Numerical analysis of orbital transfers to Mars using solar sails and attitude control

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Abstract. Solar sails present a promising alternative method of propulsion for the coming phases of the space exploration. With the recent advances in materials engineering, the construction of lighter and more resistant materials capable of impelling spaceships with the use of solar radiation pressure has become increasingly viable technologically and economically. The studies, simulations and analysis of orbital transfers from Earth to Mars proposed in this work were implemented considering the use of a flat solar sail. Maneuvers considering the delivery of a sailcraft from a Low Earth Orbit to the border of the Earth's sphere of influence and interplanetary trajectories to Mars were investigated. A set of simulations were implemented varying the attitude of the sail relative to the Sun. Results show that a sailcraft can carry out transfers with final velocity with respect to Mars smaller than the interplanetary Patched-conic approximation, although this requires a longer time of transfers, provided the attitude of the sailcraft relative to the Sun can be controlled in some points of the trajectories.

1. Introduction

Even though the concept of solar sails was presented in the 60's, material sciences was not developed enough to make sailcrafts available back then. The first successful launch only took place in 2010, when IKAROS mission (JAXA) started its operation [1] and opened the space sailing era. Other well-known initiatives are the Solar Sail Demonstrator, or Sunjammer, from NASA [2], and the LightSail initiative proposed by the Planetary Society [3]. The Sunjammer project was concluded in 2014, prior to flight testing, with interesting results. LightSail initiative is meant to be attached to a 3U CubeSat. LightSail I was launched in 2015 with the objective of testing the deployment of the sail, while in 2017, LightSail II is to be launched and fully tested.

Two objectives are proposed in this paper. The first one is to study the physical concepts and mathematical models for solar sails in order to implement numerical algorithms to simulate orbital transfers with such type of propulsion system. Secondly, to implement these simulations considering transfers from Earth to Mars with attitude control.

Following [4], the mathematical model considers the lightness vector \( L \) to be the thrust acceleration normalized to the local solar gravitational acceleration and resolved in the heliocentric inertial reference frame. The magnitude of the lightness vector can be defined by

\[
L = \left( \frac{1}{2} \frac{\sigma}{\sigma} \right) n_x \left[ (2r_{spec} + n_x \chi_f r_{diff} + \kappa a_f)n + (a_f + r_{diff})u \right].
\]
Where $\sigma_c$ is the critical sail loading and $\sigma$ is the sailcraft sail loading, $r_{spec}$ is the sailfront-side specular reflectance, $r_{diff}$ is the sailfront-side diffuse reflectance, $\chi_f$ is the sailfront-side emission/diffusion coefficient, $\kappa$ is the emission/diffusion net thrust dimensionless factor, $\alpha_f$ is the sailfront-side total absorptance. $n$ is the sail's orientation unit vector, normal to its surface, and $u$ is the direction of the incident sunlight on the sail, both resolved in the sailcraft orbital frame $x, y, z$. Two angles are important to define the orientation and the attitude of the sail in this frame, they are the azimuth $\alpha$ (in the orbital plane) and elevation $\delta$ (in the plane perpendicular to the orbital plane) – these quantities are shown in Figure 1. The critical sail loading is a constant defined by

$$\sigma_c \equiv 2 \frac{I_{1AU}}{c g_{1AU}} \equiv 1.5368 \frac{a}{m^2}, \quad (I_{1AU} = 1366 \frac{w}{m^2}),$$

and where $g_{1AU} = 5.930 \times 10^{-3}$ m/s$^2$ is the gravitational acceleration at 1 AU from the Sun, and $c = 299792458$ m/s denotes the speed of light. The sailcraft sail loading is determined by

$$\sigma \equiv \frac{m}{S},$$

where $m$ is the sail's mass and $S$ is the sail's surface area. Finally, the emission/diffusion net thrust dimensionless factor is defined by

$$\kappa \equiv \frac{\chi_f \epsilon_f(T) - \chi_b \epsilon_b(T)}{\epsilon_f(T) + \epsilon_b(T)},$$

where $T$ is the sail's temperature, $\chi$ is the emission/diffusion coefficient and $c$ is the emittance as a function of $T$. The subscripts $f$ and $b$ indicate whether the coefficient corresponds to the front-side or back-side of the sail, respectively. Let $\Xi$ be the transformation matrix from the sailcraft orbital frame to the heliocentric inertial frame, which is defined by

$$\Xi = (r \ h x r \ h),$$

where $r$ is the outward radial direction and $h$ is the direction of the sailcraft's orbital angular momentum per unit of mass. From equations (1) and (5), it is possible to calculate the sailcraft's acceleration resolved in the heliocentric inertial frame considering:

$$A_{HIF} = \Xi g L,$$

where $g$ is the local solar gravitational acceleration.

