Optomechanical Lasing and Domain Walls Driven by Exciton-Phonon Interactions

A. V. Yulin\textsuperscript{a}, A. V. Poshakinskiy\textsuperscript{b}, and A. N. Poddubny\textsuperscript{a,b,*}

\textsuperscript{a} ITMO University, St. Petersburg, 197101 Russia
\textsuperscript{b} Ioffe Institute, St. Petersburg, 194021 Russia
* e-mail: poddubny@coherent.ioffe.ru
Received July 26, 2021; revised October 4, 2021; accepted October 4, 2021

Abstract—We study theoretically interaction of optically-pumped excitons with acoustic waves in planar semiconductor nanostructures in the strongly nonlinear regime. We start with the multimode optomechanical lasing regime for optical pump frequency above the exciton resonance and demonstrate broadband chaotic-like lasing spectra. We also predict formation of propagating optomechanical domain walls driven by optomechanical nonlinearity for the optical pump below the exciton resonance. Stability conditions for the domain walls are examined analytically and are in agreement with direct numerical simulations. Our results apply to nonlinear sound propagation in the arrays of quantum wells or in the plane of Bragg semiconductor microcavities hosting excitonic polaritons.

DOI: 10.1134/S1063776122010058

1. INTRODUCTION

Semiconductor optomechanics with excitonic polaritons, hybrid half-light half-matter quasiparticles, exhibits now a rapid progress driven by the success of planar nanofabrication technology [1]. Polariton-driven phonon lasing [2] and dynamically tuned arrays of polariton parametric oscillators [3] have been recently demonstrated. The special feature of the polaritonic platform is the resonant photoelastic interaction mechanism, that is present in addition to the usual geometric one [4]: both excitonic and optical component can interact with the acoustic wave, which can enhance the overall optomechanical coupling [5]. The optomechanical polaritonic systems have been predicted to amplify sound in a parity-time-symmetric fashion [6] and should feature strong acoustic non-reciprocity [7]. However, most studies and observations of the nonlinear optomechanical dynamics for polaritons have been so far been performed in the cavity optomechanics regime, when just several localized excitonic, photonic and acoustic modes interact with each other (see [8] and references therein). Given recent tremendous progress in understanding of collective nonlinear optomechanical effects in arrays of coupled cavities [9–14], including synchronization phenomena [12, 15], formation of solitons [14] and chimera states [11], it is quite instructive to examine distributed polaritonic systems, where acoustic waves interact with excitons and light.

Here, we study theoretically the nonlinear optomechanical interaction between optically pumped excitons and propagating acoustic waves. We assume that excitons have large mass and thus are quasi-localized in space. Such situation is typically realized in periodic semiconductor superlattices where the excitons are confined inside the quantum wells while the longitudinal acoustic wave can propagate freely along the structure normal [5, 16]. Such structure is schematically illustrated in Fig. 1. Alternatively, our results can be applied to laterally propagating acoustic waves in the planar Bragg semiconductor microcavities [3]. We demonstrate, that the nonlinear optomechanical dynamics can be quite intricate depending on the pump frequency, length of the structure and boundary conditions. In addition to the well-known optomechanical lasing regime [17] we predict formation of propagating subsonic optomechanical domain walls.

2. MODEL

We consider propagation of interacting light, excitons and acoustic waves along the normal $z$ of a planar periodic semiconductor nanostructure, such as an array of quantum wells, shown in Fig. 1. When the optical and acoustic wave lengths are greater than the structure period along the growth axis $z$, the system can be considered as a continuous medium. For example, the experiment on Brillouin scattering of excitonic polaritons in [5] has been performed for 40 GaAs wells with a thickness of 17.1 nm separated by AlAs barriers.
of 7.5 nm thickness, when such continuous approximation is well justified. The Lagrangian density for such system has the form

\[ L = \frac{1}{8\pi} \left[ \left( \partial_z A \right)^2 - \left( \partial_t A \right)^2 \right] - \frac{1}{c} P \partial_t A + \frac{e}{2} \left( \partial_z A \right)^2 + \frac{G}{2} \left( \partial_z A \right)^2. \]  

(1)

