Special Theory for Superluminal Particle

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Abstract

The OPERA collaboration reported evidence for muonic neutrinos travelling faster than light in vacuum. In this paper, an extended relativity theory is proposed. We think all particles can be divided into three kinds: The first kind of particle is its velocity in the range of \(0 \leq v < c\), e.g. electron, atom, molecule and so on (\(c\) is light velocity, i.e., the limit velocity of the first kind of particle). The second kind of particle is its velocity in the range of \(0 \leq v < c_{m1}\), e.g. photon (\(c_{m1}\) is the limit velocity of the second kind of particle). The third kind of particle is its velocity in the range of \(c \leq v < c_{m2}\), e.g. tachyon, and muonic neutrinos (\(c_{m2}\) is the limit velocity of the third kind of particle). The first kind of particle is described by the special relativity. With the extended relativity theory, we can describe the second and third kinds particles, and can analysis the OPERA experiment results and calculate the muonic neutrinos mass.

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1. Introduction

Recently the OPERA collaboration reported evidence for muonic neutrinos slightly faster than light in vacuum [1]. The CERN Neutrino beam to Gran Sasso (CNGS) consists of $\nu_\mu$, with small contaminations of $\bar{\nu}_\mu$ (2.1%) and of $\nu_e$ or $\bar{\nu}_e$ (together less than 1%). At the average neutrino energy 17 GeV, the relative difference of the velocity of the muon neutrinos $v$ with respect to light quoted by OPERA is: $(v - c)/c = (2.48 \pm 0.28(stat) \pm 0.30(sys)) \times 10^{-5}$. The velocity measurement of the muon neutrinos were also reported for muon neutrino beams produced at Fermilab. Dealing with energies peaked at 3 GeV, the MINOS Collaboration [2] found in 2007 that $(v - c)/c = (5.1 \pm 2.9) \times 10^{-5}$. The measurement results above are seemingly in conflict with special relativity in 4 dimensions, a number of possible explanations for Lorentz violation exist in the literature [3-12]. The neutrino velocity is higher than the velocity of light, but we think there is a limit velocity in the nature. In the paper, we propose a full relativity theory, which is based on the two postulates:

1. All particles can be divided into three kinds: The first kind is its velocity in the range of $0 \leq v < c$, e.g. electron, atom, molecule and so on, the light velocity $c$ is their limit velocity. The second kind is its velocity in the range of $0 \leq v < c_{m1}$ ($c < c_{m1}$), e.g. photon, the velocity $c_{m1}$ is its limit velocity. The third kind is its velocity in the range of $c \leq v < c_{m2}$, e.g. muonic neutrinos and tachyon. The velocity $c_{m2}$ is their limit velocity.

2. The first kind of particle is described by Einstein’s special relativity.

In the paper, we shall study the second and third kinds of particles by the extended relativity theory. With the extended relativity theory, we give new results about photon, and calculate the limit velocity $c_{m1}$ and $c_{m2}$. Otherwise, we analysis the OPERA experiment data and calculate the muonic neutrinos mass.

2. The space-time transformation and mass-energy relation for the second kind of particle ($0 \leq v < c_{m1}$)

For the first kind of particle, its velocity is in the range of $0 \leq v < c$, e.g. electron, atom, molecule and so on, and they can be described by the special relativity. In 1905, Einstein gave the space-time transformation and mass-energy relation which are based on his two postulates, i.e., the invariant principle of light velocity and the relativity principle. The
space-time transformation is

\[
x = \frac{x' + ut'}{\sqrt{1 - \frac{u^2}{c^2}}}, \\
y = y', \\
z = z', \\
t = \frac{t' + \frac{u}{c^2}x'}{\sqrt{1 - \frac{u^2}{c^2}}},
\]

(1)

where \(x, y, z, t\) are space-time coordinates in \(\Sigma\) frame, \(x', y', z', t'\) are space-time coordinates in \(\Sigma'\) frame, \(c\) is the speed of light, and \(u\) is the relative velocity between \(\Sigma\) and \(\Sigma'\) frame, which move along \(x\) and \(x'\) axes. The velocity transformation is

\[
u_x = \frac{u'_x + u}{1 + \frac{uu_x}{c^2}}, \\
u_y = \frac{u'_y \sqrt{1 - \frac{u^2}{c^2}}}{1 + \frac{uu_x}{c^2}}, \\
u_z = \frac{u'_z \sqrt{1 - \frac{u^2}{c^2}}}{1 + \frac{uu_x}{c^2}},
\]

(2)

where \(u_x, u_y\) and \(u_z\) are a particle velocity projection in \(\Sigma\) frame, \(u'_x, u'_y\) and \(u'_z\) are the particle velocity projection in \(\Sigma'\) frame.

