Anapole moment of an exotic nucleus

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Abstract

We consider the anapole moment of $^{11}$Be and demonstrate that the contribution to it of the $1p_{1/2}$ level, which is anomalously close to the ground state, is essentially compensated for by the contribution of the continuum. Our estimate for this anapole moment is $\kappa(^{11}\text{Be}) \simeq (0.07 - 0.08)g_n$. 
1. The anapole moment is a special magnetic multipole arising in a system which has no definite parity \[1\]. The corresponding magnetic field looks like that created by a current in toroidal winding.

For many years the anapole remained a theoretical curiosity only. The situation has changed due to the studies of parity nonconservation (PNC) in atoms. Since these tiny PNC effects increase with the nuclear charge \(Z\), all the experiments are performed with heavy atoms. The main contribution to the effect is independent of nuclear spin and caused by the parity-violating weak interaction of electron and nucleon neutral currents. This interaction is proportional to the so-called weak nuclear charge \(Q\) which is numerically close (up to the sign) to the neutron number \(N\). Thus, in heavy atoms the nuclear-spin-independent weak interaction is additionally enhanced by about two orders of magnitude. Meanwhile, the nuclear-spin-dependent effects due to neutral currents not only lack the mentioned coherent enhancement, but are also strongly suppressed numerically in the electroweak theory. Therefore, the observation of nuclear-spin-dependent PNC phenomena in atoms had looked absolutely unrealistic.

However, it was demonstrated \[2, 3\] that these effects in atoms are dominated not by the weak interaction of neutral currents, but by the electromagnetic interaction of atomic electrons with nuclear anapole moment (AM). Since the magnetic field of an anapole, like that of a toroidal winding, is completely confined inside the system, the electromagnetic interaction of an electron with the nuclear AM occurs only as long as the electron wave function penetrates the nucleus. In other words, this electromagnetic interaction is as local as the weak interaction itself, and in this sense they are indistinguishable. The nuclear AM is induced by PNC nuclear forces and is therefore proportional to the same Fermi constant \(G = 1.027 \times 10^{-5} m^{-2}\) (we use the units \(\hbar = 1, c = 1; m\) is the proton mass), which determines the magnitude of the weak interactions in general and that of neutral currents in particular. The electron interaction with the AM, being of the electromagnetic nature, introduces an extra small factor into the effect discussed, the fine-structure constant \(\alpha = 1/137\). Then, how it comes that this effect is dominating?

The answer follows from the same picture of a toroidal winding. It is only natural that the interaction discussed is proportional to the magnetic flux through such a winding, and hence in our case is proportional to the cross-section of the nucleus, i.e. to \(A^{2/3}\), where \(A\) is the atomic number. Indeed, a simple-minded model calculation leads to the following analytical result for the dimensionless effective constant \(\kappa\) which characterizes the anapole interaction in the units of \(G\) \[3\]:

\[
\kappa = \frac{9}{10^7} \frac{g \alpha \mu}{mr_0} A^{2/3}.
\]

Here \(g\) is the effective constant of the P-odd interaction of the outer nucleon with the nuclear core, \(\mu\) is the magnetic moment of the outer nucleon, \(r_0 = 1.2\) fm. In heavy nuclei the enhancement factor \(A^{2/3}\) is close to 30 and compensates essentially for the smallness of the fine-structure constant \(\alpha\). As a result, \(\kappa\) is not so small in heavy atoms, it is numerically close to 0.3.

The nuclear anapole moment was experimentally discovered in 1997 \[4\]. This result for the total effective constant of the PNC nuclear-spin-dependent interaction in \(^{133}\text{Cs}\) is

\[
\kappa_{\text{tot}}^{(^{133}\text{Cs})} = 0.44(6).
\]

If one subtracts from this number the nuclear-spin-dependent contribution of neutral currents, as well as the result of the combined action of the “weak” charge \(Q\) and the usual hyperfine interaction, the answer for the anapole constant is

\[
\kappa_{\text{exp}}^{(^{133}\text{Cs})} = 0.37(6).
\]
Thus, the existence of an AM of the \(^{133}\)Cs nucleus is reliably established.

The discussed result brings valuable information on PNC nuclear forces. Of course, to this end it should be combined with reliable nuclear calculations. The most detailed theoretical predictions for this AM can be reasonably summarized, at the so-called “best values” for the parameters of P-odd nuclear forces [5], as follows [6, 7]:

\[
\kappa_{\text{theor}}^{(133\text{Cs})} = 0.15 - 0.21. \tag{4}
\]

There are good reasons to consider this prediction sufficiently reliable, at the accepted values of the P-odd nuclear constants.

