GENERATION OF SEMIGROUPS FOR THE THERMOELASTIC PLATE EQUATION WITH FREE BOUNDARY CONDITIONS

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Abstract. We consider the linear thermoelastic plate equations with free boundary conditions in uniform $C^4$-domains, which includes the half-space, bounded and exterior domains. We show that the corresponding operator generates an analytic semigroup in $L^p$-spaces for all $p \in (1, \infty)$ and has maximal $L^q$-$L^p$-regularity on finite time intervals. On bounded $C^4$-domains, we obtain exponential stability.

1. Introduction. Let $\Omega \subset \mathbb{R}^N$ be a domain with boundary $\Gamma$. We consider the linear thermoelastic plate equations

\begin{align}
 u_{tt} + \Delta^2 u + \Delta \theta &= f_1 \quad \text{in } (0, \infty) \times \Omega, \\
 \theta_t - \Delta \theta - \Delta u_t &= f_2 \quad \text{in } (0, \infty) \times \Omega
\end{align}

(1)

with initial conditions

\begin{align}
 u|_{t=0} &= u_0 \quad \text{in } \Omega, \\
 u_t|_{t=0} &= u_1 \quad \text{in } \Omega, \\
 \theta|_{t=0} &= \theta_0 \quad \text{in } \Omega.
\end{align}

(2)

System (1) serves as a standard simplified model for thin elastic plates with thermoelastic effects, see [10], Chapter 2, or [1], for a discussion of this and similar models. In (1), $u(t,x)$ stands for the vertical displacement at time $t \geq 0$ and at position $x = (x_1, \ldots, x_N) \in \Omega$, while $\theta(t,x)$ denotes the temperature (relative to some reference temperature) at time $t$ and position $x$. Note that we omitted all physical constants for simplicity.

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Among the physically relevant boundary conditions, the maybe most complicated
are the so-called free boundary conditions
\[
\Delta u - (1 - \beta)\Delta' u + \theta = g_1 \quad \text{on } (0, \infty) \times \Gamma,
\]
\[
\partial_\nu (\Delta u + (1 - \beta)\Delta' u + \theta) = g_2 \quad \text{on } (0, \infty) \times \Gamma,
\]
\[
\partial_\nu \theta = g_3 \quad \text{on } (0, \infty) \times \Gamma.
\]
which will be considered in the present paper. In (3), \(\Delta\) and \(\Delta'\) stand for the Laplace
operator in \(\Omega\) and the Laplace-Beltrami operator on the boundary \(\Gamma\), respectively,
\(\beta \in (0, \frac{1}{2})\) is Poisson’s ratio, and \(\partial_\nu\) denotes the derivative in outer normal direction.
For a survey on other types of boundary conditions and generation of semigroups for
them, we refer, e.g., to [11]. The physically relevant situation is the two-dimensional
case \(N = 2\), but we can consider (1)–(3) in any dimension. We remark that for
\(N = 2\) the boundary conditions (3) differ from the physically relevant boundary
conditions by lower-order operators (see [10], p. 2), but we will show in Lemma 3.5
below that all results also hold in this case, see Corollaries 3.7 and 3.9 for details.
One of the standard approaches to (1)-(3) is to set \(v := \partial_t u\) and obtain the
first-order system acting on \(U := (u, u_t, \theta)\top\) and being of the form
\[
U_t - A(D)U = 0 \quad \text{in } (0, \infty) \times \Omega
\]
with an operator-matrix \(A(D)\) of mixed order. This equation is augmented by
boundary conditions of the form
\[
B(D)U = 0 \quad \text{on } (0, \infty) \times \Gamma.
\]
Here \(A(D)\) and \(B(D)\) are given by
\[
A(D) := \begin{pmatrix}
0 & 1 & 0 \\
-\Delta^2 & 0 & -\Delta \\
0 & \Delta & \Delta
\end{pmatrix}, 
B(D) := \begin{pmatrix}
\Delta - (1 - \beta)\Delta' & 0 & 1 \\
\partial_\nu (\Delta + (1 - \beta)\Delta') & 0 & 0 \\
0 & 0 & \partial_\nu
\end{pmatrix}.
\]
The natural space for the \(L^p\)-realization of the mixed-order boundary value problem
\((A(D), B(D))\) is given by \(E^{(0)}_p(\Omega)\) and its solution space by \(E^{(2)}_p(\Omega)\), where for
\(j \in \{0, 1, 2\}\) we set
\[
E^{(j)}_p(\Omega) := H^{2+j}(\Omega) \times H^j_p(\Omega) \times H^j_p(\Omega).
\]
More precisely, we define \(A_{p,\Omega}\) as an unbounded operator in \(E^{(0)}_p(\Omega)\) with domain
\[
D(A_{p,\Omega}) := \{U \in E^{(2)}_p(\Omega) : B(D)U = 0\}
\]
anacting as \(A_{p,\Omega}U := A(D)U \ (U \in D(A_{p,\Omega}))\). We consider uniform \(C^4\)-domains, see
Definition 3.1 below. The main result of the present paper shows that for all \(p \in (1, \infty)\), the operator \(A_{p,\Omega}\) generates an analytic \(C_0\)-semigroup. This is a consequence
of the stronger result that \(A_{p,\Omega}\) has maximal \(L^q-L^p\)-regularity (Theorem 3.4). On
bounded \(C^4\)-domains, we obtain exponential stability (Theorem 3.8).

The thermoelastic plate equations has been studied by many authors, mostly
in an \(L^2\)-setting. Many results deal with exponential stability of the associated
semigroup, e.g., [8], [18], [17], [11], [22]. For the analyticity of the semigroup, we refer to [15], [14], and [16] in the \(L^2\)-setting. For the treatment of nonlinear
problems, corresponding results in \(L^p\) are of relevance. In the whole-space case,
analyticity of the generated semigroup in \(L^p\) was shown in [3]. In the case of
the half-space and of bounded domains, equations (1) with Dirichlet (clamped) boundary conditions

\[ u = \partial_x u = \theta = 0 \quad \text{on } (0, \infty) \times \Gamma \]

were studied in [20] and [19]. In the paper [13], a rather complete analysis in the \( L^p \)-setting can be found for hinged boundary conditions \( u = \Delta u = \theta = 0 \).

