Full-Coupled Channel Approach to Doubly Strange s-Shell Hypernuclei

H. Nemura, S. Shinmura, Y. Akashi, and Khin Swe Myint

1 Institute of Particle and Nuclear Studies, KEK, Tsukuba 305-0801, Japan
2 Department of Information Science, Gifu University, Gifu 501-1193, Japan
3 College of Science and Technology, Nihon University, Funabashi 274-8501, Japan
4 Department of Physics, Mandalay University, Mandalay, Union of Myanmar

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We describe ab initio calculations of doubly strange, \( S = -2 \), s-shell hypernuclei (\( \Lambda^4\Lambda \), \( \Lambda^5\Lambda \), and \( \Lambda^6\Lambda \)) as a first attempt to explore the few-body problem of the full-coupled channel scheme for these systems. The wave function includes \( \Lambda \Lambda \), \( \Lambda \Sigma \), \( N \Xi \), and \( \Sigma \Sigma \) channels. Minnesota \( NN \), \( DZ' \), \( YN \), and simulated \( YY \) potentials based on the Nijmegen hard-core model, are used. Bound-state solutions of these systems are obtained. We find that a set of phenomenological \( B_0B_8 \) interactions among the octet baryons in \( S = 0 \), \(-1 \) and \(-2 \) sectors, which is consistent with all of the available experimental binding energies of \( S = 0 \), \(-1 \) and \(-2 \) s-shell (hyper-)nuclei, can predict a particle stable bound-state of \( \Lambda^4\Lambda \). For \( \Lambda^4\Lambda \) and \( \Lambda^5\Lambda \), \( \Lambda N - \Sigma N \) and \( \Xi N - \Lambda \Sigma \) potentials enhance the net \( \Lambda \Lambda - N \Xi \) coupling, and a large \( \Xi \) probability is obtained even for a weaker \( \Lambda \Lambda - N \Xi \) potential.

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Both recent experimental and theoretical studies of doubly strange \( (S = -2) \) s-shell hypernuclei (\( \Lambda^4\Lambda \), \( \Lambda^5\Lambda \), and \( \Lambda^6\Lambda \)) are the utmost exciting topics in the field of hypernuclei. An experimental report on a new observation of \( \Lambda^6\Lambda \) has had a significant impact on strangeness nuclear physics. The Nagara event provides unambiguous identification of \( \Lambda^6\Lambda \) production, and suggests that the \( \Lambda \Lambda \) interaction strength is rather weaker than that expected from an older experiment.

The BNL-AGS E906 experiment has conjectured a formation of \( \Lambda^4\Lambda \), in accordance with our earlier predictions that \( \Lambda^4\Lambda \) would exist as a particle stable bound-state against strong decay. If this is the case, the \( \Lambda^4\Lambda \) would be the lightest bound state among doubly strange hypernuclei. However, a theoretical study of the weak-decay modes from \( \Lambda^4\Lambda \) does not support this conjecture, and our earlier studies should be reanalyzed by taking account of the new datum, Nagara event.

A recent Faddeev-Yakubovskyy search for \( \Lambda^4\Lambda \) found no bound-state solution over a wide range of \( \Lambda \Lambda \) interaction strengths, although this conclusion has been in conflict with the result calculated by authors using a variational method. The total binding energy is more sensitive to the \( ^3S_1 \) channel of the \( \Lambda \Xi \) interaction than to the \( ^1S_0 \) \( \Lambda \Lambda \) interaction, because the number of \( ^3S_1 \) \( \Lambda \Xi \) pairs is three times larger than the number of the \( ^1S_0 \) \( \Lambda \Lambda \) pair, as discussed in Ref. 4. Therefore, the spin-dependent part of the \( \Lambda \Xi \) interaction has to be determined very carefully. The algebraic structure of the \( (\sigma_\Lambda \cdot \sigma_\Xi) \) interaction for the \( S = -2 \) system is similar to the structure for the \( \Lambda^4\Lambda \). Namely, the \( \Lambda \Xi \) interaction, which is utilized in the theoretical search for \( \Lambda^4\Lambda \), has to reproduce the experimental \( B_\Lambda(\Lambda^4\Lambda) \) as well as the \( B_\Lambda \)’s of \( A = 3, 4 \) \( S = -1 \) hypernuclei. However, there is a long-standing problem known as the \( \Lambda^4\Lambda \) anomaly, since the publication by Dalitz et al. in 1972.

