Rough Surface Effect on Meissner Diamagnetism in Normal-layer of N-S Proximity-Contact System

Kotaro Yamada, Seiji Higashitani and Katsuhiko Nagai

Department of Engineering, Hiroshima University, Higashi-hiroshima 739-8527
1Faculty of Integrated Arts and Sciences, Hiroshima University, Higashi-hiroshima 739-8521

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Rough surface effect on the Meissner diamagnetic current in the normal layer of proximity contact N-S bi-layer is investigated in the clean limit. The diamagnetic current and the screening length are calculated by use of quasi-classical Green’s function. We show that the surface roughness has a sizable effect, even when a normal layer width is large compared with the coherence length $\xi = v_F/\pi T_c$. The effect is as large as that of the impurity scattering and also as that of the finite reflection at the N-S interface.

KEYWORDS: Meissner effect, proximity effect, rough surface, screening length, quasi-classical Green’s function

§1. Introduction

A normal metal in contact with a superconductor is known to bear superconducting properties even if the normal metal has no pairing interaction. The superconducting order in the normal layer induced by the proximity effect produces a finite diamagnetic current to expel the magnetic field. The quantity of interest is the screening fraction, i.e., how great part of the normal layer contributes to the Meissner effect. Theories of diamagnetic response of N-S proximity contact system in both the dirty and the clean limit have been already reported. While early experimental results were found to be in agreement with the dirty limit theory, some data of more recent experiments failed to be described satisfactorily by either the clean or the dirty limit theory.

Belzig et al. proposed a quasi-classical theory of diamagnetic response of a normal layer with impurities and pointed out that, even in such a clean system that has a longer mean free path than the layer width, temperature dependence of the screening fraction can deviate considerably from that of the clean limit theory. Müller-Allinger et al. tried to fit the quasi-classical theory with their experimental data using the mean free path $l_N$ as a fitting parameter. Although they found

* Present address: Institute for Research in Humanities, Kyoto University, Kyoto 606-8501.
fairy good agreement in the temperature dependence of the screening fraction with the experimental
data of the samples with various impurity concentration, there remain some samples which exhibit
considerably different behavior that cannot be fitted by the theory of ref. 7. Moreover, best fits
were obtained by using the mean free path a few times smaller than the measured value. 8)

In the theory by Belzig et al. 7 it is assumed that the N-S interface is transparent, i.e., electrons
pass through the interface without reflection. It is also assumed that the surface scatters electrons
specularly. The effect of the finite reflection at the interface was examined by Higashitani and
Naga 5 for clean systems and by Hara et al. 15 for systems with impurities. They showed that the
finite electron reflection at the N-S interface has strong influence on the screening fraction.

The purpose of this paper is to study the effect of surface roughness on the screening fraction
using the quasi-classical Green’s function method. 2, 3, 4 The proximity induced order parameter
extends from the N-S interface into the normal layer with the length scale of the N side coherence
length $\xi_N(T)$. At low temperatures, it will reach the end surface of the normal layer and feel
the surface roughness. It is expected, therefore, the surface roughness effect is important at low
temperatures. To focus on the surface roughness effect, we study a clean system and neglect the
effect of finite reflection at the interface. We show that the surface roughness has a quantitatively
large effect on the screening fraction.

This paper is organized as follows. In the next section, we derive a linear response of the
quasi-classical Green’s function to the external vector potential. In §3, we briefly review the quasi-
classical theory of rough surface effect by Nagato et al. 17, 18 In §4, explicit form of the quasi-classical
Green’s function in the normal layer of our model system is given. Diamagnetic current and the
screening fraction are discussed in §5 and 6. The last section is devoted to summary and discussion.
Throughout this paper, we use the unit $\hbar = k_B = 1$.

§2. Quasi-classical Theory of Diamagnetic Current

In this paper, we consider a clean N-S double layer system as shown in Fig. 1. The end wall at
$z = 0$ of the normal layer has roughness, but we assume for simplicity that the N-S interface has a
translational symmetry in the $x$-$y$ plane.

We calculate the linear response of the diamagnetic current to the vector potential using quasiclassical Green’s function method proposed by Ashida et al. and Nagato et al. 17 The vector
potential is taken as $A = (A(z), 0, 0)$, so that the magnetic field is in the $y$-direction.

