DIAGRAMMATIC ANALYSIS OF THE TWO-STATE QUANTUM HALL SYSTEM WITH CHIRAL INVARIANCE

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Abstract

The quantum Hall system in the lowest Landau level with Zeeman term is studied by a two-state model, which has a chiral invariance. Using a diagrammatic analysis, we examine this two-state model with random impurity scattering, and find the exact value of the conductivity at the Zeeman energy $E = \Delta$. We further study the conductivity at the another extended state $E = E_1$ ($E_1 > \Delta$). We find that the values of the conductivities at $E = 0$ and $E = E_1$ do not depend upon the value of the Zeeman energy $\Delta$. We discuss also the case where the Zeeman energy $\Delta$ becomes a random field.
1 Introduction

The critical behavior around the extended state in the two dimensional quantum Hall system has been studied by various methods. Recently, the spin degenerate case attracted interests. The spin-up state and the spin-down state almost degenerate when the Zeeman energy is small. It is considered that these two states can be mixed by the impurity scattering.

One of author, Shirai and Wegner [1] (HSW) considered a two-state model in the lowest Landau level, in which the impurity scattering occurs only between different spin states. This model corresponds to the strong spin-orbit scattering limit, in which the spin should be changed at each impurity scattering. Remarkably there appear three extended states in this model, one is at the band center \( E = 0 \), and the other two at \( E = \pm E_1 \). The conductivity at \( E = 0 \) has been obtained exactly by a diagrammatic analysis and becomes \( \sigma = e^2/2\pi^2\hbar \). This model has a chiral invariance; the energy eigenvalues always appear in the positive and negative pairs. Then the state at \( E = 0 \) becomes a special state, which can hybridizes itself due to this chiral invariance. [2] The density of state near \( E = 0 \) is not broaden so much by the impurity scattering. Therefore, the density of state at \( E = 0 \) is enhanced and could be singular. The \( E = 0 \) state is a resonant state. At \( E = 0 \), all scattering effects are remarkably cancelled out for the conductivity, and the localization effect is smeared out. This cancellation occurs not only for the Gaussian white noise distribution but also for the general local non-Gaussian random distribution. [1]

This model has been examined further by the numerical method [3, 4] and the localization exponents have been estimated for these extended states. The localization length exponent at \( E = 0 \) seems different from the usual quantum Hall system and belongs a new universality class, although other two extended states at \( E = E_1 \) belongs to the conventional quantum Hall universality class with the localization length exponent \( \nu \approx 2.3 \) [4].

The state at \( E = 0 \) in this model has been suggested to be relevant to the chiral Dirac fermion model with a random vector potential [5], which gives a singularity for the density of state. The value of the conductivity for this random vector potential model agrees with the value of HSW model.

In the previous paper [4], the Zeeman term has been included, which does not break a chiral invariance. It has been shown that the density of state has a gap less than the Zeeman energy \( \Delta \), and the extended state shifts from
In this paper, we further consider this extended HSW model with a Zeeman term by a diagrammatic method. We evaluate the exact value of the longitudinal conductivity at $E = \Delta$. Also we will discuss the extended state at $E = \pm E_1 \ (E_1 > \Delta)$, which is believed to belong the conventional quantum Hall universality class. We argue that the value of the conductivity at $E = E_1$ becomes same as the conductivity for no-Zeeman case $\Delta = 0$ by the diagrammatic analysis. This is consistent with the numerical result[4]. We show exactly that the inclusion of the Zeeman term does not alter the values of the conductivities of the extended state. This result may be expected but we verify it by a diagrammatic expansion method. When the Zeeman energy becomes a random variable, the situation will be changed. We briefly discuss this random Zeeman energy case by the diagrammatic method.

