A Frequent Closed Itemsets Lattice-based Approach for Mining Minimal Non-Redundant Association Rules

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Abstract

There are many algorithms developed for improvement the time of mining frequent itemsets (FI) or frequent closed itemsets (FCI). However, the algorithms which deal with the time of generating association rules were not put in deep research. In reality, in case of a database containing many FI/FCI (from ten thousands up to millions), the time of generating association rules is much larger than that of mining FI/FCI. Therefore, this paper presents an application of frequent closed itemsets lattice (FCIL) for mining minimal non-redundant association rules (MNAR) to reduce a lot of time for generating rules. Firstly, we use CHARM-L for building FCIL. After that, based on FCIL, an algorithm for fast generating MNAR will be proposed. Experimental results show that the proposed algorithm is much faster than frequent itemsets lattice-based algorithm in the mining time.

Keywords: Data mining, frequent closed itemsets, frequent closed itemsets lattice, frequent itemsets lattice, minimal generators, minimal non-redundant association rules.

1. Introduction

Mining association rules is divided into two phases: i) Mining FI/FCI and ii) Generating association rules from FI/FCI. There have been many algorithms developed for the phase i) such that Apriori-based [2, 14-15], FP-tree-based [5-7, 16, 23], and IT-tree-based [25-27], etc. However, the algorithms deal with the phase ii) have received little attention. In 1993, Agrawal et al developed a method for mining traditional association rule (TAR) [1]. After that, Apriori algorithm has been proposed [2]. Because TAR contains a lot of redundancies, therefore, minimal non-redundant association rule (MNAR) concept has been proposed [3, 14-15]. The set of MNAR is more compact than TAR in number of generated rules. Besides, the number of FCI is often much smaller than the number of FI, so the time for generating rules from FCI reduces significantly.

Recent years, lattice-based approaches for fast mining association rules have been proposed. In 2009, we proposed an algorithm for mining TAR based on frequent itemsets lattice (FIL) [20]. This work saves a lot of time for generating association rules. Because based on the lattice, we can determine all child nodes of a given node and need not traverse all FI. After that, a modification of FIL (MFIL) for generating MNAR has been proposed in [22]. MNAR only mines from X to Y, where X is a minimal generator, Y is an frequent closed itemset and X ⊆ Y. FIL is modified by

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adding one field to determine whether a lattice node is a minimal generator or not and
one field to determine whether a lattice node is a closed itemset or not. After building
the lattice, we can generate MNAR easily.

The purpose of this paper is to mine MNAR based on frequent closed itemsets
lattice and compare it with the algorithm based on MFIL. In section 2, we introduce
some basic concepts and related works. Section 3 presents an algorithm for mining
MNAR using FCIL. Section 4 discusses our experimental results. Conclusion and future
work are in section 5.

2. Concepts and Related Works

2.1. Transaction Database

Let $I = \{i_1, i_2, ..., i_n\}$ be a set of items, $T = \{t_1, t_2, ..., t_m\}$ be a set of transaction
 identifiers (tids or tidset) in a database D. The input database is a binary relation $\delta \subseteq I \times T$. If an item $i$ occurs in a transaction $t$, we write it as $(i,t) \in \delta$ or $i \delta t$.

Example: Consider database in Table 1

| TID | Item bought          |
|-----|----------------------|
| 1   | A, C, T, W           |
| 2   | C, D, W              |
| 3   | A, C, T, W           |
| 4   | A, C, D, W           |
| 5   | A, C, D, T, W        |
| 6   | C, D, T              |

The second transaction can be represented as $\{C \delta 2, D \delta 2, W \delta 2\}$.

2.2. Support

Let $D$ be a transaction database and an itemset $X \subseteq I$. The support of $X$, denoted
$\sigma(X)$, is number of transactions in $D$ containing $X$.

2.3. Frequent Itemset and Frequent Closed Itemset

Itemset $X \subseteq I$ is called to be frequent if $\sigma(X) \geq \text{minSup}$ ($\text{minSup}$ is a minimum
support threshold). Let $X$ be a frequent itemset, $X$ is called a frequent closed itemset if
there have not any frequent itemset $Y$ such that $X \subseteq Y$ and $\sigma(X) = \sigma(Y)$.

2.4. Minimal Generators [22, 25-26]

Let $X$ be a frequent closed itemset, $X' \neq \emptyset$ is called a generator of $X$ if and only if:

i) $X' \subseteq X$ and
ii) $\sigma(X) = \sigma(X')$.

Let $G(X)$ denote the set of generator of $X$. We say that $X' \in G(X)$ is a minimal
generator if it has no subset in $G(X)$. Let $mGs(X)$ denote the set of all minimal
generators of \( X \). By definition, \( mGs(X) \neq \emptyset \) since if there is no proper generator then \( X \) is a \( mG \) of \( X \).

