Gluonium nature of the $\sigma/f_0(600)$ from its coupling to $K\bar{K}$

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We extract the $K^+K^-$ couplings of the isoscalar scalar mesons $\sigma/f_0(600)$ and $f_0(980)$ from $\pi\pi\rightarrow\pi\pi/K\bar{K}$ scatterings and found: $|g_{\sigma K+K^-}|/|g_{f_0+K^-}| \approx 0.8$ and $|g_{f_0+K^-}|/|g_{f_0+\pi-}| \approx 1.7$. These results, together with the tiny “direct” $\gamma\gamma$ width of the $\sigma$ and its large hadronic width, are a strong indication for the gluonium/glueball nature of the $\sigma$-meson, as predicted by QCD spectral sum rules (QSSR) \textsuperscript{2} some low-energy theorems (LET), while some other assignments ($\bar{\pi}\pi$ molecule, tetraquark state and ordinary $q\bar{q}$ meson) do not satisfy simultaneously these requirements from the data. These properties suggest that the $\sigma$ can be a scalar meson associated to the $U(1)_V$ conformal anomaly like is the $\eta'$-meson for the $U(1)_A$ anomaly.

1. Introduction

Understanding the nature of scalar mesons in terms of quark and gluon constituents is a long standing puzzle in QCD \textsuperscript{12}. The problem here is that some states are very broad (e.g. $\eta'$, $\eta''$) and some other assignments (e.g. $\bar{\pi}\pi$ molecule, tetraquark state and ordinary $q\bar{q}$ meson) do not satisfy simultaneously these requirements from the data. These properties suggest that the $\sigma$ can be a scalar meson associated to the $U(1)_V$ conformal anomaly like is the $\eta'$-meson for the $U(1)_A$ anomaly.

2. Gluonium nature of the $\sigma$ from $\pi\pi/\gamma\gamma \rightarrow \pi\pi$

The existence of glueballs/glubions is a characteristic prediction of QCD and some scenarios have been developed already back in 1975 \textsuperscript{21}. Today, there is agreement that such states exist in QCD and the lightest state has quantum numbers $J^{PC} = 0^{++}$. QSSR \textsuperscript{5,6,7,8,9,2} determinations of its mass found\textsuperscript{1}:

\[ M_{\sigma} \approx 1 \text{ GeV}, \] (1)

confirmed recently by lattice simulation using dynamical fermions \textsuperscript{10} and a strong coupling calculation \textsuperscript{11}.

Some phenomenological implications of scalar glueball have been studied in the literature \textsuperscript{5,6,7,8,12,13,14,15,16}. The analysis of the $\gamma\gamma \rightarrow \pi\pi$ data \textsuperscript{2} using an improved model of \textsuperscript{17} leads to the (model-dependent) “partial” $\gamma\gamma$ widths:

\[ \Gamma_{\sigma \rightarrow \gamma\gamma}^{\text{dir}} \approx (0.13 \pm 0.05) \text{ keV}, \]
\[ \Gamma_{\sigma \rightarrow \gamma\gamma}^{\text{resc}} \approx (2.7 \pm 0.4) \text{ keV}, \] (2)

where $\text{dir}$ and $\text{resc}$ refer respectively to the direct coupling of the $\sigma$ resonance and to the rescattering term obtained using an unitarized Born amplitude. This leads to the (model-independent) total $\gamma\gamma$ width (direct + rescattering):

\[ \Gamma_{\sigma \rightarrow \gamma\gamma}^{\text{tot}} \approx (3.9 \pm 0.6) \text{ keV}, \] (3)

which is in agreement with the results from the existing fits in \textsuperscript{18,19,20,21,22,23,24,25,26,27}.

