The effects of disorder and interactions on the Anderson transition in doped graphene

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Abstract

We undertake an exact numerical study of the effects of disorder on the Anderson localization of electronic states in graphene. Analyzing the scaling behaviors of inverse participation ratio and geometrically averaged density of states, we find that the Anderson metal–insulator transition can be introduced by the presence of quenched random disorder. In contrast with the conventional picture of localization, four mobility edges can be observed for the honeycomb lattice with specific disorder strength and impurity concentration. Considering the screening effects of interactions on disorder potentials, the experimental findings of the scale enlargements of puddles can be explained by reviewing the effects of both interactions and disorder.

(Some figures in this article are in colour only in the electronic version)

1. Introduction

As a two-dimensional (2D) allotrope of carbon [1, 2], graphene holds great promise for replacing conventional semiconductors on account of its unique electronic properties. In graphene, the $\sigma$-band formed by the $sp^2$ hybridized orbitals determines the robustness of the honeycomb structure, and the half-filled $\pi$-band provided with Dirac-like electronic excitations is responsible for the unusual transport properties [3]. It has been observed in some experiments [4–9] that disorder has remarkable effects on the unusual electronic properties of graphene, and thus there has been a great deal of interest in recent years concerning the role of disorder in graphene.

Some experiments have confirmed the existence of a metal–insulator transition (MIT) in disordered graphene [4–6], which suggests that the one-parameter scaling theory [10] may break down [11]. In accordance with the prediction of the Klein paradox [12], the Dirac fermions are found to be robust against disorder in the 2D single valley Dirac model. In addition, the one-parameter scaling in the single valley Dirac Hamiltonian [13, 14] shows that the logarithmic derivative $\frac{d\ln \sigma}{d\ln L} = \beta(\sigma)$ is a positive function and the scaling flow has no fixed point, indicating that the system always scales towards a metal. This scenario is different from what would be expected for conventional 2D electron systems, where all states are localized for arbitrary weak disorder [10].

Recently, some numerical methods have been adopted in the study of the massless Dirac model with random potential [14–18] or the Anderson tight-binding model [19–27]. It is already clear that the single valley Dirac approximation is valid for graphene with weak long-range impurities, but the inter-valley scattering and chirality breaking scattering should be considered in the presence of strong short-range disorder. Therefore, the full tight-binding description of graphene with Anderson disorder has become a popular alternative model.

Some very recent simulations [22–24] have provided new evidence for the existence of an MIT in graphene with short-range disorder by obtaining the mobility edges near the Fermi energy. However, a contrary view has been supported by the transfer-matrix approach [25] and the kernel polynomial method [27] that all states in the Anderson tight-binding model are localized for arbitrary weak disorder. A possible reason for this disagreement to arise is that different measures and scaling rules are adopted in the above studies in an effort to distinguish localized states from delocalized ones.

In this paper, we introduce a new approach to scale the inverse participation ratio (IPR), which overcomes the instability and unreliability problems caused by the primitive scaling of the IPR with the negative first or second power of lattice size. We study the tight-binding Hamiltonian of the honeycomb lattice with randomly fluctuated disorder potentials, and a unique picture of the Anderson MIT has been obtained. As expected, we find that all electronic states...
in graphene are localized by strong disorder. However, four mobility edges can be observed when we decrease the disorder strength until under a critical value $W_c$, and the electronic states within two nearest neighbor mobility edges are entirely extended or localized alternatively. Therefore, the MIT can be introduced by changing the carrier density to move the Fermi surface across a certain mobility edge. Our results are in agreement with the very recent experimental suggestions, where a transition from localization to anti-localization has been achieved by decreasing the carrier density in graphene samples [6].

On the other hand, the sources of the coexistence of electron and hole puddles observed by a scannable single-electron transistor (SET) have been proved to be both the substrate-induced structural distortion as well as chemical doping, while the length scale of the density fluctuations, which is more than 150 nm [7], is a great deal larger than the disorder length scale introduced by the short-range scatters. In this paper, we show that the unexpected large disorder length scale observed by an SET can be explained by the screening effect of interactions on disorder, suggesting that the electron–electron interactions should be taken into account. In addition, the delocalization effect of interactions through screening disorder potentials can also introduce the zero-bias anomaly at the Fermi surface [28–30].

