Analysis of Detection Methods in Massive MIMO Systems

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Abstract. Large-scale Multiple Input Multiple Output (MIMO) system is significant in our daily life such as used in hundreds of antennas at the base station work together at the same time. Matrix inversion can be used to solve this problem and calculate the result of formulas in this system, but this method is difficult for some people, and it may easy to make mistakes. In this paper uses some low-complexity signal detection algorithm, which are the Gauss-Seidel (GS) method, successive over relaxation (SOR) method, Minimum mean square error (MMSE) signal detection, in order to avoid the complicated matrix inversion. Can also use a systolic array and show reference FPGA implementation results for various system configurations. Steepest descent algorithm is employed to obtain an efficient searching direction for the following Jacobi iteration to speed up convergence is also a method to solve this problem. Firstly, we have to prove a special property that the MMSE filtering matrix. Then, we prove the convergence of a low-complexity iterative signal detection algorithm. The result shows that the method can reduce the computational complexity from K3 to K2 (K is the number of users). Finally, the result of the experiment prove that the method is more advantage than the recently proposed Neumann series approximation algorithm, and achieved the near-optimal performance of the classical MMSE algorithm.

1. Introduction
Large-scale Multiple Input Multiple Output (MIMO) system is different from small-scale MIMO technology. For instance, small-scale MIMO technology at most 8 antennas in LTE-A. Large-scale MIMO can achieve a very large number of antennas at the base station, maximum at 128 antennas. In theory, large-scale MIMO can provide potential opportunity to increase the spectrum and energy efficiency by orders of magnitude. But in practice, some challenging problems in MIMO systems. One of them is the practical signal detection algorithm in the up-link have to be solved. With the increase of the number of transmit antennas, the complexity of detector are increased too, which makes it impractical for large-scale MIMO systems.

One obviously classify is based on the sphere decoding (SD) algorithm. This kind of algorithms uses the underlying lattice structure of the received signal and considers the most promising approach to achieve the maximum likelihood (ML) detection performance with reduced complexity. Another classify must be taken into account is based on the tabu search (TS) algorithm derived from artificial intelligence. This kind of algorithms utilizes the idea of local neighborhood search to estimate the transmitted signal and limits the selection of neighborhood by a tabu list. Nevertheless, these methods must use unfavorable inversion of a matrix of large size. In order to reduce the complexity of matrix inversion, proposed the Neumann series approximation algorithm to convert the matrix inversion into a series of matrix-vector multiplications. This method can only reduce a few complexity.

This paper introduced five easy methods, such as follows:
Gauss-Seidel (GS) method [1]: Gauss-Seidel (GS) method realizes the MMSE estimate without matrix inversion based on the special property that the MMSE filtering matrix of large-scale MIMO systems is Hermitian positive definite. Then, this method can use the diagonal component of the MMSE filtering matrix to obtain a diagonal approximate. Finally, this paper advice an approximated method to calculate the channel gain and the noise-plus-interference (NPI) variance for log-likelihood ratios (LLRs) computation initial solution to the GS method.

Successive over relaxation (SOR) method [2]: This method detection algorithm with low complexity based on the successive over relaxation (SOR) method for large-scale MIMO systems. For up-link large-scale MIMO systems, we would better to use the SOR method to avoid the complicated matrix inversion. Meanwhile, we prove the convergence of the proposed.

Richardson method [3]: This method used the Richardson method to avoid the complicated matrix inversion. Gather for any initial solution when the relaxation parameter is appropriate. In order to up speed the convergence rate and reduce the computational complexity, the method propose a zone-based initial solution to the Richardson method, which can get a fast convergence rate and consequently reduces the required number of iterations.

Neumann Series Approximation [4]: A new method for matrix inversion: The method told a novel systolic VLSI architecture for carrying out the inversion at high throughput for the high-dimensional problems arising in large-scale MIMO systems.

The Proposed Joint Algorithm [5]: The method introduce an approach is proposed based on joint steepest descent algorithm and Jacobi iteration. The steepest descent is the best way to find an efficient searching direction. Besides, in order to further improve, we offer two optimization strategies, which are initial estimation and hybrid iteration.

The results of these methods are different. They all have their advantages and disadvantages. Compared with method one and method two, not only does method three reduce the difference of solve the problem, but also use a high speed to save time to calculate. Method one and two use a way of linear algebra, this way may need more time. Method four use an architecture model, can be more interesting. The last method use two exist optimization strategies, it can also improve the speed of answering. In conclusion, method three can be the way of recommendation.

