Approximate Lorentz invariance of the vacuum: a physical solution of the ‘hierarchy problem’?

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Abstract

In the ‘condensed phase’ of effective quantum field theories one expects deviations from exact Lorentz invariance at ultralow momenta $|k| < \delta$ where the shell $\delta$ should only vanish in the strict local limit of the theory when the ultraviolet cutoff $\Lambda \to \infty$. I explore this idea for the Higgs condensate suggesting that, in this case, the resulting relation connecting $\delta$, $\Lambda$ and the Fermi scale might provide a simple physical solution of the ‘hierarchy problem’. In this picture, the Planck scale is not a purely ultraviolet quantity but embodies in its numerical value the peculiar infrared-ultraviolet connection that is realized in the scalar condensate.
1. Introduction

1.1 Following analogies with condensed matter, the idea of a ‘condensed vacuum’ is now playing a very important role in modern particle physics. Indeed, in many different contexts one introduces a set of elementary quanta whose perturbative ‘empty’ vacuum state $|\phi\rangle$ is not the physical ground state of the interacting theory. In the physically relevant case of the Standard Model, the situation can be summarized saying [1] that "What we experience as empty space is nothing but the configuration of the Higgs field that has the lowest possible energy. If we move from field jargon to particle jargon, this means that empty space is actually filled with Higgs particles. They have Bose condensed.”

This type of conclusion is also favoured by the experimental observation that bodies can flow in the vacuum without any apparent friction. This leads to the physical picture of a superfluid, a result that can easily be understood in the Standard Model where the restoring critical temperature is so high. Thus, for all practical purposes, the scalar condensate might be considered a zero-temperature Bose liquid.

Clearly, this type of medium is not the ether of classical physics. However, it is also different from the ‘empty’ space-time of Special Relativity, assumed at the base of axiomatic quantum field theory. Thus, it is not un conceivable that the macroscopic occupation of the same quantum state can represent, in some appropriate sense, the operative construction of a preferred frame, a ‘quantum ether’. This might account for that particular type of non-locality which is required by a ‘realistic’ description of EPR experiments [2, 3] and by the observed violations of Bell’s inequalities [4].

In connection with the idea of ether, it should be better underlined, perhaps, that the original Einstein’s point of view had been later reconsidered with the transition from Special Relativity to General Relativity. Probably, he realized that Riemannian geometry (with its covariant derivatives, its Christoffel symbols,...) is also the natural framework to describe the dynamics of deformable media (see the appendix of ref.[5]). In fact, for this or other reasons, in 1919, in the ‘Morgan Manuscript’ [6, 7], he wrote "..in 1905 I was of the opinion that it was no longer allowed to speak about the ether in physics. This opinion, however, was too radical as we will see later when we consider the general relativity theory. It is allowed much more than before to accept a medium penetrating the whole space and to regard the electromagnetic fields and the matter as well as states of it. But it is not allowed to attribute to this medium, in analogy to ponderable matter, a state of motion in any point. This ether must not be conceived as composed of particles the identity of which can be followed in time...
One can thus say that the ether is resurrected in the general theory of relativity, though in a more sublimated form.”

This ‘resurrection’ of ether in Einstein’s mind is confirmed by a large amount of published and unpublished manuscripts that have now been collected in a book by Kostro [8]. I just observe that, in a picture of the vacuum as a quantum Bose liquid, the last statements would be easy to understand. In fact, the properties of such a medium depend in an essential way on the quantum nature of its constituents. This requires a form of quantum hydrodynamics, of the type originally considered by Landau [9], where the local density of the fluid $n(r)$ and the current density vector $J(r)$ have canonical commutation relations as in quantum mechanics for the position and momentum operators.

Therefore, following the original evolution of Einstein’s thought, one may attempt to introduce an underlying quantum ether in connection with gravity by choosing the particle physics vacuum as the most natural candidate. To this end, however, we have to start to consider the Higgs condensate as a real physical medium and improve on the simplest approximation where it is treated as a purely classical $c$-number field.

To better appreciate the potentiality of this approach, I observe that the idea of generating gravity from the excitations of a ‘self-sustaining’ superfluid is particularly appealing since it leads to the conclusion that only its departures from equilibrium contribute to the space-time curvature [10]. This provides a simple physical argument to understand why the huge energy density of the unperturbed vacuum plays no role and, in this sense, the gravity-superfluid connection follows naturally from Feynman’s indication ”...the first thing we should understand is how to formulate gravity so that it doesn’t interact with the energy in the vacuum” [11].

The analogies between gravity and the density fluctuations of some physical medium have been explored by several authors in different frameworks (for a comprehensive list see ref.[12]). I just observe that a model of gravity from an underlying scalar ether has been proposed in ref.[13]. In a different context, an acoustic analogy of gravity has been constructed in refs.[14, 15, 16] when studying the propagation of density fluctuations in a moving fluid. This requires the introduction of an ‘acoustic metric’ modelled on Galilei covariance, and leads to the acoustic equivalent concepts of black holes, Hawking radiation,... In this framework, the analog of gravitational black holes, as structures that can be formed in dilute Bose-Einstein condensates, has also been explicitly considered [17].

However, if the density fluctuations of the physical vacuum are really governed by a Galilei-covariant acoustic metric, why Lorentz covariance works so well ? Equivalently, why gravity is so weak in ordinary experimental conditions as compared to the other effects (say
electromagnetism) for which an exact Lorentz-covariant description is possible?

As a possible answer to these questions, one can exploit the analogy with an infinite isotropic elastic medium. Its infinitesimal deformation at a given point \( r \) at the time \( t \), say \( Z(r,t) \), is governed by the source-free partial differential equation [5, 18]

\[
(c^2_t \Delta - \frac{\partial^2}{\partial t^2})Z + \frac{c^2_t}{1 - 2\nu} \nabla(\nabla \cdot Z) = 0
\]  

(1)

In the above equation, I have introduced the square velocity \( c^2_t \equiv \frac{Y^2}{2\rho(1+\nu)} \), where \( \rho \) denotes the density of the medium, \( Y \) the Young modulus and \( \nu \) the Poisson ratio (\( 0 \leq \nu \leq 1/2 \) with \( \nu \to 1/2 \) defining the incompressibility limit).

Now, by expressing \( Z = S + T \) such that \( \nabla x S = 0 \) and \( \nabla \cdot T = 0 \), and introducing \( \tau \equiv c_t t \), Eq.(1) implies the propagation of two type of waves: transverse waves of distortion

\[
(\Delta - \frac{\partial^2}{\partial \tau^2})T = 0
\]  

(2)

and longitudinal waves of dilatation

\[
(\eta \Delta - \frac{\partial^2}{\partial \tau^2})S = 0
\]  

(3)

whose velocity is

\[
c^2_s = \eta c^2_t > c^2_t
\]  

(4)

with

\[
\eta = (1 + \frac{1}{1 - 2\nu}) > 1
\]  

(5)

For any value of \( \nu \) no linear transformation \( (r, \tau) \to (r', \tau') \) can preserve the form-invariance of both the differential operators in Eqs.(2) and (3). In this sense, there are two separate forms of ‘Lorentz-covariance’ (associated with \( c = c_s \) or \( c = c_t \)) and no unified description is possible.

The physically relevant situation arises in the limit \( \nu \to 1/2 \) where \( \eta \to \infty \). In this case, \( c_s \)-Lorentz-covariance reduces to Galilei covariance. Further, any observer with speed \( |v| \ll c_t \ll c_s \) sees no appreciable Doppler shift of the frequencies or of the wave vectors of the longitudinal waves. Indeed, with respect to the corresponding effect for the transverse waves, these are suppressed by the very small number \( c_t/c_s \to 0 \). Finally, under the action of an external force \( f \equiv \nabla Q \), Eq.(3) becomes

\[
(\eta \Delta - \frac{\partial^2}{\partial \tau^2})S = \nabla Q
\]  

(6)
so that, by defining $S = \nabla \varphi$, one gets

$$\Delta \varphi(r, t) = \frac{1}{\eta} Q(r, t)$$

up to infinitesimal higher order $1/\eta^2$ corrections. Thus, the longitudinal waves of dilatation are seen as an instantaneous effect and the presence of the huge velocity $c_s$ remains ‘hidden’ in the suppression factor $1/\eta$ of the strength of $Q$.

In this paper, starting from a microscopic description of the scalar condensate, I’ll construct the equivalent of the $c_s/c_t \to \infty$ limit, with $c_t = c$, thus obtaining a possible solution of the so called ‘hierarchy problem’ between the Fermi scale of electroweak interaction and the Planck scale. In this sense, it turns out that gravity can be induced by the phenomenon of spontaneous symmetry breaking. Some peculiar aspects of this idea, however, have no obvious counterpart in the traditional ‘induced-gravity’ approach [19, 20, 21]. For this reason, I’ll suggest in the end a possible way to relate the two descriptions.

1.2 For my analysis, I’ll start from scratch, i.e. from the more conventional point of view that the phenomenon of vacuum condensation is just a convenient way to rearrange the set of original degrees of freedom and can peacefully coexist with Lorentz invariance. Actually, some authors [22] realized that a non-trivial vacuum structure has to imply some form of non-locality but concluded that it should be possible to reabsorb all differences between the physical vacuum and empty space into some basic parameters such as the particle masses and few physical constants.

However, on a general ground, the coexistence of exact Lorentz invariance and vacuum condensation in effective quantum field theories is not so trivial. In fact, as a consequence of the violations of locality at the energy scale fixed by the ultraviolet cutoff $\Lambda$, one may be faced with non-Lorentz-covariant modifications of the infrared energy spectrum that depend on the vacuum structure [23]. To indicate this type of infrared-ultraviolet connection, originating from vacuum condensation in effective quantum field theories, Volovik [24] has introduced a very appropriate name: reentrant violations of special relativity in the low-energy corner. These are expected in a small shell of three-momenta, say $|k| < \delta$, that only vanishes in the strict local limit where $\Lambda \to \infty$ and an exact Lorentz-covariant energy spectrum is re-obtained in the whole range of momenta.

To understand the physical nature of these effects, let us assume that the spontaneously broken phase of a $\lambda \Phi^4$ theory is a real Bose condensate of physical spinless quanta. In the cutoff theory, these are treated as ‘hard spheres’ with a repulsive core $a \sim 1/\Lambda$ analogously
to the molecules of ordinary matter. Thus, in the limit of very long wavelengths $k \to 0$, one expects the lowest excitations to arise from small displacements of the condensed quanta, that already ‘pre-exist’ in the ground state, giving rise to density fluctuations whose energy vanishes when $k \to 0$.

Notice the difference with the usual empty vacuum state of a massive theory. There, a state with non vanishing three-momentum $k \neq 0$ can only be obtained after the creation of one or more massive quanta thus providing the physical argument for the existence of a mass-gap in the energy spectrum. On the other hand, in the case of a medium, the existence of density fluctuations is a very general experimental fact that depends on the coherent response of the elementary constituents to disturbances whose wavelength becomes larger than their mean free path. This leads to an universal description, the ‘hydrodynamic regime’, that does not depend on the details of the underlying molecular dynamics and even on the nature of the elementary constituents. For instance, the same low-energy picture is expected in superfluid fermionic vacua [10] that, as compared to the Higgs vacuum, bear the same relation of superfluid $^3$He to superfluid $^4$He.

Now, the basic macroscopic properties of Bose liquids can be understood in terms of their long-wavelength excitations: phonons. In fact, [25] "Any quantum liquid consisting of particles with integral spin (such as the liquid isotope $^4$He) must certainly have a spectrum of this type...In a quantum Bose liquid, elementary excitations with small momenta $k$ (wavelengths large compared with distances between atoms) correspond to ordinary hydrodynamic sound waves, i.e. are phonons. This means that the energy of such quasi-particles is a linear function of their momentum”.

Therefore, quite independently of the Goldstone phenomenon, the energy spectrum of a Higgs condensate should terminate with an ‘acoustic’ branch, say $E(k) = c_s |k|$ for $k \to 0$, i.e. for momenta $|k| < \delta$, where the associated wavelength $\frac{2\pi}{\delta}$ is larger than $r_{\text{mfp}}$, the mean free path for the condensed scalar quanta.

