Calculation of Heat-Bearing Agent’s Steady Flow in Fuel Bundle

E V Amosova¹², G G Guba¹

¹Department of Mechanics and Mathematical Modeling, Far Eastern Federal University, 8, Sukhanova St., Vladivostok 690090, Russia
²Institute of Applied Math, Far Eastern Department of RAS, 7, Radio St., Vladivostok 690041, Russia

E-mail: el_amosova@mail.ru

Abstract. This paper introduces the result of studying the heat exchange in the fuel bundle of the nuclear reactor’s fuel magazine. The article considers the fuel bundle of the infinite number of fuel elements, fuel elements are considered in the checkerboard fashion (at the tops of a regular triangle a fuel element is a plain round rod. The inhomogeneity of volume energy release in the rod forms the inhomogeneity of temperature and velocity fields, and pressure. Computational methods for studying hydrodynamics in magazines and cores with rod-shape fuel elements are based on a significant simplification of the problem: using basic (averaged) equations, isobaric section hypothesis, porous body model, etc. This could be explained by the complexity of math description of the three-dimensional fluid flow in the multi-connected area with the transfer coefficient anisotropy, curved boundaries and technical computation difficulties. Thus, calculative studying suggests itself as promising and important. There was developed a method for calculating the heat-mass exchange processes of inter-channel fuel element motions, which allows considering the contribution of natural convection to the heat-mass exchange based on the Navier-Stokes equations and Boussinesq approximation.

1. Introduction

Today, nuclear power industry is rapidly growing in the world. The most significant tasks engineer face during designing a reactor, include enhancing its reliability and efficiency in comparison with existing reactors. The current methods for determining distribution of temperature and velocity fields are based on problem simplification: either porous body models are applied, or channel models are used [1,2]. For channels formed by spiral fuel element bundles and tubes [3,4] methods for solving steady- and unsteady-state problems, and studies of the flow’s turbulence pattern were carried out. The [5] introduces the three-dimensional calculation of the thermohydralic problem for the spiral-finning and equally-oriented infinite fuel bundle.

The paper presents the flow of heat transfer viscous incompressible fluid in between fuel elements forming the fuel magazine. A heat-bearing agent located in the primary circuit of the reactor, exercises heat abstraction from fuel elements’ surface, and can act as an inhibitor regulating fuel elements’ temperature. In this paper, we assume the entire heat release is concentrated in fuel elements, a heat-bearing agent is heated by the fuel element’ surface by virtue of thermal conductivity, heat flow density on a fuel element’s surface is not time-dependent and considered given.
2. Math modeling
Channels formed by cylindric-element bundles are characterized by the complex three-dimensional flow of the heat-bearing agent. The paper presents the study of the heat-bearing agent’s flow structure with variable properties in the isolated inter-channel area of the nuclear reactor’s core, with consideration of inhomogeneity of heat release distribution across the core and during the campaign. Flat section of the core part distant from boundaries is shown in Figure 1.

Figure 1. Fuel Assembly Cross-Section.

Figure 2. Calculation Area Cross-Section with Boundary Marking.

It is expected that the heat release capacity of each fuel element and distances between them are equal, while velocity and temperature fields in the bundle repeat themselves in some area. Let us assume the flow velocity vector is parallel to the bundle axis, we shall consider the distribution of the natural convection. As a calculation area, we select the area between fuel elements, bounded by the lateral surfaces of three adjacent fuel elements and surfaces constituting the mirror plane of the flow – see Figure 2, \( V \subset R^3 \) will stand for the 3D calculation area, \( G \subset R^2 \) for the cross-section of the volume at \( z = 0 \), Figure 2. A lateral boundary of the volume \( V \) will be denoted by \( \Gamma = \Gamma_1 \cup \Gamma_2 \cup \Gamma_3 \cup G_1 \cup G_2 \cup G_3 \), where \( G_1, G_2, G_3 \) are parts of lateral surfaces of three adjacent fuel elements belonging to the calculation area, while \( \Gamma_1, \Gamma_2, \Gamma_3 \) are free surfaces.

