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Geometrical and computer modeling of mechanical engineering surfaces products intersection line

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Abstract

In the design and manufacture of engineering products geometrical problem is known by shaping the surface of the product. An important element of general solution of this problem is to define the lines of surfaces intersection, forming the shape of designed product. Existing possibilities of modern CAD systems do not allow achieving fullness of the result in this direction. For example, control points and change point of visibility is difficult to identify in the product drawings. In addition, there are no possibilities of detecting imaginary points which are necessary for a complete analysis of intersection surfaces, and mapping these points in the drawing. The aim of the study is to develop a geometric algorithm of constructive determining the intersection line and is devoid of these shortcomings. The objectives of the study are testing the obtained algorithm by experimental verification with geometric modeling solutions to specific problems by tools CAD. This study adopted the method, which is based on quotient of geometric sets, which are regarded as intersecting surfaces in space $E^3$. One area of practical use of surface engineering products geometric algorithm - shaping is based on their intersection line.

1. Introduction

At all stages of new engineering products creation - from design to manufacture, have to solve geometric problems of forming surfaces of the product. The geometric form of modern engineering products is a set of merged into one elementary and compound surfaces: polyhedron, spherical, conical, cylindrical, toroidal, helical, spline and others (Fig. 1). When shaping geometric shape products of mechanical engineering we must take into account technological and technical (choice of equipment) the possibility of their production - metal casting, stamping, forging, machining, pressure, welding and others. It is also necessary to take into account the pragmatic (functional) and esthetical requirements for the form of the designed product, which in some cases may reach full unity. For example, in perfect shape airliner or supersonic fighter. Often there is the predominance of one requirements over the other. Accounting for of technological and technical possibilities, pragmatic and aesthetic requirements leads eventually to formation of a specific geometric shape certain engineering products. At the same time, some of the combined in one form and geometric shape of the surface can
form edge - intersection of the line representing the boundary between two or more surfaces (Fig. 1, a). In another case, it may move one surface to another at the contact line (special case line of intersection) (Fig. 1, a), or on the surface rounding formed along the intersection line of surfaces (Fig. 1, b). Therefore, whenever it is possible, to synthesize the geometric shape of the product in the form of a set of elementary surfaces has to deal with the lines of intersection of these surfaces.

![Figure 1. Examples of the creation of complex surfaces of engineering products:](image)

a) 1 – the line of intersection, 2 – the contact line; b) 3 – the surface rounding.

There are plenty of methods for determining the line of intersection of surfaces, each of which solves the positional problem of intersection for a certain limited class of surfaces [1-4]. The purpose of this work is to develop a common and universal of geometric algorithm of definition constructive line of surfaces intersection with the possibility of its implementation by means of modern CAD.

It should be noted that the problem of constructing the intersection line of surfaces is basic and, at the same time, one of the most complex functions in the geometric modeling by means of modern CAD. It is noted, for example, by the authors: Liang Sun, Christopher M. Tiemey, Cecil G. Armstrong, Trevor T. Robinson [5], V. A. Korotkiy, E. A. Usmanova, L. I. Khmarova [6]. Depending on the mathematical content of the geometric modeler the considered problem has different levels and degrees of complexity solutions. Solving such problems by means of CAD begins with the creation of models of surfaces, some of which are simple primitives CAD. The result of the intersection of surfaces, depending on the choice of the way of modeling (the surface or the solid), can be a line, as a geometric definition of the object, or the boundary between the surfaces, which is a certain vicinity of the intersection line. In the latter case the line is determined only approximately. In modern a CAD it is possible to allocate the line of intersection from this boundary. In the transition from 3D-model of the intersecting surfaces to drawing it is required the positions that are defined intermediate and anchor points of the line of intersection. For example, these are the points of change of visibility. These points form the projection of lines of intersection on projection planes. Furthermore, in some cases it is necessary to consider the presence an imaginary component of the intersection line. It is necessary for completeness of representation of the complete picture of the surfaces intersection. The thing is that the imaginary part of the required line of intersection should be considered and have a real figurative representation on the planes of projections. The modern CAD-systems do not have yet the means and opportunities for electronic geometric realization of the imaginary part of the solution.

2. **Formulation of the Problem**

Based on the above information it is necessary to develop a general geometrical algorithm for determining the intersection line of surfaces. The possibilities of this algorithm should describe, on the one hand, accurately and completely the geometry of the final result - the line of intersection, and on
the other hand, should provide various approaches. Under the variability the existence of several directions in solving the problem of intersection with the opportunity of selection of optimal solutions should be understood.

