Brane intersections, anti-de Sitter spacetimes and dual superconformal theories

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Abstract

We construct a class of intersecting brane solutions with horizon geometries of the form $adS_k \times S^l \times S^m \times E^n$. We describe how all these solutions are connected through the addition of a wave and/or monopoles. All solutions exhibit supersymmetry enhancement near the horizon. Furthermore we argue that string theory on these spaces is dual to specific superconformal field theories in two dimensions whose symmetry algebra in all cases contains the large $\mathcal{N} = 4$ algebra $\mathcal{A}_4$. Implications for gauged supergravities are also discussed.
1 Introduction

During the last few years a new unifying picture of all string theories has emerged. All of them can be viewed as special limits of an eleven-dimensional theory, M-theory \( [1] \). M-theory at low energies is described by eleven-dimensional supergravity (11d SUGRA) \( [2] \). This has led to an extensive study of the latter. However, most of the studies were confined to toroidal compactifications of 11d SUGRA. One reason for this is that in this case the connection with string theories is rather direct. These compactifications lead to Poincaré supergravities in lower dimensions. From the point of view of M-theory, however, there is no \textit{a priori} reason that distinguishes toroidal compactifications from other ones. A large class of compactifications that received considerable attention in the mid-eighties are compactifications on spheres. These compactifications lead to (maximally supersymmetric) gauged supergravities in lower dimensions \( [3, 4, 5] \). The latter contain a negative cosmological constant in the action and in many cases admit a stable anti-de Sitter (adS) vacuum. A famous example is the compactification of eleven-dimensional supergravity on \( S^7 \), which yields \( N = 8 \) adS4 gauged supergravity \( [6] \).

A natural question is what is the rôle of gauged supergravities in M-theory. Since the asymptotic group associated to these theories is different from the one associated to Poincaré supergravities one may consider them as different sectors of M-theory. However, one of the lessons that our experience with dualities teaches us is that differently looking theories may actually be equivalent. Indeed, it has been known for some time that dualities can change the asymptotic geometry of spacetime \( [7] \). We have recently \( [8] \) provided a connection between Poincaré and gauged supergravities by giving a set of duality transformations (such duality transformations have also appeared in \( [9] \) and were recently also studied in \( [10] \) that map solutions of the former to solutions of the latter. In particular, we have shown that certain brane configurations and intersections thereof which are asymptotically flat are mapped by the so-called shift transformation \( [8] \) to solutions that are locally isometric to adS\(_k\) \( \times E^l \times S^m \), where adS\(_k\) is the \( k \)-dimensional anti-de Sitter space, \( E^l \) is the \( l \)-dimensional Euclidean space and \( S^m \) is the \( m \)-dimensional sphere. Based on these considerations, it was further suggested in \( [11] \) that the actual symmetry group of M-theory is bigger than what one usually assumes, and allows for connections between spacetimes with different asymptotic group and different number of (non-compact) dimensions. The fact that these considerations lead to a microscopic derivation of the Bekenstein-Hawking entropy formula for 4d and 5d non-extremal black holes and organize many results on black hole entropy in a unifying way \( [11] \) strongly supports this point of view.

The proposed new dualities are rather surprising from the spacetime point of view. In a given spacetime, the particle states carry unitary irreducible representations (UIRs) of the spacetime group. If the spacetime is asymptotically flat then the particle states carry UIRs of the Poincaré group. After the duality transformation the spacetime changes to one which is asymptotically anti-de Sitter. So, now the particle states should carry UIRs of the anti-de Sitter group. Therefore, a necessary condition for the duality to work is that there are multiplets of the anti-de Sitter group that can carry the original degrees of freedom. This is a very non-trivial condition since from the point of view of classical relativity asymptotically flat spacetimes are different from asymptotically anti-de Sitter spacetimes. It is string theory that makes such a connection possible.

To make the discussion concrete consider the case of a Dirichlet three-brane (D3) in IIB string theory. The duality transformation maps the supergravity solution describing the D3 brane to a space which is locally isometric to adS\(_5\) \( \times S^5 \). In the limit of decoupling gravity one is left with the D3 worldvolume fields. For a single D3 brane the latter belong to a \( U(1) \) \( N = 4 \) super Yang-Mills (SYM) multiplet. For the duality to work there should exist a UIR of the adS\(_5\) supergroup that contains precisely these.
fields. Indeed, it turns out that such a multiplet exists. It is the so-called doubleton multiplet of $adS_5$. The latter is the most fundamental multiplet of $adS_5$. All other UIRs can be constructed by a tensoring procedure. The doubleton multiplet appeared in the compactification of IIB supergravity on $S^5$. However, these degrees of freedom could be gauged away everywhere but in the boundary of $adS_5$. The anti-de Sitter group $SO(d - 1, 2)$ coincides with the conformal group in one dimension lower. It follows that the singleton field theory is a superconformal field theory. Similar remarks apply for the M-theory branes. The supergravity solution describing a membrane $M2$ (a fivebrane $M5$) is mapped to a spacetime which is locally isometric to $adS_4 \times S^7$ ($adS_7 \times S^4$), and furthermore the worldvolume fields of $M2$ ($M5$) are the same as the ones in the singleton (doubleton) multiplet of $adS_4$ ($adS_7$). (From now on, we shall not distinguish between singletons and doubletons. We shall call both singletons.) These facts have led to an association of branes to singletons.  

A puzzle arises, however, when one considers multiple coincident branes. In the case of D-branes there are new massless states that arise from strings that become massless when one moves the branes on top of each other. This leads to enhanced gauge symmetry $\mathcal{N} = 4$. In particular, the worldvolume theory of $N$ coincident $D3$ branes is an $SU(N) \mathcal{N} = 4$ SYM theory. The shift transformation though does not depend on the number of $D3$ branes. So, by the same argument as before one expects to find the $SU(N)$ degrees of freedom in the $adS$ side. However, as we argued in footnote 4 it is rather unclear whether non-abelian singletons exist. So these degrees of freedom should appear in a more subtle way.

In a parallel development Maldacena [21] (for earlier related work see [22]) conjectured that the large $N$ limit of $\mathcal{N} = 4$ $SU(N) \mathcal{N} = 4$ SYM is described by IIB supergravity on $adS_5 \times S^5$. This conjecture was further sharpened in [23, 24], where a precise correspondence between the conformal field theory and the supergravity was proposed. In particular, it was shown that the chiral operators of $\mathcal{N} = 4$ $SU(N) \mathcal{N} = 4$ SYM correspond to the Kaluza-Klein modes on $S^5$. This provides the resolution of the puzzle raised in the previous paragraph. Notice that the large $N$ limit is needed in our case in order to trust the spacetime picture. Similar remarks hold for the cases of $M2$ and $M5$. These developments have led to a series of papers [25-35].

Most of the recent papers were devoted to the study of $adS_4$, $adS_5$ and $adS_7$. However, in these cases the corresponding superconformal theories to which the string theory (in the anti-de Sitter background) is dual to are rather poorly understood. This prevents detailed quantitative tests. In this paper we shall formulate similar conjectures based on intersections of branes. The corresponding dual superconformal field theories are two-dimensional and depending on the particular intersection they may be chiral or not. For the intersections we discuss the near-horizon limits correspond to exact conformal field theories. This opens the possibility to perform a detailed and quantitative study of the conjectural relations between string theories on anti-de Sitter backgrounds and superconformal field theories.

The new set of solutions of 11d and 10d supergravities are of the form $adS_k \times S^l \times S^m \times E^n$, with $k, l, m = 2, 3$ and $n$ is such that the dimensions sum up to 10 or 11 depending on the solution. These solutions are actually exact solutions of string theory, i.e. there is a conformal field theory (CFT) associated to them. For $S^3$ this is an $SO(3)$ WZW model and for $adS_3$ an $SL(2,R)$ WZW model. The one associated with $S^3$ follows from the fact that $S^3$ is a $U(1)$ bundle over $S^2$ (Hopf fibration). The CFT associated with $adS_2$ follows in a similar way through an appropriate quotient of the $SL(2,R)$ WZW model [27]. The solutions that we construct have specific gauge fields and antisymmetric tensors

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4 Actually the doubleton multiplet consists of six real scalars, four complex spinors and a complex antisymmetric tensor. A tensor in five dimensions is dual to a vector. Since the tensor is pure gauge in the bulk, so is the dual vector field. So, at the end one obtains an $\mathcal{N} = 4$ $U(1)$ SYM multiplet on the boundary. As far as we know, it is not known how to combine antisymmetric tensor gauge invariance with Yang-Mills gauge invariance. Thus, it is not clear whether one can associate $\mathcal{N} = 4$ $SU(N)$ SYM theory with singletons.

5 Singletons were discovered by Dirac (for $adS_4$), and named singletons by Fronsdal [14].
needed in order to have a CFT. In some cases, however, one may T-dualize the solution to obtain a new one with the same spacetime geometry but field strengths which are not the canonical ones from the CFT point of view. This form of the solution is sometimes more natural from the M-theory point of view.

The developments in the last few years suggest that the basic solutions of 11d SUGRA, namely the membrane (M2), the fivebrane (M5), the wave (W) and the Kaluza-Klein monopole (KK) solution may form a “basis” in the space of all solutions of 11d SUGRA. I.e. one may eventually obtain all solutions by appropriate (extremal or non-extremal) intersections of the basic ones, dimensional reduction and use of dualities. In support of this supposition we shall present intersecting brane configurations that correspond to the new solutions. These intersections are obtained by appropriately combining solutions built according to the “standard” intersection rules[38, 39] with the solution of [40, 41]. In this way we obtain supersymmetric solutions that contain up to 6 different charges.

In all cases the intersection of branes interpolates between different stable vacua; Minkowski at infinity, $adS_k \times S^l \times S^m \times E^n$ close to all branes and $adS_k \times S^l \times E^{n+m}$, $adS_k \times S^m \times E^{n+l}$, close to some branes but far from the others. This is analogous to the case of M2, M5, D3, and NS5 studied in [17] and for “standard” intersections of branes studied in [8, 42].

By explicitly computing the Killing spinors we find that in all cases the near-horizon solution exhibits doubling of supersymmetry. Knowing how the Killing spinors transform under the isometry group allows us to determine the $adS$ supergroup that organizes the spectrum of the theory. The same supergroup can be interpreted as a conformal supergroup in one dimension lower. Based on these facts we argue that string theory on $adS_k \times S^l \times S^m \times E^n$ backgrounds is dual to superconformal field theories in two dimensions. For $k = 3$ we find (4,4) or (4,0) SCFTs. For $k = 2$ one would naively expect one-dimensional $\mathcal{N} = 8$ superconformal theories, but we argue that they are more properly viewed as Kaluza-Klein reductions of chiral SCFTs. In all cases the SCFT contains the $A_\gamma$ algebra. Our results are summarized in Table 3.

One may wonder whether one may use the shift transformation to reach these solutions. In order to perform the shift transformation one needs to dualize each brane to a wave. For this to be possible each brane should be wrapped on a torus. However, the sphere parts of the solutions come from the worldvolume part of certain branes. So, if we want to end up with a solution that contains spheres we should not wrap the brane on a torus. Spheres and also $adS$ spaces have non-abelian isometries. So, it may still be possible to use non-abelian dualities to obtain an appropriate version of the shift transformation.

There is a simple rule (which we shall call the wave/monopole rule) that leads to solutions with $adS_2$ and $S^2$’s once a solution with $adS_3$ and $S^3$’s is given. To get an $adS_2$ one adds a wave to $adS_3$, whereas to get an $S^2$ one adds a monopole to $S^3$. One may anticipate these rules by looking at the corresponding CFTs. To get an $S^2$ one views $S^3$ as a Hopf fibration over $S^2$. The monopole solution precisely supplies the needed $U(1)$ monopole gauge field. Similarly, for $adS_2$ one needs a $U(1)$ gauge field to support the $adS_2$ part. Putting a wave and reducing one obtains a $D0$ brane that precisely provides the required gauge field. The wave/monopole rule also allows for a determination of the killing spinors of all solutions once the killing spinors of the solutions containing only $adS_3$ and $S^3$’s are given.

