Some Properties of Domain Wall Solution in the Randall-Sundrum Model

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Abstract
Properties of the domain wall (kink) solution in the 5 dimensional Randall-Sundrum model are examined both analytically and numerically. The configuration is derived by the bulk Higgs mechanism. We focus on 1) the convergence property of the solution, 2) the stableness of the solution, 3) the non-singular property of the Riemann curvature, 4) the behaviours of the warp factor and the Higgs field. It is found that the bulk curvature changes the sign around the surface of the wall. We also present some exact solutions for two simple cases: a) the no potential case, b) the cosmological term dominated case. Both solutions have the (naked) curvature singularity. We can regard the domain wall solution as a singularity resolution of the exact solutions.

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1 Introduction

There exist two standpoints in the treatment of the domain world physics. One is that the geometry should be singular and the domain is regarded as a defect. In the original work by Randall and Sundrum[1], the walls stand on the fixed points of the $S_1/Z_2$ orbifold in the form of $\delta$-function. Recently the renormalization of a bulk-boundary system is discussed in this standpoint[2] where the conical singularity at the center of the extra 2 dim space is regarded as a domain of the vortex type. The other standpoint is that the geometry should be non-singular. The domain is regarded as a soliton. This approach looks natural if the domain world is "derived" from the more fundamental theory of the D-brane. Similar situations occurred in the past literature, such as the relation between the Dirac string[3] and 'tHooft-Polyakov string[4, 5]. (For the situation about the (topological) defect and the soliton in relation to the domain world, see a good review [6].) At present both standpoints look important to understand the brane world physics. We take here the latter standpoint.

In ref.[7], the domain wall configuration of the RS-model is realized as a soliton (kink) solution in the bulk (5D) Higgs potential. It has some advantageous points, compared with the $\delta$-function description, such as non-singularity and stability. The solution is obtained in the form of the infinite power-series of some hyperbolic function. The convergence of the coefficient-series is crucial for the boundary condition (b.c.) to be satisfied. It was checked by explicitly calculating the coefficients at the 2nd order. We present here the 6th order calculation result, and reconfirm the convergence property further strongly. The main purpose of this paper is to strengthen the content of ref.[7] by presenting the various results in a concrete way. Some physical quantities, such as the bulk scalar curvature, are obtained. Very interestingly, the curvature changes its sign near the "surface" of the domain wall. In order to clarify the structure of the solution, we first present some exact solutions for simple cases. They clearly show the origin of some integration constants and free parameters. These exact solutions have (naked) singularities. They tell us the Higgs potential is important for the non-singular property of the configuration. It plays the role of singular resolution.

We consider one-wall model which was considered in [8]. An interesting stable (kink) solution exists for a family of vacua. As explained in [7], the solution does not miss the key points of the original one. Similar analysis
was successfully done in the 6 dim model[9].

As some recent related works, we find [10, 11].

In Sec.2, the RS domain wall model is explained. The exact solution is presented for the no-potential case in Sec.3, and for the cosmological-constant dominated case in Sec.4. The general case of the Higgs potential is examined in Sec.5, where the domain-wall solution is obtained as a oneparameter family of kink solutions. It is the analytical solution. The concrete values of parameters and coefficients are obtained in the 6th order calculation. Numerical analysis also confirms the obtained solution.

2 Randall-Sundrum model with the bulk Higgs field

We consider, as the brane world, the following 5D gravity-Higgs theory.

\[
S[G_{AB}, \Phi] = \int d^5X \sqrt{-G} \left( -\frac{1}{2}M^3 \hat{R} - \frac{1}{2}G^{AB} \partial_A \Phi \partial_B \Phi - V(\Phi) \right),
\]

\[
V(\Phi) = \frac{\lambda}{4}(\Phi^2 - v_0^2)^2 + \Lambda,
\]

where \(X^A(A = 0, 1, 2, 3, 4)\) is the 5D coordinates and we also use the notation \((X^A) \equiv (x^\mu, y)\), \(\mu = 0, 1, 2, 3\). \(X^4 = y\) is the extra axis which is taken to be a space coordinate. The signature of the 5D metric \(G_{AB}\) is \((-++++)\). \(\Phi\) is a 5D Higgs (scalar) field, \(G = \det G_{AB}\), \(\hat{R}\) is the 5D Riemannian scalar curvature. \(M\) and \(V(\Phi)\) are the 5D Planck mass and the Higgs potential respectively. The three parameters \(\lambda, v_0\) and \(\Lambda\) in \(V(\Phi)\) are called here vacuum parameters. \(\lambda(> 0)\) is a coupling, \(v_0(> 0)\) is the Higgs field vacuum expectation value, and \(\Lambda\) is the 5D cosmological constant.

