INTRODUCTION

The measure of certain characteristics of individual micro-scale entities as they pass under a detection source associated with a microdevice is important for many engineering applications including separation and characterization (Chen et al., 2014; Piyasena & Graves, 2014). Thus, it is important to focus micro-scale entities prior to any other operation in microdevices such as separation/sorting and characterization (Xuan, Zhu, & Church, 2010). Focusing is the ordering of randomly scattered micro-scale entities (Xuan et al., 2010); their arrangement along a single line is referred to as 3D focusing (Xuan et al., 2010). 3D focusing leads to reduced variation in velocity/position distributions of micro-scale entities inside the microchannel which in turn leads to the reduced occurrence of coincidental events compared with unfocused or 2D focused micro-scale entities (Piyasena & Graves, 2014; Simonnet & Groisman, 2005). The drawback of 2D focusing is the spreading of micro-scale entities on the vertical plane in which it is focused (Simonnet & Groisman, 2005). Simonnet and Groisman (2005) proposed 2D focusing of micro-scale entities on a vertical plane for micro-flow cytometers; however, micro-scale entities of the focused stream did not pass across the detection zone individually even when the width of the focused stream and the size of micro-scale entity are similar. This is because the micro-scale entities are located at different positions along the microchannel’s depth as it is greater than the size of micro-scale entity (Xuan et al., 2010). Many 2D focusing approaches result in a large number of undetected micro-scale entities due to the vertical spread (Xuan et al., 2010). Thus, for individually perusing micro-scale
entities it is important to simultaneously focus them along both vertical and horizontal directions, that is 3D focusing is required.

To achieve focusing, the micro-scale entities need to be acted upon by actuation forces. Several types of actuation forces are employed in microdevices to realize focusing and these include dielectrophoresis (DEP), magnetophoresis, acoustophoresis and hydrophoresis (Xuan et al., 2010). The choice of actuation mechanism to activate focusing depends on the application. DEP is widely used for manipulation of micro-scale entities and compared with other actuation mechanisms it neither requires specialized wafer nor sheath flow; moreover, DEP scales well with miniaturization, that is a low applied voltage can create a high electric field due to the short electrode-to-electrode distances.

Dielectric micro-scale entities scattered in a conductive medium and exposed to non-uniform electric field are attracted to positions of either the strongest or weakest gradient of the same electric field, and this phenomenon is DEP (Pethig, 2010; Qian et al., 2014; Zhang, Khoshmanesh, Mitchell, & Kalantar-Zadeh, 2010). DEP is thus an effective means of manipulating a micro-scale entity’s trajectory (Pethig, 2010; Qian et al., 2014; Zhang et al., 2010). The DEP forces’ direction and strength are dependent on the conductivity and permittivity, of the micro-scale entity and the medium, in addition to the frequency of operation of the applied electric signal; the DEP force is provided in Equation (1) (Pethig, 2010; Qian et al., 2014; Zhang et al., 2010).

\[
F_{DEP} = 2\pi \varepsilon_r \mu \Re \left[ \varepsilon_{CM} \left( \varepsilon_p \sigma_p \varepsilon_m \sigma_m \omega \right) \right] |\nabla |E_{rms}|^2
\]

(1)

\[
\text{Re}[\varepsilon_{CM}] = \frac{\varepsilon_r^* - \varepsilon_m^*}{\varepsilon_r^* + 2 \varepsilon_m^*}
\]

(2)

\[
\varepsilon^* = \varepsilon - \frac{\sigma}{\omega j}
\]

(3)

Depending on the polarity of \(\text{Re}[\varepsilon_{CM}]\), DEP attracts micro-scale entities towards the position of strongest or weakest electric field gradient; most often this results in micro-scale entities being attracted to or repelled from the electrodes (Pethig, 2010; Qian et al., 2014; Zhang et al., 2010). When micro-scale entities are drawn towards the position of the strongest gradient of electric field, DEP is termed as positive DEP or pDEP, while attraction of micro-scale entities in the direction of the weakest gradient of the electric field is labelled as negative DEP or nDEP (Pethig, 2010; Qian et al., 2014; Zhang et al., 2010). Changing the frequency of operation changes the behaviour of the micro-scale entity from nDEP to pDEP or vice versa (Pethig, 2010; Qian et al., 2014; Zhang et al., 2010). This enables realizing electrically controlled manipulation of micro-scale entities for trapping, focusing and separation (Pethig, 2010; Qian et al., 2014; Zhang et al., 2010). The dependence of \(\text{Re}[\varepsilon_{CM}]\) on frequency determines the dependence of the DEP force on frequency, and this dependency is distinctive to a particular combination of micro-scale entity and medium.