Table 1

| Process | $M_\sigma - i \Gamma_\sigma/2$ (MeV) |
|---------|--------------------------------|
| $\pi\pi \rightarrow \pi\pi/K\bar{K}$ | 422$^{+16}_{-10}$ - i 290$^{+19}_{-15}$ |
| $J/\psi \rightarrow \omega \pi\pi$ | 541$^{+39}_{-30}$ - i 222$^{+42}_{-40}$ |

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which is in agreement with the other results given there. These values of the $\sigma$ meson parameters in the complex plane, when appropriately translated into the real axis, become at the on-shell mass $M_{\sigma}^{os} \approx 0.92$ GeV (see section 4):

\[ \Gamma_{\sigma \rightarrow \pi\pi}^{\text{os}} \approx 1.02 \text{ GeV}, \]
\[ \Gamma_{\sigma \rightarrow \pi\pi}^{\text{os,dir}} \approx 1.0 \pm 0.4 \text{ keV}, \] (5)

which are in a remarkable agreement with the QSSR \textsuperscript{27,28} and LET predictions obtained in the real axis \textsuperscript{2} for a “bare/unmixed” gluonium/glueball $\sigma_B$ state having the mass \textsuperscript{3}:

\[ M_{\sigma_B} \approx (0.95 \pm 1.10) \text{ GeV}, \] (6)

and the widths \textsuperscript{6,7,8} from the couplings in Table 2:

\[ \Gamma_{\sigma_B \rightarrow \pi^+\pi^-} \approx 0.5 \text{ GeV}, \]
\[ \Gamma_{\sigma_B \rightarrow \gamma\gamma} \approx (0.2 \sim 0.6) \text{ keV}. \] (7)

\textsuperscript{2}These LET have been used earlier in \textsuperscript{29,30}.
The large $\sigma$-width into $\pi\pi$ indicates a strong violation of the OZI rule in this channel and signals large non-perturbative effects in its treatment. This large hadronic width also disfavours its $\bar{q}q$ interpretation.

Table 2

| Meson            | $g_{S\pi^+\pi^-}$ | $g_{SK^+K^-}$ | $\Gamma_{S\gamma\gamma}$ |
|------------------|------------------|--------------|--------------------------|
| $\sigma_B \equiv gg$ | 5 $\sigma_B \pi^+\pi^-$ | 0.2 $\sim$ 0.6  |
| $S_2 \equiv 1/\sqrt{2}(\bar{u}u + \bar{d}d)$ | 2.5 $g_{S\pi^+\pi^-}$ | $6.2 \Gamma_{a_0\gamma\gamma}$ |
| $S_3 \equiv \bar{q}q$ | $g_{S\pi^+\pi^-}$ | 2.7 $\pm$ 0.5  | $4.1 \Gamma_{a_0\gamma\gamma}$ |

In fact, using the hadronic couplings in Table 2, QSSR predicts for a $S_2 \equiv 1/\sqrt{2}(\bar{u}u + \bar{d}d)$, with mass of 1 GeV:

$$\Gamma_{S_2 \rightarrow \pi^+\pi^-} \equiv \left| \frac{g_{S\pi^+\pi^-}}{16\pi M_{\sigma_B}} \right|^2 \left[ 1 - \frac{4m_q^2}{M_{S_2}^2} \right] \simeq 120 \text{ MeV}. \quad (8)$$

Using the $a_0 \rightarrow \gamma\gamma$ width of about 2 keV from a QSSR analysis of a quark triangle loop vertex including the $\langle \bar{q}q \rangle$ condensate contribution [31], one predicts a $S_2 \rightarrow \gamma\gamma$ width of about 5 keV (an analogous result has been obtained in [33] and a $S_3 \rightarrow \gamma\gamma$ width of about 0.2 keV (see also [34]). In the same way, QSSR also predicts, for a four-quark state, having the same mass of 1 GeV [35,34], a $\gamma\gamma$ width of about 0.4 eV [31]. These QSSR predictions and the value of the $\sigma \rightarrow \gamma\gamma$ direct coupling from the data do not favour the $\bar{q}q$ and 4-quark scenarios. However, the conclusion is not sharp as some other approaches may still allow the possibility to have a four-quark state.

