Optical appearance of Hayward black hole binary as possible evidence of wormholes

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Abstract. A wormhole may be formed in the spacetime of a black hole (BH) binary system. We investigate the optical appearance of the Hayward BH binary system for reveal possible evidence of the wormhole. Adopting the ray-tracing method, we find that the contralateral spacetime may reflect a significant fraction of the lights back to their original spacetime. We derive the effective potential, light deflection and azimuthal angle of the system and found that the light trajectory and deflection angle depend on the BH mass ratio of the system. Assuming that the BHs in the system is surrounded by a thin accretion disk, we present the optical appearance of the system. It shows that the image has additional photon rings and a lensing band appear in comparison with the image of a single BH. We propose that such features could provide critical information of the wormhole.
1 Introduction

Since the photons travel along null geodesics of the spacetime metric, the geodesic equation provides valuable information about light bending. Synge proposed the black hole (BH) shadow as the result to describe the deflection of light around the BH and derived the contour of the spherical BH (Schwarzschild) shadow as a standard circle [1]. Extending to the rotating BH (Kerr) situation, Bardeen argued that the BH angular momentum carries the deformation information of the BH shadow [2]. The shadow of the rotating BH is no longer circular but rather flattened on one side, which is the consequence of the dragging of lightlike geodesics by the BH. Since then, the geometry of BHs shadows under the various modified gravity context have been extensively investigated [3–24].

An observable astrophysical BH is generally surrounded by luminous accretion matters. Luminet showed that the emergence of the BH shadow and ring depend on the accretion position [25]. Assuming the spherical accretion scenario, Narayan et al. investigated the optical appearance of the Schwarzschild BH and illustrated that the BH shadow size is independent of the spherical accretion radius [26]. If the Schwarzschild BH is surrounded by a thin accretion disk, Gralla et al. proposed an elegant description of the BH shadow, lensing ring, and photon ring [27]. They found that the shadow brightness is diverged logarithmically at the photon ring and the BH shadow is affected by the details of the accretion. The effect of the location and morphology of accretion flow surrounded the single BH spacetime was extensively studied in various modified gravity theories [28–38].

Utilizing the numerical simulations for ray-tracing around a BH, Falcke et al. showed that the BH shadow could be observable [39]. The BH image is firstly obtained with the Event Horizon Telescope (EHT) collaboration for the supermassive BH in the elliptical galaxy center of Messier 87* (M87*) [40–45]. The image shows a bright ring-shaped lump surrounded the BH shadow. Recently, the EHT obtained the first horizon-scale radio observations image of the Sagittarius A* (Sgr A*) in our own Milky Way. The derived ring size of Sgr A* is consistent with the shadow critical curve predicted by the general relativity (GR) within 10% [46–51]. These results illustrate that the BH shadow encodes valuable spacetime information near the BH and provides direct and powerful experimental verification for the GR in the regime of strong gravity [52–54]. These researches focus on the single BH shadow.
It is well known that BH binaries widely exist in the Universe. The Laser-Interferometer Gravitational Wave-Observatory (LIGO) and Virgo detectors have observed the gravitational wave signals from $\sim 100$ coalescences of BH binary system, double neutron stars system, and BH-neutron star system [55]. Most of them are from BH binaries. Using the gravitational wave data, the potential deviations of the BH binary system and GR predictions is revealed [56]. Abdolrahimi et al. investigated the shadow of a BH binary system and illustrated the configuration of the system [57]. Considering an extremal charged BH binary in the static equilibrium, Shipley et al. investigated the qualitative features of the BH binary system [58]. They showed that the shadow of the BH binary system depends on the chaotic scattering of the spacetime. Patil et al. discussed the gravitational lensing effect of the BH binary system and found that the BH binary system has two photon spheres and two shadows, indicating that there may be a channel for transmitting information between the two BHs [59]. Interestingly, the wormhole is considered as a bridge connecting the two spacetimes. In the framework of the wormhole, Shaikh et al. showed that the compact star binary system can contain two photon spheres since its relativistic system is independent of each other [60]. Wielgus et al. studied the Schwarzschild wormhole and its shadow, finding that the wormhole can be used as a channel for photons traveling between two spacetimes [61]. Hence, the shadow of a BH binary system would be a practical approach to the exploration of the wormhole.

