Quantum mechanics without statistical postulates

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Abstract

The Bohmian formulation of quantum mechanics is used in order to describe the measurement process in an intuitive way without a reduction postulate in the framework of a deterministic single system theory. Thereby the motion of the hidden classical particle is chaotic during almost all nontrivial measurement processes. For the correct reproduction of experimental results, it is further essential that the distribution function $P(x)$ of the results of a position measurement is identical with $|\Psi|^2$ of the wavefunction $\Psi$ of the single system under consideration. It is shown that this feature is not an additional assumption, but can be derived strictly from the chaotic motion of a single system during a sequence of measurements, providing a completely deterministic picture of the statistical features of quantum mechanics.

\textit{Key words:} Foundations of Quantum mechanics, Measurement process, Bohmian Interpretation of Quantum mechanics, Statistical postulates in Quantum mechanics, Chaos

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1 Introduction

Since the invention of quantum mechanics overwhelming experimental support for this basic theory of nonrelativistic physics has been accumulated. In contrast to this great success, quantum reality created paradoxes or counter-intuitive behaviour from the very beginning, one of the most prominent being

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"Schrödinger’s cat" and the problem of the reality of a quantum state in the absence of an observer. Most of these problems have their origin in the lack of a microscopic description of the measurement process, where an ad hoc reduction of the wavefunction $\Psi$ is assumed.

In order to avoid this assumption, a deterministic formulation of quantum mechanics has been suggested by David Bohm [1]. In this theory the dynamics of a nonlocal hidden variable is derived from the wavefunction $\Psi$. It is equivalent to the "standard" formulation of quantum mechanics with respect to the prediction of experimental results, but allows for an continuous and conceptually clear analysis of the measurement process without additional assumptions. Up to now it seemed that an additional statistical assumption concerning the distribution $P(x, t)$ of the particles in an ensemble $\{x_i\}$ has to be made, in order to reproduce the experimental results. This is one of the main reasons, why this intuitive classical interpretation of quantum mechanics had been abandoned in the early days, as this assumption is in contrast to a purely deterministic formulation [2,3].

In the following it will be proven that this statistical assumption can be derived strictly from the properties of this purely deterministic theory by considering the chaotic dynamics of the Bohmian particle. This shows that quantum mechanics can be understood completely on the basis of a nonstatistical formulation.

The paper is organized as follows: After a short review of von Neumann’s description of the measurement process in standard quantum mechanics, this approach is reconsidered from the point of view of the Bohmian interpretation. By the investigation of the chaotic motion of the particle it will be demonstrated that quantum equilibrium will be established intrinsically during a sequence of measurements of a single system.

2 Standard quantum mechanics

Within the standard interpretation of quantum mechanics the state of a system is completely described by the wavefunction $\Psi(x, t)$ of the system [4]. The time evolution of the wavefunction is not only determined by the unitary process according to the Schrödinger equation

$$i\hbar \frac{\partial}{\partial t} \Psi(x, t) = -\frac{\hbar^2}{2m} \frac{\partial^2}{\partial x^2} \Psi(x, t) + V(x, t)\Psi(x, t) \quad ,$$

but also by the ad hoc reduction of the wavefunction in case of a measurement. In this process the original wavefunction $\Psi(x) = \sum_n c_n \Psi_n(x)$ which is a
superposition of eigenfunctions $\Psi_n(x)$ of the Observable $\hat{A}_x$, is replaced with probability $|c_n|^2$ by one eigenfunction $\Psi_{n_0}(x)$ with the eigenvalue $a_{n_0}$. For the case of simplicity a discrete spectrum with nondegenerate eigenvalues has been assumed.

In von Neumann’s approach to the measurement process the measurement device is described by a wavefunction $\Phi_0(y)$ with a standard deviation $\sigma_{\Phi_0(y)} > 0$ of $|\Phi_0(y)|^2$ and is considered as an integral part of the total quantum system [4]. As there is no connection between the measurement device and the system to be measured before the measurement, the total initial wavefunction

$$
\Psi_0(x, y) = \Psi(x)\Phi_0(y)
$$

is per definition a product state (Fig. 1(a)). During the measurement an interaction