We begin with the Gor’kov Green’s function. According to ref. 13, the Gor’kov Green’s function
is given by a product of the slowly varying part and the rapidly oscillating part with the period of
the Fermi wave length. In the geometry of present interest, the Gor’kov Green’s function can be
expanded in a form

$$G(\mathbf{r}, \mathbf{r'}, \omega_n) = \sum_{K, K'} \sum_{\alpha = \pm} \sum_{\beta = \pm} G_{\alpha \beta}(K, K', z, z') e^{iK \cdot s - iK' \cdot s'} e^{i\alpha k z - i\beta k' z'},$$  (2.1)
where $\omega_n$ is the Matsubara frequency, $K$ is the $s = (x, y)$ component and $k = \sqrt{2m^*E_F - K^2}$ is the $z$-component of the Fermi momentum. Note that the slowly varying Green's functions $G_{\alpha\beta}(K, K', z, z')$ are not diagonal in the directional ($\alpha$) space because of the presence of the surface reflection. They are also not diagonal in $K$ space because of the surface roughness. They obey the first order differential equation

$$
(i\omega_n + i\alpha v_K \rho_3 \partial_z + \frac{e}{mc} A(z) K_x - \hat{\Delta}_\alpha) G_{\alpha\beta}(K, K', z, z') = \delta_{\alpha\beta} \delta_{KK'} \delta(z - z'),
$$

(2.2)

where $v_K = k/m$ is the $z$-component of the Fermi velocity, $\rho_3$ is a Pauli matrix in the particle-hole space and $\hat{\Delta}$ is the order parameter matrix given by

$$
\hat{\Delta}_\alpha = \begin{pmatrix}
0 & \Delta_\alpha(K, z) \\
\Delta_\alpha^*(K, z) & 0
\end{pmatrix}.
$$

(2.3)

It is convenient to introduce a one-point function $\hat{G}_{\alpha\beta}(K, K', z)$ defined by

$$
\hat{G}_{\alpha\beta}(K, K', z) \pm i\alpha \delta_{KK'} - 2\sqrt{v_K v_K'} \rho_3 G_{\alpha\beta}(K, K', z \pm 0, z),
$$

(2.4)

where we have taken into account the fact that $G_{\alpha\beta}(K, K', z, z')$ has a jump at $z = z'$ because of the delta function in the right hand side of eq.(2.2). The one-point function obeys an equation

$$
\partial_z \hat{G}_{\alpha\beta}(K, K', z) = i\alpha \hat{\varepsilon}_{K\alpha} \rho_3 \hat{G}_{\alpha\beta}(K, K', z) - i\beta \hat{G}_{\alpha\beta}(K, K', z) \hat{\varepsilon}_{K'\beta} \rho_3,
$$

(2.5)

$$
\hat{\varepsilon}_{K\alpha} = i\omega_n + \frac{e}{mc} A(z) K_x - \hat{\Delta}_\alpha.
$$

(2.6)

When $K = K'$ and $\alpha = \beta$, this equation coincides with the Eilenberger equation. Thus the one-point function $\hat{G}_{\alpha\beta}(K, K', z)$ can be interpreted as generalized quasi-classical Green’s function.

Since eq.(2.5) is a first-order differential equation, the spatial dependence of $\hat{G}_{\alpha\beta}(K, K', z)$ can be described by

$$
\hat{G}_{\alpha\beta}(K, K', z) = U_\alpha(K, z, z') \hat{G}_{\alpha\beta}(K, K', z') U_\beta(K', z'),
$$

(2.7)

where $U_\alpha(K, z, z')$ is the evolution operator that satisfies

$$
iv_K \partial_z U_\alpha(K, z, z') = -\alpha \hat{\varepsilon}_{K\alpha} \rho_3 U_\alpha(K, z, z')
$$

(2.8)

and the initial condition

$$
U_\alpha(K, z, z) = 1.
$$

(2.9)

