On Theoretical Uncertainties of the 
$W$ Boson Mass Measurement at LEP2

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Abstract

We discuss theoretical uncertainties of the measurement of the $W$ boson mass at LEP2 energies, reconstructed with the help of the tandem of the Monte Carlo event generators KoralW and YFSWW3. Exploiting numerical results obtained with these programs, and the existing knowledge in the literature, we estimate that the theoretical uncertainty of the $W$ mass due to electroweak corrections, as reconstructed at LEP2 with the help of these programs, is $\sim 5$ MeV. Since we use certain idealized event selections and a simple $M_W$-fitting procedure, our numerical exercises can be (should be) repeated for the actual “$M_W$ extraction methods” of the LEP2 measurements, using KoralW and YFSWW3 or other Monte Carlo programs.

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In this work we would like to present our estimate of the theoretical uncertainties (TUs) related to electroweak corrections in the measurement of the mass of the $W$ heavy boson in the LEP2 experiments. The estimate will be based on new numerical results of our own and on the best results available in the literature. One important reason for writing this paper is that the discussion of the electroweak TUs in the $W$ mass measurement, including complete $O(\alpha)$ electroweak (EW) corrections, is not available in the literature. On the other hand, it is becoming a burning issue, as the error on the combined LEP2 result of the $W$ mass measurement approaches 30 MeV, while the total TU should be limited to $<15$ MeV.

Since the $W$ mass measurement has a very specific character, very different from the measurement of the total cross section, let us characterize it briefly. The actual way in which $M_W$ is measured by the LEP2 experiments is complicated, see e.g. refs. [2–5]. In particular, it seems to be beyond the reach of the simple fit to a one-dimensional $W$ invariant mass distribution (having integrated over the invariant mass of the second $W$). This is due to direct inobservability of the neutrino (from the $W$ decay) and of most of the initial-state radiation (ISR) photons, loss of a fraction of the hadronic final state in the beam pipe and the non-trivial dependence on the invariant masses of both $W$’s that certain corrections may have.

Let us give the reader at least a rough idea of how the $W$ mass $M_W$ is measured by the LEP2 experiments. In a nutshell, this is done with the help of a two-level fitting procedure. At the first level, with the help of the so-called kinematic fit, an entire multi-momentum event, either experimental or of Monte Carlo (MC) origin, is reduced to a point in much fewer dimensions than the total dimension of the original set of four-momenta. This space consists typically of the two fitted $W$ masses and of an auxiliary parameter controlling the detector energy resolution. In this way one gets 3-dimensional histograms with $\sim 10^4$ experimental events. On the other hand, one gets the analogous 3-dimensional histograms from a Monte Carlo simulation with $\sim 10^7$ events. The latter one is obtained typically from the combined KoralW [6–8] and YFSWW3 [9–13] programs, which we shall refer to as K-Y. The actual “$M_W$-extraction” is done by fitting $M_W$ such that the difference between the above two 3-dimensional histograms is minimized (typically using a likelihood function). The K-Y prediction for every bin in the 3-dimensional histogram is of course dependent on $M_W$. This dependence is calculated/recalculated in the above fitting procedure by means of averaging the “correcting weight” [8,13] corresponding to a variation of $M_W$, over the entire $\sim 10^7$ sample of MC events stored on a computer disk. All complications of the experimental detector and data analysis are therefore taken into account without any approximation. In this way the multidifferential distribution implemented in the K-Y MC ensures a direct unbiased link between the $M_W$ of the electroweak Lagrangian and the experimental LEP2 data, assuming perfect detector simulation.

