Lattice understanding of the Delta I=1/2 rule & some implications

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After decades of intensive efforts, lattice methods finally revealed one clear source of the large enhancement of the ratio \( ReA_0/ReA_2 \), which has been a puzzle in particle physics for about sixty years. Lattice studies of direct \( K \to \pi\pi \) in the \( I = 2 \) channel show that in fact this channel clearly suffers from a severe suppression due to a significant cancellation between the two amplitudes for the original, charged current (tree) operator. One of these amplitudes goes as \( N \) and the other one goes as \( N^2 \), where \( N = 3 \) for QCD. For physical pion masses the cancellation between the two contributions towards \( ReA_2 \) is about 70%. This appreciable cancellation suggests that expectations from large \( N \) for QCD may be amenable to receiving significant corrections. The penguin operators seem to make a small contribution to \( ReA_0 \) at a scale \( \gtrsim 1.5 \text{GeV} \). Possible repercussions of the lattice observation for other decays are briefly discussed.

1 Introduction

Quantitative understanding of the long-standing \( \Delta I = 1/2 \) puzzle and more importantly a reliable calculation of the important direct CP-violation parameter in \( K \to \pi\pi \), \( \epsilon'/\epsilon \) were in fact the primary motivation for my entry into lattice methods for calculating weak matrix elements, about thirty years ago\(^\text{[23,24,20]}\). In fact to tackle this difficult problem and bring it to our current level of understanding and progress has so far taken at least six Ph D theses\(^\text{[7,10,11,12]}\). Indeed, at the time the experimental measurement of \( \epsilon' \) was a huge challenge and it took close to 20 years to completely nail it down experimentally. For the lattice there were numerous obstacles that had to be overcome. First and foremost was lack of chiral symmetry of Wilson fermions entailing mixing with lower dimensional operators\(^\text{[13,14]}\). While this severe difficulty thwarted early attempts for all application to kaon physics (even for kaon-mixing parameter, \( B_K \)\(^\text{[15]}\)), it motivated us to consider applications to heavy-light physics as it was felt that therein chiral symmetry will be less of an issue\(^\text{[16,17,18,19]}\). Many of the important applications to observables relevant to the Unitarity Triangle are in fact off springs of these efforts.

While the primary focus of this article is on developments exclusively from the lattice perspective, we want to use the opportunity to mention some prominent studies of \( K \to \pi\pi \), the

\(^\text{aInvited talk at the EW Moriond 2013}\)
In 1996-97 the first simulations with domain wall quarks (DWQ) demonstrated the feasibility of using this 5-dimensional formulation, even with a modest extent of about 10 sites in the 5th dimension, Domain Wall Quarks (DWQ) exhibited excellent chiral symmetry as the first application to kaon matrix element, in the quenched approximation, showed. With the formation of RIKEN-BNL-Columbia (RBC) Collaboration around 1998 first large scale simulations, in the quenched approximations, with domain wall quarks to $K \to \pi\pi$, $\Delta I = 1/2$ and $\epsilon'$ began. These continued to use chiral perturbation theory (as was the case with the previous attempts with Wilson fermions) to reduce the problem to a calculation of $K \to \pi$ and $K \to \text{vac}$ following. The first results from this approach showed that for $\epsilon'$, quenched approximation is highly pathological. In particular, the QCD penguin operator $Q_6$ which is an $(8,1)$ suffers from mixing with the $(8,8)$ operators such as $Q_8$ emphasizing to us the need for full QCD in so far as the calculation of $\epsilon'$ is concerned.

It took several years to finish the first calculation of $K \to \pi\pi$ with DWQ in full $(2 + 1)$ flavor QCD again using ChPT only to discover that the kaon is simply too heavy for ChPT to be reliable; the systematic errors for matrix elements of many of the key operators were O(50%) or even more.

That brings us to the efforts of the past $\approx 6$ years jointly by RBC and UKQCD collaborations to go instead for direct calculations of $K \to \pi\pi$ using finite volume correlation functions as suggested by Lellouch-Luscher. The results reported in this talk are primarily using three different lattices (see Tab.1) accumulated over the past several years. The $16^3$ and $32^3$ lattices only allow for threshold studies, whereas the $32^3$ lattice of volume $(4.5\text{fm})^3$ is used to study $K \to \pi\pi$ with physical kinematics. While all three lattices have been used already for the simpler $I=2$ final state, for the more challenging $I=0$ final state studies at physical kinematics on the $32^3$ lattice are still not complete. Fortunately, as will be explained, for the $\Delta I = 1/2$ puzzle, it turns out that understanding the simpler $I=2$ channel proves to be crucial.

