Reheating temperature in non-minimal derivative coupling model

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Abstract

We consider the inflaton as a scalar field described by a non-minimal derivative coupling model with a power law potential. We study the slow roll inflation, the rapid oscillation phase, the radiation dominated and the recombination eras respectively, and estimate e-folds numbers during these epochs. Using these results and recent astrophysical data we determine the reheating temperature in terms of the spectral index and the amplitude of the power spectrum of scalar perturbations.

1 Introduction

To solve some dilemmas in the standard model of cosmology such as the flatness, the horizon, the monopoles problems and so on, inflation as an accelerated expansion era in the early universe was introduced by [1]. This scenario is now dubbed as old model of inflation, in which the universe underwent a de-Sitter expansion in a supercooled unstable false vacuum. Afterwards, by proposing a scalar field (inflaton) as the source of inflation, a new inflationary model was introduced in [2]. In this context, inflation was driven by the inflaton which slowly rolled down towards the minimum of its effective potential. To provide enough e-folds number, the potential must be nearly flat near its minimum.

The nature of this scalar field has not yet been identified, but a simple possible candidate might be the Higgs boson [3]. To adapt the inflaton to the Higgs boson, a non-minimally derivative coupling model in which the kinetic term of the inflaton is coupled to the Einstein tensor, was proposed in [4]. This model does not suffer from unitary violation problem and is safe from quantum corrections. Besides, slow roll inflation can be described by

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steep potentials in this framework[5, 6]. More general non-minimal derivative coupling model has also been considered in the literature to study the accelerated expansion of the universe in the early universe as well as in the late time [7].

After the end of the inflation, the universe was cold and dominated by the inflaton scalar field energy. This energy had to be converted to relativistic particles to reheat the universe via a procedure called the reheating process [8]. A proposal for reheating, is the decay of the inflaton to ultra-relativistic particles (radiation) during a rapid coherent oscillation phase about the minimum of the potential. These particles interacted rapidly to become in thermal equilibrium characterized by the reheating temperature $T_{reh}$, and the universe entered on the radiation dominated era.

Although the exact value of $T_{reh}$ has not yet been known, but some upper and lower bounds for this temperature have been obtained in the literature. By considering that the reheating process occurred before the big bang nucleosynthesis (BBN), and by combining constraints on light elements abundance and data obtained from large scale structure and cosmic microwave background (CMB), one can find a lower bound for $T_{reh}$, $4MeV \lesssim T_{reh}$ [9]. An upper bound may be taken as the energy scale at the end of inflation which is around the GUT scale $T_{reh} \lesssim 10^{16}GeV$. These assumptions give a wide range for the reheating temperature. In [10] by involving supersymmetry and considering the gravitino production, and on the base of cosmic microwave background (CMB) radiation data, this range was tightened to $6TeV \lesssim T_{reh} \lesssim 10^{4}TeV$.

A more accurate method to determine $T_{reh}$ in terms of CMB data was introduced in [11]. This method is based on determining the number of e-folds during the evolution of the universe from the inflation until the present time. Although in this context $T_{reh}$ may be determined in terms of spectral index and amplitude of power spectrum of scalar perturbations, but due to uncertainties of these quantities in WMAP7 data [12], a large relative uncertainty for $T_{reh}$ is arisen: $\sigma(T_{reh})/T_{reh} \approx 53$, where $T_{reh} = 3.5 \times 10^{6}GeV$.

In this paper we assume that the inflaton is a scalar field described by non-minimal derivative coupling model introduced in [4]. We study the inflationary era, the rapid oscillation phase of the inflaton, the radiation dominated epoch respectively and employ the method proposed in [11] to determine the reheating temperature in terms of the spectral index and power spectrum of scalar perturbations. Finally, the value of $T_{reh}$ and its relative uncertainty are computed from recent astrophysical data such as WMAP9 and Planck 2013 results.

