Energy Dispersion of Westward Propagating Rossby Waves in Tropical Easterlies

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Abstract

The energy dispersion process of westward propagating Rossby waves in tropical easterlies are investigated in the linear nondivergent barotropic atmosphere. The variations in wave energy and amplitude along energy dispersion paths are calculated by solving the wave action conservation equation. The results suggest that a westward marching ray can form a cycle-like path near the turning latitude that is located in easterlies. Waves with shorter periods propagate between two turning latitudes, which are located in either the easterlies or westerlies and have the largest meridional propagation range. Waves with longer periods propagate between a turning latitude in westerlies and a critical latitude in easterlies. Both wave energy and amplitude can simultaneously increase to their maximum values at the turning latitudes that are located in easterlies. This implies that waves may develop significantly. Wave energy and amplitude do not always have an in-phase variation when the ray moves toward the turning latitude that is located in westerlies. The oscillating ranges of wave energy and amplitude are also limited. In this case, waves may not develop significantly.

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1. Introduction

Rossby waves generally propagate westward in tropical easterlies, therefore they are also called easterly waves. As early as 1948, they are known as a precursor of tropical cyclogenesis (Riehl 1948). Yanai (1961) made the one of the earliest observational analyses and revealed the transition of an easterly wave into typhoon Doris which appeared in the Marshall Islands area in 1958 (see Takayabu et al. 2016). Since then, the relationship between easterly waves and tropical cyclogenesis has been widely studied (e.g., Thorncroft and Hodges 2001; Russell et al. 2017; Patricola et al. 2018). The westward propagating Rossby waves also play important roles in various local weather phenomena, such as the modulation of convection and rainfall over West Africa (e.g., Kiladis et al. 2006) and India (e.g., Sanap et al. 2018) and North American monsoon circulation (e.g., Adams and Stensrud 2007). The energy dispersion process of westward propagating Rossby waves also has been a topic of many studies. For example, Diaz and Aiyer (2013) found the eastward energy dispersion of African easterly waves by applying single-point regressions. Hoskins and Yang (2016) developed a general one-dimensional theory of waves propagating in a zonally varying flow from the basic wave theory and derived equations for variations in wavenumber and energy along ray paths.

The physical mechanism responsible for the propagation of Rossby waves is virtually the same—the conservation of the potential vorticity (e.g., Gill 1982). The commonly applied β plane approximation can be reduced to the equatorial β plane approximation with \( f_0 = 0 \). Therefore, it is also suitable to extend the methodology applied in mid-latitudes to the tropics. For example, Karoly (1983) discussed the conditions that allow the cross-equatorial propagation of westward propagating Rossby waves. He concluded that waves should have phase speeds greater than the easterly jet maximum and high propagation speeds so that they propagate freely between the hemispheres. Liu and Nathan (1994) suggested that westward moving, low-frequency Rossby waves could propagate through tropical easterlies into the extratropics. Yang and Hoskins (1996) further pointed out that westward propagating waves tended to have larger total wavelengths and increased meridional group velocities, which are beneficial to meridional and cross-equatorial propagation. These works briefly investigated the ray paths, amplitude, and energy dispersion patterns of westward propagating Rossby waves. However, the discrepancies in ray paths between westward and eastward propagating waves are still poorly understood. In addition, the discussion on variations in wave energy and amplitude along the ray paths is also insufficient. Therefore, this study aims to further investigate the energy dispersion process of westward propagating Rossby waves.

2. Wave ray theory

The linearized barotropic vorticity equation in the Mercator projection of the sphere (Hoskins and Karoly 1981) is written as

\[
\frac{\partial}{\partial t} + \beta_y \frac{\partial}{\partial x} \nabla \psi + \beta_x \frac{\partial \psi}{\partial x} = 0,
\]

where

\[
\beta_y = \frac{2\Omega}{a} \cos^2 \phi - \frac{1}{\cos \phi} \frac{\partial}{\partial y} \left( \frac{\partial}{\partial y} \psi \right),
\]

are the zonal wind and meridional gradient of absolute vorticity in Mercator projection, \( x \) and \( y \) are the x-axis and y-axis in Mercator projection respectively, \( \psi \) is the geostrophic streamfunction, \( \nabla \psi \) is the geostrophic wind vorticity and \( \nabla^2 \) is the Laplace operator, \( \Omega \) is the angular velocity of the Earth’s rotation, and \( a \) is the radius of the Earth. The local dispersion relation is

