On the Error Performance of LoRa-Enabled Aerial Networks Over Shadowed Rician Fading Channels

Lina Bariah, Senior Member, IEEE, Wael Jaafar, Senior Member, IEEE, Sami Muhaidat, Senior Member, IEEE, Hany Elgala, Member, IEEE, and Halim Yanikomeroglu, Fellow, IEEE

Abstract—UAVs can be used as aerial relays to provide communication services in remote uncovered areas or dense environments with occasional high capacity demands. However, due to the low power of Internet-of-Things (IoT) devices, UAV-based IoT applications, such as precision agriculture and environment monitoring, may experience high shadowing or equipment failure, which degrades the communications’ quality between IoT devices and their gateways. To tackle this issue, we consider the long range (LoRa) communication technology. Specifically, we investigate the performance of LoRa-enabled aerial communications, where a LoRa gateway communicates with a distant IoT device through the assistance of an amplify-and-forward (AF) aerial relay. Under the assumption of shadowed Rician fading channels, we characterize at first the end-to-end LoRa communication link. Then, we derive an exact symbol error rate expression for the underlying system model. Finally, numerical results are presented to corroborate the efficacy of our derived expressions and provide valuable insights into the error performance of LoRa-enabled aerial networks.

Index Terms—Long range communication, LoRa, unmanned aerial vehicle, UAV.

I. INTRODUCTION

RECENTLY, the deployment of embedded systems, such as wireless sensor networks and Internet-of-Things (IoT) devices, has seen an exponential growth due to their ability to collect valuable data in different types of applications, such as precision agriculture, traffic management, and industrial faults detection [1]. In low power networks, IoT devices are typically characterized by their low battery power and thus, can communicate only at low-power levels. Subsequently, this may degrade the communications’ quality between IoT devices and a gateway, particularly in urban and dense urban areas. Recently, it has been demonstrated that aerial relays, such as unmanned aerial vehicles (UAVs), can provide strong wireless links towards ground devices. The UAV technology has matured over the last years, and its use for civil applications is actively explored in the research and industry fields [2], [3]. In our context, thanks to their deployment flexibility and mobility, UAVs can be placed at strategic 3D locations to both strengthen the gateway-IoT device communication link and provide extended coverage for gateways. This approach is expected to be more cost-effective than deploying additional fixed gateways in the targeted IoT servicing area.

Although the use of UAVs in the context of IoT applications is interesting, its efficiency depends on the communication protocol. Long range (LoRa) differs from existing protocols, such as Wi-Fi, Bluetooth and ZigBee [4], by its ability to achieve communications over long distances at descent (typically low) data rates. Specifically, it relies on the advantages of spread spectrum, chirp orthogonality, and the specific characteristics of kHz and MHz spectrum bands. A LoRa network exploits a fixed infrastructure of gateways for devices connecting with low power and over a long distance range. Nevertheless, due to environmental conditions, LoRa signals transmitted over long distances experience corruption or loss. In this regard, dedicated channel coding mechanisms, e.g., DaRe, are utilized to ensure reliable LoRa signals transmission [5], [6].

Several works investigated the use of LoRa for UAV-assisted IoT networks. The authors of [7] and [8] proposed to use UAVs as flying LoRa gateways to extend the coverage of IoT networks to rural and remote areas. They presented a number of use cases and preliminary results on the received LoRa signal quality at the UAVs, based on a practical implementation. In [9], the authors reported signal strength measurements for LoRa-enabled UAV networks in urban and sub-urban environments. They demonstrated that the UAV altitude and antenna orientation significantly impact the communication range. Additionally, the authors in [10] designed a LoRa-based mesh network protocol where communication between an IoT device and a gateway is realized in a multi-hop UAV network. The proposed protocol extends LoRa transmission range compared with the one-hop links, as well as being robust against network dynamic topology changes. Considering a UAV as an aerial gateway, the authors of [11] optimized UAV’s altitude to efficiently collect data from ground IoT devices, in a precision agriculture use case. Finally, the authors in [12] supported a disaster management application where LoRa is used for the UAV-IoT device link, while Wi-Fi handles the UAV-gateway communication. The authors evaluated the system’s performance in terms of average end-to-end packet
reception rate, found to be above 80% when a sufficient number of UAVs is deployed in the targeted area.

