Numerical Solution of Unsteady Ekman Equation Modified by Wave and Linear Friction Term

Jia Chen*, Wenli Qiao^b and Jinbao Song*
Ocean College Zhejiang University, Zhejiang Zhoushan, China

*Corresponding author e-mail: songjb@zju.edu.cn, ^b18374883087@163.com, ^eqiaowli@zju.edu.cn

Abstract. Based on the assumption that taking the KPP model as the vertical vortex viscosity coefficient, the unsteady Ekman equation modified by wave and linear friction terms is solved numerically with the finite difference method and the influence of wave and linear friction terms on the unsteady Ekman current are studied. Based on the measured data of AWAC, the Stokes drift is calculated by the Stokes drift formula; and on the basis of the measured data of wind speed, the average wind speed of the 3 hours is divided into three different sections: low wind speed (0~4) m/s, medium-low wind speed (4~8) m/s and medium-high wind speed (8~12) m/s. The results show that: Stokes drift has the greatest impact on Ekman current under the low wind speed, which is accounted for 27.4%, the second is under the medium-low wind speed of 7.5%, and the smallest effect is under the medium-high wind speed of 4.2%. In addition, the effect of linear friction term on Ekman current is also studied. It is shown that under the medium-high wind speed and the medium-low wind speed, together with the ratio of the linear friction term to the Coriolis force is, the influence of the linear friction term on the Ekman current has exceeded that of the Stokes drift on the Ekman current; while under the low wind speed, the effect of linear friction term on Ekman current can not exceed that of Stokes drift on Ekman current until. Furthermore, the comparison between the numerical solution and the actual measured data also shows different characteristics. In the perspective of relevance, the correlation between the numerical simulation results and the measured data is above 0.7 under the medium-high wind speed, which is higher than the correlation between the both of 0.4~0.5 under the low wind speed and the medium-low wind speed; and in the perspective of the root mean square deviation degree, the root mean square deviation between the numerical simulation results and the measured data under the medium-high wind speed is lower than that under the low wind speed and the medium-low wind speed. The above results show that the influence of Stokes drift, linear friction term and the unsteady wind on Ekman model can not be ignored.

1. Introduction
Ekman (1905) [1] first proposed the Ekman equation, which demonstrated the formation of the ocean current driven by wind stress in the steady state, and explained the phenomenon that the net transport in a marine friction layer caused by the wind stress in the northern hemisphere pointed to the right side of
the wind stress. Based on the Ekman theory, the theoretical research framework of the upper boundary layer of the ocean has been developed in succession. The representative results are as follows: Sverdrup (1947) [2] considered the pressure gradient force based on the Ekman drifting theory, that is, the pressure gradient force, the Coriolis force and the friction force balance each other, and he had put forward the Sverdrup equation and demonstrated that the essence of the North Equatorial Countercurrent is the windward geostrophic current; based on the Sverdrup theory, Stommel (1948) [3] considered the sea bottom friction effect and proposed the Westward intensification of wind-driven ocean currents; Munk (1950) [4] proposed a quantitative theory about the main features of the wind-driven circulation, and comprehensively elaborated the main features of the wind-driven ocean circulation. However, with the continuous development of observation technology, it is found that there are significant differences between the traditional Ekman theory and the actual observation. In particular, several hypotheses of the Ekman theory are different from the actual ocean situations to a great extent.

