Distortion of the acoustic peaks in the CMBR due to a primordial magnetic field

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\textbf{Abstract}

In this paper we study the effect of a magnetic field on the fluctuation spectrum of the cosmic microwave background. We find that upcoming measurements might give interesting bounds on large scale magnetic fields in the early Universe. If the effects are seen, it might be possible to establish the presence of different fields in different patches of the sky. Absence of any effect, will provide by one order of magnitude a better limit for a primordial field, now given by nucleosynthesis.

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1 Introduction

Very little is known about cosmic magnetic fields, both those that exist today and those in the early Universe. Even the stability of large fields is open to conjecture\cite{1}. In the galaxy one measures a field of the order of $10^{-6}$ Gauss but its origin remains a mystery \cite{2}. If it is primordial, it could have resulted from a compression of a cosmological field corresponding to around $10^{-9}$ Gauss today. This is comparable to limits set for fields on the horizon scale using Faraday rotation on faraway galaxies and quasars. When traced back in time such a field becomes quite strong since $B \sim 1/a^2$ where $a$ is the scale factor.

The presence of primordial fields is a hotly debated issue. For a long time the dynamo mechanism with small seed fields was favored, but the recent discovery of damped Ly\alpha lines in QSO’s indicates that primordial fields existed at early times. Moreover, there are problems with the dynamo mechanism. For a short discussion and further references see \cite{3}.

The QSO measurements are consistent with having $\mu$G fields at $z^{\text{abs}} = 2$. It is not unreasonable to expect that such fields might have had measurable effects on physics in the early Universe. One such possibility was studied in \cite{4} where it was found that nucleosynthesis bounded the field to $10^{11} - 10^{12}$ Gauss (lower limit for fields homogeneous on the horizon scale) at a time when $T = 10^9 K$. This corresponds to between $10^{-6}$ and $10^{-7}$ Gauss today. Another way to set limits, this time at last scattering, is to study Faraday rotation directly in the CMB. In \cite{3} it is claimed that it should be possible to reach a field equivalent to $10^{-9}$ Gauss today in this way. Existence of these fields may also have a large impact on structure formation \cite{7}.

In this paper we will discuss the possibility of taking advantage of the many upcoming precision measurements of CMB anisotropies. These measurements, involving satellites, ground interferometry, and balloons\cite{6}, promise to provide us with accurate values of many cosmological parameters.

When primordial density fluctuations, perhaps generated by inflation, enter the horizon some time before last scattering, they initiate acoustic oscillations in the plasma. These oscillations distort the primordial spectrum of fluctuations and their effect can be studied today. Clearly the result will be very sensitive to the physics of the plasma and this is the reason for the present optimism.

As we will argue in this paper, magnetic fields of reasonable magnitude will also affect the plasma leaving a possibly measurable imprint on the CMB. There are several exciting possibilities that may be detectable: a) different types of waves (see below) depending on the properties of the primordial fluid creating different displacements of acoustic peaks and changing their magnitudes, b) anisotropies (at the level of $10^{-6}$) that maybe different in different areas of the sky, signaling the presence of magnetic field patches in the early Universe.

The paper is organized as follows. In section 2 we recall some elementary magnetohydrodynamics describing the kind of waves which might be occurring in the plasma. In section 3 we discuss the various types in detail and give some qualitative
and quantitative predictions on how they might affect the CMB. Section 4 contains our conclusions.

2 Some magnetohydrodynamics

A rigorous analysis of the effects of the magnetohydrodynamics modes on the CMB requires the introduction of a multifluid theory and a general relativistic treatment. However, a brief description of the main features of the magnetohydrodynamics of a nonrelativistic one component charged fluid is physically illuminating and will occupy this section.

We will be considering a magnetic field homogenous on scales larger than the scale of plasma oscillations. We will therefore assume a background magnetic field $B_0$ constant in space. The actual field is $B_0 + B_1$ where $B_1$ is a small perturbation. We assume that the electric conductivity of the medium is infinite, thus the magnetic flux is constant in time. Then, due to the expansion of the Universe, $B_0 \propto a^{-2}$. We neglect here any dissipative effect, due for example to a finite viscosity and heat conductivity \[7\]. In other words we are assuming that $\lambda = \frac{2\pi}{k} \gg l_{diss}$. This is justified for the large scale fields that we are considering.