System (1)–(3), i.e. the thermoelastic plate equations with free boundary conditions in the \( L^p \)-setting, has been studied recently by the authors in [5]. It was shown that the second-order (in time) system (1)–(3) has maximal \( L^q - L^p \)-regularity. However, this does not imply that the first-order system (4)–(5) generates an analytic \( C_0 \)-semigroup. This was also observed in the case of the structurally damped plate equation with clamped boundary conditions in [4]. In fact, in the situation of [4], we have maximal regularity, but no generation of semigroup unless additional conditions are included in the basic space. Roughly speaking, this is due to the fact that the standard resolvent estimates hold only for right-hand sides with vanishing first component, and the reformulation of (1) as a first-order system in fact leads to such a right-hand side.

In the present paper, however, we show that the operator related to the first-order system (7) generates an analytic \( C_0 \)-semigroup without additional conditions on the basic space \( E(0)^p(\Omega) \). The proofs are based on Fourier multiplier methods on one hand and on the results from [5] on the other hand. If the domain \( \Omega \) is bounded, we obtain exponential stability apart from the kernel of the operator. In particular, we obtain generation of an analytic semigroup and exponential stability for the two-dimensional system which was studied in [12], in this way generalizing the results in [12] from the \( L^2 \)-case to the \( L^p \)-case.

2. The whole space case. In this section, we consider the whole-space case, i.e. system (1)–(2) with \( \Omega = \mathbb{R}^N \). Our approach is based on the Fourier transform and results on vector-valued Fourier multipliers. In particular, the proof of maximal regularity in the sense of well-posedness in \( L^q - L^p \)-Sobolev spaces make use of the concept of \( \mathcal{R} \)-boundedness and variants of Michlin’s theorem. As standard references, we mention [2] and [9].

The Fourier transform \( \mathcal{F} \) in \( \mathbb{R}^N \) is given by

\[ (\mathcal{F} \varphi)(\xi) := (2\pi)^{-N/2} \int_{\mathbb{R}^N} \varphi(x) e^{-ix\xi} dx \quad (\xi \in \mathbb{R}^N) \]

for Schwartz functions \( \varphi \) and extended by duality to tempered distributions. A symbol \( m \in L^\infty(\mathbb{R}^N) \) is called a Fourier multiplier if \( \mathcal{F}^{-1} m(\mathcal{F}) \) defines a bounded linear operator in \( L^p(\mathbb{R}^N) \). One of the key ingredients to show \( \mathcal{R} \)-sectoriality will be the vector-valued version of Michlin’s theorem on Fourier multipliers due to Weis [23] and Girardi and Weis [7].

The following definition is a variant of [5], Definition 3.2.

**Definition 2.1.** Let \( \Sigma \subset \mathbb{C} \) be a set, let \( m : (\mathbb{R}^N \setminus \{0\}) \times \Sigma \to \mathbb{C}, (\xi, \lambda) \mapsto m(\xi, \lambda) \), be \( C^\infty \) with respect to \( \xi \). Let \( s \in \mathbb{R} \). Then \( m \) is called a multiplier of order \( s \) in \( \Sigma \) if the estimates

\[ |\partial_\xi^\alpha m(\xi, \lambda)| \leq C_\alpha (|\lambda|^{1/2} + |\xi|)^s |\xi|^{-|\alpha|} \]

hold for any multi-index \( \alpha \in \mathbb{N}^N_0 \) and \( (\xi, \lambda) \in (\mathbb{R}^N \setminus \{0\}) \times \Sigma \) with some constant \( C_\alpha \) depending only on \( \alpha \) and \( \Sigma \). The set of all multipliers of order \( s \) in \( \Sigma \) will be denoted by \( M_s(\Sigma) \).
It is easily seen that $M_s(\Sigma)$ is a complex vector space and that for $m_1 \in M_{s_1}(\Sigma)$ and $m_2 \in M_{s_2}(\Sigma)$ we have $m_1 m_2 \in M_{s_1 + s_2}(\Sigma)$ (see [5], Lemma 3.3).

**Example 2.2.** We mention some examples which will be useful below. Let $\theta \in (0, \pi)$, and let

$$\Sigma_\theta := \{ \lambda \in \mathbb{C} \setminus \{0\} : |\arg \lambda| < \theta \}$$

be the open sector in the complex plane.

a) Directly from the definition it can be seen that $\lambda \in M_2(\Sigma_\theta)$ (where $\lambda$ stands for the constant mapping $(\xi, \lambda) \mapsto \lambda$) and $|\xi|^{2k} \in M_{2k}(\Sigma_\theta)$ for all $k \in \mathbb{N}$.

b) Let $s \in \mathbb{R} \setminus \{0\}$, and let $m(\xi, \lambda) := (\lambda + |\xi|^2)^{s/2}$. Then $m \in M_s(\Sigma_\theta)$. This can be seen by homogeneity: As $m$ is quasi-homogeneous of order $s$ in the sense that

$$m(\rho \xi, \rho^s \lambda) = \rho^s m(\xi, \lambda) \quad (\rho > 0, \xi \in \mathbb{R}^N \setminus \{0\}, \lambda \in \Sigma_\theta),$$

the derivative $\partial_\xi^2 m$ is quasi-homogeneous of degree $s - |\alpha|$. By a compactness argument,

$$|\partial_\xi^2 m(\xi, \lambda)| \leq c_\alpha (|\lambda|^{1/2} + |\xi|)^{s-|\alpha|} \leq c_\alpha (|\lambda|^{1/2} + |\xi|)^{s} |\xi|^{-|\alpha|}$$

which shows $m \in M_s(\Sigma_\theta)$.

c) By a similar homogeneity argument, we see that $(\xi, \lambda) \mapsto \frac{|\xi|}{(1 + |\xi|^2)\lambda^s} \in M_0(\Sigma_\theta)$.

d) Let $\lambda_0 > 0$. Then $(\xi, \lambda) \mapsto 1 \in M_2(\lambda_0 + \Sigma_\theta)$ due to

$$1 \leq c_\alpha |\lambda| \leq c_\alpha (|\lambda|^{1/2} + |\xi|)^2.$$ 

Therefore, $1 + |\xi|^2 \in M_2(\lambda_0 + \Sigma_\theta)$. Note that $1 \notin M_2(\Sigma_\theta)$.

The following result is one main tool for the results below and was shown in [6], Theorem 3.3.