1.1 The Puzzle

Let’s briefly recapitulate the so-called $\Delta I = 1/2$ puzzle. The issue boils down to the huge (factor of about 450) disparity in the life-times of neutral (i.e $K_S$) and that of $K^\pm$. Thus, basically the spectator u-quark in $K^+$ is changing to d-quark in $K_S$ resulting in this huge change in their life-times. Their main decay mode is just to two pions. However, whereas $\pi^+\pi^0$ (resulting from the decays of $K^+$) is in a pure $I=2$ final state, $\pi^+\pi^-$ or $\pi^0\pi^0$ are mixtures of $I = 0$ and $I = 2$; thus the ratio of the two relevant amplitudes $ReA_0/ReA_2 \approx 22$, for the $I = 0$ and $I = 2$ is a lot bigger than unity. Since for the charged K the change in isospin, $\Delta I = 3/2$ whereas for the neutral K its either 1/2 or 3/2, it implies that the $\Delta I = 1/2$ amplitude is significantly larger than the $\Delta I = 3/2$ and this is the long-standing puzzle (see e.g.32). While its long been speculated that QCD corrections may be responsible for this huge enhancement, at this scale highly non-perurbative effects are anticipated; of course, over the years there have been numerous suggestions, including new physics (see e.g.33) as the cause for this large enhancement.

2 Weak Effective Hamiltonian and 4-quark operators

Using the OPE apparatus, one arrives at the effective Hamiltonian for $\Delta S = 1$ weak decays. Using the OPE apparatus, one arrives at the effective Hamiltonian for $\Delta S = 1$ weak decays.
\[ H_{\Delta S=1} = \frac{G_F}{\sqrt{2}} V^\ast_{ud} V_{us} \sum_{i=1}^{10} \left( \left[ (z_i(\mu) + \tau y_i(\mu)) \right] Q_i \right) . \]  

(1)

Here, \( Q_i \) are the well-known 4-quark operators,

\begin{align*}
Q_1 &= (\bar{s}_a d_a)_{V-A}(\bar{u}_\beta u_\beta)_{V-A}, \\
Q_2 &= (\bar{s}_a d_\beta)_{V-A}(\bar{u}_\beta u_a)_{V-A}, \\
Q_3 &= (\bar{s}_a d_a)_{V-A} \sum_{q=u,d,s} (\bar{q}_\beta q_\beta)_{V-A}, \\
Q_4 &= (\bar{s}_a d_\beta)_{V-A} \sum_{q=u,d,s} (\bar{q}_\beta q_\alpha)_{V-A}, \\
Q_5 &= (\bar{s}_a d_\alpha)_{V-A} \sum_{q=u,d,s} (\bar{q}_\beta q_\beta)_{V+A}, \\
Q_6 &= (\bar{s}_a d_\alpha)_{V-A} \sum_{q=u,d,s} (\bar{q}_\beta q_\alpha)_{V+A}, \\
Q_7 &= \frac{3}{2} (\bar{s}_a d_\alpha)_{V-A} \sum_{q=u,d,s} e_q (\bar{q}_\beta q_\beta)_{V+A}, \\
Q_8 &= \frac{3}{2} (\bar{s}_a d_\beta)_{V-A} \sum_{q=u,d,s} e_q (\bar{q}_\beta q_\alpha)_{V+A}, \\
Q_9 &= \frac{3}{2} (\bar{s}_a d_\alpha)_{V-A} \sum_{q=u,d,s} e_q (\bar{q}_\beta q_\beta)_{V-A}, \\
Q_{10} &= \frac{3}{2} (\bar{s}_a d_\beta)_{V-A} \sum_{q=u,d,s} e_q (\bar{q}_\beta q_\alpha)_{V-A}.
\end{align*}

(2a)-(2j)

where \( \alpha, \beta \) are color indices and \( (V - A) \) means \( \gamma_\mu(1 - \gamma_5) \).

