Single-Carrier Modulation for Large-Scale Antenna Systems

Yinsheng Liu, Geoffrey Ye Li, Zhenhui Tan, and Deli Qiao.

Abstract

Large-scale antenna (LSA) has gained a lot of attention due to its great potential to significantly improve system throughput. In most existing works on LSA systems, orthogonal frequency division multiplexing (OFDM) is presumed to deal with frequency selectivity of wireless channels. Although LSA-OFDM is a natural evolution from multiple-input multiple-output OFDM (MIMO-OFDM), several drawbacks of LSA-OFDM are inevitable. In this paper, we investigate single-carrier (SC) modulation for LSA systems (LSA-SC). In LSA-SC, the multipath propagation can be suppressed using antenna array when the antenna number is large enough and the channels at different antennas are independent. In that case, the transmit symbol can be recovered by directly sampling the received waveform if the waveform in SC modulation is designed to satisfy the Nyquist condition. In practical systems, however, the antenna number is always finite and the channels at different antennas may be also correlated when placing hundreds of antennas in a small area. Therefore, we will analyze the performance of LSA-SC in such practical environments.

Index Terms

Large-scale antenna, massive MIMO, single-carrier, OFDM.

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I. Introduction

Large-scale antenna (LSA) has gained a lot of attention recently [11–4]. In an LSA system, a base station (BS) is equipped with hundreds of antennas. It can be considered as an extension of traditional multiple-input multiple-output (MIMO) systems, which has been widely studied during the last couple of decades [5]. The properties of LSA make it a potential technique for future wireless systems.

Through the employment of a large number of antennas at the BS, the channel vectors or matrix between the BS and different users become asymptotically pairwisely orthogonal. In this case, the matched filter (MF) becomes the optimal detector [6]. The asymptotical orthogonality of channel vectors or matrices corresponding to different users also allows multiusers to work in the same bandwidth, and thus can improve the spectrum efficiency of the network. It is shown that the transmit power for each user can be scaled down by the number of antennas or the square root of the number of antennas, depending on whether accurate channel parameters are available or not [7]. As a result, the transmit powers of users can be arbitrarily small when the number of antennas is large enough, and therefore the energy efficiency of users can be also significantly improved.

Traditional MIMO systems are usually combined with orthogonal frequency division multiplexing (OFDM), where the latter is used to convert frequency selective channels into flat fading channels. As a natural evolution of MIMO-OFDM, LSA-OFDM has been presumed in most existing works in LSA systems [3], [6], [7]. Although straightforward, LSA-OFDM has several drawbacks. The BS needs hundreds of discrete Fourier transforms (DFTs) for OFDM demodulation, resulting in a heavy computation burden. The guard band and cyclic prefix in OFDM reduce the spectral efficiency. The peak-to-average power ratio (PAPR) of OFDM signal is also very high, resulting in low efficient power amplifier for users.

In view of the drawbacks of OFDM, single-carrier (SC) modulation has been proposed for LSA systems [8], [9]. In [8], SC is used for downlink precoding with an MF precoding matrix. The work in [8] can be easily extended to uplink according to the duality [5]. In [9], we have considered SC in an LSA system over Rician fading channels. In that case, an equalizer is
required at the receiver to suppress multiuser interference caused by the line-of-sight path.

In this paper, we consider SC for uplink transmission in an LSA system. Traditionally, the SC receiver is usually designed to recover the transmit symbol. In this paper, we propose to recover the waveform, rather than the symbol, for SC receiver design. When the waveform in SC modulation is designed to satisfy the Nyquist condition [10], the symbols can be easily recovered with baud-rate sampling once the waveform is obtained. In LSA-SC, the multipath propagation can be suppressed through LSA array. Hence, there is no need for an equalizer in the receiver and only an MF, which has been widely used in traditional SC receivers [10], is enough for SC demodulation.