1.3 As anticipated, these gap-less modes, are quite unrelated to the usual Goldstone modes of spontaneously broken continuous symmetries and would exist even in a Bose condensate of strictly neutral bosons, such as the atoms of $^4$He. In fact, in the quantum field theoretical case, the translation from ‘field jargon to particle jargon’, amounts to establish a well defined functional relation (see ref.[26] and Sect.2) $n = n(\phi^2)$ between the average particle density $n$ in the $k = 0$ mode and the average value of the scalar field $\phi$. Thus, Bose condensation is just a consequence of the minimization condition of the effective potential $V_{\text{eff}}(\phi)$. This has
absolute minima at some values $\phi = \pm v \neq 0$ for which $n(v^2) = \bar{n} \neq 0$ [26].

In spite of its intuitive nature, this conclusion seems to be in conflict with the standard analysis of the broken phase in a one-component $\Phi^4$ theory. Indeed, this standard analysis predicts a purely massive single-particle energy spectrum $\sqrt{k^2 + M_H^2}$. Here the parameter $M_H$, the Higgs boson mass, is defined from the quadratic shape of $V_{\text{eff}}(\phi)$ at $\phi = \pm v$ and should be non-zero.

However, as reviewed in Sect. 3, no real conflict with the intuitive picture of the broken phase as a quantum liquid exists if one takes into account the results of more formal analyses [27, 28] of the zero-4-momentum inverse connected propagator $G^{-1}(k = 0)$ in the broken phase. Once the scalar condensate is not treated as a purely classical c-number field, $G^{-1}(k = 0)$ is a two-valued function and includes the case $G^{-1}(k = 0) = 0$, as in a gap-less theory.

More precisely, the existence of both a $G_a^{-1}(k = 0) = M_H^2$ and a $G_b^{-1}(k = 0) = 0$ implies that there are two possible types of excitations with the same quantum numbers but different energies when the 3-momentum $k \to 0$: a single-particle massive one, with $E_a(k) \to M_H$, and a collective gap-less one with $E_b(k) \to 0$. ‘A priori’, they can both propagate (and interfere) in the broken-symmetry phase as in superfluid $^4$He, where the observed energy spectrum is due to the peculiar transition from the ‘phonon branch’ to the ‘roton branch’ at a momentum scale $|k_o|$ where

$$E_{\text{phonon}}(k_o) \sim E_{\text{roton}}(k_o)$$

This analogy supports the view that a scalar condensate is like a real physical medium and can propagate different types of excitations. At the same time, deducing the detailed form of the energy spectrum that interpolates between $E_a(k)$ and $E_b(k)$ is a formidable task. In fact, the same problem in superfluid $^4$He, after more than fifty years and despite the efforts of many theorists, notably Landau and Feynman, has not been solved in a satisfactory way.

Nevertheless, even without knowing the spectrum in full detail, one can draw a certain number of conclusions. For instance, the gap-less, acoustic branch, say $E_b(k) = c_s |k|$, dominates for $k \to 0$ and gives rise to a weak, attractive long-range force proportional to $1/c_s^2$ [29]. To this end, let us consider the standard Fourier transform

$$D(r) = \int \frac{d^3k}{(2\pi)^3} \frac{e^{ik\cdot r}}{E^2(k)}$$

that, in QED, after replacing $E^2(k) = |k|^2$ as the appropriate value for photon propagation, gives rise to the $1/r$ Coulomb potential. In our case, assuming an energy spectrum with the following limiting behaviours:
a) \( E(k) \rightarrow E_a(k) = \sqrt{k^2 + M_H^2} \) when \( |k| \rightarrow \infty \)

b) \( E(k) \rightarrow E_b(k) = c_s |k| \) when \( k \rightarrow 0 \)

and using the Riemann-Lebesgue theorem on Fourier transforms [30], one deduces the leading asymptotic behaviour for \( r \rightarrow \infty \)

\[
D(r) = \frac{1}{4 \pi c_s^2 r} \left( 1 + O(1/M_H r, 1/\delta r) \right)
\]  

(10)

\( \delta \) being the momentum scale associated with the transition between the two regimes a) and b).

To understand the value of \( c_s \), we shall use in Sect.4 the formalism of ref.[26]. This leads to a well defined hierarchy of scales \( \delta \ll M_H \ll \Lambda \) that decouple in the continuum limit. Indeed, they are related through

\[
\Lambda \delta \sim M_H^2
\]  

(11)

so that the sound velocity is

\[
c_s \sim \frac{M_H}{\delta} \sim \frac{\Lambda}{M_H}
\]  

(12)

(in units of the light velocity \( c = 1 \)).

Therefore, the continuum limit can be defined in two equivalent ways. On one hand, one can require an exact Lorentz-covariant spectrum down to \( k = 0 \). This means \( r_{\text{mfp}} \sim \frac{2 \pi}{\delta} \rightarrow \infty \) (in units of \( \xi_H = 1/M_H \)) so that the phonon wavelengths become larger than any finite scale. When taken at face value, this corresponds to phonons of ‘infinite’ wavelengths that, with periodic boundary conditions, become indistinguishable from the zero mode of the scalar field that defines the unperturbed condensate itself.

On the other hand, the continuum theory can also be defined as \( \Lambda/M_H \sim c_s \rightarrow \infty \). In this other limit, the scalar condensate become incompressible so that the associated long-range force becomes instantaneous and of vanishingly small strength. Thus the acoustic branch becomes unphysical since \( c_s |k| \) exceeds the energy of the massive mode \( \sqrt{k^2 + M_H^2} \) for any finite value of \( |k| \) (i.e. with the exception of the zero-measure set \( k = 0 \)).

1.4 At the same time, in the cutoff theory, where the rigidity of the vacuum is very large but not infinite, density fluctuations for \( |k| < \delta \) will propagate at a superluminal speed. Qualitatively, this same effect can be understood [31, 29] without looking at the energy spectrum but considering the relation between the pressure \( P \) and the energy-density \( \mathcal{E} \). In this case, treating the scalar condensate as a zero-temperature perfect fluid with
\[ \mathcal{P} = -\mathcal{E} + n d\mathcal{E}/dn, \]  
\[ \frac{c_s^2}{c^2} = \frac{dP}{dE} = \frac{nd^2\mathcal{E}/dn^2}{d\mathcal{E}/dn} \]  

Using the result \( n = n(\phi^2) \sim \phi^2 \) (see ref.[26] and Sect.2) and the ‘Maxwell construction’ for the energy density, i.e. \( \mathcal{E}(n) = V_{\text{eff}}(\phi) \) for \( \phi^2 > v^2 \) and \( \mathcal{E}(n) = V_{\text{eff}}(\pm v) \) for \( \phi^2 \leq v^2 \), the value of \( c_s^2/c^2 \) can become arbitrarily large near the symmetry-breaking minima. For instance, approximating \( V_{\text{eff}}(\phi) \) with a standard double-well form \( \sim (\phi^2 - v^2)^2 \), one finds

\[ \frac{c_s^2}{c^2} = 1 + \frac{v^2}{\phi^2 - v^2} \]  

Although Eq.(13) cannot be used at \( \phi = \pm v \), where collisional effects from a finite mean free path are essential to obtain a finite value of \( c_s \) [31], the previous argument confirms that the condensed vacuum of a \( \lambda\Phi^4 \) theory represents a very peculiar form of matter and can support nearly instantaneous density fluctuations.

In this context, I observe that theoretical and experimental motivations [32, 33] show that the idea of waves with a superluminal group velocity is physically meaningful. In particular, the possible existence of superluminal density fluctuations in a condensed medium poses no conceptual problem. For this reason, several authors [34, 35, 36] have considered the possibility of media whose long-wavelength compressional modes have phase and group velocity \( \frac{\mathcal{E}}{|k|} = \frac{dE}{dk} = c_s > c \). In this sense, ”...it is an open question whether \( c_s^2 \) remains less than unity when nonelectromagnetic forces are taken into account” [37].

In some case [35], superluminal sound arises when a large negative bare mass and a large positive self-energy combine to produce a small physical mass, just the situation expected for the quanta of the scalar condensate. More precisely, the physical origin of the superluminal sound is traced back to the asymmetric role of mass renormalization: it subtracts out self-interaction energy without altering the tree-level interparticle interactions that contribute to the pressure. Therefore, for (nearly) point-like particles in three spatial dimensions and sufficiently small values of their mass, matter can become superluminal, i.e. \( d\mathcal{P}/d\mathcal{E} > 1 \).

1.5 As the condensate compressional modes give rise to very weak interactions suppressed by a \( 1/c_s^2 \) factor, and in connection with the hierarchical pattern of scales \( \delta \ll M_H \ll \Lambda \), I shall try to combine these ingredients to connect the Fermi and Planck scales, thus providing a possible solution of the ‘hierarchy problem’.

To this end, the first minimal requirement will consist in deriving in Sect.5 Newton’s theory of gravity from the underlying physics of the Higgs condensate. In fact, up to now, ”How
relevant the Planck energy is to elementary particle physics has not really been established. It’s merely a number with the dimension of a mass that comes out of Newton’s theory of gravity. Call it Planck mass if you wish. It may or may not play a fundamental role” [38].

In addition, as a second basic requirement, the phonon dynamics should be ‘geometrizable’, i.e. re-absorbable into an effective geometry that agrees, at least for weak field, with the known metric structure of General Relativity. This analysis will be presented in Sect. 6.

Finally, Sect. 7 will contain a summary of the paper and a discussion of other possible consequences of my approach.

2. The Bose-condensate picture of symmetry breaking

I shall now start by resuming the main results of ref.[26] in the case of a one-component $\lambda \Phi^4$ theory, a system where the condensing quanta are just neutral spinless particles, the ‘phions’.

One starts by quantizing the scalar field $\Phi(x)$ in terms of $a_k, a_k^\dagger$, the annihilation and creation operators for the elementary phions whose ‘empty’ vacuum state $|o\rangle$ is defined through $a_k|o\rangle = \langle o|a_k^\dagger = 0$.

The phion system is assumed to be contained within a finite box of volume $V$ with periodic boundary conditions. There is then a discrete set of allowed modes $k$. In the end one takes the infinite-volume limit and the summation over allowed modes goes over to an integration: $\sum_k \to \int d^3k/(2\pi)^3$. In this way, the scalar field is expressed as

$$\Phi(x) = \sum_k \frac{1}{\sqrt{2VE_k}} \left[ a_k e^{ik.x} + a_k^\dagger e^{-ik.x} \right],$$

where $E_k = \sqrt{k^2 + m^2}$, $m$ being the physical, renormalized phion mass.

Bose condensation means that in the ground state there is an average number $N_0$ of phions in the $k = 0$ mode, where $N_0$ is a finite fraction of the total average number $N$

$$N = \langle \sum_k a_k^\dagger a_k \rangle$$

At zero temperature, if the gas is dilute, almost all the particles are in the condensate; $N_0(T = 0) \sim N$. In fact, the fraction which is not in the condensate (‘depletion’)

$$D = 1 - \frac{N_0}{N} = \mathcal{O}(\sqrt{na}) \ll 1$$

is a phase-space effect that becomes negligible for a very dilute system. In Eq.(17) we have introduced the phion density

$$n = \frac{N}{V}$$
and the phion-phion scattering length

\[ a = \frac{\lambda}{8\pi m_{\Phi}} \]  \hspace{1cm} (19)

defined, in the limit of zero-momentum scattering, from the dimensionless scalar self-coupling \( \lambda \) and the phion mass.

