Online Electric Vehicle Charging Control with Multistage Stochastic Programming

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Abstract—With the increasing adoption of plug-in electric vehicles (PEVs), it is critical to develop efficient charging coordination mechanisms that minimize the cost and impact of PEV integration to the power grid. In this paper, we consider the optimal PEV charging scheduling problem in a practical scenario where the non-causal information about future PEV arrivals is not known in advance, but its statistical information can be estimated. This leads to an “online” charging scheduling problem that is formulated as a multistage stochastic programming (MSP) problem in this paper. Instead of solving the MSP using the standard approaches such as sample average approximation (SAA), we show that solving a deterministic counterpart of the stochastic problem yields significant complexity reduction with negligible performance loss. Compared with the SAA approach, the computational complexity of the proposed algorithm is drastically reduced from $O(\epsilon^T)$ to $O(T^3)$ per time stage, where $T$ is the total number of the time stages and $\epsilon$ is a constant related to the accuracy of the solution. More importantly, we show that the proposed algorithm can be made scalable when the random process describing the arrival of charging demands is first-order periodic. That is, the complexity of obtaining the charging schedule at each time stage is $O(1)$ and is independent of $T$. Extensive simulations show that the proposed online algorithm performs very closely to the optimal online algorithm. The performance gap is smaller than 0.4% in most cases. As such, the proposed online algorithm is very appealing for practical implementation due to its scalable computational complexity and close to optimal performance.

I. INTRODUCTION

A. Background and Contributions

The massive introduction of plug-in electric vehicles (PEVs) imposes great challenges to power system operation, such as voltage deviation, increased power losses, distribution network overloading, and higher peak load demands. It is of critical importance to design PEV charging mechanisms that minimize the cost and impact of PEV integration [3]. Previously, PEV charging coordination has been extensively studied to minimize power loss, minimize load variance, or maximize load factor, etc [4]–[11]. It has been shown that the optimal charging schedules are the ones that flatten the electricity load demand (or the total PEV charging rate if the base load is not considered) over time [12].

Ideally, the load demand can be flattened as much as possible if the information about future charging demand is known non-causally when calculating the scheduling. For instance, [5] assumes that all PEVs negotiate with the charging station about their charging demands one day ahead. However, in practice, a PEV charging station knows the charging demand of a PEV only after it arrives at the station. Assuming that the charging station has no information about the incoming charging demand in the future, [2], [9]–[13] studied “online” charging scheduling mechanisms that rely on the causal information only. In particular, our work in [2] achieves a “best-known-so-far” competitive ratio, which guarantees the performance of the algorithm in the worst case. In practice, the online charging algorithms in [2], [8]–[13] are overly conservative in the sense that they are based on zero information of the future charging demand. Thus, they mainly focus on the worst-case performance guarantee, so that they are able to perform reasonably well for any unpredictable sequences of incoming charging demands. In practice, however, the statistical information of the future charging demands can often be acquired through historic data. For example, the demand profile at a particular location could be statistically identical at the same time every day during weekdays (or weekends). Moreover, we are typically more interested in the average performance (e.g., average charging cost) of an algorithm rather than the worst case performance.

In this paper, we consider the optimal PEV charging scheduling, assuming that the future charging demand is not known a priori, but its statistical information can be estimated. In particular, we define the cost of PEV charging as a general strictly convex increasing function of the instantaneous load demand. Minimizing such a cost leads to a flattened load demand, which is highly desirable for many reasons [4]–[11]. The online PEV charging scheduling problem is formulated as a multistage stochastic programming (MSP) problem that minimizes the average cost given that all the charging demands can be fulfilled before their predetermined deadlines.
MSP problems are typically solved by scenario approximation approaches, such as the sample average approximation (SAA) approach based on Monte Carlo sampling techniques [14]–[19]. To guarantee a solution with accuracy $1 - \epsilon$, SAA needs to generate a large amount of scenarios and requires a computational complexity of $O(e^T)$ at each time stage, where $T$ is the total number of the time stages [15]. That is, the computational complexity grows exponentially with the number of stages $T$, and quickly becomes intractable when $T$ is modestly large. Thus, it is of great interest to design tractable approximations that achieve efficient computation of near-optimal solutions [16]–[19]. In this paper, we argue that solving a deterministic counterpart that replaces all random variables with their statistical means can greatly reduce the complexity to $O(T^3)$ without noticeable performance loss for the online PEV charging scheduling problem. More importantly, we rigorously analyze a standard performance metric, the Value of the Stochastic Solution (VSS), which represents the gap between the solution of the approximate approach and that of MSP problem [16], [18], and show that it is bounded regardless of the distribution of random variables. Furthermore, we show that the proposed online algorithm can be made scalable when the random process describing the arrival of charging demands is first-order periodic. That is, the complexity of obtaining the charging schedule at each time stage is reduced to $O(1)$ and is independent of $T$. Extensive simulations show that the proposed algorithm performs very closely to the optimal online algorithm. The performance gap is smaller than 0.4% in most cases. Besides, when the variance of random variables increases 8 times, the performance gap between the proposed online algorithm and optimal online algorithm increases by at most 0.1%.

The rest of the paper is organized as follows. We introduce the offline optimal PEV charging problem in Section II. In Section III, we formulate the online PEV problem as MSP problem. In Section IV, we propose a polynomial-time algorithm and analyze its performance gap. The $O(1)$-complexity algorithm is given when the arrival process is first-order periodic in Section V. Simulation results are presented in Section VI. Finally, the paper is concluded in Section VII.

II. Offline Optimal PEV Charging Problem

A. Problem Formulation

We consider the PEV charging scheduling problem, in which PEVs arrive at the charging station at random instants with random charging demands that must be fulfilled before a random departure time. The entire system time is divided into $T$ equal-length time slots. Let $\mathcal{N}$ denote the set of PEVs that arrive during the system time. Notice that for a given time slot $t$, the charging rate $x_{it}, \forall i \in \mathcal{I}(t)$. To satisfy the demand $d_i$, $x_{it}$ must satisfy

$$\sum_{t=t_i^{(s)}}^{t_i^{(c)}} x_{it} = d_i.$$  \hspace{1cm} (1)

Let $s_t$ be the total charging rate of time slot $t$, i.e.,

$$s_t = \sum_{i \in \mathcal{I}(t)} x_{it}, \forall t = 1, 2, \cdots, T,$$  \hspace{1cm} (2)

which is also called charging load at time $t$. The total load consists of both the charging load and the inelastic base load in the same location. The base load, denoted by $l_t$, represents the load of other electricity consumptions at time $t$ except for PEV charging. Then, the total load at time $t$, denoted by $y_t$, is given by

$$y_t = s_t + l_t = \sum_{i \in \mathcal{I}(t)} x_{it} + l_t.$$  \hspace{1cm} (3)

Suppose that the charging cost at time $t$ is a strictly convex increasing function of the total load, denoted by $f(s_t + l_t)$. Then the total cost over time $T$ is computed as $\sum_{t=1}^{T} f(s_t + l_t)$. In the ideal case when all PEVs’ charging demands, including $t_i^{(s)}, t_i^{(c)}$, and $d_i$ are known non-causally at the beginning of the system time, the charging station can solve (4) and obtain the optimal charging schedule for all time $t$ before the system time starts. Such a solution is referred to as an “optimal offline solution”.

