Beyond the soft photon approximation in radiative production and decay of charged vector mesons.

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Abstract

We study the effects of model-dependent contributions and the electric quadrupole moment of vector mesons in the decays $V^- \rightarrow P^- P^0 \gamma$ and $\tau^- \rightarrow \nu V^- \gamma$. Their interference with the amplitude originating from the radiation due to electric charges vanishes for photons emitted collinearly to the charged particle in the final state. This brings further support to our claim in previous works, that measurements of the photon energy spectrum for nearly collinear photons in those decays are suitable for a first measurement of the magnetic dipole moment of charged vector mesons.

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In some recent papers [1, 2, 3], we have studied the possibility to obtain information about the experimental value of the magnetic dipole moment (MDM) of light charged vector mesons. We have focused our interest on the angular and energy distribution of photons emitted during the radiative decay [1] or production [2] processes of vector mesons. In Ref. [3] we have extended our analysis to include the finite width effects of these unstable particles. Just to illustrate the interest of the problem, let us emphasize that none of the magnetic dipole moments of resonances, neither elementary (as the W gauge boson) nor hadronic particles, have been measured yet. The only exception is the $\Delta^{++}$ resonance, where the determinations of the MDM spread over a wide range [1]: $3.5 < \mu_{\Delta^{++}} < 7.5$ in units of nuclear magnetons. This particular case makes inconclusive any comparison with theoretical predictions based on quark models [3]. Therefore, observables that could provide
information about the MDM of unstable particles, open up the possibility to test additional features of the dynamics of bound states in strong interactions.

The energy spectrum of photons emitted at small angles with respect to final state charged particles in the decays $V^- \rightarrow P^- P^0 \gamma$ [1] and $\tau^- \rightarrow \nu V^- \gamma$ [2] ($V(P)$ denotes a vector (pseudoscalar) meson), was found to be particularly sensitive to the effects of the MDM of the vector meson. We should recognize, however, the challenges that reconstructing these channels from multi-photon final states may pose to experimentalists.

In our analysis of Refs. [1, 2, 3] we have neglected model-dependent contributions and the effects coming from the electric quadrupole moment of the vector meson. We have argued, based on the fact that their lowest order contributions in the photon energy vanish for collinear photons, that these effects are expected to be small for the particular kinematical region studied in those papers [1, 2]. In order to have a quantitative estimate of these effects, in the present paper we focus on the details of our proof that they are indeed small compared to the dominant effects coming from the MDM of the vector meson.

Let us first discuss the general structure of the decay amplitude for the emission of a low-energy photon. The decay amplitude for the process $i \rightarrow f + \gamma$ (either, $i$ and/or $f$ states containing charged particles) can be expressed as a power expansion in the photon energy $\omega$ as follows [6]:

$$M = \frac{A}{\omega} + B\omega^0 + C\omega + \cdots$$

(1)

Low’s theorem [3] states that the first two terms of this amplitude are model-independent. This means that the coefficients $A$ and $B$ can depend only on the parameters that describe the corresponding non-radiative process $i \rightarrow f$ and on the static properties (electric charges and magnetic dipole moments) of the particles involved in $i$ and $f$. Furthermore, the piece of the amplitude involving terms up to order $\omega^0$ (also called Low’s amplitude) must be gauge-invariant. The electric charges contribute to the amplitude at order $\omega^{-1}$ while the effects of the MDM enter at order $\omega^0$. In the rest frame of decaying particles, Low’s amplitude is expected to give a very good approximation to the radiative process for photon energies below a typical energy scale of the final particles (let’s say, the pion mass for the $\rho^-$ meson decay considered below).

In equation (1), the terms of order one and higher in $\omega$ involve, in general, model-dependent contributions and the effects coming from the electric quadrupole moment of
vector mesons. Since Low’s amplitude is gauge-invariant, the terms that enter the amplitude (1) at $O(\omega)$ must be gauge-invariant by themselves. On another hand, by including all the model-dependent contributions in Eq. (1) one would expect to obtain an amplitude valid for the whole range of photon energies. In practice (as is the case of the present work), one is limited to include only a few sources of dominant model-dependent contributions. The final judgment on the validity of these approximations corresponds to the experiment.

