Modularity Measurement as a Crucial Design Element

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Abstract. This paper aims to explore the problem of measurement of assembly process modularity, and to offer a new approach to quantify the relative modularity of different assembly process models. Specifically, it seeks to resolve the problem of the relation between relative modularity measures and optimal modularity measures. This relation brings us more close to find an effective measure for determining whether the process network is optimally modular. In our approach, process modularity expresses the extent to which processes can be decomposed into modules to be executed in parallel, and/or in series. Moreover, it also considers important feature of modular systems - that the great majority of interactions occur within modules and only a few interactions occur between modules. Consequently, the effect of a change in a given module is confined to that module only. This advantage of modular design clearly corresponds with Axiomatic Design theory, especially with its first axiom.

1. Introduction and related work

In general, modularly organized networks are characterized, apart from other properties, by functional segregation and integration [1]. Manufacturing assembly processes that are of interest in this paper belongs to this network type. System modularity problems, equally as system complexity issues are parts of general systems theory, since one can apply them to different kinds of systems, including technical, social, and biological [2, 3]. According to Ulrich [4], a modular architecture is based on a one-to-one mapping functional elements to the physical components of the product, and specifies de-coupled connections between components. Such architecture follows the first axiom in Axiomatic Design defined by Suh [5] specifying that each system function or functional requirement has to be satisfied by an independent design parameter. Subsequently, design parameters are projected onto construction documents, and transformed through the manufacturing process variables into process output. An important feature of the system modularity is that complexity of technical systems can be effectively managed through their modular design [6, 7]. Tate [8] categorized modularity from Axiomatic Design theory perspective into three types: resource, operational, and interfacial one. According to him, the resource modularity can be defined as 'ease of manufacturing'. This definition at least shows a certain connection between process modularity and complexity in system design. According to Mehrsai et al. [9], extension of modularity into processes and resources allows to generate alternative structures of organizations or supply chain networks by splitting their performances into modules and adjust them as required.

Product and process modularity becomes increasingly important over the last decades, especially due to diffusion of mass customization that is considered as an important competition strategy of a lot of businesses [10]. In this nexus, modularity measurement plays a vital role in the product and process
design, and need to be continually developed. Existing literature offers several approaches to measure product modularity. A comprehensive overview of them has been offered by Ulrich [11]. From more recent works it can be mentioned work by Hölttä-Otto and De Weck [12] who proposed a relative product modularity metric called the Singular Value Modularity Index. On the other hand, there is a lack of process modularity measures or metrics to quantify a degree of manufacturing system modularity [13]. Selected relevant approaches on process modularity measurement, which in some way relate to our research can be found in works of [14-19].

2. Classification framework of assembly process networks

First, have a look at the simplest structural model of manufacturing assembly process (MAP). As shown in Figure 1, the structure consists of the three obligatory elements, which are inputs elements - at least two, process operation(s), and output element(s). Such a structure also represents simplest assembly module.

![Figure 1. The simplest structural model of manufacturing assembly network.](image)

As topology of assembly process structures varies case by case and depends upon specific factors, it seems to be useful to create working classification of process structures by the number of process layers, and the number of output elements in order to bring out the relevant modularity measures more clearly. Accordingly, we propose to divide assembly process structures into the following classes and subclasses:

1. **Single Layer Multi-Product** (SLMP) assembly network. This class of MAP structures includes all single-step assembly networks with different numbers of input components, parallel single assembly operations, and output components. Selected alternative MAP structures, when the number of input components equals four, and the number of output components equals six are shown in Figure 2 (see the structures of the type 1). Modularization followed here is based only on vertical fragmentation of the network modules into submodules.