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1 For instance, it is missing in the CERN Report of the 2000 LEP2 MC Workshop.
2 Even more inappropriate is trying to characterize the TU of the $W$ mass by introducing some kind of an “error band” in the one-dimensional $W$ distribution – see also the discussion below.
3 The above description of the “$M_W$-extraction” tries to summarize the methods used by ALEPH, L3 and OPAL; the DELPHI method is slightly different, see. ref. [1].
All this sounds like a strong argument to show that the theoretical uncertainties, coming from higher-order corrections and other imperfections of the theoretical calculations, can be studied only within the programming environment used in the actual LEP2 experiments, with the help of the K-Y MC tandem. Nevertheless, mainly because all effects under the following discussion are small, it makes sense to compromise and apply a “simplistic approach” based on the one-dimensional fit of a single W effective mass (integrating over the second one). This is what we shall do in the following. The main danger in the use of a fitting procedure like the one described here is that almost any physical effect in the W effective mass distribution may feature strong correlations as a function of the two effective masses, which may lead to an underestimate of the effect by a factor of 2. Our fitting method provides, therefore, a valuable but rough estimate of the size of the effects under discussion in terms of the $M_W$ measured using LEP2 data. Consequently, if some effect turns out to be sizeable, that is at least 1/3 of the experimental error on $M_W$ ($\sim 10$ MeV), then it should be reanalysed within a full-scale “$M_W$-extraction” procedure of the relevant LEP2 data analysis. In such a case, our paper can be used as a guideline for a more complete study to be performed by the experiments.

Keeping all the above warnings and restrictions in mind, let us characterize more precisely our aims and adopted methodology. During the 2000 LEP2 MC Workshop, the main emphasis was on the TU for the total cross section ($\sigma_{\text{tot}}$) of the W-pair production process $[14]$. Since a variation of $M_W$ is not related to the overall normalization of the distribution $\rho(M_1, M_2) = d\sigma/(dM_1 dM_2)$ at $M_1 = M_2 = M_W$, but rather to the derivatives $D = (\partial/\partial M_i)\rho(M_1, M_2)|_{M_i=M_W}$, the discussion of TUs on $M_W$ is almost completely independent of the discussion of TUs on $\sigma_{\text{tot}}$. The higher order corrections, which strongly influence $\sigma_{\text{tot}}$, may be completely unimportant for $M_W$ and vice versa! In particular, it is inappropriate to try to translate our knowledge of TU in $\sigma_{\text{tot}}$, in terms of a certain “error band” in the distribution $d\sigma/dM_1$, into an error estimate of $M_W$ – obviously it may easily lead to a huge overestimate of the TU of $M_W$ and to overlooking effects which really contribute to it.

In the following, we consider the semileptonic process $e^+e^- \rightarrow u\bar{d}\mu^-\bar{\nu}_\mu$, which belongs to the so-called CC11 class of Feynman diagrams constituting the gauge-invariant subset of the 4-fermion final-state processes, see e.g. ref. $[15]$ for more details. We shall study only the leptonic W mass, i.e. the one reconstructed from the four-momenta of the $\mu^-$ and $\bar{\nu}_\mu$ (in the actual experiments, the neutrino four-momentum is reconstructed from the constrained kinematic fit, see e.g. refs. $[2, 5]$). The input parameters are the same as in the 2000 LEP2 MC Workshop studies $[14]$. All the results in this paper are given for the centre-of-mass energy $E_{\text{CMS}} = 200$ GeV and for the input W mass $M_W = 80.350$ GeV. The fitting function (FF) in all cases was taken from the semi-analytical program KorWan $[6, 7, 16]$. All the relevant distribution will be available in the next release of KorWan/KoralW.  

$^{4}$The relevant distribution will be available in the next release of KorWan/KoralW.
In the first preparatory step, we construct a simple fitting procedure of the $M_W$ using the 1-dimensional distribution of the $W$ effective mass $M_1$, and we “calibrate” with the help of the MC data in which we switch the same effects on and off, typically the ISR and the non-factorizable corrections (NF) in the inclusive approximation (denoted by INF in the following) of the so-called screened Coulomb ansatz by Chapovsky & Khoze [18], just to see whether we get agreement in the case of the same effect in the MC data and in the fitting function. The other immediate profit is that we also quantify these effects as a shift of $M_W$. The results of the first exercise are shown in Fig. 1. Let us explain briefly the notation: Born denotes the Born-level results, ISR the ones including the $O(\alpha^3)$ LL YFS exponentiation for the ISR as well as the standard Coulomb correction [19], INF the above plus the INF correction, and Best denotes the best predictions from YFSWW3, i.e. all the above plus the $O(\alpha^1)$ electroweak non-leading (NL) corrections.