A highly effective method for describing or mathematically constructing a class of wormholes using the cut-and-paste technique is proposed by Visser [62]. The construction calls for grafting two BHs spacetimes together, resulting in a thin-shell wormhole (TSW). Tsukamoto discussed the gravitational lensing effect of the Schwarzschild TSW, finding that the position of the light source and observer determine the image formation [63]. By investigating the influence of the unstable photon sphere on the TSW shadow, Wang et al. obtained that the size of the TSW shadow is always smaller than that of the BH shadow [64]. The observational characteristics of an accretion disk around an asymmetric Schwarzschild TSW is investigated by [65], who showed that the shadow structure of the TSW system is different from the BH system. By considering the wormhole geometries, Guerrero et al. argued that there might be other light rings in the intermediate region between the critical curves of the BH binary system [66].

Different from irregular BHs, which have intrinsic singularity in the origin of spacetime, Bardeen presented the BH solution without spacetime singularity [67]. Furthermore, Hayward obtained the regular Hayward BH solution by coupling the nonlinear electrodynamics with the Einstein field equation [68]. We investigated the effects of accretion flow and magnetic charge on the features of observed shadow and photon rings of the single Hayward BH [69]. It is found that the geometric appearance of the Hayward BH shadow is related to the spacetime geometry, while the optical appearance is affected by the accretion flow property and BH magnetic charge. In this analysis, we generalize the study of the single BH shadow to the case of the BH binary system. On the one hand, our research motivation is based on the optical characteristics of the disk structure around the BH. On the other hand, we aim to discuss some key information of the wormholes through studying the BH binaries model. Hence, we investigate the optical appearance of the Hayward BH binary system.

The organization of this work is as follows. Section 2 construct a Hayward BH binary system and discuss the effective potential. In section 3, we present light deflection of the Hayward BH binary system. In Section 4, when a thin accretion disk surrounded the Hayward BH binary system, we analyze the ring classification and transfer functions as well as the corresponding optical appearance. We draw the conclusions in section 5.
Figure 1. Cartoon of the TSW configuration of a BH binary system. Two spacetimes are connected by a throat.

2 Hayward BH binary system and the effective potential

The Hayward BH metric can be written as [68]

\[ ds^2 = -f(r)dt^2 + f(r)^{-1}dr^2 + r^2(d\theta^2 + \sin^2 \theta d\phi^2), \]

where \( f(r) \) is the metric potential,

\[ f(r) = 1 - \frac{2Mr^2}{r^3 + g^3}, \]

in which \( M \) is the mass, and \( g \) is the magnetic charge of the BH. Defining the two spacetimes as \( \mathcal{M}_1 \) and \( \mathcal{M}_2 \), a new manifold can be written as \( \mathcal{M} \equiv \mathcal{M}_1 \cup \mathcal{M}_2 \), where the two spacetimes is connected by a throat (Fig.1). A Hawyard TSW is constructed by cutting the interior region of the two Hayward spacetimes and gluing them together at the hypersurface.

According to Eq. (2.1), the two spherically symmetric metrics are connected by the throat, we have

\[ ds_i^2 = -f_i(r_i)dt_i^2 + f_i(r_i)^{-1}dr_i^2 + r_i^2(d\theta_i^2 + \sin^2 \theta_i d\phi_i^2), \]

where \( i = 1, 2 \) represent two spacetimes \( \mathcal{M}_1 \) and \( \mathcal{M}_2 \). The \( f_i(r_i) \) is

\[ f_i(r_i) = 1 - \frac{2M_i r_i^2}{r_i^3 + g_i^3}, \quad r_i \geq R, \]

in which \( R \) is the throat radius, satisfying \( R > \max\{r_{+1}, r_{+2}\} \) (\( r_+ \) is the event horizon radius). In case of the magnetic charge in both the spacetime is zero, the throat connects the two Schwarzschild BHs. If the magnetic charge in one of the spacetime is zero, the throat connects the Schwarzschild-Hayward BHs in the binary system. If the magnetic charge in both spacetimes are non-zero, the throat connects the two Hayward BHs in the binary system.

