Adaptive Digital Channelization of Sparse Wideband Analog Signals at Sub-Nyquist Rate

Peng-zhi QIAN, Bo PENG*, Zhen LIU and Yong-xiang LIU
School of Electronic Science, National University of Defence Technology, Changsha, China
*Corresponding author

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Abstract. Considering that the conventional digital channelized receiver requires a high sampling rate for a wideband signal and produces a cross-channel problem due to unknown prior information of the received signals, a method of adaptive digital channelization based on compress sensing (CS) theory is proposed in this paper. The method consists of three parts, i.e., modulated wideband converter (MWC) sampling system, sparse recovery algorithm, synthesis filter banks. MWC sampling system is applied to receive sparse wideband analog signals at sub-Nyquist sampling rate. Sparse recovery algorithm such as sparsity adaptive matching pursuit (SAMP) is chosen to recover the channelized signals. Based on the adjacent channels containing signals, the complete wideband signal can be obtained by the synthesis filter banks adaptively, thus solving the cross-channel problem. Finally, the numerical experiments demonstrate that the proposed method can implement adaptive channelization of sparse wideband signal at sub-Nyquist sampling rate.

Introduction

Digital channelized receiver is widely used in Electronic Warfare application, because they have the characteristics of wide instantaneous frequency coverage, high sensitivity and large dynamic range [1, 2]. The bandwidth demand of digital channelized receivers is becoming wider for a higher reconnaissance capability, it increases the requirement of analog-to-digital converters (ADCs) for a higher sampling rate which may even exceeds the specifications of the current best ADCs [3]. This greatly limits the received bandwidth of digital channelized receiver. At the same time, when the received signals is channelized by the conventional analysis filter banks, like the polyphase discrete Fourier transform (PDFT) filter banks [4], the cross-channel problem will easily arise. Because the received signal is non-cooperative, whose prior information is unknown, and the channels with equal bandwidth channelized by conventional analysis filter banks are fixed and narrow, a single complete wideband signal is often divided into a series of adjacent channels causing the cross-channel problem.

In recent years, non-uniform channelization technologies are proposed to solve the cross-channel problem. In [5], Wajih proposed that when modulated perfect reconstruction (PR) filter banks are applied in the reverse direction, it can be used to synthesize a set of channels with nonequal bandwidths into a single wideband signal. In [6] and [7], R. Mahesh presented two reconfigurable non-uniform filter banks architectures based on frequency response masking (FRM) and coefficient decimation for non-uniform channelization respectively. However, these methods all require the prior information of the received signals and cannot solve the problem of high sampling rate.

Actually, the bandwidth of the wideband signals may occupy for only a small portion of the extraordinarily wide received bandwidth, so the received signals can be considered as spectrum-sparse. Compress sensing (CS) theory [8] indicates that the sparse signals can be recovered by few measurements, which provides a solution to the problem of high sampling rate. In [9], Modulated Wideband Converter (MWC) was proposed based on CS by M. Mishali, which implements sub-Nyquist sampling of multiband signals. In [10], MWC was applied to wideband spectrum sensing. Thus, in our situation we intend to reduce the sampling rate and obtain the prior information of signals by MWC.
Combining the MWC and the non-uniform channelization technologies, we propose an adaptive channelization method. The front end uses the MWC sampling system to reduce the sampling rate. Then, SAMP recovery algorithm are adopted to recover the support set and channelized signals. According to the support set, efficient DFT polyphase synthesis filter banks are used to select and synthesize the adjacent channelized signals into one channelized signal, which contains the complete wideband signal at a low data rate. Thus, the cross-channel problem is solved. In the remainder of this paper, the signal model and the three parts of adaptive channelization method are described. At last, the experimental results are summarized and the conclusion is concluded.

