Constraints on the topology of the Universe derived from the 7-yr WMAP data

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ABSTRACT
We impose constraints on the topology of the Universe determined from a search for matched circles in the temperature anisotropy patterns of the 7-yr Wilkinson Microwave Anisotropy Probe (WMAP) data. We pay special attention to the sensitivity of the method to residual foreground contamination of the sky maps and show that for a full-sky estimate of the CMB signal (the Internal Linear Combination map) such residuals introduce a non-negligible effect on the statistics of matched circles. In order to reduce this effect, we perform the analysis on maps for which the most contaminated regions have been removed. A search for pairs of matched back-to-back circles in the higher resolution WMAP W-band map allows tighter constraints to be imposed on topology. Our results rule out universes with topologies that predict pairs of such circles with radii larger than $\alpha_{\text{min}} \approx 10^\circ$. This places a lower bound on the size of the fundamental domain for a flat universe of about 27.9 Gpc. This bound is close to the upper limit on the size of Universe possible to be detected by the method of matched circles, i.e. the diameter of the observable Universe is 28.3 Gpc.

Key words: cosmic background radiation – cosmology: observations.

1 INTRODUCTION
According to general relativity, the pseudo-Riemannian manifold with signature (3,1) is a mathematical model of space–time. The local properties of space–time geometry are described by the Einstein gravitational field equations. However, they do not specify the global spatial geometry of the universe, i.e. its topology. This can only be constrained by observations.

The concordance cosmological model assumes that the universe possesses a simply connected topology, yet recently detected anomalies on large angular scales in the cosmic microwave background (CMB) anisotropy suggest that it may be multiply connected. Evidence of such anomalies comes from the suppression of the quadrupole moment, alignment of the quadrupole and octopole and asymmetry in the temperature anisotropy observed in two hemispheres on the sky (Copi, Huterer & Starkman 2004; de Oliveira-Costa et al. 2004; Eriksen et al. 2004; Hansen, Banday & Gorski 2004; Schwarz et al. 2004).

The present constraints on topology were placed by studying two signatures of multiconnectedness in the CMB maps: the large-scale damping of the power in the direction of a small dimension of the domain, which causes a breakdown of statistical isotropy (de Oliveira-Costa et al. 2004; Kunz et al. 2006, 2008), and the distribution of matched patterns (Cornish et al. 2004; Aurich, Lustig & Steiner 2005, 2006; Then 2006; Key et al. 2007).

In this work, we constrain the topology of the Universe using the method of matched circles proposed by Cornish, Spergel & Starkman (1998) and apply it to the 7-yr Wilkinson Microwave Anisotropy Probe (WMAP) data (Jarosik et al. 2010). In contrast to the majority of previous studies, we will pay special attention to the impact of Galactic foreground residuals on the constraints. Some consideration of this problem was made by Then (2006) in their analysis of the first-year WMAP data release. We will use Monte Carlo (MC) simulations also for the estimation of the false detection level of the statistic. The method is applied to higher resolution maps than previously, which implies a lower level of false detection and therefore tighter constraints on the size of the Universe. As a result of computational limitations, we will restrict the analysis to a search for back-to-back circle pairs.$^1$

In the following two sections, we describe data used in analysis and simulations of the CMB maps for a flat universe with the topology of a three-torus which were used to test the reliability of our implementation. The statistic adopted in our studies is presented in Section 4. The results and conclusions are described in the last two sections.

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$^1$ Pairs of circles centred around antipodal points.
2 DATA

The search for matched circles was performed on the 7-yr WMAP data (Jarosik et al. 2010). The WMAP satellite observes the sky in five frequency bands denoted $K, Ka, Q, V$ and $W$, centred on the frequencies of 22.8, 33.0, 40.7, 60.8, 95.3 GHz with angular resolutions of 52.8, 39.6, 30.6, 21 and 13.2 arcmin, respectively. The maps\(^2\) are pixelized in the HEALPix\(^3\) scheme (Görski et al. 2005) with a resolution parameter $N_{\text{side}} = 512$, corresponding to 3145728 pixels with a pixel size of $\sim 7$ arcmin.

