tightening is the real cause for a sequential decrease in economic activity. Bachmann and Sims (2012) studies how an unexpected fiscal stimulus leads to increased economic activity. They document that it is mainly through increasing "confidence" of economic actors that fiscal policy boosts GDP, especially in highly uncertain times like recessions. Running back to our Arctic concerns, we can deploy the same methodology to find and quantify the most important channels through which $CO_2$ and temperature shocks impacts SIE. In the absence of a consensus for the name of this procedure, we will refer to it as IRF Decomposition.

### 4.3.2 Amplification of $CO_2$ Shocks

For the VARCTIC 8, Figure 4 shows the responses of SIE to an unexpected increase in one standard-deviation of $CO_2$. The blue line pictures the case of the baseline VARCTIC 8 with 90% credible region. The remaining 6 lines show the response of SIE to the same shock, but shutting down key transmission channels. In terms of implementation, it consists of imposing hypothetical shocks to one of the other variables which off-sets their own response to a $CO_2$ shock.\(^{25}\)

Figure 4a reveals – without great surprise – the importance of Temperature (especially Air Temperature) in translating $CO_2$ anomalies into decreasing SIE. That is, we observe that shutting down these channels leads to a smaller absolute response which means that those variables can be considered as amplification channels. Given the atypical shape of the $CO_2$ IRF, the scale of Figure 4 makes less visible the action of channels that only alter the longer-run effect. Since those effects are permanently negative (at different levels), their cumulative effect will more clearly reveal their relative importance. Thus, Figure 4b displays the cumulative impact of selected (more important) channels. The two temperature channels are responsible for approximately one fourth of the cumulative effect of $CO_2$ on SIE after 3 years. More precisely, restricting temperature variables to not respond to a positive $CO_2$ shock, decreases

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\(^{25}\)See Bachmann and Sims (2012) for details.
(in absolute terms) the after-3-years impact from $-0.13 \times 10^6$ km$^2$ to $-0.1 \times 10^6$ km$^2$. Of course, it was expected that temperature should be a major conductor of such shocks. We also observe similar quantitative effects for both thickness and the albedo effect in isolation. Most strikingly, we find out that the conjunction of the Albedo and Thickness amplification channels is responsible for amplifying the effect of CO$_2$ shocks by a non-negligible 50%. In the case of thickness, this goes in line with the view that thinner ice has a harder time withstanding atmospheric forcings or abnormally warm ocean currents (Parkinson and Comiso (2013)), which can create a positive feedback loop. Furthermore, the amplification by Albedo also matches evidence reported in several studies (see Perovich and Polashenski (2012), Björk et al. (2013), Parkinson and Comiso (2013)) that use very different methodologies.

This IRF Decomposition exercise provides statistical evidence to suggest that (i) anthropogenic CO$_2$ anomalies are a key driver behind the trajectory of SIE and (ii) Arctic feedback loops (Albedo effect & the thinning of ice) eventually doubles CO$_2$ initial impact on SIE. These findings, obtained from a complete statistical model, broadly go in line with the emerging consensus that the Arctic evolution is driven by a combination of anthropogenic forcing variables and the inherent dynamics of the Arctic itself (Stroeve and Notz (2018)). This section focused on how and why SIE responds to CO$_2$ shocks. In section 4.4, we rather look at the effect of the systematic increase of CO$_2$ level.

### 4.3.3 Amplification of Air Temperature Shocks

Air Temperature (AT) shocks are movements in AT that are not predictable given the past state of the system and are orthogonal to other shocks in the system, most notably CO$_2$. In other words, we are looking at the effect of unexpected higher/lower AT that are uncorrelated with other shocks in the system. As we saw in Figure 3, such AT anomalies have a pronounced
short-run effect on trend SIE for about a year after the shocks. This means that unlike CO$_2$, the cumulative effect of AT disturbances stabilizes about 1.5 year after the event.

In Figure 5a, we clearly observe (again) an important role for the thinning of ice and the Albedo effect amplifying the response of SIE to AT shocks. In fact, we see in Figure 5b that without them, the long-run impact is the same as the instantaneous one. Thus, this is evidence to suggest that the AT shock’s long-run cumulative impact of $-0.24 \times 10^6$ km$^2$ is mostly a result of the action of feedback loops.

4.4 Forecasting SIE Conditional on CO$_2$ Emissions Scenarios

If CO$_2$’s trend is mostly or solely affected by factors outside of those considered in the VAR, the forecast of SIE can be improved by treating CO$_2$ forcing as exogenous and using an external forecast rather than the one internally generated by the VAR. Conditional forecasting can be achieved in VARs following the approach of Waggoner and Zha (1999). As we will see, this will markedly sharpen the bands around our forecasts, suggesting that a great amount of uncertainty is related to the future path of CO$_2$ emissions. It is also more common practice in climate science to provide conditional rather than purely unconditional forecasts (Stroeve et al. (2012), Stroeve and Notz (2018)).

