An overview on internal geared mechanisms with small difference between teeth number

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Abstract. Internal gears with small difference between teeth number are most commonly encountered in planetary and differential mechanisms (eg. on automatic transmissions for vehicles). One of the advantages of this gear is the high transmission ratio. Their parallel axes make this gear perfect for the cases when a small deviation of the distance between axes is required. Automatic transmission allows internal gear function to high speed. In this, we present a short overview of internal gear with small difference between teeth number.

1. Introduction

Generally, a gear is a mechanism that consists from two or more gears, which send the rotational movement and torque between two shafts, by means of the teeth that are continuously and sequentially in contact [1].

Classification of the parallel axes gears is extensive, but we are interested only on the type of engagement classification [2], presented in figure 1:

- spur gear – consisting in cylindrical shaped gear, with teeth parallel to the axis;
- internal gear- cylindrical type of gear, where the teeth of one of the wheels are inside the circular ring;
- helical gear- the teeth of this gear have a helicoid shape.

![Figure 1](https://via.placeholder.com/150)

Figure 1. Different types of gears [2]: a) spur gear; b) internal gear; c) helical gear.

The gears are widely use on mechanical transmissions, due their advantages: constant transmission ratio, safety in operation, high durability, high efficiency, reduced size, wide range of powers, speeds
and gear rations usage. They are found in industrial applications such as: speed reducers and multipliers, gearboxes, differentials, toys etc. As disadvantages can be mentioned: high accuracy on execution and assembly; complicated technology; noise and vibration during operation.

The planetary gears, that especially use internal gears, are highly found in industrial application. We can classify planetary gears as [3]:
- planetary gears for transmitting power (figure 2a,b);
- planetary gears for motion transmission, with a high transmission ration (figure 2c,d).

Figure 2. Kinematics of some planetary mechanisms [3]: a), b) for power transmission; c), d) for motion transmission.

Lately research on gears made possible the development of new technologies of mechanical transmissions. Improving gears impose requirements as:
- high load capacity;
- high kinematic precision;
- possibility to transmit motion in sealed environments;
- reduced mass and dimensions.

Challenges encountered by engineers in development of internal gear systems are concerning geometry calculus and damage prevention [4].

One can analyze the internal gear from several points of view: interference phenomenon; contact ratio; transmission ratio. These are aspects that should be very careful analyzed in the case of an internal gear with a small teeth number difference.

We can use functional and kinematic parameters for determining the tooth profile on mechanical transmissions, and also the operation mode of the transmission. Simplifying the calculus of transmission design is a current issue in mechanical engineering.

2. Internal gears

2.1. Interference Phenomenon

Inner cylindrical gears are commonly constructed with straight teeth and consist of two cylindrical wheels, one with external teeth and the other with internal teeth. The directions of rotation for these two wheels are the same. The engagement takes place between a concave profile (of the internal teeth) and a convex one (of the external teeth). This kind of engagement is advantageous in terms of stress for contact teeth. On the internal gear system, the real segment of action is bigger than the external gear’s one, this leads to an increase in coverage degree, which is particularly advantageous.

For internal gears with minimum teeth number, the correct engagement occurs only inside of the engagement line, see figure 3 [2]. The connection between the involute base circle to the inner circle requires a special form, so that the teeth tips of the smaller wheel do not intersect the base pinion teeth.
Interference can occur on three different types within internal gears: involute interference, tip interference and fillet interference. Involute interference occurs between the dedendum of the external gear and the addendum of the internal gear, and it is prevalent when the number of teeth of the external gear is small. In other words, the system gearing must begin on the tangent line to the two base circles. This interference can be avoided by respecting the condition [5]:

$$\tan \alpha_{a_2} \leq \left( 1 + \frac{z_1}{z_2} \right) \cdot \tan \alpha_{w}$$  \hspace{1cm} (1)

where:
- \( \alpha_{a_2} \) is the pressure angle of a tip of the internal gear tooth;
- \( \alpha_{w} \) is the working pressure angle.

The fillet interference appears at the addendum of the external gear and the dedendum of the internal gear and tends to happen when the difference between the numbers of teeth of the two gears is small [2]. In other words, this appears when the tips of the teeth of one gear interfere with the fillets at the roots of the teeth of the other gear. To avoid this interference one must calculate very accurate the diameters of the corresponding circles of active profile.

Tip interference appears when the teeth tip of the exterior gear collides with the teeth tip of the interior gear. This type of interference can take place both during engagement and disengagement of gears, see figure 4 [6]. The tip interference can be avoided, in both cases, by satisfying this condition:

$$\theta_p > \left( \frac{z_g}{z_p} \right) \cdot \theta_g$$  \hspace{1cm} (2)

2.2. Contact Ratio

The engagement continuity is verified by the contact ratio. For a spur gear the contact ratio depends only by teeth number, and increases with the increase of this numbers, according to V. Merticaru et al. [5]. The complete contact ratio is determined by approach contact and recess contact. Their expression, established by R. Maiti et al. [6] are:
Figure 4. Tip interference [6]: a) during engagement; b) during disengagement.

\[ a_c = \sqrt{r_{ap}^2 - r_{bp}^2} - r_p \sin \alpha \]  \hspace{1cm} (3)

\[ r_c = r_g \sin \alpha - \sqrt{r_{ag}^2 - r_{bg}^2} \]  \hspace{1cm} (4)

\[ C_c = (\alpha_c + r_c) / (\pi \cdot m \cdot \cos \alpha_0) \]  \hspace{1cm} (5)

where \( r_p \) and \( r_g \) are the working pitch circle radius, \( r_{ap} \) and \( r_{ag} \) are the tip radius, \( r_{bp} \) and \( r_{bg} \) are the base circle radius of the pinion and gear respectively, \( \alpha \) and \( \alpha_0 \) are the working and standard pressure angles respectively, and \( m \) is the standard module.