Let us summarize observations resulting from Fig. 1:

- The fitted $M_W$ exactly agrees with the input $M_W$ in the case when the same ISR and INF are included both in the fitting function (FF) and the MC.

- If one is interested only in the shift of $M_W$, then any of the three FFs could be used. In the following, in a single exercise we shall typically use one or two of them only.

- The dependence on the fitting range is sizeable; it points out, albeit in a crude way, the fact discussed above, that the ultimate precise fit of $M_W$ should always be done as in the LEP2 experiments, using a multidimensional fitting procedure.

Figure 1: The introductory exercise, see more discussion in the text.
• The size of the ISR effect is about $-10$ MeV, that of the INF about $+5$ MeV, and the size of the NL corrections seems to be negligible, $\sim 1$ MeV.

Note that there were no cuts and we used the true parton-level $W$ invariant masses in all the above exercises.

In the following exercises we shall examine the influence of various effects/corrections on $M_W$ for various cuts and acceptances. Not all these effects can be included in the FF. Besides, only a very limited menu of cuts and acceptances can be applied in the FF. Therefore, our estimates of the TU will be based not on absolute values of the fitted $M_W$ but on relative differences of $M_W$'s corresponding to various effects. This is justified by our “calibration” exercise.

![Figure 2: Effects of the ISR and the FSR on $M_W$.](image)

In the second exercise, depicted in Fig. 2, we switch on and off various orders/variants of the ISR and of the final-state radiation (FSR), and finally the NL correction. The FSR was generated by the program PHOTOS [22]. While the previous exercise was without any cuts and for the so-called BARE$_{4\pi}$ acceptance (the subscript $4\pi$ means the full solid-angle coverage), we employ here the semi-realistic acceptance CALO5$_{4\pi}$, where all the photons for which the invariant mass with a final-state charged fermion was $< 5$ GeV were recombined with that fermion (also for the full solid-angle coverage). We compare the results for FF representing the Born and Born+ISR (no FSR) levels.

**Observations:**

• Changing the type of the ISR from $\mathcal{O}(\alpha^3)_{exp}$ LL to $\mathcal{O}(\alpha^2)_{exp}$ LL induces a negligible, $< 1$ MeV, effect in fitted $M_W$.

• The FSR effect is large for BARE$_{4\pi}$, $\sim 60$ MeV, and much smaller, $\sim 7$ MeV, for calorimetric CALO5$_{4\pi}$, as expected.
• Switching from the single-photon (FSR\textsubscript{1}) to the double-photon (FSR\textsubscript{2}) option in PHOTOS results in a \( \sim 4 \) MeV change of \( M_W \) for BARE\textsubscript{4\pi} and no change for CALO5\textsubscript{4\pi}.

• The INF+NL correction is \( \sim 6 \) MeV and seems to cancel partly with the FSR, see CALO5\textsubscript{4\pi}.

Before we go to the next exercise, let us describe briefly the acceptances and cuts that were used in the MC simulations for the following calculations.

1. We required that the polar angle of any charged final-state fermion with respect to the beams be \( \theta_{f_{ch}} > 10^\circ \).

2. All photons within a cone of \( 5^\circ \) around the beams are treated as invisible, i.e. they were not included in the calculation of the \( W \) invariant masses.

