Diamagnetic blob interaction model of TTauri variability

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and

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ABSTRACT

Assuming a diamagnetic interaction between a stellar-spot originated localized magnetic field and gas blobs in the accretion disk around a T- Tauri star, we show the possibility of ejection of such blobs out of the disk plane. Choosing the interaction radius and the magnetic field parameters in a suitable way gives rise to closed orbits for the ejected blobs. A stream of matter composed of such blobs, ejected on one side of the disk and impacting on the other, can form a hot spot at a fixed position on the disk (in the frame rotating with the star). Such a hot spot, spread somewhat by disk shear before cooling, may be responsible in some cases for the lightcurve variations observed in various T-Tauri stars over the years. An eclipse-based mechanism due to stellar obscuration of the spot is proposed. Assuming high disk inclination angles it is able to explain many of the puzzling properties of these variations. By varying the field parameters and blob initial conditions we obtain variations in the apparent angular velocity of the hot spot, producing a constantly changing period or intermittent periodicity disappearance in the models.

\textit{Subject headings:} stars:formation — stars: TTauri — accretion, accretion disks — magnetic fields

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1. Introduction

Most T-Tauri stars (TTS) exhibit irregularities in their luminosity, observable photometrically and spectroscopically. This variability is observed on a variety of timescales and in a wide frequency band. Although the short-term variability can be explained as variable chromospheric eruption/emission episodes (Worden et al. 1981), the long-term variability is not yet fully explained. An extreme example is BP Tau, whose lightcurve, monitored over 50 days (Simon et al., 1990), shows periodic brightness fluctuations with a period of 7.6d, then an almost constant brightness level for 2 weeks, and finally the periodic behavior resumes - but with a period of 6.1d. Variability of this type is well documented for a variety of stars (e.g. Herbig 1962, Bouvier et al. 1995, Rydgren & Vrba 1983). The puzzling property of a number of the classical TTS (CTTS) is that their variability period changes on a time scale of weeks to months (BP-Tau, mentioned above, is the only case where the period change was actually continuously monitored).

One type of explanations of this effect involves the stellar magnetic field, following the idea of Ghosh & Lamb (1978) (see also Königl, 1991). An oscillation of sorts is assumed to occur in the Alfvén radius and thus the magnetic boundary layer there changes its size and position, causing the variability (Smith et al. 1995, Armitage 1995). The weakness of these models lies in the need for an organized strong dipole magnetic field in TTS, which is rather doubtful observationally (Menard 1996). Another promising model is that of cool and hot spots on the stellar surface, with cool spots analogous to sunspots and hot spots being the by-product of the accretion process (Vrba et al. 1986, Bouvier & Bertout 1989, Bouvier et al. 1993, 1995).

A recent series of observations, the COYOTES[1] II, were obtained over a 2-month monitoring of the light variations of TTS of the Taurus-Auriga dark cloud. The results provide further evidence for temporal variations of the photometric periods of CTTS. For example, IQ Tau and GM Aur exhibit relatively flat lightcurves with narrow minima, while the other lightcurves are more sinusoidal. DR Tau is a special case, as both the length and depth of the minima vary, and the lightcurve is neither sinusoidal nor flattened, but exhibits mixed traits. In the CW Tau case, the star becomes bluer in U-V and B-V when fainter than V=14.0m. It also exhibits large photometric variability, with amplitude variations ranging from almost 3m in the U-band to 1.5m in the I-band.

Several photometric periods were reported for DF Tau at different epochs: 8.5d (Bouvier & Bertout 1989), 7.9d (Richter et al. 1992), and 9.8d in the COYOTES II observations. DR Tau and DG Tau may be also candidates for this category. In the COYOTES I campaign two possible periods were found for DR Tau (2.8d and 7.3d), while the COYOTES II data points to 7.23d; the DG Tau data from COYOTES I/II reveals a possible period of 6.3d, while a period of 4.3d is claimed by Guenther(1994, priv. comm with J.Bouvier). From the data for TAP 57NW it also seems that the period and the overall shape of the lightcurve vary with time. In addition to these period variations, the same stars sometimes do not exhibit any periods in their lightcurve (e.g. DF Tau - Rydgren et al. 1984, Bouvier & Bertout 1989, BP Tau- Simon et al. 1990), indicating that the period not only changes but may disappear temporarily.

