Influence of a strong magnetic field on paramagnetic fluid’s flow in cubical enclosure

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Abstract. The fluid behaviour in thermo-magnetic convection of paramagnetic fluid in a strong magnetic field was studied. The fluid was 50\% volume aqueous solution of glycerol with an addition of gadolinium nitrate hexahydrate (Gd(NO\textsubscript{3})\textsubscript{3} \cdot 6H\textsubscript{2}O). Experimental enclosure – a vessel with aspect ratio (AR=height/width) equal to 1.0 – was heated from the bottom, and cooled from the top. Temperature difference between top and bottom walls was kept constant at $\Delta T = 5$ and 11 $^\circ$C. The magnetic induction was increased stepwise from 1 to 10 [T] and thermocouples placed inside the enclosures measured temperature changes of the fluid. On the basis of temperature measurements, analysis of the fluid flow was performed.

1. Introduction
Thermo-magnetic convection of a paramagnetic fluid – convective motion of a magnetic fluid with an external magnetic field present – is in scientists field of interest since superconducting magnets could be produced with relatively low costs, that is since superconductors working above boiling point of liquid nitrogen were invented. Braithwaite [1] studied influence of a non-uniform magnetic field on a paramagnetic fluid convection and presented mathematical description of forces acting in such a system. Huang and Edwards [2] presented research about intensification and suppression of convective motion in thermo-magnetic convection and Fornalik [3] published thermodynamic description of phenomena occurring in thermo-magnetic convection. Another field of interest, besides intensification and suppression of heat transfer in weakly magnetic fluids, is analysis of the flow in thermo-magnetic convection [4].

One of the ways to “look inside” a convection phenomenon is by a spectral analysis. An interest in a distribution of passive scalar quantities, such as temperature, was born with a development of metrology and a need to better understanding of turbulence mechanisms. One of the first works published in this field was by Obuchow [5], who applied Kolmogorow’s local isotropy concept to describe temperature distribution in atmosphere. A decade later Batchelor [6] conducted an analysis of two extreme cases – one where momentum diffusion coefficient was much lower than heat diffusion confident and the opposite one. For the first instance he concluded that spectral function of temperature is located in inertial-convective subregime. In the second case convection subregime was more extensive and it depended more on fluid viscosity. He derived a spectrum correlation for this subregion which is fitted to a reverse wave number.

In presented work Authors conducted an experimental analysis of a magnetic field influence on a paramagnetic’s fluid convection and performed spectral analysis on temperature signal changes during measurements.
2. Experimental stand

2.1. Experimental enclosure

For the purpose of analysing the magnetic field influence on a paramagnetic fluid flow, experimental research was conducted. As a geometry, cubical enclosure of dimensions 0.032x0.032x0.032 [m], shown in Figure 1, was chosen. It consisted of two thermally active walls – the bottom one built from a copper plate heated with a nichrome wire connected to a DC supply and the top one with built in cooling chamber. Six thermocouples were inserted at all times into the thermally active walls to measure the temperature. The enclosure was made from Plexiglas and in a front vertical wall six small holes were made and thermocouples were inserted through the wall into the fluid to measure temperature changes during experimental procedure. The experimental setup worked in Rayleigh-Benard configuration, and two temperature differences were applied – 5 and 11 [°C] with the top cooled wall temperature set to [18 °C] which was the ambient temperature in the magnet working section.

![Figure 1. Schematic view of experimental enclosure with positions of the thermocouples.](image)

2.2. Position in the magnet

In natural convection fluid motion is a result of gravitational force $f^g$ acting on a fluid particles:

$$f^g = g \rho$$

(1)

Because density $\rho$ of the fluid depends on a temperature $T$, gravitational force acts with different force on a particles with various temperature creating a gravitational buoyancy force $f^G$:

$$f^G = g(\rho - \rho_0)$$

(2)

where $g$ is gravity acceleration and $\rho_0$ is a reference density. Noting the Boussinesq model, which assumes that the density of the fluid is constant except the equation for buoyancy force, and that it can be applied only to small temperature differences, fluid density can be expressed as:

$$\rho = \rho_0(1 - \beta(T - T_0))$$

(3)

where $\beta$ is thermal expansion coefficient and $T_0$ is a reference temperature. Using the above relation to eq. (2), gravitational buoyancy force can be stated as:

$$f^G = -g \rho_0 \beta(T - T_0)$$

(4)

The effect of a gravitational buoyancy force is a downward cold fluid motion, while the hot fluid particles are lifted.

