ANTIGRAVITATING BUBBLES

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Abstract

We investigate the gravitational behavior of spherical domain walls (bubbles) arising during the phase transitions in the early Universe. In the thin-wall approximation we show the existence of the new solution of Einstein equations with negative gravitational mass of bubbles and the reversed direction of time flow on the shell. This walls exhibit gravitational repulsion just as the planar walls are assumed to do. The equilibrium radius and critical mass of such objects are found for realistic models.

1 Introduction.

Topological structures such as domain walls, strings and monopoles could be produced at phase transitions in the Universe as it cooled [4, 13, 17, 37, 48, 49]. Within the context of general relativity they are assumed to be an unusual sources of gravity. Cosmic strings do not produce any gravitational force on the surrounding matter locally, while global monopoles, global strings and planar domain walls are repulsive [7, 10, 24, 25, 28, 39, 47, 48, 49].

We shall consider domain walls, produced through the breakdown of discrete symmetry. Their stress-energy is composed of surface density \( \sigma \) and strong tension \( p \) in two spatial directions with the magnitude [48, 49]

\[
\sigma = -p = \text{const.} \tag{1}
\]

This state equation corresponds to de Sitter expansion in the wall-plane and the borders of the wall run away with the horizon. We can speak about the gravitational field of the wall only in normal direction to the wall. If one assumes that for such objects it is possible to use Newtonian approximation with the mass described by Tolman’s formula [15].
\[ M = \int (T^0_0 - T^1_1 - T^2_2 - T^3_3) \cdot \sqrt{-g} dV = \int (\sigma + 2p) \cdot \sqrt{-g} dV = -\int \sigma \cdot \sqrt{-g} dV \tag{2} \]

as it is usually assumed \[\text{[48, 49]}, \text{ then that tension acts as a repulsive source of gravity and the planar domain wall has a negative gravitational mass and exhibits repulsive gravitational field \[\text{[10, 28, 39, 48, 49]}.\]

It is natural to think that the same behavior (gravitational repulsion) must occur for the spherical domain walls (bubbles), since it is assumed that they are described by the same state equations (1) (e.g. see \[\text{[10, 28, 39]}\]), different aspects of bubble-dynamics was investigated also in papers \[\text{[1, 2, 3, 4, 5, 8, 9, 10, 12, 14, 18, 21, 24, 26, 34, 37, 38, 39, 40, 43, 46]}\). On the other hand, according to Birkhoff’s theorem, the empty space, surrounding any spherical body (including bubbles), is described by Schwarzschild metric. This metric contains parameter \(m\) (corresponding to the mass of gravitating body) which is described through the integral over energy density of the body

\[ m = \int T^0_0 \cdot \sqrt{-g} dV + \text{const.} \tag{3} \]

While for planar domain walls (which stretch the horizon) the negative gravitational mass (2) can be admissible, for bubbles the negativeness of mass (3) from the first glance looks surprising since \(T^0_0\) is positively defined everywhere. Thus independently on the state equation (1) the mass (3) usually is considered to be positive \[\text{[36, 41, 45]}\].

However there can be no contradiction since in the case of spherical domain walls (in difference to isolated matter for which the condition of energodominance is valid) it is impossible to surround the full source by any boundary inside the horizon (just as it is for planar domain walls). The domain wall is only the "part" of scalar field solution which fills the whole Universe up to horizon and which has nonzero vacuum expectation value (VEV) even in infinity. The result is that the quantity \(\int T_{\mu \nu} \cdot dS^\nu\) is not a 4-vector of energy-momentum and one can not define the energy simply as \(\int T_{00} \cdot dx dy dz\) \[\text{[44]}\]. For example, the energy density of expanding spherical domain wall remains constant (see (1)) despite increasing of its surface i.e. this object “takes” the energy from vacuum, while the energy of vacuum depends on VEV of scalar field in the whole space including infinity. This means that for the case of topological objects we can not neglect the boundary terms at infinity since the scalar field forming the wall does not vanish there. The same situation is for global monopoles which also can exhibit the gravitational repulsion \[\text{[22]}\].

The other solution of the negative mass problem can be the fact that the domain walls are not described by the state equation (1). One must take into account the flux out from the volume of integration or some external forces stabilizing the domain wall. As a result the state equation can have a principally different form and both the spherical and planar walls can be gravitationally attractive.

The other possible reason of disagreement may be that planar domain walls can be described by the state equation (1) while the bubbles can not.

In this paper we argue that if nevertheless one assumes that spherical domain walls are described by the state equation (1) (as it is usually considered, see e.g. \[\text{[10, 28, 39]}\]), then they must have a negative gravitational mass and must be repulsive \[\text{[6]}\].
To start with, one has to note that Schwarzschild parameter $m$ in formula (3) contains arbitrary integration constant which must be fixed from the boundary conditions [36, 41, 45]. For the ordinary matter the boundary conditions both in the center of the body and at the infinity lead to zero value of this constant. For the topological objects surface singularities in gravitational quantities allow us to use only one boundary condition dependently on the region we are interested. When investigating the outer gravitational field one has to use the boundary condition at a large distance, where the gravitational potential must be expressed by Newton’s formula. Fixing the constant from this condition it is easy to see that the mass of topological objects is determined by Tolman’s formula (2) (on the other hand when investigating field inside the object one must take $m = 0$, because otherwise the metric will have a singularity at the center $r = 0$). Thus the constant in formula (3) takes into account the pressure (which can not be negligible for topological objects) and so the results from formulae (2) and (3) coincide. For the bubbles this formulae give negative value of $m$ and these objects appear to be repulsive (as the planar walls do). This leads us to conclusion that the space around such physically admissible objects as spherical domain walls are can be described by Schwarzschild solution with negative parameter $m$ [6].

Here we would like to emphasize that though Tolman’s formula is valid only for static objects, nevertheless even for the expanding (or collapsing) spherical domain walls the expression for active gravitational mass of the bubble (obtained, for example, in the thin-wall formalism, see sections 7 and 9) must contain the term corresponding to the gravitational energy (2) and in static Newton’s limit (when the velocity of bubble expansion tends to zero) this expression must coincide with Tolman’s formula. Since the exact solution of Einstein’s equations for thick bubbles is unknown, bubble dynamics usually is analyzed in the thin-wall formalism [15, 19, 29, 32, 41]. The above-mentioned problem emerged there too. In this formalism it also was obtained that active gravitational mass of the spherical domain wall is positive, i.e. its gravitational field is attractive [10, 28, 39]. The disagreements in gravitational properties of planar and spherical domain walls were explained by instability of the latter [28], or by existence of positive energy source stabilizing the bubble [33].

We shall investigate the bubble dynamics in the thin-wall formalism and show that the space outside the bubble can be described by Schwarzschild solution with negative mass-parameter i.e. spherical domain walls are repulsive. We shall see that this solution requires the reversal of time flow on the wall-surface. Note that the solutions of Einstein equations where in different space-regions time flows in opposite directions are well known. For example in Reissner-Nordstrom metric in the region between the upper and Cauchy radii the time coordinate changes its direction to the opposite.

Gravitational repulsion of bubbles can solve different paradoxes (for example the blueshift instead of redshift) appearing in models with large pressure [22]. It is worth to mention another contradicting example — a planar domain wall stretched by a static cosmic string hoop [28]. Such system must repel a test particle placed next to the domain wall (the domain wall is repulsive, while cosmic string does not act gravitationally [18, 19]), whereas for a distant observer it must behave as a bubble, i.e. according to [10, 28, 39] it must be gravitationally attractive. This paradox can be solved only if bubbles are repulsive.

