We investigate the strong gravitational lensing in a Kaluza-Klein black hole with squashed horizons. We find the size of the extra dimension imprints in the radius of the photon sphere, the deflection angle, the angular position and magnification of the relativistic images. Supposing that the gravitational field of the supermassive central object of the Galaxy can be described by this metric, we estimated the numerical values of the coefficients and observables for gravitational lensing in the strong field limit.

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I. INTRODUCTION

It is well known that string theory is a promising candidate for the unified theory. The extra dimension is one of the most important predictions in the string theory. Therefore, the detection of the extra dimension has attracted a lot of attention recently because it can present the signature of the string and the correctness of string theory. Recent investigations indicate that the extra dimension can imprint in the quasinormal modes originated from the perturbation around high dimensional black holes [1–4], which could be tested in the gravitational wave probe in the near future. Moreover, the spectrum of Hawking radiation from the high dimensional black holes could provide another possible way to observe the extra dimension, which is expected to be detected in particle accelerator experiments [5–13].

The strong gravitational lensing is another possible way to detect the extra dimension. According to general relativity, photons would be deviated from their straight paths as they pass close to a compact and massive body. The phenomena resulting from the deflection of light rays in a gravitational field are called gravitational lensing and the object causing a detectable deflection is usually named a gravitational lens. The strong gravitational lensing is caused by a compact object with a photon sphere, such as a black hole, black brane, and so on. As photons pass close to the photon sphere, the deflection angles become so large that an observer would detect two infinite sets of faint relativistic images on each side of the black hole which are produced by photons that make complete loops around the black hole before reaching the observer. These relativistic images can provide us not only information about black holes in the Universe, but also profound verification of alternative theories of gravity in their strong field regime [14–27]. Thus, the study of the strong gravitational lensing by high dimensional black holes can help us to detect the extra dimension in astronomical observation in the future.

The Kaluza-Klein black holes with squashed horizons are a kind of interesting Kaluza-Klein type metrics [28–34]. In the vicinity of the black hole horizon, the spacetime has a structure of the five-dimensional black holes. In the region far from the black holes, it is locally the direct product of the four-dimensional Minkowski spacetime and the circle. Recently, the Hawking radiations have been studied in these squashed Kaluza-Klein black holes, which indicates that the luminosity of Hawking radiation can tell us the size of the extra dimension which opens a window to observe extra dimensions [35, 36]. Moreover, quasinormal modes in the background of the Kaluza-Klein black hole with squashed horizons have been investigated in [37, 38], which implies that the quasinormal frequencies contain the information of the size of the extra dimension. Matsuno
et al. [39] have studied the precession of a gyroscope in a circular orbit in the squashed Kaluza-Klein black hole spacetime and find that the correction is proportional to the square of (size of extra dimension)/(gravitational radius of central object). These results are very useful for us to understand the properties of the squashed Kaluza-Klein black holes and to extract information about the extra dimension in future observations. The main purpose of this paper is to study the gravity lens in the strong field limit in the squashed Kaluza-Klein black hole spacetime and see that effect of the size of the extra dimension on the coefficients and observables of gravitational lensing in the strong field limit.

The plan of our paper is organized as follows. In Sec.II we adopt to Bozza’s method [23, 24] and obtain the deflection angles for light rays propagating in the squashed Kaluza-Klein black hole. In Sec.III we suppose that the gravitational field of the supermassive black hole at the center of our Galaxy can be described by this metric and then obtain the numerical results for the observational gravitational lensing parameters defined in Sec.II. Then, we make a comparison between the properties of gravitational lensing in the squashed Kaluza-Klein and four-dimensional Schwarzschild metrics. To conclude, we present a summary.

II. DEFLECTION ANGLE IN THE SQUASHED KALUZA-KLEIN BLACK HOLE SPACETIME

The five-dimensional neutral static squashed Kaluza-Klein black hole is described by [28, 31]

\[ ds^2 = -F(\rho)dt^2 + \frac{K}{F(\rho)}d\rho^2 + K\rho^2(d\theta^2 + \sin^2\theta d\phi^2) + \frac{r_\infty^2}{4K}(d\psi + \cos^2\theta d\phi)^2, \]

with

\[ F(\rho) = 1 - \frac{\rho_H}{\rho}, \quad K = 1 + \frac{\rho_0}{\rho}, \]

