Hadronic sizes and observables in high-energy scattering

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Abstract

The functional dependence of the high-energy observables of total cross section and slope parameter on the sizes of the colliding hadrons predicted by the model of the stochastic vacuum and the corresponding relations used in the geometric model of Povh and Hüfner are confronted with the experimental data. The existence of a universal term in the expression for the slope, due purely to vacuum effects, independent of the energy and of the particular hadronic system, is investigated.

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1. The Model of the Stochastic Vacuum and the Geometric Models

Diffractive high energy scattering is largely determined by the nonperturbative regime of QCD. The extended character of the interaction, involving correlation properties of the gauge field, determines the phenomenological properties of the observables, which are fixed by the sizes and global structures of the colliding systems, rather than by the number of their pointlike constituents and their couplings. These features have led to models of geometric nature for high-energy scattering [1,2], which give natural account of the relations between total cross sections of different hadronic systems and, through hadronic form factors appropriately introduced, describe the shapes of the diffractive peaks. QCD must provide the fundamental framework in which this phenomenology should arise naturally, and efforts have been made in this direction. Some treatments based on perturbative QCD have also lead to dependence of the observables on hadronic sizes [3].

A nonperturbative QCD description of the main features of high-energy scattering is given by the model of the stochastic vacuum [4,5], which combines QCD quantities (gluon condensate and correlation length) and hadronic sizes in an eikonal framework, leading to a unified description of the data for different hadronic systems. The calculations lead to definite size dependence of the observables of total cross section $\sigma_T$ and forward slope parameter B, and provide a dynamical scheme that explains the intuitive and Regge-based geometric treatments. In the present work we explore further the results of the model for the pp, $\bar{p}p$ and other hadronic systems, and compare them with those from the geometric model of B.Povh and J.Hüfner [2], confronting all predictions with the experimental data.

In the calculation with the model of the stochastic vacuum the hadronic structures enter in the form of transverse wavefunctions (two-dimensional wavefunctions in the plane perpendicular to the direction of the colliding hadrons). Taking into account the results of the previous analysis of different hadronic systems [4] we here only consider for the proton a diquark structure where, with respect to the relevant color degrees of freedom, the proton is described as a meson, in which the diquark replaces the antiquark. Thus we can treat on
equal footing meson-meson, meson-baryon and baryon-baryon scattering. Other structures for the baryons have been explored \[4,6\] in investigations with the model of the stochastic vacuum, and the similarity of results obtained with different structures demonstrates the role of the extended nature of the nonperturbative dynamics.

For the hadron transverse wavefunction we take the simple ansatz

\[
\psi_H(R) = \sqrt{2/\pi} \frac{1}{S_H} \exp \left( -\frac{R^2}{S_H^2} \right),
\]

where \(S_H\) is a parameter for the hadron size. The dimensionless scattering amplitude \(T_{H_1H_2}\) is given in terms of the dimensionless profile function \(\hat{J}_{H_1H_2}\) for hadron-hadron scattering by

\[
T_{H_1H_2} = ig[(g^2 FF)\alpha^4]a^2 \int d^2\vec{b} \exp (i\vec{q} \cdot \vec{b}) \hat{J}_{H_1H_2}(\vec{b}, S_1, S_2),
\]

where the impact parameter vector \(\vec{b}\) and the hadron sizes \(S_1, S_2\) are here written in units of the correlation length \(a\), and \(\vec{q}\) is the momentum transfer projected on the transverse plane, in units of \(1/a\), so that the momentum transfer squared is \(t = -|\vec{q}|^2/a^2\). For short, from now on we write \(J(b)\) or \(J(b/a)\) to represent \(\hat{J}_{H_1H_2}(\vec{b}, S_1, S_2)\). The normalization of \(T_{H_1H_2}\) is such that total and differential cross sections are given by

\[
\sigma_T = \frac{1}{s} \text{Im} T_{H_1H_2}, \quad \frac{d\sigma^{\text{el}}}{dt} = \frac{1}{16\pi s^2} |T_{H_1H_2}|^2.
\]

