Type III seesaw under $A_4$ modular symmetry with leptogenesis

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Abstract We make an attempt to study neutrino phenomenology in the framework of type-III seesaw by considering $A_4$ modular symmetry in the super-symmetric context. In addition, we have included a local $U(1)_{B-L}$ symmetry which eventually helps us to avoid certain unwanted terms in the superpotential. Hitherto, the seesaw being type-III, it involves three fermion triplet superfields $\Sigma_R$, along with which, we have included a singlet weighton field ($\rho$). In here, modular symmetry plays a crucial role by avoiding the usage of excess flavon (weighton) fields. Also, the Yukawa couplings acquire modular forms which are expressed in terms of Dedekind eta function $\eta(\tau)$. However, for numerical analysis we use $q$ expansion expressions of these couplings. Therefore, the model discussed here is triumphant enough to accommodate the observed neutrino oscillation data and also successfully explains observed baryon asymmetry of the universe through leptogenesis.

1 Introduction

Decades ago when standard model (SM) was built it seemed impeccable, but its foundation was again questioned when some unresolved puzzles came into existence. To name a few, it does not provide any satisfactory explanation to the tininess of neutrino mass [1], neutrino oscillation, strong CP problem, matter–antimatter asymmetry, the nature of dark matter and dark energy, etc. To resolve the issue regarding smallness of neutrino masses within the context of SM, Weinberg operator [2,3] helps to an extent. However, to demonstrate other phenomena, we need to go beyond standard model (BSM), so introducing right handed (RH) neutrinos becomes a necessity. This becomes the basis of canonical seesaw mechanism, i.e., as soon as these RH neutrinos come into picture, they allow Dirac mass terms for neutrinos. In this regard, type-I seesaw [4–7] is the simplest one, which includes singlet heavy $(\simeq 10^{14}$ GeV) RH neutrinos and brings down the mass scale of active neutrinos to 0.1 eV range, as observed from experimental data. Also, there exists other variants of seesaw i.e., type-II [8–12] which incorporates scalar triplets, type-III [13,14] involving fermion triplets, linear seesaw [15–19] and inverse seesaw [20–27] which are modified type-I seesaw. In this work, we intend to study the case of type-III seesaw in the context of discrete $A_4$ modular symmetry as it has not been studied earlier in this framework. In general, it is presumed that type III seesaw is more complicated compared to the canonical type I seesaw due to the involvement of triplet fermions. However, it has been shown in Refs. [28,29] that, in some cases, e.g., realistic $SO(10)$ model, type III seesaw may have less difficulty in reproducing realistic neutrino masses and mixings than the conventional type-I seesaw. Therefore, in this work we would like to investigate the implications of $A_4$ modular symmetry in the context of type III seesaw for describing the observed neutrino oscillation data.

It is interesting to notice that many non-abelian discrete symmetries i.e., $S_3$ [30–33], $A_4$ [34–37], $S_4$ [38–40] etc. and continuous symmetries like $U(1)_{B-L}$ [41–47], $U(1)_{L_R-L_L}$ [48–51], $U(1)_H$ [52–55] etc. come to our rescue to develop the model and generate neutrino mass matrix, which gives results in-accordance with experimental data. Implementation of the discrete non-abelian symmetries demand the usage of excess flavon fields. These flavon insertions make the Yukawa interaction terms non-renormalizable and bring down the predictability of the model. Therefore, a clever approach of modular symmetry [56–60] is introduced to breach the scenario of flavon fields. These flavon insertions make the Yukawa interaction terms non-renormalizable and bring down the predictability of the model. Therefore, a clever approach of modular symmetry [56–60] is introduced to breach the scenario of flavon fields. In here, the approach involves discrete symmetry group because they are isomorphic to finite modular groups, for example, $\Gamma_2 \simeq S_3$ [61–
Table 1  Particle content of the model and their charges under $SU(2)_L \times U(1)_Y \times U(1)_{B-L} \times A_4$, where, $k_I$ is the modular weight

| Fields          | $E^c_{R_1}$ | $E^c_{R_2}$ | $E^c_{R_3}$ | $L$      | $\Sigma^c_{R_1}$ | $H_{u,d}$ | $\rho$ |
|-----------------|-------------|-------------|-------------|---------|------------------|-----------|-------|
| $SU(2)_L$       | 1           | 1           | 1           | 2       | 3                | 2         | 1     |
| $U(1)_Y$        | 1           | 1           | 1           | $-\frac{1}{2}$ | 0     | $\frac{1}{2}, -\frac{1}{2}$ | 0     |
| $U(1)_{B-L}$    | 1           | 1           | 1           | $-1$    | 1                | 0         | -2    |
| $A_4$           | 1           | 1$'$        | 1$''$       | 1, 1$''$, 1$'$ | 3    | 1                 | 1     |
| $k_I$           | 0           | 0           | 0           | 0       | $-2$             | 0         | 2     |

Table 2  Charge assignment to Yukawa coupling under $A_4$ and its modular weight

| Yukawa couplings | $A_4$ | $k_I$ |
|------------------|-------|-------|
| $Y = (y_1, y_2, y_3)$ | 3     | 2     |

64]. $\Gamma_3 \simeq A_4$ [65–73], $\Gamma_4 \simeq S_4$ [74–80], $\Gamma_5 \simeq A_5$ [81–83], $\Gamma_5 \simeq A_4^* [84–87]$ etc. We make an attempt to use $A_4$ modular symmetry, which is isomorphic to $\Gamma_3$. The alluring feature of modular symmetry is that, it transforms Yukawa couplings i.e., it makes them modular in nature. Therefore, the flexibility to fine tune the Yukawa couplings is lost and now it is governed by the modulus $\tau$. The involvement of modulus $\tau$ is seen in the expression of Dedekind eta function, as shown in Eq. (A12), and further the acquisition of VEV by it, helps in the symmetry breaking of the $A_4$ group. As, $N = 3$ for $\Gamma_3 \simeq A_4$ is finite, hence, they can be constructed using $k = 1$ being the lowest weight. The dimension of $\Gamma_3$ being $2k + 1$ (see appendix D of [59]) yielding three linearly independent $Y_i(\tau)$ shown in Eqs. (A9–A11). These Yukawa couplings are utilised in curating the neutrino mass matrices after applying the $A_4$ product rules and are implicitly governed by the range of modulus $\tau$, as will become more clear while performing the analysis numerically. Further, we are able to explain the baryon asymmetry of the universe through leptogenesis [88, 89], because of presence of heavy RH neutrino, which yields the order of lepton asymmetry to be $\sim 10^{-10}$.

