Evidence of enhanced radius of Hoyle rotational state in $^{12}$C inelastic scattering

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Abstract. Nuclear radius of three $\alpha$ rotational state in $^{12}$C with a life time of $10^{-21}$ second, which has been expected to have much more extended radius than the ground $^{12}$C nucleus, is speculated from systematic analysis of the differential cross section of the $\alpha + ^{12}$C inelastic scattering. Present analysis predicts about $0.6 \sim 1.0$ fm enhancement in the matter radius of the three $\alpha$ rotational state in comparison to the normal radius of the ground state. The spatial extension of the three $\alpha$ rotational state is comparable to the extended radius observed in the neutron halo phenomena.

1. Introduction

Cluster structures, in which a nucleus is decomposed into several sub-units, are realized in the low excited states of light mass nuclei [1]. In the cluster structure, the sub-units are weakly coupled each other, and their matter radius is prominently extended in comparison to the radius of the ground state, which obeys the law of $\propto A^{1/3}$. In particular, such the enhancement of the matter radius is extensive in the $3\alpha$ structure of the $^{12}$C nucleus [2, 3]. The ground state of $^{12}$C has a radius of 2.40 fm [2] with the spin-parity of $0^+ (0^+_1)$, while the excited $0^+_2$ state at $E_x = 7.65$ MeV, which is called the Hoyle state, is considered to have a radius of 3.47 fm with the well developed $3\alpha$ cluster structure [2]. The extension of the radius in $0^+_2$ is predicted to be about 1 fm, which is comparable to the extension of the neutron halos. In this article, we propose a new method to obtain a signature of the enhanced matter radius of the $3\alpha$ cluster state.

Unfortunately, a direct measurement of the radius of the Hoyle $0^+_2$ states with the developed $3\alpha$ cluster structure is impossible due to its short life time, but there are several attempts to get an evidence of the enhanced matter radius of the $3\alpha$ Hoyle states from the inelastic scattering of $^{12}$C, which excites the Hoyle $0^+_2$ state as a final state. In the inelastic scattering of light ions by $^{12}$C, an oscillating pattern in the differential cross section of the scattered ion is discussed in connection to the enhanced radius of the final $0^+_2$ state with the $3\alpha$ structure [4, 5, 6]. However, the relation of the extended radius and the oscillating pattern of the cross section of the $^{12}$C inelastic scattering still remains unclear [7, 8].

In this article, we focus on the $2^+_2$ state, which corresponds to the $3\alpha$ rotational state of the Hoyle $0^+_2$ state [2, 3] with a life of $10^{-21}$ sec. (width of $\Gamma \sim 1$ MeV), and demonstrate that a direct evidence of the enhanced matter radius in the $2^+_2$ state clearly appears in the $\alpha + ^{12}$C inelastic scattering. In a modern theory, the Hoyle rotational $2^+_2$ state, which has been recently identified in experiment [9], is interpreted in terms of an $\alpha$ halo state with a dilute $3\alpha$ structure [3]. The differential cross section of $\alpha + ^{12}$C$(0^+_1) \to \alpha + ^{12}$C$(2^+_2)$ is compared with the reference
reaction of $\alpha + ^{12}\text{C}(0^+_1) \rightarrow \alpha + ^{12}\text{C}(2^+_1)$, in which the $2^+_1$ state is a rotational state of the ground $0^+_1$ state with a spatially compact structure. In the inelastic scattering to the $2^+_1$ states, only the matter radii are prominently different, say 2.4 fm for $2^+_1$ and 4.0 fm for $2^+_2$ [2]. Thus, the comparison of these two inelastic scattering is expected to give the evidence of the extended matter radius of the $2^+_2$ state.

