Particle Scale Dynamics in Granular Impact

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We perform an experimental study of granular impact, where intruders strike 2D beds of photoelastic disks from above. High-speed video captures the intruder dynamics and the local granular force response, allowing investigation of grain-scale mechanisms in this process. We observe rich acoustic behavior at the leading edge of the intruder, strongly fluctuating in space and time, and we show that this acoustic activity controls the intruder deceleration, including large force fluctuations at short time scales. The average intruder dynamics match previous studies using empirical force laws, suggesting a new microscopic picture, where acoustic energy is carried away and dissipated.

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The penetration of a dense granular material by a high-speed intruder occurs routinely in meteor and ballistic impacts. Many previous studies [1–8], both recent and dating back to Euler and Poncelet, have used variations of a macroscopic force law:

\[ F = m \ddot{z} = mg - f(z) - h(z) \dot{z}^2. \] (1)

Here, \( z \) is the intruder depth relative to the top of the original, unperturbed surface (i.e., \( z = 0 \) at initial impact), \( mg \) is the of gravity force, \( f(z) \) characterizes hydrostatic effects, \( h(z) \) is often assumed constant, \( h(z) = b \), and dots denote time derivatives. In Eq. (1), \( h(z) \dot{z}^2 \) represents a coarse grained collisional stress. We note that other effects, including a depth-dependent Coulomb friction term have been proposed [9,10]. Despite the success of extensive previous studies [1–8], the connections between the local granular response, the microscopic processes responsible for dissipating kinetic energy, and the dynamics of the intruder are still subjects of debate, largely due to experimental difficulties in obtaining sufficiently fast data at small scales.

In this Letter, we address this issue experimentally by high-speed imaging of an intruder of mass, \( m \), which impacts a quasi-two-dimensional system of photoelastic particles (bidisperse, larger particle diameter \( d \)) at speeds \( v_0 \leq 6.5 \text{ m/s} \), yielding both the intruder dynamics and the force response of individual grains (Fig. 1). Here, as in many previous experiments, \( v << C \), where \( C \approx 300 \text{ m/s} \) is the granular sound speed, measured from photoelastic space-time plots, as in Fig. 1(b). The frame rates of \( \sim C/d \) capture the microscopic granular response. The primary intruder energy loss mechanism in these experiments is due to intense, intermittent acoustic pulses traveling at speeds \( \sim C \) along networks of grains, transmitting energy from the intruder into the medium. These pulses decay roughly exponentially with distance from the intruder. The force on the intruder is strongly fluctuating, due to the intermittency of the force network/ acoustic activity, but the mean behavior is consistent with empirical models used previously [1–8].

Experimental Techniques.—The experimental apparatus consists of two thick Plexiglas sheets (0.91 m \( \times \) 1.22 m \( \times \) 1.25 cm), separated by a thin gap (3.3 mm) which is filled by photoelastic disks (thickness of 3 mm) of two different diameters (6 mm and 4.3 mm). These disks are cut from PS-1 material, (Vishay Precision Group; bulk density of 1.28 g/cm\(^3\), elastic modulus of 2.5 GPa, Poisson’s ratio of 0.38). Intruders are machined from bronze sheet (bulk density of 8.91 g/cm\(^3\), thickness of 0.23 cm) into disks of diameters \( D = 6.35 \text{ cm}, 10.16 \text{ cm}, 12.7 \text{ cm} \), and 20.32 cm (data for \( D = 12.7 \text{ cm} \) intruder used in images and time-series data shown here is typical for all \( D \)). These intruders are dropped from a height, \( H \leq 2.2 \text{ m} \), through a shaft connected to the top of the thin gap containing the particles, producing an impact speed, \( v_0 \approx (2gH)^{1/2} \). Results are recorded with a Photron FASTCAM SA5, at a resolution of 256\( \times \)584 pixels (\( \sim 10 \) pixels per \( d \)), at 40,000 frames per second. To locate and track the intruder, we use a circular Hough transform at each frame. Velocity, \( v \), and acceleration, \( a \) are calculated by numerical differentiation, with a low-pass filter, cut-off frequency of 133 Hz \( \approx (7.5 \text{ ms})^{-1} \approx v_0/D \), applied with each derivative to reduce noise amplification. The frequency cutoff is as large as possible while maintaining a signal-to-noise ratio of 10:1. This yields intermediate time scale data for \( v \) (\( v_{\text{int}} \)) and \( a \) (\( a_{\text{int}} \)) which are still strongly fluctuating in time. Photoelastic images are normalized by a calibration image, taken before the intruder is dropped, to account for inhomogeneities in the light source. After this, the discrete gradient-squared \( (G^2 = |\nabla I|^2) \) of the image is computed using the spatial variation of the image intensity, \( I \); the sum of the \( G^2 \) in a particular region measures the local force response [10] (i.e., beneath the intruder, as in Fig. 1). A static calibration, covering the full range of \( G^2 \) encountered in any impact, was performed by placing a weighted piston on a box of about 100 particles that are subject to the same light intensity as used in the experiments. As shown in
Fig. 1(b), it is essentially linear.

