Azimuthal asymmetries in semi-inclusive DIS with polarized beam and/or target and their nuclear dependences

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Abstract

Using the formalism obtained from collinear expansion, we calculate the differential cross section and azimuthal asymmetries in semi-inclusive deeply inelastic lepton-nucleon (nucleus) scattering process $e^- + N(A) \rightarrow e^- + q + X$ with both polarized beam and polarized target up to twist-3. We derive the azimuthal asymmetries in terms of twist-3 parton correlation functions. We simplify the results by using the QCD equation of motion that leads to a set of relationships between different twist-3 functions. We further study the nuclear dependence of azimuthal asymmetries and show that they have similar suppression factors as those in the unpolarized reactions.

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I. INTRODUCTION

Azimuthal asymmetries in semi-inclusive deeply inelastic lepton-nucleon scattering (SIDIS) play an important role in the study of partonic structure of nucleon, attracting much effort in both theory [1–15] and experiment [16–28]. In such studies, spin and nuclear dependences are often important and provide an useful tool to investigate these effects. Also because of this, higher twist contributions are often significant and need to be taken into account precisely. Besides, such higher twist effects usually depend on new higher twist parton correlation functions hence the studies of them provide a new window to learn about the structure of nucleon.

One of the most important issues in these studies is to establish the relationships between the experimentally measurable quantities and different parton distribution and/or correlation functions that describe the partonic structure of the nucleon and the properties of the hadronic interaction in a consistent theoretical framework. Collinear expansion seems to be the most promising technique that leads to such a framework. It was proposed in 1980s and has been successfully applied to the inclusive processes [29–33]. It has been shown that, after collinear expansion, the differential cross section can be expressed as a convolution of the collinear expanded hard parts with the parton distribution and/or correlation functions in nucleon. While the hard parts are calculable, the parton distribution and/or correlation functions can be defined in terms of gauge invariant matrix elements of the nucleon state. These matrix elements contain the information about parton distributions inside the nucleon. The gauge link inside the gauge invariant matrix elements is a result of multiple gluon scattering within the collinear expansion scheme. Within this scheme, one performs Taylor expansion of the hard parts around the collinear momenta. The leading twist contributions come from the zeroth order in the collinear expansion allowing all momenta taking their collinear values. Higher twist contributions from the higher orders of the Taylor expansion can be calculated consistently.

Higher twist effects in semi-inclusive deeply inelastic lepton-nucleon scattering have also been studied in literature and calculations of the differential cross section up to twist-3 level have been carried out [5, 6, 8, 9]. However, most of these studies do not consider the application of collinear expansion. Instead, they usually start from the expressions obtained directly from the Feynman diagrams, extract the leading (twist-2) and the sub-
leading (twist-3) twist contributions by making appropriate approximations, and insert the
gauge link whenever needed to guarantee the gauge invariance of the parton distribution
and/or correlation functions. It is thus unknown whether, if yes, how the collinear expansion
is applicable here. It is not obvious where the gauge link comes from and which form it takes.
It is also not known whether the calculations extend to even higher twist. A systematic study
leading to a consistent formalism is necessary but still lacking.

In Ref. [12], we made the first step towards this goal by applying the collinear expansion
technique to the semi-inclusive deep inelastic scattering (SIDIS) process $e^- + N \rightarrow e^- + q + X$,
where $q$ denotes a quark which is equivalent to a jet in experiment. We showed that the
collinear expansion technique is applicable for this process and derived a formalism suitable
for studying leading as well as higher twist contributions to $e^- + N \rightarrow e^- + q + X$ in a
systematic way. This formalism is similar to that we have for inclusive process and similar
expressions can be obtained for the differential cross section or the hadronic tensor as a con-
volution of the hard parts and the un-integrated or transverse momentum dependent (TMD)
parton correlation functions. We carried out the calculations of the azimuthal asymmetries
in the unpolarized cases up to twist-4 [14] and those in the case with transversely polarized
targets up to twist-3 [12]. Furthermore, we also showed that the multiple gluon scattering
contained in the gauge link leads to a significant nuclear dependence of the azimuthal
asymmetries which can be studied experimentally [13, 34].

In this paper, we present calculations of azimuthal asymmetries in the semi-inclusive
process $e^- + N(A) \rightarrow e^- + q + X$ with beam and target in different polarizations up to twist-3
using the formalism derived in [12]. For completeness, we summarize the formalism in Sec.
II and present the results of the hadronic tensor. In Sec. III, we present the results of the
differential cross sections and the azimuthal asymmetries. We study the nuclear dependence
in Sec. IV and conclude in Sec. V.

II. THE HADRONIC TENSOR

The formalism that we use in our calculations are derived in [12] for the semi-inclusive
process $e^- + N \rightarrow e^- + q + X$ and are summarized in [14]. It is obtained by applying
collinear expansion and contains the contributions from the multiple gluon scattering. For
completeness and also for comparison with other approaches such as those in [5, 9], we
summarize the most related results of this formalism in part A and present the results for the hadronic tensors in different polarized cases up to twist-3 in other parts of this section.

A. The formalism

We consider the SIDIS process $e^- + N \rightarrow e^- + q + X$ and use $l$, $l'$, $p$, $k$ and $k'$ to denote the four-momenta of electron, nucleon and parton respectively, those with primes are for the final state. The polarization vectors are denoted by $s_t$ and $s$ and are taken as $s_t^\mu = \lambda_l l^\mu/m_c + s_{t\perp}^\mu$, and $s^\mu = \lambda p^\mu/M + s_{\perp}^\mu$, where $\lambda_l$ and $\lambda$ are the helicities. We use light-cone coordinate $k^\mu = (k^+, k^-, \vec{k}_\perp)$ and take unit vectors as $\vec{n}$ such that $\vec{n} = (1, 0, \vec{0}_\perp)$, $n = (0, 1, \vec{0}_\perp)$, and $n_{\perp} = (0, 0, \vec{n}_{\perp})$. We choose the coordinate system such that, $p = p^+ n$, $q = -x_B p + n Q^2/(2 x_B p^+)$, $l_{\perp} = |\vec{l}_{\perp}| n_{\perp 1}$, where $x_B = Q^2/2 p \cdot q$ and define $y = p \cdot q/p \cdot l$.

The differential cross section is given by,

$$d\sigma = \frac{2 e_q^2 e_{\gamma}^2}{s Q^4} L^{\mu\nu}(l, l', s_t) \frac{d^2 W_{\mu\nu}^l d^3 l' d^2 k'_\perp}{2 E_{l'}}$$

where $L^{\mu\nu}(l, l', s_t)$ is the leptonic tensor, and the hadronic tensor is given by,

$$\frac{d^2 W_{\mu\nu}}{d^2 k'_{\perp}} = \int \frac{dk'}{(2\pi)^3 2 E_{k'}} W^{(\text{si})}_{\mu\nu}(q, p, s, k'),$$

$$W^{(\text{si})}_{\mu\nu}(q, p, s, k') = \frac{1}{2\pi} \sum_X \langle p, s | j_{\mu}(0) | k', X \rangle \langle k', X | j_{\nu}(0) | p, s \rangle (2\pi)^4 \delta^4(p + q - p_X).$$

As discussed in [29, 30] for inclusive DIS and in [12] for semi-inclusive DIS, to obtain the gauge invariant form of the hadronic tensor including leading and higher twist contributions in a systematic way, at the tree level, we need to consider the Feynman diagram series as illustrated in Fig.1. The hadronic tensor is given by a sum of the contribution from each diagram $W^{(\text{si})}_{\mu\nu} = \sum_j W^{(j,\text{si})}_{\mu\nu}$. For example, for $j = 0, 1$ and 2,

$$W^{(0,\text{si})}_{\mu\nu} = \frac{1}{2\pi} \int \frac{d^4 k}{(2\pi)^4} \text{Tr}[\hat{H}^{(0)}_{\mu\nu}(k, q) \hat{\phi}^{(0)}(k, p, S) 2E_{k'}(2\pi)^3 \delta^3(\vec{k}' - \vec{k} - \vec{q})],$$

$$W^{(1,\text{si})}_{\mu\nu} = \frac{1}{2\pi} \int \frac{d^4 k_1}{(2\pi)^4} \frac{d^4 k_2}{(2\pi)^4} \sum_{c=L,R} 2E_{k'}(2\pi)^3 \delta^3(\vec{k}' - \vec{k}_c - \vec{q}) \text{Tr}[\hat{H}^{(1,c)\rho}_{\mu\nu}(k_1, k_2, q) \hat{\phi}^{(1)}_{\rho}(k_1, k_2, p)],$$

$$W^{(2,\text{si})}_{\mu\nu} = \frac{1}{2\pi} \int \frac{d^4 k_1}{(2\pi)^4} \frac{d^4 k_2}{(2\pi)^4} \frac{d^4 k}{(2\pi)^4} \sum_{c=L,R,M} 2E_{k'}(2\pi)^3 \delta^3(\vec{k}' - \vec{k}_c - \vec{q}) \text{Tr}[\hat{H}^{(2,c)\rho\sigma}_{\mu\nu}(k_1, k_2, k, q) \hat{\phi}^{(2)}_{\rho\sigma}(k_1, k_2, k, p, S)].$$

