Coupled-Channel-Induced $S - D$ mixing of Charmonia and Possible Assignments for $Y(4260)$ and $Y(4360)$

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We calculate the $S - D$ mixing of the $J^{PC} = 1^{--}$ charmonium states induced by the coupled-channel effects. The mass shifts, the open-charm decay widths and the di-electron decay widths are evaluated. We find that it is possible to assign $Y(4260)$ and $Y(4360)$ as the mixtures of $4S$ and $3D$ states.
I. INTRODUCTION

In 2005, BaBar observed a broad structure, $Y(4260)$, near 4.26 GeV in the initial state radiation $e^+e^- \rightarrow \gamma_{ISR}\pi^+\pi^- J/\psi$. Its quantum numbers are $J^{PC} = 1^{--}$, the same as a normal vector charmonium. Nevertheless, this state challenges the standard charmonium assignment, because its mass and decay properties seem to be in conflict with expectations. Conventionally, $\psi(4040)$, $\psi(4160)$ and $\psi(4415)$ are usually assigned as the $\psi(3S)$, $\psi(2D)$ and $\psi(4S)$ states, respectively, so a resonance with the mass near 4.26 GeV cannot fit in. In addition, the already confirmed excited charmonia usually decay into open-charm states dominantly once they exceed the open-charm threshold, but $Y(4260)$ has never been found in the open-charm decay channels [2]. Partly due to these reasons, many authors considered it as a non-$q\bar{q}$ state, such as hybrid [3, 4], tetraquark state [4], molecular state [5, 7] and so on. Another resonance, $Y(4360)$, is in a quite similar situation as $Y(4260)$. It was first reported by BaBar in 2007 as a broad structure at an invariant mass of 4.32 GeV [8]. This state has never been seen in the open-charm decay channels, same as $Y(4260)$. It is also considered hard to be accommodated into the conventional charmonium family, and has been considered as a non-$q\bar{q}$ state by many researchers [5, 6, 11]. (We refer the readers to Ref. [12] for a comprehensive review on $Y(4260)$ and $Y(4360)$.)

Although there is a tendency to consider $Y(4260)$ and $Y(4360)$ as non-$q\bar{q}$ states, it is still possible to assign them as conventional charmonium states. Ref. [13] studied the heavy quarkonium spectrum based on Martin-like potential and found that $Y(4260)$ could be interpreted as the 4S vector charmonium. In Ref. [14], $Y(4260)$ is assigned as the 3D charmonium. Ref. [15] studied higher charmonia with a screened potential, it was found that the screening effect lowers the mass spectra of highly excited states, so that in their model, $Y(4260)$ and $Y(4360)$ can be assigned as $\psi(4S)$ and $\psi(3D)$ states respectively. In Ref. [16], $Y(4360)$ is interpreted as the 3D $c\bar{c}$ state. And in Ref. [17], the authors studied the masses and decay properties of vector charmonia and assigned $Y(4360)$ to be the 4S state. The mass spectrum is not the only concern in identifying these states, their absence in the open-charm decay channels is another challenge. And this is a challenge not just for the conventional charmonia interpretations of $Y(4260)$ and $Y(4360)$, but also for non-$q\bar{q}$ interpretations. So it is fair to say that the conventional charmonium interpretation is still in the market and worth further explorations. One may find the following two facts helpful in explaining the unusual decay properties of $Y(4260)$ and $Y(4360)$. First, it is well known that the excited vector charmonia are actually mixtures of S and D wave states as implied by the di-electron decay widths. Enlightened by the $S-D$ mixing in charmonium states, Ref. [18] suggested that the non-observation of $Y(4260)$ in $DD$ decay channels may be due to the interference between the S wave and D wave. The $S-D$ mixing in vector charmonia are also studied by fitting experimental data in Refs. [13, 19]. Second, the node structures of highly excited charmonia also affect the decay properties. For the two-body open-charm decays, the essential part of the a decay amplitude is the integration of the overlapped wave functions. The node structures of wave functions transmit into the decay amplitudes, so at some particular values of $|k|$, i.e., the magnitude of the momentum of one final particle, the decay amplitude vanishes. So the decay properties of excited states may exhibit some irregular patterns.

There is one well-known effect inducing the $S-D$ mixing in vector charmonia, i.e., the coupled-channel effect. It takes into account the interactions between an eigenstate of a simple quark model and the continuum multi-particle states which the quarkonium can decay into [20, 21]. So the physical states are actually mixtures of all bare quarkonium states and the continuum multi-particle states. The coupled-channel effects with all possible mixtures were studied in Refs. [20, 21], either using Cornell model or using the $3P_0$ model to evaluate the relevant decay amplitudes. For vector charmonia, it is expected that the mixing between the $(n+1)S$ and $nD$ states are most significant compared to mixing among other excitations, which was justified in Refs. [22]. As a result of the coupled-channel effects, the masses of the physical states are shifted. The dominant contributions of mass corrections are usually received from the continuum states. So many studies ignored the $S-D$ mixing in the vector charmonium sector when focusing on the mass shift effect [23, 24].

In light of the new observed resonances, we find it is interesting to study the coupled-channel induced $S-D$ mixing in the vector charmonium sector. The models we used in this study is different from Ref. [21] where the Cornell model is used to evaluate both the wave functions and the decay amplitudes. In our research, we use the instantaneous Bethe-Salpeter (BS) equation and the $3P_0$ model to do the job. The decay channels we studied are $D^{(*)}\bar{D}^{(*)}$, $D^{(*)}\bar{D}^{(*)}$. For the reason mentioned above, only the mixing between the $(n+1)S$ and $nD$ states are considered. Based on our calculations, the possibility of assigning $Y(4260)$ and $Y(4360)$ as the lower and higher states of the mixture of $\psi(4S)$ and $\psi(3D)$ is discussed.

