Surrogate-assisted Meta-Heuristic method for Aerodynamic Design of an Aircraft Wing

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Abstract
This paper presents the aerodynamic design of an aircraft wing using a surrogate-assisted meta-heuristic (MH) method. The optimization problem is posed to find the wing shape in order to maximize its lift-to-drag ratio. Two surrogate-assisted MHs are presented which are the use of a surrogate model to predict the objective function directly and using a surrogate model to predict lift coefficient and drag coefficient separately before calculating the objective function. Computational fluid dynamic analysis is used for calculating the values of lift and drag coefficients while a Differential Evolution (DE) algorithm and a radial basis function are respectively used as MH and a surrogate model. From this study, the performance of the surrogate-assisted design approaches for aerodynamic optimization of an aircraft wing is obtained. The attained results are said to be the baseline for future study of surrogate-assisted aerodynamic optimization.

Keywords: Evolutionary algorithms, Metamodel, Aerodynamic of aircraft wing, Computational Fluid Dynamics

1. Introduction
In an aircraft design, aerodynamic shape optimization is necessary while Computational Fluid Dynamics (CFD) and optimization method involve important parts in the design activity. The optimization method that is commonly used for real engineering design activity and is divided into two groups as gradient-based and non-gradient-based methods. The gradient-based method is efficient with high convergence speed and searches consistency. However, they require function derivatives and usually can be trapped at a local optimum. For the gradient-free methods, the best-known method is meta-heuristics (MHs), which is said to be a global optimization method (they can avoid a local optimum trap) while function derivatives are not required. As a result, they can deal with any kind of objective function and design variables. However, most of MHs have a lack of search consistency and a large number of function evaluations required. It is difficult to use MHs for a problem with computational expense such as CFD based aerodynamic shape design [1][2] Therefore, the development of an optimization technique for a particularly computational expensive problem is an interesting research direction.
Nowadays, the use of surrogate-assisted MHs is presented for optimization of many computational expensive problems. The main idea is to use a surrogate model for estimating the real computational expensive function values and perform optimization by MHs [3][4][5]. The surrogate model is built based on many techniques such as polynomial regression, radial basis function, Kriging, support vector regression [3] and neural network. For the aircraft wing design for aerodynamic optimization, various researchers have studied surrogate assisted MHs and applied the approaches to various aerodynamic design optimization problems [6][7][8][9][3]. However, most of the work usually use the surrogate model for predicting the objective and constraint function values directly. The use of surrogate models for predicting a part of the parameter in the objective and constraint is rarely studied in the field of aerodynamic design.

In this study, a surrogate assisted differential evolution algorithm for 3D shape aerodynamic optimization of an aircraft wing is presented. The objective function is posed to maximize the lift to drag ratio while the design variables determine wing shape. Two surrogate-assisted DE are presented which are the use of a surrogate model to predict the objective function directly and using a surrogate model to predict lift coefficient and drag coefficient separately before calculating the objective function. Section 2 shows a short explanation of the procedures applied in this paper. Next section presents numerical simulations of CFD flow solver and optimization process. In Section 4, the results are expressed. Finally, conclusions are shown in Section 5.

2. Problem Statement /Statement of Problem
In general, aerodynamic performance optimization is one of the main challenges in aircraft design. It can be taken into consideration in all design stages i.e. conceptual, preliminary, and detail design. The most commonly used design merit function for aircraft aerodynamics is the lift-to-drag ratio since it determines aircraft range and endurance. Such a design objective can be predicted using a panel method for medium fidelity computation; however, it has been shown in the literature that the use of CFD simulation spectacularly agrees with wind tunnel test. Thus, aircraft aerodynamic design using CFD is one of the research issues in aerospace engineering. The optimization problem of a wing can be expressed in the following mathematical model

\[
\text{Maximize } f(\mathbf{x}) = \frac{C_l}{C_d},
\]

\[
s.t. \quad \mathbf{x}_l \leq \mathbf{x} \leq \mathbf{x}_u
\]