The set of equations describing the flow of the viscous heat transfer flue with consideration of natural convection’s effect for non-dimensional variables, is of the following form [6]:

\[
- (u \cdot \nabla)u + \mu \Delta u = \nabla Eu + Gr \cdot Re^{-2} \cdot (T - T_0) \cdot k,
\]

\[
div u = 0,
\]

\[
(u \cdot \nabla T) = \lambda \Delta T, \quad (x,y,z) \in V,
\]

where \( u = (u_1; u_2; u_3) \) stands for non-dimensional velocity vector, \( T \) for non-dimensional flow temperature, \( Eu \) for Euler number. The temperature scale is denoted by \( \Theta_q = q_c \cdot d / \lambda_2 \), which has the temperature dimension, velocity vector scale shall be the velocity of approach flow – \( u_s = 1 m/s \).

Let us lay the boundary conditions. At the channel entrance at \( z = 0 \), velocity vector has the Poiseuille condition. Velocity vector is given by \( u = (0; 0; u_3) \), where \( u_3 \) is the Poisson’s problem solution:

\[
\Delta u_3 = - \Delta p / (2 \cdot \mu \cdot L), \quad (x,y) \in G,
\]
\[ u_j \mid_{\Gamma_1 \cup \Gamma_2 \cup \Gamma_3} = 0, \quad \text{Rot} \ u_j \times n \mid_{\Gamma_1 \cup \Gamma_2 \cup \Gamma_3} = 0. \]  

Temperature on the entrance boundary G at \( z = 0 \) is given as a constant:

\[ T(x, y, 0) = 300^{\circ}C, (x, y) \in G. \]  

On boundaries \( \Gamma_1, \Gamma_2, \Gamma_3 \) velocity vector has periodic conditions:

\[ u_{\mid_{\Gamma_1}} = u_{\mid_{\Gamma_2}} = u_{\mid_{\Gamma_3}}. \]

For temperatures on boundaries \( \Gamma_1, \Gamma_2, \Gamma_3 \) impermeability conditions are laid:

\[ (\nabla T \cdot n)_{\mid_{\Gamma_1}} = 0, \quad (\nabla T \cdot n)_{\mid_{\Gamma_2}} = 0, \quad (\nabla T \cdot n)_{\mid_{\Gamma_3}} = 0. \]

On boundaries \( G_1, G_2, G_3 \) no-slip conditions are given for velocity:

\[ u_{\mid_{G_1}} = u_{\mid_{G_2}} = u_{\mid_{G_3}} = 0. \]

On the boundary of a heat-bearing agent and fuel element, the heat flow density on the case surface is given. For temperature on boundaries \( G_1, G_2, G_3 \), second-order boundary conditions are laid:

\[ -\lambda \left( \nabla T \cdot n \right)_{\mid_{G_1 \cup G_2 \cup G_3}} = Q_v (\varphi, z). \]

We shall consider the reactor without end reflectors. In this case, heat exchange density in the examined magazine, according to [7], will be as follows:

\[ Q_v = q_v (\varphi) \cdot \left( \frac{\pi}{2} \right) \cdot \sin \left[ \left( \frac{\pi}{2} \right) \cdot z \right], \]

where, according to [7],

\[ q_v (\varphi) = 1 - a_6 \cdot 6 \cdot \varepsilon \cdot \cos(6 \cdot \varphi) \]

\[ \varepsilon = \lambda_i \cdot \left( 1 - m \cdot R_i \cdot R_1^{-1} \right) / \lambda_2 \cdot \left( 1 + m \cdot R_i \cdot R_2^{-1} \right), \quad m = (\lambda_2 - \lambda_1) / (\lambda_2 - \lambda_4). \]

Here, \( \lambda_i = 10.13 \left[ W / (m \cdot C) \right] \) (thermal conductivity of fuel pellet), \( \lambda_2 = 19 \left[ W / (m \cdot C) \right] \) (thermal conductivity of case), \( R_i = 0.02 \left[ m \right] \) (fuel pellet radius), \( R_e = 0.0205 \left[ m \right] \) (fuel element radius), \( a_6 = 0.01 \). Figure 3 shows the law of variation of the heat release density along the channel length.