The first term of the Lagrangian (1) corresponds to the free electromagnetic field described by vector potential \( A(z, t) \), while the second term represents its interaction with exciton polarization \( P(z, t) \). The second line of the Lagrangian (1) governs the field of mechanical displacement \( u(z, t) \), with \( \rho \) and \( \bar{\varepsilon} \) being the medium density and Young’s modulus, respectively. The last line of Eq. (1) is the Lagrangian of the harmonic oscillator describing the exciton polarization. The oscillator frequency shifts linearly with the deformation \( \partial_z u \); the deformation potential constant is \( \Xi = 10 \) eV. We suppose that exciton cannot move along \( z \) direction being strongly confined inside individual quantum wells. The normalization constant \( f \) is related to longitudinal-transverse splitting \( \omega_{LT} \) [18] as \( f = 2\pi/(\omega_{LT}) \). In a GaAs/AlGaAs quantum well array this splitting is on the order of \( \hbar \omega_{LT} \approx 0.1 \) meV [5].

From the Lagrangian (1) the equation for the evolution of the fields \( A(z, t) \), \( u(z, t) \), and \( P(z, t) \) are derived:

\[ \partial_t^2 A - c^2 \nabla^2 A = \frac{4\pi e \partial_t P}{\hbar}, \]  

(2)

\[ \partial_t^2 u - s^2 \partial_z^2 u = (f \omega_z \Xi/\rho) \partial_t P^2, \]  

(3)

\[ \partial_t^2 P + \omega_z^2 P = -2\omega_z \Xi P \partial_t u - (1/\rho) \partial_t A, \]  

(4)

where \( s = \sqrt{\bar{\varepsilon}/\rho} \) is the (longitudinal) sound velocity and we neglected the term quadratic in \( \Xi \).

The exciton frequency being much higher than acoustic one, we suppose \( P(z, t) = P_1(z, t) e^{-i\omega_1 t} + c.c. \) and \( A(z, t) = A_1(z, t) e^{-i\omega_1 t} + c.c. \). Then, exploiting the rotating wave approximation, we obtain from Eqs. (3), (4)

\[ \partial_t^2 \xi + 2\Gamma \partial_t \xi - s^2 \partial_z^2 \xi = \epsilon \partial_t |P|^2, \]  

(5)

\[ i\partial_t P = \Xi \partial_t u - \frac{\omega_{LT}}{4\pi} E_1, \]  

(6)

where \( E_1(z, t) = (i\omega_1/c) A_1(z, t) \) is the electric field. Instead of the displacement fields \( u(z, t) \), it is convenient to use a combined field \( \xi(z, t) = \Xi \partial_t u + \epsilon |P|^2 \), where \( \epsilon = 4\pi \Xi^2/\omega_{LT} \). That field is proportional to stress and accounts for both mechanic and excitonic contributions.

The typical pump fluences in the state-of-the-art probe-probe reflection experiment [16] were equal...
to $J \sim 0.1 \text{ mJ/cm}^2$ and nonlinear acoustic effects have been observed in for $J \geq 3 \text{ mJ/cm}^2$.

3. LINEARLY STABILITY ANALYSIS

First, we focus on the solution with spatially uniform stress $\xi(z, t) = \xi_0$ and exciton polarization $P_1(z, t) = b_0$, where

$$b_0 = \frac{G}{\omega_p - \omega_x - \xi_0 + i \Gamma_x}. \quad (9)$$

To analyze its stability, we investigate the dynamics of small corrections $\zeta$ and $a = a_1 + i a_2$ for the deformation field $\xi$ and the exciton field $P_1$. Their evolution is governed by the equations

$$\partial_t^2 \xi + 2 \Gamma_x \partial_t \xi - s^2 \partial_z^2 \xi = 2 e |b_0|^2 a_1, \quad (10)$$

$$\partial_t a_1 = - (\omega_p - \omega_x - \xi_0) a_2 - \Gamma_x a_1, \quad (11)$$

$$\partial_t a_2 = (\omega_p - \omega_x - \xi_0) a_1 - \Gamma_x a_2 - |b_0|^2 \xi. \quad (12)$$

Looking for the solution that depends on time and coordinate as $e^{-i \omega_p t + i k z}$ we obtain the solvability condition of Eqs. (10)–(12) in the form

$$(s^2 k^2 - \Omega^2 - 2i \Gamma_x \Omega)(\Delta^2 + \Gamma_x^2 - \Omega^2 - 2i \Gamma_x \Omega) + 2 \Omega^2 e |b_0|^2 \Delta = 0, \quad (13)$$

where $\Delta = \omega_p - \omega_x - \xi_0$ is the detuning of the laser frequency from the exciton resonance.

