Uncertainties in estimating the indirect production of $B_c$ and its excited states via top quark decays at CERN LHC

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1. Introduction

The $B_c$ meson is a double heavy quark–antiquark bound state and carries flavors explicitly, which provides a good platform for a systematic studies of the $b$ or $c$ quark decays. Since its first discovery at TEVATRON by CDF Collaboration [1], $B_c$ physics is attracting more and wider interests. Many progresses have been made for the direct hadronic production of $B_c$ meson at high energy colliders [2], especially, a computer program BCVEGPy for generating the $B_c$ events has been in Refs. [3–5] and has been accepted by several experimental groups to simulate the $B_c$ events. It has been estimated with the help of BCVEGPy that about $10^4$ $B_c$ events are expected to be recorded during the first year of the CMS running with a lepton trigger [6], and there are about $10^5$ $B_c$ events with $B_c \rightarrow J/\Psi + \pi$ decays in three years of ATLAS running [7].

On the other hand, the indirect production of $B_c^+$ or $B_c^-$, including its excited states, via $t$-decay or $t$-decay may also provide useful knowledge of these mesons. Without confusing and for simplifying the statements, later on we will not distinguish $B_c^+$ and $B_c^-$ (simply call them as $B_c$) and all results for $B_c^+$ and $B_c^-$ obtained in the Letter are symmetric in the interchange from particle to anti-particle. With a predicted cross section for top quark pair production hundred times larger than at TEVATRON and a much higher designed luminosity, e.g. it is expected that at CERN LHC $\sim 10^8 \bar{t}\ell$-pairs can be produced per year under the luminosity $L = 10^{34}$ cm$^{-2}$s$^{-1}$ [8], the LHC is poised to become a “top factory”. Therefore, the indirect production of $B_c$ through top quark decay will provide another important way to study the properties of $B_c$ meson in comparison to that of the direct hadronic production at CERN LHC. © 2008 Elsevier B.V. Open access under CC BY license.
caused by the value of the t-quark mass, the values of the bound state parameters \( m_c \) and \( m_b \), and the choice of the renormalization scale \( Q^2 \).

The remainder of the Letter is organized as follows. Section 2 gives the calculation technology for the indirect production of \((b\bar{c})\)-quarkonium states through the top quark decays. Section 3 is devoted to present the numerical results and to discuss the corresponding uncertainties with the help of the formulae given in Ref. [11]. And Section 4 is reserved for a summary.

2. Calculation technology

Within the non-relativistic QCD (NRQCD) framework [10], the dimensionless reduced decay width for the production of \((b\bar{c})\)-quarkonium through the channel \( t(p_0) \rightarrow (b\bar{c})(p_1) + c(p_2) + W^+(p_3) \) takes the following factorization form:

\[
\Gamma = \sum_n \Gamma_n = \sum_n \left[ \frac{1}{I_{t\rightarrow W^+b}} H_n(t \rightarrow b\bar{c} + c + W^+) \times \frac{(O_n)}{N_{col}} \right],
\]

(1)

where \( \Gamma_n \) stands for the reduced decay width for a particular \((b\bar{c})\)-quarkonium state, and the sum is over all the \((b\bar{c})\)-quarkonium states up to \( O(v^4) \), which includes six color singlets \(|(b\bar{c})^0 S^0\rangle\), \(|(b\bar{c})^1 S^1\rangle\), \(|(b\bar{c})^1 P^1\rangle\) and \(|(b\bar{c})^2 P^2\rangle\) (with \( J = (1, 2, 3) \)), and two color octets \(|(b\bar{c})^1 S^0 h\rangle\) and \(|(b\bar{c})^1 S^1 h\rangle\) respectively. \( N_{col} \) refers to the number of colors, \( n \) stands for the involved states of \((b\bar{c})\)-quarkonium. \( N_{col} = 1 \) for singlets and \( N_{col} = N^2 - 1 \) for octets. \( \langle Ch_n \rangle \) stands for the decay matrix element that can be related with the wave function at zero \( R(0) \) or the derivative of the radial wave function at origin \( R(0) \) through the saturation approximation [10]. The overall factor \( 1/I_{t\rightarrow W^+b} \) is introduced to cut off the uncertainty from the electroweak coupling. The decay width of the two body decay process \( t(p_1) \rightarrow b(p_2) + W^+(p_3) \) that is dominant for the \( t \)-quark decays can be written as

\[
I_{t\rightarrow W^+b} = \frac{G_F m_t^7 |[p_2]|}{4\sqrt{2\pi}} \left[ (1 - y^2)^2 + x^2 (1 + y^2 - 2x^2) \right],
\]

(2)

where \( |[p_2]| = \frac{m_w}{2} \sqrt{(1 - (1 - x - y^2))(1 - (1 + x + y^2)), x = m_u/m_t \text{ and } y = m_b/m_t} \).

