Quantum Hall Ferromagnetic-Paramagnetic Transition in p-Si/SiGe/Si Quantum Wells in a Tilted Magnetic Field

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Abstract. Magnetoconductance components ρxx and ρxy were measured in two p-Si/SiGe/Si quantum well samples with an anisotropic g-factor in a tilted magnetic field of up to 18T as a function of temperature (20 mK-2 K) and tilt angle. We analyzed dependences of the conductivity, the activation energy ΔE, and the filling factor ν on the tilt angle Θ. In the sample with density p=2×10¹¹ cm⁻² in the vicinity of ν=2 ΔE(Θ) undergoes a minima at Θ≈60°, while ν(Θ) shows a sharp jump. These facts allowed us to conclude that at Θ≈60° and ν≈2 a crossing of the Landau levels 0↑ and 1↓ occurs. It leads to the first order ferromagnetic-paramagnetic (F-P) phase transition. A coexistence of two phases at the transition point also supports the idea. However, in another sample, with p=7.2×10¹⁰ cm⁻², no transition was observed. For both samples we have obtained the dependences of the effective g-factor on the tilt angle, which led us to the conclusion that the F-P transition in the p-Si/SiGe/Si structure in a tilted magnetic field is a result of a violation of the g*-factor axial symmetry due to disorder.

1. Introduction
The quantum well in p-Si/Si₁₋ₓGeₓ/Si is located in the strained Si₁₋ₓGeₓ layer; therefore, the threefold degenerate valence band of SiGe splits into three bands due to spin–orbit coupling and mechanical stresses. The charge carriers are heavy holes, which band is formed by atomic states with quantum numbers L=1, S=1/2, and J=3/2. This result in a strong anisotropy of the effective g*-factor: g*≈4.5 if a magnetic field is normal to the quantum well (QW) plane, and g*≈0 if a field is parallel to the QW plane [1]. In such structures if a magnetic field is normal to the QW plane, the values of effective mass m* and g*-factor are such that the relationship g*μBB≈ωc holds true; here, μB is the Bohr magneton, ωc=eB/m*ec is the cyclotron frequency. As a result, there is the dominance of minima associated with odd filling factors for ν>2 in the SdH oscillations pattern. An unusual phenomenon, the so-called reentrant metal–insulator transition, was also found at ρxx(T,Θ) in this structure in a magnetic field at a filling factor ν = 3/2. This anomaly was explained either by presence of smooth large scale potential fluctuations with an amplitude comparable with the Fermi energy [2] or by the crossing of Landau levels (LLs) with opposite spin directions (0↑, 1↓) with the magnetic field change [3,4].

In this paper we study the conductivity in p-Si/Si₁₋ₓGeₓ/Si in a tilted magnetic field to determine the dependence of the g*-factor on the angle of magnetic field tilt to the normal to the plane of a two
dimensional (2D) channel. With this dependence, we can analyze the possible cause of the anomalies in conductivity that appear at the filling factor $\nu = 2$ in a p-Si/SiGe/Si sample. The asymmetric quantum well in our samples was 30 nm wide, and the sample structure was described in [5].

2. Experimental results and discussion

The magnetoresistance $\rho_{xx}$ and $\rho_{xy}$ were measured in the tilted magnetic field of up to 18 T, at temperatures (0.02-2) K in the samples with concentrations $p = 7.2 \times 10^{10}$ cm$^{-2}$ and $2 \times 10^{11}$ cm$^{-2}$, mobility of $7 \times 10^5$ cm$^2$/Vs, with IQHE. Indeed, the dependence $\rho_{xx}$ on the perpendicular component of the magnetic field $B_\perp$ near $\nu = 3/2$ shows the anomaly: $\rho_{xx}$ increases by more than 5 times when the angle $\Theta$ changes from $0^\circ$ to $70^\circ$. However, the dependence $\sigma_{xx}(\Theta) = \rho_{xx}(\Theta)/(\rho_{xx}^2(\Theta) + \rho_{xy}^2(\Theta))$ does not undergo any anomaly at $\nu = 3/2$. In addition, as seen in Fig.1a for the sample with $p = 2 \times 10^{11}$ cm$^{-2}$ the position of the minima $\sigma_{xx}$ in the magnetic field for $\nu = 3$ do not depend on the tilt angle, for $\nu = 2$ for $\Theta > 50^\circ$ minima begin to shift toward lower magnetic fields. When the angle reaches the value of $\Theta \approx 59.5^\circ$, two oscillations appear on the curve: the former, which shifts to the left with the angle increase, and the new one which emerges at $B_\perp = 4$ T. With further increase of $\Theta$ this new oscillation shifts left and grows in amplitude while the former oscillation disappears. There is a range of angles $\Theta = (59.5-61)^\circ$, in which both types of oscillations coexist. In the sample with $p = 7.2 \times 10^{10}$ cm$^{-2}$ at $\Theta = 0^\circ$ the oscillations at $\nu = 2$ are practically non-observant. Only at $\Theta > 70^\circ$ this oscillation starts to manifest itself. Moreover, its position in the magnetic field shifts to lower fields with the increase of $\Theta$. It is shown in Fig.1b.