The observables are written in terms of dimensionless moments of the profile function (as before, with \(b\) in units of the correlation length \(a\))

\[
I_k = \int d^2\vec{b} b^k J(b), \quad k = 0, 1, 2, ...
\]

which depend only on \(S_1/a, S_2/a\), and the Fourier-Bessel transform

\[
I(t) = \int d^2\vec{b} \ J_0(ba\sqrt{|t|}) \ J(b),
\]

where \(J_0(ba\sqrt{|t|})\) is the zeroth–order Bessel function. Then

\[
T_{H_1H_2} = ig[(g^2 FF)\alpha^4]a^2 I(t).
\]

Since \(J(b)\) is real, \(\sigma_T\) and the slope parameter \(B\) are written
\[
\sigma^T = I_0 \left[ \langle g^2 FF \rangle a^4 \right]^2 a^2, \quad B = \frac{d}{dt} \left( \ln \frac{d\sigma^T}{dt} \right) \Bigg|_{t=0} = \frac{1}{2} \frac{I_2}{I_0} a^2 \equiv Ka^2. \tag{7}
\]

It is important to observe that these results conveniently factorize the dimensionless QCD strength \( \langle g^2 FF \rangle a^4 \) in the expressions for the observables. The correlation length \( a \), which is an intrinsic parameter of the correlation function of the QCD field, appears as the natural length scale for the observables and for the geometric aspects of the interaction. These aspects are concentrated on the quantities \( I_0(S_1/a, S_2/a) \) and \( I_2(S_1/a, S_2/a) \), which depend on the hadronic structures. These quantities are mainly determined by the values of the profile functions in the range of impact parameters up to about 2.5 fm. \( \sigma^T \) measures the strength, while the slope \( B \) has the strength cancelled out and is only related to the hadron geometry. The explicit formula for the slope is

\[
B = \frac{1}{2} \frac{\int d^2 b b^2 J(b)}{\int d^2 b J(b)} a^2 = \frac{1}{2} \langle b^2 \rangle a^2, \tag{8}
\]

where it is seen as related to the average value of the square of the impact parameter in the collision, with \( J(b) \) as weight function. We recall that here \( b \) is dimensionless and that \( \langle b^2 \rangle \) depends on the hadronic sizes.

### 2. pp and \( \bar{p}p \) systems

We first discuss pp and \( \bar{p}p \) systems, with \( S_1 = S_2 = S \). The curves for \( I_0 = \sigma^T / \left[ \langle g^2 FF \rangle a^{10} \right] \) and \( K = B/a^2 \) can be parametrized as simple powers of \( S/a \) with good accuracy, the convenient expressions being

\[
I_0 = \alpha \left( \frac{S}{a} \right)^\beta, \quad K = \eta + \gamma \left( \frac{S}{a} \right)^\delta. \tag{9}
\]

The values of the parameters result from integrations over correlation functions \[4\], are intrinsic to the model of the stochastic vacuum, and do not contain any dependence on experimental quantities. For the present purpose of analysis of data in a limited energy range, and for easier comparison to the geometric model we take their values as \( \eta = 2.03, \beta = 8/3, \gamma = 3/8, \delta = 2, \delta/\beta = 3/4, \alpha = 0.76 \times 10^{-2} \).
The proton radius can be eliminated from Eqs. (7) and (9), and we obtain a relation between the observables $\sigma_T$ and $B$ at a given energy

$$B - \eta a^2 = \frac{a^2}{<g^2FF> a^4 \alpha^{\delta/\beta}} \frac{\gamma}{\alpha^{\delta/\beta}} \left( \frac{\sigma_{pom}^T}{a^2} \right)^{\delta/\beta} .$$

The two QCD parameters, $<g^2FF>$ and $a$, can be determined using this expression and the experimental data for $\sigma_T$ and $B$ at two different energies.