The remaining part of this paper is organized as follows. The theoretical tools are introduced in Section II. We use the instantaneous Bethe-Salpeter equation with a screened Cornell potential to calculate the wave functions of relevant mesons. Then the $3P_0$ model is used to evaluate decay amplitudes. The coupled channel dynamics is reviewed in this section. In Section III, with best fitted model parameters, the numerical calculations are performed. The results and discussions are presented in this section. The last section is devoted to the conclusion.
II. THEORETICAL SETTINGS

A. Coupled-Channel Dynamics

In naive quark models, mesons are bound states of a quark and an anti-quark bounded by a QCD-inspired potential. The masses and wave functions can be obtained by solving the eigenvalue problem formally expressed as:

\[ H_0 |\psi_n\rangle = M_{0,n} |\psi_n\rangle, \]

(1)

where the \( M_0 \) and \( |\psi_n\rangle \) are usually referred as the bare mass and the bare state. The Hamilton \( H_0 \) only includes the interaction described by the potential binding the quark-anti-quark pair. In coupled-channel formulism, the Hilbert space under consideration is enlarged to include the continuum states which the bare states can decay into. We are focusing on vector charmonium decaying into two-body open-charm channels, and in this situation, the Hamilton can be formally written as [23]:

\[ H = \begin{pmatrix} H_0 & H_{\text{QPC}}^\dagger \\ H_{\text{QPC}} & H_{\text{BC}} \end{pmatrix}, \]

(2)

where \( H_{\text{BC}} \) is the free Hamilton (by "free", we mean that the interactions between the two mesons are neglected) for the continuum states with two particles \( B \) and \( C \):

\[ H_{\text{BC}}|B,C;P_B,P_C\rangle = E_{BC}|B,C;P_B,P_C\rangle, \]

(3)

where

\[ E_{BC} = \sqrt{M_B^2 + P_B^2} + \sqrt{M_C^2 + P_C^2}. \]

(4)

The quark-pair creation Hamilton \( H_{\text{QPC}} \) induces the decays. With the presence of the non-diagonal elements in the Hamilton, the physical states become mixtures of all bare states:

\[ |\psi'_n\rangle = \sum_i a_{ni} |\psi_i\rangle + \sum_{BC} \int dk c_{n,BC}|B,C;P_B,P_C\rangle, \]

(5)

where \( \int dk \) denotes an integration over all three-momenta of \( B \) and \( C \). The problem now turns into the eigenvalue problem [20, 21]:

\[ \det |(M - M_{0,n})\delta_{mn} - \Pi_{mn}(M)| = 0, \]

(6)

where

\[ \Pi_{mn}(M) = \sum_{BC} \int dk \frac{\langle m|H_{\text{QPC}}|B,C;P_B,P_C\rangle\langle B,C;P_B,P_C|H_{\text{QPC}}|n\rangle}{M - E_{BC} + \imath\epsilon}. \]

(7)

Above the open-charm threshold, \( \Pi_{mn} \) develops an imaginary part, so in general, the equation allows complex value solutions. In this case, the physical mass and the width of a physical state \( |\psi'_n\rangle \) are related to the corresponding eigenvalue \( M_n \) as

\[ M_{\text{phys},n} = \text{Re}M_n, \]

(8)

\[ \Gamma_n = -2\text{ Im}M_n. \]

(9)

For real \( M_n \)'s, of course, \( M_n \) is just the physical mass of the corresponding state, and these \( M_n \)'s should satisfy the condition \( M_n < 2M_D \), where \( 2M_D \) denotes the open-charm threshold.

In vector charmonium sector, the mass of \( nD \) state is closest to the \((n+1)S\) state. So it is expected that these two states should mix each other the most, which is verified in Ref. [22] and taken as granted by many authors [19, 27]. In this work, we follow these researches and only take into account the mixing between the \( nD \) and the \((n+1)S\) states, which means \( H_{nD,(n+1)S} \) and \( \Pi_{(n+1)S,nD} \) are the only non-vanishing non-diagonal elements. With this simplification, Eq. (6) decomposes into several sectors. For 1S sector, we have

\[ (M - M_{0,1}) - \Pi_{1S1S}(M) = 0. \]

(10)
For (n + 1)S − nD sector, we have

\[
\det \begin{vmatrix} M - M_{0,S} - \Pi_{SS}(M) & -\Pi_{SD}(M) \\ -\Pi_{DS}(M) & M - M_{0,D} - \Pi_{DD}(M) \end{vmatrix} = 0, \tag{11}
\]

where the principle quantum numbers in front of S and D are omitted. Since we are focusing on the \( S − D \) mixing in this work, and the continuum components in the physical states are irrelevant, we drop off these components in the physical states as Refs. 22, 27 did. So a mixed physical state is expressed as

\[ |\psi\rangle = a_S |S\rangle + a_D |D\rangle, \tag{12} \]

with coefficients \( a_S \) and \( a_D \) satisfying \(|a_S|^2 + |a_D|^2 = 1\).

### B. Instantaneous Bethe-Salpeter Equation and the \( ^3P_0 \) Model

Now the remaining problem is to solve the naive quark model to obtain the bare masses and the wave functions, and to calculate the open-charm decay amplitudes \( (B, C; P_B, P_C|H_{QPC}|n) \). To this end, we make use of the instantaneous Bethe-Salpeter equation and the \( ^3P_0 \) model. The instantaneous BS equation, also known as the Salpeter equation, is a well developed relativistic two-body bound state equation, and is very suitable to apply on the heavy quarkonium system. The instantaneous BS wave function \( \varphi(q^µ_{P,±}) \) may be decomposed into positive and negative energy parts:

\[ \varphi = \varphi^{++} + \varphi^{--}, \]

where \( q_{P,±} \) is the perpendicular part of relative momentum \( q \). For any momentum \( l^µ \), we have \( l^µ_{P,±} = l - \frac{l^µ_{P,±}}{P^µ} P \) and \( l^µ_{P,±} = \frac{l^µ_{P,±}}{P^µ} P \), where \( P^µ \) is the meson’s momentum. The Salpeter equation then takes the form as coupled equations for \( \varphi^{++} \) and \( \varphi^{--} \):

\[
(M_0 - \omega_{1p} - \omega_{2p})\varphi^{++}(p_{P,±}) = \Lambda_1^+(p_{1P,±})\eta(q_{P,±})\Lambda_2^+(p_{2P,±}),
\]

\[
(M_0 + \omega_{1p} + \omega_{2p})\varphi^{--}(q_{P,±}) = -\Lambda_1^-(p_{1P,±})\eta(q_{P,±})\Lambda_2^-(p_{2P,±}),
\]