2.5. Mining FCI

Mining FCI is divided into four categories [9, 24]:
i) Test-and-generate (Close [15], A-Close [14]): Using level-wise approach to discover FCI. All of them are based on the Apriori algorithm.
ii) Divide-and-conquer (Closet [16], Closet+ [23], FPClose [6]): using compact data structure (extended from FP-tree) to mine FCI.
iii) Hybrid (CHARM [27], CloseMiner [18]): using both test-and-generate and divide-and-conquer to mine FCI. They are based on vertical data format to transform the database into item–tidlist and develop properties to prune fast non-closed itemsets.
iv) Hybrid without duplication (DCI-Close [12], LCM [19], PGMiner [13]): they differ from hybrid in that they do not use “subsume checking”. Therefore, they do not need storage of FCI in main memory and need not use hash tables as CHARM.

2.6. Mining MNAR/NAR

Mining MNAR was proposed in 1999 by Pasquier et al. [14-15]. Firstly, the authors mined all FCI by computing closure of minimal generators. After that, they mined all MNAR by generating rules with confidence = 100% from \( mGs(X) \) to \( X \) (\( X \) is a frequent closed itemset) and generating rules with the confidence < 100% from \( mGs(X) \) to \( Y \) (\( X, Y \) are frequent closed itemsets and \( X \subset Y \)). In 2000, Zaki proposed the method to mine NARs [25]. He was based on FCI and theirs \( mGs \) to mine NARs. This approach only mined the rules that their left hand side and right hand side are minimal in the set of rules that have the same support and confidence. In 2004, Zaki published his paper with some extensions [26].

2.7. Building Frequent (closed) Itemsets Lattice

Zaki and Hsiao proposed CHARM-L [27], which is an extension of CHARM, for building a frequent closed itemset lattice. We presented an extension of the Eclat algorithm [27] for building a frequent itemset lattice (FIL) [20]. A modification of the frequent itemset lattice for mining MNAR was also presented in [22].

In this paper, we extend the lattice-based approach for quickly mining MNAR. Firstly, CHARM-L algorithm for building FCIL will be applied. After that, based on FCIL, a mining approach for MNAR based on the obtained FCIL is designed.

2.8. Generating Association Rules from Frequent Itemsets Lattice

In [20], we have proposed an algorithm for mining traditional association rules from FIL. This algorithm uses the relation between two nodes in lattice for fast traversing all child nodes of a given node. This approach is more efficient than directly mining from frequent itemsets (using hash table) [7]. A modification of frequent itemsets lattice (MFIL) for mining MNAR was proposed in [22].
3. Generating Minimal Non-redundant Association Rules from FCIL

**Definition 3.1. General rule [22]**

Let two rules $R_1: X_1 \rightarrow Y_1$ and $R_2: X_2 \rightarrow Y_2$, $R_1$ is said more general than $R_2$, denoted $R_1 \propto R_2$, if and only if $X_1 \subseteq X_2$ and $Y_2 \subseteq Y_1$.

**Definition 3.2. Redundant rule [22]**

Let $R = \{R_1, R_2, \ldots, R_n\}$ be the set of rules which have the same support and confidence. Rule $R_j$ is redundant if in $R$ exists the rule $R_i$ such that $R_i \propto R_j$ ($i \neq j$).

**Theorem 3.1 [22].** MNAR with the confidence $= 100\%$ are only generated from $X' \rightarrow X$ ($\forall X' \in mGs(X), X$ is a FCI).

**Theorem 3.2 [22].** MNAR with the confidence $< 100\%$ are only generated from $X' \rightarrow Y$ ($\forall X' \in mGs(X), X, Y \in$ FCI, and $X \subseteq Y$).

Based on the theorem 3.1 and theorem 3.2, MNAR are only mined from $X$ to $Y$, where $X$ is a minimal generator, $Y$ is a frequent closed itemset and $X \subseteq Y$. Therefore, we modify CHARM-L [27] to build FCIL and mine all minimal generators of FCI by using MG-CHARM [21].

3.1. Algorithm for Generating MNAR from FCIL

![Algorithm for Generating MNAR from FCIL](image)

**Figure 1. Algorithm for mining minimal non-redundant association rules from FCIL**
Firstly, the algorithm traverses all child nodes $L_c$ of the root node $L_r$, then it calls \textsc{Extend}\_\textsc{MNAR}\_\textsc{FCIL}(L_c)$ function (line 3) to traverse all nodes in the FCIL (recursively and marks the visited nodes by turn flag on, lines 5-9). Considering \textsc{Generate}\_\textsc{Rule}(L_c) function, this function used a queue to traverse all child nodes of $L_c$ (and marking all visited nodes to reject coincide). For each child node $L$ of $L_c$, we compute the confidence of rules that will generate from $L_c$ to $L$, if the confidence satisfies $minConf$ then function \textsc{Find}\_\textsc{Rule} is called (lines 17-19) to generate all rules from minimal generators of $L_c$ itemset to $L$ itemset (lines 24-26).

