Monopole-Bubble in Early Universe

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Abstract

The nucleation and evolution of bubbles are investigated in the model of an $O(3)$-symmetric scalar field coupled to gravity in the high temperature limit. It is shown that, in addition to the well-known bubble of which the inside region is true vacuum, there exists another decay channel at high temperature which is described by a new solution such that a false vacuum region like a global monopole remains at the center of a bubble. The value of the Euclidean action of this bubble is higher than that of the ordinary bubble; however, the production rate of it can be considerable for a certain range of scalar potentials.

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I. Introduction

It is widely believed that the present universe was achieved through a number of phase transitions while the universe has expanded and cooled [1]. A particularly interesting possibility is that of a first-order phase transition which is described by the formation and growth of bubbles [2]. In the course of phase transitions in the early universe, two additional ingredients should be considered: one is the gravitational effect [3] and the other is high temperature [4, 5]. The inflationary scenario arose from the application of these ideas to cosmology is a synthetic attempt to answer the fundamental cosmological questions [6]. These are the study of new phase bubbles in interiors of an old phase, but the opposite case, old phase remnants surrounded by the new phase, has also been an interesting subject [7]. Recently inflation in the core of topological defects was analyzed in Ref. [8, 9, 10]. Except for the last case, the order parameter of the model has been basically a real scalar field, thereby ignoring the difference between the continuous symmetry breaking and the discrete one at the time of bubble nucleation. Only for the evolution and collision of bubbles, this difference has been taken into proper account. However, the various species of solitonic defects, e.g., 1. topological or nontopological, 2. global or local, 3. cosmic strings, monopoles or textures, are determined by the continuous symmetry which the theory of interest holds [11]. Although soliton spectra involve complexities, the scenario of bubble nucleation is a simple one based on one real scalar order parameter, and continuous symmetry does not play a role for the formation of bubbles [12], whether it is due to quantum tunneling at zero temperature [2] or due to thermal fluctuation at high temperature [4, 5]. Therefore it may be interesting to ask whether the structure of bubbles is affected by internal symmetry from the time of nucleation or not, and particularly how the internal symmetry coagulates a matter droplet inside the bubble.

In this paper we consider a model of a scalar field with a global $O(3)$ symmetry and study the first-order phase transition at high temperature and in the presence of gravity. We first show that, in addition to the ordinary bubble of which the center is true vacuum, the model in a curved spacetime supports the $O(3)$ bubble that includes a matter droplet in its core as obtained in the same model in a flat spacetime [13]. The formation of the matter droplet at the core region of the new bubble is due to the winding between the internal space and that of spatial rotations, so it can be interpreted as a global monopole inside the bubble. Henceforth we will call it “monopole-bubble” in order to distinguish this bubble solution from the well-known ordinary bubble solution. This new bubble with a global monopole has two bubble walls: one is the bubble wall which distinguishes the false vacuum region outside the bubble from the true vacuum region inside it, and the other is the inner bubble wall which surrounds the false vacuum core of the global monopole. The long range tail of global monopole energy density is inversely proportional to the square of the circumference radius and affects the spacetime structure inside the bubble. Actually, it renders the spacetime between the inner and outer walls similar to the spacetime around the ordinary global monopole [14]: the spacetime has a repulsive nature, which indicates that the core does not collapse into a black hole even at the Planck scale, and the far region is a flat spacetime with a solid deficit angle. Although the total energy of the ordinary global monopole is infinite, in the present case it
is finite since the outer wall plays an important role of cutoff.

The value of the Euclidean action needed to generate a bubble with a matter droplet is larger than that of an ordinary clean bubble; however, we observe that the decay rate for this monopole-bubble computed on the basis of the exponential formula can be quite considerable in comparison with that of the ordinary bubble for a certain range of the coefficients of the scalar potentials. Once bubbles are nucleated at high temperature in a curved spacetime, they start to expand because of some combination of processes when the environment keeps the temperature high \[13\] or the recovery of zero-temperature classical dynamics by the expansion of the background universe. We investigate the motion of the monopole-bubble by solving the coupled time-dependent field equations, neglecting the process of temperature changing. The outer bubble wall of the monopole-bubble grows similar to that of the ordinary bubble. The global monopole at the center of the bubble is stable when the phase transition scale is lower than the Planck scale. Moreover, this monopole-bubble also shares the same possibility of defect inflation at the Planck scale according to the arguments in Ref.\[8, 9, 10\].

The paper is organized as follows. In Sec. II we introduce the model and illustrate the nucleation of new $O(3)$ bubble solutions. In Sec. III we present the evolution of bubbles due to the gravitational force and briefly discuss the matter of defect inflation at the Planck scale at the core of the global monopole. Some concluding remarks are made in Sec. IV. In this paper we use the units $c = \hbar = k_B = 1$.

II. Nucleation of Monopole-Bubbles

Transition to the true vacuum state by quantum tunneling occurs through the nucleation of bubbles of the energetically favored phase. The nucleation rate per unit volume is $\Gamma = A e^{-B}$, where $B$ is evaluated by the Euclidean tunneling action and $A$ is a prefactor which has units of energy to the fourth power. In this section we will consider a model with global $O(3)$ symmetry and calculate the nucleation rate of bubbles at high temperature, specifically $B$ of the model in a curved spacetime.

A. Euclidean Solutions

We begin with the action of an $O(3)$ symmetric scalar multiplet in the presence of Einstein gravity. Since the quantum statistics of bosons at finite temperature is formulated in the imaginary-time method which is described by the Euclidean theory with fields periodic in the Euclidean time with period $\beta = 1/T$, the Euclidean action at finite temperature is given by

$$S = \int_0^\beta dt_E \int d^3x \sqrt{g} \left\{ -\frac{1}{16\pi G}R + \frac{1}{2} g^{\mu\nu} \partial_\mu \phi^a \partial_\nu \phi^a + V(\phi) \right\},$$

where $\phi^a = \hat{\phi}^a \phi$ is an $O(3)$-symmetric isovector ($a = 1, 2, 3$) with $\phi = \sqrt{\phi^a \phi^a}$.

The equations of motion are

$$\frac{1}{\sqrt{g}} \partial_\mu (\sqrt{g} g^{\mu\nu} \partial_\nu \phi^a) = -\frac{\partial (-V)}{\partial \phi^a}$$

(2.2)
\[ G_{\mu\nu} = 8\pi G T_{\mu\nu}, \quad (2.3) \]

where the energy-momentum tensor is
\[ T_{\mu\nu} = \partial_\mu \phi^a \partial_\nu \phi^a - g_{\mu\nu} \left( \frac{1}{2} g^{\sigma\rho} \partial_\rho \phi^a \partial_\sigma \phi^a + V \right). \quad (2.4) \]

Here we bring up the situation that the temperature is much larger than the inverse of the bubble radius, where the transition is described by time-independent fields. Both scales of symmetry breaking and temperature are usually assumed to be lower than the Planck scale \( M_{Pl} = 1/\sqrt{G} \); however, sometimes the supermassive scale when the transition scale is comparable with the Planck scale will also be considered when we examine the inner structure of bubbles in the very early universe.

