COSMIC-RAY STREAMING FROM SUPERNOVA REMNANTS AND GAMMA-RAY EMISSION FROM NEARBY MOLECULAR CLOUDS

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ABSTRACT

High-energy gamma-ray emission has been detected recently from supernova remnants (SNRs) and their surroundings. The existence of molecular clouds near some of the SNRs suggests that the gamma rays originate predominantly from $p-p$ interactions with cosmic rays (CRs) accelerated at a nearby SNR shock wave. Here we investigate the acceleration of CRs and the gamma-ray production in the cloud self-consistently by taking into account the interactions of the streaming instability and the background turbulence both at the shock front and in the ensuing propagation to the clouds. We focus on the later evolution of SNRs, when the conventional treatment of the streaming instability is valid but the magnetic field is enhanced due to Bell’s current instability and/or the dynamo generation of magnetic field in the precursor region. We calculate the time dependence of the maximum energy of the accelerated particles. This result is then used to determine the diffusive flux of the runaway particles escaping the shock region, from which we obtain the gamma spectrum consistent with observations. Finally, we check the self-consistency of our results by comparing the required level of diffusion with the level of the streaming instability attainable in the presence of turbulence damping. The energy range of CRs subject to the streaming instability is able to produce the observed energy spectrum of gamma rays.

Key words: acceleration of particles – cosmic rays – instabilities – ISM: supernova remnants – magnetohydrodynamics (MHD) – scattering – turbulence

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1. INTRODUCTION

Cosmic rays (CRs) below the knee are generally believed to originate from shock acceleration at the supernova remnants (SNRs). Although the process was proposed decades ago, it has not been easy to constrain the model from observations. Recent advances on gamma-ray observations provide us a possibility to study the acceleration and ensuing propagation processes in detail. In particular, the molecular clouds in the vicinity of an SNR are good testbeds because of their high density. Indeed, clouds illuminated by CRs accelerated at a nearby SNR can be bright gamma-ray sources (Aharonian & Atöyan 1996; Gabici et al. 2009).

Gamma rays are produced through the decay of neutral pions generated in $p-p$ collisions (hadronic origin) or through inverse Compton interactions by energetic electrons with ambient photon target fields such as the cosmic microwave background radiation (leptonic origin). It is still under debate whether the observed gamma-ray emissions are of the hadronic or leptonic origin although the hadronic scenario is favored by many authors because of the evidence of strongly amplified magnetic fields near SNRs (see the review by Blasi 2010 and references therein).

We consider the gamma-ray emission from massive molecular clouds near SNRs, motivated, in particular, by the recently detected gamma-ray emission from the SNR W28. As an old SNR, the emission from W28 is perceived to be of purely hadronic origin since the life time of high energy electrons is short. The observed gamma emission from W28 indicates that the flux of CRs is much enhanced there compared to the typical Galactic values. This places a constraint on the CR diffusion. The enhancement of scattering corresponding to a reduction (1%–10%) of the CR diffusion coefficient compared to the Galactic mean has been suggested by a few earlier models to match the observations (Fujita et al. 2009; Ohira et al. 2011; Li & Chen 2010). No physical justification on how and why the scattering is boosted has been provided, however. In addition, earlier studies made a few serious assumptions including the Bohm diffusion and a phenomenological power-law evolution of the maximum energy accelerated at a certain epoch. We shall reinvestigate this problem by incorporating a proper physical description of the relevant processes, i.e., the shock acceleration in the presence of the streaming instability and nonlinear damping processes by background turbulence. The slower diffusion implies enhanced wave perturbations, which may arise from streaming instability (see, e.g., Longair 2002). Earlier work has shown that streaming instability is limited by background turbulence (Yan & Lazarian 2002; Farmer & Goldreich 2004; Beresnyak & Lazarian 2008). This could be the determinative factor for the maximum energy attainable at the shock front as suggested by Ptuskin & Zirakashvili (2005). However, the isotropic Kolmogorov scaling for the turbulence, adopted by Ptuskin & Zirakashvili (2005), is not applicable to MHD turbulence. Moreover, if the enhanced scattering in the vicinity of SNRs as indicated by the observations are due to the increased flux of CRs there, we need to examine whether the flux of accelerated particles is sufficient to induce high enough growth rates of the streaming instability to overcome the nonlinear damping by background turbulence.

