Calculating the diffractive from the inclusive structure function

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Abstract

It is demonstrated that the global properties of the rapidity gap events at HERA can be understood based on electron-gluon scattering and a non-perturbative mechanism of colour neutralization. Using the measured inclusive structure function $F_2$ to determine the parameters of the parton model, the diffractive structure function $F_D^2$ is predicted. The ratio of diffractive and inclusive cross sections, $R_D = \sigma_D/\sigma_{incl} \simeq 1/9$, is determined by the probability of the produced quark-antiquark pair to evolve into a colour singlet state.

Résumé

Les événements à gap de rapidité observés à HERA peuvent être attribués à la diffusion d’un gluon et d’un électron combiné avec un mécanisme non-perturbative pour la neutralisation de la couleur. Nous utilisons la fonction de structure inclusive $F_2$ pour déterminer les paramètres du modèle de partons et pour prévoir la fonction de structure diffractive $F_D^2$. Le quotient des sections efficaces diffractives et inclusives, $R_D = \sigma_D/\sigma_{incl} \simeq 1/9$ correspond à la probabilité que le pair quark-antiquark produit dans ce procès évolue dans un état neutre par rapport à la couleur.

Recently, detailed analyses of rapidity gap events in deep inelastic scattering at HERA have appeared confirming the previously observed leading twist behaviour of the diffractive cross section $\sigma_D$\cite{1}-\cite{4}. The absence of a hadronic energy flow between proton remnant and current fragment suggests that in the scattering process a colour neutral part of the proton is stripped off. A leading twist behaviour is usually regarded as evidence for scattering on point-like constituents. This, however, appears to be in conflict with the fact that quarks and gluons carry colour.

In the following we shall demonstrate that this puzzle can be resolved considering the production of a quark-antiquark pair in electron-gluon scattering as basic partonic process and taking non-perturbative fragmentation effects into account \cite{5}. Our approach is related to previous work on “aligned jet models” \cite{6} and “wee parton lumps” in deep inelastic scattering \cite{7,8}. Photon-gluon fusion has been independently considered as basic process underlying rapidity gap events by Kowalski \cite{9} and Edin et al. \cite{10}.

The relevant kinematic variables are defined in figure\cite{4}. With $\bar{p}_g = \xi \bar{P}$, $-p_g^2 = m_g^2 \ll Q^2$ and the invariant mass square $M^2 = (q + p_g)^2$ one has,

$$\beta \equiv \frac{Q^2}{Q^2 + M^2} \simeq \frac{x}{\xi}.$$ \hfill (1)

The differential cross section for the inclusive production of quark-antiquark pairs is given by \cite{11}

$$\frac{d\sigma}{dx dQ^2 d\xi} = \frac{\alpha}{\pi x Q^2} g(\xi) \left( \left( 1 - y + \frac{y^2}{2} \right) (\sigma_T + \sigma_L) - \frac{y^2}{2} \sigma_L \right).$$ \hfill (2)

Here $g(\xi)$ is the gluon density, and the cross section $\sigma_T(\xi)$ is obtained by integrating the differential parton
cross section over the kinematically allowed range for the momentum transfer $t = (q - l')^2$. The differential partonic cross sections for the absorption of a virtual photon $\sigma_T$ and $\sigma_L$, calculated in the massive gluon scheme, can be found in ref. [11].

At small values of $\xi$ the quark and antiquark densities are much smaller than the gluon density. Therefore we shall neglect the quark contribution to $F_2$ in the following. Note, that in our model this essentially amounts to calculating the sea quark densities at scale $Q$ in terms of a gluon density $g(\xi)$ at a scale $m_g = O(1$ GeV).

From eq. (2) together with the virtual photon cross sections one then obtains the inclusive structure function $F_2(x, Q^2)$,

$$F_2(x, Q^2) = x\frac{\alpha_s}{2\pi} \sum_q e_q^2 \int_x^1 \frac{d\xi}{\xi} g(\xi) \times$$

$$\times \left( (\beta^2 + (1 - \beta)^2) \ln \frac{Q^2}{m_g^2 \beta^2} - 2 + 6\beta(1 - \beta) \right).$$

The virtuality $-m_g^2$ of the gluon regularizes the collinear divergence at $t = 0$.

For the gluon density at small values of $\xi$ we use the usual parameterization,

$$g(\xi) = A_g \xi^{1-\lambda},$$

where $A_g$ is a constant. Inserting eq. (3) into eq. (4) we can now evaluate the inclusive structure function $F_2$.

In the small-$x$ region the obtained expression can be further simplified for small values of the exponent $\lambda$. This yields

$$F_2(x, Q^2) \simeq x\frac{\alpha_s}{3\pi} \sum_q e_q^2 x g(x) \left( \frac{2}{3} + \ln \frac{Q^2}{m_g^2} \right).$$

For the parameters $\lambda = 0.23$, $m_g = 1.0$ GeV and $A_g\alpha_s \sum_q e_q^2 = 0.61$ the above expression provides a good description of the H1 measurement of the structure function (compare the phenomenological fit of $F_2$ in [12]).

