Dark charge vs electric charge

Duong Van Loi, Cao H. Nam, Ngo Hai Tan, Phung Van Dong

Phenikaa Institute for Advanced Study and Faculty of Basic Science, Phenikaa University, Yen Nghia, Ha Dong, Hanoi 100000, Vietnam
E-mail: loi.duongvan@phenikaa-uni.edu.vn, nam.caohoang@phenikaa-uni.edu.vn, tan.ngohai@phenikaa-uni.edu.vn, dong.phungvan@phenikaa-uni.edu.vn

ABSTRACT: We reconsider the question of electric charge quantization, which leads to the existence of a dark charge nontrivially unified with weak isospin in a novel gauge symmetry, $SU(3)_C \otimes SU(2)_L \otimes U(1)_Y \otimes U(1)_N$, where $Y$ and $N$ determine the electric and dark charges, respectively. The new model provides neutrino masses and dark matter appropriately, a direct consequence of the dark dynamics. We diagonalize the fermion, scalar, and gauge sectors as well as obtain relevant interactions, taking into account the kinetic mixing of $U(1)_{Y,N}$ gauge bosons. The new physics signals at colliders are examined. The dark matter observables are discussed.

KEYWORDS: Neutrino physics, Cosmology of theories beyond the SM, Gauge symmetry
1 Motivation

The neutrino mass [1, 2] and dark matter [3–5] are two of the leading questions in science, which cannot be addressed within the framework of the standard model. What is the mechanism that produces small neutrino masses? How does such neutrino mass generation scheme solve dark matter? Can the neutrino mass and dark matter have a common origin, described in a comprehensive framework?

A number of theories have been proposed so far in attempt to answer such questions, basically based up on the seesaw [6–14] or/and radiative [15–19] mechanisms. As a matter of fact, a violation of lepton number symmetry plus an extra symmetry,
sometimes related to lepton parity or matter parity, are necessarily to make nonzero neutrino masses and dark matter stability. However, such lepton symmetry is only approximated and anomalous, which prevents the model’s prediction at high energy. The extra symmetry, e.g. $Z_2$, that stabilizes dark matter is ad hoc included, even in supersymmetry the matter parity is not automatically conserved by the theory.

Recent attempts [20–37] made in interpreting neutrino mass and dark matter use anomaly-free gauged abelian charges, namely $B - L$ [38–40], $L_i - L_j$ [41–43], or some variant of weak hypercharge [44, 45]. The advantage is that the gauge symmetry breaking leads to appropriate neutrino masses and that the resultant model is well-defined up to high energy. But the presence of dark matter and its stabilization mechanism are optional. This may be that we have not yet had an underlying principle that governs dark matter physics, i.e. manifestly stabilizing dark matter candidates and setting their present abundance.

As originally proposed in [46], we introduce a dynamical dark charge, $D$, to be a variant of electric charge, which properly works up to Planck scale and manifestly provides a potential solution to both the questions. In contrast to the mentioned abelian charges, the dark charge neither commutes nor closes algebraically with weak isospin, analogous to electric charge. This requires a novel gauge extension to $SU(3)_C \otimes SU(2)_L \otimes U(1)_Y \otimes U(1)_N$ by symmetric principles, where $N$ determines the dark charge $D = T_3 + N$, in the same manner the hypercharge does so for electric charge, $Q = T_3 + Y$. Interestingly, the dark charge anomaly cancelation requires three right-handed neutrinos. And, the dark charge breaking induces both small neutrino masses through canonical seesaw and a dark parity responsible for dark matter stability, where the dark dynamics is important to set dark matter observables as well as the neutrino mass generation process.

To be concrete, we now point out the existence of dark charge, its gauge completion, and discussing its crucial role in settling the new physics.

2 Proposal

The electroweak theory is based up on the gauge symmetry, $SU(2)_L \otimes U(1)_Y$. The electric charge is a residual charge, embedded in neutral electroweak charges such as $Q = T_3 + Y$, where $T_3$, $Y$ coefficients are normalized to 1, using a freedom in rescaling electric charge and hypercharge. Thus, the standard model does not predict the quantization of electric charge because of $Q = T_3 + Y$, where $T_3$ is quantized due to the non-Abelian nature of $SU(2)_L$ algebra, whereas the value of $Y$ is completely arbitrary on the theoretical ground.¹

¹Indeed, $Y$ was chosen to describe observed electric charges, not explaining them.
Generally, the fermions transform under the electroweak group as
\[ l_{aL} = \left( \begin{array}{c} \nu_{aL} \\ e_{aL} \end{array} \right) \sim (2, Y_a), \quad \nu_{aR} \sim (1, Y_{\nu_a}), \quad e_{aR} \sim (1, Y_{e_a}), \]
\[ q_{aL} = \left( \begin{array}{c} u_{aL} \\ d_{aL} \end{array} \right) \sim (2, Y_{q_a}), \quad u_{aR} \sim (1, Y_{u_a}), \quad d_{aR} \sim (1, Y_{d_a}), \]
where \( a = 1, 2, 3 \) is a generation index. The right-handed neutrinos \( \nu_{aR} \) can be introduced or not, besides the standard model particles.

The symmetry breaking and mass generation are done by the Higgs doublet,
\[ \phi = \left( \begin{array}{c} \phi_1 \\ \phi_2 \end{array} \right) \sim (2, Y_\phi), \]
with \( \langle \phi \rangle \neq 0 \). The conservation of electric charge demands that \( Q(\phi) = 0 \), leading to \( Y_\phi = \pm 1/2 \). The electric charge of \( \phi \) is either \( \phi = (\phi_1^+ \phi_2^0)^T \) according to \( Y_\phi = 1/2 \) or \( \phi = (\phi_1^0 \phi_2^-)^T \) according to \( Y_\phi = -1/2 \). Since these solutions yield equivalently phenomenological models, we take \( \phi = (\phi_1^+ \phi_2^0)^T \sim (2, 1/2) \) into account.

Further, only Yukawa Lagrangian in the classical theory,
\[ L \supset h_{ab}^L \bar{l}_{aL} \phi e_{bR} + h_{ab}^\nu \bar{\nu}_{bR} \phi \nu_{aL} + h_{ab}^d \bar{d}_{aL} \phi d_{bR} + h_{ab}^u \bar{u}_{aL} \phi u_{bR} + H.c., \]
that is required for fermion mass generation and flavor mixing gives information on hypercharge. It deduces
\[ Y_{q_1} = Y_{q_2} = Y_{q_3} = Y_q, \quad Y_{l_1} = Y_{l_2} = Y_{l_3} = Y_l, \]
\[ Y_{d_1} = Y_{d_2} = Y_{d_3} = Y_d, \quad Y_{u_1} = Y_{u_2} = Y_{u_3} = Y_u, \]
\[ Y_{e_1} = Y_{e_2} = Y_{e_3} = Y_e, \quad Y_{\nu_1} = Y_{\nu_2} = Y_{\nu_3} = Y_{\nu}, \]
\[ Y_l = Y_{\phi} + Y_e = -Y_{\phi} + Y_e, \quad Y_q = Y_{\phi} + Y_d = -Y_{\phi} + Y_d. \]
At quantum level, only nontrivial anomaly is \([SU(2)_L]^2U(1)_Y\), which vanishes if
\[ 3Y_q + Y_l = 0. \]

With the aid of \( Y_\phi = 1/2 \), the above equations imply
\[ Y_e = \delta - 1, \quad Y_u = 2/3 - \delta/3, \quad Y_d = -1/3 - \delta/3, \]
\[ Y_l = -1/2 + \delta, \quad Y_q = 1/6 - \delta/3, \]
which depend on a parameter, \( \delta \equiv Y_{\nu} \). This yields the electric charge of particles,
\[ Q(\nu) = \delta, \quad Q(e) = \delta - 1, \quad Q(u) = 2/3 - \delta/3, \quad Q(d) = -1/3 - \delta/3, \]
which is not quantized.\(^2\)

Generally, we have an infinite number of hypercharge symmetries, depending on corresponding values of \( \delta \)-parameter. Two remarks are in order,
\(^2\)Of course, we are investigating the most general case in which neutrinos have Yukawa couplings similar to those of other fermions. Special restrictions, e.g. adding a mass term \( \nu_R \nu_R \) that may lead to the quantization of electric charge [47–50], are not favored in this work.
1. True electric-charge: $\delta = 0$. In this case the particles get correct electric charge and hypercharge, as observed, in which $\nu_{aR}$ are gauge singlets, which can be omitted as in the standard model. The correct electric charge and hypercharge are denoted as $Q$ and $Y$, as usual.

2. Mis electric-charge: $\delta \neq 0$. In this case the particles get abnormal electric charge (called dark charge) and abnormal hypercharge (called hyper dark-charge), in which $\nu_{aR}$ are nontrivially under dark charge symmetry, which must be included for anomaly cancelation. The dark charge and hyper dark-charge are denoted as $D$ and $N$, respectively.