For further tests of the nature of the $\sigma$, we investigate the extraction of the $\sigma$ coupling to $K^+K^-$. It is clear that, one expects a null value of this coupling in a $\pi\pi$ molecule and/or tetraquark assignments for the $\sigma$. This is in contrast with its gluonium assignment, where one, instead, expects its large (almost) universal coupling to pairs of pseudoscalar mesons [67,8] (see Table 2).

$$|g_{\sigma K^+K^-}| \sim |g_{\sigma\pi^+\pi^-}| \sim 5 \text{ GeV}. \quad (9)$$

3. The $g_{\sigma K^+K^-}$ coupling from $\pi^+\pi^- \rightarrow \pi\pi/K\bar{K}$

Unlike $g_{\sigma\pi^+\pi^-}$, this coupling cannot be measured due to phase space suppression. In the following,

- A kaon tadpole loop mechanism leads to smaller value of the $a_0 \rightarrow \gamma\gamma$ width of about 0.25 keV [22].
- However, an approach based on the scalar anomaly leads to a smaller value of about 0.2 keV [34].
- An alternative approach based on kaon loop leads to a larger value of about (0.2 $\sim$ 0.6) keV [35].
- This typical non-perturbative prediction differs from a perturbative argument (which should not apply below 1 GeV) where the coupling behaves like the current quark mass.

we extend the method in [17,3] used for elastic $\pi\pi$ scattering to determine the $\sigma$ parameters presented in the previous section. We shall also use S-matrix coupled-channel models with poles discussed in [39].

3.1. The analytic K-matrix model of [17,3]

The strong processes are expressed by a K matrix model representing the $\pi\pi \rightarrow \pi\pi/K\bar{K}$ amplitudes by a set of resonance poles [17]. In this case, the dispersion relations in the multi-channel case can be solved explicitly, which is not possible otherwise. This model can be reproduced by a set of Feynman diagrams, including resonance (bare) couplings to $\pi\pi$ and $K\bar{K}$ and 4-point $\pi\pi$ and $K\bar{K}$ interaction vertices. A subclass of bubble pion loop diagrams including resonance poles in the $s$-channel are resummed (unitarized Born). In [13], we study the elastic $\pi\pi$ scattering, where we introduce a shape function $f_0(s) \equiv f_P(s)$ which multiplies the $\sigma\pi\pi$ coupling and, for simplicity, we do not include the 4-point coupling term. Unlike approaches based on dispersion relations, this approach can provide a separation of the “direct” resonance coupling with the “rescattering” contributions which have been explicitly analyzed for $\gamma\gamma$ scattering in [31]. In the following, we discuss this approach (for a pedagogical reason) for the case of 1 channel $+ 1$ resonance. The real analytic function $f_P(s) \equiv P \equiv K$ is regular for $s > 0$ and has a left cut for $s \leq 0$. For our low energy approach, a convenient approximation, which allows for a zero at $s = -s_{AP}$ and a pole at $s_{P > 0}$ simulating the left hand cut, is:

$$f_P(s) = \frac{s - s_{AP}}{s + s_{D0}}. \quad (10)$$

The unitary $PP$ amplitude is then written as:

$$T_P^{(0)}(s) = \frac{G_P f_P(s)}{s_R - s - G_P f_P(s)/D_P} = \frac{G_P f_P(s)}{D_P(s)}, \quad (11)$$

where the index 0 corresponds to $I = 0$, $T_P^{(0)} = e^{i\delta_P^{(0)}} \sin \delta_P^{(0)}/\rho_P(s)$ with $\rho_P(s) = (1 - 4m_{\pi}^2/s)^{1/2}$; $G_P = g_{P,B}^2 \equiv g_{\sigma PP}/(16\pi)$ are the bare coupling squared and:

$$\text{Im } D_P = \text{Im } (G_P f_P) = -i(\theta_P)(\rho_P)G_P f_P, \quad (12)$$

with: $(\theta_P)(s) = 0$ below and $(\theta_P)(s) = \rho_P(s)$ above threshold $s = 4m_{\pi}^2$. The amplitude near the pole $s_0$ where $D_P(s_0) = 0$ and $D_P(s) \approx D_P'(s_0)/(s - s_0)$ is:

$$T_P^{(0)}(s) \sim \frac{g_P^2}{s_0 - s}; \quad g_P^2 = \frac{G_P f_P(s_0)}{-D_P'(s_0)}. \quad (13)$$

