Mirror, mirror: Landau-Zener-Stückelberg-Majorana interferometry of a superconducting qubit in front of a mirror

P. Y. Wen, O. V. Ivakhnenko, M. A. Nakonechnyi, B. Suri, J.-J. Lin, W.-J. Lin, J. C. Chen, S. N. Shevechenko, Franco Nori, and I.-C. Hoi

1 Department of Physics, National Tsing Hua University, Hsinchu 30013, Taiwan
2 B. Verkin Institute for Low Temperature Physics and Engineering, Kharkov 61103, Ukraine
3 Theoretical Quantum Physics Laboratory, Cluster for Pioneering Research, RIKEN, Wako-shi, Saitama 351-0198, Japan
4 Department of Instrumentation and Applied Physics, Indian Institute of Science, Bengaluru 560012, India
5 Department of Physics, National Taiwan University, Taipei 10617, Taiwan
6 V. N. Karazin Kharkiv National University, Kharkov 61022, Ukraine
7 Department of Physics, The University of Michigan, Ann Arbor, MI 48109-1040, USA
8 Center for Quantum Technology, National Tsing Hua University, Hsinchu 30013, Taiwan

(Dated: March 3, 2020)

We investigate the Landau-Zener-Stückelberg-Majorana interferometry of a superconducting qubit in a semi-infinite transmission line terminated by a mirror. The transmon-type qubit is at the node of the resonant electromagnetic (EM) field, “hiding” from the EM field. “Mirror, mirror” briefly describes this system, because the qubit acts as another mirror. We modulate the resonant frequency of the qubit by applying a sinusoidal flux pump. We probe the spectroscopy by measuring the reflection coefficient of a weak probe in the system. Remarkable interference patterns emerge in the spectrum, which can be interpreted as multi-photon resonances in the dressed qubit. Our calculations agree well with the experiments.

PACS numbers: 42.50.Gy, 85.25.Cp

I. INTRODUCTION

In recent years, superconducting artificial atoms coupled to open transmission-line waveguide have been a fast growing field, called waveguide Quantum Electrodynamics (w-QED), which provides a unique platform to investigate atom-light interaction. The uniqueness of w-QED, as compared to conventional cavity QED, is that atoms are coupled to continuum modes of the electromagnetic (EM) field in the waveguide. Exciting problems in w-QED include: resonance fluorescence of an artificial atom, photon-mediated interactions between distant artificial atoms, time dynamics in atom-like mirrors, photon routing, generation of non-classical microwaves, cross-Kerr effect, amplification without population inversion, collective Lamb shift between two distant artificial atoms, ultra strong coupling, quantum rifling, probabilistic motional averaging.

When a two-level system is driven back and forth around its resonance frequency, it will produce Landau-Zener-Stückelberg-Majorana (LZSM) interference. LZSM interferometry has been studied in atomic systems, quantum dots, charge and spin qubits, and superconducting qubit in cavity, among others. However, the effect of LZSM has not been explored with a single artificial atom in front of a mirror, where the artificial atom is coupled to a continuum of modes of the EM field in the transmission-line waveguide, and the atom interferes with its mirror image, as in Refs. [4,25].

LZSM interferometry is important for both system description and control. However, for this to be realized, one needs to have the avoided energy-level crossing in the spectrum as a function of a controlling parameter. One example of systems without this are transmon-type superconducting qubits, where the energy levels are almost independent of the bias voltage. The way to cure this was studied in Ref. [27], which studied the qubit by chirping the microwave frequency, which results in dressed states with avoided-level crossing. In this work, we study a transmon qubit driven by two fields (see also Ref. [28]). One of these dresses the qubit and creates the spectrum with the avoided-level crossing, while the other one makes the system periodically pass around the avoided-level point. This allows to study LZSM interferometry in a qubit placed in front of a mirror. “Mirror, mirror” briefly describes this system, because the qubit acts as another mirror.

II. SUPERCONDUCTING QUBIT IN FRONT OF A MIRROR

In this work, we investigate the LZSM interferometry of a superconducting qubit in a semi-infinite transmission line, at a distance terminated by a mirror. In particular, the qubit is located at the node of the resonant EM field, where it is hiding from the EM field. We then modulate the resonant frequency of the qubit by applying a sinusoidal wave through an on-chip flux pump. In addition, the coupling between the EM field and qubit is also being modulated. We then probe the spectroscopy of the system by applying a weak probe field along the
transmission line and measure the reflection coefficient. Interesting interference patterns emerge in the spectrum, which can be explained by multi-photon resonances in the dressed qubit. New features appear, as compared to conventional LZSM interference; for example, now the zero-order Rabi sideband vanishes (see also Ref. [29]).