$$\begin{align*}
\min_{x_{it}} & \quad \sum_{t=1}^{T} f(s_t + l_t) \\
\text{s.t.} & \quad s_t = \sum_{i \in \mathcal{I}(t)} x_{it}, \forall t = 1, \cdots, T, \\
\quad & \quad \sum_{t=t_i^{(s)}}^{t_i^{(c)}} x_{it} = d_i, \forall i \in \mathcal{N}, \\
\quad & \quad x_{it} \geq 0, \forall t = t_i^{(s)}, \cdots, t_i^{(c)}, \forall i \in \mathcal{N}.
\end{align*}$$  \hspace{1cm} (4a-4d)

In particular, the optimal solution to (4) is denoted as $x_{it}^*$, and the optimal total charging rate, denoted by $s_t^*$, is defined as

$$s_t^* = \sum_{i \in \mathcal{I}(t)} x_{it}^*.$$  \hspace{1cm} (5)

Then, the optimal value of (4), denoted by $\Psi_{off}$, is

$$\Psi_{off} = \sum_{t=1}^{T} f(s_t^* + l_t).$$  \hspace{1cm} (6)

B. Problem Size Reduction

Note that there are in total $O(T|\mathcal{I}(t)|)$ variables in (4), where $|\mathcal{I}(t)|$ denotes the cardinality of the set $\mathcal{I}(t)$. This number can be quite large when the number of cars present at each time slot, $|\mathcal{I}(t)|$, is large. In this subsection, we propose an equivalent transformation of (4) that drastically reduces the number of variables. In particular, the following Theorem 1
shows that as long as we find the optimal \( s^*_t \) \( \forall t \), the optimal \( x^*_i \) \( \forall i, t \) can be obtained by earliest deadline first (EDF) scheduling.

**Theorem 1:** If a set of \( s_t \)'s satisfy the following inequality for all \( n = 1, \cdots, T \)

\[
\sum_{t=1}^{n} s_t \geq \sum_{t=1}^{n} \sum_{i \in \{i_l(t) = t\}} d_i, \forall n = 1, \cdots, T. \tag{7}
\]

then there exists at least a set of \( x_i \)'s that is feasible to (4). One such set of \( x_i \)'s can be obtained by EDF scheduling, which charges, at each time \( t \), the PEV \( i \in I(t) \) with the earliest deadline at a rate \( s_t \). Moreover, when \( s_t = s^*_t \), the set of \( x_i \)'s obtained by EDF scheduling are the optimal solution, \( x^*_i \), to (4).

To see Theorem 1 note that (7) implies that the total energy charged by any time slot \( n \) is no less than the total charging demand that must be satisfied by time \( n \). On the other hand, by EDF scheduling, PEVs with earlier deadlines must be fully charged before those with later deadlines can be charged. Thus, (7) guarantees the fulfillment of the charging demands of each individual PEV.

With Theorem 1 we can transform (4) to the following equivalent problem with \( T \) variables.

\[
\min_{s_t} \sum_{t=1}^{T} f(s_t + l_t) \tag{8a}
\]

s.t.

\[
\sum_{t=1}^{n} s_t \geq \sum_{j=1}^{n} \sum_{i \in \{i_l(t) = j\}} d_i, \forall n = 1, \cdots, T. \tag{8b}
\]

\[
\sum_{t=1}^{n} s_t \leq \sum_{j=1}^{n} \sum_{i \in \{i_l(t) = j\}} d_i, \forall n = 1, \cdots, T. \tag{8c}
\]

The optimal solution \( s^*_t \) to (8) has an interesting feature: it does not change with the cost function \( f(.), \) as long as \( f(.) \) is strictly convex and increasing. This is proved in Theorem 2.

**Theorem 2:** The optimal solution \( s^*_t \) to (4) or (8) does not change with the cost function \( f(.), \) as long as \( f(.) \) is convex and increasing. Moreover, \( s^*_t \) is essentially a load flattening solution.

**Proof:** Please see the detailed proof in Appendix VIII-A.

Theorem 2 shows that the optimal solution to (4) is essentially a load flattening solution. Under mild conditions, this solution is also the one that minimizes the load factor, the load variance, and the distribution network overloading [4].

The formulations in (4) and (8) are offline in the sense that all the charging demands are known non causally at the beginning of the system time. In the next section, we will formulate the online problem, where the charging schedule at time slot \( k \) only depends on the causal information available at that time. The charging schedule at time \( k \), once determined, cannot be changed in the future.

### III. Online PEV Charging Problem

#### A. MSP Problem Formulation

In contrast to the offline algorithm that solves (8) only once at the beginning of system time, the online charging scheduling algorithm computes, at each time slot \( k \), the charging rate \( s_k \) based on the charging demands that have arrived so far. In particular, \( s_k \) is computed by solving a problem similar to (8), except that (i) the objective function is now an expectation of charging cost over the random PEV arrivals in the future, (ii) the sum over time in both the objective function and constraints starts from \( k \) instead of 1, and (iii) the charging demands at the right hand side of the constraints are replaced by the unfinished charging demands that have not yet been fulfilled by time \( k \). Specifically, the unfinished charging demand of PEV \( i \) at time \( k \) is given by

\[
\hat{d}_i^k = d_i - \sum_{t = l'(i)}^{k-1} x_{i,t} \tag{9}
\]

Note that, \( d_i = d_i^k \) for all PEVs that have not yet arrived by time \( k - 1 \). A close look at (8) suggests that the charging schedule \( s_t \) only depends on the total charging demand that needs to be finished before a certain time, but not the demand due to individual PEVs. Thus, for notational simplicity, we define

\[
\tilde{d}_i^k = \sum_{t \in \{i_l(t) = t\}} d_i^k, \forall t = k, \cdots, T, \tag{10}
\]

as the total unfinished charging demand at time \( k \) that must be completed by time \( t \). With this, we define the state of system at time \( t \) as

\[
D_t = [l_t, \tilde{d}_t^1, \tilde{d}_t^2, \cdots, \tilde{d}_t^T], \tag{11}
\]

where \( l_t \) is the base load at time \( t \), \( \tilde{d}_t^k \) is the total unfinished charging demand at time \( t \) that must be completed by time \( t' \). Let \( \xi_t \) represent the random arrival events at time \( t \). \( \xi_t \) is defined as

\[
\xi_t = [\xi_t, \eta_t^1, \eta_t^2, \cdots, \eta_t^T], \tag{12}
\]

where \( \xi_t \) is the base load at time \( t \), \( \eta_t^j \) is the total charging demand that arrive at time \( t \) and must be fulfilled by time \( t' \), \( c_t \) is the latest deadline among the PEVs that arrive at time \( t \). Then, the system state at time \( t + 1 \) is defined as

\[
D_{t+1} := g(s_t, D_t, \xi_{t+1}), \tag{13}
\]

where \( g(.) \) is the transition function between \( s_t, D_t, \xi_{t+1} \) and \( D_{t+1} \). Given \( D_t, s_t \) and \( \xi_{t+1} \), the system state \( D_{t+1} \) can be uniquely determined as follows:

\[
l_{t+1} = l_t + 1 \tag{14}
\]

and

\[
\tilde{d}_t^{k+1} = \tilde{d}_t^k - \left[ s_t - \sum_{j=t}^{k-1} \tilde{d}_j^k \right] + \eta_t^{k+1}, \forall t = t+1, \cdots, T. \tag{15}
\]
Here \( [x]^+ = \max\{x, 0\} \). With the above definitions of system state and state transition, we are now ready to rewrite (8) into its online counterpart. In particular, given \( D_k \) at a current time slot \( k \), the optimal online charging decision \( s_k \) is the solution to the following MSP problem.

