Antenna Azimuth Position Control Using Fractional Order PID Controller Based on Genetic Algorithm

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Abstract. This work shows the design procedure of fractional-order proportional–integral–derivative (FOPID) controllers for an antenna azimuth position system based on genetic algorithm optimization approach. FOPID controllers are considered as a special type of the classical PID controller in which the derivative and integral elements have orders of fractions between zero and one. Therefore, FOPID controllers comprise two additional variables (μ and λ) in comparison with the typical PID controller. The FOPID is designed to control the azimuth angle of the antenna. Genetic algorithm (GA) will be employed to find an optimal value for the five FOPID controller’s gains using different type of fitness function including mean square error (MSE), integral square error (ISE) and integral time square error (ITSE). The achieved results exhibit an excellent steady and transient state performance of system, for instant fast settling and rise times as well as low present of peak overshoot.

1. Introduction

Antenna azimuth position control typically involves elevation control so that antennas, solar panels and sensors are accurately positioned. For instance, antennas are generally perpendicularly directed to an earth location while solar panels should be placed towards the sun for maximum power generation [1]. In general, the classical PID controller is considered as the leading type of feedback control in the industry. This is mainly because of their simple design and great ability to produce reasonable transient and steady state responses [2][3]. However, classical controllers reveal some downsides that may include sensitivity to variation in system parameter, performance decrease while system order increases and finally poor functioning with nonlinear systems [4], [5]. Therefore, FOPID controllers have lately gained great attention in the area of control system design in which using a non-integer PID controller is a successful way of enhancing the response of their traditional counterparts [6]. It has been concluded that FOPID controller has more freedom in the tuning process of the control parameters. In comparing with classical PID, fractional order PID reveals a better response characteristic at high order and nonlinear systems [7]. However, the design process of fractional-order PID contains some amount of complexities when compared with typical PID controllers. This is mainly due to the fact that FOPID has two additional integral tuning parameters that should be taken into consideration [8].

Lately, a significant amount of literature has been presented to discuss the tuning approaches of FOPID controller. These techniques are mainly classified as optimized, analytical and graphical approaches [9]. In optimization-based approach, the gain parameters of the controller are calculated to achieve some preferable time domain characteristics. Genetic algorithm is considered as one of the most commonly applied optimization techniques for tuning such controllers. This tuning approach is initially depending...
on estimating the fractional order calculus, then setting other gain parameters of the controller similarly [10].

2. Antenna Model

The model of the antenna azimuth control system that has been used is shown in figure 1. This system has five main subsystems. The potentiometers used to convert the azimuth angles (in deg.) to voltages while the power amplifier is used as the drive circuit for the motor. The motor and the load are presented in one system where its equation takes all the necessary parameters. Finally, the gearbox gives the gear ratio to increase the applied torque while reducing the speed. For the preamplifier used in figure 2, it will be replaced with the FOPID controller [11].

![Figure 1. Layout of the antenna azimuth position control system [11]](image1)

In the antenna system, the schematic block diagram is represented in figure 2 where it shows the physical subsystems with their parameters and internal connection.

![Figure 2. Schematic diagram of the antenna azimuth position control system [11]](image2)

Where:

- $J_a$ and $J_L$ are motor inertial and load inertial constants respectively (kg.m²)
- $D_a$ and $D_L$ are motor dampening and load dampening constants respectively (N-m s/rad)
$R_a$: Motor resistance (ohm)

$K_b$ is back EMF constant (V.s/rad).

$K_t$ is motor torque constant (N-m/A).

$N_1$ and $N_2$ are the gear teethes.

The block diagram shown in figure 3 represents the close loop system of the antenna azimuth position control system.

By using equations (1-4), the transfer function of the motor and load can be represented in the simplified equation (5).

$$K_m = \frac{K_t}{J R_a}$$  \hspace{1cm} (1)

$$a_m = \frac{D_m R_a + K_b K_t}{J R_a}$$  \hspace{1cm} (2)

$$J = J_a + J_e (K_g)^2$$  \hspace{1cm} (3)

$$D_m = D_a + D_e (K_g)^2$$  \hspace{1cm} (4)

$$\frac{\phi_m(s)}{E_a(s)} = \frac{K_m}{s(s+a_m)}$$  \hspace{1cm} (5)

The gear ratio used is represented in equation (6) where it used to increase the speed supplied by the motor while increasing the provided torque. This is useful for the adjustment of the inertial components and the damping.

$$K_g = \frac{N_1}{N_2}$$  \hspace{1cm} (6)

The overall parameters used for this system are represented in table 1.

| Table 1. Schematic and block diagram parameters [11] |
|---------------------------------|-----------------|-----------------|-----------------|
| Schematic Parameter | Value | Block diagram parameters | Value |
| V | 10 | $K_{pot}$ | 0.318 |
| n | 10 | $K1$ | 100 |
| R_a | 8 | $a$ | 100 |
| J_a | 0.02 | $K_g$ | 0.1 |
| D_a | 0.01 | $K_m$ | 2.083 |
| $K_b$ | 0.05 | $a_m$ | 1.71 |
| $K_t$ | 0.5 | | |
After substituting all the parameters in table 1, the overall equation of the azimuth of the antenna system is given by equation (7).

