Interplay of Superconductivity and Magnetism in a \(t\)-\(t'\)-\(J\) Approach to High \(T_c\) Cuprates

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Abstract We review a recently proposed mechanism for superconductivity in hole-doped cuprates exhibiting a strong interplay between pairing and antiferromagnetism. Starting from the \(t\)-\(t'\)-\(J\) model for the CuO planes, we show that this interplay can explain in a unified framework the pseudogap phenomenology of the spectral weight of the hole, the hourglass-like structure of the magnetic excitation, the critical exponent of the superfluid density, the relation between the scale of the magnetic resonance and \(T_c\).

1 Common Origin of Short-Range AF and Pairing

In this note, we outline the basic ideas of our approach to the low-energy physics of hole-doped cuprates [1–3], emphasizing an explicit interplay between antiferromagnetism (AF) and pairing attraction leading to superconductivity (SC).

We tackle the non-perturbative constraint of no-double occupation of the \(t\)-\(t'\)-\(J\) model by decomposing at each site \(i\) the fermionic spin one half hole field \(c_\alpha\) into a product of a bosonic chargeless spin one half boson \(b_\alpha\), the spinon, and a charged spinless fermion \(h\), the holon: \(c_\alpha = b_\alpha h_i\). Then the Pauli exclusion principle for the holon enforces automatically the constraint. However, this decomposition involves an unphysical degree of freedom, because one can multiply at each site the spinon and the holon by arbitrary opposite phase factors leaving the physical hole field unchanged. This local \(U(1)\) gauge invariance can be made manifest by introducing a slave-particle gauge field \(A_\mu\). The gauge field induces an attraction between holon and spinon with \(T\)-dependent damping. The spin-charge decomposition is a useful approach only if this attraction does not lead to confinement, which in fact doesn’t occur in the considered situations. In planar (2D) and in linear (1D) systems, we have an additional option to improve a mean field (MF) treatment: gauging a global symmetry of the system, one can bind statistical fluxes to particle fields, resulting only in a change of their statistics. The introduction of these fluxes in the lagrangian formalism is realized through statistical Chern-Simons gauge fields. In our case, the holon carries both charge and spin degrees of freedom, so we can bind a statistical charge flux \(\Phi^c\) to the holon and a statistical spin flux \(\Phi^s\) to the spinon. Assuming the absolute value of the spin flux to be one half and the charge flux to be \(-1/2\) the resulting hole field \(c_\alpha = (\exp(i\Phi^s)b_\alpha)\exp(i\Phi^c)h\) is still fermionic. With this choice, the holon bound to the charge flux and the spinon bound to the spin flux obey semion statistics, implying that they acquire under exchange a phase \(\pm i\), intermediate between the bosonic \(+1\) and the fermionic \(-1\), hence the name “semion.” These semionic holons are expected to obey Haldane statistics of order 2 in momentum space, meaning that a maximum of two semions are allowed to have the same momenta. Hence, a gas of spinless semions of finite density has a (pseudo-)Fermi surface at low \(T\) coinciding with that of spin one half fermions with the same density.