Because the two solutions (according to $\delta = 0$ and $\delta \neq 0$) are linearly independent, i.e. $Y$ and $N$ are linearly independent as $Q$ and $D$ are, the full gauge symmetry of the theory must be

$$SU(2)_L \otimes U(1)_Y \otimes U(1)_N,$$

(2.13)

apart from the QCD group, where $Y$ and $N$ determine electric and dark charges,

$$Q = T_3 + Y, \quad D = T_3 + N,$$

(2.14)

respectively. In Appendix A, we use another approach that comes to the same conclusion of the gauge symmetry (2.13), as desirable. All the anomalies vanish, independent of $\delta$, as verified in Appendix B. In this work, we take $\delta = 1$ (for the case $\delta \neq 0$) into account, which manifestly determines dark matter.\(^{3}\)

Each particle possesses a pair of characteristic electric and dark charges, as collected in Table 1. The particle representations under the $SU(3)_C \otimes SU(2)_L \otimes U(1)_Y \otimes U(1)_N$ symmetry are listed in Table 2. Here the scalar $\chi$ is necessarily presented to break $U(1)_N$ and generate right-handed neutrino masses, and $Z'$ is the $U(1)_N$ gauge boson.

| Field | $\nu$ | $e$ | $u$ | $d$ | $\phi_1$ | $\phi_2$ | $\chi$ | $W$ | $A$ | $Z$ | $Z'$ | gluon |
|-------|-------|-----|-----|-----|-----------|-----------|-------|-----|-----|-----|-------|-------|
| $Q$   | 0     | -1  | 2/3 | -1/3| 1         | 0         | 0     | 1   | 0   | 0   | 0     | 0     |
| $D$   | 1     | 0   | 1/3 | -2/3| 1         | 0         | -2    | 1   | 0   | 0   | 0     | 0     |
| $D_P$ | 1     | -1  | 1   | -1  | 1         | 1         | -1    | 1   | 1   | 1   | 1     | 1     |

**Table 1.** $Q$, $D$ charges and dark parity of the model particles.

The scalars develop vacuum expectation values (vevs),

$$\langle \chi \rangle = \frac{1}{\sqrt{2}} \Lambda, \quad \langle \phi \rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v \end{pmatrix},$$

(2.15)

\(^{3}\)In the literature [45, 51], $N$ charge was discussed to be a linear combination of the hypercharge and baryon-minus-lepton number, as desirable. However, the dark charge $D$ and the following dark parity were not investigated.
such that $\Lambda \gg v = 246$ GeV to keep consistency with the standard model. The scheme of gauge symmetry breaking is

$$SU(3)_C \otimes SU(2)_L \otimes U(1)_Y \otimes U(1)_N \downarrow \Lambda$$

$$SU(3)_C \otimes SU(2)_L \otimes U(1)_Y \otimes U'(P) \downarrow v$$

$$SU(3)_C \otimes U(1)_Q \otimes D_P$$

where the dark parity $D_P$ is a residual symmetry of $SU(2)_L \otimes U(1)_N$ or $D = T_3 + N$, which transforms a field as $\Phi \to \Phi' = D_P\Phi$, where $D_P = e^{i\alpha\delta}$. Note that $D_P$ always conserves the weak vacuum, since $D\langle\phi\rangle = 0$. $D_P$ conserves the $\chi$ vacuum, if $D_P\Lambda = \Lambda$ implies $e^{-2i\alpha} = 1$, i.e. $\alpha = k\pi$ for $k = 0, \pm 1, \pm 2, \cdots$ Considering $k = \pm 3$, we obtain the dark parity

$$D_P = (-1)^{3(T_3+N)+2s} \tag{2.16}$$

after multiplying the spin parity $(-1)^{2s}$.\(^4\) The dark parity of particles is calculated, as indicated in Table 1. Note that $\phi^\pm$ is a Goldstone boson already eaten by the $W^\pm$ gauge boson.

At this step, the electron is the lightest particle that is $D_P$ odd. We introduce two simplest fields (or candidates), a vector-like fermion ($n$) and a scalar ($\eta$), transforming under the gauge symmetry $SU(3)_C \otimes SU(2)_L \otimes U(1)_Y \otimes U(1)_N$ as

$$n \sim (1,1,0,2r), \quad \eta \sim (1,1,0,2r-1) \tag{2.17}$$

for $r$ integer, which couple to $\nu_R$ through $y\bar{n}_L \eta \nu_R$.\(^5\) Note that all $n, \eta$ are $D_P$ odd. One of them should be lighter than electron, responsible for dark matter. In other words, the model predicts a dark matter mass below the electron mass.\(^6\)

\(^4\)The dark parity is related to weak isospin, different from those induced by $B-L$, 3-3-1-1, and left-right symmetries [52–60].

\(^5\)When $r = 0$, we can introduce only the left chiral component $n_L$ (i.e., omitting $n_R$), since it does not contribute to anomaly.

\(^6\)Hence, the dark matter candidate is stabilized by dark parity conservation, while the electron is always stabilized, ensured by electric charge conservation, by contrast.
The total Lagrangian is written as

\[ \mathcal{L} = \mathcal{L}_{\text{kinetic}} + \mathcal{L}_{\text{Yukawa}} - V. \]  

(2.18)

The first part contains kinetic terms and gauge interactions,

\[ \mathcal{L}_{\text{kinetic}} = \sum_F \bar{F}i\gamma^\mu D_\mu F + \sum_S (D^\mu S)^\dagger (D_\mu S) \]

\[- \frac{1}{4} G_{m\mu\nu} G_m - \frac{1}{4} A_{i\mu\nu} A_{i\mu\nu} \]

\[- \frac{1}{4} B_{\mu\nu} B^{\mu\nu} - \frac{1}{4} C_{\mu\nu} C^{\mu\nu} - \frac{\epsilon}{2} B_{\mu\nu} C^{\mu\nu}, \]

(2.19)

where \( F, S \) run over fermion and scalar multiplets, respectively. The covariant derivative and field strength tensors are defined as

\[ D_\mu = \partial_\mu + ig_s t_m G_{m\mu} + ig T_i A_{i\mu} + ig Y B_\mu + ig_N NC_\mu, \]

\[ G_{m\mu\nu} = \partial_\mu G_{m\nu} - \partial_\nu G_{m\mu} - g S f_{mpq} G_{p\mu} G_{q\nu}, \]

\[ A_{i\mu\nu} = \partial_\mu A_{i\nu} - \partial_\nu A_{i\mu} - \epsilon_{ijk} A_{j\mu} A_{k\nu}, \]

\[ B_{\mu\nu} = \partial_\mu B_{\nu} - \partial_\nu B_\mu, \quad C_{\mu\nu} = \partial_\mu C_\nu - \partial_\nu C_\mu, \]

(2.20-2.23)

where \( (g_s, g, g_Y, g_N), (t_m, T_i, Y, N) \), and \( (G_m, A, B, C) \) are coupling constants, generators, and gauge bosons according to \( (SU(3)_C, SU(2)_L, U(1)_Y, U(1)_N) \) groups, respectively; \( f_{mpq} \) and \( \epsilon_{ijk} \) are structure constants of the corresponding groups.

Note that \( \epsilon \) is a parameter that determines the kinetic mixing between the two \( U(1) \) gauge bosons, satisfying \( |\epsilon| < 1 \), in order for definitely positive kinetic energy. Such kinetic terms can be transformed into the canonical form, i.e.

\[- \frac{1}{4} B_{\mu\nu} B^{\mu\nu} - \frac{1}{4} C_{\mu\nu} C^{\mu\nu} - \frac{\epsilon}{2} B_{\mu\nu} C^{\mu\nu} = - \frac{1}{4} \hat{B}_{\mu\nu} \hat{B}^{\mu\nu} - \frac{1}{4} \hat{C}_{\mu\nu} \hat{C}^{\mu\nu}, \]

(2.24)

by basis changing,

\[
\begin{pmatrix}
\hat{B} \\
\hat{C}
\end{pmatrix} = \begin{pmatrix} 1 & \epsilon \\ 0 & \sqrt{1 - \epsilon^2} \end{pmatrix} \begin{pmatrix} B \\ C \end{pmatrix}.
\]

(2.25)

The Yukawa part consists of

\[ \mathcal{L}_{\text{Yukawa}} = h^{\nu}_{a\bar{b}L} \phi \bar{b}_R + h^{\nu}_{a\bar{b}L} \phi \bar{b}_R + h^{\nu}_{a\bar{b}L} \phi \bar{b}_R + h^{\nu}_{a\bar{b}L} \phi \bar{b}_R + \frac{1}{2} f_{\nu} \tilde{f}_{aR} \chi \nu_{aR} + y_{a\tilde{n}L} \eta_{aR} - m\tilde{n}_L \nu_{R} + H.c., \]

(2.26)

while the scalar potential takes the form

\[ V = \mu^2 \phi^\dagger \phi + \mu^2 \eta^* \eta + \mu^2 \chi^* \chi + \lambda_1 (\phi^\dagger \phi)^2 + \lambda_2 (\eta^* \eta)^2 + \lambda_3 (\chi^* \chi)^2 \]

\[ + \lambda_4 (\phi^\dagger \phi)(\eta^* \eta) + \lambda_5 (\phi^\dagger \phi)(\chi^* \chi) + \lambda_6 (\eta^* \eta)(\chi^* \chi). \]

(2.27)

Note that \( h^\nu, f^\nu, y, \) and \( \lambda \)'s are dimensionless, whereas \( m\) and \( \mu \)’s have mass dimension. Especially, when \( r = 0 \), the scalar potential might have extra triple terms, \( \mu \chi^* \eta^2 + H.c. \), but they do not affect the present results, hence being neglected.
3 Fermion mass

The spontaneous symmetry breaking will generate mass for fermions through the Yukawa Lagrangian. We first consider the charged leptons and quarks, which get

\[
[m_e]_{ab} = -h_{ab}^e \frac{v}{\sqrt{2}}, \quad [m_u]_{ab} = -h_{ab}^u \frac{v}{\sqrt{2}}, \quad [m_d]_{ab} = -h_{ab}^d \frac{v}{\sqrt{2}}. \tag{3.1}
\]

This provides appropriate masses for the particles after diagonalization, similar to the case of the standard model.

Since the vev of \( \eta \) vanishes due to dark parity conservation, the dark fermion \( n \) does not mix with right-handed neutrinos \( \nu_R \) although they couple via \( y_a \bar{n}_L \eta \nu_{aR} \).

The field \( n \) is a physical field by itself, with arbitrary mass \( m_n \).

