Quantum-Statistical Computation

Giuseppe Castagnoli* and David Ritz Finkelstein†

October 31, 2018

Abstract

Systems of spin 1, such as triplet pairs of spin-1/2 fermions (like ortho-hydrogen nuclei) make useful three-terminal elements for quantum computation, and when interconnected by qubit equality relations are universal for quantum computation. This is an instance of quantum-statistical computation: some of the logical relations of the problem are satisfied identically in virtue of quantum statistics, which takes no time. We show heuristically that quantum-statistical ground-mode computation is substantially faster than pure ground-mode computation when the ground mode is reached by annealing.

1 Introduction

1.1 Premise

To-day quantum computation is mostly algorithmical: a stored program controls a time-varying Hamiltonian that sends a set of qubits – two-valued quantum variables – through a sequence of unitary transformations effecting steps in a computation. The quantum speed-up depends essentially on maintaining coherent parallel computation paths during the entire computation, until the final measurement.

The well-known fragility of such a coherence may make algorithmic quantum computation impractical.

Quantum ground-mode computation could be an interesting alternative. One defines a time-independent quantum Hamiltonian proportional to the amount of “frustration” (violation) of the logical relations of the problem to be solved. Then any ground mode solves the problem.

An incoherent superposition of ground modes also solves the problem. This should greatly reduce the problem of decoherence.

In quantum annealing computation, one form of quantum ground-mode computation, the quantum computer is brought to minimum energy by a suitable coupling with a heat bath of programmed temperature. In quantum adiabatic

*Information Technology Division, Elsag spa, 16154 Genova, Italy
†School of Physics, Georgia Institute of Technology, Atlanta, GA 30332, USA
computation (Farhi et al.), another form, the computer is prepared in a minimum of a simplified Hamiltonian, which is then adiabatically deformed into the problem Hamiltonian, leaving the network at minimum energy.

To be sure, quantum ground-mode computation is suspected of being mathematically and physically intractable. At any rate, its computer simulations are lengthy and seriously limit research on this method; and since the numbers of energy levels, energy traps (for annealing computation), and energy-level crossings or proximities (for adiabatic computation), can grow exponentially with problem size, ground-mode computation alone may not yield any speed-up.

We ameliorate this problem here. We implement part of the Boolean relations as quantum symmetries expressing particle indistinguishability, using a quantum-statistical three-terminal network element. Satisfying such relations does not take time. There is a quantum speed-up for annealing computation in the sense that the speed of relaxation of a hard problem becomes comparable to that of an easy problem, which algorithmically can be solved in polynomial time by successively eliminating variables. Implementing part of the Boolean relations through quantum symmetries due to particle indistinguishability should give a speed-up over the case where all relations are implemented through an energy function.

1.2 Outline

We express the problem to be solved in terms of the computationally-universal relations \( q_x + q_y + q_z = 1 \), called the (sum-1) triode, and \( q_x = q_y \), called the wire. Here the \( q \) are Boolean variables and + denotes arithmetical sum. Quantum-statistical computation implements the wires through Hamiltonian terms, and the triodes through quantum statistics (see also Castagnoli 1998, Castagnoli et al. 1998, Castagnoli & Monti 1999).

We associate each wire \( q_m = q_n \) with a Hamiltonian term \((q_m - q_n)^2\) whose two-fold degenerate ground modes satisfy the wire. We associate each triode with a spin pair that satisfies the relation identically in virtue of particle statistics, without time development. The associated Hamiltonian is 0. Satisfying relations implemented by statistics does not take time or energy.

We show this here for quantum-statistical annealing (qusa) computation, by means of a special representation of the computation process.

The effective Hamiltonian \( H = H_w + H_r(t) \) consists of a Hamiltonian \( H_w \) for the wires of the network, and a time-dependent heat-bath coupling \( H_r \) to relax the network to its zero point. There is no triode Hamiltonian.

To estimate the solution time, we introduce a certain non-symmetrized comparison Hamiltonian \( H' \) that also includes a coupling to a heat bath and yields the actual network Hamiltonian \( H \) when statistics – symmetrization – is imposed. \( H' \) describes the relaxation process of an easy network, where all the Boolean constraints implemented by statistics are removed. The easy network may be realized in principle by replacing each pair of identical-indistinguishable fermions forming a triode by a pair of distinguishable (e.g. non-identical) fermions. But when we continuously project the process governed by \( H' \) on
the Hilbert space satisfying the statistics, we recover the relaxation process of the actual network.