For DEP-based focusing, electrodes can be realized in several configurations inside the microchannel (Xuan et al., 2010). Depending on the electrode configuration as well as the strength and frequency of operation of AC signal, it is possible to define the degree of movableness for micro-scale entities (Pethig, 2010; Qian et al., 2014; Zhang et al., 2010). This article analyses the electrode geometry/configuration shown in Figure 1; the configuration is composed of four electrodes in which two are located on the lower surface and other two are on the upper surface of the microchannel. All electrodes extend equally into the microchannel from the nearest sidewall. With this arrangement, electric fields are set up between the electrodes on the lower and upper surfaces of the same side of the microchannel. This arrangement enables the generation of vertical and horizontal nDEP forces necessary for realizing 3D focusing of micro-scale entities. The net-nDEP force’s horizontal component in the right and left part of the microchannel is directed towards the opposite side, Figure 1b. This implies that there exists a vertical plane in the microchannel in which the net-nDEP force’s horizontal component is zero. Thus, a micro-scale entity in the right part of microchannel will settle in this vertical plane after being pushed towards the left part of the microchannel; such behaviour can be
observed for the microparticle in the left part of the microchannel as well. The vertical plane, in which the micro-scale entity settles, divides the microchannel into the right and left part. Along the vertical direction, the forces acting on the micro-scale entity include sedimentation force and nDEP force’s vertical component. Depending on whether the micro-scale entity is in the upper or lower half of the microchannel, the nDEP force’s vertical component acts in the downward or upward vertical direction, respectively. Whenever a micro-scale entity is in the upper half of the microchannel, the sedimentation force and the nDEP force’s vertical component act in the downward direction thereby moving it towards the bottom of the microchannel. However, once the micro-scale entity crosses into the lower half of the microchannel the vertical force component of nDEP force is directed upwards which leads to it balancing the sedimentation force at a horizontal plane inside the microchannel. For those micro-scale entities starting from the lower half of microchannel, the nDEP force’s vertical component is always directed upwards to balance the sedimentation force in a horizontal plane inside the microchannel. Thus, the combined effect of nDEP and sedimentation force pushes micro-scale entities towards the intersection between a horizontal plane and a vertical plane to achieve 3D focusing.

One unique feature of the electrode configuration shown in Figure 1 is its ability to realize 3D focusing at any position along the microchannel’s width. When nDEP force profiles on the right and left parts of the microchannels are similar, the micro-scale entities would be 3D focused on the plane that pass vertically through the microchannel’s centre. Also, when the nDEP force profiles on the right and left parts are dissimilar then the micro-scale entity would be 3D focused at positions other than the centre of the microchannel. By appropriately selecting the applied voltages, it is possible to control the nDEP force profiles in the right and left parts of the microchannel. For equal applied voltages, the nDEP force profile in the right and left parts of the microchannel are similar; however, for unequal applied voltages the nDEP force profile in the right and left parts of the microchannel are dissimilar. Currently, sheath flow is used for focusing micro-scale entities away from the centre of the microchannel, particularly near sidewalls (Xuan et al., 2010); however, use of the proposed electrode configuration provides a new approach for focusing micro-scale entities at lateral positions other than the centre of the microchannel. Additionally, the fact that electrodes of the proposed device are continuous, allow for handling throughput higher than often handled in microdevices with planar electrodes.

Researchers have employed several approaches to achieve consistent 3D focusing in micro-scale entities. Morgan, Holmes, and Green (2003) developed a microdevice with straight planar electrodes, of finite length, on the bottom and top walls. Each electrode fixed on the top and bottom of the same side form a pair. The two electrode pairs generate opposing nDEP forces along the vertical and horizontal directions to 3D focus microparticles at the centre of the microchannel. Based on the work, they found that increase in applied voltage improves focusing. Lin, Lee, Fu, and Hwey (2006) created a microdevice that simultaneously employed hydrophoresis and DEP for 3D focusing micro-scale entities. In the device of Lin et al. (2006), sheath flows are initially employed for focusing all micro-scale entities in the vertical plane that passed through the middle of the microchannel. These micro-scale entities are then subjected to nDEP in the vertical direction thereby 3D focusing the same. Holmes, Morgan, and Green (2006) developed a microdevice with planar triangular electrodes on the bottom and top surfaces of the microchannel. Each electrode on the bottom and top surfaces is located next to one of the sides with the electric field established between two electrodes on the same side. The effect of these electrodes is to act as a nozzle to focus microparticles in the middle plane of the microchannel when applied voltages are equal. Yu et al. (2005) succeeded in developing a microdevice with finite sized interdigitated transducer (IDT) electrodes for DEP based for 3D focusing. This device employed circular microchannel, and the electrodes span the entire circumference of microchannel. The electrodes generate nDEP forces in the radial direction to 3D focus the micro-scale entity at the middle point of the microchannel. Yu et al. (2005) observed that increasing applied voltage will improve focusing. Wang, Flanagan, and Lee (2007) created a DEP-based microdevice with vertical IDT electrodes on both sides on the microchannel; each set of IDT electrodes is independently controllable. This arrangement allows the focusing of micro-scale entity onto the vertical plane passing through the microchannel centre; however, it cannot achieve 3D focusing as it does not generate DEP force along the height. Demierre, Braschler, Muller, and Renaud (2008) fabricated and tested a microdevice that employed insulator-based dielectrophoresis to attain 3D focusing. The device utilizes planar IDT electrodes partially surrounded by insulating structures to generate 2D electric field in the microchannel. The associated nDEP forces focus the micro-scale entities in the vertical plane passing through the microchannel centre; however, this device cannot 3D focus micro-scale particles since the electric field is two-dimensional. Huang, Weng, and Jen (2011) developed a microfluidic device that employed multiple X-shaped insulating structures and planar electrodes for generating the electric field necessary to achieve 3D focusing. Huang et al. (2011) initially modelled three different electrode configurations for their microfluidic device. In all their electrode configurations, two finite size planar electrodes, of same polarity, are at the centre of bottom and top surfaces of the microchannel while electrodes of opposite polarity are next to the sidewalls. The first electrode structure consisted of two fixed size planar electrodes of same polarity on the bottom wall near the sidewalls. In the second electrode configuration, vertical electrodes of same polarity are located on the sidewalls while the third electrode configuration consists of fixed size top and bottom planar electrodes next to each of the sidewalls. The modelling revealed that the second electrode configuration exhibited the best performance while the first electrode configuration had the worst performance. Huang et al. (2011) designed a microfluidic device with the second electrode configuration and conducted experiments to find whether the increase in electric field and reduction in flow rate improved focusing.

