Drag in a resonantly driven polariton fluid

A C Berceanu, E Cancellieri and F M Marchetti

Departamento de Física Teórica de la Materia Condensada, Universidad Autónoma de Madrid, E-28049 Madrid, Spain

E-mail: andrei.berceanu@uam.es

Received 24 February 2012, in final form 25 April 2012
Published 15 May 2012
Online at stacks.iop.org/JPhysCM/24/235802

Abstract

We study the linear response of a coherently driven polariton fluid in the pump-only configuration scattering against a point-like defect and evaluate analytically the drag force exerted by the fluid on the defect. When the system is excited near the bottom of the lower polariton dispersion, the sign of the interaction-renormalised pump detuning classifies the collective excitation spectra into three different categories (Ciuti and Carusotto 2005 Phys. Status Solidi b 242 2224): linear for zero, diffusive-like for positive and gapped for negative detuning. We show that both cases of zero and positive detuning share a qualitatively similar crossover of the drag force from the subsonic to the supersonic regime as a function of the fluid velocity, with a critical velocity given by the speed of sound found for the linear regime. In contrast, for gapped spectra, we find that the critical velocity exceeds the speed of sound. In all cases, the residual drag force in the subcritical regime depends on the polariton lifetime only. Also, well below the critical velocity, the drag force varies linearly with the polariton lifetime, in agreement with previous work (Cancellieri et al 2010 Phys. Rev. B 82 224512), where the drag was determined numerically for a finite-size defect.

(Some figures may appear in colour only in the online journal)

1. Introduction

Out-of-equilibrium quantum fluids such as polaritons in semiconductor microcavities are the subject of intensive study. Microcavity polaritons, the quasiparticles resulting from the strong coupling of cavity photons and quantum well excitons, have the prerogative of being easy to both manipulate, via an external laser, and detect, via the light escaping from the cavity [1]. In particular, resonant excitation allows the accurate tuning of the fluid properties, such as its density and current. However, the polariton lifetime being finite establishes the system as intrinsically out of equilibrium: an external pump is needed to continuously replenish the cavity of polaritons, that quickly, on a scale of tens of picoseconds, escape.

Recently, the superfluid properties of a resonantly pumped polariton quantum fluid in the pump-only configuration—i.e. where no other states aside from the pump one are occupied by, for example, parametric scattering—have been actively investigated both experimentally and theoretically [2–9]. This pumping scheme, differently from other cases such as the resonant optical parametric oscillator regime and the non-resonant pumping scheme, creates a polariton fluid that, inside the pump spot, is not characterized by a free phase. In contrast, the phase of the pump state is locked to the one of the external pumping laser. Nevertheless, it has been predicted [2, 3] and observed [4] that scattering can be suppressed below a critical velocity, where the system displays superfluid behaviour, similar to what has been predicted by the Landau criterion for equilibrium superfluid condensates. Further, a fixed phase clearly prevents the formation of phase dislocations, such as vortices and solitons. For this reason, it has been suggested [6] and experimentally realized [7] that the defect can be located just outside the pump spot, where the hydrodynamic nucleation of vortices,
vortex–antivortex pairs, arrays of vortices, and solitons can be observed when the fluid collides with the extended defect. Similarly, nucleation of vortices in the wake of the obstacle has been observed in pulsed experiments [8, 9].

In a conservative quantum liquid flowing past a small defect, the Landau criterion for superfluidity links the onset of dissipation at a critical fluid velocity with the shape of the fluid collective excitation spectrum [10]. In particular, for weakly interacting Bose gases, the dispersion of the low-energy excitation modes being linear implies that the critical velocity for superflow coincides with the speed of sound $c_s$. Clearly, this is strictly correct only for vanishingly small perturbations [11], while for a defect with finite size and strength, the critical velocity can be smaller than $c_s$ [12, 13].

