Generation of very high-order high purity Gaussian modes via spatial light modulation

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We experimentally demonstrate the conversion of a fundamental TEM$_{00}$ laser mode at 1064 nm to higher order Hermite-Gaussian modes (HG) of arbitrary order via a commercially available liquid crystal Spatial Light Modulator (SLM). We particularly studied the HG$_{5,5}$/HG$_{10,10}$/HG$_{15,15}$ modes. A two-mirror plano-spherical cavity filters the higher-order modes spatially. We analyze the cleaned modes via a three-mirror diagnosis cavity and measure a mode purity of 96%/93%/78% and a conversion efficiency of 6.6%/3.7%/1.7% respectively. The generated high-purity Hermite-Gaussian modes can be employed for the mitigation of mirror thermal noise in optical cavities for both optical clocks and gravitational wave (GW) detectors. HG modes are then converted into high order LG modes which can be of particular interest in cold atom physics.

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Lasers used for high-precision experiments typically generate an output beam in the fundamental TEM$_{00}$ mode, since an operation at higher-order HG$_{m,n}$ or LG$_{p,l}$ modes suffers from low lasing efficiency due to diffraction losses and a less stable output mode [1]. These are, however, beneficial in metrology experiments using optical cavities or laser interferometers which are fundamentally limited by thermally induced mirror surface motions that reduce the length sensing sensitivity [2]. More specifically, reference cavities used for the frequency stabilization of the optical oscillator in optical clocks are currently limited in stability by thermal noise of the highly-reflective mirror coatings with a fractional instability of $4 \times 10^{-17}$ in 1 s [3]. Advanced GW detectors are also limited by the coatings’ thermal noise at $h = 10^{-23}[1/\sqrt{Hz}]$ in the frequency band of 30-300 Hz [4,5], which will limit their sensitivity and the sensitivity of the third generation GW detectors [6]. The negative effect of thermal noise can be mitigated by using a spatially broader intensity profile for the laser beam in comparison to the fundamental TEM$_{00}$ mode. This results in a higher averaging over the mirror surface for the same cavity characteristics. Using flat-top beams was initially proposed [7,8] and its efficiency to reduce thermal and thermoelastic noises was theoretically demonstrated [9–11]. However, the experimental implementation of these modes is troublesome since it implies the use of the so-called Mexican-hat mirrors. Mitigation of thermal noise can also be achieved by higher-order modes whose efficiency increases with the total mode order $N$ ($N = m+n$ for HG$_{m,n}$ modes and $N = 2l + p$ for LG$_{p,l}$ modes) [12]. Higher-order Laguerre-Gauss modes were the first to be suggested for ground based GW detectors [7,12] and this idea was extended for reference cavities related to optical clocks [14]. For GW detection, the high order modes must be: high-power (efficiently generated), pure and robust with respect to mirrors defects and aberrations. While the LG$_{3,3}$ mode has been the object of several experimental investigations in this context [15–24], there has not yet been any experiment concerning high order HG modes. A reduction of thermal noise was recently demonstrated in a 10-cm Ultra Low Expansion (ULE) glass reference cavity using the HG$_{0,2}$ mode. Furthermore, LG$_{0,1}$ modes are of particular interest for capturing and guiding cold atoms which have a significant impact on quantum optics, quantum gases physics and metrology [25–27]. They provide a donut intensity shape, which remains unchanged while propagating. This shape allows transverse trapping of cold atoms in a potential well which is even more steep with increasing $l$ [28–31]. LG$_{0,1}$ modes can also be used to transfer multi $h$ angular momentum [31], i.e. link atomic levels with different $m_F$. For those applications, the mode purity is critically important. In this letter we investigate the conversion of the fundamental TEM$_{00}$ mode at 1064 nm to higher-order high purity HG$_{m,n}$ and LG$_{p,l}$ modes for the very first time.