In general, the multipath propagation can be only suppressed completely in the ideal environments where the antenna number is infinite and the channels corresponding to different antennas are independent. In practical scenarios, however, the antenna number is always finite and the channels corresponding to different antennas will be correlated when placing hundreds of antennas in a small area. In this case, the multipath propagation cannot be suppressed completely, leading to residual inter-symbol-interference (ISI). We will therefore analyze the impact of the non-ideal environments in this paper.

The rest of this paper is organized as follows. Section II introduces the system model. In Section III, we investigate the waveform recovery in an LSA-SC system. In Section IV, we will discuss the impact of non-ideal environments on the LSA-SC receiver. Numerical results are presented in Section V. Finally, conclusion is drawn in Section VI.

II. SYSTEM MODEL

In this section, we will first introduce the channel model, and then we will present SC modulation for an LSA system.
A. Channel Model

For a frequency-selective fading channel, the baseband channel impulse response (CIR) at the $m$-th receive antenna can be given by

$$c_m(t) = \sum_l \alpha_m[l] \delta(t - \tau_l), \quad (1)$$

where $\alpha_m[l]$ is the complex gain of the $l$-th tap on the $m$-th antenna, $\tau_l$ is the corresponding tap delay, and $\delta(\cdot)$ represents the Dirac delta function. In (1), we have implicitly assumed that the tap delays for different antennas are the same since the extra propagation delays caused by the physical size of the antenna array are quite small and thus can be omitted. If the gains corresponding to different taps are independent and complex Gaussian, then

$$E\{\alpha_m[l]^{\ast}[p]\} = \sigma_l^2 \delta[l - p], \quad (2)$$

where $\sigma_l^2$ is the power of the $l$-th tap and $\delta[\cdot]$ denotes the Kronecker delta function.

Denote $c(t) = [c_1(t), \cdots, c_M(t)]^T$ to be the CIR vector corresponding to different antennas at the BS. From (1), we have

$$c(t) = \sum_l \alpha[l] \delta(t - \tau_l), \quad (3)$$

where $\alpha[l] = (\alpha_1[l], \cdots, \alpha_M[l])^T$ denotes the complex gain vector corresponding to the $l$-th tap. If assuming the antenna number in an LSA system is infinite and the complex gains at different antennas are independent, the expected mean in (2) is then equal to the sample mean according to the law of large numbers [11], that is

$$\lim_{M \to \infty} \frac{1}{M} \alpha^H[l] \alpha[p] = E\{\alpha_m[l]^{\ast}[p]\} = \sigma_l^2 \delta[l - p], \quad (4)$$

since $\alpha_l[m]$’s with $m = 1, 2, \cdots, M$ are independent and identically distributed random variables. The last identity indicates that the complex gain vectors corresponding to different taps will be
asymptotically orthogonal.

B. SC Modulation

Fig. 1 shows uplink SC transmission in an LSA system with \( M \) receive antennas. Without loss of generality, we assume that the user is equipped with only one transmit antenna for simplicity even though our result can be directly applied to the multiple antenna case. If the transmitted symbols, \( s[n] \)'s, are independent with zero mean and unit variance, then we have

\[
E\{s[n]s^*[k]\} = \delta[n-k].
\]

Given a transmit filter, \( g_T(t) \), the transmit waveform is

\[
s_T(t) = \sum_{n=-\infty}^{\infty} s[n]g_T(t-nT),
\]

where \( T \) is the symbol period.

Denote \( r(t) = [r_1(t), \cdots, r_M(t)]^T \) to be the received signal vector at the BS, then we have

\[
r(t) = \sum_l \alpha[l]s_T(t-\tau_l) + v(t),
\]

where \( v(t) = [v_1(t), \cdots, v_M(t)]^T \) is the additive white Gaussian noise (AWGN) with a constant spectral density, \( N_0 \). After going through a receive filter, \( g_R(t) \), at each antenna, the filtered receive signal vector, \( x(t) = [x_1(t), \cdots, x_M(t)]^T \), becomes

\[
x(t) = r(t) * g_R(t)
\]

\[
= \sum_l \alpha[l]s(t-\tau_l) + z(t),
\]
where \( s(t) \) is given by
\[
    s(t) = \sum_{n=-\infty}^{\infty} s[n] g(t - nT),
\]  
(8)
with \( g(t) = g_T(t) * g_R(t) \) and \( z(t) = [z_1(t), \ldots, z_M(t)]^T \) is the filtered noise vector with \( z_m(t) = v_m(t) * g_R(t) \).