In order to explain the differences between the traditional Ekman theory and the actual observation, later researchers added different revisions to reduce the bias between theoretical models and actual observations. Wave as a medium to transmit momentum and energy in the atmosphere to the surface flow field of the ocean has an important influence on the structure of surface flow field and mixing of the upper layer of the ocean. Therefore, the study of the effect of waves on ocean circulation has always been the centre of attention. Stokes (1847) [5] first proposed a drift theory that periodic ocean surface gravity waves will produce a net displacement in the direction of wave propagation, that is, Stokes drift; Huang (1979) [6] pointed out that the ocean surface circulation can be generated not only by the driven by the wind stress in the traditional Ekman model, but also by the wave motion; Kenyon (1970) [7] believes that Stokes drift will become a major part of ocean surface current under the circumstance without wind; McWilliams and Restrepo (1999) [8] and Polton (2005) [9] calculated the Stokes drift and the Stokes transport based on the PM spectrum describing the full growth of wind waves, they have found that the Stokes transport is an important part of Ekman transport; Lewis and Belcher (2004) [10] used the single wave formula to calculate the Stokes drift, then they pointed out that the deviation of direction angle of the traditional Ekman model can be reduced by considering the Stokes drift and expounded the important influence of the sea surface wave on the structure of the flow of the Ekman layer; the study of Saetra at al (2007) [11] also shows that Stokes drift can reduce the deviation of direction angle of the traditional Ekman model; Song (2009) [12] calculated the Stokes drift with the DP spectrum describing the full growing wind and wave, and calculated the wind-to-wave input and the wave dissipation terms. He thought that the influence of the Stokes drift on the Ekman current in medium-high wind speed was much larger than the influence of the wind-to-wave input and the wave dissipation terms on the Ekman current; Zhang (2013) [13] used the wave spectrum of the ideal experimental output at a constant wind speed with the third-generation wave model and pointed out that Stokes drift, wind-to-wave input, and wave-induced momentum transfer all have a great influence on the Ekman flow.

However, the effect of unsteady wind on the Ekman model is rarely studied. In the study of Ashkenazy et al. (2015) [14], it is pointed out that after considering the influence of the unsteady wind, the Ekman model should add a linear friction term to counteract the dispersion of the sea surface flow, and they think that the linear friction term is critical to the Ekman equation of the unsteady wind action; Kim et al. (2015) [15] pointed out that the influence of the linear friction term on the Ekman model was very important in the finite depth of water, and compared with the measured data in ROMS and FVCOM models, he also pointed out that the operation result by the model which under the ratio of the linear friction term to the Coriolis force parameter was (0.1, 0.5) was more consistent with the measured data; Buffoni et al. (2015) [16] selected three different regions of the wind field and discussed the effects of unsteady wind on the Ekman model; the study of Wenegrat and Mcphaden (2016) [17] indicates that the diurnal variation of the vertical eddy viscosity will affect the flow structure of Ekman layer. The time-dependent Ekman equation modified by wave and linear friction terms is discretized by finite difference method in this paper, and the effect of Stokes drift and linear friction term on the Ekman
current under the unsteady state is discussed, and the results of the numerical simulation are tested with the measured data of AWAC.

2. Governing equations and corresponding boundary conditions

When steady winds continue to act on the sea surface, the resulting large-scale flow is also constant. At this point, the product of the equilibrium between vertical turbulent frictional force and Coriolis force is the classic Ekman drift, and its governing equation is:

\[
f_{\text{cor}} v + A_v \frac{\partial^2 u}{\partial z^2} = 0
\]

\[
-f_{\text{cor}} u + A_v \frac{\partial^2 v}{\partial z^2} = 0
\]

(1)

Where: \( u \) represents the flow velocity in the \( x \) direction, \( v \) represents the flow velocity in the \( y \) direction, \( z \) represents depth (setting upward as positive direction), \( f_{\text{cor}} \) represents Coriolis force parameter, \( A_v \) represents the vertical eddy viscosity and it is a constant.

The corresponding boundary conditions are as follows:

\[
z = 0: \quad \rho_w A_v \frac{\partial u}{\partial z} = \tau_x, \quad \rho_w A_v \frac{\partial v}{\partial z} = \tau_y
\]

\[
z \rightarrow \infty: \quad u = v = 0
\]

(2)

Where: \( \rho_w \) represents the density of water, \( \tau = (\tau_x, \tau_y) \) represents wind-stress.