**Figure 1.** Scheme of the orientation in $x_Sy_Sz_S$ frame fixed in Sail. $T$ represents the thrust on the sail due to the interaction with the solar radiation.
2. Mission parameters

The simulations were implemented considering as dynamical system the four-body problem Sun-Earth-Moon-sailcraft [5,6], with the Earth and the Moon in circular orbits. The acceleration components due to solar radiation pressure on the sailcraft are derived from the equation (5) and (6) and given by

$$A_{HIF} = \frac{GM\text{Sun}}{R^2} \begin{bmatrix} \frac{l_x}{R} x + \frac{l_y}{HR} \left[ z(zv_x - xv_z) - y(xv_y - yv_x) \right] + \frac{l_z}{H} (yv_z - xv_y) \\ \frac{l_x}{R} y + \frac{l_y}{HR} \left[ x(xv_y - yv_x) - z(yv_z - zv_y) \right] + \frac{l_z}{H} (zv_x - xv_z) \\ \frac{l_x}{R} z + \frac{l_y}{HR} \left[ y(yv_z - zv_y) - x(zv_x - xv_z) \right] + \frac{l_z}{H} (xv_y - yv_x) \end{bmatrix}.$$  

(7)

Where $G$ is gravitational constant, $M_{\text{Sun}}$ is the Sun’s mass, $R$ is the magnitude of the sailcraft’s position in the inertial reference frame, fixed at the Sun's center of mass, $(x, y, z)$ and $(v_x, v_y, v_z)$ are the components of the sailcraft's position and velocity, respectively. $H$ is the magnitude of the sailcraft’s orbital angular momentum. Equation (8) indicates the components $(l_x, l_y, l_z)$ of the lightness vector, derived from the equation (1) as a function of the angles $\alpha$ and $\delta$.

$$\begin{cases} 
    l_x = \left( \frac{1}{2} \frac{\sigma_c}{\sigma} \right) [2r_{\text{spec}} \cos^3 \delta \cos^3 \alpha + (\chi_f r_{\text{diff}} + \kappa a_f) \cos^2 \delta \cos^2 \alpha + (a_f + r_{\text{diff}}) \cos \delta \cos \alpha] \\
    l_y = \left( \frac{1}{2} \frac{\sigma_c}{\sigma} \right) [2r_{\text{spec}} \cos^3 \delta \cos^2 \alpha \sin \alpha + (\chi_f r_{\text{diff}} + \kappa a_f) \cos^2 \delta \cos \alpha \sin \alpha] \\
    l_z = \left( \frac{1}{2} \frac{\sigma_c}{\sigma} \right) [2r_{\text{spec}} \cos^2 \delta \cos \alpha \sin \delta + (\chi_f r_{\text{diff}} + \kappa a_f) \cos \alpha \sin \delta] 
\end{cases}$$

(8)

The values of the optical coefficients considered were taken from the values already achieved by the Jet Propulsion Laboratory (JPL/NASA) [7] and are presented in Table 1.

| Sail Type     | $r$     | $S$     | $c_f$ | $c_b$ | $b_f$ | $b_b$ |
|--------------|---------|---------|-------|-------|-------|-------|
| Ideal Sail   | 0.88    | 0.94    | 0.05  | 0.55  | 0.79  | 0.55  |
| Square Sail  | 0.88    | 0.94    | 0.05  | 0.55  | 0.79  | 0.55  |
| Heliogyro    | 0.88    | 0.94    | 0.05  | 0.55  | 0.79  | 0.55  |

The values were taken from the second line of the Table 1 for a square sail. The first column corresponds to the reflectance coefficient, whilst the second column is the specular reflection ratio. Nevertheless, a value of 90% specular reflection ratio was considered for the sailfront-side. By considering a null transmittance, the value of the absorptance can be defined from the value of the reflectance. The third and fourth columns are the front-side and back-side emittance values, respectively, while the fifth and sixth columns correspond to the front-side and back-side emission/diffusion coefficients.

