Hydrodynamic model of plasma evolution under heating by high-energy ion flux

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Abstract. The analytical solution is found for thermodynamic state of plasma created when the half-space is heating by high-energy ion flow. The dependence of free path length of heating ions on plasma temperature is taken into account. Besides, an analysis of characteristic parameters of shock wave and efficiency of energy transfer to shock wave from heating ion flow is given.

1. Introduction
The heating of matter by high-energy charged particles flow is effective method to create ultrahigh pressure and to generate powerful shock wave in the laboratory experiment. Applied interest is associated with development of the promising method of ignition of inertial confined fusion target - ignition of pre-compressed target fuel under the action of powerful converging shock wave (shock ignition) [1, 2], as well as with a possibility to study of equation of state of matter in laboratory conditions at pressure of gigabar level, which is more than an order of magnitude higher in comparison with recent research using shock wave [3].

Special interest of matter heating by high-energy ions is connected with development of heavy ion inertial fusion. The peculiarity of plasma heating by high-energy ions, in comparison with fast electrons, is that ion free path length depends on plasma temperature.

Non-stationary hydrodynamic model of plasma evolution during the heating a half-space by flow monoenergetic fast electrons with constant intensity was constructed in [4]. The analytical solution was found as a superposition of solutions for isothermal rarefaction wave and self-similar solution for isothermal expansion of given mass of matter [5] under conditions when the particle free path length of heating fast electrons is independent on temperature of heated region and remained unchanged during the all time of process of heating.

This paper is devoted to solution of similar problem for the heating a half-space by flow monoenergetic ions with the only difference that heating ion free path length $\lambda_j$ is not independent value on temperature (and, therefore, from time) and increases with temperature of heated region as $\lambda_j \propto T^{3/2}$.

2. Isothermal expansion of plasma heated by high-energy ion flow
To obtain an analytical solution we use simplified dependence of mass range of ion deceleration (which below is named as ion mass range) $\mu_j = \int \lambda \rho d\rho$ on its initial energy in the form of matching path dependences at large $\left( \mu_j \propto E^2_j \right)$ and low ion energies $\left( \mu_j \propto E^{3/2}_j \cdot T^{3/2} \right)$ [6]. The dependencies are matched at the so-called energy thermal threshold, the value of which is close to the characteristic value the ion energy when its velocity is equal to the velocity the thermal electron.

$$\mu_j \approx \mu_s, \quad E_j \geq E_s,$$

$$\mu_j \approx \mu_s \left( \frac{E_j}{E_s} \right)^{3/2}, \quad E_j \leq E_s,$$

where $\lambda_j$ is the length the total deceleration of the ion with the initial energy $E_j$, $T$ and $\rho$ - average values density and temperature of substance;

$$\mu_s \approx \frac{0.002 E^2_j}{Z^2_j A_j \left( 1 - 0.1 \Lambda \right)}, \quad g / cm^2,$$

$E_s$ is energy thermal threshold is equal to the ion energy at which its velocity is $1.5 \upsilon_e$ ($\upsilon_e$ - velocity of thermal electrons)

$$E_s = \frac{m_j}{m_e} \cdot T \approx 4.1 \cdot 10^3 A_j T,$$

where $Z_j$ and $A_j$ is charge and atomic weight ion; $E_j$ and $E_s$ is measured in $MeV$, $\Lambda = \ln \left( \rho \mu^2_j / E^2_j \right)$ . The approximation used is in good agreement with the data numerical calculations for the range of fast ions [7-9].

Solution [4] for fast electrons is superposition of two solutions: solution for initial heating stage - at $0 \leq t \leq t_h$, when rarefaction wave reaches the internal boundary of heated region - where matter is considered to be at rest, and the temperature increases linearly with time at the fixed intensity of the heating flow, and self-similar solution for $t \geq t_h$ for plane isothermal expansion of given mass of matter, according to which the density decreases with time, according to the law $\rho \propto t^{-3/2}$, and the temperature still grows linearly with time. The maximum pressure in the heated layer of matter is reached at the time $t_h$, after which it begins to decrease with time (according to the law $P \propto t^{-1/2}$).