2. Theoretical Framework

We calculate the differential cross sections of an $\alpha$ particle scattered by $^{12}\text{C}$ in the formulation of the microscopic coupled-channels (MCC) calculation [6, 10]. In the MCC calculation, the nuclear potential of $\alpha$ and $^{12}\text{C}$ is calculated from the double folding model, which is symbolically written as a function of the $\alpha - ^{12}\text{C}$ relative coordinate of $\mathbf{R}$,

$$V_{f,i}(\mathbf{R}) = N_R \int \int \rho_{f,i}^{(12\text{C})}(\mathbf{r}_1)\rho^{(\alpha)}(\mathbf{r}_2)v_{NN}(\mathbf{s})\,d\mathbf{r}_1d\mathbf{r}_2$$

with $\mathbf{s} = \mathbf{r}_2 - \mathbf{r}_1 - \mathbf{R}$. Here $\mathbf{r}_1$ ($\mathbf{r}_2$) denotes a coordinate measured from the center of mass in the $^{12}\text{C}$ ($\alpha$) nucleus. $\rho_{f,i}(\mathbf{r})$ represents the diagonal ($f = i$) or transition ($f \neq i$) densities of $^{12}\text{C}$, which are calculated by the microscopic $3\alpha$ cluster model, resonating group method (RGM) [2], while $\rho^{(\alpha)}(\mathbf{r})$ denotes the density of $\alpha$ particle, which reproduce the charge form factor of the electron scattering. In Eq. (1), $v_{NN}$ represents the effective nucleon-nucleon (NN) interaction which acts between a pair of nucleon contained in the $^{12}\text{C}$ nucleus and the $\alpha$ particle. In the present calculation, we adopt the DDM3Y (Density Dependent Michigan 3-range Yukawa) interaction [11].

The normalization factor $N_R$ is introduced because the folding potential contains a theoretical ambiguity in its strength. Here this factor is set to $N_R = 1.42$ over all the $\alpha$ incident energy, which is consistent to the MCC calculation in Ref. [6]. In addition to the folding potential, we introduce the absorptive potential with the Saxon-Woods form factor in the diagonal ($f = i$) transition in order to simulate other reaction process [6], and the parameter set of Saxon-Woods is tuned so as to reproduce all of the observed differential cross sections as much as possible. As for the internal excitation of $^{12}\text{C}$, we include the low-lying collective states ($2^+_1$ and $3^+_1$) and the $3\alpha$ cluster states ($0^+_2$, $0^+_3$, $2^+_2$) in addition to the ground $0^+_1$ state.

3. Results

We have solved the coupled-channels equation for the $\alpha + ^{12}\text{C}$ scattering at $E_{\alpha} = 386$ MeV [9] with the nuclear interaction in Eq. (1). The MCC calculation reproduce the differential cross sections of the scattering to $0^+_1$, $2^+_1$, $3^+_1$ states. Furthermore, the theoretical calculation nicely reproduce the inelastic scattering to the excited state observed at $E_x \sim 10$ MeV, which is considered as the incoherent mixture of $0^+$ and $2^+$ strength, by taking the summation of the differential cross sections of the $0^+_2$ and $2^+_2$ states.

A comparison of the $2^+_1$ cross section (dotted curve) and the $2^+_2$ one (solid curve) is shown in Fig. 1 (a). The peak position of the $2^+_2$ cross section (solid curve) shifts to the forward angular region, and its angular distribution has the shrinkage and rapid fall-down structure in comparison to the $2^+_1$ cross section (dotted curve). The shift and shrinkage features are completely consistent to the result of the multi-pole decomposition analysis (MDA) of the experimental cross section, which is performed in the range of $\theta_{c.m.} < 15^\circ$ [9].