Within these assumptions the linearized equations of MHD in comoving coordinates are:

\[ \dot{\phi} + \frac{\nabla \cdot \mathbf{v}_1}{a} = 0, \]

\[ \dot{\mathbf{v}}_1 + \frac{\dot{a}}{a} \mathbf{v}_1 + \frac{c_s^2}{a} \nabla \delta + \frac{\nabla \phi_1}{a} + \frac{\hat{B}_0 \times (\mathbf{v}_1 \times \hat{B}_0)}{4\pi a^4} + \frac{\hat{B}_0 \times (\nabla \times \hat{B}_1)}{4\pi \rho_0 a^5} = 0, \]

\[ \partial_t \hat{B}_1 = \frac{\nabla \times (\mathbf{v}_1 \times \hat{B}_0)}{a}, \]

\[ \nabla^2 \phi_1 = 4\pi G \rho_0 \left( \delta + \frac{\hat{B}_0 \cdot \hat{B}_1}{4\pi \rho_0 a^4} \right) \]

and

\[ \nabla \cdot \hat{B}_1 = 0, \]

where $\hat{\mathbf{B}} \equiv \mathbf{B} a^2$ and $\delta = \frac{\rho_1}{\rho_0}$, $\phi_1$ and $\mathbf{v}_1$ are small perturbations on the background density, gravitational potential and velocity respectively. $c_s$ is the sound velocity. Neglecting its direct gravitational influence, the magnetic field couples to fluid dynamics only through the last two terms in eq 4. The first of these terms is due to the displacement current contribution to $\nabla \times \mathbf{B}$ whereas the latter account for the magnetic force of the current density. The displacement current term can be neglected provided that $v_A = B_0 / \sqrt{4\pi \rho} \ll c_s$, where $v_A$ is the Alfvén velocity.
Let us now discuss the basic properties of the solutions of these equations, ignoring for the moment the expansion of the Universe.\footnote{The full solutions are given in [5].} A useful reference on this subject is [5].

Without a magnetic field there is only the ordinary sound wave involving density fluctuations and longitudinal velocity fluctuations (i.e. along the wave vector). In the presence of a magnetic field, however, there are no less than three different waves:

1. \textit{Fast magnetosonic waves.}

   In the limit of small magnetic fields these waves become the ordinary sound waves. Their velocity, \( c_+ \), is given by
   \[
   c_+^2 \sim c_S^2 + v_A^2 \sin^2 \theta, \tag{6}
   \]
   where \( \theta \) is the angle between \( \mathbf{k} \) and \( \mathbf{B}_0 \). Fast magnetosonic waves involve fluctuations in the velocity, density, magnetic field and gravitational field. The velocity and density fluctuations are out-of-phase by \( \pi/2 \). Equation (6) is valid for \( v_A << c_S \). For such fields the wave is approximatively longitudinal.

2. \textit{Slow magnetosonic waves.}

   Like the fast waves, the slow waves involve both density and velocity fluctuations. The velocity is however fluctuating both longitudinally and transversely even for small fields. The velocity of the slow waves is approximatively
   \[
   c_-^2 \sim v_A^2 \cos^2 \theta. \tag{7}
   \]

3. \textit{Alfvén waves}

   For this kind of waves \( \mathbf{B}_1 \) and \( \mathbf{v}_1 \) lie in a plane perpendicular to the plane through \( \mathbf{k} \) and \( \mathbf{B}_0 \). In contrast to the magnetosonic waves, the Alfvén waves are purely rotational, thus they involve no density fluctuations. Alfvén waves are linearly polarized. Their velocity of propagation is
   \[
   c_A^2 = v_A^2 \cos^2 \theta. \tag{8}
   \]

   One should note that for \( v_A \) comparable to both \( c_S \) and the speed of light, the formula for the velocity of the Alfvén waves remains uncorrected while the velocity of the magnetosonic waves are given by
   \[
   c^2 = c_S^2(1 + v_A^2 \cos^2 \theta/c^2) + v_A^2 \pm ((c_S^2(1 + v_A^2 \cos^2 \theta/c^2) - v_A^2)^2 + 4v_A^2 c_S^2 \sin^2 \theta/c^2)^{1/2})
   \frac{1}{2(1 + v_A^2/c^2)}. \tag{9}
   \]

3 \textbf{Effects on the CMB}

The fluctuations in the CMB can be divided into primary and secondary fluctuations. The primary fluctuations involve effects coming directly from the density.
fluctuations and also from Doppler shifts from velocity fluctuations and gravitational redshifts.

We will concentrate on these primary effects and show that the presence of a magnetic field will change the predicted spectrum of fluctuations by changing the speed of sound.