We have organized the paper as follows. In the next section we present the wave/monopole rule. Using this rule we re-examine and organize the solutions with near-horizon geometry $adS_k \times S^l \times E^m$ studied in [8]. Then we present the new solutions. These contain two sphere factors. This seems to be the maximum number since one can only fit three 3-dimensional subspaces in 10 dimensions (recall that the CFT of $adS_2$ and $S^2$ comes in terms of the CFT of $adS_3$ and $S^3$, respectively). In section 3,
we analyze the supersymmetry enhancement of these configurations near the horizon. In particular, we explicitly work out the case of $\text{ads}_3 \times S^3 \times S^3 \times E^2$. We use these results in section 4 to argue that string theory on the background given by the new solutions is dual to certain superconformal theories. In section 5, we present a study of the supersymmetry near the horizon of $D$-brane solutions in arbitrary frames. We find that the dual $Dp$-frame, i.e. the frame in which the curvature and the $(8-p)$-form field strength appear in the action with the same power of the dilaton, represents a “threshold” frame for supersymmetry enhancement. In addition, in this frame the near-horizon geometry factorizes into the product $\text{ads}_{p+2} \times S^{8-p}$ for $p \neq 5$, whereas for $p = 5$ it becomes $\mathcal{M}_7 \times S^4$. In section 6, we use the new solutions to obtain several results about new vacua of gauged supergravities. We also point out the possibility of a new gauged supergravity in 5d. Section 7 contains our conclusions. Finally, in the Appendix we present a Kaluza-Klein ansatz inspired by the new solutions and we explicitly compute the Killing spinors of the $\text{ads}_2 \times S^2 \times S^2 \times E^5$ solution.

2 Intersections with anti-de Sitter near-horizon limits

In this section we describe special supersymmetric intersections of branes which in their near-horizon limit have a factorized geometry involving an anti-de Sitter spacetime and some other (compact) manifolds. Most of them are found as solutions to $D = 11$ supergravity but in some cases we have to consider $D = 10$ supergravity. In [8] such solutions based on standard intersection rules were listed. Here we also use the non-standard intersection rule which gives supersymmetric configurations in which the harmonic functions depend on relative transverse coordinates only. In $D = 11$ the three standard intersection rules are $(0|2 \perp 2)$, $(1|2 \perp 5)$ and $(3|5 \perp 5)$ [38, 39, 41], and the only non-standard rule is $(1|5 \perp 5)$ [40, 41]. The intersection rules in ten dimensions can be derived from these by dimensional reduction plus $T$ and $S$-duality. For concreteness we give the standard and non-standard intersection rules in the table below. References in which the intersection rules are derived from the equations of motion are [44] and [13] for standard and non-standard intersections, respectively.

| $D = 11$ | standard | non-standard |
|----------|----------|--------------|
| $(0|M2 \perp M2)$ | | |
| $(1|M2 \perp M5)$ | | |
| $(3|M5 \perp M5)$ | $(1|M5 \perp M5)$ | |
| $D = 10$ | $(\frac{1}{2}(p + q - 4)|Dp \perp Dq)$ | $(\frac{1}{2}(p + q - 8)|Dp \perp Dq)$ |
| $(1|F1 \perp NS5)$ | | |
| $(3|NS5 \perp NS5)$ | $(1|NS5 \perp NS5)$ | |
| $(0|F1 \perp Dp)$ | | |
| $(p - 1|NS5 \perp Dp)$ | $(p - 3|NS5 \perp Dp)$ | |

Table 1: Standard and non-standard intersections in ten and eleven dimensions.

Since we use both standard and non-standard intersection rules, one has to specify which coordinates each harmonic function depends on. In all cases this is clear by inspection of the intersection and it can be further verified by looking at the field equation(s) for the antisymmetric tensor field(s).

6 By standard intersection rules we mean the rules which give supersymmetric configurations in which the harmonic functions depend only on overall transverse coordinates.

7 What we call a non-standard intersection of two branes is often called an overlap, see [43] for a motivation of this nomenclature.

8 The notation $(q|p_1 \perp p_2)$ denotes a $p_1$-brane intersecting with a $p_2$-brane over a $q$-brane.
2.1 Standard intersections

There are three single \( p \)-branes with a near-horizon geometry \( \text{adS}_{p+2} \times S^{D-p-2} \): the \( M2 \), \( M5 \) and \( D3 \) branes. We now recall the solutions based on the standard intersections and show that they are related by a simple rule which we will call the wave/monopole rule.

| \( \) | \( M2 \perp M5 \) | \( \text{adS}_3 \times E^5 \times S^3 \) |
| \( \) | \( M2 \perp M2 \perp M2 \) | \( \text{adS}_2 \times E^6 \times S^3 \) |
| \( \) | \( M5 \perp M5 \perp M5 \) | \( \text{adS}_3 \times E^6 \times S^2 \) |
| \( \) | \( M2 \perp M2 \perp M5 \perp M5 \perp M5 \) | \( \text{adS}_2 \times E^7 \times S^2 \) |

Table 2: Standard intersection of \( M \) branes with anti-de Sitter near-horizon geometries.

Table 2 shows the intersections and their near-horizon geometries \([42, 8]\). In \([8]\) it was further demonstrated that the near-horizon limits of these intersections exhibit supersymmetry doubling. Thus, \( M2 \perp M5 \) preserves \( 1/4 \) and its near-horizon limit \( 1/2 \) of supersymmetry. The other three intersections have supersymmetry enhancement from \( 1/8 \) to \( 1/4 \).

Roughly speaking, the wave/monopole rule asserts that starting from the solution with near-horizon geometry containing \( \text{adS}_3 \) and \( S^3 \) factors (the \( M2 \perp M5 \) intersection in table 2), one can get all the others simply by adding a wave and/or monopoles. A wave effectively replaces \( \text{adS}_3 \) by \( \text{adS}_2 \) and a monopole \( S^3 \) by \( S^2 \). Let us see in some detail how this works for the wave. The wave solution is given, in suitable coordinates (light-cone coordinates), by the metric

\[
d s^2 = K d x^2 + 2 d x d t + d y_i d y_i, \tag{1}
\]

where \( K = K(y_i) \) is a harmonic function in the transverse space. One can add such a wave in the direction of a common string in the intersection (in these cases there is an \( \text{adS}_3 \) factor in the near-horizon geometry). For \( M2 \perp M5 \) plus a wave the metric is\([9]\)

\[
d s^2 = H_2^{-\frac{2}{5}} H_5^{-\frac{1}{5}} (K d x^2 + 2 d x d t) + H_2^{-\frac{2}{5}} H_5^{\frac{2}{5}} (d x_2^2) + H_2^{\frac{1}{5}} H_5^{-\frac{4}{5}} (d x_3^2 + \cdots + d x_6^2) + H_2^{\frac{2}{5}} H_5^{\frac{3}{5}} (d r^2 + r^2 d \Omega_3^2), \tag{2}
\]

where \( d \Omega_3 \) is the line element on the unit three-sphere. The harmonic functions are \( H_{2,5} = 1 + \frac{Q_{2,5}}{r^2} \) and \( K = \frac{Q_{2,5}}{r^2} \). Note that adding a constant to \( K \) amounts to a coordinate transformation \( t \to t + \alpha x \). In the near-horizon limit (or after applying the shift transformation) the constants in the harmonic functions drop out and we obtain

\[
d s^2 = d x^2 + d \rho^2 + 2 e^{2\rho} d x d t + d s_{E^5}^2 + d \Omega_3^2, \tag{3}
\]

where for convenience we have set \( Q_2 = Q_5 = Q_6 = 1 \), and we have transformed to a new radial coordinate \( \rho = \log r \). Thus the metric splits up into a five-dimensional flat space, a three-sphere and the first three terms which constitute three-dimensional anti-de Sitter spacetime\([10]\). This is exactly the same near-horizon geometry as that of the \( M2 \perp M5 \) intersection without a wave. Adding the wave takes us to a different form of the \( \text{adS}_3 \) metric (provided that no global identifications are made). To see this let us make the coordinate transformation\([9]\)

\[
x = x' - t', \quad t = \frac{1}{2} (x' + t'), \quad e^{2\rho} + 1 = r'^2. \tag{4}
\]

\(^9\)This form of the \( \text{adS}_3 \) metric was also used in \([40]\) (and also appeared in \([14]\)) with the same idea of reducing it to \( \text{adS}_2 \).

\(^{10}\)A similar coordinate transformation was considered in \([10]\).
This brings \( R \) to the form

\[
ds^2 = -\frac{(r^2 - 1)^2}{r^2} dt'^2 + \frac{r^2}{(r^2 - 1)^2} dr'^2 + r^2(dx'^2 - \frac{1}{r^2} dt'^2) \tag{5}\]

This is the standard form of a massive extremal BTZ black hole with mass and angular momentum \( M = J = 2 \) in a space of cosmological constant \( \Lambda = -1/l^2 \) equal to \(-1\). These values simply reflect the fact that we have set all charges equal to 1 in \( R \). Notice, however, that the coordinate \( x' \) in \( R \) is not periodic. It is well-known that the BTZ black hole is locally isometric to \( \text{adS}_3 \), i.e. there is a coordinate transformation that brings \( R \) to the anti-de Sitter metric. For non-extremal BTZ black holes this transformation is easy to write down. In the extremal case the situation is more complicated and one needs to consider an infinite number of patches that cover the black hole spacetime (see section 3.2.4). Nevertheless this shows that \( R \) with non-periodic coordinate \( x' \) describes anti-de Sitter spacetime. Therefore, so does \( R \). Note also that the wave does not contribute to the antisymmetric field strength. Thus the near-horizon limit of \( M_2 \perp M_5 + W \) is the same as that of \( M_2 \perp M_5 \). We conclude then also that the supersymmetry of \( M_2 \perp M_5 + W \) (1/8) is enhanced to 1/2. If we dimensionally reduce the metric \( R \) on \( x \), we get \( \text{adS}_2 \) (times \( E^6 \times S^3 \)) with a covariantly constant two-form Kaluza-Klein field strength, and zero dilaton. This type IIA solution is the near-horizon limit of a \( D0 \perp F1 \perp D4 \) intersection. It preserves 1/4 of supersymmetry since only half of the \( \text{adS}_3 \) Killing spinors survive the reduction. This can be seen from the explicit form of the Killing spinors as given in [50]. The surviving \( \text{adS}_2 \) Killing spinors in the resulting coordinate frame are the ones given in [51].

The \( D0 \perp F1 \perp D4 \) configuration can be lifted to a \( D = 11 \) solution built only out of two and five-branes after first \( T \)-dualizing along two relative transverse directions parallel to the \( D4 \) brane. This yields \( M_2 \perp M_2 \perp M_2 \) with near-horizon geometry \( \text{adS}_2 \times E^6 \times S^3 \). In the same way, \( M_5 \perp M_5 \perp M_5 \) can be transformed to \( M_2 \perp M_2 \perp M_5 \perp M_5 \) by adding a wave.

