CP violation in charged Higgs decays in the MSSM

Ekaterina Christova†, Helmut Eberl‡, Elena Ginina†, Walter Majerotto‡

†Institute for Nuclear Research and Nuclear Energy, Sofia 1784, Bulgaria
‡Institut für Hochenergiephysik der Österreichischen Akademie der Wissenschaften, A-1050 Vienna, Austria
E-mails: echristo@inrne.bas.bg, helmut@hephy.oeaw.ac.at, eginina@inrne.bas.bg, majer@hephy.oeaw.ac.at

ABSTRACT: In the MSSM with complex parameters loop corrections to the decays $H^+ \to t\bar{b}$ and $H^- \to \bar{t}b$ with $t \to bW$ and $W \to l\nu$ lead to CP-violating asymmetries: a decay rate asymmetry, a forward-backward asymmetry and an energy asymmetry. We derive explicit formulas for them and perform a detailed numerical analysis. We study the dependence on the parameters and the phases involved. In particular, the influence of the running Yukawa coupling is taken into account. The decay rate asymmetry can go up to 25%, the forward-backward and the energy asymmetry up to 10%.

KEYWORDS: Supersymmetric Standard Model, Higgs Physics, CP violation.
1. Introduction

It is well known that supersymmetric models contain new sources of CP violation if the parameters are complex. In the Minimal Supersymmetric Standard Model (MSSM) the U(1) and SU(2) gaugino mass parameters $M_1$ and $M_2$, the higgsino mass parameter $\mu$, as well as the trilinear couplings $A_f$ (corresponding to a fermion $f$) may be complex. (Usually, $M_2$ is made real by redefining the fields.) Non-vanishing phases of these parameters cause CP-violating effects. While the phase of $\mu$ may be small for a supersymmetric particle spectrum of $\mathcal{O}(100 \text{ GeV})$ due to the experimental upper bounds of the electric dipole moments (EDMs) of electron and neutron, the trilinear couplings of the third generation $A_t,b,\tau$ are not so much constrained and can lead to significant CP-violation [1, 2].

In the following, we study CP violation in the decays of the charged Higgs bosons $H^\pm$ within the MSSM. There are three possible decays of $H^\pm$ into ordinary particles: $H^+$ into $t\bar{b}$, $\tau\nu$ and $W h^0$ and the CP conjugated ones, where $h^0$ is the lightest neutral Higgs boson of the MSSM. At tree level the partial decay widths of $H^+$ and $H^-$ are equal because of CP invariance of the Higgs potential. However, including loop corrections with intermediate SUSY-particles, they become different due to the CP violation induced by the complex
phases of the MSSM parameters, essentially of $A_{t,b,τ}$. Quite generally, these phases affect the whole Higgs sector of the MSSM substantially [3, 4].

A full one-loop calculation within the MSSM was done of the decays mentioned [5, 6, 7, 8], and the CP-violating decay rate asymmetry $\delta^{CP} = [\Gamma_{H^+} - \Gamma_{H^-}] / [\Gamma_{H^+} + \Gamma_{H^-}]$ for these decays was calculated. In the case of $H^+ \to t\bar{b}$ and $H^- \to \bar{t}b$ this asymmetry can go up to $\sim 25\%$.

In this paper, we go a step further by including the decay product particles of the top quark, see Fig. 1,

$$H^+ \to \bar{b}t \to \bar{b}b'W^+, \quad H^- \to b\bar{t} \to b\bar{b}'W^-,$$

and

$$H^+ \to \bar{b}t \to \bar{b}b'W^+ \to \bar{b}b't^+\nu_l, \quad H^- \to b\bar{t} \to b\bar{b}'W^- \to b\bar{b}'l^-\bar{\nu}_l. \quad (1.1)$$

We only consider CP violation induced by the loop diagrams of $H^\pm tb$ vertex. We neglect CP violation in the $tWb'$ vertex. In the Standard model it is very small and in the MSSM for $m_{H^+} > m_t$ all two-body decays of top into SUSY partners are excluded kinematically.

In particular, we will exploit the polarization of the top quark. The top-quark decays before forming a bound state due to its large mass, so that the polarization can be measured by the angular distributions of its decay products. The polarization is very sensitive to CP violation. We will consider suitable CP violating forward-backward and energy asymmetries by using angular or energy distributions of the decay particles. The asymmetries also depend on the sensitivity of $b'$ in (1.1) or the final lepton $l^\pm$ in (1.2) to the top-quark polarization. Further we make a numerical analysis for different values of the MSSM parameters.

This paper is organized in the following order. In Section 2 we present the formalism we use. There are subsections devoted to the polarization of the top quark and the CP-violating asymmetries. Sections 3 and 4 contain the angular and energy distributions and the analytic results for the asymmetries of (1.1) and (1.2), respectively. The numerical results are discussed in Section 5. Section 6 contains the conclusions. In Appendix A the formulas used for running Yukawa couplings $h_b$ and $h_t$ are given. In Appendix B we point out an error made in an equation of [5].
2. Formalism

In order to obtain the analytic expressions for the differential partial decay rates of (1.1) and (1.2), we follow the formalism of [9]. In accordance with it, for both of the processes we write

\[ d\Gamma_{f,t}^\pm = d\Gamma_{t,t}^{f} \frac{E_{t,t}}{m_t\Gamma_t}, \quad f = b', l. \]  

(2.1)

\( E_{t,t} \) is the energy of \( t(\bar{t}) \)-quark, and \( \Gamma_t \) is the total decay width of the \( t \)-quark. \( d\Gamma_{H^\pm} \) is the differential partial decay rate of the process \( H^\pm \to tb \) when CP-violation is included:

\[ d\Gamma_{H^\pm} = \frac{1}{2m_H} |\mathcal{M}_{H^\pm}|^2 d\Phi_{H^\pm}. \]  

(2.2)

where \( d\Phi_{H^\pm} \) is the relevant phase space element and

\[ \mathcal{M}_{H^\pm} = i\bar{u}(pt)[Y^+_t + Y^+_t P_R + Y^-_t P_L]u(-p_{\bar{b}}), \]  

(2.3)

\[ \mathcal{M}_{H^-} = i\bar{u}(pb)[Y^-_t - Y^-_t P_R + Y^+_b P_L]u(-p_t), \]  

(2.4)

\[ Y^\pm_t = y_t + \delta Y^\pm_t, \quad Y^\pm_b = y_b + \delta Y^\pm_b. \]  

(2.5)

