Radiative Corrections to Triple Higgs Coupling and Electroweak Phase Transition: Beyond One-loop Analysis

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We evaluate dominant two-loop corrections to the triple Higgs coupling and strength of a first-order electroweak phase transition in the inert Higgs doublet model. It is found that sunset diagrams can predominantly enhance the former and reduce the latter. As a result, the triple Higgs coupling normalized by the standard model value at two-loop level is more enhanced than the corresponding one-loop value.

I. INTRODUCTION

Higgs mechanism is one of the fundamental footings of the standard model (SM). In spite of the new scalar particle with a mass of 125 GeV was discovered at the LHC [1], its roles as the mass giver for all the SM particles and the symmetry breaker for SU(2)_L × U(1)_Y have not been fully established yet. For the latter, measurement \( \lambda \) which constrains the mass of the heavy Higgs bosons have the nondecoupling heavy Higgs bosons play a pivotal role, and the symmetry breaker for SU(2)

In Ref. [4], \( \lambda_{hhh} \) is calculated at one-loop level in a softly Z_2-broken two Higgs doublet model (2HDM). It is found that the extra heavy Higgs boson loop corrections can significantly enhance \( \lambda_{hhh} \). As a result, \( \kappa_\lambda \) can be \( O(2-4) \). This can occur if the one-loop corrections of the heavy Higgs bosons have the nondecoupling properties, i.e., the power corrections such that \( \lambda_{hhh} \) is still relevant to the correlation between \( \kappa_\lambda \) and strength of the first-order EWPT.

II. CALCULATION SCHEME

We expand the effective \( hh \) vertex defined in the on-shell (OS) scheme in powers of momenta as

\[
\Gamma_{hhh}(p_1^2, p_2^2, p_3^2) \simeq \hat{\Gamma}_{hhh}(0, 0, 0) + \cdots \tag{1}
\]

Apart from a threshold enhancement that occurs when incoming momentum is twice as large as the masses of particles running in loops, the dominant quantum contributions come from the momentum-independent part \([4]\). Moreover, since we are interested in a deviation of the effective \( hhh \) vertex from the SM value in new physics models such as the IDM, the ratio of \( \Gamma_{hhh}^{NP}(p_1^2, p_2^2, p_3^2)/\Gamma_{hhh}^{SM}(p_1^2, p_2^2, p_3^2) \) is well approximated by \( \Gamma_{hhh}^{NP}(0, 0, 0)/\Gamma_{hhh}^{SM}(0, 0, 0) \equiv \kappa_\lambda \). Therefore, we will exclusively focus on the momentum-independent term in Eq. (1) in this Letter. Calculation of \( \hat{\Gamma}_{hhh}(0, 0, 0) \) is greatly simplified if an effective potential is used. Let us define \( \lambda_{hhh} \) as

\[
-\hat{\Gamma}_{hhh}(0, 0, 0) \equiv \lambda_{hhh} = \hat{Z}_h^{\lambda/2} \lambda_{hhh}, \tag{2}
\]

where \( \lambda_{hhh} \) is the third derivative of the effective potential \( V_{eff} \) defined in the \( \overline{\text{MS}} \) scheme and \( \hat{Z}_h = Z_h^{\overline{\text{MS}}}/Z_h^{\overline{\text{OS}}} \) with \( Z_h^{\overline{\text{OS}}} \) being the Higgs wavefunction renormalization constant in the OS scheme and \( Z_h^{\overline{\text{MS}}} \) in the \( \overline{\text{MS}} \) scheme.

Though \( \lambda_{hhh} \) is calculated up to two-loop level in supersymmetric SMs \([11]\), the analytic expression of \( \lambda_{hhh} \) in the SM seems absent in the literature. We thus start with the SM case using our calculation scheme.

