Calculation of gas flow through the flow of axial turbocompressors

A I Lubysheva 1, A V Potashev 2 and E V Potasheva 2

1 Kazan Federal University, Russian Federation
2 Kazan Cooperative Institute (branch) of the Russian University of Cooperation, Russian Federation

E-mail: pot_andrey@mail.ru

Abstract. When designing centrifugal compressors, much attention is paid to increasing the efficiency of the compression process with a wide range of changes in the operating modes of the stages. In this regard, the role of methods that make it possible to calculate the characteristics taking into account the compressibility of the gas flow through the elements of the flow path increases. In this work, the development of the method for solving the second two-dimensional problem is given, which makes it possible to calculate the subsonic gas flow. In this case, impellers with both single-row and multi-row blade elements are considered.

1. Introduction
When designing centrifugal compressors, much attention is paid to increasing the efficiency of the compression process with a wide range of changes in the operating modes of the stages (see, for example, [1-5]). In this regard, the role of methods that make it possible to calculate hydrodynamic characteristics with the most complete consideration of the properties of a liquid or gas flowing through the elements of the flow path increases.

At JSC NII turbokompressor named after V.B. Schneppe, when designing compressor impellers, calculation programs based on a quasi-three-dimensional model are widely used. According to it, the flow in the flow path of turbomachines is divided into two two-dimensional flows calculated taking into account the mutual influence on each other (see, for example, [6-9]). For the first two-dimensional problem (calculation of the averaged axisymmetric flow), the calculation methods have been implemented both for an incompressible liquid and for a subsonic gas flow (see, for example, [9, 10]). As applied to the second problem (calculation of the flow in rotating lattices of profiles located on axisymmetric current surfaces in a layer of variable thickness), the methods and programs created on the basis of the works of B.S. Rauchman (see, [11-13]), implemented mainly for an incompressible flow.

This article presents an algorithm for solving the second two-dimensional problem, which makes it possible to calculate the subsonic gas flow in the inter-blade channels. In this case, impellers with multi-row blade elements are considered. The paper is structured as follows: Section 2 presents the basic equations for calculating the compressible flow in the interscapular channel; two options for organizing the iterative solution process are presented in Section 3; Section 4 presents the results of
numerical calculations demonstrating the effect of gas compressibility on the distributions of velocity, pressure and density; Section 5 presents the conclusions drew from the results of the study.

2. Basic equations for calculating the velocity field of a compressible flow in the inter-blade channel of a multi-row lattice

The problem of a compressible flow around a multi-row lattice of profiles, as in the case of a single-row lattice, is reduced to solving in the plane of the conformal mapping the equation

\[
\frac{\partial^2 \psi}{\partial x^2} + \frac{\partial^2 \psi}{\partial y^2} - \frac{\partial \ln \rho(h)}{\partial x} \frac{\partial \psi}{\partial x} - \frac{\partial \ln \rho(h)}{\partial y} \frac{\partial \psi}{\partial y} = -2 \phi(h) \frac{r}{r_0} \frac{dr}{dy}.
\]  

(1)

Here \( \psi = \psi(x, y) \) is the stream function of the relative motion associated with the projections of the relative velocity in the plane \((x, y)\) by the relations

\[
w_x = \frac{1}{\rho} \frac{\partial \psi}{\partial y}, \quad w_y = -\frac{1}{\rho} \frac{\partial \psi}{\partial x}.
\]  

(2)

Here \( \rho \) is the density; \( h = h(y) \) – relative layer thickness; \( r \) – distance of a point from the axis of rotation; \( r_0 \) – display radius; \( \omega \) – angular velocity of rotation.

If we add to this equation (see [14]) the Bernoulli equation

\[
\frac{\kappa}{\kappa - 1} \frac{\rho}{\rho} + \frac{w^2 - (\omega r)^2}{2} = \text{const}
\]  

(3)

and the equation of state for adiabatic gas motion

\[
\rho \rho^{\kappa} = \text{const},
\]  

(4)

then we come to a closed system of equations for three unknown functions \( \psi(x, y) \), \( \rho(x, y) \) and \( p(x, y) \), where \( p \) is the pressure.