It is straightforward to calculate from eq.(2.2) the linear response of the Green’s function $\delta G_{\alpha\beta}(K, K', z, z')$ to the vector potential $A(z)$:

$$
\delta G_{\alpha\beta}(K, K', z, z') = \sum_\gamma \sum_{K''} \int_0^\infty dz'' G_{\alpha\gamma}^{(0)}(K, K'', z, z'') \left[ -\frac{e}{mc} A(z) K_x \right] G_{\gamma\beta}^{(0)}(K'', K', z'', z'),
$$

(2.10)
where $G^{(0)}_{\alpha\beta}$ is the Green’s function without magnetic field. In what follows, we omit the superscript (0). The diamagnetic current is calculated from the quasi-classical Green’s function $\delta \hat{G}_{\alpha\alpha}(K, K', z)$. Using eqs.(2.4) and (2.7) we find

$$
\delta \hat{G}_{\alpha\alpha}(K, K', z) = U_\alpha(K, z, 0) \sum_\gamma \sum_{K''} \left[ \int_0^z dz' \hat{G}_{\alpha\gamma}^{(+)}(K, K'', 0) \hat{L}_\gamma(K'', z') \hat{G}_{\gamma\alpha}^{(-)}(K'', K', 0) + \int_z^\infty dz' \hat{G}_{\alpha\gamma}^{(-)}(K, K'', 0) \hat{L}_\gamma(K'', z') \hat{G}_{\gamma\alpha}^{(+)}(K'', K', 0) \right] U_\alpha(K', 0, z),
$$

where

$$
\hat{G}_{\alpha\gamma}^{(\pm)}(K, K', 0) = \hat{G}_{\alpha\beta}(K, K', 0) \pm i \alpha \delta_{\alpha\beta} \delta_{KK'},
$$

and

$$
\hat{L}_\gamma(K, z') = U_\gamma(K, 0, z') \frac{e}{mc} A(z') K_x \mu_K \rho_\gamma U_\gamma(K, z', 0).
$$

The quasi-classical Green’s function $\delta \hat{G}_{\alpha\alpha}(K, K', z)$ is completely determined if we have a knowledge of the surface value of $\hat{G}_{\alpha\beta}(K, K', 0)$ that can be determined by solving the boundary problem.

§3. Rough Surface Boundary Condition

To treat the rough surface effect, we use the random S-matrix theory developed by Nagato et al.\cite{17, 18} Let us consider the scattering of an electron at the Fermi level by the surface. The scattering from an initial state $(K, -k)$ to a final state $(Q, q)$ is described by an S-matrix $S_{KQ}$. Since $S$ is a unitary matrix, we can rewrite $S$ using an Hermite matrix $\eta$

$$
S = -\frac{1 - i\eta}{1 + i\eta}.
$$

Nagato et al.\cite{17, 18} showed that one can have a formal solution for $\hat{G}_{\alpha\beta}(K, K', 0)$ that satisfies the boundary condition described by the S-matrix. The quasi-classical Green’s function thus obtained is that for a particular configuration of the rough surface, while the surface roughness will be distributed randomly enough over the surface. What we are interested in is the quasi-classical Green’s function averaged over such randomness.

In what follows, we treat every element of $\eta$ as random variable to describe the statistical properties of the surface. We assume that $\eta_{KQ}$’s obey the Gaussian statistics with $\overline{\eta_{KQ}} = 0$ and $\overline{\eta_{KQ} \eta_{K'Q'}} = \eta_{KQ} \eta_{K'Q'}^2 = \eta_{KQ} \delta_{Q-K, Q'-K'}$.

It is instructive to discuss here on the nature of the surface scattering in the normal state. The average of the specular reflection amplitude is evaluated within the self-consistent Born approximation to be\cite{17, 18}

$$
\overline{S_{KK}} = -\frac{(1 - i\eta)}{1 + i\eta} = -(1 - 2\eta^2 + 2\eta^4 - \cdots)_{KK}
$$

$$
= -\frac{(1 - \sigma_K)}{1 + \sigma_K},
$$

(3.2)
where $\sigma_K$ obeys an integral equation

$$\sigma_K = \sum_Q \eta_{KQ}^{(2)} \frac{1}{1 + \sigma_Q}.$$  

(3.3)