The available data on $\sigma_T$ and $B$ in pp and $\bar{p}p$ scattering at high energies consist mainly of ISR (CERN) measurements at energies ranging from $\sqrt{s} = 23$ GeV to $\sqrt{s} = 63$ GeV, of the $\sqrt{s} = 541 - 546$ GeV measurements in CERN SPS and in Fermilab, and of the $\sqrt{s} = 1800$ GeV data from the E-710 Fermilab experiment. The Fermilab CDF measurements at $\sqrt{s} = 1800$ GeV seem discrepant with the E-710 experiment at the same energy and are not used here.

Since we are here concerned with nonperturbative contributions only, at the ISR energies we take for total cross sections the values given by the Donnachie and Landshoff parametrization for the pomeron-exchange contribution

$$\sigma_{pom}^T (pp, \bar{p}p) = (21.70 \text{ mb}) s^{0.0808} ,$$

and for values of the slope we take those of the pp system (not those of the $\bar{p}p$ data). Using as input the data for the highest energies (541 and 1800 GeV), where the process is essentially nonperturbative and no separation is needed, we obtain

$$a = 0.32 \pm 0.01 \text{ fm} , \quad <g^2FF> a^4 = 18.7 \pm 0.4 , \quad <g^2FF> = 2.7 \pm 0.1 \text{ GeV}^4 .$$

The relation between the experimental values of the two observables is well represented at all energies from 23.5 to 1800 GeV with the form

$$B = B_\Delta + C_\Delta (\sigma_T)^\Delta .$$

We use this expression with $B$ and $B_\Delta$ in GeV$^2$, $\sigma_T$ in mb and $C_\Delta$ in mixed units. This form is similar to Eq. (10), with an obvious correspondence of parameters. In our calculation...
with the model of the stochastic vacuum the exponent $\Delta = \delta/\beta$ does not depend on QCD quantities and is equal to about 0.75.

Following ideas that relate hadron-hadron scattering to the shape and size of the colliding hadrons, B.Povh and J.Hüfner [2] show that the combination of Regge amplitudes with electromagnetic form factors relates the slope parameter and the sum of the squares of the radii of the colliding hadrons, and write for $H_p$ (hadron-proton) scattering

$$B_{H_p} = R_p^2 + R_H^2 , \quad (14)$$

which is to be considered as a definition of effective hadronic radii. Observing the behavior of the experimental points in a plot of the observables $\sigma_T$ and $B$ against each other, they suggest that the dependence of the total cross section on the radii is

$$\sigma_{H_p}^T = gR_H^2 R_p^2 . \quad (15)$$

Introducing electromagnetic form factors to reproduce the shape of the elastic differential cross section, B.Povh and J.Hüfner write the relation of these hadronic radii to the electromagnetic radii as $< r_{em}^2 >= 3R^2$. Deviations from these simple formulae occurring at low energies show that they should be used only for $\sqrt{s} \geq 20$ GeV.

For proton-proton scattering, with $R_H = R_p$, Eqs. (14) and (15) lead to

$$B = C_{1/2} (\sigma_T^{1/2} , \quad (16)$$

which is of the form of Eq. (13) with $\Delta = 1/2$ and $B_\Delta = 0$, and it is remarkable that, using as input the data at 541 and 1800 GeV, one obtains with $\Delta = 1/2$ the same value $B_\Delta = 0$.

In Fig. 1 we show the description of the data through Eq. (13), using for $\Delta$ the values 1, 0.75 and 0.5. The case $\Delta = 1$ is included in Fig. 1 for a numerical reference, although we do not refer to any model suggesting it. The vertical axis represents the constant $C_\Delta$, which, together with $B_\Delta$, is fixed in each case by the input data at the energies 541 and 1800 GeV. Then the five ISR data points are considered as parameter-free predictions, and we calculate a $\chi^2$ value representing the observed deviations. In the horizontal axis we mark the energy,
which here works just as an external label used to spread the information in the plot. There
are no free parameters, since $B_\Delta$ and $C_\Delta$ are fixed by the input data. The values of $\chi^2$
are also shown, and, although $\Delta = 0.75$ is favoured, we cannot say that the differences are
statistically meaningful. However, the model of the stochastic vacuum gives precise meaning
to the parameters $B_\Delta$ and $C_\Delta$ in terms of QCD quantities, successfully predicts $\Delta = 0.75$
and introduces hadronic sizes in definite form, as parameters accounting for the extensions
of the wavefunctions. It is remarkable the presence in this case of a bounding minimum $B_\Delta$
(equal to $\eta a^2$) for the slope, which is the same for all hadron-hadron systems.