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The tree-level Higgs potential is given by

$$V_0(\Phi) = -\mu_0^2 \Phi^\dagger \Phi + \lambda (\Phi^\dagger \Phi)^2, \quad \Phi = \left( \begin{array}{c} G^+ \\ (v + h + iG^0) \end{array} \right),$$

where $v$ denotes the vacuum expectation value (VEV) of the Higgs boson ($h$) and $G^{0,\pm}$ are the Nambu-Goldstone bosons. We calculate $\lambda_{hhh}$ using the $\overline{\text{MS}}$-regularized effective potential at two-loop level \cite{12, 13}, which is expanded as

$$V_{\text{eff}}(\varphi) = V_0(\varphi) + V_1(\varphi) + V_2(\varphi),$$

where $\varphi$ denotes the background classical field. With this, the Higgs mass and the triple Higgs coupling are, respectively, defined as

$$m_h^2 = \left. \frac{\partial^2 V_{\text{eff}}}{\partial \varphi^2} \right|_{\varphi = v} = 2\lambda v^2 + D_m \Delta V_{\text{eff}}(\varphi),$$

$$\lambda_{hhh}^{\text{SM}} = \left. \frac{\partial^4 V_{\text{eff}}}{\partial \varphi^4} \right|_{\varphi = v} = \frac{3m_h^2}{v} + D_\lambda \Delta V_{\text{eff}}(\varphi),$$

where

$$D_m = \left[ \frac{\partial^2}{\partial \varphi^2} - \frac{1}{v} \frac{\partial}{\partial \varphi} \right]_{\varphi = v},$$

$$D_\lambda = \left[ \frac{3}{v} \left( \frac{\partial^2}{\partial \varphi^2} - \frac{1}{v} \frac{\partial}{\partial \varphi} \right) \right]_{\varphi = v},$$

with $\Delta V_{\text{eff}}(\varphi) = V_1(\varphi) + V_2(\varphi)$. Note that $\mu_0$ is eliminated by a minimization condition so that $m_h^2$ is defined in the minimum of $V_{\text{eff}}$. Furthermore, since $\lambda$ is replaced by $m_h^2$ using Eq. (5), the leading log corrections at each loop in Eq. (4) are absorbed by the Higgs mass renormalization as explicitly demonstrated below.

The dominant two-loop contributions arise from the sunset diagrams as depicted in Fig. 1. From them, one can obtain the $O(g_3^2 y_t^2)$ and $O(y_t^6)$ corrections to $\lambda_{hhh}^{\text{SM}}$, where $g_3$ and $y_t$ are the SU(3)$_C$ and top Yukawa couplings, respectively. Combining the dominant one-loop contribution coming from the top quark, one finds

$$\lambda_{hhh}^{\text{SM}} = \frac{3m_h^2}{v} + \Delta^{(1)} \lambda_{hhh}^{\text{SM}} + \Delta^{(2)} \lambda_{hhh}^{\text{SM}},$$

where

$$\Delta^{(1)} \lambda_{hhh}^{\text{SM}} = \frac{1}{16\pi^2} \left( -\frac{48m_h^4}{v^3} \right),$$

$$\Delta^{(2)} \lambda_{hhh}^{\text{SM}} = \frac{1}{(16\pi^2)^2} \frac{m_t^4}{v^3} \left[ 768g_3^2 \left( \ell_t + \frac{1}{6} \right) - 144g_3^2 \left( \ell_t - \frac{7}{6} \right) \right],$$

with $m_t = y_t v/\sqrt{2}$ and $\ell_t = \ln(m_t^2/\bar{\mu}^2)$ with $\bar{\mu}$ being the renormalization scale. As mentioned above, the leading-log terms at each loop level are absorbed by the $m_h$ renormalization. As a result, the one-loop leading contribution becomes $O(m_t^4)$. Likewise, after absorbing the double-log terms at two-loop level, one has $O(m_t^4)$ with extra coefficients including the single log terms. Noting that all the parameters appearing in $\lambda_{hhh}^{\text{SM}}$ are the $\overline{\text{MS}}$ running parameters, the $\ell_t$ terms at two-loop level can be absorbed into $m_t$ at one-loop order using the renormalization group (RG) equations. After expressing all the $\overline{\text{MS}}$ variables evaluated at the pole top quark mass ($M_t = 173.1$ GeV \cite{15}) with the physical ones \cite{10}, one arrives at

