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Dynamic Damping-Based Terminal Sliding Mode Event-Triggered Fault-Tolerant Pre-Compensation Stochastic Control for Tracked ROV

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Abstract: Due to the unknown disturbance caused by the harsh environment in deep water, the stability of Underwater Tracked Remotely Operated Vehicle (UTROV) trajectory tracking control is affected; especially the resistance forces of random vibrations caused by non-differentiable random disturbance resistance, which has become one of the main problems in controller design. Considering engineering practice, a stochastic model and new dynamic damping-based terminal sliding mode event-triggered fault-tolerant controller were designed in this paper. Firstly, based on the random resistance pre-compensation theory for the first time, a stochastic model was designed for differential drive UTROV. Meanwhile, a new nonsingular terminal sliding mode and dynamic damping reaching law were designed to achieve global finite-time convergence and reduce chattering with better robust response speed. Furthermore, to deal with the wear and tear caused by actuator failure and fixed sampling rate transmission, a new dynamic event trigger mechanism was designed and the faults analyzed. On this basis, combined with the finite-time adaptive on-line estimation technology, it can not only better reduce the transmission frequency, but also the finite-time dynamic active fault-tolerant compensation. The control scheme has semi-globally finite-time stability in probability and is proved by theory, which is compliant with engineering requirements. Then, according to characteristics of innovation, the three groups of simulation of control methods are designed to compare the methods in this paper. Finally the advantages of the method are verified by simulation to achieve the design expectations.

Keywords: fault-tolerant; terminal sliding mode; dynamic damping reaching law; dynamic event-trigger; fault coupling; actuator saturation; stochastic model

1. Introduction

At present, the heavy work-class Underwater Tracked Remotely Operated Vehicle (UTROV) can be remotely controlled by an umbilical cable of surface vessel to dive into deep water to perform engineering tasks. This makes difficult and complex deep-sea unmanned operations (such as deep-sea cable laying, dredging or maintenance surveys) easy to complete. Among them, high-quality independent completion of tracking mobile according to control instructions is a key link in engineering practice [1].

With the aim of efficiently completing tracking movements along a reference trajectory in deep-sea environments, many mature and advanced control methods, such as proportion integration differentiation [2], adaptive control [3] and nonlinear control [4], have been applied in underwater robot control. However, for the coupling resistance disturbance caused by ocean currents, uncertain dynamics and drag resistance of heavy umbilical cable [5,6] affect UTROV, leading to high robustness requirements for the control algorithm. So the SMC [7,8] with a simple structure and strong robustness is focused on. However, the chattering problem [9] of sliding mode control is the first problem to be solved—whether it can be applied in engineering practice. In recent years, some scholars focused on the
SMC reaching-phase; the dynamic Super-Twist Reaching Law (STRL)\cite{5,10,11} and the saturation replacement method\cite{12} were proposed. These methods reduce the reaching speed near the sliding mode surface (SMS) to drop the chattering and further, for the SMC sliding-phase, to increase the order of SMS to reduce chattering from low latitudes\cite{13}. Unfortunately, because the robustness is generated by the switching of the sliding-phase, these SMCs'\cite{5–13} robust performance and sensitivity were reduced. To make up for the robustness reduction after optimizing chattering, a model reference adaptive on-line estimator\cite{14} is used to estimate the external disturbance, thereby reducing the dependence on SMC robustness and indirectly improving the overall robust performance. In addition, the intelligent algorithms (such as Radial Basis Function Neural Network (RBFNN)\cite{15,16}, fuzzy algorithm\cite{17}, etc.) were used to improve the compensation accuracy of dynamic uncertainty. However, intelligent algorithms depend on weight selection\cite{15}, and complex parameter adjustment becomes a difficult problem for further applications.

What cannot be ignored is that the above are asymptotically stable, but the tracking control needs to keep up with the task trajectory within a specified time\cite{8} in practice. Therefore, the finite-time SMC\cite{18,19} and finite-time adaptive\cite{20} have been designed by researchers. Among them, Terminal Sliding Mode Surface (TSMS)\cite{21} has become the mainstream method for achieving finite-time UTROV convergence. However, the singularity of the TSMS and the loss of sensitivity of high-order TSMS\cite{22} have become the most difficult problems in practical applications.

In the actual underwater operation, the deterioration of water tightness and potential problems of equipment may cause actuator faults\cite{23}. In addition, dragging the heavy UTROV out of the water to repair is more labor-intensive, so a Fault-Tolerant Control (FTC)\cite{24} is needed to improve the ability of continuous control underwater operation under faults. Nowadays, using observers\cite{23} or adaptives\cite{24,25} to reconfigure and compensate for faults has become the mainstream of FTC. However, the above FTCs were not applicable to Partial Loss of Effect Fault (PLOEF)\cite{26}, which is strongly coupled with the actuator. It is worth noting that SMC and FTC depended on actuator performance, but the limit of the rated power of the UTROV servomotor actuator leads to input saturation\cite{27,28}. It will affect the control effectiveness.

In engineering, the remote control of frequent transmission at a fixed sampling rate will accelerate actuator wear, so the Event-Trigger Control (ETC)\cite{29,30} to reduce communication frequency has become one of the widely accepted control optimization schemes. Some scholars combine the trigger mechanism to couple the input fault, and have designed a new adaptive event-trigger FTC\cite{30}. However, the FTC performance and communication resource saving ability depend on the setting of ETC weights.

Although the above methods comprehensively consider the influence of external disturbance and umbilical cord cable resistance, they are all based on the assumption that the disturbance is smooth and derivable. Unfortunately, a part of the ocean current is in fact Brownian motion\cite{31}. The resistance for directly generated by the ocean current or indirect produced by umbilical cable to affect UTROV tracking control\cite{5}, a part of it shows irregularly and non-differentiable random characteristics\cite{31,32}. How to deal with the Resistance Forces of Random Vibrations (RFRV) is one of the problems that must be solved to achieve high-precision trajectory tracking. In other similar fields, engineers reconstruct the random model of manipulators in the presence of random vibration resistance\cite{33}, but the differential drive wheel characteristic of UTROV makes it impossible to use this generalized resultant force method to model directly.

Combined with the engineering practice and research status, a New Dynamic Damping-based Terminal Sliding Mode Event-triggered Fault-tolerant Control (NDDTSMFC) algorithm is proposed for UTROV trajectory tracking. The contributions of this paper are as follows:

1. For the purpose of solving the problem\cite{5,31,32} that the irregularly and non-differentiable RFRV affect UTROV control stability, according to the engineering essence that the resistance disturbance is finally compensated by the force generated by the driving
wheel under robust control, the driving pre-compensation mechanical analysis idea of RFRV is used creatively, and finally the UROV Stratonovich stochastic differential equation (SSDE) model with pre-compensation RFRV term is established. As a result, it overcomes the deficiency that the SSDE modeling method of single driving source in [33] can not be directly used in UTROV with differential drive wheel. It is more in line with engineering reality and facilitates stable control of UTROV.

2. A new Dynamic Damping Reaching Law (DDRL) and a new nonsingular TSMS are designed to achieve finite-time reach and finite-time convergence. It can overcomes the problem that two phase (Reaching-phase and sliding-phase) global finite-time stability [18,19] and singularity [21], and can improves robustness response. And DDRL can sensitively extract state information for on-line dynamic adjustment to better improve robustness and reduce chattering.

3. A dynamic ETC mechanism is designed to change the conditional weights according to the characteristics of the input information. By using the ETC condition and the coupling characteristic of the system saturation input under smooth saturation hyperbolic tangent function to decouple PLOEF. The new Adaptive Dynamic Event-Trigger Fault Coupling Analytical (ADEFCA) mechanism is designed in active fault-tolerant compensator. To be end, the PLOEF, distraction and RFRV are compensated by finally the finite-time RBF adaptive on-line active fault-tolerant compensator. The problem [29,30] that static weights affect the accuracy of fault tolerance is solved.

In this paper, The work-class UTROV designed by the fund is taken as the research object. The Section 2 is the model optimization and problem formulation, the establishment of UROV SSDE model, and related assumptions and lemmas. The Section 3 is the theory design of the new TSMS, DDRL and event-trigger mechanism. The NDDTSMEFC is designed and proof. The Section 4 is the simulation comparison and analysis. The Section 5 is the discussion of the research results.

2. Model Optimization and Preliminaries

2.1. Engineering Problem Formulation and Model Optimization

Due to the complexity of underwater environment, a mathematical model that considers more actual engineering factors is more beneficial to improve the model reference control accuracy and application value in engineering. In the previous UTROV trajectory tracking controller design, the kinematic deconstruction of the trajectory based on the global generalized coordinate system is mostly performed, and a virtual kinematic model [34] (Equation (1)) is established. The UTROV plane geometry is shown in Figure 1 (I) and motion 3D schematic show as Figure 2 (I).

\[
\dot{q}(t) = s(q(t))u(t) = \begin{bmatrix}
\cos \theta(t) & \sin \theta(t) \\
\sin \theta(t) & -\cos \theta(t) \\
0 & 1
\end{bmatrix} \begin{bmatrix}
v(t) \\
w(t)
\end{bmatrix},
\]

where \(\dot{q}(t) = [\dot{x}(t), \dot{y}(t), \dot{\theta}(t)]\) is a UTROV position. The \(w(t)\) is the ROV angular velocity. The \(\dot{v}(t)\) is the ROV linear velocity.

The Figures 1 (II) and 2 (I)–(II) show that when UTROV moving in an underwater environment, it will be affected by underwater ocean current, umbilical cables, other hydrodynamics and its own negative disturbance forces, etc. When using only a conventional position outer loop controller by the kinematic model, these dynamic factors will affect the UTROV can not achieve normal velocity control, resulting in deviation from the trajectory can not complete the underwater operation. Therefore, the following will be analyzed from the point of view of UTROV dynamics that all kinds of interference resistance affect the control accuracy of acceleration. The Lagrange Ordinary Differential Equation (ODE) dynamics model [34] with time-varying underwater disturbance term is established on Equation (2).

\[
M(q(t))\ddot{q}(t) + C(q(t),\dot{q}(t))\dot{q}(t) + F(\dot{q}(t)) + G(q(t)) + \tau_d(t) = B(q(t))\tau(t),
\]
where $C(q(t), \dot{q}(t)) \in \mathbb{R}^{3 \times 3}$ is the Coriolis force term. The $F(\dot{q}) \in \mathbb{R}^{3 \times 1}$ is the dynamic uncertainty such as friction resistance, fluid uncertainty resistance and mechanical damping force, it has nonlinear characteristics. The $\tau_d \in \mathbb{R}^{3 \times 1}$ is an external bounded and differentiable unmatched disturbance. The $B(q) \in \mathbb{R}^{2 \times 2}$ is the transformation term, and $S^T(q)A(q) = 0. M(q) \in \mathbb{R}^{3 \times 3}$ is a inertia term, it has $M(q) = M^T(q)$. The $J$ is UTROV moment of inertia. $\tau(t) = [\tau_l(t), \tau_r(t)]$ is torque vector of left and right drive wheels. The UTROV two-dimensional motion nature in the process of trajectory tracking cable laying maintenance, so the gravity term $G(q)$ is ignored.