where \(f(\mathbf{x})\) is an objective function of the problem, \(\mathbf{x}_l\) and \(\mathbf{x}_u\) are lower bound and upper bound of the design variables vector \(\mathbf{x}\). In this work, the objective function is to maximize the lift-to-drag coefficient \((C_l/C_d)\) while the design variables are the Lancer wing airfoil sections. In this case, the Lancer wing is shaped using five airfoil sections as shown in Figure 1. Therefore, there are five design variables \((\mathbf{x}=[x_1, x_3, x_3, x_4, x_5])\) in this study, where \(x_i\) represent the 4 digit NACA (00**) airfoils and \(x_i \in [\text{Naca0010}, \text{Naca0011}, \text{Naca0012}, \text{Naca0013}, \text{Naca0014}, \text{Naca0015}, \text{Naca0016}, \text{Naca0017}, \text{Naca0018}, \text{Naca0019}]\). In the terminology of the four-digit NACA airfoil, the first digit is the value of the maximum camber (stated in percent of the chord) while the second digit is the maximum camber (measured from the leading edge in tenths of the chord). The last two digits represent the maximum thickness of the airfoil in percents of the chord [7]. The Lancer 200 wings (downloaded from openvsp.org) are used for the design example in this work.
3. Numerical simulation and optimization process

In this section, detail of CFD-based aerodynamic analysis and surrogate assisted MHs used in this study are given. CFD simulation is reliable and accurate for aerodynamic analysis, however, it is time-consuming to use. As a result, we have to exploit a surrogate model for an optimization process.

3.1. CFD Flow Solver and Mathematical Model

Simulation of fluid dynamics of engineering systems is achieved by numerical methods, in which a studied domain is discretized into a finite number of simple shape elements. The most popular technique for fluid dynamic analysis is a finite volume method often referred to as computational fluid dynamics (CFD) [10]. In this paper, the value of lift and drag coefficients are estimated by using CFD simulation. The details of a CFD model are as follows.

3.1.1. Aircraft Wing Geometries and Arrangement

Aircraft wings are constructed with various shapes and sizes for preferred flight characteristics of an airplane to obtain greater lift, balance or stability in flight. In this paper, we used the Lancer 200 aircraft wing [11] to demonstrate the optimal aerodynamic design.

3.1.2. Boundary Conditions

The working fluid of wing is used as air at atmospheric pressure. The inlet velocity, angle of attack and the temperature are set at 90 m/s, 5 degrees and 293.2K, respectively. Types of wall conditions were applied real wall condition to all surfaces. During the simulation process, the value of temperature is kept constant while all the walls of the inlet section, the outlet section and the wing are fixed to have an adiabatic condition.

3.1.3. Mathematical Model

Mathematical equations and physical properties can be used to solve the CFD flow solver problem. Three dimensional steady Navier-Stokes equations and governing equations are used to find the satisfied solution in this work. The governing equations of the Navier stroke equations can define the conservation of mass, momentum and energy of the incompressible fluid flow. Where the fluid properties were constant and three-dimensional laminar and turbulent flows are used in this simulation. These equations can be expressed as follows:

For laminar flow,

Continuity equation: \[
\frac{\partial (\rho u_i)}{\partial x_i} = 0
\]  (2)

Momentum Equation: \[
\frac{\partial (\rho u_j u_i)}{\partial x_j} = -\frac{\partial \rho}{\partial x_i} + \frac{\partial}{\partial x_j} \left[ \mu \left( \frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i} \right) \right]
\]  (3)
Energy Equation : \[ \frac{\partial}{\partial x_i} (\rho \mathbf{u} T) = \frac{\partial}{\partial x_j} \] (4)

For turbulent flow

Continuity equation: \[ \frac{\partial u}{\partial x} + \frac{\partial \phi}{\partial y} + \frac{\partial w}{\partial z} = 0 \] (5)

Momentum Equation: \[ \rho \left[ \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} \right] = -\frac{\partial P}{\partial x} + \mu \left( \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} \right) - \rho \frac{\partial (u'u')}{\partial x} - \frac{\partial (u'v')}{\partial y} \] (6)

Energy Equation: \[ \rho c_p \left[ \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} \right] = k \left[ \frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} \right] - \rho c_p \frac{\partial (u'T')}{\partial x} - \frac{\partial (v'T')}{\partial y} \] (7)