In general, the same root-raised-cosine function with roll-off factor, \( \beta \), is used for both transmit filter and receiver filter, and the overall impulse response thus becomes a raised-cosine function which can satisfy Nyquist condition \([10]\), that is, \( g(nT) = \delta[n] \) and thus \( s(nT) = s[n] \). In this case, the correlation matrix of the filtered noise can be expressed by
\[
    E\{z(t)z^H(t + \tau)\} = N_0 g(\tau) I,
\]  
(9)
where \( I \) denotes the identity matrix.

III. LSA-SC SYSTEM

In this section, we will first introduce the single-tap LSA-SC receiver using waveform recovery, then a multi-tap receiver will be addressed. Finally, the relationship with the traditional SC receiver and the practical implementation of the proposed LSA-SC receiver will be discussed.
A. Single-Tap Receiver

As shown in Fig. 2 (a), a single-tap receiver exploits a particular tap from the received signal to recover the Nyquist waveform, $s(t)$. To recover the waveform over the $p$-th tap with delay $	au_p$, as in Fig. 2 (a), we have

$$
\hat{s}_p(t) = \frac{1}{M}\alpha_{H[p]}x(t + \tau_p),
$$

(10)

Then, substituting (7) into (10), we can obtain

$$
\hat{s}_p(t) = \frac{1}{M}\|\alpha[p]\|_2^2 s(t) + \sum_{l \neq p} \frac{1}{M}\alpha_{H[p]}\alpha[l]s(t - \tau_l + \tau_p) + \frac{1}{M}\alpha_{H[p]}z(t + \tau_p),
$$

(11)

where the first term in (11) denotes the desired waveform over the $p$-th tap, the second and the third terms are inter-tap interference (ITI) and additive noise, respectively. Note that the ITI is in general different from the ISI since the former may still contain the desired symbols. ITI and ISI are equivalent only when tap delays are integer times of symbol period [8].

Using the LSA property in (4), the complex gain vectors are asymptotically orthogonal if assuming the antenna number is infinite and the channels at different antennas are independent. In this case, the ITI in (11) can be completely suppressed and thus (11) reduces to

$$
\hat{s}_p(t) = \sigma_p^2 s(t) + w_p(t),
$$

(12)

which is composed of only $s(t)$ except for a scaling factor, $\sigma_p^2$, and the additive noise, $w_p(t)$, with

$$
w_p(t) = \frac{1}{M}\alpha_{H[p]}z(t + \tau_p),
$$

(13)

whose correlation function is given by

$$
E\{w_p(t)w_p^*(t + \tau)\} = \frac{N_0}{M}g(\tau)\sigma_p^2.
$$

(14)

From (12), the multipath propagation can be completely suppressed using the LSA properties in (4). Therefore, an equivalent AWGN channel except for an scaling factor is obtained over the
tap of interest.

Since $g(t)$ satisfies the Nyquist condition, the transmit symbol can be easily recovered with baud-rate sampling. Therefore, the decision variable corresponding to the $p$-th tap at $t = kT$ is

$$\hat{s}_p[k] = \sigma^2_p s[k] + w_p(kT).$$

(15)

Accordingly, the signal-to-noise ratio (SNR) corresponding to the $p$-th tap will be

$$\text{SNR}_p = \frac{\sigma^2_p M}{N_0}.$$  

(16)

B. Multi-Tap Receiver

In above, the desired waveform is recovered with only the $p$-th tap. Actually, the SNR can be further improved by exploiting all taps through a linear combination of $\hat{s}_p(t)$’s with $p = 0, 1, \cdots, L$, as shown in Fig. 2(b).