\[
Q_k(D_k) = \min_{s_k} \quad f(s_k + l_k) + \mathbb{E}_{\xi_{k+1}}[Q_{k+1}(g(s_t, D_t, \xi_{t+1}))]
\]

\[
\text{s. t.} \quad d_k^i \leq s_k \leq \sum_{t=k}^T d_t^i
\]

(16a)

(16b)

where \( Q_{k+1}(g(s_t, D_t, \xi_{t+1})) \) is the optimal value of the MSP at time \( k + 1 \). The left side of (16b) ensures all charging demands to be satisfied before their deadlines, and the right side of (16b) implies that the total charging power up to a certain time cannot exceed the total demands that have arrived up to that time. By slight abuse of notation, in the rest of the paper we denote the optimal solutions to both the online and offline problems as \( s^*_k \), when no confusion arises. The actual meaning of \( s^*_k \) is the optimal solution to (16) at stage \( k \). Then, the resultant total charging cost, denoted by \( \Psi_{\text{on}} \), is

\[
\Psi_{\text{on}} = \sum_{k=1}^T f(s_k^* + l_k),
\]

(17)

### B. The SAA Method Applied to MSP Problem

Note that (16a) comprises nested expectations with respect to random PEV arrivals at each time slot. Except for few special cases, it is in general hard to compute the expectations analytically. Thus, MSP problems are typically solved by scenario approximation approaches, such as the SAA approach based on Monte Carlo sampling techniques [14]-[19]. In particular, a scenario, denoted by \( \Xi \), is defined as a possible realization of the future sequence of random data.

\[
\Xi = [\xi_2, \xi_3, \cdots, \xi_T].
\]

Here, we treat \( \xi_1 \) as deterministic information since the demand of PEVs arrived at the first stage is known by the scheduler. Through Monte Carlo sampling, the SAA approach generates a set of scenarios as follows [14]: 1) generate a sample \( \xi_2^1, \cdots, \xi_{M_1}^1 \) of \( M_1 \) realizations of random vector \( \xi_2 \); 2) conditional on each \( \xi_2^i, i = 1, \cdots, N_1 \), generate a random sample \( \xi_3^j, j = 1, \cdots, M_2 \), of \( M_2 \) realizations of \( \xi_3 \) according to the conditional distribution of \( \xi_3 \) given \( \xi_2 = \xi_2^i \), 3) so on and so force. By this sampling scheme, we establish a scenario tree. The total number of scenarios in the scenario tree is

\[
M = \prod_{i=1}^T M_i,
\]

where each scenario take with the equal probability \( 1/M \). This analysis provided by [14] shows that the total number of scenarios needed to solve the true problem with a reasonable accuracy grows exponentially with increase of the number of time stages \( T \). That is, the complexity of obtaining the optimal \( s_k \) at each time slot is \( O(\epsilon^T) \), where \( \epsilon \) is a constant related to the accuracy of the solution. As the exponential growth, the complexity quickly becomes intractable with the increase of \( T \), the existing scenario-aggregation based methods are not sustainable in the PEV charging problem. This is because a charging station is typically expected to operate for a relative long time period.

### IV. POLYNOMIAL-TIME ONLINE CHARGING ALGORITHM

Considering the exponential complexity of the SAA approach, we are motivated to solve a much simpler problem here: the one obtained by replacing all random variables by their expected values. This is referred to the expected value problem in the literature [16]-[19]. In Section IV-A we show that the load-flattening feature of the solution leads to a low-complexity online algorithm that solves the expected value problem with complexity \( O(T^3) \) at each time slot. Section IV-B proves that the solution to the expected value problem yields a bounded performance gap with the optimal online charging scheduling. Numerical results show that the performance gap is negligible \((<0.4\%)\) in most cases.

#### A. Algorithm Description

Denote the expectation of \( \xi_t \) as

\[
\mu_t = [\nu_t, \mu_t^1, \cdots, \mu_T^1],
\]

where

\[
\nu_t = \mathbb{E}[\xi_t], \mu_t^j = \mathbb{E}[\xi_t^j], \forall t = t, \cdots, T.
\]

(20)

Replacing \( \xi_t \) in (16) with \( \mu_t \), we obtain the following deterministic problem:

\[
\min_{s_k} \quad f(s_k + l_k) + \sum_{t=k+1}^T f(s_t + \nu_t)
\]

(21a)

\[
\text{s. t.} \quad \sum_{t=k}^T s_t \geq \sum_{t=k}^T d_t^k + \sum_{j=m=k+1}^T \sum_{n=m}^T \mu_n^j, \forall j = k, \cdots, T,
\]

(21b)

\[
\sum_{t=k}^T s_t \leq \sum_{t=k}^T d_t^k + \sum_{j=m=k+1}^T \sum_{n=m}^T \mu_n^j, \forall j = k, \cdots, T,
\]

(21c)

In each time \( k \), we solve problem (21) and obtain the optimal charging solution \( s_k^* \). Then problem (21) is resolved with the updated \( d_k^t \) according to the realization of the PEVs arrived in next time. So on and so force, we obtain the optimal charging solution \( s_k^* \) for stage time \( k = 2, \cdots, T \). The total cost, denoted by \( \Psi_{\text{po}} \), is defined as

\[
\Psi_{\text{po}} = \sum_{k=1}^T f(s_k^* + l_k),
\]

(22)

where \( s_k^* \) is the optimal solution to (21) at time stage \( k \). The solution to (21) is always feasible to (16) in the sense that it always guarantees fulfilling the charging demand of the current parking PEVs before their departures. This is because (21b) is a super set of the constraints in (16).

Recall that Theorem 2 states that the optimal solution to (8) is a load-flattening solution. This conclusion can be
easily extended to (21). By exploiting the load flattening feature of the solution, we present in Algorithm 1 a low-complexity algorithm that solves (21) with complexity $O(T^3)$. For notation brevity, we denote in Algorithm 1
\[
\bar{d}_{t'}^{i'} = \begin{cases} 
\bar{d}_{t'}^{i'}, & \text{for } t'' = k, \cdots, T, t' = k; \\
\bar{d}_{t'}^{i'}, & \text{for } t'' = t', \cdots, T, t' = k + 1, \cdots, T.
\end{cases}
\]
(23)
The key idea of Algorithm 1 is to balance the charging load among all time slots $k, \cdots, T$ and then assign the balanced load at time $k$ to the solution $s_k$. Specifically, step 3 - 5 is to find the time interval that has the maximum load density during time $k$ to $T$, and set the optimal charging rate for that time interval to be equal to the maximum density. The time interval is then deleted, and the process is repeated until the current time $k$ belongs to the maximum-density interval.

**Algorithm 1: Online Algorithm**

**input**: $D_k, \mu_t, k = k + 1, \cdots, T$

**output**: $s_k$

1. initialization $i = 0, j = 0$;
2. repeat
3. For all time slot $i = k, \cdots, T$, $j = i, \cdots, T$, compute
   \[
i^*, j^* = \arg \max_{i,j} \left\{ \frac{\sum_{t'=i}^{j} \bar{d}_{t'}^{i'}}{j - i + 1} + \nu_v \right\}.
\]
   (24)
4. Set
   \[
y^* = \frac{\sum_{j'=i}^{j} \bar{d}_{j'}^{i'}}{j^* - i^* + 1}.
\]
5. Delete time slot $i^*, \cdots, j^*$ and relabel the existing time slot $t > j^*$ as $t - j^* + i^* - 1$.
6. until $i^* = k$;
7. Set $s_k = y^* - l_k$.

