Gate-Tunable Lifshitz Transition of Fermi Arcs and Its Transport Signatures

Yue Zheng(郑悦)¹, Wei Chen(陈伟)¹,²*, Xiangang Wan(万贤纲)¹,², and D. Y. Xing(邢定钰)¹,²

¹National Laboratory of Solid State Microstructures and School of Physics, Nanjing University, Nanjing 210093, China
²Collaborative Innovation Center of Advanced Microstructures, Nanjing University, Nanjing 210093, China

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One hallmark of Weyl semimetals is the emergence of Fermi arcs (FAs) in surface Brillouin zones, where FAs connect the projected Weyl nodes of opposite chiralities. Unclosed FAs can give rise to various exotic effects that have attracted tremendous research interest. Configurations of FAs are usually thought to be determined fully by the band topology of the bulk states, which seems impossible to manipulate. Here, we show that FAs can be simply modified by a surface gate voltage. Because the penetration length of the surface states depends on the in-plane momentum, a surface gate voltage induces an effective energy dispersion. As a result, a continuous deformation of the surface band can be implemented by tuning the surface gate voltage. In particular, as the saddle point of the surface band meets the Fermi energy, the topological Lifshitz transition takes place for the FAs, during which the Weyl nodes switch their partners connected by the FAs. Accordingly, the magnetic Weyl orbits composed of the FAs on opposite surfaces and chiral Landau bands inside the bulk change their configurations. We show that such an effect can be probed by the transport measurements in a magnetic field, in which the switch-on and switch-off conductances by the surface gate voltage signal the Lifshitz transition. Our work opens a new route for manipulating the FAs by surface gates and exploring novel transport phenomena associated with the topological Lifshitz transition.

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In the last two decades, research on novel topological materials has seen rapid progress, involving the discoveries of topological insulators[1–3] and topological semimetals.[4–10] The latter ones possess gapless energy spectra but nontrivial band topology, which can give rise to interesting effects stemming from both bulk and surface states. According to features of band degeneracies, topological semimetals can be further classified into several types, including Weyl semimetals (WSMs), Dirac semimetals,[4–6] and nodal-line semimetals.[10–12] As counterparts of massless Weyl and Dirac fermions in condensed matter physics, quasiparticles with linear dispersion in WSMs and Dirac semimetals provide an interesting platform for exploring novel effects predicted by high-energy physics.[13–24] These effects are manifested as anomalous transport and optical properties which can be probed using a standard approach of condensed matter physics.[25–34]

Nontrivial band topologies of WSMs are embodied in monopole charges (or Chern numbers of the Berry curvature field) carried by Weyl nodes. According to the no-go theorem,[35,36] Weyl nodes of opposite chiralities must appear in pairs. The manifestation of nontrivial band topology of a WSM is the unclosed Fermi arcs (FAs) spanning between Weyl nodes of opposite chiralities projected into the surface Brillouin zone. The emergence of FAs is a unique property of WSMs, without any counterpart in high-energy physics, which can not only serve as the hallmark of WSMs,[13–23,25,26] but also lead to several novel phenomena.[14,24–51] Given that FAs are the Fermi surface of topological surface states, one may think that all their properties, especially how they connect pairs of Weyl nodes, are completely determined by the band topology of the bulk states through the bulk-boundary correspondence. Therefore, it seems that the only way to modify configurations of FAs is to change bulk properties of WSMs.

Interestingly, recent research progress shows that configurations of FAs are quite sensitive to details of sample boundary,[52–54] which opens the possibility for manipulating the FAs through surface modifications. In particular, the topological Lifshitz transition[55] of FAs can be induced by surface decoration[50] or chemical potential modification,[54] which changes the sizes and shapes of the FAs, and especially, the way they connect pairs of Weyl nodes. The existing experiments show that FAs with different configurations can be realized in different samples,[52–54,56–59] but whether it is possible to continuously modify the FAs in a given sample remains an open question. It is of great interest to explore the possibility of manipulating FAs by external fields, in which both continuous deformation and abrupt Lifshitz transition of the FAs can be achieved.

