Movement of polymers in an asymmetric nano-channel under various parameters

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Abstract. Separation is an important process in various fields in science and technology, and nanostructures are now being widely used to improve the resolution and speed of separation processes. Continuous separation occurs when the components of a mixture separate at different locations in a device; this makes it possible to inject a new sample into the device at any time during an on-going separation process. Continuous separation of nano-particles in a nano-channel composed of consecutive cones has been shown previously, while this study investigates whether a nano-channel can perform continuous separation of polymers such that polymers of different lengths are separated at the two sides of the channel. For this purpose, the motion of a charged polymer in an asymmetric nano-channel under constant and alternating applied electric fields was studied theoretically and by applying computer simulations. Theoretical formalism was used to determine the situation under which the channel could perform continuous separation, with computer simulations used to check the theoretical results. The effect of different channel parameters on the performance of the nano-channel, as a device for continuous separation, was thus determined.

Introduction

The motion and separation of polymers in different nanostructures have been considered in previous studies [1,2]. These have shown that the motion of polymer in a channel with varying diameter can be used to facilitate polymer separation [3] and for sequencing purposes [4]. The motion of polymer in corrugated channels has also been investigated theoretically [5] and under computer simulations [6]. It has thus been shown that an asymmetric channel can rectify the motion of polymer, under the application of an alternating electric field [6], and this phenomenon of rectified motion has been widely studied in the case of particles [7], being shown to be able to perform fine separation of particles to different sides of the nanochannel [8]. This idea of separating a mixture into two different sides of a device is related to continuous separation, which is of great importance for science and industry [9].

In a previous study, the authors investigated the mechanism of rectification of polymer motion in an asymmetric channel [10]. Here, the agreement between the theory and the simulations is investigated to determine the best parameters for an asymmetric channel for the separation of polymers. A theoretical approach to motion of polymers in a nano-channel was used to determine suitable parameters for the separation of polymers. Simulations were then performed to check these theoretical results. Some agreement
with regard to the value of the velocities between the theory and the simulations was observed; however, the sign inversion predicted by the theory was not observed in the simulations.

**Theory**

The translocation velocity of a Gaussian polymer across a varying section channel can be captured by an effective 1D model [5]. Accordingly, the dynamics of the centre of mass in three dimensions is controlled by the effective potential:

\[
F(x) = -Nf_0x - 2k_BT \left\{ \ln \left[ \frac{16R(x)}{R_0\pi^2} \right] + \sum_{p=1,3,\ldots}^{\infty} \frac{1}{p^2} \exp \left( -\pi^2 p^2 \left( \frac{R_G}{2h(x)} \right)^2 \right) \right\}
\]

(1)

Here, \(x\) is the position of the centre of mass of the polymer, \(R_G\) is its equilibrium gyration radius in an unbound medium, \(N\) is the number of monomers, \(R(x)\) is the channel half-section evaluated at the position of the centre of mass, \(R\) is the average half-section, \(k_B\) is the Boltzmann constant, \(T\) is the absolute temperature, and \(f_0\) is the magnitude of the external force acting on each monomer. The translocation time can thus be calculated by solving the Langevin equation governing the dynamics of the centre of mass of the polymer:

\[
\dot{x}(t) = \frac{1}{\gamma} \partial_x F(x(t)) + \xi(t)
\]

(2)

where the random force \(\xi\) is characterized by \(\langle \xi(t) \rangle = 0\) and \(\langle \xi(t)\xi(t') \rangle = \frac{2k_BT}{\gamma} \delta(t-t')\). Equation 2 can then be integrated numerically via an Euler scheme with periodic boundary conditions at the channel ends, and the translocation velocity is calculated using the average time needed to cross a channel unit. This theory is used to predict situations in which a sign inversion in the velocity of the polymer along the channel may occur, as a sign inversion in the velocity means that various polymers move in different directions and separate to the two sides of the nanostructure.