The first heating stage will always correspond to the case when heating ion range is independent from plasma temperature and is equal to the value $\mu_s$. In this there is characteristic time $t_s$, during which the substance is heated to such an extent that the thermal threshold reaches the initial energy the ion $E_s = E_j$, after which further heating occurs on the ion mass range, which increases with increasing temperature, according to the law $\mu_s \propto T^{3/2}$. The solution type and value the maximum pressure that can be reached under high-energy ion heating depends on the ratio times $t$ and $t_s$, i.e. it will be different situation in the cases when increasing of ion mass range with temperature begins before or after the beginning of hydrodynamic matter expansion.

If expansion of the heated layer occurs before increasing of ion mass range, then there are the following stages:

1) heating of immobile of given mass layer corresponding to initial and fixed ion mass range (the step continues until the rarefaction wave spreads to the entire heated mass);
2) heating of given mass layer corresponding to initial fast ions range during the isothermal expansion of the layer (the step continues until the value of thermal threshold reaches the value of the initial ion energy);  

3) heating of the layer with increasing mass, corresponding to increasing mass range of ions during the isothermal expansion of layer.

The solution in this case is:

\[ T = T_h \begin{cases} \frac{t}{t_h}, 0 \leq t \leq t_s, & t, \leq t_s \leq t \leq t_h, \\ \frac{1}{t_{h^*}}, t \leq t \leq t_s, & \rho = \rho_0, \\ 1, t \leq t_h, & \end{cases}, P = P_h \begin{cases} \frac{t}{t_h}, 0 \leq t \leq t_s, \\ \frac{t^{2/3}}{t_{h^*}}, t \leq t \leq t_s, & \rho = \rho_0, \\ \frac{t_s}{t_{h^*}}, t \geq t_s. & \end{cases} \]

\( t_h, t_s \) - are the time moments corresponding to endings of the first and second stages; \( T_h, P_h \) are defined by formulas:

\[ T_h = \frac{1}{C_v} \left[ \frac{9a_{in}^2}{4(y-1)} I_{J} \right]^{2/3} \rho_0 \] and \[ P_h = \frac{3(y-1)a_{in}^2}{2} \rho_0 \left( \frac{I_j}{\rho_0} \right)^{2/3}, \]

where \( I_j \) is ion beam intensity, \( C_v \) - specific heat, and \( a_{in} \) - fraction of thermal component of energy.

If matter's expansion stage begins after ions range begins to depend on temperature of substance being heated, then there are following stages:

1) heating of immobile layer of a given mass (the step continues until the thermal threshold energy reaches value of ion initial energy);

2) heating of immobile layer, mass of which increases with time in accordance with increases of ion mass range (the step continues until the expansion of the layer begins);

3) heating of the layer with increasing mass during its isothermal expansion.

The solution in this case is:

\[ T = T_{h'}, \begin{cases} \frac{t}{t_{h'}}, 0 \leq t \leq t_{s'}, & P = P_{h'}, \begin{cases} \frac{t_{h'}}{t_{h^*}}, 0 \leq t \leq t_{s'}, \\ \frac{t^{2/3}}{t_{h^*}}, t \leq t \leq t_{s'}, & \rho = \rho_0, \\ \frac{t_{s'}}{t_{h^*}}, t \geq t_{s'} & \end{cases} \end{cases} \]

\( t_{h'}, t_{s'} \) - are the time moments corresponding to endings of the first and second stages; \( T_{h'}, P_{h'} \) are defined by formulas:

\[ T_{h'} = T_h \left( \frac{2}{5} \right)^{2/3} \left[ 1 + \frac{5}{4} \left( \frac{E_{sh}}{E_{sh}} \right) \right]^{2/3}, P_{h'} = P_h \left( \frac{2}{5} \right)^{2/3} \left[ 1 + \frac{5}{4} \left( \frac{E_{sh}}{E_{sh}} \right) \right]^{2/3}, \]

where \( E_{sh} \) is the thermal threshold energy that corresponds to the temperature \( T_{h'} \).