Signal Model
We assume a signal model as show in Figure 1. In channelized receiver, the received spectral range is \([-f_N/2, f_N/2]\), where \(f_N = 1/T_N\) represents Nyquist rate, and contains several wideband signals, like LFM signals. The received spectral range is separated into \(M\) spectrum channels by the channelized receiver, whose bandwidth is \(B\). The number of the spectrum channels wideband signals occupy on is assumed be less than \(N\). Due to the fact that the received spectral range is extraordinarily wide and the spectrum is underutilized, the received signal is spectrum-sparse with \(NB \ll f_N\).

MWC Sampling System
The MWC sampling system structure is shown in Figure 2. The input analog signal \(x(t)\) enter \(m\) channels simultaneously. In the \(i\) th channel, the original signal \(x(t)\) is mixed by a pseudo random symbol sequence \(p_i(t)\) with period \(T_p\). The pseudo random symbol sequence \(p_i(t)\) is a piece
constant function that alternates between the level ±1 for each of $M_p$ equal time intervals. Then, the mixed signal passes a low-pass filter, whose cutoff frequency is $1/2T_p$. Finally, we can obtain the low rate discrete signal $y_i[n]$ at the sampling rate $f_s=1/T_s$ from the filtered signal. The discrete-time Fourier transform (DTFT) of the $i$th sequence $y_i[n]$ is given as:

$$Y_i(e^{j2\pi f_s}) = \sum_{n=-\infty}^{\infty} y_i[n]e^{-j2\pi f_s n} = \sum_{l=0}^{L_0} c_{il} X(f - ilf_p).$$

where $f_p=1/T_p$, and $c_{il}$ is the coefficients of Fourier series of $p_i(t)$.

$$c_{il} = \frac{1}{T_p} \int_{T_p}^0 p_i(t)e^{-j2\pi f_p t} \, dt.$$ (2)

The $L_0 = \lfloor (f_N + f_s)/2f_p \rfloor - 1$, $f \in [-f_s/2, f_s/2]$ and $f_N$ is the Nyquist rate of $x(t)$. It can be seen clearly that the spectrum after sampling becomes a linear combination of $f_p$-shifted copy of original spectrum $X(f)$ in (1). Considering all channels, it is convenient to write (1) in matrix form as

$$Y(f) = AZ(f).$$ (3)

where $Y(f)=[Y_1(e^{j2\pi f_p}), \ldots, Y_m(e^{j2\pi f_p})]^T$, and the vector $Z(f)=[Z_1(f), \ldots, Z_L(f)]^T$ is of length $L=2L_0+1$ with

$$Z_k(f) = X(f + (i-L_0-1)f_p),$$

$$f \in [-f_s/2, f_s/2], k \in \{1, 2, \ldots, L\}.$$ (4)

The $m \times L$ matrix $A$ contains the coefficients $c_{il}$

$$A_{il} = c_{i,-l} = c_{il}^*.$$ (5)

Given $Y(f)$ and $A$, by solving (3), we can obtain the vector $Z(f)$, $f \in [-f_s/2, f_s/2]$. When $L$ is even, the sampling system structure is also effective. A basic parameters configuration of the MWC is $f_s \geq f_p \geq B$ and $L > m \geq 2N$.

As shown in Figure 3, the formula (3) is depicted with $f_s = f_p = B$. The obtained vector $Z(f)$ is consistent with the channelized signals which is obtained by conventional DFT analysis filter banks, and the length $L$ of vector $Z(f)$ corresponds to the channelized channel number $M$ of channelized receiver. According to the proposed signal model, the channelized channels containing the signals can be obtained by the support set

$$S = \{k \mid Z_k(f) \neq 0\}.$$ (6)

Thus, the elements of support set $S$ represent the serial numbers of channelized channels which contain signals, and can be chosen as the prior information of synthesis filter banks. The vector $Z(f)$ and support set $S$ is obtained by the sparse recovery algorithm in the next subsection.
Sparse Recovery Algorithm

From (3) the relationship between unknown time sequence $z[n]$ and measurements $y[n]$ can be developed by inverse operation of DTFT, and we have

$$y[n] = Az[n].$$

(7)

where

$$z[n] = [z_1[n], z_2[n], \ldots, z_L[n]]^T,$$

(8)

$$y[n] = [y_1[n], y_2[n], \ldots, y_L[n]]^T.$$

(9)