In this paper, we apply our present day understanding of the interaction between the streaming instability and the background turbulence to the modeling of the gamma-ray emission from molecular clouds near SNRs. We shall treat the problem in a self-consistent way by comparing the streaming level that is allowed by the preexisting turbulence and the required diffusion...
In Section 2 we determine the maximum energy of accelerated particles at shocks, in Section 3 we calculate the spectrum of CRs in the vicinity of SNRs, in Section 4 we obtain the spectrum of gamma ray produced by the $p-p$ interactions of the escaping CRs in a nearby cloud and compare it with observations, and in Section 5 we check the self-consistency of our result by comparing the required the diffusion near SNRs and the streaming level allowed in the presence of turbulence. The discussion and summary are provided in Sections 6 and 7. In the Appendix we provide a list of notations used in the paper.

2. MAXIMUM ENERGY OF CRs ACCELERATED AT THE SHOCKS

Diffusive shock acceleration of energetic CR particles relies on the crucial process of amplification of MHD turbulence so that particles can be trapped at the shock front long enough to be accelerated to the high energy observed. One of the most popular scenarios that has been adopted in the literature is the streaming instability generated by the accelerated particles. However, in the highly nonlinear regime the fluctuations of magnetic field arising from the streaming instability get large and the classical treatment of the streaming instability is not applicable. We circumvent the problem by proposing that the field amplification we consider does not arise from the streaming instability, but is achieved earlier through other processes, e.g., the interaction of the shock precursor with density perturbations preexisting in the interstellar medium (Beresnyak et al. 2009). Due to the resonant nature of the streaming instability, the perturbations $\delta B$ arising from it are more efficient in scattering CRs compared to the large scale fluctuations produced by non-resonant mechanisms, e.g., the one in Beresnyak et al. (2009). Therefore in this paper, we limit our discussions to the regime of $\delta B \lesssim B_0$, where $B_0$ is the magnetic field that has already been amplified in the precursor region.4

When particles reach the maximum energy at a certain time, they escape and the growth of the streaming instability stops. Therefore we can obtain the maximum energy by considering the stationary state of the evolution. The steady state energy density of the turbulence $W(k)$ at the shock is determined by

$$ (U \pm v_A)\nabla W(k) = 2(\Gamma_{cr} - \Gamma_d)W(k), \quad (1) $$

where $U$ is the shock speed, and the term on the left-hand side represents the advection of turbulence by the shock flow. $v_A \equiv B_0/\sqrt{4\pi nm}$ and $n$ are the Alfvén speed and the ionized gas number density of the precursor region, respectively. The plus sign represents the forward propagating Alfvén waves and the minus sign refers to the backward propagating Alfvén waves. The terms on the right-hand side describes the wave amplification by the streaming instability and damping with $\Gamma_{cr}$, $\Gamma_d$ as the corresponding growth and damping rates of the wave. The distribution of accelerated particles at strong shocks is $f(p) \propto p^{-4}$. If taking into account the modification of the shock structure by the accelerated particles, the CR spectrum becomes harder. Assume the distribution of CRs at the shock is $f_0(p) \propto p^{-4+\epsilon}$. The nonlinear growth was studied by Ptuskin & Zirakashvili (2005).

It was demonstrated by Schlickeiser & Shalchi (2008) that waves can grow or damp at the shock precursor depending on the spatial boundary conditions. If assuming an equal amount of forward and backward waves at the shock front, the forward wave will be growing and the backward wave will be efficiently damped in the upstream region. We therefore neglect the backward moving wave mode and consider here only the growing forward moving mode, which is the one that effectively contributes to the particle scattering. The generalized growth rate of streaming instability then is

$$ \Gamma_{cr} = \frac{12\pi^2 q^2 v_A \sqrt{1 + A^2}}{c^2 k} $$

$$ \times \int_{p_{res}}^{\infty} dp \frac{1 - \left(\frac{p_{res}}{p}\right)^2} {D \frac{df}{dx}}. $$