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where $c$ denotes different cuts, $k_L = k_1$, $k_R = k_2$, $k_M = k$, and the hard parts are given by,

$$\hat{H}^{(0)}_{\mu\nu}(q, k) = \gamma_\mu(k + \hat{q})\gamma_\nu(2\pi)\delta_+((k - q)^2),$$

$$\hat{H}^{(1, L)}_{\mu\nu}(k_1, k_2, q) = \gamma_\mu(k_2 + \hat{q})\gamma^\rho \frac{k_1 + \hat{q}}{(k_1 + q)^2} - i\epsilon \gamma_\nu(2\pi)\delta_+((k_2 + q)^2),$$

$$\hat{H}^{(2, L)}_{\mu\nu}(k_1, k_2, q) = \gamma_\mu(k_2 + \hat{q})\gamma^\rho \frac{k + \hat{q}}{(k + q)^2} - i\epsilon \gamma_\nu(2\pi)\delta_+((k_2 + q)^2),$$

and the matrix elements or the correlators are defined as,

$$\hat{\phi}^{(0)}(k, p, S) = \int d^4z e^{ikz} \langle p, S|\bar{\psi}(0)\psi(z)|p, S\rangle,$$

$$\hat{\phi}^{(1)}_{\rho}(k_1, k_2, p, S) = \int d^4y d^4z e^{ik_1z + i(k_2 - k_1)y} \langle p, S|\bar{\psi}(0)gA_\rho(z)\psi(y)|p, S\rangle,$$

$$\hat{\phi}^{(2)}_{\rho\sigma}(k_1, k_2, k, p, S) = \int d^4y d^4y' d^4z e^{ik_1\cdot y + i(k - k_1)\cdot z'} + i(k_2 - k)\cdot z \times \langle p, S|\bar{\psi}(0)gA_\rho(z)gA_\sigma(z')\psi(y)|p, S\rangle.$$
\[
\frac{d\tilde{W}^{(0)}_{\mu\nu}}{d^2k_{\perp}} = \frac{1}{2\pi} \int dx d^2k_{\perp} \text{Tr} \left[ \hat{H}^{(0)}_{\mu\nu}(x) \hat{\Phi}^{(0)N}(x, k_{\perp}) \right] \delta^{(2)}(k_{\perp} - \vec{k}_{\perp}), \quad (14)
\]
\[
\frac{d\tilde{W}^{(1)}_{\mu\nu}}{d^2k_{\perp}} = \frac{1}{2\pi} \int dx_1 dx_2 d^2k_{\perp} \sum_{c=L,R} \text{Tr} \left[ \hat{H}^{(1,c)\rho}_{\mu\nu}(x_1, x_2) \omega_\rho \delta^{(1)N}(x_1, k_{\perp}, x_2, k_{\perp}) \right] \delta^{(2)}(k_{\perp} - \vec{k}_{\perp}), \quad (15)
\]

where the tilded symbols $\tilde{W}^{(j)}$'s represent results after collinear expansion and $\omega_\rho \delta^{(1)}$ is a projection operator. The matrix elements take the gauge invariant form and are given by,

\[
\hat{\Phi}^{(0)N}(x, k_{\perp}) = \int \frac{p^+ dy^- d^2y_\perp}{(2\pi)^3} e^{ixp^+ y^- - i\vec{k}_{\perp} \cdot \vec{y}_\perp} \langle \psi(0) | \mathcal{L}(0; y) \psi(y) | N \rangle,
\]
\[
\hat{\Phi}_\rho^{(1)N}(x_1, k_{\perp}, x_2, k_{\perp}) = \int \frac{p^+ dy^- d^2y_\perp p^+ dz^- d^2z_\perp}{(2\pi)^3} \times e^{ix_2 p^+ z^- - i\vec{k}_{\perp} \cdot \vec{z}_\perp + i x_1 p^+ (y^- - z^-) - i\vec{k}_{\perp} \cdot (\vec{y}_\perp - \vec{z}_\perp)} \langle \psi(0) | \mathcal{L}(0; z) D_\rho(z) \mathcal{L}(z; y) \psi(y) | N \rangle,
\]

where $\mathcal{L}(0; y)$ is the gauge link obtained in the collinear expansion and is given by,

\[
\mathcal{L}(0; y) = \mathcal{L}_\parallel(\infty, \vec{0}_\perp; 0, \vec{0}_\perp) \mathcal{L}_\perp(\infty, \vec{0}_\perp, \vec{y}_\perp) \mathcal{L}_\parallel(\infty, \vec{y}_\perp; y^-, \vec{y}_\perp),
\]
\[
\mathcal{L}_\parallel(\infty, \vec{y}_\perp; y^-, \vec{y}_\perp) = Pe^{-ig \int_0^\infty d\xi^+ A^+ (\xi^-, \vec{y}_\perp)},
\]
\[
\mathcal{L}_\perp(\infty, \vec{0}_\perp, \vec{y}_\perp) = Pe^{-ig \int_{\perp} d\xi_\perp \cdot A_\perp(\infty, \vec{\xi}_\perp)},
\]

where $P$ stands for path integral. The hard parts in these tilded $\tilde{W}^{(j)}$'s are the first terms in the Taylor expansions at $k_i = x_i p$ of the corresponding hard parts obtained directly from the Feynman diagrams. They are given by \[12\],

\[
\hat{H}^{(0)}_{\mu\nu}(x) = \frac{2\pi}{2q \cdot p} \gamma_\mu (\hat{q} + x \hat{p}) \gamma_\nu \delta(x - x_B),
\]
\[
\hat{H}^{(1,L)\rho}_{\mu\nu}(x_1, x_2) = \frac{2\pi}{(2q \cdot p)^2} \gamma_\mu (\hat{q} + x_2 \hat{p}) \gamma_\rho \gamma_\nu \delta(x_1 - x_B).
\]

These equations \[13,22\] form the basis for calculating the hadronic tensor in $e^- + N \rightarrow e^- + q + X$ at the tree level including leading and higher twist contributions. We emphasize once more that these equations are derived from the Feynman diagram series in Fig.1 using collinear expansion. They are nothing else but a reorganization of $W^{(j,si)}$ given by Eqs. \[4,16\] obtained directly from this diagram series. We also note that $\tilde{W}^{(j)}$ differs distinctly from the corresponding $W^{(j,si)}$ and shows in particular the following features.
(1) None of the tilded $\tilde{W}^{(j)}$ corresponds to one single Feynman diagram in the diagram series given in Fig.1. It contains contributions from all the infinite number of diagrams in this diagram series with exchange of $j = 0, 1, 2, \ldots$ gluon(s).

(2) The correlators acquire automatically the gauge links and are gauge invariant. The gauge link comes from the multiple gluon scattering shown in Fig.1. Furthermore, in the quark-gluon-quark correlator, covariant derivative is obtained to replace the gluon field operator in the original correlator before collinear expansion.

(3) All the parton momenta in the hard parts take only the $\bar{n}$-components, while the corresponding $n$ and $n_{\perp}$ components are taken as zero. Also there are projection operators $\omega^{\rho'}\rho$’s in the expressions for $\tilde{W}^{(j)}$ for $j > 0$ due to the collinear expansion.

Because of the features mentioned above in particular point (3), the expressions for $\tilde{W}^{(j)}$ can be further simplified to a great deal. In fact, because of (3), the hard parts reduce to the following simple form,

$$
\hat{H}^{(0)}_{\mu\nu}(x) = \pi \hat{h}^{(0)}_{\mu\nu}(x) = \pi \hat{h}^{(0)}_{\mu\nu}(x) \delta(x - x_B),
$$

$$
\hat{H}^{(1),L}_{\mu\nu}(x_1, x_2) \omega^{\rho'} = \frac{\pi}{2q \cdot p} \hat{h}^{(1)}_{\mu\nu} \omega^{\rho'} \delta(x_1 - x_B),
$$

where $\hat{h}^{(0)}_{\mu\nu} = \gamma_\mu \gamma_\nu / p^+$, and $\hat{h}^{(1)}_{\mu\nu} = \gamma_\mu \gamma_\nu / p^+$.