**Lemma 2.3.** In the situation of Definition 2.1, let $m \in M_0(\Sigma)$. For $\lambda \in \Sigma$, define the operator $m(D, \lambda)$ by $m(D, \lambda)f = \mathcal{F}_\xi^{-1} [m(\lambda, \xi) \mathcal{F}_\xi f(\xi)]$. Then, the family

$$\{m(D, \lambda) : \lambda \in \Sigma\} \subset L(L^p(\mathbb{R}^N))$$

is $R$-bounded and

$$\mathcal{R}_{L(L^p(\mathbb{R}^N))}(\{m(D, \lambda) : \lambda \in \Sigma\}) \leq C_{p, N} \max_{|\alpha| \leq N+1} C_\alpha$$

with $C_{p, N}$ depending only on $p$ and $N$.

The analysis of the operator $A_{p, \mathbb{R}^N}$ in the whole space was essentially done in [20] and [19]. We summarize some results from these papers. Define $\gamma_1, \gamma_2, \gamma_3$ by the equality

$$p(t) := t^3 + t^2 + 2t + 1 = (t + \gamma_1)(t + \gamma_2)(t + \gamma_3)$$

with $\gamma_1 \in \mathbb{R}$, $\gamma_2 = \overline{\gamma}_3$ and $\text{Im } \gamma_2 > 0$. Then $\gamma_1 \in (0, 1)$, $\text{Re } \gamma_2 = \text{Re } \gamma_3 \in (0, \frac{1}{2})$, and
det$(\lambda - A(\xi)) = \prod_{j=1}^3 (\lambda + \gamma_j |\xi|^2)$ (see [20], Lemma 2.3). We define $\vartheta_0 := \arg(-\gamma_3) \in (\frac{\pi}{2}, \pi)$ (note that $-\gamma_3$ is the root of the polynomial $p$ with positive imaginary part).

We consider the whole space resolvent

$$R(\lambda) := (\lambda - A(D))^{-1} := \mathcal{F}_\xi^{-1} (\lambda - A(\xi))^{-1} \mathcal{F}_\xi$$

with

$$A(\xi) := \begin{pmatrix} 0 & 1 & 0 \\ -|\xi|^4 & 0 & |\xi|^2 \\ 0 & -|\xi|^2 & -|\xi|^2 \end{pmatrix}.$$

For $j \in \{0, 1, 2\}$, define

$$S_j(\xi) := (1 + |\xi|^2)^{j/2} \text{diag}((1 + |\xi|^2), 1, 1),$$
where \( \text{diag}(\ldots) \) stands for the diagonal matrix with the corresponding elements on the diagonal. For the next result, we use the fact that the induced operator \( S_j(D) \) defines an isometric isomorphism

\[
S_j(D) \in L_{1\text{om}}(\mathcal{E}_j^{(p)}, L^p(\mathbb{R}^N; \mathbb{C}^3)) \quad (j \in \{0, 1, 2\}).
\]

**Lemma 2.4.** For every \( \vartheta < \vartheta_0 \), \( \lambda_0 > 0 \) and \( j \in \{0, 1, 2\} \) we have

\[
\Re L_{\mathcal{E}_j^{(p)}(\mathbb{R}^N), \mathcal{E}_j^{(p)}(\mathbb{R}^N)}(\{\lambda^{j/2} R(\lambda) : \lambda \in \lambda_0 + \Sigma_\vartheta \}) < \infty.
\]

**Proof.** Let \( j \in \{0, 1, 2\} \). In view of (9) and Lemma 2.2, we have to show that every entry of the matrix

\[
M^{(j)}(\xi, \lambda) = (m_{kl}^{(j)}(\xi, \lambda))_{k,l=1,2,3} := \lambda^{j/2} S_{2-j}(\xi)(\lambda - A(\xi))^{-1} S_0(\xi)^{-1}
\]

belongs to \( \mathcal{M}_0(\lambda_0 + \Sigma_\vartheta) \). It was shown in [20], Section 2, that for all \( \lambda \in \lambda_0 + \Sigma_\vartheta \) we have

\[
(\lambda - A(\xi))^{-1} = \frac{1}{\det(\lambda - A(\xi))} \begin{pmatrix}
\lambda(\lambda + |\xi|^2) + |\xi|^4 & |\xi|^2 & |\xi|^2 \\
-(\lambda + |\xi|^2)|\xi|^4 & \lambda(\lambda + |\xi|^2) & |\xi|^2 \\
|\xi|^6 & -\lambda|\xi|^2 & \lambda^2 + |\xi|^4
\end{pmatrix}.
\]

Moreover, \( \det(\lambda - A(\xi)) = \prod_{i=1}^3 (\frac{\lambda}{\gamma_i} + |\xi|^2) \) ([20], Lemma 2.3). As in Example 2.2 b), we see that

\[
(\xi, \lambda) \mapsto (\frac{\lambda}{\gamma_i} + |\xi|^2)^{-1} \in \mathcal{M}_2(\Sigma_\vartheta)
\]

and therefore \( \det(\lambda - A(\xi))^{-1} \in \mathcal{M}_0(\Sigma_\vartheta) \). For the left upper corner of \( M^{(j)} \), we have

\[
m_{11}^{(j)}(\xi, \lambda) = (\det(\lambda - A(\xi))^{-1} (1 + |\xi|^2)^{(2-j)/2} (\lambda^2 + \lambda|\xi|^2 + |\xi|^4).
\]

With Example 2.2 we see that

\[
\lambda^{j/2} \in \mathcal{M}_j(\Sigma_\vartheta),
\]

\[
(1 + |\xi|^2)^{(2-j)/2} \in \mathcal{M}_{2-j}(\lambda_0 + \Sigma_\vartheta),
\]

\[
\lambda^2 + \lambda|\xi|^2 + |\xi|^4 \in \mathcal{M}_4(\Sigma_\vartheta),
\]

which yields \( m_{11}^{(j)} \in \mathcal{M}_0(\lambda_0 + \Sigma_\vartheta) \). Similarly,

\[
m_{21}^{(j)}(\xi, \lambda) = -(\det(\lambda - A(\xi))^{-1} \lambda^{j/2} (1 + |\xi|^2)^{(2-j)/2} (\lambda + |\xi|^2)^{\frac{|\xi|^2}{1+|\xi|^2}}.
\]

