Formation of high-redshift objects in a cosmic string theory with hot dark matter

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ABSTRACT
Using a modification of the Zel'dovich approximation adapted to hot dark matter (HDM), the accretion of such matter on to moving cosmic string loops is studied. It is shown that a large number of $10^{12}$-M\(_{\odot}\) non-linear objects will be produced by a redshift of $z = 4$. These objects could be the hosts of high-redshift quasars.

Key words: cosmic strings – cosmology: theory – large-scale structure of Universe.

1 INTRODUCTION
Observation of high-redshift objects has emerged as a powerful tool for testing theories of structure formation. For example, in inflationary universe models, the epoch at which the first massive non-linear structures (which could be the hosts of quasars or primordial galaxies) form is a sensitive function of the fraction of hot dark matter (HDM). Recent data on the abundance of damped Ly\(\alpha\) absorption systems (DLASs) (Lanzetta et al. 1991; Lanzetta, Turnshek & Wolfe 1993; Lanzetta, Wolfe & Turnshek 1995; Storrie-Lombardi et al. 1995) and on the quasar abundance (Warren, Hewett & Osmer 1991; Irwin, McMahon & Hazard 1991; Boyle et al. 1991; Schmidt, Schneider & Gunn 1991, 1995) have provided tight constraints on inflation-based models of structure formation with adiabatic density fluctuations and mixed dark matter (MDM).

Searches for high-redshift quasars have been going on for some time. The quasar luminosity function is observed to rise sharply as a function of redshift, $z$, until $z \approx 2.5$. According to recent results from the Palomar grism survey by Schmidt et al. (1991, 1995), it peaks in the redshift interval $z \approx [1.7, 2.7]$ and declines at higher redshifts. Irwin et al. (1991) on the other hand find that the luminosity function is constant up to redshifts of about 4. Quasars (QSOs) are extremely luminous, and it is generally assumed that they are powered by accretion on to black holes. It is possible to estimate the mass of the host galaxy of the quasar as a function of its luminosity, assuming that the quasar luminosity corresponds to the Eddington luminosity of the black hole. For a quasar of absolute blue magnitude $M_B = -26$, the host galaxy mass can be estimated as (Subramanian & Padmanabhan 1994)

$$M_\odot = c_1 10^{12} M_\odot,$$

where $c_1$ is a constant which contains the uncertainties in relating the blue magnitude to the bolometric magnitude of quasars, in the baryon fraction of the universe and in the fraction of baryons in the host galaxy able to form the compact central object (taken to be $10^{-2}$). The best estimate for $c_1$ is about 1. Models of structure formation have to pass the test of producing enough early objects of sufficiently large mass to host the observed quasars.

Damped Ly$\alpha$ systems provide potentially even more powerful constraints on structure formation models. Evidence is mounting (Briggs & Wolfe 1983; Lanzetta & Bower 1992) that the absorption-line systems observed in the spectra of distant quasars are caused by progenitors of typical galaxies. Based on the number density of absorption lines per frequency interval and on the column density calculated from individual absorption lines, the fraction of bound neutral gas (denoted by $Q$) can be estimated. Recent observational results (Lanzetta et al. 1995; Storrie-Lombardi et al. 1995) indicate that

$$\Omega_\mathrm{Q}(z) > 10^{-3},$$

for $z[1, 3]$, with the highest value in fact taken on at $z = 3$!

Note that the above corresponds to a value of $\Omega$ in bound matter which is larger by a factor of $f_\Omega^{-1}$, where $f_\Omega$ is the fraction of bound matter which is baryonic. In an $\Omega = 1$ cosmology, the value of $f_\Omega$ is expected to be of the order $10^{-1}$.

The constraints coming from observation of high-redshift objects for inflation-based models of structure formation...
were studied by several groups. Efstathiou & Rees (1988) showed that the high-redshift quasar abundance is compatible with an unbiased cold dark matter (CDM) model, but that the theory predicts an exponential decrease in the quasar abundance for $z > 5$. Recently (Haehnelt 1993), this analysis has been extended to MDM models. The abundance of damped Ly$\alpha$ systems was used to further constrain MDM models (Subramanian & Padmanabhan 1994; Mo & Miralda-Escude 1994; Kaufmann & Charlot 1994; Ma & Bertschinger 1994; Klypin et al. 1995). It was found that models with a fraction of HDM exceeding $\Omega_{\Lambda}/\Omega = 0.2$ do not predict the existence of enough non-linear structures at high-redshift in order to be able to explain the data.

However, there exists a viable class of alternative theories of structure formation: the cosmic string (CS) models. In these models, density fluctuations are generated by topologically stable strings of trapped energy density (one-dimensional topological defects in a relativistic field theory describing matter), instead of originating as the result of quantum fluctuations during an early period of exponential expansion of the universe. The main purpose of this paper is to present a preliminary study of the constraints on cosmic string models which can be derived using the QSO and DLAS abundances. The model we are most interested in is a cosmic string theory in a spatially flat ($\Omega = 1$) universe with only HDM and baryons. This model is briefly reviewed in Section 2.