The new correlator $\hat{\phi}^{(1)}_{\rho}$ is defined as,

$$
\hat{\phi}^{(1)}_{\rho}(x_1, k_{\perp}) \equiv \int dx_2 d^2 k_{2\perp} \hat{\phi}^{(1)}_{\rho}(x_1, k_{1\perp}, x_2, k_{2\perp}),
$$

and is given by,

$$
\hat{\phi}^{(1)}_{\rho}(x, k_{\perp}) = \int \frac{dy^- d^2 y_{\perp}}{(2\pi)^3} e^{i p^+ y^- - i k_{\perp} \cdot \vec{y}_{\perp}} \langle N | \bar{\psi}(0) D_{\rho}(0) \mathcal{L}(0; y) \psi(y) | N \rangle.
$$

It depends only on one parton momentum $k$, the quark field operator $\bar{\psi}$ and the covariant derivative $D_{\rho}$ are at the same space-time point. Here, we may note that, unlike what we do in the current approach, the calculations presented in e.g. [9] start from the hadronic tensors $W^{(j)}$’s given by Eqs. (4) and (5) obtained directly from the Feynman diagrams given by
Up to twist-3 level, $\Phi^0(0)$ following, we calculate different contributions term by term. We first consider hadronic tensors in terms of these parton distribution and correlation functions. In the 35]. By inserting them into the above mentioned Eqs. (28) and (29), we can obtain the functions. Such decompositions are the same as those discussed in different publications [9, 35].

To proceed, we need to decompose the matrix elements involved in terms of the Lorentz covariants constructed from $p, n, k_\perp$ and $S$ multiplied by scalar functions of $x$ and $k_\perp^2$. These scalar functions are just different components of the parton distribution and/or correlation functions. Such decompositions are the same as those discussed in different publications [9, 35]. By inserting them into the above mentioned Eqs. (28) and (29), we can obtain the hadronic tensors. In the formalism obtained using collinear expansion where the Feynman diagram series is considered systematically and the gauge links are obtained automatically.

### B. Twist-3 contributions to $d^2W_{\mu\nu}/d^2k_\perp$

Up to twist-3, we need to consider the contributions from $d^2\tilde{W}^{(0)}_{\mu\nu}/d^2k_\perp$ and those from $d^2\tilde{W}^{(1,L)}_{\mu\nu}/d^2k_\perp$. Since the hard part $\hat{h}_{\mu\nu}^{(0)}$ and $\hat{h}_{\mu\nu}^{(1)}$ have odd number of $\gamma-$matrices, only $\gamma^\alpha$ and $\gamma^\alpha\gamma_5$ terms of correlation matrices contribute. We decompose the correlation matrices as $\hat{\Phi}^{(0)} = (\Phi^{(0)}_\alpha \gamma^\alpha - \tilde{\Phi}^{(0)}_\alpha \gamma_5 \gamma^\alpha)/2 + \ldots$, $\varphi^{(1)}_\rho = (\varphi^{(1)}_{\rho\alpha} \gamma^\alpha - \tilde{\varphi}^{(1)}_{\rho\alpha} \gamma_5 \gamma^\alpha)/2 + \ldots$, and obtain the hadronic tensors as,

\[
\frac{d^2\tilde{W}^{(0)}_{\mu\nu}}{d^2k_\perp} = \frac{1}{4} \text{Tr}[\hat{h}^{(0)}_{\mu\nu} \gamma^\alpha] \Phi^{(0)}_\alpha - \frac{1}{4} \text{Tr}[\hat{h}^{(0)}_{\mu\nu} \gamma_5 \gamma^\alpha] \tilde{\Phi}^{(0)}_\alpha,
\]

\[
\frac{d^2\tilde{W}^{(1,L)}_{\mu\nu}}{d^2k_\perp} = \frac{1}{8\rho \cdot q} \text{Tr}[\hat{h}^{(1)}_{\mu\nu} \gamma^\alpha] \varphi^{(1)}_{\rho\alpha} - \frac{1}{8\rho \cdot q} \text{Tr}[\hat{h}^{(1)}_{\mu\nu} \gamma_5 \gamma^\alpha] \tilde{\varphi}^{(1)}_{\rho\alpha}.
\]

Up to twist-3 level, $\Phi^{(0)}_\alpha$ and $\tilde{\Phi}^{(0)}_\alpha$ are decomposed as [35],

\[
\Phi^{(0)}_\alpha = (f_1 - \varepsilon_\perp^k f_{1T}^\perp) p_\alpha + f_\perp^\perp k_\perp \cdot s_\perp - f_T M \varepsilon_{\perp\alpha i} s_i + \ldots,
\]

\[
\tilde{\Phi}^{(0)}_\alpha = -\left(\lambda g_{1L} - \frac{k_\perp \cdot s_\perp}{M} g_{1T}^\perp\right) p_\alpha + g_\perp^\perp \varepsilon_{\perp\alpha i} k_i + g_T M s_\perp + \ldots,
\]

where $\varepsilon_\perp^{\mu\nu} \equiv \varepsilon_{\sigma\mu\nu} n_\rho n_\sigma$, $d_{\mu\nu} \equiv g_{\mu\nu} - \bar{n}_\mu n_\nu - \bar{n}_\nu n_\mu$, and $\varepsilon_\perp^{k s} \equiv (1/M)\varepsilon^{i j}_{\perp} k_i s_{\perp j} = (1/M)(\vec{k}\times$
\[
\bar{s} \cdot \hat{z}.
\]
\[
\text{Tr}[\hat{h}_{\mu\nu}^{(0)} \rho] = -4d_{\mu\nu},
\]
\[
\text{Tr}[\hat{h}_{\mu\nu}^{(0)} \gamma_5 \rho] = 4i\varepsilon_{\perp \mu\nu},
\]
\[
\text{Tr}[\hat{h}_{\mu\nu}^{(0)} \gamma_\alpha] = \frac{4}{p^+} n_{(\mu} d_{\nu)\alpha},
\]
\[
\text{Tr}[\hat{h}_{\mu\nu}^{(0)} \gamma_5 \gamma_\alpha] = \frac{4i}{p^+} n_{[\mu} \varepsilon_{\perp \nu]\alpha].
\]

where \(A_{(\mu}B_{\nu)} = A_{\mu}B_{\nu} + A_{\nu}B_{\mu}\), and \(A_{[\mu}B_{\nu]} = A_{\mu}B_{\nu} - A_{\nu}B_{\mu}\). Hence, we obtain, up to twist-3,

\[
d^2\tilde{W}^{(0)}_{\mu\nu} = -d_{\mu\nu}(f_1 - \varepsilon_{ks} f^+_T) + \frac{1}{p \cdot q} k_{\perp \mu} (q + xBp)_{\nu}\right) (f_1 - \varepsilon_{ks} f^+_T)
\]
\[
- \frac{M}{p \cdot q} (q + xBp)_{(\mu} \varepsilon_{\perp \nu)} s^i_{\perp \perp} \hat{f}_T - \frac{\lambda}{p \cdot q} (q + xBp)_{(\mu} \varepsilon_{\perp \nu)} k_{\perp \mu} \hat{f}_T
\]
\[
+ i\varepsilon_{\perp \mu\nu} \left( \lambda g_{1L} - \frac{k_{\perp \nu}}{M} g_{1T} \right) - \frac{i}{p \cdot q} k_{\perp \mu} (q + xBp)_{\nu} (g_1^+ + \varepsilon_{ks} g^+_T)
\]
\[
+ \frac{iM}{p \cdot q} (q + xBp)_{[\mu} \varepsilon_{\perp \nu]} s^i_{\perp \perp} \hat{g}_T + \frac{i\lambda}{p \cdot q} (q + xBp)_{[\mu} \varepsilon_{\perp \nu]} k_{\perp \mu} \hat{g}_T,
\]

where \(\hat{f}_T = f_T - \frac{k^2}{2M^2} f^+_T\) and \(\hat{g}_T = g_T - \frac{k^2}{2M^2} g^+_T\).