In order to investigate the deflection of lights in the BH binary system, we derive the effective potential of the Hayward BH binary system. We only consider the photons move on the equatorial plane. If the interaction between the photon and throat is only controlled by the gravity, the photon 4-momentum is invariant in crossing the throat. According to the
Table 1. The critical impact parameters of the Hayward BH binary system, where the BHs mass as $M_1 = 1$, $M_2 = 1.2$, and the magnetic charge taking as $g = 0, 0.2, 0.4, 0.6, 0.8$.

| $b_{c_i}/g$ | 0   | 0.2 | 0.4 | 0.6 | 0.8 |
|-------------|-----|-----|-----|-----|-----|
| $b_{c_1}$   | 5.19615 | 5.19461 | 5.18373 | 5.15336 | 5.09013 |
| $b_{c_2}$   | 6.23538 | 6.23431 | 6.22679 | 6.20603 | 6.16412 |

TSW definition, the metric of the spacetime $\mathcal{M}$ is continuous, i.e. $g_{\mu\nu}^{M_1}(R) = g_{\mu\nu}^{M_2}(R)$ [64], the motion equation of the null geodesic can be written as

$$\frac{\dot{p}_t^2}{f_1(r)} - \frac{\dot{p}_{\phi_i}^2}{r_i^2} = \frac{(p_{t_i}^r)^2}{f_1(r)},$$

(2.5)

where $p^\mu = dx^\mu/d\lambda$ is the photon 4-momentum, $\lambda$ is an affine parameter. $p_t$ and $p_{\phi_i}$ represent the energy ($p_t = -E_i$) and angular momentum ($p_{\phi_i} = L_i$) of the photon, which are conserved along the geodesic by considering the Killing symmetries of spacetime. Utilizing Eq. (2.5), the radial component of the null geodesic is

$$p_{t_i}^r = \pm E_i \sqrt{1 - \frac{b_i^2}{r_i^2} f_1(r)},$$

(2.6)

where $b$ is the impact parameter, defining as $b \equiv p_{\phi_i}/p_t = |L_i|/E$. From Eq. (2.6), the effective potential $V_{\text{eff}}$ of the Hayward BH binary system is obtained, i.e.

$$V_{\text{eff}} = \frac{f_1(r)}{r_i^2} = \frac{1}{r_i^2} \left(1 - \frac{2Mr_i^2}{r_i^3 + g_i^3}\right).$$

(2.7)

The photon ring orbit satisfies the effective potential critical conditions

$$V_{\text{eff}}(r_{ph_i}) = \frac{1}{b_{c_i}^2}, \quad V'_{\text{eff}}(r_{ph_i}) = 0,$$

(2.8)

in which $r_{ph_i}$ is the photon ring radius and $b_{c_i}$ is the critical impact parameter. Table ?? reports the critical impact parameters of the Hayward BH binary system with different magnetic charges. It is found that the increase of $g$ value leads to the decrease of the $b_{c_i}$ value, indicating that the photon ring is shrunk inward the Hayward BH by increasing the magnetic charge.

Figure 2 illustrates the Hayward BH binary system effective potential as a function of radius, which shows essentially different from the case of a single Hayward BH. One can observe that the throat connects two spacetimes $\mathcal{M}_1$ and $\mathcal{M}_2$. If the mass of the BHs satisfies $M_1 = M_2$, the effective potential functions are the same on both sides of the throat, implying that the critical curves of the two BHs are consistent. If the mass of the BHs satisfies $M_1 < M_2$, the effective potential of two spacetimes is equal at the throat, and the highest point of the effective potential curve in the $\mathcal{M}_2$ spacetime is greater than that in the $\mathcal{M}_1$ spacetime. It is indicated that the effective potential function of the spacetime $\mathcal{M}_2$ can reflect some lights from the spacetime $\mathcal{M}_1$. We also find that the more significant magnetic charge leads to a more substantial peak effective potential, which is similar to the single Hayward BH.
Figure 2. Hayward BH binary system effective potential as a function of radius for different magnetic charge. The blue and red curves represent two Hayward spacetimes. The BHs mass are taken as $M_1 = 1$, $M_2 = 1.2$, and the throat radius is taken as $R = 2.6$. The gray curve represents the effective potential function with the same spacetime parameters ($M_1 = M_2 = 1.2$ and $g_1 = g_2 = 0.5$).