![Figure 3. Heat Flow Density on Fuel Element Surface.](image-url)
In order to find the numerical solution of the problem state (1) – (12), the finite element method is used, implemented in Free Fem++. In order to describe non-linear addends, linearization of moment equations of the initial problem is applied. For the incompressibility equation (2) of the heat-bearing agent, a stabilizing addend is entered by the penalty function method [8]. The iteration process is implemented by using the Newton method [9]. The graph of iteration process convergence is given in Figure 4.

The iteration process is finished once the following condition is met:

\[ \delta = \int \left( |d\mathbf{u}|^2 + dT^2 + |d\rho|^2 \right) dx dy dz < 10^{-6}. \]

The suggested method allows calculating temperature, velocity fields, as well as pressure values for given parameters of the thermohydralic process:

\[ \text{Re} = u_a \cdot d_f \cdot \nu^{-1} = 41523.8, \quad \text{Pe} = u_a \cdot d_f \cdot \alpha^{-1} = 216.015, \quad \text{Gr} = 3.52439 \cdot 10^7, \]

\[ \text{Gr} \cdot \text{Re}^{-2} = 0.0204404, \quad d_f = d_0 \cdot \frac{2\sqrt{3} \cdot b^2}{(\pi \cdot d^2)} - 1 \approx 0.013703, \]

\[ \mu = d_f / (\text{Re} \cdot R_e) = 1.609810^{-5}, \quad \lambda = d_f / (\text{Pe} \cdot R_e) = 0.0031, \]

where \( R_e \) for fuel element radius, \( d_f \) for equivalent diameter, \( \text{Re} \) for Reynolds number, \( \text{Pe} \) for Peclet number, \( \text{Gr} \) for Grashoff number.

The density of fuel element bundling is \( b / d = 1.1 \), where \( b \) stands for the distance between centers of two neighboring fuel elements, \( d \) for fuel element diameter.

**Figure 4. Iteration Process.**

### 3. Findings

For conduction of the numerical experiment, liquid sodium was chosen as a heat-bearing agent. Table 1 shows properties of liquid sodium for 400°C. Liquid sodium can be used in fast-neutron as it slightly moderates neutrons. Liquid metals have a little higher thermal conductivity and higher boiling point compared to water, and the latter property provides for using the lightweight reactor body, which significantly reduces its cost.

**Table 1. Liquid Sodium Properties.**

| Metal | Melting T, \( ^{\circ}\text{C} \) | Boiling Point, \( P=1 \text{ Bar}, \^{\circ}\text{C} \) | Thermal Conductivity Coefficient, \( 400^{\circ}\text{C} \) | Heat Capacity, \( 400^{\circ}\text{C} \) | Density, \( 400^{\circ}\text{C} \), \( \text{kg/m}^3 \) | Prandtl Number, \( 400^{\circ}\text{C} \) |
|-------|----------------|------------------|---------------------|----------------|-----------------|-----------------
| Na    | 97.8           | 883              | 68.8                | 1.26           | 854             | 0.0052          |
Based on the numerical modeling, temperature and velocity fields were determined, and stress vector magnitude on the fuel element’s wall was found for various cross-sections. In case of fairly close bundling and volumetric change of heat flow density on the fuel elements’ surface, heat exchange coefficient turns out to be variable along the circle and length of the channel – see Figure 5.

![Figure 5. Heat Exchange Coefficient.](image)

The heat exchange coefficient reaches its maximum value in places of the largest distance between fuel elements.

The numerical experiment enabled to select the optimal capacity of internal heat sources in the fuel element, which allows the tolerable heat-bearing agent temperature in the inter-channel area. Capacity of internal heat sources in this calculation accounted for \( q_I = 3.5 \cdot 10^7 \text{ W/m}^3 \).

