CONFORMAL BLOCKS OF COSET CONSTRUCTION:
ZERO GHOST NUMBER

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Abstract

It is shown that zero ghost conformal blocks of coset theory G/H are determined uniquely by those of G and H theories. G/G theories are considered as an example, their structure constants and correlation functions on sphere are calculated.

1. Introduction

Conformal blocks of 2D conformal quantum field theory are chiral (holomorphic) N-point functions out of which the monodromy invariant N-point correlation function is built. Conformal blocks are usually solutions of a differential equation. For minimal models this equation is produced by null vectors of Verma module of Virasoro algebra or W algebra. Wess-Zumino-Witten (WZW) theories possess Knizhnik-Zamolodchikov equation produced by the relation between energy-momentum tensor and Kac-Moody currents. Conformal blocks are solutions of this equation with constraints imposed by null vectors of Kac-Moody algebra.

Coset models obey some special features. Goddard, Kent and Olive (GKO) proposed them as ‘quotients’ G/H of two theories in sense of factorization of state spaces. Gawedzki and Kupiainen proposed Lagrangian theory of coset construction, based on gauged WZW theories. BRST approach was used to investigate states and conformal blocks. The difficulty of coset models is that they often have not their own differential equations. For example, the simplest coset model SU(2)N/U(1) (ZN parafermions) adopts in fact the differential equation from SU(2)N theory, although it has currents. For more complex coset theories the currents turn out essentially non-local.

The other approach to coset construction consists in finding relation between coset conformal blocks and those of G and H theories. These relations can be obtained in Lagrangian approach or from duality in general consideration. Here this general way independent from Lagrangian and any other details of G and H theories is analyzed. The existence and uniqueness of coset is proved (Sec. 2). The connection with BRST approach on zero-ghost-number level is shown (Sec. 3). The results are applied to G/G coset models (Sec. 4). Another application to bosonic representation of coset theories can be found in Ref. 21.

† Supported by Landau Institute Foundation.
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Note added in February 1993. Recently I have learnt that Eq. (2.3) was first supposed by M. R. Douglas and proved and investigated by M. B. Halpern and N. Obers. I am very grateful to M. B. Halpern and N. Obers for calling my attention to this.

2. The Main Equation

Conformal block $\Phi_{\lambda_1,\cdots,\lambda_n}^{\alpha_1,\cdots,\alpha_n}(\beta_1,\cdots,\beta_n;z)$ of $H$ theory is a holomorphic ‘correlation function’ of four chiral fields $\phi_{\lambda_1}(0), \phi_{\lambda_2}(z), \phi_{\lambda_3}(1), \phi_{\lambda_4}(\infty)$ with intermediate state from module $\lambda_5$ (Fig. 1). Here $\lambda_i$ labels an irreducible representation $H^H_{\lambda_i}$ of $\hat{h}$ chiral algebra (Virasoro, Kac-Moody, $W$ etc.); $\mu_i$ labels a state in the irreducible module $\lambda_i$; $\alpha_1 = 1,\cdots, N_{\lambda_1,\alpha_2}; \alpha_2 = 1,\cdots, N_{\lambda_5,\lambda_3,\lambda_4}$, where $\lambda^+$ is conjugate representation to $\lambda$, and $N_{\lambda,\lambda',\lambda''}^H$ are multiplicities of fusion rules.

$$H^H_{\lambda} \otimes H^H_{\lambda'} = \bigoplus_{\lambda''}^{N_{\lambda,\lambda',\lambda''}^H} \bigoplus_{\alpha=1}^{N_{\lambda,\lambda',\lambda''}^H} H^H_{\lambda''}. \quad (2.1)$$

For $SU(2)_k$ model and conformal models with central charge of Virasoro algebra $c < 1$ multiplicities are 0 (fusion is forbidden) or 1 (fusion is permitted).