However, even for perturbatively weak defects, in out-of-equilibrium systems, where the spectrum of excitations is complex, the validity of the Landau criterion has to be questioned [5, 14, 15]. In the particular case of coherently driven polaritons in the pump-only configuration, it has been predicted [2, 3], and later observed [4], that scattering is suppressed at either strong enough pump powers or small enough flow velocities. Yet, on closer scrutiny, it has been shown that, despite the apparent validity of the Landau criterion, the system always experiences a residual drag force even in the limit of asymptotically large densities [5] or small velocities. This result has been proven by numerically solving the Gross–Pitaevskii equation describing the resonantly driven polariton system in the presence of a non-perturbative extended defect. Here, the drag force exerted by the defect on the fluid has been shown to display a smooth crossover from the subsonic to the supersonic regime, similarly to what has been found in the case of non-resonantly pumped polaritons [15]. In this work, we find an even richer phenomenology for the dependence of the drag force on the fluid velocity and two different kinds of crossovers from the sub- to the supercritical regime. Further, we show that the origin of the residual drag force, which, in agreement with [5], lies in the polariton lifetime only, can be demonstrated even within a linear response approximation.

More specifically, in this work, we apply the linear response theory to analytically evaluate the drag force exerted by the coherently driven polariton fluid in the pump-only configuration on a point-like defect. To simplify the formalism, we restrict our analysis to the case of resonant pumping close to the bottom of the lower polariton dispersion, where the dispersion is quadratic. Here, the properties of the collective excitation spectrum have been shown to be uniquely determined by three parameters only [3]: the fluid velocity $v_p$, the interaction-renormalised pump detuning $\Delta_p$ and the polariton lifetime $\kappa$. In particular, the sign of the detuning $\Delta_p$ determines three qualitatively different types of spectra: linear for $\Delta_p = 0$, diffusive-like for $\Delta_p > 0$ and gapped for $\Delta_p < 0$.

For both cases of linear and diffusive spectra, we find a qualitatively similar behaviour of the drag force as a function of the fluid velocity $v_p$: in particular, the drag displays a crossover from a subsonic or superfluid regime—characterized by the absence of quasiparticle excitations—to a supersonic regime—where Cherenkov-like waves are generated by the defect and propagate into the fluid. The crossover becomes sharper for increasing polariton lifetimes $1/\kappa$ and displays the typical threshold behaviour for $\kappa \to 0$ with a critical velocity given by the speed of sound of the linear regime, $v^c = c_s$, exactly as for weakly interacting equilibrium superfluids (in the case of perturbatively weak defects). This behaviour is similar to the one predicted for polariton superfluids excited non-resonantly [15], where the spectrum in that case is diffusive-like.

However, for gapped spectra at $\Delta_p < 0$, we find that the critical velocity governing the drag crossover exceeds the speed of sound, $v^c > c_s$, and we determine an analytical expression of $v^c$ as a function of the detuning $\Delta_p$. Further, for $\kappa \to 0$, the drag has a threshold-like behaviour qualitatively different from the one of weakly interacting equilibrium superfluids, with the drag jumping discontinuously from zero to a finite value at $v_p = v^c$.

We evaluate the drag as a function of the polariton lifetime $\kappa$ and find for all three cases that, in the supercritical regime, $v_p > v^c$, the lifetime tends to suppress the propagation of the Cherenkov waves away from the defect and therefore to suppress the drag. Instead, well in the subcritical regime, $v_p \ll v^c$, we find that the residual drag goes linearly to zero with the polariton lifetime $\kappa$, in agreement to what was found in [5], by making use of a non-perturbative numerical analysis for a finite-size defect. Similar to [5], here, we do also find that the residual drag in the subcritical regime can be explained in terms of an asymmetric perturbation induced in the fluid by the defect in the direction of the fluid velocity.

This paper is structured as follows. In section 2 we briefly introduce the linear response approximation. We classify the three types of collective excitation spectra in the simplified case of excitation close to the bottom of the lower polariton dispersion in section 2.1. In section 3 we derive the drag force and characterize the crossover from the subsonic to the supersonic regime in the three cases of zero, positive and negative detuning. In this section, we also evaluate the drag as a function of the polariton lifetime, interpreting therefore the results of [5]. Brief conclusions are drawn in section 4.