(2)

where $q$ is the charge of the particle, $c$ is the light speed, $p_{res} = ZeB_0\sqrt{1 + A^2}/c/k_{res}$ is the momentum of particles that resonate with the waves. $A = \delta B/B_0$ is wave amplitude normalized by the mean magnetic field strength $B_0$.

$$ D = \sqrt{1 + A^2 v_r^2 / 3 A^2 (> k_{res}) } $$

is the diffusion coefficient of CRs, $v_r$ and $r_s$ are the velocity and Larmor radius of the CRs. The distribution function of CRs is

$$ f_0 = \frac{3\xi nmU^2 H(p_{max} - p)}{4\pi c \phi(p_{max})(mc)^{\alpha} p^{4-\alpha}}, $$

$$ \phi(p_{max}) = \int_{0}^{p_{max}/mc} dy y^{\alpha} \frac{\sqrt{1 + y^2}}{\sqrt{1 + \gamma^2}}, $$

(4)

where $\xi$ measures the ratio of CR pressure at the shock and the upstream momentum flux entering the shock front, $m$ is the proton rest mass, and $p_{max}$ is the maximum momentum accelerated at the shock front. $H(p)$ is the Heaviside step function.

In the planar shock approximation, the distribution of accelerated particles is

$$ f_1(p, x) = f_0(p) \exp \left(- \frac{d}{D(p, x)} \int_{0}^{x} dx \right) $$

(5)

at the upstream of the shock ($x \geq 0$) and $f = f_0$ at the downstream. Inserting Equations (3)–(5) into Equation (2), one gets the following growth rate of the upstream forward moving wave at $x = 0$,

$$ \Gamma_{cr}(k) = \frac{C_{cr} \xi U^2 (U + v_A) k^{1-\alpha}}{1 + A^2(1-\alpha)/2 cv_A \phi(p_{max}) r_0^2}, $$

(6)

where $C_{cr} = 9/2/(4 - \alpha)/(2 - \alpha)$, $r_0 = mc^2/q/B_0$. The linear damping is negligible since the medium should be highly ionized. In fully ionized gas, there is nonlinear Landau damping, which, however, is suppressed due to the reduction of particles’ mean free path in the turbulent medium (see Yan & Lazarian 2011; Brunetti & Lazarian 2011); we therefore neglect this process here. Background turbulence itself can cause nonlinear damping to the waves (Yan & Lazarian 2002). Unlike hydrodynamic turbulence, MHD turbulence is anisotropic with eddies elongated along the magnetic field. The anisotropy increases with the decrease of the scale (Goldreich & Sridhar 1995). At the scales of the Larmor radii $r_s$ of the CRs, which are also the wavelengths $1/k$ of the waves induced by the streaming instability, the scale disparity becomes very large.
with $k_{\perp} > k_f$, where we use $k_f$ to distinguish the parallel component of the turbulence wave packet wavenumber $k_f$ from the parallel wavenumber of the growing wave $k_{\parallel}$. In MHD turbulence, wave packet cascades after it travels a distance of $1/k_{\parallel} \sim L_{\parallel}/k_{\perp} \gg 1/k_{\perp}$. On the other hand, the instability grows fastest for the most parallel wave ($k_0 \sim k_{\parallel}$) allowed in a turbulence medium with their wave numbers satisfying $k_{\perp}/k_{\parallel} \sim \delta B/B \sim (k_{\parallel}L)^{-1/3}$. Because of the scale disparity, $k_{\parallel} > k_{\perp} \gg k_f$, the nonlinear damping rate in MHD turbulence is less than the wave frequency $k_{\parallel}v_A$, and it is given by (Farmer & Goldreich 2004; Yan & Lazarian 2004; Beresnyak & Lazarian 2008)

$$\Gamma_d \sim \sqrt{k_{\parallel}/v_A},$$  \hfill (7)

where $L$ is the injection scale of background turbulence, and $k$ is set by the resonance condition $k \sim k_{\parallel} \sim 1/r_L$. Inserting Equations (6) and (7) into Equation (1) and adopting $U_dW/\delta x \approx U^2W/D$ in the case of efficient wave amplification, one gets

$$\frac{3U^2A^2}{2v(1 + A^2)} + v_A\sqrt{kL} = \frac{C_{\text{ce}}\xi U^3(U + v_A)}{c_vA\phi(p_{\text{max}})(kr_0)^2(1 + A^2)(1 + a)^2}. \hfill (8)$$