where

\[ \Lambda_j^±(p_{jP,±}) = \frac{1}{\sqrt{2\omega_{jp}}} \left[ \frac{p^j}{\sqrt{P^2}} \omega_{jp} \pm (\eta_{jP,±} + (-1)^{j+1}m_j) \right], \tag{14} \]

\[ \eta(q_{P,±}) = \int \frac{dk^3}{(2\pi)^3} V(k_{P,±}^3 - q_{P,±}^3) \varphi(k_{P,±}^3), \tag{15} \]

\[ \omega_{jp} = \sqrt{m_j^2 - p_{jP,±}^2}, \tag{16} \]

with \( j = 1 \) for quark and \( j = 2 \) for anti-quark. \( p_1 \) is the quark momentum and \( p_2 \) is the anti-quark momentum. The instantaneous interaction kernel \( V(k_{P,±}^3 - q_{P,±}^3) \) now becomes the QCD-inspired potential between quark and anti-quark. In our model, we use the Cornell potential with screening effect. The Cornell potential without screening effect can be written as:

\[ V(r) = V_s(r) + V_v(r)\gamma_0 \otimes \gamma_0 = \lambda r - \frac{4}{3} \frac{\alpha_s}{r} \gamma_0 \otimes \gamma_0. \tag{17} \]

The screening effect is introduced by modify the scalar part \( V_s \) to be [15, 29]:

\[ V_s(r) = \lambda \frac{1}{\alpha}(1 - e^{-\alpha r}). \tag{18} \]

Now the potential at sufficient long range, i.e. for \( r \gg \frac{1}{\alpha} \), is suppressed (becomes more and more flat), while for \( r \ll \frac{1}{\alpha} \), it retains the linear form. So 1/\( \alpha \) sets the distance where the screening effect becomes important. Transforming it into momentum space, we obtain:

\[ V(q) = V_s(q) + V_v(q)\gamma_0 \otimes \gamma_0, \]

\[ V_s(q) = -\left(\frac{\lambda}{\alpha} + V_0\right)\delta^3(q) + \frac{\lambda}{\pi^2} \frac{1}{(q^2 + \alpha^2)^2}, \]

\[ V_v(q) = -\frac{2}{3\pi^2} \frac{\alpha_s(q)}{(q^2 + \alpha^2)^2}. \]
where \( q \) is the three-momentum of \( q_{\mu}^{p} \), i.e., \( q_{\mu}^{p} = (0, q) \) in the meson’s rest frame. A constant \( V_0 \) has been added into \( V_s \) to adjust the ground state energy as usual. An infrared cut is introduced in the vector part of the potential to avoid the infrared divergence, which we set it equaling to \( a \) because the results are insensitive to this approximation \[29\]. \( \alpha_s(q) = \frac{12\pi}{3\alpha + 2\alpha}\beta_0 q^2/\Lambda_{QCD}^2 \) is the QCD running coupling constant; the constants \( \lambda, \alpha, a, V_0 \) and \( \Lambda_{QCD} \) are the parameters characterizing the potential.

The method of solving the full Salpeter equation is given in Ref. [30]. After solving the Salpeter equation, we obtain the bare mass spectrum and the wave functions. Then we can use them to calculate the open-charm decay amplitudes. The \( A \to BC \) open-charm decay is a typical OZI-allowed strong decay process, which in essence is a non-perturbative QCD problem. Due to our pure knowledge of QCD in its non-perturbative region, such processes are usually evaluated using models. Among others, the \( ^3P_0 \) model is a widely accepted one \[31, 32\]. This model assumes the decay takes place via creating a quark anti-quark pair from the vacuum baring the quantum number \( ^3P_0 \), so it is also known as the quark pair creation model. The \( ^3P_0 \) model is a non-relativistic model, and majority of works using this model stick on its non-relativistic form. On the other hand, the Salpeter wave function is a relativistic wave function containing the Dirac spinor of quark/anti-quark. So incorporating the Salpeter wave function into \( ^3P_0 \) model requires the relativistic extension of the original \( ^3P_0 \) model, which has been done in Ref. [33]. In this work we employ the formula derived in that reference and write the amplitude for the \( A \to BC \) open-charm decay as

\[
\mathcal{M} = g \int \frac{d^3q_A}{(2\pi)^3} \text{Tr} \left\{ \frac{P_A}{M_A} \phi_{A}^{++}(q_A^+) \frac{P_A}{M_A} \phi_{C}^{++}(q_A^+) \phi_{B}^{++}(q_A^+) \right\},
\]

(19)

where quantities referred to \( A, B \) and \( C \) are labeled with the subscript/superscript \( \alpha, \beta \) and \( \gamma \) respectively. \( g = 2m_\gamma \beta \) with \( m_\gamma \) being the mass of the created quark \( q \) or anti-quark \( \bar{q} \) and \( \gamma \) being a universal constant characterizing the strength of the decay. Negative-energy contributions have been neglected due to their smallness comparing to positive-energy contributions.

For \( J^P = 1^- \) meson, the wave function \( \phi^{++} \) can be decomposed into two parts: the \( S \)-wave part and the \( D \)-wave part, i.e.,

\[
\phi^{++}(q_{\perp}) = \sqrt{\frac{m_1 m_2}{\omega_{1\perp} \omega_{2\perp}}} \sum_{s_1, s_2, m, \tilde{M}} u_{s_1}(p_1) \bar{u}_{s_2}(p_2) \chi^{1, \tilde{M}}_{s_1, s_2} \left( \sqrt{2M_0} \phi_{q_{\perp}} \phi_{\perp} \delta_{\tilde{M}, \lambda} + \sqrt{2M_0} \phi_{q_{\perp}} \phi_{\perp} \delta_{\tilde{M}, \lambda} \right),
\]

(20)

where \( R_n \) is the radial wave function with principle number \( n \) and orbital angular momentum \( l \). \( Y_{lm} \) is the spherical harmonic function. \( m \) is the magnetic quantum number of orbital angular momentum. \( S, \tilde{M} \) represent the total spin of the quark-anti-quark pair. The \( \lambda \) appearing in Eq. \( \phi_{\perp} \) is the magnetic quantum number of the meson. \( s_1(2) \) is the spin of the quark (anti-quark) in the meson. \( S_{m, \tilde{M}, J, M} = \langle M, m | J, M \rangle \) and \( \chi_{s_1, s_2} = \langle s_1, s_2 | S, \tilde{M} \rangle \) are Clebsch-Gordan (C-G) coefficients. \( u_{s_1}(p_1) \) (\( \bar{u}_{s_2}(p_2) \)) is the Dirac spinor of the quark (anti-quark) with spin \( s \) and momentum \( p \). This expression clearly shows the \( L - S \) coupling inside the meson, and it is the same as the corresponding non-relativistic wave function except that: a) the non-relativistic spinor is replaced with the Dirac spinor; b) the radial wave function is the solution of the Salpeter equation rather than the non-relativistic Schrödinger equation.