Recently, Akashi et al. successfully resolved the anomaly by explicitly taking account of \( \Lambda \Lambda - \Sigma \Xi \) coupling.

Considering the fact that the \( \Lambda \Lambda \) system couples to the \( \Lambda \Xi \) states, and also the \( \Lambda \Lambda \) system couples to the \( \Sigma \Xi \) states, a theoretical search for \( \Lambda^4\Lambda \) should be made in a fully coupled channel formulation with a set of interactions among the octet baryons. The \( \Lambda^4\Lambda \) \( (\Lambda^4\Lambda) \) mixing due to \( \Lambda \Lambda - \Sigma \Xi \) coupling is also interesting topic, since the \( \alpha \)-formation effect could be significant. The purpose of this study is threefold: First is to describe a systematic study for the complete set of s-shell hypernuclei with \( S = -2 \) in a framework of a fully-coupled channel formulation. Second is to make a conclusion if a set of baryon-baryon interactions, which is consistent with the experimental data, predicts a particle stable bound state of \( \Lambda^4\Lambda \). The third is to explore the fully hyperonic mixing of \( \Lambda^4\Lambda \), including the \( \Lambda \Lambda - \Sigma N \) transition potential in addition to \( \Lambda \Lambda - \Sigma \Xi \).

The wave function of a system with \( S = -2 \), comprising \( A(= N + Y) \) octet baryons, has four isospin-basis components. For example, \( \Lambda^4\Lambda \) has four components as \( pppn\Lambda \), \( nnnN \Xi \), \( NNNN \Lambda \Xi \), and \( NNNN \Gamma \Sigma \). We abbreviate these components as \( \Lambda \Lambda \), \( \Lambda \Xi \), \( \Sigma \Sigma \), and \( \Xi \Xi \), referring to the last two baryons. The Hamiltonian of the system is hence given by \( 4 \times 4 \) components as

\[
H = \begin{pmatrix}
H_{\Lambda \Lambda} & V_{\Lambda \Xi - \Lambda \Xi} & V_{\Lambda \Sigma - \Lambda \Sigma} & V_{\Sigma \Xi - \Lambda \Lambda} \\
V_{\Lambda \Xi - \Lambda \Xi} & H_{\Lambda \Xi} & V_{\Lambda \Sigma - \Lambda \Sigma} & V_{\Sigma \Xi - \Lambda \Lambda} \\
V_{\Lambda \Sigma - \Lambda \Sigma} & V_{\Lambda \Xi - \Lambda \Xi} & H_{\Lambda \Sigma} & V_{\Sigma \Xi - \Lambda \Lambda} \\
V_{\Sigma \Xi - \Lambda \Xi} & V_{\Lambda \Xi - \Lambda \Xi} & V_{\Lambda \Sigma - \Lambda \Sigma} & H_{\Sigma \Xi}
\end{pmatrix},
\]

where \( H_{B_1B_2} \) operates on the \( B_1B_2 \) component, and \( V_{B_1B_2 - B'_1B'_2} \) is the sum of all possible two-body transition potential connecting the \( B_1B_2 \) and \( B'_1B'_2 \) components:

\[
V_{\Lambda \Lambda - \Lambda \Xi} = v_{\Lambda \Xi - \Lambda \Xi}.
\]
Note that we take account of full-coupled channel potentials including the $\Lambda\Lambda-\Sigma\Sigma$ and $N\Xi-\Lambda\Sigma$ transitions (Eq. (3)) in the $^3S_1$ channel, while other full-coupled channel approaches (e.g., Refs. [11]) only take $\Lambda\Lambda-N\Xi-\Sigma\Sigma$ in the $^1S_0$ channel (Eqs. (2) and (4)) into account.

