Geometric Optimisation of a Coupled Vane Compressor

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Abstract. The coupled vane compressor (CVC) is a rotary compressor with two coupled vanes which slide diametrically through the rotor during operation. As compared to most other rotary vane compressors, this CVC works well with small rotor-to-stator diameter ratio, typically less than that for all existing rotary compressors (>0.7). Hence, CVC has probably the most compact design among all rotary compressors, thus saving materials and manufacturing cost. This compressor is intended for use in room air-conditioners and refrigerators. Currently, mathematical model and measurement have been carried out to determine the working characteristics of this compressor. In this paper, a genetic algorithm with constraints handling has been devised and linked with the verified mathematical model of the CVC to search for a set of geometrical parameters which produces highest COP when working in the refrigeration cycle. During this optimisation search process, the significant design variables such as rotor radius, cylinder radius, vane thickness and valve reed thickness are allowed to vary.

In this paper, the details on the application of the optimization process, constraints handling and descriptions, objective function definitions as well as the details of optimization implementation will be presented and discussed. Results obtained will be shown and deliberated. The results show that this optimization study reconfirms the value of optimisation in compressor design.

1. Introduction

Rotary vane compressor can be found widely in their use as part of the cooling cycles in air-conditioners and refrigerators. They have a significant amount of space in their cylinders taken up by the rotor. This reduces the working volume per cylinder size for these cylinders. The CVC¹ is able to counter this with its small rotor-to-stator diameter ratio, allowing for it to be more compact. Due to this fact, it has the potential to consume less materials for its manufacturing than other rotary vane compressors.

This research aims to build upon this by optimizing the CVC. The simulator for the CVC produces a unique set of values of efficiencies for each set of geometric parameters input. A multi-objective optimization has been written whereby the efficiency values, COP, and minimized volume are the objectives, while the geometric parameters are the variables. This program will aim to reduce the amount of raw material used in manufacturing the CVC while also maximizing the performance of the CVC. Pareto optimality can be obtained for this with the plot for the inverse of volume versus the other objectives, that is the scaled operating parameters such that each is appropriately represented.
2. Coupled Vane Compressor
The coupled vane compressor is a novel compressor which improves on current designs by reducing the amount of material required for its manufacturing while ensuring that performance does not suffer as drastically if such a reduction of geometric parameters were made on other compressors. This is done by reducing the need for the rotors of the compressor to be a minimum ratio when compared to the cylinder.

2.1. Design
The compressor is designed with a twin-vane system running through the rotors diametrically, as shown in Figure 1. They slide along each other and allows for the overall vane to vary in length. To ensure that the rotors maintain contact with the cylinder walls and that there is no leakage from the compression chambers, the centrifugal force from the rotation of the vane will balance the various pressure forces acting on the vane tip and other surfaces.

![Figure 1: Coupled Vane Compressor Schematics](image)

2.2. Mathematical Model
The mathematical model is the design of the simulation program for the CVC, as created by Ooi and Shakya. It is arrived at through the derivations of the geometry of the working chamber, the thermodynamic properties of the working fluid, and analysis of the primary flows through the suction and discharge ports and of the secondary leakages through internal clearances. To allow for the program to run iterations at a faster rate, the heat transfer model for this simulation has been excluded.

3. Method
To carry out the multi-objective optimization as required by this aim of this paper, genetic algorithm, a form of evolutionary algorithm, has been chosen as it is able to handle the requirements of outputting and analyzing multiple objectives, as well as its ability to handle multiple inputs.

3.1. Genetic Algorithm
Genetic algorithm is a form of optimization in computer science which brings inspiration from the process of natural selection and Charles Darwin’s theory of evolution. It replicates the evolutionary processes of nature through operations such as selection, crossover, and mutation. It is able to obtain, through multiple iterations and generations, better results from a certain set of variables through the above-mentioned processes. The genetic algorithm chosen here is the NSGA-II.

Modifications were made to the base genetic algorithm code provided from the GreenPlan-IT planning tool, which allowed for the program to be used for the CVC simulation program. These modifications include the adjusting of the variable constraints to fit the new program and selecting the...
appropriate objectives to represent the fitness for the selection of ‘parents’ for the next generation of iterations.