![Figure 1a](image1.png) ![Figure 1b](image2.png)

**Figure 1a.** $\sigma_{xx}$ on $B_1$ for different $\Theta$ shown for two orientations of the field $B_1$ || and $B_1$ \perp; $T = 0.3$ K for the sample with $p = 2 \times 10^{11}$ cm$^{-2}$. The curves are shifted for clarity.

For the sample with $p = 2 \times 10^{11}$ cm$^{-2}$ we also measured the temperature dependence of conductivity at different tilt angles $\Theta$. Thus, we were able to determine the activation energy $\Delta E$ at various angles experimentally, as shown in Figure 2. As seen, $\Delta E$ achieves its minimum at $\Theta = 60^\circ$. The maximum of the angle dependence of the conductivity $\sigma_{xx}(\Theta)$, at the minima of oscillations at $\nu = 2$, is also observed at $\Theta = 60^\circ$. This dependence is shown in the inset of Fig. 2.

Now we analyze the ratio of the conductivity in the oscillation minimum near $\nu = 2$ at the angle $\Theta$ to the conductivity in the perpendicular field $\sigma_{xx}(\Theta)/\sigma_{xx}(0)$ in the sample with $p = 7.2 \times 10^{10}$ cm$^{-2}$. The angle dependence of this ratio is illustrated in Figure 3. Up to $\Theta = 70^\circ$ ratio $\sigma_{xx}(\Theta)/\sigma_{xx}(0)$ does not change, however, at $\Theta > 70^\circ$ it starts to decrease sharply. Such a decrease is evidently associated with the energy gap opening in the hole spectrum due to a divergence of the LLs $0\uparrow$ and $1\downarrow$ with an increasing...
tilt angle, as illustrated in the inset of Fig.3. An abrupt change in the conductivity with the angle increase suggests that the conductivity depends exponentially on the energy gap between the LLs $0^\uparrow$ and $1^\downarrow$: $\sigma_{xx} \propto \exp\left(\frac{(\hbar \omega_c - g^* \mu_B B)}{2k_B T}\right)$.

Figure 2. Activation energy on tilt angle $\Theta$. Inset: Dependence of $\sigma_{xx}$ on $\Theta$ at $\nu \approx 2$; $T=0.3$ K.

Figure 3. Ratio $\sigma_{xx}(\Theta)$ to $\sigma_{xx}(0)=6$, as in [6]. The dependence of the reduced $g^*$-factor on $\Theta$ is shown in Fig. 4. As follows from this figure, the $g^*(\Theta)$ dependence in the sample $p=2 \times 10^{11}$ cm$^{-2}$ differs from $\cos \Theta$.

Figure 4. Dependence of the reduced g-factor $g^*(\Theta)/g(0)$ on $\Theta$ for two samples with $p=7.2 \times 10^{10}$ cm$^{-2}$ and $p=2 \times 10^{11}$ cm$^{-2}$; the solid line is the $\cos \Theta$; the dash line is a guide for eye. Inset: Enlarged fragment of the same curves.

The observed effects could be associated with g*-factor behaviour. In the case of the axial symmetry the value of an anisotropic effective g*-factor in a tilted magnetic field is determined by the equation $g^*=[g_L^2 \cos^2(\Theta)+g_R^2 \sin^2(\Theta)]^{1/2}$. For a strong anisotropy, when $g_R=0$ (as should be in our structures), we have $g^*=g_L \cos \Theta$. In this case, when a field is tilted, the positions of the oscillation minima shift toward high fields but the oscillation amplitude is independent of $\Theta$. In our samples, the oscillation amplitude depends on $\Theta$; therefore, axial symmetry is likely to be violated here. The inset of Fig. 4 shows that at $\Theta>70^\circ$ for the sample with $p=7.2 \times 10^{10}$ cm$^{-2}$ dependence of $g^*(\Theta)/g(0)$ on the angle starts to deviate slightly from the curve $\cos(\Theta)$, which results in a discrepancy between the levels $0^\uparrow$ and $1^\downarrow$, the emergence of the energy gap, and a sharp decrease in conductivity.

