A Two Parameter Texture of Nearly Bi-maximal Neutrino Mixing

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We propose a texture of three generation Majorana-type neutrino mass matrix in terms of only two parameters which gives rise to nearly bi-maximal mixing angles. We also demonstrate an explicit realization of such type of neutrino mass-matrix in the context of an $SU(2)_L \times U(1)_Y$ model due to higher dimensional mass terms through the inclusion of discrete $Z_3 \times Z_4$ symmetry and two extra singlet Higgs fields.

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I. Introduction

Evidence in favour of neutrino oscillation (as well as neutrino mass) has been provided by the Super-Kamiokande (SK) atmospheric neutrino experiment [1] through the measurement of magnitude and angular distribution of the $\nu_\mu$ flux produced in the atmosphere due to cosmic ray interactions. Observed depletion of $\nu_\mu$ flux in earth has been interpreted as the oscillation of $\nu_\mu$ to some other species of neutrino. In a two flavour neutrino oscillation scenario, oscillation between $\nu_\mu - \nu_\tau$, the experimental result leads to maximal mixing between two species $\sin^22\theta \geq 0.82$ with a mass-squared difference $\Delta m^2_{atm} \sim (5 \times 10^{-4} - 6 \times 10^{-3})$ eV$^2$. Furthermore, recent result of SuperKamiokande experiment disfavours any large mixing between purely $\nu_\mu$ and $\nu_s$ (sterile neutrino) at 99% c.l.[2]. The solar neutrino experimental results [3] are also in concordance with the interpretation of atmospheric neutrino experimental result and the data provide the following values as $\Delta m^2_{e\mu} \sim (0.8 - 2) \times 10^{-5}$ eV$^2$, $\sin^22\theta \sim 1$ (Large angle MSW solution) or $\Delta m^2_{e\mu} \sim (0.5 - 6) \times 10^{-10}$ eV$^2$, $\sin^22\theta \sim 1$ (vacuum oscillation solution). Furthermore, the CHOOZ experimental result [4] gives the value of $\Delta m^2_{eX} < 10^{-3}$ eV$^2$ or $\sin^22\theta_{eX} < 0.2$. In order to reconcile with the solar and atmospheric neutrino experimental results, a possible explanation known as bi-maximal neutrino mixing is advocated [5], in which $\theta_{12}=\theta_{23} = 45^\circ$, and if, the CHOOZ experimental result is interpreted in terms of $\nu_e - \nu_\tau$ oscillation, then $\theta_{31} < 13^\circ$. Another scenario could still be possible if the solar neutrino experimental result is explained in terms of small angle MSW solution however, we have not address this scenario in the present work. In the present work, we propose a texture of Majorana-type neutrino mass matrix in terms of only two parameters considering only three generations of neutrinos. Two parameter texture of neutrino mass matrix has also been discussed earlier [6, 7, 8]. In Ref.6, with three light neutrinos, different zeroth order textures of both neutrino and charged lepton mass matrices has been proposed in view of the solar and atmospheric
neutrino experimental results advocating the implication of flavor symmetry.
A detailed analysis is found in Ref.7 where the implication of $L_e - L_\mu - L_\tau$ symmetry has been discussed to realize light neutrino mass both via see-saw mechanism and low energy effective theory. An investigation in this path has also been done in Ref.8 through the introduction of a partially conserved chiral $U(1)_{f_1} \times U(1)_{f_2}$ symmetry with Standard model gauge group to generate both quark and lepton mass matrices. Apart from the successful description of quark and lepton mass matrices, however, in this model a large value of Higgs coupling of the term of dimension greater than four is needed to avoid the conflict between the minimization condition of the Higgs potential and the choice of low value of the VEV of an $SU(2)_L$ triplet Higgs field when vacuum oscillation solution of solar neutrino problem is considered in addition with the atmospheric neutrino experimental result. This problem is avoided in the present model by discarding any hard $(\text{dim} \geq 4)$ discrete symmetry violating term in the scalar potential.

