Study on Dynamic Simulation of New Cosine Gear

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Abstract. Based on the gear meshing theory, after the process of construction is discussed, the mathematical model about profile curve equation, conjugate equation and action line equation is built. It provides some theoretical proofs for research of performance and manufacture of cosine-gear. In this article, the profile curve equation of cosine gear is introduced, and the conjugate equation is deduced. The solid model of cosine-gear is built by example with Pro/E software, and the meshing process of a pair of cosine-gears is simulated. From the result of simulating, it can detect the interference and the dynamic characteristics of a pair of cosine gears quickly, and during the meshing process, there is no meshing interference. Moreover, the cosine gear is line contact. It is significant to improve the efficiency of design and manufacturing of cosine gear.

1. Introduction
Inspired by Logix gear [1], double involute gear [2] and other new gears, in order to overcome some defects in the application of gears in gear pumps [3]. Such as low volumetric efficiency, low flow rate and oil retention, a new type of gear drive, cosine gear drive is put forward. The profile of the cosine gear is a cosine curve, it consists of two parts: convex curve and concave curve. The radius of curvature in middle of cosine tooth profile is very large, it is beneficial to improve contact strength, the normal meshing performance of conjugate tooth profile can be guaranteed, the utility model can be widely used in gear pumps and other fields, it is positive to relieve the oil loss, improve the volumetric efficiency and increase the flow and pressure of the gear pump.

Before the real cosine gear is machined, using Pro/ENGINEER software, the 3D solid model can be established accurately, it is transformed into dynamic simulation environment, add complex forces and constraints, then the movement simulation analysis is done. Through the simulation results, gear profile interference and continuous transmission during gear transmission can be detected, then the model can be modified quickly and accurately. So as to shorten the modification time and improve the design efficiency.

2. Derivation of cosine profile curve and conjugate tooth profile equation

2.1. The equation of profile curve
The profile of cosine gear is similar to the double circular arc gear, they are all concave at the top of the tooth and protruding at the root of the tooth. The tooth profile curve is shown in figure 1. In polar coordinates, the polar coordinate equation of the gear profile is:

\[ \rho = r + A \cos(z \phi) \] (1)

A is the amplitude, just the radius of the circle, and Z is the number of teeth, \( z = 2\pi r / p \)
The parameter equation is:

\[
\begin{align*}
  x &= [r + A \cos(z\phi)]\sin(\phi) \\
  y &= [r + A \cos(z\phi)]\cos(\phi)
\end{align*}
\]

(2)

2.2. Conjugate tooth profile equation

Just as figure 2, set the coordinate system \( \Sigma_0 (O : x, y) \) as the static coordinate system, its origin \( O \) coincides with the meshing point \( P \), the coordinate system \( \Sigma_1 (O_1 : x_1, y_1) \) is connected with the tooth profile \( C_1 \). The coordinate system \( \Sigma_2 (O_2 : x_2, y_2) \) is connected with the tooth profile \( C_2 \). \( O_1 \) and \( O_2 \) are the origin of \( \Sigma_1 \) and \( \Sigma_2 \) respectively. In the coordinate system \( \Sigma_1 \), the equation \( C_1 \) of the tooth profile is:

\[
\begin{align*}
  x_i &= [r_i + A \cos(z\phi)]\sin(\phi) \\
  y_i &= [r_i + A \cos(z\phi)]\cos(\phi)
\end{align*}
\]

(3)

According to the meshing principle of tooth profile, the equation of meshing motion of tooth profile \( C_1 \) is expressed in coordinate system \( \Sigma_1 \):

\[
\begin{align*}
  \overrightarrow{r}_i^{(1)} &= x_i(u)\overrightarrow{i} + y_i(u)\overrightarrow{j} \\
  \phi(u, t) &= n^{(1)} \cdot \nu^{(12)} = 0
\end{align*}
\]

(4)

In order to deduce the parametric equation of tooth profile \( C_2 \), the radius of the meshing point in the coordinate system \( \Sigma_1 \) is changed directly to the coordinate system \( \Sigma_2 \) by the coordinate system \( \Sigma_1 \):

\[
\begin{align*}
  \overrightarrow{r}_0^{(2)} &= M_{21} \overrightarrow{r}_0^{(1)} + r_0^{(2)} \\
  r_0^{(2)} &= M_{20} r_0^{(1)} + r_0^{(2)}
\end{align*}
\]

(5)

In style, \( r_0 = \begin{pmatrix} 0 \\ -r \end{pmatrix} \), \( r_0^{(2)} = \begin{pmatrix} r_0 \sin\phi_0 \\ -r_0 \cos\phi_0 \end{pmatrix} \)

\[
M_{20} = \begin{pmatrix} \cos\phi_2 & -\sin\phi_2 \\ \sin\phi_2 & \cos\phi_2 \end{pmatrix}
\]

\[
M_{21} = \begin{pmatrix} \cos(\phi_1 + \phi_2) & -\sin(\phi_1 + \phi_2) \\ \sin(\phi_1 + \phi_2) & \cos(\phi_1 + \phi_2) \end{pmatrix}
\]
Thus obtained
\[
\begin{align*}
    r_{01}^{(2)} &= \left( \begin{array}{c} \cos \varphi_2 \\ \sin \varphi_2 \end{array} \right) - \left( \begin{array}{c} r_2 \sin \varphi_2 \\ -r_2 \cos \varphi_2 \end{array} \right) = \left( \begin{array}{c} a \sin \varphi_2 \\ -a \cos \varphi_2 \end{array} \right) \\
    r^{(2)} &= \left( \begin{array}{cc} \cos(\varphi_1 + \varphi_2) & -\sin(\varphi_1 + \varphi_2) \\ \sin(\varphi_1 + \varphi_2) & \cos(\varphi_1 + \varphi_2) \end{array} \right) \left( \begin{array}{c} x_1 \\ y_1 \end{array} \right) + \left( \begin{array}{c} a \sin \varphi_2 \\ -a \cos \varphi_2 \end{array} \right)
\end{align*}
\]
(6)