For $G$ theory any representation $H^G_l$ of $\hat{g}$ chiral algebra can be decomposed into the direct sum of $H$ modules

$$H^G_l = \bigoplus_{\lambda}^{N_{\lambda}} \bigoplus_{n=1}^{N_{\lambda}} H^H_{\lambda_n}. \quad (2.2)$$

where $N_{\lambda}$ is the multiplicity (possibly infinite) of module $H^H_{\lambda}$ in $H^G_l$. Thus, one can designate conformal block of $G$ model as $F_{l_1,\cdots,l_n}^{l_3,\cdots,l_4}(l_5|_{a_1}; z)$, where $a_1 = 1,\cdots, N_{l_1,\cdots,l_n}^G; a_2 = 1,\cdots, N_{l_2,\cdots,l_n}^G; n_1 = 1,\cdots, N_{\lambda}; \mu_i$ is again a label of a state from the $H$ module $H^H_{\lambda_i}$.

A conformal block of coset model will be written as $\Psi_{l_1,\cdots,l_n}^{l_3,\cdots,l_4}(l_5|_{a_1}; z)$.

Consider the direct product of space of conformal blocks of $H$ theory and of that space of the coset theory. We will look for conformal blocks of $G$ model in this product. In other words we will search them in form

$$F_{l_1,\cdots,l_n}^{l_3,\cdots,l_4}(l_5|_{a_1}; z)$$

$$\quad = \sum_{\lambda_5,\lambda} X_{\lambda_1,\lambda_2}^{\lambda_5,\lambda}(\lambda_5|_{a_2,\cdots,\alpha_1}) \Phi_{\lambda_1,\lambda_2,\lambda_3,\alpha_1}^{\lambda_5,\lambda_4,\lambda_4}(\lambda_5|_{a_2,\cdots,\alpha_1}; z) \Psi_{l_1,\cdots,l_n}^{l_3,\cdots,l_4}(l_5|_{a_2,\cdots,\alpha_1}; z). \quad (2.3)$$

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*Four-point conformal blocks are considered for simplicity - the designations are overloaded as they are. In a sense that is sufficient.*
Here $X_{\lambda_1 \lambda_2}^{\lambda_3 \lambda_4} \left( \lambda_5 |_{\alpha_1 \alpha_2}^{\alpha_2 \alpha_2'} \right)$ are coefficients to be found. These coefficients must ensure duality of the r.h.s. or, equivalently, its right monodromy properties. Monodromy matrices are known only to depend on fractional parts of conformal dimensions and fusion rules.\textsuperscript{2,3,21} Thus, monodromy matrix of $\Psi_{l_1 \lambda_1 n_1, l_2 \lambda_2 n_2}^{l_3 \lambda_3 n_3, l_4 \lambda_4 n_4} (l_5 \lambda_5 |_{\alpha_1 \alpha_2}^{\alpha_2 \alpha_2}, z)$ is the same as that of the product

$$\Phi_{\lambda_1 \mu_1, \lambda_2 \mu_2}^{\lambda_3 \mu_3, \lambda_4 \mu_4} (\lambda_5 |_{\alpha_1 \alpha_1}^{\alpha_2 \alpha_2}) F_{l_1 \lambda_1 n_1, l_2 \lambda_2 n_2}^{l_3 \lambda_3 n_3, l_4 \lambda_4 n_4} (l_5 |_{\alpha_2}^{\alpha_2}, z), \quad (2.4)$$

where $z^*$ is the complex conjugate to $z$. Substituting it in Eq.(2.3) (understood now only as an equality between monodromy matrices rather than between conformal blocks themselves) and cancelling $F$-blocks, we find that