However, in the fact, it is impossible for steady winds to have a lasting effect on the sea surface in the ocean. After considering the influence of unsteady wind, wave and linear friction term on Ekman current, the equation (1) becomes as follows (Jenkins, 1989 [18]; Song and Xu, 2013 [19])

\[
\frac{\partial U_{\text{WE}}}{\partial t} + if_{\text{cor}} U_{\text{WE}} + r U_{\text{WE}} + if_{\text{cor}} U_{s} = \frac{\partial}{\partial z} \left( A_v \frac{\partial U_{\text{WE}}}{\partial z} \right)
\]

(3)

Where: \( U_{\text{WE}} = U_{\text{WE}}(z,t) = u_{\text{WE}}(z,t) + iv_{\text{WE}}(z,t) \) represents the horizontal velocity on the x-y complex plane, which is equivalent to the Lagrange mean flow minus Stokes drift, it can also be understood as the Euler mean flow (Jenkins, 1987 [20]), \( z \) represents depth (setting upward as positive direction), \( z = 0 \) means sea level at rest, \( t \) represents time, \( i = \sqrt{-1} \), \( U_s = U_s(z,t) = u_s(z,t) + iv_s(z,t) \) represents Stokes drift on the x-y complex plane, \( r \) represents a constant coefficient similar to Rayleigh friction (Ashkenazy et al., 2015 [14]), its physical meaning is: when the Ekman model is at a finite depth: molecular friction at smaller scales, such as the generation of internal waves, wave breaking, boundary friction, and turbulent mixing processes, causes many energy losses that are often overlooked, \( r \) is an empirical coefficient used to represent these energy losses.

Correspondingly, its boundary conditions are:

\[
z = 0, \quad A_v(z,t) \frac{\partial U_{\text{WE}}}{\partial z} = \frac{\tau_x}{\rho_w}
\]

\[
z = -h, \quad A_v(z,t) \frac{\partial U_{\text{WE}}}{\partial z} = 0
\]

(4)
Where: $\rho_w = 1025 \, \text{kg} \, \text{m}^{-3}$ represents the density of water, $\tau_a = \tau_{ax} + i \tau_{ay}$ represents complex wind-stress, and it can be calculated by 10 meters high surface wind field $U_{10}$:

$$\tau_a = (\tau_{ax}, \tau_{ay}) = \rho_u C_d \left| U_{10} \right| U_{10}$$

(5)

Where: $\rho_a = 1.2 \, \text{kg} \, \text{m}^{-3}$ represents air density, $C_d$ represents ocean-atmosphere drag coefficient, it can be calculated by the formula given below (Wu, 1982 [21]):

$$C_d = (0.8 + 0.065U_{10}) \times 10^{-3}$$

(6)

In the classical Ekman equation, the vertical eddy viscosity $A_z$ is a constant, this paper adopts the vertical eddy viscosity of the KPP model (Large et al., 1994 [22]; McWilliams and Huckle, 2005 [23]; McWilliams, 2012 [24]), that is:

$$A_z(z,t) = \begin{cases} 
\kappa u_c h G(\sigma), & 0 \leq \sigma \leq 1 \\
\kappa u_c h G(1), & \sigma > 1 
\end{cases}$$

(7)

Where: $\kappa = 0.41$ is the Carmen constant, $u_c = \sqrt{\frac{\tau_a(t)}{\rho_w}}$ is the friction velocity, $h = c_1 \frac{u_c}{f \cos}$ is the depth of the boundary layer, $c_1 = 0.7$ is the constant (McWilliams and Huckle, 2005 [23]), $\sigma = -\frac{z}{h}$ can be regarded as coordinate conversion, $G(\sigma)$ is the function related to $\sigma$, it can be calculated by the following formula:

$$G(\sigma) = \sigma(1-\sigma)^2 + \frac{(\sigma_0-\sigma)^2}{2\sigma_0} H(\sigma_0-\sigma)$$

(8)

Where: $\sigma_0 = 0.05$, $H(a) = \begin{cases} 1 & a > 0 \\
0 & \text{in other cases} \end{cases}$