In the limit of strong coupling, when $\Gamma_x$ and $\Gamma_y$ are small, the dispersion relation in the leading approximation order is given by

$$\Omega^2 = \frac{s^2 k^2 + \Delta(\Delta - 2e |b_0|^2)}{2} \pm \frac{1}{2} \sqrt{[s^2 k^2 - \Delta(\Delta - 2e |b_0|^2)]^2 - 8 \Delta^2 e |b_0|^2}. \quad (14)$$

For a positive laser detuning $\Delta > 0$, the eigenfrequencies become complex in the vicinity of $k = \pm \sqrt{\Delta(\Delta + 2e |b_0|^2)} / s$. Moreover, some eigenfrequencies have positive imaginary part, indicating that the corresponding eigenmodes grow with time.

We now examine the effect of finite losses on the instability. We assume that the losses and the pump are weak compared to the relevant sound frequencies, $\Gamma_x$, $\Gamma_y$, $e |b_0|^2 \ll |\Delta|$, $s|k|$. Then, it is natural to assume that the most unstable mode will have the wave vector $k \approx \pm |\Delta| / s$ and frequency $\Omega \approx |\Delta|$. In the vicinity of this point, Eq. (13) can be simplified to

$$(\Omega - s k + i \Gamma_x)(\Omega - |\Delta| + i \Gamma_x) = -\frac{1}{2} e |b_0|^2 \Delta. \quad (15)$$

The analysis yields that the instability persists if

$$e |b_0|^2 \Delta > 2 \Gamma_x \Gamma_y.$$  

Specifically, the acoustic waves with wave vectors in the region

$$|k| - \frac{\Delta}{s} < \frac{\Gamma_x + \Gamma_y}{s} \sqrt{\frac{e |b_0|^2 \Delta}{2 \Gamma_x \Gamma_y} - 1},$$

are unstable.
4. OPTOMECHANICAL LASING REGIME

We now study numerically the system dynamics in the case when the ground state is unstable and the structure generates self-sustained oscillations. Figures 2 and 3 show the calculation results for $\Delta/T_x = 1.5$, $\Gamma_x/T_x = 0.01$, $\epsilon G^2/T_x^3 = 0.09$, and different system lengths $L$. We assume vanishing stress at the structure edges, corresponding to the boundary conditions $\xi(\pm L/2) = 0$. The initial conditions are weak noise for $\xi$ and a stationary solution for $b_0$.

Figure 2 is calculated for a structure length $L = 3.3{s/T_x}$. Panel (a) shows the color plot of the deformation $\xi(z, t)$. Panel (b) presents the time evolution of the amplitudes of spatial spectral harmonics. We define them as $S_m = \sqrt{\left(k_m/s\right)^2 + \left|S_{\delta,0}(k_m)\right|^2}$, where $S_{\delta}$ and $S_{\delta,0}$ are spatial spectra of $\xi$ and $\partial_t \xi$ correspondingly, and $k_m = \pi m/L$. One can see that the growth rate is positive only for one mode, $m = 2$.

For a wider system, several spatial harmonics can be located within the amplification range in the linear regime. The case when two harmonics are simultaneously amplified is shown in Fig. 3. The parameters are the same as for Fig. 2 but $L = 4.5{s/T_x}$. The competition between the modes takes place and, depending on the initial condition, two different stationary states can form, where dominating is either the harmonic with $m = 3$, as in Fig. 3, or that with $m = 2$.

The dynamics becomes much richer in the case of a long system and high intensity of the pump. This ensures that a large number of spatial harmonics fall into the amplification range and get excited in the system simultaneously. At the nonlinear stage, the behavior of the system is defined by a complex interplay of the interacting modes. We performed numerical simulations for the pump $\epsilon G^2/T_x^3 = 4$ and the system length $L = 32{s/T_x}$. The evolution of the acoustic energy distribution in the system is illustrated in Fig. 4. Panels correspond to different boundary conditions imposed at the right edge,

$$\partial_t \xi(L/2) + s Z_r \partial_t \xi(L/2) = 0,$$

which describe the situation when the area $z > L/2$ is filled with a material that has different acoustic impedance characterized by parameter $Z_r = 0, 0.25, 4, \infty$, for panels (a)–(d), respectively. On the left edge we always suppose $\xi(-L/2) = 0$, i.e., $Z_l = 0$.

