Mixed Dimension Embedding with Application to Memory-Efficient Recommendation Systems

A Preprint

Antonio A. Ginart\(^{\ast,1}\), Maxim Naumov\(^{2}\), Dheevatsa Mudigere\(^{2}\), Jiyan Yang\(^{2}\), and James Zou\(^{1}\)

\(^1\)Stanford University, Palo Alto, CA, USA
\(^2\)Facebook, Menlo Park, CA, USA

September 27, 2019

Abstract

In many real-world applications, e.g., recommendation systems, certain items appear much more frequently than other items. However, standard embedding methods—which form the basis of many ML algorithms—allocate the same dimension to all of the items. This leads to statistical and memory inefficiencies. In this work, we propose mixed dimension embedding layers in which the dimension of a particular embedding vector can depend on the frequency of the item. This approach drastically reduces the memory requirement for the embedding, while maintaining and sometimes improving the ML performance. We show that the proposed mixed dimension layers achieve a higher accuracy, while using 8× fewer parameters, for collaborative filtering on the MovieLens dataset. Also, they improve accuracy by 0.1% using half as many parameters or maintain baseline accuracy using 16× fewer parameters for click-through rate prediction task on the Criteo Kaggle dataset.

Keywords embedding representations · recommendation systems · collaborative filtering · click-through rate prediction

1 Introduction

It is difficult to overstate the impact of representation learning and embedding-based models in the present AI landscape. Embedding representations power state-of-the-art applications in diverse domains, including computer vision [7, 53], natural language processing [13, 33, 31, 49, 2, 43, 32], computational biology [5], and recommendation systems [12, 41, 54].

There seems to be a fundamental trade-off between the dimension of an embedding representation and the statistical performance (i.e., accuracy or loss) of embedding-based models on a particular learning task. It is well-documented that statistical performance suffers when embedding dimension is too low [55]. Thus, we are interested in the fundamental question: Is it possible to re-architect embedding representations for a more favorable trade-off between number of parameters and statistical performance?

This challenge is particularly prominent in recommendation systems tasks such as collaborative filtering (CF) and click-through rate (CTR) prediction problems. Recommendation models power some of the most commonplace data-driven services on the internet that benefit users across the globe with personalized experiences on a daily basis [1, 23, 25, 44].

At present, the out-sized memory consumption of standard embedding layers in these recommendation systems is a major burden on the memory hierarchy – an embedding layer for a single recommendation engine can consume tens of gigabytes of space [41, 44]. This makes the engineering challenges associated with large-scale recommendation embeddings of particular importance and interest.

\(^\ast\)Work done while at Facebook
In many models, embedding layers are used to map input categorical values into a vector space. This feature mapping outputs a $d$-dimensional vector for each value, and is learned concurrently with the model during supervised training. However, the relationship between data distributions, embedding representations, model architectures, and optimization algorithms is highly complex. This makes it difficult to design efficient quantization and compression algorithms for embedding representations from information-theoretic principles.

Nevertheless, the distributions of accesses for users and items is often heavily skewed in many real-world applications. For instance, for the CF task on MovieLens dataset\(^2\) the top 10% of users receive as many queries as the remaining 90% and the top 1% of items receive as many queries as the remaining 99%. To an even greater extent, for the CTR prediction task on the Criteo AI Labs Ad Kaggle dataset\(^3\) the top 0.0003% of indices receive as many queries as the remaining 30 million, as summarized on Fig. 1. In networks science, this phenomena is referred to as *popularity* \([14, 39]\), a terminology we adopt here.

![Figure 1: Histogram of accesses for different embedding vectors](image)

We propose mixed dimension embedding layers, a novel architecture for embedding representations. The central thesis behind mixed dimension embedding layers is that the dimension of a particular embedding vector should not remain uniform, but should scale with that feature’s popularity. In particular, we show that popular embedding features are often not allotted sufficient parameters, whereas infrequent embeddings waste them. By architecting mixed dimension embedding layers such that the dimension of an embedding vector scales with its popularity, we can make significant improvements to statistical performance at small parameter budgets.

In order to illustrate that the proposed mixed dimension embedding layers greatly improve parameter-efficiency we show that mixed dimension layers achieve slightly lower loss, while using $8 \times$ fewer parameters compared to uniform dimension embeddings on the Movielens dataset \([21]\). Also, we show that mixed dimension embeddings improve accuracy by 0.1% using half as many parameters and maintain the baseline accuracy using $16 \times$ fewer parameters on the Criteo Kaggle dataset \([29]\).

We point out that even though we focus on recommendation and and event-probability prediction problems, our methods are applicable to representation learning in general. We state our main contributions next.

**Summary of Contributions**

1. We propose mixed dimension embedding layers, where the dimension of a particular embedding vector scales with said vector’s popularity.

2. We provide a simple heuristic scheme for sizing of embedding vectors given a prior distribution or training sample. The dimensions prescribed by our scheme compare favorably to uniform dimensions.

3. We identify two distinct mechanisms by which mixed dimension layers improve parameter-efficiency and achieve superior statistical performance at a given parameter budget.

**2 Background & Related Works**

In this work, we focus on explicit CF as well as CTR prediction. In explicit CF user ratings for particular items are directly observed and therefore it can be formally framed as a matrix completion problem \([9, 10, 11, 22]\).
Embedding-based approaches, such as matrix factorization (MF) \[22, 28, 46]\] or neural collaborative filtering (NCF) \[15, 24]\] are among the most popular and efficient solutions to matrix completion. The main alternative is to use a convex relaxation to find the minimum nuclear norm solution. This entails solving a semi-definite program, which takes time $O(n^4)$ \[56]\], and thus will not scale to real-world applications. Instead, embedding/factorization approaches have the drawback of explicitly requiring an embedding dimension, but in practice this can be solved with cross-validation or other hyperparameter tuning techniques. In CTR prediction tasks we are predicting the event-probability of a click, which can also be viewed as context-based CF with binary ratings. A wide variety of models has been developed over the recent years for this task, including but not limited to \[12, 20, 30, 36, 57, 58]\]. These state-of-the-art models share many similar characteristics, and all of them without exception use memory-intensive embedding layers that dwarf the rest of the model.