There are various models for the diffusive shock acceleration. We consider here the escape-limited acceleration. In this model, particles are confined in the region near the shock where turbulence is generated. Once they propagate far upstream at a distance $l$ from the shock front, where the self-generated turbulence by CRs fades away, the particles escape and the acceleration ceases. The characteristic length that particles penetrate into the upstream is $D(p)/U$. The maximum momentum is reached when $D(p)/U \approx 1/4$.\textsuperscript{5} Assuming $l \propto R_{\text{sh}}$, then the maximum momentum of particles accelerated during the Sedov phase is determined by the condition

$$D(p_{\text{max}}) = \kappa U R_{\text{sh}}, \hfill (9)$$

where $\kappa < 1$ is a numerical factor, see Table 2. From Equations (3) and (9), we get

$$\frac{p_{\text{max}}}{mc} = \frac{3\kappa A^2U R_{\text{sh}}}{\sqrt{1 + A^2}vr_0}. \hfill (10)$$

Inserting Equation (10) into Equation (8), we get for $A < 1$\textsuperscript{6}

$$\frac{p_{\text{max}}}{mc} = \left[\left(-v_A\sqrt{\frac{1}{r_0L}} + \frac{v_A^2}{r_0L} + \frac{2C_{\text{ce}}\alpha^3U^3(U + v_A)}{\kappa r_0R_{\text{sh}}c_vA}\right)^2\right]^{1/2},$$

$$A = \frac{p_{\text{max}}r_0}{\sqrt{18mcU R_{\text{sh}}} \left[1 + \sqrt{1 + \frac{2\kappa \mu U R_{\text{sh}}}{p_{\text{max}}r_0}}\right]^2}, \hfill (11)$$

where $\phi(p_{\text{max}})$ is approximated by $(p_{\text{max}}/mc)^{3/2}/a$.

\textsuperscript{5} The factor $1/4$ arises from the following reason. As pointed out by Ostrowski & Schlickeiser (1996), the spectrum is steepened for small $l$, i.e., $lU/D(p) \leq 4$.

\textsuperscript{6} We neglect a factor of $\sqrt{1 + A^2}$ here.
at a given energy $E = cp$, there is a one to one correspondence between the CR momentum and the CR escape time $t_{\text{esc}}$ (or radius $R_{\text{esc}} = (1 + \kappa) R_{\text{sh}}$) in spite of the fact that the acceleration is continuous during the shock expansion since $p_{\text{max}}(t)$ reaches $p$. If the maximum momentum has a power-law dependence on $t$, one can easily gets $t_{\text{esc}} \propto t^{-1/\delta}$. Since we consider the later stage of SNR acceleration, the shock radius cannot be neglected and the point source assumption is not applicable in general. One needs to take into account the spatial distribution of the sources. Nevertheless, in the case that the diffusion distance $R_d = 2 \sqrt{D \tau_{\text{age}} - t_{\text{esc}}}$ is larger than the radius $R_{\text{esc}}$, the distribution function of energetic proton at a distance $r$ can be described as if the CRs were from a point source (see, e.g., Ohira et al. 2011):

$$F(E) \approx \frac{f(E)}{\pi^{3/2} R_d^3} \exp \left[ -\left( \frac{r}{R_d} \right)^2 \right].$$

(16)

The diffusion coefficient $D = \chi D_{\text{ISM}}$, where $D_{\text{ISM}} = 10^{28} \sqrt{E/100 \text{GeV}} \text{cm}^2 \text{s}^{-1}$ (Gabici et al. 2009; Berezh zinskii et al. 1990). $f(E)$ is the spectrum of runaway particles that is integrated over shock expansion.

