Effect of wall-slip on natural convection in a circular annulus

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Abstract. Two-dimensional natural convection in an annulus enclosed by a hot inner cylinder and cold outer cylinder with air (Prandtl number, Pr=0.71) as the fluid is numerically investigated for Rayleigh number (Ra) = 5 \times 10^4 using OpenFOAM. For a gap width to radius ratio (L/R_i) of 1.6, the slip configurations are imposed by varying the Slip factor (SF) from 0 (no-slip) to 1 (full-slip) in steps of 0.2. The flow patterns and thermal fields for each slip factor are visualised using streamlines and isotherms and heat transfer is quantified by Nusselt number (Nu). It is observed that the natural convection patterns at full-slip boundary condition (SF=1) exhibit unique phenomena like vortex splitting, shrinkage in the thermally stagnant regions, and reduction in thermal boundary layer thickness. The heat transfer characteristics, in general, get enhanced with the increase in SF.

1. Introduction
Natural convection in an annulus has been a topic of interest due to several applications in nuclear reactors, thermal storage systems, electronic cooling, etc., to cite a few. In one of the earliest reported studies, Kuehn & Goldstein[1] experimentally and numerically investigated natural convection in the horizontal annulus between two concentric circular cylinders for a range of Rayleigh numbers (Ra). The transition of flow regime from pure-conduction to convection was discussed in detail through visual inspection of flow and thermal fields.

Boyd[2] established a correlation between mean Nusselt number and Rayleigh Number using fundamental concepts of different flow regimes and derived its governing equations in the annulus. This correlation technique was found to be applicable for the irregular annulus formed by an inner hexagonal cylinder and an outer concentric circular cylinder. The flow regimes were divided into inner and outer boundary layers, inner and outer intermediate region, plume, and stably stratified region. Rao et al.[3] investigated experimentally and numerically the natural convection and flow patterns of wide, moderate, and narrow annuli. They noticed that the number of vortices increased with an increase in Rayleigh number and for higher Rayleigh number, oscillatory flow patterns were observed. Date[4] conducted numerical investigations on natural convection heat transfer in a horizontal annulus for gap width to inner diameter ratios (\frac{L}{D_i}) = 0.8 and 0.15. The latter value was of interest for their study due to its application in horizontal heavy water reactors.

Numerous other studies discussed heat transfer in the annulus of horizontal circular cylinders by changing the location of the inner cylinder. Projahn & Beer[5] studied the influence
of Prandtl number on natural convection in eccentric and concentric annuli. Shahraki[6] employed the modified Boussinesq approximation for simulating the heat transfer characteristics in the annulus of two horizontal cylinders with a radius ratio of 2.6 and for various Rayleigh numbers from $10^3$ to $10^5$ at various eccentricities. Guj & Stella[7] numerically visualized the fluid flow and thermal fields in a two-dimensional annulus of eccentric cylinders for $Pr=0.71, 0.53 \times 10^4 \leq Ra \leq 8.27 \times 10^4$ with eccentricity from 0 to 1 and concluded that heat transfer reduced at lower gaps. Shi et al.[8] proposed a finite difference-based lattice BGK model to obtain velocity and temperature distribution in a horizontal annuli. Waheed[9] followed a heat line approach to understand how heat transfer took place in a horizontal annulus.

The previously reported experimental and numerical analyses on natural convection inside the annulus have considered no-slip boundary condition and the influence of slip boundary condition applied on the walls has not been studied so far. This paper is hence dedicated to finding out the effect of wall-slip on natural convection inside a cylindrical annulus at $Ra$ (based on gap width)$=5 \times 10^4$. The results of our studies will be helpful for industrial and academic research relating to wettability of surfaces especially in heat pipes where the presence of hydrophobic surfaces can limit the formation of residuals and thus improving heat transfer characteristics.