3. The invariant mass of a visible photon with each charged final-state fermion, \( M_{f_{ch}} \), is calculated, and the minimum value \( M_{f_{ch}}^{\min} \) is found. If \( M_{f_{ch}}^{\min} < M_{\text{rec}} \) or if the photon energy \( E_{\gamma} < 1 \text{ GeV} \), the photon is combined with the corresponding fermion, i.e. the photon four-momentum is added to the fermion four-momentum and the photon is discarded. This is repeated for all visible photons.

In our numerical tests we used three values of the recombination cut:

\[
M_{\text{rec}} = \begin{cases} 
0 \text{ GeV}: & \text{BARE}, \\
5 \text{ GeV}: & \text{CALO5}, \\
25 \text{ GeV}: & \text{CALO25}.
\end{cases}
\]

Let us remark that we have changed here the labelling of these recombination cuts from the slightly misleading bare and calo names used in Ref. \cite{14}. They correspond to our CALO5 and CALO25, respectively. This change allows us to reserve the BARE name for a “truly bare final fermion” setup (without any recombination).

In the next exercise, presented in Fig. 3, we examine once again the effect of switching on the 4\textsubscript{f}-background corrections\footnote{The complete Born-level 4\textsubscript{f} matrix element in KoralW was generated with the help of the GRACE2 package \cite{24}.} and the INF corrections, now for BARE and CALO5. The effect of the 4\textsubscript{f} background is \( \sim 1 \) MeV (it is therefore negligible for the LEP2 experiments\footnote{This smallness is due to a general smallness of the 4\textsubscript{f}-background correction in the CC11 class of channels for LPA\textsubscript{a}; it may be less pronounced for a different type of LPA, such as LPA\textsubscript{b} in YFSWW3, for example. Our conclusion is unaffected, as it is really meant for the sum of LPA\textsubscript{a} and 4\textsubscript{f}; see also the discussion below.}), and that of the INF is \( \sim 5 \) MeV. The effects of the 4\textsubscript{f} background in the non-CC11 channels can be larger, but they strongly depend on the applied experimental cuts or acceptances, so that they can be studied in detail only within the full-scale LEP2 \( M_W \) fitting framework.

The size of the NF effect of \( \sim 5 \) MeV requires some explanation, as the genuine NF effect in \( M_W \) is in fact only about \( \sim -1 \) MeV; see refs. \cite{24,27} and the discussion of the
INF ansatz in ref. [18] (see also more discussion in the following). This effect in Fig. 3 understood as a difference between our ISR and INF calculations, is blown up artificially, because the ISR includes the $\sim -6$ MeV $M_W$ shift due to the so-called “standard Coulomb effect” for historical reasons, although its derivation is not valid far away from the $WW$ threshold.

Another point to be explained is whether the genuine NF effect of $\sim -1$ MeV obtained in the INF (inclusive) approximation can be increased to higher values, say 10 MeV, due to the LEP2 experimental cuts. In principle it can be; however, as is well known, the NF correction does not include (fermion) mass logarithms, and its “energy scale”, which enters the big logarithm owing to a cut on the photon energy, is $\Gamma_W$ and not $\sqrt{s}$. Consequently, in order to get an enhancement factor of $\ln(\Gamma_W/E_{\text{max}}) \sim 10$, one would need to veto the appearance of any photon above $E_{\text{max}} = 0.1$ MeV – a very unrealistic experimental selection indeed. On the contrary, in the actual LEP2 experiments photons with energy $\leq 2$ GeV are not disturbed, directly or indirectly, by the experimental event selection. This is why any strong enhancement of the NF effect with respect to its “inclusive” treatment (INF) must be just absent.

In the Fig. 4, we examine the difference in $M_W$ fitted to the $W$ mass distribution obtained from YFSWW3 and from RacoonWW [28, 29]. The distributions used for the $M_W$ fits are exactly the same as those that were used for the plots in the CERN Report of the 2000 LEP2 MC Workshop, see [14]. The statistical error is taken into account in the fits and propagates into the fitted $W$ mass. It is merely $< 1$ MeV. We use two fitting functions, one in which the ISR is included (with the incomplete NL but with the Coulomb effect) and another one in which the INF (Chapovsky & Khoze) is also included. It is done for two kinds of calorimetric acceptances: the CALO5 and CALO25 described.
Observations concerning the results shown in Fig. 4:

- The comparison of YFSWW3 with RacoonWW is very interesting because the two calculations differ in almost every aspect of the implementation of the ISR, FSR, NL and NF corrections.