In this way we show that the solution time for the actual network is comparable with that of the easy network.

Although the interplay between relaxation and statistics proposed is a well-defined physical effect, the computation model based on it still dwells in the same abstract conceptual realm as other current literature on quantum ground-mode computation (see for example Farhi et al., among others). Our model demonstrates that a new form of quantum speed-up is possible in principle, leaving implementation problems for the future.

2 The Boolean problem

By a triode with qubits $q_x, q_y, q_z$ we mean the relation $q_x + q_y + q_z = 1$, where + denotes arithmetical sum. This is the negation of the POR (partial OR) relation used by Boole and algebraicized by C. S. Peirce [Finkelstein 1996]. It is a partial NOR (or partial Scheffer stroke) relation and can be written as $q_z = q_x \text{PNOR} q_y$.

We consider a Boolean triode network, one consisting only of $T$ triodes, $Q = 3T$ qubits, and $W$ wires between qubits.

The triode is satisfied by just the values

| $q_x$ | $q_y$ | $q_z$ |
|-------|-------|-------|
| 0     | 0     | 1     |
| 0     | 1     | 0     |
| 1     | 0     | 0     |

(1)

No value of $q_x$ is defined for $q_y = q_z = 1$; and similarly for $q_y$ and $q_z$. This triode is therefore not a gate, since it does not define an input-output function relating its variables. We may call this triode a partial gate, since it defines a partial function; in fact, three.

A toy triode network is

with six qubits $q_1, q_2, q_3, q_1', q_2', q_3'$, two triodes $q_1 + q_2 + q_3 = 1$, $q_1' + q_2' + q_3' = 1$ and three wires $q_1 = q_1'$, $q_2 = q_2'$, $q_3 = q_3'$.

The problem is to solve the network assuming that there is at least one solution. This problem has practically the same difficulty as checking whether the network is satisfiable. It is the "exact cover" problem addressed in quantum adiabatic computation (Farhi et al. 2001), and is NP-complete.
3 The network model

3.1 The qubit

The qubits used in quantum computation differ from the bits of classical computation in that a qubit $q$ has variables that do not commute with $q$, and modes that are quantum superpositions of the $q = 0$ and $q = 1$ modes. Qusa computation exploits the quantum nature of the qubit still further.

3.2 The sum-1 triode

We model the triode with a triplet proton pair identically fulfilling the triode relation $q_x + q_y + q_z = 1$ in virtue of statistics, as follows.

A quantum spin $1/2$ provides three anticommuting two-valued variables $\sigma_x, \sigma_y, \sigma_z$, each taking two values $\pm 1$, and subject to the relation $\sigma_x \sigma_y \sigma_z = 1$.

A hydrogen molecule has a triplet of ground modes and an excited singlet mode. Let $\frac{1}{2} \sigma_1, \frac{1}{2} \sigma_2$ be the respective spin vectors of protons 1 and 2 (in units with $\hbar = 1$). Then $s = \frac{1}{2}(\sigma_1 + \sigma_2)$ represents the total spin angular momentum.

Two independent spin vectors $\sigma_1$ and $\sigma_2$ define a total spin $s = (\sigma_1 + \sigma_2)/2$ with $s_z = \pm 1, 0$, and three commuting binary variables $q_x = 1 - s_x^2$, $q_y = 1 - s_y^2$, $q_z = 1 - s_z^2$ subject to the relation $q_x + q_y + q_z = 1 \mod 2$. These spins thus constitute a gate – implementing the weaker relation $q_x + q_y + q_z = 1 \mod 2$ – among the three qubits:

\[
\begin{array}{ccc}
q_x & q_y & q_z \\
0 & 0 & 1 \\
0 & 1 & 0 \\
1 & 0 & 0 \\
1 & 1 & 1 \\
\end{array}
\]  

(3)

Define the spin quantum number $S$ by

\[ s^2 = S(S + 1) \]  

(4)

as usual. The fourth line in this table is the singlet mode $S = 0$. This gate is the (non-universal) complemented-XOR gate. Since each variable is 1 (true) when and only when the other two are equal, it may also be called the EQUALS (or EQU) gate.

