Universal Voltage Fluctuations in Disordered Superconductors

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The Aharonov-Casher effect is the analogue of the Aharonov-Bohm effect that applies to neutral particles carrying a magnetic moment. This can be manifested by vortices or fluxons flowing in trajectories that encompass an electric charge. These have been predicted to result in a persistent voltage which fluctuates for different sample realizations. Here we show that disordered superconductors exhibit reproducible voltage fluctuation, antisymmetrical with respect to magnetic field, as a function of various parameters such as magnetic field amplitude, field orientations and gate voltage. These results are interpreted as the vortex equivalent of the universal conductance fluctuations typical of mesoscopic disordered metallic systems. We analyze the data in the framework of random matrix theory and show that the fluctuation correlation functions and curvature distributions exhibit behavior which is the fingerprint of Aharonov-Casher physics. The results demonstrate the quantum nature of the vortices in highly disordered superconductors both above and below $T_c$.

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The Aharonov-Casher (AC) effect is the dual effect to the Aharonov-Bohm (AB) effect. While in the AB effect a charged particle quantum mechanical phase is affected by the electromagnetic vector potential, in the AC effect a neutral particle carrying a magnetic moment quantum mechanical phase is also affected by an "charge vector potential", even when no force is exerted on the particle.

Magnetic vortices are an example of a neutral particle with a magnetic moment and the AC effect was discussed for vortices in type II superconductors. A somewhat different realization of similar ideas is for a vortex (fluxon) in a two-dimensional (2D) Josephson-junctions arrays (see Fig. 1a). Although it carries no local magnetic flux, such a fluxon's phase is influenced by the charge encompassed by the array. Indeed oscillatory behavior has been observed for transport measurements of such arrays.

The phenomenon of persistent currents is an illuminating demonstration of the AB physics. Essentially when a ring encircles a magnetic flux $\Phi$ a persistent equilibrium current proportional to $\frac{\partial E}{\partial \Phi}$, where $E$ is the energy, is predicted and measured. van Wees realized that for the dual situation of a vortex in a 2D ring shaped array circling a charge $Q$ a persistent voltage between the inner and external circumference of the ring proportional to $\frac{\partial E}{\partial Q}$ should appear. This prediction has not been yet experimentally verified.

A different direction from the neat realizations described above is to seek manifestations of AC physics in disordered samples (equivalent to mesoscopic disordered metals that exhibit universal conductance fluctuations (UCF)). Here, one gains in avoiding difficult preparation of the sample, but has much less control on the details (Fig. 1b). Specifically, a disordered 2D Josephson array composed of irregular placed and shaped superconducting islands is expected to exhibit AC physics manifested by reproducible voltage fluctuations for different sample realizations.

In this letter we describe measurements of spontaneous voltage in amorphous indium oxide ($a-InO$) films where the disorder is tuned so the samples are close to the superconductor-insulator transition (SIT). These samples, despite being morphologically uniform, have been shown to include "emergent granularity" in the form of
FIG. 2: (a) Spontaneous voltage as a function of gate voltage $V_G$ and out-of-plane magnetic field $H$ at 2K for a sample of $T_c = 3.2K$. Quasi-periodicity arises both in response to magnetic field and enclosed charge on the islands, controlled by $V_G$. (b) Spontaneous voltage as a function of out-of-plane magnetic field for a few $\alpha-InO$ films. (c) $V(H)$ as a function of angle of the applied field relative to the film plane for one of our films. Quasi-periodicity w.r.t. the magnetic field is seen be present even when the field is applied along the sample plane, indicating that effects other than the orbital effect are at work.

superconducting puddles embedded in an insulating matrix \cite{19,20}, hence, they are perfect candidates for detection of AC effect signatures. For these films we find reproducible voltage fluctuations as a function of magnetic field amplitude, field orientation, and gate voltage. We analyze the results in terms of random matrix theory and show that they exhibit universal features expected for the AC effect.

The studied samples were $\alpha-InO$ films of thickness 30nm that were e-beam evaporated on MEMpax™ borosilicate glass or silicone substrates of thickness 0.4mm. The $O_2$ partial pressure during evaporation (in the range 1-8x10^{-5} Torr) determined the initial state of the sample, superconductor or insulator. The results presented in this letter represent measurements performed on 7 samples spanning the SIT with sheet resistance $R_{T=5K}$ ranging from 600Ω to 100 kΩ.

The natural parameter to vary for obtaining different sample realizations for the AC effect is the gate voltage which controls the charges trapped between the islands and the various loops of vortex trajectories. However, for our 2D disordered samples on conventional substrates, obtaining detailed enough structure for analysis requires unattainable large voltages. Fig. 2a shows that varying the gate voltage does result in voltage fluctuations. However, utilizing the variation of magnetic field as a driving parameter gives rise to much richer structure, hence we prefer this knob for generating AC voltage fluctuations. Depending on the magnetic field, which determines the vortex arrangement in the sample, a reproducible random persistent voltage between two points on the edge of the sample can be expected due to several different origins, the most obvious being the variation of the number of fluxons. Field may also change the superconducting properties of the islands as well as the charge distribution in the normal metal areas.