The neutrinos achieve a mass matrix after the two stages of gauge symmetry breaking are taken place,

\[
\mathcal{L} \supset -\frac{1}{2} \left( \bar{\nu}_L \bar{\nu}_R \right) \left( \begin{array}{cc} 0 & m_D \\ m_D^T & m_M \end{array} \right) \left( \begin{array}{c} \nu_L \\ \nu_R^* \end{array} \right) + H.c., \tag{3.2}
\]

where \( m_D^* = -h_D^e \frac{v}{\sqrt{2}} \) is Dirac mass that couples \( \nu_L \) to \( \nu_R \) and \( m_M^* = -f_\nu \frac{\Lambda}{\sqrt{2}} \) is Majorana mass that couples \( \nu_R \nu_R \) by themselves.

Diagonalizing the mass matrix of the neutrinos with the aid of \( \Lambda \gg v \), we obtain the following mass eigenvalues

\[
m_{\nu_L'} \simeq -U^T m_D m_M^{-1} m_D^T U, \tag{3.3}
\]

\[
m_{\nu_R'} \simeq V^\dagger m_M V^*, \tag{3.4}
\]

where \( U \) is the Pontecorvo-Maki-Nakagawa-Sakata matrix, given that the charged leptons are flavor diagonal. The observed light neutrinos \( \nu_L' \) and the heavy neutrinos \( \nu_R' \) are given as follows

\[
\left( \begin{array}{c} \nu_L \\ \nu_R' \end{array} \right) \simeq \left( \begin{array}{cc} 1 & \theta \\ -\theta^\dagger & 1 \end{array} \right) \left( \begin{array}{cc} U & 0 \\ 0 & V^* \end{array} \right) \left( \begin{array}{c} \nu_L \\ \nu_R^* \end{array} \right), \tag{3.5}
\]

where \( \theta = m_D m_M^{-1} \). Due to the fact that the mixing parameter \( \theta \sim v/\Lambda \) is small, we have the following approximation, \( \nu_L \simeq U \nu'_L \) and \( \nu_R \simeq V \nu'_R \). Without loss of generality, we can take \( V = 1 \) into account.

The neutrino mass generation is presented by the diagram in Fig. 1, attached by external fields \( \phi, \chi, \phi \) with internal lines \( \nu_R, \nu_R \), happening when the dark and weak charges are broken, derived by \( \langle \chi \rangle \) and \( \langle \phi \rangle \), respectively. The large \( \nu_R \) Majorana mass is produced due to its interaction with \( \chi \) (see the middle part in Fig. 1) after dark charge breaking. It is clear that the gauge symmetry suppresses all neutrino mass types, but the dark and weak breaking supplies consistent neutrino masses through an improved Higgs mechanism. The canonical seesaw is manifestly realized since \( \nu_R \) are fundamental constituents required for dark charge anomaly cancelation and the Majorana masses arise from dark charge breaking, irrelevant to the lepton number.
Figure 1. Neutrino mass generation process induced by dark charge breaking, where $\nu_{L,R}$ carry a unit of dark charge, converted by the Higgs field $\phi$, while the coupling $\nu_R\nu_R$ breaks dark charge by two unit induced by $\chi$.

4 Scalar sector

Because the electric charge and dark parity are conserved, only the scalar fields that are electrically neutral and $D_P$ even can develop a vev, such as $\langle \phi \rangle = \frac{1}{\sqrt{2}} (0 \ v)^T$, $\langle \chi \rangle = \frac{1}{\sqrt{2}} \Lambda$, and $\langle \eta \rangle = 0$, aforementioned.

Moreover, the necessary conditions for the scalar potential (2.27) to be bounded from below as well as yielding a desirable vacuum structure are

$$\lambda_{1,2,3} > 0, \quad \mu^2_{1,3} < 0, \quad |\mu_1| \ll |\mu_3|, \quad \mu_2^2 > 0. \quad (4.1)$$

To obtain the potential minimum and physical scalar spectrum, we expand the scalar fields around vevs as

$$\phi = \begin{pmatrix} \phi^+ \\ \frac{1}{\sqrt{2}} (v + S_1 + i A_1) \end{pmatrix}, \quad \chi = \frac{1}{\sqrt{2}} (\Lambda + S_2 + i A_2), \quad \eta = \frac{1}{\sqrt{2}} (S_3 + i A_3). \quad (4.3)$$

Substituting (4.2) and (4.3) into (2.27), the potential minimum conditions are

$$\Lambda^2 = \frac{-2\lambda_3 \mu_1^2 + 4\lambda_1 \mu_3^2}{\lambda_3^2 - 4\lambda_1 \lambda_3}, \quad v^2 = \frac{-2\lambda_5 \mu_3^2 + 4\lambda_3 \mu_1^2}{\lambda_5^2 - 4\lambda_1 \lambda_3}. \quad (4.4)$$

Using the minimum conditions, we obtain physical scalar fields,

$$\phi = \begin{pmatrix} \phi^+ \\ \frac{1}{\sqrt{2}} (v + c_\xi H + s_\xi H' + i G_Z) \end{pmatrix}, \quad \chi = \frac{1}{\sqrt{2}} (\Lambda - s_\xi H + c_\xi H' + i G_{Z'}), \quad \eta, \quad (4.6)$$
where $G_W$, $G_Z$, and $G_{Z'}$ are the Goldstone bosons corresponding to the $W$, $Z$, and $Z'$ gauge bosons, respectively. $H$ is identical to the standard model Higgs boson, while $H'$ is a new Higgs boson associate to the dark charge breaking.

The $S_1$-$S_2$ mixing angle and mass eigenvalues are given by

$$t_{2\xi} = \frac{\lambda_5 v \Lambda}{\lambda_3 \Lambda^2 - \lambda_1 v^2} \simeq \frac{\lambda_5 v}{\lambda_3 \Lambda},$$

$$m^2_H = \lambda_1 v^2 + \lambda_3 \Lambda^2 - \sqrt{(\lambda_1 v^2 - \lambda_3 \Lambda^2)^2 + \lambda_5^2 v^2 \Lambda^2} \simeq (2\lambda_1 - \frac{\lambda_5^2}{2\lambda_3}) v^2,$$

$$m^2_{H'} = \lambda_1 v^2 + \lambda_3 \Lambda^2 + \sqrt{(\lambda_1 v^2 - \lambda_3 \Lambda^2)^2 + \lambda_5^2 v^2 \Lambda^2} \simeq 2\lambda_3 \Lambda,$$

which imply that $\xi$ is small, $m_H$ is at weak scale, and $m_{H'}$ is at $\Lambda$ scale.

Last, but not least, $\eta$ does not mix with other fields, being a physical field by itself, with mass

$$m^2_\eta = \mu_2^2 + \frac{1}{2} \lambda_4 v^2 + \frac{1}{2} \lambda_6 \Lambda^2.$$  

Dependent on the scalar couplings $\lambda_{4,6}$ and mass parameter $\mu_2$, the dark scalar $\eta$ can have a mass at $\Lambda$, $v$, or low scale.

## 5 Gauge sector

The gauge bosons acquire mass through their interaction with scalar fields, when the gauge symmetry breaking happens. The charged gauge boson $W^\pm = (A_1 \mp i A_2)/\sqrt{2}$ gets a mass $m_W^2 = g^2 v^2/4$, which leads to $v = 246$ GeV.

The mass matrix of the neutral gauge bosons in the canonical basis $(A_3, \hat{B}, \hat{C})$ is given by

$$M^2 = L^T_e \begin{pmatrix} \frac{g^2 v^2}{4} & -\frac{gY v^2}{4} & -\frac{g\gamma v^2}{4} \\ -\frac{gY v^2}{4} & \frac{g_1^2 v^2}{4} & \frac{g\gamma v^2}{4} \\ -\frac{g\gamma v^2}{4} & \frac{g\gamma v^2}{4} & \frac{g_2^2 v^2}{4} + 4g_3^2 \Lambda^2 \end{pmatrix} L_e,$$

where $L_e$ is not an orthogonal matrix, that relates the present basis to the original basis, $(A_3 \ B \ C)^T = L_e (A_3 \ \hat{B} \ \hat{C})^T$, obtained by

$$L_e = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & -\frac{\epsilon}{\sqrt{1-\epsilon^2}} \\ 0 & \frac{\epsilon}{\sqrt{1-\epsilon^2}} & 1 \end{pmatrix}.$$

It is easily checked that the mass matrix (5.1) provides a zero eigenvalue (photon mass) with corresponding eigenstate (photon field)

$$A = s_W A_3 + c_W \hat{B},$$

$$M^2 = \begin{pmatrix} \frac{g^2 v^2}{4} & -\frac{gY v^2}{4} & -\frac{g\gamma v^2}{4} \\ -\frac{gY v^2}{4} & \frac{g_1^2 v^2}{4} & \frac{g\gamma v^2}{4} \\ -\frac{g\gamma v^2}{4} & \frac{g\gamma v^2}{4} & \frac{g_2^2 v^2}{4} + 4g_3^2 \Lambda^2 \end{pmatrix}$$
where the Weinberg angle is defined by \( t_W = g_Y/g \).\(^7\) The \( Z_0 \) boson is defined orthogonally to \( A \), such as

\[
Z_0 = c_W A_3 - s_W \hat{B},
\]

which is identical to that of the standard model. Hence, in the new basis \((A, Z_0, \hat{C})\), the photon is decoupled, while there remains a mixing between \( Z_0 \) and \( \hat{C} \) determined by a mixing angle \( \alpha \). That said, diagonalizing the mass matrix (5.1) by an orthogonal transformation,

\[
O^T M^2 O = \text{diag}(0, m_Z^2, m_{Z'}^2),
\]

we have

\[
O = \begin{pmatrix}
s_W & c_W & 0 \\
c_W & -s_W & 0 \\
0 & 0 & 1
\end{pmatrix} \begin{pmatrix}
1 & 0 & 0 \\
0 & c_\alpha & s_\alpha \\
0 & -s_\alpha & c_\alpha
\end{pmatrix}.
\]

The \( Z_0-\hat{C} \) mixing angle is evaluated as

\[
t_{2\alpha} \simeq -\sqrt{1-\epsilon^2} \frac{1}{8g_N^2} \sqrt{g^2 + g_Y^2 (g_N - \epsilon g_Y) v^2 / \Lambda^2}.
\]

And, the mass eigenvalues read

\[
m_Z^2 \simeq \frac{g^2 + g_Y^2}{4} v^2 \left[ 1 - \frac{(g_N - \epsilon g_Y)^2 v^2}{16g_N^2 \Lambda^2} \right],
\]

\[
m_{Z'}^2 \simeq \frac{4g_N^2 \Lambda^2}{1-\epsilon^2} \left[ 1 + \frac{(g_N - \epsilon g_Y)^2 v^2}{16g_N^2 \Lambda^2} \right].
\]

Note that the \( Z-Z' \) mixing, i.e. the \( \alpha \) angle, comes from two sources, the kinetic mixing (characterized by \( \epsilon \)) and the symmetry breaking (induced by \( \Lambda \)). Such two contributions cancel out if \( \epsilon = g_N/g_Y \).