Fig. 1 - Test of the parameters of Eq. (13), comparing different models, using the experimental
quantities of the pp and $\bar{p}p$ systems. For energies up to 62.3 GeV the values of $\sigma^T$ are given by the
parametrization $\sigma^T_{pom} = (21.70 \text{ mb})s^{0.0808}$ and the values of B are those of the pp data. The values
of $B_\Delta$ and $C_\Delta$ are obtained with the 541 GeV and 1800 GeV data as inputs. The horizontal lines
represent the constant $C_\Delta$ with the choices $\Delta=1$, 0.75 (model of the stochastic vacuum) and 0.5 (geometric model). $\chi^2$ represents the average deviation of the five ISR points from the constant line.

The description given in Fig. 1 covers all pp and $\bar{p}p$ data. Extrapolating Eq. (13) to higher energies (e.g. LHC energies), where the total cross sections may be about 100 mb, we find a small, but hopefully measurable, difference in the values of the slope, with B higher by 0.2 GeV$^{-2}$ for $\Delta = 3/4$, compared with the $\Delta = 1/2$ case.

The proton radius presents a slow increase with the energy, taking values about the electromagnetic radius. In the case of the model of the stochastic vacuum, where the radius enters as a parameter of the wavefunction, the energy dependence of the radius can be parametrized in the form

$$S_p(s) = 0.671 + 0.057 \log \sqrt{s} \text{ (fm)} \quad (a),$$

or

$$S_p(s) = 0.572 + 0.123 \log \sqrt{s}^{0.75} \text{ (fm)} \quad (b).$$

In the geometric model Eq. (14) requires a power $1/2$ in the logarithm in order to yield a log$^2 s$ dependence in the cross section.

Parametrizations (a) and (b) predict for $\sqrt{s} = 14$ TeV values of the proton radius 1.215 fm and 1.240 fm respectively, which are about 40% higher than the electromagnetic radius. The pp cross section at this energy is predicted as (a) 95.5 mb and (b) 100.8 mb. These values are in good agreement with the results of the Akeno collaboration [12].

The value of $g$ is obtained by B.Povh and J.Hüfner, on the basis of Hp data, as $g=75$ fm$^{-2}$. In Fig. 1 we show that $C_{1/2} \approx 2$ GeV$^{-2}$ mb$^{-1/2}$, corresponding to $g = (2/C_{1/2})^2 \times 0.1/(0.197)^4 = 66.4$ fm$^{-2}$. This means a fair simultaneous description of pp and pH systems, which is also achieved in the calculations with the model of the stochastic vacuum, where the interaction strength and the hadronic radii appear as fundamental quantities. An important phenomenological difference between the two approaches rests in the existence of a finite universal minimum value $B_\Delta$ for the slope, which may be tested in systems where smaller hadrons collide with the proton, as we discuss below.
Eqs. (14), (15) and (16) for pp and \( \bar{p}p \) scattering can be built from a profile function of simple Gaussian shape

\[
J(b) = \frac{g R_p^2}{4\pi} e^{-b^2/4R_p^2},
\]

where \( R_p(s) \) has an energy dependence, and where the normalization for \( J \) is chosen appropriately. However, two Gaussians are needed to describe the data for \( t \neq 0 \).

3. Hadron-Proton Systems

We now consider other systems of hadrons colliding at high energies. In the treatment of the pp system we are constrained by \( \sqrt{s} \geq 20 \) GeV, and cannot observe clearly the effect of the minimum slope \( B_\Delta \). The contribution of this term could be better observed in Hp systems, where H represents hadrons of small size. We must remark that, since we deal with radii which are energy dependent quantities, we must compare different hadronic systems at the same center-of-mass energy.