To go from an \( S^3 \) to an \( S^2 \) one adds a monopole. This is based on the fact that \( S^3 \) can be written as a \( U(1) \) bundle over \( S^2 \) (Hopf fibration). The \( U(1) \) field is precisely the monopole field, explaining why we need a monopole in order to go from a solution involving an \( S^3 \) to a solution involving an \( S^2 \). Reduction or \( T \)-duality along the “Hopf isometry” of \( S^3 \) yields \( S^2 \) or \( S^2 \times S^1 \), respectively. Similar procedures were also described in the recent papers [33, 52]. To illustrate the mechanism we consider again the \( M_2 \perp M_5 \) intersection and add a monopole. This means that we replace the overall transverse space by a (single-center) euclidean Taub-NUT space (see [53] p.363). The solution is given by

\[
ds^2 = H_2^{\frac{7}{4}} H_5^{-\frac{1}{4}} (-dt^2 + dx_2^2) + H_2^{\frac{7}{4}} H_5^{-\frac{1}{4}} (dx_3^2) + H_2^{\frac{1}{2}} H_5^{-\frac{1}{4}} (dx_5^2 + \cdots + dx_8^2) + H_2^4 H_5^4 \left[ (d\psi + Q_M \cos \theta d\phi)^2 + H_M (dr^2 + r^2 d\Omega_2^2) \right]. \tag{6}\]

where \( \psi \) is a periodic coordinate with period \( 4\pi Q_M \), \( (\theta, \phi) \) are coordinates on \( S^2 \) and

\[
H_M = 1 + \frac{Q_M}{r}, \quad H_2,5 = 1 + \frac{Q_{2,5}}{r}. \tag{7}\]

For small \( r \) one may neglect the one in the various harmonic functions. The result for the metric is (putting all charges but \( Q_M \) to one)

\[
ds^2 = ds^2_{\text{adS}_3} + ds^2_{\text{E}^6} + Q_M [(d\psi' + \cos \theta d\phi)^2 + d\Omega_2^2]. \tag{8}\]

where \( \psi' \) has period \( 4\pi \). This still represents \( \text{adS}_3 \times E^6 \times S^3 \), since the two terms in the square brackets constitute \( S^3 \) in the coordinate system corresponding to the Hopf fibration. So again in the near-horizon limit nothing has changed by adding a monopole. Reducing along \( \psi' \), one obtains the near-horizon limit of \( D2 \perp NS5 \) lying inside a \( D6 \) brane. Dualizing with respect to two relative transverse coordinates of \( D2 \) and \( D6 \) one obtains a configuration of one \( NS5 \) brane and two \( D4 \) branes each of them intersecting
the NS5 brane on a three brane. This configuration can be uplifted to the standard $M5 \perp M5 \perp M5$ in 11$d$. The near-horizon geometry of the latter is $adS_3 \times E^6 \times S^2$. In the same way the $M2 \perp M2 \perp M2$ solution may be transformed to $M2 \perp M2 \perp M5 \perp M5$ by adding a monopole.

The wave/monopole rule for the standard intersections is summarized in the figure below.

Figure 1: The wave/monopole rule for standard intersections. Starting from solution (a) one can obtain solutions (b), (c), (d) by adding a wave and/or a monopole. The lower-case letters in brackets refer to the four standard intersections listed in table 2.

### 2.2 Non-standard intersections

We now turn to intersections with near-horizon geometries of the form $adS_k \times S^l \times S^m \times E^n$ with $k, l, m$ equal to 2 or 3. As we will see, all such configurations involve at least one pair of branes intersecting in the non-standard way, and therefore the harmonic functions will depend on relative transverse coordinates. It was shown in [46] that the near-horizon limit of the intersection of two ten-dimensional NS five-branes over a line has geometry $M_4 \times S^3 \times S^3$, covariantly constant field strength and a linear dilaton. Addition of a fundamental string along the common direction of the five-branes yields a solution with near-horizon geometry $adS_3 \times S^3 \times S^3 \times E^1$ and constant dilaton, provided the harmonic function of the string is the product of the five-brane harmonic functions [46]. In $D = 11$ the analogous solution is the intersection of two $M5$ branes over a line, plus a membrane:

\[
\begin{array}{cccccc}
M5_1 & 1 & 3 & 4 & 5 & 6 \\
M5_2 & 1 & 7 & 8 & 9 & 10 \\
M2 & 1 & 2
\end{array}
\]

where $H_F^{(1)}(x_7, x_8, x_9, x_{10}), H_F^{(2)}(x_3, x_4, x_5, x_6), H_T = H_F^{(1)} H_F^{(2)}$. (See [46] below for the explicit solution in terms of the metric and antisymmetric tensor field.) This solution preserves 1/4 of supersymmetry. The easiest way to check that one can take for the membrane harmonic function the product of

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The wave/monopole rule for the standard intersections is summarized in the figure below.

- $adS_3 \times S^3$ (a) # charges
- $adS_2 \times S^3$ (b)
- $adS_3 \times S^2$ (c)
- $adS_2 \times S^2$ (d)

Figure 1: The wave/monopole rule for standard intersections. Starting from solution (a) one can obtain solutions (b), (c), (d) by adding a wave and/or a monopole. The lower-case letters in brackets refer to the four standard intersections listed in table 2.

### 2.2 Non-standard intersections

We now turn to intersections with near-horizon geometries of the form $adS_k \times S^l \times S^m \times E^n$ with $k, l, m$ equal to 2 or 3. As we will see, all such configurations involve at least one pair of branes intersecting in the non-standard way, and therefore the harmonic functions will depend on relative transverse coordinates. It was shown in [46] that the near-horizon limit of the intersection of two ten-dimensional NS five-branes over a line has geometry $M_4 \times S^3 \times S^3$, covariantly constant field strength and a linear dilaton. Addition of a fundamental string along the common direction of the five-branes yields a solution with near-horizon geometry $adS_3 \times S^3 \times S^3 \times E^1$ and constant dilaton, provided the harmonic function of the string is the product of the five-brane harmonic functions [46]. In $D = 11$ the analogous solution is the intersection of two $M5$ branes over a line, plus a membrane:

\[
\begin{array}{cccccc}
M5_1 & 1 & 3 & 4 & 5 & 6 \\
M5_2 & 1 & 7 & 8 & 9 & 10 \\
M2 & 1 & 2
\end{array}
\]

where $H_F^{(1)}(x_7, x_8, x_9, x_{10}), H_F^{(2)}(x_3, x_4, x_5, x_6), H_T = H_F^{(1)} H_F^{(2)}$. (See [46] below for the explicit solution in terms of the metric and antisymmetric tensor field.) This solution preserves 1/4 of supersymmetry. The easiest way to check that one can take for the membrane harmonic function the product of

---

By near-horizon limit we mean here the asymptotic configuration in the region near both five-branes.
the five-brane harmonic functions is to consider the field equation for the antisymmetric tensor field, \( \partial_M(\sqrt{-g}F^{MNPQ}) = 0 \). Substituting the metric associated to (A) according to the harmonic function rule, one obtains (for \( N,P,Q \) in the membrane directions):

\[
\left( H^{(1)}_F(x') \partial^2_{x'} + H^{(2)}_F(x) \partial^2_{x} \right) H_T(x,x') = 0 ,
\]

where \( x \) denotes coordinates \( x_3, \ldots x_6 \) and \( x' \) denotes \( x_7, \ldots x_{10} \). Notice that (9) is the Laplace equation in the curved transverse space produced by the five-branes. Clearly, \( H_T = H^{(1)}_F H^{(2)}_F \) is a solution. What is essential though for obtaining the anti-de Sitter product geometry is that the near-horizon limit of \( H_T \) behaves as \( 1/r^2 \), so one could take a more general solution of (9) with that property. For example, \( H_T = 1 + \frac{Q}{r^2} \) would represent a membrane intersecting on a string localized within both fivebranes.

More generally, one could take \( H_T = 1 + \frac{Q^{(1)}}{r^2} + \frac{Q^{(2)}}{r^2} + \frac{Q^{(3)}}{r^2} \). In the remaining of this article we shall, for simplicity, use \( H_T = H^{(1)}_F H^{(2)}_F \). The near-horizon limit of (A) has geometry \( adS_3 \times S^3 \times S^3 \times E^2 \) and preserves 1/2 of supersymmetry as we will show in the next section. So there is supersymmetry doubling also in this case.

Now consider intersection (A) with harmonic functions \( H^{(1)}_F = 1 + \frac{Q_1}{r^2} \), \( H^{(2)}_F = 1 + \frac{Q_2}{r^2} \) and \( H_T = H^{(1)}_F H^{(2)}_F \) where \( r^2 = x_3^2 + x_4^2 + x_5^2 + x_6^2 \) and \( r'^2 = x_7^2 + x_8^2 + x_9^2 + x_{10}^2 \). We can consider various limits of this solution. First, if we go very far from one of the fivebranes, \( r \to \infty (r' \to \infty) \), we recover the standard intersection (a), i.e. \( M2 \perp M^3 \). If we go near the horizon in the latter intersection, \( r' \to 0 (r \to 0) \), we find again the near-horizon geometry \( adS_3 \times E^5 \times S^3 \). We describe in detail the interpolating structure of solution (A) in figure 2.

![Figure 2: Interpolating structure of solution (A). Keeping one of the radial coordinates fixed while the other tends to infinity we recover solution (a), which itself interpolates between \( adS_3 \times E^5 \times S^3 \) and Minkowski. In addition, in the limit \( r \) and \( r' \) go to zero, (A) approaches \( adS_3 \times E^2 \times S^3 \times S^3 \). The horizontal and vertical axes correspond to a solution ((A) with the 1 removed from one of the harmonic functions) which interpolates between the supersymmetric vacua with geometries \( adS_3 \times E^2 \times S^3 \times S^3 \) and \( adS_3 \times E^5 \times S^3 \). The subscripts denote the fractions of unbroken supersymmetry.](image)

We can now use the wave/monopole rule to find other solutions with near-horizon geometries where some or all of \( adS_3 \times S^3 \times S^3 \) are replaced by their two-dimensional versions. The scheme is as follows:

\[Q_2 = 0 \ (Q_1 = 0)\]
Figure 3: The wave/monopole rule for non-standard intersections. Starting from solution (A) one can obtain solutions (B), (C), (D), (E), (F) by adding a wave and/or monopole(s). The upper-case letters correspond to the solutions given in the text.

They correspond to the following intersections. Adding a wave to solution (A) and dimensionally reducing to type IIA supergravity in the same way as described before for $M2 \perp M5$, one finds

\[
\begin{align*}
D0 & \\
D4_1 & 2 \; 3 \; 4 \; 5 \\
D4_2 & 6 \; 7 \; 8 \; 9 \\
F1 & 1
\end{align*}
\]

where $H_f = H_0 = H_4^{(1)} H_4^{(2)}$, $H_4^{(1)}(x_6, x_7, x_8, x_9)$ and $H_4^{(2)}(x_2, x_3, x_4, x_5)$\(^{14}\). This solution is 1/8 supersymmetric as follows from the set of supersymmetry projection operators associated to the various branes in the configuration. Its near-horizon limit has geometry $adS_3 \times S^3 \times S^3 \times E^2$ and preserves 1/4 of supersymmetry. The dilaton is a constant (depending on the charges) in this limit. For illustrative purposes we shall for this case write down explicitly the solution and its near-horizon limit. According to the harmonic function rule we get

\[
ds^2 = H_f^{-1} (H_0 H_4^{(1)} H_4^{(2)})^{-\frac{1}{2}} (-dt^2) + H_f^{-1} (H_0 H_4^{(1)} H_4^{(2)})^{-\frac{1}{2}} dx_6^2 \\
+ H_0^{\frac{1}{2}} H_4^{(1)} H_4^{(2)} (dx_2^2 + \cdots + dx_5^2) + H_0^{\frac{1}{2}} H_4^{(1)} H_4^{(2)} (dx_2^2 + \cdots + dx_5^2),
\]

\(^{14}\)This schematic way of writing the harmonic functions is only intended to give the dependence on the coordinates, and one may take different charges for all harmonic functions.
where \( I \) runs over all \( m \in \{2, 3, 4, 5\} \) and \( m' \in \{6, 7, 8, 9\} \). The harmonic functions are given in terms of \( H_4^{(1)} = 1 + \frac{Q_2}{r^2} \) and \( H_4^{(2)} = 1 + \frac{Q_1}{r^2} \) with \( r^2 = x_1^2 + x_2^2 + x_3^2 + x_4^2 \) and \( r'^2 = x_5^2 + x_6^2 + x_7^2 + x_8^2 \). In the near-horizon limit \( r \to 0 \) and \( r' \to 0 \) the configuration becomes

\[
\begin{align*}
ds^2 &= (Q_1 Q_2)^{-2} (r r')^{-2} (-dt^2 + dx_1^2 + \frac{Q_2}{r^2} (dr^2 + r^2 d\Omega_3^2) + \frac{Q_1}{r'^2} (dr'^2 + r'^2 d\Omega_3^2)) \\
&= e^{-4A} (-dt^2 + d\rho^2 + d\lambda^2 + dx_1^2 + Q_2 d\Omega_3^2 + Q_1 d\Omega_3^2),
\end{align*}
\]

where in the second line the coordinate transformation

\[
\rho = -A^{-1} \log \frac{r r'}{\sqrt{Q_1 Q_2}}, \quad \lambda = A^{-1} \left[ \sqrt{Q_2} \log r - \sqrt{Q_1} \log r' \right], \quad A = \left( \frac{1}{Q_1} + \frac{1}{Q_2} \right) \frac{1}{4} \tag{12}
\]

has been used. The resulting metric describes \( adS_2 \times E^2 \times S^3 \times S^3 \). The \((t, \rho)\) part of the metric is the standard form of the \( adS_2 \) metric in horospherical coordinates. The negative cosmological constant (the “radius” \( R \) of \( adS_3 \)) is equal to \( \Lambda = -R^2 = -Q_1 Q_2 / 4(Q_1 + Q_2) \). The dilaton vanishes in the limit, \( e^{-2\phi} = 1 \). The field strengths become

\[
H_{01\rho} = F_{0\rho} = -2A \epsilon_{0\rho}, \quad F_{1\alpha'\beta'\gamma'} = -2Q_1 \epsilon_{\alpha'\beta'\gamma'}, \quad F_{1\alpha\beta\gamma} = -2Q_2 \epsilon_{\alpha\beta\gamma}, \tag{13}
\]

where \( \epsilon_{0\rho}, \epsilon_{\alpha\beta\gamma} \) and \( \epsilon_{\alpha'\beta'\gamma'} \) are volume forms on \( adS_2 \), \( S^3 \) and \( S^3 \), respectively. (We use the convention that the epsilon symbols with tangent space indices are constants.) The covariantly constant field strengths in (13) support \( adS_2 \) and the two \( S^3 \)'s, respectively. Each of the factors of the geometry together with the corresponding field strengths represents a CFT, as discussed in the introduction.