In summary, the physical states \((A, Z, Z')\) are related to the canonical states \((A_3, \hat{B}, \hat{C})\) and the original states \((A_3, B, C)\) as follows

\[
(A_3 \hat{B} \hat{C})^T = O(A Z Z')^T, \quad (A_3 B C)^T = L_\epsilon O(A Z Z')^T.
\]

### 6 Interactions

We are interested in the interaction of electroweak and dark gauge bosons with fermions. Let us expand the relevant Lagrangian,

\[
\sum_F \bar{F} i \gamma^\mu D_\mu F = \sum_F \bar{F} i \gamma^\mu \partial_\mu F - g_s \sum_F \bar{F} \gamma^\mu t_m G_{m\mu} F + \mathcal{L}^{CC} + \mathcal{L}^{NC},
\]

\(^7\)Interested reader can refer to [61–63] for diagonalizing a more-general neutral-gauge sector with/without a kinetic mixing term.
where

\[ \mathcal{L}^{CC} = -g \sum_{F_L} \bar{F}_L \gamma^\mu (T_1 A_{1\mu} + T_2 A_{2\mu}) F_L, \]  
\[ \mathcal{L}^{NC} = -g \sum_{F_L} \bar{F}_L \gamma^\mu (T_3 A_{3\mu} + t_W Y_{F_L} B_\mu + t_N N_{F_L} C_\mu) F_L \]
\[ -g \sum_{F_R} \bar{F}_R \gamma^\mu (t_W Y_{F_R} B_\mu + t_N N_{F_R} C_\mu) F_R, \]  
\[ \text{(6.3)} \]

where \( F_L \) and \( F_R \) run over the left-handed and right-handed fermion multiplets of the model, respectively, and we define \( t_N = g_N/g \).

From (6.2), we obtain the interaction of fermions with charged gauge bosons,

\[ \mathcal{L}^{CC} = -\frac{g}{\sqrt{2}} (\bar{e} L \gamma^\mu U_{\nu L} + \bar{d} L \gamma^\mu V_{\text{CKM}} u_L) W^\mu_\nu + H.c., \]  
\[ \text{(6.4)} \]

where we denote \( \nu \equiv (\nu_1 \nu_2 \nu_3)^T \), \( e \equiv (e \mu \tau)^T \), \( u \equiv (u c t)^T \), and \( d \equiv (d s b)^T \) to be mass eigenstates, without confusion.

Equation (6.3) gives rise to the interaction of fermions with neutral gauge bosons,

\[ \mathcal{L}^{NC} = -e Q(f) \bar{f} \gamma^\mu f A_\mu \]
\[ -\frac{g}{2c_W} \left\{ C^{Z}_{\nu_L} \bar{\nu}_L \gamma^\mu \nu_L + C^{Z}_{\nu_R} \bar{\nu}_R \gamma^\mu \nu_R + \bar{f} \gamma^\mu [g^Z_\mu (f) - g^Z_\mu (f) \gamma_5] f \right\} Z_\mu \]
\[ -\frac{g}{2c_W} \left\{ C^{Z'}_{\nu_L} \bar{\nu}_L \gamma^\mu \nu_L + C^{Z'}_{\nu_R} \bar{\nu}_R \gamma^\mu \nu_R + \bar{f} \gamma^\mu [g^{Z'}_\mu (f) - g^{Z'}_\mu (f) \gamma_5] f \right\} Z'_\mu, \]  
\[ \text{(6.5)} \]

where \( f \) indicates to every fermion of the model, except for neutrinos, and

\[ C^{Z}_{\nu_L} = c_\alpha - \frac{c_W t_N + s_W s_\alpha}{\sqrt{1 - \epsilon^2}} s_\alpha, \]
\[ C^{Z}_{\nu_R} = -\frac{2c_W t_N}{\sqrt{1 - \epsilon^2}} s_\alpha, \]  
\[ \text{(6.6)} \]
\[ C^{Z'}_{\nu_L} = s_\alpha + \frac{c_W t_N + s_W s_\alpha}{\sqrt{1 - \epsilon^2}} c_\alpha, \]
\[ C^{Z'}_{\nu_R} = \frac{2c_W t_N}{\sqrt{1 - \epsilon^2}} c_\alpha. \]  
\[ \text{(6.7)} \]

The vector and axial-vector couplings of \( Z, Z' \) to the remaining fermions are listed in Tables 3 and 4, respectively.

| \( f \) | \( g^Z_\mu (f) \) | \( g^{Z'}_\mu (f) \) |
|---|---|---|
| \( e, \mu, \tau \) | \( 1 - 2c_W^2 \frac{c_\alpha}{2} - \frac{c_W t_N + 3s_W}{2\sqrt{1 - \epsilon^2}} s_\alpha \) | \( -\frac{1}{2} c_\alpha - \frac{c_W t_N - 3s_W}{2\sqrt{1 - \epsilon^2}} s_\alpha \) |
| \( u, c, t \) | \( -1 + 3c_W^2 \frac{c_\alpha}{6} - \frac{3c_W t_N - 5s_W}{6\sqrt{1 - \epsilon^2}} s_\alpha \) | \( \frac{1}{2} c_\alpha + \frac{c_W t_N - 5s_W}{2\sqrt{1 - \epsilon^2}} s_\alpha \) |
| \( d, s, b \) | \( -\frac{1 + 3c_W^2}{6} + \frac{5c_W t_N - 5s_W}{6\sqrt{1 - \epsilon^2}} s_\alpha \) | \( -\frac{1}{2} c_\alpha - \frac{c_W t_N - 5s_W}{2\sqrt{1 - \epsilon^2}} s_\alpha \) |
| \( n \) | \( -\frac{4c_W t_N}{\sqrt{1 - \epsilon^2}} s_\alpha \) | 0 |

**Table 3.** Couplings of \( Z \) with fermions \( (f \neq \nu) \).
Table 4. Couplings of $Z'$ with fermions ($f \neq \nu$).

7 Electroweak precision test

7.1 $\rho$-parameter

Because the $Z$ boson mixes with the new neutral gauge boson through the kinetic mixing and the symmetry breaking, the new physics contributions to the $\rho$-parameter start from the tree-level,

$$\Delta \rho = \frac{m_W^2}{c_W^2 m_Z^2} - 1 \simeq \frac{(t_N - c_W)^2 v^2}{16 t_N^2} \frac{v}{\Lambda^2}. \quad (7.1)$$

From the global fit, the $\rho$ parameter is bounded by $0.0002 < \Delta \rho < 0.00058$ [64], which leads to the following lower bound,

$$\Lambda \gtrsim 2.553 \times \frac{|t_N - c_W|}{t_N} \text{ TeV.} \quad (7.2)$$

7.2 Total $Z$ decay width

In this subsection, we use the precision measurement of the total $Z$ decay width to impose the constraint on the free parameters of the model. The total $Z$ decay width is measured by the experiment and predicted by the standard model as, $\Gamma^{\exp}_{Z} = 2.4952 \pm 0.0023$ GeV and $\Gamma^{\text{SM}}_{Z} = 2.4942 \pm 0.0008$ GeV, respectively [64].

First, we rewrite the Lagrangian describing the $Z$ couplings to the standard model fermions given as

$$\mathcal{L}^{NC} \supset - \frac{g}{2c_W} \{ \bar{\nu}_L \gamma^\mu (1 + \Delta_{\nu_L}) \nu_L 
+ \bar{f} \gamma^\mu [g_{0V}(f)(1 + \Delta_{V,f}) - g_{0A}(f)(1 + \Delta_{A,f}) \gamma^5] f \} Z_\mu, \quad (7.3)$$

where $g_{0V}(f) = T_3(f) - 2Q(f) s_W^2$ and $g_{0A}(f) = T_3(f)$ are the standard model predictions for the vector and axial-vector couplings, respectively, $\Delta_{\nu_L}$, $\Delta_{V,f}$ and $\Delta_{A,f}$ are the coupling shifts given as follows

$$\Delta_{\nu_L} \simeq \frac{t_N^2 \Delta_{\nu_L}}{16 t_N^2} \frac{v^2}{\Lambda^2}. \quad (7.4)$$
\[ \Delta_{V,f} \simeq \frac{2 [t_N D(f) - e t_W Q(f)] - T_3(f)(t_N - e t_W) t_N - e t_W v^2}{T_3(f) - 2 Q(f) s_W^2} \ \left( t_N^2 - \epsilon t_W \right) \ \left( f \right) - \frac{T_3(f)(t_N - e t_W)}{16 t_N^2} \ \Lambda^2; \] (7.5)

\[ \Delta_{A,f} \simeq -\frac{(t_N - e t_W)^2 v^2}{16 t_N^2} \ \Lambda^2. \] (7.6)

