The stacked spectral energy distribution (SED) 24 μm Lyman break galaxies (MIPS-LBGs) detected by the Multiband Imaging Photometer for Spitzer (MIPS) is fitted by means of the spectrophotometric model GRASIL with an “educated” fitting approach which benefits from the results of chemical evolution models. The star formation rate–age–metallicity degeneracies of SED modeling are broken by using star formation history (SFH) and chemical enrichment history suggested by chemical models. The dust mass, dust abundance, and chemical pattern of elements locked in the dust component are also directly provided by chemical models. Using our new “fitting” approach, we derive the total mass \( M_{\text{tot}} \), stellar mass \( M_\star \), gas mass \( M_g \), dust mass \( M_d \), age, and star formation rate (SFR) of the stacked MIPS-LBG in a self-consistent way. Our estimate of \( M_\star \approx 8 \times 10^{10} \) of the stacked MIPS-LBG agrees with other works based on UV–optical SED fitting. We suggest that the MIPS-LBGs at \( z \sim 3 \) are young (0.3–0.6 Gyr), massive \( (M_{\text{tot}} \sim 10^{11} M_\odot) \), dusty \( (M_d \sim 10^{7} M_\odot) \), and metal-rich \( (Z \sim Z_\odot) \) progenitors of elliptical galaxies undergoing a strong burst of star formation \( (SFR \sim 200 M_\odot \text{yr}^{-1}) \). Our estimate of \( M_g \approx 7 \times 10^{7} M_\odot \) of the stacked MIPS-LBG is about a factor of eight lower than the estimated value based on single temperature graybody fitting, suggesting that self-consistent SED models are needed to estimate dust mass. By comparing with Milky Way molecular cloud and dust properties, we suggest that denser and dustier environments and flatter dust size distribution are likely in high-redshift massive star-forming galaxies. These dust properties, as well as the different types of SFHs, can cause different SED shapes between high-redshift star-forming ellipticals and local starburst templates. This discrepancy of SED shapes could in turn explain the non-detection at submillimeter wavelengths of IR luminous \((L_{\text{IR}} \gtrsim 10^{12} L_\odot)\) MIPS-LBGs.

**Key words:** dust, extinction – galaxies: high-redshift – galaxies: starburst – ISM: clouds

**Online-only material:** color figures

1. INTRODUCTION

The past decade witnessed a tremendous increase in the knowledge of high-redshift star-forming galaxies. Large samples, obtained with different selection techniques (see Shapley 2011 for a review), yielded statistical information about masses (e.g., Erb et al. 2006b; Daddi et al. 2007), star formation rates (SFRs), and dust attenuation (e.g., Hopkins & Beacom 2006; Reddy et al. 2008, 2010), as well as metallicities (e.g., Erb et al. 2006a) for \( z \approx 2 \) galaxies.

In a small subset of well-studied objects, a more accurate determination of their chemical abundance pattern, dust depletion, and kinematics, was possible due to lensing magnification (e.g., Hainline et al. 2009) and deep integral field observations (e.g., Law et al. 2012; Gnerucci et al. 2011; Genzel et al. 2011).

Taken together, these observables shaped our view of \( z = 2–3 \) redshift star-forming galaxies as relatively evolved systems, as witnessed by their metallicities (e.g., Erb et al. 2006a; Calura et al. 2009), whose SFRs are high due to the availability of large gas reservoirs as opposed to mergers (e.g., Genzel et al. 2010; Rodighiero et al. 2011; Kaviraj et al. 2012).

Among the various methods, the Lyman break technique (Steidel & Hamilton 1993) was particularly successful in yielding insight into high-redshift star-forming galaxies. Lyman break galaxies (LBGs) are observed at different redshifts: \( z \sim 1 \) (Burgarella et al. 2007), \( z \sim 1.4–2.5 \) “BX” and “BM” (Steidel et al. 2004), \( z \sim 3 \) (Steidel et al. 2003), \( z \sim 4, 5, 6 \) (Vanzella et al. 2009) by means of the photometrical technique for rest-frame 912 Å Lyman-continuum discontinuity in a dese...
et al. 2013), in most cases empirical attenuation laws are usually adopted to deconvolve the effect of the dust from the SED. This means that molecular cloud (MC) condition and dust properties, such as dust mass, dust abundance, and dust size distribution, are not directly studied. Only a few works attempt to fit the SEDs of LBGs from the UV to the IR (e.g., Magdis et al. 2010b, 2011). However, separate models are adopted in these works to fit the UV–optical and the IR parts of the SED. Furthermore, the dust temperature and mass are usually estimated by a graybody fitting, and the IR luminosity is usually estimated with template SEDs from an observational library. While these assumptions might be justified as necessary to explore uncharted territory and certainly provide a good zero-order estimate of the relevant physical quantities (masses and SFRs), we believe that more sophisticated models should be adopted to understand the physical processes acting in high-redshift star-forming galaxies in greater detail.

To this purpose, in previous works (Matteucci & Pipino 2002; Pipino et al. 2011), we focused on the chemical abundance pattern of a well-studied single LBG cB 58 (Pettini et al. 2000) and LBGs from two surveys: AMAZE (Maiolino et al. 2008) and LSD (Mannucci et al. 2009). We self-consistently derived the SFH and the dust properties for these high-redshift LBGs. Here we address in detail the dust properties of $z \sim 3$ LBGs. In particular, we make use of the stacked SED of 24 $\mu$m LBGs (MIPS-LBGs) detected with the Multiband Imaging Photometer for Spitzer (MIPS) at $z \sim 3$ (Magdis et al. 2010b), since they allow, for the first time, an entire coverage from the UV to the radio wavelengths. MIPS-LBGs, per se, are not a special class of objects. However, they are likely the most massive, dust-rich, and vigorously star-forming LBGs at a redshift of $z \sim 3$, following an SFR–stellar mass relation with a similar slope to that found at a redshift of $z = 1$ (Elbaz et al. 2007) and $z = 2$ (Daddi et al. 2008), but with a higher normalization factor (Magdis et al. 2010a). The inferred properties of the stacked MIPS-LBG SED will represent the average massive LBGs at a redshift of $z \sim 3$. High SFR and infrared luminosity suggest that MIPS-LBGs should be detected at submillimeter (submm) wavelengths. However, this occurs in a small fraction of cases (Chapman et al. 2000; Shim et al. 2007; Chapman & Casey 2009). This non-detection of far-IR counterparts of MIPS-LBGs could be caused by higher dust temperatures, different dust spatial distributions, and lower SFRs in LBGs than in submillimeter galaxies (SMGs; see discussion in Chapman et al. 2000; Rigopoulou et al. 2006; Magdis et al. 2010b). The self-consistent SED model combined with the chemical model will allow us to investigate this issue in more detail.