In this paper we address the following issues related to the effect of disorder in graphene. (1) Does there exist a mobility edge to separate the localized states from the delocalized states in disordered graphene? A distinctive scenario of the Anderson MIT with four mobility edges has been observed in graphene with quenched random disorder. (2) How do we measure the localization length of a localized electronic state by the IPR? We present a new method to set the lattice size scaling of the IPR, which can give the localization length of an electronic state precisely. (3) What is the role of interactions in disordered graphene? The screening effect of the short-range interactions on disorder potentials is found to have strong affect for the scale enlargements of puddles.

The paper is organized as follows. In section 2.1 we present the fully tight-binding description of the disordered graphene. In sections 2.2 and 2.3, we discuss the lattice size scaling of the IPR and the geometrically averaged density of states (GADOS) respectively, and show the scaling method relevant for the subsequent discussions. Next we show our findings regarding the static disorder in graphene in section 3. In addition, the screening effects of the electron–electron interactions on the disorder potentials are presented in section 4. The principal findings of this paper are summarized in section 5.

2. Theory

2.1. The Anderson tight-binding model

The Anderson tight-binding Hamiltonian [31, 32] for disordered graphene can be expressed as

$$H_{AM} = -t \sum_{i<j} c_i^\dagger c_j + \sum_i \epsilon_i c_i^\dagger c_i, \quad (1)$$

where $c_i$ ($c_i^\dagger$) are annihilation (creation) operators of a fermion at site $i$, $\epsilon_i$ represent the on-site disorder energies, and $t$ denote the hopping integrals between nearest neighbor (NN) sites, which are set to be unity in our calculations. We take the exact numerical approach to find all eigenenergies and eigenfunctions of the Hamiltonian given by equation (1) for a finite hexagonal lattice with the periodic boundary conditions. The numerical calculations are performed with regard to the following three different disordered cases.

Firstly, in the presence of quenched random disorder, we introduce the box distributed Anderson disorder, and the on-site disorder energies $\epsilon_i$ in equation (1) are assumed to be independent random variables distributed between $-W/2$ and $W/2$. Therefore, the probability distribution of $\epsilon_i$ is given by

$$P(\epsilon) = W^{-1} \Theta(W/2 - |\epsilon|), \quad (2)$$

and the disorder strength is parameterized by the width $W$.

Secondly, to take into account the influences of the impurity potentials on the adjacent sites [21], the correlation length of the disorder $\zeta$ is introduced and the on-site disorder energies $\epsilon_i$ in equation (1) can be replaced by the correlated disorder energy $\tilde{\epsilon}_i$ as

$$\tilde{\epsilon}_i = \sum_{l=1}^{N} \epsilon_j \exp(-|r_i - r_j|^2/\zeta^2), \quad (3)$$

where $\epsilon_j$ represent the ‘uncorrelated’ disorder energies of the impurities at sites $i$, which have been defined in the first case.

Finally, in order to generalize this Hamiltonian by including electron–electron correlations, one must add an interaction term $H_{int}$ into the Anderson tight-binding model:

$$H = H_{AM} + H_{int}, \quad (4)$$

with

$$H_{int} = U \sum_i n_{i \uparrow} n_{i \downarrow} + \sum_{i<j} V_{ij} n_i n_j. \quad (5)$$

Here $n_{i \sigma} = c_{i \sigma}^\dagger c_{i \sigma}$ is the local charge-density operator with $\sigma = \uparrow, \downarrow$, $U$ represent the on-site interactions, and $V_{ij}$ denote the nonlocal interactions between electrons at sites $i$ and $j$. To compare with the experimental findings about the enlargements of puddles [7], here we consider a case of the binary alloy disorder, in which the probability distribution function of the on-site disorder energies is given by

$$p(\epsilon_i) = x \delta(\epsilon_i - W) + (1-x) \delta(\epsilon_i), \quad (6)$$

where $x$ is the fraction of the lattice sites with energies $\epsilon_i = W$, and $W$ represents the on-site energy splitting.

