Nonlinear coherent state of an exciton in a wide quantum dot

M Bagheri Harouni, R Roknizadeh and M H Naderi

Quantum Optics Group, Physics Department, University of Isfahan, Iran
E-mail: m-baghreri@phys.ui.ac.ir, rokni@sci.ui.ac.ir and mhnaderi@phys.ui.ac.ir

Received 26 August 2008, in final form 2 October 2008
Published 7 November 2008
Online at stacks.iop.org/JPhysB/41/225501

Abstract
In this paper, we derive the dynamical algebra of a particle confined in an infinite spherical well using the $f$-deformed oscillator approach. We consider an exciton with definite angular momentum in a wide quantum dot interacting with two laser beams. We show that under the weak confinement condition, and quantization of the centre-of-mass motion of exciton, its stationary state can be considered as a special kind of nonlinear coherent states which exhibits the quadrature squeezing.

1. Introduction

The conventional coherent states of the quantum harmonic oscillator, defined by Glauber [1] as the right-hand eigenstates of non-Hermitian annihilation operator $\hat{a}([\hat{a}, \hat{a}^\dagger] = 1)$, have found many interesting applications in different areas of physics such as quantum optics, condensed matter physics, statistical physics and atomic physics [2]. These states, which play an important role in the quantum theory of coherence, are considered as the most classical ones among the pure quantum states, and laser light can be supposed as a physical realization of them. Due to the vast application of these states, there have been many attempts to generalize them [3]. Among all the generalizations, nonlinear coherent states (NLCSs) [4] have been paid attention in recent years because they exhibit nonclassical features such as quadrature squeezing and sub-Poissonian statistics [5]. These states are defined as the right-hand eigenstates of a deformed operator $\hat{A}$,

$$\hat{A} = \hat{a} f(\hat{n}) \quad \hat{A}[\alpha, f] = \alpha[\alpha, f], \quad (1)$$

where the deformation function $f(\hat{n})$ is an operator-valued function of the number operator $\hat{n}$. From (1) one can obtain explicit forms of NLCSs in the number state representation

$$|\alpha, f\rangle = N_f \sum_n \frac{\alpha^n}{\sqrt{n! f(n)!}} |n\rangle,$$

$$N_f = \left( \sum_n \frac{|\alpha|^{2n}}{[f(n)!]^2 n!} \right)^{-\frac{1}{2}}. \quad (2)$$

A class of NLCSs can be realized physically as the stationary state of the centre-of-mass motion of a laser-driven trapped ion [6, 7]. Furthermore, a theoretical scheme has been proposed to show the possibility of generating various families of NLCSs of the radiation field in a lossless coherently pumped micromaser within the framework of the intensity-dependent Jaynes–Cummings model.

Recently, the influences of the spatial confinement [9] and the curvature of physical space [10] on the algebraic structure of the coherent states of the quantum harmonic oscillator have been investigated within the framework of a nonlinear coherent states approach. It has been shown that if a quantum harmonic oscillator is confined within a small region of order of its characteristic length [9] or having its physical space as a sphere [10], then it can be regarded as a deformed oscillator, i.e., an oscillator whose creation and annihilation operators are deformed operators $\hat{A}$ and $\hat{A}^\dagger$ given by equation (1).

On the other hand, we can consider nanostructures as systems whose physical properties are related to the confinement effects. Thus, we expect that it is possible to realize some natural deformations in these systems [9, 11]. In addition, in nanostructures different kinds of quantum states can be prepared. One of the most applicable of these states is the exciton state. An exciton is an elementary excitation in semiconductors interacting with light, an electron in a conduction band which is bounded to a hole in a valance band and can easily move through the sample. In one of the nanosize systems, the quantum dot (QD), due to the confinement in three dimensions, energy bands reduce to quasi-energy levels. Therefore, in order to describe the interaction of a QD with light we can consider it as a few-level atom [12]. These exciton states can be used in quantum information processes. It has been shown that excitons in coupled QDs are
ideal for preparation of entangled states in solid-state systems [13]. Entanglement of the exciton states in a single QD or in a QD molecule has been demonstrated experimentally [14]. Entanglement of the coherent states of the excitons in a system of two coupled QDs has been considered [15]. Recently, coherent exciton states of excitonic nano-crystal-molecules has been considered [16]. A theoretical approach for generating Dicke states of excitons in optically driven QDs has been proposed in [17]. In a QD, the effects of exciton–phonon interaction, exciton–impurity interaction and exciton–exciton interaction play an important role. These effects are the main sources for the decoherence of exciton states [18]. Furthermore, these effects cause the exciton to undergo spontaneous recombination or be scattered to other exciton modes [19, 20].