Still, according to the JPL, current technologies allow the construction of a sail with sailcraft loading of 5.27 g/m² [7]. And taking LightSail 1 mission [3] into account, a mass of 20 kg was considered and, indirectly, a surface area of 3795 m². With that, all the coefficients and parameters of the sail were defined, making it possible to determine the acceleration in the inertial heliocentric frame at every iteration.
2.1 Constant attitude simulations

A total of 304 simulations were performed considering the following conditions:

- Eight initial positions (A, B, C, D, E, F, G, H) on the border of the Earth’s sphere of influence, in the plane of the ecliptic, were considered. For a geocentric frame, these points have a separation of 45 degrees, as illustrated in Figure 2. To reach these points, impulsive Hohmann transfers from a Low Earth Orbits (LEO) were performed. During these maneuvers, the solar radiation pressure was considered not to act on the sail. Hohmann transfers proved to be more economic to start the mission from a LEO [6] in a scenario where the sail could be opened in a point on the Earth’s sphere of influence. In terms of the magnitude of the total velocity change ($\Delta V_{\text{H/Total}}$), it is required 3.157 km/s to deliver the sailcraft from a LEO to the Earth's sphere of influence through a Hohmann transfer.

- For each initial position, indicated in Fig. 2, a group of $\alpha$ values was considered {$1^\circ, 5^\circ, 10^\circ, 15^\circ, 20^\circ, 25^\circ, 30^\circ, 35^\circ, 40^\circ, 45^\circ, 50^\circ, 55^\circ, 60^\circ, 65^\circ, 70^\circ, 75^\circ, 80^\circ, 85^\circ, 89^\circ$}, maintaining them constant along each simulation;

- Since the orbits considered are coplanar, the $\delta$ value was always 0 degrees.

At the end of each simulation, the time of transfer and the position at the which the spacecraft inserted itself in the destination's planet sphere of influence were recorded. At this position, the destination planet's velocity was calculated and the difference between its value and the sailcraft's velocity ($\Delta V$) was determined. Together with these information, it was also determined the destination planet's position relative to Earth at the beginning of the interplanetary transfer.

Figure 2. Relative position between the Sun, the Earth and the sailcraft. $r_{14}$ and $r_{24}$ are the position of the sailcraft relative to the Sun and the Earth, respectively. $f_0$ is the angular displacement (45 degrees) of the initial positions occupied by the sailcraft on the border of the Earth’s sphere of influence (points: A, B, C, D, E, G, G, H).

2.2 Simulations with attitude control

To improve the results, especially regarding to the relative velocity between the sailcraft and the destination planet ($\Delta V$) and the time of transfer, a procedure to change the attitude of the sailcraft has been implemented. This procedure consists in a few number of changes in the parameter $\alpha$ during the flight between the Earth’s and the destination planet’s sphere of influences. In general, as consequence, the solar radiation pressure will act to accelerate and then to deaccelerate the sailcraft reducing its velocity on arrival in the destination planet. However, other phases with acceleration and deacceleration can happen.

To consider the attitude change, the final state vector values of all the bodies (Sun, Earth, Moon and sailcraft) of the interrupted simulation were saved and inserted into a new simulation as the initial state vector of all the bodies. This new simulation would proceed with a different sail attitude.

Simulations starting from all positions on the Earth’s sphere of influence (Fig. 2) were performed. The values of the attitude changes were chosen based on the results of the constant attitude simulations and the moment of the changes were evaluated considering the behavior of the value of the sailcraft's specific orbital energy.
3 Interplanetary transfers to Mars

The results achieved from the simulations were gathered in form of data plots and trajectory graphs are shown and discussed in this section.

3.1 Results for the transfers with constant attitudes α

For each transfer with α constant analyzed, the time of transfer and final relative velocity between the sailcraft and Mars (ΔV) were calculated, recorded and plotted. The behavior of a sailcraft's transfers to Mars is illustrated in Figure 3 where a clear relation between the time of transfer, ΔV and α can be seen for fo = 0 degrees (Figure 2). The value of the constant attitude α that minimizes the time of transfer also maximizes the ΔV. The shortest time of transfer, 122.098 days, is found for α = 20 degrees with ΔV = 12.296 km/s. However, this is not acceptable for missions where the objective is, for example, to insert a space vehicle into an orbit around a planet. On the other hand, transfers with constant attitude α which offer low ΔV have higher time of transfer, as example, for α = 88 degrees, the time of transfer is equal 14409.86 days (39.48 years) and ΔV = 0.473 km/s, but this time of flight is also not acceptable.