The comparison of the theoretical value (4) for the cesium AM with the experimental result (3) indicates that the “best values” of [5] somewhat underestimate the magnitude of P-odd nuclear forces. In no way is this conclusion trivial. The point is that the magnitude of parity-nonconserving effects found in some nuclear experiments is much smaller than that following from the “best values” (see review [8]). In all these experiments, however, either the experimental accuracy is not high enough, or the theoretical interpretation is not sufficiently convincing. The experiment [4] looks much more reliable in both respects. Still, further experimental investigations of nuclear AMs are certainly of great interest.

2. In principle, the AM can be enhanced not only due to large \(A\), but also in the case when anomalously close to the ground state of a nucleus there is an opposite-parity level of the same angular momentum. In this connection, attention was attracted in [9, 10] to exotic halo nuclei. In particular, the exotic neutron-rich halo nucleus \(^{11}\)Be was considered therein. In this nucleus the outer odd neutron is in the state \(2s_{1/2}\), its only bound excited level being \(1p_{1/2}\) (the well-known “inversion of levels”). The anomalously small energy separation between these two levels of opposite parity,

\[
|\Delta E| = E(1p_{1/2}) - E(2s_{1/2}) = 0.32 \text{ MeV}, \tag{5}
\]

enhances by itself their P-odd mixing and thus the AM of this nucleus. As pointed out in [9, 10], the small binding energy of the odd neutron,

\[
|\Delta E_0| = 0.50 \text{ MeV}, \tag{6}
\]

affects the AM additionally, but in two opposite directions. On one hand, it suppresses the overlap of the odd-neutron wave function with the core, and thus suppresses the mixing of the \(2s_{1/2}\) and \(1p_{1/2}\) levels due to the weak interaction operator which looks as

\[
W = \frac{G}{\sqrt{2}} \frac{g_n}{2m} \{\sigma \mathbf{p}, \rho(r)\}; \tag{7}
\]

here \(g_n\) is the effective constant of the P-odd interaction of the outer neutron with the nuclear core, \(\sigma\) and \(\mathbf{p}\) are the momentum and spin operators of the outer neutron, and \(\rho(r)\) is the spherically symmetric core density. On the other hand, the small binding energy enhances the matrix element of \(\mathbf{r}\) in the anapole operator of the neutron

\[
a = \frac{\pi e \mu_n}{m} \mathbf{r} \times \sigma; \tag{8}
\]

here \(\mu_n = -1.91\) is the neutron magnetic moment.
The detailed calculation which takes into account the P-odd mixing of the ground state with the 1p_{1/2} level only, results in the following value for the effective anapole constant [10]:

\[ \kappa_{1}(^{11}\text{Be}) = 0.17g_n. \] (9)

Indeed, this value is 15 times larger than that given by the estimate [11] for A = 11 (the neutron constant \(g_n\) is poorly known by itself, most probably \(g_n \lesssim 1\)). Certainly, this enhancement of an AM in a light nucleus would be of a serious interest, even if its possible experimental implications are set aside.

However, such a strong enhancement of AM, as given in [11], in a loosely bound nucleus does not look natural. In particular, nothing of the kind happens in the deuteron. Even in the limit of vanishing binding energy, when the energy interval between the deuteron \(s\) state and the continuum \(p\) states tends to zero, the deuteron AM in no way is enhanced [11] (see also [12]).

As to the problem of \(^{11}\text{Be}\) discussed here, we argue below that a strong cancellation between the contribution of the bound 1p_{1/2} state (accounted for in [9]) and that of the continuum (omitted therein) takes place, resulting in a serious suppression of the estimate [9].

3. We start with the general expression for the anapole moment, as induced by operators (7) and (8):

\[
\langle 0|a|0 \rangle = \frac{G}{\sqrt{2}} \frac{\pi e \mu g}{2m^2} \sum_n \frac{\langle 0|\mathbf{r} \times \mathbf{\sigma}|n \rangle \{\mathbf{\sigma} \mathbf{p}, \mathbf{\rho}(r)\}|0 \rangle + \langle 0|\{\mathbf{\sigma} \mathbf{p}, \mathbf{\rho}(r)\}|n \rangle \langle n|\mathbf{r} \times \mathbf{\sigma}|0 \rangle}{E(2s_{1/2}) - E_n} \tag{10}
\]

To estimate the sum we use at first the closure approximation, which is facilitated here by the same (negative) sign of all energy denominators. After extracting some average value of denominators, \(-\Delta\) (\(\Delta > 0\)), and using the completeness relation, the sum (10) reduces to

\[
\langle 0|a|0 \rangle = -\frac{G}{\sqrt{2}} \frac{\pi e \mu g}{2m^2 \Delta} \langle 0|\{\mathbf{r} \times \mathbf{\sigma}, \{\mathbf{\sigma} \mathbf{p}, \mathbf{\rho}(r)\}\}|0 \rangle \tag{11}
\]

The thus arising effective operator transforms as follows:

\[
\{\mathbf{r} \times \mathbf{\sigma}, \{\mathbf{\sigma} \mathbf{p}, \mathbf{\rho}(r)\}\} = 4(\mathbf{1} + \mathbf{\sigma}) \mathbf{\rho}(r); \tag{12}
\]

de here \(\mathbf{1}\) is the orbital angular momentum of the valence nucleon. (It is rather amusing that we arrive here at the same combination \(\mathbf{1} + \mathbf{\sigma}\) which enters the expression for the magnetic moment of a bound electron.)