Due to the condition of $m < L$, the matrix equations system is underdetermined, so we cannot obtain the unique solution directly by inverse operations. When the support set $S$ is found, the signals can be recovered by

$$z_S[n] = A_S^\dagger y[n],$$

(10)

$$z_l[n] = 0, \quad l \notin S.$$

(11)

where $z_S[n]$ contains only the entries of $z_l[n]$ indexed by $S$, and $A_S^\dagger = (A_S^H A_S)^{-1} A_S^H$ is the pseudoinverse of $A_S$ which contains the columns of $A$ indexed by $S$.

We suggest using the SAMP algorithm in [11] as the recovery algorithm, because the main feature of the SAMP is its capability of signal reconstruction without prior information of the sparsity. Thus, the support set $S$ can be recovered without the prior information of the sparse wideband by the SAMP recovery algorithm.

Synthesis Filter Banks

As mentioned above, when $f_s = f_p = B$, these recovered $z_l[n]$ ($l \in \{1, 2, \ldots, L\}$) is consistent with channelized signals with channelized channel number $M = L$. Thus, the conventional synthesis filter banks are applicable in this method to synthesize channelized signals for solving the cross-channel problem. According to the support set $S$, we consider that adjacent channelized signals at spectrum belong to the same wideband signal. Thus, we can respectively synthesize the adjacent channelized channels with signals instead of synthesizing all channels at once for solving the cross-channel problem. There are only a few channels that need to be synthesized at one time, thus the channelized
signals are not required to be interpolated to the Nyquist rate, and the requirement of subsequent data processing rate are reduced.

Assume that the number of wideband signals in the received spectral range is \( N_s \). The \( N_s \) wideband signals are respectively channelized into multiple adjacent channelized channels, whose serial numbers are consecutive integer. Assume that these channelized channel numbers correspond to the signal support set \( S_j \) \( ( j \in \{1,2,\ldots,N_s\} ) \), and there are \( D_j \) elements in \( S_j \) \( ( S_j = \{S_j[0],S_j[1],\ldots,S_j[D_j-1]\} ) \). The recovered support set \( S \) can be considered as

\[
S = S_1 \cup S_2 \cup \cdots \cup S_{N_s}.
\]

The reconstructed signal \( u_j[m] \) can be obtained by

\[
u_j[m] = \sum_{k=0}^{D_j-1} (r_{j,k}[m] * g_j[m])e^{j\omega_km}, \quad \omega_k = \frac{2\pi k}{D_j} + \frac{\pi}{D_j}
\]

where

\[
r_{j,k}[m] = \begin{cases} z_{S_j[k]}[m] & m = nD_j \\ 0 & \text{otherwise} \end{cases}
\]

The \( r_{j,k}[m] \) is the \( z[n] \) indexed by \( S_j[k] \) and interpolated by \( D_j \) times. The \( g_j[m] \) is the lowpass filter with normalized cutoff frequency \( 1/2D_j \). Thus, the \( j \) th reconstructed signal \( u_j[m] \) is obtained by synthesizing the \( D_j \) adjacent channelized signals which are indexed by \( S_j \), and is reconstructed with \( D_j/M \) times of Nyquist rate \( f_N \). Finally, the recovered channelized signals can be reconstructed into \( N_s \) non-uniform channelized signals, and the cross-channel problem is solved. The PDFT filter banks architecture can be used as the synthesis filter banks to synthesize the channelized signals efficiently. We rewrite (13) as

\[
u_j[m] = \sum_{k=0}^{D_j-1} \left[ \sum_{l=-\infty}^{\infty} r_{j,k}[lD_j]g_j[m-lD_j] \right] e^{j\omega_km} = \sum_{k=0}^{D_j-1} \left[ \sum_{l=-\infty}^{\infty} z_{S_j[k]}[l]g_j[m-lD_j] \right] e^{j\omega_km}.
\]

