Mathematical Modeling of dry Sliding Wear Behaviour of Stir – Squeeze cast Aluminium Matrix Composite

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Abstract: An attempt was made through this investigation to establish relationship between the input parameter and the outcome of the process by generating expression using Dimensional analysis for dry sliding wear behaviour of aluminium matrix composites reinforced with 2.5, 5 and 7.5 weight percentage of fly-ash particles synthesized using stir-squeeze cast method. Then the produced composite specimens were subjected to dry sliding wear test using pin-on-disc setup. Empirical modeling using Minitab -17 statistical tool was carried out to find the effectiveness and contribution of input parameter over the process outcome. Validation trials were also carried out for the intermediate values of input parameters to justify the results. Finally theoretical wear values generated by the analytical and empirical modeling methods were compared with the experimental values to justify the validity of the model. Finally validation trials were modeled using R- Tool program.

Keywords: Aluminium matrix composite, Stir – squeeze casting, Sliding wear, Mathematical modeling, Dimensional Analysis, Minitab-17 analysis, R- Tool program

1. Introduction

Investigation on composites gained priority in terms of their processing methods, enhancement of properties, improvement of tribological behavior, better understanding of wear mechanisms, optimization of process parameters & modeling. The enormous applications of metal matrix composites are based on their wear behavior which in turn depends on a number of operational and material related conditions in a complex manner such as chemical composition, microstructure and shape, size, content and mode of distribution of the micro constituents, hardness, strength, cracking tendency, lubricating characteristics, load bearing capability as well as thermal stability and interactions between them so that they can be tailored for specific applications by proper selection of parameters. The sliding wear behavior is controlled by a number of experimental parameters such as sliding velocity, normal load, sliding distance, density, hardness, counterpart and other mechanical properties. So material properties and test parameters need to be optimized for better wear behavior of composites [1]

A modeling is a method of establishing a physical system so as to conveniently represent the various process parameters which is linked to the final output. Through modeling, the better understanding of real world systems and problems can happen which is a simplified representation of some real object or physical situation which serves a particular purpose. Modeling enables to forecast the effect of changes to the system. A good model is always an optimized balance between reality and simplicity. There are different types of modeling techniques. Empirical models are used for the huge collection of
data whereas analytical modeling involves basic physics of the process to establish mathematical equations to connect the inputs and outputs of the process. Simulation method involves the virtual validation of the process.

3D finite-element models were used as multiscale approach to Al₂O₃ fibers reinforced aluminum matrix composite (AMC) failure in order to analyze the load transfer from broken to unbroken fibers as a function of matrix yield strength. These investigations demonstrated that the local critical damage level must be attained to drive global failure. The analytical models for strength provided information on reliability versus size and were used in stochastic finite element models [2]. A low cycle fatigue model was developed to predict the fatigue life of both the AA6061 aluminum alloy and Al₂O₃/Saffilshort-fiber reinforced AMC, based on crack initiation and propagation assuming that fatigue-damaged zone ahead of the crack tip promotes actual degradation of the material. It was found that the low-cycle fatigue crack growth was controlled by amount of cyclic plasticity generated within the fatigue-damaged zone ahead of the crack tip [3]. The potential of Ziegler’s continuum principles for the development of processing maps considering instability condition was demonstrated on the published flow stress data of Al 6061 -10 vol. % Al₂O₃ particulate reinforced metal matrix composite. The ‘stable’ and ‘unstable’ regions in the processing maps were identified & the optimum hot working conditions were suggested for the composites [4]. An attempt was made to model the experimental values of mechanical properties and wear resistance of Al 2024 alloy reinforced with different vol. % of coated boron carbide particles by using the combined effects of FEM method, Artificial neural networks and parallel repetitive genetic algorithm method to predict optimal solidification conditions in the production of the composite with minimum wear and maximum strength [5].

In this regard, it has gained greater importance in raising new dimension in the modeling of characterisation and dry sliding wear behaviour of AMCs. Hence an attempt has been made to model the dry sliding wear behaviour of stir – squeeze cast AMC,s using dimensional analysis, Minitab – 17 analysis, followed by modeling of validation trials using R-Tool program.