Comparing to previous models. An important question is whether the observed dynamics are consistent with existing models, i.e. Eq. (1). To address this, we consider the intruder trajectory, $z(t)$, and the filtered derivatives $v_{int}$ and $a_{int}$. As noted, the derivatives, particularly $a_{int}$, are strongly fluctuating, and these fluctuations are a physical aspect of the dynamics, as discussed below. Plots of $a_{int}$ vs. $v_{int}^2$ data from different impacts with varying $v_0$, show good agreement, within fluctuations, with Eq. (1). This analysis yields $f(z)$ and $h(z)$: a constant value for $h(z)$ (i.e. $h(z) = b \simeq 5D$) after an initial transient at impact, and $f(z)$ which is nearly linearly increasing in depth.

However, for any individual trajectory, we measure large fluctuations in $a_{int}$ (Fig. 2), on a scale that is comparable to the mean acceleration. These fluctuations are absent in the “slow-time” models discussed above, and their large amplitude is both a novel observation and a potential weak point of the models. That is, the braking of the intruder is not a smooth steady process, but a series of events where the intruder is subjected briefly to large accelerations, followed by more quiescent periods that can be close to acceleration-free.

Connecting acoustic activity to intruder deceleration. As noted, during an impact, we observe complex propagating force networks (known as force chains) generated intermittently at the leading edge of the intruder as it moves through the medium, as shown in Fig. 1(a), as well as in Supplementary Videos 1 and 2. To quantify the photoelastic response, we consider the angular region extending radially outward from the bottom half of the intruder over a length $\sim 10d$, forming a half-annulus. Comparing $a_{int}$ to the photoelastic response, $G^2$, requires time-filtering the photoelastic data such that the time scale matches that of $a_{int}$. This gives a comparison at the intermediate time scale; a plot, Fig. 2, of $a_{int}$ and filtered $G^2$ data gives the same curve, showing that the two are virtually identical. For this comparison, we first normalized $G^2$ by a constant to obtain the optimum agreement between filtered $G^2$ and $a_{int}$, but this normalization matches well with the static calibration of $G^2$ discussed above. (We used this double comparison to be sure that the static calibration matched well with the dynamics measurements.) We conclude that the large photoelastic events are the main force mechanism acting on the intruder. By inference, the energy loss for the intruder is tied to these acoustic events, rather than, e.g., to frictional drag with the intruder.

Acoustic dissipation. Once the acoustic pulses have moved ahead of the intruder, there must be a loss mecha-
cumulative photoelastic response, versus depth. The normalization for each pulse is the cumulative response in a long, thin angular slice, centered directly beneath the intruder. We use 40 different pulses from different impacts of a single intruder ($D = 21.17d$), where the intruder velocity at the pulse emission varies between 2 and 6 m/s. We then plot the natural logarithm of $G^2$ per area as a function of depth for each pulse, normalized by the total intensity in the pulse (wave intensity will decrease as $1/r$ moving away from a point source in 2D, and this effect has already been accounted for in this plot). Imposed fit (thick, red line) is $\exp(-r/L)$, where $L$ is the decay length, roughly 10 particle diameters.