Let $\eta[p]$ with $p = 0, 1, \cdots, L$ be the coefficients for the linear combination, then the overall recovered waveform from all taps can be expressed as

$$\hat{s}(t) = \sum_{p=0}^{L} \eta[p] \hat{s}_p(t)$$

$$= \sum_{p=0}^{L} \eta[p] \sigma^2_p s(t) + \sum_{p=0}^{L} \eta[p] w_p(t).$$

(17)

In this case, decision variable after sampling at $t = kT$ can be expressed as

$$\hat{s}[k] = \sum_{p=0}^{L} \eta[p] \sigma^2_p s[k] + \sum_{p=0}^{L} \eta[p] w_p(kT).$$

(18)

Accordingly, the SNR after combination will be

$$\text{SNR} = \frac{M \left( \sum_{p=0}^{L} \sigma^2_p \eta[p] \right)^2}{N_0 \sum_{p=0}^{L} \sigma^2_p \eta^2[p]}.$$  

(19)
Direct calculation from (19) yields that the maximum SNR can be achieved when

$$\eta[0] = \eta[1] = \cdots = \eta[L],$$

and the maximum SNR, if we assume that the total channel power is unit, will be

$$\text{SNR} = \frac{M}{N_0} \geq \text{SNR}_p,$$

for any $p$. As expected, the SNR of the multi-tap receiver can be improved compared to the single-tap receiver.

C. Practical Implementation

Without loss of generality, we assume that $\eta[p] = 1$ for simplicity. Then, by substituting (10) into (17), we can obtain

$$\hat{s}(t) = \frac{1}{M} e^{H}(-t) * x(t),$$

where (1) is used for equation. The receiver structure for (22) is shown as in Fig. 3. From the figure, the multi-tap receiver is similar to the traditional SC receiver except that there is no need for the equalizer and only MFs are enough for SC demodulation [12].

Actually, the receiver structure in Fig. 3 is only a theoretical one, and it is in general difficult to implement since the tap delays and gains (or equivalently, CIR, $c_m(t)$) are usually unknown in advance [10]. For practical receiver implementation, we can adopt a twice oversampling based
receiver structure as in Fig. 4 because it can achieve satisfied performance with lower complexity than the ones with higher sampling rates [12], [13]. In the figure, \( c_m[k] \) denotes the discrete CIR, that is

\[
c_m[k] = \int c_m(t) \cdot \text{sinc}\left(\frac{2\pi t}{T} - k\pi\right) dt. \tag{23}
\]

For each antenna, the top and the bottom branches correspond to samples at \( t = kT \) and \( t = kT + T/2 \), respectively, the combination of which forms a twice oversampling receiver. From the figure, the LSA-SC receiver can be implemented equivalently through discrete CIR, \( c_m[k] \), which can be obtained through channel estimation techniques [14].

**IV. IMPACT OF NON-IDEAL ENVIRONMENTS**

The derivation of an LSA-SC system in Section III assumes that the antenna number is infinite and the channels at different antennas are independent. In practical systems, however, the antenna number is always finite and the channels at different antennas will be also correlated when placing hundreds of antennas in a small area. Therefore, we will investigate the impact of non-ideal environments on the LSA-SC system in this section.
A. Spatial Correlation

Without loss of generality, we consider a typical uniform-linear-array (ULA), where \( M \) antennas are placed along a line with length \( D \) as in Fig. 5. The spatial correlation analysis here is also valid for any other form of antenna arrays, and the correlation function will then depend on the geometry of the array. In the ULA, the distance between the \( m \)-th and the \( m_0 \)-th antennas will be \( d = (m - m_0)D/(M - 1) \). Denote the spatial correlation between any two antennas with distance, \( d \), to be

\[
\rho(d/\lambda) = \rho \left( \frac{m - m_0}{M - 1} \cdot \frac{D}{\lambda} \right).
\]

The particular form of \( \rho(\cdot) \) depends on the surrounded environments. If the power azimuth spectrum is a constant from 0 to \( 2\pi \)~\cite{15}, then

\[
\rho(d/\lambda) = J_0(2\pi d/\lambda),
\]

where \( J_0(\cdot) \) is the zeroth-order Bessel function of the first kind.

