Do stellar interferometry observations indicate UV modification of gravity?

Michael Maziashvili†

Institute of High Energy Physics, Tbilisi State University, 9 University Str., Tbilisi 0186, Georgia

Recently it was demonstrated that the theoretical predictions for the Planck scale space-time fluctuations derived by the simultaneous consideration of quantum mechanics and general relativity fail by many orders of magnitude against the stellar interferometry observations. We propose the explanation of this puzzle based on the short distance modification of gravity. Roughly speaking the criterion of this modification to fit the astronomical observations is that the gravitational radius of the mass as large as \(10^{26}\text{kg}\) should not exceed \(10^{-36}\text{cm}\). From this point of view we consider the modification of Newtonian inverse square law due to Yukawa type correction and the modification due to quantum gravitational corrections obtained in the framework of nonperturbative RG approach. The latter one does not satisfy the required criterion for modification.

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I. INTRODUCTION

After the invention of quantum mechanics, the need to make it consistent with special relativity led to creation of relativistic quantum mechanics and quantum field theory subsequently [1]. In (relativistic) quantum field theory, due to the possibility of pair creation, a single particle cannot be localized more closely than its Compton wavelength without losing its identity as a single particle [1]. The vexing problem of unifying the general relativity with quantum mechanics remains a holy grail of theoretical physics. From the very outset one sees that there is an important difference between the quantum mechanical space-time and the general relativistic one. In quantum mechanics the space-time is regarded as an essentially absolute, not affected by the processes taking place inside it. While in general relativity the space-time is given in terms of gravitational field that undergoes some evolution during the physical process and therefore becomes dynamical. Frequently we speak of length and time intervals, i.e., space and time, without asking critically how one actually measures them. It is clear that the coordinate system is defined only by explicitly carrying out the space-time distance measurements. Wigner and subsequently Salecker and Wigner considered first this problem by applying simultaneously the principles of quantum mechanics and general relativity [2]. The analysis based on the gedankenexperiment for space-time measurement shows that the quantum fluctuations of geometry becomes of the same order as the geometry itself at the Planck scale resulting therefore in the notion of minimal observable length beneath of which the geometrical properties of space-time may be lost [3]. This would give space-time a foam-like structure at sub-Planckian distances, for instance it may show up a fractal structure at this length scale while on larger scales it would look smooth and with a well-defined metric structure.

Planck length, \(l_p\), beneath of which the very concepts of space and time lose their meaning may play a role analogous to the speed of light in special relativity. If so, it means that Planck length sets an observer independent minimum value of wavelength. A quantum uncertainty in the position of a particle implies an uncertainty in its momentum and due to gravity-energy interaction leads to uncertainty in the geometry introducing in its turn an additional position uncertainty for the particle. Due to gravitational effects the minimum uncertainty in the position of a particle with energy \(E\) takes the form

\[
\delta x \gtrsim \max\left(E^{-1}, l_p^2 E\right),
\]

showing therefore that the Planck scale determines the lower bound for particle localization no matter what its Compton wavelength, \(E^{-1}\), is [3]. (Throughout this paper we set \(\hbar = c = 1\). By taking into account the gravitational effects one can also argue that it is impossible to accelerate a particle to post-Planckian energies [4].

The above conclusions concerning the Planck scale effects rely on the simultaneous consideration of usual principles of quantum theory and general relativity. Loosely speaking in the framework of general relativity the Newtonian inverse square law is understood to be valid for length scales from infinity to roughly the Planck scale. Thus far the direct experimental tests for the short distance behavior of gravity are at sub-millimeter scale [2], while, loosely speaking, quantum theory (electroweak interactions) is probed at distances approaching \(10^{-16}\text{cm}\). Therefore it is important to notice that the fundamental role attributed to the Planck length is based on the assumption that the gravity is unmodified over the 30 orders of magnitude between where it is measured at \(10^{-2}\text{cm}\) down to the Planck length \(10^{-33}\text{cm}\). For the direct test of gravitational interaction at short distances is highly nontrivial problem, it is interesting therefore if indirect experimental measurements can shed some light on the question and guide us to get some understanding of the gravity behavior at relatively short distances. In what follows we discuss the UV modification of gravity in light of the discrepancy between the stellar interferometry observations and the theoretical predictions of Planck

*Electronic address: maziashvili@hepi.edu.ge
scale space-time fluctuations demonstrated in [6, 7]. We think UV modification of gravity is most natural (and perhaps most exciting) possible outcome of the puzzle.