Using this Lagrangian, one can write the total \( Z \) decay width predicted in this model as

\[ \Gamma_Z = \Gamma_{Z}^{\text{SM}} + \Delta \Gamma_Z, \] (7.7)

where \( \Gamma_{Z}^{\text{SM}} \) is the standard model value and the shift \( \Delta \Gamma_Z \) is given by

\[
\Delta \Gamma_Z \simeq \frac{m_Z^{\text{SM}}}{6 \pi} \left( \frac{g}{2 c_W} \right)^2 \left\{ \sum_f N_C(f) \left[ (g_{0V}^Z(f))^2 \Delta_{V,f} + (g_{0A}^Z(f))^2 \Delta_{A,f} \right] + \frac{3 \Delta m_{\nu L}}{2} \right\} + \frac{\Delta m_Z}{12 \pi} \left( \frac{g}{2 c_W} \right)^2 \left\{ \sum_f N_C(f) \left[ (g_{0V}^Z(f))^2 + (g_{0A}^Z(f))^2 \right] + \frac{3}{2} \right\}, \] (7.8)

where \( m_Z^{\text{SM}} \) is the standard model value of the \( Z \) gauge boson mass, \( N_C(f) \) is the color number of the fermion \( f \), the sum is taken over the standard model charged fermions, and the mass shift of the gauge boson \( Z \) is given by

\[ \Delta m_Z \simeq -\frac{g}{2 c_W} \frac{(t_N - e t_W)^2 v^2}{32 t_N^2} \ \Lambda^2. \] (7.9)

Note that if allowed kinetically the gauge boson \( Z \) can decay into the dark matter candidate pairs \( \bar{n}n \) and \( \eta^* \eta \) but these two-body decays are highly suppressed by \( v^4/\Lambda^4 \). From the experimental and theoretical values of \( \Gamma_Z \) as mentioned above, we require \( |\Delta \Gamma_Z| < 0.0041 \text{ GeV} \), which leads to the following bound

\[ \Lambda \gtrsim 1.14 \times \frac{\sqrt{[t_N - 1.62 \epsilon](t_N - 0.55 \epsilon)}}{t_N} \text{ TeV}. \] (7.10)

8 Collider bounds

8.1 LEPII constraint

The on-shell new gauge boson \( Z' \) would not be produced at the existing \( e^+e^- \) colliders if its mass is in the TeV region or higher. But, below the resonance, \( Z' \) would manifestly contribute to viable observables that deviates them from the standard model predictions. Hence, the new gauge boson \( Z' \) can be indirectly searched at the LEPII experiment through the processes \( e^+e^- \rightarrow \bar{f}f \) with \( f = e, \mu, \tau \).

The considering processes that are induced by the exchange of the new gauge boson \( Z' \) can be described by the following effective Lagrangian,

\[ \mathcal{L}_{\text{eff}} = \frac{1}{1 + \delta_{e,f}} \left( \frac{g}{2 c_W m_{Z'}} \right)^2 \bar{e} \gamma_{\mu} [g_{0V}^Z(e) - g_{0A}^Z(e) \gamma_5] e \bar{f} \gamma_{\mu} [g_{0V}^Z(f) - g_{0A}^Z(f) \gamma_5] f. \] (8.1)
where $\delta_{ef} = 1(0)$ for $f = e$ ($f \neq e$).

By using the relevant data of the LEPII experiment [65], we impose the constraint,

$$\frac{4\sqrt{\pi}c_Wm_{Z'}}{g\sqrt{[g^Z_V(e)]^2 + [g^Z_A(e)]^2}} \gtrsim 24.6\ \text{TeV},$$

which leads to

$$\Lambda \gtrsim 1.23 \times \sqrt{(t_N + \epsilon t_W)^2 + 4\epsilon^2 t_W^2} \text{ TeV.}$$

### 8.2 LHC dilepton constraint

The new gauge boson $Z'$ can be resonantly produced at the LHC via the quark fusion $\bar{q}q \rightarrow Z'$ and subsequently it would decay into the standard model fermions as well as the exotic particles such as the dark matter candidate $n(\eta)$. The most significant decay channel of $Z'$ is given by $Z' \rightarrow l^+l^-$ with $l = e, \mu$, which have well-understood backgrounds and measure a $Z'$ that owns both couplings to quarks and leptons.

The cross-section for this process is approximately computed in the case of the very narrow $Z'$ decay width as

$$\sigma(pp \rightarrow Z' \rightarrow l^+l^-) \simeq \frac{\pi}{3} \left( \frac{g}{2c_W} \right)^2 \sum_q L_{q\bar{q}}(m_{Z'}^2) \left\{ [g^Z_V(q)]^2 + [g^Z_A(q)]^2 \right\} \frac{\Gamma(Z' \rightarrow l^+l^-)}{\Gamma_{Z'}},$$

where the parton luminosities $L_{q\bar{q}}$ is given by

$$L_{q\bar{q}}(m_{Z'}^2) = \int_{m_{Z'}^2}^{1} \frac{dx}{xs} \left[ f_q(x, m_{Z'}^2) f_{\bar{q}}(\frac{m_{Z'}^2}{xs}, m_{Z'}^2) + f_{\bar{q}}(x, m_{Z'}^2) f_q(\frac{m_{Z'}^2}{xs}, m_{Z'}^2) \right]$$

with $\sqrt{s}$ to be collider center-of-mass energy and $f_q(x, m_{Z'}^2)$ to be the parton distribution function of the quark $q$ (antiquark $\bar{q}$) evaluated at the scale $m_{Z'}$, and the total $Z'$ decay width reads

$$\Gamma_{Z'} \simeq \frac{m_{Z'}}{12\pi} \left( \frac{g}{2c_W} \right)^2 \sum_f N_C(f) \left\{ [g^Z_V(f)]^2 + [g^Z_A(f)]^2 \right\}$$

$$+ \frac{m_{Z'}}{24\pi} \left( \frac{g t_N}{\sqrt{1 - \epsilon^2}} \right)^2 \sum_{a=1}^{3} \left( 1 - \frac{4m_{\nu_a R}^2}{m_{Z'}^2} \right)^{3/2} \theta \left( \frac{m_{Z'}}{2} - m_{\nu_a} \right)$$

$$+ \frac{m_{Z'}}{48\pi} \left[ g(2r - 1)t_N \frac{1}{\sqrt{1 - \epsilon^2}} \right]^2 \left( 1 - \frac{4m_{\eta}^2}{m_{Z'}^2} \right)^{3/2} \theta \left( \frac{m_{Z'}}{2} - m_{\eta} \right),$$

where $f$ refers to the standard model fermions and dark matter candidate $n$, and $\theta(x)$ is the step function.
In Figure 2, we show the dilepton production cross-section $\sigma(pp \rightarrow Z' \rightarrow l^+l^-)$ as a function of the new neutral gauge boson mass for various values of $t_N$ and $\epsilon$, with $r = 1$, $m_{\nu_1R} = m_{\nu_2R} = m_{\nu_3R} = m_{Z'}/3$ and $m_\eta = m_{Z'}/4$. In addition, we include the upper limits on the cross-section of this process at 95% CL using 36.1 fb$^{-1}$ of $pp$ collision at $\sqrt{s} = 13$ TeV by the ATLAS experiment [66]. In the left panel, the lower bounds on the new neutral gauge boson mass are determined about $2.1$, $2.8$, $3.5$ and $3.7$ TeV, corresponding to $t_N = 0.1$, $0.3$, $0.6$, and $0.8$, respectively. Whereas, in the right panel for $\epsilon = -0.5$, $-0.1$, $0.4$, and $0.8$, the lower bounds are $3.9$, $2.3$, $3.4$ and $4.5$ TeV, respectively.

**Figure 2.** The cross-section for the process $pp \rightarrow Z' \rightarrow l^+l^-$ as a function of the new neutral gauge boson mass. The solid and dashed black curves refer to the observed and expected limits, respectively, while the green and yellow bands refer to $1\sigma$ and $2\sigma$ for the expected limit. The top and bottom panels correspond to $\epsilon = 0.1$ and $t_N = 0.2$, respectively.

In Figure 3, we combine the lower bounds, which are obtained from the current LHC limits of the dilepton production, $\rho$-parameter, precision measurement of the $Z$
decay width, and the LEPII constraint, to find the allowed parameter space in the \( t_N - \Lambda \) and \( \epsilon - \Lambda \) planes. The top panel of this figure indicates that with \( \epsilon = 0.1 \) the current LHC limits of the dilepton production impose the most stringent bound for the sufficiently small/intermediate values of \( t_N \). On the contrary, for the sufficiently large values of \( t_N \), the LEPII data is the most strong constraint. Whereas, the bottom-left and -right panels suggest that with \( t_N = 0.2 \) the current LHC limits of the dilepton production impose the most stringent bound for the whole region of \( \epsilon \) under consideration.

\[ \text{Figure 3.} \text{ The allowed parameter space is determined by the green regions. The black, blue, red, and purple curves correspond to the lower bounds obtained from the current LHC limits of the dilepton production, precision measurement of the Z decay width, \( \rho \)-parameter, and the LEPII constraint, respectively. The regions which are below each of these curves are excluded. The top panel corresponds to } \epsilon = 0.1, \text{ while the bottom-left and -right panels correspond to } t_N = 0.2. \]

9 Dark matter abundance

Comparing the predicted neutrino mass in (3.3) with the data, we obtain

\[ \Lambda \sim \left[ (h^\nu)^2 / f^\nu \right] \times 10^{14} \text{ GeV.} \]

This leads to two scenarios for the seesaw scale with corresponding dark matter production mechanisms, as presented in [46].
Large seesaw scale: $\Lambda \sim 10^{14}$ GeV, given that $(h^\nu)^2/f^\nu \sim 1$. First, this scenario can explain the cosmic inflation driven by the $U(1)_N$ dynamics \cite{67–70}. Further, it can supply the baryon asymmetry via leptogenesis due to the CP-violating decay of $\nu_R$ to normal matter $\nu_R \rightarrow e\phi^+$ \cite{71}. The new observation is that $\nu_R$ also decays to dark matter $n\eta^*$ via the CP-violating coupling $y_i\bar{n}_L\eta\nu_R$. Taking $y_{2,3} = e^{-i\theta}y_1$, with $y_1 \sim 1$ and $\theta \sim \pi/4$, this scenario yields $m_{DM} \sim \frac{10^{-4}m_p}{y_1^2\sin(2\theta)} \sim 0.1$ MeV. Since $\eta$ should have a sizable mass from (4.10) to be at the weak scale, the dark matter mass is associate to the dark fermion $n$.