It is important to recognize that \( Q_2 \) is the original 4-quark (tree) operator of the basic charged current weak decay, \([s_\alpha(\gamma_\mu(1 - \gamma_5)u_\alpha)]\bar{u}_\beta\gamma_\mu(1 - \gamma_5)d_\beta\), conventionally written here in the Fierz transformed basis. When you switch on QCD, \( Q_2 \) is not multiplicatively renormalizable and as was realized long ago \[^{39}^{40}\] it mixes with another tree operator \( Q_1 \). On the other hand, \( Q_3 \) to \( Q_6 \) are the QCD penguin operators \[^{38}\] and \( Q_7 \) to \( Q_{10} \) are the electroweak (EW) penguin operators \[^{39}^{40}\].

On the lattice, in the absence of exact chiral symmetry, each of these dim-6, 4-quark operator of the \( \Delta S = 1 \) Hamiltonian can mix with lower dimensional operators, e.g. \( \bar{s}d, \bar{s}\gamma_5d, \) etc. The effects of these mixings are purely unphysical and need to be subtracted away. As you make the lattice spacing finer and move towards the continuum limit, these unphysical contributions tend to become huge and it can become a very demanding and delicate subtraction, quite akin to fine tuning. The chiral behavior of wilson fermions was so bad that original methods \[^{39}^{40}^{41}\] that were proposed to deal with such subtraction issues proved to be quite inadequate. Because of the excellent chiral symmetry of DWQs, this became by and large a non-issue provided the extent of the 5th dimension is not too small.

Matrix element \( \langle \pi \pi | Q_i | K^0 \rangle \) for each operator entail an evaluation of 48 different Wick contractions which can be grouped into four different types \[^{41}^{41}^{41}\]. Of these, type-4 involve disconnected diagrams and are therefore, computationally the most demanding. Type-3 contain “eye” contractions \[^{2}\] type-2 correspond to “figure-eight” diagrams, and type-1 correspond to original
weak interaction tree graphs; see fig. In particular, it is to be stressed that only type-1 contributes to the $\Delta I = 3/2$ transitions and the corresponding $I = 2$ final state of the two pions whereas the $\Delta I = 1/2$ transitions for $I = 0$ final state receive contributions from all four types and consequently are much more intricate and challenging to tackle than the $\Delta I = 3/2$ case.

![Diagram of quark flow diagrams](image)

Figure 1: Four general type of quark flow diagrams contribution to $K^0 \rightarrow \pi^+ \pi^-$; (a) corresponds to spectator types in the continuum literature, (b) and (d) to annihilation and (c) to penguins; (d) though requires disconnected contributions which on the lattice are extremely demanding. Taken from [15]

3 $\Delta I = 3/2$

As indicated already, ironically at the end of the day, it turned out that it is the simpler $\Delta I = 3/2, K \rightarrow \pi\pi$ that is very revealing in so far as the enhancement of the ratio is concerned. The $3/2$ amplitude involves simply type-1 contractions [14]. The Wick contractions for $\langle \pi\pi|Q_{1,2}|K^0\rangle$, for the dominant operators $Q_2$ or $Q_1$, entail two contributions, one goes as product of two traces in color space ($N \times N$) and the other is a single trace in color space ($N$), where for QCD, $N = 3$.

As is well known, continuum folklore says that $N^2$ term dominates and the two terms add [36,45,46,47]. Our data using three different lattices [1] collected over the past few years allows us to study these contributions as a function of the pion mass (with $m_K \approx 2m_\pi$). In fact the relative sign between the terms is negative and the cancellation between the two terms increases as the pion masses is lowered. Indeed at physical kinematics with $m_\pi = 142MeV$ and $m_K = 520MeV$, the single trace contribution is around -0.7 of the trace $\times$ trace term. So, the observed amplitude is only around $2.7/12 \approx 0.25$ of naive expectations, assuming $N = 3$. In other words, out of the observed enhancement in the ratio of the two amplitudes of a factor of around 22, as much as a factor of 4 may simply be coming from the fact that there is this cancellation making the $3/2$ amplitude only about 0.25 of naive expectations.

Another notable feature of the $I = 2$ channel is that its amplitude, $ReA_2$, shows a significant dependence on $m_\pi$. We attribute this largely to the cancellation mentioned above. From Tab.1 we see that as the pion mass decreases from about 420 MeV to 140 MeV, $ReA_2$ decreases by about a factor of 3.5 and with physical $\pi, K$ masses it is in good agreement (within $\approx 15\%$) with its measured value from experiments [23].

