Electron Scattering from Freely Moveable spin-1/2 fermion in Strong Laser Field

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We study the electron scatter from the freely movable spin-1/2 particle in the presence of a linearly polarized laser field in the first Born approximation. The dressed state of electrons is described by a time-dependent wave function derived from a perturbation treatment (of the laser field). With the aid of numerical results we explore the dependencies of the differential cross section on the laser field properties such as the strength, the frequency, as well as on the electron-impact energy, etc. Due to the targets are movable, the DCS of this process reduced comparing to the Mott scattering, especially in small scattering angles.

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Physics related to the radiative processes experienced by free electrons inside a strong electromagnetic field were studied since the advent of laser sources in the early 1960’s [1,2]. An overview on this field can be found in the textbooks by Mittleman [3] and Fedorov [4] and some other recent reviews [5,7]. Most of these studies are carried out in the regime of non-relativistic collisions and for low- or moderate-field intensities. There are also some studies have been carried out to investigate theoretically the relativistic potential. In the presence of ultrastrong lasers, a relativistic treatment becomes imperative (even for slow electrons). In the treatments of Refs.[6-10], effects related to the electron spin have been neglected and the electron has been considered as a Klein-Gordon particle. Based on the theory of Refs.[11,12], Szymanowski et al. [13,14] investigated the spin effect in the relativistic potential scattering in the presence of a circularly polarized field, however, as they stated, the resulting expression for circularly polarized field turned out to be more tractable than for the general case of elliptical or linear polarizations, and then Li et al. for the case of linearly polarized field [15] and Attaouarti et al., for the cases of circularly and elliptically polarized fields [16]. Manaut et al. investigated the case of polarized electrons [17].

The present study addresses the problem of an electron scattering off the freely moveable target in the presence of a monochromatic linearly polarized homogeneous laser field. The aim of this study is to add some physical insight and to show the modification of differential cross section (DCS) due to the movable target and compare to the case of Mott scattering. We investigate, to be specific, the relativistic scattering of an electron from freely movable proton/positive-muon and its recoil effect. A differential cross section is derived by the trace procedure with the aid of Feynman Calcu, Mathematica, and a simplified form is given for specific. Unless specifically stated, atomic units (a.u.) $\hbar = m = e = 1$ are used throughout, and the matric tensor is $g^{\mu\nu} = diag(1, -1, -1, -1)$.

In the regime of laser field intensity as considered in this paper, the field can be treated as classically that does not allow pair creation [18], its four-potential that satisfies the Lorenz condition $\partial A(x) = 0$ is described by (linear polarization): $A(x) = a \cos(kx)$, where $a(0,a)$, and $a$ is the amplitude of vector potential of the field. The four wave vector of field is $k = (\omega/c, k)$, where $\omega$ and $k$ being the frequency and wave number, respectively.

This scattering process involves two fermions, including an electron($e^-$), a proton ($p$) or positive-muon ($\mu^+$).

$$p(\mu^+) + e^- \rightarrow p(\mu^+) + e^-. \quad (1)$$

The relativistic, asymptotic electron state in laser field can be described by Dirac-Volkov function [19], when normalized in volume $V$, considering the linearly polarized field, it reads:

$$\psi_q(x) = \psi_p(x) = \left(1 + \frac{kA}{2c(kp)}\right) \frac{u(p,s)}{\sqrt{2Q}} e^{iS(x)} \quad (2)$$

$$S(x) = -qx - \frac{a \cdot p}{c(k \cdot p)} \sin(kx), \quad (3)$$

where $u$ represents a bispinor for the free electron which us normalized as $\overline{uu} = 2c^2$, and $q^\mu = (Q/c, q)$ is the averaged four-momentum of electron in the presence of the laser field $q'^\mu = p'^\mu - \frac{a^2}{2c^2(k \cdot p)} k^\mu$, where $\overline{a^2}$ is time-averaged square of the four-potential of the laser field. The square of this momentum $q^\mu q_\mu = m^2_c e^2$. The parameter $m_c = \sqrt{1 - \frac{q^2}{c^2}}$ is an effective mass of the electron in the radiative field.

In a first approximation proton/positive-muon will be treated as a structureless, spin-1/2 Dirac particle. Comparing to the electron, it is much heavier, therefore its modification in the
presence of laser is not so notable and we can use plane wave \( \psi_p(x) \) to describe it.

Then the S-matrix element for scattering process takes the form

\[
S_{fi} = -i \int d^4x \bar{\psi}_f(x) A(x) \psi_i(x) = i \int d^4x d^4y \bar{\psi}_f(x) \gamma^\mu \psi_i(x) D_F(x-y) \bar{\psi}_p(y) \gamma_\mu \psi_p(y). 
\]  

(4)

with \( D_F \) is the Feynman propagator for electromagnetic radiation.