Therefore, for a very dilute system, where, to a first approximation, one neglects the residual operator part of \( a_{k=0} \), one gets \( a_{k=0}^\dagger a_{k=0} \sim N \) and so, \( a_{k=0} \) can be identified with the c-number \( \sqrt{N} \) (up to a phase factor). In this way

\[ \phi = \langle \Phi \rangle = \frac{1}{\sqrt{2Vm_{\Phi}}} \langle (a_{k=0}^\dagger + a_{k=0}) \rangle \sim \sqrt{\frac{2N}{Vm_{\Phi}}} \]  \hspace{1cm} (20)

or

\[ n(\phi^2) \sim \frac{1}{2} m_{\Phi} \phi^2 \]  \hspace{1cm} (21)

With this identification, setting \( a_{k=0}^\dagger = a_{k=0} = \sqrt{N} \) is equivalent to shifting the quantum field \( \Phi \) by a constant term \( \phi \).

Finally, using Eq.(21), the energy density \( E = E(n) \) can be translated into the effective potential

\[ V_{\text{eff}}(\phi) = E(n) \]  \hspace{1cm} (22)

Therefore, provided the effective potential of the (cutoff) \( \lambda \Phi^4 \) theory exhibits non-trivial absolute minima \( \phi = \pm v \neq 0 \) for real and non-negative values of \( m_{\Phi} \), spontaneous symmetry breaking is equivalent to Bose condensation with an average density \( \bar{n} = n(v^2) \). In other words, Bose condensation requires spontaneous symmetry breaking in (cutoff) \( \lambda \Phi^4 \) theories to be a first-order phase transition, differently from the second-order picture of standard renormalization-group-improved perturbation theory.

The problematic aspects associated with standard perturbation theory in ‘trivial’ \( \lambda \Phi^4 \) theories [39] have been discussed in [40] and the whole issue has been reviewed in detail in ref.[26]. The result of those analyses, as well as of refs.[41, 42], is the following. There is a class of ‘triviality compatible’ approximations to \( V_{\text{eff}} \) (one-loop potential, gaussian and post-gaussian [43] calculations of the Cornwall-Jackiw-Tomboulis [44] effective potential for composite operators), i.e. where the fluctuation field is described by an effective quadratic hamiltonian, in which spontaneous symmetry breaking is described as an infinitesimally weak first-order phase transition. This means that, in this class of approximations, differently from the standard perturbative predictions, the phase transition is first-order in the cutoff theory and becomes asymptotically second-order when approaching the continuum limit [45].
The physical ingredient to understand this result consists in the observation [26] that the phion-phion interaction is not always repulsive. Besides the tree-level contact $+\lambda \delta^{(3)}(r)$ potential, there is an induced attraction $-\lambda^2 e^{-2m_\Phi r}/r^3$ from the ultraviolet-finite parts of the one-loop graphs (see also ref.[47]) that, differently from the standard ultraviolet divergences, cannot be reabsorbed into a perturbative redefinition of the tree-level coupling. Actually, the two effects replicate themselves to all orders [26] since, for each ultraviolet divergent higher-order graph that redefines the strength of the tree-level potential, say $\lambda \to \lambda_{\text{eff}}$, there is a corresponding effect that redefines in the same way, $\lambda^2 \to \lambda^2_{\text{eff}}$, the strength of the attractive tail.

Thus, the long-range attraction persists to all orders and is responsible for the failure of ordinary renormalization-group-improved perturbation theory when approaching the phase transition. In fact, when making $m_\Phi$ smaller and smaller, the long-range attractive tail extends over regions that become larger and larger in units of the length scale associated with the repulsive core $+\lambda_{\text{eff}} \delta^{(3)}(r)$. As a consequence, the corresponding graphs, when taken into account consistently in the effective potential, can compensate for the effects of both the short range repulsion and of the non-zero phion mass. In this situation, the perturbative empty vacuum state $|o\rangle$, although locally stable, is not globally stable and, in a regime where $m_{\Phi}^2 = O(\lambda)$, the lowest energy state becomes a state with a non-zero density of phions that are Bose condensed in the zero-momentum state.

I emphasize that this weakly first-order scenario of symmetry breaking is discovered in a class of approximations to the effective potential, just those that are consistent with the assumed exact ‘triviality’ properties of the theory in 3+1 dimensions. In any case, it can be objectively tested against the standard perturbative predictions based on a second-order phase transition. In fact, one can run numerical simulations near the phase transition region and compare the predictions of refs.[41, 42] with the conventional existing two-loop or renormalization-group-improved forms of the effective potential. As the existing lattice data [48, 49] support unambiguously the alternative picture of refs.[41, 42, 26], I shall take this result as an additional motivation for pursuing further the Bose-condensate picture of symmetry breaking.

3. The excitations of the scalar condensate

3.1 As anticipated in the Introduction, and on the base of refs.[27, 28], the excitation spectrum of the broken phase has a two-branch structure due to the existence of two different
solutions, $G_a^{-1}(k = 0) = M^2_H$ and $G_b^{-1}(k = 0) = 0$, for the connected zero-4-momentum inverse propagator $G^{-1}(k = 0)$. In this way, even without solving the integral equation for $G(k)$, one can deduce that for $k \to 0$, there are two excitations of the scalar condensate: a massive one with energy $E_a(k) \to M_H^2$ and a gap-less one with energy $E_b(k) \to 0$. They will be analyzed in the following.

3.2 The parameter $M_H$, entering the massive solution, is obtained after introducing the absolute minima $\phi = \pm v$ of the effective potential $V_{\text{eff}}(\phi)$. As such, $M_H$ is extracted from the propagator as defined from the Dyson sum of one-particle irreducible graphs only. In this way, using simple tree-level relations, one can relate $M_H$ to $n$ and $a$ through ($\bar{n} = n(v^2)$)

$$M^2_H \sim \lambda v^2 \sim \bar{n} a$$

In this context, one should notice a peculiar point concerning the relation between $M_H$ and the physical vacuum field, say $v_R^2$, that in the Standard Model is then related to the Fermi constant ($G_F \sim 1/v_R^2$). Up to now, the normalization of $\phi$ has been determined from the vacuum expectation value of the bare scalar field $\Phi(x)$ defined at the cutoff scale. On the other hand, when using the standard condition

$$\left. \frac{d^2 V_{\text{eff}}(\phi_R)}{d\phi_R^2} \right|_{\phi_R = \pm v_R} = M^2_H$$

(24)

to define the physical normalization of the vacuum field, say

$$v_R = \frac{v}{\sqrt{Z_\phi}}$$

(25)

one has to account for the non-trivial renormalization effect discussed in refs.[41, 42, 26] and observed in lattice computations [50, 51] of the zero-momentum susceptibility $\chi_{\text{latt}}$ in the broken phase. This effect is a direct consequence of Bose condensation, that gives a special role to the vacuum condensate. In fact, differently from the rescaling of the fluctuation field, as defined from the \text{K\"allen-Lehmann} decomposition, say $Z_{\text{prop}} = 1 - |O(\lambda)|$, the vacuum field re-scales non-perturbatively as

$$Z_\phi \sim \frac{1}{\lambda}$$

(26)

Therefore, one always ends up with the cutoff-independent relation

$$M^2_H \sim v_R^2$$

(27)

even in the limit of a vanishingly small coupling $\lambda$. For this reason, the existence of $Z_\phi$ represents the basic ingredient to reconcile spontaneous symmetry breaking with the generally
accepted ‘triviality’ of \( \lambda \Phi^4 \) theory in 4 space-time dimensions [39]. In this way, the continuum limit \( \lambda \to 0 \) corresponds to free-field fluctuations with \( Z_{\text{prop}} = 1 \) but finite values of \( M_H \) and \( v_R \). Physically, these two quantities can be thought as arising from equivalent phion condensates with smaller and smaller scattering lengths \( a \) but increasingly large particle density \( \bar{n} \) such that \( \bar{n}a \) remains constant.

### 3.3

Let us now consider the gap-less solution \( G_b^{-1}(k = 0) = 0 \). Following the discussion presented in the Introduction, it would be naturally understood as the acoustic branch, i.e. \( E_b(k) = c_s|k| \), associated with the long-wavelength oscillations of the scalar condensate. This dominates in the infrared region and represents the physical mechanism for a new long-range force [29].

I observe that the gap-less solution is found when treating the zero-mode of the scalar field as a genuine quantum degree of freedom [27] over which to perform the last functional integration to compute the zero-four-momentum propagator. However, it can also be discovered diagrammatically [28] provided one includes in the analysis of the propagator a new class of graphs: the one-particle reducible zero-momentum tadpole graphs. These originate from the vacuum source \( J(\phi) = \frac{dV_{\text{eff}}(\phi)}{d\phi} \) and are connected to the other parts of the diagrams through zero-momentum propagators. As such, this class of graphs can be considered a manifestation of the quantum nature of the scalar condensate and, traditionally, is not included in the standard perturbative expansion at \( \phi = \pm v \) where \( J(\pm v) = 0 \).

However, in an all-order analysis for arbitrary \( \phi \), where the limit \( \phi \to \pm v \) is only taken at the end of the calculation, the one-particle reducible tadpole graphs have to be included to obtain the correct propagator. In an intuitive mechanical analogy, \( J(\phi) \) represents an infinitesimal driving force. Thus it will not produce any observable effect, unless the mass of a body vanishes when \( J(\phi) \to 0 \). In our case, the tadpole graphs are attached to the other parts of the diagrams through zero-momentum propagators so that the ‘mass’ of our body is just the inverse zero-momentum propagator. In this sense, ignoring such contributions (or including their effect in a pure perturbative way as \( G(k = 0) = \frac{1}{M_H^2} + ... \)) one neglects the potentially singular nature of \( G(k = 0) \) and only finds the massive solution.

Addressing to ref.[28] for the details, the tacit assumption at the base of the standard approach is better illustrated with a very simple example. Consider the quadratic equation

\[
f^{-1}(x) = 1 + x^2 - g^2x^2f(x)
\]

for \( g^2 \ll 1 \). The analogy with the problem of the one-particle reducible zero-momentum tadpole graphs is established when comparing \( f(x) \) with \( G(k = 0) \) at a given value of \( \phi \).
and comparing the limit $x \to 0$ with the limit $J(\phi) \to 0$. Standard perturbation theory is based on the iterative structure $f_{\text{reg}} = 1/(1 + x^2) + O(g^2)$ that provides a class of solutions that are regular for $x \to 0$ where $f_{\text{reg}}(0) = 1$. This corresponds to the massive propagator as defined from the one-particle irreducible graphs only. On the other hand, the singular solution $f_{\text{sing}} \sim 1/g^2 x^2$ corresponds to a divergent zero-momentum propagator when $\phi \to \pm v$. This can only be discovered by retaining the full non-linearity of the problem where the zero-momentum propagators joining to the vacuum sources are not approximated perturbatively.

Since the existence of two solutions for $G(k = 0)$ depends on how one treats the general case $\phi \neq \pm v$, and since we know that one can translate from $\phi^2$ to $n$, it may be interesting to establish an analogy with the possible ways of defining the theory at an arbitrary particle density $n \neq \bar{n}$. In this way, one can obtain an explicit form for a two-branch energy spectrum without any need to include the second-quantized counterpart of the one-particle reducible zero-momentum tadpole graphs.

To this end, I observe that for $n \neq \bar{n}$, the phion condensate cannot sustain itself. In fact, strictly speaking $\mu_c = d\mathcal{E}/dn = 0$ only for $n = \bar{n}$. Thus, to consider the general case $n \neq \bar{n}$, one may introduce a fictitious external chemical potential $\mu_c$ that will be sent to zero at the end of the calculation. In this case, still within the standard Bogolubov approximation, one can use Eq.(4.17) of ref.[26] and obtain the energy spectrum

$$E(k) = \sqrt{k^2 + m_\Phi^2} - \mu_c \left[ 1 + \frac{8\pi n a}{\sqrt{k^2 + m_\Phi^2}} \right]$$

(29)

Now, in the semi-classical approximation of ref.[52] for a temperature $T = 0$, one would predict $m_\Phi \leq \mu_c$ as the condensation condition with a positive physical mass of the scalar quanta (compare with Fig.3 of ref.[52]). Therefore, by requiring a physical non-negative $E(k = 0)$, i.e. $\mu_c = m_\Phi$, and taking the limit $m_\Phi \to 0$ one obtains

$$E(k) = \sqrt{k^2 + 8\pi n a}$$

(30)

On the other hand, by first taking the limit $k \to 0$, for any arbitrarily small but non-zero $m_\Phi$, one gets

$$\lim_{k \to 0} E(k) \sim \frac{|k|}{2m_\Phi} \sqrt{16\pi n a}$$

(31)

while still

$$E(k) \sim \sqrt{k^2 + 8\pi n a}$$

(32)

at larger $|k|$. In this case the infrared limit is different and one finds, even for $n = \bar{n}$, an explicit realization of a two-branch energy spectrum.
As in the case of the zero-momentum tadpole graphs, there is a subtlety associated with two non-commuting limits. In the latter case, the limit \( m_\Phi \to 0 \) yields a massive spectrum with mass \( M_H^2 \sim \bar{n}a \) with the exception of the zero-measure set \( \mathbf{k} = 0 \) or, more precisely, with the exception of a region \( |\mathbf{k}| < \delta \) where

\[
\delta \sim m_\Phi
\]  

(33)
is infinitesimal in units of \( M_H = \sqrt{8\pi n a} \).