In this paper we propose a theoretical scheme for generating excitonic NLCSs. We will show that under certain conditions the quantized motion of a wave packet of centre-of-mass of an exciton can be considered as a special kind of NLCS. Our scheme is based on the interaction of a QD with two laser beams. By using the approach considered in [6], we propose a theoretical scheme for generation of an NLCS of an exciton in a wide QD.

In section 2, we consider different confinement regimes in a QD, and the explicit forms of the creation and annihilation operators for a particle confined in an infinite well are derived by using the deformed quantum oscillator approach. In section 3, we consider an exciton in a wide QD which interacts with two laser beams. We shall show that under the weak confinement condition, the stationary state of the exciton centre-of-mass motion can be considered as an NLCS.

2. Algebraic approach for a particle in an infinite spherical well

In nanostructures and confined systems, there are three different confinement regimes. The criteria for this classification are based on the comparison between the excitation Bohr radius and the spatial dimensions of the system under consideration. In the case of a QD, these regimes are defined as follows [21].

We first introduce three quantities \( \Delta E_c, \Delta E_v \) and \( V_{exc} \) which, respectively, denote the electron energy due to the confinement, the hole energy due to the confinement and Coulomb energy between correlated electron–hole (exciton).

1. \( V_{exc} > \Delta E_c - \Delta E_v \): in this case, the exciton energy is greater than the confinement energies of the electron and hole. If we show the system size by \( L \) and the exciton Bohr radius by \( a \), then in this regime \( L > a \). This regime corresponds to the weak confinement (in some literature the weak confinement is characterized by the situation in which the electron and the hole are not in the same matter, for example, the hole is in the QD and the excited electron in the host matter. In this paper, by the weak confinement regime we mean \( L > a \) and the excitations in the same matter). In this regime due to the confinement, the centre-of-mass motion of the exciton is quantized and the confinement does not affect the electron and hole separately. Hence, the confinement affects the exciton motion as a whole [22].

2. \( V_{exc} < \Delta E_c, \Delta E_v \): this regime, in contrast to the previous one, is associated with the cases where \( L < a \). In this regime the exciton is completely localized, and the confinement affects both the electron and the hole independently and their states become quantized in the conduction and valance bands. This regime is called strong confinement.

3. \( \Delta E_c > V_{exc}, \Delta E_v \): this condition is equivalent to the situation \( a_c < a < a_v \), where \( a_c \) and \( a_v \) are, respectively, the Bohr radii of the electron and hole. Here, due to the different effective masses of the electron and hole, the hole which has heavier effective mass than the electron is localized and the electron motion will be quantized. This regime is called intermediate confinement.

In the first case (weak confinement), in a wide QD, an exciton can move due to its centre-of-mass momentum, and because of the presence of the barriers, its centre-of-mass motion is quantized. Therefore, it moves as a whole between energy levels of an infinite well. We consider a wide spherical QD whose energy levels are equivalent to the energy levels of a spherical well

\[
E_{nl} = \frac{\hbar^2}{2M} \frac{a_{nl}^2}{R^2},
\]

where \( a_{nl} \) is the \( n \)th zero of the first kind Bessel function of order \( l \), \( j_l(x) \). In this energy spectrum according to the azimuthal symmetry around the \( z \) axis, we have a degenerate spectrum. As mentioned before, in the weak confinement regime, the Coulomb potential plays an essential role and its spectrum is given by

\[
E_{b,\mu}^k = \frac{\mu}{2\hbar^2 k^2} \left( \frac{1}{k} \right), \quad \mu = \frac{m_e m_h}{m_e + m_h},
\]

