Estimation for Unit Root Testing*

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Abstract
We revisit estimation and computation of the Dickey Fuller (DF) and DF-type tests. Firstly, we show that the usual one step approach, based on the "DF autoregression", is likely to be subject to misspecification. Secondly, we clarify a neglected two step approach for estimation of the DF test. (In fact, we introduce a new two step DF autoregression.) This method is always correctly specified and efficient under the circumstances. However, it is either neglected or misused in unit root testing literature. The commonly employed hybrid of the (correct) two step method is shown to be inefficient, even asymptotically. Finally, we further improve/robustify the proposed two step method by employing the missing initial observations. Our finally proposed method is to be used in unit root testing, since it is a new DF autoregression that retains the missing observations.

Keywords: Linear regression; Autoregressive error; Deterministic component; Dickey Fuller autoregression; Two step autoregression; Unit root.

JEL classification: C12, C13, C15.

1 Introduction
Econometricians are often accused of trying to discover electricity by playing the radio. In the case of estimation for unit root testing, some econometricians seriously believe they have discovered electricity. That is they believe that their estimation methods are genuine, fully efficient, and that they do not rely on previous literature. Unfortunately, these beliefs are not true in general. It turns out, that existing estimation methods for unit root testing quite likely misuse...
existing estimation methods, or, even worse, efficient methods of the literature are neglected. In general less, than needed, attention is paid to estimation. Jansson and Nielsen (2012) and DeJong et al. (1992 a) clearly imply the main estimation problem for unit root testing. This is estimation of a linear, in the parameters, regression with autocorrelated error. Of course, this is a problem a suitable version of the Cochrane-Orcutt (CO) method, that allows for AR\(p\) (ignoring the first \(p\) (say) observations), can handle. Or in its place any similar, fully iterated Gauss-Newton (GN) (or any other) algorithm. However, due to the nature of employed regressors (purely deterministic), one round CO type methods are suitable for estimation and unit root inference. Dickey and Fuller (DF) (1979) are clearly aware of the estimation problem, and their autoregression solves this problem computationally very cheaply\(^1\). This is what we call the one step approach.\(^2\) However, the DF approach is liable to misspecification, and can be very easily used for con business! Especially, if the deterministic component is anything but a trend polynomial. In addition, there is no obvious way to amend the DF approach to incorporate the first \(p\) missing values.

In this paper, we clarify and recommend a neglected estimation method that calculates the DF and DF-type tests correctly. It is immune to misspecification and any potential "saucy" business. This is the two step method, we discuss and fully develop. It relies on work by Durbin (1970), and subsequent research by Breusch (1978), and Godfrey (1978 a, b) and (1988). No one has used this method for unit root testing before.\(^3\) The two step method has certain advantages. It guarantees correct specification of the deterministic component, and can provide inference about the structural parameters of the data generating process (DGP). In addition, by proving the proposed method, we expose an inefficient version of the method which is used in the literature. This method crudely calculates the "DF" test from the least squares (LS) residuals of the DGP. Such a resulting "DF" test is an inefficient variant of the original DF test. The DF test exhibits particularly low power, and size distortion that can be large in certain cases, see Schwert (1989), Agiakloglou and Newbold (1992), De-Jong et al. (1992 b). More recent studies on the DF test include amongst others Leybourne et al. (1998), Leybourne and Newbold (1999, 2000), and Harvey et al. (2009). In view of these findings, it is of importance to amend the proposed two step estimation method in line with findings in Belsley (1996). Our proposed amended two step method, uses zero padded lags, so that the first \(p\) rows of the DGP are implicitly used. This new approach increases estimation efficiency and power, and robustifies the resulting efficient DF test with respect to a neglected break. In addition, the amended procedure alleviates the size problems of the original DF test. This new DF test is to be called the efficient DF test.

\(^1\)See also Nelson and Plosser (1982), Dickey and Said (1981), Said and Dickey (1984, 1985), and Fuller (1996).

\(^2\)In effect, this method owes to Durbin (1960).

\(^3\)Or when used, an inefficient version of the estimation method is employed.
The paper is organised as follows: Section 2 discusses potential pitfalls arising from estimation using the one step DF autoregression, while Section 3 shows an alternative two step DF autoregression, which is corrected to retain the first observations. Finally, Section 4 concludes.