This article describes the mathematical model of a microdevice employing DEP for 3D focusing. This model is subsequently used for a
parametric analysis to recognize the influence of geometric and operating parameters on 3D focusing. The developed mathematical model is a dynamic model, and this allows for determining the axial distance and time demanded by micro-scale entities to be 3D focused. The developed model is thus superior compared with existing static models associated with DEP-based microdevices (Huang, Wang, Becker, & Gascoyne, 1997); static models cannot predict the axial distance or the time demanded by micro-scale entities to reach the final position and thus not ideal for the purposes of design of continuous flow devices. This work is an extended version of a previous work by the Hilal-Alnaqbi, Dagher, Alazzam and Mathew (2017).

2 | MATHEMATICAL MODEL

Newton’s 2nd law as provided in Equation (4) describes the trajectory of micro-scale entities (Crowe, Sommerfeld, Tsui, & Schwarzkopf, 2011).

\[
m_p \frac{d^2 \begin{bmatrix} x_p \\ y_p \\ z_p \end{bmatrix}}{dt^2} = \begin{bmatrix} \sum F_{\text{ext,x}} \\ \sum F_{\text{ext,y}} \\ \sum F_{\text{ext,z}} \end{bmatrix}
\]

The inertial force associated with the micro-scale entity is exhibited on the left-hand side of Equation (4) while the right-hand side of the same expresses the sum of all external forces acting on the micro-scale entity. The movement of micro-scale entities under continuous flow conditions in a microdevice is influenced by various forces including that related to buoyancy \((F_b)\), gravity \((F_g)\), drag \((F_d)\), virtual mass \((F_{vm})\) and DEP \((F_{DEP})\) (Crowe et al., 2011). Therefore, these forces need to be accounted in the modelling and design of the microdevice as presented in Equation (5). The mathematical expression of each of the above-mentioned external forces is given in Equation (6). The drag \((F_d)\), virtual mass \((F_{vm})\) and DEP \((F_{DEP})\) forces act in all directions; however, the force related to gravity \((F_g)\) and buoyancy \((F_b)\) acts only in the vertical directions.

\[
\sum F_{\text{ext}} = F_b - F_d + F_g + F_{vm} + F_{DEP}
\]

by rearrangement of the terms to obtain Equation (7) which allows for determining the displacements of the micro-scale entity at any future time. The parameter \(n\) in Equation (7) represents the time step count; \(n = 0\) represents the start of displacement for the micro-scale entity. The time step is fixed for this study, that is at \(\Delta t = 0.0001\) s. Smaller time steps is found to have no effect on the accuracy of the results. Moreover, the computational time for the selected time step is found to be adequate.

\[
\begin{bmatrix} x_{p,n+1} \\ y_{p,n+1} \\ z_{p,n+1} \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} + \Delta t \begin{bmatrix} \sum F_{\text{ext,x}} \\ \sum F_{\text{ext,y}} \\ \sum F_{\text{ext,z}} \end{bmatrix} + \Delta t^2 \begin{bmatrix} \sum F_{\text{ext,x}} \\ \sum F_{\text{ext,y}} \\ \sum F_{\text{ext,z}} \end{bmatrix} + \Delta t^2 \begin{bmatrix} \sum F_{\text{ext,x}} \\ \sum F_{\text{ext,y}} \\ \sum F_{\text{ext,z}} \end{bmatrix}
\]

The initial conditions \((x_{p,0}, y_{p,0}, z_{p,0})\) represent the initial displacements of the micro-scale entity, and they are available from the initial conditions as provided in Equation (8). The micro-scale entity is assumed to occupy any position over the microchannel’s cross-section at its inlet as depicted in Equation (8).

\[
\begin{bmatrix} x_p \\ y_p \\ z_p \end{bmatrix} = \begin{bmatrix} 0 \\ Y \\ Z \end{bmatrix} \quad \text{Where} \quad 0 \leq Y \leq W_{ch} \quad \text{and} \quad 0 \leq Z \leq H_{ch}
\]

Equation (4) is solved by Finite Difference Method (FDM) (Morton & Mayers, 2005). To employ FDM, the second-order derivative terms of the above equations are substituted by second-order central difference scheme while the first-order derivative terms are replaced with first-order central difference scheme. This is followed
Due to the initial velocity of the micro-scale entity being equal to that of medium, the corresponding drag is zero. This fact as well as the information in Equation (8) and Equation (10) needs to be taken into account for calculating \((x_p,1, y_p,1, z_p,1)\); a simplified version of Equation (7) is used for calculating \((x_p,1, y_p,1, z_p,1)\).

