Giant Goos-Hänchen effect at the reflection from left-handed metamaterials

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We study the beam reflection from a layered structure with a left-handed metamaterial. We predict a giant lateral (Goos-Hänchen) shift and splitting of the beam due to the resonant excitation of surface polaritons with vortex-like energy flow between the right- and left-handed materials.

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As is known, a totally reflected beam experiences a lateral displacement from its position predicted by the geometric optics because each of its plane-wave components undergoes a different phase change \[ \Delta \]. For the beam reflected from an interface, the lateral beam shift (the so-called Goos-Hänchen effect) is usually much less than the beam width. However, larger beam shifts may occur in the layered systems supporting surface waves, which are able to transfer energy along the interface. Such surface waves are not excited at a single interface because the phase-matching condition between the incident and surface waves is not satisfied. However, a surface polariton can be excited in a layered structure, when the phase-matching condition is satisfied. Such surface waves can transfer energy along the interface leading to a great enhancement of the lateral beam shift.

The Goos-Hänchen shift \[ \Delta \] can be calculated analytically as \[ \Delta = -d\Phi/dk_x\ ] when the phase \[ \Phi \] of the reflection coefficient is a linear function of the wave vector component \[ k_x \] along the interface, across the spectral width of the beam. The (standard) lateral beam shift has been calculated for the reflection from layered structures with left-handed materials, for the cases of a single interface \[ \text{[6]} \] and for a periodic structure of alternating right- and left-handed layers \[ \text{[8]} \].

In our geometry [see Fig. 1(a)], the reflection coefficient \[ R = R(k_x) \] for the TE-polarized plane monochromatic \[ \sim \exp(i\omega t) \] wave can be found as

\[ R = \frac{(\alpha_1 + 1)(\alpha_2 + 1) - (\alpha_1 - 1)(\alpha_2 - 1)e^{2ik_xd}}{(\alpha_1 - 1)(\alpha_2 + 1) - (\alpha_1 + 1)(\alpha_2 - 1)e^{2ik_xd}}, \tag{1} \]

where \( \alpha_{1,2} = k_{3,1,3}\mu_2/k_{3,2}\mu_1,3, k_{3,2} = (\omega^2\epsilon_i\mu_i/c^2 - k_x^2)^{1/2} \) is the gap thickness, and \( c \) is the speed of light in vacuum. Here, we consider the TE polarized waves, but the results are qualitatively similar for the TM polarized waves.

We consider an incident beam of the Gaussian shape, \[ E_i(x) = \exp(-x^2/4a^2 - ik_{x0}x), \] where \( a \) is the width of the beam, \( k_{x0} \) determines the angle of incidence \( \phi \), \( k_{x0} = k_1 \sin \phi \), and \( k_1 \) is the propagation constant in the first medium, \( k_1 = \omega\sqrt{\epsilon_1\mu_1}/c \). The reflected field is found in a standard way

\[ E_r(x) = \frac{1}{2\pi} \int_{-\infty}^{\infty} R(k_x)E_i(k_x)\,dk_x, \tag{2} \]

where \( E_i \) is the Fourier spectrum of the incident beam.

Larger lateral beam shifts are expected at the angles of incidence such that the beam spectrum contains the components with the same \( k_x \) as the propagation constant of the surface waves. As has been shown recently, both forward and backward surface polaritons can exist at the left-handed interface \[ \text{[6]} \], depending on the parameters \( X = |\epsilon_3|/|\epsilon_2 | \) and \( Y = |\mu_3|/|\mu_2 | \). Excitation of the forward surface waves results in the energy transfer in the direction of incidence. A negative shift of the beam is obtained when the backward surface waves are excited, this corresponds to the parameters \( XY > 1 \) and \( Y < 1 \).
The relation between the electric field of the reflected beam, $\Delta$, and the propagation constant of surface polaritons can be found from the phase matching condition $\Delta = \Delta_1$, where

$$\Delta_n = \alpha^n \int_{-\infty}^{\infty} x^n |E_r(x)|^2 dx \left( \int_{-\infty}^{\infty} |E_r(x)|^2 dx \right)^{-1}.$$  \hspace{1cm} (3)

The values $\Delta < 1$ correspond to the beam shift much smaller than the beam width, while $\Delta \geq 1$ corresponds to a giant Goos-Hänchen shift. The second momentum $\Delta_2$ characterizes a change of the beam width during the reflection, $W = \Delta_2^{1/2}$.