2. System Model

This section told a model of MIMO. This system assumes N is the number of antennas work at the base station at the same time, K is the number of signal antennas. Usually, the number of N is significantly more than the number of K. For instance, N=128, K=16 have been considered. Hc is denote the flat Rayleigh fading channel matrix.

The transmitted bit streams from different users are first encoded by the channel encoder and then mapped to symbols by taking values from a modulation alphabet. Sc=[Sc,1,Sc,2,...,Sc,K]T this signal vector stands for all users. yc=[yc,1,yc,2... , yc,N]T this signal vector stands for the number of receivers.

At the base station, these vectors can be seen as:

\[ y_c = H_c s_c + n_c. \]  

For check a signal, this complex-valued function can transform a real-valued function, such as the following one:

\[ y = Hs + n. \]  

In this function, H is the slope, it has to be solved. y=[Re\{yc\} I_m\{yc\}]T, s = [Re\{sc\} I_m\{sc\}]T, n = [Re\{nc\} I_m\{nc\}]T. Then, H can be known as:

\[ H = \begin{bmatrix} \text{Re\{He\}} & -\text{Im\{He\}} \\ \text{Im\{He\}} & \text{Re\{He\}} \end{bmatrix} 2N \times 2K. \]  

At the base station, time-domain and frequency-domain train pilots can gets channel matrix H. Recovering the transmitted signal vector s from the received signal vector y is the mission of signal detection. The estimate of the transmitted signal vector ŝ can be obtained by
\[ \hat{y} = \left(H^H H + \sigma^2 I_{2K}\right)^{-1} H^H \hat{y} = W^{-1} \hat{y} \times 2K. \]  

In this function, \( \hat{y} = H^H y \), and through MMSE filtering matrix, \( W \) can be known as:

\[ W = G + \sigma^2 I_{2K}. \]

where \( G = H^H H \) is the Gram matrix. It is difficult to calculate the direct matrix, especially for large-scale MIMO systems.

### 3. Low-Complexity Detection For Uplink Scale MIMO System

#### 3.1. Gauss-Seidel (GS) method

This method first proposed a low-complexity signal detection algorithm which uses GS method to get the MMSE estimate without matrix inversion. In order to accelerate the convergence rate and reduce the complexity, we also propose a diagonal-approximate initial solution to the GS method. Then, an approximated method to calculate the channel gain and use the LLRs computation without matrix inversion.

1). Signal detection algorithm based on Gauss-Seidel method

The GS method is used to solve the N-dimension linear equation \( Ax = b \). Compared with the traditional method which uses \( A^{-1}b \) to get \( x \), this method is easy to solve. MMSE filtering matrix \( W \) is also Hermitian positive definite as mentioned above, \( W \) can be decomposed as:

\[ W = D + L + L^H. \]

In this formula, \( D, L, L^H \) are the diagonal component, it can be seen as an upper triangular determinant or a lower triangular determinant. Then, GS method can calculate an approximate signal vector \( s \) as follow:

\[ s^{(i)} = \left( D + L \right)^{-1} (y - L^H s^{(i-1)}), i = 1, 2, ... \]

2). Diagonal-approximate initial solution

For up-link large-scale MIMO systems, the channel matrix \( H \) is asymptotically orthogonal when \( N \gg K \), a relationship can be get:

\[ \frac{h_{mk}^H h_k^H}{N} \to 0, m \neq k, m, k = 1, 2, ..., K. \]

It can be observed that the domination of the diagonal elements of \( W^{-1} \) becomes more obvious with the increasing value of \( N/K \), and the difference between the diagonal matrix \( D^{-1} \) and the non-diagonal matrix \( W^{-1} \) becomes smaller. The special method can use \( D^{-1} \) to get \( W^{-1} \) with the lowest mistake. Then, the initial solution \( s^{(0)} \) can be approximated as

\[ s^{(0)} = D^{-1} y. \]

Since diagonal-approximate initial solution will be closer to the final, a faster convergence rate can be achieved.