If no approximations are made, all the slave-particle approaches to the \(t\)-\(t'\)-\(J\) model are strictly equivalent. However, as soon as MF approximations are made, this approach is a new slave-semion approach, distinct from slave boson or slave fermion approaches. The semionic statistics is the same found in the solution of the 1D model [4, 5]. In the improved semionic mean field approximation (MFA), the spinon configurations in the presence of moving holons are
first optimized, leading to a new bosonic spinon denoted by $s$, describing fluctuations around the optimized spinon background and still satisfying the constraint $s_{\alpha}\sigma_{\alpha} = 1$ (summation over repeated spin indices is understood). From now on, it is this spinon that we refer to. In the adopted MFA, we neglect the holon fluctuations in $\Phi^h$ and the spinon fluctuations in $\Phi_s$. This leads to a much simpler form of the two statistical fluxes. The charge one is actually static and it provides a $\pi$-phase factor per plaquette. This flux converts, via Hofstadter mechanism, the low-energy modes of the spinless holons $h$ into Dirac fermions with dispersion defined in the magnetic Brillouin zone (BZ) and a small Fermi surface (FS) $\epsilon_F \sim t\delta$, where $\delta$ is the doping concentration, characterizing the “pseudogap phase” (PG) of the model. Increasing doping or temperature, one reaches a crossover line $T^*$, identified with the experimental inflection point of in-plane resistivity. Crossing this line, we enter in the “strange metal phase” (SM) in which the effect of the charge flux is screened by the optimal $b$-spinon configuration in MFA and we recover a “large” FS for the holons with $\epsilon_F \sim t(1 + \delta)$. For the SU(2) spin flux in MFA only the $\sigma_z$-component survives:

$$
\Phi_s(x) = \sigma_z \sum_i h^*_i h_j (-1)^{|i|} \frac{(x)}{2} \arg(x - i),
$$

and its gradient can be viewed as the potential of spin-vortices with the axis along the direction of the AF background. These vortices appear in the U(1) subgroup of the spin group complementary to the coset labeling the directions of the spin. Fluctuations of such directions describe the spin-waves, viewed as composites of spinons generated by gauge attraction between spinon and antispinon. Therefore, the spin-vortices have a purely quantum origin, somewhat analogous to the Aharonov-Bohm effect. Note that there is no holon density in the right hand side of (1), so that the spin-vortices are always centered on the holes. The vorticity or chirality of the vortices is $(-1)^{|i|}$, with $|i| = I_x + I_y$. The effect of the optimal spin flux is then to attach a spin-vortex to the holon, with opposite chirality on the two Néel sublattices. These vortices take into account the long-range quantum distortion of the AF background caused by the insertion of a dopant hole. They play also a crucial role in the development of superconductivity and their chirality structure is a consequence of antiferromagnetic interaction, showing a first manifestation of the interplay between SC and magnetism in this approach.

The optimization of the $b$-spinons discussed above involves also a spin-flip associated to every holon jump between different Néel sublattices. Therefore, in the $t$, $t^*$-terms, the spinons appear in the “ferromagnetic” Affleck-Marston form $A M_{ij} = (s_i e^{-i\Phi_s})_{\alpha}(e^{i\Phi_s})_{\alpha}$, whereas in the $J$-term, they appear in the “antiferromagnetic” resonating valence bond (RVB) form $R V B_{ij} = e^{\alpha\beta}(e^{i\Phi_s})_{\alpha}(e^{i\Phi_s})_{\beta}$. The above AM/RVB dichotomy is characteristic of the slave-semon approach involving the SU(2) spin rotation group even in 1D, where it has been rigorously derived [4, 5]. It has the appealing feature that optimize the hopping $|AM_{ij}| = 1$ we optimize also the Heisenberg term. In fact, neglecting $A$-fluctuations, the leading terms of the Hamiltonian can be written as:

$$
H = \sum_{nn} \epsilon_{n} a_{n+1}^\dagger a_n + \sum_{nn} \epsilon_{n} a_{n-1}^\dagger a_n + h.c.
$$

where in the second to last term, we used the identity holding for a bosonic spinon $|AM_{ij}|^2 + |RVB_{ij}|^2 = 1$. A long wavelength treatment of this term, neglecting the spin-vortices, leads to a CP1 spinon nonlinear $\sigma$-model.