Here \( y_t \) and \( y_b \) are the DR running couplings, see Appendix A, \( \delta Y^\pm_{t,b} \) are the SUSY-induced loop corrections, which most generally have CP-invariant and CP-violating parts:

\[ \delta Y^\pm_{t,b} = \delta Y^\text{inv}_{t,b} \pm \frac{1}{2} \delta Y^\text{CP}_{t,b}. \]  

(2.6)

\( d\Gamma_{f,t}^f \) is the differential partial rate of the process \( t \to b'\pm \) or \( t \to b'\nu \) when the \( t \)-quark is polarized and its polarization is determined in the former process \( H^\pm \to tb \):

\[ d\Gamma_{t,t}^f = \Gamma_0^f \left[ 1 \pm \alpha_f \frac{m_t (\xi_{t,t} p_f)}{(p p_f)} \right] d\Phi_{t,t}^f. \]  

(2.7)

\( \xi^o_t \) is the polarization vector of the \( t \)-quark and \( d\Phi_{t,t}^f \) are the phase space elements. The index \( f \) stands for the corresponding fermion (\( f = b', l \)) and \( \alpha_f \) determines its sensitivity to the polarization of the \( t \)-quark:

\[ \alpha_b = \frac{m_t^2 - 2m_W^2}{m_t^2 + 2m_W^2}, \quad \alpha_l = 1. \]  

(2.8)

In the kinematics of both of the processes (1.1) and (1.2) we work in the approximation

\[ m_t^2/m_W^2 \simeq m_b^2/m_t^2 \simeq m_b^2/m_W^2 \simeq 0, \]  

(2.9)

but we keep \( m_b \neq 0 \) in the couplings, where it is multiplied by \( \tan \beta \).
2.1 The t-quark polarization vector

The polarization four-vectors $\xi^\alpha_t$ and $\xi^\alpha_{\bar{t}}$ for the considered processes are covariantly given by the expressions [9]:

$$\xi^\alpha_t = \left( g^{\alpha\beta} - \frac{p_t^\alpha p_t^\beta}{m_t^2} \right) \frac{\text{Tr} [M_{H^+} (\Lambda(-p_b)) \overline{M} p_t^\alpha \Lambda(p_t)]}{\text{Tr} [M_{H^+} (-1) \Lambda(-p_b) \overline{M} p_t^\alpha \Lambda(p_t)]},$$

(2.10)

$$\xi^\alpha_{\bar{t}} = \left( g^{\alpha\beta} - \frac{p_{\bar{t}}^\alpha p_{\bar{t}}^\beta}{m_{\bar{t}}^2} \right) \frac{\text{Tr} [M_{H^-} (\Lambda(-p_t)) \overline{M} p_{\bar{t}}^\alpha \Lambda(p_t)]}{\text{Tr} [M_{H^-} (-1) \Lambda(-p_t) \overline{M} p_{\bar{t}}^\alpha \Lambda(p_t)]},$$

(2.11)

where

$$M_{H^+} = Y_b^+ P_R + Y_{t^+} P_L, \quad M_{H^-} = Y_{\bar{t}}^- P_R + Y_t^- P_L,$$

(2.12)

$$\overline{M} = \gamma_0 M^+ \gamma_0, \quad \Lambda(p_t) = \not{p_t} + m_t.$$

(2.13)

Thus we obtain:

$$\xi^\alpha_{t,\bar{t}} = m_t P^\pm Q^\alpha_{b,\bar{b}}, \quad Q^\alpha_{b,\bar{b}} = \left( \frac{p_b p_{\bar{b}}}{m_t^2} - \frac{(p_t p_{\bar{t}})}{m_{\bar{t}}^2} \right) .$$

(2.14)

Notice that the four-vector $Q^\alpha_{b,\bar{b}}$ is the only four-vector in $H^\pm \to tb$ that can be constructed so that it satisfies the orthogonal condition $(\xi^\alpha_t p_t) = 0$. The polarization vectors (2.14) have CP-invariant and CP-violating parts, and the CP-violating parts are only due to the loop corrections:

$$P^\pm = \pm P^{\text{inv}} + P^{\text{CP}},$$

(2.15)

$$P^{\text{inv}} = \frac{y_t^2 - y_b^2}{(y_t^2 + y_b^2)(p_t p_{\bar{b}}) - 2 m_t m_b y_t y_b},$$

(2.16)

$$P^{\text{CP}} = \frac{y_t \text{Re}(\delta Y_t^{\text{CP}}) - y_b \text{Re}(\delta Y_b^{\text{CP}})}{(y_t^2 + y_b^2)(p_t p_{\bar{b}}) - 2 m_t m_b y_t y_b}.$$

(2.17)

The explicit forms of the individual contributions to Re$(\delta Y_{t,b}^{\text{CP}})$ are taken from [5].

2.2 CP-violating asymmetries

The CP-violating decay rate asymmetry $\delta^{\text{CP}}$ is given by the expression

$$\delta^{\text{CP}} = \frac{\Gamma_+ - \Gamma_-}{\Gamma_+ + \Gamma_-}.$$

(2.18)

In (2.18) $\Gamma_\pm$ are the partial decay widths of $H^\pm$.

Next, we construct a CP-violating forward-backward (FB) asymmetry $\Delta A^{\text{CP}}$ from the FB asymmetries $A^{\text{FB}}_\pm$ using the angular distributions of the processes

$$\Delta A^{\text{CP}} = A^{\text{FB}}_+ - A^{\text{FB}}_-,$$

(2.19)

where

$$A^{\text{FB}}_\pm = \frac{\Gamma_\pm - \Gamma_\mp}{\Gamma_+ + \Gamma_-},$$

(2.20)

$$\Gamma_\pm = \int_0^{\frac{\pi}{2}} \frac{d\Gamma_\pm}{d\cos \theta} d\cos \theta \quad \text{and} \quad \Gamma_\mp = \int_{\frac{\pi}{2}}^\pi \frac{d\Gamma_\mp}{d\cos \theta} d\cos \theta.$$

(2.21)
i.e. $\Gamma_\pm^F$ are the number of particles /antiparticles measured in the forward direction of the decaying $t/\bar{t}$ quarks, etc. Analogously, a CP-violating energy asymmetry $\Delta R^{CP}$ can be defined, using the energy distributions of the processes

$$\Delta R^{CP} = R_+ - R_-,$$

where $R_\pm$ are

$$R_\pm = \frac{\Gamma_\pm(x > x_0) - \Gamma_\pm(x < x_0)}{\Gamma_\pm(x > x_0) + \Gamma_\pm(x < x_0)}. \quad (2.23)$$

$x$ is a dimensionless variable proportional to the energy, and $x_0$ is any fixed value in the energy interval.