In the present calculations, we use the Minnesota potential [18] for the $NN$ interaction and $D2'$ for the $YN$ interaction. The Minnesota potential reproduces reasonably well both the binding energies and sizes of few-nucleon systems, such as $^3H$, $^3H$, $^3He$ and $^4He$. [19]. The $D2'$ potential is a modified potential from the original $D2$ potential [17]. The strength of the long-range part ($V_l$ in Table I of Ref. [17]) of the $D2'$ potential in the $\Lambda\Lambda-\Lambda\Lambda$ $^3S_1$ channel is reduced by multiplying by a factor (0.954) in order to reproduce the experimental $B_N(\Lambda\Lambda)$ value. The calculated $B_N$ values for the $\Lambda$ hypernuclei ($^3H$, $^3H$, $^3He$, $^3He$, and $^3He$) are 0.056, 2.23, 2.17, 0.91, 0.89, and 3.18 MeV, respectively. For the $YY$ interaction, we use a full-coupled channel potential among the octet baryons in both the spin triplet and the spin singlet channels. We assume that the $YY$ potential consists of only the central component, and the effect due to the non-central force (e.g., tensor force) should be included into the central part effect. The force has Gaussian form factors, whose parameters are set to reproduce the low-energy $S$ matrix of the Ni-jmegen hard-core model [ND] or $F(NF)$ [20]. We take the hard-core radius to be $r_c = 0.56271(0.44915)$ fm in the spin singlet (triplet) channel for the ND, whereas $r_c = 0.52972(0.52433)$ fm is used in the singlet (triplet) channel for the NF. Each number is the same as the hard-core radius of the $NN$ sector in each channel for each model. The strength parameters are firstly determined on a charge basis, and then the strength parameters on an isospin basis are constructed from the charge-basis parameters. We denote $NDS$ ($NF_S$) for the simulating ND (NF) potential.

The calculations are made by using the stochastic variational method [21, 22]. This is essentially along the lines of Ref. [22], except for the isospin function. The isospin function consists of four components, in accordance with Eq. (4). The reader is referred to Refs. [22, 23] for the details of the method.

Table I lists the $B_{\Lambda\Lambda}$ values for $S = -2$ hypernuclei. Using $ND_S$ or $NF_S$ $YY$ potential, we have obtained the bound-state solutions of $^4\Lambda\Lambda$, $^5\Lambda\Lambda$, $^6\Lambda\Lambda$, and $^6\Lambda\Lambda$. In the case of $ND_S$ $YY$ potential, we have

$$\Delta B_{\Lambda\Lambda}^{(calc)}(^6\Lambda\Lambda) = B_{\Lambda\Lambda}^{(calc)}(^6\Lambda\Lambda) - 2B_{\Lambda}^{(calc)}(\Lambda)$$

Note that $\Delta B_{\Lambda\Lambda}^{(calc)}(^6\Lambda\Lambda)$ is explicitly due to the $\Lambda\Lambda$ channel.

| YY   | $B_{\Lambda\Lambda}(^4\Lambda\Lambda)$ | $B_{\Lambda\Lambda}(^5\Lambda\Lambda)$ | $B_{\Lambda\Lambda}(^6\Lambda\Lambda)$ | $B_{\Lambda\Lambda}(^6\Lambda\Lambda)$ |
|------|-------------------------------------|-------------------------------------|-------------------------------------|-------------------------------------|
| ND_S | 0.107                               | 4.04                                | 3.96                                | 7.93                                |
| mND_S| 0.058                               | 3.74                                | 3.66                                | 7.53                                |
| NF_S | 0.127                               | 3.84                                | 3.77                                | 7.52                                |
| Exp  | 7.25 ± 0.19 [0.18]                   | 1.01 ± 0.20 [0.11]                  | 1.01 ± 0.20 [0.11]                  | 1.01 ± 0.20 [0.11]                  |

The scattering length and effective range parameters for the $ND_S$, $mND_S$, and $NF_S$ are (given in units of fm): $(a, r_s) = (-1.37, 4.98)$, $(-0.91, 6.25)$, and $(-0.40, 12.13)$, respectively. The scattering length for the $mND_S$ or $NF_S$ is consistent with the other analyses [21, 23] concerning the Nagara event. We should note that the $mND_S$ potential predicts the particle stable bound state of $^4\Lambda\Lambda$; The obtained energy is very close to, but still (0.02 MeV) lower than, the $^3\Lambda\Lambda + \Lambda$ threshold. Therefore, due to the result for $mND_S$ or $NF_S$, we should come to the following novel conclusion: A set of phenomenological baryon-baryon interactions among the octet baryons in $S = 0$, $-1$ and $-2$ sectors, which is consistent with the Nagara event as well as all the experimental binding energies of $S = 0$ and $-1$ $s$-shell (hyper-) nuclei, can predict a particle stable bound state of $^4\Lambda\Lambda$.

