Abstract

This paper proposes a new robust update rule of the target network for deep reinforcement learning, to replace the conventional update rule, given as an exponential moving average. The problem with the conventional rule is the fact that all the parameters are smoothly updated with the same speed, even when some of them are trying to update toward the wrong directions. To robustly update the parameters, the t-soft update, which is inspired by the student-t distribution, is derived with reference to the analogy between the exponential moving average and the normal distribution. In most of PyBullet robotics simulations, an online actor-critic algorithm with the t-soft update outperformed the conventional methods in terms of the obtained return.

Keywords: Deep reinforcement learning, Target network, Student-t distribution

1. Introduction

Reinforcement learning (RL) (Sutton and Barto, 2018) and its extension using deep neural networks (DNNs) (Krizhevsky et al., 2012) to approximate policy and value functions, named deep reinforcement learning (DRL) (Silver et al., 2016; Levine et al., 2018), is one of the promising methodologies for controlling complicated problems. In practice, only applying DNNs as function approximators would make learning process unstable due to their high nonlinearity. To stably learn the optimal policy, therefore, techniques to reduce the variance of learning signals have been developed for DRL. For example, the experience replay (Lin, 1992) method can statistically mitigate the effects of anomalies by allowing DRL to transform into mini-batch learning; and various regularization techniques for the policy (Schulman et al., 2017; Haarnoja et al., 2018; Kobayashi, 2019; Parisi et al., 2019) yield a conservative learning algorithm that avoids updates into the wrong directions; also, by learning two ensemble value functions, either one can be selected to minimize the approximation bias (Fujimoto et al., 2018). Alternatively, several heuristic ways, such as reward and/or gradient clipping, have also been employed in many cases.

As one of such techniques employed for stable learning, target networks have been proposed (Mnih et al., 2015). The target network generates the reference signals for the main network, and "slowly" updates its parameters (i.e., weights and biases in the network) toward the parameters of the main network. In this way, the reference signals would not fluctuate frequently, thereby making it easier to learn more stably, although the learning speed of models employing a target network is basically decreased (Kim et al., 2019). When the target network was first introduced, the "hard" update strategy, which copies, every few steps, the main network into the target one, was used. However, after that, a new strategy, the "soft" update, by which the new parameters for the target network are interpolated, through a fixed ratio, between the current parameters of the target network and the parameters of the main network, became the mainstream in DRL libraries (Stooke and Abbeel, 2019).

As another problem different from the learning speed slowdown, the soft update is basically sensitive to noise and outliers in the updates of the parameters for the main network. For example, even if parts of the parameters for the main network are updated largely along some wrong directions, the soft update will approve all of them without any checks, and copy them into the target network using the fixed ratio. A naive solution for this problem would be to make the ratio for copy as small as possible, but it will slow down the learning speed further. This means that a trade-off between the sensitivity to noise and outliers and learning speed is given by the fixed ratio employed for copying.

As pointed out in the literature (Ilboudo et al., 2020), this problem comes from the exponential moving average (EMA), which can be regarded as the update rule of the mean of a normal distribution with a fixed number of samples. Therefore, based on that literature, this paper proposes a new update rule of the target network, named "t-soft" update. It is inspired by the student-t distribu-
tion, which is well known to be a distribution robust to outliers (Tipping and Lawrence, 2005; Shah et al., 2014; Kobayashi, 2019). The EMA in the soft update is replaced with a moving average derived from the mean of the student-t distribution. In addition, its computation and memory costs are minimized as much as possible by assuming a simple stochastic model while keeping the performance of the t-soft update.

We verify the superiority of the t-soft update through four kinds of dynamical simulations for DRL. In three of four benchmarks, the t-soft update outperforms the conventional soft update and the case without the target network. Even in the remaining one benchmark, the t-soft update succeeds in mitigating the slow down of the learning speed in comparison with the conventional soft update.