2. Linear response

The description of cavity polaritons resonantly excited by an external laser is usually formulated in terms of a classical nonlinear Schrödinger equation (or Gross–Pitaevskii equation) [16] for the lower polariton (LP) field $\psi_{\text{LP}}(r, t)$ ($\hbar = 1$):

$$i \partial_t \psi_{\text{LP}} = [\omega_{\text{LP}}(r) - \omega_{\text{c}}^0] \psi_{\text{LP}} + g |\psi_{\text{LP}}|^2 \psi_{\text{LP}} + F(r, t).$$ (1)

The LP dispersion is expressed in terms of the photon $\omega_{\text{c}}^0(k) = \omega_{\text{c}}^0 + \frac{\hbar k^2}{2m_\text{c}}$ and exciton $\omega_{\text{X}}^0$ energies, the photon mass $m_\text{c}$ and the Rabi splitting $\Omega_\text{R}$ [1]:

$$\omega_{\text{LP}}(k) = \frac{1}{2} [\omega_{\text{c}}^0(k) + \omega_{\text{X}}^0] - \frac{1}{2} \sqrt{[\omega_{\text{c}}^0(k) - \omega_{\text{X}}^0]^2 + \Omega_\text{R}^2}. \quad (2)$$
Because polaritons continuously decay at a rate $\kappa$, the cavity is replenished by a continuous wave resonant pump $F(r,t)$ at a wavevector $k_p$ (we will later assume $k_p$ directed along the $x$ direction, $k_p = (k_p, 0)$) and frequency $\omega_p$:

$$\mathcal{F}(r,t) = f_0 e^{i(k_p \cdot r - \omega_p t)}. \quad (3)$$

Note that, as discussed in the appendix, equation (1) is a simplified description of the polariton system: This implies that the interaction nonlinearities are small enough not to mix the lower and upper polariton branches. Moreover, starting from a formulation in terms of coupled exciton and photon fields, the polariton lifetime would be momentum-dependent and, similarly, the polariton–polariton interaction strength $g$ is not contact-like as instead assumed in equation (1). However, as shown in the appendix, these simplifications do not affect our results qualitatively, rather, it allows us to write them in terms of simpler expressions. Further, we have checked that, whenever the system is excited near the bottom of the lower polariton dispersion, the results for the drag force reported in section 3 coincide with those obtained by using an exact photon–exciton coupled field description.

The potential $V(r)$ in equation (2) describes a defect, which can be either naturally present in the cavity mirror [4] or can be created by an additional laser [17]. Later on, we will assume the defect to be point-like $V(r) = g_V \delta(r)$ and weak, so that we can apply the linear response approximation [11]. In this treatment, we divide the response of the LP field in a mean-field component $\psi_0$ corresponding to the case when the perturbing potential is absent and a fluctuation part $\delta \psi (r, t)$ reflecting the linear response of the system to the perturbing potential:

$$\psi_{LP}(r, t) = e^{-i \omega_p t} [e^{i k_p \cdot r} \psi_0 + \delta \psi (r, t)]. \quad (4)$$

By substituting (4) into (1), we obtain a mean-field equation and by retaining only the linear terms in the fluctuation field and the defect potential, the following first-order equation in $\delta \psi (r, t)$:

$$i \hbar \delta \psi (r, t) \delta \psi (r, t) + V(r) \begin{pmatrix} \psi_0 e^{i k_p \cdot r} \\ - \psi_0^* e^{-i k_p \cdot r} \end{pmatrix}, \quad (5)$$

where the operator $\hat{L}$ is given by

$$\hat{L} = \begin{pmatrix} \omega_{LP}^0 (-i \nabla) - i \kappa & g \psi_0^2 e^{2ik_p \cdot r} \\ -g \psi_0^2 e^{-2ik_p \cdot r} & -\omega_{LP}^0 (-i \nabla) - i \kappa \end{pmatrix}. \quad (6)$$

with $\omega_{LP}^0 = \omega_{LP} - \omega_p + 2g^2 |\psi_0|^2$. We are not interested here in solving the complex cubic mean-field equation for $\psi_0$, as this has been already widely studied [1]. Rather, we want to study the response of the system to the presence of the defect and how different behaviours of the onset of dissipation can be described in terms of the different excitation spectra one can get for polaritons resonantly pumped close to the bottom of the LP dispersion.

