Measurement of muon polarization
in $K^+ \rightarrow \pi^0 \mu^+ \nu_\mu$ at a $\phi$ factory

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Abstract

The potentiality of an experiment measuring the muon polarization in $K^+ \rightarrow \pi^0 \mu^+ \nu_\mu$ at a $\phi$ factory is discussed, with particular attention to the possibility of revealing unexpected violation of Time reversal invariance through the transverse polarization component. An experimental method is proposed which is based on the reconstruction of the $K^+ \rightarrow \pi^0 \mu^+ \nu_\mu$ decay products after stopping the charged kaons in a target placed around the beam pipe. The muon polarization is measured by observing the asymmetry in the angular distribution of positrons from the $\mu^+ \rightarrow e^+ \nu \bar{\nu}$ decay of muons stopped in a polarization analyser. It is shown that an error $\sigma_P \approx 5 \cdot 10^{-4}$ on each muon polarization component can be obtained, which represents a factor ten improvement with respect to current limits on T invariance violation in the $K^+ \rightarrow \pi^0 \mu^+ \nu_\mu$ decay and a precise measurement of the $Re\xi$ parameter at the level of $2 \cdot 10^{-3}$. 
1 Introduction

A precision study of the CP violation parameters in the $K^0 - \bar{K}^0$ system is the central item of the physics program at a $\phi$ factory [1]. On the other hand, a $\phi$ factory will produce a large statistics ($\approx 10^{10}$ per year) of $K^+K^-$ pairs. Significant improvements in the understanding of the decay and interaction of charged kaons are expected from the approved experiments [2] [3] at the Frascati $\phi$ factory [4].

In this context, a measurement of muon polarization in the semileptonic decay $K^+ \rightarrow \pi^0\mu^+\nu_\mu$ ($K^+_{\mu3}$) would be very interesting. In fact, the component of the muon polarization transverse to the decay plane is sensitive to possible violation of Time reversal (T) invariance, as first noticed by Sakurai [5]. The transverse muon polarization

$$P_T = \frac{\vec{s}_{\mu^+} \cdot (\vec{p}_{\pi^0} \times \vec{p}_{\mu^+})}{|\vec{p}_{\pi^0} \times \vec{p}_{\mu^+}|},$$

(1)

in which $\vec{s}_{\mu^+}$ is the muon spin vector, $\vec{p}_{\mu^+}$ and $\vec{p}_{\pi^0}$ are the muon and neutral pion momenta, changes sign under the T-reversal operation. A non-zero value of $P_T$ would evidentiate a T violation.

In the standard model, where CP violation is introduced with an imaginary phase in the Cabibbo-Kobayashi-Maskawa matrix elements [6], $P_T$ is expected to be very small ($\approx 10^{-6}$). Also, the presence of electromagnetic interaction (even if T invariant) in the final state can produce a non-zero $P_T$, estimated to be $10^{-6}$ for $K^+_{\mu3}$ [4]. Thus, the measurement of a non-zero value ($> 10^{-6}$) of $P_T$ would be a definite sign of physics beyond the standard model. On the other hand, there are several models [8] [9] containing charged Higgs particles or other non-standard scalar interactions like lepto-quarks that do not violate existing experimental constraints from the neutron dipole moment and $\epsilon'/\epsilon$ and allow a transverse muon polarization as large as $P_T \approx 10^{-3}$.

The magnitude of the other two components of the muon polarization is determined by the form factors describing the decay $K^+ \rightarrow \pi^0\mu^+\nu_\mu$. A precise measurement of the total muon polarization would allow the verification of predictions of the modern chiral symmetry theory [10].

In this paper, the feasibility of an Experiment for T Invariance in KAon decay (ETIKA) at a $\phi$ factory is analysed. Section 2 is devoted to the description of the theoretical framework of the $K^+ \rightarrow \pi^0\mu^+\nu_\mu$ decay. The achievable accuracy of the muon polarization measurement at a $\phi$ factory is analysed in Section 3. In Section 4 the essential properties and requirements of the ETIKA experiment are discussed. Other physics topics are presented in Section 5. Lastly, the conclusions are given.