The time flow reversal can explain also the problems mentioned in [12]. Consider
the finite region of false vacuum with nonzero energy density (and thus with negative pressure) separated by a domain wall from an infinite region of true vacuum with zero energy density. In this case an observer placed into the false-vacuum region (described by de Sitter metric) would expect to see inflation and thus increasing of bubble radius. At the same time an outer observer on the true-vacuum side (described by Schwarzschild metric) would discover that the pressure forces are inward and bubble must collapse, thus he would not see an increase of the radius of curvature. This problem was explained by assumption, that false-vacuum region does not move out into true-vacuum region and this two areas expand separately [12]. However, if we take into account the time reversal in the region with the strong pressure and the negative mass (i.e. the false vacuum inside the bubble) this paradox can be explained in the frames of the standard scenario of phase transitions, when the false-vacuum region expand into true vacuum region.

In the next section we describe some features of domain walls.

The section 3 is dedicated to review of the thin-wall formalism. The motion equations of thin shells are given.

In section 4 we consider the surface stress-energy tensors for thin shells.

In section 5 we write the motion equations for spherical shells.

In section 6 the sign ambiguity of motion equations is discussed. It is shown the possibility of existence of a new solution of Einstein equations for bubbles (Schwarzschild solution with negative mass) in a thin-wall formalism. The negative value of the bubble mass in the motion equations leads to the time flow reversal on the bubble.

In sec. 7 the simplest examples of spherical dust and domain walls in vacuum are considered. It is shown that gravitational fields around this objects are described by Schwarzschild metric with the opposite signs of mass parameter.

In sec. 8 some unusual properties of repulsive spheres are mentioned. The embedding of Schwarzschild metric with different signs of mass parameter in 6-dimensional space-time is investigated.

In sec. 9 the dynamic of repulsive bubbles in general case of charged bubbles and nonzero vacuum energy density is discussed. The equilibrium radius and critical mass of static bubbles are found for different symmetry violation scales.

In sec. 10 the problem of stability is considered.

2 The domain wall.

A first-order phase transitions which take place in the most of cosmological models proceed through the nucleation of the new phase bubbles (see e.g. [13, 26, 37]). Such processes take place at the very beginning of the phase transition. At the final stage of transition the old phase fragments which are left up to that moment also take the spherical form (for example due to surface tension effects or dissipation). The surface of phase separation in the case of the first-order phase transition is represented by the so-called domain wall. Such objects are created when the vacuum manifold for the order parameter or scalar field \( \varphi \) driving the symmetry breaking has a discrete symmetry [48, 49]. At the time of transition there can be both infinite and closed surface walls.

To model the essential features of a domain wall, we will consider a scalar field with the Lagrangian
\[ L = \frac{1}{2} g^{\mu \nu} \partial_\mu \varphi \partial_\nu \varphi - V(\varphi), \]

where \( V(\varphi) \) has two minima at nonzero \( \varphi \). Here the greek indices take the values 0,1,2,3 and the metric has the signature (+,—,—,—). The example of such potential with discrete symmetry \( \varphi \to -\varphi \) is

\[ V(\varphi) = -\frac{M_G^2}{2} \varphi^2 + \frac{\lambda}{4} \varphi^4, \]

\( M_G^2/\lambda \) representing the symmetry breaking scale and \( \lambda \) — the self coupling constant.

In the case of a flat wall in Minkowski space the equations of motion

\[ \partial_\mu \partial_\nu \varphi - \frac{\partial V(\varphi)}{\partial \varphi} = 0 \]

admit the classical ”domain wall” or ”membrane” solution, depending on one coordinate:

\[ \varphi(x) = \frac{M_G}{\sqrt{\lambda}} \cdot \tanh \left[ \frac{M_G}{\sqrt{2}} (x - x_0) \right], \]

where \( x \) is the normal direction to the wall and \( x_0 \) is the wall position.

In this case the VEV of scalar field \( < \varphi > \to \pm < \varphi_0 > = \pm M_G/\sqrt{\lambda} \) on either side of the wall and thus there is defined the wall surface \( \Sigma \) — the three dimensional surface on which \( \varphi = 0 \).

The energy-momentum tensor

\[ T_{\mu \nu} = \partial_\mu \varphi \cdot \partial_\nu \varphi - g_{\mu \nu} L \]

for this solution takes the form:

\[ T_{00} = \left( \frac{\partial \varphi}{\partial x} \right)^2 = \frac{M_G^4}{2\lambda} \cdot \cosh^{-4} \left[ \frac{M_G}{2} (x - x_0) \right]; \]

\[ T_{xx} = 0; \]

\[ T_{yy} = T_{zz} = -T_{00}. \] (4)

From the first equation of (4) it is easy to see, that the thickness and energy per unit area of this wall are respectively:

\[ \Delta \sim \frac{1}{M_G}; \]

\[ \sigma = \int_{-\infty}^{+\infty} dx T_{00} \sim \frac{M_G^3}{\lambda}. \]

From the third equation of (4) one can see that the wall solution has a high pressure along the wall, which plays an essential role in gravitational behavior of such objects (see Tolmans formula (3)).
3 Thin wall approximation.

In the limit of vanishing thickness of the domain wall it can be considered as an infinitely thin three-dimensional hypersurface $\Sigma$ with the energy-momentum tensor $T_\mu^\nu$ having, in general, singularities on it. This hypersurface divides the Riemannian manifold $^{(4)}V$ into two parts $V^+$ and $V^-$. Each of them contains $\Sigma$ as a part of its boundary.

We intend to apply the Einstein equations

$$G_\mu^\nu = R_\mu^\nu - \frac{1}{2} \cdot g_\mu^\nu R = 8\pi G \cdot T_\mu^\nu$$

(5)

to such thin boundaries of phase separation and investigate their dynamics and gravitational behavior using the thin-wall approximation. In (5) $R_\mu^\nu$ is the Ricci tensor, expressed in terms of Christoffel connections $\Gamma^\lambda_\mu_\nu$ in the usual way:

$$R_\mu^\nu = \partial_\rho \Gamma^\rho_\mu_\nu + \Gamma^\sigma_\rho_\mu \Gamma^\rho_\sigma_\nu$$

(here the square brackets denote the antisymmetrization) and

$$\Gamma^\lambda_\mu_\nu = \frac{1}{2} \cdot g^{\lambda_\sigma} (\partial_\nu g_{\sigma_\mu} + \partial_\mu g_{\sigma_\nu} - \partial_\sigma g_{\mu_\nu})$$

$G = M_{Pl}^{-2}$ is the gravitational constant, $M_{Pl} = 1.2 \cdot 10^{19} GeV$ is the Planck mass, $\partial_\mu$ is usual derivative. (One must realize, that when speaking about a thin shell we always keep in mind that the thickness of the shell must not be smaller then $1/M_{Pl}$, provided gravity is described just by the classical Einstein equations).

We shall restrict ourselves to the case of time-like hypersurfaces $\Sigma$ with the energy-momentum tensor having singularities no stronger, then those given by $\delta$-functions. Then the first derivatives of $g_{\mu\nu}$ are discontinuous on $\Sigma$, while metric itself is still continuous on $\Sigma$. Gravitational formalism for such boundaries is considered in details in papers [10,15,19,29,32,41].

Let $\{y_\mu\}^+ (\{y_\mu\}^-)$ be an arbitrary coordinate system in $V^+$ ($V^-$) region, and $\{\xi^i\}$ be an arbitrary coordinate system on $\Sigma$ (here the latin indices run through 0,1,2). The coordinate charts $\{y_\mu\}^+$ and $\{y_\mu\}^-$ need not join smoothly on $\Sigma$.