Using

\[
\lambda^{j/2} \in \mathcal{M}_j(\Sigma_\vartheta),
\]

\[
(1 + |\xi|^2)^{(2-j)/2} \in \mathcal{M}_{2-j}(\lambda_0 + \Sigma_\vartheta),
\]

\[
(\lambda + |\xi|^2) \in \mathcal{M}_2(\Sigma_\vartheta),
\]

\[
\frac{|\xi|^2}{1+|\xi|^2} \in \mathcal{M}_0(\Sigma_\vartheta),
\]

we obtain \( m_{21} \in \mathcal{M}_0(\lambda_0 + \Sigma_\vartheta) \). All other entries of the matrix \( M^{(j)} \) can be estimated similarly. Therefore, \( M^{(j)} \in \mathcal{M}_0(\lambda_0 + \Sigma_\vartheta) \) which finishes the proof. \( \square \)

**Corollary 2.5.** a) For all \( \lambda \in \Sigma_{\vartheta_0} \), the operator \( \lambda - A_{p,\mathbb{R}^N} : \mathcal{E}_p^{(2)} \to \mathcal{E}_p^{(0)} \) is invertible.

b) The operator \( A_{p,\mathbb{R}^N} \) is not sectorial for any angle and therefore does not generate a bounded \( C_0 \)-semigroup on \( \mathcal{E}_p^{(0)} \).
c) For any $\lambda_0 > 0$, the operator $A_{p;\mathbb{R}^N} - \lambda_0$ is $\mathcal{R}$-sectorial with $\mathcal{R}$-angle $\vartheta_0$. Therefore, $A_{p;\mathbb{R}^N} - \lambda_0$ has maximal $L^3$-$L^2$-regularity in $(0, \infty)$, and $A_{p;\mathbb{R}^N}$ has maximal $L^3$-$L^2$-regularity in $(0, T)$ with $T < \infty$. In particular, $A_{p;\mathbb{R}^N}$ generates an analytic $C_0$-semigroup.

Proof. a) Let $\lambda \in \Sigma_{\vartheta_0}$ and choose $\vartheta < \vartheta_0$ and $\lambda_0 > 0$ such that $\lambda \in \lambda_0 + \Sigma_{\vartheta}$. By Lemma 2.4 with $j = 0$, we have $R(\lambda) \in L(E^{(0)}_{p_1}, E^{(2)}_{p_2})$. Obviously, $R(\lambda)$ is the inverse of $\lambda - A_{p;\mathbb{R}^N}$, and therefore $\lambda$ is in the resolvent set of $A_{p;\mathbb{R}^N}$.

b) Assume that $\|\lambda(\lambda - A_{p;\mathbb{R}^N})^{-1}\|_{L(E^{(0)}_{p_1})} \leq C (\lambda \in (0, \infty))$ holds. Then the operator $M_0(D, \lambda) \in L(L^p(\mathbb{R}^N; C^3))$ is uniformly (with respect to $\lambda$) bounded, where $M_0(\xi, \lambda) := \lambda S_1(\xi)(\lambda - A(\xi))^{-1} S_1(\xi)^{-1}$. In particular, every entry of $M_0(\xi, \lambda)$ is an $L^\infty$-function (see Prop. 3.17 in [2]). For the last entry in the first row of $M_0(\xi, \lambda)$, we obtain

$$\left| \frac{\lambda(1 + |\xi|^2)|\xi|^2}{\prod_{j=1}^{3}(\lambda + \gamma_j|\xi|^2)} \right| \leq C < \infty \quad (\lambda \in (0, \infty), \xi \in \mathbb{R}^N).$$

However, setting $\lambda = k^{-2}$ and $|\xi| = k^{-1}$, we see that the left-hand side is unbounded for $k \to \infty$.

c) The $\mathcal{R}$-sectoriality follows from Lemma 2.4 with $j = 2$, and the other statements are consequences of the general theory on $\mathcal{R}$-sectorial operators. □

3. The case of a uniform $C^4$-domain.

Definition 3.1. A domain $\Omega$ is called a uniform $C^4$-domain if there exist positive constants $\alpha$, $\beta$ and $K$ such that for any $x_0 \in \Gamma$ there exist a coordinate number $j$ and a $C^4$-function $h(x')$ defined on $B'_\alpha(x_0')$ such that $\|h\|_{H^4_{\mathcal{R}}(B'_\alpha(x_0'))} \leq K$ and

$$\Omega \cap B_\alpha(x_0) = \{ x \in \mathbb{R}^N \mid x_j > h(x') \ (x' \in B'_\alpha(x_0')) \cap B_\alpha(x_0),$$

$$\Gamma \cap B_\beta(x_0) = \{ x \in \mathbb{R}^N \mid x_j = h(x') \ (x' \in B'_\alpha(x_0')) \cap B_\beta(x_0).$$

Here, $x'$ has been defined by $x' = (x_1, \ldots, x_{j-1}, x_{j+1}, \ldots, x_N)$ for $x = (x_1, \ldots, x_N)$,

$$B'_\alpha(x_0') = \{ x' \in \mathbb{R}^{N-1} \mid |x' - x_0'| < \alpha \}, \quad B_\beta(x_0) = \{ x \in \mathbb{R}^N \mid |x - x_0| < \beta \}.$$

Let $\Omega \subset \mathbb{R}^N$ be a uniform $C^4$-domain with boundary $\Gamma$, and let $p \in (1, \infty)$. To show that the operator $A_p : E^{(0)}_{p_1}(\Omega) \supset D(A_p) \to E^{(0)}_{p_1}(\Omega)$ generates an analytic $C_0$-semigroup, we first consider the boundary value problem

$$(\lambda - A(D))U = 0 \quad \text{in} \ \Omega,$$

$$B(D)U = G \quad \text{on} \ \Gamma. \quad (10)$$

Here, $G = (g_1, g_2, g_3)^T$ is defined in the whole of $\Omega$. Similarly to [5], (1.8), we define the spaces

$$\mathcal{G}_p(\Omega) := H^2_{p_1}(\Omega) \times H^1_{p_1}(\Omega) \times H^1_{p_1}(\Omega),$$

$$\mathcal{X}_p(\Omega) := \mathcal{G}_p(\Omega) \times (H^1_{p_1}(\Omega) \times L^p(\Omega) \times L^p(\Omega)) \times L^p(\Omega).$$

