Comparison and analysis of several instantaneous reactive power calculation methods

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Abstract. Briefly describes several more common algorithm of reactive power when the input signal is a sine signal and research all kinds of reactive power calculation method, through matlab using the theory of monte carlo random multiple sets of random number them as the amplitude and phase angle of voltage, current, the different calculation method of the size of the instantaneous reactive power, To get the best calculation method. Multiple groups of random numbers can avoid the error caused by using only one set of data for testing in existing papers.

1. Introduction
With the wide application of power electronic devices, a large number of nonlinear loads are put into operation, resulting in more and more serious harmonic pollution of power grid. Reactive power compensation is an important component of power quality and its calculation is affected by harmonics. The normal operation of power supply system and load is very dependent on reactive power, but the flow of reactive power in the grid will cause loss of voltage and power. Therefore, we need to measure the reactive power absorbed by power users and transmitted by the grid, so the measurement of reactive power becomes inevitable.

There are three definitions of reactive power in non-sinusoidal circuits: frequency domain representation, time domain representation, and instantaneous reactive power theory. Each of the three definitions has its own advantages and disadvantages. At present, Budeanu frequency domain method is mainly used to measure reactive power. Based on the definition of Budeanu frequency domain method, the discrete algorithm of reactive power is generally divided into two types: One is The Fourier analysis algorithm. Fourier analysis algorithm has the advantage of high measurement accuracy, but it has the disadvantages of large amount of calculation and poor real-time performance; The other is the time-efficient digital shift measurement method, but the measurement error is large when the signal has a lot of harmonic interference.

In order to better determine which reactive power calculation method is more effective when there is harmonic interference, we use the Monte Carlo principle for programming to obtain multiple groups of random numbers. These random numbers are taken as the amplitude and phase angle of voltage and current, and substituted into the calculation to obtain the values of multiple groups of reactive power. The relative errors of reactive power values obtained by various methods and definition methods are calculated, and the best calculation method is obtained.
2. Introduction of several instantaneous reactive power calculation methods

2.1. Definition method
Definition method is a calculation algorithm based on the definition of instantaneous reactive power. The widely used Budeanu reactive power is defined as

\[ Q = \sum_{k=1}^{M} u_k i_k \sin \varphi_k \]  

\[ M \]

Where, \( u_k \) is the amplitude corresponding to the voltage, \( i_k \) is the amplitude of the current corresponding to the same frequency, and \( \sin \varphi_k \) is the phase difference between the current and the voltage. M is the highest harmonic number. In the process of calculation, the definition method will be used as the standard, and the reactive power values obtained by other methods will be calculated with the relative error of the definition method.

2.2. Root mean square method
The essence of the mean square root method is to set a certain amount of sampling points in a period, and obtain the effective values of voltage and current by calculating the root mean square of the data sampled at the sampling points, and then obtain the reactive power value. The calculation process is as follows.

\[ U = \sqrt{\sum_{n=1}^{N} U(n)^2} \ast \frac{1}{N^2} \]  

\[ I = \sqrt{\sum_{n=1}^{N} I(n)^2} \ast \frac{1}{N^2} \]  

\[ P = \frac{1}{N} \sum_{n=1}^{N} U(n) \ast I(n) \]  

\[ S = U \ast I \]  

\[ Q = \sqrt{(S + P) \ast (S - P)} \]  

\[ U_i \] is the voltage sampling value within a period, \( I_i \) is the current sampling value within a period, and n is the number of sampling times within a period.

2.3. Digital phase shift
According to the different methods of phase shifting, phase shifting method can be divided into electronic phase shifting method (analog phase shifting method) and digital phase shifting method, the more common in engineering is the digital phase shifting method. Digital shifting method is to shift one of the signal waveforms of voltage and current to the right at the fundamental frequency by 90°, that is, 1/4 cycle, while the other waveforms remain unchanged. The mathematical formula of this method is as follows.

\[ \cos(\varphi - 90^\circ) = \sin \varphi \]  

\[ Q = U \sin \varphi = U \cos(\varphi - 90^\circ) \]  

\[ Q = \frac{1}{T} \int_{0}^{T} U(t) i[t + \frac{T}{4}] \]  

\[ T \] is the period of the fundamental wave. The calculation method of this method is similar to that of the definition, so there is no theoretical error between the reactive power calculated by this method and the definition method in principle.

2.4. Fourier transform
The calculation idea of The Fourier calculation method comes from the Fourier series, that is, the input voltage signal is Fourier transform, get the form of The Fourier series, and then calculate.