According to the polyphase principle, set \( m = nD_j + q \), \( u_{j,q}[n] = u_j[nD_j + q] \), \( g_{j,q}[n] = g_j[nD_j + q] \), thus

\[
u_{j,q}[n] = (-e^{j\omega_kD_j}) \sum_{l=-\infty}^{\infty} g_{j,q}[n-l] \left( \sum_{k=0}^{D_j-1} z_{S_j[k]}[l] e^{j2\pi kl/D_j} \right).
\]

Set \( a_q[l] = \left( \sum_{k=0}^{D_j-1} z_{S_j[k]}[l] e^{j2\pi kl/D_j} \right) \), and the (16) can be described as

\[
u_{j,q}[n] = (-e^{j\omega_kD_j}) \sum_{l=-\infty}^{\infty} g_{j,q}[n-l]a_q[l] = (g_{j,q}[n] * a_q[n]) (-e^{j\pi k/D_j})^q.
\]

Thus, we can get the efficient \( D_j \) channel synthesis filter banks architecture as shown in the Figure 4. The \( g_{j,q}[n] \) is the \( q \)th polyphase filter of the low-pass prototype filter \( g_j[m] \).
Results

To evaluate the performance of the proposed adaptive channelization method, we simulate the system on spectrum-sparse test signals, which consists of LFM signal, with the bandwidth $B = 200$ MHz, chirp rate $K = 20$ MHz/μs and the initial frequency $f_i = 500$ MHz. The received spectral range of the system is $[-f_N / 2, f_N / 2]$ with $f_N = 2$ GHz. The parameters of MWC are configured as $f_s = f_p = 62.5$ MHz, $M = L = 32$, $m = 8$.

The time domain and spectrum of signals sampled by MWC sampling system are shown in the Figure 5. The results show that the signals are mixed by pseudo random symbol sequences, and sampled at 62.5 MHz. The 32 channelized signals can be recovered from the sampling signals of 8 channels. The schematic diagram of channelization is shown in Fig. 6, and these numbers represent the serial number of the channelized channels of the recovered signals. The time domain and spectrum of recovered channelized signals (channel 9-12) are respectively shown in the Fig. 7 and Fig. 8. These results demonstrate that the SAMP algorithm can recover the channelized signals from the sampling signals, and show that the cross-channel problem arises. As shown in Fig. 9, the signal reconstructed by 4 channel synthesis filter banks demonstrates that the cross-channel problem is solved and complete wideband signal can be obtained in a low rate. Due to the notches of non-ideal DFT synthesis filter banks, there are some imperfection effects in Fig. 9.

![Diagram](image-url)

Figure 4. Efficient Dj channel synthesis filter banks architecture.

![Graphs](image-url)

Figure 5. The time domain and Spectrum of sampling signals. (a) sampling channel 1. (b) sampling channel 2. (c) sampling channel 3. (d) sampling channel 4. (e) sampling channel 5. (f) sampling channel 6. (g) sampling channel 7. (h) sampling channel 8.
Figure 6. Schematic diagram of channelization.

Figure 7. The time domain of recovered channelized signals. (a) Recovered channel 9 signal. (b) recovered channel 10 signal. (c) Recovered channel 11 signal. (d) Recovered channel 12 signal.

Figure 8. The Spectrum of recovered channelized signals. (a) Recovered channel 9 signal. (b) recovered channel 10 signal. (c) Recovered channel 11 signal. (d) Recovered channel 12 signal.

Figure 9. Reconstructed signal. (a) The time domain of reconstructed signal. (b) The Spectrum of reconstructed signal.

**Conclusion**

In this paper, a novel adaptive channelization method based on MWC is proposed. The digital receiver consists of 3 parts: MWC sampling system, sparse recovery algorithm and synthesis filter.
banks. Numerical experiments demonstrate that proposed adaptive channelization method can solve the cross-channel problem and implement adaptive channelization of sparse wideband signal at sub-Nyquist sampling rate.

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