3 Light trajectory and deflection angle of the Hayward BH binary system

The light deflection is divided into three situations for the single BH. In the case of $b > b_c$, the light ray is reflected the observer area under gravity action. In the case of $b < b_c$, the light ray enters the BH inside and falls into the singularity. The light ray rotates many times on the BH photon orbit and generates a bright ring for the $b = b_c$ scenario [27]. Due to the behavior of the photons in the BH binary system is rather differently from the single BH system, we must re-consider the light deflection in this analysis.

Assuming that the light enter the spacetime $\mathcal{M}_1$ from infinity and the impact parameter is $b_1$. The light deflection is divided into three situations according to the mass relationship of the BH binary:

- **Case 1** $M_1 = M_2$: The critical impact parameter satisfies $b_{c_1} = b_{c_2}$. The light passes through the throat into the spacetime $\mathcal{M}_2$ within the necessary condition $b_1 < b_{c_1}$, and then returns to the spacetime $\mathcal{M}_1$.

- **Case 2** $M_1 > M_2$: The critical impact parameter satisfies $b_{c_1} > b_{c_2}$. If $b_1 < b_{c_2}$, the light passes through the throat into the spacetime $\mathcal{M}_2$, and then returns to the spacetime $\mathcal{M}_1$. If $b_1 = b_{c_2}$, the light will provide additional brightness for the BH photon ring in the spacetime $\mathcal{M}_2$. If $b_1 > b_{c_2}$, the light goes to infinity in the spacetime $\mathcal{M}_2$.

- **Case 3** $M_1 < M_2$: The critical impact parameter satisfies $b_{c_1} < b_{c_2}$. The light passes through the throat into the spacetime $\mathcal{M}_2$, and then returns to the spacetime $\mathcal{M}_1$. Note that some additional lights from $\mathcal{M}_2$ will enter $\mathcal{M}_1$ and reach infinity in this scenario.

The impact parameters of two spacetimes satisfy [64]

$$\frac{b_1}{b_2} = \sqrt{\frac{f_2(R)}{f_1(R)}} \equiv H. \quad (3.1)$$

In summary, if the impact parameter satisfies $Hb_{c_2} < b_1 < b_{c_1}$, the lights enter the spacetime $\mathcal{M}_1$ from infinity and fall to the spacetime $\mathcal{M}_2$ through the throat, then return, enter the
spacetime $\mathcal{M}_1$ through the throat and return to infinity. Hence, we focus on Case 3 in the following discussion since it can fully display how the photons are passing through the throat move.

Employing the ray-tracing code, we reveal the light trajectory in the BH binary system. The light trajectory is determined by the orbit equation (2.5), which can be rewritten as

$$\frac{1}{b_i^2} - \frac{f_i(r_i)}{r_i^2} = \frac{1}{r_i^2} \left(\frac{dr_i}{d\phi_i}\right)^2. \quad (3.2)$$

By introducing a parameter $u \equiv 1/r$, one can get

$$\Omega_i(u_i) \equiv \frac{du_i}{d\phi_i} = \sqrt{\frac{1}{b_i^4} - u_i^2 \left[1 - \frac{2M_i}{u_i^2(g^3 + \frac{1}{u_i})}\right]}. \quad (3.3)$$

Taking the several representative magnetic charge values as an example, Fig.3 shows the light trajectories in the Hayward (Schwarzschild) BH binary system. Assuming that the lights are coming from far right in $M_1$, one can observe that the smaller impact parameter leads the lights to have more motion regions on the spacetime $M_2$.

