A Theory of the Longitudinal and Hall Conductivities of the Cuprate Superconductors

Branko P. Stojković and David Pines

Department of Physics and Materials Research Laboratory,
1110 West Green Street, University of Illinois, Urbana, IL 61801
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Abstract

We establish the applicability to transport phenomena in the cuprate superconductors of a nearly antiferromagnetic Fermi liquid (NAFL) description of the magnetic interaction between planar quasiparticles by using it to obtain the doping and temperature dependent resistivity and Hall conductivity seen experimentally in the normal state. Following a perturbative calculation of the anisotropic (as one goes around the Fermi surface) quasiparticle lifetimes which are the hallmark of a NAFL, we obtain simple approximate expressions for the longitudinal, $\sigma_{xx}$, and Hall, $\sigma_{xy}$, conductivities which reflect the magnetic crossovers seen experimentally as one varies the doping level and temperature. We present a simple phenomenological model for the variation in mean free path around the Fermi surface, and use this to extract from experiments on $\sigma_{xx}$ and $\sigma_{xy}$ quasiparticle lifetimes in the hot (strongly coupled quasiparticle) and cold (weakly coupled quasiparticle) regions of the Fermi surface which are consistent with the perturbation theory estimates. We improve upon the latter by carrying out direct numerical (non-variational) solutions of the Boltzmann equation for representative members of the YBa$_2$Cu$_3$O$_{6+x}$ and La$_{2-x}$Sr$_x$CuO$_4$ systems, with results for transport properties in quantitative agreement with experiment. Using the same numerical approach we
study the influence of CuO chains on the a-b plane anisotropy and find results in agreement with experimental findings in YBa$_2$Cu$_4$O$_8$. 
I. INTRODUCTION

The peculiar normal state properties of High Temperature Superconductors (HTS) have been extensively studied over the past decade in the expectation that their understanding will likely reveal the mechanism of superconductivity. Yet no consensus has emerged as to their physical origin. Indeed, it is striking that so much work has been done on understanding a variety of complicated experimental situations, while rather basic measurements, such as the d.c. transport, are not well understood. So far it has been clearly established and well documented in the literature that practically all normal state properties in these materials are anomalous with respect to the behavior of standard Landau Fermi Liquids (FLs). This, in turn, poses the question of whether Landau FL theory is applicable to HTS. A variety of proposals based on non-Fermi Liquid (NFL) or marginal FL theories have emerged which have had considerable success in explaining a number of specific experimental results. However, at present no single theory has been able to account for all the anomalies found in the normal state properties of the cuprates.

In this paper we focus on the planar resistivity and the Hall effect, which have drawn particular attention in the HTS community, due to their arguably “contradictory” behavior. The temperature dependences of the Hall angle, $\theta_H$, and the longitudinal resistivity, $\rho_{xx}$, are so vastly different that the Hall coefficient, $R_H$, in practically all cuprates is a strong function of temperature, especially near optimal doping. This phenomenon by itself, is rarely observed in ordinary FLs. It is also found that all HTS near optimal doping have $\rho_{xx}$ proportional to the temperature, $T$, in a wide region of interest. Hence it appears that the transport properties in the presence of magnetic field as well as in its absence require two separate scattering mechanisms, and presumably two separate relaxation rates. In fact this is the only common scheme among the competing theories for the explanation of the anomalous transport in HTS materials.

We begin with a brief review of the theoretical proposals. Anderson, who first drew attention to the problem of explaining the unusual magnetotransport in the normal state,
has proposed a scenario based on a NFL ground state. In his scenario the two rates observed in experiment are attributed to scattering of two distinct many-body excitations, spinons and holons. The existence of these excitations is well established in 1D Luttinger liquids and Anderson has put forth a non-trivial generalization of this concept to the quasi-two dimensional system found in cuprate superconductors. Strictly speaking one can talk about a NFL in any system at, or near half-filling, which has an interaction sufficiently strong to produce the Neél instability, but the validity of this approach at large doping levels is somewhat questionable. In the Anderson scenario both spinons and holons contribute to the resistivity, but it is often tacitly assumed that the scattering of holons dominates, while only spinons, which effectively commute with the magnetic field, $B$, appear in the temperature variation of the Hall angle. Although commonly used to analyze experiments, this proposal has yet to be verified by an explicit calculation of the transport coefficients.