In our approach (see details below), we study the properties of stellar population and ISM by the spectrophotometric model GRASIL (Silva et al. 1998). Parameter degeneracy is a problem for any galactic SED fitting (see reviews by Gawiser 2009; Walcher et al. 2011). Therefore, our self-consistent chemical evolution model (Pipino et al. 2011), which can reproduce most of the chemical properties of ellipticals of different mass, is adopted to reduce the degree of freedom as much as possible in our SED modeling by GRASIL. The combination of chemical evolution models and GRASIL has already been adopted by Schurer et al. (2009) to model SEDs of different morphological-type galaxies using the chemical and dust evolution models from Calura et al. (2008).

In this article, we aim at (1) deriving the dust mass, and (2) investigating dust composition, temperature, and grain size distribution of MIPS-LBGs. We will also assess the robustness of previously derived galaxy masses and SFR, compare MIPS-LBGs with other $z \sim 3$ LBGs, confirm the nature of LBGs at $z \sim 3$ from previous chemical models, and try to solve the challenge of non-detection far-IR counterparts of MIPS-LBGs. The paper is organized as follows: in Section 2, we describe our models and fitting approach; our results and discussions are presented in Section 3; and our conclusions are drawn in Section 4. Throughout the paper, we adopt a $0.7, 0.3, 0.7$ cosmology.

2. THE MODELS

We combine the chemical evolution models for elliptical galaxies (Pipino et al. 2011) and the spectrophotometric model GRASIL (Silva et al. 1998) to model the SED of the stacked MIPS-LBG. The combination approach is fully described in Schurer et al. (2009) for galaxies of different morphological types. We direct the reader to the above articles for equations and details. For the sake of convenience, the two models will be briefly summarized in the following sections. In particular, the details of our “fitting” approach will be described.

2.1. The Chemical Evolution Model

The chemical abundance ratios versus metallicity are among the best indicators to constrain the SFH of a galaxy. However, there is no chemical data of the average MIPS-LBGs at $z \sim 3$ to constrain the SFH. We adopt the SFHs of elliptical models (Pipino et al. 2011) that have successfully reproduced the chemical abundance properties of normal ellipticals (see Pipino & Matteucci 2004) as well as those of low-mass LBGs at $z \sim 3$ (see Pipino et al. 2011 for details). In this way, the SFH, metallicity enrichment and dust evolution history of a galaxy are obtained from a self-consistent chemical evolution model. The impact of this assumption is reviewed in the discussion section.

In these models, both the infall and the star formation timescale are assumed to decrease with galactic mass in order to reproduce the “chemical downsizing” (e.g., Thomas et al. 2005). The initial conditions for ellipticals allow for the formation by either the collapse of a gas cloud into the potential well of a dark matter halo or, more realistically, by merging several gas clouds. In any case, the timescale for both processes should be shorter than 0.5 Gyr, so that the ellipticals form very rapidly. The rapid gas assembly triggers an intense and rapid star formation process that lasts until a galactic wind, powered by the thermal energy injected by stellar winds and SNe (Ia, II) explosions, occurs. At that time, the thermal energy is equal to or larger than the binding energy of gas, and all residual gas is assumed to be lost. After the wind, star formation ceases and the galaxies evolve passively. The evolution of the global metallicity as well as of 21 single chemical elements are studied in detail. Those elements are produced by single low- and intermediate-mass stars (asymptotic giant branch (AGB) stars), SNe Ia, single massive stars (SNe II), and Type Ia SNe (white dwarfs in binary systems). The yields used in this paper are as follows: (1) for single low- and intermediate-mass stars ($0.8 \leq M/M_\odot \leq 8$) we make use of the yields by van den Hoek & Groenewegen (1997) as a function of metallicity and (2) for SNe Ia and SNe II we adopt the empirical yields of François et al. (2004). These yields are revised versions of the Woosley & Weaver (1995, for SNe II) and Iwamoto et al. (1999, for SNe Ia) calculations adjusted to best fit chemical abundances in the Milky Way. We consider that the dust producers are AGB stars, SNe Ia, and SNe II. We also take into account the dust accretion and
Figure 1. SFR vs. time in different mass galaxies. M511, M11, and M310 represent $5 \times 10^{11}$, $10^{11}$, and $3 \times 10^{10} M_\odot$ galaxies, respectively. The red horizontal line corresponds to an SFR = 200 $M_\odot$ yr$^{-1}$, which is the estimated value for the average MIPS-LBGs.

Figure 2. Output of the chemical evolution models adopted by GRASIL. The values in the star-forming phase are shown since we only focus on this phase. (a) Total (red solid line), graphite (black dotted line) and silicate (blue dashed line) dust mass of the $10^{11} M_\odot$ galaxy as a function of galactic age. (b) Dust to gas ratio of the $10^{11} M_\odot$ galaxy as a function of galactic age. (c) Metallicity of the $10^{11} M_\odot$ galaxy as a function of galactic age.

(A color version of this figure is available in the online journal.)