### 3. The $H^\pm \rightarrow W^\pm bb'$ process

Following the formalism of Section 2 for the differential partial decay rate of the process (1.1) in the rest frame of $H^\pm$, we obtain

$$d\Gamma_b^\pm = \Gamma_{H^\pm}^0 \left[ 1 \pm \frac{\alpha_b m_t (\xi_t, p_{t,y'})}{(p_t p_y)} \right] \frac{E_t E_y}{m_t m_y} d\Phi_{\nu', \bar{\nu}}. \quad (3.1)$$

$\Gamma_{H^\pm}$ is the partial decay width of the process $H^\pm \rightarrow tb$, assuming CP-violation in its vertex

$$\Gamma_{H^\pm} = C_H (\Gamma^{inv} \pm \Gamma^{CP}), \quad (3.2)$$

where

$$C_H = \frac{3\alpha \lambda^{1/2}(m_H^2, m_t^2, m_b^2)}{4m_H^2 m_Y^2}, \quad \alpha = \frac{g^2}{4\pi}, \quad (3.3)$$

$$\Gamma^{inv} = (y_t^2 + y_b^2)(p_t p_b) - 2m_t m_y y_t y_b, \quad (3.4)$$

$$\Gamma^{CP} = [y_t \text{Re}(\delta Y_t^{CP}) + (y_b \text{Re}(\delta Y_b^{CP})] (p_t p_b) - m_t m_b [y_t \text{Re}(\delta Y_t^{CP}) + y_b \text{Re}(\delta Y_b^{CP})], \quad (3.5)$$

$$\lambda(x, y, z) = x^2 + y^2 + z^2 - 2x y - 2x z - 2y z, \quad \lambda^{1/2}(m_H^2, m_t^2, m_b^2) \approx m_H^2 - m_t^2, \quad (3.6)$$

$$\Gamma_b^0 = \frac{g^2(m_H^2 - m_t^2)(m_b^2 + 2m_W^2)}{8m_W^2 E_t}, \quad \text{and} \quad d\Phi_{\nu', \bar{\nu}} = -\frac{(m_t^2 - m_b^2) d\cos \theta_{\nu', \bar{\nu}}}{16\pi E_t^2 (1 - \beta_t \cos \theta_{\nu', \bar{\nu}})} \quad (3.8)$$

#### 3.1 Angular distributions

For the angular distributions of $b' (\bar{b}')$ when the 3-momentum of the $t(\bar{t})$-quark is along the $z$-axis one gets

$$\frac{d\Gamma_b^\pm}{d\cos \theta_{\nu', \bar{\nu}}} = \frac{C_b}{(1 - \beta_t \cos \theta_{\nu', \bar{\nu}})^2} \left\{ \Gamma^{inv} \pm \Gamma^{CP} + \right. \quad (3.9)$$

$$\left. \alpha_b m_t^2 [\Gamma^{inv} \pm (\Gamma^{CP} \cdot \Gamma^{inv} + \Gamma^{inv} \cdot \Gamma^{CP})] \left( \frac{E_b (1 + \cos \theta_{\nu', \bar{\nu}})}{E_t (1 - \beta_t \cos \theta_{\nu', \bar{\nu}})} - \frac{(p_t p_b)}{m_t^2} \right) \right\},$$

$$\Delta R^{CP} \approx R_+ - R_-,$$
where
\[ C_b = -\frac{3\alpha_0^2|\vec{p}|(m_b^2 - m_W^2)^2(m_t^2 + 2m_W^2)}{64m_H^2 m_W^4 E_t^2 m_t \Gamma_t} . \]  
(3.10)

\[ \beta_t = \frac{|\vec{p}|}{E_t}, \quad \vec{p} = \frac{\lambda^{1/2}(m_b^2, m_t^2, m_b^2)}{2m_H} \approx \frac{m_b^2 - m_t^2}{2m_H} , \]  
(3.11)

\[ E_t = \frac{m_b^2 + m_t^2 - m_H^2}{2m_H} \approx \frac{m_b^2 + m_t^2}{2m_H}, \quad E_b = \frac{m_b^2 + m_t^2 - m_H^2}{2m_H} \approx \frac{m_b^2 - m_t^2}{2m_H} . \]  
(3.12)

We are interested in the CP-violating contributions to the loop corrections of the \( H^\pm bt \) vertex (2.6). The quantities \( \delta Y_t^{CP} \) and \( \delta Y_b^{CP} \) enter the two independent combinations \( \Gamma^{CP} \) and \( P^{CP} \). One therefore needs two measurements to determine them. The decay rate asymmetry \( \delta_b^{CP} \) for process (1.1) measures \( \Gamma^{CP} \) given in (3.5),
\[ \delta_b^{CP} = \frac{N_{b'} - N_{\bar{b'}}}{N_{b'} + N_{\bar{b'}}} = \frac{\Gamma^{CP}}{\Gamma^{inv}}, \]  
(3.13)

where \( N_{b'(\bar{b}')} \) are the total number of \( b' (\bar{b}') \) in \( H^\pm \to bb'W^\pm \) decay.

The CP-violating angular asymmetry \( \Delta A^{CP} \) measures the other combination \( P^{CP} \) given in (2.17). We have
\[ \Delta A_b^{CP} = 2\alpha_b m_t^2 m_H^2 \frac{m_b^2 - m_t^2}{(m_H^2 + m_t^2)^2} P^{CP} . \]  
(3.14)

The FB asymmetries are given by
\[ A_b^{FB} = \beta_t + \alpha_b m_t^2 m_H^2 \frac{m_b^2 - m_t^2}{(m_H^2 + m_t^2)^2} \frac{(\Gamma^{inv} P^{inv} \pm \Gamma^{CP} P^{inv} \pm \Gamma^{inv} P^{CP})}{\Gamma^{inv} \pm \Gamma^{CP}} . \]  
(3.15)

Using the expansion
\[ \frac{(\Gamma^{inv} P^{inv} \pm \Gamma^{CP} P^{inv} \pm \Gamma^{inv} P^{CP})}{\Gamma^{inv} \pm \Gamma^{CP}} = P^{inv} \pm P^{CP} + \text{higher orders}, \]  
(3.16)

we get at one-loop level
\[ A_b^{FB} = \beta_t + \alpha_b m_t^2 m_H^2 \frac{m_b^2 - m_t^2}{(m_H^2 + m_t^2)^2} (P^{inv} \pm P^{CP}) . \]  
(3.17)

