Stocks and Cryptocurrencies: Anti-fragile or Robust?

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Abstract

Antifragility was recently defined as a property of complex systems that benefit from disorder. However, its original formal definition is difficult to apply. Our approach has been to define and test a much simpler measure of antifragility for complex systems. In this work we use our antifragility measure to analyze real data from the stock market and cryptocurrency prices. Results vary between different antifragility interpretations and for each system. Our results suggest that the stock market favors robustness rather than antifragility, as in most cases the highest and lowest antifragility values are reached either by young agents or constant ones. There are no clear correlations between antifragility and different ‘good-performance’ measures, while the best performers seem to fall within a robust threshold. In the case of cryptocurrencies, there is an apparent correlation between high price and high antifragility.

Introduction

Antifragility was defined by N. Taleb [1] as a property of complex systems that benefit from disorder. The original formal definition [2], however, is difficult to use in practical terms. We have recently proposed a pragmatic antifragility measure for complex systems, provided one can define measures of “satisfaction” for each of its agents and of “perturbations” for the whole system. Under this definition, an agent is robust if its antifragility is close to 0 and fragile if it is negative. Such a definition of antifragility has been studied in the context of random Boolean networks (RBNs) [3], Multi-layer RBNs [4], Convolutional Neural Networks [5], and ecosystems [6].

In this work we use our antifragility measure to study real data from the stock market and cryptocurrency prices, considering several different perturbation measures (involving the difference of the price, normalized price and volume of each agent, and the volatility index VIX, in two consecutive observations) and the difference of the price of an agent as a
satisfaction measure. Each of these perturbation measures defines a different antifragility measure that we tested in daily, weekly, monthly and annually time-scales.

The rest of the paper is organized as follows. The next section includes some background on the study of physical systems that “gain from disorder”. Afterwards we explain our definition of antifragility both in an abstract way and for our case study. In the fourth section we describe the data and the variables employed. Section 5 contains the results, and finally we include some discussion and concluding remarks.

Background

The idea of studying nonlinear systems that use noise, an external stochastic signal or some form of disorder, has been explored for several decades. These ideas encompass different phenomena.

Stochastic resonance

The idea of stochastic resonance is to add noise to a signal and measure the signal to noise ratio SNR. Contrary to what is observed in a linear system, where the addition of noise decreases the SNR, for some non-linear systems we find that the SNR increases as the intensity of the noise increases, until we reach a maximum after which the SNR starts to decrease. In this way, we obtain a SNR, as a function of the intensity of the noise, that has a maximum, similar to a resonance in a physical system; thus, the name of stochastic resonance. This phenomenon can be observed in physical and biological systems alike. The idea was introduced in the early 1980s by G. Parisi and collaborators. For a thorough review see [7].

Noise-induced transport and Brownian motors

These phenomena refer to the counterintuitive idea of incorporating noise or a stochastic signal to enhance and generate transport in an asymmetric physical system. The asymmetry, that can be spatial or temporal, rectifies a symmetric noise generating a finite current. The phenomenon was studied by Richard Feynman in the early 1960s and gained attention in the 1990s with the study of thermal ratchets and Brownian motors that use Brownian motion on top of asymmetric ratchet potentials. One of the key motivations was to understand the mechanism behind motor proteins inside the cell and applications to nanoscience. For a review, see [8]. For a combination of Brownian motors and stochastic resonance, see [9].

Optimization by simulated annealing

This is an important phenomenon where one implements simulated annealing, as in Statistical Mechanics, using thermal noise, to escape from local minima in a landscape to arrive at a global minimum; thereby finding an optimization function. There is a whole literature around this idea. See for instance the original paper [10].
There are other phenomena related to the idea of antifragility, like noise-induced synchronization in non-linear physical and biological systems [11], the “order from noise” [12], and the “complexity from noise” [13] principles.