From Equation (7), it is clear that the velocity of medium needs to be known for calculating unknown displacements. In microdevices handling micro-scale entities, the Reynolds number is very small, that is \(Re \ll 1\); that is Stokes flow. For Stokes flow, the flow is fully developed and this reduces the continuity equation and Navier-Stokes equations to the form shown in Equation (11) and Equation (12), respectively (Deen, 2011). Thus, for Stokes flow the medium velocity along the width and depth of the microchannel are non-existent. In addition, the velocity of the medium along the axial direction is independent of the axial coordinate; it varies only with respect to the coordinates along the width and depth of the microchannel.

\[
\frac{d}{dx}v_{mx} = 0 \tag{11}
\]

\[
\left(\frac{\partial}{\partial y^2} + \frac{\partial}{\partial z^2}\right)v_{mx} = \frac{1}{\mu m} \frac{\partial p}{\partial x} \tag{12}
\]

The solution of Equation (12) provides the mediums’ velocity in the flow direction, and this is listed in Equation (13). Using this velocity, it is possible to determine the derivatives of the velocity required in Equation (7).

\[
v_{mx}(y,z)=48 \frac{Q_m}{
\zeta / 4WH \sum_{i=1,3,5} \left(1 - \frac{i}{p} \right) \cos \left[ \frac{\pi}{2} \left( \frac{y}{p} - 1 \right) \right] + \frac{1 - \frac{1}{12} \frac{W_m}{L_m} \sum_{i=1,3,5} \cosh \left( \frac{z_i}{p} \right)}{1 - \frac{1}{12} \frac{W_m}{L_m} \sum_{i=1,3,5} \tanh \left( \frac{z_i}{p} \right)} \right) \tag{13}
\]

Another parameter that is of utmost importance for determining the displacements is the electric field in the microchannel. The microchannel’s electric field is described using Equation (14). However, to resolve the electric field inside the microchannel it is important to establish the voltage inside the microchannel. Voltage in the microchannel is mathematically described as in Equation (15).

\[
\begin{bmatrix}
E_x \\
E_y \\
E_z
\end{bmatrix}
= -
\begin{bmatrix}
\frac{\partial}{\partial x} \\
\frac{\partial}{\partial y} \\
\frac{\partial}{\partial z}
\end{bmatrix}
\varphi \tag{14}
\]

\[
\left(\frac{\partial}{\partial x^2} + \frac{\partial}{\partial y^2} + \frac{\partial}{\partial z^2}\right)\varphi = 0 \tag{15}
\]
RESULTS AND DISCUSSIONS

As mentioned, this model aids examining the effects of geometric and operating parameters on the proposed microdevice’s working. The geometric parameters include radius of micro-scale entity, microchannel height, initial positions and electrode protrusion width while the operating parameters include volumetric flow rate and applied voltages. Studies are performed with the microdevice using water (medium) and the polystyrene microparticle (micro-scale entity). The dielectric constant of water is 78.5 while the viscosity and density of water are $10^{-3}$ Pa·s and 998 kg/m$^3$, respectively (Deen, 2011). The polystyrene microparticle’s dielectric constant is 2.55, and its density is 1,050 kg/m$^3$, respectively (Deen, 2011).

From Equation (2) and Equation (3), it is evident that the DEP force depends on the operating frequency. The operating frequency of polystyrene microparticles needs to be very high for realizing nDEP (Zhang et al., 2010). In addition, the influence of conductivities on Re[CM] is negligible at operating frequencies higher than 1MHz and it is considered that at these high frequencies it is dependent only on the permittivity as shown in Equation (16).

$$\text{Re}[f_{CM}] = \frac{\epsilon_p - \epsilon_m}{\epsilon_p + 2\epsilon_m}$$

(16)

4.1 Equal applied voltages

Figure 3 presents the trajectory of micro-scale entity for several radii, and these trajectories are projected onto the horizontal and vertical planes as observed in the figure. The Figure 3 illustrates that the final steady-state positions (vertical and lateral positions) of the polystyrene microparticle is independent of its radius. The equilibrium position of the polystyrene microparticle in the direction of microchannel height is regulated by sedimentation force (combination of forces related to gravity and buoyancy) and the vertical components of the force associated with DEP. The equilibrium position along the height is where the sedimentation force equals the upward acting nDEP force. Since all of these forces acting on polystyrene particle at the steady-state vertical position have equal dependence on radius, it is clear that the same is independent of the radius. The equilibrium position along the width is where the horizontal component of nDEP force from the pair of electrode on one side of the microchannel is equal to the horizontal component of nDEP force from the electrode pair on opposite side. Since the electrodes are powered by equal applied voltages, the opposing horizontal components of nDEP force became equal on the vertical plane passing that through the geometric centre at the microchannel. In addition, it could be noticed that the trajectories of microparticles are influenced by the radius; the axial distance required by microparticles to attain the equilibrium position reduces with increase in radius of the same. During transient phase, the influence of radius is not nullified due to which the microparticles with large radius attains the steady-state position faster than microparticles with smaller radius.