where superscript \( b \) shows binding energy related to the Coulomb interaction and \( k \) shows dielectric constant of the system. As is usual, we interpret the Coulomb part as an exciton and another degree of freedom (motion between energy levels of the well) as the exciton centre-of-mass motion. Therefore, in a wide QD an exciton has two different kinds of degrees of freedom: internal degrees of freedom due to the Coulomb potential and external degrees of freedom related to the quantum confinement. Here we consider the lowest exciton state, the 1s exciton, because this exciton state has the largest oscillator strength among the other exciton states. Then the energy of the exciton in a wide QD can be written as

\[
E_{nlm, \mu} = E_{b,\mu}^k + \frac{\hbar^2}{2MR^2} a_{nl}^2,
\]

where \( E_{b,\mu}^k \) is the energy gap of QD, \( E_{b,\mu}^k = \left| E_{b,\mu}^k \right|_{k=\infty} \) is the exciton binding energy, \( M = m_e + m_h \) is the total mass of exciton, and \( R \) is the radius of the QD. Due to the relation of quantum numbers \( l \) and \( m \) with the angular momentum and the selection rules for optical transitions, we can fix \( l \) and \( m \) (by choosing a certain condition), and hence the energy of the exciton depends only on a single quantum number

\[
E_n = E_{b,\mu}^k + \frac{\hbar^2}{2MR^2} a_{nl}^2.
\]

Therefore, we can prepare the conditions under which the exciton centre-of-mass motion has a one-dimensional degree
of freedom. Due to the quantization of the exciton centre-of-mass motion, we can describe the exciton motion between the energy levels by the action of a special kind of ladder operator. In order to find these operators we use the $f$-deformed oscillator approach [4].

As mentioned elsewhere [9], if the energy spectrum of the system is equally spaced, such as a harmonic oscillator, its creation and annihilation operators satisfy the ordinary Weyl–Heisenberg algebra. However, in this treatment we note that there is no common between the harmonic oscillator potential and the conventional oscillator algebra should be reduced to the conventional oscillator algebra. The energy spectrum of a particle with mass $M$ confined in an infinite spherical well can be written as (3). According to the conservation of angular momentum, we assume that the particle has been prepared with definite angular momentum (for example by measuring its angular momentum). Then $l$ becomes completely determined, i.e., in the energy spectrum the number $l$ is a constant. By determining the number $l$ and considering the rotational symmetry of the system around the $z$ axis, the angular part of the spectrum becomes completely determined, and the radius part is described by (3). Now we use a factorization method and write the Hamiltonian of the centre-of-mass motion of the system as follows:

$$ H = \frac{1}{2} (\hat{A} \hat{A}^\dagger + \hat{A}^\dagger \hat{A}), $$

(7)

where $\hat{A}$ and $\hat{A}^\dagger$ are definded through the relation (1). Therefore the spectrum of $\hat{H}$, after straightforward calculation, is obtained as

$$ E_n = \frac{1}{2} [(n + 1) f^2(n + 1) + n f^2(n)]. $$

(8)

By comparing (8) with equation (3) we arrive at the following expression for the corresponding deformation function $f_1(n)$,

$$ f_1(n) = \frac{\hbar^2}{MR^2} \sum_{i=1}^{n} (-1)^i \alpha_i^2 L_{-i}^{i}. $$

(9)

Then, the ladder operators associated with the radial motion of a confined particle in a spherical infinite well are given by

$$ \hat{A} = \hat{a} \left[ \frac{\hbar^2}{MR^2} \sum_{i=1}^{n} (-1)^i L_{-i}^{i} \right], $$

$$ \hat{A}^\dagger = \hat{a}^\dagger \left[ \frac{\hbar^2}{MR^2} \sum_{i=1}^{n} (-1)^i \alpha_i^2 L_{-i}^{i} \right]. $$

(10)

These two deformed operators obey the following commutation relation:

$$ [\hat{A}, \hat{A}^\dagger] = -nf_1^2(n) + \frac{\hbar^2}{MR^2} \alpha_i^2 \alpha_i^2. $$

(11)

As is usual in the $f$-deformation approach, for a particular limit of the corresponding deformation parameter, the deformed algebra should be reduced to the conventional oscillator algebra. However, in this treatment we note that there is nothing common between the harmonic oscillator potential and an infinite spherical well. Only in the limit $R \rightarrow \infty$, the system reduces to a free particle which has a continuous spectrum.