The $\lambda$-dependent map $H(\lambda) : \mathcal{G}_p(\Omega) \to \mathcal{X}_p(\Omega)$ is defined by

$$H(\lambda)((g_1, g_2, g_3)^T) := ((g_1, g_2, g_3), \lambda^{1/2}(g_1, g_2, g_3), \lambda g_1)^T.$$
Remark 3.2. We note that we could consider the boundary data \( G \) in (10) to be defined on the boundary \( \Gamma \) only. In this case the space for \( G|\Gamma \) would be given by

\[
G|\Gamma \in W^{2-1/p}(\Gamma) \times W^{1-1/p}(\Gamma) \times W^{1-1/p}(\Gamma). \tag{11}
\]

Here, \( W^{s-1/p}(\Gamma) = B^{s-1/p}_p(\Gamma) \), \( s = 1, 2 \), denotes the Sobolev-Slobodeckii space which coincides with the Besov space. However, due to standard trace results, there exists a continuous extension operator (co-retraction) from the space (11) to the space \( \mathcal{G}_p(\Omega) \). Therefore we may assume that \( G \) is already given on \( \Omega \). In fact, in the proof of Theorem 3.4 below, we will have \( G = -B(D)U_1 \) which is already defined on the whole domain.

The following result on the existence of \( \mathcal{R} \)-bounded solution operators was shown in [5], Theorem 1.4.

Theorem 3.3. There exist a number \( \vartheta_1 \in (\frac{\pi}{2}, \pi) \), a positive number \( \lambda_0 \) and an operator family

\[
L(\lambda) = (L_1(\lambda), \lambda L_1(\lambda), L_2(\lambda))^\top \in L(\mathcal{X}_p(\Omega), \mathcal{E}_p^{(2)}(\Omega))
\]

such that for every \( \lambda \in \lambda_0 + \Sigma_{\vartheta_1} \) and every \( G \in \mathcal{G}_p(\Omega) \), problem (10) admits a unique solution \( U \in \mathcal{E}_p^{(2)}(\Omega) \) given by \( U = L(\lambda)H(\lambda)G \). Moreover,

\[
\mathcal{R}_{L(\mathcal{X}_p(\Omega), H^2_{-1}(\Omega))}(\{\lambda^{j/2}L_1(\lambda) : \lambda \in \lambda_0 + \Sigma_{\vartheta_1}\}) \leq C \quad (j = 0, \ldots, 4),
\]

\[
\mathcal{R}_{L(\mathcal{X}_p(\Omega), H^2_{-1}(\Omega))}(\{\lambda^{j/2}L_2(\lambda) : \lambda \in \lambda_0 + \Sigma_{\vartheta_1}\}) \leq C \quad (j = 0, 1, 2).
\]

Let \( R_\Omega : f \mapsto f|\Omega \) denote the restriction of a function defined on \( \mathbb{R}^N \) to \( \Omega \). Obviously, we have \( r_\Omega \in L(H^1_0(\mathbb{R}^N), H^1_0(\Omega)) \) with norm 1 for any \( i \in \mathbb{N}_0 \). In fact, \( r_\Omega \) is a retraction as a corresponding co-retraction (extension operator) exists for uniform \( C^2 \)-domains. In the following, we fix an extension operator \( e_\Omega : L^1_{\text{loc}}(\Omega) \to L^1_{\text{loc}}(\mathbb{R}^N) \) with the property that for any \( p \in (1, \infty) \) and \( f \in H^1_0(\Omega) \), we have \( e_\Omega f \in H^1_p(\mathbb{R}^N) \), \( r_\Omega e_\Omega f = f \), and \( \|e_\Omega\|_{L(\mathcal{X}_p(\Omega), H^1_p(\mathbb{R}^N))} \leq C_p \) for \( i = 0, \ldots, 4 \). For the existence of such an extension operator, we refer to [21], Appendix A.

The following theorem is the main result of the present paper.

Theorem 3.4. There exist \( \lambda_0 > 0 \) and \( \vartheta > \frac{\pi}{2} \) such that the operator \( A_{p, \Omega} - \lambda_0 \) is \( \mathcal{R} \)-sectorial with \( \mathcal{R} \)-angle \( \vartheta \). Therefore, \( A_{p, \Omega} \) has maximal \( L^p \)-\( L^p \)-regularity in every finite time interval. In particular, \( A_{p, \Omega} \) generates an analytic \( C_0 \)-semigroup in \( \mathcal{E}_p^{(0)}(\Omega) \).

Proof. We first obtain a description of the resolvent \( (\lambda - A_{p, \Omega})^{-1} \). For this, let \( F \in \mathcal{E}_p^{(0)}(\Omega) \) be given. We apply the extension operator \( e_\Omega \) from above to every component of \( F \) and obtain \( e_\Omega F \in \mathcal{E}_p^{(0)}(\mathbb{R}^N) \). We set \( U_1 := r_\Omega R(\lambda)e_\Omega F \) for \( \lambda \in \Sigma_{\vartheta_0} \) with \( R(\lambda) \) being the whole space resolvent defined in (8).

To solve

\[
(\lambda - A(D))U = F \quad \text{in } \Omega,
\]

\[
B(D)U = 0 \quad \text{on } \Gamma,
\]

we set \( U = U_1 + U_2 \) and obtain the boundary value problem

\[
(\lambda - A(D))U_2 = 0 \quad \text{in } \Omega,
\]

\[
B(D)U_2 = -B(D)U_1 \quad \text{on } \Gamma.
\]
for \( U_2 \). Due to Theorem 3.3, there exist \( \lambda_0 \) and \( \theta_1 \) such this equation is uniquely solvable for \( \lambda \in \lambda_0 + \Sigma_{\theta_1} \), and its solution is given by

\[
U_2 = -L(\lambda)H(\lambda)B(D)U_1.
\]

Therefore, for \( \theta \in (\frac{\pi}{2}, \min\{\theta_0, \theta_1\}) \) and \( \lambda \in \lambda_0 + \Sigma_{\theta} \), the boundary value problem (12) is uniquely solvable with solution

\[
U = U_1 + U_2 = r_\Omega R(\lambda)\epsilon_\Omega F - L(\lambda)H(\lambda)B(D)r_\Omega R(\lambda)\epsilon_\Omega F.
\]

Consequently, we have to show the \( \mathcal{R} \)-boundedness of the operator family

\[
\lambda(\lambda - A_{p,\Omega})^{-1} = r_\Omega \lambda R(\lambda)\epsilon_\Omega - \lambda L(\lambda)H(\lambda)B(D)r_\Omega R(\lambda)\epsilon_\Omega \quad (\lambda \in \lambda_0 + \Sigma_{\theta}).
\]