For $b_1 > b_{c1}$, the inflection point of the light in the spacetime $\mathcal{M}_1$ corresponds to the minimally positive real root of $\Omega_1(u_1)$, defining as $u_1^{\text{min}}$. Utilizing Eq. (3.3), the total change of azimuthal angle in the spacetime $\mathcal{M}_1$ can be written as

$$\phi_1(b_1) = 2 \int_0^{u_1^{\text{min}}} \frac{du_1}{\sqrt{\Omega_1(u_1)}}, \quad b_1 > b_{c1}. \quad (3.4)$$

This situation is similar to the single BH. For $Hb_{c2} < b_1 < b_{c1}$, the change of the azimuthal angle (throat outside) in the spacetime $\mathcal{M}_1$ can be given as

$$\phi_1^*(b_1) = \int_0^{1/R} \frac{du_1}{\sqrt{\Omega_1(u_1)}}, \quad b_1 < b_{c1}. \quad (3.5)$$

Then, the light falls to the spacetime $\mathcal{M}_2$ through the throat. The inflection point of the light in the spacetime $\mathcal{M}_2$ corresponds to the maximally positive real root of $\Omega_2(u_2)$, defining as $u_2^{\text{max}}$. According to Eq. (3.3), the change of azimuthal angle in the spacetime $\mathcal{M}_2$ is obtained, i.e.

$$\phi_2(b_2) = 2 \int_{u_2^{\text{max}}}^{1/R} \frac{dw_2}{\sqrt{\Omega_2(w_2)}}, \quad b_2 > b_{c2}. \quad (3.6)$$

4 Optical appearance of the Hayward BH binary system

4.1 Light rays classification

Gralla et al. proposed that the radiation originates from an optically and geometrically thin accretion disk in the equatorial plane of the BH [27]. According to the number of times the light intersects the accretion disk and the number of light orbits ($n \equiv \phi/2\pi$), the light rays are classified as: $i$) $n > 1/4$, the light falls on the front of the disk and it intersects the equatorial plane just once; $ii$) $n > 3/4$, the light breaks through the thin disk and falls on the back of the disk, and it intersects the equatorial plane twice; $iii$) $n > 5/4$, the light arriving at the
Figure 3. Light trajectories in the polar coordinates \((r, \phi)\) with the impact parameters in the range \(Hb_c < b_1 < b_c\). The blue curves represent the light trajectories in the spacetime \(M_1\) \((M_1 = 1)\), and the green dashed curves represent the light trajectories in the spacetime \(M_2\) \((M_2 = 1.2)\). Left Panel—magnetic charge \(g = 0\) (Schwarzschild scenario), Middle Panel—magnetic charge \(g = 0.5\) and Right Panel—magnetic charge \(g = 0.8\). We take throat radius \(R = 2.6\).

front side of the accretion disk once again, and it intersects the equatorial plane more than three times. These additional intersections contribute extra brightness to an observer.

For the BH binary system, we assume that a static observer and thin accretion disk are located in the spacetime \(M_1\). The static observer is in the north pole, the accretion disk is in the equatorial plane, and the disk emits isotropically in the rest frame of static worldlines. Considering the lights enter the spacetime \(M_1\) and fall to spacetime \(M_2\) through the throat, the number of orbits of the BH binary system are defined as [65]:

\[
n_1(b_1) = \frac{\phi_1(b_1)}{2\pi},
\]

\[
n_2(b_2) = \frac{\phi_1'(b_1) + \phi_2(b_1/H)}{2\pi},
\]

\[
n_3(b_1) = \frac{2\phi_1'(b_1) + \phi_2(b_1/H)}{2\pi}.
\]

The case of orbits number \(n_1\) is equivalent to the single BH situation. When \(n_2 < 3/4\) and \(n_3 > 3/4\), the reflected outgoing lights in the spacetime \(M_1\) fall on the back of the disk.
Figure 4. Total orbit number as a function of the impact parameter for the Hayward BH binary system. The BHs mass are taken as $M_1 = 1$, $M_2 = 1.2$ and the throat radius is taken as $R = 2.6$. The blue, green, and pink curves represent the magnetic charges $g = 0$ (Schwarzschild BH binary scenario), $g = 0.5$ and $g = 0.8$, respectively.