TeV seesaw scale: $\Lambda \sim 1–10$ TeV, if $(h^\nu)^2/f^\nu$ is suitably small. In this case $Z'$ can pick up a mass at TeV, as given above. The dark matter is produced via a freeze-in mechanism \cite{72} with decay $\nu_{1R} \rightarrow n\eta^*$, where $\nu_{1R}$ and $\eta$ are in thermal equilibrium with standard model plasma, maintained by the interaction with $Z'$, i.e. $SM + SM \leftrightarrow Z' \leftrightarrow \nu_{1R}\nu_{1R}(\eta\eta^*)$. The relic density is \cite{46}

$$\Omega_{DM}h^2 \sim 0.1\left(\frac{y_1}{10^{-8}}\right)^2 \left(\frac{300 \text{ GeV}}{m_{\nu_{1R}}}\right) \left(\frac{m_{DM}}{0.1 \text{ MeV}}\right),$$

yielding a dark fermion ($n$) mass about 0.1 MeV, given that $y_1 \sim 10^{-8}$ and the smallest $\nu_R$ mass $\sim 300$ GeV.

10 Conclusion

We have proved that a dynamical dark charge may arise to be a variant of the usual electric charge. The dark dynamics interprets the right-handed neutrinos as fundamental fields, charged under dark charge, and possessing large Majorana masses from the dark charge breaking. This yields suitable observed neutrino masses in terms of a canonical seesaw and the resultant dark parity, which implies a dark matter candidate to be lighter than an electron.

The large scale scenario for dark charge breaking generates appropriate asymmetric dark fermion with a mass around 0.1 MeV, similar to the lepton asymmetry, which all arise from a standard leptogenesis. By contrast, the TeV scale scenario for dark charge breaking is appropriate to the production of freeze-in dark fermions, which possess a mass around 0.1 MeV. Such two dark matter generation mechanisms are manifestly governed by the dark dynamics or $U(1)_N$.

For the latter case with accessible new physics regime, we have examined the new physics effects through the $\rho$-parameter, the $Z$ width, the LEPII and LHC dilepton searches, indicating that the new physics scale for dark charge breaking, i.e. $\Lambda$, and the $Z'$ mass are in several TeVs.

Acknowledgments

This research is funded by Vietnam National Foundation for Science and Technology Development (NAFOSTED) under grant number 103.01-2019.353.
A Current algebra approach

Consider the $SU(2)_L$ symmetry of weak isospin $T_i \ (i = 1, 2, 3)$ regarding the $V$–$A$ theory. The fermions transform as isodoublets $l_L = (\nu_L \ e_L)^T$ and $q_L = (u_L \ d_L)^T$, plus corresponding right-handed fermion singlets, where the generation index is suppressed. The vector-like fermion $n$ is not counted, without loss of generality. Further, assume that $Q(\nu, e, u, d) = 0, -1, 2/3, -1/3$ for electric charge and $D(\nu, e, u, d) = \delta, \bar{\delta} - 1, 2/3 - \delta/3, -1/3 - \delta/3$ for dark charge, respectively. Here, the latter can be referred from (2.12).

The covariant derivative relevant to $SU(2)_L$ is

$$D_\mu = \partial_\mu + igT_\alpha A_\mu^\alpha = \partial_\mu + ig[(T_\mu W^\mu_\mu + H.c.) + T_3 A_3^\mu],$$  \hspace{1cm} (A.1)

where $T_\pm \equiv (T_1 \pm iT_2)/\sqrt{2}$ and $W^\pm \equiv (A_1 \mp iA_2)/\sqrt{2}$. Thus, the fermion gauge interaction takes the form,

$$\mathcal{L} \ni \bar{\psi}_i \gamma^\mu D_\mu D_\nu \psi_j + \cdots + \bar{\psi}_i \gamma^\mu (T_\mu W^\mu_\mu + H.c.) - g \bar{\psi}_i \gamma^\mu T_3 F L A_3^\mu, \hspace{1cm} (A.2)$$

which implies weak currents appearing in Lagrangian, $-gJ_\mu^L V_\mu$, such that $J_\mu^L = \bar{\psi}_i \gamma^\mu T_3 F L$ and $J_3^\mu = \bar{\psi}_i \gamma^\mu T_3 F L$. This gives rise to corresponding weak charges,

$$T_+(t) \equiv \int d^3x J^0_+(t) = \frac{1}{\sqrt{2}} \int d^3x (\nu_L^\dagger \nu_L + u_L^\dagger d_L),$$

$$T_3(t) \equiv \int d^3x J^0_3(t) = \frac{1}{2} \int d^3x (\nu_L^\dagger \nu_L - e_L^\dagger e_L + u_L^\dagger u_L - d_L^\dagger d_L), \hspace{1cm} (A.3)$$

and $T_-(t) = [T_+(t)]^\dagger$. Using canonical anticommutation relation, $\{f(\vec{x}, t), f^\dagger(\vec{y}, t)\} = \delta^{(3)}(\vec{x} - \vec{y})$, the weak charges obey $SU(2)_L$ algebra,

$$[T_+(t), T_-(t)] = T_3(t), \hspace{0.5cm} [T_3(t), T_\pm(t)] = \pm T_\pm(t), \hspace{1cm} (A.4)$$

as expected.

The $Q(t)$ and $D(t)$ charges are

$$Q(t) = \int d^3xF^\dagger QF = \int d^3x \left[ -e_L^\dagger e_L + \frac{2}{3}u_L^\dagger u_L - \frac{1}{3}d_L^\dagger d_L + (RR) \right], \hspace{1cm} (A.5)$$

$$D(t) = \int d^3xF^\dagger DF = \int d^3x \left[ \delta\nu_L^\dagger \nu_L + (\delta - 1)e_L^\dagger e_L + \frac{2 - \delta}{3}u_L^\dagger u_L - \frac{1 + \delta}{3}d_L^\dagger d_L + (RR) \right], \hspace{1cm} (A.6)$$

which are not proportional to $T_3(t)$, since they have the right currents. $Q, D$ and weak isospin do not form a closed algebra. Further, we derive $[Q(t), T_\pm(t)] = \pm T_\pm(t)$ and $[D(t), T_\pm(t)] = \pm T_\pm(t)$, implying that $Q, D$ do not commute with weak isospin.
We obtain
\[ Q(t) - T_3(t) = \int d^3x \left[ -\frac{1}{2} l^\dagger_L l_L + \frac{1}{6} q^\dagger_L q_L - e^\dagger_R e_R + \frac{2}{3} u^\dagger_R u_R - \frac{1}{3} d^\dagger_R d_R \right] \]
\[ \equiv \int d^3x F^\dagger YF, \] (A.7)
\[ D(t) - T_3(t) = \int d^3x \left[ \left( \delta - \frac{1}{2} \right) l^\dagger_L l_L + \left( \frac{1}{6} - \frac{\delta}{3} \right) q^\dagger_L q_L \right. \]
\[ + \left( \delta - 1 \right) e^\dagger_R e_R + \frac{2 - \delta}{3} u^\dagger_R u_R - \frac{1 + \delta}{3} d^\dagger_R d_R \right] \]
\[ \equiv \int d^3x F^\dagger NF, \] (A.8)
which yield two new Abelian charges, \( Y \) and \( N \), with values for multiplets coinciding those in the body text, respectively. It is easily to check that \( Y(t) \) and \( N(t) \) commute with weak isospin and linearly independent.

We conclude that the manifest gauge symmetry must be
\[ SU(2)_L \otimes U(1)_Y \otimes U(1)_N, \] (A.9)
where \( Y \) and \( N \) define the electric charge and dark charge,

\[ Q - T_3 = Y, \quad D - T_3 = N, \] (A.10)
respectively. It is noteworthy that the weak isospin theory contains in it two conserved, noncommutative charges, \( Q \) and \( D \), and the requirement of algebraic closure yields the \( SU(2)_L \otimes U(1)_Y \otimes U(1)_N \) gauge model, describing the electroweak and dark interactions. Interestingly, the weak and dark interactions are unified in the same manner the electroweak theory does so for weak and electromagnetic interactions.

