Error Analysis and Accuracy Assessment of GPS Absolute Velocity Determination without SA

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Abstract  Error sources which decrease the accuracy of GPS in absolute velocity determination have been changed since SA was turned off. Firstly, quantities of all kinds of error sources that influence velocity determination are analyzed. The potential accuracy of GPS absolute velocity determination is derived from both theory and field GPS data simulation. After that, two tests were carried out to evaluate the performance of GPS absolute velocity determination in the case of a static and an airborne GPS receiver and INS (Inertial Navigation System) instrument in kinematic mode. In static mode, the receiver velocity has been estimated to be several mm/s with the carrier-phase derived Doppler measurements, and several cm/s with the receiver generated Doppler measurements. In kinematic mode, GPS absolute velocity estimates are compared with the synchronized measurements from the high accuracy INS. The root mean square statistics of the velocity discrepancies between GPS and INS come up to dm/s. Moreover, it has a strong correlation with the acceleration or jerk of the aircraft.

Keywords  GPS absolute velocity determination; Doppler measurement; error; accuracy analysis

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Introduction

High accuracy velocity determination is important for missions involving satellite orbit rendezvous, airborne survey etc. GPS is an ideal technology for providing these real-time accurate velocity estimates at a low cost. Before SA was turned off, GPS absolute velocity determination could only achieve an accuracy of dm/s\(^1\). In order to improve the accuracy of GPS velocity determination, researchers have made some investigations on differential GPS velocity determination\(^{[2-6]}\). However, these results did not coincide with each other on the accuracy of DGPS velocity determination. For example, the results of kinematic airborne experiments in China showed that the velocity accuracies of mm/s could be achieved with the Doppler shift measurement\(^{[2-4]}\), but the scholars in the University of Calgary found that the accuracies of relative velocity determination depended on dynamic conditions according to many experimental results using DGPS simulation system\(^{[5,6]}\). The velocity accuracy with mm/s could be achieved from GPS measurements under low dynamic conditions with constant velocity, but it decreased to 0.2 m/s, or even few m/s in the worst case under high dynamic conditions with high accelerations or jerks.

1 Principle of GPS absolute velocity determination

The observation equation for velocity determina-
tion can be derived from the ranging rate between the receiver and the satellite j as following:[1]

\[ \lambda \Phi_j = \frac{(r-r'^j)(\dot{r}-\dot{r}'^j)}{\rho'^j} + c \delta i - c \delta i'^j + \dot{t}^j + \epsilon^j \] (1)

where \( \Phi \) is the receiver-generated Doppler measurement, which is called the raw Doppler measurement in the following sections; superscript \( j \) represents the PRN number of the GPS satellite; \( \rho'^j \) stands for the geometric range between the receiver and the satellite \( j \); \( r' \) and \( \dot{r}' \) represent the position and velocity vector of the receiver, respectively; \( r'^j \) and \( \dot{r}'^j \) represent the position and velocity vector of the satellite \( j \); \( \delta i \) is the receiver clock drift; \( \delta i'^j \) is the clock drift of the satellite \( j \); \( \dot{t}^j \) is the tropospheric delay rate; \( \epsilon^j \) is the noise of the observation.

The raw Doppler measurement can be replaced by the carrier-phase-derived Doppler measurement, which is derived from the difference in carrier phase observations in the time domain. The equation is expressed as follows:

\[ \Phi'_k = \frac{\Phi_{k+1} - \Phi_{k-1}}{2 \Delta t} \] (2)

Where \( \Phi \) is the carrier-phase observation; subscripts \( k, k+1, k-1 \) stand for the observation epoch; \( \Delta t \) is the interval of the observation data.

The algorithm of the absolute velocity determination is similar to that of GPS single point positioning. The velocity of a receiver as well as the clock shift can be estimated provided that four satellites are tracked by a receiver simultaneously. If the number of satellites tracked is bigger than four, the velocity solution can be obtained by the least squares method.

