Three–dimensional oscillator in magnetic field: The de Haas–van Alphen effect in mesoscopic systems.

N.K. Kuzmenko\textsuperscript{a,*}, V.M. Mikhailov\textsuperscript{b}

\textsuperscript{a}V.G.Khlopin Radium Institute, 194021 St.-Petersburg, Russia
\textsuperscript{b}Institute of Physics St.–Petersburg State University 198904, Russia

Abstract

The theoretical investigation of the cluster de Haas - van Alphen (dHvA) oscillations in three-dimensional systems performed for the first time. Applying a three-dimensional oscillator model to systems with electron numbers $10 < N \leq 10^5$ we predict distinctive size effects: the dHvA oscillations can be observed only within a certain temperature range determined by $N$; the lower size limit for $N$ is $\approx 20$; the amount of the dHvA oscillations is reduced with decreasing $N$ which is accompanied by stretching the period of the oscillations.

Key words: Mesoscopic de Haas-van- Alphen oscillations; Three-dimensional oscillator; Size effects

\textit{PACS:} 71.18, 75.20.-g, 75.75.+a

1 Introduction

With increasing a uniform magnetic field applied to a macroscopic body its electron characteristics can reveal low-temperature oscillations manifesting themselves e.g. in the de Haas - van Alphen (dHvA) or Schubnikov-de Haas effects [1]. Oscillations in magnetization of metal thin films, the period of which increases with decreasing thickness, were theoretically predicted in Refs. [2,3]. The Shubnikov-de Haas oscillations were experimentally observed in antimony plates (thickness is of order of $10^2 \div 10^3$ nm) [4]. Now the dHvA effect is

* Corresponding author.

\textit{Email address: Kuzmenko@NK9433.spb.edu} (N.K. Kuzmenko).
rather intensively studied in two-dimensional mesoscopic systems (e.g. [5,6,7] and works cited therein).

However the possibility of the dHvA oscillations in atomic clusters has not been established so far. In this Letter we predict the properties of the dHvA oscillations in three-dimensional (3D)-mesoscopic systems in the absence of the disorder and interactions assuming that noninteracting electrons are confined in an oscillator well. We find that in 3D-mesoscopic systems, where electrons confined in all three directions, along with the dHvA oscillations there exist much more types oscillations than in bulk metals, thin films and two-dimensional mesoscopic systems. We consider the finite size effects in the dHvA oscillations in clusters with electron numbers $10 < N \leq 10^5$ in a wide range of arbitrarily oriented magnetic fields and find that number of the dHvA oscillations is reduced with decreasing $N$ and these oscillations appear at a temperature which depends on $N$. From our calculations follows that grains of materials with small the Fermi energy and the effective electron mass are best suited to observe the cluster dHvA oscillations.

2 Model

The harmonic oscillator model is widely exploited in describing structure of quantal systems including mesoscopic ones [3,8,9,10]. This is caused by the possibility of obtaining simple analytical solutions for oscillator energy eigenvalues and wave functions. Electron motion in the anysotropic 3D - oscillator potential and a uniform magnetic field $\vec{B}$ is described by the Hamiltonian

$$H = \sum_{\nu=x,y,z} \left\{ \frac{1}{2m^*}(p_{\nu} - \frac{e}{c} A_{\nu})^2 + \frac{m^*}{2}\Omega_{\nu}^2 r_{\nu}^2 + \frac{g}{2}\mu_B^* \sigma_{\nu} B_{\nu} \right\},$$