\[
\frac{\theta_d(s)}{\theta_i(s)} = \frac{318}{s^3 + 101.7s^2 + 171s + 318}
\]  

(7)

3. Fractional PID Controller Design

The fractional Order PID controller, also referred to as PI λ D μ, depends on the fractional calculus that includes two added parameters, which are (λ, and μ), with regards to the classical PID controller. These two additional parameters tend to increase the flexibility and robustness of the controller and enhance the time domain performance of the system. The main structure of the fractional order PID controller applied to closed loop systems is illustrated in figure 4.

The main equation of FOPID controller that defines the control action \( u(t) \) is given in equation (8).

\[
u(t) = kp \ e(t) + ki \ D^{-\lambda} \ e(t) + kd \ D^\mu \ e(t)
\]  

(8)

The FOPID controller is designed depending on the fractional order theory. The fractional differential and integral can be written by using as \( aD^\mu_t \) and \( at^\lambda \). The symbol μ represents the differential order and λ is the integral order while a and t are the lower and upper bounds. There are mainly two kinds of fractional calculus. These are Riemann-Liouville and Grunwald-Letnikov calculus [12], [13]. Under zero initial condition, the Riemann-Liouville calculus can be defined in the S-domain as in equation (9)

\[
\int_0^\infty e^{-st} \ aD^x_t f(t) \ dt = S^x F(s)
\]  

(9)
By applying Riemann-Liouville calculus equation with Laplace transform to the control action equation, the controller output will be as represented in equation (10).

\[ u(s) = \left( k_p + \frac{k_i}{s} + k_d s^\mu \right) e(s) \text{ Where } (\lambda & \mu > 0) \]  

(10)

4. Genetic Algorithm

Genetic Algorithm (GA) is one of the most applied optimization search engines that imitates the chromosome natural development process. It starts with the representation of the chromosome structure that is formed by all the elements that needs to be optimized to achieve the required response. Fractional order PID design procedure requires a selection of five gains values (Kp, Ki, Kd, \(\mu\) and \(\lambda\)) in order to achieve an optimal output response. In general, GA comprises three main stages: Mutation, Crossover and Selection to enhance the initial population and hence produce a better response. The validity of the obtained solution is evaluated using a fitness function which is the measurement of the chromosome’s quality. The objective functions employed in this particular design process are the Integral Time Square Error (ITSE), Integral Square Error (ISE) and Mean Square Error (MSE) that have the following equations [14]:

\[ \text{ITSE} = \int_0^T t(e(t))^2 \, dt = \int_0^T t(r(t) - y(t))^2 \, dt \]  

(11)

\[ \text{ISE} = \int_0^T (e(t))^2 \, dt = \int_0^T (r(t) - y(t))^2 \, dt \]  

(12)

\[ \text{MSE} = \frac{1}{n} \sum_{i=1}^n (e(t))^2 \]  

(13)

The genetic algorithm process can be summarized in the flowchart shown in figure 5.

As per design requirements, maximum overshoot (Mp), Rise time (Tr) and Rise time (Ts) are required to be minimized, the fitness function selection was depending on that. ISE has the square function were used for dealing with the positive and negative values of the error and makes the system’s performance faster while the integration is to eliminate the steady state error when times goes large. ITSE has an extra element over ISE, which is the time; this is for further improvement of the system’s speed; however,
it could slightly increase the maximum overshoot. Finally, MSE has advantage over ISE and ITSE regarding the overshoot where it further decreases the maximum overshoot with no effect on the other specifications (Tr and Ts).

The main configuration of genetic algorithm tuning process for this particular design is implemented using a population size of 50 and 100 maximum number of generations with 0.05 and 0.2 selections and crossover probabilities respectively.

5. Simulation and Results
The first step in the simulation is to check the stability of the open loop system using Bode plot as shown in figure 6. As the gainmargin (Gm) and phase margin (Pm) are positive, the system is stable.

![Bode Plot](image)

**Figure 6.** Stability analysis of the open loop system using Bode plot.

The parameters of GA-FOPID controllers are shown in figures 7, 8 and 9 for MSE, ISE and ITSE fitness functions respectively, while the step responses of GA-FOPID controllers for MSE, ISE and ITSE fitness functions are plotted in figure 10.
Figure 7. Generation’s number of $K_p$, $K_i$, $K_d$, $\mu$ and $\lambda$ value GA- FOPID controller for MSE fitness function.

Figure 8. Generation’s number of $K_p$, $K_i$, $K_d$, $\mu$ and $\lambda$ value GA- FOPID controller for ISE fitness function.
Figure 9. Generation’s number of Kp, Ki, Kd, \( \mu \) and \( \lambda \) value GA-FOPID controller for ITSE fitness function.