### 3.1 The fast magnetosonic waves

The simplest, and most important, case is the fast wave. Let us consider the equations describing the oscillating baryon and photon fluid in conformal Newtonian gauge using conformal time, see e.g. [8] for the case without magnetic field. They are (for small \( v_A \)):

\[
\begin{align*}
\dot{\delta}_b + V_b - 3\phi &= 0, \\
\dot{V}_b + \frac{\dot{a}}{a} V_b - c_b^2 k^2 \delta_b + k^2 \psi + \frac{6\sigma_T(V_b - V_\gamma)}{R} - \frac{1}{4\pi\rho_b a} k \cdot (\dot{\mathbf{B}}_0 \times (k \times \dot{\mathbf{B}}_1)) &= 0, \\
\dot{\delta}_\gamma + \frac{4}{3} V_\gamma - 4\dot{\phi} &= 0
\end{align*}
\]

and

\[
\dot{V}_\gamma - k^2 \left(\frac{1}{4}\delta_\gamma - \sigma_\gamma\right) - k^2 \psi - 6\sigma_T(V_b - V_\gamma) = 0,
\]

where \( V = i k \cdot \mathbf{v} \) and \( R = \frac{3\rho_b}{4\rho_\gamma} \). \( c_b \) is the baryon sound velocity in the absence of interactions with the photon gas. We have also for convenience defined \( \rho_b = \frac{\hat{\rho}_b}{a^3} \) and \( \mathbf{B} = \frac{\hat{\mathbf{B}}}{a^3} \). The terms with \( \sigma_T \) are due to Thompson scattering and couple the photons and the baryons. This term can be eliminated between the equations. If furthermore tight coupling is assumed (implying e.g. \( V_b \sim V_\gamma \)), one can derive an equation for the density fluctuations only. If \( c_b \sim 0 \) one finds that in the absence of magnetic fields the effective sound velocity is

\[
c_{s}^2 = \frac{1}{3} \frac{1}{1 + R}.
\]

Thus through tight coupling the photons provide the baryon fluid with a pressure term and a non-zero sound velocity arises.

With a magnetic field we need one more equation:

\[
\dot{\mathbf{B}}_1 = i(\dot{\mathbf{B}}_0 \cdot \mathbf{k})\mathbf{v}_b - i(\mathbf{k} \cdot \mathbf{v}_b)\dot{\mathbf{B}}_0.
\]

Assuming longitudinal waves we find the last term of equation (11) to be

\[
- v_A^2 \sin^2 \theta k^2 \delta_b,
\]

as expected from the previous section.
Hence we find, to this order of approximation, that the only effect of the magnetic field is a change in the speed of sound. A simple way to account for a magnetic field is therefore to change

\[ c^2_s \rightarrow c^2_s + v_A^2 \sin^2 \theta. \] (17)

We have computed the microwave background spectrum with this adjustment of the sound velocity using the code of [9].

An extra step in the calculation of the CMB anisotropy arises due to the fact that the velocity of the fast waves depends on the angle between the wave-vector and the magnetic field. As mentioned previously we are assuming a magnetic field that is varying in direction on scales larger than the scale of the fluctuation. Hence we should sum over all wave-vectors with the angle between the magnetic field and the line of sight fixed. Different patches of the sky might therefore show different fluctuation spectra depending on this angle. In this paper we will only be considering an all-sky average assuming a field that is varying in direction on very large scales. For this reason we also sum over the angle between the field and the line-of-sight.

In practice, it is easier to reverse the order of the sum and the calculation of the microwave background anisotropy. The result of this procedure is shown in figure 1. We have assumed a magnetic field that gives a maximum increase in \( c^2_s \) of 0.05\( c^2 \) at last scattering, i.e. \( v_A^2 \sim 0.05 c^2 \). This corresponds to \( 2 \times 10^{-7} \) Gauss today. For a comparison consider figure 2 which shows the effect of a 20% decrease of baryons. Around the first peak the effects are comparable. This allows us to obtain a rough estimate for the magnitude of the magnetic fields which should be able to be detected by future measurements of the microwave background anisotropy. The process of parameter determination using a maximum likelihood fit of the observed multipole coefficients is discussed in [3, 10]. Assuming knowledge of the other cosmological parameters which affect the microwave background spectrum, a prediction of \( \Omega_b \) accurate to the order of a percent or so should be obtainable. This translates into a limit on the current strength of magnetic fields which were present in the early Universe, of the order of \( 5 \times 10^{-8} \) Gauss.

On very large scales, larger than the characteristic scale of the magnetic field, the effect will presumably be averaged out and the precise shape of the curve will depend on this scale. The curve in figure 1 is therefore not applicable for the very lowest values of \( l \), if we assume a field varying on, say, the horizon scale.