**LG versus HG** The thermal noise at low frequencies in the case of an optical cavity operating with axisymmetric LG modes is computed in [13,32] using the Bondy-Hello-Vinet (BHV) technique [33](amended in [34]). This calculation is based on Levin’s theory [35] and is applied on finite mirrors. HG modes’ related thermal noise was computed analytically in [12] only in the case of infinite mirrors. Based on the latter reference, we compare in Tab. I the mitigation efficiency of thermal noise for the LG$_{5,5}$, a frequent example in theoretical computations [13,33,36], the LG$_{3,3}$ and the HG modes presented in this letter: HG$_{5,5}$, HG$_{10,10}$. Coating Brownian thermal noise is currently limiting both reference cavities and GW detectors, whereas the thermoelastic noise limit, which undergoes similar reduction factors, lies below that and is not considered here. LG modes are more efficient to mitigate thermal noise for equivalent clipping losses. However, it was found that the LG
modes pose strong requirements on the mirror surface quality since they exhibit a mode pseudo-degeneracy \[22\] which greatly decreases their coupling efficiency to linear cavities. Residual astigmatism of the mirrors turns out to be the main defect responsible for the intracavity mode degradation \[18, 23\] and its thermal compensation was demonstrated to improve the mode quality \[19\]. Astigmatism also prevents LG modes from resonating in a 4 mirror pre-mode cleaner unless in a non-planar configuration \[24\]. Note that LG modes do not resonate in triangular cavities \[17\] which are largely used in GW detectors.

### TABLE I. Thermal noise limit reduction for higher order modes with respect to the Gaussian mode for both substrate and coatings. \(a\) represents the mirror radius and \(w\) designates the beam radius. The ratio \(a/w\) considered here corresponds to a clipping loss of 1 ppm.

| Order | Substrate | Coatings | \(a/w\) |
|-------|-----------|----------|--------|
| HG    | 5,5       | 0.576    | 0.493  | 4.379  |
|       | 10,10     | 0.491    | 0.392  | 5.513  |
| LG    | 3,3       | 0.559    | 0.378  | 3.313  |
|       | 5,5       | 0.496    | 0.310  | 4.968  |

Whereas astigmatic LG modes are not a mathematical solution of the free propagation equation, astigmatic HG modes are, and they are likely to couple into a 2-mirror astigmatic cavity (the case of 3-mirror cavities will be described below). To that end, one needs to angle the two mirrors around the optical axis in order to match their own axes to those of the injected HG mode. To confirm these assumptions, FFT based simulations using the software DarkF \[27\] on a 3km long cavity of a finesse 1000 involving an Advanced Virgo like mirror (surface roughness 0.5 nm rms, radius of curvature 3465 m) and a perfect flat mirror were conducted. LG\(_{3,3}\) and LG\(_{5,5}\) mode coupling efficiency does not exceed 70%. On the other hand, the HG\(_{5,5}\) and HG\(_{10,10}\) mode coupling increases from 77% to 99% and from 62% to 96% respectively when one rotates the mirrors by the right angle (see supplementary material). This renders higher-order HG modes as promising candidates for applications in GW detectors and reference cavities for optical clocks, although they grant a slightly smaller mitigation effect of thermal noise.