($\vec{A} = [\vec{r} \times \vec{B}] / 2, m^*$ is the effective electron mass, $\Omega_{\nu}$ are oscillator frequencies at $B = 0, g$ is the effective Lande factor, $\mu_B^* = e\hbar/2m^*c, \sigma_{\nu}$ is the Pauli matrix). Eq. (1) being expressed through oscillator quanta, $b_{\mu}^\dagger, b_{\nu} (\mu, \nu = x, y, z)$ is a bilinear form of these operators ($b^\dagger b, b^\dagger b^\dagger, bb$). We use the boson Bogolubov transformation to remove terms with two creation and annihilation bosons and at the same time to diagonalize the rest part, i.e. to reduce it to $\sum W_\alpha b_\alpha^\dagger b_\alpha, \alpha = 1, 2, 3$ where $W_\alpha$ are the eigen frequencies of new oscillator quanta in arbitrarily oriented field. As known both fermion and boson Bogolubov transformations result in equations quadratic in eigenvalues. Therefore we arrive at the cubic equation in $W^2$ since Eq. (1) deals with three spatial degrees of freedom. Relative simplicity of the Hamiltonian (1) is embodied in
a rather compact equation for eigenvalues:

\[
\prod_{\nu=x,y,z} (\Omega^2_{\nu} - W^2) - W^2 \sum_{\nu=x,y,z} \omega^2_{\nu}(\Omega^2_{\nu} - W^2) = 0,
\]  

(2)

\(\bar{\omega}\) is the cyclotron frequency, \(\bar{\omega} = 2\mu_B B\).

Three solutions of Eq. (2), \(W_+, W_-, W_0\), determine the single electron energies (hereafter \(\hbar = 1\))

\[
\varepsilon(n_+, n_-, n_0, \omega) = W_+(n_+ + \frac{1}{2}) + W_-(n_- + \frac{1}{2}) + W_0(n_0 + \frac{1}{2}) \pm \frac{g}{4} \omega.
\]  

(3)

\(n_+, n_0\) are integers. The spin contributes to the last term in Eq. (3). Solutions \(W_+, W_0\) vary with increasing \(\omega\): \(W_+\) grows, \(W_-\) falls down and \(W_0\) depends mainly on the direction of \(\vec{B}\). If the field is directed along one of the symmetry axes of the spheroid, e.g. \(\omega_z = \omega, \omega_x = \omega_y = 0\), \(W_0 = \Omega_z\) and \(W_+\) are found straightforwardly. For axial symmetry \((\Omega_x = \Omega_y)\) the solutions coincide with those given by Fock [11]. In high fields \((\omega^2 \gg \sum_{\nu} \Omega^2_{\nu})\) solutions of Eq. (2) gain the values:

\[
W^2_+ = \omega^2; \quad W^2_- = \frac{D}{\omega^2}; \quad W^2_0 = \Omega^2_0;
\]

(4)

\[
\Omega^2_0 = \sum_{\nu} \omega^2_{\nu}\Omega^2_{\nu}/\omega^2; \quad D = \prod_{\nu} \Omega^2_{\nu}/\Omega^2_0, \quad \nu = x, y, z.
\]

To model single–electron levels we adopt the relation [12] between semiaxes of ellipsoidal 3D - clusters \((a_x, a_y, a_z)\) and their oscillator frequencies

\[
\Omega_x a_x = \Omega_y a_y = \Omega_z a_z; \quad \Omega = \frac{\varepsilon_F}{(3N)^{1/3}},
\]  

(5)

\(a_xa_ya_z = R^3, R = r_s N^{1/3}, r_s\) is the radius of a sphere with the volume \(V/N\). \(\Omega^3 = \Omega_x \Omega_y \Omega_z = W_+W_-W_0\) is an invariant. \(\varepsilon_F\) is the Fermi energy. Eq. (5) is true for comparable semiaxes and small frequencies, \(\Omega_{x,y,z} \ll \varepsilon_F\).

3 Results of calculations

Our calculations of the electronic susceptibility \(\chi\) and heat capacity \(C\) are performed for ellipsoidal clusters. Values of \(C\) are presented here in order to demonstrate the temperature averaged level density near the Fermi level,
which is practically proportional to $C$. Calculations involve so wide single-electron spectra that their enlarging has no effect on results presented below.