Although there is no rigorous proof in a curved spacetime when the \( O(3) \) symmetric solutions with respect to spatial coordinates saturate the solutions with the lower Euclidean action, we choose to consider the bubbles which possess spherical symmetry. The form of the metric compatible with \( O(3) \) symmetry can be written as
\[
d s^2 = \left( 1 - \frac{2GM(r)}{r} \right) e^{2\delta(r)} dt^2 + \left( 1 - \frac{2GM(r)}{r} \right)^{-1} dr^2 + r^2(d\theta^2 + \sin^2\theta d\varphi^2). \quad (2.5)\]

Using these coordinates the scalar field takes the following form,
\[
\phi^a = \hat{\phi}^a(\theta, \varphi) \phi(r) = (\sin n\theta \cos m\varphi, \sin n\theta \sin m\varphi, \cos n\theta) \phi(r), \quad (2.6)
\]
where the spherical symmetry and the regularity at the origin allow only two cases: \( n = 0 \), and \( n = m = 1 \).

Writing down the equations of motion by use of Eqs. (2.5) and (2.6), we have three independent field equations:
\[
(1 - \frac{2GM(r)}{r}) \frac{d^2\phi}{dr^2} + (1 - \frac{2GM(r)}{r}) \frac{d}{dr} \ln \left( r^2 e^\delta \left( 1 - \frac{2GM(r)}{r} \right) \right) \frac{d\phi}{dr} - \frac{2\delta_{n1}}{r^2} \phi = \frac{dV}{d\phi}, \quad (2.7)
\]
\[
\frac{1}{r} \frac{d\delta}{dr} = 4\pi G \left( \frac{d\phi}{dr} \right)^2, \quad (2.8)
\]
\[
\frac{1}{r^2} \frac{dGM}{dr} = 4\pi G \left[ \frac{1}{2} \left( 1 - \frac{2GM}{r} \right) \left( \frac{d\phi}{dr} \right)^2 + \frac{\delta_{n1}}{r^2} \phi^2 + V \right], \quad (2.9)
\]

where \( \delta_{n1} \) denotes the Kronecker delta. For the actual calculation, let us choose a sixth-order scalar potential such as
\[
V(\phi) = \frac{\lambda}{v^2} (\phi^2 + \alpha v^2)(\phi^2 - v^2)^2 \text{ with } 0 < \alpha < 1/2, \quad (2.10)
\]
which describes the potential in a broken phase. Although we choose a specific shape of the scalar potential, our argument in the following does not depend on the detailed form of the scalar potential and the existence of the new bubble solution with \( n = 1 \) is guaranteed under
any potential as long as it includes one false vacuum and one true vacuum. Here we only consider the transition from a symmetric vacuum to the broken vacuum, i.e. from de Sitter spacetime with the horizon $H^{-1} \equiv (8\pi GV(0)/3)^{-\frac{3}{2}}$ to Minkowski spacetime.

The boundary conditions for nonsingular solutions of Eqs. (2.7), (2.8) and (2.9) are

$$\frac{d\phi}{dr}(r = 0) = 0 \text{ for } n = 0 \quad \text{or} \quad \phi(r = 0) = 0 \text{ for } n = 1,$$

$$\phi(r \to H^{-1}) = 0, \quad \text{and} \quad M(r = 0) = 0,$$  \hspace{1cm} (2.11)

and we choose the normalization of $t$ by setting $\delta(r \to H^{-1}) = 0$. The scalar field rapidly approaches the false vacuum for large $r$ when the radius of the bubble is smaller than the de Sitter horizon $H^{-1}$. The above is our main interest; however, we will also comment on the cases in which the radius of the bubble is comparable to or larger than the de Sitter horizon.

To find the behavior near the origin, we expand the variables by a power series, finding

$$\phi(r) \approx \phi_{esc} - \frac{1}{6} \frac{dV}{d\phi} \phi_{esc}^2 \quad r^2, \quad \phi_{esc} \equiv \phi(0)$$

$$\delta(r) \approx \delta_{esc} + \frac{\pi}{9} G \left( \frac{dV}{d\phi} \right)_{\phi_{esc}}^2 r^4, \quad \delta_{esc} \equiv \delta(0)$$

$$GM(r) \approx \frac{4}{3} \pi GV(\phi_{esc}) r^3$$  \hspace{1cm} (2.12)

for the $n = 0$ configuration, and

$$\phi(r) \approx \phi_0 r \left[ 1 - \left( \frac{1}{2} \lambda (2 - \alpha) - \pi G \phi_0^2 \right) r^2 \right], \quad \phi_0 \equiv \frac{d\phi}{dr}(0)$$

$$\delta(r) \approx \delta_0 + 2\pi G \phi_0^2 r^2, \quad \delta_{esc} \equiv \delta(0)$$

$$GM(r) \approx 2\pi G \phi_0^2 r^3$$  \hspace{1cm} (2.13)

for the $n = 1$ configuration. The constants $(\phi_{esc}, \delta_{esc})$ and $(\phi_0, \delta_0)$ are determined by the proper behavior of the fields for large $r$. If the transition scale $m_H = \sqrt{4\lambda (3 + 2\alpha)} v$ is smaller than the Planck scale and the size of the bubble is smaller than that of the de Sitter horizon $H^{-1}$, we may meet no coordinate singularity inside the bubble. Therefore, the scalar amplitude $\phi(r)$ and a metric function $\delta(r)$ approach their boundary values exponentially in their asymptotic region, and Eq.(2.3) says

$$GM(r) \approx \frac{4}{3} \pi G \lambda v^4 r^3,$$  \hspace{1cm} (2.14)

since the first two terms in the right-hand side of Eq.(2.3) are negligible and the last cosmological constant term is dominant at the false vacuum region outside the bubble. Figure 1 shows that $\delta(r)$ and $GM(r)$ are monotonically increasing functions of $r$; however, the behavior of the scalar field $\phi$ near the origin depends on whether $n = 0$ or $n = 1$. 