The real part of $D_P$ is obtained from a dispersion relation with subtraction at $s = 0$ and one obtains:

$$f_P(s) = \frac{1}{\pi} \left[ h_P(s) - h_P(0) \right], \quad (14)$$

$h_P(s) = f_P(s)\tilde{L}_3(s) - (\sigma_{NP}((s + s_{DP}))\tilde{L}_3(s) - \sigma_{DP})$, $\sigma_{NP}$ is the residue of $f_P(s)$ at $-\sigma_{DP}$ and: $\tilde{L}_3(s) = \frac{A}{\sigma_{NP}}$ A separation of the direct and rescattering term can also be studied by measuring $C$ asymmetry in $e^+e^-$. 38.
\[
\left( s - 4m^2_{\pi}\right) / m^2_{\pi} \bar{L}_1(s, m^2_{\pi}) \text{ with } \bar{L}_1 \text{ from [17]. This analysis can be generalized to the case of 2 channels } \oplus 2 \text{ resonance poles. A priori, the shape functions differ for the } \pi\pi \text{ and } K^+K^- \text{ channels. However, evoking } SU(3) \text{ symmetry, we can assume (to a first approximation) that they are equal.}
\]

3.2. Coupled channel model of [39]

In the 2- and 3-coupled channel approach presented in [39] resonances correspond to the closest to physical region poles of the S-matrix in complex energy plane. For each resonance such a pole is chosen among 2' poles (i is number of coupled channels) what is in contrast with K-matrix models and those using Breit-Wigner formulae. In this way, the unitary S-matrix approach in [39] delivers us a spectrum of scalar mesons below 1.6 GeV together with their couplings and branching ratios fitted to experimental data on the \( \pi\pi \) and \( K\bar{K} \) phase shifts and inelasticities. For example, for the Solution A in [39], the ratio of couplings to the \( K \) and \( \pi\pi \) channels for \( \sigma \) and \( f_0(980) \) states were 0.25 and 2.24 respectively. These fits can be improved by using the method in [24] based on dispersive analysis of experimental data. The authors have shown that theoretical constraints given by Forward Dispersion Relations (FDR), sum rules and by once and twice subtracted dispersion relation (GKPY and the Roy’s equations respectively) allow to determine the \( \pi\pi \) scattering amplitudes consistent with analyticity, unitarity and crossing symmetry. Forward Dispersion Relations (calculated at \( t = 0 \)) are used in [24] for three isospin combinations of \( \pi\pi \) amplitudes: for \( \pi^0\pi^+ \), \( \pi^0\pi^0 \) and for \( l \)-channel one with isospin \( I_l = 1 \). For example, the FDR for the former two combinations which need two subtractions read:

\[
\text{Re} F_i(s,0) - F_i(4m^2_{\pi},0) = \frac{2(s' - 4m^2_{\pi})Im F_i(s',0)ds'}{s'(s'-s)(s'-4m^2_{\pi})(s'+s-4m^2_{\pi})} \quad (15)
\]

where \( F_i \) stands for \( \pi^0\pi^+ \) or \( \pi^0\pi^0 \) amplitudes, while FDR for the latter one do not need subtractions. In the ideal situation, the difference on the left hand side of Eq. (15) equals to zero but for realistic amplitudes this difference is minimized in the fitting procedure described in [24].

In the case of nonforward dispersion relation (the Roy’s and GKPY equations) derived with imposed crossing symmetry condition, a difference between “output” and “input” partial wave amplitudes is minimized together with that for FDR. The “output” ones are calculated for three partial waves \( Jl: S0, P1 \) and \( S2 \) up to almost 1 GeV and are given by

\[
\text{Re } f_{i}^{(output)}(s) = ST(a_0^0, a_2^2) + \sum_{l'} \sum_{I'} \int \frac{ds'K_{ll'}^i(s,s')Im f_{ll'}^i(s')}{{4m^2_{\pi}}} + d_{i}^l(s, s_{max}). \quad (16)
\]