**Materials and Methods**

**Definition of antifragility**

We consider systems where, at every instant of time, there are values of satisfaction \( S \) for each of its agents and perturbation \( P \) for the whole system. Let us assume that \(-1 \leq S \leq 1\), for every agent, and \(0 \leq P \leq 1\). More concretely if

\[ S(x, i) \text{ is a satisfaction measure of agent } x \text{ at time } i, \]

\[ P(i) \text{ is a perturbation measure of the whole system at time } i, \]

the antifragility for the agent \( x \) at time \( i \), is defined as:

\[ A(x, i) = S(x, i) * P(i). \] (1)

We define the global *antifragility* of agent \( x \) as the mean value of \( A(x, i) \) over the whole time interval under consideration, *i.e.*

\[ A(x) = \frac{\sum_{i=1}^{n} A(x, i)}{n}, \] (2)

where \( n \) is the total number of observations.

**Satisfaction measure**

Throughout this study we considered two different systems: the stock market and the cryptocurrencies’ market. Intuitively, an agent (either a stock or a cryptocurrency) has positive satisfaction at an instant in time if its price rose from the previous observation. So, the satisfaction measure we used is the (normalized between -1 and 1) difference of the (normalized between 0 and 1) price of the stock over two consecutive time intervals:

\[ S(x, i) = p(x)_i - p(x)_{i-1}, \] (3)

where \( p(x)_i \) is the normalized price of the agent \( x \) at time \( i \).

**Perturbation Measures**

We focused on four different\(^1\) perturbation measures for each system. Some perturbation measures are defined for each agent and the perturbation of the whole system is the mean perturbation over all of the agents. In the case of cryptocurrencies all of the perturbation measures are taken this way. But in the case of stocks we also considered the volatility index with symbol VIX, and a mean of the volatility of Nasdaq, Dow Jones and S&P 500 indexes. More concretely, let \( o(x)_i \), \( v(x)_i \), \( m(x)_i \), be the (open) price, volume, and market capitalization of agent \( x \) at time \( i \). Then, in the case of stocks, we defined:

- \[ P_p(x, i) = |o(x)_i - o(x)_{i-1}|, \]

\(^1\) A brief explanation on how we chose these four measures takes place in the discussion section.
\[ P_v(x, i) = \frac{|S(x, i) + v(x, i) - v(x, i-1)|}{2}, \]

- \( P_x(i) \) is the (normalized between 0 and 1) value of the volatility index VIX. 
- \( P_{3m}(i) \) is the (normalized) mean of absolute differences of the Nasdaq, Dow Jones and S&P 500 indexes between time \( i - 1 \) and time \( i \).

And in the case of cryptocurrencies:

- \( P_p(x, i) = |o(x, i) - o(x, i-1)|, \)
- \( P_v(x, i) = |v(x, i) - v(x, i-1)|, \)
- \( P_m(x, i) = |m(x, i) - m(x, i-1)|, \)
- \( P_n(x, i) = |S(x, i)|. \)

The measures \( P_n(x, i) \) and \( P_p(x, i) \) are different in the sense that in \( P_n(x, i) \) the prices were normalized before taking the mean, and for \( P_p(x, i) \) afterwards. So the perturbation recorded by a stock in the former measure is the same for two agents of different prices if they changed the same percentage of their price. While in the latter the perturbation recorded by an expensive cryptocurrency will almost always be greater than the perturbation of a cheap cryptocurrency.

When the perturbation measure is defined for agents (as in all of the cases except for \( P_x(i) \) and \( P_{3m}(i) \) ), the perturbation measure of the system is defined as the mean value over all of the agents. That is, if \( P(x, i) \) is the perturbation suffered by agent \( x \) at time \( i \), then

\[ P(i) = \frac{\sum_{x} P(x, i)}{k}, \] (4)

where \( x \) varies over the set of agents and \( k \) is the total number of them.

Thus, each perturbation measure is used to define an antifragility measure \( A \) which we shall identify by the sub-index of the perturbation measures. We denoted them by \( af_p, af_v, af_x, af_{3m} \), in the case of stocks, and by \( af_p, af_v, af_n, af_{m} \), in the case of cryptocurrencies. For each of these measures, we computed three different antifragility values for each agent, each of them associated to a different time-scale: daily, weekly, monthly. We used a suffix which represents the time-scale under consideration (0: daily, 1: weekly, and 2: monthly).