In this work, we propose an explicit pattern of two parameter texture of neutrino mass matrix which gives rise to nearly bi-maximal neutrino mixing and also can accommodate the required mass-squared differences to explain the solar (by large angle MSW solution or by Vacuum oscillation) and atmospheric neutrino experimental results. Next, we demonstrate an explicit realization of the proposed texture within the framework of an $SU(2)_L \times U(1)_Y$ model with an extended Higgs sector and discrete symmetry. The plan of the paper is as follows: Section II contains the proposed neutrino mass-matrix and its phenomenology. A model accomplishes the proposed mass matrix is presented in Section III. Section IV contains summary of the present work.

II. Neutrino Mass Matrix

Before going into the details, first of all, we consider the charged lepton mass
matrix is diagonal in flavor space. Consider now the following Majorana-type neutrino mass matrix with the basis of the leptonic fields \((l_1L, l_2L, l_3L)\) (where \(l_iL\) have (2,1) quantum numbers under \(SU(2)_L \times U(1)_Y\) gauge group, \(i\) is the generation index)

\[
M_\nu = \begin{pmatrix}
0 & a & a \\
 a & 0 & b \\
 a & b & 0 \\
\end{pmatrix}
\]

(1)

where \(a\) and \(b\) are two real model independent parameters and we consider \(a \neq b\) so that \(M_\nu\) contains at least two parameters. Also it is to be noted that the absence of \(\nu_e\nu_e\) mass term in the above neutrino mass matrix evades the bound on the Majorana-type neutrino due to \(\beta\beta_0\) decay. Moreover, the above texture admits no observable CP violating effect in the leptonic sector as the number of parameters is only two. The phases of \(a\) and \(b\) could easily be rotated away by redefining the leptonic fields. The elements of \(M_\nu\) can be generated either by radiative mechanism or by non-renormalizable mass operators. We have not addressed here the see-saw type mass generation because in that case a judicious choice of Dirac-type neutrino mass matrix is necessary. Diagonalizing the neutrino mass matrix \(M_\nu\) by an orthogonal transformation as \(O^T M_\nu O = M_D = \text{Diag}(-m_{\nu_1}, m_{\nu_2}, m_{\nu_3})\) where

\[
O = \begin{pmatrix}
c_{31}c_{12} & c_{31}s_{12} & s_{31} \\
-s_{23}s_{12} - s_{23}s_{31}c_{12} & c_{12}c_{23} - s_{23}s_{31}s_{12} & s_{23}c_{31} \\
s_{23}s_{12} - c_{23}s_{31}c_{12} & -s_{23}c_{12} - s_{31}s_{12}c_{23} & c_{23}c_{31} \\
\end{pmatrix},
\]

(2)

we obtain the following values of the mixing angles as

\[
\theta_{23}^\nu = -\frac{\pi}{4}, \ \theta_{31}^\nu = 0, \ \tan^2\theta_{12}^\nu = \frac{m_{\nu_1}}{m_{\nu_2}}
\]

(3)

and the eigenvalues of the above mass matrix comes out as

\[
-m_{\nu_1} = \frac{b - x}{2}
\]

\[
m_{\nu_2} = \frac{b + x}{2}
\]
\[-m_{\nu_3} = b\]  \hspace{1cm} (4)

where \(x = \sqrt{b^2 + 8a^2}\). The sign of \(m_{\nu_1}\) and \(m_{\nu_2}\) can be made positive by re-defining lepton doublet fields. Furthermore, in terms of the three eigenvalues \(m_{\nu_1}, m_{\nu_2}\) and \(m_{\nu_3}\), the mixing matrix \(O\) can be written as

\[
O = \begin{pmatrix}
-c_{12} & s_{12} & 0 \\
-\frac{1}{\sqrt{2}}s_{12} & \frac{1}{\sqrt{2}}c_{12} & -\frac{1}{\sqrt{2}} \\
-\frac{1}{\sqrt{2}}s_{12} & \frac{1}{\sqrt{2}}c_{12} & \frac{1}{\sqrt{2}} \\
\end{pmatrix} = \begin{pmatrix}
\sqrt{\frac{m_{\nu_2}}{m_{\nu_1} + m_{\nu_2}}} & \sqrt{\frac{m_{\nu_1}}{m_{\nu_1} + m_{\nu_2}}} & 0 \\
-\frac{1}{\sqrt{2}}\sqrt{\frac{m_{\nu_1}}{m_{\nu_1} + m_{\nu_2}}} & \frac{1}{\sqrt{2}}\sqrt{\frac{m_{\nu_2}}{m_{\nu_1} + m_{\nu_2}}} & -\frac{1}{\sqrt{2}} \\
-\frac{1}{\sqrt{2}}\sqrt{\frac{m_{\nu_1}}{m_{\nu_1} + m_{\nu_2}}} & \frac{1}{\sqrt{2}}\sqrt{\frac{m_{\nu_2}}{m_{\nu_1} + m_{\nu_2}}} & \frac{1}{\sqrt{2}} \\
\end{pmatrix} \hspace{1cm} (5)
\]

