Yang-Mills thermodynamics: The preconfining phase

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Abstract

We summarize recent nonperturbative results obtained for the thermodynamics of an SU(2) and an SU(3) Yang-Mills theory being in its preconfining (magnetic) phase. We focus on an explanation of the involved concepts and derivations, and we avoid technical details.
Introduction. This is the second one in a series of three papers summarizing the thermodynamics of an SU(2) and an SU(3) Yang-Mills theory as it is analysed non-perturbatively in [1]. The goal is to give a nontechnical presentation of the concepts involved in quantitatively describing the preconfining or magnetic phase. In the phase diagram of either theory this phase is sandwiched inbetween the deconfining phase (bosonic statistics) and the absolutely confining (fermionic statistics) phase. While fundamental, heavy, and fermionic test charges are confined by condensed magnetic monopoles dual gauge modes are massive but propagate.

To derive the phenomenon of quark confinement from a microscopic Lagrangian based on an SU(3) gauge symmetry, that is, from Quantum Chromodynamics (QCD), is a major challenge to human thinking. This is due to the theory being strongly interacting at large distances which disqualifies a perturbative approach to this problem. The difficulties are overwhelming even in the somewhat simplified situation, where quarks are considered heavy and nondynamical test charges that are immersed into pure gluodynamics.

Two main proposals for the confining mechanism are discussed in the literature: The dual Meissner effect, which takes place within a condensate of massless magnetic monopoles [2], and the condensation of infinitely mobile magnetic center-vortex loops [3] implying a dielectricity of the ground state which strongly increases with distance. In its electrically dual form the former mechanism is observed in Nature if a superconducting material is subjected to an external, static magnetic field. Within the Cooper pair condensate the field lines of the latter are forced into thin flux tubes [4]: an immediate consequence of infinitely mobile, that is, stiffly correlated electric charges. In the hypothetical situation, where the magnetic field is sourced by a pair of a heavy magnetic monopole and antimonopole at distance $R$ the squeezing-in of flux lines leads to a linear potential at large $R$ and thus to monopole confinement. A similar situation would hold if the condensate of Cooper pairs would be replaced by infinitely mobil magnetic dipoles. The magnetic dual of this phenomenon is that a condensate of electric dipoles confines a heavy electric charge and its anticharge.

But an electric dipole originates from a circuit of magnetic flux. In an SU(N) Yang-Mills theory the latter is naturally provided by a magnetic center-vortex loop. The confinement mechanism involving a dual superconductor and a center-vortex condensate are mutually exclusive. One of the results in [1] is, however, that either of the two mechanisms belongs to one of the two separate phases with test-charge confinement in SU(2) and SU(3) gluodynamics.

The objective of the present paper is the discussion of a phase of SU(2) and SU(3) gluodynamics whose ground state is a dual superconductor (magnetic phase). We focus on the SU(2) case and mention generalizations to the SU(3) case and the associated results in passing only. In discussing the properties of the ground state and those of its (noninteracting) quasiparticle excitations we pursue the following program:

First, we remind the reader why pairs of nonrelativistic and screened Bogomol’nyi-Prasad-Sommerfield (BPS) [5] magnetic monopoles and antimonopoles arise as spa-
tially isolated defects in the deconfining (electric) phase.

Second, a continuous parameter, namely, the magnetic flux originating from a zero-momentum pair of a monopole and its antimonopole, both located at spatial infinity, is computed in the limit of total screening. The limit of total magnetic charge screening is dynamically reached at the electric-magnetic phase boundary [1 6]. Recall, that total screening means masslessness for each individual monopole or antimonopole and that the projection onto zero three-momentum corresponds to a spatial coarse-graining. The obtained magnetic flux, being an angular parameter, is identified with the euclidean time $\tau$. Since only charge-modulus one monopoles and antimonopoles contribute to the flux the winding number, associated with the phase of a complex scalar field $\varphi$, is $\pm$ unity. In the (at this stage hypothetical but in a later, intermediate step shown to be selfconsistent) absence of interactions between the monopoles and antimonopoles in the condensate the field $\varphi$ is energy- and pressure-free. That is, its euclidean time dependence is BPS saturated. Assuming the existence of an externally provided mass scale $\Lambda_M$, this determines $\varphi$’s BPS equation. The latter, in turn, uniquely fixes $\varphi$’s potential $V_M$. Similar to the case of the adjoint scalar field $\phi$ in the electric phase the statistical and quantum mechanical inertness of the complex field $\varphi$ is established by comparing $V_M$’s curvature with the squares of temperature $T$ and the maximal resolution $|\varphi|$. Notice that $|\varphi|$ is the maximal spatial resolution after a coarse-graining up to a length scale $|\varphi|^{-1}$ has been performed.