\[
\omega = \pi_{\mu}(\gamma k - \frac{\beta_{\mu}(\mu)k}{k^2 + i(\gamma)}) \to \omega = \frac{\beta_{\mu}(\mu)k}{(\pi_{\mu} - c)},
\]

where \( k, l, \) and \( \omega \) are the zonal wavenumber, meridional wavenumber, and frequency, respectively. One can set negative frequencies or negative zonal wavenumbers to feature westward propagating Rossby waves. Here, we choose a negative zonal wavenumber \( k < 0 \) to do so. According to Eq. (3), the meridional propagation of the Rossby waves requires

\[
it^2 = \frac{\beta_{\mu}(\mu)k}{(\pi_{\mu} - c)} - k^2 \geq 0,
\]

where \( c = \omega / k < 0 \) is the zonal phase velocity. Notice that it is negative since the waves propagate westward. When \( t^2 = 0 \), Eq. (4) becomes

\[
\pi_{\mu} - c = \frac{\beta_{\mu}(\mu)}{k^2}.
\]

For westward propagating waves in tropical easterlies, both \( \pi_{\mu} \) and \( c \) are negative values. Therefore, Eq. (5) can be rewritten as

\[
\pi_{\mu} = \frac{\beta_{\mu}(\mu)}{k^2} - c > 0.
\]
The second equation in Eq. (8) indicates that the meridional group velocity will be greater than zero for an initial leading wave structure \((k'l > 0)\). Since \(k < 0\) for the westward waves, it requires \(l < 0\). Similarly, an initial trailing wave structure \((k'l < 0)\) will lead to a negative meridional group velocity. The fact that there exists a region where the zonal group velocity is greater than zero means that a ray with an initial westward zonal group velocity will form a cycle-like path near the turning latitude: a northwest moving ray arrives at a westernmost point where the zonal group velocity equals to zero and then changes its direction to the northeast until it arrives at the northernmost point at the turning latitude where the meridional group velocity decreases to zero. After being reflected by the turning latitude, it moves southeast to arrive at an easternmost point and then changes its direction to southeast. This cycle-like path is another major difference with the eastward propagating waves, the ray of which move northeastward toward the turning latitude and then is reflected southeastward, namely, not a cycle-like path (e.g., see Li et al. 2021).

The wave action density equation is

\[
\frac{\partial A}{\partial t} + \nabla \cdot (c_e A) = 0,
\]

where \(A = \frac{E}{2} K^2 A_e^2\) is the wave energy density, \(A_e\) is the amplitude of the geostrophic streamfunction, \(\omega^e = \omega - \tilde{u}_e k\) is the intrinsic frequency, and \(c_e\) the the group velocity vector. Eq. (9) can be rewritten along the ray

\[
\frac{D A}{Dt} = -A \nabla \cdot c_e,
\]

where \(\frac{D}{Dt} = \frac{\partial}{\partial t} + c_e \cdot \nabla\) means the derivative along the ray. Eq. (10) denotes that the wave action density variation along the ray is determined by the divergence of group velocity. A convergent group velocity \((\nabla \cdot c_e < 0)\) will increase the wave action density while a divergent group velocity \((\nabla \cdot c_e > 0)\) will decrease the wave action density. Following the method proposed by Li et al. (2021), the divergence of group velocity is expressed as

\[
\nabla \cdot c_e = \lim_{\Delta t \to 0} \left( \frac{c_e(e^{-dS})_{t+\delta t} - (c_e dS)}{\Delta t} \right) \left( \frac{c_e}{\Delta S} \right),
\]

where subscript \(t\) denotes time, \(\Delta t\) represents a short time interval, \(dS\) is the cross-sectional area of a thin ray tube. Applying the method, we can easily solve the divergence of group velocity and hence the wave action density.