2. **Multi-Layer Single-Product** (MLSP) assembly network. This class of MAP structures is further divided into two sub-classes, which are:
   a) MLSP, when order of input components entering into the assembly operations is not important. All possible alternative MAP structures, when the number of input components equals four, and the number of output component equals one are shown in Figure 2 – see structures of the type 2a. These networks are modeled as single-rooted tree graphs.
   b) MLSP, when order of input components entering into the assembly operations is determined. All possible alternative MAP structures, when the number of input components equals four, and the number of output component equals one are shown in Figure 2 – see structures of the type 2b. As it can be seen, the number of all alternative structures is in such case is lower than in case of type 2a. It is specifically because, the structure No. 2.4a does not satisfied the rule of determined order of assembly operations. These networks follow the pattern of single-rooted tree graphs with only single branch of nodes.

3. **Multi-Layer Multi-Product** (MLMP) assembly network. This class of MAP structures is important in terms of mass customization. Let’s say graph No. 3.1. (see Figure 2) represents originally designed
MAP structure. Moreover, initial elements c, d and e entering the second assembly operation in this structure are optional components. Then, it is topical to create alternative structures (see structures No. 3.2. and No. 3.3.), which better satisfy the requirements of assembly process in terms of mass customization. It is because, assembly processes in terms of mass customization have to be as modular as possible. These networks present multi-rooted tree graphs.

Figure 2. The assembly process networks classification.

3. Proposition of the relative and optimal process modularity measures

3.1. Modularity typology and working definitions

Our interest lies in investigating assembly manufacturing processes, which are generalized as technical systems. According to Li et al. [20] a technical system modularity needs to be divided at least into three basic domains: product, service and supply chain processes, and these three domains fuse into an organic whole, interacting and restraining each other. The same authors argue, that in spite of well-developed product classification, there is absence of service modularity classification and, supply chain modularity classification. In order to identify the object, we want to explore, we classify supply chains into two groups, i.e., general supply chains (interconnecting a group of organizations), and manufacturing assembly supply chains (assembly processes). As there are important differences between the two groups, our research is focused only on the assembly processes modularity.

In generally, modularization of assembly process means subdividing one integral operation into several separate operations/modules, and assembly process modularity indicates the degree of decomposability of a given process into sub-processes without changing the number of inputs and outputs. When process modularity is defined as the degree of decomposability of a given process, then, it reflects so called relative modularity. Efatmaneshnik, and Ryan [21] stated, that modularization remains more of an art than a science, to the extent that the notion of optimal modularity is not applied.
For this reason, the general concept of optimal modularity developed by Newman and Girvan [22] is employed in our approach as complementary indicator along with the proposed relative measure of process modularity. However, adoption of the optimality modularity measure needed in our research to verify its applicability for MAP networks. Its validity for given purpose has been tested through selected alternative MAP structures, while obtained results are presented in section 4 of this paper.

3.2. Description of proposed relative modularity measure
A construct of the proposed process modularity measure is based on a realistic assumption that structural network complexity directly relates to network modularity [6-8]. Then, we further assume that adequate structural complexity measure, which inherently captures the modularity features could be transformed to measure the degree of process modularity. In this order, so called the vertex degree distribution of a graph (Ivd) will be used for this purpose, since it intrinsically covers network modularity signs. This indicator was developed by Bonchev and Buck [23], and it measures structural complexity of a general networks. It is formalized as a graph G consisting of a set of V vertices \( \{V\} = \{v_1, v_2, \ldots, v_V\} \), and expressed by formula:

\[
Ivd = \sum_{i=1}^{V} deg(v_i) \log_2 deg(v_i),
\]

where \( deg(v_i) \) is the degree of vertex \( (v_i) \) in G.

To outline mutual relation between network complexity and network modularity, the first type of assembly process networks (SLMP), namely process structures No. 1.1., 1.2., 1.3. and 1.4 will be used. Subsequently, complexity levels of these structures are enumerated by employing formula (1), and obtained values are available in the first line of Table 1.