This work is organised as follows. In Sect. 2, we discuss the model framework containing particles contributing towards expressing the superpotential for type-III seesaw and its associated mass matrices. Further, in Sect. 3, we perform the numerical analysis where a common parameter space along with best-fit data set are extracted using chi-square minimization technique using the data of all the phenomena discussed in our model. Additionally, Sect. 4 sheds light on lepton asymmetry generated through leptogenesis, in the context of our model and collider bound on the mass of new gauge boson $Z'$ is presented in Sect. 5. Finally, in Sect. 6, we conclude our results.

2 Model framework

In order to fulfil our desired goal, we incorporate new particles and assign them suitable charges under extended symmetries (i.e., modular $A_4$ and $U(1)_{B-L}$), as presented in Table 1, such that the superpotential remains invariant. The idea behind the inclusion of $U(1)_{B-L}$ symmetry along with $A_4$ modular symmetry is to avoid certain unwanted terms in the superpotential which is not possible by $A_4$ modular symmetry. The suitability to go beyond standard model (BSM) paves the way to include heavy RH neutrinos $\Sigma_R$ in our model, which transform as triplet under $SU(2)_L$, and accompanying these, we have also included a weighton ($\rho$). These symmetries are broken at a very high scale, much greater than the scale of electroweak symmetry breaking. The $U(1)_{B-L}$ symmetry is spontaneously broken by assigning non-zero VEV to the singlet weighton $\rho$ and the $Z'$ boson associated with it acquires its mass by the singlet VEV $v_\rho$. We will show in Sect. 5 that its mass and gauge coupling satisfy the present experimental bounds. Moreover, the non-zero VEV acquired by the singlet weighton helps heavy RH neutrinos to gain mass. We implement modular symmetry because it restricts the usage of excess flavon fields, which otherwise, overfill the particle gamut and reduces the predictability of the model while working in BSM. This becomes possible only because Yukawa couplings acquire modular form and also take over the job performed by extra flavon fields. In addition, the complete superpotential of our model is represented below.

$$W_{III} = W_{M_L} + W_{M_D} + W_{M_R},$$  \hspace{1cm} (1)

where, the terms $W_{M_L}$, $W_{M_D}$ and $W_{M_R}$ are responsible for generating the mass term for the charged leptons, Dirac mass term for the neutrinos and Majorana mass term for the RH neutrinos and their explicit forms are provided in the following subsections.

Masses of charged leptons

We urge to have a simplified form of charged lepton mass matrix for which we assign $U(1)_{B-L}$ charge to the right-
hand (RH) charged leptons i.e., $E^c_R$, as $+1$, and three generations of left-handed (LH) charged leptons have the value $-1$. While under $A_4$ symmetry, RH and LH charged leptons transform as $\{1, 1', 1''\}$ and $\{1, 1''', 1''''\}$. In addition, the modular weight assigned to the charged leptons is zero. The Higgsinos $H_u, d$ are given charges 0 and 1 under $U(1)_{B-L}$ and $A_4$ symmetry respectively, with zero modular weight. The VEVs of Higgsinos i.e., $(v_u, v_d)$ are related to the SM Higgs $v_H$ by a simple equation $v_H = \frac{1}{2} \sqrt{v_u^2 + v_d^2}$. The ratio of Higgsinos VEV is written as $\tan \beta = (v_u/v_d) \simeq 5$ (used in our analysis) [90–92].

The admissible superpotential term for the charged lepton sector is given below:

$$W_{M_{L}} = y_{ij} E^c_{R_i} H_d L_j.$$  \hspace{1cm} (2)

After the electroweak symmetry breaking the mass matrix for the charged leptons takes the diagonal form:

$$M_{\ell} = \frac{v_d}{\sqrt{2}} \begin{bmatrix} y_{ee} & 0 & 0 \\ 0 & y_{e\mu} & 0 \\ 0 & 0 & y_{e\tau} \end{bmatrix}.  \hspace{1cm} (3)$$

**Dirac mass term**

The neutral lepton sector gets mass as and when $H_u$ acquires non-vanishing VEV. To keep Dirac term invariant under $A_4$ modular group, we need fermion triplets to have charge 3 as Yukawa couplings are triplet ($Y=(y_1, y_2, y_3)$) (Table 2). Hence, the Dirac interaction term of neutral multiplet of fermion triplet with the SM left-handed neutral leptons can be written as:

$$W_{M_{D}} = -(G_D)_{ij} \left[ H_u \Sigma^c_{R_i} \sqrt{2} Y L_j \right]. \hspace{1cm} (4)$$

with $G_D = \text{diag}(\alpha_1, \alpha_2, \alpha_3)$, which gives the mass matrix

$$M_{D} = v_u \begin{bmatrix} \alpha_1 & 0 & 0 \\ 0 & \alpha_2 & 0 \\ 0 & 0 & \alpha_3 \end{bmatrix} \begin{bmatrix} y_1 & y_3 & y_2 \\ y_2 & y_1 & y_3 \\ y_3 & y_2 & y_1 \end{bmatrix}. \hspace{1cm} (5)$$