Inserting the \( S \)-wave part or the \( D \)-wave part of the wave function into Eq. \( \phi_{\perp} \), one arrives at the familiar form of the decay amplitude of the \( ^3P_0 \) model:

\[
\mathcal{M} = g \int \frac{d^3q_A}{(2\pi)^3} \text{Tr} \left\{ \frac{P_A}{M_A} \phi_{A}^{++}(q_A^+) \phi_{C}^{++}(q_A^+) \phi_{B}^{++}(q_A^+) \right\},
\]

(21)

where \( \psi_{nlm} = \sqrt{2M_0} \phi_{q_{\perp}} \phi_{\perp} \) and \( \omega_{q_{\perp}} = \sqrt{\frac{q_{\perp}^2 + m_\gamma^2}{2}} \). The decay width is then given by

\[
\Gamma = \frac{|B_C|}{8M^2 A_\pi} \frac{1}{3} \sum_{pol} |\mathcal{M}|^2 = \frac{2\pi|B_C|E_B E_C}{M_A} \frac{1}{3} \sum_{pol} |f_{A \to BC}|^2,
\]

(22)

where \( f_{A \to BC} \) is introduced to make connection with the convention which is widely used in literature (such as Ref. [34]). \( \sum_{pol} \) means summation over polarizations. \( f_{A \to BC} \) is related to the matrix element as

\[
\langle B; C | P_B, P_C | H_{QPC} | A \rangle = \delta^3(P_A - P_B - P_C) f_{A \to BC}.
\]

(23)
TABLE I. Our theoretical results of bare masses, physical masses and mass shifts of vector charmonia. Experimental data and some theoretical predictions from literature are given for comparison. All quantities are given in the unit of MeV.

| State | $M_0$ of ours (MeV) | $M_{phys}$ of ours (MeV) | $M$ of Ref. [15] (MeV) | $M_{ex}$ [2] (MeV) | $\Delta M$ of ours | $\Delta M$ of Ref. [20] |
|-------|---------------------|--------------------------|------------------------|---------------------|------------------|------------------|
| $\psi(1S)(\psi'(1S))$ | 3170 | 3103 | 3097 | 3096.900 ± 0.006 | -67 | -159 |
| $\psi(2S)(\psi'(2S))$ | 3764 | 3666 | 3673 | 3686.097 ± 0.025 | -98 | -227 |
| $\psi(1D)(\psi'(1D))$ | 3870 | 3768 | 3757 | 3773.13 ± 0.35 | -102 | -231 |
| $\psi(3S)(\psi'(3S))$ | 4092 | 4018 | 4022 | 4039 ± 1 | -74 |
| $\psi(2D)(\psi'(2D))$ | 4152 | 4089 | 4089 | 4191 ± 5 | -63 |
| $\psi(4S)(\psi'(4S))$ | 4311 | 4285 | 4273 | 4230 ± 8 ($Y(4260)$) | -26 |
| $\psi(3D)(\psi'(3D))$ | 4390 | 4319 | 4317 | 4368 ± 13 ($Y(4360)$) | -31 |

Given these decay amplitudes, whenever $\Pi_{mn}$ develops imaginary part, $\Pi_{mn}$ can be calculated as

$$\begin{align*}
\text{Im} \Pi_{mn}(E) &= -\sum_{BC} \left\{ \frac{1}{3} \sum_{pol} f_{m-BC}^* f_{n-BC} \times \frac{\pi |P_B| E_B E_{\gamma}^*}{E} \theta(E - E_{th}^{BC}) \right\}, \\
\text{Re} \Pi_{mn}(E) &= -\frac{1}{\pi} \int dE' \frac{\text{Im} \Pi_{mn}(E')}{E - E'},
\end{align*}$$

where $E_{th}^{BC}$ represents the threshold energy of $BC$ channel. $\theta(x)$ is the step function. The expressions of $\text{Im} \Pi_{mn}$ for each decay channels are given in Appendix A.

III. RESULTS AND DISCUSSIONS

Previous section gives the formula for calculating the charmonium spectrum and the coupled-channel effects. To obtain the bare masses and the bare states of vector charmonia, we use the following parameters:

$$m_c = 1.71 \text{ GeV}, \quad a = e = 2.7183, \quad \alpha = 0.12 \text{ GeV},$$

$$\lambda = 0.202 \text{ GeV}^2, \quad \Lambda_{QCD} = 0.40 \text{ GeV}, \quad V_0 = -0.204 \text{ GeV}.$$  

The value of $\alpha$ in this work is comparable to the corresponding parameter in Ref. [12]. While all the other parameters lie in reasonable ranges, the parameter $\Lambda_{QCD}$ is a bit larger than its typical value. At two-loop order in the $\overline{MS}$ scheme, $\Lambda_{QCD}^{(4)} = 325$ MeV corresponds to $\alpha_s(M_c) = 0.118$, where the superscript of $\Lambda_{QCD}^{(4)}$ represents $n_f = 4$. On the other hand, $\Lambda_{QCD}^{(4)} = 413$ MeV corresponds to $\alpha_s(M_c) = 0.123$ [32]. Since in our model $\Lambda_{QCD}$ is in essence a model parameter, we do not require it to strictly binding to $\Lambda_{QCD}^{(4)}$. So our value of $\Lambda_{QCD}$ is still acceptable.