**Figure 4: Interpolating structure of solution (B).**

The interpolation diagram for intersection (B) is shown in figure 4. It is now understood that (b) corresponds to the standard intersection \( D0 \perp D4 \perp F1 \) obtained from \( M2 \perp M2 \perp M2 \) by a dimensional reduction and two T-dualities. Since the reduction and T-dualities are along relative
transverse directions, one still has the adS near-horizon geometry and the same number of unbroken supersymmetries.

The next solution is found by adding instead of a wave a monopole to solution (A) and reducing to $D = 10$, in the same way as illustrated before for $M2 \perp M5$. Notice, however, that one of the $M5$ branes lies within the worldvolume directions of the additional monopole and the other intersects on a string with it. The latter is a non-standard intersection of a monopole and a five-brane. These and other intersection rules including waves and monopoles are described in [54]. The result in ten dimensions is

\[
\begin{array}{cccccccc}
D6 & 1 & 5 & 6 & 7 & 8 & 9 \\
D4 & 1 & 2 & 3 & 4 \\
NS5 & 1 & 5 & 6 & 7 & 8 \\
D2 & 1 & 9 \\
\end{array}
\]

where $H_6 = H_5(x_2, x_3, x_4)$ and $H_2 = H_5 H_4, H_4(x_5, x_6, x_7, x_8)$. This solution is 1/8 supersymmetric. The near-horizon limit has geometry $adS_3 \times S^2 \times S^3 \times E^2$ and preserves 1/4 of supersymmetry. The dilaton is a constant in the limit. The interpolating structure of intersection (C) is as follows ($r^2 = x_2^2 + x_3^2 + x_4^2$ and $r'^2 = x_5^2 + x_6^2 + x_7^2 + x_8^2$):

\[
(\text{adS}_3 \times E^5 \times S^2)_{1/4} \quad (c)_{1/8} \quad (M^{10})_1
\]

\[
(\text{adS}_3 \times E^2 \times S^2 \times S^3)_{1/4} \quad (a)_{1/4}
\]

\[
(\text{adS}_3 \times E^4 \times S^3)_{1/2}
\]

Figure 5: Interpolating structure of solution (C). Notice that in this case one obtains different standard intersections ((a) and (c)) depending on which radial coordinate is sent to infinity.

The type IIA solutions (B) and (C) cannot be lifted to $D = 11$ configurations with the same near-horizon geometry. In the standard intersections this was always possible, but here there is not enough freedom to dualize away the Kaluza-Klein two-form gauge field without changing the horizon geometry.

Whereas the four-charge configurations above are type IIA solutions, we find that the five-charge configurations, if we want to write them in terms of NSNS and/or RR branes only (without Kaluza-Klein monopoles or wave), such that the near-horizon geometries are as indicated in the scheme above, can only be found as solutions of type IIB supergravity. Adding a monopole to intersection (B) and
applying $T$-duality along the extra isometry direction of the monopole we find:

\[
\begin{array}{llllll}
D1 & 1 & & & & \\
D5 & 1 & 5 & 6 & 7 & 8 \\
F1 & & & 9 & & (D) \\
NS5 & 5 & 6 & 7 & 8 & 9 \\
D3 & 2 & 3 & 4 & & \\
\end{array}
\]

where $H_f = H_1 = H_3 H_5$ and $H_{s5} = H_5$, $H_3(x_5,x_6,x_7,x_8)$, $H_5(x_2,x_3,x_4)$. This solution can also be obtained by adding a wave to intersection (C) and doing the $T$-duality along the wave direction $x$ as in \(^{[3]}\). It is 1/8 supersymmetric, and the near-horizon limit has geometry $adS_2 \times S^2 \times S^3 \times E^3$ and preserves 1/4 of supersymmetry. Moreover, the dilaton is constant in this limit. The interpolating structure of intersection (D) is as follows ($r^2 = x_2^2 + x_3^2 + x_4^2$ and $r'^2 = x_5^2 + x_6^2 + x_7^2 + x_8^2$):

\[
\begin{array}{ccc}
(adS_2 \times E^6 \times S^2)_{1/4} & (d)_{1/8} & (M^{10})_{1/4} \\
D_{1/8} & (D)_{1/8} & (b)_{1/8} \\
(adS_2 \times E^3 \times S^2 \times S^3)_{1/4} & r \end{array}
\]

Figure 6: Interpolating structure of solution (D). In this case, similar to solution (C), one obtains different standard intersections ((b) and (d)) depending on which radial coordinate is sent to infinity.

Adding a monopole to (C) and $T$-dualizing one gets the type IIB intersection

\[
\begin{array}{llllllll}
D5_1 & 1 & & & 6 & 7 & 8 & 9 \\
D5_2 & 1 & 2 & 3 & 4 & 5 & & \\
NS5_1 & 1 & 5 & 6 & 7 & 8 & & (E) \\
NS5_2 & 1 & 2 & 3 & 4 & & 9 \\
D3 & 1 & & 5 & & 9 \\
\end{array}
\]

where $H_5^{(1)} = H_{s5}^{(1)}(x_2,x_3,x_4)$, $H_5^{(2)} = H_{s5}^{(2)}(x_6,x_7,x_8)$, $H_4 = H_5^{(1)} H_5^{(2)}$. It is 1/8 supersymmetric. Its near-horizon limit has geometry $adS_3 \times S^2 \times S^2 \times E^3$ and preserves 1/4 of supersymmetry. The dilaton is a constant in this limit. Note also that (D) and (E) are self-dual with respect to type IIB $S$-duality. The interpolating structure of intersection (E) is given in figure 7 ($r^2 = x_2^2 + x_3^2 + x_4^2$ and $r'^2 = x_5^2 + x_6^2 + x_7^2 + x_8^2$).

Finally, one can add a wave to (E) and $T$-dualize or add a monopole to (D) and $T$-dualize to obtain the solution with near-horizon geometry $adS_2 \times S^2 \times S^2 \times E^4$. Here there is again enough freedom to $T$-dualize in certain relative transverse directions (corresponding to flat directions in the near-horizon limit) such that we can uplift the solution to an intersection of $M2$ and $M5$ branes with the same

\[\text{See also } [33] \text{ for a recent application of } T\text{-dualities along isometries of odd-dimensional spheres.}\]
horizon geometry. The resulting eleven-dimensional intersection has already appeared in \cite{55} and the configuration is as follows:

\[
\begin{array}{cccccccc}
M_{21} & 1 & 2 \\
M_{22} & 3 & 4 \\
M_{51} & 2 & 4 & 5 & 6 & 7 \\
M_{52} & 1 & 3 & 5 & 6 & 7 \\
M_{53} & 2 & 3 & 8 & 9 & 10 \\
M_{54} & 1 & 4 & 8 & 9 & 10 \\
\end{array}
\]

where \( H_F^{(1)} = H_F^{(2)}(x_8, x_9, x_{10}) \), \( H_F^{(3)} = H_F^{(4)}(x_5, x_6, x_7) \), \( H_T^{(1)} = H_T^{(2)} = H_F^{(1)} H_F^{(3)} \). This solution is 1/8 supersymmetric as follows from the set of supersymmetry projection operators associated to the various branes in the configuration. Its near-horizon limit has geometry \( \text{adS}^3 \times S^2 \times S^2 \times E^5 \) and preserves 1/4 of supersymmetry. The interpolating structure of intersection (E) is shown in figure 8 \((r^2 = x_8^2 + x_9^2 + x_{10}^2)\) and \( r'^2 = x_5^2 + x_6^2 + x_7^2 \).

It is easy to check that all pairs of branes in the configurations (A)-(F) satisfy either standard or non-standard intersection rules, see table 1. This concludes our presentation of the intersections with \( \text{adS} \) near-horizon geometry. We have seen that adding a wave and/or monopoles does not change the near-horizon limit of the intersection and it is only after an additional dimensional reduction or T-duality that one gets a different near-horizon geometry. For example, this means that solution (A) with a wave and two monopoles added still has supersymmetry enhancement to 1/2 at the horizon. A similar phenomenon also can be seen in the non-extremal generalizations of certain intersections. Let us illustrate this for the standard intersection \( M_5 \perp M_5 \perp M_5 \). In the limit of keeping the non-extremality parameter \( \mu \) fixed and taking large charges, the non-extremality function \( f(r) = 1 - \frac{\mu}{r} \) remains unchanged but the 1’s in the harmonic functions of the five-branes become negligible\footnote{This limit can be reached by the shift transformation without taking large charges \cite{16}. However, in that case the \( \text{adS} \) part is really the non-extremal BTZ black hole.}. The geometry then becomes \( \text{adS}^3 \times E^6 \times S^2 \) (where \( \text{adS}^3 \) is the non-extremal BTZ black hole without the identification, which is just \( \text{adS}^3 \)), and the field strengths are still covariantly constant w.r.t. this metric. This is the same as the near-horizon limit of the extreme version and therefore supersymmetry is (partially) restored in this limit.
3 Supersymmetry enhancement

A by now well-known phenomenon of certain solutions with anti-de Sitter near-horizon geometry is supersymmetry enhancement [56]. For example, the $M_2$, $M_5$ and $D_3$ branes break one half of supersymmetry, whereas their near-horizon limits are maximally supersymmetric vacua of $d=11$ supergravity. These branes are therefore solitons which interpolate between maximally supersymmetric vacua at the horizon and at infinity [17]. A few other examples of supersymmetry enhancement for static $p$-brane solutions in different dimensions are known, and in all these cases the near-horizon geometry contains a factor $adS_k \times S^m$. All solutions of table 2 exhibit supersymmetry enhancement at the horizon. It turns out [8] that the condition for unbroken supersymmetry, $\delta \psi_M = 0$ where $\psi_M$ is the eleven-dimensional gravitino, in the background of the intersections in table 2, reduces to the geometric Killing spinor equations on the anti-de Sitter, flat and spherical factors of the geometry. In the case of $M_2 \perp M_5$ there is one additional projection, whereas for the intersections with three and four charges there are two projections needed. As one projection reduces the supersymmetry by a factor one half and since anti-de Sitter, flat and spherical geometries all admit the maximal number of Killing spinors, one concludes that the solutions in the right column of table 2 have double the amount of supersymmetry as compared to their brane counterparts in the left column.

Furthermore, for the configurations in table 2, a dimensional reduction over one or more of the relative transverse directions (which correspond to the flat directions in the near-horizon limit) will always give rise to lower dimensional solutions which also exhibit supersymmetry enhancement at the horizon. This is because all Killing spinors are independent of these coordinates. Further applications of $T$-duality in the relative transverse directions or $S$-duality lead to more solutions with supersymmetry enhancement. The thus obtained class of solutions exhibiting supersymmetry enhancement includes all previously known ones, such as the four and five-dimensional extremal black holes with nonzero entropy. There are some other interesting features that all these solutions have in common. They have regular (i.e. finite) dilaton at the horizon (or no dilaton in eleven dimensions), and in the shifted solutions the dilaton is a constant everywhere. Besides, the antisymmetric field strengths become covariantly constant in the shifted solutions, as in the Bertotti-Robinson solution. Also, these solutions are non-singular [57].