The benefits of such an analysis are in fact twofold. The first, as we have seen, is obtaining potentially better forecasts. The second is that when a specific scenario corresponds to a policy choice, we can evaluate the effect of such policies. In the spirit of Sigmond et al. (2018) who constrain the levels of AT in their climate model, we will look at CO$_2$ emissions under three

\[\text{Footnote: For the sake of clarity, the VARCTIC considered in this section treats CO}_2\text{ as an exogenous variable for which the out-of-sample path is known. This is a natural approach given that long-run CO}_2\text{ increase is undoubtedly anthropogenic.}\]
different representative concentration pathways (RCP) and investigate their impact on the evolution of Arctic sea ice. RCPs are those trajectories of greenhouse gases, which were estimated by Integrated Assessment Models and selected by the Intergovernmental Panel on Climate Change (IPCC) for its Fifth Assessment Report as a basis for projecting the near- and long-term evolution of our climate. Those pathways are standard and have been used in a wide range studies to display specific models’ properties conditional on different CO$_2$ emissions scenarios. For instance, Stroeve and Notz (2018) considers the evolution of SIE conditional on those using a simple linear relationship between the two variables in levels and find that CO$_2$ emissions can be a decisive factor between having the Arctic ice-free in the next 50 years or not. Hence, it is of interest to see if the more complete VARCTIC 8 produces results in line with their simpler statistical analysis. Additionally, in our case, we will be able to decompose such projections and assess the impact of internal variability in section 4.5.

Figure 1 shows a steady increase in CO$_2$ emissions over the last three decades, but several RCPs paint different pictures for the trajectory of carbon emissions until the end of the century. Therefore, we now consider the evolution of SIE under three different representative concentration pathways: the RCP 2.6, the RCP 6 and the RCP 8.5. The RCP 2.6 represents a low-mitigation scenario, in which CO$_2$ emissions peak around mid-century (van Vuuren et al., 2011). Following this trajectory would be necessary to comply with the Paris Agreement (UNFCCC, 2015). The second scenario projects CO$_2$ emissions – measured in gigatones of carbon per year – to peak during mid-century and taper off thereafter. This is very much in line with levels of CO$_2$, projected by models in the absence of any climate-policies (van Vuuren et al., 2011). RCP 8.5 serves as the most pessimistic scenario. Figure 6a shows the different paths of CO$_2$ under the three different RCP scenarios, as well as the projected path following VARCTIC 8. Most interestingly, we find our completely endogenous and unconditional forecast of CO$_2$ to lay somewhere between the "very bad" RCP 8.5 scenario and the "business-as-usual" RCP 6 one.

Figure 6b shows the variation of Arctic SIE in the VARCTIC when conditioning the out-of-sample path of CO$_2$ on the three different RCP scenarios, as well as under the scenario of letting the model determine the future path of CO$_2$ endogenously. The pictured effect is dramatic. If emissions were reduced as to follow the RCP 2.6 scenario, whose CO$_2$ emissions are still at the higher boundary of what the Paris Agreement demands, the Arctic would be far from blue and even recover earlier losses by the end of the century. If emissions follow the more likely RCP 6, SIE would vanish later than projected by the VARCTIC 8, but still be completely gone during the 2070’s. In the worst case scenario, RCP 8.5, we obtain an ice-free September by the mid-2050’s. Interestingly, this result is very close to what Stroeve and Notz (2018) reported using a very different methodology (extrapolating a linear relationship). Their bivariate (SIE and CO$_2$) analysis suggests the Arctic summer months to be ice-free by 2050. However, in contrast, our results are much more optimistic than theirs in terms of SIE conditional on the (rather unlikely) RCP 2.6 scenario. Such analysis is not conditional on the identification scheme since it is based
Figure 6: VARCTIC Projections & Different RCPs

(a) Evolution of CO$_2$ emissions until the End of the Century under different Scenarios

(b) Evolution of SIE under different Scenarios of CO$_2$

solely on the reduced form. Overall, these results reinforce the view that anthropogenic CO$_2$ is the main driver behind the current melting of SIE as well as the main source of uncertainty around the future SIE path. Furthermore, the optimistic RCP 2.6 results suggest that internal variability by itself cannot lead to the complete melting of SIE, even when starting from today’s level. Overall, it is interesting to see that the VARCTIC yields similar conclusions about the importance of CO$_2$ to that of Dai et al. (2019) and Notz and Stroeve (2016). It is reassuring to see that climate models conclusions can be corroborated by a transparent approach that relies

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27 Important to note is the fact that the very last in-sample observations for CO$_2$ even range above the RCP 8.5 values, which generates the slight upward jump in case of the latter scenario.
solely and directly on the multivariate time series properties the observational record.

Given the very smooth look of deseasonalized CO\textsubscript{2} in Figure 1, one could worry that it merely acts as a proxy for an omitted linear trend. We view the use of trends as undesirable in our multivariate setup as it would undermine the capacity of the VARCTIC to be a "complete" model. Including a trend would make it rely on a unknown/unexplained latent force – which is at odds with the main goal of our modeling strategy.\textsuperscript{28} Nevertheless, for the sake of completeness, we estimated such models to find out that the inclusion of an exogenous time trend is in fact not preferred by the data according to the Deviance Information Criterion (DIC, a generalization of the well-known AIC). The VARCTIC 8 has a DIC of -6894.35 and the VARCTIC 8 with the exogenous trend has -6817.32. The smallest value being preferred, this justifies on a data-driven basis the exclusion of the trend. While seemingly technical of point, this carries great meaning: the specification of the VARCTIC 8 system, based solely on dynamic relationships of observable data, can generate/simulate the observed SIE downward path. Additionally, the role CO\textsubscript{2} as a central driving (downward) force is unmistakable in Figure 6b.