### 3.2. Illustration

Using CHARM-L [27] and MG-CHARM [21], we have the frequent closed itemsets lattice of the database in Table 1 with $minSup = 50\%$ as follow:

![Frequent closed itemsets lattice and their minimal generators of the database in Table 1.](image)

Consider the process of generating MNAR with the $minConf = 80\%$ from node $\{CW\}$ of the lattice (in Figure 2), we have:

- At first, Queue = $\emptyset$. After that, the algorithm will call \textsc{Find}\_\textsc{Rule} ($\{CW\}$, $\{CW\}$, 1.0). This function will generate rule from $mG$ of CW (i.e., W) to CW with the confidence is 100%, we have rule $W \xrightarrow{5.1.0} C$.
- The child nodes of $\{CW\}$ are $\{\{CDW\}, \{ACW\}\}$, so they are added to Queue $\Rightarrow$ Queue = $\{\{CDW\}, \{ACW\}\}$.
- Because Queue $\neq \emptyset$:
  - $L = \{CDW\}$ (Queue = $\{\{ACW\}\}$).
  - The confidence of rules from CW to CDW is $3/5 < \text{minConf}$, the algorithm will not call function \textsc{Find}\_\textsc{Rule}.
  - Next, because Queue $\neq \emptyset$: ...
\[ L = \{\text{ACW}\} \ (\text{Queue} = \emptyset). \]
\[ \text{The confidence of rules from CW to ACW is } 4/5 \geq \minConf, \text{ the algorithm will call function FIND\_RULE (\{CW\}, \{ACW\}, 4/5). This function will generate rule } W^{4.4/5} \rightarrow \text{AC}. \text{ After that, algorithm will add all child nodes of ACW to Queue } \Rightarrow \text{Queue} = \{\{\text{ACTW}\}\}. \]
\[ \text{Next, because Queue } \neq \emptyset:\]
\[ \bullet \ L = \{\text{ACTW}\} \ (\text{Queue} = \emptyset). \]
\[ \bullet \text{The confidence of rules from CW to ACTW is } 3/5 < \minConf, \text{ the algorithm will not call the function FIND\_RULE.} \]
\[ \bullet \text{Next, because Queue } = \emptyset, \text{ stop.} \]

4. Experimental Results

All experimental results described below have been performed on a Centrino core 2 duo \(2 \times 2.53\ \text{GHz}\), 4GBs RAM memory, Windows 7. Algorithms were coded in C# (2008). The experimental databases from http://fimi.cs.helsinki.fi/data/ (downloaded on April 2005) were downloaded to perform the test with theirs features displayed in Table 2.

| Databases  | #Trans | #Items |
|------------|--------|--------|
| Chess      | 3196   | 76     |
| Mushroom   | 8124   | 120    |
| Pumsb      | 49046  | 7117   |
| Connect    | 67557  | 130    |
| Accidents  | 340183 | 468    |

Figures from 3 to 7 present the execution time of two algorithms for mining MNAR based on MFIL and FCIL.

![Chess](image)

**Figure 3. Execution time of the two algorithms for Chess under different minSup values**
Figure 4. Execution time of the two algorithms for Mushroom under different minSup values

Figure 5. Execution time of the two algorithms for Pumsb under different minSup values

Figure 6. Execution time of the two algorithms for Connect under different minSup values
Figure 7. Execution time of the two algorithms for Accidents under different minSup values

Results from figures 3-7 show that the execution time of mining MNAR based on FCIL is faster than that of based on MFIL [22]. Especially, when the number of FI in MFIL is much larger than the number of FCI in FCIL. For example, consider Mushroom with minSup = 20%, the number of FI is 53583 while the number of FCI is only 1200.

If we only measure the time of generated rules (without considering the time of mining FCI and building lattice), the results for the five databases are shown from Figure 8 to Figure 12.

Figure 8. Execution time of the two algorithms for Chess under different minSup values (without mining FCI and building lattice)
Figure 9. Execution time of the two algorithms for Mushroom under different minSup values (without mining FCI and building lattice)

Figure 10. Execution time of the two algorithms for Pumsb under different minSup values (without mining FCI and building lattice)

Figure 11. Execution time of the two algorithms for Connect under different minSup values (without mining FCI and building lattice)
Results from Figure 8 to Figure 12 show that generating MNAR based on FCIL is always faster than that of based on MFIL in all databases if we do not consider the time of mining FCI and building lattice.

5. Conclusion and Future Work

This paper has proposed a method for mining minimal non-redundant association rules based on frequent closed itemsets lattice. Experimental results show that the proposed algorithm is more efficient than that of mining MNAR from a modification frequent itemsets lattice. Although building lattice consumes a bit of time for updating parent-child relationship between nodes and memory for the storage these relations, but the algorithm saves a lot of time in generating rules.

In future, we will study how to build FCIL faster. Besides, a method for mining efficient association rules will be discussed.

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