The structure of the presentation of our findings in the rest of the article is as follows. Solution methodology in Section 2 describes the numerical method, computational domain, and boundary conditions. Results and discussion in Section 3 include the grid sensitivity studies and validation of the code followed by visual inspection of flow and thermal fields using streamlines and isotherms. Heat transfer characteristics for various slip configurations have been explained using the variation in local Nusselt number along the hot and cold walls and average Nusselt
number. Conclusions in Section 4 summarises the overall numerical results.

2. Solution methodology

2.1. Numerical method

Numerical simulations are performed using the open-source computational fluid dynamics toolbox OpenFOAM. The generic solver \textit{buoyantBoussinesqPimpleFoam} is used to solve the steady, laminar and incompressible governing equations in two-dimensional form as given below. A second-order central differencing \textit{Gausslinear} scheme is used for the spatial discretization of convective and diffusive terms.

\begin{align}
\text{Continuity:} \quad & \frac{\partial U}{\partial X} + \frac{\partial V}{\partial Y} = 0 \\
\text{X - momentum:} \quad & U \frac{\partial U}{\partial X} + V \frac{\partial U}{\partial Y} = -\frac{\partial P}{\partial X} + Pr \left( \frac{\partial^2 U}{\partial X^2} + \frac{\partial^2 U}{\partial Y^2} \right) \\
\text{Y - momentum:} \quad & U \frac{\partial V}{\partial X} + V \frac{\partial V}{\partial Y} = -\frac{\partial P}{\partial Y} + Pr \left( \frac{\partial^2 V}{\partial X^2} + \frac{\partial^2 V}{\partial Y^2} \right) + RaPr \Theta \\
\text{Energy:} \quad & U \frac{\partial \Theta}{\partial X} + V \frac{\partial \Theta}{\partial Y} = \frac{\partial^2 \Theta}{\partial X^2} + \frac{\partial^2 \Theta}{\partial Y^2}
\end{align}

The non-dimensional variables in the governing equations are defined as follows

\begin{align}
X &= \frac{x}{L}, \quad Y = \frac{y}{L}, \quad U = \frac{uL}{\alpha}, \quad V = \frac{vL}{\alpha}, \quad \Theta = \frac{T - T_c}{T_h - T_c}, \quad P = \frac{pL^2}{\rho \alpha^2}
\end{align}

Rayleigh Number (Ra) based on the gap width L is given by \( Ra = \frac{g \beta L^3 (T_h - T_c)}{\nu \alpha} \) and Prandtl number (Pr) is defined as \( Pr = \frac{\nu}{\alpha} \). \( \rho \) and \( \nu \) are density and kinematic viscosity of the fluid and \( \beta \) is the coefficient of thermal expansion.

The fluid properties are assumed to be constant, except for the variation in density which is obtained by the Boussinesq approximation. The radiation heat transfer is considered negligible and the acceleration due to gravity (g) acts downwards in the negative y-direction.

The heat transfer from the walls is quantified in terms of \( Nu_i \) and \( Nu_{avg} \) along the walls using

\begin{align}
Nu_i &= \left. \frac{\partial \Theta}{\partial n} \right|_{\text{wall}}, \quad Nu = \frac{1}{W} \int_0^W Nu_i \, dS
\end{align}

where \( n \) is the unit normal to the wall, \( W \) is the surface area of the wall, \( dS \) is the infinitesimal surface area of the wall. The term \( Nu_{avg} \) is the surface averaged Nusselt number along both the walls given by

\begin{align}
Nu_{avg} &= \frac{Nu_o + Nu_i}{2}
\end{align}

where \( Nu_o \) and \( Nu_i \) are \( Nu \) for outer and inner walls respectively.
2.2. Computational domain and boundary conditions
A schematic of the computational domain is shown in Figure 1(a). The gap width is chosen as 1 unit and the ratio of gap width to the inner radius ($L/R_i$) is 1.6. The inner and outer cylinder walls are kept at uniform hot ($\Theta = 1$) and cold ($\Theta = 0$) temperatures respectively.