where \( 0 < \theta < \pi, 0 < \phi < 2\pi \) and \( 0 < \psi < 4\pi \). \( \rho_H \) is the radius of the black hole event horizon. The positive parameters \( r_\infty, \rho_0 \) and \( \rho_H \) are related by \( r_\infty^2 = 4\rho_0(\rho_H + \rho_0) \). The Komar mass of this black hole is given by \( M = \pi r_\infty \rho_H/G_5 \) [39, 40], where \( G_5 \) is the five-dimensional gravitational constant. It is found that in such a squashed Kaluza-Klein black hole spacetime the relationship between \( G_5 \) and \( G_4 \) (the four-dimensional gravitational constant) can be expressed as \( G_5 = 2\pi r_\infty G_4 \) [39, 40]. Thus, the radius of the black hole event horizon \( \rho_H \) can be written as \( \rho_H = 2G_4 M \) and the Hawking temperature can be expressed as

\[ T_H = \frac{1}{4\pi \rho_H} \sqrt{\frac{\rho_H}{\rho_H + \rho_0}}. \]

As \( \rho_0 \to 0 \), the function \( K = 1 \) and the metric (1) reduces to a spacetime which is described as the four-dimensional Schwarzschild black hole with a constant twisted \( S_1 \) fiber. As \( \rho_H \ll \rho_0 \), the function \( K \) becomes
important, and the metric (1) tends to a spherical symmetrical five-dimensional Schwarzschild black hole spacetime (28, 31, 39). Thus, the parameter $\rho_0$ can be regarded as a scale of transition from five-dimensional spacetime to an effective four-dimensional one.

The geodesics equations in the curved spacetime are

$$\frac{d^2x^\mu}{d\lambda^2} + \Gamma^\mu_{\nu\tau} \frac{dx^\nu}{d\lambda} \frac{dx^\tau}{d\lambda} = 0,$$

(4)

where $\lambda$ is an affine parameter along the geodesics. For the squashed Kaluza-Klein black hole spacetime (1), the geodesics equations obey

$$\frac{d^2t}{d\lambda^2} + \frac{F'(\rho)}{F(\rho)} \frac{dt}{d\lambda} \frac{d\rho}{d\lambda} = 0,$$

(5)

$$\frac{d^2\theta}{d\lambda^2} = \frac{(K\rho^2)' \theta}{K \rho^2} \frac{d\rho}{d\lambda} - \cos \theta \sin \theta \left( 1 - \frac{r_\infty^2}{4K^2\rho^2} \right) \frac{d\phi}{d\lambda}^2 + \frac{r_\infty^2 \sin \theta}{4K^2\rho^2} \frac{d\phi}{d\lambda} \frac{d\psi}{d\lambda} = 0,$$

(6)

$$\frac{d^2\phi}{d\lambda^2} + \frac{(K\rho^2)' \phi}{K \rho^2} \frac{d\rho}{d\lambda} + \cos \theta \sin \theta \left( 2 - \frac{r_\infty^2}{4K^2\rho^2} \right) \frac{d\phi}{d\lambda} \frac{d\theta}{d\lambda} - \frac{r_\infty^2}{4K^2\rho^2} \frac{d\theta}{d\lambda} \frac{d\psi}{d\lambda} = 0,$$

(7)

$$\frac{d^2\psi}{d\lambda^2} - \frac{2(K\rho)' \cos \theta \phi}{K \rho} \frac{d\phi}{d\lambda} \frac{d\rho}{d\lambda} - \frac{K'}{K} \frac{d\psi}{d\lambda} \frac{d\rho}{d\lambda} + \frac{1}{\sin \theta} \left[ 1 + \left( 1 - \frac{r_\infty^2}{4K^2\rho^2} \right) \cos^2 \theta \right] \frac{d\phi}{d\lambda} \frac{d\theta}{d\lambda} + \frac{r_\infty^2}{4K^2\rho^2} \cos \theta \frac{d\theta}{d\lambda} \frac{d\psi}{d\lambda} = 0,$$