II. SPACE-TIME MEASUREMENT

Let us consider a gedankenexperiment for distance measurement proposed in [8]. Our measuring device is composed of a spherical clock localized in the region \( \delta x = 2r_c \) and having a mass \( m_c \), which also serves as a light emitter and receiver, and a spherical mirror of radius \( l/2 \) and mass \( m \) surrounding the clock, \( r_c \) denotes the radius of the clock), Fig.1. We are measuring a distance \( l \) by sending a light signal to the mirror under assumption that at the moment of light emission the centers of clock and mirror coincide. However, quantum uncertainties in the positions of the clock and mirror introduce an inaccuracy \( \delta l \) during the measurement. (Throughout this paper we set \( \hbar = c = 1 \).

Assuming the minimal uncertainty the spread in velocity of the clock can be found as \( \delta v_c = \delta p_c/m_c = 1/2m_c\delta x \) and analogously for the mirror as \( \delta v_m = 1/2(m+m_c)l \).

After the time \( t = l \) elapsed by light to travel along the closed path clock-mirror-clock the total uncertainty during the measurement takes the form

\[
\delta l = \delta x + \frac{t}{2m_c\delta x} + \frac{t}{2(m+m_c)l},
\]

which after minimization with respect to \( \delta x \) gives

\[
\delta x = \sqrt{\frac{l}{2m_c}} \quad \Rightarrow \quad \delta l_{\text{min}} = \sqrt{\frac{2l}{m_c}} + \frac{1}{2(m+m_c)} \quad (1)
\]

This uncertainty diminishes with increasing masses of the mirror and clock. But the masses are limited by the requirement the clock and the whole device as well not to collapse into the black hole. In other words the size of the clock/mirror should be greater than twice its gravitational radius. So by equating the size of the clock, \( \delta x \), to its gravitational radius taken with factor 2, one gets the maximum allowed mass of the clock

\[
m_c^{\text{max}} = \left( \frac{l}{32l_p^4} \right)^{1/3},
\]

and then equating the size of the measuring device, \( l \), to its gravitational radius taken again with factor 2 one finds the maximal allowed mass of the mirror

\[
m^{\text{max}} = \frac{1}{4l_p^2}
\]

In this way from Eq. (1) one gets the minimum error in measuring a length \( l \) to be

\[
\delta l_{\text{min}} = (16l_p^2l)^{1/3} + 2l_p l^{-1} \quad (2)
\]

From this equation it follows immediately that there exists the minimal observable length of the order of \( l_p \).

Loosely speaking, in Eq. (2) one can consider the first and second terms as the uncertainties contributed to the measurement by the clock and mirror respectively. Due to maximal symmetry of the measuring device considered here one can hope it provides minimal uncertainty in length measurement.

The above discussion to be self-consistent let us notice that due to Eq. (3) the wavelength of photon used for a measurement may be known with the accuracy \( \delta \lambda \sim (l_p^2 \lambda)^{1/3} \) that results in additional uncertainty

\[
\delta l_{\text{min}} \gtrsim 2^{4/3} (l_p^2 \lambda)^{1/3}, \quad \delta l_{\text{min}} \gtrsim 2^{4/3} (l_p^2 \lambda)^{1/3}. \quad (3)
\]