2 Diagrammatic analysis of the two-state quantum Hall system

The Hamiltonian for the two-spin state may be described by $2 \times 2$ matrix[4]

$$H = \frac{1}{2m}(p - eA)^2 + \begin{pmatrix} \Delta & v^\dagger(r) \\ v(r) & -\Delta \end{pmatrix}$$

(2.1)

where $v^\dagger(r)$ and $v(r)$ are random potentials at the spacial point $r$. The constant $\Delta$ represents the Zeeman energy. In the Landau quantization, the up-spin state and the down-spin state acquires the Zeeman energy $\pm \Delta$. The matrix of the second term of (2.1) acts on the spin state, which eigenstate is represented by a vector of two components. The distribution of these random potential $v(r)$ is assumed as a Gaussian white noise distribution, i. e.

$$< v(r) >_{av} = < v^\dagger(r) >_{av} = 0$$

(2.2)

$$< v^\dagger(r)v(r') >_{av} = w\delta(r - r')$$

(2.3)

The diagrammatic expansions for the one particle Green function and the two-particle Green function for the lowest Landau level has been investigated [3, 4, 5]. In the case of no-Zeeman term, a useful expansion for the
diffusion constant $D$ was derived, and indeed by this expression, the exact value of the conductivity was obtained [1]. Note that we mainly consider the Gaussian white noise distribution in this paper but the exact evaluation of the conductivity is also applied to any local non-Gaussian random potential as shown in [1].

In the two dimensional case, the Green function for the lowest Landau level is simply expressed by

$$G(r) = \langle\langle r | \frac{1}{E - H} | r \rangle\rangle_{av} = \frac{1}{A_1 + iA_2}$$

(2.4)

where $G(r)$ has a translational invariant, and $A_1$ and $A_2$ become real numbers independent of $r$. This is due to the quantization under a strong magnetic field. For example, the density of state $\rho(E)$ becomes simply as $-A_2/\pi (A_1^2 + A_2^2)$.

When the two-spin state model is considered, we have two different Green functions $G_A(r)$ and $G_B(r)$. The notation A and B are the spin-up state and the spin down state, respectively. Using the self-energy $\Sigma$ for A and B, we obtain by definition,

$$A_1 = 2\pi(E - \frac{1}{2}\hbar \omega_c - \Delta - \frac{1}{2\pi} \text{Re}\Sigma_A)$$

(2.5)

$$A_2 = 2\pi(\frac{\epsilon}{2} - \frac{1}{2\pi} \text{Im}\Sigma_A)$$

(2.6)

$$B_1 = 2\pi(E - \frac{1}{2}\hbar \omega_c + \Delta - \frac{1}{2\pi} \text{Re}\Sigma_B)$$

(2.7)

$$B_2 = 2\pi(\frac{\epsilon}{2} - \frac{1}{2\pi} \text{Im}\Sigma_B)$$

(2.8)

The diagrammatic expansion follows the previous studies and the convenient method for obtaining the coefficients of each orders may be found in [1], [2]. As the first exercise, let us approximate the self-energy $\Sigma$ by the Green function itself. Then we have $\Sigma_A = 2\pi w/(B_1 + iB_2)$ and $\Sigma_B = 2\pi w/(A_1 + iA_2)$. 4
It may be convenient to represent two Green functions by $G_A = C_A e^{i\theta A}$, $G_B = C_B e^{i\theta B}$, and also $x = C_A C_B$. From (2.6) and (2.8), in the limit $\epsilon \to 0$, we obtain $\theta_A = \theta_B$, $x = 1$. We represent the energy $E - \frac{1}{2} \hbar \omega_c$ simply by $E$.

From (2.5) and (2.7), using $x^2 = 4\pi^2 w^2/(A_1^2 + A_2^2)(B_1^2 + B_2^2) = 1$, we obtain

$$A_1 = 2\pi(E - \Delta) - \frac{1}{2\pi w} B_1(A_1^2 + A_2^2)$$

$$= 2\pi(E - \Delta) - \frac{1}{w} (E + \Delta)(A_1^2 + A_2^2) + A_1 \quad (2.9)$$

Thus we obtain $A_1^2 + A_2^2 = 2\pi w(E - \Delta)/(E + \Delta)$. Similarly, we get $B_1^2 + B_2^2 = 2\pi w(E + \Delta)/(E - \Delta)$.