In modern machine learning, embeddings are often used to represent categorical features. Embedding vectors are mined from data, with the intention that certain semantic relationships between the categorical concepts represented by the vectors are encoded by the spatial or geometric relationships and properties of the vectors \[33, 34]\]. Thus, large embeddings are a natural choice for recommendation systems, which require models to understand the relationships between users and items.

Many techniques have been developed to decrease the amount of memory consumed by embedding layers. They can roughly be split into two high-level classes: (i) compression algorithms and (ii) compressed architectures. Compression algorithms usually involve some form of additional processing of the model beyond standard training. They can be performed offline, when they only involve post-training processing, or online, when the compression process is interleaved with or otherwise materially alters the training process. Simple offline compression algorithms include post-training quantization, pruning or low-rank SVD \[4, 8, 47, 51]\]. Model distillation techniques, such as compositional coding \[50]\] and neural binarization \[52]\] are also a form of complex offline compression in which autoencoders are trained to mimic uncompressed, pre-trained embedding layers. Online compression algorithms include quantization-aware training, gradual pruning, and periodic regularization \[3, 16, 17, 35, 40]\]. We note that many of these compression algorithms are not unique to embedding layers, and are widely used in the model compression literature.

On the other hand, we also have compressed architectures, which attempt to architect embedding representations of comparable statistical quality with fewer parameters. Compressed architectures have the advantage of not only reducing memory requirements for inference time, but also at training time. This is the approach followed by hashing-based and tensor factorization methods \[6, 18, 26, 27, 48]\], which focus on reducing the number of parameters used in an embedding layer by re-using parameters in various ways. These techniques stand in contrast with our approach, which focuses on non-uniform reduction of the dimensionality of embedding vectors based on embedding popularity. In principle, nothing precludes the compound use of our proposed technique and most other compression algorithms or compressed architectures. This is an interesting direction for future investigation.

Finally, we note that non-uniform and deterministic sampling has been addressed in the matrix completion literature in a line of work \[37]\, but only in so far as how to correct for popularity so as to improve statistical recovery performance, or build theoretical guarantees for completion under non-uniform sampling \[39]\. As far as we know, we are the first to leverage popularity to actually reduce parameter counts in large-scale embedding layers.

### 3 Problem and Model Formulation

We pose the problem formulations for CF and CTR tasks. For each task, we also describe the relevant models.

#### 3.1 Collaborative Filtering Problem

Inspired by the apparent inefficiencies of using uniform dimension vectors in the non-uniform sampling regime, we formally state the central formulation of study in the noisy setting, similar to that of \[37]\.

Let $M^* \in \mathbb{R}^{n \times m}$, for $n \geq m$, be an unknown target matrix. Let $\Omega \subseteq [n] \times [m]$ denote a sample of indices. Define $M := M^* + \mathcal{E}$ for some noise term $\mathcal{E}$ with $\|\mathcal{E}\| \leq \epsilon$. Our observation, denoted $\hat{\Omega}$, is the set of 3-tuples:

$$\hat{\Omega} := \{(i, j, M_{ij}) : (i, j) \in \Omega\}$$
We refer to a collaborative filter \( A \) as an algorithm that inputs the observation \( \Omega \) and outputs an estimate of \( M^* \), denoted \( \hat{M} \leftarrow A(\Omega) \). The goal of the filter is to minimize the \( Q \)-weighted Frobenius loss:

\[
||\hat{M} - M^*||_Q^2 = \sum_{i,j \in \Omega} \pi_{ij}(\hat{M}_{ij} - M^*_{ij})^2
\]

### 3.1.1 Matrix Factorization

In MF we define \( A \) as solving the following optimization problem

\[
\min_{W,V} \sum_{i,j \in \Omega} (w_i^T v_j - M_{ij})^2
\]

where user \( W^T = [w_0, ..., w_{n-1}] \) and item \( V^T = [v_0, ..., v_{m-1}] \) embedding matrices correspond to an \( r \)-dimensional embedding layer. When the rank of \( M \) is a known priori, it can be used to set the dimension of the embedding layer, otherwise it is treated as a hyperparameter.

### 3.1.2 Neural Collaborative Filtering

Although NCF lacks the nice theoretical guarantees offered by MF, it empirically performs mildly better on real-world datasets [24]. Since we are primarily concerned with the embedding layer, we adopt the simplest NCF model where user and item embeddings are concatenated to form an input to the following multilayer perceptron (MLP).

In NCF we define \( A \) as solving the following optimization problem

\[
\min_{W,V,\theta} \sum_{i,j \in \Omega} (F([w_i, v_j], \theta) - M_{ij})^2
\]

where \( W \) and \( V \) are embeddings, while \( \theta \) denotes additional parameters of the MLP, denoted by function \( F \).

### 3.2 Click-Through Rate Prediction Problem

CTR prediction tasks can be interpreted as event-probability prediction problem or as context-based CF with binary ratings. Compared to canonical CF, these tasks also include a large amount of context which can be used to better predict user and item interaction events. Therefore, this problem can be viewed as restricting targets \( M_{ij} \in \{0,1\} \), while allowing user and items to be represented by multiple features, often expressed through sets of indices (categorical) and floating point values (continuous).

