Influence of Taylor waves on standing windows of oblique detonation wave

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Abstract: An oblique detonation wave stabilized over a body has been studied due to the ongoing development of high-speed propulsion systems, such as oblique detonation wave engines and ram accelerators. Standing window of oblique detonation wave reflects the degree of standing difficulty. Firstly, Shock relations coupled with chemical equilibrium are solved to draw detonation polar diagrams between oblique detonation angle and wedge angle by iterative algorithm. In these diagrams, the wedge angle detaching from the wedge nose is referred as the maximum wedge angle $\theta_{\text{max}}$ and forming CJ detonation in the direction normal to the oblique detonation wave is referred as the minimum wedge angle $\theta_{\text{CJ}}$. The region between the two wedge angles is regarded as the standing windows. G. Emanuel holds that Taylor waves follow the oblique detonation wave as the wedge angle is smaller than $\theta_{\text{CJ}}$. Based on reactive Euler equation, wedge-induced oblique detonation is simulated in this paper. For the case of the wedge angle below $\theta_{\text{CJ}}$, Taylor waves change post-detonation wave flow field to maintain standing oblique detonation wave. Therefore, the standing window becomes wider.

1. Introduction

An oblique detonation wave (ODW) stabilized over a body has been studied due to greater efficiency in comparison with traditional scramjets for high-speed propulsion systems [1,2], such as ODW engines and ram accelerators. In addition, ODW in a combustor can reduce the length of the engine and minimize drag losses.

A series of extensive experimental and numerical studies were conducted in the last decades [3-8]. Their researches aimed at the unstable oblique detonation structure. In this concept, the key challenge is the ability to initiate an ODW at the desired location and to stabilize it over the entire flight time. If prompt combustion in ODW can be achieved via shock-induced combustion without large total pressure losses, stabilization condition is crucial. Pratt et al [9] constructed detonation polar relation by relating the detonation wave angle to the deflection angle (wedge angle). The polar curve represents the solution of the Rankine-Hugoniot relations. Based on solution analysis, Pratt concluded that the
deflection angle must be in the range $\theta_{CJ} < \theta < \theta_{\text{detach}}$ to stabilize the ODW. The range is referred as a standing window. For $\theta < \theta_{CJ}$, weak underdriven ODW cannot be realized. However, Emanuel [10, 11] discussed theoretically and computationally the steady, oblique detonation wave and suggested that an ODW exists at the CJ angle for deflection angle $\theta < \theta_{CJ}$. Verreault [12] presented the formation and structure of oblique detonation waves of semi-wedge and cones using the method of characteristics and numerical simulations and concluded that the wave structure for wedge angles less than $\theta_{CJ}$ is composed of an induction zone, the onset of a CJ ODW, and expanding waves that follow the end of the ODW reaction zone.

For the above standing window, the specific heat ratio is constant. However, the specific heat ratio is related to temperature and component of gas mixture. In this article, the detonation polar relation and standing windows are calculated theoretically and numerically for real gas effect. Influence of Taylor wave on the standing windows is discussed numerically.

2. Theoretical method of standing windows for oblique detonation

An oblique detonation wave coupled with chemical equilibrium in multi-species gas mixtures is theoretically solved in this study. For hydrogen-air mixture, 9 species ($H_2$, $O_2$, $H$, $O$, $OH$, $HO_2$, $H_2O_2$, $H_2O$, $N_2$) can be considered. Figure 1 is the sketch of an oblique detonation wave in multi-species gas mixture induced by a wedge. All parameters, including pressure, temperature, velocity and composition, before the oblique shock wave and the composition behind the shock wave are assumed to be known as well as the wedge angle $\theta$, and other parameters behind the shock wave are to be solved, such as pressure, temperature, velocity and the oblique detonation wave angle $\beta$. The governing equations for the normal and tangential coordinate system can be expressed as follow.

![Figure 1. An oblique detonation wave with multi-species gas mixture induced by a wedge.](image)

Conservation of mass:

$$\rho_1 u_{1n} = \rho_2 u_{2n},$$

Conservation of normal momentum:

$$P_1 + \rho_1 u_{1n}^2 = P_2 + \rho_2 u_{2n}^2,$$

Conservation of tangential momentum:

$$(\rho_1 u_{1n})u_{1t} = (\rho_2 u_{2n})u_{2t},$$
or simplified via (1) as:

\[ u_{1t} = u_{2t} = u_i , \]  

(4)

Conservation of energy:

\[ h_1 + \frac{1}{2} u_{1n}^2 = h_2 + \frac{1}{2} u_{2n}^2 , \]  

(5)

Equation of state:

\[ p_1 = \rho_1 R T , \]  

(6)

\[ p_2 = \rho_2 R T_2 , \]  

(7)

From equation (5) the enthalpy can be written as a sum of the specific and chemical enthalpies.

\[ c_{p1} T_1 + \frac{1}{2} u_{1n}^2 = c_{p2} T_2 + \frac{1}{2} u_{2n}^2 , \]  

