Screening in Hot SU(2) Gauge Theory and Propagators in 3d Adjoint Higgs model

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We investigate the large distance behavior of the electric and magnetic propagators of hot SU(2) gauge theory in different gauges using lattice simulations of the full 4d theory and the effective, dimensionally reduced 3d theory. A comparison of the 3d and 4d data for the propagators suggests that dimensional reduction works surprisingly well down to temperatures \( T \sim 2T_c \). A detailed study of the volume dependence of magnetic propagators is performed. The electric propagators show exponential decay at large distances in all gauges considered and a possible gauge dependence of the electric screening mass turns out to be statistically insignificant.

The poles of the finite temperature gluon propagator can be related to screening of chromoelectric and chromomagnetic fields \([1]\). The static chromoelectric (Debye) screening mass was calculated in leading order of perturbation theory long ago and was found to be gauge independent to this order \([1]\). The existence of a static chromomagnetic screening mass generated non-perturbatively was postulated by Linde to render the perturbative expansion of different thermodynamic quantities finite \([2]\). Beyond the leading order also the Debye screening mass is non-perturbative \([3]\), i.e. depends explicitly on the magnetic mass. Static electric and magnetic propagators and the corresponding screening masses were studied in SU(2) gauge theory in Landau gauge \([4]\). In Ref. \([5]\) it was shown that static propagators could be studied in the effective dimensionally reduced version of finite temperature SU(2) gauge theory, the 3d adjoint Higgs model. Here we will discuss the determination of the screening masses from propagators calculated in different gauges and their gauge dependence. Results calculated within full 4d SU(2) gauge theory as well as the effective 3d theory will be presented.

In four dimensions (4d) all our calculations are performed with the standard Wilson action for SU(2) lattice gauge theory. We will use the notation \( \beta_4 = 4/g_4^2 \) for the lattice gauge coupling. In three dimensions the standard dimensional reduction process leads us to consider the 3d adjoint Higgs model

\[
S = -\beta_3 \sum_P \frac{1}{2} \text{Tr} U_P - \beta_3 \sum_{x, \mu} \frac{1}{2} \text{Tr} A_0(x) U_{\mu}(x) A_0(x + \mu) U_{\mu}^\dagger(x) + \frac{1}{4} \beta_3 \sum_x \left[ (6 + h) \text{Tr} A_0^2(x) + x (\text{Tr} A_0^2(x))^2 \right]. \tag{1}
\]

where \( \beta_3 \) now is related to the dimensionfull 3d gauge coupling and the lattice spacing \( a \), i.e. \( \beta_3 = 4/g_3^2 a \), and the adjoint Higgs field is parameterized by hermitian matrices \( A_0 = \sum_{a} \sigma^a A_0^a \) (\( \sigma^a \) are the usual Pauli matrices). Furthermore, \( x \) parameterizes the quartic self coupling of the Higgs field and \( h \) denotes the bare Higgs mass squared. We also note that the indices \( \mu, \nu \) run from 0 to 3 in four dimensions and from 1 to 3 in three dimensions.

As we want to analyze properties of the gluon propagator, which is a gauge dependent quantity, we have to fix a gauge on each configuration on which we want to calculate this observable. In the past most studies of the gluon propagator have been performed in Landau gauge. Here we will consider a class of \( \lambda \)-gauges introduced in Ref. \([6]\). In the continuum these gauges correspond to the gauge condition

\[
\lambda \partial_0 A_0 + \partial_i A_i = 0. \tag{2}
\]

Here the index \( i \) runs from 1 to 3. The case \( \lambda = 1 \) corresponds to the usual Landau gauge. In addition to the \( \lambda \)-gauges we also consider the Maximally Abelian gauge (MAG) \([6]\). In this case
one has to fix a residual gauge degree of freedom which we do by imposing an additional U(1)-Landau gauge condition. In the 4d SU(2) gauge theory we also consider the static time averaged Landau gauge (STALG) introduced in Ref. [9]. In continuum it is defined by

$$\partial_0 A_0(x_0, x) = 0, \quad \sum_{x_0} \sum_{i=1}^{3} \partial_i A_i = 0. \quad (3)$$

While the notion of Landau and Maximally Abelian gauges carries over easily to the 3d case we have to specify our notion of \(\lambda\)-gauges in 3d. We have considered two versions of \(\lambda\)-gauge,

$$\partial_1 A_1 + \partial_2 A_2 + \lambda_3 \partial_3 A_3 = 0, \quad (4)$$

$$\lambda_1 \partial_1 A_1 + \partial_2 A_2 + \partial_3 A_3 = 0, \quad (5)$$

which we will refer to as \(\lambda_1\)-gauge and \(\lambda_3\)-gauge correspondingly. Furthermore, we consider in 3d the Coulomb gauge, which fixes the gauge in a plane transverse to the \(z\)-direction, \(\partial_1 A_1 + \partial_2 A_2 = 0\). The residual gauge freedom is fixed by demanding that \(\sum_{x,y} U_3(x) = u_{30}\) should be constant.