In Figure 5 we now draw the energy levels $0^\downarrow$, $0^\uparrow$ and $1^\downarrow$ as a function of angle, while including the dependence $g^*(\Theta)$ obtained from the experiment as $E(0^\uparrow)=\hbar \omega_c / 2 + g^* \mu_B B / 2$; $E(1^\downarrow)=3\hbar \omega_c / 2 + g^* \mu_B B / 2$. 

From this equation we can calculate the dependence of $g^*(\Theta)$ at $\nu=2$ and $g^*(0)=6$, as in [6]. The dependence of the reduced $g^*$-factor on $\Theta$ is shown in Fig. 4. As follows from this figure, the $g^*(\Theta)$ dependence in the sample $p=2 \times 10^{11}$ cm$^{-2}$ differs from $\cos \Theta$.

Then the ratio of the conductivities can be expressed as

$$\frac{\sigma_{xx}(\theta)}{\sigma_{xx}(0)} = \exp \left[ \frac{g^*(\theta) \mu_B B_{\perp} \cos(\theta) - g^*(0) \mu_B B_{\parallel}}{2k_B T} \right]$$

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In the sample with $p=2\times10^{11} \text{ cm}^{-2}$, at $\theta=0^\circ$ a ferromagnetic order should be in the system, since two filled LLs with the same spin direction lie below the Fermi level. Then, at $\theta=60^\circ$, LLs intersect each other and, at $\theta>60^\circ$, change their relative position, breaking the ferromagnetic order. In the range from $59.5^\circ$ to $61^\circ$, both ferromagnetic and paramagnetic states coexist. At the point of crossing the value of filling factor $\nu$ calculated from the position of the oscillation minima in the magnetic field axis sharply jumps. However, the conductivity at the oscillation minima does not jump and reaches a maximum at the transition point. It confirms that the observed F-P transition is indeed associated with the crossing of the LLs $0^\uparrow$ and $1^\downarrow$ at $60^\circ$. F-P transition is expected to be accompanied by the formation of ferromagnetic domains, which, as on [7], should be manifested by the magnetoresistance anisotropy. We tilted the sample in two orientations keeping the field projection $B_\parallel$ parallel and perpendicular to the current, but did not observe any anisotropy in the vicinity of the transition as shown in Fig.1a.

In the sample with $p=7.2\times10^{10} \text{ cm}^{-2}$, as seen from Fig.5b, the complete F-P transition in tilted field was not observed. In a range of angles $\theta=(0-70)^\circ$ the LLs $0^\uparrow$ and $1^\downarrow$ still coincided, and only for $\theta>70^\circ$ there was a gap in the hole spectrum arising as a result of the LLs divergence. Ambiguity in the results observed by various authors [1-4] on different p-Si/GeSi/Si samples, in our opinion, is due to dissimilar dependences of the $g^*$-factors on the tilt angle. Moreover, the absolute value of $g^*$-factor could be possibly different. Both these characteristics in p-Si/GeSi/Si samples are likely determined by a disorder in the sample and the hole concentration. This conclusion is demonstrated in Fig.4. Indeed, the violation of the axial symmetry in the sample with an anisotropic $g^*$-factor is determined by the system disorder and a corresponding deviation of the dependence $g^*(\Theta)$ from the functional $g^*\sim \cos \Theta$. In Fig.4 it is evident that the large deviations of the dependence $g^*(\Theta)$ from $\cos \Theta$, as it is in the sample with $p=2\times10^{11} \text{ cm}^{-2}$, lead to the F-P transition which occurs at $\Theta=60^\circ$. With just a slight decline from $g^*\sim \cos \Theta$, as in the sample with $p=7.2\times10^{10} \text{ cm}^{-2}$, the LLs $0^\uparrow$ and $1^\downarrow$ are coinciding in a wide range of angles $\Theta=0-70^\circ$. With further angle increase the LLs diverge, thus resulting in the energy gap opening in the hole spectrum, and there is a transition to the paramagnetic state. Therefore, if there is no deviation from the dependence of $g^*\sim \cos \Theta$, there is no transition.

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3. References
[1] Dorozhkin S I 1994, *JETP Lett.* **60** 595
[2] Dorozhkin S I et al. 1995, *JETP Lett.* **62** 534
[3] Coleridge P T et al. 1997, *Sol. State Commun.* **102** 755
[4] Sakr M R et al. 2001, *Phys. Rev. B* **64**, 161308
[5] Drichko I L et al. 2009, *Phys. Rev. B* **79**, 205310
[6] Drichko I L et al. 2010, *JETP* **111**, 495
[7] Chalker J T et al. 2002, *Phys. Rev. B* **66** 161317