Our main result is that for reasonable values of the parameters of the model, the CS plus HDM theory is compatible with the present observational constraints on the quasar abundance of the Palomar grism survey by Schmidt et al. (1991, 1995), which are plotted in Fig. 1. We will compare these observations for the comoving space density of quasars brighter than $M_H = -26$ with the abundance of objects of mass greater than $M_\odot$ (given in 1.1) in the CS plus HDM model. We also comment on the implications of the DLAS abundance (as given by 1.2) for cosmic strings.

Our result is quite non-trivial. The reason why inflationary MDM models with a large fraction of HDM do not predict a sufficient abundance of non-linear objects at high redshifts is that the spectrum of density perturbations is suppressed at small wavelengths (those which first become non-linear in the standard CDM inflationary model) by neutrino free-streaming (Bond, Efstathiou & Silk 1980; Bisnovatyi-Kogan & Novikov 1980). The primordial power spectrum in a cosmic string theory is scale-invariant (Zel'dovich 1970) which in principle is the only free parameter in a cosmic string model (Vilenkin 1991), as in inflation-based models. The reason why a CS plus HDM model is viable at all is that cosmic strings survive free-streaming (Vilenkin & Shafi 1983; Brandenberger et al. 1987; Brandenberger, Kaiser & Turok 1987). Since the strings are long-lived seed perturbations (as opposed to adiabatic dark matter fluctuations), accretion of dark matter on small scales -- wavelength $\lambda < \lambda_{\text{ps}}$ -- where $\lambda_{\text{ps}}$ is the maximal neutrino free-streaming scale (whose value is given later) -- is delayed but not prevented. Thus, the spectrum of density perturbations is not cut off exponentially below $\lambda_{\text{ps}}$ as it is for an inflationary HDM theory. It is (Vilenkin & Shafi 1983; Albrecht & Stebbins 1992), however, suppressed by a power of $\lambda/\lambda_{\text{ps}}$ compared with that of an inflationary CDM model. It also has a smaller amplitude than the MDM model with $\Omega = 0.2$. Hence, it would seem that the CS plus HDM model would be unable to explain the abundance of high-redshift QSOs and DLAS. However, as explained in the following paragraph, the above reasoning misses a crucial point.

In the cosmic string theory -- in contrast to inflation-based models -- the density field is not a Gaussian random field. There are localized high-density peaks even when the average density contrast is small. Hence, knowledge of the density power spectrum is insufficient to calculate the number density of non-linear objects.

In particular, cosmic string loops seed large-amplitude local density contrasts. In this paper, we study the accretion of HDM on to moving string loops and use the results to compute the number density of high-redshift objects as a function of a parameter $v$ which determines the number density of loops in the scaling solution (see Section 2). We demonstrate that for realistic values of $v$, the number of massive non-linear objects at redshifts $\leq 4$ satisfies the recent observational constraint of quasar abundances (see Fig. 1). We also comment on the implications of equation (1.2).

The next section of this paper contains a brief review of the CS plus HDM theory of structure formation. In Section 3 we summarize the methods used: a Zel'dovich approximation (Zel'dovich 1970) technique modified (Perivolaropoulos, Brandenberger & Stebbins 1990; Brandenberger, Perivolaropoulos & Stebbins 1990) to HDM and its adaptation to moving seed perturbations (Bertschinger 1987). Section 4 contains the main calculations, and in Section 5 we discuss the results. Units in which $h = = c = k_0 = 1$ are used throughout, and a Hubble constant of $H = 50 h_0$ km s$^{-1}$ Mpc$^{-1}$ and a redshift at equal matter and radiation of $z_{\text{eq}} = 5750 h_0^{-1}$ are used.

2 BRIEF REVIEW OF THE COSMIC STRING AND HOT DARK MATTER THEORY

Cosmic strings (Vilenkin & Shellard 1994; Brandenberger 1994; Hindmarsh & Kibble 1995) are one-dimensional topological defects which are predicted in many relativistic field theories describing matter. In such theories, a network of strings forms during a phase transition in the very early Universe. These strings are characterized by a mass per unit length $\mu$ (which in principle is the only free parameter in a cosmic string model) which determines their gravitational effects.