$$
\hat{H}_{WW} = \lambda \hat{A}_x \hat{p}_y
$$

($\hat{p}_y = -i\hbar \frac{\partial}{\partial y}, \lambda = \text{const.}$) between the detector (with coordinate $y$) and the system ($x$) is assumed, which is strong and short compared with the interaction $V(x, t)$ in the unperturbed Schrödinger equation 1. Therefore the dynamic of the total system during the measurement process is governed by $\hat{H}_{WW}$ alone and the time evolution of the total wavefunction is given by

$$
\Psi(x, y, t) = \sum_n c_n \Psi_n(x)\Phi_0(y - \lambda a_n t).
$$

This shows that the modulus $|\Psi(x, y, t)|^2$ of the total wavefunction separates into disjunct wavepackets along the detector coordinate $y$ during the measurement process (Fig. 1(b)), provided that $\lambda \Delta a \Delta t > \sigma_{\Phi_0(y)}$ ($\Delta t =$ duration of the measurement, $\Delta a = \min_n (a_n - a_{n-1})$). As each wavepacket corresponds to an eigenvalue $a_n$ of the observable $\hat{A}_x$, the choice of the measured quantity influences the modification of the total wavefunction during the measurement process.

Although this formalism is statistically in perfect agreement with all known experiments, for the description of a single measurement also the reduction process has to be understood. This process is intrinsically statistical in the sense that the statistical distribution $|c_n|^2$ of the results $a_n$ can be calculated, but the outcome of a single measurement cannot be predicted in principle. This is the origin of the randomness in the usual interpretation of quantum mechanics. In addition to this, the fact that the behaviour of the system during the measurement cannot be analysed further leads to the famous quantum paradoxes.
Fig. 1. Schematic picture of a von Neumann measurement process: (a) At the beginning of the measurement the system is prepared in the product state \( \Psi(x, y, 0) = \Psi(x)\Phi_0(y) \) of the system \( \Psi(x) \) and the detector \( \Phi_0(y) \) (cf. equ. 2), which is localized around \( y = 0 \). (b) During the measurement process the wavepacket \( |\Psi(x, y, t)| \) splits into different wavepackets along the \( y \)-direction (cf. equ. 4), corresponding to different eigenvalues \( a_n \) of the measured observable \( \hat{A}_x \) of the system.

3 Bohmian quantum mechanics

On the other hand, the Bohmian quantum mechanics is not a statistical, but a single system theory with a principal lack of these problems. Within the Bohmian mechanics, a state of a system is completely determined not only by the wavefunction \( \Psi \), but also by the position \( x(t) = (x_1, \ldots, x_N)(t) \) of a hidden particle in the configuration space of the whole system [1,5–9].

The dynamics of the wavefunction \( \Psi \) is determined from the Schrödinger equation 1 in the usual way, while the dynamic of the particle is deduced from the wavefunction \( \Psi(x, t) \). By introducing the modulus \( R(x, t) \) and the phase \( S(x, t) \) of the wavefunction \( \Psi(x, t) = R(x, t)e^{\frac{i}{\hbar}S(x,t)} \) the Schrödinger equation (1) can be rewritten as

\[
\frac{\partial}{\partial t} S(x, t) = \frac{(\frac{\partial}{\partial x} S(x, t))^2}{2m} - \frac{\hbar^2}{2m} \frac{\partial^2}{\partial x^2} R(x, t) + V(x, t) ,
\]

\[
\frac{\partial}{\partial t} R(x, t)^2 + \frac{\partial}{\partial x} \left( R(x, t)^2 \frac{\partial}{\partial x} S(x, t) \right) = 0 .
\]

While equation (6) represents a continuity equation for \( |\Psi(x, t)|^2 \), equation (5) can be interpreted as a Hamilton Jacobi equation of a classical particle with coordinate \( x \) in the potential \( -\frac{\hbar^2}{2m} \frac{\partial^2}{R(x, t)} + V(x, t) \). As a consequence, the momentum \( p(t) \) and the energy \( E(t) \) of the particle are determined by the phase \( S(x, t) \) of the wavefunction \( \Psi(x, t) \) at its position \( x(t) \):

\[
p(t) := \frac{\partial}{\partial x} S(x, t)|_{x(t)} , \quad E(t) := -\frac{\partial}{\partial t} S(x, t)|_{x(t)} .
\]
3.1 Measurement Process

The interpretation of the quantum mechanical reality in the framework of Bohmian quantum mechanics allows for an elegant and conceptually clear description of a single measurement without the necessity of a reduction process \[6,8,10\]. In the following a Bohmian extension of the von Neumann measurement process explained above will be developed. Thereby not only the additional particle has to be taken into account, but the whole concept of the measurement process has to be reconsidered.