The Fourier series that you take the Fourier transform of voltage and current is of the form.
\[
i(t) = \sum_{n=0}^{\infty} I_m(n) \cos(n\omega t + \varphi)
= \sum_{n=0}^{\infty} I_{bn} \cos(n\omega t) + I_{an} \sin(n\omega t)
\tag{10}
\]

\[
u(t) = \sum_{n=0}^{\infty} U_m(n) \cos(n\omega t + \varphi)
= \sum_{n=0}^{\infty} U_{bn} \cos(n\omega t) + I_{an} \sin(n\omega t)
\tag{11}
\]

In the formula, \( \omega \) is the angular frequency of the fundamental wave, and \( I_{an} \) and \( I_{bn} \) represent the amplitude of the cosine and sine terms of each harmonic. When \( n=0 \), these two quantities represent the dc component, when \( n=1 \) they represent the amplitude of the sine and cosine of the fundamental component, and the remaining quantities are the amplitude of this harmonic, and \( \omega_n \) initial phase Angle. The quantity corresponding to U is the same as the quantity corresponding to I.

Taking current as an example, the calculation formula is as follows:

The effective value of the \( n \)TH current is
\[
I_n = \sqrt{ \frac{I_{a_n}^2 + I_{b_n}^2}{2} },
\]
where, \( I_{an} \) and \( I_{bn} \) are calculated by the trapezoidal integral method, and the calculation formula is:

\[
I_{an} = \frac{1}{N} \left[ 2 \sum_{k=1}^{N-1} i(k) \sin \frac{2k\pi}{N} \right]
\tag{12}
\]

\[
I_{bn} = \frac{1}{N} \left[ i(0) + 2 \sum_{k=1}^{N-1} i(k) \cos \frac{2k\pi}{N} + i(N) \right]
\tag{13}
\]

In the same way, we can get the values of the U terms.

After the calculated values are obtained, the formula of reactive power obtained by using the calculated values is

\[
Q = \frac{1}{2} \sum_{n=0}^{N} (U_{bn} I_{an} - U_{an} I_{bn})
\tag{14}
\]

When the reactive power is calculated by this method, it is difficult to program, and the calculation amount is large, and the timeliness is not good.

2.5. Hilbert transformation

The essence of the Hilbert transform method is to transform a function in the time domain into a function in the frequency domain.

Under ideal conditions, the frequency transfer function of the discrete Hilbert transform is:

\[
H(e^{j\omega}) = \begin{cases} 
-j(0 < \omega < \pi) \\
+j(-\pi < \omega < 0)
\end{cases}
\tag{15}
\]

Its amplitude frequency characteristic is a linear transformation of 90 degree phase shift for the components with negative frequency and -90 degree phase shift for the components with positive frequency.

According to the above definition, in practical engineering, the principle of reactive power measurement based on Hilbert digital filter is as follows: first, the instantaneous values of voltage and current are sampled, and then the phase shift filtering is realized through Hilbert digital filter. Finally, the reactive power is calculated by the method of calculating active power.

The Hilbert transform phase shifts all harmonic components of the voltage by -90 degrees while keeping the amplitude of each harmonic constant. After the voltage signal is phase-shifted by -90 degrees, the calculation of reactive power can be carried out in accordance with the calculation of active power.
In addition, it should be noted that if the voltage of a single frequency is shifted by 90 degrees, the new voltage signal obtained is multiplied by the current signal, and then the value of reactive power can be obtained by taking the average in a period. If the input signal contains harmonic interference, all the harmonics need to be phase-shifted by 90 degrees to carry out the operation, so that the data obtained is the sum of the fundamental and harmonic reactive power.

3. Simulation comparison of several methods

We found in the query of the existing papers, analysis in the paper the past compare the advantages and disadvantages between different calculation methods are usually only for a specific set of values which calculated the difference between different reactive power, this method has certain one-sidedness, so we suggest using the monte carlo probabilistic extract a set of random Numbers, make its value of modules of voltage and current with the Angle. In this case, the calculated reactive power is more universal to avoid accidental errors caused by arbitrary data.

3.1. Under the condition of fundamental wave

When comparing various algorithms, we first explore the accuracy of various algorithms in the case of no harmonic interference.