We use units $\hbar = c = 1$ through the paper.
2 Evolution of the universe and the reheating temperature

We consider the spatially flat Friedmann-Lemaître-Robertson-Walker (FLRW) space-time
\[ ds^2 = -dt^2 + a^2(t)(dx^2 + dy^2 + dz^2), \]
and choose an arbitrary length scale, \( \lambda_0 \), crossing the Hubble radius \( R_H := \frac{1}{H} = \frac{a}{\dot{a}} \) at some time, denoted by \( t_* \), during the inflation [13]. By using the red-shift relation
\[ \frac{\lambda(t_*)}{\lambda_0} = \frac{a(t_*)}{a_0}, \]
where subscript "0" denotes the present time and by taking \( a_0 = 1 \), we obtain
\[ \lambda_0 = \frac{1}{a(t_*)H(t_*)}. \]
This reference time will be used in division of the evolution of the universe into four parts as follows:

I- From \( t_* \) until the end of slow roll, denoted by \( t_e \).

II- From \( t_e \) until the reheating or beginning of the radiation dominated epoch, denoted by \( t_{reh} \).

III- From \( t_{reh} \) until recombination era, denoted by \( t_{rec} \).

IV- From \( t_{rec} \) until the present time \( t_0 \).

The number of e-folds from \( t_* \) until \( t_0 \) is then given by
\[ N = \ln \left( \frac{a_0}{a_*} \right) = \ln \left( \frac{a_0}{a_{recc}} \right) = \ln \left( \frac{a_{recc}}{a_{reh}} \right) = \ln \left( \frac{a_{reh}}{a_e} \right) = \ln \left( \frac{a_e}{a_*} \right) = N_{IV} + N_{III} + N_{II} + N_I, \]
where the subscripts denote the value of the parameter at their corresponding times.

In the following we will try to use eq.(4) to determine the reheating temperature in a non-minimal derivative coupling model in which the inflaton kinetic term is non-minimally coupled to Einstein tensor. This inflationary model is described by the action [4]
\[ S_\phi = \int d^4x \sqrt{-g} \left[ \frac{M_P^2}{2} R - \frac{1}{2} g^{\mu\nu} \partial_\mu \phi \partial_\nu \phi + \frac{1}{2M^2} G^{\mu\nu} \partial_\mu \phi \partial_\nu \phi - V(\phi) \right], \]
where \( G^{\mu\nu} = R^{\mu\nu} - \frac{1}{2} g^{\mu\nu} R \) is the Einstein tensor, \( R \) is the scalar curvature, \( M_P \) is the reduced planck mass given by \( M_P = \sqrt{\frac{1}{8\pi G}} = 2.4 \times 10^{18} \text{GeV} \), and \( M \) is a scale with mass dimension.
2.1 Slow roll

In the era (I), the universe is dominated by the inflaton scalar field. The Friedmann equation is

\[ H^2 = \frac{1}{3M_P^2} \rho_\phi, \]  

(6)

where

\[ \rho_\phi = \frac{1}{2} \left( 1 + 9 \frac{H^2}{M^2} \right) \dot{\phi}^2 + V(\phi), \]  

(7)

is the energy and the upper dot is \( \dot{\cdot} = \frac{d}{dt} \). The pressure is obtained as

\[ P_\phi = \frac{1}{2} \left( 1 - 3 \frac{H^2}{M^2} \right) \dot{\phi}^2 - V(\phi) - \frac{1}{M^2} \frac{d(H\dot{\phi}^2)}{dt}. \]  

(8)

With the help of continuity equation

\[ \dot{\rho}_\phi + 3H(P_\phi + \rho_\phi) = 0, \]  

(9)

one can derive the equation of motion of the inflaton as

\[ \left( 1 + 3 \frac{H^2}{M^2} \right) \ddot{\phi} + 3H \left( 1 + \frac{3H^2}{M^2} + \frac{2\dot{H}}{M^2} \right) \dot{\phi} + V'(\phi) = 0. \]  

(10)