Figure 4 shows the trajectory of polystyrene microparticles for various electrode protrusion widths. The steady-state vertical position increased with expands of electrode protrusion width while the steady-state lateral position remained same. In the proposed microdevice, Figure 1, the voltage inside the microchannel increases with the enlargement of electrode protrusion width and
this subsequently leads to increased nDEP forces inside the microchannel thereby leading to the observed rise in the steady-state vertical position. The steady-state lateral position remains persistent, in the vertical plane crossing the centre of the microchannel, irrespective of the electrode protrusion width. This is being induced by the minimum gradient of the electric field towards the y-direction which is always in the vertical plane that passes through the microchannel centre. Additionally, it is evident that the electrode’s protrusion width influences the trajectory. The enlargement of electrode protrusion width will reduce the axial distance travelled by microparticles before reaching its steady-state positions (vertical and horizontal). This imputes the increased nDEP force acting on the microparticle with the increase of electrode protrusion width.

An important parameter that has direct influence on the trajectory of polystyrene microparticle is applied voltages as visualized in Figure 5. The steady-state vertical position of the polystyrene microparticle increases with increase in applied voltages. The steady-state vertical position is where the upward and downward forces balance. The downward force is the sedimentation force while the upward force is the vertical nDEP force. With increase in applied voltages, the nDEP forces increased while the sedimentation force remains constant and this leads to the steady-state vertical position of polystyrene microparticle assuming a higher position along the microchannel height as observed in the Figure 5. As the net-nDEP force along the y-direction is zero only in the vertical plane crossing the microchannel centre for equal applied voltages, steady-state lateral position of polystyrene microparticle does not change with the increment of applied voltages. It is observed from Figure 5 that applied voltages influence the trajectory of microparticles in such a way that the axial distance demanded for reaching the steady-state position decreases with increase in applied voltages. This is because of the associated increase in nDEP forces with increase in applied voltages as can be inferred from Equation (1).

Changes in microchannel height influence the steady-state vertical position of micro-scale entities. It can be seen in Figure 6 that the steady-state vertical position increases with initial increase in microchannel height. This can be explained using the gradient of the vertical nDEP force with respect to the microchannel height. The gradient becomes steeper with the increment in microchannel height due which to the vertical nDEP force that is needed for balancing the sedimentation force is raised which leads to the increment in steady-state vertical position. Figure 7 shows the variation of the vertical nDEP force, in the vertical plane crossing through the microchannel centre, with microchannel heights. It is evident from Figure 6 is that there is a threshold value for microchannel height beyond which no increment in steady-state vertical position occurs. According to the fluctuation of the vertical nDEP force associated the microchannel height shown as in Figure 7, it is clear that the variation becomes primarily concentrated near the electrodes with increase in microchannel height, and additionally, this variation near the electrodes is independent of the microchannel height especially at high microchannel heights. These reasons lead to the stagnation of the steady-state vertical position when the height of the microchannel is increased above a certain value. Variation of the microchannel height will change the velocity of the medium and it in turn actuates the polystyrene microparticle’s trajectory because it is the fluid flow which drags the microparticle along the axial direction. The axial direction travelled by polystyrene microparticle to reach its steady-state position reduces with the increase in height of microchannel.

Additional parameters such as volumetric flow rate and initial positions influence the trajectory of polystyrene microparticle; however, they do not show any influence on the steady-state vertical and lateral positions. The supplementary material (Supporting Information Figures S1 and S2) provides the plots that represent these parameters influence on the trajectory and steady-state position. Volumetric flow rate influences the trajectory as it is the fluid flow which drags the microparticle along the axial direction. Thus, with increase of volumetric flow rate the axial distance travelled by a micro-scale entity before reaching its steady-state position increases. Unlike some other electrode configurations, the electrode configuration considered in this article does not have threshold value for volumetric flow rate below which it is not
achievable to levitate polystyrene microparticles. This is because the electrode configuration considered does not have the DEP force component along the axial location as the electrodes are continuous. Changes in the initial position alter the trajectory of polystyrene microparticles. Farther the micro-scale entity is from its steady-state position, longer the axial displacement travelled to reach the steady-state position. The initial position is not an influencing parameter of the trajectory of polystyrene microparticles as can be observed from Equation (7).