4.5 Amplification Effects in the Projection of SIE under different RCPs

The previous section documented the evolution of SIE conditional on several CO\textsubscript{2} trajectories, treating it as an exogenous driver. This section seeks to quantify the importance of internal variability when it comes to translating a RCP path into SIE loss. That is, we attempt to quantify to which extent the albedo effect and thickness effects can be held responsible for amplifying the effect of CO\textsubscript{2} forcing and thus accelerating the melting of SIE.

Following the findings of section 4.3, in which we identified Thickness and Albedo to show potential for mitigating the adverse influence of CO\textsubscript{2} on SIE, we ask the question about how SIE would evolve, if Thickness and Albedo were to remain constant at a certain level over the forecasting period. In particular, we repeat the forecasting exercise of the previous section for all three RCP scenarios, but keep Thickness and Albedo constant until the end of the forecasting period. For both variables we set the level equal to the value, which is given by the series’ deterministic component at the end of the sample period. By doing so, we create artificial shocks to both Thickness and Albedo in each forecasting step, which off-set their response to the external forcing variable. As we are modeling a dynamic and interconnected system, these shocks do affect all the other variables (except for CO\textsubscript{2} on which we condition our forecast).

Figure 7 documents the corresponding results for RCP 8.5, RCP 6 and RCP 2.6. For each scenario we show three different cases: (i) the projection of SIE under the respective RCP; (ii) the evolution of SIE under the respective RCP while keeping albedo constant at its last in-sample

\textsuperscript{28}This approach makes more sense in macroeconomics where the trend is usually seen as an exogenous increasing productivity process which is the object of a different field of study (Growth). In the other words, there is a strong belief that the trend and deviations from it arise from two very different models. This is not the case in our application to the Arctic where the trend is not a nuisance, but rather part of the essence.
deterministic value; (iii) the projection of SIE while keeping both albedo and thickness constant at their last respective deterministic value. Of course, these are rather radical assumptions, but they primarily serve the purpose of describing the properties of the deterministic part of the VARCTIC. For instance, the relative importance of both channels in accelerating SIE loss as visualized in figure 7 depends on how radical it is for both quantities to be respectively set around their last recorded value while letting the rest of the system evolve.\textsuperscript{29} We view this exercise more as of a way to illustrate the dynamic properties of the Arctic cryosphere as estimated by our VAR, in opposition to a definitive quantitative assessment of a potentially implementable counterfactual.

The role of internal variability as described by both Albedo and Thickness is undeniable. First, fixing Albedo to its 2019 value and thus shutting down this particular long-run amplification effect postpones the zero-SIE date by a bit less than a decade in both RCP 8.5 and RCP 6. Thickness of Arctic sea ice plays a major role for the reaction and resilience of SIE to anthropogenic forcing. Figure 7 re-enforces this view by showing that preventing both Thickness and Albedo from further decay could in fact postpone the zero-SIE event to the next century under RCP 6. Under RCP 8.5, shutting down both amplification channels starting from 2020 leads to SIE crossing the bar of $1 \times 10^6$ km$^2$ about a decade later.\textsuperscript{30} Overall, this suggest a moderate contribution of both sources of feedback: shutting them down does not prevent SIE from heading towards 0 quickly. Nevertheless, this feeds into the pictured non-linearity and acceleration of SIE loss. In fact, for the two more realistic RCPs, shutting down both channels immediately makes the trend look more linear. Of course, those eventually accelerate, in accordance with CO$_2$, but it is much later. Under the RCP 6, we would still see a blue Arctic before the turn of the century, but the decrease flattens out in the very long-run. This provides a potential justification for the finding in Diebold and Rudebusch (2019) that a quadratic trend is a preferable approximation of long-run summer months’ SIE evolution. Finally, by symmetry of the VAR, we expect feedback loops that accelerate melting to also accelerate the reverse phenomena after the 2050’s under RCP 2.6. However, this happens with a long delay, which explains why the curves constraining thickness and albedo are in fact above the green line for all the forecasting period.

\textsuperscript{29}In that line of thought, fixing Thickness while letting SIE decrease is more likely to necessitate shocks of a size that has not been observed in our sample.