**Majorana mass term**

The superpotential for Majorana mass term for right handed neutrinos is given as,

$$W_{M_{\nu}} = -\frac{M^c}{2} \left( \beta_S \text{Tr} \left[ \Sigma^c_{R_i} Y \Sigma^c_{R_i} \right]_{\text{sym}} + \gamma_S \text{Tr} \left[ \Sigma^c_{R_i} Y \Sigma_{R_i} \right]_{\text{asym}} \right) \frac{\rho}{\Lambda}. \hspace{1cm} (6)$$

where, $M^c_{\nu}$ is the free mass parameter and $\Sigma^c_{R_i}$ with $(i = 1, 2, 3)$, which can be represented in $SU(2)$ basis as,

$$\Sigma^c_{R_i} = \begin{pmatrix} \Sigma^0_{R_i} / \sqrt{2} & \Sigma^{-}_{R_i} \\ \Sigma^{+}_{R_i} & -\Sigma^0_{R_i} / \sqrt{2} \end{pmatrix}. \hspace{1cm} (7)$$

Applying $A_4$ symmetry product rule to Eq. (6), yields both symmetric and anti-symmetric parts with $\beta_S = \text{diag}(\beta_1, \beta_2, \beta_3)$ and $\gamma_S = \text{diag}(\gamma_1, \gamma_2, \gamma_3)$ being the associated free parameter matrices respectively:

$$M_{R} = \frac{v_u}{\Lambda \sqrt{2}} \left( \frac{M^c_{\nu}}{2} \right) \times \left( \frac{\beta_S}{3} \begin{pmatrix} 2 y_1 - y_3 - y_2 \\ -y_3 + 2 y_2 - y_1 \\ -y_2 - y_1 + 2 y_3 \end{pmatrix} + \gamma_S \begin{pmatrix} 0 & y_3 & -y_2 \\ -y_3 & 0 & y_1 \\ y_2 & -y_1 & 0 \end{pmatrix} \right). \hspace{1cm} (8)$$

The active neutrino mass matrix in the framework of type-III seesaw is given as,

$$m_{\nu} = -M_{D} M_{R}^{-1} M_{D}^T. \hspace{1cm} (9)$$

**3 Numerical analysis**

The global fit neutrino oscillation data at $3\sigma$ interval from [93] is used for numerical analysis, as given in Table 3. The neutrino mass matrix calculated using Eq. (9) is numerically diagonalized using the relation $U^\dagger M U = \text{diag}(m_{\nu 1}^2, m_{\nu 2}^2, m_{\nu 3}^2)$, where, $M = m_{\nu} m_{\nu}^T$ and $U$ is a unitary matrix, from which the neutrino mixing angles can be derived using the conventional relations:
We consider the free mass parameter (Table 4) Best-fit of model parameters by under constraint of experimentally observed data

| Model parameters | \( \alpha_1 \) | \( \alpha_2 \) | \( \alpha_3 \) | \( \beta_1 \) | \( \beta_2 \) | \( \beta_3 \) |
|------------------|-------------|-------------|-------------|-------------|-------------|-------------|
| Best-fit values  | \( 3.83 \times 10^{-7} \) | \( 1.61 \times 10^{-6} \) | \( 5.73 \times 10^{-7} \) | \( 4.44 \times 10^{-2} \) | \( 0.824 \) | \( 1.05 \times 10^{-3} \) |

Fig. 1 Left(right) panel shows the plane of the mixing angle i.e., \( \sin^2 \theta_{13} \) (\( \sin^2 \theta_{12}, \sin^2 \theta_{23} \)) with sum of neutrino mass for the best fit values of model parameters while grid-lines represent the 3\( \sigma \) range of mixing angles

\[
\sin^2 \theta_{13} = |U_{13}|^2, \quad \sin^2 \theta_{12} = \frac{|U_{12}|^2}{1 - |U_{13}|^2}, \\
\sin^2 \theta_{23} = \frac{|U_{23}|^2}{1 - |U_{13}|^2}.
\]

Other observables related to the mixing angles and phases of PMNS matrix are

\[
J_{CP} = \text{Im}[U_{ei}U_{ej}^*U_{ej}^*\mu_1^*], \quad \langle m_{ee} \rangle = |m_{\nu_1}|^2 \theta_{12} \cos^2 \theta_{13} + m_{\nu_2}^2 \sin^2 \theta_{12} \cos^2 \theta_{13} \cos \alpha_{21} \\
+ m_{\nu_3}^2 \sin^2 \theta_{13} \sin \alpha_{31} \sin \alpha_{32} \sin \alpha_{31} - 2 \alpha_{CP}||. 
\]

The effective Majorana mass parameter \( \langle m_{ee} \rangle \) is expected to have improved sensitivity measured by KamLAND-Zen experiment in coming future [94]. Further, we chose the following model parameter ranges to fit the present neutrino oscillation data:

\[
\text{Re}[\tau] \in [-0.5, 0.5], \quad \text{Im}[\tau] \in [0.75, 2], \\
M_\Sigma^G \in [10^4, 10^9] \text{ eV}, \quad v_\rho \in [10^3, 10^5] \text{ eV}, \\
\Lambda \in [10^4, 10^5] \text{ eV}, \quad G_D \in [10^{-8}, 10^{-5}], \\
\beta_\Sigma \in [10^{-5}, 10^{-1}], \quad \gamma_\Sigma \in [10^{-9}, 10^{-10}].
\]