In this Letter, we show that FAs of WSMs can be continuously tuned by a surface gate voltage. Because the penetration length of the surface state depends on the in-plane momentum, the surface gate voltage acts unequally on these surface modes and induces a momentum-dependent potential energy, or effectively, an additional dispersion of the surface band.[57] As a result, a continuous deformation of the surface band and then the FAs can be achieved by simply tuning the gate voltage. It is shown that the existence of the saddle point in the surface

*Corresponding author. Email: pchenweis@gmail.com
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band is responsible for the topological Lifshitz transition of the FAs. Specifically, the transition takes place when the saddle point coincides with the Fermi energy. At the same time, the Weyl nodes switch their partners that are connected by the FAs. A direct physical result is that the magnetic Weyl orbits composed of the FAs on opposite surfaces and the chiral Landau bands inside the bulk change their configurations. We show that such an effect can be probed by the transport measurements in a magnetic field, in which the current can be switched on and off by the gate voltage, thus providing a clear signature of the gate-voltage induced Lifshitz transition. Moreover, we provide some remarks on the experimental implementation of our proposal. Our work not only uncovers the scenario of the Lifshitz transition of FAs but also paves the way for continuous manipulation by a surface gate voltage.

Lifshitz Transition Induced by Surface Gate. We start with an effective model of WSMs with four Weyl nodes, \[ H(k) = M_1(k_x^2 - k_y^2)\sigma_x + v_y k_y \sigma_y + M_2(k_y^2 - k_y^2)\sigma_z, \] \[ \text{where} \quad v_y \text{ is the velocity in the y direction, } k_{0,1} \text{ and } M_{1,2} \text{ are parameters, } \sigma_{x,y,z} \text{ are the Pauli matrices that operate on the pseudo-spin (orbital) space. The two bands are degenerate at four Weyl nodes } k_w = (\pm k_0, 0, \pm k_0) \text{ with two FAs spanning between them respectively. Expanding } H(k) \text{ around the Weyl points yields four Weyl equations } h(k) = 2M_1k_x k_z \sigma_x + v_y k_y \sigma_y \pm 2M_2 k_y k_z \sigma_z. \]

We are interested in the FA surface states, which can be solved under the open boundary condition in the y direction. Consider a semi-infinite WSM that occupies the \( y \)-direction. The edge states \( \phi \) for the eigenvalue \( H(k_y) - \varepsilon \) \( \psi(k_\parallel, y, k_z) = 0 \) under the open boundary condition \( \psi(y = 0) = 0 \), we obtain the dispersion and wave function of the surface state as \[ \varepsilon_0 = M_1(k_0^2 - k_z^2), \] \[ \psi(k) = f_{k_z}(y) e^{i k_x x + i k_z z} \begin{pmatrix} a \\ b \end{pmatrix}, \] \[ M_2 > 0, \quad f_{k_z}(y) = \eta(e^{\lambda_1 y} - e^{\lambda_2 y}), \] \[ \eta = \sqrt{\frac{2M_2(k_0^2 - k_z^2)}{1 - 4M_2^2(k_0^2 - k_z^2)}}, \] \[ \lambda_{1,2} = -\frac{1}{2M_2} \pm \sqrt{\frac{1}{4M_2^2} + (k_0^2 - k_z^2)}, \] where \( f_{k_z}(y) \) being the spatial distribution function and \( \eta \) the normalization coefficient.