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Figure 1: A plot of a bubble solution for $\lambda = 1$, $\alpha = 0.1$ and $v/M_{Pl} = 0.1$. The solid, dotted and dashed lines correspond to $\phi(r)$, $\delta(r)$ and $GM(r)$, respectively. (a) $n = 0$, (b) $n = 1$. 
Figure 2: $T_t^l$ profiles for fixed $\lambda = 1$ and $v/M_{pl} = 0.1$. (a) thick-wall bubble of $\alpha = 0.3$, (b) thin-wall bubble of $\alpha = 0.01$. The dotted and solid lines correspond to an $n = 0$ bubble and an $n = 1$ bubble, respectively.
As shown in Fig. 2, the $n = 0$ solution is the well-known vacuum bubble solution where the energy is accumulated at the bubble wall. Although there remains some matter inside the thick-wall bubble (see the dotted line in Fig. 2-(b)), contrary to the case of the thin-wall bubble (see the dotted line in Fig. 2-(a)), such matter does not form an aggregate inside the bubble. The energy density of $n = 1$ bubbles,

$$T^t_t = \frac{1}{2} \left( 1 - \frac{2GM}{r} \right) \left( \frac{d\phi}{dr} \right)^2 + \delta_{n1} \frac{\phi^2}{r^2} + V,$$  \hspace{1cm} (2.15)

is expressed by solid lines in Fig. 2. We easily read that a matter droplet is formed at the center of the $n = 1$ bubble due to the nontrivial local mapping between internal $O(3)$ symmetry and spatial $O(3)$ symmetry. From the boundary conditions of the scalar field in Eq.(2.11) together with the ansatz (2.6), we can interpret this matter droplet as a global monopole. In a flat spacetime of dimension more than two, there is a no-go theorem that says the scalar fields described by the standard relativistic form of the Lagrangian do not support non-trivial static soliton solutions of finite energy [16]. Therefore, when we consider global vortices or global monopoles in the presence of gravity [17, 14], the introduction of a cutoff scale, for example, the horizon length, provides us a way to control the divergent quantities [18]. However, the global monopole created inside the $n = 1$ bubble is a finite energy static configuration since the long-range tail of the global monopole is tamed by the outer bubble wall.
The features described above do not depend on whether the bubbles lie in a flat spacetime or a curved spacetime; however, other characteristics due to gravity are also worth noting. We can read the gravitational effects on the shapes of bubbles in the weak gravity limit as follows. If we think of $r$ as a time and $\phi$ as a position of a hypothetical particle in a one-dimensional motion, the scalar field equation (2.7) can be interpreted as the Newton equation for a particle of variable mass \((1 - \frac{2GM}{r})\) subject to several forces: the first one is the conservative force from \(-V(\phi)\), the second one a friction with time-dependent coefficient \(\left(1 - \frac{2GM}{r}\right)\frac{d}{dr} \ln \left(r^2 e^{\delta(1 - \frac{2GM}{r})}\right)\), and the last one only for \(n = 1\) is a time-dependent repulsion whose strength is extremely large at \(r = 0\). Although it is difficult to prove analytically the existence of solutions in the presence of gravity, we can understand how and what kind of bubble solutions are supported by use of the terminology of Newtonian mechanics as was done in the flat spacetime case [19]. Let us consider the returning motion of a hypothetical particle both for \(n = 0\) and \(n = 1\) solutions, i.e., from \(\phi(r = 0) = \phi_{\text{esc}}\) for the \(n = 0\) solution (or \(\phi(r = r_{\text{turn}}) = \phi_{\text{turn}}\) for the \(n = 1\) solution) to \(\phi(r = \infty) = 0\). Since \(GM(r)\) is an increasing function of \(r\), the variable mass of a hypothetical particle \(\left(1 - \frac{2GM}{r}\right)\) decreases as time \(r\) elapses. Furthermore, the gravitational effect due to the sum of \(\delta(r)\) and \(GM(r)\) decreases the time-dependent coefficient at the starting point for the \(n = 0\) solution and at the turning point for the \(n = 1\) solution. Both effects involve the decrement of energy gained during the rolling of a hypothetical particle down to the minimum point of the effective potential \(-V(\phi)\), hence \(\phi_{\text{esc}}\) (or \(\phi_{\text{turn}}\)) should become small after the inclusion of gravity (see Fig. 3). For an \(n = 0\) bubble, it makes the radius of the bubble smaller. For the behavior

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**Figure 3:** A plot of bubble solutions with and without gravity coupling for \(\lambda = 1\) and \(\alpha = 0.25\). The solid line denotes the bubble with gravity and the dotted line without gravity. (a) \(n = 0\), (b) \(n = 1\).
of the $n = 1$ bubble, the hypothetical particle reaches the returning point, $\phi_{\text{turn}}$, at a time earlier than in the flat case. In order to reach the smaller turning point $\phi_{\text{turn}}$ in spite of the larger acceleration for a given $\phi_0$ and for small $r$ as shown in Eq.\((2.13)\), the initial velocity $\phi_0$ of the particle for $n = 1$ bubbles should be smaller than the value in a flat spacetime. Hence, the radius of the bubble becomes small, although the size of the matter droplet is increased by the gravity. The contraction of the outer walls of both $n = 0$, $1$ bubbles can be understood through the attractive nature of gravity; however, the explanation for the larger matter droplet will be given later by mentioning the repulsive nature of gravity on the matter core. Looking back at Fig. 2, we can compare the radius of an $n = 0$ bubble and of an $n = 1$ bubble. In a flat spacetime the radius of an $n = 0$ bubble is always larger than that of an $n = 1$ bubble \([\text{13]}\), and it is also true for a thick-wall bubble in a curved spacetime (see Fig. 2-(b)). For a thin-wall bubble in a curved spacetime where the global monopole stretches a long-range tail; however, the radius of an $n = 1$ bubble becomes smaller than that of an $n = 0$ bubble. This fact will be justified by our analytic discussion under the thin-wall approximation in Sec. III.

For an $n = 1$ bubble, let us look at the spacetime structure in the neighborhood of the point $r = r_{\text{turn}}$, where $\phi$ takes the maximum value $\phi_{\text{turn}}$, i.e., $\left.\frac{d\phi}{dr}\right|_{r=r_{\text{turn}}} = 0$. Equations \((2.8)\) and \((2.9)\) around $r = r_{\text{turn}} = 0$ have approximate solutions such as

$$\delta(r) \approx \delta_{\text{turn}}$$

$$GM(r) \approx GM_{\text{turn}} + 4\pi G \phi_{\text{turn}}^2 r + \frac{4}{3}\pi G V(\phi_{\text{turn}}) r^3.$$  

The constants $\delta_{\text{turn}}$ and $M_{\text{turn}}$ are fixed by

$$\delta_{\text{turn}} \approx -4\pi G \int_{r_{\text{turn}}}^{H^{-1}} dr r \left(\frac{d\phi}{dr}\right)^2$$

$$GM_{\text{turn}} \approx 4\pi G \int_0^{r_{\text{turn}}} dr r^2 \left\{\frac{1}{2} \left(1 - \frac{2GM}{r}\right) \left(\frac{d\phi}{dr}\right)^2 + \frac{\phi^2 - \phi_{\text{turn}}^2}{r^2} + (V - V(\phi_{\text{turn}}))\right\}.$$  