2. Experimentation

LM – 25 aluminium alloy reinforced with 2.5, 5 & 7.5 wt. % of fly ash reinforced stir squeeze cast AMC’s were fabricated using three –phase electrical resistance furnace & 20 T hydraulic squeeze press. Squeeze pressure was 50 kg/cm² and the squeeze time was 15 minutes. 8-10 mm dia and 30 mm length cylindrical specimens were prepared from the produced composite and dry sliding wear tests were carried out using pin-on-disc setup by varying normal load, sliding velocity & sliding distance. The regular experiments were framed as per Taguchi’s full factorial method and the details are given in table1. The amount of wear was recorded in terms of weight loss. The experiments were carried out for all three composites along with the base metal for the comparison purpose & the results were tabulated. Validation trials were conducted for the intermediate values of the process parameters to assess the precision & accuracy of the regular results as per Taguchi’s full factorial method and the details are given in table 2.

| Table 1. Process parameters with their values at three levels for regular trials |
|---------------------------------------------------------------|
| **Factors** | **Unit** | **Level 1** | **Level 2** | **Level 3** |
| Load | Newton | 4.5 | 9.8 | 14.7 |
| Sliding velocity | m/s | 0.9423 | 1.88 | 2.8274 |
| Sliding Distance | m | 1000 | 2000 | 3000 |
Table 2. Process parameters with their values & levels for validation trials

| Factors          | Unit   | Level 1 | Level 2 |
|------------------|--------|---------|---------|
| Load             | Newton | 7.15    | 12.25   |
| Sliding velocity | m/s    | 1.41    | 2.354   |
| Sliding Distance | m      | 1500    | 2500    |

2.1 A Mathematical modeling using Dimensional Analysis for dry sliding wear behaviour of AMC’s

Dimensional analysis is an analytical modeling technique involving identification of repeating, non-repeating & independent variables of the process. Then number of Π-terms were decided on the basis of Buckingham pi-theorem. The next step was the formation of pi-terms by multiplying one of the non-repeating variables by the product of the repeating variables, each raised to an exponent such that the combination remain dimensionless. Formed Π-terms were solved for exponential constants & new Π-term was formed by combining all the Π-terms.

The final expression linking the process parameters with the process outcome was generated & checked for dimensional homogeneity.

The following six parameters were taken into consideration to establish the theoretical model for dry sliding wear.

\[ \Delta w = f( L, V, D, H, \rho) \]

\( \Delta w \) – Amount of wear / wear rate in kg
\( \rho \) – Density in kg / m³
\( D \) – Sliding distance in m
\( V \) - Sliding velocity in m / s
\( L \) - Applied load in N (kg.m / s²)
\( H \) - Hardness of the material in kg / m – s²

where \( \rho \) – Fluid property
\( V \) – Kinematic property
\( D \) - Geometrical property

Hence \( \rho, V \) and \( D \) are repeating variables and the basic dimensions considered were M, L & T

Total number of parameters ‘n’ = 6
Total number of basic dimensions = ‘m’ = 3
Therefore total number of Π terms = ‘n – m’ = 6 – 3 = 3 terms.

\( \Pi_1 = (\rho^{a_1}, V^{b_1}, D^{c_1}, \Delta w) \)
\( \Pi_2 = (\rho^{a_2}, V^{b_2}, D^{c_2}, L) \)
\( \Pi_3 = (\rho^{a_3}, V^{b_3}, D^{c_3}, H) \)

On solving for \( \Pi_1 \), we have,

\[ M^0 L^0 T^0 = (ML^{-3})^{a_1} (LT^{-1})^{b_1} (L)^{c_1} \]

\( M \rightarrow 0 = a_1 + 1 \Rightarrow a_1 = -1 \)
\( L \rightarrow 0 = -3a_1 + b_1 + c_1 \Rightarrow 0 = -3(-1) + 0 + c_1 \Rightarrow c_1 = -3 \)
\( T \rightarrow 0 = -b_1 \)

Hence \( a_1 = -1, b_1 = 0 \) & \( c_1 = -3 \)

\[ \Pi_1 = \rho^{-1} V^0 D^{-3} \Delta w \Rightarrow \Pi_1 = [\Delta w / (\rho^* D^3)] \] (1)
On solving for $\Pi_2$, we have,

$$M^0 L^0 T^0 = (ML^3)^{a_2} (LT^{-1})^{b_2} (L)^{c_2} M L T^{-2}$$

$M \rightarrow 0 = a_2 + 1 \Rightarrow a_2 = -1$

$L \rightarrow 0 = -3a_2 + b_2 + c_2 + 1 \Rightarrow 0 = 3(-1) - 2 + c_2 + 1 = 0 \Rightarrow c_2 + 2 = 0 \Rightarrow c_2 = -2$