3. Numerical discrete method and data sources

3.1. Numerical discrete method

When the influences of unsteady wind, linear friction term and wave are considered, the Ekman equation with the vertical eddy viscosity of the KPP model becomes extremely complex, and it is difficult to obtain the theoretical solution. Therefore, compared with the theoretical solution of the modified Ekman equation, numerical discrete solution becomes a more effective method. The governing equation (3) is discretized by the finite difference method. The upwind scheme is used for the time term and the central difference scheme for the diffusion term is adopted. The discrete form is as follows:
\[
\frac{(U_{we})_{j}^{n+1} - (U_{we})_{j}^{n}}{\Delta t} + (r + if_{cor})(U_{we})_{j}^{n+1} + if_{cor}(U_{s})_{j}^{n+1} = \]
\[
\frac{(A_{v})_{j+1/2}^{n+1} \frac{\partial U_{we}}{\partial z} \bigg|_{j+1/2} - (A_{v})_{j-1/2}^{n+1} \frac{\partial U_{we}}{\partial z} \bigg|_{j-1/2}}{\Delta z} \]
\[
= \frac{(A_{v})_{j+1}^{n+1} + (A_{v})_{j}^{n+1} (U_{we})_{j+1}^{n+1} - (U_{we})_{j}^{n+1} + (A_{v})_{j-1}^{n+1} (U_{we})_{j-1}^{n+1} - (U_{we})_{j}^{n+1}}{2}\Delta z \]
\[
(9)
\]

Move the item \(U_{we}\) at time \(n+1\) to the left of the equation, and the rest to the right of the equation:
\[
a_{j} (U_{we})_{j+1}^{n+1} + b_{j} (U_{we})_{j}^{n+1} + c_{j} (U_{we})_{j+1}^{n+1} = (U_{we})_{j}^{n+1} - if_{cor}(U_{s})_{j}^{n+1}
\]
\[
(10)
\]

Where:
\[
a_{j} = -((A_{v})_{j}^{n+1} + (A_{v})_{j-1}^{n+1}) \frac{\Delta t}{2\Delta z^{2}}
\]
\[
b_{j} = 1 + ((A_{v})_{j}^{n+1} + 2(A_{v})_{j}^{n+1} + (A_{v})_{j-1}^{n+1}) \frac{\Delta t}{2\Delta z^{2}} + (r + if_{cor}) \Delta t
\]
\[
c_{j} = -((A_{v})_{j+1}^{n+1} + (A_{v})_{j}^{n+1}) \frac{\Delta t}{2\Delta z^{2}}
\]
\[
(11)
\]

Since it is a second kind of boundary condition, the boundary conditions need to be separately discretized.

When \(z = 0\), i.e. on the sea surface:
\[
\frac{(U_{we})_{0}^{n+1} - (U_{we})_{1}^{n}}{\Delta t} + (r + if_{cor})(U_{we})_{1}^{n+1} + if_{cor}(U_{s})_{1}^{n+1} = \]
\[
\frac{(A_{v})_{1/2}^{n+1} \frac{\partial U_{we}}{\partial z} \bigg|_{1/2} - (A_{v})_{1/2}^{n+1} \frac{\partial U_{we}}{\partial z} \bigg|_{1/2}}{\Delta z} \]
\[
= \frac{(A_{v})_{1}^{n+1} + (A_{v})_{1}^{n+1} (U_{we})_{1}^{n+1} - (U_{we})_{1}^{n+1} + (A_{v})_{0}^{n+1} (U_{we})_{0}^{n+1} - (U_{we})_{0}^{n+1}}{2}\Delta z \]
\[
(12)
\]

The above formula makes use of the boundary condition that \(\frac{(A_{v})_{1}^{n+1} + (A_{v})_{1}^{n+1} (U_{we})_{1}^{n+1} - (U_{we})_{0}^{n+1}}{2}\Delta z = \frac{\tau_{w}}{\rho_{w}}\).

Move the item \(U_{we}\) at time \(n+1\) to the left of the equation, and the rest to the right of the equation:
\[
(b_{2} + c_{2})(U_{we})_{0}^{n+1} + a_{2}(U_{we})_{2}^{n+1} = (U_{we})_{1}^{n} - \frac{\tau_{w}\Delta t}{\rho_{w}\Delta z} - if_{cor}\Delta t(U_{s})_{1}^{n+1}
\]
\[
(13)
\]
When $z = -h$, i.e. at the bottom of the sea:

$$\frac{(U_{we})^n_{j+1} - (U_{we})^n_j}{\Delta t} + (r + if_{cor})(U_{we})^n_{j+1} + if_{cor}(U_s)^n_{j+1}$$

$$= (A_j)^n_{j+1/2} \frac{\partial U_{we}}{\partial z} |_{j+1/2} - (A_j)^n_{j-1/2} \frac{\partial U_{we}}{\partial z} |_{j-1/2}$$