### B Anomaly checking

For convenience in reading, let us indicate the \( U(1)_{Y,N} \) quantum numbers in Table 5.

| Multiplet | \( l_L \) | \( q_L \) | \( \nu_R \) | \( e_R \) | \( u_R \) | \( d_R \) | \( n \) |
|-----------|----------|----------|----------|--------|--------|--------|------|
| \( Y \)   | \(-\frac{1}{2}\) | \( \frac{1}{6} \) | 0        | \(-1\) | \( \frac{2}{3} \) | \(-\frac{1}{3}\) | 0    |
| \( N \)   | \( \delta - \frac{1}{2} \) | \( \frac{1}{3} - \frac{\delta}{3} \) | \( \delta \) | \( \delta - 1 \) | \( \frac{2 - \delta}{3} \) | \( -\frac{1 + \delta}{3} \) | 2\( r \) |

**Table 5.** \( Y,N \) quantum numbers of fermion multiplets in the general case.

Indeed, all the anomalies are cancelled in each generation, independent of \( \delta \). Let’s see.

\[
[SU(3)_C]^2U(1)_Y \sim \sum_{\text{quarks}} (Y_{f_L} - Y_{f_R}) = 3(2Y_q - Y_u - Y_d)
\]
\[ [SU(3)_C]^2 U(1)_N \sim \sum_{\text{quarks}} (N_{fL} - N_{fR}) = 3(2N_q - N_u - N_d) = 3[2(1/6) - (2/3) - (-1/3)] = 0. \quad (B.1) \]

\[ [SU(2)_L]^2 U(1)_Y \sim \sum_{\text{doublets}} Y_{fL} = Y_l + 3Y_q = (-1/2) + 3(1/6) = 0. \quad (B.2) \]

\[ [SU(2)_L]^2 U(1)_N \sim \sum_{\text{doublets}} N_{fL} = N_l + 3N_q = (-1/2 + \delta) + 3(1/6 - \delta/3) = 0. \quad (B.3) \]

\[ [\text{Gravity}]^2 U(1)_Y \sim \sum_{\text{fermions}} (Y_{fL} - Y_{fR}) = 2Y_l + 2 \times 3Y_q + Y_{nL} - Y_{nR} - Y_\nu - Y_\tau - 3Y_u - 3Y_d = 0 \quad (B.4) \]

\[ [\text{Gravity}]^2 U(1)_N \sim \sum_{\text{fermions}} (N_{fL} - N_{fR}) = 2N_l + 2 \times 3N_q + N_{nL} - N_{nR} - 3N_u - 3N_d - N_{nL} = 2(-1/2 + \delta) + 6(1/6 - \delta/3) + 2r - \delta - (\delta - 1) - 3(2/3 - \delta/3) - 3(-1/3 - \delta/3) = 0. \quad (B.5) \]

\[ [U(1)_Y]^2 U(1)_N = \sum_{\text{fermions}} (Y_{fL}^2 N_{fL} - Y_{fR}^2 N_{fR}) = 2Y_l^2 N_l + 2 \times 3Y_q^2 N_q + Y_{nL}^2 N_{nL} - Y_{\nu}^2 N_\nu - Y_{\tau}^2 N_\tau - 3Y_u^2 N_u - 3Y_d^2 N_d - Y_{nR}^2 N_{nR} = 2(-1/2)^2(-1/2 + \delta) + 6(1/6)^2(1/6 - \delta/3) + 0^2 \times 2r - 0^2 \times \delta - (-1)^2(\delta - 1) - 3(2/3)^2(2/3 - \delta/3) - 3(-1/3)^2(-1/3 - \delta/3) \]

\[ U(1)_Y [U(1)_N]^2 = \sum_{\text{fermions}} (Y_{fL} N_{fL} - Y_{fR} N_{fR}) = 2Y_l N_l + 2 \times 3Y_q N_q + Y_{nL} N_{nL} - Y_{\nu} N_\nu - Y_{\tau} N_\tau - 3Y_u N_u - 3Y_d N_d - Y_{nR} N_{nR} = 2(-1/2)(-1/2 + \delta)^2 + 6(1/6)(1/6 - \delta/3)^2 + 0 \times (2r)^2 - 0 \times \delta^2 - (-1)(\delta - 1)^2 - 3(2/3)(2/3 - \delta/3)^2 - 3(-1/3)(-1/3 - \delta/3)^2 \]

\[ U(1)_Y^3 = \sum_{\text{fermions}} (Y_{fL}^3 - Y_{fR}^3) \]

\[ U(1)_Y^3 = \sum_{\text{fermions}} (Y_{fL}^3 - Y_{fR}^3) \]
\[ \begin{aligned}
&= 2Y_l^3 + 2 \times 3Y_q^3 + Y_{nL}^3 - Y_\nu^3 - Y_e^3 - 3Y_u^3 - 3Y_d^3 - Y_{nR}^3 \\
&= 2\left(-1/2\right)^3 + 6\left(1/6\right)^3 + 0^3 - 0^3 - (-1)^3 \\
&- 3\left(2/3\right)^3 - 3\left(-1/3\right)^3 - 0^3 = 0. \\
\end{aligned} \] (B.9)

\[ 
\left[ U(1)N \right]^3 = \sum_{\text{fermions}} \left( N_{fL}^3 - N_{fR}^3 \right) \\
= 2N_l^3 + 2 \times 3N_q^3 + N_{nL}^3 - N_{\nu}^3 - N_e^3 - 3N_u^3 - 3N_d^3 - N_{nR}^3 \\
= 2\left(-1/2 + \delta\right)^3 + 6\left(1/6 - \delta/3\right)^3 + (2r)^3 - \delta^3 - \left(\delta - 1\right)^3 \\
- 3\left(2/3 - \delta/3\right)^3 - 3\left(-1/3 - \delta/3\right)^3 - (2r)^3 = 0. \] (B.10)

The dark fermion \( n \) is vector-like, not contributing to the anomalies, which can be skipped from the beginning.

Additionally, in the case of the model that contains several dark charges, \( SU(2)_L \otimes U(1)_Y \otimes U(1)_{N_1} \otimes U(1)_{N_2} \otimes \cdots \otimes U(1)_{N_p} \), the anomalies of above types are still canceled. For the remainders, it is sufficient to consider,

\[ 
\left[ U(1)N \right]^2U(1)_{N'} = \sum_{\text{fermions}} \left( N_{fL}'^2N_{fL}^3 - N_{fR}^3N_{fR}' \right) = 2N_l^2N_l' + 2 \times 3N_q^2N_q' - N_{\nu}^2N_{\nu}' \\
-N_e^2N_e' - 3N_u^2N_u' - 3N_d^2N_d' \\
= 2\left(\delta - 1/2\right)^2\left(\delta' - 1/2\right) + 6\left(1/6 - \delta/3\right)^2\left(1/6 - \delta'/3\right) - \delta^2 \times \delta' \\
- \left(\delta - 1\right)^2\left(\delta' - 1\right) - 3\left(2/3 - \delta/3\right)^2\left(2/3 - \delta'/3\right) \\
- 3\left(-1/3 - \delta/3\right)^2\left(-1/3 - \delta'/3\right) = 0. \] (B.11)

Hence the model of multi dark charges is viable, attracting attention.

References

[1] T. Kajita, Nobel Lecture: Discovery of atmospheric neutrino oscillations, Rev. Mod. Phys. 88 (2016) 030501.

[2] A. B. McDonald, Nobel Lecture: The Sudbury Neutrino Observatory: Observation of flavor change for solar neutrinos, Rev. Mod. Phys. 88 (2016) 030502.

[3] PLANCK collaboration, Planck 2015 results. XIII. Cosmological parameters, Astron. Astrophys. 594 (2016) A13 [1502.01589].

[4] G. Jungman, M. Kamionkowski and K. Griest, Supersymmetric dark matter, Phys.Rept. 267 (1996) 195 [hep-ph/9506380].

[5] G. Bertone, D. Hooper and J. Silk, Particle dark matter: Evidence, candidates and constraints, Phys. Rept. 405 (2005) 279 [hep-ph/0404175].

[6] P. Minkowski, \( \mu \rightarrow e\gamma \) at a Rate of One Out of \( 10^9 \) Muon Decays?, Phys. Lett. 67B (1977) 421.
[7] M. Gell-Mann, P. Ramond and R. Slansky, *Complex Spinors and Unified Theories*, Conf. Proc. C790927 (1979) 315 [1306.4669].

[8] T. Yanagida, *Horizontal gauge symmetry and masses of neutrinos*, Conf. Proc. C7902131 (1979) 95.

[9] S. L. Glashow, *The Future of Elementary Particle Physics*, NATO Sci. Ser. B 61 (1980) 687.

[10] R. N. Mohapatra and G. Senjanovic, *Neutrino Mass and Spontaneous Parity Nonconservation*, Phys. Rev. Lett. 44 (1980) 912.

[11] R. N. Mohapatra and G. Senjanovic, *Neutrino Masses and Mixings in Gauge Models with Spontaneous Parity Violation*, Phys. Rev. D23 (1981) 165.

[12] G. Lazarides, Q. Shafi and C. Wetterich, *Proton Lifetime and Fermion Masses in an SO(10) Model*, Nucl. Phys. B181 (1981) 287.

[13] J. Schechter and J. W. F. Valle, *Neutrino Masses in SU(2) x U(1) Theories*, Phys. Rev. D22 (1980) 2227.

[14] J. Schechter and J. W. F. Valle, *Neutrino Decay and Spontaneous Violation of Lepton Number*, Phys. Rev. D25 (1982) 774.

[15] A. Zee, *A Theory of Lepton Number Violation, Neutrino Majorana Mass, and Oscillation*, Phys. Lett. B 93 (1980) 389.

[16] A. Zee, *Quantum Numbers of Majorana Neutrino Masses*, Nucl. Phys. B 264 (1986) 99.

[17] K. Babu, *Model of 'Calculable' Majorana Neutrino Masses*, Phys. Lett. B 203 (1988) 132.

[18] L. M. Krauss, S. Nasri and M. Trodden, *A Model for neutrino masses and dark matter*, Phys. Rev. D 67 (2003) 085002 [hep-ph/0210389].

[19] E. Ma, *Verifiable radiative seesaw mechanism of neutrino mass and dark matter*, Phys. Rev. D 73 (2006) 077301 [hep-ph/0601225].

[20] N. Okada and O. Seto, *Higgs portal dark matter in the minimal gauged U(1)B−L model*, Phys. Rev. D82 (2010) 023507 [1002.2525].

[21] J. C. Montero and B. L. Sanchez-Vega, *Neutrino masses and the scalar sector of a B-L extension of the standard model*, Phys. Rev. D84 (2011) 053006 [1102.0321].