2 Theoretical Framework

In the usual V-A theory of the $K^+_{\mu3}$ decay, the matrix element governing the decay is proportional to $< \pi | J_\lambda | K >$, in which $J_\lambda$ is the strangeness-changing hadronic current. The hadronic vertex associated to the current is a function of three four-vectors, only two
of which are independent by energy-momentum conservation. The combinations \( P_K - P_\pi \) and \( P_K + P_\pi \) are usually chosen as basis vectors, where \( P_K \) and \( P_\pi \) are the kaon and pion four-vectors, respectively. Also, there is only one independent scalar, apart from the kaon and pion masses, which can be formed from the basis vectors. It is usually chosen to be \( q^2 = (P_K - P_\pi)^2 \). Being \( < \pi | J_\lambda | K > \) a four-vector which represents the hadronic vertex, it must be expressible as a linear combination of the basis vectors, with coefficients at most depending on \( q^2 \). The standard expression is

\[
< \pi | J_\lambda | K > = f_+(q^2)(P_K + P_\pi)_\lambda + f_-(q^2)(P_K - P_\pi)_\lambda.
\]

In this formalism, all experimental quantities except absolute rates can be written as a function of only one complex parameter \( \xi(q^2) \) given by the ratio of \( f_-(q^2) \) to \( f_+(q^2) \). The Dalitz plot density in the kaon rest frame is then given by

\[
\frac{d^2N}{dE_\pi dE_\mu} \propto |f_+(q^2)| (A + BRe\xi + C|\xi|^2),
\]

in which

\[
A = m_K(2E_\mu E_\nu - m_K E'_\pi) + m_\mu^2 \left( \frac{1}{4} E'_\pi - E_\nu \right),
\]

\[
B = m_\mu^2 (E_\nu - \frac{1}{2} E'_\pi),
\]

\[
C = \frac{1}{4} m_\mu^2 E'_\pi,
\]

\[
E'_\pi = E'^{\text{max}}_\pi - E_\pi, \quad E^{\text{max}}_\pi = \frac{m_K^2 + m_\pi^2 - m_\mu^2}{2m_K},
\]

in which \( m_i \) and \( E_i \) are the energy and mass of particle \( i \). The Dalitz plot density of the \( K^+ \rightarrow \pi^0 \mu^+ \nu_\mu \) decay, corresponding to equation (3) and the Particle Data Group (PDG) values \( Re\xi = -0.35 \pm 0.15 \) and \( Im\xi = -0.017 \pm 0.025 \), is plotted in Fig. \[1\].

Expressions for the muon polarization in \( K_{\mu3}^+ \) decays have been given by several authors \[13\]. Since the neutrino from the decay has unit elicity, the muon moving with a given angle and momentum relative to the neutrino has a unique spin orientation. In the K rest frame, the muon polarization is then a unit vector whose components are uniquely specified by the decay kinematics. The expression given by Cabibbo and Maksymowics \[13\] for the muon polarization in the kaon rest frame is \( \vec{P} = \vec{A}/|\vec{A}| \), in which

\[
\vec{A} = a_1(\xi)\vec{p}_\mu - a_2(\xi) \left[ \frac{p_\mu}{m_\mu} \left( (m_K - E_\pi) + \frac{p_\mu \cdot (E_\mu - m_\mu)}{|p_\mu|^2} \right) + \vec{p}_\pi \right] + m_K Im\xi (\vec{p}_\pi \times \vec{p}_\mu),
\]

with

\[
a_1(\xi) = 2 \frac{m_K^2}{m_\mu} [E_\mu + Reb(q^2)E'_\pi],
\]

\[
a_2(\xi) = m_K^2 + 2 Reb(q^2)m_K E_\mu + |b(q^2)|^2 m_\mu^2,
\]

\[
b(q^2) = \frac{\xi(q^2) - 1}{2},
\]

\[1\] For the sake of simplicity, the notation \( K, \mu \) and \( \pi \) instead of \( K^+, \mu^+ \) and \( \pi^0 \) will be used in the following.
in which $\vec{p}_\mu$ and $\vec{p}_\pi$ are the momenta of the muon and pion, respectively, in the kaon rest frame.

Notice that $\vec{P} = \vec{P}(P_\mu, P_\pi, \xi)$. Having measured the kinematics of an event (i.e. the four-vectors $P_\mu$ and $P_\pi$), the actual value of $\vec{P}$ is determined by $\xi$. In particular, for each event a reference frame can be defined having axes $\hat{z} = \vec{p}_\mu/|\vec{p}_\mu|$, $\hat{x} = \vec{p}_\pi \times \vec{p}_\mu/|\vec{p}_\pi \times \vec{p}_\mu|$, and $\hat{y} = \hat{z} \times \hat{x}$. The longitudinal component of the muon polarization in this reference frame is then $P_L = \vec{P} \cdot \hat{z}$, the perpendicular component is $P_T = \vec{P} \cdot \hat{y}$ and the transverse component is $P_T = \vec{P} \cdot \hat{x}$. The $P_T$ component is exactly the T-odd observable identified in equation (1). According to equation (1), $P_T$ is proportional to $|\Im \xi|$, and if T invariance is conserved one expects $P_T$ to be zero. On the other hand, the $P_L$ and $P_T$ components are sensitive to $\Re \xi$. The values averaged over the Dalitz plot are $<P_L> \simeq -0.75$, $<P_T> \simeq -0.50$ and $<P_T> \simeq 0.22|\Im \xi|$.