FIG. 2: (color online.) (a) Comparing the intruder trajectory to photoelastic response from Fig. 1(c) shows that the intruder acceleration is very well correlated to the photoelastic/acoustic fluctuations in high-speed videos. We time-average the photoelastic response (thick, blue line) to match the time scale of the acceleration measurement, $a_{\text{int}}$ (black, dashed line), which has limited time resolution. Rescaling the photoelastic measurement gives extremely close agreement with the measured deceleration (both the mean and fluctuations). The calibrated photoelastic force measurement without time-filtering (thin, red line) shows much larger fluctuations at a much shorter time scale. (b) Inset shows calibration of photoelastic response vs. 2D pressure (force per width) from experiment (black dashed line) and from a static test (blue circles), with good agreement.

Large fluctuations in the photoelastic response averages over multiple events shows an exponential decay (Fig. 3), with a decay length of $\sim 10$ particle diameters, which is short enough that reflections from the bottom or sides of the container are not important. It is unclear which grain-scale interactions are responsible for this decay, but it could be explained by force-chain splitting, grain-grain friction, restitution losses for each “collision”, or other dissipative mechanisms.

Fluctuation statistics and stochastic description. Large fluctuations in the photoelastic response (Fig. 2) suggest a stochastic description, which captures mean behavior as well as short-time fluctuations. For example, one might modify Eq. (1) to:

$$F(z, \dot{z}, t) = mg - [f(z) + h(z)\dot{z}^2]\eta(t).$$

(2)

Here, $\eta(t)$ is a multiplicative stochastic term, which should follow directly from microscopic physics and have a mean of unity. A multiplicative term is chosen here since rescaling by the mean photoelastic behavior yields a statistically stationary fluctuating term, as discussed below, and since fluctuations in dense granular systems often scale with the mean (as here).

To experimentally characterize the fluctuations in Eq. (2), we write $\eta(t) \sim G^2(t)/G^2_{\text{avg}}(t)$, where $G^2_{\text{avg}}(t)$ is the photoelastic time series used to measure force (e.g. bottom of Fig. 1), and $G^2_{\text{avg}}(t)$ is the mean behavior, obtained by fitting a low-order polynomial to $G^2(t)$. This yields a fluctuating term which appears statistically stationary throughout the duration of an impact, as shown in Fig. 4. Typically, $\eta(t)$ has an autocorrelation decay time of $\sim 1$ ms, and a probability distribution function (PDF) that is nearly exponential. The PDF describes the likelihood of the large events which dominate the decelerating force. Such a PDF is typical for forces in static dense granular systems and is presumably related to the probability of generating force-chain-like structures.

Surprisingly, the fluctuation statistics show almost no dependence on intruder size. One might expect that the contact forces or force chains generated from two sufficiently separated points along the bottom of the intruder are uncorrelated. If so, increasing the intruder size...
would include more of these independent forces, which, by the central limit theorem, would yield smaller and more Gaussian-like fluctuations, regardless of the statistics of each one. However, this does not occur, suggesting a more subtle collective mechanism. One possibility is that spatially separated intruder-particle contacts often excite the same persistent force network.

**Conclusion.** In this Letter, we present a new microscopic picture of the force on an intruder moving through a granular material, which focuses on acoustic activity and fluctuations due to the generation of force-chain-like pulses. We observe consistency with established impact force models, but with substantial fluctuations in the measured deceleration of the intruder during the impact process. We have shown that the acceleration profiles, including these fluctuations, are a direct consequence of acoustic pulses transmitted along networks of particles. Other recent studies have indicated an important role for granular force networks in intruder impacts [17] and acoustic transmission [18]. The microscopic description presented here should also help connect granular impact experiments with differing microstructure, such as more dilute or compacted [8, 19], or anisotropic (e.g. sheared) systems, or even more general experiments on granular flow around an obstacle. Strong force fluctuations suggest a stochastic model, which gives a natural way to separate the slowly varying macroscopic response from fast-time fluctuations. We believe that the granular sound speed is critical in our description, so we expect substantial differences when intruder speeds are close to sonic or even supersonic. This could be achieved by increasing intruder velocity or reducing the granular sound speed by using softer material. Also of interest is how these effects translate to three-dimensional systems, or systems with much larger ratio of intruder size to particle size.

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