From practical demand of the completeness and accuracy of solving the problem on the intersection of the surfaces it is necessary to perform a theoretical rationale and develop an algorithm for a constructive solution of this problem, implemented by means tools 2D and 3D geometric modeling in CAD.

3. Theory and Methods

In the work on the set theory, the authors K. Kuratowski and A. Mostowski [7] described how to perform the operation of intersection of two sets $A^m$ and $B^q$ over the set $I^n$ at the space of intersection $E^p$, where $m, q, p$ and $n$ – the dimension of nonintersecting sets, the set of intersection and Euclidean space respectively:

$$p = m + q - n$$  \hspace{0.5cm} (1)

where $m + q \geq n$.

Then, for the space $E^t$ formula of intersection will be:

$$p = m + q - 3.$$  \hspace{0.5cm} (2)

It is obviously, that $m + q \geq 3$ and, hence for the two surfaces ($m = q = 2$) has the equality $p = 1$, that is the intersection of surfaces happens on a line or a finite number of lines.

According to the theory of sets [7] between the elements of sets $A$ and $B$, a binary equivalence relation $\Delta$ can be installed. Then, for every element $a_1$, $a_2$ and $a_3$ of set $A$ the conditions will be performed:

1) reflexivity: $a \Delta a$, for any $a \in A$;

2) symmetry: $a_1 \Delta a_2 \Rightarrow a_2 \Delta a_1$;

3) transitivity: $(a_1 \Delta a_2, a_2 \Delta a_3) \Rightarrow a_1 \Delta a_3$.

Elements of the set $A$, which are equivalent to an element $a$, form a class of equivalence. Thus, the equivalence attitude $\Delta$ allows dividing, the set $A$ into nonintersecting classes of equivalence. All the set of equivalence classes, which fills the set $A$, is called factor set of the set $A$. Operation of the division of set $A$ classes of equivalence is called factorization of this set. Factorization of set can be expressed as follows:

$$A^m = \sum_r r^{(t)} = \bigcup_r \sigma^t,$$  \hspace{0.5cm} (3)

Where $r + t = m$, $r$ – dimension of the set $\Sigma$ of equivalence classes, $t$ – the dimension of the class $\sigma$ of equivalence. For surface $A$ of space $E^t$ formula (3) takes the form:

$$A^2 = \sum_1 ^{l(1)} = \bigcup_1 \sigma^1 .$$  \hspace{0.5cm} (4)

Therefore, the surface may be divided into a plurality of lines $\sigma^j$, that is a one-parameter set, where $\Sigma^1$ – the factor-set of surface.

For the space $E^t$, both for the set, there are the following factorization:

$$E^3 = \sum_2 ^{2(1)} = \bigcup_2 \sigma^1 ,$$  \hspace{0.5cm} (5)

$$E^3 = \Lambda ^{l(2)} = \bigcup_1 \Lambda^2 .$$  \hspace{0.5cm} (6)

In the first case, the space is filled with a two-parameter set of equivalence classes – nonintersecting lines, in the second case – it is the one-parameter set of nonintersecting surfaces.

Division (5) of the space $E^t$ allows installing different conformity between the sets $A^2$ and $B^2$, which are the surfaces. Data conformity is such that a finite number of elements from $A^2$ and $B^2$ will belong
to the same equivalence class $\sigma^1$. If in the case of entering the third set $C^2$ in the space $E^3$ (surface or plane), then the factor-set
\[
\sum_{1}^{2} = \bigcup_{1}^{2} \sigma^1
\]
can be taken as projecting and review the operation (5) as an opportunity to perform the projection (mapping) the surface $A^2$, as well as the $B^2$, to the set $C^3$.

Thus, known in Descriptive Geometry the projection operation takes the new interpretation, which can be the basis for solving the problem of the definition of line of intersection surfaces $A^2$ and $B^2$.

We considered the kinematic surface known in descriptive geometry, i.e., surfaces, which are formed by the movement of the lines of constant or variable forms. This surface may be factorized, i.e., it could be represented by the formula (4). We have introduced the concept «the expansion of kinematic surfaces». The operation of expansion based on the possibility of «propagation» of each equivalence class (lines) $\sigma^1$ from the factor-set $\bigcup_1 \sigma^1$ of surface $A^2$ in a the classes of one type. For example, one-parameter set of circles of a cylindrical surface of revolution «propagated» in the two-parameter set of circles centered on the axis of the surface and in planes perpendicular to the axis.