One can see that the dynamics varies significantly depending on the impedance at the right edge. To further investigate this, we calculated the temporal spectra of the velocity $v(t) = \partial_t \xi(z_0, t)$, see Fig. 5 where the top row corresponds to the impedances $Z_r = 0$, the lower three rows correspond to $Z_r = 0.25 s$, $Z_r = 4s$, $Z_r = \infty$ correspondingly. The velocity was calculated at the right end ($z_0 = L/2$) for all panels excepting the top ones (Fig. 5a). The choice to measure the velocity at the right edge is made because this value defines the intensity of the radiation of the acoustic waves to the media contacting to the structure from the right. In the case when $Z_r = 0$ the velocity at the right edge is exactly equal to zero, that is why we measured this value in the middle of the structure at $z_0 = 0$.

One can see that all the velocity spectra in Figs. 5a, 5c, 5e, 5g are quite broad but have different width and structure. The correlation functions $\gamma(\tau) = \int v(t - \tau) v(t) dt$, shown in the right column of Fig. 5, decay rapidly which is in good accordance with the width of the spectra. The wide spectra and rapidly decaying correlation functions confirm that the dynamics of the acoustic field is very complex, probably chaotic. It is interesting to note that in the linear regime the absolute values of the acoustic wave reflection coefficients for $Z_r = 0.25 s$ and $Z_r = 4s$ are the
same, but in the nonlinear lasing regime the dynamics is different for these impedances. This can happen because the excitons affect the reflection of the acoustic mode at the edges of the system.

It is instructive to look at the fine structure of the spectra and on the behavior of the correlation functions at the times comparable to the travelling time of the acoustic waves through the system. This is illustrated in Fig. 6. It is seen that in the case of \( Z_r = 0 \) the correlation function has a sharp maximum at \( \tau \approx \frac{64}{\Gamma_x} \) which corresponds to the round trip of the acoustic wave in the system. Another maximum is seen at \( \tau \approx \frac{32}{\Gamma_x} \) which corresponds to the travelling time of the acoustic waves from the left to the right. \( Z_r = 0.25 \) no maxima of the correlation function except the maximum at \( \tau = 0 \) are seen. For \( Z_r = 4 \) and \( Z_r = \infty \) the first maxima is situated at \( \tau \approx \frac{64}{\Gamma_x} \). These features are also revealed in the fine structure of the spectra. The separation between the neighbouring modes corresponds to the above mentioned times: it is \( \Delta \omega \approx 0.2 \Gamma_x \) in the top panel and \( \Delta \omega = 0.1 \Gamma_x \) in the two bottom panels.

5. OPTOMECHANICAL DOMAIN WALLS

We now consider the situation when the pump is negatively detuned from the excitonic resonance, \( \omega_p < \omega_x \). In this case the background is stable and no optomechanical lasing occurs, but we show below that optomechanical domain walls can form.

We start with the case when \( \Gamma_s = 0 \) and rewrite the master Eqs. (7), (8) in the reference frame moving with the velocity \( v \),

\[
(\partial_t^2 - 2v \partial_{\eta} \partial_{\eta})\xi + \epsilon|b|^2 = (s^2 - v^2)\partial_{\eta}^2 \xi + \epsilon v^2 \partial_{\eta}|b|^2,
\]

\[
\partial_t b = v \partial_{\eta} b - i(\omega_x - \omega_p) b - \Gamma_x b - i b \xi - i G,
\]

where \( b(\eta, t) = P_1(z, t) \) with \( \eta = z - vt \). The stationary-solutions in the moving reference frame are governed by the coupled equations

\[
\partial_{\eta}^2 \xi = -\frac{\epsilon v^2}{v^2 - s^2} \partial_{\eta}|b|^2,
\]

\[
\nu \partial_{\eta} b = i(\omega_x - \omega_p) b + \Gamma_x b + i b \xi + i G.
\]  (17)

Equation (17) can be solved as \( \xi = \frac{\epsilon v^2}{v^2 - s^2} |b|^2 + \xi_0 \), where \( \xi_0 \) is a constant. Using this we obtain the
equation for the stationary distribution of the exciton field

\[ \nu \partial_t b = i \hat{\Omega} b + \Gamma_x b + i \mu |b|^2 b + iG, \]  

where \( \hat{\Omega} = \omega_x - \omega_p + \xi_0 \) and \( \mu = \epsilon v^2/(\nu^2 - s^2) \).