In the continue, we restrict ourselves to high friction regime \[4\]

\[ \frac{H^2}{M^2} \gg 1. \]  

(11)

This choice, as we will see, by enhancing the slow roll procedure, enables us to consider more general steep potentials. During the slow roll, we have \( \frac{H^2}{M^2} \dot{\phi}^2 \ll V(\phi), \dot{\phi} \ll H\dot{\phi} \), and therefore the Friedmann equation and inflaton equation of motion reduce to

\[ H^2 \approx \frac{1}{3M_P^2} V(\phi), \]  

\[ \dot{\phi} \approx -\frac{M^2 V'(\phi)}{9H^3} \]  

(12)

respectively. The slow roll parameters \( \epsilon, \delta \) satisfy

\[ \epsilon := -\frac{\dot{H}}{H^2} \approx \frac{M_P^2 M^2}{2} \frac{V''(\phi)}{3H^2 V'(\phi)} \ll 1, \quad \delta := \frac{\ddot{\phi}}{H\dot{\phi}} \ll 1. \]  

(13)

\( \delta \) can be written as

\[ \delta \approx -\eta + 3\epsilon \]  

(14)
where $\eta$ is defined with
\[ \eta := \frac{M^2 \, M^2 \, V''(\phi)}{3 \, H^2 \, V(\phi)}. \] (15)

Eqs. (13), imply that by choosing an appropriate $M$ in high friction regime (11), slow roll conditions do not oblige us to adopt approximately flat potentials.

Hereafter we will restrict ourselves to power law potential
\[ V(\phi) = v \phi^n, \] (16)
where $v$ is a real number, and $n$ is an even positive integer to guarantee that the potential has a minimum, about which the rapid oscillation of the inflaton occurs after slow roll.

The number of e-folds in the era (I) is
\[ N_I = \ln \left( \frac{a_e}{a_s} \right) = \int_{t_s}^{t_e} H \, dt = \frac{1}{M_H^2 M^2} \int_{\phi_s}^{\phi_e} \frac{V^2(\phi)}{V'(\phi)} \, d\phi \]
\[ \simeq \frac{v}{n(n+2)M^4 P^2 \phi_s^{n+2}}. \] (17)

To obtain the above equation we have used (16), and $\phi_s \gg \phi_e$. To estimate $\phi_s$, we consider the spectral index $n_s$ [5],
\[ n_s - 1 \simeq -2\epsilon - 2\delta \approx \frac{M_H^2 M^2}{H^2 s} \left[ -\frac{4}{3} \frac{V''(\phi_s)}{V'(\phi_s)} + \frac{2}{3} \frac{V''(\phi_s)}{V(\phi_s)} \right]. \] (18)

For the power law potential, this equation reduces to
\[ 1 - n_s = \frac{2}{v} \frac{M^4 P^2 M^2 n(n+1)}{\phi_s^{(n+2)}}. \] (19)

The number of e-folds in the time interval I is then obtained by substituting $\phi_s$ from (19) into (17):
\[ N_I = 2 \frac{(n+1)}{(n+2)(1 - n_s)}. \] (20)

### 2.2 Reheating era

At the end of slow roll we have $\epsilon(\phi_e) \simeq 1$, which yields
\[ \phi_{e+2}^n = \frac{M_H^4 M^2 n^2}{2v}. \] (21)

At this time the energy density is approximately given by
\[ \rho_e \simeq V(\phi_e) = \frac{v}{2} \left( \frac{M^2 M_H^4 n^2}{2v} \right)^{\frac{n-2}{n+2}}, \] (22)

5
Figure 1: $\varphi := \frac{\dot{\phi}}{M_P}$ in terms of dimensionless time $\tau = mt$, for $\frac{M^2}{M_P^2} = 10^8$ with initial conditions $\{\varphi(1) = 0.056, \dot{\varphi}(1) = 0\}$, for the quadratic potential.