\textsuperscript{30}The graphs are cut at the $1 \times 10^6$ km$^2$ bar as keeping Thickness constant (which the thought experiment suggests) is untenable as SIE approaches 0: Thickness cannot be constrained to be positive if SIE is 0.
Figure 7: Conditional Forecasts with and without Feedback

(a) RCP 8.5

(b) RCP 6

(c) RCP 2.6
5 Conclusion and Directions for Future Research

We proposed the VARCTIC as a middle ground alternative to purely theoretical or statistical modeling. It generates long-run forecasts that embody the interaction of many key variables without the inevitable opacity of climate models. First, we focus our attention on exogenous impulses which in the context of a structural VAR, have a meaning of quasi-experimental disturbances. Our results show that CO$_2$ anomalies have a permanent effect on SIE which takes about a year to settle in. It is the only impulse that has the property of permanently affecting SIE. Other impulses usually die out after a year and a half. We show that Albedo and Thickness play an important role in amplifying the response of SIE to CO$_2$ and Air Temperature shocks. In both cases, the conjunction of the two effects can double the cumulative impact of such shocks after two years.

Second, we focus on the systematic/deterministic part of the VARCTIC and conduct conditional forecasting experiments that again seek to quantify the effect of anthropogenic CO$_2$ and how internal variability can amplify it. We condition on the future path of CO$_2$ and show that, within the context of our model, it is the prime source of uncertainty for the long-run forecast of SIE. RCP 8.5 implies 0 September SIE around 2054, RCP 6 says so around 2075 and finally, RCP 2.6 (~ Paris Accord) implies that such an event would never happen. We conclude the analysis by evaluating to which extent internal variability can amplify the long-run effect of CO$_2$ forcing. Overall, our results provide statistical backing for the view that internal variability (as characterized here by Albedo and Thickness) can indeed transform relatively linearly trending CO$_2$ emissions into a non-linearly melting SIE.

Regarding future research on the Arctic ecosystem, there are many methodological extensions available within the VAR paradigm that could be of some interest for climate scientists. For instance, there is a class of models called Smooth-Transition VAR (with a popular application in Auerbach and Gorodnichenko (2012)) that could be put to use to accommodate for dynamics (read VAR coefficients) evolving over the seasonal cycle. Moreover, there is an abundant literature that allows for structural breaks (immediate discrete change of parameters) and time-varying parameters (slow/smooth change). For instance, Screen and Deser (2019) remark the importance of changing weather phenomena that transition through decadal cycles, such as the pacific oscillation. Extensions that would allow parameters to evolve through time could evaluate the quantitative relevance of such phenomena. Furthermore, some recent attention (Chavas and Grainger (2019)) has been given to the potentially non-linear relationship between CO$_2$ and SIE. Methods that blend time series econometrics, Machine Learning and abundant data of the like in Goulet Coulombe (2020) could reveal interesting insights on complex/time-varying relationships in the Arctic. In sum, the VAR methodology and time series econometrics still offer a rather unexploited potential for research on the Arctic.
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A Appendix

A.1 Data Details

Table 1: List of Variables

| Abbreviation | Description                                      | Data Source            |
|--------------|--------------------------------------------------|------------------------|
| Age          | Gridded monthly mean of Sea Ice Age              | nsidc.org              |
| AirTemp      | Gridded monthly mean of Air Temperature          | esrl.noaa.gov          |
| Albedo       | Gridded monthly mean of Surface Albedo           | disc.gsfc.nasa.gov     |
| CO₂          | Global monthly mean of CO₂                       | esrl.noaa.gov          |
| LWGAB        | Gridded monthly mean of Surface Absorbed Longwave Radiation | disc.gsfc.nasa.gov     |
| LWGEM        | Gridded monthly mean of Longwave Flux Emitted from Surface | disc.gsfc.nasa.gov     |
| LWGNT        | Gridded monthly mean of Surface Net Downward Longwave Flux | disc.gsfc.nasa.gov     |
| LWTUP        | Gridded monthly mean of Upwelling Longwave Flux at TOA | disc.gsfc.nasa.gov     |
| PrecipCMAP   | Gridded monthly mean of Precipitation            | esrl.noaa.gov          |
| SeaSurfTemp  | Median northern-hemispheric mean Sea-Surface Temperature anomaly (relative to 1961-1990) | metoffice.gov.uk      |
| SIE          | Gridded monthly mean of Sea Ice Extent           | nsidc.org              |
| SWGNT        | Gridded monthly mean of Surface Net Downward Shortwave Flux | disc.gsfc.nasa.gov     |
| SWTNT        | Gridded monthly mean of TOA Net Downward Shortwave Flux | disc.gsfc.nasa.gov     |
| TAUTOT       | Gridded monthly mean of In-Cloud Optical Thickness of All Clouds | disc.gsfc.nasa.gov     |
| Thickness    | Gridded monthly mean of Sea Ice Thickness        | psc.apl.uw.edu         |
| TotCloudCover| Gridded monthly mean of Total Cloud Cover        | esrl.noaa.gov          |
| TS           | Gridded monthly mean of Surface Skin Temperature | disc.gsfc.nasa.gov     |

A.2 Transmission Mechanism Analysis for a Shock to SIE

The purpose of the TMA analysis is to assess how the response of variable $i$ to a shock on variable $j$ changes, if a third variable $z$ were immune to the shock generated by variable $j$. Here we follow Sims (2012) by differentiating between the direct and indirect effect. The former is variable $i$’s own response to the shock hitting variable $j$. However, the shock also affects variable $z$, which itself transmits the shock further to variable $i$. This channel is the indirect effect of a shock to variable $j$ on the response of variable $i$. Hence, it is the latter that will explain the role of variable $z$ within the transmission channel of a shock to $j$ on $i$. To do so,
Sims (2012) introduce artificial shocks to variable $z$, which offset its own response to a shock to $j$. These artificial shocks have two effects: (i) the IRF of variable $z$ will be zeroed over the whole IRF horizon; (ii) the indirect channel transmits the artificial shock onto variable $i$ and allows to identify the direct effect of $j$ on $i$.