2.1. Spectrum of collective excitations

The spectrum of the collective excitations can be obtained by diagonalizing the operator $\hat{L}$ in the momentum space representation:

$$\mathcal{L}_{k,k_p} = \begin{pmatrix} \omega_{LP}(\delta k + k_p) - i \kappa & g \psi_0^2 \\ -g \psi_0^2 & -\omega_{LP}^0(\delta k - k_p) - i \kappa \end{pmatrix}. \quad (7)$$

where $\delta k = k - k_p$. The description of the spectrum simplifies in the case when the pumping is close to the bottom of the LP dispersion, which can be approximated as parabolic:

$$\omega_{LP}(\delta k \pm k_p) \approx \omega_{LP}(0) + \frac{k_p^2}{2m} \pm \delta k \cdot v_p. \quad (8)$$

where $v_p = k_p/m$ is the fluid velocity and $m$ is the LP mass, $m = 2m_C [1 - (\omega_C^0 - \omega_X^0)/\sqrt{(\omega_C^0 - \omega_X^0)^2 + \Omega_p^2}]$. This simplification allows one to describe the complex spectrum in terms of three parameters only, namely the fluid velocity $v_p$, the interaction-renormalised pump detuning:

$$\Delta_p = \omega_p - \left[ \omega_{LP}(0) + \frac{k_p^2}{2m} + g^2 |\psi_0|^2 \right] \quad (9)$$

and the LP lifetime $\kappa$:

$$\omega_{LP}(k) = \delta k \cdot v_p - i \epsilon \pm \sqrt{\epsilon (5 \kappa)} \left[ \epsilon (\delta k) + 2g^2 |\psi_0|^2 \right]. \quad (10)$$

where $\epsilon(k) = \frac{k^2}{2m} - \Delta_p$. For energies that are measured in units of the mean-field energy blueshift $g |\psi_0|^2$ (we will use the notation $\Delta_p = \Delta_p/g |\psi_0|^2$ and $\kappa' = \kappa/g |\psi_0|^2$), then the fluid velocity $v_p$ is measured in units of the speed of sound $c_s = \sqrt{g |\psi_0|^2}/m$. In order to make a connection with the current experiments, note that, for blueshifts in the range $g |\psi_0|^2 \approx 0.1-1$ meV, typical values of the speed of sound $c_s$ are $0.8-2.7 \times 10^6$ m s$^{-1}$. Similarly, for common values of the LP mass, the range in momenta in figure 1 comes of the order of $\kappa_s \approx 0.2-0.8 \mu$m$^{-1}$. The spectrum (10) can be classified according to the sign of the interaction-renormalised pump detuning $\Delta_p$ [2, 3]—see figure 1. For $\Delta_p < 0$ (panels (a), (b)), the real part of the spectrum is gapped while the imaginary part is determined by the polariton lifetime $\kappa$ only. If one applies the Landau criterion making reference to the real part of the spectrum only, then one finds a critical velocity

$$\frac{v_c}{c_s} = \sqrt{1 + |\Delta_p|^2 + \sqrt{|\Delta_p|^4 + (2 + 2)|\Delta_p|^2}} > 1, \quad (11)$$

always larger than the speed of sound for $\Delta_p < 0$. If the fluid velocity is subcritical, $v_p < v_c$ (see the black) solid lines in figure 1(a)), then no quasiparticles can be excited and thus, for infinitely living polaritons $\kappa \to 0$, the fluid would experience no drag when scattering against the defect. For supercritical velocities instead, $v_p > v_c$ see the (red) dashed lines in figure 1(a), one expects dissipation in the form of radiation of Cherenkov-like waves from the defect into the fluid. In the supercritical regime, the set of wavevectors $k$ for which $\text{Re}[\omega^{-1}(k)] = 0$ form a closed curve in the $k$ space with no singularity of the derivative, i.e. in other words, the radiation can be emitted in all possible directions around the defect. This, as we will see in the next section, will imply
that the drag force for $\kappa \to 0$ goes abruptly, rather than continuously, from zero at $v_p < v^c$ to a finite value at $v_p = v^c$.