In the aforementioned works, UAVs operate as aerial gateways rather than relays. This mechanism requires additional investment and processing at the UAV, which increases the communication and energy costs. Moreover, state-of-the-art works lack performance evaluation studies and theoretical investigations. Within this context, we propose in this letter a system model for LoRa-enabled aerial communication networks. By leveraging a dedicated LoRa relay node, we achieve an overall outage rate of above 80% when a sufficient number of UAVs is deployed, which is a significant improvement over existing works.

Fig. 1. Considered system model for LoRa-assisted UAV network.

Ⅱ. SYSTEM MODEL

We consider a network in which two LoRa-enabled IoT devices communicate with each other, namely a source (S) and a destination (D), through an AF aerial relay (R), as depicted in Fig. 1. Each node is equipped with a single antenna. The transmitted chirp signals from S use bandwidth $B$, i.e., a LoRa sample is transmitted every $T_b = 1/B$. Recalling that LoRa relies on the chirp spread spectrum (CSS) technology, with spreading factor $SF \in \{7, 8, \ldots , 12\}$, LoRa signals are modulated by spreading the frequency of chirp signals over $M = 2^{SF}$ samples, i.e., symbol duration is $T_s = MT_b$. Note that the spreading factor indicates the number of chips per bit and hence, it controls the chirp rate. Also, $M$ denotes the total number of LoRa samples. The $n$th sample of the $m$th LoRa symbol can be written as [16]

$$x_m[n] = \frac{1}{\sqrt{M}} \exp \left( j \pi \frac{n^2 + m^2 + 2nm}{M} \right).$$

(1)

Note that $x_m[n]$, $m = 0, \ldots , M-1$, $n = 0, \ldots , M-1$, represents the LoRa orthonormal basis functions. Therefore, the received LoRa signal at $R$ is expressed as follows

$$r_m^{(R)}[n] = \sqrt{\frac{P_T}{\sigma_1^2}} g_1 x_m[n] + z_m^{(R)}[n],$$

(2)

where $P_T$ is the transmit power of the LoRa gateway, $\alpha$ is the path-loss exponent, and $L_1 = \sqrt{d_1^2 + d_2^2}$ is the S-R distance, with $H$ and $d_1$ denoting the aerial relay altitude and the distance between S and the projection of R on the ground, respectively. The additive white Gaussian noise (AWGN) at $R$ is modeled by $z_m^{(R)}[n] \sim \mathcal{CN}(0, N_0/M)$, and $N_0$ represents the noise power spectral density. Also, $g_1$ denotes the small-scale fading of the S-R link, which is modeled as a shadowed Rician random variable. In this work, we consider the channel model derived in [17], in which the multi-path fading is modeled as a Rayleigh random variable, while the shadowing effect is modeled using the Nakagami-$\tilde{m}_1$ distribution. It is worth noting that such a versatile model provides an accurate characterization of the random propagation environment, and facilitates the investigation of the system performance under different scenarios.

The probability density function (PDF) of $|g_1|^2$ is given by [17]

$$f_{|g_1|^2}(g) = a_1 \sum_{j=0}^{\tilde{m}_1 - 1} \zeta(j) g^j \exp(- (\beta_1 - \kappa_1) g),$$

(3)

where $\tilde{m}_1$ is the fading severity,

$$a_1 = \left( \frac{\Omega_{i,1} \tilde{m}_i}{\Omega_{i,1} \tilde{m}_i + \Omega_{i,2}} \right)^{\tilde{m}_1} \Omega_{i,1}^{-1},$$

(4)

$$\zeta(j) = \frac{(-1)^j \kappa_j}{(j)!^2(1 - \tilde{m}_1)_j},$$

(5)

with $(.)_j$ denoting the Pochhammer symbol and

$$\kappa_i = \frac{\Omega_{i,2}}{\Omega_{i,1} (\Omega_{i,1} \tilde{m}_i + \Omega_{i,2})}.$$ 

(6)

and $\beta_1 = 1/\Omega_{i,1}$. Note that $\Omega_{i,1}$ and $\Omega_{i,2}$ represent the average power of the multi-path and LoS components, respectively. Assuming blind relaying, the aerial relay amplifies the received LoRa waveform by a scaling factor $A$ and forwards it to D. Subsequently, the normalized received signal at the IoT destination can be written as

$$r_m^{(D)}[n] = A g_2 r_m^{(R)}[n] + z_m^{(D)}[n] = \sqrt{\gamma_1} A g_2 g_1 x_m[n] + A g_2 z_m^{(R)}[n] + z_m^{(D)}[n],$$

