Rate-Splitting Multiple Access for Downlink MIMO: A Generalized Power Iteration Approach

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Abstract—Rate-splitting multiple access (RSMA) is a general multiple access scheme for downlink multi-antenna systems embracing both classical spatial division multiple access and more recent non-orthogonal multiple access. Finding a linear precoding strategy that maximizes the sum spectral efficiency of RSMA is a challenging yet significant problem. In this paper, we put forth a novel precoder design framework that jointly finds the linear precoders for the common and private messages for RSMA. Our approach is first to approximate the non-smooth minimum function part in the sum spectral efficiency maximization problem as a form of the log-sum of Rayleigh quotients to convert it into a tractable non-convex optimization problem. By interpreting the first-order optimality condition of the reformulated problem as an eigenvector-dependent nonlinear eigenvalue problem, we reveal that a leading eigenvector is a local optimal solution. To find the leading eigenvector, we propose a computationally efficient algorithm inspired by a power iteration method. Simulation results show that the proposed RSMA transmission strategy provides significant improvement in the sum spectral efficiency compared to the state-of-the-art RSMA transmission methods, while requiring considerably less computational complexity.

Index Terms—Rate-splitting multiple access (RSMA), multi-user MIMO, imperfect channel state information (CSI), sum spectral efficiency maximization, generalized power iteration.

I. INTRODUCTION

Multi-user multiple-input multiple-output (MU-MIMO) downlink transmissions can provide extensive gains in spectral efficiency by serving multiple users with a shared time-frequency resource [2]–[4]. Assuming perfect channel state information at the transmitter (CSIT), a transmitter is able to send information symbols along with multiple linear precoding vectors to different users simultaneously by mitigating inter-user interference. In practice, however, the theoretical gains of downlink MU-MIMO transmissions can greatly vanish due to the inaccuracies of the CSIT. In particular, considering the frequency division duplex (FDD) systems, acquiring the accurate CSIT is infeasible; the downlink channel has to be estimated at the receiver first and sent back to the transmitter via a finite-rate feedback link [5], [6]. For this reason, in order to attain the de facto MU-MIMO spectral efficiency gains, it is crucial to design a downlink MU-MIMO transmission strategy that achieves high spectral efficiency under imperfect CSIT.

Rate-splitting multiple access (RSMA) is a robust downlink multiple access technique, especially when a transmitter has inaccurate knowledge for downlink CSI. Unlike the conventional spatial division multiple access (SDMA), in RSMA [7]–[10], the transmitter harnesses the rate-splitting strategy that breaks user messages into common and private parts for encoding. The transmitter constructs a common message by jointly encoding the common parts of the users’ split messages. The rate for this common message is carefully controlled so that all the users can decode it. The transmitter also encodes the private parts of the users’ messages to generate private information symbols. Then, the transmitter sends the common and private information symbols along with linear precoding vectors in a non-orthogonal manner. Each user decodes and eliminates the common message by performing successive interference cancellation (SIC) while treating the residual interference as noise. It then decodes the desired private message. Thanks to the rate-splitting encoding and SIC decoding, RSMA has been shown to outperform dirty paper coding (DPC) when partial CSIT is given [11].

To clearly understand the gains of RSMA over SDMA, it is instructive to consider a simple case of a two-user multi-antenna broadcast channel with partial CSIT. From an information-theoretical viewpoint, when applying linear precoding with partial CSIT in a two-user multi-antenna broadcast channel, the channel can be interpreted as a virtual two-user interference channel with transmitter cooperation, in which the channel gains of desired and interfering links are determined by the precoding vectors and the vector channels. In this equivalent interference channel, the quasi-optimal transmission strategy is the Han-Kobayashi scheme [12], i.e., splitting messages in common and private parts and allocate the power according to the relative channel gains between the interfering and the desired link. RSMA mimics this near capacity-achieving strategy to reap the sum spectral efficiency gains under a partial CSIT condition. In this way, the key feature of RSMA is that the power of the private information of each user should be set such that it is received at the noise level at the non-intended user after applying linear precoding vectors. In multi-antenna RSMA, finding the optimal linear precoding solution is a challenging yet significant problem. Although a simple zero-forcing (ZF) beamforming is optimal in the sense of degrees of freedom (DoF) [13], this method causes
the loss in the sum spectral efficiency for finite signal-to-noise-ratio (SNR) regimes. Unlike the sum-spectral efficiency maximization problem for SDMA relying on private messages only, the problem for RSMA has an additional unique challenge induced by the common message rate, which is the minimum of all the achievable rates for the common message at the users. This minimum function is non-smooth, which makes the sum-spectral efficiency maximization problem for RSMA intricate. In this work, we put forth a new approach for designing precoder to maximize the sum spectral efficiency of multi-antenna RSMA with partial CSIT.

A. Related Works

The rate-splitting technique was originally proposed for interference channels. In early work, a seminal Han-Kobayashi scheme was developed [12] and it was shown to achieve the capacity of a two-user Gaussian interference channel within one bit [14]. Recently, to cope with imperfect CSIT in downlink MIMO systems, the idea of rate-splitting has been re-explored as a multiple access technique, i.e., RSMA [15]. In [13], it was shown that RSMA provides sum DoF gains in multi-antenna broadcast channels where erroneous CSIT is given. Exploiting the idea of [13], in [16], the achievable spectral efficiency was analyzed while fixing the precoder for the common message as random precoding and the precoder for the private messages as ZF.

Besides the theoretical analysis, there exist several prior works that have developed practical linear precoding designs for RSMA multi-antenna systems. In [17], it was shown that hierarchical RSMA can be well-harmonized in massive MIMO systems thanks to its spatial covariance separability [18]. Relaxing a non-convex sum spectral efficiency maximization problem into a weighted mean squared error (WMSE) minimization problem, [7] proposed a linear precoding design based on the weighted minimum mean square (WMMSE) approach [3]. Specifically, a non-convex original problem was transformed into a quadratically constrained quadratic program (QCQP) by using the equivalence between the sum spectral efficiency maximization and the sum MSE minimization problem; thereafter an interior point method was used to solve the QCQP. Employing the same idea, in [8], [19], [20], a max-min fairness problem with RSMA was addressed. In [19], a linear precoding method for general RSMA was proposed by exploiting a concave-convex procedure (CCCP) [21] that successively approximates the original problem into convex forms. To evaluate the performance of RSMA in practical settings, e.g., finite constellation and implementable channel coding, [22] performed a link-level simulation in RSMA downlink MIMO systems. In [23], considering a single-antenna downlink channel, a power control method was proposed by incorporating the SIC constraint. Beyond the sum spectral efficiency maximization, other variants also exist. For example, energy efficiency maximization [24] and hardware impairments mitigation [25] have been studied in the context of optimization for RSMA. Further, multi-antenna RSMA in interference channels [26] and uplink channels [27], [28] were presented.

A key obstacle of the RSMA linear precoding design arises from the common message rate that should be determined as the minimum of all the achievable rates. To resolve this, a conventional approach commonly used in the prior methods is convex relaxation. Namely, an original non-convex problem is relaxed into a convex problem first, and then this convexified problem is put into an off-the-shelf optimization toolbox such as CVX to obtain a solution. A limitation of such approaches is that the optimization toolbox is hard to implement in limited hardware due to its extremely high complexity [29]. For this reason, the existing optimization methods for RSMA are rarely used in practice.

B. Contributions

The contributions of this paper are listed as follows.

- Considering a partial CSIT model, in which the CSI error statistic is modeled as complex Gaussian with zero-mean and a certain covariance matrix, we derive a lower bound of the instantaneous sum spectral efficiency for RSMA. In contrast to the sum spectral efficiency maximization for SDMA with partial CSIT, this lower bound entails the non-smooth minimum function for the common message rate. To convert the non-smooth minimum function in a tractable form, we take the LogSumExp technique, which offers tight approximation of the minimum function to a smooth function. Then, by representing all optimization variables onto a higher dimensional vector, we reformulate the lower bound of the instantaneous sum spectral efficiency for RSMA into a tractable non-convex function in the form of the log-sum of Rayleigh quotients.

- Using the derived lower bound with the smooth function approximation, we establish the first-order optimality condition for the sum spectral efficiency maximization problem. It is shown that the derived condition is cast as an eigenvector-dependent nonlinear eigenvalue problem [30], where the optimization variable behaves as an eigenvector and the objective function behaves as an eigenvalue. Accordingly, if we find a leading eigenvector that ensures the derived condition, the best local optimal solution is obtained, which indeed maximizes the approximate lower bound of the instantaneous sum spectral efficiency for RSMA.

- To obtain the leading eigenvector of the derived condition, we put forth a novel algorithm referred to as generalized power iteration for rate-splitting (GPI-RS). Using the power iteration principle, the idea of GPI-RS is to iteratively compute the leading eigenvector. The solution obtained by GPI-RS jointly provides the precoding directions and power allocation for the common and private messages. Notably, we do not rely on CVX in the proposed algorithm. We also extend the proposed GPI-RS for a case in which users have multiple receive antennas; we present a two-phase optimization framework that determines the precoders at the transmitter and the combiners at the receivers in an alternating manner.