Coleman et al. have developed a phenomenological transport theory based on the assumption that the dominant scattering in cuprates is mediated by an interaction which distinguishes quasiparticle wave functions with respect to their charge conjugation symmetry. Such interactions are absent in ordinary FLs and hence this can be regarded as a NFL model, although in this case there is no spin-charge separation and hence the model is more correctly categorized as an unusual FL. In this model quasiparticles with different symmetry have vastly different scattering rates, which in turn have different temperature dependences, ultimately leading to the correct behavior of all transport coefficients. Although the origin of the unusual interaction has not been identified, this is the only model to date which appears to be consistent with the recent experimental results on optical reflectivity.

The anomalous Hall effect can also be understood in terms of the bipolaron theory, developed by Mott and Alexandrov. Anomalous behavior in their model arises from Anderson localization of bipolarons: at low temperatures bipolarons in a random potential lose their mobility, producing an enhanced value of $R_H$. At sufficiently high temperature, larger than the random potential binding energy, the mobility is recovered, and $R_H$ saturates, as seen in experiment. The bipolaron model is also consistent with a number of photoemission
experiments.

A variety of FL-based models have also been utilized to explain the anomalous transport in cuprates. Clearly, in order to account for experiments, one must introduce the two scattering rates without violating the FL ground state. This is done either by introducing a certain anisotropy of scattering or by assuming a different mechanism of scattering in the presence of magnetic field. Initially, the anomalous Hall effect was attributed to the well-known skew scattering present in many materials with strong spin-orbit coupling; the anomalous temperature variation of $\rho_{xy}$ arises from anisotropic scattering of conduction electrons by local magnetic moments. This approach naturally produces the observed temperature dependence of $\rho_{xy}$, provided the resistivity is assumed linear in $T$. However, one runs into problems for systems with doping levels for which the resistivity is not proportional to temperature. Moreover, skew scattering should saturate with increasing magnetic field, due to the complete polarization of localized moments. Such saturation has not been observed, although some discrepancy from the linear in $B$ Hall resistivity does occur in extremely high pulsed fields.\textsuperscript{17}

Levin and Quader\textsuperscript{18} focused on a very important subgroup, the bilayer cuprates. In their approach the anomalous behavior of the Hall effect is due to the multiband structure of bilayer cuprates. Their main assumption is that one of the bands is very close to the Fermi level and responsible for the anomalous resistivity, while the other is responsible for the Hall angle. The former is thermally activated at higher temperatures in underdoped materials so that the model can explain qualitatively the behavior of resistivity in the underdoped bilayers. Although their approach appears reasonable, the striking similarity between the experimental results on single layer and bilayer materials, which we discuss in Sec. IV, indicates that this approach may have to be revised before it possesses general applicability.

Another class of models is based on anisotropic scattering, which can arise from either anomalous band structure in the cuprates or spin-fluctuation scattering. Newns\textsuperscript{19} has proposed a model based on an anomalously large density of states near the Fermi surface (FS) for quasiparticles in the vicinity of the so called Van Hove singularities: angle resolved
photoemission (ARPES) experiments in a number of samples show extended flat band features near \( k = (\pi, 0) \) and symmetry related points in the Brillouin zone, which give rise to highly anomalous quasiparticle scattering. The two scattering rates necessary to explain the measured transport properties are those for quasiparticles near and far away from these singularities. Although the model can produce a temperature dependent Hall coefficient, \( R_H \equiv \rho_{xy} \), it is unclear whether it can simultaneously account for the anomalous behavior of the resistivity.

We next consider models based on spin-fluctuation scattering. One of the first contributions has been provided by Carrington, Cooper and their collaborators\(^2\) who used a phenomenological model of the spin-fluctuations and a realistic FS and calculated a number of transport coefficients in cuprates. They assumed that large parts of the FS have an anomalous, linear in \( T \) scattering rate, while smaller parts have a Fermi liquid, quadratic in \( T \) scattering rate. Although the anomalous quasiparticle scattering rate, assumed to be due to spin-fluctuations, has been taken as an input to the theory, they obtained a remarkable agreement not only with the Hall effect, but with the thermoelectric power as well. This pioneering work motivated a number of authors to explore more particular choices of spin-fluctuation spectra and their relationship with transport\(^2\)–\(^4\).