The stream function \( \psi \) must satisfy the following boundary conditions:

- at infinity in front of the lattice \((y = y_1 \to \infty)\) (figure 1)

\[
\left. \left( \frac{\partial \psi}{\partial x} \right) \right|_{y = y_1} = -h_i \rho_i w_{1y}, \quad \left. \left( \frac{\partial \psi}{\partial y} \right) \right|_{y = y_1} = h_i \rho_i w_{1x},
\]

in addition, in this section, the given values are \( \rho_1, \rho_1, w_{1x} \) and \( w_{1y} \),

- at infinity behind bars \((y = y_2 \to -\infty)\)

\[
\left. \left( \frac{\partial \psi}{\partial x} \right) \right|_{y = y_2} = -h_2 \rho_2 w_{2y}, \quad \left. \left( \frac{\partial \psi}{\partial y} \right) \right|_{y = y_2} = h_2 \rho_2 w_{2x},
\]

If we take into account the equation of continuity at infinity \( h_i \rho_i w_{2x} = h_i \rho_i w_{1x} \), then we get

\[
\left. \left( \frac{\partial \psi}{\partial x} \right) \right|_{y = y_1} = \left. \left( \frac{\partial \psi}{\partial x} \right) \right|_{y = y_2}.
\]

From the equation of the absence of a vortex it follows \( w_{2x} = w_{1x} - \Gamma / t + \omega (r_1^2 - r_2^2) / r_0 \), where \( \Gamma = \sum_{k=1}^{n} \Gamma_i \) is the total circulation of the velocity along the contours of the profiles of one lattice period; \( \Gamma_i, k = \overline{1,n} \) is the velocity circulation along the profile contour of the \( k \)th row (tier)
For a multi-row or multi-tiered lattice of profiles (profiles), as in the case of an incompressible fluid \([15]\), on the contours of the profiles \(l_k\) \((k = 1, n)\) impermeability conditions are set

\[
\psi|_{l_k} = C_k
\]

Moreover, the constants \(C_k\) are different for each of the profiles.

To determine the circulations \(\Gamma_k\) the conditions for equalizing the velocity at two selected points near the trailing edge of each of the blades are used

\[
|w_{k+}| = |w_{k-}|
\]  

(5)

2.1. Integral equation

For a further solution, the function

\[
\Psi = \psi / \sqrt{h \rho}
\]

(6)

Then equation (1) can be written in the form

\[
\Delta \Psi = m_1(x, y)\Psi - 2\omega \sqrt{h \rho} \frac{r}{r_0} \frac{d r}{d y}
\]

where \(\Delta = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}\) is the Laplace differential operator;

\[
m_1 = \sqrt{h \rho} \left[ \frac{\partial^2}{\partial x^2} \left( \frac{1}{\sqrt{h \rho}} \right) + \frac{\partial^2}{\partial y^2} \left( \frac{1}{\sqrt{h \rho}} \right) \right]
\]

Further, the function \(\Psi\) is represented as the sum

\[
\Psi(x, y) = \Psi_0(x, y) + \Psi_1(x, y)
\]  

(7)

Here

\[
\Psi_0 = c_{0m} x + v_{0z} g_1(y) - \omega r_0 g_2(y) \frac{h \rho}{r_0} \frac{1}{\sqrt{2}}; \quad c_{0m} = w_1 h_0 \rho_1; \quad v_{0z} = v_{1z} + v_{2z} / 2; \quad g_1(y) = \int \sqrt{h \rho} d\eta; \quad g_2(y) = \int \frac{r}{r_0} \sqrt{h \rho} d\eta; \quad \bar{\rho}(y) = \int \rho(x, y) dx + \delta_i(y) \rho_1\text{ the density averaged over}
\]
the step in the direction of the $x$ axis under the assumption that inside the contours of the profiles $\rho = \rho_1$; $\delta_1(y) = \sum_{k=1}^{n} \delta_{ak}(y)$ is the total circumferential thickness of all blades of the same cascade period (figure 1).