(8)

$$\frac{d^2\rho}{d\lambda^2} + \frac{F(\rho)F'(\rho)}{2K} \left( \frac{dt}{d\lambda} \right)^2 + \frac{F'(\rho)}{F(\rho)} \left( \frac{K'}{K} \right) \frac{d\rho}{d\lambda} \left( \frac{dt}{d\lambda} \right)^2 - \frac{F(\rho)(K\rho^2)'}{2K} \left( \frac{d\theta}{d\lambda} \right)^2 - \frac{F(\rho)}{2K} \left( \frac{(K\rho^2)'}{K} \sin^2 \theta \right) \left( \frac{d\phi}{d\lambda} \right)^2 + \frac{r_\infty^2}{8K^3} \frac{K'}{\rho^2} \left( \frac{d\phi}{d\lambda} \cos \theta + \frac{d\psi}{d\lambda} \right)^2 = 0,$$

(9)

where a prime represents a derivative with respect to $\rho$. In this paper, we consider only the orbits with $\theta = \pi/2$. With this condition, we find from (6) that

$$\frac{d\phi}{d\lambda} \frac{d\psi}{d\lambda} = 0.$$

(10)

This implies that either $\frac{d\phi}{d\lambda} = 0$ or $\frac{d\psi}{d\lambda} = 0$. Here we set $\frac{d\phi}{d\lambda} = 0$, so that we can compare with the results obtained in the four-dimensional black hole spacetime. Obviously, Eq. (8) is satisfied naturally as $\frac{d\phi}{d\lambda} = 0$.

Therefore, the equations of motion for this case can be simplified as

$$\frac{dt}{d\lambda} = \frac{1}{F(\rho)},$$

$$K\rho^2 \frac{d\phi}{d\lambda} = J,$$

$$\frac{d^2\rho}{d\lambda^2} + \frac{F'(\rho)}{2KF(\rho)} \left( \frac{dt}{d\lambda} \right)^2 + \frac{F'(\rho)}{2KF(\rho)} \left( \frac{K'}{F(\rho)} \right) \frac{d\rho}{d\lambda} \left( \frac{dt}{d\lambda} \right)^2 - \frac{F(\rho)(K\rho^2)'}{2K^2\rho^3} \left( \frac{d\theta}{d\lambda} \right)^2 = 0.$$

(11)

Following Refs. [14, 20], we can obtain the deflection angle for the photon coming from infinite in the squashed Kaluza-Klein black hole spacetime

$$\alpha(\rho_\ast) = I(\rho_\ast) - \pi,$$

(12)
where $\rho_s$ is the closest approach distance and $I(\rho_s)$ is

$$I(\rho_s) = 2 \int_{\rho_s}^{\infty} \frac{\sqrt{Kd\rho}}{\sqrt{F(\rho)C(\rho)} \sqrt{\frac{C(\rho)^2F(\rho)}{C(\rho_s)F(\rho)}} - 1},$$

(13)

where $C(\rho) = K \rho^2 = \rho(\rho + \rho_0)$. As in the four-dimensional black hole spacetime, the deflection angle increases when parameter $\rho_s$ decreases. When the deflection angle becomes $2\pi$, the light ray makes a complete loop around the compact object before reaching the observer. If $\rho_s$ is equal to the radius of the photon sphere, one can find that the deflection angle diverges and the photon is captured. In the squashed Kaluza-Klein black hole spacetime, one can find from Eq.(13) that the deflection angle contains the information about the scale of transition $\rho_0$. This implies that we could detect the extra dimension by the gravitational lens.

For the squashed Kaluza-Klein black hole spacetime, the photon sphere equation is given by

$$\frac{C(\rho)'}{C(\rho)} = \frac{F(\rho)'}{F(\rho)},$$

(14)

The radius of the photon sphere is the largest real root of Eq. (14), which can be expressed as

$$\rho_{hs} = \frac{3\rho_H - \rho_0 + \sqrt{\rho_0^2 + 10\rho_0\rho_H + 9\rho_H^4}}{4},$$

(15)

Obviously, it also depends on the scale of transition $\rho_0$. As $\rho_0$ approaches zero, the radius of the photon sphere $\rho_{hs} = \frac{3}{2}\rho_H$, which is consistent with that in the four-dimensional Schwarzschild black hole spacetime. As $\rho_0$ tends to infinite, we find that $\rho_{hs} \rightarrow 2\rho_H$. According to the coordinate translation $\rho = \rho_0 \sqrt{r_H^2 - r^2}$, one can obtain easily that $r_{hs} \rightarrow \sqrt{2r_H}$ in the case that $\rho_0$ tends to infinite, which agrees with the photon sphere in the five-dimensional Schwarzschild black hole spacetime. In Fig.(1), we plot the variety of the ratio between the radius of the photon sphere $\rho_{hs}$ and the radius of the black hole event horizon $\rho_H$ with the parameter $\rho_0/\rho_H$. From fig.(1), one can obtain that $\rho_{hs}/\rho_H$ increases with $\rho_0/\rho_H$.