Fig. 2b depicts the voltage as a function of perpendicular magnetic field for a number of $\alpha-InO$ film spanning the SIT. We note that for any realistic experimental setup one can not avoid some stray voltage, $V_0$, which is present even at zero magnetic field. By subtracting this stray voltage, we obtain an anti-symmetric with magnetic field ($V(H) = -V(-H)$), reproducible structure. The anti-symmetric nature of the fluctuations indicates that they originate from vortex motion. The structure is a true "fingerprint" of the sample microscopics in the sense that it is very reproducible for different magnetic field scans of a single sample but it changes from sample to sample. Hence, we attribute these results to the AC equivalent of the UCF originating from the AB effect in disordered metallic systems.

It should be noted that unlike AB effect, AC physics does not depend directly on the magnetic flux penetrating the sample and thus is not expected to be washed out when the orientation of the field is varied from perpendicular to parallel to the film. Indeed, Fig. 2c, that shows the $V(H)$ curves as a function of magnetic field angle, demonstrates that though the voltage measurements depend on the magnetic field orientation, the amplitude and the typical field scale of the fluctuations do not vary much as a function of field orientation.

Fig 3 shows the monotonous suppression of the effect with growing temperature. As with the AB effect, decoherence, as result of e.g. temperature should reduce the observed effect. Nevertheless, the interpretation of effects of temperature on coherence should be done carefully, since it also effects the superconductivity of the grains composing the network and thus the vortices. A similar situation occurs also for the magnetic field.

Since in these conditions one can not model (nor control) the details of the sample one must look for global and statistical properties of the reproducible voltage in order to verify its origin. The experience of mesoscopic physics has taught us that even for disordered systems certain universal properties can be teased out of the data,
which attest to the physics of the system [30]. As in other cases of disordered mesoscopic systems [31,34] we assume that the universal part of the behavior of the system may be captured by a random matrix Hamiltonian. This assumption works well for the description of diverse phenomena from correlations in the conductance at different magnetic fields [35] to the fluctuations in position and height of conductance peaks in the Coulomb blockade regime [36,39].

The basic logic is similar here. We start with a rather general random matrix Hamiltonian describing a disordered 2D Josephson array composed of irregular placed and shaped superconducting islands. Depending on the magnetic field and on the charges trapped between the islands a reproducible random persistent voltage between two points on the edge of the sample is expected. Keeping track of all the influences of magnetic field on the vortex motion is a rather herculean task. Nevertheless, we can exploit the theory developed for correlations [40–43] and curvature distribution [44–47] of the spectral response to external parameters which show universal behavior of the derivative of the energies of the system with respect to an external parameter. Specifically, for the $i$-th eigenvalue $\epsilon_i(x)$ (where $x$ is the value of the external parameter) of the Hamiltonian one may define a “velocity” $j_i = \partial_x \epsilon_i(x) / \delta$ (where $\delta$ is the mean level spacing, i.e. a system dependent parameter) and a “curvature” $K_i = \partial^2_x j_i(x)$ characterizing the response of the spectrum to the external parameter. This corresponds to the identification of the persistent voltage with the current and its derivative with the curvature. The correlation

$$C_i(\delta x) = \langle j_i(x) j_i(x + \delta x) \rangle, \quad (1)$$

where $\langle \ldots \rangle$ is an average over different systems and ranges of $x$. After a renormalization of the parameter $X = \sqrt{C_i(0)} x$, a universal behavior emerges [40,43]

$$C_i(\delta X) = -\frac{2}{(\beta \pi^2 \delta X)^2}, \quad (2)$$

as long as $\delta X$ is larger than some non-universal value and $\beta = 1$ for the orthogonal ensemble (GOE) and $\beta = 2$ for the unitary one (GUE). It is important to note that since $x$ depends on a non-universal parameter of the system $\delta$, which is hard to obtain, one can determine $X$ only up to a factor. This leads to an unusual correlation curve since the correlation should be maximum at $\delta X = 0$, and approaches zero from below as $\delta X$ gets large, which means that there is a negative minimum of the correlation at some intermediate value of $\delta X$. For the curvature, a universal distribution is expected [44–47]. Defining $k = K_i/\langle |K_i| \rangle$, one obtains the distribution

$$P(k) = \frac{A_\beta}{(1 + k^2)^{(2+\beta)/2}}, \quad (3)$$

where $A_1 = 1, A_2 = (4/\pi)$. Corrections to this distribution, which are especially notable at low values of $k$ as result of non-universal features have also been discussed [45–47]. Since here one normalizes the curvature by its absolute averaged value, the dependence on $\delta$ disappears.

The negative minimum of the correlation as well as the absolute averaged value, the dependence on $\delta$ disappears. This assumption works well for the description of diverse phenomena from correlations in the conductance as well as the identification of the fluctuation in voltage with the derivative of the energy, in agreement with the Aharonov-Casher picture.