After the time of formation, the network of strings rapidly approaches a 'scaling solution', a distribution of defects whose statistical properties are time independent when lengths are rescaled by dividing them by the Hubble radius. The mean separation of the strings increases by having strings interconnect and chop off loops. The scaling solution of the string network is characterized by a fixed number $N$ of long strings crossing each Hubble volume at any time $t$, and the presence of loops with a distribution

$$n(l, t) = v l^{-3} t^{-2}, \quad (2.1)$$

where $l$ is the length of the loop and $v$ is a constant. The quantity $n(l, t) dl$ gives the number per unit physical volume at time $t$ of loops with lengths in the interval between $l$ and

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Comoving space density of quasars

10^{-3} c^{-1} \rho^{0.5} \sim 1 \ (\text{Mpc})^{-3} \text{ for static loops}

10^{-4} = \cdots \ \text{ for moving loops}

\frac{nG}{nQ} \text{ from Palomar survey}

Figure 1. Comparison of the observed number density \( n_Q \) of quasars brighter than \( M_B = -26 \) mag from Schmidt et al. (1991, 1995) (‘ \times ‘ marks) with the number density \( n_G \) of protogalaxies of mass greater than \( 10^{12} M_\odot \) predicted in the cosmic string theory with HDM, for the parameters discussed in the text, and \( h_{00} = 1 \). The horizontal axis is the redshift.

The loops are remnants of string intercommunications at times \( t' < t \). Loops oscillate and decay slowly by emitting gravitational radiation. Hence, there is a lower cutoff value of \( l \) for the distribution (2.1) given by

\[ l_{\text{min}} \sim G \mu, \]

\( G \) being Newton's constant. Below \( l_{\text{min}} \), the distribution \( n(l, t) \) becomes constant. We will not discuss this point since it will not be relevant to our computations.

In principle, the properties of the scaling solution and hence also of the value of the constant \( v \) is calculable, albeit only numerically. In practice, however, the dynamics of the defect network is quite complicated and the numerical resolution inadequate to solve this problem. Thus, we must treat \( v \) as an undetermined constant. There are two more such constants which are denoted by \( \alpha \) and \( \beta \). The first constant determines the mean radius \( R_i \) of a string loop at the time of formation

\[ R_i(l) = \alpha t, \]

(2.3)

The second relates the mean radius \( R \) to the length \( l \) of a loop:

\[ l = \beta R. \]

(2.4)

Numerical simulations (Bennett & Bouchet 1988; Albrecht & Turok 1989; Allen & Shellard 1990) indicate that \( \alpha \lesssim 10^{-2} \) and \( \beta \approx 10 \). They also indicate that \( N \sim 10 \). From these values it follows that – unless \( v \) is extremely large – most of the mass of the string network resides in long strings (where long strings are defined operationally as strings which are not loops that have radius smaller than the Hubble radius).

In the cosmic string theory there are two basic mechanisms which seed structures, loops and wakes. String loops act gravitationally almost like point mass objects when viewed from distances larger than \( R \). Long straight strings, on the other hand, lead to planar overdense regions called wakes (Silk & Vilenkin 1984). On distance scales smaller than its curvature radius, the local gravitational attraction of a string vanishes. However, space perpendicular to the string is conical with deficit angle \( 8\pi G \mu \). Hence, a string moving with transverse velocity \( v \) will impart a velocity perturbation

\[ \delta v = 4\pi G \mu \gamma(v), \]

(2.5)

towards the plane behind the string, where \( \gamma(v) \) is the relativistic \( \gamma \) factor. This develops into a planar overdense region behind the string, the wake.

Since most of the mass in the string network is in long strings, the wake mechanism will be responsible for the formation of most of the present structure in the Universe. The thickest and most numerous wakes are those created at the time of equal matter and radiation \( t_{eq} \) (Vachaspati 1986; Stebbins et al. 1987). The cosmic string theory hence predicts a distinguished scale and topology of the large-scale structure in encouraging agreement with the data from the
The accretion of HDM on to cosmic string wakes was considered in detail by Perivolaropoulos et al. (1990) and Brandenberger et al. (1990). It was found that the first comoving scale to become non-linear about a wake caused by a string at time $t_{eq}$ is

$$q_{max}(t_{eq}) = v_{eq} z_{eq} t_{eq} \simeq v_{eq} \times 50 \ h^{-2} \ Mpc,$$

where $v_{eq}$ is the mean HDM velocity at $t_{eq}$. In an $\Omega = 1$ universe, $v_{eq}$ is about 0.1. Hence, the distance $q_{max}$ is in good agreement with the observed thickness of the CFA galaxy sheets (de Lapparent, Geller & Huchra 1986, 1991; Geller & Huchra 1989; Vogeley et al. 1994).

Note, however, that the first dark matter non-linearities around wakes form only at late times, at a redshift (Perivolaropoulos et al. 1990; Brandenberger et al. 1990)

$$z_{max} = \frac{24\pi}{5} G \mu v^2(t_{eq}) v_{eq}^{-1} z_{eq},$$

which for $\gamma(v_{eq}) \simeq 1$ and for the value of $G \mu$ from equation (2.6) corresponds to a redshift of about 1. Before this redshift, no HDM non-linearities form as a consequence of accretion on to a single uniform wake.