In the same approximation, we can calculate the average of the scattering probability $|S_{KQ}|^2$:

$$|S_{KQ}|^2 = |S_{KK}|^2 \delta_{KQ}$$

$$+ 4 \left( \frac{1}{1 + \sigma_K} \right)^2 \Gamma_{KQ} \left( \frac{1}{1 + \sigma_Q} \right)^2,$$

(3.4)

$$\Gamma_{KQ} = \eta_{KQ}^{(2)} + \sum_P \eta_{KP}^{(2)} \left( \frac{1}{1 + \sigma_P} \right)^2 \Gamma_{PQ}.$$  

(3.5)

We adopt a simplest model and put $\eta_{KQ}^{(2)}$ constant:

$$\eta_{KQ}^{(2)} = 2W \sum_Q 1.$$  

(3.6)

When $W = 1$, it simulates a completely diffusive surface. The specular reflection amplitude $S_{KK}$ vanishes and the scattering probability $|S_{KQ}|^2$ is independent of the outgoing momentum $Q$. Nagato et al. have shown that this model reproduces the diffusive wall boundary condition for the Ginzburg-Landau equation of the $p$-wave state given by Ambegaokar, de Gennes and Rainer.

The case with $W = 0$ corresponds to the specular surface. Thus, by changing $W$ from $W = 0$ to $W = 1$, we can discuss the surface scattering from the specular limit to the diffusive limit.

Let us consider the random average of $\delta \hat{G}_{\alpha\alpha}(K, K', z)$ of eq.(2.11). It is convenient to write

$$\delta \hat{G}_{\alpha\alpha}(K, K', z) = U_{\alpha}(K, z, 0) \sum_{\gamma} \sum_{K''} \left[ \int_0^z dz' \hat{G}_{\alpha\gamma}^{(+)}(K, K'', 0) \hat{L}_\gamma(K'', z') \hat{G}_{\gamma\alpha}^{(-)}(K'', K', 0) 

+ \int_0^\infty dz' \hat{G}_{\alpha\gamma}^{(-)}(K, K'', 0) \hat{L}_\gamma(K'', z') \hat{G}_{\gamma\alpha}^{(+)}(K'', K', 0) 

+ \int_0^\infty dz X_{\alpha\gamma}(K, K'', K', z') \right] U_{\alpha}(K', 0, z),$$

(3.7)

where $X_{\alpha\gamma}(K, K'', K', z')$ is defined by

$$X_{\alpha\gamma}(K, K'', K', z) = \left( \hat{G}_{\alpha\gamma}(K, K'', 0) - \hat{G}_{\alpha\gamma}(K, K'', 0) \right) \times \hat{L}_\gamma(K'', 0, z) \left( \hat{G}_{\gamma\alpha}(K'', K', 0) - \hat{G}_{\gamma\alpha}(K'', K', 0) \right).$$

(3.8)

The average $\hat{G}_{\alpha\beta}(K, K', 0)$ in the random S-matrix model has been given by Nagato et al.

$$\hat{G}_{\alpha\beta}(K, K', 0) = \hat{G}_{\alpha\beta}(K, 0) \delta_{KK'},$$

(3.9)

$$\hat{G}_{\alpha\beta}(K, 0) = \left( \hat{G}^S(K, 0)^{-1} + \frac{i}{2}(\alpha - \beta) \right) 

+ \left( \hat{G}^S(K, 0)^{-1} - i\alpha \right) \frac{1}{\hat{G}^S(K, 0)^{-1} - \sigma_K} \left( \hat{G}^S(K, 0)^{-1} + i\beta \right),$$

(3.10)
where $\hat{G}^S(K,0)$ is the Green’s function for the specular surface

$$\hat{G}^S(K,0) \equiv \hat{G}^S_{++}(K,0) = \hat{G}^S_{--}(K,0)$$

and $\sigma_K$ is the surface self energy determined by an integral equation

$$\sigma_K = \sum_Q \eta^{(2)}_{KQ} \frac{1}{G^S(Q,0)^{-1} - \sigma_Q}. \quad (3.11)$$

The average of $X_{\alpha\gamma}(K, K''', K', z)$ that corresponds to the vertex correction in the theory of impurity scattering can be performed in the same way as in the calculation of the scattering probability of eq.(3.4).

