Numerical study of a gas coupled VM-PT hybrid cryocooler using $^3$He as the working fluid

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Abstract. The two-stage Vuilleumier gas-coupling pulse tube cryocooler (VM-PT) is one kind of novel low-frequency cryocoolers. In this gas-coupled form, the single stage Vuilleumier cryocooler serves as both pressure wave generator and a pre-cooler for coaxial pulse tube. Compared with the most commercialized GM and GM pulse tube cryocooler, the two-stage VM-PT cryocooler is characterized by its high stability, compact size and thermal actuation which are indispensable for space application. It has already been verified experimentally that this cryocooler can obtain 9.75mW@4.2K and the lowest no-load temperature 3.39K when $^4$He as the working fluid. However, such refrigerating capacity seems not enough for further application. $^3$He as a more potential substitution of $^4$He has better physical properties to improve performance, which has been studied in GM type and Stirling pulse tube cryocooler. For further optimization, a numerical study on the specific performance of two-stage VM-PT cryocooler using $^3$He is carried out in the present paper though Sage software. Working at the frequency of 1.0Hz and the pressure of 0.8MPa, the two-stage VM-PT cryocooler with $^3$He obtained 50mW@4.06K. The usage of $^3$He was 0.0038kg, about 30L under STP. At 4.2K, using $^3$He can obtain 58mW cooling power and 0.49% relative Carnot efficiency, about 1.6 times higher than using $^4$He.

1. Introduction

The requirements of cryocoolers operating at liquid helium temperature with a cooling power about 10-500 mW are becoming urgent due to their extensive applications in aerospace exploration, materials technology, physics and medical science. G-M cryocoolers and G-M pulse tube cryocoolers working at 4.2 K have been commercially applied for many years. However, low efficiency and rather big bulk limit their application, especially in the space science. Recently, multi-stage Stirling pulse tube cryocooler (SPTC) working at liquid helium temperature attracted many efforts on multi-stage structure optimization, effective phase shifter design and thermodynamic optimization and analysis by its reliability and compactness [1-5]. But its high-frequency operation makes SPTC difficult to attain high efficiency at liquid helium temperature due to the large irreversible losses in the regenerator.

Vuilleumier (VM) cryocooler is one kind of low-frequency Stirling cryocooler driven by a thermal compressor. Compared with GM-type cryocooler, the VM-type cryocooler has more compact size and smaller weight for avoiding such large compressor of the GM-type cryocooler. Also, there is no oil inside of the VM-type system, which ensures the stability of performance in complicated situations. VM cryocooler was first patented by Rudolph Vuilleumier in 1918 [6] and it had been applied in aerospace exploration in the 1970s [7]. Chellis et al. developed VM cryocooler and attained 15 K by using liquid...
nitrogen cooling the middle cavity [8]. Zhou optimized the main parameters of this kind of cryocooler and obtained the no-load temperature of 10.7K. After coupled one stage Simon expansion refrigerators, this hybrid cryocooler could get 40ml/h liquid 4He [9]. Recently, a series of theoretical analyses and experimental improvements of VM cryocooler charging 4He had been conducted [10-11]. For single stage VM cryocooler, a terminal temperature of 7.35 K was attained by using rare earth materials in cold regenerator [12]. Pan et.al reported a novel VM-PT cryocooler capable of attaining 4.4K when using a room temperature phase shifter, the cooling power at 5.6K is 40mW [13]. For avoiding the heat leakage caused by room temperature phase shifter, Zhang et.al studied the cold phase shifter through numerical and experimental methods finally obtained the lowest no-load temperature of 3.39K and cooling power of 9.75mW at 4.2K [14].

Figure 1 The comparison of thermal properties between 3He and 4He at 1.0MPa under 10K

However, such cooling capacity of the present VM-PT cryocooler at 4.2K obtained with 4He is not enough for practical applications if the cooling rate and extra losses are considered. 3He as a more potent substance has already been proved to achieve better performance effectively in GM type cryocoolers. De Waele et al. indicated that 3He could produce the cooling effect through expansion at lower temperature than 4He because the volume expansion coefficient of 3He dropped to zero at 1K, in the contrast, the volume expansion coefficient of 4He was very close to zeros at 2K and even became negative at lower temperature [15-16]. Fig.1 provides a more intuitive expression of the theory above. It is clear that the volumetric heat capacity of 3He is lower than that of 4He, which is good for the regenerator efficiency, and when the temperature drops below 7K, the volume expansion coefficient of 3He is always higher than 4He. At 2.17K, the value of volume expansion coefficient of 3He is 0.02, over twice of 4He, furthermore, the value of 3He will reduce to 0.01 under 1.5K where the value of 4He is very close to zeros or even negative. They verified the superiority of 3He by a three-stage GM pulse tube cryocooler which had achieved 1.87K with 3He and 2.19K with 4He in 1999 [16]. The lowest temperature of two-stage GM cryocooler was 1.47K with 3He by Numazawa et al. [17]. So far, the two-stage GM pulse tube cryocooler could obtain 1.27K with 3He, regarded as the lowest temperature of regenerative cryocoolers [18].

