Evaluation of pion-nucleon sigma term in Dyson-Schwinger equation approach of QCD

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We calculate the variation of the chiral condensate in medium with respect to the quark chemical potential and evaluate the pion-nucleon sigma term via the Hellmann-Feynman theorem. The variation of chiral condensate in medium are obtained by solving the truncated Dyson-Schwinger equation for quark propagator at finite chemical potential, with different ansätze for the quark-gluon vertex and gluon propagator. We obtain the value of the sigma term $\sigma_{\pi N} = 62(1)(2)$ MeV, where the first represents the systematic error due to our different ansätze for the quark-gluon vertex and gluon propagator and the second represents a statistical error in our linear fitting procedure. Our result favors a relatively large value and is consistent very well with the recent data obtained by analyzing the pion-nucleon scattering and pionic atom experiments.

I. INTRODUCTION

The pion-nucleon sigma term $\sigma_{\pi N}$ is of fundamental importance for understanding the chiral symmetry breaking effects in nucleon [1, 2], and the origin of mass of the observable matter [3, 4]. Recently, special attentions have been paid to the $\sigma_{\pi N}$, since it is also significant in searching for the Higgs boson, supersymmetric particles and cold dark matter [5–7]. $\sigma_{\pi N}$ can be obtained indirectly in experiments, such as the pion-nucleon scattering or pionic atom experiments [8–10]. Several recent analysis [9–11] give 50 MeV $< \sigma_{\pi N} < 70$ MeV, which is relatively larger than the widely used value $\sigma_{\pi N} \simeq 45$ MeV [12]. Especially Refs. [9, 10] give the value around 60 MeV with quite small error bars. Theoretically the pion-nucleon sigma term could be calculated in chiral perturbation theory [13–16], lattice QCD [17–22], Dyson-Schwinger Equations (DSE) approach of QCD [23, 24] and various other models [25–28]. However, theoretical results varies largely with different methods. Notably, the values from lattice QCD are around 30 to 40 MeV, which are much smaller than the above experimental analyses. Conversely, chiral perturbation theory give relatively large values, even up to 80 MeV. Thus still further efforts are needed in the theoretical calculations of the sigma term. In this work, we evaluate the pion-nucleon sigma term in the DSE approach of QCD, via the Hellmann-Feynman theorem.

Theoretically, the pion-nucleon sigma term $\sigma_{\pi N}$ is usually written via the Hellmann-Feynman theorem as

$$\sigma_{\pi N} = m_q \frac{\partial M_N}{\partial m_q},$$

where $M_N$ is the nucleon mass, $m_q$ is the averaged current quark mass for u,d quarks (see Sec. III for details).

It has been known that the nucleon mass $M_N$ comes almost all from the dynamical chiral symmetry breaking (DCSB) (see, e.g., Ref. [3]). It has also been known that the DSEs of QCD provide a natural approach to investigate the DCSB and the chiral symmetry restoration in vacuum (see, e.g., Refs. [29–32]), in hot medium (see, e.g., Refs. [33–38]), in cold dense matter (see, e.g., Refs. [39–44]), and the properties of hadrons (see, e.g., Refs. [45–50]).

Inspired by the above mentioned successes of the DSE approach, we restudy the pion–nucleon sigma term in the DSE approach with both the widely used rainbow approximation and the Ball-Chiu vertex [11, 51] for the effective quark-gluon vertex, and two different infrared dominant models for the effective interaction.

The paper is organized as follows. In Sec. II, the truncation scheme of DSE for quark propagator in vacuum and at finite chemical potential is given. In Sec. III, we briefly describe the method for evaluating the pion-nucleon sigma term $\sigma_{\pi N}$ via the DCSB in medium (more explicitly, the quark condensate in medium). Then, the numerical results are given in Sec. IV. Finally, we summarize our work and give a brief remark in Sec. V.