Here we analyze the results of seven different samples, for one of them, we analyze the data for 7 different angles of the magnetic field with respect to the sample plain, and for another sample at a range of temperatures between $T = 0.42 \, K$ and $3.26 \, K$. After subtracting the stray voltage background $V_0$ we antisymmetrize the data in the following way:

$$V_{AS}(H) = \frac{V(H) - V(-H)}{2} - \bar{V}, \quad (4)$$

where $\bar{V}$ is the average over $(V(H) - V(-H))/2$ for the whole range of measurement so $\langle V_{AS}(H) \rangle = 0$.

In order to substantiate these observation we calculate the correlation:

$$C(\delta H) = \langle V_{AS}(H + \delta H) V_{AS}(H) \rangle_H / \langle V_{AS}^2(H) \rangle_H, \quad (5)$$

where $\langle \ldots \rangle_H$ denotes an average over the available values of the magnetic fields. The samples show a peak in the correlation which then turns into negative values and

FIG. 3: Temperature dependent V-H curves for an $a-InO$ sample in the range $T = 0.39 \, K$ (purple) to $4.00 \, K$ (brown). Inset: Temperature dependence of the ‘amplitude’ $\Delta V$ (black symbols) and resistance (blue line).
FIG. 4: Top: The correlation $C(\delta X)$ for seven different samples where $\delta X = a \delta H$ and $a$ is a sample dependent rescaling constant. The dashed curve corresponds to $-e/(\delta X)^2$, with $c = 2/\pi^2$ (GOE). As discussed in the text plotting the GUE curve ($c = 1/\pi^2$) will result with same figure with rescaled values of $a$ by half. A numerical generated sequence on $1/f$ noise for a short ($10^2$) and long ($10^4$) sequence are shown for comparison. Bottom: The distribution $P(k)$ aggregated over seven different samples, as well as the distribution for seven different angles of the external magnetic field, and for low $T < 2.3K$ and high $T > 2.3K$ temperatures ($T_c$, defined at the temperature at which the resistance drops to 10% of its normal value, equals 2.3K). For comparison, the distribution for a numerical sequence of $1/f$ noise is presented. The GOE ($\beta = 1$) and GUE ($\beta = 2$) distributions are plotted.

then returns to values around zero. Recalling the correlations such that $\delta X = a \delta H$, with $a$ a sample specific constant results in a very similar correlation curve for all samples (see Fig. 4a). It is important to note that since $a$ depends on a microscopic scale which is unknown to us, here it is not possible to distinguish between GOE and GUE ($\beta = 1, \beta = 2$). Nevertheless, the width of the correlation for all samples is similar after rescaling, and the correlation follows the expected $\propto (\delta X)^{-2}$ behavior.

In Fig. 4b the experimental distribution for several different ensembles are plotted. Since here we can avoid sample specific fit parameters by dividing the curvature by its sample average over different magnetic fields, we can pool together data from several samples, angles of the external magnetic field and temperatures. All of these distributions have several common features. They follow quite closely the GOE distribution, especially if one compares distribution from numerical generated $1/f$ noise sequence. The deviations are most pronounced for small values of $k$ as has been been expected from non-universal corrections to the distribution $[45, 47]$. It is also notable that the measurement of 7 different angles treated as independent samples yields similar results as for 7 different samples. This lends further support to the fluxon interpretation which is not based on orbital effects.

Although as seen in Fig. 4 temperature suppresses the amplitude of the voltage fluctuation it hardly wipes out the curvature distribution although at temperatures above $T > 2.3K$ the global superconductance is strongly suppressed. This hints that there is no need for a global coherent superconductivity, and remnant local superconductivity suffices. Similarly, we obtain repeatable voltage structure for samples in the insulating phase though these have not been analyzed here. Indeed, local superconductivity, manifested by a finite gap, was observed both above $T_c$, $[48]$ and in the insulating phase $[49]$ of $a-InO$ films. This may also address another puzzle. A magnetic field breaks time reversal symmetry and therefore one expects the appropriate random matrix ensemble to be GUE. Here, the distribution is GOE, which can be understood if the mechanism for the voltage fluctuation is short ranged and therefore time reversal symmetry is not broken on that length scale.

In summary, granular samples around the superconducting transition are promising candidates for experimental studies of additional features of random matrix theory. In our disordered superconductors, that incorporate "electronic granularity", the correlations and curvature of the voltage fluctuations fit much better the expectation for a system that follows the universal features of random matrix theory than those of non-correlated noise (e.g. $1/f$ noise). Nevertheless, non-universal features are prevalent and need further study. Although some features such as level spacings and wave function properties have been extensively studied experimentally in systems such as quantum dots $[54]$, features such as level velocity and curvature to the best of our knowledge have not. The identification of the voltages fluctuations as signatures of the AC effect emphasize the quantum nature of the vortices close to the SIT, below and above $T_c$.

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