Notice that similar algorithms have been proposed in the literature of the speed scaling problem [20], [21] and the PEV charging problem [2]. The optimality of the algorithm has been proved therein, and hence omitted here. The algorithm presented here, however, paves the way for further complexity reduction to $O(1)$ in Section V.

**B. Optimality Analysis**

In this subsection, we analyze the optimality of the solution to (21). A well-accepted metric, *Value of the Stochastic Solution* (VSS) is adopted to evaluate optimality gap between the optimal online solution and the solution to the expected value problem [16]-[19]. To evaluate the VSS, the previous study, e.g., [17]-[19] mainly focus on the numerical simulations. Whereas in this subsection we derive an upper bound of the VSS through the rigorous theoretical analysis.

Let $\Phi_{on}$ be the expectation of the optimal value of the MSP in (16), i.e.,
\[
\Phi_{on} = E_{\Xi} [\Psi_{on}(\Xi)],
\]
(26)
where the expectation is taken over the random scenarios $\Xi$ and $\Psi_{on}$ was defined in (17). Likewise, let $\Phi_{po}$ be the expectation of the optimal value of the expected value problem (21), i.e.,
\[
\Phi_{po} = E_{\Xi} [\Psi_{po}(\Xi)],
\]
(27)
where $\Psi_{po}$ was defined in (22). Let $\Phi_{off}$ be the expectation of optimal offline value, i.e.,
\[
\Phi_{off} = E_{\Xi} [\Psi_{off}(\Xi)],
\]
(28)
where $\Psi_{off}$ is defined in (6). It has been proved previously [16]-[18] that
\[
\Phi_{off} \leq \Phi_{on} \leq \Phi_{po}.
\]
(29)
To assess the benefit of knowing and using the distributions of the future outcomes, the VSS is defined as
\[
VSS = \Phi_{po} - \Phi_{on}.
\]
(30)
To show that the proposed online algorithm yields a bounded VSS, we need to bound $\Phi_{po}$ and $\Phi_{on}$. Generally, it is hard to calculate $\Phi_{on}$ or analyze the lower bound of $\Phi_{on}$ directly [17]-[19]. Thus, we choose to analyze the lower bound of $\Phi_{off}$ instead, since (29) shows that the lower bound of $\Phi_{off}$ is also the bound of $\Phi_{on}$. In what follows, we will derive in Lemma 1 and Proposition 1 the lower bound of $\Phi_{off}$. Likewise, we will also derive the upper bound of $\Phi_{po}$ in Proposition 2.

Let us first introduce a new formulation (31), which is obtained by replacing all the random variables by their expected values and solving a deterministic program, with $\xi_t = \mu_t, t = 1, \cdots, T$:

\[
\min_{s_t} \sum_{t=1}^{T} f(s_t + \nu_t)
\]
(31a)
s. t.
\[
d_1^t \leq s_t \leq \sum_{t=1}^{c_1} d_1^t,
\]
(31b)
\[
\sum_{t=1}^{n} d_1^t + \sum_{t=2}^{n} \sum_{j=t}^{n} \mu_j^t \leq \sum_{t=1}^{n} s_t \leq \sum_{t=1}^{n} \sum_{t=2}^{n} \mu_j^t,
\]
(31c)
n = 2, \cdots, T.

We denote by $\Phi_{ev}$ the optimal value of (31). Now we are ready to show that $\Phi_{off}$ is always no smaller than $\Phi_{ev}$ by Lemma 1.

**Lemma 1:** The inequality
\[
\Phi_{ev} \leq \Phi_{off}
\]
(32)
always holds.

**Proof:** Please see the detailed proof in Appendix VIII-B.

Combine (29) and (32), there is
\[
\Phi_{ev} \leq \Phi_{off} \leq \Phi_{on} \leq \Phi_{po}.
\]
(33)
In other words, for the gap VSS, we have
\[
VSS = \Phi_{po} - \Phi_{on} \leq \Phi_{po} - \Phi_{ev}.
\]
(34)
Thus, in order to bound VSS, we need to give the lower bound of $\Phi_{ev}$ and the upper bound of $\Phi_{po}$. First we provide the lower bound of $\Phi_{ev}$ by the following proposition.
Section V-B.

For example, the arrival of charging demands at a particular arrival process of the charging demands are usually periodic, and thus the algorithm is perfectly scalable. In practice, the implication that it does not increase with the system time $T$.

When the arrival process is first-order stationary, i.e., $\forall t \geq 2$, $\mu_t$ in (19) is no longer a function of $t$, and can be represented as

$$\mu = [\mu, \mu_1, \mu_2, \cdots, \mu_c, 0, \cdots, 0],$$

where $\bar{c}$ is the maximum parking time of a PEV. To find the subinterval $[i^*, j^*] \subseteq [k, T]$ with the maximum density, we decompose the search region $\{i, j|i = k, \cdots, j = i, \cdots, T\}$ into three sub-regions, i.e., $\{i, j|i = k, j = k, \cdots, k + \bar{c}\}$, $\{i, j|i = k, j = k + \bar{c} + 1, \cdots, T\}$ and $\{i, j|i = k + 1, j = i, \cdots, T\}$. Let $[i_1, j_1]$ be the maximum-density subinterval within the first sub-region $\{i, j|i = k, j = k, \cdots, k + \bar{c}\}$, and the corresponding maximum density is denoted as $X$. Likewise, let $[i_2, j_2]$ be the maximum-density subinterval of the second sub-region $\{i, j|i = k, j = k + \bar{c} + 1, \cdots, T\}$, with the maximum density denoted as $Y$, and $[i_3, j_3]$ be the maximum-density subinterval within the third sub-region $\{i, j|i = k + 1, j = i, \cdots, T\}$, with the maximum density denoted as $Z$.

By definition, $i_1 = i_2 = k$. The maximum density $X$ of the first sub-region can be obtained by searching $j_1$ over $\{k, \cdots, k + \bar{c}\}$:

$$X = \max_{k \leq n \leq k + \bar{c}} \left\{ \frac{\sum_{i=k}^{n} \bar{d}_i + \sum_{i=1}^{n} (n - k + j + 1) \mu_j}{n - k + 1} \right\}.$$  

The searching complexity is limited by $\bar{c}$ instead of the system time $T$. Moreover, we will show in the following Lemma 2 that $Y$ and $Z$ can be calculated in closed form. That is, the complexity of obtaining the maximum densities over the second and third sub-regions is very low.

**Lemma 2:** The maximum density of $\{i, j|i = k, j = k + \bar{c} + 1, \cdots, T\}$ is achieved by setting $j_2 = T$, and calculated by

$$Y = \frac{\sum_{i=k}^{k+\bar{c}} \bar{d}_i + \sum_{j=1}^{k+\bar{c}} (T - k - j + 1) \mu_j}{T - k + 1}.$$  

Moreover, the maximum density of $\{i, j|i = k + 1, j = i, \cdots, T\}$ is achieved by setting $i_3 = k + 1, j_3 = T$, and calculated by

$$Z = \frac{\sum_{j=1}^{k+\bar{c}} (T - k - j + 1) \mu_j}{T - k}.$$  

**Proof:** Please see the detailed proof in Appendix VIII-F.

The largest of $X, Y$, and $Z$ is the maximum density of the interval $[i^*, j^*] \subseteq [k, T]$ over all possible pairs $i, j \in \{i = k, \cdots, T, j = i, \cdots, T\}$. Specially, if $X$ or $Y$ is the largest one, then $k$ is already contained in the maximum-density interval, and thus $X$ or $Y$ is the optimal charging rate at time $k$. On the other hand, if $Z$ is the largest, then the maximum-
density interval, i.e., \([k + 1, T]\), does not include \(k\). Following Algorithm\(^{1}\) we will delete the maximum-density interval and repeat the process. Now, time slot \(k\) is the only remaining time slot after deletion. This implies that all charging demands that have arrived by time slot \(k\) should be fulfilled during time slot \(k\). These arguments are summarized in Proposition\(^{3}\) which provides the closed form solution to \((21)\).