We consider the free mass parameter (\( M_\Sigma^G \)), real and imaginary part of \( \tau \), VEV of weighton (\( v_\rho \)) and cut-off parameter (\( \Lambda \)) randomly in the range given in Eq. (13). The range of \( \tau \) is taken to be \([-0.5, 0.5]\) for the real part and \([0.75, 2]\) for the imaginary part, which provides the validity of model to follow normal hierarchy (NH). Considering these ranges, we arbitrarily scrutinise the input values of parameters and extract the best-fit values of those by applying chi-square minimization technique. The approach followed here by considering the general chi-square formula [95,96], which is utilized for calculating the \( \chi^2 \) values for all the available observables of the neutrino sector, like two mass squared differences and three mixing angles, further yielding cumulative \( \chi^2 \) minimum allowing us to get the values of the free parameters corresponding to the minimum i.e., best-fit values [87,97]. As there are a large number of free parameters involved in this framework, i.e., total number of free parameters are much larger than the number of observed neutrino oscillation parameters, it is not possible to get a constrained a correlated plot. Therefore, we calculate minimal chi-square and the the associated values of free parameters are considered as the best-fit values of the free parameters. Hence, Table 4 is obtained by keeping the experimentally observed oscillation parameters along with the cosmological bound for sum of neutrino masses \( \sum m_\nu \leq 0.12 \text{ eV} \) [98]. We have not mentioned the best-fit values of \( \gamma_\Sigma \) here in the Table 4, since it gives negligible contribution to the observables as compared to \( G_D \) and \( \beta_\Sigma \), hence, conventionally we deal with total six free parameters. As a consequence, the left panel of Fig. 1 projects the correlation between \( \sin^2 \theta_{13} \) w.r.t. \( \sum m_\nu \), where, the sum of neutrino mass is above its lower bound i.e., 0.058 eV [99], while the right panel shows the interdependence of \( \sum m_\nu \) with \( \sin^2 \theta_{12}, \sin^2 \theta_{23} \) with grid-lines showing their respective 3\( \sigma \) ranges. Moreover, in Fig. 2 the left panel shows an interdependence of \( \sin^2 \theta_{13} \) with Jarlskog invariant \( |J_{CP}| \) whose value is constrained to be \( |J_{CP}| \leq 7.1 \times 10^{-3} \). As can be seen in the plot of \( \delta_{CP} \) against \( \sin^2 \theta_{13} \) on the right panel of Fig. 2, \( \delta_{CP} \) is varying.
in the range [202°–211°] while constrained by 3σ bound of $\sin^2 \theta_{13}$.

The process of neutrinoless double beta decay (NDBD) involves the simultaneous conversion of two neutrons into two protons and two electrons without any emission of neutrinos [100–103], as shown in the left panel of Fig. 3. In the presence of new heavy neutral fermions, the additional contribution to the NDBD is linked to the mixing between active and heavy neutrinos and is expected to be rather small. The mixing of active and sterile neutrinos is generally described by the parameters $\Theta_{ai}$ which plays a crucial role [104] in the description of neutrinoless double beta decay. Thus, the $6 \times 6$ neutrino mass matrix takes the form

$$\hat{M} = \begin{pmatrix} 0 & M_D \\ M_D^T & M_\Sigma \end{pmatrix}. \quad (14)$$

We can diagonalize it by using the unitary matrix $\hat{U}$ as $\hat{U}^\dagger \hat{M} \hat{U}^* = \hat{M}^{\text{diag}}$. The seesaw mechanism shows that $\hat{U}$ at the leading order takes the form [104]

$$\hat{U} = \begin{pmatrix} U & \Theta \\ -\Theta^\dagger U & 1 \end{pmatrix}. \quad (15)$$

where, $U$ is the PMNS matrix, diagonalizing the light active neutrino mass matrix as

$$U^\dagger M_\nu U^* = \text{diag}(m_1, m_2, m_3), \quad (16)$$

with $M_\nu = -M_D M_\Sigma^{-1} M_D^T$ as the light neutrino mass matrix obtained from Type-III seesaw. The eigenstates related to masses $m_i$ and $M_\Sigma$ are $\nu_i$ and $\Sigma_R$. The neutrino mixing in the charged current is then induced through

$$\nu_{Li} = U_{ai} \nu_i + \Theta_{ai} \Sigma_{Ri} \quad (17)$$

where, the $3 \times 3$ mixing matrix $\Theta$ is found to at the leading order as

$$\Theta_{ai} = \frac{[M_D]_{ai}}{M_\Sigma} \quad (18)$$

The vertex coupling, thus given as $V^{\nu\Sigma} = \frac{1}{v_s} M_D U^{-1} M_\nu^{-1}$ [105].

The right panel of Fig. 3 represents the Feynman diagram due the exchange of the heavy neutrinos $\Sigma_R$, consisting all the relevant vertex couplings [105] whose relevance is seen in numerical deductions, due to which the effective mass parameter $\langle m_{ee} \rangle$ receives additional contribution. We showcase the
Fig. 4 Upper left panel shows the correlation in the plane of effective mass parameter $\langle m_{ee} \rangle$ and $\sum m_{\nu}$, whereas, upper right panel projects the correlation of $\langle m_{ee} \rangle$ and with lightest neutrino mass $m_{\text{min}}$ (i.e., $m_{\text{min}}$). Lower panel shows the correlation Majorana phases $\alpha_{21}$ and $\alpha_{31}$.

results in Fig. 4, wherein the upper left (right) panel reflects the behaviour of $\langle m_{ee} \rangle$ \[106,107\] w.r.t. sum of neutrino mass ($\sum m_{\nu}$) (lightest neutrino mass $m_{\text{min}}$) \[108,109\] abiding the KamLAND-Zen bound \[110\] and the bottom panel shows the correlation between Majorana phases i.e., ($\alpha_{21}$ and $\alpha_{31}$).