$T \rightarrow 0 = -b_2 - 2 \Rightarrow b_2 = -2$

Hence $a_2 = -1$, $b_2 = -2$ and $c_2 = -2$

$$\Pi_2 = \rho^{-1} \frac{V^{-1} D^2 L}{\Pi_2 = \frac{L}{(\rho * V^2 D^4)}}$$

(2)

On solving for $\Pi_3$, we have,

$$M^0 L^0 T^0 = (ML^3)^{a_3} (LT^{-1})^{b_3} (L)^{c_3} M L T^{-2}$$

$M \rightarrow 0 = a_3 + 1 \Rightarrow a_3 = -1$

$L \rightarrow 0 = -3a_3 + b_3 + c_3 - 1 \Rightarrow 0 = 3(-1) - 2 + c_3 - 1 = 0 \Rightarrow c_3 + c_2 - 3 = 0 \Rightarrow c_3 = 0$

$T \rightarrow 0 = -b_3 - 2 \Rightarrow b_3 = -2$

Hence $a_3 = -1$, $b_3 = -2$ and $c_3 = 0$

$$\Pi_3 = \rho^{-1} \frac{V^{-1} D^4 H}{\Pi_3 = \frac{H}{(\rho * V^2)}}$$

(3)

$$\Pi_{(new)} = \phi (\Pi_2 * \Pi_3) \Rightarrow [\Delta w / (\rho * D^4)] = \phi [ L / (\rho * V^2 D^4)] * \frac{H}{(\rho * V^2)}$$

$$\Rightarrow [\Delta w / (\rho * D^4)] = \phi [L^*H^* / (\rho * V^2 D^4)]$$

$$\Delta w = \phi \frac{[L^*H^* / (\rho * V^2 D^4)]}{[L^*H^* / (\rho * V^2 D^4)]}$$

$$\Rightarrow \Delta w = \phi [L^*H^* / (\rho * V^2 D^4)]$$

(4)

On applying basic units for all dimensions to both LHS & RHS of the above equation 4,

To verify the dimensional homogeneity, we have

$$\Delta w = \phi \frac{[L^*H^* / (\rho * V^2 D^4)]}{[L^*H^* / (\rho * V^2 D^4)]}$$

$$M = \phi \frac{[M^*L^*T^4 / (M^*L^*T^4)]}{[M^*L^*T^4 / (M^*L^*T^4)]}$$

$$\Rightarrow M = \phi$$

$$\Rightarrow \phi = 1$$

2.2 Density & Hardness tests

Densities of the prepared composite specimen were calculated by measuring mass in an accurate electronic balance and calculating the volume of the cylindrical specimen by using the mathematical formula $V = \pi r^2 h$ where $r$ is the radius of the circular face and $h$ is the height of the specimen. Then density is calculated by dividing the mass [gm] by volume [cm$^3$].

Brinell hardness tests were carried out on cylindrical test specimen of 15 mm diameter and 20 mm length in order to investigate the influence of fly ash particles on the matrix hardness by applying a load of 500 kg through the steel ball indenter of 10 mm diameter.

2.3 Modeling studies

i) Dimensional analysis was carried out to determine the theoretical wear and the same was compared with the experimental wear values to validate the model.

ii) Minitab-17 statistical tool was used on the generated experimental wear data to carry out ANOVA to assess the percentage contribution of each input parameter over the output.

iii) Validation trials were modeled using regression equations generated by the R – tool Program and theoretical wear values were calculated to validate the model.
3. Results & discussion

3.1 Results of dry sliding wear test for regular trials

Fig. 1 represents bar chart analysis for dry sliding wear behaviour of the base metal, 2.5, 5 & 7.5 % fly ash reinforce AMC’s. From the bar chart, it is evident that wear resistance is very low for the base metal without reinforcement and got increased continuously with the increase in the percentage reinforcement of fly ash. The common observation recorded was that the wear being least for the composite with 7.5% fly ash reinforcement and high for the base metal. This is because; hardness of the fly ash particles embedded in the soft ductile aluminium matrix contributes for the enhancement of the wear resistance of the composite with its continuous contributes for the enhancement of the wear resistance of the composite with its continuous addition. It also facilitates in bearing load transferred by the matrix material.