The mass, momentum and energy of a particle of rest mass \(m_0\) and velocity \(\vec{v}\) are:

\[
m = \frac{m_0}{\sqrt{1 - \frac{\vec{v}^2}{c^2}}},
\]

(3)

\[
\vec{p} = \frac{m_0 \vec{v}}{\sqrt{1 - \frac{\vec{v}^2}{c^2}},}
\]

(4)

and

\[
E = mc^2 = \frac{m_0 c^2}{\sqrt{1 - \frac{\vec{v}^2}{c^2}}},
\]

(5)

and they satisfy the dispersion relation

\[
E^2 = m_0^2 c^4 + p^2 c^2.
\]

(6)

For the second kind of particle, its velocity is in the range of \(0 \leq \nu < c_{m1}\) \((c < c_{m1})\), e.g. photon \((c_{m1}\ is the limit velocity of the second kind of particle). Recently, a series
of experiments revealed that electromagnetic wave was able to travel at a group velocity faster than $c$. These phenomena have been observed in dispersive media [13, 14], in electronic circuits [15], and in evanescent wave cases [16, 17]. It is about 40 years before, that O.M.P. Bilaniuk, V.K. Deshpande and E.S.G. Sudarshan have studied the space-time relation for superluminal reference frames within the framework of special relativity [18, 19]. They assumed that the space-time and velocity transformation of special relativity are suitable for superluminal reference frames. They obtained the new results that the proper length $L_0$ and proper time $T_0$ must be imaginary so that the measured quantities, such as length $L$ and time $T$, are real. We think there is superluminal photon, but its velocity can not be infinity. So, we think there is a limit velocity $c_{m1}$ for the superluminal photon and give two postulates for the second kind of particle (photon) as follows:

1. The Principle of Relativity: All laws of nature are the same in all inertial reference frames.

2. The Universal of Limit Velocity: There is a limit velocity $c_{m1}$, and the $c_{m1}$ is invariant in all inertial reference frames.

From the two postulates, we can obtain the space-time transformation and velocity transformation for the second kind of particle ($0 \leq v < c_{m1}$). When we replace $c$ with $c_{m1}$, we can obtain the new space-time transformation from the Lorentz transformation, they are

$$
\begin{align*}
    x &= \frac{x' + ut'}{\sqrt{1 - \frac{u^2}{c_{m1}^2}}} \\
    y &= y' \\
    z &= z' \\
    t &= \frac{t' + \frac{u}{c_{m1}}x'}{\sqrt{1 - \frac{u^2}{c_{m1}^2}}},
\end{align*}
$$

(7)

where $x, y, z, t$ are space-time coordinates in $\Sigma$ frame, $x', y', z', t'$ are space-time coordinates in $\Sigma'$ frame, $c$ is the speed of light, and $u$ is the relative velocity between $\Sigma$ and $\Sigma'$ frame, which move along $x$ and $x'$ axes. The velocity transformation is

$$
u_x = \frac{u_x' + u}{1 + \frac{uu_x'}{c_{m1}^2}}$$
\[
\begin{align*}
  u_y &= \frac{u_y^' \sqrt{1 - \frac{u^2}{c_{m1}^2}}}{1 + \frac{uu_x^'}{c_{m1}^2}}, \\
  u_z &= \frac{u_z^' \sqrt{1 - \frac{u^2}{c_{m1}^2}}}{1 + \frac{uu_z^'}{c_{m1}^2}},
\end{align*}
\]

(8)

where \( u_x, u_y, u_z \) and \( v = \sqrt{u_x^2 + u_y^2 + u_z^2} \) \((0 \leq v < c_{m1})\) are a particle velocity component and velocity amplitude in \( \Sigma \) frame, \( u_x^', u_y^', u_z^' \) and \( v^' = \sqrt{u_x'^2 + u_y'^2 + u_z'^2} \) \((0 \leq v^' < c_{m1})\) are the particle velocity component projection and velocity amplitude in \( \Sigma' \) frame. Now, we can discuss the problem of the speed of light. For two inertial reference frames \( \Sigma \) and \( \Sigma' \), the \( \Sigma' \) frame is a rest frame for light, i.e., the two reference frames relative velocity \( v \) is equal to \( c \). At the time \( t = 0 \), a beam of light is emitted from the origin \( O \). When \( u_x = c \), we obtain the result from Eq. (8),

\[
  u_x^' = 0,
\]

(9)

when \( u_x = -c \), we have

\[
  u_x^' = \frac{-c - c}{1 + \frac{c^2}{c_{m1}^2}} = -2 \frac{cc^2_{m1}}{c^2 + c_{m1}^2} > -2c.
\]

(10)

It shows that the invariant principle of light velocity is violated for the second kind particle in the inertial reference of light velocity movement, but the limit velocity \( c_{m1} \) is invariant.