- Simulation results show that the proposed GPI-RS provides spectral efficiency gains over the existing methods, including the conventional convex relaxation-based
WMMSE method (7) in various system environments. Especially, more performance gains are achieved in i) time division duplex (TDD), ii) mild CSIT error, and iii) correlated users’ channels. In addition to this performance improvement, a crucial benefit of our method is that no particular optimization toolbox such as CVX is used to solve the problem. Thanks to this feature, significantly less computational complexity are required in the proposed method. To be more specific, comparing the CPU computation time to (7), only 1 ~ 2% computation time is consumed in our method. As a result, our method is useful to realize RSMA gains in practical wireless systems.

**Notation:** The superscripts \((-)^T\), \((-)^H\), and \((-)^{-1}\) denote the transpose, Hermitian, and matrix inversion, respectively. \(I_N\) is the identity matrix of size \(N \times N\). Assuming that \(A_1, \ldots, A_N \in \mathbb{C}^{K \times K}\), \(A\) = blockdiag \((A_1, \ldots, A_n, \ldots, A_N)\) is a block diagonal matrix defined as

\[
A = \begin{bmatrix}
A_1 & 0 & \cdots & 0 \\
\vdots & \ddots & \cdots & \vdots \\
0 & \cdots & A_n & \cdots & 0 \\
\vdots & \ddots & \ddots & \ddots & \ddots \\
0 & \cdots & 0 & \cdots & A_N
\end{bmatrix} \in \mathbb{C}^{NK \times NK}. \tag{1}
\]

To present vectors and matrices conveniently, we adopt the MATLAB-style notation. For example, assuming a matrix \(H\), its \(i\)-th column vector is written as \(H[i, \cdot]\). For a vector \(h\), its \(i\)-th element is presented as \(h[i]\).

### II. SYSTEM MODEL

#### A. Channel Model

We consider a single-cell downlink MU-MIMO system, where a base station (BS) equipped with \(N\) antennas serves \(K\) single-antenna users. We denote a user set as \(\mathcal{K} = \{1, \ldots, K\}\). The channel vector between the BS and user \(k\) is denoted as \(h_k \in \mathbb{C}^N\) for \(k \in \mathcal{K}\), where \(h_k\) is generated based on the spatial covariance matrix \(R_k\), i.e., \(R_k = \mathbb{E}[h_k h_k^H]\). For constructing the channel covariance matrix, we adopt the one-ring model (18). Specifically, we assume that the BS is equipped with uniform circular array with radius \(\psi D\) where \(\psi\) denotes a signal wavelength and \(D = \sqrt{(1 - \cos(2\pi/N))^2 + \sin^2(2\pi/N)}\). Then, the channel correlation coefficient between the \(n\)-th antenna and \(m\)-th antenna corresponding to user \(k\) is defined as

\[
[R_k]_{n,m} = \frac{1}{2\Delta k} \int_{\theta_k + \Delta k}^{\theta_k - \Delta k} e^{-j\frac{\pi}{\psi} \Psi(x)(r_n - r_m)} dx, \tag{2}
\]

where \(\theta_k\) is angle-of-arrival (AoA) of user \(k\), \(\Delta k\) is the angular spread of user \(k\), \(\Psi(x) = [\cos(x), \sin(x)]\), and \(r_n\) is the position vector of the \(n\)-th antenna. By employing the Karhunen-Loeve model as in (17), (18), the channel vector \(h_k\) is represented as

\[
h_k = U_k \Lambda_k^{1/2} g_k, \tag{3}
\]

where \(\Lambda_k \in \mathbb{C}^{K \times K}\) is a diagonal matrix that contains the non-zero eigenvalues of \(R_k\), \(U_k \in \mathbb{C}^{N \times K}\) is a collection of the eigenvectors of \(R_k\) corresponding to the eigenvalues in \(\Lambda_k\), and \(g_k \in \mathbb{C}^K\) is a independent and identically distributed channel vector. We assume that each element of \(g_k\) is drawn from \(\mathbb{C}N(0, 1)\). We consider a block fading model, where \(g_k\) keeps constant within one transmission block. Over the two consecutive transmission blocks, \(g_k\) changes independently.

#### B. CSIT Acquisition Model

In this subsection, we explain the CSIT estimation and the corresponding error model used throughout this paper. We assume perfect channel state information at the receiver (CSIR), which is achieved via downlink pilots planted in the data packet as described in LTE and 5G NR.

Depending on the duplex types of the system, the CSIT acquisition is performed differently. In TDD, the CSIT is estimated via uplink pilots thanks to the channel reciprocity. In FDD, however, the channel reciprocity cannot be used so that the user quantizes the CSIR first and reports the feedback bits to the BS. In what follows, we explain the CSIT error model for each system in detail.

- **TDD:** As mentioned above, the BS is able to exploit the channel reciprocity to obtain the CSIT. Specifically, the BS estimates the CSIT from the uplink training sent from the users using the MMSE estimation (31), (32). The estimated CSIT is written as

\[
\hat{h}_k = h_k - e_k, \tag{4}
\]

where \(e_k\) is the CSIT estimation error vector. Since \(h_k\) is distributed as Gaussian, \(\hat{h}_k\) and \(e_k\) are also Gaussian that is independent to each other. The error covariance is obtained as

\[
\mathbb{E}[e_k e_k^H] = \Phi_k = R_k - R_k \left(R_k + \frac{\sigma^2}{\tau_d p_{ul}}\right)^{-1} R_k, \tag{5}
\]

where \(\tau_d\) and \(p_{ul}\) are uplink training length and uplink transmit power, respectively. As the uplink training length and power increases to infinity, the error covariance \(\Phi_k = 0\) and the CSIT error \(e_k\) also vanishes.

- **FDD:** In FDD, the channel reciprocity cannot be exploited; thereby, the CSIT should be estimated via limited feedback (5). (6). Once the user gets the CSIR, the obtained \(h_k\) is quantized with finite bits, and these bits are sent to the BS via the uplink feedback link. The quantized CSI is expressed as

\[
\hat{h}_k = U_k \Lambda_k^{1/2} \sqrt{1 - \kappa_k^2} g_k + v_k = \sqrt{1 - \kappa_k^2} h_k + e_k, \tag{6}
\]

where \(v_k\) are drawn from IID \(\mathbb{C}N(0, 1)\), \(e_k\) is the CSIT quantization error vector. The amount of feedback is implicitly incorporated into the parameter \(\kappa_k, 0 \leq \kappa_k \leq 1\). For instance, as the amount of feedback goes to infinity, \(\kappa_k = 0\) and no quantization error is induced. The error covariance of the quantization error \(e_k\) is

\[
\mathbb{E}[e_k e_k^H] = \Phi_k = U_k \Lambda_k^{1/2} \left(2 - 2\sqrt{1 - \kappa_k^2}\right) \Lambda_k^{1/2} U_k^H. \tag{7}
\]

In both systems, we assume that the BS has perfect knowledge on the long-term channel covariance \(R_k, k \in \mathcal{K}\), which can be obtained by using the covariance estimation methods (33). (34). A detailed description regarding the channel covariance acquisition is omitted since it is beyond our scope.
C. RSMA Signal Model

Using RSMA, the message for user \( k \) is split into the common message \( s_c \) and the private message \( s_k \). The common message \( s_c \) is drawn from a public codebook so that any user associated with the BS can decode it. On the contrary, the private message \( s_k \) comes from an individual codebook. Therefore it cannot be decoded at non-intended users.

One common message and \( K \) private messages are linearly precoded and then transmitted in a superimposed manner, so that the transmit signal \( x \in \mathbb{C}^N \) is given by

\[
x = f_c s_c + \sum_{k=1}^{K} f_k s_k,
\]

where \( f_c \in \mathbb{C}^N \) and \( f_k \in \mathbb{C}^N \) are the precoding vectors for the common and private messages respectively with the transmit power constraint: \( ||f_c||^2 + \sum_{k=1}^{K} ||f_k||^2 \leq 1 \). Then, the received signal at user \( k \) for \( k \in K \) is written as

\[
y_k = h_k^H f_c s_c + h_k^H f_k s_k + \sum_{\ell=1, \ell \neq k}^{K} h_k^H f_{\ell} s_{\ell} + z_k,
\]

where \( z_k \sim \mathcal{CN}(0, \sigma^2) \) is additive white Gaussian noise. We also assume that \( s_c \) and \( s_k \) are drawn from an independent Gaussian codebook, i.e., \( s_c, s_k \sim \mathcal{CN}(0, P) \).

D. Performance Metrics and Problem Formulation

Each user first decodes the common message \( s_c \) by treating all the other private messages as noise. Once the common message is successfully decoded, using SIC, the users remove the common message from the received signal and decode the private messages with a reduced amount of interference.