3. Results

Westward propagating Rossby waves generally have a wavelength of several thousands of kilometers and a wave period of several days in the lower troposphere (e.g., Thornthwaite and Hoskins 1994; Kiladis et al. 2006). Therefore, we take 600 hPa zonal wind from the NCEP reanalysis data provided by the NOAA/OAR/ESRL PSD, Boulder, Colorado, USA, from their website at https://www.esrl.noaa.gov/psd/ as the basic zonal flow. The zonal wavenumber is set to -12 (equivalent wavelength is approximately 3200 km). The tropical easterly region is located around 20°S–15°N in the December-January-February (DJF) season (Fig. 1a). The dispersion region (Fig. 1b) enclosed by turning latitudes and critical latitudes shows some interesting features. When the wave period is shorter than 4 days, waves cannot propagate. When the wave period varies from 4 to 6 days, the energy dispersion region is enclosed by two turning latitudes that are located in easterlies. This means that the ray is reflected between the two turning latitudes. When the wave period is longer than 7 days, the energy dispersion regions are enclosed by a turning latitude and a critical latitude. With the longer wave period, the turning latitudes move into westerlies. There also exists an energy dispersion region enclosed by two critical latitudes (e.g., wave period equals 16 days). This type of region is caused by the
shape of the easterly, at a lower latitude of approximately 7°S. The situation is quite similar for the June-July-August (JJA) season when the tropical easterly moves northward to approximately 15°S–25°N (Fig. 2a). The energy dispersive region (Fig. 2b) is also similar to that in the DJF season. These findings from the DJF and JJA seasons show that there are also three types of energy dispersive regions enclosed by the different combinations of turning latitude and critical latitude. This is the same as the eastward propagating Rossby waves (e.g., Li et al. 2021). However, there are also significant differences. The first is that for a westward propagating wave with a shorter period (a westward propagating wave generally has a short period), the area of the major dispersive region enclosed by a critical latitude and a turning latitude is limited. The second is that for a westward propagating wave with a longer period (an eastward propagating wave generally has a relatively long period), the major region is enclosed by a critical latitude and a turning latitude. The second is that for a westward propagating wave, a shorter period wave has a larger dispersive region, while for an eastward propagating wave, a longer period wave has a larger dispersive region.

Based on the above discussion, we calculate two cases in JJA season to show the variations in wave energy and amplitude along the ray trajectories. The first case (Fig. 3a) corresponds to an initial leading wave with a period of 5 days and starts at 15°N. The ray (Fig. 3b) begins to move northeast toward a westermost place (red vertical straight line in Fig. 3b) at approximately 20°N within approximately 2 days. Then, it changes its direction to move northeast to arrive at the northernmost turning latitude (the red horizontal straight line in Fig. 3b) on approximately the 3rd day. Next, the ray is reflected toward the southeast and arrives at the easternmost place (see red vertical straight line in Fig. 3c). After leaving from the easternmost place, the ray moves southwest and changes its direction to northeast to arrive at the turning latitude. The wave energy can increase more than 3 times its initial value. The situation is similar when the ray moves close to the another turning latitude (e.g., from around 15 days to 17 days). A wave may develop significantly if both its energy and amplitude increase. Of course, it must be noted that the Wenzel-Kramers-Brillouin (WKB) approximation is invalid at the turning latitude where the small perturbation approximation is invalid. Besides, the ray forms a small cycle-like trajectory, which also seems to violate the WKB approximation, which requires a slowly varying medium and rays. However, despite the WKB approximation is invalid, we can still take other form of approximation (e.g., see Gill 1982). Therefore, the characteristics near the turning latitudes still can provide qualitative information. Of course, one should be extra careful when referring the knowledge near the turning latitude. The variations in wave energy and amplitude for waves propagating in two turning latitudes are also significantly different with the corresponding eastward propagating waves in westerlies, the wave energy and amplitude of which do not always have phase variation trends (e.g., Li et al. 2021).