Thus, in the CS plus HDM theory, a different mechanism is required in order to explain the origin of high-redshift objects. Possible mechanisms related to wakes are early structure formation at the crossing sites of different wakes (Hara & Miyoshi 1989; Hara, Moriga & Miyoshi 1990; Hara et al. 1994), small-scale structure of the strings giving rise to wakes (Volklick 1992; Vachaspati & Vilenkin 1991; Aguirre & Brandenberger 1995; Sornborger et al. 1995), inhomogeneities inside of wakes (Sornborger et al. 1995), and non-linear baryonic perturbations in wakes formed after the time of decoupling. In this paper, however, we will explore a different mechanism, namely the accretion of HDM on to loops.

In earlier work (Brandenberger et al. 1987a; Brandenberger, Kaiser & Turok 1987b), the accretion of HDM on to static cosmic string loops was studied. It was found that in spite of free-streaming, the non-linear structure seeded by a point-mass grows from the inside out, and that the first non-linearities form early on (accretion on to string filaments proceeds similarly; Aguirre & Brandenberger 1995). In the context of the ‘old’ cosmic string scenario (wakes unimportant), this mechanism was used by Brandenberger & Shellard (1989) to derive the mass function of galaxies. Since loop accretion leads to non-linear structures at high redshift, we will now investigate this mechanism in detail to see whether or not there form enough high-redshift massive objects to satisfy the QSO constraints and equation (1.2).

### 3 Modified Zel’dovich Approximation

We will use the Zel’dovich approximation (Zel’dovich 1970) and modifications thereof to study the accretion of HDM on to moving string loops. The Zel’dovich approximation is a first-order Lagrangian perturbation theory technique in which the time evolution of the comoving displacement $\psi$ of a dark matter particle from the location of the seed perturbation is studied.

The physical distance of a dark matter particle from the centre of the cosmic string loop is

$$h(q, t) = a(t)[q - \psi(q, t)].$$

The scalefactor $a(t)$ is normalized such that $a(t_0) = 1$, where $t_0$ is the present time. The Zel’dovich approximation is based on combining the Newtonian equation for $h$

$$\ddot{h} = -\nabla_h \Phi$$

with the Poisson equation for the Newtonian gravitational potential $\Phi$ and linearizing in $\psi$. For a point-like seed mass of magnitude $m$ located at the comoving position $q' = 0$ the resulting equation is

$$\ddot{\psi} + 2 \dot{a} \dot{\psi} + 3 \frac{\dot{a}}{a} \psi = \frac{Gm}{a^2 q},$$

This equation describes how as a consequence of the seed mass, the motion of the dark matter particles away from the seed (driven by the expansion of the universe) is gradually slowed down. If the seed perturbation is created at time $t_i$ and the dark matter is cold, then the appropriate initial conditions for $\psi$ are

$$\psi(q, t_i) = \psi(q, t_i) = 0,$$

leading to for $a(t) \sim t^{2/3}$ appropriate in the matter-dominated epoch $t > t_{eq}$ to the solution

$$\psi(q, t) = \frac{9}{10} \frac{Gm}{a^2} \left[ \frac{t}{t_i} \right]^{2/3}.$$  

As formulated above, the Zel’dovich approximation only works for CDM, particles with negligible thermal velocities. The theory of interest to us, however, is based on HDM. Luckily, the Zel’dovich approximation can be modified for HDM (Perivolaropoulos et al. 1990; Brandenberger et al. 1990). HDM particles have large thermal velocities. At time $t$, the free-streaming length in comoving coordinates is

$$\lambda_i(t) = v(t) z(t) t,$$

where $v(t) \sim z(t)$ is the HDM velocity. The length $\lambda_i(t)$ is the mean distance an HDM particle will move in one expansion time. Free-streaming erases density perturbations on scales $q < \lambda_i(t)$, a scale which decreases as $t^{-1/3}$ as $t$ increases.

A simple prescription\(^2\) for taking into account free-
streaming in the Zel'dovich approximation – for a fixed comoving scale \( q \) – is to only let the perturbation start to develop at time \( t_s(q) \), when

\[
\lambda_s(t_s(q)) = q.
\]  

(3.7)
i.e., replace the initial conditions (3.4) by

\[
\psi(q, t_s(q)) = 0,
\]

(3.8)
with

\[
\tilde{t}_s(q) = \max\{t_i, t_s(q)\}.
\]  

(3.9)
The result for the comoving displacement \( \psi(q) \) then becomes

\[
\psi(q, t) = \frac{9}{10} \frac{Gm}{q^2} \left[ \frac{t}{t_s(q)} \right]^{2/3}.
\]  

(3.10)