**Proposition 3:** The optimal charging schedule to \((21)\) is given by the following close-form

\[
s^*_k = \begin{cases} 
X, & \text{if } X = \max\{X, Y, Z\}, \\
Y, & \text{if } Y = \max\{X, Y, Z\}, \\
\sum_{t=k}^{k+\hat{\epsilon}} d_t^k, & \text{otherwise.}
\end{cases}
\]

It is obvious that the complexity of calculating \(s^*_k\) is independent of \(T\). In other words, the computational complexity is reduced to \(O(1)\).

### B. First-Order Periodic Process

In this subsection, we extend Proposition\(^{3}\) to the case when the arrival process is first-order periodic. By first-order periodic, we mean that \(\mu_t\) in \((19)\) repeats itself periodically. Suppose that the period is \(p\). Then, instead of considering \(\mu_t\) for \(t = k + 1, \ldots, T\), we only need to consider \(\mu_t\) for one period, i.e., for \(t = k + 1, k + p, \ldots\)

\[
\mu_{k+1} = \mu_{k+1}, \mu_{k+2}, \ldots, \mu_{k+p}, 0, 0, 0],
\]

\[
\vdots
\]

\[
\mu_{k+p} = \mu_{k+p}, \mu_{k+p}, \mu_{k+p}, \ldots, \mu_{k+p}, 0, 0, 0].
\]

Here, \(e_n \leq T, n = 1, \ldots\), is the maximum parking time for PEVs arriving at time \(k + n\). Specially, we define \(\hat{\epsilon}\) as

\[
\hat{\epsilon} = \max\{e_{k+1}, e_{k+2}, \ldots, e_{k+p}\}.
\]

Similar to the first-order stationary case, we decompose the search region \(\{i,j|i = k, \ldots, T, j = i, \ldots, T\}\) into three sub-regions, i.e., \(\{i,j|i = k, j = k, \ldots, k+\hat{\epsilon}\}\), \(\{i,j|i = k, j = k+\hat{\epsilon}+1, \ldots, T\}\) and \(\{i,j|i = k+1, j = i, \ldots, T\}\). Let \([\hat{i}_1, \hat{j}_1]\) be the subinterval with the maximum density, denoted by \(X\), over the first sub-region \(\{i,j|i = k, j = k, \ldots, k+\hat{\epsilon}\}\), \([\hat{i}_2, \hat{j}_2]\) be the subinterval with the maximum density, denoted by \(Y\), over the second sub-region \(\{i,j|i = k, j = k+\hat{\epsilon}+1, \ldots, T\}\) and \([\hat{i}_3, \hat{j}_3]\) be the subinterval with the maximum density, denoted by \(Z\), over the third sub-region \(\{i,j|i = k+1, j = i, \ldots, T\}\). By definition, \(\hat{i}_1 = \hat{i}_2\). Similar to the stationary case, \(\hat{X}\) can be calculated by searching \(\hat{j}_1\) over \([k, \ldots, k+\hat{\epsilon}]\). That is,

\[
\hat{X} = \max_{k \leq i \leq k+\hat{\epsilon}} \left\{ \frac{\sum_{n=k}^{i} (d_n^k + \nu_n) + \sum_{n=k}^{i} \nu_n}{n-k+1} \right\}.
\]

Moreover, Lemma\(^{5}\) shows that \(\hat{Y}\) and \(\hat{Z}\) can be calculated once the maximum density of \([k+1, k+\hat{\epsilon}]\) has been obtained. Here, we define \([\hat{i}, \hat{j}]\) to be the maximum-density interval within \([k+1, k+\hat{\epsilon}]\).

\[
\hat{i}, \hat{j} = \arg \max_{k+i \leq j \leq k+\hat{\epsilon}} \frac{\sum_{n=k+i}^{j} (d_n^k + \nu_n) + \sum_{n=k+i}^{j} \nu_n}{j-i+1}.
\]

Now we are ready to present Lemma\(^{5}\)

**Lemma 3:** The maximum density of \(\{i,j|i = k, j = k+\hat{\epsilon}+1, \ldots, T\}\) is calculated by

\[
\hat{Y} = \frac{\sum_{n=k+i}^{\hat{j}+1} (d_n^k + \nu_n)}{\hat{j}-k+1},
\]

where

\[
\hat{j} = \begin{cases} 
\max(j, k+\hat{\epsilon}+1), & \text{if } j < \hat{i}+p, \\
(j+(r-1)p), & \text{otherwise.}
\end{cases}
\]

![Graph](image-url)

**Proof:** Please see the detailed proof in Appendix\(^{VIII-C}\)

Based on Lemma\(^{3}\) we can modify the searching region of step 3 in Algorithm\(^{1}\) as follows:

- If \(j < \hat{i}+p\), the interval with the maximum density during time stages \([k+1, T]\) is \([\hat{i}, \hat{j}]\). Then, for the step 3 of Algorithm\(^{1}\) the search region of \(i, j\) is reduced from \(\{i,j|i = k, \ldots, T, j = i, \ldots, T\}\) to \(\{i,j|i = k, \ldots, \hat{i}, j = \hat{i}, \hat{j}, \ldots, T\}\), where \(\hat{i} \in [k+1, k+\hat{\epsilon}]\).
- If \(j \geq \hat{i}+p\), the interval with the maximum density during time stages \([k+1, T]\) is \([\hat{i}_3, \hat{j}+(r-1)p]\). Then, for the step 3 of Algorithm\(^{1}\) the search region of \(i, j\) can be reduced from \(\{i,j|i = k, \ldots, \hat{i}, j = \hat{i}, \hat{j}, \ldots, T\}\) to \(\{i,j|i = k, \ldots, \hat{i}, j = \hat{i}, \hat{j}, \ldots, \hat{i}, \hat{j}, \hat{j}+(r-1)p\}\), where \(\hat{i} \in [k+1, k+\hat{\epsilon}]\).

Hence, the searching region of Algorithm\(^{1}\) is only related to \([k+1, k+\hat{\epsilon}]\) instead of \(T\). Thus, the computation complexity is \(O(1)\) instead of \(O(T^3)\).

### VI. Simulations

In this section, we investigate the performance of the proposed polynomial-time online algorithm through numerical simulations. For comparison purpose, we also simulate the performance of the optimal offline solution, the optimal online solution obtained by SAA method introduced in Section\(^{II-B}\) as well as the online algorithm ORCHARD proposed in\(^{2}\). The average cost denoted by \(\Phi_{ORC}\). Define the average performance ratio as the ratio of the average cost of the proposed polynomial-time online algorithm to that of offline optimal algorithm, i.e., \(\Phi_{ORC}/\Phi_{OPT}\). Likewise, the relative performance loss of the proposed polynomial-time online algorithm and ORCHARD algorithm compared with the optimal online
algorithm are $\Phi_{on} - \Phi_{off}$ and $\Phi_{on} - \Phi_{max}$, respectively. Similar to [2] [10], we adopt a quadratic cost function in the simulations, i.e., $f(s_t + l_t) = (s_t + l_t)^2$.