From the experimental point of view, it is important to identify the regions of the Dalitz plot where the best sensitivity to $\xi$ is obtained from the measurement of the muon polarization. For example, one can use the variation of the polarization $|\Delta \hat{P}|$ in a given point of the Dalitz plot $(E_\mu, E_\pi)$ for a fixed change of $\Re \xi$ or $\Im \xi$. Since the error on the measurement of a polarization scales as $\simeq 1/\sqrt{N}$, where $N$ is the number of events in the point $(E_\mu, E_\pi)$ of the Dalitz plot, the ratio $|\Delta \hat{P}|/(1/\sqrt{N}) = |\Delta \hat{P}|\sqrt{N}$ is a good estimator of the sensitivity. A measure of the sensitivity to $\Im \xi$ is obtained by $|\Delta \hat{P}_T|\sqrt{N}$. A measure of the sensitivity to $\Re \xi$ is given by $|\Delta \hat{P}_{\text{plane}}|\sqrt{N}$, where $\hat{P}_{\text{plane}} = P_L\hat{z} + P_T\hat{y}$ is the component of the muon polarization lying on the decay plane. The sensitivity over the Dalitz plot is almost identical for the two cases, and contour lines corresponding to this estimator are shown in Fig. 2. It is clear that the best sensitivity to both $\Im \xi$ and $\Re \xi$ is obtained in the region of low $\pi^0$ energy and high $\mu^+$ energy. However, the sensitivity to $\xi$ is still significant over most of the Dalitz plot area, allowing a non-critical definition of the experimental acceptance.

### 3 The Achievable Accuracy

The DAΦNE $\phi$ factory is expected to collide $e^+e^-$ beams with a luminosity of $5 \cdot 10^{32}\text{cm}^{-2}\text{sec}^{-1}$, which corresponds to $\simeq 2.2 \cdot 10^{10}$ $\phi$ produced in a year of $10^7$ seconds. Thus, $\simeq 1.1 \cdot 10^{10}$ $K^+K^-$ from the $\phi$ decay will be produced in one year of running. Assuming a branching ratio of 3.18% [12] for $K_{\mu3}^+$, a number $N = 3.5 \cdot 10^8$ of $K^+ \rightarrow \pi^0 \mu^+\nu_\mu$ will be available. In order to measure the muon polarization, the subsequent spin analysing decay $\mu^+ \rightarrow e^+\nu\bar{\nu}$ must be used. After integration over the positron energy spectrum, the decay angular distribution in the muon rest frame is:

$$f(x, P) = \frac{1}{2} \left( 1 + \frac{P}{3} x \right),$$

in which $P$ is the muon polarization, and $x = \hat{P} \cdot \vec{p}_{e^+}$ is the cosinus of the angle between the polarization unit vector $\hat{P}$ and the unit vector along the positron momentum. An estimate of the error on the muon polarization obtained using the polarization sensitivity
of equation (5) is given by [14]:

\[
\sigma_P = \frac{1}{\sqrt{N}} \left[ \int_{-1}^{+1} \frac{1}{f} \left( \frac{\partial f}{\partial P} \right)^2 \, dx \right]^{-\frac{1}{2}} = \frac{1}{S\sqrt{N}}.
\]  

(6)

For example, in the case of the T violating polarization \( P_T \), which is \( \simeq 0 \), \( S \simeq 0.19 \) and the expected error is:

\[
\sigma_P \simeq \frac{1}{0.19\sqrt{3.5 \times 10^8}} \simeq 2.8 \times 10^{-4}.
\]  

(7)

Similar errors are also expected for the \( P_L \) and \( P_P \) components.

It should be noticed that an improved sensitivity is obtained if the full \( f(E_{e^+}, x, P) \) distribution is used, once appropriate cuts on the positron energy \( E_{e^+} \) and angle are applied. In fact, it can be shown [13] that \( S \) can be as large as 0.26, thus reducing the error quoted in equation (7) to 2.0 \( \times 10^{-4} \).