Third, having derived the coarse-grained monopole physics in the absence of interactions the full effective theory is obtained by a minimal coupling of coarse-grained dual plane waves to the inert monopole sector. (The electric-magnetic transition is survived only by gauge fields transforming under $U(1)_D$ (SU(2)) and $U(1)^2_D$ (SU(3)) [1]). A pure-gauge solution $a^{\mu,bg}_D$ to the dual gauge-field equations of motion emerges in the effective theory. This is the coarse-grained manifestation of monopole and antimonopole interactions mediated by plane-wave quantum fluctuations in the dual gauge fields. (The averaged-over quantum fluctuations are of off-shellness larger than $|\varphi|^2$.) By virtue of the pure-gauge configuration $a^{bg}_D$ the vanishing pressure and the vanishing energy density of a (hypothetical) condensate of noninteracting monopoles and antimonopoles are shifted to $\rho^{gs} = -P^{gs} = \pi \Lambda_M^3 T$ (SU(2)). (For SU(3) two monopole condensates $\varphi_1$, $\varphi_1$ and two pure-gauge configurations $a^{\mu,bg}_1 = a^{\mu,bg}_2$ exist, and one has $\rho^{gs} = -P^{gs} = 2\pi \Lambda_M^3 T$.) The negative ground-state pressure can, on a microscopic level and at finite magnetic coupling $g$, be understood in terms of magnetic flux loops which collapse as soon as the are created. The collapse takes place under the influence of negative pressure occuring away from the vortex cores [1]. The coarse-grained manifestation of dual gauge modes scattering off the magnetic monopoles or antimonopoles along a zig-zag like path within the condensate is the abelian Higgs mechanism. The latter gives rise to quasiparticle masses.

Fourth, the required invariance of the Legendre transformations for thermodynamical quantities under the applied coarse-graining yields a first-order differential
equation which governs the evolution of the magnetic coupling $g$ with temperature. This evolution allows to analyse the electric-magnetic phase transition and the transition to the absolutely confining (center) phase where center-vortex loops emerge as fermionic particles from the decaying ground state. Moreover, it determines the evolution of thermodynamical quantities. This evolution is exact on the one-loop level.

**Screened BPS magnetic monopoles.** To understand the origin of isolated and screened magnetic monopoles and antimonopoles in the deconfining phase [7] we recall some properties of certain, topologically nontrivial, (anti)selfdual solutions to the euclidean Yang-Mills equations at finite temperature: calorons and anticalorons of nontrivial holonomy, topological charge modulus $|Q| = 1$, and no net magnetic charge [9]. As it turns out [11, 8] only these configurations are relevant for a coarse-grained re-formulation of the fundamental theory at high temperatures. (The term holonomy refers to the behavior of the Polyakov loop when evaluated on a gauge-field configuration at spatial infinity. A configuration is associated with a trivial holonomy if its Polyakov loop is in the center of the group ($Z_2$ for SU(2) and $Z_3$ for SU(3)) and associated with a nontrivial holonomy otherwise.) On a microscopic level nontrivial holonomy is excited out of trivial holonomy by gluon exchanges between calorons and anticalorons. On the classical level the (anti)selfdual nontrivial-holonomy solution possesses a pair of a BPS monopole and its antimonopole, which do not interact, as constituents. Switching on one-loop quantum fluctuations, a situation investigated for an isolated (anti)caloron in [10], the monopole and its antimonopole either attract (small holonomy) or repulse (large holonomy) each other. The former process is much more likely than the latter and leads to a negative ground-state pressure after spatial coarse-graining: a monopole and its antimonopole attract each other so long until the annihilate and subsequently get re-created elsewhere. The repulsion between a monopole and its antimonopole, which both originate from a quantum blurred large-holonomy caloron, fades with an increasing number of small-holonomy caloron fluctuations taking place inbetween these particles. This facilitates the life of screened monopoles and antimonopoles in isolation. The screening of the magnetic charge $g = \frac{4\pi}{e}$ by small-holonomy calorons is, on average, described by the gauge coupling $e$ in the coarse-grained theory. A fact, important when considering the limit $e \to \infty$ below, is that the sum