To successfully perform SIC, the common message \( s_c \) should be decodable to every user without any error. To this end, the channel coding rate of the common message \( s_c \) is set as the minimum of the ergodic spectral efficiencies among the users. Accordingly, under the premise that the BS has imperfect CSIT, \( \hat{h}_k = h_k + e_k \) for \( k \in K \), the ergodic spectral efficiency of the common message is obtained as [4]. [7]

\[
R_c = \min_{k \in K} \left[ \mathbb{E}_{\{e_k\}} \left[ \log_2 \left( 1 + \frac{|h_k^H f_c|^2}{\sum_{\ell=1, \ell \neq k}^{K} |h_k^H f_{\ell}|^2 + \sigma^2/P} \right) \right] \right],
\]

where the inner expectation is taken over the randomness associated with the CSIT error (\( \mathbb{E}_{\{e_k\}}[\cdot] \)) and the outer expectation is taken over the randomness associated with the imperfect knowledge of the channel fading process (\( \mathbb{E}_{\{h_k\}}[\cdot] \)).

Assuming that the channel code length spans an infinite number of channel blocks and we set the channel coding rate of the common message \( s_c \) less than or equal to \( R_c \), no decoding error for \( s_c \) occurs. Similar to [10], the ergodic spectral efficiency of the private message \( s_k \) after cancelling the common message is obtained as [4]. [7]

\[
R_k = \mathbb{E}_{\{h_k\}} \left[ \mathbb{E}_{\{e_k\}} \left[ \log_2 \left( 1 + \frac{|h_k^H f_c|^2}{\sum_{\ell=1, \ell \neq k}^{K} |h_k^H f_{\ell}|^2 + \sigma^2/P} \right) \right] \right],
\]

where the inner expectation is regarding to the randomness associated with the CSIT error (\( \mathbb{E}_{\{e_k\}}[\cdot] \)) and the outer expectation regarding to the randomness associated with the channel fading process (\( \mathbb{E}_{\{h_k\}}[\cdot] \)). Since we assume that the common message is successfully cancelled, it is observed that there is no interference from the common message in (11). Under the assumption that the channel code length spans an infinite number of channel blocks and the channel coding rate of the private message \( s_k \) is less than or equal to \( R_k \), the users can successfully decode \( s_k \).

Our main goal is to optimize the precoder with imperfect CSIT per fading block. When the BS has knowledge of \( \hat{h}_k = h_k + e_k \) for \( k \in K \) for every channel block, the instantaneous spectral efficiency of the private message \( s_k \) is defined as

\[
R_k^{\text{ins}} = \mathbb{E}_{\{e_k\}} \left[ \log_2 \left( 1 + \frac{|\hat{h}_k^H f_c|^2}{\sum_{\ell=1, \ell \neq k}^{K} |\hat{h}_k^H f_{\ell}|^2 + \sigma^2/P} \right) \right],
\]

where the expectation is taken over the randomness associated with the CSIT error. The instantaneous spectral efficiency \( R_k^{\text{ins}} \) in (12) differs from the ergodic spectral efficiency \( R_k \) in (11). On the one hand, the ergodic spectral efficiency \( R_k \) is the long-term performance that can be achieved when the channel code length spans very long channel blocks. On the other hand, the instantaneous spectral efficiency \( R_k^{\text{ins}} \) is the short-term rate expression when taking into account the channel estimation error effect per channel realization.

Computing a closed-form expression of the instantaneous spectral efficiency \( R_k^{\text{ins}} \) is challenging because the distribution of the residual interference by imperfect CSIT, \( e_k^H f_{\ell} \) for \( i \in K \), is unknown. By treating the residual interference induced by the channel estimation error as an additional Gaussian noise as in (4), we obtain a lower bound on \( R_k^{\text{ins}} \) as follows:

\[
R_k^{\text{ins}} \geq \mathbb{E}_{\{e_k\}} \left[ \log_2 \left( 1 + \frac{|\hat{h}_k^H f_c|^2}{\sum_{\ell=1, \ell \neq k}^{K} |\hat{h}_k^H f_{\ell}|^2 + \sum_{\ell=1}^{K} |e_k^H f_{\ell}|^2 + \sigma^2/P} \right) \right]
\]

\[
\geq \mathbb{E}_{\{e_k\}} \left[ \log_2 \left( 1 + \frac{|\hat{h}_k^H f_c|^2}{\sum_{\ell=1, \ell \neq k}^{K} |\hat{h}_k^H f_{\ell}|^2 + \sum_{\ell=1}^{K} |e_k^H f_{\ell}|^2 + \sigma^2/P} \right) \right]
\]

\[
= \bar{R}_k^{\text{ins}},
\]

where (a) follows Jensen’s inequality and the CSIT error covariance. Note that \( \Phi_k \) is the CSIT error covariance matrix determined depending on the used duplex type, i.e., TDD or FDD. For better understanding, let’s make a connection between the ergodic spectral efficiency \( R_k \) and a lower bound (13). If we use the channel coding rate with (13) in each channel block and make the channel code length span an infinite number of channel blocks, the achievable ergodic spectral efficiency for the private message \( s_k \) is \( \mathbb{E}_{\{h_k\}}[\bar{R}_k^{\text{ins}}] \), where the expectation is taken over the randomness associated with
the channel fading process. Since $\hat{R}_k^{\text{ins}}$ is a lower bound on the instantaneous spectral efficiency of each channel block, we have

$$R_k = \mathbb{E}_{\{h_k\}}[\hat{R}_k^{\text{ins}}] \geq \mathbb{E}_{\{h_k\}}[R_k^{\text{ins}}].$$

Next, similar to the above characterization, we obtain a lower bound on the spectral efficiency of the common message $s_C$. Defining $R_c^{\text{ins}}(k)$ as the instantaneous spectral efficiency of the common message that can be achieved at user $k$, we derive a lower bound on $R_c^{\text{ins}}(k)$ as follows.

$$R_c^{\text{ins}}(k) = \mathbb{E}_{\{e_k\}} \left[ \log \left( 1 + \frac{\|h_k^H f_k\|^2}{\sum_{\ell=1, \ell \neq k}^{K} \|h_k^H f_{\ell}\|^2 + \sigma^2 / P} \right) \right]_{h_k} \geq \mathbb{E}_{\{e_k\}} \left[ \log \left( 1 + \frac{\|h_k^H f_{\ell}\|^2}{\sum_{k=1}^{K} \|h_k^H f_{k}\|^2 + \sigma^2 / P} \right) \right] \geq \log \left( 1 + \frac{\|h_k^H f_{\ell}\|^2}{\sum_{k=1}^{K} \|h_k^H f_{k}\|^2 + \|e_k\|^2 + \sigma^2 / P} \right)$$

$$= \tilde{R}_c^{\text{ins}}(k),$$

where (a) comes from treating the residual interference caused by the channel estimation error as an additional Gaussian noise [4] and (b) comes from Jensen’s inequality. Using the very long channel code with the coding rate (15) in each channel block, the achievable ergodic spectral efficiency for the common message is $\min_{k \in K} \mathbb{E}_{\{h_k\}} \left[ \tilde{R}_c^{\text{ins}}(k) \right]$, which is a lower bound on the ergodic spectral efficiency of the common message $R_c$ as follows:

$$R_c = \min_{k \in K} \mathbb{E}_{\{h_k\}}[\hat{R}_c^{\text{ins}}(k)] \geq \min_{k \in K} \mathbb{E}_{\{h_k\}}[\tilde{R}_c^{\text{ins}}(k)] \geq \mathbb{E}_{\{h_k\}}[\tilde{R}_c^{\text{ins}}(k)],$$

where the expectation is taken over the channel fading process and the inequality follows from putting the minimum operator into the expectation does not increase the value. Combining (14) and (17), we finally complete the following lower bound on the ergodic sum spectral efficiency $R_{\Sigma}$:

$$R_{\Sigma} \geq \tilde{R}_{\Sigma} = \mathbb{E}_{\{h_k \in K\}} \left[ \min_{k \in K} \{\tilde{R}_c^{\text{ins}}(k)\} + \sum_{k=1}^{K} \tilde{R}_c^{\text{ins}} \right].$$

Note that the expectation is regarding to the channel fading process. Since the BS is able to calculate $\tilde{R}_k^{\text{ins}}$ and $\min_{k \in K} \{\tilde{R}_c^{\text{ins}}(k)\}$ in a closed form by using the estimated CSIT, maximizing $R_k^{\text{ins}}$ and $\min_{k \in K} \{\tilde{R}_c^{\text{ins}}(k)\}$ is feasible in each transmission block. Further, maximizing $\tilde{R}_c^{\text{ins}}$ and $\min_{k \in K} \{\tilde{R}_c^{\text{ins}}(k)\}$ per each transmission block given $\{h_k \in K\}$ is equivalent to maximizing $\tilde{R}_c^{\text{ins}}$, i.e., a lower bound on the ergodic sum spectral efficiency. Motivated by this, we formulate an optimization problem as follows:

$$\begin{aligned}
&\text{maximize} & & \min_{k \in K} \{\tilde{R}_c^{\text{ins}}(k)\} + \sum_{k=1}^{K} \tilde{R}_c^{\text{ins}} \\
&\text{subject to} & & \|f_k\|^2 + \sum_{k=1}^{K} \|f_k\|^2 \leq 1.
\end{aligned}$$

We tackle [19] as our main problem. Finding the global solution of [19] is still challenging due to its non-convexity and non-smoothness. In the next section, we resolve these difficulties and propose a new algorithm.