In addition, R. Radebaugh et al. studied the packed sphere regenerators through REGEN3.3 operating with 3He less than 0.3 to 1.5MPa and between 4 K and 10 K, also a detailed comparison was made with 4He. It indicated that both for 3He and 4He, the COP would increase and regenerator loss would decrease with the decreasing of porosity. For 4He, the regenerator performance was optimal when pressure below 1 MPa, whereas such pressure did not benefit 3He regenerators. The COP of a 3He regenerator with 0.2
porosity operating at 30Hz and 0.5 MPa is about 3.8 times higher than a helium-4 regenerator using packed spheres with 0.38 porosity. So, it can be inferred that $^3$He is much helpful in the mechanical cryocoolers at 4K even lower temperature [19].

Therefore, this paper presented a numerical investigation of two-stage VM-PT cryocooler charging with $^3$He. The cooling capacity of two-stage charging with $^3$He and $^4$He were compared at the different operating frequency and working pressure. For further analyzing the difference, the cooling capacity and relative Carnot efficiency were also taken into consideration.

![Figure 2 The schematic structure of two-stage VM-PT hybrid cryocooler](image)

### 2. Physical parameters and simulation models

#### 2.1. Physical parameters

The schematic structure of two-stage VM-PT hybrid cryocooler was showed in Figure 2. In its gas-coupled form, the Vuilleumier cryocooler serves as both pressure wave generator and pre-cooler for coaxial pulse tube cryocooler. For avoiding the heat leakage from room temperature, the cold phase shifter was selected and fixed on the VM cold end. It included two pieces of capillary and an annular reservoir, which also worked as the radiation shield of pulse tube. Based on the previous experimental experience with $^4$He, the VM cold regenerator was filled with two layers of regenerative materials and the pulse tube regenerator was filled with three layers. The detailed physical parameters of two-stage VM-PT cryocooler were listed in table 1.

| Components | Parameters |
|------------|------------|
| VM stage   |            |
| Hot displacer (Annular) | Diameter: 95 mm, length: 165 mm, stroke: 32 mm |
| Hot regenerator (Annular) | Diameter: 23.6/45 mm, length: 128 mm, filling pattern: 80# SS |
| Cold displacer | Diameter: 26 mm, length: 190 mm, stroke: 20 mm |
| Cold regenerator | Diameter: 23.6 mm, length: 125 mm, filling pattern: 200# SS*45mm+lead sphere (0.4-0.45 mm)*85mm |
| PTC        |            |
| Regenerator (Annular) | Diameter: 8.9/18 mm, length: 70 mm, filling pattern: Er$_3$Ni sphere (0.2-0.25 mm)*20mm+ HoCu$_2$ sphere (0.2-0.25 mm)*50mm |
| Pulse tube | Diameter: 8.5 mm, length: 70 mm |

#### 2.2. Mathematical model
The SAGE software was a general 1-D solver to analyze the performance and inner state of cryocoolers. In order to operate with greater ease and get results faster, spectral element method and modular packaging were adopted in this software. The conventional governing equations were translated into the connection of energy, mass flow and pressure for all component modules.

The 1-D simulation model of two-stage VM-PT hybrid cryocooler was showed in Figure 3. The entire model consisted of single stage VM cryocooler and pulse tube cryocooler with cold phase shifter. The gas-coupled form was realized by building energy and mass connection between two stages. The boundary conditions of both hot cavity and middle cavity were isothermal, exchanging heat at 300K and 77K respectively. Because the solid properties like specific heat and thermal conductivity under 3K were not offered in Sage, the no-load temperature of this cryocooler could not be studied directly. So, a constant heat flux of 50mW was attached at the cold end of pulse tube to promise the results were convergent and reliable.

![Figure 3 1-D simulation model of two-stage VM-PT hybrid cryocooler in Sage](image)

3. Numerical results and discussion

3.1. Operating frequency

Figure 4 showed the temperature of both VM and pulse tube cold end and the heat consumption at 77K as a function of operating frequency from 0.6 to 2.0Hz at 0.9MPa. With the growing of frequency, the heat consumption at 77K increased both for charging $^3$He and $^4$He, and using $^4$He were little higher than $^3$He. For the PT cold end, using $^4$He obtained the lowest 4.68K with 50mW load at 0.8Hz while $^3$He got...
the lowest 4.06K with 50mW load at 1.0Hz. If the frequency deviated from the optimal value, the temperature would increase with the variation of frequency. Using $^3$He could always provide lower temperature of PT cold end with the growth of the frequency. Meanwhile, for the VM cold end, using $^4$He could obtain the lowest temperature of 10.91K at 1.1Hz and using $^3$He could get the lowest temperature of 11.19K at 1.2Hz. The numerical results illustrated that the optimal frequency of temperature of PT cold end is lower than that of VM cold end, no matter using $^3$He or $^4$He. Overall, using $^3$He provided better cooling capacity of PT cold end than $^4$He among 0.6 to 2.0Hz.