**Variable Definitions**

Throughout our analysis we regarded the change of price of an agent as a satisfaction measure. This means that we classify antifragile, robust and fragile agents as those whose price rises, stands still and decreases, respectively, when the system is perturbed. Moreover, we compared quantitatively the antifragility values of every agent with several different “performance measures” listed next:

- **age**: age of the agent in days.
- **pct_dlt-pr**: maximum price minus minimum price divided by the mean price.
- **pct_dlt-mk**: same as before but using market capitalization instead of price.
- **pct_dlt-vl**: same as before but using volume.
- **pct_pr-f-i**: final price minus initial price divided by the mean price.
- **pct-mk-f-i**: same as before but using market capitalization instead of price.
- **pct-vl-f-i**: same as before but using volume.
- **pr_mea**: mean price.
- **pr_std**: price standard deviation.
- **mk_mea**: mean market capitalization.
- **vl_mea**: mean volume.

The variables which involve the market capitalization were only used in the case of cryptocurrencies as we do not have this data for stocks. Instead of them, in the case of stocks, we used lists of the best stocks of the year (from 2010 to 2017) according to different stock market analysts such as Forbes, Yahoo Finance, Stock Market Watch, among others. We will often refer to the stocks belonging to such lists as the ‘top-performers’ and to the variables listed above as the ‘good-performance’ measures.

**Results and Discussion**

**Stocks**

We carried on several different comparisons between our variables and the obtained values of $A$. On a first analysis, plotting all of our variables vs all types of antifragility in eight consecutive years starting in 2010, we come up with the following observations:

Among the following plots, colors represent a time scale: daily in blue, weekly in green, monthly in red.

1. The large scale behavior among all different $A$ definitions, years, and time scales do not change drastically (see Fig. 2).
2. There are no linear correlations between any of the ‘good-performance’ measures and the antifragility measures (see Fig. 8).
3. The higher values of the ‘good-performance’ measures concentrate around the robust ($A=0$) axis (see Fig. 1).
4. The behavior from Observation 3 is clearer and more consistent for different definitions and time scales among the ‘good-performance’ measures $age, vl_mea, pct_dlt-vl$ (see Fig. 3).
5. The probability distributions of the antifragility measures are close to normal distributions, while the probability distributions of the antifragility values of the ‘top-performers’ skew from the former to higher values of antifragility. The mean $A$ of the top performers is greater than the mean $A$ of all of them 56% of the times. In these cases, the sum of the differences of the mentioned mean values is almost two times bigger than the sum in the rest of the cases (see Fig. 4).
Figure 1: Left frame: \texttt{pct\_dlt-vl}, Right frame: \texttt{vl\_mea}. Each column of such frames correspond to a year (2010-2017), and each row to an \( A \) measure from up to bottom: \texttt{af3m, afp, afv, afx}. Each plot compares the corresponding \( A \) measure (x-axis) to the ‘good-performance’ measure under consideration.

Figure 2: There are 6 frames in the image, each of them corresponds to one ‘good-performance’ measure, from up-left to bottom-right: \texttt{pct\_dlt-vl, pct\_pr-f-i, pct\_dlt-pr, pr\_mea, vl\_mea, pr\_std}. Each column of such frames correspond to a year (2010-2017), and each row to an \( A \) measure from up to bottom: \texttt{af3m, afp, afv, afx}. Each plot compares the corresponding \( A \) measure (x-axis) to the ‘good-performance’ measure under consideration.
Figure 3: Columns: years from 2010 to 2017; rows: \textit{af3m, afp, afv, afx}. Each frame compares the maximum, mean, and minimum of ‘good-performance’ measures (y-axis) \textit{age} (a) and \textit{vl_mea} (b), among five bins containing the same amount of stocks sorted in low, mid-low, middle, mid-high and high values of the corresponding \textit{A} measure.

Figure 4: Columns: years from 2010 to 2017; rows: \textit{af3m, afp, afv, afx}. Each frame shows the probability distribution of \textit{A} values of all stocks (lines) and ‘top-performers’ (dots).
Cryptocurrencies
We carried on basically the same analysis as in the case of stocks. Even though the first two observations from before also hold in this case, there is way less structure to spot from the data. Among the following plots, colors represent a time scale: daily in green, weekly in blue, monthly in red. Still:

6. The large scale behavior among all different $A$ definitions, years, and time scales do not change drastically. (See Fig. 5).