(7)

where $g_2$ represents the R-D channel coefficient, which follows the shadowed Rician distribution. Since we consider AF relaying, the scaling factor at the relay, $A$, can be given by

$$A = \sqrt{L_2^{-\alpha} \frac{P_R}{\mathbb{E} \left[ t_m^{(R)} \left( t_m^{(R)} \right)^* \right]}} = \sqrt{(L_1 L_2)^{-\alpha} \tilde{\gamma}_2 M / \left( L_1^{-\alpha} \tilde{\gamma}_1 M \sigma_1^2 |g_1|^2 + 1 \right)}.$$ 

(8)

where $L_2 = \sqrt{H^2 + d_2^2}$ denotes the R-D distance, $d_2$ represents the distance between D and the projection of R on the ground, $P_R$ is the transmission power of R, and $\tilde{\gamma}_1 = P_T/N_0$ and $\tilde{\gamma}_2 = P_R/N_0$ are the average received signal-to-noise ratios (SNRs) at R and D, respectively. Finally, $\mathbb{E}\{\cdot\}$ and $(\cdot)^*$ are the expectation and conjugate operators, respectively.
Given that the normalized AWGN at $D\ z_{m}^{(D)}[n] \sim \mathcal{CN}(0, \sigma_{g1}^{2})$, the total received noise can be modeled as $\hat{z}_{m}[n] \sim \mathcal{CN}(0, \sigma_{g1}^{2}|g_{2}|^{2} + 1)$. The variance of the channel $g_{1}$, denoted by $\sigma_{g1}^{2}$, can be expressed as [17]

$$\sigma_{g1}^{2} = \Omega_{i,1} \left( \frac{\Omega_{m} \Omega_{m}^{\ast}}{1 + \tau_{m,1}^{2}} \right) m_{m} \Omega_{m}^{\ast} \Omega_{m} \Omega_{m}^{\ast} .$$

LoRa demodulation is achieved by exploiting the orthogonality property between the $M$ LoRa basis functions, and therefore, a correlator is utilized to evaluate the correlation between the received signal and LoRa basis functions. Based on this, and assuming perfect time and frequency synchronization, the detected LoRa symbol can be evaluated as

$$\hat{x}_{m,l}[n] = \sum_{n=0}^{M-1} r_{m}^{(D)}[n] x_{l}^{\ast}[n],$$

which, alternatively, can be written as

$$\hat{x}_{m,l}[n] = \sqrt{\gamma_{1}} A_{g} g_{1} \exp \left( \frac{j\pi}{M}(m^{2} - l^{2}) \right) \delta[m - l] + \hat{z}_{m}[n] .$$

where $\delta[.]$ is the Kronecker delta function, and $\hat{z}_{m}[n] \sim \mathcal{CN}(0, |g_{2}|^{2} + 1)$. Therefore, the output of the LoRa demodulator can be written as

$$\hat{x}_{m,l}[n] = \begin{cases} \sqrt{\gamma_{1}} A_{g} g_{1} + \hat{z}_{m}[n] & l = m \\ \hat{z}_{m}[n] & l \neq m. \end{cases}$$

Assuming an energy detection-based receiver, the detected LoRa symbol is given by

$$\hat{x}_{m,l}[n] = \arg \max_{i \in \{0, \ldots, M-1\}} \left( \sqrt{\gamma_{1}} A_{g} g_{1} + \hat{z}_{i}[n] \right).$$

Kindly note that such a detection scheme is considered sub-optimal.

III. ERROR RATE PERFORMANCE ANALYSIS

In this section, we derive an exact symbol error rate (SER) expression for the underlying system model, in order to quantify the error rate performance of LoRa modulation in aerial networks.

The probability that the $m^{th}$ LoRa symbol is incorrectly detected can be given as

$$P_{e} = \Pr \left( \max_{l \neq m} |\hat{Z}_{l}[n]| > |\sqrt{\gamma_{1}} A_{g} g_{1} + \hat{Z}_{m}[n]| \right)$$

$$= 1 - \Pr \left( \max_{l \neq m} |\hat{Z}_{l}[n]| \leq |\sqrt{\gamma_{1}} A_{g} g_{1} + \hat{Z}_{m}[n]| \right) .$$

(14)

Given that $|\hat{Z}_{l}[n]|$ are independent and identically distributed for $l = 0, \ldots, M - 1$ and $l \neq m$, then it can be concluded that

$$|\hat{Z}_{l}[n]| \leq |\sqrt{\gamma_{1}} A_{g} g_{1} + \hat{Z}_{m}[n]|, \forall l = 0, \ldots, M - 1. \quad (15)$$