#### Deep Learning Recommendation Model

Facebook's state-of-the-art deep learning recommendation model (DLRM) [36] allows for \( \kappa + 1 \) categorical features, which can represent arbitrary details about the context of an on-click or personalization event. The \( i \)-th categorical feature can be represented by an index \( x_i \in \{0, ..., n_i - 1\} \) for \( i = 0, ..., \kappa \). In addition to \( \kappa + 1 \) categorical features, we also have \( s \) scalar features, together producing a dense feature vector \( \mathbf{x}' \in \mathbb{R}^s \). Thus, given some \( (x_0, ..., x_\kappa, \mathbf{x}') \in ([n_0] \times \ldots \times [n_\kappa]) \times \mathbb{R}^s \), we would like to predict \( y \in \{0,1\} \) which denotes an on-click event in response to a particular personalized context \( (x_0, ..., x_\kappa, \mathbf{x}') \).

In DLRM we define \( A \) as solving the following optimization problem

\[
\min_{E^{(0)}, ..., E^{(\kappa)}, \theta} \sum_{i,j} (G(x', e_0^{(i)}, ..., e_{n_\kappa}^{(i)}, \theta) - y)^2
\]

where \( E^{(i)} = [e_0^{(i)}, ..., e_{n_i-1}^{(i)}] \) for \( i = 0, ..., \kappa \) are embeddings, while \( \theta \) denotes any additional parameters (mostly related to MLPs) in the model \( G \). Note that we often use binary cross-entropy instead of MSE loss in practice.
4 Mixed Dimension Embedding Layer

Let us now define the mixed dimension embedding layer and describe the equipartition- as well as popularity-based schemes used to determine its dimensionality. We will also discuss how we can apply it to CF and CTR prediction tasks.

Let a mixed dimension embedding layer $\bar{E}$ consist of $k+1$ blocks and be defined by $2k+1$ matrices, so that

$$\bar{E} = (\bar{E}^{(0)}, \bar{E}^{(1)}, ..., \bar{E}^{(k)}, P^{(1)}, ..., P^{(k)})$$

with $\bar{E}^{(i)} \in \mathbb{R}^{n_i \times d_i}$ and $P^{(i)} \in \mathbb{R}^{d_i \times d_0}$ for $i = 0, ..., k$, where $P^{(0)} \in \mathbb{R}^{d_0 \times d_0}$ is implicitly defined as identity.

Let us assume that the dimensions of these blocks are fixed. Then, forward propagation for a mixed dimension embedding layer takes an index $x$ in the range from 1 to $n = \sum_{i=0}^{k} n_i$, and produces an embedding vector $e_x$ as defined in Alg. 1. The steps involved in this algorithm are differentiable, therefore we can perform backward propagation through this layer and update matrices $\bar{E}^{(i)}$ and $P^{(i)}$ accordingly during training. We note that Alg. 1 may be generalized to support multi-hot lookups, where embedding vectors corresponding to some query indices are fetched and reduced by a differentiable operator, such as add, multiply or concatenation.

Algorithm 1 Forward Propagation

| Input: Index $x \in [n]$ | Output: Embedding vector $e_x$ |
|--------------------------|--------------------------------|
| $i \leftarrow 0$ and $t \leftarrow 0$ | $\bar{E}^{(i)}[x-t]P^{(i)}$ if $i > 0$ else $\bar{E}^{(i)}[x-t]$ |
| while $t + n_i < x$ do | $\triangleright$ Find offset $t$ and sub-block $i$ for index $x$ |
| $t \leftarrow t + n_i$ | $\triangleright$ Construct a single embedding vector corresponding to it |
| $i \leftarrow i + 1$ | |
| end while |

Note that we return projected embeddings for all but the first embedding matrix and that all embedding vectors $e_j$ have the same base dimension $\bar{d} := d_0$. Therefore, models based on a mixed dimension embedding layer should be sized with respect to $\bar{d}$. We illustrate the matrix architecture of the mixed dimension embedding layer with two blocks on Fig. 2 where the parameter budget (total area) consumed by uniform and mixed dimension matrices is the same, but allocated differently.

![Figure 2: Matrix Architecture for Uniform and Mixed Dimension Embedding Layers](image-url)

Let us now focus on how to find the block structure in the mixed dimension architecture. This includes the row count $n_i$ as well as dimension $d_i$ assigned to each block in the mixed dimension embedding layer. We restrict ourselves to use of popularity information for sizing the mixed dimension embedding layer (i.e. frequency $f$ of access of a particular feature; assumed here to be mostly consistent between training and test samples). We note that in principle, one could also use a related but distinct notion of importance, that refers
to how statistically informative a particular feature is, on average, to the inference of the target variable. Importance could be determined either by domain experts or in data-driven manner at training time.

4.1 Blocking Scheme for Mixed Dimensions

There is some amount of malleability and choice in the re-architecture of a uniform dimension embedding layer into a mixed dimension layer. With appropriate re-indexing, multiple embedding matrices may be stacked into a single block matrix, or a single embedding matrix may be row-wise partitioned into multiple block matrices. The point of the partitioned blocking scheme is to map the $n$ total embedding rows into blocks, with block-level row counts given by $(n_0, ..., n_k)$ and offset vector $t \in \mathbb{N}_{k+1}$ with $t_i := \sum_{j=0}^{i-1} n_j$.

Blocking for Collaborative Filtering In CF tasks we only have two features – corresponding to users and items – with corresponding embedding matrices $W \in \mathbb{R}^{n \times d}$ and $V \in \mathbb{R}^{m \times d}$, respectively. To size the mixed dimension embedding layer we apply mixed dimensions within individual embedding matrices by partitioning them. First, we sort and re-index the rows based on row-wise frequency: $i < i' \implies f_i \geq f_{i'}$. Then, we partition each embedding matrix into $k+1$ blocks such that the total popularity (area under the curve) in each block is constant, as shown in Alg. 2. For a given frequency $f$ the $k$-equipartition is unique and is simple to compute. In our experiments, we saw that setting $k$ anywhere in the $(8, 16)$ range is sufficient to observe the effects induced by mixed dimensions, with diminishing effect beyond that.