The change of the temperature field of the heat-bearing agent in the inter-channel area of the nuclear reactor, found through numerical modeling, is shown in Figure 6.

![Figure 6. 3D Distribution of Temperature Field.](image)

Inhomogeneity of distribution of temperature fields across the fuel element perimeter is shown in Figure 7. The point \( Z_{\text{max}} \) corresponds to the fuel element height, where the max flow temperature is reached.
Figure 7. Temperature Fields Change Across Fuel Element Perimeter.

Figure 8 depicts that the extremum of case temperature is reached at some distance from the fuel element end, being shifted from the center to the right, where heat flow density on the fuel element surface reaches its maximum on the channel length.

With that, channel-length max temperatures are reached in places of the smallest gap between fuel elements – $\varphi = 0^\circ$, $\varphi = 60^\circ$, see Figure 1. When reaching the maximum, temperature falls, which could be explained by decrease of the heat flow density in the upper part of the fuel element. At the max gap, fuel element surface temperature will fall, and then rise being affected by diffusion processes – see Figure 8.

Inhomogeneity of the temperature field affects pressure changes in the flow. Hydrostatic pressure is taken as $p_0 = 5\text{[MPa]}$. Based on calculation velocity fields and pressure, circle- and length-dependent changes of stress vector magnitude are determined.

Figure 8. Change of Temperature Fields along the Channel Length.

The graph in Figure 9 shows the rangeability of stress vector magnitude for the fuel element surface is not significant ($\approx 0.58\text{[MPa]}$) and changes with the fuel element height. At the smallest gap between fuel elements, stresses are equalized to the constant value. At the largest gap, stress vector magnitude reaches its max and min value – at that, max value is reached on the fuel element surface at a height of the channel length’s max temperature.

Changes of the section-average pressure in the channel is given in Figure 10. The graph shows the pressure falls along the channel length.
Results of solving the heat-mass exchange problem showed increase of the mass-average temperature along the channel length by almost 11°C – see Figure 11.

Based on the suggested calculations, we may infer that this method for solving 3D problems for Navier-Stokes equations provides for modeling the complex movement of the viscous incompressible heat transfer fluid, considering the impact of axial symmetry of the channel.

References
[1] Minashin V E, Sholokhov A A and Gribanov Yu I 1971 Thermal physics of liquid-metal cooling nuclear reactors (Moscow: Atomizdat)
[2] Osmachkin V S 1964 Specific of heat exchange in nuclear reactors cooled by incompressible non-boiling fluids Third Int. Conf. on Peaceful Use of Nuclear Energy (Geneva)
[3] Dzyubenko B V, Kuzma-kichta Yu A, Leont'ev A I, Fedik I I and Kholpanov L P 2008 Macro-, micro- and nano-scale heat-mass exchange augmentation (Moscow: Tsniatominform) p 532
[4] Dzyubenko B V, Sakalauskas A V, Ashmantas A V and Segal' M D 1995 Turbulent flow and heat exchange in energy unit channels (Vilnius: Pradai) p 300
[5] Chukhlov A G, Smirnov V P and Afonin S Yu 2010 Using periodic boundary conditions in thermohydraulic calculation of finned fuel bundles Zhurnal.ape.relarn 31 pp 363–70
[6] Subbotin V I and Ibragimov M Kh 1975 Hydrodynamics and heat exchange in atomic energy units (Moscow: Atomizdat) p 407
[7] Petukhov B S, Genin L G and Kovalev S A 1974 Heat exchange in nuclear energy units (Moscow: Atomizdat) p 408
[8] Roberts J E and Thomas J-M 1989 Mixed and Hybrid Methods in Handbook of Numerical Analysis vol II (North Holland)
[9] Kazufumi I and Kunisch K 2003 Semi Smooth Newton Methods for Variational Inequalities of The First Kind ESAIM: Math. Mod. and Numerical Anal. vol 37 1 pp 41–62