At very low temperatures ($T < \Delta$, $\Delta$ is a mean level spacing) $\chi$ and $C$ are strongly oscillating functions of $\varepsilon_F/\omega$ revealing no evident periodicity v.s. the field (Fig. 1). Numerous peaks in $\chi$ and $C$ result from crossings of the Fermi level and an upper level nearest to it that gives at low temperatures a maximum in $\chi$ and simultaneously a minimum in $C$ [13] as shown in the insert in Fig. 1. These low temperature oscillations, being absent in bulk metals and thin films, are caused by the discreteness of the electronic level spectrum. Increasing temperature drastically changes this picture damping high frequency oscillations. Starting with a temperature $T_{\text{start}}$ one can observe the emerging of regularities near the quantum limit, $\varepsilon_F/\omega \sim 1$ (Figs. 2). The final suppression of high frequency oscillations occurs at an optimum temperature $T_{\text{opt}}$ (Figs. 3-6) exposing a maximum amount ($n_{\text{max}}$) of periodic oscillations. They are analogous with the dHvA oscillations in bulk metals and are determined by the common physical reason: the crossings of the Landau levels with the Fermi surface in a strong magnetic field. Further increasing the temperature ($T \gg T_{\text{opt}}$) leads to damping of the cluster dHvA oscillations (Fig. 3).

To interpret these results we have found analytical expressions for the $3D$ oscillator level density $\rho_0(\varepsilon)$ and twice integrated level density $\rho_2(\varepsilon)$ which determine the grand canonical $C$ and $\chi$ at a fixed particle number $N$

$$C = k_B \beta^2 \int d\varepsilon \rho_0(\varepsilon) \left[ (\varepsilon - \lambda)^2 - \left( \frac{\partial \lambda}{\partial \beta} \right)^2 \right] \Phi(\varepsilon); \quad (6)$$

$$\frac{\chi}{\chi_L} = \frac{8 \varepsilon_F}{N \beta} \int d\varepsilon \left[ \frac{\partial^2 \rho_2(\varepsilon)}{\partial \omega^2} - \rho_0(\varepsilon) \left( \frac{\partial \lambda}{\partial \omega} \right)^2 \right] \Phi(\varepsilon); \quad (7)$$

$$\Phi(\varepsilon) = e^{\beta(\varepsilon-\lambda)} / \left[ 1 + e^{\beta(\varepsilon-\lambda)} \right]^2, \quad \beta = 1/k_B T,$$

$$\chi_L = -\mu_B^2 N/2V \varepsilon_F, \quad V \text{ being the volume of the system.} \quad \lambda \text{ is the chemical potential practically equal to } \varepsilon_F \text{ and weakly oscillating with } \omega \text{ in a wide region of } \omega \text{ up to } \omega \sim \varepsilon_F \text{ where } \lambda \text{ markedly decreases.}$$

$$\rho_n(\varepsilon) = \frac{1}{16\pi(i)^{3+n}} \int_{\mathcal{L}} dq q^n e^{i\varepsilon q} \left[ \sin \frac{W_+ q}{2} \sin \frac{W_- q}{2} \sin \frac{W_0 q}{2} \right]^{-1}. \quad (8)$$

The integration path ($\mathcal{L}$) envelops all poles including $q = 0$. The residue of $\rho_n(\varepsilon)$ in $q = 0$ gives (after integrating over $\varepsilon$ in Eqs. (6), (7)) smooth functions of $\omega$ for $C$ and $\chi$ ($C_S$, $\chi_S$)
\[ C_S = k_B^2 \pi^2 TN \left( \frac{\lambda}{\varepsilon_F} \right) \left[ 1 + \left( \frac{g\omega}{4\lambda} \right)^2 - \left( \frac{\sum_{a=x,y,z} \Omega_a^2 + \omega^2}{12\lambda^2} \right) \right], \]  

\[ \chi_S/|\chi_L| = \left( -2 + \frac{3}{2} g^2 \right) \left( \frac{\lambda}{\varepsilon_F} \right)^2. \]