We adopt the routine from Ohira et al. (2010) here. If the accelerated particles at a given time is

$$f(p, t) dp dt = K(t) \left( \frac{p}{m_c} \right)^{-2 + \alpha} \exp \left[ -\frac{p}{p_{\text{max}}(t)} \right] d \ln t dp,$$

then the general spectrum of protons dispersed from the accelerator is

$$f(p) \propto \frac{p^{-1+\alpha} K(p_{\text{max}}^{-1}(p))}{p_{\text{max}}^{-1}(p) [dp_{\text{max}}/dt]_{t = p_{\text{max}}^{-1}(p)}}$$

(18)

where $p_{\text{max}}^{-1}(p)$ is the inverse function of $p_{\text{max}}(t)$. $K(t)$ is a normalization factor and can be estimated from

$$P_{\text{cr}} \approx \frac{4 \pi}{3} \int_{p_{\text{min}}}^{p_{\text{max}}} dp \nu(p) f_0(p) = \frac{\xi n U_0^2}{3}.$$  

(19)

In the case of flatter spectrum at the shock front, i.e., $a > 0$, $K(t) \propto U^2 R_{\text{sh}}^2 p^{-a}$. Evaluation of $dp_{\text{max}}/dt$ from Equation (11) shows that it can be well represented by a power law in spite of the fact that $p_{\text{max}}$ does not follow an exact power law. Inserting $K(t)$ and $dp_{\text{max}}/dt \approx p_{\text{max}}/t$ into Equation (18), we get a universal power spectrum $F(E) \propto \eta E^{\delta} E^{-\alpha}$ with $\delta = 2$ regardless of the flatter original spectrum at the shock front. $\eta$ is the fraction of SN energy converted into CRs.

5 The influence of CR cooling on the spectrum index is negligible (Ohira et al. 2010).

6 It is a good approximation for $E_{\text{max}} \lesssim 1000 \text{ GeV}$ (or $t > 4 \text{ kyr}$) in the case we consider (see Figure 1).

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**Table 1**

| Physical Parameters of W28 |
|-----------------------------|
| $r$ (kpc) | $E_{\text{SN}}(10^{51})$ (erg) | $M_{\text{e}}$ ($M_\odot$) | $t_{\text{age}}$ (kyr) | $U_1$ (km s$^{-1}$) | $L$ (pc) | $n$ (cm$^{-3}$) | $R_{\text{c}}$ (pc) | $T$ (K) | $B_0$ ($\mu$G) | $B_{\text{cov}}$ ($\mu$G) |
|-----------------------------|
| 1.8 | 0.5 | $4 \times 10^4$ | 50 | 5500 | 30 | 8 | 12 | $10^6$ | 200 | 2 |

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**Table 2**

| Model Parameters Adopted in the Paper |
|---------------------------------------|
| $a$ | $\chi$ | $\eta$ | $\kappa$ | $\xi$ | $\alpha$ |
|---------------------------------------|
| 0.1 ~ 0.3 | $-0.05$ | $-0.3$ | 0.04 ~ 0.1 | 0.2 ~ 0.4 | 0.5 |

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![Figure 2](image-url)  

**Figure 2.** Spectrum of CRs at a distance $r = 12$ pc after 1800 (solid line), 6000 (dotted line), 12,000 (dash-dotted line), and 50,000 years (cross line). The Galactic mean is plotted as a reference (dashed line). (A color version of this figure is available in the online journal.)
shown as dotted points (Abdo et al. 2010), and the H.E.S.S. data are plotted as “•” points (Aharonian et al. 2008) with error bars. Solid line is our result. (A color version of this figure is available in the online journal.)