The squared amplitude obtained after summation over polarization of particles in $i$ and $f$, exhibits other interesting properties. This unpolarized probability has the following structure in terms of the expansion in $\omega$:

$$\sum_{pols\ i,f} |\mathcal{M}|^2 = \frac{\alpha_0}{\omega^2} + \alpha_0 \omega^0 + O(\omega),$$

i. e., the interference terms of order $\omega^{-1}$ vanish. This is the well known Burnett and Kroll’s theorem [7] (see also [8]).

As it will be shown below, $\alpha_2$ is proportional to $\sin^2 \theta$ ($\theta$ being the angle of the photon emission with respect to a given charged particle of the process) and it comes from the radiation off external charged particles. On another hand, $\alpha_0$ receives two contributions: the term proportional to $|B|^2$ and the interference term $\text{Re} \{AC^*\}$. For the cases of our interest here, the angular dependence of the $|B|^2$ term is very mild as it can be appreciated from the plots in our previous papers [1, 2]. Therefore, the decay probability (2) for $i \to f + \gamma$ at order $\omega^0$ is in general model-dependent. A reliable information about the magnetic dipole moment can be obtained provided the contributions coming from model-dependent and electric quadrupole moment effects are small. This can be explicitly proven for radiative decay and production of charged vector mesons. In the following we will show that the interference term $\text{Re}\{AC^*\}$ is also suppressed for the kinematical configurations chosen in Refs. [1, 2].

We divide our analysis into two parts: first we consider the decay $V^- \to P^- P^0 \gamma$ and then we focus on the production $\tau^- \to V^- \nu \gamma$ processes of charged vector mesons $V^- \,(P \text{ denote a pseudoscalar meson}).$

Let us first introduce some useful notation. Let $q$ and $p$ be the four-momenta of the decaying and the charged particle in the final state, respectively, and $k$ ($\epsilon$) the four-momentum.

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1In this paper we will consider decay process involving charged spin 1 particles. We will assume exact CP symmetry, which implies the vanishing of the electric dipole and magnetic quadrupole moments of the vector particles.
(polarization) of the photon. We define the four-vector:

\[ L^\mu \equiv \left( \frac{p^\mu}{p \cdot k} - \frac{q^\mu}{q \cdot k} \right) \]  

(3)

which satisfies the property \( L \cdot k = 0 \). In the rest frame of the decaying particle, we have

\[ L^2 = L \cdot L = -\frac{|\vec{p}|^2}{(p \cdot k)^2} \sin^2 \theta \]  

(4)

where \( \theta \) is the angle between the photon three-momentum and \( \vec{p} \).

**Vector meson decays: \( V^- \rightarrow P^- P^0 \gamma \)**

We first consider the decay process of vector mesons and, for definiteness, we choose the radiative decay \( \rho^-(q, \eta) \rightarrow \pi^-(p) \pi^0(p') \gamma(k, \epsilon) \) (Latin and Greek letters within parenthesis denote four-momenta and polarization four-vectors, respectively). The Low’s amplitude can be written as \[ \mathcal{M}_L \]

\[ \mathcal{M}_L = ieg(p - p') \cdot \eta L \cdot \epsilon^* + ieg(k \cdot \eta \frac{q \cdot \epsilon^*}{q \cdot k} - \epsilon^* \cdot \eta) F_1 + ieg(k \cdot \eta \frac{p \cdot \epsilon^*}{p \cdot k} - \epsilon^* \cdot \eta) F_2 \],  