Trugman\(^5\) calculated the band structure renormalization in the presence of strong antiferromagnetic (AF) fluctuations within the context of the \( t - t' - J \) model and obtained a single particle band structure which has the form of a precursor to a spin-density-wave (SDW) state. In this case one finds flat regions in the band, somewhat similar to those mentioned above, which are characterized by an energy scale which is much smaller than the fermionic bandwidth. In essence the size of the Fermi energy is effectively reduced to an energy \( \Omega \ll t \), and hence, just as in ordinary FLs, the resistivity deviates from its usual \( T^2 \) dependence, while the Hall coefficient is temperature dependent. However we note that this anomalous temperature dependence of the Hall effect is primarily due to the strong anisotropy of scattering rates and densities of states in different regions of the Brillouin zone. Similar models were also provided by Kim et al\(^6\) and by Dagotto et al\(^7\).
Lercher and Wheatley also studied a spin-fluctuation model of magnetotransport in the cuprates. They concluded that the behavior of several quantities of interest is significantly altered depending on whether one incorporates Umklapp processes. At high temperatures they find a highly $T$ dependent Hall coefficient, with a resistivity approximately linear in $T$. As in the above cases, their calculation is based on the anisotropy of scattering rates in different regions of the Brillouin zone, although here the anisotropy originates in the specific structure of the effective interaction.

The nearly antiferromagnetic Fermi liquid (NAFL) model for transport in the cuprates resembles the model studied by Lercher and Wheatley, as well as the earlier Hubbard model calculations of the resistivity by Bulut, Scalapino and White, with one important difference: the choice of the effective interaction. Although in both cases the effective quasiparticle interaction, $V_{\text{eff}}$, assumed to be of magnetic origin, is a strong function of momentum transfer, and is highly peaked near the antiferromagnetic wavevector $Q$, Lercher and Wheatley calculate $V_{\text{eff}}$ using an RPA formalism with a local restoring force, while in the NAFL model one takes the momentum and frequency dependence of the interaction to be that of the spin-spin response function which provides a quantitative account of the NMR experiments, as discussed in the next Section. While an RPA formalism can provide insight into the physical origin of the spin-fluctuations, it neglects changes in the effective interaction brought about by the spin-fermion vertex corrections, which lead to a sequence of crossovers in the low frequency magnetic behavior in the normal state. Thus it is not capable of accounting for the rich morphology of the magnetic properties seen in NMR experiments, a morphology which we shall see is reflected in transport experiments.

Pioneering calculations of the resistivity of YBa$_2$Cu$_3$O$_7$ using a NAFL model were carried out by Monthoux and Pines, who found, in a strong coupling (Eliashberg) calculation which incorporated vertex corrections, that a good quantitative account of experiment could be obtained using a spin-fluctuation spectrum taken from NMR experiments and a coupling constant which yielded a superconducting transition at 90K. This work provided an important bridge between the anomalous normal state properties and the superconducting
transition. There remained the important question of whether an NAFL model could also account for the anomalous Hall effect. The present authors addressed this question by using standard Boltzmann transport theory to calculate both the resistivity, \( \rho_{xx} \), and the Hall conductivity, \( \sigma_{xy} \) of \( \text{YBa}_2\text{Cu}_3\text{O}_7 \). A direct (nonvariational) numerical solution of the BE showed that the highly anisotropic quasiparticle scattering at different regions of the Fermi surface (brought about by the highly anisotropic magnetic quasiparticle interaction which characterizes a NAFL) could give rise qualitatively, and in some cases quantitatively, to the measured anomalous temperature dependence of the resistivity and Hall coefficient while yielding the quadratic temperature dependence of the Hall angle. The present paper extends these calculations in a number of ways. First, we use perturbation theory to calculate, for a broad range of doping and temperature regimes, the anisotropic quasiparticle scattering rates around the Fermi surface; we then use these results to calculate the longitudinal and Hall conductivities. Our perturbation theory approach not only makes explicit the difference in quasiparticle behavior at hot spots (those regions of the FS for which the magnetic interaction is anomalously large) and in cold regions (the remaining parts of the FS), but enables one to examine in detail the changes in quasiparticle lifetimes in both regions brought about by both the crossovers in the spin-fluctuation spectra measured in NMR experiments and the changes in FS shape measured in experiments using angle resolved photoemission techniques. On taking the relevant averages of quasiparticle lifetimes for the longitudinal and Hall conductivities, we are then able to obtain simple approximate expressions for these quantities as a function of doping and temperature which are in good agreement with experiment.