Only stars whose lightcurves are consistent with the presence of hot spots were found so far to exhibit period changes and disappearances. This suggests that the cause of these phenomena is to be found in the accretion process itself. This is also suggested by the spectroscopic signature of mass inflow at free fall velocities (Hartmann et al. 1994, Edwards et al. 1987), which provide support for the magnetic accretion mechanism.

The period variations cannot be attributed to true changes in the stellar rotation, as the response time of the stellar angular velocity to non-steady accretion is of the order of 10^5y (Pudritz & Patel 1994) while for example the timescale over which the period changes are observed to occur is two weeks for BP Tau (Simon et al. 1990). Thus one is bound to conclude that the cause of the period variations lies in changes of the angular velocity of the hot spots themselves.

2. The model

King & Regev (1994) (hereafter KR) proposed that the low rotation rates and outflows in TTS could be explained given the presence of a magnetic loop structure on the central star in a T Tauri-disk system. The loop is assumed to interact with the disk material via a magnetic surface drag force exerted on diamagnetic gas blobs, following the scheme of Drell, Folley and

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[1] Coordinated Observations of Young stellar ObJects from Earthbound Sites.
Ruderman (1965). These authors showed that the energy-loss timescale for a diamagnetic object of mass \( m \) crossing the lines of a magnetic field \( B \) is

\[
t_{\text{drag}} = \frac{c_A m}{B^2 l^2},
\]

with \( l \) a typical length scale for the object and \( c_A \) the Alfvén velocity in the plasma outside the object. The minimum conductivity condition for this equation to hold is

\[
\sigma > \frac{e^2}{2\pi l c_A}.
\]

This reduces to a condition on the blob lengthscale with the following functional dependence on position:

\[
\sigma > \frac{e^2}{M_{\odot}},
\]

where \( M_{\odot} \) is the mass of the central star. Therefore it is a function of position. In addition, it depends also on the individual blob parameters. In our simple model, we wish to avoid repeated blob-flux tube interaction resulting from a stellar magnetic loop crossing the disk, and thus the field, \( r_{co} = (GM/\omega)^{1/3} \), the time by the rotational velocity \( \omega = \Omega_\star \) and the rotational velocity by \( \Omega_\star \).

The full expression for the drag coefficient, \( k \), is obtained by substituting \( c_A = B/\sqrt{4\pi \rho} \) and \( m = \rho_0 l \) in the original definition. \( \rho_0 \) and \( \rho \) are the densities of the blob and the interblob plasma in the disk, respectively. Thus

\[
k = 2\sqrt{\pi} B \rho_{0}^{3/2} \frac{1}{\rho_0 l}.
\]

The equations of motion of a blob are written in a frame rotating with the star, with angular velocity \( \vec{\omega} \) as

\[
\dot{\vec{r}} = -\frac{GMr}{r^3} - 2\vec{\omega} \times \vec{r} - \vec{\omega} \times (\vec{\omega} \times \vec{r}) - k \left( \vec{r} - (\vec{r} \cdot \hat{b}) \hat{b} \right),
\]

with \( \vec{r} \equiv (x, y, z) \) and \( \hat{b} = (b_x, b_y, b_z) \) defined as

\[
b_x = \sin \theta \cos \phi, \quad b_y = \sin \theta \sin \phi, \quad b_z = \cos \theta.
\]

The vertical cutoff (Gaussian) velocity equals \( \epsilon \) the perpendicular velocity component. The drag coefficient, \( k \), depends on the magnetic field strength and on the local interblob plasma density (through \( c_A \)). Therefore it is a function of position. In addition, it depends also on the individual blob parameters.

We shall use a magnetic field of a flux tube resulting from a stellar magnetic loop crossing the disk, with the following functional dependence on position:

\[
B(x, y, z) = B_0 e^{-\frac{(x-x_0)^2+(y-y_0)^2}{2w_x^2}} e^{-\frac{z^2}{2w_z^2}},
\]

where \( x, y, z \) are Cartesian coordinates (the origin is at the star’s center and \( z = 0 \) is the disk plane), \( B_0 \) is a constant and \( w_x, w_z \) are the Gaussian spreads characterizing the loop.