Accordingly, the error rate expression in (14) can be rewritten as

$$P_{e} = 1 - \prod_{l \neq m}^{M-1} \Pr \left( |\hat{Z}_{l}[n]| \leq \frac{|\sqrt{\gamma_{1}} A_{g} g_{1} + \hat{Z}_{m}[n]|}{\Delta_{m}} \right) . \quad (16)$$

Theorem 1: The exact SER of LoRa modulation in the underlying system model is given by (17), as shown at the bottom of the page,

where $\Gamma(\cdot)$ and $\Psi(\cdot, \cdot, \cdot)$ are the complete Gamma and the Tricomi functions, respectively [18], and

$$q = \tilde{\gamma}_{1} A_{g}^{2} + 4(\beta_{1} - \kappa_{1})A^{2} . \quad (18)$$

Proof: In order to evaluate the closed-form expression of the SER, we first evaluate the following probability expression

$$\phi = \Pr \left( |\hat{Z}_{l}[n]| \leq \frac{|\sqrt{\gamma_{1}} A_{g} g_{1} + \hat{Z}_{m}[n]|}{\Delta_{m}} \right) . \quad (19)$$

Conditioned on $g_{1}$, $g_{2}$, and $\hat{Z}_{m}[n]$, and given that $\hat{Z}_{l}[n]$ follows the complex Gaussian distribution, (19) can be expressed in terms of the cumulative distribution function of a Rayleigh random variable as

$$\phi = 1 - \exp \left( -\frac{\Delta_{m}^{2}}{2|g_{2}|^{2} + 1} \right) . \quad (20)$$

To obtain an unconditional SER expression, in the following, we first derive the PDF of $\Delta_{m}$. Conditioned on $|g_{1}|$ and $|g_{2}|$, $\Delta_{m}$ is modeled as a Rician random variable with non-centrality parameter equals to $\tilde{\gamma}_{1} A_{g}^{2} |g_{2}|^{2} |g_{1}|^{2}$ and variance $|A^{2} |g_{2}|^{2} + 1$. Therefore, the PDF of $\Delta_{m}$ can be represented as

$$f_{\Delta_{m}}(x) = \frac{x}{|A^{2} |g_{2}|^{2} + 1} \exp \left( -\frac{x^{2} + \tilde{\gamma}_{1} A_{g}^{2} |g_{2}|^{2} |g_{1}|^{2}}{2|A^{2} |g_{2}|^{2} + 1} \right) I_{0} \left( \sqrt{2 \tilde{\gamma}_{1} A_{g}^{2} |g_{2}|^{2} |g_{1}|^{2}} \right) . \quad (21)$$

Using the PDF of $|g_{1}|^{2}$, the PDF of $\Delta_{m}$, given in (21), can be rewritten as (22), shown at the bottom of the next page. To evaluate the integral in (22), we implement the change of variables approach with the aid of [19, eq. 2.15.5.4], which results in (23), as shown at the bottom of the next page. In (23), $F_{1}(\cdot, \cdot)$ denotes the confluent hypergeometric function. The derived PDF in (23) is further utilized to evaluate an exact SER

$$P_{e} = 1 - \prod_{l \neq m}^{M-1} \left[ 1 - a_{1} a_{2} \sum_{j=0}^{m_{l}-1} \sum_{i=0}^{m_{l}-1} \sum_{\tau=0}^{j+1} \binom{j+1}{\tau} 2^{2i+2\tau} \frac{\zeta(j) \zeta(i) \Gamma(j+1) \Gamma(i+\tau+1)}{q^{\tau+\tau+1} (\beta_{1} - \kappa_{1})^{\tau+\tau+1} A^{-2\tau}} \times \Psi \left( i + \tau + 1; i + \tau - j + 1; \frac{4(\beta_{1} - \kappa_{1})(\beta_{2} - \kappa_{2})}{q} \right) \right] . \quad (17)$$
expression. Conditioned on $|g_1|^2$, the SER of the $m^{th}$ LoRa symbol can be evaluated by integrating (16) over the PDF in (23), yielding (24), as shown at the bottom of the page. Note that, similar to the PDF of $|g_1|^2$, the PDF of $|g_2|^2$ is given by (3), by replacing $m_1 \rightarrow m_2$, $\Omega_{1,1} \rightarrow \Omega_{2,1}$, and $\Omega_{1,2} \rightarrow \Omega_{2,2}$. To further simplify the expression in (24), we first resort to the binomial expansion theorem as follows

$$[A^2|g_2|^2 + 1]^{j+1} = \sum_{\tau=0}^{j+1} \binom{j+1}{\tau} A^{2\tau}|g_2|^{2\tau}. \quad (25)$$

Finally, by leveraging (25) and [20, eq. 2.3.6.9], the conditional SER in (24) can be averaged over the PDF of $|g_2|^2$, and subsequently, a closed-form expression of the SER can be obtained as (17).