The above integrals are estimated as $\delta_{\text{turn}} \sim 4\pi G \phi_{\text{turn}}^2$ and $|GM_{\text{turn}}| \sim |Gm_H|$. If we consider an $n = 1$ bubble with a thin wall when the phase transition scale $m_H = \sqrt{4\lambda(3 + 2\alpha)}v$ is much smaller than the Planck scale $M_P$, the first and the third terms in Eq.\((2.17)\) can be neglected since $V(\phi_{\text{turn}}) \approx 0$ and $vr_{\text{turn}} \gg 1$. Substituting the above results into Eq.\((2.5)\) and rescaling the variables $t$ and $r$ as

$$t \rightarrow (1 - 8\pi G \phi_{\text{turn}}^2 r^2)^{-\frac{1}{2}} e^{-\delta_{\text{turn}} t}$$

$$r \rightarrow (1 - 8\pi G \phi_{\text{turn}}^2 r^2)^{\frac{1}{2}} r,$$

we obtain a metric after a Wick rotation, which describes the region inside the outer wall but outside the global monopole:

$$ds^2 = -dt^2 + dr^2 + r^2(1 - 8\pi G \phi_{\text{turn}}^2)(d\theta^2 + \sin^2 \theta d\phi^2).$$  

Although the actual metric is not completely flat due to the additional small terms in Eq.\((2.17)\), the observer inside the $n = 1$ bubble feels no gravitational force exerted by the
global monopole apart from the tiny effects from the monopole core and the energy difference between the true vacuum $v$ and the maximum value of the $n = 1$ bubble $\phi_{\text{turn}}$, $V(\phi_{\text{turn}}) - V(v)$. This phenomenon can be explained by a Newtonian gravitational potential. The radial component of tension $-T^r_r$ also has a long range term such as

$$T^r_r = -\frac{1}{2}\left(1 - \frac{2GM}{r}\right)\left(\frac{d\phi}{dr}\right)^2 + \delta_n \frac{\phi^2}{r^2} + V,$$  \hspace{1cm} (2.23)

which cancels the energy density $T^t_t$ in (2.13) in the Newtonian limit of the Einstein equations: $\nabla^2 \Phi = 8\pi G(T^t_t - T^\nu_\nu) \approx 0$ at $r \approx r_{\text{turn}}$. However, since the metric (2.22) describes a space with a deficit solid angle, if we consider a light signal propagating from a source to an observer, the observer inside the $n = 1$ bubble must notice the light bending due to the deficit solid angle $\Delta = 8\pi G v^2$, as is the case of a straight cosmic string. Thus a rough evaluation gives the angular separation $\delta \varphi \sim 8\pi G v^2 \sim \text{few arcsec}$ at a typical grand unification scale $v \sim 10^{16}$ GeV, which can be observable.

For the phase transition in the supermassive scale, the absolute value of $M_{\text{turn}}$ in Eq. (2.19) is of the order of the Planck scale, $|M_{\text{turn}}| \sim M_{\text{Pl}}$, and then the first term in Eq. (2.17) becomes considerably large. Let us discuss the structure of the spacetime in this case. Assuming the outer wall is extremely thin, i.e., the third term in the right hand side of Eq. (2.17) is negligible inside the bubble, we obtain a metric around $\phi(r) \sim \phi_{\text{turn}}$ as

$$ds^2 = -\left(1 - 8\pi G v^2 - \frac{2GM_{\text{turn}}}{r}\right)e^{2\delta_0} dt^2 + \left(1 - 8\pi G v^2 - \frac{2GM_{\text{turn}}}{r}\right)^{-1}dr^2 + r^2(d\theta^2 + \sin^2 \theta d\varphi^2).$$ \hspace{1cm} (2.24)

Here we make a crude assumption that the region between the inner wall and the outer wall ($R_m \leq r \leq R_{n=1}$, see Fig. 2) is described by the above metric in Eq. (2.24), and the region inside the inner wall ($r \leq R_m$) is approximated to be de Sitter spacetime:

$$ds^2 = -\left(1 - \frac{8}{3}\pi GV(0)r^2\right)e^{2\delta_0} dt^2 + \left(1 - \frac{8}{3}\pi GV(0)r^2\right)^{-1}dr^2 + r^2(d\theta^2 + \sin^2 \theta d\varphi^2).$$ \hspace{1cm} (2.25)

Equation (2.8) says that the change of $\delta(r)$ can be neglected inside the monopole, which is supported by Fig. 1(b), and thus we take $\delta_0 \approx \delta_{\text{turn}}$ under our assumption. From the continuity of the metric and its first derivative with respect to $r$, we estimate the size of the global monopole (or equivalently the position of the inside bubble wall) as $R_m = 1/\sqrt{4\lambda(3 + 2\alpha)}v$ and $M_{\text{turn}} = -\sqrt{4\lambda(3 + 2\alpha)} v < 0$. It is confirmed by envisaging the integral in (2.19): if we divide the integration domain into $(0, R_m)$ and $(R_m, r_{\text{turn}})$, and then substitute the values of the scalar amplitude, i.e., $\phi = 0$ in $(0, R_m)$ and $\phi = \phi_{\text{turn}}$ in $(R_m, r_{\text{turn}})$, the integral of the core region has only the negative contribution to $GM_{\text{turn}}$. This result is consistent with the known result that the global monopole inside the $n = 1$ bubble does not form a black hole even at the Planck scale [14]. It can also be checked by the radial motion of a test particle governed by the geodesic equation

$$\frac{d^2r}{d\tau^2} = \frac{d}{dr}\left(\frac{GM}{r}\right) - \left(1 - \frac{2GM}{r}\right)\frac{d\delta}{dr},$$ \hspace{1cm} (2.26)
where $\tau$ is the proper time of a test particle. Equation (2.26), together with (2.13), tells us that the acceleration increases in proportion to the radius around the center of the global monopole. Although it is a weak gravity case ($v/M_{Pl} = 0.1$), Fig. 4 shows a typical example consistent with the above argument. Here a question arises: what is the structure of a spacetime manifold which is formed when the deficit solid angle is equal to or greater than $4\pi$? In the case of local cosmic strings, when the deficit angle is equal to or greater than $2\pi$, a possible two-dimensional spatial manifold is described by cylinder or two sphere [20]. However, it is an open question for the global monopole.

