A new concept of QM-models and truth in quantum mechanics (QM), the new hybrid-epistemic model of QM and its observer independence, the proof of the invalidity of no-go theorems

Jiří Souček

Charles University, Faculty od Arts; Prague, Czech Republic, U Kříže 8, 15800
E-mail: jiri.soucek@ff.cuni.cz

Abstract. We define a concept of a QM-model. The true theorem in QM is a statement which is true in all QM-models. In some QM-models the Bell's theorem can be proved (e.g. in the standard model of QM) while in other QM-models the Bell's theorem cannot be proved (e.g. in the hybrid-epistemic model of QM). The same situation is true when other no-go theorems are considered. Thus no-go theorems are not true in QM and the proof of the non-locality of QM is invalid.

Then the axiomatic definition of the hybrid-epistemic (HE) model of QM is presented in all details. At the end the recent proof of the inconsistency of the standard QM-model is discussed.

Our main program is to start the axiomatic study of QM (in the sense of the Hilbert’s sixth problem), to prove the invalidity of no-go theorems and to identify the right (acceptable) QM-model.

1. Introduction

The situation in the foundations of quantum mechanics (QM) is now quite confused (see the recent inconsistency proofs ([1], [2]). This situation requires the exact study which can be done only by using the axiomatic method. We propose to start the systematic study of different models and different interpretations of QM and comparisons among them using systematically the axiomatic method. Each proposed model/interpretation should be transformed into the exactly defined axiomatic theory. This direction of study makes a part of the famous sixth Hilbert’s problem – this problem proposes the axiomatization of physical theories in general (the axiomatization of QM is a special, but very important, part of this problem). Here we present the first steps in this program and the first results:

- The concept of an axiomatic QM-model
- The new concept of truth in QM
- The discovery of the hybrid-epistemic (HE) model and its axiomatic formulation
- The impossibility to prove the no-go theorems in the HE model
- The impossibility to prove no-go theorems in QM and the non-locality of QM
- The proof of the locality of EPR correlations in the HE model of QM
We think that the exact axiomatic study of these questions is completely necessary for the true understanding of the contemporary situation in the foundations of QM.

2. QM-models, truth in QM

We shall assume the standard QM model formalized by von Neumann ([3]) – its formulation has not the exact axiomatic form but it is an “almost” axiomatic theory. Moreover we shall assume that

- QM is a true description of the physical world
- We shall consider only finite dimensional quantum systems, i.e. quantum systems which
  Hilbert space has finite dimension (for simplicity)

Definition 2.1. The QM-model is the axiomatic system such that the set of its empirical predictions is identical to the set of empirical predictions of the standard model of QM.

This means that all QM-models are empirically equivalent among them. But this does not imply that there cannot exist theorems which are provable in one model and non-provable in another model.

Let us assume that there exists an axiomatic theory T such that (we shall show below that such a theory T exists)

- T is a QM-model
- no-go theorems (and thus the non-locality of QM) cannot be proved in T.

This is a quite new situation which requires the new definition of truth in QM.

Definition 2.2. The true statement in QM is the statement which is true in all QM-models.

For example, all empirical predictions of the standard QM are the true QM statement. The negative reformulation of this definition is the following theorem.

Theorem 2.1. Let us assume that a statement S is not proved in a certain QM-model R. Then the statement S cannot be proved as a true statement of QM.

Proof 2.1. We are not able to find which QM-model is the “true” model representing the Nature. Thus R is a possible candidate for a “true” model describing the Nature. Then S cannot be a true statement of QM.

The exact statement is the following: the empirical content of QM does not imply the statement S.

Theorem 2.2. The following statements are not the true statements in QM:

(i) No-go theorems cited above
(ii) The non-locality of QM

The basic fact is that such a theory T (mentioned above) exists and, in fact, it is, for example, the hybrid-epistemic model (HE model) defined below.

The intuition under this theorem is the fact that all QM-models are (from our empirical point of view) equally possible candidates for the “true” model of the Nature. We usually prefer the standard model of QM but there is no real reason to do this. From the empirical point of view all QM-models are equally good candidates.

The true meaning of the Bell’s theorem is not a non-locality of QM but the inconsistency of the standard (von Neumann’s) model of QM (see below).

Remark 2.1. The exact content of Theorem 2.2 can be illustrated by following statements.
• “The locality of QM is incompatible with the empirical content of QM”. This statement is false (since the HE model is local and respects the empirical content of QM).