For the second kind particle, the mass \( m \), momentum \( \vec{p} \) and energy \( E \) of a particle of rest mass \( m_0 \) and velocity \( \vec{v} \) are

\[
  m = \frac{m_0}{\sqrt{1 - \frac{v^2}{c_{m1}^2}}},
\]

(11)

\[
  \vec{p} = \frac{m_0}{\sqrt{1 - \frac{v^2}{c_{m1}^2}}} \vec{v},
\]

(12)

and

\[
  E = mc_{m1}^2,
\]

(13)

and the dispersion relation is

\[
  E^2 = m_0^2 c_{m1}^4 + p^2 c_{m1}^2.
\]

(14)

\[
\]
In the following, we shall study the nature of photon, and obtain some new results. From Eqs. (11)-(14), we obtain the photon mass, momentum and energy at light velocity

\[ m_c = \frac{m_0}{\sqrt{1 - \frac{c^2}{c_{m1}^2}}} \]  

and

\[ \vec{p}_c = \frac{m_0}{\sqrt{1 - \frac{c^2}{c_{m1}^2}}} \vec{c}, \]  

\[ E = m_v c_{m1}^2 = \frac{m_0}{\sqrt{1 - \frac{v^2}{c_{m1}^2}}} c_{m1}^2 = h \nu_v, \]  

where \( m_c \) is photon mass, \( \nu_v \) is photon frequency at velocity \( v \) (\( 0 \leq v < c_{m1} \)). When \( v = 0 \), we obtain the photon rest energy \( E_0 \), it is

\[ E_0 = m_0 c_{m1}^2 = h \nu_0, \]  

where \( m_0 \) is photon rest mass, \( \nu_0 \) is photon rest frequency, we find photon has rest mass and rest frequency when \( v = c \), we obtain the photon energy at light velocity

\[ E_c = m_c c_{m1}^2 = \frac{m_0}{\sqrt{1 - \frac{c^2}{c_{m1}^2}}} c_{m1}^2 = h \nu_c, \]  

where \( m_c \) and \( \nu_c \) are photon mass and frequency at light velocity \( c \). From Eq. (19), we can obtain the photon rest mass \( m_0 \) and light velocity mass \( m_c \), they are

\[ m_0 = \frac{h \nu_c}{c_{m1}^2} \sqrt{1 - \frac{c^2}{c_{m1}^2}}, \]  

and

\[ m_c = \frac{h \nu_c}{c_{m1}^2}, \]  

From Eqs. (17) and (18), we have

\[ \nu_v = \frac{m_v c_{m1}^2}{h \sqrt{1 - \frac{v^2}{c_{m1}^2}}} = \frac{\nu_0}{\sqrt{1 - \frac{v^2}{c_{m1}^2}}}. \]  

The Eq. (22) gives the relation of a photon’s frequency with its velocity, and we find photon movement frequency \( \nu_v \) is larger than its rest frequency \( \nu_0 \).
When $v = c$, we obtain the photon frequency $\nu_c$ at light velocity $c$

$$\nu_c = \frac{\nu_0}{\sqrt{1 - \frac{c^2}{c_{m1}^2}}}. \quad (23)$$

From Eq. (22) and (23), we obtain the ratio between the photon frequency at arbitrary velocity $v$ and the frequency at light velocity $c$

$$\frac{\nu_v}{\nu_c} = \frac{\sqrt{c_{m1}^2 - c^2}}{\sqrt{c_{m1}^2 - v^2}}. \quad (24)$$

Substituting Eq. (24) into the relation $\nu_v \lambda = v$, i.e.,

$$\frac{m_0 c_{m1}^2}{h} \frac{1}{\sqrt{1 - \frac{v^2}{c_{m1}^2}}} \lambda = v, \quad (25)$$

we obtain

$$\lambda = \frac{h \nu_j}{m_0 c_{m1}^2} \sqrt{1 - \frac{v^2}{c_{m1}^2}}, \quad (26)$$

The Eq. (26) gives the relation between photon wavelength and its velocity $v$. When $v = 0$, the light wavelength $\lambda = 0$, i.e., the photon hasn’t wavelength when it rests, but it has rest mass $m_0$ and rest frequency $\nu_0$.

From Eq. (17) and (19), we have

$$c_{m1} = \sqrt{\frac{h \nu_v}{m_{v}},} \quad (27)$$

and

$$c_{m1} = \sqrt{\frac{h \nu_c}{m_c}} \quad (28)$$

From Eqs. (27) and (28), we can calculate the limit velocity $c_{m1}$ if we can measure the photon mass $m_v(m_c)$ when its frequency is $\nu_v(\nu_c)$ at arbitrary velocity $v$ (light velocity $c$).