EQU is a functional relation. It may be solved for any of its qubits, say $q_x$:

\[ q_x = q_y + q_z + 1 \mod 2. \]  

(5)

Networks of triodes and wires are universal for Boolean computation; it is easy to construct NOT from two triodes and NOR from three triodes, with several wires.

Networks of EQU gates and wires are not universal. A Boolean network made of just EQU gates and wires is a system of modulo-2 arithmetical equations that is quickly solvable.
In particular there is always the solution where all qubits are 1 and all triodes are singlet.

When \( q_z = 0 \) the total spin is “up-or-down,” which is doubly degenerate. When \( q_z = 1 \) it is “not up-or-down,” which is also doubly degenerate. The qubits \( q_x \) and \( q_y \) have similar meanings relative to the \( x \) and \( y \) axes.

In the singlet mode, \( q_x = q_y = q_z = 1 \).

If we restrict the system to its triplet mode \( S = 1 \), the three commuting qubits \( (q_x, q_y, q_z) \) obey the PNOR relation

\[
q_x + q_y + q_z = 1
\]

by (4). Then “not up-or-down” becomes (approximately) “sideways” and is non-degenerate. "Up-or-down" remains doubly degenerate.

In the comparison network constructed later we drop the restriction to the triode relation. All triodes are replaced by EQU gates of truth table (3).

### 3.3 The error metric

We define an “error metric” measuring the distance of the network from a solution. From now on we use the following notation. We index the network wires with \( \omega = 1, \ldots, W \) and the triodes with \( \tau = 1, \ldots, T \). For each wire \( \omega \), with terminal qubits \( q(\omega, 0) \) and \( q(\omega, 1) \), we define

\[
\epsilon_\omega := [q(\omega, 0) - q(\omega, 1)]^2,
\]

\( \epsilon_\omega = 0 \) (1) for a satisfied (frustrated) wire. Then the error metric is

\[
\epsilon_w := \sum_{\omega=1}^{W} \epsilon_\omega,
\]

the number of frustrated wires. We do not engineer the error metric operator here. Solving the given problem requires minimizing the wire error subject to the triode relations.

Each triode is associated with three orthogonal eigenmodes \( |\theta\rangle \) defined so that the binary variable \( s^2_\theta \) has the value 1 for the mode \( |\theta\rangle \) and 0 in the other two modes.

The three-dimensional Hilbert space of the triplet modes of triode \( \tau \) we designate by \( \mathcal{H}_\tau \). We define an auxiliary mode space of \( T \) disconnected triodes as the tensor product \( \bigotimes_{\tau=1}^{T} \mathcal{H}_\tau \), a Hilbert space of dimension \( 3^T \).

The total error form \( \epsilon_w \) of the network is a lower bound on the number of wires that have to be changed to attain the solution. The number could be as great as \( W \) even if \( \epsilon_w = 1 \).

We turn the error function into the Hamiltonian \( H_w = g\epsilon_w \) diagonal in the qubit basis, with a coefficient \( g \) to provide the dimension of energy.
4 Computation model

For qusa computation we take the effective Hamiltonian \( H_w + H_r(t) \), where \( H_r(t) \) is a small effective Hamiltonian term (possibly non-Hermitian) representing relaxation processes that bring the network to the ground mode of \( H_w \). \( H_r(t) \) is discussed in the following.

4.1 Continuous statistical projection

To estimate qusa speed-up, we represent a development subject to statistics as a continuous projection of a dynamical development not subject to statistics.

Consider a pair of identical protons 1 and 2. We freeze their spatial mode to a fixed antisymmetric wave-function \( \psi_{12}(x_1, x_2) \) so that only the spin degrees of freedom need be considered. The individual spin modes form two-dimensional Hilbert spaces \( \mathcal{H}_1 \) and \( \mathcal{H}_2 \). We consider:

- an unsymmetrized tensor-product ("comparison") Hilbert space \( \mathcal{H}' = \mathcal{H}_1 \otimes \mathcal{H}_2 = \mathcal{H}_t \oplus \mathcal{H}_s \), the direct sum of the physical triplet subspace \( \mathcal{H} \equiv \mathcal{H}_t \) and the singlet subspace \( \mathcal{H}_s \),
- a particle exchange operator \( X_{12} : \mathcal{H}' \to \mathcal{H}' \),
- a projection \( P_{12} := \frac{1}{2} (1 + X_{12}) \) on the physical triplet subspace \( \mathcal{H} \equiv \mathcal{H}_t \subset \mathcal{H}' \)
- a symmetrized Hamiltonian \( H_{12} : \mathcal{H}_t \to \mathcal{H}_t \) invariant under proton exchange: \( X_{12} H_{12} = H_{12} X_{12} \), so that \( X_{12} \) is a constant of the motion.
- the extension \( H \) of the spin Hamiltonian operator \( H_{12} \) from the triplet subspace \( \mathcal{H}_t \subset \mathcal{H}' \) to the entire space \( \mathcal{H}' \), with \( H = 0 \) on the singlet space for convenience.

The orthohydrogen modes are symmetric in the spin variables. They vary over the effective three-dimensional Hilbert space of triplet modes \( \mathcal{H}_t \subset \mathcal{H}' \).

In the Hilbert space \( \mathcal{H}' \), the time development of the orthohydrogen spins is then governed by a Hamiltonian \( H \) that maps the triplet subspace into itself:

\[
|dt\rangle := (1 - iHdt) |0\rangle,
\]

where \( |0\rangle \) is a symmetric mode in \( \mathcal{H}' \). We now develop an equivalent representation of the time development. The aim is to free the Hamiltonian from the mathematical conditions representing proton indistinguishability.

We start from a symmetric initial mode \( |0\rangle \) of \( \mathcal{H}' \) at time 0 and let it evolve for an infinitesimal amount of time \( dt \) according to a different comparison Hamiltonian \( H' \):

\[
|dt\rangle' := (1 - iH'dt) |0\rangle.
\]
$H'$ is not subject to exchange symmetry, but we choose it so that its symmetrization yields the actual Hamiltonian:

$$P_{12}H'P_{12} = H. \quad (11)$$

In general $|dt\rangle'$ is not symmetric under proton exchange. We restore particle indistinguishability by projecting $|dt\rangle'$ on $\mathcal{H}_t$. This means symmetrizing $|dt\rangle'$ to form

$$P_{12}|dt\rangle' \equiv (1 - iH dt)|0\rangle. \quad (12)$$

The projection of the infinitesimal development (10) on $\mathcal{H}_t$ yields the actual development (9), up to higher order infinitesimals. We “continuously project” on $\mathcal{H}_t$ the development governed by $H'$. That is, we project after each interval $\Delta t$ and take the limit $\Delta t \to 0$. This recovers the actual development generated by the symmetrized Hamiltonian $H$.

This mathematical artifice of asymmetric time-development accompanied with continuous symmetrization permits us to estimate the speed-up due to quantum statistical computation.

### 4.2 The speed-up due to statistics

For quas computation, we apply the continuous symmetrization of Section 4.1 to the relaxation of the triode network; see for example (2). For a comparison network we work in the unsymmetrized tensor product Hilbert space $\mathcal{H}' \supset \mathcal{H}^T$, suspending proton indistinguishability and removing all the consequent statistical relations.

This means dropping the triode condition $q_x + q_y + q_z = 1$ for the weaker condition $q_x + q_y + q_z = 1 \mod 2$. The latter holds independently of statistics due to the composition of angular momentum alone, so it survives.

$H_w$ usually has traps (local minima relative to all immediately adjacent energy levels) that slow classical annealing computation. We eliminate these for the comparison computation by redefining $H_w$ in a way that does not change the ground mode:

$$H_w = g\epsilon_w + g'\epsilon_w \sum_{\tau} \left\{ [1 - q(\tau, 1)]^2 + [1 - q(\tau, 2)]^2 + [1 - q(\tau, 3)]^2 \right\}, \quad (13)$$

where $q(\tau, 1), ...$ are the three qubits of triode $\tau$.

Since for the triode network each triode has only one qubit equal to 1, we have merely multiplied the previous Hamiltonian by $\left( 1 + 2T^2 \frac{g}{g'} \right)$. This does not change the ground mode.