Moreover, recall that $ReA_2$ is closely related to $B_K$, the neutral Kaon mixing operator, as has been long known since the famous work of [10], who obtained $B_{KLOchPT} \approx 0.3$ by exploiting its relationship with the experimentally measured value of $ReA_2$ from the charged Kaon lifetime, assuming SU(3) and lowest order chiral perturbation theory. Lattice studies for a long time of course also have shown that $B_K$ changes from about 0.3 to 0.6 as you move from the chiral limit to $m_K$ [20,21].
4 Implications for \(ReA_0\) and the \(\Delta I = 1/2\) Rule

What is even more striking is how this cancellation that is responsible for the suppression of \(ReA_2\) actually also ends up enhancing \(ReA_0\). First let’s just look at the dominant operator, \(Q_2\). Its contribution to \(ReA_2\) and to \(ReA_0\) is as follows:

\[
ReA_{2,2} = i \sqrt{\frac{2}{3}} (ST + TSQ), \quad (3)
\]

\[
ReA_{0,2} = i \sqrt{\frac{1}{3}} (-ST + 2TSQ) \quad (4)
\]

where \(A_{i,j}\) notation means \(i = 0\) or \(2\), (depending on the isospin of the pion final state) and \(j = 1, 2\) and \(j = 2\), for example, means \(Q_2\) and \(ST\) means single trace over color indices and TSQ means trace \(\times\) trace. Thus, recalling that at physical kinematics, \(ST/TSQ \approx -0.7\), the ratio, \(ReA_0/ReA_2 \approx 6.4\). So far we only looked at the contribution of the dominant tree operator \(Q_2\). Let us next, also retain the next most important operator, which happens to be the tree operator, \(Q_1\). One finds,

\[
ReA_{2,1} = i \sqrt{\frac{2}{3}} (ST + TSQ), \quad (5)
\]

\[
ReA_{0,1} = i \sqrt{\frac{1}{3}} (2ST - TSQ). \quad (6)
\]

Thus, incorporating the Wilson coefficients \((Z_j, \text{with} \ j = 1, 2)\) for these two operators, \(Z_1 = -0.30\) and \(Z_2 = 1.14\), one gets,

\[
ReA_i = Z_j A_{i,j} \quad (7)
\]

for \(i = 0, 2\) corresponding to \(I = 0, 2\) for the two final states. Then given \(ST \approx -0.7 \times TSQ\), we get \(ReA_0/ReA_2 \approx 10.8\); thus accounting for almost half of the experimental number \(\approx 22.5\).

Note also that the cancellation between the single color trace and trace square term ends up causing not only a further suppression of \(ReA_2\) because of the fact that the sign of Wilson coefficient \(Z_1\) is negative to that of \(Z_2\), but in addition as an interesting coincidence, it ends up enhancing \(ReA_0\). This is easily understood from the above simple eqns (4, 6) as the relative signs between single trace and the squared trace switch from \(ReA_2\) to \(ReA_0\).

While our calculation of \(ReA_0\) at physical kinematics is not yet complete, there are several interesting features of the existing calculations summarised in Tab. that are noteworthy. One item to note is the ratio \(ReA_0/ReA_2\) resulting from our two completed calculations on the 16.
and $24^3$ lattices. It is 9 and 12 respectively. These numbers are for amplitudes calculated at threshold. As commented before the corresponding $ReA_2$ on these lattices are factors of $\approx 3.5$ and $\approx 2$ times the value of $ReA_2$ at physical kinematics. This is mostly the result of significant mass dependence of $ReA_2$. In contrast, our numbers for $ReA_0$ show milder mass dependence and infact the value we obtain on our $24^3$ lattice (whose volume is about three times bigger compared to the smaller lattice) at threshold is quite consistent with experiment; whether this feature will remain true at physical kinematics or not remains to be seen.

Table 1: Reproduced from \cite{1} Summary of simulation parameters and results obtained on three domain wall fermion ensembles. The errors with the Iwasaki action are statistical only, the second error for $ReA_2$ at physical kinematics from the IDSDR simulation is systematic and is dominated by an estimated 15% discretization uncertainty as explained in \cite{48}.

\begin{center}
\begin{tabular}{|c|c|c|c|c|c|c|}
\hline
 & $a^{-1}$ & $m_\pi$ & $m_K$ & $ReA_2$ & $ReA_0$ & $ReA_0$\nnote{\textsuperscript{ReA_2}} \\
\hline
16$^3$ Iwasaki & 1.73(3) & 422(7) & 878(15) & 4.911(31) & 45(10) & 9.1(2.1) \\
24$^3$ Iwasaki & 1.73(3) & 329(6) & 662(11) & 2.668(14) & 32.1(4.6) & 12.0(1.7) \\
IDSDR & 1.36(1) & 142.9(1.1) & 511.3(3.9) & 1.38(5)(26) & - & - \\
Experiment & & 135-140 & 494-498 & 1.479(4) & 33.2(2) & 22.45(6) \\
\hline
\end{tabular}
\end{center}