Without loss of generality it is assumed that in a first step (i) there will be an interaction of the investigated system and a microscopic detector with one or a few degrees of freedom. After that there will be a macroscopic measurement (ii) of the state of the microscopic detector after the interaction.

i) The detector is described by a particle at the position \(y(t)\) and a wavefunction \(\Phi_0(y)\) with a standard deviation \(\sigma_{\Phi_0(y)}\) of \(|\Phi_0(y)|^2\). At all times during the interaction an additional particle in the configuration space of system \(x\) and detector \(y\) at the position \((x, y)(t)\) is present in the Bohmian theory. The evolution of the wavefunction \(\Psi(x, y, t)\) during this process is the same as in the standard von Neumann theory described above (cf. chapter 2). From the knowledge of \((x, y)(0)\) and \(\Psi(x, y, 0)\) the values of \((x, y)(t)\) can be calculated with equation (7) at any time \(t \in [0, \Delta t]\) during the measurement. Due to the equation (7) the dynamics of the particle is exclusively determined by the local behaviour of the wavefunction \(\Psi(x, t)\) at the position \(x(t)\) of the particle.

The crucial point is that after the different wavepackets of \(|\Psi(x, y, t)|^2\) are separated in the configuration space of the detector and the system at the time \(\Delta t\) (cf. equ. 4), the particle \((x, y)(\Delta t)\) is influenced only locally by one wavepacket \(|\Psi_{\lambda a_n}(y - \lambda a_n \Delta t)|^2\) belonging to the single eigenfunction \(\Psi_{\lambda a_n}(x)\) and the eigenvalue \(a_{\lambda a_n}\) respectively.

As a consequence of the separation of the wavefunction \(\Psi\) into disjunct wavepackets, the result \(a_{\lambda a_n}\) of the measurement of \(\hat{A}_x\) is coded in the position \((x, y)(\Delta t)\) of the particle in one of the subsets

\[
M_{\lambda a_n} := \{(x, y) \mid |\Psi_{\lambda a_n}(x)\Phi_0(y - \lambda a_n \Delta t)|^2 \neq 0\}
\]

of the total phase space.

A special, but very important observable is the position measurement, as any experiment involves at least the reading out of the position of some detector coordinate, e.g. of a pointer on a display \[4,11\]. The accuracy \(\Delta x > 0\) of a position measurement is thereby in principle limited by the (unavoidable)
Fig. 2. Schematic picture of a position measurement with finite accuracy $\Delta x = \frac{1}{3}$ in the Bohmian formulation: (a) At the beginning of the position measurement the situation is the same as in Fig. 1(a). (b) During the interaction of system and detector there is a separation of wavepackets in $y$-direction. Each corresponds to one measurement interval $[0, \frac{1}{3}], [\frac{1}{3}, \frac{2}{3}]$ or $[\frac{2}{3}, 1]$. The deviation of the wavepacket is proportional to the position eigenvalue $x_i$. The position $(x, y)(\Delta t) \in M_{x_i}$ (cf. equ. 8) of the particle at the end of the measurement in one of the wavepackets determines the result $x_i$ of the experiment.

An important point is that the $x$-coordinate of the particle $(x, y)(t)$ does not change during the measurement, as the dominating interaction $\hat{H}_{WW}$ only affects the wavefunction in the direction $y$. As a consequence, the position $x(t)$, where the particle is detected during the measurement, coincides with the initial position

$$x(0) = x(t) = x(\Delta t) \quad \forall t \in [0, \Delta t] \quad .$$

This is a nontrivial feature of the position measurement, which is not true for arbitrary observables.

ii) For a complete measurement the information about the state of the microscopic detector with a few degrees of freedom has to be read out by a macroscopic device. This is described by a particle $z(t) = (z_1, \ldots, z_N)(t)$ and a wavefunction $\chi_0(z)$ with a standard deviation $\sigma_{\chi_0(z)}$, $N$ being of the order of $10^{23}$.