4.2 Unequal applied voltages

The behaviour of polystyrene microparticles subjected to unequal applied voltages is investigated in this section. Figure 8 shows the trajectory and steady-state positions of polystyrene microparticles, subjected to unequal applied voltages, for different radii. It is noticeable that the lateral position at which the polystyrene microparticles are focused is different from the centre of the microchannel; it is focused close to the pair of electrodes with the smaller applied voltage. This is due to the fact that the horizontal nDEP force from the electrode pair with smaller applied voltage is lower than the horizontal nDEP force of the electrode pair on the opposite side. By altering the difference between the applied voltages, it is possible to 3D focus micro-scale entities, including polystyrene microparticles, at any lateral position along the microchannel width.

From Figure 8, it is evident that even when the applied voltages are different, the steady-state position for the polystyrene micro-scale entity is independent of its radius. In addition, the trajectory of the microparticle depends on the radius of the same. The reason for these trends is same as that mentioned earlier in relation to equal applied voltages.

The influence of protrusion width of electrodes on the trajectory and steady-state positions of polystyrene microparticle is shown in Figure 9. It is observed that the enlargement of electrode protrusion width actsuates the movement of polystyrene microparticle faster to its steady-state position. The nDEP forces acting on the microparticle are observed to increase with the enlargement of electrode protrusion width thereby quickening its approach towards the steady-state position, and it is due to the voltage inside the microchannel increasing with increase of protrusion width of electrodes. It can be visualized from Figure 9 that the increase in protrusion width of electrode pairs does not alter the steady-state lateral distance of polystyrene microparticles. With the increment of electrode protrusion width, the voltage inside the microchannel increased which in turn increased the horizontal nDEP force component from both side that acting on
With increase in microchannel height, there is reduction in sedimentation force with increase in the variation between the applied voltages. This is due to the reduction in vertical component of the net-nDEP force with increase in the difference between the applied voltages. Regarding the steady-state vertical position, it rises with increase in protrusion width of electrode on the steady-state lateral position. Furthermore, increase in width of electrode protrusion increases the steady-state vertical position. As steady-state position is where the sedimentation force and the nDEP force’s vertical component balance, increase in the negative DEP force associated with the increase in protrusion width of electrodes, in turn increases the steady-state vertical position.

Figure 10 represents the variation of the trajectory of polystyrene microparticles with applied voltages. It is observed that with increase in the variation between the applied voltages, the steady-state lateral position moves near the electrode pair with the smaller electric voltage. With increase in the variation between the applied voltages, net-nDEP force’s horizontal component acting towards the electrode pair with the smaller voltage increases thereby shifting the steady-state lateral position of the microparticle towards the same. Regarding the steady-state vertical position of the microparticle, it reduces with increase in the difference between the applied voltages. This is due to the reduction in vertical component of the net-nDEP force with increase in the variation between the applied voltages.

The influence of microchannel height on the achievement of 3D focusing in the microdevice is presented in Figure 11. Evidently, increase in microchannel height leads to the steady-state lateral position being pushed near the electrode pair with the lower applied voltage. With increase in microchannel height, there is reduction in the electric fields associated with electrode pairs which in turn affects the horizontal nDEP force component. However, the reduction in horizontal component of the negative DEP force, due to extension of microchannel height, associated with the electrode pair with the higher applied voltage is smaller than the electrode pair with the lower applied voltage. These reasons move the steady-state lateral position of the microparticles to the proximity of the electrode pair with lower applied voltage. This shifting of the steady-state lateral position of the microparticle, towards the electrode pair with lower applied voltage resulting from the increase in microchannel height, ultimately takes it to the sidewall near the electrode pair with lower applied voltage. Thus, no focusing occurs when the microchannel height is greater than the height for which the steady-state lateral position is at the sidewall near to the electrode pair with lower applied voltage. Regarding the steady-state vertical position, it rises with increase in microchannel height. In the proximity of the electrode pair with the lower applied voltage, the vertical nDEP force experienced by the microparticles increases thereby leading to the observed increment in steady-state vertical position. Figure 11 plots the variation of the vertical nDEP force component on the vertical plane passing through the steady-state lateral position of the microparticle.

Even though the volumetric flow rate influences the trajectory of microparticles, it has little influence on the steady-state

**FIGURE 9** Influence of electrode protrusion width on trajectory for unequal applied voltages (a) \( V_1 = 3 V_{pp} \) and \( V_2 = 2 V_{pp} \), and (b) \( V_1 = 2 V_{pp} \) and \( V_2 = 3 V_{pp} \). \( \mu = 2 \mu m, 4 \mu m, 6 \mu m, 8 \mu m \); \( R_p = 3 \mu m, Q_m = 5 \mu l/hr, H_{ch} = 60 \mu m, W_{ch} = 60 \mu m, Y = 5 \mu m, Z = 10 \mu m \)