This procedure requires the transformation of the structural VAR, given in equation (1) into the reduced form VAR of equation (2), which reads as follows:

$$y_t = c + \sum_{p=1}^{p} A^{-1}\Psi_p y_{t-p} + A^{-1}\varepsilon_t,$$  \hspace{1cm} (A.1)

where $A^{-1}$ is the Choleski decomposition of matrix $A$ in equation (1). This imposes the necessary restrictions in order to identify the contemporaneous relationships of the variables. In particular, it assumes higher ordered variables to have an immediate effect on variables that are ranked below, but not vice versa. As CO$_2$ is ordered first in all of our models, an exogenous shock to carbon dioxide in period $t$ will have an immediate effect on all of the other variables.

The companion form of equation (A.1) is

$$Y_t = c + \Phi Y_{t-1} + A^{-1}\varepsilon_t,$$  \hspace{1cm} (A.2)

where $Y_t = [y_t \ y_{t-1} \cdots y_{t-p-1}]'$ and the corresponding companion matrix is

$$\Phi = 
\begin{pmatrix}
A^{-1}\Psi_1 & A^{-1}\Psi_2 & \cdots & A^{-1}\Psi_p \\
I & 0 & 0 & \cdots & 0 \\
0 & I & 0 & \cdots & 0 \\
\vdots & \vdots & \ddots & \vdots & \vdots \\
0 & \cdots & \cdots & I & 0
\end{pmatrix}.$$  \hspace{1cm} (A.3)

An equivalent way (to what laid out in section 3.4) of constructing IRFs, i.e. the response of variable $i$ to a structural shock on variable $j$ over all horizons $h = 0, \ldots, H$, is to proceed iteratively. Hence, for a given period $h$, the response of $i$ to a shock hitting $j$ is given by

$$IRF(j \rightarrow i, h) = e_i \Phi^h A_{\bullet,j}^{-1},$$  \hspace{1cm} (A.4)

where $e_i$ is a selection vector of dimension $1 \times M$ with 1 at entry $i$ and 0 otherwise. $A_{\bullet,j}^{-1}$ elicits the $j^{th}$ column of $A^{-1}$. Following Sims (2012), switching off the indirect effect of a shock to variable $j$ on $i$ via variable $z$ amounts to $IRF(j \rightarrow z, h) = 0 \ \forall \ h = 0, \ldots, H$. That requires the artificial shocks, $\varepsilon_{z,h}$, to be calibrated such that the response of variable $z$ to a shock to variable $j$ is zero over the whole IRF period. Hence, at $h = 1$ the artificial shock $\varepsilon_{z,1}$ is
\[ \varepsilon_{z,1} = -\frac{A_{j,1}^{-1}}{A_{z,1}^{-1}}. \] 

(A.5)

As these shocks are transmitted through time, the artificial response \( \varepsilon_{z,h} \) has to account for all the past shocks, \( \varepsilon_{z,h-1} \), for any periods beyond \( h = 1 \):

\[ \varepsilon_{z,h} = \frac{\text{IRF}(j \rightarrow z,h) + \sum_{h'=0}^{h-1} \varepsilon_{z,h'} A_{j,1}^{-1} \varepsilon_{z,h'}}{\varepsilon_{z} A_{z,1}^{-1}}. \] 

(A.6)

The altered IRFs (that omits the transmission channel \( z \)) for all the variables in the model to a shock to \( j \) is

\[ \text{IRF}_{-z}(j \rightarrow i,h) = \text{IRF}(j \rightarrow i,h) + \sum_{h'=0}^{h} \varepsilon_{z,h'} A_{j,1}^{-1} \varepsilon_{z,h'}. \] 

(A.7)

So far, we have reviewed how IRF decomposition works when one is interested in shutting down a single channel at a time. In contrast to Sims (2012), our VAR comprises more than three variables. Therefore, in some cases, it is desirable to shut-down not only one, but a group \( Z \in M \setminus \{ i, j \} \) of indirect channels. To do so, equations (A.5) and (A.6) need to be generalized. At impact, the artificial response of variable \( z \) to a shock to \( j \) does not only have to offset the direct effect of \( j \), but also the indirect effect of a shock \( j \) via the indirect effect of all the other artificial responses \( \varepsilon_{z,1}^{+} \) of those variables in \( Z \) which are ordered above \( z \).\(^{31}\) This amounts to the following extension of equation (A.5):

\[ \varepsilon_{z,1} = -\left( \frac{A_{j,1}^{-1}}{A_{z,1}^{-1}} + \frac{\sum_{m \in z^{+}} \varepsilon_{m,1}}{A_{z,1}^{-1}} \right). \] 