Then, we calculate the contributions from \(d^2\tilde{W}^{(1)}_{\mu\nu} / d^2 k_{\perp}\). Up to twist-3, in the correlation matrix \(\varphi^{(1)}_{\rho\alpha}\) and \(\tilde{\varphi}^{(1)}_{\rho\alpha}\), we need to consider the \(p_{\alpha}\)-terms as given in the following,

\[
\varphi^{(1)}_{\rho\alpha} = p_{\alpha} \left[ \varphi^{(1)}_{\perp \rho} k_{\perp \rho} - \varphi_T M \varepsilon_{\perp \rho i} s^i_{\perp \perp} - \frac{\bar{\varphi}^{(1)}_{\perp \rho}}{M} \left( k_{\perp \alpha} k_{\perp \beta} - \frac{1}{2} k_{\perp \alpha}^2 d_{\alpha\beta} \right) \varepsilon_{\perp \rho i} s^i_{\perp \perp} - \lambda \varphi^{(1)}_{\perp \rho} \varepsilon_{\perp \rho i} k_{\perp \rho} \right] + ...
\]
\[
\tilde{\varphi}^{(1)}_{\rho\alpha} = i\rho_{\alpha} \left[ - \varphi^{(1)}_{\perp \rho} k_{\perp \rho} - \tilde{\varphi}_T M s_{\perp \rho} - \frac{\tilde{\varphi}^{(1)}_{\perp \rho}}{M} \left( k_{\perp \alpha} k_{\perp \beta} - \frac{1}{2} k_{\perp \alpha}^2 d_{\alpha\beta} \right) s^i_{\perp \perp} + \lambda \tilde{\varphi}^{(1)}_{\perp \rho} k_{\perp \rho} \right] + ...
\]

The corresponding hard factors are given by,

\[
\text{Tr}[\hat{h}_{\mu\nu}^{(1)\rho} \varphi] = -8p_{\mu} d_{\nu}^\rho,
\]
\[
\text{Tr}[\hat{h}_{\mu\nu}^{(1)\rho} \gamma_5 \varphi] = -8i p_{[\mu} \varepsilon_{\perp \nu]}^\rho.
\]

We insert them into Eq. (29), and obtain,

\[
\frac{d^2\tilde{W}^{(1,L)}_{\mu\nu}}{d^2 k_{\perp}^{(1)}} = - \frac{p_{\mu}}{p \cdot q} \left[ (\varphi^{(1)}_{\perp \rho} - \varepsilon_{ks} \varphi^+_T) k_{\perp \nu} - \tilde{\varphi}_T M \varepsilon_{\perp \nu i} s^i_{\perp \perp} - \lambda \varphi^{(1)}_{\perp \rho} \varepsilon_{\perp \nu i} k^i_{\perp \perp} \right]
\]
\[
- \frac{p_{\mu}}{p \cdot q} \left[ (\tilde{\varphi}^{(1)}_{\perp \rho} + \varepsilon_{ks} \tilde{\varphi}^+_T) k_{\perp \nu} + \tilde{\varphi}_T M \varepsilon_{\perp \nu i} s^i_{\perp \perp} + \lambda \tilde{\varphi}^{(1)}_{\perp \rho} \varepsilon_{\perp \nu i} k^i_{\perp \perp} \right],
\]

where \(\varphi_T = \varphi_T - \frac{k^2}{2M^2} \varphi^+_T\) and \(\tilde{\varphi}_T = \varphi_T - \frac{k^2}{2M^2} \tilde{\varphi}^+_T\).
C. Simplifying $d^2W_{\mu\nu}/d^2k_\perp$ with QCD EOM relations

The quark field operator $\psi(y)$ satisfies the QCD equation of motion (EOM) for massless quark $\gamma \cdot D(y)\psi(y) = 0$. Hence, the correlation functions defined in Eqs. (30), (31), (37), (38) are not independent from each other. We have in particular, for $\rho = 1, 2$,

$$x\Phi^{(0)}_{\rho} = -\frac{n^{\epsilon}}{p^{z}}(\Re\phi_{\rho\alpha}^{(1)} - \epsilon_{\perp}^{\beta} \Im\tilde{\phi}_{\sigma\alpha}^{(1)}),$$  

(42)  

$$x\tilde{\Phi}^{(0)}_{\rho} = -\frac{n^{\epsilon}}{p^{z}}(\Re\tilde{\phi}_{\rho\alpha}^{(1)} + \epsilon_{\perp}^{\beta} \Im\phi_{\sigma\alpha}^{(1)}).$$  

(43)

We make Lorentz contractions of both sides of Eq. (42) with $k_\perp^{\rho}$ and $\epsilon^{\alpha}_{\perp} k_\perp^{i}$, and obtain,

$$xf^{\perp}_{\perp} = -\Re(\phi^{\perp}_{\perp} - \tilde{\phi}^{\perp}_{\perp}),$$  

(44)  

$$xf_{T} = -\Re(\phi_{T} + \tilde{\phi}_{T}),$$  

(45)  

$$xf_{L}^{\perp} = -\Re(\phi_{L}^{\perp} + \tilde{\phi}_{L}^{\perp}),$$  

(46)  

$$xf_{T}^{\perp} = -\Re(\phi_{T}^{\perp} + \tilde{\phi}_{T}^{\perp}).$$  

(47)

Similarly, after Lorentz contractions of both sides of Eq. (43) with $k_\perp^{\rho}$ and $\epsilon^{\alpha}_{\perp} k_\perp^{i}$, we obtain,

$$xg^{\perp}_{\perp} = \Im(\phi^{\perp}_{\perp} - \tilde{\phi}^{\perp}_{\perp}),$$  

(48)  

$$xg_{T} = -\Im(\phi_{T} + \tilde{\phi}_{T}),$$  

(49)  

$$xg_{L}^{\perp} = -\Im(\phi_{L}^{\perp} + \tilde{\phi}_{L}^{\perp}),$$  

(50)  

$$xg_{T}^{\perp} = -\Im(\phi_{T}^{\perp} + \tilde{\phi}_{T}^{\perp}).$$  

(51)

We note that, similar relations have also been obtained earlier in e.g. [9]. However, the twist-3 parton correlation functions in the corresponding equations in [9] are defined using the quark-gluon-quark correlater where gluon field $A_{\rho}$ is used instead of $D_{\rho}$ used here. We see clearly the similarities and differences between those relations obtained there and those that listed above.

Using these relations, we re-write the contributions from $\tilde{W}_{\mu\nu}^{(1)}$ as,

$$2\Re \frac{d^2\tilde{W}_{S,\mu\nu}^{(1,L)}}{d^2k_\perp} = \frac{x_B}{p \cdot q} \left\{ p_{\mu k_\perp} \left( f^{\perp} - \epsilon^{k\perp}_{\perp} f^{\perp}_{T} \right) - M p_{(\mu \epsilon_\perp \nu) i} s^{i}_{\perp} \hat{f}_{T} - \lambda p_{(\mu \epsilon_\perp \nu) i} k^{i}_{\perp} f^{\perp}_{L} \right\},$$  

(52)  

$$2\Im \frac{d^2\tilde{W}_{A,\mu\nu}^{(1,L)}}{d^2k_\perp} = \frac{x_B}{p \cdot q} \left\{ p_{\mu k_\perp} \left( g^{\perp} + \epsilon^{k\perp}_{\perp} g^{\perp}_{T} \right) + M p_{(\mu \epsilon_\perp \nu) i} s^{i}_{\perp} \hat{g}_{T} + \lambda p_{(\mu \epsilon_\perp \nu) i} k^{i}_{\perp} g^{\perp}_{L} \right\}. $$  

(53)

It is very interesting to see that, up to twist-3, all the contributions can be expressed by the coefficient functions of $\hat{\Phi}^{(0)}$.  

10
We add the contributions from $\tilde{W}_{\mu\nu}^{(1)}$ to those from $\tilde{W}_{\mu\nu}^{(0)}$, and obtain the final result for the hadronic tensor up to twist-3 as,

$$
\frac{d^2W_{\mu\nu}}{d^2k_\perp} = -d_{\mu\nu}(f_1 - \epsilon_{\perp}^k f_T^1) + \frac{1}{p \cdot q}k_{\perp\nu}(q + 2x_Bp)\nu\left(f^1_{\perp} - \epsilon_{\perp}^k f_T^1\right)
$$

$$
- \frac{M}{p \cdot q}(q + 2x_Bp)(\mu\epsilon_{\perp\nu})i\epsilon_{\perp}^i f_T^i - \frac{\lambda}{p \cdot q}(q + 2x_Bp)(\mu\epsilon_{\perp\nu})k_{\perp\nu} f_L^i
$$

$$
+ i\epsilon_{\perp\mu\nu}(\lambda g_{iL} - \frac{k_{\perp\nu}^i s_{\perp}^i g_{iL}}{M}) - \frac{i}{p \cdot q}k_{\perp\mu}(q + 2x_Bp)\mu\left(g^1_{\perp} + \epsilon_{\perp}^k g_T^1\right)
$$

$$
+ \frac{iM}{p \cdot q}(q + 2x_Bp)(\mu\epsilon_{\perp\nu})i\epsilon_{\perp}^i g_T^i + \frac{i\lambda}{p \cdot q}(q + 2x_Bp)(\mu\epsilon_{\perp\nu})k_{\perp\nu} g_L^i.
$$

(54)

We see that the result satisfies the electromagnetic gauge invariance $q^\mu d^2W_{\mu\nu}/d^2k_\perp = q^\nu d^2W_{\mu\nu}/d^2k_\perp = 0$ explicitly. The result is expressed in terms of 12 transverse momentum dependent (TMD) parton distribution and/or correlation functions. They contain the information from hadron structure and that from the multiple gluon scattering. We discuss them briefly in the following section.