![Figure 3. Transfer to Mars plot: time of transfer × α and ΔV × α for fo = 0°.](image)

3.2 Earth-Mars transfer considering changes in the attitude α

An example of an Earth–Mars transfer can be observed in Figure 4. Four attitude changes of 5 degrees each are made during the transfer, the locations of the changes in the transfer trajectory are indicated by the little black dashes. According to Figure 3, if α is maintained constant and equal 60 degrees and for fo = 0 degrees, it is possible to transfer a sailcraft to Mars in 194 days, and the relative velocity between both is 7.210 km/s on arrival at a point on the Mars sphere of influence. Considering four attitude changes, it is possible to reduce this relative velocity to 2.090 km/s also at the same point on the Mars sphere of influence, but increasing the time of transfer to 305.87 days. The changes in the attitude are:
1) $\alpha = 60$ degrees during the first 24.44 days.
2) First change $\alpha = 65$ degrees from 24.44 days to 59.81 days, i. e., for 35.37 days.
3) Second change $\alpha = 70$ degrees from 59.81 days to 103.71 days, i. e., for 43.9 days.
4) Third change $\alpha = 75$ degrees from 103.71 day to 147.61 days, i. e., for 43.9 days.
5) Fourth change $\alpha = 80$ degrees from 147.61 days to 305.87 days, i. e., for 158.26 days, when the sailcraft reaches the Mars sphere of influence.

In comparison with the interplanetary Patched-conic approximation [8], for which the minimal relative velocity between the Mars and a space vehicle found is 3.300 km/s at the same point and with a time of transfer of 177 days, in this example, the four changes in the angle $\alpha$ provide a reduction of 36% in the relative velocity between de Mars and the sailcraft against an increase of 129 days in its trip.

Even though this is not an optimal result essentially, but the strategy of changing $\alpha$ shows that the main obstacle to the use of sailcrafts in interplanetary exploration, that is, the constant increase in its velocity, can be solved if an onboard control system could provide a few minor changes in the $\alpha$ values during its journey.

![Figure 4. Transfer trajectory found considering four changes in the angle $\alpha$ of 5 degrees each.](image)

### 4. Conclusion
The constant increase in the velocity can be considered as a problem to be overcome with regarding the use of sailcrafts in the planetary exploration. In this context, the possibility of changing the attitude of a sailcraft through few variations in the angle $\alpha$ during its journey across the space could reverse this constraint. This strategy allows to accelerate the spaceship at the beginning and reduce the relative velocity between the it the target celestial body at the end, although the time of transfer can increase. In the example presented here, in a transfer to Mars was possible to find a trajectory in which the relative velocity at the Mars sphere of influence is 2.090 km/s against 3.300 km/s found for a conventional impulsive transfer as determined by the interplanetary Patched-conic approximation.

Although it is important to note that these values were obtained without the use of any optimization algorithms, as they are the results of an introductory study on the application of solar sails on interplanetary transfers. Therefore, these are values with potential for refinement and improvement to reduce its dependency of chemical propulsion in the final trajectory adjustments.
References
[1] http://www.isas.jaxa.jp/en/missions/spacecraft/current/ikaros.html Last access: 26 Apr. 2017
[2] https://www.nasa.gov/mission_pages/tdm/solarsail/index.html Last access: 26 Apr. 2017
[3] The Planetary Society LightSail: a solar sailing spacecraft from the Planetary Society Available at: <http://sail.planetary.org/> Last access: 26 Apr. 2017.
[4] Vulpetti G, Johnson L and Matloff G L Solar Sails: a novel approach to interplanetary travel 2nd edition New York: Copernicus Books 2015 277 p.
[5] Murray, C.D. and Dermott, S.F. 1999 Solar System Dynamics, UK: Cambridge University Press.
[6] Meireles L G Análise de Transferências Orbitais com uso de Velas Solares Belo Horizonte: 2016.
[7] McInnes C R Solar Sailing: Technology, Dynamics and Mission Applications 2nd edition London: Springer Praxis Books 2004 296 p.
[8] Bate, R.R.; Mueller, D.D.; White, J.E. Fundamentals of Astrodynamics. USA: Dover Publications Inc., 1971. 455p.