Here, the key point is that the wave function \( \psi(k) \) of the surface state relies on the in-plane momentum \( k_\parallel = (k_x, k_z) \). Specifically, the wave function for different \( k_z \) possesses unequal spatial spreading in the y direction as shown in Fig. 1(b). This property opens a new route for manipulating FA surface states by a surface potential, which can be induced by a surface gate voltage. Consider an electric potential imposed on the surface of the WSM, which can be captured by \( U_s(y) = U \delta(y - L_0) \). It induces an energy shift of the surface states, which can be evaluated by the overlap integral

\[ \delta \varepsilon = \int_0^\infty U \delta(y - L_0)|f_{k_z}(y)|^2 dy. \]

The energy shift has a \( k_z \) dependence and can be expanded as \( \delta \varepsilon \approx (-\kappa_2^2 + B) \varepsilon_0 \). For the second order of \( k_z \), we have \( A, B > 0 \) for \( M_1, M_2 > 0 \). Physically, such potential energy induces an effective surface dispersion, thus the full energy of the surface states becomes

\[ \varepsilon(k_z) = \varepsilon_0 + \delta \varepsilon \approx M_1(k_0^2 - k_z^2) + (-\kappa_2^2 + B) \varepsilon_0, \]

\[ A = 4\varepsilon_0 \frac{2 \eta}{\sinh(\xi L_0)} + 2M_2 k_0^2 \eta \sinh(\xi L_0), \]

\[ B = 4\varepsilon_0 \frac{2 \eta}{\sinh(\xi L_0)} (1 - \cosh(\xi L_0)), \quad \xi = \sqrt{1 - 4\kappa_2^2}. \]

This means that the band shape of the surface states and then the FAs can be continuously tuned by the surface electric potential.

![Fig. 1.](attachment:image.png)

(a) Schematic of the electrostatic potential imposed on the top surface of WSM. (b) Dispersion of \( f_{k_z} \) along the y direction, the color of curves is corresponding to solid squares in (a) for different \( k_z \) channels, here \( k_y = k_0 \).

The interesting case occurs for \( U > 0 \), such that the coefficients before \( k_0^2 \) and \( k_z^2 \) have opposite signs. This means that the surface band bends towards opposite directions for \( k_x \) and \( k_z \) and becomes a hyperbolic paraboloid, see Figs. 2(e) and 2(f). The hyperbolic paraboloid surface band contains a saddle point, which is the key to understand the Lifshitz transition of the FAs. Setting the Fermi energy to zero, the FAs defined by the intersection curves between the surface band and Fermi surface undergo continuous modifications as the surface potential \( U \) increases. The critical point for the Lifshitz transition takes place when the saddle point of the surface band lies exactly at the Fermi energy [see Fig. 2(e)], where the two FAs cross each other [see Fig. 2(b)]. The critical value \( U_c \) for Lifshitz transition can be obtained by \( U_c = M_1 k_0^2 / B \). As \( U \) increases further, the saddle point is lifted above the Fermi energy and accordingly, the two FAs split again but change their way connecting the Weyl nodes [see Figs. 2(c) and 2(f)]. Moreover, with \( k_z \to k_0 \), the normalization factor \( \eta \to 0 \), ensuring the energy shift at the Weyl nodes to be zero. Here the Delta-function potential is chosen for...
simplicity. Adoption of a surface potential extended to a finite region will not change the result.

In addition to the analysis based on the continuous model, we investigate the effect of the surface potential on the FAs by the lattice model of Eq. (1). To study the boundary states, we consider the system with a slab geometry, namely, the size of the system is finite in a certain direction. By substituting \( k = x, y, z \rightarrow a \sin (k, a) \), \( k_y^2 \rightarrow 2a^{-2}(1 - \cos (k, a)) \) and performing the partial Fourier transformation in the \( y \) direction, we obtain

\[
H_{k_y} (l_y) = \sum_{l_y} \psi_{k_y} (l_y) \left[ M_l \left( \frac{2}{a^2} \cos k_x + k_y^2 - \frac{2}{a^2} \right) \sigma_x \right.
+ M_2 \left( 2 \sin k_y \cos k_x + k_y^2 - \frac{4}{a^2} \right) \sigma_y \psi_{k_y} (l_y)
\]

\[
H_{k_y} (l_y) = \sum_{l_y} \psi_{k_y} (l_y) \left[ \frac{v_0 \sigma_y}{2a} + \frac{M_2 \sigma_y}{a^2} \right] \psi_{k_y} (l_y + 1)
\]