Following the method developed by Bozza [23], we can define a variable

$$z = 1 - \frac{\rho_s}{\rho},$$

(16)

and rewrite Eq.(13) as

$$I(\rho_s) = \int_0^1 R(z, \rho_s) f(z, \rho_s) dz,$$

(17)

with

$$R(z, \rho_s) = \frac{2\rho_s^2 \sqrt{KC(\rho_s)}}{\rho_s C(\rho)} = 2 \sqrt{\frac{\rho_s + \rho_0}{\rho_s + \rho_0(1 - z)}},$$

(18)
FIG. 1: Variety of the quantity $\rho_{hs}/\rho_H$ with $\rho_0/\rho_H$ in the squashed Kaluza-Klein black hole spacetime.

\[ f(z, \rho_s) = \frac{1}{\sqrt{F(\rho_s) - F(\rho)C(\rho_s)/C(\rho)}}. \]  

(19)

Obviously, the function $R(z, \rho_s)$ is regular for all values of $z$ and $\rho_s$. While the function $f(z, \rho_s)$ diverges as $z$ tends to zero. Thus, we split the integral (17) into the divergent part $I_D(\rho_s)$ and the regular one $I_R(\rho_s)$

\[ I_D(\rho_s) = \int_0^1 R(0, \rho_{hs}) f_0(z, \rho_s) dz, \]  

\[ I_R(\rho_s) = \int_0^1 [R(z, \rho_s) f(z, \rho_s) - R(0, \rho_{hs}) f_0(z, \rho_s)] dz. \]  

(20)

In order to find the order of divergence of the integrand, we expand the argument of the square root in $f(z, \rho_s)$ to the second order in $z$

\[ f_s(z, \rho_s) = \frac{1}{\sqrt{p(\rho_s)z + q(\rho_s)z^2}}. \]  

(21)

FIG. 2: The ratio $u_{hs}/\rho_H$ (between the minimum impact parameter $u_{hs}$ and $\rho_H$) changes with $\rho_0/\rho_H$ in the squashed Kaluza-Klein black hole spacetime.
where
\[
p(\rho_s) = 2 - \frac{2\rho_H}{\rho_s} - \frac{\rho_H + \rho_0}{\rho_s + \rho_0},
\]
\[
q(\rho_s) = \frac{(\rho_0^2 + 3\rho_0 \rho_s) \rho_H + (3\rho_H - \rho_s) \rho_0^2}{\rho_s (\rho_0 + \rho_s)^2}.
\]  
(22)

If \(\rho_s\) tends to the radius of the photon sphere \(\rho_{hs}\), one can find that the coefficient \(p(\rho_s)\) approaches zero and the leading term of the divergence in \(f_s(z, \rho_s)\) is \(z^{-1}\), which means that the integral (17) diverges logarithmically. Close to the divergence, the deflection angle can be expanded in the form
\[
\alpha(\theta) = -\tilde{a} \log \left( \frac{\theta D_{OL}}{u_{hs}} - 1 \right) + \tilde{b} + O(u - u_{hs}),
\]
(23)

with
\[
\tilde{a} = \frac{R(0, \rho_{hs})}{\sqrt{q(\rho_{hs})}} = \left[ \frac{1}{2} \left( 1 + 3 \sqrt{\frac{\rho_0 + \rho_H}{\rho_0 + 9\rho_H}} \right) \right]^{1/2},
\]
\[
\tilde{b} = -\pi + b_R + \tilde{a} \log \frac{\rho_{hs}^2 [C''(\rho_{hs}) F(\rho_{hs}) - C(\rho_{hs}) F''(\rho_{hs})]}{u_{hs} \sqrt{F'(\rho_{hs}) C'(\rho_{hs})}} = -\pi + b_R + \log \left[ 2 \frac{\rho_0 + 9\rho_H}{\rho_0 + \rho_H} \right],
\]
\[
b_R = I_R(\rho_{hs}),
\]
\[
u_{hs} = \sqrt{\frac{C(\rho_{hs})}{F'\rho_{hs}}} = \sqrt[4]{\frac{27}{4} \left[ 27\rho_H^2 + 18\rho_0 \rho_H - \rho_0^2 + \sqrt{\rho_0 + \rho_H} (\rho_0 + 9\rho_H)^{3/2} \right]}^{1/2}.
\]  
(24)

Here \(D_{OL}\) is the distance between observer and gravitational lens. Making use of Eqs. (15), (23) and (24), we can study the properties of strong gravitational lensing in the squashed Kaluza-Klein black hole spacetime.

