Analysis L_{1/2} Regularization: Iterative Half Thresholding Algorithm for CS-MRI

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ABSTRACT Recently, the L_{1/2} regularization has shown its great potential to eliminate the bias problems caused by the convex L_1 regularization in many compressive sensing (CS) tasks. CS-based magnetic resonance imaging (CS-MRI) aims at reconstructing a high-resolution image from under-sampled k-space data, which can shorten the imaging time efficiently. Theoretically, the L_{1/2} regularization-based CS-MRI will reconstruct the MR images with higher quality to investigate and study the potential and feasibility of the L_{1/2} regularization for the CS-MRI problem. In this paper, we employ the nonconvex L_{1/2}-norm to exploit the sparsity of the MR images under the tight frame. Then, two novel iterative half thresholding algorithms (IHTAs) for the analysis of the L_{1/2} regularization are introduced to solve the nonconvex optimization problem, namely, smoothing-IHTA and projected-IHTA. To evaluate the performance of the L_{1/2} regularization, we conduct our experiments on the real-world MR data using three different popular sampling masks. All experimental results demonstrate that the L_{1/2} regularization can improve the L_1 regularization significantly and show the potential and feasibility for future practical applications.

INDEX TERMS L_{1/2} regularization, compressive sensing, analysis model, iterative half thresholding algorithm, tight frame, smoothing, magnetic resonance imaging.

I. INTRODUCTION

Recently, compressive sensing (CS) [1], [2] has shown its great potential for various sparse signal recovery problems, e.g., signal estimation and detection in various signal acquisition and processing system [3]–[11], localization [12]–[16], image restoration [17], [18], and particularly in magnetic resonance imaging (MRI) [19]. The CS theory based MRI can accelerate the imaging speed via reconstructing MR images from only a small of under-sampled k-space data. To make it possible, we need to fit the k-space under-sampled k-space date in a sparse transform domain. The under-sampled model for CS-MRI can be described as

\[ y = UFx + n \] (1)

where \( x \in \mathbb{C}^N \) denotes a latent MR image or rearranged vector, and \( F \in \mathbb{C}^{M \times N} \) is the discrete Fourier transform operator.

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U ∈ \( \mathbb{C}^{M \times N} \), (\( M < N \)) represents the under-sampling matrix, \( n \in \mathbb{C}^M \) is the additive Gaussian noise, and \( y \in \mathbb{C}^M \) is the observed under-sampled k-space MR data.

According to the CS and sparse representation theory, if the desired image or signal is sparse in some certain sparse transform domain, it will be possible to reconstruct the image \( x \) using the image sparsity. Generally, it is assumed that \( x \) can be a sparse and linear combination of given dictionary elements, where these elements are columns of the dictionary matrix \( \Phi \in \mathbb{R}^{N \times p} \), where \( p \geq N \). Then \( x = \Phi \alpha \) with the vector \( \alpha \) is sparse, then the reconstruction problem of (1) can be formulated as

\[ \min_\alpha \frac{1}{2} \| y - UF\alpha \|^2_2 + \lambda \| \alpha \|_1 \] (2)

where \( \lambda \) is called regularization parameter which can make a tradeoff between the sparsity prior knowledge and the L_2-norm based fidelity term. The minimization model of (2) is also called synthesis approach for CS recovery problem.
Recently, another more popular alternative model is proposed and also shown its great potential for CS reconstruction problem, namely the analysis model [20]. Under the analysis based framework, we usually need to provide an analysis dictionary $\psi \in \mathbb{R}^{N \times P}$, under which the $\psi \mathbf{x}$ is sparse, and the analysis approach can be described as

$$\min_{\mathbf{x}} \frac{1}{2} \| \mathbf{y} - \mathbf{U} \mathbf{F} \mathbf{x} \|_2^2 + \lambda \| \psi \mathbf{x} \|_{1/2}$$

(3)

where $\psi$ denotes an analysis operator. Compared with the synthesis method, the analysis based model can work under a wider range of dictionary, and can often achieve better performance [6]. For example, recently, a multiple-prespecified-dictionary based compressive sensing image reconstruction is proposed with nonconvex regularization by using the different sparsity under different given analysis dictionary, and hence can further exploit the sparse structure information as prior knowledge for sparse minimization problem [21], [22].