![Figure 4: The acceleration of a test particle $d^2r/d\tau^2$ versus the radius $r$ for $\lambda = 1$ and $\alpha = 0.1$. The acceleration denoted by the solid line is always positive and increases at the core of the global monopole for a weak gravity case ($v/M_{Pl} = 0.1$).](image)

**B. Nucleation Rate**

We now turn to the evaluation of the nucleation rate of bubbles; specifically the values of the Euclidean action at two stationary points of $n = 0$ ($B_0$) and of $n = 1$ ($B_1$) bubbles are of our interest. Suppose that there exists a barrier between a local minimum of $B_0$ and $B_1$, and the maximum value of $B$ between $B_0$ and $B_1$ can be set to be $B_{\text{top}}$. When each valley is so deep that the height of the barrier at the hilltop $B_{\text{top}}$ evaluated from the ordinary bubble point is larger than the difference of two local minima at the bottoms of valleys, i.e., $B_{\text{top}} - B_1 \gg B_1 - B_0$, then the total decay rate from a metastable phase into a stable phase per unit volume can be estimated to be the sum of the nucleation rate of each bubble

$$\Gamma = \Gamma^{(0)} + \Gamma^{(1)}. \quad (2.27)$$

It means that both $n = 0$ and $n = 1$ solutions give distinct decay channels where each solution describes the nucleation of bubbles with the critical size. Moreover, if the tunneling
action for each bubble is larger than unity, the nucleation rate for the $n$-th bubble takes the exponential form

$$\Gamma^{(n)} = A_n e^{-B_n}.$$  \hspace{1cm} (2.28)

On the other hand, if the tunneling action is of the order of unity or smaller, which corresponds to thick-wall bubbles in the high temperature limit, the exponential formula (2.28) is no longer valid. Here we take a heuristic viewpoint and keep our analysis on the basis of the above formula in Eq. (2.28), notwithstanding the above possibility. When the first-order phase transition is considered in a curved spacetime, the background spacetime itself experiences time evolution, e.g., inflation in the metastable vacuum region and this expansion of spatial volume induces a sudden drop in temperature. Therefore, the shapes of bubbles or equivalently the configurations of the scalar field change, and hence the formula (2.28) should not be applied. Here we discuss the probability to nucleate bubbles when the system is initially in the metastable phase and the temperature change can be neglected \cite{21}. Under this restriction $B_n$ can be approximated to be the values of action (2.1) for a given $n = 0$ or $n = 1$ bubble since we consider only the decay from a spacetime with a positive cosmological constant, $V(0) > 0$, into a spacetime with zero cosmological constant, $V(v) = 0$.

We plot a dimensionless value $B'_n \equiv (T/v)B_n$ for various $v/M_{Pl}$ and $\alpha$ in Figs. 5 and 6, respectively. Figure 5 shows that $B'_n$ becomes small for large $v/M_{Pl}$, which implies that the materialization of both $n = 0$ and $n = 1$ bubbles is more likely at higher energy scales. This result is consistent with the fact that the gravitational contribution to the total energy is always negative, as we have already seen in Fig. 3.

![Figure 5: The values of $(T/v)B_n$, shown versus the vacuum expectation value of the scalar field in unit of the Planck scale, $v/M_{Pl}$](image)

$n = 0 \cdots \cdots$

$n = 1$

Figure 5: The values of $(T/v)B_n$, shown versus the vacuum expectation value of the scalar field in unit of the Planck scale, $v/M_{Pl}$. The solid and dotted lines correspond to an $n = 0$ bubble and an $n = 1$ bubble, respectively.
When we have a model of a first-order phase transition with several decay channels, an interesting question may be which decay channel is dominant, which is illustrated by the ratio of the two decay rates, $\Gamma^{(1)}/\Gamma^{(0)}$. The prefactors $A_n$ usually are assumed to be of the order of $m_{H}^{3}T$ at high temperature. If we neglect the difference between $A_0$ and $A_1$, then the relative decay rate is determined by the exponential factor,

$$\frac{\Gamma^{(1)}}{\Gamma^{(0)}} \sim \exp \left[ -\frac{v}{T} (B'_1 - B'_0) \right].$$

(2.29)

As expected, the $n = 1$ bubble solutions take higher values of action, i.e., $B_1 > B_0$, regardless of the shape of the scalar potential (see Fig. 6) and of the strength of gravitation (see Fig. 5). From Fig. 6, we read that $B'_1 - B'_0$ is large in the thin-wall limit (small $\alpha$), because the leading contribution of action difference $(v/T)(B'_1 - B'_0)$ can be understood as the energy to support the global monopole in an $n = 1$ bubble, of which the long-range tail of the global monopole, $T_i \sim v^2/r^2$, consumes the energy proportional to the radius of the $n = 1$ bubble. On the other hand, if the bubble wall is relatively thick, $B'_1 - B'_0$ becomes small, and then the nucleation of $n = 1$ bubbles is not negligible. When the relative ratio of the exponentials (2.29) is not far from the order of unity, we should take into account the ratio of prefactors $A_1/A_0$ in order to determine the dominant decay channel. It is difficult to compute $A_n$ even for an $n = 0$ bubble in a flat spacetime. In a curved spacetime it has units of energy to the fourth power and is expected to be of the order of $m_{H}^{3}T$. To get some information on the
ratio of the prefactors, let us consider bubbles in a flat spacetime \[\frac{\Gamma^{(1)}}{\Gamma^{(0)}} \sim \left(\frac{B_1'}{B_0'}\right)^6 \exp\left[-\frac{v}{T}(B_1' - B_0')\right],\] (2.30)

where the system for the fluctuations around each classical solution includes six zero modes (three from spatial translations and another three from spatial rotations). As explained before, \(B_1/B_0 - 1\) represents the ratio of energy to make a global monopole to that to generate a bubble, thus \(B_1/B_0\) tends to one in the thin-wall limit and a few in the thick-wall limit. If we just replace the values of the action in a flat spacetime to those in a curved spacetime, we obtain several values of \(\Gamma^{(1)}/\Gamma^{(0)}\), as displayed in Table 1.

| \(\alpha \backslash \frac{v}{T}\) | 1.0     | 2.0     |
|----------------|---------|---------|
| 0.3            | 3.32\times10^{-2} | 1.98\times10^{-9} |
| 0.1            | 3.43\times10^{-9}  | 1.59\times10^{-18}  |
| 0.03           | 5.23\times10^{-13} | 1.47\times10^{-29}  |

Table 1. Values of \(\Gamma^{(1)}/\Gamma^{(0)}\) for \(\lambda = 1\) and \(v/M_{Pl} = 0.1\).

We find that monopole-bubbles become more likely to be nucleated at high temperature and in the relatively thick-wall case. (Remember that larger \(\alpha\) corresponds to a smaller potential barrier, which creates a bubble with a thicker wall.) Although \(B_1\) is always larger than \(B_0\), there may exist some parameter region of the scalar potential where \(n = 1\) bubbles cannot be neglected.

### III. Evolution of Monopole-Bubbles

When we consider a first-order phase transition in the framework of a finite-temperature field theory with imaginary time, high-temperature bubbles are given by static solutions of the Euclidean equations, and they are also static solutions of the Lorentzian equations. It is obvious that the bubbles start to evolve immediately after their nucleation, so a way of description is to borrow the physics of combustion processes when the environment keeps the temperature high enough \([13]\), which is indeed the case in a flat spacetime. Once the gravity is taken into account in the early universe, the background universe is expanding and then it is rapidly cooled down to zero temperature. The motion of bubbles eventually follows the zero-temperature classical dynamics. An accurate bubble dynamics may be as follows. When the bubble larger than the critical size is nucleated, the detonation process induces the growth of it and simultaneously the region outside the bubble begins to expand due to the gravitational effect. In the next step the variation of temperature requires the inclusion of whole complex ingredients into the evolution procedure of bubbles; for instance, the temperature dependence of the the combustion processes, the effect of gravity on the classical evolution of bubbles, the change of the effective potential due to temperature and quantum
corrections, possible reheating, the generation of Goldstone bosons and so on. However, we already know what actually happens to $n = 0$ bubbles: if the expansion rate of the universe is large enough, the motion of an $n = 0$ bubble turns out to be that of a bubble governed by the classical equations of motion at zero temperature \[2, 22, 23\]. Here we suppose that the above simplification for $n = 0$ bubbles is applied to the case of $n = 1$ bubbles in a similar manner and concentrate our interest only on the evolution of $n = 1$ bubbles due to classical effects.