$$M_{m+a} \equiv \frac{8\pi^2}{e^3} = \frac{8\pi^2}{e^3}.$$  

of the masses of a screened monopole and its screened antimonopole, both originating from a dissociating large-holonomy caloron, is independent of the holonomy [9]. Notice that $M_{m+a} \to 0$ for $e \to \infty$ and that this limit dynamically takes place at the electric-magnetic transition, where $e \sim -\log(\lambda E - \lambda_{c,E})$ ($\lambda_{c,E} \equiv \frac{2\pi T}{\Lambda}$), through a total screening of the magnetic charge of an isolated monopole and its antimonopole by intermediate small-holonomy caloron fluctuations.
Monopole condensate(s) as spatial average(s). Just as performed in the deconfining phase, we need to derive the \( \tau \)-dependence of the phase of a now complex, spatially homogeneous field \( \varphi \) (SU(2)) or fields \( \varphi_1, \varphi_2 \) (SU(3)) describing the monopole condensate(s) after spatial coarse-graining down to a resolution \(|\varphi|\) (SU(2)) or \(|\varphi_1|, |\varphi_2|\) (SU(3)). Again, we spatially average over large (topological) quantum fluctuations provided by massless, noninteracting, and stationary monopoles and antimonopoles and, subsequently, over quantum fluctuations carried by plane-waves.

The average needs to be performed in a physical (unitary) gauge where the magnetic flux emanating of each isolated monopole or antimonopole is compensated for by an associated Dirac string. A continuous dimensionless parameter, eventually to be identified with \( \beta \), arises when considering the magnetic flux in the limit \( e \to \infty \) which belongs to a pair of noninteracting, stationary (with respect to the heat bath) monopole and antimonopole situated outside of an \( S^2_\infty \) with infinite radius \( R = \infty \). The latter acts as a heat bath to the pair. Notice that a monopole-antimonopole pair situated inside the \( S^2_\infty \) does not contribute to the flux. (To readers having trouble distinguishing inside from outside for an \( S^2_\infty \) we propose to consider \( R < \infty \) first and then take the limit \( R \to \infty \).)

More specifically, we are interested in the average flux through \( S^2_\infty \) as a function of the angle \( \delta \) between the monopole’s and the antimonopole’s Dirac string. If there were no coupling to the heat bath of the monopole-antimonopole pair outside of \( S^2_\infty \) the mean flux (average over the absolute orientation of the Dirac strings) would read [1]

\[
\bar{F}_\pm = \pm \frac{\delta}{2\pi} \frac{4\pi}{e} = \pm \frac{2\delta}{e}, \quad (0 \leq \delta \leq \pi).
\]

(2)

In the limit \( e \to \infty \) we have \( \bar{F}_\pm \to 0 \) and no continuous parameter determining the \( \tau \) dependence of \( \varphi \)'s phase (SU(2)) would arise. After the coupling to the heat bath is switched on and after performing a spatial average the mean occupation number for a massless monopole-antimonopole pair (with vanishing spatial momenta of its constituents) diverges in such a way that the mean magnetic flux \( \bar{F}_{\pm,\text{th}}(\delta) \) is finite. (The total spatial momentum of the monopole and antimonopole system vanishes such that each individual momentum is zero. Notice that in the absence of a dynamically generated scale \( \Lambda_M \) the volume \( V \), over which the spatial average is performed, is undetermined: The only available mass scale \( T \), which could determine \( V \), cancels in the Bose-distribution for \( p \to 0 \) and \( e \to \infty \) since \( M_{m+a} \)'s explicit \( T \) dependence is linear, see Eq. (1).)