In particular, we used the Internal Linear Combination (ILC) map and the $W$-band map corrected for Galactic emission outside of the applied mask using a template fitting scheme (Gold et al. 2010) and smoothed with a Gaussian beam profile of full width at half-maximum (FWHM) = 20 arcmin to decrease noise level. The former has been used for a number of full-sky analyses, and versions available with earlier data releases have already been exploited in the search for matched circles. The latter is the highest angular resolution data measured by the WMAP satellite. Since the false detection rate for such a map is lower, it provides the opportunity to estimate tighter constraints on topology. We elected not to use data pre-processed as in Cornish et al. (2004), i.e. a noise-weighted combination of the $Q$, $V$ and $W$-band maps smoothed to $O$-band map resolution outside of the Kp2 Galactic cut, and the ILC map within this region. Such a map has complex properties since it consists of two regions with different angular resolutions; moreover, as we will see the ILC map remains too contaminated by Galactic foreground residuals inside the Kp2 cut to be used for cosmological analysis.

3 SIMULATIONS OF THE CMB ANISOTROPY MAPS IN THE MULTICONNECTED UNIVERSE

To test the reliability of codes used to search for the signature of a multiconnected topology in the Universe, we performed simulations of CMB skies for a flat universe with the topology of a three-torus (Riazuelo et al. 2004a). The simulations can be made in two ways: one can compute the covariance matrix\(^4\) of the spherical harmonic coefficients $d_{lm}$, $M_{lm} = \{d_{lm}, d_{lm}^{*}\}$, (Riazuelo et al. 2004b; Phillips & Kogut 2006) and then generate correlated $a_{lm}$ coefficients using the Choleski decomposition of the covariance matrix $M = L L^\dagger$; and uncorrelated unit variance Gaussian variable $x$, $a = L x$, or one can directly compute the $a_{lm}$ coefficients using equation

$$a_{lm}^\gamma = -\frac{(2\pi)^3}{V} i^l \sum_k \Delta_\gamma^\tau(k, \Delta \tau) \frac{P(k)}{k^2} \sum_{|k|=r} Y_{lm}(\Omega_k) \tilde{e}_k,$$

(1)

where $\Delta \tau = \tau_0 - \tau_i$ is the difference between the present conformal time $\tau_0$ and the time of recombination $\tau_i$, $P(k)$ is the density perturbation power spectrum, $\Delta_\gamma^\tau (k, \Delta \tau)$ is the response function for temperature, $\tilde{e}_k$ is a Gaussian random variable which satisfies $\langle \tilde{e}_k \tilde{e}_k^* \rangle = \frac{1}{2\pi k^3} 2\pi V k^3$ and $V$ is the volume of the fundamental domain. The sum over wavevectors $k$ was split into two sums $\sum_k = \sum_{|k|=r} \sum_{|k|<r}$ – the sum over all possible magnitudes of the $k$ vector and over all $k$ of magnitude $k$.

The advantage of the latter approach is a less-aggressive scaling in the computation time, $\ell_{\text{max}}^5 V$ (number of $a_{lm}$ coefficients $\propto \ell_{\text{max}}^5 V$) in comparison with the former\(^5\) that goes as $\ell_{\text{max}}^5 V$. On the other hand, the disadvantage of the latter approach is that for each simulation we have to repeat the computations.\(^6\) Nevertheless, we decided to adopt this method since the difference in scaling is particularly significant for the high-resolution maps of interest in our studies. We also do not need more than one map for a given topology to evaluate the possibility of detection.