[22] N. Okada and Y. Orikasa, *Dark matter in the classically conformal B-L model*, Phys. Rev. D85 (2012) 115006 [1202.1405].

[23] T. Basak and T. Mondal, *Constraining Minimal U(1)B−L model from Dark Matter Observations*, Phys. Rev. D89 (2014) 063527 [1308.0023].

[24] B. L. Sánchez-Vega, J. C. Montero and E. R. Schmitz, *Complex Scalar DM in a B-L Model*, Phys. Rev. D90 (2014) 055022 [1404.5973].
[25] N. Okada and S. Okada, *Z'-portal right-handed neutrino dark matter in the minimal U(1)X extended Standard Model*, Phys. Rev. **D95** (2017) 035025 [1611.02672].

[26] W. Rodejohann and C. E. Yaguna, *Scalar dart matter in the B-L model*, JCAP **1512** (2015) 032 [1509.04036].

[27] N. Okada and O. Seto, *Inelastic extra U(1) charged scalar dark matter*, Phys. Rev. **D101** (2020) 023522 [1908.09277].

[28] N. Okada, D. Raut and Q. Shafi, *SMART U(1)X − Standard Model with Axion, Right handed neutrinos, Two Higgs doublets and U(1)X gauge symmetry*, 2002.07110.

[29] A. Dasgupta, S. K. Kang and O. Popov, *Radiative Dirac neutrino mass, neutrinoless quadruple beta decay, and dark matter in B-L extension of the standard model*, Phys. Rev. D **100** (2019) 075030 [1903.12558].

[30] A. Biswas, D. Borah and D. Nanda, *Type III seesaw for neutrino masses in U(1)B-L model with multi-component dark matter*, JHEP **12** (2019) 109 [1908.04308].

[31] J. Gehrlein and M. Pierre, *A testable hidden-sector model for Dark Matter and neutrino masses*, JHEP **02** (2020) 068 [1912.06661].

[32] C. Han, M. LÚpez-IbGHUEZ, B. Peng and J. M. Yang, *Dirac dark matter in U(1)B−L with Stueckelberg mechanism*, 2001.04078.

[33] D. Choudhury, K. Deka, T. Mandal and S. Sadhukhan, *Neutrino and Z' phenomenology in an anomaly-free U(1) extension: role of higher-dimensional operators*, 2002.02349.

[34] S. Mahapatra, N. Narendra and N. Sahu, *Verifiable type-II seesaw and dark matter in a gauged U(1)B−L model*, 2002.07000.

[35] J. Leite, A. Morales, J. W. Valle and C. A. Vaquera-Araujo, *Scotogenic dark matter and Dirac neutrinos from unbroken gauged B − L symmetry*, 2003.02950.

[36] N. Okada, S. Okada and Q. Shafi, *Light Z' and Dark Matter from U(1)X Gauge Symmetry*, 2003.02667.

[37] H. Motz, H. Okada, Y. Asaoka and K. Kohri, *Cosmic-Ray Signatures of Dark Matter from a Flavor Dependent Gauge Symmetry Model with Neutrino Mass Mechanism*, 2004.04304.

[38] A. Davidson, *B − L as the fourth color within an SU(2)L × U(1)R × U(1) model*, Phys. Rev. **D20** (1979) 776.

[39] R. N. Mohapatra and R. E. Marshak, *Local B-L Symmetry of Electroweak Interactions, Majorana Neutrinos and Neutron Oscillations*, Phys. Rev. Lett. **44** (1980) 1316.

[40] R. E. Marshak and R. N. Mohapatra, *Quark - Lepton Symmetry and B-L as the U(1) Generator of the Electroweak Symmetry Group*, Phys. Lett. **91B** (1980) 222.
[41] R. Foot, *New Physics From Electric Charge Quantization?, Mod. Phys. Lett. A* 6 (1991) 527.

[42] R. Foot, X. He, H. Lew and R. Volkas, *Model for a light Z-prime boson*, Phys. Rev. D 50 (1994) 4571 [hep-ph/9401250].

[43] X.-G. He, G. C. Joshi, H. Lew and R. Volkas, *Simplest Z-prime model*, Phys. Rev. D 44 (1991) 2118.

[44] B. Holdom, *Two U(1)’s and Epsilon Charge Shifts*, Phys. Lett. B 166 (1986) 196.

[45] T. Appelquist, B. A. Dobrescu and A. R. Hopper, *Nonexotic Neutral Gauge Bosons*, Phys. Rev. D68 (2003) 035012 [hep-ph/0212073].

[46] P. Van Dong, *Flipping principle for neutrino mass and dark matter*, 2003.13276.

[47] K. Babu and R. Mohapatra, *Is There a Connection Between Quantization of Electric Charge and a Majorana Neutrino?*, Phys. Rev. Lett. 63 (1989) 938.

[48] R. Foot, G. C. Joshi, H. Lew and R. Volkas, *Charge quantization in the standard model and some of its extensions*, Mod. Phys. Lett. A 5 (1990) 2721.

[49] K. Babu and R. N. Mohapatra, *Quantization of Electric Charge From Anomaly Constraints and a Majorana Neutrino*, Phys. Rev. D 41 (1990) 271.

[50] C. A. de Sousa Pires and O. Ravinez, *Charge quantization in a chiral bilepton gauge model*, Phys. Rev. D 58 (1998) 035008 [hep-ph/9803409].

[51] S. Oda, N. Okada and D.-s. Takahashi, *Classically conformal U(1)? extended standard model and Higgs vacuum stability*, Phys. Rev. D92 (2015) 015026 [1504.06291].

[52] P. V. Dong, H. T. Hung and T. D. Tham, *3-3-1-1 model for dark matter*, Phys. Rev. D87 (2013) 115003 [1305.0369].

[53] P. V. Dong, D. T. Huong, F. S. Queiroz and N. T. Thuy, *Phenomenology of the 3-3-1-1 model*, Phys. Rev. D90 (2014) 075021 [1405.2591].

[54] A. Alves, G. Arcadi, P. V. Dong, L. Duarte, F. S. Queiroz and J. W. F. Valle, *Matter-parity as a residual gauge symmetry: Probing a theory of cosmological dark matter*, Phys. Lett. B772 (2017) 825 [1612.04383].

[55] P. V. Dong, *Unifying the electroweak and B-L interactions*, Phys. Rev. D92 (2015) 055026 [1505.06469].

[56] P. V. Dong and D. T. Huong, *Left-right model for dark matter*, Commun. Phys. 28 (2018) 21 [1610.02642].

[57] P. V. Dong, D. T. Huong, D. V. Loi, N. T. Nhuan and N. T. K. Ngan, *Phenomenology of the SU(3)_C \otimes SU(2)_L \otimes SU(3)_R \otimes U(1)_X gauge model*, Phys. Rev. D95 (2017) 075034 [1609.03444].

[58] P. V. Dong, D. T. Huong, F. S. Queiroz, J. W. F. Valle and C. A. Vaquera-Araujo, *The Dark Side of Flipped Trinification*, JHEP 04 (2018) 143 [1710.06951].
[59] D. T. Huong, P. V. Dong, N. T. Duy, N. T. Nhuan and L. D. Thien, *Investigation of Dark Matter in the 3-2-3-1 Model*, Phys. Rev. D98 (2018) 055033 [1802.10402].

[60] D. T. Huong, D. N. Dinh, L. D. Thien and P. Van Dong, *Dark matter and flavor changing in the flipped 3-3-1 model*, JHEP 08 (2019) 051 [1906.05240].

[61] P. V. Dong and D. T. Si, *Kinetic mixing effect in the 3-3-1-1 model*, Phys. Rev. D93 (2016) 115003 [1510.06815].

[62] P. V. Dong and H. N. Long, *U(1)(Q) invariance and SU(3)(C) x SU(3)(L) x U(1)(X) models with beta arbitrary*, Eur. Phys. J. C 42 (2005) 325 [hep-ph/0506022].

[63] D. Van Loi, P. Van Dong and L. X. Thuy, *Kinetic mixing effect in noncommutative B – L gauge theory*, 1906.10577.

[64] PARTICLE DATA GROUP collaboration, *Review of Particle Physics*, Phys. Rev. D98 (2018) 030001.

[65] ALEPH, DELPHI, L3, OPAL, LEP ELECTROWEAK collaboration, *Electroweak Measurements in Electron-Positron Collisions at W-Boson-Pair Energies at LEP*, Phys. Rept. 532 (2013) 119 [1302.3415].

[66] ATLAS collaboration, *Search for new high-mass phenomena in the dilepton final state using 36 fb⁻¹ of proton-proton collision data at √s = 13 TeV with the ATLAS detector*, JHEP 10 (2017) 182 [1707.02424].

[67] D. T. Huong, P. V. Dong, C. S. Kim and N. T. Thuy, *Inflation and leptogenesis in the 3-3-1-1 model*, Phys. Rev. D91 (2015) 055023 [1501.00543].

[68] D. T. Huong and P. V. Dong, *Neutrino masses and superheavy dark matter in the 3-3-1-1 model*, Eur. Phys. J. C77 (2017) 204 [1605.01216].

[69] P. Van Dong, D. Huong, D. A. Camargo, F. S. Queiroz and J. W. Valle, *Asymmetric Dark Matter, Inflation and Leptogenesis from B – L Symmetry Breaking*, Phys. Rev. D 99 (2019) 055040 [1805.08251].

[70] P. Van Dong and D. Van Loi, *Asymmetric matter from B – L symmetry breaking*, 2001.03862.

[71] M. Fukugita and T. Yanagida, *Baryogenesis Without Grand Unification*, Phys. Lett. B 174 (1986) 45.

[72] L. J. Hall, K. Jedamzik, J. March-Russell and S. M. West, *Freeze-In Production of FIMP Dark Matter*, JHEP 03 (2010) 080 [0911.1120].