4. GAMMA-RAY FLUX IN THE VICINITY OF SNRs

In the case \( F(E) \) follows a power-law distribution, the pion gamma-ray emissivity is given by (see Aharonian & Atoyan 1996)

\[
q(E_\gamma) \approx \frac{16\pi f_\gamma^{-2}}{s^2} \sigma_{pp} F(E_\gamma) c \eta_A, \tag{20}
\]

where \( E_\gamma \approx E_\gamma/10 \) is the corresponding \( \gamma \) ray energy, \( \sigma_{pp} \approx 30 \times [0.95 + 0.06 \ln(E_p/\text{GeV})] \text{mb} \) is the cross section for \( \pi^{-} \) collisions at \( E_p \), \( f_\pi \) is the fraction of energy that is transferred from parent protons to secondary pions, \( \eta_A \approx 1.4-1.5 \) is a parameter to account for the contribution from both CRs and the interstellar gas (Dermer 1986). The total flux then is

\[
dN_{\gamma}/dE_{\gamma} = \frac{M_c q(E)}{4\pi d^2 m}, \tag{21}
\]

where \( M_c, d \) are the mass and the distance of the cloud. Combining Equations (16), (20), and (21), we obtain the flux of gamma-ray emission as shown in Figure 3, where our result is plotted against both Fermi and High Energy Stereoscopic System (H.E.S.S.) data. The GeV and TeV data are adopted from Abdo et al. (2010) and Aharonian et al. (2008), respectively. Our result produces a power-law spectral index \(~2.75\), showing that the steepening of the spectrum of particle can be naturally explained by the propagation effect. Indeed, similar fits have been also obtained by other models, e.g., Ohira et al. (2011), Li & Chen (2010). We did not need, however, to assume either Bohm diffusion and phenomenological power-law evolution of the momentum of the escaping particles as in Ohira et al. (2011) or a steeper spectrum for the escaping particles as in Li & Chen (2010). A decrease of the spatial diffusion coefficient by a factor of \( \chi = 0.05 \) comparable to earlier works (Gabici et al. 2011; Li & Chen 2010) is inferred here, which can originate from streaming instability. Estimates will be provided below to demonstrate that the instability can grow for the energy range we consider.

5. ENHANCED SCATTERING AND STREAMING INSTABILITY NEAR SNRs

Our results show that the local scattering of CRs has to be enhanced by an order of magnitude \( \chi = 0.05 \) in order to produce the amount of \( \gamma \) ray emission observed. A natural way to increase the scattering rate is through the streaming instability. We provide here a self-consistency check by examining whether the streaming instability operates in the presence of nonlinear damping by the background turbulence. We envisage a local low density, high temperature cavity with \( B_{\text{env}} = 2 \mu G \) surrounding the supernova subjected to strong UV radiation and stellar wind (see Fujita et al. 2009), so that there is no ion-neutral damping in this case. Only nonlinear damping exists (see Equation (7)). The growth rate in the linear regime is.

\[
\Gamma_{gr} = \Omega_0 \frac{N(\geq E)}{n} \left( \frac{v_i}{v_A} - 1 \right). \tag{22}
\]

where \( v_i \) is the streaming speed of CRs. The growth rate should overcome the damping rate (Equation (7)) for the instability to operate. The condition \( \Gamma_{gr} > \Gamma_d \) leads to

\[
v_i > v_A \left( 1 + \frac{n v_A}{N \Omega_0 \sqrt{r_g L}} \right). \tag{23}
\]

The spatial diffusion coefficient adopted here, \( D = v_i L = \chi D_{\text{ISM}} \), satisfies this requirement. The growth and damping rates are compared in Figure 4. We see that the streaming instability works in the energy range needed to produce the observed \( \gamma \) ray emission, proving that our results are self-consistent.

Note that the case we consider here is different from the general interstellar medium discussed in Yan & Lazarian (2004) and Farmer & Goldreich (2004), namely, the local CR flux near SNRs is much enhanced (see Figure 2). Consequently, the growth rate of the streaming instability becomes high enough to overcome the damping rate by the preexisting turbulence in the considered energy range.