The topology does not affect local physics, so the equations governing the evolution of cosmological perturbations are left unchanged. Thus, to compute the response functions, $\Delta_\gamma^\tau (k, \Delta \tau)$, one can use the publicly available CAMB code (Lewis, Challinor & Lasenby 2000). A change of topology translates into a change of the modes that can exist in the universe. Therefore, we need to compute the response function only for a set of allowed wavenumbers $k$. To speed up computations, we use in our code pre-computed response functions. Then, the most time-consuming part is the computation of the sum $\sum_{k=1,|k|=r} Y_{lm}(\Omega_k) \tilde{e}_k$.

The simulations do not include gravitational lensing of the CMB as the lensing deflection angle is small (Seljak 1996), $\sim 3$ arcmin, compared to the minimal angular scale taken into account in the simulations, $\sim 20$ arcmin. The effect of the finite thickness of the last scattering surface is included in the simulations through the response function, $\Delta_\gamma^\tau (k, \Delta \tau)$, computed by CAMB. However, the effect starts to be important at angular scales smaller than $\sim 4$ arcmin and will therefore not be studied here.

3.1 Requirements and numerical implementation

To study the signatures of a given topology, a CMB map is required with resolution comparable to the angular size of the beam profile used to smooth the WMAP W-band data (i.e. $\sim 20$ arcmin) to be analysed here (see Section 2). We have adopted $\ell_{\text{max}} = 500$ in our simulations. The dimension of the fundamental domain of the three-torus was $L = 2 c/H_0$, which is about three times less than the diameter of the observable Universe i.e. $\sim 6.6 c/H_0$. In such a universe, there are many pairs of matched circles of different radii. The time needed for the generation of one such CMB map on a single processor with clock speed of 3 GHz is about 42 h.

4 SEARCHING FOR THE CIRCLES IN THE SKY

If light had sufficient time to cross the fundamental cell, an observer would see multiple copies of a single astronomical object. To have the best chance of seeing ‘around the universe’, we should look for multiple images of the furthest reaches of the universe. Searching for multiple images of the last scattering surface – the edge of the visible universe – is then a powerful way to constrain topology. Because the surface of last scattering is a sphere centred on the observer, each copy of the observer will come with a copy of the last scattering surface, and if the copies are separated by a distance less than the diameter of the last scattering surface, then they will intersect along circles. These are visible by both copies of the observer, but from opposite sides. The two copies are really one observer, so if space

\(^2\) Available at http://lambda.gsfc.nasa.gov.

\(^3\) http://healpix.jpl.nasa.gov

\(^4\) Assuming zero mean of the CMB maps.

\(^5\) The parity and symmetry relations can reduce by an order of magnitude the number of different elements in the matrix and computation time.

\(^6\) In the former case, one need only generate a vector of Gaussian random numbers and multiply by the Choleski decomposed matrix. It is much faster than the full computation.
is sufficiently small, the CMB radiation from the last scattering surface will demonstrate a pattern of hot and cold spots that match around the circles.

The idea of using these circles to study topology is due to Cornish et al. (1998). Therein, a statistical tool was developed to detect correlated circles in all-sky maps of the CMB anisotropy — the circle comparison statistic

\[ S^\pm_{p,r}(\alpha, \phi_*) = \frac{1}{2} \left( \sum_{n,m} T_{p,m} \exp(i m \phi) \right) \]

\[ \times \sum_{n,m} T_{r,m} \exp(-i m \phi) \]

\[ \left( \sum_{n,m} |T_{p,m}|^2 + |T_{r,m}|^2 \right)^{-1/2} , \]

where \( T_{p,m} \) and \( T_{r,m} \) are the temperature fluctuations around two circles of angular radius \( \alpha \) centred at different points, \( p \) and \( r \), on the sky with relative phase \( \phi_* \). The sign \( \pm \) depends on whether the points along both circles are ordered in a clockwise direction (phased, sign +) or alternately whether along one of the circles the points are ordered in an anticlockwise direction (antiphased, sign −). This allows the detection of both orientable and non-orientable topologies. For orientable topologies, the matched circles have antiphased correlations, while for non-orientable topologies, they have a mixture of antiphased and phased correlations. The statistic has a range over the interval \([-1, 1]\). Circles that are perfectly matched have \( S = 1 \), while uncorrelated circles will have a mean value of \( S = 0 \). To find matched circles for each radius \( \alpha \), the maximum value \( S^\pm_{p,r}(\alpha, \phi_*) \) is determined.