2.2. The lattice size scaling of the IPR

One of the widely used quantities to measure the Anderson localization of electronic states in disordered systems is called the IPR, which is defined as [33, 34]

$$I_2(E_n) = \sum_{i=1}^{N} |\Psi_n(r_i)|^4, \quad (7)$$
where $E_n (n = 1, \ldots, N)$ and $\Psi_n (r_i)$ are the eigenenergies and eigenfunctions of a disordered finite lattice with $N$ sites. Since the eigenenergies of a finite lattice are always discrete, it is more convenient to introduce a continuous energy-dependent IPR as

$$I_2(\omega, N) = \frac{\sum_{i=1}^{N} \Theta(\frac{\omega}{2} - |\omega - E_n|)I_2(E_n)}{\sum_{i=1}^{N} \Theta(\frac{\omega}{2} - |\omega - E_n|)},$$  \hspace{1cm} (8)

where $\omega$ represents energy, and $\gamma$ is a very small energy bin for the average. Since the IPRs of extended states are proportional to $1/L^d$ and go to zero in the infinite lattice limit $L \rightarrow \infty$, it is not difficult to identify extended states by carrying out lattice size scaling of IPRs. Here $d$ denotes the dimension of the system, and $L$ represents the lattice size with $N = L^d$. However, the lattice size scaling of the IPR for a localized state is found to be more complicated than expected [35–38].

The wavefunction of Anderson localized states decays exponentially as $|\psi(r)\rangle \sim \exp(-|r - r_0|/\xi_{\text{loc}})$, where $\xi_{\text{loc}}$ is the localization length. When the lattice size $L$ is much larger than the localization length, the IPR is a size-independent constant $I_2(\infty)$, and the localization length can be obtained easily by $\xi_{\text{loc}} = (I_2(\infty))^{-1/d}$ [38]. However, we cannot always perform calculations satisfying the condition $L \gg \xi_{\text{loc}}$ on account of the limited capacity of the numerical calculations. When the lattice sizes are comparable to or even smaller than the localization length $\xi_{\text{loc}}$, $L$ becomes a variable to the function of the IPR. Later, a simple lattice size scaling formula $1/L^\alpha$ is suggested. Unfortunately, this IPR scaling can mistakenly regard a localized state as an extended state in some cases. For example, the exact scaling formula of the IPR in one-dimensional (1D) systems with box distributed disorder has been obtained as[38]

$$I_2(\omega, L) = I_2(\omega, \infty) \coth(L/2\xi_{\text{loc}}(\omega)), \hspace{1cm} (9)$$

with $\xi_{\text{loc}}(\omega) = I_2^{-1}(\omega, \infty)$, It is obvious that the simple formula $1/L^\alpha$ is not a proper approximation in the considerably large lattice size region to scale the IPR. Certainly, there are also exceptions for the conditions with $L \gg \xi_{\text{loc}}$ and $L \ll \xi_{\text{loc}}$, where we can choose $\alpha = 0$ or $\alpha = 2$, respectively.

Unlike the 1D disordered system, the exact expression for the IPR has not been obtained for the 2D finite systems. Therefore, we introduce a Taylor series to scale the IPR by

$$I_2(\omega, L) = I_2(\omega, \infty) + \frac{a_1(\omega)}{L} + \frac{a_2(\omega)}{L^2} + \frac{a_3(\omega)}{L^3} + \cdots,$$  \hspace{1cm} (10)

where $a_n(\omega)$ is the $n$th Taylor parameter. The minimum radius of convergence is found to be $R_{\text{min}} = 0.1$ in our studies, and to make all Taylor series adopted convergent, we have to perform calculations when lattices meet the condition $L > 1/R_{\text{min}} = 10$. For convenience, we employ a polynomial formula consisting of the first five terms in equation (10) since the contributions of high-order terms are negligible. As shown in figure 1, the new fitting is reasonable and practical since the intercept of an extended state is found to be zero or even a very small negative number, whereas the fitting curve of a localized state has a positive intercept in the infinite lattice limit. We have checked the above fitting method by the disordered cubic lattice [39], and good agreement has been achieved with the accepted scaling theories [10]. Especially, we find that the fitting curves of extended states can be approximated by $I_2(\omega, L) \propto 1/L^2$, in accordance with the prediction of some other theories. On the other hand, when the electronic state is localized, the five parameters are all found to have non-zero values, but the low-order terms play the main roles. As show by the solid line in figure 1, there exists a finite intercept $I_2(\omega, \infty)$ for a localized state, and its localization length can be obtained by

$$\xi_{\text{loc}}(\omega) = \frac{1}{\sqrt{I_2(\omega, \infty)}}. \hspace{1cm} (11)$$

To sum up, the new scaling method of the IPR has the advantage of distinguishing explicitly the localized states with extended states, and in addition we can acquire the localization length precisely by the intercept obtained in the infinite-size limit.