Let us show this in a more explicit way. The thermal average to be performed is [1]

\[
\bar{F}_{\pm,\text{th}}(\delta) = 4\pi \int d^3p \delta^{(3)}(p) n_B(\beta|E(p)|) \bar{F}_\pm = \pm \frac{8\pi \delta}{e} \int d^3p \frac{\delta^{(3)}(p)}{\exp \left[ \frac{\beta \sqrt{M_{m+a}^2 + p^2}}{T} \right] - 1}.
\]

(3)
After setting $p = 0$ in \( \exp \left[ \beta \sqrt{M_{a+b}^2 + p^2} \right] - 1 \) and by appealing to Eq. (1), the expansion of this term reads

\[
\lim_{p \to 0} \left( \exp \left[ \beta \sqrt{M_{m+a}^2 + p^2} \right] - 1 \right) = \frac{8\pi^2}{e} \left( 1 + \frac{1}{2} \frac{8\pi^2}{e} + \frac{1}{6} \left( \frac{8\pi^2}{e} \right)^2 + \cdots \right). \quad (4)
\]

The limit $e \to \infty$ can now safely be performed in Eq. (3), and we have

\[
\lim_{e \to \infty} \bar{F}_{\pm,\text{th}}(\delta) = \pm \frac{\delta}{\pi}, \quad (0 \leq \delta \leq \pi). \quad (5)
\]

This is finite and depends on the angular variable $\delta$ continuously. Now $\frac{\delta}{\pi}$ is a (normalized) angular variable just like $\frac{\tau}{\beta}$ is. Thus we may set $\frac{\delta}{\pi} = \frac{\tau}{\beta}$. Since $\varphi$ (SU(2)) is spatially homogeneous (spatial average, $p \to 0$!) its phase $\hat{\varphi} \equiv |\varphi|$ depends on $\frac{\tau}{\beta}$ only and this in a periodic way. Moreover, since the physical flux situation for the thermalized monopole-antimonopole pair does not repeat itself for $0 \leq \frac{\delta}{\pi} \leq 1$ we conclude that this period is $\pm$ unity:

\[
\hat{\varphi}(\tau) = \exp \left[ \pm 2\pi \frac{\tau}{\beta} \right]. \quad (6)
\]

To derive $\varphi$’s modulus, which together with $T$ determines the length scale $|\varphi|^{-1}$ over which the spatial average is performed, we assume the existence of an (at this stage) externally provided mass scale $\Lambda_M$. Since the weight for integrating out massless and noninteracting monopole-antimonopole systems in the partition function is $T$ independent and since the cutoff in length for the spatial average defining $|\varphi|$ is $|\varphi|^{-1}$ an explicit $T$ dependence ought not arise in any quantity being derived from such a coarse-graining. That is, in the effective action density any $T$ dependence (still assuming the absence of interactions between massless monopoles and antimonopoles when performing the coarse-graining) must appear through $\varphi$ only. Moreover, since integrating massless and momentum-free monopoles and antimonopoles into the field $\varphi$ means that this field is energy- and pressure-free $\varphi$’s $T$ dependence (residing in its phase) must be BPS saturated. On the right-hand side of $\varphi$’s or $\bar{\varphi}$’s BPS equation the requirement of analyticity (because away from a phase transition the monopole condensate should exhibit a smooth $T$ dependence) and linearity (because the $T$ dependence of $\varphi$’s phase, see Eq. (6), needs to honoured) in $\varphi$ or $\bar{\varphi}$ yields the following first-order equation of motion

\[
\partial_\tau \varphi = \pm i \frac{\Lambda_M^3}{|\varphi|^2} \varphi = \pm i \frac{\Lambda_M^3}{\varphi}. \quad (7)
\]

Notice that Eq. (7) is not invariant under Euclidean boosts: A manifestation of the existence of a singled-out rest frame – the spatially isotropic and spacetime homogeneous heat bath. Substituting $\varphi = |\varphi| \hat{\varphi}$ into Eq. (7) and appealing to Eq. (5),
we derive $|\varphi| = \sqrt{\frac{\Lambda M}{2\pi}}$. Notice that the ‘square’ of the right-hand side in Eq. (7) uniquely defines $\varphi$’s potential $V_M$. (In contrast to a second-order equation of motion, following from an action by means of the variational principle, Eq. (7) does not allow for a shift $V_M \rightarrow V_M + \text{const}$.) For the case SU(3) a BPS equation (7) arises for each of the two independent monopole condensates $\varphi_1$ and $\varphi_2$.

Finally, one shows by comparing the curvature of their potentials with the square of temperature and the squares of their moduli that the field $\varphi$ (SU(2)) and the fields $\varphi_1, \varphi_2$ (SU(3)) do neither fluctuate on-shell nor off-shell, respectively [1]: Spatial coarse-graining over nonfluctuating, classical configurations generates nonfluctuating macroscopic fields.