The process of the NSGA-II code begins with the random creation of an initial population within the constraints as mentioned above. The variables chosen in this optimization process are the cylinder radius, rotor radius, cylinder height, vane length, vane thickness, and discharge valve reed thickness. This initial population will then be fed into the simulation program. The cylinder volume, COP, and overall efficiency are the outputs. These values form the two objectives which we will be maximizing and are used for ranking of the solution through non-dominated and crowding distance sorting. The two objectives chosen are the inverse of cylinder volume, and the product of the COP and the overall efficiency. The inverse of cylinder volume is chosen as the main goal of this optimization program is to reduce the size of the novel compressor and the amount of raw material required for its manufacturing. The product of COP and overall efficiency ensures that the overall performance is also pushed to be improved despite a decreasing compressor size.

\[
\text{maximise } Obj_i = f(x_{raw}, x_{rCr}, x_{slr}, x_{vt}, x_{rr}), \quad i = 1, 2
\]

\[
\text{where } Obj_1 = \frac{1}{\text{Cylinder Volume}}
\]

\[
\text{and } Obj_2 = \text{COP} \times \text{overall efficiency}
\]

Through ranking of the objectives from the first generation of results \(^5\), a ‘parent’ population is created which ‘favors’ the members which have a higher value. Selection for a reduced mating pool is done based on a tournament selection of randomly selected members using their ranking.

From the reduced pool, a next generation is created through crossover and mutation of the binary-coded chromosomes. This forms the ‘child’ population which is the same size as the initial ‘parent’ population.

![Halving of Global Population](image)

**Figure 2: Halving of Global Population\(^3\)**

The ‘child’ population is then combined with the ‘parent’ population to form a global population. The global population is ranked, after which a halving process occurs, as shown in Figure 2. A new ‘parent’ population is created.
This whole process is repeated over multiple iterations, as shown in Figure 3, until the maximum number of generations is reached or if the user chooses to end the program. Results for the variables and their objective values are recorded.

The constraints implemented for the iterations to be discussed is as follows:

\[ x_{ul} \leq x_{var} \leq x_{ul} \] represents the general constraint equation where \( x_{ul} \) and \( x_{ul} \) are the lower and upper limits of the variable respectively, and \( x_{var} \) is the value of the parameter to be evaluated.

\[
0.0150\,mm \leq x_{raw} \leq 0.0350\,mm
\]

\[
0.55 \leq x_{rcr} \leq 0.80
\]

\[
0.50 \leq x_{slr} \leq 2.00
\]

\[
0.20 \leq x_{vtr} \leq 0.30
\]

\[
0.0030 \leq x_{rr} \leq 0.0050
\]

Where \( x_{raw} \) is the cylinder radius, \( r_{cr} \) is the rotor to cylinder radius ratio, \( slr \) is the slenderness ratio, \( vtr \) is the ratio of vane thickness to rotor diameter, and \( rr \) is the ratio of discharge valve reed thickness to cylinder thickness. Conversion from these decision variables to the raw geometrical parameters of the CVC are shown in Figure 4. The reason for the storage of variables in mostly ratio is to prevent a situation in which the output optimized compressor has an impossible geometry. One example of an impossible geometry created through mating would be a solution in which the rotor radius is larger than the cylinder radius. This can easily be solved by setting \( r_{cr} < 1 \).
These values are then converted to the actual values of the rotor radius, cylinder radius, cylinder height, vane length, vane thickness, and reed thickness. Calculations had to be done to obtain the maximum and minimum allowable values of vane length based on the rotor and cylinder radius, and clearance gap length. Vane length is then taken to be the average of the two.

\[ L_{\text{min}} \leq L_{\text{vane}} \leq L_{\text{max}} \]  
\[ L_{\text{max}} = \sqrt{r_{\text{cyl}}^2 - \left( r_{\text{cyl}} - r_{\text{rot}} + \text{gap} + \frac{vane_{\text{t}} \times \cos \beta}{2} \right)^2} - \frac{vane_{\text{t}} \times \sin \beta}{2} \]  
\[ \beta = \cos^{-1} \left( \frac{r_{\text{cyl}} - r_{\text{rot}} + \text{gap}}{r_{\text{cyl}} \times \frac{vane_{\text{t}}}{2 \times r_{\text{cyl}}}} \right) \]  
\[ L_{\text{min}} = 1.01 \times (2 \times r_{\text{cyl}} - 2 \times r_{\text{rot}} + \text{gap}) \]  
\[ L_{\text{vane}} = \frac{L_{\text{max}} + L_{\text{min}}}{2} \]

There are 30 generations iterated, each with a population size of 20. The value for the probability of mutation is the inverse of the length of a chromosome, in this case 1/150 or 0.667%. Crossover probability is fixed at 90%. The variables for this genetic algorithm are binary coded into the chromosomes.