By Corollary 2.5 c), \( \lambda \epsilon_\Omega R(\lambda) \) is \( \mathcal{R} \)-bounded. As \( \epsilon_\Omega \) and \( r_\Omega \) are continuous and \( \lambda \)-independent, we obtain

\[
\mathcal{R}_{L(E_p^{(0)}(\Omega))}(\{ \lambda r_\Omega R(\lambda)\epsilon_\Omega : \lambda \in \lambda_0 + \Sigma_{\theta} \}) < \infty. \tag{14}
\]

Similarly, by Theorem 3.3 with \( j = 2 \) and \( j = 4 \) we see

\[
\mathcal{R}_{L(X_{p}(\Omega), E_p^{(0)}(\Omega))}(\{ \lambda L(\lambda) : \lambda \in \lambda_0 + \Sigma_{\theta} \}) < \infty. \tag{15}
\]

It remains to show that the family \( H(\lambda)B(D)r_\Omega R(\lambda)\epsilon_\Omega \) is \( \mathcal{R} \)-bounded. By the definition of the matrix \( B(D) \) and the spaces, we see that the operators

\[
\begin{align*}
B(D) & : E_p^{(2)}(\Omega) \to C_p(\Omega), \\
B(D) & : E_p^{(1)}(\Omega) \to H^1_p(\Omega) \times L^p(\Omega) \times L^p(\Omega), \\
B_1(D) & : E_p^{(0)}(\Omega) \to L^p(\Omega)
\end{align*}
\]

are continuous, where \( B_1(D) \) stands for the first row of \( B(D) \), i.e. \( B_1(D)(u, v, \theta)^\top = (\Delta - (1 - \beta)\Delta')u + \theta \). Thus,

\[
\begin{pmatrix}
B(D) \\
B(D) \\
B_1(D)
\end{pmatrix} : E_p^{(2)}(\Omega) \times E_p^{(1)}(\Omega) \times E_p^{(0)}(\Omega) \to X_p(\Omega) \tag{16}
\]

is continuous (and independent of \( \lambda \)). By Corollary 2.5 c), the family

\[
\left\{ \begin{pmatrix}
R(\lambda) \\
\lambda^{1/2} R(\lambda) \\
R(\lambda)
\end{pmatrix} : \lambda \in \lambda_0 + \Sigma_{\theta} \right\} \subset L(E_p^{(0)}(\Omega), E_p^{(2)}(\Omega) \times E_p^{(1)}(\Omega) \times E_p^{(0)}(\Omega))
\]

is \( \mathcal{R} \)-bounded. In combination with

\[
H(\lambda)B(D)r_\Omega R(\lambda)\epsilon_\Omega F = \begin{pmatrix}
B(D)r_\Omega R(\lambda)\epsilon_\Omega F \\
B_1(D)r_\Omega \lambda R(\lambda)\epsilon_\Omega F \\
B_1(D)r_\Omega \lambda^{1/2} R(\lambda)\epsilon_\Omega F
\end{pmatrix}
\]

and (16), this yields

\[
\mathcal{R}_{L(E_p^{(0)}(\Omega), X_p(\Omega))}(H(\lambda)B(D)r_\Omega R(\lambda)\epsilon_\Omega : \lambda \in \lambda_0 + \Sigma_{\theta}) < \infty. \tag{17}
\]

From (14), (15), and (17), the first statement of the theorem follows by the description of the resolvent in (13). As before, the other statements follow by the general theory of \( \mathcal{R} \)-boundedness. \( \square \)
The results of Theorem 3.4 are preserved under lower-order perturbations of the operators \( A(D) \) and \( B(D) \). More precisely, we consider perturbation matrices of the form

\[
A'(D) = \begin{pmatrix}
0 & 0 & 0 \\
0 & 0 & a_{32}(x, D) \\
a_{21}(x, D) & 0 & 0
\end{pmatrix}, \quad B'(D) = \begin{pmatrix}
b_{11}(x, D) & 0 & 0 \\
b_{21}(x, D) & 0 & 0 \\
0 & 0 & b_{33}(x, D)
\end{pmatrix}.
\]

Here \( a_{ij}(x, D) \) and \( b_{ij}(x, D) \) are linear differential operators. With respect to the orders of the operators, we assume \( \operatorname{ord} a_{ij}(x, D) \leq s_{ij} \) and \( \operatorname{ord} b_{ij}(x, D) \leq t_{ij} \) with \( s_{21} = 3, s_{23} = s_{32} = s_{33} = 1 \) and \( t_{11} = 1, t_{21} = 2, t_{33} = 0 \). The coefficients of \( a_{ij}(x, D) \) are assumed to belong to \( H_{\infty}^{s_{ij}-1}(\Omega) \), while the coefficients of \( b_{11}(x, D), b_{21}(x, D), \) and \( b_{33}(x, D) \) are assumed to belong to \( H_{\infty}^{2}(\Omega), H_{\infty}^{1}(\Omega), \) and \( H_{\infty}^{1}(\Omega) \), respectively.

**Lemma 3.5.** Let \( (A'(D), B'(D)) \) be a lower-order perturbation as described above. Define the perturbed operator \( \tilde{A}_{p, \Omega} : E_{p}^{0}(\Omega) \to E_{p}^{0}(\Omega) \) by

\[
D(\tilde{A}_{p, \Omega}) := \{ U \in E_{p}^{2}(\Omega) : \tilde{B}(D)U = 0 \}, \quad \tilde{A}_{p, \Omega}U := \tilde{A}(D)U,
\]

where \( \tilde{A}(D) := A(D) + A'(D) \) and \( \tilde{B}(D) := B(D) + B'(D) \).

Then there exist \( \lambda_0 > 0 \) and \( \vartheta > \frac{\pi}{2} \) such that the operator \( \tilde{A}_{p, \Omega} - \lambda_0 \) is \( \mathcal{R} \)-sectorial with \( \mathcal{R} \)-angle \( \vartheta \). In particular, \( \tilde{A}_{p, \Omega} \) has maximal \( L^p \)-\( L^p \)-regularity in every finite time interval and generates an analytic \( C_0 \)-semigroup in \( E_{p}^{0}(\Omega) \).