As shown in Fig. 1, there are two Feynman diagrams for the concerned process \( t(p_0) \rightarrow (b\bar{c})(p_1) + c(p_2) + W^+(p_3) \). Due to the involved massive quarks, the calculation of the process is very complicated and lengthy, to simplify the calculation, we have improved a so-called ‘new trace technology’ to calculate the process [11]. Under such approach, we first arrange the whole amplitude into several orthogonal sub-amplitudes \( M_{\alpha \nu} \) according to the spins of the \( t \)-quark (\( s' \)) and \( c \)-quark (\( s \)) and then do the trace of the Dirac \( \gamma \) matrix strings at the amplitude level by properly dealing with the massive spinors, which results in explicit series over some independent Lorentz-structures, and finally, we obtain the square of the amplitude. All the necessary formulae together with its subtle points for the square of the hard scattering amplitude \( H_{\text{int}}(t \rightarrow (b\bar{c}) + c + W^+) \) can be found in Ref. [11], so we shall only present the main results here and the interesting reader may turn to Ref. [11] for more detailed calculation technology.

The involved color-singlet and color-octet matrix elements provide systematical errors for the NRQCD framework itself. Their values can be determined by global fitting of the experimental data or directly related to the wave functions at the zero point \( R(0) \) (or the derivative of the wave function at the zero point \( R'(0) \)) derived from certain potential models for the color-singlet case, some potential models can be found in Refs. [14–17]. A model dependent analysis of \( R(0) \) and \( R'(0) \) can be found in Ref. [18], where the spectrum of \( B_c \) under the Cornell potential [14], the Buchmüller–Tye potential [15], the power-law potential [16] and the logarithmic potential [17] have been discussed respectively in their discussions, which shows that \( R(0)^2 = [1.508, 1.710] \text{ GeV}^4 \) and \( R'(0)^2 \in [0.201, 0.327] \text{ GeV}^5 \). Since the model-dependent \( R(0) \) and \( R'(0) \) emerge as overall factors and their uncertainties can be conveniently discussed when we know their possible ranges well, so we shall not discuss such uncertainties in the present Letter. More explicitly, we shall fix their values to be \( R(0)^2 = 1.642 \text{ GeV}^4 \) and \( R'(0)^2 = 0.201 \text{ GeV}^5 \), which is derived under the Buchmüller–Tye potential [18]. Secondly, although we do not know the exact values of the two decay color-octet matrix elements, \(|\langle b\bar{c}(^1 S_0 h) |O_0^1 (^1 S_0)| b\bar{c}(^1 S_0) \rangle |^2 \) and \(|\langle b\bar{c}(^3 S_1) |O_1^3 (^3 S^1)| b\bar{c}(^3 S_1) \rangle |^2 \), we know that they are one order in \( v^2 \) higher than the S-wave color-singlet matrix elements according to NRQCD scale rule. More specifically, based on the velocity scale rule [10], we have

\[
\langle b\bar{c}(^1 S_0 h) |O_0^1 (^1 S_0)| b\bar{c}(^1 S_0) \rangle \sim \Delta_S(v^2) \cdot \langle b\bar{c}(^3 S_1) |O_1^3 (^3 S_1)| b\bar{c}(^3 S_1) \rangle,
\]

(3)

and

\[
\langle b\bar{c}(^3 S_1) |O_1^3 (^3 S_1)| b\bar{c}(^3 S_1) \rangle \sim \Delta_S(v^2) \cdot \langle b\bar{c}(^1 S_0 h) |O_0^1 (^1 S_0)| b\bar{c}(^1 S_0) \rangle.
\]

(4)

where the second equation comes from the vacuum-saturation approximation. \( \Delta_S(v) \) is of order \( v^2 \) or so, and we take it to be within the region of 0.10–0.30, which is in consistent with the identification: \( \Delta_S(v) \sim \alpha_s(Mv) \) and has covered the possible variation due to the different ways to obtain the wave functions at the origin (S-wave) and the first derivative of the wave functions at the origin (P-wave), etc.