With this choice, the function $\Psi_0$ defines a certain unperturbed flow with velocity components $w_0 = v_0, w_v = c_{ov} / h$, that depend only on the coordinate $y$. Then the function $\Psi_1$ (taking into account (7)) will be described by the equation

$$\Delta \Psi_1 = m_1(x, y) - m_2(y)\Psi_0 - 2\omega_0 \sqrt{\rho - \rho_0} \frac{d}{dy} \left( \frac{1}{\sqrt{\rho_0}} \right).$$

If now we also take into account that on the contours of the profiles $\frac{\partial \Psi}{\partial s} = 0, \frac{\partial \Psi}{\partial n} = \rho_0 w(x)$, and write down the Green's formula for the function $\Psi_1$, then we get

$$\Psi_1(x, y) = \int A(x, y; \sigma)w(\sigma)d\sigma + \omega_0\int g_1(\eta)(L_1 d\xi + K_1 d\eta) - \int g_2(\eta)(L_2 d\xi + K_2 d\eta) +$$

$$+ \frac{1}{\sqrt{h(y)\rho(y)}} \int \left( r^2 \right)^{n} (L_2 d\xi + K_2 d\eta) - 2\int \sqrt{h(y)\rho(y)} \frac{d}{dy} \left( \frac{1}{\sqrt{h(y)\rho(y)}} \right)$$

$$+ \int m_1(\xi, \eta)(\Psi_0 + \Psi_1) L_2 d\xi d\eta - \int m_2(\eta)(\Psi_0 + \Psi_1) L_2 d\xi d\eta +$$

$$+ \sum_{k=1}^{n} C_k \left[ \int \left( \frac{1}{\sqrt{h(y)\rho(y)}} \right) (L_2 d\xi + K_2 d\eta) + \int \left( \frac{1}{\sqrt{h(y)\rho(y)}} \right) (K_2 d\xi - L_2 d\eta) \right],$$

where $A(x, y; \sigma) = \sqrt{h(y)\rho(y)} L_1 - \sqrt{h(y)\rho(y)} L_2 - h_0 w(y)^{-1/2}$;

$$L_1(x, y; \xi, \eta) = \frac{1}{4\pi} \ln \left[ \frac{2\pi(\eta - y) - \cos \frac{2\pi(\xi - x)}{t}}{t} \right];$$

$$L_2(x, y; \xi, \eta) = \frac{[g_1(\eta) - g_2(\eta)]}{2t};$$

$$K_1(x, y; \xi, \eta) = \frac{\text{sign}(y - \eta)(h(y)\rho(y)(x - \xi))}{2t};$$

$$K_2(x, y; \xi, \eta) = \frac{\text{sign}(y - \eta)(x - \xi)}{2t};$$

$$g_1(\eta) = \sqrt{h(y)\rho(y)} \frac{r^2(\eta)}{r^2 \rho};$$

$$\Psi_1(y) = \frac{1}{\sqrt{h_0}} \sum_{k=1}^{n} \int w(\sigma) L_2 d\sigma + \omega_0 \int \left( r^2 \right)^{n} (L_2 d\xi + K_2 d\eta)$$

$-$ step-averaged function $\Psi_1(x, y)$; $D'$ $-$ one period of the flow area; $D$ $-$ region of one lattice period together with the interiors of the profiles $d_k$, that is $D = D' \bigcup \left( \bigcup_{k=1}^{n} d_k \right)$ (figure 1).

Taking advantage of the conditions of non-leakage of the contours of the profiles

$$\Psi_1|_{d_k} = C_k \ h_0^{-1/2} - c_{ov} x + v_0 g_1(y) - \omega_0 g_2(y) \ h_0^{-1/2},$$
we arrive at integral equations for the functions \( w_k(s), \) \( k = \overline{1, n} \) – the distributions of the relative velocity on each of the profile contours