All photon possess a finite mass and their physical implications have been discussed by many theories and experiments [20, 21, 22]. In Ref. [21, 22], the experiment were made by laser, and it determined the range of the photon mass is $10^{-6}eV < m_\nu < 10^{-4}eV$. We know the laser frequency is in the range of $8.9 \times 10^{13}Hz \sim 9.23 \times 10^{14}Hz$. From Eq. (28), we can estimate the limit velocity $c_{m1}$. It is in the range of:

$$\sqrt{\frac{6.626 \times 10^{-34} \times 8.9 \times 10^{13}}{10^{-4} \times 1.6 \times 10^{-19}}}c \leq c_{m1} \leq \sqrt{\frac{6.626 \times 10^{-34} \times 9.23 \times 10^{14}}{10^{-6} \times 1.6 \times 10^{-19}}}c, \quad (29)$$
\[ 60c \leq c_{m1} \leq 2000c. \]  

(30)

If we take the middle experiment value, i.e., \( m_c = 10^{-5}\text{eV}, \nu_c = 5.5 \times 10^{14}\text{Hz} \), the limit velocity is

\[
c_{m1} = \sqrt{\frac{\hbar \nu_c}{m_c}} = \sqrt{\frac{6.626 \times 10^{-34} \times 5.5 \times 10^{14}}{10^{-5} \times 1.6 \times 10^{-19}}} = 480c. \tag{31}
\]

In experiment [23], E. Fomalont et al. have found light of frequency \( \nu = 43\text{GeV} \) passing near the solar limb, the photon mass upper limit is \( 3.5 \times 10^{-11}\text{MeV} \). We can estimate the lower limit of limit velocity \( c_m \), it is

\[
c_{m1} = \sqrt{\frac{\hbar \nu_c}{m_c}} > \sqrt{\frac{6.626 \times 10^{-34} \times 43 \times 10^9}{3.5 \times 10^{-11} \times 10^6 \times 1.6 \times 10^{-19}}} = 22c. \tag{32}
\]

In Ref. [24], the experiment measured the superluminal velocity is 310c. In Refs. [25-26], the experiments measured signal velocity were 4.7c for the microwave and 1.7c for single photon.

3. The space-time transformation and mass-velocity relation for the third kind of particle \( (c \leq \nu < c_{m2}) \)

In the following, we will give the space-time relation in two inertial reference frames \( \Sigma \) and \( \Sigma' \) for the third kind of particle, which its velocity is in the range of \( c \leq \nu < c_{m2} \). We think the muonic neutrinos and tachyon travels faster than light, but its velocity can not be infinity. So, we can assume there is a limit velocity \( c_{m2} \) for the third kind of particle, and we also give two postulates:

1. The Principle of Relativity: All laws of nature are the same in all inertial reference frames.

2. The Invariant Principle of Limit Velocity: There is a limit velocity \( c_{m2} \) in nature, and the \( c_{m2} \) is invariant in all inertial reference frames.

From the two postulates, we can obtain the new space-time transformation and velocity transformation for the third kind of particle. When we replace \( c \) with \( c_{m2} \), we can obtain the new transformation relation from the Lorentz transformation Eq. (1), they are

\[
x = \frac{x' + u't'}{\sqrt{1 - \frac{u'^2}{c_{m2}^2}}}
\]
Figure 1: The \( \Sigma \) is the laboratory system, \( \Sigma' \) is the mass-center system for two particles \( m_1 \) and \( m_2 \), and the two inertial frames relative velocity is \( v \).

\[
\begin{align*}
y &= y' \\
z &= z' \\
t &= t' + \frac{u}{c_{m2}}x', \\
\end{align*}
\]

(33)

where \( x, y, z, t \) are space-time coordinates in \( \Sigma \) frame, \( x', y', z', t' \) are space-time coordinates in \( \Sigma' \) frame, \( c_{m2} \) is the limit velocity, and \( u \) is the relative velocity between \( \Sigma \) and \( \Sigma' \) frame, which move along \( x \) and \( x' \) axes. The velocity transformation is

\[
\begin{align*}
u_x &= \frac{u_x' + u}{1 + \frac{u^2}{c_{m2}^2}} \\
u_y &= \frac{u_y' \sqrt{1 - \frac{u^2}{c_{m2}^2}}}{1 + \frac{u^2}{c_{m2}^2}} \\
u_z &= \frac{u_z' \sqrt{1 - \frac{\nu^2}{c_{m2}^2}}}{1 + \frac{u^2}{c_{m2}^2}},
\end{align*}
\]

(34)

where \( u_x, u_y, u_z \) and \( v = \sqrt{u_x'^2 + u_y'^2 + u_z'^2} (c \leq v < c_{m2}) \) are a particle velocity component and velocity in \( \Sigma \) frame, \( u_x', u_y', u_z' \) and \( v' = \sqrt{u_x'^2 + u_y'^2 + u_z'^2} (c \leq v' < c_{m2}) \) are the particle velocity component and velocity in \( \Sigma' \) frame.