In order to speed up the computations, one can use the fast Fourier transform (FFT) along the circles. \( T_{p,m} \) depends on the ratio of the signal rms \( \sigma_s \) to the noise rms \( \sigma_n \) ratio, denoted \( \xi \) for the map, thus the efficiency of the statistic to detect matched circles is increased by smoothing the data. A smoothing scale should therefore be adopted, that is a reasonable trade-off between these two requirements.

To eliminate the regions most contaminated by Galactic foreground residuals, we utilize the masks defined by the WMAP team. However, the statistic is very sensitive to the masking, particularly if a significant fraction of one or both circle pairs lies in the masked region. In this case, there is a significant probability to find a chance correlation of the temperature pattern between unmasked parts of the circles, thus increasing the false detection level. The effect is the most pronounced for circles with the largest radii, close to 90°, as well with very small radii. To avoid this, we restrict our analysis to those pairs for which less than half the length of each circle is masked. Though the statistic computed in this way is not optimal, it is much more robust with respect to masking. It should be noted that the dependence of the statistic on the cut could be avoided if the statistic were expressed as a function of the number of coherent structures along the circles. However, because our principal goal is to constrain the size of the Universe, it is better to use the statistic determined as a function of circle radius.
Finally, Key et al. (2007) have suggested that the performance of the statistic (3) may be further improved by bandpass filtering the input map to minimize the anisotropy contribution from the ISW and Doppler terms which blur any signatures of topology present on the last scattering surface. We chose not to apply this kind of filtering because of possible complications arising from the interaction of the mask and the filter. Our analysis is enhanced by the use of high-resolution maps rather than filtering.

5 RESULTS

Before beginning the search for pairs of matched circles in the WMAP data, we validated our algorithm using simulations of the CMB sky for a universe with three-torus topology for which the dimension of the cubic fundamental domain $L = 2c/H_0$, and with cosmological parameters corresponding to the Lambda cold dark matter ($\Lambda$CDM) model (see Larson et al. 2010; Table 3) determined from the 7-yr WMAP results. In particular, we verified that our code is able to find all pairs of matched circles in such map. The statistic $S_{\text{max}}(\alpha)$ for the map is shown in Fig. 1. Note that the peak amplitudes in the statistic, corresponding to the temperature correlation for matched circles, decrease with radius of the circles. Cornish et al. (2004) noted that this is primarily caused by the Doppler term which becomes increasingly anticorrelated for circles with radius smaller than $45^\circ$. The temperature match on the last scattering surface will be also diluted by the ISW effect which comes from the evolution of the structures close to the observer. However, to a large extent the $m$ weighting used in the statistic (3) mitigates against this and prevents the statistic from being dominated by the large scales.

The intersection of the peaks in the matching statistic with the false detection level estimated for the W-band data (the same as in Fig. 3) defines the minimum radius of the correlated circles which can be detected for this map. The height of the peak with the smallest radius seen in Fig. 1 indicates that the minimum radius is about $\alpha_{\text{min}} \approx 10^\circ$. To emphasize the advantage of using higher resolution data in the analysis, we also show the false detection level estimated for the ILC map (the same as in Fig. 2). In this case, the minimum radius for detectable matched circles is about $\alpha_{\text{min}} \approx 25^\circ$.

The statistics for the WMAP ILC map are shown in Fig. 2. The map was analysed on both the full sky and after applying the KQ85y7 mask (Gold et al. 2010). In the case of the full-sky analysis, the $S_{\text{max}}$ statistic shows some excess for pairs of circle with large radii. Nevertheless, after removing the most foreground-contaminated regions the statistic does not reveal any unusually large values. This indicates that residuals of the Galactic emission, especially in the Galactic plane, cannot be neglected in the analysis of matched circles.