2 Errors analysis of GPS absolute velocity determination

In order to evaluate the effect of each error on the accuracy of GPS absolute velocity determination, the magnitude of each error affecting the ranging rate between a receiver and a satellite is first discussed one by one. By differentiating Eq.(1) we can derive the following equation:

\[ \lambda d \Phi'_j = \frac{(\dot{r}-\dot{r}'^j) \cdot (dr-d\dot{r}'^j) - (r-r'^j) \cdot d\dot{r}'^j}{\rho'^j} + c d \delta i + d \dot{t}^j - \frac{(r-r'^j) \cdot (\dot{r}-\dot{r}'^j) r-r'^j}{(\rho'^j)^2} d\dot{r}'^j \] (3)

The last two terms in Eq.(3) are negligible because of its insignificant effects.

2.1 Errors from satellite orbit and clock

On one hand errors from satellite orbit and clock produce an indirect effect on velocity determination by decreasing the positioning accuracy of a receiver. On the other hand, the error of satellite orbit introduces a computational error of the design matrix. According to Eq.(3) the impact of the satellite orbit error on the ranging rate can be expressed as \( \lambda d \Phi'_j = \frac{(\dot{r}-\dot{r}'^j) \cdot d\dot{r}'^j}{\rho'^j} \). If a magnitude of the satellite orbit error is 10 m, its impact on the ranging rate will only be 1.6 mm/s when \( \rho'^j \) and \( \dot{r}'^j \) are approximately 20 000 km and 3.2 km/s respectively. At present the accuracy of GPS broadcast ephemerides is 1.6 m from the website of IGS central bureau[7], therefore its effect on the accuracy of GPS absolute velocity determination will be insignificant.

2.2 Error from satellite velocity

An investigation made by Serrano[8], who had summarized the deviations between two results of velocities of many satellites computed with IGS post-processed ephemeris and broadcast ephemeris respectively, showed that the accuracy of satellite velocity from broadcast ephemeris is smaller than 2 mm/s. Its magnitude is almost equal to the noise level of measurement.

2.3 Error from satellite clock drift

The stability of an atomic clock assembled in GPS satellite keep to \( 10^{-12} - 10^{-13} \) in a period of one day. The residual of that has negligible influence on velocity determination after making the correction by using navigation message.

2.4 Relativistic effect

The relativistic effect caused by the orbital eccen-
tricity will affect the accuracy of velocity estimation of a receiver that can be modeled as follows<sup>[9]</sup>:

$$\delta \dot{t} = -4.433 \times 10^{-10} e \sqrt{a} \cos E \frac{dE}{dt}$$  \hspace{1cm} (4)

Where $e$ is the orbital eccentricity; $a$ is the semi-major axis of the orbit; $E$ is the eccentric anomaly; $\frac{dE}{dt}$ is the rate of $E$. The results showed that the magnitude of the relativistic effect would be up to 0.01 ns/s on the satellite clock drift. It can be modeled by Eq.(4) in the processing of velocity determination.

### 2.5 Ionospheric and tropospheric delay rate

The ionospheric and tropospheric delay rate are dependent on the local atmospheric disturbance and the satellite elevation. In general, the atmosphere in most regions of the world is fairly calm, and the recording interval of observables is short such as 1 sec, so the effects of atmospheric delay rate are negligible except for the satellite at a low elevation. But this can be resolved by masking the satellite with low elevation in the velocity estimations.

### 2.6 Error of positioning

The error of the receiver positioning will decrease the accuracy of velocity determination because it introduces computational errors of the design matrix same as the error of satellite orbit. If the receive positioning error is about 10 m, the effect of ranging rate will reach 1.6 mm/s. It is relevant to the ranging rates of all satellites tracked.

In order to estimate influence of position error on velocity determination, a numerical simulation was carried out with static test data. In this simulation, an extra bias of 100 m was intentionally added to the position component of the receiver. Then the velocities were estimated with the carrier-phase-derived Doppler measurements in each case. Table 1 shows the statistical results of each case. The accuracy of velocity determination is about 3.4 mm/s if the position of the receiver has no bias. And 100 m position bias will degrade the accuracy to about 1 cm/s, which coincide with the previous analysis.