Let us assume, that equation of hypersurface $\Sigma$ in the chosen coordinates $y_\mu$ has the form

$$F(y_\mu) = 0$$

(6)

(Since all equations in the regions $V^+$ and $V^-$ differ only by the indices ”+” and ”−”, we, for simplicity, will not write them separately if not necessary).

We can introduce the new function

$$n(y_\mu) = \varepsilon \frac{F(y_\mu)}{\sqrt{g_{\lambda_\nu} \partial^\lambda F \cdot \partial^\nu F}}$$

(7)

which describes the surface $\Sigma$. Here the sign function $\varepsilon$ depends on the orientation of the (1+2)-surface $\Sigma$. If a displacement vector $dy_\mu$ lies on the hypersurface $n = \text{const}$, then

$$dn = \partial_\mu n \cdot dy_\mu = 0,$$
\( \partial_\mu \) being the usual derivative. Therefore a vector

\[ N_\mu \equiv \partial_\mu n |_\Sigma \]  

(8)
is the unit normal vector to the hypersurface \( \Sigma \):

\[ N_\mu N^\mu = -1 \]

(recall that we consider the timelike \( \Sigma \)).

We can also introduce the unit vectors, tangential to \( \Sigma \):

\[ e^i_\mu = \frac{\partial y^\mu}{\partial \xi^i} \]

The tetrad field \( (N^\mu, e^i_\mu) \) is thereby defined on \( \Sigma \).

For our further calculations it is convenient to split the field equations into their components orthogonal and tangential to the wall surface \( \Sigma \) [10, 15, 19, 29, 32, 41].

For this purpose the interval, which in \( y^\mu \) chart has the form

\[ ds^2 = g_{\mu\nu}dy^\mu dy^\nu \]

(\( g_{\mu\nu} \) is the metric tensor, which determines the geometry in \( V \) region), can be written in Gaussian normal coordinates in the form

\[ ds^2 = -dn^2 + \gamma_{ij}(\xi^k, n)d\xi^id\xi^j. \]  

(9)

From (6) and (7) it is clear, that \( n = 0 \) is the equation of hypersurface \( \Sigma \) and thus the interval

\[ ds^2 = \gamma_{ij}(\xi^k)d\xi^id\xi^j \]

determines the 3-geometry on \( \Sigma \).

Any vector and tensor naturally is splitted into its components orthogonal and tangential to \( \Sigma \):

\[ A^\nu = A^\mu N^\nu + A^i e^i_\nu, \]

\[ Q^{\mu\nu} = Q^{mn}N^\mu N^\nu + Q^{mi}e^i_\mu N^\nu + Q^{jn}N^\mu e^j_\nu + Q^{ij}e^\mu_\nu e^\nu_i \]

Using the Gaussian coordinates the Einstein equations (3) also can be decomposed into scalar, 3-vector and 3-tensor parts in respect to the coordinate transformations on Hypersurface \( \Sigma \):

\[ G^m_n = -\frac{1}{2} \cdot \left( (3^R + (K^i_i)^2 - K^l_i K^l_i) \right) = 8\pi G \cdot T^m_n; \]

\[ G^m_i = D^m_nK^l_i - D_iK^l_i = 8\pi G \cdot T^m_i; \]

\[ G^i_j = (3^G_j) - N^\mu D_\mu(K^j_i - \delta^j_i K^l_i) + K^l_i K^j_i - \frac{1}{2} \cdot \delta^j_i \left( (K^l_j)^2 + K^l_m K^l_i \right) = 8\pi G \cdot T^i_j. \]  

(10)

Here the \( D_\mu \) denotes covariant differentiation with respect of the connection in \( V \), while \( D_i \) denotes 3-dimensional covariant differentiation with respect to the connection on \( \Sigma \) , \((3^R)\) is three dimensional scalar curvature, \( (3^G) \) is three dimensional...
Einstein tensor and $K^i_j$ is the extrinsic curvature tensor of the hypersurface $\Sigma$, which is defined in the following way:

$$K_{ij} = -\epsilon^i_{\mu} \epsilon^j_{\nu} D_{\nu} N_{\mu}. \tag{11}$$

In the equations (10) we have used the well-known Gauss-Kodazzi equations [10, 15, 19, 29, 32, 41].

Note, that in the Gaussian coordinates (9) the tensor of extrinsic curvature (11) acquires the simple form:

$$K_{ij} = -\Gamma^n_{ij} = -\frac{1}{2} \cdot \partial_n \gamma_{ij}. \tag{12}$$

Since the 3-geometry on the surface $\Sigma$ is, by assumption, well defined, the components of Christoffel symbols (in the intrinsic coordinates $\xi^i$), not containing indices $n$ are regular. Components with two or three indices $n$ are equal to zero, while with one $n$ — are discontinuous and have the step-function behavior when crossing $\Sigma$. However $g_{\mu\nu}$ is assumed to be regular on $\Sigma$, and $g^+_{\mu\nu}$ and $g^-_{\mu\nu}$ have to match continuously on the shell. Three-curvature tensor $(^3)R_{ij}$ of hypersurface $\Sigma$ also does not contain singularities and is expressed in terms of 3-metric tensor $\gamma_{ij}$ in the usual way just as $R_{\mu\nu}$ is expressed by $g_{\mu\nu}$.

Integrating the $(ij)$ component of equation (10) in the normal direction by the proper distance $dn$ through $\Sigma$ we can get the so-called Lanczos equation (see [10, 15, 19, 29, 32, 41]):

$$[K^i_j] - \delta^i_j [K^i_i] = 8\pi G \cdot S^i_j, \tag{13}$$

where

$$[K^i_j] \equiv \lim_{\epsilon \to 0} \left( K^i_j(n = +\epsilon) - K^i_j(n = -\epsilon) \right)$$

is the discontinuity of the outer curvature tensor and

$$S^i_j = \lim_{\epsilon \to 0} \int_{-\epsilon}^{+\epsilon} T^i_j dn$$

is the intrinsic surface energy tensor on the $\Sigma$.

Noting (12), writing the $(nn)$ and $(ni)$ components of Einstein equations in the regions $V^+$ and $V^-$ and subtracting the corresponding equations for the $V^+$ region from those for the $V^-$ region one yields (using (13)):

$$D_j S^i_j + [T^n_i] = 0$$

{\{K^i_j\}} S^i_j + [T^n_i] = 0, \tag{14}$$

where \{\{K^i_j\}\} = \frac{1}{2} \cdot \lim_{\epsilon \to 0} \left( K^i_j(n = +\epsilon) + K^i_j(n = -\epsilon) \right).

Now to describe completely the gravitational behavior and dynamics of bubbles one needs to investigate the outer curvature and energy-momentum tensors of these objects, insert these quantities into motion equations (13) and (14) and solve them.
4 The surface stress-energy tensor.

We shall restrict ourselves to a pure vacuum case, i.e. when there are no particles at the either sides off the shell.

An observer, which moves with an element of the shell finds that the momentum of the matter all the time lies on the surface of the shell i.e.

\[ T_{\mu\nu} \cdot N^\mu = 0 \]

on \( \Sigma \) and

\[ T_{\mu\nu} = 0 \]

outside \( \Sigma \). Thus in the intrinsic coordinates

\[ T^n_n = T^i_i = 0 \] (15)

and the surface energy-momentum tensor for an observer on \( \Sigma \) is represented by the tensor \( S_{ij} \).

It is easy to see that with the condition (15) the motion equations (14) are satisfied automatically (they become identities) and one is left only with Lanczos equations (13).