These expressions are true at \( T \ll \varepsilon_F \). The first negative term in \( \chi_S/|\chi_L| \) is the “3D-oscillator” Landau diamagnetism and the second (\( \sim g^2 \)) term is the “3D-oscillator” Pauli paramagnetism. Their absolute values are two times more than those for free electrons, nevertheless their ratio at \( g = 2 \) is equal to the well known value (\( \chi_L/\chi_P = -1/3 \)). \( C_S \) and \( \chi_S \) are those middle levels on which all types of oscillations are superimposed. At high enough temperatures (but \( T/\varepsilon_F \ll 1 \)) the most part of oscillations are suppressed and \( C \) and \( \chi \) become equal to \( C_S \) and \( \chi_S \). Fig. 3 shows that at \( 4T_{\text{opt}} \) the oscillations in the system with \( N = 10^5 \) disappear in weak fields (\( \varepsilon_F/\omega \geq 25 \)) and \( \chi \) becomes equal to \( \chi_S \).

Oscillating components of \( C \) and \( \chi \) (which are determined by residues in points \( q \neq 0 \)) can be divided into several constituent parts. The large scale oscillation, which are denote as the cluster dHvA oscillations by analogy with bulk metal oscillations, are described by functions \( \cos(2\pi \lambda n_+ / W_+) \cos(\pi g\omega n_+ / 2W_+) \)\(^n_+^2 \). For the dHvA oscillations frequencies \( W_+ \), \( W_- \), \( W_0 \) must not be multiple, i.e. \( W_+/W_- \neq n_+/n_- \) and \( W_+/W_0 \neq n_+/n_0 \). Since \( \lambda \leq \varepsilon_F \) and \( W_+ \simeq \omega \) for \( \omega \gg \Omega \) the period \( (t_+) \) of the main “tone” of these oscillations \( (n_+ = 1) \) on the scale \( \varepsilon_F/\omega \) is almost equal to 1:

\[ \cos(2\pi \lambda / W_+) \equiv \cos \left( \frac{2\pi}{t_+} \frac{1}{\omega} \right), \quad t_+ = \frac{\varepsilon_F}{\lambda} W_+, \]

though with decreasing \( \omega \) the period \( t_+ \) is stretched, and the smaller is the particle number the stronger is the stretching. In Fig. 4 this stretching is quite evident for \( N = 10^3 \). For instance in a simple case \( \omega_z = \omega, \omega_x = \omega_y = 0 \)

\[ W_+/\omega \simeq 0.5 \left\{ \left[ 4(3N)^{-2/3} \left( \frac{\varepsilon_F}{\omega} \right)^2 + 1 \right]^{1/2} + 1 \right\}. \]

The role of the effective electron Lande factor, entering into \( \cos(\pi g\omega n_+ / 2W_+) \), turns out to be similar to that in the bulk dHvA oscillations [1] (Fig. 5): At \( g = 2 \) the oscillations are paramagnetic while at \( g = 0 \) they are diamagnetic. At \( g = 1 \) the main tone \( (n_+ = 1) \) disappears in strong fields that leads to lowering amplitudes and doubling frequencies. Thus experimental studying of the cluster dHvA oscillations could give information about cluster \( g \)-factors. Functions in Fig. 1-4 are calculated with \( g = 2 \).
The temperature damping factor \( D_\chi(T, n_a) \) of the cluster dHvA oscillations \((n_a = n_+)\) and the other types of oscillations considered below has for the susceptibility the same form as for the bulk metal oscillations [1]

\[
D_\chi(T, n_a) = \frac{x_a}{\sinh x_a}; \quad x_a = 2\pi^2 k_B T n_a/W_a, \quad (11)
\]

\( D_\chi(T, n_+) \)-factor causes attenuation of the dHvA amplitudes with decreasing \( \omega \) as seen e.g. in Fig. 3. At \( T \ll \omega \) the temperature damping of the dHvA oscillations is inessential that allows summing over \( n_+ \) to be performed. It leads to parabolic segments in \( \chi \) and \( C \) which are well illustrated in Fig. 2 by the example of \( \chi \).