The operation of «propagation» can be represented symbolically as follows:
\[
A^2 = \sum_{1}^{1} = \bigcup_{1}^{1} \sigma^1 \Rightarrow \bigcup_{1}^{2} \sigma^1 = \Phi_{2(1)}.
\]

Obviously, the original surface $A^2$ belongs to the set $\Phi_{2(1)}$. In addition, the set $\Phi_{2(1)}$ can be viewed on the dimension of as a factor set of the space $E^3$.

On the basis of the given information we can propose the following algorithm of constructive determine intersection line of surfaces $A^2$ and $B^2$:

1) Perform a factorization of one of the kinematic surfaces $A^2$ or $B^2$, for example, $A^2$: $A^2 = \sum_{1}^{1(1)} = \bigcup_{1}^{1} \sigma^1$.

2) Perform the extension of the surface $A^2$: $\bigcup_{1}^{1} \sigma^1 \Rightarrow \bigcup_{2}^{2} \sigma^1 = \Phi_{2(1)}$.

3) The set should be taken as projecting factor-sets of space $E^3$: $E^3 = \Phi_{2(1)}$.

4) Assign a plane of mapping (the plane of projection) $C^2$.

5) Perform a factorization of surface $B^2$: $B^2 = \sum_{1}^{1(1)} = \bigcup_{1}^{1} \lambda^1$.

6) Determine a confluent projection of the surface $A^2$ (its factor-set is included in the projecting the factor-set) on a the projection plane $C^2$: $\bigcup_{1}^{1} \sigma^1 \rightarrow \delta_{C}^1$.

7) Determine the set of projections of equivalence classes (the lines $\lambda^1$ of surface $B^2$) in the plane $C^2$: $\bigcup_{1}^{1} \lambda^1 \rightarrow \bigcup_{1}^{1} \lambda_C^1$.

8) Determine the set of projections $\psi_{C}^0$ - the images of points at the required lines of intersection: $\delta_{C}^1 \cap \lambda_C^1 = \psi_{C}^0$; $\bigcup_{1} \psi_{C}^0 = \delta_{C}^1$.

9) Determine the required line of intersection $\Psi^1$ as a set of points of inverse images: $\psi_{C}^0 \rightarrow \Psi^1$; $\bigcup_{1} \Psi^0 = \Psi^1$.

The proposed algorithm can be used for finding of intersection of surfaces positions of the physical object that simulates the value of radio navigation parameters. It is known that when determining the
location of the physical object according to the set of radio navigation parameters in radiolocation and radio navigation to such surfaces, as noted in this work [8], belong spheres, ellipsoids, hyperboloids.

In the first example (Fig. 2) the intersection of the sphere \( A(0, R) \) and the hyperboloid of rotation \( B'(j, g) \) are defined on the basis of 2D-modeling tools.

According to the proposed algorithm, we adopted one surface, for example a hyperboloid \( B' \) as the base for the factorization of space \( E^3 \). According to the first stage of the algorithm we have presented the surface \( B' \) as a continuous set of circles with variable radius, each of which is disposed in the plane which is perpendicular to the axis \( j (j_1, j_2) \) of the surface. According to stage 2 of the algorithm we have done the expansion of the set of circles. In expanding each circle is transformed into the bundle of circles, and all the set of circles of surfaces \( B' \) is converted into congruence (two-parameter set) circles. We accept the congruence as projecting factor-set of space \( E^3 \). Then we appoint the plane of projection \( C^2 \). As a sample, we chose the frontal plane - the overall plane symmetry of surfaces \( A \) and \( B' \). Next, we perform the factorization of surface of the sphere \( A \).

![Figure 2. 3D- and 2D-modeling of the intersection of the sphere and the surface of the hyperboloid.](image)

One option of the factorization of the surface is the presentation of the sphere as a continuous set of circles in the profile planes. One of these circles is a circle \( e(e_2) \) with a diameter \( 1_2 \). Then, on the sixth stage of the algorithm, we have drawn the confluent of the hyperboloid \( B' \) on the plane of the projection \( C^2 \).

