Measurement of Low-Activity Uranium Contamination using Bayesian Statistical Decision Theory

Hanan Arahmane\textsuperscript{a},* \textadd superscript{a,} Jonathan Dumazert\textsuperscript{b}, Eric Barat\textsuperscript{a}, Thomas Dautremer\textsuperscript{a}, Frédérick Carrel\textsuperscript{a}, Nicolas Dufour\textsuperscript{a}, Maugan Michel\textsuperscript{a}

\textsuperscript{a}Université Paris-Saclay, CEA, List, F-91120 Palaiseau, France
\textsuperscript{b}CEA-DAM, DIF, F-91297 Arpajon, France

Abstract

Amongst the various technical challenges in the field of radiation detection is the need to carry out accurate low-level radioactivity measurements in the presence of large fluctuations in the natural radiation background, while lowering the false alarm rates. Several studies, using statistical inference, have been proposed to overcome this challenge. This work presents an advanced statistical approach for decision-making in the field of nuclear decommissioning. The results indicate that the proposed method allows to adjust the confidence degree in the stationarity of the background signal. It also ensures an acceptable tradeoff between the True Detection Rate (TDR), the False Alarm Rate (FAR) and the response time, and is consistent with the user’s requirements.

Keywords: Radioactivity Detection, Statistical Inference, Gamma-Ray Spectrometry

1. Introduction

Accurate detection of radioactivity in low count rates is crucial for many practical radiation-detection applications. The particular approach considered here is that of statistical analysis that has recently provided new horizons in the area of radioactive detection. It is one of the reliable and accurate analysis for determining the most effective detection strategy with reasonable operational requirements. In this paper, we aim to measure a low-activity uranium contamination on concrete surfaces, with varying enrichment encountered levels within a basic nuclear facility. Our surface contamination limit was taken as $S_A = 500 \text{ Bq}/\text{0.1 m}^2$ \cite{1}.

In this context, we have developed an advanced Bayesian method based on a Kibble bivariate gamma distribution, referred to as the Bayesian Kibble Dirichlet (BKD) test. It allows to adjust the confidence degree in the stationarity of the background signal. In this perspective, the BKD test combines the properties of an absolute and a relative hypothesis statistical test, that were both developed with the same application at hand \cite{1}, and respectively referred to as the Bayesian Absolute test by mixture of Multivariate Poisson laws (BAM) and the Bayesian Relative test by mixture of Multinomial laws (BRM). Note that unlike the BKD test, the modeling of the BAM test was based on the univariate gamma

*Corresponding Author

Email address: hanan.arahmane@gmail.com (Hanan Arahmane)
distribution. The implementation of the Bayesian approach were based on a priori vectors constructed from the coupling of experimental data acquired within a basic nuclear facility using a high-purity germanium detector (HPGe), as well as simulated data with Monte Carlo N-Particles 6 transport code (MCNP6.1) [2]. The performance evaluation and characterization of the Bayesian method were performed using classical receiver operating characteristic curves (ROC) [3] with the study of the background signal variations effect.

2. Materials and Methods

2.1. Uranium gamma peaks of interest and background signal

Detection of low levels of uranium contamination is to determine its presence by means of an HPGe detector. To do so, we used the main gamma-ray emission lines from the decay chains of U-235 and U-238 [1]: (i) from the U-235 chain, one main line at 185.7 keV, and an additional triplet at 143.8; 163.4; 205.3 keV, and (ii) from the U-238 chain, one main line at 1001.0 keV.

Regarding the background signal, it should be noted that the detection of a low-surface activity uranium contamination on concrete surfaces, based on gamma-ray emission, can be disturbed by the presence of the background signatures, in particular K-40, and the decay chains of U-238 and Th-232. In order to quantify this variation within nuclear facility that is subdivided into three subunits (SUs), we performed the acquisitions over source-detector distance $d = 22.36 \, cm$ for a surface of $0.1 \, m^2$. Then, we calculated the relative deviation between two separate spots inside each SU.

2.2. Simulation of the detector response

The performance evaluation of the Bayesian method needs prior knowledge of the expected spectral response from a uranium contamination on concrete surfaces, based on gamma-ray emission, can be disturbed by the presence of the background signatures, in particular K-40, and the decay chains of U-238 and Th-232. In order to quantify this variation within nuclear facility that is subdivided into three subunits (SUs), we performed the acquisitions over source-detector distance $d = 22.36 \, cm$ for a surface of $0.1 \, m^2$. Then, we calculated the relative deviation between two separate spots inside each SU.