$$\sum_{\lambda_5 \alpha} X_{\lambda_1 \lambda_2}^{\lambda_3 \lambda_4} (\lambda_5 |_{\alpha_1 \alpha_2}^{\alpha_2 \alpha_2}) \Phi_{\lambda_1 \mu_1, \lambda_2 \mu_2}^{\lambda_3 \mu_3, \lambda_4 \mu_4} (\lambda_5 |_{\alpha_1 \alpha_1}^{\alpha_2 \alpha_2}, z) \Phi_{\lambda_1 \mu_1, \lambda_2 \mu_2}^{\lambda_3 \mu_3, \lambda_4 \mu_4} (\lambda_5 |_{\alpha_2}^{\alpha_2}, z^*)$$

is monodromy invariant. But it means that the expression (2.5) can be considered as a correlation function, and coefficients $X_{\lambda_1 \lambda_2}^{\lambda_3 \lambda_4} (\lambda_5 |_{\alpha_1 \alpha_2}^{\alpha_2 \alpha_2})$ are found by solving Dotsenko-Fateev equation.\textsuperscript{3,21}

Now we will show that Eq. (2.3) with given $X$-coefficients permits to find conformal blocks uniquely. We will use the following matrix designations. $F$ will be considered as a matrix $(F_{l_1 \lambda_1 n_1, l_2 \lambda_2 n_2}^{l_3 \lambda_3 n_3, l_4 \lambda_4 n_4} (z))_{i,j}$ with $i = (\mu_1 \mu_2 \mu_3 \mu_4)$, $j = (l_5 a_1 a_2)$; $X$ as a matrix $(X_{\lambda_1 \lambda_2}^{\lambda_3 \lambda_4} (z))_{i,j}$, $i = (\lambda_5 \alpha_1 \alpha_2)$, $j = (\lambda_5 \alpha_1' \alpha_2')$, diagonal over $\lambda_5 \lambda_5'$; $\Phi$ as $(\Phi_{\lambda_1 \lambda_2}^{\lambda_3 \lambda_4} (z))_{i,j}$, $i = (\mu_1 \mu_2 \mu_3 \mu_4)$, $j = (\lambda_5 \alpha_1 \alpha_2)$; $\Psi$ as $(\Psi_{l_1 \lambda_1 n_1, l_2 \lambda_2 n_2}^{l_3 \lambda_3 n_3, l_4 \lambda_4 n_4} (z))_{i,j}$, $i = (\lambda_5 \alpha_1 \alpha_2)$, $j = (l_5 a_1 a_2)$. Eq. (2.3) can be represented in matrix form as follows

$$F_{l_1 \lambda_1 n_1, l_2 \lambda_2 n_2}^{l_3 \lambda_3 n_3, l_4 \lambda_4 n_4} (z) = \Phi_{\lambda_1 \lambda_2}^{\lambda_3 \lambda_4} (z) X_{\lambda_1 \lambda_2}^{\lambda_3 \lambda_4} (z) \Psi_{l_1 \lambda_1 n_1, l_2 \lambda_2 n_2}^{l_3 \lambda_3 n_3, l_4 \lambda_4 n_4} (z). \quad (2.3a)$$

If it will not lead to confusion we will omit non-matrix indices too:

$$F(z) = \Phi(z) X(z). \quad (2.3b)$$

Recall that conformal blocks $\Phi(z)$ are solutions of linear differential equation with some linear constraints. Index $j = (\lambda_5 \alpha_1 \alpha_2)$ numerates all linearly independent solutions. Consider minimal set of rows of $\Phi$-matrix forming some matrix $\Phi(z)$, such that $\Phi(z)$ is expressed algebraically through this minimal set:

$$\Phi_{\lambda_1 \lambda_2}^{\lambda_3 \lambda_4} (z) = A_{\lambda_1 \lambda_2}^{\lambda_3 \lambda_4} (z) \hat{\Phi}_{\lambda_1 \lambda_2}^{\lambda_3 \lambda_4} (z), \quad (2.6)$$

where $A_{\lambda_1 \lambda_2}^{\lambda_3 \lambda_4} (z)$ is some matrix function. The matrix $\hat{\Phi}_{\lambda_1 \lambda_2}^{\lambda_3 \lambda_4} (z)$ is evidently square and $A_{\lambda_1 \lambda_2}^{\lambda_3 \lambda_4} (z)$ is of maximal rank.