The additional interaction term between spinons and spin-vortices is then of the form

$$
J(1 - 2\delta)(\nabla \Phi_s(x))^2 s^* s(x),
$$

and it is the source of both short-range AF and charge pairing, providing a second interplay between SC and AF. In fact, from a quenched treatment of spin-vortices, we derive the MF expectation value $\langle (\nabla \Phi_s(x))^2 \rangle = m^2 \approx 0.5\delta |\log \delta|$, which opens a mass gap for the spinon, consistent with AF correlation length at small $\delta$ extracted from the neutron experiments [6]. Thus, propagating in the gas of slowly moving spin-vortices, the AF spinons, originally gapless in the undoped Heisenberg model, acquire a finite gap, leading to a short range AF order. By averaging instead the spinons in (3), we obtain an effective interaction:

$$
J(1 - 2\delta)(s^* s) \sum_{i,j} (-1)^{|i|+|j|} \Delta^{-1}(i - j) h^*_i h_j h^*_j h_i, \quad (4)
$$

where $\Delta$ is the 2D lattice Laplacian. From (4), we see that the interaction mediated by spin-vortices on holons is of 2D Coulomb type. From the known behaviour of planar Coulomb systems, we derive that below a crossover temperature $T_{ph} \approx J(1 - 2\delta)(s^* s)$, which turns out to be greater than $T^*$, a finite density of incoherent holon pairs appears. Therefore, the origin of the charge-pairing is magnetic, but it is not due to exchange of AF spin fluctuations.

More in detail, if we describe the magnetic Brillouin Zone (BZ) with the upper half of the BZ, in PG, the Bardeen-Cooper-Schrieffer (BCS) treatment of this pairing on the two small FS centered at $(\pm \pi/2, \pi/2)$ yields two
p-wave orders for the holon, recombining to give a d-wave order in the full BZ for the hole, as first suggested in a different setting in Ref. [7]. Since the pairing distinguishes the two Néel sublattices, in SM (but below $T_{ph}$) it produces also a folding of the holon FS into the magnetic BZ, inducing the formation of two hole-like FS around $(\pm \pi/2, \pi/2)$ and an electron-like FS around $(\pm \pi, 0) = (0, \pi)$. In BCS approximation, we have again p-wave order in the two hole-like FS and s-wave order in the electron-like FS for the holon, reproducing finally a d-wave order for the hole in the full BZ [3]. The pairing turns out to be not far from the BCS-BEC crossover, but still in the BCS side. Since the pairing originates from spin-vortices it is independent of nesting features of the Fermi surface, used in most spin-wave approaches.

2 Consequences of the Interplay Between Magnetism and Pairing

However, we do not have condensation of holon pairs because the fluctuations of the phase of the pairing field are too strong. In fact, the scattering of the phase of the holon-pair field, with a gap $\sim T$, against holons destroys the holon-pair coherence of BCS approximation. Via spinon-holon binding, it produces the phenomenology of Fermi arcs coexisting with gap in the antinodal region [8], i.e., lowering $T$ one finds a decrease of the hole spectral weight on the FS and the formation of superconducting-like peaks even in the normal state, starting from the antinodal region, but with a strong overdamping due to the strong interaction of the spinon with gauge fluctuations. The final outcome [3] agrees qualitatively with experimental data on tunneling, ARPES, and conductivity. The aspects of pseudogap phenomenology related to the gradual reduction of the spectral weight are therefore a consequence of the AF responsible for the pairing.

This charge pairing does not yet lead to hole-pairing, since the spins are still unpaired. The hole-pairing is achieved by the gauge attraction between holon and spinon which, using the holon-pairs as sources of attraction, induces in turn the formation of short-range spin-singlet (RVB) spinon pairs, see Fig. 1 (left).