In the case of the EQU network, all the three qubits of a triode can be 1. If $g' \gg g$, each frustrated network mode ($\epsilon_w \geq 1$) has a gradient toward the solution where all qubits are 1. This ground mode is quickly reachable by the EQU network even in classical annealing.
We define the effective Hamiltonians of the actual and comparison networks,

\begin{align}
H &= H_w + H_r, \tag{14} \\
H' &= H'_w + H'_r. \tag{15}
\end{align}

Each describes a network with symmetric wire Hamiltonian \(H_w\) or \(H'_w = H_w\), coupled to a heat bath by an effective coupling Hamiltonian respectively \(H_r\) and \(H'_r\). \(H'_r\) is not symmetrized; its symmetrization yields \(H_r\). \(H\) describes the relaxation process of the actual triode network. \(H'_r\) describes the relaxation process of a comparison network obtained by replacing all pairs of identical indistinguishable spin 1/2 particles with pairs of distinguishable (e.g. non-identical) spin 1/2 particles. Correspondingly, all triode relations are replaced by EQU gate relations.

We model the actual heat bath coupling \(H_r\) by coupling each proton spin \(\vec{\sigma}\) to a small Gaussian random time-varying magnetic field \(\vec{B}\) at the site of that spin. \(\vec{B}\) might be polarized along the principal direction \(x + y + z\). We index the sites with the triode index \(\tau = 1, \ldots, T\) and a binary index \(\beta = 1, 2\). While \(\tau\) enumerates the triodes (proton pairs), \(\beta\) distinguishes the two protons in each triode.

To preserve statistics we must demand that the two protons \(\beta = 1, 2\) of each triode \(\tau\) experience the same magnetic field \(\vec{B}(\tau)\). We may then write the actual relaxing coupling as

\[ H_r = g \sum_{\tau, \beta} \vec{B}(\tau) \cdot \vec{\sigma}(\tau, \beta) \tag{16} \]

The comparison heat bath is a random magnetic field at each proton site. Unlike the actual heat bath coupling, the comparison heat bath coupling \(H'_r\) is not invariant under proton exchange. Different protons in the same triode see different magnetic fields \(\vec{B}(\tau, \beta)\):

\[ H'_r = g \sum_{\tau, \beta} \vec{B}(\tau, \beta) \cdot \vec{\sigma}(\tau, \beta) \tag{17} \]

\(\vec{B}(\tau, \beta)\) too might be polarized along the principal direction \(x + y + z\).

Let \(P\) be the symmetrization operator for all the relevant proton permutations; it is not necessary to permute protons between triodes. We may arrange that the projected heat-bath coupling is the actual coupling,

\[ PH'_r P = H_r, \tag{18} \]

by identifying the random magnetic field \(\vec{B}(\tau)\) of the actual heat bath with the average of the two random magnetic fields of the comparison heat bath:

\[ \vec{B}(\tau) \equiv \frac{\vec{B}(\tau, 1) + \vec{B}(\tau, 2)}{2} \tag{19} \]

The sum of two Gaussian random variables is also a Gaussian random variable.
Summing up, we have $PH'_w P = H_w$, $PH'_r P = H_r$, and thus $PH'P = H$. Therefore we can apply the method of Section 4.1.

The fact that $H'_w = H_w$ is already symmetric with respect to proton exchange (as all its qubits are) does not introduce any unwanted constraint in the comparison relaxation process. It does not prevent the generation of triode violations by $H'_r$. $H_w$ symmetry only reflects the composition of angular momentum, not statistics.

Here we do not tackle the difficult problem of estimating the relaxation time of the actual triode network, or the comparison EQU network. We just know that $H'$ will eventually drive the EQU network to its ground mode, and compare the relaxation times of the two networks.

For this comparison, decompose the actual computation time $\Delta T$ into $N = \Delta T/\Delta t$ consecutive time slices $\Delta t_i : t_i \leq t \leq t_{i+1}$ of equal length $\Delta t$, with $i = 1, 2, ..., N$, $t_i = i\Delta t$. Take the relaxation within each $\Delta t_i$ to be that of the comparison EQU network. At the end of each $\Delta t_i$ project the network mode on the Hilbert space $\mathcal{H}_T$. Then take the limit $\Delta t \to 0$. This reproduces the actual computation.