Figure 1 shows (a) a sketch of the device, (b) the image of the device, and (c) the measurement setup. A transmon qubit is embedded in a semi-1D transmission line with characteristic impedance \( Z_0 \approx 50 \, \Omega \), with the ground (excited) state \( |0\rangle \) (|1\rangle). The \( |0\rangle \leftrightarrow |1\rangle \) transition energy is \( \hbar \omega_{10}(\Phi) \approx \sqrt{8E_j(\Phi)E_C - E_C} \), which is determined by the single-electron charging energy \( E_C = e^2/2C_S \), where \( C_S \) is the total capacitance of the qubit, and the flux-dependent Josephson energy \( E_j(\Phi) = E_{j,max} |\cos(\pi \Phi/\Phi_0)|; \Phi_0 = \hbar/2e \) is the magnetic flux quantum. The \( E_C \) determines the anharmonicity of the qubit.

In Fig. 1(c), a probe field of frequency \( \omega_p \) is fed into the transmission line. The pump field of frequency \( \omega_{pump} \) is applied to the on-chip flux line, sinusoidally modulating the transition frequency of the qubit. The key parameters are summarized in Table I.

### Table I: Table of controllable parameters. Here \( \omega_{node} = 4.75 \, \text{GHz} \, 2\pi \).

| Value      | Description                  | Range                  |
|------------|------------------------------|------------------------|
| \( \omega_{10} \) | qubit frequency, \( \omega_{10} = \omega_{10}(V) \) | \( \approx \omega_{node} \) |
| \( \delta \) | pump amplitude; \( \delta = \delta(P_{pump}) \) | \( \sim 0.1 \, \text{GHz} \, 2\pi \) |
| \( \omega_{pump} \) | pump frequency               | \( < 0.1 \, \text{GHz} \, 2\pi \) |
| \( \omega_p \) | probe frequency              | \( \approx \omega_{node} \) |

### III. THEORETICAL DESCRIPTION

Let us now consider the qubit Hamiltonian, with details presented in Appendix A. Thanks to the mirror, the transmission-line voltage at the point of coupling the qubit, \( x = L \), is proportional to \( \cos(\omega_p L/v) \). When this factor is zero, this gives the node frequency \( \omega_{node} \), with \( \cos(\omega_{node} L/v) = 0 \). For small offset, \( \Delta \omega = \omega_p - \omega_{node} \ll \omega_p \), instead of \( \cos(\omega_{p} L/v) \), we have \( \Delta \omega/\omega_{node} \), which shows that at \( \Delta \omega = 0 \) the qubit is “hidden” or “decou-
of the Bloch equations, for this value we have (see e.g. Refs. [19,33]):

\[ P_1 = \frac{1}{2} \sum_{k=-\infty}^{\infty} G_k^2 + [(\omega_p - \omega_{10}) - k\omega_{\text{pump}}]^2 \frac{\Gamma_1}{\Gamma_2} + \Gamma_1 \Gamma_2, \]

where the renormalized driving amplitude \( G_k = GJ_k(\delta/\omega_{\text{pump}}) \) follows the oscillating Bessel function \( J_k \) of the first kind; \( \Gamma_1 \) and \( \Gamma_2 = \Gamma_1/2 + \Gamma_\phi \) are the relaxation and decoherence rates with the pure dephasing rate \( \Gamma_\phi \) being much smaller than \( \Gamma_1 \). With this formula, Eq. (5), we plot theoretical graphs in Figs. 2, 3, and 4. For this we use \( G_0 = 0.1 \text{ GHz}\cdot2\pi, \Gamma_1 < 40 \text{ MHz}\cdot2\pi, \) \( \Gamma_2 \approx \Gamma_1/2 \) (i.e. \( \Gamma_\phi \ll \Gamma_1 \)).