4. Results
Fitness of each generation are obtained and plotted as shown. Note that the opacity of the marker shows the density of data that falls on that point, while the size of the marker represents the generation number – the larger the marker, the greater the generation. Obj2 is the product of overall efficiency and COP. Inverse of Cylinder Volume represents the inverse of the size of the compressor.
The graphs show the change in each objective by dividing its value with the one of the lowest overall fitness values obtained – in this case, a datapoint obtained in generation 1, population member 7 (Inverse Volume = 0.00337, Obj2 = 0.500).
Objective values are plotted here against the generations to show their improvement through the iterations. Obj2 can be seen to be increasing while Cylinder Volume decreases, as is the purpose of the optimization.

Table 1: Final Objective Values and Geometrical Parameters

| 10th Generation | Solution 4 | Solution 20 |
|-----------------|------------|-------------|
| Objectives      | Inverse Cylinder Volume | 0.071783 | 0.076182 |
|                 | Obj2       | 1.769241 | 1.659964 |
| Decision Variables | raw       | 0.01613   | 0.01582   |
|                 | rcr        | 0.55156   | 0.66425   |
|                 | slr        | 0.52836   | 0.52794   |
|                 | vtr        | 0.23078   | 0.21320   |
|                 | rr         | 0.00321   | 0.00321   |

Table 1 shows the final optimized values obtained which belongs to solution 4 and 20 of the last generation. Each solution does not dominate the other for the objective values used. The final decision variables highlight that compressor design tends towards one with a generally smaller rcr. Solution 4 has a rcr value close to the minimum value of 0.55 and it has Obj2 being maximized. Solution 20 minimizes the volume of the cylinder with a lower Obj2 value. During this trade-off, the rcr value is observed to be higher at 0.664.

The general decrease in the rcr value could be due to the maximization of the maximum working chamber volume to cylinder size of the CVC. This maximization is shown to be beneficial to the defined performance of the compressor.

Another interesting parameter change to take note of during the optimization of the CVC is the tendencies for the other decision variables to approach each of their minimum set limits. These decreases brought about a better value for both the objectives identified.

It is important to note that results obtained from the optimization are from the mathematical simulation of the compressor. Physical testing should be carried out to confirm the accuracy of the calculations carried out.

5. Discussion

The results show that there is a general increase in the fitness of the algorithm, with the compressor size decreasing and Obj2 increasing. As seen from Figure 5, the front for each generation can be seen to be tending towards the upper right corner where the fitness can be maximized. The fitness for the inverse of volume can be seen to have increased in value by about 23 times when compared to the weakest member (gen 1, member 7) across all 30 generations. This means that volume decreased by
around 95% as can be seen from Figure 6. Figure 7 shows that Obj2 increased by around 3.5 times when compared to the same weakest member.

Both these results are significant in showing that NSGA-II optimization, through the varying of geometrical parameters, provides great value towards the creation of better compressor designs.

A general decrease is observed for the final geometrical parameters of the optimized CVC. This decrease is less distinct in the rcr as the final value tends to fluctuate between around 0.66 to 0.55. This could be due to the trade-off between the two objectives identified where a lower value of rcr favors a greater value for Obj2 and a lower value of rcr favors a greater value for the inverse of cylinder volume.

These results show that an increase in the working chamber volume of the compressor with respect to its cylinder size generally improves the performance of the CVC. This study shows that an optimized geometry for the CVC should be one where the rcr is relatively smaller and the working chamber to cylinder size is larger. Reduction in vane thickness, which increases working chamber volume without a change in cylinder size, is also seen to contribute towards a better performance.

6. Conclusion

The decrease of the size of the CVC without a decrease in its performance, could mean the reduction in the amount of raw materials and cost required for manufacturing. This can be achieved with a design optimization software. NSGA-II has proven to be able to achieve this and reconfirms the value of such multi-objective optimization in compressor design.

In conclusion, the geometry of an optimized CVC tends towards one with a smaller rotor-to-cylinder ratio and larger working chamber to cylinder size ratio. This might be because a larger working chamber volume allows the compressor to work at a higher efficiency.

The user-defined optimization parameters such as mutation and crossover probability can be tested at other values to evaluate how they affect diversity and convergence of the results. This will allow fitter and more diverse Pareto fronts to be obtained. As optimization is carried out through computer simulations from the mathematical model of the CVC, further work can be done through the manufacturing and physical testing of a prototype to check the accuracy of the simulation program used for the optimization.

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