and the scalar field commences a rapid coherent oscillation around the bottom of the potential (see fig (1), depicted for the quadratic potential $V(\phi) = \frac{1}{2}m^2\phi^2$ via numerical methods). To obtain the equation of state of the universe in this era, we follow the steps used in [14]. In this high frequency regime, the behavior of the scalar field is opposite to the slow roll, and its quasi-periodic evolution may be described as [14] [15]

$$\phi(t) = \Phi(t) \cos \left( \int W(t) dt \right).$$ \hspace{1cm} (23)

$W(t)$ is some function of time and the time dependent amplitude, $\Phi(t)$, is given by

$$V(\Phi(t)) = v\Phi^n(t) = \rho_\phi.$$ \hspace{1cm} (24)

The rapid oscillation of the scalar field $\phi$ occurs after the slow roll. This high frequency regime is characterized by (for more details, see the first reference in [14] and also [15])

$$\left| \frac{\dot{\Phi}(t)}{\Phi(t)} \right| = \left| \frac{\dot{\varphi}}{\varphi} \right| \ll W(t),$$ \hspace{1cm} (25)

implying that the energy density and the Hubble parameter decrease very slowly (insignificantly) in one period of oscillation of the scalar field [14]. This can be seen numerically in fig (1) which shows that although $\dot{\phi}$ oscillates rapidly, but $\frac{\delta\Phi(t)}{\Phi(t)} \ll 1$, where $\delta\Phi(t)$ is the change of $\Phi(t)$ during one period of $\phi$ oscillation.

With the help of (7), (8), and (11), the continuity equation of the scalar field can be written as

$$\dot{\rho}_\phi = -2H (\rho_\phi - V(\phi)) + \frac{3H}{M^2} \frac{d}{dt} \left( H\dot{\phi}^2 \right).$$ \hspace{1cm} (26)
Now let us take the time average of both sides of the above equation over one oscillation of the scalar field ($\langle ... \rangle = \frac{\int_{t}^{t+T} \rho dt}{T}$ is the time average over an oscillation whose period is $T$). The left hand side of (26) gives

$$\langle \dot{\rho} \rangle = \frac{1}{T} \int_{t}^{t+T} \rho \dot{\rho} dt = \frac{\delta \rho(t)}{T} \approx \dot{\rho}(t).$$  \hspace{1cm} (27)$$

This approximation is valid only on time scales large with respect to the short period of high frequency quasi-oscillation. Converting time integration to $\phi$ integration and using

$$\dot{\phi}^2 = \frac{2M^2}{9H^2} (\rho - V(\phi)),$$  \hspace{1cm} (28)$$

and (24), we obtain the time average over an oscillation of the right hand side of (26) as

$$- \frac{2}{3T} (\rho - V(\phi)) \left|_{-\Phi}^{\Phi} \right. + \frac{2}{3} \rho \left( \rho - V(\phi) \right) \right|_{-\Phi}^{\Phi} \hspace{1cm} \approx - \frac{2}{3} \rho \phi.$$  \hspace{1cm} (29)$$

To obtain the above relation we have used the evenness of $V$, (24), and also (25) which implies that the variables in the integral except $\phi$ may be replaced by their averaged values in one oscillation of $\phi$ in rapid oscillation phase. So finally we get

$$\dot{\rho} \approx - \frac{2n}{n+2} H \rho.$$  \hspace{1cm} (30)$$

In (29) and (30), $\rho(t)$ and $H$ are the averaged values in the sense explained above and (30) holds for time scales large with respect to $T$ ($t \gg T$). Therefore on time scales much larger than the period of rapid oscillation, the effective equation of state parameter of the scalar field is $w = \gamma - 1$ where

$$\gamma = \frac{2n}{3n+6}.$$  \hspace{1cm} (31)$$

If the non-minimal derivative coupling was absent we would have $\gamma = \frac{2n}{n+2}$ [14]. In this regime, following our analysis and by using (30), the Hubble parameter can be approximated by [15]

$$H = \sqrt{\frac{1}{3M^2_P}} \rho \approx \frac{2}{3\gamma t}.$$  \hspace{1cm} (32)$$
Figure 2: $\Pi := \frac{\Pi}{m}$ in terms of dimensionless time $\tau = mt$, with initial conditions \{\varphi(1) = 0.056, \dot{\varphi}(1) = 0\}, where $\varphi = \frac{\phi}{M_p}$, for the quadratic potential $\frac{1}{2}m^2\phi^2$ and $\frac{m^2}{M_p^2} = 10^8$.