The substantial anisotropy of quasiparticle mean free paths found in the perturbation theory approach led us to develop a phenomenological model for the variation in mean free path as one goes around the FS, and to use this to extract from experiments on \( \sigma_{xx} \) and \( \sigma_{xy} \) the behavior of the quasiparticle lifetimes, \( \tau_{\text{hot}} \) and \( \tau_{\text{cold}} \), as one varies temperature and doping level for the \( \text{YBa}_2\text{Cu}_3\text{O}_{6+x} \) (123), \( \text{La}_{2-x}\text{Sr}_x\text{CuO}_4 \) (214), and Tl-doped systems. Again, good agreement is found between the perturbation theory estimates and the experimental
Guided in part by the perturbation theory results and our phenomenological model, we then carry out direct numerical solutions of the BE for representative members of the 123 and 214 systems, and for a highly simplified model of a system, such as YBa$_2$Cu$_4$O$_8$ (124), in which the presence of chains brings about considerable planar anisotropy. For all three cases considered, good agreement is found between theory and experiment.

The paper is organized as follows: in the next section we provide the necessary background on the signatures of NAFL model in NMR experiments and define the relevant crossover temperatures. In Sec. II we analyze the properties of NAFLs using a simple perturbative approach. In Sec. IV we review experimental results found for the 123, 214, and overdoped Tl systems, and call attention to the regularities (or lack thereof) which $\rho_{xx}(T)$ and $\rho_{xy}(T)$ exhibit as a function of doping and temperature. We then present our phenomenological analysis of $\sigma_{xx}$ and $\sigma_{xy}$ in terms of a highly anisotropic mean free path, and use this analysis to extract from experiment the quasiparticle lifetimes in the hot and cold regimes. In Sec. V, we present numerical calculations for a realistic set of band and spin-fluctuation parameters, and compare our results to experiment. Finally, in Sec. VI we summarize our conclusions.

II. BACKGROUND ON PHYSICAL PROPERTIES OF NAFLS

In the NAFL description of the normal state properties of the superconducting cuprates it is the magnetic interaction between planar quasiparticles which is responsible for the anomalous spin and charge behavior. The magnetic properties of the system are specified by a dynamical spin-spin response function of fermionic origin, $\chi(q, \omega)$, which near a peak at $Q_i$, in the vicinity of the commensurate wavevector, $Q = (\pi, \pi)$, takes the form:

$$\chi(q, \omega) = \sum_i \frac{\chi_Q}{1 + (q - Q_i)^2 \xi^2 - i\omega/\omega_{sf} - \omega^2/\Delta^2}$$

Here $\chi_{Q_i} \gg \chi_0$ is the magnitude of the static spin susceptibility at a (possibly) incommensurate wavevector, $Q_i$, near $Q$, $\xi$ is the antiferromagnetic correlation length, $\omega_{sf}$ specifies
the low frequency relaxational mode, brought about by the near approach to antiferromagnetism, and \( \Delta \) allows for the possible presence of a spin gap at large energies. We have assumed a system of units in which the lattice spacing \( a = 1 \). The quasiparticle spectrum is assumed to take the tight-binding form,

\[
\epsilon_k = -2t(\cos k_x + \cos k_y) - 4t' \cos k_x \cos k_y - 2t''(\cos(2k_x) + \cos(2k_y))
\]  

(2)

where \( t, t' \) and \( t'' \) are the appropriate hopping integrals, while the effective magnetic interaction between the planar quasiparticles is specified by

\[
V_{\text{eff}}(q, \omega) = g^2 \chi(q, \omega).
\]  

(3)

For a given system the parameters \( \chi_Q, \xi, \omega_{sf} \) which determine \( \chi(q, \omega) \) are taken from fits to NMR and INS (inelastic neutron scattering) experiments, while the effective coupling constant, \( g \), is assumed to be momentum independent for the wavevector near \( Q_i \). As discussed by Chubukov et al., in principle the effective interaction, Eq. (3), can be derived microscopically starting with, e.g., a one-band Hubbard model, but for a number of reasons it has not yet been possible to carry through such a derivation.