### A. Average Performance Evaluation

In this subsection, we evaluate the performance of the proposed online algorithm under three different traffic patterns, i.e., light, moderate, and heavy traffics. In particular, the running time $T$ is set to be 24 hours. The PEV arrivals follow a Poisson distribution and the parking time of each PEV follows an exponential distribution [13] [2]. The mean arrival and parking durations of the three traffic patterns are listed in Table I. The main difference lies in the arrival rates at the two peak hours, i.e., 12:00 to 14:00 and 18:00 to 20:00. The settings of the peak hour match with the realistic vehicle trips in National Household Travel Survey (NHTS) 2009 [22]. We choose the base load profile of one day in the service area of South California Edison from [12]. Each PEV’s charging demand is uniformly chosen from [25, 35] kWh.

For each scenario, we simulate $10^5$ independent instances and plot the base load as well as the total load over time in Fig. 1. In addition, the VSS, the relative performance loss and the average performance ratio are shown in Table II.

From Fig. 1 we notice that the curve of total load output by the proposed online algorithm follows closely to that of optimal offline algorithm. Table II shows that the proposed online algorithm has on average less than 7% extra cost compared with the optimal offline algorithm throughout three scenarios. Moreover, the proposed online algorithm performs very close to the optimal online algorithm. The VSS and the relative performance loss are no more than 0.1536 and 0.38% respectively. In contrast, ORCHARD largely deviate from the curve of the optimal offline algorithm in Fig. 1. Compared with ORCHARD, the proposed online algorithm always performs better in three scenarios, which produces half of the extra cost compared with the optimal offline algorithm than that produced by ORCHARD. This is because ORCHARD does not rely on any future information that leads to a larger fluctuation of the total load curve.

### B. Discussion the Influence of Variance

Note that only the statistic means of the random variables are required in the proposed online algorithm. Intuitively, the variance of the random variables may influence the results. In this subsection, we discuss how the variance of charging demands affects the performance of our online algorithm. Let the parking durations be the same as scenario 1 in Table I. For comparison, we simulate two cases where each PEV’s charging demand is uniformly chosen from different intervals:

- case 1: [25, 35] kWh, where the variance is $\frac{2}{3} (kWh^2)$;
- case 2: [5, 35] kWh, where the variance is $\frac{2}{3} (kWh^2)$.

Thus, the variance of the charging demands in case 2 is 9 times of that in case 1.

For each scenario, we simulate $10^5$ independent instances and plot the average total load over time in Fig. 2 and Fig. 3.

### Table I

| Time of Day | Arrival Rate (PEVs/hour) S. 1 | Arrival Rate (PEVs/hour) S. 2 | Arrival Rate (PEVs/hour) S. 3 | Mean Parking Time (hour) |
|-------------|--------------------------------|--------------------------------|--------------------------------|--------------------------|
| 08:00-10:00 | 7                              | 7                              | 7                              | 10                       |
| 10:00-12:00 | 5                              | 5                              | 5                              | 1/2                      |
| 12:00-14:00 | 10                             | 30                             | 50                             | 2                        |
| 14:00-16:00 | 5                              | 5                              | 5                              | 1/2                      |
| 16:00-18:00 | 10                             | 30                             | 50                             | 2                        |
| 18:00-20:00 | 5                              | 5                              | 5                              | 10                       |
| 20:00-22:00 | 0                              | 0                              | 0                              | 0                        |
| 22:00-24:00 | 0                              | 0                              | 0                              | 0                        |

### Table II

| Scenario | VSS ($) | $\Phi_{on} - \Phi_{off}$ | $\Phi_{on} - \Phi_{max}$ | $\Phi_{on} - \Phi_{off}$ |
|----------|---------|--------------------------|---------------------------|--------------------------|
| 1        | 0.1178  | 0.19%                    | 3.50%                     | 1.031                    |
| 2        | 0.1319  | 0.28%                    | 4.46%                     | 1.057                    |
| 3        | 0.1536  | 0.38%                    | 5.82%                     | 1.070                    |
In this subsection, we verify the iteration complexity of the $O(1)$-complexity online algorithm introduced in Section VII-B when the random pattern of the charging demands is first-order periodic. We also compare the proposed $O(1)$-complexity online algorithm with the optimal online algorithm and ORCHARD algorithm. Specifically, the SAA method introduced in Section VI-B is adopted as the optimal online algorithm. We simulate $10^8$ times and compute the average number of iterations and operations for 7 cases respectively. The results are plotted in Fig. 4 which shows the iteration number of the proposed $O(1)$-complexity online algorithm hardly varies with the running time, while the iteration numbers of the optimal online algorithm and the ORCHARD algorithm grow fast as the running time increases. This observation is consistent with our analysis in Section VII-B which proves the scalable complexity of the proposed online algorithm.

**VII. Conclusions**

In this paper, we formulate the optimal PEV charging scheduling problem as a MSP problem. Instead of solving the MSP using standard methods such as SAA, we rigorously prove that solving a deterministic counterpart of the stochastic problem yields the bounded performance loss. Meanwhile, the computational complexity is drastically reduced to $O(T^3)$ per time stage, where $T$ is the total number of the time stages. Moreover, we show that the algorithm can be made scalable with $O(1)$—complexity when the arrival of charging demands is first-order periodic. Extensive simulations show that the proposed online algorithm performs very closely to the optimal
online algorithm. The performance gap is smaller than 0.4% in most cases. Besides, when the variance of random variables increases 8 times, the gap only increases at most 0.1%, and the iteration complexity does not increase with $T$ when the arrival process is first-order periodic.

VIII. APPENDIX

A. Proof of Theorem 2

First, we show that if there exists a PEV parking in the station at both time $t_1$ and $t_2$, i.e.,

$$t_1, t_2 \in \{t_1^{(s)}, \ldots, t_1^{(e)}\},$$

and

$$x_{it_1}^* \geq 0, x_{it_2}^* > 0,$$

then the optimal total loads at time $t_1$ and $t_2$ must satisfy that

$$s_{it_1}^* + l_{t_1} \geq s_{it_2}^* + l_{t_2}. \quad (54)$$

The Karush-Kuhn-Tucker (KKT) conditions to the convex problem (4) are

$$f'(s_{it_1} + l_{t_1}) - \lambda_i - \omega_{ik} = 0, i \in N, t = t_1^{(s)}, \ldots, t_1^{(e)}, \quad (55a)$$

$$\lambda_i(d_i - \sum_{t=t_i^{(s)}}^{t_i^{(e)}} x_{it}) = 0, i \in N, \quad (55b)$$

$$\omega_{it}x_{it} = 0, i \in N, t = t_1^{(s)}, \ldots, t_1^{(e)}, \quad (55c)$$

where $\lambda, \omega$ are the non-negative optimal Lagrangian multipliers corresponding to (4e) and (4d), respectively. We separate our analysis into the following two cases:

1) If $x_{it_1}^* = 0$ for a particular PEV $i$ at a time slot $t_1 \in \{t_1^{(s)}, \ldots, t_1^{(e)}\}$, then, by complementary slackness, we have $\omega_{it_1} = 0$. From (55a),

$$f'(s_{it_1} + l_{t_1}) = \lambda_i \quad (56)$$

2) If $x_{it_2}^* > 0$ for PEV $i$ during a time slot $t_2 \in \{t_1^{(s)}, \ldots, t_1^{(e)}\}$, we can infer from (55c) that $\omega_{it_2} = 0$. Then,

$$f'(s_{it_2} + l_{t_2}) = \lambda_i \quad (57)$$

On the other hand, since $f(s_{it})$ is a strictly convex function of $s_{it}$, then $f'(s_{it})$ is an increasing function. From the above discussions, we get the following two conclusions:

1) If $x_{it_1}^* > 0, x_{it_2}^* > 0$, then by (57),

$$f'(s_{it_1} + l_{t_1}) = f'(s_{it_2} + l_{t_2}) = \lambda_i \quad (58)$$

Due to the monotonicity of $f'(s_{it})$, we have $s_{it_1}^* + l_{t_1} = s_{it_2}^* + l_{t_2}$. 