To confirm and justify our results, based on eqs. (6), (7), and (10), the behavior of $H(\propto \sqrt{\rho})$ is depicted via numerical methods in fig. (2). This shows a good agreement between numerical result and (32) for large time scales.

We assume that in this epoch the inflaton decays to ultra-relativistic particles (whose the energy density is denoted by $\rho_r$), to reheat the universe. From the beginning of rapid oscillation, i.e. $\rho_r = 0$, until $\rho_r = \rho_{reh} \approx \rho_\phi$, which is the beginning of radiation dominated era, the universe is approximately dominated by the rapidly oscillating scalar field. Therefore, in this era the Hubble parameter can be approximated by [32][15], and the scale factor during this era scales as

$$\frac{a_{reh}}{a_e} \simeq \left(\frac{\rho_e}{\rho_{reh}}\right)^{\frac{n+2}{2n}}.$$

At $t_{reh}$ we can estimate the energy density as

$$\rho_{reh} \simeq \frac{g_{reh}}{30} \pi^2 T_{reh}^4,$$  

where $T_{reh}$ is the temperature of ultra relativistic particles at $t_{reh}$, and $g_{reh}$ is the number of (massless) degrees of freedom corresponding to the ultra-relativistic particles present in the model at $t_{reh}$ [16]. Collecting all together, we can estimate the number of $e$-folds during rapid oscillation

$$N_{II} = \ln \left(\frac{a_{reh}}{a_e}\right) = \ln \left(\sqrt{\frac{M_p^4 M^2 M_{reh}^2}{2a_e^2}} \left(\frac{30}{g_{reh} \pi^2 T_{reh}^4}\right)^{\frac{n+2}{2n}} \pi \right).$$
To be more specific we must determine \( v \). To do so, consider the power spectrum of the scalar perturbation
\[
\mathcal{P}_s = \frac{H^2}{8\pi^2 M_P^2 \epsilon},
\]
which is computed at the horizon crossing \( k = k_0 = a_s H_s \) (see (2) and (3)), where \( k_0 = \frac{1}{k_0} \) is a pivot scale. Thus
\[
H_s = 2\pi M_P \sqrt{2\epsilon \mathcal{P}_s(k_0)}.
\]
Using this equation together with (19) and \( H^2 \gtrsim \frac{1}{3M_P} V(\phi_s) \), and after some computations we find that
\[
v = \left( \frac{1 - n_s}{1 + n} \right)^{1+n} \left( 6\pi^2 \mathcal{P}_s(k_0) \right)^{\frac{n-2}{2}} \left( 2M^2 \right)^{-\frac{n}{2}} M_P^4 n.
\]
Substituting (38) into (35) yields
\[
\mathcal{N}_{II} = \ln \left[ \frac{1}{2} \left( \frac{n(1 - n_s)}{1 + n} \right)^{\frac{n-2}{2}} \left( 180M_P^4 \mathcal{P}_s(k_0) \right)^{\frac{n-2}{2n}} \frac{g_{reh} T_{reh}^4}{g_{reh}} \right].
\]
So far, in our computations we have implicitly assumed that \( \frac{H^2}{M^2} \gg 1 \) holds during inflation and reheating. But as the inflaton oscillation amplitude and consequently \( H^2 \) (see eq.(24)) decrease, the validity of this assumption must be investigated. At the end of the reheating era (beginning of radiation dominated era) \( \rho_\phi \simeq \rho_{reh} \), we have \( H_{reh}^2 \simeq \frac{1}{3M_P} \rho_{reh} \). As \( H \) decreases, \( H_{reh} \) is the the minimum of the Hubble parameter in the era I and II; so if
\[
\frac{H_{reh}^2}{M^2} \gg 1,
\]
then the assumption \( \frac{H^2}{M^2} \gg 1 \) is safe in our computations. In terms of the reheating temperature, (40) may be written as
\[
\frac{\pi^2 g_{reh}}{90M^2 M_P^2} T_{reh}^4 \gg 1.
\]
\[2.3 \quad \text{Radiation dominated and recombination eras}\]
In the radiation dominated era the universe is constituted of ultra-relativistic particles which are in thermal equilibrium, and undergoes an adiabatic expansion during which the entropy per comoving volume is conserved: \( dS = 0 \) [16]. In this era the entropy density, defined by \( s = Sa^{-3} \), is obtained as (16)
\[
s = \frac{2\pi^2}{45} g_{reh} T^3.
\]
So we have
\[ \frac{a_{\text{rec}}}{a_{\text{reh}}} = \frac{T_{\text{reh}}}{T_{\text{rec}}} \left( \frac{g_{\text{reh}}}{g_{\text{rec}}} \right)^{\frac{1}{3}}. \] (43)