In addition to the color-singlet and color-octet matrix elements, the quark mass values \( m_t \), \( m_c \) and \( m_b \) also ‘generate’ uncertainties for the hadronic production. At present, these parameters cannot be completely fixed by fitting the available data of the heavy quarkonium. Furthermore, since the \((b\bar{c})\)-quarkonium state is the non-relativistic and weak-binding bound state, we approximately have \( M_{bc} \sim m_b + m_c \), which also is the requirement from the gauge invariance of the hard scattering amplitude. To choose the renormalization scale \( Q^2 \) is a tricky problem for the estimates of the LO pQCD calculation. If \( Q^2 \) is chosen properly, the results may be quite accurate. In the present case with three-body final state, there is ambiguity in choosing the renormalization scale \( Q^2 \) and various choices of \( Q^2 \) would generate quite different results. Such kind of ambiguity cannot be justified by the LO calculation itself, so we take it as the uncertainty of the LO calculation, although when the NLO calculation of the subprocess is available, the uncertainty will become under control a lot. While the NLO calculation is very complicated and it cannot be available in the foreseeable future, so here we take \( Q^2 \) as the possible characteristic momentum of the hard subprocess being squared. According to the factorization formulae, the running of \( \alpha_s \) should be of leading logarithm order, and the energy scale \( Q^2 \) appearing in the

\[\text{\footnotesize Since the Cornell potential has stronger singularity in spatially smaller states [18], so we do not include its corresponding values for } R(0) \text{ and } R'(0).\]
calculation should be taken as one of the possible characteristic energy scales of the hard subprocess. As a default choice, we take $Q^2 = 4m_c^2$, since the intermediate gluon should be hard enough to produce $a$ and $c$ pair as shown in Fig. 1.

3. Numerical results and discussions

Firstly, we study the uncertainties of $m_c$, $m_b$ and $m_b$ in 'a factorizable way' by fixing the renormalization scale $Q^2 = 4m_c^2$. For instance, when focussing on the uncertainties from $m_c$, we let it be a basic 'input' parameter varying in a possible range $m_c = 1.5 \pm 0.3$ GeV with all the other factors, including the $t$-quark mass, $b$-quark mass and, etc., being fixed to their center values. The Particle Data Group value for the top quark mass is $m_t = 172.5 \pm 2.7$ GeV [19]. And the $b$-quark mass $m_b$ varies within the region of $m_b = 4.9 \pm 0.4$ GeV. The reduced decay width $\Gamma_n$ for the indirect production of $B_c$ through top quark decays with varying $m_c$, $m_b$ and $m_t$ is shown in Table 1, where $n$ stands for a particular color singlet $(cb)$-quarkonium state. The results for the two $S$-wave color octet can be conveniently obtained from that of color singlet $S$-wave $(cb)$-quarkonium states and by setting $\Delta S(v) \in [0.10, 0.30]$. The second column of Table 1 is for the center values of all these parameters, the third and fourth columns setting the upper and the lower limit for $m_t$ varying within the region of $[1.2, 1.8]$ GeV, the fifth and the sixth columns setting the upper and the lower limit for $m_c$ varying within the region of $[4.5, 5.3]$ GeV, and the seventh and eighth columns setting the upper and the lower limit for $m_t$ varying within the region of $[170, 175]$ GeV respectively.

From Table 1, it is found that the reduced decay width $\Gamma_n$ is very sensitive to $m_c$. $\Gamma_n$ decreases with the increment of $m_c$, and more definitely, when $m_c$ increase by steps of $0.1$ GeV, $\Gamma_n$ decreases by $10$–$20\%$ for $S$-wave states and by $25$–$35\%$ for $P$-wave states. This condition is similar to the direct hadronic production [12], which is caused by the fact that a larger $m_c$ leads to a smaller allowed phase space. Summing up all the mentioned Fock states' contribution, we obtain $\sum_n \frac{\Gamma_n}{m_b} = (1.038^{+0.037}_{-0.022}) \times 10^{-3}$ for $m_b \in [4.5, 5.3]$ GeV and $\Delta S(v) \in [0.10, 0.30]$.

The reduced decay width $\Gamma_n$ slightly increases with the increment of $m_t$, which is due to the larger phase space for a larger $m_t$. To check the results of Ref. [11], we also calculate the results for $m_t = 176$ GeV, which shows a good agreement with those of Ref. [11]. Summing up all the mentioned Fock states' contribution, we obtain $\sum_n \frac{\Gamma_n}{m_b} = (1.038^{+0.046}_{-0.029}) \times 10^{-3}$ for $m_b \in [170, 175]$ GeV and $\Delta S(v) \in [0.1, 0.3]$.

Secondly, we study the uncertainties caused by the various choices of $Q^2$, where for consistency, the leading order $\alpha_s$, running is adopted, i.e. $\alpha_s(Q^2) = 4\pi/(11 - 2n_f) \ln(Q^2/A_{QCD}^2)$), where $n_f = 3$ and $A_{QCD} = 200$ MeV. We choose three typical renormalization scale $Q^2$: Type A: $Q^2 = 4m_c^2$; Type B: $Q^2 = 4m_t^2$; Type C: $Q^2 = E_{B_c}^2$, where $E_{B_c}$ stands for the $B_c$ meson energy in the top quark rest frame, and by setting $s_2 = (p_2 + p_3)^2$ for the channel $t(p_0) \rightarrow (b)(p_1) + (c)(p_2) + W^+(p_3)$, we have $E_{B_c} = (m_d^2 + (p_2 + s_2)/2m_b)$. The uncertainties for the reduced decay width $\Gamma_n$ with three typical choices of $Q^2$ are given in Table 2, where $m_c = 1.5$ GeV, $m_b = 4.9$ GeV and $m_t = 172.5$ GeV. It is found that the reduced width for $Q^2 = 4m_d^2$ is only about half of that of $Q^2 = 4m_c^2$, which is a comparatively large effect.