IV. MEASUREMENTS

We first perform single-tone spectroscopy of the qubit-mirror system. In Fig. 2, the resonant frequency of the qubit is tuned by voltage. As the voltage increases, the linewidth of the qubit decreases from a finite linewidth to zero, and then increases back to a finite linewidth. At the frequency where the linewidth vanishes, around \( \omega_{10} = \omega_{\text{node}} \approx 4.75 \text{ GHz}\cdot2\pi \), the qubit is located at the node of the EM field, as indicated by the vertical dashed line, where it is hidden from the EM field.

By using two-tone spectroscopy \([10]\), we know that \( \omega_{10}/h \approx 324 \text{ MHz} \). For \( \omega_{10}/2\pi = 4.75 \text{ GHz} \), the corresponding Josephson energy is \( E_J/h \approx 9.9 \text{ GHz} \).

After the basic characterization of the system, we want to study the spectrum as a function of the following parameters: qubit frequency \( \omega_{10} \), pump amplitude \( \delta \), pump frequency \( \omega_{\text{pump}} \), and probe frequency \( \omega_p \). For spectroscopy, we always use a weak probe amplitude in experiments.

To start with, we set the qubit frequency corresponding to the node as a working point, where \( \omega_{10} = \omega_{\text{node}} \). We then apply a sinusoidal flux pump at a fixed power to the qubit, and sweep the pump frequency from 1 MHz to 100 MHz. At the same time, we probe the spectroscopy of the system using a weak field \( \omega_p \) near the qubit frequency. We show the amplitude reflection coefficient \( |r| \) in Fig. 3(a,b) as a function of \( \omega_{\text{pump}} \) and \( \omega_p \) in (a) for \( P_{\text{pump}} = -45 \text{ dBm} \), in (b) for \( P_{\text{pump}} = -38 \text{ dBm} \). We observed LZSM interference fringes. These interference fringes can be interpreted as multi-photon resonances in the dressed qubit. Multi-photon resonances appear at \( \omega_p = \omega_{10} \pm k\omega_{\text{pump}} \), where \( k \) is the order, as indicated in the figures. The zero order, where \( k = 0 \), is missing, which is a key feature here, different from conventional LZSM interference fringes. In Fig. 3(b), we can clearly see the order \( k \) up to \( \pm 4 \). We increase the pump power in Fig. 3 from \(-45 \text{ dBm} \) in (a) to \(-38 \text{ dBm} \) in (b), and the gap between negative \( k \) and positive \( k \) fringes becomes wider. Indeed, the stronger the pump power, the wider they separate (data not shown). The power in Fig. 3 (a)
Figure 3: Sinusoidal modulation of the qubit by flux pumping with the resonance frequency at a fixed pump power $P_{\text{pump}}$. Amplitude reflection coefficient $|r|$ for a weak coherent probe as a function of the probe frequency $\omega_p$ and pump frequency $\omega_{\text{pump}}$; (a,b,c) are experimental data, while (d,e,f) are our theoretical calculations. (a) Qubit biased at $\omega_{\text{node}} = 4.75\,\text{GHz}\cdot2\pi$, the flux pump power is fixed at $P_{\text{pump}} = -45\,\text{dBm}$. (b) Qubit biased at the node around $4.75\,\text{GHz}$ with $P_{\text{pump}} = -38\,\text{dBm}$. Note that the $k$-dependent multi-photon resonances emerge: $\Delta\omega = k\omega_{\text{pump}}$. In (b), we can see Rabi sidebands at $k$ from −4 to 4. In (a) and (b), the $k = 0$ Rabi sideband disappears whereas in (c) the $k = 0$ Rabi sideband appears. For (a) and (b), the positive $k$ and negative $k$ fringes are symmetric, whereas, in (c) the interference fringes are not symmetric along $k = 0$. As the fringes approach the node regime, near $4.75\,\text{GHz}$, they become weaker. In (d-f) we show the respective calculated qubit upper-level occupation probabilities $P_1$.

and (b) differs by 7 dB, meaning that their pump amplitude $\delta$, differ by a factor of 2.2. This is also what happens in the theory calculation plots. In this sense, this separation can be used to calibrate the pump power. In Fig. 3(c), we bias the qubit at around $4.58\,\text{GHz}$, red detuned from the node, and we then see asymmetric interference fringes. At this bias point, when the qubit is pumped in the negative part of the sinusoidal, the qubit is pumped toward the larger linewidth regime, see Fig. 2. However, when the qubit is pumped in the positive part of the sinusoidal, the qubit is pumped towards the zero-linewidth regime; therefore, we can see that the interference fringes vanish near $\omega_{\text{node}}$. In addition, as compared to (a) and (b), the $k = 0$, zero Rabi sideband appears in (c).