Table 1. Process complexity values of the selected assembly process networks.

| No. 1.1. | No. 1.2. | No. 1.3. | No. 1.4. |
|---------|---------|---------|---------|
| Ivd     | 33,22   | 41,16   | 52,25   | 47,6    |
| AMC     | 33,22   | 20,58   | 17,42   | 7,93    |

By visual comparison of the SLMP models from Figure 2, it can be seen that the structure No. 1.4. is the most modular, and the network No. 1.1 is non-modular. Then, one would expect that complexity of the structure No. 1.4. will be the smallest one and the complexity of the structure no. 1.1. will be the highest one. However, Ivd indicator, which is based on information entropy, expresses absolute complexity measure by prioritization of the highest magnitudes of vertices against overall sum of vertex degrees.

Therefore, Ivd does not satisfy this expectation. For the sake to find out the most appropriate coincidence between the networks complexity and the networks modularity we propose to modify formula (1) to count relative network complexity as the average complexity of its modules. Then, the modified indicator further named as Average Module Complexity (AMC) can be expressed by the equation:

\[
AMC = \frac{\sum_{i=1}^{V} deg(v_i) \log_2 deg(v_i)}{V}
\]

An example of enumeration of Ivd and AMC values for the model No. 1.2 is depicted in Figure 3.
By applying AMC indicator for all the SLMP networks from Figure 2, we obtain network complexity values, which are shown in Table 1 – see the second line of the values. Now, we can see that the most modular network is with the lowest complexity and vice versa. Equally, this relation is actual also for networks No. 1.2. and 1.3. Later, it will be showed that this relation is also valid for other types of assembly process structures. Based on these assumptions it is reasonable to quantify relative network modularity through the inverse value of AMC. Then, relative network modularity (RNM) can be expressed by formula:

$$RNM = \frac{V}{\sum_{i=1}^{V} deg(v_i) log_2(deg(v)_i)}.$$

In the next section, it will be shown how this formula works for other types of networks, i.e., MLSP and MLMP.

3.3. Adaption of optimal modularity measure for assembly process networks

According to [24], network modularity is the degree to which it can be separated into nearly independent sub-networks. Newman and Girvan [22] developed a quantitative measure of optimal modularity (Q). They developed an algorithm finding the division of the nodes into modules that maximizes a measure Q. The formula to quantify optimal level of modularity Q is expressed as follows:

$$Q = \sum_{s=1}^{K} \frac{ls}{L} - \left( \frac{ds}{2L} \right)^2$$

where:

- K - is the number of modules,
- L - is the number of edges in the network,
- ls - is the number of edges between nodes in module s,
- ds - is the sum of the degrees of the nodes in module s.

Adoption of this modularity measure for assembly process networks is explained through the following model of structure No.1.2. (see Figure 4).
In this way, one can employ formula (4) for our reference models of assembly process networks and compare them with RNM values (see Table 2).

Table 2. The Obtained RNM and Q values.

| Type | RNM | Q  |
|------|-----|----|
| 1.   | 0.03 | 0  |
| 1.1. | 0.049| 0.24|
| 1.2. | 0.057| 0.25|
| 1.3. | 0.126| 0.54|
| 2a   | 0.086| 0  |
| 2.1a | 0.157| 0.32|
| 2.2a | 0.157| 0.32|
| 2.3a | 0.21 | 0.38|
| 2.4a | 0.21 | 0.38|
| 2.5a | 0.21 | 0.38|
| 2b   | 0.086| 0  |
| 2.1b | 0.157| 0.32|
| 2.2b | 0.157| 0.32|
| 2.3b | 0.21 | 0.38|
| 2.4b | 0.21 | 0.38|
| 3.   | 0.12 | 0.425|
| 3.1. | 0.155| 0.459|
| 3.2. | 0.185| 0.462|

As it can be seen, the models No. 1.1., 2.1a. and 2.1b. report zero values of Q. In fact, it is because that these structures are non-modular ones. The remaining Q values of the structures numbered as 1.2., 1.3. and 1.4. have the same tendency as RNM values. It is due to the fact, that optimal network modularity according to indicator Q is dominantly affected by the number of modules in the network. Therefore, SLMP process models are from the optimality viewpoint quite specific, since the most modular structures are at the same time considered as optimal.