2. Preliminaries

2.1. Reinforcement learning

RL enables an agent to learn the optimal policy, which can achieve the maximum sum of rewards from an environment (Sutton and Barto, 2018). In RL, Markov decision process (MDP) with the tuple \((S, A, \mathcal{R}, p_0, p_T, \gamma)\) is assumed. After getting the initial state \(s_0 \in S, s_0 \sim p_0(s_0)\), the agent decides the action at the time step \(t, a_t \in A\) using the policy \(a_t \sim \pi(a_t \mid s_t)\). By performing the action \(a_t\) on the environment, the state is transited to the next according to the transition probability, \(s_{t+1} \sim p_T(s_{t+1} \mid s_t, a_t)\). At the same time, the agent gets a reward according to the reward function: \(r_t = r(s_t, a_t, s_{t+1}) \in \mathcal{R}\). The sum of rewards is converted to produce the return defined as \(R_t = \sum_{k=0}^{\infty} \gamma^k r_{t+k}\) where \(\gamma \in [0, 1)\) for finiteness. As already mentioned, the purpose of the agent is to acquire the optimal policy \(\pi^*\) that maximizes \(R_t\). To this end, numerous algorithms have been proposed, and in this paper, we employ an actor-critic algorithm (Kobayashi, 2020a), which is suitable for robotics due to the capability of directly optimizing the policy in continuous action space.

2.2. Target network with soft update

In most of DRL methods using value function, \(Q(s, a)\) and/or \(V(s)\), the target network is employed to reduce the variance of learning signals (Mnih et al., 2015; Stooke and Abbeel, 2019). For example, when the advantage function \(A(s, a) = Q(s, a) - V(s) = r + \gamma V(s') - V(s)\) is introduced as the learning signal, the following loss is minimized by optimizing the parameters set for the main network \(\theta\):

\[
\mathcal{L} = (r + \gamma V(s'; \phi) - V(s; \theta))^2
\]

where \(\phi\) denotes the parameters set for the target network. That is, the target network outputs the supervision for the bootstrap learning. Note that, even for \(\pi\), a baseline policy similar to the target network is sometimes prepared for sampling actions and for smoothly optimizing the policy, although this paper omits that for simplicity.

The above minimization problem does not update \(\phi\) directly, and therefore, an alternative update rule for \(\phi\) is needed. While its optimal value is expected to be \(\theta\) after its update, it should be noted that the update is unstable in the optimization of nonlinear function approximations like deep learning. Hence, instead of immediately following \(\theta\), we use the soft update rule, which smoothly updates \(\phi\) to \(\theta\):

\[
\phi \leftarrow (1 - \tau)\phi + \tau\theta
\]

where \(\tau\) denotes the smoothness, and if \(\tau = 1\), this update rule becomes the hard update.

3. Proposal

3.1. Analogy between EMA and normal distribution

The soft update employs the EMA of \(\theta\), i.e., \(\phi\) is regarded as its moving average. As explained in the literature (Ilboudo et al., 2020), the EMA extracts the same ratio from the new value \(\theta\) even if it contains noise and outliers (i.e., sudden changes by extreme gradients). To avoid the adverse effects of noise and outliers during the update, we should understand the EMA more deeply.

To this end, let us focus on the analogy between the EMA and normal distribution as well as the above literature did. Specifically, given the \(N\) sampled data \(\{x_n\}_{n=1}^{N}\), the maximum likelihood estimation of normal distribution derives its mean \(\mu\) as follows:

\[
\mu_N = \frac{1}{N} \sum_{n=1}^{N} x_n = \left(\frac{N-1}{N}\right)^{1/2} \frac{1}{N-1} \sum_{n=1}^{N-1} x_n + \frac{1}{N} x_N
\]

where \(\tau_N = 1/N\). When the effective number of sampled data is fixed \((N = \text{const.})\), eq. (3) matches the EMA (i.e., the soft update). Since normal distribution is well known as a distribution sensitive to outliers, we agree that the sensitivity of the EMA is taken over from the normal distribution.

3.2. Moving average based on student-t distribution

The concept of our proposal stands on replacing normal distribution to the distribution robust to outliers. As such distribution, student-t distribution is employed in this paper following the previous studies (Tipping and Lawrence, 2005; Shah et al., 2014; Kobayashi, 2019). Hence, to derive a new update rule based on student-t distribution, we derive the alternative moving average formula from the maximum likelihood estimation of student-t distribution.