### 3.2 Energy distributions

We write the energy distribution of \( b'(\bar{b}') \) as a function of \( x = E_{b'}/m_H \) (\( x = E_{\bar{b}'}/m_H \)):
\[ \frac{d\Gamma^\pm}{dx} = C_E[c_0^\pm + c_1^\pm x], \]  
(3.18)

where
\[ C_E = \frac{3\alpha_0^2(m_b^2 - m_W^2)(m_t^2 + 2m_W^2)}{2^6 m_W^4 m_H^2 m_t \Gamma_t}, \quad c_0^\pm = c_{0}^{inv} \pm c_{0}^{CP}, \quad c_1^\pm = c_{1}^{inv} \pm c_{1}^{CP} , \]  
(3.19)

\[ c_0^{inv} = \Gamma^{inv} - \alpha_b \frac{m_t^2 + m_b^2}{2} \Gamma^{inv} P^{inv} , \]  
(3.20)
\[ c_{0CP} = \Gamma_{CP} - \alpha_0 \frac{(m_H^2 + m_t^2)}{2} (\Gamma_{\text{inv}pCP} + \Gamma_{CPp\text{inv}}), \]  
\[ c_{1\text{inv}} = 2\alpha_0 \frac{m_H^2 m_t^2}{(m_t^2 - m_W^2)} \Gamma_{\text{inv}p\text{inv}}, \quad c_{1CP} = 2\alpha_0 \frac{m_H^2 m_t^2}{(m_t^2 - m_W^2)} (\Gamma_{\text{inv}pCP} + \Gamma_{CPp\text{inv}}). \]  

The asymmetry \( \Delta R_{CP} \) also measures \( \mathcal{P}_{CP} \). We choose \( x_0 = (x_{\min} + x_{\max})/2 \). Inserting the one-loop result

\[ R_b = \frac{1}{4} \alpha_b (m_H^2 - m_t^2) (\mathcal{P}_{\text{inv}} \pm \mathcal{P}_{CP}) \]  
into eq.(2.23) gives

\[ \Delta R_{bCP} = \frac{1}{2} \alpha_b (m_H^2 - m_t^2) \mathcal{P}_{CP}. \]  

4. The \( H^\pm \to bb't^{\pm} \nu \) process

In order to obtain the differential partial decay rate of (1.2), we fix the coordinate system such that the t-quark points in the direction of the z-axis and the 3-momenta of \( t \) and \( l \) determine the yz-plane:

\[ \bar{p}_i = |\bar{p}_i| |0,0,1|, \quad \bar{p}_l = |\bar{p}_l| |0, \sin \theta_l, \cos \theta_l|, \]

\[ \bar{p}_{l', \bar{t}'} = |\bar{p}_{l', \bar{t}'}| |\sin \theta_{l', \bar{t}'}, \sin \phi_{l', \bar{t}'}, \cos \theta_{l', \bar{t}'}| \]  
(4.1)

The angular distributions of \( l^\pm \) are then given by

\[ d\Gamma_l = \Gamma_l^0 \left[ 1 \pm \alpha l m_t \frac{\xi_{l,l'}}{(p_l p_l')} \right] \frac{E_{l,l'}^2 d\Omega_{l,l'}}{m_l m_{l'}}, \]  
(4.2)

where

\[ \Gamma_l^0 = \frac{g^4 \pi [m_l^2 - 2(p_l p_{l'}) (p_l p_{l'})]}{2E_l m_W E_{l'}}, \quad \delta(p_{l'}^2 - m_W^2) d\Phi_l = \frac{1}{(2\pi)^4} \frac{E_{l'}^2 E_l^2 d\Omega_{l,l'} d\cos \theta_{l,l'}}{4 (m_l^2 - m_W^2) m_{l'}^2}, \]  
(4.3)

\[ (p_l p_{l'}) = E_l E_{l'} (1 - \beta_l \cos \theta_{l,l'}), \]  
(4.4)

\[ E_l = \frac{m_W^2}{2[E_l (1 - \beta_l \cos \theta_l) - E_{l'} (1 - \cos \theta_{l'})]} \quad \text{and} \quad E_{l'} = \frac{m_l^2 - m_{l'}^2}{2E_l (1 - \beta_l \cos \theta_{l,l'})}. \]  
(4.5)

The angle \( \theta_{l,l'} \) is between \( \bar{p}_l \) and \( \bar{p}_{l'} \):

\[ \cos \theta_{l,l'} = \sin \theta_l \sin \phi_{l'} + \cos \theta_l \cos \theta_{l'} \]  
(4.6)

The angular distributions of \( l^\pm \) then reads

\[ \frac{d\Gamma_l}{d\cos \theta_{l,l'}} = \frac{C_l}{(1 - \beta_l \cos \theta_{l,l'})^2} \left\{ (\Gamma_{\text{inv}} \pm \Gamma_{CP}) \right. \]

\[ \left. \alpha l m_t^2 (\Gamma_{\text{inv}} \mathcal{P}_{\text{inv}} \pm (\Gamma_{\text{inv}} \mathcal{P}_{CP} + \Gamma_{CP} \mathcal{P}_{\text{inv}})) \right\} \left( \frac{E_{l'} (1 + \cos \theta_{l,l'})}{E_l (1 - \beta_l \cos \theta_{l,l'})} - \frac{(p_l p_{l'})}{m_l^2} \right). \]  
(4.7)
where
\[
C_l = -\alpha^3 m_W |\bar{p}_t|(m_t^2 - m_W^2)^2 \times \frac{(1 - \beta_t^2)^2 E_t^2 + 3m_t^4 - 5m_t^2 m_W^2 + 2m_W^4 - 3m_t^2 - 2m_W^2)}{2^8 m_t^2 m_t \Gamma_t \Gamma_W E_t^2 (m_t^2 - m_W^2 - (1 - \beta_t^2) E_t^2)^3}.
\]

(4.8)

\(\Gamma_W\) is the total decay width of the W boson.

As there is no CP violation in \(t \to bW\) decay, the decay rate asymmetry \(\delta_l^{CP}\) for process (1.2) will measure the same combination \(\Gamma^{CP}\):

\[
\delta_l^{CP} = \frac{N_{l^+} - N_{l^-}}{N_{l^+} + N_{l^-}} = \frac{\Gamma^{CP}}{\Gamma^{inv}},
\]

(4.9)

where \(N_{l^\pm}\) are the total number of \(l^\pm\) in \(H^\pm \to bb'l^\pm\nu\) decay.