As mentioned in Sect.2, an infinitesimal value of the ratio \( m_\Phi / \sqrt{n a} \) is found in the weakly first-order scenario of refs.[41, 42, 26]. Actually, as it will be shown in Sect.4, relation (33) is precisely the one expected when \( \delta \) is taken to represent the inverse mean free path for the condensed phions. Therefore, the infrared trend Eq.(31) provides an explicit realization of the ‘acoustic regime’ mentioned in the Introduction.

3.4 I conclude this section with some remarks concerning the eigenmode expansion of the scalar field in the broken phase. Beyond perturbation theory, due to the different physical status of the vacuum field and of its quantum fluctuations, the field re-scaling cannot be given as an overall ‘operatorial’ condition [48] (of the type, say, \( \Phi_R(x) = \sqrt{Z}\Phi(x) \)). In fact, Eqs.(24) and (26), used to define the physical normalization of the vacuum field, coexist with a free-field normalization for the massive branch of the fluctuating field (i.e a value \( Z_{\text{prop}} = 1 \)). Actually, Eq.(24) represents the standard condition for a smooth zero-4-momentum limit of a free-field propagator \( 1/(k^2 + M_H^2) \) with mass \( M_H \). Ignoring for a moment the existence of the gap-less branch, such physical representation of the scalar field amounts to write (phys='physical')

\[
\Phi_{\text{phys}}(x) = v_R + H(x)
\]

(34)

where

\[
H(x) = \sum_k \frac{1}{\sqrt{2}\sqrt{E_k}} \left[ \bar{H}_k e^{i(k,\mathbf{x} - E_k t)} + (\bar{H}_k)^* e^{-i(k,\mathbf{x} - E_k t)} \right]
\]

(35)

with \( E_k = \sqrt{k^2 + M_H^2} \).

In the usual approach, where the Higgs vacuum is treated as a purely classical c-number field and the Higgs boson is represented as a purely massive quantum field, the decomposition Eq.(35) is adopted even when dealing with wavelengths \( 2\pi/|\mathbf{k}| \) (say one meter) that are infinitely larger than \( \xi_H = 1/M_H \). In our case, however, to account for the existence of two types of fluctuations, it is convenient to introduce two fields, \( h(x) \) and \( H(x) \), whose Fourier components are separated by the end of the acoustic branch, say \( |\mathbf{k}| = \delta \). In this simplified
approach, where the energy spectrum is approximated by two independent branches and no attempt is made to find an appropriate form of matching condition, Eqs.(34) and (35) are replaced by the alternative expression

\[ \Phi_{\text{phys}}(x) = v_R + h(x) + H(x) \]  

(36)

with

\[ h(x) = \sum_{|k|<\delta} \frac{1}{\sqrt{2V_E}} [\tilde{h}_k e^{i(k \cdot x - E_k t)} + (\tilde{h}_k)^\dagger e^{-i(k \cdot x - E_k t)}] \]  

(37)

and

\[ H(x) = \sum_{|k|>\delta} \frac{1}{\sqrt{2V_E}} [\tilde{H}_k e^{i(k \cdot x - E_k t)} + (\tilde{H}_k)^\dagger e^{-i(k \cdot x - E_k t)}] \]  

(38)

where \( E_k = c_s |k| \) for \(|k| < \delta\), \( E_k = \sqrt{k^2 + M_H^2} \) for \(|k| > \delta\), with \( c_s \delta \sim M_H \). The more conventional Eqs.(34) and (35) are reobtained in the limit \( \delta \to 0 \) (or \( c_s \to \infty \) for a fixed \( M_H \)) where \( h(x) \) disappears and the broken phase has only massive excitations.

4. A hierarchy of scales in the condensate

Let us now consider the pattern of scales associated with the phion condensate. In condensed matter, the transition between acoustic branch and single-particle spectrum corresponds to their matching at a momentum scale set by the inverse mean free path for the elementary constituents. As anticipated, in the scalar condensate, this defines a momentum \( \delta \) where \( E_a(\delta) \sim E_b(\delta) \) or

\[ c_s \delta \sim \sqrt{\delta^2 + M_H^2} \]  

(39)

with \( \delta \sim \frac{1}{r_{\text{mfp}}} \), \( r_{\text{mfp}} \) being the phion mean free path \([53, 31]\)

\[ r_{\text{mfp}} \sim \frac{1}{n a^2} \]  

(40)

Now, assuming ‘triviality’ the scattering length \( a \) should vanish in the continuum limit of the theory. In this limit, the associated momentum scale

\[ \Lambda \equiv 1/a \]  

(41)

diverges in units of \( M_H \) and, therefore, can be used to define a far ultraviolet scale of the theory up to which phions can be treated as ‘hard spheres’. This yields

\[ t = \frac{\Lambda}{M_H} \sim \sqrt{\frac{1}{n a^3}} \]  

(42)
\[ \frac{1}{c_s} \sim \frac{\delta}{M_H} \sim \sqrt{\bar{n}a^3} \]  

(43)

Therefore, when \( t \to \infty \) one gets an infinitely dilute Higgs condensate where \( \bar{n}a^3 \to 0 \) and a hierarchy of scales \( \delta \ll M_H \ll \Lambda \) with

\[ \Lambda \delta \sim M_H^2 \]  

(44)

As anticipated in the Introduction, the order of magnitude of the ‘sound’ velocity

\[ c_s \sim \frac{M_H}{\delta} \sim \frac{\Lambda}{M_H} \]  

(45)

is much larger than unity (in units of \( c = 1 \)). Actually, \( c_s \) must diverge in the continuum limit where the vacuum has an infinite rigidity and the energy spectrum becomes Lorentz covariant down to \( k = 0 \). In fact, formally, \( O(\frac{\delta}{M_H}) \) vacuum-dependent corrections are equivalent to \( O(\frac{M_H}{\Lambda}) \) effects and these have always been neglected when discussing [54] how Lorentz covariance emerges in effective theories when removing the ultraviolet cutoff.

Finally, I observe that the order of magnitude relation Eq.(33) is found naturally when the momentum scale \( \delta \) is identified with the inverse mean free path for the condensed phions. In fact, from refs.[41, 42, 26], one deduces that spontaneous symmetry breaking sets in for infinitesimally small values of the ratio \( m_\Phi/M_H \), precisely

\[ \frac{m_\Phi^2}{\bar{n}a} \sim \lambda \sim m_\Phi a \]  

(46)

Therefore, using Eq.(40), one finds

\[ m_\Phi \sim \bar{n}a^2 \sim \frac{1}{r_{\text{mfp}}} \]  

(47)

as anticipated.

Summarizing the previous results, we find that in the local limit of the theory, where \( \Lambda/M_H \to \infty \), the energy spectrum \( E(k) \) reduces to \( \sqrt{k^2 + M_H^2} \) in the whole range of \( |k| \) (with the exception of the zero-measure set \( k = 0 \)). In the cutoff theory, however, one should expect infinitesimal deviations in an infinitesimal region of three-momenta. Therefore, it is natural to ask what the word _infinitesimal_ might actually mean in the physical world. For instance, assuming \( \Lambda = 10^{19} \) GeV and \( M_H = 250 \) GeV, a scale \( \delta \sim \frac{M_H^2}{\Lambda} \sim 10^{-5} \) eV, for which \( \frac{\delta}{M_H} \sim 4 \cdot 10^{-17} \), might well represent the physical realization of a formally infinitesimal quantity. If this were the right order of magnitude, the collective density fluctuations of the Higgs vacuum have wavelengths \( > \frac{2\pi}{\delta} \) thus ranging from about a centimeter up to infinity.
At the same time, the associated long-range force Eq.(10) would correspond to a nearly instantaneous interaction (since \( c_s \sim \frac{M_H}{a} = \mathcal{O}(10^{16}) \) in units of the light velocity) with a typical strength \( \frac{1}{c_s^2} \sim \frac{M_H^2}{\Lambda^2} = \mathcal{O}(10^{-33}) \). However small, this strength is non-vanishing and these interactions can play a physical role over distances that are infinitely larger than any elementary particle scale.

5. A weak attractive long-range force

I shall now exploit the idea that \( \Lambda = 1/a \) can be used to mark the Planck scale, \( M_H \sim v_R \) can be taken to indicate the Fermi scale \( G_F \equiv 1/v_R^2 \) so that \( \delta \sim 10^{-5} \) eV. As anticipated in the Introduction, for the validity of this identification, the long-range interactions associated with the longitudinal waves Eq.(37) should represent the physical mechanism at the base of Newtonian gravity. The possibility that, as it happens in rotating liquids [55] or in superfluid \(^4\)He [56, 57], there might be other kind of waves (as transverse waves that cannot be described in terms of a single scalar function) goes beyond the scope of this paper and will only be briefly addressed as a concluding remark at the end of Sect.6.

Starting from the simple representation based on Eq.(36)-(38), with \( \delta \sim 1/r_{\text{mfp}} \), one should find an effective macroscopic interaction to describe the coupling of \( h(x) \). By macroscopic, I mean that, starting, as in the Standard Model, from a model field theory for \( \Phi_{\text{phys}} \), one has to fill a gap of many orders of magnitude and find the global coupling of \( h(x) \) to matter over scales of linear size \( r_{\text{mfp}} \) or larger. As \( r_{\text{mfp}} = \mathcal{O}(1) \) centimeters, these lengths correspond to classical physics. Therefore, one has to take a suitable limit where the relevant degrees of freedom are expressed in terms of classical motions. To derive such a global coupling, one can argue as follows.

Let me first observe that in an exact Lorentz-invariant theory the simplest possible coupling between ordinary matter and a (dimensionless) scalar field \( \varphi(x) \) is through \( T_{\mu}^{\mu}(x) \), the trace of the energy momentum tensor. In this case, up to terms in the derivatives of \( \varphi \) and up to higher powers in the \( \varphi \)-field, one gets the simple interaction lagrangian

\[
\mathcal{L}_{\text{int}} = -T_{\mu}^{\mu}(x)\varphi(x)
\]

with

\[
T_{\mu}^{\mu}(x) = \sum_{n} \frac{E_n^2 - \mathbf{p}_n \cdot \mathbf{p}_n}{E_n} \delta^3(x - \mathbf{x}_n(t))
\]
The normalization in Eq.(48) corresponds to the standard form of the action

$$S_{\text{int}} = \int d^4x L_{\text{int}} = -\sum_n M_n \int ds_n \varphi(x)$$

(50)

for point-like particles interacting with a scalar field $\varphi(x)$ where $x_\mu = x_\mu(s_n)$ and $ds_n = dt_n \sqrt{1 - v_n^2}$ denotes the proper-time element of the n-th particle.

Now, I shall assume that the field $\varphi(x)$ represents the long-wavelength component of the full scalar field $\Phi_{\text{phys}}(x)$ in Eqs.(36) and (37), i.e.

$$\varphi(x) \equiv \frac{h}{v_R}$$

(51)

so that its Fourier decomposition will only contain wavelengths that are larger than one centimeter or so. I shall also assume that $\varphi(x)$ is the only non-Lorentz-invariant effect in the theory and that the fundamental couplings of $\Phi_{\text{phys}}(x)$ are formally Lorentz invariant.