II. DYSON-SCHWINGER EQUATION FOR QUARK PROPAGATOR

The quark propagator at finite chemical potentials $S(p; \mu)$ satisfies the Dyson-Schwinger equation

$$S^{-1}(p; \mu) = Z_2(i\gamma \cdot \hat{p} + m_q) + Z_1 q^2(\mu) \int \frac{d^4q}{(2\pi)^4} \times D_{\rho\sigma}(k; \mu) \frac{\lambda^a}{2} \gamma_\mu S(q; \mu) \Gamma^a_{\rho\sigma}(q, p; \mu),$$

where $\hat{p} = (\tilde{p}, p_4 + i\mu)$, $k = p - q$, $D_{\rho\sigma}(k; \mu)$ is the full gluon propagator, $\Gamma^a_{\rho\sigma}(q, p; \mu)$ is the dressed quark-gluon vertex, $Z_1$ is the renormalization constant for quark-gluon vertex, $Z_2$ is the quark wave-function normalization constant. The general structure of the quark propagator at finite chemical potential can be written as

$$S^{-1}(p; \mu) = i\gamma_5 \cdot \tilde{p} A(\tilde{p}^2, p_4; \mu) + i\gamma_4 p_4 C(\tilde{p}^2, p_4; \mu) + B(\tilde{p}^2, p_4; \mu),$$

where $A(\tilde{p}^2, p_4; \mu)$, $B(\tilde{p}^2, p_4; \mu)$, $C(\tilde{p}^2, p_4; \mu)$ are scalar functions of $\tilde{p}^2$ and $p_4$, while in vacuum $A(\tilde{p}^2, p_4; \mu = 0) = A(\tilde{p}^2; \mu) = 0$, $B(\tilde{p}^2, \mu) = 0$, $C(\tilde{p}^2, \mu) = 0$.

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\[ 0) = C(p^2, p_4; \mu = 0) = A_0(p^2), \quad B(p^2, p_4; \mu) = B_0(p^2). \]

\[ S^{-1}(p) = i\gamma \cdot p A(p^2) + B(p^2). \quad (4) \]

The gluon propagator and the quark-gluon vertex in vacuum are usually taken as

\[ Z_1 g^2 D_{p\sigma}(k) \Gamma_\sigma(q, p) = G(k^2) D_{p\sigma}(k) \frac{\lambda^a}{2} \Gamma_\sigma(p, q), \quad (5) \]

where \( D_{p\sigma}(k) = \frac{1}{k^2} \left[ \delta_{p\sigma} - \frac{k_p k_\sigma}{k^2} \right] \) is the Landau-gauge free gauge-boson propagator, \( G(k^2) \) is a model effective interaction, and \( \Gamma_\sigma(q, p) \) is the effective quark-gluon vertex. In the following, we carry out our investigation by taking two widely used Ansätze for the effective quark-gluon vertex: \( \Gamma_\sigma(q, p) = \gamma_\sigma \), and the Ball-Chin (BC) vertex \([11, 51]\), which describes meson properties well in the symmetry-preserving Dyson-Schwinger equation and Bethe-Salpeter Equation (BSE) scheme (see, e.g., Refs. \([52-54]\)). The extended form for the BC vertex at finite chemical potentials is given in Ref. \([40]\)

\[ i\Gamma^{BC}_\sigma(q, p; \mu) = i\Sigma_A(q, p; \mu) \gamma^\parallel_\sigma + i\Sigma_C(q, p; \mu) \gamma^\perp_\sigma \]

\[ + (\bar{q} + p) \sigma \frac{i}{2} \gamma^\parallel \cdot (\bar{q} + p) \Delta_A(q, p; \mu) \]

\[ + (\bar{q} + p) \sigma \frac{i}{2} \gamma^\perp \cdot (\bar{q} + p) \Delta_C(q, p; \mu) \]

\[ + (\bar{q} + p) \sigma \Delta_B(q, p; \mu), \quad (7) \]

where \( \gamma^\parallel = (\bar{0}, \gamma_4), \gamma^\perp = \gamma - \gamma^\parallel, \quad F = A, B, C. \)

\[ \Sigma_F(q, p; \mu) = \frac{1}{2} \left[ F(q^2, q_4; \mu) + F(p^2, p_4; \mu) \right], \]

\[ \Delta_F(q, p; \mu) = \frac{F(q^2, q_4; \mu) - F(p^2, p_4; \mu)}{\sqrt{q^2 - p^2}}. \]