Figure 5 shows the dependence of Yukawa couplings on the real and imaginary parts of $\tau$, while keeping the model parameters at their best-fit values. Finally, in Fig. 6 we show the hierarchical nature of the heavy neutrinos which follow the pattern $M_{\Sigma R_1} \ll M_{\Sigma R_2} \ll M_{\Sigma R_3}$.

4 Leptogenesis

Considering the fact that the universe had started from an initially symmetric state of baryons and antibaryons, the present baryon asymmetry can be explained, as suggested by Sakharov \[111\], if the following three criteria are satisfied: Baryon number violation, C and CP violation and departure from thermal equilibrium during the evolution of the universe. Though the SM assures all these criteria for an expanding universe akin ours, the extent of CP violation found in the SM is quite small to accommodate the observed baryon asymmetry of the universe. Therefore, additional sources of CP violation are absolutely essential for explaining this asymmetry. The most common new sources of CP violation possibly could arise in the lepton sector, which is however, not yet firmly established experimentally. Leptogenesis is the phenomenon that furnishes a minimal setup to correlate the CP violation in the lepton sector to the observed baryon asymmetry, as well as imposes indirect constraints on the CP phases from the requirement that it would yield the correct baryon asymmetry. In here, we explore leptogenesis in type-III seesaw model with fermion triplets, where, the lightest heavy fermion is in TeV scale. The general expression for CP asymmetry is mentioned below \[112\]

$$\epsilon_{CP} = -\sum_{j} \frac{\Gamma_{\Sigma R_j}}{2 M_{\Sigma R_j}} \left( \frac{V_j - 2S_j}{3} \right) \sqrt{\text{Im}(\tilde{Y}_\Sigma \tilde{Y}_\Sigma^\dagger \text{ij})^2 \frac{\tilde{\Sigma}_j}{(\tilde{Y}_\Sigma \tilde{Y}_\Sigma^\dagger \text{ij})(\tilde{Y}_\Sigma \tilde{Y}_\Sigma^\dagger \text{jj})}}$$, with $\tilde{Y}_\Sigma = Y_\Sigma U_R$. \[19\]
where, $Y_\Sigma = (M_D/v_u)$ is the Yukawa matrix of Dirac mass term with its corresponding free parameters given in Eq. (5) and $U_R$ being the eigenvector matrix of $M_R$ used for its diagonalization i.e., $U_R M_R U_R^T \simeq \text{diag}(M_{\Sigma R_1}, M_{\Sigma R_2}, M_{\Sigma R_3})$. From Eq. (26) it is evident that vertex ($V_j$) and self-energy ($S_j$) diagrams [112] must contribute to CP asymmetry significantly. However, in the hierarchical limit (i.e., $M_{\Sigma R_1} \ll M_{\Sigma R_2,3}$ and $M_{\Sigma R_2} \neq M_{\Sigma R_3}$) they attain the value unity i.e., ($S_j = V_j = 1$). As, we don’t have the hold on fine tuning of the Yukawa couplings, in order to calculate correct lepton asymmetry, we utilize the following benchmark values as shown in Table 5. Moreover, we also show in Fig. 7 the correlation between the one flavor CP asymmetry i.e., $\epsilon_{CP} \mathcal{O}(10^{-4})^1$ with the Yukawa couplings within their corresponding ranges i.e., $0.99 \lesssim y_1 \lesssim 1.015$ (upper left panel),

Fig. 5 Left (right) panel shows the correlation of the Yukawa couplings i.e., ($y_1, y_2, y_3$) w.r.t. $\text{Re}[\tau]$ ($\text{Im}[\tau]$), where $y_1, y_2$ and $y_3$, are shown in red, blue and green colours respectively.

![Fig. 5](image1)

![Fig. 6](image2)

Fig. 6 The left (right) plots show the correlation between the heavy neutrino masses i.e., $M_{\Sigma R_1}$ and $M_{\Sigma R_2}$ ($M_{\Sigma R_2}$ and $M_{\Sigma R_3}$) in TeV scale.

Table 5 Benchmark values of the Yukawa couplings and CP asymmetry utilized to generate the correct lepton asymmetry

| $y_1$ | $y_2$ | $y_3$ | $M_{\Sigma R_1}$ | $\epsilon_{CP}$ |
|-------|-------|-------|-----------------|-----------------|
| 1.0051 | 0.574 | 0.312 | $6.53 \times 10^3$ GeV | $9.713 \times 10^{-4}$ |

0.4 $\lesssim y_2 \lesssim 1.3$ (upper right panel) and 0.1 $\lesssim y_3 \lesssim 0.8$ (bottom panel).

4.1 Boltzmann equations

The dynamics of applicable Boltzmann equations determine the evolution of particle number densities. The Sakharov conditions [111] necessitate the decay of the parent heavy fermion, which must be out of equilibrium in order to generate the lepton asymmetry. To do so, one must compare the Hubble expansion rate to the decay rate, as shown below:

$$K_{\Sigma R_i} = \frac{\Gamma_{\Sigma R_i}}{H(T = M_{\Sigma R_i})}.$$  (20)
Fig. 7 In above we show the correlation of the Yukawa couplings i.e. \((y_1, y_2, y_3)\) w.r.t CP asymmetry i.e., \(\epsilon_{\text{CP}}\).