### III. Precoder Optimization with Generalized Power Iteration

In this section, we explain the key ideas to solve the optimization problem [19]. We first approximate the non-smooth minimum function as a smooth function by using the LogSumExp technique. Subsequently, we represent the optimization variable onto a higher dimensional vector to reformulate the problem [19] into a tractable non-convex optimization problem expressed as a function of Rayleigh quotients. By deriving the first-order KKT condition for the reformulated problem, we show that the first-order optimality condition is cast as an eigenvector-dependent nonlinear eigenvalue problem (NEPv) [30], and finding a leading eigenvector is equivalent to finding the best local optimal point of the reformulated problem. Consequently, to find a leading eigenvector, we propose a computationally efficient generalized power iteration algorithm.

#### A. Reformulation to a Tractable Form

At first, we approximate the non-smooth minimum function by using the LogSumExp technique. With the LogSumExp, the minimum function is approximated as [35]

$$\min_{i=1, \ldots, N} \{x_i\} \approx -\alpha \log \left( \frac{1}{N} \sum_{i=1}^{N} \exp \left( \frac{x_i}{-\alpha} \right) \right),$$

where the approximation becomes tight as $\alpha \to +0$. Leveraging (21), we approximate

$$\min_{k \in K} \{\tilde{R}_c^{\text{ins}}(k)\} \approx -\alpha \log \left( \frac{1}{K} \sum_{k=1}^{K} \exp \left( \frac{\tilde{R}_c^{\text{ins}}(k)}{-\alpha} \right) \right).$$

To help understand the LogSumExp approximation technique, we draw an illustration in Fig. 1. In Fig. 1 assuming the $N = 1, K = 2$, a landscape of the minimum spectral efficiency between two users is depicted. In addition to that, the approximate minimum spectral efficiency using the LogSumExp technique is also presented. As shown in the figure, the true maximum value of the non-smooth minimum function is tightly approximated by the LogSumExp technique.

Now we rewrite the precoding vectors $f_c, f_1, \ldots, f_K$ in a higher dimensional vector $\tilde{f}$ by stacking each vector as follows:

$$\tilde{f} = [f_c^T, f_1^T, \ldots, f_K^T]^T \in \mathbb{C}^{N(K+1) \times 1}.$$
With (23), we express the instantaneous spectral efficiency regarding the common message $s_c$ as

$$R_{\text{ins}}^c(k) = \log_2 \left( \frac{\hat{f}^H A_c(k) \hat{f}}{\hat{f}^H B_c(k) \hat{f}} \right),$$

(24)

where

$$A_c(k) = \text{blkdiag} \left( \hat{h}_k \hat{h}_k^H + \Phi_k, \ldots, \hat{h}_k \hat{h}_k^H + \Phi_k \right) + I_{N(K+1)} \sigma^2,$$

(25)

$$B_c(k) = A_c(k) - \text{blkdiag} \left( \hat{h}_k \hat{h}_k^H, 0, \ldots, 0 \right).$$

(26)

Note that we implicitly assume $||\hat{f}||^2 = 1$ to have (24). This assumption does not hurt the optimality since the spectral efficiency is monotonically increasing with the transmit power. It is also worthwhile to mention that (24) is presented as a function of Rayleigh quotient terms. Similar to this, we also write the spectral efficiency of the private message $s_k$ as

$$R_{\text{ins}}^k = \log_2 \left( \frac{\hat{f}^H A_k \hat{f}}{\hat{f}^H B_k \hat{f}} \right),$$

(27)

where

$$A_k = \text{blkdiag} \left( 0, (\hat{h}_k \hat{h}_k^H + \Phi_k), \ldots, \hat{h}_k \hat{h}_k^H + \Phi_k \right) + I_{N(K+1)} \sigma^2,$$

(28)

$$B_k = A_k - \text{blkdiag} \left( 0, \ldots, 0, \hat{h}_k \hat{h}_k^H, 0, \ldots, 0 \right).$$

(29)

With this Rayleigh quotients representation, the problem is transformed to

$$\text{maximize} \quad \log \left( \frac{1}{K} \sum_{k=1}^K \exp \left( \frac{1}{\alpha} \log_2 \left( \frac{\hat{f}^H A_c(k) \hat{f}}{\hat{f}^H B_c(k) \hat{f}} \right) \right)^{-\alpha} \right),$$

$$+ \sum_{k=1}^K \log_2 \left( \frac{\hat{f}^H A_k \hat{f}}{\hat{f}^H B_k \hat{f}} \right)$$

subject to $||\hat{f}||^2 = 1.$

(30)

(31)

We note that in (30), the obtained precoding vector $\tilde{f}$ can always be normalized by dividing the numerator and the denominator of each Rayleigh quotient with $||\hat{f}||$, while not affecting the objective function. Thanks to this feature, the constraint $||\hat{f}||^2 = 1$ can vanish from (31). Now we are ready to tackle the problem (30).

### B. First-Order Optimality Condition

To approach the solution of the transformed problem (30), we derive a first-order optimality condition of (30). The following lemma shows the main result in this subsection.

**Lemma 1.** The first-order optimality condition of the optimization problem (30) is satisfied if the following holds:

$$\text{B}_{\text{KKT}}^{-1}(\hat{f}) A_{\text{KKT}}(\hat{f}) \hat{f} = \lambda(\hat{f}) \hat{f},$$

(32)

where

$$A_{\text{KKT}}(\hat{f}) = \lambda_{\text{num}}(\hat{f}) \times \sum_{k=1}^K \left[ \exp \left( \frac{1}{\alpha} \log_2 \left( \frac{\hat{f}^H A_c(k) \hat{f}}{\hat{f}^H B_c(k) \hat{f}} \right) \right) A_c(k) + A_k \right]$$

(33)

$$\text{B}_{\text{KKT}}(\hat{f}) = \lambda_{\text{den}}(\hat{f}) \times \sum_{k=1}^K \left[ \exp \left( \frac{1}{\alpha} \log_2 \left( \frac{\hat{f}^H A_c(k) \hat{f}}{\hat{f}^H B_c(k) \hat{f}} \right) \right) B_c(k) + B_k \right],$$

(34)

with

$$\lambda(\hat{f}) = \left( \frac{1}{K} \sum_{k=1}^K \exp \left( \frac{1}{\alpha} \log_2 \left( \frac{\hat{f}^H A_c(k) \hat{f}}{\hat{f}^H B_c(k) \hat{f}} \right) \right) \right)^{-\frac{\alpha}{2}}$$

(35)

$$\prod_{k=1}^K \left( \frac{\hat{f}^H A_k \hat{f}}{\hat{f}^H B_k \hat{f}} \right) = \frac{\lambda_{\text{num}}(\hat{f})}{\lambda_{\text{den}}(\hat{f})}.$$

**Proof.** See Appendix A.

Now we interpret the derived optimality condition (32). We first observe if a precoding vector $\tilde{f}$ satisfies the condition (32), then it also satisfies the first-order optimality condition, which means that the corresponding $\hat{f}$ is a stationary point of the problem (30) whose gradient is zero. If the problem (30) has multiple stationary points, it is possible to exist multiple $\tilde{f}$ satisfying (32). Next, we see that (32) is presented as a form of the eigenvector problem for the matrix $B_{\text{KKT}}^{-1}(\hat{f}) A_{\text{KKT}}(\hat{f})$. More rigorously, (32) is cast as a NEPv (30). As described in
Algorithm 1 GPI-RS

initialize: \( \tilde{f}_{(0)} \)

Set the iteration count \( t = 1 \).

while \( \| \tilde{f}_{(t)} - \tilde{f}_{(t-1)} \| > \epsilon \) do

Construct the matrices \( A_{\text{KKT}}(\tilde{f}_{(t-1)}) \) and \( B_{\text{KKT}}(\tilde{f}_{(t-1)}) \) by using (33) and (34).

Update \( \tilde{f}_{(t)} \leftarrow \frac{B_{\text{KKT}}(\tilde{f}_{(t-1)})A_{\text{KKT}}(\tilde{f}_{(t-1)})\tilde{f}_{(t-1)}}{\|B_{\text{KKT}}(\tilde{f}_{(t-1)})A_{\text{KKT}}(\tilde{f}_{(t-1)})\tilde{f}_{(t-1)}\|} \).

\( t \leftarrow t + 1 \).

end while

\[ \[ \]

Proposition 1. Denoting that the local optimal point for the problem (30) as \( \tilde{f}^* \), \( \tilde{f}^* \) is the leading eigenvector of \( B_{\text{KKT}}^{-1}(\tilde{f})A_{\text{KKT}}(\tilde{f}) \) satisfying

\[ B_{\text{KKT}}^{-1}(\tilde{f}^*)A_{\text{KKT}}(\tilde{f}^*)\tilde{f}^* = \lambda^* \tilde{f}^*, \]

where \( \lambda^* \) is the corresponding eigenvalue.

Finding \( \tilde{f}^* \) is, however, not straightforward due to the intertwined nature of the problem. In the next subsection, we propose a novel method called GPI-RS. GPI-RS is able to find the leading eigenvector of the matrix \( B_{\text{KKT}}(\tilde{f})A_{\text{KKT}}(\tilde{f}) \) in a computationally efficient fashion.