**Algorithm 2** Blocking Number of Vectors Assignment Scheme

| Input: Desired number of blocks $k$             |
| Input: Row-wise frequencies vector $\bar{f}$  |
| Output: Offsets vector $t$                     |
| $f \leftarrow \text{sort}(\bar{f})$           |
| $\text{Find offsets } t_i \text{ such that } \sum_{l=t_i}^{t_{i+1}} f_l = \sum_{l=t_j}^{t_{j+1}} f_l \text{ for } \forall j \text{ and } i = 0, ..., k$ |

Yields a unique $k$-equipartition

Blocking for CTR Prediction In CTR prediction tasks, we have several categorical features, with $k+1$ corresponding embedding matrices $E^{(i)} \in \mathbb{R}^{n_i \times d}$. To size the mixed dimension embedding layer for CTR prediction applications we apply mixed dimensions across different embedding matrices by stacking them. Therefore, the problem structure defines the number of blocks and number of vectors in each original embedding defines the row counts $n_i$ in the corresponding block in the mixed dimension embedding layer.

4.2 Popularity-Based Mixed Dimensions

We now assume that the number of vectors $n_i$ in each block of the mixed dimension embedding layer $\bar{E}$ is already fixed. Therefore, it only remains to assign the dimensions $d := (d_0, ..., d_k)$ to completely specify it.

We propose a popularity-based scheme that operates on a block-level and that is based on a heuristic: Each embedding should be assigned a dimension proportional to some fractional power of its popularity. Note that here we distinguish block-level probability $\mathbf{p}$ from row-wise frequency $\mathbf{f}$. Given $\mathbf{f}$, we define $a_i = \sum_{j=t_i}^{t_{i+1}} f_j$ as the area under the frequency curve in the interval $[t_i, t_{i+1}]$ and total $\tau = \sum_{j=0}^{n} f_j$. Then, we let block-level probability vector $\mathbf{p} \in \mathbb{R}^{k+1}$ be defined by its elements $p_i = a_i / \tau$. We formalize the popularity-based scheme in Alg. 3 with an extra hyperparameter temperature $\alpha > 0$.

**Algorithm 3** Popularity-Based Dimension Assignment Scheme

| Input: Desired baseline dimension $d$ and fixed temperature $0 \leq \alpha \leq 1$ |
| Input: Probability vector $\mathbf{p}$                        |
| Output: Dimension assignment vector $\mathbf{d}$              |
| $\lambda \leftarrow d p_0^{-\alpha}$                        |
| $\mathbf{d} \leftarrow \lambda \mathbf{p}^\alpha$          |

$\triangleright$ Compute scalar scaling factor

$\triangleright$ Perform component-wise power operation

The proposed technique requires knowledge of probability vector $\mathbf{p}$ that governs the feature popularity. When such distribution is unknown, we may easily replace it by the empirical distributions from the data sample.\(^4\)

\(^4\)For instance, in CF for practical purposes we can approximate $f_i$ as number of samples pertaining to $i$-th user.
Alternatively, we can set \( \lambda \leftarrow B \sum_i p_i^{\alpha-1} \) to approximately constrain the embedding layer sizing to a total (number of parameters) budget \( B \). Furthermore, to get crisp sizing, one might also elect to round \( d \), perhaps to the nearest power of two, after applying Alg. 3.

Notice that we finally have all the tools to size the mixed dimension embedding layers as well as to perform forward and backward propagation through them, using Alg. 1-3.

5 Experiments

We proceed to present our experiments validating mixed dimension embedding layers. All implementations described herein are done in the Pytorch framework [42] using NVIDIA V100 32GB GPUs [38]. In total, we run experiments on two datasets with corresponding models summarized in Table 1. We note that we always use the MLP variation of the NCF model.

| Dataset      | Tasks | # Samples | # Categories | Models Used |
|--------------|-------|-----------|--------------|-------------|
| MovieLens    | CF    | 27M       | 300K         | MF, NCF     |
| Criteo Kaggle| CTR   | 40M       | 32M          | DLRM        |

Table 1: Datasets, tasks and models

We explore the trade-off between memory and statistical performance for mixed dimension embeddings, parameterized by varying \( \alpha \). For a given \( \alpha \), we vary the budget by varying the base dimension, \( \bar{d} \). In our setting, we measure memory in terms of number of 32-bit floating point parameters in the embedding layer. We measure performance in terms of MSE loss for CF tasks and accuracy for CTR prediction tasks. We choose to use loss rather than accuracy for CF because we would like to differentiate when the quality of our predictions are closer to the target (e.g. we want to differentiate between predicted ratings of 0 and 3 for a target of 4) in our results. We do not have to address this issue for the binary targets in the CTR prediction problem and therefore use more natural accuracy as a metric for it. In all experiments, we use the Xavier uniform initialization [19], for all matrices in our models.

The parameters are allocated to embedding layers according to the \( \alpha \)-parameterized rule proposed in Alg. 3. Notice that for models with uniform dimension embedding layers (\( \alpha = 0 \)), the number of parameters directly controls the embedding dimension because the number of vectors per embedding is fixed. On the other hand, for mixed dimension layers (\( \alpha > 0 \)) the fixed parameters lead to skewed dimensions. In fact the more alpha increases the more skewed dimension assignments become based on popularity. For instance, we illustrate how different choices of \( \alpha \) assign embedding dimensions in Fig 3.

![Embedding Dimension Sorted by Popularity](a) MF/NCF on MovieLens (items) dataset

![Embedding Dimension Sorted by Popularity](b) DLRM model on Criteo Kaggle dataset

Figure 3: Embedding dimensions allocated with different choices of \( \alpha \) for the same budget.