\[
+ U \psi_{k_y} (l_y = 0) \psi_{k_y} (l_y = 0) + H.c.
\]  

with \( a \) being the lattice constant and \( \psi_{k_y} (l_y) = [\psi_{1,k_y} (l_y), \psi_{2,k_y} (l_y)] \) the two-component Fermi operator. \( L_y \) and \( i_y \) are the number and coordinate of sites along the \( y \) direction, respectively. An onsite potential \( U \) is introduced to the outmost layer (\( l_y = 0 \)) of the lattice. Solving the effective 1D chain in the \( y \) direction under the open boundary condition for each in-plane momentum \( k_y \) yields the surface states. The surface bands for different electrostatic potentials \( U \) are plotted in Figs. 2(d) and 2(e).

As is expected, a surface potential can induce a severe deformation of the surface band, and most importantly, a saddle point emerges. For our parameters, the critical value of \( U \) for Lifshitz transition is \( U_c \approx 0.297 \text{ eV} \). The FAs can be revealed by the spectral function \( \mathcal{A}(E, k_y) = -(1/\pi) \text{Im} \mathcal{G}(E) \) at the Fermi energy \( E = 0 \) with the surface Green function defined as

\[
\mathcal{G}^{\parallel}(E) = \sum_{n} \frac{u_n u_n^*}{E - E_n + \text{i} \delta},
\]

where \( u_n \) is the surface component (\( l_y = 0 \)) of the eigenvector corresponding to the \( n \)th eigenvalue \( E_n \). The configurations of FAs in Figs. 2(a)–2(c) are consistent with those of the surface bands in Figs. 2(d)–2(f).

We have established the relation between the topological Lifshitz transition of the FAs and the underlying saddle points in the surface band, the latter ones indicate the existence of the van Hove singularity in the density of states (DOS) of the surface band. Therefore, the Lifshitz transition of the FAs indicates that there exists the van Hove singularity at the Fermi energy. In Figs. 2(g)–2(i), we plot the DOS calculated by \( \text{DOS}(E) = \sum_{k_y} \mathcal{A}(E, k_y) \).

One can see that the energy of the van Hove singularity tracks that of the saddle point and equals the Fermi energy at the critical point of Lifshitz transition. Given that a variety of physical effects such as the enhancement of electron-electron interaction and the disturbance of many-body ground states are closely related with the van Hove singularity of the band, our work opens the possibility to realize interesting effects in the surface states of the

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**Fig. 2.** (a)-(c) Fermi arcs spectra for different surface potentials \( U \). (d)-(f) Corresponding energy dispersion of FA surface states. The blue planes represent the Fermi energy. (g)-(i) Corresponding density of states for FA surface states. The calculation parameters are \( a = 1 \text{ nm}, L_y = 80, M_1 = 0.2 \text{ eV nm}^2, M_2 = 1.2 \text{ eV nm}^2, v_y = 0.66 \text{ eV nm}, k_0 = k_1 = 0.4 \text{ nm}^{-1} \).
WSM by the surface gate induced Lifshitz transition.

Transport Signatures. Here we investigate quantum transport in the device sketched in Fig. 3(a) under a magnetic field and show that the conductance can provide a decisive signature of the Lifshitz transition of the FAs. The device is fabricated by depositing multiple strip electrodes (I–IV) on both the top and bottom surfaces of the WSM, see Fig. 3(a). General orientations of the FAs and the stripe electrodes are considered. Without loss of generality, we set the normal of the stripe electrodes to the $x$ direction and perform a rotation to the WSM or Hamiltonian (1) by an angle $\theta$ about the $y$ axis, or explicitly, $\tilde{H}(k) = H(U_y^{-1}k)$ with
\[
U_y(\theta) = \begin{pmatrix}
\cos \theta & 0 & -\sin \theta \\
0 & 1 & 0 \\
\sin \theta & 0 & \cos \theta
\end{pmatrix},
\]
see Sec. I of the Supplemental Material for details.