In passing let us note that from SU(2) ChPT description of $K \rightarrow \pi \pi$ one also finds a significant dependence on pion mass of $ReA_2$ than of $ReA_0$ \cite{52} in qualitative agreement with the lattice observations.

\begin{center}
\begin{tabular}{c}
\includegraphics[width=\textwidth]{Figure3.pdf}
\end{tabular}
\end{center}

Figure 3: Contractions $\mathcal{C}_2$ (that goes as trace $\times$ trace in color space and is also call TSQ in here), $\mathcal{C}_1$ (that goes as single trace in color space and is also called ST in here) and $\mathcal{C}_1 + \mathcal{C}_2$ as functions of $t$ from the simulation at physical kinematics. Taken from \cite{1}.

4.1 The role of penguins in the $\Delta I = 1/2$ Puzzle

From Tab.\cite{2} we see that at a scale of $\approx 2.15$ GeV, the tree operators $Q_2$ and $Q_1$ account for almost 97% of $ReA_0$ so the contribution of the remaining operators, in particular the QCD penguins, is only a few % and the EW penguins around 0.1%. In fact roughly similar conclusions were arrived previously when we used the chiral perturbation approach both in the quenched approximation\cite{25} as well as in dynamical 2+1 flavor QCD\cite{30,10}.
Figure 4: Contractions $\{1\}$, $\{-2\}$ and $\{1\} + \{2\}$ as functions of $t$ from the simulation at threshold with $m_\pi \simeq 330$ MeV; see also fig. Taken from.

We stress again that these calculations Tab.2 for $ReA_0$ are not at physical kinematics so the relative importance of the penguin to tree contributions may well change to some degree; however, the fact remains that the cancellation and suppression of $ReA_2$ and enhancement of $ReA_0$, in the tree contributions, which are the new aspects being reported here, imply a diminished role for the penguin contributions at least at a renormalization point around 2 GeV.

Table 2: Contributions from each operator to $ReA_0$ for $m_K = 662$ MeV and $m_\pi = 329$ MeV. The second column contains the contributions from the 7 linearly independent lattice operators with $1/a = 1.73(3)$ GeV and the third column those in the 10-operator basis in the $\overline{\text{MS}}$-NDR scheme at $\mu = 2.15$ GeV. Numbers in parentheses represent the statistical errors. Taken from.

$$\begin{array}{c|cc}
  i & Q_{i}^{\text{lat}} [\text{GeV}] & Q_{i}^{\overline{\text{MS}}-\text{NDR}} [\text{GeV}] \\
\hline
  1 & 8.1(4.6) \times 10^{-8} & 6.6(3.1) \times 10^{-8} \\
  2 & 2.5(0.6) \times 10^{-7} & 2.6(0.5) \times 10^{-7} \\
  3 & -0.6(1.0) \times 10^{-8} & 5.4(6.7) \times 10^{-10} \\
  4 & - & 2.3(2.1) \times 10^{-9} \\
  5 & -1.2(0.5) \times 10^{-9} & 4.0(2.6) \times 10^{-10} \\
  6 & 4.7(1.7) \times 10^{-9} & -7.0(2.4) \times 10^{-9} \\
  7 & 1.5(0.1) \times 10^{-10} & 6.3(0.5) \times 10^{-11} \\
  8 & -4.7(0.2) \times 10^{-10} & -3.9(0.1) \times 10^{-10} \\
  9 & - & 2.0(0.6) \times 10^{-14} \\
  10 & - & 1.6(0.5) \times 10^{-11} \\
  \text{Re}A_0 & 3.2(0.5) \times 10^{-7} & 3.2(0.5) \times 10^{-7} \\
\end{array}$$

4.2 The role of disconnected diagrams for $ReA_0$

Our calculation of $A_0$ being discussed here is not yet at physical kinematics. It is actually at threshold and perhaps more importantly the (valence) pion masses $\approx$ are relatively heavy. As Tab.1 shows we completed the threshold calculation of $ReA_0$ with two different lattices ($16^3$ and $24^3$) with pion mass around 420 MeV and 330 MeV attaining statistical accuracies around 25%
and 15% respectively. These calculations include the contribution from disconnected diagrams as well. Within the stated accuracy, we do not seem to see any discernible contribution from the disconnected diagrams in so far as \( ReA_0 \) is concerned. Again we emphasize that this is with pion mass around 330 MeV and not with physical pion masses.