The parametrization of the results obtained with the model of the stochastic vacuum for general Hp systems is

\[
\sigma_{\text{pom}}^T = I_0 \left[ \langle g^2 FF \rangle a^4 \right] a^2 = \left[ \langle g^2 FF \rangle a^4 \right] a^2 \alpha \left( \frac{S_p S_H}{a} \right)^{\beta/2},
\]

and

\[
B = \frac{1}{2} \frac{I_2}{I_0} a^2 = \eta a^2 + \frac{1}{2} \gamma \left( S_p^2 + S_H^2 \right).
\]

With \( a = 0.32 \) fm, we have \( \eta a^2 = 5.38 \) GeV\(^{-2} \). In the treatment of B.Povh and J.Hübner the corresponding relations are given by Eqs. (14) and (13).

In order to compare the models, it is important to eliminate the influence of specific values of radii, since they have different definitions. Thanks to the convenient factorization in the final expressions, we may actually build relations involving only the observables, or involving only the ratios of radii, which we may assume to follow the ratios of electromagnetic radii. We thus have for \( \sigma_{Hp}/\sigma_{H\bar{p}} \) the ratios \((r_H/r_{H\bar{p}})^{4/3}\) and \((r_H/r_{H\bar{p}})^2\) in the stochastic vacuum and geometric models respectively. Entering with the known values for the radii of the
proton (0.862 ± 0.012 fm), of the pion (0.66 ± 0.01 fm) and of the kaon (0.58 ± 0.04 fm) we obtain the results shown in Table I. The experimental ratio refers to the pomeron exchange contribution, taken from the parametrization of Donnachie and Landshoff. We observe that the value 2/3 given for the ratio $\sigma_{\pi p}/\sigma_{pp}$ by the quark additivity rule is here obtained as a simple consequence of the sizes of the hadrons. Also the ratio $\sigma_{Kp}/\sigma_{\pi p}$ is consistently obtained with a value close to the data, without need for different couplings of the pomeron to strange and non-strange quarks, as must be the case with quark additivity rules. Since the analysis of the proton structure in HERA (DESY) shows that the proton is better characterized as a sea rather than as a valence structure, the explanation of the high-energy phenomenology through the hadronic sizes is more legitimate. The factorization relation $\sigma_{\pi\pi} = \sigma_{\pi p}^2/\sigma_{pp}$ is identically satisfied in both cases considered here.

Table I - Ratios of the pomeron exchange contributions to total cross sections for different hadronic systems. The experimental values are taken from the parametrization of Donnachie and Landshoff.

| Cross section ratios | stochastic vacuum | geometric model | Experimental values |
|----------------------|-------------------|-----------------|---------------------|
| $\sigma_{\pi\pi}/\sigma_{pp}$ | 0.69 ± 0.02 | 0.59 ± 0.02 | 0.63 |
| $\sigma_{pK}/\sigma_{\pi\pi}$ | 0.83 ± 0.08 | 0.77 ± 0.08 | 0.87 |

Considering all Hp systems at a given energy, Eqs. (19) and (20) lead to a nonzero minimum possible value for the slope, given by

$$B_{\text{Hp}}^\text{min}(s) = \eta a^2 + \frac{\gamma}{2} s_p^2 = \frac{1}{2} \eta a^2 + \frac{1}{2} B_{pp}(s) = 2.69 \text{ GeV}^{-2} + \frac{1}{2} B_{pp}(s).$$

(21)

The existence of this minimum slope that can be observed in the scattering of any hadron by a proton is characteristic of the model of the stochastic vacuum. To relate the observables for different Hp systems at a given energy, we call $G = a[\langle g^2 FF \rangle a^4]^2 a^2$ and write
\[
\frac{B_{pp} - B_{Hp}}{\sigma_{pp}^{4/\beta} - \sigma_{Hp}^{4/\beta}} = \frac{(\gamma/2)(S_p^2 - S_H^2)}{G^{4/\beta}(S_p^2/a^2)^2 - G^{4/\beta}(S_pS_H/a^2)^2} = \frac{(\gamma/2) a^2}{G^{2/\beta} \sigma_{pp}^{2/\beta}}. \tag{22}
\]