In the following, we will give the new relation of particle mass \( m \) with its velocity \( v \). We can consider the collision between two identical particle. It is shown in Figure 1.
The $\Sigma$ is the laboratory system, and $\Sigma'$ is the mass-center system of two particles $m_1$ and $m_2$. In $\Sigma$ system, the velocity of two particles $m_1$ and $m_2$ are $\vec{v}_1$ and $\vec{v}_2$ ($c \leq v_2 < v_1 < c_{m_2}$), which are along with $x(x')$ axis, and they are $v'$ and $-v'$ in $\Sigma'$ system. After collision, the two particles velocities are all $v$ ($c \leq v < c_{m_2}$ in $\Sigma$ system. The momentum was conserved in this process:

\[ m_1v_1 + m_2v_2 = (m_1 + m_2)v. \]  \hfill (35)

According to equation (34), there is

\[ v_1 = \frac{v' + v}{1 + \frac{vv'}{c_{m_2}^2}}, \]
\[ v_2 = \frac{-v' + v}{1 - \frac{vv'}{c_{m_2}^2}}. \]  \hfill (36)

From equations (35) and (36), we get

\[ m_1(1 - \frac{vv'}{c_{m_2}^2}) = m_2(1 + \frac{vv'}{c_{m_2}^2}). \]  \hfill (37)

From equation (34), we can obtain

\[ 1 - \frac{u'v}{c_{m_2}^2} = \frac{\sqrt{1 - \frac{v^2}{c_{m_2}^2}} \sqrt{1 - \frac{v^2}{c_{m_2}^2}}}{\sqrt{1 - \frac{v'^2}{c_{m_2}^2}}}, \]  \hfill (38)

where $v = \sqrt{u_x^2 + u_y^2 + u_z^2}$ and $v' = \sqrt{u'_x^2 + u'_y^2 + u'_z^2}$. For the particle $m_1$, the equation (38) becomes

\[ 1 + \frac{v'v}{c_{m_2}^2} = \frac{\sqrt{1 - \frac{v'^2}{c_{m_2}^2}} \sqrt{1 - \frac{v^2}{c_{m_2}^2}}}{\sqrt{1 - \frac{v'^2}{c_{m_2}^2}}}, \]  \hfill (39)

For the particle $m_2$, the equation (38) becomes

\[ 1 - \frac{v'v}{c_{m_2}^2} = \frac{\sqrt{1 - \frac{v'^2}{c_{m_2}^2}} \sqrt{1 - \frac{v^2}{c_{m_2}^2}}}{\sqrt{1 - \frac{v'^2}{c_{m_2}^2}}}. \]  \hfill (40)

By substituting equations (39) and (40) into (37), we get

\[ m(v_1) \sqrt{1 - \frac{v_1^2}{c_{m_2}^2}} = m(v_2) \sqrt{1 - \frac{v_2^2}{c_{m_2}^2}} = m(c) \sqrt{1 - \frac{c^2}{c_{m_2}^2}} = \text{constant}, \]  \hfill (41)
where \( m(c) \) is the particle mass when its velocity is light velocity \( c \).

For arbitrary velocity \( v \) (\( c \leq v < c_{m2} \)), we have

\[
m(v) \sqrt{1 - \frac{v^2}{c_{m2}^2}} = m(c) \sqrt{1 - \frac{c^2}{c_{m2}^2}},
\]

and hence

\[
m(v) = m_c \sqrt{\frac{c_{m2}^2 - c^2}{c_{m2}^2 - v^2}},
\]

where \( m_c = m(c) \). The equation (43) is the relation between tachyon mass \( m \) and its velocity \( v \) (\( c \leq v < c_m \)).

From Eqs. (11)-(14), we can obtain the third kind particle energy, momentum and dispersion relation by replacing \( c \) with \( c_{m2} \), and replacing \( m_0 \) with \( m_c \), they are

\[
E = \frac{m_c c_{m2}^2}{\sqrt{1 - \frac{v^2}{c_{m2}^2}}},
\]

\[
\vec{p} = \frac{m_c \vec{v}}{\sqrt{1 - \frac{v^2}{c_{m2}^2}}},
\]

and

\[
E^2 - p^2 c_{m2}^2 = m_c^2 c_{m2}^4.
\]

By substituting Eq. (43) into (44), we can obtain the relation of mass-energy.

\[
E = \frac{m(v)}{\sqrt{c_{m2}^2 - c^2}}.
\]

From Eqs. (43) and (47), we can analysis the OPERA muonic neutrino experiment. and estimated the velocity limit \( c_{m2} \) for the third kind particle. Otherwise, we can calculate the muonic neutrino mass.

4. The Lorentz group and extended Lorentz group

For the first kind particle, we can obtain the invariant interval \( ds \) by the invariant principle of light velocity, it is

\[
ds^2 = c^2 dt^2 - dx^2 - dy^2 - dz^2,
\]

\[
ds^* \rho^2 = c^2 dt^2 - dx^2 - dy^2 - dz^2,
\]
and
\[ ds^2 = ds'^2. \] (50)

The Lorentz transformation is a line transformation in 4-dimension space-time, which satisfies the invariant of internal, it is
\[ x'_\mu = a_{\mu\nu}x_\nu \] (51)
where \( \mu, \nu = 0, 1, 2, 3 \), and \( x_0 = ct, \ x_1 = x, \ x_2 = y, \ x_3 = z \).