2 Pitfalls of the One Step DF Autoregression

An observed time series \( y_t \) is generated via a deterministic component \((x_t)\) and a stochastic process \((z_t)\)

\[
y_t = \gamma' x_t + z_t, \quad t = 1, ..., T. \tag{1}
\]

The model is linear in the parameters. In addition

\[
z_t = \alpha z_{t-1} + u_t, \tag{2}
\]

with

\[
u_t = \xi(L)\varepsilon_t, \quad \xi(L) = \sum_{i=0}^{\infty}\xi_i L^i, \quad \xi_0 = 1, \quad \sum_{i=0}^{\infty}|\xi_i| < \infty, \quad \xi(1) \neq 0. \tag{3}
\]

\(z_0\) is either an unknown constant or stochastically bounded, \(O_p(1)\), and \(\varepsilon_t\) is a martingale difference sequence with \(E\varepsilon_t^2 = \sigma^2\) and \(\sup_t E\varepsilon_t^4 < \infty\), see Stock (1991). The inverse of \(\xi(L)\), say \(b(L)\),

\[
b_k(L)u_t = \varepsilon_{kt}, \quad b_k(L) = 1 - b_1 L - ... - b_k L^k,
\]

with some decay in \(k\), say \(T^{-1/3}k \to 0\) as both \(T\) and \(k\) increase. Chan and Park (2002) discuss alternative truncation orders. From Eq. (2) and (4), the long AR\((p)\) \((p = k + 1)\) representation for \(z_t\) is

\[
z_t = \sum_{j=1}^{p}\rho_j z_{t-j} + \varepsilon_{kt}, \quad t = p + 1, ..., T, \tag{5}
\]

where

\[
\rho(L) = 1 - \rho_1 L - ... - \rho_k L^k - \rho_{k+1} L^{k+1} = (1 - \alpha L)b(L). \tag{6}
\]

Note that \(\rho(1) = 0\), if and only if \(\alpha = 1\).

The commonly used one step approach to calculate the DF or DF-type tests

\[\text{approximately, } \varepsilon_t \text{ and } \varepsilon_{kt}, \text{ have similar properties.}\]

\[\text{See Fuller (1996) and Dickey and Fuller (1979, 1981), Nelson and Plosser (1982), Dickey and Said (1981), and Said and Dickey (1984, 1985).}\]
combines Eq. (1) and (5), and employs the so-called DF transformation. It owes to Durbin (1960), and derived by lagging Eq. (1), \( j = 1, \ldots, p \) times, multiplying each resulting equation by \( \rho_j \), and subtracting each outcome from Eq. (1). The analysis below in effect extends the work of DeJong et al. (1992 a). That is, one employs

\[
y_t = \sum_{j=1}^{p} \rho_j y_{t-j} + \gamma' x_t - \rho_1 \gamma' x_{t-1} - \ldots - \rho_p \gamma' x_{t-p} + \varepsilon_t, \quad t = p+1, \ldots, T. \tag{7}
\]

Eq. (7) is always correctly specified, and for feasibility the term \( \gamma' x_t - \rho_1 \gamma' x_{t-1} - \ldots - \rho_p \gamma' x_{t-p} \) must be correctly expanded. This may not be an easy task in general. For \( x_t = \{1, t, \ldots, t^r\}' \) (full \( r \)-th order polynomial trend, with no power missing), with \( \gamma = \{\gamma_0, \gamma_1, \ldots, \gamma_r\}' \) being the vector of associated parameters, we obtain

\[
y_t = \sum_{j=1}^{p} \rho_j y_{t-j} + \mu' x_t + \varepsilon_t = \rho y_{t-1} + \sum_{j=1}^{k} \beta_j \Delta y_{t-j} + \mu' x_t + \varepsilon_t, \quad t = p+1, \ldots, T. \tag{8}
\]

The elements of \( \mu = \{\mu_0, \mu_1, \ldots, \mu_r\}' \) are complicated functions of the elements of \( \gamma = \{\gamma_0, \gamma_1, \ldots, \gamma_r\}' \), \( \alpha \) and \( b_1, \ldots, b_k \), and

\[
\rho = \sum_{j=1}^{k+1} \rho_j, \quad \beta_j = -(\rho_{j+1} + \ldots + \rho_{k+1}), \quad j = 1, \ldots, k, \quad p = k + 1. \tag{9}
\]

In fact only for a full trend polynomial, \( \gamma' x_t - \rho_1 \gamma' x_{t-1} - \ldots - \rho_p \gamma' x_{t-p} \) reduces to \( \mu' x_t \) after manipulation. This invariance does not hold for any other set of regressors in \( x_t \), or if some time powers are missing from \( x_t \). However, researchers may incorrectly assume the invariance is correct for all variables. They may falsely employ it, creating what we shall call ”spurious efficiencies”, by throwing important variables out of the DF autoregression. To make it clear, Eq. (8) is only valid for a trend polynomial. Eq. (7) is correctly specified, but it may be cumbersome to make it feasible. To give an example of potential misspecification, note that Eq. (7) dictates that lags of the break dummy variables must be included in the DF autoregression. Perron (1989) ignores this fact, although Kim and Perron (2009) correct this mistake. In addition, the DF approach loses the first \( p \) observations of the autoregression, and there is no obvious way to recover the information they contain.