For the CP-violating FB asymmetry \(\Delta A^{CP}_{l}\) of the process (1.2) we obtain

\[
\Delta A^{CP}_{l} = 2\alpha_l m_t^2 m_H^2 \frac{(m_H^2 - m_l^2)}{(m_H^2 + m_l^2)^2} \frac{\Gamma^{CP}}{\Gamma^{inv}},
\]

(4.10)

and the FB asymmetries are at one-loop level

\[
A^{FB}_{l} = \beta_l + \alpha_l m_t^2 m_H^2 \frac{(m_H^2 - m_l^2)}{(m_H^2 + m_l^2)^2} \left( \mathcal{P}^{inv} \pm \mathcal{P}^{CP} \right).
\]

(4.11)

Notice, that the only difference between the expressions (3.14) and (4.10) is the coefficient \(\alpha_b\) in (3.14) and \(\alpha_l\) in (4.10). These coefficients are only connected to the polarization of the t quark. Because of the fact that \(\alpha_b = 0.38\) and \(\alpha_l = 1\), one would expect a bigger effect measuring (4.10).

5. Numerical Results

Here we present a numerical analysis of the discussed asymmetries. First we analyze the CP-violating asymmetries \(\delta^{CP}_{b,l}\), \(\Delta A^{CP}_{b,l}\) and \(\Delta R^{CP}_{b}\). Further, we study the FB asymmetries \(A^{FB}_{b,l}\) and \(R_{b\pm}\) needed for \(\Delta A^{CP}_{b,l}\) and \(\Delta R^{CP}_{b}\).

5.1 The CP-violating asymmetries

The expressions \(\mathcal{P}^{CP}\) and \(\Gamma^{CP}\), (2.17) and (3.5), are linear combinations of the CP violating form factors \(\text{Re}(\delta Y^{CP}_t)\) and \(\text{Re}(\delta Y^{CP}_b)\). Therefore, we need to measure: 1) the decay rate asymmetries \(\delta^{CP}_{b,l}\) which are proportional to \(\Gamma^{CP}\) and 2) the angular and/or energy asymmetries which are proportional to \(\mathcal{P}^{CP}\).

As there is no CP violation in \(t \to bW\), the decay rate asymmetries for (1.1) and (1.2) are equal to the decay rate asymmetry for \(H^\pm \to tb\). We denote it, following ref.[5], by \(\delta^{CP}\):

\[
\delta^{CP} = \delta^{CP}_b = \delta^{CP}_l = \frac{\Gamma^{CP}}{\Gamma^{inv}}.
\]

(5.1)
The angular and energy asymmetries are not independent either. As seen from (3.14) and (4.10), the angular asymmetries for leptons and $b$-quarks are related by:

$$\Delta A_{\mu}^{CP} \approx 2.6 \Delta A_{b}^{CP}. \quad (5.2)$$

Further, (3.14) and (3.24) lead to a simple relation between the $b$-quark energy and angular asymmetries:

$$\Delta R_{b}^{CP} = \frac{(m_{H}^{2} + m_{t}^{2})}{4m_{H}^{2}m_{t}^{2}} \Delta A_{b}^{CP}, \quad (5.3)$$

which implies that for $m_{H} > m_{t}$, $\Delta R_{b}^{CP}$ is bigger than $\Delta A_{b}^{CP}$. Thus, in general, $\Delta R_{b}^{CP}$ is the biggest asymmetry of 2) for $m_{H} > 490 \text{ GeV}$. Fig. 2 illustrates the relative size of the asymmetries $\Delta A_{b,\mu}^{CP}$ and $\Delta R_{b}^{CP}$ as a function of $m_{H}$. Note that the relations (5.2) and (5.3) are independent of $\tan \beta$.

![Figure 2: The ratio $\Delta R_{b}^{CP}/\Delta A_{b}^{CP}$ as a function of $m_{H}$](image)

Therefore, in the following we shall discuss only the decay rate asymmetry $\delta^{CP}$ and the energy asymmetry $\Delta R_{b}^{CP}$. (Because of a conjugation error in our paper [5] in the formula for the $t\bar{b}g$ vertex, see the Appendix B, we have redone the numerical analysis for $\delta^{CP}$.) The purpose of our analysis is to determine the size of the asymmetries as functions of $m_{H}$ and $\tan \beta$, being the most important parameters of the Higgs sector in MSSM, for different values of the CP-violating phases.

The sources of CP violation in our processes are the one-loop corrections to the $H^{+}tb$ vertex with intermediate SUSY particles, see Fig. 1a, 1b, 1d, 1e of [5] and the self-energy graph with $\tau \bar{\nu}_{\tau}$. (The corrections due to Fig. 1c and Fig. 1f of [5] are of higher order and we do not consider them here.) In order not to deal with too many phases we assume the GUT relation between $M_{1}$ and $M_{2}$ which fixes $\phi_{M_{1}} = 0$. According to the experimental limits on the electric and neutron EDM’s, we take $\phi_{\mu} = 0$ or $\phi_{\mu} = \pi/10$. Thus, the remaining CP-violating phases in our study are the phases of $A_{t}$, $A_{b}$ and $A_{\tau}$ which we shall vary. If not specified otherwise, we fix the following values for the other MSSM parameters:

$$M_{2} = 300 \text{ GeV}, \quad M_{3} = 745 \text{ GeV}, \quad M_{\tilde{U}} = M_{\tilde{Q}} = M_{\tilde{D}} = M_{E} = M_{L} = 350 \text{ GeV},$$
\[ \mu = -700 \text{ GeV}, \quad |A_t| = |A_b| = |A_\tau| = 700 \text{ GeV} \] (5.4)

The relevant masses of the sparticles for the choice (5.4) and \( \tan \beta = 5 \) or 30 are given in Table 1. For the case with \( \phi_\mu = \pi/10 \) and the other parameters unchanged, all masses do not change by more than 1 GeV from those given in Table 1, except for \( m_{\tilde{t}_1} = 187 \) GeV and \( m_{\tilde{t}_2} = 515 \) GeV for \( \tan \beta = 5 \), and \( m_{\tilde{t}_1} = 176 \) GeV for \( \tan \beta = 30 \). Note that \( \phi_\mu = \pi/10 \) implies \( \mu = -700 e^{i\pi/10} \) GeV. We have used running top and bottom Yukawa couplings, calculated at the scale \( Q = m_{H^+} \), see Appendix A. We have checked that the asymmetries have only a very weak dependence on the scale \( Q \).

![Figure 3](image-url)

**Figure 3:** The asymmetries \( \delta^{CP} \) and \( \Delta R_b^{CP} \) as a function of \( m_{H^+} \) for \( \phi_{A_t} = \pi/2, \phi_{A_b} = \phi_\mu = 0 \). The red, blue, and green lines are for \( \tan \beta = 5,10, \) and 30.

shown that the most important CP-violating phase is \( \phi_{A_t} \). There is only a very weak dependence on \( \phi_{A_b} \) and \( \phi_{A_\tau} \). We therefore take them zero.