C. Generalized Power Iteration for Rate-Splitting

The basic process of the proposed GPI-RS follows that of a typical power iteration. Given \( \tilde{f}_{(t-1)} \) obtained in the \( (t - 1) \)th iteration, we construct the matrices \( B_{\text{KKT}}(\tilde{f}_{(t-1)}) \) and \( A_{\text{KKT}}(\tilde{f}_{(t-1)}) \) using (33) and (34). Then, we update the precoding vector for the current iteration as

\[ \tilde{f}_{(t)} \leftarrow \frac{B_{\text{KKT}}^{-1}(\tilde{f}_{(t-1)})A_{\text{KKT}}(\tilde{f}_{(t-1)})\tilde{f}_{(t-1)}}{\|B_{\text{KKT}}^{-1}(\tilde{f}_{(t-1)})A_{\text{KKT}}(\tilde{f}_{(t-1)})\tilde{f}_{(t-1)}\|}. \]

We repeat this process until the convergence criterion is met. In this paper, we use \( \| \tilde{f}_{(t)} - \tilde{f}_{(t-1)} \| < \epsilon \) for small enough \( \epsilon \).

We summarize this process in Algorithm 1. For an initial point \( \tilde{f}_{(0)} \), we use maximum ratio transmission (MRT). In the later section, we will show that MRT works well as an initial point of the GPI-RS.

Remark 1. (Algorithm complexity) One essential advantage of the proposed GPI-RS is low complexity. The total computational complexity of the proposed GPI-RS is dominated by the calculation of \( B_{\text{KKT}}^{-1}(\tilde{f}) \). The matrix \( B_{\text{KKT}}^{-1}(\tilde{f}) \) is the sum of the block diagonal matrices as presented in (32). Specifically, \( K + 1 \) number of \( N \times N \) submatrices are concatenated, so that the total size is \( (K + 1)N \times (K + 1)N \). For this reason, the inverse matrix \( B_{\text{KKT}}^{-1}(\tilde{f}) \) is obtained by computing the inverse of each submatrix, and this requires the complexity with the order of \( O\left(\frac{1}{2}(K + 1)N^3\right) \). This results in that the complexity of the proposed GPI-RS per iteration is with the order of \( O\left(\frac{1}{2}KN^3\right) \).

We note that this is substantially small compared to the existing methods. For example, the conventional methods based on QCQP [7], [30] need the complexity order of \( O(KN^4) \). Further, the CCCP based method [10] is associated with the complexity order of \( O(N^6K^{0.5}2^{3K}) \). In particular, it is noteworthy that the proposed GPI-RS has the linear order complexity with the number of user \( K \). Additionally, our algorithm is easy to implement in limited hardware in that any off-the-shelf optimization solver such as CVX is not needed.

Later, we numerically compare the computation time of the proposed method and the existing method based on QCQP [7].

Remark 2. (Selection of the parameter \( \alpha \)) The parameter \( \alpha \) causes a trade-off between the performance and the convergence of the proposed GPI-RS. On the one hand, if \( \alpha \) is small, the LogSumExp approximation [22] becomes tight so that the minimum spectral efficiency regarding the common message is more accurately counted to the GPI-RS. As a result, the sum spectral efficiency performance can increase. However, small \( \alpha \) increases the possibility that the GPI-RS does not converge. This is because, as \( \alpha \) decreases, the LogSumExp becomes more similar to the minimum function and its shape turns to be non-smooth. In this case, no stationary point is characterized; thus, the algorithm cannot find the converging point. On the other hand, if \( \alpha \) is large, the algorithm converges in most cases as the objective function becomes smooth. Nevertheless, the algorithm cannot reflect the accurate minimum spectral efficiency, incurring some performance degradation.

Identifying the optimal \( \alpha \) in an analytical manner is very challenging. To find the proper \( \alpha \) numerically, we heuristically modify the GPI-RS. The key idea is to obtain the smallest \( \alpha \) that makes the GPI-RS algorithm converge. Specifically, we start the GPI-RS with small \( \alpha \). If the iteration loop of the GPI-RS does not converge within the predetermined number of iterations, then we enforce to terminate the loop, increase \( \alpha \), and newly start the algorithm again. We repeat this process until the algorithm converges before the predetermined number. To reduce the algorithm time, we can empirically adapt the starting \( \alpha \) value and the increasing ratio depending on the system configuration, i.e., the number of the antennas \( N \) and the users \( K \).

IV. EXTENSION TO MULTIPLE USER ANTENNAS

This section presents an algorithm to extend the proposed GPI-RS for the case of multiple receive antennas at users. If a user has multiple antennas, it is possible to deliver more than one private messages to each user by using a combiner. To maximize the sum spectral efficiency in this setup, we propose a two-phase optimization framework that alternatively determines the precoders and the combiners. Specifically, in the first
phase, we design the precoders by using the proposed GPI-RS algorithm under the premise that all the combiners are fixed. Subsequently, in the second phase, with the fixed precoders conveyed from the first phase, we compute the combiners by employing the linear minimum mean-squared error (LMMSE) techniques, known as the optimal linear combiner. Finally, we repeat this process until the outer loop termination criterion is satisfied.

Now we formally develop the alternating optimization framework. We assume that $N_k$ number of antennas are equipped in user $k$. For user $k$, the BS sends the private message vector $s_k \in \mathbb{C}^{N_k}$ through the preceding matrix $F_k \in \mathbb{C}^{N \times N_k}$. For rate-splitting, the BS also transmits the common message $s_c$ through the preceding vector $f_c \in \mathbb{C}^{N \times 1}$. Then the transmitted signal vector $x$ is expressed as

$$x = f_c s_c + \sum_{k=1}^{K} F_k s_k.$$

The received signal at user $k$, denoted as $y_k \in \mathbb{C}^{N_k \times 1}$, is

$$y_k = H_k^H f_c s_c + H_k^H F_k s_k + \sum_{\ell=1,\ell \neq k}^{K} H_k^H F_{\ell} s_{\ell} + z_k,$$

where $H_k \in \mathbb{C}^{N \times N_k}$ is the channel matrix between the BS and user $k$. We first determine the LMMSE combiner while fixing the precoders $f_c$ and $F_k$, $k \in \mathcal{K}$. For ease of notation, we rewrite the signal model as

$$y_k = g_{k,c}(s_c) + G_{k,s_k} + \sum_{\ell=1,\ell \neq k}^{K} G_{k,\ell} s_{\ell} + z_k,$$

where $g_{k,c} = H_k^H f_c \in \mathbb{C}^{N_k \times 1}$ and $G_{k,\ell} = H_k^H F_{\ell} \in \mathbb{C}^{N_k \times N_k}$ are the effective channel vectors or matrices. Since the precoders are fixed, the effective channel $g_{k,c}$ and $G_{k,\ell}$ for $\ell \in \mathcal{K}$ are also given. User $k$ uses two linear combiners successively, $v_k^{(c)} \in \mathbb{C}^{N_k}$ and $v_k \in \mathbb{C}^{N_k \times N_k}$ where

$$v_k = \left[ v_k^{(1)}, v_k^{(2)}, \ldots, v_k^{(N_k)} \right]$$

for decoding $s_c$ and $s_k$, respectively. Following the well-known LMMSE criteria, the LMMSE for the common message $s_c$ is designed as

$$\hat{s}_c = \left( I - \sigma^2 \sum_{\ell=1}^{K} G_{k,\ell} G_{k,\ell}^H \right)^{-1} v_k^{(c)}.$$

Provided that the spectral efficiency of $s_c$ is designed less than $\min_{k \in \mathcal{K}} R_c^{\text{ins}}(k)$, the common message can be perfectly eliminated. Then the LMMSE $V_k$ is constructed as follows.

$$v_k^{(i)} = \left( \hat{s}_c + \frac{\sigma^2}{P} \sum_{\ell=1}^{K} G_{k,\ell} G_{k,\ell}^H + \sum_{j=1, j \neq i}^{N_k} \hat{G}_{k,k} \hat{G}_{k,k}^H \right)^{-1} G_{k,k} \hat{G}_{k,k}^H.$$

Armed with this, the spectral efficiency for the private information symbol $s_k[i]$ is given by

$$R_{k,i}^{\text{ins}} = \log_2 \left( 1 + \frac{|\langle v_k^{(i)} \rangle^H G_{k,k} \hat{G}_{k,k}^H \rangle^2}{\text{IUI} + \text{ISI} + \frac{\sigma^2}{P}} \right),$$

where IUI means the inter-user interference and ISI means the inter-stream interference. Each term is defined as

$$\text{IUI} = \sum_{\ell=1, \ell \neq k}^{K} ||v_k^{(i)} G_{k,\ell}^H||^2,$$

$$\text{ISI} = \sum_{j=1, j \neq i}^{N_k} ||v_k^{(i)} G_{k,k}||^2.$$

Next, we assume that the the combiners $v_k^{(c)}$ and $V_k$ for $k \in \mathcal{K}$ are fixed and obtain the precoders. When decoding the common message $s_c$, the received signal at user $k$ is

$$y_k = g_{k,c}^H f_c s_c + \sum_{i=1}^{N_k} g_{k,c}^H F_{k}[:,i] s_k[i] + \sum_{\ell=1, \ell \neq k}^{K} g_{k,\ell}^H F_{\ell}[:,i] s_{\ell}[i] + z_k,$$

where $g_{k,c} = H_k^H f_c \in \mathbb{C}^{N_k \times 1}$ is the effective channel vector incorporating the effect of the combiner. Then the instantaneous spectral efficiency of the common message $s_c$ achieved at user $k$ is

$$\bar{R}_c^{\text{ins}}(k) = \log_2 \left( 1 + \frac{|g_{k,c}^H c|^2}{\frac{\sum_{k=1}^{K} \sum_{i=1}^{N_k} |g_{k,c}^H F_{k}[:,i]|^2 + \sigma^2}{P}} \right),$$

