The charge asymmetry from Pomeron-Odderon interference in hard diffractive $\pi^+\pi^-$-electroproduction

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The interference of Pomeron and Odderon amplitudes gives rise to a charge asymmetry in the diffractive electroproduction of a $\pi^+\pi^-$-pair. We calculate this charge asymmetry in perturbative QCD (pQCD) in the Born approximation and on the leading $Q^2$-level. The numerical evaluation shows a sizeable asymmetry in an experimentally accessible kinematical region. We find a characteristic $m_{\pi^+\pi^-}$-dependence mainly dictated by the relevant Breit-Wigner-amplitudes and the corresponding phase-shifts.

Pomeron and Odderon exchanges in pQCD are to lowest order given by colour singlet exchanges in the $t$-channel with two and three gluons, respectively. While the Pomeron is described by the well known solutions of the Balitsky-Fadin-Kuraev-Lipatov (BFKL) equation \cite{2}, only in recent years progress in solving the corresponding Bartels-Kwiecinski-Praszalowicz (BKP) equation \cite{3} for the Odderon has been reported (see \cite{4} and \cite{5}).

Although the Pomeron and the Odderon exchange both theoretically induce dominant contributions to high energy hadronic cross sections, so far only the Pomeron exchange has been confirmed in the comparison of theory and experiment. Pure Odderon exchange, which e.g. has been considered in the case of diffractive $\eta_c$-meson photo- and electroproduction in the Born approximation in QCD, introduces rather small cross sections \cite{6,7}. However, the status of Odderon induced reactions is not settled. The inclusion of evolution following from the BKP equation \cite{8} leads to an increase of the predicted $\eta_c$-meson photoproduction cross section by one order of magnitude. Unfortunately recent experimental studies at HERA of exclusive $\pi^0$ photoproduction \cite{9} point towards a very small cross section, which confusingly stays in disagreement with theoretical predictions based on the stochastic vacuum model \cite{10}.

In order to push the hunt for the Odderon forward, it seems therefore to be highly valuable to study Odderon effects at the amplitude level by means of asymmetries, utilising the different (even and odd) charge conjugation properties of the Pomeron and the Odderon. Such asymmetries $A$ are approximately proportional to the ratio of the amplitude of the Odderon induced reaction $\mathcal{M}_O$ and the corresponding amplitude for the Pomeron ex-

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change $\mathcal{M}_\mathcal{P}$, $A \bar{\mathcal{M}}_\mathcal{O}/|\mathcal{M}_\mathcal{P}|$, in contrast to seemingly much smaller cross-section ratios $\mathcal{R}$ in pure Odderon mediated reactions which would behave like $\mathcal{R} \bar{\mathcal{M}}_\mathcal{O}/|\mathcal{M}_\mathcal{P}|^2$.

Brodsky et al. [11] suggested in this context to study the Odderon by the use of asymmetries in open charm production. Since the final state charm-anticharm system has no definite charge parity both Pomeron and Odderon exchanges contribute to this process, and the asymmetry allows to project out the pure interference contribution. Similarly Ivanov et al. [12] studied recently the charge asymmetry in the soft photoproduction of a $\pi^+\pi^-$-pair. In both approaches ([11] and [12]) the Pomeron and the Odderon are treated non-perturbatively.

We carry over the idea of Nikolaev et al. to the case of the charge asymmetry in diffractive $\pi^+\pi^-$-electroproduction to search for the QCD-Odderon at the amplitude level. In contrast to the previous works we investigate the process $\gamma^* + P \to \pi^+\pi^- + P$ at high energies squared $s$ but small momentum transfer squared $t$ and large photon virtuality squared $Q^2$. This permits us in the first place to apply the $k_\perp$-factorization, secondly to adopt the collinear approximation for the subprocess $q\bar{q} \to \pi^+\pi^-$ (with $q = u, d$) and thirdly to treat the Pomeron and the Odderon as two respectively three gluon exchanges within QCD perturbation theory. Since we limit ourself to leading contributions in $Q^2$, only a longitudinal virtual photon is considered. A scheme of our approximations is shown in fig. 1. The above explained approach allows us to write the corresponding amplitudes for the Pomeron and Odderon exchange in the following impact-parameter-representation

\begin{equation}
\mathcal{M}_\mathcal{P} = -i s \int \frac{d^2k_1 d^2k_2 \delta^{(2)}(k_1 + k_2 - \vec{p}_{\pi^+\pi^-})}{(2\pi)^2 k_1^2 k_2^2} J^{\pi^+\pi^-}P(\vec{k}_1, \vec{k}_2) \cdot J^{N \to N'}_N(\vec{k}_1, \vec{k}_2),
\end{equation}

\begin{equation}
\mathcal{M}_\mathcal{O} = \frac{8 \pi^2 s}{3!} \int \frac{d^2k_1 d^2k_2 d^2k_3 \delta^{(2)}(k_1 + k_2 + k_3 - \vec{p}_{\pi^+\pi^-})}{(2\pi)^6 k_1^2 k_2^2 k_3^2} J^{\pi^+\pi^-}_O \cdot J^{N \to N'}_O. \quad (2)
\end{equation}
The nucleon impact factors phase factors times the corresponding absolute values of the Breit-Wigner-amplitudes. The GDAs have to be modelled and in our case are proportional to the s, p QCD for isospin Ipling discussed in Ref. [8].