(8)

For real gas mixture, the specific heats and enthalpy of a particular gas species can be expressed as fourth- and fifth-order polynomial fits, respectively, in the NASA-Lewis format [13]:

\[ \frac{C_{pi}^0}{R} = a_{i1} + a_{i2} T + a_{i3} T^2 + a_{i4} T^3 + a_{i5} T^4 , \]  

(9)

\[ \frac{H_{i}^0}{R} = a_{i1} + \frac{1}{2} a_{i2} T + \frac{1}{3} a_{i3} T^2 + \frac{1}{4} a_{i4} T^3 + \frac{1}{5} a_{i5} T^4 + a_{i6} \frac{1}{T} , \]  

(10)

As shown in figure 1, the velocity of the gas flow before and behind the oblique detonation wave can be broken into components normal and tangential to the wave. These components are written:

\[ u_{1n} = u_i \sin \beta \quad u_{1t} = u_i \cos \beta , \]  

(11)

\[ u_{2n} = u_2 \sin(\beta - \theta) \quad u_{2t} = u_2 \cos(\beta - \theta) , \]  

(12)

By combining the equations (1)-(12), the relation between the oblique shock wave angle \( \beta \) and the wedge angle \( \theta \) is expressed:

\[ \frac{\tan(\beta - \theta)}{\tan \beta} = \frac{\sqrt{\gamma_1 (\gamma_2 + \eta + N)}}{\gamma_1 (\gamma_2 + 1)} , \]  

(13)

where:

\[ N = \sqrt{(\gamma_1 / \gamma_2 - \eta)^2 - K \eta} , \]  

(14)

\[ \eta = \frac{1}{M_{in}^2} , \]  

(15)

\[ K = \frac{2 \gamma_1 (\gamma_2 + 1)}{\gamma_2^2} \left[ \frac{\gamma_2 - \gamma_1}{\gamma_1 - 1 + \gamma_1 (\gamma_2 - 1)/a_i^2} \right] , \]  

(16)
In equation (13), the parameters $\gamma_2$ and Q are related to the temperature and the equilibrium composition behind the shock which are solved by the specific heat $\gamma_2$. Thus, the detonation polar curve (13) is solved by iterative mechanism. The solving process is expressed in details as follow:

(1) The parameters $\gamma_2$ and Q are obtained by CJ detonation of gas mixture before the oblique shock wave. According to equation (13), the wave angle $\beta$ is assumed and the wedge angle $\theta$ can be calculated. The parameters, such as $p_2$, $T_2$, $\rho_2$ and $u_2$, behind the oblique detonation wave are calculated by equations (1)-(3);

(2) Equilibrium composition and the specific heat ratio $\gamma_2$ behind the oblique detonation wave are calculated with pressure $p_2$ and temperature $T_2$ in the step (1) by Chemical Equilibrium with Applications (CEA) [13];

(3) According to the convergence criteria, if the difference between the specific heat ratio $\gamma_2$ in the step (2) and the initial value $\gamma_2$ in the step (1) is smaller than a minimum value, the calculation is achieved; otherwise, a new iteration through step (1) and (2) will be processed.

The detonation polar diagram of stoichiometric hydrogen-air mixture for Mach number 9 is shown in figure 2. The curve corresponding to the symbol “+” in the numerator of equation (13) is the dashed line, which is the weak underdriven detonation. This branch is a violation of the second law and does not occur. The curve corresponding to the symbol “-” in the numerator of equation (13) is the solid line, which is the overdriven detonation. This branch is divided into two branches by the detached angle $\theta_{\text{max}}$. The detached angle is the maximum angle which the oblique detonation wave can be attached with the wedge. The upper branch is strong overdriven detonation, and the lower branch is weak overdriven detonation. Due to the instability of an attached, strong oblique shock wave, the weak overdriven detonation is the only stable solution for the shock that is attached to the wedge. When the variable N in the numerator of equation (13) is zero, CJ detonation occurs. The corresponding angle $\theta_{\text{CJ}}$ is the minimum attached angle. Thus, the ranges between $\theta_{\text{CJ}}$ and $\theta_{\text{max}}$ is referred as a standing window, which the weak overdriven detonation exits. The detonation polar diagrams for different Mach numbers are shown in figure 3. As Mach number is larger, the standing window is wider. The curves connect the minimum and maximum angle for different Mach numbers, separately, and the standing windows is shown in figure 4.

![Figure 2](image)

**Figure 2.** Detonation polar diagram of stoichiometric hydrogen-air mixture for Mach number 9 ($p_1$=1atm, $T_1$=300K).
3. Influence of Taylor waves on standing windows for oblique detonation

For the deflection angle smaller than CJ wedge angle, Taylor waves follow closely the oblique detonation wave and adjust the flow field behind the wave, which is discussed numerically by solving the reactive Euler equations.