![Figure 1. Bar chart for dry sliding wear behaviour of base metal and 2.5, 5 & 7.5 wt. % fly ash reinforced AMC’s for regular trials](image)

3.2 Results of dry sliding wear test for validation trials

Confirmation trials were carried out for the intermediate values of process parameters to verify the accuracy, precision and repeatability of the sliding wear process for base metal LM – 25 and 2.5, 5 & 7.5 wt. % fly ash reinforced stir – squeeze cast AMC’s. The results justified the regular trials by following the same trend. This is reflected in fig 2.
3.3 Results of Density test

Fig. 3 represents variation of densities with the varying fly ash percentage of the aluminium matrix composite. Calculated densities of the produced composites clearly indicate that % reinforcement of the fly ash is inversely proportional to the density of the composite. So higher the % reinforcement, lower will be the density of the composite. Extended study of this revealed that there is 30% increase in the density of squeeze cast composite compared with stir cast composite for the similar reinforcement (5%) conditions [6]. Slight decrease in the density was observed with compared Al-Cu Fly ash squeeze cast composites [7].

3.4 Results of Hardness test

Fig. 4 represents variation of BHN with the varying fly ash percentage of the aluminium matrix composite. BHN tests on the base metal and the squeeze cast Al–fly ash composites revealed that incorporation of fly ash particles into the LM-25 matrix increases the hardness. LM – 25 matrix contains silicon needles and fly ash also comprised of silicon content; hence contribute to the increased hardness of the final composites. Higher the % reinforcement of fly ash particles, greater will be the hardness. The percentage reinforcement and hardness are directly proportional to one another since fly ash particles are hard and rigid. Also silicon content present in the matrix as well
as in fly ash contributes in enhancing the hardness of the composite. Extended study of this revealed that there is 5.7% increase in hardness of squeeze cast composite compared with stir cast composite for the similar matrix-reinforcement (7.5%) combination [6]. The stir cast aluminium composite with 25% reinforcement of SiCp showed 45.5 BHN [8] whereas this present research work with 7.5% reinforced fly ash squeeze cast composite showed 74 BHN which stands much superior and hence proves squeeze cast technique. Al–10 vol.% Al₂O₃–5 vol.% Graphite hybrid composite produced by stir casting method exhibits lower hardness (47.5 BHN) compared with the present squeeze cast composites [9]. Al - 9% fly ash - 9% SiC reinforced stir cast hybrid composite ended with a BHN value of 71 which is lower than the present investigation hardness result for only fly ash reinforcement of 7.5% [10]. Al-SiC-fly ash hybrid composite produced through stir casting method showed very low BHN values with the incorporation of 5 & 10% of fly ash compared with the present research work [11]. In stir–squeeze casting, the molten mixture (matrix and reinforcement) is squeezed with predetermined squeeze pressure. The squeezing results in densification of composite structure thereby reducing the porosity. This enhances the bonding strength between the matrix and the reinforcement particles which resists the penetration of indenter. Hence, densification increases hardness of the final composite material.

![Figure 4. Variation of BHN with fly ash percentage](image)

**Figure 4.** Variation of BHN with fly ash percentage

### 3.5 Results of Dimensional Analysis

Through dimensional analysis and using equation 4 of section 2 discussed earlier, the constant ‘ϕ’ value for all trials for base metal LM-25 and 2.5, 5 & 7.5 wt. % fly ash reinforced AMC’s for dry sliding wear studies were calculated. Then for a specific speed & particular load and sliding distance ranges, a common ‘ϕ’ values were calculated. Then, using this common ‘ϕ’ values, the theoretical wear values were computed and compared with experimental wear to calculate the percentage error. The deviations were well within ±15%. Fig. 5 represents the comparison of experimental wear values with the theoretical wear values obtained by dimensional analysis for all four materials.
Figure 5. Bar chart representation of experimental v/s theoretical wear values through Dimensional analysis for base metal & three composites.
3.6 Results of Minitab -17 Analysis

Table 3. Analysis of variance for Base metal & three composites using Minitab Analysis