Figure 1. Schematic diagram of UTROV geometric kinematics and underwater environmental resistance force analysis.

Figure 2. (I) Schematic of UROV trajectory tracking in global generalized coordinate system. (II) force analysis of random resistance acts on umbilical cable.
Considering the conservation of energy and the engineering practice that the UTROV has a motor power rating limit [28], it leads to a physical saturation characteristic of the actuator. The actual saturation input model of this model is shown in Equation (3).

\[
\tau_s(t) = \text{sat}(\tau(t)) = \begin{cases} 
\tau & \tau_{\text{min}} \leq \tau \leq \tau_{\text{max}} \\
\text{sign}(\tau) \tau_{\text{max}} & \tau_{\text{max}} \leq \tau \end{cases},
\]

where \(\tau_s(t) \in \mathbb{R}^{2 \times 1}\) is the torque input. It has bound of the actuator \(\tau_{\text{max}}^+ \in \mathbb{R}^{2 \times 1}\), \(\tau_{\text{max}} > 0\), \(\tau_{\text{min}} \in \mathbb{R}^{2 \times 1}\), \(\tau_{\text{min}} < 0\) and \(\|\tau_{\text{min}}\| = \|\tau_{\text{max}}\| = \tau_M\).

The actuator faults of wear, equipment life loss and performance become common with long time full load drives or frequent remote regulation control commands. In particular, when working in deep sea, that dragging the UTROV to land for maintenance will delay work efficiency. Therefore, considering the fault situation, the actuator fault model is established as shown in Equation (4).

\[
\tau_f(t) = F_f \tau_s(t) + F_b,
\]

where \(F_f = \text{diag}(F_{f1}, F_{f2})\) is the PLOEF [26] state coefficient diagonal matrix of actuator. The \(\tau_f(t) \leq \tau_M\). The PLOEF denotes the actuator can not fully execute the control input instructions, that is \(0 < F_f < 1\), and there is a certain proportion of loss in each actually drive. The \(F_b = \text{diag}(F_{b1}, F_{b2})\) is bias failure of the deviation between the actual drive and the control command \(\|F_b\| > 0\).

Considering underwater GPS positioning is more difficult, the dynamics are converted into the form of velocity and acceleration representation. Using the nonholonomic constraint of eccentric centroid \(A(q) = [\sin \theta(t), -\cos \theta(t), -d][x, y, \dot{\theta}]^T = 0\), generalized coordinate transformation and simplification [9], it can be obtained

\[
\ddot{u}(t) = \ddot{\theta} - \bar{F}(q(t)) - \tau_d(t),
\]

where \(\bar{F}(q(t)) = M^{-1}S^T(q(t))\bar{F}(q(t))\) is modelled dynamic uncertainty terms consisting of unknown friction and centrifugal forces. The \(\ddot{\theta} = \begin{bmatrix} \dot{\theta}/mr, \dot{\theta}/mr, \dot{\theta}/mr \end{bmatrix}^T\), \(\tau_d = M^{-1}S^T(q)\tau_d\) and \(M^{-1} = (mr)^{-1}[J, 0; 0, m]^T\). The \(\tau_d\) is the total differentiable resistance of UTROV due to underwater environment disturbance, umbilical cable, and so on, in the direction of the left and right differential wheels.

**Remark 1.** Different from the terrestrial environment, a part of the ocean current in the marine environment shows Brownian motion (Wiener process) [31,35]. Therefore, from Figure 2 (II) can know that a part of the resistance forces by the ocean current that affects UTROV trajectory tracking is irregularly, non-differentiable and it shows Brownian motion, that is \(F_{\text{CR}}\). However, the model (Equation (2)) is defined in the Lipschitz compact set, and the disturbance \(\tau_d\) is assumed to be a time-dependent smooth differentiable function, that is, the disturbance \(\tau_d\) has the property of smooth and low frequency variation [4]. So the ODE model (Equation (2)) can not fully describe the underwater disturbance resistance of UTROV. However, not all of these perturbations that are not modeled in the ODE affect the trajectory tracking control. Since the controller is the focus of this paper, the RFRV only needs to be studied from the point of view of affecting the stability of UTROV underwater trajectory tracking control.

**Definition 1.** According to the Brownian motion characteristics of resistance, these kinds of irregularly and non-differentiable resistance forces of affecting UTROV motion can be regarded as excitation of the random vibration force to the UTROV [33] in the process of trajectory tracking, and the RFRV is the vibration force generated by the random excitation that affects the UTROV motion. Furthermore, because of the need for underwater operations such as submarine cable laying detection, the UTROV always operates within the design depth and has been in contact with the
seabed. Therefore, the positive direction of RFRV is the opposite direction of the linear velocity direction at t moment.

Next, the generalized random force theory is used to establish irregularly non-differentiable terms and a stochastic differential equation (SDE) model that is more in line with underwater operation.

**Step 1.** Generalized force analysis of resistance forces of random vibrations.

Based on the conservation of energy and Newton’s third law, using trajectory tracking plane motion essence in process of UTROV underwater working, this paper establishes the plane generalized random force decomposition coordinate system to study RFRV. From Figure 1 (I) and Figure 2 (II), According to plane generalized force decomposes theory [33], the RFRV $F_{CR}$ act on UTROV by the ocean current is decomposed by generalized force into $F_{Cx}$ and $F_{Cy}$ respectively, as follows:

$$
\begin{align*}
F_{Cx} &= \cos \theta(t) F_{CR} \\
F_{Cy} &= \sin \theta(t) F_{CR} .
\end{align*}
$$

As can be seen from Figure 2 (I), not only the UTROV will be affected by the ocean current, but the umbilical cable will also be shaken by the current and that umbilical cable of great mass [5], in the random shaking process, has a part of the force to drag UTROV.

**Remark 2.** According to particularity of marine environment of Remark 1 and the complexity of hydrodynamics of umbilical cable [5,6], the umbilical cable flexible irregular coupling force $F_{Ui}$ created by non-differentiable ocean current force acting on umbilical cable is variably irregular, non-differentiable and many force points (That is $i = 1, 2, \ldots, n$). However, the purpose of this paper is to design a trajectory tracking controller, it is only necessary to study the forces that affect the motion of the UTROV. Based on the plane generalized force decomposition method [36,37] of the total RFRV coupling force, the $F_{Ui}$ is planarly decomposed into $F_{Upi}$ and $F_{Uzi}$. Because the z-axis component force $F_{Uzi}$ does not affect the tracking motion of the UTROV, according to Remark 1 and Definition 1, it is not a part of the RFRV. So the plane force $F_{Upi}$ contains the RFRV that affects the UTROV. After the force acting on the umbilical cable is simplified on particle of umbilical cable [36], can obtain $F_{Upi} = F_{Ut} + F_{U}$. The RFRV that umbilical cable dragging UTROV along the opposite direction of track motion, it is denoted $F_{Ut} = (F_{Utx}^2 + F_{Uty}^2)^{0.5}$. The $F_{U} = (F_{Ux}^2 + F_{Uy}^2)^{0.5}$ is a plane force that only acts on the umbilical cable by Brownian motion ocean current and does not affect the UTROV motion, it does not produce elastic deformation force on UTROV. That is, $F_{U}$ is the force acting on the umbilical cable by the ocean current except the force $F_{Ut}$ of dragging UTROV in the Figure 2 (II).

Based on the theory of generalized relative motion and Definition 1, the total RFRV of affecting UTROV motion is denoted $F_{R}$ (See Figure 1 (II)) at geometric center C. Then the components of the X-axis and Y-axis of the total generalized RFRV suffered by the UTROV are denoted $F_x$ and $F_y$ respectively, and they have the following relationship:

$$
\begin{align*}
F_x &= F_{Cx} + F_{Utx} \\
F_y &= F_{Cy} + F_{Uty} .
\end{align*}
$$

What cannot be ignored is that a part of the $F_R$ is the resistance of the centripetal force, which is the force in the direction of vertical line velocity (See Figure 3). The $\theta_R(t)$ is angle of the generalized force, and it is not necessarily the position angle $\theta(t)$ of UTROV at t time. Therefore, the resultant force of the generalized force has the relationship as $F_R = (F_x^2 + F_y^2)^{0.5}$. 
Figure 3. (I) Schematic of Force Analysis of Pre-compensated RFRV of Left and Right Drive Wheel in Generalized Coordinate System. (II) Schematic diagram of servomotor actuator components.

Step 2. Drive pre-compensated RFRV term analyze and modeling.

Remark 3. It can be seen from Figure 2 (II) that the essence of trajectory tracking control is a process of remotely adjusting the system input torque generated by the servomotor actuator, it drives the differential drive wheel to complete the desired task. Thus, the anti-disturbance control is to counteract the disturbance resistance that affects the motion of the UTROV through the drive force. However, when UTROV turning or forward movement is affected by irregular RFRV, the compensation force (Driving acceleration velocity) generated by each driving wheel is different, so the total random force design theory of single drive source in Refs. [5,6] can not be directly used for design UTROV model. The conversion between the total generalized random resistance of a single drive source and the differential driving force of a double drive source requires more parameters. To reduce the complexity of engineering practice, the following will use reverse thinking, from the essence of drive wheels are controlled to counteract RFRV to design the generalized random force analysis method of driving pre-compensated RFRV. That is, the force that needs to be generated to resist the RFRV that affects the accuracy of UTROV trajectory tracking control. Therefore, according to the principle of interaction force and matching interference design idea, all the RFRV that need to be compensated by driving wheel are abstracted as the resistance $F_r$ and $F_l$ in Figure 3.