We can now define the mass that has become non-linear about a seed perturbation as the rest mass inside of the shell which is ‘turning around’, i.e. for which

\[
h(q, t) = 0.
\]  

(3.11)
This yields an equation

\[
q = 2\psi(q, t)
\]

(3.12)
for the scale \( q_{nl}(t) \) which is turning around at time \( t \). For CDM, equation (3.5) can be combined with equation (3.12) to obtain \( q_{nl}(t) \) as well as the corresponding mass

\[
M_{\text{CDM}}(t) = \frac{2}{5} m \left( \frac{t}{t_i} \right)^{2/3}.
\]  

(3.13)
Note the similarity of this result to what can be obtained in linear perturbation theory: \((t/t_i)^{2/3}\) is precisely the growth factor of linear cosmological perturbations on small scales.

For HDM, equations (3.8), (3.10) and (3.12) can be combined to yield

\[
M_{\text{HDM}}(t) = \frac{8}{125} \frac{m}{M_{\text{eq}}} \left( \frac{q_t}{q_{i,t}} \right)^{2/3}.
\]  

(3.14)
with

\[
M_{\text{eq}} = \frac{2}{9} \frac{v_{\text{eq}}^2 t_{\text{eq}}}{G}.
\]  

(3.15)
A further complication is caused by the finite velocity of the cosmic string loops. This implies that we must extend the Zel'dovich approximation technique to moving sources. For CDM, this was done by Bertschinger (1987) with the interesting result that there is, to a first approximation, no change in the total mass accreted. The suppression of the growth of perturbations in the direction perpendicular to the direction of motion of the seed mass is cancelled by the larger length of the non-linear region in the direction of motion. For HDM, however, there will be a net suppression of growth if the seed mass is moving. It will be important for us to take this effect into account.

Given a moving point source, the basic Zel'dovich approximation equation (3.3) becomes a vector equation:

\[
\ddot{\psi} + 2 \frac{a}{a} \dot{\psi} + \frac{\ddot{a}}{a} \psi = \frac{Gm(q - q')}{a^2 |q - q'|^3},
\]

(3.16)
where \( q'(t) \) indicates the comoving position of the source. Without loss of generality we can take the source to move along the \( z \)-axis with initial velocity \( v_i \) at time \( t_i \), so that (Bertschinger 1987)

\[
q'(t) = 3v_i t_i \left[ 1 - \left( \frac{a(t)}{a(t_i)} \right)^{1/2} \right] e_z(t_i),
\]

(3.17)
e, being the unit vector along the \( z \)-axis. In the matter-dominated epoch equation (3.16) can be solved exactly (Bertschinger 1987) (taking \( q'=0 \) without loss of generality):

\[
\psi(q, t) = b(t) \left[ f_z(q, t) e_z + f_x(q, t) e_x \right],
\]

(3.18)
with

\[
d_i = 3v_i t_z(t),
\]

(3.19)
and where \( f_x \) and \( f_z \) are known functions of \( q, t \) and \( d_i \) which at late times become independent of time and contain the information about the geometry of the pattern of accretion on to the moving loop. In particular, the transverse displacement at late times approaches

\[
f_z(q) = 1 \left( \frac{d_i}{q_t} \right)^2 \left( R_{eq} - R_{x} - \frac{q_t d_x}{R_{eq} d_z} \right) \left( \frac{1}{2} \frac{d_t}{q_t} \right)^2 k(q).
\]

(3.20)
In the above,

\[
R(q, t) = \left[ q_t^2 + (q_t - q(t))^2 \right]^{1/2},
\]

(3.21)

\[
R_{eq} = R(q, t_i),
\]

(3.22)
and

\[
R_x = R(q, t_{eq}),
\]

(3.23)
k(q) is the factor by which the accretion at \( q \) is suppressed owing to the motion of the source.

It is easy to check that for \( q^2 \gg d_i^2 \) the factor \( k(q) \) tends to 1 (for \( q^2 \gg q_t^2 \)) and that the result for \( \psi(q, t) \) from equation (3.18) becomes the result of equation (3.5). For rapidly moving loops we are interested in the opposite limit, \( d_i^2 \gg q^2 \). In this case, evaluated at \( q_t = 0 \), the suppression factor becomes

\[
k(q_t) \approx 2 \frac{q_t}{d_i}.
\]

(3.24)
In this case, the ‘turn-around’ condition

\[
q_t = 2\psi_x
\]

(3.25)
for transverse accretion yields

\[
q_t^2 \approx \frac{18}{5} \frac{Gm t_i}{v_{eq} t_z(t_i)}.
\]

(3.26)
For a cosmic string loop formed at time \( t_i \), whose mass is (see equations 2.3 and 2.4)

\[
m = \alpha \mu t_i,
\]

(3.27)
this gives

\[ q_i(t, t) = \left( \frac{5}{6} \beta \right)^{1/2} (G \mu)^{1/2} v_i^{-1/2} t_o^{-1/2} (t). \]  

(3.26)

From this result we can immediately recover Bertschinger's result (1987) that accretion of CDM on to a moving loop is not suppressed: the total accreted mass \( M(t) \) is proportional to

\[ M(t) \sim q_i^2(t) d \rho_c \]  

(3.27)

where \( \rho_c \) is the background comoving energy density. The factors of \( v_i \) evidently cancel!