The last quantity is fixed, for a given energy. As we go from a hadron H to another, we obtain in a plot of \(B_{Hp}\) against \(\sigma_{Hp}\) a line from the point representing the observables of the pp system to the limit point \(\sigma = 0, B = B^{\text{min}}\) given by Eq. (21). With \(\beta = 8/3\) we have

\[
B_{Hp} = B_{Hp}^{\text{min}} + \frac{B_{pp} - B_{Hp}^{\text{min}}}{\sigma_{pp}^{1.5}} \sigma_{Hp}^{1.5}. \tag{23}
\]

Using the data for the pp system at 23.5 GeV we obtain \(B_{Hp} = 8.59 + 0.014775 \sigma_{Hp}^{1.5}\). This plot is shown in Fig. 2, together with data of the pp, \(\pi p\) and Kp systems \([10]\) at \(\sqrt{s} \approx 20\) GeV. The limit point is shown inside a square window in the figure.

In the case of the geometric model we have

\[
B_{Hp}^{\text{min}}(s) = R_p^2 = \frac{1}{2} B_{pp}(s) \tag{24}
\]

and a straight line in a plot of \(B_{Hp}\) against \(\sigma_{Hp}\)

\[
\frac{B_{pp} - B_{Hp}}{\sigma_{pp} - \sigma_{Hp}} = \frac{1}{(\sigma_{pp}g)^{1/2}}. \tag{25}
\]

\(g\) is fixed by pure pp data putting \(\sigma_{Hp} = 0, B_{Hp} = B_{pp}/2\), and using Eq. (14). Then

\[
B_{Hp} = \frac{1}{2} B_{pp} \left[ 1 + \frac{\sigma_{Hp}}{\sigma_{pp}} \right]. \tag{26}
\]

Comparing the two lines shown in Fig. 2 we may tell, for now with some subjective
judgement, if either model describes better the data.

Fig. 2 - Observables for different hadronic systems at 20 GeV. The straight line is the prediction of the geometric model, the square window showing the minimum value for the slope predicted for small hadrons colliding with protons at this energy. The upper curve and the upper square window represent the predictions of the model of the stochastic vacuum.

The model of the stochastic vacuum predicts that the slope B for the \( \pi \pi \) system at about \( \sqrt{s} \approx 20 \) GeV is \( B_{\pi\pi} = \eta a^2 + \gamma 2 \ S^2_\pi \approx 9.6 \) GeV\(^{-2} \), while the geometrical model predicts \( B_{\pi\pi} = (2/3) \ r_\pi^2 \approx 7.5 \) GeV\(^{-2} \). This is not a trivial difference, as it tests the contribution of a nonperturbative QCD effect.

4. Conclusions

The results presented in this work exhibit the simplicity of the connections between
hadronic high-energy observables determined by the hadronic sizes and stress the importance of geometric relations as indicative of properties of nonperturbative QCD dynamics.

Both the model of the stochastic vacuum and the geometric description of Povh and Hüfner give fair account of the present data, although it seems to us that the relations provenient of the QCD calculation are more accurate. The differences between the two descriptions are important and interesting, and once they can be fully tested by the data, may become crucial. The existence or not of the universal term $\frac{1}{2}\eta a^2$, of unique value for all energies, representing a pure nonperturbative QCD contribution to hadronic scattering, is a question of fundamental importance. Direct hadronic data on hadronic systems with small mesons, such as $\phi p$ and $\psi p$ would be very interesting for the study of nonperturbative QCD effects. Hopefully the $\phi$ factory in Frascati will create the opportunity for these studies.