Now let us suppose, that the surface energy-momentum tensor has the structure of an ideal fluid

\[ S^{ij} = (\sigma + p)u^iu^j - p\gamma^{ij}, \] (16)

where \( u^i \) is the 3-velocity of an element of hypersurface \( \Sigma \), while \( \sigma \) and \( p \) are the surface energy density (energy per unit area) and surface tension respectively.

The energy-momentum conservation in the intrinsic coordinates implies that equation

\[ D_i S^{ij} = 0 \] (17)

expresses the energy-momentum balance of matter on the shell. For tensor (16) this conservation equation becomes

\[ u^i \cdot D_i \left[ (\sigma + p)u^i + (\sigma + p)u^iD_iu^j - \gamma^{ij} \cdot D_ip \right] = 0. \] (18)

Multiplying (18) by \( u_j \) we yield

\[ D_i \left[ (\sigma + p)u^i \right] - u^i \cdot D_i p = 0. \] (19)

Feeding the last relation back into (18) we obtain

\[ (\gamma^{ij} - u^iu^j) \cdot D_i p - (\sigma + p)u^iD_iu^j = 0. \] (20)

The conservation equations (19) and (20) are easily solved in two cases.

(a). For the dust wall

\[ p = 0 \]

and we have
\[ D_i(\sigma u^i) = 0, \]
that states that the total amount of dust is conserved.

(b). For the domain wall

\[ p = -\sigma \]
and from (19) it follows immediately that

\[ \sigma = \text{const}. \]

This two examples are considered in section 7 in spherically symmetrical case.

5 The geometry of spherical vacuum shells.

Here we consider the bubbles of spherical form. For the spherical shells the Lanczos equations (13) (the only nontrivial equation of motion for the vacuum case) takes the form (see [10]):

\[
\begin{align*}
[K_2^2] &= 4\pi G S_0^0, \\
[K_0^0] + [K_2^2] &= 8\pi G S_2^2.
\end{align*}
\]

(21)

For the spherical shell the most convenient choice for the outer region coordinates \( y^\mu \) is the ordinary spherical coordinate system. The symmetry features of the problem allows us to choose the coordinates \( \vartheta \) and \( \varphi \) to be continuous across \( \Sigma \), i.e.

\[
\vartheta^+ = \vartheta^- = \vartheta;
\]

\[
\varphi^+ = \varphi^- = \varphi.
\]

The metric off the shell in the \( V^\pm \) regions must be the solution of spherically symmetrical Einstein equations. According to Birkhoff’s theorem such metric has the form [36, 41, 45]

\[
(ds^\pm)^2 = f^\pm \cdot (dt^\pm)^2 - \frac{1}{f^\pm} \cdot (dr^\pm)^2 - (r^\pm)^2 \cdot d\Omega^2,
\]

(22)

where

\[ d\Omega^2 = d\vartheta^2 + \sin^2 \vartheta \cdot d\varphi^2. \]

For the intrinsic coordinates \( \xi^i \) of the shell we shall use the proper time \( \tau \) and the spherical angles \( \vartheta, \varphi \). Then the metric on \((1+2)\)-surface \( \Sigma \), induced both by exterior \((ds^+)\) and interior \((ds^-)\) metrics (22), can be expressed as

\[
ds^2 = d\tau^2 - R^2(\tau) d\Omega^2,
\]

(23)

where \( R(\tau) \) is the shell radius.

Generally speaking, the time and radial coordinates are not continuous on the shell, but some restrictions on these coordinates is obtained from the junction of metrics (22), (23):
\[(ds^+)^2 = (ds^-)^2 = ds^2_{\Sigma}. \] (24)

The radius of the shell \(R(\tau)\) can be described in the coordinate invariant way, so the (24) gives only two conditions. Identification of the two-spheres \((r,t = \text{const})\) on \(\Sigma\) yields

\[r^+ = r^- = R(\tau)_{|\Sigma}, \] (25)

while the time coordinate can be discontinuous on the shell: \((t^+ \neq t^-)_{|\Sigma}\). Comparison of timelike lines \((\varphi, \vartheta = \text{const})\) gives

\[d\tau^2 = f^+ \cdot (dt^+)^2 - \frac{1}{f^+} \cdot dR^2 = f^- \cdot (dt^-)^2 - \frac{1}{f^-} \cdot dR^2. \] (26)

From these relations it is easy to find that

\[\left( f^\pm \dot{t}^\pm \right)^2 = f^\pm + \dot{R}^2, \]

\[\frac{1}{f^\pm} \cdot \left( \frac{dR}{dt^\pm} \right)^2 = f^\pm - \frac{1}{(\dot{t}^\pm)^2} = \frac{f^\pm \cdot \dot{R}^2}{f^\pm + \dot{R}^2}, \] (27)

where the overdot denotes the derivation with respect of proper time \(\tau\). Thus, after the procedure of junction (24), there remains only one unknown function \(R(\tau)\) which must obey the Einstein equations on the shell. It means that from the two equations of motion (21) only one is independent. We shall choose and solve the first one (then the second one will be satisfied automatically). Now we have to find the outer curvature tensor.

It is easy to see, that equation (23) is the equation of motion of the shell (compare with (13)):

\[F = r - R(\tau) = 0. \]

Noting, that

\[dF/dt = -dR/dt; \]
\[dF/dr = 1\]

and using equations (22),(27) we can compute the outer normal to \(\Sigma\) (see (17) and (18)):

\[N_0 = - \left( \frac{dR}{dt} \right) \cdot N_1, \]
\[N_1 = \varepsilon \cdot |\dot{t}|, \]
\[N_2 = N_3 = 0. \]

Here \(\varepsilon\) is a sign function, which depends on the direction of an outer normal \(N_\mu\), i.e. on the orientation of \((1+2)\)-surface \(\Sigma\):

\[\varepsilon = \text{sign} \left( \dot{t}, \frac{\partial r}{\partial q} \right), \] (28)
where \( q \) is any coordinate increasing along the direction from the bubble-center.

Using formulae (11) one can find the components of extrinsic curvature tensor:

\[
K^0_0 = K^2_2 + \frac{R}{\dot{R}} \cdot \dot{K}^2_2, \\
K^2_2 = -\varepsilon \frac{|f\dot{t}|}{R}.
\]

(29)

Note that the first equation of (27) admits both signs for \( f\dot{t} \). The sign of \( \varepsilon \) is equivalent to the sign of \( f\dot{t} \), thus instead of the second equation (29) one can write

\[
K^2_2 = -\frac{f\dot{t}}{R}.
\]

(30)

Using (27) one can yield from (29):

\[
(K_{\pm})^0_0 = -\varepsilon_{\pm} \frac{\dot{R} + \frac{i}{2} \cdot (f^{\pm})'}{\sqrt{f^{\pm} + \dot{R}^2}}, \\
(K_{\pm})^2_2 = -\varepsilon_{\pm} \frac{\sqrt{f^{\pm} + \dot{R}^2}}{R},
\]

(31)

where \( f' \) denotes the derivation by \( R \).

Thus we have shown that the dynamics of spherically symmetrical shell is completely described by the first equation (21)

\[
K_+ - K_- = 4\pi G \sigma,
\]

(32)

where \( \sigma = S^0_0 \) is the energy density of the shell and \( K \) is the only independent component \( K^2_2 \) of the extrinsic curvature tensor described by (30) or (31).

The fact that the first equation of (27) admits both signs for \( f\dot{t} \) leads to the new class of solutions of Einstein equations for bubbles with outer \( \varepsilon_+ = -1 \). This was missed in earlier works (for example [10]), where it was considered that for bubbles, lying above the Schwarzschild horizon, in outer space always \( \varepsilon_+ = 1 \). The correct sign must be chosen from boundary conditions. We shall discuss this subject below.