$$= \frac{(A_j)^n_{j+1} + (A_j)^n_{j-1} (U_{we})^n_{j+1} - (U_{we})^n_{j+1}}{2\Delta z} - \frac{(A_j)^n_{j} + (A_j)^n_{j-1} (U_{we})^n_{j} - (U_{we})^n_{j-1}}{2\Delta z}$$

$$= \frac{(A_j)^n_{j+1} + (A_j)^n_{j+1} (U_{we})^n_{j} - (U_{we})^n_{j}}{2\Delta z}$$

The above formula makes use of the boundary condition that \( \frac{(A_j)^n_{j+1} + (A_j)^n_{j} (U_{we})^n_{j} - (U_{we})^n_{j}}{\Delta z} = 0 \).

Move the item $U_{we}$ at time $n + 1$ to the left of the equation, and the rest to the right of the equation:

$$a_j (U_{we})^n_{j+1} + (b_j + c_j) (U_{we})^n_j = (U_{we})^n_j - if_{cor} \Delta \tau \Delta \rho$$

Equations (10), (13), and (15) are expanded and linked together, and the discrete equations can be converted into tri-diagonal matrix and solved by using the chasing method:

$$\begin{pmatrix}
(b_2 + c_2 & a_2 \\
 a_2 & b_2 & c_2 \\
 a_3 & b_3 & c_3 \\
 \vdots & \vdots & \vdots \\
 a_{j-1} & b_{j-1} & c_{j-1} \\
 a_j & b_j + c_j & \end{pmatrix}
\begin{pmatrix}
(U_{we})^n_1 \\
(U_{we})^n_2 \\
(U_{we})^n_3 \\
\vdots \\
(U_{we})^n_{j-1} \\
(U_{we})^n_j \\
(U_{we})^n_{j+1} \\
\vdots \\
(U_{we})^n_{j+10} \\
\vdots \\
\end{pmatrix}
= \begin{pmatrix}
(U_{we})^n_1 - if_{cor} \Delta \rho \Delta \tau \\
(U_{we})^n_2 - if_{cor} \Delta \rho \Delta \tau \\
(U_{we})^n_3 - if_{cor} \Delta \rho \Delta \tau \\
\vdots \\
(U_{we})^n_{j-1} - if_{cor} \Delta \rho \Delta \tau \\
(U_{we})^n_j - if_{cor} \Delta \rho \Delta \tau \\
(U_{we})^n_{j+1} - if_{cor} \Delta \rho \Delta \tau \\
\vdots \\
(U_{we})^n_{j+10} - if_{cor} \Delta \rho \Delta \tau \\
\end{pmatrix}$$

3.2. Data sources

3.2.1. Wind speed data. Wind speed data are derived from the marine meteorological observation platform in Maoming, Guangzhou. The time span is measured every ten minutes from February 1, 2012 to October 22, 2012. In total, the wind speeds at different altitudes 13.4, 16.4, 20, and 23.4 meters were measured.

This paper takes the wind speed at 13.4 meters and converts it to the wind speed at 10 meters by the following formula:

$$U_z = U_{10} (1 + \frac{C_{10}^{1/2}}{\kappa} \ln \frac{z}{10})$$
Where: $\kappa = 0.41$ is the Carmen constant, $C_{10}$ is the ocean-atmosphere drag coefficient corresponding to $U_{10}$ which can be calculated by the formula (6).

As the AWAC data is measured in a period of 3 hours, in order to study the influences of the different wind speeds on the solution of the time-dependent Ekman equation modified by wave and linear friction terms with the vertical eddy viscosity of the KPP model, so the wind speed is averaged every three hours. The 3 hour mean wind speed was divided into three groups: low wind speed ($0, 4$) m/s, medium-low wind speed ($4, 8$) m/s and medium-high wind speed ($8, 12$) m/s.