(5)

where \( F_1 \equiv \frac{m_{\rho}^2 - m_{\pi}^2}{m_{\rho}^2} (1 - \frac{\beta(0)}{2}) - \frac{\beta(0)}{2} \), \( F_2 \equiv -1 + \frac{\beta(0)p \cdot k}{q \cdot k} \), and \( g \approx 6.01 \) denotes the \( \rho \pi \pi \) coupling, \( e \) is the electric charge of the positron, \( m_\rho \) is the \( \rho \) meson mass and \( m_\pi \, (m'_\pi) \) is the mass of the charged (neutral) pion. The first term of this amplitude, hereafter called \( \mathcal{M}_e \), is proportional to \( L \) and can be identified with the term \( A/\omega \) in Eq. (1). This term is gauge invariant by itself and \( \sum_{\text{pols}} |\mathcal{M}_e|^2 \), is proportional to \( L^2 \). The remaining terms of order \( \omega^0 \) in Eq. (5) are also gauge invariant, contain the effects of the magnetic dipole moment \( \beta(0) \) (in units of \( e/2m_\rho \)), and can be identified with the term \( B \) in Eq. (1). \( \beta(0) = 2 \), corresponds to the canonical value for point particles.

Now let us consider the model-dependent contributions to the term \( C \) in Eq. (1). They are dominated by those terms coming from intermediate mesons through the sequence \( \rho^- \rightarrow \pi^- X^0 \rightarrow \pi^- \pi^0 \gamma \) where \( X^0 \) is an isoscalar vector meson \( (\omega, \phi) \) or a neutral axial meson \( (a_1) \). The contribution from the \( a_1^0 \) meson is not possible because \( C \) parity, while that from the
φ meson is suppressed by the Okubo-Zweig-Iizuka rule. Thus, we are left with the ω vector meson as the dominant identified source of model-dependent contributions

The contribution to the amplitude coming from the ω meson intermediate state is given by:

\[
M_d = -\frac{g_{\omega\pi^0\gamma} g_{\rho\omega\pi}}{m_\rho^2 + m_\pi^2 - m_\omega^2 - 2q \cdot p} \epsilon^{*\nu} \eta^\rho \epsilon_{\mu\nu\alpha\delta} \epsilon_{\lambda\theta\tau\delta} k^\mu (q - p)^\alpha q^\lambda (q - p)^\tau
\]  

(6)

where \( m_\omega \) is the mass of the ω meson and \( g_{\omega\pi^0\gamma} \approx 0.811 \, GeV^{-1} \), \( g_{\rho\omega\pi} \approx 13.5 \, GeV^{-1} \) are their corresponding coupling constants to final and initial states, respectively. Note that this amplitude is gauge-invariant and of \( \mathcal{O}(\omega) \) in the photon energy.

The interference of this amplitude with the term of \( \mathcal{O}(\omega^{-1}) \) in the Low’s amplitude, \( M_e \), summed over polarizations is

\[
\sum_{pol} M_d M_e^* = -\frac{2e g g_{\omega\pi^0\gamma} g_{\rho\omega\pi} k \cdot p (k \cdot q)^2}{m_\rho^2 + m_\pi^2 - m_\omega^2 - 2q \cdot p L^2}.
\]  

(7)

Thus, the contribution of model-dependent terms, at their leading order in \( \omega \), will be suppressed for small angles of photon emission since they are proportional to \( L^2 \).

We focus now on the contribution of the electric quadrupole moment of the \( \rho \) meson to the term \( C \) in Eq. (1). This multipole enters the electromagnetic vertex \( \rho^- (q) \rightarrow \rho^- (q') \gamma(k) \) as follows \( \llbracket \) (Lorentz indices \( \nu (\lambda) \) refers to incoming (outgoing)) \( \rho^- \) meson and \( \mu \) to the photon external lines):

\[
ie \Gamma_Q^{\mu\nu\lambda} = ie \gamma(0) \{(2q - k)^\mu k^\nu k^\lambda - q^\mu (k^\nu g^{\mu\nu} + k^\nu g^{\mu\lambda})\},
\]  

(8)

where \( \gamma(0) \) denotes the electric quadrupole moment \( \llbracket \) in units of \( e/2m_\rho^2 \). Note that \( k_\mu \Gamma_Q^{\mu\nu\lambda} = 0 \), namely the vertex is gauge-invariant on its own.