For the model effective-interaction, we employ two infrared dominant models, noted as the “GS” and the “QC” model, which only express the long-range behavior of the renormalization-group-improved Maris-Tanday model \([55]\), and the Qin-Chang (QC) model \([56]\). The two models are expressed as:

\[ G^{GS}(k^2) = \frac{4\pi^2}{\omega^6} D k^2 e^{-k^2/\omega^2}, \quad (8) \]

\[ G^{QC}(k^2) = \frac{8\pi^2}{\omega^4} D e^{-k^2/\omega^2}. \quad (9) \]

Eq. (8) and Eq. (9) deliver an ultraviolet-finite model gap equation. Hence, the regularization mass scale can be removed to infinity and the renormalization constants can be set to 1. For the corresponding ansatz at finite chemical potential, we follow that in Ref. \([40]\), neglecting the dependence of the effective interaction \( G \) and the gluon propagator on the chemical potential at low densities.

There are only two main parameters \( D \) and \( \omega \) in our model. We choose the set of values that can fit meson properties in vacuum well \([52]\) or fit the chiral quark condensate and the pion decay constant \( f_\pi \) in vacuum approximately. We use the approximate formula for calculating \( f_\pi \) which is accurate to within 5% in chiral limit with rainbow approximation \([57, 58]\).

\[ f^2_\pi = \int \frac{ds}{8\pi^2} N_c s B(s)^2 \sigma(s)_V, \]

\[ -2\sigma(s)_S \sigma(s)_V - 2\sigma(s)_V \sigma(s)_V' \]

\[ -\sigma(s)_S \sigma(s)_V'' + s \sigma(s)_V'^2 \]

\[ -s^2 (\sigma(s)_V \sigma(s)_V' - (\sigma(s)_V)^2), \quad (10) \]

with the primes denoting the differentiation with respect to \( s = p^2 \), and

\[ \sigma_V = \frac{A(p^2)}{p^2 A(p^2) + B(p^2)}; \quad (11) \]

\[ \sigma_S = \frac{B(p^2)}{p^2 A(p^2) + B(p^2)}. \quad (12) \]

III. THE PION-NUCLEON SIGMA TERM

It has been known that the pion-nucleon sigma term can be determined by the chiral susceptibility \( \frac{\partial M_N}{\partial m_N} \), and the current quark mass in form of Eq. (1). However, it is very complicated to calculate the nucleon mass \( M_N \), which depends on the four-dimensional Poincaré invariance Paddeev equations in the DSE approach of QCD, and the results are still robust to get the dependence of nucleon mass on the current quark mass \([17, 24]\). Meanwhile it is oversimplified to regard nucleon as three non-interacting constituent quarks \([23, 59]\). Therefore, we do not perform the calculation from Eq. (1) directly.

It has been well known that, in the QCD Hamiltonian \( \hat{H}_{QCD} \), the mass term \( \hat{H}_{mass} \) is

\[ \hat{H}_{mass} = \int d^3x (m_u \bar{u}u + m_d \bar{d}d + \cdots), \quad (13) \]

where \( u, d \) denotes the up, down quark with current quark mass \( m_u, m_d \), respectively, \( \cdots \) denotes the contributions from heavier quarks. It is useful to reorganize the up- and down-quark contribution to \( \hat{H}_{mass} \) in order to isolate the isospin breaking effects. Defining \( \bar{q}q = \frac{1}{2}(\bar{u}u + \bar{d}d) \), \( m_q = \frac{1}{2}(m_u + m_d) \), Eq.(13) can be rewritten as