However, the $L_1$-norm regularization based model often caused biased problem, recent work reveal that the non-convex based optimization can enhance the reconstruction performance [23]–[26]. Recently, Z. Xu propose a iterative-half thresholding fast algorithm (IHTA) for $L_{1/2}$ regularization model for sparse modeling and CS [27], which is able to yield sparser approach than convex $L_1$-norm regularization, and has been widely used in various CS recovery applications [28]–[34], such as radar imaging, wireless communications, and particularly in image restoration problems. In this paper, we will give an investigation and study on IHTA for analysis CS-MRI, the model of $L_{1/2}$ regularization for MRI can be generally modeled as

$$\min_{\mathbf{x}} \frac{1}{2} \| \mathbf{y} - \mathbf{U} \mathbf{F} \mathbf{x} \|_2^2 + \lambda \| \psi \mathbf{x} \|_{1/2}$$

(4)

where $\| \mathbf{a} \|_{1/2} = \sum_i \sqrt{|a_i|}$ denotes a quasi-norm. To make our proposed model more robust and tractable, we employ two typical framework for our proposed model (4), namely projected framework and smoothing framework, respectively. Experimental results demonstrate that the $L_{1/2}$ regularization based scheme can improve the reconstructed image quality significantly, and has shown its great potential in CS-MRI problem.

The contribution of this paper can be summarized as following. Firstly, to exploit the sparsity of MR images for reconstruction, we employ a more accurate surrogate function of $L_0$-norm, namely $L_{1/2}$-norm to regularize the sparsity. Secondly, considering that the MR images are not sparse and the traditional $L_{1/2}$ regularization cannot be applied to MRI problems directly, we employ a tight frame representation system which can exploit the sparsity of the MR images efficiently. Thirdly, to solve the nonconvex analysis $L_{1/2}$ based CS-MRI problem, we introduce two analysis IHTA, namely smoothing-IHTA and the projected-IHTA. Finally, to evaluate the performance, numerous experiments are conducted using different MR datasets and sampling masks. This paper will give an intuitive investigation and study of $L_{1/2}$ regularization for CS-MRI problem.

II. TIGHT FRAMES REPRESENTATION BASED ANALYSIS $L_{1/2}$ REGULARIZATION

A. TIGHT FRAMES REPRESENTATION THEORY

Considering the fact that the MR images is not sparse directly, therefore, it is crucial that how to design and construct the sparse transform and representation system for sparsifying and represent the MR images. In this section, we will introduce a redundant representation system of tight frame for our analysis $L_{1/2}$ regularization, and then proposed a smoothing based iterative-half thresholding algorithm and projected iterative-half thresholding algorithm for CS-MRI. It is noticed that any given signal of $\mathbf{x} \in \mathbb{C}^N$ can be represented by a frame in the same space, where $\mathbb{C}^N$ denotes the Hilbert space. Let $\{\mathbf{d}_j\}_{j=1,2,\ldots,J}$ denotes a frame in the Hilbert space $\mathbb{C}^N$, if there are two equal positive real number $a$, $b$, and for any given $\mathbf{x} \in \mathbb{C}^N$ we have

$$a \| \mathbf{x} \|_2^2 \leq \sum_j |\langle \mathbf{x}, \mathbf{d}_j \rangle|^2 \leq b \| \mathbf{x} \|_2^2$$

(5)

then the set of vectors $\{\mathbf{d}_j\}$ is called tight frame, where $\langle \mathbf{x}, \mathbf{d}_j \rangle = \mathbf{d}_j^H \mathbf{x}$ denotes the inner product, and $(\cdot)^H$ denotes the Hermitian transpose operator.

B. ITERATIVE HALF THRESHOLDING ALGORITHM FOR ANALYSIS $L_{1/2}$ REGULARIZATION

Considering the optimization problem of (4), in this paper, we will introduce two novel IHTA based on analysis $L_{1/2}$ regularization for CS-MRI, called Smothing IHTA, and projected [35] IHTA by replacing the well-known fast iterative shrinkage-thresholding algorithm (FISTA) using the IHTA, shown as

Smoothing IHTA:

$$\mathbf{x}^{k+1} = \left( 1 - \frac{\gamma}{\lambda} \right) \mathbf{x}^k - \left( \frac{\gamma}{\lambda} \right) \psi^* H_{\lambda \gamma} \left( \psi \mathbf{x}^k \right) + \gamma \mathbf{F}^\ast \mathbf{U}^T \left( \mathbf{y} - \mathbf{U} \mathbf{F} \mathbf{x}^k \right)$$

(6)