We chose the following parameters of the media: $\epsilon_1 = 12.8$, $\mu_1 = \epsilon_2 = \mu_2 = 1$, $\epsilon_3 = -3$, $\mu_3 = -0.5$. The propagation constant of surface polaritons can be found from the relation $\hbar^2 = (\omega/c)^2 \epsilon_2 \mu_2 Y(Y - X)/(Y^2 - 1)$, and for these parameters the surface waves at the interface are backward propagating. Figures (b,c) show the dependence of $\Delta$ and $W$ on the angle of incidence at $\omega a/c = 100$ and $\omega d/c = 3$, respectively. We observe a distinctive resonant dependence of the beam shift, and the maximum corresponds to the phase matching condition $k_{z0} = h$.

In the inset (2) of Fig. (c), two distinctive peaks in the reflected beam are observed. The first peak corresponds to a mirror reflection, while the second peak is shifted from the incident beam and it appears due to the excitation of surface waves. At the resonance, the lateral beam shift becomes larger than the beam width. The double-peak structure appears only for narrow beams, when the beam spectrum is wider than the spectrum width of the surface mode, the later can be found as a width of the resonance shown in Fig. (b). The components of the beam spectrum outside this region are reflected in a mirror-like fashion, while the spectrum components near the resonance transform into a surface wave, being responsible for the second peak in the shifted reflected beam. For wider beams, such that their spectrum is completely falls into the surface mode excitation line, only the shifted peak appears. With an increase of the beam width, though, the relative beam shift decreases due to the fact that the absolute shift of the beam grows slower than the beam width.

Figures (a-c) show the beam shift and width vs. the normalized gap thickness and the structure of the energy flow. The resonances observed in Figs. (a,b) have simple physical explanation. Indeed, when the gap is absent ($d = 0$) or small, no surface wave is excited, and the beam shift is negligible. Increasing the gap width, one increases the quality factor of the surface mode, thus increasing its propagation distance and the reflected beam shift. Similarly, for large $d$ the surface wave is not excited, and the...
The strongly curved flow lines in the upper medium correspond to the interference of the incident and mirror-like reflected beams. The finite-extension surface wave excited in a slab along the interface has a distinctive vortex-like structure. This surface wave transfers the energy in the negative direction, and then the energy is reflected from the interface as a shifted beam.

To make our predictions more realistic, we study the effect of losses always present in left-handed materials. We introduce losses by adding the imaginary parts to the dielectric permittivity \( \epsilon_3 \) and magnetic permeability \( \mu_3 \). In particular, we take \( \text{Im}(\epsilon_3) = -2 \times 10^{-5} \) and vary the imaginary part of \( \epsilon_3 \). We notice that the losses in the left-handed medium affect mostly the surface waves and, therefore, the major effect produced by the losses is observed for the strongly shifted beam component.

We distinguish two limiting cases. When the beam is narrow, i.e. its spectral width is large, only a part of the beam energy is transferred to surface waves, while the other part is reflected. This case is shown in Figs. (a,b). In this case the increase of losses, i.e. the increase of the absolute value of \( \text{Im}(\epsilon_3) \), results in the suppression of the second peak in the reflected beam, which is due to the surface wave excitation. The growth of the amplitude of the mirror-like reflected beam can be explained by a detuning from the resonance between the incident wave and the surface polariton.

For a wide beam with a narrow spectrum, almost all energy of the beam is transferred into a surface wave and, therefore, the lateral shift becomes strongly affected by the losses in the left-handed medium, as shown in Fig. (c). Figure (d) show the dependence of the reflection coefficient on the imaginary part of the dielectric permittivity in the third medium.

In conclusion, we have described the Goos-Hänchen shift and splitting of a beam totally reflected from a layered structure with a left-handed metamaterial. The giant lateral shift of the beam is explained by the resonant excitation of surface vortex-like polaritons at the interface between the right- and left-handed materials. The beam shift can be both positive and negative, depending on the type of the surface waves excited at the interface. We believe that the giant enhancement of the lateral shift of the reflected beam can be employed for direct measurements of the parameters of left-handed materials.

FIG. 3: (a,c) Relative beam shift and (b,d) reflection coefficient vs. the imaginary part of the dielectric permittivity, for narrow \( d\omega/c = 100 \) (a,b) and wide \( d\omega/c = 600 \) (c,d) beams, for \( d\omega/c = 3 \). Insets show the profiles of the reflected beam.

To understand deeper the physical mechanism for the giant Goos-Hänchen shift, we calculate the energy flow for the beam reflection and compare it with the results for the right-hand media. Figure (c) shows the structure of the energy flow for the case of a negative lateral shift. The strongly curved flow lines in the upper medium correspond to the interference of the incident and mirror-like reflected beams. The finite-extension surface wave excited in a slab along the interface has a distinctive vortex-like structure. This surface wave transfers the energy in the negative direction, and then the energy is reflected from the interface as a shifted beam.

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