Fig. 8 Left panel exhibits the comparison of interaction rates with Hubble expansion rate \((H)\) represented by red solid line, where, green solid line corresponds to decay \(\left(\Gamma_D = \Gamma \frac{\chi_1}{\chi_{2c}^2}\right)\) [118], inverse decay \((\Gamma_{I0} = \Gamma_D (\chi_{2c}^2/\chi_{1c}^2))\) [119] is shown by dotted orange line and annihilation rate \((\Gamma_A)\) by dotted purple line. Right panel projects the evolution of \(Y_{B-L}\) (green solid line) as a function of \(z = \frac{M_{X_1}}{f}\).
The Hubble rate is defined as $H = \frac{1.67 \sqrt{\frac{g_*}{M_{Pl}}} T^2}{s}$, where, $g_* = 106.75$ is the number of relativistic degrees of freedom in the thermal bath and $M_{Pl} = 1.22 \times 10^{19}$ GeV is the Planck mass. The size of the couplings between the triplet fermions and leptons become the determining factor, guaranteeing that inverse decay does not approach thermal equilibrium. For example, if the value is less than or equal to $10^{-7}$, it gives $K_{\Sigma_{Rj}} \sim 1$. The Boltzmann equations associated with evolution of the number densities of right-handed fermion field and lepton can be articulated in terms of the yield parameters, i.e., the ratio of number densities to entropy density, and are expressed as [113–116]

$$
\frac{dY_\Sigma}{dz} = -\frac{\varepsilon}{sH(M\Sigma)} \left( \begin{array}{c} Y_\Sigma \gamma_{\Sigma - 1} - 1 \gamma_D + \left( \frac{Y_\Sigma}{\gamma_{\Sigma}} - 1 \right) \gamma_A \end{array} \right),
$$

$$
\frac{dY_{\ell - L}}{dz} = -\frac{\varepsilon}{sH(M\Sigma)} \left( \begin{array}{c} Y_{\ell - L} \gamma_{\ell - 1} - \varepsilon_{CP} \frac{Y_\Sigma}{\gamma_{\Sigma}} - 1 \gamma_p \end{array} \right),
$$

(21)

where $\varepsilon = M_{\Sigma_{Rj}} / T$, $s$ is the entropy density, and the equilibrium number densities have the form [88]

$$
Y_\Sigma^{eq} = \frac{135 g_\Sigma}{16 \pi^2 g_*} \frac{z^2 K_2(z)}{K_1(z)}; \quad Y_\ell^{eq} = \frac{345 \xi(3)}{4 \pi^2 g_*}. \quad (22)
$$

$K_{1,2}$ in Eq. (22) represent the modified Bessel functions, the lepton and RH fermion degrees of freedom take the values $g_\ell = 2$ and $g_{\Sigma_{Rj}} = 2$ and the decay rate $\gamma_{D}$ is given as

$$
\gamma_{D} = s Y_\ell^{eq} \Gamma_\Sigma K_1(z); \quad \Gamma_\Sigma = \frac{1}{8 \pi} M_{\Sigma_{Rj}} (\tilde{\gamma}_\Sigma \tilde{\gamma}_\Sigma)_{ii},
$$

$$
\gamma_A = \frac{M_{\Sigma_{Rj}} T^3}{32 \pi^3} e^{-2z} \left[ \frac{111 g^4}{8 \pi} + \frac{3}{2 \pi} \left( \frac{111 g^4}{8 \pi} + \frac{51 g^4}{16 \pi} \right) + O(1/z^2) \right],
$$

(23)

wherein $\gamma_A$ process [112–117], with $g$ being the typical gauge coupling. The comparison of the interaction rates with Hubble expansion rate ($H$) is displayed in the left panel of Fig. 8, while the solution of Boltzmann Eq. (21) is presented in the right panel. For coupling strength of around ($\simeq 10^{-7}$), $Y_\Sigma$ (magenta solid curve) with $|Y_\Sigma - Y_\Sigma^{eq}|$ (blue dashed curve) are shown where the charged lepton asymmetry is around ($\simeq 10^{-10}$) (green thick curve). The lepton asymmetry thus obtained can be converted into baryon asymmetry through the sphaleron transition process, and is given as [113, 120, 121]

$$
Y_B = 3 \left( \frac{5 n_f + 4 n_H}{22 n_f + 13 n_H} \right) Y_{B - L},
$$

(24)

where, $n_f$ represents the number of triplet fermion generations, $n_H$ denotes the no. of Higgs doublets and the factor of 3 comes from the three $SU(2)_L$ degrees of freedom of the triplets. The observed baryon asymmetry of the universe generally expressed in terms of baryon to photon ratio as [98]

$$
\eta = \frac{n_B - n_\gamma}{n_\gamma} = 6.08 \times 10^{-10}.
$$

(25)

The current bound on baryon asymmetry [122] can be procured from the relation $Y_B = \eta / 7.04$ as $Y_B = (8.6 \pm 0.1) \times 10^{-11} \equiv Y_B^{obs}$. Using the asymptotic value of the lepton asymmetry as $(8.77 \times 10^{-10})$ from Fig. 8, we obtain the value of baryon asymmetry as $Y_B = \frac{24}{23} Y_{B - L} \sim 10^{-10}$.

4.2 A note on flavor consideration

When ($T > 10^{12}$ GeV), one flavor approximation suffices in leptogenesis, indicating that all Yukawa interactions are out of equilibrium. However, at temperatures $\ll 10^6$ GeV, various charged lepton Yukawa couplings (i.e., each for three generations) come into equilibrium, making flavor effects a crucial factor in determining the final lepton asymmetry. All Yukawa interactions occur in equilibrium at temperatures below $10^6$ GeV, and the asymmetry is encoded in the individual lepton flavor. Numerous studies on flavor effects in type-I leptogenesis can be found in the literature [123–128]. The lower bounds on heavy Majorana masses are relaxed when flavour effects are taken into account, giving more room for low scale leptogenesis [129–131]. Given the significance of flavour effects in low scale leptogenesis, we briefly examine their implications in the current framework in relation to the CP asymmetry for each particular lepton flavour $(\alpha = e, \mu, \tau)$ given below [132, 133]

$$
\epsilon_{\Sigma}^\alpha = \frac{1}{2} \sum_j \frac{M_{\Sigma_{Rj}} \Gamma_{\Sigma_{ij}}}{M_{\Sigma_{Rj}} \Sigma_{Rj}} \left[ \Im \left( \tilde{Y}_\Sigma \tilde{Y}_\Sigma^\dagger \right)_{ij} \tilde{Y}_{\alpha j} \tilde{Y}_{\alpha j} \right],
$$