Its contribution to the decay amplitude is given by the gauge-invariant term:

\[
M_Q = -\frac{ie g \gamma(0)}{2} \left\{(2p - q) \cdot k \left(\frac{q \cdot \epsilon}{q \cdot k} k \cdot \eta - \eta \cdot \epsilon\right) - 2k \cdot \eta p \cdot k L \cdot \epsilon\right\}.
\]  

(9)

\( ^2 \)The contribution of the intermediate ω meson to the branching ratio of \( \rho^- \rightarrow \pi^- \pi^0 \gamma \) lies two orders or magnitude below the model-independent contribution given by Low’s amplitude (see Bramon et al in Ref. \( \llbracket \).

\( ^3 \)Quark model predictions \( \llbracket \) for the quadrupole moment cluster around 0.055 \( fm^2 \). We will use a value \( \gamma(0) = 2 \) in our numerical estimations which corresponds to 0.065 \( fm^2 \), i.e. not very far from theoretical expectations.
The interference term coming from $\mathcal{M}_Q$ and $\mathcal{M}_e$ is:

$$\sum_{\text{pol}} \mathcal{M}_Q \mathcal{M}_e^* = -(eg)^2 \gamma(0) \left\{ 2 - \frac{q \cdot k}{m^2_\rho} \right\} p \cdot (2p - q) \cdot k L^2 . \quad (10)$$

In summary, since interference terms of order $\omega^0$ coming from model-dependent contributions and from the electric quadrupole moment behave as

$$2 \text{Re}(AC^*) = \text{Re}\{\mathcal{M}_e \cdot (\mathcal{M}_d^* + \mathcal{M}_Q^*)\} \propto L^2 , \quad (11)$$

and using its proportionality to $\sin^2(\theta)$ ($\theta$ the angle between the corresponding photon and charged pion 3-momentum), we conclude that model-dependent and quadrupole moment contributions to $\rho^- \to \pi^- \pi^0 \gamma$, at leading order in $\omega$ are suppressed when $\pi^-$ and $\gamma$ are emitted nearly to the collinear configuration.

**Production of vector mesons: $\tau^- \to V^- \nu \gamma$**

Let us consider the decay $\tau^-(q) \to \rho^-(p, \eta) \nu(p') \gamma(k, \epsilon)$ as an example of radiative production of charged vector mesons [4]. The corresponding Low’s amplitude is given by [4]

$$\mathcal{M}_L = \frac{eg_\rho G_F V_{ud}}{\sqrt{2}} \left\{ \overline{\varphi}(p') O^\alpha u(q) \eta^*_\alpha L \cdot \epsilon^* + \overline{\varphi}(p') \frac{\alpha}{2g_{\rho}} \frac{k^*}{k} u(q) \eta^*_\alpha + \overline{\varphi}(p') \frac{1 + \gamma_5}{p \cdot k} G^\alpha u(q) (k_\alpha \epsilon^*_\lambda - \epsilon^*_\alpha k_\lambda) \eta^\lambda \right\} . \quad (12)$$

where $G^\alpha \equiv \frac{\beta(0)}{2} \gamma^\alpha + (1 - \frac{\beta(0)}{2}) \frac{m_\tau}{m^2_\rho} \rho^\alpha$, $G_F$ is the Fermi constant, $V_{ud}$ the Cabibbo-Kobayashi-Maskawa $ud$ mixing, $g_\rho \simeq 0.166$ GeV$^2$ the $\rho - W$ coupling, and $O^\alpha \equiv \gamma^\alpha(1 - \gamma_5)$. As in the case of vector meson decays, the first term of this amplitude ($\mathcal{M}_L$), of order $\omega^{-1}$, is proportional to the four-vector $L$ and identified with the electric charge radiation amplitude. Thus, its associated probability will be suppressed for small values of $\sin \theta$ ($\theta$ here is the angle between the three-momenta of the $\rho^-$ and the photon in the rest frame of the $\tau$ lepton). The second and third terms are of $O(\omega^0)$ and correspond to the term $B$ in Eq. (1).