The main contributions to both \( \delta^{CP} \) and \( \Delta R_b^{CP} \) come from the self-energy graph with stop-sbottom. The vertex graph with stop-sbottom-gluino also gives a non-zero contribution. Their contributions are shown in Fig. 4. The contribution of the rest of the graphs is negligible. This justifies the use of the GUT relation which fixes \( \phi_{M_1} = 0 \), and it explains the weak dependence on \( \phi_{A_\tau} \).
Figure 4: The contribution of the $\tilde{t}\tilde{b}$ self-energy (red line), $\tilde{t}\tilde{b}\tilde{g}$ vertex contribution (blue line) and the sum of the other (green line) diagrams to $\delta^{CP}$ and $\Delta R_{b}^{CP}$ as a function of $m_{H^\pm}$ for $\tan\beta = 5$ and $\phi_{A_t} = \pi/2$, $\phi_{A_b} = \phi_{\mu} = 0$.

Up to now, all the analyses are done for $M_3 = m_{\tilde{g}} = 744$ GeV. In Fig. 5, we show the dependence of $\delta^{CP}$ on the gluino mass. In general, $\delta^{CP}$ gets small with increasing gluino mass.

Figure 5: $\delta^{CP}$ as a function of the gluino mass for $\tan\beta = 5$. The red line is for $m_{H^+} = 600$ GeV, the blue line is for $m_{H^+} = 800$ GeV and the green line is for $m_{H^+} = 1000$ GeV.

Let us now allow a non-zero phase of $\mu$. We take a very small phase, $\phi_{\mu} = \pi/10$, in order not to be in contradiction with the experimental data. As can be seen in Fig. 6, the asymmetries can increase up to 25% for $\delta^{CP}$ and 10% for $\Delta R_{b}^{CP}$. The discussed asymmetries $\delta^{CP}$ and $\Delta R_{b}^{CP}$ show a very strong dependence on the sign of $\mu$. As noted above, our analysis is done for $\mu = -700$ (see (5.4)), however if $\mu$ changes sign, $\mu = 700$, all asymmetries become extremely small.

5.2 The P-violating asymmetries

When discussing the possibilities to measure $\Delta A_{b,l}^{CP}$ and $\Delta R_{b}^{CP}$, it is also important to know the size of the FB asymmetries $A_{b,l}^{FB}$ (3.17) and (4.11), and of the energy asymmetry $R_{b,\pm}$, (3.23), that enter the corresponding CP-violating asymmetries.
Figure 6: The asymmetries $\delta^\text{CP}$ and $\Delta R^\text{CP}_b$ as a function of $m_{H^+}$ for $\phi_{A_t} = \pi/2$, $\phi_{A_b} = 0$ and a non zero phase of $\mu$, $\phi_{\mu} = \pi/10$. The red, blue and green lines are for $\tan \beta = 5, 10, \text{and } 30$.

$A_{b,l}^{FB}$ and $R_{b}^{\pm}$ are determined by the polarization $P^\pm$ of the $t$-quark in $H^\pm \rightarrow tb$ decays. As the Lagrangian violates parity, these asymmetries appear already at tree level and thus should be rather large.

Neglecting the loop induced CP-violating part $P^{CP}$ in (3.17), (4.11), and (3.23), we get

$$A_{b,l}^{\text{inv}} = \frac{1}{2} (A_{b,l}^{FB} + A_{b,l}^{FB}) = \beta_t + \alpha_{b,l} m_t^2 m_{H^+}^2 \frac{(m_{H^+}^2 - m_t^2)}{(m_{H^+}^2 + m_t^2)^2} P^{\text{inv}},$$

$$(5.5)$$

$$R_{b}^{\text{inv}} = \frac{1}{2} (R_{b} + R_{b} -) = \frac{1}{4} \frac{\alpha_b (m_{H^+}^2 - m_t^2) P^{\text{inv}}}{y_t^2 + y_b^2} \approx \frac{\alpha_b y_t^2 - y_b^2}{2 (y_t^2 + y_b^2)}.$$  

Thus $\Delta A_{b,l}^{CP}$, (3.14, 4.10), and $\Delta R_{b}^{CP}$, (4.10), are determined by $P^{CP}$, while $A_{b,l}^{FB}$ and $R_b$ are determined by $P^{\text{inv}}$, and there is no a priori reason to expect that their $\tan \beta$ and $m_{H^+}$ dependences will be the same. $A_{b,l}^{\text{inv}}$ and $R_{b}^{\text{inv}}$ are tree-level quantities. Including the one-loop corrections to these would require the full renormalization of the process which is beyond the scope of this paper.

In Fig. 7 we present $A_{b}^{\text{inv}}$ and $A_{l}^{\text{inv}}$ as a function of $m_{H^+}$ for $\tan \beta = 5, 10, \text{and } 30$, and in Fig. 8 we show $R_{b}^{\text{inv}}$ as a function of $\tan \beta$. Note that at tree-level $A_{b,l}^{FB}$ depends only on $\tan \beta$ and $m_{H^+}$, and $R_b$ only on $\tan \beta$.

6. Conclusions

We have calculated the CP-violating decay rate, forward-backward and energy asymmetries between $H^+ \rightarrow \bar{b} t \rightarrow \bar{b} b' W^+ (\rightarrow b b' l^+ \nu_l)$ and $H^- \rightarrow b \bar{t} \rightarrow b \bar{b'} W^- (\rightarrow b b' l^- \nu_l)$. They are induced by loop corrections in the $H^\pm tb$- vertex. The CP violating forward-backward and energy asymmetries are determined by the polarization of the top quark and are therefore related. We have shown that it is necessary to measure both the decay rate asymmetry $\delta^{CP}$ and the forward-backward or the energy asymmetry to get the maximal information on the CP-violating parts of the decay amplitude. We have performed a detailed numerical
Figure 7: The forward-backward asymmetries $A_{inv}^{b}$ and $A_{inv}^{l}$ as a function of $m_{H^+}$ for $\tan \beta = 5$ (red), 10 (blue), and 30 (green).

Figure 8: The energy asymmetry $R_{inv}^{b}$ as a function of $\tan \beta$

analysis of these quantities. An important improvement with running Yukawa couplings at the $m_{H^+}$ scale has been made. The asymmetries are most sensitive to the phase $\phi_{A_t}$. The asymmetries reach their maximum for $\tan \beta = 5$ and $\mu = -700$ GeV. The decay rate asymmetry can go up to 25%, the others up to 10%. The main contribution comes from the self-energy diagram with stop and sbottom exchange. We have also calculated the $P$-violating asymmetries at tree level.