D. TMD quark distribution/correlation functions

As can be seen from Eq. (54), up to twist-3, 12 TMD parton distribution and/or correlation functions are involved for the semi-inclusive DIS scattering process $e^- + N \rightarrow e^- + q + X$. Six of them are from the expansion of $\Phi_\alpha^{(0)} = Tr[\gamma_\alpha \tilde{\Phi}_\alpha^{(0)}]/2$ and six from $\tilde{\Phi}_\alpha^{(0)} = Tr[\gamma_5\gamma_\alpha \tilde{\Phi}_\alpha^{(0)}]/2$. They are defined in Eqs. (30) and (31). By reversing these two equations, we can obtain the operator expressions for these quark distribution and correlation functions. Four of them are leading twist parton distribution functions are quite familiar with us and can be found in different literature, e.g., in [36]. They all have clear physical interpretations and have attracted much attention and have been with much efforts both theoretically [9, 39–54] and experimentally [23–25, 27, 55–63]. As can been seen in Sec II.B, in the jet production process $e + N \rightarrow e + q + X$ where only one hadron state is involved, the hard parts contain odd number of $\gamma-$matrices. Hence, in the decomposition of correlation matrices, chiral-odd distribution and/or correlation functions such as transversity and Boer-Mulders functions, involve even number of $\gamma-$matrices and will not contribute. Such functions can be studied in the hadron production process $e + N \rightarrow e + h + x$ or the Drell-Yan process $p + p \rightarrow \ell\bar{\ell} + X$, where two hadron states are involved.
The other 8 are twist-3 and have the following operator expressions\[35],

\[
k_\perp^2 f^+(x, k_\perp) = \int \frac{p^+ dy^- d^2y_\perp}{2(2\pi)^3} e^{ixp^+ y^- - i k_\perp \cdot \vec{y}_\perp} \langle p|\bar{\psi}(0) k_\perp \mathcal{L}(0; y)\psi(y)|p\rangle, \tag{55}
\]

\[
k_\perp^2 g^+(x, k_\perp) = -\int \frac{p^+ dy^- d^2y_\perp}{2(2\pi)^3} e^{ixp^+ y^- - i k_\perp \cdot \vec{y}_\perp} \langle p|\bar{\psi}(0) \epsilon^i_{\perp j} k_\perp \gamma_j \gamma_5 \mathcal{L}(0; y)\psi(y)|p\rangle, \tag{56}
\]

\[
\epsilon^{k_\perp}_\perp (k_\perp \cdot s_\perp) f_T(x, k_\perp) = -\frac{1}{M^2} \int \frac{p^+ dy^- d^2y_\perp}{2(2\pi)^3} e^{ixp^+ y^- - i k_\perp \cdot \vec{y}_\perp} \times \langle p, s_\perp \rangle |\bar{\psi}(0) (k_\perp^i k_\perp^j - \frac{1}{2} k_\perp^2 \delta^{ij}) \gamma_\perp i s_\perp j \mathcal{L}(0; y)\psi(y)|p, s_\perp \rangle, \tag{57}
\]

\[
\epsilon^{k_\perp}_\perp (k_\perp \cdot s_\perp) g_T(x, k_\perp) = \frac{1}{M^2} \int \frac{p^+ dy^- d^2y_\perp}{2(2\pi)^3} e^{ixp^+ y^- - i k_\perp \cdot \vec{y}_\perp} \times \langle p, s_\perp \rangle |\bar{\psi}(0) (k_\perp^i k_\perp^j - \frac{1}{2} k_\perp^2 \delta^{ij}) \gamma_\perp i s_\perp j \mathcal{L}(0; y)\psi(y)|p, s_\perp \rangle, \tag{58}
\]

\[
\epsilon^{k_\perp}_\perp (k_\perp \cdot s_\perp) f_T^+(x, k_\perp) = -\int \frac{p^+ dy^- d^2y_\perp}{2(2\pi)^3} e^{ixp^+ y^- - i k_\perp \cdot \vec{y}_\perp} \langle p, s_\perp \rangle |\bar{\psi}(0) k_\perp \mathcal{L}(0; y)\psi(y)|p, s_\perp \rangle, \tag{59}
\]

\[
\epsilon^{k_\perp}_\perp (k_\perp \cdot s_\perp) g_T^+(x, k_\perp) = -\int \frac{p^+ dy^- d^2y_\perp}{2(2\pi)^3} e^{ixp^+ y^- - i k_\perp \cdot \vec{y}_\perp} \langle p, s_\perp \rangle |\bar{\psi}(0) \epsilon^i_{\perp j} k_\perp \gamma_j \gamma_5 \mathcal{L}(0; y)\psi(y)|p, s_\perp \rangle, \tag{60}
\]

\[
k_\perp^2 f_L^+(x, k_\perp) = -\int \frac{p^+ dy^- d^2y_\perp}{2(2\pi)^3} e^{ixp^+ y^- - i k_\perp \cdot \vec{y}_\perp} \langle p, + |\bar{\psi}(0) \epsilon^i_{\perp j} k_\perp \gamma_j \mathcal{L}(0; y)\psi(y)|p, + \rangle, \tag{61}
\]

\[
k_\perp^2 g_L^+(x, k_\perp) = \int \frac{p^+ dy^- d^2y_\perp}{2(2\pi)^3} e^{ixp^+ y^- - i k_\perp \cdot \vec{y}_\perp} \langle p, + |\bar{\psi}(0) k_\perp \gamma_5 \mathcal{L}(0; y)\psi(y)|p, + \rangle. \tag{62}
\]

Among them, \( f^+ \) and \( g^+ \) are related to the unpolarized case; \( f_T, g_T, f_T^+ \), and \( g_T^+ \) are related to the transverse polarization and \( f_L^+ \) and \( g_L^+ \) are related to the longitudinal polarization.

These twist-3 quark correlation functions have no simple probabilistic interpretation. In fact, as we can see from the derivations that lead to these results, these twist-3 correlation functions come from the interference terms between amplitudes for scattering without multiple gluon scattering and that with one gluon scattering.

If we integrate over \( \int d^2k_\perp \), we obtain the hadronic tensor \( W_{\mu\nu} \) from \( d^2W_{\mu\nu}/d^2k_\perp \). Since all parton distribution and/or correlation functions \( f's \) and \( g's \) are scalar functions of \( k_\perp \), all the terms that are linearly dependent on \( k_\perp \) vanish after the integration and we obtain from Eq. \[51\] that,

\[
W_{\mu\nu} = -d_{\mu\nu}f_1(x) - \frac{M}{p \cdot q} (q + 2xBp)\langle \mu \varepsilon_\perp \nu \rangle s_\perp^i f_T(x)
\]
\[ + i \varepsilon_{\perp \mu \nu} \lambda g_{1L}(x) + \frac{i M}{2 p \cdot q} (q + 2 x B p)_{[\mu} \varepsilon_{\nu]} i s^I g_T(x), \]

where \( f_1(x) \equiv \int d^2 k_\perp f_1(x, k_\perp), \ g_{1L}(x) \equiv \int d^2 k_\perp g_{1L}(x, k_\perp), \) and,

\[ f_T(x) \equiv \int d^2 k_\perp f_T(x, k_\perp), \]
\[ g_T(x) \equiv \int d^2 k_\perp g_T(x, k_\perp). \]

The \( g_T(x) \) term is the only twist-3 contribution to the structure function in inclusive DIS with longitudinally polarized lepton beam and transversely polarized nucleon target, as discussed in [64]. The \( f_T(x) \) term is a time-reversal-odd term corresponding to the T-odd term \( p_{[\mu} \varepsilon_{\nu] \rho} \sigma p^\rho q^\sigma p^\tau \) in \( W_{\mu \nu} \). It can be shown that, under time reversal invariance, \( f_T(x) = 0 \).