A schematic of the channel is shown in figure 1. Here, the conical units are composed of two different cones, which define \(R(x)\) in equation 1. The lengths of left and right cones in the structural unit of the structure are taken as \(L_1\) and \(L_2\), respectively, and the length unit of the channel is equal to \(L = L_1 + L_2\). The small and large diameters of the cones are \(R_1\) and \(R_2\), respectively. A polymer of chain length \(N\) is considered inside the channel, under the simultaneous action of constant and oscillating electric fields, \(f_{\text{fix}}\) and \(f_{\text{osc}}\), respectively. The oscillating force has the form of a square wave function, and the total force on the polymer (\(f_0\) in equation 1) thus receives contributions from both constant and oscillating forces. Throughout this work, the size of a monomer and the thermal energy are taken as the length and the energy units; in addition, the friction coefficient is set such that the diffusion coefficient of a single monomer is equal to unity, which also sets the time unit as equal to unity.
Figure 1. Schematic of the asymmetric nanostructure. Definition of different sizes of the nanostructure and the direction of the z-axis are shown.

Figure 2 shows various sets of parameters for the system that may give a sign inversion to the polymer velocity. In this figure, for the blue, orange and cyan lines, \( L = 100, L_1 = 2.5 \). For the blue, cyan and orange lines, \( \{R_2 = 42, R_1 = 2\}, \{R_2 = 41, R_1 = 3\} \) and \( \{R_2 = 42.75, R_1 = 1.125\} \), respectively. For the red and grey lines, \( L = 50 \) and \( L_1 = 1.25 \). For the red and grey lines, \( \{R_2 = 42.6, R_1 = 1.4\} \) and \( \{R_2 = 42.75, R_1 = 1.25\} \), respectively. The time period of the alternating electric field is \( T = 500 \) so that the polymer can cross several units of the structure in one period.

Figure 2. Theoretical values for the velocity of the polymer versus the polymer chain length for different parameters of the system.

In all curves, the constant force is smaller than the oscillating force; this is a requirement to observe sign inversion. The strength of the force determines the order of the maximum magnitude of the velocity of the polymers of various chain lengths inside the nanostructure. Comparing the curves obtained at the same length of the channel unit (red with grey and blue with cyan and orange) shows that reduction in the diameter of the structure has a dramatic effect on the separation process; overall, decreasing \( R_1 \) lowers the onset of sign inversion.

According to these theoretical results, three sets of parameters were checked with simulations:

1. \( L = 50, L_1 = 1.25, R_2 = 42.75, R_1 = 1.4, f_{fIx} = -0.75, f_{osc} = 1.0 \).
2. \( L = 50, L_1 = 1.25, R_2 = 42.75, R_1 = 1.25, f_{fIx} = -0.75, f_{osc} = 1.0 \). This set of parameters was chosen to test the effect of the small diameter cone on the motion of a charged polymer in an asymmetric nano-channel.
3. \( L = 100, L_1 = 2.5, R_2 = 42.75, R_1 = 1.25, f_{fIx} = -0.35, f_{osc} = 1.0 \). This set of parameters was chosen to test the effect of the unit length of the structure.

Simulation Method
Langevin Dynamics (LD) simulations were used to check the accuracy of the theory. In these simulations, the nanostructure was composed of \( n = 31 \) consecutive conical units of length \( L \). A simulation box of size
n × L was taken, equal to the size of the whole structure, and a flexible charged polymer of chain length N was located in the middle of the nano-structure. The chain length of the polymer varied, with values 20, 45, 100, 224, and 500. Monomers of the polymer were connected to each other via harmonic potentials of high stiffness. The shifted-truncated Lennard-Jones potential was used for the excluded volume interaction between monomers, and between monomers and the walls of the structure. The simulations were then performed at a constant temperature using a Langevin thermostat in ESPResSo [11].

At the start of the simulation, the monomers of the polymer were arranged on the axis of the nano-structure. The polymer was equilibrated for a timespan of order $N \tau_0$, where $\tau_0$ is the time unit of the simulations. The middle monomer of the polymer was fixed during this equilibration. After equilibration, the middle monomer was released and two constant and alternating electric fields were applied simultaneously to the polymer. To test the effect of the time period, simulations with two time periods $T = 500\tau_0$ and $T = 1,000\tau_0$ were performed. Snapshots from the simulation are obtained using Visualize Molecular Dynamics (VMD) [12]. The system was integrated for a time equal to 5T and 2.5T for $T = 500$ and 1000, respectively, and the results of the simulations (the centre of mass of the polymer) were averaged over the different simulation instances. The average velocity of the polymer in the nanochannel was obtained from the change between the initial and final positions of the centre of mass of the polymer, with the initial position of the polymer taken at time 500 to eliminate the effect of the initial configuration of the polymer. The average velocity was also averaged over the different simulation instances.