FIG. 3: Variation of the coefficients of the strong field limit \(\tilde{a}\) (left) and \(\tilde{b}\) (right) with parameter \(\rho_0/\rho_H\) in the squashed Kaluza-Klein black hole spacetime.

In the Figs. (2)-(4), we plot the variations of the ratio \(u_{hs}/\rho_H\), the coefficients \(\tilde{a}\) and \(\tilde{b}\), and deflection angle \(\alpha(\theta)\) with the parameter of the extra dimension \(\rho_0/\rho_H\). Obviously, as \(\rho_0\) tends to zero, these quantities reduce
FIG. 4: Deflection angles in the squashed Kaluza-Klein black hole spacetime evaluated at $u = u_{hs} + 0.003$ as functions of $\rho_0/\rho_H$.

to those in the four-dimensional Schwarzschild black hole spacetime. From Fig.(2), we can see that with the increase of $\rho_0/\rho_H$, the ratio $u_{hs}/\rho_H$ increases, which is different from that originated from the charge in the Reissner-Norström black hole spacetime. Moreover, from Fig.(3), we can find that the coefficients $\bar{a}$ and $\bar{b}$ increase with $\rho_0/\rho_H$. We also show the deflection angle $\alpha(\theta)$ evaluated at $u = u_{hs} + 0.003$ in Fig.(4). It indicates that the presence of $\rho_0$ increases the deflection angle $\alpha(\theta)$ for the light propagated in the squashed Kaluza-Klein black hole spacetime. Comparing with those in the four-dimensional Schwarzschild black hole spacetime, we could extract information about the size of the extra dimension by using strong field gravitational lensing.

III. OBSERVATIONAL GRAVITATIONAL LENSING PARAMETERS

Let us now study the effect of the scale parameter $\rho_0$ on the observational gravitational lensing parameters. We start by assuming that the gravitational field of the supermassive black hole at the Galactic center of Milky Way can be described by the squashed Kaluza-Klein black hole spacetime, and then estimate the numerical values for the coefficients and observables of gravitational lensing in the strong field limit.

When the source, lens, and observer are highly aligned, the lens equation in strong gravitational lensing can be approximated as

$$\beta = \theta - \frac{D_{LS}}{D_{OS}} \Delta \alpha_n,$$

where $D_{LS}$ is the distance between the lens and the source, $D_{OS} = D_{LS} + D_{OL}$, $\beta$ is the angular separation between the source and the lens, $\theta$ is the angular separation between the imagine and the lens, $\Delta \alpha_n = \alpha - 2n\pi$ is
the offset of deflection angle, and \( n \) is an integer. The \( n \)-th image position \( \theta_n \) and the \( n \)-th image magnification \( \mu_n \) can be approximated as

\[
\theta_n = \theta_0 + \frac{u_{hs}(\beta - \theta_0)e^{\frac{\theta_0 - 2n\pi}{\bar{a}D_{LS}D_{OL}}}}{\bar{a}D_{LS}D_{OL}},
\]

\[
\mu_n = \frac{u_{hs}^2(1 + e^{\frac{\theta_0 - 2n\pi}{\bar{a}D_{LS}D_{OL}^2}})}{\bar{a}\beta D_{LS}D_{OL}^2},
\]

respectively. The quantity \( \theta_0 \) is the image positions corresponding to \( \alpha = 2n\pi \). In the limit \( n \to \infty \), the relation between the minimum impact parameter \( u_{hs} \) and the asymptotic position of a set of images \( \theta_\infty \) can be expressed as

\[
u_{hs} = D_{OL}\theta_\infty.
\]