(26)
The Boltzmann equation describing the generation of \((B-L)\) asymmetry for each lepton flavor is \([124]\)

\[
\frac{dY_{B-L}^{\alpha}}{dz} = -\frac{\varepsilon}{sH(M_\Sigma)} \times \left[ \epsilon_\Sigma^{\alpha} \left( \frac{Y_\Sigma^{\alpha}}{Y_\Sigma^B} - 1 \right) - \left( \frac{Y_D^{\alpha}}{2} - \frac{A_{\alpha\alpha}Y_D^{\mu} - L_{\alpha\mu}}{Y_\ell^{\alpha}} \right) \right]. \tag{27}
\]

where, \(\epsilon_\Sigma^{\alpha}\) i.e., \((\alpha = e, \mu, \tau)\) represents the CP asymmetry in each lepton flavor

\[
\gamma_D^{\alpha} = sY_\Sigma^{\alpha} \Gamma_\Sigma^{\alpha} \frac{K_1(z)}{K_2(z)}, \quad \gamma_D = \sum_{\alpha} \gamma_D^{\alpha}. \tag{28}
\]

The matrix \(A\) is given by \([125]\),

\[
A = \begin{pmatrix}
-\frac{21}{16} & \frac{16}{7} & \frac{16}{7} \\
\frac{16}{7} & -\frac{21}{16} & \frac{16}{7} \\
\frac{16}{7} & \frac{16}{7} & -\frac{21}{16}
\end{pmatrix}. \tag{29}
\]

In addition to which we have expressed a plot to show the interdependence of each flavor on \(\delta_{CP}\) in Fig. 9. Subsequently, for the flavor case, benchmark values of CP asymmetry associated with \((e, \mu, \tau)\) flavors are \(\epsilon_\Sigma^e = 4.7 \times 10^{-4}\), \(\epsilon_\Sigma^\mu = 5.6 \times 10^{-4}\) and \(\epsilon_\Sigma^\tau = 7.2 \times 10^{-4}\) respectively. Therefore, we estimate the \(B-L\) yield with flavor consideration in the left panel of Fig. 10. It is quite obvious to notice that the enhancement in \(B-L\) asymmetry is obtained in case of flavor consideration (green dashed line) over the one flavor approximation (orange solid line), as displayed in the right panel. This is because, in one flavor approximation the decay of the heavy fermion to a particular lepton flavor final state can get washed away by the inverse decays of any flavor unlike the flavored case \([126]\).

5 Collider bound on \(Z'\) mass

As previously mentioned in Sect. 2, the \(U(1)_{B-L}\) gauge symmetry is spontaneously broken by assigning the vacuum expectation value \(v_\rho\) to the singlet scalar \(\rho\). Consequently, the neutral gauge boson \(Z'\) associated with this symmetry becomes massive by absorbing the massless pseudoscalar component of \(\rho\) and its mass is given as,

\[
M_{Z'} = g_{BL}v_\rho. \tag{30}
\]

where, \(g_{BL}\) is the gauge coupling constant of \(U(1)_{B-L}\). The LEP-II provides the constraint on the ratio of mass of \(Z'\) boson to its coupling as \(M_{Z'}/g_{BL} > 6.9\) TeV \([134]\). Hence, in this work we have considered the range of the \(v_\rho\) as \([10^3 - 10^4]\) TeV \((13)\), consistent with the LEP-II bound.

The ATLAS and CMS collaborations have performed extensive searches for the new resonances in both dilepton and dijet channels. In the absence of any excess events over the SM background, they put lower bounds on the mass of \(Z'\) boson. These bounds are usually limited to a specific model, and typically the experiments report their results assuming simplified models, like the Sequential Standard Model (SSM) or GUT-inspired \(E_6\) models.

Recent results from ATLAS \([135]\), provide the lower limits on the \(Z'\) mass from the dilepton search using Run 2 data, collected with the center of mass energy \(\sqrt{s} = 13\) TeV. In this work, we use CalcHEP \([136]\) to compute the production cross section of \(Z'\), i.e., \(pp \rightarrow Z' \rightarrow ee(\mu\mu)\). In Fig. 11, we show the \(Z'\) production cross section times the branching fraction of \(Z'\) decaying to dilepton \((ee, \mu\mu)\) signal as a function of \(M_{Z'}\), for some representative values of the gauge coupling \(g_{BL}\) = 0.03, 0.08, 0.11. The black dashed line denotes the dilepton bound from ATLAS \([135]\). It can be noticed from the figure that the region below \(M_{Z'} \approx 1.3\) TeV is excluded for \(g_{BL} = 0.03\) in red color. For \(g_{BL} = 0.08, M_{Z'} < 2.47\) TeV.
in blue color is ruled out and the mass region of $M_{Z'} > 2.69$ TeV is allowed for $g_{BL} = 0.11$ in orange color. Thus, one can generalize these observations as the lower limits on $M_{Z'}$ increases with the increase of the gauge couplings.