The functional form of the field should approximate the spatial dependence of a magnetic field localized in a flux tube with a horizontal spread \( w_x \) around the point \( x_0, y_0 \). A Gaussian form is chosen as a convenient possibility. The vertical cutoff (Gaussian decline in the field strength, with spread \( w_z \)) reflects our wish to avoid repeated blob-flux tube interaction in our simple model. In any case, a (Gaussian) vertical cutoff in \( k(r) \) will follow from its dependence on the disk density (see below), which is usually assumed to have a Gaussian decay with height.

The equations of motion, scaling the physical variables by the following characteristic units: distances are scaled by the value of the corotation radius, defined here to be the radius at which the local Keplerian angular velocity equals \( \omega \) the angular velocity of the star and thus the field, \( r_{co} = (GM/\omega^2)^{1/3} \), the time by the rotational velocity \( \omega = \Omega_\star \) and the rotational velocity by \( \omega \). The following nondimensional equation of motion then results:

\[
\ddot{\vec{r}} = -\frac{\vec{r}}{r^3} - 2\vec{\omega} \times \dot{\vec{r}} - \vec{\omega} \times (\vec{\omega} \times \vec{r}) - \kappa(\vec{r}) \left( \dot{\vec{r}} - (\dot{\vec{r}} \cdot \hat{b}) \hat{b} \right),
\]

where the nondimensional drag coefficient is

\[
\kappa(\vec{r}) = \frac{B(\vec{r})}{B_0} \sqrt{\frac{\rho(z)}{\rho_0}}.
\]

Here we have assumed that the interblob disk density depends on \( z \) only and \( \rho_0 \) is the midplane value. As mentioned before this causes the appropriate decline of \( \kappa \) with height. The constant parameter \( \alpha \) is

\[
\alpha = \pi^{-1/2} P_b B_0 \rho_0^{1/2} \frac{1}{\rho_0 l},
\]
where \( P_\star = 2\pi/\omega \) is the rotation period. With the various physical variables expressed in their typical values the parameter \( \alpha \) can be written as

\[
\alpha = 0.4875 \frac{B_0}{100G} \left( \frac{\rho_0}{10^{-10}[g/cm^3]} \right)^{1/2} P_\star[\text{days}] \\
\times \left( \frac{l}{10^9[cm]} \frac{\rho_b}{10^{-7}[g/cm^3]} \right)^{-1}.
\] 

(11)

The interaction is thus parametrized by the interaction strength \( \alpha \), the flux tube width \( w_p \) and the tube direction \( \theta, \phi \). \( w_z \) should be approximately the disk half-thickness. The motion of a blob after suffering an interaction with a magnetic loop will obviously depend also on the initial blob velocity relative to the field line. As a rule the blobs will be taken to orbit the central star with a local Keplerian velocity plus a small inward drift, hence the importance of the interaction radius \( r_0 = \sqrt{x_0^2 + y_0^2} \).

Eq. (8) describes the blob motion in a rotating frame, with the Coriolis and centrifugal terms included as appropriate. It can be solved numerically for a variety of initial conditions and loop and blob parameters. We shall refer to the solutions of this equation as ballistic orbits if the trajectory leaves the disk and crosses its plane again.

3. Ballistic orbit calculations

3.1. General considerations

We start the integration of the blob motion from a point close to the interaction point \((x_0, y_0)\) in the disk plane, but far enough from it so that the magnetic drag term is negligible. A distance of several \( w_p \) is appropriate and thus with \( w_p = c r_0 \) and \( \epsilon \ll 1 \), we obtain a sharply (with respect to \( r_0 \)) increasing field.

For \( r_0 < r_{co} \) the blobs are slowed down by the magnetic drag (see KR). If no vertical velocity component is acquired this will merely increase the accretion rate. However, significant vertical velocity component may often be produced by the interaction; in such cases the blob leaves the disk - only to fall back on it (close to the star in most cases), after a very inclined orbit.

In our calculations the blob is started with a Keplerian velocity (in the inertial frame) plus a small inward drift. For the case \( r_0 < r_{co} \) the blob is started behind of flux tube, so as to let the blob overtake the tube. In this work we shall concentrate only on this case, since \( r_0 > r_{co} \) implies almost always ejection of the blob out of the disk plane without subsequent impact, see KR and Pearson & King (1996). As pointed out in the Introduction, we focus here on the possibility of blob ejection out of the disk plane, but with less than escape energy and with the blob supposed to land quite close to the star (see below). Indeed we were able to obtain such a situation, for a large fraction of the parameter space, in the case \( r_0 < r_{co} \).