2.3. Bayesian Methodology

The construction of Bayesian statistical tests, either BAM, BRM and BKD needs some prior Knowledge. The spectral signatures of both the background signal, and an expected uranium contamination inside a nuclear facility, are the priors retained in this study. We
recall here that the prior spectra of the background signal, and of an expected uranium contamination are respectively provided by in-situ acquisitions, and Monte Carlo simulations based on MCNP6.1.

The construction of the Bayesian test leading to the acceptance of one of the two hypotheses $H_0$ or $H_1$ makes use of Bayes' theorem as follows:

$$P(H_0 | \text{m}_{\text{ref}}, \text{m}_{\text{test}}) = \frac{P(\text{m}_{\text{ref}}, \text{m}_{\text{test}} | H_0) P(H_0)}{P(\text{m}_{\text{ref}}, \text{m}_{\text{test}} | H_1) P(H_1)}$$

(1)

where $m_{\text{ref}}$ and $m_{\text{test}}$ the multichannel countings, $P(H_0)$ and $P(H_1)$ are priors on the respective likelihoods of hypotheses $H_0$ and $H_1$.

Without any a priori information of the presence of a uranium contamination, these probabilities are taken equal by default: $P(H_0) = P(H_1) = 0.5$. Moreover, we note that: $P(H_1 | \text{m}_{\text{ref}}, \text{m}_{\text{test}}) = 1 - P(H_0 | \text{m}_{\text{ref}}, \text{m}_{\text{test}})$. As a result, by taking into account these equalities in equation (1) the posterior probability of hypothesis $H_0$, that will be ultimately compared to a decision threshold, can be expressed as:

$$P(H_0 | \text{m}_{\text{ref}}, \text{m}_{\text{test}}) = \frac{1}{1 + \kappa_{\text{BAM}, \text{BRM}, \text{BKD}}}$$

(2)

with the Bayes factor expressed as:

$$\kappa_{\text{BAM}, \text{BRM}, \text{BKD}} = \frac{P(\text{m}_{\text{ref}}, \text{m}_{\text{test}} | H_1)}{P(\text{m}_{\text{ref}}, \text{m}_{\text{test}} | H_0)}$$

(3)

From the last three equations, we can see that the construction of the Bayesian tests is the comparison of $P(H_0 | \text{m}_{\text{ref}}, \text{m}_{\text{test}})$ and $P(H_1 | \text{m}_{\text{ref}}, \text{m}_{\text{test}})$ through the computation of the Bayes factor $\kappa_{\text{BAM}, \text{BRM}, \text{BKD}}$.

It should be noted that the process of adjusting the properties of the BKD test in order to retrieve the limiting behaviors of the absolute (BAM) and relative (BRM) tests, relies essentially upon the parametrization of the BKD test with respect to the correlation coefficient of Kibble law $\rho$ [4]. Therefore, the implementation of the BKD test requires the setting of a parameter $\rho$ which governs the behavior of BKD between that of BRM ($\rho = 0$) and that of BAM ($\rho \to 1$).

3. Results and Conclusions

In this section, we present the comparative performance study of the BKD test with the absolute (BAM) and relative (BRM) ones by using a ROC-based analysis. The scenarios involving these three settings of $\rho$ in both cases of a stationary or non-stationary background signal, in amplitude as well as in shape, yield the ROC curves in Figures 1 and 2.

These ROCs allow to identify explicit trends. From the figures, we can see that the BKD test, whether the background signal is stationary and non-stationary, has the same behavior as the relative BRM test (Figures 1a and 2a) in the case of $\rho = 0$, while it presents the properties of the absolute BAM test in the case of $\rho \to 1$ (Figures 1b and 2b). Furthermore, the results showed that a reliable detection of uranium was reachable with a significantly higher TDR/FAR tradeoff. However, we can observe that the variation of the background signal, affected the detection performance of the Bayesian tests, which is normal, due to an increase in the statistical fluctuations in the spectra.
Figure 1: ROC curves for an enrichment level of 0.7 wt%, an integration time $t = 3000$ s, a stationary background signal, and $\rho$: (a) $\rho = 0$, (b) $\rho \rightarrow 1$.

Figure 2: ROC curves for an enrichment level of 0.7 wt%, an integration time $t = 3000$ s, a non-stationary background signal, and $\rho$: (c) $\rho = 0$, (d) $\rho \rightarrow 1$.

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