It is interesting that similar behaviour is not seen for a full-sky analysis of the first-year WMAP ILC map. This probably explains why the importance of masking the most contaminated regions was not recognized in previous papers using the matched circles statistic, the only exception being the paper by Then (2006). However, because they studied only the first-year WMAP data, they also came to the conclusion that the ILC map is good enough for the full-sky analysis. We checked that the excess is not related to the bias correction (for residual foreground emission) applied to the 7-yr ILC map by the WMAP team since this was not implemented for the first-year data. Indeed, the excess remains present for the 7-yr ILC constructed by simply co-adding the individual frequency channels with the weights provided in Gold et al. (2010). Deeper studies of this problem are beyond the scope of this paper. Nevertheless, it provides a further warning against the naive use of a full-sky ILC map for cosmological analysis, and particularly with respect

![Figure 1](https://academic.oup.com/mnras/article-abstract/412/3/2104/1058423)
Figure 2. $S_{\pm \text{max}}$ statistic for the 7-yr WMAP ILC map. The solid and dotted lines show the statistics $S_{\text{max}}^-$ and $S_{\text{max}}^+$, respectively, for the ILC map masked with the KQ85y7 mask. The dot-dashed and dashed lines show the statistic $S_{\text{max}}^-$ for the 7-yr and the first-year ILC unmasked maps, respectively. The false detection levels estimated from 100 MC simulations of the ILC map and by scrambling the $a_{\ell m}$ coefficients of the map are marked by dash–three dotted and long-dashed lines, respectively. The peak at $90^\circ$ corresponds to a match between two copies of the same circle of radius $90^\circ$ centred around two antipodal points.

to the two-point correlation function which is closely related to the statistic used here.\(^7\)

The false detection level shown in Fig. 2 was estimated on the basis of 100 simulations of the ILC map with the KQ85y7 mask applied, assuming a simple-connected universe and established from the requirement that fewer than one in 100 simulations should yield a false match. We did not correct the simulated maps for the bias coming from the random correlation between the CMB and the Galactic foreground,\(^8\) since the effect is small outside of the Galactic plane region. Moreover, such a correction does not appear to be reliably evaluated as demonstrated with simulations that have been generated using the same foreground templates as for the simulated ILC maps. Maps corrected in this way do not properly reflect uncertainties concerning details of the Galactic emission, especially in the Galactic plane.

For comparison, we show also the false detection level estimated on the basis of a ‘scrambled’ version of the ILC map (i.e. by randomly exchanging the $n$ spherical harmonic coefficients, $a_{\ell m}$, of the map at every $\ell$) as in Cornish et al. (2004). However, we still applied the KQ85y7 mask for each ‘scrambled’ map to be consistent with simulations of the ILC data. As can be seen, the false detection level is slightly lower in this case. It is not surprising because the scrambling generates maps with the same two-point function and only different phase correlations that result in a smaller variance of the $S$-statistic than for an ensemble of simulated ILC maps.

In order to decrease the false detection level and be able to detect matched circles with smaller radius, we analysed also the WMAP data with the highest angular resolution, i.e. the $W$-band map, corrected for Galactic foregrounds and smoothed with a Gaussian beam profile of FWHM = 20 arcmin to decrease the noise level. The statistic for this map analysed with the KQ85y7 mask is shown in Fig. 3. As for the ILC map, the false detection level was estimated from 100 simulations of the $W$-band data.

We did not find any statistically significant correlation of circle pairs in either map. As seen in Fig. 1, the minimum radius at which the peaks expected for the matching statistic are larger than the false detection level is about $a_{\text{min}} \approx 10^\circ$ for the $W$-band map. Thus, we can exclude any topology that predicts matching pairs of back-to-back circles larger than this radius. This implies that in a flat universe described otherwise by the best-fitting 7-yr WMAP cosmological parameters, a lower bound on the size of the fundamental domain is $d = 2R_{\text{LSS}} \cos (a_{\text{min}}) \approx 27.9$ Gpc, where $R_{\text{LSS}}$ is the distance to the last scattering surface.