Assuming the Poincaré invariance in the brane, the line element can be written as

\[
ds^2 = e^{-2\sigma(y)}\eta_{\mu\nu}dx^\mu dx^\nu + dy^2 \equiv G_{AB}dX^AdX^B,
\]

where \(\eta_{\mu\nu} = \text{diag}(-1, 1, 1, 1)\). \((G_{AB})\) is explicitly written as

\[
(G_{AB}) = \begin{pmatrix}
e^{-2\sigma} \eta_{\mu\nu}, & 0 \\
0, & 1
\end{pmatrix}, \quad \sqrt{-G} = e^{-4\sigma}.
\]
The 5D Einstein equation gives us

\[-6M^3(\sigma')^2 = \frac{1}{2}(\Phi')^2 + V,\]  
\[3M^3\sigma'' = (\Phi')^2,\]  

where \(\prime=d/dy\) and we consider the case that \(\Phi\) depends only on the extra coordinate \(y; \Phi = \Phi(y)\). The above equations are 1) translation invariant \((y \to y + \text{const.})\); 2) \(Z_2\) symmetric \((y \to -y, \sigma' \to -\sigma', \Phi \to \pm \Phi)\); 3) even with respect to the \(\Phi\)-reflection \((\Phi \leftrightarrow -\Phi)\). Besides they are 4) global scale invariant, when some vacuum parameters change appropriately:

\[y \to ky, \quad \lambda \to \frac{\lambda}{k^2}, \quad \Lambda \to \frac{\Lambda}{k^2}, \quad v_0 \to v_0,\]  

where \(k\) is a constant. This invariance says the scale of \(y\) can be adjusted by the scaling of \(\lambda\) and \(\Lambda\). Note that the scaling power is independent of their mass-dimensions.

Eq.(5) gives an important positivity relation,

\[\sigma'' = \frac{1}{3M^3}(\Phi')^2 \geq 0,\]  
\[3M^3\{\sigma'|_{y=y_2} - \sigma'|_{y=y_1}\} = \int_{y_1}^{y_2}(\Phi')^2 dy \geq 0, \quad y_1 < y_2,\]  

where non-singularity of \(\sigma''\) is assumed in the region \(y_1 < y < y_2\). This relation will serve as a consistency check of the solutions. We will also focus on the (non)singularity of the bulk curvature: \(\tilde{R} = -8\sigma'' + 20\sigma'^2\).

As the extra space (the fifth dimension), we take the real number space \(\mathbb{R} = (-\infty, +\infty)\). This is a simplified version of the original RS-model\(^2\) and was examined in the subsequent work\(^3\).

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\(^2\) The mass-dimensions of the vacuum parameters, \(\lambda, \Lambda\) and \(v_0\), are \((\text{mass})^{-1}, \text{mass}^5\), and \((\text{mass})^{3/2}\) respectively.

\(^3\) For a general argument about the consistency of the domain world configuration, see \([12]\).

4
3 Exact Solution for the No Potential Case

Let us consider the case of no potential, $V = 0$; $\lambda = \Lambda = 0$. Then eq.(4,5) reduce to

$$-6M^3(\sigma')^2 = -\frac{1}{2}(\Phi')^2 , \quad 3M^3\sigma'' = (\Phi')^2 .$$

(8)

\(\sigma(y)\) is solved as

$$\sigma' = \frac{1}{A - 4y} , \quad \sigma = -\frac{1}{4} \ln \frac{|A - 4y|}{B} , \quad B > 0 ,$$

(9)

where $A(-\infty < A < \infty)$ and $B(>0)$ are integration constants. The constant, $A$, comes from the translation invariance of (8). $B$ comes from the global scale invariance of (8). The line element is given by

$$ds^2 = \sqrt{|A - 4y|}B \eta_{\mu\nu}dx^\mu dx^\nu + dy^2 .$$

(10)