![Fig. 3](image_url)

**Fig. 3.** The trajectories of electrons in real and momentum spaces, with the surface potential $U = 0.15$ eV [(a), (b)] and $U = 0.4$ eV [(c), (d)] at the top surface, the azimuthal angle is taken as $\theta = \pi/4$. [(a), (c)] Schemes of the planar junction consisting of multiple normal metal (N) electrodes deposited on top and bottom of the WSM and the scattering of particles at the surface. The trajectories of electrons are sketched as colored arrows, and the channels provided by chiral Landau bands in the bulk is marked by dashed lines, respectively. [(b), (d)] The momentum of electrons slides along the FAs and transmit between top and bottom surfaces, driven by the Lorentz force. The curves (circles) in blue or red represent FAs (electrons) on the top or bottom surfaces, respectively. The thicknesses of the WSM and the N electrodes along the $y$ direction are 100 nm and 50 nm, respectively. The length of the coupling area between the WSM and the N electrodes is $W = 20$ nm, the separation between two electrodes is $L = 80$ nm.

Accordingly, the configurations of the FAs shown in Fig. 2 undergo the same rotation, see Figs. 3(b) and 3(d). The stripe electrodes are just normal metals which can be described by the effective Hamiltonian $H_S(k) = CK^2 - \mu_S$ with $C$ being the parameter determined by the effective mass and $\mu_S$ the chemical potential. We here focus on the surface transport with a negligible density of the bulk states and then set the chemical potential of the WSM to zero for simplicity.

Consider an electron injected from electrode I into the surface of the WSM. Without a magnetic field, it propagates straightforwardly into electrode II. Interesting situations take place when a magnetic field $B$ is imposed in the $y$ direction. Due to the Lorentz force, electrons are driven to slide along the FAs, see pictures in the momentum space [Figs. 3(b) and 3(d)] and the corresponding curved trajectories in the real space [Figs. 3(a) and 3(c)]. Because the FAs are terminated at the Weyl nodes, a closed loop for electron transport is composed of the FAs on both surfaces and the chiral Landau bands inside the bulk, i.e., the so-called Weyl orbit.\[3\] As a result, the trajectory of an incident electron is completely determined by the configuration of the Weyl orbit. As discussed above, a surface gate deposited on the top surface can modify the FAs therein, while the chiral Landau band and the FAs on the bottom surface are unaffected. A direct result is that the Weyl orbit possesses distinctive configurations and topology before and after the Lifshitz transition of the top FAs, compare Fig. 3(b) and Fig. 3(d), which can obviously affect the $B$-dependence of conductance between different electrodes.

On this basis, we perform numerical calculation of the quantum transport in the device sketched in Fig. 3(a) to verify the semiclassical picture discussed above. The calculation is conducted by discretizing the effective Hamiltonian $\tilde{H}(k)$ and $H_S(k)$ on a cubic lattice (see Sec. I of the Supplemental Material for details). It is assumed that the size of the strip electrodes in the $z$ direction is much larger than the Fermi wavelength and their contacts with the WSM ensure the conservation of the momentum $k_z$. Then by taking $k_z$ as a parameter, the 3D system can be decomposed into a series of 2D slices labeled by $k_z$, which effectively accelerates the numerical calculation. For the magnetic field effect, the Landau gauge $A = (0, 0, -Bx)$ is adopted so that the Peierls substitution $k \rightarrow -i\nabla \pm \epsilon A/h$ (taking $\epsilon > 0$) retains the conservation of $k_z$.