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9 We neglect the nonlinear Landau damping, which is suppressed in turbulence due to decrease of mean free path.
6. DISCUSSION

Gamma rays have been detected from molecular clouds near SNRs. They are believed to be the result of interactions with the energetic particles accelerated at SNR shocks, and thus provide the best probe to both the shock acceleration and propagation processes of CRs near SNRs. To determine the spectrum of CRs that generates the gamma-ray emissions, it is necessary to establish the time dependence of the energy of escaping particles. Earlier it has been done assuming that only CRs with $p > p_{\text{max}}(t)$ can escape and that the time dependence is a power law (see Gabici et al. 2010; Ohira et al. 2011). In this paper, we derive our results on the basis of the well motivated and tested physical model of the streaming instability in the presence of background turbulence. With this model we obtain $p_{\text{max}}(t)$ which is determined by the interaction of streaming instability at the upstream and the background turbulence. We limit the discussions to the later evolution of SNR acceleration which is stage of the observed gamma-ray sources. In this case, the classical formula for the growth of streaming is valid.

The spectral fit at high gamma-ray energies with H.E.S.S. data is less optimal. High energy CRs are generated earlier when strong magnetic amplification is needed. As we pointed out earlier, the classical treatment of streaming is invalid and moreover, the field amplification may well be due to other processes, e.g., the current instability (Bell 2004) and/or turbulent density perturbations interacting with the shock precursor (Beresnyak et al. 2009).

Another crucial ingredient that determines the CR flux at the clouds is the spatial diffusion coefficient of CRs escaping from SNRs. It has been speculated recently that this spatial diffusion is suppressed due to CR induced instabilities to match the observed level of gamma-ray fluxes (see Gabici et al. 2010). No specific study has been provided so far. Here we place a constraint to the level of streaming by applying our present understanding of turbulence damping of waves (Yan & Lazarian 2004; Farmer & Goldreich 2004; Beresnyak & Lazarian 2008). By balancing the growth rate of the streaming instability with the turbulence damping rate, we obtain not only the energy range at which the streaming instability operates, but also the lower limit of the CR spatial diffusion coefficient. These results are found to be consistent with the required enhanced scattering indicated from the interpretation of the gamma-ray data.

For the spectral index of the CR power-law spectrum, we took into account the hardening of the accelerated particles at the shock front. This does not contradict the fact that the spectrum of runaway particles is steeper as it is a time-averaged result. For the same reason, we do not consider that the decrease of the compression ratio would contribute to the steepening of the power-law spectrum of the accelerated CRs. In fact, as long as $s < 2$, the spectral index of runaway particles becomes uniformly 2. The main steepening at a distance $r$ from the SNR is due to the propagation effect. In the case when $r \gtrsim R_d > R_{\text{esp}}$, the point source is a good approximation for SNR, and the spectrum is steepened by $p^{\alpha/2}$ if $D \propto p^\alpha$.

The diffusion coefficient $D$ near the shock is assumed to be of the form $D = \chi 10^{28} \sqrt{E/10^{16} \text{GeV}} \text{cm}^2 \text{s}^{-1}$ following earlier treatments (see Gabici et al. 2010; Ohira et al. 2011). Undoubtedly, solving the diffusion coefficient surrounding SNRs taking into account both streaming instability and background turbulence is an important step toward a complete self-consistent picture. It will be one of our future endeavors. Another simplification we made concerns the magnetic field strength at the shock. For the later stage of Sedov phase evolution, we assumed an already increased magnetic field strength $\sim 100 \mu \text{G}$, and did not consider the specifics of earlier amplification process, which is widely believed to be present. This process is important for earlier shock evolution and acceleration of ultra high energy CRs. It, however, is a subject of intensive debates and is definitely beyond the scope of our current paper.

The set of physical parameters and model parameters we used are listed, respectively, in Tables 1 and 2. The physical condition we adopted for W28 is close to that used in the literature (Fujita et al. 2009; Gabici et al. 2010; Li & Chen 2010). As for the model parameters, we have tight constraints only for the diffusion coefficient $(\alpha, \chi)$, similar to those obtained in earlier studies (Gabici et al. 2010; Li & Chen 2010) and that for the energy of the SN explosion $\eta$. The results are not so sensitive to the other model parameters, as shown in Table 2.

7. SUMMARY

We investigated the acceleration of particles at the later stage of SNR evolution and the escape of these particles. We calculated the flux of the escaping CRs at a molecular cloud in the vicinity of SNRs. We quantitatively took into account the competition of turbulence generation by the streaming instability of CRs and background turbulence damping, both, near the shock waves and in the ensuing propagation of CRs from SNRs. The resulting gamma-ray spectrum from the $p-p$ interactions is compared with both Fermi and H.E.S.S. data. Our main results are the following.