Figure I schematically displays the present results of the full-coupled channel calculations of $A = 3 - 6, S = -1, -2$ hypernuclei, using the $mND_S$. Since the present calculation has been made on the isospin basis, the results for $^3\Lambda\Lambda$ are qualitatively similar to the results for $^3\Lambda\Lambda$, so that we omit the explicit result for $^3\Lambda\Lambda$. Fig. I also displays the probabilities of the $N\Xi$, $\Lambda\Sigma$ and $\Sigma\Sigma$ components for the $S = -2$ hypernuclei. In the case of $NF_S$, the probabilities are (given in percentage): $(P_{N\Xi}, P_{\Lambda\Sigma}, P_{\Sigma\Sigma}) = (0.56, 0.37, 0.03)$ for $^4\Lambda\Lambda$, $(3.10, 2.10, 0.10)$ for $^5\Lambda\Lambda$, and $(1.33, 1.13, 0.10)$ for $^6\Lambda\Lambda$, respectively. In the present calculations, the $\Lambda\Lambda$ component is the main part of the wave function. No unrealistic bound states were found for the $YY$ subsystem, since the hard-core model hardly incorporates an unrealistic strong attractive force in the short-range region, in contrast to the soft core model, such as $NSC97i$ or $NSC97f$. This is one of the reasons why we used the $YY$ potential constructed from the hard-core model, for the first attempt.
FIG. 1: Λ and ΛΛ separation energies of core nucleus, \(m_{ND}\) between the ΛΛ and ΣΣ channels (of the complicated full coupling dynamics of the kind of model is useful to make a clear explanation to the full-coupled channel calculation.

We should emphasize that the present calculation assumes no simplification structures, such as \((t+\Lambda+\Lambda)\) and \((\alpha+\Xi^-)\) two-channel model. Although the present calculation assumes no simplified structures, this kind of model is useful to make a clear explanation of the complicated full coupling dynamics of the \(A=5, S=-2\) hypernucleus. Let us consider a set of simple core nucleus + \((Y+Y)\) model wave functions for the \(\Lambda^5\)H:

\[
|\Lambda^5\Lambda\rangle = \psi_t \times \psi_{\Lambda \Lambda} \times \psi_{\Lambda \Lambda - t}, \tag{7}
\]

\[
|\Lambda^5\Sigma\rangle = \psi_h \times \psi_{\Sigma -} \times \psi_{-\Sigma -}\alpha, \tag{8}
\]

\[
|\Lambda^5\Sigma\rangle_{S_{\Sigma\Sigma}} = \sqrt{\frac{1}{3}} \left[ |\psi_t \times |\psi_{\Lambda \Sigma}\rangle_{S_{\Lambda \Sigma}} \times \psi_{\Lambda \Sigma^o - t} - \sqrt{\frac{2}{3}} |\psi_h \times |\psi_{\Lambda \Sigma^-}\rangle_{S_{\Lambda \Sigma}} \times \psi_{\Lambda \Sigma^- - h}\right] (for \(S_{\Lambda \Sigma} = 0\) or 1), \tag{9}
\]

where \(\psi_t (t, h, \alpha)\) is the wave function (WF) of the core nucleus, \(\psi_{YY} (YY = \Lambda \Lambda, \Xi^-, \Lambda \Sigma)\) is the WF of the hyperon(s), and \(\psi_{YY - c}\) is the WF that describes the relative motion between \(YY\) and \(c\). We assume that all of the baryons occupy the same \((0s)\) orbit. For the \(\Lambda^5\)H state, we have two independent states for the WF \(\psi_{\Lambda \Lambda}\), that the spin of two hyperons \((S_{\Lambda \Sigma})\) is either a singlet or a triplet. Since the ΣΣ component plays a minor role, we omit the \(\Xi^5\)H state. Using these WFs, we can obtain the algebraic factors for each averaged coupling potential of the allowed spin state, \(\vec{v}\) or \(\vec{v}'\):