For the numerical discretization of the governing equations, the second-order upwind scheme was used to the advection term in these problems. It has been found that the mesh generated contains 200,255 nodes between the computational time and the accuracy of the calculation for optimization purpose [12]. In the steady-state flow, lift coefficient and drag coefficient equations can be written as follows

Coefficient of Lift,

\[ C_l = \frac{2 \times L}{\rho \times v^2 \times A} \] (8)

Coefficient of Drag,

\[ C_d = \frac{2 \times D}{\rho \times v^2 \times A} \] (9)

Where, \( \rho \) = density of air (kg/m\(^3\))

\( D \) = Drag Force (N)

\( L \) = Lift Force (N)

\( A \) = Area of Wing (m\(^2\))

3.1.4. CFD Analysis of Wing

Before solving the flow analysis, we control the mesh of the wing to get the accurate and efficient multiphysics solutions. The simulation is started by using with above equation to solve the problem iteratively as a steady-state condition. Finally, a postprocessor is applied for the analysis and visualization of the resulting solution. Inflow simulation, the solve time is extremely dependent on the number of cells in the computational domain. Figure 2 shown a three-dimensional CFD model of the Lancer wing used in this study. Constant speed of 95m/s air-fluid is set to flow to the wing in x-direction while the K-epsilon turbulent model is used for the CFD simulation.
3.2. Design of Experiment
In building a surrogate model, design of experiment (DoE) is a procedure which is frequently employed to produce places of sample points within the design space [13]. A surrogate model is generally used to decide the locations of the simple points in the design space. There are two types of DoE methods: classic DoE methods and modern DoE method. The modern DoE method include Latin Hypercube Sampling (LHS), Optimum Latin Hypercube Sampling(OLHS), Orthogonal Array design (OA) and uniform design (UD) [14]. In this paper, the LHS method is used to create the number of initial sampling points in the design space. Each sampling point represents a candidate aerodynamic shape. These sample points are made for the computational grid and analysed to find the corresponding aerodynamic data needed to estimate the objective function and constraints. Then, the data were saved to be database [10].

3.3. Surrogate Modelling
Surrogate modeling is an approximation of functions of design objective and constraints. The main purpose is to reduce computational time in cases that the functions are computationally too costly to evaluate [15]. The model will be constructed based on a set of sampling data which are calculated from the real expensive functions. We consider an m dimensional problem and n design variables to predict and evaluate aerodynamic function $y: \mathbb{R}^m \rightarrow \mathbb{R}$. Assume that the function $y$ is sampled at sites

$$S=\{x^{(1)}, \ldots, x^{(n)}\} \in \mathbb{R}^{n \times m} \quad (10)$$

With the corresponding responses

$$f_i = \{f_1, \ldots, f^n\} \quad (11)$$

Here $n$ is the number of sampling points [14]. The pair $(S, y_i)$ represents the sample data sets in the vector space[14]. There are many types of surrogate models such as Kriging, Polynomial response surface model (RSM), radial basis function (RBF) and support regression. A polynomial surface method has some disadvantages which need statistical treatment for getting the suitable design of experiment and degree of the polynomial based on the available data. RBF, on the other hand, can be best for consideration of acceptable accuracy, very fast and high robustness with small sample size. By comparing several surrogate model methods, the radial basis function network can be used easily which having good generalization, being flexible to input noise, behaving well even with unknown data and having online learning ability. It is a real value and an alternative interpolation method for a meta-model which is a distributed data and contains a very great application because it can interpolate all places in the space. It can be specified that the RBF method can fit distributed exactly multivariate data, thus preserving the CFD sample points. This method usually applies linear combinations of a radially symmetrical function based on Euclidean distance to estimate the response functions. It has been found

Figure 2. (a) Enclosure of Lancer wing and (b) Meshing of wing model for one sampling
that it is very accurate for interpolation in high dimensions and is ideal for interpolation of distributed data. [16].