\[ \hat{H}_{mass} = \int d^3x [2m_q \bar{q}q + \frac{1}{2} (m_u - m_d)(\bar{u}u - \bar{d}d) + \cdots]. \quad (14) \]
Making use of the Hellmann-Feynman theorem, one obtains (see, for example, Ref. [60])

\[ 2m_q \langle \Psi | \int d^4x \bar{q} \gamma \Psi \rangle = m_q \langle \Psi | \frac{dH_{\text{mass}}}{dm_q} \rangle \Psi \]

\[ = m_q \frac{d}{dm_q} E_{\Psi}, \tag{15} \]

where \(| \Psi \rangle\) represents a normalized eigenvector of QCD Hamiltonian and \(E_{\Psi}\) stands for the energy of the state \(| \Psi \rangle\).

Considering the case in which \(| \Psi \rangle\) is the state of hadron matter at rest with baryon number density \(n_B\), and also the vacuum, one has

\[ 2m_q[\langle \bar{q}q \rangle_n - \langle \bar{q}q \rangle_0] = m_q \frac{de}{dm_q}, \tag{16} \]

where \(e\) is the energy density of the baryon matter, and can be written as

\[ e = M_N n_B + \delta e, \tag{17} \]

where \(\delta e\) denotes the contributions from the kinetic energy of baryons and baryon-baryon interactions. \(\delta e\) is of high order in density and is empirically small at low densities — the binding energy per nucleon in nuclear matter saturation density is only 16 MeV. Therefore, neglecting \(\delta e\) and implementing Eq. (16), one obtains

\[ 2m_q[\langle \bar{q}q \rangle_n - \langle \bar{q}q \rangle_0] = m_q \frac{dM_q}{dm_q} n_B = \sigma_{\pi N} n_B. \tag{18} \]

Replacing baryon number density \(n_B\) with quark number density \(n_q = 3n_B\), we obtain the linear dependence of the variation of chiral condensate on the quark number density

\[ [\langle \bar{q}q \rangle_n - \langle \bar{q}q \rangle_0] = \frac{\sigma_{\pi N}}{6m_q} n_q = k n_q, \tag{19} \]

where \(k\) is the slope and can be obtained from the linear fitting of the relation in Eq. (19). Conversely, we can evaluate the pion-nucleon sigma term as

\[ \sigma_{\pi N} = 6m_q k = 6m_q \frac{\langle \bar{q}q \rangle_n - \langle \bar{q}q \rangle_0}{n_q}. \tag{20} \]

For the light u, d quark, we can take advantage of the Gell-Mann–Oakes–Renner (GOR) relation [61], which is accurate within 5% [62]:

\[ m_{\pi}^2 f_{\pi}^2 = -2m_q \langle \bar{q}q \rangle_0^0, \tag{21} \]

where \(m_{\pi} = 138\) MeV and \(f_{\pi} = 93\) MeV are well established in experiments, \(\langle \bar{q}q \rangle_0^0\) is the quark condensate in chiral limit (represented by the superscript ’0’) in vacuum. We can then obtain

\[ \sigma_{\pi N} = 3k \left( -\frac{m_{\pi}^2 f_{\pi}^2}{\langle \bar{q}q \rangle_0^0} \right), \tag{22} \]

with which the pion-nucleon sigma term \(\sigma_{\pi N}\) can be well evaluated from the the quark condensate in vacuum and that in medium.

The quark number density \(n_q\) can be calculated from the quark propagator at finite chemical potential, \(S(p; \mu)\), with the definition:

\[ n_q = N_c N_f Z_2^* Tr[\gamma_4 S(p; \mu)], \tag{23} \]

For light quarks, we can further approximate the chiral quark condensate in Eq. (19) with that in the chiral limit, which can be well defined from the quark propagator in chiral limit:

\[ -\langle \bar{q}q \rangle_n^0 = N_c Z_2 Z_m Tr[S^0(p; \mu)], \tag{24} \]

where \(Tr\) represents the trace in color and Dirac space and integration in momentum space, \(Z_2, Z_m\) are renormalization constants for quark wave function and quark mass, respectively.