3). Approximated method to compute LLRs

It can be also utilized to obtain the estimate of the matrix inversion \( W^{-1} \). According to formulas (4) and (7), a new function can be get

\[ (W_{inv})^{(i)} = \left( D + L \right)^{-1} (I_k - L^H (W_{inv})^{(i-1)}). \]

Let \( E^{(i)} = (W_{inv})^{(i)} G \) and \( U^{(i)} = (W_{inv})^{(i)} G(W_{inv})^{(i)} \), then an equivalent channel and a NPI variance can be get

\[ \mu_k^{(i)} = E_k^{(i)}, \]

\[ (v_k^{(i)})^2 = \sum_{m,k} |E_{mk}^{(i)}|^2 + U_{kk}^{(i)} \sigma^2. \]

According to the above two formulas, the exact mag-log LLRs for soft-input channel decoding can be obtained.
3.2. Successive over relaxation (SOR) method

Last method proved the MMSE filtering matrix is symmetric positive definite. In this section, the SOR method to iteratively achieve the MMSE estimate without matrix inversion will be illustrated. Since the complex-valued MIMO system model has been converted into the real-valued one, it is the same as the conjugate transpose of matrix. So there are

\[ G^T = (H^T H)^T = H^T H = G. \]  

(13)

It is obvious that matrix G is symmetric and equation \( Hq = 0 \) has an unique solution. The 2K×1 zero vector. Thus, for any 2K×1 non-zero real-valued vector \( r \), we have

\[ (Hr)^T Hr = r^T Gr > 0. \]  

(14)

It implies that G is positive definite. Thus, it can concluded formulas (5) and (13). These two functions are symmetric positive definite. Due to the MMSE filtering matrix \( W \) is symmetric positive definite for up-link large-scale MIMO systems as proved in , we can also decompose \( W \) as formula (6).

The SOR method is used to solve N-dimension linear equation \( Ax = b \). In this equation, \( A \) is the N×N symmetric positive definite matrix, \( x \) is the N×1 solution vector, and \( b \) is the N×1 measurement vector. The SOR method can efficiently solve the linear equation. Then, the relationship between them can be get

\[ x^{(i+1)} = (L_D + \frac{1}{\omega} D_A)^{-1}[((\frac{1}{\omega} - 1)D_A - L^T_A)x^{(i)} + b] \]  

(15)

Then we can use the SOR method to estimate the transmitted signal vector \( s \) as follow:

\[ s^{(i+1)} = (L + \frac{1}{\omega} D)^{-1}[((\frac{1}{\omega} - 1)D - L^T)x^{(i)} + y]. \]  

(16)

In order to solve the signal detection problem, it can be solved by the SOR method according to

\[ (L + \frac{1}{\omega} D)s^{(i+1)} = y + ((\frac{1}{\omega} - 1)D - L^T)s^{(i)}. \]  

(17)

In this formula, \((L + \frac{1}{\omega} D)\) is a lower triangular matrix. It can solve the formula (16) to obtain \( s \) with low complexity.

3.3. Richardson method

The MMSE filtering matrix in large-scale MIMO systems is symmetric positive definite has been proved in the last two methods, it won’t be tell again.

1). Signal detection based on Richardson method

Based on the MMSE filtering matrix symmetric positive definite in large-scale MIMO systems and formulas (13) and (14), The Richardson iteration can be described as

\[ x^{(i+1)} = x^{(i)} + \omega(b - Ax^{(i)}), i = 1,2,... \]  

(18)

In this formula, \( i \) is the number of iterations, and \( w \) is the relaxation parameter. \( W \) is symmetric positive definite, it has to be illustrated that the Richardson method can estimate the transmitted signal vector \( \hat{s} \) without matrix inversion as follow

\[ s^{(i+1)} = s^{(i)} + \omega(y - Ws^{(i)}), i = 0,1,2,... \]  

(19)

The necessary and sufficient conditions for convergence of Richardson method is \( 0 < \omega < 2 / \lambda_1 \), \( \lambda_1 \) is the largest eigenvalue for symmetric positive definite matrix A.

2). Zone-based initial solution

For up-link large-scale MIMO systems, \( \hat{s}_i \hat{y}_j > 0 \), Let \( W_{ij}^{-1} \) denote the ith row and jth column element of \( W^{-1} \), an new formula can be get as
\[ s_i y_i = \left( \sum_{j=1}^{2K} W^{-1}_{ij} y_j \right) y_i. \]  

(20)

Diagonal entries of \(W^{-1}\) are all positive. Thus, formula (20) can be approximated as follow

\[ \hat{s}_i y_i = \left( \sum_{j=1}^{2K} W^{-1}_{ij} y_j \right) y_i \approx W^{-1}_{ii} y_i y_i > 0. \]

(21)

Based on that, the range of the initial solution \(s^{(0)}\) can be narrowed. For instance, there is

\[ y = y - W \times \left( z, z, \ldots, z \right)^T. \]

(22)