The solution for the efficiency of energy transfer - the fraction of energy transferred to shock wave from heating ion flow - for the two cases are defined by formulas:
Comparative analysis of the thermodynamic state of the plasma when it heated by fluxes of fast ion and electron

We introduce a time scale \( t_{sh} \) independent of the ion energy (the loading time \( t_h \) for an ion with energy equal to the energy thermal threshold at the temperature of the heated region \( T_h \)):

\[
\eta = \eta_h \sim \begin{cases} 
\left( \frac{t}{t_h} \right)^{3/2}, & t \leq t_h \\
10 \left( \frac{t_h}{t} \right)^{3/4} - 9 \left( \frac{t_h}{t} \right)^{5/6}, & t_h \leq t \leq t_s \\
\left( \frac{t_{sh}}{t_{sh}} \right)^{9/10} \left( \frac{t}{t_{sh}} \right)^{3/2}, & 0 \leq t \leq t_{sh} \\
\frac{25}{16} \left( \frac{t}{t_{sh}} \right)^{3/5} - \frac{9}{16} \left( \frac{t_{sh}}{t_{sh}} \right)^{1/10}, & t_{sh} \leq t \leq t_{sh} \\
\frac{25}{7} \left( \frac{t_{sh}}{t} \right)^{3/10} - \frac{5}{2} \left( \frac{9}{40} \left( \frac{t_{sh}}{t_{sh}} \right)^{1/5} + \frac{45}{56} \right), & t \geq t_{sh}
\end{cases}
\]

\[
\eta_h = \frac{3a_h(\gamma - 1)}{5} \left( \frac{2}{\gamma + 1} \right)^{1/2};
\]

where \( \eta \) - is the mass of ion, \( m_e \) - is the mass of electron.

Figures 1-4 show graphs dependences of the temperature, density, and pressure normalized to \( T_h \), \( P_h \) and \( \rho_h \) and the energy transfer efficiency on the dimensionless time \( t / t_{sh} \).

\[
T_{sh} = \frac{9}{4(\gamma - 1)a_{sh}} \left( \frac{0.002 m_j^2 T_h^2}{Z_j^2 A_j \rho_0 T_{sh}^{1/3} m_e^2} \right)^{1/3},
\]

where \( m_j \) - is the mass of ion, \( m_e \) - is the mass of electron.
Fig. 1-4. Dependence of the temperature (Fig. 1), pressure (Fig. 2), density (Fig. 3) normalized to T and energy transfer efficiency (Fig. 4) on the dimensionless time. Line 1 correspond to the ion energy $E_i = 3\, MeV$, line 2 $E_i = 2.3\, MeV$, line 3 - energy $E_i = 1.69\, MeV$ (energy thermal threshold), line 4 - $E_i = 1.1\, MeV$, line 5 - $E_i = 0.5\, MeV$, line 6 - the beam of fast electrons with mass range is equal to mass range of fast ion with energy equal to energy of thermal threshold.
The graphs show the features of the found solutions, connected to the variation of the dependence of the mass range of ion on the temperature of the heated region. The loading time at which the maximum pressure \( P = P_0 \) is reached and the expansion of the substance begins to increase with increasing of ion energy, and this growth occurs faster for ions with energy \( E_j > E_{sh} \). For ions with energy \( E_j \geq E_{sh} \) and fast electrons, the maximum achievable pressure is the same. For ions with energy \( E_j < E_{sh} \) the maximum achievable pressure slightly decreases with decreasing ion energy.

Change to slow decrease of pressure with time \( (P \propto t^{-1/5}) \) for ions with energy \( E_j \leq E_{sh} \) begins immediately after reaching the maximum pressure \( P = P_0 \). The duration this change for ions with energy \( E_j > E_{sh} \) than longer that greater the ratio \( E_j / E_{sh} \).

When heating of matter by an equivalent beam of fast electrons, there is no stage of a slow decreases pressure - after pressure reaches the maximum, it decrease like \( P \propto t^{-1/2} \). The time to reach the maximum value of the energy transfer efficiency increases with increase of ions beam energy. The maximum value of energy transfer efficiency for fast electrons (25%) is 1.2 times smaller than the maximum value of the maximum for ions with an energy equal to the energy of the thermal threshold (30%).

4. Conclusion
The found solution describes the thermodynamic state of a plasma created when a flat target is heated by a stream of high-energy ions, when the free path length of ions does not depend on temperature of the heated region at the beginning of heating and increases with the temperature as \( T^{3/2} \) when the material reaches the temperature of the thermal threshold of the ion. The solution shows that the heating of substance by a stream of ions with an energy equal to the energy thermal threshold is the most advantageous regime for the formation of a quasi-stationary shock wave. Other things being equal, this regime corresponds to attainment of the limiting pressure when the substance is heated by the flow of charged particles, and the slowest subsequent pressure drop with time. Pressure of gigabar level is achieved by heating ions a flat target of aluminum by a proton beam with a particle energy about 4 MeV with a beam intensity of about \( 5 \times 10^{15} \) W/cm².

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