Incorporating Chiral Symmetry in Extrapolations of Octet Baryon Magnetic Moments

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Abstract

We explore methods of extrapolating lattice calculations of hadronic observables to the physical regime, while respecting the constraints of chiral symmetry and heavy quark effective theory. In particular, we extrapolate lattice results for magnetic moments of the spin-1/2 baryon octet to the physical pion mass and compare with experimental measurements. The success previously reported for extrapolations of the nucleon magnetic moments carries over to the Σ baryons. A study of the residual discrepancies in the Ξ baryon moments suggests that it is important to have new simulation data with a more realistic strange quark mass.

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I. INTRODUCTION

One of the key goals of lattice QCD is to confront experimental data with the predictions of QCD. However, computational limitations mean that hadronic observables, such as masses and magnetic moments, are calculated at quark masses much larger than their physical values. Although improvements in algorithms and computer speed will allow lattice calculations of hadronic observables to be performed much closer to the physical regime, these improvements will proceed over many years. In the meantime it is imperative that one has an understanding of how to extrapolate lattice results, obtained at large quark masses, to the physical world.

A difficult problem encountered in calculating hadronic observables at heavy quark masses on the lattice is that chiral perturbation theory is not applicable in this heavy quark mass regime. However, chiral perturbation theory predicts that near the chiral limit there are important non-analytic terms as a function of the quark mass, \( m_q \) (or equivalently of \( m_{\pi}^2 \), as \( m_q \propto m_{\pi}^2 \) in this range), in the expansion of a physical observable. This non-analytic behaviour must be taken into account in any extrapolation to the physical regime.

Here we focus on linking lattice calculations of magnetic moments of the spin-1/2 baryon octet to the physical world. In particular, we follow an extrapolation procedure first introduced for the nucleon magnetic moments which builds in the non-analytic behaviour of the magnetic moments near the chiral limit, as well as the correct heavy quark behaviour [1]. In the case of the nucleon, this extrapolation procedure predicts \( \mu_p = 2.85(22) \mu_N \) and \( \mu_n = -1.90(15) \mu_N \) (see Fig. 5 of Ref. [1]). This agrees well with the experimental measurements of \( \mu_p = 2.793 \mu_N \) and \( \mu_n = -1.913 \mu_N \). Here we explore the application of this procedure to octet baryons in general.

The magnetic moment results used here are extracted from the lattice QCD calculations of Ref. [2]. While the results are now quite old, they continue to be the only lattice estimates of the spin-1/2 baryon octet magnetic moments available at the moment. These results were all obtained at pion masses above 600MeV. We extrapolate these results as functions of the pion mass, \( m_{\pi} \), to the physical pion mass of 140MeV, to obtain the physical magnetic moment predictions. Because the lattice calculations are quenched, we expect that there are errors in the lattice data which we have been unable to take into account. However, as explained in Ref. [2], these errors are expected to be on the scale of the statistical errors. Nevertheless, an ideal extrapolation of magnetic moments would use full QCD lattice results which are unavailable at the moment.

II. EXTRAPOLATIONS

To extrapolate the lattice calculations of the magnetic moments we use the Padé approximant:

\[
\mu_i(m_{\pi}) = \frac{\mu_0}{1 - \frac{\lambda}{\mu_0} m_{\pi} + c m_{\pi}^2}
\]  

(1)
where $\chi_i$, corresponding to the $i$th baryon, is fixed model-independently by chiral perturbation theory and $\mu_0$ and $c$ are allowed to vary to best fit the data [1]. This formula builds in the chiral behaviour at small $m_\pi$, governed by $\chi_i$, as well as the correct heavy quark behaviour, as discussed in the following.

The Goldstone boson loops resulting from dynamical chiral symmetry breaking mean that the baryon magnetic moments exhibit certain model independent, non-analytic behaviour in the quark masses. Using an expansion about the chiral SU(3) limit, one finds that the magnetic moments of the octet baryons (in nuclear magnetons, $\mu_N$) are given by

$$\mu_i = \gamma_i + \sum_{X=\pi,K} \beta_i^{(X)} \frac{m_N}{8\pi f^2} m_X + \ldots$$

(2)

where the ellipses represent higher order terms, including logarithms [3]. Here $f$ is the pion decay constant in the chiral limit (93 MeV) and $m_N$ is the nucleon mass. For our purposes, namely extrapolating lattice data at fixed strange quark mass ($m_s$) as a function of the light quark mass ($m_q$), it is preferable to expand about the SU(2) chiral limit. The cloudy bag calculations in Ref. [1] showed that Goldstone boson loops are suppressed like $m_X^{-4}$ at large $m_X$ (comparable to $m_K$). Although this result is model dependent, the lattice simulations themselves do not show a rapid variation with $m_X$ at values of order $m_K$ or higher, thus supporting the general conclusion. One therefore expects that the kaon loops should be relatively small and slowly varying as a function of $m_q$ [4]. They can therefore be absorbed in the fit parameters $\mu_0$ and $c$. On the other hand, the rapid variation of $m_\pi$ with $m_q$ means that the leading non-analytic behaviour in $m_\pi$ must be treated explicitly.