It can be known that there is a relationship as follows:

$$F_R = \sqrt{F_x^2 + F_y^2} = \sqrt{(F_{Rx} + F_{Lx})^2 + (F_{Ry} + F_{Ly})^2}$$

$$= \sqrt{(\cos \theta(t)F_r + \cos \theta(t)F_l)^2 + (\sin \theta(t)F_r + \sin \theta(t)F_l)^2},$$

(8)
where $F_{Rx}$ and $F_{Ry}$ is the generalized RFRV $F_r$ along the X-axis and Y-axis generalized coordinates that the right drive wheel needs to compensate. Similarly, it can be seen that $F_{Lx}$ and $F_{Ly}$ are also the X-axis and Y-axis component forces after the generalized coordinate decomposition of the $F_t$. To sum up, it is known that there is relationship $F_y = F_{Ry} + F_{Ly}$ and $F_x = F_{Rx} + F_{Lx}$, that is generalized random force after decomposition by left and right drive wheels.

**Remark 4.** Because UTROV do not has the turning wheels, the driving direction at $t$ time is parallel to the linear velocity direction at $t$ time. The driving torque of pre-compensated RFRV of the left driving wheel $\tau_{fl}(t) = F_l(t) \cdot r$ and right driving wheel $\tau_{fr}(t) = F_r(t) \cdot r$. According to the principle of interaction force, the $\tau_{fl}(t) + \tau_{fr}(t) + \tau_{dl}(t) = \tau_r(t) = F_r(t) \cdot r$, where $\tau_{dl}(t)$ and $\tau_r(t)$ are the moments that drive UTROV motion except for compensating resistance. Actually, the turning force is derived from the centripetal force by the difference between $F_{fl}(t)$ and $F_{fr}(t)$ forces. Therefore, he actual resultant force of the force perpendicular to the linear velocity direction is 0, that is $F_{lx}(t) + F_{rx}(t) - F_{ly}(t) - F_{ry}(t) = 0$.

Finally, the generalized random force relationship of differential drive pre-compensation RFRV which affects the stability of UTROV control is as follows:

$$
\begin{align*}
F_l &= \cos \theta F_{lx} + \sin \theta F_{ly} \\
F_r &= \cos \theta F_{rx} + \sin \theta F_{ry}.
\end{align*}
$$

(9)

According to Remarks 3 and 4, Equation (9) and Definition 1, the drive pre-compensated RFRV term in SDE model is

$$
F_u = \Lambda_L(\theta(t))\Xi_L + \Lambda_R(\theta(t))\Xi_R
\begin{bmatrix}
\cos(\theta(t)) & \sin(\theta(t)) \\
0 & 0
\end{bmatrix}
\begin{bmatrix}
F_{lx} \\
F_{ly}
\end{bmatrix}
+ \begin{bmatrix}
0 & \cos(\theta(t)) & \sin(\theta(t))
\end{bmatrix}
\begin{bmatrix}
F_{Rx} \\
F_{ Ry}
\end{bmatrix},
$$

(10)

where the $\Xi_L$ and $\Xi_R$ are generalized RFRV matrix with mean value of 0 generated by Brownian motion of ocean current after decomposition by left and right drive wheels.

**Step 3.** The UTROV SSDE model establishment.

In order to make SDE dynamic system have similar properties to general differential equation system, it is convenient for engineering understanding and application, the SSDE system is modelled below. To begin with, based on Steps 1 and 2, the SDE model with random force is

$$
du(t) = \left[\bar{B}\tau_f(t) - \bar{F}(\dot{q}(t)) - \tau_d\right]dt + \bar{B}[\Lambda_L(\theta(t))\Xi_L + \Lambda_R(\theta(t))\Xi_R]dt.
$$

(11)

Based on the theory of random excitation force, the zero mean white noise $G_L(t) \in \mathbb{R}^{2 \times 1}$ and $G_R(t) \in \mathbb{R}^{2 \times 1}$ are introduced as random force excitation, can get

$$
du(t) = \left[\bar{B}\tau_f(t) - \bar{F}(\dot{q}(t)) - \tau_d\right]dt + \bar{B}[\Lambda_L(\theta(t))G_L]dt + \left[\bar{B}\Lambda_R(\theta(t))G_R\right]dt,
$$

(12)

where the correlation function of white noise $G_L(t)$ and $G_R(t)$ are $E[G_L(t)G_L(t + T_{gL})] = \Lambda_L[\delta_l(T_{gL})]$ and $E[G_R(t)G_R(t + T_{gR})] = \Lambda_R[\delta_R(T_{gR})]$ respectively. The $T_{gL}$ and $T_{gR}$ are occurrence time of white noise.

Here, the Brownian motion (Wiener process) term $B_{AL}$ and $B_{AR}$ after driving pre-compensation decomposition is introduced. According to Langevin equation, the $\frac{dB_{AL}}{dt}$ and $\frac{dB_{AR}}{dt}$ are used instead of $G_L$ and $G_R$ respectively, can get

$$
du(t) = \left[\bar{B}\tau_f(t) - \bar{F}(\dot{q}(t)) - \tau_d\right]dt + \bar{B}\sqrt{2\pi\Lambda_L(\theta(t))} \circ dB_{AL} + \bar{B}\sqrt{2\pi\Lambda_R(\theta(t))} \circ dB_{AR},
$$

(13)
where $2\pi\Delta_R$ and $2\pi\Delta_L$ are intensity of white noise. The $B_L$ and $B_R$ are standard Brown process (Wiener process), and $B_L$ and $B_R$ are defined on complete probability space $\{\Omega, \mathcal{F}, P\}$, where $\Omega$ as sample space, $\mathcal{F}$ is denoted $\sigma$-field and $P$ denotes the probability measure, can obtain $\sqrt{2\pi}\Delta dB_{AL} = dB_L$ and $\sqrt{2\pi}\Delta dB_{AR} = dB_R$. The $E[dB_RdB_R^T] = \Theta_R^T\Theta_R$ and $E[dB_RdB_L^T] = \Theta_L^T\Theta_L$, where $\Theta_R$ and $\Theta_L$ are constant values.

According to the method [33] of transforming SDE into SSDE, the Equation (13) is rewritten as:

$$
\frac{du(t)}{dt} = [\hat{B}\tau_f(t) - \hat{F}(\hat{q}(t)) - \hat{\tau}_d]dt + \hat{\Lambda}_L(q,u)dB_L + \hat{\Lambda}_R(q,u)dB_R,
$$

(14)

where $\hat{\Lambda}_L(q, u) = B\Lambda_L(\theta(t))$ and $\hat{\Lambda}_R(q, u) = B\Lambda_R(\theta(t))$.

Considering the effect of parametric excitation on the stochastic equation, the Itô stochastic differential equation (See Equation (15)) excited by white noise.

$$
\frac{du(t)}{dt} = [\hat{B}\tau_f(t) - \hat{F}(\hat{q}(t)) - \hat{\tau}_d + \frac{1}{2}\hat{\Lambda}_L(q,u)\frac{\partial\hat{\Lambda}_L(q,u)}{\partial q}dt + \hat{\Lambda}_L(q,u)dB_L + \hat{\Lambda}_R(q,u)dB_R,
$$

(15)

where $\frac{1}{2}\hat{\Lambda}_R(q, u)\frac{\partial\hat{\Lambda}_R(q,u)}{\partial u}$ and $\frac{1}{2}\hat{\Lambda}_L(q, u)\frac{\partial\hat{\Lambda}_L(q,u)}{\partial u}$ are Wong–Zakai modified terms, and they are all equal to zero. Using the Wong–Zakai modified term approximation theory, the $\hat{\Lambda}_L(q, u)$ and $\hat{\Lambda}_R(q, u)$ do not depend on $u(t)$. Since the diffusion matrix of subsystem equals 0, it can be seen that the SDE and SSDE are equivalent to Equation (14). The existence and uniqueness of the solution of SSDE with this kind of correction term 0 are the same as those in Ref. [33].

The SSDE global model with random force is:

$$
\begin{align*}
\frac{dq(t)}{dt} &= S(q(t))u(t)dt \\
\frac{du(t)}{dt} &= [\hat{B}\tau_f(t) - \hat{F}(\hat{q}(t)) - \hat{\tau}_d]dt + \hat{\Lambda}_L(q,u)dB_L + \hat{\Lambda}_R(q,u)dB_R.
\end{align*}
$$

(16)

**Definition 2.** As can be seen from Figure 1, UTROV trajectory tracking is a process of remotely controlling UTROV to track the reference trajectory $q_*$ of engineering task requirements. The trajectory tracking controller is to make the error $q_e$ (See Equation (17)) to 0 in finite-time, so as to achieve stable tracking. The UTROV trajectory position state error is defined as $q_e = q_T - q$. In addition, the trajectory tracking controller needs to make the UTROV velocity reach the reference velocity to meet the requirements of the task, that is velocity state error $u_d - u = u_e \rightarrow 0$.

The UTROV position error denotes as:

$$
q_e = q_T - q = \begin{bmatrix}
x_e \\
y_e \\
\theta_e
\end{bmatrix} = \begin{bmatrix}
x_r - x \\
y_r - y \\
\theta_r - \theta
\end{bmatrix}
$$

(17)

The velocity state error is:

$$
u_e(t) = u_d(t) - u(t) = \begin{bmatrix}
v_d(t) - v(t) \\
w_d(t) - w(t)
\end{bmatrix} = \begin{bmatrix}
v_d(t) \\
w_d(t)
\end{bmatrix},
$$

(18)

where $u_d(t)$ is the reference velocity provided by virtual outer loop (Outer loop controller law).

### 2.2. Preliminaries

To facilitate the interpretation of Lemmas and Assumptions, simplified SSDE model (Equation (19)) as follows:

$$
\frac{du(t)}{dt} = u_d(u(t))dt + \Lambda_{pR}(u(t))dB_{pR} + \Lambda_{pL}(u(t))dB_{pL},
$$

(19)
where \( f_a(u(t)) = \tilde{B}\tau_f(t) - \tilde{F}(\dot{q}(t)) - \tilde{\tau}_d \), \( \Lambda_{pR}(u(t)) = \widehat{\Lambda}_R(\dot{q}, u) \) and \( \Lambda_p(u(t)) = \widehat{\Lambda}_{L}(\dot{q}, u) \) are nonlinear terms.