In the following we consider the contributions of order $\omega$ to the decay amplitude of $\tau^- \to \rho^- \nu \gamma$. First, we can identify the model-dependent contribution given by the $\pi^-$ intermediate state: $\tau^- \to \pi^- \nu \to \rho^- \nu \gamma$. This contribution would be expected to provide the
dominant model-dependent channel, given the fact that couplings of other hadrons to the weak charged current are strongly suppressed.

The corresponding gauge-invariant decay amplitude is given by:

$$\mathcal{M}_\pi = \frac{iG_F f_\pi g_{\rho\pi\gamma}}{\sqrt{2}} \epsilon_{\mu\nu\alpha\lambda}(p+k)\epsilon^\mu p^\nu \eta^\lambda \bar{u}(p') O^\beta u(q)(p+k)_\beta,$$

where $g_{\rho\pi\gamma} \approx 0.239$ GeV$^{-1}$ is the $\rho\pi\gamma$ coupling constant and $f_\pi \approx 93$ MeV is the pion decay constant. It is straightforward to show that the interference of this model-dependent amplitude with the term of order $\omega^{-1}$, namely $\sum_{pol} \mathcal{M}_e \mathcal{M}_\pi^*$, vanishes identically. The reason for this is the following: in the amplitudes that contribute to $\mathcal{M}_e$, the $\rho^-$ meson couples to the spin-1 component of the virtual $W$ gauge-boson, while the intermediate $\pi^-$ meson in the $\mathcal{M}_\pi$ amplitude is produced from the spin-0 component of the $W$ gauge-boson. Thus, the interference of both amplitudes vanishes owing to this orthogonality.

Another possible model-dependent contribution arises from the $a_1$ meson intermediate state in the process $\tau \rightarrow \nu a_1 \rightarrow \nu\gamma\rho$. The gauge-invariant amplitude (of order $\omega$) corresponding to this process is:

$$\mathcal{M}_{a_1} = \frac{-ig_{a_1\rho\gamma} G_F g_{a_1 V_{ud}}}{\sqrt{2}} \eta^\lambda \epsilon^{\mu\nu\alpha\lambda} (p+k)\epsilon^\mu (p') \bar{u}(p') O_\phi u(q)$$

where $g_{a_1} \approx 0.22$ GeV$^2$ is the $a_1$-$W$ coupling [10], and $g_{a_1\rho\gamma} \approx 0.789\epsilon$ ($\epsilon = \sqrt{4\pi\alpha}$ in the electric charge of the proton) can be estimated from vector-meson dominance relations using the fact that the $a_1$ meson in the orbital excitation of the pion. We can compute the unpolarized interference of this amplitude with the one that arises from electric charge radiation:

$$\sum_{pol} \text{Re} \mathcal{M}_{a_1} \mathcal{M}_e^* = \frac{4g_{a_1\rho\gamma} g_\rho g_{a_1 V_{ud}}^2}{(2k \cdot p - m_{a_1}^2 + m_\rho^2)} (k \cdot p)(k \cdot q) \left( \frac{k \cdot q}{m_\rho^2} - 2 \right) L^2$$

i.e. this interference is proportional to the factor $L^2$, as in the case of the $\rho^-$ meson decay. Therefore, the leading contribution to the decay probability coming from model-dependent pieces vanishes in the limit of collinear photons.

On another hand, using the Feynman rule given in Eq. (8), we can compute the contribution of the electric quadrupole moment to the $\tau$ lepton decay amplitude. We obtain:

$$\mathcal{M}_Q = \frac{eg_\rho G_F V_{ud} \gamma(0)}{2\sqrt{2}} \bar{u}(p') O^\alpha u(q) \left( \frac{k \cdot \epsilon}{p \cdot k} (k\beta - \epsilon\beta) \right) \left[ k \cdot \eta g_{\alpha\beta} + \left( k - \frac{p \cdot k}{m_\rho^2} q_{\alpha} \right) \eta_{\beta} \right] \left( \frac{k \cdot p}{m_\rho^2} - 2 \right) L^2,$$
where \( q' = p + k \). This amplitude is gauge-invariant and of order \( \omega \).