As a result, in this section we conclude that the radial motion of a particle confined in a three-dimensional infinite spherical well can be described by an $f$-deformed Weyl–Heisenberg algebra.

3. Exciton dynamics in a QD

Now we consider the formation of an exciton and its dynamics in a wide QD during the exciton lifetime. As mentioned before, in this situation the centre-of-mass motion of the exciton is quantized. The exciton is created during the interaction of a QD with light, and because of the angular momentum conservation, the exciton has a well-defined angular momentum. The exciton is a quasiparticle composed of an electron and a hole and thus the exciton spin state can be in a singlet state or a triplet state. According to the optical transition selection rules, the triplet state is optically inactive and is called a dark exciton [23]. By adding spin and angular momentum of absorbed photons, the angular momentum of the exciton state can be determined. Hence, the exciton behaves like a particle in a spherical well with definite angular momentum. According to the previous section, the centre-of-mass motion of the exciton in the QD and the barriers of the QD can be described by an oscillator-like Hamiltonian expressed in terms of the $f$-deformed annihilation and creation operators given by equation (10)

$$ H_{\text{well}} = \frac{1}{2} (\hat{A} \hat{A}^\dagger + \hat{A}^\dagger \hat{A}), $$

(12)

where we interpret the operator $\hat{A}(\hat{A}^\dagger)$ as the operator whose action causes the transition of exciton centre-of-mass motion to a lower (upper) energy state. In fact the Hamiltonian (12) is related to the external degree of freedom of the exciton. On the other hand, one can imagine the QD as a two-level system with the ground state $|g\rangle$ and the excited state $|e\rangle$ (associated with the presence of exciton). Thus, for the internal degree of freedom we can consider the following Hamiltonian:

$$ H_{\text{ex}} = \hbar \omega_{\text{ex}} \hat{S}_{z2}, $$

(13)

where $\hat{S}_{z2} = |e\rangle \langle e| - |g\rangle \langle g|$ and $\hbar \omega_{\text{ex}} = E_g - E_t$ is the exciton energy.

We consider a single exciton of frequency $\omega_{\text{ex}}$ confined in a wide QD interacting with two laser fields, respectively, tuned to the internal degree of freedom of the frequency $\omega_{\text{ex}}$ and to the non-equal spaced energy levels of the infinite well. It is necessary that the second laser has special conditions, because it should give rise to the transitions between energy levels whose frequencies depend on intensity. The interacting system can be described by the Hamiltonian

$$ H = H_0 + H_{\text{int}}, $$

(14)

where $H_0 = \hat{H}_{\text{well}} + \hat{H}_{\text{ex}}$ and

$$ H_{\text{int}} = g [E_0 e^{-i(k_0 x - \omega_0 t)} + E_1 e^{-i(k_1 x - \omega_1 t)}] \hat{S}_{z1} + \text{h.c.}, $$

(15)

in which $g$ is the coupling constant, $k_0$ and $k_1$ are the wave vectors of the laser fields, $\hat{S}_{z1} = |g\rangle \langle e|$ is the exciton annihilation operator, and $\omega_{\text{tr}}$ is the frequency of exciton transition between energy levels of the QD due to the spatial confinement. Here, we consider the transition between specific side-band levels, hence we show the frequency transition with definite dependence to $n$. We denote this by a $c$-number quantity $\bar{\omega}$.

The exciton has a finite lifetime that in systems with small dimension is increased [24]. The interaction with phonons is
the main reason for damping of the exciton [25]. Phonons in bulk matter have a continuous spectrum while in a confined system such as a QD their spectrum becomes discrete. Hence in a QD, the resonant interaction between the exciton and phonons decreases and in this system the exciton lifetime will increase. Therefore during the lifetime of an exciton, its dynamics is under the influence of a bath reservoir, and its damping play an important role. We assume that during the presence of the exciton in the QD, it interacts with the reservoir and hence we can consider its steady state. We consider an exciton in the dark state. Experimental preparation methods of such an exciton have been described in [23]. In this situation the lifetime of the exciton will increase and it has no spontaneous recombination radiation. However, its interaction with phonons gives it a finite lifetime.