The spectrum gap closes to zero in the resonant situation at $\Delta_p = 0$, when the two branches $\omega^\pm(k)$ touch at $\delta k = 0$ (panels (c, d) of figure 1): here, the real part of the spectrum displays the standard linear dispersion at small wavevectors as for the weakly interacting bosonic gases, with the slope given by $c_s \pm v_p$. The imaginary part, as in the previous case, is constant and equal to $-\kappa$. It is clear therefore that in this case, when $\kappa \to 0$, one recovers the equilibrium results valid for weakly interacting gases [11, 18], where the critical velocity for superfluidity equals the speed of sound, $v^c = c_s$, and the drag displays a threshold-like behaviour. Here, in the supersonic regime $v_p > v^c$, the close curve $\text{Re}[\omega^+(k)] = 0$ has instead a singularity, resulting in the standard Mach cone of aperture $\theta$, $\sin \theta = c_s/v_p$, inside which radiation from the defect cannot be emitted [18].

Finally, for $\Delta_p > 0$, the real parts of the particle $\omega^+(k)$ and hole $\omega^-(k)$ branches of the spectrum touch together in either one ($\Delta_p \leq 2$, see panels (e, f)) or two ($\Delta_p > 2$, see panels (g, h)) separate regions in momentum space. In the same regions, the corresponding imaginary parts split instead.

With a somewhat abuse of language, we call these kinds of spectrum diffusive-like. We note that, clearly, these spectra have no correspondence in equilibrium systems, because a finite polaron lifetime $\kappa$ is needed in order for these modes to be stable, $\text{Im}[\omega^+(k)] < 0$. We also note that for these spectra, even if considering only the real part of the collective excitation spectrum, as soon as the fluid is in motion $v_p > 0$, dissipation in the form of waves is possible. However, we will see that similar to the case of polaritons non-resonantly pumped [15], when decreasing $\kappa$ (and accordingly $\Delta_p$ in order to have stable solutions), this situation connects continuously to the previous case, where a threshold-like behaviour with $v^c = c_s$ was found.

We will see in section 3 how these different spectra imply only two qualitatively different types of crossover of the drag force as a function of the fluid velocity, for either $\Delta_p < 0$ or $\Delta_p \geq 0$ pump detunings.

3. Drag force

The steady state response of the system to a static and weak defect can be evaluated starting from equation (5):

$$\begin{align*}
\left( \begin{array}{c}
\delta \psi_s(r) \\
\delta \psi^*_s(r)
\end{array} \right) &= \hat{L}^{-1} \left( V(r) e^{ik_p r} \psi_0 \right) - \left( V(r) e^{-ik_p r} \psi_0 \right).
\end{align*}$$

For a point-like defect, this can be written in momentum space as

$$\left( \begin{array}{c}
\delta \psi_s(k + k_p) \\
\delta \psi^*_s(k_0 - k)
\end{array} \right) = -g_V \psi_0 \left( \varepsilon(k) - k \cdot \psi_0 + ik \right) \delta(k - k_p),$$

while the other component $\delta \psi^*_s(k_0 - k)$ can be obtained by complex conjugation and by substituting $k \to -k$. The drag force exerted by the defect on the fluid is given by [11]

$$\mathbf{F} = -\int d^3r \left| \psi_L \mathbf{v}(r) \right|^2 \nabla \mathbf{v}(r),$$

and, in the steady state linear response regime, we obtain

$$\mathbf{F} = g_V \int \frac{dk}{(2\pi)^2} ik \left[ \psi_0^* \delta \psi_s(k + k_p) + \psi_0 \delta \psi^*_s(k_0 - k) \right]$$

$$= 2g_V^2 |\psi_0|^2 \int \frac{dk}{(2\pi)^2} \frac{ik \varepsilon(k)}{\omega^+(k) \omega^+(k) - \varepsilon(k)}.$$  (13)

The drag is clearly oriented along the fluid velocity $v_p$, i.e. $\mathbf{F} = F \mathbf{v}_p$. If $\kappa \to 0$, then the integral in equation (13) is finite only if poles exist when $\text{Re}[\omega^+(k)] = 0$, i.e. when quasiparticles can be excited, in agreement with the Landau criterion. For finite polaron lifetimes, however, it is clear that the integral will always be different from zero for $v_p > 0$. We now analyse the behaviour of the drag force as a function of the fluid velocity for the three ($\Delta_p = 0$, $\Delta_p > 0$ and $\Delta_p < 0$) different spectra illustrated in section 2.1.