Notice that

$$F_{l_1 \lambda_1 n_1, l_2 \lambda_2 n_2}^{l_3 \lambda_3 n_3, l_4 \lambda_4 n_4} (z) = A_{\lambda_1 \lambda_2}^{\lambda_3 \lambda_4} (z) \hat{F}_{l_1 \lambda_1 n_1, l_2 \lambda_2 n_2}^{l_3 \lambda_3 n_3, l_4 \lambda_4 n_4} (z), \quad (2.7)$$
where \( \hat{F}(z) \) is the matrix consisting of rows with the same indices \( i = (\mu_1 \mu_2 \mu_3 \mu_4) \) as in \( \hat{\Phi}(z) \). Indeed, all currents act on \( \Phi(z) \) as algebraical or differential operators. Taking into account the differential equation we can present them as purely algebraical operators forming \( A(z) \) in Eq. (2.6). These very operators are used to decompose \( H^G_i \) in \( H^H \) in Eq. (2.2) and act on \( F(z) \) (Eq. (2.7)). This claim is induced in the accurate definition of a coset model. Particularly, if \( \hat{g} \) and \( \hat{h} \) are Kac-Moody algebras, induced by WZW currents, this claim is satisfied automatically. Indeed, any current, \( J(z) \), acts on a correlation function of primary fields as

\[
\langle J(z)\phi_1(z_1)\ldots\phi_N(z_N) \rangle = \sum_{i=1}^{N} \frac{t_i}{z-z_i} \langle \phi_1(z_1)\ldots\phi_N(z_N) \rangle,
\]

where \( t_i \) is the representation of primary field \( \phi_i(z_i) \). If there are more than one currents, the formula is a bit more complex because of central terms, but in any case these Ward equations have just the same form both in H subalgebra of G WZW theory and in H WZW theory.

From Eqs. (2.3), (2.6) and (2.7) we find

\[
A(z)\hat{F}(z) = A(z)\hat{\Phi}(z)X\Psi(z),
\]

or, taking into account maximal rank of \( A(z) \) in generic point,

\[
\hat{F}(z) = \hat{\Phi}(z)X\Psi(z). \tag{2.8}
\]

We admit that \( X \) is indegenerate, and it remains to prove the same about \( \Phi(z) \). But \( \det \hat{\Phi}(z) \) is simply Wronskian of linearly independent solutions. Therefore

\[
\det \hat{\Phi}(z) \neq 0 \tag{2.9}
\]

for all \( z \neq 0, 1, \infty \), and solution of Eq. (2.8) is unique:

\[
\Psi(z) = X^{-1}\hat{\Phi}^{-1}(z)\hat{F}(z). \tag{2.10}
\]

This function has not any additional singular points except 0, 1 and \( \infty \). Moreover, the differential equation with constraints can be reduced to form

\[
\partial^n y(z) - \left(\frac{\alpha}{z} + \frac{\beta}{1-z} \right)\partial^{n-1} y(z) + \cdots = 0,
\]

where \( y(z) \) is one of the components of \( \Phi(z) \) and dots mean lower derivatives. From elementary theory of differential equations we find

\[
\det \hat{\Phi}(z) = Cz^\alpha(1-z)^\beta, \tag{2.11}
\]

where \( C \) is a constant. We see that if \( F(z) \) and \( \Phi(z) \) are functions of hypergeometric type,\( ^{2,3} \) \( \Psi(z) \) is also of hypergeometric type.
3. Conjugate Theory

Consider the matrix
\[
\hat{M}_{\lambda_1\lambda_2}(z) = (X_{\lambda_1\lambda_2}^{\lambda_3\lambda_4})^{-1} (\hat{\Phi}_{\lambda_1\lambda_2}(z))^{-1}.
\] (3.1)

Then
\[
\Psi(z) = \hat{M}(z) \hat{F}(z).
\] (3.2)

The matrix \(\hat{M}(z)\) can be supplemented by zero rows to a matrix \(M(z)\) so as
\[
\Psi(z) = M(z) F(z)
\] (3.3)

(without hats). The matrix \(M\) depends on choice of the minimal set of rows of \(\Phi(z)\), but \(\Psi(z)\) does not. The choice of minimal set is a kind of gauge fixing.