For a given choice of the parameters, each ejected blob will follow exactly the same path and thus its center of mass will impact the disk at the same position (in the rotating frame) as all the preceding ones. If we envisage a succession of blobs being ejected one after the other and forming during their flight a continuous stream of matter, this stream will impact the disk at a fixed (in the rotating frame) position. The impact point with the aforementioned hot spot created by the stream will thus move on the disk, around the star, with an angular velocity \( \omega \). If the spot is close enough to the star for a given inclination angle, it will be, at least partly, eclipsed by the star. The observed luminosity variability due to this spot will then have the fixed frequency \( \omega \). The stream may also impact the star and the observational result, regarding time variability, should be the same. Now, if we allow the field parameters to vary in time, the spot is bound to move, in the rotating frame, on a timescale of the order of the magnetic field variation timescale. We shall assume the this timescale approximately of the order \( t_B \sim 5 P_\star \), where \( P_\star = 2\pi/\omega \) is a rotation period of the field. Thus we postulate that the field configuration changes (for example due to the interaction with the disk plasma and the blobs, or due to internal changes), and this happens on a time scale of \( \sim 5 \) rotations. A spot moving in the rotating frame can explain all the unusual features in the observed light curves. The period can change on this time scale. The variability may disappear (if the spot moves far enough out as not be eclipsed any more) and reappear again, if the spot moves back in. A systematic study of all the possibilities calls for a many body particle numerical simulation for the blobs in the disk. In addition we need to simulate numerically the stream of the ejected matter and its impact on the disk. Such a project is now in progress, while here we would like only to demonstrate the idea by describing various cases of individual blobs.

If we fix all the parameters but start the calculation with different initial conditions (due to blobs coming from slightly different places), we expect a spread in
the impact point in the rotating frame, giving rise to an extended hot-spot. The typical spread of impact points we have found is of the order of the stellar radius, \( R_\star \) for the cases described below, with the initial blob conditions around the magnetic flux tube set to give a ballistic orbit.

### 3.2. Results for varying field parameters

The canonical values of the parameters for our calculations are: \( \theta = \pi/4, \phi = 3\pi/4, r_0 = 0.85, \alpha = 100 \) in non-dimensional units, with time scaled by \( P_\star \) and lengths by the value of the co-rotation radius \( r_{co} \). We chose always a single blob initial condition, remembering that a spread in the spot size is expected due to blobs with close initial conditions. We then vary (linearly in time) each of these parameters in turn, keeping the others fixed. The timescale of change is of the order of \( 5P_\star \). The Gaussian spreads are always kept fixed at \( w_p = w_z = 0.05 \).

The variation of \( \alpha \) has a relatively small effect on the impact point position in the rotating frame. Thus the spread in blob sizes and masses as well as the actual absolute value of the field matter little in this model (A variation of \( \alpha \) between 1 and \( 10^6 \) resulted in a change of only \( \sim 2\% \) in spot angular velocity, see figure 2(a).

Significant change occurred when we have varied the value of \( r_0 \), the radius of the point where the loop crosses the disk (in units of \( r_{co} \)). A linear change \( r_0 = 0.6 + 0.08t \) (variation in the range \( 0.6 - 0.92 \) during 4 rotations) resulted in changes of the spot center position in polar coordinates \(( R_{imp}, \Theta_{imp})\), shown as a function of time in figure 2(b). Unlike in the variable \( \alpha \) case, (figure 2(a)), here the apparent angular velocity of the spot changed drastically; combined with the spot grazing the star itself part of the time \( (R_\star \approx 0.23r_{co}) \) this produced a lightcurve very much resembling that of DR Tau (see fig. 3 of Bouvier et al. 1995). The model lightcurve exhibits variation in shape, period and depth of the minima (figures 3, 4).