(A.8)

Also equation (A.6) has to be adjusted accordingly. However, at horizons \( h > 1 \) the artificial response \( \varepsilon_{z,h} \) will not only have to offset the contemporaneous effects of \( z^{+} \), but also compensate for the artificial responses of all other variables in \( Z \) over the period \( h' = 0 \cdots h - 1 \):

\[ \varepsilon_{z,h} = -\frac{\text{IRF}(j \rightarrow z,h) + \sum_{h'=1}^{h-1} \varepsilon_{z,h'} A_{j,1}^{-1} \varepsilon_{z,h'} + \sum_{h'=1}^{h-1} \sum_{n \in Z} \varepsilon_{n,h'} + \sum_{m \in z^{+}} \varepsilon_{m,1}}{\varepsilon_{z} A_{z,1}^{-1}}. \] 

(A.9)

Equation (A.7) for the modified IRF \( \text{IRF}_{-z}(j \rightarrow i,h) \) remains intact.

\(^{31}\) \( z^{+} \) denotes all those variables in \( Z \) which are ordered above \( z \).
A.3 Bayesian Estimation Details

The Minnesota prior is a specific structure for values of both $\beta_0$ and $\Sigma_0$. In words, it allows concisely to parameterize heterogeneity in both the prior mean and variance. It consists of three major elements: the first one is about $\beta_0$ and the last two concern $\Sigma_0$.

1. For any equation $y_{m,t}$ with $m = 1, \ldots, M$ – where $M$ is the total number of observed variables in the VAR – all parameters are shrunk to 0 except for its first own lag $y_{m,t-1}$. The latter is usually shrunk to a value $b_{AR}$ between 0.5 and 1. This can be interpreted as shrinking each VAR equation to the much simpler and parsimonious AR(1) process. Given the persistent nature of time-series data, this structure for $\beta_0$ is much more appropriate than that of Ridge regression (or LASSO), which shrinks all coefficients homogeneously towards 0.

2. It is often observed in multivariate time series models that $y_{m,t-1} \rightarrow y_{m,t}$ will be way stronger than almost any of the $y_{m,t-1} \rightarrow y_{m,t}$ relationships. $\lambda_2$ therefore calibrates the intensity of shrinking dynamic cross-correlations rather than auto-correlations.

3. Distant partial lag relationships (say $y_{m,t-12} \rightarrow y_{m,t}$) are expected to be of smaller magnitude than close ones like $y_{m,t-1} \rightarrow y_{m,t}$, and $y_{m,t-2} \rightarrow y_{m,t}$. $\lambda_3$ is in charge of determining the intensity of shrinking the coefficients of more distant lags.

The overall tightness of the whole prior apparatus is determined by $\lambda_1$.

Since we wish to be as agnostic as possible when tuning our model, we optimize/estimate hyperparameters $\{b_{AR}, \lambda_1, \lambda_2, \lambda_3\}$ within some grid to optimally balance bias and variance. To do so, we use the methodology developed in Giannone et al. (2015). This data-driven way of setting the priors’ tightness can be understood as analogous (at a philosophical level) to setting tuning parameters using cross-validation in Machine Learning. Details on the exact values of hyperparameters used can be found in section A.3. Finally, we fix the number of lags in the VARCTIC 8 to $P = 12$ and to $P = 3$ in VARCTIC 18 respectively. The total number of posterior draws is 2000. The benchmark VARCTIC 8 is obtained with the following prior variances:

- Autoregressive Coefficient: $= 0.9$;
- Overall tightness is $\lambda_1 = 0.3$;
- Cross-variable weighting is $\lambda_2 = 0.5$;
- Lag decay is $\lambda_3 = 1.5$;
- Exogenous variable tightness: $\lambda_4 = 100$.

32 For further details, explicit mathematical formulation of the prior and additional discussion on priors for VARs, the reader is referred to (Dieppe et al., 2016).
It is worth remembering that the different \( \lambda \)'s are prior variances. A larger value of \( \lambda_1 \) and \( \lambda_2 \) implies a looser prior, whereas a higher \( \lambda_3 \) assigns less importance to lagged values. To draw a parallel to penalized regression (like Ridge and LASSO), a small \( \lambda_1 \) in a Bayesian VAR increase regularization in way analogous to how increasing the \( \lambda_{\text{RIDGE}} \) – that is, by pushing the BVAR estimate \( \hat{\Phi} \) away from \( \hat{\Phi}_{\text{OLS}} \).

Put shortly, \( \lambda_1 \) guides the overall level of shrinkage in the model. \( \lambda_2 \) is indicative of how much the cross-variable dynamic relationship are shrunk to zero relative to own lags. It is often observed in multivariate time series models that \( Y_{t-1} \rightarrow Y_t \) will be way stronger than much of the \( X_{t-1} \rightarrow Y_t \) relationships. Thus, \( \lambda_2 \) defines how we \textit{a priori} think that autocorrelations should be more important to explain \( Y_t \) than dynamic cross-correlations. Finally, \( \lambda_3 \) is yet another hyperparameter in charge of determining the tightness of a reasonable prior: far away lag relationships (say \( Y_{t-12} \rightarrow Y_t \)) are expected to be less important than close ones like \( Y_{t-1} \rightarrow Y_t \) and \( Y_{t-2} \rightarrow Y_t \).