2) If $x_{it_1}^* = 0, x_{it_2}^* > 0$, then by (56) and (57), there is

$$f'(s_{it_1} + l_{t_1}) = \lambda_i \quad (59)$$

Since $f'(s_{it})$ is an increasing function, we have $s_{it_1}^* + l_{t_1} \geq s_{it_2}^* + l_{t_2}$. 

Let $\tilde{s}_{it}^*$ denote the optimal solution to (4) with another convex increasing objective function $\bar{f}(s_{it} + l_{t_1})$, and $\bar{x}_{it}, t = 1, \ldots, T$, 

denote the optimal solution to (4) with another convex increasing objective function $\bar{f}(s_{it} + l_{t_1})$. Define $\tilde{s}_{it}^*, \bar{s}_{it}^*$ as

$$\tilde{s}_{it}^* = \sum_{i \in I(t)} \hat{x}_{it}^*, \bar{s}_{it}^* = \sum_{i \in I(t)} \bar{x}_{it}^* \quad (60)$$

respectively. Suppose that there exists a time slot $t_1$ such that

$$\tilde{s}_{it_1}^* < \bar{s}_{it_1}^*. \quad (61)$$

Since

$$\sum_{i=1}^{T} \tilde{s}_{it}^* = \sum_{i=1}^{T} \bar{s}_{it}^* = \sum_{i \in N} d_i, \quad (62)$$

there must exist another time slot $t_2$ such that

$$\tilde{s}_{it_2}^* > \bar{s}_{it_2}^*. \quad (63)$$

Thus, we can find a PEV $i \in N$ such that

$$\tilde{x}_{it_1}^* < \bar{x}_{it_1}^*, \tilde{x}_{it_2}^* > \bar{x}_{it_2}^*. \quad (64)$$

As a result,

$$\tilde{x}_{it_2}^* > 0 \quad (65)$$

since $\tilde{x}_{it_2}^* \geq 0$. Based on (54), there is

$$\tilde{s}_{it_2}^* + l_{t_2} \leq \tilde{s}_{it_1}^* + l_{t_1}. \quad (66)$$

Combining (61)-(63)-(66), we get

$$\tilde{s}_{it_2}^* + l_{t_2} < \tilde{s}_{it_2}^* + l_{t_2} \leq \tilde{s}_{it_1}^* + l_{t_1} < \tilde{s}_{it_1}^* + l_{t_1}. \quad (67)$$

Due to the convexity of function $\bar{f}(s_{it} + l_{t_1})$, we have

$$\bar{f}(\tilde{s}_{it_1}^* + l_{t_1}) + \bar{f}(\tilde{s}_{it_2}^* + l_{t_2}) > \bar{f}(\tilde{s}_{it_1}^* + l_{t_1}) + \bar{f}(\tilde{s}_{it_2}^* + l_{t_1}). \quad (68)$$

This contradicts with the fact that the $s_{it}^*$ is the optimal total charging rate for objective function $\bar{f}(s_{it} + l_{t_1})$. Therefore, the optimal charging solution is the same for any convex increasing function $\bar{f}(s_{it} + l_{t_1})$. Next, we show that optimal solution $s_{it}^*$ is also a load flattening solution under the condition that $f(.)$ is a convex increasing function. Let $\tilde{s}_{it_1}^*$ and $\tilde{s}_{it_2}^*$ be the optimal total charging rates at $t_1$ and $t_2$ respectively, where

$$\tilde{s}_{it_1}^* < s_{it_2}^*. \quad (69)$$

Suppose there exist another group of total charging rates $\tilde{s}_{it_1}$ and $\tilde{s}_{it_2}$, which satisfy

$$\tilde{s}_{it_1} < s_{it_2} < \tilde{s}_{it_2} < \tilde{s}_{it_2}. \quad (70)$$

Thus, due to the convexity of function $f(s_{it} + l_{t_1})$, we have

$$f(\tilde{s}_{it_1} + l_{t_1}) < f(s_{it_1} + l_{t_1}) < f(s_{it_2} + l_{t_2}) < f(\tilde{s}_{it_2} + l_{t_2}), \quad (71)$$

which implies that the optimal solution $s_{it}^*$ is always more flatten than other solutions.

B. Proof of Lemma 7

For any $\Xi'$, define $s_{it}^*(\Xi')$ as optimal solution that minimizes $\Psi(\Xi')$ subject to (6b) - (6c). Likewise, we define $s_{it}^*(\Xi'')$ for any $\Xi''$. Now let

$$\Xi''' = \lambda \Xi' + (1 - \lambda) \Xi'', \lambda \in [0, 1]. \quad (72)$$
Then, there must exist a feasible solution \( s_t(\Xi'') \) such that
\[
s_t(\Xi'') = \lambda s_t'(\Xi') + (1 - \lambda) s_t''(\Xi''). \tag{73}
\]
Note that \( s_t(\Xi'') \) still satisfies (8b) - (8c) due to the linearity of the constraints. Meanwhile, based on the convexity of \( f(s_t + \lambda l_t) \), we have
\[
\sum_{t=1}^{T} f(s_t(\Xi'') + l_t) \\
\leq \lambda \sum_{t=1}^{T} f(s_t'(\Xi') + l_t) + (1 - \lambda) \sum_{t=1}^{T} f(s_t''(\Xi'') + l_t),
\]
which holds for all \( \lambda \in [0, 1] \). On the other hand, for \( \Xi''' = \lambda \Xi' + (1 - \lambda) \Xi'' \), let \( s_t'(\Xi''') \) be the optimal solution that minimizes \( \sum_{t=1}^{T} f(s_t + l_t) \) under \( \Xi'''. \) Then
\[
\sum_{t=1}^{T} f(s_t'(\Xi''') + l_t) \leq \sum_{t=1}^{T} f(s_t(\Xi'') + l_t), \tag{75}
\]
Combining (74) and (75), we have
\[
\Psi(\Xi''') = \sum_{t=1}^{T} f(s_t'(\Xi''')) + l_t) \\
\leq \lambda \sum_{t=1}^{T} f(s_t'(\Xi') + l_t) + (1 - \lambda) \sum_{t=1}^{T} f(s_t''(\Xi'') + l_t) \\
= \lambda \Psi(\Xi') + (1 - \lambda) \Psi(\Xi'').
\]
Thus, we have established the convexity of \( \Psi(\Xi) \) over the set of \( \Xi \). Additionally, \( \Psi(\Xi) \) is also a continuous function. Then, by Jensen’s inequality, we have
\[
E[\Psi(\Xi)] \geq \Psi(E[\Xi]),
\]
That is,
\[
\Phi_{off} \geq \Phi_{ev}. \tag{77}
\]
This completes the proof.