In the recombination era, \( g_{\text{rec}} \) corresponds to photons degrees of freedom and consequently \( g_{\text{rec}} = 2 \). Hence
\[ N_{III} = \ln \left( \frac{T_{\text{reh}}}{T_{\text{rec}}} \left( \frac{g_{\text{reh}}}{2} \right)^{\frac{1}{3}} \right). \] (44)

The temperature decreases by the expansion of the universe via \( T(z) = T(z = 0)(1 + z) \), where \( z \) is the redshift parameter. Therefore we can express \( T_{\text{rec}} \) in terms of \( T_{\text{CMB}} \) as
\[ T_{\text{rec}} = (1 + z_{\text{rec}})T_{\text{CMB}}. \] (45)

We have also
\[ \frac{a_0}{a_{\text{rec}}} = (1 + z_{\text{rec}}) \] (46)

Thus
\[ N_{III} + N_{IV} = \ln \left( \frac{T_{\text{reh}}}{T_{\text{CMB}}} \left( \frac{g_{\text{reh}}}{2} \right)^{\frac{1}{3}} \right). \] (47)

### 2.4 Reheating temperature

So far, we have determined the number of e-folds in the the right hand side of (4). To obtain the reheating temperature we need also to determine \( N \) in (4). Taking \( a_0 = 1 \), the number of e-folds from the horizon crossing until the present time is derived as \( N = \exp(\Delta) \), where
\[ \Delta = \frac{1}{a_s} = \frac{H_s}{k_0} = \frac{2\pi M_P}{k_0} \sqrt{\frac{2n(1 - n_s)}{1 + n}} \mathcal{P}_s(k_0). \] (48)

To obtain the above relation we have made use of (57) and
\[ \epsilon \simeq \frac{n(1 - n_s)}{4(n + 1)}, \] (49)

which is derived from [13,15] and (19).

Using (4), (20), (39), (47), and (48) we finally arrive at
\[ T_{\text{reh}} = \alpha T_{\text{CMB}}^{-\frac{n}{n+4}}, \] (50)

where \( \alpha \) is defined through
\[ \alpha = \frac{\frac{2n+6}{2n+2} M_P}{\frac{\pi}{2}} \left( \frac{k_0}{\pi} \right)^{\frac{n+1}{2}} \left( \frac{180n(1 - n_s)}{n + 1} \right) \times \exp \left( \frac{2n(n + 1)}{(n + 2)(n + 4)(1 - n_s)} \right) \mathcal{P}_s^{\frac{1}{n+4}}(k_0). \] (51)
\( \alpha \), up to this order of approximation, is independent of \( v \) and \( M \). Note that, following (11), validity of the high friction assumption (11) requires that \( M \) satisfies

\[ M^2 \ll \frac{\pi^2 g_{\text{reh}}}{90 M_P^2} \alpha^2 T_{\text{CMB}}^2. \]  