As a definite physical framework to obtain Eq.(48), I shall follow ref.[29], by considering a microscopic scalar-fermion Yukawa coupling

$$L_{\text{Yukawa}} = -g_f \bar{\psi}_f \psi_f \Phi_{\text{phys}}$$

(52)

In this case, using Eq.(36) (and dropping the $H$-part), one obtains

$$L_{\text{Yukawa}} = -m_f \bar{\psi}_f \psi_f (1 + \varphi)$$

(53)

where

$$m_f = g_f v_R$$

(54)

Finally, if the field $\psi_f$ describes a sharply localized wave packet with momentum $p$, i.e. such that $\int d^3x \langle \bar{\psi}_f \psi_f \rangle = m_f / \sqrt{p^2 + m_f^2}$ and such that $\varphi$ does not vary appreciably over the localization region, the interaction in Eq.(52) produces the classical action in Eq.(50) for a particle of mass $m_f$.

In this particular case, where the elementary fermions have no interaction with other fields, the macroscopic global coupling of $\varphi(x)$ will be described as a formally Lorentz-invariant coupling of the type in Eqs.(48) and (50) and the mass parameters in Eq.(50) coincide with the mass in Eq.(54).

An analogous result follows when the fermions in Eq.(52) have additional couplings to other fields. Provided these other interactions are Lorentz-invariant, $\varphi(x)$ will always couple to a classical, Lorentz-invariant, dimension-four variable depending on all positions and velocities needed to describe the classical system. In this way, again, the basic coupling is through

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$T_{\mu}$. However, in this more general situation, there is no reason for the mass of the classical particles in Eq.(50) to coincide with any of the mass parameters $m_f$ of the microscopic Yukawa interactions in Eq.(54).

To better understand this result, I’ll concentrate on quarks, whose relevant additional couplings are due to their QCD interactions, starting from the unperturbed situation where $\varphi = 0$. In this case, one can assume that the mutual interactions among the various condensates give rise to suitable relations arising from the minimization of the overall energy density. One can express these relations as

$$\alpha_s \langle F^a_{\mu\nu} F^a_{\mu\nu} \rangle_o = c_1 v_R^4$$ (55)

and

$$m_q^{(o)} \langle \bar{\psi}_q \psi_q \rangle_o = c_2 v_R^4$$ (56)

where $c_1$ and $c_2$ are dimensionless numbers.

Now, let us consider an external perturbation that induces long-wavelength oscillations of the scalar condensate so that $v_R \rightarrow v_R(1 + \varphi)$. Let us also assume that QCD has only short-range fluctuations whose wavelengths are infinitely smaller than $r_{mfp} \sim O(1)$ centimeters. In this situation, $v_R(1 + \varphi)$ can be considered a new ‘local vacuum’ that, however, extends over an infinitely large region on the QCD scale and to which the quark masses and the relevant expectation values of the gluon and quark operators have to be adjusted. This amounts to write

$$\alpha_s \langle F^a_{\mu\nu} F^a_{\mu\nu} \rangle = c_1 v_R^4 (1 + \varphi)^4$$ (57)

and

$$m_q \langle \bar{\psi}_q \psi_q \rangle = c_2 v_R^4 (1 + \varphi)^4$$ (58)

The overall result is equivalent to a rescaling of the QCD scale parameter

$$\Lambda_{\text{QCD}} \rightarrow \Lambda_{\text{QCD}}(1 + \varphi)$$ (59)

of the chiral condensate

$$\langle \bar{\psi}_q \psi_q \rangle \rightarrow \langle \bar{\psi}_q \psi_q \rangle (1 + \varphi)^3$$ (60)

and of the quark masses

$$m_q \rightarrow m_q(1 + \varphi)$$ (61)

so that the nucleon mass $m_N$, arising from the expectation value of both quarks and gluon condensates, undergoes the same rescaling

$$m_N \rightarrow m_N (1 + \varphi)$$ (62)
In this way, regardless of the detailed relation of the classical $T^\mu_\mu$ Eq.(49) to the fundamental fields, the only remnant of the non-Lorentz-invariant nature of the vacuum is represented by the free lagrangian of the $\varphi$-field entering the effective lagrangian

$$\mathcal{L}_{\text{eff}} = \frac{v_R^2}{2} \varphi \left[ c_s^2 \Delta - \frac{\partial^2}{\partial t^2} \right] \varphi - T^\mu_\mu \varphi$$

(63)

In Eq.(63) higher-order terms in the $\varphi$-field (or higher derivatives of $\varphi$) have been neglected and the trace of the energy-momentum tensor of ordinary matter is treated as an external source for $\varphi(x)$. This structure holds true regardless of the spectral decomposition of $\varphi(x)$ for $k \to 0$. Of course, by choosing $c_s = 1$ in Eq.(63) and assuming the wavelengths of $\varphi(x)$ to cover the full range from zero to infinity, the resulting theory would be exactly Lorentz invariant and correspond to a standard retarded interaction.

On the other hand, for the very large values of $c_s$ that are suggested by the properties of the vacuum, the effects of $\varphi$ have practically no retardation. In this case, one gets an instantaneous interaction

$$\Delta \varphi = \frac{T^\mu_\mu}{c_s^2 v_R^2}$$

(64)

of vanishingly small strength when $c_s \to \infty$ (in units of $c = 1$). On the base of Sect.4, we know this limit to correspond to the continuum theory.

As discussed by Dicke [58], when averaged over sufficiently long times (e.g. with respect to the atomic times), by the virial theorem [59], the spatial integral of $T^\mu_\mu$ represents the total energy of a bound system, i.e. includes the binding energy. Therefore, for microscopic systems whose components have large $v_n/c^2$ but very short periods, this definition becomes equivalent to the rest energy. On the other hand, for macroscopic systems, that have long periods but small $v_n/c^2$, there are no observable differences from the mass density

$$\rho(x) = \sum_n M_n \delta^3(x - x_n(t))$$

(65)

and, in the latter case, one re-obtains formally the Poisson equation

$$\Delta U_N = G_N \rho(x)$$

(66)

To this end, the Newton potential $U_N$ is here identified as

$$U_N = \varphi$$

(67)

and the Newton constant $G_N$ is expressed as

$$G_N = \frac{G_F}{c_s^2}$$

(68)
If $G_F$ is taken to be the Fermi constant, this gives $c_s \sim 4 \cdot 10^{16}$ (in units of $c$) as anticipated.

I observe that the structure of the vacuum suggests the wavelengths of $\varphi$ to be larger than a centimeter or so. Therefore, the identification in Eq.(67), with $U_N$ being a solution of Eq.(66), can only hold for suitable forms of the mass density, i.e. such that their generated Newton potential does not vary appreciably over distances of a centimeter or smaller. Experimentally, all known gravitational fields fulfill this condition.

Notice also that the Planck mass $M_{\text{Planck}} = \frac{1}{\sqrt{G_N}}$ is here identified with $c_s v_R$ and is not directly related to $\Lambda = 1/a$. For $M_H \sim v_R$, however, one gets $c_s v_R \sim \Lambda$ and the two concepts become equivalent. In this sense, the Planck scale is not a purely ultraviolet quantity but embodies in its numerical value the peculiar infrared-ultraviolet connection that is realized in the scalar condensate.

6. Geometrizing the phonon dynamics

6.1 After having discussed how the leading effects of phonon dynamics can generate Newtonian gravity, one may ask whether these phonon effects can be ‘geometrized’, i.e. re-absorbed into an effective metric structure that depends parametrically on $\varphi(x)$, in agreement with the hydrodynamic treatment [15, 12, 10] of superfluid media that exhibits general covariance. In this section, I shall outline a simple answer to this question. My derivation, however, rather than a ‘proof’, should be considered a heuristic argument that can be used as a first approximation.

As a starting point, let me consider the very peculiar space-time picture associated with the gravitational red-shift. This is usually interpreted as a modification of ‘time’. Namely, the phenomenon is seen as if the period $T$ of any clock placed in a gravitational potential ($U_N < 0$) were increased, by the precise amount $1 - U_N$ with respect to the corresponding value $T_o$ of Special Relativity. This last quantity can be then defined as the general-relativistic value in the limit $U_N \to 0$.

Now, to accept this interpretation, i.e. in order to really consider the gravitational red-shift as the effect of a modification of ‘time’, one should restrict the class of admissible clocks that can be used to define a measure of time. In fact, one should only define a clock through periodic processes that exhibit the expected slowing down. For instance, a pendulum, whose period $T_{\text{pendulum}} \to \infty$ in the limit of vanishing gravitational field, would not be an admissible clock. For the same reason sandglasses, water clocks,...whose existence is solely due to the gravitational force, should also be discarded.
Therefore, it is reasonable to conclude that the general-relativistic notion of ‘clock’ has to be restricted to periodic processes whose existence does not depend on the gravitational field and for which there is a well defined zero-gravity limit [60]. Among these, we find the atomic clocks, whose period is associated with the spectral lines of an atomic transition, or their nuclear counterpart, whose period is defined in terms of some corresponding nuclear transition. Actually, due to their highest precisions, this class of clocks is the only one used so far to detect the tiny effect of a weak gravitational field. In any such case, however, the observed effect on the spectral lines can simply be understood in terms of the modification of the relevant particle mass (electron or nucleon) in the given gravitational field.

To this end, I shall restrict to ordinary non-relativistic matter (where \( T^{\mu}_{\mu} \) reduces to the mass density) and look for the effects of a non-zero \( \varphi = U_N \) through a re-definition of any mass placed in the external \( U_N \) with the tree-level substitution

\[
m_o \rightarrow m_o(1 + U_N)
\]

I shall also take into account that \( U_N \) changes so slowly that its variation on the atomic scale can be totally neglected. In this situation, the effects on the energy levels of a hydrogen-like atom simply amounts to a re-definition of the electron mass with an average constant value \( m = m_o(1 + U_N) \) in the Dirac Hamiltonian

\[
H_D = \alpha \cdot \mathbf{p} + \beta m - \frac{Z e^2}{r}
\]

This changes the energy levels and the frequencies \( \omega_o \rightarrow \omega_o(1 + U_N) \). Therefore, the natural period of an atomic clock \( T = \frac{2\pi}{\omega_o} \) is changed, \( T = T_o(1 - U_N) \), with respect to the value \( T_o = \frac{2\pi}{\omega_o} \) associated with \( U_N = 0 \). Analogously, the Bohr radius \( r_B = \frac{\hbar}{Ze^2m_o} \) is changed into \( r_B(1 - U_N) \) thus producing a symmetric re-scaling of the length of the rods. Since all masses are affected in the same way, and the units of length and time scale as inverse masses, the overall effect is equivalent to a conformal re-scaling of the metric tensor. To the \( O(U_N) \) accuracy, we get

\[
g_{\mu\nu}(x) = (1 - 2U_N)\eta_{\mu\nu}
\]

where \( \eta_{\mu\nu} \) denotes the Minkowski metric.

However, besides the modifications induced on physical rods and clocks, the idea of a non-zero \( \varphi = U_N \) to describe the fluctuations of a (‘non-dispersive’) medium, suggests another physical effect: the introduction of a refractive index in matter-free space. In this case, before the conformal re-scaling, the Minkowski metric would be replaced by

\[
\hat{\eta}_{\mu\nu} = \left( \frac{1}{N^2}, -1, -1, -1 \right)
\]
with a refractive index $N = N(U_N)$ and a normalization such that $N = 1$ when $U_N = 0$ (when no confusion can arise, besides the speed of light in the ‘vacuum’ $c = 1$, I shall also set to unity the Newton constant $G_N$).