Even though, the RNM values and Q values of MLSP and MLMP selected structures (see Table 2), have the same tendency, it is not the rule, but just the specific cases. This situation can be explained by the small number of alternative MAP structures, what is limited in case of MLSP process structures by the number of input components (i = 4). In case of MLMP process networks, other MAP alternatives are not taken into account. For this reason, in the next section, the analysis of alternatives process structures with input components i = 5 and i = 6 will be provided. The aim of this analysis is to prove or disprove our above mentioned assumption that structural network complexity directly relates to network modularity in this way that higher network modularity reduces its complexity.

4. Testing of the modularity indicators
In this section, MAP structures of type 2 - MLSP with five and six input components for both subclasses, will be tested using the selected modularity indicators with the aim to find the optimal modularity values.

4.1. Testing MAP structure with five input components
Firstly, we start with the first subclass of MLSP type of the MAP structures - when order of input components entering into the assembly operations is not important. Then, it is possible to deterministically generate twelve possible alternative MAP structures (see Figure 5a).
Figure 5. All possible MLSP process structures with five input components a) when order of input components entering into the assembly operations is not important, b) when order of input components entering into the assembly operations is important.

After generating of all possible MAP structures for $i = 5$, RNM values using equation (3) and $Q$ values using equation (4) are calculated, ordered by the increasing indicator RNM, and shown using graphical representation in Figure 6a.

Figure 6. MLSP process structure with five input components a) when order of input components entering into the assembly operations is not important, b) when order of input components entering into the assembly operations is important.

As can be seen from Figure 6a, two optimal modular MAP structures, namely, No. 6 and No. 8 are found.

Now, we continue with the second subclass of MLSP of the MAP structures - when order of input components entering into the assembly operations is important. Then, it is possible to exactly generate eight possible alternative MAP structures (see Figure 5b).

After generating of all possible MAP structures for $i = 5$, RNM values using equation (3) and $Q$ values using equation (4) are calculated, and ordered by the increasing indicator RNM as shown in
Figure 6b. As can be seen in this figure, the MAP structure No. 6 is accordingly identified as optimal modular structure.

4.2. Testing MAP structure with six input components

Let us start in this subsection with the first subclass of MLSP type of the MAP structures - when order of input components entering into the assembly operations is not important. Then, it is possible to generate 33 possible alternative MAP structures (see Figure 7a).

![Figure 7](image)

**Figure 7.** All possible MLSP process structures with six input components a) when order of input components entering into the assembly operations is not important, b) when order of input components entering into the assembly operations is important.

Subsequently, RNM values and Q values are calculated for all possible MAP structures. Then, the MAP structures are ordered by the increasing indicator RNM, as shown in Figure 8a.

![Figure 8](image)

**Figure 8.** MLSP process structure with six input components a) when order of input components entering into the assembly operations is not important, b) when order of input components entering into the assembly operations is important.

Accordingly, the optimal modular MAP structures are identified, which are namely structures No. 10 and No. 19.
In the case of the second subclass of MLSP type of the MAP structures - when order of input components entering into the assembly operations is important, it is possible to generate 16 possible alternative MAP structures (see Figure 7b).

Then, RNM values and Q values are calculated and presented in Figure 8b, where the structures are ordered by the increasing values of RNM indicator. The MAP structures No. 27 and No. 28 are considered as optimal ones.

5. Conclusions
The presented results from computational experiments in section 4 indicate promising potential of the two modularity measures, namely, optimal level of modularity Q and relative modularity network RNM to manage modularity issues in early stage of assembly process designing. This two complementary indicators allowing better understanding of relation between the relative modularity concept and the optimal modularity method.