Figure 1 shows the average wind speed at different time periods. The wind speed data at different time periods are used because the wind speeds in successive time periods are mainly distributed in the medium-low wind speed, which is not conducive to the study of the influences of different wind speed intervals on the Ekman equation modified by wave and linear friction terms. From Figure 2, we can see that the wind speed varies with time in different intervals, in which there are 99 sets of data in the low wind speed, and 107 sets of data in the medium-low wind speed, and only 41 sets of data are available in the medium-high wind speed in which lack of the measured data.

![Figure 1](image)

**Figure 1.** Figure of wind speed over time, in which a), b), c) that the wind speed at different time periods.

3.2.2. *The AWAC data.* The wave data was derived from AWAC measurements placed at the edge of the Maoming Tower in Guangzhou. The time span spanned from January 12, 2012 to July 4, 2012. However, due to the battery of the instrument, the effective data measurement ended on June 13, 2012.
The AWAC instrument measures in a 3-hour period, where wave data is measured for 10 minutes and current data is measured in the remaining time. The data measured by the AWAC instrument is measured under the corresponding wind data. Roll and Pitch are the parameters that reflect the stability and accuracy of the data measured by AWAC, and they refer to the left and right sides and the up and down sides swing indicators of AWAC respectively. For the general case, the swing indicators of Roll and Pitch can not be greater than 100, and the values of Roll and Pitch of the AWAC instrument used in this time fluctuated between [00, 50], indicating that the measured data is good. This paper selects 99 sets of AWAC data at low wind speed, 107 sets of AWAC data at medium-low wind speed and 41 sets of AWAC data at medium-high wind speed.

4. Results

4.1. Model validation

In order to verify the accuracy of the numerical model, we compared with figure 1 of Song (2009) and the parameter setting is the same as that set in Song (2009). In Figure 3, $u, v$ respectively represent the Ekman current after the situation that only Stokes drift is considered. The solid black line in Figure 2 indicates the result of the numerical simulation, and the dashed black line indicates that the theoretical solution in the paper of Song (2009). It can be seen from the picture that there is little difference between the numerical model and the theoretical solution, that is, the accuracy of the numerical model meets the requirements.

![Figure 2. Comparison of numerical model and theoretical solution.](image)

4.2. The effect of Stokes drift on Ekman current

On the basis of Webb and Fox-Kemper (2015) [25], Stokes drift can be calculated using the following formula:

$$U_S = (\cos \theta + i \sin \theta) \frac{16 \pi^3}{g} \int_0^\infty f^3 S_f(f) \exp \left( \frac{8 \pi^2 f^2 z}{g} \right) df$$

(18)

Where: $U_S = U_S(z,t) = u_S(z,t) + iv_S(z,t)$ represents the complex Stokes drift, $\theta$ represents the direction angle, $f$ represents the frequency, $z$ represents the depth, $S_f(f)$ and represents the energy spectrum.
Numerical simulation results at low wind speed, medium-low wind speed and medium-high wind speed are shown in Figure 3-5 respectively. Among them, $U_{E1}, U_{E2}, U_{E3}, U_{WE1}, U_{WE2}, U_{WE3}$ indicate that Stokes drift is not considered and the influence of Stokes drift on Ekman current respectively, in addition, $U_{Ei} = \sqrt{u_{Ei}^2 + v_{Ei}^2}, U_{WEi} = \sqrt{u_{WEi}^2 + v_{WEi}^2} (i = 1, 2, 3)$.

From Figure 3-5, it can be seen that the impact on the Ekman current is greater after Stokes drift is considered. Among them, the effects of Stokes drift have the most influence on Ekman current are 27.4% in low wind speed, 7.5% in medium-low wind speed and 4.2% in medium-high wind speed.

The results show that considering the influence of the wave, adding the Stokes drift into the Ekman model can significantly affect the current velocity of the Ekman flow, and the effect of Stokes drift on the Ekman flow in the wind speed range of (0, 12) decreases with the increase of wind speed.
4.3. The effect of linear friction term on Ekman current

In order to study the effect of linear friction terms on Ekman model, take $\lambda = r / f_{cor} = 0.3, 0.5, 0.7, 1$, among them $f_{cor} = 10^{-4}$, $\lambda$ represents the ratio of the linear friction term to the Coriolis force, the value of it represents the proportion of the linear friction term and the Coriolis force in the Ekman model.