The configurations (A) to (F), based on non-standard intersections, turn out to have the same...
properties as those listed above for the standard intersections. Near the horizon the dilaton has a fixed finite value and the field strengths become covariantly constant. All of them have supersymmetry doubling as we will argue below. We explicitly checked the Killing spinor equations for (A) and (F). Solution (F) will be discussed in detail in the appendix. Here we verify the supersymmetry enhancement of (A).

The explicit solution belonging to configuration (A) is [58, 43]

\[
ds^2 = (H_T)^4 \left( H_F^{(1)} H_F^{(2)} \right)^* \left[ \left( H_T H_F^{(1)} H_F^{(2)} \right)^{-1} (dt^2 + dx_7^2) \right] + (H_T)^{-1} dx_3^2 + (H_F^{(1)})^{-1} (dx_3^2 + \cdots + dx_6^2) + (H_F^{(2)})^{-1} (dx_2^2 + \cdots + dx_1^{10}) \right],
\]

where \( I \) runs over all \( m \in \{3, 4, 5, 6\} \) and \( m' \in \{7, 8, 9, 10\} \). Going near the horizon, one can neglect the ones in the harmonic functions, so \( H_F^{(1)} = \frac{Q_1}{2r^2} \), \( H_F^{(2)} = \frac{Q_2}{r^2} \) and \( H_T = \frac{Q_1 Q_2}{2 r^2} \), where \( r^2 = x_7^2 + \cdots + x_6^2 \) and \( r'^2 = x_7^2 + \cdots + x_{10}^2 \). The geometry then becomes

\[
ds^2 = (Q_1 Q_2)^{-1} r'^2 \left( -dt^2 + dx_1^2 \right) + dx_2^2 + \frac{Q_2}{r^2} \frac{dr^2}{r^2} + \frac{Q_1}{r^2} \frac{dr'^2}{r'^2} + Q_2 d\Omega_3^2 + Q_1 d\Omega_1^2
= e^{-2A(r)}(-dt^2 + dx_1^2) + dp^2 + dx_2^2 + dx_3^2 + Q_2 d\Omega_3^2 + Q_1 d\Omega_1^2,
\]

where in the last line the change of coordinates (12) has been performed, as in [46]. This is a metric for \( adS_3 \times E^2 \times S^3 \). The field strengths become

\[
\begin{align*}
F_{\mu \nu \rho} &= 2 A \epsilon_{\mu \nu \rho}, \quad F_{\alpha \beta \gamma 2} = 2 Q_2 \epsilon_{\alpha \beta \gamma}, \quad F_{\alpha' \beta' \gamma' 2} = 2 Q_1 \epsilon_{\alpha' \beta' \gamma'},
\end{align*}
\]

where \( \mu \in \{0, 1, \rho\} \) is the \( adS_3 \) index and \( \alpha \) and \( \alpha' \) are indices for the two \( S^3 \) factors, respectively. The field strengths are covariantly constant. Killing spinors are solutions of

\[
\delta \psi_M = D_M \epsilon + \frac{1}{288} (\Gamma_M^{NPQR} - 8 \delta_M^N \Gamma^{PQR}) F_{NPQR} \epsilon = 0.
\]

The \( \Gamma \)-matrices can be chosen

\[
\begin{align*}
\Gamma^\mu &= \gamma^\mu \otimes \gamma^3 \otimes 1 \otimes 1 \otimes \sigma_2, \\
\Gamma^s &= 1 \otimes 1 \otimes 1 \otimes 1 \otimes 1 \otimes \sigma_2, \\
\Gamma^\alpha &= 1 \otimes 1 \otimes 1 \otimes 1 \otimes 1 \otimes \sigma_1, \\
\Gamma^{\alpha'} &= 1 \otimes 1 \otimes 1 \otimes 1 \otimes 1 \otimes \sigma_3,
\end{align*}
\]

where the index \( s \) is used for the flat directions \( x_2, \lambda \), and \( \gamma^3 = i \gamma^2 \gamma^\lambda \). The matrices \( \gamma^\mu, \gamma^s, \gamma^\alpha, \gamma^{\alpha'} \) are gamma-matrices in \( adS_3, E^2, S^3 \) and \( S^3 \), respectively, and \( \sigma_i \) are the Pauli spin matrices. Substituting in (17), one finds for the \( E^2 \) directions [7]

\[
\begin{align*}
\delta \psi_s &= \partial_s \epsilon + \frac{\delta s \lambda}{6} \left( -\sqrt{2} i 1 \otimes 1 \otimes 1 \otimes 1 \otimes \sigma_2 + 1 \otimes \gamma^3 \otimes 1 \otimes 1 \otimes (\sigma_1 + \sigma_3) \right) \epsilon \\
&\quad + \frac{\delta x_2}{3} \left( \sqrt{2} 1 \otimes \gamma^3 \otimes 1 \otimes 1 \otimes 1 \otimes i 1 \otimes 1 \otimes 1 \otimes (\sigma_1 + \sigma_3) \right) \epsilon.
\end{align*}
\]

Taking the spinors independent of \( x_2, \lambda \), we must require

\[
\sqrt{2} \gamma^3 \xi \otimes \sigma_2 \chi + i \xi \otimes (\sigma_1 + \sigma_3) \chi = 0,
\]

\footnote{For notational convenience we have set \( Q_1 = Q_2 = 1 \).}
where we also wrote \( \epsilon = \eta \otimes \xi \otimes \rho \otimes \rho' \otimes \chi \). This can also be written as

\[
P(\xi \otimes \chi) = \frac{1}{2} (1 + \Gamma) (\xi \otimes \chi) = \frac{1}{2} \left( 1 - \frac{1}{\sqrt{2}} \gamma^3 \otimes (\sigma_1 - \sigma_3) \right) (\xi \otimes \chi) = 0.
\] (21)

Since \( \Gamma \) is traceless and \( \Gamma^2 = 1 \) this is a projection which breaks \( 1/2 \) of supersymmetry. For the \( adS_3 \) components of (17) we find

\[
D_\mu \epsilon - \frac{1}{6} \left( \gamma_\mu \otimes \gamma^3 \otimes 1 \otimes (\sigma_1 + \sigma_3) + 2\sqrt{2} \gamma_\mu \otimes \gamma^2 \otimes 1 \otimes 1 \otimes \sigma_2 \right) \epsilon = 0.
\] (22)

Using (20) one can rewrite this as

\[
D_\mu \epsilon - \frac{1}{\sqrt{2}} \left( \gamma_\mu \otimes \gamma^2 \otimes 1 \otimes 1 \otimes \sigma_2 \right) \epsilon = 0.
\] (23)

Similarly, the \( \alpha \) and \( \alpha' \) components of (17) give rise, together with the projection (20), to the following equations:

\[
D_\alpha \epsilon - \frac{i}{2} \left( 1 \otimes \gamma^2 \otimes \gamma_\alpha \otimes 1 \otimes \sigma_2 \right) \epsilon = 0,
\] (24)

\[
D_\alpha' \epsilon - \frac{i}{2} \left( 1 \otimes \gamma^2 \otimes 1 \otimes \gamma_\alpha' \otimes \sigma_2 \right) \epsilon = 0.
\] (25)

Now observe that \( \gamma^2 \otimes \sigma_2 \) commutes with the projection operator \( P \) in (21). This means that we can still decompose our projected spinor space into \( \pm \) eigenspinors of \( \gamma^2 \otimes \sigma_2 \). Thus for these eigenspinors equations (23) to (25) reduce to the geometric Killing spinor equations on \( adS_3 \) and the three-spheres,

\[
D_\mu \eta \pm \frac{1}{\sqrt{2}} \gamma_\mu \eta = 0,
\] (26)

\[
D_\alpha \rho \pm \frac{i}{2} \gamma_\alpha \rho = 0,
\] (27)

\[
D_\alpha' \rho' \pm \frac{i}{2} \gamma_\alpha' \rho' = 0.
\] (28)

Since anti-de Sitter spacetimes and spheres admit the maximal number of Killing spinors, we conclude that the only projection is (21) and thus the near-horizon limit of solution (A) is \( 1/2 \) supersymmetric. Thus there is supersymmetry doubling as compared to the intersection itself.

Now for the other configurations (B) to (F), one has to reduce or \( T \)-dualize along one or more directions, and some Killing spinors will not survive this procedure. In the transition from \( adS_3 \) to \( adS_2 \) one reduces along the coordinate \( x \) as in (3) and only half of the Killing spinors survive this reduction. In fact, from the analysis in (5) it is clear that this half corresponds to the Killing spinors which solve the Killing spinor equation (26) with one definite choice of sign, say minus. Thus the additional projection operator is effectively \( \frac{1}{2} (1 - \gamma^2 \otimes \sigma_2) \). This is the same for the spheres and therefore the supersymmetry is reduced by one factor of \( \frac{1}{2} \) relative to (A) in all cases. Thus, the near-horizon limits of (B) to (F) preserve \( 1/4 \) of supersymmetry.

### 4 Dual superconformal field theories

We have argued in the introduction that the shift transformation implies that the degrees of freedom of \( D3, M2 \) and \( M5 \) should organize themselves into multiplets of \( adS_5, adS_4 \) and \( adS_7 \), respectively.\(^{18}\)

\(^{18}\) Our calculation also seems to show that if one changes the orientation of the membrane (corresponding to the other sign in the membrane’s field strength contribution), in which case the intersection breaks all supersymmetry, the near-horizon limit is still \( 1/2 \) supersymmetric.
Supporting evidence has been provided by many recent papers. The conjecture of \cite{21} may now be reformulated as stating that in the large $N$ limit ($N$ is the number of coincident branes) only the worldvolume degrees of freedom of $D3$, $M2$ and $M5$, respectively, are relevant. Then M-theory (or string theory) on the corresponding background is equivalent to a SCFT. This is very similar to the case of Matrix theory \cite{3} where in the large $N$ limit strings and other branes decouple and only the $D0$ degrees of freedom are relevant. Actually this may be more than just a similarity. We have argued \cite{1} that the actual symmetry of M-theory contains elements that connect compactifications on tori to compactifications on spheres. Thus, the Matrix formulation of M-theory on some tori may be equivalent to M-theory on some spheres.

Most of the recent papers have studied the cases of $adS_5$, $adS_4$ and $adS_7$. However, in these cases it is difficult to test the conjecture of \cite{21} since our knowledge of the corresponding superconformal theories is rather limited. In contrast to the $d > 2$ cases, two dimensional SCFTs are rather well understood. In this section we formulate conjectures similar to the ones in \cite{21} but involving the solutions presented in this article. This leads to $d = 2$ SCFTs. In addition, in our case the $\alpha'$ corrections are under control since we have an exact conformal field theory for each solution. This means that one does not need the large $N$ limit for the correspondence to hold. Hence, one may perform a detailed quantitative analysis.

In the last section we computed the number of supersymmetries that remain unbroken in the near-horizon limit of our new solutions. We shall now use these results to obtain the superalgebra that organizes the spectrum of the near-horizon theory. The master solution from which all others follow is the one with near-horizon geometry $adS_3 \times S^3 \times S^3 \times E^2$. All others can be obtained from it by a combination of adding a wave, monopoles and sending the radius of one of the spheres to infinity. In addition, one can follow what happens to the supersymmetries after these operations.

The isometry group of the near-horizon limit of solution (A) contains $SO(2,2) \times SO(4) \times SO(4)$ where the first factor is associated to $adS_3$ and the other two to the two spheres. The spectrum is thus organized by a superalgebra which is a superextension of this bosonic part. There are a number of different $d = 3$ anti-de Sitter supergroups. The latter have been classified in \cite{60}. Since $SO(2,2)(= SL(2,R) \times SL(2,R))$ is not simple the corresponding supergroup is in general a direct product $G_1 \times G_2$, where $G_1$ and $G_2$ are both superextensions of $SL(2,R)$. Thus, giving only the bosonic algebra is not sufficient to fix the corresponding superalgebra. However, in most cases the superalgebra is uniquely determined if one knows how the supercharges transform under the bosonic algebra.