\(\Phi'\) and $\Phi$ are solved as

$$\Phi' = \pm \frac{\sqrt{12M^3}}{A - 4y} , \quad \Phi = \mp \frac{\sqrt{3M^3}}{2} \ln \frac{|A - 4y|}{C} , \quad C > 0 ,$$

(11)

where $C(>0)$ is another integration constant. The plural signs come from the evenness of (8) under the ”$\Phi$-reflection”: $\Phi \leftrightarrow -\Phi$. The Higgs field, $\Phi(y)$, does not go to a constant in the asymptotic region $|y| \to \infty$, which should be compared with other solutions obtained later.

The 5D Riemann scalar curvature is obtained as

$$\hat{R} = \frac{-12}{(A - 4y)^2} < 0 ,$$

(12)

which is 1) negative definite, 2) singular at \(y = A/4\) and 3) vanishes for $|y| \to \infty$. The metric (10) has no horizon, hence this curvature singularity is a naked one.

The obtained solution has unwanted properties and can not be used as the brane world model. This model is too simple. It has, however, some common or comparative features to the more realistic solutions of later sections in some points such as 1) the appearance of some integration constants
in relation to some symmetries of the field equations, 2) the plural signs, 3) (naked) curvature singularity, 4) no horizon. Furthermore a suggestive relation to the R-S metric can be found by considering the region: $|A| \gg |y|$. In this case the metric (10) reduces to

$$ds^2 \approx \sqrt{|A|} e^{-\frac{1}{2}y} \eta_{\mu\nu} dx^\mu dx^\nu + dy^2, \quad |A| \gg |y|,$$

which looks like the RS-type metric although we cannot take $|y| \to \infty$.

## 4 Exact Solution for the Cosmological Term Dominated Case

Let us examine a little more general case, that is, the cosmological term dominated case: $\lambda = 0$.

$$-6M^3 (\sigma')^2 = -\frac{1}{2}(\Phi')^2 + \Lambda,$$

$$3M^3 \sigma'' = (\Phi')^2.$$  \hfill (14)

$$3M^3 \sigma'' = (\Phi')^2.$$  \hfill (15)

We have interest in the case: $\Phi \to \text{const. as } y \to \pm \infty$. Then, from (14), $\Lambda \leq 0$.

From above equations, we get $3M^3 \sigma''/(12M^3 \sigma'^2 + 2\Lambda) = 1$, which can be integrated as

$$\sigma' = \frac{1 + Ae^{8\omega y}}{1 - Ae^{8\omega y}} \omega = \sqrt{-\frac{\Lambda}{6M^3}}$$

where $A$ is an integration constant ($-\infty < A < \infty$) which comes from the translation invariance of (14,15).

### (i) $A < 0$

The solution (16) can be written as

$$\sigma' = -\omega \tanh 4\omega (y - y_*) \quad |A| \equiv e^{-8\omega y_*}.$$  \hfill (17)

It is attractive that this solution is non-singular. But it contradicts with the positivity relation (7). Hence we conclude, in this case (i), there does not
exist a consistent solution.

(ii) $A = 0$
This case is solved as

$$\sigma' = \omega, \quad \sigma = \omega y + B, \quad \Phi = v_0,$$  \hspace{1cm} (18)

where $B$ and $v_0$ are another integration constants. The 5D curvature is a positive constant everywhere.

$$\hat{R} = \frac{10}{3} \frac{(-\Lambda)}{M^3} > 0.$$  \hspace{1cm} (19)

The geometry is Anti de Sitter space.