The results of bare masses are listed in Table I. With the obtained wave functions, the coupled-channel effects are calculated. The decay strength parameter of $^3P_0$ model is fitted to be $\gamma = 0.43$, and we also set the strength parameter of creating $s\bar{s}$ to be $\gamma_s = \frac{\gamma}{\sqrt{2}}$ as usual [30, 37]. For consistency, the wave functions of $D$ and $D_s$ are also calculated with the Salpeter equation. With constituent quark mass $m_u = m_d = 0.305$ GeV and $m_s = 0.500$ GeV, we obtain $M_D = 1.865$ GeV, $M_{D^*} = 2.008$ GeV, $M_{D_s} = 1.968$ GeV and $M_{D_s^*} = 2.112$ GeV. Now the physical states are mixtures of the bare $\psi((n+1)S)$ and $\psi(nD)$ states. For $\psi(1S)$, we assume that it does not mix with other bare states. We denote the physical state as $\psi'(nS)/\psi'(nD)$, where $nS/nD$ indicates the dominant component in this physical state. The physical masses are shifted by the coupled-channel effects and can be compared with the experimental data. The mass shifts are represented by $\Delta M$. We also listed some results from literature for comparison. These information are gathered in Table I. One can see that from the $\psi'(1S)$ through $\psi'(3S)$, the masses are comparable with the experimental data, which justifies our model calculations. ($\psi'(2D)$ is one exception, which is also a problem of another screened potential model calculation [12].) We find that the masses of the two physical states $\psi'(4S)$ and $\psi'(3D)$ are roughly comparable with the masses of $Y(4260)$ and $Y(4360)$ respectively.

In Table II we give the coefficients $a_S$ and $a_D$ as in Eq. (12) for different states. We also extracted the mixing angles defined as in

$$|\psi(S)| = \cos \theta |S| + \sin \theta |D|, \quad |\psi(D)| = -\sin \theta |S| + \cos \theta |D|.$$

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TABLE II. The coefficients $a_S$ and $a_D$ characterizing the $S - D$ mixing for different states and the extracted mixing angles.

| $\psi(2S)$ | $\psi(1D)$ | $\theta_{2S-1D}$ |
|------------|------------|------------------|
| $\psi(3S)$ | $\psi(2D)$ | $\theta_{1S-2S}$ |
| $\psi(3S)$ | $\psi(2D)$ | $\theta_{1S-3S}$ |

In extracting the mixing angles, we neglect the phase of the complex number $a_{S/D}$ and set the sign equal to the sign of its real part. The magnitude of mixing angles for $2S - 1D$ states in our model are smaller than $10^\circ$, the result from Ref. [20]. The difference may due to different model settings in these two works. The magnitude of mixing angles for $3S - 2D$ states are larger than those for $2S - 1D$ states as expected. For $4S - 3D$ states, the mixing angles are around $-6^\circ \sim -5^\circ$.

To provide more details, we plot the real and imaginary parts of $\Pi_{mn}$ in Figs. 1-3. The real parts of diagonal elements roughly reflect the mass shifts at given energy scale. The imaginary part of diagonal elements roughly reflect half-widths at given energy scale. And non-diagonal elements may reflect how much the $S - D$ mixing at given energy scale. One can find that the non-diagonal elements are in general smaller than the diagonal ones. Actually, the diagonal elements $Im\Pi_{SS}^{BC}$ and $Im\Pi_{DD}^{BC}$ for each channel $BC$ always $\leq 0$, so the contributions to $Im\Pi_{SS}$ and $Im\Pi_{DD}$ add up. But the non-diagonal elements $Im\Pi_{SD}^{BC}$ for different channels may have different signs, so $Im\Pi_{SD}$ is generally smaller than $Im\Pi_{SS}$ and $Im\Pi_{DD}$, and oscillates around zero as $E$ varies. This in turn results in $Re\Pi_{SD} < Re\Pi_{SS}$ and $Re\Pi_{DD}$ in general. The diagonal elements $Im\Pi_{SS}$ and $Im\Pi_{DD}$ also oscillate but without changing signs. The oscillation behavior of the diagonal elements of $\Pi$ is a reflection of the node structures of initial states, so this behavior becomes more frequent (against $E$) for higher excitation states.

![FIG. 1. Real and imaginary parts of $\Pi_{mn}$ in 2S - 1D sector and Real part of $\Pi_{nn}$ in 1S sector.](image)

In general, oscillations make $\Pi$ and relevant observables sensitive to the energy scale $E$. In view of this, we calculate the decay widths of physical states at their real mass scales, i.e., the experimental masses. The decay widths of physical states are given in Table III. $\Gamma_{tot}^{th}$ denotes the summation of those in $D_s^{(*)}\bar{D}_s^{(*)}, D_s^{(*)}\bar{D}_s^{(*)}$ channels which are allowed by energy conservation. The decay widths of $Y(4260)$ and $Y(4360)$ are given under the assumption that they are the $\psi(4S)$ and $\psi(3D)$ states respectively. The width of $\psi(3770), \psi(4040)$ and $\psi(4160)$ are dominated by the two-body
open-charm decays. So for these states, $\Gamma_{\text{theo.}}$ should be comparable to the observed width of the corresponding particle. From Table III one can see that our results are consistent with the experimental data. For $\psi'(4S)$ and $\psi'(3D)$, it is interesting to find that their $\Gamma_{\text{theo.}}$ are notably smaller than those of $\psi'(3S)$ and $\psi'(2D)$. Especially $\Gamma_{D\bar{D}}$ of $\psi'(4S)$ is less than 1 MeV. If we use the experimental data of total widths, we can find the branching ratios for these channels:

$$\text{Br}(Y(4260) \to D\bar{D}) = 0.758\% \quad \text{Br}(Y(4260) \to D^*\bar{D} + \text{c.c.}) = 12.2\% \quad \text{Br}(Y(4260) \to D^*\bar{D}^*) = 35.6\% \quad (28)$$

$$\text{Br}(Y(4360) \to D\bar{D}) = 8.10\% \quad \text{Br}(Y(4360) \to D^*\bar{D} + \text{c.c.}) = 1.28\% \quad \text{Br}(Y(4360) \to D^*\bar{D}^*) = 5.76\% \quad (29)$$

The small branching ratios of $Y(4260)$ and $Y(4360)$ in these channels may be the reason why we haven’t seen them in open-charm channels.