Figure 1: Schematic of (a) computational domain (b) grid structure used for the present study

Slip conditions on the walls are varied using the partialSlip boundary condition available in OpenFOAM. partialSlip represents a mixed boundary condition. For a 2-D flow, the first-order approximation for the slip-velocity is given by

$$U + \lambda \frac{dU}{dY} = 0$$ (8)

where $\lambda$ is the slip length [10].

Let $U_0$ be the tangential velocity on the wall face boundary and $U_1$ the cell centre velocity on boundary layer. Equation (8) can be re-written in terms of $U_0$, $U_1$ and $D$ as follows

$$U_0 + \lambda \frac{(U_0 - U_1)}{D/2} = 0$$ (9)

where $D$ is the size of the adjacent cell. The above equation after re-arranging becomes

$$U_0 = \frac{2\lambda}{D + 2\lambda} U_1$$ (10)

Here, we introduce a new term Slip factor($SF$) defined as

$$SF = \frac{2\lambda}{D + 2\lambda}$$ (11)

Hence equation (10) becomes

$$U_0 = (SF) U_1$$ (12)
For $\lambda \to 0$, we have $SF = 0$, which indicates no-slip condition.
For $\lambda \to \infty$, we have $SF = 1$, which indicates full-slip condition.

Conditions imposed at the walls other than the slip boundary conditions are as follows:

- **Inner cylinder wall**: Dirichlet boundary condition, $\Theta = 1$
- **Outer cylinder wall**: Dirichlet boundary condition, $\Theta = 0$

### 3. Results and discussion

#### 3.1. Grid sensitivity study

Grid sensitivity studies are conducted to minimise the influence of grid size on the computational results and to optimise the computational time. To accurately capture the boundary layer phenomenon, a finer mesh is adopted near the walls and a coarser mesh for other regions. Table 1 shows the variation in $Nu_{avg}$ while increasing the grid size. The percentage deviation of the successive values of $Nu_{avg}$ is indicated in the bracket. It can be observed that the % difference keeps decreasing. However, since the variation between grid sizes 5400 and 9600 is less than 0.3%, the latter grid size is selected for the present study.

| Number of cells | $Nu_{avg}$   |
|-----------------|-------------|
| 2400            | 1.387       |
| 3750            | 3.71 (167.48%) |
| 5400            | 3.688 (0.592%) |
| 9600            | 3.677 (0.297%) |
| 12150           | 3.673 (0.111%) |
| 15000           | 3.672 (0.026%) |

#### 3.2. Validation

The computational code used for the present study is compared and validated with the previous literature. Comparison is done by plotting the local Nusselt number ($Nu_l$) values along the hot and cold walls for $Ra = 5 \times 10^4$ (Figures 2(a) and(b)). The curve representing the present study is seen to follow other curves of the previous studies very closely.
Table 2 shows the comparison of the average Nusselt number ($Nu_{avg}$) for different Rayleigh numbers. The percentage deviation of the calculated value of $Nu_{avg}$ from the data in the literature is indicated in brackets (absolute values) and the results are found to be in accordance with the existing data. The above observations conclude that the present values are in good agreement with the previously reported results.

| Ra     | Present study | Shahrai[6] | Kuehn & Goldstein[1] | Shi et al.[8] |
|--------|---------------|------------|----------------------|---------------|
| $1.02 \times 10^5$ | 3.677 | 3.647 (0.808%) | 3.66 (0.464%) | 3.531 (4.134%) |
| $6.19 \times 10^4$ | 3.276 | 3.309 (1%) | 3.32 (1.325%) | 3.361 (2.53%) |
| $9.5 \times 10^3$ | 2.052 | 1.990 (3.11%) | 2.01 (2.089%) | 1.999 (2.651%) |

3.3. Streamlines
Figure 3 shows the streamlines for various values of SF at $Ra=5 \times 10^4$. In the region around the hot inner cylinder, the fluid gets heated up, becomes less dense, and begins to rise. As it reaches the vicinity of the cold outer cylinder, it interacts with the cold surface, becomes heavier, and moves down. While descending, some of the cold fluid again warms up due to the interaction with the hot surface and its neighbourhood thus giving rise to a pair of recirculating convection currents manifested as vortices.
For no-slip condition, SF=0 (Figure 3(a)), counter-rotating crescent-shaped vortices can be seen on either side of the vertical axis of symmetry, similar to the ones observed by Kuehn & Goldstein[1] and Date[4].