When $n_2 < 5/4$ and $n_3 > 5/4$, the reflected outgoing lights in the spacetime $M_1$ fall on the front of the disk.

Figure 4 shows the total number of orbits as a function of the impact parameter for the Hayward BH binary system. One can see that the orbits function $n_1$ of the Hayward BH binary system is similar to the single Hayward BH, illustrating that the BH binary system image seen by an observer contains the single BH image. Note that the additional orbits functions $n_2$ and $n_3$ appear in the BH binary situation, which indicates the image of the BH binary system should contain additional ring structure. It is found that the increase of $g$ value leads to the decrease of the $b$ range in comparison with the Schwarzschild BH binary system ($g = 0$), implying that the additional ring is shrunk inward the BH by increasing the magnetic charge.

4.2 Observed intensity and transfer functions

As discussed above, the light can pick up additional brightness from the multiple intersection between the light and the accretion disk. The total observed intensity ($\text{ergs}^{-1}\text{cm}^{-2}\text{str}^{-1}\text{Hz}^{-1}$) should be the sum of those intensities. Based on the Liouville’s theorem, $I_\text{e}/(\nu_\text{e})^3$ is conserved in the direction of light propagation, where $I_\text{e}$ is the radiation specific intensity and $\nu_\text{e}$ is the radiation frequency in a static frame. An observer in infinity receive the specific intensity $I_\text{o}$ with red-shifted frequency $\nu_\text{o} \equiv \sqrt{f}\nu_\text{e}$. Considering $I/\nu^3$ is conserved along a ray, we have

$$\frac{I_\text{o}}{\nu_\text{o}^3} = \frac{I_\text{e}}{\nu_\text{e}^3}. \quad (4.4)$$

For a single frequency, the observed specific intensity is

$$I_\text{o}(r) = f(r)^{3/2}I_\text{e}(r). \quad (4.5)$$

By integrating over the whole range of received frequencies, the total observed intensity can be written as

$$I_\text{S}(r) = \int I_\text{o}(r)\,d\nu_\text{o}$$

$$= \int f(r)^2I_\text{e}(r)\,d\nu_\text{e} = f(r)^2I_\text{R}(r), \quad (4.6)$$
Figure 5. Transfer functions of the Hayward BH binary system with magnetic charges of $g = 0$ (Schwarzschild BH binary scenario) [Panel (a)], $g = 0.5$ [Panel (b)], and $g = 0.8$ [Panel (c)]. The black, blue, and green curves are for the direct emission, lensing band, and photon ring group, respectively. The BHs mass are taken as $M_1 = 1$, $M_2 = 1.2$, and the throat radius is taken as $R = 2.6$.

where $I_R(r) \equiv \int I_e(r) dv_e$ is the total radiation intensity from the thin accretion disk. The total observed intensity is

$$I_{\text{obs}}(b) = \sum_n f(r)^2 I_R|_{r=r_n(b)}, \quad (4.7)$$

in which $r_n(b)$ is the transfer function, denoting the radial coordinate of the $n$th intersection between the light with impact parameter $b$ and the accretion disk. The slope of the $r_n(b) - dr/db$ - is defined as the (de)magnification factor [27]. For the single BH, the first transfer function ($n = 1$) represents “direct emission”, the second ($n = 2$) and third ($n = 3$) transfer functions represent “lensing ring” and “photon ring”, respectively.

Figure 5 shows the transfer function $r_n(b)$ as a function of impact parameter $b$ for several representative magnetic charges. The black lines correspond to the first transfer function ($n = 1$), which is similar to the single Hayward BH. One can observe that $r_1(b)$ is proportional to $b$ and the slope approximately equal to 1, indicating the the direct emission contributes the most to the total observed flux. The blue lines correspond to the second transfer function ($n = 2$). It is found that an irregular curve with a slightly wider range of the impact parameter range connected at the lower end of the asymptotic curve, illustrating that the observer sees a “lensing band” in the BH binary situation. The green lines correspond to the third transfer function ($n = 3$). In addition to the usual third transfer function (an almost vertical line near $b_{c1}$), two new third transfer functions present. One of them is close to $Hb_{c2}$ and has the same slope as the usual third transfer function. The other one is on the left of $b_{c1}$, and its slope is less than the usual third transfer function but greater than the usual second transfer function. Thus, the observer sees a “photon ring group”.