Since in practice the effective damping of the magnetic excitations is large, the term proportional to \( \omega^2 \) in the denominator of Eq. (3) can be neglected, so that we may write:

\[
\chi(q, \omega) = \frac{\alpha \omega_0}{\omega_q - i\omega}.
\]  

(4)

where we have introduced the scale factor, \( \alpha \), which relates \( \chi_Q \) to \( \xi^2 \),

\[
\chi_Q = \alpha \xi^2,
\]  

(5a)

\[
\omega_q = \omega_{sf} + \omega_0(q - Q_i)^2.
\]  

(5b)

and

\[
\omega_0 = \omega_{sf} \xi^2,
\]  

(5c)
In the next Section we shall for the most part assume that the spin fluctuation spectrum possesses only a single peak at $Q$; we consider, in Sec. V, the effects of incommensuration on the transport properties.

The effective interaction $V_{eff}$, Eq. (3), has an obvious property: for sufficiently large correlation lengths it is highly peaked for momentum transfers in the vicinity of the antiferromagnetic wavevector $Q$. The importance of this fact is hard to overestimate: if the FS of the system of fermions, defined by the quasiparticle dispersion (2), is such that it intersects the magnetic Brillouin zone $Z$ (see Fig. 1), then quasiparticles in the vicinity of these intersection points on the FS, often referred to as hot spots, are much more strongly scattered by the spin-fluctuations than those which are on other parts of the FS. This is the main source of the anisotropy of quasiparticle behavior in this model, since the temperature variation of the quasiparticle scattering rates at and far away from hot spots is in general very different. We shall return to this point in the following Section.

Barzykin and Pines (BP) have utilized Eq. (1) in their analysis of the NMR results in HTS. Because their work, subsequently extended to an analysis of INS results, provides an important insight into the morphology of the magnetic spectrum in these materials, we briefly review their main conclusions. BP find that for underdoped systems the low frequency magnetic behavior possesses three distinct temperature regimes, with a crossover at $T_{cr}$ from non-universal mean field (MF) behavior with dynamical exponent $z = 2$ to $z = 1$ pseudoscaling (PS) behavior. For $T \geq T_{cr}$, $\omega_{sf} \sim 1/\xi^{2}$, and the product, $\chi_{Q}\omega_{sf} \sim \omega_{sf}\xi^{2} = \omega_{0}$ is independent of temperature, while between $T_{cr}$ and $T_{s}$, it is $\omega_{sf}\xi$ which is independent of temperature. NMR experiments show that above $T_{cr}$, in the MF regime, $\omega_{sf}$ and $\xi^{2}$ scale linearly with $T$, i.e., $\omega_{sf} = A + BT$, while between $T_{s}$ and $T_{cr}$, the PS regime, it is $\omega_{sf}$ and $\xi^{-1}$ which scale linearly with $T$, albeit with a somewhat different slope and intercept of $\omega_{sf}$ than that found above $T_{cr}$. At $T_{cr}$, the temperature dependent uniform susceptibility, $\chi_{0}(T)$, possesses a maximum, while $\xi \sim 2$, as has now been verified experimentally for the YBa$_2$Cu$_3$O$_{6.92}$, YBa$_2$Cu$_4$O$_8$, and La$_{2-x}$Sr$_x$CuO$_4$ compounds. Below $T_{s}$, in the pseudogap (PG) regime, $\xi$ becomes independent of temperature while $\omega_{sf}$, after exhibiting a minimum
near $T_\ast$, rapidly increases as $1/T$ as $T$ decreases toward $T_c$. In addition to the change in the magnetic fluctuation spectrum, the PS regime is characterized by a strong temperature variation of the quasiparticle band structure, resulting in a FS evolution. This evolution has non-trivial consequences for the transport, as discussed in the next section.

From a magnetic perspective, the so called optimally doped systems (e.g., YBa$_2$Cu$_3$O$_6$$_{93}$ and La$_{1.85}$Sr$_{0.15}$CuO$_4$) are a special case of the underdoped systems, in which $T_\ast$ is comparatively close to $T_c$. As may be seen in Table I, $T_{cr}$ varies rapidly with doping, and approaches $T_c$ as the system approaches the overdoped regime, while $T_\ast$, which varies relatively weakly with doping possesses a shallow maximum ($\sim 200K$ in the 123 system, $\sim 100K$ in the 214 system) as one varies the hole content in the planes.