(iii) $A > 0$

(16) is singular at $y = -(\ln A)/8\omega \equiv y_*$. The solutions are obtained as

$$\sigma' = -\omega \coth 4\omega (y - y_*),$$
$$\Phi' = \pm \frac{\sqrt{-2\Lambda}}{\sinh(4\omega (y - y_*))},$$

$$\Phi = \pm \frac{\sqrt{3M^3}}{2} \ln |\tanh(2\omega(y - y_*))| + v_0,$$  \hspace{1cm} (20)

where $v_0$ is an integration constant. Fields asymptotically behave as $\sigma' \to \mp \omega, \quad \Phi \to v_0$ when $y - y_* \to \pm \infty$. The 5D scalar curvature is obtained as

$$\hat{R} = \frac{-\Lambda}{M^3} \left\{ \frac{10}{3} - \frac{2}{\{\sinh(4\omega (y - y_*))\}^2} \right\},$$  \hspace{1cm} (21)

which is singular at $y = y_*$. It asymptotically behaves as $\hat{R} \to -\frac{10}{3} \frac{\Lambda}{M^3}$ when $|y - y_*| \to \infty$. Integrating $\sigma'$ in eq.(20), the line element is obtained as

$$ds^2 = B \sqrt{\sinh(4\omega (y - y_*))} \eta_{\mu\nu} dx^\mu dx^\nu + dy^2,$$  \hspace{1cm} (22)

where $B$ is another integration constant. From this result, we see there is no horizon. The curvature singularity at $y = y_*$ is the naked one. In the region far from the singularity ($4\omega |y - y_*| \gg 1$), the line element can be
approximated as \( ds^2 \approx (B/\sqrt{2})e^{2\omega|y-y^*|} \eta_{\mu\nu}dx^\mu dx^\nu + dy^2 \), which is similar to the RS metric except the sign of the exponent.

We can check the solution of (iii) goes to (ii) as \( A \to +0(y\to \infty) \). We can further check the solution (iii) is continuously connected to the no potential case of Sec.3 by taking \( \Lambda \to -0 \). The parameter \( A \) in Sec.3 (≡ \( A^{\text{no-pot}} \)) is related to \( A \) of (13) as \( 1/A^{\text{no-pot}} = \omega(1+A)/(1-A) \). In the point that \( \Phi, \sigma' \) and \( \hat{R} \) become (non-zero) constants in the asymptotic region, the solution of this section approaches, compared with Sec.3, a realistic one.

In Sec.3 and 4, we have obtained the exact solutions. They become RS-type solutions for special regions of \( y \). (It suggests that further ”deformation” of the potential makes us find the domain wall solution. Indeed it will do so.) The solutions, however, still have a bad property of (naked) curvature singularity. To seek a non-singular solution, we must take into account all vacuum parameters \( \lambda, v_0 \) and \( \Lambda \). That is the following subject.

5 Domain Wall Solution

Let us solve the 5D Einstein equations (4,5) for the general case of vacuum parameters. We impose the following asymptotic behaviour (boundary condition) for the Higgs field \( \Phi(y) \).

\[
\Phi(y) \to \pm v_0 \quad , \quad y \to \pm \infty
\]

This means \( \Phi' \to 0 \), and from (13), \( \sigma'' \to 0 \). From this result and (4), we are led to \( \sigma' \to \pm \omega, \sigma \to \omega|y| \) as \( y \to \pm \infty \), where \( \omega(>0) \) is some constant. It can be fixed, by considering \( y \to \pm \infty \) in (13), as

\[
\omega = \sqrt{-\Lambda/6} M^{-3/2}, \quad \Lambda \leq 0
\]

where we see the sign of \( \Lambda \) must be negative, that is, the 5D geometry must be anti de Sitter in the asymptotic regions. We may set \( M = 1 \) without ambiguity. (Only when it is necessary, we explicitly write down \( M \)-dependence.)

We notice, in the results of previous sections, \( \sigma' \) and \( \Phi \) behave in a comparative way. Let us take the following form for \( \sigma'(y) \) and \( \Phi(y) \) as a solution.

\[
\sigma'(y) = \omega \sum_{n=0}^{\infty} \frac{c_{2n+1}}{(2n+1)!}\{\tanh(ky+l)\}^{2n+1}
\]
\[ \Phi(y) = v_0 \sum_{n=0}^{\infty} \frac{d_{2n+1}}{(2n+1)!} \{ \tanh(ky + l) \}^{2n+1}, \quad (25) \]

where \( c \)'s and \( d \)'s are coefficient-constants to be determined. \(^4\) The odd-power terms are only taken here using the \( Z_2 \) symmetry of (4,5) and the boundary conditions explained above. The free parameter \( l \) comes from the translation invariance of (4) and (5). A new mass scale \( k(>0) \) is introduced here as a free parameter to make the quantity \( ky \) dimensionless. The freedom comes from the global scale invariance of (4) and (5). The physical meaning of \( 1/k \) is the thickness of the domain wall. The parameter \( k \) plays a central role in the dimensional reduction scenario\(^7\). The distortion of 5D space-time geometry by the existence of the domain wall should be small so that the quantum effect of 5D gravity can be ignored and the present classical analysis is valid. This requires the condition\(^7\)

\[ k \ll M. \quad (26) \]