When the slip factor is increased from SF=0 to SF=0.6, no noticeable changes are observed, except that the center of rotation of the vortices moves slightly upward. Visual inspections, however, reveals that these cores remain in the upper half, which points to strong convection currents in this region. For SF=0.8 (Figure 3(e)), a new pair of secondary vortices appear near to the bottom of the annulus which indicates boundary layer separation from the cold outer cylinder. Significant transitions are seen in the flow structures when the boundary condition changes to full-slip (Figure 3(f)). The centres of the counter-rotating primary vortices on either side of the vertical axis split into two, creating two additional inner vortices. However, the outer recirculating envelope remains in position. It is also observed that the pair of secondary vortices near the cold wall which appears at the bottom of the annulus for SF=0.8 totally disappears for the full-slip condition.

3.4. Isotherms
The isotherms for various slip configurations are shown in Figure 4. It is observed from previous literature that as Rayleigh number nears $10^4$, radial temperature inversion appears, indicating the separation of thermal boundary layers.
Due to the domination of convection currents, the thermal boundary layer separation occurs on the upper half of the hot cylinder which develops into a plume in the upward direction before the flow attains a steady state. The isotherms observed for the no-slip condition (Figure 4(a)) are in good agreement with previously published literature (Kuehn & Goldstein[1] and Date[4]). The isotherms are seen to be distorted as a result of the migration of the plumes after impinging the cold surface. Owing to the formation of plumes in the upper half, the thermal gradients in this region are very high compared to the other half. This could be the reason why the core of the primary vortices are present in the upper half as found in the streamlines (Figures 3(a)-(e)).

The isotherms for SF values ranging from 0 to 0.6 are similar however, for SF=0.8 (Figure 4(e)), the hot plume originating from the top half of the inner cylinder slightly extends upwards which contributes to the further distortion of isotherms. This upward movement influences the thermal distribution throughout the annulus and close visual inspection reveals the wings of the bottom-most isotherm becoming slightly closer to each other which subsequently reduces the thermal stagnation at the bottom of the annulus. As full-slip condition is imposed, the plumes above the hot wall shrink and slightly extend vertically upwards towards the cold wall. The influence of this change is also visible in the lower half as the isotherms expand more towards the bottom wall of the outer cylinder. The thermally stagnant region shrinks considerably and is seen to inhabit a narrow space between the lowermost sections of the inner and outer cylinders.

**Figure 4**: Isotherms for various slip configurations plotted at $Ra = 10^5$
3.5. Variation in Nusselt number

Figures 5(a) and (b) show variation in local Nusselt number ($Nu_l$) along the right half of hot and cold walls for various configurations of SF. Due to the symmetric nature of flow and thermal fields, the heat transfer characteristics too are the same on either side of the vertical axis hence, the local Nusselt number variation along the left wall (of both inner and outer cylinders) is the same as that of the right side of which the latter is considered for analysis.

Moving from $\theta=0^\circ$ to $\theta=180^\circ$ in the clockwise direction, for no-slip (SF=0) condition, the $Nu_l$ along the hot surface increases monotonically up to $\theta=130^\circ$ after which the local Nusselt number does not vary appreciably. The boundary layer thickness reduces while moving down the surface which enhances the heat transfer rate. However, between $\theta=130^\circ$ and $\theta=180^\circ$, the growing influence of the thermally stagnant region at the bottom of the annulus actively opposes the plume dynamics thus maintaining uniformity in the local Nusselt number. A similar trend is observed for all slip configurations up to SF=0.4. For SF=0.6 and 0.8, after the initial monotonic increase up to $\theta=130^\circ$, the influence of partial-slip is seen to dominate the effect of plume which could be the reason for the slightly decreasing trend (negative slope) in the $Nu_l$-$\theta$ curve between $\theta=130^\circ$ and $\theta=180^\circ$. Significant transitions occur when full-slip (SF=1) is imposed on the walls. The shrinkage of the hot plume decreases the boundary layer thickness in its vicinity thereby producing an enhancement in heat transfer rate between $\theta=0^\circ$ and $\theta=30^\circ$. Between $\theta=30^\circ$ and $\theta=120^\circ$, the increase in $Nu_l$ is monotonic however beyond $\theta=120^\circ$ the combined effect of full-slip and the shrinkage in the thermally stagnant region produces a steep rise in the $Nu_l$-$\theta$ curve.