In the following subsections, we report basic results – namely, the long run forecast and IRFs – with tighter and looser priors. In the latter case the point estimates approach what would have been obtained by classical Maximum Likelihood. Results remain both qualitatively and quantitatively unchanged. Additionally, we experiment with alternative prior specifications and again similar results.

A.3.1 Altering the Priors

The hyperparamter \( \lambda_1 \), is tuned to be more relaxed than the ones of the benchmark VARCTIC 8 in section 4, whereas the lag decay is strengthened and the AR coefficient is slightly reduced. Lags remain at 12. The present specification reads as follows:

- Autoregressive Coefficient: \( = 0.8 \);
- Overall tightness is \( \lambda_1 = 1 \);
- Cross-variable weighting is \( \lambda_2 = 1 \);
- Lag decay is \( \lambda_3 = 3 \);
- Exogenous variable tightness: \( \lambda_4 = 100 \).

In this specification, SIE is projected to hit the zero-lower bound in 2061. The DIC of -6828.57 is higher than its counterpart of the VARCTIC 8 (-6894.35).
Figure 8: Trend Sea Ice Extent

![Graph showing trend in Sea Ice Extent](image)

Figure 9: IRFs: Response of Sea Ice Extent

![Graphs showing response of Sea Ice Extent to different shocks](images)
A.4 Different Ordering

In this section, we check the sensitivity of the responses of SIE to a shock of any of the other variables when varying the ordering of variables compared to the benchmark VARCTIC 8 in section 4. The priors and lags remain unaltered to the specification outlined in section A.3. The ordering now reads: \( \text{CO}_2, \text{AirTemp}, \text{SeaSurfTemp}, \text{TotCloudCover}, \text{PrecipCMAP}, \text{SIE}, \text{Thickness}, \text{ALBEDO} \).

Figure 10: IRFs: Response of Sea Ice Extent

A comparison of the responses of the benchmark VARCTIC 8 in Figure 3 and the IRFs after reordering the model (Figure 10) documents the robustness of results to different identification schemes. A second – more radical – variation in the model set-up locates Albedo at position two. Hence, a shock to Albedo will contemporaneously affect all the other variables except \( \text{CO}_2 \): \( \text{CO}_2 \), ALBEDO, TotCloudCover, PrecipCMAP, AirTemp, SeaSurfTemp, SIE, Thickness.
Figure 11: IRFs: Response of Sea Ice Extent

For most of the effects, the shapes remain robust in comparison with Figure 3. Only the response to air temperatures deviates visibly with the statistically significant impact in the short-run now vanishing.

A.5 Results of VARCTIC 18

VARCTIC 18, including all the variables displayed in Figure 15, tests the robustness of the VARCTIC 8 projection of SIE. The ordering of variables in the VARCTIC 18 reads as follows: SWGNT, SWTNT, CO\textsubscript{2}, LWGNT, TotCloudCover, TAUTOT, PrecipCMAP, TS, AirTemp, SeaSurfTemp, LWGAB, LWTUP, LWGEM, SIE, Age, Thickness, EMIS and Albedo. Due to the increased number of variables, the lags were reduced to 3 and the estimation period starts in 1984 due to some series unavailability. With more parameters to estimate, the prior-specification slightly tightens and reads as follows:

- Autoregressive Coefficient: $\rho = 0.8$;
- Overall tightness is $\lambda_1 = 0.5$;
- Cross-variable weighting is $\lambda_2 = 0.5$;
- Lag decay is $\lambda_3 = 3$;
• Exogenous variable tightness: $\lambda_4 = 100$.

The VARCTIC 18 predicts Arctic sea ice to reach the zero-lower bound by the year 2062, which is in the very neighborhood of the VARCTIC 8 (see Figure 12). This result suggests the VARCTIC 8 to comprise the key variables for a proper and robust projection of Arctic sea ice. With the key-mechanisms for forecasting SIE apparently being captured by the benchmark specification, we conduct the investigation of the main feedback-channels and amplification mechanisms of sections 4.3 through 4.5 by using the VARCTIC 8.

Figure 12: Trend Sea Ice Extent

For completeness, the impulse response of SIE to a shock to any of the other variables is shown in Figure 13. IRFs of key variables remains roughly unchanged in VARCTIC 18. Most interestingly, in the VARCTIC 18, not only the CO$_2$ shock has the effect of triggering a permanently decreasing SIE, but also LWGAB, which measures the longwave radiation absorbed by the surface. Great many other shocks have statistically significant impacts in the short run. However, none has the lasting (and damaging) impact of CO$_2$ and LWGAB. These latter two shocks have the unique property of permanently pushing the system out of the former equilibrium.

The forecast of SIE under the specification of the VARCTIC 18 is shown in Figure 14. The projected ice-free dates under the RCP 6 and RCP 8.5 scenarios are consistent with the results reported by the VARCTIC 8 in Figure 6a. The trajectory of SIE under RCP 2.6, however, slightly changes and seems to stabilize rather than recover by the end of the century.