Let $|0\rangle$ be an initial symmetrical preparation, where all triodes must be satisfied and wires can be frustrated. Let this evolve into $|t\rangle$ at time $t$, with random phases as required. To consider the development of $|t\rangle$ inside the interval $\Delta t_i$, we resolve $|t\rangle$ as follows:

$$|t\rangle = |S,t\rangle + |F,t\rangle + |V,t\rangle.$$  \hspace{1cm} (20)

$|S,t\rangle$ denotes a superposition of tensor products of triode eigenmodes (terms) which are solutions of the triode network (each with a random phase to represent incoherence as necessary); its terms have satisfied triodes and satisfied wires. Most probably, relaxation randomly generates a solution with probability $p_S(t) := \langle S,t|S,t\rangle$ of the order of $1/2^Q$ in poly($Q$) time. We assume this is the case at time $t_i$. Since $H_w |S,t\rangle = 0$, \hspace{1cm} (21)

we can assume that $\langle S,t|S,t\rangle$ remains approximately constant inside $\Delta t_i$.

$|F,t\rangle$ is the component of $|t\rangle$ whose terms have satisfied triodes and at least one frustrated wire. Its probability $p_F(t) := \langle F,t|F,t\rangle$ is initially close to 1 (see further below).

$|V,t\rangle$ is the component whose terms have at least one violated triode; wires can be either satisfied or frustrated. Its probability is $p_V(t) := \langle V,t|V,t\rangle$. $|V,t\rangle$ is generated by the relaxation of the EQU network inside each $\Delta t_i$; it goes to zero with $\Delta t$ and is annihilated by the projection at the end of $\Delta t_i$.

We compare the rate of relaxation of the triode network to that of the EQU network as follows.

Inside $\Delta t_i$, the evolution is that of the comparison network where all triodes are replaced by EQU gates. Therefore $p_F(t)$ goes down at the rate of relaxation of the EQU network, building up the mode $|V,t\rangle$. We are particularly interested in the “take off” of the solution probability $p_S(t)$ from $O\left(1/2^Q\right)$ to $O(1)$, say to $p_S(t) = 1/10$. During take off, we can
assume a constant rate of relaxation $k$ of the EQU network with $p_F(t) \approx 1$. That is, $p_F(t)$ changes from $p_F(t_i) \approx 1$ at the beginning of $\Delta t_i$ to

$$p_F(t_{i+1}) \approx (1 - k\Delta t) p_F(t_i) \approx (1 - k\Delta t)$$

at the end of $\Delta t_i$. Correspondingly, $p_V(t)$ changes from 0 to about $k\Delta t$.

The projection at $t_i + \Delta t_i$ therefore reduces $\langle t | t \rangle$ by about $k\Delta t$. Renormalizing then multiplies $p_S(t)$ by about $(1 - k\Delta t)$ at each $\Delta t_i$. After a time $N\Delta t = \Delta T$ we have

$$p_S(t_i + \Delta T) = p_S(t_i) \lim_{\Delta t \to 0} (1 + k\Delta t) = p_S(t_i)e^{k\Delta T}. \quad (22)$$

Thus $p_S(t)$ becomes $O(1)$ in a time $\Delta T$ such that $e^{k\Delta T} = O(2^Q)$. This means $k\Delta T = O(Q)$, the number of qubits. The relaxation time $\Delta T$ of the actual network is comparable with that of the easy EQU network.

Let us check that assuming a constant (average) relaxation rate does not introduce unwanted restrictions. We show that the continuous projection method works in the same way in presence of fluctuations of the expected energy of the EQU network. Assume this energy goes up in the time interval $[t_i, t_j]$, with $t_j > t_i$, namely $\langle t_j | H_w | t_j \rangle > \langle t_i | H_w | t_i \rangle$. On the basis of (22), this brings $p_S(t)$ down but, as readily checked, what is lost is exactly regained when the expected energy goes back to $\langle t_i | H_w | t_i \rangle$ at some time $t_h > t_j$.

Summing up: (i) We have defined an effective symmetrized Hamiltonian $H$ for the relaxation of the “hard” triode network. (ii) We have defined a comparison non-symmetrical Hamiltonian $H'$ whose symmetrization yields $H$: $PH'P = H$. (iii) Furthermore $H'$ describes the relaxation process of an easy EQU network, obtained by replacing all triode relations by EQU relations. (iv) The relaxation of the hard triode network can be obtained by continuously projecting that of the easy EQU network on the symmetrical subspace $H^T$. By eq. (22), this yields comparable relaxation times for the hard and easy networks.