We are interested in the accretion of HDM on to moving loops. Provided that the turnaround distance \( q_i \) of equation (3.26) exceeds the initial comoving free-streaming distance, i.e.

\[ q_i(t_1, t) > \lambda_i(t), \]  

(3.28)

then the above analysis can also be applied to HDM. We will check this condition in our calculations in the following section.

### 4 COMPUTATIONS

At this point, we are able to use the methods described in the previous section to compute the number density \( n_{C_i}(> M_i, t) \) in non-linear objects heavier than \( M_i \), and the fraction \( \Omega_{C_i} \) of the critical density in such objects at high-redshift \( z(t) \) in the CS plus HDM model. By considering only the accretion on to string loops we will be underestimating these quantities.

It can be shown (Brandenberger & Shellard 1989) that in an HDM model string loops accrete matter independently, at least before the large-scale structure turns non-linear at the redshift given by equation (2.8), which is about 1, i.e., later than the times of interest in this paper. Hence the number density \( n(l, t) \) of loops of length \( l \) given by equation (2.1) can be combined with the mass \( M(l) \) accreted by an individual loop (which follows from equations (3.13) and (3.14)) to give the mass function \( n(M, t) \). Here, \( n(M, t) dM \) is the number density of objects with mass in the interval between \( M \) and \( M + dM \) at time \( t \). This in turn determines the comoving density in objects of mass \( M > M_i \),

\[ n_{C_i}(> M_i, t) = z^{-3} \int_{M_i}^{M} \frac{dM n_C(M)}{M}. \]  

(4.1)

and the fraction of the critical density in objects of mass greater than \( M_i \), \( \Omega_{C_i}(t) \):

\[ \Omega_{C_i}(t) = 6 \pi G t_i^2 \int_{M_i}^{M} \frac{dM M n_C(M)}{M}. \]  

(4.2)

If we want to compare with the observational results from QSO counts we must use appropriate integration limits \( M_i \) and \( M_C \) in equations (4.1) and (4.2). The mass \( M_C \) is the mass limit corresponding to the limiting QSO luminosity of the observational sample given in equation (1.1), \( M_C = c_l 10^{12} M_{\odot} \). For comparison with damped Ly\( \alpha \) systems, the lower cut-off mass \( M_i \) is somewhat smaller, but we will not need the exact value. For large loop radii, the approximation of treating the loop as a point-mass breaks down. This will lead to a time-dependent upper mass cut-off \( M'_i(t) \). A rough criterion for \( M'_i(t) \) can be obtained by demanding that the mass \( M(t) \) accreted on to a loop exceeds the mass in a sphere of radius equal to the loop radius at \( t \),

\[ M(t) > \frac{1}{6 \pi G t_i^2} \frac{4 \pi}{3} (z t_i)^3. \]  

(4.3)

Using the CDM mass formula (3.13) with initial mass (3.25) yields the estimate

\[ M'_i(t) \approx \frac{2}{3} \left( \frac{G \mu}{\beta} \right)^{1/2} \frac{t_0}{G} z(t)^{-3/2}. \]  

(4.4)

Since

\[ \frac{t_0}{G} \approx 8 \times 10^{22} h_{50}^{-1} M_{\odot}, \]  

(4.5)

the value of \( M'_i(t) \) is much greater than both \( M_i \) and the maximal neutrino Jeans mass \( M'(t) \), the largest mass which is affected by free-streaming. This justifies the use of the CDM mass formula (3.13).

As indicated above, there is another mass which is crucial for the computation of \( n_{C_i}(t) \) and \( \Omega_{C_i}(t) \), namely \( M'_i(t) \). The easiest way to determine \( M'_i(t) \) is to find the value of \( M \) for which the mass formulas (3.13) and (3.14) intersect:

\[ \frac{2}{5} m \left( \frac{t}{t_i} \right)^{2/3} = \frac{2}{5} \left( \frac{t}{t_i} \right)^{3/2} \frac{m^3}{M_{eq}^2} \left( \frac{t}{t_{eq}} \right)^2. \]  

(4.6)

Using expression (3.25) for \( m \) and taking account of equation (3.15), one finds

\[ M'(t) = \frac{2}{5} \left( \frac{t}{t_i} \right)^{1/4} \left( x \beta G \mu \right)^{1/2} \frac{t_{eq}^3}{M_{eq}^2} \left( z_{eq}^2 \right)^{1/2} \frac{t_0}{G}, \]  

(4.7)

from which it follows that – at least for the redshifts we are interested in – \( M'(t) \) is smaller than \( M'_i(t) \).