**FIGURE 10** Influence of applied voltages on trajectory for unequal applied voltages (a) \( V_1 \geq V_2 \); \( V_1 = 6 V_{pp} \) and \( V_2 = 1 V_{pp} \), \( V_1 = 6 V_{pp} \) and \( V_2 = 2 V_{pp} \), \( V_1 = 6 V_{pp} \) and \( V_2 = 3 V_{pp} \), \( V_1 = 6 V_{pp} \) and \( V_2 = 4 V_{pp} \), \( V_1 = 6 V_{pp} \) and \( V_2 = 5 V_{pp} \), \( V_1 = 6 V_{pp} \) and \( V_2 = 6 V_{pp} \) and (b) \( V_1 \neq V_2 \); \( V_1 = 1 V_{pp} \) and \( V_2 = 6 V_{pp} \), \( V_1 = 2 V_{pp} \) and \( V_2 = 6 V_{pp} \), \( V_1 = 3 V_{pp} \) and \( V_2 = 6 V_{pp} \), \( V_1 = 4 V_{pp} \) and \( V_2 = 6 V_{pp} \), \( V_1 = 5 V_{pp} \) and \( V_2 = 6 V_{pp} \), \( V_1 = 6 V_{pp} \) and \( V_2 = 6 V_{pp} \); \( R_p = 3 \mu m, d_w = 2 \mu m, Q_m = 5 \mu l/hr, H_{ch} = 60 \mu m, W_{ch} = 60 \mu m, Y = 5 \mu m, Z = 10 \mu m \)
positions; the supplementary material provided in Supporting Information Figure S3. The steady-state positions are not influenced by volumetric flow rate since the drag forces, along the vertical and horizontal directions, are non-existent at steady-state position. However, the volumetric flow rate influences the trajectory of microparticles since it is the fluid that carries the microparticle along the axial direction. It could be noticed that the axial displacement required for the microparticle to reach its steady-state position increases with increase in volumetric flow rate. Another parameter that is relevant with regard to focusing devices is initial positions. The initial positions do not influence the steady-state position; however, they influence the microparticle's trajectory as shown in Supporting Information Figure S4 of the supplementary material. The initial positions do not affect the steady-state position since the forces relevant at steady-state, that is DEP, gravity and buoyancy, are not dependent on initial positions. On the other hand, the initial positions influence the trajectory such that microparticles starting at positions far from the steady-state position require greater axial distance to reach the same (Figure 12).

5 | CONCLUSION

This study discusses the mathematical model of a microdevice for 3D focusing of microparticles based on dielectrophoresis; the device employs planar electrodes on the top and bottom of the microchannel. The model helps to predict the motion of micro-scale entities under the influence of external forces and recognize the influence of operating and geometric parameters on the performance of the microdevice. Based on the model the following, conclusions are made:

1. 3D focusing achieved with the proposed electrode geometry can be at any lateral position along the microchannel width. When the applied voltages are equal, the micro-scale entity is focused at the microchannel centre, and when the electric voltages applied are unequal, then the micro-scale entity will focus at positions other than the centre of the microchannel but near the electrode pair with smaller applied voltage.
2. The trajectory of micro-scale entity is dependent on all geometric and operating parameters such as micro-scale entity's radius, electrode protrusion width, microchannel height, initial positions, volumetric flow rate and applied voltages.
3. Steady-state vertical and lateral positions of the micro-scale entity are not dependent on the radius of the micro-scale entity, initial positions and volumetric flow rate for both equal and unequal applied voltages.
4. The steady-state vertical position of the micro-scale entity is dependent on applied voltages; it increases with increase in applied voltages. When the applied voltages are equal, there is no effect on the steady-state lateral position. On the other hand, when the applied voltages are unequal, it influences the steady-state lateral position.
5. Steady-state vertical position is influenced by microchannel height for both equal and unequal applied voltages. For equal applied voltages, the steady-state vertical position increases with increase in microchannel height before becoming independent of the same. In the case of unequal applied voltages, the steady-state vertical position increases with increase in microchannel height until the steady-state lateral position of the microparticle is on the sidewall near the electrode with lower applied voltage.
6. The steady-state vertical position of micro-scale entities is dependent on electrode protrusion widths for both equal and unequal applied voltages; however, the steady-state lateral position is
independent of the electrode protrusion width for both equal and unequal applied voltages.

**NOMENCLATURE**

| Symbol | Quantity |
|--------|----------|
| d      | electrode protrusion width (µm) |
| E      | RMS electric field (V/m) |
| H      | height (m) |
| m      | mass (kg) |
| p      | pressure (Pa) |
| Q      | volumetric flow rate (m³/s) |
| R      | radius (m) |
| v      | velocity (m/s) |
| V      | volume (m³) |
| W      | width (m) |
| x      | x-displacement (m) |
| y      | initial y-displacement (m or µm) |
| y'     | y-displacement (m) |
| z      | initial z-displacement (m or µm) |
| z'     | z-displacement (m) |

**GREEK ALPHABETS**

| Symbol | Quantity |
|--------|----------|
| Δt     | time step (s) |
| μ      | viscosity (Pa·s) |
| ρ      | density (kg/m³) |
| ϕ      | applied electric potential (V) |
| ε      | electrical permittivity (F/m) |
| σ      | electrical conductivity (S/m) |
| ω      | operating frequency (rad/s) |

**SUBSCRIPTS**

| Subscript | Description |
|-----------|-------------|
| eq        | equivalent |
| m         | medium      |
| p         | particle    |
| pp        | peak-to-peak |

**REFERENCES**

Chen, Y., Li, P., Huang, P.-H., Xie, Y., Mai, J. D., Wang, L., … Huang, T. J. (2014). Rare cell isolation and analysis in microfluidics. *Lab on a Chip*, 14, 626–645. https://doi.org/10.1039/C3LC90136J

Crowe, C. T., Sommerfeld, M., Tsuji, Y., & Schwarzkopf, J. D. (2011). *Multiphase flows with droplets and particles*. Boca Raton, FL: CRC Press.