In order to analyze the difference of the differential cross sections shown in Fig. 1 (a) more deeply, we have performed the partial wave analysis, in which the angle-integrated cross sections of Fig. 1 (a) are decomposed into the individual components of the incident orbital spin $L$. The results of the decomposition are shown in Fig. 1 (b). The dotted curve with the crosses shows the partial wave distribution ($L$-distribution) of the angle-integrated $2^+_1$ cross section, while the $L$-distribution of $2^+_2$ is plotted by the solid curve with the open circles.
Figure 1. Panel (a): Theoretical differential cross sections of the final state of $^{2+}_{1}$ (dotted curve) and $^{2+}_{2}$ (solid curve) in the $\alpha + ^{12}\text{C}$ inelastic scattering. The magnitudes of both the cross sections are normalized by the maximum value around $\theta_{c.m.} \sim 5^\circ$. Panel (b): Partial wave decomposition of the angle integrated cross sections calculated as the function of the incident orbital spin $L$ ($L$-distribution). The open circles with the dotted curve and the solid circles with the solid curve show the partial cross sections of the $^{2+}_{1}$ and $^{2+}_{2}$ final states, respectively. The magnitudes of all the partial cross section is normalized by the total cross section, which is the summations of the partial cross sections.

In the comparison of $^{2+}_{1}$ with $^{2+}_{2}$, we can clearly see the extended $L$-distribution of $^{2+}_{2}$ to the higher $L$ region. This extension of $^{2+}_{2}$ in the $L$-space (Fig. 1 (b)) is just opposite to the shrinkage in the $\theta$-space, the differential cross section (Fig. 1 (a)). This is nothing but the uncertainty relation of $L \cdot \theta \sim 1$, which can be hold in the diffraction scattering. According to the classical relation of $L = kb$, where $k$ and $b$ denote the incident wave number and the impact parameter, respectively, the extension of the $L$-distribution in Fig. 1 (b) clearly means that the production area of the $^{2+}_{2}$ state is more extended than the area of the $^{2+}_{1}$ production.

The distribution in the $L$-space can be transformed into a size of the production area of the final state by applying the method of the scattering radius [12]. In this method, the effective orbital spin $\hat{L}$ for the transition of $0^+ \rightarrow 2^+$ is derived according to the following expression:

$$\hat{L}_{2^+_{0^+}} = \sqrt{\sum_{L} \frac{L^4 \sigma_{2^+_{0^+}}(L)}{L^2 \sigma_{2^+_{0^+}}(L)}}$$

with a definition of $\hat{L} = \sqrt{L(L+1)}$. Here $\sigma_{2^+_{0^+}}(L)$ represents the partial cross section for the $0^+ \rightarrow 2^+$ transition at the incident orbital spin $L$. The radius of the final-state production, which is called the scattering radius ($R_{SC}$), is simply obtained according to the classical relation of $L = kR_{SC}$. $R_{SC}$ naturally goes to the matter radius of a target nucleus in the high energy limit of the proton elastic scattering [5, 12, 13]. The resultant scattering radius for $^{2+}_{1}$ is $R_{SC}(^{2+}_{1}) = 4.58$ fm, while the radius for $^{2+}_{2}$ is $R_{SC}(^{2+}_{2}) = 5.20$ fm. The difference of the scattering radius is $\Delta R_{SC} = R_{SC}(^{2+}_{2}) - R_{SC}(^{2+}_{1}) = 0.62$ fm.

We have extended the MCC calculation with the method of the scattering radius to the lower $\alpha$ incident energy of $E_{\alpha} \leq 240$ MeV. In all the energy region considered, $\Delta R_{SC}$ ranges from 0.6
fn to 1 fm, which means that $R_{SC}(2^+_2)$ is larger than $R_{SC}(2^+_1)$ over the entire incident energy. From the values of $\Delta R_{SC}$, we can speculate the lower bound of the matter radius of the $2^+_2$ state. In a naive consideration, we can image the relation of $\Delta R_{SC}$ and the matter radius $R_m$, such as

$$R_m(2^+_2) \geq R_m(2^+_1) + \Delta R_{SC}.$$  (3)

In the present analysis, $\Delta R_{SC} \sim 1$ fm is predicted in the lower energy region of $E_\alpha \leq 240$ MeV. The matter radius of $2^+_1$ is still unknown but we can safely assume $R_m(2^+_1) \sim R_m(0^+_1)$. This assumption is because the $2^+_1$ state is the first excited state of the ground $0^+_1$ state and hence, we can expect that the spatial size of $2^+_1$ is not drastically changed from the size of the ground $0^+_1$. This assumption is also supported by the $3\alpha$ RGM calculation [2]. Therefore, the matter radius of the Hoyle rotational state is speculated to $R_m(2^+_2) \geq 3.7$ fm if we employ the experimental value of $R_m(0^+_1) = 2.7$ fm [14].