From this same magnetic perspective, overdoped cuprates are defined as those for which $T_{cr} < T_c$. For these systems, then, since $\xi < 2$, the antiferromagnetic correlations are comparatively weak, $\chi_0(T)$ is at most weakly temperature dependent, while $\omega_{sf} \propto \xi^2$ follows the linear in $T$ behavior found in the underdoped systems above $T_{cr}$. Examples of overdoped systems are the $T_c \sim 40K$ Tl 2212 system and La$_{2-x}$Sr$_x$CuO$_4$ for $x \geq 0.2$.

As we shall see, the crossovers seen in the low frequency magnetic behavior possess to a remarkable extent their charge counterparts in magnetotransport and optical experiments.

### III. PERTURBATION THEORY APPROACH

In this section we estimate the transport coefficients in NAFLs using a simple perturbation theory. We calculate the scattering rates for quasiparticles near the FS, assuming the effective interaction $V_{eff}$, Eq. (3), and then use these to calculate the longitudinal and Hall conductivities. We discuss the behavior of these quantities in the relevant doping and temperature regimes.

Within the Boltzmann-Bloch formalism the relaxation rate for a particle near the FS is generally given by:

$$
\frac{1}{\tau_k} = \int \frac{d^2k'}{(2\pi)^2} M(k, k')(1 - \Phi_{k'}/\Phi_k)
$$

(6)
where \( \Phi_k \) is the quasiparticle distribution function displacement obtained by solving the linearized Boltzmann equation. To second order in the coupling constant the scattering rate between points \( k \) and \( k' \) is given by

\[
M(k, k') = 2g^2 \text{Im} \chi(k - k', \epsilon' - \epsilon) \left[ n(\epsilon' - \epsilon) + f(\epsilon') \right] \tag{7}
\]

where \( \epsilon \equiv \epsilon(k) \), \( \epsilon' \equiv \epsilon(k') \), \( n(\epsilon) \), and \( f(\epsilon) \) are the Bose and Fermi distribution functions respectively. Note that Eq. (7) is obtained by summing the Matsubara frequencies and assuming a retarded scattering potential. No lifetime effects are included. One can, of course, easily add the self-energy corrections to the bare band spectrum \( \epsilon_k \), but the life-time effects are considerably harder to include and in principle should be obtained self-consistently. In the relaxation time approximation one replaces \( \Phi_k \), the quasiparticle distribution displacement, by \( \varphi(T)(v \cdot n) \), where \( \varphi \) is a function of temperature independent of \( k \). The assumption that \( \varphi \) does not vary greatly in the Brillouin zone is most certainly invalid for HTS given the strong momentum dependence of the effective interaction \( V_{eff} \), Eq. (3). However, it is still quite reasonable, as long as the temperature dependence of \( \varphi \) is the same in each of the hot spots, which is almost always the case. Since the inclusion of \( (v \cdot n) \) produces only a geometric factor (vertex correction), which is large mostly for small momentum transfer, and no additional temperature dependence, we can approximate the relaxation rate by the scattering rate, which, in the second order Born approximation reads,

\[
\frac{1}{\tau_k} = \int \frac{d^2 k'}{(2\pi)^2} M(k, k'). \tag{8}
\]

One can make a change of variables in the integral in Eq. (8):

\[
\int d^2 k' \rightarrow \int d\epsilon' \int \frac{ds'}{|v|} \tag{9}
\]

and solve the integral over \( \epsilon' \) analytically for \( k \) near the FS, \( \epsilon \approx 0 \), using:

\[
\int dx n(-x) \frac{-x}{\Omega^2 + x^2} f(x) = \frac{\pi}{2\Omega} + \pi \sum_n \frac{(-1)^n}{\Omega + n\pi} = \frac{\pi}{2\Omega} - \hbar \left( 1 + \frac{\Omega}{\pi} \right) \tag{10}
\]

where
\[ h(x) = \frac{1}{2} \left[ \psi \left( \frac{x+1}{2} \right) - \psi \left( \frac{x}{2} \right) \right] \]  

(11)

which has the following limiting behavior:

\[ h(1 + x) \approx \ln 2 - \frac{\pi}{2} x \quad x \to 0 \]  

(12)

\[ h(x) \approx \frac{1}{2x+1} \quad x \to \infty \]  

(13)

Thus the integral in Eq. (10) can be approximated by:

\[ \frac{\pi}{2} \left( \frac{1}{\Omega} - \frac{1}{\Omega + \pi} \right) \]  