In Figure 6-8, $U_{WE1}, U_{WE2}, U_{WE3}$ represent that Stokes drift is considered but does not consider the influence of linear friction term respectively, $U_{E1}, U_{E2}, U_{E3}$ represent that Stokes drift and linear friction term are not considered respectively. And the different lines indicate that the cases where the linear friction term is considered but the Stokes drift is not taken into account. In order to compare the influence of Stokes drift and linear friction term on Ekman current, the following formula is used to calculate:

$$\gamma = \left| \frac{b-a}{b} \right|$$

Where: $b$ indicates that considered the effect of Stokes drift or linear friction term, $a$ means with no consideration of the effects of Stokes drift and linear friction term.

### Table 1. The influence of linear friction term about Ekman current

| Wind speed         | The Influence of Different Items on Ekman Current |
|--------------------|-----------------------------------------------|
|                    | $\lambda = 1$ | $\lambda = 0.7$ | $\lambda = 0.5$ | $\lambda = 0.3$ | $U_{WE}$ |
| low                | 30.99%        | 19.8%           | 12.89%          | 6.78%           | 27.41%   |
| medium-low         | 21.93%        | 13.81%          | 8.85%           | 4.5%            | 7.5%     |
| medium-high        | 19.2%         | 11.82%          | 7.32%           | 3.54%           | 4.2%     |

The value of $\gamma$ is shown in Table 1. From Figure 6-8 and Table 1, it can be seen that under low wind speed, the impact of the linear friction term on the Ekman current is less significant than that of the Stokes drift on the Ekman current. At that time, the linear friction term has an effect on the Ekman current by 30.99%. The effect of Stokes drifting on the Ekman current is 27.41%, indicating that the effect of linear friction term exceeds the effect of Stokes drift on the Ekman current.

At that time, the effect of linear friction on Ekman flow is 8.85% and 7.32% under middle and low wind speed, and the effect of Stokes drift on Ekman flow is 7.5% and 4.2% respectively, indicating that the linear friction term exceeds the effect of Stokes drift on Ekman flow at this time.

When $\lambda = 0.5$, the impacts of the linear friction term on the Ekman flow were 8.85% at medium-high wind speeds and 7.32% at medium-low wind speeds, respectively. The influences of Stokes drift
on the Ekman flow were 7.5% and 4.2%, respectively, indicating that the influence of the linear friction term on the Ekman current has exceeded that of the Stokes drift this moment.

The above situation shows that the linear friction term has different effects on the Ekman current under different wind speeds, and with the increasing of $\lambda$, the linear friction term affects more and more on the Ekman current. When $\lambda = 1$, the Coriolis force and the linear friction term occupy the same proportion in the Ekman model, and the linear friction term plays an important role at this time.

![Figure 6](image6.png)

**Figure 6.** The case of low wind speeds, influence of linear friction term on Ekman current at Sea Surface. Among them, the dashed black line indicates that Stokes drift is considered but does not consider the influence of linear friction term, and the dashed black line shows that Stokes drift and linear friction term are not considered, the yellow line, the blue line, the red line, and the green line represent the cases where the linear friction term is considered but the Stokes drift is not taken into account when $\lambda = 0.3, 0.5, 0.7, 1$, respectively.

![Figure 7](image7.png)

**Figure 7.** Like Figure 6, but corresponds to medium-low wind speeds.
5. The comparison with measured data

The AWAC is placed on the coast and affected by many factors. Therefore, only 540 groups of measured AWAC current data at medium-high wind speeds were selected, and 1008 groups of measured AWAC current data at medium-low wind speeds, and 900 groups at low wind speeds. These data are averaged every three hours. The comparison between measured data and numerical simulation is shown in Figure 9-11.