For the two-dimensional lattice with fixed $k_z$, the scattering process can be described by
\[
\begin{align*}
\psi_{i,\alpha}^{(\text{out})} &= \sum_j \left[ t_{i,\alpha}^{\beta} \psi_{j,\beta}^{(\text{in})} + r_{i,\alpha}^{\beta} \psi_{j,\beta}^{(\text{out})} \right], \\
\psi_{i,\alpha}^{(\text{in})} &= \sum_j \left[ t_{i,\alpha}^{\beta} \psi_{j,\beta}^{(\text{in})} + r_{i,\alpha}^{\beta} \psi_{j,\beta}^{(\text{out})} \right],
\end{align*}
\]
where the $\psi_{i,\alpha}^{(\text{in/out})}$ represents the income (outgoing) wave amplitudes of electrons at the $i$ electrode, $t_{i,\alpha}^{\beta}$ describes the scattering amplitude from electrons of channel $\beta$ in electrode $j$ to channel $\alpha$ in electrode $i$. The transmission probability can be calculated by taking trace of the transmission matrix $T_{ij}^{\alpha\beta}(E) = \text{Tr}[\tilde{T}_{ij}(E,k_z)\tilde{T}_{ij}(E,k_z)] = \sum_{\alpha,\beta} |t_{ij}^{\alpha\beta}(E,k_z)|^2$, using the KWANT code.\[64\] Then the differential conductance $G_{ij} = \partial I_i/\partial V_i$ at zero temperature can be calculated using the Landauer–Büttiker formula by summing up the contributions of all the $k_z$ channels as
\[
G_{ij} (\text{eV}) = \frac{e^2}{h} \sum_{k_z} T_{ij}^{k_z}.
\]

Considering a nonzero bias $V_i$ applied to electrode I, we plot the conductances between electrode I and the other three electrodes versus the surface potential $U$ and magnetic field $B$ in Fig. 4. The unit of the magnetic field is set to the critical value $B_0 = hK_z/(eL)$ with $K_z$ being the
span of the FA in the $k_x$ direction [Fig. 3(b)] and $L$ the separation between electrodes I and II [Fig. 3(c)]. $B_0$ is the critical field strength to drive all incident electrons to the Weyl node and to penetrate into the bulk before they reach electrode II (see Sec. II of the Supplemental Material for details). As discussed above, a surface gate potential $U$ can induce the Lifshitz transition in the FAs on the top surface. We denote the gate voltage corresponding to the transition point by $U_c$. Rich information is involved in the conductance patterns in Fig. 4, which can be interpreted by looking at the $B$-dependence of the conductances before and after the Lifshitz transition. We will next discuss these results in the context of semiclassical paths of the electrons.

For the Weyl orbit in Fig. 3(b), since the FAs on the top and bottom surfaces form two independent Weyl orbits, the right-moving electrons from electrode I can only slide along the right loop in Fig. 3(b). Moreover, the states of the FAs on the top and bottom surfaces possess opposite group velocities. As a result, electrons injected from electrode I may have two kinds of trajectories. For the first one, electrons propagate only on the top surface until they enter electrode II, see the blue arrowed line in Fig. 3(a). In the second case, electrons first slide along the FAs on the top surface and reach the Weyl node, then transfer through the chiral Landau band to the bottom surface, propagate backward and finally reach electrode IV, see the yellow arrowed lines in Fig. 3(a). Based on this picture, one can infer that as electrode I is biased, the injected electrons can only contribute to current flowing in electrodes II and IV, because no electrons can reach electrode III. The distribution of the current in II and IV relies on both the magnetic field and the distance $L$ between electrodes I and II. Such discussion on semiclassical picture is consistent with the $U < U_c$ part of Fig. 4, as $B$ increases from zero, more incident electrons take the latter trajectory. As a result, $G_{41}$ increases while $G_{21}$ decreases for a larger $B$ as shown in Figs. 4(a) and 4(c). Meanwhile, no electrons can reach electrode III, thus the conductance $G_{31}$ vanishes in this regime, see Fig. 4(b).