Then (2.5) becomes

$$A_1 = 2\pi(E - \Delta) - B_1(E - \Delta) \quad (E + \Delta) \quad (2.10)$$

From (2.3), we have $A_2 = 2\pi w B_2/(B_1^2 + B_2^2) = B_2(E - \Delta)/(E + \Delta)$. Further noting that $A_1/A_2 = B_1/B_2$, and from (2.10) we obtain the following solution,

$$A_1 = \pi(E - \Delta) \quad (2.11)$$

Similarly we get

$$B_1 = \pi(E + \Delta) \quad (2.12)$$

The imaginary parts $A_2$ and $B_2$ are obtained from $A_1^2 + A_2^2 = 2\pi w(E - \Delta)/(E + \Delta)$. They become

$$A_2 = \frac{1}{2} \sqrt{\frac{E - \Delta}{E + \Delta}} \sqrt{4w - (E^2 - \Delta^2)} \quad (2.13)$$

$$B_2 = \frac{1}{2} \sqrt{\frac{E + \Delta}{E - \Delta}} \sqrt{4w - (E^2 - \Delta^2)} \quad (2.14)$$

The density of state $\rho_A$ and $\rho_B$ are given by the $\rho_A = -A_2/\pi(A_1^2 + A_2^2)$, $\rho_B = -B_2/\pi(B_1^2 + B_2^2)$. Since $A_1^2 + A_2^2 = 2\pi w(E - \Delta)/(E + \Delta)$, $B_1^2 + B_2^2 = 2\pi w(E + \Delta)/(E - \Delta)$, the density of state $\rho(E)$ has a gap between $-\Delta < E < \Delta$, and has the inverse square root singularity at $E = \pm \Delta$. This behavior resembles to the density of state of the superconductor. Note that
we fix the Zeeman energy parameter $\Delta$. Later, we consider the average over this $\Delta$ for the density of state.

We now go beyond this approximation by expanding the self-energy in the power series of $w$. In this two-state model with the zeeman term, the diagrams become same as the two-state model without Zeeman term [1]. Using the notations $A_2/A_1 = -\tan \theta_A$ and $B_2/B_1 = -\tan \theta_B$, and $x = C_A e^{\theta_A}$ and $G_B = C_B e^{\theta_B}$, we have

$$\pi \epsilon / A_2 = 1 - x \sin \theta_B / \sin \theta_A - 1/4 x^3 \sin(3\theta_B + 2\theta_A) - 2/5 x^4 \sin(4\theta_B + 3\theta_A) + \cdots.$$  

(2.15)

$$\pi \epsilon / B_2 = 1 - x \sin \theta_A / \sin \theta_B - 1/4 x^3 \sin(3\theta_A + 2\theta_B) - 2/5 x^4 \sin(4\theta_A + 3\theta_B) + \cdots.$$  

(2.16)

Up to order $x^{10}$, the expansion coefficients are given in (4.1) of Ref.[1].

At $E = \pm \Delta$, the phases $\theta_A$ and $\theta_B$ become $-\pi/2$. This is evident within the first order approximation by (2.12) and (2.14); $A_2/A_1 = \tan \theta_A = 0$ and $B_2/B_1 = \tan \theta_B = 0$. Beyond this order, it remains also true. We have evaluated the real part of the Green function numerically by the same method of Ref.[4], and find that the real part vanishes at $E = \pm \Delta$.