Consider the case of Kac-Moody algebra \(\hat{h}\). Knizhnik-Zamolodchikov equation (with constraints) has the form
\[
(\partial - \frac{1}{k+c_V} \theta(z)) \hat{\Phi}(z) = 0,
\] (3.4)

where \(k\) is the central charge\(^b\) of algebra \(\hat{h}\), \(c_V\) is defined by structure constants \(f^\alpha_\gamma\) and basic inner metric \(g^{\alpha\beta}\) as
\[
f^\alpha_\gamma f^\beta_\delta = -c_V g^{\alpha\beta};
\]
\(\theta(z)\) is a matrix which only depends on representations \(\lambda_i\) and minimal set. Then
\[
0 = \partial(\hat{\Phi} X \hat{M}) = \partial \hat{\Phi} X \hat{M} + \hat{\Phi} X \partial \hat{M} = \frac{1}{k+c_V} \theta + \hat{\Phi} X \partial \hat{M} = \hat{\Phi} X (\partial \hat{M} + \frac{1}{k+c_V} \hat{M} \theta),
\]
and
\[
\partial \hat{M} + \frac{1}{k+c_V} \hat{M} \theta(z) = 0,
\]
or
\[
(\partial + \frac{1}{k+c_V} \theta(z)) \hat{M}^+(z) = 0.\] (3.5)

The sign \(+\) designates here conjugation of all representations (see above) with transposition of matrix. Eq. (3.5) differs from Eq. (3.4) by the sign before \(1/(k+c_V)\). Therefore, Eq. (3.5) is the Knizhnik-Zamolodchikov equation for Kac-Moody algebra \(\hat{h}\) with central charge \(\tilde{k}\) such that
\[
\frac{1}{k+c_V} = -\frac{1}{k+c_V},
\]

\(^b\) For simplicity we consider a simple Kac-Moody algebra. Generalization to semisimple algebras is evident.
or

\[
\tilde{k} = -2c_V - k. \tag{3.6}
\]

On the other hand, one can consider an arbitrary solution \( \tilde{\Phi}^{\lambda_5\mu_3,\lambda_4\mu_4}(\lambda_5|\alpha_2; z) \) of Knizhnik-Zamolodchikov equation for Kac-Moody algebra \( \tilde{h} \) with central charge \( \tilde{k} \) (conjugate theory\(^{12,14-16} \)). If \( k > 0 \), then \( \tilde{k} < 0 \) and the conjugate theory is not rational. Thus, there is no constraints produced by null vectors in the conjugate theory. Diagonalizing \( \tilde{\Phi} \) matrix on primary fields, it is easy to prove that the solutions, which are not linear combinations of columns of \( M^+ \), do not contribute in expression \( \tilde{\Phi}^+ F \) (matrices are restricted on primary fields), and

\[ MF = \tilde{\Phi}^+ F. \]

Finally,

\[
\Psi_{l_1\lambda_1n_1,l_2\lambda_2n_2}(l_5|\alpha_2; z) = \sum_{\mu_1\ldots\mu_4} F_{l_1\lambda_1n_1\mu_1,l_2\lambda_2n_2\mu_2}(l_5|\alpha_2; z) \tilde{\Phi}_{\lambda_5\mu_3,\lambda_4\mu_4}^{\mu_1^+,\mu_2^+}(\lambda_5|\alpha_2; z), \tag{3.7}
\]

where the summation is done over components of primary with respect to \( H \) fields. Eq. (3.7) holds evidently in general case if conjugate theory is irrational, and not just in the case of Kac-Moody algebra \( \tilde{h} \).