Let us rewrite the bosonic algebra as $(SL(2,R)_1 \times SU(2)_{1a} \times SU(2)_{1b}) \times (SL(2,R)_2 \times SU(2)_{2a} \times SU(2)_{2b})$. Then from our calculation in the previous section we know that the supercharges transform as $(2_1, 2_{1a}, 2_{1b}, 0_2, 0_{2a}, 0_{2b})$ and $(0_1, 0_{1a}, 0_{1b}, 2_2, 2_{2a}, 2_{2b})$. It follows \cite{61} that the corresponding superalgebra is $D^1(2,1,\alpha) \times D^1(2,1,\beta), 0 < \alpha, \beta \leq 1$. The bosonic subalgebra of $D^1(2,1,\alpha)$ is $SL(2,R) \times SU(2) \times SU(2)$. The superalgebra $D^1(2,1,\alpha)$ has 8 fermionic generators. In addition, $D^1(2,1,1)$ is isomorphic to $Osp(4|2)$. For $\alpha \neq 1$, $D^1(2,1,\alpha)$ differ from $Osp(4|2)$ in the way the $SU(2)$ generators enter in the right hand side of the anti-commutator of supersymmetries.

The group $D^1(2,1,\alpha)$ is also a (finite dimensional) $d = 2$ superconformal group. The question is then whether it admits an infinite dimensional extension. Indeed, this turns out to be the case. The corresponding infinite dimensional SCFT, the $\mathcal{A}_\alpha$ algebra ($\alpha = \gamma/(1 - \gamma)$), was introduced in \cite{62,63}. The algebra contains an affine subalgebra corresponding to $SU(2) \times SU(2) \times U(1)$ and a set of four dimension-1/2 fields. For unitary representations, the parameter $\alpha$ is related to the levels $k_+, k_-$ of the two affine $SU(2)$ algebras as $\alpha = k_-/k_+$. The central charge is given by $c = 6k_-k_+/(k_- + k_+)$. For $\alpha = 1$ the algebra reduces to the standard large $\mathcal{N} = 4$ algebra. For arbitrary $\alpha$ the algebra contains two $\mathcal{N} = 4$ subalgebras. Each of them realizes the small $\mathcal{N} = 4$ algebra (which contains only one $SU(2)$ factor).
We therefore conjecture that M-theory on $adS_3 \times S^3 \times S^3 \times T^2$ is equivalent to a $(4,4) \ A_\gamma \times A_{\gamma'}$ SCFT. There is a canonical way to split the superalgebra into a left and a right moving part. The full isometry group is generated by left and right translations on the group manifolds $SL(2,R)$ and $SU(2)$(s). So, we take for the left superalgebra the superextension of the isometry subalgebra generated by left translations and for the right one the superextension of the isometry subalgebra generated by right translations. In abelian groups, however, left and right translations coincide. So there are only two $U(1)$’s coming from the torus $T^2$, but we also only need two $U(1)$’s; one for each $A_{\gamma'}$. Thus in this way we have completely geometrized the $N = 4$ SCFT algebra. In addition, it seems natural that left and right affine $SU(2)$’s have the same level $k_\pm$, since these $SU(2)$’s come from the same sphere in the $adS$ picture. Then there is one parameter in the SCFT for each modulus of the solution (A). In particular, the radii of the two spheres correspond to the levels of the two affine $SU(2)$’s. In addition, deformations of the spheres should correspond to marginal operators on the SCFT side. In particular, by “squashing” the sphere one may lose one $SU(2)$ and gain a $U(1)$ factor in the isometry group. On the SCFT side the marginal operator would move us from the first entry in Table 3 to the second one, and so on.

Each isometry gives rise to a gauge symmetry after dimensional reduction (we keep all massive modes, so the issue of consistent truncation does not arise). According to the analysis of \cite{[24]}, the bulk gauge fields should couple to the global currents of the SCFT in the boundary. In our case, we precisely have one global current for each gauge field. Thus, following \cite{[24]}, we propose

$$\langle \exp(\int_{M^2} j_a A_0^a) \rangle_{CFT} = Z_S(A_0)$$

(29)

where $j^a$ denotes collectively all currents, $A_0$ is the boundary value of the bulk gauge field $A$ (this can also be a graviton), and $M^2$ is either (a conformal completion of) two-dimensional Minkowski space or $S^2$ depending on whether we consider $adS_3$ or its Euclidean version. $Z_S(A_0)$ denotes the string theory partition function in the background specified by the boundary values $A_0$.

Let us emphasize that the near-horizon geometries we consider correspond to exact CFTs. So one can go beyond the supergravity approximation, and prove or disprove \cite{[29]} by explicitly computing both sides. We hope to report a detailed analysis in a future publication.

All other cases can now be obtained by using the monopole/wave rule. We tabulate these results below.

| $adS_3 \times S^3 \times S^3 \times T^2$ | $A_\gamma \times A_{\gamma'}$ |
|--------------------------------------|-------------------------------|
| $adS_3 \times S^2 \times S^3 \times T^2$ | $A_\gamma \times (Vir \times SU(2) \times U(1))$ |
| $adS_3 \times S^2 \times S^2 \times T^3$ | $A_\gamma \times (Vir \times U(1))_{\gamma}$ |
| $adS_2 \times S^3 \times S^3 \times T^2$ | $A_\gamma \times SU(2) \times SU(2) \times U(1)$ |
| $adS_2 \times S^2 \times S^3 \times T^3$ | $A_\gamma \times SU(2) \times SU(2) \times U(1)$ |
| $adS_2 \times S^2 \times S^2 \times T^4$ | $A_\gamma \times U(1)$ |

Table 3: Near-horizon geometries of solutions (A)-(F) and the corresponding SCFTs.

Vir denotes the Virasoro algebra and the hat an affine algebra. To obtain the result in the second entry one adds a monopole to the one in the first entry and then reduces over the nut direction. This has the effect of projecting out the spinor $2_{2a}$. Thus, one is left with $(4,0)$ supersymmetry. In addition, adding a monopole and reducing yields $S^2$ instead of $S^3$. Thus, the isometry group of the new solution loses an $SU(2)$ factor and gains a $U(1)$ (that used to be the fiber). Similar remarks apply to the third entry. Now, in addition to $2_{2a}$, the $2_{2b}$ spinor is projected out, but this yields the same overall projection. The theory still has $(4,0)$ supersymmetry. The isometry group loses again one $SU(2)$ and gains a $U(1)$.
To obtain the last three entries one adds a wave to the solutions involving \( adS_3 \). This has the effect of projecting out the spinor 2. This projection eliminates half of the supersymmetries; the same ones that the projections corresponding to transitions from \( S^3 \) to \( S^2 \) eliminate. Following the discussion of the \( adS_3 \times S^3 \times S^3 \times T^2 \) case one expects string theory on the background given in the left column of the last three entries of Table 3 to be equivalent to a superconformal quantum mechanical model with global symmetry given in the right column. However, the way we get \( adS_2 \) from \( adS_3 \) suggests that the corresponding quantum mechanical model is a Kaluza-Klein reduction of a chiral CFT. Indeed, the entries in the right column correspond to chiral \((4,0)\) SCFTs. Similar conjectures may also be formulated for the standard intersections \([21]\).

In some cases one may consistently truncate the massive modes. This leads to gauged supergravities. In particular, one may consistently gauge only an \( SU(2) \) part of the \( SO(4) \) isometry group of \( S^3 \). In this case one would need only one \( SU(2) \) global current in the boundary. Indeed, the \( \mathcal{A}_r \) algebra can be consistently truncated to the small \( N = 4 \) algebra that contains only one \( SU(2) \) factor. Thus we expect that gauged supergravities arising from \( S^3 \) compactifications are related to the small \( N = 4 \) algebra.

Let us finish this section with some further remarks about the AdS/CFT correspondence. The right hand side of (29) seems to depend only on the near-horizon geometry. In general, one may have two different brane configurations (not related by dualities) whose near-horizon limits are related through dualities. For example, one can connect through \( T \)-dualities all the entries in the left column in Table 3 but this cannot be done for the corresponding brane intersections (they have different number of charges). On the other hand, the SCFTs in the right column are different. (Presumably the connection between the different SCFTs by marginal operators is related to the connection of the near-horizon geometries by \( T \)-dualities.) So, one may need to refine the proposal (29) so it distinguishes between different brane configurations that have equivalent near-horizon limit. For instance, if one only considers the supergravity limit all entries in table 3 are different. So, the adS/CFT equivalence may only hold in the large \( N \) limit, even though one may reliably calculate the right hand side of (29) for any \( N \).

## 5 Branes in a dual frame

We would like to discuss now the issue of supersymmetry enhancement in 10 dimensions, in arbitrary frames, for several brane configurations. We will start with the case of one single \( Dp \) brane (with \( p \leq 6 \)), which for \( p = 1, 2, 3 \) was already discussed in \([12]\) in two specific frames, the string frame and the Einstein frame. We will find agreement with their results in these special cases. The metric, dilaton and field strengths for a single \( Dp \) brane are, in the string metric, given by \([14]\)

\[
\begin{align*}
  ds^2 &= H_p(r)^{-1/2}(-dt^2 + dx_1^2 + \cdots + dx_p^2) + H_p(r)^{1/2}(dx_{p+1}^2 + \cdots + dx_9^2), \\
  A_{01\ldots p} &= H_p(r)^{-1} - 1; \quad e^{-2\phi} = H_p^{\frac{p-3}{2}}, \\
  H_p(r) &= 1 + \frac{Q}{r^7 - p}; \quad r^2 = x_{p+1}^2 + \cdots + x_9^2.
\end{align*}
\]

The reason that we study supersymmetry enhancement in arbitrary frames, is the fact that, in the limit \( r \to 0 \), the exponent of the dilaton either vanishes or diverges after the shift transformation (or equivalently when one approaches the horizon). Therefore rescalings of the metric by powers of the string coupling will lead to different behaviour depending on the rescaled metric. Only for \( p = 3 \) the dilaton itself vanishes, the metric factorizes as \( adS_5 \times S^5 \), and we get supersymmetry enhancement. In the following we will take \( p \neq 3 \).

In order to study the number of supersymmetries preserved by the background configurations \([30]\), one has to find the number of solutions to the vanishing of the supersymmetry variations of the dilatino
and the gravitino, which in the string frame are given by (see e.g. [33])

$$\delta \lambda = (\gamma^\mu \partial_\mu \phi) \varepsilon + \frac{3 - p}{4(p + 2)!} \phi^F F_{\mu_1 \cdots \mu_{p+2}}^{\alpha_{\mu_1} \cdots \alpha_{\mu_{p+2}} \varepsilon^\prime}$$

$$\delta \psi_\mu = D_\mu \varepsilon + \frac{(-1)^p}{8(p + 2)!} \phi^F F_{\mu_1 \cdots \mu_{p+2}}^{\alpha_{\mu_1} \cdots \alpha_{\mu_{p+2}} \gamma_\mu \varepsilon^\prime}.$$  \[31\]

Let us define

$$g_{(\alpha)} = e^{-\alpha \phi} g_s,$$  \[32\]

where $g_s$ is the metric in the string frame (30). First of all, it is easy to see that, in order to go to the ‘dual Dp brane frame’, in which both the curvature and the $(8 - p)$-form field strength $\frac{1}{(p+2)!} \phi^F F_{\mu_1 \cdots \mu_{p+2}}^{\alpha_{\mu_1} \cdots \alpha_{\mu_{p+2}}}$ appear in the action with the same power of the dilaton exponential, one has to set $\alpha = \alpha_p = \frac{2}{p-3}$. Furthermore, only in this metric, after performing the shift transformation, the geometry factorizes into the product $adS_{p+2} \times S^{8-p}$ for $p \neq 5$, whereas for $p = 5$ it becomes $M_7 \times S^3$, similar to the case of the solitonic fivebrane in the string metric [17].