(For the sake of simplicity we assume that the wave length of the photon is not affected by the gravitational field of the clock). For \( \lambda \) can not be greater than \( l \) in order to measure this distance the latter term in Eq. (3) can be minimized by taking \( \lambda \sim l \). In this case the latter term becomes comparable to the first one and therefore justifies the Eq. (3). So, in general one can say that our precision in space-time distance measurement is inherently limited by the measurement process itself such that

\[
\delta l_{\text{min}} \gtrsim 2^{4/3} \frac{l_p^2 \lambda}{\lambda^{1/3}}, \quad \delta p^{(\lambda)} \gtrsim 2^{4/3} \left( l_p \frac{\lambda}{\lambda} \right)^{2/3}.
\]

The quantum with momentum \( p \) has the wavelength
\[ \lambda = 2\pi p^{-1} \text{ and from Eq.}\, 3 \text{ for the energy-momentum uncertainties one gets} \]

\[ \delta p \gtrsim \frac{2^{4/3} p^{5/3}}{(2\pi)^{2/3} m_p^{2/3}} \quad \delta E = pE^{-1} \delta p \, . \tag{5} \]

III. ASTRONOMICAL OBSERVATIONS

An interesting idea proposed in \cite{3} is to consider the phase incoherence of light coming to us from extragalactic sources. Since the phase coherence of light from an astronomical source incident upon a two-element interferometer is necessary condition to subsequently form interference fringes, such observations offer by far the most sensitive and uncontroversial test. The interference pattern when the source is viewed through a telescope will be destroyed if the phase incoherence, \( \delta \varphi \), approaches \( 2\pi \). In other words, if the light with wavelength \( \lambda \) received from a celestial optical source located at a distance \( l \) away produces the normal interference pattern, the corresponding phase uncertainty should satisfy the condition

\[ \delta \varphi < 2\pi \, . \]

The energy-momentum uncertainties, Eq.\,(5), result in uncertainties of phase and group velocities of the photon

\[ \delta v_p = \frac{2\delta E}{E} \quad \delta v_g = \frac{\delta E}{p} \, . \]

The light with wavelength \( \lambda \) traveling over a distance \( l \) accumulates the phase uncertainty

\[ \delta \varphi = \frac{2\pi l}{\lambda} (v_p / v_g) = \frac{2\pi l}{\lambda} (\delta v_p + \delta v_g) \, , \tag{6} \]

where \( \delta (v_p / v_g) \) stands for maximal deviation of the ratio \( v_p / v_g \) due to fluctuations \( \delta v_p \), \( \delta v_g \) taking a \( \pm \) sign with equal probability. Using the Eqs.\,3,\,4 one finds

\[ \delta \varphi \approx 58.0513 \left( \frac{E}{\mu} \right)^{2/3} \lambda^{5/3} \, . \tag{7} \]

Consider the examples used in \cite{4}. The Young’s type of interference effects were clearly seen at \( \lambda = 2.2 \mu m \) light from a source at 1.012 kpc distance, viz. the star S Ser, using the Infra-red Optical Telescope Array, which enabled a radius determination of the star \cite{4}. Using these values of wavelength and distance from Eq.\,7 one gets

\[ \delta \varphi \approx 6.74 \times 10^{20/3} \, . \]

Airy rings (circular diffraction) were clearly visible at both the zeroth and first maxima in an observation of the active galaxy PKS1413+135 \((l = 1.216 \text{ Gpc})\) by the Hubble Space Telescope at 1.6\( \mu m \) wavelength \cite{10}. Correspondingly, using the Eq.\,7 one gets the phase incoherence

\[ \delta \varphi \approx 1.37 \times 10^{41/3} \, . \]

This idea was further elaborated in \cite{7} by observing that the uncertainty in length measurement will lead to apparent blurring of distant point sources observed through the telescope confirming the above huge discrepancy between the theoretical predictions and the astronomical observations.