Thus I obtain the metric structure

$$\hat{g}_{\mu\nu} \equiv \left(\frac{1 - 2U_N}{N^2}, -(1 - 2U_N), -(1 - 2U_N), -(1 - 2U_N)\right)$$

(73)

that re-absorbs the local, isotropic modifications of Minkowski space into its basic ingredients: the value of the speed of light and the space-time units. Equivalently, the same metric structure can be interpreted as arising from separate local changes of the space and time units. In fact, such a transformation is known to represent one of the many possible ways, perhaps the most fundamental, to introduce the concept of curvature [61, 62, 63]. In particular, there exist definitions of units, depending on a scalar field, for which a general curved space-time becomes flat, all the Riemannian invariants being zero [64].

The idea of introducing a refractive index in connection with gravity is well known (see e.g. [65]) and very natural, at least when comparing with any known medium with definite physical properties. For instance, when considering a condensate of spinless quanta, and these are treated as hard-spheres, Lenz [66] first showed that such a system behaves like a medium with a refractive index. In this approach, the refractive index is not derived from some dynamical coupling but is determined by the geometrical constraints that are placed on the propagation of waves of a given wavelength by the presence of the hard spheres.

Rather than attempting a microscopic derivation from first principles, it is much easier to deduce a possible form of $N(U_N)$ using very general arguments. To this end, I first observe that, for a time-independent $U_N$, the metric Eq.(73) is just a different way to write the general isotropic metric

$$\hat{g}_{\mu\nu} \equiv (A, -B, -B, -B)$$

(74)

Now, one may ask when the local light velocity $c(x, y, z) \equiv \sqrt{A \over B}$, defined as a ‘particle’ velocity from the condition $ds^2 = \hat{g}_{\mu\nu}dx^\mu dx^\nu = 0$, agrees with the curved-space equivalent of the phase velocity of light pulses. These are solutions of the D’Alembert wave equation with the metric $(A, -B, -B, -B)$ [67]

$$\frac{1}{A} \frac{\partial^2 F}{\partial t^2} - \frac{1}{B} (\frac{\partial^2 F}{\partial x^2} + \frac{\partial^2 F}{\partial y^2} + \frac{\partial^2 F}{\partial z^2}) F - \frac{1}{\sqrt{AB^3}} (\nabla \sqrt{AB}) \cdot (\nabla F) = 0$$

(75)

so that, by introducing the 3-vector $\mathbf{g} \equiv \sqrt{A \over B} (\nabla \sqrt{AB})$ we obtain

$$\frac{\partial^2 F}{\partial t^2} = \frac{A}{B} \Delta F + \mathbf{g} \cdot (\nabla F)$$

(76)
By identifying $\frac{1}{F} \frac{\partial^2 F}{\partial t^2}$ as the equivalent of $-\omega^2$ and $\frac{1}{F} \Delta F$ as the corresponding of $-k^2$, we find that particle velocity and phase velocity $c_{\text{ph}} \equiv \frac{\omega}{k}$ agree with each other only when $g = 0$, i.e. when $AB$ is a constant. This product can be fixed to unity with flat-space boundary conditions at infinity and, therefore, the resulting value

$$AB = 1 \quad (77)$$

or, in our case

$$N = (1 - 2U_N) \quad (78)$$

can be considered a consistency requirement on the physical medium, if this has to preserve, at least to the $O(U_N)$ accuracy, the observed particle-wave duality which is intrinsic in the nature of light. On the other hand, if $N \neq (1 - 2U_N)$, one should specify the operative definition used for the local speed of light: a) the time difference for a light pulse to go forth and back between two infinitesimally close objects at relative rest, b) the value obtained combining frequency and wavelength of a given radiation source,..

With this choice, and to the $O(U_N)$ accuracy, Eq.(73) becomes

$$\hat{g}_{\mu\nu} \equiv ((1 + 2U_N), -(1 - 2U_N), -(1 - 2U_N), -(1 - 2U_N)) \quad (79)$$

thus yielding the first approximation to the line element of General Relativity [65, 68] in isotropic form

$$ds^2 = (1 + 2U_N)dt^2 - (1 - 2U_N)(dx^2 + dy^2 + dz^2) \quad (80)$$

As anticipated, in this description, the space-time curvature is equivalent to suitable local re-definitions of the space and time units. Thus, for instance, the gravitational red-shift is explained through the behaviour of clocks in the gravitational field whereas the energy of the propagating photon does not change with the height [69] and there is no need to introduce the concept of an effective gravitational mass for the propagating photons. Analogously, the deflection of light can be explained by converting the units of time and length that define the local speed of light into those that are used for the plotted speed [70].

\textbf{6.2} Let us now try to include the effect of the higher-order $O(U_N^2)$ terms that have been neglected so far by replacing the tree-level relation Eq.(69) with the more general structure

$$m_o \rightarrow m_o e^{-\sigma} \quad (81)$$

where the unknown scalar function $\sigma = \sigma(x)$ in Eq.(81) accounts for the higher powers of $\varphi = U_N$. 
The extension cannot be done in an universal way. For instance, the Schwarzschild metric in isotropic form [71]
\[ ds^2 = \frac{(1 + U_N/2)^2}{(1 - U_N/2)^2} dt^2 - (1 - U_N/2)^4 (dx^2 + dy^2 + dz^2) \] (82)
violates the condition \( AB = 1 \) to \( \mathcal{O}(U_N^2) \). On the other hand, insisting on the condition \( AB = 1 \) and repeating the same steps as before, the metric structure Eq.(79) is replaced by
\[ \hat{g}_{\mu\nu} \equiv (e^{-2\sigma}, -e^{2\sigma}, -e^{2\sigma}, -e^{2\sigma}) \] (83)
Now, assuming that \( \sigma \) can be expanded in powers of \( \varphi = U_N \)
\[ \sigma = a_1 U_N + a_2 (U_N)^2 + \ldots \] (84)
and comparing Eq.(83) with Eq.(79) we obtain the following results. The first term in \( \sigma \) is just (minus) the Newton potential, i.e. \( a_1 = -1 \). At the same time, by comparing with all experimental results in a weak centrally symmetric gravitational field, one finds \( a_2 = 0 \). Therefore, up to \( \mathcal{O}(\frac{M^3}{r^3}) \) terms, one finds
\[ \sigma_{\text{exp}} = \frac{M}{r} \] (85)
(or \( \sigma_{\text{exp}} = \frac{GMN}{4\pi c^2 r^4} \) by expressing \( M \) in grams and \( r \) in centimeters). If one ignores the cubic and higher-order terms, this leads to the experimental identification
\[ \sigma_{\text{exp}} = -U_N \] (86)
suggesting that the tree-level redefinition of the particle mass \( m_0 \rightarrow m_0 (1 + U_N + \ldots) \) might actually be turned into its exponentiated form \( m_0 \rightarrow m_0 e^{U_N} \) after inclusion of the higher-order effects. Although not totally unexpected, dealing with infrared effects, a full understanding of this result requires the all-order evaluation of the effective lagrangian for the \( \varphi \)-field in the presence of the external source \( T_{\mu}^{\mu} \).

To further appreciate the meaning of Eqs.(83) and (85) let us compare with the solutions of Einstein equations. To this end, one can consider the class of metrics that are solutions of the following field equations
\[ R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = \gamma (\sigma_\mu \sigma_\nu - \frac{1}{2} g_{\mu\nu} \sigma^\alpha \sigma_\alpha) \] (87)
These can be considered Einstein equations in the presence of an energy-momentum tensor for a scalar field \( \sigma(x) \), for various values of the parameter \( \gamma \). Indeed, if curvature arises
from the fluctuations of a medium, it is natural to take into account its energy-momentum tensor. Now, for a centrally symmetric field $\sigma = \sigma(r)$ Tupper [72] has shown that all solutions of Eqs.(87) are consistent with the three classical weak-field tests and the Shapiro planetary radar reflection experiment. Depending on the value of the parameter $\gamma$ one gets the Schwarzschild metric, for $\gamma = 0$, or the Yilmaz metric [73], for $\gamma = 2$. For any $\gamma$, the solutions are conformal transformations of solutions of the Brans-Dicke theory [74].

However, only for $\gamma = 2$ the resulting metric tensor depends parametrically on $\sigma$. In this case, Eqs.(87) become algebraic identities consistently with the point of view that the introduction of the metric tensor is only an auxiliary tool to effectively account for the dynamics of the more fundamental field $\sigma(x)$. In this case, for $\gamma = 2$, where $\sigma = \frac{M}{r}$ represents the Newton potential, the Yilmaz metric in isotropic form (‘Y’=Yilmaz)

$$g_{\mu\nu}^Y \equiv (e^{-\frac{2M}{r}}, -e^{\frac{2M}{r}}, -e^{\frac{2M}{r}}, -e^{\frac{2M}{r}})$$ (88)

is formally identical to Eq.(83).

In connection with the choice $\gamma = 2$, it might be interesting that the metric Eq.(83), differently from all other metrics, is not just a one-body solution but is also valid for a gravitational many-body system where $T_{\mu\nu} \equiv (\rho, 0, 0, 0)$ with $\rho$ defined in Eq.(65). Indeed, when replacing the Newton potential

$$\sigma(x) = \sum_n \frac{M_n}{|x - x_n|}$$ (89)

in the metric Eq.(83), the field equations

$$R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R = 2(\sigma_\mu \sigma_\nu - \frac{1}{2}g_{\mu\nu} \sigma^\alpha \sigma_\alpha) - 2T_{\mu\nu}$$ (90)

become algebraic identities. In this sense, if one wants to compare with real many-body gravitational systems, Eq.(83) represents a very convenient starting point for any time-dependent approximation. Indeed, for slow motions, the situation is similar to the conventional adiabatic Born-Oppenheimer approximation where one approaches the time-dependent problem by expanding in the eigenfunctions of a static 2-center, 3-center,.. n-center hamiltonian.

In connection with a many-body gravitational system, I observe that the peculiar factorization properties of the Yilmaz metric, $e^{\sum \frac{M_i}{r}} = e^{\frac{M_1}{r}} e^{\frac{M_2}{r}}..$, provide an alternative explanation for the controversial huge quasar red-shifts, a large part of which could be interpreted as being of gravitational (rather than cosmological) origin [75]. This might represent a ‘fifth’ test of gravity outside of the weak-field regime.
Before concluding, I observe that the metric structure Eq.(83) with $\sigma = M/r$ was also obtained by Dicke [58] in a stimulating remake of Lorentz’s electromagnetic ether. This was based on the simultaneous replacements of the particle mass

$$m_o \rightarrow m_o f(\epsilon, \mu)$$ (91)

and of the light velocity

$$c^2 \rightarrow \frac{c^2}{\epsilon \mu}$$ (92)

where $\epsilon$ and $\mu$ are respectively the space-time dependent dielectric function and magnetic permeability of the ether. Consistency with the experimental results (the Eötvös experiment, velocity independence of the electric charge,...) requires $\epsilon = \mu$ leading to the effective metric structure (‘LD’=Lorentz-Dicke)

$$g_{\mu\nu}^{LD} \equiv \left( \frac{f^2}{\epsilon^4}, -\frac{f^2}{\epsilon^2}, -\frac{f^2}{\epsilon^2}, -\frac{f^2}{\epsilon^2} \right)$$ (93)

Finally, a comparison with the classical tests in a weak gravitational field gives

$$f^2 = \epsilon^3$$ (94)

and

$$\epsilon = 1 + 2 \frac{M}{r} + .. = \epsilon^{2\sigma}$$ (95)

so that Eq.(93) reduces to Eq.(83) (to compare with Eq.(81) one has to rescale $m_o c^2$).

6.3 This does not mean that all possible phenomena that occur in the scalar condensate can be ‘geometrized’ in terms of Eq.(83). In fact, this metric structure arises naturally in connection with the longitudinal density fluctuations. However, not all excitation states of a superfluid medium, even made up of neutral spinless quanta, can be described in terms of a single scalar function.

For instance, it is experimentally known [56] that, as a consequence of vortex lines formation, superfluid $^4$He can also support circularly polarized transverse waves, the so called ‘Kelvin modes’ [57]. In this sense, vortex line formation in the scalar condensate might represent the key to obtain genuine graviton states as quantized transverse oscillations of a string. Clearly, if such new states were substantially excited, there is no reason for the associated space-time description to fit with Eq.(83).