It is simple to see that the Padé approximant, Eq. (1), guarantees the correct behaviour of the magnetic moments in the chiral SU(2) limit. Expanding Eq. (1) about $m_\pi = 0$ we find

$$\mu_i = \gamma_i + \chi_i m_\pi + \left( \frac{\chi_i^2}{\mu_0} - \mu_0 c \right) m_\pi^2 + \ldots$$

(3)

In order to reproduce the leading non-analytic behaviour of the chiral expansion in our fit we fix $\chi_i$ to the value $\beta_i^{(\pi)} (m_N/8\pi f^2)$ for the $i$th octet baryon. The one-loop corrected estimates [3] of the coefficients $\beta_i^{(\pi)}$ and $\chi_i$ are given in Table I.

The Padé approximant, Eq. (1), also builds in the expected behaviour at large $m_\pi$. At heavy quark masses we expect that the magnetic moment should fall off as the Dirac moment

$$\mu = \frac{e_q}{2m_q} \propto \frac{1}{m_\pi^2}$$

(4)

as $m_\pi$ becomes moderately large. This is clearly the case in the Padé approximant. Therefore, the Padé approximant has been chosen to reproduce physical phenomena at the small 1

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1Recall that $m_K^2 \propto m_s + m_q$ and $m_s$ is fixed and large.
TABLE I. One-loop corrected estimates of $\beta_1^{(\pi)}$ (in Eq. (2)) and $\chi_i = \beta_i^{(\pi)} (m_N/8\pi f^2)$

|   | $p$   | $n$   | $\Lambda$ | $\Sigma^+$ | $\Sigma^0$ | $\Sigma^-$ | $\Xi^0$ | $\Xi^-$ |
|---|-------|-------|------------|------------|------------|------------|----------|---------|
| $\beta_1^{(\pi)}$ | $-(F + D)^2$ | $(F + D)^2$ | 0 | $-\frac{2}{3}D^2 - 2F^2$ | 0 | $\frac{2}{3}D^2 + 2F^2$ | $-(D - F)^2$ | $(D - F)^2$ |
| $\beta_i^{(\pi)}$ | -1.02 | 1.02 | 0 | -0.57 | 0 | 0.57 | -0.04 | 0.04 |
| $\chi_i$ | -4.41 | 4.41 | 0 | -2.46 | 0 | 2.46 | -1.91 | 1.91 |

and large $m_\pi$ scales. It also succinctly describes the excellent phenomenology of the Cloudy Bag Model [1,4]. The Padé approximant has already been used successfully in the extrapolation of lattice results of magnetic moments of the nucleon, which we include here for completeness [1].

III. RESULTS

In the following graphs, Figs. 1–4, lattice calculations of the baryon magnetic moments are fitted as a function of $m_\pi$, according to the Padé approximant given in Eq. (1), with coefficients, $\chi_i$, from Table I. In each case the solid lines are Padé approximant fits to the magnetic moment lattice results. Experimental measurements are indicated at the physical pion mass by an asterisk (*). The magnetic moment predictions of the Padé approximant are compared with experimental values in Table II. The fit parameters, $\mu_0$ and $c$, for the solid lines are also indicated.

In the case of the nucleon, the fits given here (Figs. 1 and 2) are slightly different from those given in Ref. [1], as we omit the second set of lattice results (these were extracted from Ref. [6] which dealt with the nucleon only) in order to produce a consistent set of graphs for the entire baryon octet. However, the nucleon fits shown here still give excellent agreement with experimental data. The physical magnetic moment predictions for the $\Sigma^+$ and $\Sigma^-$ are also in good agreement with experiment.

Using magnetic moment values predicted by the Padé approximant we can calculate the ratio of the $\Xi^-$ and $\Lambda$ magnetic moments. The simple quark model predicts that this ratio is given by

$$\frac{\mu_{\Xi^-}}{\mu_\Lambda} = \frac{1}{3} \left( 4 - \frac{\mu_d}{\mu_s} \right)$$  \hspace{1cm} (5)

which becomes

$$\frac{\mu_{\Xi^-}}{\mu_\Lambda} = \frac{1}{3} \left( 4 - \frac{m_s}{m_d} \right)$$  \hspace{1cm} (6)

if we take each quark magnetic moment to be given by the Dirac moment of its constituent mass. In this case the ratio is less than 1 for $m_s > m_d$. This disagrees with the experimentally
FIG. 1. Fits to lattice results of the proton magnetic moment. The physical value predicted by the fit is also indicated, as is the experimental value, denoted by an asterisk.