The initial leading wave period is set to 10 days, and the ray starts at 25°N in the second case (Fig. 4). With increasing wave period, the energy dispersive region is limited. The ray marches between a northern turning latitude at approximately 30°N in westerlies and a critical latitude at approximately 18°N in easterlies (Fig. 4a). The initial position is still in the easterly wind region but with a very weak wind speed. The ray starts with a very weak negative zonal group velocity, moves northwest slightly and later changes its direction to northeast to arrive at the turning latitude. Then, it is reflected to move southeast and then southwest toward the critical latitude. When the ray marches toward the turning latitude, the wave action density (blue solid line in Fig. 3d) approaches to the extreme value due to huge divergence of group velocity caused by the considerable curvature near the cycle-like structure. The intrinsic frequency (red solid line in Fig. 3d) also approaches to the extreme value at the turning latitude. Therefore, both increasing wave action density and intrinsic frequency determine that wave energy density (blue dashed line in Fig. 3d) also approaches to the extreme value at the turning latitude. Since total wavenumber decreases to the minimum value at the turning latitude, we can find out that amplitude (blue dotted line in Fig. 3d) also approaches to the extreme value at the turning latitude. The wave energy can increase more than 3 times its initial value, while the amplitude is approximately 2.5 times the initial value. The situation is similar when the ray moves close to the another turning latitude (e.g., from around 15 days to 17 days). A wave may develop significantly if both its energy and amplitude increase. Of course, it must be noted that the Wenzel-Kramers-Brillouin (WKB) approximation is invalid at the turning latitude where the small perturbation approximation is invalid. Besides, the ray forms a small cycle-like trajectory, which also seems to violate the WKB approximation, which requires a slowly varying medium and rays. However, despite the WKB approximation is invalid, we can still take other form of approximation (e.g., see Gill 1982). Therefore, the characteristics near the turning latitudes still can provide qualitative information. Of course, one should be extra careful when referring the knowledge near the turning latitude. The variations in wave energy and amplitude for waves propagating in two turning latitudes are also significantly different with the corresponding eastward propagating waves in westerlies, the wave energy and amplitude of which do not always have phase variation trends (e.g., Li et al. 2021).

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latitude. The variation in amplitude (Fig. 4d) is the same as the wave energy. It means that the wave energy plays a dominant role in controlling amplitude. After the ray is reflected by the turning latitude and moves toward the critical latitude, the wave action density, wave energy and amplitude eventually decrease. It means that the wave declines and is eventually absorbed by the basic flow.

4. Conclusion and discussion

This paper investigates the characteristics of energy dispersion for westward propagating Rossby waves in tropical easterlies. The results suggest that the energy dispersive regimes for westward propagating waves in tropics are the same as those for eastward
propagating waves in midlatitudes. There are also three types of energy dispersive regions. The first type is enclosed by two turning latitudes. The second type is enclosed by a higher turning latitude and a lower critical latitude. The third type is enclosed by two critical latitudes. For waves with shorter periods, the first type is the major energy dispersive form, while for waves with longer periods, the second type is the major form.

It is easy to derive that the zonal group velocity is larger than zero when the ray arrives at the turning latitude, regardless of whether the turning latitude is located in easterlies or westerlies. This means that an initially northwestward ray will arrive at a westemmost place where the zonal group velocity equals zero. After that, the ray will move northeastward until it arrives at the turning latitude, which is also the northemmost place a ray can arrive. When the ray leaves from its turning latitude, it moves southeastward to an easternmost place where the zonal group velocity equals zero again. After that, the ray will move southwestward. The spatial trajectory implies a cycle-like structure close to a turning latitude. The spatial scale of the cycle-like path depends on the location of the turning latitude. If it is in the westerly (easterly), the cycle-like path will have a relatively large (small) spatial scale. This cycle-like structure in a ray trajectory is the most significant difference from a ray of an easterward propagating Rossby wave.

If the turning latitude is located in the easterly, both the wave energy and amplitude approach their maximum values at the turning latitude. Therefore, the wave may develop significantly. If the turning latitude is located in the westerly, the wave energy approaches the maximum value before it arrives at the turning latitude. Actually, it approaches the maximum value at the westernmost and easternmost locations. The wave energy decreases to a local minimum (slightly smaller than the initial value) at the turning latitude. Amplitude does not always have a variation trend consistent with that of the wave energy. Both wave energy and amplitude have limited variation ranges, which means that the wave may not develop significantly.

The cycle-like structure in the ray trajectory leads to some interesting implications. First, the westward waves that propagate between two turning latitudes (especially both two turning latitudes are located in easterlies) seem to easily develop significantly even though the basic flow does not satisfy the necessary conditions for barotropic instability. Additionally, this behavior is quite different from eastward propagating waves in westerlies, where the energy dispersive region enclosed by two turning latitudes acts as a waveguide to permit long-distance propagation without significant development or decay. This warrants further investigation on wave development and even instability in easterlies. Second, the relevant wave ray theory is basically an extension of wave ray theory for midlatitudes. Waves in tropical regions have some different features, such as discrete meridional modes due to equatorial trapped boundary conditions (e.g., Matsumo 1966; Gill 1980). Therefore, a comprehensive conclusion can be summarized when both methods have been applied in analyzing the energy dispersion of westward propagating waves in easterlies.

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References

Adams, J. L., and D. J. Stensrud, 2007: Impact of tropical easterly waves on the north American monsoon. J. Climate, 20, 1219–1238.

Diaz, M., and A. Aiyyer, 2013: Energy dispersion in African easterly waves. J. Atmos. Sci., 70, 130–145.

Enomoto, T., and Y. Matsuda, 2000: Numerical experiments on the behavior of Rossby waves in the critical layer. Fluid. Dyn. Res., 26, 257–279, doi:10.1016/S0169-5983(99)00028-3.

Gill, A. E., 1980: Some simple solutions for heat-induced tropical circulation. Quart. J. Roy. Meteor. Soc., 106, 447–462.

Gill, A. E., 1982: Atmosphere-Ocean Dynamics. Academic Press, 662 pp.

Hoskins, B. J., and D. J. Karoly, 1981: The steady linear response of a spherical atmosphere to thermal and orographic forcing. J. Atmos. Sci., 38, 1179–1196.

Hoskins, B. J., and G. Yang, 2016: The longitudinal variation of equatorial waves due to propagation on a varying zonal flow. J. Atmos. Sci., 73, 605–620.

Karoly, D. J., 1983: Rossby wave propagation in a barotropic atmosphere. Dyn. Atmos. Oceans, 7, 111–125.

Kiladis, G. N., C. D. Thorncroft, and N. M. J. Hall, 2006: Three-dimensional structure and dynamics of African easterly waves. Part I: Observations. J. Atmos. Sci., 63, 2212–2230.

Killworth, P. D., and M. E. McIntyre, 1985: Do Rossby-wave critical layers absorb, reflect, or over-reflect? J. Fluid. Mech., 1611, 449–492, doi:10.1017/S0022112085003019.

Li, L., and T. R. Nathan, 1994: The global atmospheric response to Low-Frequency tropical forcing: Zonally averaged basic states. J. Atmos. Sci., 51, 3412–3426.

Li, Y. K., J. P. Chao, and Y. Y. Kang, 2021: Variations in wave energy and amplitudes along the energy dispersion paths of nonstationary barotropic Rossby waves. Adv. Atmos. Sci., 38, 49–64.

Matsumo, T., 1966: Quasi-geostrophic motions in the equatorial area. J. Meteor. Soc. Japan, 44, 25–43.

Patricola, C. M., R. Saravanan, and P. Chang, 2018: The response of Atlantic tropical cyclones to suppression of African easterly waves. Geophys. Res. Lett., 45, 471–479.

Riehl, H., 1948: On the formation of typhoons. J. Atmos. Sci., 5, 247–265, doi:10.1175/1520-0469(1948)005<0247:OTFOT>2.0.CO;2.

Russell, J. O., A. Aiyyer, J. D. White, and W. Hannah, 2017: Revisiting the connection between African easterly waves and Atlantic tropical cyclogenesis. Geophys. Res. Lett., 44, 587–595.

Sanan, S. D., P. Priya, G. K. Sawaisarje, and K. S. Hosalikar, 2018: Heavy rainfall events over southeast peninsular India during northeast monsoon: Role of El Niño and easterly wave activity. Int. J. Climatol., 39, 1954–1968.

Schopf, P. S., D. L. T. Anderson, and R. Smith, 1981: Beta-dispersion of low-frequency Rossby waves. Dyn. Atmos. Oceans, 5, 187–214.

Takayabu, Y. N., G. N. Kiladis, and V. Magaña, 2016: Michio Yanai and tropical waves. Meteor. Monogr., 56, 1–3, doi: 10.1175/AMS-MONOGRAHS-D-15-0019.1.

Thornicroft, C., and K. Hodges, 2001: African easterly wave variability and its relationship to Atlantic tropical cyclone activity. J. Climate, 14, 1166–1179.

Thornicroft, C. D., and B. J. Hoskins, 1994: An idealized study of African easterly waves. I: A linear view. Quart. J. Roy. Meteor. Soc., 120, 953–982.

Yang, G., and B. J. Hoskins, 1996: Propagation of Rossby waves of nonzero frequency. J. Atmos. Sci., 53, 2365–2378.

Yanai, M., 1961: A detailed analysis of typhoon formation. J. Meteor. Soc. Japan. Ser. II, 39, 187–214, doi:10.2151/jmsj192339.4_187.