The operator of the centre-of-mass motion position  
\[ \hat{x} = \frac{\kappa}{k_{\text{ex}}} (\hat{A} + \hat{A}^\dagger), \]  
where \( \kappa \) is a parameter similar to the Lamb–Dicke parameter in ion trapped systems and is defined as the ratio of QD radius to the wavelength of the driving laser (because of the spatial confinement of exciton, its wavefunction width is determined by the barriers of the QD), and we assume \( k_0 \approx k_1 \approx k_{\text{ex}} (k_{\text{ex}} \) is the wavevector of the exciton). The operators \( \hat{A} \) and \( \hat{A}^\dagger \) are defined in equation (10). The interaction Hamiltonian (15) can be written as

\[ H_{\text{int}} = \hbar \omega_{\text{m}} \Omega_1 \left[ \frac{\Omega_0}{\Omega_1} + e^{-i\omega_{\text{m}}t} \right] \exp[i\kappa (e^{-i\omega_{\text{m}}t} \hat{A} + \hat{A}^\dagger e^{i\omega_{\text{m}}t})] + \text{h.c.}, \]

where \( \Omega_0 = \frac{\hbar E_0}{\kappa} \) and \( \Omega_1 = \frac{\hbar E_1}{\kappa} \) are the Rabi frequencies of the lasers, respectively, tuned to the electronic transition of QD (internal degree of freedom) and the first centre-of-mass motion transition of exciton. Since the external degree of freedom is definite, then \( \omega_{\text{m}} \) depends on a special value of \( n \) such that it can be considered as a \( c \)-number quantity. The frequency \( \omega_{\text{m}} \) depends on the number of quanta for each transition and hence the laser tuned to the centre-of-mass motion must be so strong that it causes transition. This allows us to treat the interaction of the confined exciton in a wide QD with two lasers separately, by using a nonlinear Jaynes–Cummings Hamiltonian [26] for each coupling. The interaction Hamiltonian in the interaction picture can be written as

\[ H_I = \hbar \Omega_1 \hat{S}_{12} \left[ \Omega_0 \Omega_1 + e^{-i\omega_{\text{m}}t} \right] \exp[i\kappa (e^{-i\omega_{\text{m}}t} \hat{A} + \hat{A}^\dagger e^{i\omega_{\text{m}}t})] + \text{h.c.}, \]

where \( \omega_{\text{m}} = \frac{1}{2} [(\hat{n} + 2)f_i(\hat{n} + 2) - \hat{n} f_i(\hat{n})] \). By using the vibrational rotating wave approximation [6], applying the disentangling formula introduced in [27], and using the fact that in the present case the Lamb–Dicke parameter is small, the interaction Hamiltonian (18) is simplified to

\[ H_I^{(1)} = \hbar \Omega_1 \hat{S}_{12} \left[ F_0(\hat{n}, \kappa) \frac{\Omega_0}{\Omega_1} + i\kappa F_1(\hat{n}, \kappa) \hat{a}\right] + \text{h.c.}, \]

where the function \( F_i(\hat{n}, \kappa)(i = 0, 1) \) is defined by

\[ F_i(\hat{n}, \kappa) = e^{-\frac{\kappa^2}{2}} (\hat{n} + i) f_i(\hat{n} + i) 
\times \sum_{l=0}^{\infty} \left( \frac{\kappa^2}{2}\right)^l f_i(\hat{n} + i) \frac{\Gamma_l(\hat{n} + i)^l}{\Gamma_l(\hat{n})^l} (\hat{a}^\dagger)^l \hat{a}^l. \]

It should be noted that this function in the limit \( f_i(\hat{n}) \rightarrow 1 \) (which is equivalent to the harmonic confinement) is proportional to the associated Laguerre polynomials

\[ F_i(\hat{n}, \kappa) = e^{-\frac{\kappa^2}{2}} \frac{\Gamma_i}{\hat{n} + i} L_i^{(1)}(\kappa^2). \]

Now we write the function \( F_i(\hat{n}, \kappa) \)

\[ F_i(\hat{n}, \kappa) = e^{-\frac{\kappa^2}{2}} \frac{\Gamma_i}{\hat{n} + i} \times f_i(\hat{n}) f_i(\hat{n} + i) L_i^{(1)}(\kappa^2), \]

where the function \( L_i^{(1)}(x) \) is defined as

\[ L_i^{(1)}(x) = \sum_{l=0}^{\infty} \frac{1}{\Gamma_i(\hat{n} - l)!^2} (\hat{n} + i)^l f_i(\hat{n} + i) L_i^{(1)}(-x)^l. \]

This function is similar to the associated Laguerre function.