IV. NUMERICAL RESULTS

In this section, we present numerical results to validate the derived analytical framework, and to demonstrate the error rate performance of LoRa-enabled aerial networks, when shadowed Rician fading is experienced over the dual-hop link. Unless stated otherwise, simulation parameters are presented in Table I. Without loss of generality, we assume that S-R and R-D links are balanced, i.e., $\bar{m}_1 = \bar{m}_2 = \bar{m}$, $\bar{\gamma}_1 = \bar{\gamma}_2 = \bar{\gamma}$, $\Omega_{1,1} = \Omega_{2,1} = \Omega_1$, and $\Omega_{1,2} = \Omega_{2,2} = \Omega_2$. Note that in all presented results, we consider the SNR per chirp.

In Fig. 2, we study the SER performance versus $\Omega_2$, for $\text{SF} \in \{7, 9, 12\}$, and $\bar{m} \in \{2, 4\}$. From the figure, it can be observed that the error rate performance of the underlying system model improves as SF decreases. It can be further noticed that SF has no effect on the achievable diversity order of the system, rather, the diversity order is determined by the fading severity, quantified by $\bar{m}$. However, it is noted that the effect of the considered SF value has less impact on the system performance as $\bar{m}$ value increases. This is due to the fact that as $\bar{m}$ increases, the LoS link quality improves, and hence, the effect of the channel becomes dominant.

The results obtained in Fig. 3 demonstrate the advantage of LoRa in aerial networks. In particular, Fig. 3 investigates the error rate performance of LoRa-enabled UAV system versus the UAV altitude, $H$, for $\Omega_2 \in \{5, 15\}$ dB. Although it shows that the system performance is highly affected by the quality of the LoS link, it can be noticed that the system performs reliably even at high UAV altitudes. Specifically, it is observed that the system experiences a relatively acceptable error rate when the UAV is flying at altitudes above 1 Km, at all SF values. This further motivates the use of LoRa in aerial networks as an efficient scheme to extend the transmission distance. Finally, obtained results for the no shadowing scenario justify the performance analysis of the considered system under the shadowed Rician channel model.

| Parameter | Value | Parameter | Value |
|-----------|-------|-----------|-------|
| $H$       | 100 m | $N_0$     | -109 dB |
| $\alpha$  | 2.7   | $P_t = P_R$ | 0 dB |
| $d_1 = d_2$ | 60 m | $\Omega_{1,1} = \Omega_{2,1}$ | 5 dB |
| $\bar{m}_1 = \bar{m}_2$ | 2 | $\Omega_{1,2} = \Omega_{2,2}$ | 10 dB |

| $P_e$ = $1 - \prod_{i=0}^{M-1} \left[ 1 - \alpha_i \sum_{j=0}^{m_i-1} \frac{2^{|g_2_i|^2 + 1} [\bar{\gamma}_1 A^2 |g_2_i|^2 + 4(|\beta_1 - \kappa_1|) A^2 |g_2_i|^2 + 1]^{j+1}}{\bar{\gamma}_1 A^2 |g_2_i|^2 + 4(|\beta_1 - \kappa_1|) A^2 |g_2_i|^2 + 1]^{j+1}} \right]$. | (24) |
the assumption of shadowed Rician fading. The obtained results confirm the advantages of exploiting LoRa in order to improve the reliability and coverage of UAV-assisted IoT applications.

**REFERENCES**

[1] A. Al-Fuqaha, M. Guizani, M. Mohammadi, M. Aledhari, and M. Ayyash, “Internet of Things: A survey on enabling technologies, protocols, and applications,” IEEE Commun. Surveys Tuts., vol. 17, no. 4, pp. 2347–2376, 4th Quart., 2015.

[2] M. Mozaffari, W. Saad, M. Bennis, Y.-H. Nam, and M. Debbah, “A tutorial on UAVs for wireless networks: Applications, challenges, and open problems,” IEEE Commun. Surveys Tuts., vol. 21, no. 3, pp. 2334–2360, 3rd Quart., 2019.