This phenomenon occurs, however, only when the density of holon-pairs is sufficiently high, since this attraction has to overcome the original AF-repulsion of spinons caused by the Heisenberg $J$-term which is positive in the slave-fermion approach (see (2)). Summarizing at an intermediate crossover temperature lower than $T_{ph}$ and denoted by $T_{ps}$, a finite density of incoherent spinon RVB pairs appears; combined with the holon pairs, it gives rise to a gas of incoherent preformed hole pairs. The lowering of free energy allowing the formation of spinon pairs is due to the fact that the spinon gap originates from screening due to unpaired vortices, the short-range vortex-antivortex pairs essentially not contributing to it. Therefore, the spinon gap lowers proportionally to the density of spinon pairs, implying a lowering of the kinetic energy of spinons, a mechanism definitely not BCS-like. More precisely, for a finite density of spinon pairs, there are two (positive energy) spinon excitations, with different energies, but the same spin and momenta. For example, for spin up, they are obtained by creating a spinon up unpaired or destroying a spinon down in one of the RVB pairs. The corresponding dispersion relation, thus exhibits two (positive) branches:

\[
\omega(\mathbf{k}) = \sqrt{(m_s^2 - |\Delta|^2) + (|\mathbf{k}| \pm |\Delta|)^2},
\]

where $\Delta$ is (proportional to) the RVB spinon pair order parameter. The lower branch exhibits a minimum with an energy lower than $m_s$ corresponding to the reduction of

![Fig. 1](image-url)
the spinon gap discussed above, and it implies a backflow of the gas of spinon-pairs dressing the “bare” spinon. The spinon-antispinon attraction mediated by gauge fluctuations induces a similar structure for the magnon dispersion around the AF wave vector [9], see Fig. 2, reminiscent of the hourglass shape of spectrum found in neutron experiments [10], with an energy at the AF wave vector approximately twice the spinon gap $J(1 - 2\delta)/\delta \ln \delta)^{1/2}$.

This feature is evident in the superconducting phase, where the damping induced by gauge fluctuations in the normal phase is strongly suppressed. Hence, the modification of the magnetic excitation leading to the magnetic resonance is ultimately due to the charge pairing interaction.

The gradient of the phase of the low-energy treatment of the RVB-singlet hole-pair field $\hbar^2 h^\alpha e^{\alpha\beta} (s_i/e^{\alpha\beta})_\alpha (e^{\beta\delta}s_j)_\delta$ describes magnetic vortices (not to be confused with quantum spin-vortices). Between $T_{ps}$ and the superconducting transition, these magnetic vortices are in the plasma phase because the hole pairs are incoherent. Therefore, we propose to identify $T_{ps}$ with the experimental crossover corresponding to the appearance of the diamagnetic and (vortex) Nernst signal [11, 12]. This interpretation is reinforced by the computation of the equi-level plot of the spinon pair density in the $\delta-T$ plane [2], resembling the contour-plot of the Nernst/diamagnetic signal. The doping density enters in the above mechanism through two factors: the density of hole pairs and the strength of the attraction behaving like $\approx J(1 - 2\delta)$ from (4). These two effects act in an opposite way increasing doping, thus yielding a “dome” shape to $T_{ps}(\delta)$, starting from a non-zero doping concentration.

Superconductivity occurs by condensation of hole pairs at a temperature $T_c$ lower than $T_{ps}$ inheriting the dome structure, see Fig. 1 (right).

Below $T_{ps}$, the effective action obtained integrating out holons and spinons is a $A$-gauged 3D XY model, where the angle-field of the XY model is the phase of the hole-pair field. At the SC transition, the gauge field $A$ is gapped by the Anderson-Higgs mechanism and the gauge fluctuations are suppressed. Hence, the superconducting transition is almost of classical 3D XY type, driven by condensation of magnetic vortices related to phase coherence as in BEC systems and providing another interplay between SC and magnetism. The transition is therefore not BCS-like, as shown by the critical exponent of the superfluid density which is 2/3 [13], characteristic of the 3D XY model. This value turns out to agree with the experimental data in underdoped-optimally doped cuprates [14].

Finally, the common origin of pairing and short-range AF from the same term in the representation of the $t$-$t’$-$J$ model (3) produces an intrinsic approximately linear relation between the energy of the magnon resonance and $T_c$ [2], as observed in neutron scattering experiments [15].

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