The temperature damping of the heat capacity oscillations is governed by \( D_C(T, n_a) \)

\[
D_C(T, n_a) = \frac{3x_a}{\sinh x_a} \left[ 1 - 2(\coth x_a)^2 + 2x_a \coth x_a \right], \quad (12)
\]

\( x_a \) takes the same values as in Eq. (11). When \( x_a \to 0 \) \( D_C(T, n_a) \to 1 \), however this function decreases with \( x_a \) much faster than \( D_\chi(T, n_a) \). Therefore with decreasing \( \omega \) a stronger damping of the oscillations in \( C \) is observed as compared with \( \chi \). Besides, the amplitude of the dHvA oscillations in \( C \) is proportional to \( W_+^2 \) that additionally smoothes these oscillations with increasing \( \varepsilon_F/\omega \).

Another type of high frequency oscillations is connected with \( W_- \)-frequency through the function \( \cos(2\pi \lambda n_-/W_-) \). (Here frequencies must not be multiple again.) Thus the period of this oscillations rapidly falls down with increasing \( \omega \). Therefore the temperature damping becomes stronger at large \( \omega \). Hence one can find \( T_{\text{start}} \) supposing that this temperature is such that \( D_\chi(T_{\text{start}}, n_-) \approx 0.1 \)

i.e. this type of oscillation is essentially suppressed at \( T_{\text{start}} \). As \( D_\chi = 0.1 \) at \( x \approx 4.5 \) and \( W_- = \Omega^2/\omega \) near the quantum limit one gains:

\[
T_{\text{start}} \approx \frac{1}{3} (3N)^{1/3} \Delta. \quad (13)
\]

In the 3D-case there could be exist one more type of oscillations: \( \cos(2\pi \Lambda W_0/n_0) \)

\( (W_0/W_+ \neq n_0/n_+; \ W_0/W_- \neq n_0/n_-) \). However these oscillations can really reveal themselves when the direction of \( \vec{B} \) does not coincide with any of oscillator symmetry axes and only at small \( \omega \) because near the quantum limit \( W_0 \) does not depend on \( \omega \) and these oscillations do not arise.

In all three cases considered above we marked in brackets that \( W_+, W_-, W_0 \) should not be multiple to each other. Nevertheless, if \( \omega \) varies continuously
a pair of frequencies or even all three can become multiple. When $\omega$ reaches such values that

$$\frac{n_+}{W_+} = \frac{n_-}{W_-} \neq \frac{n_0}{W_0} \text{ or } \frac{n_0}{W_0} = \frac{n_-}{W_-} \neq \frac{n_+}{W_+}$$

these types oscillations are suppressed at $T = T_{\text{start}}$ and $\omega \leq \varepsilon_F$. The third type of such oscillations $n_+/W_+ = n_0/W_0 \neq n_-/W_-$ appears in such points of $\omega$ where the ratio of $W_+$ and $W_0$ is equal to an integer, if $n_0 = 1$, i.e. these points are divided by an interval approximately equal to $W_0 \sim \Omega$. Therefore this type of oscillations can be damped by the temperature if $D_\chi(T_{\text{opt}}, n_0) \approx 0.1$. The parameter $x$ in this case at $n_0 = 1$ is $2\pi^2 T_{\text{opt}}/W_0 = 4.5$:

$$T_{\text{opt}} \approx \frac{1}{4}(3N)^{2/3}\Delta. \quad (14)$$

The last type of oscillations $W_+/W_0/W_- = n_+/n_0/n_-$ is also damped because the minimum value of $n_-$ is 1 and at $\omega > 2\Omega$ the frequency $W_- \ll W_0$, i.e. $n_0 \gg 2$ and $D_\chi \ll 0.1$. Amplitudes of all oscillations with multiple frequencies increase with decreasing $\omega$ and at small $\omega \leq 2\Omega$ the temperature damping is neutralized. This is confirmed by our calculations indicating that in a wide range of $N$ the cluster dHvA oscillations can be separated from others at $T = T_{\text{opt}}$ when the cyclotron frequency $\omega_{\text{min}}$ is about two times larger than $\Omega$ Eq. (5):

$$\omega_{\text{min}} \simeq 2\Omega, \quad n_{\text{max}} \simeq \frac{1}{2}(3N)^{1/3}. \quad (15)$$