Finally, we discuss the combined effects of all the above mentioned uncertainty sources by varying $m_c \in [1.2, 1.8]$ GeV, $m_b \in [4.5, 5.3]$ GeV, $m_t \in [170, 175]$ GeV and by taking one of the three typical renormalization scales simultaneously, where all the mentioned Fock states' contributions shall be summed up. Additionally, the value of $m_b + m_t$ cannot be too small, as has been found both experimentally and theoretically that the mass of the ground state $(bc)$-quarkonium is around 6.30 GeV [20,21], so we imply $m_b + m_t \geq 6.20$ GeV as an extra constraints. Summing up all the mentioned Fock states' contribution, we obtain $\sum_n \frac{\Gamma_n}{m_b} = (1.038^{+1.153}_{-0.782}) \times 10^{-3}$. Let us show some more characteristics of the decay $t \rightarrow (b)(c) + c + W^+$. The differential distributions of the reduced decay width versus the invariant masses $s_1 = (p_1 + p_2)^2$ and $s_2 = (p_2 + p_3)^2$, i.e. $d\Gamma/ds_1$ and $d\Gamma/ds_2$ are shown in Fig. 2. While the differential distributions of the reduced decay width ver-

| $m_c$ (GeV) | $Q^2 = 4m_b^2$ | $Q^2 = 4m_t^2$ | $Q^2 = 4m_d^2$ |
|------------|---------------|---------------|---------------|
| 1.5        | 1.2           | 1.8           | 4.5           |
| 4.9        |               |               | 5.3           |
| 172.5      | 172.5         |               | 170           |
| $\Gamma_n$ ($\times 10^{-3}$) |            |               |               |
| $\sum_n \frac{\Gamma_n}{m_b}$ | (1.038$^{+0.037}_{-0.022}$) | (1.038$^{+0.046}_{-0.029}$) | (1.038$^{+1.153}_{-0.782}$) |
sus cos θ₁₁ and cos θ₁₂, i.e. \(d\bar{\Gamma}/d\cos \theta_{12}\) and \(d\bar{\Gamma}/d\cos \theta_{13}\) are shown in Fig. 3, where \(\theta_{11}\) is the angle between \(\vec{p}_1\) and \(\vec{p}_3\), and \(\theta_{12}\) is the angle between \(\vec{p}_1\) and \(\vec{p}_2\) respectively in the \(t\)-quark rest frame \((\vec{p}_0 = 0)\). In drawing the curves, all the mentioned Fock states’ contribution have been summed up for convenience. The shaded band shows the corresponding uncertainty, with the upper edge of the band is obtained by setting \(m_c = 1.2\) GeV, \(m_b = 5.0\) GeV, \(m_t = 175\) GeV and \(Q^2 = 4m_c^2\) and the lower edge of the band is obtained by setting \(m_c = 1.8\) GeV, \(m_b = 5.3\) GeV, \(m_t = 170\) GeV and \(Q^2 = 4m_c^2\), and the center solid line is for \(m_c = 1.5\) GeV, \(m_b = 4.9\) GeV, \(m_t = 172.5\) GeV and \(Q^2 = 4m_c^2\).

4. Summary

In the Letter we have presented quantitative studies on the main uncertainties in estimating the indirect production of the \((b\bar{c})\)-quarkonium via top quark decays, \(t \rightarrow (b\bar{c}) + c + W^+\). It is found that the reduced decay width \(\bar{\Gamma}_n\) is very sensitive to the \(c\)-quark mass, while the uncertainty from the \(b\)-quark and \(t\)-quark masses are small. The renormalization scale also affects the decay width to a certain degree. A comparative study on the similarity and difference of the direct and indirect production has also been presented in due places. About \(10^8\) \(t\bar{t}\)-pairs shall be produced per year at CERN LHC, if adopting the assumption that all the higher Fock states decay to the ground state with 100% probability, then we may have \((1.038^{+1.352}_{-0.782}) \times 10^5\) \(B^- (\bar{B}^+)\) events per year. So the indirect production is another important way to study the properties of \(B_c\) meson in comparison to that of the direct hadronic production at LHC. Further more, the contribution from the \(P\)-wave states together with the two color-octet Fock states can be about 20% in total, so the \(P\)-wave production itself is worthwhile to study the possibility of directly measuring the \(P\)-wave \(B_c\) states.

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