### Generation of HG modes

In our experiment we achieved the conversion of a fundamental TEM\(_{00}\) mode to higher-order Hermite-Gaussian modes via the reflection on a commercially available computer addressed liquid crystal phase-only SLM by Hamamatsu Photonics. The same technique was used by several groups to generate LG beams for different purposes \[29, 30, 38–43\]. The amplitude distribution of the reflected beam is then conserved whereas the phase distribution is exclusively defined by the phase map \(\varphi_{\text{SLM}}(x, y)\). The higher-order mode is then obtained out of the field distribution \(E_{\text{out}}\) in the image focal plane of a Fourier lens. This field distribution is thus related to the incident beam according to \(F[E_{\text{out}}(X, Y)] = |\text{TEM}_{00}(x, y)| \exp[i\varphi_{\text{SLM}}(x, y)]\) where \(F\) denotes the Fourier transform. Since a HG\(_{m,n}\) mode is invariant by \(F\), the last equation shows that a fundamental Gaussian mode cannot be converted into an arbitrary HG\(_{m,n}\) mode with 100% efficiency. In \[44, 45\] it is proposed to generate the HG\(_{m,n}\) mode in a limited area of the Fourier plane while no condition is imposed on the rest of the plane. The approach is inspired from the Gerchberg-Saxton algorithm \[46\]. It consists of consecutive iterations between the SLM and the Fourier planes using respectively the Fourier and inverse Fourier transforms while imposing on each iteration the fundamental mode amplitude on the SLM and the HG\(_{m,n}\) mode in the restricted area of the Fourier plane. Originally, the Gerchberg-Saxton algorithm imposes an amplitude condition in the Fourier plane and was successfully used for different purposes (see for example \[27\] and the references therein). Different conditions can be imposed provided enough degrees of freedom are respected \[47\]. In our case, both amplitude and phase are imposed in the Fourier plane but in a restricted area. This iterative approach was generalized by Levi et. al. and received the name of generalized projection method \[48\]. In iterative Fourier transform algorithms, vortex stagnation \[49\] issues might occur which correspond to a very slow convergence or its absence. Relaxation parameters can be introduced to the algorithm to speed up its convergence \[45\] and was used for the generation.
of a HG$_{1,0}$ mode in refs. 44, 45. No such effect was observed in our case and relaxation parameters were tested to be unnecessary. 20 iterations were sufficient to obtain an arbitrary order HG$_{m,n}$ mode within less than 1% error and with a theoretical conversion efficiency spreading from 45% for the HG$_{5,5}$ mode to 20% for the HG$_{15,15}$ mode. The SLM diffraction efficiency decreases with the spatial frequency of the phase map, a non-diffracted light was superimposed to the HG mode in the Fourier plane. A blazed grating was then added to the SLM phase map in order to obtain a free diffraction area. Unwanted light (undiffracted and diffracted in the free area) was spatially filtered with an iris placed in the Fourier plane.

A simplified schematic picture of the experimental setup is given in Fig. 1. The laser source is a continuous-wave single frequency Nd:YAG non-planar ring oscillator (NPRO) at 1064 nm. The laser beam is spatially filtered via a single-mode polarization maintaining fiber. A fiber-based electro-optical modulator generates phase modulated sidebands at 100 MHz for a Pound-Drever-Hall (PDH) cavity stabilization scheme. The phase via a single-mode polarization maintaining fiber. A (NPRO) at 1064 nm. The laser beam is spatially filtered

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**Beam purity measurement** As pointed out by Fulda et al. 17, one can inject the mode into a non-degenerate diagnosis cavity and measure its coupling efficiency in order to quantify its purity. This method was identified to be non ideal to test the purity of LG modes because the cavity modes themselves are not pure LG modes. A partial measurement of the LG mode purity was made by fitting the intensity profile with the corresponding analytical function at some spatial location 15, 17. Unless the mirrors are angled with respect to each other and to the incoming mode as described above, testing HG modes faces the same difficulty. However, at some level of the cavity astigmatism, the degeneration lifting between modes of constant total order $N = m + n$ exceeds the cavity bandwidth, which then cleans up the cavity mode. FFT based simulations were conducted on the same Virgo-like cavity by artificially adding a curvature to the end mirror either along $x$ or the $y$ axis. Coupling efficiencies exceeding 97% for both HG$_{5,5}$ and HG$_{10,10}$ are obtained (see supplementary material). We analyzed the mode-purity via a three-mirror triangular cavity whose mode is naturally astigmatic. It consists of two plane mirrors and a high-reflective spherical back mirror having a cavity Finesse about 709. The cavity back-mirror is attached to a piezo-electric actuator to tune the cavity length. The roundtrip length of the cavity is 42.5 cm, resulting in a free spectral range of 706 MHz. The beam impinges on the 50 cm radius of curvature spherical mirror with an angle of 4°. This makes the degeneracy lifting between modes of constant
FIG. 3. (a) Intensity profile measurements of the \( \text{HG}_{5,5} \), \( \text{HG}_{10,10} \), and \( \text{HG}_{15,15} \) modes (bottom) using a Dataray WinCamD beam camera. The mode purity is given by the measurement of the coupling efficiency on the reflected signal of the three-mirror diagnosis cavity. The red resonance curves depict a low amplitude length scan over the diagnosis cavity resonance. (b) Beam camera measurement showing a \( \text{HG}_{25,25} \) mode. The picture was taken directly after the generation of the mode without spatial filtering, since the mode’s low purity excluded a sufficient mode-matching and mode cleaner cavity stabilization.