The connection with string theory can be established within a hydrodynamical treatment of the scalar condensate. Independently of the nature of the elementary constituents of a
fluid, there is a basic phenomenon in hydrodynamics: the formation of vortices. To describe
the most general case one needs four scalar fields, the fluid density \( n \) and the three Clebsch
potentials \( (\varphi_1, \varphi_2, \varphi_3) \) \([77, 78, 79, 80]\), in terms of which the fluid velocity is expressed as
\[
\mathbf{u} = \varphi_1 \nabla \varphi_2 + \nabla \varphi_3
\]  
(96)

For instance, when \( \varphi_1 = \varphi_2 = 0 \) and \( \varphi_3 = \varphi \), a given configuration of the fluid can be
specified through a complex field
\[
\psi = \sqrt{n} e^{i\varphi}
\]  
(97)

In this way, \(|\psi|^2 = n\) gives the density of the fluid and the phase \( \varphi \) defines the fluid velocity
through \( \mathbf{u} = \nabla \varphi \) so that vortices can only occur in multiply connected regions. Notice that,
in this formalism, although dealing with a complex field, there is no fundamental ‘electric
charge’ and all dynamical effects arise from the possible states of motion of the fluid.

In this ‘abelian’ case, i.e. when the four-dimensional system with vortices is modeled
as a spontaneously broken U(1) symmetry \([81]\), it is known that the resulting theory can
be mapped into a string theory through a duality transformation \([82]\). This series of steps,
based on the analogy with fluid mechanics, leads to the logical possibility to obtain higher
spin states as excitations of a simple scalar condensate, i.e. of a medium whose ultimate
constituents are just neutral spinless quanta (of size \( a \sim 10^{-33} \text{ cm} \)).

The potential implications of applying the fluid analogy to the scalar condensate cannot
be easily predicted. For instance, fluid mechanics, through the implementation of invariance
under volume-preserving transformations, provides a simple and elegant motivation \([83]\) for
the introduction of non-commuting gauge fields that are interpreted in terms of the Lagrange
coordinates used to label the evolving portions of the fluid.

As anticipated at the beginning of Sect.5, the problem of transverse graviton states lies
outside the scope of this paper. Nevertheless, at least in the weak-field regime, one expects
the resulting classical space-time picture to agree with General Relativity. This expectation,
quite independently of my proposal, has been well illustrated in ref.\([12]\). According to these
authors, what we call ‘Einstein gravity’ is a kind of universal picture that arises in a large
variety of systems (dielectric media, flowing fluids, Bose condensates, media with a refractive
index, non-linear electrodynamics, thermal vacua,...). In this sense, General Relativity (or
more precisely general covariance) arises in a coarse-grained description of the world, as
hydrodynamics. Both, concentrating on the properties of matter at scales that are larger
than the mean free path for the elementary constituents, are insensitive to the details of the
underlying molecular dynamics.
Of course, even within a general-covariant description, there are some features, such as the structure of the energy-momentum tensor or the presence of a cosmological term $\lambda_c$, that cannot be determined ‘a priori’ but may depend on the properties of the underlying medium. For instance, as mentioned above in connection with the Yilmaz metric, a different energy-momentum tensor can change qualitatively the space-time description in the strong-field regime. Analogously, a non-zero $\lambda_c$ might modify the causal structure of the space-time and give rise to closed time-like loops as solutions of Einsten equations [84]. Finally, there can be other effects if string formation takes place in the scalar condensate. For instance, by considering a straight isolated vortex filament along the z-axis, of given mass-density $\rho$ and given spin-density $J$ per unit length, even with a vanishing cosmological term, the resulting geometry supports time-like loops [84]. These can be seen as circular paths around the filament in the $(x,y)$ plane of sufficiently small radius

$$r < \frac{4G_N J}{c^3 - 4G_N \rho c}$$

(98)

7. Summary and outlook

7.1 In this paper I have proposed a solution of the ‘hierarchy problem’ based on the picture of spontaneous symmetry breaking as a real condensation phenomenon of elementary quanta, the phions [26]. Skipping the more technical details, this physical scenario can be summarized in a series of simple basic steps:

i) taking into account the two-valued nature [27, 28] of the zero-4-momentum connected propagator in the broken phase, one gets the idea of a true physical medium that can propagate two types of excitations: a massive one, $H(x)$, whose energy $E_a(k) \to M_H$ when $k \to 0$, and that corresponds to the usual Higgs boson field, and a gap-less one, $h(x)$, whose energy $E_b(k) \to 0$. The latter is naturally interpreted in terms of the collective density fluctuations of the system so that its wavelengths are larger than $r_{\text{mfp}}$, the mean free path for the condensed phions. The overall picture is very similar to the coexistence of phonons and rotons in superfluid $^4$He that, in fact, is usually considered the condensed-matter analogue of the Higgs condensate.

ii) following the formalism of ref.[26], the scalar condensate emerges as a highly hierarchical system characterized by two basic parameters, the density $\bar{n}$ and the phion-phion scattering length $a$, that, approaching the continuum limit, combine to produce length scales that differ by many orders of magnitude. Indeed, $a$, the length scale $\xi_H = 1/M_H \sim 1/\sqrt{\bar{n} a}$
and the phion mean free path $r_{\text{mfp}} \sim 1/(\bar{n}a^2) \sim 1/\delta$ are related by inverse powers of the diluteness factor $\sqrt{\bar{n}a^3} \to 0$ and satisfy the relation

$$\xi_H^2 \sim ar_{\text{mfp}}$$  (99)

iii) the mechanism is such that when the ultraviolet cutoff is removed, i.e. when

$$t = \frac{\xi_H}{a} \sim \frac{1}{\sqrt{\bar{n}a^3}} \to \infty$$  (100)

the acoustic branch $h(x)$ gives rise to infinitesimally weak (of strength $\epsilon^2 = \frac{1}{t^2}$ in units of the Fermi constant) attractive and nearly instantaneous interactions (with $c_s/c \sim t$). Therefore, the natural interpretation of $h(x)$ is in terms of the Newton potential $U_N$, thus also providing the underlying rationale for its traditional interpretation as an ‘action at distance’. As a consequence, my proposal might represent a simple physical solution of the ‘hierarchy problem’ between the Fermi scale $v_R \sim M_H$ and the Planck scale $M_{\text{Planck}} \sim tv_R$. The latter is not a purely ultraviolet quantity but embodies in its numerical value the infrared-ultraviolet connection that is realized in the scalar condensate. Numerically one finds $M_{\text{Planck}} \sim 1/a$ in terms of the phion-phion scattering length $a \sim 10^{-33} \text{ cm}$ that marks the scale up to which phions can be considered as elementary quanta.

On the other hand, if $h(x)$ were not related to $U_N$, one should, nevertheless, find some dynamical interpretation. In fact, rejecting the model presented in this paper without an alternative scenario, the existence of $h(x)$ leads to unexplained long-range forces. This might represent a serious consistency problem for any physical theory based on the phenomenon of spontaneous symmetry breaking, such as the Standard Model.

iv) by further exploiting the scenario where $h(x) = U_N$, I have suggested that the phonon effects can be ‘geometrized’, i.e. re-absorbed into the space-time metric Eq.(79) that agrees with General Relativity in weak gravitational fields. The extension to strong fields cannot be done unambiguously and there is more than one scenario. For instance, by implementing the particle-wave duality which is intrinsic in the nature of light, a variant of standard General Relativity, Eq.(83), known in the literature as Yilmaz metric [73], might also be considered. Finally, the analogy with superfluid $^4\text{He}$ suggests the idea of string formation in the scalar condensate. In this case, their quantized transverse excitations might represent a physical model of genuine graviton states.

7.2 The comparison with General Relativity requires some additional comments. In the picture presented in this paper, Newtonian gravity originates from the long-wavelength
density fluctuations of the scalar condensate. These propagate as longitudinal waves, starting at wavelengths that are larger than the mean free path $r_{\text{mfp}}$ for the elementary phions and phenomenology requires $r_{\text{mfp}}$ to be an $O(1)$-centimeter length scale. As there can be no variation of the gravitational potential between two points whose distance is smaller than $r_{\text{mfp}}$, the collective oscillations of the condensate average over distances that are much larger than the atomic size. Thus all quantum interference effects disappear. As these depend on the relative phases of the wave functions and, through the particle momentum, on the particle mass, one understands the origin of the so called ‘weak equivalence principle’. This states that physical effects in an external gravitational field do not depend on the particle mass and, as such, is at the base of the idea of gravity as an overall modification of the space-time geometry.

An exception is represented, however, by those particular experiments where the coherence of the wave-functions can be maintained over distances where $h(x)$ can vary appreciably, as for the neutron diffraction experiments in the earth’s gravitational field [85]. In such cases, the interference pattern depends on the particle mass. Therefore, the ‘weak equivalence principle’ is no longer valid at the quantum level. However, for each given type of particle and as a consequence of the exact equality of inertial and gravitational mass, the interference pattern obtained in the gravitational field coincide with that obtained in the equivalent accelerated frame. In this sense, one can conclude that a gravitational field is still equivalent to an accelerated frame (‘strong equivalence principle’ [85]).

One more comment is also needed about the possible connection with the traditional induced-gravity approach [19, 20, 21] mentioned in the Introduction. Part of this discussion was presented in ref.[26]. However, for the convenience of the reader, I’ll reproduce here the basic steps.

The original induced-gravity approach was motivated by the observation that for a scalar field $\Phi$ there is a direct coupling $R\Phi^2$ in the curved-space lagrangian density, $R$ being the curvature scalar. Thus, if the field $\Phi$ has a non-vanishing expectation value, it was proposed [19, 20, 21] that the Einstein-Hilbert lagrangian could emerge from spontaneous symmetry breaking, namely

$$\mathcal{L}_{\text{Einstein–Hilbert}} \sim R\langle\Phi\rangle^2$$  \hspace{1cm} (101)

Clearly, it goes without saying that the vacuum expectation value in Eq.(101) cannot be the same $v_R$ related to the Fermi constant through $G_F \equiv 1/v_R^2$. In fact, in this case, gravity would have a strength that is $10^{33}$ larger than allowed by experiments. Therefore, in the original
approach, one was concluding that the hypothetical inducing-gravity scalar field could not be the same scalar field that induces spontaneous symmetry breaking in the Standard Model.

As discussed in Sect.3, however, the non-trivial renormalization effect Eqs.(24)-(26), predicted in refs.[41, 42, 26] and observed in lattice computations [50, 51] of the zero-momentum susceptibility, shows that one is naturally faced with two possible definitions of the vacuum expectation value: a ‘bare’ value \( v \) and a physical value \( v_R \). They are connected through their relation to \( M_H \), namely

\[
M_H^2 \sim \lambda v^2 \sim v_R^2
\]

so that, using Eqs.(19), (23), and (40) one finds

\[
v^2 \sim \frac{M_H^2}{am_\Phi} \sim \frac{1}{m_\Phi r_{\text{mfp}}} \frac{1}{a^2}
\]

Therefore, within this paper, where \( a \approx 10^{-33} \text{ cm} \), the induced-gravity approach can be recovered with two conditions. First, \( \langle \Phi \rangle \equiv v \) is the appropriate value to be used in Eq.(101). Second, the phion Compton wavelength has to be of the same order as the phion mean free path, i.e.

\[
m_\Phi \sim \frac{1}{r_{\text{mfp}}}
\]

Notice that this condition was found in Sect.4 (see Eq.(47)) on the base of the analysis of the scalar condensate. Therefore, the present paper can also provide a convenient framework to recover the original induced-gravity approach.

7.3 The mechanism I have proposed to explain the relative size of the Fermi and Planck scales, through the introduction of a third infinitesimal momentum scale \( \delta \), bears some analogy to other approaches [86] based on extra space-time dimensions with a ‘large’ compactification size \( r_c \) (in a regime where \( r_c \sim r_{\text{mfp}} \sim 1/\delta \)). In fact, these other approaches, besides solving the ‘hierarchy problem’ through the same kind of relation \( L_1/L_2 = L_2/L_3 \) among three length scales \( L_1, L_2, L_3 \), predict, typically, a superluminal 4D effective propagation of gravity [87].