Figure 10. Antenna azimuth position response of GA-FOPID controller with MSE, ISE and ITSE fitness functions.
The Parameters of GA-FOPID controllers and the transient response specifications are summarized in table 2.

| GA-FOPID Controller | KP   | Ki   | Kd   | λ    | μ    | Ts   | Tr   | % Mp |
|---------------------|------|------|------|------|------|------|------|------|
| MSE                 | 28.151 | 37.51 | 10.51 | 0.875 | 0.0451 | 0.2534 | 0.0455 | 0.0455 |
| ISE                 | 27.151 | 37.51 | 9.851 | 0.843 | 0.0513 | 0.2651 | 0.0470 | 3.4458 |
| ITSE                | 26.51  | 35.51 | 9.515 | 0.85  | 0.0752 | 0.2567 | 0.0457 | 3.4526 |

6. Conclusion
In this study, the antenna system has been modeled and analyzed in detail while its subsystems were described and transfer functions have been found for each of them. After that, this system is controlled by a FOPID controller with genetic algorithm optimization method. Three different fitness functions, MSE, ISE and ITSE, have been used. Results showed that FOPID controller with ISE and ITSE gives very similar results in term of %Mp and Tr; only Ts is further reduced with ITSE. However, the MSE fitness function gives better performance than ISE and ITSE in term of settling and rise times as well as low present of maximum peak overshoot.

7. References
[1] A. Zayed, “A New Discrete Self-tuning LQG Controller Applied to Geostationary Satellite System,” cetj.edu.ly, Accessed: Dec. 16, 2020. [Online]. Available: http://www.cetj.edu.ly/magazine/number2/18.docx.
[2] A. Saleem, H. Soliman, S. Al-Ratrout, and M. Mesbah, “Design of a fractional order PID controller with application to an induction motor drive,” Turkish Journal of Electrical Engineering & Computer Sciences, vol. 26, no. 5, pp. 2768-2778, 2018, doi: 10.3906/elk-1712-183.
[3] S. Ibrahim Khather, M. Almaged, and A. I. Abdullah, “Fractional order based on genetic algorithm PID controller for controlling the speed of DC motors,” International Journal of Engineering & Technology, vol. 7, no. 4, pp. 5386-5392, 2018, doi: 10.14419/ijet.v7i4.25601.
[4] M. A. Rahimian and M. S. Tavazoei, “Improving integral square error performance with implementable fractional-order PI controllers,” Optimal Control Applications and Methods, vol. 35, no. 3, pp. 303-323, 2014, doi: 10.1002/oca.2069.
[5] P. Cominos and N. Munro, “PID controllers: Recent tuning methods and design to specification,” IEE Proceedings-Control Theory and Applications, vol. 149, no. 1, pp. 46–53, 2002, doi: 10.1049/ip-cta:20020103.
[6] I. Podlubny, L. Dorcak, and I. Kostial, “On fractional derivatives, fractional-order dynamic systems and PIλDμ-controllers,” Proceedings of the 36th IEEE Conference on Decision and Control, vol. 5, pp. 4985-4990, 1997, doi: 10.1109/cdc.1997.649841.
[7] R. Duma, P. Dobra, and M. Trusca, “Embedded application of fractional order control,” Electronics Letters, vol. 48, no. 24, pp. 1526-1528, 2012, doi: 10.1049/el.2012.1829.
[8] D. Valério and J. S. da Costa, “Tuning of fractional PID controllers with Ziegler-Nichols-type rules,” Signal Processing, vol. 86, no. 10, pp. 2771-2784, 2006, doi: 10.1016/j.sigpro.2006.02.020.
[9] F. Merrikh-Bayat and M. Karimi-Ghartemani, “Method for designing PIλDμ stabilisers for minimum-phase fractional-order systems,” IET control theory & applications, vol. 4, no. 1, pp. 61-70, 2010, doi: 10.1049/iet-cta.2008.0062.
[10] V. Singh and V. K. Garg, “Tuning of PID controller for speed control of DC motor using soft computing techniques - A review,” International Journal of Applied Engineering Research, vol. 9, no. 9, pp. 1141-1148, 2014.
[11] Nise, Norman S. “Control systems engineering 6th edition.” New York (2011).
[12] Y. Q. Chen, I. Petráš, and D. Xue, “Fractional order control - A tutorial,” American control conference. IEEE, pp. 1397-1411, 2009, doi: 10.1109/ACC.2009.5160719.

[13] H. Zhao, W. Deng, X. Yang, X. Li, and C. Dong, “An optimized fractional order PID controller for suppressing vibration of AC motor,” Journal of Vibroengineering, vol. 18, no. 4, pp. 2205-2220, 2016, doi: 10.21595/jve.2016.16652.

[14] M. Almaged, S. I. Khather, A. I. Abdulla, and M. R. Amjed, “Comparative Study of LQR, LQG and PI Controller Based on Genetic Algorithm Optimization for Buck Converters,” 2019 11th International Conference on Electrical and Electronics Engineering (ELECO). IEEE, pp. 1012–1017, 2019, doi: 10.23919/ELECO47770.2019.8990572.