• “The non-locality of QM is a consequence of the empirical content of QM together with the assumption of the standard model of QM”. This statement is true.

• Some people assert that the “local realism” implies the Bell’s inequality. It seems that this statement is true. But the “local realism” is not the same as “locality”. The violation of the “local realism” does not imply the violation of “locality”.

3. The hybrid-epistemic (HE) model of QM
This model was for the first time published in [4]. The presentation given here is slightly different but equivalent to the formulation given in [4]. The main point is the meaning of the wave function, i.e. of the quantum state (we shall not consider in this paper mixed states since this is not necessary). There are two possibilities:

• The wave function describes the state of the individual system – this is the main idea of the standard model of QM (called the ontic model in [4]).

• The wave function describes only states of ensembles of systems – this is the basic postulate of the epistemic model of QM (this assumption is also a base of the so-called statistical interpretation of QM).

The HE model of QM is a certain mixture of these two positions: each wave function can be a state of an ensemble but only some wave functions in some special situations can be considered as states of individual systems (they are then called individual states).1 On the other hand the properties are associated with individual systems. The concept of properties is the fundamental new element in the HE model of QM.

Definition 3.1. The structured Hilbert space is a couple \((H, L)\) where \(H\) is a (finite-dimensional) Hilbert space and \(L\) is an orthogonal decomposition of \(H\), i.e. \(H = \oplus\{L(v) | v \in I\}\).

Axiom 3.1. We shall assume that to each system \(S\) there is associated a complex \(n\) – dimensional structured Hilbert space \((H_S, L_S)\), \(n \geq 2\), (together with its decomposition \(H_S = \oplus\{L_S(v) | v \in I_S\}\)).

Definition 3.2. An ensemble is a set of systems \(E = \{S_1, \ldots, S_N\}\) which are created by some preparation procedure and satisfy conditions \(H(S_1) \cong \ldots \cong H(S_N) \cong H(E), L(S_1) \cong \ldots \cong L(S_N) \cong L(E)\), where \(\cong\) denotes the isomorphisms. It is assumed that the isomorphism between \(H(S_i)\) and \(H(S_j)\) generates the isomorphism between \(L(S_i)\) and \(L(S_j)\). Moreover \(I(S_1) \cong \ldots \cong I(S_N) \cong I(E)\). It is also assumed that systems \(S_1, \ldots, S_N\) are independent.

Axiom 3.2.

(i) The state of an ensemble (in a given moment of time) is described by some element of \(H_E\) – more precisely by a ray \([\Psi] = \{\alpha \Psi | |\alpha| = 1, \alpha \in C\}\), where \(||\Psi|| = 1\). The state of \(E\) is denoted by \(St(E)\).

(ii) The time evolution of the state \([\Psi(t)]\) of an ensemble \(E\) is given by the unitary group \(\{U_t\}\) in \(H_E\) and we have \([\Psi(t)] = [U_t(\Psi(0))]\).

1 Each individual state is also a state of an ensemble. This ensemble consists of systems where each of these systems is in a given individual state (this ensemble is called the homogeneous ensemble). In this sense each individual state is also a collective state.
Axiom 3.3. Let ensembles \( E \) and \( F \) be such that for some system \( S \) we have \( S \in E, S \in F \). We assume that \( St(E) = \{\Psi, \Psi \in L^v_S \} \) for some \( v \in I_S = I_E \) and that \( St(F) = \{\Phi, \Phi \in L^w_S \} \) for some \( w \in I_S = I_F \). Then \( v = w \). (I.e. the property of the individual system \( S \) depends only on \( S \).)

Axiom 3.4. For each system \( S \) there exists the ensemble \( E \) and \( v \in I_S \) such that \( S \in E \), \( St(E) = \{\Psi\} \) and \( \Psi \in L^v_S \).

Theorem 3.1. For each system \( S \) there exists a unique \( v \in I_S \) such that for each ensemble \( E \) satisfying \( S \in E \) it is true that \( St(E) = \{\Psi\} \) implies \( \Psi \in L^v_S \).

Definition 3.3. For each system \( S \) we denote \( v_S \in I_S \) such \( v \) which satisfies the conditions from Theorem 3.1 Then we also define a function \( v : I_S \to \{1, 0\} \) by \( v(S) = 1 \) if \( v = v_S \) and \( v(S) = 0 \) if \( v \neq v_S \).