**Proof.** (i) First, we consider boundary perturbations, i.e. \( A'(D) = 0 \). As in the proof of Theorem 3.4, we have to find a solution \( \tilde{U}_2 \) of the boundary value problem

\[
(\lambda - A(D))\tilde{U}_2 = 0 \quad \text{in } \Omega, \\
\tilde{B}(D)\tilde{U}_2 = G \quad \text{on } \Gamma,
\]

where \( G := -\tilde{B}(D)r_{12}(\lambda)e_{\Omega}F \). We set \( U := L(\lambda)H(\lambda)G \). Then \( U \) solves

\[
(\lambda - A(D))U = 0 \quad \text{in } \Omega, \\
\tilde{B}(D)U = G - B'(D)U = (1 - B'(D)L(\lambda)H(\lambda))G =: \tilde{G} \quad \text{on } \Gamma.
\]

Let \( \lambda_0 \) and \( \vartheta \) be as in the proof of Theorem 3.2. We show that the operator family

\[
\{ \lambda^{1/2}H(\lambda)B'(D)L(\lambda) : \lambda \in \lambda_0 + \Sigma_{\vartheta} \} \subset L(X_p(\Omega))
\]

is \( \mathcal{R} \)-bounded. In fact, due to the assumptions on \( B'(D) \), we have

\[
b_{11}(x, D) \in L(H_{p}^{4-j}(\Omega), H_{p}^{3-j}(\Omega)) \quad (j = 1, 2, 3), \\
b_{21}(x, D) \in L(H_{p}^{4-j}(\Omega), H_{p}^{2-j}(\Omega)) \quad (j = 1, 2), \\
b_{33}(x, D) \in L(H_{p}^{2-j}(\Omega), H_{p}^{2-j}(\Omega)) \quad (j = 1, 2).
\]

By Theorem 3.3, the families

\[
\{ \lambda^{j/2}L_1(\lambda) : \lambda \in \lambda_0 + \Sigma_{\vartheta} \} \subset L(X_p(\Omega), H_{p}^{4-j}(\Omega)) \quad (j = 0, \ldots, 4), \\
\{ \lambda^{j/2}L_2(\lambda) : \lambda \in \lambda_0 + \Sigma_{\vartheta} \} \subset L(X_p(\Omega), H_{p}^{2-j}(\Omega)) \quad (j = 0, 1, 2)
\]

are \( \mathcal{R} \)-bounded. Therefore, \( \{ \lambda^{1/2}H(\lambda)B'(D)L(\lambda) : \} \subset L(X_p(\Omega)) \) is \( \mathcal{R} \)-bounded.
are \( \mathcal{R} \)-bounded. By composition, we see that the family
\[
\left\{ \lambda^{1/2}H(\lambda)B'(D)L(\lambda) = \begin{pmatrix} \lambda^{1/2}B'(D)L(\lambda) \\ AB'(D)L(\lambda) \\ \lambda^{3/2}b_{11}(x, D)L_1(\lambda) \end{pmatrix} : \lambda \in \lambda_0 + \Sigma_\vartheta \right\} \subset L(\mathcal{X}_p(\Omega))
\]
is \( \mathcal{R} \)-bounded. Choosing \( \lambda_1 > \lambda_0 \) sufficiently large, we obtain
\[
\mathcal{R}_{L(\mathcal{X}_p(\Omega))}\left( \{ H(\lambda)B'(D)L(\lambda) : \lambda \in \lambda_1 + \Sigma_\vartheta \} \right) \leq \frac{1}{2}.
\] (19)
Therefore, \( 1 - H(\lambda)B'(D)L(\lambda) \in L(\mathcal{X}_p(\Omega)) \) is invertible for all \( \lambda \in \lambda_1 + \Sigma_\vartheta \). A simple algebraic calculation shows that this implies that also \( 1 - B'(D)L(\lambda)H(\lambda) \in L(\mathcal{G}_p(\Omega)) \) is invertible, and that we have
\[
H(\lambda)(1 - B'(D)L(\lambda)H(\lambda))^{-1} = (1 - H(\lambda)B'(D)L(\lambda))^{-1}H(\lambda).
\] (20)
Setting \( \tilde{U}_2 := L(\lambda)H(\lambda)(1 - B'(D)L(\lambda)H(\lambda))^{-1}G \), we obtain \( (\lambda - A(\lambda))\tilde{U}_2 = 0 \) and
\[
\tilde{B}(D)\tilde{U}_2 = (1 - B'(D)L(\lambda)H(\lambda))(1 - B'(D)L(\lambda)H(\lambda))^{-1}G = G,
\]
i.e., \( \tilde{U}_2 \) is a solution of (18). As in the proof of Theorem 3.4, the solution \( \tilde{U} \) of the resolvent equation is now given by \( \tilde{U} = U_1 + \tilde{U}_2 \) with \( U_1 := r_\Omega R(\lambda)e_{\Omega} \) as in the proof of Theorem 3.4. Therefore, the resolvent of \( \tilde{A}_{\rho, \Omega} \) is given by
\[
(\lambda - \tilde{A}_{\rho, \Omega})^{-1} = r_\Omega R(\lambda)e_{\Omega} - L(\lambda)H(\lambda)(1 - B'(D)L(\lambda)H(\lambda))^{-1}\tilde{B}(D)r_\Omega R(\lambda)e_{\Omega}
= r_\Omega R(\lambda)e_{\Omega} - L(\lambda)(1 - H(\lambda)B'(D)L(\lambda))^{-1}H(\lambda)\tilde{B}(D)r_\Omega R(\lambda)e_{\Omega},
\]
where we used (20) for the last equality. We have already seen in the proof of Theorem 3.4 that the operator families \( \lambda L(\lambda) \) and \( H(\lambda)\tilde{B}(D)r_\Omega R(\lambda)e_{\Omega} \) are \( \mathcal{R} \)-bounded. Using (19) and a Neumann series argument, we see that
\[
\mathcal{R}_{L(\mathcal{X}_p)}\left( \{ (1 - H(\lambda)B'(D)L(\lambda))^{-1} : \lambda \in \lambda_1 + \Sigma_\vartheta \} \right) \leq 2.
\]
Now the statements of the lemma follow in the same way as in the proof of Theorem 3.4.