2.2. The Spectrophotometric Model

Using the SFH and the chemical and dust evolution of a galaxy extracted from the chemical evolution model (e.g., as shown in Figures 1 and 2(a)-(c)), we synthesize the SED of that galaxy with the spectrophotometric code GRASIL (Silva et al. 1998). Briefly, emissions from stellar populations are calculated by an evolutionary population synthesis technique (Bressan et al. 1994) using the chemical evolution model and the Padova simple stellar population model (Bertelli et al. 1994). Younger stellar generations are more affected by dust obscuration in their birthplace (MCs) than older ones in diffuse ISM. The effect of the age selective extinction of stellar populations was modeled for the first time by a parametric approach adopting the parameter $t_{esc}$ in GRASIL (Silva et al. 1998). This is described in GRASIL assuming that the fraction of starlight radiated inside the clouds by stars is a function of the star age. In practice, if $t_{esc}$ is the timescale for the process, 100% of the stars younger than $t_{esc}$ are considered to radiate inside the MCs, and this percentage increases linearly to 0% in $2t_{esc}$. The dust composition in GRASIL consists of graphite and silicate grains, and polycyclic aromatic hydrocarbons (PAH) molecules, with a distribution of grain sizes for each composition. The optical properties of silicate and graphite grains are taken from Laor & Draine (1993). The optical properties of PAH molecules are taken from Draine & Li (2007). Dust abundance and dust composition relative abundances are obtained directly from the chemical evolution model. For the MCs, a full radiative transfer calculation is performed. The radiative transfer of starlight through dust is computed along the required line of sight yielding the emerging SED (Granato & Danese 1994) based on lambda-iteration.
numerical method (Efstathiou & Rowan-Robinson 1990; Collison & Fix 1991). However, for the diffuse dust, the effects of scattering are only approximated by assuming an effective optical depth related to the true absorption and scattering optical depths by $\tau_{\text{eff}} = \left[\tau_{\text{abs}}(\tau_{\text{abs}} + \tau_{\text{sca}})\right]^{1/2}$ and assuming that there is no re-absorption of the radiation emitted by the cirrus.

2.3. The Difference between the Chemical and Spectrophotometric Model

The adopted chemical evolution models contain only one gas phase whereas GRASIL takes into account two gas phases: MCs and diffuse gas. We therefore assume that the chemical and dust-to-gas ratio and dust abundances are the same for those two gas phases. Ca and S are not contained in GRASIL’s dust composition. However, those two elements give a negligible contribution to dust-to-gas ratio. In this paper, we only consider the following refractory elements: C, Si, Fe, Mg, and O. That is, quantities of the total dust mass will be computed neglecting other elements (see Figure 2(b)). The effect caused by the difference between ISM (ISM ≡ gas + dust) metallicity and gas phase metallicity on SED is also negligible. The Salpeter IMF (Salpeter 1955) is adopted in the two models. The baryonic matter (i.e., stars plus gas) is assumed to follow the Jaffe (1983) spatial distribution in the chemical evolution model, while GRASIL adopts the King profile (King 1966) spatial distribution. The spatial distribution, which determines the potential well, only directly affects the galactic wind time in chemical evolution models (see discussion in Section 2.1). The geometry effect is not important in the elliptical galaxy SED (e.g., see Silva et al. 1998; Piovan et al. 2006b for details). The effects of various parameters on the SED will be discussed in further work in detail. In this paper, we only describe the general effects of parameters on deriving physical properties of galaxies.

2.4. The Fitting Approach

In this work, with the help of the SFH, chemical pattern, and dust properties from the chemical evolution models (Pipino et al. 2011) and pan-SED data of the stacked MIPS-LBG (Magdis et al. 2010b), we break the SFR–age–metallicity degeneracies for SED modeling. Basically, the flux derived by median stacking analysis represents the average properties of many undetected individual objects. We will not give “ad hoc” parameters but the general properties of the average MIPS-LBG by modeling the stacked SED without taking into account the filter widths in SED fitting. We will adopt an “educated fitting” approach to estimate the physical parameter of the galaxy. This approach is guided by answering these questions: (1) which parameter dominates the overall level of SED of a galaxy? In other words, from a galaxy SED, which estimated property is most reliable? (2) How do other parameters affect the SED in detail, such as the shape and the peak of some parts of a SED? By means of this approach, we use SFHs suggested by self-consistent chemical evolution models. Moreover, chemical and dust properties, such as metallicity and dust mass, which are the basic ingredients in SED modeling, are given by the same chemical evolution model which gives the SFH. The “fitting” approach is as follows: first, since the mass is the most robust parameter in SED fitting (e.g., see Shapley et al. 2005), the total mass of the average MIPS-LBG is estimated by comparing the predicted overall level of SED with data, and confirmed by further fitting steps. This treatment effectively reduces the computation time since we do not need to test the MC and dust parameters in GRASIL to estimate the total mass (step1; see Figure 3). Once the galaxy is selected by total mass estimate, the SFH, chemical, and dust evolution history are given by the chemical evolution model with that total mass. Second, the age of the average MIPS-LBG is estimated by comparing the total mass-selected overall level of SED with the data. At this step, the SFR, metallicity, stellar mass, and dust mass of the average MIPS-LBG are derived by the SFH adopted in the chemical evolution model (step 2; see Figure 4). Third, the “best-fitting” dust parameters (shown in Tables 3 and 4) are derived by the min-$\chi^2$ method based on a grid of testing parameters in GRASIL (see an example of grid models in Figure 5). At this step, the galactic parameters estimated in previous steps are confirmed by the “best-fitting” SED (step 3; see Figure 6).

To fit the SED, we have tested in GRASIL many SFHs originating from elliptical models. These models cover a large mass range (from $10^9 M_\odot$ to $10^{12} M_\odot$; see Pipino et al. 2011 for details). The parameters and results of three examples of tested models with total mass $3 \times 10^{10} M_\odot$ (M310), $10^{11} M_\odot$ (M11), and $5 \times 10^{11} M_\odot$ (M511) are shown in Table 1. SFHs of these three models and the estimated SFR of the average MIPS-LBG are shown in Figure 1. Dust mass, dust-to-gas ratio, and metallicity as a function of galactic age of a typical elliptical of $10^{11} M_\odot$ (the “best-fitting” galaxy; see below) adopted in the GRASIL are shown in Figures 2(a)–(c), respectively.

The free parameters in GRASIL, which will be investigated by the min-$\chi^2$ method in this work, are the ones related to MCs and dust size distribution. There are four parameters (in Table 3) related to MCs: (1) the timescale $t_{\text{esc}}$ of young stars escaping from their birthplaces (MCs), (2) the fraction of gas content in MCs $f_{\text{inc}}$, (3) the mass of a single MC $M_{\text{mc}}$, and (4) the radius of a single MC $R_{\text{mc}}$. Note that the predicted SED depends on $M_{\text{mc}}$ and $R_{\text{mc}}$ only through the combination $M_{\text{inc}}/R_{\text{mc}}^2$, which is the true free parameter. The dust size distribution for each dust component is described by the power laws

![Figure 3. Rest-frame SEDs of different models. M511, M11, and M310 represent $5 \times 10^{11}$, $10^{11}$, and $3 \times 10^{10} M_\odot$ galaxies with starburst SFH at 0.5 Gyr with the MW dust parameters shown in Tables 3 and 4, respectively. Data are from Magdis et al. (2010b): for the SED we use the median UGR+BVIK+IRAC+MIPS24 photometry of MIPS-LBGs, and the values (or upper limits) derived from stacking 100 μm, 160 μm AzTEC and radio, divided by 1+e. The upside down triangles are upper limits. The MIPS data are indicated with a red point. At the first fitting step, the $10^{11} M_\odot$ model is empirically selected for next fitting step. (A color version of this figure is available in the online journal.)](image-url)
3. RESULTS AND DISCUSSION

The zero-order estimates of the relevant physical quantities (masses and SFRs) are typically derived by global or integral properties, such as overall level SED and IR luminosity, therefore estimates of those quantities are robust. Other quantities (such as UV slope and the predicted flux at a particular band) depend on the shape of the SED, and could be different.