The time evolution of the system under consideration is characterized by the master equation

\[ \frac{d\hat{\rho}}{dt} = -i \left( [\hat{H}_I^{(1)}, \hat{\rho}] + \Sigma \hat{\rho}, \right) \]

where \( \Sigma \hat{\rho} \) defines damping of the system due to the different kinds of interactions which leads to annihilation of the exciton. We assume a bosonic reservoir that causes damping of the exciton system. Due to the properties of the dark exciton, the rate of spontaneous recollection and hence spontaneous emission is decreased. On the other hand, interactions of exciton–phonon and exciton–impurities cause the exciton to be damped. In fact in low temperatures it is possible to ignore the phonon effects and by assuming a pure system we neglect the impurity effects. Hence we can write

\[ \Sigma \hat{\rho} = \frac{\Gamma}{2} \left[ 2\hat{b}\hat{b}^\dagger - \hat{b}^\dagger \hat{b} - \hat{b}^\dagger \hat{b} \right], \]

where \( \Gamma \) is the energy relaxation rate, \( \hat{b} \) and \( \hat{b}^\dagger \) are the annihilation and creation operators of the reservoir. Due to the confinement and dark state properties, spontaneous recombination of the exciton decreases and hence its lifetime becomes so long that we can consider the stationary solution of equation (24). We assume a finite lifetime for the exciton, and during this time we neglect damping effects. The stationary solution of the master equation (24) in the time scales of our interest is

\[ \hat{\rho} = |\psi\rangle\langle \psi|, \]

where \( |\psi\rangle \) is the electronic excited state corresponding to the presence of exciton and \( |\psi\rangle \) is the centre-of-mass motion state of the exciton, which can be considered as a right-hand eigenstate of the deformed operator \( \hat{A} = \frac{F_i(\hat{n}, \kappa)}{F_0(\hat{n}, \kappa)} \hat{A} \)

\[ F_i(\hat{n}, \kappa) \hat{a}|\psi\rangle = \frac{i\kappa}{\Omega_0} |\psi\rangle. \]
According to equation (22) the corresponding deformation function reads as

$$f(\hat{n}) = \frac{F_1(\hat{n} - 1, \kappa)}{F_0(\hat{n} - 1, \kappa)} = \frac{f_1(\hat{n})L_{f,\hat{n}-1}^1(\kappa^2)}{nL_{f,\hat{n}-1}^0(\kappa^2)} e^{-\frac{1}{\Omega_1} f_{1(f(n+1))} - f_{1(f(n-1))}}. \quad (28)$$

Hence, we can express the state $|\psi\rangle$ in the Fock space representation as

$$|\psi\rangle = N_f \sum_n \frac{\chi^n}{\sqrt{n!f(n)!}} |n\rangle,$$  

where $\chi = i \Omega_0 / \Omega_1$. According to definition (2), it is evident that the state $|\psi\rangle$ can be regarded as a special kind of NLCS. As is seen from equation (27), the eigenvalue of the deformed operator $\hat{A}$ depends on some physical parameters such as the Rabi frequencies, the parameter $\kappa$ and radius of the QD.

As is clear from equation (28), the deformation function $f(\hat{n})$ depends on the quantum number $\hat{n}$ and physical parameters such as QD radius and $\kappa$ which characterizes the confinement regime. In the limit $f_1(\hat{n}) \rightarrow 1$, (harmonic confinement), which corresponds, for example, to a QD in lens shape [28], the function $L_{f,\hat{n}}^0$ reduces to the ordinary associated Laguerre polynomials, its argument tends to $\kappa^2$ and therefore, the deformation function (28) takes the following form:

$$f(\hat{n}) = e^{-\chi^2} L_{\hat{n}}^1(\kappa^2)[(\hat{n} + 1) L_{\hat{n}}^0(\kappa^2)]^{-1}. \quad (30)$$

This is the deformation function that appears in the centre-of-mass motion of a trapped ion confined in a harmonic trap [6].