We will now show that this metric also corresponds to what we shall call ‘threshold supersymmetry enhancement’, that is, we get supersymmetry enhancement in frames with $\alpha < \alpha_p$ ($\alpha > \alpha_p$) for $p < 3$ ($p > 3$), and not when $\alpha \geq \alpha_p$ ($\alpha \leq \alpha_p$). As $\alpha = \frac{1}{2}$ corresponds to the metric in the Einstein frame, we see that the results of [42] are contained in ours. To this end, we first work out the supersymmetry variation of the dilatino, when we plug in the solution (30), written in an arbitrary frame, and after performing the shift transformation. We find

$$\delta \lambda \sim (3 - p) r^{-\frac{3 - p}{8}(p - 7)\alpha + 2} \mathcal{P} \varepsilon.$$  \[33\]

Here $\mathcal{P}$ is a projection operator that projects out half the number of components of the spinor $\varepsilon$. From this equation we can already conclude that indeed there will be no supersymmetry enhancement (in the limit $r \to 0$) in frames with $\alpha \geq \alpha_p$ for $p < 3$, and $\alpha \leq \alpha_p$ for $p > 3$. In the frames where we rescaled with $\alpha < \alpha_p$ ($p < 3$), or $\alpha > \alpha_p$ ($p > 3$), we do find the supersymmetry variation of $\lambda$ to vanish without use of the projection operator in the limit $r \to 0$. In order to check that we really get enhancement of supersymmetry in these frames in this limit, we should also consider the supersymmetry variation of the gravitino. One can show that also the $r$-dependence of this variation is proportional to an identical factor, so that this is indeed the case.

We will not discuss here intersections of several $Dp$ branes in general, but restrict ourselves to some interesting observations. In particular, we shall discuss supersymmetry enhancement for an arbitrary number of asymptotically flat and orthogonally intersecting $D3$ branes.

When there is only one $D3$ brane present, the geometry factorizes as $adS_5 \times S^5$ (after making the shift transformation), and we do get supersymmetry enhancement. The case with two $D3$ branes, intersecting along a string, can be related to the intersection of an $M2$ brane and an $M5$ brane that we considered before. These intersections are related by $T$-duality transformations and a compactification along relative transverse directions only, so the conclusions on supersymmetry enhancement from the M-theory perspective directly apply. So also in this case the geometry factorizes, as $adS_4 \times S^3 \times E^4$, and we do get supersymmetry enhancement. For three $D3$ branes we do not get a factorization of the geometry, and also no supersymmetry enhancement occurs (so such a configuration always preserves only $1/8$ of the supersymmetry). One can show this either by a direct computation, or by observing that this configuration can only be obtained from an M-theory configuration which does have supersymmetry enhancement (three $M5$ branes or three $M2$ branes) by compactifying or $T$-dualizing along one of the $adS$ directions; also in the latter case we need to compactify this direction, thereby reducing the number of geometric Killing spinors by one half [51], so there will be no supersymmetry enhancement.
When we consider a configuration of four $D3$ branes, the result can be related to the $M2\perp M2\perp M5\perp M5$ configuration we studied before; now this relation is again by making $T$-duality transformations (and also compactifying) only along relative transverse directions. Therefore the geometry factorizes, as $adS_2 \times S^2 \times E^6$, and we do get supersymmetry enhancement (to leave 1/4 of the supersymmetry unbroken), unless the orientations of the branes are such that all supersymmetry is broken. Finally, solutions with more than four $D3$ branes, pairwise intersecting over strings, are not asymptotically flat and there is no indication of supersymmetry enhancement.

6 Compactifications and (gauged) supergravity

In this section we comment on the implications of our results for supergravity theories in various dimensions. First we consider dimensional reductions along the flat coordinates in the near-horizon limits of the $adS$ solutions. Each of the $D$-dimensional $adS_k \times E^l \times S^m \times S^n$ solutions ($D = 11$ or $D = 10$) described in this paper can be dimensionally reduced along $p \leq l$ of the flat directions, giving a solution with geometry $adS_k \times E^{l-p} \times S^m \times S^n$ and the same amount of supersymmetry (counting the number of spinor components) in $(D-p)$-dimensional maximal Poincaré supergravity. For $p = l$ and $n = 0$ (standard intersections) these can be interpreted as the near-horizon limits of $4d$ and $5d$ extreme black holes and $5d$ and $6d$ extreme black strings. For example, the $M2 \perp M5$ intersection gives rise, upon reduction along the five relative transverse directions, to dyonic string solutions in six dimensions with near-horizon geometry $adS_3 \times S^1$. The solution $adS_3 \times S^3$ is itself a 1/2 supersymmetric vacuum configuration of $N = 4$ $6d$ supergravity. If the $M2$ and $M5$ charges are taken to be equal one gets the self-dual string which can also be embedded into $6d$ $N = 2$ chiral supergravity where it breaks only 1/2 of supersymmetry, and hence its near-horizon limit $adS_3 \times S^3$ is a maximally supersymmetric vacuum.

As an example of reducing non-standard intersections along flat directions, we note that the near-horizon limit of solution (F) reduces to a 1/4 supersymmetric $adS_2 \times S^2 \times S^2$ solution in six dimensions. However, in this case it is not clear whether this is the near-horizon geometry of some $6d$ black hole (or if there exists a solution that interpolates between this horizon geometry and Minkowski spacetime at infinity), since we can reduce the intersection (F) itself only along four directions. The fifth flat direction in the horizon geometry is not an isometry of the full intersection. Reducing (F) along the four relative transverse directions $(1, 2, 3, 4)$, one obtains a solution in $d = 7$ with two radial coordinates and near-horizon geometry $adS_2 \times S^2 \times S^2 \times S^1$, and it would be interesting to see whether this solution can be interpreted as a black hole.

In addition we can deduce the existence of solutions with a certain amount of supersymmetry after spontaneous compactification on spheres. These compactifications are expected to give rise to solutions of gauged supergravities. Several of these results are well-known, such as the spontaneous compactification of $11d$ supergravity on $S^7$, giving gauged $N=8$ supergravity in $d=4$, and the $adS_7 \times S^4$ and $adS_5 \times S^5$ solutions of $d=11$ supergravity and type IIB supergravity. The anti-de Sitter parts of these solutions are maximally supersymmetric vacua of gauged maximal supergravities in seven and five dimensions. Below we list some further results and some predictions for supersymmetric vacua of gauged supergravities. We hope to report on these issues in more detail in a future publication. For the intersections discussed here we always find either $S^2$ or $S^3$ in the horizon geometry. It seems there is not much known about gauged supergravities that might result from Kaluza-Klein reductions over two-spheres, probably due to the problem of making a consistent Kaluza-Klein ansatz for compactifications on even dimensional spheres (see e.g. the discussion in [3]). A consistent KK ansatz for $S^3$ compactification can be made.

\[\text{See also [3] and [10].}\]
by introducing non-abelian vector fields for the $SU(2)$ group manifold. The compactification of 10$d$ type I supergravity on $S^3$ then leads to the $SU(2)$ gauged $\mathcal{N} = 2$ 7$d$ supergravity of \cite{35}. One can associate certain solutions of this gauged supergravity to known spontaneous compactifications of type I supergravity having an $S^3$ factor in the geometry. So, for example, the $M_7 \times S^3$ solution \cite{35} of type I supergravity corresponding to the fivebrane with shifted harmonic function (the harmonic function without the 1) corresponds to the 1/2 supersymmetric domain wall solution with linear dilaton of gauged $\mathcal{N} = 2$ $d = 7$ supergravity found in \cite{35}. Of the intersections (a) through (F), only (a) and (A) can be embedded into the type I theory\cite{35}. From solution (a) one deduces the existence of a 1/2 supersymmetric $adS_3 \times E^4$ solution of 7$d$ gauged supergravity, which is indeed known \cite{35}. Furthermore, solution (A) predicts the existence of a 1/2 supersymmetric $adS_3 \times S^3 \times E^1$ solution. Once we dimensionally reduce $\mathcal{N} = 2$ 7$d$ gauged supergravity to lower-dimensional gauged supergravity there will be many more such vacuum solutions, since as we have seen this reduction can be done along isometries of $adS_3$ or $S^3$ yielding $adS_2$ or $S^2$, respectively, with a covariantly constant field strength but without introducing a dilaton. For example, in six-dimensional gauged supergravity we predict the existence of the following supersymmetric vacua: $adS_2 \times E^3$, $adS_3 \times E^5$, $adS_3 \times S^3 \times E^1$, $adS_3 \times S^3 \times E^1$ and $adS_3 \times S^3$.

For the maximally supersymmetric theories, a Kaluza-Klein reduction of 11$d$ supergravity on the $S^3$ group manifold yields $SU(2)$ gauged $\mathcal{N} = 2$ 8$d$ supergravity \cite{29}. Recalling the near-horizon geometries in $D = 11$ containing an $S^3$ factor, we predict the existence of $adS_3 \times E^5$, $adS_2 \times E^6$ and $adS_3 \times S^3 \times E^2$ vacua, preserving 1/2, 1/4 and 1/2 of supersymmetry, respectively. Dimensional reduction of this theory gives rise to maximally supersymmetric gauged supergravities in lower dimensions. Similar to the discussion in the previous paragraph, we can construct vacuum solutions of these supergravities. For instance, in seven dimensions we obtain the following supersymmetric vacua: $adS_3 \times E^4$, $adS_2 \times E^5$, $adS_3 \times S^3 \times E^1$, $adS_3 \times S^3 \times E^2$, $adS_2 \times S^3 \times E^3$ and $adS_2 \times S^2 \times E^3$ (after an extra dimensional reduction and T-duality).

Finally we consider compactifications on $S^3 \times S^3$. An explicit reduction of 10$d$ type I supergravity on the group manifold $S^3 \times S^3$ was done in \cite{35}. The resulting 4$d$ theory is $\mathcal{N} = 4$ $SU(2) \times SU(2)$ gauged supergravity \cite{35}. As observed in \cite{16}, solution (A), whose type I supergravity analogue represents a string localized on the intersection of two fivebranes, implies that there is an $adS_3 \times E^1$ vacuum. The half gauged version of $\mathcal{N} = 4$ gauged supergravity, where one of the $SU(2)$ coupling constants is set to zero, corresponds to a compactification on the group manifold $S^3 \times T^3$, and is therefore just a dimensional reduction of $SU(2)$ gauged 7$d$ supergravity \cite{35}. This version should have in addition the supersymmetric vacua $adS_2 \times E^2$ and $adS_2 \times S^2$. It would be interesting to see how these brane inspired solutions are related to several known solutions of 4$d$ gauged supergravity \cite{35,14}. The geometries do match, but the number of unbroken supersymmetries appear not to. For example, the $adS_2 \times S^2$ Freedman-Gibbons solution breaks all supersymmetry and is also a solution in the fully gauged $SU(2) \times SU(2)$ model. In general, one expects more anti-de Sitter vacua to exist, but there may not be so many supersymmetric (and therefore stable) ones. For example, one easily finds $adS_2 \times S^2 \times E^6$, $adS_2 \times S^2 \times S^2 \times E^4$ and $adS_2 \times S^2 \times S^2 \times S^2 \times E^2$ solutions with constant dilaton of type I supergravity, by taking appropriate covariantly constant field strengths supporting the different Einstein spaces. A preliminary analysis suggests that these solutions break all supersymmetry. This could be related to the fact that they do not seem to have a simple brane interpretation.

Finally, let us remark that an $S^3 \times S^3$ reduction of 11$d$ supergravity is expected to give rise to a maximally supersymmetric $SU(2) \times SU(2)$ gauged 5$d$ supergravity, which to the best of our knowledge has not been constructed. This theory should also be obtainable after a consistent reduction on $S^3$ of the maximal $\mathcal{N} = 2$, $d = 8$ gauged supergravity.

\text{Of course, one can add a wave and/or monopoles but this will not change the near-horizon geometry, as we have argued. That will happen only after reduction or T-duality.}
7 Conclusions

In this paper we constructed several intersections of branes with the special property that the near-horizon geometry has the form $adS_k \times E^m \times S^n$ where $k, m, n$ can take the values 2, 3. All these configurations involve the non-standard intersection rule in which the number of common worldvolume directions is two less than in the corresponding standard intersection. The two spheres in the geometry are associated with two sets of relative transverse coordinates. One can derive all these solutions from the $1/4$ supersymmetric 11d intersection with near-horizon geometry $adS_3 \times E^2 \times S^3 \times S^3$ describing a membrane intersecting on the common string of two fivebranes. Adding a wave does not change the near-horizon limit but if one reduces (or in $d=10$ T-dualizes) along an appropriate isometry direction of $adS_3$ one obtains an intersection with near-horizon geometry in which $adS_3$ is replaced by $adS_2$. In the same way adding a monopole (plus reduction or T-duality) replaces one $S^3$ by an $S^2$ near the horizon. Further solutions may be obtained from the ones that contain two monopoles by replacing the two-monopole part with a toric Hyperkähler manifold\[55].