Table 1

| Model Name | \( M_{\text{sun}} \) (\( M_\odot \)) | \( R_{\text{eff}} \) (kpc) | \( \nu \) (Gyr\(^{-1}\)) | \( \tau \) (Gyr) | \( t_{gw} \) (Gyr) | SW Yields | SNe Ia and SN II Yields | Dust Elements |
|-----------|-------------------------------|----------------|----------------|----------------|----------------|----------|-------------------------|---------------|
| M310      | \( 3 \times 10^{10} \)        | 2              | 3              | 0.5            | 0.8            | V&G      | François C, Si, Fe, Mg, O, (S, Ca) |
| M11       | \( 10^{11} \)                | 3              | 10             | 0.4            | 0.7            | V&G      | François C, Si, Fe, Mg, O, (S, Ca) |
| M511      | \( 5 \times 10^{11} \)       | 6              | 15             | 0.3            | 0.7            | V&G      | François C, Si, Fe, Mg, O, (S, Ca) |

Notes. \( M_{\text{sun}} \), the total mass of the galaxy at galactic wind time \( t_{gw} \); \( R_{\text{eff}} \), the effective radius of the galaxy; \( \nu \), the star formation efficiency. \( \tau \), the infall timescale. \( t_{gw} \), the galactic wind time. It is determined only by \( M_{\text{sun}} \) (and hence \( \nu \) and \( \tau \)). Note that it is shorter than in Pipino et al. (2011). This is because the \( t_{gw} \) refers to the central region of the galaxy in Pipino et al. (2011), while it refers to the whole galaxy in this work. This is a feature of the outside-in formation scenarios (Pipino & Matteucci 2004). Single low- and intermediate-mass stars yields (SW yields) are from van den Hoek & Groenewegen (1997; V&G). SNe Ia and SNe II yields are from François et al. (2004). C, Si, Fe, Mg, and O are the dust elements adopted in GRASIL, while S and Ca are also the dust elements in chemical models.

Figure 4. Rest-frame SED of a \( 10^{11} M_\odot \) galaxy at different ages with the MW dust parameters as in Tables 3 and 4. The parameter space of age is <0.7 Gyr. The model with age 0.3–0.6 Gyr could fit the overall-level SED. At the second fitting step, the \( 10^{11} M_\odot \) galaxy at 0.5 Gyr is the model adopted for a further fitting step. This model predicts comparable SFR and stellar mass with other models. The zero-order estimates of the relevant physical quantities (such as UV slope and the predicted flux at a particular band) depend on the shape of the SED, and could be different. Table 2 shows the main fitting properties of the average MIPS-LBG by different models. These models fit the overall level SED and predict consistent IR luminosities. Stellar mass and IR-derived SFR in all models are comparable, while other quantities are not necessarily consistent with each other. We will discuss these properties in more detail below.

3.1. Stellar Populations

The overall level rest-frame SED, like the value of the luminosity density or flux density, of a galaxy is dominated by its total galaxy mass. Particularly, the total stellar mass dominates the UV–optical part of the SED and the total dust mass dominates the IR part of the SED. In general, stars, such as AGB stars and SNe, are the main sources of dust in a galaxy. A more massive star-forming galaxy contains more stars as well as more dust (e.g., see the case of young elliptical galaxies in Pipino et al. 2011). The more massive star-forming galaxy will show a higher overall level of rest-frame SED. Since our estimations are based on the UV–radio overall-level SED fitting, our results of the stellar population are derived by a complementary approach compared with the normal approach, which is usually based on optical to near-IR SED fitting.
can obtain another “best-fitting” model using another galaxy model (such as a dust and MCs and should not be considered as the real ones. Note that one is given by the min-

\[ \chi^2 \text{ fitting approach based on the grid models shown in Figure 5.} \]

\[ \text{The mean differences of dust and MC properties is the same for those models with different ages.} \]

\[ \text{Figure 6. “Best-fitting” rest-frame SED. Red solid line: the “best-fitting” SED of a 10^{11} M_\odot galaxy at 0.5 Gyr with WM-like parameters (see Tables 3 and 4).} \]

\[ \text{b 10^{11} M_\odot model (M11) at 0.5 Gyr with MIPS parameters (see Tables 3 and 4).} \]

\[ \text{c From Magdis et al. (2010b).} \]

\[ \text{d From Rigopoulou et al. (2010).} \]

\[ \text{e From G. E. Magdis (2012, private communication).} \]

Table 2
Results from SED Fitting for the Average MIPS-LBG

| Models                     | MW     | MIPS   | Other Works |
|----------------------------|--------|--------|-------------|
| IR luminosity: \( \log(L_{IR}/L_\odot) \) | 12.13  | 12.11  | 12.21^c     |
|                            |        |        | 12.44^d     |
| Stellar mass: \( M_\star (M_\odot) \) | \( 8 \times 10^{10} \) | \( 8 \times 10^{10} \) | \( 7.9 \times 10^{10} \) |
| Age (Gyr)                  | 0.5    | 0.5    | 1^e         |
| SED-derived SFR (M_\odot yr^{-1}) | 200    | 200    | \ldots     |
| IR-derived SFR             | 233    | 222    | 275^g       |
| Dust mass: \( M_d (M_\odot) \) | \( 7 \times 10^7 \) | \( 7 \times 10^7 \) | \( 5.5 \times 10^6 \) |
| Observational frame 850 \( \mu \)m flux density: \( f_{50}(\text{mJy}) \) | 0.16   | 0.34   | 1.36^d      |
| UV slope: \( \beta \)       | -1.6   | -2.1   | \ldots     |
| Extinction: \( E(B-V) \)    | 0.213  | 0.127  | 0.1^f       |
| Luminosity weighted metallicity: \( <Z>_{\text{lum}} \) | 0.017  | 0.017  | \ldots     |
| ISM metallicity: \( Z \)    | 0.02   | 0.02   | 0.02^e      |
| Dust-to-gas ratio: \( D_g \) | 0.0025 | 0.0025 | \ldots     |

Notes.