$$\lambda_{hhh}^{\text{SM}} = 3\frac{m_t^2}{v_{\text{phys}}} \left[ \frac{1}{16\pi^2} \left( -\frac{16M_t^4}{M_t^2 v_{\text{phys}}^2} + \frac{7}{2} \frac{M_t^2}{v_{\text{phys}}^2} \right) \right] + \frac{1}{(16\pi^2)^2} \frac{16M_t^4}{M_t^2 v_{\text{phys}}^2} \left[ 24g_3^2 + \frac{7M_t^2}{v_{\text{phys}}^2} \right]$$

$$= (190.38 \text{ GeV}) \times \left[ 1 - 8.5\% + 2.2\% \right] = 178.36 \text{ GeV},$$

where $v_{\text{phys}} = 1/(\sqrt{2}G_F)$ with $G_F(\approx 1.166 \times 10^{-5}$ GeV$^{-2}$) being the Fermi coupling constant, $M_h = 125.0$ GeV is the pole mass of the Higgs boson and $g_3(M_t) = 1.167$. One can see that $\lambda_{hhh}$ gets enhanced compared to the one-loop result. Note that the additional one-loop correction arises when converting the $\overline{\text{MS}}$ parameters into the OS ones, which has the $+1.1\%$ contribution. Even though it is subleading at one loop order, it is comparable to the two-loop corrections so that it is not negligible. In our numerical study, we also take the leading one-loop corrections of the gauge and Higgs bosons into account. Our numerical calculations show that $\lambda_{hhh}^{\text{SM}} = 176.23$ GeV and 180.17 GeV at one and two-loop levels, respectively \cite{17}, and the analytic formula \cite{12} gives a good approximation. We have checked that the corresponding one-loop value of H-COUP \cite{18} is $\lambda_{hhh}^{\text{SM}} = 177.96$ GeV, so the relative error is 0.9%. The difference may come from subleading gauge bosons contributions that are missing in our calculation.

III. MODEL

As a benchmark model, we consider the inert two Higgs doublet model (IDM) in which a $Z_2$-odd Higgs
doublet ($\eta$) is added to the SM [19 21]. It is known that the model can accommodate both the strong first-order EWPT and the dark matter (DM) candidate simultaneously [22 24]. We quantify the leading two-loop corrections of the extra Higgs bosons to $\lambda_{hhh}$ in a cosmologically interesting region.

As a result of the $Z_2$ parity ($\Phi \rightarrow \Phi$ and $\eta \rightarrow -\eta$), the Higgs potential is cast into the form

$$V_0^{\text{IDM}}(\Phi, \eta) = \mu_1^2 \Phi^4 + \mu_2^2 \eta^4 + \lambda_1 \Phi^2 \eta^2 + \lambda_2 \Phi^4 + \lambda_3 \Phi \eta^4,$$

where $\Phi$ is the same as in the SM given in Eq. (3) and $\eta$ is parametrized as

$$\eta = \left( \frac{\sqrt{H^+}}{v_2} \right).$$

The Higgs boson masses are expressed at tree level as $m_h^2 = \lambda_1 v^2$ and $m_\phi^2 = \mu_1^2 + \lambda_{h\phi\phi} v^2/2$ for $\phi = H, A, H^\pm$, where $\lambda_{hHH} = \lambda_3 + \lambda_4 + \lambda_5$, $\lambda_{hAA} = \lambda_3 + \lambda_4 - \lambda_5$, and $\lambda_{hH^+H^-} = \lambda_3$. In our analysis, $H$ is assumed to be the DM. The pole Higgs masses are denoted as $M_h$, $M_H$, $M_A$ and $M_{H^\pm}$, respectively, and we trade $\{m_h^2, m_\phi^2, \lambda_1, \lambda_2, \lambda_3, \lambda_4, \lambda_5\}$ with $\{v_{\text{phys}}, M_h, M_H, M_A, M_{H^\pm}, \lambda_{hHH}, \lambda_{hAA}, \lambda_{hH^+H^-}\}$ as the input parameter set.