The conductivity in the lowest Landau level is obtained from Kubo-formula by the diagrammatic expansion. As an equivalent method, we have Einstein relation $\sigma = e^2 D \rho$, where $D$ is a diffusion constant. Here we use Einstein relation, since a diagrammatic expansion is simpler. The diffusion constant $D$ is defined as the coefficient of $q^2$ in the inverse of the two-particle correlation function $K(q)$;

$$K(q) = \int \langle \langle r | E - H + i0 \rangle | r' \rangle \langle \langle r' | E - H - i0 \rangle | r \rangle > a_v e^{-iq(r-r')} d^2 r.$$  

(2.17)

This $K(q)$ is expanded in the power series of $w$. The Feynman rule for this expansion may be seen in the previous literatures. We have for the
small momentum $q$,
\[
\frac{K(q = 0)}{K(q)} = 1 + \frac{D}{\epsilon} q^2
\] (2.18)

Since we have two different propagator $G_A$ and $G_B$, the two-particle correlation function $K(q)$ is also divided into two parts, $K_A(q)$ and $K_B(q)$. And the diffusion constant $D$ also is defined differently by (2.18). The denominator $\epsilon$ in (2.18) can be expressed by (2.15) and (2.16). Then finally, we obtain the following equations which are the modification of the previous expression[1].

\[
\frac{2\pi D_B}{B_2} = 1 - \frac{x^3}{4} (\cos(2\theta_B + 2\theta_A) + \cos(\theta_A + \theta_B)) + \cdots
\] (2.19)

\[
\frac{2\pi D_A}{A_2} = 1 - \frac{x^3}{4} (\cos(2\theta_B + 2\theta_A) + \cos(\theta_A + \theta_B)) + \cdots
\] (2.20)

The imaginary part of $G_A$ and $G_B$ are proportional to the density of state $\rho$. The conductivity $\sigma_{xx}$ is given by Einstein relation,
\[
\sigma_{xx} = \frac{1}{2} (e^2 D_A \rho_A + e^2 D_B \rho_B)
\] (2.21)

At $E = \Delta$, and $E = -\Delta$, we have $A_1/A_2 = B_1/B_2 = 0$ as explained before. Thus, we have $\theta_A = \theta_B = -\pi/2$. Remarkably all corrections cancels out in (2.19) and (2.20) except one. These cancellations are essentially same as the previous case without Zeeman term[1]. The conductivity $\sigma_{xx}$ at $E = \pm \Delta$ becomes $e^2/2\pi^2\hbar$, which is same value for the no-Zeeman term at $E = 0$.

Thus, we have found the exact value of the longitudinal conductivity at $E = \pm \Delta$. In the previous numerical work[4], this value was obscure, although it suggested the similar value. We find no particular difference for the conductivity between the extended state at $E = \Delta$ and the $E = 0$ in the $\Delta = 0$ case.

The effect of our Zeeman term on the density of state can be also discussed by the matrix model. As an analogous matrix model to HSW model, a complex block matrix model has been studied and the universal oscillation of the density of state near $E = 0$ has been obtained in the large $N$ limit, where $N$ is a size of the matrix[9, 10]. The simple matrix model is given by
\[
M = \begin{pmatrix}
\Delta & v^\dagger \\
v & -\Delta
\end{pmatrix}
\] (2.22)
where $\Delta$ is a unit matrix multiplied by $\Delta$, and $v^\dagger$ is a $N \times N$ complex matrix. It is a straightforward exercise to evaluate the density of state for the finite $N$ through Kazakov method\,[10, 11], since this matrix model has a chiral invariance; the eigen values appear always in a pair of positive and negative one. The effect of this Zeeman term $\Delta$ is just a shift of the energy $E$. When we take the large $N$ limit first in this model, the density of state coincides with (2.13) and (2.14). However, there is a crossover to the oscillatory behavior near $\Delta$ in the small region of order $1/N$\,[10].

3 Extended state at $E = E_1$

As pointed out by the numerical works\,[3, 4], there are extended states at $E = \pm E_1$, which is greater than $\Delta$. It was suggested that the conventional universality class of the quantum Hall effect with a localization exponent $\nu \simeq 2.3$ is realized at $E = E_1$\,[3, 4]. The shift of the energy from $E = 0$ to $E = E_1$ is due to the effective magnetic field effect due to the off-diagonal random potential $v$.

The shift of the conventional extended state of quantum Hall system to $E = E_1$ has been observed by several models. The Chalker-Coddington network model\,[12] was extended to include the spin-scattering, and the shift of the extended state is shown with the same localization exponent \,[13, 14, 15].