The best existing measurement of \( P_T \) [16] is consistent with zero with an error of 5 \( \times 10^{-3} \). Measurement of \( P_L \) and \( P_P \) are quite old [12] and have large errors. An error \( \sigma_P \simeq 5 \times 10^{-4} \) would represent a factor ten improvement on the sensitivity to T violating effect in \( K_{\mu3}^+ \). Also, it corresponds to a precise measurement of \( Re\xi \), with an error \( \simeq 2 \times 10^{-3} \), through the \( P_L \) and \( P_P \) components. In order to reach such precision, the overall acceptance of the ETIKA experiment at the \( \phi \) factory should be in the range of \( \epsilon = 15 - 30\% \) (depending on the actual value of \( S \)), which includes the fraction of accepted \( K^+ \) from the \( \phi \) decay, the efficiency of reconstructing the \( K^+ \rightarrow \pi^0\mu^+\nu_\mu \) decay, and the acceptance over the spin analysing decay \( \mu^+ \rightarrow e^+\nu\bar{\nu} \).

If a luminosity of \( 10^{33} \text{cm}^{-2}\text{sec}^{-1} \) should be reached, the quoted errors will improve by a factor \( \sqrt{2} \). Alternatively, the overall acceptance needed to reach a precision of \( \simeq 5 \times 10^{-4} \) would reduce to 8 \( - 15\% \).

4 A \( \phi \) Factory ETIKA Experiment

The measurement of the muon polarization is performed with the standard technique of stopping the positive muons in a non-depolarizing material, like Carbon or Aluminium, and then observing the asymmetry in the angular distribution of the positrons from the \( \mu^+ \rightarrow e^+\nu\bar{\nu} \) decay. It should be noted that the decay \( \phi \rightarrow K^+K^- \) allows in principle also the study of \( K^- \rightarrow \pi^0\mu^-\bar{\nu}_\mu \), when the \( K^- \) decay is observed in flight. However, stopped negative muons are rapidly captured in atomic orbits and almost completely depolarized. Thus, the \( K^- \rightarrow \pi^0\mu^-\bar{\nu}_\mu \) decay cannot be used for the measurement of the muon polarization in charged kaon semileptonic decays, and the ETIKA experiment must be able to distinguish between \( K^+ \) and \( K^- \) decay products.

A possible solution is the kaon charge identification by tracking particles curvature in a magnetic field. On the other hand, the presence of a magnetic field will cause the spin of stopped muons to precess. The net effect will be a significant reduction of the
sensitivity, with a corresponding increase by a factor of three of the expected error $\sigma_P$ given by equation (6). Moreover, the presence of radial components of the field which are typically at the level of a few gauss can introduce systematic effects which are difficult to control. The polarization analyser could be placed in a magnetic shield, but, due to the high value of the magnetic field (0.5-1 Tesla), the level of 0.1 gauss inside the shield needed to reduce the systematic uncertainties would be hardly reachable [17]. In conclusion, this approach looks very difficult and, even if feasible, complicated and costly.

A different experimental method is proposed, based on stopping charged kaons in a target surrounding the $e^+e^-$ interaction point, which provides a good acceptance with detectors of reasonable dimensions and the separation of $K^+$ and $K^-$ without a magnetic field. Charged kaon pairs produced at the DAΦNE factory will have $\approx 16$ MeV kinetic energy, and stop in about 0.5 g/cm$^2$ thickness of material. As a matter of fact, a Carbon target surrounding the DAΦNE beam pipe is the key element for the study of hypernuclear states proposed by the FINUDA experiment. A schematic picture of the ETIKA experiment is shown in Fig. 3. The Beryllium beam pipe is surrounded by segmented scintillators which represent the active target. A positively charged kaon stops into the scintillator target, and its subsequent decays can then be observed. On the other hand, the $K^-$ brought to rest into the target suffers nuclear capture, giving rise to characteristic "stars" from nuclear evaporation. Low energy heavy fragments, $\Lambda$ and $\Sigma$ particles emitted in the process of $K^-$ absorption give distinct signals in scintillators and tracking detectors placed around the target, when compared with the single muon track coming from the $K^+$ decay. Also, the prompt signal ($\leq \Lambda$ lifetime) from the products of $K^-$ absorption can be used to efficiently tag the $K^+$ decay having a 12 ns lifetime. The energy deposited in the target scintillators by the charged kaons and the back to back topology of the $K^+K^-$ events is used for the first level trigger of the experiment. Also, a tracking detector is placed between the scintillator target and the beam pipe, in order to precisely determine the $K^+$ stopping point into the target.