Simulation Results

Figure 3(a) and 3(b) show the schematics of the polymer as it moves in the negative and positive directions along the channel, respectively. When the polymer moves in the negative direction, it becomes compressed behind the narrow parts of the channel (figure 3(a)). However, when the polymer moves in the positive direction, no compression behind the narrow openings occurs (figure 3(b)).

Figure 3. Snapshots of a simulation with $N = 100, L = 50, L_1 = 1.25, R_1 = 1.4, R_2 = 42.75, n and T = 1000\tau_0$. The nanostructure is not visible in the snapshots. (a) The polymer moves in the negative direction. A short length of the polymer has passed through the narrow openings (red arrow) and the other monomers are compressed behind it. (b) As the polymer moves in the positive direction, no compression behind the narrow openings is observed due to the funnelling effect of the longer cones of length L_2.
The polymer develops a globular shape after the equilibrium time (figure 4(a)). After equilibration, when the electric fields are applied, the polymer takes an elongated shape as it moves through the nanostructure (figure 4(b)).

Figure 4. Snapshots of the polymer configuration in the simulation with \( R_1 = 1.25 \) and \( L = 50 \). The nanostructure is not visible in the snapshots. (a) The polymer has a globular structure after equilibration. (b) The polymer takes an extended configuration after the application of electric fields as it starts moving along the nanostructure.

Figure 5(a) shows the position of the centre of mass of the polymer as a function of time, for simulations with \( T = 1000 \tau_0 \). Counter-intuitively, the overall motion of the polymer is toward the negative direction along the channel; motion is slow in the positive direction and fast in the negative direction. Figure 5(b) is the plot of the polymer velocity versus the chain length of the polymer for the same simulations. The velocity is minimised in the polymer chain length \( L = 2 \); the velocity decreases with the unit length of the nanostructure and as the diameter of the narrow openings decreases.

Figure 5. (a) Position of the centre of mass of the polymer versus time. Solid, dashed and dashed-dotted lines correspond to simulations with \( L = 100, R_1 = 1.25 \), \( L = 50, R_1 = 1.25 \) and \( L = 50 \) and \( R_1 = 1.4 \), respectively. The time period of the oscillating force is \( T = 1000 \). (b) Averaged velocity of the polymer versus the polymer chain length for the same simulations.

Figure 6(a) shows the position of the centre of mass of the polymer as a function of time for simulations with \( T = 500 \tau_0 \). The time period has no significant effect on the results, and the simulation results are not in agreement with the theory, as no sign inversion is observed in the results.
Figure 6. (a) Position of the centre of mass of the polymer versus time. Solid, dashed and dashed-dotted lines correspond to simulations with \([L = 100, R_1 = 1.25], [L = 50, R_1 = 1.25] \) and \([L = 50 \text{ and } R_1 = 1.4]\), respectively. The time period of the oscillating force is \(T = 500\). (b) Averaged velocity of the polymer versus the polymer chain length for the same simulations.

To identify the reason behind the disagreement between the theory and simulations, longer runs were performed to ensure the elimination of any effects from the initial configuration of the polymer. The results were similar, however (not shown here). A closer investigation of the theory shows that there are two assumptions in the theoretical framework which are not upheld in the simulations. (1) The length of the channel should be smaller than the radius of gyration of the polymer; and (2) The force should satisfy the condition that \(f_{\text{fix}} \times L < 1\) [5]. Both of these assumptions are not satisfied in the parameters chosen for the simulations, where longer polymers extend over several length units of the nanochannel and the constant force \(f_{\text{fix}} = 0.75\) multiplied by the length unit of the channel \((L = 50.100)\) is much larger than unity.

Figure 7. The effect of different system parameters on the separation process. For all simulations \(\frac{L_4}{L} = \frac{1.25}{50}\).
(a) For all simulations, \(f_{\text{fix}} = -0.1875\) and \(f_{\text{osc}} = 1.0\) and \(R_2 = 42.75\), unless otherwise stated. (b) For all simulations, \(L = 50, R_1 = 1.4, R_2 = 42.75\).
To examine separation of various polymer lengths, extra simulations were performed. Figure 6(b) shows that the constant force which is negative is strongest, with a smaller value of constant force required for the separation process. To explore this, further simulations were performed with smaller magnitudes for the constant force (not shown here). Figure 7(a) shows that separation was observed for a lower magnitude of the constant force $f_{\text{fix}} = -0.1875$. Smaller polymers ($N = 5,10$) move in the positive direction, while longer polymers ($N > 20$) move in the negative direction. Polymers of various lengths separate at the two sides of the channel.