\[
\langle V_{\Lambda \Lambda - N\Xi} \rangle = \sqrt{\frac{1}{2}} \vec{v}_{\Lambda \Lambda - N\Xi}, \tag{10}
\]

\[
\langle V_{\Lambda \Lambda - \Lambda \Sigma} \rangle = \begin{cases} 
\sqrt{\frac{1}{2}} \vec{v}_{\Lambda \Lambda - N\Xi} + \sqrt{\frac{1}{2}} \vec{v}_{\Lambda \Lambda - \Lambda \Sigma} & (for \(S_{\Lambda \Sigma} = 0\)), \\
\sqrt{\frac{1}{2}} \vec{v}_{\Lambda \Lambda - N\Xi} - \sqrt{\frac{1}{2}} \vec{v}_{\Lambda \Lambda - \Lambda \Sigma} & (for \(S_{\Lambda \Sigma} = 1\))
\end{cases}, \tag{11}
\]

\[
\langle V_{\Lambda \Lambda - \Lambda \Sigma} \rangle = \begin{cases} 
-\frac{1}{\sqrt{3}} \vec{v}_{\Lambda \Lambda - \Lambda \Sigma} - \frac{1}{\sqrt{3}} \vec{v}_{\Lambda \Lambda - \Lambda \Sigma} & (for \(S_{\Lambda \Sigma} = 0\)), \\
\frac{2}{\sqrt{3}} \vec{v}_{\Lambda \Lambda - \Lambda \Sigma} & (for \(S_{\Lambda \Sigma} = 1\)).
\end{cases} \tag{12}
\]

The \(\vec{v}_{\Lambda \Lambda - N\Xi}\) potential is suppressed by a factor of \(\sqrt{1/2}\) for the \(A = 5\) hypernucleus. The \(\vec{v}_{\Lambda \Lambda - \Lambda \Sigma}\) potentials, particularly in the spin triplet channel, play significant roles instead. Namely, these equations imply that the \(\Lambda \Sigma\) component strongly couples both to the \(\Lambda\Lambda\) and to the \(N\Xi\) components, and the \(\Lambda \Sigma\) component plays a crucial role in the hypernucleus.

The normalized energy expectation values of the Hamiltonian \(H \) for \(\Lambda^5\)H are (given in units of MeV),