\[
\sum_{m=1}^{n} \int A(x, y; \sigma)w_m(\sigma)d\sigma - \frac{C_k}{\sqrt{h\rho}} = \frac{c_{m\infty} - v_{\infty} g_1(y)}{\sqrt{h\rho}} + \omega r_0 \left[ \frac{g_2(y)}{\sqrt{h\rho}} - \sum_{m=1}^{n} \int g_5(\eta)(L_4 d\xi + K_4 d\eta) - \int g_3(\eta)(L_4 d\xi + K_4 d\eta) - \int g_3(\eta)(L_4 d\xi + K_4 d\eta) + \frac{1}{\sqrt{h\rho}} \int r^2(\eta) (L_2 d\xi + K_2 d\eta) \right] + \frac{1}{\sqrt{h\rho}} \int \left[ r^2 - \frac{r^2}{r_0^2} L_4 d\xi d\eta \right] - \frac{1}{\sqrt{h\rho}} \int m_1(\xi, \eta)(\Psi_0 + \Psi_1)L_4 d\xi d\eta - \frac{1}{\sqrt{h\rho}} \int m_2(\eta)(\Psi_0 + \Psi_1)L_4 d\xi d\eta - C_k \left[ \sum_{m=1}^{n} \int \frac{\partial}{\partial \eta} \left( \frac{1}{\sqrt{h}} \right) (L_4 d\xi + K_4 d\eta) + \int \frac{\partial}{\partial \xi} \left( \frac{1}{\sqrt{h\rho}} \right) (K_4 d\xi - L_4 d\eta) \right] + \frac{1}{\sqrt{h\rho}} - \frac{1}{\sqrt{h\rho}} \right],
\]

where \( k = \overline{1, n} \), which together with relations (3) - (5) and circulation equations

\[
\int w_k(\sigma)d\sigma = \Gamma_k - \omega r_0 \int \frac{r^2}{r_0^2} d\sigma, \quad k = \overline{1, n}
\]

form a closed system.

2.2. Calculation of distributions of density, pressure, velocity and stream function in the interblade channel

To calculate the gas density inside the flow region, one should use formulas (3), (4), rewritten in the form

\[
\rho(x, y) = \rho_i \left[ \left( H + \frac{\kappa - 1}{\kappa p_i} \right) \frac{r^2}{2} \right]^{\frac{1}{\kappa - 1}},
\]

\[
w = \sqrt{w_x^2 + w_y^2}, \quad w_x = \frac{1}{\sqrt{h\rho}} \left( \frac{\partial \Psi}{\partial y} + \frac{1}{\sqrt{h\rho}} \frac{\partial \sqrt{h\rho}}{\partial y} \Psi \right), \quad w_y = \frac{1}{\sqrt{h\rho}} \left( \frac{\partial \Psi}{\partial x} + \frac{1}{\sqrt{h\rho}} \frac{\partial \sqrt{h\rho}}{\partial x} \Psi \right),
\]

where \( H = \frac{\kappa}{\kappa - 1} \frac{p_i}{p_i} + \frac{w_x^2 - \omega^2 r_0^2}{2} \); \( w_x^2 = w_{x_1}^2 + w_{x_2}^2 \).

It can be seen from the above formulas that to determine the density, it is necessary to calculate the velocity distribution on the contours of the blade profiles and the velocity field inside the channel. This calculation is carried out by formulas (2), where the stream function is related by formula (6) to the new function. If we substitute expressions (6) and (7) in the right-hand sides of formulas (2), then we find that to calculate the velocities it is necessary to calculate the values of the functions \( \Psi_0(x, y) \) and \( \Psi_1(x, y) \) and their partial derivatives. For this, it is convenient to represent these functions as the sums

\[
\Psi_1 = \epsilon_{00} \Psi_{11} + \epsilon_{11} \Psi_{12} + \sum_{k=1}^{n} \frac{\Gamma_k}{\kappa} \Psi_{1k} + \omega r_0 \Psi_{14}, \quad l = 1, 2,
\]

where \( \Psi_{01} = \chi h\bar{\rho}^{-1/2}; \quad \Psi_{02} = g_1(y) h\bar{\rho}^{-1/2}; \quad \Psi_{03} = -\Psi_{02}/2; \quad \Psi_{04} = -g_2(y) h\bar{\rho}^{-1/2}; \)
\[ \Psi_{ij} = \sum_{k=1}^{n} \left[ \sqrt{\hat{h}p} L_1 - \sqrt{\hat{h}p} L_2 \right] w_j(\sigma) d\sigma + \frac{1}{\sqrt{\hat{h}p}} \sum_{k=1}^{n} \int_{L_2}^{L_1} \left( L_2 w_j(\sigma) d\sigma + \Psi_{l_{ij}}(x, y) \right) \]

\[ \Psi_{l_{ij}} = 0 \text{ at } j = 1, 2, 3; \]