IV. DISCUSSION

From the preceding section one sees that the theoretical prediction for minimum lengths uncertainty in measuring the length of the order of \( \sim \mu m \) is about 14 orders of magnitude greater in comparison with astronomical observations. The extent to which a rigorous physical meaning can be attributed to Eq.\,(6) must be founded on a direct appeal to experiments and measurements. The only ingredients used in derivation of minimum length uncertainty are position-momentum uncertainty relation and the standard expression of gravitational radius. Therefore the contradiction between theoretical results and experiments can be understood as a failure of at least one of these ingredients. From section II one sees that the size and the mass of the clock giving the minimal uncertainty in measuring a distance \( l \) are given by

\[ r_c \approx l^{2/3} \lambda^{1/3} \quad m_c \approx l^{-4/3} \lambda^{1/3} \, . \]

By taking into account that \( r_c \) is comparable to the gravitational radius of the clock evaluated on the bases of inverse square law and experimentally established lower limit to date for the Newtonian inverse square law is \( \sim 10^{-2} \text{cm} \), one can say for sure that the Eq.\,(6) describes physical reality for \( l \gtrsim 10^{16}\text{cm} \) but can not be considered as rigorously established result beneath this length scale. In analyzing astronomical observations considered in \cite{4,7} we are evaluating the uncertainty for the photon wavelength being of the order of \( \sim \mu m \) by means of Eq.\,(6) assuming thereby the validity of Newtonian inverse square law and the Heisenberg position-momentum uncertainty relation at the lengths scale \( \sim 10^{-23}\text{cm} \). Simply speaking the position-momentum uncertainty relation is based on the de Broglie relation plus simple properties common to all waves. If we want to measure the localization of some particle with accuracy \( \delta x \) we need to use the quantum (photon for instance) the wavelength of which \( \lambda \lesssim \delta x \). Due to de Broglie relation, such a quantum is characterized with momentum

\[ p \sim \lambda^{-1} \, , \]

and during the scattering on the particle being observed imparts to it the momentum \( \delta p \) evaluated roughly as \( \delta p \sim \)}
\( p \) leading thereby to the position-momentum uncertainty relation
\[
\delta x \delta p \gtrsim 1 .
\]
The length scale \( \sim 10^{-23} \) is corresponding to the ultra-high energy cosmic rays, let us assume the position-momentum uncertainty relation is valid at least up to this scale. If position-momentum uncertainty relation is maintained, then as a weak spot in evaluation of minimum length uncertainty should be considered the use of Newtonian inverse square law at the length scale \( \sim 10^{-23} \) cm.

Considering the modified gravity law
\[
g_{00} = g_{rr}^{-1} = 1 - \frac{f(r, m)}{r} ,
\]
for the gravitational radius one finds
\[
\delta l_{min} = 4 r_g(l) + \frac{1}{2(m_{max} + m_{c_{max}})} ,
\]
which together with the Eq.(8) determines gravitational radius of the clock giving the minimal error in measurement of a given length \( l \) as a function of \( l \)
\[
r_g = f(r_g, l/8r_g) .
\]
Knowing the maximal allowed mass of the clock one can determine the maximal allowed mass of the mirror as a function of \( l \) through the equation
\[
l = 2f(l/2, m_{max} + m_{c_{max}}) ,
\]
and estimate the minimum length uncertainty by means of Eq.(9) as
\[
\delta v_p = \frac{8 r_g(\lambda)}{\lambda} , \quad \delta v_g = \frac{8 r_g(\lambda)}{\lambda} - 4 \frac{dr_g(\lambda)}{d\lambda} .
\]
Correspondingly the phase incoherence accumulated along a distance \( l \) takes the form
\[
\delta \varphi = \frac{2 \pi l}{\lambda} \left\{ \frac{8 r_g(\lambda)}{\lambda} - \frac{4 dr_g(\lambda)}{d\lambda} \right\} .
\]
This equation combined with the second example of astronomical observation (HST image of PKS1413+135) puts the following limit
\[
r_g(\lambda = 1.6 \mu m) < 0.85 \times 10^{-36} \text{cm} .
\]
From Eq.(9) one sees that the clock mass corresponding to this gravitational radius satisfy
\[
m_c > 0.12 \times 10^{27} \text{kg} .
\]
Besides this restriction on \( r_g(\lambda) \), from Eq.(12) one sees that \( dr_g(\lambda)/d\lambda \) is also subject to the constraint due to experimental observations.