The interference of the radiation amplitudes due to the electric charges, \( M_e \), and the electric quadrupole moment, \( M_Q \), is given by:

\[
\sum_{pol} M_e M_Q^* = 2 \left( \frac{e g_{\rho} G_F V_{ud}}{\rho} \right)^2 (0) q \cdot k \left( -4q.k + \frac{p.k}{m_\rho^2} \left( 2[m_\rho^2 + m_\pi^2 - q.k] + p.k \right) \right) L^2 \tag{17}
\]

i.e., proportional to \( L^2 \). Thus, as in the previous case, the interference term \( 2\text{Re}\{A C^*\} \propto L^2 \) will be suppressed for small values of \( \theta \).

Finally, let us illustrate the effects of the model-dependent and the electric quadrupole moment contributions to the distributions of photons studied in Refs. [1, 2]. In Figs. 1 and 2 we plot the angular and energy decay distributions of photons normalized to the corresponding non-radiative decay rate \( \langle 1/\Gamma_{nr} \rangle d^2\Gamma/dxdy \) with \( x \equiv 2\omega/M \) (\( M \) is the mass of the decaying particle) and \( y = \cos\theta \) in the decays \( \rho^- \to \pi^-\pi^0\gamma \) and \( \tau^- \to \rho^-\nu\gamma \), respectively. Figure 2 includes a close up (right sided plot) to show the behavior of the interference terms around \( x = 0.5 \).

These distributions are plotted as functions of the photon energy and for a small value of the photon angle emission with respect to the charged particle in the final state (\( \theta = 10^0 \)). The short– and long–dashed lines in both Figures correspond to the electric charge and the magnetic dipole moment (taken at the canonical value \( \beta(0) = 2 \)) contributions, respectively. Also displayed are the curves corresponding to the model-dependent (long-short–dashed line) and electric quadrupole moment (solid line with \( \gamma(0) = 2 \)) contributions, which come from their interference with the electric charge amplitudes. We observe that, in both cases, these additional contributions are very suppressed at small values of photon energies and grow monotonically as \( x \) increases until they become as important as the magnetic dipole moment contribution at the end of the spectrum. In the \( \rho^- \to \pi^-\pi^0\gamma \) decay, the presence of the model-dependent and the electric quadrupole moment effects reduce the region where the magnetic dipole moment plays the most important role (now reduced to the region \( x \approx 0.5 \sim 0.65 \)). In the case of the \( \tau^- \to V^-\nu\tau\gamma \) decay, this region remains almost unaffected by the small contribution arising from the electric quadrupole moment and the \( a_1 \) intermediate state. However, these contributions can be important only at the end of the photon spectrum.

The effects of model-dependent and electric quadrupole moment contributions to the radiative production and decay of vector mesons may be viewed as an obstacle for the deter-
mination of the MDM of the charged vector mesons as suggested in Refs. [1, 2]. However, the present analysis shows that their contributions are suppressed for the same configurations studied in our previous papers and that they are under good control in the kinematic region of interest. Since model-dependent contributions are fixed from independent phenomenological sources, one can look at them as background effects to be subtracted from the observable spectra. Following the same method used to determine the triple gauge-boson vertices at the Tevatron and LEPII colliders, measurements of the photon distributions of our interest can provide information about one electromagnetic moment (say the MDM) by fixing the other one to a reference (canonical) value.

In conclusion, the energy spectrum of photons in the processes \( V^- \to P^- P^0 \gamma \) and \( \tau^- \to V^- \nu_\tau \gamma \) is sensitive to the effects of the magnetic dipole moment of vector mesons \( V^- \) when photons are emitted at small angles with respect to the charged particle in the final state [1, 2]. In this paper we have provided further support to this statement by proving that the model-dependent and electric quadrupole moment effects in this observable are indeed small. We have drawn this conclusion from the fact that the interference of the radiation amplitude off electric charges and any other gauge-invariant amplitude is always proportional to the Lorentz-invariant quantity \( L^2 \) defined in Eq. (4) (see the appendix for a proof in the case of two-body plus photon decays; the possible generality of this result in an arbitrary decay process is under study). If the difficulty of reconstructing those particular kinematical configurations with multiple final state photons is surmount, this method would allow the first determination of the magnetic dipole moment of charged vector mesons.