\[ J_{P-\pi^+\pi^-}^\gamma (k_1, k_2) = -i e g^2 \delta^{ab} Q \frac{2 N_C}{z^2 p_{\pi^+\pi^-}^2 + \mu^2} \int_{0}^{1} dz z \bar{z} P_P(k_1, k_2) \Phi^{I=1}(z, \zeta, m_{\pi^+\pi^-}^2), \tag{3} \]

\[ P_P(k_1, k_2) = \frac{1}{z^2 p_{\pi^+\pi^-}^2 + \mu^2} + \frac{1}{z^2 P_{\pi^+\pi^-}^2 + \mu^2} - \frac{1}{(k_1 - z \bar{p}_{\pi^+\pi^-})^2 + \mu^2} \]

for the Pomeron and

\[ J_{O-\pi^+\pi^-}^\gamma (k_1, k_2, k_3) = -i e g^2 d_{abc} Q \frac{4 N_C}{z^2 P_{\pi^+\pi^-}^2 + \mu^2} \int_{0}^{1} dz z \bar{z} P_O(k_1, k_2, k_3) \frac{1}{3} \Phi^{I=0}(z, \zeta, m_{\pi^+\pi^-}^2), \tag{4} \]

\[ P_O(k_1, k_2, k_3) = \frac{1}{z^2 P_{\pi^+\pi^-}^2 + \mu^2} - \frac{1}{z^2 P_{\pi^+\pi^-}^2 + \mu^2} - \frac{1}{(k_1 - z \bar{p}_{\pi^+\pi^-})^2 + \mu^2} \]

for the Odderon. The variable \( \mu \) is defined by \( \mu^2 = m_q^2 + z \bar{z} Q^2 \), where \( \bar{z} \equiv 1 - z \), \( m_q \) is the quark mass, and we put \( m_u \approx m_d \). The 2πGDAs \( \Phi^I(z, \zeta, m_{\pi^+\pi^-}^2) \) (see e.g. [13-15]) for isospin \( I = 0, 1 \) depend on the longitudinal momentum fraction \( z \), the invariant mass \( m_{\pi^+\pi^-} \) of the \( \pi^+\pi^- \)-system and the variable \( \zeta \), which is directly related to the polar decay angle \( \theta \) of the \( \pi^+ \) in the dipion rest frame by \( \beta \cos \theta = 2 \zeta - 1 \) and \( \beta \equiv \sqrt{1 - 4m_{\pi^+}^2/m_{\pi^+\pi^-}^2} \).

The GDAs have to be modelled and in our case are proportional to the s, p and d wave phase factors times the corresponding absolute values of the Breit-Wigner-amplitudes. The nucleon impact factors \( J_{P,O-\pi^+\pi^-}^N \) likewise cannot be rigorously calculated. The details of our choices for the functions \( \Phi^I(z, \zeta, m_{\pi^+\pi^-}^2) \) and \( J_{P,O-\pi^+\pi^-}^N \) can be found in [1] and the references therein. The charge asymmetry is finally defined by [12]

\[ A(Q^2, t, m_{\pi^+\pi^-}^2) = \frac{\int \cos \theta d\sigma(s, Q^2, t, m_{\pi^+\pi^-}^2, \theta) d\sigma(s, Q^2, t, m_{\pi^+\pi^-}^2, \theta)}{\int d\sigma(s, Q^2, t, m_{\pi^+\pi^-}^2, \theta)} = \frac{\int \cos \theta d\sigma \cos \theta 2\text{Re} \left[ M_P^\gamma (M_O^\gamma)^* \right]}{\int \cos \theta \left[ |M_P^\gamma|^2 + |M_O^\gamma|^2 \right]}. \]

Numerical results for the \( m_{\pi^+\pi^-}^2 \)-dependence of the charge asymmetry are shown in fig.2. The error-band results from a combined variation of \( \Lambda_{QCD} \) and the nonperturbative coupling \( \alpha_{soft} \) (representing the coupling of the gluons in the nucleon impact factor). The asymmetry is sizeable in the vicinity of the \( f_0 \) and the \( f_2 \)-resonance. The t-dependent plot in fig.3 shows a characteristic zero around \( |t| = 0.1 \text{ GeV}^2 \), which has been already discussed in Ref. 3.

We learnt at this conference that preliminary results for the charge asymmetry are available by now from the HERMES collaboration 4. Although the interference process

\[^{2}\text{Thanks to N.Bianchi, private communication}\]
at work at medium values of $x_{Bj}$ is different from our case \[16\], it is still quite surprising that there is no sign of the $f_0$-resonance in the preliminary data.

We plan to extend the present study to include transversely polarized photons for the charge asymmetry and to perform calculations of the single spin asymmetry induced by the Pomeron-Odderon-interference \[17\].

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