Low mass tracking chambers are placed between the target scintillators and the polarization analyser in order to measure precisely the $\mu^+$ direction of flight. The polarization analyser detector, in its simplest configuration, is composed by layers of non-depolarising absorber, interleaved by tracking detector planes, which track the muon and determine its range and also measure the positron from the $\mu^+ \rightarrow e^+\nu\bar{\nu}$ decay. A resolution of $\approx 3$ MeV on the muon energy is obtained on the basis of the stopping layer by using thin absorbers (for example 5 mm thick Carbon layers). Photons from the $\pi^0$ decay are reconstructed in the electromagnetic calorimeter (not shown in Fig. 3) placed around the polarization analyser detector. The ETIKA detector would fit in a cylinder of $\approx 3$ m diameter and $\approx 3$ m length.

A simulation [18], in which the polarization analyser is formed by 70 Carbon layers with 5 mm thickness, showed that an overall acceptance $\epsilon \approx 20\%$ can be obtained. It is clear that this value for the efficiency $\epsilon$ is only a reasonable estimate, and a detailed Monte Carlo simulation of the individual detectors is needed in order to be more precise. Nevertheless, this estimate of the acceptance is well within the range quoted in Section 3 ($\epsilon \approx 15 - 30\%$), and it shows that an error $\sigma_P \approx 5 \cdot 10^{-4}$ on the $T$ violating muon polarization is achievable with the experimental method proposed.

The effect of detectors resolutions was estimated by a Monte Carlo simulation. Energy
and angular resolutions were chosen on the basis of the performances of available detector techniques. In particular, a resolution of 10 mrad was assumed for both the polar ($\theta$) and azimuthal ($\phi$) angle reconstruction of positive muons, together with 3 MeV resolution in energy. $\sigma_{E_{\gamma}}/E_{\gamma} = 5%/\sqrt{E_{\gamma}}$ and a granularity corresponding to 20 mrad angular resolution in $\theta$ and $\phi$ was taken for the electromagnetic calorimeter. The direction of photons interacting in the polarization analyser was assumed to be reconstructed with 70 mrad precision on both angles. The resolution on the positron direction was taken to be 120 mrad in $\theta$ and $\phi$. The simulation showed that detector resolutions and the procedure of reconstruction of the $K^+ \rightarrow \pi^0 \mu^+ \nu_\mu$ kinematics do not spoil the sensitivity to the polarization measurement and do not introduce significant systematic effects.

A large part of the systematic uncertainties on the muon polarization measurement can be estimated from the data themselves. For example, the decay $K^+ \rightarrow \mu^+\nu$ ($K^+_{\mu2}$) provides a copious source of monoenergetic muons, which will be univocally identified by their long range into the polarization analyser. The characteristics of $K^-$ interaction into the target will be precisely determined looking at the detector signals in events tagged by a $K^+ \rightarrow \mu^+\nu$ decay. Also, these events allow a calibration of the muon polarization measurement, since muons from the $K^+_{\mu2}$ decay have $P_L = -1$ and zero transverse components, and any systematic effect will show up in the polarization sensitive positron angular distribution. The identification and kinematics reconstruction of the $\pi^0$ from the $K^+ \rightarrow \pi^0 \mu^+\nu_\mu$ decay can be studied with the two-body decay $K^+ \rightarrow \pi^+\pi^0$, which features a monoenergetic $\pi^+$ and two photons reconstructed in the electromagnetic calorimeter or polarization analyser. The energies of the photons can be predicted by applying momentum conservation, the photons directions being measured by the detectors. This will allow a calibration of the detectors energy response to photons. Also, the procedure of kinematics reconstruction of events in which one photon is measured in the electromagnetic calorimeter and the other photon is measured in the polarization analyser will be tested.

Background events in the sample of $K^+ \rightarrow \pi^0 \mu^+\nu_\mu$ candidates come mainly from other $K^+$ decays. In fact, a pion stopping in the range detector will decay into a muon of 4 MeV kinetic energy, which will stop typically in the same layer where the originating pion stopped, and then decay through $\mu^+ \rightarrow e^+\nu\bar{\nu}$. Thus, $K^+ \rightarrow \pi^+\pi^0$ or $K^+ \rightarrow \pi^+\pi^0\pi^0$ (in which two of the photons coming from the decays of the neutral pions are not reconstructed) will mimic a $K^+ \rightarrow \pi^0 \mu^+\nu_\mu$ event. However, these events can be in large part rejected by kinematics constraints and requirements on the momentum of the observed particles. In any case, expected backgrounds will at most reduce the sensitivity to the muon polarization measurement, but they do not produce any significant spurious component of the polarization.