\[
h = \begin{pmatrix}
\langle H_{\Lambda \Lambda} \rangle \\
\langle V_{\Lambda \Lambda - N\Xi} \rangle \\
\langle V_{\Lambda \Lambda - \Lambda \Sigma} \rangle
\end{pmatrix} = \begin{pmatrix}
\sqrt{\frac{1}{2}} P_{\Lambda \Lambda} P_{\Lambda \Lambda} \\
\sqrt{\frac{1}{2}} P_{\Lambda \Lambda} P_{\Lambda \Lambda} \\
\sqrt{\frac{1}{2}} P_{\Lambda \Lambda} P_{\Lambda \Lambda}
\end{pmatrix} \begin{pmatrix}
P_{\Lambda \Lambda} \\
P_{\Lambda \Lambda} \\
P_{\Lambda \Lambda}
\end{pmatrix} \begin{pmatrix}
\langle H_{\Lambda \Lambda} \rangle \\
\langle V_{\Lambda \Lambda - N\Xi} \rangle \\
\langle V_{\Lambda \Lambda - \Lambda \Sigma} \rangle
\end{pmatrix}.
\]
Here, we display only the 3 × 3 components of the Hamiltonian \( \mathbf{H} \), comprising \( \Lambda \Lambda \), \( N \Xi \), and \( \Lambda \Sigma \), since the contributions from the \( \Sigma \Sigma \) component are not large. If we solve the eigenvalue problem, \( \det(\mathbf{H} - \lambda \mathbf{I}) = 0 \), we obtain the ground state energy, \( E = -11.82 \text{ MeV} \) (\(-11.82 \text{ MeV}\)), and the probability, \( P_{N \Xi} = 3.99\% (2.83\%) \), for the mND \((\text{NF})\). The first 2×2 components for the mND\( S \) are quite different from those for the NF\( S \), while the last row and the last column are qualitatively similar to each other. If we solve the eigenvalue problem of only the first 2×2 subspace, including the \( \Lambda \Lambda \) and the \( N \Xi \), we obtain the ground state energy, \( E = -9.35 \text{ MeV} \) (\(-9.46 \text{ MeV}\)), and the probability, \( P_{N \Xi} = 1.57\% (2.62\%) \), for the mND \((\text{NF})\). This clearly means that the couplings between the \( \Lambda \Lambda \) and \( N \Xi \) components have to be taken into account in order to obtain an accurate solution. In the case of mND\( S \), the large coupling potentials, \( \langle \Lambda \Lambda - \Lambda \Sigma \rangle \) and \( \langle N \Xi - \Lambda \Sigma \rangle \), also enhance the \( P_{N \Xi} \) probability. On the other hand, for NF\( S \), these coupling potentials hardly enhance \( P_{N \Xi} \), though the total binding energy significantly increases. The large repulsive energy in the \( N \Xi \) component, which mainly comes from the \( N \Xi - N \Xi \) diagonal potential in the \( I=0, ^{1}S_{0} \) channel, suppresses the net effect of the \( \Lambda \Lambda - N \Xi \) coupling for NF\( S \). On the other hand, the \( N \Xi - N \Xi \) potential of mND\( S \) is weakly attractive, which is consistent with recent experimental data [24,25,26].

In summary, we have performed full-coupled channel \emph{ab initio} calculations for the complete set of doubly strange s-shell hypernuclei. Two kinds of \( YY \) interactions, mND\( S \) and NF\( S \), reproduce the \( \Delta B_{\Lambda \Lambda}(\Lambda \Lambda \text{He}) \) of the Nagara event. We obtained bound-state solutions for \( \Lambda \Lambda \text{He}, \Lambda \Sigma \text{He} \) and \( \Lambda \Xi \text{He} \) by using these \( YY \) interactions. We thus conclude that a set of phenomenological \( B_{8}B_{8} \) interactions among the octet baryons in \( S = 0, -1, -2 \) sectors, which is consistent with all of the available experimental binding energies of the \( S = 0, -1, -2 \) s-shell (hyper-) nuclei, can predict a particle stable bound state of \( \Lambda \Lambda \text{He} \). For the \( \Lambda \Lambda \text{He} \) (and \( \Lambda \Xi \text{He} \)), the probability \( P_{N \Xi} \) by using the mND\( S \) is larger than the \( P_{N \Xi} \) by using the NF\( S \). We found that the \( \Lambda \Lambda - N \Xi \) and \( N \Xi - \Lambda \Sigma \) potentials make a larger \( P_{N \Xi}(\Lambda \Lambda \text{He}) \) for the weak \( \Lambda \Lambda - N \Xi \) and attractive \( N \Xi - N \Xi \) potentials of mND\( S \), whereas the net effect of the stronger \( \Lambda \Lambda - N \Xi \) coupling potential of NF\( S \) is suppressed in \( \Lambda \Xi \text{He} \) due to the repulsive \( N \Xi - N \Xi \) potential in the \( I = 0, ^{1}S_{0} \) channel. The one-boson-exchange potential models for the \( B_{8}B_{8} \) interactions have sound bases of the SU(3) symmetry and are widely accepted, though uncertainties of the interactions in the \( S = -2 \) sector are still large because of limitation of experimental information. The NSC97 models have a crucial defect in \( \Lambda \Lambda - N \Xi - \Sigma \Sigma \) couplings. Therefore, we have attempted only two possible cases with the models ND and NF, which have different characters in the \( S = -2 \) sector (strengths of \( \Lambda \Lambda - N \Xi \) coupling and \( N \Xi - N \Xi \) diagonal potentials). We do hope that a future experimental facility (e.g., J-PARC) develops our knowledge of the \( S = -2 \) interactions.

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