In order to investigate the nonclassical behaviour of the NLCS $|\psi\rangle$ we consider the quadrature squeezing of the centre-of-mass motion. For this purpose, we define the deformed quadratures operators as follows:

$$\hat{X}_1 = \frac{1}{2}(\hat{A}e^{i\phi} + \hat{A}^\dagger e^{-i\phi}), \quad \hat{X}_2 = \frac{1}{2i}(\hat{A}e^{i\phi} - \hat{A}^\dagger e^{-i\phi}).$$  \quad (31)

In the limiting case $f(\hat{n}) \rightarrow 1$, these two operators reduce to the conventional (non-deformed) quadrature operators [29]. The commutation relation of $\hat{X}_1$ and $\hat{X}_2$ is

$$[\hat{X}_1, \hat{X}_2] = \frac{i}{2}[(\hat{n} + 1) f^2(\hat{n} + 1) - \hat{n} f^2(\hat{n})]. \quad (32)$$

The variances $\langle(\Delta \hat{X}_1)^2\rangle = \langle\hat{X}_1^2\rangle - \langle\hat{X}_1\rangle^2 (i = 1, 2)$ satisfy the uncertainty relation

$$\langle(\Delta \hat{X}_1)^2\rangle \cdot \langle(\Delta \hat{X}_2)^2\rangle \geq \frac{1}{16} \langle(\hat{n} + 1) f^2(\hat{n} + 1) - \hat{n} f^2(\hat{n})\rangle. \quad (33)$$

A quantum state is said to be squeezed when one of the quadratures components $\hat{X}_1$ and $\hat{X}_2$ satisfies the relation

$$\langle(\Delta \hat{X}_i)^2\rangle < \frac{1}{4} \langle(\hat{n} + 1) f^2(\hat{n} + 1) - \hat{n} f^2(\hat{n})\rangle \quad i = 1 \text{ or } 2. \quad (34)$$

The degree of squeezing can be measured by the squeezing parameters $s_i (i = 1, 2)$ defined by

$$s_i = 4 \langle(\Delta \hat{X}_i)^2\rangle - \frac{1}{4} \langle(\hat{n} + 1) f^2(\hat{n} + 1) - \hat{n} f^2(\hat{n})\rangle. \quad (35)$$

![Figure 1. Plots of the squeezing parameter $s_1$ versus the ratio $\frac{R}{\alpha_0}$. The solid line corresponds to $\frac{\Omega_0}{\Omega_1} = 0.5$, the dashed line corresponds to $\frac{\Omega_0}{\Omega_1} = 0.2$. In both plots the Lamb–Dicke parameter is chosen as $\kappa = 0.3$. (This figure is in colour only in the electronic version).](image)

Then the condition for squeezing in the quadrature component can be simply written as $s_i < 0$. In figure 1 we plot the squeezing parameter $s_1$ versus the parameter $\frac{R}{\alpha_0}$ defined as the ratio of the QD radius to the Bohr radius of the exciton for two different values of ratio $\frac{\Omega_0}{\Omega_1}$. As is clear from figure 1, for small values of the parameter $\frac{R}{\alpha_0}$ the state shows quadrature squeezing and by increasing this parameter the quadrature squeezing disappears.

4. Conclusion

In this paper, we first considered a particle confined in a spherical infinite well and we found the explicit forms of its creation and annihilation operators by using the $f$-deformed oscillator approach. Then we considered an exciton in a wide QD interacting with two laser beams. We showed that under the weak confinement condition, the exciton is influenced as a whole and its centre-of-mass motion will be quantized. Within the framework of the $f$-deformed oscillator approach, we found that under certain circumstances of exciton–laser interaction the stationary state of the exciton centre-of-mass motion is a nonlinear coherent state which exhibits quadrature squeezing.