### 2.7 Noise of the Doppler measurement

The Doppler measurement can be directly generated from the receiver (named the raw Doppler measurement) or derived from the carrier-phase measurements (called the derived Doppler measurement). The noise of the two kinds of measurement isn’t identical, which has been discussed in some references.

The noise of the derived Doppler measurement can be derived from Eq.(2) according to the law of variance-covariance propagation. Suppose the noise of the carrier-phase measurement is about 1 ~ 2 mm, and the recording interval of observation data is 1 sec, then the noise of the derived Doppler measurement can be computed from the following equation:

$$\sigma_\phi = \frac{\sqrt{2}}{2} \sigma_\phi$$

where $\sigma_\phi$ is the noise of the derived Doppler measurement; $\sigma_\phi$ is the noise of the carrier-phase measurement. Therefore the noise of the derived Doppler measurement is about 0.7 ~ 1.4 mm/s.

The noise of the raw Doppler measurement is argued in some references. Someone showed that it was superior to the derived Doppler measurement<sup>[2]</sup>, however the other’s opinion is opposite<sup>[8]</sup>.

According to the previous analyses and numerical simulations of the major error sources, it is obvious that the integrated effects of errors on the ranging rate between a satellite and a receiver is about 3.7 mm/s using the derived Doppler measurement in the case without the SA.
3 Test and results

3.1 Static test

In order to verify previous analyses and evaluate the accuracy of GPS absolute velocity determination, a series of tests were carried out. The first test was performed in static mode on the roof of the building of the school of Geodesy and Geomatics in the Wuhan University on Feb 24, 2006. GPS observations were recorded at every 1 Hz by a Leica 1230 receiver for 1 hour. The GPS antenna was fixed on the mark. The errors of velocity determination came from integrated effects of all error sources.

Table 2 shows the statistical accuracy of velocity determination using the raw Doppler measurement and the derived Doppler measurement respectively.

|                      | Raw Doppler Measurement | Derived Doppler Measurement |
|----------------------|-------------------------|-----------------------------|
|                      | N | E | U | N | E | U |
| Mean                 | 3.2| 2.3| 7.7| 0.3| -0.9| -1.1 |
| Std                  | 13.1| 10.9| 25.9| 1.4| 1.1| 2.4 |
| RMS                  | 13.5| 11.1| 27.1| 1.5| 1.4| 2.7 |

It is obvious that the velocity accuracies of the derived Doppler measurement are better than that of the raw Doppler measurement. Similar to GPS point positioning, the accuracy of absolute velocity determination can be expressed as:

\[ m_v = V_{PDOP} \cdot m_p \]  

Where \( m_v \) denotes the accuracy of velocity determination, \( V_{PDOP} \) is the value of the positional dilution of precision; \( m_p \) is the integrated effect on ranging rate from all error sources. In the static test, the average PDOP is about 2.5, so the \( m_p \) values of both the raw Doppler and the derived Doppler measurement may be estimated as 12.9 mm/s and 1.4 mm/s, respectively. Therefore the noise of the raw Doppler measurement should amount to a magnitude of 1 cm/s. It is further proven that the major error source degrading the accuracy of velocity determination comes from the Doppler measurement on the basis of the results of the static test.

3.2 Kinematic test

The second test was performed in kinematic mode. A Javad LEGACY dual-frequency receiver was set on the aircraft together with an IMU. Dual-frequency pseudo-range, carrier phase, raw Doppler measurements were recorded at every 1 Hz, and the velocity and acceleration measurements were recorded by the INS at 10 Hz. The time system of INS was synchronized with PPS pulses of the GPS receiver.

At the beginning of the kinematic test, the aircraft stayed on the ground while the engine had been starting for a half hour, then took off and flew in a variety of dynamic conditions. Fig.1 shows the trajectory of the flight.

