A Virtual State in a Two-Body System Using the Complex Scaling Method

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Abstract. In association with the property of the $1/2^+$ state observed just above the $^8\text{Be}(0^+)+n$ threshold energy in $^9\text{Be}$, we investigate the photo-disintegration cross section of an $s$-state in a schematic two-body system using the complex scaling method.

1. Introduction  
The $^9\text{Be}$ nucleus is one of the most interesting objects in nuclear cluster studies, because this nucleus is considered a typical non-alpha nucleus which has a neutron in addition to the $^8\text{Be}$ core consisting of two $\alpha$-clusters. In particular, it has been a long-standing problem to clarify the structure of the excited $1/2^+$ state in $A=9$ nuclei ($^9\text{Be}$ and $^9\text{B}$). Recently, applying the complex scaling method (CSM) \cite{1, 2, 3} to the $\alpha+\alpha+n$ three-body model for $^9\text{Be}$, we have shown that the sharp peak of the photo-disintegration cross section observed just above the $^8\text{Be}(0^+)+n$ threshold is well explained as a $1/2^+$ virtual state of the $^8\text{Be}(0^+)+n$ two-body configuration \cite{4}.

In this report, we discuss the virtual state nature in more detail using a schematic two-body model and calculate the cross section of the virtual state. We investigate the peak in the cross section, which has a different shape from the Breit-Wigner form. Furthermore, we observe that the phase shift behavior of the virtual state can also be described using the eigenvalue solutions of the CSM. In this contribution, we discuss that the CSM is very useful to study the virtual state.

2. $1/2^+$ state in $^9\text{Be}$  
The $\alpha+\alpha+n$ system is one of the Borromean systems. There is only one bound state, which is corresponding to the $^9\text{Be}$ ground state with $J^\pi = 3/2^-$. The first excited state is a $1/2^+$ unbound state above the $\alpha+\alpha+n$ threshold, and has been observed as a sharp peak in the
photo-disintegration cross sections [5, 6]. The photo-disintegration cross section is observed above but not below the $^8$Be($0^+$) + $n$ threshold. The behavior of the cross sections suggests a dominant two-body component of $^8$Be($0^+$) plus $s$-wave neutron, while there is no barrier to keep the neutron inside a potential.

We performed the complex scaling calculation of an $\alpha + \alpha + n$ model to look for a three-body resonance or a $^8$Be($0^+$, $2^+$) + $n$ coupled channel resonance for the $1/2^+$ state. In the results, we could not find a sharp resonance corresponding to the peak of the photo-disintegration cross section. As for the experimental sharp peak, we can well reproduce its position and shape as shown in Figure 1 without resonance solutions. The conclusion of our study is that the peak of the photo-disintegration cross section is due to the virtual state of $^8$Be($0^+$) + $n$.

3. Virtual state in two-body model with the complex scaling method

To understand the photo-disintegration cross section due to the virtual state, we investigate a simple two-body potential model for $^8$Be+$n$ system in the CSM. The CSM has been believed not possible to describe virtual states corresponding to $S$-matrix poles on the negative imaginary axis on the complex momentum plane. However, we show that continuum solutions of the CSM describe the virtual state through the phase shift of the $s$-wave.

The simple two-body potential model is expressed by the Hamiltonian $H = -\frac{1}{2}\nabla^2 + V(r)$, where $V(r) = V_1 \exp (-0.16r^2)$. Using the complex-scaled Hamiltonian $H(\theta)$, we solve the complex-scaled Schrödinger equation $H(\theta)\Psi_{J^\pi}(\theta) = E^\theta \Psi_{J^\pi}(\theta)$.

The energy levels of $J^\pi = 0^+$ and $1^-$ obtained by solving the complex-scaled Schrödinger equation are presented schematically in Figure 2. The potential strength $V_1$ is taken so as to reproduce one bound $0^+$ of $s$-waves. But, this $0^+$ solution is assumed as the Pauli forbidden state, because this model simulates $^8$Be($0^+$) + $n$. So, in this model, the $1^-$ solution describes the ground state, and there is no $0^+$ bound state. To see the transition of the excited $0^+$ solution from unbound to bound states, we change the strength $V_1$ for $0^+$ state.

We can calculate the scattering phase shift using the eigenvalue solutions of $H(\theta)$ and $H_0(\theta)$.
Figure 3. Calculated phase shifts at $V_1 = -1.42$ MeV and $-1.43$ MeV. The dashed red and open black curves are phase shift calculated at $V_1 = -1.42$ MeV and $-1.43$ MeV, respectively.

Figure 4. Calculated photo-disintegration cross sections for the Gaussian potential with the strength $V_1 = -1.42$ MeV. The arrow indicates the threshold energy.

in the CSM [7], where $H_0(\theta)$ is the free-Hamiltonian without potentials.

$$
\delta(E) = N_b\pi + \sum_{r=1}^{N_b^\theta} \left\{ - \cot^{-1}\left( \frac{E - E^{res}_r}{\Gamma_r/2} \right) \right\} + \sum_{c=1}^{N_c^\theta} \left\{ - \cot^{-1}\left( \frac{E - \epsilon_c^{Re}}{\epsilon_c^{Im}} \right) \right\} - \sum_{k=1}^{N} \left\{ - \cot^{-1}\left( \frac{E - \epsilon_{0k}^{Re}}{\epsilon_{0k}^{Im}} \right) \right\},
$$

(1)

where the eigenvalues are classified into the numbers of $N_b$ bound states, $N_r^\theta$ resonant states and $N_c^\theta$ continuum states for a given $\theta$; $N = N_b + N_r^\theta + N_c^\theta$ for total number of eigenstates. The energy eigenvalues of $H_0(\theta)$ are given as $\epsilon_{0k}^{Re} - i\epsilon_{0k}^{Im}$ with $k = 1, \cdots, N$. In Figure 3, the calculated phase shifts are shown for $V_1 = -1.42$ and $-1.43$ MeV, where an additional bound state appears suddenly at $V_1 = -1.43$ MeV. The shape of phase shifts of $V_1 = -1.42$ MeV indicates a virtual state behavior.

Using the virtual state solution with $V_1 = -1.42$ MeV, we also calculate the $E1$ strength and the photo-disintegration cross section. The result is shown in Figure 4, and the peak is obtained at a lower energy region in the similar way as was observed in $^9$Be.

4. Conclusion
To understand the structure and the origin of the peak of the photo-disintegration cross section for the $1/2^+$ state in $^9$Be, we investigated the simple two-body potential model in detail in the CSM. It was shown that the virtual state can be described by the CSM through calculations of the phase shift. The $E1$ strength is also calculated to show the similar peak as observed for the $1/2^+$ state in $^9$Be.

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