A force, acting on a paramagnetic fluid in a magnetic field can be described as [7]:

$$f^{me} = \mu_0 (M \cdot \nabla)H$$

(5)

where $M$ is magnetization, $H$ is the intensity of the magnetic field and $\mu_0$ is a permeability of the vacuum. Noting that a physical value speaking about magnetic properties of the fluid is magnetic susceptibility $\chi$: 
Equation (5) can be rearranged into:

\[ \chi = \frac{M}{H} \]  

(6)

Equation (5) can be rearranged into:

\[ f^{me} = \mu_0 \chi (\mathbf{H} \cdot \nabla) \mathbf{H} = \frac{\mu_0 \chi}{2} \nabla \mathbf{b}^2 \]  

(7)

Also knowing that magnetic induction \( \mathbf{b} \) depends on an intensity of magnetic field and on a magnetic permeability of the material and that for a paramagnetic fluid \( \chi \ll 1 \) eq. (7) can be transformed into:

\[ f^{me} = \frac{\chi}{2 \mu_0} \nabla \mathbf{b}^2 \]  

(8)

Using above relations, Curie’s law and Boussinesq approximation, Tagawa [8] developed an equation describing magnetic buoyancy force:

\[ f^M = - \left( 1 + \frac{1}{\beta T_0} \right) \frac{\chi \rho_0 (T - T_0)}{2 \mu_0} \nabla \mathbf{b}^2 \]  

(9)

where \( \chi \rho_0 \) is mass magnetic susceptibility at reference temperature.

Equation (5) concludes that fluid, which temperature \( T \) is lower than the reference temperature \( T_0 \), is attracted to a square of magnetic field gradient \( \nabla \mathbf{b}^2 \). Taking into account both the gravitational and magnetic buoyancy forces, experimental enclosure was placed in the superconducting helium free magnet (HF10-100VHT-B Sumito Heavy Industries Ltd.) in an upper half position where the square of the magnetic field gradient is highest and intensification of fluid motion could be obtained.

**Figure 2.** Schematic view of experimental setup (left) and configuration of the gravitational and magnetic buoyancy forces acting on a fluid (right).

### 2.3. Working fluid

A fluid chosen for the experiments was a non-electrically conducting 50% volume solution of distilled water and glycerol with an addiction of 0.8 [mol/kg of solution] concentration of gadolinium(III) nitrate hexahydrate (Gd(NO₃)₃·6H₂O) to make it paramagnetic. Properties of the fluid are listed in Table 1 and were measured experimentally.
Table 1. Properties of the working fluid at 25 [°C].

| Property                        | Symbol | Value       | Unit          |
|--------------------------------|--------|-------------|---------------|
| Heat capacity                  | \(c_p\) | 2.92 \times 10^3 | [J/kg·K]     |
| Thermal diffusivity            | \(\alpha\) | 9.13 \times 10^{-8} | [m²/s]       |
| Thermal expansion coefficient  | \(\beta\) | 4.78 \times 10^{-4} | [1/K]        |
| Dynamic viscosity              | \(\mu\) | 1.30 \times 10^{-2} | [kg/m·s]    |
| Thermal conductivity           | \(\lambda\) | 0.376       | [W/m·K]      |
| Kinematic viscosity            | \(v\) | 9.25 \times 10^{-6} | [m³/s]       |
| Density                        | \(\rho\) | 1411        | [kg/m³]      |
| Mass magnetic susceptibility   | \(\chi_m\) | 2.39 \times 10^{-7} | [m³/kg]     |

2.4. Experimental procedure

Because of the way a Nusselt number was calculated in this study, the first step in experimental analysis was to measure the heat losses of the experimental enclosure. To do that, experimental enclosure was filled with water, rotated 180° from the Rayleigh-Benard configuration, so the bottom wall was cooled and the top one heated, and placed in the designed position in the magnet’s working section. This allowed, after the chosen temperature difference was set, to obtain a temperature stratification in the fluid and then calculation of the heat losses of the system, which is described in section 3.1.