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**Appendix**

In this appendix we show that for a radiative decay \( A^- (q) \to B^- (p) C^0 (p') \gamma (\epsilon, k) \) \( (q, p, p' \) and \( k \) are four-momentum vectors, and the superscripts denote the charges of particles \( A, B \) and \( C \)), the interference of the electric charge amplitude with any other gauge-invariant amplitude for this process is proportional to the Lorentz scalar \( L^2 \) given in Eq. (4). The
charged particles in this reaction can be spinor, scalar or vector particles. The generality of this property for an arbitrary radiative process will be considered elsewhere.

The total amplitude for the above process can be written as:

\[ M = \epsilon^\mu (L_\mu M_0 + M_{1,\mu}) . \]

The first term in the r.h.s. of this equation, \( L \cdot \epsilon M_0 \), is the electric charge amplitude of order \( \omega^{-1} \), while \( M_{1,\mu} \) is a gauge-invariant amplitude \( (k \cdot M_1 = 0) \) starting at order \( \omega \), such that.

The interference term between the electric charge and the remaining amplitude, after summing over photon polarizations, is given by:

\[ I = 2 \text{Re} \left[ (-g^{\mu\nu}) L_\mu \left\{ \sum_{\text{pol}:A,B,C} M_0^\ast M_{1,\nu} \right\} \right] . \quad (18) \]

Now, the most general form of the factor within curly brackets in this equation is:

\[ V_\mu = \sum_{\text{pol}:A,B,C} M_0^\ast M_{1,\mu} \]

\[ = a_1 k_\mu + a_2 p_\mu + a_3 q_\mu + \epsilon_{\mu\alpha\beta\delta}(b_1 k^\alpha q^\beta p^\delta + b_2 p^\alpha k^\beta q^\delta + b_3 p^\alpha q^\beta k^\delta) \]

where \( a_i, b_i \) are Lorentz-invariant coefficients.

The gauge-invariance condition \( k \cdot V = 0 \) imposes \( a_3 = -(p \cdot k/q \cdot k)a_2 \). Thus, we obtain:

\[ V_\mu = a_1 k_\mu + a_2' L_\mu + \epsilon_{\mu\alpha\beta\delta}(b_1 k^\alpha q^\beta p^\delta + b_2 p^\alpha k^\beta q^\delta + b_3 p^\alpha q^\beta k^\delta) \]

Therefore, we can complete our proof by observing that after contracting with the four-vector \( L_\mu \), Eq. (18) becomes:

\[ I \sim -a_2' L^2 . \]

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Figure 1: Angular and energy decay distributions of photons in the process $\rho^+ \rightarrow \pi^+\pi^0\gamma$, normalized to the non-radiative rate, as a function of the photon energy ($x \equiv 2\omega/m_\rho$) for $\theta = 10^0$ ($y \equiv \cos\theta$). The short– and long–dashed lines correspond to the electric charge and the magnetic dipole moment ($\beta(0) = 2$) contributions, respectively. The long-short–dashed and solid lines correspond to the model-dependent and the electric quadrupole moment ($\gamma(0) = 2$) effects, respectively.
Figure 2: Same as Figure 1 for the decay $\tau^- \to \rho^- \nu_{\tau}\gamma$. In this case $x \equiv 2\omega/m_\tau$ ($\omega$ is the photon energy). The solid line correspond to the electric quadrupole moment contribution when $\gamma(0) = 2$ (see text) and the long-short-dashed line is the corresponding by $a_1$. The figure at the right hand side is a close up of the photon spectrum around $x = 0.5$. 