We want to add a few remarks on the measurability of these asymmetries. In principle, the production rate for $H^\pm$ at LHC is not so small being 0.2 pb for $m_{H^+} = 500$ GeV and $\tan \beta = 30$ [10, 11]. The main production process is due to $g \bar{b} \rightarrow H^+ \bar{t}$. Because of the large background, the actual signal production rate is strongly reduced. According to [10], one can expect $N = 733$ signals with $N/\sqrt{B} = 12.6$ for $m_{H^+} = 500$ GeV, $\tan \beta = 50$ for a luminosity $L = 100$ fb$^{-1}$. The statistical significance $\sqrt{N}$ $A$ to measure an asymmetry $A$ of several percent might be too low for a clear observation of CP violation in $H^+$ decays at LHC in the first stage. However, at SLHC for which a luminosity of 1000-3000 fb$^{-1}$ is designed, such a measurement would be worth of being performed. Here we have only considered CP violation in the $H^+$ decays. However, similar graphs are also present in the production process $g \bar{b} \rightarrow H^+ \bar{t}$ [8]. One would expect a CP-violating asymmetry.
of similar size. The total asymmetry in production and decay would be approximately additive, $A_{\text{tot.}} = A_{\text{prod.}} + A_{\text{decay}}$.

Acknowledgements

We thank Jennifer Williams for finding the conjugation error in [5]. The authors acknowledge support from EU under the MRTN-CT-2006-035505 network programme. This work is supported by the "Fonds zur Förderung der wissenschaftlichen Forschung" of Austria, project No. P18959-N16.
A. Running Yukawa couplings

For clarity, we present all formulas used for programming the running top and bottom Yukawa couplings, $h_b$ and $h_t$, respectively. The Lagrangian for the $H^\pm t\bar{b}$ interactions reads

$$\mathcal{L}_{Hqq} = H^+ \bar{t} (y_b^* P_R + y_t P_L) b + H^- \bar{b} (y_t^* P_R + y_b P_L) t + \ldots ,$$  \hspace{1cm} (A.1)

with the $\overline{\text{DR}}$ running top and bottom Yukawa couplings in the MSSM,

$$y_b = h_b \sin \beta , \quad y_t = h_t \cos \beta ,$$  \hspace{1cm} (A.2)

given at the scale $Q = m_{H^\pm}$ in our studied case, and

$$h_b = \frac{g m_{b, \overline{\text{DR}}}(Q)}{\sqrt{2} m_W \cos \beta} , \quad h_t = \frac{g m_{t, \overline{\text{DR}}}(Q)}{\sqrt{2} m_W \sin \beta} .$$  \hspace{1cm} (A.3)

In [4] it is shown that within an effective theory approach large finite scale independent parts can be resummed, which in case of complex MSSM input parameters leads to complex $h_b$ and $h_t$. Effective means that the masses of the particles in the loops are much bigger than those of the in- and outgoing particles so that these states can be integrated out in the Lagrangian. In our case, we are interested in additional open channels, e.g. $H^+ \to \tilde{t}\bar{b}$. This implies that the resummation is not applicable here. But we can improve our calculation by using full one-loop running quark masses with some higher order improvements of the gluonic part. Note, that $m_{q, \overline{\text{DR}}}(Q)$ can always be made real by field redefinition [4] and therefore also $h_q$ is real in our case.

We take as input set the bottom mass $m_{b, \overline{\text{MS}}}(m_b) = 4.2$ GeV, the mass for the top quark is the pole mass, $m_{t, \text{pole}} = 171.4$ GeV, the strong coupling is $\alpha_{s, \overline{\text{MS}}}(m_Z) = 0.1176$, $m_Z = 91.1876$ GeV, and $m_W = 80.406$ GeV.

First we want to have the $\overline{\text{DR}}$ bottom mass of the Standard Model at the scale $Q$ (see for comparison eq. (26) in [12]),

$$m_{b, \overline{\text{DR}}}^{\overline{\text{MS}}}(Q) = m_{b, \overline{\text{MS}}}(Q) \left[ 1 - \frac{\alpha_{s, \overline{\text{DR}}}}{3 \pi} - \frac{23 \alpha_s^2}{72 \pi^2} \right]$$  \hspace{1cm} (A.4)

with $m_{b, \overline{\text{MS}}}(Q) \equiv m_b(Q)_{\overline{\text{SM}}}$ given in [13, 14]. Adding the loop contributions due to supersymmetric and heavy SM particles, denoted by $\Delta m_b^{\text{extra}}$ (calculated in the $\overline{\text{DR}}$ renormalization scheme), we get the full one-loop $\overline{\text{DR}}$ running bottom mass (with some higher-order improvements) within the MSSM,

$$m_{b, \overline{\text{DR}}}(Q) = m_{b, \overline{\text{SM}}}(Q) + \Delta m_b^{\text{extra}}(Q).$$  \hspace{1cm} (A.5)

The $\overline{\text{DR}}$ running top mass we get from

$$m_{t, \overline{\text{DR}}}(Q) = m_{t, \text{pole}} \left[ 1 + \frac{\Delta m_t^{(1)}}{m_t} + \frac{\Delta m_t^{(2, g)}}{m_t} \right].$$  \hspace{1cm} (A.6)
where $\Delta m_t^{(1)}$ is the full one-loop contribution to $m_t$ (calculated in the DR renormalization scheme) and $\Delta m_t^{(2,g)}$ is the gluonic two-loop contribution,

$$\frac{\Delta m_t^{(2,g)}}{m_t} = - \left( \frac{\alpha_s(Q)}{4\pi} \right)^2 \left( \frac{8\pi^2}{9} + \frac{2011}{18} + \frac{16\pi^2 \log(2)}{9} - \frac{8\xi(3)}{3} + 82L + 22L^2 \right),$$  \hspace{1cm} (A.7)

with $L = \log(Q^2/m_t^2)$, see [15].