In the limit \(b \to 0\), \(\theta_{12} \to \frac{\pi}{4}\), the two eigenvalues \(m_{\nu_1}\) and \(m_{\nu_2}\) become degenerate and we can achieve the exact bi-maximal neutrino mixing. In this situation, although we obtain the exact bi-maximal neutrino mixing however, the obtained eigenvalues \(m_{\nu_1} = m_{\nu_2}\) and \(m_{\nu_3} = 0\), can be fitted with either the solar or the atmospheric neutrino experimental result. Removal of degeneracy between the two eigenvalues require further higher order corrections.

For our analysis, we set the value of \(\Delta m^2_{21} = \Delta m^2_{\text{sol}}\) which in turn sets the value of \(\theta_{12}^{\nu}\). The value of \(x\) depends on the hierarchical relation between \(a\) and \(b\) parameters which is manifested from the values of

\[
\Delta m^2_{21} = bx \hspace{1cm} (6)
\]

and

\[
\Delta m^2_{23} = \frac{1}{4}(3b + x)(x - b). \hspace{1cm} (7)
\]

Now, if \(b^2 \gg 8a^2\), then the value of \(x\) comes out as \(x \simeq b\) and \(\Delta m^2_{23} \simeq 0\), \(\Delta m^2_{21} \simeq b^2\), hence, in this case it is not possible to accommodate both the results of solar and atmospheric neutrino experiments. Thus, for a phenomenologically viable model, we have to consider the hierarchy \(8a^2 \gg b^2\) and in this case \(m_{\nu_1}\) is also become positive. The pattern of neutrino mass is presented in Figure I. In this situation, we obtain, \(\Delta m^2_{21} \simeq 2\sqrt{2}ab\), \(\Delta m^2_{23} \simeq 2a^2\). For a typical value of \(\Delta m^2_{23} \simeq 4 \times 10^{-3} \text{ eV}^2\) which can explain the atmospheric neutrino deficits, we obtain \(2a^2 \simeq 4 \times 10^{-3} \text{ eV}^2\). For a typical value
of $\Delta m_{21}^2 \simeq 4 \times 10^{-10}$ eV$^2$ which can explain the solar neutrino deficits due to vacuum oscillation, the value of $b^2$ comes out as $b^2 \sim 10^{-17}$ eV$^2$ whereas for the large angle MSW solution a typical value of $\Delta m_{21}^2 \sim 10^{-5}$ eV$^2$ the value of $b^2$ comes out of the order of $10^{-9}$ eV$^2$. The mixing angle $\theta_{12}^\nu$ comes out as $\tan^2 \theta_{12}^\nu \sim \frac{2a\sqrt{2} - b}{2a\sqrt{2} + b}$ and since $a \gg b$, $\theta_{12}^\nu \to 45^\circ$, and, hence, there is no conflict to satisfy the value of $\theta_{12}^\nu$ well within the allowed range of the experimental value.