- It is quite striking that the results of YFSWW3 and RacoonWW differ, in terms of the fitted mass, by only $\leq 3$ MeV, slightly more for CALO5 than for CALO25.

- The difference between YFSWW3 and RacoonWW is definitely smaller than the size of the INF correction, roughly by a factor of 2 (the INF is of order 3–5 MeV for these two acceptances).

The most important result of the comparison between YFSWW3 and RacoonWW is that it reconfirms the smallness of the NF corrections in the $W$ mass. Its size is well below the 10 MeV precision target of the TU for the measurement of $M_W$ at LEP2. It would be interesting to repeat the above exercise for the true LEP2 acceptance, using the full-scale fitting procedure. In our opinion the above difference between YFSWW3 and RacoonWW in terms of the fitted $M_W$ cannot be attributed to some dominant source. Most likely it consists of several contributions and the leading candidates are ordinary factorizable QED corrections and/or some purely technical/numerical problems.

In order to gain better understanding of the above numerical results it is worth while to estimate them semi-quantitatively, in terms of some “scale parameters” representing various QED or EW corrections to $M_W$. We shall do it in the following. In addition, we shall discuss certain effects not included in K-Y or RacoonWW. The corrections to $M_W$
Table 1: Estimation of the missing effects in the K-Y MC tandem.

| Error Type | Scale Param. $\Delta M_W = \Gamma \times \epsilon$ | Numerical cross-check | $\Delta M_W$ |
|------------|-----------------------------------------------|-----------------------|-------------|
| $\delta M_W$ | $\epsilon \simeq \frac{1}{8} \alpha \beta (\frac{1}{4} - \beta^2) \ln(M_W^{2}/\rho^{2})$ | $\sim 10^{-4}$ | $< 2$ MeV |
| $\delta M_W$ | $\epsilon \simeq \frac{1}{2} \alpha \beta (\frac{1}{4} - \beta^2) \ln(M_W^{2}/\rho^{2})$ | $\sim 10^{-7}$ | $< 1$ MeV |

are generally of the type $\frac{\delta M_W}{M_W} \sim \Gamma W M_W$, or $\frac{\delta M_W}{M_W} \sim \left(\frac{1}{\Gamma W M_W}\right)^2 \epsilon$, where $\epsilon$ is a small parameter of the perturbative expansion. We divide them into three types:

**Case (a):** A mildly mass-dependent correction to the $W$ mass distribution $\rho(M) = \frac{d\sigma}{dM} \simeq |BW(M)|^2 \times f(M^2)$, which leads to $\delta M_W \simeq \frac{1}{8} \Gamma W \frac{d\ln f(M^2)}{dM} |_{M=M_W}$, where $BW(M)$ denotes the Breit–Wigner resonance function and $f(M^2)$ is a mild function in the vicinity of the resonance (in the semi-quantitative discussion, we usually take $M = (M_1 + M_2)/2$). The most trivial example is the kinematic factor $f(M^2) = \beta_M = (1 - 4M^2/s)^{1/2}$, yielding $\delta_{\text{kin}} M_W \simeq -\Gamma W \frac{\Gamma W M_W}{2\beta_W^2}$, where $\beta_W = \beta_M |_{M = M_W}$. It is not visible in our fits (always taken into account in the FF); however, the $\beta_W$-factor gets modified by the ISR, giving rise to $\delta M_W \simeq \delta_{\text{kin}} M_W \times 2\beta W \Delta \delta \simeq -6$ MeV ($L_e = 2 \ln \frac{s}{\rho^2}$). This effect is responsible for most of the $M_W$ shift when switching from the Born to the ISR in Fig. 1. It vanishes at high energies. The response of $\delta M_W$ to a more general variation: $f(x) \rightarrow f(x) + \epsilon f_1(x)$, where $\epsilon f_1$ is due to the higher-order ISR correction, is in general negligible, $< 1$ MeV. It can be estimated using $\delta M_W \simeq \epsilon \frac{1}{8} \Gamma^2 W \frac{d\ln f_1(M^2)}{dM} |_{M=M_W} \simeq \epsilon \frac{\Gamma^2 W}{4s} M_W$ (here, we exploit the fact that $f_1$ has a derivative of $O(1)$ as a function of $M^2/s$). For instance, the missing $O(\alpha^2)$ NLL ISR is proportional to $\epsilon \sim \alpha^2 L_e \sim 10^{-3}$, giving rise to $\delta M_W \sim 10^{-3}$ MeV.