We explore the trade-off between memory and statistical performance for mixed dimension embeddings, parameterized by varying \( \alpha \). For a given \( \alpha \), we vary the budget by varying the base dimension, \( \bar{d} \). In our setting, we measure memory in terms of number of 32-bit floating point parameters in the embedding layer. We measure performance in terms of MSE loss for CF tasks and accuracy for CTR prediction tasks. We choose to use loss rather than accuracy for CF because we would like to differentiate when the quality of our predictions are closer to the target (e.g. we want to differentiate between predicted ratings of 0 and 3 for a target of 4) in our results. We do not have to address this issue for the binary targets in the CTR prediction problem and therefore use more natural accuracy as a metric for it. In all experiments, we use the Xavier uniform initialization [19], for all matrices in our models.

The parameters are allocated to embedding layers according to the \( \alpha \)-parameterized rule proposed in Alg. 3. Notice that for models with uniform dimension embedding layers (\( \alpha = 0 \)), the number of parameters directly controls the embedding dimension because the number of vectors per embedding is fixed. On the other hand, for mixed dimension layers (\( \alpha > 0 \)) the fixed parameters lead to skewed dimensions. In fact the more alpha increases the more skewed dimension assignments become based on popularity. For instance, we illustrate how different choices of \( \alpha \) assign embedding dimensions in Fig 3.

\( ^5 \)This does not account for integer rounding or for the parameters in the projection matrices \( P^{(i)} \), but the projection matrices should only add a small number of parameters relative to the total when \( n \gg d \), which generally holds.
5.1 Matrix Factorization

MF is a canonical algorithm for CF. From the perspective of this work, pure MF model for CF tasks is important because it offers the most distilled, simplest setting for which we can investigate the effects of mixed dimension embeddings. In our experiments we use the MovieLens dataset [21]. We train at learning rate $10^{-2}$ using Amsgrad optimizer [45] for 100 epochs, taking model that scored the lowest validation loss. We always run 3 replicates per experiment.

In Fig. 4a, we report learning curves for MF with three different embedding layers, plotting the number of training epochs against the validation loss for each epoch. We plot the learning curve for $d = 16$ uniform dimension embeddings, which has the lowest validation loss (under early termination). We also plot the learning curve for $d = 4$ as a baseline. Third, we plot the learning curve for mixed dimension embedding layers at $\alpha = 0.6$, which uses a number of parameters equivalent to uniform dimension embedding layers at $d = 4$.

![Learning curves](image)

Figure 4: Loss under different parameters budgets

Notice that for uniform dimension $d = 4$ (orange line) the model underfits the data, because the attained loss is significantly higher than the one achieved with other hyper-parameters. Also, notice that the model with uniform dimension $d = 16$ (blue line) severely overfits after about 10 epochs. On the other hand, mixed dimension embeddings (green line) train well and achieve the lowest validation loss among other hyper-parameters.

This evidence that at dimension $d = 4$, the popular embedding vectors are underfitting, and that at dimension $d = 16$ the unpopular embedding vectors are overfitting. At the $d = 4$ parameter budget, the mixed dimension embeddings at $\alpha = 0.6$ use a baseline dimension $\bar{d} = 32$ and result in an embedding layer architecture that can more adequately fit both popular and unpopular embedding vectors.

In Fig. 4b, we report the test loss for different $\alpha$ at varying total parameter budgets using optimal early termination. We can see that using mixed dimension embedding layers generally improves the memory-performance frontier at each parameter budget. We point out a simple trend illustrated by the uniform dimension (blue line): performance improves with number of parameters, until we reach a critical point, after which performance decreases with increasing parameters. Notice that at all memory budgets there is an $\alpha>0$ that produces mixed embeddings that are equivalent or better than uniform dimension embeddings ($\alpha = 0$).

Finally, we point out that the optimal $\alpha$ is dependant on the total memory budget. Thus, at any given budget, one should tune $\alpha$ as a hyperparameter. Ultimately, we are able to achieve approximately 0.02 lower MSE while using approximately $4\times$ fewer parameters by using mixed dimension embeddings.

5.2 Neural Collaborative Filtering

NCF models represent more modern approaches for CF. From the perspective of this work, NCF models are interesting because they add a moderate degree of realism and show how the presence of the non-linearity in neural layers affects the results.
In our experiments we use NCF with a 3-layer MLP with dimension 128. We train at learning rate $10^{-3}$ using Amsgrad optimizer for 100 epochs, taking model that scored the lowest validation loss. We still use the same dataset and run 3 random seeds for each point in the experiments.

In Fig. 5a, we report learning curves for the NCF model based on three different embedding layers, plotting the number of training epochs against the validation loss for each epoch. We plot the learning curve for $d = 32$ uniform dimension embeddings, which has the lowest validation loss (under early termination). We also plot the learning curve for $d = 4$ as a baseline. Third, we plot the learning curve for mixed dimension embedding layers at $\alpha = 0.6$, which uses a number of parameters equivalent to uniform dimension embedding layers at $d = 4$.

Similarly to the MF setting, the optimally terminated model using the $d = 32$ embeddings has slightly higher loss than the chosen mixed dimension embeddings. However, unlike in MF, here, we do see that all three models overfit as training progresses, and thus early termination is essential for all three models.

Under prolonged training, the uniform $d = 32$ based NCF model overfit severely. The mixed dimension based NCF model overfits significantly, but not as drastically as the optimal $d = 32$ NCF model. The NCF model with $d = 4$ embeddings overfits only slightly. Given that the two latter embedding layers, when identically architected and initialized, do not cause the model overfit when the embedding vectors as used for MF, we can attribute the overfitting directly to presence of the MLP layers in the NCF model.

In Fig 5b, we report the test loss for different $\alpha$ at varying total parameter budgets using optimal early termination. Overall, the NCF models attain slightly lower test losses than their MF counterparts, generally a decrease in the range of $0.03 - 0.01$ MSE. Interestingly, the optimally terminated NCF models actually suffered from less overfitting than the MF models did at higher parameter counts. The critical points corresponding to the optimal total parameter also increased, for each $\alpha$. Yet, $\alpha = 0.6$ still performed the best, albeit at a larger higher budget.