| Material                        | Source                    | DF  | Adj SS   | Adj MS  | F- value | P- value | % C |
|---------------------------------|---------------------------|-----|----------|---------|----------|----------|-----|
| LM - 25                         | Load (N)                  | 2   | 0.000244 | 0.000122| 10.43    | 0.001    | 10.7|
|                                 | Sliding velocity (m/s)    | 2   | 0.000858 | 0.000429| 36.69    | 0.000    | 40.7|
|                                 | Sliding distance (m)      | 2   | 0.000716 | 0.000358| 30.65    | 0.000    | 33.7|
|                                 | Error Total               | 26  | 0.002034 | 0.000122| Interaction of all Total | 14.9 | 100 |
| 2.5 % fly ash reinforced        | Load (N)                  | 2   | 0.000158 | 0.000079| 9.37     | 0.001    | 10.7|
| aluminium matrix composite      | Sliding velocity (m/s)    | 2   | 0.000491 | 0.000245| 29.19    | 0.000    | 35.7|
|                                 | Sliding distance (m)      | 2   | 0.000510 | 0.000255| 30.31    | 0.000    | 37.2|
|                                 | Error Total               | 26  | 0.001368 | 0.000008| Interaction of all Total | 16.4 | 100 |
| 5 % fly ash reinforced          | Load (N)                  | 2   | 0.000221 | 0.000111| 18.57    | 0.000    | 22.5|
| aluminium matrix composite      | Sliding velocity (m/s)    | 2   | 0.000224 | 0.000112| 18.81    | 0.000    | 22.8|
|                                 | Sliding distance (m)      | 2   | 0.000364 | 0.000182| 30.5     | 0.000    | 37.9|
|                                 | Error Total               | 26  | 0.000119 | 0.000006| Interaction of all Total | 16.8 | 100 |
| 7.5 % fly ash reinforced        | Load (N)                  | 2   | 0.000126 | 0.000063| 12.88    | 0.000    | 19.0|
| aluminium matrix composite      | Sliding velocity (m/s)    | 2   | 0.000103 | 0.000051| 10.52    | 0.000    | 15.3|
|                                 | Sliding distance (m)      | 2   | 0.000283 | 0.000141| 29.01    | 0.000    | 44.8|
|                                 | Error Total               | 26  | 0.000098 | 0.000005| Interaction of all Total | 20.9 | 100 |

ANOVA is used to estimate the relative percentage contribution of each process parameter and their interactions on the measured response and to estimate the error variance. The statistical model can be constructed using this data to forecast the probable wear rate for various combinations for 95 % confidence level. The table 3 reflects analysis of variance for base metal LM – 25 and 2.5, 5 & 7.5 % fly ash reinforced stir – squeeze cast aluminium matrix composites respectively under dry sliding situation. ANOVA analysis for dry sliding wear for base metal and different composites reveal that the all three process parameters have their influence on the wear rate as P-value less than 0.5. It also indicates that the percentage contribution of applied load on wear slightly increases from base metal to 7.5 % fly ash reinforcement composite whereas the percentage contribution of speed over wear drastically reduces from base metal LM – 25 to 7.5 % fly-ash reinforcement composite. Similarly, the percentage contribution of sliding distance over wear keeps on increases from base metal to 7.5 % fly ash reinforcement composite. This result presents that sliding distance is a more dominating factor on the wear
process and applied load is the second influential factor in regulating wear but speed is inversely proportional to wear as it moves from base metal to 7.5% percentage fly ash reinforced composites. Addition of fly ash resists wear against the increased speed. Effectiveness of interaction of all process parameters got increased with increased fly ash content of the composite.

3.7 Results of R-Tool program

Regression equations were obtained by using R-tool program for the dry sliding wear response of base metal LM-25 and 2.5, 5 & 7.5% fly ash reinforced stir squeeze cast aluminium matrix composites. The wear values obtained through these regression equations were compared with the experimental wear values. Error analysis was carried out and the percentage error was calculated to assess the validity of the obtained results. It was found that all the trials of the base metal and three composites pertaining to dry sliding wear behaviour fell within –15% to +15%. Hence the validity of the model is justified. This is reflected in fig 6.
4. Conclusions

- The Stir – Squeeze cast fabrication of fly ash reinforced aluminium matrix composites were successfully carried out to derive the benefits of both stir and squeeze cast techniques & the densification of the fabricated composites got improved. Increase of 30 % in terms of density was achieved when compared with stir – cast composite of similar reinforcement under same operating conditions.

- BHN value of the squeeze cast composite got increased by 5.7 % compared with stir – cast composite of similar reinforcement under same operating conditions. Investigations revealed that fabricated squeeze cast composite found much superior in terms of hardness compared with composites with higher percentage reinforcement of SiCp and also hybrid composites.

- Stir - squeeze cast aluminium fly ash composites exhibited higher wear resistance under dry sliding compared with stir – cast composite for the similar reinforcement and similar operating conditions.

- Mathematical / analytical (Dimensional Analysis) and empirical modeling (Minitab and R-Tool) approaches were carried out to realize the validity of the experimental results.

- All three modeling results varied between -15 % to +15 % and hence found satisfactory.
Out of three modeling approaches, the priority order found for this investigation was Mini-
tab analysis, followed by dimensional analysis and R – tool programming respectively.

This approach can be applied to any manufacturing activity to forecast the output for the
future considering various input factors and their levels and also by using different mode-
ing methods. The evaluation of the alternative modeling methods along with cost analysis
can be done to attain optimization

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