Provided that the spectral efficiency of the common message is properly determined, it is completely removed by exploiting SIC. Under this premise, the received signal for decoding the private message $s_k[i]$ is

$$y_k = g_{k,c}^H F_{k}[:,i] s_k[i] + \sum_{j=1, j \neq i}^{N_k} g_{k,c}^H F_{k}[:,j] s_k[j] + \sum_{\ell=1, \ell \neq k}^{K} g_{k,\ell}^H F_{\ell}[:,i] s_{\ell}[m] + z_k,$$

where $g_{k,c}^H = H_k^H f_c \in \mathbb{C}^{N_k \times 1}$ is the effective channel vector. Subsequently, the instantaneous spectral efficiency of the private message $s_k[i]$ is expressed as

$$\bar{R}_{k,i}^{\text{ins}} = \log_2 \left( 1 + \frac{|g_{k,c}^H F_{k}[:,i]|^2}{\text{IUI'} + \text{ISI'} + \frac{\sigma^2}{P}} \right),$$
where

\[
\text{IUI}' = \sum_{\ell=1}^{K} \sum_{k=1}^{N_k} |\mathbf{g}^{\ell}_k \mathbf{F}_\ell [:, m]|^2, \tag{52}
\]

\[
\text{ISI}' = \sum_{j=1, j \neq k}^{N_k} |\mathbf{g}^{j}_k \mathbf{F}_j [:, i]|^2. \tag{53}
\]

Then the sum spectral efficiency maximization problem is formulated as follows.

\[
\text{maximize}_{k, \mathbf{F}_1, \ldots, \mathbf{F}_K} \min_{k \in \mathcal{K}} \{R_{\text{c}}^{\text{ins}}(k)\} + \sum_{k=1}^{K} \sum_{i=1}^{N_k} R_{k,i}^{\text{ns}}, \tag{54}
\]

subject to \( \sum_{k=1}^{K} \sum_{i=1}^{N_k} \|\mathbf{F}_k [:, i]\|_2^2 + \|\mathbf{f}_c\|_2^2 \leq 1 \). \tag{55}

To solve \((54)\), we follow the approach of the proposed GPI-RS. To do this, we first stack the precoding vectors such as

\[
\mathbf{\tilde{f}} = \left[ \mathbf{\tilde{f}}_1^T, \mathbf{\tilde{f}}_2^T, \ldots, \mathbf{\tilde{f}}_K^T \right]^T, \tag{56}
\]

with \( \mathbf{\tilde{f}} \in \mathbb{C}^{N(\Sigma_{\ell,i} + N_t + 1)} \). Then we express the instantaneous spectral efficiency of the common message \( s_c \) achieved at user \( k \) as

\[
R_{\text{c}}^{\text{ins}}(k) = \log_2 \left( \frac{\mathbf{\tilde{f}}_k^H \mathbf{A}_c(k) \mathbf{\tilde{f}}}{\mathbf{\tilde{f}}_k \mathbf{B}_c(k) \mathbf{\tilde{f}}} \right), \tag{57}
\]

where

\[
\mathbf{A}_c(k) = \text{blkdiag}(\mathbf{g}'_{k,c} \mathbf{\hat{g}}_{k,c}^H, \mathbf{g}'_{k,c} \mathbf{\hat{g}}_{k,c}^H, \ldots, \mathbf{g}'_{k,c} \mathbf{\hat{g}}_{k,c}^H) + \mathbf{I}_N(\Sigma_{\ell,i} + N_t + 1) \sigma^2 \theta. \tag{58}
\]

\[
\mathbf{B}_c(k) = \mathbf{A}_c(k) - \text{blkdiag}(\mathbf{g}'_{k,c} \mathbf{\hat{g}}_{k,c}^H 0, \ldots, 0). \tag{59}
\]

Note that we use the estimated CSIT in design of the precoders, so \( \mathbf{\hat{g}}_{k,c} = \mathbf{\hat{H}}_k \mathbf{V}_k^{(c)} \). Similarly, we also present the spectral efficiency of the private message \( s_k[i] \) as

\[
R_{k,i}^{\text{ins}} = \log_2 \left( \frac{\mathbf{\tilde{f}}_k^H \mathbf{A}_{k,i} \mathbf{\tilde{f}}}{\mathbf{\tilde{f}}_k \mathbf{B}_{k,i} \mathbf{\tilde{f}}} \right), \tag{60}
\]

where

\[
\mathbf{A}_{k,i} = \text{blkdiag}(0, \mathbf{\hat{g}}_{k,i}^H \mathbf{\hat{g}}_{k,i}^H, \ldots, \mathbf{\hat{g}}_{k,i}^H \mathbf{\hat{g}}_{k,i}^H) + \mathbf{I}_N(\Sigma_{\ell,i} + N_t + 1) \sigma^2 \theta. \tag{61}
\]

\[
\mathbf{B}_{k,i} = \mathbf{A}_{k,i} - \text{blkdiag}(0, \mathbf{\hat{g}}_{k,i}^H 0, \ldots, 0). \tag{62}
\]

We have \( \mathbf{\hat{g}}_{k,i} = \mathbf{\hat{H}}_k \mathbf{V}_k^{(i)} \). With this form, by using the proposed method, the problem \((54)\) is reformulated as

\[
\text{maximize } \log_2 \left( \frac{1}{K} \sum_{k=1}^{K} \exp \left( \log_2 \left( \frac{\mathbf{\tilde{f}}_k^H \mathbf{A}_c(k) \mathbf{\tilde{f}}}{\mathbf{\tilde{f}}_k \mathbf{B}_c(k) \mathbf{\tilde{f}}} \right) \right)^{1/\alpha} \right) + \sum_{k=1}^{K} \sum_{i=1}^{N_k} \log_2 \left( \frac{\mathbf{\tilde{f}}_k^H \mathbf{A}_{k,i} \mathbf{\tilde{f}}}{\mathbf{\tilde{f}}_k \mathbf{B}_{k,i} \mathbf{\tilde{f}}} \right). \tag{63}
\]

\[\text{Algorithm 2: Alternating Optimization for Multi-Antenna Users}\]

\[\text{initialize: } \mathbf{f}(0)\]

\[\text{Set initial the LMMSE combiners for } \mathbf{V}_k \text{ and } \mathbf{v}_k^{(c)} \text{ as (41) and (42) for } k \in \mathcal{K}.\]

\[\text{Set the iteration count } t = 1.\]

\[\text{while } |R_{c}^{(t)} - R_{c}^{(t-1)}| > \tau \text{ do}\]

\[\text{(Phase 1) Obtain } \mathbf{f}(t) \text{ using Algorithm 1 while fixing } \mathbf{V}_k \text{ and } \mathbf{v}_k^{(c)} .\]

\[\text{(Phase 2) Obtain } \mathbf{V}_k \text{ and } \mathbf{v}_k^{(c)} \text{ using (41) and (42) for } k \in \mathcal{K} \text{ while fixing } \mathbf{f}(t).\]

\[t \leftarrow t + 1.\]

\[\text{end while}\]

Since the newly formulated problem \((63)\) is equivalent to the problem \((30)\), the GPI-RS is applied to obtain the precoders.

As explained above, we iterate the two inner loops (phase 1 and phase 2) until the outer loop termination condition is met. For the outer loop termination condition, we check the increment of the sum spectral efficiency per iteration. If the increasing sum spectral efficiency is less than the predetermined threshold, we terminate the outer loop. Finally, we summarize the whole process of the proposed alternative framework in Algorithm 2.

\[\text{V. Numerical Results}\]

In this section, we evaluate the ergodic sum spectral efficiency to demonstrate the proposed GPI-RS. For the baseline methods, we considered the followings:

- **MRT**: The precoding vectors are designed by matching the estimated channel while ignoring the inter-user interference. Specifically, we have \( \mathbf{f}_k = \mathbf{h}_k^c, k \in \mathcal{K}, \) and \( \mathbf{f}_c = 0.\)

- **RZF**: The precoding vectors are designed by following the ZF rule, while regularizing it depending on SNR:

\[\mathbf{f}_k = \left( \mathbf{H}^H + \frac{\sigma^2}{P} \right) \mathbf{h}_k^c, k \in \mathcal{K}, \text{ and } \mathbf{f}_c = 0. \tag{64}\]

As SNR goes to infinity, RZF is equal to ZF.

- **Sum SE Max with no RS**: In this method, we use the method proposed in [4] to maximize the sum spectral efficiency without considering RSMA. If we enforce \( \mathbf{f}_c = 0 \) so as to prevent transmitting a common message, then the proposed GPI-RS is reduced to this case.