6 Conclusion

We have curated a model involving $A_4$ modular symmetry and $U(1)_{B-L}$ gauged symmetry using type-III seesaw mechanism in super-symmetric context in order to realize the neutrino phenomenology and to explain the observed oscillation data. We have incorporated $SU(2)_L$ triplet fermions ($\Sigma$) along with a singlet weighton field ($\rho$). The Yukawa couplings acquire modular forms under $A_4$ modular discrete symmetry, where, acquisition of VEV by modulus $\tau$ breaks $A_4$ symmetry. This discrete symmetry is useful in procuring a definite neutrino mass matrix structure. Here, in analysis section numerical diagonalization technique lessens the burden and the predicted results are in accordance to the $3\sigma$ bound as obtained through several experiments. We can extract the best fit values for the model parameters using the Chi-square minimization approach, which helps us find strong correlations between the observables. As a consequence, we obtain the sum of active neutrino masses $\sum m_{\nu_i}$ within $[0.058 - 0.12]$ eV and mixing angles are seen to be within their $3\sigma$ ranges. The model engenders neutrinoless double beta decay mass parameter ($m_{ee}$) between 0.0039 and 0.0087, which assures the limit coming from KamLAND-Zen experiment. Also, Majorana phases $\alpha_{21}$ and $\alpha_{31}$ are revealed in the range $[0^\circ, 80^\circ]$ and $[0^\circ, 360^\circ]$ respectively. Proceeding further, the results for $\delta_{CP}$ and Jarlskog invariant $J_{CP}$ is seen to be within $[202^\circ, 211^\circ]$ and $[5.5, 7.1] \times 10^{-3}$ respectively establishing a strong correlation. Further, as there is an hierarchical mass difference between the heavy fermions ($M_{\Sigma}$) in the model with $M_{\Sigma R_1}, M_{\Sigma R_2}$ and $M_{\Sigma R_3}$ are found to be within the range $[6 - 8.5]$ TeV, $[50 - 110]$ TeV and $[2000 - 4500]$ TeV respectively, hence, the decay of lightest one gives rise to non-zero CP asymmetry. The lepton asymmetry coming from Boltzmann equation is $\simeq 10^{-10}$, and hence explains the baryon asymmetry of the Universe and also we have discussed the flavor effects as our lightest heavy fermion is in TeV scale. Additionally, we have discussed the mass of the new neutral $Z'$ gauge boson associated with $U(1)_{B-L}$ symmetry which is within the present experimental collider bounds.

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Appendix A: $A_4$ modular symmetry

$A_4$ group is the alternating group of even permutations of four entries. It is isomorphic to the tetrahedral symmetry. The generators of the group $S$ and $T$, following the relations,

\[ S^2 = (TS)^3 = (ST)^3 = 1. \]  

(A1)

Group formed by the generators $S$ and $T$ is the inhomogeneous modular group $\Gamma$ and the transformations are abbreviated as follows [59,60]

\[ S : \tau \rightarrow \frac{-1}{\tau}, \quad T : \tau \rightarrow \tau + 1 \]  

(A2)
Representation of $S$ and $T$ in the $\text{SL}(2,\mathbb{Z})$ group is,

$$S = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}, \quad T = \begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix}. \quad (A3)$$

A group of linear fractional transformations forms the modular group, which transforms the modulus $\tau$ in the upper half-plane $[\text{Im}(\tau) > 0]$,

$$\tau \rightarrow \frac{c\tau + d}{a\tau + b}, \quad (a, b, c, d \text{ are integers, } cb - da = 1) \quad (A4)$$

and the mapping,

$$\frac{c\tau + d}{a\tau + b} \rightarrow \left(\begin{array}{cc} c & d \\ a & b \end{array}\right), \quad (A5)$$

is an isomorphism from the modular group. Following is the series of groups $\Gamma(N)$, where $N = 1, 2, 3, \ldots$,

$$\Gamma(N) = \left\{ \left(\begin{array}{cc} c & d \\ a & b \end{array}\right) \in \text{SL}(2,\mathbb{Z}), \quad \left(\begin{array}{cc} c & d \\ a & b \end{array}\right) = \left(\begin{array}{cc} 1 & 0 \\ 0 & 1 \end{array}\right) \right\}. \quad (A6)$$

where, $\Gamma = \text{SL}(2,\mathbb{Z})$ is homogeneous modular group and $\Gamma(N)$. The group $\Gamma(N)$ operates on the complex modulus $\tau$, in the upper half plane as the linear fractional transformation,

$$\gamma \rightarrow \frac{c\tau + d}{a\tau + b}. \quad (A7)$$

A significant modular invariant element is the modular function $f(\tau)$ which is holomorphic function of $\tau$ with level $N$ and modular weight $2k$, under $\Gamma(N)$ is,

$$f\left(\frac{c\tau + d}{a\tau + b}\right) = (a\tau + b)^{2k}f(\tau), \quad \forall \left(\begin{array}{cc} c & d \\ a & b \end{array}\right) \in \Gamma(N). \quad (A8)$$

Here, $N$ can vary according to the symmetry group $A_4$, $S_3$, $S_4$, or $A_5$. In reference [60], it is given that for $N = 2, 3, 4,$ and $5$; $\Gamma_2, \Gamma_3, \Gamma_4$ and $\Gamma_5$ are isomorphic to $S_3, A_4, S_4$, and $A_5$ respectively. The modular forms of $A_4$ triplet Yukawa couplings read as,

$$Y_1(\tau) = \frac{i}{2\pi} \left[ \eta\left(\frac{\tau}{2}\right) + \eta\left(\frac{\tau+1}{2}\right) + \eta\left(\frac{\tau+2}{2}\right) - 27\eta(3\tau) \right] \quad (A9)$$

$$Y_2(\tau) = \frac{-i}{\pi} \left[ \eta\left(\frac{\tau}{2}\right) + \omega^2 \eta\left(\frac{\tau+1}{2}\right) + \omega^4 \eta\left(\frac{\tau+2}{2}\right) \right] \quad (A10)$$

where $\eta(\tau)$ is Dedekind eta-function which can be defined in the upper half plane of the complex plane.

$$\eta(\tau) = q^{1/24} \prod_{m=1}^{\infty} (1 - q^m) \quad q = e^{i\pi \tau} \quad (A12)$$

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