Eq. (15) implies that these oscillations begin at such magnetic field, the cyclotron radius $R_\text{c}$ of which turns out to be equal to the minimal size of a cluster in the plane perpendicular to the magnetic field. Thus $\omega_{\text{min}}$ is determined by the same conditions as in bulk metals. The existence of $n_{\text{max}}$ depending on $N$ limits the cluster particle number $N_{\text{min}}$ at which the dHvA oscillations might be still observed. Since the minimum value of $n_{\text{max}}$ is 1 (the only parabolic segment shows up in $\chi$) Eq. (15) gives $N_{\text{min}} \simeq 20$ (Fig. 6). Fig. 6 and our calculations for $N \sim 10^4$ show that for an array of clusters with wide enough size distribution (and consequently with wide distribution in particle numbers) one can observe practically the same number of the oscillations as for a single cluster with $N_0$.

We have analyzed the dependence of our results on variations of cluster shapes and the direction of the magnetic field. For this purpose at a fixed particle number we have altered the cluster shape from oblate to prolate ($0.1 < a_z/a_x < 3$) with different degree of nonaxiality ($1 < a_y/a_x < 2$) and varied the direction of a magnetic field. Our calculations have shown that the properties of the dHvA
oscillations in 3D finite systems are rather insensitive to the variations of the cluster shape if the size of a cluster in the direction of the field is not too small as against the size in the perpendicular plane (e.g. $a_z$ should be greater than $0.5a_x$). Thus reasonably large variations of cluster shape and direction of the magnetic field cannot hinder measurements of the cluster dHvA oscillations.

Values of minimal magnetic fields required to observe cluster dHvA oscillations at $T_{\text{opt}}$ can be assessed from Eq. (15) (obviously the upper limit is $\varepsilon_F$)

$$\omega_{\text{min}} \sim \frac{\varepsilon_F}{2(3N)^{1/3}}; \quad B_{\text{min}}(T) \sim 10^4 \frac{m^* \varepsilon_F (eV)}{m \ 2(3N)^{1/3}}. \quad (16)$$

These equations indicate that reasonable fields can be used for large grains of such materials for which the Fermi energy ($\varepsilon_F$) and effective electron mass ($m^*$) are rather small quantities. In addition, at a small $\varepsilon_F$ and large $N$ the temperatures can be low enough, $T_{\text{opt}} \sim 1K$, to neglect electron scattering effects. For GaAs systems ($m^* = 0.067m$, $\varepsilon_F \approx 10 meV$)

$$B_{\text{min}}(T) \sim 2/N^{1/3}; \quad T_{\text{opt}}(K) \sim 20/N^{1/3}. \quad (17)$$

4 Summary

In conclusion using the three-dimensional oscillator model we predict the dHvA oscillations in grains of materials with small $\varepsilon_F$ and $m^*$. Analogously with bulk metals the cluster dHvA oscillations arise when the cyclotron radius is less than the size of a system. Size effects are clearly observed for $N \leq 10^5$: the number of the oscillations is reduced with decreasing $N$; the size limit for $N$ is $\sim 20$; the period of the oscillations stretches with increasing $\varepsilon_F/\omega$. The properties of the dHvA oscillations (the number of oscillations, the amplitude, the period) are weakly sensitive to the significant variations of cluster shapes and the direction of the magnetic field especially near the quantum limit. The characteristic feature of the cluster dHvA oscillations is a special temperature regime at which high frequency oscillations caused by the discreteness of electronic level spectra are suppressed. The oscillator model predicts that the 3D-mesoscopic dHvA oscillations appear at $T_{\text{start}} \simeq \varepsilon_F/(3N)^{2/3}$ and the most favorable temperature for the observation of their maximum number is $T_{\text{opt}} \simeq \varepsilon_F/(4(3N)^{1/3})$, while an excess of $T_{\text{opt}}$ results in smoothing and subsequent disappearing the cluster dHvA oscillations.