Then the main experiment took place. The experimental enclosure was rotated back to Rayleigh-Benard configuration and temperature difference was set and the setup was left to stabilize, which took about two hours. Then the first measurement took place – the neutral case – natural convection in the system, without external magnetic field present. The measurement was a 15-min record of temperature changes of the fluid. Next step was connected with thermo-magnetic convection - magnetic field was applied - and because magnetic buoyancy force was acting toward intensification of fluid motion, the voltage and current supplied to the heater had to be corrected to maintain chosen temperature difference between horizontal walls. After stabilization of the setup, measurement was made. The same procedure took place for magnetic inductions from 1 [T] to 10 [T] with s step of 1 [T].

3. Analysis of the thermocouples signals

Temperature signals obtained during experimental part can be analysed two ways – one to acquire information about heat transfer in the system and two – to calculate spectral functions.

3.1. Heat transfer analysis

To analyse an influence of a thermo-magnetic convection on a heat transfer in the system, a method proposed by Ozoe and Churchill [9] was utilized to calculate a Nusselt number, which is ratio of net convection heat flux \(Q_{\text{net,conv}}\) and net conduction heat flux \(Q_{\text{net,cond}}\):

\[
\text{Nu} = \frac{Q_{\text{net,conv}}}{Q_{\text{net,cond}}} \tag{10}
\]

Where the net conduction heat flux is:

\[
Q_{\text{net,cond}} = Q_{\text{cond}} - Q_{\text{loss}} \tag{11}
\]

and the net convection heat flux is:

\[
Q_{\text{net,conv}} = Q_{\text{conv}} - Q_{\text{loss}} \tag{12}
\]

In proposed research it was assumed that the heat losses depend only on the temperature of the heated wall – hence the conduction state measurements. The heat losses were estimated from:

\[
Q_{\text{loss}} = Q_{\text{cond}} - Q_{\text{Fourier's law}} \tag{13}
\]

where theoretical conduction through the fluid was calculated from Fourier’s law:
And since \( a = d = 0.032 \text{ [m]} \) (14) takes form:

\[
Q_{\text{theor, cond}} = \frac{a^2 \lambda \Delta T}{d}
\]  

(14)

Heat flux was calculated for conduction area of 0.032 [m] x 0.032 [m]. The estimated heat loss was approximated linearly:

\[
Q_{\text{loss}} = 0.08 \cdot \Delta T
\]  

(16)

Applying equations 11, 13 and 14 to equation 10, Nusselt number can be expressed as:

\[
Nu = \frac{Q_{\text{conv}} - Q_{\text{loss}}}{a \lambda \Delta T}
\]  

(17)

where:

\[
Q_{\text{conv}} = UI
\]  

(18)

where \( U \) is electric voltage and \( I \) is current.

3.2. Spectral analysis

Temperature signals, which were recorded during experiments, allowed examination of the flow structure. This was obtained through Fast Fourier Transform (FFT), which calculates discrete Fourier transform (DFT) of a sequence:

\[
F_n = \sum_{i=0}^{N-1} x_i e^{-\frac{2\pi in}{N}}
\]  

(19)

where \( x_i \) is a sequence of length \( N \), and \( F_n \) is its discrete Fourier transform.

Calculated results, in the form of amplitude versus frequency, were used to analyse the fluid behaviour in thermo-magnetic convection.

Spectral functions of a scalar field (i.e. temperature), in general, are very useful tool to analyze a turbulent transport mechanisms. So, with an assumption that the turbulence is homogenous, power density \( P_{xx} \) or spectrum can be calculated with the utilization of FFT:

\[
P_{xx}(e^{j\omega}) = \sum_{m=-\infty}^{\infty} r_{xx}(m)e^{-j\omega m}
\]  

(20)

and with the use of Peridogram method, which estimates the power as time-integral square amplitude from amplitude obtained with FFT:

\[
\text{Power density (TISA)} = \frac{\Delta \left( \text{Re}^2 + \text{Im}^2 \right)}{n}
\]  

(21)

In general, temperature spectral functions depend on the energy dissipation, thermal diffusivity, kinematic viscosity and temperature. But in some ranges, spectral function does not depend on the diffusion processes, and therefore does not rely on kinematic viscosity and thermal diffusivity. This subrange is called intertial-convective [10] and the spectral function has an inclination of wave number \( k \) with -5/3 exponent. When thermal diffusivity becomes more significant, the spectral function has an inclination of reverse wave number, and this subrange is called viscous-diffusive [10].