Since the previous work\,[1] did not discuss this extended state at $E = E_1$, we first consider this state for the no-Zeeman term $\Delta = 0$ case. The diagrammatic expansion for $D/A_2$ was given up to order $x^8$\,[1]. The series for the diffusion constant $D$ without Zeeman term becomes

\[
\frac{2\pi D}{A_2} = 1 - \frac{1}{4} (\cos 4\theta + \cos 2\theta)x^3 - (0.32 \cos 6\theta + 0.16 \cos 4\theta + 0.16)x^4 \\
- (1.14279155188 \cos 8\theta + 0.715564738292 \cos 6\theta) \\
+ 0.180555555555 \cos 4\theta + 0.75195133149 \cos 2\theta \\
+ 0.144168962351)x^5 \\
- (4.01604212958 \cos 10\theta + 2.10780216729 \cos 8\theta) \\
+ 0.228564968429 \cos 6\theta + 1.65837390674 \cos 4\theta \\
+ 0.61362486859 \cos 2\theta + 1.0920589084)x^6
\]
(16.8938594252 \cos 12\theta + 8.85669612784 \cos 10\theta + 1.34798158141 \cos 8\theta + 1.75117610591 \cos 4\theta + 6.74855019206 \cos 2\theta + 1.10403646547)x^7
\quad - (79.7915118420 \cos 14\theta + 40.5552408026 \cos 12\theta + 5.99939335079 \cos 10\theta + 20.19686645487 \cos 8\theta + 4.4215378158747 \cos 6\theta + 23.475132715585 \cos 4\theta + 6.4926520588331 \cos 2\theta + 12.477855103819)x^8 + \cdots. \quad (3.1)

where the variable \( x \) is solved by the asymptotic expansion of (2.15). Putting \( \theta_A = \theta_B, \epsilon = 0 \), we have up to the third order of \( x \),

\[ x \simeq 1 - \frac{1}{4} \sin 5\theta \sin \theta \] \quad (3.2)

This approximation shows the maximum of \( x \) at \( \theta \approx -0.9 \). The maximum value of \( x \) becomes 1.3. The value of \( x \) becomes zero for \( \theta \to 0 \) from (2.15).

This is quite similar to the case of the conventional quantum Hall case: the exact value of \( x \) at the band center is \( x = 4/\pi = 1.2732 \) \cite{16}. And \( x \) becomes zero for \( \theta \to 0 \). Thus the point \( \theta = -0.9 \) for this two-state quantum Hall system corresponds to the band center of the one-state quantum Hall system. The shift appears due to the off-diagonal two-state random potential. Up to order \( x^3 \), from (3.1), we obtain by inserting the value of \( x \),

\[ \frac{2\pi D}{A_2} = 1 - \frac{1}{4} (\cos 4\theta + \cos 2\theta)(1 - \frac{1}{4} \sin 5\theta)^3 \] \quad (3.3)