Besides \( M = 1 \), we can also take \( k = 1 \) without ambiguity (keeping the relation (23) in mind). The coefficient-constants \( c \)'s and \( d \)'s have the following constraints,

\[ 1 = \sum_{n=0}^{\infty} \frac{c_{2n+1}}{(2n+1)!}, \quad 1 = \sum_{n=0}^{\infty} \frac{d_{2n+1}}{(2n+1)!}, \quad (27) \]

which are obtained by considering the asymptotic behaviours \( y \to \pm \infty \) in (25).

We first obtain the recursion relations between the expansion coefficients, from the field equations (4) and (5). For \( n \geq 2 \), they are given by \(^7\)

\[ \frac{c_{2n+1}}{(2n)!} - \frac{c_{2n-1}}{(2n-2)!} = \frac{v_0^2}{\sqrt{-3\Lambda/2}} (D'_n - 2D'_{n-1} + D'_{n-2}) , \]

\[ C_{n-1} = -\frac{v_0^2}{2\Lambda} (D'_n - 2D'_{n-1} + D'_{n-2}) + \frac{\lambda v_0^4}{4 \Lambda} (E_{n-2} - 2D_{n-1}) , \quad (28) \]

where

\[ D_n = \sum_{m=0}^{n} \frac{d_{2n-2m+1}d_{2m+1}}{(2n-2m+1)!(2m+1)!}, \quad D'_n = \sum_{m=0}^{n} \frac{d_{2n-2m+1}d_{2m+1}}{(2n-2m)!(2m)!}, \]

\(^4\) Normalization of \( c_{2n+1} \) is different from that of ref.\(^6\). The relation is \( c_{2n+1} = (k/\omega) \times c_{2n+1} \) of ref.\(^6\).
\[ C_n = \sum_{m=0}^{n} \frac{c_{2n-2m+1}c_{2m+1}}{(2n-2m+1)!(2m+1)!} \quad , \quad E_n = \sum_{m=0}^{n} D_{n-m}D_m \]  \hspace{1cm} (29)

The first few terms, \((c_1, d_1), (c_3, d_3)\), are explicitly given as

\[ d_1 = \pm \sqrt{2} \sqrt{\Lambda + \frac{\lambda v_0^4}{4}}, \quad c_1 = \frac{2}{3 \sqrt{\Lambda / 6}} (\Lambda + \frac{\lambda v_0^4}{4}) \]

\[ \frac{d_3}{d_1} = 2 + \left\{ \frac{8}{3} (\Lambda + \frac{\lambda v_0^4}{4}) - \lambda v_0^2 \right\}, \quad \frac{c_3}{c_1} = 2 + \left\{ \frac{16}{3} (\Lambda + \frac{\lambda v_0^4}{4}) - 2\lambda v_0^2 \right\} \]  \hspace{1cm} (30)

where \pm sign in \(d_1\) reflects \(\Phi \leftrightarrow -\Phi\) symmetry in (1) and (3). We take the positive one in the following. We can confirm that the above relations, (28) and (30), determine all \(c\)'s and \(d\)'s recursively in the order of increasing \(n\). In (28) and (30), \(M = k = 1\) is taken for simplicity. Their dependence is easily recovered by \(\Lambda \rightarrow \Lambda / k^2 M^3\), \(\lambda \rightarrow \lambda M^3 / k^2\), \(v_0 \rightarrow v_0 / \sqrt{M^3}\). Note that all coefficients, derived above, are solved and are described by the three (dimensionless) vacuum parameters. Among the 3 parameters, there exist 2 constraints from (27). Hence the present solution is one-parameter family solution.