It is easy to obtain the one-loop contributions of the extra particles to $\lambda_{hhh}$, which takes the form [21]

$$\Delta^{(1)}\lambda_{hhh}^{\text{IDM}} = \sum_{\phi=H,A,H^\pm} n_\phi \frac{4m_\phi^4}{16\pi^2 v^2} \left( 1 - \frac{\mu_1^2}{m_\phi^2} \right)^3,$$

where $n_H = n_A = 1$ and $n_{H^\pm} = 2$. As found in Refs. [4 21], the one-loop correction can grow with $m_\phi^2$ if $\mu_1^2 \ll m_\phi^2$, i.e., $m_\phi^2 \approx \lambda_{h\phi\phi} v^2/2$ (nondecoupling regime). In the opposite limit of $m_\phi^2 \approx \mu_1^2$, on the other hand, $\Delta^{(1)}\lambda_{hhh}^{\text{IDM}}$ would be suppressed in the large $m_\phi$ limit (decoupling regime). As discussed below, the nondecoupling regime is exactly the condition that EWPT is strongly first order. In our work, we consider $M_H \approx M_h/2$ so that the former limit applies only for $A$ and $H^\pm$. Furthermore, $M_A = M_{H^\pm}$ is taken to avoid the $\rho$ parameter constraint [19]. In this case, one has $\lambda_{hAA} = \lambda_{hH^+H^-} = \lambda_3$.

At two-loop order, the dominant corrections to $\lambda_{hhh}$ come from the sunset diagrams with $O(\lambda_3^2)$ in magnitude. For illustrative purpose, we consider the case of $m_\phi \ll m_A, m_{H^\pm}$ in the nondecoupling regime. In this limit, $m_h$ can be considered as a small perturbation and hence the $O(\lambda_3^2)$ contributions have the simple form

$$\Delta^{(2)}\lambda_{hhh}^{\text{IDM}} \approx \sum_{\phi=A,H^\pm} n_\phi \frac{8n_\phi \lambda^2_{h\phi\phi} m_\phi^2}{(16\pi^2)^2 v} \left( \frac{\ell_\phi}{2} - 1 \right),$$

where $\ell_\phi = \ln(m_\phi^2/\mu_1^2)$. As is the top quark contribution, the log terms are absorbed into $m_\phi^2$ in Eq. (15) by use of the RG equations. Putting all together, one arrives at

$$\Delta^{(1)}\lambda_{hhh}^{\text{IDM}} + \Delta^{(2)}\lambda_{hhh}^{\text{IDM}} \approx \sum_{\phi=A,H^\pm} \frac{4m_\phi}{16\pi^2 v^2} m_\phi^4 \left[ m_\phi^2 (m_\phi) - \frac{4m_\phi^6}{16\pi^2 v^2} \right] + \cdots,$$

where $m_\phi(m_\phi)$ are the MS-running masses of $\phi$ evaluated at $m_\phi$. We choose $M_Z = 91.1876$ GeV as the input scale for all the running parameters as in Refs. [23 24]. Modification due to the two-loop contributions mainly comes through the RG running effects in the first term, which enhances $\lambda_{hhh}$.

It is found that the $O(M_h^2 m_\phi^2/(16\pi^2 v^2))$ terms appear when expressing $\lambda_{hhh}^{\text{IDM}}$ with the physical parameters. However, they are actually cancelled for $M_A = M_{H^\pm}$. Use of the analytic expression (17) yields overestimated results for nonzero $m_\phi^2$ so that we evaluate $\Delta^{(2)}\lambda_{hhh}^{\text{IDM}}$ numerically for our quantitative studies [17].

In Fig. 2 $\kappa_\lambda$ at one and two-loop levels are plotted as functions of $M_A$ with the red-dashed and solid-blue curves, respectively. We take $M_H = 62.7$ GeV, $\lambda_2 = 0.02$ and $\lambda_{hHH} = 4.6 \times 10^{-3}$ at the $M_Z$ scale.
be greater than a certain value due to the occurrence of \( \mu_2^2 < 0 \) that generates a nontrivial minimum along the inert doublet field direction, which could be deeper than the prescribed electroweak vacuum and thus excluded. In the chosen parameter set, it is found that \( \mu_2^2 \lesssim 0 \) for \( M_A \gtrsim 300 \text{ GeV} \). Within the allowed range, \( \kappa_\lambda \) at two-loop level can be enhanced up to about 2\%. It should be noted, however, that the further enhancement could be possible if the requirement of \( \mu_2^2 > 0 \) were absence. In the ordinary 2HDM, for instance, \( \lambda_{hhh} \) can receive \( O(100)\% \) corrections at one-loop level with increasing \( M_A \) as mentioned above. In this case, one may ask whether the power correction of the \( m_\phi^6 \) term in Eq. (17) can compete with the one-loop ones. Here, we give a simple argument that it would not happen. On the grounds of the dimensional analysis, the dominant power corrections to \( \lambda_{hhh} \) at \( \ell \)-loop order may be cast into the form