**Case (b):** This is the case of the QED effects in the decays, the so-called FSR. In this case the mass distribution gets distorted according to $\frac{d\sigma}{dM^2}(M^2) \simeq \int dz \gamma_{\text{FSR}}(1 - z)^{\gamma_{\text{FSR}} - 1} \frac{d\sigma}{dM^2}(z M^2)$, where $\gamma_{\text{FSR}} \simeq \left(\frac{4}{\pi}\right) \ln(1 + \frac{m_W^2}{m_\mu^2}) \simeq 0.03$ for the BARE and

$$\frac{d\ln}{dM} \left[ \int f dz \frac{d\sigma}{dM^2}(z)^{\gamma_{\text{ISR}}}(1 - z)^{\gamma_{\text{ISR}} - 1} \right] |_{M=M_W},$$

where $\gamma_{\text{ISR}} = \frac{4}{\pi} L_e$.
\[ \gamma_{\text{FSR}} \simeq \left( \frac{\alpha}{\pi} \right) \ln \left( \frac{M_W^2}{m_{\text{CALO}}^2} \right) \simeq 0.01 \text{ for CALO-type acceptance (with } m_{\text{CALO}} = 5 \text{ GeV). The mass shift } \delta M_W \simeq \Gamma_W \varepsilon, \varepsilon \simeq -\frac{\pi}{4} \gamma_{\text{FSR}} \simeq -0.012, \text{ is accounted for in the complete } \mathcal{O}(\alpha) \text{ calculation; see also refs. } [8, 13]. \] 

In the case of PHOTOS the missing \( \mathcal{O}(\alpha) \) is related mainly to high-\( p_T \) photons, and from tests of this program listed in ref. [22] one can conclude that it corresponds to \( \sim 0.2 \times \varepsilon, \varepsilon \simeq -\frac{\pi}{4}. \) The missing \( \mathcal{O}(\alpha^2) \) FSR effect we estimate as follows: \( \Delta \delta M_W \sim \Gamma_W \frac{1}{2} \varepsilon^2 \ll 1 \text{ MeV}. \)

**Case (c):** The influence of the NF QED interferences on the \( W \) mass is characterized by the correction function \( \delta_{\text{NF}} \), which is strongly dependent on \( M_W \) in the vicinity of the resonance:

\[ \frac{d\sigma}{dM} \simeq |BW(M)|^2 f(M) \left[ 1 + \alpha \delta_{\text{NF}} \left( \frac{M^2 - M_W^2}{M_W^2 \Gamma_W} \right) \right]. \]