Acknowledgments

The authors wish to thank the Office of Graduate Studies of the University of Isfahan and Iranian Nanotechnology initiative for their support.

References

[1] Glauber R J 1963 Phys. Rev. 130 2529
[2] Glauber R J 1963 Phys. Rev. 131 2766
[3] Glauber R J 1963 Phys. Rev. Lett. 10 84
[2] Klauder J R and Skagerstam B S 1985 Coherent States, Applications in Physics and Mathematical Physics (Singapore: World Scientific)

[3] Perelomov P A 1986 Generalized Coherent States and Their Applications (Berlin: Springer)

[4] Man’ko V I, Marmo G, Sudarshan E C G and Zaccaria F 1997 Phys. Scr. 55 528

[5] Man’ko V I, Marmo G, Sudarshan E C G, Zaccaria F and Atakishiev N M (ed) 1996 Proc. 4th Wigner Symp. (Guadalajara, Mexico, July 1995) (Singapore: World Scientific) p 427

[6] Ali S T, Antoine J-P and Gazeau J-P 2000 Coherent States, Wavelets and Their Generalization (New York: Springer)

[7] Man’ko V I, Marmo G, Sudarshan E C G, Zaccaria F and Atakishiev N M (ed) 1996 Proc. 4th Wigner Symp. (Guadalajara, Mexico, July 1995) (Singapore: World Scientific) p 427

[8] Man’ko V I, Marmo G, Porzio A, Solimeno S and Zaccaria F 2000 Phys. Rev. A 62 053407

[9] Naderi M H, Soltaolkotabi M and Roknizadeh R 2004 J. Phys. A: Math. Gen. 37 5649

[10] Filho R L deMatos and Vogel W 1996 Phys. Rev. A 54 4560

[11] Man’ko V I, Marmo G, Porzio A, Solimeno S and Zaccaria F 2000 Phys. Rev. A 62 053407

[12] Naderi M H, Soltaolkotabi M and Roknizadeh R 2005 Eur. Phys. J. D. 32 397

[13] Harouni M B, Roknizadeh R and Naderi M H 2004 J. Phys. A: Math. Gen. 37 3225

[14] Chen G, Bonadeo N H, Steel D G, Gammon D, Katzer D S, Park D and Sham L J 2000 Science 289 1906

[15] Liu Y X, Özdemir S K, Koashi M and Imoto N 2002 Phys. Rev. A 65 042326

[16] Hong S-K, Nam S W and Yeon K-H 2007 Phys. Rev. B 76 115330

[17] Zou X, Pahlke K and Mathis W 2003 Phys. Rev. A 68 034306

[18] Zhu K-D, Wu Z-J, Yuan X-Z and Zhen H 2005 Phys. Rev. B 71 235312

[19] Tassone F and Yamamoto Y 1999 Phys. Rev. B 59 1003

[20] Bondarev I V, Maksimenko S A, Slepyan G Ya, Krestnikov I L and Hoffmann A 2003 Phys. Rev. B 68 073310

[21] Hanamura E 1988 Phys. Rev. B 37 1273

[22] D’Andrea A and Sole R Del 1990 Phys. Rev. B 41 1413

[23] Jazirl S, Bastard G and Bennaceur R 1993 Semicond. Sci. Tech. 8 670

[24] Nirmal M, Norris D J, Kuno M, Bawendi M G, Efros Al L and Rosen M 1995 Phys. Rev. Lett. 75 3728

[25] Sugawara M 1995 Phys. Rev. B 51 10743

[26] Califano M, Franceschetti A and Zunger A 2007 Phys. Rev. B 75 115401

[27] Machikowski P and Jacak L 2005 Phys. Rev. B 71 115309

[28] Vogel W and Filho R L de Matos 1995 Phys. Rev. A 52 4214

[29] Meekhof D M, Monroe C, King B E, Itano W M and Wineland D J 1996 Phys. Rev. Lett. 76 1796

[30] Feynman R P 1951 Phys. Rev. 84 108

[31] Wjos A, Hawrylak P, Fafard S and Jacak L 1996 Phys. Rev. B 54 5604

[32] Scully M O and Zubairy M S 1997 Quantum Optics (Cambridge: Cambridge University Press)