Suppose \(d\)-dimensional diagonal student-t distribution with model parameters, i.e., location \(\mu \in \mathbb{R}^d\), scale \(\sigma \in \mathbb{R}^d\), and shape \(\nu\) parameters. The mean of the student-t distribution is \(\mu\), and its covariance matrix is \(\nu / (\nu + d) \sigma \sigma^T\). The maximum likelihood estimation of \(\nu\) and \(\sigma\) are:

\[
\nu_{\text{MLE}} = \sum_{n=1}^{N} (x_n - \mu)^T (x_n - \mu)
\]

\[
\sigma_{\text{MLE}} = \sqrt{\frac{1}{N-d} \sum_{n=1}^{N} (x_n - \mu)(x_n - \mu)^T}
\]

The student-t distribution is a scale mixture of normal distributions, and its mean is estimated by the EMA of the sample mean with a moving average based on the MLE of \(\nu\) and \(\sigma\) in place of normal distribution in EMA. The result is:

\[
\mu_{\text{MLE-EMA}} = \left(\frac{N-1}{N}\right)^{1/2} \frac{1}{N-1} \sum_{n=1}^{N-1} x_n + \frac{1}{N} x_N
\]
\( \mathbb{R}^d \), and degrees of freedom \( \nu \in \mathbb{R}_+ \). Given the \( N \) sampled data \( \{x_n\}_{n=1}^N \), the estimated \( \mu_N \) is derived as follows:

\[
\mu_N = \frac{1}{W_N} \sum_{n=1}^N w_n x_n
\]

(4)

where

\[
W_N = \sum_{n=1}^N w_n
\]

(5)

\[
w_n = \frac{\nu + d}{\nu + (x_n - \mu_{N-1})^2} \sigma^2
\]

(6)

That is, since \( \mu_{N-1} \) remains inside of the formula, \( \mu_N \) has to be computed recursively.

This fact requires us to approximate the derivation of moving average as follows:

\[
\mu_N \simeq \frac{W_{N-1}}{W_{N-1} + w_N \mu_{N-1}} \sum_{n=1}^{N-1} w_n x_n + \frac{w_N}{W_{N-1} + w_N} x_N
\]

\[
\simeq \frac{W_{N-1}}{W_{N-1} + w_N} \mu_{N-1} + \frac{w_N}{W_{N-1} + w_N} x_N
\]

(7)

where \( \tau_w = w_N/(W_{N-1} + w_N) \in (0, 1) \). We notice that \( W_N \simeq W_{N-1} + w_N \) and \( W_{N-1} \sum_{n=1}^{N-1} w_n x_n \simeq \mu_{N-1} \) are the approximated terms. If the update of the model parameters are slow enough, these approximations are with high precision.

As can be seen in eq. (7), it becomes the same form as eq. (5) with a different definition of \( \tau \). While \( \tau \) in the case of normal distribution is fixed to be constant, \( \tau \) in the case of student-t distribution is adaptive even if the effective number of samples \( N \) is fixed. For example, if the new sample \( x \) is far away from the current \( \mu \), \( w \) will be small with large \( D \), and \( x \) hardly affects the update of the \( \mu \). This behavior is desired for the robust update of the target network.

### 3.3. Practical design of t-soft update

Although the new moving average in eq. (7) is effective to ignore outliers, to employ it as the update rule of target network, we have to design the parameters \( \sigma, \nu, d \), and \( W \). In particular, unlike the literature (Ilboudo et al., 2020), where \( \sigma \) is already estimated via another EMA, the update rule of \( \sigma \) is additionally required. It is also desirable to minimize the computation and memory costs to the same level as the conventional soft update. To this end, this paper proposes a practical design of the t-soft update.

First of all, we define two hyperparameters, \( (\tau, \nu) \). Suppose that the update is performed for each subset of the parameters set (a.k.a., weights or a bias of each layer). In that case, the \( i \)-th subset has \( \sigma_i \), \( d_i \), and \( W_i \).

#### Algorithm 1 Proposed t-soft update with hyperparameters \( \nu \in \mathbb{R}_+ \) and \( \tau \in (0, 1) \)

1. Initialize \( \theta \)
2. \( \phi \leftarrow \theta \)
3. Initialize optimizer SGD with learning rate \( \alpha \)
4. \( W = (1 - \tau)\tau^{-1} \)
5. while True do
   6. Compute \( \mathcal{L}(s, a, s', r; \theta, \phi) \)
   7. \( \theta \leftarrow \theta - \alpha \text{SGD}(\nabla_\theta \mathcal{L}) \)
   8. for \( \theta_i, \phi_i \subset \phi \) do
      9. \( \Delta_i^2 = \frac{1}{\nu} \sum_{n=1}^{N_i} (\theta_{i,n} - \phi_{i,n})^2 \)
     10. \( W_i = (1 + \Delta_i^2)(\nu + \Delta_i^2)^{-1} \)
     11. \( \tau_i = \frac{w_i}{W_i + w_i} \)
     12. \( \sigma_i^2 \leftarrow (1 - \tau_i)\sigma_i^2 + \tau_i \Delta_i^2 \)
     13. \( \phi_i \leftarrow (1 - \tau_i)\phi_i + \tau_i \theta_i \)
   14. end for
15. end while