- **WMMSE-SAA**: This case indicates the WMMSE with sample average approximation (SAA) [7], which uses a convex relaxation technique for the sum spectral efficiency maximization with rate-splitting. Specifically, by tuning the MSE weights, this method transforms the original sum spectral efficiency maximization problem into the corresponding WMMSE minimization problem with multiple constraints. For incorporating the CSIT imperfection, the corresponding MSE is approximated by using the sample average approximation. The WMMSE minimization problem is convex; therefore, we can solve this by using the CVX toolbox. This algorithm iteratively performs until convergence.
A. Ergodic Sum Spectral Efficiency

First, we compare the ergodic sum spectral efficiency of the proposed GPI-RS and the other baseline methods. The considered simulation setups are as follows: \( N = 6, K = 4 \), \( \kappa_k = 0.4 \) (FDD), \( r_{ul} p_{ul} = 2 \) (TDD), and \( \sigma^2 = 1 \). Each user’s location is randomly determined, so that the AoA \( \theta_k \) is drawn from the uniform distribution. The angular spread is fixed as \( \Delta_k = \pi/6 \) for \( k \in \mathcal{K} \). For updating \( \alpha \), we set the initial \( \alpha \) value as 0.1 if \( \text{SNR} < 15\text{dB} \), and 0.5 for the rest of the cases. We note that this initial setup is designed empirically. If the GPI-RS loop is not terminated within 50 iterations, we increase \( \alpha \) by 50% and repeat the algorithm. The performance comparison is illustrated in Fig. 2.

As shown in Fig. 2, the proposed GPI-RS provides meaningful spectral efficiency gains over the baseline methods in both TDD and FDD systems. In particular, compared to the WMMSE-SAA at \( \text{SNR} = 40\text{dB} \), the GPI-RS obtains around 19% spectral efficiency gains in the TDD system and 10% spectral efficiency gains in the FDD system. The rationale that the GPI-RS provides more gains in the TDD system is as follows. As shown in (13) and (15), our method uses the error covariance \( \Phi_k \) to regularize the precoding design coping with the CSIT error. In TDD, the error covariance \( \Phi_k \) is more accurately reflected into the precoding design by using MMSE estimation. This leads to the performance improvement.

In addition, we also compare the ergodic sum spectral efficiency by increasing \( N \) and \( K \) to \( N = 12 \) and \( K = 8 \). The other simulation setups are: \( \kappa_k = 0.5 \) (FDD), \( r_{ul} p_{ul} = 5 \) (TDD), and \( \sigma^2 = 1 \). For \( \alpha \) update, we set the initial \( \alpha \) value as 0.75 if \( \text{SNR} < 20\text{dB} \), and 1 for the rest of the cases. Similar to the previous simulation, this is empirically determined. The results are depicted in Fig. 3. In this case, compared to the WMMSE-SAA at \( \text{SNR} = 40\text{dB} \), around 20% spectral efficiency gains are achieved in TDD. In FDD, the spectral efficiency performances of the GPI-RS and the WMMSE-SAA are comparable. As in Fig. 2, we observe that the GPI-RS provides more spectral efficiency performance gains in TDD.

B. Complexity

Even though the algorithm complexity is analyzed in Remark 1 in terms of big-O notation, we also need to study the practical complexity for the proposed GPI-RS. This subsection compares the computational complexity between the GPI-RS and the WMMSE-SAA by measuring the CPU time consumed in each precoder optimization process. The optimization is performed by using MATLAB with the CPU spec: AMD Ryzen Threadripper 2990WX 32-core, 3.00GHz, with 128GB RAM. Considering the 6 \( \times \) 4 case and the 12 \( \times \) 8 case \( (N \times K) \), we plot the average CPU time in Fig. 4. As observed in the figure, the average CPU time for the GPI-RS is 1.0038 (sec) for the 6 \( \times \) 4 case and 3.3545 (sec) for the 12 \( \times \) 8 case, which are only 2.3% \((6 \times 4)\) and 1.9% \((12 \times 8)\) of the WMMSE-SAA. This significant reduction comes from the fact that the proposed GPI-RS does not rely on any optimization toolbox such as CVX, which is the main source of consuming the CPU time. Further, as explained in Remark 1, the complexity of the proposed GPI-RS can be more reduced by computing the matrix inversion separately by exploiting the block-diagonal structure of \( B_{\text{KKT}}(\bar{F}) \). From this result, we conclude that the proposed GPI-RS is more suitable to implement in limited hardware compared to the WMMSE-SAA.

C. Correlation on User Locations

In this subsection, we compare the ergodic sum spectral efficiency under the assumption that the users are located at the same position. By this setup, the AoA \( \theta_k \) are equal and the channel of each user is generated from the same covariance matrix. This makes a favorable environment to deliver a common message since the channel shares the same subspace \( \mathcal{F} \). For the simulation setups, we assume: \( N = 6, K = 4 \), \( \kappa_k = 0.4 \) (FDD), \( \sigma^2 = 1 \), and \( \Delta_k = \pi/6 \) for \( k \in \mathcal{K} \). The \( \alpha \) update is same with Section V-A. We illustrate the results in Fig. 5. In Fig. 5, we observe that around 22% spectral efficiency gains are obtained by using the proposed GPI-RS compared to the WMMSE-SAA. One noticeable point is that the relative gains of the GPI-RS increase compared to the random user location case (recall that 10% gains are obtained in Fig. 2(b)). From this investigation, we claim that the GPI-RS brings more performance gains if the users’ locations are correlated.
D. CSIT Error

Now, we depict the spectral efficiency depending on the CSIT accuracy. Assuming the FDD system, we depict the ergodic sum spectral efficiency as $\kappa_k$ increases. The simulation setups are: $N = 6, K = 4, \sigma^2 = 1, \theta_k \sim \text{Unif}[0, 2\pi], \text{and } \Delta_k = \pi / 6$ for $k \in \mathcal{K}$. The parameter $\kappa_k$ is assumed to be equal to all the users. The plot is in Fig. 6, wherein we drop the subscript $k$ from $\kappa$. The figure shows that more spectral efficiency gains are obtained by the GPI-RS as $\kappa$ decreases, i.e. less CSIT error. At $\kappa = 0.1$, the GPI-RS provides around 20% spectral efficiency gains over the WMMSE-SAA. On the contrary, at $\kappa = 0.6$, the spectral efficiencies of the GPI-RS and the WMMSE-SAA are similar. From this observation, the following conclusion is extracted: the GPI-RS provides more gains when the CSIT error is not severe. This is because our lower bound (13) and (15) might become loose as the CSIT estimation accuracy decreases.

E. Increasing BS Antennas

We also study the spectral efficiency as the number of antennas $N$ increases. As shown in Fig. 7, the proposed GPI-RS brings the performance improvement compared to the baseline mtddtos in all the assumed antenna numbers. Specifically, around 5.5% spectral efficiency gains are obtained compared to the WMMSE-SAA when the number of the antennas $N = 4, 8, 16, 32, 64$. Measuring the average CPU computation time at $N = 64$, the GPI-RS consumes $105 \times 10^3$ (sec), while the WMMSE-SAA consumes $3826 \times 6$ (sec). As a result, the GPI-RS needs only 2.7% of the computation time compared to the WMMSE-SAA. This result implies that the proposed GPI-RS is particularly useful in massive MIMO systems where a large scale antenna array is equipped in the BS [17].
Ergodic Sum Spectral Efficiency (bps/Hz)

method. Thanks to this, the proposed GPI-RS is suitable to
tional complexity is significantly reduced. Specifically, only
performance gains, the remarkable point is that the compu-
error, Especially, more gains are achieved in
optimization properly supports the multiple user antenna cases.
more antennas. This leads to that the proposed alternating
delivered messages increases if each user is equipped with
are dominant in a high SNR regime, and the number of the
users is
\[ N \] as the number of the user antenna increases. In particular,
we observe that more spectral efficiency gains are attained
\[ t \] = 40 dB. This makes sense since multiplexing gains
\[ k \] for \[ N_k = 2 \], and the FDD system is assumed. In Fig. 8,
we observe that more spectral efficiency gains are attained
while the associated complexity is reduced compared to the
existing convex-relaxation based method.

For future work, it is of interest to extend the proposed
method in various ways. For example, it is possible to consider
physical layer security in an RSMA MU-MIMO system [37],
[38]. Investigating the feasibility of RSMA in unconventional
wireless systems, such as the finite blocklength regime where
a non-zero decoding probability is induced [39] or Terahertz
line-of-sight MIMO environments [40] is also promising.