By varying \( \phi \) and \( \theta \) we again obtain significant motion of the spot in the rotating frame. Figure 4 depicts these cases as well: spot’s coordinates evolution with \( \theta \) varying as \( \theta = 0.5 + 0.2t \)- curve (c), and the coordinate evolution with \( \phi = 1.9 + 0.3t \)- curve (d). We can see that a spot moves away from the star, and so the possibility of period disappearance and its consequent return is present, although with a different apparent angular frequency. It should be also noted that although \( \Omega \) (spot’s angular velocity as seen by an observer in an inertial frame) varies some \( 10 - 15\% \), the angular velocity deduced from the spectrogram is just the average. In both cases the apparent period differed by about \( 10\% \) from \( P_\star \) (fig. 3).

### 3.3. Model predictions

In order to be able to predict whether the variability can be observed, one needs to know the stellar radius, \( R_\star \), the angle of inclination of the disk and the spot size. The apparent angular velocity of the spot is clearly \( \Omega = \omega + \Theta_{imp} \). This can be calculated with no further assumptions. We assume that the spot size is very small \( (\sim 0.2R_\star) \) and the inclination angle is such that eclipses are always occurring for the cases studied. The star is assumed to have \( R_\star = 2R_\odot \), \( \Omega_\star = \omega = 0.1\Omega_K \) (a tenth of the breakup value). The results should thus be considered as illustrative only. In figures 2, 3 we see the angular velocity of the spot in the inertial frame, i.e. as should be observed, and the energy spectrum of the predicted intensity variation-- for the three cases of varying different parameters (corresponding to the results depicted in figure 4).

### 4. Discussion

The above model is an extremely simplified view of the whole problem. In reality, a process for the creation of the diamagnetic blobs would have to be specified; the interaction between blobs should be included; and the disk thickness has to be taken into account, as it is unreasonable to expect that a blob flung out from the downward side upwards will traverse the disk without colliding with the disk and other blobs inside it. Here we do not specify the MHD instability responsible for the creation of blobs, but only note that in magnetic Cataclysmic Variables there is probably observational evidence for a blobby accreted plasma (see Wynn and King 1995). In addition, we need the blobs to exist only for a time sufficient for the magnetic interaction in the disk.

If the conditions confining the blobs are specific to the disk, they will expand on the thermal time-scale after ejection. Expanding, they will merge together into a stream-like structure; thus the above picture of single blobs falling into the disk is useful only in the context of checking the whole idea; the results should be taken as no more than qualitative and representative of the general scales.
Still, the model looks promising, as it might explain the variability of the observed period in certain TTS; for example, the disappearance of the period altogether can be explained by the hot spot moving farther away from the star, thus evading being eclipsed. The model works only for highly inclined systems. It will be thus interesting to look for an observational correlation between period variability and inclination in CTTS. We do not rule out stellar spots as causing variability in the case when disks are absent (then this is the only effect) or when they are present (then the star and disk spots are both present, but only the disk spots are responsible for period changes).

In the next step of this research we intend to perform a three dimensional SPH simulation of the accretion disk and the stream resulting from the magnetic interaction taking into account in the mass infall rates as well (which shall affect the general spot luminosity).

Concluding, we make the following remarks:

1. The optical thickness of the stream and the penetration depth in the disk are functions of the stream density. We expect that if the stream density gets smaller, the observed temperature will get closer to the shock temperature; meanwhile the total luminosity will decline. This could explain the observed blue shift during the low luminosity phase in some TTS.

2. In some instances the observed period disappears altogether, and reemerges afterwards. In our model, for low disk inclination angles $i$, the spot will not be eclipsed for $R_{mp} > R_*/\sin(i)$. As we see in the examples, $R_{mp}$ varies in a wide range, so this is quite plausible. Another possible explanation is that the stream disappears altogether (for example if the loop lies in the disk plane), and reappears on the other side of the disk, its luminosity greatly diminished by the disk thickness. In addition, although improbable, the spot angular velocity could be very close to 0 if a suitable parameter variation occurs. The spot could thus be visible for a long duration.

3. We do not take here into account the scenario where the spot falls on the star itself. This occurrence is quite possible, especially when taking into account the spot size. At least a part of the spot may be on the star, while the other part is still on the disk. For instance, a disk spot changing into a star spot will have the following influences on the lightcurve: as a growing proportion of the stream falls on the star, the apparent area of the spot will grow; thus the luminosity will rise. Being on the stellar surface, the spot apparent size will vary as $\sin(\Omega_{spot} t)$, and the luminosity will vary sinusoidally. We may expect that the lightcurve will look somewhat similar to the one we obtained in figure 4.