(52)

For the quartic potential, \( n = 4 \), (50) reduces to

\[ T_{\text{reh}} = 1.927 g_{\text{reh}} M_P (1 - n_s) \frac{3}{2} P_s^{\frac{1}{2}} (k_0) \exp \left( \frac{5}{6 (1 - n_s)} \right) \left( \frac{k_0}{T_{\text{CMB}}} \right)^\frac{1}{2}, \]  

(53)

and for quadratic potential, \( n = 2 \), it reduces to

\[ T_{\text{reh}} = 2.205 g_{\text{reh}} M_P (1 - n_s) \frac{3}{2} P_s^{\frac{1}{2}} (k_0) \exp \left( \frac{1}{2 (1 - n_s)} \right) \left( \frac{k_0}{T_{\text{CMB}}} \right)^\frac{3}{2}, \]  

(54)

In the minimal model, for the quadratic potential, the reheating temperature was obtained in [11]:

\[ T_{\text{reh}} = 0.017 M_P (1 - n_s) \frac{3}{2} P_s^{\frac{1}{2}} (k_0) \exp \left( \frac{6}{1 - n_s} \right) \left( \frac{k_0}{T_{\text{CMB}}} \right)^3. \]  

(55)

Note that in contrast to the non-minimal derivative coupling model, \( T_{\text{reh}} \) in (55) does not depend on relativistic degrees of freedom \( g_{\text{reh}} \).

By taking \( g_{\text{reh}} = 106.75 \), which is the ultrarelativistic degrees of freedom at the electroweak energy scale, the reheating temperature in minimal model for quadratic potential was computed in [11] as \( T_{\text{reh}} = 3.5 \times 10^8 \text{GeV} \). This result was based on WMAP7 data [12] which, for the pivot scale \( k_0 = 0.002 \text{Mpc}^{-1} \), imply (for 68% CL, or 1\( \sigma \) error)

\[ P_s(k_0) = 2.441^{+0.088}_{-0.092} \times 10^{-9} \quad n_s = 0.963 \pm 0.012. \]  

(56)

The relative uncertainty, \( \frac{\sigma(T_{\text{reh}})}{T_{\text{reh}}} \) up to first order Taylor expansion, where

\[ \sigma(T_{\text{reh}}) = \sqrt{\left( \frac{\partial T_{\text{reh}}}{\partial n_s} \right)^2 \sigma^2(n_s) + \left( \frac{\partial T_{\text{reh}}}{\partial P_s} \right)^2 \sigma^2(P_s)} \]  

(57)

was derived as \( \frac{\sigma(T_{\text{reh}})}{T_{\text{reh}}} \approx 53 \) [11]. In our nonminimal model, for quadratic potential, and by using (53), we find \( T_{\text{reh}} = 6.53 \times 10^{12} \text{GeV} \) which is much larger than what was obtained in the minimal case. The relative uncertainty is now

\[ \frac{\sigma(T_{\text{reh}})}{T_{\text{reh}}} = 4.275. \]  

(58)

Due to exponential dependence of \( T_{\text{reh}} \) on \( n_s \), the uncertainty in determining the spectral index has a large effect on the reheating temperature uncertainty. Fortunately recent results from WMAP9, ACT, SPT and Planck...
2013, may be employed to obtain more exact value for reheating temperature with less relative uncertainty. These results are quoted in the table, for $68\% \, \text{CL}$, or $1\sigma$ error. The adopted pivot scale for the two first column is $k_0 = 0.002\,\text{Mpc}^{-1}$, while for the two last columns $k_0 = 0.05\,\text{Mpc}^{-1}$.