6 The sign ambiguity of extrinsic curvature.

Investigating the dynamics of vacuum spherical shells by means of the motion equation (32) one has to be careful when choosing the sign of \( \varepsilon \) and thus of \( f\dot{t} \). For the given the inner and the outer metrics \((ds^\pm)^2\) the \( \varepsilon_\pm \) determines the global geometry, i.e. how the inner geometry is stuck with the outer one. In some cases the junction is impossible.

As we have mentioned above, the sign of \( \varepsilon \) depends on the direction of an outer normal \( N_{\mu} \), i.e. on the orientation of (1+2)-surface \( \Sigma \):

\[
\varepsilon = \text{sign} \left( i \cdot \frac{\partial r}{\partial q} \right),
\]

(33)

where \( q \) is any coordinate increasing along the direction from the bubble-center.
Generally speaking, the sign of $\varepsilon$ may change at the shell motion, for example, when the shell passes the horizon. If the initial conditions allow the shell to collapse, then in the case $\varepsilon_+ > 0$ the shell forms the black hole, while in the case $\varepsilon_+ < 0$ — the warmhole.

In paper [10] the classification of possible geometries for different signs of $\varepsilon_+$ and $\varepsilon_-$ was done. However in this paper the dependence of the sign-function (33) on the mutual orientation of time coordinates on and off the shell, i.e. on the sign of $\dot{t}$ was overlooked. The bubble is the (1+2)-surface embedded in the 4-dimensional space-time $\mathcal{V}$ and not only the 2-sphere as it appears if we do not consider the direction of time coordinate. Thus we get the more general classification of bubble geometries compared to those presented in previous papers.

Generally speaking, we have four different cases of equation (32), depending on the signature of $\varepsilon_+$ or $(\dot{f}t)_\pm$ and, therefore, on the signature of $K_\pm$. The sign of $\varepsilon$ must be chosen from the motion equation (32) and the boundary conditions mentioned in the Introduction. One has to take into account the positiveness of the surface energy density $\sigma$ and the fact that there must not be the contradiction between the Newton’s limit of Einstein equations (32) and Tolman’s formula (3).

Let us discuss this problem and examine the signatures of $\varepsilon$ or $\dot{f}t$.

a) The signature of $f^\pm (\dot{t}_\pm > 0)$.

Let us assume for a while that $\dot{t}_\pm > 0$. The the signature of $f\dot{t}$ depends on the sign of $f$. The general form of $f$ for the spherically symmetrical source is

$$f = 1 - \frac{2Gm}{r} + \frac{Ge^2}{r^2} - G\Lambda r^2,$$  \hspace{1cm} (34)

where $m$ is the mass of the source, $e$ is its charge, $\Lambda = (8\pi/3) \cdot \rho$ and $\rho$ is the vacuum energy density. In the simplest case

$$e = \Lambda = 0$$

and

$$f_+ = 1 - \frac{2Gm}{R};$$

$$f_- = 1.$$  \hspace{1cm} (35)

The signature of $f_+$ is positive if the radius of the shell lies above the Schwarzschild horizon, i.e. when $R > 2m$ and is negative beyond it. So, $f\dot{t}$ changes its signature, when the radius of shell passes the horizon. The standart metric (34) has an unphysical singularity at the horizon caused by a poor choice of the coordinate system. To be more correct in correspondence between the signature of $K$ and the radius of shell, one can use another coordinate system, for example the isotropic coordinates (see [12]), which have no singularities at the horizon. In ref. [12] it was investigated (in Kruscal-Szekares coordinates) the case when the value $f\dot{t}$ becomes negative for the bubbles crossing the horizon, the systematical catalog of possible solutions of the equation of motion was obtained and was shown, that two signatures of $f$ correspond to the two different, but equally acceptable trajectories of shell.

b). The signature of $\dot{t}_\pm (f^\pm > 0)$.

Here we shall examine the mostly interesting case of macroscopic bubbles we shall assume, that the shell always lies outside the horizon and thus the unusual features
of the space geometry are avoided. In this case \( f^\pm > 0 \) and the signature of \( \dot{t} \) is determined by the sign of \( \dot{t} \).

Note that the example, when time flows in opposite directions is the case of Reissner-Nordstrom metric, when in the region between the upper and Cauchy radii the time coordinate changes its direction to the opposite. In this case (in difference to the ours) the metric covers the whole space-time \((r > 0)\) and such feature was easy to notice. The problem of sign ambiguity entered due to taking a square root in equation (27). Similar situation is in Dirac theory when one has states with negative energy that correspond to antiparticles.

We shall choose the positive direction for the 2-spheres in the way that the radii increase in the direction of the outer normal. Then orientation of \( \Sigma \) is determined by the direction of time flow on the shell and

\[ \varepsilon = \text{sign} \dot{t} \]  \hfill (36)

It is clear, that the signature of \( \dot{t}^\pm \) depends on the direction of time flowing in \( V^\pm \) regions and on the shell. It is natural to assume, that in \( V^\pm \) regions time flows in the same direction i.e.

\[ \text{sign}(t^+) = \text{sign}(t^-) \]  \hfill (37)

while the direction of intrinsic time \( \tau \) has to be found from the equation of motion. From (33) and (37) it is clear that \( \varepsilon^+ = \varepsilon^- \equiv \varepsilon = 1 \) when \( t_\pm \) and \( \tau \) flow in the same direction and \( \varepsilon = -1 \) in the opposite case. Assumption (37) leaves only two cases of equation (32):

\[ \sqrt{f_+ + \dot{R}^2} - \sqrt{f_- + \dot{R}^2} = \pm 4\pi\sigma GR \]  \hfill (38)

which depend on the values of \( f^\pm \).

If \( f_- > f_+ \), than \(|K_+| < |K_-|\) and we have to take the lower sign \((-\cdot)\) in (38), because the energy density of the shell is positive (see (32)). According to (30) and (37) this means that \( \dot{t}_\pm > 0 \) and thus the time off and on the shell flows in the same direction.

Similarly if \( f_- < f_+ \) we have \( \dot{t}_\pm < 0 \), that means that the time on the shell flows in the opposite direction to the time in \( V^\pm \) regions.

Note, that for the simplest case (33) the signature of \( \dot{t} \) is connected with the sign of constant \( m \), which is the integration constant of solution of Einstein equations in \( V^+ \) region. This constant is fixed by matching of \( f_+ \) to the solution of Einstein equations on the shell in the same coordinates. In the Newton approximation

\[ f_+ = 1 - 2G \cdot \int \left( T^0_0 - \frac{1}{2} \cdot T \right) \cdot \frac{1}{R} \cdot dV. \]

The value and the signature of \( m \) is determined by this formula and it can be positive as well as negative dependently on boundary conditions. In the next section we shall see that the correct choice of sign in (38) cancels the contradictions between Tolman’s formula and Newton’s limit of (38).

### 7 Different kinds of shells

Here we shall consider two cases of macroscopic bubbles and show how one has to choose the different signs of \( \dot{t} \).
a. The dust walls in vacuum.