Without a loss of generality we can fix that at \( \eta = -\infty \) the state is characterized by \( \xi = 0 \). This means that \( b(\eta = -\infty) = G/(\omega_p - \omega_x + i\Gamma_x) \) and so \( \xi_0 = -\mu G^2/|\Gamma_x^2 + (\omega_x - \omega_p)^2| \). This state we further refer as a basic state.

Let us find out if the basic state can be connected by a domain wall to another spatially uniform state. This is possible only if Eq. (18) has three spatially uniform solutions. It is straightforward to write the algebraic equation for the intensity of the spatially uniform states

\[ |b|^2 = \frac{1}{2} \left[ \frac{G^2}{(\omega_x - \omega_p)^2 + \Gamma_x^2} \right] + \frac{2(\omega_x - \omega_p)}{\mu} \left[ \frac{G^4}{(\omega_x - \omega_p)^2 + \Gamma_x^2} - \frac{4(\omega_x - \omega_p)G^2}{\mu |(\omega_x - \omega_p)^2 + \Gamma_x^2|} - \frac{4\Gamma_x^2}{\mu^2} \right]^{1/2}. \]

Three solutions exist in the areas of parameters \( \omega_x - \nu \) right of the curves in Fig. 7a and left of the curves in Fig. 7b. Our analysis demonstrates that the supersonic domain walls can never connect dynamically-stable backgrounds. That is why we focus our attention on the subsonic domain walls. First of all we examine how the steady states changes when we vary the parameter \( \nu \) for a fixed value of \( \omega_x \) within the area of multiple solutions, this path is shown by the dashed line in panel (a) of Fig. 8. The solution shown by the blue line corresponds to the basic solution \( \xi = 0 \). The other two solutions are shown by the green and the red lines. Panels (b) and (c) show how the excitonic components of the steady states vary with the speed \( \nu \). The analogous bifurcation diagram for the mechanical component \( \xi \) is presented in panel (d). It can be seen that at certain value of \( \nu \) the basic steady state collides with the another steady state solution, the collision point is marked as “bf” in Fig. 8. We now proceed to
classification of the steady states. This can be done by finding the eigenvalues of the linearized problem governing the spatial evolution of field $b$ in the vicinity of the equilibrium points. The real and imaginary parts of the corresponding eigenvalues are presented in Fig. 9. The calculation shows that for the velocities close to 1 the basic state is a saddle, the other two are focuses. The corresponding phase plane for $\nu = 0.85s$ is shown in Fig. 10. It demonstrates that the basic state is a saddle indeed and it has heteroclinic connections to all other steady states, these heteroclinic connections are shown by thick red lines. This tells us that there exist two different domain walls moving with the velocity $\nu = 0.85s$. These domain walls connect the
basic state to the states with different mechanical $\xi$ and excitonic $b$ components.

The spatial profile of the domain wall connecting the basic state to the state marked as “f2” is illustrated in Fig. 11a. Since one of the steady states is a spiral, the tail decaying to the steady state is oscillating in space. This is why there exists an additional maximum in the spatial spectrum of the domain wall, see panel (b). This maximum becomes more pronounced for higher velocities when the oscillations become faster and the decay rate gets slower, see panels (c) and (d) of Fig. 9.

For the lower value of the frequency $\omega_x$ the bifurcation diagram looks quite differently, as shown in Fig. 12. In this case the basic steady state collides with the other solution than for $\omega_x - \omega_p = \Gamma_x$. The dependences of the eigenvalues defining the relaxation of the $b$ are shown in Fig. 13. At high velocities, the basic steady state is a saddle, the other two states are spirals. Then one of the spirals transforms to a knot. Then it no longer collides with the basic state and a transcritical bifurcation takes place: the basic state becomes a knot and the second state becomes a saddle. As in the previous case, after the bifurcation point the basic state can be connected to only one of other two states. At even smaller velocities, the second focus state transforms into a knot.

For the lower value of the frequency $\omega_x$ the bifurcation diagram looks quite differently, as shown in Fig. 12. In this case the basic steady state collides with the other solution than for $\omega_x - \omega_p = \Gamma_x$. The dependences of the eigenvalues defining the relaxation of the $b$ are shown in Fig. 13. At high velocities, the basic steady state is a saddle, the other two states are spirals. Then one of the spirals transforms to a knot. Then it no longer collides with the basic state and a transcritical bifurcation takes place: the basic state becomes a knot and the second state becomes a saddle. As in the previous case, after the bifurcation point the basic state can be connected to only one of other two states. At even smaller velocities, the second focus state transforms into a knot.