The intersections we obtained have an interesting structure in the sense that they interpolate between three or four different vacua. Besides flat spacetime at infinity and the near-horizon limit which corresponds to the limit of small radii in both relative transverse spaces, one can also let one radius become small and the other large in which case the solution becomes a vacuum solution whose geometry is the near-horizon geometry with the appropriate sphere replaced by flat space. In fact, letting one of the radii go to infinity reproduces one of the four standard intersections with near-horizon geometry $adS_k \times E^m \times S^n$.

The near-horizon limits of the intersections that we obtained imply the existence of certain vacua of gauged supergravities, as we described in section 6. Also, they provide an intersecting brane interpretation of certain known such vacua. It would be interesting to look for brane interpretations of other known solutions too.

By an explicit computation of the Killing spinors we have shown that all $adS$ intersections exhibit a doubling of unbroken supersymmetry near the horizon. This is of importance for the association of a superconformal theory to the string theory on the $adS$ background. By considering the transformation properties of the unbroken supersymmetries under the isometry group of the background geometry, we argued what the superconformal groups of the dual superconformal theories should be. In all cases the superconformal algebra contains the large $\mathcal{N}=4$ algebra. We argued that the maximally symmetric case $adS_3 \times S^3 \times S^3 \times T^2$ corresponds to $\mathcal{A}_r \times \mathcal{A}_{r'}$. The spacetime geometry provides a geometric realization of this algebra.

In a similar way we proposed a dual superconformal theory for all other solutions. Actually these results also follow from the monopole/wave rule once one starts from the case of $adS_3 \times S^3 \times S^3 \times T^2$. In the case that the solutions contain $adS_2$ one would expect the dual theory to be a $0+1$ dimensional superconformal theory. However, we argued that these quantum mechanical models are just reductions of chiral SCFTs.

Our results open the possibility to explicitly check the conjectural equivalence between string theory on an $adS$ background and superconformal field theories. In the case at hand one may proceed as follows. As a first step the conformal dimensions of the “massless” representation of the $\mathcal{A}_r$ should match the masses of the Kaluza-Klein harmonics. This would fix the relations between the moduli of our solutions and the parameters of the SCFT. Once this is done one may proceed to explicitly compute both sides in (29). This calculation seems tractable since on the left hand side we have the partition function of some $\mathcal{N}=4$ SCFT and on the right hand side there are also known CFTs associated with the $adS$ background. We hope to report on this and related issues in the near future.
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A Kaluza-Klein ansatz and supersymmetry

Here we discuss the near-horizon limit of solution (F). We start from a Kaluza-Klein ansatz for an eleven-dimensional supergravity solution with covariantly constant four-form field strength and we wish to obtain the geometry $adS_2 \times E^5 \times S^2 \times S^2$. The ansatz for the field strength is as follows:

$$ F_{\mu\nu12} = c_1 \epsilon_{\mu\nu}, \quad F_{\mu\nu34} = c_3 \epsilon_{\mu\nu}, $$

$$ F_{\alpha\beta13} = c_2 \epsilon_{\alpha\beta}, \quad F_{\alpha\beta24} = -c_2 \epsilon_{\alpha\beta}, $$

$$ F_{\alpha'\beta'14} = c_3 \epsilon_{\alpha'\beta'}, \quad F_{\alpha'\beta'23} = c_3 \epsilon_{\alpha'\beta'}, $$

(34)

where $\mu, \nu$ are $adS_2$ indices, 1, 2, 3, 4 are flat directions (note that the fifth flat direction does not appear in the ansatz), and $\alpha, \beta; \alpha', \beta'$ are indices for the two $S^2$'s. The opposite sign of $F_{\alpha\beta24}$ is chosen in order to obtain a supersymmetric configuration as will be shown below. Substituting (34) in the Einstein equation of $D = 11$ supergravity,

$$ R_{MN} = \frac{1}{12} (F_{MPQR} F_N^{PQR} - \frac{1}{12} g_{MN} F^2), $$

(35)

we get for the flat components of the Ricci tensor

$$ R_{mn} = 2 \delta_{mn}(-c_1^2 + c_2^2 + c_3^2), $$

(36)

where $m, n$ run over 1, 2, 3, 4 and we have assumed that $g_{mn} = \delta_{mn}$. Then we require that $R_{mn} = 0$ and thus

$$ c_1^2 = c_2^2 + c_3^2. $$

(37)

The other nonzero components of the Ricci tensor then become

$$ R_{\mu\nu} = -c_1^2 g_{\mu\nu}, $$

$$ R_{\alpha\beta} = c_2^2 g_{\alpha\beta}, $$

$$ R_{\alpha'\beta'} = c_3^2 g_{\alpha'\beta'}. $$

(38)

It is now clear that we can take $adS_2$ and two $S^2$ factors to solve the above Einstein equations. The field equation for $F$ is automatically satisfied because $F$ is covariantly constant and in (34) there is always at least one common index in any pair of $F$'s so that the topological term $F \wedge F$ does not contribute to the field equation (see also the footnote referred to just above equation (9)).

If one includes a sign for all six field strengths the supersymmetry calculation gives two relations for the signs in order that supersymmetry is not completely broken. This is directly related to the fact that the corresponding intersection is supersymmetric not for any possible orientations of the branes.

Note that the above ansatz shows that a four-form field strength not only can support a four-dimensional Einstein space, or its dual a seven dimensional one, as in [77, 31], but that by taking some of the components along flat directions we can also obtain products of lower-dimensional Einstein spaces.
Next we check the supersymmetry of this solution. The factorization of $\Gamma$-matrices can be takes as

$$
\Gamma^\mu = \gamma^\mu \otimes 1 \otimes 1 \otimes 1,
$$

$$
\Gamma^s = \gamma^3_{adS} \otimes \gamma^s \otimes \gamma^3_S \otimes \gamma^3_{S'},
$$

$$
\Gamma^\alpha = \gamma^3_{adS} \otimes 1 \otimes \gamma^\alpha \otimes 1,
$$

$$
\Gamma^\alpha' = \gamma^3_{adS} \otimes 1 \otimes \gamma^3_S \otimes \gamma^\alpha',
$$

where $\gamma^3_{adS} = \frac{1}{2} \epsilon_{\mu \nu} \gamma^{\mu \nu}$, $\gamma^3_S = \frac{i}{2} \epsilon_{\alpha \beta} \gamma^{\alpha \beta}$ and $\gamma^3_{S'} = \frac{i}{2} \epsilon_{\alpha' \beta'} \gamma^{\alpha' \beta'}$ such that $(\gamma^3_{adS})^2 = (\gamma^3_S)^2 = (\gamma^3_{S'})^2 = 1$. The index $s$ is used for all five flat directions. Substituting this and (34) into the supersymmetry variation of the gravitino (17), we find for the flat components

$$
\delta \psi_s = \partial_s \epsilon + \frac{1}{12} \left\{ c_1 1 \otimes (\gamma^{12} + \gamma^{34}) \otimes \gamma^3_S \otimes \gamma^3_{S'} + ic_2 \gamma^3_{adS} \otimes (\gamma^{13} - \gamma^{24}) \otimes 1 \otimes \gamma^3_{S'} + ic_3 \gamma^3_{adS} \otimes (\gamma^{14} + \gamma^{23}) \otimes 1 \otimes \gamma^3_S \right\} \epsilon.
$$

Since we take $\partial_s \epsilon = 0$, this equation reduces for $s = 5$, the flat direction which does not appear in the ansatz (34), to

$$
(c_1 1 \otimes (\gamma^{12} + \gamma^{34}) \otimes \gamma^3_S \otimes \gamma^3_{S'} + ic_2 \gamma^3_{adS} \otimes (\gamma^{13} - \gamma^{24}) \otimes 1 \otimes \gamma^3_{S'})
$$

$$
+ ic_3 \gamma^3_{adS} \otimes (\gamma^{14} + \gamma^{23}) \otimes \gamma^3_S \otimes 1 \epsilon = 0.
$$

Taking $s = 1, 2, 3, 4$ in (34) one gets four equations which reduce, after some algebra, to only two independent conditions:

$$
P_1 \xi = \frac{1}{2} (1 + \Gamma_1) \xi = (1 + \gamma^{1234}) \xi = 0,
$$

$$
P_2 \epsilon = \frac{1}{2} (1 + \Gamma_2) \epsilon = (1 + c_2 \gamma^3_{adS} \otimes \gamma^{23} \otimes \gamma^3_S \otimes 1 + c_1 \gamma^3_{adS} \otimes \gamma^{24} \otimes 1 \otimes \gamma^3_{S'}) \epsilon = 0,
$$

where $\xi$ is the four-component spinor factor associated with $E^5$. Both $\Gamma_1$ and $\Gamma_2$ square to one (using (37)) and are traceless. The two projectors moreover commute and therefore break 3/4 of supersymmetry. Equation (11) is fulfilled after these projections. One can also show that $P_2$ corresponds to the projection operator found for configuration (A), equation (21), and $P_1$ corresponds to the extra projection due to the reduction along isometries of $adS_3$ or $S^3$.

The other components of the Killing spinor equation are

$$
\delta \psi_{\mu} = D_{\mu} \epsilon + \frac{1}{12} \left\{ ic_2 \gamma_{\mu} 1 \otimes (\gamma^{13} - \gamma^{24}) \otimes \gamma^3_S \otimes 1 \right\} \epsilon = 0,
$$

$$
\delta \psi_{\alpha} = D_{\alpha} \epsilon + \frac{1}{12} \left\{ c_1 1 \otimes (\gamma^{12} + \gamma^{34}) \otimes \gamma^3_S \otimes 1 \right\} \epsilon = 0,
$$

$$
\delta \psi_{\alpha'} = D_{\alpha'} \epsilon + \frac{1}{12} \left\{ c_1 1 \otimes (\gamma^{12} + \gamma^{34}) \otimes \gamma^3_S \otimes \gamma_{\alpha'} \right\} \epsilon = 0.
$$

One can now reduce these equations to the respective geometric Killing spinor equations by using identity (11). One finds:

$$
D_{\mu} \epsilon - \frac{1}{2} c_1 (\gamma^3_{adS} \otimes \gamma^{12} \otimes 1 \otimes 1) \epsilon = 0,
$$

26
\[ D_\alpha \epsilon - \frac{i}{2} c_2 (1 \otimes \gamma^{13} \otimes \gamma_\alpha \gamma^3_3 \otimes 1) \epsilon = 0, \quad (47) \]
\[ D_{\alpha'} \epsilon - \frac{i}{2} c_3 (1 \otimes \gamma^{14} \otimes 1 \otimes \gamma_{\alpha'} \gamma^3_{3'} \otimes 1) \epsilon = 0. \quad (48) \]

Now the operators inside the brackets in the above three equations commute with both projection operators \( P_1 \) and \( P_2 \), and therefore we can decompose these equations according to their \( \pm i \) eigenvalues with respect to \( \gamma^{12} \), \( \gamma^{13} \) and \( \gamma^{14} \), to obtain the geometric Killing spinor equations on \( adS_2 \), \( S^2 \) and \( S^2 \), respectively,

\[ D_\mu \eta \pm \frac{i}{2} c_1 \gamma_\mu \gamma^3_{adS} \eta = 0, \quad (49) \]
\[ D_\alpha \rho \pm \frac{1}{2} c_2 \gamma_\alpha \gamma^3_{S^2} \rho = 0, \quad (50) \]
\[ D_{\alpha'} \rho' \pm \frac{1}{2} c_3 \gamma_{\alpha'} \gamma^3_{S^2'} \rho' = 0, \quad (51) \]

where we wrote \( \epsilon = \eta(x^\mu) \otimes \xi \otimes \rho(z^\alpha) \otimes \rho'(z^{\alpha'}). \) We conclude that the solution preserves \( 1/4 \) of supersymmetry. The corresponding brane intersection, configuration (F), is \( 1/8 \) supersymmetric.

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