All our 4d simulations have been performed at temperature \(T = 2T_c\). The values for the 4d coupling \(\beta_4\) corresponding to this temperature were taken from Ref. [4], \(\beta_4 = 2.52\) for \(N_t = 4\) and \(\beta_4 = 2.74\) for \(N_t = 8\). The 3d adjoint Higgs model simulations were done for three sets of parameters, \(\beta_3 = 11\), \(x = 0.099\), \(h = -0.395\) corresponding to \(T = 2T_c\), and \(\beta_3 = 16\), \(x = 0.03\), \(h = -0.2085\) as well as \(\beta_3 = 24\), \(x = 0.03\), \(h = -0.1510\) which correspond to \(T \sim 9200T_c\). For the 3d gauge coupling \(g_3^2\) we always use the 1-loop relation \(g_3^2 = g^2(T)T\) with \(g(T)\) being the 4d 1-loop running coupling constant in \(\overline{MS}\)-scheme with \(\mu = 18.86T\) [12]. The detailed procedure for fixing the parameters of the 3d effective theory is described in Ref. [12].

We note that for static fields the gauge condition for 4d \(\lambda\)-gauges, \(\lambda \partial_0 A_0 + \partial_i A_i = 0\) is equivalent to 3d Landau gauge. The same is true for STALG. One would expect that if dimensional reduction works, propagators calculated in different \(\lambda\)-gauges and STALG agree with each other, and of course, with the propagators calculated in the

3d effective theory in Landau gauge. A comparison of the corresponding data in 4d SU(2) gauge theory and the 3d effective theory shows that this is indeed the case. Furthermore, we have compared the electric and magnetic propagators calculated in 4d SU(2) gauge theory and 3d effective theory in MAG. Good agreement between 3d and 4d data was found also here.

Previous lattice calculations of the magnetic propagator in hot SU(2) gauge theory in 4d [4] and in the 3d adjoint Higgs model [1] gave evidence for its exponential decay in coordinate space and thus suggested the existence of a magnetic mass. In was also found that the magnetic propagators calculated in the 3d adjoint Higgs model are quite insensitive to the scalar couplings and are quite close to the corresponding propagators of 3d pure gauge theory. In Ref. [11] the Landau gauge gluon propagator of 3d gauge theory was studied in momentum space and was found to be infrared suppressed for large volumes. Such a behavior clearly rules out the existence of a simple pole mass. To clarify the picture of magnetic screening a detailed study of finite size effects is necessary. In what follows we will mainly concentrate on a discussion of the magnetic propagators in the limit of the 3d pure gauge theory. Where appropriate, comparison with the results from 4d SU(2) gauge theory will be made. In Figure [1] we show the magnetic propagators in coordinate space on different volumes at \(T = 2T_c\). Calculations were done in 4d SU(2) gauge theory at \(\beta_4 = 2.52\), 2.74 and in 3d pure gauge theory at \(\beta_3 = 8\). On small volumes the propagator indeed shows exponential decay but it continues to drop faster as the volume increases. For volumes \(VT^3 \gtrsim 330\) we have clear evidence that the propagators become negative at \(zT \gtrsim 2\). A similar strong volume dependence was observed in other \(\lambda\)-gauges and also in Coulomb gauge.

Because of the strong volume dependence of the magnetic propagators in \(\lambda\)-gauges simulations on large lattices are necessary to get control over finite size effects. We have analyzed the magnetic propagators in Landau gauge in the 3d pure gauge theory for \(\beta_3 = 5\) and on lattices of size \(L^3\) with \(L = 32, 40, 48, 56, 64, 72\) and 96. In what follows all dimensionfull quantities will be scaled by ap-
appropriate powers of the 3d gauge coupling $g_3$. Using the relation of $g_3$ to the renormalized 4d gauge coupling (see above) it is straightforward to express dimensionfull quantities in units of $T$. The magnetic propagators at $\beta_3 = 5$ and for different volumes are shown in Figure 1b and Figure 1c. As one can see from the figure the volume dependence of the coordinate space propagator is small on these large lattices. The propagator becomes negative for $z g_3^2 \gtrsim 4$. The strong volume dependence translates into infrared sensivity of the momentum space propagators. For $p/g_3^2 < 0.3$ the momentum space propagator is sensitive to the volume, while for large momenta ($p/g_3^2 > 0.3$) it is essentially independent of the lattice size. Moreover, we note that for small momenta the finite volume effects lead to a decrease of $D(p)$ with increasing volume while the volume dependence is already negligible for $p/g_3^2 \approx 0.3$. Thus the magnetic propagators in Landau gauge are infrared suppressed. The propagators in other $\lambda$-gauges show similar behavior. Let us note that the infrared suppression of the Landau gauge propagator was observed also $T = 0$ 4d SU(N) gauge theory [13].

In contrast to the complicated structure found in Landau gauge the propagator calculated in MAG does show a simple exponential decay at large distances and does not show any significant volume dependence. We find for the magnetic screening mass in MAG $m_M = 0.5(5)g_3^3$.

Contrary to the magnetic propagators the electric propagators show little volume dependence and decay exponentially in all gauges considered. We have investigated in detail the gauge dependence of the electric propagators in the 3d effective theory at temperature $T \sim 9200T_c$. The results are summarized in Figure 1. As one can see from this figure any possible gauge dependence of the local electric masses in the region where they reach a plateau is statistically insignificant.

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![Figure 1](image-url)
Figure 2. Local electric masses calculated in $\lambda$-gauges, Coulomb gauge and MAG at $T \sim 9200T_c$. Calculation were done in the 3d effective theory at $\beta_3 = 24$, $x = 0.03$ and $h = -0.1510$ except in the case of $\lambda_1 = 0.1$ where they were done at $\beta_3 = 16$, $x = 0.03$, $h = -0.2085$.

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