The effect of different channel parameters on the separation process was also investigated, specifically with regard to the separation threshold (the critical length at which the velocity is zero). As shown Fig. 7(a), the constant and oscillating forces were generally held at $f_{\text{fix}} = -0.1875$ and $f_{\text{osc}} = 1.0$ and $R_2 = 42.75$, and the ratio $L_1/L$ was kept constant for all simulations. Two different values for the length unit $L$, three values for $R_1$, two values for $R_2$ and two values for the constant force were thus examined. It was observed that these changes did not have a dramatic effect on the separation threshold, however. The parameters that affected the separation threshold were decreases in $R_1$ and the constant force. Decreasing $R_1$ lowers the separation threshold, but it is not possible to decrease $R_1$ lower than unity. Thus, the best parameter for changing the separation threshold is the value of the constant force.

The effect of the magnitude of the oscillating force on the separation process was investigated, as shown in Fig. 7(b). For all simulations, $f_{\text{osc}} = 1.0$, $R_1 = 1.4$, and $R_2 = 42.75$. It was observed that lowering the magnitude of the oscillating force was not suitable for separating the polymers, while increasing the magnitude of the oscillating force increased the velocity of the polymers moving in the positive direction, facilitating a faster separation process.

Conclusion

This study examined the theoretical formalism intended to decrease the problem of polymer motion in an asymmetric channel to one dimension using computer simulations. The importance of the assumptions in the theoretical formalism in generating agreement with simulation results was discussed. It was shown that separation is possible with the asymmetric channel based on a combination of constant and alternating forces. The results showed, however, that the channel parameters do not have any significant effect on the separation threshold while the ratio $L_1/L$ is kept constant. Changing the magnitude of the constant force can increase the separation threshold, encouraging separation at longer polymer lengths. The magnitude of the oscillating force can also be tuned to encourage faster separation.

References

[1] Yasui, Takao, et al. "DNA manipulation and separation in sublithographic-scale nanowire array." *ACS nano* 7.4 (2013): 3029-3035.
[2] Wu, Lingling and Stephen Levy. "Fluctuations of DNA mobility in nanofluidic entropic traps." *Biomicrofluidics* 8.4 (2014): 044103.000
[3] Magill, Martin, Ed Waller and Hendrick W. de Haan. "A sequential nanopore-channel device for polymer separation." *The Journal of chemical physics* 149.17 (2018): 174903.
[4] Milon, Nicolas, et al. "A tunable filter for high molecular weight DNA selection and linked-read sequencing." *Lab on a Chip* 20.1 (2020): 175-184.
[5] Bianco, Valentino and Paolo Malgaretti. "Non-monotonous polymer translocation time across corrugated channels: Comparison between Fick-Jacobs approximation and numerical simulations." *The Journal of Chemical Physics* 145.11 (2016): 114904.
[6] Mondal, Debasish and M. Muthukumar. "Ratchet rectification effect on the translocation of a flexible polyelectrolyte chain." The Journal of Chemical Physics 145.8 (2016): 084906.
[7] Reguera, David, et al. "Entropic splitter for particle separation." Physical Review Letters 108.2 (2012): 020604.
[8] Li, Yongge, et al. "Fine separation of particles via the entropic splitter." Physical Review E 96.2 (2017): 022152.
[9] Pamme, Nicole. "Continuous flow separations in microfluidic devices." Lab on a Chip 7.12 (2007): 1644-1659.
[10] Heidari, Maedeh, Mahdieh Mikani and Narges Nikoofard. "Mechanism of rectification of polymer motion in an asymmetric nano-channel." Journal of Nanostructure in Chemistry (2020): 1-12.
[11] Limbach, Hans-Jörg, et al. "ESPResSo an extensible simulation package for research on soft matter systems." Computer Physics Communications 174.9 (2006): 704-727.
[12] Humphrey, William, Andrew Dalke and Klaus Schulten. "VMD: visual molecular dynamics." Journal of molecular graphics 14.1 (1996): 33-38.