The variation in the $Nu_l$-$\theta$ curve for the cold wall is monotonically decreasing up to $\theta=150^\circ$ for all slip configurations. Between SF=0 and SF=0.8, the variation in $Nu_l$ between $\theta=150^\circ$ and $\theta=180^\circ$ is negligible as this zone falls in the thermally stagnant region located at the bottom of the annulus. However, for full-slip configuration, since the thermally inactive region shrinks, the local Nusselt number keeps decreasing up to $\theta=180^\circ$. It is worth noting that with an increase in SF, local Nusselt number keeps increasing at all points on the hot and cold walls.

Figure 5(c) illustrates the variation in the average Nusselt number for the entire range of slip configurations. As evidenced, the $Nu_{avg}$ variation is gradual and monotonically increasing up to SF=0.8 however, between SF=0.8 and SF=1, the curve witnesses a steep increase in the heat transfer rate which could be the effect of nullified viscous retardation in combination with the narrow stagnant region. The presence of twin vortices on either side of the vertical axis is also seen to enhance the heat transfer characteristics for the full-slip condition.

4. Conclusions

Numerical analysis is conducted to study the effect of slip boundary condition on natural convection in the annulus between concentric isothermal cylinders for $Ra=5 \times 10^4$ and $Pr=0.71$. It is observed that an increase in slip factor from SF=0 (no-slip) to SF=1 (full-slip) improves the overall heat transfer characteristics. The other inferences are as follows:

- For SF values between 0 and 0.6, a pair of primary vortices are seen on either side of the vertical axis whose cores are found in the upper half of the annulus. When SF reaches 0.8, a pair of secondary vortices appears at the bottom of the annulus in addition to the primary vortices. For full-slip configuration, the secondary vortices cease to exist and the cores of primary vortices break into two inner vortices enclosed by a single recirculating envelope on either side of the vertical axis.
- For all values of SF, the thermal boundary layer on the upper half of the annular region develops into a plume projecting upwards. An increase in SF from 0.6 to 0.8 is accompanied by a slight contraction of the thermally stagnant zone located at the bottom of the annulus. When full-slip (SF=1) is applied, the plume originating from the top of the hot wall shrinks and rises more, while the stagnant zone occupies the narrow region at the bottom of the annulus.
Figure 5: Nusselt number variation: (a) $N_u_l$ along hot wall (b) $N_u_l$ along cold wall and (c) $N_{u_{avg}}$ with SF

- Between SF=0 and SF=0.4, local Nusselt number along the hot wall monotonically increases up to $\theta = 130^\circ$ beyond which $N_u_l$ remains constant. For SF=0.6 and 0.8, after the monotonic increase up to $\theta = 130^\circ$, $N_u_l$ reduces which could be due to the increased influence of partial-slip. The combined effect of full-slip and the shrinkage in the thermally stagnant region produces a steep rise in the $N_u_l$-$\theta$ curve beyond $\theta = 120^\circ$. Along the cold wall, $N_u_l$ continuously decreases up to $\theta=150^\circ$ for all slip configurations. Beyond $\theta=150^\circ$, the local Nusselt number remains constant up to SF=0.8. For full-slip configuration, the local Nusselt number monotonically decreases throughout the cold wall.

- $N_{u_{avg}}$ variation is gradual and monotonically increasing up to SF=0.8 however, between SF=0.8 and SF=1, the curve witnesses a steep increase in the average Nusselt number.
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