Definition 3.4. Let us consider an ensemble \( E = \{S_1, \ldots, S_N\} \) and \( v \in I_E \). Then we define

\[
p(v|E) = \lim_{N} 1/N \sum_{i=1}^{N} v(S_i) = \lim_{N} 1/N |\{S \in E | v(S) = 1\}|^2
\]

Axiom 3.5. Let \( E \) and \( F \) be two ensembles such that \( St(E) = St(F) = \{\Psi, \Psi \in H_E \} \). Then for each \( v \in I_E \) we have \( p(v|E) = p(v|F) \). This means that the probability of observing the property \( v \) depends only on the state of an ensemble.

On the basis of Axiom 3.5 the probability \( p(v|E) \) depends only on the state of \( E \). Thus we can define

Definition 3.5. We set \( p(v|\{\Psi\}) = p(v|E) \) where \( St(E) = \{\Psi\} \).

Thus we have defined the probability distribution \( p(.|\{\Psi\}) \) on \( I_E \) depending only on \( \{\Psi\} \).

Axiom 3.6. (The composed systems.) Let us consider two ensembles \( M = \{M_1, \ldots, M_N\}, S = \{S_1, \ldots, S_N\} \), where \( St(M) = \{\Psi, \Psi \in H_M, St(S) = \{\Phi, \Phi \in H_S \} \). Let us consider the composed ensemble \( E = M \oplus S = \{M_1 \oplus S_1, \ldots, M_N \oplus S_N\} \). Then

(i) \( H_E = H_M \otimes H_S \)
(ii) \( I_E = I_M \times I_S, L_{M \oplus S}^{(v,w)}(v,w) = L_{M}^{(v)} \otimes L_{S}^{(w)} \)
(iii) If \( M \) and \( S \) are independent ensembles and if \( St(M) = \{\Psi, \Psi \in H_M, St(S) = \{\Phi, \Phi \in H_S, \) then \( St(M \oplus S) = \{\Psi \otimes \Phi\} \).

Definition 3.6. Let \( P : H_M \to H_M \) be a projection and \( Id(H_S) \) be an identity map on \( H_S \). Then the projection \( P \otimes Id(H_S) : H_E \to H_E \) is defined by

\[
(P \otimes Id(H_S))(\Psi \otimes \Phi) = P(\Psi) \otimes \Phi \text{ for each } \Psi \in H_M, \Phi \in H_S.
\]

Axiom 3.7. (The Born’s rule.) Let \( E = M \oplus S = \{M_1 \oplus S_1, \ldots, M_N \oplus S_N\} \) and let \( St(E) = \{\Psi, \Psi \in H_M \oplus H_S = H_M \otimes H_S \} \). Let \( P^{(v)}_M : H_M \to L^{(v)}_M, v \in I_M \) be a standard projection operator. Let us assume that the individual system \( M_k \oplus S_k \) be a composition of two individual systems. Then the probability that the individual system \( M_k \) will have the property \( v \) (i.e. \( v(Mk) = 1 \)) will be

\[
p(v|\Psi)) = ||(P^{(v)}_M \otimes Id(H_S))(\Psi)||^2.
\]

\(^2\) We assume the stability (i.e. the existence of the limit) of this relative frequency.
Axiom 3.8. (The up-date rule.) Let $E = M \oplus S = M_1 \oplus S_1, \ldots, M_N \oplus S_N$ and let $St(E) = |\Psi\rangle, \Psi \in H_{M \oplus S} = H_M \otimes H_S$. Let $P_M^{(v)} : H_M \rightarrow L_M^{(v)}, v \in I_M$ be a standard projection operator. Let us assume that the individual system $M_k \oplus S_k$ be a composition of two individual systems. Let us assume that the observation $M_k$ gives that $v(M_k) = 1$, i.e. that the subsystem $M_k$ has that property $v$. Then the individual system $M_k \oplus S_k$ will be an element of an up-dated ensemble $E^{(v)} = \{ M_I \oplus S_I | v(M_I) = 1 \}$ which is in the state 

$$/(P_M^{(v)} \otimes I d(H_S))(\Psi)/1/(P_M^{(v)} \otimes I d(H_S))(\Psi).$$

Axiom 3.9. (The existence of measuring systems.) For each $n \geq 2$ there exists at least one ensemble $E = \{ M_1, \ldots, M_N \}$ such that $|I(E)| = n$.