4.3 Optical appearance of Hayward BH binary system

Assuming two radiation functions of the thin accretion disk, we investigate the optical appearance of the Hayward BH binary system according to the total observed intensity function and the transfer function. It is well known that the radiation of the accretion disk in the Universe satisfies Gaussian distribution [69]. We parameterize the radiation function of the accretion disk as a Gaussian function.
Figure 6. The radiation intensity as a function of the radius. The green and red curves are for the Model I and Model II.

Taking the Hayward BH mass as 1, the radius of innermost stable circular orbit is derived as \( r_{\text{isco}} \approx 6 \). Firstly, the Model I is assumed to \( r = 6 \) is inner radii at which the accretion stops radiating. We assume that the radiation function is

\[
I_{R_1}(r) = \begin{cases} 
\exp \left[ -\frac{(r-6)^2}{8} \right] & r > 6, \\
0 & r \leq 6.
\end{cases}
\tag{4.8}
\]

Secondly, we assume that the radiation function of the Model II falls to zero more smoothly than the Model I, so as to form a contrast with the Model I. In this scenario, the event horizon radius of the Hayward BH \( (r_+ \approx 2) \) is assumed to the inner radii at which the accretion stops radiating. We have

\[
I_{R_2}(r) = \begin{cases} 
\frac{\pi}{2} - \arctan(r-5) & r > 2, \\
\frac{\pi}{2} - \arctan(-3) & r \leq 2.
\end{cases}
\tag{4.9}
\]

Figure 6 illustrates the radiation intensity as a function of the radius for two models. For Model I, the total observed intensity \( I_{\text{obs}} \) as a function of the impact parameter \( b \), the two-dimensional image in the celestial coordinates, and the local density image are displayed in Fig. 7. It is found that the regions of the direct emission, lensing band, and photon ring group are separated. Taking the magnetic charge \( g = 0.5 \) (Fig. 7 middle panel) as an example, the direct emission starts at \( b \approx 7.04M_1 \) and peaks at \( b \approx 7.56M_1 \). Its maximum intensity is about 0.47. The lensing band is limited to a small range of \( b \approx 5.57M_1 \sim 5.83M_1 \). The photon ring group appears at \( b \approx 5.17M_1 \), \( b \approx 4.93M_1 \), and \( b \approx 3.69M_1 \). In the two-dimensional image, the boundary of the black disk corresponds to the direct radiation starting position. A bright lensing band is shown within the black disk, while the position of the photon ring group continue to move toward the central region and appears as several significantly weaker ring. One can observe that one of the new photon ring is near \( Hb_{c_2} \), and the other new photon ring is inside the lensing band and outside \( b_{c_1} \). Note that the new second transfer function is insensitive to the observed function in this model. The top and bottom panels of Fig. 7 shows the Schwarzschild BH binary scenario and the Hayward BH binary system under magnetic...
Figure 7. Model I—The total observed intensity as a function of the impact parameter (left panel) for the several representative magnetic charges and the corresponding two-dimensional images (middle panel) together with zooming in part of the image for illustrating their local density (right panel). The top, middle, and bottom panels are for \( g = 0 \) (Schwarzschild BH binary scenario), \( g = 0.5 \), and \( g = 0.8 \), respectively. The BHs mass are taken as \( M_1 = 1 \), \( M_2 = 1.2 \), and the throat radius is taken as \( R = 2.6 \).

charge \( g = 0.8 \). One can observe that the increase of the magnetic charge value leads to the new photon ring is shrunk inward the Hayward BH.