Next, we keep the pump frequency $\omega_{\text{pump}}$ constant. We change the power of the pump $P_{\text{pump}}$ from $-70\,\text{dBm}$ to $-30\,\text{dBm}$, and probe the system with a weak field near the resonance frequency of the qubit. In Fig. 4, we show in (a) and (b) $\omega_{\text{pump}}/2\pi = 10\,\text{MHz}$ and $100\,\text{MHz}$ for the qubit frequency at node $\omega_{\text{node}}$, and $\omega_{\text{pump}}/2\pi = 10\,\text{MHz}$ for the qubit frequency red detuned from $\omega_{\text{node}}$ in (c). These plots show the amplitude reflection coefficient $|r|$ as a function of the probe frequency $\omega_p$ and the pump power $P_{\text{pump}}$. In Fig. 4(a,b), when the flux pump is weak (this corresponds to a small change of the qubit resonance frequency) there are no interference fringes in the interference pattern, as indicated in (a) and (b). This can be explained using Fig. 2, where the “node regime” corresponds to $4.7\,\text{GHz}$ to $4.8\,\text{GHz}$, and there is no response for a weak flux pump. When the pump power increases, this corresponds to larger changes of the resonance frequency, we see the Rabi-splitting-like behavior in (a). In (b), Rabi sidebands at $k = -2, -1, +1, +2$, ...
Figure 4: Sinusoidal modulation of the qubit by flux pumping at the resonance frequency at a fixed pump frequency $\omega_{pump}$. The plots show the amplitude of the reflection coefficient $|r|$ for a weak coherent probe versus the probe frequency $\omega_p$ and pump power $P_{pump}$. (a,b,c) correspond to experiments and (d,e,f) to theory. (a) and (b) are at the node around 4.75 GHz with $\omega_{pump}/2\pi = 10$ MHz and 100 MHz, respectively. Note the Rabi sidebands at $\Delta \omega = k \omega_{pump}$, with $k = -2, -1, +1, +2$. The higher the pump power, the more resolved sidebands are visible. Note the onset of the Rabi sidebands for $k = \pm 1$ and $k = \pm 2$, for $-45$ dBm and $-35$ dBm, respectively. (c) red detuned from the node at 4.56 GHz. In (a,c), we observe Rabi-like splittings as the pump power increases. In (a), we see a symmetric splitting, whereas in (c) there are several asymmetric splittings. In (c), as the fringes approach $\omega_{node}$, they become weaker. In (d-f) we show the respective calculated qubit upper-level occupation probabilities $P_1$.

appear. These match the condition $\Delta \omega = k \omega_{pump}$. The higher the pump power, the more resolved the sideband $k$ becomes. In Fig. 4(c), when we bias the qubit frequency away from $\omega_{node}$, the interference fringes become weaker as the probe frequency approaches the node frequency at 4.75 GHz.

V. CONCLUSIONS

In conclusion, we investigate the LZSM interferometry of a superconducting qubit in a semi-infinite transmission line terminated by a mirror. When the qubit frequency is set to the node of the EM field, after flux pumping the qubit frequency, remarkable interference patterns emerge, which can be interpreted as multi-photon resonances in the dressed qubit. We see multi-photon resonances up to 4th order. The zero-order photon resonance disappears. One of the advantages of this atom-mirror arrangement is that we can effectively manipulate the absorption properties of the two-level atom, providing a novel way to manipulate the quantum states.

Acknowledgments

I.-C.H. and J.C.C. would like to thank I. A. Yu and C.-Y. Mou for fruitful discussions. This work was financially supported by the Center for Quantum Technology from the Featured Areas Research Center Program within the framework of the Higher Education Sprout Project by the Ministry of Education (MOE) in Taiwan. I.-C.H. acknowledges financial support from the MOST of Taiwan under project 109-2636-M-007-007. J.C.C. ac-
knowledges financial support from the MOST of Taiwan under project 107-2112-M-007-003-MY3. O.V.I., M.A.N., and S.N.S. acknowledge partial support by the Grant of the President of Ukraine (Grant No. F84/185-2019). F.N. is supported in part by the MURI Center for Dynamic Magneto-Optics via the Air Force Office of Scientific Research (AFOSR) (FA9550-14-1-0040), Army Research Office (ARO) (Grant No. Grant No. W911NF-18-1-0358), Japan Science and Technology Agency (JST) (via the Q-LEAP program, and the CREST Grant No. JPMJCR1676), Japan Society for the Promotion of Science (JSPS) (JSPS-RFBR Grant No. 17-52-50023, and JSPS-FWO Grant No. VS.059.18N), the RIKEN-AIST Challenge Research Fund, the Foundational Questions Institute (FQXi), and the NTT PHI Laboratory.