In the presence of spatial correlation, we have

\[
E \left\{ \alpha_m[l] \alpha_{m_0}^*[p] \right\} = \sigma_l^2 \rho \left( \frac{m - m_0}{M - 1} \cdot \frac{D}{\lambda} \right) \delta[l - p].
\]

Accordingly, the correlation matrix for the tap vectors, \( \alpha[l] \), is given by

\[
E \left\{ \alpha[l] \alpha_H^*[p] \right\} = \sigma_l^2 \delta[l - p] R,
\]
where $R$ is the spatial correlation matrix, with $(m, m_0)$-th entry being

$$[R]_{(m, m_0)} = \rho \left( \frac{m - m_0}{M - 1} \cdot \frac{D}{\lambda} \right).$$

(28)

Since the channels at different antennas are not independent, the LSA property in (4) is not valid any more and thus the ITI cannot be completely removed, leading to residual ISI to the desired symbol.

**B. Residual ISI**

Similar to (11), the overall recovered waveform after combination can be rewritten as

$$\hat{s}(t) = \frac{1}{M} \sum_p \|\alpha[p]\|^2 s(t) + \frac{1}{M} \sum_p \sum_{p \neq l} \alpha^H[p] \alpha[l] s(t - \tau_l + \tau_p) + \sum_p w_p(t),$$

(29)

where the first term in (29) is the desired waveform, the second term is the overall ITI, and the third term is the additive noise.

If the antenna number is infinite and the complex tap gains corresponding to different antennas are independent, the ITI can be completely suppressed, as we have discussed in Section III. However, in the case of finite antenna number or in the presence of spatial correlation, ITI cannot be suppressed completely, causing residual ISI to the desired symbol.

To analyze residual ISI, we rewrite the received symbol, from Fig. 3, in a symbol spaced form as

$$\hat{s}[k] = f[0]s[k] + \sum_{n \neq 0} f[n]s[k - n] + w[k],$$

(30)

where $f[n]$ is the overall discrete impulse response observed by the receiver, that is,

$$f[n] = \frac{1}{M} \sum_l \sum_p \alpha^H[p] \alpha[l] g(nT - \tau_l + \tau_p),$$

(31)

and $w[k]$ is given by

$$w[k] = \sum_p w_p(kT).$$

(32)
In (30), the first term is the desired symbol, the second term is the residual ISI, and the third term is the additive noise.

In the ideal case, that is, the antenna number $M \to \infty$ and the channels at different antennas are independent, the complex gain vectors for different taps will be asymptotically orthogonal and thus $f[n] = \delta[n]$. As a result, the ISI in (30) can be completely suppressed.

In the non-ideal environments, the LSA property in (4) is not available any more. In this case, $f[n] \neq \delta[n]$ and there exists ISI. The residual ISI can be expressed as

$$I[k] = \sum_{n \neq k} f[n]s[k - n].$$  \hspace{1cm} (33)

Direct calculation from the Appendix shows that the average power of residual ISI can be expressed as

$$P_{\text{ISI}} = \mathbb{E}\{ |I[k]|^2 \}$$

$$= P_0 \sum_{n \neq 0} \sum_p \sum_l \sigma_p^2 \sigma_l^2 g^2 (nT - \tau_l + \tau_p),$$  \hspace{1cm} (34)

where

$$P_0 = \frac{\text{tr}\{ R^2 \}}{M^2}. $$  \hspace{1cm} (35)

In (34), $P_{\text{ISI}}$ is determined by the spatial correlation through $P_0$ and the power delay profile. To get more insights, we consider the following three different cases.

1) Spatially Independent Taps: If the channels at different antennas are spatially independent, we have $R = I$ and thus $P_0 = 1/M$. Therefore,

$$P_{\text{ISI}} = \frac{1}{M} \sum_{n \neq 0} \sum_p \sum_l \sigma_p^2 \sigma_l^2 g^2 (nT - \tau_l + \tau_p),$$  \hspace{1cm} (36)

From (36), the power of residual ISI is inverse proportional to the number of antennas when the channels at different antennas are independent, and therefore $P_{\text{ISI}} = 0$ when $M \to \infty$. 