(14)

and the scattering rate, Eq. (8), is given by:

\[ \frac{1}{\tau_k} = \frac{\alpha g^2 \omega_0}{4} \int_{FS} dk' \tilde{M}(k, k') \]  

(15)

where

\[ \tilde{M}(k, k') = \frac{1}{|\mathbf{v}|} \frac{T^2}{\omega_{kk'}(\omega_{kk'} + \pi T)}. \]  

(16)

Note that in a typical FL one has an identical expression; however \( \omega_{kk'} \) is large, temperature independent and only weakly dependent on \( k, k' \). Then for \( T \ll \omega_{kk'} \) the scattering rate has the usual \( T^2 \) temperature dependence which ceases only when \( T \sim \omega_{kk'} \). In a realistic metal, however, this energy scale can be considerably reduced for certain \( k \) due to the presence of band singularities at these points. In a NAFL, however, \( \omega_{kk'} \) is a strong function of momentum transfer \( \mathbf{q} \), i.e., \( \omega_{kk'} \) has a minimum for \( \mathbf{q} = \mathbf{Q} \). Thus, the scattering rate \( \tilde{M} \) is maximized for points \( k \) and \( k' \) on the FS such that \( \mathbf{q} = \mathbf{k} - \mathbf{k}' = \mathbf{Q} \).

We now calculate the scattering rate for a quasiparticle near the FS as a function of \( T \) and \( \Delta k \), where \( \Delta k \) is the distance to a nearby hot spot along the FS. For the commensurate case and a large FS the hot spots are usually distributed along the FS in such a way that one can write, for \( \mathbf{k} - \mathbf{k}' \approx \mathbf{Q} \), that

\[ (\mathbf{k} - \mathbf{k}' - \mathbf{Q})^2 \approx (\Delta k)^2 + (\Delta k')^2, \]

where \( \Delta k \) and \( \Delta k' \) are the displacements of \( k \) and \( k' \) from the nearby hot spots (see Fig. 1) (the adjoint hot spots correspond to \( \Delta k, \Delta k' = 0 \)). This basically means that there is no strong nesting at \( \mathbf{q} \approx \mathbf{Q} \),

...
and the FS and its shadow FS, obtained by translating the FS by $Q$, intercept at a high angle $\Omega \sim \pi/2$. Then one easily finds:

$$\frac{1}{\tau_k} = \frac{\alpha g^2 T \sqrt{\omega_0}}{4v_f} \left( \frac{1}{\sqrt{\omega_{sf} + \omega_0(\Delta k)^2}} - \frac{1}{\sqrt{\omega_{sf} + \pi T + \omega_0(\Delta k)^2}} \right)$$  \hspace{1cm} (17)

where we have assumed that the integration over $\Delta k'$ in Eq. (16) can be extended to infinity. Eq. (17) can be rewritten as:

$$\tau_k = \frac{4v_f Q_k}{\alpha \pi g^2 T^2 \sqrt{\omega_0}}$$  \hspace{1cm} (18)

where

$$Q_k = \sqrt{\omega_{sf} (1 + (\xi \Delta k)^2)} \sqrt{\omega_{sf} (1 + (\xi \Delta k)^2) + \pi T} \times \left( \sqrt{\omega_{sf} (1 + (\xi \Delta k)^2)} + \sqrt{\omega_{sf} (1 + (\xi \Delta k)^2) + \pi T} \right)$$  \hspace{1cm} (19)

Note that this result is easily generalized to arbitrary $\Omega$ by replacing $\Delta k$ with $(\Delta k) \sin \Omega$.

Before proceeding with the transport coefficients it is instructive to calculate the scattering rates for different values of $\Delta k$. Eq. (19) suggests that there are primarily two regions of interest: we call a region of momenta $k$ hot if

$$\Delta k \leq 1/\xi$$  \hspace{1cm} (20)

and we call a region cold if

$$\Delta k \geq 1/\xi$$  \hspace{1cm} (21)

Clearly the relaxation times in these two regions are different. In the hot region one has

$$Q_k \approx \sqrt{\omega_{sf}} \sqrt{\omega_{sf} + \pi T} \left( \sqrt{\omega_{sf}} + \sqrt{\omega_{sf} + \pi T} \right)$$  \hspace{1cm} (22)