The matrix form of Eq. (51) is
\[
\begin{pmatrix}
  x'_0 \\
  x'_1 \\
  x'_2 \\
  x'_3
\end{pmatrix} =
\begin{pmatrix}
  a_{00} & a_{01} & a_{02} & a_{03} \\
  a_{10} & a_{11} & a_{12} & a_{13} \\
  a_{20} & a_{21} & a_{22} & a_{23} \\
  a_{30} & a_{31} & a_{32} & a_{33}
\end{pmatrix}
\begin{pmatrix}
  x_0 \\
  x_1 \\
  x_2 \\
  x_3
\end{pmatrix},
\] (52)

and Lorentz transformation \( a_{\mu\nu} \) satisfies the orthogonal relation
\[ a_{\mu\nu}a_{\mu\rho} = \delta_{\nu\rho}, \] (53)
the aggregate of orthogonal transformation \( a_{\mu\nu} \) constitutes a group, which is Lorentz group.

For \( \nu \) in the x-direction, the special Lorentz transformation is
\[
a =
\begin{pmatrix}
  \frac{1}{\sqrt{1-v^2/c^2}} & 0 & 0 & i\frac{v}{c}\frac{1}{\sqrt{1-v^2/c^2}} \\
  0 & 1 & 0 & 0 \\
  0 & 0 & 1 & 0 \\
  -i\frac{v}{c}\frac{1}{\sqrt{1-v^2/c^2}} & 0 & 0 & \frac{1}{\sqrt{1-v^2/c^2}}
\end{pmatrix}.
\] (54)

For the second and third kind particles, we can also obtain the invariant interval \( ds \) by the invariant principle of limit velocity, they are
\[ ds^2 = c_{m_i}^2dt^2 - dx^2 - dy^2 - dz^2, \] (55)
\[ ds'^2 = c_{m_i}^2dt'^2 - dx'^2 - dy'^2 - dz'^2, \] (56)
and
\[ ds^2 = ds'^2. \] (57)

Where \( c_{m_i}(i = 1, 2) \) is the limit velocity, and \( c_{m_1}, \ c_{m_2} \) are the limit velocity of second and third kind particles. The transformation is
\[ x'_\mu = b_{\mu\nu}x_\nu, \] (58)
the matrix form of Eq. (58) is

\[
\begin{pmatrix}
x_0' \\
x_1' \\
x_2' \\
x_3'
\end{pmatrix} =
\begin{pmatrix}
b_{00} & b_{01} & b_{02} & b_{03} \\
b_{10} & b_{11} & b_{12} & b_{13} \\
b_{20} & b_{21} & b_{22} & b_{23} \\
b_{30} & b_{31} & b_{32} & b_{33}
\end{pmatrix}
\begin{pmatrix}
x_0 \\
x_1 \\
x_2 \\
x_3
\end{pmatrix},
\tag{59}
\]

and the transformation \( b_{\mu\nu} \) satisfies the orthogonal relation

\[ b_{\mu\nu} b_{\nu\rho} = \delta_{\mu\rho}. \tag{60} \]

The aggregate of orthogonal transformation \( b_{\mu\nu} \) constitute a group, which is extended Lorentz group.

For \( v \) in the x-direction, the special extended Lorentz transformation is

\[
b = \begin{pmatrix}
\frac{1}{\sqrt{1 - \frac{v^2}{c^2}}} & 0 & 0 & i \frac{v}{c m_i} \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}}
\\
0 & 1 & 0 & 0
\\
0 & 0 & 1 & 0
\\
-i \frac{v}{c m_i} \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}} & 0 & 0 & \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}}
\end{pmatrix}.
\tag{61}\]

5. The relativistic dynamics for the second and third kinds particles

For the first kind particle (\( 0 \leq v < c \)), we know the 4-force is defined as:

\[ K_\mu = \frac{dp_\mu}{d\tau} = (\vec{K}, iK_4), \tag{62} \]

the "ordinary" force \( \vec{K} \) is

\[ \vec{K} = \frac{d\vec{p}}{dt} d\tau = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}} \frac{d\vec{p}}{dt}, \tag{63} \]

while the fourth component

\[
K_4 = \frac{dp_4}{d\tau} = \frac{1}{c} \frac{dE}{d\tau}
= \frac{1}{c} \vec{v} \cdot \vec{K}, \tag{64}
\]

so

\[ K_\mu = (\vec{K}, -\frac{i}{c} \vec{v} \cdot \vec{K}), \tag{65} \]
the covariant equation for a particle are

\[
\vec{K} = \frac{d\vec{p}}{d\tau}, \quad (66)
\]

\[
\vec{K} \cdot \vec{v} = \frac{dE}{d\tau}. \quad (67)
\]

\[
\sqrt{1 - \frac{v^2}{c^2}} \vec{K} \cdot \vec{v} = \frac{dE}{dt}, \quad (68)
\]

we define force \( \vec{F} \) as

\[
\vec{F} = \sqrt{1 - \frac{v^2}{c^2}} \vec{K}. \quad (69)
\]