$$\sum_{\gamma} \sum_{K''} X_{\alpha\gamma}(K, K''', K', z) = X_{\alpha}(K, z) \delta_{KK'}, \quad (3.12)$$

$$X_{\alpha}(K, z) = \left(G^S(K,0)^{-1} - i\alpha\right) \left(Y_K \sum_Q \eta^{(2)}_{KQ} \Pi_Q(z) Y_K\right) \left(G^S(K,0)^{-1} + i\alpha\right), \quad (3.13)$$

where

$$Y_K = \frac{1}{G^S(K,0)^{-1} - \sigma_K}, \quad (3.14)$$

$$\Pi_K(z) = Y_K \left(\hat{M}(K, z) + \sum_Q \eta^{(2)}_{KQ} \Pi_Q(z)\right) Y_K, \quad (3.15)$$

$$\hat{M}(K, z) = \sum_{\gamma} \left(G^S(K,0)^{-1} + i\gamma\right) \hat{L}_\gamma(K, 0, z) \left(G^S(K,0)^{-1} - i\gamma\right). \quad (3.16)$$

Thus the averaged quasi-classical Green’s function becomes diagonal in $K$ space:

$$\delta\hat{G}_{\alpha\alpha}(K, K', z) = \delta\hat{G}_{\alpha\alpha}(K, z) \delta_{KK'}. \quad (3.17)$$

§4. Quasi-classical Green’s Function in the Normal Layer

In this section, we discuss the quasi-classical Green’s function in the normal layer of N-S proximity contact system. In order to focus on the rough surface effect, we assume that the superconducting layer is an s-wave semi-infinite superconductor and the normal layer has no pairing interaction, i.e.,

$$\Delta_\alpha(K, z) = \begin{cases} 0 & \text{for } 0 \leq z \leq d, \\ \Delta(z) & \text{for } d \leq z \leq \infty. \end{cases} \quad (4.1)$$

We also adopt the simplest model of eq.(3.6) for surface roughness, as a result the surface self energy $\sigma_K$ is a constant independent of $K$.

Under these assumptions, one finds from eq.(3.15) that $\sum_Q \eta^{(2)}_{KQ} \Pi_Q(z) = 0$, because $\hat{L}_\gamma(K, 0, z)$ is an odd function of $K$. It follows that $X_{\alpha}(K, z) = 0$ and the third term in eq.(3.7) can be omitted.
We further assume that the N-S interface at \( z = d \) is transparent. Then, we have for the averaged Green’s function in the normal layer

\[
\delta \hat{G}^N_{\alpha\alpha}(K, z) = U^N_{\alpha}(K, z, 0) \sum_{\gamma} \left[ \int_0^z dz' \hat{G}^{(+)}_{\alpha\gamma}(K, z', z') \hat{G}^{(-)}_{\gamma\alpha}(K, z') \right. \\
+ \int_z^d dz' \hat{G}^{(-)}_{\alpha\gamma}(K, z') \hat{G}^{(+)}_{\gamma\alpha}(K, z') + \int_{\infty}^d dz' \hat{G}^{(-)}_{\alpha\gamma}(K, z') \hat{G}^{(+)}_{\gamma\alpha}(K, z') \right] U^N_{\alpha}(K, 0, z), \tag{4.2}
\]

where

\[
\hat{G}^{(\pm)}_{\alpha\gamma}(K, 0) = \hat{G}^{\mp}_{\alpha\gamma}(K, 0) \pm i\delta_{\alpha\gamma}, \tag{4.3}
\]

\[
\hat{L}^N_{\gamma}(K, z') = U^N_{\gamma}(K, 0, z') \frac{e}{mc} A(z') K_x v_F^\omega \rho_3 U^N_{\gamma}(K, z', 0), \tag{4.4}
\]

\[
\hat{L}^S_{\gamma}(K, z') = U^S_{\gamma}(K, 0, d) U^S_{\gamma}(K, d, z') \frac{e}{mc} A(z') K_x v_F^\omega \rho_3 U^S_{\gamma}(K, z', d) U^N_{\gamma}(K, d, 0). \tag{4.5}
\]

Here, \( U^{N(S)} \) is the evolution operator in the N (S) layer.