FIG. 4. (a) LG modes generated from corresponding HG modes. The HG mode shown here are imaged after non tilted cylindrical lenses. (b) LG modes generated from pure \( \text{HG}_{5,5} \) and \( \text{HG}_{10,10} \) modes. The central peak is typical of \( \text{LG}_{p,0} \) modes.

**HG to LG conversion** The high purity HG modes we generate here are converted into LG modes using a simple combination of two cylindrical lenses [51–53]. HG modes form a complete sets of solutions on which any propagation mode can be decomposed. Interestingly, 45° tilted \( \text{HG}_{m,n} \) mode have the same decomposition on the straight HG modes set as the \( \text{LG}_{\min(m,n),m-n} \) modes except for a multiple of \( \pi/2 \) phase for each term [54]. When the 45° tilted mode crosses two identical cylindrical lenses placed around its waist, with the conditions \( f = \sqrt{2}d \) and \( z_R = f + d \) (\( f \) is the focal length of the...
lenses, $d$ is the distance separating them and $z_R$ is the Rayleigh range of the impinging mode), each term of the decomposition on the straight modes set accumulates a Gouy phase shift corresponding to the required multiple of $\pi/2$ allowing the conversion of the $45^\circ$ tilted HG mode to the corresponding LG mode [22]. In Fig.4a, we show a LG_{0,15} generated from a HG_{0,15} mode which, to our knowledge, is the highest domut mode order ever generated offering an important angular momentum transfer. LG_{5,5} and LG_{10,10} are also generated and we are currently studying their behavior in a middle finesse cavity and comparing them with HG modes. This technique can also be used the other way around: LG modes can be generated by the SLM and converted into HG mode before being injected into the mode cleaner. This profile from high efficiency generation of LG modes (tens of % [23]) to the corresponding LG mode [52]. In Fig.4a, we show a LG_{p,0} tilted HG mode [24] to overcome the lack of efficiency in direct generation of HG modes. In Fig.4b, both LG_{5,0} and LG_{10,0} are generated from pure HG_{5,5} and HG_{10,10} to show the match between these particular modes. Except for this, no clear application is known for LG_{p,0} modes since they do not carry any angular momentum and they have high intensity central peaks preventing the thermal noise mitigation.

**Conclusion** We experimentally demonstrated the transformation of the fundamental TEM_{00} mode to higher-order HG laser modes as high as HG_{25,25} via a liquid crystal spatial-light modulator. The generated HG_{5,5}/HG_{10,10}/HG_{15,15} modes were filtered via a two-mirror modecleaner cavity and analyzed by a three-mirror triangular cavity. The experiment achieved a conversions efficiency of 6.6/3.7/1.7% and a mode purity of 96/93/78% for the HG_{5,5}/HG_{10,10}/HG_{15,15} modes. High order HG modes were converted into axisymmetric LG modes using a system of two 45° tilted cylindrical mirrors. To our knowledge, this is the first time modes of such high order mode and purity were published.

Following these results, we conclude that the generated HG_{l,m} modes are compatible with the frequently used triangular three-mirror cavities used in GW detectors. Whereas three mirror cavities accommodate pure, rather slightly astigmatic, HG eigenmodes, linear two-mirror cavities suffer from mode pseudo degeneracy related to mirrors aberration. In comparison to LG modes, HG modes exhibit the advantage of adapting themselves to the cavity astigmatism provided the cavity mirrors are properly angled to each other and to the impinging mode. HG modes are then less sensitive to mirrors aberrations which is highly beneficial when implemented in metrology experiments [23,24]. Our results thus pave the way for further investigating higher-order HG modes for the reduction of thermal noise in optical cavities.