The maximum of \( 2\pi D/A_2 \) becomes 1.6 at \( \theta = -0.9 \). The conductivity \( \sigma \) is obtained by multiplying \( e^2 \sin^2 \theta/2\pi^2 \hbar \) to the value of \( 2\pi D/A_2 \). We have analysed up to order \( x^3 \). We think the maximum peak of the conductivity remains finite for the higher order analysis. And also we think that the state at \( \theta = -0.9 \) corresponds to the band center of the one-state quantum Hall system, and becomes extended. This is consistent with the previous numerical result \cite{14}, which shows there is an extended state at \( E = E_1 \) except \( E = 0 \). The states of the energy \( 0 < E < E_1 \) and \( E > E_1 \) are considered to be localized. For the investigation of the localization, we need the renormalization group analysis via \( 1/N \) expansion \cite{5}, and we do not discuss it here.
For the Zeeman case ($\Delta \neq 0$), the series of (3.1) is modified as (2.19) and (2.20), where $2\theta$ is replaced by $\theta_A + \theta_B$. In general, $\theta_A$ is not equal to $\theta_B$. The range of these angles are between $-\pi/2$ and 0. We assume that there is an extended state at $E = E_1$ for the Zeeman case. Then, we find that if $\theta_A + \theta_B$ is same as the critical value $\theta_c$ in (3.1), we have the same expression for (2.19) and (2.20). Since there is one extended state, we have $\theta_A = \theta_B$ at $E = E_1$. This is a duality between A-state and B-state. Then we find that the same conductivity as the no-Zeeman case at $E = E_1$. The conductivity is obtained from (2.19) by multiplying a factor $\sin^2 \theta$, which is $A_2^2/(A_1^2 + A_2^2)$ for the case $A_1 \neq 0$. Indeed our previous numerical result shows this behavior. This argument of the equivalence does not determine the absolute value of the conductivity, but it verifies that the value of the conductivity at $E = E_1$ does not depend upon the Zeeman energy $\Delta$.

4 Random Zeeman energy model

In the previous sections, we assumed that the Zeeman energy $\Delta$ is a fixed constant. When this $\Delta$ in (2.1) is a random field, which depends upon the spacial coordinate $r$, the situation becomes different. The distribution of this random field $\Delta(r)$ is Gaussian. We will discuss this random Zeeman energy model by a diagrammatic expansion method.

Instead of the Zeeman energy $\Delta$, we represent it by a random field $u(r)$. Then the second term of (2.1) becomes

$$V(r) = \begin{pmatrix} u(r) & v^\dagger(r) \\ v(r) & -u(r) \end{pmatrix}$$

(4.1)

where $r$ is a place of the impurity scattering. This model represents the spin flip at $r$ due to the random field $v$ and the random Zeeman energy by $u(r)$. There is no correlation between $v(r)$ and $u(r)$. The matrix $V(r_1)$ does not commute with the matrix $V(r_2)$. We have to consider the successive operation of the random scattering at $r_1, r_2, \cdots, r_N$ for the eigenstate. The eigenstate is represented by a vector of two components. The random variable $u$ has the following average,

$$< u(r)u(r') >_{av} = w'\delta(r - r')$$

(4.2)
Then the diagrammatic expansions of (2.15) and (2.16) become a series of the scattering strength $w$ and $w'$. Note that some terms have a negative sign due to the minus sign in (4.1) in the matrix element.

In this random Zeeman energy model, the chiral invariance is broken. The scattering appears between a state A and a state B but also between the same spin state due to the diagonal random field $u(r)$.

We find that after the average of the multiplication of the matrix $V(r_i)$ over the random distribution, the non-vanishing diagrams can be expressed by assigning the indecis A and B for the Green function.

The self-energy of $\Sigma_A$ becomes by the diagrammatic expansion,

$$\Sigma_A = w'G_A + wG_B - w w' G_A G_B$$

$$- \frac{1}{4}(w^3 G_A^5 + w^3 G_A^2 C_B^3 + 3 w w'^2 G_A^2 G_B^3)$$

$$- \frac{1}{3}(3w^3 G_A^5 + 3w w'^2 G_A^2 G_B^3 - 2w w' G_A G_B^4 + w' w^2 G_A G_B^4)$$

$$- 4w^2 w' G_A G_B^2 + 2w' w^2 G_A G_B^2 + \cdots \quad (4.3)$$

From this equation, we have

$$\frac{\pi \epsilon}{A_2} = 1 - \frac{1}{\sin \theta_A} (w' C_A^2 \sin \theta_A + w C_B C_A \sin \theta_B)$$

$$+ w w' C_A^2 C_B^2 \frac{\sin(\theta_A + 2\theta_B)}{\sin \theta_A}$$

$$- \frac{1}{4}(w^3 C_A^6 + w^3 + 3w'^2 w) C_A^3 C_B^3 \frac{\sin(2\theta_A + 3\theta_B)}{\sin \theta_A}$$