FIG. 2. Fits to lattice results of the neutron, Λ and Σ⁻ magnetic moments. The physical values predicted by the fits are indicated, as are the experimental values, which are denoted by asterisks.
TABLE II. Magnetic moments of the octet baryons (in nuclear magnetons) predicted by lattice QCD compared with experiment. The fit parameters $\mu_0$ and $c$ of the Padé approximant are also indicated in units of $\mu_N$ and GeV$^{-2}$ respectively. The column entitled “Averaged Lattice” reports magnetic moments from extrapolations of lattice calculations averaged to better describe the strange quark mass, as discussed in the text.

The experimental value for $\Sigma^0$ is taken from the average of $\Sigma^+$ and $\Sigma^-$ experimental results, which is valid in the limit of isospin symmetry.

measure value of $1.13(7)$. However, using the predictions of the Padé approximant, we obtain a value of $1.15$ for this ratio, which is in excellent agreement with the experimental data. This is a good indication that meson cloud effects must be included in an extrapolation of lattice results to the physical regime.

The lattice calculations of baryon magnetic moments used in this letter were made with a strange quark mass of approximately $250$ MeV. This is much heavier than the physical mass of the strange quark of $115 \pm 8$ MeV at a scale $2$ GeV, taken from a careful analysis of QCD sum rules for $\tau$ decay. The contribution of the strange quark to the $\Sigma$ baryon magnetic moments is very small. Lattice QCD calculations indicate that the contribution of a singly represented quark in a baryon is half that anticipated by SU(6) spin-flavour symmetry. Hence the heavy strange quark mass will have a subtle effect on the $\Sigma$ moments. By contrast, the strange quarks dominate the $\Lambda$ and $\Xi$ magnetic moments. Thus the heavy strange quark produces a large error in the lattice data for these baryons, which so far we have not taken into account. This is reflected in the predictions of the $\Lambda$, $\Xi^0$ and $\Xi^-$ magnetic moments which are smaller in magnitude than the experimental measurements in all cases.

In an attempt to correct for the effect of the large strange quark mass considered in the lattice calculations, we average the magnetic moment lattice results of each $S \neq 0$ baryon with magnetic moment results of a light-quark equivalent baryon. This procedure

$Lattice calculations of a light-quark equivalent $\Lambda$ are not available.
interpolates between magnetic moment lattice results produced with heavy strange quarks and those produced with zero strange quark mass. These averaged results have an effective strange quark mass closer to the physical strange quark mass. We have also used the Padé approximant to extrapolate the averaged results. The effect on the Σ moments is subtle (see Table II). However, in the case of Ξ−, this method is sufficient to reproduce the empirical Ξ− moment (as shown by the dashed line in Fig. 4). There is a remaining discrepancy in the value predicted for the Ξ0. Clearly the present estimate of the correction for the heavy strange quark mass is somewhat crude. We therefore regard it as very important to have new simulation data with a realistic strange quark mass. At that stage it may also be necessary to include kaon loop effects, because the transition Ξ0 → Σ+ + K− is energetically favoured, and will make a negative contribution to the Ξ0 magnetic moment.

IV. CONCLUSION

We have shown that the Padé approximant which was introduced to extrapolate lattice results for the magnetic moments of the nucleon, is also successful in predicting magnetic moments for the spin-1/2 baryon octet. The magnetic moment values predicted by the fits for the p, n, Σ+, and Σ− compare well with experimental data. As a first estimate of the correction to be expected if a more realistic strange quark mass were used, we averaged lattice results for the S ≠ 0 baryons with the magnetic moments of the corresponding light-quark baryons. This had a small effect on the predictions for the Σ baryon magnetic moments, but significantly improved the Ξ baryon results. In the case of Ξ−, the averaging
FIG. 4. Fits to lattice calculations of the $\Xi^0$ and $\Xi^-$ magnetic moments. The upper two lines are fits for $\Xi^-$ results and the lower two lines for $\Xi^0$ results. Solid lines represent fits to the magnetic moment results, whereas dashed lines represent fits to averaged results (denoted by open symbols which are offset for clarity), as described in the text. The physical values predicted by the fits are indicated, as are the experimental values, which are denoted by asterisks.

procedure produced good agreement with the experimental results. In the future we hope to perform a similar extrapolation procedure using more precise magnetic moment lattice data, calculated with realistic strange quark masses. At that stage it may also be necessary to include the kaon loop corrections, especially for the doubly strange $\Xi$ hyperons.

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