III. A Model

In this section, we demonstrate an explicit realization of the above neutrino mass matrix as well as a flavor diagonal charged lepton mass matrix within the framework of an SU(2)$_L \times$ U(1)$_Y$ model with two singlet Higgs fields and discrete $Z_3 \times Z_4$ symmetry. The charged masses are arising in a similar way to Standard Model (SM) whereas neutrino masses are generated through non-renormalizable operators. We have also discussed the situation when the mixing is exactly bi-maximal. Instead of three almost degenerate neutrinos $[9, 10]$, we obtain a hierarchical pattern of neutrino masses. To obtain a realistic low energy phenomenological model, several attempts have been made through the inclusion of discrete symmetry $[11]$. Recently, it has been shown $[12]$ that non-abelian discrete groups (such as dihedral groups $D_n$, dicyclic groups $Q_{2n}$) plays an attractive role to obtain required mixing pattern in the fermionic sector. A recent work in this path has been done $[13]$ through the inclusion of $U(1) \times Z_2$ symmetry in the flavor space to explain both the quark and leptonic sector mixing angles. Although the question of embedding such symmetries under a large symmetry is still open, nevertheless, to understand from the low energy point of view, inclusion of discrete symmetry and extra matter fields is an attractive way. The discrete $Z_3 \times Z_4$ symmetry prohibits unwanted mass terms in the charged lepton and neutrino mass matrices in the present model. We consider soft discrete symmetry breaking terms in the scalar potential, which are also responsible to obtain non-zero values of the
Table 1: Representation content of the lepton and Higgs fields considered in the present model. The generators of $Z_3$ and $Z_4$ groups are $\omega$ and $i$, respectively.

| Fields   | $SU(2)_L \times U(1)_Y$ | $Z_3$ | $Z_4$ |
|----------|-------------------------|-------|-------|
| leptons  |                         |       |       |
| $l_{1L}$ | (2, -1)                 | 1     | 1     |
| $l_{2L}$ | (2, -1)                 | $\omega$ | $-i$ |
| $l_{3L}$ | (2, -1)                 | $\omega$ | $i$  |
| $e_R$    | (1, -2)                 | $\omega^*$ | 1     |
| $\mu_R$ | (1, -2)                 | 1     | $-i$ |
| $\tau_R$| (1, -2)                 | 1     | $i$  |
| Higgs    |                         |       |       |
| $h$      | (2,1)                   | $\omega$ | 1     |
| $\rho$  | (1,0)                   | 1     | $i$  |
| $\xi$   | (1,0)                   | $\omega$ | $-1$ |

VEV’s of the Higgs fields upon minimization of the scalar potential. It is to be noted that in order to avoid conflict between the choice of VEV’s of the Higgs fields ($\rho$ and $\xi$) with the minimization condition of the Higgs potential, we discard any hard discrete symmetry breaking term in the Higgs potential. Discrete symmetry invariant soft or hard terms will not cause hierarchical problem as addressed in Ref.8. The Majorana neutrino masses are obtained due to explicit breaking of lepton number through higher dimensional terms. The representation content of the leptonic fields and Higgs fields considered in the model is given in Table I. Apart from the standard model doublet $h$ Higgs field, we introduced another two singlet Higgs $\xi$ and $\rho$ fields to obtain two independent parameters for the neutrino sector.

The most general lepton-Higgs Yukawa interaction in the present model gen-
erating Majorana neutrino masses is given by

\[ \mathcal{L}_\nu = \frac{(l_{1L}l_{2L})hh\rho}{M_f^2} + \frac{(l_{1L}l_{3L})hh\rho^*}{M_f^2} + \frac{(l_{2L}l_{3L})hh\xi^2}{M_f^3} \]  

(8)

and the Yukawa interaction which is responsible for generation of charged lepton masses is given by

\[ \mathcal{L}_E^Y = f_1\bar{l}_1^c l_R^e h + f_2\bar{l}_2^c l_R^\mu h + f_3\bar{l}_3^c l_R^\tau h + H.c.. \]  

(9)

We consider \( \rho \) is a complex scalar field whereas \( \xi \) is a real scalar field. The present model contains a large mass scale \( M_f \), and for our analysis we set \( M_f \sim M_{GUT} \) which is the highest scale considered in the present model. The VEV’s, \( \langle \xi \rangle \) and \( \langle \rho \rangle \) are constrained by the solar and atmospheric neutrino experimental results.