Our task is now reduced to solving the Einstein equations and the scalar field equation by use of numerical analysis. In order to examine the evolution of $n = 1$ bubbles after the temperature decreases to zero, we should prepare the initial configurations. However, they should be different from our static $n = 1$ bubble solutions of the Euclidean equations because they are also static solutions of equations of the motion after a Wick rotation. The effect due to the change of temperature must be reflected when we prepare the initial conditions. Suppose that the various effects mentioned above give rise to the evolution of bubbles and then the structure of the spacetime manifold undergoes changes, there are too many directions of perturbations to include such effects into the initial conditions. Even under this complicated situation, we may have several disciplines which make the problem consistent and tractable:

1. We keep the spherical symmetry;
2. The initial configuration for the scalar fields and the corresponding gravitational fields keeps more or less the characteristics of those for $n = 1$ static bubble solutions. Furthermore, previous work tells us that the initial size of an O(4)-symmetric bubble at zero temperature is larger than that of an O(3)-symmetric static bubble in the high temperature limit, and that the O(4)-symmetric bubble expands after a Wick rotation. (If a bubble is smaller than the static one, the amount of surface energy required to grow is larger than the released bulk energy so that it is energetically favorable to shrink.) Thus we assume two initial configurations of the scalar amplitude $\phi$. One is a scaled $n = 1$ bubble solution in which the size of the global monopole is also increased:

\[
\phi(0, r) = \phi_{n=1}\left(\frac{r}{c_1}\right) \quad (c_1 > 1), \tag{3.1}
\]

and the other involves the initial shrinking of monopole radius but the expansion of the outer bubble wall:

\[
\phi(0, r) = \phi_{n=1}\left(\frac{r}{1 + (c_2 - 1) \tanh(r/r_{\text{turn}} - 1)}\right) \quad (c_2 > 1). \tag{3.2}
\]

Here we solve the equations of motion by numerical calculation. As a coordinate system in a Lorentzian spacetime, we adopt the following form,

\[
d s^2 = -d t^2 + A^2(t, \chi) d \chi^2 + B^2(t, \chi) \chi^2 (d \theta^2 + \sin^2 \theta d \varphi^2). \tag{3.3}
\]

With the metric (3.3), we write down the equations of motion (2.2) and (2.3) as

\[
\ddot{\phi} - K \dot{\phi} - \frac{\phi''}{A^2} - \left(-\frac{A'}{A} + \frac{2B'}{B} + \frac{2}{\chi}\right) \frac{\phi'}{A^2} + \frac{2\delta_{n1}\phi}{\chi^2 B^2} + \frac{dV}{d\phi} = 0, \tag{3.4}
\]
\[-G_t^t \equiv (2K - 3K_\theta^\theta)K_\theta^\theta - \frac{2}{A^2}\frac{B''}{B} + \frac{B'}{A^2B^2}\left(\frac{2A'}{A} - \frac{B'}{B}\right) + \frac{2}{\chi A^2}\left(\frac{A'}{A} - 3\frac{B'}{B}\right)
\]
\[-\frac{1}{\chi^2}\left(\frac{1}{A^2} - \frac{1}{B^2}\right) = 8\pi G\left(\frac{\dot{\phi}^2}{2} + \frac{\dot{\phi}^2}{2A^2} + \frac{\delta_{m1}\phi^2}{\chi^2B^2} + V\right)\]  \hspace{1cm} (3.5)

\[\frac{1}{2}G^t_t \equiv K_\theta^\theta' + \left(\frac{B'}{B} + \frac{1}{\chi}\right)(3K_\theta^\theta - K) = 4\pi G\dot{\phi}\dot{\phi}' \]  \hspace{1cm} (3.6)

\[\frac{1}{2}(G^\chi_\chi + G^\theta_\theta + G^\phi_\phi - G^t_t) \equiv \ddot{K} - K^2 + 4KK_\theta^\theta - 6K_\theta^2 = 8\pi G(\ddot{\phi}^2 - V), \]  \hspace{1cm} (3.7)

where the overdot \(\dot{}\) and the prime \(\prime\) stand for the partial derivative with respect to \(t\) and \(\chi\) in Eq. (3.3), respectively. Following Ref. [24], we have introduced the extrinsic curvature tensor of a \(t = \text{const}\) hypersurface, whose components are given by

\[K^\chi_\chi = -\frac{\dot{A}}{A}, \hspace{1cm} K^\theta_\theta = K^\phi_\phi = -\frac{\dot{B}}{B}, \]  \hspace{1cm} (3.8)

and we have denoted its trace by \(K \equiv K^i_i\).

For initial data for the metric, we assume an asymptotically flat de Sitter metric just for a technical reason. Although the metric of a Euclidean spacetime is asymptotically closed de Sitter spacetime, the effect of the spatial curvature is not so important as long as a bubble is smaller than the horizon, and hence our treatment can be verified. Specifically, we suppose \(A(t = 0, r) = B(t = 0, r) = 1\) and solve the constraint equations (3.5) and (3.6) to determine \(K(t = 0, r)\) and \(K_\theta^\theta(t = 0, r)\). This treatment is suitable for this system because we obtain

\[-\frac{K}{3} \approx -K_\theta^\theta \approx \sqrt{\frac{8\pi G}{3}}(-T_t^t), \]  \hspace{1cm} (3.9)

which approaches zero as \(r\) increases; we can construct an asymptotically flat spacetime without iterative integration. We have also assumed \(K(t = 0, r) < 0\), which means that every point in the spacetime is locally expanding. As for the configuration of the scalar field, we have supposed Eq. (3.1) or (3.2) and \(\Phi(t = 0, r) = 0\).

In order to solve the dynamical equations, we use a finite difference method with 1000 meshes. The evolution of a bubble is depicted by 5 dynamical variables, \(A, B, K, K_\theta^\theta\) and \(\phi\). Equations (3.8), (3.7) and (3.4) provide the next time-step of \(A, B, K\) and \(\phi\), respectively. At each step, we integrate (3.6) in the radial direction to obtain \(K_\theta^\theta\). In this way we have reduced spatial derivatives appearing in the equations, which may become seeds for numerical instability. The Hamiltonian constraint equation (3.5) remains unsolved during the evolution and is used for checking numerical accuracy. We keep numerical error less than 1 percent throughout calculations.