2) \textit{Spatially Correlated Taps:} If the channels at different antennas are correlated, \( \text{tr}\{R^2\} \) can be rewritten, from the Appendix, as

\[
\text{tr}\{R^2\} = \sum_{m=-(M-1)}^{M-1} \rho^2 \left( \frac{m}{M-1} \cdot \frac{D}{\lambda} \right) (M - |m|).
\] (37)

Therefore, when \( M \to \infty \), \( P_0 \) can be rewritten as

\[
P_0 = \int_{-1}^{+1} \rho^2 \left( \frac{x D}{\lambda} \right) (1 - |x|) dx > 0.
\] (38)

It means that in the presence of spatial correlation, the ISI cannot be completely suppressed even when the antenna number is large enough.

For the typical Jakes correlation in (25), direct calculation of (38) yields that \[16\]

\[
P_0 = 2 \cdot 2F_3 \left( \frac{1}{2}, \frac{1}{2}; 1, 1, \frac{3}{2}; -\frac{4\pi^2 D^2}{\lambda^2} \right) - J_0^2 \left( \frac{2\pi D}{\lambda} \right) - J_1^2 \left( \frac{2\pi D}{\lambda} \right)
\approx 2 \cdot 2F_3 \left( \frac{1}{2}, \frac{1}{2}; 1, 1, \frac{3}{2}; -\frac{4\pi^2 D^2}{\lambda^2} \right),
\] (39)

where \( 2F_3(\cdot;\cdot;\cdot) \) is the generalized hypergeometric function and \( J_1(\cdot) \) denotes the first order Bessel function of the first kind. The last identity is due to the fact that \( J_0(\cdot) \) and \( J_1(\cdot) \) are very small when \( D/\lambda \) is large enough. It is easy to verify that \( 2F_3(\cdot;\cdot;\cdot) \) increases as the decreasing of \( D/\lambda \), and therefore the ISI power can be reduced by using longer ULA.

3) \textit{Integer Tap Delays:} In this case, we assume that the tap delays are integer times of symbol duration, that is, \( \tau_i = lT \). Then, we have \( g^2(nT - \tau_i + \tau_p) = \delta[n - l + p] \), and substituting this identity into (34), we can obtain

\[
P_{\text{ISI}} = P_0 \left( 1 - \sum_{i=0}^{L} \sigma_i^4 \right),
\] (40)

which shows that the ISI power, \( P_{\text{ISI}} \), is always less than \( P_0 \).

V. Simulation Results

In this section, numerical results are presented to evaluate the proposed LSA-SC system. In the simulation, we considered a \textit{quadrature-phase-shift-keying} (QPSK) modulated SC signal with
Fig. 6. SER versus SNR with different taps.

roll-off factor $\beta = 0.25$ and symbol duration $T = 0.2 \mu s$. The BS is equipped a ULA with $M = 100$ antennas without specification. A normalized extended typical urban (ETU) channel model [17] is used in the simulation, which has 9 taps with maximum delay spread 5 $\mu s$. Both spatially independent and correlated channels are considered in the simulation. For practical implementation of the LSA-SC receiver, we adopt a twice oversampling based approach where the MF front-end can be implemented in a discrete manner [13].

Fig. 6 shows the symbol-error rate (SER) performance versus the SNR with independent channels where $L_0$ is the number of the strongest discrete taps used for demodulation at the receiver. The SERs for the matched filter bound (MFB) [18] and AWGN channel are both included in the figure as benchmarks. From the figure, the LSA-SC can work in small SNR zone due to the SNR gain of antenna array [6]. By adopting more discrete taps, the SER performance will be improved as expected. From the figure, using only five strongest discrete taps can result in a performance degradation within about 0.25 dB compared to the MFB when $\text{SER} = 1\%$ because most power of channel response is concentrated on those taps. On the other hand, the
remain taps contain little powers and thus cause little improvement.