The resulting \( M_W \)-shift is \( \delta M_W \simeq \frac{1}{8} \Gamma_W^2 a \frac{d\delta_{\text{NF}}(M)}{dM} |_{M=M_W}. \) The simple INF formula for \( \delta_{\text{NF}} \) of ref. [15], leads to \( \delta M_W \simeq -\Gamma_W \frac{a}{\pi} \frac{(1-\beta_W)^2}{\beta_W} \sim -1 \text{ MeV}, \) in perfect agreement with the complete calculations of the NF corrections, see refs. [24–27]. If \( \mathcal{O}(\alpha^1) \) NF is accounted for, then we estimate the missing \( \mathcal{O}(\alpha^2) \) NF contribution at \( \Delta \delta M_W \sim \Gamma_W \frac{1}{2} \left( \frac{\alpha^2 (1-\beta_W)^2}{\beta_W} \right)^2 \ll 1 \text{ MeV}. \)

The above discussion confirms that all our numerical results are consistent with expectations based on the “scale parameters” analysis and semi-quantitative calculations, and provides some estimates of the effects not accounted for in our MC programs. It is also summarized in Table 1 together with the relevant numerical estimates.

Let us finally discuss a question of the TU due to the so-called “ambiguity of a definition of the LPA”. In YFSWW3, we implemented two different definitions of the LPA, called LPAa and LPAb [13]. Differences in \( M_W \) reconstructed from the results obtained in these two options can give us a hint of the TU due to the LPA. Actually, we need to check only the variation of \( M_W \) caused by the NL corrections. This is because KoralW implements the full 4f-process at the so-called ISR level; hence, the ambiguity due the LPA is reduced from \( \mathcal{O}(\frac{\Gamma_W}{M_W}) \) to \( \mathcal{O}(\frac{a \Gamma_W}{\pi M_W}) \) and is located only in the NL part. We performed numerical tests of the dependence of NL on the choice of the LPA with the help of YFSWW3, finding the variation of the \( M_W \) induced by the change from LPAa to LPAb to be \( \leq 1 \text{ MeV} \) (the ISR and NL parts are always defined as in refs. [8, 13]). Another uncertainty in the LPA is due to the missing higher orders in the NL part. This can be estimated by comparing the predictions of the so-called schemes (A) and (B) [13] in YFSWW3. These two schemes account for some higher-order effects by the use of the effective couplings in two different ways – in fact, the scheme (B) follows the prescription employed in RacoonWW. We have checked that the change from the scheme (A) to the scheme (B) results in the fitted \( W \) mass shift of \( \leq 1 \text{ MeV} \) (as expected, the results of the latter scheme are slightly closer to the ones of RacoonWW). Consequently, we attribute \( \Delta M_W = 1 \text{ MeV} \) to the TU of \( M_W \) due to the LPA.

In the above, we have considered only the leptonic \( W \) mass coming from the \( W^- \rightarrow \mu^- \bar{\nu}_\mu \) decay. For the other leptonic decays, the results should be similar when one applies the calorimetric-type acceptance – as was shown in ref. [11]. For the hadronic \( W \) masses at the parton level, the results would be analogous to the ones presented here with the FSR.
switched off. More realistic estimates of the TU for the hadronic $W$ mass would require taking into account the QCD effects, hadronization, jet definitions, etc. This should be done in the full-scale experimental data analysis, which is beyond the scope of this paper.

From the above numerical exercises and the accompanying discussion, we come to the following conclusions:

- The electroweak theoretical uncertainty in $M_W$ of the K-Y MC tandem at LEP2 energies is $\sim 5$ MeV.

- The above conclusion is strengthened by the smallness of the differences between YFSWW3 and RacoonWW, which we attribute to the standard factorizable corrections (ISR, FSR, etc.) and purely technical/numerical effects.

- In the above estimate we included a “safety factor” of 2, corresponding to the fact that our fits of $M_W$ were done for 1-dimensional effective $W$ mass distributions. In order to eliminate it, our analysis should be repeated for the realistic measurements of the LEP2 experiments.

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References

[1] Reports of the Working Groups on Precision Calculations for LEP2 Physics, eds. S. Jadach, G. Passarino and R. Pittau (CERN 2000-009, Geneva, 2000).

[2] R. Barate et al., Eur. Phys. J. C17, 241 (2000).