### 3 Two Step and Extended Two Step DF Autoregression

Hopefully, there is an alternative, although neglected, approach to calculate DF and DF-type tests. Note that although the method is known, it has never been employed for unit root testing. We term it the two step approach. It relies on

\[\text{Only an inefficient variant of the method is sometimes used in the literature.}\]
work by Durbin (1970), Breusch (1978), and Godfrey (1978 a, b) and (1988), see also Davidson and MacKinnon (1993). To this end, from Eq. (1) and (5), one obtains the infeasible regression

\[ y_t = \gamma'x_t + \sum_{j=1}^{p} \rho_j z_{t-j} + \varepsilon_{kt}, \quad t = p + 1, ..., T. \] (10)

For feasibility, we apply LS to Eq. (1) to obtain estimator \( \hat{\gamma} \) for \( \gamma \), and (current and lagged) residual \( \hat{z}_{t-j} = y_{t-j} - \hat{\gamma}'x_{t-j} \) or \( z_{t-j} = (\hat{\gamma} - \gamma)'x_{t-j} + \hat{z}_{t-j} \) for \( j = 0, 1, ..., p \). Secondly, substituting these relationships into Eq. (10), one obtains the feasible form

\[ y_t = \gamma'x_t + \sum_{j=1}^{p} \rho_j \hat{z}_{t-j} + \varepsilon_{kt}, \quad t = p + 1, ..., T. \] (11)

\[ \varepsilon_{kt} = \sum_{j=1}^{p} \rho_j (\hat{\gamma} - \gamma)'x_{t-j} + \varepsilon_{kt}, \quad t = p + 1, ..., T. \]

Eq. (11) is always correctly specified, and delivers the DF test. When the feasible version of Eq. (7) is correctly specified, Eq. (11) and (7) give identical values for the DF test. This is so because both methods provide alternative reduced form specifications of the same structural model of Eq. (1) and (5). However, only Eq. (11) is always correctly specified, and this is the reason we recommend it. An additional feature of Eq. (11) is that it transforms to

\[ \hat{z}_t = (\gamma - \hat{\gamma})'x_t + \sum_{j=1}^{p} \rho_j \hat{z}_{t-j} + \varepsilon_{kt}^* = \]

\[ (\gamma - \hat{\gamma})'x_t + \rho \hat{z}_{t-1} + \sum_{j=1}^{k} \beta_j \Delta \hat{z}_{t-j} + \varepsilon_{kt}^*, \] (12)

after subtracting \( \hat{\gamma}'x_t \) from both of its sides. Eq. (12)/(11) is the correctly specified residual based autoregression to calculate the DF test. Sometimes, an inefficient variant of the "DF" test is calculated from an autoregression similar to Eq. (12) but with \( x_t \) omitted. It is easy to show inefficiency. The employed misspecified autoregression is

\[ \hat{z}_t = \sum_{j=1}^{p} \rho_j \hat{z}_{t-j} + \varepsilon_{kt}^{*'} = \rho \hat{z}_{t-1} + \sum_{j=1}^{k} \beta_j \Delta \hat{z}_{t-j} + \varepsilon_{kt}^{*'}, \varepsilon_{kt}^{*'} = (\gamma - \hat{\gamma})'x_t + \varepsilon_{kt}^*. \] (13)

It is apparent that

\[ \text{var}(\varepsilon_{kt}^{*'}) > \text{var}(\varepsilon_{kt}^*). \] (14)

This is true because \( \gamma - \hat{\gamma} \) is stochastic. Hence, omitting \( x_t \) from Eq. (12) increases the error variance of the resulting regression, and this misspecification
induces inefficiency in finite samples. Note that this inefficiency does not vanish asymptotically, if \( z_t \) has a unit root and \( x_t \) contains an intercept. It is known (see Durlauf and Phillips (1988)) that, in this case, the intercept estimator in \( \hat{\gamma} \) is inconsistent. Although Eq. (11) and (12) are identical, Eq. (11) is to be preferred. This is because it identifies and estimates the structural parameters \( \gamma \) (better than \( \hat{\gamma} \)), since the LS estimator for \( \gamma \), say \( \tilde{\gamma} \), takes into account all serial correlation. We may denote \( \tilde{\rho} \) the corresponding estimator for \( \rho \). Eq. (11) definitely provides the correct DF test, avoiding misspecification. Nevertheless, it also ignores the first \( p \) observations, neglecting useful information.

Either for robustification, or further efficiency gains, or even at the cost of some minimal efficiency loss, we propose to augment Eq. (11) with the first \( p \) observations. This comes from employing zero padded, instead of truncated, lags in estimation. Belsley (1996) provides a Monte Carlo study that uses zero padded lags in serial correlation testing. We increase the sample in Eq. (11) (or (12)) from \( T - p \) to \( T \) by zero padding the lags. That is by using 0 in place of the missing values \( \hat{z}_0, \hat{z}_{-1}, ..., \hat{z}_{-p+1} \). Zero padding (and increasing the sample size) is expected to do very little efficiency harm. Nevertheless, it allows useful information about the initial observations to be utilised, via (roughly) employing the first \( p \) rows of Eq. (1). We thus recommend the following regression for calculating the new or efficient DF test:

\[
y_{t} = \gamma'x_{t} + \sum_{j=1}^{p} \rho_{j}\hat{z}_{t-j} + \varepsilon_{kt}^{*}
\]

\[= \gamma'x_{t} + \rho\hat{z}_{t-1} + \sum_{j=1}^{k} \beta_{j}\Delta\hat{z}_{t-j} + \varepsilon_{kt}^{*}, \quad \hat{z}_{0}, \hat{z}_{-1}, ..., \hat{z}_{-p+1} = 0, t = 1, ..., T.
\]

We give an example to clarify our discussion above. Perron and Yabu (2009), among others, employ \( y_{t} = \gamma_{0} + \gamma_{1}t + z_{t} \) with \( z_{t} = \alpha z_{t-1} + u_{t} \) and \( u_{t} \) iid. The LS estimator for \( \alpha \), \( \hat{\alpha} \), is obtained from the autoregression \( \hat{z}_{t} = \alpha z_{t-1} + u_{t}' \), \( t = 2, ..., T \). (\( \hat{z}_{t} \) is the LS residual of the DGP.) This estimator is inefficient. The efficient estimator must be obtained from either \( y_{t} = \gamma_{0} + \gamma_{1}t + \alpha z_{t-1} + u_{t}' \) or equivalently \( \hat{z}_{t} = \gamma'_{0} + \gamma'_{1}t + \alpha \hat{z}_{t-1} + u_{t}'', \quad t = 2, ..., T \). (Note that \( \gamma'_{0} \) and \( \gamma'_{1} \), in the latter regression, do not estimate true \( \gamma_{0} \) and \( \gamma_{1} \), only the former regression does this. In fact, \( \gamma'_{0} = \gamma_{0} - \tilde{\gamma}_{0} \) and \( \gamma'_{1} = \gamma_{1} - \tilde{\gamma}_{1} \), where \( \tilde{\gamma}_{0} \) and \( \tilde{\gamma}_{1} \) are DGP LS estimators for \( \gamma_{0} \) and \( \gamma_{1} \), respectively.) (Also it is true that \( u_{t}' = \gamma'_{0} + \gamma'_{1}t + u_{t}'' \).) The resulting estimator for \( \alpha \), from both preceding regressions, denoted \( \hat{\alpha}_{TR} \), is the efficient estimator. (Both regressions give common standard error for \( \hat{\alpha}_{TR} \).)

Note that this estimator is identical to the DF estimator \( \hat{\alpha}_{DF} \) \( (\hat{\alpha}_{TR} \equiv \hat{\alpha}_{DF}) \) obtained from the DF autoregression \( y_{t} = \mu_{0} + \mu_{t}t + \alpha y_{t-1} + u_{t}, \quad t = 2, ..., T \). The same is true for its standard error. That is the DF estimator is efficient. Since all

7This is well known and avoided in the literature of serial correlation testing (the Breusch-Godfrey test). Unfortunately, unit root testing literature ignores this and employs inefficient estimators.
discussed estimators are derived from autoregressions that lose the first observation, we propose the zero padded estimator \( \hat{\alpha}_{ZP} \) for \( \alpha \). Setting \( \hat{z}_0 = 0 \), either \( y_t = \gamma_0 + \gamma_1 t + \alpha \hat{z}_{t-1} + u''_t \) or equivalently \( \hat{z}_t = \gamma'_0 + \gamma'_1 t + \alpha \hat{z}_{t-1} + u''_t \), \( t = 1, \ldots, T \), delivers \( \hat{\alpha}_{ZP} \). This way the first observation is retained via \( y_1 = \gamma_0 + \gamma_1 + u''_1 \) and robustifies inference. (This approach cannot be applied to the DF autoregression.) Note that \( \hat{\alpha}_{ZP} \) and \( \hat{\alpha}_{DF}/\hat{\alpha}_{TR} \) have very close efficiency properties. However, only \( \hat{\alpha}_{ZP} \) employs the first observation, and is recommended.

Eq. (11), or (12), definitely improves efficiency over Eq. (13). However, Eq. (15) must be recommended, even if it is slightly less efficient than Eq. (11) or (12). This is so because of its utilisation of the initial observations. There is no need to prove theoretically that zero padding increases efficiency. The recommendation is on grounds of utilisation of the first \( p \) observations. We denote Eq. (15) estimators for \( \gamma \) and \( \rho \) as \( \bar{\gamma} \) and \( \bar{\rho} \), respectively.

We focus on Eq. (15), which gives the efficient DF test, but this also applies to Eq. (11), which gives the correctly specified, original/usual DF test. The DF \( t \) test is the signed squared root of the Wald/F test for excluding \( \hat{z}_{t-1} \) from the regression

\[
y_t - \hat{z}_{t-1} = \gamma' x_t + (\rho - 1) \hat{z}_{t-1} + \sum_{j=1}^{k} \beta_j \Delta \hat{z}_{t-j} + \varepsilon_{kt}^{**}, \tag{16}
\]

\( \hat{z}_0, \hat{z}_1, \ldots, \hat{z}_{p+1} = 0, t = 1, \ldots, T \).

All tests: Wald/F, LR, and LM have the same size controlled finite sample, size adjusted power. However, they may give different decision, when applied to an empirical time series. It is of importance to construct (t versions) of the LR and LM versions of the unit root test. This is so because the number of excluded variables is one. We denote the DF \( t \) test of Eq. (15/16) as the \( t_{DF}^* \). Since the LR test has properties close to the Wald test, we only focus on the LM test. The new LM \( t \) test corrects for the sample size and the number of variables included in the autoregression. We denote the new LM \( t \) test as \( t_{LM}^* \). Similar notation, without the star, is assigned to the corresponding tests of Eq. (11). Let \( F \) denote the \( F \)-test for \( \rho - 1 = 0 \) in Eq. (15/16) (the \( F \) test for the exclusion of \( \hat{z}_{t-1} \)). The \( t_{DF}^* \) is also derived as

\[
t_{DF}^* = \text{sign}(\bar{\rho} - 1)F^{1/2} \tag{17}
\]

where \( \bar{\rho} \) is the LS estimator of \( \rho \). There is a clear, well known relationship between the Wald/F test, \( F \), and the LM test, denoted \( \chi \). It is well known that

\[
F = (T - m)\chi/(T - \chi), \tag{18}
\]

where \( m \) is the total number of regressors in Eq. (15/16). Solving for \( \chi \), results in

\[
\chi = \frac{TF}{(T - m) + F}. \tag{19}
\]
The corresponding $t$ version of the new LM test, $t^{*}_{LM}$, is

$$t^{*}_{LM} = \text{sign}(\bar{\rho} - 1)\chi^{1/2} = \text{sign}(\bar{\rho} - 1)\left\{(TF / \{(T - m) + F\})^{1/2} \right\}. \quad (20)$$

Note that as $T$ gets large, $t^{*}_{DF}$ and $t^{*}_{LM}$ become close. That is asymptotically, the two tests are going to be the same, but not in finite samples.

### 4 Conclusions

We have revisited estimation theory for (parametric) unit root testing. The original/usual DF test, obtained from the one step approach, is quite likely to be subject to potential misspecification. Especially, when a complicated trend function is to be employed. In addition, it cannot retain the first observations. An inefficiency criticism applies to rough calculation of the "DF" test from an autoregression of the LS residuals of the DGP (that does not include the original levels regressors). This criticism is shown to be true, after demonstrating and exemplifying the correct two step procedure to obtain the DF test that correctly utilises LS residuals. This neglected/new method is always correctly specified, it gives the original DF test, and provides inference about the structural parameters of the DGP. (It is an alternative for the original, one step DF approach.) Furthermore, we also propose to improvise this method by (roughly) employing the first $p$ rows of the DGP. This is done by zero padding, instead of truncating, lagged residuals. The sample size increase and utilisation of the first $p$ rows of the DGP robustify estimation and inference. No or very minimal efficiency harm is to be expected by zero padding. Or even if there is some minor efficiency harm, there is associated size robustness of resulting tests to compensate. This gives what we call the efficient/new DF or DF-type test, with associated new DF autoregression.

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