Deen, W. M. (2011). *Analysis of transport phenomena* (2nd ed.). Oxford, UK: Oxford University Press.

Demierre, N., Braschler, T., Muller, R., & Renaud, P. (2008). Focusing and continuous separation of cells in a microfluidic device using lateral dielectrophoresis. *Sensors and Actuators B: Chemical*, 132, 388–396. https://doi.org/10.1016/j.snb.2007.09.078

Hilal-Alnaqbi, A., Alazzam, A., Dagher, S., & Mathew, B. (2017). Analysis of dielectrophoresis based 3D-focusing in microfluidic devices with planar electrodes. In 2017 39th Annual International Conference of the IEEE Engineering in Medicine and Biology Society (EMBC), Jeju Island, South Korea (pp. 3588–3591). https://doi.org/10.1109/EMBC.2017.8037633

Huang, Y., Wang, X.-B., Becker, F. F., & Gascoyne, P. (1997). Introducing dielectrophoresis as a new force field for field-flow fractionation. *Biophysical Journal*, 73, 1118–1129. https://doi.org/10.1016/S0006-3495(97)81444-X

Lin, C.-H., Lee, G.-B., Fu, L.-M., & Hwey, B.-H. (2006). Vertical focusing device utilizing dielectrophoretic force and its application on microflow cytometry. *Journal of Microelectromechanical Systems*, 13, 923–932. https://doi.org/10.1109/JMEMS.2004.838352

Morgan, H., Holmes, D., & Green, N. G. (2003). 3D focusing of nanoparticles in microfluidic channels. *IEEE Proceedings Nanobiotechnology*, 150, 76–80. https://doi.org/10.1049/ip-ntb:20031090

Morton, K. W., & Mayers, D. F. (2005). *Numerical solution of partial differential equations* (2nd ed.). Cambridge, UK: Cambridge University Press. https://doi.org/10.1017/CBO9780511812248

Pethig, R. (2010). Review Article—Dielectrophoresis: Status of the theory, technology, and applications. *Biomicrofluidics*, 4, art. no. 022811. https://doi.org/10.1063/1.3456626

Piyasena, M. E., & Graves, S. W. (2014). The intersection of flow cytometry with microfluidics and microfabrication. *Lab on a Chip*, 14, 1044–1059. https://doi.org/10.1039/C3LC51152A

Qian, C., Huang, H., Chen, L., Li, X., Ge, Z., Chen, T., … Sun, L. (2014). Dielectrophoresis for bioparticle manipulation. *International Journal of Molecular Sciences*, 15, 18281–18309. https://doi.org/10.3390/ijms151018281

Simonnet, C., & Groisman, A. (2005). Two-dimensional hydrodynamic focusing in a simple microfluidic device. *Applied Physics Letters*, 87, art. no. 114104. https://doi.org/10.1063/1.2046729

Wang, L., Flanagan, L., & Lee, A. P. (2007). Side-wall vertical electrodes for lateral field microfluidic applications. *Journal of Microelectromechanical Systems*, 16, 454–461. https://doi.org/10.1109/JMEMS.2006.889530

Xuan, X., Zhu, J., & Church, C. (2010). Particle focusing in microfluidic devices. *Microfluidics and Nanofluidics*, 9(2010), 1–16. https://doi.org/10.1007/s10404-010-0602-7

Yu, C., Vyvoukal, J., Vyvoukal, D. M., Schwartz, J., Shi, L., & Gascoyne, P. R. C. (2005). A three-dimensional dielectrophoretic particle focusing channel for microcytometry applications. *Journal of Microelectromechanical Systems*, 14, 480–487. https://doi.org/10.1109/JMEMS.2005.844839

Zhang, C., Khoshmanesh, K., Mitchell, A., & Kalantar-Zadeh, K. (2010). Dielectrophoresis for manipulation of micro/nano particles in microfluidic systems. *Analytical and Bioanalytical Chemistry*, 396, 401–420. https://doi.org/10.1007/s00216-009-2922-6

**SUPPORTING INFORMATION**

Additional supporting information may be found online in the Supporting Information section at the end of the article.

How to cite this article: Alnaimat F, Krishna S, Hilal-Alnaqbi A, Alazzam A, Dagher S, Mathew B. 3D focusing of micro-scale entities in dielectrophoretic microdevice. *Med Devices Sens*. 2019;2:e10028. https://doi.org/10.1002/mds3.10028