4. Results

Figure 3 presents results of spectral analysis for \( \Delta T=5 \text{ [°C]} \) as a amplitude versus frequency and power spectrum diagrams. For natural convection case (\( |b_0|_{\text{max}}=0 \text{ [T]} \)) power spectrum shows a partial match to -5/3 slope for low frequencies 0.01-0.05 [Hz] and a flat power spectrum for frequencies higher than 0.05 [Hz] which is specific to regular and stable flows. For this case two characteristic frequencies can be observed 0.00706 [Hz] and 0.00823 [Hz]. Applying magnetic induction to the system causes significant changes in flow character. For \( |b_0|_{\text{max}}=3 \text{ [T]} \) only one characteristic frequency appears – 0.07411 [Hz] and power spectrum shows a good fit to viscous-diffusive subrange for small frequencies. Further increase of magnetic field causes rapid change in fluid behaviour. For magnetic
induction of 10 [T] signals from thermocouples show high and irregular oscillations, indicating that magnetic field results in flow destabilization. For those cases power spectrum points towards inertial-convective regime of the flow.

![Power spectrum and amplitude versus frequency diagrams](image)

**Figure 3.** Power spectrum (left) and amplitude versus frequency (right) diagrams for $\Delta T=5$ [$^\circ$C].

Figure 4 presents results of spectral analysis for $\Delta T=11$ [$^\circ$C]. Natural convection case exhibits flat power spectrum indicating that the flow is stable and regular. Frequency analysis shows that only one characteristic frequency with small amplitude appears for that case – 0.00335 [Hz].
Figure 4. Power spectrum (left) and amplitude versus frequency (right) diagrams for $\Delta T=11$ °C.

A rapid change in flow behaviour appears for smaller magnetic induction than in the previous case. For magnetic induction 3 [T] in the centre of the magnet flow changes significantly. Power spectrum displays sharp inclination with steeper slope than $-5/3$. Many characteristic frequencies with relatively high amplitudes for this case point toward chaotic flow and transitioning regime. Further increase of
magnetic induction indicates that flow is in viscous-diffusive subregime for small frequencies and in inertial-convective subregime for frequencies up to 1 [Hz].

Figure 5 presents results of heat transfer analysis as a Nusselt number ratio $\frac{Nu}{Nu_0}$ where $Nu_0$ is a Nusselt number value for a neutral case – without magnetic induction applied to the system. Intensification of heat exchange in the system is clearly seen. For small values of magnetic induction ($|b_0|_{\text{max}} = 1$ [T]) Nusselt number value is about 3-5% higher than for natural convection. For maximal values of magnetic inductions, intensification of heat transfer reaches 290% for $\Delta T = 11$ [°C] and over 310% for $\Delta T = 5$ [°C]. This is caused by intensification of fluid motion with the magnetic induction increase, as was explained in section 2.2.

![Figure 5. Ratio of $\frac{Nu}{Nu_0}$ versus magnetic induction in the centre of the magnet.](image)

5. Summary
Spectral analysis of magnetic field, with different magnetic inductions, influence on natural convection was conducted for two temperature differences between thermally active walls. Presented research allowed to draw following conclusions: a) for natural convection in both temperature differences temperature field is stable state – a flat power spectrum can be observed; b) applying magnetic induction to the system significantly changes character of the flow; c) for temperature difference of 5 [°C] changes in flow behaviour appear for magnetic inductions higher than 3 [T] and for high magnetic inductions flow is in viscous-diffusive subregime; d) for higher temperature differences destabilization of the flow field appears for smaller magnetic induction values – 3 [T] and power spectrum functions have inclination sharper than $-5/3$; e) magnetic induction applied to convection of paramagnetic field, where magnetic field source is below experimental setup, causes intensification of fluid motion and heat transfer in the system; f) for values of $|b_0|_{\text{max}} > 5$ [T] intensification of heat transfer is more than 200%; g) Fast Fourier Transform is a good tool in analyzing temperature signals from the flows and h) spectral analysis allows an insight into phenomena occurring during thermo-magnetic convection processes.

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