In order to be experimentally observable the domain walls have to be dynamically stable. The spectral analysis shows that this is the case provided that the backgrounds are stable. As it is discussed before, the spatially uniform states are stable if $(\omega_x - \omega_p) + \xi > 0$. Thus, the stable domain walls are the connections between the states situated above the thin horizontal black lines in panel (d) of Figs. 8 and 12.

For the lower value of the frequency $\omega_x$ the bifurcation diagram looks quite differently, as shown in Fig. 12. In this case the basic steady state collides with the other solution than for $\omega_x - \omega_p = \Gamma_x$. The dependences of the eigenvalues defining the relaxation of the $b$ are shown in Fig. 13. At high velocities, the basic steady state is a saddle, the other two states are spirals. Then one of the spirals transforms to a knot. Then it no longer collides with the basic state and a transcritical bifurcation takes place: the basic state becomes a knot and the second state becomes a saddle. As in the previous case, after the bifurcation point the basic state can be connected to only one of other two states. At even smaller velocities, the second focus state transforms into a knot.

In order to be experimentally observable the domain walls have to be dynamically stable. The spectral analysis shows that this is the case provided that the backgrounds are stable. As it is discussed before, the spatially uniform states are stable if $(\omega_x - \omega_p) + \xi > 0$. Thus, the stable domain walls are the connections between the states situated above the thin horizontal black lines in panel (d) of Figs. 8 and 12.
domain walls can form in the presence of mechanical losses provided that the losses are not too high.

The formation of the domain walls was observed in direct numerical simulation of the master equations. We have checked numerically that the propagating waves are described well by the developed theory. Our numerical simulations also indicate that the two domain walls form after the left edge of the system is shifted by $\Delta \xi_1$ at $t = t_1$ and then at $t = t_2 > t_1$ by $\Delta \xi_2$. In general the domain walls propagate with different velocities. If the second domain wall is faster than the first one, then at some moment they collide forming a new domain wall with its own velocity.

6. SUMMARY

To summarize, we have studied theoretically the strongly nonlinear regime of optomechanical interaction between optically pumped excitons, localized in a semiconductor superlattice, and propagating acoustic phonons. We have examined different regimes of interaction depending on the detuning of the pumping frequency from the excitonic resonance.

When the structure is pumped above the excitonic resonance it is in the optomechanical lasing regime. However, contrary to the usual situation of single-mode optomechanical laser [8, 17, 23], we reveal an intricate competition between lasing acoustic modes that happens when the structure is long enough so that multiple spatial harmonics fall into the amplification range simultaneously and get excited. The wide spectra of the acoustic field and rapidly decaying time-dependent correlation functions indicate quite complicated and probably chaotic lasing dynamics. We also demonstrate that the spectra significantly depend on the boundary conditions for the acoustic wave at the edge of the structure.

Fig. 9. The evolution of the real and imaginary parts of the eigenvalues governing the spatial relaxation to the basic steady states is shown in panels (a) and (b) correspondingly. The same for the other two steady states is shown in panels (c)–(f). The types of the steady states are marked as “s” for saddles, “n” for knots and “f” for spirals (focuses).
Fig. 10. Phase plane for $v = 0.85s$ and $\omega_x = \omega_p + \Gamma_x$ is shown in panel (a). The parts of the phase plane in the vicinity of the steady states are shown in panels (b–d).

Fig. 11. The evolution of the excitonic field $b$ in the domain wall connecting the basic states “s” and the steady state “f2.” Panel (b) and (c) show the normalized mechanical components $\xi$ and $\partial_z \xi$ in the domain wall. The normalized spatial spectrum of $\xi_z$ for $v = 0.85s$ is shown in panel (d). The thinner blue and green lines show the spectra for the domain wall velocities $v = 0.8s$ and $v = 0.9s$. 
Fig. 12. The same as Fig. 8 but for $\omega_x = \omega_p + 0.2\Gamma_x$.

Fig. 13. The same as Fig. 9 but for $\omega_x = \omega_p + 0.2\Gamma_x$. 