The explicit form of the Green’s function \( \hat{G}^{++} \) is given by

\[
\hat{G}^{++}(K, 0) = i \left( \frac{\hat{A}}{\frac{1}{2} \text{Tr} \hat{A}} - 1 \right), \tag{4.6}
\]

where

\[
\hat{A} = U^N_{++}(K, 0, d) U^S_{++}(K, d, \infty) U^S_{++}(K, \infty, d) U^N_{++}(K, d, 0) \frac{1 - i\sigma}{1 + i\sigma} \tag{4.7}
\]

and \( \text{Tr} \) denotes the trace in particle-hole space. We note that the surface self energy \( \sigma_K = \sigma \) is \( K \)-independent in the present model of eq.(3.6). The \( U^S_{\pm}(z, z') \) carries all the information on the superconducting layer. It depends in general on the spatial profile of the order parameter \( \Delta(z) \).

But we assume for simplicity that \( \Delta(z) \) is a constant and is equal to the bulk BCS value \( \Delta(T) \). By this simplification, we can have an analytic solution for \( U^S_{\pm}(z, z') \) as well as for \( U^N_{\pm}(z, z') \):

\[
U^N_{\alpha}(z, z') = \exp \left[ -\alpha \frac{\omega_\n}{v_F} (z - z') \right], \tag{4.8}
\]

\[
U^S_{\alpha}(z, z') = \exp \left[ -\alpha \frac{\omega_\n}{v_F} (\omega_\n \rho_3 + \Delta \rho_2) \right] (z - z'). \tag{4.9}
\]

Substituting them into eq.(4.7), we find

\[
\hat{A} = \frac{1}{2\Omega} \begin{pmatrix} (\Omega + \omega_\n)e^{\omega_\n d} & -i\Delta \\ i\Delta & (\Omega - \omega_\n)e^{-\omega_\n d} \end{pmatrix} \frac{1 - i\sigma}{1 + i\sigma}, \tag{4.10}
\]

where \( \Omega = \sqrt{\omega_\n^2 + \Delta^2} \) and \( \kappa = 2\omega_\n/v_F \). The surface self energy is determined from eq.(3.11). This has to be done numerically for each Matsubara frequency \( \omega_\n \).

The other Green’s functions \( \hat{G}^{+-}, \hat{G}^{-+} \) and \( \hat{G}^{--} \) are related to \( \hat{G}^{++} \) via eq.(3.10).
§5. Diamagnetic Current

In this section, we calculate the diamagnetic current using the quasi-classical Green’s function derived in the previous section. The diamagnetic current \( j(z) \) which flows in the \( x \)-direction in the present geometry is given by

\[
j(z) = (-e) T \sum_{\omega_n} \sum_K v_{F_x} \text{Tr} \rho_3 \sum_\alpha \delta \hat{G}_{\alpha\alpha}(K, z) \frac{\pi}{2} \sum_\omega \int_0^{\pi/2} d\theta \sin \theta \int_0^{2\pi} \frac{d\varphi}{2\pi} v_{F_x} \text{Tr} \rho_3 \sum_\alpha \delta \hat{G}_{\alpha\alpha}(K, z), \quad (5.1)
\]

where \( N(0) \) is the density of states at the Fermi level, \( \theta \) is the polar angle of the Fermi momentum and \( v_{F_x} = v_F \sin \theta \cos \varphi \) is the \( x \)-component of the Fermi velocity.

As was shown by Zaikin and Higashitani and Nagai, the diamagnetic current \( j(z) \) is constant throughout the normal layer and the current-field relation is completely nonlocal. This is characteristic to the clean system and happens because the normal state evolution operator \( U_N^{\alpha}(K, z, z') \) commutes with \( \rho_3 \), therefore \( \text{Tr} \rho_3 \sum_\alpha \delta G_{\alpha\alpha}^N(K, z) \) is independent of \( z \).