This is hyperbole \( g_1, g_2 \). Then on the seventh step of the algorithm the set of projections on the plane \( C^2 \) of the circles in the selected variant the factorization of sphere \( A \) is defined. Construction of
projections is based on the following obvious fact: the rotation of the circle about the point on the perpendicular to the plane of the circle passing through center forms a sphere. For example, circle $e(e_2)$ while rotating about the point $F(F_1, F_2)$ on the perpendicular $OF(O_1, O_2)$ forms the sphere for which the contour circle $e'_2$ will be the projection of the circle $e(e_2)$ on the plane $C^2$ under mapping by the projecting factor-set (the congruence of circles). As a result, all the circles of factor-set of the sphere $A^2$ is displayed on the plane $C^2$ in the form of a bundle $(F)$ of circles centered at the point $F(F_1, F_2)$. A bunch of circles $(F)$ is the image factor-set circles of sphere $A^2$. The intersection points $3', 4'$ of circle $e(e_2)$ and contour hyperbole $g_2$ represent images of points 3($3_1$, $3_2$) and 4($4_1$, $4_2$) of the required intersection line $s(s_1, s_2)$. Performing such actions for each circle of factor-set of sphere $A^2$ allows to get a discrete set of points. Interpolation of the set allows us to construct the intersection line $s$. Factorization of sphere $A^2$ and projecting the factor-set for given conditions of the problem are basis and evidence base of the method of spheres, which is known in descriptive geometry. This method, as applied to the present problem of the surfaces intersection, is the only one which allows to determine pivots points $K(K_1, K_2)$ and $N(N_1, N_2)$ of the intersection line $s$ and the points its imaginary of continuation, which are real projections, for example, points $A(A_1, A_2)$ and $B(B_1, B_2)$.

For the sphere $A^2$ you can select another variant of factorization in the form the continuous set of circles in horizontal planes. In this case, any circle of a new factor-set of the sphere, for example, the circle $k(k_2)$ with the diameter $C_2D_2$ is displayed on plane $C^2$ in the form of a horizontal line that intersects with frontal contour $g_2$ of surface $B^2$ at points 5’, 6’ - the images of points 5($5_1$, $5_2$) and 6($6_1$, $6_2$) of the required intersection line $s$. Moving from the images 5’ and 6’ to the prototype of 5 and 6 is performed in the circle $k'(k'_1, k'_2)$ to the intersection with the circle $k(k_2)$. The new version of factorization of sphere $A^2$ with the same projected factor-set is the basis and the evidence base of other known method in Descriptive Geometry, the method of auxiliary planes. In the example in Fig. 3 the intersection line of two ellipses of the private location by the method auxiliary planes is determined using 2D-modelling.

**Figure. 3** 2D-modeling of the surfaces of intersection of ellipsoids private location.
If the intersecting surfaces (ellipsoids, hyperboloids) have more complex geometry and general location, then the search of the intersection line by means of 2D-modeling becomes difficult. In this case, it is advisable to use 3D-modeling tools that are available in CAD (Fig. 4, 5).

![3D-models of the intersection of the surfaces of the two ellipsoids general arrangement.](image1)

**Figure 4.** 3D-models of the intersection of the surfaces of the two ellipsoids general arrangement.

![3D-models of the intersection of the three surfaces of ellipsoids general arrangement.](image2)

**Figure 5.** 3D-models of the intersection of the three surfaces of ellipsoids general arrangement.

Wherein there is the possibility of surface or solid modeling to obtain the form and coordinates of the points of the intersection line with the necessary accuracy.

4. **Results and Discussion**

The line of intersection of two surfaces of a set of spherical, ellipsoid and hyperboloid, which are used in forming products engineering, may consist only of the real points (for pairs of a set of spheres and ellipsoids) and may have other than the real part of the imaginary extension (for the pairs of surfaces, one of which - hyperboloid).

Obviously, the spheres method provides a more complete picture of the intersection as opposed to the auxiliary planes method, which is used to solve the same problem. The auxiliary planes method is more universal and allows determining the intersection line of surfaces of simple and complex geometric forms in the most general case their location. Both methods are a consequence of the proposed factorization algorithm. They can be used with the use of CAD means on the basis 2D- and 3D-modeling for the tasks at the intersection, which is required to obtain the most complete picture of the intersection of surfaces, forming a geometric shape of the product engineering.
5. Conclusion
We propose a factorization algorithm for determining the intersection line of surfaces, forming a geometric shape of the product engineering. The algorithm is based on the quotient of surfaces and can be implemented by means of a CAD on the basis 2D- and 3D-modeling.

Methods of surface and solid modeling in CAD should be used to get an idea of the line of intersection of surfaces as a whole. A more complete and accurate picture of the formation of the line of intersection can be obtained using the factorization algorithm, allowing to make the choice of optimal solutions of the problem of the intersection.

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