For this reason, independently of my proposal, it becomes more and more important to have an experimental determination of the speed of gravity. However, before attempting any experimental determination, one should first try to clarify the issue from a theoretical point of view. For instance, can the ‘speed of Newtonian gravity’, \( c_N \equiv c_s \), as defined in this paper through Eq.(10) or Eq.(63), be measured experimentally? Or, is there any obvious counterpart in General Relativity?
Clearly, \( c_N \) is not the same concept of the ‘speed of gravitational waves’. This is defined within General Relativity in the weak-field approximation by expanding the metric tensor around Minkowski space-time as \( g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu} \). Further, imposing the transversality condition and using the formula of quadrupole gravitational radiation, gravitational waves are emitted from rapidly varying gravitational systems that deviate from spherical symmetry. In this framework, their propagation speed has the same value as the speed of light. Their emission, when properly taken into account in the energy balance, leads to tiny deviations from Keplerian orbits that are consistent with the observed slowing down of binary pulsars [88].

On the other hand, it is also true that, within the presently accepted expanding-universe scenario, a cosmological term \( \lambda_c \) is now needed to match the experimental observations with General Relativity. In this case, by using the alternative expansion of the metric tensor \( g_{\mu\nu} = \bar{g}_{\mu\nu} + h_{\mu\nu} \), the linearized problem for gravitational waves should be cast in the form [89]

\[
\dot{M}^{\alpha\beta}_{\mu\nu}(\bar{g}, \lambda_c)h_{\alpha\beta} = 0
\]  

(105)

where the graviton mass matrix depends on \( \lambda_c \) and on the actual background metric \( \bar{g}_{\mu\nu} \) used to fit the cosmological data. Thus, since the sign of the effective graviton mass squared is not positive-definite [89], independently of any experiment to detect gravitational waves, the simple idea of massless gravitons propagating, just as ordinary real photons, at the speed of light is not entirely obvious. For instance, some eigenvalue might correspond to a complex graviton mass \( m_g \). This problem is usually ignored, probably, because a value \( |m_g| \sim \sqrt{|\lambda_c|} \) is so small that the associated ‘Compton wavelength’ is as large as the visible universe. However, also the closed time-like loops mentioned in Sect.6 in connection with a cosmological constant have a typical radial size \( 1/\sqrt{|\lambda_c|} \) [84]. Therefore, the two problems might be related.

Returning to the speed of Newtonian gravity, an experimental value of \( c_N \) should be extracted from the properties of the orbits of gravitationally bound systems. In this respect, there is no difference between Newtonian gravity and a static Schwarzschild metric. The well-known differences, such as the precession of perihelia, are genuine \( \mathcal{O}(v^2/c^2) \) effects that exist independently of the speed of gravity.

Rather, one should study the actual motions in the solar system by replacing the absolute-time description of Newtonian gravity with a description where the gravitational influence at \((\mathbf{r}, t)\) from all possible \( \mathbf{r}' \) is evaluated at a retarded time \( t' = t - |\mathbf{r} - \mathbf{r}'|/c_N \). This leads to modifications in the parameters of the orbits [90] that can be compared with experiments.
When this is done, for a large set of data, Van Flandern finds [91] a lower limit \((c_N/c)_{\text{exp}} > 2 \cdot 10^{10}\).

Van Flandern’s analysis has been criticized [92] on the base of the following observation. In ref.[91] a finite \(c_N\) is equivalent to introduce an ‘aberration’. Namely, the acceleration felt by a test particle points towards the retarded position of the gravitational source rather than towards its instantaneous position. Within Newtonian theory, where gravity is a purely central force, the experimental evidence for the absence of any gravitational aberration leads to the above lower limit on \(c_N\). However, it is known from the Lienard-Wiechert potentials of electromagnetism, that aberration can be cancelled by additional interactions that are proportional to the velocity of the particle. In this case, despite the potentials are retarded (i.e. evaluated at a time \(t - r/c\)), the acceleration, as defined from the Lorentz force, is always directed along the line that instantaneously connects source and test particle. Therefore, in principle, introducing suitable velocity-dependent interactions, the absence of aberration can become consistent with a value \(c_N = c\). This is certainly true and, in this sense, any direct determination of \(c_N\) contains a model dependence due to the theoretical framework that is used to compare with experimental data. Only having a more or less rigid theoretical scenario (as in my case, where \(c_N \equiv c_s\) determines as well the strength of the central interaction \(1/c_s^2 r\)) one can try to obtain an unambiguous answer. On the other hand, it is also true that, in a perspective where gravity is the excitation of a physical medium, there is no conceptual compelling reason for \(c_N = c\) (as with the elastic medium considered in the Introduction for which \(c_s \neq c_t\)).

In this context, there are now other attempts to extracts the speed of gravity from recent measurements of radio signals past Jupiter in the experimental conditions of last September 8th, 2002. The idea is that a finite speed of propagation for gravity, affecting the standard Shapiro time delay obtained in the static limit, should introduce extra effects. As it will be clear in the following, it is not evident that this other ‘speed of gravity’ is the same \(c_N \equiv c_s\) considered in this paper. Therefore, I’ll introduce one more symbol, say \(c_g\), to indicate the relevant value in this framework.

To discuss the present situation, it is essential to understand how \(c_g\) is introduced. The author of ref.[93] starts from the weak-field Einstein equations

\[
(\Delta - \frac{1}{c^2} \frac{\partial^2}{\partial t^2}) \gamma^{\alpha\beta}(\mathbf{r}, t) = - \frac{16\pi G_N}{c^4} T^{\alpha\beta}(\mathbf{r}, t)
\]  

(106)

and argues that to replace a value \(c_g \neq c\) in the l.h.s. of Eq.(106) violates Einstein’s equations thus leading to a different theory of gravity. Thus, to remain consistent with General
Relativity, but consider the possibility that $c_g \neq c$, a re-scaling of time $t \rightarrow \tau$, such that $c_g \tau = ct$, has to be introduced. By defining $\epsilon = c/c_g$, the energy-momentum tensor is then re-scaled as $\Theta^{00} = T^{00}$, $\Theta^{i0} = \epsilon T^{i0}$, $\Theta^{ij} = \epsilon^2 T^{ij}$ and Eqs.(106) are replaced by

$$ (\Delta - \frac{1}{c_g^2} \frac{\partial^2}{\partial \tau^2}) \gamma^{\alpha \beta} (r, \tau) = - \frac{16\pi G_N}{c^4} \Theta^{\alpha \beta} (r, \tau) $$

(107)

where finally $c_g$ appears. Within this framework, the author of ref.[93] claims that a measurement of the extra effects imply an experimental result $(c_g/c)_{\text{exp}} = 1.06 \pm 0.21$.

Another author [94] concludes, however, that this type of extra effects is merely due to the time-delay associated with light propagation (and not to gravity propagation) thus denying that one can obtain a measurement of $c_g$.

Finally, a third author [95] concludes that ”...recent measurements of the propagation of radio signals past Jupiter are sensitive to $\alpha_1$ but are not directly sensitive to the speed of gravity”. The same author concludes, however, that the existing measurements of the post-newtonian parameter $|\alpha_1| < 2 \cdot 10^{-4}$ should provide, by themselves, experimental evidence for $c_g = c$. To find the relation between $\alpha_1$ and $c_g$, the relevant sources are the papers of ref.[96]. There the parameter $\alpha_1$ was introduced to take into account deviations from Lorentz-invariance in the post-newtonian metric through a dependence on the velocity $w$ of a given observer with respect to the mean rest-frame of the Universe, identified with the frame of the CMBR. As for velocities $|w| \sim 300$ km/sec, no dependence on $|w|$ is observed, $|\alpha_1|$ has to be vanishingly small and gravity has to be Lorentz-invariant, i.e. $c_g = c$.

On the other hand, if gravity were exactly Lorentz-invariant, say as electromagnetism, it is not clear why the dynamics could not be formulated in ordinary Minkowski space-time. Therefore, some genuine Lorentz-non-invariant effect has to be included. The mechanism, however, does not give rise to observable asymmetries among Lorentz-equivalent frames. In this sense, my proposal introduces the ‘minimal’ departures from Lorentz invariance. These are only due to the vacuum structure while, for the rest, the source of $h(x)$ is the Lorentz-invariant trace of the energy-momentum tensor in Eq.(63). In such a framework, there is no way to generate any observable asymmetry.

Finally, following the loose analogy between EPR experiments and gravity mentioned in the Introduction, it is interesting that the lower limit for $(c_N/c)_{\text{exp}}$ quoted by Van Flandern [91] has a counterpart in a similar lower limit for the speed of the ‘quantum information’ $c_Q$. In fact, for this quantity, an analysis of long-distance EPR experiments [3] provides the lower limit $(c_Q/c)_{\text{exp}} > 2 \cdot 10^4$ in the CMBR frame.
7.4 Before concluding, I observe that, in general, in a superfluid medium one expects non-linear corrections to the free-phonon approximation. As this represents, in my picture, the mechanism for Newtonian gravity, I shall try to estimate the scale where non-linearity shows up by evaluating a mean free path $\zeta_{\text{mfp}}$, associated with phonon propagation in the phion condensate.

In superfluid $^4$He, for temperature $T \to 0$, the phonon mean free path becomes larger than the size of the container. Indeed, the typical order of magnitude relation is [97]

$$\zeta_{\text{mfp}}(T) \sim 1/f_{\text{norm}}(T)$$

where $f_{\text{norm}}(T)$, the fraction of ‘normal’ fluid in the superfluid system at a given temperature $T$, is known to become vanishingly small when $T \to 0$. For this reason, the phonon mean free-path $\zeta_{\text{mfp}}$ is much larger than the phion mean free path $r_{\text{mfp}} \sim \frac{1}{n a^2}$ that we have considered so far.

Actually, $f_{\text{norm}}(T = 0)$ is non-zero but infinitesimally small. In fact, even at zero-temperature, a pure Bose condensate cannot be dynamically stable in the presence of interactions among the elementary spinless quanta. In the Bogolubov approximation, these produce a tiny populations of the lowest $(k, -k)$ excited states giving rise to the depletion Eq.(17).

In the phion condensate where $\sqrt{\bar{n}a^3} = \mathcal{O}(10^{-16})$, this fraction $f_D \sim \sqrt{\bar{n}a^3}$ is infinitesimal and can be taken as a measure of the residual phonon-phonon interactions. For this reason, the relevant density to determine the phonon mean free path is $f_D \bar{n}$ and not $\bar{n}$. Therefore, I would tentatively estimate a phonon mean free path

$$\zeta_{\text{mfp}} \sim \frac{1}{f_D \bar{n} a^2} \sim 4 \cdot 10^{16} r_{\text{mfp}}$$

(109)

to mark the distance over which non-linear effects might modify the free-phonon propagation. Using the previous estimate $r_{\text{mfp}} \sim 1$ centimeter, I find

$$\zeta_{\text{mfp}} \sim 4 \cdot 10^{16} \text{ cm} \sim 2 \cdot 10^3 \text{ AU}$$

(110)

Using this value, in the relation for the gravitational acceleration due to the sun

$$\frac{G_N M_{\text{sun}}}{4\pi \zeta_{\text{mfp}}^2} \sim 10^{-8} \text{ cm} \cdot \text{sec}^{-2}$$

(111)

one gets a very good agreement with Milgrom’s critical acceleration value [98]

$$g_o \sim 10^{-8} \text{ cm} \cdot \text{sec}^{-2}$$

(112)
at which one should find deviations from a pure Newtonian behaviour. In solar-system conditions, this type of situation occurs when studying the long-term comets, those with a period $> 200$ yr and semimajor axis larger than $\sim 10^2$ AU. To describe some of their features one has to introduce some ‘ad hoc’ assumptions [99], and this might be indicative of deviations from a pure Newtonian behaviour [100]. In this sense, the idea of gravity as a long-wavelength oscillation of a superfluid medium leads naturally to the existence of a non-linear regime. Its precise characteristics, however, are not easy to predict.
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