We thank V.E. Bunakov for useful discussions.
References

[1] D. Shoenberg, F. R. S., *Magnetic Oscillations in Metals* (Cambridge University Press, 1984).

[2] A. M. Kosevitch and I. M. Lifshitz, Zh. Eksp. Teor. Fiz. 29, 743 (1955) [translated in Sov. Phys. JETP].

[3] D. Childers and P. Pinkus, Phys. Rev. 117, 1036 (1969).

[4] J. P. Gajdukov and E. M. Goljamina, Pis’ma v ZhETF, 23, 336 (1976) [translated in Letters to JETP].

[5] D. Yoshioka and H. Fukuyama, J. Phys. Soc. Japan 61, 2368 (1992).

[6] E. N. Bogachek, A. G. Scherbakov and U. Landman, Phys. Rev. B63, 115323 (2001).

[7] V. Gudmundsson, S. Erligsson and A. Manolescu, Phys. Rev. B61, 4835 (2000).

[8] D. U. Felderhof, S. P. Raval, Physica 82A, 151 (1976).

[9] W. D. Heiss, R. G. Nazmitdinov, Phys. Lett. A222, 309 (1996) [cond-mat/9704216 (1997)].

[10] L. P. Kouwenhoven, D. G. Austing and S. Tarucha, Rep. Prog. Phys. 64, 701 (2001).

[11] V. A. Fock, Z. Physik, 47, 446 (1928).

[12] A. Bohr, B. Mottelson, *Nuclear Structure* (New York, Amsterdam, 1969), Vol.2.

[13] N. K. Kuzmenko and V. M. Mikhajlov, Phys. Lett. A296, 49 (2002).
Fig. 1. Low temperature oscillations of the electron susceptibility $\chi$ (a) and heat capacity $C$ (b) in a tilted magnetic field ($\omega_z = \omega \cos \theta$, $\omega_x = \omega \sin \theta \cos \varphi$, $\omega_y = \omega \sin \theta \sin \varphi$, $\theta = \pi/4$, $\varphi = \pi/4$) vs $\varepsilon_F/\omega$ for $N = 10^5$ electrons moving in an anisotropic oscillator potential ($a_y/a_x = 1.33$, $a_z/a_x = 1.55$). $\chi_L = -\mu_B^2 N/2V\varepsilon_F$, $\Delta = \varepsilon_F/3N$, $g = 2$. 
Fig. 2. Oscillations of the magnetic susceptibility vs $\varepsilon_F/\omega$ for a cluster with $N = 10^5$ electrons at $T = T_{\text{start}} = 22\Delta$. The oscillator anisotropy and the direction of the magnetic field are the same as in Fig. 1, $g = 2$. 
Fig. 3. Influence of the temperature on the dHvA oscillations in a system with $N = 10^5$. The oscillator anisotropy and the direction of the magnetic field are the same as in Fig. 1.
Fig. 4. Comparison of the dHvA oscillations in systems with $N = 10^5$ and $N = 10^3$ at $T = T_{\text{opt}}$. $T_{\text{opt}} = 10^3\Delta$ for $N = 10^5$ and $T_{\text{opt}} = 50\Delta$ for $N = 10^3$. The oscillator anisotropy and the direction of the magnetic field are the same as in Fig. 1, $g = 2$. 
Fig. 5. Influence of the effective Lande factor on the dHvA oscillations in systems with $N = 10^4$ at $T = T_{\text{opt}} = 250\Delta$. The oscillator anisotropy and the direction of the magnetic field are the same as in Fig. 1.

Fig. 6. N-averaged dHvA oscillations of spherical clusters with $N = N_0 \pm 3\Delta N$ ($N_0 = 20$, $\Delta N$ is the averaging width parameter), $g = 2$. 