Projected IHTA:

$$\mathbf{x}^{k+1} = \psi^* H_{\lambda \gamma} \left( \psi \left( \mathbf{x}^k + \gamma \mathbf{F}^\ast \mathbf{U}^T \left( \mathbf{y} - \mathbf{U} \mathbf{F} \mathbf{x}^k \right) \right) \right)$$

(7)

where $\gamma$ denotes the step size parameter, and $\psi^*$ is a canonical dual tight frame of $\psi$, and the operator of $H_{\lambda \gamma} (\cdot)$ denotes the half-thresholding operator, which is given by

$$H_{\lambda \gamma} (\cdot) = (h_{\lambda \gamma} (\theta_1), h_{\lambda \gamma} (\theta_2), \ldots, h_{\lambda \gamma} (\theta_N))$$

(8)

where

$$h_{\lambda \gamma} (\theta_i) = \begin{cases} \phi_{\lambda \gamma} (\theta_i), & \left| \theta_i \right| > T \\ 0, & \text{otherwise} \end{cases}$$

(9)

and

$$\phi_{\lambda \gamma} (\theta_i) = \frac{2}{3} \theta_i \left( 1 + \cos \left( \frac{2}{3} \pi - \frac{2}{3} \cos^{-1} \left( \frac{\lambda \gamma}{4} \left( \left| \theta_i \right| \right)^{1/2} \right) \right) \right)$$

(10)
in which, $T = \frac{3\sqrt{2}}{2} (2\lambda\gamma)^{2/3}$ is the thresholding value. The whole algorithm process is summarized in the Algorithm 1 and Algorithm 2, respectively.

**Algorithm 1** Smoothing Iteration Half Thresholding Algorithm (Smoothing-IHTA)

- **Problem:** $x \leftarrow \min_{x} \frac{1}{2} \| y - UFx \|^2 + \lambda \| \psi x \|^\frac{1}{2} / \| \psi \|^\frac{1}{2}$
- **Input:** Measured data $y$, measurement matrix $UF$, the parameter $\lambda$, $\gamma$
- **Initialization:** $x^0 = 0$, $\hat{x}^0$
- **While not converge,** do
  1. Compute $x^{k+1} = \left(1 - \frac{\gamma}{\lambda}\right)x^k - \left(\frac{\gamma}{\lambda}\right)\psi^*H_{\lambda\gamma}\left(\psi x^k\right)$
  2. Compute $\mu^{k+1} = \frac{1 + \sqrt{1 + 4T^2}}{2}$
  3. Compute $x^{k+1} = x^k + \frac{\mu_k - 1}{\mu_k + 1} (x^{k+1} - x^k)$
  4. **Preparing for next iteration** $x^{k+1} \leftarrow \hat{x}^{k+1}$
- **Output** $x$.

**Algorithm 2** Projected Iteration Half Thresholding Algorithm (Projected-IHTA)

- **Problem:** $x \leftarrow \min_{x} \frac{1}{2} \| y - UFx \|^2 + \lambda \| \psi x \|^\frac{1}{2} / \| \psi \|^\frac{1}{2}$
- **Input:** Measured data $y$, measurement matrix $UF$, the parameter $\lambda$, $\gamma$
- **Initialization:** $x^0 = 0$, $\hat{x}^0$
- **While not converge,** do
  1. Compute $x^{k+1} = \psi^*H_{\lambda\gamma}\left(\psi\left(x^k\right)\right)$
  2. Compute $\mu^{k+1} = \frac{1 + \sqrt{1 + 4T^2}}{2}$
  3. Compute $x^{k+1} = x^k + \frac{\mu_k - 1}{\mu_k + 1} (x^{k+1} - x^k)$
  4. **Preparing for next iteration** $x^{k+1} \leftarrow \hat{x}^{k+1}$
- **Output** $x$.

**C. CONVERGENCE ANALYSIS**

There are two important parameters in both smoothing IHTA and projected IHTA, namely, the regularization parameter $\lambda$ and the step-size parameter $\gamma$. The parameter $\gamma$ play a key role for convergence of IHTA, according to the convergence theory of IHTA, the IHTA can converge to a fixed and stationary local minimizer, if the step size $\gamma$ satisfies the following condition as

$$0 < \gamma < \frac{1}{\|UF\|^2}$$  \hspace{1cm} (11)

However, it is not tractable to compute $L_{1/2}$-norm of the measure matrix of $UF$ accurately. To reduce the impact of parameter $\gamma$, we empirically set $\gamma = 1$ for all the experiments in our work.