In order to obtain the coefficients \( \bar{a} \) and \( \bar{b} \), one needs to separate at least the outermost image from all the others. As in Refs. [23, 24], we consider here the simplest case in which only the outermost image \( \theta_1 \) is resolved as a single image and all the remaining ones are packed together at \( \theta_\infty \). Thus the angular separation between the first image and other ones can be expressed as

\[
s = \theta_1 - \theta_\infty,
\]

and the ratio of the flux from the first image and those from the all other images is given by

\[
\mathcal{R} = \frac{\mu_1}{\sum_{n=2}^{\infty} \mu_n}.
\]

For highly aligned source, lens, and observer geometry, these observables can be simplified as [23, 24]

\[
s = \theta_\infty e^{\frac{\theta_0 - 2n\pi}{\bar{a}D_{LS}D_{OL}}},
\]

\[
\mathcal{R} = e^{\frac{2\pi}{\bar{a}D_{LS}D_{OL}^2}}.
\]

Thus, through measuring \( s \), \( \mathcal{R} \), and \( \theta_\infty \), we can obtain the strong deflection limit coefficients \( \bar{a} \), \( \bar{b} \) and the minimum impact parameter \( u_{hs} \). Comparing their values with those predicted by the theoretical models, we could detect the size of the extra dimension. With the help of Eqs. (24), (28) and in combination with

\[
\rho_H = 2G_4 M,
\]

we can find that

\[
\theta_\infty = \frac{u_{hs}}{D_{OL}} = \frac{\sqrt{2}}{2} \left[ 27 + 18 \frac{\rho_0}{\rho_H} - \frac{\rho_0^2}{\rho_H^2} + \sqrt{\left( \frac{\rho_0}{\rho_H} + 1 \right) \left( \frac{\rho_0}{\rho_H} + 9 \right)^{3/2}} \right]^{1/2} \frac{G_4 M}{D_{OL}}.
\]

The mass of the central object of our Galaxy is estimated to be \( 2.8 \times 10^6 M_\odot \) and its distance is around \( 8.5 kpc \), so that the ratio of the mass to the distance \( G_4 M / D_{OL} \approx 1.574 \times 10^{-11} \). Here \( D_{OL} \) is the
FIG. 5: Gravitational lensing by the Galactic center black hole. Variation of the values of the angular position $\theta_\infty$ and the relative magnitudes $r_m$ with parameter $\rho_0/\rho_H$ in the squashed Kaluza-Klein black hole spacetime.

distance between the lens and the observer in the $\rho$ coordination rather than that in $r$ coordination because that in the five-dimensional spacetime the dimension of the black hole mass $M$ is the square of that in the polar coordination $r$. Making use of Eqs. (32), (24) and (31) we can estimate the values of the coefficients and observables for gravitational lensing in the strong field limit. For the different $\rho_0/\rho_H$, the numerical value for the relative minimum impact parameter $u_{hs}/\rho_H$, the angular position of the relativistic images $\theta_\infty$, the angular separation $s$ and the relative magnification of the outermost relativistic image with the other relativistic images $r_m$ (which is related to $R$ by $r_m = 2.5 \log R$) are listed in the Table I. The dependence of these observables on the parameter $\rho_0/\rho_H$ are also shown in Fig. (5). Obviously, our results reduce to those in the four-dimensional Schwarzschild black hole spacetime as $\rho_0 = 0$. From Table I and Fig. (5), we find that with the increase of $\rho_0$, the relative minimum impact parameter $u_{hs}/\rho_H$, the angular position of the relativistic images $\theta_\infty$ and the angular separation $s$ increase, while the relative magnitudes $r_m$ decrease. This information could help us to detect the extra dimension in the future.

| $\rho_0/\rho_H$ | $\theta_\infty$ (arcsec) | $s$ (arcsec) | $r_m$ (magnitudes) | $u_{hs}/\rho_H$ | $\bar{a}$ | $\bar{b}$ |
|-----------------|------------------|-------------|-----------------|----------------|--------|--------|
| 0               | 16.870           | 0.02112     | 6.8219          | 2.598          | 1.000  | -0.4002 |
| 0.1             | 17.420           | 0.02346     | 6.7497          | 2.683          | 1.011  | -0.3974 |
| 0.2             | 17.949           | 0.02588     | 6.6838          | 2.764          | 1.021  | -0.3938 |
| 0.3             | 18.458           | 0.02835     | 6.6234          | 2.843          | 1.030  | -0.3895 |
| 0.4             | 18.950           | 0.03088     | 6.5678          | 2.918          | 1.039  | -0.3847 |
| 0.5             | 19.427           | 0.03346     | 6.5162          | 2.992          | 1.047  | -0.3794 |

TABLE I: Numerical estimation for main observables and the strong field limit coefficients for the black hole at the center of our Galaxy, which is supposed to be described by the squashed Kaluza-Klein black hole spacetime. $\rho_0$ is the parameter of the extra dimension, $r_m = 2.5 \log R$. 
IV. SUMMARY

Gravitational lensing in the strong field limit provides a potentially powerful tool to identify the nature of black holes in the different gravity theories. The extra dimension is one of the important predictions in string theory, which is believed to be a promising candidate for the unified theory of everything. In this paper we have investigated strong field lensing in the squashed Kaluza-Klein black hole spacetime and found that the size of the extra dimension imprints in the radius of the photon sphere, the deflection angle, the angular position and magnification of the relativistic images. The model was applied to the supermassive black hole in the Galactic center. Our results show that with the increase of the parameter $\rho_0/\rho_H$ the relative minimum impact parameter $u_{hs}/\rho_H$, the angular position of the relativistic images $\theta_\infty$ and the angular separation $s$ increase, while the relative magnitudes $r_m$ decrease. This may offer a way to detect the extra dimension by astronomical instruments in the future.

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[1] J. Y. Shen, B. Wang and R. K. Su, Phys. Rev. D 74, 044036 (2006).
[2] E. Abdalla, B. Cuadros-Melgar, A. B. Pavan and C. Molina, Nucl.Phys. B 752, 40 (2006).
[3] S. B. Chen, B. Wang, R. K. Su, Phys. Lett. B 647, 282 (2007).[arXiv:hep-th/0701209]
[4] P. Kanti, R. A. Konoplya, A. Zhidenko, Phys. Rev. D 74, 064008 (2006); P. Kanti, R. A. Konoplya, Phys. Rev. D 73, 044002 (2006); D. K. Park, Phys. Lett. B 633, 613 (2006).
[5] P. Kanti, [hep-ph/0310162].
[6] C. M. Harris and P. Kanti, JHEP 0310 014 (2003) ; P. Kanti, Int. J. Mod. Phys. A 19 4899 (2004); P. Argyres, S. Dimopoulos and J. March-Russell, Phys. Lett. B441 96 (1998); T. Banks and W. Fischler, [hep-th/9906038] R. Emparan, G. T. Horowitz and R. C. Myers, Phys. Rev. Lett. 85 499 (2000).
[7] E. Jung and D. K. Park, Nucl. Phys. B 717 272 (2005); N. Sanchez, Phys. Rev. D 18 1030 (1978); E. Jung and D. K. Park, Class. Quant. Grav. 21 3717 (2004).
[8] A. S. Majumdar, N. Mukherjee, Int. J. Mod. Phys. D 14 1095 (2005) and reference therein; G. Kofinas, E. Papantonopoulos and V. Zamarias, Phys. Rev. D 66, 104028 (2002); G. Kofinas, E. Papantonopoulos and V. Zamarias, Astrophys. Space Sci. 283, 685 (2003); A. N. Aliev, A. E. Gumrukcuoglu, Phys. Rev. D 71, 104027 (2005); S. Kar, S. Majumdar, Int. J. Mod. Phys. A 21, 6087 (2006); S. Kar, S. Majumdar, Phys. Rev. D 74, 066003 (2006); S. Kar, Phys. Rev. D 74, 126002 (2006).

[9] E. Jung, S. H. Kim and D. K. Park, Phys. Lett. B 615 273 (2005); E. Jung, S. H. Kim and D. K. Park, Phys. Lett. B 619 347 (2005); D. Ida, K. Oda and S. C. Park, Phys. Rev. D 67 064025 (2003); G. Duffy, C. Harris, P. Kanti and E. Winstanley, JHEP 0509 049 (2005); M. Casals, P. Kanti and E. Winstanley, JHEP 0602 051 (2006); E. Jung and D. K. Park, Nucl. Phys. B 731 171 (2005); A. S. Cornell, W. Naylor and M. Sasaki, JHEP 0602, 012 (2006); V. P. Frolov, D. Stojkovic, Phys. Rev. Lett. 89, 151302 (2002); Valeri P. Frolov, Dejan Stojkovic, Phys. Rev. D 66, 084002 (2002); D. Stojkovic, Phys. Rev. Lett. 94, 011603 (2005).