4. If the spot grows beyond $2R_*$, we shall encounter shallower minima as only a part of the spot will be eclipsed. Taking into account the fact that a star is spherical, even for a constant spot size the eclipsed part shall decline as the spot falls further from the star, again resulting in shallower (and shorter) eclipses.

REFERENCES
Armitage K.J., 1995, MNRAS, 274,1242
Bertout C., 1989, ARA&A, 27,351
Bouvier J., Bertout C., 1989, A&A, 211,99
Bouvier J., Cabrit C., Fernandez M., Martin E.L., Matthews J., 1993, A&A, 272, 176
Bouvier J., Covino E., Kovo O., Martin E.L., Matthews J.M., Terranegra L., Beck S.G., 1995, A&A, 299, 89
Drell, S.D., Foley H.M., Ruderman, M.A., 1965, J. Geophys. Res., 70, 3131
Edwards S., Cabrit S., Strom S.E., Heyer I., Strom K.M., Anderson E. 1987, ApJ, 321, 473
Ghosh P., Lamb F.K., 1978, ApJ, 223, L83
Herbig, G.H., 1962, Adv. in A&A., 1,47
Hartmann L., Hewett R., Calvet N. 1994, ApJ, 426, 669
King, A.R., Regev, O., MNRAS, 268, L69-L73 (1994)
Königl A., 1991, ApJ, 370, L39
Lin, D.N.C., Pringle, J.E., 1976, IAU Symp. No.7, p183, eds. Eggleton,P., et al., Reidel, Dordrecht, Holland.
Menard, F., 1996, Magnetic Effects in Accretion Workshop, Technion, Haifa.

Pearson, K.J., King, A.R., 1995, MNRAS, 276, 1303

Pudritz R.E., Patel K., 1994, ApJ, 424, 688

Richter M., Basri G., Perlmutter S., Pennypacker C., 1992, PASP, 104, 1144

Rydgren A.E., Vrba F.J., 1983, ApJ, 267, 191

Rydgren A.E., Zak D.S., Vrba F.J., Chugainov P.F., Zajtseva G.V., 1984, AJ, 89, 1015

Simon T., Vrba F.J., Herbst W., 1990, AJ, 100(6), 1957

Smith, K.W., Lewis, G.F., Bonnell, I.A., MNRAS, 276, L5-L8 (1995)

Vrba F.J., Rydgren A.E., Chugainov P.F., Shakovskaya N.I., Zak D.S., 1986, ApJ, 306, 199

Worden S.P., Schneeberger T.J., Kuhn J.R., Africano J.L., 1981, ApJ, 244, 250

Wynn G.A. and King A.R., 1995, MNRAS, 275, 9

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Fig. 1.— Spot coordinates (in the rotating frame) as a function of time (a) for varying $\alpha$ ($\alpha$ in the range $1-10^6$), (b) $r_0$ (range 0.6-0.92), (c) $\phi$ (range 1.9-3.1 rad), (d) $\theta$ (in the range 0.5-1.3 rad). $t$ – time in $P_*^\star$ units

Fig. 2.— Apparent spot angular velocity, with the same parameters as figure 1. We can see that the maximal variation occurs in the $r_0$ case (b), while varying $\alpha$ hardly influences the results (a). Variation of $\phi$ (c) and $\theta$ (d) produce a comparable variability of some 10-15%. ($t$ – time in $P_*^\star$ units)

Fig. 3.— Power spectrum of the calculated intensity variations. (a) In the varying $\alpha$ case the frequency obtained equals $0.9 \times 2\pi/\Omega_*$. (b) Varying $R_0$ case. The deduced frequency is half the actual one, as one of the minima would be inobservable. (c) Varying $\phi$ case. The power spectrum suggests a frequency that is 10% less than the actual. (d) Varying $\theta$ case. The deduced frequency is about 15% more than the actual. ($t$ – time in $P_*^\star$ units)

Fig. 4.— Modelled spot luminosity (variable $r_0$ case). Both the period and the eclipse length vary, as does the overall shape of the light curve. The second minimum (arrow) is too shallow to influence the power spectrum.
Figure 1:
Figure 2:
Figure 3:
