Fermion masses in the economical 3-3-1 model

P. V. Dong$^a$, D. T. Huong$^{a,b}$, T. T. Huong$^a$ and H. N. Long$^{a,c}$

$^a$ Institute of Physics, VAST, P. O. Box 429, Bo Ho, Hanoi 10000, Vietnam
$^b$ Department of Physics and Astronomy, Aichi University of Education, Kariya 448-8542, Japan
$^c$ The Abdus Salam International Centre for Theoretical Physics, 34014 Trieste, Italy

Abstract

We show that, in frameworks of the economical 3-3-1 model, all fermions get masses. At the tree level, one up-quark and two down-quarks are massless, but the one-loop corrections give all quarks the consistent masses. This conclusion is in contradiction to the previous analysis in which, the third scalar triplet has been introduced. This result is based on the key properties of the model: First, there are three quite different scales of vacuum expectation values: $\omega \sim O(1) \, \text{TeV}$, $v \approx 246 \, \text{GeV}$ and $u \sim O(1) \, \text{GeV}$. Second, there exist two types of Yukawa couplings with different strengths: the lepton-number conserving couplings $h$'s and the lepton-number violating ones $s$'s satisfying the condition in which the second are much smaller than the first ones: $s \ll h$. With the acceptable set of parameters, numerical evaluation shows that in this model, masses of the exotic quarks also have different scales, namely, the $U$ exotic quark ($q_U = 2/3$) gains mass $m_U \approx 700 \, \text{GeV}$, while the $D_\alpha$ exotic quarks ($q_{D\alpha} = -1/3$) have masses in the TeV scale: $m_{D\alpha} \in 10 \div 80 \, \text{TeV}$.

PACS number(s): 12.15.Lk, 12.15.Ff, 11.30.Qc
Keywords: Electroweak radiative corrections, Quark and lepton masses and mixing, Spontaneous and radiative symmetry breaking.

1 Introduction

The recent experimental results of SuperKamiokande Collaboration [1], KamLAND [2] and SNO [3] confirm that neutrinos have tiny masses and oscillate. This implies that the standard model (SM) must be extended. Among the beyond-SM extensions, the models based on the $SU(3)_C \otimes SU(3)_L \otimes U(1)_X$ (3-3-1) gauge group [4, 5] have some intriguing features: First, they can give partial explanation of the generation number problem. Second, the third quark generation has to be different from the first two, so this leads to the possible explanation of why top quark is uncharacteristically heavy.

In one of the 3-3-1 models, the scalar sector is minimal with just two Higgs triplets; hence it has been called the economical model [6, 7]. The general Higgs sector of this model is very simple (economical) and consists of three physical scalars (two neutral and one charged) and eight Goldstone bosons - the needed number for massive gauge ones [8].
At the tree level, the mass matrix for the up-quarks has one massless state and in the
down-quark sector, there are two massless ones. This calls radiative corrections. To solve
this problem, the authors in Ref. [9] have introduced the third Higgs triplet. Therefore, it
was though that the economical 3-3-1 model with such only two Higgs triplets is not realistic.

In the present work we will show that this is a mistake! Without the third one, at the
one loop level, the fermions in this model, with the given set of parameters, gain a consistent
mass spectrum. A numerical evaluation leads us to conclusion that in this model, there are
two scales for masses of the exotic quarks.

The rest of this paper is organized as follows: In Section 2, we give a brief review of
the economical 3-3-1 model. Sec. 3 is devoted for the fermion mass spectrum. In Sec. 4
we present some details on the one-loop quark masses. We summarize our result and make
conclusions in the last section - Sec. 5.

2 A review of the economical 3-3-1 model

The particle content in this model, which is anomaly free, is given as follows:

$$\psi_{iL} = \left( \begin{array}{c}
\nu_i \\
 e_i \\
\nu_i^c
\end{array} \right) \sim \left(3, -\frac{1}{3}\right), \quad e_{iR} \sim (1, -1), \quad i = 1, 2, 3,$$

$$Q_{1L} = \left( \begin{array}{c}
u_i \\
 u_i \\
 d_i
\end{array} \right) \sim \left(3, \frac{1}{3}\right), \quad Q_{\alpha L} = \left( \begin{array}{c}d_\alpha \\
 -u_\alpha
\end{array} \right) \sim (3^*, 0), \quad \alpha = 2, 3,$$

$$u_{iR} \sim \left(1, \frac{2}{3}\right), \quad d_{iR} \sim \left(1, -\frac{1}{3}\right), \quad U_R \sim \left(1, \frac{2}{3}\right), \quad D_{\alpha R} \sim \left(1, -\frac{1}{3}\right).$$

Here, the values in the parentheses denote quantum numbers based on the \((SU(3)_L, U(1)_X)\)
symmetry. In this case, the electric charge operator takes a form

$$Q = T_3 - \frac{1}{\sqrt{3}} T_8 + X,$$

where \(T_a\) \((a = 1, 2, ..., 8)\) and \(X\) stand for \(SU(3)_L\) and \(U(1)_X\) charges, respectively. Electric
charges of the exotic quarks \(U\) and \(D_{\alpha}\) are the same as of the usual quarks, i.e., \(q_U = 2/3\)
and \(q_{D_{\alpha}} = -1/3\).