There is no doubt that the problem of detecting structural modularity is one of the most challenging issues in the study of network systems. Even though, the method of optimal modularity developed by Newman and Girvan [22] is outstanding algorithm for the optimization of the network modularity that is applicable to a many real-world networks, its application to the new fields requires certain adaptation and at least theoretical validation. Hopefully, this paper contributes to the knowledge in this domain, and after further investigation will offer a useful tool for manufacturing process designers.

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References
[1] Hilger K, Ekman M, Fiebach C J, Basten U 2017 Intelligence is associated with the modular structure of intrinsic brain networks Scientific reports 7(1) 1-12
[2] Schilling M A 2000 Toward a general modular systems theory and its application to interfirm product modularity Academy of management review 25(2) 312-334
[3] Baldwin C Y and Clark K B 2003 Managing in an age of modularity Managing in the modular age: Architectures, networks, and organizations 149 84-93
[4] Ulrich K 1995 The role of product architecture in the manufacturing firm Research policy 24(3) 419-440
[5] Suh N P 1990 The Principles of Design New York, Oxford University Press 418
[6] Ethiraj S K and Levintal D 2004 Modularity and innovation in complex systems Management science 50(2) 159-173
[7] Parker D B 2010 Modularity and complexity: An examination of the effects of product structure on the intricacy of production systems Michigan State University 119
[8] Tate D 1999 A Roadmap for Decomposition: Activities, Theories, and Tools for System Design Ph.D. Thesis Massachusetts Institute of Technology 210
[9] Mehrsai A, Karimi H R, Thoben K D 2013 Integration of supply networks for customization with modularity in cloud and make-to-upgrade strategy An Open Access Journal Systems Science & Control Engineering 1(1) pp 28-42
[10] Modrak V (Ed.) 2017 Mass customized manufacturing: theoretical concepts and practical approaches CRC Press 313
[11] Ulrich K 1994 Fundamentals of Product Modularity Management of Design Dordrecht: Springer 219–231
[12] Holitta K, Suh E S, de Weck O 2005 Tradeoff between modularity and performance for engineered systems and products ICED 05: 15th International Conference on Engineering Design: Engineering Design and the Global Economy Engineers Australia 449-450
[13] Calcagno M 2002 Dynamics of Modularity: A Critical Approach *Euram Conference* 9(11) 1-11
[14] Vickery S K, Koufieros X, Dröge C, Calantone R 2016 Product modularity, process modularity, and new product introduction performance: does complexity matter? *Production and Operations management* 24(4) 751-770
[15] Blackenfelt M 2001 Managing complexity by product modularisation *Doctoral dissertation, Maskinkonstruktion* 100
[16] Maier J F, Eckert C M, Clarkson P J 2017 Model granularity in engineering design–concepts and framework *Design Science* 3 1-29
[17] Frenken K and Mendritzki S 2012 Optimal modularity: a demonstration of the evolutionary advantage of modular architectures *Journal of Evol Econ* 22 935–956
[18] Modrak V and Soltysova Z 2018 Process modularity of mass customized manufacturing systems: principles, measures and assessment *Procedia CIRP* 67 36-40
[19] Kusiak A 2002 Integrated product and process design: a modularity perspective *Journal of Engineering Design* 13(3) 223-231
[20] Li H, Yang M, Evans S 2019 Classifying different types of modularity for technical system *International Journal of Technology Management* 81(1-2) 1-23
[21] Efatmaneshnik M and Michael J R 2016 On optimal modularity for system construction *Complexity* Complexity 21 176-189
[22] Newman M E J and Girvan M 2004 Finding and evaluating community structure in networks *Physical Review E* 69(2) 1-16
[23] Bonchev D and Buck G A 2005 Quantitative measures of network complexity *Complexity in chemistry, biology, and ecology* 191-235
[24] Alon U 2019 An introduction to systems biology: design principles of biological circuits *CRC press* 343