The \( a_0^0 \) and \( a_2^2 \) are the \( S0 \) and \( S2 \) scattering lengths, \( K_{ll'}^i(s, s') \) are kernels and \( d_{i}^l(s, s_{max}) \) are the so-called driving terms. The subtracting terms (ST) are linear combinations of scattering lengths in the GKPY equations and do not depend on \( s \). In the Roy’s equations, these are also combinations of \( a_0^0 \) and \( a_2^2 \) but depend linearly on \( s - 4m^2_{\pi} \). The integrals with kernels \( K_{ll'}^i(s, s') \) are calculated for partial waves with \( l' < 2 \) for the Roy’s equations and with \( l' < 4 \) for GKPY ones. The maximal value of the effective two pion mass squared \( s_{max} \) up to which experimental phase shifts and inelasticities for waves with \( l' < 2 \) for the Roy’s equations and with \( l' < 4 \) for GKPY ones. The maximal value was chosen to be 1.42 GeV. The driving terms \( d_{i}^l(s, s_{max}) \) describe the influence of higher partial waves in the whole \( s \) range and of the lower ones above \( s = s_{max} \) where Regge model was applied. The “input” amplitudes are those of which imaginary parts were used in once and twice subtracted dispersion relations in kernel and driving terms.

As was shown in [16] the GKPY equations give output amplitudes with much smaller errors than those from the Roy’s ones. It means that the GKPY equations impose stronger constraints on studied amplitudes. In addition to the Roy’s and GKPY equations, two sum rules which relate high energy (Regge) parameters to low energy \( P \) and \( D \) waves were also considered.

Following this method, we have performed similar fit for the \( S0 \) amplitude from 2- and 3-coupled channel model of [39]. All other \( \pi\pi \) partial waves were the same as in [24] and fixed. As a result, the ratio of couplings to the \( K\bar{K} \) and \( \pi\pi \) channels becomes 0.75 for \( \sigma \) and 1.98 for \( f_0(980) \), which we report in Table 3. The change of this ratio for \( \sigma \) in comparison with that for solution A of [39] confirms the general conclusion from [24] and [41] that available \( \pi\pi \) experimental data sets, (also this from [42] used in fits of [39]) do not fulfill enough well theoretical constraints from dispersion relations. It shows how important are theoretical demands given by dispersion relations and sum rules. Such constraints should be used in the fits of the experimental data of \( \pi\pi \) amplitudes.

3.3. Data input and results

We have used the data of inelasticities and phase shifts from [43,44,45,39]. More details of the analysis will be published elsewhere. In Table 3 we give the different couplings coming from the analyses based on the models in [17] and [39]. We shall use the normalization in Eq. (8), implying that \( |g_{pp}|^2 \) defined in section 3.1 is equal to \( |g_{pp}|^2/(16\pi) \) in the narrow width approximation. We shall also use the relations:

\[
|g_{S\pi\pi^-}|^2 = \frac{2}{3}|g_{S\pi\pi}|^2, \quad |g_{SK+K^-}|^2 = \frac{1}{2}|g_{SKK}|^2. \quad (17)
\]

We compare in the same Table 3 the results with the ones obtained in [22] from the use of dispersion relations in \( \pi\pi \) and \( K\bar{K} \) coupled-channel analysis in [16]. We also compare the results with the ones from \( \phi \to \sigma f_0(980) \gamma \) [47] and \( J/\psi \to \phi \pi\pi \) \( K\bar{K} \) [15] decays. One can notice that the different predictions for the \( \sigma \) couplings are quite stable. The \( \sigma \) couplings results using the model of [3] have been obtained in the case with 1 resonance \( \oplus 2 \) channels. In this case, one has obtained two output
poles but the 2nd one stays in an unphysical region. The results for the absolute value of the $f_0(980)$ couplings depend on the different models but the ratio of couplings is also quite stable. These results indicate that the $\sigma$ and $f_0(980)$ have important couplings to $KK$, where the one for the $\sigma$ is more remarkable.