**Effective theory: Thermal ground state and dual quasiparticles.** To obtain the full effective theory the spatially coarse-grained and topologically trivial (dual) gauge fields $a_D^{\mu}$ (SU(2)) and $a_{\mu,1}^{D,2}$ (SU(3)) are minimally coupled (with a universal effective magnetic coupling $g$) to the inert fields $\varphi$ (SU(2)) and $\varphi_1, \varphi_2$ (SU(3)). Since the effective theory is abelian with (spontaneously broken) gauge group U(1)$_D$ (SU(2)) and U(1)$_{\beta 2}$ (SU(3)) and since the monopole fields do not fluctuate thermodynamical quantities are exact on the one-loop level. Before we discuss the spectrum of quasiparticles running in the loop we need to derive the full ground-state dynamics in the effective theory. The classical equations of motion for the dual gauge field $a_D^{\mu}$ are

$$\partial_\mu G_{\mu\nu}^D = ig\left[\mathcal{D}_\nu \varphi - \varphi \mathcal{D}_\nu \varphi\right]$$

(8)

where $G_{\mu\nu}^D = \partial_\mu a_D^{\nu} - \partial_\nu a_D^{\mu}$ and $\mathcal{D}_\mu \equiv \partial_\mu + ig a_D^{\mu}$. (For SU(3) the right-hand sides for the two equations for the dual gauge fields $a_{\mu,1}^{D,2}$ can be obtained by the substitutions $\varphi \rightarrow \varphi_1$ or $\varphi \rightarrow \varphi_2$ in Eq. (8).) The pure-gauge solution to Eq. (8) with $\mathcal{D}_\mu \varphi \equiv 0$ is given as

$$a_{\mu, bg}^{D,2} = \pm \delta_\mu^4 \frac{2\pi}{g\beta}.$$  

(9)

In analogy to the deconfining phase, the coarse-grained manifestation $a_{\mu, bg}^{D,2}$ of monopole-antimonopole interactions, mediated by dual, off-shell plane-wave modes on the microscopic level, shifts the energy density $\rho^{gs}$ and the pressure $P^{gs}$ of the ground state from zero to finite values: $\rho^{gs} = -P^{gs} = \pi \Lambda_M^3 T$ (SU(2)) and $\rho^{gs} = -P^{gs} = 2\pi \Lambda_M^3 T$ (SU(3)). In contrast to the deconfining phase, where the negativity of $P^{gs}$ arises from monopole-antimonopole attraction, the negative ground-state pressure in the preconfining phase originates from collapsing and re-created center-vortex loops [1]. (There are two species of such loops for SU(3) and one species for SU(2)). The core of a given center-vortex loop can be pictured as a stream of the associated monopole species flowing oppositely directed to the stream of their antimonopoles [1]. Since by Stoke’s theorem the magnetic flux carried by the vortex is determined by the dual gauge field $a_{\mu}^{D,\nu\tau}$ transverse to the vortex-tangential and since $a_{\mu}^{D,\nu\tau}$ is in a covariant gauge – invariant under collective boosts of the streaming monopoles or antimonopoles in the vortex core it follows that the magnetic flux solely depends on the monopole charge and not on the collective state of monopole-antimonopole
Figure 1: The evolution of the effective magnetic gauge coupling \( g \) in the preconfining phase for SU(2) (thick grey line) and SU(3) (thick black line) where \( \lambda_M \equiv \frac{2\pi T}{\Lambda_M} \). At \( \lambda_{c,M} = 7.075 \) (SU(2)) and \( \lambda_{c,M} = 6.467 \) (SU(3)) \( g \) diverges logarithmically, 
\[ g \sim -\log(\lambda_M - \lambda_{c,M}). \]

motion. This, in turn, implies a center-element classification of the magnetic fluxes carried by the vortices justifying the name center-vortex loop.) Viewed on the level of large-holonomy calorons an unstable center-vortex loop is created within a region where the mean axis for the dissociation of several calorons represents a net direction for the monopole-antimonopole flow. Notice that each so-generated vortex core must form a closed loop due to the absence of isolated magnetic charges within the monopole condensate. In contrast to the deconfining phase a rotation to unitary gauge \( a^{D,bg}_{\mu} = 0, \varphi = |\varphi| \) is facilitated by a smooth, periodic gauge transformation which leaves the value 1 of the Polyakov loop invariant: the electric \( \mathbb{Z}_2 \) degeneracy, observed in the deconfining phase, is lifted in the ground-state. (For SU(3) it is an electric \( \mathbb{Z}_3 \) degeneracy that is lifted.)