That is
\[
\begin{align*}
    x_2 &= x_1 \cos(\varphi_1 + \varphi_2) - y_1 \sin(\varphi_1 + \varphi_2) + a \sin \varphi_2 \\
    y_2 &= x_1 \sin(\varphi_1 + \varphi_2) + y_1 \cos(\varphi_1 + \varphi_2) - a \cos \varphi_2
\end{align*}
\]
(7)

Among them, \(\varphi_1\), \(\varphi_2\) respectively stands for the angle of gear 1 and gear 2 is turning, \(\varphi_2 = i \times \varphi_1\), \(i\) stands for the ratio of gear transmission, \(a\) stands for the distance of gear center.

As shown in figure 3, the instantaneous center line of gear 1 is a circle with a radius \(r_1\), P is the instantaneous center point, its position in the certain coordinate system \(\Sigma_1\) is fixed, the known tooth profile a-a is tied together with the coordinate system \(\Sigma_1\), the angle between the tangent and the axis \(x_1\) at any point \(m_1(x_1, y_1)\) on the tooth setting a-a is \(\beta\), draw the perpendicular line of \(m_1p_1\) through \(o_1\), and the angle between it and \(o_1p_1\) is \(\psi\), then we can see by the diagram:

Figure 3. Calculation parameters of conjugate tooth profile.

Equation (8): \(p\) -he normal of the point on a-a and the intersection of \(Y_1\), \(p_1\) - The intersection of the normal point \(m_1\) of the tooth profile and the node.

Simultaneous equations (3), (7) and (8) can obtain the equation of tooth profile \(C_2\). The shape of the conjugate tooth profile is also a curve of approximately cosine shape.

3. Establishment of gear solid model
The part of gear teeth is the most complicated in the gear, the cosine gear is the same as other gears, in order to meet the needs of kinematics simulation analysis and finite element analysis, the 3D solid model is required to be mapped accurately. In this paper, the number of teeth \(z_1=13\), modulus \(m = 3mm\), pressure angle \(\alpha = 20^\circ\), width of the cosine gear \(b = 30mm\) (as specified in gear 1) as an example, mating gear \(z_2=28\) according to the method of establishing 3D model of involute gear [6], describe the construction of the solid model of gear 1 and its conjugate gear (set as gear 2).

3.1. The solid model of gear 1
Use the command of Curve/From equation to enter the following equation in the Cartesian coordinate system, and obtain a cosine curve equation of gear 1.
\[
\begin{align*}
\phi &= t + 9 \\
x &= [20 + 2 \times \cos(20 \times \phi)] \sin(\phi) \\
y &= [20 + 2 \times \cos(20 \times \phi)] \cos(\phi) \\
z &= 0
\end{align*}
\]

Obtain another cosine curve by the Mirror command, in order to ensure that the gear pair does not appear meshing interference in the meshing process. The teeth of the gear 1 and 2 are moved by a circular arc that is tangent to the cosine curve, considering the oil film thickness [7] between the gear pairs, the radius of the arc is 1.

Generate a single gear with the Extrude command, and the Pattern command generates the blank model of gear 1, the Cut command generates the key groove, and a 3D solid model of gear 1 is obtained, as shown in figure 4.

3.2. The solid model of gear 2
Because the curve equation of conjugate tooth profile is complex, it is difficult to generate cosine curve according to gear 1. Using Matlab software to take 20 points on the conjugate tooth profile, the tooth profile of gear 2 is obtained by connecting the points. Finally, according to the same method as gear 1, a three-dimensional solid model of gear 2 is established, as shown in figure 5.

4. Meshing simulation analysis of cosine gear pair
Taking a pair of gear pairs (\(z_1 = 13\), \(z_2 = 30\)) as an example, to mesh simulation analysis of gear pair is carried out. Figure 6 shows the virtual assembly of the gear pair. Figure 7 shows the shape of the contact area of a single tooth profile.

We can know in figure 6-7:
1. The simulation results show that the gear pair has no mesh interference and can continuously drive.
2. The contact shape of gear pair is straight line, so it can be deduced that the form of cosine gear transmission is line contact.
5. Conclusion
The equation of tooth profile curve of cosine gear is established, using the gear meshing principle and differential geometry method, the conjugate tooth profile equation is derived, by using the modeling function of Pro/E, a pair of cosine gear pairs is generated, and the meshing process is simulated. Meshing simulation of cosine gear pair, the actual assembly effect of gear pair can be detected, the assembly relation and motion relation among the parts are tested. It is very convenient to observe whether the cosine tooth profile formed by the selected parameters can cause tooth profile interference and whether the gear pair can be continuously transmitted, and the shape of the contact area can also be observed dynamically, so that the design, assembly and transmission of the cosine gear are reasonable. It is of great value to instruct designers to modify design models directly and quickly, to improve design efficiency, and to shorten the design and manufacture time of gears. During the meshing process, there is no meshing interference. Moreover, the cosine gear is line contact.

6. References
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