For a useful result we calculated the \( d\sigma \) in the laboratory frame of reference in which the initial proton/muon is at rest and set \( q_f = (Q, q), q_i = (Q, q_i) \) and \( P_i = (Mc, 0) \). To get the unpolarized cross section we must average over initial states and sum over final ones. Then using the relation \( d^2 q_f = \frac{1}{Q} dQ dQ d\Omega \) and integrating over the final state energy \( Q_f \), we get for the scattering DCS [20]:

\[
\frac{d\sigma}{d\Omega} = \sum_f \frac{1}{(2\pi)^2} \frac{1}{16Mc^3} \sum_i \frac{q_i}{q} \frac{1}{q} \frac{1}{M^2 + Q + i\omega - \frac{q}{c_q}(q_i \cos \theta + lk \sin \theta \cos \phi)}.
\]

(5)

where

\[
|M_f|^2 = \frac{1}{4} \sum_{l=-\infty}^{\infty} Tr[(c\gamma_f + c^2\Gamma)^l (c\gamma_i + c^2\Gamma^0) Tr[(c\gamma_f + c^2\gamma^0)(c\gamma_i + c^2\gamma^0)^l],
\]

(6)

with \( \Gamma = \Delta_0\gamma^\mu + \Delta_1\gamma^\mu\lambda_k + \Delta_2\gamma^\mu\lambda_k + \Delta_3\gamma^\mu\lambda_k\lambda_k\). Here the \( \Delta_0, \Delta_1, \Delta_2, \) and \( \Delta_3, \) have the same meaning as those of Ref. [15].

The energy conservation derived from the \( \delta \)-function is

\[
Q(Mc^2 + Q + i\omega - c^2(q \cdot q_i + q \cdot k)) = m^2e^4 + Mc^2 Q + i\omega (M\omega - q \cdot k).
\]

(7)

In the limit case of \( Q \ll Mc^2 \), the scattering potential is approach the fixed Coulomb potential, and there is laser field assisted. The energy conservation here become

\[
Q = Q + i\omega
\]

(8)

This is exactly the Mott scattering that has been discussed in Ref. [15].

In table I we display the laser-assisted \( e-p \) scattering differential cross section (DCS) for laser-assisted at the field strength \( E_0 = 1.0 \times 10^9 \text{V/cm} \) and photon energy \( h\omega = 1.17 \) eV, for both geometries \( E_0 \perp p \) and \( E_0 \parallel p \) respectively (at the scattering angle \( \theta = 90^\circ \) the azimuthal angle is \( \phi = 0^\circ \)). It is shown that the DCS for scattering is greatly enhanced with the application of laser field. Due to the target is moveable, there is small modification on the DCS, the cross section is a little smaller that that of laser assisted Mott scattering [15]. With the impact energy increasing, the modifications become larger.

In this work we study the electrons scattering from protons in the presence of a radiation field. The theoretical results for the linear polarization case show that the DCS of scattering is greatly enhanced by the presence of the laser field, and reduced compared the Mott scattering of the same situation. The treatment can be readily extended to the case of a general polarized of the field, and even the cases of Müller scattering and Bhabha scattering.

| \( \theta(\circ) \) | \( E_{f1} \) | \( \log_{10} \frac{d\sigma}{d\Omega} \) | \( \log_{10} \frac{d\sigma}{d\Omega} \) | \( \log_{10} \frac{d\sigma}{d\Omega} \) |
|-----------------|-------------|-----------------|-----------------|-----------------|
| 90              | 1           | -7.1035          | -7.0904          | -7.0907          |
| 10              | -10.9225    | -8.8490          | -8.8515          | -8.8515          |
| 20              | -11.4943    | -9.4169          | -9.4218          | -9.4218          |
| 40              | -12.0821    | -10.0000         | -10.0094         | -10.0094         |
| 80              | -12.6826    | -10.6378         | -10.6558         | -10.6558         |
| 160             | -13.4947    | -11.2346         | -11.4219         | -11.4219         |
| 320             | -13.9197    | -11.7881         | -11.8469         | -11.8469         |
| 180             | -10.1032    | -8.7878          | -8.8789          | -8.8789          |
| 10              | -13.3037    | -11.9318         | -11.9282         | -11.9282         |
| 20              | -14.3907    | -13.0591         | -13.0139         | -13.0139         |
| 40              | -15.1959    | -14.2225         | -13.8190         | -13.8190         |
| 80              | -15.4298    | -15.4539         | -14.1010         | -14.1010         |
| 160             | -15.5149    | -16.6473         | -14.1861         | -14.1861         |
| 320             | -15.6363    | -17.7977         | -14.2591         | -14.2591         |
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