^a 10^{11} M_\odot model (M11) at 0.5 Gyr with WM-like parameters (see Tables 3 and 4).

^b 10^{11} M_\odot model (M11) at 0.5 Gyr with MIPS parameters (see Tables 3 and 4).

^c From Magdis et al. (2010b).

^d From Rigopoulou et al. (2010).

^e From Magdis et al. (2010a).

^f From G. E. Magdis (2012, private communication).

We compare the predicted SEDs of different massive galaxies at different ages (see an example of different massive galaxies at moderate age 0.5 Gyr in Figure 3). Since short and intense SFHs are yielded by chemical evolution models, the parameter space of age is not large (age <0.7 Gyr). From Figures 3 and 4, we can estimate that the galaxy with a total mass of \( \sim 10^{11} M_\odot \) could reproduce the overall level SED of the average MIPS-LBG. Note that this mass is the total mass of an elliptical galaxy at the wind time given by the chemical evolution model. In fact, after the wind, no addition stars are formed and the galaxy evolves passively.

From the modeling point of view, the total stellar mass is dominated by SFH, IMF, and the age of stellar populations. Age is the only free parameter of stellar populations when SFH, chemical enrichment history, and the IMF of a galaxy are fixed. With a given SFH in a chemical model, the SFR is a predictable parameter. By comparing with the value estimated from observational indicators, SFR could constrain the age of a galaxy. UV and IR indicators for SFR (e.g., Kennicutt1998) have been adopted in the literature for high-redshift galaxies. Besides, depending on stellar population synthesis models, the UV-derived SFR depends on the treatment of extinction for deriving the extinction-correct fluxes, and the IR-derived SFR depends on the SED library for converting observed flux (e.g., the flux at 24 \( \mu \)m) from 8–1000 \( \mu \)m IR luminosity \( L_{IR} \). This approach has the drawback of assuming that the low-redshift template SEDs accurately represent the SED of high-redshift galaxies. For example, the excessive high 850 \( \mu \)m fluxes predicted by an empirical UV-based relationship and the unreasonable high \( L_{IR} \) estimated by MIR flux–IR luminosity correlation in SED templates have caused the problem of the non-detection of z \( \sim 3 \) LBGs in 850 \( \mu \)m band, as reported in Chapman et al. (2000) and Shim et al. (2007), respectively. Additionally, various dust intrinsic properties strongly affect the UV–FIR fluxes (see Figure 5). Therefore, we need other indicators to estimate the SFR in our SED fitting approach. In GRASIL, the non-thermal radio emission, which is not affected by dust intrinsic properties, is proportional to the SN II rate. Since the lifetime of SNe II is very short (\(< 30 \) Myr), the SN II rate is proportional to the SFR. Thus the rest-frame radio flux.
should be a good estimator of SFR. This is reflected by the fact that both the radio flux and SFR decrease (as shown in Figures 1 and 4) between ages of 0.2 and 0.6 Gyr. For the $10^{11} M_\odot$ model, the phase with SFR $\sim 200 M_\odot$ yr$^{-1}$, corresponding to the age of 0.3–0.6 Gyr (see Figure 1), fits the rest-frame radio flux well, and this SFR agrees with the estimation ($\sim 250 M_\odot$ yr$^{-1}$) of Magdis et al. (2010b). Since we have already selected the $10^{11} M_\odot$ model based on overall level SED, the SFR derived here benefits from the merits of radio-derived and SED-derived approaches and is remarkably consistent with the IR-derived SFR (see Table 2 and discussion below).

In our approach, the age is estimated by fitting the overall level SED and the radio flux, namely, it is a luminosity weighted age confirmed by the “observed” SFR. The SFR and total mass suggest that the ages of MIPS-LBGs should be $\sim 0.3$–0.6 Gyr (see Figure 4), and younger than 0.7 Gyr since LBGs are star-forming galaxies. Note that the estimated SFR $= 200 M_\odot$ yr$^{-1}$ is a typical SFR in the age range of 0.3–0.6 Gyr. The SFR is 221 $M_\odot$ yr$^{-1}$ and 140 $M_\odot$ yr$^{-1}$ at 0.3 and 0.6 Gyr for the $10^{11} M_\odot$ model, respectively. The total stellar mass is $\sim 6 \times 10^{10} M_\odot$ and $\sim 1 \times 10^{11} M_\odot$ at 0.3 and 0.6 Gyr for the $10^{11} M_\odot$ model, respectively. We adopt 0.5 Gyr for the next fitting step. This age corresponds to SFR $\sim 185 M_\odot$ yr$^{-1}$ and total stellar mass $\sim 8 \times 10^{10} M_\odot$, which agrees with the value ($\sim 7.9 \times 10^{10} M_\odot$) of Magdis et al. (2010b). Our age estimate of 0.3–0.6 Gyr is not consistent with the median age $\sim 1$ Gyr derived by the synthesis population model in Magdis et al. (2010a). In particular, our elliptical model does not allow an LBG to be older than 1 Gyr. It is, however, worth noting that while the SFH predicted in chemical evolution models is modulated by the gas inflow and self-consistently quenched due to SN-driven wind, this does not happen in the template SFHs adopted in standard SED fitting, and this might cause the disagreement in the age estimate.