By the dual (abelian) Higgs mechanism the mass of (noninteracting) quasiparticle modes is
\[ m = g|\varphi| = g|\varphi_1| = g|\varphi_2|. \]

Evolution of magnetic coupling. The requirement that Legendre transformations between thermodynamical quantities need to be invariant under the applied spatial coarse-graining determines the evolution of the magnetic coupling \( g \) with temperature in terms of a first-order differential equation [1]. In Fig. 1 the temperature evolution of \( g \) is shown for SU(2) and SU(3). By virtue of Eq. (10) and Fig. 1 it is clear that inside the preconfining phase dual gauge modes are massive. Thus the full Polyakov-loop expectation is extremely suppressed as compared to the deconfining phase: Infinitely heavy, fundamental, and fermionic test charges are confined by the dual Meissner effect while dual gauge modes still propagate with a maximal off-shellness \( |\varphi|^{-1} \). Notice the logarithmic singularity at \( T_{c,M} \).

Since the energy \( E_v \) and the pressure \( P_v(r) \) of a (nonselfintersecting) center-vortex loop, whose center of mass is at rest with respect to the heat bath, scale like \( +g^{-1} \) and \( -g^{-2} \), respectively, this
soliton starts to be generated as a massless and stable (spin-1/2) particle at $T_{c,M}$. Moreover, all (dual) gauge modes decouple by a diverging mass: For $T \leq T_{c,M}$ no continuous gauge symmetry can be observed in the system at an extraneously applied spatial resolution smaller than $\varphi(T_{c,M})$.

**Results and summary.** In Figs. 2 and 3 we present the results for the temperature evolution of the pressure and the energy density for both the deconfining (electric) and preconfining (magnetic) phase. To express the magnetic scale $\Lambda_M$ in terms of the electric scale $\Lambda_E$ continuity of the pressure is demanded across the electric-magnetic transition. We have $\Lambda_M \sim (\frac{1}{4})^{-1/3} \Lambda_E$ (SU(2)) and $\Lambda_M \sim (\frac{1}{2})^{-1/3} \Lambda_E$ (SU(3)).

Notice the increasing negativity of the pressure with decreasing temperature in the magnetic phase. On the microscopic level this is understood in terms of an increasingly large caloron-holonomy, implying an increasingly large repulsion of its constituent monopole and antimonopole, which, in turn, means an increasingly large collimation of the monopole-antimonopole motion in the condensate. The effect is an increasing rate for the creation of (unstable) center-vortex loops implying, after spatial coarse-graining, an increasingly negative ground-state pressure and, at the same time, an increase of the quasiparticle masses. At $T_{c,M}$ all quasiparticles are infinitely heavy and the equation of state is $P = -\rho$, compare Figs. 2 and 3. The (continuous) order parameter of mass-dimension one for the spontaneous breakdown of the dual gauge symmetry $U(1)_D$ (SU(2)) and $U(1)^2_D$ (SU(3)) across the (second-order like) electric-magnetic phase transition is the ‘photon’ mass in Eq. (10). The associated critical exponents $\nu$ are mean-field ones, $\nu = 0.5$, for both SU(2) and SU(3) \cite{1}. The transition is, however, not strictly second order because of a negative latent heat, see Fig. 3. The reason for this discontinuous increase of the energy density across the electric-magnetic transition is the discontinuous increase of the number of polarizations from two to three due to the dual ‘photon’ becoming massive. This effect is important because it stabilizes the temperature of the cosmic...
microwave background $T_{\text{CMB}}$, described by an SU(2) pure gauge theory of Yang-Mills scale $\Lambda_{\text{CMB}} \sim T_{\text{CMB}}$, against gravitational expansion. The latter increasingly liberates a formerly locked-in, spatially homogeneous Planck-scale axion field $[1,12]$ which, eventually, will drive the Universe out of thermal equilibrium globally.

The Polyakov-loop expectation, which is an order parameter associated with the spontaneous breaking of the electric $Z_2$ (SU(2)) and the electric $Z_3$ (SU(3)) symmetry, though strongly suppressed on the magnetic side, remains finite across the electric-magnetic transition. This happens despite the fact that the magnetic ground state is nondegenerate with respect to these symmetries.

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