The \(SU(3)_L \otimes U(1)_X\) gauge group is broken spontaneously via two steps. In the first step,
it is embedded in that of the SM via a Higgs scalar triplet

$$\chi = \left( \begin{array}{c}
\chi_1^0 \\
\chi_2^0 \\
\chi_3^0
\end{array} \right) \sim \left(3, -\frac{1}{3}\right).$$

\(2\)
acquired with a vacuum expectation value (VEV) given by

$$\langle \chi \rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} u \\ 0 \\ \omega \end{pmatrix}.$$  \hspace{1cm} (2.4)

In the last step, to embed the gauge group of the SM in $U(1)_Q$, another Higgs scalar triplet

$$\phi = \begin{pmatrix} \phi_1^+ \\ \phi_2^0 \\ \phi_3^+ \end{pmatrix} \sim (3, \frac{2}{3})$$  \hspace{1cm} (2.5)

is needed with the VEV as follows

$$\langle \phi \rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v \\ 0 \end{pmatrix}.$$  \hspace{1cm} (2.6)

The Yukawa interactions which induce masses for the fermions can be written in the most general form as follows

$$\mathcal{L}_Y = \mathcal{L}_{LNC} + \mathcal{L}_{LNV},$$  \hspace{1cm} (2.7)

where the subscripts LNC and LNV, respectively indicate to the lepton number conserving and lepton number violating ones as shown below. Here, each part is defined by

$$\mathcal{L}_{LNC} = h^U Q^{1L} \chi U_R + h^D_{\alpha \beta} Q^{\alpha L} \chi^* D_{\beta R}$$
$$+ h^{\epsilon}_{ij} \bar{\psi}_{iL} \phi j_R + h^{\epsilon}_{ij} \epsilon_{abc} \bar{\psi}_{iL} (\psi_{jL})_a (\psi_{kL})_b (\phi)_c$$
$$+ h^{i}_{ij} Q^{1L} \phi d_{iR} + h^u_{\alpha i} Q^{\alpha L} \phi^* u_{iR} + H.c.,$$  \hspace{1cm} (2.8)

$$\mathcal{L}_{LNV} = s^U_{i} Q^{1L} \chi u_{iR} + s^d_{a i} Q^{\alpha L} \chi^* d_{iR}$$
$$+ s^D_{a i} Q^{1L} \phi D_{aR} + s^u_{a i} Q^{\alpha L} \phi^* U_R + H.c.,$$  \hspace{1cm} (2.9)

where $a, b$ and $c$ stand for SU(3)$_L$ indices.

The VEV $\omega$ gives mass for the exotic quarks $U$ and $D_\alpha$, $u$ gives mass for $u_1, d_1$ quarks, while $v$ gives mass for $u_\alpha, d_\alpha$ and all ordinary leptons. In the next sections we provide more details on analysis of fermion masses. As mentioned, the VEV $\omega$ is responsible for the first step of symmetry breaking, while the second step is due to $u$ and $v$. Therefore the VEVs in this model have to be satisfied the constraint

$$u, v \ll \omega.$$  \hspace{1cm} (2.10)

The Yukawa couplings of Eq.(2.8) possess an extra global symmetry [10, 11] which is not broken by VEVs $v, \omega$ but by $u$. From these couplings, one can find the following lepton symmetry $L$ as in Table 1 (only the fields with nonzero $L$ are listed; all other fields have vanishing $L$). Here $L$ is broken by $u$ which is behind $L(\chi^0_1) = 2$, i.e., $u$ is a kind of the
Table 1: Nonzero lepton number $L$ of the model particles.

| Field | $\nu_iL$ | $e_{iL,R}$ | $\nu^c_{iL,R}$ | $\chi_1^0$ | $\chi^-_2$ | $\phi^+_3$ | $U_{L,R}$ | $D_{2L,R}$ | $D_{3L,R}$ |
|-------|-----------|-------------|----------------|------------|------------|------------|------------|------------|------------|
| $L$   | 1         | 1           | -1            | 2          | 2          | -2         | -2         | 2          | 2          |

lepton number violating parameter. It is interesting that the exotic quarks also carry the lepton number; therefore, this $L$ obviously does not commute with gauge symmetry. One can construct a new conserved charge $L$ through $L$ by making the linear combination $L = xT_3 + yT_8 + LI$. Applying $L$ on a lepton triplet, the coefficients will be determined

$$L = \frac{4}{\sqrt{3}}T_8 + LI.$$  \hspace{1cm} (2.11)

Another useful conserved charge $B$ which is exactly not broken by $u$, $v$ and $\omega$ is usual baryon number

$$B = BI.$$  \hspace{1cm} (2.12)

Both the $L$ and $B$ charges for the fermion and Higgs multiplets are listed in Table 2.

Table 2: $B$ and $L$ charges of the model multiplets.

| Multiplet | $\chi$ | $\phi$ | $Q_{1L}$ | $Q_{\alpha L}$ | $u_{iR}$ | $d_{iR}$ | $U_R$ | $D_{\alpha R}$ | $\psi_{iL}$ | $e_{iR}$ |
|-----------|--------|--------|----------|----------------|----------|----------|-------|----------------|------------|----------|
| $B$-charge| 0      | 0      | $\frac{1}{3}$ | $\frac{1}{3}$ | $\frac{1}{3}$ | $\frac{1}{3}$ | $\frac{1}{3}$ | 0      | 0          |
| $L$-charge| $\frac{4}{3}$ | $-\frac{2}{3}$ | $-\frac{2}{3}$ | $\frac{2}{3}$ | 0      | 0      | -2   | 2          | $\frac{1}{3}$ | 1        |

Let us note that the Yukawa couplings of (2.9) conserve $B$, while violate $L$ with $\pm2$ units which implies that these interactions are much smaller than the first ones, i.e.,

$$s^u_i, \ s^d_{ai}, \ s^D_{a}, \ s^U_{a} \ll h^U, \ h^D_{a\beta}, \ h^d_{i}, \ h^u_{ai}.$$  \hspace{1cm} (2.13)

In the previous studies [9, 12], the lepton number violating terms of this kind have often been excluded, commonly by the adoption of an appropriate discrete symmetry. There is no reason within the 3-3-1 models why such terms should not be present. Let us see next.
In this model, the most general Higgs potential has very simple form

\[
V(\chi, \phi) = \mu^2 \chi^\dagger \chi + \mu^2 \phi^\dagger \phi + \lambda_1 (\chi^\dagger \chi)^2 + \lambda_2 (\phi^\dagger \phi)^2 + \lambda_3 (\chi^\dagger \chi)(\phi^\dagger \phi) + \lambda_4 (\chi^\dagger \phi)(\phi^\dagger \chi). \tag{2.14}
\]

Note that there is no trilinear scalar coupling and this makes the Higgs potential much simpler than the previous ones of the 3-3-1 models \cite{10, 11, 13} and closer to that of the SM. The analysis in Ref. \cite{8} shows that after symmetry breaking, there are eight Goldstone bosons - needed number for massive gauge ones, and three physical scalar fields. One of two physical neutral scalars is the SM Higgs boson.

The non-zero values of \(\chi\) and \(\phi\) at the minimum value of \(V(\chi, \phi)\) can be obtained by

\[
\begin{align*}
\chi^+ \chi &= \frac{\lambda_3 \mu^2 - 2 \lambda_2 \mu^2_1}{4 \lambda_1 \lambda_2 - \lambda^2_3} \equiv \frac{u^2 + \omega^2}{2}, \tag{2.15} \\
\phi^+ \phi &= \frac{\lambda_3 \mu^2_1 - 2 \lambda_1 \mu^2_2}{4 \lambda_1 \lambda_2 - \lambda^2_3} \equiv \frac{v^2}{2}. \tag{2.16}
\end{align*}
\]

Any other choice of \(u, \omega\) for the vacuum value of \(\chi\) satisfying (2.15) gives the same physics because it is related to (2.4) by an SU(3) \(\otimes U(1)_X\) transformation. It is worth noting that the assumed \(u \neq 0\) is therefore given in a general case. This model of course leads to the formation of majorons \cite{8}. The constraint from the \(W\) decay width gives an upper limit on \(t_\theta = u/\omega\) which leads to the fact that this LNV parameter can exceed, say \(t_\theta = 0.08\) \cite{6}. Such a ratio of the scales for the lepton number breaking is much larger than those in Refs. \cite{11, 14}. Moreover, the Higgs potential (2.14) conserve both the mentioned global symmetries which implies that the considering model explicitly differs from those in Refs. \cite{10, 11, 13}.

For the sake of convenience in further reading, we present the relevant Yukawa couplings in terms of Feynman diagrams in the figures (1) and (2), where the Hermitian-conjugated couplings are not displayed.

The Higgs boson self-couplings are depicted in the figure (3).

Let us note that in Ref. \cite{9}, the authors have considered the fermion mass spectrum under the \(Z_2\) discrete symmetry which discards the LNV interactions. Here the couplings of the figure (2) in such case are forbidden. Then it can be checked that some quarks remain massless up to two-loop level. To solve the mass problem of the quarks, the authors in Ref. \cite{9} have shown that one third scalar triplet has to be added to the resulting model.

In the following we show that it is not necessary. The \(Z_2\) is not introduced and thus the third one is not required. The LNV Yukawa couplings are vital for the economical 3-3-1 model.
Figure 1: The relevant lepton-number conserving couplings to quarks

Figure 2: The lepton-number violating couplings
3 Fermion masses

The VEV \( \omega \) breaks \( SU(3)_L \otimes U(1)_X \) down to \( SU(2)_L \otimes U(1)_Y \) and gives masses of the exotic quarks as well as non-SM gauge bosons \( X, Y \) and \( Z' \). The VEV \( u \) gives mass for \( u_1, d_\alpha \) quarks, while \( v \) gives mass for \( u_\alpha, d_1 \) and all ordinary leptons. The SM gauge bosons gain mass both from \( u \) and \( v \). The fermions gain mass terms via Yukawa interactions given in (2.8) and (2.9).

3.1 Lepton masses

The charged leptons \( (l = e, \mu, \tau) \) gain masses via the following couplings

\[
\mathcal{L}_Y^e = h_{ij}^e \overline{\psi}_i L \phi e_j R + \text{H.c.}
\]  

(3.1)

The mass matrix is therefore followed by

\[
M_l = -\frac{v}{\sqrt{2}} \begin{pmatrix}
    h_{11}^e & h_{12}^e & h_{13}^e \\
    h_{21}^e & h_{22}^e & h_{23}^e \\
    h_{31}^e & h_{32}^e & h_{33}^e
\end{pmatrix},
\]

(3.2)

which of course gives consistent masses for the charged leptons [9].

Interesting physics is of the neutrino sector. At the tree level, the neutrinos gain Dirac masses via

\[
M_\nu = -\sqrt{2}v \begin{pmatrix}
    0 & h_{12}^\nu & h_{13}^\nu \\
    -h_{12}^\nu & 0 & h_{23}^\nu \\
    -h_{13}^\nu & -h_{23}^\nu & 0
\end{pmatrix},
\]

(3.3)
which gives the mass pattern 0, \(-m_u, m_u\), here \(m_u \equiv v \sqrt{2(h_{12}^2 + h_{13}^2 + h_{23}^2)}\). This means that one neutrino is massless and two are degenerate in mass. This pattern is clearly not realistic under the current data; however, it is severely changed by the quantum effect. We will return on this topic in the future publication.

### 3.2 Quark masses

The Yukawa couplings in (2.8) and (2.9) give the mass Lagrangian for the up-quarks (quark sector with electric charge \(q_{\text{up}} = 2/3\))

\[
\mathcal{L}_{\text{mass}}^{\text{up}} = \frac{h_u^U}{\sqrt{2}} \left( \overline{u_L}u + \overline{U}_L \omega \right) U_R + \frac{s_u^U}{\sqrt{2}} \left( \overline{u_L} u + \overline{U}_L \omega \right) u_{iR} - \frac{v}{\sqrt{2}} \overline{u\alpha L} \left( h_{\alpha iR} u_{iR} + s_{\alpha} U_{iR} \right) + H.c. \tag{3.4}
\]

Consequently, we obtain the mass matrix for the up-quarks (\(u_1, u_2, u_3, U\)) as follows

\[
M_{\text{up}} = \frac{1}{\sqrt{2}} \begin{pmatrix}
-s_1^u u & -s_2^u u & -s_3^u u & -h^U u \\
\hbar^u_{11} v & \hbar^u_{22} v & \hbar^u_{23} v & s_2^v u \\
\hbar^u_{31} v & \hbar^u_{32} v & \hbar^u_{33} v & s_3^v u \\
-s_1^u \omega & -s_2^u \omega & -s_3^u \omega & -h^U \omega
\end{pmatrix} \tag{3.5}
\]

Because the first and last rows of the matrix (3.5) are proportional, the tree level up-quark spectrum contains a massless one!

Similarly, for the down-quarks (\(q_{\text{down}} = -1/3\), we get the following mass Lagrangian

\[
\mathcal{L}_{\text{mass}}^{\text{down}} = \frac{h_{\alpha \beta}^D}{\sqrt{2}} \left( \overline{d_{\alpha L}} u + \overline{D_{\alpha L}} \omega \right) D_{\beta R} + \frac{s_{\alpha}^D}{\sqrt{2}} \left( \overline{d_{\alpha L}} u + \overline{D_{\alpha L}} \omega \right) d_{iR} + \frac{v}{\sqrt{2}} \overline{d_{iL}} \left( h_{\beta iR} u + s_{\alpha} D_{iR} \right) + H.c. \tag{3.6}
\]

Hence we get mass matrix for the down-quarks (\(d_1, d_2, d_3, D_2, D_3\))

\[
M_{\text{down}} = -\frac{1}{\sqrt{2}} \begin{pmatrix}
h_{11} d & h_{12} d & h_{13} d & s_{1}^D v & s_{2}^D v \\
\hbar_{11}^d u & \hbar_{12}^d u & \hbar_{13}^d u & \hbar_{22}^D u & \hbar_{23}^D u \\
\hbar_{31}^d u & \hbar_{32}^d u & \hbar_{33}^d u & \hbar_{22}^D u & \hbar_{23}^D u \\
-s_{1}^D \omega & s_{2}^D \omega & s_{3}^D \omega & h_{2}^D \omega & h_{3}^D \omega \\
h_{31}^d \omega & h_{32}^d \omega & h_{33}^d \omega & h_{32}^D \omega & h_{33}^D \omega
\end{pmatrix} \tag{3.7}
\]

We see that the second and fourth rows of matrix in (3.7) are proportional, while the third and the last are the same. Hence, in this case there are two massless eigenstates.

The masslessness of the tree level quarks in both the sectors calls radiative corrections (the so-called mass problem of quarks). These corrections start at the one-loop level. The diagrams in the figure (7) contribute the up-quark spectrum while the figure (8) gives the down-quarks. Let us note the reader that the quarks also get some one-loop contributions in the case of the \(Z_2\) symmetry enforcing [9]. The careful study of this radiative mechanism shows that the one-loop quark spectrum is consistent.
4 Typical examples of the one-loop corrections

To provide the quarks masses, in the following we can suppose that the Yukawa couplings are flavor diagonal. Then the $u_2$ and $u_3$ states are mass eigenstates corresponding to the mass eigenvalues:

$$m_2 = h_{22}^u v \sqrt{2}, \quad m_3 = h_{33}^u v \sqrt{2}. \tag{4.1}$$

The $u_1$ state mixes with the exotic $U$ in terms of one sub-matrix of the mass matrix (3.4)

$$M_{uU} = -\frac{1}{\sqrt{2}} \begin{pmatrix} s_{1u}^u & h_{1U}^u \\ s_{1\omega}^u & h_{1\omega}^U \end{pmatrix} \tag{4.2}$$

This matrix contains one massless quark $\sim u_1$, $m_1 = 0$, and the remaining exotic quark $\sim U$ with the mass of the scale $\omega$.

Similarly, for the down-quarks, the $d_1$ state is a mass eigenstate corresponding to the eigenvalue:

$$m_1' = -h_{1d}^d v \sqrt{2}. \tag{4.3}$$

The pairs $(d_2, D_2)$ and $(d_3, D_3)$ are decouple, while the quarks of each pair mix via the mass sub-matrices, respectively,

$$M_{d_2D_2} = -\frac{1}{\sqrt{2}} \begin{pmatrix} s_{2d}^d u & h_{2D_2}^d \\ s_{2\omega}^d & h_{2\omega}^{D_2} \end{pmatrix}, \tag{4.4}$$

$$M_{d_3D_3} = -\frac{1}{\sqrt{2}} \begin{pmatrix} s_{3d}^d u & h_{3D_3}^d \\ s_{3\omega}^d & h_{3\omega}^{D_3} \end{pmatrix}. \tag{4.5}$$

These matrices contain the massless quarks $\sim d_2$ and $d_3$ corresponding to $m_2' = 0$ and $m_3' = 0$, and two exotic quarks $\sim D_2$ and $D_3$ with the masses of the scale $\omega$.

With the help of the constraint (2.10), we identify $m_1$, $m_2$ and $m_3$ respective to those of the $u_1 = u$, $u_2 = c$ and $u_3 = t$ quarks. The down quarks $d_1$, $d_2$ and $d_3$ are therefore corresponding to $d$, $s$ and $b$ quarks. Unlike the usual 3-3-1 model with right-handed neutrinos, where the third family of quarks should be discriminating [15], in the model under consideration the first family has to be different from the two others.

The mass matrices (4.2), (4.4) and (4.5) remain the tree level properties for the quark spectra - one massless in the up-quark sector and two in the down-quarks. From these matrices, it is easily to verify that the conditions in (2.10) and (2.13) are satisfied. First, we consider radiative corrections to the up-quark masses.

4.1 Up quarks

In the previous studies [9, 12], the LNV interactions have often been excluded, commonly by the adoption of an appropriate discrete symmetry. Let us remind that there is no reason
within the 3-3-1 model to ignore such interactions. The experimental limits on processes
which do not conserve total lepton numbers, such as neutrinoless double beta decay [16],
require them to be small.

If the Yukawa Lagrangian is restricted to \( \mathcal{L}_{\text{LNC}} \) [9], then the mass matrix (4.2) becomes

\[
M_{uU} = -\frac{1}{\sqrt{2}} \begin{pmatrix}
0 & h_U u \\
0 & h_U \omega
\end{pmatrix}.
\]

(4.6)

In this case, only the element \((M_{uU})_{12}\) gets a oneloop correction defined by the figure (4). Other elements remain unchanged under this one-loop effect.

The Feynman rules gives us

\[
-i(M_{uU})_{12} P_R = \int \frac{d^4 p}{(2\pi)^4} (i h_U P_R) \frac{i(p + M_U)}{p^2 - M_U^2} (-i M_U P_L) \frac{i(p + M_U)}{p^2 - M_U^2} (i h_U P_R)
\]

\[
\times \frac{-1}{(p^2 - M_{\chi_1}^2)(p^2 - M_{\chi_3}^2)} (4\lambda_1) \frac{u \omega}{2}
\]

Thus, we get

\[
(M_{uU})_{12} = -2i u \omega \lambda_1 M_U (h_U)^2 \int \frac{d^4 p}{(2\pi)^4} \frac{p^2}{(p^2 - M_U^2)^2(p^2 - M_{\chi_3}^2)(p^2 - M_{\chi_1}^2)}
\]

\[
\equiv -2i u \omega \lambda_1 M_U (h_U)^2 I(M_U^2, M_{\chi_3}^2, M_{\chi_1}^2). \quad (4.7)
\]

The integral \(I(a, b, c)\) with \(a, b \gg c\) is given in the Appendix A. Following Ref. [8], we conclude that in an effective approximation, \(M_U^2, M_{\chi_3}^2 \gg M_{\chi_1}^2\). Hence we have

\[
(M_{uU})_{12} \approx -\frac{\lambda_1 t_\theta \mu^2}{4\pi^2} \left[ \frac{M_U^2 - M_{\chi_3}^2 + M_{\chi_3}^2 \ln \frac{M_{\chi_1}^2}{M_U^2}}{(M_U^2 - M_{\chi_3}^2)^2} \right] \sim u,
\]

\[
\equiv -\frac{1}{\sqrt{2}} R(M_U). \quad (4.8)
\]
The resulting mass matrix is given by
\[ M_{uU} = -\frac{1}{\sqrt{2}} \begin{pmatrix} 0 & h_U u + R \\ 0 & h_U \omega \end{pmatrix}. \] (4.9)

We see that one quark remains massless as the case of the tree level spectrum. This result keeps up to two-loop level, and can be applied to the down-quark sector as well as in the cases of non-diagonal Yukawa couplings. Therefore, under the $Z_2$, it is not able to provide consistent masses for the quarks.

If the full Yukawa Lagrangian is used, the LNV couplings must be enough small in comparison with the usual couplings [see (2.13)]. Combining (2.10) and (2.13) we have
\[ h_U \omega \gg h_U u, \quad s_1^u \omega \gg s_1^u u. \] (4.10)

In this case, the element $(M_{uU})_{11}$ of (4.2) gets the radiative correction depicted in Fig.(5).

Figure 5: One-loop contribution to the up-quark mass matrix (4.2)

The resulting mass matrix is obtained by
\[ M_{uU} = -\frac{1}{\sqrt{2}} \begin{pmatrix} s_1^u(u + \frac{R}{h_U}) & h_U u \\ s_1^u \omega & h_U \omega \end{pmatrix}. \] (4.11)

In contradiction with the first case, the mass of $u$ quark is now non-zero and given by
\[ m_u \simeq \frac{s_1^u}{\sqrt{2 h_U}} R. \] (4.12)

Let us note that the matrix (4.11) gives an eigenvalue in the scale of $\frac{1}{\sqrt{2}} h_U \omega$ which can be identified with that of the exotic quark $U$. In effective approximation [8], the mass for the Higgs $\chi_3$ is defined by $M_{\chi_3}^2 \simeq 2 \lambda_1 \omega^2$. Hereafter, for the parameters, we use the following values $\lambda_1 = 2.0$, $t_\theta = 0.08$ as mentioned, and $\omega = 10$ TeV. The mass value for the $u$ quark is as function of $s_1^u$ and $h_U$. Some values of the pair $(s_1^u, h_U)$ which give consistent masses for the $u$ quark is listed in Table 3.

Note that the mass values in the Table 3 for the $u$ quark are in good consistence with the data given in Ref. [17]: $m_u \in 1.5 \div 4$ MeV.
Table 3: Mass for the $u$ quark as function of $(s_1^u, h^U)$.

| $h^U$ | 2   | 1.5 | 1   | 0.5 | 0.1 |
|-------|-----|-----|-----|-----|-----|
| $s_1^u$ | 0.0002 | 0.0003 | 0.0004 | 0.001 | 0.01 |
| $m_u$ [MeV] | 2.207 | 2.565 | 2.246 | 2.375 | 2.025 |

### 4.2 Down quarks

For the down quarks, the constraint,

$$h^D_{aa} \omega \gg h^D_{aa} u, \quad s^d_{aa} \omega \gg s^d_{aa} u,$$

(4.13)

should be applied. In this case, three elements $(M_{d_D}a)_11$, $(M_{d_D}a)_12$ and $(M_{d_D}d)_21$ will get radiative corrections. The relevant diagrams are depicted in figure (6).

It is worth noting that diagram 6(c) exists even in the case of the $Z_2$ symmetry. The contributions are given by

$$(M_{d_D}a)_11 = -\frac{s^d_{aa}}{\sqrt{2}h^D_{aa}} R(M_{D_D}),$$

(4.14)

$$(M_{d_D}d)_21 = -4i\lambda_1\frac{s^d_{aa}}{h^D_{aa}} M^3_{D_D} I(M^2_{D_D}, M^2_{\chi_3}, M^2_{\chi_3})$$

$$= \frac{\lambda_1 s^d_{aa} M^3_{D_D}}{4\pi^2 h^D_{aa}} \left[ \frac{M^2_{D_D} + M^2_{\chi_3}}{(M^2_{D_D} - M^2_{\chi_3})^2} - \frac{2M^2_{D_D} M^2_{\chi_3}}{(M^2_{D_D} - M^2_{\chi_3})^3} \ln \frac{M^2_{D_D}}{M^2_{\chi_3}} \right]$$

$$\equiv -\frac{1}{\sqrt{2}} R'(M_{D_D}),$$

(4.15)

$$(M_{d_D}d)_12 = -\frac{1}{\sqrt{2}} R(M_{D_D}).$$

(4.16)

We see that two last terms are much larger than the first one. This is responsible for the masses of the quarks $d_2$ and $d_3$. At the one-loop level, the mass matrix for the down-quarks is given by

$$M_{d_D} = -\frac{1}{\sqrt{2}} \begin{pmatrix} s^d_{aa} (u + \frac{R}{h^D_{aa}}) & h^D_{aa} u + R \\ s^d_{aa} \omega + R' & h^D_{aa} \omega \end{pmatrix}.$$ 

(4.17)
Figure 6: One-loop contributions to the down-quark mass matrix (4.4) or (4.5).
We remind the reader that a matrix (see also [18])

\[
\begin{pmatrix}
  a & c \\
  b & D
\end{pmatrix}
\] (4.18)

with \(D \gg b, c \gg a\) has two eigenvalues

\[
x_1 \simeq \left[ a^2 - \frac{2bca}{D} + \frac{b^2c^2 - (b^2 + c^2)a^2}{D^2} \right]^{1/2},
\]
\[
x_2 \simeq D.
\] (4.19)

Therefore the mass matrix in (4.17) gives an eigenvalue in the scale of

\[
D \equiv \frac{1}{\sqrt{2}} h^{D}_{\alpha\alpha} \omega
\]

which is of the exotic quark \(D'_{\alpha}\). Here we have another eigenvalue for the mass of \(d'_{\alpha} \)

\[
m_{d'_{\alpha}} = \frac{h^{D}_{\alpha\alpha} u + R}{\sqrt{2} h^{D}_{\alpha\alpha} \omega} \left\{ R'^2 - \frac{(s^{d}_{\alpha\alpha})^2}{(h^{D}_{\alpha\alpha})^2} \left[ (s^{d}_{\alpha\alpha} \omega + R')^2 + (h^{D}_{\alpha\alpha} u + R)^2 \right] \right\}^{1/2}.
\] (4.20)

Let us remember that \(M_{\lambda_3}^2 \simeq 2\lambda_1 \omega^2\), and the parameters \(\lambda_1 = 2.0, t_b = 0.08\) and \(\omega = 10\ \text{TeV}\) as given above are used in this case. The \(m_{d_{\alpha}}\) is function of \(s^{d}_{\alpha\alpha}\) and \(h^{D}_{\alpha\alpha}\). We take the value \(h^{D}_{\alpha\alpha} = 2.0\) for both the sectors, \(\alpha = 2\) and \(\alpha = 3\). If \(s^{d}_{32} = 0.015\) we get then the mass of the so-called \(s\) quark

\[
m_s = 99.3\ \text{MeV}.
\] (4.21)

For the down quark of the third family, we put \(s^{d}_{33} = 0.7\). Then, the mass of the \(b\) quark is obtained by

\[
m_b = 4.4\ \text{GeV}.
\] (4.22)

We emphasize again that Eqs. (4.21) and (4.22) are in good consistence with the data given in Ref. [17]: \(m_s \sim 95 \pm 25\ \text{MeV}\) and \(m_b \sim 4.70 \pm 0.07\ \text{GeV}\).

## 5 Summary and Conclusions

In this paper, we have presented the answer to one of the most crucial questions: whether within the minimal scalar Higgs content, all fermions including quarks can gain the consistent masses.

In Ref. [6] we have shown that, in the considered model, there are three quite different scales of vacuum expectation values: \(\omega \sim O(1)\ \text{TeV}, v \approx 246\ \text{GeV}\) and \(u \sim O(1)\ \text{GeV}\). In this paper we have added a new characteristic property, namely, there are two types of Yukawa couplings with different strengths: the LNC coupling \(h\)'s and the LNV ones \(s\)'s satisfying the condition: \(s \ll h\). With the help of these key properties, the mass spectrum of quarks is consistent without introducing the third scalar triplet.

With the given set of parameters, the numerical evaluation shows that in this model, masses of the exotic quarks also have different scales, namely, the \(U\) exotic quark \((q_U = 2/3)\)
gains mass \( m_U \approx 700 \text{ GeV} \), while the \( D_\alpha \) exotic quarks \( (q_{D, \alpha} = -1/3) \) have masses in the TeV scale: \( m_{D, \alpha} \in 10 \div 80 \text{ TeV} \).

Let us summarize our results:

1. **At the tree level**
   
   (a) All charged leptons gain masses similar to that in the SM.
   (b) One neutrino is massless and two are degenerate in masses.
   (c) All exotic quarks gain masses proportional to the \( \omega \) - the VEV of the first step of symmetry breaking.
   (d) The quarks \( u_1, d_2, d_3 \) are massless

2. **At the one-loop level**

   (a) All above-mentioned quarks gain masses.
   (b) The light-quarks gain masses proportional to \( u \) - the VEV of lepton-number violation
   (c) There exist two types of Yukawa couplings: the LNC and LNV with quite different strengths.
   (d) The masses of the exotic quarks are separated too:

\[
m_U \approx 700 \text{ GeV}, \quad m_{D, \alpha} \in 10 \div 80 \text{ TeV} \tag{5.1}
\]

With the positive answer, the economical version becomes one of the very attractive models beyond the SM.

This is the time to mention some developments of the considered model. The idea to give VEVs at the top and bottom elements of \( \chi \) triplet was given in Ref. [7]. Some consequences such as the atomic parity violation, \( Z - Z' \) mixing angle and \( Z' \) mass were studied [19]. However, in the above-mentioned works, the \( W - Y \) and \( W_4, Z, Z' \) mixings were excluded. To solve the difficulties such as the SM coupling \( ZZh \) or quark masses, the third scalar triplet was introduced. Thus, the scalar sector did not to be minimal anymore and the economical was though to be unrealistic!

In the beginning of this year, there is a new step in development of the model. In Ref [6], the correct identification of non-Hermitian bilepton gauge boson \( X^0 \) was established. The \( W - Y \) mixing as well as \( W_4, Z, Z' \) one were also entered into couplings among fermions and gauge bosons. The scalar sector was studied in Ref. [8] and all gauge-Higgs couplings were presented and all similar ones in the SM were recovered. The Higgs sector contains eight Goldstone bosons - the needed number for massive gauge ones of the model. Interesting to note that, the \( CP \)-odd part of Goldstone associated with the neutral non-Hermitian bilepton gauge boson \( G_{X^0} \) is decouple, while its \( CP \)-even counterpart has the mixing by the same way in the gauge boson sector.
In Ref. [20], the deviation $\delta Q_W$ of the weak charge from its SM prediction due to the mixing of the $W$ boson with the charged bilepton $Y$ as well as of the $Z$ boson with the neutral $Z'$ and the real part of the non-Hermitian neutral bilepton $X^0$ is established.

In this model, the lepton-number violating interactions exist in both charged and neutral gauge boson sectors. However, the lepton-number violation happens only in the neutrino and exotic quarks sectors, but not in the charged lepton sector. In this model, lepton-number changing ($\Delta L = \pm 2$) processes exist but only in the neutrino sector. Consequently, neutrinos get Majorana masses at the one-loop level.

It is worth mentioning on the advantage of the considered model: the new mixing angle between the charged gauge bosons $\theta$ is connected with one of the VEVs $u$ - the parameter of lepton-number violations. There is no new parameter, but it contains very simple Higgs sector, hence the significant number of free parameters is reduced.

The model contains new kinds of interactions in the neutrino sector, and the exotic quark sector is rich too. Hence the model deserves further studies.

Acknowledgement

Financial support from Swedish International Development Cooperation Agency (SIDA) through the Associateship Scheme of the Abdus Salam International Centre for Theoretical Physics, Trieste, Italy is acknowledged (H. N. L.). This work was supported in part by National Council for Natural Sciences of Vietnam.

References

[1] SuperKamiokande Collaboration, Y. Fukuda et al., Phys. Rev. Lett. 81, 1158 (1998); Phys. Rev. Lett. 81, 1162 (1998); Phys. Rev. Lett. 82, 2644 (1999); Phys. Rev. Lett. 85, 3999 (2000); Y. Suzuki, Nucl. Phys. B, Proc. Suppl. 77, 35 (1999); S. Fukuda et al., Phys. Rev. Lett. 86, 5651 (2001); Y. Ashie et al., Phys. Rev. Lett. 93, 101801 (2004).

[2] KamLAND Collaboration, K. Eguchi et al., Phys. Rev. Lett. 90, 021802 (2003); T. Araki et al., Phys. Rev. Lett. 94, 081801(2005).

[3] SNO Collaboration, Q. R. Ahmad et al., Phys. Rev. Lett. 89, 011301 (2002); Phys. Rev. Lett. 89, 011302 (2002); Phys. Rev. Lett. 92, 181301 (2004); B. Aharmim et al., Phys. Rev. C 72, 055502 (2005).