The results of our numerical computations are summarized as follows. Figure 7 shows that the outer wall of an \(n = 1\) bubble starts to expand, whether the bubble wall is thin or thick. Figure 7 also tells us that the velocity of the outer wall increases as time elapses. As the bubble grows, the wall becomes thinner and the energy accumulated inside the bubble moves out to support the expansion of the outer bubble wall (see Fig. 7-(b)). Therefore, after time
elapses sufficiently, the motion of the outer wall can be modeled by that of the extremely thin wall even for any bubble with initially a thick wall. We trace the time-evolution of the position of $\phi = 0.5v$ for the outer wall, as shown in Fig. 8. Its trajectory looks like a hyperbola, similar to that of the $n = 0$ bubble wall.

![Graph](image_url)

Figure 7: $n = 1$ bubble profiles for fixed $\lambda = 1$ and $v/M_{Pl} = 0.1$ associated with the classical evolution ($c_1 = 1.2$). $t/H^{-1} = 0, 0.5, 1.0$ are shown as dotted, dashed and solid lines, respectively. (a) thin-wall bubble of $\alpha = 0.1$, (b) thick-wall bubble of $\alpha = 0.3$. 

An interesting physical quantity at the moment is the terminal velocity in terms of the outer expanding coordinates. For an $n = 0$ bubble it is computed in the thin-wall approximation and, because of the gravitational effect, the terminal velocity of the wall is found to be smaller than the light velocity \[22\]. For an $n = 1$ bubble, because the spacetime between the inner wall and the outer wall is described by Eq.\(2.24\) and the spacetime outside the
outer wall is de Sitter spacetime \((2.25)\), we obtain a junction condition,
\[
e^{+}\sqrt{\left(\frac{dR}{d\tau}\right)^{2}} + 1 - \frac{8}{3}\pi GV(0)R^{2} - e^{-}\sqrt{\left(\frac{dR}{d\tau}\right)^{2}} + 1 - 8\pi Gv^{2} + \frac{2G|M_{\text{turn}}|}{R} = -4\pi G\sigma R, \tag{3.10}
\]
where \(R\), \(\tau\), and \(\sigma\) are the circumference radius, the proper time, and the surface energy density of the shell, respectively. \(\epsilon^{+}\) and \(\epsilon^{-}\) are the signs of the angular component of the extrinsic curvature of the 2+1-world-hypersurface. In general \(\epsilon^{+}\) and \(\epsilon^{-}\) take +1 or −1, but in the present case they are always positive. Neglecting the term of \(2GM_{\text{turn}}/r\) in Eq. (3.10), we obtain the initial radius of the \(n = 1\) bubble,
\[
R_{n=1}(0) = \frac{R_{n=0}(0)}{2}\left(\sqrt{1 - 8\pi Gv^{2}} + \sqrt{1 - 8\pi Gv^{2} + \frac{4v^{2}}{R_{n=0}(0)}\sigma}\right), \tag{3.11}
\]
where \(R_{n=0}(0) \equiv 3\sigma/(V(0) + 6\pi G\sigma^{2})\) is the initial radius of the \(n = 0\) bubble. When the size of a bubble is sufficiently large, Eq. (3.11) reduces to
\[
\frac{R_{n=1}(0)}{R_{n=0}(0)} \approx \sqrt{1 - 8\pi Gv^{2}}, \tag{3.12}
\]
which shows why the size of \(R_{n=1}(0)\) is smaller than \(R_{n=0}(0)\) (Fig. 2-(a)). We numerically solve Eq. (3.10) and obtain an approximate formula for the terminal velocity:
\[
v_{\text{terminal}} \approx 1 - \frac{4\pi GV(0)}{3|R_{n=0}(0)|^{2}}, \tag{3.13}
\]
which agrees with that of an \(n = 0\) bubble. Note that we did not consider the temperature effect here in Eq. (3.13), and the inclusion of the temperature effect can decrease the terminal velocity further \([15]\). Now, let us explain why the long-range term does not change the terminal velocity \(v_{\text{terminal}}\). When the velocity of the outer wall reaches its terminal velocity, an \(n = 1\) bubble can be approximated as a thin-wall bubble and the energy density \(T_{t}^{\phi}\) contributed from the expansion of the long-range tail of the global monopole is
\[
T_{t}^{\phi} = \frac{1}{2}\ddot{\phi}^{2} + \frac{1}{2A^{2}}\dot{\phi}^{2} + \frac{1}{\chi^{2}B^{2}}\dot{\phi}^{2} + V
\approx \frac{v^{2}}{\chi^{2}B^{2}}, \tag{3.14}
\]
since the scalar field \(\phi\) stays at the vacuum \(v\). This contribution is considerable at the initial stage of the evolution; however, it subsides to zero as the \(n = 1\) bubble grows (\(B\chi\) increases). Therefore, the energy difference per unit surface area of the bubble becomes the same as that of the \(n = 0\) bubble. We thus conclude that the terminal velocity of an \(n = 1\) bubble is equal to that of an \(n = 0\) bubble.

Another important motion for the \(n = 1\) bubble is of course that of the inner wall. Figure 8 shows the trajectory of the position of \(\phi = 0.5v\) for the inner wall. We find that the inner wall just oscillates, and we can expect that this oscillation will be damped gradually. Since we
assumed initial configurations with both enlarged (Eq.(3.1)) and shrunken (Eq.(3.2)) cores of the global monopole, the above result implies that the global monopole inside an \( n = 1 \) bubble is stable against the perturbations of the scalar amplitude,

\[
\dot{\phi}^a = \hat{\phi}^a(\chi) \to \hat{\phi}^a(\phi(t, \chi) + \delta\phi(t, \chi)).
\] (3.15)

From the above discussion, we summarize the evolution of bubbles as follows. When the spherical symmetry is assumed for the scalar field, the motion of both \( n = 0 \) and \( n = 1 \) bubbles is represented by the expansion of the bubble wall. The global monopole formed in the \( n = 1 \) bubble remains to be stable and its long-range energy tail in Eq.(3.14) keeps growing before bubble percolation by consuming a part of the false vacuum energy (proportional to the increment of bubble radius) obtained from the growth of the true vacuum bubble (proportional to the increment of spatial volume). It explains why one need not worry about the huge amount of energy necessary to maintain a large-size global monopole if it is formed through the first-order phase transition. As a monopole-bubble is larger, the ratio of the energy for keeping the global monopole to the energy obtained through the growth of the bubble is smaller. Furthermore, this ratio finally becomes negligible for an extremely large bubble. We have also shown that the global monopole is a stable object during the evolution of a spherically symmetric \( n = 1 \) bubble; however, the stability and its physical implication due to the distortion of the bubble or the collision of two bubbles remain topics for future work.