The macroscopic limit of large $N$ has two important implications: Firstly, a macroscopic experiment can be realized (without loss of generality) by a position measurement of the position $y$ via an interaction $\hat{H}_{WW}' = \lambda' \hat{y} \hat{p}_z$. Due to the special property (9) of the position measurement the result $a_{m_0}$ of the measurement (i) contained in the coordinate $y$ (i.e. $(x, y) \in M_{m_0}$) will not be affected by the manipulation (ii) along the $z$-coordinate described in the
Secondly on the macroscopic level the information about a measurement result can be stored for a sufficiently long time, while in step (i) the wavepackets separated along $y$ can overlap again in the course of the future dynamics. If the overlap $\int dz_n \Psi_1(z_n)\Psi_2(z_n) \sim \epsilon < 1$ of the wavepackets $\Psi_n$ in one dimension $z_i$ is small, the interference $\int d^N z \Psi_1(z)\Psi_2(z)$ of the complete wavefunctions in $N$ dimensions will be suppressed $\sim \epsilon^N \ll 1$. If the interference is missing in at least one dimension, the total overlap is even exactly zero. This is a necessary condition for the storage of the information of the measurement (i) in a detector by the position of the particle $z(t)$, as in this case the particle trajectory is unable to leave the support of the wavepacket corresponding to the eigenvalue $a_{no}$ (at least in $z$-direction) anymore (cf. equ. 8).

Thus the Bohmian quantum mechanics describes the complete measurement in a very elegant and clear way without the need of a reduction process.

### 3.2 Deterministic Chaos

Chaotic phenomena within the Bohmian quantum mechanics have been studied repeatedly [12–14]. As it is possible to construct a trajectory $(x(t), p(t))$ of the particle in the phase space of position and momentum coordinates, deterministic chaos can be defined with the well known Lyapunov exponent in the same way as in classical mechanics [12]. In the Bohmian quantum mechanics also the measurement is a deterministic process, whose chaotic properties can be studied [8,13,14]. During the first part (i) of the measurement process presented above the dynamics of the system $x(t)$ and the detector $y(t)$ are given by the system

\begin{equation}
\dot{x}(t) = \frac{1}{m_x} \frac{\partial}{\partial x} S(x,y,t)\big|_{(x,y)(t)} , \quad \dot{y}(t) = \frac{1}{m_y} \frac{\partial}{\partial y} S(x,y,t)\big|_{(x,y)(t)} . \quad (10)
\end{equation}

of differential equations. During a nontrivial measurement, which is connected with a modification of $\Psi(x,y,t)$ according to equ. 4, the phase $S(x,y,t)$ is time dependent. As the dimension of both the system and the detector is at least one, the Poincaré-Bendixson-Theorem [12], which excludes chaos in autonomous systems of dimension $n \leq 2$, is not applicable. Therefore in the general case chaotic dynamics of the hidden variable during the measurement process can be expected.

This behaviour becomes more explicit if a sequence of measurements of a single system is considered, where the information about the experimental results is stored in a macroscopic device and the system is repeatedly prepared in the
initial states an infinite number of times.

For simplicity a sequence of position measurements with only two intervals 
\([0,1]\) and \([1,2]\), for a wavefunction \(\Psi(x)\) with a constant value of \(|\Psi(x)|^2\) in each interval is considered (cf. Fig. 3(a)). Because of the interaction of

the system with the detector the two wavepackets corresponding to the two
intervals separate from each other in the direction of the detector coordinate

\(y\) (Fig. 3(b)) due to equation (4).

Note that in the framework of Bohmian quantum mechanics the result \(x_i\)
of a measurement is coded in the position of the particle \((x,y) \in M_{x_i}\) in
one of the separated wavepackets corresponding to the interval \([x_i, x_i+\Delta x]\).

It is pointed out that this information will not be affected by the following
measurements, if it is preserved by the process (ii) in the \(z\)-coordinate of a
macrocopic device. As the overlap of the wavepackets vanishes in at least
one dimension and the state \((x,y)\) of the system cannot leave this sector of
phase space anymore, the other part of configuration space is irrelevant for
all future dynamics and can therefore be neglected (Fig. 3(c)). Thereby the
measurement process breaks the ergodicity of the trajectory \((x,y)(t)\). For a
new preparation in the initial state (Fig. 3(a)) this remaining wavepacket has
to flow into the form \(|\Psi(x,y,t_0)|\) at \(t = t_0\) (Fig. 3(d)). This means that the
phase space accessible for particle trajectories \((x,y)(t)\) will be enlarged along
the \(x\) coordinate from Fig. 3(c) to 3(d).