As for the setting of regularization parameter $\lambda$, generally, there are three typical strategies for $\lambda$, e.g.,

1. **strategy 1:** When $\gamma^k = \gamma^0$, the parameter $\lambda$ can be design by the cross-validation method.
2. **strategy 2:** If $\gamma^k \in (0, \gamma^0)$,
   $$\lambda^k = \frac{\sqrt{96}}{96} \| UF \|^2 \left[ B_{\gamma^0} \left( x^k \right) \right]_{K+1} \}$$  \hspace{1cm} (12)
3. **strategy 3:** If $\gamma^k = 0$, the parameter can be chosen as
   $$\lambda^k = \min \left\{ \lambda^{k-1}, \frac{\sqrt{96}}{96} \| UF \|^2 \left[ B_{\gamma^0} \left( x^k \right) \right]_{K+1} \} \}$$  \hspace{1cm} (13)

where $\gamma^0 = \frac{1}{\|UF\|^2}$ with a very small content $\epsilon \in (0, 1)$, and $B_{\gamma^0} (x^k) = x^k + \gamma^0 F^* UF (y - UF x^k)$, the operator of $[\cdot]$$_K+1$ means the $K$-sparsity degree.

According to the strategy 2 and 3, to obtain the sparsity degree $K$ is the core step, however, it is very hard to compute the $K$ value accurately for practical various images. In this paper, we employ the strategy 1 to set the regularization parameter $\lambda$ empirically.

**III. EXPERIMENTAL RESULTS**

In this section, we conduct numerical experiments using our analysis $L_{1/2}$ regularization approaches with different MR datasets and sampling masks, as is shown in the figure 1 and figure 2. To evaluate the superior performance of our $L_{1/2}$ regularization approaches, we employ their corresponding convex algorithms of smoothing-FISTA [20] and projected-FISTA [35] for comparison, and analyze the achieved image qualities, the convergence. All the test MR data are acquired on the real-world MR scanner from a healthy using T2-weighted spin echo sequence. As shown in Fig. 2, we employ typical brain MR data (can be downloaded from the link of http://csrc.xmu.edu.cn/xiaobo/), and the cardiac data [36] for our experiments. Both the size of MR data and sampling mask are $256 \times 256$. To evaluate the reconstructed quality quantitatively, we employ the PSNR and the relative $L_{1/2}$-norm error (RLNE), defined as

$$PSNR = \frac{20 \log_{10} \left( \frac{\text{MAX}}{\sqrt{MSE}} \right)}{\text{}$$  \hspace{1cm} (14)
where $\text{MAX}$ denotes the possible maximum value of all pixels in the image, and $MSE$ denotes the mean squared error.

$$RLNE = \frac{\|x - \hat{x}\|_2}{\|x\|_2}$$ (15)

where $\hat{x}$ denotes the reconstructed MR image, and $x$ is the original MR image.

A. SMOOTHING IHTA VERSUS SMOOTHING FISTA

The smoothing FISTA is a recent state-of-the-art algorithm for analysis sparse recovery. In this section, we will first conduct experiments using smoothing FISTA and our smoothing IHTA. The parameter $\lambda$ as set as $10^{-4}$ and $5 \times 10^{-3}$ for smoothing-FISTA and smoothing-IHTA, respectively. And the thresholding value of $T_{1,\lambda \gamma} = \lambda \gamma$ for $L_1$-norm based algorithm for all competing experiments.

1) CARTESIAN SAMPLING

In this subsection, we first evaluate our algorithm under a popular cartesian sampling method. The experimental results are presented in Figs. 3-5. Fig. 3 presents the sampling mask, and the corresponding original MR image and two reconstructed MR images, and Fig. 4 is two reconstructed amplitude error. Form the results we can see that the our $L_{1/2}$ regularization can achieved higher PSNR values and higher quality image. To demonstrate the convergence of our smoothing-IHTA approach, Fig. 5 gives the RLNE curve and PSNR curve versus the iteration number. It can be observed that the RLNE curve and the PSNR show excellent convergence properties with the increasing of iteration number.