B. Squark–quark–gluino contribution

In Appendix B of [5], eq. (62) is incorrect and therefore also eqs. (14, 15). For clarification, the definition of the squark rotation matrix $\tilde{R}$ is essential. In this work and in [5], one has

$$\tilde{q}_\alpha = R^{\alpha}_{\alpha i} \tilde{q}_i \quad \text{with} \quad \alpha = L, R \quad \text{and} \quad i = 1, 2.$$  \hspace{1cm} (B.1)

Hence, the squark–quark–gluino interaction (eq. (62) of [5]) is given by

$$\mathcal{L}_{q\tilde{q}\tilde{g}} = -\sqrt{2} g_s T^{a}_{st} \left[ \tilde{g}^a (R^{\tilde{q}^*}_{1i} e^{-\frac{i}{2} \phi_3} P_L - R^{\tilde{q}^*}_{2i} e^{\frac{i}{2} \phi_3} P_R) \tilde{q}_s \tilde{q}^*_{i,t} \right.$$  

$$+ \tilde{q}_s (R^{\tilde{g}}_{1i} e^{\frac{i}{2} \phi_3} P_R - R^{\tilde{g}}_{2i} e^{-\frac{i}{2} \phi_3} P_L) \tilde{g}^a \tilde{q}_{i,t} \left]. \hspace{1cm} (B.2)

The contribution from the diagram with a stop, a sbottom, and a gluino (in [5] eqs. (14,15)) is

$$\text{Re} \delta Y^{CP}_b (\tilde{t}_i \tilde{b}_j \tilde{g}) = -\frac{4}{3} \frac{\alpha_s}{\pi} \left[ m_{\tilde{g}} \text{Im}(G_{4ij} R^{\tilde{t}_i}_{1i} R^{\tilde{b}_j}_{1j} e^{i\phi_3}) \text{Im}(C_0) \right.$$  

$$+ m_t \text{Im}(G_{4ij} R^{\tilde{t}_i}_{2i} R^{\tilde{b}_j}_{2j} \text{Im}(C_1) + m_b \text{Im}(G_{4ij} R^{\tilde{t}_i}_{1i} R^{\tilde{b}_j}_{1j} \text{Im}(C_2)) \right),$$  \hspace{1cm} (B.3)

$$\text{Re} \delta Y^{CP}_t (\tilde{t}_i \tilde{b}_j \tilde{g}) = -\frac{4}{3} \frac{\alpha_s}{\pi} \left[ m_{\tilde{g}} \text{Im}(G_{4ij} R^{\tilde{t}_i}_{2i} R^{\tilde{b}_j}_{1j} e^{-i\phi_3}) \text{Im}(C_0) \right.$$  

$$+ m_t \text{Im}(G_{4ij} R^{\tilde{t}_i}_{1i} R^{\tilde{b}_j}_{1j}) \text{Im}(C_1) + m_b \text{Im}(G_{4ij} R^{\tilde{t}_i}_{2i} R^{\tilde{b}_j}_{2j}) \text{Im}(C_2)) \right],$$  \hspace{1cm} (B.4)

with $C_X = C_X(m_{\tilde{t}_i}^2, m_{H^+_i}^2, m_{\tilde{b}_j}^2, m_{\tilde{g}}^2, m_{\tilde{t}_i}^2, m_{\tilde{b}_j}^2)$.
References

[1] P. Nath, Phys. Rev. Lett. 66 (1991) 2565; Y. Kizukuri and N. Oshimo, Phys. Rev. D 46 (1992) 3025; R. Garisto and J. D. Wells, Phys. Rev. D 55 (1997) 1611 [hep-ph/9609511]; Y. Grossman, Y. Nir and R. Rattazzi, Adv. Ser. Direct. High Energy Phys. 15 (1998) 755 [hep-ph/9701231].

[2] T. Ibrahim and P. Nath, Phys. Lett. B 418 (1998) 98 [hep-ph/9707409]; M. Brhlik, G. J. Good and G. L. Kane, Phys. Rev. D 59 (1999) 115004 [hep-ph/9810457]; A. Bartl, T. Gajdosik, W. Porod, P. Stöckinger and H. Stremnitzer, Phys. Rev. D 60 (1999) 073003 [hep-ph/9903402].

[3] A. Pilaftsis, Phys. Rev. D 58 (1998) 096010 [hep-ph/9805373] and Phys. Lett. B 435 (1998) 88 [hep-ph/9805373]; A. Pilaftsis and C. E. Wagner, Nucl. Phys. B 553 (1999) 3 [hep-ph/9902371]; D. A. Demir, Phys. Rev. D 60 (1999) 055006 [hep-ph/9901389].

[4] M. Carena, J. R. Ellis, A. Pilaftsis and C. E. Wagner, Nucl. Phys. B 586 (2000) 92 [hep-ph/0003180].

[5] E. Christova, H. Eberl, S. Kraml and W. Majerotto, Nucl. Phys. B 639 (2002) 263 [hep-ph/0205227]; Erratum-ibid. B 647 (2002) 359.

[6] E. Christova, H. Eberl, S. Kraml and W. Majerotto, JHEP 0212 (2002) 021 [hep-ph/0211063].

[7] E. Christova, E. Ginina, M. Stoilov JHEP 11 (2003) 027 [hep-ph/0307319].

[8] J. Williams, contribution to CPNSH Report, CERN-2006-009 [hep-ph/0608079].

[9] S. M. Bilenky, E. Christova, N. Nedelcheva, Bulg. J. Phys.13 (1986) 283
S. M. Bilenky, Introduction to the Physics of Electroweak Interactions (Pergamon, Oxford 1992).

[10] A. Belyaev, D. Garcia, J. Guasch, J. Sola, JHEP 0206 (2002) 059 [hep-ph/0203031].

[11] V. D. Barger, R. J. N. Phillips, D. P. Roy, Phys. Lett. B 324 (1994) 236 [hep-ph/9311372]; J. Alwall, J. Rathsman, JHEP 50 (2004) 050 [hep-ph/0409094]; E. L. Berger, T. Han, J. Jiang, T. Plehn, Phys. Rev. D 71 (2005) [hep-ph/0312286].

[12] J. A. Aguilar-Saavedra et al., EPJ C 46 (2006) 43 [hep-ph/0511344].

[13] H. Eberl, K. Hidaka, S. Kraml, W. Majerotto, Y. Yamada, Phys. Rev. D 62 (2000) 055006, [hep-ph/9912463].

[14] S.G. Gorishny, A.L. Kataev, S.A. Larin, and L.R. Surguladze, Mod. Phys. Lett. A5 (1990) 2703; Phys. Rev. D 43 (1991) 1633;
A. L. Kataev, V. T. Kim, Mod. Phys. Lett. A 9 (1994) 1309;
A. Djouadi, M. Spira, and P.M. Zerwas, Z. Phys. C 70 (1996) 427;
A. Djouadi, J. Kalinowski, and M. Spira, Comput. Phys. Commun. 108 (1998) 56;
M. Spira, Fortschr. Phys. 46 (1998) 203.

[15] A. Bednyakov, A. Onishchenko, V. Velizhanin, O. Veretin, Eur. Phys. J. C 29 (2003) 87, [hep-ph/0210258].