3.4. Neumann Series Approximation

To arrive at low-complexity, start by computing the matched-filter output \(y^{MF} = H^H y\) and \(G = H^H H\). Then, a regularized matrix as follow

\[ A = GE_S + N_0 I_M. \]

(23)

Then, an estimate of the transmit vector can then be computed as

\[ s = A^{-1} y^{MF} = A^{-1} G s + A^{-1} n. \]

(24)

In order to check out soft-output, using vector \(i\) can illustrate more clearly, put it into formula (24), there are

\[ s_i = a_i^T g_i s_i + \sum_{j \neq i} a_i^T g_j s_j + a_i^T n = \mu s_i + \omega_i. \]

(25)

To reduce the complexity of inverse matrix, Neumann Series Approximation is a good method that can be recommended. Clearly, \(A\) is close to an invertible matrix \(X\)

\[ \lim_{n \to \infty} (I - X^{-1} A)^n = 0. \]

(26)

Matrix \(A^{-1}\) can be written as

\[ A^{-1} = \sum_{n=0}^{\infty} (X^{-1}(X - A))^n X^{-1}. \]

(27)

Since \(A\) is close to \(D\) in (6) for large-scale MIMO, let \(X = D\) by using Neumann Series Approximation. Assume that \(D\) is invertible, \(A = D + E\) can be rewritten as

\[ A^{-1} = (D + E)^{-1} = \sum_{n=0}^{\infty} (-D^{-1} E)^n D^{-1}. \]

(28)

By keeping only the first \(K\) terms of the Neumann Series, \(A^{-1}\) is about

\[ A^{-1}_k = \sum_{n=0}^{K-1} (-D^{-1} E)^n D^{-1}. \]

(29)

Finally, \(A^{-1}\) in place of \(A^{-1}_k\), which can reduce the complexity of linear detection in large-scale MIMO systems.

3.5. The Proposed Joint Algorithm

This method can be divided into three steps. Firstly, set up the model of equation. Secondly, perform steepest descent algorithm once. Thirdly, employ (K-1)-time Jacobi iterations.

1). Initial Estimation

Due to the diagonally dominant property of \(A\), the diagonal-approximating can obtain considerable performance improvement while causing little additional complexity, which is

\[ x^{(0)} = D^{-1} b = D_{inv} b. \]

(30)

2). Hybrid Iteration
Rewriting the first Jacobi iteration, the time in the formula has to be replaced, change 0 to 2. There
are:
\[
\begin{align*}
x^{(2)} &= D_{\text{inv}}[(D - A)x^{(1)} + b] \\
&= x^{(1)} + D_{\text{inv}}(b - Ax^{(1)}) = x^{(1)} + D_{\text{inv}}r^{(1)}, \\
x^{(1)} &= x^{(0)} + ur^{(1)}, \\
r^{(1)} &= b - Ax^{(1)}.
\end{align*}
\]
Combining with formula (31), (32) and (33), we can get a new relationship
\[
r^{(1)} = b - A(x^{(0)} + ur^{(0)}) = b - Ax^{(0)} - uAr^{(0)} = r^{(0)} - up^{(0)}.
\]
Combining with formula (31), (32), (33) and (34), we can get
\[
x^{(2)} = x^{(0)} + ur^{(0)} + D_{\text{inv}}(r^{(0)} - up^{(0)}).
\]
Therefore, within K iterations, the improved joint algorithm realizes 1-time steepest descent algorithm
and K-time Jacobi iterations.

4. Results Analysis
In method one, it is obvious that the traditional way about MMSE is difficult to calculate and it can
loss the accurate when the channel correlation becomes serious. The GS method can converge to the
MMSE algorithm, but more serious channel correlation may down the speed of convergence rate. On
the other hand, the GS reduce the complexity of calculation. In method two, the SOR method is also
less complexity than the traditional way. And simulation results show that it can achieve the near-
optimal performance of the classical MMSE algorithm with a small number of iterations. Richardson
method uses can achieve the near-optimal BER performance of the MMSE, got an approximate answer.
Neumann Series Approximation used a small number of Neumann series. Reduced the complex of terms.
Method five used FPGA to increase the parallelism. And it can also reduce the time to calculate.

5. Conclusion
This paper told five easy methods to achieve the MIMO system, instead of using inverse matrix.
Compared with method one and method two, not only does method three reduce the difference of solve
the problem, but also use a high speed to save time to calculate. Method one and two use a way of linear
algebra, this way may need more time. Method four use an architecture model, can be more interesting.
The last method use two exist optimization strategies, it can also improve the speed of answering. In
conclusion, method three can be the way of recommendation.

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