\[ \Psi_{l_{ij}} = \sum_{k=1}^{n} \left\{ g_j \left[ (L_1 - L_2) d\xi + (K_1 - K_2) d\eta \right] - \frac{1}{\sqrt{\hat{h}p}} \int_{L_0}^{L_2} \left( L_2 d\xi + K_2 d\eta \right) \right\}; \]

\[ Q_j(x, y) = \left[ \int_{L_0}^{L_2} \left[ f_{i_{1j}}(\xi, \eta) - f_{i_{2j}}(\xi, \eta) \right] L_2 d\xi d\eta - \sum_{k=1}^{n} \int_{L_0}^{L_2} f_{i_{2j}} L_2 d\xi d\eta + \right. \]

\[ \left. + \sum_{k=1}^{n} C_i \left[ \frac{\partial}{\partial \eta} \left( \frac{1}{\sqrt{\hat{h}p}} \right) (L_2 d\xi + K_2 d\eta) + \left( \frac{\partial}{\partial \xi} \left( \frac{1}{\sqrt{\hat{h}p}} \right) (K_2 d\xi - L_2 d\eta) \right) \right] + \frac{\partial Q}{\partial \eta} \right]; \]

\[ Q_{m_{ij}} = 0 \text{ at } j = 1, 2, 3; \quad Q_{m_{i4}} = -2 \int_{L_0}^{L_2} \sqrt{\hat{h}p} - \sqrt{\hat{h}p} \cdot \frac{dr}{\sqrt{\hat{h}p}} L_2 d\xi d\eta; \]

\[ f_{i_{1j}}(x, y) = m_1(x, y) \Psi_j(x, y); \quad f_{i_{2j}}(x, y) = m_2(y) [\Psi_{o_{ij}}(x, y) + \Psi_{t_{ij}}(y)]; \]

\[ \Psi_{t_{ij}}(y) = \frac{1}{\sqrt{\hat{h}p}} \sum_{k=1}^{n} \left[ L_2 w_j(\sigma) d\sigma + \Psi_{t_{ij}}(y) \right]; \quad \Psi_{o_{ij}}(y) = 0 \text{ at } j = 1, 2, 3; \quad \Psi_{t_{ij}} = \int_{L_0}^{L_2} (L_2 d\xi + K_2 d\eta). \]

Then their derivatives are defined as

\[ \frac{\partial \Psi_{a}}{\partial \alpha} = \frac{c_{am}}{\sqrt{\hat{h}p}}, \quad \frac{\partial \Psi_{0}}{\partial \alpha} = \sqrt{\hat{h}p} \left( v_{o_{a}} - \omega r^2 + \frac{d}{dy} \frac{1}{\sqrt{\hat{h}p}} \Psi_{a} \right), \] (13)

\[ \frac{\partial \Psi_{i}}{\partial \alpha} = \sum_{k=1}^{n} \left[ \sqrt{\hat{h}p} \frac{\partial L_2}{\partial \alpha} w_j(\sigma) d\sigma + \omega r^2 \left( \frac{\partial L_2}{\partial \alpha} - \frac{\partial L_3}{\partial \alpha} \right) d\xi + \left( \frac{\partial L_2}{\partial \alpha} - \frac{\partial L_3}{\partial \alpha} \right) d\eta \right] + \frac{\partial Q}{\partial \alpha}, \] (14)

\[ \frac{\partial \Psi_{t_{j}}}{\partial \alpha} = \sum_{k=1}^{n} \left[ \int_{L_0}^{L_2} \left( \sqrt{\hat{h}p} \frac{\partial L_2}{\partial \alpha} \right) d\sigma + \frac{1}{\sqrt{\hat{h}p}} \int_{L_0}^{L_2} \left( \sqrt{\hat{h}p} \frac{\partial L_3}{\partial \alpha} \right) d\sigma + \right. \]

\[ \left. + \frac{d}{dy} \left( \frac{1}{\sqrt{\hat{h}p}} \right) \int_{L_0}^{L_2} \left( \sqrt{\hat{h}p} \frac{\partial L_3}{\partial \alpha} \right) d\sigma + \omega r^2 \left( \frac{\partial L_3}{\partial \alpha} - \frac{\partial L_4}{\partial \alpha} \right) d\xi + \left( \frac{\partial L_3}{\partial \alpha} - \frac{\partial L_4}{\partial \alpha} \right) d\eta \right] + \frac{\partial Q}{\partial \alpha} \] (15)