VI. Conclusions

In this paper, we have proposed a novel precoding optimi-
ization method for downlink MU-MIMO systems with rate-
splitting. Aiming to maximize the sum spectral efficiency of
the considered system, we have formulated an optimization
problem, while a sum spectral efficiency maximization prob-
lem regarding linear precoding vectors is infeasible to solve
due to its non-convexity and non-smoothness. To resolve this,
we have approximated a non-smooth minimum function by
using the LogSumExp technique and have reformulated the
problem as a form of Rayleigh quotients by an optimization
variable (precoding vectors) onto a higher dimensional vector.
With this form, we have shown that the first-order optimality
condition is cast as a NEPv, where the corresponding
eigenvalue is equivalent to the objective function. In order to
find a leading eigenvector for the derived condition, we have
proposed the GPI-RS. Simulations have demonstrated that the
proposed method brings significant spectral efficiency gains
antenna users are also well incorporated into the proposed

F. Multiple User Antennas

Lastly, we depict the ergodic sum spectral efficiency where
each user is equipped with multiple antennas. We investigate
the ergodic sum spectral efficiency at \( \text{SNR} = 40 \text{dB}, 50 \text{dB} \)
assuming that each user is equipped with \( N_k = 1, 2, 4 \)
antennas. The BS antenna number is \( N = 8 \), the number of
the users is \( K = 2 \), and the FDD system is assumed. In Fig. 8,
we observe that more spectral efficiency gains are attained
as the number of the user antenna increases. In particular,
the relative performance gains in the \( N_k = 4 \) case increase
at \( \text{SNR} = 50 \text{dB} \). This makes sense since multiplexing gains
are dominant in a high SNR regime, and the number of the
delivered messages increases if each user is equipped with
more antennas. This leads to that the proposed alternating
optimization properly supports the multiple user antenna cases.

In summary, the proposed GPI-RS provides considerable
spectral efficiency gains over the existing WMMSE-SAA. Especially,
more gains are achieved in i) TDD, ii) less CSIT
error, iii) when users’ locations are correlated. Besides these
performance gains, the remarkable point is that the computa-
tional complexity is significantly reduced. Specifically, only
1% ~ 2% of the WMMSE-SAA’s CPU time is needed in our
method. Thanks to this, the proposed GPI-RS is suitable to
use in massive MIMO systems. Furthermore, multiple receive

\[ f \]

\[ \bar{f} \]

\[ \hat{A} \]

\[ \hat{B} \]

\[ \hat{A} \bar{f} \]

\[ \hat{B} \bar{f} \]

\[ \frac{\hat{A} \bar{f}}{\hat{B} \bar{f}} \]

\[ \frac{\hat{A} \bar{f}}{\hat{B} \bar{f}} \]

\[ \frac{\hat{A} \bar{f} - \hat{B} \bar{f}}{\hat{B} \bar{f}} \]

APPENDIX A

PROOF OF LEMMA 1

We first derive the KKT condition of the problem [30]. The
corresponding Lagrangian function is defined as

\[
L(\bar{f}) = \log \left( \frac{1}{K} \sum_{k=1}^{K} \exp \left( \log_2 \left( \frac{\hat{A}_c(k)\bar{f}}{\hat{B}_c(k)\bar{f}} \right) \right)^{-\alpha} + \sum_{k=1}^{K} \log_2 \left( \frac{\hat{A}_k\bar{f}}{\hat{B}_k\bar{f}} \right) \right).
\]

To find a stationary point, we take the partial derivatives of
\( L(\bar{f}) \) with respect to \( \bar{f} \) and set it to zero. For simplicity, we
denote the first and the second part of the Lagrangian function
as \( L_1(\bar{f}) \) and \( L_2(\bar{f}) \), respectively. Since we have

\[
\hat{f} \left( \frac{\hat{A} \bar{f}}{\hat{B} \bar{f}} \right) / \hat{B} \bar{f} = \left( \frac{\hat{A} \bar{f}}{\hat{B} \bar{f}} \right) - \frac{\hat{A} \bar{f} - \hat{B} \bar{f}}{\hat{B} \bar{f}} \right).
\]

\[
\hat{f} \left( \frac{\hat{A} \bar{f}}{\hat{B} \bar{f}} \right) / \hat{B} \bar{f} = \left( \frac{\hat{A} \bar{f}}{\hat{B} \bar{f}} \right) - \frac{\hat{A} \bar{f} - \hat{B} \bar{f}}{\hat{B} \bar{f}} \right).
\]
the partial derivative of $L_1(\mathbf{f})$ is obtained using the above calculation:

$$\frac{\partial L_1(\mathbf{f})}{\partial \mathbf{f}^H} = \sum_{k=1}^{K} \left[ \exp \left( \frac{1}{\alpha} \log_2 \left( \frac{\mathbf{P}^H \mathbf{A}_k(\mathbf{f})}{\mathbf{P}^H \mathbf{B}_k(\mathbf{f})} \right) \right) \mathbf{A}_k(\mathbf{f}) \right] \times \left( \log_2 \left( \frac{\mathbf{P}^H \mathbf{A}_k(\mathbf{f})}{\mathbf{P}^H \mathbf{B}_k(\mathbf{f})} \right) \right) \frac{\partial}{\partial \mathbf{f}^H}$$

$$= \frac{1}{\log 2} \sum_{k=1}^{K} \left[ \exp \left( \frac{1}{\alpha} \log_2 \left( \frac{\mathbf{P}^H \mathbf{A}_k(\mathbf{f})}{\mathbf{P}^H \mathbf{B}_k(\mathbf{f})} \right) \right) \mathbf{A}_k(\mathbf{f}) \right]$$

$$= \left[ \frac{\mathbf{A}_k(\mathbf{f})}{\mathbf{P}^H \mathbf{A}_k(\mathbf{f})} - \frac{\mathbf{B}_k(\mathbf{f})}{\mathbf{P}^H \mathbf{B}_k(\mathbf{f})} \right] + \sum_{k=1}^{K} \left[ \frac{\mathbf{A}_k(\mathbf{f})}{\mathbf{P}^H \mathbf{A}_k(\mathbf{f})} - \frac{\mathbf{B}_k(\mathbf{f})}{\mathbf{P}^H \mathbf{B}_k(\mathbf{f})} \right] = 0.$$  

(67)

Similar to this, we calculate $L_2(\mathbf{f})/\partial \mathbf{f}^H$ as follows.

$$\frac{\partial L_2(\mathbf{f})}{\partial \mathbf{f}^H} = -\log 2 \sum_{k=1}^{K} \left[ \frac{\mathbf{A}_k(\mathbf{f})}{\mathbf{P}^H \mathbf{A}_k(\mathbf{f})} - \frac{\mathbf{B}_k(\mathbf{f})}{\mathbf{P}^H \mathbf{B}_k(\mathbf{f})} \right].$$  

(68)

The first-order KKT condition holds when

$$\frac{\partial L_1(\mathbf{f})}{\partial \mathbf{f}^H} + \frac{\partial L_2(\mathbf{f})}{\partial \mathbf{f}^H} = 0$$

(69)

$$\Leftrightarrow \sum_{k=1}^{K} \left[ \exp \left( \frac{1}{\alpha} \log_2 \left( \frac{\mathbf{P}^H \mathbf{A}_k(\mathbf{f})}{\mathbf{P}^H \mathbf{B}_k(\mathbf{f})} \right) \right) \mathbf{A}_k(\mathbf{f}) \right] + \sum_{k=1}^{K} \left[ \frac{\mathbf{A}_k(\mathbf{f})}{\mathbf{P}^H \mathbf{A}_k(\mathbf{f})} - \frac{\mathbf{B}_k(\mathbf{f})}{\mathbf{P}^H \mathbf{B}_k(\mathbf{f})} \right] = 0.$$  

(70)

Defining $\mathbf{A}_{KKT}(\mathbf{f})$, $\mathbf{B}_{KKT}(\mathbf{f})$, and $\lambda(\mathbf{f})$ as

$$\mathbf{A}_{KKT}(\mathbf{f}) = \lambda_{\text{num}}(\mathbf{f}) \times \sum_{k=1}^{K} \left[ \exp \left( \frac{1}{\alpha} \log_2 \left( \frac{\mathbf{P}^H \mathbf{A}_k(\mathbf{f})}{\mathbf{P}^H \mathbf{B}_k(\mathbf{f})} \right) \right) \mathbf{A}_k(\mathbf{f}) \right],$$

$$\mathbf{B}_{KKT}(\mathbf{f}) = \lambda_{\text{den}}(\mathbf{f}) \times \sum_{k=1}^{K} \left[ \exp \left( \frac{1}{\alpha} \log_2 \left( \frac{\mathbf{P}^H \mathbf{A}_k(\mathbf{f})}{\mathbf{P}^H \mathbf{B}_k(\mathbf{f})} \right) \right) \mathbf{B}_k(\mathbf{f}) \right],$$

$$\lambda(\mathbf{f}) = \left( \frac{1}{K} \sum_{k=1}^{K} \left[ \exp \left( \frac{1}{\alpha} \log_2 \left( \frac{\mathbf{P}^H \mathbf{A}_k(\mathbf{f})}{\mathbf{P}^H \mathbf{B}_k(\mathbf{f})} \right) \right) \mathbf{P}^H \mathbf{A}_k(\mathbf{f}) \right] \right) \times \left( \frac{\lambda_{\text{num}}(\mathbf{f})}{\lambda_{\text{den}}(\mathbf{f})} \right).$$

(71)

(72)

(73)

the first-order KKT condition is rearranged as

$$\mathbf{A}_{KKT}(\mathbf{f}) = \frac{\lambda(\mathbf{f})}{\lambda_{\text{den}}(\mathbf{f})} \mathbf{B}_{KKT}(\mathbf{f})$$

(74)

This completes the proof. □
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