While the geometric relations of Povh and Hüfner are basically empirical, the quantitative details (the form of the functional relations and the intrinsic values of parameters) in the predictions of the model of the stochastic model determine fundamental QCD quantities using only a small amount of data. The model explains the energy dependence of the observables in terms of the energy variation of the hadronic sizes, and relates experimental quantities for different hadronic systems, exhibiting properties of the extended nature of the interaction, which is determined by the structure of the QCD vacuum.
REFERENCES

[1] C. Bourrely, J. Soffer and T.T. Wu, Nucl. Phys. B247, 15 (1984); Phys. Rev. Lett. 54, 757 (1985); Phys. Lett. B196, 237 (1987); J. Dias de Deus and P. Kroll, Nuovo Cimento A37, 67 (1977); Acta Phys. Pol. B9, 157 (1978); J. Phys. G9, L81 (1983); P. Kroll, Z. Phys. C15, 67 (1982); T.T. Chou and C.N. Yang, Phys. Rev. 170, 1591 (1968); ibid. D19, 3268 (1979); Phys. Lett. B128, 457 (1983); ibid. B244, 113 (1990).

[2] B. Povh and J. H"ufner, Phys. Rev. Lett. 58, 1612 (1987); Phys. Lett. B215, 772 (1988); ibid. B245, 653 (1990); Phys. Rev. D46, 990 (1992); Zeit. Phys. C63, 631 (1994).

[3] J.F. Gunion and D.E. Soper, Phys. Rev. D15, 2617 (1977); E. Levin and M.G. Ryskin, Sov. J. Nucl. Phys. 34, 619 (1981).

[4] H.G. Dosch, E. Ferreira and A. Kr"amer, Phys. Lett. B289, 153 (1992); ibid. B318, 197 (1993); Phys. Rev. D50, 1992 (1994).

[5] E. Ferreira and F. Pereira, Phys. Rev. D55, 130 (1997).

[6] H.G. Dosch and M. Rueter, Phys. Lett. B205, 117 (1996).

[7] Data on pp and $\bar{p}p$ systems. (a) N. Amos et al., Nucl. Phys. B262, 689 (1985); (b) R. Castaldi and G. Sanguinetti, Ann. Rev. Nucl. Part. Sci. 35, 351 (1985); (c) C. Augier et al., Phys. Lett. B316, 448 (1993); (d) M. Bozzo et al., Phys. Lett. B147, 392 (1984); M. Bozzo et al., ibid. B147, 385 (1984); (e) N. Amos et al., Phys. Lett. B247, 127 (1990); Phys. Rev. Lett. 68, 2433 (1992);

[8] F. Abe et al., Phys. Rev. D50, 5550 (1994); ibid. D50, 5518 (1994).

[9] A. Donnachie and P.V. Landshoff, Phys. Lett. B296, 227 (1992).

[10] Data on slopes of $\pi p$ and Kp systems. (a) J.B. Burq et al., Nucl. Phys. B217, 285 (1983); (b) A. Schiz et al., Phys. Rev. D24, 26 (1981); (c) L.A. Fajardo et al., Phys. Rev. D24, 46 (1981); (d) N. Adamus et al., Phys. Lett. B186, 223 (1987).
[11] Electromagnetic radii. (a) Proton: G.G. Simon et al., Z. Naturforschung 35A, 1 (1980); (b) Pion: S.R. Amendolia et al., Nucl. Phys. B277, 168 (1986); (c) Kaon: S.R. Amendolia et al., Phys. Lett. B178, 435 (1986).

[12] M. Honda et al., Phys. Rev. Lett. 70, 525 (1993).
FIGURE AND TABLE CAPTIONS

**Fig. 1** - Test of the parameters of Eq. (13), comparing different models, using the experimental quantities of the pp and \( \bar{p}p \) systems. For energies up to 62.3 GeV the values of \( \sigma^T \) are given by the parametrization \( \sigma^T_{\text{pom}} = (21.70 \text{ mb})s^{0.0808} \) and the values of B are those of the pp data. The values of \( B_\Delta \) and \( C_\Delta \) are obtained with the 541 GeV and 1800 GeV data as inputs. The horizontal lines represent the constant \( C_\Delta \) with the choices \( \Delta=1, 0.75 \) (model of the stochastic vacuum) and 0.5 (geometric model). \( \chi^2 \) represents the average deviation of the five ISR points from the constant line.

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