We show below that rough surface effect is large even if the normal layer width \( d \) is larger than the coherence length \( \xi \) and the penetration depth \( \lambda \). In such a case, we can neglect the third term in eq.(4.2), because the vector potential \( A(z) \) decays exponentially. Now we have an expression for the diamagnetic current in the normal layer:

\[
j(z) = -\frac{e^2}{mc} n_N^* \bar{A}, \quad (5.2)
\]

where \( \bar{A} \) is the vector potential averaged over the normal layer and

\[
n_N^* = n_N d\pi T \sum_{\omega_n} \langle \langle K / v_{F_x} \rangle \rangle \quad (5.3)
\]

is the effective superfluid density, i.e., the density of proximity-induced superconducting electrons in the normal layer. Here,

\[
K = \frac{1}{2} \text{Tr} \rho_3 \sum_{\alpha\gamma} \hat{G}^{(+)}_{\alpha\gamma} \rho_3 \hat{G}^{(-)}_{\gamma\alpha}, \quad (5.4)
\]

\( n_N = p_F^3 / 3\pi^2 \) is the electron number density in the normal layer and the double bracket means the angle average over the polar angle

\[
\langle \langle \cdots \rangle \rangle = \frac{3}{2} \int_0^{\pi/2} d\theta \sin^3 \theta \langle \cdots \rangle. \quad (5.5)
\]

Combining eq.(5.2) with the Maxwell equation \( j_x = -\frac{c}{4\pi} \frac{\partial^2}{\partial z^2} A(z) \), we find the magnetic field \( B(z) \) in the normal layer when the applied external magnetic field is \( H \):

\[
B(z) = H (1 - z/\lambda), \quad (5.6)
\]
where
\[ \lambda = d \left( \frac{2\lambda_{NL}^2 n_N^*}{d^2 n_N^*} + \frac{2}{3} \right) \] (5.7)
and \( \lambda_{NL} \) is the London penetration depth of the normal layer defined by
\[ \lambda_{NL} = \sqrt{\frac{mc^2}{4\pi e^2 n_N}}. \] (5.8)

In Fig. 2, we show the surface roughness dependence of \( n_N^*/n_N \) for \( d/\xi =20 \) and 50 (\( \xi = v_F/\pi T_c \) the coherence length). The solid curve represents the temperature dependence of \( n_N^*/n_N \) in the diffusive limit and the dashed one represents the specular limit. In both the limits \( n_N^* \) shows similar temperature dependence but the magnitude is considerably different. As was shown in ref. 5, \( n_N^* \) is exponentially small at high temperatures and begins to grow below temperature \( \sim (\xi/d)T_c \) at which the order parameter extends up to the normal layer end. The magnitude of \( n_N^* \) is strongly reduced, in particular, at low temperatures by the diffusive scattering effect. The largest reduction is obtained at \( T = 0 \). We see that \( n_N^*/n_N \) at \( T = 0 \) reaches to unity in the specular case but is reduced to about 0.65 in the diffusive limit. We can also see that the suppression rate of \( n_N^*/n_N \) does not depend upon the normal layer width \( d \). This means that, at low temperatures, the rough surface effect has to be taken into account even in such thick normal layers that satisfy the conditions \( d > \xi \) or \( d > \lambda_{NL} \).

§6. Screening Length

In this section, we discuss the screening length \( \rho \) defined by
\[ \rho = \frac{1}{H} \int_0^d dz \left( H - B(z) \right). \] (6.1)
This length is equivalent to total diamagnetic susceptibility of the normal layer. The explicit expression for the screening length \( \rho \) in clean N-S systems was first given in ref. 5. The same result holds for the present N-S system and is given by
\[ \rho/d = \frac{3}{4} \frac{1}{1 + 3(\lambda_{NL}/d)^2(n_N/n_N^*)}. \] (6.2)