**Assumption 1.** The UROV system state is Borel measurable. The initial system state error \( u_e(0) \) and \( q_e(0) \) are defined on compact set. The desirably reference trajectory \( \dot{q}_r = [x_r, y_r, \theta_r]^T \) and reference positive scalar speed \( u_r = [v_r, w_r]^T \), their derivatives are smooth and bounded. The UROV also works only within the effective engineering range, so the UROV system state is assumed as \( \|u_e(t)\| \leq \bar{u}_e \) and \( \|q_e(t)\| \leq \bar{q}_e \), where \( \bar{u}_e \) and \( \bar{q}_e \) are normal values.

**Assumption 2.** Since the energy of real-world disturbance is limited. Thus, the unknown bounded low frequency time-varying disturbance \( \tau_d \) is \( \leq \bar{T}_d \), and energy losses from actuator faults is also bounded.

**Assumption 3.** For facilitating the matrix operation, all constant terms are in the form of diagonal matrix.

**Assumption 4.** Because the engineering practice that performs GPS underwater becomes worse with deeper water, the speed sensor is used as the output signal of the system that is transmitted through the umbilical cable, and the on-line soft analysis is carried out at the remote control terminal. Then the position information is obtained, from which the virtual controller of the position outer loop is constructed.

**Lemma 1** ([30]). For \( \chi \in \mathbb{R} \), when \( D_1 > 0 \) exists, the following relationship holds:

\[
- \chi \tanh(D_1\chi) \leq -\|\chi\| + \frac{0.2785}{D_1}.
\]

**Lemma 2** ([38]). Assuming the system is controllable, for the \( e_{\chi} \in \mathbb{R} \) and \( e_{\chi} \neq 0 \), the Lyapunov function \( V(e_{\chi}) \in \mathbb{C}^2 \) exists. The finite-time stability condition relationship holds:

\[
\dot{V}(e_{\chi}) + \kappa_1 V(e_{\chi}) + \kappa_2 V^o(e_{\chi}) - \Delta_M \leq 0,
\]

where \( \Delta_M > 0, \kappa_1 > 0, \kappa_2 > 0 \) and \( 0 < \kappa_1 < 1 \). The stable time depending on the initial state \( e_{\chi 0} \) is given \( T_V \leq \beta_2^{-1}(\beta_1 - \beta_1 o)^{-1}(\beta_2 \cdot V^{1-o}(e_{\chi 0}) + \beta_1) \).

**Lemma 3** ([39]). For any \( V(u(t)) \in \mathbb{C}^2 \) is positive definite function, it is related to the model (Equation (19)) the infinitesimal differential operator \( i \) denotes as:

\[
i\dot{V}(u(t)) = \frac{\partial V(u(t))}{\partial u(t)} f_a(u(t)) + \frac{1}{2} \text{Tr} \left\{ \Lambda_{pR}^T(u(t)) \frac{\partial^2 V(u(t))}{\partial (u(t))^2} \Lambda_{pR}(u(t)) \right\} + \frac{1}{2} \text{Tr} \left\{ \Lambda_{pL}^T(u(t)) \frac{\partial^2 V(u(t))}{\partial (u(t))^2} \Lambda_{pL}(u(t)) \right\}.
\]

**Lemma 4** ([15]). The RBFNN is cited because of the nonlinear characteristics of system uncertainty. If there exists \( m \)-dimensional compact set \( \Xi^m \subseteq \mathbb{R}^m \rightarrow \mathbb{R} \), and there is an unknown nonlinear function \( f(Q) \) with initial value 0 defined on \( \Xi^m \). Then RBFNN approximator is used to fit the dynamic values of \( f(Q) \).

\[
f(Q) = W^*^T Z(Q) + e_Z(Q), \quad \forall Q \in \Xi^m,
\]

where \( e_Z(Q) \) is the bounded RBF fitting error, which is defined on the compact set \( \Xi^m \), \( |e_Z(Q)| \leq \bar{e}_Z \), and \( \bar{e}_Z \) is the maximum nuclear distance. To improve the nonlinear local approximation ability, the Gaussian function \( Z(Q) = \exp((Q - \kappa)^T(Q - \kappa) / -l^2) \) is selected as the smooth kernel function. The \( \kappa \) is approaching the center column distance vector. The \( l \) is varying constant value. \( W^* \) is the order \( m \)-dimensional weight row vector of optimally fitted, as \( W^* = \ldots \).
Lemma 5 ([33]). For any scalars or vectors, there is an inequality relationship that is $h^T j \leq \frac{\mu_a}{\delta_a} ||h||^2 + \frac{1}{\delta_a \sigma_p} ||j||^2$, where $h \in \mathbb{R}^n$, $j \in \mathbb{R}^n$, $\delta_a > 1$, $\mu > 0$, and $\delta_b = \frac{\delta_a}{\delta_a - 1}$.

Lemma 6 ([39]). For $a_i (i = 1, 2, \ldots, n)$ and $l > 0$, it is known that the Cauchy-Schwarz theorem holds, its form as $(\sum_{i=1}^{n} |a_i|^l)^{\frac{1}{l}} \leq \sum_{i=1}^{n} |a_i|^l$.

Lemma 7 ([39]). Consider system (Equation (19)) and, based on Assumption 1, it is obvious that $f_a(u(t))$, $\Lambda_{pL}(u(t))$ and $\Lambda_{pR}(u(t))$ are locally bounded and locally Lipschitz continuous in $u(t) \in \Omega$. For any $u(0) \in \mathbb{R}^n$, the $C^2$ function $V(x) \in \mathbb{R}^n$ is a positive definite. There are $K_\infty$ class functions $\vartheta_a$ and $\vartheta_p$, for $u(t) \in \mathbb{R}^n$ and $t \geq 0$, based on Lemma 3, it holds:

$$
\vartheta_a(||u(t)||) \leq V(x) \leq \vartheta_p(||u(t)||) \quad \forall V(x) \leq -c_\Delta V^\sigma(x) + \Delta_c .
$$

As a result, the random stable time $T_{\Delta}$ satisfies $E(T_{\Delta}) \leq \frac{EV^{1-\sigma}(u(0)) - (\frac{\Delta_c}{c_\Delta(1-\sigma)}) L_{\max}}{c_\Delta(1-\sigma)}$, where positive real number $c_\Delta > 0$, $\Delta_c > 0$, $1 > \varsigma > 0$ and $0 < \sigma < 1$. The system is semi-globally finite-time stability in probability.

3. Controller Design and Proof

In this part, according to the engineering practice and Definition 1, the actual velocity inner loop and position virtual outer loop control strategy are used. The virtual outer loop controller is designed by finite-time theory to meet the engineering requirements of trajectory tracking, and it is designed as the velocity inner loop reference velocity.

A TSMS is designed to ensure the time requirement of improving the velocity inner loop control, which can reach the sliding surface in a finite-time in Section 3.1. Based on this, to compensate for various random disturbances and time-varying actuator faults, a new DDRL and a new ADEFCA are designed to better control the input torque of the differential wheels. Meanwhile, the coupling characteristic of event-trigger weight is used to decouple unknown time-varying actuator faults in Section 3.2. Then, a neural network adaptive on-line estimator is designed to compensate fault, resistance and disturbance in Section 3.3. To the end, the NDDTSMEFC is designed and proof in Section 3.3.

3.1. Terminal Sliding Mode Surface and New Dynamic Damping Reaching Law

3.1.1. The New Terminal Sliding Mode Surface

According to Definition 1, the TSMS is designed to:

$$
S_a(t) = a_1 u_e(t) + a_2 \int \| u_e(\partial) \|^{\frac{1}{2}} \text{sign}(u_e(\partial)) d\partial + a_3 \| u_e(t) \|^{\frac{9}{2}} \text{sign}(u_e(t)) ,
$$

where $a_1 = \text{diag}\{a_{11}, a_{12}\} > 0$, $a_2 = \text{diag}\{a_{21}, a_{22}\} > 0$ and $a_3 = \text{diag}\{a_{31}, a_{32}\} > 0$ are constant diagonal matrices.

Remark 5. Compared with the traditional integral sliding surface $S_i(t) = a_{i1} u_e(t) + a_{i2} \int u_e(\partial)d\partial$, the TSMS Equation (25) can ensure that the system state can converge in finite time after the system state is controlled to the sliding surface (that is, system state in the sliding stage of sliding mode control). Moreover, the TSMS will not be troubled by the singularity in Ref. [21]. Further, through Equation (25), it is known that the sliding surface design accords with the sliding mode design criteria.
Thus, TSMS (Equation (25)) proved finite-time convergence.

The derivative of Equation (25) is:

\[
dS_u(t) = \left[ a_1 + \frac{9a_3}{4} \|u_e(t)\|^{\frac{3}{2}} \text{sign}(u_e(t))du_e(t) + a_2 \|u_e(t)\|^{\frac{1}{2}} \text{sign}(u_e(t)) \right] dt. \tag{27}\]

It can be known from Equation (28) that \(\dot{S}_u(t)\) has no singularity. When \(\frac{dS_u(t)}{dt} = 0\), Equation (28) can be transformed into:

\[
\frac{a_1 + \frac{9a_3}{4} \|u_e(t)\|^{\frac{3}{2}} \text{sign}(u_e(t))}{a_2 \|u_e(t)\|^{\frac{1}{2}} \text{sign}(u_e(t))} \dot{u}_e(t) = -dt. \tag{28}\]

Integrating both sides to Equation (29), we can obtain:

\[
a_2^{-1} \int_{u_e(0)}^{u_e(T_u)} \frac{a_1 + \frac{9a_3}{4} \|u_e(t)\|^{\frac{3}{2}} \text{sign}(u_e(t))}{a_2 \|u_e(t)\|^{\frac{1}{2}} \text{sign}(u_e(t))} dt = \int_{u_e(0)}^{u_e(T_u)} a_2 \|u_e(t)\|^{\frac{1}{4}} \text{sign}(u_e(t)) dt
\]