Figure 5 The comparison of acoustic power distribution in pulse tube regenerator

The comparison acoustic power distribution in pulse tube regenerator between $^3$He and $^4$He at 1Hz were showed in Figure 5. When $^3$He was charged, the input acoustic power of pulse tube regenerator was 2.28W, the output is 1.125W. For $^4$He, the input and output were 1.839W and 0.981W, respectively. Although the acoustic power dissipated 0.297W more along regenerator compared with $^4$He, the input and output acoustic power of $^3$He were both higher than $^4$He, which contributed to the higher performance and lower cooling temperature of $^3$He.

Figure 6 The influence of operating pressure

3.2. Operating pressure
As shown in Figure 4, the optimal frequency for $^3\text{He}$ was 1.0Hz. For further optimization, the influence of operating pressure was studied at 1.0Hz, then the results were shown in Figure 6. Similar with influence of frequency, the heat consumption at 77K of using $^3\text{He}$ or $^4\text{He}$ were increased with the growth of pressure. Under the similar cooling capacity, the high consumption means low efficiency. Also, the $^3\text{He}$ is very rare so the working pressure should be lower to reduce the usage amount of $^3\text{He}$. For the VM cold end, the temperature increased with the growth of pressure both for using $^3\text{He}$ and $^4\text{He}$. For the PT cold end, the optimal pressure of $^3\text{He}$ was around 0.8-0.9MPa, lower than that of $^4\text{He}$, around 1.2-1.3MPa. At the optimal pressure, the usage of $^3\text{He}$ was about 0.0038kg in system, equivalent to the volume of 30L gas under STP. So, it indicated that using $^3\text{He}$ can obtain 50mW cooling power on PT cold end reliably.

![Image](image1.png)

**Figure 7** The distribution of acoustic power

![Image](image2.png)

**Figure 8** Cooling capacity and relative Carnot efficiency of using $^3\text{He}$ and $^4\text{He}$

Figure 7 showed the pressure ratio and time-averaged enthalpy flow of the cold end of the pulse tube regenerator changed with the working pressure. Both two parameters were positively correlated with the working pressure. The increasing of pressure ratio contributed to improving the cooling performance. However, with the growth of working pressure, the heat storage capacity of pulse tube regenerator showed inadequate to fit the larger mass flow rate, so the time-averaged enthalpy out of the pulse tube.
regenerator would increase. That resulted there was an optimal pressure to the lowest cooling temperature, as shown in Figure 6. Because the $^3$He had lower density than $^4$He below 10K, under the same working pressure, using $^3$He had lower pressure ratio.

3.3. **Comparison of cooling capacity and efficiency**

Figure 8 showed the comparison of cooling capacity and relative Carnot efficiency of using $^3$He and $^4$He at 4.2K, 5K and 6K. The both cooling power curves and efficiency curves increased with the increasing of cooling temperature, and the growth rate of using $^3$He was bigger than $^4$He. At 4.2K, the cryocooler with $^3$He can offer 58mW cooling capacity, but it with $^4$He can only offer 33mW, nearly half of the former. The relative Carnot efficiency of charging $^3$He and $^4$He are 0.49% and 0.19% respectively. At 5K and 6K, the VM-PT with $^3$He can provide 115mW and 217mW cooling power and the input power were all less than 1000W. For relative Carnot efficiency, the $^3$He must be an ideal substitution of $^4$He for two-stage VM-PT cryocooler.

4. **Conclusion**

This paper reported a numerical study of a gas coupled VM-PT hybrid cryocooler using $^3$He as the working fluid and a detailed comparison with using $^4$He. By optimizing the frequency and working pressure and comparing with $^4$He, the advantage of using $^3$He in this type cryocooler became obvious. The lowest temperature with 50mW was 4.06K at 1Hz and 0.8MPa, which indicated that charging with $^3$He, the two-stage VM-PT cryocooler can obtain 50mW@4.2K reliably. At last, the relative Carnot efficiency and cooling power changed with cooling temperature were studied. At 4.2K, it can reach 0.49% by $^3$He and was over twice than the highest efficient obtained by $^4$He at 4.2K.

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