As we have mentioned above, for the shell of dust

\[ p = 0 \] (39)

and the surface energy tensor of layer \( \mathbf{10} \) is equal to

\[ S^{ij} = \sigma u^i u^j. \]

Conservation law \( \mathbf{19} \) now has the form

\[ D_i (\sigma u^i) = 0 \] (40)

stating, that the total amount of dust is conserved. For the spherical shells in the intrinsic coordinates \( \xi^i \) with the interval \( \mathbf{23} \) we have

\[ u^0 = 1; \]
\[ u^1 = u^2 = 0. \]

Therefore

\[ D_i u^i = \Gamma^i_{0i} = \frac{2 \dot{R}}{R}. \]

So that equation \( \mathbf{10} \) reduces simply to

\[ \dot{\sigma} + 2\sigma \frac{\dot{R}}{R} = 0. \]

This equation is easily solved and yields

\[ \sigma = \frac{\text{const}}{R^2}. \] (41)

To fix the constant in \( \mathbf{41} \) one can use the Tolman’s formula for the proper mass of the shell in the state of rest (see \( \mathbf{2} \)). For the ideal fluid

\[ M = 8\pi \cdot \int \left[ \sigma - \frac{1}{2} \cdot (\sigma - 2p) \right] \cdot \delta(r - R) \cdot r^2 dr = 4\pi R^2 \sigma = 4\pi \cdot \text{const}. \] (42)

Using \( \mathbf{39} \) and \( \mathbf{41} \) for the \( \text{const} \) from \( \mathbf{42} \) we obtain

\[ \text{const} = \frac{M}{4\pi}. \]

In the simplest case of uncharged shells of dust in vacuum when the metric is given by the formulae \( \mathbf{33} \) the motion equation \( \mathbf{38} \) takes the form

\[ \sqrt{1 - 2Gm/R + \dot{R}^2} - \sqrt{1 + \dot{R}^2} = \pm 4\pi \sigma GR = \pm G \frac{M}{R}. \] (43)

We can rewrite \( \mathbf{43} \) in the form

\[ m = \mp M \cdot \sqrt{1 + \dot{R}^2 - G \frac{M^2}{2R}}. \] (44)
The term \( -GM^2/2R \) represents the gravitational interaction energy according to the Newton’s law, and \( M \cdot \sqrt{1 + \dot{R}^2} \) represents the internal mass of the shell (the root is the analog of the Lorentz factor and is equal to 1 in equilibrium, when \( R = 0 \)).

From this relation it is easy to find that one has to choose in (43) the lower sign ”−” corresponding to \( f \dot{t} > 0 \) (\( \varepsilon = 1 \)). Certainly, in this case \( m > 0 \), \( f_+ < f_- \) and the condition of surface energy density positiveness is satisfied. Besides, it is easy to see that with this choice of sign the static Newton’s limit of (44) coincides with Tolman’s formula (42) (though Tolman’s formula is valid only for static states, which, as we’ll see, do not occur in this simplest model, the term corresponding to gravitational energy of the shell must enter the expression for the shell mass with the correct sign. Then in static Newton’s limit \( (\dot{R} = 0; \ G = 0) \) the mass of the shell will be described by Tolman’s formula).

Note that if one would choose the upper sign ”+” in (43) then he should receive the contradiction between the static Newtons limit of (44) and Tolman’s formula (42).

Now let us show that in this simplest model there is no equilibrium state for the dust shell. The conditions

\[
\dot{R} = 0, \quad \ddot{R} = 0
\]

must be satisfied for the spherical shell to be in equilibrium state. The first of them leads from (44) to the equation

\[
m = M - GM^2/2R.
\]

Taking \( d/d\tau \) of (44) we obtain

\[
\ddot{R} = \frac{1}{M^2} \cdot \left( m + GM^2/2R \right) \cdot \frac{dm}{dR},
\]

so the second condition (43) gives the stipulation

\[
\frac{dm}{dR} = 0.
\]

Using (44) and (46) one can see, that the radius of stability of the shell \( R_{\text{stab}} \rightarrow \infty \), i.e. there is no equilibrium configuration for the dust wall in vacuum.

b. Domain wall in vacuum.

In the case of domain wall one has the picture opposite to the previous case. For the domain wall

\[
p = \sigma
\]

and the surface energy tensor of layer (10) is

\[
S^{ij} = \sigma \gamma^{ij}.
\]

From the conservation law (19)

\[
D_i \sigma = 0
\]

we obtain
\[ \sigma = \text{const.} \]

From the Tolman’s formula (2) we have:

\[ M = 8\pi \cdot \int \left[ \sigma - \frac{1}{2} \cdot (\sigma - 2\rho) \right] \cdot \delta(r - R) \cdot r^2 dr = -4\pi \sigma R^2. \]  

(47)

One can see that the proper mass of layer is negative, if the energy density of domain wall \( \sigma \) is positive.

Considering again the simplest case (35) one obtains the motion equation

\[ \sqrt{1 - \frac{2Gm}{R}} + \dot{R}^2 - \sqrt{1 + \dot{R}^2} = \pm 4\pi \sigma GR = \mp \frac{GM}{R}, \]

(48)

where \( M \) is described by (47).

Again let us rewrite (48) in the form

\[ m = \pm M \cdot \sqrt{1 + \dot{R}^2} - \frac{GM^2}{2R}. \]

(49)

This formula as well as (44) corresponds to the energy balance equation and must contain the Newton’s gravitational energy of the shell.

In opposite to dust-wall case, now one has to choose in (48) the upper sign ”+” corresponding to \( f \dot{t} < 0 \) (\( \varepsilon = -1 \)). Certainly, in this case \( m < 0 \), \( f_+ > 1 \) and the surface energy density positiveness condition is satisfied. Besides, for this choice of sign the static Newton’s limit of (49) coincides with Tolman’s formula (17).

We emphasize once more that though Tolman’s formula is valid only for static states, the term corresponding to gravitational energy of bubble must enter the expression for the bubble mass with the correct sign. Then in static Newton’ limit the mass of the bubble will be described by Tolman’s formula.

One can see, that when one chooses the upper sign in (48) then

\[ m = - \left( 4\pi \sigma R^2 \sqrt{1 + \dot{R}^2} + 8\pi^2 \sigma^2 GR^3 \right). \]

In this expression the mass is negative and we can believe, that distant observer will be repelled by the domain wall.

If one considers the equation (48) with different sign in the right side (see [10, 28, 39]), one will obtain the positive value for mass

\[ m = \kappa R^2 \sqrt{1 + \dot{R}^2} - \frac{G\kappa^2 R^3}{2}. \]

This expression contains the rest mass (17) with the wrong sign (the first term with \( \dot{R} = 0 \)) and contradicts with Tolman’s formula (17). Thus Tolman’s formula helps us to choose the correct sign in a motion equation (18).

Thus the correct choice of the sign in the motion equation (18) leads to the new solution of Einstein equations for bubbles (Schwarzschild solution with negative mass) which shows that spherical domain walls are repulsive. This fact requires a further investigation of bubble-dynamics and their creation in cosmological models.
8 Schwarzschild space with negative mass.

Let us investigate the properties of Schwarzschild metric

\[ ds^2 = \left( 1 - \frac{b}{r} \right) \cdot dt^2 - \frac{1}{1 - b/r} \cdot dr^2 - r^2 \cdot (d\theta^2 + \sin^2 \theta \cdot d\varphi^2) \]  

(50)

in the case, when constant \( b \) (which is related to the active gravitational mass of the source), can be negative.

First of all, for such spaces \( g_{00} > 1 \) and the velocity of light exceeds its velocity measured in Minkowski space. However this fact does not cause appearance of takhions.