Table 3

| Processes | $g_{\sigma\pi\pi} - r_{\sigma\pi\pi} f_{S\pi\pi} - r_{f_0KK}$ Models |
|-----------|---------------------------------------------------------------|
| $\pi\pi \rightarrow \pi\pi/K\bar{K}$ | 1.55 $\approx$ 1 1.57 1.8 [17,3] 2.5 0.75 $-$ 1.98 [39] |
| Others | 2.5 0.62 1.55 1.2 [22] $-$ 0.67 $-$ $-$ [17] |
| $\phi \rightarrow \sigma / f_0(980)$ | $\gamma$ $-$ $-$ $-$ [17] |
| $J/\psi \rightarrow \phi \pi\pi/K\bar{K}$ | $-$ $-$ 2.35 1.8 [18] |
| Average | 2.2 0.8 1.8 1.7 |

4. Comparison with the LET results

4.1. LET for the $\sigma$ couplings to Goldstone bosons

These couplings can be obtained from the vertex function:

$$V(q^2) \equiv (q_1 - q_2)^2 \equiv \langle \pi_1 | \theta_\mu^\alpha | \pi_2 \rangle ,$$  \hspace{1cm} (18)

where

$$\theta_\mu^\alpha \equiv \frac{1}{4} \beta(\alpha_s) G_\mu^\nu \bar{G}_\nu^\rho + \sum_{i=u,d,s} (1 + \gamma_m) m_i \bar{v}_i v_i ,$$  \hspace{1cm} (19)

is the conformal anomaly (trace of the energy-momentum tensor) with $\beta$ and $\gamma_m$ are the QCD $\beta$-function and mass-anomalous dimension while $G_\mu^\nu$ is the gluon field strength and $v_i$ is the quark field. $V(q^2)$ obeys a once subtracted dispersion relation [6]:

$$V(q^2) = V(0) + q^2 \int_{4m_z^2}^{\infty} dt \frac{1}{t - q^2 - i\epsilon} \frac{1}{\pi} \text{Im} V(t) ,$$  \hspace{1cm} (20)

with the condition: $V(0) = \mathcal{O}(m_z^2) \rightarrow 0$ in the chiral limit. Using also the fact that $V'(0) = 1$, one can then derive the two sum rules:

$$\sum_{S=\sigma B, \ldots} g_{S\pi\pi} - \sqrt{2} f_S = 0, \quad \sum_{S=\sigma B, \ldots} g_{S\pi\pi} - \frac{\sqrt{2} f_S}{M_S^2} = 1 ,$$  \hspace{1cm} (21)

where $f_S$ is the decay constant analogue to $f_\pi$. The 1st sum rule requires the existence of at least two resonances coupled strongly to $\pi\pi$. Considering the $\sigma_B$ and $\sigma'_B$ (its first radial excitation) but neglecting the small $G$-coupling to $\pi\pi$ as indicated by GAMS [39] and some other data [30], one predicts in the chiral limit [47] the almost universal couplings in Eq. (19) (see Table 2), which would correspond to a large width [\[9\].

$$\Gamma_{\sigma_B - \pi^+\pi^-} \approx 0.5 \text{ GeV} .$$  \hspace{1cm} (22)

This large width into $\pi\pi$ is a typical OZI-violation due to non-perturbative effects expected to be important in the region below 1 GeV, where perturbative arguments (like a vanishing of the hadronic glueball coupling in the chiral limit [51]) are valid in the region of the $G(1.5-1.6)$ cannot be applied.

4.2. Evidences that the $\sigma$ is a gluonium

In section 2 we have given several indications that the $\sigma$ can be a gluonium from the values of its small $\gamma_\gamma$ and large hadronic widths and of its on-shell mass. In the previous section, the value of its $K^+\bar{K}^-$ coupling, which is about 0.7 times of its $\pi^+\pi^-$ one (see Table 3), gives a further support on its gluonium structure due to the fact that a $\bar{\pi}\pi$ molecule and a tetraquark assignments would lead to a null value of the $K^+\bar{K}^-$ coupling. This gluonium nature of the $\sigma$ can indicate that the effective $KK\bar{\pi}\pi$ four-vertex used in a kaon loop model for successfully explaining e.g. $\phi \rightarrow \gamma\pi\pi$ can be induced by a $s$-channel exchange of a gluonium state. Indeed, using a gluonium picture, one can predict correctly this observed radiative width [27,18]. For better comparing the results obtained in the complex plane with the QSSR results obtained in the real axis, we introduce like in [3] the on-shell meson masses and hadronic widths [52], where the amplitude is purely imaginary at the phase $90^\circ$:

$$\text{Re} D((M_{\pi^+\pi^-}^2))^2 = 0 \implies M_{\pi^+\pi^-}^\text{os} \approx 0.92 \text{ GeV} .$$  \hspace{1cm} (23)