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Supplementary Material: FFT based simulations of high order propagation modes behavior in a Virgo like real cavity

We present here the FFT based simulations of a 3 km plano-concave cavity with a finesse 1000. The input mirror is plane and has a transmission $T = 3137$ ppm. Its diameter is 30 cm. The end mirror has an identical transmission and has a radius of curvature $R = 3464$ m. Hence, the $g$ factor of the cavity is $g \simeq 0.11$, the cavity mode has its waist on the input mirror with a beam radius $w_0 \simeq 1.88$ cm and the beam radius on the end mirror is $w_1 \simeq 5.72$ cm for a laser wavelength $\lambda = 1064$ nm. The theoretical power enhancement factor (PEF) is $F/\pi \simeq 318$. The simulations presented below were conducted considering the input mirror as perfect whereas the end mirror presents the distortions displayed on Fig.5 which represent a Virgo like mirror surface residual when piston, tilts and curvature are subtracted. The surface error considered here is 0.5 nm rms corresponding to the requirements of the Advanced Virgo project.

This cavity is simulated using the DarkF software which uses an FFT algorithm to compute the Fresnel propagation of the laser beam from one mirror to the other. The injected beams used here are LG$_{3,3}$, LG$_{5,5}$, HG$_{5,5}$ and HG$_{10,10}$ whose intensity distributions are represented in Fig.6.

![Fig. 5. Typical coated mirror distortion when piston, tilts and curvature are subtracted in the AdvancedVirgo project.](image1)

![Fig. 6. Intensity distribution of the high order modes which are injected into the cavity.](image2)

| TABLE II. Coupling efficiency of considered HG and LG modes for different end mirrors angles. |
|--------------------------------------|--------|----------|--------|
| angle ($^\circ$) | PEF   | Matching (%) | RTL (ppm) |
|------------------|-------|-------------|-----------|
| LG$_{3,3}$       | 218   | 68          | 0.27      |
| LG$_{5,5}$       | 207   | 65          | 0.3       |
| HG$_{5,5}$       | 0     | 246         | 77        |
| HG$_{10,10}$     | 29    | 315         | 99        |
| LG$_{10,10}$     | 50    | 305         | 96        |

**End mirror orientation** The end mirror is angled around the optical axis in order to align the astigmatism axis along with the HG modes. The angle is optimized so to maximize the mode matching into the cavity. Obviously, this procedure has no effect on the LG modes coupling efficiency. The PEF, the mode matching and the Round Trip Losses (RTL) are given in Tab.II for the optimum angles, and show a clear increase in HG modes coupling efficiencies above 95% whereas LG modes matching does not exceed 70%. PEF also increases accordingly. Note that the optimum angles for HG$_{5,5}$ and HG$_{10,10}$ are different since the latter explores a wider surface of the mirror than the former which shows that aberration determination is mode dependent. The simulated intensity profiles are intracavity modes are displayed in Fig.6. A clear improvement of the HG modes profiles appears between $0^\circ$ angle and optimum angles for which distortions are no more observed.

**Astigmatic cavity** To mimic the astigmatism of a triangular cavity, we compute the mode behavior while changing the radius of curvature independently along
the HG mode axis, namely $R_x$ and $R_y$ in the range [3444 m, 3484 m]. Results of the mode matching are displayed on the top of Fig. 8. On the diagonal, where only residual astigmatism exists which is not aligned with the $x$ and $y$ axis, the matching remains below 70% for both HG$_{5,5}$ and HG$_{10,10}$. When the additional astigmatism is important enough, the matching exceed 95%. On the bottom of Fig. 8 the intensity profiles of intracavity modes are displayed for the maximum astigmatism considered here $R_x/R_y \simeq 0.99$.

**FIG. 7.** Simulated intensity profiles of intracavity modes for different angles.

**FIG. 8.** Top: Matching as a function of mirror curvature $R_x$ and $R_y$. Yellow stars correspond to the astigmatism maximum. Bottom: Intensity profile of the intracavity mode for the astigmatism maximum.