For simplicity, \( W_i \) is shared in all the subsets, i.e., \( W_i = W \). \( W \) is designed as the function of \( \tau \) as follows:

\[
W = \frac{1 - \tau}{\tau}
\]

(8)

By designing \( W \) as above, when \( \nu \rightarrow \infty \) converges the student-t distribution to the normal distribution, the t-soft update also reverts to the soft update.

To reduce the memory cost to store \( \sigma \) for all the parameters, each \( i \)-th subset is assumed to have a common \( \sigma_i \in \mathbb{R}_+ \). This means that for each subset, a \( d_i = d = 1 \)-dimensional student-t distribution is assumed. Following this assumption, eq. (6) is redefined as follows:

\[
w_i = \frac{\nu + 1}{\nu + \Delta_i^2} \sigma_i^2
\]

(9)

where

\[
\Delta_i^2 = \frac{1}{N_i} \sum_{n=1}^{N_i} (\theta_{i,n} - \phi_{i,n})^2
\]

(10)

where \( N_i, \theta_i, \) and \( \phi_i \) denote the number of parameters in \( i \)-th subset, the main and target network’s parameters in \( i \)-th subset, respectively.

Based on eq. (7), the t-soft update is derived as follows:

\[
\phi_i \leftarrow (1 - \tau_i)\phi_i + \tau_i \theta_i
\]

(11)

where

\[
\tau_i = \frac{w_i}{W_i + w_i}
\]

(12)

That is, the update of \( i \)-th subset is suppressed if the mean of the differences between \( \theta_i \) and \( \phi_i \) (i.e., \( \Delta_i \)) is larger than the threshold implied by \( \sigma_i \). Note that, due to \( \tau_i \in (0, 1) \),
Table 1: Simulation environments provided by Pybullet Gym (Brockman et al., 2016; Coumans and Bai, 2016)

| ID                          | Name                        | State space $d_s$ | Action space $d_a$ | Episode $E$ |
|-----------------------------|-----------------------------|-------------------|-------------------|-------------|
| InvertedPendulumBulletEnv-v0 | InvertedPendulum            | 5                 | 1                 | 150         |
| InvertedPendulumSwingupBulletEnv-v0 | Swingup        | 5                 | 1                 | 150         |
| HalfCheetahBulletEnv-v0      | HalfCheetah                 | 26                | 6                 | 1500        |
| AntBulletEnv-v0              | Ant                         | 28                | 8                 | 2000        |

Table 2: Common hyperparameters for the simulations

| Symbol | Meaning                | Value          |
|--------|------------------------|----------------|
| $N$    | Number of neurons      | 100            |
| $L$    | Number of layers       | 5              |
| $\gamma$ | Discount factor     | 0.99           |
| $\alpha$ | Learning rate       | 5e-4           |
| $(\lambda_{1\text{max}}, \lambda_{2\text{max}}, \kappa)$ | Hyperparameters for (Kobayashi 2020a) | (0.5, 0.9, 10) |
| $\epsilon$ | Threshold for clipping | 0.1            |
| $\beta_{DE}$ | Gain for entropy regularization | 0.025 |
| $\beta_{TD}$ | Gain for TD regularization | 0.025 |

the target network can eventually converge to the main network even with the t-soft update.

After that, to adaptively adjust $\sigma_i$ according to the observed $\Delta_i$, the following EMA update is employed.

$$\sigma_i^2 \leftarrow (1 - \tau)\sigma_i^2 + \tau\Delta_i^2$$  \hspace{1cm} (13)

Although $\tau_i$ can replace $\tau$ in this formula, we found that it makes the update of $\phi$ too slow. Note that the initial $\sigma_i^2$ is desired to be sufficiently large to keep the learning speed.

Here, a pseudo algorithm of the t-soft update is summarized in Alg. 1. Note that a stochastic gradient descent (SGD) optimizer and a loss function for RL $\mathcal{L}$ can be arbitrarily selected.