$$- \frac{1}{3}(3w^3 C_A^6 + 3w w'^2 C_A^3 C_B^3 \sin(2\theta_A + 3\theta_B))$$

$$+ (w' w^2 - 2w w'^2) C_A^2 C_B^4 \frac{\sin(\theta_A + 4\theta_B)}{\sin \theta_A}$$

$$+ (2w' w^2 - 4w w'^2) C_A C_B^6 \frac{\sin(4\theta_A + 2\theta_B)}{\sin \theta_A}$$

$$+ \cdots \quad (4.4)$$

By the symmetry between A and B states, we are able to put $\theta_A = \theta_B$, $C_A = C_B$. Then, (4.4) becomes simpler.

It may be interesting to consider the three different cases: 1) $w' << w$, 2) $w' \sim w$, 3) $w' >> w$. The case 1) corresponds to the 2-state model, which
we have discussed previously as $\Delta = 0$. The perturbation of the parameter $w'$ can be obtained. The case 2) shows the strong effect of the random field $u(r)$. When, for example, $w' = \frac{1}{2}w$, the series of (2.13) has alternative sign, and when $\theta_A = \theta_B = -\pi/2$ at $E = 0$, the density of state is suppressed. This behavior is similar to the gap state for the non-vanishing Zeeman energy $\Delta$, for which we have discussed in the section 2. The case 3) is similar to the conventional quantum Hall sysytem, since two state $A$ and $B$ can be decoupled completely in the limit $w \to 0$. The extended energy $E = E_1$ approaches to $E = 0$.

The case 1) can be studied by the perturbation of $1/N$. We need to generalize the model to the N-orbital model. The random field $u(r)$ in (4.1) is changed to $u(r) \times I$ where $I$ is $N \times N$ unit matrix. $v$ is also a complex $N \times N$ matrix. The density of state in the $1/N$ expansion shows the logarithmic singularity at order $1/N^2$ for $w' = 0$. In the lowest order of $w'$, (2.13) becomes at $E = 0$,

$$\frac{\pi \epsilon}{A_2} = 1 - wC^2 - \frac{w'}{N} \left( \frac{wC^2}{1 - wC^2} \right) + \frac{d_1}{N^2} \ln^2 (1 - wC^2) + \cdots \quad (4.5)$$

The $1/N$ term is obtained from the diagrams of Fig. 4. Up to order of $1/N$, solving the equation, we obtain

$$wC^2 = 1 + \frac{w'}{2Nw} \pm \sqrt{\frac{w'}{Nw}} \quad (4.6)$$

Thus the logarithmic divergence is smeared out for small $w'$, since $\ln^2 \epsilon$ is changed to $\ln^2 w'$.

In the presence of $w'$, the conductivity $\sigma = e^2/2\pi^2\hbar$ is also changed. The logarithmic terms in the diffusion constant in $1/N^2$ order is cancelled by the vertex correction for $w'$, indeed each diagram cancels completely not only for the logarithmic term, as we have seen in (3.1). When $w' \neq 0$, this cancellation by the vertex part is not complete at $E = 0$, and the logarithmic term exists. Then, there appears a localization for $E = 0$ when $w' \neq 0$. By the same reason, $E \neq 0$ state is also localized.
5 Discussion

In this paper, we have evaluated the exact value of the conductivity of the two state model with the fixed Zeeman term at the Zeeman energy and find that this result is consistent with the previous numerical result. We also observed that the conductivity at $E = E_1$ for the case of non-vanishing Zeeman energy is same as the conductivity at $E = E_1$ for the no-Zeeman energy. Thus the effect of the Zeeman term does not alter the values of the conductivities at $E = \Delta$, $E = E_1$.

We discussed further how the situation is modified when the Zeeman energy becomes a random field, which obeys the Gaussian white noise distribution. Then, the diagrammatic expansion becomes two parameters $w$ and $w'$. We find, in the first order of $w'$, the cut-off of the singularity of the density of state appears, and it leads to the localization at $E = 0$.

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