Finally, concerning the comparison between mixed and uniform dimension embeddings, we see similar trends and draw the same conclusions as in the MF setting. Ultimately, we achieve approximately 0.01 lower MSE while using approximately $8\times$ fewer parameters by using mixed dimension embeddings.

### 5.3 Deep Learning Recommendation Model

CTR prediction tasks can be interpreted as event-probability prediction problem or as context-based CF with binary ratings. From the perspective of this work, using mixed dimension embedding layers on real CTR prediction data with state-of-the-art deep recommendation models is an important experiment that shows how mixed dimension embedding layers might scale to real-world recommendation engines.

In our experiments we use state-of-the-art Facebook’s DLRM [36] and the Criteo Kaggle dataset [29]. We train at learning rate $10^{-3}$ using Amsgrad optimizer for a single epoch. For each $\alpha$ we increase the embedding layer’s parameter budget up to 32GB (GPU memory limit). Notice that because of this limitation we do
not see the overfitting behavior from earlier sections. Also, as usual we perform 3 replicates per experiment. However, for this task we report accuracy and not loss as discussed earlier in this section.

In Fig. 6a, we plot learning curves for the DLRM model based on three different embedding layers. We show that a mixed dimension embeddings with $\alpha = 0.3$ (orange line) produces a learning curve equivalent to that of a $d = 32$ uniform dimension embeddings (blue line) using a total parameter count equivalent to that of $d = 4$ uniform dimension (green line).

In Fig. 6b, we present test accuracy for DLRM using mixed dimension embedding layers at various $\alpha$ with varying parameter budgets. It is evident that mixed dimension embedding layers improve the memory-performance frontier. In fact, we see a very similar trend compared to the two classical CF settings. Ultimately, we achieve approximately 0.01% higher accuracy while using approximately half as many parameters and achieve on par accuracy with $16\times$ fewer parameters by using mixed dimension embeddings.

![Learning curves: Mixed vs. Uniform Dimension](image1)

![Number of Parameters vs. Test Accuracy](image2)

(a) Validation accuracy convergence (under a budget)  (b) Test accuracy final (with different $\alpha$)

Figure 6: Accuracy under different parameters budgets

To summarize we identify two distinct mechanisms by which mixed dimension embedding layers improve upon uniform-dimension layers in our experiments.

i) Generalization – At training time, embeddings learn to represent categorical features such as users or products. We find that at a uniform-dimension, frequently-accessed vectors are often under-fitting, whereas infrequently-accessed vectors are often over-fitting. Thus, mixed dimension embeddings actually learn in a more parameter-efficient manner.

ii) Allocation – Learning aside, at inference time, for a sufficiently constrained parameter budget, one faces a resource allocation trade-off. Even with an oracle factorization of the target matrix, under an expected distortion metric at a given parameter budget, it is more efficient to allocate more parameters to frequently-accessed vectors than to infrequently-accessed vectors.

6 Conclusion

We propose mixed dimension embedding layers in which the dimension of a particular embedding vector is based on its popularity. This approach addresses severe inefficiencies in the number of parameters used in uniform dimension embedding layers. It offers a superior trade-off between number of parameters and statistical performance of the model. In general, it improves the memory-performance frontier across memory budgets, but is particularly effective at small parameter budgets. For instance, in our experiments we were able to attain the same baseline accuracy with about $8\times$ and $16\times$ smaller models for CF on MovieLens and CTR prediction on Criteo Kaggle datasets, respectively. In the future we would like to investigate the composition of mixed dimension embeddings technique with other compression algorithms and architectures.

Acknowledgements The authors would like to thank Shubho Sengupta, Michael Shi, Jongsoo Park, Jonathan Frankle and Misha Smelyanskiy for helpful comments and suggestions about this work.
References

[1] Charu C. Aggarwal. Recommender Systems: The Textbook. Springer, 2016.

[2] Alan Akbik, Duncan Blythe, and Roland Vollgraf. Contextual string embeddings for sequence labeling. In Proceedings of the 27th International Conference on Computational Linguistics, pages 1638–1649, 2018.

[3] Jose M Alvarez and Mathieu Salzmann. Compression-aware training of deep networks. In Proc. Advances in Neural Information Processing Systems, pages 856–867, 2017.

[4] Martin Andrews. Compressing word embeddings. CoRR, abs/1511.06397, 2015.

[5] Ehsaneddin Asgari and Mohammad R.K. Mofrad. Continuous distributed representation of biological sequences for deep proteomics and genomics. PLOS ONE, 10: e0141287, 2015.

[6] Josh Attenberg, Kilian Weinberger, Anirban Dasgupta, Alex Smola, and Martin Zinkevich. Collaborative email-spam filtering with the hashing trick. In Proceedings of the Sixth Conference on Email and Anti-Spam, 2009.

[7] Björn Barz and Joachim Denzler. Hierarchy-based image embeddings for semantic image retrieval. In 2019 IEEE Winter Conference on Applications of Computer Vision, pages 638–647. IEEE, 2019.

[8] Prasad Bhavana, Vikas Kumar, and Vineet Padmanabhan. Block based singular value decomposition approach to matrix factorization for recommender systems. Pattern Recognition Letters, abs/1907.07410, 2019.

[9] Emmanuel J Candes and Yaniv Plan. Matrix completion with noise. Proceedings of the IEEE, 98(6):925–936, 2010.

[10] Emmanuel J Candès and Benjamin Recht. Exact matrix completion via convex optimization. Foundations of Computational mathematics, 9:717, 2009.

[11] Emmanuel J Candès and Terence Tao. The power of convex relaxation: Near-optimal matrix completion. IEEE Transactions on Information Theory, 56(5):2053–2080, 2010.