Figure 5: There are 10 frames in the image, each of them corresponds to one ‘good-performance’ measure, from up-left to bottom-right: $\text{pct\_dlt\_pr, pct\_dlt\_mk, pct\_dlt\_vl, pct\_pr\_f\_i, pct\_mk\_f\_i, pct\_vl\_f\_i, pr\_mea, pr\_std, mk\_mea, vl\_mea}$. Each column of such frames corresponds to a year (2013-2018), and each row to an $A$ measure from up to bottom: $\text{afm, afn, afp, afv}$. Each plot compares the corresponding $A$ measure (x-axis) to the ‘good-performance’ measure under consideration.

7. There are no linear correlations between any of the ‘good-performance’ measures and the antifragility measures (See Fig. 8).

8. The higher values of the ‘good-performance’ measures are more distributed along the $A$ axis than in the case of stocks. (See Fig. 5).

9. Contrary to what happens in the case of stocks as in Observation 4, higher values of $A$ measures are achieved by cryptocurrencies with higher values of ‘good-performance’ measure. Such behaviour is more clearer for the measures $\text{pr\_mea}$ and $\text{pr\_std}$. 

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Figure 6: Columns: years from 2013 to 2018 and the last column contains the mean over all columns; rows: \texttt{afm, afn, afp, afv}. Each frame compares the maximum, mean, and minimum of the corresponding $A$ measure (y-axis) with the ‘good-performance’ measure $pr\_std$, among five bins containing the same amount of stocks sorted in low, mid-low, middle, mid-high and high.

10. The probability distributions of the antifragility measures are close to skew-normal distributions, more commonly towards fragility but sometimes the other way around. The probability distributions of the antifragility values of the ‘top-performers’ does not seem to improve from the rest. The mean $A$ of the top performers is greater than the mean $A$ of all of them 58\% of the times, but the differences of such mean values in these cases add up to 50\% more than the rest of the cases.

Figure 7: Columns: years from 2013 to 2018; rows: \texttt{afm, afn, afp, afv}. Each frame shows the probability distribution of $A$ values of all stocks (lines) and ‘top-performers’ (dots).
Discussion

Among the several different definitions for perturbation measures we first considered, the four measures we analyzed further were selected because of their better correlations both between them and each of them among the different time scales.

![Correlation heatmaps between A and good-performance measures. Left, stocks; right, cryptocurrencies.](image)

Conclusions

Results are different for the systems considered, and although they also vary between our different definitions of antifragility, there are indications suggesting that the Stock Market do not show antifragility explicitly, as in most of the cases the highest and lowest antifragility values are reached either by young agents or those with fewer transaction volume. There are no clear correlations between our antifragility measures and several different ‘good-performance’ measures, while the best performer agents according to different market analysts seem to fall within a robust threshold.

The case of cryptocurrencies seems to be different, and the tools from the previous case seem limited. Not only there is way less information but also the market seems to be governed by a single agent. Even so, observations suggest that the more expensive cryptocurrencies are also the more antifragile.

Data Availability

We analysed a data-set with daily historical data (open price and volume) of more than 7,000 stocks from the US Stock Market obtained in this [link](https://www.kaggle.com/borismarjanovic/price-volume-data-for-all-us-stocks-etfs). We truncated the data starting from

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2 [https://www.kaggle.com/borismarjanovic/price-volume-data-for-all-us-stocks-etfs](https://www.kaggle.com/borismarjanovic/price-volume-data-for-all-us-stocks-etfs)
1990. Such a dataset considered only active stocks by the end of 2017 and there is no data from stocks that disappeared from the market before that date.

In the case of cryptocurrencies, the data consisted of daily historical data (open price, market capitalization and volume) of around 1800 agents, the eldest of them starting in 2013 and up to November 2018. Instructions for accessing the processed data-sets may be found here.

Conflicts of Interest

The authors declare that there is no conflict of interest regarding the publication of this paper.

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