It should be noted that \( \lambda_5 \) in Eq. (3.5) must only sweep over modules that are present in fusion rules of \( \tilde{h} \) theory. For other \( \lambda_5 \) which are present in fusion rules of \( \tilde{h} \) model, the r.h.s. of Eq. (3.7) vanishes.

The r.h.s. is annihilated by any current \( J_n^\alpha + \tilde{J}_n^\alpha, n \geq 0 \) \( (J_n^\alpha \) means a current of \( H \) theory and \( J_n^\alpha \) of the conjugate theory \( \tilde{H} \), acting at any point \( 0, z, z_1 \), and \( \infty \). This can be expressed in BRST form\(^{14-17} \) by using ghosts \( b_\alpha(z) = \sum_n b_\alpha^nz^{-n-\Delta(b_\alpha)} \) and \( c_\alpha(z) = \sum_n c_\alpha n z^{-n-\Delta(c_\alpha)} \) with conformal dimensions related with those of currents, \( \Delta(J^\alpha) \), as \( \Delta(b_\alpha) = \Delta(J^\alpha) \) and \( \Delta(c_\alpha) = 1 - \Delta(J^\alpha) \). Their non-zero anticommutation relations are given by

\[
b_\alpha(z')c_\beta(z) = \frac{\delta_\beta^\alpha}{z' - z} + O(1), \tag{3.8a}
\]

\[
\{b_\alpha^m, c_\beta^n\} = \delta_\beta^\alpha \delta_{m+n,0}. \tag{3.8b}
\]

Vacuum, \( ^c \langle \rangle^\text{gh} \), satisfies the equations

\[ b_n^\alpha \langle \rangle^\text{gh} = 0, \ n \geq 0; \]

\(^c \text{In Ref. 17 another ghost vacuum (type II according to Hu-Yu classification}^{16} \).
\[ c_{\alpha n} | \phi^{\text{gh}} \rangle = 0, \ n > 0. \] (3.9)

Ghost energy-momentum tensor is given by
\[ T_{\text{gh}}(z) = \sum_\alpha \left( (\Delta(J^\alpha) - 1) : b^\alpha \partial c_\alpha : - \Delta(J^\alpha) : c_\alpha \partial b^\alpha : \right). \] (3.10)

The normal ordering is consisted with vacuum (3.9).

Current algebra \( \hat{h} \) has the form
\[ J^\alpha(z') J^\beta(z) = f_{\gamma}^{\alpha \beta}(z', z) J^\gamma(z) \]
\[ + \{\text{central term} \} + \{\text{regular terms}\}, \] (3.11a)

or
\[ [J^\alpha_m, J^\beta_n] = f_{mn, \gamma}^{\alpha \beta, m+n} J^\gamma_{m+n} + \{\text{central term}\}. \] (3.11b)

Particularly, in the case of Kac-Moody algebra
\[ f_{\gamma}^{\alpha \beta}(z', z) = f_{\gamma}^{\alpha \beta}, \ f_{mn, \gamma}^{\alpha \beta, m+n} = f_{\gamma}^{\alpha \beta}. \] (3.12)

The BRST charge is given by
\[ Q = \oint \frac{dz}{2\pi i} \left( c_\alpha(z) (J^\alpha(z) + \bar{J}^\alpha(z)) - \frac{1}{2} \oint \frac{dz'}{2\pi i} f_{\gamma}^{\alpha \beta}(z', z) : c_\alpha(z') c_\beta(z) b^\gamma(z) : \right) \]
\[ = \sum_m c_{\alpha, -m} (J^\alpha_m + \bar{J}^\alpha_m) - \frac{1}{2} \sum_{m,n} f_{mn, \gamma}^{\alpha \beta, m+n} : c_{\alpha, -m} c_{\beta, -n} b^\gamma_{m+n} :, \] (3.13)

and the ghost number operator
\[ U = \oint \frac{dz}{2\pi i} : c_\alpha(z) b^\alpha(z) : = \sum_m : c_{\alpha, -m} b^\alpha_m : . \] (3.14)