3.1 Governing equation and numerical method

The two-dimensional Euler equations with a chemical reaction are used in terms of computational coordinate $(\xi, \eta)$

\[
\frac{\partial Q}{\partial t} + \frac{\partial F}{\partial \xi} + \frac{\partial G}{\partial \eta} = S , \tag{17}
\]

where subscripts $t$, $\xi$, and $\eta$ denote partial derivatives, $Q$ is the solution vector, $F$ and $G$ are the convective flux vectors in the $\xi$ and $\eta$ directions, respectively, $S$ is the chemical source vector. These
vectors are expressed as

\[
Q = \begin{pmatrix}
\rho_1 \\
\vdots \\
\rho_{in} \\
\rho u \\
\rho v \\
E
\end{pmatrix},
F = \begin{pmatrix}
\rho_1 \vec{U} \\
\vdots \\
\rho_{in} \vec{U} \\
\rho u + p \vec{v} \\
\rho v + p \vec{v} \\
\vec{U}(p + E)
\end{pmatrix},
G = \begin{pmatrix}
\rho_1 \vec{V} \\
\vdots \\
\rho_{in} \vec{V} \\
\rho u + p \eta \\
\rho v + p \eta \\
\vec{V}(p + E)
\end{pmatrix},
S = \begin{pmatrix}
\rho_1 \vec{V} \\
\vdots \\
\rho_{in} \vec{V} \\
\rho u + p \eta \\
\rho v + p \eta \\
\vec{V}(p + E)
\end{pmatrix}
\]

The governing equation (17) are solved by the fractional step method [14]. The convective and reaction terms are integrated separately. The Godunov-splitting scheme is employed to merge two different physical processes. The solutions of \( Q \) at the \((n+1)\)th time step can be expressed as

\[
Q^{n+1} = L_S L_G L_F Q^n ,
\]

where the terms \( L_F \) and \( L_G \) are the operators of convection computations in the \( \xi \) and \( \eta \) directions, respectively; the terms \( L_S \) are the operators of chemical reaction computation.

For the convection process, the fifth-order weighted essentially non-oscillatory (WENO) scheme [15] is adopted to capture shock and contact discontinuity. The reactive source term is solved by the LSODE solver based on the implicit Gear algorithm. The code is validated in our previous researches [16, 17].

A sketch of oblique detonation wave induced wedge is shown in figure 5. The incoming gas flow is the stoichiometric hydrogen-air mixture with the initial pressure of 1 atm and temperature of 300K.

![Figure 5](image_url) A sketch of oblique detonation of stoichiometric hydrogen-air mixture induced wedge.

3.2 Results and discussion
From the above discussion, the standing window is \( 12.5^\circ \leq \theta \leq 47.9^\circ \) for the Mach 9 gas flow. Taylor wave only exists for the wedge angle \( \theta \) smaller than \( \theta_{CJ} \). We choose the wedge angle of 11.5°. When the calculation is convergent, the temperature contour is shown in figure 6, where figure 6a is the overall flow field, and figure 6b is the enlarge flow field. In figure 6b, white line represents the streamline and black line represents the wedge surface. Passing through the detonation wave, the gas flow has a deflection of \( \theta_{CJ} \) which is larger than wedge angle. Taylor waves following the detonation wave make the gas flow deflect again, which causes the flow parallel to the wedge surface. For
comparison, we choose a $20^\circ$ wedge. Figure 7 is the temperature contour. There are not Taylor waves. Passing through the detonation wave, the gas flow is parallel to the wedge surface. Thus, Taylor waves make the standing windows of oblique detonation wave widen, as shown in figure 8, in which red hot is the enlarged lower limit.

![Figure 6](image1.png)  
(a) Overall flow field  
(b) Enlarge flow field  

**Figure 6.** Temperature contour of oblique detonation of stoichiometric hydrogen-air mixture with wedge angle of $11.5^\circ$ for the Mach 9 flow.

![Figure 7](image2.png)  
(a) Overall flow field  
(b) Enlarge flow field  

**Figure 7.** Temperature contour of oblique detonation of stoichiometric hydrogen-air mixture with wedge angle of $20^\circ$ for the Mach 9 flow.

![Figure 8](image3.png)  

**Figure 8.** Standing windows of stoichiometric hydrogen-air mixture under Taylor wave $(p_1=1\text{atm}, T_1=300\text{K})$. 

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4. Conclusion

The relation between the detonation angle and wedge angle is deduced considering the real gas effect. By iteration method, the detonation polar curve is solved and drawn. There are three branches. For reality, the weak overdriven detonation is the only stable solution, in which the corresponding wedge angle range \([\theta_{CJ}, \theta_{max}]\) is referred as the standing windows. When the wedge angle is smaller than the minimum angle \(\theta_{CJ}\), Taylor waves appear and make the flow behind the detonation wave deflect to parallel the wedge surface. Thus, the standing windows are widen.

Acknowledgments

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