The situation as considered by in [2] and [4] can be recovered by putting \( g = 0 \). In this case, there is no multiple gluon scattering and \( \mathcal{L} = 1 \). Consequently the T-odd TMD distribution and/or correlation functions must be zero. The twist-3 quark correlation functions reduces to,

\[ x f_1^\perp(x, k_\perp)|_{g=0} = f_1(x, k_\perp)|_{g=0}, \]
\[ f_T(x, k_\perp)|_{g=0} = f_T^\perp(x, k_\perp)|_{g=0} = f_T^\perp(x, k_\perp)|_{g=0} = 0, \]
\[ x g_{1L}(x, k_\perp)|_{g=0} = g_{1L}(x, k_\perp)|_{g=0}, \]
\[ x g_T(x, k_\perp)|_{g=0} = - \frac{k_\perp^2}{2 M^2} x g_T^\perp(x, k_\perp)|_{g=0} = - \frac{k_\perp^2}{2 M^2} g_T^\perp(x, k_\perp)|_{g=0}, \]
\[ g_T^\perp(x, k_\perp)|_{g=0} = 0. \]

The hadronic tensor reduces to,

\[ \frac{d^2 \tilde{W}_{\mu \nu}}{d^2 k_\perp}|_{g=0} = - \left[ d_{\mu \nu} - \frac{1}{x B p \cdot q} k_{\perp} \varepsilon_{\perp \mu \nu} (q + 2 x B p)_{\nu} \right] f_1(x, k_\perp)|_{g=0} \\
+ i \lambda \left[ \varepsilon_{\perp \mu \nu} + \frac{1}{x B p \cdot q} (q + 2 x B p)_{[\mu} \varepsilon_{\nu]} k^I_\perp \right] g_{1L}(x, k_\perp)|_{g=0} \\
- i \frac{k_\perp \cdot s}{M} \left[ \varepsilon_{\perp \mu \nu} + \frac{1}{x B p \cdot q} (q + 2 x B p)_{[\mu} \varepsilon_{\nu]} k^I_\perp \right] g_T^\perp(x, k_\perp)|_{g=0}. \]

This just corresponds to the results obtained using the simple parton model with intrinsic transverse momentum as discussed in [2] and [4] for the unpolarized and the longitudinally polarized case respectively. The deviations from this result come from the multiple gluon scattering.
III. CROSS SECTIONS AND AZIMUTHAL ASYMMETRIES

Making the Lorentz contraction of the hadronic tensor $d^2 W_{\mu\nu}/d^2 k_\perp$ as given by Eq. (54) with the leptonic tensor $L_{\mu\nu}(l,l')$, we obtain the differential cross section of the process $e^-(l,s) + N(p,s) \rightarrow e^-(l') + q(k') + X$ as,

$$
\frac{d\sigma}{dx_B dy d^2 k_\perp} = \frac{2\pi\alpha^2 e^2}{Q^2 y} \left( W_{UU} + \lambda_l W_{LU} + s \lambda W_{UT} + \lambda_l \lambda W_{LL} + \lambda_l s \lambda W_{LT} \right),
$$

(72)

where $W_{s_1s}$ represents the contribution in the different polarization case, and we use the superscript $s_1 = U$ or $L$ to denote whether the lepton is unpolarized or longitudinally polarized, while $s = U, L$ or $T$ denotes whether the nucleon is unpolarized, longitudinally or transversely polarized [66]. These different contributions are given by,

$$
W_{UU} = A(y) f_1 - \frac{2x_B |\vec{k}_\perp|}{Q} B(y) f^\perp \cos \phi,
$$

(73)

$$
W_{UT} = \frac{|\vec{k}_\perp|}{M} A(y) f^\perp_1 \sin (\phi - \phi_s) - \frac{2x_B M}{Q} B(y) \left[ f_T \sin \phi_s - \frac{k^2}{2M^2} f^\perp_T \sin(2\phi - \phi_s) \right],
$$

(74)

$$
W_{UL} = -\frac{2x_B |\vec{k}_\perp|}{Q} B(y) f^\perp L \sin \phi,
$$

(75)

$$
W_{LU} = -\frac{2x_B |\vec{k}_\perp|}{Q} D(y) g^\perp \sin \phi,
$$

(76)

$$
W_{LL} = C(y) g_{1L} - \frac{2x_B |\vec{k}_\perp|}{Q} D(y) g^\perp L \cos \phi,
$$

(77)

$$
W_{LT} = \frac{|\vec{k}_\perp|}{M} C(y) g^\perp_{1T} \cos (\phi - \phi_s) - \frac{2x_B M}{Q} D(y) \left[ g_T \cos \phi_s - \frac{k^2}{2M^2} g^\perp_T \cos(2\phi - \phi_s) \right].
$$

(78)

where $A(y) = 1 + (1 - y)^2$, $B(y) = 2(2 - y)\sqrt{1 - y}$, $C(y) = y(2 - y)$, $D(y) = 2y\sqrt{1 - y}$, $\cos \phi = \vec{l}_\perp \cdot \vec{k}_\perp / ||\vec{l}_\perp|| ||\vec{k}_\perp||$, $\sin \phi = (\vec{l}_\perp \times \vec{k}_\perp) \cdot \vec{e}_z / ||\vec{l}_\perp|| ||\vec{k}_\perp||$, $\cos \phi_s = \vec{l}_\perp \cdot \vec{s}_\perp / ||\vec{l}_\perp|| ||\vec{s}_\perp||$, and $\sin \phi_s = (\vec{l}_\perp \times \vec{s}_\perp) \cdot \vec{e}_z / ||\vec{l}_\perp|| ||\vec{s}_\perp||$.

We note that, except the slightly different notations [66], these results are almost the same as those obtained in [9] for jet production. They have the same structures and the forms of the coefficients in each term are the same [67]. This is expected since the kinematics is the same and the approximations made in the hard parts in [9] should be equivalent to keep the leading and sub-leading terms in the collinear expansion. Also, due to the relationship
given by Eqs. (18,51) obtained from equation of motion, all the results are expressed by
the different components of \( \hat{\phi}^{(0)} \) defined by Eq. (16) which is identical to the original \( \hat{\phi}^{(0)} \)
given by Eq. (16) except for the gauge link. Hence, the difference in defining higher twist
correlators such as that between \( \hat{\phi}^{(1)} \) given by Eq. (27) and \( \hat{\phi}^{(1)} \) given by Eq. (14) does not
show up in the final results. However, it seems not the case for even higher twist [14].
Other features of the results are summarized in the following.

We see that there are leading twist contributions in the \( UU, UT, LL \) and \( LT \) cases, while
there are twist-3 contributions in all cases. The different azimuthal asymmetries are defined
by the average values of the corresponding sine or cosine of the angles. There are two leading
twist azimuthal asymmetries as given by,

\[
\langle \sin(\phi - \phi_s) \rangle_{UU} = s_{\perp} \frac{|\vec{k}_{\perp}| f^+_{LT}(x, k_{\perp})}{2M f_1(x, k_{\perp})},
\]

\[
\langle \cos(\phi - \phi_s) \rangle_{LT} = \lambda_1 s_{\perp} \frac{|\vec{k}_{\perp}| C(y) g^+_{LT}(x, k_{\perp})}{2M A(y) f_1(x, k_{\perp})}.
\]

The azimuthal asymmetry \( \langle \cos \phi \rangle \) exists at twist-3 for the unpolarized case. It receives also
a twist-3 contribution in the \( LL \) case but also a leading twist contribution in the \( LT \) case, i.e.,

\[
\langle \cos \phi \rangle_{UU} = -\frac{|\vec{k}_{\perp}| B(y) x_B f^+_{LT}(x, k_{\perp})}{Q A(y) f_1(x, k_{\perp})},
\]

\[
\langle \cos \phi \rangle_{LL} = -\frac{|\vec{k}_{\perp}| B(y) x_B f^+_{LT}(x, k_{\perp}) + \lambda_1 \lambda D(y) x_B g^+_{LT}(x, k_{\perp})}{Q A(y) f_1(x, k_{\perp}) + \lambda_1 \lambda C(y) g_{LT}(x, k_{\perp})},
\]

\[
\langle \cos \phi \rangle_{LT} = \frac{|\vec{k}_{\perp}| \lambda_1 s_{\perp} C(y) g^+_{LT}(x, k_{\perp}) \cos \phi_s - \frac{2M}{Q} B(y) x_B f^+_{LT}(x, k_{\perp})}{2M A(y) f_1(x, k_{\perp}) - \lambda_1 s_{\perp} \frac{2M}{Q} x_B g_{LT}(x, k_{\perp}) \cos \phi_s}.
\]

We note in particular that there exist a twist-3 asymmetry \( \langle \sin \phi \rangle \) for the \( LU \) or \( UL \) case,
i.e. when lepton or nucleon is longitudinally polarized while the other is unpolarized. It is
given by,

\[
\langle \sin \phi \rangle_{LU} = -\lambda_1 \frac{|\vec{k}_{\perp}| D(y) x_B g^+_{LT}(x, k_{\perp})}{Q A(y) f_1(x, k_{\perp})},
\]

\[
\langle \sin \phi \rangle_{UL} = -\lambda_1 \frac{|\vec{k}_{\perp}| B(y) x_B f^+_{LT}(x, k_{\perp})}{Q A(y) f_1(x, k_{\perp})}.
\]

They are determined by the TMD parton correlation \( g^\perp \) and \( f^+_T \) respectively.