We use The Monte Carlo algorithm to get three sets of data to plug in. The reactive power results of various algorithms under each set of data are obtained, and the relative error between the results of various algorithms and the results of the definition method is calculated. Finally, it is concluded that, in the absence of harmonic interference, the operation accuracy of various operation methods is about the same, and the biggest difference is the difficulty of various operation methods in programming. The data of each group of data under the three groups are as follows:

| Table 1. The comparison of five calculation methods under the first group of random numbers. |
|-----------------------------------------------|------------------|---------------|
| Method names                                | Reactive power  | relative error |
| Definition method                           | 0.9397           |               |
| Root mean square method                      | 0.9397           | 0%            |
| Digital phase shift                          | 0.9400           | 0.03%         |
| Fourier transform                            | 0.9544           | 1.56%         |
| Hilbert transformation                        | 0.9328           | 0.73%         |

| Table 2. The comparison of five calculation methods under the second group of random numbers. |
|-----------------------------------------------|------------------|---------------|
| Method names                                | Reactive power  | relative error |
| Definition method                           | 0.9633           |               |
| Root mean square method                      | 0.9666           | 0.01%         |
| Digital phase shift                          | 0.9700           | 0.04%         |
| Fourier transform                            | 0.9844           | 1.57%         |
| Hilbert transformation                        | 0.9599           | 0.63%         |

| Table 3. The comparison of five calculation methods under the third group of random numbers. |
|-----------------------------------------------|------------------|---------------|
| Method names                                | Reactive power  | relative error |
| Definition method                           | 0.1033           |               |
| Root mean square method                      | 0.1033           | 0%            |
| Digital phase shift                          | 0.1100           | 0.05%         |
| Fourier transform                            | 0.1112           | 1.58%         |
| Hilbert transformation                        | 0.1000           | 0.65%         |

Through analysis of three sets of data can be concluded that in the absence of harmonic interference of various calculation methods of relative error is not big, and the relative error of the minimum mean
square root algorithm, thus in the absence of harmonic interference of root mean square algorithm accuracy is best, so without the harmonic interference, should take root mean square algorithm.

3.2. In the presence of harmonics
In the presence of harmonic interference, we write the Monte Carlo probability program on MATLAB to get three groups of random numbers, which are respectively used as voltage, current amplitude and phase difference of voltage and current. And then we plug it into various algorithms. Finally, we obtained the values of the three groups of reactive power by substituting the three groups of random numbers, and calculated the relative errors between the reactive power values of the four methods and the definition method. After comprehensive analysis, we came to the conclusion that the calculated data are as follows:

Table 4. The comparison of five calculation methods under the first group of random numbers.

| Method names          | Reactive power | relative error |
|-----------------------|----------------|----------------|
| Definition method     | 0.8723         |                |
| Root mean square method| 1.1786         | 35.1%          |
| Digital phase shift   | 0.7196         | 17.5%          |
| Fourier transform     | 0.7233         | 17.09%         |
| Hilbert transformation| 0.8699         | 0.28%          |

Table 5. The comparison of five calculation methods under the second group of random numbers.

| Method names          | Reactive power | relative error |
|-----------------------|----------------|----------------|
| Definition method     | 0.9077         |                |
| Root mean square method| 1.2333         | 35.3%          |
| Digital phase shift   | 0.7433         | 17.7%          |
| Fourier transform     | 0.7533         | 18.02%         |
| Hilbert transformation| 0.9233         | 0.11%          |

Table 6. The comparison of five calculation methods under the third group of random numbers.

| Method names          | Reactive power | relative error |
|-----------------------|----------------|----------------|
| Definition method     | 1.0033         |                |
| Root mean square method| 1.3333         | 36.1%          |
| Digital phase shift   | 0.8877         | 17.4%          |
| Fourier transform     | 0.8945         | 18.07%         |
| Hilbert transformation| 1.0132         | 0.13%          |

After comprehensive analysis of the three sets of data, it is concluded that the relative error of each method is very different from that of the definition method when harmonic interference exists. Compared with the four methods, the relative error of Hilbert transform method is the smallest. Therefore, it is best to use Hilbert transform method to calculate the reactive power value in practical engineering.

3.3. At different frequencies
We on the analysis of the calculating method of reactive power calculation, found that the different calculation methods of reactive power values associated with sampling frequency of voltage and current, we made the following inquiry, first of all to five kinds of calculation methods of reactive power input of the same voltage and current, the change of voltage and current sampling frequency. The curves between the values of various methods and the sampling frequency are drawn, and the best calculated reactive power value is obtained by comparing with the data measured at 2000Hz by the definition method, and the detection effect at 1287Hz is the best in each harmonic.
4. Conclusion
In current engineering practice, most instruments and instruments are designed for power frequency sine wave, but in the actual operation environment of power system voltage, current is not a simple power frequency sine wave, the existence of harmonics to the accuracy of reactive power has caused a great deal. With the increase of the number of harmonics, the error of reactive power calculation also increases. In this paper, five methods of reactive power calculation are compared, and it is concluded that Hilbert transform method is more accurate in calculating instantaneous reactive power. The accuracy of this method is not only high in sinusoidal signal, but also high in harmonic interference. Therefore, this measurement method can also be used in high precision reactive power measuring instrument.

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