Based on Assumptions 1 and 2 and integral valuation theorem, we know that \(-m_{ue}(T_u - 0) \geq -\int_{u_e(0)}^{u_e(T_u)} \|u_e(t)\|^{\frac{3}{2}} dt \geq -M_{ue}(T_u - 0)\) holds. The \(m_{ue}\) and \(M_{ue}\) are the minimum and maximum values of \(\frac{9a_3}{4} \|u_e(t)\|^{\frac{3}{2}}\), respectively. By substituting \(m_{ue}(T_u - 0)\) into Equation (30), we can obtain:

\[
-T_u \geq a_2^{-1} \int_{u_e(0)}^{u_e(T_u)} a_1 \|u_e(t)\|^{\frac{3}{4}} dt \geq a_2^{-1} a_1 \|u_e(0)\|^{\frac{3}{2}}. \tag{30}\]

In the end, because the \(u_e(0) \neq 0\), the finite-time available is \(T_u \leq a_2^{-1} a_1 \|u_e(0)\|^{\frac{3}{2}}\). Thus, TSMS (Equation (25)) proved finite-time convergence. \(\square\)

3.1.2. The New Dynamic Damping Reaching Law

To realize the global finite-time convergence of SMC and optimize the chattering and robust response ability. According to the characteristics of SMS error information, the online damper \(D(S_u(t))\) is designed on Equation (31), and a new DDRL \(\dot{S}_D(t)\) is obtained as shown in Equation (32) based on Equation (25). When \(u_e(t) \to 0, \dot{S}_D(t) \to 0\) denotes the time-varying existence of DDRL.

\[
D(S_u(t)) = \tanh \left( \left( 2 + S_u^2(t) \right) \ln \left( 2 + S_u^2(t) \right) \right) \ln \left( 2 + S_u^2(t) \right) - 2 \ln(2). \tag{31}\]

\[
d\dot{S}_D(t) = [-\varepsilon_u \text{sign}(S_u(t))D(S_u(t))] dt. \tag{32}\]

**Remark 6.** From Figure 4, DDRL shows that the closer to the SMS (that is, the small system error), the smaller the reaching law value (that is, small sliding mode reaching speed), and the farther away from the SMS (that is, the larger system error), the greater the reaching law value (that is, the large sliding mode reaching speed). From the point 0.2 selected near 0, it can be seen that the reaching speed of DDRL and STRL decreases gradually, which avoids the sudden decrease of ERL and effectively reduces the chattering caused by the speed exceeding the sliding mode surface. In addition, when systems have a large error, the greater the gain of DDRL than STRL and ERL.
The DDRL can effectively shorten the approach time. From Figure 4 and Equation (31), \(D(S_u(t))\) is changed according to the size of the SMS value \(S_u(t)\) to avoid the problem of gain variation difference between STRL and ERL. Further, the DDRL has the characteristic of making the system approach the sliding surface in a finite-time.

\[V_u(t) = \frac{1}{2} S_u^2(t) \ln(2 + S_u^2(t)) - 2 \ln(2) - \left(2 + S_u^2(t)\right) \ln(2 + S_u^2(t)).\]  

Using the Lemma 1 logarithmic inequality and Young’s inequality, can know:

\[-D(S_u(t)) \leq -\left[2 + S_u^2(t)\right] \ln(2 + S_u^2(t)) - 2 \ln(2) - \left(2 + S_u^2(t)\right) \ln(2 + S_u^2(t)).\]  

Figure 4. Schematic diagram of different reaching laws.

**Proof of finite-time reaching SMS of DDRL.** The Quartic Lyapunov function \(V_S(t) = (S_u^2(t)S_u(t))^2\) is established. After differentiating and replacing \(S_u(t)\) with DDRL \(S_D(t)\) (Equation (32)), can get:

\[dV_S(t) = [S_u^2(t)S_u(t)S_u^T(t)S_u(t)]dS_D(t)dt,\]

\[= [-\varepsilon u S_u^2(t)S_u(t)S_u^T(t)\text{sign}[S_u(t)]D(S_u(t))]dt.\]  

(33) Using the Lemma 1 logarithmic inequality and Young’s inequality, can know:

Equation (34) can be transformed into:

\[V_S(t) \leq -\varepsilon u S_u^2(t)S_u(t)S_u^T(t)[(0.75 + \|S_u(t)\|)] - (0.2785 - 2 \ln(2))|\text{sign}[S_u(t)]|\]

\[\leq -\varepsilon u V_S(t) - \varepsilon u^3 V_S^3(t),\]

(35) where \(\varepsilon u^3 = \varepsilon u(0.4715 + 2 \ln(2))\). In accordance with Lemma 2, the DDRL can achieve finite-time reaching to TSMS.

3.2. New Smooth Saturated Input Function and New Dynamic Event-Trigger Mechanism

The traditional control signal (system input) is transmitted through the umbilical cable according to the designed sampling frequency. However, from Remark 1 and engineering practice, the umbilical cable is shaken or even deformed, which will affect the quality of signal transmission. In addition, the continuous transmission of system input may cause
the weight trigger mechanism that can be automatically adjusted online, the event-trigger rule as shown in Equation (36).

\[
\tau_e (t) = \tau_e (t_k), \forall t \in [t_k, t_{k+1})
\]

\[
\tau_e (t_{k+1}) = \inf \{\tau_e (t), \|\xi_t (t)\| > D_e (t)\|\tau_e (t_k)\| + \ell_k\},
\]

where \(\xi_t (t) = \tau_e (t) - \tau_e (t_k)\) is the difference between the updated transmission input and the input calculated by the control terminal. The \(D_e (t) = 1 - \frac{1}{1 + \text{tanh}(a_k \|\xi_t (t)\|)}\) is an on-line dynamic ETC weight damper and \(\ell_k = D_e (t)\ell_u\) is ETC threshold reservation error, where positive parameters satisfy \(\ell_u > 0\) and \(a_k > 0\). The \(\tau_e (t)\) is control value continuously updated by the remote control terminal. The \(\tau_e (t_k)\) is the updated value at the trigger node \(t_k\), when the trigger threshold is not met. If the trigger threshold condition is still met in the subsequent time period \([t_k, t_{k+1}), \tau_e (t_k)\) is still as the system input. During this period, the remote control terminal will not transmit a new control signal to the UTROV until it is updated to \(t_{k+1}\) again after the condition (Equation (36)) is not met. Then, repeat the above steps.

**Remark 7.** Because of the physical limit of the input of UTROV (See Equation (3)), the threshold Equation (36) design also considers this situation. When the input is saturated, the absolute value of the input difference \(\xi_t (t)\) will not exceed the saturation value, which is very consistent with the engineering practice. Therefore, the on-line dynamic ETC weight damper \(D_e (t)\) satisfy the essence of weight value [29], that is \(0 \leq D_e (t) < 1\). Furthermore, when \(\xi_t (t) \to 0, \ell_k = 0\) avoid excessive ETC sensitivity and further save transmission resources. Next, it will be proved that this trigger mechanism avoids the Zeno phenomenon.

**Proof of without Zeno problem in ETC.** In the field of mathematics, the essence of Zeno phenomenon is that the difference of each trigger node tends to 0. Therefore, it is necessary to prove that the interval between two adjacent trigger nodes is not equal to 0. According to the input saturation of UTROV, it can be known that the following relationship exists:

\[
\frac{d\|\xi_t (t)\|}{dt} = \text{sign}(E_r (t)) \frac{dE_r (t)}{dt} \leq \|\xi_t (t)\| \leq \frac{d_{\text{max}}}{dt},
\]

where \(\tau_M\) is the actuator saturation value. Using proof by contradiction, first assume that in the ETC mechanism (Equation (36)) exists the Zeno phenomenon, that is, \(t_{k+1} - t_k = \Delta t = 0\). So \(\frac{d\|\xi_t (t)\|}{dt} = \lim_{t \to t_{k+1}} \frac{\|\xi_t (t)\| - \|\xi_t (t)\|}{t_{k+1} - t_{k+1}}\) does not exist. However, Equation (37) is present and \(\Delta t = 0\). It can be seen that there is no Zeno phenomenon. □

**Remark 8.** It should be noted that the control law \(\tau_f (t)\) we should be designed in remotely control terminal, which is updated according to the status information, and \(\tau_f (t_k)\) that is actually executed at the remotely control terminal. Furthermore, because the zero-order keeper in UTROV make servomotor actuator can only execute \(\tau_e (t_k)\) \((t \in [t_k, t_{k+1})\), the SSDE dynamical system’s actual input is \(\tau_f (t_k) = F_f \tau_e (t_k) + F_d\), can obtain:

\[
du (t) = \left[\tilde{B} (F_f \tau_e (t_k) + F_d) - \tilde{F} (q (t)) - \tau_d\right] dt + \tilde{\Lambda}_L (q, u) dB_L + \tilde{\Lambda}_R (q, u) dB_R.
\]
Next, we will decouple and analyze the actuator faults by using the ETC weight coupling mechanism (see Remark 9) of the event-trigger condition (see Equation (36)), and realize the active pre-compensation for the fault.

**Remark 9.** When UTROV is controlled, the event-trigger rule is always held. Based on reciprocity of inequality and equality theory, Equation (36) existed as \( \xi(t) = D_e(t) \tau(t_k) + \xi_k - B_\xi \), and \( B_\xi = B_{21}(\xi_0, \tau_0) - B_{22}(\xi_0, \tau_0) \) is the conversion constant. Then we can obtain \( -\xi(t_k) = -\beta_\xi \tau(t) + \beta_\xi B_\xi \), where \( \beta_\xi = \xi_k - B_\xi \). From \( D_e(t) \), we can know that it is a coupled dynamic weight damper of \( \| \xi(t) \| \). Because fault \( 0 < F_f < 1 \), by using the coupling characteristic of \( \beta_\xi \) and \( \eta_f \), we can know:

\[
-F_f \tau(t_k) - F_d = -\eta_f \tau(t) + \gamma_f B_\gamma,
\]

where \( \eta_f = F_f \beta_\xi B_\gamma = \beta_\xi^2 B_\xi - F_d \) and \( \gamma_f = \eta_f^{-1} \). The PLOEF \( F_f \) is coupled by \( \beta_\xi \) to build the dynamic fault coupling filter term \( \eta_f \).