For the metric (50) the nonzero components of curvature tensor are

\[ R_{trtr} = \frac{b}{r^3}, \]
\[ R_{t\theta\theta} = \frac{R_{t\varphi t\varphi}}{\sin^2 \theta} = -\frac{b(r - b)}{2r^2}, \]
\[ R_{r\theta r\theta} = \frac{R_{r\varphi r\varphi}}{\sin^2 \theta} = -\frac{b}{2(r - b)}, \]
\[ R_{\theta\varphi \theta \varphi} = -br \cdot \sin^2 \theta. \]

In this case the complex invariants of gravitational field have the form

\[ I_1 = \frac{1}{48} \cdot R_{\alpha\beta\gamma\delta} \cdot \left( R_{\alpha\beta\gamma\delta} - \frac{i}{2} \cdot \varepsilon_{\alpha\beta \mu\nu} \cdot R^{\gamma\delta}_{\mu\nu} \right) = \left( \frac{b}{2r^3} \right)^2, \]
\[ I_2 = \frac{1}{96} \cdot R_{\alpha\beta\mu\nu} R^{\mu\nu\rho\sigma} \cdot \left( R_{\sigma\rho} - \frac{i}{2} \cdot \varepsilon_{\sigma \rho \lambda \kappa} \cdot R^{\lambda \kappa \alpha \beta} \right) = -\left( \frac{b}{2r^3} \right)^3. \]

One can see, that for the negative \( b \) the second invariant changes its sign. The result of this fact is that the 3-space \( t = \text{const} \) Gauss curvature for the plains, normal to radius also changes its sign:

\[ k = \frac{P_{\theta\varphi \theta \varphi}}{\gamma_{\theta \theta} \cdot \gamma_{\varphi \varphi}} = -P_r = \frac{b}{r^3}. \]

(51)

Here \( P_{\theta\varphi \theta \varphi}, P_r, \gamma_{\theta \theta}, \gamma_{\varphi \varphi} \) are 3-curvatures and 3-metric tensors of space (50) in the case \( t = \text{const} \). Equation (51) means, that for the different signs of \( b \) this subspaces belong to the different types of surfaces. To show this let us embed metric (50) into Euclidean space with more then 4 dimensions.

An embedding of the linear element (50) in 5 dimensions is impossible (see [20, 42]). Embedding of (50) with positive \( b \) in 6 dimensions with signature 2+4

\[ ds^2 = dz_1^2 + dz_2^2 - dz_3^2 - dz_4^2 - dz_5^2 - dz_6^2 \]

is given by
\[ z_1 = \left(1 - \frac{b}{r}\right) \cdot \cos t, \]
\[ z_2 = \left(1 - \frac{b}{r}\right) \cdot \sin t, \]
\[ z_3 = \int \left( \frac{b(b + 4r^3)}{4r^3(r - b)} \right)^{1/2} \cdot dr, \]
\[ z_4 = r \cdot \cos \theta, \]
\[ z_5 = r \cdot \sin \theta \cdot \cos \varphi, \]
\[ z_6 = r \cdot \sin \theta \cdot \sin \varphi. \]

(52)

It is possible to eliminate the coordinates

\[ z_1^2 + z_2^2 = 1 - \frac{b}{r}, \]
\[ z_3 = \int \left( \frac{b(b + 4r^3)}{4r^3(r - b)} \right)^{1/2} \cdot dr, \]
\[ z_4^2 + z_5^2 + z_6^2 = r^2. \]

We note, that this surface in \( z_1 z_2 \) plane is the 1-sphere, in the \( z_4 z_5 z_6 \) is the 2-sphere and \( z_3 \) is space-like. Time-like coordinates \( z_1 \) and \( z_2 \) are periodic functions of \( t \) so that embedding (52) identifies distinct points of the original manifold. This suggests replacing the trigonometrical functions by the hyperbolic functions and embedding of (50) for the positive \( b \) is possible for the signature 1+5:

\[ ds^2 = dz_1^2 - dz_2^2 - dz_3^2 - dz_4^2 - dz_5^2 - dz_6^2, \]

where

\[ z_1 = \left(1 - \frac{b}{r}\right) \cdot \sinh t, \]
\[ z_2 = \left(1 - \frac{b}{r}\right) \cdot \cosh t, \]
\[ z_3 = \int \left( \frac{b(4r^3 - b)}{4r^3(r - b)} \right)^{1/2} \cdot dr, \]
\[ z_4 = r \cdot \cos \theta, \]
\[ z_5 = r \cdot \sin \theta \cdot \cos \varphi, \]
\[ z_6 = r \cdot \sin \theta \cdot \sin \varphi. \]

(53)

We note that in \( z_1 z_2 \) plane this surface now is hyperbola

\[ z_2^2 - z_1^2 = \left(1 - \frac{b}{r}\right). \]

For the negative mass

\[ b < 0 \]
we can see from (52) and (53), that coordinate $z_3$ became complex and embedding (52) now takes place in the space with signature 3+3

$$ds^2 = dz_1^2 + dz_2^2 + dz_3^2 - dz_4^2 - dz_5^2 - dz_6^2$$

where

$$z_1 = \left(1 - \frac{b}{r}\right) \cdot \cos t,$$

$$z_2 = \left(1 - \frac{b}{r}\right) \cdot \sin t,$$

$$z_3 = \int \left(\frac{b(b - 4r^3)}{4r^3(r - b)}\right)^{1/2} \cdot dr,$$

$$z_4 = r \cdot \cos \theta,$$

$$z_5 = r \cdot \sin \theta \cdot \cos \varphi,$$

$$z_6 = r \cdot \sin \theta \cdot \sin \varphi.$$  

Now the coordinate $z_3$ became time-like. Embedding (53) is possible for the signature 2+4

$$ds^2 = -dz_1^2 + dz_2^2 + dz_3^2 - dz_4^2 - dz_5^2 - dz_6^2$$

where

$$z_1 = \left(1 - \frac{b}{r}\right) \cdot \sinh t,$$

$$z_2 = \left(1 - \frac{b}{r}\right) \cdot \cosh t,$$

$$z_3 = \int \left(\frac{b(b - 4r^3)}{4r^3(r - b)}\right)^{1/2} \cdot dr,$$

$$z_4 = r \cdot \cos \theta,$$

$$z_5 = r \cdot \sin \theta \cdot \cos \varphi,$$

$$z_6 = r \cdot \sin \theta \cdot \sin \varphi.$$  

In this case the coordinate $z_1$ is space-like and we can not identify the surface in $z_1 z_2 z_3$ space. $z_4 z_5 z_6$ surface is the 2-sphere $r = R$ in all the cases and is identified in our work with the surface of domain bubble.

To conclue this section we can say that the space (50) with $b < 0$ can be embedded into a 6-dimensional space with signature (2+4) or (3+3), while the usual Schwarzschild space ($b > 0$) — into the space with signature (1+5) or (2+4) respectively.
9 The model.

After we have formulated our idea how to avoid the disagreements mentioned at the beginning of this paper let us investigate a more general case of a spherically symmetrical charged bubble in vacuum, when the metric outside the bubble is

\[ f_+ = 1 - \frac{2Gm}{r} + \frac{G\epsilon^2}{r^2} - G\Lambda_+ r^2, \]

while inside —

\[ f_- = 1 - G\Lambda_- r^2, \]

where \( \Lambda_\pm \equiv \frac{8\pi}{3} \cdot \rho, \rho \) being the vacuum energy density in \( V^\pm \) regions, and \( \epsilon \) is the charge on the shell.