During this whole process the position \(2x_n\) of the particle during the \(n\)–th
measurement can be formally mapped onto a Bernoulli-shift \(x_{n+1} = 2x_n \mod 1\).

Thereby the rescaling with the factor 2 is due to the flow of the wavepacket
– and of the ergodic particle trajectory \(x(t)\) respectively – during the prep-
eration of the initial state for the next measurement. The calculation \(\mod 1\)
corresponds to the fact that without loss of generality only the wavepacket
containing the particle is kept after the measurement and that the support
\(M_{x_n}\) of this wavepacket can be identified with the original phase space. In
other words: the series of measurements and repreparations leads to a Bäcker-
Transformation (closely connected to a Bernoulli-shift) of the accesible area
in phase space, in which the trajectory \(x(t)\) is ergodic.

The results obtained in the simple model presented here also hold true for
a general position measurement with finite measurement intervals, where a
generalized form of the Bernoullishift is to be used. As the Bernoulli-shift is the
standard example and basic ingredient of chaotic motion [15], the motion of
the hidden variable shows deterministic chaos during any (position) measurement.

Although the position of the particle evolves deterministicaly from the initial
value \(x(0)\), the intrinsic inaccuracy \(\Delta x\) of any measurement together with
the mixing property of the dynamics prevents from the complete knowledge
Fig. 3. Measurement and repreparation: (a) At the initial time $t = t_0$ an arbitrary wavefunction $\Psi(x, y, t_0)$ is to be measured by a detector with coordinate $y = 0$. (b) During the interaction between system and detector there is a separation of two parts of the wavefunction corresponding to each interval $[0, 1], [1, 2]$. (c) After the registration of the result of a measurement the wavepacket without the particle (here $[0, 1]$) does not influence the particle dynamics anymore and can therefore be neglected. (d) For a new measurement with the same initial state (a) the remaining wavepacket has to flow from one to both intervals into the original form $|\Psi(x, y, t_0)|$.

of the system. Therefore in this theory the result of a future measurement can in principle not be predicted from the history of the system, although no stochastic features have been introduced in the theory. In this sense it might be that God does not play at dice, but we do not look closely enough to discover.

3.3 Quantum equilibrium

3.3.1 Problem

First consider an ensemble $\{x_i\}$ of independent systems with the same wavefunction $\Psi(x, t)$, but different positions $x(t)$ of the particles (cf. Fig. 4), according to the distribution function $P(x, t)$. The relation $P(x, t) = |\Psi(x, t)|^2$ is called "quantum equilibrium" [16], while the alternative $P(x, t) \neq |\Psi(x, t)|^2$ is ruled out by experimental results (cf. Fig. 4).

Because of the analogous form of the continuity equations
Let an ensemble be a collection of \( N \) independent systems, which have a wavefunction \( \Psi(x_i, t_0) \) with the same \( |\Psi(x_i, t_0)|^2 \) for all \( i \in \{1, \ldots, N\} \), the position \( x(t_0) \) of the particle being distributed according to the position density \( P(x, t) \). The case \( P(x, t_0) = |\Psi(x, t_0)|^2 \) is called quantum equilibrium, which is derived in chapter 3.3.2 from the properties of the measurement process.

\[
0 = \frac{\partial}{\partial t} P(x, t) + \frac{\partial}{\partial x} \left( P(x, t) \frac{\partial}{\partial x} S(x, t) \right) \quad \text{or} \quad \frac{|\Psi(x, t)|^2}{m},
\]

\[
0 = \frac{\partial}{\partial t} |\Psi(x, t)|^2 + \frac{\partial}{\partial x} \left( |\Psi(x, t)|^2 \frac{\partial}{\partial x} S(x, t) \right) \quad \text{or} \quad \frac{|\Psi(x, t)|^2}{m}
\]

for \( P(x, t) \) and \( |\Psi(x, t)|^2 \) it is sufficient to assume or derive \( P(x, t_0) = |\Psi(x, t_0)|^2 \) at an arbitrary time \( t_0 \), in order to guarantee quantum equilibrium for all times.