[10] D.K. Park, Class. Quant. Grav. 23, 4101 (2006).

[11] Eylee Jung and D. K. Park, [hep-th/0612043]; V. Cardoso, M. Cavaglia, L. Gualtieri, Phys. Rev. Lett. 96, 071301 (2006); V. Cardoso, M. Cavaglia, L. Gualtieri, JHEP 0602, 021 (2006).

[12] D. Dai, N. Kaloper, G. Starkman and D. Stojkovic, Phys. Rev. D 75, 024043 (2007).

[13] L.H. Liu, B. Wang, G.H. Yang, Phys. Rev. D 76, 064014 (2007).

[14] C. Darwin, Proc. of the Royal Soc. of London 249, 180 (1959).

[15] K. S. Virbhadra, D. Narasimha and S. M. Chitre, Astron. Astrophys. 337, 18 (1998).

[16] K. S. Virbhadra and G. F. R. Ellis, Phys. Rev. D 62, 084003 (2000).

[17] C. M. Claudel, K. S. Virbhadra and G. F. R. Ellis, J. Math. Phys. 42, 818 (2001).

[18] K. S. Virbhadra and G. F. R. Ellis, Phys. Rev.D 65, 103004(2002).

[19] S. Frittelly, T. P. Kling and E. T. Newman, Phys. Rev. D 61, 064021 (2000).

[20] V. Bozza, S. Capozziello, G. Iovane and G. Scarpetta, Gen. Rel. and Grav. 33, 1535 (2001).

[21] E. F. Eiroa, G. E. Romero and D. F. Torres, Phys. Rev. D 66, 024010 (2002); E. F. Eiroa, Phys. Rev. D 71, 083010 (2005); E. F. Eiroa, Phys. Rev. D 73, 043002 (2006).

[22] R. Whisker, Phys. Rev. D 71, 064004 (2005).

[23] V. Bozza, Phys. Rev. D 66, 103001 (2002).

[24] V. Bozza, Phys. Rev. D 67, 103006 (2003); V. Bozza, F. De Luca, G. Scarpetta, and M. Sereno, Phys. Rev. D 72, 08300 (2005); V. Bozza, F. De Luca, and G. Scarpetta, Phys. Rev. D 74, 063001 (2006).

[25] A. Bhadra, Phys. Rev. D 67, 103009 (2003).

[26] T. Ghosh and S. Sengupta, arXiv: 1001.5129.

[27] A. N. Aliev and P. Talazan, Phys. Rev. D80, 044023 (2009), arXiv:0906.1465

[28] H. Ishihara and K. Matsuno, Prog. Theor. Phys. 116, 417 (2006).

[29] S. S. Yazadjiev, Phys. Rev. D 74, 024022 (2006).

[30] Y. Brihaye and E. Radu, Phys. Lett. B 641, 212 (2006).

[31] H. Ishihara, M. Kimura, K. Matsuno, and S. Tomizawa, Phys. Rev. D 74, 047501 (2006); Class. Quantum Grav. 23, 6919 (2006).

[32] T. Harmark, V. Niarchos and N. A. Obers, Class. Quant. Grav. 24, R1-R90 (2007).

[33] T. Wang, Nucl. Phys. B 756, 86 (2006).

[34] V. Frolov and D. Stojkovic, Phys. Rev. D 67, 084004 (2003); H. Nomura, S. Yoshida, M. Tanabe and K. Maeda,
Prog. Theor. Phys. **114**, 707-712 (2005).

[35] H. Ishihara and J. Soda, Phys. Rev. D **76**, 064022 (2007).

[36] S. Chen, B. Wang and R. Su, Phys. Rev. D **77**, 024039 (2008).

[37] X. He, B. Wang and S. Chen, Phys. Rev. D **79**, 084005 (2009); X. He, S. Chen, B. Wang, R. G. Cai, C. Lin, Phys. Lett. B **665**, 392 (2008).

[38] H. Ishihara, M. Kimura, R. A. Konoplya, K. Murata, J. Soda and A. Zhidenko, Phys. Rev. D **77**, 084019 (2008).

[39] K. Matsuno and H. Ishihara, Phys. Rev. D **80**, 104037 (2009).

[40] Y. Kurita and H. Ishihara, Class. Quant. Grav. **24**, 4525 (2007); Y. Kurita and H. Ishihara, Class. Quant. Grav. **25**, 085006 (2008).