Wherein

\[ \frac{\partial Q}{\partial \alpha} = \int_{L_0}^{L_2} \left[ f_{i_{1j}}(\xi, \eta) - f_{i_{2j}}(\xi, \eta) \right] \frac{\partial L_2}{\partial \alpha} d\xi d\eta - \sum_{k=1}^{n} \int_{L_0}^{L_2} f_{i_{2j}}(\xi, \eta) \frac{\partial L_2}{\partial \alpha} d\xi d\eta + \]
3. Calculation algorithm

3.1. The first option for organizing the iterative process

It can be seen from the above formulas that to solve the problem, it is necessary to organize an iterative process associated with determining the density distribution both on the contours of the blade profiles and in the interblade channel. When compiling the calculation algorithm, two options for organizing the iterative process were considered. The essence of the first is as follows:

1. as an initial approximation, it is assumed that the density is constant at all points of the flow region and is equal to the density at the entrance to the lattice, that is \( \rho(x, y) = \rho_1 \), and \( \overline{\rho}(y) = \rho_t \);
2. taking into account the obtained density distribution, the velocities are calculated on the contours of the blade profiles \( w_{k}(s) \), \( k = 1, n \); using formulas (11) – (15) at the calculated points of the region, the values of the velocity \( w(x, y) \) are calculated;
3. according to the found values of the velocities according to the formula (10), the density values on the contours of the blade profiles and in the interblade channel, as well as the averaged density distribution, are found;
4. from the obtained density distributions on the contours of the blades and in the interblade channel, taking into account formula (3), the corresponding pressure distributions are found \( p = p_{i}(\rho / \rho_t)^{k} \);
5. the obtained density distributions are compared with the distributions from the previous iteration; if the difference exceeds a predetermined small value, the process is repeated from step 2;
6. when the convergence condition is met, the streamlines of the relative motion are constructed. To do this, using formulas (6), (7), (12) at the calculated points of the region, the values of the stream function \( \psi \) are found, these values are interpolated along the lines \( y = \text{const} \), and the points corresponding to some given values of the stream function are constructed. After connecting the points with the same values, the shape of the streamlines in the interblade channel is found.
3.2. The second option for organizing the iterative process

The disadvantage of the first version of the iterative process is its high labor intensity, since at each iteration step it is necessary to calculate the velocity and density distributions in the interblade channel. The computation time can be significantly reduced by organizing the iteration process according to the following scheme:

1. as an initial approximation, it is assumed that the density is constant at all points of the flow region and is equal to the density at the entrance to the lattice, that is \( \rho(x, y) = \rho_1 \) and \( \bar{\rho}(y) = \rho_1 \);
2. taking into account the obtained density distribution, the velocities are calculated on the contours of the blade profiles \( w_k(s), k = \frac{1}{n} \);
3. according to a method similar to that described in \([15]\), the averaged moment of velocity \( \bar{w}_c(y) \) calculated and the velocity of the averaged flow \( \bar{w}(y) \) is sought;
4. from the formula (10) according to the distributions of velocities \( w_k(s), k = \frac{1}{n} \) the density values on the contours of the blade profiles are found, and the distribution of the averaged density is found from the velocity of the averaged flow \( \bar{w}(y) \);
5. the obtained density distributions are compared with the distributions found at the previous iteration; if the difference exceeds the specified small value, the process is repeated from step 2;
6. when the convergence condition is met, the streamlines of relative motion in the interblade channel are constructed and the pressures are calculated.

The second version of the iterative process makes it possible to reduce the computation time, however, the reliability of the results obtained in this case must be checked by comparing the averaged density distributions obtained by the formula

\[
\bar{\rho}(y) = \frac{1}{T} \int_{0}^{T} \rho(x, y) dx + \bar{\delta}_c(y) \rho_1
\]

and by the method for steps 3 and 4. If there are large discrepancies, the solution should be refined according to the first version of the iterative process.

It should also be keep in mind that the method used is applicable for subsonic flow around the lattices. In the presence of supersonic regions, the iterative process is not always convergent. Therefore, other options for completing the iterative process should be foreseen.