Now the motion equation (32) takes the form

\[
\sqrt{\dot{R}^2 + 1 - \Lambda_+ GR^2 - \frac{2Gm}{R} + \frac{G\epsilon^2}{R} - \sqrt{\dot{R}^2 + 1 - \Lambda_- GR^2}} = \pm G\kappa R, \tag{54}
\]

where

\[ \kappa \equiv 4\pi\sigma. \]

Finding \( m \) from this equation we yield:

\[
m = \frac{\Lambda_- - \Lambda_+}{2} \cdot R^3 - \frac{G\kappa^2}{2} \cdot R^3 + \frac{\epsilon^2}{2R} \mp \kappa R^2 \cdot \sqrt{\dot{R}^2 + 1 - \Lambda_- GR^2} \tag{55}
\]

It is easy to understand the meaning of the terms in (55). The first term is the volume energy of the bubble (a difference between the old and new vacuum energy densities). The second term represents an energy of gravitational self-interaction of the shell (the surface-surface binding energy). The third term is the electrostatic energy lying in the three-space outside the bubble. The last term contains the kinetic energy of the shell and surface-volume binding energy. As we have mentioned above, we have to choose the upper sign in equations (54) and (55) to avoid the disagreements with Tolman’s formula (47).

To examine dynamics of the bubble let us rewrite the equation (55) in the following way:

\[
\dot{R}^2 - \left[ \frac{1}{\kappa^2} \cdot \left( -\frac{m}{R^2} - \frac{a}{2} \cdot R + \frac{\epsilon^2}{2R^3} \right)^2 + \Lambda_- GR^2 \right] = -1, \tag{56}
\]

where \( a \equiv \Lambda_+ - \Lambda_- + G\kappa^2 \). It is worth to note that in this equation the sign ambiguity disappears due to squaring. Introducing new dimensionless variables

\[
z \equiv \frac{R}{(-2m)^{1/3}} \cdot (a^2 + 4\kappa^2 \Lambda_- G)^{1/6},
\tau' \equiv \frac{\tau}{2\kappa} \cdot (a^2 + 4\kappa^2 \Lambda_- G)^{1/2} \tag{57}
\]

and dimensionless parameters...
\[ A \equiv a \cdot (a^2 + 4\kappa^2\Lambda G)^{-1/2}, \]
\[ E \equiv -\frac{4\kappa^2}{(-2m)^{2/3}} \cdot (a^2 + 4\kappa^2\Lambda G)^{-2/3}, \]
\[ Q^2 \equiv \frac{e^2}{(-2m)^{4/3}} \cdot (a^2 + 4\kappa^2\Lambda G)^{1/6}, \]

we can represent the motion equation (56) as
\[ \left( \frac{dz}{d\tau} \right)^2 + U(z) = E, \]
which is identical to that of the point-like particle with energy \( E \), moving in one dimension under the influence of the potential
\[ U(z) = -\left[ z^2 - \frac{2A}{z} \cdot \left( 1 + \frac{Q^2}{z} \right) \right] \cdot \left( 1 + \frac{Q^2}{z} \right)^2 ] \quad (58) \]
For real trajectories \( U \) must be negative since \( E < 0 \). Such potentials (but for the case of uncharged shells (\( Q = 0 \)) with \( m > 0 \) were discussed in [4, 12].

In the equilibrium state \( \dot{z}_{z=\dot{z}_0} = 0 \), where \( z_0 \) is the equilibrium point, \( U(z_0) = E \) and one can find the critical mass of the bubble:
\[ m_0 = -\frac{4\kappa^3}{(a^2 + 4\kappa^2G\Lambda_\perp) \cdot U_0^{3/2}} \quad (59) \]
where \( U = |U(z_0)| > 0 \). From (57) for the equilibrium radius we have
\[ R_0 = \frac{2\kappa z_0}{U_0^{1/2} \cdot (a^2 + 4\kappa^2G\Lambda_\perp)^{1/2}}. \quad (60) \]
From this formula it is easy to notice that for the de Sitter spaces (\( \Lambda > 0 \)) the radius is maximal, when \( a \) is minimal. So to get the macroscopic bubbles it is worth to set \( \Lambda_+ = \Lambda_- = \Lambda \). In this case the domain wall separates the phases with the same vacuum energy. Then \( a \sim G^2\kappa^4 \) which is negligible compared to the second term in brackets. The charge enters the formulae (59), (60) only through \( U_0 \) and has very small influence on the values \( m_0 \) and \( R_0 \). So we can set \( Q = 0 \). With this assumptions one finds that \( U \) reaches its maximum \( U_0 \sim 1 \) at the point \( z_0 \sim 1 \). This implies that
\[ m_0 \sim -\frac{\kappa}{G\Lambda}, R_0 \sim \frac{1}{\sqrt{G\Lambda}}. \]

Considering a scenario for domain wall production in models with spontaneous breaking of some gauge symmetry group, it is easy to see that \( \Lambda \sim \alpha^{-1} M_G^4, \kappa \sim \alpha^{-1} M_G^3 \) and the thickness of the wall \( d \sim \frac{1}{M_G} \), where \( M_G \) is the symmetry breaking scale and \( \alpha \sim 10^{-2} \) is the coupling constant [10, 48, 49].

For the different scales of particle physics we yield:

a). The Electro-Weak scale: \( M_G \sim 10^2 GeV \). Then \( \Lambda \sim 10^{10} GeV^4, \kappa \sim 10^8 GeV^3 \) and \( R_0 \sim 10^{14} GeV^{-1}, m_0 \sim -10^{36} GeV \). The radius of such bubble is about 1 cm, and its negative mass is about million tons.
b). The scale of family symmetry \[16\]: \(M_G \sim 10^4 \div 10^{10} GeV\). Then \(\Lambda \sim 10^{18} \div 10^{42} GeV^4\), \(\kappa \sim 10^{14} \div 10^{32} GeV^3\) and \(R_0 \sim 10^{10} \div 10^{-2} GeV^{-1} \sim 10^{-4} \div 10^{-16} cm\), \(m_0 \sim -(10^{24} \div 10^{28}) GeV \sim -(10^{10} \div 10^4) g\).

\[\]

c). The scale of Grand Unification: \(M_G \sim 10^{15} GeV\). Then \(\Lambda \sim 10^{62} GeV^4\), \(\kappa \sim 10^{47} GeV^3\) and thus \(R_0 \sim 10^{-12} GeV^{-1}\), \(m_0 \sim -10^{23} GeV \sim 0,1 g\). This is a very small radius and there arises the question of the validity of the thin-wall approximation \(d \sim 10^{-15} GeV^{-1}\).

We see that in all the cases the mass of the bubble is negative and it exhibits a strong gravitational repulsion.

10 The problem of stability.

Unfortunately this equilibrium state seems to be unstable. Potential \(58\) has the single maximum. In contrary to the case with positive \(m\) \([10]\), the charge does not stabilize the bubble. Perhaps one could find the stable equilibrium states of bubble with negative mass in some nonvacuum models or in models with rotating bubble.

In \[23\] it is suggested that the inclusion of a nonvanishing angular momentum might stabilize the shell. The surface energy-momentum density tensor of a charged rotating shell is given by \[30, 38\]

\[
T^i_j = -\sigma u^i u_j + \delta^i_j \sigma, \quad i, j = 0, 2, 3
\]  

(61)

It consists of a mixture of two perfect fluids. The first term can be interpreted as "dust" particles with negative energy density. The second one represents a domain wall. The negative mass "dust" increases the repulsive character of the bubble and can keep it static. Exterior for of a spinning charged shell is the Kerr-Newman metric.

Our simple analysis based on spherical symmetrical equations is not valid in this case and model \([61]\) requires further investigations.

If the problem of stability will be solved there will be an interesting possibility of creation of the static singular shell of macroscopic size and with repulsive gravitational field. Radius and repulsive features of such objects depend on the scale of symmetry violation and can be varied slowly by changing the forces stabilizing the bubble.

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