In order for this solution to make sense, as seen from the expression for \(d_1\), the 5D cosmological term \(\Lambda\) should be bounded both from below and from above.

\[ -\frac{\lambda v_0^4}{4} < \Lambda < 0 \]  \hspace{1cm} (31)

The presence of this relation says the non-singular solution presented here cannot continuously connect with the singular solutions of Sec. 3 (\(\lambda = \Lambda = 0\)) and of Sec. 4 (\(\lambda = 0\)).

6 Evaluation of Coefficients and Numerical Check of Analytic Results

We present here the results of concrete values of \(c\)'s and \(d\)'s for two input values \(v_0 = 1.0\) and \(v_0 = 1.6\). We solve constraints (27) by taking the first 7 terms (up to \(n=6\)th order). The most important point is to confirm the convergence of the infinite series \(\sum_{n=0}^{\infty} \frac{c_{2n+1}}{(2n+1)!}\) and \(\sum_{n=0}^{\infty} \frac{d_{2n+1}}{(2n+1)!}\), which
Fig. 1 The values of $c_{2n+1}/(2n+1)!$ (blob), $d_{2n+1}/(2n+1)!$ (small circle). The large circles show $(0.25)^n$. ($n = 0, 1, \ldots, 6$) $v_0 = 1.0$ (input).

guarantees the present boundary condition. The 6th-order approximation calculation gives the vacuum parameters as

\[ v_0 = 1.0 \text{ (input)} \quad , \quad \Lambda = -0.272 \quad , \quad \lambda = 2.88 \quad , \]
\[ v_0 = 1.6 \text{ (input)} \quad , \quad \Lambda = -1.50825 \quad , \quad \lambda = 1.49925 \quad . \tag{32} \]

The input value $v_0 = 1.6$ is quite near the case of \[7\], and the above results of $\Lambda$ and $\lambda$ are consistent with the previous results. Both cases give similar behaviours, hence we present further results only for $v_0 = 1.0$. The obtained values of the coefficients are shown in Fig. 1. In the figure we also plot the data from the geometrical series: $1/(1 - x) = 1 + x + \cdots$ at $x = 0.25$. Comparing them, we can recognize the (rapid) convergence of the coefficient-series (up to this approximation order). Note that two series, \{ $c_{2n+1}/(2n+1)!$ \} and \{ $d_{2n+1}/(2n+1)!$ \}, are 'oscillating'. Using these results, the analytical results of the scalar field $\Phi(y)$ and the warp factor $\sigma'(y)$, (23), are shown in Fig. 2.

\[ 5 \] The constraint is satisfied as $(1 - \sum_{n=0}^{6} \frac{c_{2n+1}}{(2n+1)!})^2 + (1 - \sum_{n=0}^{6} \frac{d_{2n+1}}{(2n+1)!})^2 < 1.1 \times 10^{-7}$ ($v_0 = 1.0$), $1.6 \times 10^{-9}$ ($v_0 = 1.6$). In the search of the solution ($\Lambda, \lambda$), the mesh-size of the parameter-space determines the "precision". The high precision for the $v_0 = 1.6$ case shows the mesh-size is taken much smaller than that of $v_0 = 1.0$.

\[ 6 \] We use the truncated version of the expressions (23) by the first seven terms.
Fig. 2 The analytic result of $\sigma'/\omega$ (dashed line) and $\Phi/v_0$ (normal line). Both are odd with respect to $y \leftrightarrow -y$. The graphs are depicted by using (25) with the 6-th order approximation. $v_0 = 1.0$ (input). The horizontal axis is $ky$.

Scalar curvature is also shown in Fig. 3. It shows the curvature is negative inside the wall, whereas positive outside. There exist two points, $ky \approx \pm 1$, where the curvature vanishes. We see the present solution is non-singular everywhere. The presence of a dip around $y = 0$ (Fig. 3) clearly says that the domain wall exists there.