Satisfying relations implemented by particle statistics does not take time in annealing computation.

If quantum annealing can reach the solution 111... of a general EQU network in polynomial time, then in qusa computation NP=P.

Qusa computation survives decoherence as well as general annealing computation does. They both avoid this basic difficulty of reversible quantum computation.

5 Discussion

5.1 The origin of the qusa speed-up

What makes quantum computation more efficient than classical computation has been called a “most pressing” question for the advancement of the field (Mahler 2001). The speed-up of the quantum algorithms (Deutsch 1985, Shor...
1994, Grover 1996, among others) stems from the fact that a quantum transition is jointly influenced by an initial and a final extra-dynamical selection. It may be regarded as extra-dynamical in origin (Castagnoli & Finkelstein 2001). This kind of process is richer than classical computation, which is a dynamical development of one selection alone.

Quasa computation can be seen as a dynamical development generated by a symmetric time-varying Hamiltonian $H$ with a time-dependent magnetic field $\vec{B}$ (varying randomly for annealing). The extra-dynamical origin of its speed-up is $H$ symmetry itself. This originates extra-dynamically in proton indistinguishability, not dynamically. By the way, one can see from equations (10) and (12) that symmetrizing the network mode and symmetrizing $H'$ are equivalent.

It is as if proton indistinguishability provides for free an extra-dynamical symmetrization engine continuously acting on an un-symmetrized $H'$ or the development generated by it (relating the easy EQU network). The comparison with the quantum algorithms is easier if we consider the development.

Quasa speed-up comes from the fact that the Boolean statistical relations are satisfied through the projection of a development that is unaffected by them. Correspondingly, we have seen that in each $\Delta t_i$ there is a quantum transition jointly influenced by an initial and a final extra-dynamical selection. This is as in the quantum algorithms, but for the fact that quasa computation always selects a predetermined subspace, the one that satisfies statistics.

5.2 Conclusions

Quasa speeds quantum ground mode computation up by implementing the gates (or partial gates) of a Boolean network with statistical symmetries, and only the wires through an energy function which is zero when the wire is satisfied. Logical relations associated with statistical symmetries do not slow down the development of the initial mode toward a ground mode where both gates and wires are satisfied. This is unlike classical ground mode computation, where logical relations reduce relaxation rate.

Quasa computation develops quantum parallelism through the incoherent superposition of parallel computation paths (i.e. through mixtures). It abandons the delicate superposition of coherent parallel computation paths in reversible quantum computation but introduces the almost indestructible superposition of different permutations of identical particles subject to a given statistics. This greatly reduces the problem of decoherence.

This synthesis of ground mode computation and quantum statistics appears to be a promising architecture for robust quantum computing.

A natural next step in this research is to design physical systems exhibiting the quasa effect.

ACKNOWLEDGMENTS

The ideas propounded in this work were developed through discussions with
REFERENCES

1. Castagnoli, G. 1998 Physica D 120, 48.

2. Castagnoli, G., Ekert, A. & Macchiavello, C. 1998 Int. J. Theor. Phys. 37, 463.

3. Castagnoli, G. & Monti, D. 1999 Chaos, Solitons & Fractals 10, 1665.

4. Castagnoli, G. & Finkelstein, D.R. 2001 Proc. R. Soc. Lond. A 457, 1799.

5. Deutsch, D. 1985 Proc. R. Soc. Lond. A 400, 97.

6. Farhi, E., Goldstone, J., Gutmann, S., Lapan, J., Lundgren, A. & Preda, D. 2001 Science 292, 472.

7. Finkelstein, D. 1996 Quantum Relativity. Springer.

8. Grover, L. 1996 In Proc. 28th A. ACM Symp. on Theory of Computing, P. 212. Philadelphia, PA: ACM Press.

9. Kirkpatrick, S. & Selman, D. 1994 Science 264, 1297.

10. Mahler, G. 2001 Science 292, 57.

11. Shor, P. 1994 In Proc. 35th A. Symp. of the Foundation of Computer Science, Los Alamitos, CA, P. 124. Los Alamitos, CA: IEEE Computer Society Press.