2.3. Measuring Anderson localization by the GADOS

It has been proposed that Anderson localization can be measured by an order parameter $\rho_s(\omega)$, called the GADOS, which is obtained by geometrically averaging the local DOS (LDOS) of each site as

$$\rho_s(\omega) = \frac{1}{N_s} \sum_{m=1}^{N_s} \left[ \prod_{i} \rho(r_i, \omega) \right]^{1/N_s}, \hspace{1cm} (12)$$

with

$$\rho(r_i, \omega) = \frac{\sum_{n=1}^{N} \Theta(\sqrt{\gamma} - |\omega - E_n|)\rho(r_i, E_n)}{\sum_{n=1}^{N} \Theta(\sqrt{\gamma} - |\omega - E_n|)}, \hspace{1cm} (13)$$

where $\rho(r_i, E_n) = |\Psi_n(r_i)|^2$ is the LDOS at site $i$ for the $n$th eigenstate $\Psi_n$, $N_s$ represents the number of disorder configurations to be averaged, and the energy bin $\gamma$ plays the same role as in section 2.2.
For the infinite-dimensional disordered system, it has been defined that the Anderson transition happens when \( \rho_L(\epsilon_F) \) vanishes completely at the Fermi surface \( \epsilon_F \). This criterion has been introduced to investigate the competition between the Anderson transition and the Mott MIT in the infinite-dimensional systems within the dynamical mean-field theory [40, 41], while, in the numerical calculation for a finite lattice of a low-dimensional disordered system, a finite energy bin \( \gamma \) has to be introduced. As a result, the GADOS is a function of lattice size, and its lattice size scaling should be introduced to detect the Anderson MIT.

In [38], we have examined the lattice size scale of the GADOS and found that, for any non-zero energy bin \( \gamma \), \( \rho_L(\omega) \) decays exponentially with the increase of lattice size for a localized electronic state. On the contrary, there is no significant variation of \( \rho_L \) with the increase of lattice size for a delocalized state. Therefore, it is not difficult to distinguish localized states from extended states by the lattice size scaling of the GADOS. In this paper, we use GADOS scaling to check our results obtained by the IPR scaling about whether the electronic states are localized or not, and good agreement has been achieved.

3. Localization in disordered graphene

The scaling theories of localization have proved that all electronic states should be localized in the conventional 2D systems with arbitrary weak disorder [10]. Compared with the parabolic DOS of a conventional 2D system, graphene has a special band structure with zero weight at Dirac points. Applying the Dirac model with random potentials to the weakly disordered graphene, some studies have confirmed the existence of anti-localized electronic states [15, 14] because of the insensitivity of the Dirac fermions to the disordered external electrostatic potentials. On the other hand, strong disorder can manifestly enhance the DOS at Dirac points and strongly affect the electronic properties. For this reason, our study will concentrate firstly on the evolutions of electronic states in the whole energy band with the increase of disorder strength.

We study the Anderson tight-binding model on a hexagonal lattice by exact numerical simulations, where the largest lattices could be \( 120 \times 120 \). Applying the IPR scaling method introduced in section 2.2, the localization lengths of the localized electronic states are obtained for different disorder strengths \( W \). As shown in figure 2, it is obvious that disordered graphene has a quite different scenario of Anderson localization from conventional 2D systems with disorder. When the disordered strength is \( W = 2 \) or 3, we find that the energy regions consist of localized electronic states separated by mobility edges from regions of extended ones, where the physical observables are the finite localization lengths of the localized states. Close to the top and bottom of the whole energy band, we obtain two symmetrical mobility edges at \( \omega = \pm \omega_F \). In addition, two mobility edges appear near the Fermi surface \( (\epsilon_F = 0) \) with \( \omega = \pm \omega_2 (\omega_3 < \omega_2) \), suggesting that the electronic states around the Fermi surface are also more easily localized by the scattering with the disorder potentials. Furthermore, all electronic states can be localized when the disorder strength is increased to more than \( W_c = 3.10 \).

The region close to the Dirac points is very important, and great interest has been shown in the possible existence of anti-localized electronic states there. However, the scaling results are affected greatly by the relatively large errors for the IPR since the DOS is quite low in the vicinity of Dirac points. In addition, it is well known that to distinguish an extended state from a weakly localized state with large localization length is very difficult. To make our finding more convincing, we also use the scaling of the GADOS to measure Anderson localization in disordered graphene. We find very good agreement between the scalings of the GADOS and IPR.