It is also interesting to see that, if we integrate over \( \phi \), we obtain,

\[
\frac{d\sigma}{|\vec{k}_{\perp}| dx_B dy d|\vec{k}_{\perp}|} = \frac{4\pi^2 a^2 e^2}{Q^2 y} \left\{ A(y) f_1 - s_{\perp} \frac{2x_B M}{Q} B(y) f_T \sin \phi_s \right\}
\]
\[ + \lambda t \lambda C(y) g_{1L} - \lambda t s_\perp \frac{2x_B M}{Q} D(y) g_T \cos \phi_s \]. 

(87)

We see that the transverse spin asymmetry exists for the semi-inclusive process \( e^- + N \rightarrow e^- + q + X \) at the twist-3 level both in the target singly polarized case \( UT \) and in the case \( LT \) where the lepton is also longitudinally polarized. But the asymmetries in these two cases are different and are given by,

\[
\langle \sin \phi_s \rangle_{UT} = -s_\perp \frac{M B(y) x_B f_T(x_B, k_\perp)}{Q A(y) f_1(x_B, k_\perp)},
\]

(88)

\[
\langle \cos \phi_s \rangle_{LT} = -\lambda t s_\perp \frac{M D(y) x_B g_T(x_B, k_\perp)}{Q A(y) f_1(x_B, k_\perp)}.
\]

(89)

We also note that, in experiments, we usually measure for a given \( |\vec{k}_\perp| \) interval. In this case, we need to carry out the integration over \( |\vec{k}_\perp| \). For example, if we integrate over the whole \( |\vec{k}_\perp| \) region, we obtain,

\[
\langle \langle \sin \phi \rangle \rangle_{LU} = -\lambda t \frac{B(y)}{A(y)} \frac{2\pi}{Q^2 y} \int \frac{d|\vec{k}_\perp|}{|\vec{k}_\perp|} x_B g^+(x_B, k_\perp) f_1(x_B).
\]

(90)

By carrying out the integration over \( d^2k_\perp \), we obtain the differential cross section \( d\sigma/dx_B dy \) for the inclusive DIS process \( e^- + N \rightarrow e^- + X \) as,

\[
\frac{d\sigma}{dx_B dy} = \frac{2\pi \alpha_s^2 e_q^2}{Q^2 y} \left\{ A(y) f_1(x_B) + \lambda t \lambda C(y) g_{1L}(x_B) - \lambda t s_\perp \frac{2x_B M}{Q} D(y) g_T(x_B) \cos \phi_s \right\},
\]

(91)

where we see clearly that the only twist-3 contribution exists for the case that the lepton is longitudinally polarized and the nucleon is transversely polarized.

At \( g = 0 \), the cross section reduces to the result obtained in the simple parton model with intrinsic transverse momentum [2, [4]. By inserting the results given by Eqs. 66 - 70 into Eqs. 73 - 78, we obtain,

\[
W_{UU}|_{g=0} = \left[ A(y) - \frac{2|\vec{k}_\perp|}{Q} B(y) \cos \phi \right] f_1(x, k_\perp)|_{g=0},
\]

(92)

\[
W_{UT}|_{g=0} = W_{UL}|_{g=0} = W_{LU}|_{g=0} = 0,
\]

(93)

\[
W_{LL}|_{g=0} = \left[ C(y) - \frac{2|\vec{k}_\perp|}{Q} D(y) \cos \phi \right] g_{1L}(x, k_\perp)|_{g=0},
\]

(94)

\[
W_{LT}|_{g=0} = \frac{|\vec{k}_\perp|}{M} \left[ C(y) - \frac{2|\vec{k}_\perp|}{Q} D(y) \cos \phi \right] g_{1T}(x, k_\perp)|_{g=0} \cos (\phi - \phi_s).
\]

(95)

Correspondingly, for the azimuthal asymmetries discussed above, we obtain,

\[
\langle \sin(\phi - \phi_s) \rangle_{UT}|_{g=0} = 0,
\]

(96)
\[ \langle \cos(\phi - \phi_s) \rangle_{LT} |_{g=0} = \lambda l \lambda D(y) g_{LT}^{\perp}(x, k_\perp) \]

Clearly, a systematic study of these asymmetries should provide very important information on the structure of the nucleon and the properties of strong interaction. In particular, the deviations from the results given by Eqs. (96) - (102) tell us the influences from the multiple gluon scattering.

### IV. NUCLEAR DEPENDENCE

The above mentioned calculations apply to \( e^- + N \rightarrow e^- + q + X \) as well as \( e^- + A \rightarrow e^- + q + X \), i.e. for reactions using a nucleus target. Similar results are obtained with only a replacement of the state \( |N\rangle \) by \( |A\rangle \) in the definitions of the parton distribution and/or correlation functions. It has also been shown [34] that the multiple gluon scattering contained in the gauge link leads to a strong nuclear dependence for these TMD parton distribution and/or correlation functions. Such nuclear dependences can manifest themselves in the azimuthal asymmetries in SIDIS [13, 14]. In this section, we present the results for the parton distributions and azimuthal asymmetries given in last section.

#### A. A-dependence of the parton correlation functions

If we replace the state \( |N\rangle \) by \( |A\rangle \), the multiple gluon scattering in the gauge link can be connected to different nucleons in the nucleus \( A \) thus gives rise to nuclear dependence. It has been shown that, under the “maximal two gluon approximation” [34], a TMD quark
distribution $\Phi^A_A(x, k_\perp)$ in nucleus defined in the form,

$$\Phi^A_A(x, k_\perp) \equiv \int \frac{p^+dy^-d^2y_\perp}{(2\pi)^3}e^{ip^+ y^- - i\vec{k}_\perp \cdot \vec{y}_\perp}\langle A | \bar{\psi}(0) \Gamma_\alpha \mathcal{L}(0; y) \psi(y) | A \rangle,$$  \hspace{1cm} (103)

is given by a convolution of the corresponding distribution $\Phi^N_N(x, k_\perp)$ in nucleon and a Gaussian broadening \[34\], i.e.,

$$\Phi^A_A(x, k_\perp) \approx \frac{A}{\pi \Delta_{2F}} \int d^2\ell_\perp e^{-(\vec{k}_\perp - \vec{\ell}_\perp)^2/\Delta_{2F}} \Phi^N_N(x, \ell_\perp),$$  \hspace{1cm} (104)

where $\Gamma_\alpha$ is any gamma matrix, $\Delta_{2F}$ is the broadening width, $\Delta_{2F} = \int d\xi \hat{\Delta}_{F}(\xi_N)$, and $\hat{\Delta}_{F}(\xi_N) = (2\pi^2\alpha_s/N_c)\rho^A_A(\xi_N)[x f^N_g(x)]_{x=0}$ is the quark transport parameter, where $\rho^A_A(\xi_N)$ is the spatial nucleon number density inside the nucleus and $f^N_g(x)$ is the gluon distribution function in nucleon, the superscript $A$ or $N$ denotes that it is for the nucleus or the nucleon.