For the linear spectrum, at $\Delta_p = 0$, in the equilibrium limit, $\kappa \to 0$, we recover for the drag the known result of weakly interacting Bose gases in two dimensions [11]:

$$\frac{F}{(mc_s)^3 \xi_0^2 / g} = \frac{(v_p/c_s)^2 - 1}{v_p/c_s} \Theta(v_p - c_s),$$

with a threshold-like behaviour at a critical fluid velocity equal to the speed of sound $c_s$. This limiting result is plotted as a bold grey line in panels (b, c) of figure 2. For $\Delta_p = 0$ and finite lifetimes $\kappa$, we find a smooth crossover from the subsonic to the supersonic regime, with the drag being closer to the equilibrium threshold behaviour for decreasing $\kappa$ (see figure 2(b)). A finite lifetime tends to increase the value of the drag in the subsonic region $v_p \ll v^c$, giving place

![Figure 1. Collective excitation spectra for the subsonic (thick solid) line at $v_p = 0.2c_s$, with $c_s = \sqrt{|\varepsilon|} / |\varphi_0|^2 / m$ and supersonic (dashed red) line at $v_p = 1.9c_s$ regimes and for an interaction-renormalised pump detuning $\Delta_p = -0.3g|\varphi_0|^2$ ((a), (b)), $\Delta_p = 0$ ((c), (d)), $\Delta_p = 0.3g|\varphi_0|^2$ ((e), (f)) and $\Delta_p = 2.3g|\varphi_0|^2$ ((g), (h)). Real parts of the spectra are plotted in the left panels and the corresponding imaginary parts in the right panels for $\kappa = 1.1g|\varphi_0|^2$. Note that in our description the spectrum imaginary parts do not depend on the fluid velocity $v_p$.](image-url)
to a residual drag force, similar to what was found in the numerical simulations of [5]. Instead, in the supersonic region \( v_p \gg v^* \), the finite lifetime tends to decrease the value of the drag. In the case of diffusive-like spectra at \( \Delta p > 0 \) the situation is qualitatively very similar to the resonant case (see figure 2(c)), with the difference that now, in order to have stable solutions, we can decrease the value of the lifetime only by also decreasing accordingly the value of the pump detuning \( \Delta p \). The crossover for both \( \Delta p = 0 \) and \( \Delta p > 0 \) is also qualitatively very similar to the case of non-resonantly pumped polaritons [15], where the spectrum of excitation is diffusive-like in that case.

In the case of gapped spectra, the situation is, however, qualitatively different (see figure 2(a)). For infinitely living polaritons, \( \kappa \to 0 \), the drag force can also be evaluated analytically and its expression is similar to equation (14), but with a critical velocity larger than the speed of sound, which expression is given in equation (11):

\[
\frac{F}{(mc^2)^3 g_1^2 / g} = \left( \frac{v_p / c_s}{v^* / c_s} \right)^2 - 1 \Theta(v_p - v^*). 
\]

Therefore now the drag experiences a jump for \( v_p = v^* \), rather than a continuous threshold as for the resonant case \( \Delta p = 0 \). As already mentioned in the previous section, this discontinuous behaviour of the drag for the gapped spectra is connected to the fact that, as soon as quasi-particles can be excited by the defect at \( v_p \geq v^* \), Cherenkov-like waves can be immediately emitted in all directions, rather than being restricted in a region outside the Mach cone as before. For \( \Delta p = 0 \), the cone was gradually closing with increasing the fluid velocity.