The correlation coefficient (Cor) and the root-mean-square deviation (RMSE) are used to express the deviation between the numerical simulation results and the measured data as follows:

\[
Cor(x, y) = \frac{Cov(x, y)}{\sqrt{D(x)D(y)}}
\]

(20)

\[
RMSE = \sqrt{\frac{1}{n} \sum (H_x - H_y)^2}
\]

(21)

Table 2. Correlation analysis of numerical simulation results and the measured data

| Wind speed    | Cor | \(\lambda = 1\) | \(\lambda = 0.7\) | \(\lambda = 0.5\) | \(\lambda = 0.3\) | \(U_{WE}\) | \(U_E\) |
|---------------|-----|----------------|----------------|----------------|----------------|---------|--------|
| Low           |     | 0.4618        | 0.4615         | 0.4613         | 0.4611         | 0.3858  | 0.4607 |
| Medium-low    |     | 0.5458        | 0.5457         | 0.5455         | 0.5453         | 0.5630  | 0.5453 |
| Medium-high   |     | 0.7709        | 0.7708         | 0.7707         | 0.7705         | 0.7277  | 0.7700 |

Table 3. The root mean square deviation analysis of numerical simulation results and measured data

| Wind speed    | RMSE | \(\lambda = 1\) | \(\lambda = 0.7\) | \(\lambda = 0.5\) | \(\lambda = 0.3\) | \(U_{WE}\) | \(U_E\) |
|---------------|------|----------------|----------------|----------------|----------------|---------|--------|
| Low           |      | 0.0235         | 0.0219         | 0.0210         | 0.0203         | 0.0247  | 0.0197 |
| Medium-low    |      | 0.0184         | 0.0163         | 0.0155         | 0.0153         | 0.0163  | 0.0159 |
| Medium-high   |      | 0.013          | 0.0086         | 0.0075         | 0.0084         | 0.0130  | 0.0107 |

From Figure 9-11 and Table 2-3, it can be seen that, in the aspect of correlation, the correlation coefficient between the numerical simulation results under medium-high wind speed and the measured data is higher than that under low wind speed and medium-low wind speed; root mean square deviations...
between the numerical simulation results at low wind speed and medium-low wind speed and the measured data are higher than those of at medium-high wind speed. It shows that the numerical simulation results are closer to the measured values in the case of medium-high wind speed.

Figure 9. The case of low wind speeds, comparison between numerical simulation and measured data at Sea Surface. Among them, the dashed black line indicates that Stokes drift is considered but does not consider the influence of linear friction term, and the dashed black line shows that Stokes drift and linear friction term are not considered, the yellow line, the blue line, the red line, and the green line represent the cases where the linear friction term is considered but the Stokes drift is not taken into account when $\lambda = 0.3, 0.5, 0.7, 1$, respectively.

Figure 10. Like Figure 9, but corresponds to medium-low wind speeds.
6. Conclusion

In this paper, under the unsteady state, the time-dependent Ekman equation modified by wave and linear friction terms with the vertical eddy viscosity of the KPP model is discretized by finite difference method. Based on the measured data of the wind speed, the 3 hour mean wind speed was divided into three groups: low wind speed (0, 4) m/s, medium-low wind speed (4, 8) m/s and medium-high wind speed (8, 12) m/s and the influence of Stokes drift on the Ekman current under unsteady conditions is studied. The results show that the effects of Stokes drift has the most influence on Ekman current are 27.4% in low wind speed, 7.5% in medium-low wind speed and 4.2% in medium-high wind speed. The influence of linear friction term on Ekman current in the unsteady state is also studied. At $\lambda = r / f_{cor} = 0.5$, the influence of the linear friction term on the Ekman current has exceeded that of the Stokes drift in medium-high and medium-low wind speeds. However, at low wind speeds, the influence of the linear friction term on the Ekman current has exceeded that of the Stokes drift at $\lambda = 1$.

The comparison between the numerical solution and the AWAC measured data also shows different characteristics: in the aspect of correlation, the correlation coefficient between the numerical simulation results under medium-high wind speed and the measured data is higher than that under low wind speed and medium-low wind speed; root mean square deviations between the numerical simulation results at low wind speed and medium-low wind speed and the measured data are higher than those of at medium-high wind speed. It shows that the numerical simulation results are closer to the measured values in the case of medium-high wind speed. In the unsteady state, with the increase of the proportion of linear friction term in the Ekman flow, the root mean square deviation from the actual measured data is also increasing, which shows that the proportion of linear friction term is more consistent with the observed results in the interval of (0.3, 0.7). The above results show that Stokes drift, linear friction term and unsteady wind forcing should not be ignored in the Ekman model.

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