Figure 3 shows the temperature dependence of the screening fraction \( \rho/d \) when \( d/\xi = 20 \) and \( d/\xi = 50 \). The solid curves represent the diffusive limit and the dashed curves represent the specular limit. The temperature dependence of \( \rho/d \) comes only from \( n_N^* \). When \( n_N^*/n_N \ll 1 \) at high temperatures, the screening fraction can be approximated by \( \rho/d \simeq \frac{1}{3}(n_N^*/n_N)/\lambda_{NL}/d)^2 \) and is proportional to the effective superfluid density \( n_N^* \). Accordingly, \( \rho/d \) increases exponentially with decreasing temperature. At low temperatures, since \( n_N^*/n_N \) becomes of order unity and we are considering thick normal layers such that satisfy \( \lambda_{NL}/d \ll 1 \), we may neglect the term \( 3(\lambda_{NL}/d)^2(n_N/n_N^*) \) in the denominator of eq.(6.2), so that \( \rho/d \) exhibits saturation to 3/4. These
high and low temperature behaviors of $\rho/d$ are common to the specular and the diffusive limits. The difference between the two results is therefore visible only in the intermediate temperature range, though the superfluid density $n_s^*$ has significant difference at all temperatures.

Belzig et al. discussed the impurity scattering effect on $\rho/d$ in a same system with our’s but with specular surface. Hara et al. considered the effect by finite reflection at the interface in addition to the impurity effect. The finite reflection has a similar effect to the rough surface effect, namely, that dose not alter much the qualitative temperature dependence of $\rho$ but reduces considerably its magnitude. The impurity effect is more involved. In dirty systems, the screening fraction $\rho/d$ increases rather slowly with decreasing temperature ($\rho \propto T^{-1/2}$ in the dirty limit) and reaches to a larger value than the clean limit one $3/4$ at $T = 0$. The detailed structure of the temperature dependence of $\rho$ is sensitive to the impurity concentration. The difference in $\rho$ between clean and dirty systems originates from the fact that the impurity scattering makes the magnetic response be local.

When compared with the results of Belzig et al. and Hara et al., the present analysis suggests that the rough surface scattering effect is quantitatively as large as those by impurity scattering and by finite reflection at the interface. This means that when a system is comparatively clean we have to take into account the rough surface effect simultaneously with those by the impurity scattering and by the finite reflection at the interface.

§ 7. Summary and Discussion

We have studied the rough surface effect on the diamagnetic current in the N-S proximity-contact system using the quasi-classical theory. The linear response of the quasi-classical Green’s function to the vector potential has been calculated correctly taking into account the rough surface effect.

We studied the temperature dependence of the diamagnetic current in the clean normal layer and found that the diamagnetic current is significantly suppressed by the diffusive surface scattering even in a thick normal layer of which the width is larger than the coherence length $\xi = v_F/\pi T_c$. We have shown that in comparatively clean systems the rough surface scattering effect is as significant as those by the impurity scattering and by the finite reflection at the interface. Those effects should be considered simultaneously for the quantitative comparison with experiments.

Before concluding this report it is worth mentioning the paramagnetic re-entrant behavior of a N-S proximity contact system reported by Mota and co-workers. In the present calculation, no such behavior was found. Bruder and Imry suggested that the glancing current along the surface of normal metal cylinder contributes to the paramagnetic susceptibility. But the theory was criticized for the predicted paramagnetism being too small to explain the experiment. Fauchère et al. pointed out that when the pairing interaction in the normal metal is repulsive, a zero energy bound state is formed at the interface and that zero-energy bound state contributes to the paramagnetic current. The bound state energy, however, is not always equal to zero but depends
on the reflection coefficient $R$ at the interface. More recently, Maki and Haas proposed that the normal metals (noble metals) may become $p$-wave superconductor and generate a counter-current in the normal metal. The problem of the paramagnetic re-entrant behavior is still to be studied theoretically as well as experimentally.

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Fig. 1. A geometry of N-S double layer system with a rough surface. The solid and the dashed curves represent the order parameter on the S side and on the N side, respectively.

Fig. 2. The effective superfluid density $n_N^*$ in the normal layer as a function of temperature $T/T_c$ for layer widths $d/\xi = 20$ and $d/\xi = 50$. Solid curves represent the diffusive limit. Dashed curves represent the specular limit.

Fig. 3. Temperature dependence of screening length $\rho$ for layer widths $d/\xi = 20$ and $d/\xi = 50$. Solid curves represent the diffusive limit. Dashed curves represent the specular limit.
Fig. 1

Yamada et al.
Fig. 1

Yamada et al.
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Yamada et al.
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