[3] O. Ghdiri, W. Jaafar, S. Alfattani, J. B. Abderrazak, and H. Yanikomeroglu, “Offline and online UAV-enabled data collection in time-constrained IoT networks,” IEEE Trans. Green Commun. Netw., vol. 5, no. 4, pp. 1918–1933, Dec. 2021.

[4] J. P. S. Sundaram, W. Du, and Z. Zhao, “A survey on LoRa networking: Research problems, current solutions, and open issues,” IEEE Commun. Surveys Tuts., vol. 22, no. 1, pp. 371–388, 1st Quart., 2020.

[5] Y. Fang, Y. Bu, P. Chen, F. C. M. Lau, and S. A. Otaibi, “Irregular-mapped protograph LDPC-coded modulation: A bandwidth-efficient solution for 6G-enabled mobile networks,” IEEE Trans. Intell. Transp. Syst., early access, Nov. 3, 2021, doi: 10.1109/TITS.2021.3122994.

[6] L. Dai, Y. Fang, Z. Yang, P. Chen, and Y. Li, “Protograph LDPC-coded BICM-ID with irregular CSK mapping in visible light communication systems,” IEEE Trans. Veh. Technol., vol. 70, no. 10, pp. 11033–11038, Oct. 2021.

[7] M. Marchese, A. Mcheddine, and F. Patrone, “Towards increasing the LoRa network coverage: A flying gateway,” in Proc. Int. Symp. Adv. Electr. Commun. Technol. (ISAECT), Nov. 2019, pp. 1–4.

[8] A. Mcheddine, F. Patrone, and M. Marchese, “UAV and IoT integration: A flying gateway,” in Proc. 26th IEEE Int. Conf. Electron., Circuits Syst. (ICECS), Nov. 2019, pp. 121–122.

[9] V. A. Dambal, S. Mohadi, A. Kumbhar, and I. Guvenc, “Improving LoRa signal coverage in urban and suburban environments with UAVs,” in Proc. Int. Wkshp. Ant. Technol. (iWAT), 2019, pp. 210–213.

[10] M. Pan et al., “UAVs-aided emergency environmental monitoring in infrastructure-less areas: LoRa mesh networking approach,” IEEE IoT J., vol. 9, no. 4, pp. 2918–2932, Feb. 2022.

[11] A. Caruso, S. Chessa, S. Escolar, J. Barba, and J. C. Lopez, “Collection of data with drones in precision agriculture: Analytical model and LoRa case study,” IEEE IoT J., vol. 8, no. 22, pp. 16692–16704, Nov. 2021.

[12] O. A. Saraer et al., “Performance evaluation of UAV-enabled LoRa networks for disaster management applications,” Sensors, vol. 20, no. 8, pp. 1–18, Apr. 2020.

[13] A. AliHamdani, L. Bariah, S. Naser, S. Muhaidat, M. Al-Qutayri, and P. C. Sofotasios, “Analysis of superimposed LoRa in multi-user networks,” in Proc. 4th Int. Conf. Adv. Commun. Technol. Netw. (CommNet), Dec. 2021, pp. 1–6.

[14] A. S. Ali, L. Bariah, S. Muhaidat, P. Sofotasios, and M. Al-Qutayri, “Performance analysis of LoRa-backscatter communication system in AWGN channels,” in Proc. 4th Int. Conf. Adv. Commun. Technol. Netw. (CommNet), Dec. 2021, pp. 1–5.

[15] T. Elishabrawy and J. Robert, “Closed-form approximation of LoRa modulation BER performance,” IEEE Commun. Lett., vol. 22, no. 9, pp. 1778–1781, Sep. 2018.

[16] M. Hanif and H. H. Nguyen, “Slope-shifting key LoRa-based modulation,” IEEE IoT J., vol. 8, no. 1, pp. 211–221, Jan. 2020.

[17] A. Abdi, W. C. Lau, M.-S. Alouini, and M. Kaveh, “A new simple model for land mobile satellite channels: First-and second-order statistics,” IEEE Trans. Wireless Commun., vol. 2, no. 3, pp. 519–528, May 2003.

[18] I. S. Gradshteyn and I. M. Ryzhik, *Table of Integrals, Series, and Products*. Moscow, Russia: Academic, 1943.

[19] A. P. Prudnikov, Y. A. Brychkov, and O. I. Marichev, *Integrals and Series: Special Functions*, vol. 2. New York, NY, USA: CRC Press, 1986.