1. The streaming instability plays a crucial role for CR acceleration, particularly at the later stage of SNRs. The competition of turbulence generation by the streaming instability and turbulence damping determines the maximum energy of the accelerated particles.
2. The spectrum of runaway particles follows a universal power law of $f(E) \propto E^{-2}$ if the original spectrum index $s$ at the shock front is $s \leq 2$ for the later Sedov phase. The main steepening at a distance from the SNR is due to the propagation effect.
3. The flux of CRs is increased by several orders of magnitude compared to the mean Galactic value, creating enough turbulence by the streaming instability in the vicinity of the shock which overcomes the damping by background turbulence for the energy range considered.
4. The enhanced scattering by the instability is enough to reproduce the observed gamma-ray flux from massive molecular clouds near the SNR.

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APPENDIX

LIST OF NOTATIONS

The notations used in this paper are listed in Table 3.
### Table 3

Notations Used in the Paper

| Symbol | Description |
|--------|-------------|
| A      | Normalized wave amplitude $\delta B / B_0$ |
| $a$    | Hardening of the CR spectrum at the shock front |
| $B_0$  | Mean magnetic field at the shock in the later Sedov phase |
| $B_{\text{cav}}$ | Inercloud magnetic field strength |
| $\delta B$ | Wave amplitude |
| $c$    | Light speed |
| $d$    | Distance of the molecular cloud from observer |
| $D$    | Diffusion coefficient of CRs |
| $E$    | CR energy |
| $E_{\text{SN}}$ | Supernova explosion |
| $f$    | Distribution function of CRs |
| $f_{\pi}$ | Fraction of energy transferred from parent protons to pions |
| $k$    | Wave number |
| $K(t)$ | Normalization factor of CR distribution function |
| $L$    | The injection scale of background turbulence |
| $m$    | Proton rest mass |
| $M_c$  | Cloud mass |
| $n$    | Intercloud number density |
| $N_\gamma$ | $\gamma$ ray flux |
| $p$    | CR's momentum |
| $p_{\text{max}}$ | The maximum momentum accelerated at the shock front |
| $P_{\text{cr}}$ | CR pressure |
| $q$    | Charge of the particle |
| $r$    | Distance from SNR center |
| $R_e$  | The distance of the molecular cloud from the SNR center |
| $r_\Omega$ | Larmor radius of CRs |
| $R_d$  | Diffusion distance of CRs |
| $R_{\text{sh}}$ | Shock radius |
| $R_{\text{exp}}, t_{\text{exp}}$ | The escaping distance/time of CRs |
| $s$    | One-dimensional spectrum index of CR distribution |
| $t$    | Time since supernova explosion |
| $t_{\gamma e}$ | The age of SNR |
| $t_{\text{sed}}$ | The time at which SNR enters the Sedov phase |
| $U$    | Shock speed |
| $U_i$  | Initial shock velocity |
| $v$    | Particle speed |
| $v_{s}$ | Streaming speed of CRs |
| $W$    | Wave energy |
| $\alpha$ | Power index of $D$ with respect to particle momentum $p$ |
| $\chi$ | Reduction factor of $D$ with respect to $D_{\text{ISM}}$ |
| $\delta$ | Power index of $p_{\text{max}}$ with respect to $t$ |

### Table 3 (Continued)

| Symbol | Description |
|--------|-------------|
| $\eta$ | Fraction of SN energy converted into CRs |
| $\eta_A$ | A numerical factor in Equation (20) |
| $\Gamma_{\text{cr}}$ | The growth rate of streaming instability |
| $\Gamma_d$ | Wave damping rate |
| $\kappa$ | Ratio of diffusion length to shock radius |
| $\Omega_0$ | The Larmor frequency of nonrelativistic protons |
| $\sigma_{pp}$ | Cross section for $p-p$ collision |
| $\xi$ | The ratio of CR pressure to fluid ram pressure |

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