**Remark 10.** Under the limitation of actuator saturation, how to carry out saturation compensation in advance in a remotely controlled terminal is the key step of input saturation control. To eliminate the non-smooth point of the traditional saturation function \( \text{sat}(\tau(t)) \) and prevent the stability from being affected in the actual execution process, hyperbolic tangent function \( \tanh(\tau(t)) \) is used for fitting saturation pre-compensation. The \( \tanh(\tau(t)) \) can realize nonlinear input when unsaturated and improve the robustness of input signal, and \( \epsilon_f = \tau(t) - \tau(t) \).

3.3. New Dynamic Damping-Based Terminal Sliding Mode Event-Triggered Fault-Tolerant Controller

This section designs and proves the virtual outer loop finite-time controller and velocity inner loop NDDTSMEFC. The UTROV control flow is shown in Figure 5.

**Step 1.** Kinematics virtual outer loop controller design and proof.

From Definition 1 and the two-loop UTROV control strategy, it is clear that the outer loop controller needs to be designed according to the kinematic model. By way of the generalized coordinate transformation method and the kinematic model, Equation (18) is transformed to:

\[
\dot{q}_e = \begin{bmatrix} \dot{x}_e \\ \dot{y}_e \\ \dot{\theta}_e \end{bmatrix} = \begin{bmatrix} dw_v \sin(\theta_e(t)) + v_r \cos(\theta_e(t)) + y_e w_v - v \\ v_r \sin(\theta_e(t)) - dw_v \cos(\theta_e(t)) + dw - x_e w_v \\ w_r - w \end{bmatrix}.
\]

(40)

Then, the filtering error \( Q_x(t), Q_y(t), \) and \( Q_\theta(t) \) are designed based on Equation (40). The virtual outer loop control law \( u_d \) providing to the reference value of speed for velocity inner loop, and it can converge in finite time, the outer loop control law is:

\[
u_d = \begin{bmatrix} v_d \\ w_d \end{bmatrix} = \begin{bmatrix} k_2 Q_x(t) - v_r Q_\theta(t) - w \theta_e + v_r \\ a k_3 Q_y(t) + \beta_\nu \sin(\theta_e(t)) + w_r \end{bmatrix},
\]

(41)

where \( Q_x(t) = x_e(t) - d + d \cos(\theta_e(t)) \), \( Q_\theta(t) = 1 - \sin(\theta_e(t) + 0.5\pi) \) and \( Q_y(t) = y_e(t) + \theta_e(t) + d \sin(\theta_e(t)) \).

**Proof of finite-time convergence of the outer loop control.** According to the system (Equation (16)), Definition 2 and Remark 2, design Lyapunov as \( V_D(t) = \frac{1}{2} Q_x^2(t) + \frac{1}{2} Q_y^2(t) + Q_\theta(t) \).

Using Lemma 3 infinitesimal differential operator, we can obtain:

\[
V_D(t) = \left[ L_x(t)(-d \sin \theta_e(t) + x_e(t)) + L_y(t)(\dot{\theta}_e(t) + \dot{y}_e(t) + \dot{\theta}_r(t) \cos(\theta_e(t))) \right] dt
\]

\[
+ \dot{\theta}_e(t) \sin(\theta_e(t)) dt
\]

\[
= \left[ L_x(t)(-v(t) + v_r \cos(\theta_e(t) + w(t) L_y(t) - w(t) \theta_e(t)) \right] + L_y(t)(-w(t) L_x(t) + v_r(t) \sin(\theta_e(t) + \theta_r(t)) + \dot{\theta}_e(t) \sin \theta_e(t)) \right] dt.
\]

(42)
The control law Equation (41) is carried into Equation (42) and combined with Young’s inequality, follow as:

\[
iV_D(t) \leq \left[-t_1 ||L_x(t)|| - t_2 L_2^2(t) - t_3 ||L_y(t)|| - t_4 L_2^2(t) - t_5 (1 - \cos \theta_e(t)) \right. \\
\left. - t_6 (1 - \cos \theta_e(t))^{0.5} + 2 \right] dt \\
\leq -L_{d1} V_D(t) - L_{d2} V_D^2(t) + 2,
\]

where \( L_{d1} = \min \{t_2, t_4, t_5 \} \) and \( L_{d2} = \min \{t_1, t_3, t_6 \} \) are positive numbers. According to Leman 3, the virtual control law (Equation (41)) makes kinematics system stable in finite-time. □

Figure 5. Schematic diagram of UTROV control flow.

**Step 2.** UROV dynamics system EDDSMSFCP.

According to Definition 1, Remark 9, Equation (19) and system (See Equation (16)), the state error of the dynamic system is:

\[
du_e(t) = \left[ \dot{u}_d(t) - \ddot{B} \left( F_f \tau_e(t_k) + F_d \right) + \ddot{F}(\dot{q}(t)) + \tau_d \right] dt + \dot{\Lambda}_L(q, u) dB_L + \dot{\Lambda}_R(q, u) dB_R. \tag{44}
\]

For Equation (44), differential of TSMS (Equation (25)) is designed as follows:

\[
dS_u(t) = \left[ A_1 + A_2 \left( \dot{u}_d(t) - \ddot{B} \left( F_f \tau_e(t_k) + F_d \right) + \ddot{F}(\dot{q}(t)) + \tau_d \right) \right] dt + \dot{\Lambda}_L(q, u) dB_L + \dot{\Lambda}_R(q, u) dB_R, \tag{45}
\]

where the \( A_1 = a_2 ||u_e(t)||^{\hat{\gamma}} \) \( \hat{\gamma} \) \((u_e(t)) \) and \( A_2 = a_1 + \frac{9a_2}{4} ||u_e(t)||^{\hat{\gamma}} \) \( \hat{\gamma} \) \((u_e(t)) \).

Combining DDRFL (Equation (32)), the velocity inner loop NDDTSMEFC control law \( \tau(t) \), adaptive law \( \dot{\hat{\tau}}(t) \) and ADEFCA pre-compensated compensator \( \dot{\hat{\tau}}(t) \) are obtained respectively:

\[
\tau(t) = \dot{\hat{\tau}}(t) + \ddot{B}^{-1} \dot{u}_d(t) + \dddot{B} A_2^{-1} c_{tr} [c_{tU} \text{sign} \{S_u(t)\} D(S_u(t)) + \Gamma_7 + \Gamma_8 + \dddot{B} S_u(t) \left( (\Lambda_2^{\hat{\gamma}}(\hat{\theta}(t))) + \Lambda_2^{\hat{\gamma}}(\hat{\theta}(t)) \right) \right] (46)
\]

\[
\dot{\hat{\tau}}(t) = G_T S^2_u(t) \text{tanh}(D_1 S_u(t)) - b_1 \hat{\tau}(t) + G_H ||S_u(t)||^2 H^2_3(t), \tag{47}
\]

\[
\dddot{\tau}(t) = G_T S^2_u(t) S^T_u(t) \text{tanh}(S^2_u(t)) H_5(t) - b_2 \dot{\dot{\tau}}(t) - A_2 (S^2_u(t) S_u(t))^2 - A_2 \hat{\tau}^2(t) ||S_u(t)||^3, \tag{48}
\]

where the \( G_T, G_H, D_1, G_H, b_1, b_2 \) and \( b_2 \) are bounded positive numbers. The \( \Gamma_7 = \text{tanh}(D_1 S^2_u(t)) \hat{\tau}(t) G_T + S_u(t) G_H \hat{\tau}(t) H^2_3(t) \) and \( \Gamma_8 = -A_2 \dot{\hat{\tau}}^2(t) \hat{\dot{\tau}}(t) \text{sign} \{S_u(t)\} - S_u(t) A_2 \dot{\hat{\tau}}^2(t) + \dddot{\tau}(t) \text{tanh}(S_u(t)) H_5(t) \).

The control law Equation (41) is carried into Equation (42) and combined with Young’s inequality, follow as:

\[
\Vert \dot{B}_{\text{max}} \Vert + 5 + \Vert Z(Q) \Vert + \Vert u_e(t) \Vert + \Vert H_P \Vert + \Vert S_u(t) \Vert + \Vert \hat{\tau}(t) \Vert + \Vert H_D(t) \Vert = \Vert A_2 \Vert \left[ 4 + \frac{1}{4} \Vert Z(Q) \Vert + (2 + \Vert S_u(t) \Vert^3) \tau_M + 0.8355 + 0.557 \Vert S_u(t) \Vert^3 \right] + \Vert A_1 \Vert.
\]
Proof. Based on Equations (44) and (45) and the system (see Equation (16)), the Lyapunov function is designed as follows:

$$V_u(t) = \frac{1}{2} \left( S_u^T(t) S_u(t) \right) + \frac{1}{2} \phi_i^T(t) \phi_i(t) + \frac{1}{2} \gamma^T(t) \gamma(t),$$

(49)

where $\phi_i(t) = \phi_i(t) - \phi_i(t)$ and $\gamma(t) = \gamma(t) - \gamma(t)$ are adaptive fitting error. Using the Lemma 3 for the Equation (49), the following can be obtained:

$$\dot{V}_u(t) = 0.5 Tr \left\{ \frac{\partial^2 V_u(t)}{\partial (S_u(t))^2} \right\} + 0.5 Tr \left\{ \frac{\partial^2 V_u(t)}{\partial (S_u(t))^2} \right\} + S_u^T(t) S_u(t) \left[ A_1 + A_2 \left( \tilde{u}_d(t) - B_l \eta_f \tau_e(t) + B \gamma(t) H_p \right) \right]$$

+ $\| S_u(t) \|^2 \langle S_u(t) \rangle \left[ A_1 + A_2 \left( W^*Z(q) + e_2(z) + \tilde{\tau} \right) \right] + Q_\Lambda,$

where $Q_\Lambda = \left[ B^2 A_R^2(\theta(t)) \| S_u(t) \|^2 \right] + \left[ B^2 A_L^2(\theta(t)) \| S_u(t) \|^2 \right].$

According to Remark 8, Equation (48), $H_p = B_\gamma$ and $\| W^* \| \| Z(q) \| \leq \gamma_\tau \| Z(q) \| + 0.5 \| Z(q) \| \| W^* \| \| \eta_f \| /$ and $\| \tau(t) \| \leq \tau_M$ and $\| \tau(t) \| \leq \tau_M$, we can obtain:

$$\dot{V}_u(t) \leq \| S_u(t) \|^2 S_u^T(t) A_2 \left[ \tilde{u}_d(t) - \eta_f \tau_e(t) \right] + \| S_u(t) \|^2 S_u^T(t) A_2 \gamma(t) H_d(t)$$