Appendix A: Hamiltonian

In this Appendix we describe how we obtain the Hamiltonian (1) for a qubit in front of a mirror, schematically shown in Fig. 5.

The transmission line is described by the voltage $V(x,t)$ and current $I(x,t)$:

$$V(x,t) = V(x)e^{i\omega_p t}, \quad I(x,t) = I(x)e^{i\omega_p t}, \quad (A1)$$

with

$$V(x) = V_+ e^{ik x} + V_- e^{-ik x}, \quad (A2)$$

$$I(x) = \frac{V_+}{Z_0} e^{ik x} + \frac{V_-}{Z_0} e^{-ik x}, \quad (A3)$$

where $k = \omega_p/v$. Thanks to the mirror at $x = 0$, we have $I(0) = 0$, $V_+ = V_-$, and $V(x) = 2V_+ \cos(kx)$ for $x \in (0, L)$.

The transmon is described by the number of Cooper pairs on it $\langle n \rangle$, with the number operator given by the Pauli matrix

$$n = \left( \frac{E_J}{32EC} \right)^{1/4} \sigma_x. \quad (A4)$$

If we take here $\hbar \omega_{10} \approx \sqrt{8ECF_J}$, we have

$$n = \sqrt{\frac{\hbar \omega_{10}}{EC}} \sigma_x. \quad (A5)$$

Then, writing down the charges of the respective capacitor plates, we obtain the island voltage

$$V_I = \frac{2e}{C_\Sigma} \langle n \rangle - \frac{C_c}{C_\Sigma} V(L,t), \quad (A6)$$

where $C_\Sigma = C_J + C_B + C_c$. With this we can write the Hamiltonian of the transmon qubit coupled to the transmission line, which can be rewritten (omitting e-numbers) as follows

$$H_c = \frac{1}{2} C_c [V(L,t) - V_I]^2 \rightarrow C_\Sigma V(L,t) V_I \rightarrow \quad (A7)$$

$$\rightarrow eV_+ \frac{C_c}{C_\Sigma} \sqrt{\frac{\hbar \omega_{10}}{EC}} \cos \left( \frac{\omega_p}{v} L \right) \sin (\omega_p t) \sigma_x.$$  

For a small frequency offset, $\Delta \omega = \omega_p - \omega_{\text{node}} \ll \omega_p$, we have

$$\cos \left( \frac{\omega_p}{v} L \right) \approx \pi \frac{\Delta \omega}{2 \omega_{\text{node}}}, \quad (A8)$$

with $\cos (\omega_{\text{node}} L/v) = 0$. Then the Hamiltonian (A7) describes the off-diagonal part of the transmon Hamiltonian (1) with

$$G = G_0 \frac{\Delta \omega}{\omega_{\text{node}}}, \quad (A9)$$

$$G_0(V_+) = \pi \frac{C_c}{\hbar} \sqrt{\frac{\hbar \omega_{10}}{EC}} eV_+.$$  

This is written in the main text as Eq. (3).

Consider next the diagonal part of the transmon Hamiltonian given by the energy-level splitting

$$\hbar \omega_{10} = \sqrt{8ECF_J} - EC. \quad (A10)$$

The flux contains the dc and ac components, $\Phi = \Phi_{dc} + \Phi_{ac} \sin (\omega_{\text{pump}} t)$. Assuming the latter being a small value, we obtain

$$\hbar \omega_{10} = \hbar \omega_{10} (\Phi_{dc}) + \hbar \delta (\Phi_{ac}) \sin (\omega_{\text{pump}} t), \quad (A11)$$

where $\delta (\Phi_{ac}) \propto \Phi_{ac}$ is the driving amplitude. This is written in the main text as Eq. (2).
D. S. Karpov, V. Y. Monarkha, D. Szombati, A. Gomez Frieiro, A. N. Omelyanchouk, E. Il’ichev, A. Fedorov, and S. N. Shevchenko, “Resonance fluorescence of a single artificial atom,” Science 327, 840–843 (2010).