[12] Heng-Tze Cheng, Levent Koc, Jeremiah Harmsen, Tal Shaked, Tushar Chandra, Hrishi Aradhya, Glen Anderson, Greg Corrado, Wei Chai, Mustafa Ispir, et al. Wide & deep learning for recommender systems. In Proc. 1st workshop on deep learning for recommender systems, pages 7–10. ACM, 2016.

[13] Billy Chiu, Gamal Crichton, Anna Korhonen, and Sampo Pyysalo. How to train good word embeddings for biomedical nlp. In Proceedings of the 15th workshop on biomedical natural language processing, pages 166–174, 2016.

[14] Kideok Cho, Munyoung Lee, Kunwoo Park, Ted Taekyoung Kwon, Yanghee Choi, and Sangheon Pack. Wave: Popularity-based and collaborative in-network caching for content-oriented networks. In 2012 Proceedings IEEE INFOCOM Workshops, pages 316–321. IEEE, 2012.

[15] Maurizio Ferrari Dacrema, Paolo Cremonesi, and Dietmar Jannach. Are we really making much progress? A worrying analysis of recent neural recommendation approaches. CoRR, abs/1907.06902, 2019.

[16] Ahmed T. Elthakeb, Prannoy Pilligundla, and Hadi Esmaeilzadeh. SinReQ: Generalized sinusoidal regularization for automatic low-bitwidth deep quantized training. CoRR, abs/1905.01416, 2019.

[17] Jonathan Frankle and Michael Carbin. The lottery ticket hypothesis: Finding sparse, trainable neural networks. CoRR, abs/1803.03635, 2018.

[18] Jinyang Gao, Beng Chin Ooi, Yanyan Shen, and Wang-Chien Lee. Cuckoo feature hashing: Dynamic weight sharing for sparse analytics. In IJCAI, pages 2135–2141, 2018.

[19] Xavier Glorot and Yoshua Bengio. Understanding the difficulty of training deep feedforward neural networks. In Proceedings of the thirteenth international conference on artificial intelligence and statistics, pages 249–256, 2010.

[20] Huifeng Guo, Ruiming Tang, Yuning Ye, Zhenguo Li, and Xiuqiang He. DeepFM: a factorization-machine based neural network for ctr prediction. CoRR, abs/1703.04247, 2017.

[21] F. Maxwell Harper and Joseph A. Konstan. The MovieLens datasets: History and context. ACM Transactions on Interactive Intelligent Systems, 5, 2015.

[22] Trevor Hastie, Rahul Mazumder, Jason D Lee, and Reza Zadeh. Matrix completion and low-rank svd via fast alternating least squares. The Journal of Machine Learning Research, 16:3367–3402, 2015.
[23] Kim Hazelwood, Sarah Bird, David Brooks, Soumith Chintala, Utku Diril, Dmytro Dzhulgakov, Mohamed Fawzy, Bill Jia, Yangqing Jia, Aditya Kalro, James Law, Kevin Lee, Jason Lu, Pieter Noordhuis, Misha Smelyanskiy, Liang Xiong, and Xiaodong Wang. Applied machine learning at Facebook: A datacenter infrastructure perspective. In *Proc. IEEE Int. Symposium on High Performance Computer Architecture*, pages 620–629, 2018.

[24] Xiangnan He, Lizi Liao, Hanwang Zhang, Liqiang Nie, Xia Hu, and Tat-Seng Chua. Neural collaborative filtering. In *Proc. 26th international conference on world wide web*, pages 173–182, 2017.

[25] Xinran He, Junfeng Pan, Ou Jin, Tianbing Xu, Bo Liu, Tao Xu, Yanxin Shi, Antoine Atallah, Ralf Herbrich, Stuart Bowers, and Joaquin Quionero Candela. Practical lessons from predicting clicks on ads at Facebook. In *Proc. 8th Int. Workshop on Data Mining for Online Advertising*, pages 1–9, 2014.

[26] Alexandros Karatzoglou, Alex Smola, and Markus Weimer. Collaborative filtering on a budget. In *Proceedings of the Thirteenth International Conference on Artificial Intelligence and Statistics*, pages 389–396, 2010.

[27] Valentin Khurulkov, Oleksei Hrinchuk, Leyla Mirvakhabova, and Ivan V. Oseledets. Tensorized embedding layers for efficient model compression. *CoRR*, abs/1901.10787, 2019.

[28] Yehuda Koren, Robert Bell, and Chris Volinsky. Matrix factorization techniques for recommender systems. *Computer*, 2009.

[29] Criteo AI Labs. Kaggle Display Advertising Challenge dataset. 2014.

[30] Jianxun Lian, Xiaohuan Zhou, Fuzheng Zhang, Zhongxia Chen, Xing Xie, and Guangzhong Sun. xDeepFM: Combining explicit and implicit feature interactions for recommender systems. In *Proc. 24th International Conference on Knowledge Discovery & Data Mining*, pages 1754–1763. ACM, 2018.

[31] Yang Liu, Zhiyuan Liu, Tat-Seng Chua, and Maosong Sun. Topical word embeddings. In *Twenty-Ninth AAAI Conference on Artificial Intelligence*, 2015.

[32] Yinhan Liu, Myle Ott, Naman Goyal, Jingfei Du, Mandar Joshi, Danqi Chen, Omer Levy, Mike Lewis, Luke Zettlemoyer, and Veselin Stoyanov. Roberta: A robustly optimized bert pretraining approach. *arXiv preprint arXiv:1907.11692*, 2019.

[33] Tomas Mikolov, Ilya Sutskever, Kai Chen, Greg S Corrado, and Jeff Dean. Distributed representations of words and phrases and their compositionality. In *Advances in neural information processing systems*, pages 3111–3119, 2013.

[34] Maxim Naumov. On the dimensionality of embeddings for sparse features and data. *CoRR*, abs/1901.02103, 2019.

[35] Maxim Naumov, Utku Diril, Jongsoo Park, Benjamin Ray, Jedrzej Jablonski, and Andrew Tulloch. On periodic functions as regularizers for quantization of neural networks. *CoRR*, abs/1811.09862, 2018.