From Eqs. (66)-(68), we have the relativistic dynamics equations for a particle are

\[
\vec{F} = \frac{d\vec{p}}{dt}, \quad (70)
\]

\[
\vec{K} \cdot \vec{v} = \frac{dE}{dt}. \quad (71)
\]

For the second kind particle \((0 \leq v < c_{m1})\), the relativistic dynamics equations are Eqs. (70) and (71), but some physical quantities should be modified

The 4-momentum and 4-force are

\[
p_\mu = m_0 U_\mu = (\vec{p}, \frac{i}{c_{m1}} E), \quad (72)
\]

and

\[
K_\mu = (\vec{K}, iK_4), \quad (73)
\]

with \( \vec{p} = \frac{m_0 \vec{v}}{\sqrt{1 - \frac{v^2}{c_{m1}^2}}}, p_4 = \frac{m_0 c_{m1}}{\sqrt{1 - \frac{v^2}{c_{m1}^2}}}, \vec{K} = \frac{d\vec{p}}{dt} \frac{1}{\sqrt{1 - \frac{v^2}{c_{m1}^2}}} \) and \( K_4 = \frac{1}{c_{m1}} \vec{\sigma} \cdot \vec{K}. \)

For the third kind particle \((c \leq v < c_{m2})\), the relativistic dynamics equations are also Eqs. (70) and (71), and some physical quantities also should be modified

The 4-momentum and 4-force are

\[
p_\mu = m_c U_\mu = (\vec{p}, \frac{i}{c_{m2}}), \quad (74)
\]

and

\[
K_\mu = (\vec{K}, iK_4), \quad (75)
\]

with \( \vec{p} = \frac{m_c \vec{v}}{\sqrt{1 - \frac{v^2}{c_{m2}^2}}}, p_4 = \frac{m_c c_{m2}}{\sqrt{1 - \frac{v^2}{c_{m2}^2}}}, \vec{K} = \frac{d\vec{p}}{dt} \frac{1}{\sqrt{1 - \frac{v^2}{c_{m2}^2}}} \) and \( K_4 = \frac{1}{c_{m2}} \vec{\sigma} \cdot \vec{K}. \)
6. The quantum wave equation for the second and third kinds particles

For the first kind particle \((0 \leq v < c)\), we express \(E\) and \(\vec{p}\) as operators:

\[
E \to i\hbar \frac{\partial}{\partial t}, \quad \vec{p} \to -i\hbar \nabla,
\]

we can obtain the quantum wave equation of spin 0 particle from Eq. (6)

\[
\left[ \frac{\partial^2}{\partial t^2} - c^2 \nabla^2 + \frac{m_0^2 c^4}{\hbar^2} \right] \Psi(\vec{r}, t) = 0,
\]

and we can obtain the quantum wave equation of spin \(\frac{1}{2}\) particle

\[
\hbar \frac{\partial}{\partial t} \Psi = [-i\hbar c \vec{\alpha} \cdot \nabla + m_0 c^2 \beta] \Psi,
\]

where \(\vec{\alpha}\) and \(\beta\) are matrixes

\[
\vec{\alpha} = \begin{pmatrix} 0 & \vec{\sigma} \\ \vec{\sigma} & 0 \end{pmatrix},
\]

and

\[
\beta = \begin{pmatrix} I & 0 \\ 0 & -I \end{pmatrix},
\]

where \(\vec{\sigma}\) are Pauli matrixes, and \(I\) is unit matrix of \(2 \times 2\).

For the second kind particle \((0 \leq v < c_{m_1})\), we can obtain the quantum wave equation of spin 0 particle from Eq. (14)

\[
\left[ \frac{\partial^2}{\partial t^2} - c_{m_1}^2 \nabla^2 + \frac{m_{m_1}^2 c_{m_1}^4}{\hbar^2} \right] \Psi(\vec{r}, t) = 0,
\]

and we can obtain the quantum wave equation of spin \(\frac{1}{2}\) particle

\[
\hbar \frac{\partial}{\partial t} \Psi = [-i\hbar c_{m_1} \vec{\alpha} \cdot \nabla + m_0 c_{m_1}^2 \beta] \Psi.
\]

For the third kind particle \((c \leq v < c_{m_2})\), we can obtain the quantum wave equation of spin 0 particle from Eq. (46)

\[
\left[ \frac{\partial^2}{\partial t^2} - c_{m_2}^2 \nabla^2 + \frac{m_{m_2}^2 c_{m_2}^4}{\hbar^2} \right] \Psi(\vec{r}, t) = 0,
\]

and we can obtain the quantum wave equation of spin \(\frac{1}{2}\) particle

\[
\hbar \frac{\partial}{\partial t} \Psi = [-i\hbar c_{m_2} \vec{\alpha} \cdot \nabla + m_c c_{m_2}^2 \beta] \Psi.
\]
7. Numerical analysis for OPERA experiment