![Fig. 4. The differential conductance $G_{ij}$ between (a) electrodes I and II, (b) electrodes I and III, (c) electrodes I and IV, for different magnetic fields and surface potential. Here $C = 1 \text{ eV}$, $\mu_L = 2.2 \text{ eV}$, $V_1 = 1 \text{ meV}$, other parameters are the same as those in Fig. 2.](image)

For $U > U_c$, the Lifshitz transition takes place for the FAs on the top surface of the WSM, which gives rise to a drastic modification of the trajectories of electrons and accordingly, different $B$-dependence of the conductances. At this circumstance, the two isolated Weyl orbits in Fig. 3(b) merge into a single but larger one as shown in Fig. 3(d). The electrons injected from electrode I change their trajectories accordingly. Apart from the direct propagation from electrode I to II, electrons can also transfer to the bottom surface and reach electrode III, see the green arrowed lines in Fig. 3(c). The main difference between this regime and that in Fig. 3(a) is that the electrons transferred to the bottom surface do not reverse their velocity in the $x$ direction. It can be expected that the conductance between electrode I and II will exhibit a switch-on effect during the Lifshitz transition of the FAs, which provides a decisive signal for its detection. Moreover, once the magnetic field exceeds a critical value, the electrons can propagate along a more complicated trajectory and enter electrode IV, see the purple arrowed line in Fig. 3(c). Similarly, all the above analysis can also be reflected in the conductance patterns in Fig. 4. First, $G_{21}$ always decreases for a stronger magnetic field, which is similar to the situation for $U < U_c$. The main difference is the $B$-dependence of $G_{31}$ and $G_{41}$. Due to the unique trajectory sketched by the green arrowed lines in Fig. 3(c), $G_{31}$ is switched on by the Lifshitz transition and increases with $B$ for $B < B_0$, see Fig. 4(b).

Accordingly, $G_{41}$ is switched off in this region [Fig. 4(c)] because the yellow trajectory in Fig. 3(a) is absent. Instead, the electrons need to take more complex trajectory sketched by the purple arrowed lines in Fig. 3(c) to reach electrode IV, which needs a stronger magnetic field. Such abrupt changes of $G_{31}$ and $G_{41}$ with $U$ manifest the topological Lifshitz transition of the FAs and the Weyl orbits. As $B$ reaches the critical value $B_0$, all electrons penetrate to the bottom surface and lead to a maximum $G_{31}$, see Fig. 4(b). As $B$ increases further, the purple trajectory in Fig. 3(c) comes into play, which results in an increase of $G_{41}$ accompanied by a decrease of $G_{31}$, see Figs. 4(b) and 4(c).

**Discussions and Summary.** Some remarks are made below about the experimental implementation of our proposal, the stripe electrodes can be fabricated on the WSM by state-of-the-art techniques.9,65,66 FAs with regular shapes are profitable to our proposal, as the spatial trajectories of electrons possess regular configurations accordingly. Moreover, a big separation between Weyl points in the momentum space67–72 is also beneficial for our proposal, in which visible transport signatures can be expected. The minimal WSM model we adopt in the main text contains a single pair of the FAs on each surface, which has been reported in NbIrTe4 (TaIrTe4),54,66,73,74 WP2,79 MoTe2,75 and YbMnBi2.81 In other WSMs however, more than one pair of FAs exist. We argue that
our main conclusion of the gate tunable Lifshitz transition of the FAs should maintain in all these realistic cases. First, the feature of the surface bands that can sustain FAs generally allow the existence of the saddle points, which is the key scenario of the Lifshitz transition. Second, the surface bands can be generally tuned by the surface gate voltage, which is the other ingredient of our scheme. To show the feasibility of our proposal, we calculate the surface electrostatic potential induced by the gate voltage in a specific WSM ZrTe using the first-principles calculations (see Sec. II of the Supplemental Material). It shows that the gate voltage can induce large potential energy which is sufficient for driving the Lifshitz transition of FAs. In addition to the transport signature, the gate-induced Lifshitz transition can also be detected using the quantum oscillation approach, where the Lifshitz transition is revealed by the change in the oscillation frequency.[53,63,76]

To conclude, we show that topological Lifshitz transition of FAs can be induced by a surface gate voltage. Such a transition is attributed to the existence of saddle points in the surface bands, which meets the Fermi energy at the critical transition point. A direct result due to the Lifshitz transition is the abrupt change of the magnetic Weyl orbits composed of both the FAs and bulk Landau bands. Such an effect can be detected by transport signatures, and the Lifshitz transition can be visibly revealed by the switch-on and switch-off conductances.

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