3.3.1. Radial Basic Function Predictor and approximation

Radial basis functions are used as interpolation models or regression models. In this study, we typically build up the function in approximations of the form. The structure of RBF-based interpolation is completed as reported in [10]. Given a set of sampling design variables, \( \mathbf{x} = \{x^1, \ldots, x^n\} \), which are the input vector corresponding to the function values, \( f = (f^1, \ldots, f^n)^T \), an approximate function \( \tilde{f} \) of any \( \mathbf{x} \) can be expressed by RBF models have shown to provide a higher modeling accuracy and higher robustness (3)

\[
\tilde{f}(\mathbf{x}) = \sum_{i=1}^{n} c_i \phi(\|\mathbf{x} - \mathbf{x}^i\|) 
\]

(12)

where \( \|\mathbf{x} - \mathbf{x}^i\| \) distance between \( \mathbf{x} \) and \( \mathbf{x}^i \) and \( c_i \) is interpolation coefficients which can be calculated by solving the linear equation of

\[
\sum_{i=1}^{n} c_i \phi(\|\mathbf{x}_i - \mathbf{x}_j\|) = f_j \quad \text{for } j=1,\ldots,n 
\]

(13)

where \( \phi \) is a kernel function that determines the smoothness properties of the estimation scheme. Examples of radial basis kernel functions are

- Linear: \( \phi(r) = r \),
- Cubic: \( \phi(r) = r^3 \),
- Gaussian: \( \phi(r) = e^{-\theta_i r^2}, 0 \leq \theta_i \leq 1 \)
- Quadratic: \( \phi(r) = \sqrt{r^2 + \theta_i^2}, 0 \leq \theta_i \leq 1 \)

where \( \theta_i \) represent the shape parameters and \( r = ((x-c_i)^T + (x-c_i))^2 \) is the radial distance. In this paper, the linear kernel function method is used.

3.4. Optimization search with Differential Evolution

Surrogate-assisted differential evolution was generally used to reduce computational time in evolutionary optimization problems, such as aerodynamic optimization [17]. Figure 3 shows the optimization procedure which involves overall steps in this work. The process starts with the generation of a set of sampling points using LHS. The CFD is performed for real \( C_l \) and \( C_d \) calculation. After that, the optimization search is performed using a Differential Evolution (DE) algorithm and the optimum results obtained are used to calculate real function values based on CFD again. The number of sampling points used in this study is 20 while population size and number of iterations for DE are set to be 100 and 200, respectively.
4. Results
Figure 4 shows the 20 sampling wings from LHS, which are used for constructing 2 surrogate models after performing CFD analyses. Table 1 shows the values of $C_l/C_d$ from 20 sampling points before performing optimization based on CFD. After performing optimization based on surrogate-assisted DE, the optimum results are reported in Table 2 and Figure 5 (a) and (b). Optimum $C_l/C_d$ of Case A is based on the surrogate model that is used to directly predict $C_l/C_d$, while optimum $C_l/C_d$ of Case B is based on the surrogate model that is used to separately predict $C_l$ and $C_d$. For the Case-A, the optimum $C_l/C_d$ obtained based on the surrogate model and CFD are 22.02 and 22.51 where the percentage of error of $C_l/C_d$ between surrogate model and CFD is 2.23%. For the Case-B, the optimum $C_l/C_d$ obtained based on the surrogate model and CFD are 20.382 and 22.07 where the percentage of error of $C_l/C_d$ between surrogate model and CFD is 8.2%. It is found that the optimum $C_l/C_d$ obtained from both Case-A and Case-B is better than all initial sampling points. The Case-A which performs optimization based on using a surrogate model to predict the objective function ($C_l/C_d$) directly obtain significantly better optimum results than the Case B which performs optimization based on using the surrogate model to predict $C_l$ and $C_d$ separately. Figure 5 (a) and (b) shows the optimum wing obtained for Case-A and Case-B in this paper.

Based on this study, for optimization of the aerodynamic shape of an aircraft wing with an objective to maximize $C_l/C_d$ using surrogate-assisted DE, it was found that using the surrogate model to predict the objective function ($C_l/C_d$) directly is the more accurate and obtains the better optimum results than using the surrogate model to predict $C_l$ and $C_d$ separately.
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Figure 4. Twenty initial sampling point of the wing by LHS method