The problem is that this assumption originally made by D. Bohm [2,3] has a statistical character, which is in contrast to the rest of the Bohmian quantum mechanics, which is a single system theory without any statistical inputs. Several propositions have been made to justify this: random collisions [17], coarse graining [18], subquantum fluctuations [2,6,19] or properties of the total wavefunction of the universe [16,20,21].
3.3.2 Derivation from the measurement process

In the following this problem will be solved by considering the results of a sequence of measurements of a single system, which exactly coincides with the real physical situation to be described.

Before each measurement at the times \( t_i \) the wavefunction is prepared again in the state \( \Psi(x, t_0) = \Psi(x, t_i) \). As already discussed in chapter 3.1 the registration of the detector state involves at least one position measurement. As it has also been proven that the particle trajectory during any nontrivial position measurement is chaotic (cf. chapter 3.2), it can be concluded that the motion of the particle during almost all nontrivial measurements is ergodic [22].

Let \( P(x) \) be the distribution of the position \( x_i = x(t_i) \) determined in a sequence of measurements of a single system. Due to ergodicity of the trajectory \( x(t) \) the distribution \( P(x) \), which is obtained along one trajectory \( x(t) \) at different times \( t_i \), can also be expressed by the probability distribution \( \mathcal{P}(x, t_0) \) of an appropriate fictive ensemble of particles \( \{x_i\} \) at a fixed time \( t_0 \):

\[
P(x) = \mathcal{P}(x, t_0) .
\]

(13)

Formally this identity can be concluded from the coincidence of the time average \( \langle x^k \rangle_t := \frac{1}{M} \sum_i x^k(t_i) \) and the ensemble average \( \langle x^k \rangle_e := \frac{1}{V} \int x^k dx \) in ergodic systems for all \( k \in \mathbb{Z} \). Note that here the ensemble \( \{x_i\} \) is not introduced by an additional statistical assumption, but follows from the proven ergodicity of the dynamical system under investigation.

It will now be demonstrated that the distribution function of any ensemble of particles, which move according to equation (7), is determined uniquely by the restriction posed by the continuity equations 11 and 12 for \( \mathcal{P}(x, t) \) and \( |\Psi(x, t)|^2 \). For technical details see [8,23].

Note that the continuity equation

\[
\frac{\partial}{\partial t} \mathcal{P}(x, t) + \frac{\partial}{\partial x} \left( \mathcal{P}(x, t) \frac{\partial S(x, t)}{m} \right) = 0,
\]

(14)

is formally identical to equation (12). Defining \( f(x, t) \) implicitly by

\[
\mathcal{P}(x, t) = f(x, t)|\Psi(x, t)|^2
\]

(15)

and inserting (15) and (12) into (14) we get \( \frac{df}{dt}f(x(t), t) = 0 \) for particle trajectories \( x(t) \), i.e. \( f(x, t) \) is constant along trajectories. As the trajectory is ergodic, it is a dense subset of phase space and \( f(x, t) \) is constant in the whole
accessible area. Because of the normalisation \( \int \mathcal{P}(x, t) dx = \int |\Psi(x, t)|^2 dx = 1 \) it follows that \( f(x, t) = 1 \) and

\[
\mathcal{P}(x, t) = |\Psi(x, t)|^2 .
\] (16)

Finally we can conclude that the distribution \( P(x) \) of the real experimental results during a sequence of measurements in a single system is given by

\[
P(x) \equiv_{13} \mathcal{P}(x, t_0) \equiv_{16} |\Psi(x, t_0)|^2 = |\Psi(x, t_i)|^2 ,
\] (17)

where the last step follows from the consecutive repreparation of the system at times \( t_i \) in the initial state.

This means that the density \( P(x) \) of particle positions in a sequence of measurements is identical to \( |\Psi(x, t)|^2 \) of the wavefunction of the single systems which is prepared before each measurement.

This central result indicates that the microscopic, deterministic dynamics of a single system during a sequence of measurements produces the statistical predictions of the standard quantum mechanics without any statistical assumptions. Thereby it turns out that deterministic chaos is responsible for the statistics of quantum mechanics in a similar way as it provides a microscopic foundation of (classical) statistical mechanics via the proof of Boltzmann’s \( H \)-theorem.

4 Conclusion

Within the Bohmian quantum mechanics the whole measurement process including its statistical properties can be described for a single system as a deterministic process without the assumption of a reduction collapse of the wavefunction.

Deterministic chaos can be introduced to Bohmian quantum mechanics in the same way as in classical mechanics. In particular it has been demonstrated that the motion of the Bohmian particle during the measurement is intrinsically chaotic.