[3] P. Abreu et al., Phys. Lett. B511, 159 (2001).

[4] M. Acciarri et al., Phys. Lett. B454, 386 (1999).

[5] G. Abbiendi et al., Phys. Lett. B507, 29 (2001).

[6] M. Skrzypek, S. Jadach, W. Placzek and Z. Was, Comput. Phys. Commun. 94, 216 (1996).

[7] S. Jadach et al., Comput. Phys. Commun. 119, 272 (1999).

9 Including the photon radiation from quarks without QCD effects is too crude an approximation and we do not consider such a scenario here.
[8] S. Jadach et al., The Monte Carlo Program KoralW version 1.51 and The Concurrent Monte Carlo KoralW & YFSWW3 with All Background Graphs and First-Order Corrections to W-Pair Production, preprint CERN-TH/2001-040, UTHEP-01-0102, February 2001; [hep-ph/0104049]; to appear in Comput. Phys. Commun.

[9] S. Jadach, W. Placzek, M. Skrzypek and B. F. L. Ward, Phys. Rev. D54, 5434 (1996).

[10] S. Jadach et al., Phys. Lett. B417, 326 (1998).

[11] S. Jadach et al., Phys. Rev. D61, 113010 (2000).

[12] S. Jadach et al., Precision predictions for (un)stable W⁺W⁻ production at and beyond LEP2 energies, preprint CERN-TH/2000-337; [hep-ph/0007012]; submitted to Phys. Lett. B.

[13] S. Jadach et al., The Monte Carlo Event Generator YFSWW3 version 1.16 for W-Pair Production and Decay at LEP2/LC Energies, preprint CERN-TH/2001-017, UTHEP-01-0101, January 2001; [hep-ph/0103163]; Comput. Phys. Commun. (2001), in print.

[14] M. Grünwald et al., Four-fermion production in electron-positron collisions, in Ref. [1], p. 1.

[15] Physics at LEP2, eds. G. Altarelli, T. Sjöstrand and F. Zwirner (CERN 96-01, Geneva, 1996), 2 vols.

[16] M. Skrzypek et al., Phys. Lett. B372, 289 (1996).

[17] R. Stuart, Nucl. Phys. B498, 28 (1997), and references therein.

[18] A. P. Chapovsky and V. A. Khoze, Eur. Phys. J. C9, 449 (1999).

[19] V. Fadin, V. Khoze, A. Martin and W. Stirling, Phys. Lett. B363, 112 (1995).

[20] J. Fleischer, F. Jegerlehner and M. Zrałek, Z. Phys. C42, 409 (1989).

[21] J. Fleischer, K. Kołodziej and F. Jegerlehner, Phys. Rev. D49, 2174 (1994).

[22] E. Barberio, B. van Eijk and Z. Was, Comput. Phys. Commun. 66, 115 (1991), ibid. 79, 291 (1994).

[23] J. Fujimoto et al., GRACE User’s manual, version 2.0, MINAMI-TATEYA collaboration.

[24] W. Beenakker, A. Chapovsky and F. Berends, Nucl. Phys. B508, 17 (1997).

[25] W. Beenakker, A. Chapovsky and F. Berends, Phys. Lett. B411, 203 (1997).
[26] A. Denner, S. Dittmaier and M. Roth, Nucl. Phys. B519, 39 (1998).
[27] A. Denner, S. Dittmaier and M. Roth, Phys. Lett. B429, 145 (1998).
[28] A. Denner, S. Dittmaier, M. Roth and D. Wackeroth, Nucl. Phys. B587, 67 (2000).
[29] A. Denner, S. Dittmaier, M. Roth and D. Wackeroth, Nucl. Phys. Proc. Suppl. 89, 100 (2000).
[30] W. Beenakker, F. A. Berends and A. P. Chapovsky, Phys. Lett. B435, 233 (1998).
[31] W. Beenakker, F. A. Berends and A. P. Chapovsky, Nucl. Phys. B548, 3 (1999).