Fig. 7 shows SER versus SNR when channels are spatially correlated. In this figure, we use $L_0 = 3$ discrete branches for demodulation. From the figure, SER improves by increasing the array length since the tap gains corresponding to different antennas are less correlated for larger array length, which coincides with our analysis in Section IV.

Fig. 8 shows SER versus array length. From the figure, SER can be improved by increasing the array length. When $D = 50\lambda$, the SER can be hardly improved by further increasing the array length since the tap gains corresponding to different antennas are sufficiently independent. This corresponds to a distance of $\lambda/2$ between adjacent antennas, which is exactly the Nyquist sample distance in space domain \[19\].

Fig. 9 shows SER versus antenna numbers with $L_0 = 3$ and $E_s/N_0 = -10$ dB. From the figure, SER reduces as the increasing of the antenna numbers. However, severe performance degradation can be observed due to spatial correlation.
Fig. 8. SER versus array length with $E_s/N_0 = -8$ dB.

Fig. 9. SER versus antenna number with $E_s/N_0 = -10$ dB with $L_0 = 3$. 
VI. CONCLUSIONS

We have investigated SC modulation for an LSA system in this paper and analyzed its performance degradation due to non-ideal environments. The multipath propagation in frequency selective channels can be suppressed through antenna array, and therefore the equalizer is not needed in the LSA-SC receiver. We have proposed to recover the waveform, rather than the symbol, for receiver design. If the waveform is designed to satisfy the Nyquist condition, the symbols can be easily recovered with baud-rate sampling once the waveform is obtained. In practical systems where the antenna number is finite and the channels are spatially correlated, we have also analyzed the resulted impacts of those non-ideal environments. Our work shows that the LSA-SC is a good combination of LSA and SC under the criterion of waveform recovery, and it can be used in future wireless systems to deal with the drawbacks of OFDM.

APPENDIX

Assuming the transmit symbols are independent, $P_{\text{ISI}}$ can be given by

$$P_{\text{ISI}} = \sum_{n \neq 0} E \{|f[n]|^2\}. \quad (A.1)$$

From (31), $E \{|f[n]|^2\}$ can be expressed by

$$E \{|f[n]|^2\} = \frac{1}{M^2} \sum_{p_1} \sum_{l_1} \sum_{p_2} \sum_{l_2} g(nT - \tau_{l_1} + \tau_{p_1}) g(nT - \tau_{l_2} + \tau_{p_2}) \cdot \sum_{m_1=1}^{M} \sum_{m_2=1}^{M} E\{\alpha_{m_1}[l_1]\alpha_{m_1}^*[p_1]\alpha_{m_2}^*[l_2]\alpha_{m_2}[p_2]\}. \quad (A.2)$$

Using the moment and cumulant relation [20], we have

$$E\{\alpha_{m_1}[l_1]\alpha_{m_1}^*[p_1]\alpha_{m_2}^*[l_2]\alpha_{m_2}[p_2]\} =$$

$$\text{Cum}\{\alpha_{m_1}[l_1], \alpha_{m_1}^*[p_1], \alpha_{m_2}^*[l_2], \alpha_{m_2}[p_2]\} + E\{\alpha_{m_1}[l_1]\alpha_{m_1}^*[p_1]\} E\{\alpha_{m_2}^*[l_2]\alpha_{m_2}[p_2]\} +$$

$$E\{\alpha_{m_1}[l_1]\alpha_{m_2}^*[l_2]\} E\{\alpha_{m_1}^*[p_1]\alpha_{m_2}[p_2]\} + E\{\alpha_{m_1}[l_1]\alpha_{m_2}[p_2]\} E\{\alpha_{m_1}^*[p_1]\alpha_{m_2}^*[l_2]\}. \quad (A.3)$$
Since $\alpha_{m_1}[l_1], \alpha_{m_1}^*[p_1], \alpha_{m_2}^*[l_2], \alpha_{m_2}[p_2]$ are Gaussian distributed random variables, we have