\[
\Delta^{(\ell)} \lambda_{hhh} \sim (-1)^{\ell+1} \frac{m_\phi^2}{v} \left( \frac{4 m_\phi^2}{16 \pi^2 v^2} \right)^\ell,
\]

where \( \mu_2^2 = 0 \) and combinatorial factors are ignored. If the expansion parameter \( m_\phi^2/(4\pi^2 v^2) \) is close to unity, one obtains \( m_\phi \simeq 2\pi v = 1546 \text{ GeV} \) which corresponds to \( \lambda_{h\phi\phi} \simeq 8\pi^2 \). This is clearly far beyond the perturbativity bound. Conversely, if we require \( \lambda_{h\phi\phi} = 2 m_\phi^2/v^2 < 4\pi \) as a crude perturbativity criterion, \( m_\phi^2/(4\pi^2 v^2) < 1/(2\pi) \simeq 0.16 \). Thus, the maximal two-loop power corrections amount to about \(-16\% \) of the one-loop ones.

**IV. \( \lambda_{hhh} \)-EWPT CORRELATION**

It is known that the remnant of the strong first-order EWPT can appear in \( \lambda_{hhh} \). Before conducting the two-loop analysis, we briefly outline the \( \lambda_{hhh} \)-EWPT correlation at one-loop. The criterion for the strong first-order EWPT is given by [2]

\[
\frac{\nu_C}{T_C} > \zeta_{sph},
\]

where \( T_C \) is a temperature at which there are two degenerate vacua in the effective potential, \( \nu_C \) is the Higgs VEV at \( T_C \), and \( \zeta_{sph} \) depends on the sphaleron profile etc, and typically, \( \zeta_{sph} \simeq 1 \). Use of the high-\( T \) expansion (HTE) of the one-loop thermal function [27] makes it easy to see the \( \lambda_{hhh} \)-EWPT correlation. AT \( T_C \) the effective potential is cast into the form

\[
V_{\text{eff}}(\varphi; T_C) = \frac{\lambda_{T_C}}{4} \varphi^2(\varphi - v_C)^2, \quad \nu_C = \frac{2E T_C}{\lambda_{T_C}},
\]

where \( E \) denotes the coefficient of the \( \varphi^3 \) term. In the SM, \( E_{\text{SM}} \simeq 0.01 \) coming from the gauge bosons. In the IDM model, on the other hand, the extra Higgs bosons yield the contributions of \( -T(m_\phi^2)^{3/2}/(12\pi) \) in \( V_{\text{eff}}(\varphi; T) \). As is the gauge boson case, the \( \varphi^3 \) term can be generated if \( m_\phi^2 \simeq \lambda_{h\phi\phi} \varphi^2/2 \), which contributes to \( E \). Remarkably, this is exactly the case that \( \Delta^{(1)} \lambda_{hhh}^{\text{IDM}} \) is enhanced, i.e., nondecoupling regime. As mentioned above, only \( A \) and \( H^\pm \) can have such a limit and play an essential role in achieving the strong first-order EWPT. The additional contributions in \( E \) are found to be \( \Delta E \simeq (m_A^2 + 2m_{H^\pm}^2)/(12\pi v^2) \). One can find that the minimum values of \( m_A \) and \( m_{H^\pm} \) satisfying the criterion [19] sets the minimum deviation of \( \Delta^{(1)} \lambda_{hhh}^{\text{IDM}}/\lambda_{hhh}^{\text{SM}} \). In this way, the strong first-order EWPT inevitably leads to the significant deviation in \( \lambda_{hhh} \). Detailed knowledge of \( \zeta_{sph} \) is of great importance in order to quantify the amount of the deviation precisely (for an improvement of \( \zeta_{sph} \) and its impact on \( \lambda_{hhh} \) in the SM with a real singlet scalar, see Ref. [28]).