To illustrate the discussion let us consider the running Newtonian constant of the form
\[
G(r) = l_p^2 \left( 1 - \alpha e^{-r/r_0} \right) ,
\]
the direct experimental test for which in the case \( |\alpha| \geq 1 \) sets \( r_0 < 197 \times 10^{-4} \text{cm} \). Correspondingly, the equation for the gravitational radius takes the form
\[
r_g = 2l_p^2 m \left( 1 - \alpha e^{-r_g/r_0} \right) .
\]
We notice that the Yukawa type correction to the Newtonian inverse square law with negative \( \alpha \) is inconsistent for our purposes. From the above discussion one finds that gravitational radius of the clock giving the minimal error in measuring of a given length \( l \) is determined by the equation
\[
r_g^3 - \frac{l^2}{4} \left( 1 - \alpha e^{-r_g/r_0} \right) = 0 .
\]
Taking the upper bounds for the parameters \( r_g(\lambda = 1.6 \mu m), r_0 \), for parameter \( \alpha \) one finds
\[
\alpha = \left( 1 - \frac{4r_g^3}{l_p^2 \lambda} \right) e^{r_g/r_0} \approx (1 - 0.58 \times 10^{-38}) e^{0.43 \times 10^{-34}} .
\]
So, in this case \( \alpha \) equals 1 with a great accuracy. Denoting \( r_g' = dr_g/dl \) one finds
\[
r_g' = -\frac{4r_0 r_g^3}{l \left( 12 r_0 r_g^2 - \alpha l_p^2 e^{-r_g/r_0} \right)} .
\]
Using the above parameters, from Eq.(14) one gets
\[
r_g'(\lambda = 1.6 \mu m) \approx 72.06 \times 10^{-38} .
\]
Therefore the running Newtonian constant \( r_g'(\lambda \approx 1) \) satisfies the experimental constraints on Eq.(12).

Another example of running Newton constant obtained by means of the Wilson-type effective action has the form
\[
G(r) = \frac{l_p^2 r^3}{r^3 + 2.504l_p^2 (r + 4.5l_p^2 m)} ,
\]
where \( m \) is the mass of the source. By taking the quantum corrected equation for gravitational radius as
\[
r_g = 2G(r_g)m ,
\]
one finds the following expression for outer horizon
\[
\frac{r_p}{l_p^2 m} = 0.667 + 0.265 \left( 16 - 139.5 t + 10.392 \sqrt{t(t-0.204)(176.392 + t)} \right)^{1/3} - \frac{0.41(3t-4)}{(16 - 139.5 t + 10.392 \sqrt{t(t-0.204)(176.392 + t)})^{1/3}},
\]

where \( t \equiv 2.504 m_p^2/m^2 \). (An interesting feature of this running Newton constant is that there is a critical value of mass, \( t_{cr} = 0.204 \), \( m_{cr} = 3.503 m_p \), below which the horizon disappears and leads therefore to the black hole remnants of about Planck mass \([11, 12]\).) From Eq. (15) one sees that this running Newtonian constant gives practically the standard value of gravitational radius for \( m \sim 10^{26}\)kg and therefore does not satisfy the criterion for explaining the astronomical observations of the preceding section.

The basic conceptual point coming from the gedanken-experiment for space-time measurement is that quantum fluctuations of measuring device in the process of making measurement does not allow the space-time intervals to be precisely determined. In other words the space-time is disturbed by the measurement process itself limiting thereby our precision in space-time measurement. Therefore, one should take note of difficulty in evaluating of space-time disturbance during the real measurement stemming from the fact that in the real measurements we are not using optimal devices minimizing the space-time uncertainties caused by the measurement process itself.

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