They form the usual BRST algebra
\[ \{Q, Q\} = 0, \ \{Q, U\} = -Q, \ \{U, U\} = 0. \] (3.15)

The condition
\[ (J^\alpha_n + \bar{J}^\alpha_n) |\phi\rangle = 0, \ n \geq 0 \] (3.16)

is equivalent to
\[ Q|\phi\rangle = 0, \ b^\alpha_n |\phi\rangle = c_{\alpha, n+1}^\alpha |\phi\rangle = 0, \ n \geq 0. \] (3.17)

Hence
\[ Q|\phi\rangle = 0, \ U|\phi\rangle = 0. \] (3.18)
This condition is, in the general case, weaker than Eq. (3.17), and the coset state space is a subspace of the BRST cohomology with zero ghost number

$$H_{\text{coset}} = (\text{Ker } Q / \text{Im } Q) \cap \bigcap_{n \geq 0} (\text{Ker } b_n^\alpha \cap \text{Ker } c_{n+1}^\alpha) \subseteq H^0(H^G \otimes H^\tilde{H} \otimes H^{\text{gh}}, Q) = (\text{Ker } Q / \text{Im } Q) \cap \text{Ker } U. \quad (3.19)$$

Here $H^G$ is the state space of G theory, $H^\tilde{H}$ of the theory conjugate to H, $H^{\text{gh}}$ of ghosts.

### 4. A Simple Example: G/G Theory

In the case of G/G model the whole G module $\lambda$ turns into a single field of coset construction. Eq. (2.3) takes the form

$$F(\lambda_1 \mu_1, \lambda_2 \mu_2 \mid \lambda_3 \mu_3, \lambda_4 \mu_4) = \sum_{\beta_1, \beta_2, \gamma_1, \gamma_2} X_{\lambda_1 \lambda_2}^{\lambda_3 \lambda_4} (\lambda_5^{|\alpha_2 ; \gamma_2}) F(\lambda_1 \mu_1, \lambda_2 \mu_2 \mid \lambda_5^{|\beta_2 ; \beta_1}) (\lambda_5^{|\beta_2 ; \beta_1}) F(\lambda_1 \lambda_2 \mid \lambda_5^{|\alpha_2 ; \gamma_2}) \Psi(\lambda_1 \lambda_2 \mid \lambda_5^{|\beta_2 ; \beta_1}), \quad (4.1)$$

or

$$F = FX \Psi. \quad (4.1a)$$

Hence, conformal block of the coset construction

$$\Psi(z) = X^{-1} \quad (4.2)$$

does not depend on $z$. It means that the theory is topological. With the monodromy properties of coset conformal blocks (see (2.4)) we find correlation functions $d$

$$\langle \lambda_1 \lambda_2 \lambda_3 \lambda_4 \rangle$$

$$= \sum_{\lambda_5 \alpha \beta} X_{\lambda_5 \lambda_2}^{\lambda_3 \lambda_4} (\lambda_5^{|\alpha_2 \alpha_1}) (X_{\lambda_5 \lambda_2}^{\lambda_3 \lambda_4} (\lambda_5^{|\beta_2 \beta_1}) \Psi_{\lambda_1 \lambda_2}^{\lambda_3 \lambda_4} (\lambda_5^{|\beta_2 \beta_1}) \Psi_{\lambda_1 \lambda_2}^{\lambda_3 \lambda_4} (\lambda_5^{|\beta_2 \beta_1})^*)$$

$$= \text{Tr} (\Psi X \Psi^\dagger X^\dagger) = \text{Tr} 1.$$

Finally,

$$\langle \lambda_1 \lambda_2 \lambda_3 \lambda_4 \rangle = N_{\lambda_1 \lambda_2}^{\lambda_3 \lambda_4} \equiv \sum_{\lambda_5} N_{\lambda_1 \lambda_2 \lambda_5}^+ N_{\lambda_5 \lambda_3 \lambda_4}, \quad (4.3)$$