Fig.1 Flight trajectory of aircraft

3.2.1 Stage before takeoff

The body of the aircraft vibrated slightly as the engine started before the aircraft took off. The high rate of outputs from the INS instrument, especially the vertical component of velocity, shows sinusoidal wave signal whose amplitude was at several mm/s. Table 3 shows the accuracy of velocity determination from the raw Doppler measurements (marked with RV), the derived Doppler measurements (DV) and INS instrument (AV) as well. The accuracies of each component of velocity estimated from two kinds of Doppler measurements are approximately the same as those in static mode. The RMS values of INS velocity components such as N, E and U are 3.0, 2.6 and 4.4
mm/s, respectively. So the accuracy of INS velocity determination is very accurate, which will be used as basis when comparing velocity determination accuracy during the entire flight.

Table 3  Statistics of velocity determination before the aircraft took off

| RV/(mm·s⁻¹) | DV/(mm·s⁻¹) | AV/(mm·s⁻¹) |
|-------------|-------------|-------------|
| N           | E           | U           | N           | E           | U           | N           | E           | U           |
| Mean        | 0           | 0           | 0           | 0           | -1.9       | -0.1       | 0.2         | -1.1       | -0.8       | -0.5       | 0           |
| Std         | 11.0        | 9.4         | 27.5        | 1.3         | 1.1        | 2.9        | 2.8         | 2.6        | 4.4        |
| RMS         | 11.0        | 9.4         | 27.5        | 1.3         | 1.1        | 3.1        | 3.0         | 2.6        | 4.4        |

3.2.2  Stage after takeoff

Fig.2 shows discrepancies of velocity among the three results epoch by epoch, and their accuracy statistics are summarized in Table 4. The coincidence of velocities from both kinds of Doppler measurement is better than that between GPS and INS. Moreover, the discrepancies of velocity between GPS and INS show a strong correlation with respect to the acceleration of the aircraft movement in the horizontal direction.

Table 4  Statistics of velocity determination during the kinematic flight

| AV-RV/(mm·s⁻¹) | AV-DV/(mm·s⁻¹) | RV-DV/(mm·s⁻¹) |
|----------------|----------------|----------------|
| N              | E              | U              | N              | E              | U              | N              | E              | U              |
| Mean           | 12.9           | 90.1           | 30.9           | 13.4           | 90.5           | 30.2           | 0.5           | 0.4           | -0.6          |
| Std            | 98.4           | 107.0          | 60.7           | 106.2          | 112.0          | 74.9           | 32.2          | 31.1          | 73.6          |
| RMS            | 99.2           | 139.9          | 68.1           | 107.0          | 144.0          | 80.8           | 32.2          | 31.1          | 73.6          |

The derived Doppler measurement, derived from differentiating carrier phase observations in the time domain, is only used to estimate the average velocity over a short period of time. The more dramatic the dynamic conditions vary, the greater the discrepancy between the average velocity and the instantaneous velocity receiver will be. The raw Doppler observation at every epoch is measured in an instant, but some receivers, such as NovAtel Millennium™, have an average output observation in an interval of 50~100 ms as the raw Doppler measurement[10].

The experimental results using velocity estimated with the observations generated by a DGPS simulator system during various dynamics, show that the errors of velocity determination were correlated with either the acceleration or jerk of the vehicle and can be expressed as:

$$\sigma_v(t) = a \times A(t)$$

Where, $A(t)$ is the acceleration or jerk of the vehicle; $a$ is the coefficient. If the velocities determined by the INS are taken as truth herein, the horizontal velocities estimated with the raw Doppler measure-
ment are correlated with the accelerations where the coefficient is approximately 0.08. Fig.3 shows the correlation between the velocity errors and the accelerations of the aircraft during flight.

![Fig.3 The correlation between errors of GPS velocity estimation and the flight acceleration](image)

4 Conclusions

Some error sources such as GPS satellite orbit, satellite clock error and drift have decreased to a magnitude of mm/s after SA was turned off.

The accuracy of GPS absolute velocity determination will be at the level of cm/s when the positioning accuracy of the receiver is better than 100m. So it is possible to obtain high velocity accuracy with the single frequency receiver at a low cost.

The errors of the epoch-by-epoch velocity estimations with both the receiver-generated Doppler measurement and the carrier-phase derived Doppler measurement at 1Hz data rate show a strong correlation with the acceleration or the jerk of the receiver during high dynamic conditions.

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