4. Summary and discussions

In summary, we have shown that the enhanced radius of the $2^+_2$ state can be probed via the differential cross section by comparing with the respective cross section of the $2^+_1$ state. The inelastic differential cross section of the $2^+_2$ state is more shrunk than that of the $2^+_1$ state, and this shrinkage is a first evidence of the extended matter radius of the final $2^+_2$ state in the inelastic scattering of $\alpha + {^{12}}C(0^+_1) \rightarrow \alpha + {^{12}}C(2^+_2)$. According to the prescription of the partial wave decomposition, the shrunk structure of the differential cross section corresponds to about $0.6 \sim 1$ fm enhancement in the size of the production area of the final $2^+_2$ state. The comparison of the yrast $2^+_2$ state and the Hoyle rotational $2^+_2$ state is a new insight in the discussion of the inelastic scattering to the Hoyle $0^+_2$ state [4, 5, 6, 7, 8].

About 1 fm enhancement in the matter radius of the Hoyle rotational state is comparable to the extended radius in $^{11}$Li [15], which largely deviates from the systematics of $\propto A^{1/3}$. $^{11}$Li corresponds to the excited state from $^{11}$B, in which the isospin degrees of freedom is excited. The excitation energy of $^{11}$Li, (B.E.$(^{11}$Li) − B.E.$(^{11}$B)) is about 34 MeV, while the excitation energy of $^{12}$C$(2^+_2)$ is just about 10 MeV. Thus, the 1 fm enhancement of the matter radius in $^{12}$C$(2^+_2)$ is exotic phenomena, which occurs in much lower excitation energy than the neutron-excess nucleus.

References

[1] Ikeda K et al 1980 Prog. Theor. Phys. 68 1
[2] Fukushima Y and Kamimura M 1977 Proc. Int. Conf. Nuclear Structure, J. Phys. Soc. Jpn. 44 225; Kamimura M 1981 Nucl. Phys. A 351 146
[3] Funaki Y, Thosaki A, Horiuchi H, Schuck P and Reöpke G 2005 Eur. Phys. J. A 24 321 and references therein
[4] Danilov A N, Belyaeva T L, Ganchalov S A and Ogloblin A A 2009 Phys. Rev. C 80 054603
[5] Iida K, Koide S, Kohama A and Oyamatsu K 2012 Mod. Phys. Lett. A 27 1250020
[6] Ohkubo S and Hirabayashi Y 2004 Phys. Rev. C 70 041602(R)
[7] Takashina M and Sakuragi Y 2006 Phys. Rev. C 74 054606
[8] Takashina M 2008 Phys. Rev. C 78 014602
[9] Itoh et al 2011 Phys. Rev. C 84 054308 (2011)
[10] Ito M, Hirobayashi Y and Sakuragi Y 2002 Phys. Rev. C 66 034307 and references therein
[11] Kobos A M, Brown B A, Hodgson P E, Satchler G R and Budzanowski A 1982 Nucl. Phys. A 384 65
[12] Tomita M, Iwasaki M, Otani R and Ito M 2014 Phys. Rev. C 89 034619
[13] Kohama A, Iida K and Oyamatsu K 2016 J. Phys. Soc. Jpn 85 094201 and references therein
[14] Kim J C, Hicks R S, Yen R, Auer I P, Caplan H S and Bergstrom J C 1978 Nucl. Phys. A 297 301
[15] Tanihata I, Suwajilos H and Kanungo R 2013 Proc. Part. Nucl. Phys. 68 215