The derivations in \[34\] apply to any nucleon and nucleus in the unpolarized case. Since both $\Phi^N_A(0)$ and $\Phi^0_A(0)$ defined in Eqs. \[30\] and \[31\] are of the form given by Eq. \[104\], Eq. \[103\] applies and derive the $A$-dependences of different parton distribution and/or correlation functions in the unpolarized case. For those involved in the differential cross section up to twist-3, we obtain,

$$f^1_A(x, k_\perp) \approx \frac{A}{\pi \Delta_{2F}} \int d^2\ell_\perp e^{-(\vec{k}_\perp - \vec{\ell}_\perp)^2/\Delta_{2F}} f^N_1(x, \ell_\perp),$$  \hspace{1cm} (105)

$$|\vec{k}_\perp|^2 f^{+A}_1(x, k_\perp) \approx \frac{A}{\pi \Delta_{2F}} \int d^2\ell_\perp e^{-(\vec{k}_\perp - \vec{\ell}_\perp)^2/\Delta_{2F} (\vec{k}_\perp \cdot \vec{\ell}_\perp)} f^{+N}_1(x, \ell_\perp),$$  \hspace{1cm} (106)

$$|\vec{k}_\perp|^2 g^{+A}_1(x, k_\perp) \approx \frac{A}{\pi \Delta_{2F}} \int d^2\ell_\perp e^{-(\vec{k}_\perp - \vec{\ell}_\perp)^2/\Delta_{2F} (\vec{k}_\perp \cdot \vec{\ell}_\perp)} g^{+N}_1(x, \ell_\perp).$$  \hspace{1cm} (107)

To illustrate the dependence more clearly, we take the Gaussian ansatz for the transverse momentum dependence, i.e.,

$$f^1_N(x, \ell_\perp) = \frac{1}{\pi \alpha} f^N_1(x)e^{-\vec{\ell}_\perp^2/\alpha},$$   \hspace{1cm} (108)

$$f^{+N}_1(x, \ell_\perp) = \frac{1}{\pi \beta} f^{+N}_1(x)e^{-\vec{\ell}_\perp^2/\beta},$$   \hspace{1cm} (109)

$$g^{+N}_1(x, \ell_\perp) = \frac{1}{\pi \gamma} g^{+N}_1(x)e^{-\vec{\ell}_\perp^2/\gamma},$$   \hspace{1cm} (110)

and obtain immediately,

$$f^1_A(x, k_\perp) \approx \frac{A}{\pi \alpha_A} f^N_1(x)e^{-\vec{k}_\perp^2/\alpha_A},$$   \hspace{1cm} (111)

$$f^{+A}_1(x, k_\perp) \approx \frac{A}{\pi \beta_A} f^{+N}_1(x)e^{-\vec{k}_\perp^2/\beta_A},$$   \hspace{1cm} (112)
\[ g_{A}^{\perp}(x, k_{\perp}) \approx \frac{A}{\pi \gamma_A \gamma} g_{N}^{\perp}(x) e^{-\vec{k}_{\perp}^2/\gamma_A}, \tag{113} \]

where \( \alpha_A = \alpha + \Delta_{2F} \), \( \beta_A = \beta + \Delta_{2F} \) and \( \gamma_A = \gamma + \Delta_{2F} \). We see that all the TMD distribution/correlation functions have \( p_T \)-broadening with the magnitude \( \Delta_{2F} \), but the twist-3 parton correlation function \( f_{A}^{\perp}(x, k_{\perp}) \) or \( g_{A}^{\perp}(x, k_{\perp}) \) has an extra suppression factor \( \beta/\beta_A \) or \( \gamma/\gamma_A \).

### B. \( A \)-dependence of the azimuthal asymmetry

Having the nuclear dependences of the TMD parton distribution and correlation functions and the expressions for the azimuthal asymmetries presented in the previous sections, we can calculate the nuclear dependence of the azimuthal asymmetries in a straight forward manner with the Gaussian ansatz for the TMD distributions and/or correlations.

For reactions with unpolarized target, the results are just the same as those for the unpolarized reaction as discussed in [13]. This applies to the asymmetry \( \langle \sin \phi \rangle_{LU} \) given by Eq. (84), for which we obtain,

\[
\frac{\langle \sin \phi \rangle_{LU}^{A}}{\langle \sin \phi \rangle_{LU}^{N}} \approx \frac{\alpha_A}{\alpha} \left( \frac{\gamma}{\gamma_A} \right)^2 \exp \left[ \left( \frac{1}{\alpha_A} - \frac{1}{\alpha} - \frac{1}{\gamma_A} + \frac{1}{\gamma} \right) \vec{k}_{\perp}^2 \right]. \tag{114} \]

For \( \alpha = \gamma \), it simply reduces to

\[
\frac{\langle \sin \phi \rangle_{LU}^{A}}{\langle \sin \phi \rangle_{LU}^{N}} \approx \frac{\alpha}{\alpha + \Delta_{2F}}. \tag{115} \]

Integrated over \( |\vec{k}_{\perp}| \), we have,

\[
\frac{\langle \langle \sin \phi \rangle \rangle_{LU}^{A}}{\langle \langle \sin \phi \rangle \rangle_{LU}^{N}} \approx \sqrt{\frac{\gamma}{\gamma + \Delta_{2F}}}. \tag{116} \]

We see that, also in this case, the asymmetry is suppressed in reactions using the nucleus target in similar manner as in the unpolarized case discussed in [13, 14].

The width \( \gamma \) can in general be different from \( \alpha \). Hence, we present as an example in Figs. (1a) and (1b) the ratio as a function of \( k_T \)-broadening parameter \( \Delta_{2F} \). We see that it is very similar to \( \langle \cos \phi \rangle_{UU} \) discussed in [13]. We also plot the \( k_T \)-dependence of the ratio in Figs. (2a) and (2b). It is easy to see that for \( \gamma/\alpha < 1 \), the ratio of \( \langle \sin \phi \rangle_{LU} \) is quite sensitive to the value of \( \gamma/\alpha \) in the large \( k_{\perp} \) region.
FIG. 2. Ratio of $\langle \sin \phi \rangle^e_{LU} / \langle \sin \phi \rangle^N_{LU}$ as a function of $\Delta_{2F}/\alpha$ for different $k_\perp$ and $\gamma$.

V. SUMMARY

We present a systematic calculation of the hadronic tensor and azimuthal asymmetries in the semi-inclusive deep-inelastic scattering $e^- + N \rightarrow e^- + q + X$ with polarized beam and/or polarized target based on the collinear expansion formalism in LO pQCD and up to twist-3 contributions. The results depend on a number of new TMD parton correlation functions. We showed that measurements of the corresponding azimuthal asymmetries and their $k_\perp$-dependence can provide much information on these TMD correlation functions which in turn can shed light on the properties of multiple gluon interaction in hadronic processes. We presented the results also for reactions with nucleus target $e^- + A \rightarrow e^- + q + X$ and discuss the nuclear dependence. We show that the relationship between these TMD correlation
functions inside large nuclei and that of a nucleon under two-gluon correlation approximation. One can study the nuclear dependence of the different azimuthal asymmetries which are determined by the corresponding parton distribution and correlation functions. With the Gaussian ansatz for the TMD parton correlation functions inside the nucleon, we also illustrate numerically that the asymmetries are suppressed in the corresponding SIDIS with nuclear target.

Experimental studies of the azimuthal asymmetries have been carried out in both unpolarized and polarized semi-inclusive deep inelastic scattering with nucleon target [16–27]. More results are expected from CLAS at JLab and COMPASS at CERN. The available data seem to be consistent with the Gaussian ansatz for the transverse momentum dependence of the TMD matrix elements in the unpolarized case [65]. However these data are

FIG. 3. Ratio of $\langle \sin \phi \rangle_{LU}^{eA}/\langle \sin \phi \rangle_{LU}^{eN}$ as a function of $k_\perp$ for different $\gamma$ and $\Delta_{2F}$. 
still not adequate enough to provide any precise constraints on the form of the higher twist matrix elements. Our calculations of the azimuthal asymmetries are most valid in the small transverse momentum region where NLO pQCD corrections are not dominant. The high twist effects are also most accessible in intermediate region of $Q^2$. One expects that future experiments such as those at the proposed Electron Ion Collider (EIC) [54] will be better equipped to study these high twist effects in detail.

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[66] The notations that we use here are slightly different from those in [9]. The W’s defined here are equivalent to the F’s in [9] multiplied by the corresponding sine or cosine.

[67] We see also that the first and the second terms in $W_{UT}$, the $W_{LU}$, the $W_{UL}$, and the third term in $W_{LT}$ have different signs from the corresponding terms of the results presented in [9]. This is caused by the difference in the definition of $\phi$ and $\phi_s$ in the two papers. In [9], when defining $\sin \phi$ and $\sin \phi_s$, z-direction is taken as the direction of the momentum of the virtual photon. However, in defining $x = k^+/p^+$, the 4-momentum $p$ of the incident nucleon is taken in a way that the $+$ component in the lightcone coordinate dominates which implies that the z-direction is taken as the direction of motion of the nucleon. In this paper, we unify the notation by taking z-direction as the direction of motion of the incident nucleon both in defining $x$ and $\sin \phi$ thus we define $\sin \phi$ and $\sin \phi_s$ as mentioned after Eq.(69) and they have a sign difference with those in [9].