As a self-consistency check we note that \( M_i \) is larger than the mass accreted on to a loop created at \( t_{eq} \). Hence, it is consistent to restrict our attention to the matter epoch \( t > t_{eq} \).

Since the functional form of \( M(m) \) and hence \( M(l) \) changes at \( M=M'_i \), the functional forms of the mass function \( n(M) \) will be different above and below \( M'_i \). Thus

\[ n_{C_i}(> M_i, t) = z^{-3} \int_{M_i}^{M} \frac{dM n_{H}(M)}{M} + \int_{M'_i}^{M} \frac{dM n_{C}(M)}{M}. \]  

(4.8)

Similarly, for \( \Omega_{C_i}(t) \),

\[ \Omega_{C_i}(t) = \left[ \int_{M_i}^{M} \frac{dM n_{H}(M)}{M} + \int_{M'_i}^{M} \frac{dM n_{C}(M)}{M} \right] 6 \pi G t_i^2, \]  

(4.9)

where \( n_{H}(M) \) and \( n_{C}(M) \), respectively, refer to the mass functions computed with the HDM and CDM mass formulas (3.13) and (3.14) respectively. Combining (2.1) with
(3.13) and (3.14), we obtain

\[ n_c(M, t) = \frac{24}{125} (\alpha \beta)^2 \frac{\mu^3}{M^5} \]  

(4.10)

and

\[ n_H(M, t) = \frac{2}{15} \frac{z_{eq}}{\beta} \frac{1}{t^2 M^2_{eq} M^{15/4}}. \]  

(4.11)

As a consistency check, we note that for \( M = M'(t) \) the above two expressions coincide.

By inspection of equations (4.8)-(4.11) it is clear that the integrals for \( n_G(t) \) and \( \Omega_\omega(t) \) are dominated at \( M = M'(t) \) and that a reasonable approximation to (4.8) will be

\[ n_G(M > M_1, t) \sim n_c(M', t) M' z^{-3}(t) \approx \frac{1}{5} (\alpha \beta)^2 \frac{\mu^3}{M'(t)^5} z^{-3}(t), \]  

(4.12)

which gives

\[ n_G(M > M_1, t) \sim 5 (\alpha \beta)^{-1} (G\mu)^{1/4} \left( \frac{z_{eq}}{v_{eq}} \right)^{1/4} z^{-3}(t) t_0^{-3} \]

\[ \sim 2 \times 10^{-4} v z^{-3}(t) h_50^2 (h_{50}^{-1} \text{Mpc})^{-3} \]  

(4.13)

when inserting \( v_{eq} = 0.1, G\mu = 10^{-6} \) and the values of \( \alpha = 10^{-2} \) and \( \beta = 10 \) from Section 2. This result is plotted in Fig. 1, for \( h_{50} = 1 \) and \( v = 1 \). Similarly, equation (4.9) can be approximated by

\[ \Omega_\omega(t) \sim n_c(M', t) M' \frac{6\pi G t^2}{\alpha \beta} \approx \frac{1}{5} (\alpha \beta)^2 \frac{\mu^3}{M'(t)^5} \frac{6\pi G t^2}{\alpha \beta}, \]  

(4.14)

which gives

\[ \Omega_\omega(t) \sim \frac{15\pi}{2} \left( \frac{9}{5} \right)^{1/2} v (\alpha \beta)^{1/2} \left( \frac{z_{eq} G\mu}{v_{eq}} \right)^{1/2} z(t)^{-3/4} \]

\[ \sim 10^{-1} h_50^2 v z(t)^{-3/4} \]  

(4.15)

for the same values of the parameters as above.

At this point it looks as if the CS plus HDM model will produce too many quasar host galaxies at redshift \( z \sim 5 \). However, so far loop velocities have been neglected.

Loop velocities can be taken into account by incorporating condition (3.28). Loops which do not satisfy this criterion will not be able to accrete much mass. Note that as \( t_i \) increases, \( q(t_i) \) remains constant, whereas \( \lambda_i(t_i) \) decreases. Hence condition (3.28) corresponds to a low-mass cut-off \( M_i \) in the integrals (4.11) and (4.2). Using (3.6) and (3.26) it follows that the inequality (3.28) becomes

\[ t z(t_i) v(t_i) < \left( \frac{5}{6} \alpha \beta \right)^{1/2} (G\mu)^{1/2} v_i^{-1/2} t_0 z(t)^{-1/2}, \]  

(4.16)

where \( v(t_i) \) is the HDM velocity at time \( t_i \) and \( v_i \) is the loop velocity at the same time. After some algebra, (4.16) translates to

\[ \frac{6}{5} \beta G \mu v_{eq}^{-1} v_i^{-1} > z(t). \]  

(4.26)
which is marginally satisfied for $z=4$ if $v_i=0.25$, $v_{eq}=0.1$ and
\[ z(t) < 3h_{100}^2. \] (4.27)

For values of $z$ larger than 4, the values of $n_G(t)$ and $\Omega_m(t)$ are suppressed beyond the results (4.20) and (4.22) since only the tail of the loop ensemble with velocities smaller than the mean velocity $v_i=0.25$ manage to accrete a substantial amount of mass.