Now we extend the above discussion to two-loop level. As far as the first-order EWPT is concerned, the sunset diagrams are more relevant than the figure-8 diagrams [30]. In the IDM, the relevant contributions are

\[
V_2(\varphi; T) \equiv -\frac{1}{4} \sum_{\phi=A,H^\pm} n_\phi \left[ \lambda_{h\phi\phi}^2 \frac{H(T)m_\phi^2}{m_\phi^2} \right] \simeq \sum_{\phi} \frac{T^2 \lambda_{h\phi\phi}^2}{128\pi^2} \ln \frac{\bar{m}_{\phi}^2}{T^2},
\]

where \( H(T) \) is the finite-temperature part of the sunset diagram [29]. In the second line, the HTE as well as \( \bar{m}_h \simeq 0 \) are assumed. It is known that \( \varphi^2 \ln(\bar{m}/T^2) \) with positive (negative) coefficient would weaken (strengthen) the first-order EWPT [30], and the dominant scalar sunset diagrams in the IDM correspond to the former. From this simple argument, one can infer that strength of the first-order EWPT would get smaller than those at one-loop level while the other way around for \( \lambda_{hhh} \). In what follows, we evaluate the \( \lambda_{hhh} \)-EWPT correlation without using the HTE approximation of Eq. (21) (details are given in a separate paper [17]).

Following the thermal resummation and renormalization schemes adopted in Refs. [31, 32], we study \( \nu_C/T_C \) numerically. Previous two-loop analysis of EWPT in the IDM can be found in Ref. [24], and our results are consistent with them within theoretical uncertainties if we use their input parameters.

In the left panel of Fig. 3 \( \nu_C/T_C \) at one and two-loop levels against \( M_A \) are shown by the red-dashed and blue-solid curves, respectively. We take the same input parameters as in Fig. 2 and set \( \bar{m} = M_A \). As expected from the qualitative discussion above, \( \nu_C/T_C \) in both cases grow with increasing \( M_A \) due to the nondecoupling effects of \( A \) and \( H^\pm \). However, \( \nu_C/T_C \) at two-loop level is reduced by around (7-16)\%, which is due mostly to the sunset diagrams involving \( A \) and \( H^\pm \).

In the right panel of Fig. 3 the correlations between \( \nu_C/T_C \) and \( \kappa_\lambda \) are represented at one and two-loop levels. The style and color schemes of the curves are the same as those in the left panel. It is found that \( \kappa_\lambda \) at two-loop
V. CONCLUSION AND DISCUSSION

We have quantified the two-loop effects on the triple Higgs coupling and strength of the first-order EWPT in the IDM. We found that the sunset diagrams can alter both of the one-loop results by about +2% and −(7-16)%, respectively. The magnitudes of the corrections are restricted by the requirement of $\mu_2^2 > 0$. Correspondingly, at two-loop level $M_A (= M_{H^\pm})$ is shifted upward by about 10 GeV and $\kappa_\lambda$ rises up to around 4% in the region where EWPT is strongly first order.

We finally make some comments on the $\bar{\mu}$ and gauge dependences of $v_C/T_C$. It is found that the former can reach about 5% under the variation of $0.5 \leq \bar{\mu}/M_A \leq 1.5$, which is predominantly originated from the thermal loop functions. In spite of this, it still holds that $v_C/T_C|_{2\text{-loop}} < v_C/T_C|_{1\text{-loop}}$ so that the inequality of $\kappa_\lambda^{1\text{-loop}} < \kappa_\lambda^{2\text{-loop}}$ remains intact. For the later, our calculation method is not gauge invariant, and the Landau gauge is adopted. Since the dominant two-loop contributions are the scalar loops, their effects are not spoiled by the gauge artifact. However, to solve those two theoretical problems in a satisfactory manner, more refined calculation scheme is needed. We defer this to future work.

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