Both the increase of the value of the drag in the subcritical region as a function of the polariton lifetime and the decrease in the supercritical region, are behaviours common to all the types of spectra. We plot the drag force as a function of \( \kappa \) in figure 3 for two values of the fluid velocity \( v_p \) and a specific value of the pump detuning \( \Delta p \), though we have checked that the following results are generic. For \( v_p < v^* \), we find that the residual drag is a finite lifetime effect only and, in agreement with the results of [5], we find that, well below the critical velocity, the drag force goes linearly to zero for \( \kappa \to 0 \). In the resonant case \( \Delta p = 0 \), the slope of the drag for \( v_p < c_s \) can be evaluated analytically starting from the expression (13)

\[
\frac{F}{(mc^2)^3 g_1^2 / g} \sim \frac{2c_s}{{\sqrt{1 - (v_p / c_s)^2}}} \frac{1}{\kappa g|\psi_0|^2}.
\]

The residual drag in the subsonic regime is an effect of the broadening of the quasiparticle energies: Even when the spectrum real part does not allow any scattering against the defect (e.g. for \( \Delta p \leq 0 \), the broadening produces some scattering close to the defect. This results in a perturbation of the fluid around the defect, asymmetric in the direction of the fluid velocity (see panel (a) of figure 3), similar to what was obtained in [5]. Instead, in the supersonic regime, the drag force is weaker in the non-equilibrium case with respect to the equilibrium one. This is caused by the finite lifetime tending to suppress the propagation of the Cherenkov waves away from the defect, as shown in panel (b) of figure 3.

4. Conclusions and discussion

To conclude, we have analysed the linear response to a weak defect of resonantly pumped polaritons in the pump-only
state and we have been able to determine two different kinds of threshold-like behaviours for the drag force as a function of the fluid velocity. In the case of either zero or positive pump detuning, one can continuously connect to the case of equilibrium weakly interacting gases, where the drag displays a continuous threshold with a critical velocity equal to the speed of sound. However, for negative pump detuning, where the spectrum of excitations is gapped, the drag shows a discontinuity with a critical velocity larger than the speed of sound. In this sense, the case of coherently driven microcavity polaritons in the pump-only configuration displays a richer phenomenology than the case of polariton superfluids non-resonantly pumped. It would be interesting to perform a similar analysis in the case of polaritons in superfluids non-resonantly pumped. It would be interesting to perform a similar analysis in the case of polaritons in superfluids non-resonantly pumped. It would be interesting to perform a similar analysis in the case of polaritons in superfluids non-resonantly pumped. It would be interesting to perform a similar analysis in the case of polaritons in superfluids non-resonantly pumped. It would be interesting to perform a similar analysis in the case of polaritons in superfluids non-resonantly pumped.

Acknowledgments

We are grateful to C Tejedor and M Szymanska for useful discussions. The authors acknowledge the financial support from the Spanish MINECO (MAT2011-22997), CAM (S-2009/ESP-1503) and FP7 ITN ‘Clermont4’ (AB), and from the program Ramón y Cajal (FMM).

Appendix. Gross–Pitaevskii equation for the lower polariton field

If one starts from a description of polaritons in terms of separate exciton and cavity photon fields, a rotation into the lower and upper polariton basis, followed by neglecting the occupancy of the upper polariton branch, results in the following Gross–Pitaevskii equation for the lower polariton (LP) field in momentum space $\psi_{\text{LP}}(\mathbf{r}, t) = \sum_k e^{i\mathbf{k}\cdot\mathbf{r}} \psi_{\text{LP}, k}(t)$ [16]:

$$i\hbar \frac{\partial \psi_{\text{LP}, k}}{\partial t} = \hbar\omega_{\text{LP}}(k) \psi_{\text{LP}, k} + \sum_{k_1, k_2} g_{k, k_1, k_2} \psi_{\text{LP}, k_1} \psi_{\text{LP}, k_2} \psi_{\text{LP}, k} + s_\gamma \sum_{k_1} V_{k_1} \psi_{\text{LP}, k_1} \psi_{\text{LP}, k},$$

(A.1)

where $\kappa(k) = \kappa_c \xi^2 + \kappa_s \xi^2$ is the effective LP decay rate, $g_{k, k_1, k_2} = g^2 \xi \xi \xi^{k_1 + k_2 - k} \xi \xi$ is the interaction strength and where $V(\mathbf{r}) = \sum_k e^{i\mathbf{k}\cdot\mathbf{r}} V_k$. In these expressions, the coefficients:

$$c^2_k, s^2_\gamma = \frac{1}{2} \left( 1 \pm \frac{\omega_c(k) - \omega_k^0}{\sqrt{(\omega_c(k) - \omega_k^0)^2 + \Omega_R^2}} \right)$$