A.6 Stochastic De-seasonalization

As discussed in section 2, the raw data (see 15) is highly seasonal except those that have been pre-treated by the data provider.
As a robustness check, we verify that our main results hold if we employ a radically different technique to take out seasonality. In this subsection, we adopt the approach of structural
time series (Harvey (1990) and Harvey and Koopman (2014)) where $y_{raw}^t$ is split in three somewhat intuitive parts:

$$y_{raw}^t = \mu_t + \gamma_t + \eta_t$$

a trend component $\mu_t$; a seasonality component $\gamma_t$ and a (possibly autocorrelated) noise component $\eta_t$. Each of them are stochastic and have their own law of motion. The structure and law of motions we use follow the well-established Harvey Basic Structural Model (Harvey and Todd (1983)). The model reads as follows.

$$\mu_t = \mu_{t-1} + \beta_t + u_t$$

$$\beta_t = \beta_{t-1} + v_t$$

$$\gamma_t = -\sum_{m=1}^{11} \gamma_{t-m} + w_t$$

$$(\eta_t, u_t, v_t, w_t) \sim iid N(0, \Sigma)$$
\[
\Sigma = \begin{pmatrix}
\sigma_{\eta\eta}^2 & 0 & 0 & 0 \\
0 & \sigma_{uu}^2 & 0 & 0 \\
0 & 0 & \sigma_{vv}^2 & 0 \\
0 & 0 & 0 & \sigma_{ww}^2
\end{pmatrix}
\]

The law of motion is that of Harvey and Todd (1983) and fits in a traditional state space model. The trend \( \mu_t \) is a random walk with a stochastic drift. The drift \( \beta_t \) is itself evolving according to a random walk. For instance, this means that \( \mu_{SIE,t} \), the trend of SIE, is trending down stochastically at a rate \( \beta_{SIE,t} \). That (negative) growth rate is itself allowed to evolve. A quick look at a flexibly modeled trend of SIE suggests that all owing for a time-varying growth rate is necessary given the acceleration and deceleration of the melting of SIE in the 2000’s. Figure 16 shows the complete set of stochastic trends resulting from the BSM.

Figure 16: Basic Structural Model: 8 Variables

Extracted Trends adjusted for average September-Seasonality

The extraction of trends as a first step and their subsequent modeling as a second step is analogous to standard practice in macroeconomics, but not similar. In macroeconomics, it is customary in a strand of empirical work to filter the data as a pre-processing step. The VAR is then estimated on the extracted cycles, which is simply the difference of the raw data and the estimated trend. Here, we are indeed doing the filtering step first but using trends components – rather than seasonality and short-run noise – for the second step. However, our trend components \( \mu_t \) are rather stochastic with respect to what is usually seen in economics.

A.6.1 The Benchmark Specification and Results

Following Giannone et al. (2015), we obtain the optimal hyperparameters:

- Autoregressive Coefficient: = 1;
• Overall tightness is $\lambda_1 = 0.3$;
• Cross-variable weighting is $\lambda_2 = 0.5$;
• Lag decay is $\lambda_3 = 1.51$;
• Exogenous variable tightness: $\lambda_4 = 100$;

The date of the zero-lower bound of the stochastic de-seasonalized version remains in the neighborhood of the benchmark model. In this specification, the Arctic would be ice-free by the year 2062.

Figure 17: Trend Sea Ice Extent
Stochastic De-seasonalization
Figure 18: IRFs: Response of Sea Ice Extent

*Stochastic De-seasonalization*

![IRFs: Response of Sea Ice Extent](image1)

Figure 19: Evolution of SIE under different Scenarios of CO$_2$

*Different Scenarios*

*Stochastic De-seasonalization - Extracted trend adjusted for mean September-seasonality*

![Evolution of SIE under different Scenarios of CO$_2$](image2)

The BSM specification allowing for evolving seasonality, we can also use it to obtain more
flexible month-specific VARCTICs. The benchmark specification implies that we can transform our series into a string of "synthetic" Septembers or Marchs by simply adding or substracting a constant. In the evolving seasonality model, one can rewrite a slowly widening seasonal pattern as the expression of heterogeneous trends across seasons. Thus, rather than adding back the mean (over time) of $\gamma_t, September$ to $\mu_t$ to fit the model on static synthetic Septembers, we can add back

$$\tilde{\gamma}_t, September = \sum_{t' = 1}^{T} I(t' = t) \gamma_{t', September}$$

to model evolving synthetic Septembers (or any month of interest). Unlike our benchmark specification, this approach allow for summer vs non-summer months to have different trends. Figure 20 reports results of our conditional forecasting analysis conducted for two radically different months. While March’s SIE is linearly trending in-sample, these results suggest a potential acceleration of melting in the second half of the century – with widening uncertainty.

Figure 20: Evolution of SIE under different Scenarios of CO$_2$

Different Scenarios
Stochastic De-seasonalization:
Extracted trend adjusted for yearly September- & March-seasonality