### 3.2. Dust Properties

As MCs are the birthplace of stars, studying the properties of MCs at high redshift is important for our understanding of star formation in galaxies (see review by Riechers 2011). In this paper, we assume that all the MCs have the same mass $M_{mc}$ and spherical radius $R_{mc}$. MCs make many more contributions in extinction than diffuse ISM in a starburst galaxy with a fractionary gas content in the MCs $f_{mc} \geq 0.5$. The parameter $t_{esc}$ shown in Table 3 controls how long the young stars remain in their birth clouds and roughly controls the fraction of extinct stellar light, therefore it controls the total level of rest-frame UV–optical SED and the slope of the rest-frame UV–optical SED. The optical depth of dust in an MC is determined by the dust-to-gas ratio $\delta$ times $M_{mc}/R_{mc}^2$. This parameter moderately controls the total level of rest-frame UV–optical SED and also affects the slope of the rest-frame UV–optical SED. The “best-fitting” parameters of MCs for the average MIPS-LBGs are shown in Table 4. The larger value of $t_{esc}$ and the smaller value of $R_{mc}$ compared with the MW ones imply that MCs in the average MIPS-LBG are likely to be in more dense dusty environments. The same trend is found in local- and high-redshift starburst galaxies (Silva et al. 1998; Schurer et al. 2009; Swinbank et al. 2011). Yan et al. (2010) suggested that the average molecular gas mass of 24 $\mu$m detected ultraluminous infrared galaxies (ULIRGs; $L_\text{IR} > 10^{12} L_\odot$) at $z = 1.6$–2.5 is $1.7 \times 10^{10} M_\odot$ by observed CO $J = 2 \rightarrow 1$ or $J = 3 \rightarrow 2$ emission. Their total gas mass $M_g$ in the $10^{11} M_\odot$ galaxy at 0.5 Gyr is $1.6 \times 10^{10} M_\odot$. With fractionary gas content in the MCs $f_{mc} = 0.5$, the total mass in MCs in our “best-fitting” model is $0.8 \times 10^{10} M_\odot$. However, the lack of emission lines makes it difficult to break the degeneracy of the MC conditions and dust size distribution.

Once the emission from stellar populations is set, the peak in the emission in far-IR (FIR) is dominated by the total cold dust mass. The total dust mass $M_d$ is $\sim 7 \times 10^7 M_\odot$ in “best-fitting” $10^{11} M_\odot$ model at 0.5 Gyr. It is about a factor of eight less than the value ($M_d = 5.5 \pm 1.6 \times 10^8 M_\odot$) in Rigopoulou et al. (2010) based on single temperature graybody fitting (Hildebrand 1983). This may be partly caused for three reasons: (1) the uncertainty in the galaxy mass estimate, and therefore the dust mass value in this work. Since the dust mass is a consequence of the galaxy evolution in our chemical models, the estimated total galactic mass affects the total dust mass. For example, a $2 \times 10^{11} M_\odot$ galaxy at 0.5 Gyr produces a dust mass $M_d \sim 2 \times 10^8 M_\odot$. (2) The well-known degeneracy of dust temperature $T_d$ and

### Table 3
Adopted Values of MC in GRASIL

|       | MW   | MIPS |
|-------|------|------|
| $t_{esc}$ | 2    | 200  |
| $f_{mc}$ | 0.5  | 0.5  |
| $M_{mc}$ | 10^6 | 10^6 |
| $R_{mc}$ | 40   | 16   |

**Notes.**

a Timescale for the evaporation of MCs in Myr.
b Fraction of gas content in the MCs.
c Total gas mass in each MC in $M_\odot$.
d Radius of each MC in pc. The value of $R_{mc}$ in the MW is provided to respond to the lower dust-to-gas ratio in MW than in starburst galaxies (see the text in Section 3.2).

### Table 4
Parameters for Size Distribution of the Dust Components (Equations (1) and (2)) in the MW and MIPS-LBGs

|       | MW             | MIPS-LBG          |
|-------|----------------|-------------------|
| X     | $3.3 \times 10^{-5} \text{ cm}^2/\text{H}$ | $5 \times 10^{-5} \text{ cm}^2/\text{H}$ |

**Notes.** The MW size distribution was derived by Silva et al. (1998) to match observations from the MW. The MIPS-LBGs size distributions are calculated in this paper to match the SED.
slope $\beta_d$ in a single temperature graybody fitting (Blain et al. 2003). A higher $\beta_d$ will result in a lower dust temperature derived from a single temperature graybody fitting (Sajina et al. 2006), therefore a higher dust mass will be estimated. (3) The uncertainty of single temperature graybody fitting parameter, such as the rest-frame dust mass absorption coefficient $\kappa$ (e.g., a factor $\sim 7$ at 800 $\mu$m estimated by Hughes et al. 1997). Note that the single temperature graybody fitting could predict a higher IR flux than the value by full-SED fitting. An example is shown in Magdis et al. (2010b). The value of the far-IR emission peak predicted by the single temperature graybody fitting shown in their Figure 4 is larger than 1 mJy, while the peak value shown in their Figure 2 is smaller than 1 mJy predicted by SED fitting.

Prominent PAH features are observed in MIPS-LBGs (Huang et al. 2007). In starburst galaxies, the mid-IR (MIR) flux is contributed by small hot dust and PAH (see, e.g., Laurent et al. 2000 and references therein). The abundance of PAHs of our model is calculated from the chemical composition of the dust as predicted by the chemical evolution model, and their abundance is proportional to the total abundance of carbon molecules in the dusty component of the ISM. The PAH size distribution adopted in GRASIL is

$$ \frac{dn}{da} = X a^{-3.5} \text{cm}^{-1}. \quad (1) $$

PAH is needed in the “best-fitting” model to fit the average MIPS-LBG MIR flux (see Figure 6). However, we cannot put more constraints on the PAH without emission lines. The “best-fitting” model gives $X = 5 \times 10^{-25} \text{cm}^2/\text{H}$ ($X = 3.3 \times 10^{-28} \text{cm}^{2.5}/\text{H}$ in Silva et al. 1998).

### 3.3. Dust Intrinsic Properties

For many years, great effort has been made to investigate the dust size distribution through three approaches: (1) theoretical approach (e.g., Birnstiel et al. 2011), (2) fitting extinction curve approach (e.g., Mathis et al. 1977; Weingartner & Draine 2001; Clayton et al. 2003; Zubko et al. 2004), and (3) by means of the SED approach (e.g., Carciofi et al. 2004; Takeuchi & Ishii 2004; Piovan et al. 2006a). However, the nature of dust size distribution is still not clear, especially for high-redshift objects.