(A.2)

are the Hopfield coefficients used to diagonalize the free polariton Hamiltonian. We want here to justify the simplified description done in equation (1). If we follow the linear response expansion as in (4), the operator $\hat{L}$ in momentum space analogous to (7) is

$$\hat{L}_{k, k'} = \left( \omega_{\text{LP}}(\mathbf{k} + \mathbf{k'}) - i \kappa(k + k') \right) \frac{g^2 \xi \xi \xi \xi \xi \xi}{\sqrt{2\xi \xi \xi \xi \xi \xi}} \left( \phi_{\text{LP}}(\mathbf{k} + \mathbf{k'}) - i \kappa(k + k') \phi_{\text{LP}}(\mathbf{k} - \mathbf{k'}) \right),$$

(A.3)

where now $\omega_{\text{LP}}(\mathbf{k} \pm \mathbf{k'}) = \omega_{\text{LP}}(\mathbf{k} \pm \mathbf{k'}) - \omega_0 + 2g^2 \xi \xi \xi \xi \xi \xi |\phi_{\text{LP}}(\mathbf{k} \pm \mathbf{k'})|^2$. It is easy to show that the eigenvalues of this operator coincide with our approximated expressions (10) in the limit of $\delta k \ll k_p$, when $c^2_{k, k} \approx c^2_{k_p}, s^2_{\gamma \pm k_p} \approx s^2_{\gamma_p}$ and when we can simply rename $g = g^2 \xi \xi \xi \xi \xi \xi$ and $\kappa = \kappa(k_p)$. It is interesting to note that, even if we would retain the linear terms in $k_p \cdot \delta k$ in the expansion of $c^2_{k, k}$, this would result in a renormalization of the fluid velocity $v_p$ in the expression (10) which takes into account the blueshift of the lower polariton dispersion due to the interaction.

References

[1] Kavokin A V, Baumberg J J, Malpuech G and Laussy F P 2007 Microcavities (Oxford: Oxford University Press).
[2] Carusotto I and Ciuti C 2004 Phys. Rev. Lett. 93 166401
[3] Ciuti C and Carusotto I 2005 Phys. Status Solidi b 224 2224
[4] Amo A, Lefrêre J, Pigeon S, Adriados C, Ciuti C, Carusotto I, Houédré R, Giacobino E and Bramati A 2009 Nature Phys. 5 805
[5] Cancellieri E, Marchetti F M, Szymańska M H and Tejedor C 2010 Phys. Rev. B 82 224512
[6] Pigeon S, Carusotto I and Ciuti C 2011 Phys. Rev. B 83 144513
[7] Amo A et al 2011 Science 332 1167
[8] Nardin G, Grosso G, Leger Y, Pietka B, Morier-Genoud F and Deveaud-Plédran B 2011 Nature Phys. 7 635
[9] Sanvitto D et al 2011 Nature Phys. 6 527
[10] Pitaevskii L P and Stringari S 2003 Bose–Einstein Condensation (Oxford: Clarendon)
[11] Astrakharchik G E and Pitaevskii L P 2004 Phys. Rev. A 70 013608
[12] Onofrio R, Raman C, Vogels J M, Abo-Shaeer J R, Chikkatur A P and Ketterle W 2000 Phys. Rev. Lett. 85 22228
[13] Iasenelli S, Menotti C and Smerzi A 2006 J. Phys.: Condens. Matter 18 S279
[14] Szymańska M H, Keeling I and Littlewood P 2006 J. Phys. B: At. Mol. Opt. Phys. 39 S135
[15] Ciuti C, Schwindtman P and Quattropani A 2003 Semicond. Sci. Technol. 18 S279
[16] Amo A, Pigeon S, Adriados C, Houédré R, Giacobino E, Ciuti C and Bramati A 2010 Phys. Rev. B 82 081301
[17] Carusotto I, Hu S X, Collins L A and Smerzi A 2006 Phys. Rev. Lett. 97 260403
[18] Wouters M and Carusotto I 2007 Phys. Rev. A 76 043807