2) PSEUDO-RADIO SAMPLING

In this subsection, we evaluate our algorithm using the pseudo-Radio sampling method, see as the middle image of Fig. 1. The experimental results are presented in Figs. 6-8.
3) 2D GAUSSIAN RANDOM SAMPLING
In this third experiment for smoothing-IHTA, we reconstruct the MR images from 2D Gaussian random undersampled MR data. Fig. 9 presents the sampling mask, and the corresponding original MR image and two reconstructed MR images, and Fig. 10 is two reconstructed amplitude error. From the results we can see that our $L_1/2$ regularization can achieve higher PSNR values and higher quality image. To demonstrate the convergence of our smoothing-IHTA approach, figure 11 gives the RLNE curve and PSNR curve versus the iteration number. It can be observed that the RLNE curve and the PSNR show excellent convergence properties with the increasing of iteration number.

B. PROJECTED IHTA VERSUS PROJECTED FISTA
The projected FISTA is another recent state-of-the-art algorithm for analysis sparse recovery. In this section, we will conduct experiments using projected FISTA and our projected-IHTA. The parameter $\lambda$ as set as $10^{-4}$ and $10^{-3}$ for projected-FISTA and projected-IHTA in all experiments, respectively.

1) CARTESIAN SAMPLING
We first evaluate our algorithm under cartesian sampling method. The experimental results are presented in the Figs. 12-14. Fig. 12 presents the sampling mask, and the corresponding original MR image and two reconstructed MR images, Fig. 13 is two reconstructed amplitude error. From the results we can see that the our $L_1/2$ regularization can achieve higher PSNR values and higher quality image. To demonstrate the convergence of our projected-IHTA approach, Fig. 14 gives the RLNE curve and PSNR curve versus the iteration number, and the convergence behavior of our $L_1/2$ regularization can be observed visually.

2) PSEUDO-RADIO SAMPLING
This subsection evaluates our algorithm under another popular cartesian sampling method compared with the competing
3) 2D GAUSSIAN RANDOM SAMPLING

In this subsection, we conduct experiments under cartesian sampling mask. The experimental results are presented in Figs 15-17. Fig. 15 presents the sampling mask, and the corresponding original MR image and two reconstructed MR images, and Fig. 16 is two reconstructed amplitude error. Form the results we can see that the our $L_{1/2}$ regularization can achieved higher PSNR values and higher quality image. To demonstrate the convergence of our projected-IHTA approach, Fig. 17 gives the RLNE curve and PSNR curve versus the iteration number. It can be observed that the RLNE curve and the PSNR show excellent convergence properties.

FIGURE 15. From left to right: the Pseudo-Radio sampling mask of 20%, the original MR image, the reconstructed MR image by projected-FISTA algorithm (PSNR = 29.18 dB), and the reconstructed MR image by projected-IHTA (PSNR = 30.74 dB).

FIGURE 16. The reconstructed amplitude error. Left: Projected-FISTA; Right: Projected-IHTA.

FIGURE 17. The achieved RLNE and PSNR curves.

FIGURE 18. From left to right: the 2D Gaussian random sampling mask of 20%, the original MR image, the reconstructed MR image by projected-FISTA algorithm (PSNR = 30.47 dB), and the reconstructed MR image by projected-IHTA (PSNR = 32.18 dB).

FIGURE 19. The reconstructed amplitude error. Left: Projected-FISTA; Right: Projected-IHTA.

FIGURE 20. The achieved RLNE and PSNR curves.

in Figs. 18-20. Fig. 18 presents the sampling mask, and the corresponding original MR image and two reconstructed MR images, and Fig. 19 is two reconstructed amplitude error. Form the results we can see that the our $L_{1/2}$ regularization can achieved higher PSNR values and higher quality image. To demonstrate the convergence of our Projected-IHTA approach, Fig. 20 gives the RLNE curve and PSNR curve versus the iteration number. It can be observed that the RLNE curve and the PSNR show excellent convergence properties.

IV. CONCLUSION AND FUTURE WORK

In this paper, we have investigated the analysis $L_{1/2}$ regularization for tight frame based CS-MRI problem. Two different algorithm have been developed for analysis $L_{1/2}$ regularization by replacing the FISTA with IHTA, namely smoothing-IHTA and projected-IHTA. We have evaluated the smoothing-IHTA and projected-IHTA for CS-MRI problem from three different sampling patterns. All experimental results have demonstrated that the $L_{1/2}$ regularization can improve the convex $L_1$ based regularization significantly. In future work, we aim to develop some deep learning for further improving some CS algorithms [37]–[39]. In addition, it is also great opportunity to bridge gap between CS and deep learning in practical applications.

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