In the same way as for the mass, one can define an “on-shell width” [see Eqs. (12) and (13)] evaluated at $s = (M_{\pi^+\pi^-}^2)^2$:

$$M_{\pi^+\pi^-}^\text{os} \Gamma_{\pi^+\pi^-}^\text{os} \approx \frac{\text{Im} D}{-\text{Re} D} \implies \Gamma_{\pi^+\pi^-}^\text{os} \approx 0.7 \text{ GeV} ,$$  \hspace{1cm} (24)

which are comparable with the Breit-Wigner mass and width [45,53,54]:

$$M_{BW} \approx \Gamma_{BW} \approx 1 \text{ GeV} .$$  \hspace{1cm} (25)

These values lead to the on-shell coupling:

$$|g_{\pi^+\pi^-}^\text{os}| \approx 6 \text{ GeV} , \quad r_{\sigma\pi\pi} \equiv \frac{g_{\pi^+\pi^-}}{g_{\sigma\pi\pi}} \approx 0.8 .$$  \hspace{1cm} (26)

These fitted values and the one predicted in Eq. (19) strongly indicate a large gluonium component in the $\sigma$ wave function.

5. Summary and conclusions

- We have extracted the $\sigma\pi^+\pi^-$ and $\sigma K^+\bar{K}^-$ couplings from $\pi\pi \rightarrow \pi\pi/K\bar{K}$ scatterings using different models...
of K- and S-matrices. We have shown in Table the different results and have compared them with the ones from $\phi \to \sigma/f_0(980)\gamma$ and $J/\psi \to \phi\pi\pi/K\bar{K}$ decays. A comparison of these results with the predictions from LET+QSSR favours a large gluonium/glueball content in the $\sigma$ wave function.

- This feature can explain the splitting of its mass from the pion one, which is similar to the $\pi-\eta'$ mass splitting occuring in the well-known $U(1)_A$ anomaly [55]. The similarity with the $\eta'$ also signaled by the affinity of the $\sigma$ to couple to Goldstone bosons indicating a large OZI violation in its decay into $\pi\pi$ and a large value of its $K\bar{K}$ coupling. In this way, the $\sigma$ associated to the trace of the energy-momentum tensor $\theta^i_0$ [$U(1)_V$ conformal anomaly] can be considered as the partner of the $\eta'$ of the $U(1)_A$ anomaly. This non-$\bar{q}q$ property of the $\sigma$ may also go in lines with the 1-peak counting done in [56] using unitarized ChPT partial waves for describing $\pi\pi$ scatterings.

- Therefore, we consider that the assignement for the $\sigma$ as a non-strange partner of the $\kappa/K_0^*(800)$ (if confirmed), like often advocated in the four-quark literature (see e.g. [57]), may not be appropriate. The latter having an isospin 1/2 cannot have a gluonium in its wave function. Indeed, the $\kappa$ can likely be the isoscalar partner of the $\omega(980)$, where, in the standard classification of current algebra, they are respectively the $\bar{u}s$ and $\bar{u}d$ mesons having a mass around 1 GeV [37] which are associated to the divergence of the vector currents. Also, within this description, a successful determination of the running light quark mass differences from QSSR have been achieved [28].

- Finally, with the gluonium nature of the $\sigma$ and its large coupling to pseudoscalar boson pairs, the effective four-meson vertex $K\bar{K}\pi\pi$ used e.g. for explaining the $\phi \to \gamma\pi\pi$ process can be due to a $s$-channel exchange of a glue rich state. We plan to come back to this point as well as to the study of the gluonium-quarkonium mixing below 1 GeV which has been initiated in [127].

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