The position of the peak in FIR is mainly affected by the total dust mass, dust component relative abundances, and dust size distribution, but not by the stellar populations (see Figure 3; Slater et al. 2011). With the help of chemical evolution models, which predict the total dust mass and dust component relative abundance, we can study the dust size distribution in a more reliable way. The dust size distribution of graphite and silicate grains strongly affect the slope of the rest-frame UV–optical SED. For the sake of simplicity, a simple power-law dust size distribution form is adopted in this paper. The dust size distribution for the $i$ (graphite or silicate) composition follows a broken power law defined with a threshold $a_b$:

$$ \frac{dn_i}{da} = \begin{cases} A_i n_{i0} a_i^{\beta_i}, & \text{if } a_b < a < a_{\text{max}}, \\ A_i n_{i0} a_i^{\beta_i-a_{\beta_i}}, & \text{if } a_{\text{min}} < a < a_b. \end{cases} \quad (2) $$

For the sake of simplicity, dust in MIPS-LBGs only uses Equation (2) (see Table 4). The dust mass $m_i$ and dust-to-gas ratio $\delta$ are provided by the chemical evolution model:

$$ m_i = \int_{a_{\text{min}}}^{a_{\text{max}}} \frac{4\pi a^3 \rho_i}{3n_\text{H}} \frac{dn_i}{da} \, da, \quad (3) $$

where $n_{\text{H}}, m_{\text{H}},$ and $\rho_i$ are the gas number density, gas mass, and grain mass density, respectively. The normalized parameter $A_i$ is calculated by Equations (2)–(4). Compared with the dust size distribution in the MW (see Table 4), a larger amount of dust with a greater size is needed to fit the slope of rest-frame UV–optical SED (see Figures 5 and 6). With the theoretical indication of a few micrometer sizes (e.g., Tanaka et al. 2005; Birnstiel et al. 2009) and an observational suggestion of up to centimeter sizes (e.g., Rodmann et al. 2006; Ricci et al. 2010), our estimation of maximum dust size $a_{\text{max}} = 2.2 \mu$m is acceptable.

With indications from both the peak and the position of FIR and the slope of the rest-frame UV–optical SED, we suggest that the dust size distribution of the average MIPS-LBG is top-heavy (flatter) compared with that in the MW. The reliability of this trend for all high-redshift LBGs requires more data to provide more reliable constraints for the models in the future.

### 3.4. The Impact of the Adopted SFH

The adopted SFH is the main input ingredient that produces different predictions on detailed physical processes by means of population synthesis modeling. By the standard SED fitting, we cannot rule out other SFHs than the ones adopted. For example, Figure 7 shows two models adopting a self-consistent star formation history (SFH of M11 the case of Figure 1; SSF for short) and a constant star formation history (CSF). These two models can produce similar overall level SEDs by fine-tuning the ages corresponding to comparable stellar masses. In this paper, to improve the reliability of the adopted SFH for the $z \sim 3$ MIPS-LBGs, we adopted chemical model constrained SFHs, which have been successfully reproduced the chemical data of several $z \sim 3$ LBGs.

The stellar mass derived by the elliptical-type starburst SFH in this work is consistent with other works, although we used a different SED model and fitting approach to fit the SED. Again, this is because the stellar mass is a more robust parameter than
other parameters, such as age, which have been found in other SED fitting works (e.g., Papovich et al. 2001; Shapley et al. 2005; Magdis et al. 2010a). Recently, Lo Faro et al. (2013) found discrepancies between their results and those based on optical-only SED fitting for the same objects. By fitting observed SEDs with their physical model, they found higher stellar masses (by $\Delta M_*$ $\sim$ 1.4 and 2.5 dex) resulting from higher extinction (the average total extinction in the rest-frame V-band $A_V \sim$ 0.81 and 1.14) for $z \sim$ 1 and $z \sim$ 2 dusty normal star-forming (e.g., SFR $\lesssim 10 M_\odot$ yr$^{-1}$) galaxies, respectively. With only one average $z \sim$ 3 dusty star-forming galaxy in our paper, it is difficult to test whether or not there is also the same systematic discrepancy, which is mainly caused by extinction for $z \sim$ 3 dusty starburst (e.g., SFR $\sim$ 100 $M_\odot$ yr$^{-1}$) galaxies. We point out that the dusty star-forming galaxies at different redshifts are not necessarily linked from an evolutionary point of view, therefore the types of SFH and the evolution phases in which galaxies are observed might be different. Therefore, one might obtain comparable stellar masses by full-SED fitting and optical-only SED fitting with different ages and extinction values (laws). For example, by playing with dust parameters in the CSF case shown in Figure 7, we would expect to fit the optical-only SED at 2 Gyr estimating a stellar mass of $\sim$9 $10^{10}$ which is comparable to the stellar mass of $\sim$8 $10^{10}$ at 0.5 Gyr in the “best-fitting” model by full-SED fitting.

Unlike the stellar mass, our estimated age ($\sim$0.5 Gyr) is younger than other works ($\sim$1 Gyr). Our intense starburst SFH allows the galaxy to form enough stars and dust to produce the overall level SED at that age. Dust is the key link between the UV–optical SED and IR SED. In the standard SED fitting approach, dust mass cannot be derived, therefore it does not depend on the adopted SFH. In our approach, dust mass is a consequence of galaxy and dust evolution in the chemical models and depends on the SFH. By assuming reasonable detailed dust properties, the dust in our model interacts with stellar light and reproduces the SED of the average MIPS-LBG. The value of the dust mass in our work is about a factor of eight less than the value in Rigopoulou et al. (2010) based on a single temperature graybody fitting (see discussion in Section 3.2). Note that our predictions on dust environment and dust intrinsic properties correspond to the “best-fitting” SED of the average MIPS-LBG, and further work/data regarding these MIPS-LBGs are needed to confirm this result.

Clearly, the high SFR shown in the LBGs could be triggered by galaxy merger, which means completely different SFH and physical processes. This scenario of galaxy evolution is not considered in this work. We hope that more data in the future, such as galaxy morphology and chemical abundances, will reveal the nature of all LBGs.

Having discussed the limitations in our approach, we will compare our “best-fitting” model for the average MIPS-LBG with other objects that show similar observed features in some aspects.