Table 1: Reheating temperature and its relative uncertainty

|               | WMAP9                | WMAP9 + $\epsilon\text{CMB}$ | Planck (only)          | Planck + WP + highL + BAO |
|---------------|----------------------|-------------------------------|------------------------|---------------------------|
| $n_s$         | 0.972 ± 0.013        | 0.9608 ± 0.0080               | 0.9616 ± 0.0094        | 0.9608 ± 0.0054           |
| $10^3 P_s$    | 2.41 ± 0.1           | 2.464 ± 0.072                 | 2.23 ± 0.16            | 2.200 ± 0.056             |
| $T_{\text{reh}}(n=2)\,(\text{GeV})$ | $4.57 \times 10^{14}$ | $3.12 \times 10^{12}$        | $1.16 \times 10^{13}$  | $8.96 \times 10^{12}$     |
| $T_{\text{reh}}(n=4)\,(\text{GeV})$ | $2.42 \times 10^{15}$ | $5.58 \times 10^{11}$        | $4.26 \times 10^{12}$  | $2.75 \times 10^{12}$     |
| $\frac{\sigma(T_{\text{reh}})}{T_{\text{reh}}}$ | $(n=2)$ 8.13         | 2.53                          | 1.062                  | 0.585                     |
| $\frac{\sigma(T_{\text{reh}})}{T_{\text{reh}}}$ | $(n=4)$ 13.64        | 4.26                          | 1.044                  | 0.57                      |

Note that in the last column the relative uncertainty is less than one. This could occur in the context of WMAP9 results provided that $\sigma(n_s) \leq 0.003$.

To compute the above uncertainties, a first order Taylor expansion was employed (see (57)) which, because of exponential dependence of temperature on $\frac{1}{n_s}$, is insufficient. To obtain more accurate bounds for $T_{\text{reh}}$ one can insert $n_s$ and $P_s$ directly in (55). For example for quadratic potential and using Planck+WP+highL+BAO data, at 2 sigma error ($95\% \, \text{CL}$), we obtain

$$ 6.12 \times 10^{11} \text{GeV} < T_{\text{reh}} < 1.04 \times 10^{15} \text{GeV}. \quad (59) $$

## 3 Conclusion

Non-minimal derivative coupling model with a power law potential was employed to describe the inflation (see (5)). In this context, the slow roll inflationary phase of the inflaton was discussed in high friction regime (see (11)). The rapid oscillation phase after the slow roll, during which the inflaton decays to ultra-relativistic particles, was studied. From the beginning of this rapid oscillation until the radiation dominated epoch, the equation of state parameter of the universe can be approximated by a constant (see (31)), which enabled us to compute the number of e-folds in this era (see (55)). We also estimated the number of e-folds number in radiation dominated, and recombination era, in the same way as the minimal model (see (17)). By gathering all these results together, we obtained the reheating temperature in terms of $T_{\text{CMB}}$, spectral index and the amplitude of the power spectrum of scalar perturbations (see (50)). Finally according to recent astrophysical data, we determined the value of $T_{\text{reh}}$ which is much bigger than the reheating temperature obtained in minimal model. This is due to high
friction regime adopted in this paper, which enhances the decay of the inflaton. We also showed that the uncertainty in our result is very smaller with respect to the minimal model.

Due to the transparency of the universe to gravitational wave the detection of primordial gravitational wave may also be used as a powerful tool to study the history of the universe evolution. Expansion rate of the universe and his thermal history after the inflationary phase have direct imprints on the gravitational wave spectrum and its detectability \cite{18}. In \cite{19}, depending on the tensor-to-scalar ratio $r$, and the reheating temperature, the required sensitivity of some experiments to detect gravitational wave was discussed. Indeed there was shown that $r$ and the reheating temperature, $T_{\text{reh}}$, are the main parameters for determining the gravitational wave spectrum. As an outlook, in future studies, these results may be extended to our non-minimal derivative coupling model for which $r$ was computed in \cite{20}.

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