To be more accurate in the case of physical u, d and even s quark, one can take the current quark mass better fitting the meson properties obtained from the BSE. However, the formula in Eq. (24) are divergent in the case of finite current quark mass. Though some different subtraction points are introduced to give finite values, it is still an open question to define the chiral quark condensate from quark propagator with finite quark mass, see, for example, Ref. [63, 64]. Fortunately, we only need the variation of the chiral quark condensate in medium, which is independent from a fixed subtraction point. Therefore, we also investigate the variation of the chiral quark condensate in medium with finite current quark mass, defined as:

\[ \Delta \langle \bar{q}q \rangle_n^{m_q} = \langle \bar{q}q \rangle_n^{m_q} - \langle \bar{q}q \rangle_0^{m_q} = Z_2 Z_m Tr[S(p; \mu) - S(p; \mu = 0)], \tag{25} \]

where the quark propagator are calculated with finite current quark mass \(m_q\).

### IV. NUMERICAL CALCULATIONS AND RESULT

To carry out the numerical calculations, we need the parameters \(D\) and \(\omega\) in the effective interaction. Usually the parameters are determined by fitting meson properties by the BSE approach. The parameters and some characterized results at \(\mu = 0\) are listed in Table 1. “DSE1” represents the results with the rainbow approximation and the ‘GS’ model for the effective interaction. “DSE2” represents the results with the rainbow approximation and the ‘QC’ model for the effective interaction. “DSE3” represents the results with BC vertex and the ‘GS’ model for the effective interaction. “DSE4” represents the results with the rainbow approximation and the ‘GS’ model for the effective interaction, but the variation of the chiral condensate in medium are calculated with Eq. (25) beyond chiral limit.
TABLE I. Parameters and some characterized numerical results (dimensional quantities in unit of MeV). DES1, DSE2 and DSE3 are in chiral limit $m_q = 0$, while DSE4 investigates chiral condensate beyond chiral limit, see the text for details.

| DSE  | vertex | interaction | $\omega$ | $D$    | $-(\bar{q}q)_0^{1/3}$ | $m_q$  | $k$       | $\sigma_{\pi N}$ |
|------|--------|-------------|----------|--------|------------------------|--------|-----------|------------------|
| DSE1 | RB     | GS          | 500      | 1.00   | 252                    | 5.2    | 1.95 ± 0.03| 61 ± 1          |
| DSE2 | RB     | QC          | 678      | 1.10   | 253                    | 5.2    | 2.04 ± 0.03| 63 ± 1          |
| DSE3 | BC     | GS          | 678      | 0.50   | 258                    | 4.7    | 2.22 ± 0.01| 63 ± 1          |
| DSE4 | RB     | GS          | 500      | 1.00   | −                      | 5.2    | 1.94 ± 0.05| 61 ± 2          |

The parameters of DSE1 and DSE3 are taken from the Ref. [52], and that of the DSE2 are obtained by fitting the pion decay constant $f_\pi = 93$ MeV with Eq. (10) and the chiral quark condensate $-(\bar{q}q)_0 = (250 \text{ MeV})^3$. Notably we get $f_\pi = 93$ MeV with DSE1 and Eq. (10). In DSE4, we take the same parameter values as in DSE1.

With the above settled parameters and the ansatz described in last section, we solve the Dyson-Schwinger equation of the quark propagator and calculate the chemical potential dependence of the chiral quark condensate and the quark number density. The obtained results in chiral limit are illustrated in Fig. 1.

Fig. 1 shows that, when the quark chemical potential $\mu < M_1$, where $M_1$ is the first constituent-mass-like pole of the quark propagator, the chiral quark condensate keeps the same value as that in vacuum (i.e., at $\mu = 0$) and the quark number density maintains zero, i.e. the system remains that in the vacuum and no matter emerges[40]. When $\mu > M_1$, the quark number density becomes nonzero and simultaneously the chiral quark condensate decreases gradually. It indicates that dynamical chiral symmetry is partially restored in the medium at low density [41, 65]. With the above results, we get the variation of the quark condensate $\Delta\langle\bar{q}q\rangle_\mu$ in the medium with respect to the quark number density $n_q$ of the system. The obtained result is displayed in Fig. 2.