The presence of a magnetic field can introduce systematic effects in the polarization measurement. In fact, the muon polarization will precess around the field, generating spurious components of the polarization. The expected effects in the case of the ETIKA experiment are small, since, given the presence of a magnetic field along a fixed direction, the muon polarization of muons from the $K^+ \rightarrow \pi^0 \mu^+\nu_\mu$ decay will be isotropically distributed around this direction, and any spurious component will tend to cancel. This statement was checked with a Monte Carlo simulation, which showed that enclosing the ETIKA detector in a magnetic shield in order to reduce the earth magnetic field (typically
a few tenths of gauss) and any other spurious field to less than 0.1 gauss, will bring the systematic uncertainty to a negligible level.

5 Other Physics Topics

The specificity of the ETIKA experiment can be used to measure the muon polarization in the $K^+ \rightarrow \mu^+ \nu_\mu \gamma$ decay. Also in this case the transverse component of the muon polarization, proportional to $(\vec{p}_\mu \times \vec{p}_\gamma) \cdot s^*_\mu$, can reveal a possible violation of the T invariance. The transverse muon polarization in $K^+ \rightarrow \mu^+ \nu_\mu \gamma$ is negligible within the standard model, but the electromagnetic final state interaction can mimic T violation effects as large as $10^{-3}$ [19]. However, the contributions from final state interaction can in principle be precisely calculated, thus leaving the possibility of searching for T violation below the $10^{-3}$ level. Also, the knowledge of the expected value of the transverse polarization in the $K^+ \rightarrow \mu^+ \nu_\mu \gamma$ decay can be used to calibrate the polarization measurement. The expected sensitivity of the ETIKA experiment for the $K^+ \rightarrow \mu^+ \nu_\mu \gamma$ decay mode is similar to that of the $K^+ \rightarrow \pi^0 \mu^+ \nu_\mu$ mode.

Apart from the specific capability of measuring the muon polarization, the detector is able to reconstruct the kinematics of the kaon decay. For example, the Dalitz plot density of $K^+ \rightarrow \pi^0 \mu^+ \nu_\mu$ given in equation (3) can be studied, which will provide an independent measurement of $Re\xi$. Precise measurements of the branching ratios and Dalitz plot densities of the kaon decay modes will be performed, as well as a determination of the charged kaon lifetime.

6 Conclusions

The potentiality of an experiment measuring the muon polarization in $K^+ \rightarrow \pi^0 \mu^+ \nu_\mu$ at a $\phi$ factory was discussed. Particular attention was paid to the possibility of revealing unexpected violation of Time reversal invariance through the transverse polarization component. An experimental method is proposed which is based on the reconstruction of the $K^+ \rightarrow \pi^0 \mu^+ \nu_\mu$ decay products after stopping the charged kaons in a target placed around the beam pipe. The muon polarization is measured by observing the asymmetry in the angular distribution of positrons from the $\mu^+ \rightarrow e^+ \nu \bar{\nu}$ decay of muons stopped in a polarization analyser. The experiment does not use a magnetic field, which limits the cost and dimensions of the detector. It was shown that an error $\sigma_P \simeq 5 \cdot 10^{-4}$ on each muon polarization component can be obtained, which represents a factor ten improvement with respect to current limits on T invariance violation in the $K^+ \rightarrow \pi^0 \mu^+ \nu_\mu$ decay and a precise measurement of the $Re\xi$ parameter at the level of $2 \cdot 10^{-3}$. The systematic uncertainties are expected to be negligible. The detector is well suited to perform also other precise measurements, like the muon polarization in $K^+ \rightarrow \mu^+ \nu_\mu \gamma$ and the study of the charged kaon decay modes. The ETIKA experiment could be a very interesting option for the long term physics program at a $\phi$ factory.
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Figure 1: Contours indicating the relative phase space population of the $K^{+}_{\mu3}$ Dalitz plots
Figure 2: Contours indicating the sensitivity to $\xi$ over the Dalitz plot from muon polarization measurement.
Figure 3: Schematic picture of the ETIKA experiment