Table 1. The value of $C_d/C_l$ of 20 sampling points from CFD

| Sampling No. | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
|--------------|---|---|---|---|---|---|---|---|---|----|
| $C_l/C_d$    | 11.19 | 11.52 | 10.17 | 12.81 | 9.548 | 14.9 | 7.68 | 8.14 | 11.16 | 11.15 |

| Sampling No. | 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 | 19 | 20 |
|--------------|---|---|---|---|---|---|---|---|---|----|
| $C_l/C_d$    | 16.63 | 13.73 | 8.311 | 11.65 | 19.84 | 16.8 | 13.19 | 11.21 | 9.723 | 11.57 |

Table 2. Comparison of the predicted and the validated $C_d/C_l$ at the obtained optimal point

| Case  | By Surrogate Model | By CFD | Error |
|-------|--------------------|--------|-------|
|       | $C_l$               | $C_d$  | $C_l/C_d$ | $C_l$ | $C_d$  | $C_l/C_d$ | $C_l$ | $C_d$  | $C_l/C_d$ |
| Case-A| 22.02              | 0.07337 | 0.00326  | 22.51 | -      | -         | 2.23% |
| Case-B| 0.07584            | 0.00372 | 0.00724  | 20.382| 0.00328| 13.4%     | 8.2%  |

Figure 5. The Optimum Aircraft wing shape of (a) Directly Predict and (b) separately predict from Differential Evolution
5. Conclusions
The aerodynamic shape optimization of an aircraft wing using surrogate-assisted DE was successfully conducted. The optimization problem is posed to find wing shape in order to maximize its lift-to-drag ratio. Two surrogate-assisted DE are presented which are the use of a surrogate model to predict the objective function directly and using a surrogate model to predict lift coefficient and drag coefficient separately before calculating the objective function. The results obtained found that using the surrogate model to predict the objective function ($C_l/C_D$) directly is the more accurate and obtains the better optimum results than using the surrogate model to predict $C_l$ and $C_D$ separately. For further work, the development of the more effective and efficient surrogate-assisted MHs for such a problem in order to get better results are the main focus.

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Reference
[1] Jameson A Fatica M 2002 Icas 1–24
[2] Li J Cai J Qu K 2019 Struct. Multidiscip. Optim. 59(2) 403–419
[3] Skinner S N Zare-Behtash H 2018 Appl. Soft Comput. J. 62 933–962
[4] Acerbi L Ma W J 2017 Advances in Neural Information Processing Systems 30 1834–1844
[5] He X Li J Mader C A Yildirim A Martins J R R A2019 Aerosp. Sci. Technol. 87 48–61
[6] Koreanschi A Gabor O S Acotto J Brianchon G Portier G Botez R M Mamou M Mebarki Y 2017 Chinese J. Aeronaut. 30(1) 164–174
[7] Jouhaud J C Sagaut P Montagnac M Laurenceau J 2007 Comput. Fluids 36(3) 520–529
[8] Tesfahunegn Y A Koziel S Leifsson L Bekasiewicz A 2015 Procedia Comput. Sci. 51(1) 795–804
[9] Maruyama D Liu D Görtz S 2016 Proc. VII European Congress on Computational Methods in Applied Sciences and Engineering M Papadrakakis V Papadopoulos G Stefanou V Plevris (eds.) (Greece) 8787–8800
[10] Jakobsson S Patriksson M Rudholm J Wojciechowski A 2010 Optim. Eng. 11(4) 501–532
[11] All-aero 2019 http://all-aero.com/index.php/53-planes-l-m-n-o/7237-neico-lancair-200--320--360
[12] Wang H Zhu X Du Z 2010 Int. Commun. Heat Mass Transf. 37(8) 998–1003
[13] Han Z H Zhang K S Song W P Qiao Z D 2010 J. Aircr. 47(2) 603–612
[14] Han Z H Abu-Zurayk M Görtz S Ilic C 2018 Notes Numer. Fluid Mech. Multidiscip. Des. 138 257–282
[15] Koziel S Yang X S Zhang Q J Koziel S Leifsson L Yang X S 2013 Simulation-Driven Des. Optim. Model. Microw. Eng. (Imperial College Press:London) 41–79
[16] Mukesh R Lingadurai K Selvakumar U 2014 J. King Saud Univ. - Eng. Sci. 26(2) 191–197
[17] Jin Y 2011 Swarm Evol. Comput. 1(2) 61–70