---

$d$ The four-point correlation function can be built from conformal blocks not only by the same $X$-constants, but by any solution of Dotsenko-Fateev equation. Nevertheless, we will analyze only the simplest case with the same $X$-constants in both coset factorizing and pairing chiral fields.
where \( N_{\lambda_3 \lambda_4} \) is the number of ‘intermediate states’ \( \lambda_5 \) (including multiplicities) according to fusion rules of initial G model. These correlation function are dual (field algebra is associative). Indeed, the number in the r.h.s. of Eq. (4.3) is simply the number linearly independent solutions of the differential equation with constraints.\(^1\) It does not, of course, depend on choice of basic functions (see also Ref. 5):

\[
\sum_{\lambda_5} N_{\lambda_1 \lambda_2 \lambda_5} N_{\lambda_5 \lambda_3 \lambda_4} = \sum_{\lambda_6} N_{\lambda_1 \lambda_3 \lambda_6} N_{\lambda_6 \lambda_2 \lambda_4}.
\]

Structure constants are given by

\[
\langle \lambda_1 \lambda_2 \lambda_3 \rangle = N_{\lambda_1 \lambda_2 \lambda_3}.
\]

At last, \( n \)-point correlation functions on sphere are given by (Fig. 2)

\[
\langle \lambda_1 \cdots \lambda_n \rangle = \sum_{\Lambda_1 \cdots \Lambda_{n-3}} N_{\lambda_1 \lambda_2 \Lambda_1} N_{\lambda_3 \lambda_4 \Lambda_2} \cdots N_{\lambda_{n-3} \lambda_{n-1} \lambda_n}.
\]

5. Conclusion

It has been shown that conformal blocks of coset model G/H are uniquely determined by those of G and H models. This construction of conformal blocks is equivalent to BRST construction in the ghostless sector.

As an example of application of this approach G/G coset models are considered. These theories are topological ones and their correlation functions turn out simply numbers of intermediate states.

To conclude, we present two conjectures concerning non-zero ghost numbers.

**Conjecture 1.** All nonvanishing correlation functions of states with non-zero ghost numbers coincide with those of corresponding zero-gost states.

**Conjecture 2.** For SU(2)_k/SU(2)_k coset for rational \( k \) and Virasoro/Virasoro for minimal models states are labelled by pairs \((\lambda, h)\), where \( h \) is ghost number. Structure constants are given by

\[
\langle (\lambda_1, h_1)(\lambda_2, h_2)(\lambda_3, h_3) \rangle = N_{\lambda_1 \lambda_2 \lambda_3} \delta_{h_1 + h_2 + h_3, 0},
\]

and correlation functions are

\[
\langle (\lambda_1, h_1) \cdots (\lambda_n, h_n) \rangle
\]

\[
= \sum_{\Lambda_1 \cdots \Lambda_{n-3}} \langle (\lambda_1, h_1)(\lambda_2, h_2)(\Lambda_1, -h_1 - h_2) \rangle \langle (\Lambda_1, h_1 + h_2)(\lambda_3, h_3)(\Lambda_2, -h_1 - h_2 - h_3) \rangle
\]

\[
\cdots \langle (\Lambda_{n-3}, h_1 + \cdots + h_{n-2})(\lambda_{n-1}, h_{n-1})(\lambda_n, h_n) \rangle.
\]

\[9\]
The author is grateful to Vl.S. Dotsenko for very fruitful discussions.

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