In figure 3, we show the lattice size dependences of the GADOS for some representative energies when \( W = 2 \). In figure 3(a), the GADOS of \( \omega = 3.25 \) increases manifestly with the increase of \( 1/L \), suggesting that the state is localized. Contrary to this, the GADOS of the other two extended states with energy \( \omega = 0.5 \) and 2.5 are both lattice size independent as shown in figure 3(b). When \( \omega = 0.0125 \), we have performed the scaling of the GADOS very carefully and precisely by averaging more than two hundred disorder configurations for a lattice with \( L = 120 \). As shown by the inset in figure 3(b), the lattice size independent behavior of the GADOS indicates clearly that the electronic states quite close to the Dirac points are still extended, in accordance with the result obtained by the IPR scaling. Our result suggests that the prediction of the single valley model, i.e. Dirac fermions cannot be localized by disorder, is reasonable in weakly disordered graphene since no inter-valley scattering can be stimulated by very weak disorder. In addition, it is obvious that the Anderson MIT can be introduced by only changing the carrier density to move the Fermi surface from an extended region to a localized region with fixed disorder strength. Recently, a transition between localization and anti-localization has been observed experimentally in graphene when the carrier density is decreased [6]. Therefore, our result...
Figure 3. The lattice size scaling of the GADOS of electronic states with different energies: (a) $\omega = 3.175$ (black circles), and (b) $\omega = 0.5$ (black triangles) and 2.5 (blue squares). Inset: the scaling of the GADOS close to the Dirac point with $\omega = 0.0125$ (green circles). The disorder strength of the Anderson disorder is $W = 2.0$.

Figure 4. (a) The energy dependences of the critical disorder strength $W_c(\omega)$ of graphene with correlated disorder; (b) the effects of the correlations of disorder on the DOS around the Fermi surface $\omega = 0$. The solid (black), dashed (blue), and dashed–dotted (red) lines represent, respectively, the results of point defects, NN scatters and both NN and NNN scatters. The correlation length $\zeta$ is set as $1.5a$, which is the same as the parameters chosen in [42], and the blue dashed and red dash-dotted lines reveal the energy dependence of the critical disorder strength $W_c(\omega)$ for graphene with the NN scatters only and with both NN and NNN scatters, respectively. For convenience of comparison, we also show the results of uncorrelated point defects (black solid line). It is obvious that the picture of the Anderson MIT for short-range scatters is about Anderson localization with four mobility edges helps to clarify this experimental observation of MIT in graphene. In addition, our findings suggest that the delocalization is closely related to the linear dispersion near the Dirac points which may suppress the inter-valley scattering. Further investigations will be made in a future study.

The electrostatic Coulomb potentials of the surface absorptions or the adatoms in the substrate for graphene should be described by equation (3), in which the correlations between the disorder potentials of individual sites are considered. Owing to the screening effect of electrons, the long-range correlations of disorder could be neglected. Here we name the uncorrelated and short-range correlated disorder as point defects and short-range scatters, respectively. In figure 4, we show the effects of the nearest neighbor (NN) and next NN (NNN) scatters on the localization of electronic states. The correlation length $\zeta$ is set as $1.5a$, which is the same as the parameters chosen in [42], and the blue dashed and red dash-dotted lines reveal the energy dependence of the critical disorder strength $W_c(\omega)$ for graphene with the NN scatters only and with both NN and NNN scatters, respectively.
the same as that of the point defects, while the short-range correlations of disorder have strong effects to localize the electronic states. As to why the critical strengths of disorder \( W_c \) for localizing the states are greater for uncorrelated disorder than for correlated disorder, the effective disorder strength is actually considerably enhanced as we add the influences of the impurity potentials of the adjacent sites into the uncorrelated on-site energy of a certain site directly. As a result, the localization of electronic states is enhanced accordingly when the re-scaling is not introduced to reduce the original disorder strength.

4. The interactions and electron–hole puddles in disordered graphene

In pure graphene, the influence of short-range interactions on electronic properties has been investigated by dynamical mean-field theory (DMFT) [43]. The Dirac sea state is found to remain stable against local many-body interactions since the interactions in graphene are much smaller than the Mott critical value \( U_c = 13.3t \) [43]. In the presence of disorder, the combined effects of interactions and disorder will have strong influences on the LDOS and also the localization of electronic states. It has been proved recently by statistic DMFT calculations [38] that the Hartree–Fock approximation (HFA) could give reasonable results for the conventional 2D Anderson–Hubbard model when the interactions \( U \) are smaller than the energy bandwidth \( D \). Based on the predictions for the many-body effect in graphene given by [3], we study the Anderson–Hubbard mode with weak interactions in the region of \( U = 0.5t-4.0t \) within the HFA.