Fig. 2 manifests apparently a well linear relation (in Eq. (19) ) between the difference of the quark condensate in low density medium with respect to that in vacuum and the baryon number density, even thought the lines of chemical potential dependence of quark number density and chiral quark condensate are not in linear as shown in Fig. 1. Though adopting different models for the quark-gluon vertex and the effective interaction, and the dependence of the chiral quark condensate and quark number density on the quark chemical potential are quite different, Fig. 2 show that the dependence of the chiral quark condensate on the quark number density are similar. The fitted value of the slope of the lines and deduced values of the sigma term are listed in Table. I. It shows evidently that the result of the $\sigma_{\pi N}$ depends only very weakly on the choice of the ansatz for the quark-gluon vertex and effective interaction.

In DSE4, we investigate the effect of finite current quark mass with Eq. (25), one can find that the result of $k$ and sigma term is not sensitive to such a change. Therefore, we can be quite confident on the chiral limit approximation Eq. (22) for pion-nucleon sigma term and on the validation of Eq. (25) in the case of finite current quark mass.

With the above results, we estimate that the pion-nucleon sigma term $\sigma_{\pi N}$ is about $6k \approx 11$ times the current quark mass $m_q$. It deduces apparently a larger value of $\sigma_{\pi N}$ than the result given in Ref. [23], which estimates $\sigma_{\pi N}$ at chiral limit in the vacuum is $9/2$ times $m_q$. By determining the value of $m_q$ with the the Gell-Mann–Oakes–Renner relation (in Eq. (21)), we obtain the pion-nucleon sigma term $\sigma_{\pi N} = 62(1)(2)$ MeV, where the first represents the systematic error due to our different ansatz for the quark-gluon vertex and gluon propagator, and the second represents the statistical error in our linear fitting procedure. Finally, we compare our results with recent experimental data and theoretical results in Fig. 3. One can notice easily from Fig. 3 that our present result is ev-
Identically consistent with the recent experimental results.

V. SUMMARY AND REMARK

In summary, with the Dyson-Schwinger equations approach of QCD, we calculate the chiral quark condensate in strong interaction matter at low density, and then evaluate the pion-nucleon sigma term \( \sigma_{\pi N} \) via the Hellmann-Feynman theorem. In our work, we adopt different ansätze for the gluon propagator and quark-gluon vertex, and find that our evaluated value of the pion nucleon sigma term depends on the model very weakly. We obtain the result \( \sigma_{\pi N} = 62(1)(2) \text{ MeV} \) in the Dyson-Schwinger equation approach of QCD. Our results are consistent very well with the relatively large value given in recent experimental analysis.

In solving the Dyson-Schwinger equation of the quark propagator, we adopt models of the effective interaction (gluon propagator) which is independent on the quark chemical potential. However, the gluon propagator should depend on the quark chemical potential via the quark loop diagram in its vacuum polarization. It is reasonably to expect this to be a lower order effect on the pion-nucleon sigma term. However, the situation may be different in calculating the sigma term for...
the strange quark $\sigma_s$, which is important for dark matter searches [7]. Since the strange quark chemical potential is zero at low baryon number densities, such an effect is the leading order effect for the variation of the strange quark condensate. It is then necessary to consider this effect in calculating $\sigma_s$. We would investigate it to calculate the $\sigma_s$ and further improve our results on the $\sigma_{sN}$. On the other hand, more sophisticated even the full dressed quark-gluon interaction vertex has also been established (e.g., Refs. [47, 66, 67]). Calculating the $\sigma_{sN}$ and the $\sigma_s$ with the full dressed quark-gluon vertex is also interesting. The related works are in progress.

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