At OPERA muonic neutrino experiment, when the average neutrino energy 17 GeV, the relative difference of the velocity of the muon neutrinos \( v \) is: \( (v - c)/c = (2.48 \pm 0.28(\text{stat}) \pm 0.30(\text{sys})) \times 10^{-5} \), and the neutrinos produced at CERN are tachyons with mass \( m \), after having travelled a distance \( L \approx 730 \text{ km} \), their associated early arrival time is \( \delta t = \frac{L}{c} \frac{v - c}{c} \), with \( \frac{L}{c} \approx 2.4 \text{ ms} \). Consider two tachyonic neutrino beams of energy \( E_1 \) and \( E_2 \), with \( E_1 < E_2 \) for definiteness. The OPERA Collaborations considers two sample neutrino beams with mean energy equal to \( E_1 = 13.9 \text{ GeV} \) and \( E_2 = 42.9 \text{ GeV} \) respectively. The experimental values of the associated early arrival times are respectively \( \delta t_1 = (53.1 \pm 18.8(\text{stat}) \pm 7.4(\text{sys})) \text{ ns} \) and \( \delta t_2 = (67.1 \pm 18.2(\text{stat}) \pm 7.4(\text{sys})) \text{ ns} \). The OPERA experiment data can be concluded as follows:

\[
E_1 = 13.9 GeV, \quad v_{1\text{min}} = (3 \times 10^8 + 3362) m/s, \quad v_{1\text{max}} = (3 \times 10^8 + 9913) m/s, \quad (83)
\]

\[
E_2 = 17 GeV, \quad v_{2\text{min}} = (3 \times 10^8 + 5700) m/s, \quad v_{2\text{max}} = (3 \times 10^8 + 9180) m/s, \quad (84)
\]

\[
E_3 = 42.9 GeV, \quad v_{3\text{min}} = (3 \times 10^8 + 5188) m/s, \quad v_{3\text{max}} = (3 \times 10^8 + 11588) m/s, \quad (85)
\]

Using the experiment data above, we can calculate the limit velocity \( c_{m2} \) and muonic neutrinos mass by Eqs.(43)-(47).

With Eq. (44), we have

\[
\frac{E_i}{E_j} = \frac{\sqrt{c_{m2}^2 - v_j^2}}{\sqrt{c_{m2}^2 - v_i^2}}, \quad (i, j = 1, 2, 3) \quad (86)
\]

Table 1: Different energy muonic neutrino according to its velocity, mass and the limit velocity.

| E (GeV) | \( v_{\text{min}} \) (m/s) | \( v_{\text{max}} \) (m/s) | \( m_v \) (GeV \( s^2/m^2 \)) | \( m_c \) (GeV \( s^2/m^2 \)) |
|--------|-----------------|-----------------|-----------------|-----------------|
| 13.9   | \( 3 \times 10^8 + 3363 \) | \( 3 \times 10^8 + 9912 \) | (0.48 \( \sim \) 1.82)(\( \times 10^{-18} \)) | (0.772 \( \sim \) 1.671)(\( \times 10^{-18} \)) |
| 17     | \( 3 \times 10^8 + 5700 \) | \( 3 \times 10^8 + 9180 \) | (1.03 \( \sim \) 2.23)(\( \times 10^{-18} \)) | (0.705 \( \sim \) 1.527)(\( \times 10^{-18} \)) |
| 42.9   | \( 3 \times 10^8 + 5188 \) | \( 3 \times 10^8 + 11588 \) | (2.60 \( \sim \) 5.63)(\( \times 10^{-18} \)) | (1.161 \( \sim \) 2.511)(\( \times 10^{-18} \)) |

Substituting Eqs.(83)-(85) into (86), we can calculate the limit velocity \( c_{m2} \), it is in the range of \( (3 \times 10^8 + 15534)^{+5390}_{-1091} \text{ m/s} \). From Eq. (47), we can calculate the muonic neutrinos mass \( m(v) \) at different energy. Otherwise, we can calculate the muonic neutrinos mass
$m(c)$ at light velocity by Eq. (43). All the calculation results are shown in Table 1, where the first column is muonic neutrinos energy, the second and third column are corresponding to different energy neutrinos minimum and maximum velocity, the forth column is corresponding to different energy neutrinos mass, the final column is muonic neutrinos mass at light velocity $c$. From Table 1, we can find the muonic neutrinos mass $m(c)$ is in the range of $(1.161 \sim 1.527) \times 10^{-18} GeV s^2/m^2$, and the muonic neutrinos mass increase when its energy increase.

8. Conclusion

In this paper, an extended relativity theory is proposed. We think all particles can be divided into three kinds: The first kind of particle is its velocity in the range of $0 \leq v < c$, e.g. electron, atom, molecule and so on. The second kind of particle is its velocity in the range of $0 \leq v < c_{m1}$, e.g. photon. The third kind of particle is its velocity in the range of $c \leq v < c_{m2}$, e.g. tachyon, and muonic neutrinos. The first kind of particle is described by the special relativity. With the extended relativity theory, we can describe the second and third kinds particles, and obtain some new results. Otherwise, we analysis the OPERA experiment data and calculate the muonic neutrinos mass.
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