$$\text{Cum}\{\alpha_{m_1}[l_1], \alpha_{m_1}^*[p_1], \alpha_{m_2}^*[l_2], \alpha_{m_2}[p_2]\} = 0, \quad (A.4)$$

and thus

$$E\{\alpha_{m_1}[l_1]\alpha_{m_1}^*[p_1]\alpha_{m_2}^*[l_2]\alpha_{m_2}[p_2]\} =$$

$$E\{\alpha_{m_1}[l_1]\alpha_{m_1}^*[p_1]\} E\{\alpha_{m_2}^*[l_2]\alpha_{m_2}[p_2]\} +$$

$$E\{\alpha_{m_1}[l_1]\alpha_{m_2}^*[l_2]\} E\{\alpha_{m_1}^*[p_1]\alpha_{m_2}[p_2]\} +$$

$$E\{\alpha_{m_1}[l_1]\alpha_{m_2}[p_2]\} E\{\alpha_{m_1}^*[p_1]\alpha_{m_2}^*[l_2]\}. \quad (A.5)$$

From (26), we have

$$E\{\alpha_{m_1}[l_1]\alpha_{m_1}^*[p_1]\} E\{\alpha_{m_2}^*[l_2]\alpha_{m_2}[p_2]\} = \sigma_{l_1}^2 \sigma_{l_2}^2 \delta[l_1 - p_1] \delta[l_2 - p_2], \quad (A.6)$$

$$E\{\alpha_{m_1}[l_1]\alpha_{m_2}^*[l_2]\} E\{\alpha_{m_1}^*[p_1]\alpha_{m_2}[p_2]\} = \sigma_{l_1}^2 \sigma_{p_1}^2 \rho^2 \left( \frac{m_1 - m_2}{M - 1} \cdot \frac{D}{\lambda} \right) \delta[l_1 - l_2] \delta[p_1 - p_2]. \quad (A.7)$$

Meanwhile, note that

$$E\{\alpha_{m_1}[l_1]\alpha_{m_2}[p_2]\} = E\{\alpha_{m_1}^*[p_1]\alpha_{m_2}^*[l_2]\} = 0. \quad (A.8)$$

Substituting (A.6) to (A.8) into (A.5),

$$E\{\alpha_{m_1}[l_1]\alpha_{m_1}^*[p_1]\alpha_{m_2}^*[l_2]\alpha_{m_2}[p_2]\}$$

$$= \sigma_{l_1}^2 \sigma_{l_2}^2 \delta[l_1 - p_1] \delta[l_2 - p_2] + \sigma_{l_1}^2 \sigma_{p_1}^2 \rho^2 \left( \frac{m_1 - m_2}{M - 1} \cdot \frac{D}{\lambda} \right) \delta[l_1 - l_2] \delta[p_1 - p_2]. \quad (A.9)$$

and thus

$$E \{ |f[n]|^2 \} = g^2(nT) + \frac{\text{tr} \{ R^2 \}}{M^2} \sum_p \sum_l \sigma_{p_l}^2 \sigma_{l}^2 g^2(nT - \tau_l + \tau_p). \quad (A.10)$$
where we used the identity

$$\sum_{m_1=1}^{M} \sum_{m_2=1}^{M} \rho^2 \left( \frac{m_1 - m_2}{M - 1} \cdot \frac{D}{\lambda} \right) = \text{tr}\{ \mathbf{R}^2 \},$$  \hspace{1cm} (A.11)$$

for equation (A.10).

Finally, substituting (A.10) into (A.1), $P_{\text{ISI}}$ is obtained as

$$P_{\text{ISI}} = \frac{\text{tr}\{ \mathbf{R}^2 \}}{M^2} \sum_{n \neq 0} \sum_{p} \sum_{l} \sigma_p^2 \sigma_l^2 g^2(nT - \tau_l + \tau_p),$$  \hspace{1cm} (A.12)$$

since $g(nT) = 0$ for $n \neq 0$, which is exactly (34).

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