K. Lahiri, C. B. Sanders, A. F. van Loo, A. Fedorov, A. Wallraff, and A. Blais, “Input-output theory for waveguide QED with an ensemble of inhomogeneous atoms,” Phys. Rev. A 88, 043806 (2013).

I.-C. Hoi, A. F. Kockum, L. Tornberg, A. Pourkabirian, G. Johansson, P. Delsing, and C. M. Wilson, “Probing the quantum vacuum with an artificial atom in front of a mirror,” Nat. Phys. 11, 1045–1049 (2015).

M. Mirkosseimi, E. Kim, X. Zhang, A. Sipahigil, P. B. Diederer, A. J. Keller, A. Asenjo-Garcia, D. E. Chang, and O. Painter, “Cavity quantum electrodynamics with atom-like mirrors,” Nature 569, 692 (2019).

I.-C. Hoi, C. M. Wilson, G. Johansson, T. Palomaki, B. Peropadre, and P. Delsing, “Demonstration of a single-photon router in the microwave regime,” Phys. Rev. Lett. 107, 073601 (2011).

I.-C. Hoi, T. Palomaki, J. Lindkvist, G. Johansson, P. Delsing, and C. M. Wilson, “Generation of nonclassical microwave states using an artificial atom in 1D open space,” Phys. Rev. Lett. 108, 263601 (2012).

I.-C. Hoi, A. F. Kockum, T. Palomaki, T. M. Stace, B. Fan, L. Tornberg, S. R. Suthyamoorthy, G. Johansson, P. Delsing, and C. M. Wilson, “Giant Cross-Kerr effect for propagating microwave fields induced by an artificial atom,” Phys. Rev. Lett. 111, 053603 (2013).

P. Y. Wen, A. F. Kockum, H. Ian, J. C. Chen, F. Nori, and I.-C. Hoi, “Reflective amplification without population inversion from a strongly driven superconducting qubit,” Phys. Rev. Lett. 120, 053601 (2018).

P. Y. Wen, K.-T. Lin, A. F. Kockum, B. Suri, H. Ian, J. C. Chen, S. Y. Mao, C. C. Chiu, P. Delsing, F. Nori, G.-D. Lin, and I.-C. Hoi, “Large collective Lamb shift of two distant superconducting artificial atoms,” Phys. Rev. Lett. 123, 233602 (2019).

P. Forn-Diaz, J. J. Garcia-Ripoll, B. Peropadre, J.-L. Or- giauzzi, M. A. Yurtalan, R. Belyansky, C. M. Wilson, and A. Lupascu, “Ultrastrong coupling of a single artificial atom to an electromagnetic continuum in the nonperturbative regime,” Nat. Phys. 13, 39–43 (2017).

D. Szombati, A. Gomez Frieiro, C. Müller, T. Jones, M. Jerger, and A. Fedorov, “Quantum rifling: Protecting a qubit from measurement back action,” Phys. Rev. Lett. 124, 070401 (2020).

D. S. Karpov, V. Y. Monarkha, D. Szombati, A. G. Frieiro, A. N. Omelyanchouk, E. Il’ichev, A. Fedorov, and S. N. Shevchenko, “Probabilistic motion averaging,” arXiv:1912.04557 (2019).

W. D. Oliver, Y. Yu, J. C. Lee, K. K. Berggren, L. S. Levitov, and T. P. Orlando, “Mach-Zehnder interferometry in a strongly driven superconducting qubit,” Science 310, 1655–1657 (2005).

M. Sillanpää, T. Lehtinen, A. Paila, Y. Makhlin, and P. Hakonen, “Continuous-time monitoring of Landau-Zener interference in a Cooper-pair box,” Phys. Rev. Lett. 96, 187002 (2006).

S. N. Shevchenko, S. Ashhab, and F. Nori, “Landau-Zener–St"uckelberg interferometry,” Phys. Rep. 492, 1–30 (2010).

C. S. E. van Ditzhuijzen, A. Tauschinsky, and H. B. van Linden van der Heuvell, “Observation of St"uckelberg oscillations in dipole-dipole interactions,” Phys. Rev. A 80, 063407 (2009).