Of course, the above constraints do not apply to those universes for which the orientation of the matched circles is impossible to detect due to partial masking on the sky. Examples of topologies which can be overlooked are so-called slab and chimney spaces (Riazuelo et al. 2004a). For such topologies and appropriate configuration, all pairs of matched circles could lie in the Galactic plane that is removed by the mask. The probability of overlooking the circles depends on the specific topology and radii of the circles, so it is difficult to give a general expression. Nevertheless, one can suppose that it decreases as the fraction of the masked sky decreases.

6 CONCLUSIONS

We have studied constraints on the topology of the Universe using the method of matched circles as applied to the 7-yr WMAP ILC map.

\(^7\) The remarks of Efstathiou, Ma & Hanson (2010) regarding the lack of evidence for Galactic foreground residuals at low Galactic latitudes are only relevant for the ILC map when smoothed to 10$^\circ$ resolution.

\(^8\) so-called ‘cosmic covariance’
and the foreground-reduced $W$-band map. We paid special attention to three aspects of the analysis that have been neglected in previous studies – the application of a mask, the use of high-resolution data and the estimation of the false detection level on the basis of detailed MC simulations of the sky maps.

The necessity for the application of a mask is due to the presence of residual Galactic foreground emission present even in the ILC map. These introduce a non-negligible effect on the matched circles statistic that is used for constraining topology. However, the possibility to apply the analysis to masked maps yields the opportunity to more tightly constrain topology by using higher resolution, foreground-corrected $W$-band data. Constraints on the topology depend significantly on the threshold for a significant match of the circle pairs. In order to estimate this correctly, we used 100 MC simulations of the ILC and $W$-band maps assuming a simply connected universe. The level of false detection calibrated in this manner is slightly higher than that derived in Cornish et al. (2004) using a method in which the analysed maps are resampled by shuffling their spherical harmonic coefficients. Although the difference is not very big, it is worth noting that the constraints on the size of the Universe are overestimated if one uses a lower level of false detection.

The analysis of the $W$-band map, after correction using templates of Galactic foreground emission, did not reveal any significant correlations for pairs of back-to-back circles with a radius greater than $\sim 10^\circ$. This substantially extends the previous constraint on the minimum radius of detectable matched circles given by Key et al. (2007) of $20^\circ$. It also places a lower bound on the size of the fundamental domain of about $27.9$ Gpc for a flat universe described by the best-fitting 7-yr $W$-band cosmological parameters. Although this constraint concerns only those universes with such dimensions and orientation of the fundamental domain with respect to the mask that allows the detection of pairs of matched circles, the probability of overlooking circle pairs is rather low for the KQ85y7 mask that removes only a relatively small fraction of the sky.

Of course, observations of the CMB with higher angular resolution and significantly lower noise level by the Planck satellite may yield even tighter constraints on the topology of the Universe. However, one should bear in mind that the possible improvement in the lower bound on the size of the Universe will not be substantial. The current constraint is not much smaller than the diameter of the observable Universe $2R_{\text{LSS}} = 28.3$ Gpc, which imposes a limit on the size of the fundamental domain that it is possible to detect using the method of matched circles. The only significant improvement might be related to improved modelling of the Galactic emission allowing the application of a smaller mask approaching closer to the Galactic plane. This would minimize the probability of overlooking topologies with matched circles that are hidden within the masked region of the sky.

Finally, as in Cornish et al. (2004), the studies could also be extended to search for nearly back-to-back circle pairs. However, the much higher computational requirements for the analysis of high-resolution maps make such studies difficult and extremely time consuming. Nevertheless, we can hope that the steadily increasing speed of processors and availability of larger computational resources will make such computations feasible in the coming years, thus allowing a final resolution of the problem of the topology of our Universe.

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9 http://camb.info/
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