Given that the dominant contribution to \( ReA_0 \) seems to come from tree operators, which do not receive contribution from any disconnected diagrams, it is understandable that the disconnected diagrams contribution to \( ReA_0 \) is most likely rather small.

4.3 The role of disconnected diagrams for \( ImA_0 \)

Tree level operators cannot contribute to \( ImA_0 \). Only eye-contractions and disconnected diagrams make contributions to \( ImA_0 \); thus one expects an enhanced role for disconnected graphs in \( ImA_0 \). This is why our calculation of \( ImA_0 \), even with \( m_\pi \approx 330 MeV \) has statistical errors of around 50%.

4.4 Status of \( \epsilon' \)

As is well known contributions to \( \epsilon' \) can be divided into two categories: QCD penguins and EW penguins originating respectively from \( Q_3, Q_4, Q_5 \) and \( Q_6 \) and \( Q_7, Q_8, Q_9 \) and \( Q_{10} \). Amongst these \( Q_6 \) and \( Q_8 \) are the dominant players.

RBC and UKQCD have already finished their computation of \( ImA_2 \) as indicated in Tab.\[\] at physical kinematics with an estimated statistical error of \( \approx 20\% \) and roughly similar error for systematics. This means the EWP contributions to \( \epsilon' \) has already been completed; indeed improved calculations of \( ImA_2 \) are well underway and are expected rather soon with appreciable reduction in errors.

From a purely personal perspective, \( \epsilon' \) has always been the main focus of the \( K \to \pi\pi \) effort from the very beginning. The calculation of \( ImA_0 \) relevant to \( \epsilon' \) is even more challenging than that of \( ReA_0 \) relevant for the \( \Delta I = 1/2 \) rule. This is because \( ImA_0 \) does not receive any contribution from the tree operators. This is understandable as in the SM all three generations have to participate to make a non-vanishing contribution to any CP violation phenomena. Thus penguin graphs and consequently eye contractions become essential on the lattice. While that renders the calculation quite challenging, perhaps another order of magnitude in the complexity is added by the fact that the \( I = 0 \) channel receives contributions from disconnected diagrams. The error on our \( \epsilon' \) calculation is around 100% at present.

4.5 Possible implications for other weak decays

Our lattice studies of direct \( K \to \pi\pi \) seem to show that for QCD \( i.e, N=3, \) large N approximation is amenable to rather large corrections. Since its use, as well as that of the closely related notion of factorization, is so pervasive in weak decays, perhaps, D and B decays ought to be re-examined in light of these lattice findings. Moreover, because the cancellation discussed above results in a significant fraction of the enhancement of \( ReA_0/ReA_2 \) and also because the penguin contribution to \( ReA_0 \) seems to be so small, it tells us that the penguin contribution in D-decays (in the \( I=0 \) channel) is bound to be quite small as

\[
\frac{P_D}{T_D} \approx \delta_{U\text{spin}} \times \frac{P_K}{T_K} \tag{8}
\]

where, subscript D or K means exclusive \( D \to PP \) or \( K \to PP \), respectively, with \( P=\text{pseudoscalar} \) and \( \delta_{U\text{spin}} \) is indicative of \( U\text{spin} \) violation reflecting the cancellation between the s and d virtual quarks in the penguin.
5 Summary & Outlook for the near future

Summarizing, lattice studies of direct $K \to \pi\pi$ show that in the simpler $I = 2$ channel, at physical kinematics, the contributing amplitude from the original, tree, 4-quark $\Delta S = 1$ weak operators that goes as $N^2$ cancels significantly with the one that goes as $N$, causing an appreciable suppression of the $\Delta I = 3/2$ transition. This seems to lead to a considerable fraction of the enhancement in the ratio of $\text{Re} A_0/\text{Re} A_2$. These results suggest that expectations from large $N$ may receive large corrections for QCD in weak decays.

Understanding $K \to \pi\pi$ decays and calculation of $\epsilon'$ remains a very important goal of the RBC and UKQCD collaborations. I am hopeful that the first calculation of $\text{Re} A_0$ and $\epsilon'$ in full QCD with physical kinematics would be completed in about two years.

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