A. Bogan, S. Studenikin, M. Korkusinski, and L. Gaudreau, “Landau-Zener-St"uckelberg-Majorana interferometry of a single hole,” Phys. Rev. Lett. 120, 207701 (2018).

K. Ono, S. N. Shevchenko, T. Morii, S. Moriyama, and F. Nori, “Quantum interferometry with a $g$-factor-tunable spin qubit,” Phys. Rev. Lett. 122, 207703 (2019).

R. M. Otxoa, A. Chatterjee, S. N. Shevchenko, S. Barraud, F. Nori, and M. F. Gonzalez-Zalba, “Quantum interference capacitor based on double-passage Landau-Zener-St"uckelberg-Majorana interferometry,” Phys. Rev. B 100, 205425 (2019).

J. Li, M. P. Silveri, K. S. Kumar, J.-M. Pirkkalainen, A. Veps"al"ainen, W. C. Chien, J. Tuorila, M. A. Sillanp"a"a, P. J. Hakonen, E. V. Thuneberg, and G. S. Paraoanu, “Motional averaging in a superconducting qubit,” Nat. Commun. 4, 1420 (2013).

M. Silveri, K. Kumar, J. Tuorila, J. Li, A. Veps"al"ainen, E. Thuneberg, and G. Paraoanu, “St"uckelberg interference in a superconducting qubit under periodic latching modulation,” New J. Phys. 17, 043058 (2015).

J. Pan, Y. Fan, Y. Li, X. Dai, X. Wei, Y. Lu, C. Cao, L. Kang, W. Xu, J. Chen, G. Sun, and P. Wu, “Dynamically modulated Autler-Townes effect in a transmon qubit,” Phys. Rev. B 96, 024502 (2017).

T. Bera, S. Majumder, S. K. Sahu, and V. Singh, “Large flux-mediated coupling in hybrid electromechanical system with a transmon qubit,” arXiv:2001.05700.

J. Eschner, C. Raab, F. Schmidt-Kaler, and R. Blatt, “Light interference from single atoms and their mirror images,” Nature 413, 495 (2001).

T. Wu, Y. Zhou, Y. Xu, S. Liu, and J. Li, “Landau-Zener-St"uckelberg interference in nonlinear regime,” Chin. Phys. Lett. 36, 124204 (2019).

M. Gong, Y. Zhou, D. Lan, Y. Fan, J. Pan, H. Yu, J. Chen, G. Sun, Y. Yu, S. Han, and P. Wu, “Landau-Zener-St"uckelberg-Majorana interference in a 3D transmon driven by a chirped microwave pulse,” Applied Phys. Lett. 110, 112602 (2016).

A. M. Satanin, M. V. Denisenko, A. I. Gelman, and F. Nori, “Amplitude and phase effects in Josephson qubits driven by a biharmonic electromagnetic field,” Phys. Rev. B 90, 104516 (2014).

G. Giavaras and Y. Tokura, “in preparation,” (2020).

J. Q. You, J. S. Tsai, and F. Nori, “Hybridized solid-state qubit in the charge-flux regime,” Phys. Rev. B 73, 014510 (2006).

J. Q. You, X. Hu, S. Ashhab, and F. Nori, “Low-decoherence flux qubit,” Phys. Rev. B 75, 140515 (2007).

A. F. Kockum and F. Nori, “Quantum Bits with Josephson Junctions,” in Fundamentals and Frontiers of the Josephson Effect (Springer International Publishing, 2019) pp.
703–741.

33 O. V. Ivakhnenko, S. N. Shevchenko, and F. Nori, “Simulating quantum dynamical phenomena using classical oscillators: Landau-Zener-Stückelberg-Majorana interferometry, latching modulation, and motional averaging,” Sci. Rep. 8, 12218 (2018).

34 J. Koch, T. M. Yu, J. Gambetta, A. A. Houck, D. I. Schuster, J. Majer, A. Blais, M. H. Devoret, S. M. Girvin, and R. J. Schoelkopf, “Charge-insensitive qubit design derived from the Cooper pair box,” Phys. Rev. A 76, 042319 (2007).

35 S. N. Shevchenko and D. S. Karpov, “Thermometry and memcapacitance with qubit-resonator system,” Phys. Rev. Applied 10, 014013 (2018).