The kink b.c. for the Higgs field (23) guarantees the stability of the present solution. It reflects the b.c. for $\sigma'$ as specified by the parameter value (24). We can also see the stability from the behaviour of $\hat{R}$ (Fig. 3) as follows. From the expression (25) and the b.c. (27), $\hat{R}$ has the following b.c. in the IR region.

$$|\sigma'| \to \omega \ , \ \sigma'' \to 0 \ , \ \frac{\dot{R}}{\omega^2} \to 20 \ , \ \text{as} \ |y| \to \infty \ . \quad (33)$$

We can also see, from (25), the b.c. in the UV region is

$$\sigma' \to 0 \ , \ \sigma'' \to \omega kc_1 \ , \ \frac{\ddot{R}}{\omega^2} \to -\frac{8k}{\omega}c_1 \ , \ \text{as} \ y \to 0 \ . \quad (34)$$
Fig. 3 (5D) Riemann scalar curvature $\hat{R}/\omega^2$ in the 6th order approximation. It even with respect to $y \leftrightarrow -y$. The horizontal axis is $ky$. $v_0 = 1.0$ (input).

where $c_1 > 0$ from the b.c. of $\sigma'$. Due to the continuity, $\hat{R}$ must have a dip around the origin with a finite thickness.

As the coupled differential equations for $\Phi(y)$ and $\sigma'(y)$, the equations (4,5), have the standard form of the numerical analysis, that is, Runge-Kutta method. We can numerically solve them without any ansatz about the form of the solution. In this numerical analysis, the following two points are important: 1) the choice of three parameters $v_0, \lambda$, and $\Lambda$; 2) the choice of the initial conditions, $\sigma'(y = 0)$ and $\Phi(y = 0)$. As for the point 1) we can borrow the values obtained in the 6th order approximation (32). As for 2), due to the required $Z_2$ symmetry (the oddness under $y \rightarrow -y$) for $\Phi(y)$ and $\sigma'(y)$ (25), we can take $y = 0$ as the initial point of $y$ and the initial conditions $\Phi(0) = \sigma'(0) = 0.0$. The numerical result is shown in Fig. 4. It shows the analytic solution of Fig. 2, based on Sec. 5, is reproduced very well. The numerical output data stop at $ky \sim 3.0$ with producing imaginary values. This occurs because keeping the positivity, $\Phi'(y)^2 \geq 0$, in the numerical analysis becomes so stringent in the infrared region. The quantity becomes so small in that region and vanishes at $k|y| = \infty$. We understand that further

\footnote{For the input $v_0 = 1.0$, the obtained values, $\Lambda/k^2M^3 = -0.272, c_1 = 1.4$, give $-8kc_1/\omega = -52.6$ which is shown in Fig. 3.}
Fig. 4 The numerical results for $\sigma'/\omega$(up) and $\Phi/v_0$(down). They are obtained by Runge-Kutta method. One step value along $ky$-axis is 0.05. About 65 points are plotted for each line in the figure. The initial point is $y = 0$. The horizontal axis is $ky$. 
higher order calculation is required for the values of $\lambda$ and $\Lambda$ (for an input value $v_0$) in order to extend the valid region furthermore.

7 Discussion and conclusion

The assumption about the convergence of the series (25), which was assumed in [7], is strongly confirmed. The present result of the 6th order calculation does not so much deviate from that of the 2nd order one in [7]. It says the truncation approximation of (27) is valid.

We point out some results which are potentially important in phenomenology. The cosmological term has both the upper and the lower bound (31). It is expected to be useful when we fix the parameters $\lambda, v_0$ and $\Lambda$. The sign change of the curvature near the "surface" $k|y| \approx 1$ (Fig.3) could become an important check point of the confirmation of this model.

From the point of singularity resolution[13], the curvature singularity appearing in Sec.3 and Sec.4 is resolved in Sec.5 by a sort of "deformation" (adding the potential terms appropriately). In the procedure two constraints appear among the three vacuum parameters.

The present standpoint is that the domain configuration should be realized in the non-singular geometry. The approach based on the singular geometry, such as the original one[1], can be regarded as a temporary stage of the development. The singularity, often expressed by the $\delta$-function introduced by hand, is expected to be derived by some definite limiting procedure, say, the thickness goes to zero: $k^{-1} \rightarrow 0$. If the string or the D-brane is the fundamental constituents of nature, such extended objects are strongly expected to behave smoothly in the UV-region. Because the domain world physics can be regarded as a transitive approach from the field theory to the string-brane theory, we believe seeking non-singular solutions is important in the development.

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