4. Simulations

4.1. Benchmark tasks

Four benchmark tasks for DRL simulated by Pybullet Gym (Brockman et al., 2016; Coumans and Bai, 2016) are prepared, and listed in Table 1. Their rewards are designed based on the following purposes:

(a) InvertedPendulum: A cart keeps a pole standing.
(b) Swingup: A cart swings up a pole and keeps it standing.
(c) HalfCheetah: A two-dimensional cheetah with two legs walks as fast as possible.
(d) Ant: A three-dimensional quadruped walks as fast as possible.

After learning, the agent performs the learned task 50 times to compute the sum of rewards for each, and their median is used as the score. In total, 20 trials are performed for each condition and for each trial, the random seed is set as the trial number.

4.2. Conditions

Basic network architecture, which is implemented by PyTorch (Paszke et al., 2017), has $L$ fully connected layers with $N$ neurons with layer normalization (Ba et al., 2016) and Swish activation function (Ramachandran et al., 2017; Elfwing et al., 2018). Using this network architecture, the actor-critic algorithm with the adaptive eligibility traces (Kobayashi, 2020a) and the student-t policy (Kobayashi, 2019) is implemented. Therefore, the state value function $V$ in the critic requires the target network. Note that this method has the hyperparameters $(\lambda_{1\text{max}}, \lambda_{2\text{max}}, \kappa)$.

To stably learn the tasks, the latest regularization techniques are also combined: PPO (Schulman et al., 2017) with a clipping value $\epsilon$; the policy entropy regularization based on SAC (Haarnoja et al., 2018) with a regularization weight $\beta_{DE}$; and the TD regularization (Parisi et al., 2019) with a regularization weight $\beta_{TD}$. In addition, a robust SGD, i.e., LaProp (Ziyin et al., 2020) with t-momentum (Ilboudo et al., 2020) and d-AmsGrad (Kobayashi, 2020b), is employed with their default parameters except the learning rate.

Table 2 summarizes the common hyperparameters to be used in the simulations. Note that the learning rate is set higher than the value in the literature (Kobayashi, 2020a) in order to confirm the benefit of learning stabilization by the target network. Three different conditions are compared throughout the simulations. Their hyperparameters for the target network, $(\tau, \nu)$, is summarized below.

1. $(1.0, \inf)$ as no target network
2. $(0.3, \inf)$ as the conventional soft update
3. $(0.3, 1.0)$ as the proposed t-soft update

Note that the scale of $\tau$ is dependent on the learning method.
4.3. Results

Learning curves are illustrated in Fig. 1. The test results after learning are also depicted in Fig. 2. From the first three tasks (a)–(c), we agree that the target network could stabilize the learning. The t-soft update additionally improved the task performances by conservatively updating the target network.

On the other hand, the last task (d), i.e., Ant task, implies that, if the task itself is stable enough, the adverse effect of the target network, i.e., deterioration of learning speed, is dominant (suggested in the literature (Kim et al., 2019)). However, even in that case, the proposed t-soft update could acquire the sub-optimal policy. This implies that the stable task leads to stable updates of the main network, thereby yielding larger $\tau_i > \tau$ in eq. (11) on average.

To confirm this expectation that the t-soft update mitigates slowdown in learning, the means of absolute differences between the main and target networks, $\|\Delta\|$, after learning are summarized in Fig. 3. As can be seen in this figure, the t-soft update succeeded in making the target network close to the main network. This result suggests that the t-soft update made $\tau_i$ larger than $\tau$ when the updates were along the appropriate direction while ignoring the noise in the updates of the main network.

Overall, in any tasks, we can conclude that the target network should employ the t-soft update rather than the conventional soft update for a better learning stability and speed.

5. Conclusion

This paper proposed a new robust update rule of the target network for deep reinforcement learning, so-called the t-soft update. By focusing on the fact that the conventional soft update is based on the EMA, which can be replaced with the robust version derived from the student-t distribution, we designed the proposed t-soft update. In practice, its computation and memory costs were reduced by assuming a fixed $W$ and a one-dimensional model for each subset of parameters. In PyBullet robotics simu-
lations, the actor-critic algorithm with the t-soft update outperformed the conventional methods. Even in the task where the adverse effects of slowing down the learning speed by the target network are pronounced, the t-soft update enabled the agent to achieve the sub-optimal policy similar to that obtained without the target network by recovering the learning speed to some extent.

In future work, we will investigate the versatility of the t-soft update by integrating with other learning algorithms. In addition, the best algorithm with the t-soft update will be applied to complicated robotic tasks.

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