In conventional 2D disordered systems, the zero-bias anomaly at the Fermi surface [28–30] arises from the interplay between disorder and interactions, which indicates the delocalization effects of interactions on localized electronic states [44]. In the same manner, we observe the DOS of disordered graphene at the Fermi surface to show the delocalization effects of the short-range interactions. Firstly, we only consider the effects of the on-site interactions \( U \), and, as shown in figure 5(a), significant decrease of the DOS around the Fermi surface appears when we increase \( U \) from 0.5 to 4.0. On the contrary, the long-range interactions are found to have less effect on the zero-bias anomaly at the Fermi surface. In figure 5(b), we plot the DOS for graphene with different NN interactions and fixed \( U \), and it is obvious that even the effects of NN interactions are still weak and negligible. Therefore, we predict that the on-site interaction is the key point in the study of the screening effects of interactions in disordered graphene. The local effect of interactions can be understood by calculating the screened potential \( u_i = \epsilon_i(1-U\chi_{ii}) \), where the local charge susceptibility is defined as \( \chi_{ii} = -\partial n_i/\partial \epsilon_i \).

It is obvious that the screening effect in the localized phase is expected to be less than in the metallic phase since \( \chi_{ii} \) is restrained for a site with small LDOS at the Fermi level. We find in the paramagnetic phase that the on-site interactions have strong effects to delocalize the electronic states by increasing the localization lengths accordingly.

Since the LDOS at the Fermi surface \( \epsilon_F \) is detectable, to compare with the experimental results, we calculate the evolution of \( \rho(r_i, \epsilon_F) \) with the on-site interactions \( U \) for a particular binary disordered configuration. As shown in figures 6 and 7, the length scale of density variations of LDOS for \( U = 0.0 \) is much smaller than that for \( U = 2.0 \) and 4.0. Compared with the quarter-filling case, the delocalization effects of interactions are not remarkable when the system is half-filled. It is obvious that the states at Dirac points are strongly localized at half-filling with \( W = 4 \). This further proves the results we obtained in section 3 from a different perspective. The regions of electron-rich and hole-rich puddles have been observed by the experiment of using a

![Figure 5](image-url) The evolutions of the DOS of graphene with disorder strength \( W = 4 \) and short-range interactions: (a) there are only on-site interactions \( U = 0.5 \) (black dashed line), 1.0 (red dashed line), and 2.0 (blue solid line), and (b) the on-site interactions are fixed as \( U = 2.0 \), and the NN interactions are given as \( V = 0.0 \) (black dashed line), 0.5 (red dashed line) and 1.0 (blue solid line).
Figure 6. (a) The disorder configuration of a $108 \times 108$ lattice with binary disorder. The on-site energy splitting is $W = 4$. The corresponding LDOS of an eigenstate right at the Fermi level for different on-site interactions: (b) $U = 0.0$, (c) 2 and (d) 4.0. The systems are all half-filled with impurity concentration $x = 0.2$.

Figure 7. The disorder configuration and the corresponding LDOS are shown for the same parameters as in figure 6, except that the systems are all quarter-filled.
scanning SET to map the LDOS of graphene sheet [7]. The smallest length scale on which density variations are observed is roughly 150 nm, which is apparently significantly larger than the intrinsic disorder length scale of approximately 30 nm in the graphene samples. There arises the question of why there exists such a large difference between the length scales of the LDOS and intrinsic impurities. Our results predict that the interactions play an important role in the electron–hole puddles observed by experiment, where the screening effects of the on-site interactions on disorder potentials can enlarge the length scale of the LDOS significantly.

5. Conclusions

We have studied numerically the Anderson tight-binding model of finite hexagonal lattices and found that, unlike conventional 2D disordered systems, there is a unique picture of the Anderson localization with four mobility edges in disordered graphene. We predict that the Anderson MIT can be achieved in graphene by changing the carrier density to make a move of the Fermi surface across the mobility edges. In addition, we have found that the length scale of the LDOS is considerably enhanced by the on-site interactions when the screening effects of interactions on the disorder potentials are also taken into consideration.

The measure of Anderson localization by the IPR has also been discussed, and a polynomial formula has been introduced for the lattice size scaling of the IPR. As a result, the precise localization length of the localized electronic state can be obtained by the intercept found in the infinite-size limit.

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