4. Results of numerical calculations for the gas flow

To illustrate the possibilities of the program for calculating the gas flow, an impeller with a profile lattice in the conformal mapping plane shown in figure 2 (a) was considered. The shape of an axisymmetric stream surface \( r(y) \) and a layer of variable thickness \( h(y) \) are shown in figure 2 (b) and figure 2 (c). The front view of the corresponding lattice on the axisymmetric stream surface is shown in figure 2 (d).

As a result of the calculations, the distributions of the average velocity and velocity on the contour of the blade profile (figure 3 (a)) and the corresponding distributions of densities (figure 3, (b)) and pressures (figure 3 (c)) were constructed.

Comparison of the velocity distributions on the blade profile in gas and incompressible flows is shown in figure 3 (d). It is seen that taking into account the compressibility leads to a decrease in the level of velocities, which is a consequence of the compression of the gas flow in the channel. In addition, the pressure ratio in the gas flow takes on a higher value \( q_0 = 0.8217 \) than in an incompressible \( q_0 = 0.8092 \). This fact is also due to an increase in the density at the outlet from the lattice and, as a consequence, to a decrease in the relative velocity at the outlet with \( w_2 = 0.2502 \) in an incompressible flow to \( w_2 = 0.2336 \) in a gas flow to.
Figure 2. Initial data for the calculation: (a) the shape of the profile lattice in the plane of conformal mapping; (b) dependence $r(-y)$; (c) dependence $h(-y)$; (d) front view of the wheel

Figure 3. Calculation results: (a) distribution $w(-y)$; (b) distributions $\rho(-y)$; (c) distributions $p(-y)$; (d) comparison of the velocity distributions on the contour of the blade profile in the gas flow (solid curve 1) and in the incompressible flow (dashed curve 2).

5. Conclusion
As a result of the work carried out, the method and algorithm for calculating the flow of the gas flow in the inter-blade channels of any row are implemented in the form of computer programs. Taking into account the compressibility makes it possible to significantly improve the design process, since it leads to a more accurate calculation of the velocity field and, as a consequence, to more accurate
values of the loss coefficient and efficiency. As a result, it becomes possible not only for a qualitative assessment of the characteristics of the impellers, but also for an accurate quantitative calculation.

Acknowledgments
The study was carried out with the financial support of the Russian Foundation for Basic Research and the Republic of Tatarstan as part of a research project No. 18-41-160026.

References
[1] Viktorov G V 1969 Hydrodynamic lattice theory (Moscow: High School) p. 368 (in Russ)
[2] Seleznov K P and Galerkin Yu B 1982 Centrifugal Compressors (Leningrad: Mechanical engineering) p. 271 (in Russ)
[3] Gostelow J P 1984 Cascade Aerodynamics (Pergamon Press) p. 270
[4] Cumpsty N A 1989 Compressor Aerodynamics (Longman Scientific & Technical) p. 509
[5] Elizarov A M, Il’inskiy N B and Potashev A V 1997 Mathematical methods of airfoil design: inverse boundary-value problems of aerohydrodynamics (Akademie Verlag) p. 292
[6] Wu Chung-Hua 1952 Transactions of the ASME, 74 (8) pp. 1363-80
[7] Stepanov G Yu 1962 Hydrodynamics of turbomachine grids (Moscow: Fizmatgiz) p. 512 (in Russ)
[8] Etinberg I E and Raukhman B S 1978 Hydrodynamics of hydraulic turbines (Leningrad: Mechanical engineering) p. 280 (in Russ)
[9] Potashev A V, Potasheva E V and Khisameev I G 2018 Compressors and pneumatics 2 pp. 23-33 (in Russ)
[10] Potashev A V. and Potasheva E V 2017 Compressors and pneumatics 6 pp. 4-8 (in Russ)
[11] Raukhman B.S. 1965 Proceedings of CKTI 61 (in Russ)
[12] Raukhman B S 1971 Proceedings of CKTI 106 pp. 9-33 (in Russ)
[13] Raukhman B S 1972 RTM no. 24.023.07. Hydraulic turbines (Leningrad: TsKTI name I I Polzunova) p. 55 (in Russ)
[14] Loitsyansky L G 1987 Mechanics of liquid and gas (Moscow: Nauka) p. 840 (in Russ)
[15] Potashev A V and Potasheva E V 2019 Compressors and pneumatics 3 pp. 9-18 (in Russ)