[36] Maxim Naumov, Dheevatsa Mudigere, Hao-Jun Michael Shi, Jianyu Huang, Narayanan Sundaraman, Jongsoo Park, Xiaodong Wang, Udit Gupta, Carole-Jean Wu, Alisson G Azzolini, et al. Deep learning recommendation model for personalization and recommendation systems. *CoRR*, abs/1906.00091, 2019.

[37] Sahand Negahban and Martin J Wainwright. Restricted strong convexity and weighted matrix completion: Optimal bounds with noise. *Journal of Machine Learning Research*, 13:1665–1697, 2012.

[38] Nvidia. Tesla V100 GPU architecture white paper. 2017.

[39] Fragkiskos Papadopoulos, Maksim Kitsak, M Ángeles Serrano, Marián Boguná, and Dmitri Krioukov. Popularity versus similarity in growing networks. *Nature*, 489:537, 2012.

[40] Eunhyeok Park, Sungjoo Yoo, and Peter Vajda. Value-aware quantization for training and inference of neural networks. In *Proc. European Conference on Computer Vision*, pages 580–595, 2018.

[41] Jongsoo Park, Maxim Naumov, Protonu Basu, Summer Deng, Aravind Kalaiah, Daya Shanker Khudia, James Law, Parth Malani, Andrey Malevich, Nadathur Satish, Juan Pino, Martin Schatz, Alexander Sidorov, Viswanath Sivakumar, Andrew Tulloch, Xiaodong Wang, Yiming Wu, Hector Yuen, Utku Diril, Dmytro Dzhulgakov, Kim M. Hazelwood, Bill Jia, Yangqing Jia, Lin Qiao, Vijay Rao, Nadav Rotem, Sungjoo Yoo, and Mikhail Smelyanskiy. Deep learning inference in Facebook data centers: Characterization, performance optimizations and hardware implications. *CoRR*, abs/1811.09886, 2018.

[42] Adam Paszke, Sam Gross, Soumith Chintala, Gregory Chanan, Edward Yang, Zachary DeVito, Zeming Lin, Alban Desmaison, Luca Antiga, and Adam Lerer. Automatic differentiation in PyTorch. 2017.
[43] Matthew Peters, Mark Neumann, Luke Zettlemoyer, and Wen-tau Yih. Dissecting contextual word embeddings: Architecture and representation. In Proceedings of the 2018 Conference on Empirical Methods in Natural Language Processing, pages 1499–1509, 2018.

[44] Qi Pi, Weijie Bian, Guorui Zhou, Xiaqiang Zhu, and Kun Gai. Practice on long sequential user behavior modeling for click-through rate prediction. CoRR, abs/1905.09248, 2019.

[45] Sashank J. Reddi, Satyen Kale, and Sanjiv Kumar. On the convergence of Adam and beyond. In Proc. International Conference on Learning Representations, 2018.

[46] Steffen Rendle, Li Zhang, and Yehuda Koren. On the difficulty of evaluating baselines: A study on recommender systems. CoRR, abs/1905.01395, 2019.

[47] Prasanna Sattigeri and Jayaraman J. Thiagarajan. Sparsifying word representations for deep unordered sentence modeling. In Proc. 1st Workshop on Representation Learning for NLP, pages 206–214.

[48] Kaiyu Shi and Kai Yu. Structured word embedding for low memory neural network language model. In Proc. Interspeech, 2018.

[49] Mohammad Shoeybi, Mostofa Patwary, Raul Puri, Patrick LeGresley, Jared Casper, and Bryan Catanzaro. Megatron-lm: Training multi-billion parameter language models using gpu model parallelism. arXiv preprint arXiv:1909.08053, 2019.

[50] Raphael Shu and Hideki Nakayama. Compressing word embeddings via deep compositional code learning. CoRR, abs/1711.01068, 2017.

[51] Fei Sun, Jiafeng Guo, Yanyan Lan, Jun Xu, and Xueqi Cheng. Sparse word embeddings using l1 regularized online learning. In Proc. 25th International Joint Conference on Artificial Intelligence, 2016.

[52] Julien Tissier, Amaury Habrard, and Christophe Gravier. Near-lossless binarization of word embeddings. CoRR, abs/1803.09065, 2018.

[53] Mariya I. Vasileva, Bryan A. Plummer, Krishna Dusad, Shreya Rajpal, Ranjitha Kumar, and David Forsyth. Learning type-aware embeddings for fashion compatibility. In Proceedings of the European Conference on Computer Vision, pages 390–405, 2018.

[54] Xiaorui Wu, Hong Xu, Honglin Zhang, Huaming Chen, and Jian Wang. Saec: Simplicity-aware embedding compression in recommendation systems. CoRR, abs/1903.00103, 2019.

[55] Zi Yin and Yuanyuan Shen. On the dimensionality of word embedding. In Proc. Advances in Neural Information Processing Systems, pages 887–898, 2018.

[56] Yongbin Zheng, Yuzhuang Yan, Sheng Liu, Xinsheng Huang, and Wanying Xu. An efficient approach to solve the large-scale semidefinite programming problems. Mathematical Problems in Engineering, 2012, 2012.

[57] Guorui Zhou, Na Mou, Ying Fan, Qi Pi, Weijie Bian, Chang Zhou, Xiaqiang Zhu, and Kun Gai. Deep interest evolution network for click-through rate prediction. CoRR, abs/1809.03672, 2018.

[58] Guorui Zhou, Xiaqiang Zhu, Chenru Song, Ying Fan, Han Zhu, Xiao Ma, Yanghui Yan, Junqi Jin, Han Li, and Kun Gai. Deep interest network for click-through rate prediction. In Proc. 24th International Conference on Knowledge Discovery & Data Mining, pages 1059–1068. ACM, 2018.