The reason of the smallness of decay widths of $\psi'(4S)$ and $\psi'(3D)$ are the oscillation behavior of decay amplitudes and the mixing between $S$ and $D$ wave components. The node structures of initial states make the decay amplitude of

FIG. 2. Real and imaginary parts of $\Pi_{mn}$ in $3S - 2D$ sector.

FIG. 3. Real and imaginary parts of $\Pi_{mn}$ in $4S - 3D$ sector.
TABLE III. Decay widths of $D^*(s)\bar{D}^*(s), D_s^*(s)\bar{D}_s^*(s)$ channels for states above the open-charm threshold in the unit of MeV. $\Gamma^\text{theo.}$ denotes the summation of those in $D^*(s)\bar{D}^*(s), D_s^*(s)\bar{D}_s^*(s)$ channels which are allowed by energy conservation.

| $\psi(nl)$ | $\Gamma_{DD}$ | $\Gamma_{D\bar{D}^*+c.c.}$ | $\Gamma_{D^*\bar{D}^+}$ | $\Gamma_{D_s\bar{D}_s}$ | $\Gamma_{D_s\bar{D}_s^*+c.c.}$ | $\Gamma_{D_s^*\bar{D}_s^*}$ | $\Gamma^\text{theo.}$ | $\Gamma^\text{ex.}$ |
|-----------|--------------|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|
| $\psi(3770)$ | 18.4 |  |  |  |  |  | 18.4 | 27.2 ± 1.0 |
| $\psi(4040)$ | 2.75 | 21.4 | 56.6 | 6.15 |  |  | 86.9 | 80 ± 10 |
| $\psi(4160)$ | 11.7 | 7.98 | 61.4 | 0.0349 | 8.75 |  | 89.9 | 70 ± 10 |
| $\psi(4250)$ | 0.417 | 6.73 | 19.6 | 0.773 | 0.126 | 0.261 | 27.9 | 55 ± 19 |
| $\psi(4360)$ | 7.78 | 1.23 | 5.53 | 0.0990 | 1.09 | 1.13 | 16.9 | 96 ± 7 |

each channel oscillate, which can be reflected by $\text{Im}\Pi^{BC}_{SS\bar{S}\bar{S}D\bar{D}}$ in each channel. Fig. 4 shows that the decay amplitudes (actually, the square of decay amplitudes) approach zero at particular energies. $\psi(3D) \rightarrow D^*\bar{D}^*$ dose not reach zero but is close to zero at some energy scales. From Fig. 4 we can see that in the range $E \simeq [4.2, 4.4]$ GeV, where $Y(4260)$ and $Y(4360)$ lie in, the decay amplitude of each channel, except $\psi(4S) \rightarrow D^*\bar{D}^*$, has one trough, and this exception has one crest. So, except $\psi(4S) \rightarrow D^*\bar{D}^*$ process, $\text{Im}\Pi^{BC}_{SS\bar{S}\bar{S}D\bar{D}}$ of all channels at the mass scales of $Y(4260)$ and $Y(4360)$ have good chance to be small. One may wonder that the locations of the zeros of these decay amplitudes may vary as model parameters vary. But as long as we require a reasonable spectrum to be able to accommodate $Y(4260)$ and $Y(4360)$, the model parameters can not vary too far, and the locations of zeros are not sensitive to small variations in parameters.

![Fig. 4. Im$\Pi^{BC}_{SS\bar{S}\bar{S}D\bar{D}}$ and Im$\Pi^{BC}_{D\bar{D}}$ of $D\bar{D}$, $D\bar{D}^*$ + c.c. and $D^*\bar{D}^*$ channels.](image)

The $S - D$ mixing then affect the final results in the following ways. For the case when the $S$ wave amplitude and the $D$ wave amplitude are not comparable to each other, for example in $D^*\bar{D}^*$ channel the $S$ wave amplitude is larger than the $D$ wave amplitude, the decay width of the mixed state dominated by $S$ wave is depressed by mixing with the smaller $D$ wave amplitude and vice versa. For the case when the $S$ wave amplitude is comparable to the $D$ wave amplitude, the decay widths of mixed states either enhanced by the constructive interference or depressed by the destructive interference, $Y(4260) \rightarrow D\bar{D}$ is an example of the constructive interference case and $Y(4260) \rightarrow D\bar{D}^* + c.c.$ is an example of the destructive interference case. In Ref. [13], the authors noticed that the $S$ wave amplitude and the $D$ wave amplitude of $Y(4260) \rightarrow D\bar{D}$ decay are of opposite signs, but since the mixing angle is negative, the two amplitudes actually interfere constructively.

Finally, we indicate that from the di-electron decay widths of vector charmonia, one may expect larger mixing angles than the coupled-channel-induced mixing angles presented in this work. To show this we calculate the $\Gamma_{ee}$ for each physical states, and the results are shown in Table [15]. In these calculations, the QCD correction factor $1 - \frac{\delta}{\alpha_s}$ with $\alpha_s = 0.3$ is included. On the other hand, by fitting the experimental data of $\Gamma_{ee}$ under the assumption of Eqs. [20] and [27], we obtain $\theta_{2S-1D} = -11.5^\circ$ and $\theta_{3S-2D} = -30.7^\circ$. This may imply that there are some unrevealed
TABLE IV. Di-electron decay widths, i.e., $\Gamma_{ee}$ in the unit of keV

| State | $\Gamma_{\text{theo}}$ | $\Gamma_{\text{ex}}$ |
|-------|------------------------|----------------------|
| $J/\psi$ | 5.98 | 5.55 ± 0.14 ± 0.02 |
| $\psi(2S)$ | 2.47 | 2.33 ± 0.04 |
| $\psi(3S)$ | 0.0861 | 0.202 ± 0.018 |
| $\psi(4S)$ | 1.30 | 0.86 ± 0.07 |
| $\psi(4160)$ | 0.161 | 0.48 ± 0.22 |
| $Y(4260)$ | 0.969 |
| $Y(4360)$ | 0.0447 |