One way to understand the stability of the inner bubble wall against the perturbation of the scalar amplitude in Eq.(3.15) is to count the number of negative modes by considering the small fluctuation around a given \( n = 1 \) bubble solution. Since it is too difficult to calculate the negative modes with the inclusion of gravity even for a \( n = 0 \) bubble, let us attempt to do it in a flat spacetime. Under small static fluctuations around a spherically-symmetric bubble solution \( \delta\phi^a_n = \phi^a_n \sum c_k \psi_k(x^i) \), we obtain a Schrödinger-type equation by varying the scalar equation (2.2):

\[
\left( -\nabla^2 + \frac{d^2V}{d\phi^2} \right) \hat{\phi}^a_n \psi_k(x^i) = \lambda_k \hat{\phi}^a_n \psi_k(x^i),
\] (3.16)

where \( x^i \) \((i = 1, 2, 3)\) denote spatial Cartesian coordinates and a subscript \( n \) takes 0 or 1, corresponding to an \( n = 0 \) or \( n = 1 \) bubble. For an \( n = 0 \) bubble with \( \hat{\phi}^a_0 = (0, 0, 1) \), Eq.(3.16) contains a unique negative mode of which the wave function is that of a nodeless \( s \)-wave \[2, 12\]. In the case of \( n = 0 \) bubbles, Eq.(3.16) becomes

\[
\left( -\nabla^2 + \frac{2}{\chi^2} + \frac{d^2V}{d\phi^2} \right) \psi_k(x^i) = \lambda_k \psi_k(x^i),
\] (3.17)

and then the lowest mode is not the nodeless \( s \)-wave mode but the \( l = 1 \) mode with a single node at \( \chi = 0 \). For \( n = 0 \) bubbles, the unique negative mode was used for the explanation of the motion of its bubble wall. We already showed through the numerical computation of \( n = 1 \) bubbles that the inner bubble wall is stable but the outer bubble wall starts to evolve as time goes. Thus we are likely to interpret this unique negative mode obtained from radial
perturbation as the one related with the motion of the outer bubble wall, albeit we need further study to reach a definite conclusion. However, for perturbations in all directions, it is extremely difficult to count the number of zero modes for an $n = 1$ bubble in a curved spacetime and the $n = 1$ bubble can also have the possibility of containing more than one zero modes as happens for the $n = 0$ bubble [24].

Finally, let us discuss the case where the scale of symmetry breakdown $v$ approaches the Planck scale $M_{Pl}$. When the de Sitter horizon is comparable to or smaller than the radius of a bubble, the procedure of a first-order phase transition may not follow the scenario in Ref. [8] but drastic change occurs. For $n = 0$ bubbles, one may bring up two proposed scenarios of phase transitions: One is the one-bubble universe formed inside a thin-wall bubble [23] and the other is the Hawking-Moss type phase transition [27]. If we follow the viewpoint of Ref. [23] for the $n = 1$ bubble, it is probable that a bubble with a super-horizon-sized monopole is nucleated. Once such a configuration is formed, interesting phenomena are expected: the evolution of global monopoles [20] or the defect inflation at the monopole site [8, 9]. If we consider the model of interest with the $SO(3)$ gauge coupling, the nucleated false vacuum island is dressed by gauge fields, and consequently shows defect inflation and the creation of a Schwarzschild-like wormhole [7, 28, 10].

IV. Conclusion and Discussions

In this paper we have studied a first-order phase transition in an $O(3)$-symmetric model in a curved spacetime and at high temperature. We found a new bubble solution which describes another possible decay channel. Different from an ordinary bubble, it contains a matter lump at the center of the bubble, which is nothing but a global monopole supported by the winding between the internal group $O(3)$ space and the real space. It manifestly shows how the continuous internal symmetry of the theory can play an important role from the beginning of bubble nucleation.

The obtained monopole-bubble ($n = 1$ bubble) has the following characteristics. First, in addition to the outer bubble wall which distinguishes the true vacuum region inside the bubble from the false vacuum environment, there is another inner bubble wall which divides the core of the global monopole and the true vacuum region inside the bubble with the long-range energy tail of the global monopole. Since the formation of the global monopole consumes a part of the energy obtained by the difference between false vacuum energy and true vacuum energy, the size of the monopole-bubble is slightly larger than that of the ordinary bubble for thick-wall bubbles. However, strikingly enough, the opposite is true for the monopole-bubbles with sufficiently thin walls. Second, the long-range tail of the global monopole is terminated by the cutoff “outer bubble wall”, and the total energy to form the global monopole is proportional to the radius of the monopole-bubble. Furthermore, while the outer wall expands, the global monopole itself is a stable configuration before bubble collisions. It suggests a new mechanism for the production of global monopoles through a first-order phase transition despite its infinite energy. The spacetime inside the monopole-bubble is flat with solid deficit angle, but the black hole is not produced at the center even at the Planck scale. Extending our analysis to general models, we expect that other topological
defects or non-topological solitons can also be created through a first-order phase transition as \textit{solitonic-bubble} solutions. We should take into account a new possibility “soliton production at the bubble nucleation era” in addition to the soliton production by the horizon or bubble collisions [29, 25, 30].

Since the action of a monopole-bubble is larger than that of an ordinary bubble, the production rate of monopole-bubbles is exponentially suppressed in comparison with that of ordinary bubbles at low temperature; however, it is enhanced considerably at high temperature. We showed that the production rate of monopole-bubbles can be comparable to that of ordinary bubbles for some parameter range of the scalar potential, so the first-order phase transition at high temperature described by such models can proceed through this new decay channel by the nucleation of monopole-bubbles. If we look at the evolution of the monopole-bubble by classical dynamics after the background universe is cooled down to zero temperature, we can easily notice the following: 1. The outer bubble wall immediately starts to expand and reaches the terminal velocity smaller than light velocity, just as the case of an ordinary bubble; 2. The inner bubble wall remains stable and the long-range tail of the global monopole grows as the outer wall expands.

It is well known that topological defects are produced by the Kibble mechanism or bubble collisions in the early universe [29, 30], this monopole-bubble nucleation can be a new mechanism of producing global monopoles through a first-order phase transition in the early universe. If we introduce the gauge coupling, the character of a monopole inside the bubble changes from the global one to the \textquoteleft t Hooft-Polyakov monopole [31]. It may be interesting to apply this mechanism to the production of local monopoles and compare with the results obtained in other ways [25]. If the production rate of monopole-bubbles is too large, then one must worry about monopole abundance in the early universe, though it can be diluted by inflation. For computing the number of monopoles which survive after the completion of the first-order phase transition, further study of bubble collisions is needed, particularly between $n = 1$ and $n = 0$ bubbles, or between $n = 1$ and $n = 1$ bubbles. Finally, we should emphasize again that the above procedure to nucleate new bubbles involving a soliton in its center is due to continuous symmetry and can be generalized for any continuous global or local symmetry which has an appropriate winding between internal group space and spacetime. However, it is unclear for the discrete symmetry case in (3+1) dimensions [32] and needs further study.

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