$|\beta|^2$ 

Wall velocity, v/s 

Wall velocity, v/s 

Wall velocity, v/s 

Wall velocity, v/s 

Wall velocity, v/s 

Wall velocity, v/s 

Deformation, $\xi$
In case when the structure is pumped below the excitonic resonance, optomechanical lasing is not possible, but optomechanical domain walls can form instead. We perform a detailed analysis of steady states in the structure and demonstrate, that only subsonic domain walls can be stable and can connect dynamically stable backgrounds. The dependence of the wall velocity and deformation amplitude on the pump frequency has been calculated.

We hope that our results provide useful insights in the rapidly developing field of resonant optomechanics and could be experimentally verified in the state-of-the-art structures. A natural extension of this work would be a study of the phononitonic regime [7, 24–26], when not only excitons and acoustic waves, as in the current study, but three types of excitations, light, exciton and acoustic waves, experience a nonlinear interaction.

ACKNOWLEDGMENTS
This work has been funded by the Russian Science Foundation, grant no. 20-42-04405.

CONFLICT OF INTEREST
The authors declare that they have no conflicts of interest.

REFERENCES
1. P. Delsing et al., J. Phys. D: Appl. Phys. 52, 353001 (2019).
2. D. L. Chafatinos et al., Nat. Commun. 11, 4552 (2020).
3. A. S. Kuznetsov, G. Dagvadorj, K. Biermann, M. H. Szymanska, and P. V. Santos, Optica 7, 1673 (2020).
4. C. Baker et al., Opt. Express 22, 14072 (2014).
5. B. Jusserand, A. N. Poddubny, A. V. Poshakinskiy, A. Fainstein, and A. Lemaître, Phys. Rev. Lett. 115, 267402 (2015).
6. A. V. Poshakinskiy, A. N. Poddubny, and A. Fainstein, Phys. Rev. Lett. 117, 224302 (2016).
7. A. V. Poshakinskiy and A. N. Poddubny, Phys. Rev. Lett. 118, 156801 (2017).
8. E. S. Vyatkin and A. N. Poddubny, Phys. Rev. B 104, 075447 (2021).
9. G. Heinrich, M. Ludwig, J. Qian, B. Kubala, and F. Marquardt, Phys. Rev. Lett. 107, 043603 (2011).
10. A. Xuereb, C. Genes, and A. Dantan, Phys. Rev. Lett. 109, 223601 (2012).
11. E. A. Martens, S. Thutupalli, A. Fournière, and O. Hallatschek, Proc. Natl. Acad. Sci. U. S. A. 110, 10563 (2013).
12. M. Bagheri, M. Poot, L. Fan, F. Marquardt, and H. X. Tang, Phys. Rev. Lett. 111, 213902 (2013).
13. B. J. Eggleton, C. G. Poulton, and R. Pant, Adv. Opt. Photon. 5, 536 (2013).
14. J.-H. Gan, H. Xiong, L.-G. Si, X.-Y. Lü, and Y. Wu, Opt. Lett. 41, 2676 (2016).
15. M. H. Matheny et al., Phys. Rev. Lett. 112, 014101 (2014).
16. M. Kobacki et al., arXiv: 2106.07019 (2021).
17. M. Aspelmeyer, T. J. Kippenberg, and F. Marquardt, Rev. Mod. Phys. 86, 1391 (2014).
18. E. I. Ivchenko, Optical Spectroscopy of Semiconductor Nanostructures (Alpha Science Int., Harrow, UK, 2005).
19. D. V. Vishnevsky, D. D. Solnyshkov, G. Malpuech, N. A. Gippius, and I. A. Shelykh, Phys. Rev. B 84, 035312 (2011).
20. N. Bobrovska, M. Matuszewski, T. C. H. Liew, and O. Kyriienko, Phys. Rev. B 95, 085309 (2017).
21. A. V. Yulin, V. K. Kozin, A. V. Nalitov, and I. A. Shelykh, Phys. Rev. A 100, 043610 (2019).
22. I. A. Mogunov et al., Phys. Rev. Mater. 4, 125201 (2020).
23. H. Wu, G. Heinrich, and F. Marquardt, New J. Phys. 15, 123022 (2013).
24. A. L. Ivanov and L. Keldysh, Sov. Phys. JETP 57, 234 (1982).
25. L. Hanke, D. Frohlich, A. L. Ivanov, P. B. Littlewood, and H. Stolz, Phys. Rev. Lett. 83, 4365 (1999).
26. S. Latini et al., Phys. Rev. Lett. 126, 227401 (2021).