3.5. Comparison with Lower Mass $z \sim$ 3 LBGs

The gravitational lensed LBG cB58 is the best-studied typical LBG. This object is at $z \sim$ 2.73 and has a luminous mass $\sim$10$^{10}$ $M_\odot$, an SFR $\sim$ 40 $M_\odot$ yr$^{-1}$ (Pettini et al. 2002), and an effective radius of $r_e \sim$ 2 kpc (Seitz et al. 1998) for a $\Omega_m = 0.3$, $\Omega_\Lambda = 0.7$, and $h = 0.70$ cosmology. The average MIPS-LBG, with a total mass of $\sim$10$^{11}$ $M_\odot$ estimated in this work, is more massive than cB58 (cB58 is 10$^{10}$–3 $\times$ 10$^{10}$ $M_\odot$ in Pipino et al. 2011). The model suggests an SFR $\sim$ 200 $M_\odot$ yr$^{-1}$ for the average MIPS-LBG, which is higher than that of cB58. The metallicity of MIPS-LBGs ($\sim Z_\odot$) is higher than cB58 ($\sim$0.25 $Z_\odot$). These differences imply that MIPS-LBGs and cB58 are at opposite ends of the observed mass–SFR (Magdis et al. 2010a) and mass–metallicity (Maiolino et al. 2008) relations at $z \sim$ 3.

The shape of a galaxy SED depends on many physical processes as we have shown above. We compare the average MIPS-LBG in this work with other 24 $\mu$m MIPS-detected $z \sim$ 3 LBGs (Rigopoulou et al. 2006; Shim et al. 2007) in Figure 8 by plotting the estimated extinction $E(V - B)$ versus the observed 24 $\mu$m flux density $f_{24}$. As shown in Figure 8, the observed frame $f_{24}$ does not correlate with the $E(V - B)$ for these samples. We could not derive reliable IR properties based only on fitted parameters, which describe UV–optical properties. Therefore, we suggest that self-consistent SED models are needed when deriving detailed properties of galaxies from their UV–radio multi-band data.

3.6. Comparison with Submillimeter Galaxies (SMGs)

MIPS-LBGs with IR luminosity of $L_{IR} = 1.3 \times 10^{12} L_\odot$ are in the class of ULIRGs ($L_{IR} > 10^{12} L_\odot$). Since MIPS-LBGs could be the most massive, dusty, star-forming, and young ellipticals as discussed in the previous section, we suggest that these ellipticals contribute a significant population to ULIRGs at a redshift of $z \sim$ 3. The non-detection of infrared luminous LBGs in the submm band is puzzling, given their large $L_{IR}$ and high UV-derived SFR, as well as substantial dust component (see details in Chapman et al. 2000 and Shim et al. 2007). There are only two detections of FIR counterparts of MIPS-LBGs reported in the literature (Chapman et al. 2000; Chapman & Casey 2009). Higher dust temperature, different dust spatial distribution, and lower SFR in LBGs than in SMGs have been suggested to explain this question (e.g., Chapman et al. 2000; Rigopoulou et al. 2006; Magdis et al. 2010b). Based on our “best-fitting” model, our predicted observational frame flux density of MIPS-LBGs at 850 $\mu$m is 0.3 mJy, which is under the confusion/detection limit of current submm surveys.

Figure 8. Observed 24 $\mu$m flux density $f_{24}$ vs. estimated extinction $E(B - V)$ for 24 $\mu$m MIPS-detected LBGs at $z \sim$ 3. The asterisks, crosses, and square represent data from Shim et al. (2007), Rigopoulou et al. (2006) and the average MIPS-LBG, respectively. The $f_{24}$ is not correlated with $E(B - V)$ for these objects.

(A color version of this figure is available in the online journal.)
CONCLUSIONS

In this paper, we modeled the rest-frame UV–radio SED of stacked MIPS-LBGs at $z \sim 3$ by a new “fitting” approach. In this self-consistent approach, we derived the average galactic-wide properties of MIPS-LBGs at $z \sim 3$ considering the stacked MIPS-LBG as a proxy of all MIPS-LBGs at $z \sim 3$. Our findings are summarized as follows.

1. The new “fitting” approach, which combines the chemical evolution model and GRASIL, can reproduce the rest-frame UV–radio SED of the stacked MIPS-LBG at $z \sim 3$. This approach suggests that the MIPS-LBGs at $z \sim 3$ are likely young (0.3–0.6 Gyr) massive ($\sim 10^{11} M_\odot$) elliptical galaxies with a fast starburst regime of star formation. Chemical enrichment and dust evolution history are provided by chemical evolution models, calibrated at both low and high redshifts.

2. We estimated that the average stellar mass of MIPS-LBGs is in the range of $\sim 6 \times 10^{10} M_\odot$ to $\sim 1 \times 10^{11} M_\odot$. The “best-fitting” stellar mass and SFR of the stacked MIPS-LBG are $8 \times 10^{10} M_\odot$ and $200 M_\odot$ yr$^{-1}$, respectively. The stellar mass is in agreement with estimates in other works and based on UV–optical SED fitting. This confirms that the derived stellar mass is robust in SED fitting.

3. The “best-fitting” model of the stacked MIPS-LBG is the 0.5 Gyr $10^{11} M_\odot$ elliptical galaxy, which has a dust mass of $M_d = 7 \times 10^7 M_\odot$ and a gas mass of $M_g = 1.6 \times 10^{10} M_\odot$. Our “best-fitting” dust mass of the stacked MIPS-LBG is about a factor of eight less than the value based on single temperature greybody fitting ($M_d = 5.5 \pm 1.6 \times 10^8 M_\odot$ in Rigopoulou et al. 2010).

4. The parameters of MCs and dust of the Milky Way cannot fit the stacked MIPS-LBG SED. Our “best-fitting” parameters reflect that (1) MCs of the stacked MIPS-LBG are denser dusty environments than in the Milky Way, (2) the dust size distributions in stacked MIPS-LBG may be flatter than in the Milky Way, (3) both small and large sizes of dust are needed to reproduce the stacked MIPS-LBG SED, and (4) non-negligible PAH also make an important contribution to MIR flux. More observational data, such as chemical abundances and emission lines, are needed to make a more realistic prediction of the properties of all MIPS-LBGs, especially for dust intrinsic properties.

5. We suggest that high-redshift star-forming ellipticals make a significant contribution to the population of ULIRGs. The dust properties and morphology of high-redshift starburst galaxies could be different from local starburst galaxies. We argue that self-consistent SED models are needed to investigate the detailed properties of high-redshift LBGs.

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REFERENCES

Bertelli, G., Bressan, A., Chiosi, C., Fagotto, F., & Nasi, E. 1994, A&AS, 106, 275

Birnstiel, T., Dullemond, C. P., & Brauer, F. 2009, A&A, 503, L5

Birnstiel, T., Ormel, C. W., & Dullemond, C. P. 2011, A&A, 525, A11

Blain, A. W., Barnard, V. E., & Chapman, S. C. 2003, MNRAS, 338, 733