In the reference [4] there are proved the basic properties of the HE model:

(i) The HE model is empirically equivalent to the standard model of QM (see [4], sect. 7, p.36, where the standard model of QM is called the ontic model of QM)

(ii) The measurement process is the standard process in QM and uses the concept of an observation of the fact that the measuring system has certain property $v$.

(iii) The EPR correlations are explicitly local in the HE model of QM.

Thus the HE model of QM satisfies the requirements for the model T considered in the preceding section and thus theorems stated in the preceding section are proved.

The existence of different QM-models is the basic new fact presented in [4]. The invalidity of no-go theorems in QM and invalidity of the non-locality of QM are consequences of this discovery.

4. The impossibility to prove no-go theorems (and the non-locality) in the HE model of QM. The inconsistencies in the standard model of QM.

The simplest sub-case of the HE model is the hybrid model (introduced in [5] under the name of the modified QM). The hybrid system is such system $S$ for which $dim L_S^{(v)} = 1$ for each $v \in I_S$. The hybrid model of QM is the model such that it contains only hybrid systems.

In the hybrid model each state $|\Psi\rangle, \Psi \in L_S^{(v)}$ can be considered as an individual state since it can be associated with the individual system $S$ (through the property $v(S) = 1$).

Each proof of any no-go theorem is based on the existence and the use of individual states. In general at least at one part (Alice’s or Bob’s) the set of considered individual states must contain at least two orthogonal bases. I.e. at least two settings of measuring apparatus is needed. But in the hybrid model we have in a disposition only one base composed of individual states for a given system. Thus the standard way of proving the no-go theorem cannot be applied. Thus the no-go theorems cannot be proved in the hybrid model. In the HE model the situation with individual states is the same. As a consequence also the proof of the non-locality of QM is impossible in the HE model of QM (details can be found in [4]).

The essential feature of the HE model of QM is its observer independence. This means that the concept of the observer does not take a part in the formulation of the model. In the HE model the measurement process is the one of standard processes in QM. Instead of the concept of a measurement there is a concept of an observation that the individual measuring system has (or does not have) a given property. The detailed description of the internal measurement process in the HE model can be found in [4].

The recently discovered inconsistency of the standard model ([1], [2]) was proved by constructing a rather sophisticated example based on the paradox of “Wigner’s friend”. Such an example cannot be constructed in the HE model since the HE model is observer independent.
and the paradox of “Wigner’s friend” does not exist in this model. The observer independence creates the basic difference between the standard model and the HE model of QM (i.e. the HE model is a completely new type of a model of QM).

The inconsistency mentioned above implies that the HE model is the unique realistic local possibility for QM (the Bohm’s model of QM is explicitly non-local).

5. Conclusion

(i) There exist different QM-models, e.g., the standard model, the HE model and others. The true theorem of QM must be true in all QM models.

(ii) The no-go theorems (and consequently the non-locality of QM) are not the consequences of the empirical content of QM. The standard proof of no-go theorems requires the existence of “individual states” which do not exist in the HE model of QM (or, at least, do not exist in the sufficiently big amount).

(iii) It can be proved that the EPR correlations are explicitly local in the HE-model. All details of formulations and proofs can be found in [4]

(iv) The HE model of QM is defined in the exact form of the axiomatic theory

(v) The HE model does not contain the axiomatic definition of the measurement process (in contrast to the standard model). Thus this model is observer independent. The concept of an observer does not enter into this model.

(vi) As a consequence, the Wigner’s friend paradox does not exist in the HE model of QM (since there is no role of an observer in the HE model of QM).

(vii) In the light of the recent proofs of the inconsistency of the standard model ([?], [2]), the HE model of QM is the unique surviving local possibility for QM.

References

[1] Frauchiger D and Renner R 2016 Quantum theory cannot consistently describe the use of itself Preprint arXiv:1604.07422

[2] Frauchiger D and Renner R 2018 Nature Communications 9 3711

[3] von Neumann J 1932 Mathematische Grundlagen der Quantenmechanik (Berlin: Springer-Verlag)

[4] Soucek J 2018 A new model for quantum mechanics and the invalidity of no-go theorems (Berlin: LAP LAMBERT Academic Publishing)

[5] Soucek J 2014 The principle of anti-superposition in QM and the local solution of the Bell’s inequality problem Preprint http://www.nusl.cz/ntk/nusl-177617