Figure 8 shows the total observed intensity as a function of the impact parameter, the two-dimensional image, and local density image for Model II. One can observe that direct emission, lensing band, and photon ring group regions are overlapped. Taking the magnetic charge \( g = 0.5 \) (Fig. 8 middle panel) as an example, the direct emission starts at \( b \simeq 2.85M_1 \), and the photon ring group embed in the lensing band, presenting the bright and unique multilayer ring structure. Note that the lensing band consists of the usual lensing ring and an additional lensing band. The usual lensing band is limited in the range of \( b \simeq 5.24M_1 \sim 5.51M_1 \) and the additional lensing band between \( Hb_{c2} \) and \( b_{c1} \), implying that the new second transfer function contributed to the observed intensity of the Hayward BH binary system in this radiation model.
Figure 8. Model II– The total observed intensity as a function of the impact parameter (left panel) for the several representative magnetic charges and the corresponding two-dimensional images (middle panel) together with zooming in part of the image for illustrating their local density (right panel). The top, middle, and bottom panels are for $g = 0$ (Schwarzschild BH binary scenario), $g = 0.5$, and $g = 0.8$, respectively. The BHs mass are taken as $M_1 = 1$, $M_2 = 1.2$, and the throat radius is taken as $R = 2.6$.

5 Conclusions

In this analysis, we generalized the study of the single BH shadow to the case of the BH binary system and investigated the optical appearance of the Hayward BH binary system. By constructing the Hayward TSW model, we derived the effective potential of the Hayward BH binary system and plotted the potential function curve. It was shown that a throat connects two BHs spacetimes, and the effective potential function of the contralateral spacetime may reflect a significant fraction of the light back to the spacetime of its origin.

From the null geodesic equation, we obtained the specific expression of the radial component. The critical impact parameter with different magnetic charges were acquired. We found that the increase of $g$ value leads to the decrease of the $b_{ci}$ value, indicating that the photon ring is shrunk inward the Hayward BH by increasing the magnetic charge. The motion of photons of the Hayward BH binary system is investigated, which found that the light deflection is divided into three situations according to the mass relationship of the BH binary.
If the impact parameter satisfies $H b_c^2 < b_1 < b_c$, the lights enter the spacetime $\mathcal{M}_1$ from infinity and fall to the spacetime $\mathcal{M}_2$ through the throat, then return, enter the spacetime $\mathcal{M}_1$ through the throat and return to infinity. For lights with different impact parameters, we calculated the total deflection angle $\phi$ of the Hayward BH binary system.

By comparing the number of orbits of the single BH and BH binary system, we discussed the behavior of photons with different total number $n$ of orbits. The behavior of the total orbit number as a function of the impact parameter is shown in Fig 4. It is observed that the orbits function $n_1$ of the Hayward BH binary system is equivalent to the single Hayward BH, illustrating that the BH binary image seen by an observer contains the single BH. In particular, the additional orbits functions $n_2$ and $n_3$ appear in BH binary situation, which indicates that the image of the BH binary system should contain additional ring structure. To obtain the observed intensity, we discussed its relationship to the radiation intensity. With the ray-tracing procedure, the light can pick up additional brightness from the multiple intersection between the light and the accretion disk. Hence we investigated the transfer function $r_n(b)$ of the BH binary system. It is found that the first transfer function of the Hayward BH binary system is similar to the single Hayward BH. The second transfer function has a monotonic function and an irregular segment that appears before the monotone increasing part, meaning that corresponding to the resulting image should be a “lensing band”. In addition to the usual third transfer function (single BH), it also has two new third transfer functions, leading the observer sees a “photon ring group” in BH binary situation.

Basing on above discussions, the optical appearance of the Hayward BH binary system is derived. We parameterized the radiation function of the accretion disk as a Gaussian function and investigated the observed function as well as the two-dimensional image. By considering two Gaussian radiation models, we found that the regions of the direct emission, lensing band, and photon ring group are separated for Model I. A bright lensing band is shown within the black disk, while the position of the photon ring group continue to move toward the central region and appears as several significantly weaker ring. Besides, the regions of the direct emission, lensing band, and photon ring group are overlapped for Model II. The photon ring group embed in the lensing band, presenting the bright and unique multilayer ring structure. These results suggest that the optical appearance of the BH binary can provide a potentially qualitatively observational signature to observe wormholes in the future.

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