physical sources inducing $S - D$ mixing. (The contributions in $S - D$ mixing from tensor force in potential model are smaller than coupled-channel effects \cite{21, 27}.) So we expect a larger mixing in the $4S - 3D$ sector too. With large mixing angle, the mass splitting of $\psi'(4S)$ and $\psi'(3D)$ becomes larger. Actually, with the mixing angle being as large as $\theta_{4S-3D} = -38.0^\circ$, one may obtain $M_{\psi'(4S)} = 4.233$ GeV and $M_{\psi'(3D)} = 4.371$ GeV perfectly matching the masses of $Y(4260)$ and $Y(4360)$. The $S - D$ mixing effects would be more significant as the mixing angle becomes larger. So we plot the decay widths of $Y(4260)$ and $Y(4360)$ under the assumption of Eqs. (26) and (27) against $|\theta|$ in Fig. 5. We find that the decay widths of $Y(4360) \rightarrow D\bar{D}$ and $Y(4260) \rightarrow D\bar{D}^{*} + c.c.$ decrease as $|\theta|$ increases as expected, because the $S$ wave and $D$ wave amplitudes interfere destructively in these two channels. The summation of these channels $\Gamma_{\text{tot}}$, for both the resonances decrease as $|\theta|$ becomes larger, which means they would be harder to be observed in these open-charm decay channels.

![FIG. 5. Decay widths of Y(4260) and Y(4360) as functions of the mixing angle. The widths of D\(\ast\)\(\ast\)D\(\ast\)\(\ast\) and D\(\ast\)\(\ast\)D\(\ast\)\(\ast\) channels.](image)

IV. CONCLUSION

In this work, we studied the coupled-channel effects in $J^{PC} = 1^{--}$ charmonium states up to $\psi(3D)$. The mass shifts are found to be from tens MeV up to 100 MeV. We focused on the mixing between $(n + 1)S$ and $nD$ states induced by the coupled-channel effects. The mixing angles are extracted. We find that the mixing in $3S - 2D$ sector is larger than those in $2S - 1D$ sector and in $4S - 3D$ sector. The two-body open-charm decay widths and the di-electron decay widths are calculated. Most of the widths and the masses are consistent with the corresponding experimental data for states from $J/\psi$ to $\psi(4160)$. The calculations are performed using the instantaneous BS equation with a
screened Cornell potential and the $^3P_0$ model which has been reexpressed in the form suitable for the Salpeter wave functions.

Based on these calculations, we discussed the possibility of assigning the resonant state $Y(4260)$ as the mixture of $4S - 3D$ with lower mass and $Y(4360)$ as the mixture of $4S - 3D$ with higher mass. The predicted masses of these two states are 4285 MeV and 4319 MeV respectively. The branching ratios of $Y(4260) \to D\bar{D}$ and $Y(4360) \to D\bar{D}$ are found to be 0.758% and 8.10% respectively. Such small branching ratios may be the reason of non-observation of these two states in $D\bar{D}$ channels. Combining the predictions on mass and on the open-charm decays and comparing them to the experimental observations, although not perfect, we find it is still possible for $Y(4260)$ and $Y(4360)$ to have conventional charmonium interpretations. Of course, the present study does not exclude other possible exotic interpretations. So, we expect further studies on these resonances.

**Appendix A: Expressions of $\text{Im}\Pi_{mn}$ for Each Channel**

The decay channels we considering here are $D^{(*)}\bar{D}^{(*)}$, $D^{(*)}\bar{D}'^{(*)}$, so we need to give the decay amplitudes for $^3S_1 \to ^1S_0^1S_0$, $^3D_1 \to ^1S_0^1S_0$, $^3S_1 \to ^3S_1^1S_0$, $^3D_1 \to ^3S_1^1S_0$, $^3S_1 \to ^3S_1^1S_1$ and $^3D_1 \to ^3S_1^1S_1$ processes. Now we denote the imaginary part of $\Pi_{mn}$ for each channel as $\text{Im}\Pi_{mn}$, where $\tau$ represents a particular final state, i.e., $\tau = ^1S_0^1S_0$, $^3S_1^1S_0$ or $^3S_1^1S_1$. Recalling that we only consider the mixing between $m$ and $(n+1)$S states, so we can set $m, n$ take the value $S$ or $D$ without any confusion. For $\tau = ^1S_0^1S_0$ or $^3S_1^1S_0$, $\text{Im}\Pi_{mn}$ can be written as

$$\text{Im}\Pi_{mn}(E) = -h_m(|P_B|)h_n(|P_B|) \times \frac{\pi |P_B| E_B E_C}{E} \theta(E - E_{B/C}^b),$$

where

$$h_S^{^1S_0^1S_0} = \frac{|P_B| |P_B|}{\sqrt{3\pi}} \int \int \frac{d^3q}{(2\pi)^3} R_A(|q|) R_B(|q - q^B|) R_C(|q + \alpha_2 C|) \left( \frac{q \cdot P_B}{|P_B|^2} - 1 \right) \frac{m_q}{\omega_{qP}},$$

$$h_S^{^3S_1^1S_0} = \frac{\sqrt{2} |P_B| |P_B|}{\sqrt{3\pi}} \int \int \frac{d^3q}{(2\pi)^3} R_A(|q|) R_B(|q - q^B|) R_C(|q + \alpha_2 C|) \left( \frac{q \cdot P_B}{|P_B|^2} - 1 \right) \frac{m_q}{\omega_{qP}},$$

$$h_D^{^1S_0^1S_0} = -\frac{|P_B| |P_B|}{\sqrt{3\pi}} \int \int \frac{d^3q}{(2\pi)^3} R_A(|q|) R_B(|q - q^B|) R_C(|q + \alpha_2 C|) \left( 1 + 2 \frac{q \cdot P_B}{|P_B|^2} - 3 \frac{(q \cdot P_B)^2}{|q|^2 |P_B|^2} \right) \frac{m_q}{\omega_{qP}},$$