In our case of \(^{11}\text{Be}\), \(\mathbf{1} = 0\) and \(\mathbf{\sigma} = 2\mathbf{I}\), where \(\mathbf{I}\) is the spin of the nucleus. Thus, here the expression for AM reduces to

\[
\langle 0|a|0 \rangle = -\frac{G}{\sqrt{2}} \frac{4\pi e \mu_n g_n}{m^2 \Delta} \langle 0|\mathbf{\rho}(r)|0 \rangle \mathbf{I}. \tag{13}
\]

With the standard prescription (see [3]) of deleting from the expression for \(\langle 0|a|0 \rangle\) the factors \((G/\sqrt{2})\mathbf{I}\) and multiplying the rest by \(eI(I + 1)(-1)^{I+1/2-1}/(I + 1/2)\), we arrive finally at the following expression for the effective anapole constant:

\[
\kappa = \frac{3\pi \alpha g_n}{m^2 \Delta} \langle 0|\mathbf{\rho}(r)|0 \rangle. \tag{14}
\]

The expectation value \(\langle 0|\mathbf{\rho}(r)|0 \rangle\) was calculated by us with the same ground-state wave function

\[
R_{2s}(r) = \frac{2^{3/2}a^{2}[1 - (r/a)^2]\exp(-r/r_0)}{r_0^{3/2} \sqrt{45r_0^4 + 2a^4 - 12a^2r_0^2}}, \quad r_0 = 1.45 \text{ fm}, \quad a = 2 \text{ fm},
\]
and core density
\[ \rho(r) = \rho_0 \exp(-r^2/R_c^2), \quad \rho_0 = 0.20 \text{ fm}^{-3}, \quad R_c = 2 \text{ fm}, \]
as those used in [10]. Thus obtained expectation value is
\[ \langle 0|\rho(r)|0 \rangle = 0.052 \rho_0 = 0.01 \text{ fm}^{-3}. \] (15)

Now, if \( \Delta \) is identified with the smallest energy interval \( E(1p_{1/2}) - E(2s_{1/2}) = 0.32 \text{ MeV} \), the numerical result is
\[ \kappa(\Delta = 0.32 \text{ MeV}) = -0.036 g_n. \] (16)

The comparison of (16) with (9) demonstrates that in the last estimate the negative contribution of continuum states outweighs the positive one of \( 1p_{1/2} \), with a small net result which only slightly exceeds, if any, the typical value of \( \kappa \) as given by (1).

As expected, the small binding energy strongly suppresses \( \langle 0|\rho(r)|0 \rangle \) as compared to \( \rho_0 \) itself (see (15)). However, the expected enhancement of the matrix element of \( r \) in the anapole operator (8) is not operative in (16) since on average this \( r \) is eaten up by \( p \) in the weak interaction operator (7). And the strong suppression of \( \langle 0|\rho(r)|0 \rangle \) compensates in (16) for the enhancement due to small energy intervals.

Estimate (16) can be improved considerably in the following way. Its comparison with (9) demonstrates that with \( \Delta = 0.32 \text{ MeV} \) the contribution of the continuum to \( \kappa \) constitutes
\[ \kappa_c(\Delta = 0.32 \text{ MeV}) = -0.036 g_n - 0.17 g_n = -0.206 g_n. \] (17)

With the continuum threshold at \( \Delta = |\Delta E_0| = 0.50 \text{ MeV} \), the continuum contribution is certainly overestimated by (17). However, \( \kappa_c \) can be easily recalculated for more reasonable values of \( \Delta \) just by multiplying (17) by \( 0.32/\Delta \).

Combining thus obtained improved values of \( \kappa_c \) with (9), we arrive at the following estimates for the anapole moment of \( ^{11}\text{Be} \):
\[ \Delta, \text{ MeV} \quad \begin{array}{l} 0.6 \quad 0.7 \quad 0.8 \end{array} \]
\[ \kappa(^{11}\text{Be}) \quad \begin{array}{l} 0.060 g_n \quad 0.076 g_n \quad 0.088 g_n \end{array} \] (18)

We believe that with all the uncertainties of our estimates (18) for the anapole moment of \( ^{11}\text{Be} \), they are more reliable than (9). Most probably the real value of \( \kappa(^{11}\text{Be}) \) is around \((0.07 - 0.08)g_n\), i.e., it is 2 - 3 times smaller than (9).

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