C. Proof of Proposition 2

Based on the definition, we have
\[
\sum_{t=1}^{T} s_t = \sum_{t=1}^{T} d^t_{\text{v}} + \sum_{t=2}^{T} \sum_{j=t}^{T} \mu_j.
\]
By Jensen’s inequality,
\[
\Phi_{ev} = \min_{s_t} \sum_{t=1}^{T} f(s_t + \nu_t) \geq \sum_{t=1}^{T} f \left( \sum_{t=1}^{T} d^t_{\text{v}} + \sum_{t=2}^{T} \sum_{j=t}^{T} \mu_j + \sum_{t=1}^{T} \nu_t \right) \geq T f \left( \sum_{t=1}^{T} d^t_{\text{v}} + \sum_{t=2}^{T} \sum_{j=t}^{T} \mu_j + \sum_{t=1}^{T} \nu_t \right). \tag{80a}
\]
This completes the proof.

D. Proof of Proposition 2

For any stage \( t \), the following inequality holds.
\[
s_t \leq \sum_{t=n}^{T} \hat{d}_{t_n}^t. \tag{81}
\]
Then,
\[
\sum_{t=n}^{T} \hat{d}_{t_n}^t = \sum_{t=n}^{T} \hat{d}_{t_n}^{t-1} - s_{t-1} + \sum_{t=n}^{T} \eta_{t_n}^t \leq \sum_{t=n}^{T} \hat{d}_{t_n}^{t-1} - \hat{d}_{t_n}^{t-1} + \sum_{t=n}^{T} \eta_{t_n}^t = \sum_{t=n}^{T} \hat{d}_{t_n}^{t-1} + \sum_{t=n}^{T} \eta_{t_n}^t \leq \sum_{m=\{m\mid e_m \geq t, m = 1, \ldots, t, n = t, \ldots, e_m \}}^{e_m} \sum_{n=t}^{e_m} \eta_{n_m}^{e_m} + \sum_{n=t}^{e_m} \eta_{n}^t, \tag{82c}
\]
where the second inequality is due to the fact that
\[
s_{t-1} \geq \hat{d}_{t_n}^{t-1}. \tag{83}
\]
Let \( O(t) \) be the set that
\[
O(t) = \{(m, n)\mid e_m \geq t, m = 1, \ldots, t, n = t, \ldots, e_m \}. \tag{84}
\]
Then we get
\[
s_t \leq \sum_{(m, n) \in O(t)} \eta_{n_m}^{e_m}. \tag{85}
\]
Since \( O(t) \) is a bounded set for \( t = 1, \ldots, T \), then
\[
E \left[ \sum_{t=1}^{T} f \left( \sum_{(m, n) \in O(t)} \eta_{n_m}^{e_m} + \nu_t \right) \right] \tag{86}
\]
is also bounded. Thus, (86) is an upper bound of \( \Phi_{po} \).

E. Proof of Theorem 2

By Proposition 1 and Proposition 2, for any distribution of \( \hat{d}_{t_n}^t \) and \( \nu_t \), \( t = 1, \ldots, T, n = t, \ldots, T \), we have
\[
VSS = \Phi_{po} - \Phi_{on} < \Phi_{po} - \Phi_{off} < \Phi_{po} - \Phi_{ev} \leq \sum_{t=1}^{T} f \left( \sum_{(m, n) \in O(t)} \eta_{n_m}^{e_m} + \nu_t \right) - T f \left( \frac{\Gamma}{T} \right), \tag{87}
\]
where \( \Gamma = \sum_{t=1}^{T} \hat{d}_{t_n}^t + \sum_{t=2}^{T} \sum_{j=t}^{T} \mu_j + \sum_{t=1}^{T} \nu_t \).
given by
\[
\rho(i, j) = \frac{\sum_{k=1}^{j-i} \sum_{t=1}^{j-i+2-k} \mu_t + \sum_{t=k}^{j-i+2} \hat{d}_t}{j - i + 1}.
\]

To prove that the maximum density is achieved by setting \( j = T \), we only need to show \( \rho(i, j) \) is a non-decreasing function of \( j \) for each given \( i \), i.e.,
\[
\rho(i, j) \leq \rho(i, j + 1), \forall k + \bar{e} + 1 \leq j \leq T - 1.
\]

Since
\[
\sum_{t=1}^{j-i} (j - k + 1 - t) \mu_t + \sum_{t=k}^{j-i} \hat{d}_t 
\]
\[
\leq \sum_{t=1}^{j-i} (j - k + 1 - t) \mu_t + \sum_{t=k}^{j-i} \hat{d}_t,
\]
we have
\[
\rho(i, j + 1) = \frac{\sum_{k=1}^{j-i+1} \sum_{t=1}^{j-i+2-k} \mu_t + \sum_{t=k}^{j-i+2} \hat{d}_t}{j - i + 1} 
\]
\[
= \rho(i, j),
\]
which implies (99). Hence, \( Y \) is the maximum density of \([k, j], j = k + \bar{e} + 1, \ldots, T\). Next, we show that \( Z \) is the maximum density of \([k + 1, T]\). For any \( k + 1 \leq i \leq j \leq T \), the density of interval \([i, j]\) is given by
\[
\rho(i, j) = \frac{\sum_{k=1}^{j-i+1} \sum_{t=1}^{j-i+2-k} \mu_t + \sum_{t=k}^{j-i+2} \hat{d}_t}{j - i + 1} 
\]
\[
= \frac{\sum_{t=1}^{j-i+1} (j - i + 2 - t) \mu_t}{j - i + 1}.
\]

To prove that the maximum density is achieved by setting \( i = k + 1, j = T \), we only need to show \( \rho(i, j) \) is a non-decreasing function of \( j \) for each given \( i \), i.e.,
\[
\rho(i, j) \leq \rho(i, j + 1), \forall k + 1 \leq i \leq j \leq T - 1,
\]
and a non-increasing function of \( i \) for each given \( j \), i.e.,
\[
\rho(i, j) \geq \rho(i + 1, j), \forall k + 1 \leq i + 1 \leq j \leq T.
\]

On one hand, since
\[
\sum_{t=1}^{j-i+1} (j - i + 2 - t) \mu_t 
\]
\[
\leq \sum_{t=1}^{j-i+2} \mu_t, \forall k + 1 \leq i \leq j \leq T,
\]
we have
\[
\rho(i, j + 1) = \frac{\sum_{k=1}^{j-i+1} \sum_{t=1}^{j-i+2-k} \mu_t + \sum_{t=k}^{j-i+2} \hat{d}_t}{j - i + 1} 
\]
\[
= \rho(i, j),
\]
which implies (93). On the other hand, as
\[
\sum_{t=1}^{j-i} (j - i + 1 - t) \mu_t 
\]
\[
\leq \sum_{t=1}^{j-i+1} \mu_t, \forall k + 1 \leq i \leq j \leq T,
\]
then
\[
\rho(i + 1, j) = \frac{\sum_{k=1}^{j-i} \sum_{t=1}^{j-i+1} \mu_t + \sum_{t=k}^{j-i+1} \hat{d}_t}{j - i + 1} 
\]
\[
\leq \frac{\sum_{t=1}^{j-i} (j - i + 1 - t) \mu_t + \sum_{t=k}^{j-i+1} \hat{d}_t}{j - i + 1} 
\]
\[
= \rho(i, j),
\]
which implies (94).}

\[\]
Otherwise, $\hat{j}_2 = \hat{j}$, and
\[
\hat{Y} = \frac{\sum_{n=1}^{j} \left( \sum_{m=1}^{k} \mu_m + \nu_n \right)}{j - k + 1}.
\] (106)

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