$$h_D^{^3S_1^1S_0} = \frac{2 |P_B| |P_B|}{\sqrt{3\pi}} \int \int \frac{d^3q}{(2\pi)^3} R_A(|q|) R_B(|q - q^B|) R_C(|q + \alpha_2 C|) \left( 1 + 2 \frac{q \cdot P_B}{|P_B|^2} - 3 \frac{(q \cdot P_B)^2}{|q|^2 |P_B|^2} \right) \frac{m_q}{\omega_{qP}}.$$
and
\[ U_{D1} = \int \frac{d^3q}{(2\pi)^3} R_A(|q|) R_B(|q - \alpha_1^B P_B|) R_C(|q + \alpha_2^C P_C|) \left( 1 - \frac{q \cdot P_B}{|P_B|^2} \right)^2 - 3 \left( \frac{q \cdot P_B}{|q|^2 |P_B|^2} \right)^3 \frac{m_q}{\omega_{qp}} \] (A10)
\[ U_{D2} = \int \frac{d^3q}{(2\pi)^3} R_A(|q|) R_B(|q - \alpha_1^B P_B|) R_C(|q + \alpha_2^C P_C|) \left( 1 - \frac{q \cdot P_B}{|P_B|^2} \right)^2 - 3 \left( \frac{q \cdot P_B}{|q|^2 |P_B|^2} \right)^3 \frac{m_q}{\omega_{qp}} \] (A11)
\[ U_{D3} = \int \frac{d^3q}{(2\pi)^3} R_A(|q|) R_B(|q - \alpha_1^B P_B|) R_C(|q + \alpha_2^C P_C|) \left( 1 - \frac{q \cdot P_B}{|P_B|^2} \right)^2 - 3 \left( \frac{q \cdot P_B}{|q|^2 |P_B|^2} \right)^3 \frac{m_q}{\omega_{qp}} \] (A12)

\[ U_{S1} = \int \frac{d^3q}{(2\pi)^3} R_A(|q|) R_B(|q - \alpha_1^B P_B|) R_C(|q + \alpha_2^C P_C|) \left( \frac{q \cdot P_B}{|P_B|^2} \right)^4 \frac{m_q}{\omega_{qp}} \] (A13)
\[ U_{S2} = -U_{S1}, \]
\[ U_{S3} = 0. \]

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[1] B. Aubert, et al., (BaBar Collaboration), Phys. Rev. Lett. 95, (2005) 142001.
[2] M. Tanabashi, et al., (Particle Data Group), Phys. Rev. D98, (2018) 030001.
[3] S.-L. Zhu, Phys. Rev. B625, (2005) 212.
[4] E. Kou and O. Pene, Phys. Lett. B631, (2005) 164.
[5] D. Ebert, R. N. Faustov, V. O. Galkin, Eur. Phys. J. C58, (2008) 399.
[6] F. Close, C. Downum, C. E. Thomas, Phys. Rev. D81, (2010) 074033.
[7] M. Cleven, Q. Wang, F.-K. Guo, C. Hanhart, U.-G. Meissner and Q. Zhao, Phys. Rev. D90, (2014) 074039.
[8] B. Aubert, et al., (BaBar Collaboration), Phys. Rev. Lett. 98, (2007) 212001.
[9] Yu. S. Kalashnikova and A. V. Nefediev, Phys. Rev. D77, (2008) 054025.
[10] Y. D. Chen and C. F. Qiao, Phys. Rev. D85, (2012) 034034.
[11] X. Li and M. B. Voloshin, Mod. Phys. Lett. A29, (2014) 1450060.
[12] H.-X. Chen, W. Chen, X. Liu, S.-L. Zhu, Phys. Rept. 639, (2016) 1.
[13] M. Shah, A. Parmar, P. C. Vinodkumar, Phys. Rev. D86, (2012) 034015.
[14] A. Zhang, Phys. Lett. B647, (2007) 140.
[15] B.-Q. Li and K.-T. Chao, Phys. Rev. D79, (2009) 094004.
[16] G.-J. Ding, J.-J. Zhu, M.-L. Yan, Phys. Rev. D77, (2008) 014033.
[17] J. Segovia, A. M. Yasser, D. R. Entem and F. Fernandez, Phys. Rev. D78, (2008) 114033.
[18] F. J. Llanes-Estrada, Phys. Rev. D72, (2005) 031503.
[19] A. M. Badalian, B. L. G. Bakker and I. V. Danilkin, Phys. Atom. Nucl. 72, (2009) 638.
[20] E. Eichten, K. Gottfried, T. Kinoshita, K. D. Lane and T.-M. Yan, Phys. Rev. D17, (1978) 3090.
[21] E. Eichten, K. Gottfried, T. Kinoshita, K. D. Lane and T.-M. Yan, Phys. Rev. D21, (1980) 203.
[22] K. Heikkila, N. A. Tornqvist and S. Ono, Phys. Rev. D29, (1984) 110.
[23] S. Ono and N. A. Tornqvist, Z. Phys. C23, (1984) 59.
[24] E. J. Eichten, K. Lane, C. Quigg, Phys. Rev. D73, (2006) 014014.
[25] Y. S. Kalashnikova, Phys. Rev. D72, (2005) 034010.
[26] B.-Q. Li, C. Meng and K.-T. Chao, Phys. Rev. D80, (2009) 014012.
[27] Y. Lu, M. N. Anwar and B.-S. Zou, Phys. Rev. D94, (2016) 034021.
[28] E. E. Salpeter, Phys. Rev. 87, (1952) 328.
[29] C.-H. Chang and G.-L. Wang, Sci. China Phys. Mech. Astron. 53, (2010) 2005.
[30] G.-L. Wang, Phys. Lett. B633, (2006) 492.
[31] L. Micu, Nucl. Phys. B10, (1969) 521.
[32] A. Le Yaouanc, L. Oliver, O. Pene and J. Raynal, Phys. Rev. D8, (1973) 2223.
[33] H.-F. Fu, X.-J. Chen, G.-L. Wang and T.-H. Wang, Int. J. Mod. Phys. A27, (2012) 1250027.
[34] E. S. Ackleh, T. Barnes and E. S. Swanson, Phys. Rev. D54, (1996) 6811.
[35] A. J. Buras, hep-ph/9906471.
[36] A. Le Yaouanc, L. Oliver, O. Pene, and J. C. Raynal, Phys. Lett. B72, (1977) 57.
[37] Z.-F. Sun and X. Liu, Phys. Rev. D80, (2009) 074037.