(ii) In the case \( A'(D) \neq 0 \), we consider \( (\tilde{A}(D), \tilde{B}(D)) \) as a perturbation of \( (A(D), B(D)) \). Let \( \tilde{A}_B \) and \( A_B \) denote the corresponding operators, respectively. Note that we have \( D(\tilde{A}_B) = D(A_B) \). By the interpolation inequality, for every \( \varepsilon > 0 \) there exists \( C_\varepsilon > 0 \) such that
\[
\|\tilde{A}_Bu\|_{E_p^0(\Omega)} \leq \varepsilon\|A_Bu\|_{E_p^0(\Omega)} + C_\varepsilon\|u\|_{E_p^0(\Omega)} \quad (u \in D(A_B)).
\]
Due to part (i) of the proof, \( A_B \) is \( \mathcal{R} \)-sectorial, and by an abstract perturbation result on \( \mathcal{R} \)-sectorial operators ([9], Corollary 6.7), the same holds for \( \tilde{A}_B \). \( \square \)

**Remark 3.6.** Whereas the lower-order perturbation of the operator \( A(D) \) could be handled by an abstract perturbation result on \( \mathcal{R} \)-boundedness, to our knowledge there is no such theorem on boundary perturbation which could be applied to our situation. Therefore, the proof of Lemma 3.5 directly uses the structure of the solution operators.

The results above were formulated in a general setting in \( \mathbb{R}^N \) with \( N \geq 2 \). In the physically relevant case \( N = 2 \), the modelling can be found in [10], Chapter 2.
Theorem 3.8. Let

\[ \text{Corollary 3.9.} \]

Apart from physical constants, the equation in a uniform C⁴-domain \( \Omega \subset \mathbb{R}^2 \) is given by

\[
\begin{align*}
u_t + \Delta^2 u + \Delta \theta &= 0 & \text{in } (0, \infty) \times \Omega, \\ \theta_t - \Delta \theta - \Delta u_t &= 0 & \text{in } (0, \infty) \times \Omega
\end{align*}
\]

(21)

with boundary conditions

\[
\begin{align*}
\Delta u + (1 - \mu) B_1 u + \theta &= 0 & \text{on } (0, \infty) \times \Gamma, \\ \partial_\nu \Delta u + (1 - \mu) B_2 u + \partial_\nu \theta &= 0 & \text{on } (0, \infty) \times \Gamma, \\ \partial_\nu \theta &= 0 & \text{on } (0, \infty) \times \Gamma.
\end{align*}
\]

Here, the operators \( B_1 \) and \( B_2 \) are given by

\[
\begin{align*}
B_1 u := 2\nu_1 \nu_2 u_{xy} - \nu_1^2 u_{yy} - \nu_2^2 u_{xx}, \\ B_2 u := \partial_\tau \left[ (\nu_1^2 - \nu_2^2) u_{xy} + \nu_1 \nu_2 (u_{yy} - u_{xx}) \right],
\end{align*}
\]

where \( \nu = \left( \nu_1 \nu_2 \right) \) denotes the outer normal vector and \( \tau := (-\nu_1) \). In [12], the following variant of boundary conditions was considered:

\[
\begin{align*}
\Delta u + (1 - \mu) B_1 u + \theta &= 0 & \text{on } (0, \infty) \times \Gamma, \\ \partial_\nu \Delta u + (1 - \mu) B_2 u - u + \partial_\nu \theta &= 0 & \text{on } (0, \infty) \times \Gamma, \\ \partial_\nu \theta + b \theta &= 0 & \text{on } (0, \infty) \times \Gamma
\end{align*}
\]

(23)

with \( b > 0 \).

**Corollary 3.7.** Let \( N = 2 \), and let \( \Omega \subset \mathbb{R}^N \) be a uniform C⁴-domain. Then the statements of Theorem 3.4 hold for the operators related to the boundary value problems (21), (22) and (21), (23).

**Proof.** A straight-forward calculation shows that \( B_1 u = -\Delta' u \) and \( B_2 u = \partial_\nu \Delta' u \) holds up to lower-order terms. Therefore, we can apply Lemma 3.5 to both boundary value problems.

Finally, we study exponential stability in the case of a bounded domain.

**Theorem 3.8.** Let \( \Omega \subset \mathbb{R}^N \), \( N \geq 2 \), be a bounded C⁴-domain, and let \( (T(t))_{t \geq 0} \subset L(\mathbb{R}_p(\Omega)) \) be the \( C_0 \)-semigroup generated by \( A_{p,\Omega} \), see Theorem 3.4. Let \( P_{p,\Omega} \subset L(\mathbb{R}_p(\Omega)) \) be the spectral projection corresponding to the eigenvalue 0 of \( A_{p,\Omega} \), and let \( (T_0(t))_{t \geq 0} \subset L(\ker P_{p,\Omega}) \) be the part of \( T(t) \) in \( \ker P_{p,\Omega} \), i.e., \( T_0(t) := T(t)_{|_{\ker P_{p,\Omega}}} \).

Then \( (T_0(t))_{t \geq 0} \) is exponentially stable, i.e., there exist \( C > 0 \) and \( \varepsilon > 0 \) such that \( \|T(t)\|_{L(\ker P_{p,\Omega})} \leq Ce^{-\varepsilon t} \) \( (t \geq 0) \). The same holds for the perturbed problem \( \tilde{A}_{p,\Omega} \) as in Lemma 3.5.

**Proof.** As \( \Omega \) is bounded, the operator \( A_{p,\Omega} \) has compact resolvent and discrete spectrum. Moreover, the spectrum is independent of \( p \in (1, \infty) \). It was shown in [12] that \( A_{2,\Omega} \) is dissipative which implies \( \sigma(A_{2,\Omega}) \subset \{ \lambda \in \mathbb{C} : \text{Re} \lambda \leq 0 \} \). Moreover, 0 is the only eigenvalue on the imaginary axis. Now the statements of the theorem follow from general semigroup theory.

**Corollary 3.9.** Let \( N = 2 \), and let \( \Omega \subset \mathbb{R}^2 \) be a bounded C⁴-domain. Then the analytic semigroup related to the boundary value problem (21), (22) is exponentially stable in the space \( \ker P_{p,\Omega} \), and the analytic semigroup related to (21), (23) is exponentially stable in the whole space \( \mathbb{R}_p(\Omega) \).
Proof. This is a particular case of Theorem 3.8 where we note that in the case of (21), (23) there is no eigenvalue on the imaginary axis due to [12].

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