We are now ready to calculate the diffractive structure function. The main idea is that the quark-antiquark pair, originally produced in a colour octet state, changes its colour through further soft interactions with the colour field of the proton remnant. Hence, the quark-antiquark pair evolves into a parton cluster which separates from the proton remnant with some probability $P_k$ in a colour octet state, and with probability $P_l = 1 - P_k$ in a colour singlet state. In the first case a colour flow between proton remnant and current fragment leads to the typical hadronic final state. In the latter case, however, the colour singlet final state fragments independently of the proton remnant, yielding a gap in rapidity. For a sufficiently fast rotation of the colour spin of quark and antiquark, the probabilities should simply be given by the statistical weight factor accounting for the possible states of the quark-antiquark pair, i.e., $P_l \simeq 1/9$, $P_k \simeq 8/9$.

Similar ideas concerning the rotation of quarks in colour space have been discussed by Nachtmann and Reiter [13] in connection with QCD-vacuum effects on hadron-hadron scattering. Another approach describing the colour rotation by a soft gluonic field is based on the eikonal approximation to the quark propagator [14]. Here a non-abelian phase is introduced through a path ordered integral along the fermion line.

The diffractive structure functions are defined as

$$\frac{d\sigma_D}{dx dQ^2 d\xi} = \frac{4\pi\alpha^2}{xQ^4} \left( \left( 1 - y + \frac{y^2}{2} \right) F_2^D - \frac{y^2}{2} F_L^D \right).$$

$F_2^D(x, Q^2, \xi)$ is easily obtained from eq. (3). With $x = \beta\xi$, and including the statistical weight factor $P_1$, one obtains

$$F_2^D(x, Q^2, \xi) \simeq \frac{1}{9} x\frac{\alpha_s}{2\pi} \sum_q e_q^2 g(\xi) \tilde{F}_2^D(\beta, Q^2),$$

where

$$\tilde{F}_2^D(\beta, Q^2) = \beta \left( (\beta^2 + (1 - \beta)^2) \ln \frac{Q^2}{m_g^2 \beta^2} - 2 + 6\beta(1 - \beta) \right).$$

Since the gluon density $g(\xi)$ and the mass scale $m_g$ have been determined by the fit to the inclusive structure function $F_2$, the diffractive structure function is unambiguously predicted, including its normalization.

As an immediate consequence the ratio of diffractive and inclusive cross sections follows from eqs. (3) and (7),

$$R_D = \int_1^\infty d\xi F_2^D(x, Q^2, \xi) / F_2(x, Q^2) \simeq \frac{1}{9}. $$

This ratio directly measures the probability of forming a colour singlet parton cluster in the scattering process.
Figure 2. Dependence of the diffractive structure function on $\beta$ and $Q^2$.

The form of the diffractive structure function $F_D^2(\beta, Q^2)$ is identical with expressions obtained based on the idea of a “pomeron structure function” \cite{15, 16}. The interpretation of the ingredients, however, is rather different. The “pomeron flux factor” is replaced by the density of gluons inside the proton, which factorizes.

The “pomeron structure function” for partons with momentum fraction $\beta$ inside the “pomeron” is identified as the differential distribution for the production of a quark-antiquark pair with invariant mass $M^2$.

As it can be seen from figure 2, the function $F_D^2(\beta, Q^2)$ is rather flat for intermediate values of $\beta$ between 0.2 and 0.6. Approximating $F_D^2(\beta, Q^2)$ in this interval by $F_D^2(0.4, Q^2)$, a comparison of eqs. (3) and (7) yields the scaling relation \cite{8}

$$F_D^2(x, Q^2, \xi) \simeq \frac{0.04}{\xi} F_2(\xi, Q^2).$$

(9)

This scaling relation provides a rather accurate description of recent measurements of the diffractive structure function by the H1 collaboration \cite{2}.

Let us finally verify the appearance of rapidity gaps within the above model. The rapidity of the antiquark with momentum $l$ in the $\gamma^* p$-rest frame is related to other kinematical variables by

$$\eta = \frac{1}{2} \ln \left[ \frac{(1 - \beta) u + m_q^2 \beta}{t + m_q^2 \beta} \right].$$

(10)

Using this relation the differential cross section as a function of the rapidity of the antiquark can be calculated from eqs. (2) and (10). The total diffractive cross section for a maximum rapidity $\eta_{\max}$ can now be obtained by integrating over the kinematic domain where the rapidity of the antiquark is larger than the rapidity of the quark. The reverse configurations yield the same contribution. Using the kinematic boundaries of ref. \cite{1} the $\eta_{\max}$-distribution has been calculated in the described manner \cite{6}. Above $\eta_{\max} \sim 2$ the diffractive cross section is found to be negligible.

In summary, we have demonstrated that electron-gluon scattering can account for the global properties of the rapidity gap events observed at HERA provided the following two hypotheses are correct: First, the initially produced quark-antiquark pair evolves with a probability $P_1 \approx 1/9$ into a colour singlet parton cluster; second, the rapidity range of the diffractive hadronic final state is essentially given by the rapidity interval spanned by the produced quark-antiquark pair.

This simple picture appears to provide a rather accurate description of the observed diffractive events, including the total rate, the $\xi$-dependence and the $Q^2$-dependence. However, a number of theoretical issues still remain to be settled. Our results indicate, that a semiclassical approach to the small-$x$ region might be appropriate, where “wee partons” are treated as a classical colour field.

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