The approximations we have used can only be trusted for large scales, that means late times for the kind of fields we are considering. For earlier times the fields are too strong and the Alfvén velocity too high. It is therefore possible that an accurate treatment of the waves might turn up even more pronounced effects at small scales.

### 3.2 The slow waves

These waves are a little bit more complicated to handle than the fast ones, even at low magnetic fields because the equations do not decouple in a simple way. The reason is that they involve both longitudinal and transverse velocity fluctuations.
It is interesting to note, however, that depending on initial conditions they should be excited with an amplitude fixed relative to the fast waves. To illustrate this point we will consider a rather naive toy model. Using the initial conditions \( \dot{\delta}(0) = 0 \) and \( \mathbf{v} = 0 \) we find (using WKB)

\[
\rho \sim \alpha_+ \cos \omega_+ t + \alpha_- \cos \omega_- t + \text{constant},
\]

where \( \omega_{\pm} = c_{\pm} k \). To fix the ratio \( \alpha_- / \alpha_+ \) we need one further initial condition on \( \mathbf{B}_1 \). It is reasonable to assume

\[
\mathbf{B}_1(0) = 0,
\]

i.e., all fluctuations of the magnetic field (on this scale) are due to fluctuations of the plasma initiated when entering the horizon. Using [5] one can show that

\[
\frac{\alpha_-}{\alpha_+} \sim \frac{v_\perp^2}{c_s^2}.
\]

Since the velocity of the slow waves are much smaller than the velocity of the fast waves for small fields, we conclude that the Doppler peaks should have a long period modulation. Further details will be presented in a future publication.

### 3.3 Alfvén waves

As discussed in the previous section the Alfvén waves are purely rotational and involve no fluctuations in the density of the photon and baryon fluids.

With initial conditions like the ones above one sees that the Alfvén waves will not be excited. However, one could reverse the reasoning and use these waves to probe the initial conditions. They should be well suited for the detection of turbulent, rotational velocity perturbations in the early Universe such as those that might be generated from primordial phase-transitions. Isocurvature initial conditions are probably the most suitable to excite the Alfvén waves.

The equation describing the waves are

\[
\delta_b = 0,
\]

\[
\dot{\mathbf{v}}_b + \frac{\dot{a}}{a} \mathbf{v}_b + \frac{a n_e \sigma_T (\mathbf{v}_b - \mathbf{v}_\gamma)}{\mathcal{R}} - i \frac{(k \cdot \hat{\mathbf{B}}_0)}{4\pi \rho_b a} \hat{\mathbf{B}}_1 = 0,
\]

\[
\delta_\gamma = 0
\]

and

\[
\dot{\mathbf{v}}_\gamma - a n_e \sigma_T (\mathbf{v}_b - \mathbf{v}_\gamma) = 0.
\]

As expected, in this case the photon velocity is only affected by the baryon velocity through Thompson scattering.

It is evident that Alfvén waves give rise only to a Doppler effect on the CMB. As with the slow waves, we do not present any numerical estimate of the effects of
the Alfvén waves. This will be done in detail in a forthcoming paper. Here we only wish to point out that since we do not have any cancellation between Doppler and gravitational effects for this kind of waves, they could provide a more clear signature of the presence of magnetic fields at the last scattering surface.

4 Conclusions

In this paper we have taken some preliminary steps towards understanding the effects of magnetic fields on the CMB.

We have found that the limits one can set are comparable, or better than what can be achieved by other means, for example nucleosynthesis \[1\]. Fields below \(10^{-7}/a^2\) Gauss should be accessible in planned experiments. The possibility of finding anisotropies in different sectors of the sky and determine their nature is a possibility that is exciting. Depending on the scales this may yield information on the age of these fields and their spatial extent.

We have been considering magnetic fields on scales larger than the characteristic wavelengths of the acoustic waves. It is also important to investigate the possible effects due to random fields on smaller scales.

Clearly it is important to study these possible effects in more detail and thereby take advantage of the upcoming precision measurements of the cosmic microwave background.

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Figure Captions

Figure 1: The effect of a cosmic magnetic field on the multipole moments. The solid line shows the prediction of a standard CDM cosmology ($\Omega = 1, h = 0.5, \Omega_B = 0.05$) with an $n = 1$ primordial spectrum of adiabatic fluctuations. The dashed line shows the effect of adding a magnetic field equivalent to $2 \times 10^{-7}$ Gauss today.

Figure 2: The effect of lowering the baryon fraction by 20%
Fig 1.
Fig 2.