+ $\| S_u(t) \|^2 \langle S_u(t) \rangle \left[ A_1 + A_2 \left( \frac{3}{2} \| Z(q) \| \| W^* \| \| \eta_f \| + \| e_2(z) \| + \| \tau_d \| \right) \right]$,

(52)

Combined with the saturation characteristic of Remark 10, Lemma 5 and $\| S_u(t) \|^3 \| \phi(t) \| H_D(t) \leq \| S_u(t) \|^3 \| \phi(t) \| H_D^2(t) + Q_\Lambda + \frac{3}{2} \| \phi(t) \| \| S_u(t) \| ^3 + \frac{3}{2} \| \phi(t) \| ,$ Equations (46) and (47) bring into Equation (42):

$$\dot{V}_u(t) \leq - \phi_i^T(t) G_T \langle S_u(t) \rangle \tan \left( D_s \tau_e(t) \right) + G_H \| S_u(t) \|^4 \phi(t) H_D^2(t)$$

+ $\phi_i^T(t) b_r \phi(t) - \phi_i^T(t) G_H \| S_u(t) \|^4 H_D^2(t) + \| S_u(t) \|^3 S_u^T(t) A_2 \tilde{u}_d(t)$

- $\| S_u(t) \|^2 S_u^T(t) B \bar{A}_2 \gamma(t) \left( \bar{A} A_2 \right)^{-1} \left[ e_u \text{sign}(S_u(t)) D(S_u(t)) \right]$,

+ $\phi(t) G_T \langle S_u(t) \rangle \tan \left( D_s \tau_e(t) \right) + S_u(t) G_H \phi(t) H_D^2(t) + Q_\Lambda \| S_u(t) \|^3 + Q_\Lambda + \frac{3}{2} \phi(t)$

+ $\gamma^T(t) b_r \gamma(t) + G_T \phi(t) \| S_u(t) \|^3$$

(53)

where $Q_\Lambda = B S_u(t) \left( \Lambda^4_T(\theta(t)) \right) \tan \left( \Lambda^4_T(\theta(t)) \right) + \Lambda^4_H(\theta(t)) \tan \left( \Lambda^4_H(\theta(t)) \right)$.

$$\dot{V}_u(t) \leq l_c + \gamma^T(t) b_r \gamma(t) + \phi_i^T(t) b_r \phi_i(t) - \| S_u(t) \|^2 S_u^T(t) \left[ e_u \text{sign}(S_u(t)) D(S_u(t)) \right],$$

(54)

where $l_c = 0.2785 + 0.5 + \frac{3}{2} \phi(t).$ The $\phi(t)$ and $\gamma(t)$ are the maximum value of $\| S_u(t) \|$ and $\| \gamma(t) \||.
Based on \( \dot{\varphi}_r(t) \dot{\varphi}_r(t) \leq \dot{\varphi}_r^T(t) (\varphi_r(t) - \dot{\varphi}_r^T(t)) \leq \frac{1}{2} \varphi_r^2(t) - \frac{1}{2} \dot{\varphi}_r^T(t) \dot{\varphi}_r(t) \), and Lemma 1, simplifying Equation (54), we obtain:

\[
\begin{align*}
&\quad \lambda_1 \left[ \left| S_u(t) \right|^2 + \left| \dot{\varphi}_r(t) \right|^2 + \left| \ddot{\varphi}_r(t) \right|^2 + \left| Q_x(t) \right|^2 + \left| Q_y(t) \right|^2 + \left| Q_y(t) \right|^2 \right] + \xi \\
&\quad - \lambda_2 \left[ \left| S_u(t) \right|^2 + \left| \dot{\varphi}_r(t) \right|^2 + \left| \gamma_r(t) \right|^2 + \left| Q_x(t) \right|^2 + \left| Q_y(t) \right|^2 + \left| Q_y(t) \right|^2 \right] + \xi \\
&\quad \leq -\lambda_1 m V(0) - \lambda_2 m V(0) + \xi,
\end{align*}
\]

where \( \lambda_1 = \min \{ G_s \xi, 0.25, L_d \} \) and \( \lambda_2 = \min \{ \frac{1}{2} G_s \xi, 0.25 b_1 - \frac{1}{2}, L_d \} \).

The \( \xi = \max \{ \xi, G_s \xi, 0.25 b_1 - \frac{1}{2}, L_d \} \).

Theorem 1. The velocity inner loop NDDTSMEC control law (Equation (46)), adaptive controller (Equation (47)), ADEFCA compensator (Equation (48)) and event-trigger mechanism (Equation (36)) for the system (Equation (16)) under Assumptions 1–4, which meets the actual engineering requirements, has the following holds:

(a) The UTROV system can keep the signal semi-globally finite-time converging to the bounded neighborhood near 0 in probability in time \( T \). The convergence time \( T_{as} \) is

\[
E(0) = (0.25 \lambda_m S_u)^{-1} \left[ EV(0) - \left( \frac{\xi}{(1 - 0.5 \lambda_m \xi)} \right)^{0.5} \right] = T_{as}.
\]

(b) It can be known from Equations (48)–(57) that the system can converge regardless of whether it is affected by fault or disturbance. Moreover, the system error converges to the boundary as

\[
\lim_{t \to \infty} E \left\| U_{RE} \right\|^2 \leq \left( \frac{\xi}{(1 - \lambda_m \xi)} \right)^{0.5}.
\]

(c) There are unique solutions of the system (Equation (16)).

Proof of Theorem 1. First of all, Equation (56) can be obtained \( \dot{V}(t) \leq -\lambda_m V(t) + \xi \) by transformation based on Young’s inequality. For \( V(t) \), can know \( V(t) \in C^2 \) (Satisfies Lemmas 2 and 3) it has the property Lipschitz condition. Through (Equation (56)), we know that there is a unique existence in Ref. [33], which satisfies:

\[
EV^{0.5}(t) \leq \left( 1 - \exp(-0.5 \lambda_m T_{as}) \right) EV(0) + \left( \frac{\xi}{(1 - \lambda_m \xi)} \right)^{0.5}.
\]

Then, based on Jensen’s inequality, we can obtain \( \lim_{t \to \infty} E \left\| U_{RE} \right\|^2 \leq \left( \frac{\xi}{(1 - \lambda_m \xi)} \right)^{0.5} \). We can know system signals are definite in compact set \( \Omega = \{ U_{RE} | EV \leq \left( \frac{\xi}{(1 - \lambda_m \xi)} \right)^{0.5} \} \). From this, we can know that the system (Equation (16)) is semi-globally stable, and the system error is probabilistic bounded in the probability space \( \Omega \). According to Lemmas and Assumptions, the system states are semi-globally finite-time stability in probability. In addition, it is proved that the system is bounded and convergent in finite-time.

4. Simulation Verification and Analysis Discussion

4.1. Simulation Verification

The heavy work-class UTROV in this paper is a multifunctional work underwater robot developed by laboratories and manufacturing development company depending on the funding. The structural design model of the UTROV (see Figures 2 and 3) and
the experimentally engineering prototype (see Figure 6). The main task of the UTROV is cable laying and inspection, etc. So the UTROV is equipped with speed sensor, GPS module, operating equipment, bathymetry sensor, acceleration sensor, pressure sensor and gyroscope, etc. Through hydrodynamic experiments and empirical measurement, the relevant characteristics of this UTROV are obtained. The specification parameters are: The dimensions are $1.7 \times 1.2 \times 1.3$ m; The weight is 1200 kg; The operation depth is 100 m; The controller SOC is SIMATIC S7-200; The communication is optical fiber umbilical cable. Therefore, the relevant parameters of simulation are $m = 1200$ kg, $r = 0.1$ m, $b = 0.6$ m, $J = 500$ kg $\cdot$ m$^2$, $d = 0.1$ m, $\tau_M = [500, 500]^T$ N $\cdot$ m. Selecting a reference trajectory with abrupt curvature to verify the control performance, the design is

$$x_r = t, \quad \theta_r = \arctan\left(\frac{\dot{y}_r}{\dot{x}_r}\right)$$

and

$$y_r = \begin{cases} 10 & t \leq 47 \\ \sqrt{100 - (t - 47)^2} & 47 < t \leq 53 \\ \frac{65-t}{15} & 53 < t \leq 62 \\ 10 - \sqrt{100 - (t - 68)^2} & 62 < t \leq 68 \\ 0 & 68 < t \leq 112 \\ \frac{10 - \sqrt{100 - (t - 112)^2}}{15} & 112 < t \leq 118 \\ \frac{10 - \sqrt{100 - (t - 133)^2}}{15} & 118 < t \leq 127 \\ \sqrt{100 - (t - 133)^2} & 127 < t \leq 133 \\ 10 & 133 < t \leq 200 \end{cases}$$

(58)

The Dynamic Damping-based Sliding Mode Event-Triggered Fault-tolerant Controller (DDSMEFC) (using traditional sliding surface instead of TSMS in NDDTSMEFC) and Super-Twisted Terminal Sliding Mode Event-Triggered Fault-tolerant Controller (STTSMEFC) (using super-twisted reaching law [5] instead of DDRL in NDDTSMEFC) are designed to compare the finite-time SMC two-stages global stability and chattering reduction of DDRL and new TSMS, but they still has ADEFCA mechanism. The ADEFCA mechanism is not used to design a Dynamic Damping-based Terminal Sliding Mode Controller (DDTSMC) to verify that the ADEFCA mechanism can effectively carry out active fault compensation.

To scientifically compare the control effect of NDDTSMEFC, the same set of control parameters are designed, and the relevant parameters are $x(0) = 1, y(0) = 8, \theta(0) = 53, k_1 = 60, k_2 = 0.4, \alpha = 60, \beta = 1.5, a_1 = diag[1.5, 5.5], a_2 = diag[0.1, 0.01], a_3 = diag[5, 0.01], \varepsilon_u = diag[0.015, 1.35], G_T = diag[0.01, 0.002], G_H = diag[0.015, 0.01], G_t = diag[10, 10], \ldots$
To verify the compensation performance of FTC, the extreme dynamic time-varying fault is considered as the basis of simulation design. $H_f = \text{diag}(0.2 + 0.8\exp(0.5t), 0.3 + 0.7\exp(0.5t))$, time-varying faults $H_d = \text{diag}(6 + 3.5\sin(0.1t), 5 + 2.5\cos(0.1t))$ and $f_t = N_{w}$ to simulate. The $N_{w}$ is Gaussian white noise with the noise power of $\sqrt{2}\pi$. The external unknown disturbances is $\tau_d(t) = [1 + N_{w}, 10\sin(1.5t) + 2\cos(1.5t), 3\sin(t) + 3\cos(t)]$. The step size of the simulation is 0.01 s. The total simulation time of the control system is 200 s.

In order to quantitatively compare the performance of the three groups of simulations, the Mean Integration Absolute Control (MIAC) is used to calculate the input energy, and the Mean Integration Square Error (MISE) is used to calculate the error control accuracy. The Man Integration Total Variation (MITV) is designed to verify the input communication transmission frequency. $\text{MIAC} = \left( t_f - t_0 \right)^{-1} \int_{t_0}^{t_f} \| \tau_e(\mu) \| \text{d\mu}$, $\text{MISE} = \left( t_f - t_0 \right)^{-1} \int_{t_0}^{t_f} \| u_e(\mu) \|^2 \text{d\mu}$ and $\text{MITV} = \left( t_f - t_0 \right)^{-1} \int_{t_0}^{t_f} \| \tau_e(\mu + 1) - \tau_e(\mu) \| \text{d\mu}$, are shown in Table 1.

### Table 1. UROV quantitative analysis index.

| Control Scheme | MIAC | MITV | MISE |
|----------------|------|------|------|
| NDDTSMEFC      | 0.7028 | 1.2860 | 0.3861 | 0.9210 | 0.6341 | 0.6926 | 0.2907 |
| DDTSMC         | 0.7600 | 1.3600 | 0.4363 | 1.1190 | 0.6409 | 0.6927 | 0.2906 |
| DDSMEFC        | 0.8413 | 1.3750 | 0.3136 | 0.7708 | 0.5900 | 1.5500 | 0.6139 |
| STTSMEFC       | 0.8071 | 1.4400 | 0.3568 | 0.7813 | 0.7561 | 0.7975 | 0.7975 |

#### 4.2. Analysis Discussion

From Figure 7 and the value of MISE (Table 1), we can see that NDDTSMEFC is more stable, and its posture is all stable in about 12 s. When the large error caused by the sudden change of motion to the trajectory (See Figure 7. in the time period of 45–70 s and 125–140 s), the error state of DDTSMC is not easily converged quickly, although it is easier to tend to zero than DDSMEFC, but it can not be stable for too long. The robust performance of NDDTSMEFC is better, especially, when the error suddenly changes (See Figure 7. in the time period of 45–70 s), the sensitivity of the stable response is better and the stability is restored more quickly.

The input of the DDTSMC with poor compensate performance and cannot restore smooth control. Although the error control stability (see MISE in Table 1) of DDTSMC is similar to NDDTSMEFC, DDTSMC completely depends on the robustness of sliding mode for passive fault-tolerant control, which requires large drive energy compensation and input adjustment frequency (see Figure 8). However, this also makes the MIAC worse, not only fluctuating frequently, but also because there is no event trigger mechanism for input allocation, the control performance is wasted and the control stability cannot be guaranteed.

In addition, in Figures 7 and 9, although DDSMEFC and STTSMEFC are stable, there is no optimization of terminal sliding mode or new reaching law, and their chatter phenomenon and convergence speed are slow. Especially in Figure 9, we can see that the tracking stability of STTSMEFC is poor when switching between straight lines and curves. Combined with Figures 8 and 10–11, we can see that although the ADEFCA pre-compensated compensator compensates for the fault, sliding mode chattering is obviously more frequent than that of the controller using DDRL, which results in poor tracking stability. Therefore, not only is the effect of active fault-tolerance proved, but also the chattering optimization of DDRL and the ability of fast recovery of sliding mode robust response are verified under RFRV affecting.

$D_t = \text{diag}[0.1, 0.013], b_T = \text{diag}[215, 190], b_f = \text{diag}[200, 185], c_{tu} = \text{diag}[150, 5]$. The simulation result is shown in Figures 6–10.
Table 1. UROV quantitative analysis index.

| Control Scheme | MIAC | MITV | MISE | \(\tau_v\) | \(\tau_w\) | \(x_e\) | \(Y_e\) | \(\theta_e\) |
|----------------|------|------|------|----------|----------|-------|-------|-------|
| NDDTSMEFC      | 0.7028 | 1.2860 | 0.3861 | 0.9210  | 0.6341  | 0.6926 | 0.2907 |       |
| DDTSMC         | 0.7600 | 1.3600 | 0.4363 | 1.1190  | 0.6409  | 0.6927 | 0.2906 |       |
| DDSMEFC        | 0.8413 | 1.3750 | 0.3136 | 0.7708  | 0.5900  | 1.5500 | 0.6139 |       |
| STTSMEFC       | 0.8071 | 1.4400 | 0.3568 | 0.7813  | 0.7561  | 0.7975 | 0.7975 |       |

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Figure 7. Schematic diagram of UTROV position error.

In addition, in Figures 7 and 9, although DDSMEFC and STTSMEFC are stable, there is no optimization of terminal sliding mode or new reaching law, and their chatter phenomenon and convergence speed are slow. Especially in Figure 9, we can see that the tracking stability of STTSMEFC is poor when switching between straight lines and curves. Combined with Figures 8 and 10–11, we can see that although the ADEFCA pre-compensated compensator compensates for the fault, sliding mode chattering is obviously more frequent than that of the controller using DDRL, which results in poor tracking stability. Therefore, not only is the effect of active fault-tolerance proved, but also the chattering optimization of DDRL and the ability of fast recovery of sliding mode robust response are verified under RFRV affecting.

It can be seen that chattering is caused by the algorithm itself, not the fault, so active fault tolerance cannot play a role. Especially in the case of time-varying faults, the chattering is even worse. The active fault tolerance mechanism can compensate for the time-varying control loss caused by time-varying faults, but the robustness of the control algorithm needs to be guaranteed by DDRL.

Figure 8. Schematic diagram of UTRO system input.

Figure 12 shows the UROV input signal transmission sampling interval. The y-axis represents the sampling interval of the input transmission, that is, the interval between two adjacent input signal transmissions. The sampling interval of NDDTSMEFC is dynamically adjusted due to the dynamic event-trigger mechanism, and the sampling interval is no longer a fixed 10 ms. The maximum sampling interval is 9.72 s. On the other hand, the sampling interval of the DDTSMC is still the input transmission every 0.01 s (see Figure 12 green triangle). It can be seen that NDDTSMEFC not only ensures the control stability under the influence of faults and all kinds of disturbances, but also saves communication resources, that is, reduces the sampling frequency. It is more convenient to be used in engineering.
Figure 9. Schematic diagram of UTROV trajectory tracking.

Figure 10. Schematic diagram of UTROV system output (Velocity).
It can be seen that chattering is caused by the algorithm itself, not the fault, so active fault tolerance cannot play a role. Especially in the case of time-varying faults, the chattering is even worse. The active fault tolerance mechanism can compensate for the time-varying control loss caused by time-varying faults, but the robustness of the control algorithm needs to be guaranteed by DDRL.

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Through the analysis of the simulation, we can know that through the remodeling of the unknown non-smooth random force, after improving the accuracy of the model, the new TSMS and DDRL meet the original intention of the design. At the same time, the event-trigger adaptive fault-tolerant mechanism can also be used for active fault compensation, and the stability and effectiveness of the remote control of UTROC trajectory tracking in extreme underwater environment are verified.

**Figure 11.** Schematic diagram of UROV sliding mode surface value.

**Figure 12.** Schematic diagram of UROV input signal transmission sampling interval.
Through the analysis of the simulation, we can know that through the remodeling of the unknown non-smooth random force, after improving the accuracy of the model, the new TSMS and DDRL meet the original intention of the design. At the same time, the event-trigger adaptive fault-tolerant mechanism can also be used for active fault compensation.

5. Conclusions

In this paper, comprehensively considering a part of disturbance resistance caused by irregular Brownian motion of ocean current, the pre-compensated RFRV analysis method is used through reverse thinking to solve the problem that the dual drive source differential drive wheel does not easily perform random force analysis. Then, the SSDE model that is more suitable for the actual underwater is established. Based on the SSDE model, considering actuator faults, dynamic uncertainty and disturbance with differentiable or irregularity randomness in engineering practice, the NDDTSMEFC is designed to control UROV to complete the underwater trajectory tracking operation. In NDDTSMEFC, a new type of nonsingular TSMS is designed to improve the finite-time convergence ability of sliding mode control and ensure the robust response in the sliding phase. Moreover, the DDRL that is dynamically adjusted according to the SMS information is designed to weaken the chattering problem of SMC and improve the reaching speed, so that it has better robustness under the mutation error. The RBF adaptive on-line estimation technique and the new dynamic event-trigger mechanism are combined to design the ADEFCA compensator, it can decouple PLOEF to actively fault compensate for stability control. To facilitate the practical application, through simulation we verify that the control system in this paper meets the design expectations. And, the SSDE model is helpful to the later pool experiment and the actual deep-sea engineering practice. Furthermore, the problems have been found in the study that complete the loss of effect actuator faults and not being able to completely contact the seabed, which is also of great research value, and will be further studied in future work. The applicability of this control scheme will be further studied and optimized.

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**Abbreviations**

The following abbreviations are used in this manuscript:

- ADEFCA: Adaptive Dynamic Event-Trigger Fault Coupling Analytical
- DDRL: Dynamic Damping Reaching Law
- DDSMEFC: Dynamic Damping-based Sliding Mode Event-Triggered Fault-tolerant Controller
- DDTSMC: Dynamic Damping-based Terminal Sliding Mode Controller
- ETC: Event-Trigger Control
- FTC: Fault-Tolerant Control
- PLOEF: Partial Loss of Effect Fault
- NDDTSMEFC: New Dynamic Damping-based Terminal Sliding Mode Event-triggered Fault-tolerant Control
- Control ODE: ordinary differential equation
- RBFNN: Radial Basis Function Neural Network
- RFRV: Resistance Forces of Random Vibrations
- STTSMEFC: Super-Twisted Terminal Sliding Mode Event-Triggered Fault-tolerant Controller
- TSMS: Terminal Sliding Mode Surface
- UTROV: underwater tracked remotely operated vehicle

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