5 DISCUSSION

We have studied the accretion of HDM on to moving cosmic string loops and made use of the results to study early structure formation in the CS plus HDM model. Our main results are expressed in equations (4.20), (4.22) and (4.26).

The loop accretion mechanism is able to generate non-linear objects which could serve as the hosts of high-redshift quasars much earlier than the time cosmic string wakes start turning around (which for $G\mu=10^{-6}$ and $v_i=1/2$ occurs at a redshift of about 1). However, there is an upper cut-off to the redshift of large-mass objects which form by this mechanism given by equation (4.26), and for $v_i=0.25$ it corresponds to a redshift of about 4. For larger redshifts, only the loops with velocities sufficiently small compared to the mean loop velocity will be able to seed non-linear objects. Note that this redshift cut-off is independent of the parameters $x$ and $v$ of the cosmic string scaling distribution which must be obtained from numerical simulations and are still quite uncertain. Some of the other possible mechanisms for forming early objects, mentioned at the end of Section 2, might produce quasars at redshifts larger than 4 and remove the upper redshift cut-off determined from the mechanism investigated in this paper.

The fraction $\Omega_G(z)$ of the total mass accreted on to non-linear objects by string loops unfortunately depends very sensitively on $x$ and $v$. On the other hand, this is not surprising since the power of the loop accretion mechanism depends on the number and initial sizes of the loops, and the scaling relation $\Omega_m \sim v$ is what should be expected from physical considerations.

For the values $v=1$ and $x_{eq}=1$, which are indicated by recent cosmic string evolution simulations (Bennett & Bouchet 1988; Albrecht & Turok 1989; Allen & Shellard 1990), we conclude from equation (4.20) that the loop accretion mechanism produces enough large-mass protogalaxies to explain the observed abundance of $z \leq 4$ quasars (see Fig. 1). Note that the amplitude of the predicted protogalaxy density curves depends sensitively on the parameters of the cosmic string scaling solution, which are still poorly determined. Hence, the important result is that there are parameters for which the theory predicts a sufficient number of protogalaxies. Since not all protogalaxies will actually host quasars, and since the string parameters are still uncertain, it would be wrong to demand that the amplitude of the $n_G$ curve agree with that of the observed $n_Q$.

It is more difficult to make definite conclusions regarding the abundance of damped Ly$\alpha$ systems. In the form of equation (1.2), the condition for the cosmic string loop accretion mechanism to be able to explain the data is also satisfied. However, equation (1.2) refers to the value of $\Omega$ in baryonic matter. The corresponding constraint on the total matter collapsed in structures associated with damped Ly$\alpha$ systems is
\[ \Omega_{ob}(z<3) > f_b^{-1} 10^{-3}, \] (5.1)

where $f_b$ is the local fraction of the mass in baryons. From equation (4.22) it follows that the above constraint is only marginally satisfied, and only if the local baryon fraction $f_b$ exceeds the average value for the whole universe of about $f_b = 0.1$. This could be another manifestation of the 'baryon crisis' for galaxy clusters (White et al. 1993; Steigman & Felten 1994), the fact that $f_b$ in clusters seems to exceed what is expected based on nucleosynthesis constraints in a $\Omega = 1$ universe. On the other hand, in the CS plus HDM model we expect $f_b$ in non-linear objects to be enhanced over the average $f_b$ because baryons are able to cluster during the time that the HDM is prevented from accreting by the free-streaming. Thus, cosmic strings may be able to explain the baryon excess in clusters and restore agreement with (1.2) in a natural way. More calculations are required to resolve this issue (Moessner 1996, in preparation).

In conclusion, we have established that in addition to being in agreement with large-scale structure and CMB data, the CS plus HDM model is also able to produce a sufficient number of protogalaxies at redshifts $z<4$ which could explain the observed abundance of quasars. A prediction of the model is that the distribution of these quasars should be less correlated with today's large-scale structure that in inflation-based models, since the loops giving rise to quasars are not correlated with the long strings present at the time of equal matter and radiation, which give the dominant contribution to today's large-scale structure. However, some correlation might still be present since the loops are correlated with the long strings from which they were split off, and since the quasar host galaxies evolve into present-day galaxies and may fall into the potential wells created by the string wakes.

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