In order to avoid any zero values of the VEV’s of the Higgs fields upon minimization of the scalar potential, we have to consider discrete symmetry breaking terms. Without going into the details of the scalar potential, this feature can be realized in the following way. In general, the scalar potential can be written as (keeping up to \( \text{dim}=4 \) terms)

\[ V = Ay^4 + By^3 + Cy^2 + Dy + E \]  

(10)

where \( y \) is the VEV of any Higgs field and A, B, C, D, E are generic couplings of the terms contained in the scalar potential. Minimizing the scalar potential w.r.t. \( y \), we obtain

\[ V' = A'y^3 + B'y^2 + C'y + D \]  

(11)

Eqn.(10) reflects the fact that as long as \( D \neq 0 \), and \( A' \) or \( B' \) or \( C' \) is not equal to zero, we will get non-zero solutions for \( y \). Thus, in order to obtain \( y \neq 0 \) solution, it is necessary to retain the terms with generic coefficients
D and $A'$ or $B'$ or $C'$. In the present model, both the discrete symmetry breaking terms soft and hard, correspond to the term with coefficient D. Discarding hard symmetry breaking terms, we retain soft discrete symmetry breaking terms, and, hence, none of the VEV is zero upon minimization of the scalar potential.

Let us look at the leptonic sector of the present model. Substituting the VEV’s of the Higgs fields appeared in Eqn.(9), we obtain flavor diagonal charged lepton mass matrix as

$$M_E = \begin{pmatrix} d & 0 & 0 \\ 0 & e & 0 \\ 0 & 0 & f \end{pmatrix}$$  \hspace{1cm} (12)

where $d = f_1 \langle h \rangle$, $e = f_2 \langle h \rangle$ and $f = f_3 \langle h \rangle$ and substituting the VEV’s of $\xi$, $h$ and $\rho$ Higgs fields in Eqn.(8), we get the Majorana-type neutrino mass matrix as follows:

$$M_\nu = \begin{pmatrix} 0 & a & a \\ a & 0 & b \\ a & b & 0 \end{pmatrix}$$  \hspace{1cm} (13)

where $a = \frac{(b^2)^2}{M_f^2}$, $b = \frac{\langle \xi \rangle^2 \langle h \rangle^2}{M_f^2}$. The parameter a can fitted with the value $\Delta m_{23}^2 \simeq 2 a^2 \simeq 4 \times 10^{-3} \text{ eV}^2$ which explains atmospheric neutrino experimental data by setting $M_f \sim 10^{12} \text{ GeV}$, $\langle h \rangle \simeq 174 \text{ GeV}$ and $\langle \rho \rangle \simeq 10^{11} \text{ GeV}$. Using the same values of $M_f$ and $\langle h \rangle$, it is possible to set the value of $b$ as $b^2 \simeq 10^{-17}\text{ eV}^2$ through the choice of $\langle \xi \rangle \simeq 10^7 \text{ GeV}$ in order to explain the solar neutrino experimental results due to vacuum oscillation solution. For both the cases, the mixing angle $\theta_{12}$ (given in Eqn.(3)) comes out as nearly maximal. For large angle MSW solution, a typical value $\Delta m_{21}^2 \sim 10^{-5} \text{ eV}^2$ gives rise to $b^2 \simeq 10^{-9}\text{ eV}^2$ for $\langle \xi \rangle \simeq 2 \times 10^9 \text{ GeV}$.

**IV. Summary**

The discrete symmetry invariant $\nu_e \nu_e$ mass term appears in the present model at $M_f^5$ order which is naturally vanishingly small.
In summary, we propose a texture of Majorana-type neutrino mass-matrix which gives rise to nearly bi-maximal neutrino mixing in a natural way as well as required mass-squared differences in order to explain the solar and atmospheric neutrino experimental results. The elements of the mass-matrix could be generated either by radiative mechanism or by the use of non-renormalizable operators and, thus, those elements are model independent. The proposed neutrino mass-matrix gives rise to the eigenvalues of the three neutrino masses as $m_{\nu_1} \simeq m_{\nu_2} \gg m_{\nu_3}$ which ends up to an hierarchy between three neutrino mass-squared differences as $m^2_{23} \gg m^2_{21}$. We demonstrate an explicit realization of the proposed mass-matrix due to non-renormalizable mass operators in the context of an $SU(2)_L \times U(1)_Y$ model through the inclusion of two extra singlet Higgs fields and discrete $Z_3 \times Z_4$ symmetry. With a suitable choice model parameters the required mass-squared differences can be accommodated in order to explain the solar (both large angle MSW solution and Vacuum oscillation) and atmospheric neutrino experimental results.

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FIG. I. Neutrino mass spectrum in the present model.