2.3. Transmission Ratio

Transmission ratio for an internal-external gear pair can be calculated as the ratio between the teeth number of the ring gear (\( z_g \)) or pinion (\( z_p \)) and the difference in the teeth number (\( z_g - z_p \)) [6].

When the output motion is from the pinion, the transmission ratio is:

\[ i = \frac{z_p}{z_g - z_p} \]  \hspace{1cm} (6)

while when the output motion is from the ring gear, we can write:

\[ i = \frac{z_g}{z_g - z_p} \]  \hspace{1cm} (7)

One can observe that the transmission ratio increases as the difference in teeth number decreases, and reaches its maximum when the difference is equal to unity. The goal of many researches is to minimize the difference in teeth number, trough different gear corrections, avoiding tip interference.
3. New concepts in improving internal gears

An attempt to lower the difference of teeth number, as much as possible, in the internal-external involute gear pair was realized by Maiti, R. and Roy, A.K [6]. This work aimed to accomplish the reducing of tooth difference with avoiding tooth tip interference between involute internal-external gear pair. For this, they had done tooth truncation and center distance modification, keeping the contact ratio within a satisfactory limit. The addendum modification was considerate for avoiding the tip interference, and the results confirmed that maximum contact ratio is achieved if only the addendum of pinion is truncated. The results showed that for 20 and 22.5 involute tooth, the tooth difference cannot be lower than 5. Also, for the harmonic drives, a mathematical relation has been established, for avoiding tip interference when the pinion rim is deflected.

For calculating planetary gear transmission parameters, Muravev developed a scheme that calculates the tooth interference by the means of the Mathematica program [7]. The planetary gear transmission is considered to be used in the reduction gear drive for a worm centrifuge (figure 5 a). For his calculation scheme, he assumed that the known values are number of teeth, displacement coefficient, the tooth modulus and the tooth profile angle. The algorithm incorporates the possibility of evolvent intersection only on the working parts of the meshing teeth. Thus, we can conclude that the interference in meshing can be determined not only by calculation, but also by means of computer-aided design systems. This requires a three-dimensional model for the transmission and the exact specification of the shapes and sizes of the teeth for the wheel.

![Figure 5](image)

**Figure 5.** Kinematic designs :a) reduction gear in a worm centrifuge [7]; b) eight-speed planetary gearbox [8].

A new eight-diameter stepped planetary gear box was proposed by Salakhov, I.I. *et al.* that offer benefits regarding energy consumption and efficiency. The planetary gear box, presented in Figure 5b, implements eight passes of direct gearing and one reverse gear and high efficiency is received due to short kinematic sequences. No interruptions appear in torque delivery, on gear changing, thanks to minimal number of synchronously switching control elements. The minimal number of main links and the less number of control elements resulted into reduced dimensions of the gear box.

Plekhanov, F.I. *et al.* [9] proposed an analytical method, for determining the load capacity of a planetary transmission with internal gear engagement (figure 6 a). The planetary transmissions with small difference in the number of teeth are characterized by large load capacity, large gear ratio and relatively low frictional power losses. The profile of the gear teeth corresponds to a large reduced radius of curvature, therefore is important to calculate the flexural strength of the satellite teeth. From the formula of flexural strength, the authors determine the permissible normal linear load on the satellite tooth. Because on torque application the small gaps between the teeth are shifted, and the load appears simultaneously in several tooth pairs, the authors determined the load capacity of the planetary transmission. Based on their results, it is possible to estimate the load capacity of a planetary transmission and select the number of teeth required for the selected gear ratio.
A dynamic characteristics analysis was performed by C. Huang et al. [10] on a new internal mesh planetary gear with small tooth number difference (figure 6b). The new planetary gear apparatus was analyzed in terms of natural frequencies, vibration modes, dynamic response and acceleration noise. The displacement field and stress field were obtained through static Finit Element simulation. The rotational frequencies of the eccentric shaft were calculated through Finite Element modal analysis and the results showed a lower value. The experimental results confirmed the obtained simulation values.

Shuting Li has presented a theoretical study of the contact problem of internal gears [11]. The calculation model was realized within the limits of the elasticity theory, by considering a pair of elastic bodies, that come in contact one with each other. The assumption made in this model is that deformations are small, the two bodies obey the laws of linear elasticity and the contact surfaces are smooth. The mathematical model can be use for mathematical programming method to find the contact load under the condition of knowing the deformation, and the external forces. Also the Finite Element Model analysis was realized by applying an external torque T on the internal gear and assuming pairs of contact teeth and pairs of contact points. In this paper only FEM calculation results are presented, the experimental tests are still needed to confirm the theoretical results.

4. Conclusions
As general advantages of internal gears we can mention the high transmission ratio for small overall dimensions. It means, they may be very compact for high transmission ratios, which make them candidates for using in industrial manipulators structure. Interference phenomenon; contact ratio; transmission ratio are aspects that should be very careful analyzed in the case of an internal gear with a small teeth number difference. In this paper a short overview on the current existing tendencies into calculating and studying the small teeth number difference gears has been presented.

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