[4] F. Pisano and V. Pleitez, Phys. Rev. D 46, 410 (1992); P. H. Frampton, Phys. Rev. Lett. 69, 2889 (1992); R. Foot, O. F. Hernandez, F. Pisano and V. Pleitez, Phys. Rev. D 47, 4158 (1993).
[5] M. Singer, J. W. F. Valle and J. Schechter, Phys. Rev. D 22, 738 (1980); R. Foot, H. N. Long and Tuan A. Tran, Phys. Rev. D 50, 34(R) (1994); J. C. Montero, F. Pisano and V. Pleitez, Phys. Rev. D 47, 2918 (1993); H. N. Long, Phys. Rev. D 54, 4691 (1996); Phys. Rev. D 53, 437 (1996).

[6] P. V. Dong, H. N. Long, D. T. Nhung, and D. V. Soa, Phys. Rev. D 73, 035004 (2006).

[7] W. A. Ponce, Y. Giraldo, and L. A. Sanchez, Phys. Rev. D 67, 075001 (2003).

[8] P. V. Dong, H. N. Long, and D. V. Soa, Phys. Rev. D 73, 075005 (2006).

[9] D. A. Gutierrez, W. A. Ponce, and L. A. Sanchez, Int. J. Mod. Phys. A 21, 2217 (2006).

[10] D. Chang and H. N. Long, Phys. Rev. D 73, 053006 (2006).

[11] M. B. Tully and G. C. Joshi, Phys. Rev. D 64, 011301 (2001).

[12] R. Foot et al. in Ref. [4]; M. D. Tonasse, Phys. Lett. B 381, 191 (1996); M. B. Tully and G. C. Joshi, Phys. Lett. B 466, 333 (1999); N. T. Anh, N. A. Ky and H. N. Long, Int. J. Mod. Phys. A 15, 283 (2000).

[13] R. A. Diaz, R. Martinez, F. Ochoa, Phys. Rev. D 69, 095009 (2004).

[14] F. Pisano, S. Shelly Sharma, Phys. Rev. D 57, 5670 (1998); J. C. Montero, C. A. de S. Pires and V. Pleitez, Phys. Rev. D 60, 098701 (1999).

[15] H. N. Long and V. T. Van, J. Phys. G 25, 2319 (1999).

[16] Alex G. Dias, A. Doff, C. A. de S. Pires, P. S. Rodrigues da Silva, Phys. Rev. D 72, 035006 (2005); J. C. Montero, C. A. de S. Pires, V. Pleitez, Phys. Rev. D 64 096001 (2001).

[17] Particle Data Group, S. Eidelman et al., Phys. Lett. B 592, 1 (2004).

[18] Ta-Pei Cheng and Ling-Fong Li, Gauge theory of elementary particle physics, Clarendon Press, Oxford (2004), p. 357.

[19] A. Carcamo, R. Martinez, F. Ochoa, Phys. Rev. D 73, 035007 (2006); F. Ochoa, R. Martinez, Phys. Rev. D 72, 035010 (2005).

[20] P. V. Dong, H. N. Long and D. T. Nhung, hep-ph/0604199, to appear in Phys. Lett. B (2006).
Appendix A  Feynman integration

With the help of
\[
\int \frac{d^4p}{(2\pi)^4} \frac{1}{(p^2 - a)(p^2 - b)(p^2 - c)} = \frac{-i}{16\pi^2} \left\{ \frac{a \ln a}{(a - b)(a - c)} + \frac{b \ln b}{(b - a)(b - c)} + \frac{c \ln c}{(c - b)(c - a)} \right\},
\]
(A.1)
and by differentiating two sides with respect to \(a\), we have
\[
\int \frac{d^4p}{(2\pi)^4} \frac{1}{(p^2 - a)^2(p^2 - b)(p^2 - c)} = \frac{-i}{16\pi^2} \times \left\{ \ln a + \frac{1}{(a - b)(a - c)} - \frac{a(2a - b - c) \ln a}{(a - b)^2(a - c)^2} + \frac{b \ln b}{(b - a)^2(b - c)} + \frac{c \ln c}{(c - a)^2(c - b)} \right\}.
\]
(A.2)
Hence we get
\[
I(a, b, c) = \int \frac{d^4p}{(2\pi)^4} \frac{p^2}{(p^2 - a)(p^2 - b)(p^2 - c)} = \int \frac{d^4p}{(2\pi)^4} \left[ \frac{1}{(p^2 - a)(p^2 - b)(p^2 - c)} \right.
\]
\[
+ \frac{a}{(p^2 - a)^2(p^2 - b)(p^2 - c)} \left. \right\} = \frac{-i}{16\pi^2} \left\{ \frac{a(2 \ln a + 1)}{(a - b)(a - c)} - \frac{a^2(2a - b - c) \ln a}{(a - b)^2(a - c)^2} 
\]
\[
+ \frac{b^2 \ln b}{(b - a)^2(b - c)} + \frac{c^2 \ln c}{(c - a)^2(c - b)} \right\}.
\]
(A.3)
In the case of \(b = c\), we have also
\[
I(a, b, b) = -\frac{i}{16\pi^2} \left[ \frac{a + b}{(a - b)^2} - \frac{2ab}{(a - b)^3} \ln \frac{a}{b} \right].
\]
(A.4)
If \(a, b \gg c\) we have an approximation:
\[
I(a, b, c) \simeq -\frac{i}{16\pi^2} \left[ \frac{a - b + b \ln \frac{b}{a}}{(a - b)^2} \right].
\]
(A.5)

B  One-loop corrections
+ 16 graphs with smaller contributions

Figure 7: One-loop contributions to the up-quark mass matrix (3.5).
+ 16 graphs with smaller contributions

Figure 8: One-loop contributions to the down-quark mass matrix (3.7).