From this it can be concluded that Bohmian quantum mechanics gives the usual statistical prediction of quantum mechanics without any statistical assumptions within a single system theory. The uncertainty in the result of a quantum mechanical measurement follows from the interplay of the chaotic motion of the hidden variable \( x(t) \) and the finite accuracy \( \Delta x \) of any real measurement. Also the experimentally confirmed probability postulate \( P(x) = \)
\[ |\Psi(x, t)|^2 \] can be derived as a time average of a sequence of measurements of the same system. In the standard formulation of quantum mechanics this feature is not derived from a physical process, but ad hoc introduced by the reduction collapse of the wavefunction. Our result shows that the appearance of deterministic chaos allows for the derivation of all statistical properties of quantum mechanics within a causal formulation for a single system.

More detailed investigations of the presented results and implications for the classical limit of Quantum mechanics will be presented in a forthcoming publication [23].

References

[1] D. Bohm, A suggested interpretation of the quantum theory in terms of ‘hidden’ variables I/II, Phys. Rev. 85, (1952) 166 and 180.

[2] D. Bohm, Model of the causal interpretation of quantum theory in terms of a fluid with irregular fluctuations, Phys. Rev. 96 (1954) 208.

[3] J. B. Keller, Bohm’s interpretation of the quantum theory in terms of ‘hidden’ variables, Phys. Rev. 89 (1953) 1040.

[4] J. von Neumann, Mathematische Grundlagen der Quantenmechanik (Springer-Verlag, Berlin, 1932).

[5] D. Albert, Bohm’s alternative to quantum mechanics, Scientific American, 5 (1994) 32.

[6] D. Bohm, B. Hiley, The undivided universe – an ontological interpretation of quantum mechanics, (Routledge, 1993).

[7] J. Cushing, A. Fine, S. Goldstein, Bohmian mechanics and quantum theory: an appraisal (Kluwer Academic Publishers, Dordrecht, 1996).

[8] H. Geiger, Quantenmechanik ohne Paradoxa – Messprozeß und Chaos aus der Sicht der Bohmschen Quantenmechanik (Mainz Verlag, Aachen, 1998), Ph.D. thesis.

[9] P. Holland, The quantum theory of motion (Cambridge University press, Cambridge, 1993).

[10] D. Bohm, B. Hiley, Measurement understood through the quantum potential approach, Found. Phys. 14 (1984) 255.

[11] E. Squires, Why is position special?, Foundations of Physics Letters 3 (1990) 87.

[12] U. Schwengelbeck, F. Faisal, Definition of Lyapunov exponents and KS entropy in quantum dynamics, Phys. Lett. A 199 (1995) 281.
[13] D. Dürr, S. Goldstein, N. Zanghi, Quantum chaos, classical randomness and Bohmian mechanics, Journal of Statistical Physics 68 (1992) 259.

[14] C. Dewdney, Z. Malik, Measurement decoherence and chaos in quantum pinball, Phys. Lett. A 220 (1996) 183.

[15] R. Hilborn, Chaos and nonlinear Dynamics (Oxford University Press, 1994).

[16] D. Dürr, S. Goldstein, N. Zanghi, Quantum equilibrium and the origin of absolute uncertainty, Journal of Statistical Physics 67 (1992) 843.

[17] D. Bohm, Proof that the probability density approaches $|\Psi|^2$ in causal interpretation of the quantum theory, Phys. Rev. 89 (1953) 458.

[18] A. Valentini, Signal-locality, uncertainty, and the subquantum $H$-theorem I/II, Phys. Lett. A 156 (1991) 5.

[19] D. Bohm, B. Hiley, Non-locality and locality in the stochastic interpretation of quantum mechanics, Phys. Rep. 172 (1989) 93.

[20] D. Dürr, S. Goldstein, N. Zanghi, Quantum mechanics, randomness, and deterministic reality, Phys. Lett. A 172 (1992) 6.

[21] D. Dürr, S. Goldstein, N. Zanghi, A global equilibrium as the foundation of quantum randomness, Found. Phys. 23 (1993) 721.

[22] H. Schuster, Deterministic Chaos – An Introduction, 2nd edition (VCH, Weinheim, 1988).

[23] H. Geiger, G. Obermair, Measurement process and chaos in the Bohmian Quantum mechanics, in preparation.