The fits to NMR experiments of Barzykin and Pines show that in all HTS one can assume for practically all temperatures of interest that, to a good approximation,

$$\omega_{sf} \ll \pi T$$  \hspace{1cm} (23)
and hence in the hot region we have

\[ \frac{1}{\tau_k} \approx \frac{\alpha g^2 T \xi}{4v_F}. \]  \hspace{1cm} (24)

On the other hand, in the limit \( \xi \gg 1 \), in the cold region one has

\[ Q_k \approx \sqrt{\omega_0(\Delta k)^2} \sqrt{\pi T + \omega_0(\Delta k)^2} \left( \sqrt{\omega_0(\Delta k)^2} + \sqrt{\pi T + \omega_0(\Delta k)^2} \right). \]  \hspace{1cm} (25)

Here we distinguish two different cases, depending on the value of \( \Delta k \), yielding:

\[ \frac{1}{\tau_k} = \frac{\alpha g^2 T}{4v_F(\Delta k)} \quad \pi T \gg \omega_0(\Delta k)^2 \]  \hspace{1cm} (26)

\[ \frac{1}{\tau_k} = \frac{\alpha \pi g^2 T^2}{8v_F(\Delta k)^2 \omega_0} \quad \pi T \ll \omega_0(\Delta k)^2 \]  \hspace{1cm} (27)

Obviously, the scattering rates depend on the particular temperature regime for the magnetic fluctuations, through the temperature dependence of \( \omega_{sf} \) and \( \omega_0 \). In the \( z = 2 \) MF regime \( \omega_0 = \omega_{sf} \xi^2 = \text{const} \) and \( \omega_{sf} = A + BT \), and therefore it is easy to verify (see Eq. (19)) that the behavior of the scattering rate for any \( \Delta k \) can be obtained from the \( \Delta k = 0 \) case by replacing \( A \) in \( \omega_{sf} \) with \( A + \omega_0(\Delta k)^2 \). In other words, every point on the FS has a similar shape of \( 1/\tau_k \); the hot region is enlarged with increasing temperature, as can be verified from Eq. (24).

In the \( z = 1 \) PS regime the situation is somewhat different, due to the temperature dependence of \( \omega_0 \), \( \omega_0 \sim 1/T \). First, from Eq. (24) it is clear that the scattering rate due to spin-fluctuation scattering is roughly temperature independent in the hot region. The scattering rate in Eq. (26), valid for \( \xi \Delta k \ll \sqrt{\pi T/\omega_{sf}} \), is applicable in this case as well. Finally, far away from hot spots, Eq. (27), the scattering rate is proportional to \( T^3 \). Note that the condition \( \omega_0(\Delta k)^2 \gg \pi T \) is satisfied for significant portions of the FS at very low temperatures. The scattering rates as functions of temperature for several different values of \( \Delta k \), in MF and PS regimes, are given in Figs. 2a and 3a.

In the PG case the situation is even more complicated and somewhat unclear, due to the FS evolution mentioned in Sec. II. Here \( \Delta k \) does not correspond to a fixed point on the FS, since the FS itself migrates with decreasing temperature. Nevertheless, provided that
the FS is still relatively large, the scattering rates are nominally the same as in the PS case, since $\omega_0 = \omega_{sf} \xi_c^2 \sim 1/T$, where $\xi_c \sim \text{const}$ is the correlation length at $T_c$, and $\omega_{sf} \sim 1/T$. However, the scattering rate at a particular point in the Brillouin zone is non-trivial and we do not present it here.

So far we have neglected the fact that in deriving Eq. (17) the integral over $\Delta k'$ was performed from $(-\infty, \infty)$. This is quite reasonable provided that the system is far from the PG regime. However, as mentioned in Section I, as the temperature is lowered toward $T_c$, the quasiparticle spectral weight is removed from the hot spots due to strong spin-fermion vertex corrections and at $T \sim T_c$ the system actually starts to lose parts of the FS. Hence, one must assume a finite limit of integration over $\Delta k'$, yielding a somewhat more cumbersome form of Eq. (17) which we do not quote here. However, it is straightforward to show that although this does not affect the cold regions very much, the scattering rate in the hot region is considerably modified, as shown in Figs. 2b and 3b. Although somewhat surprising, this result is natural in view of the FS modulation: the hot spots migrate with decreasing temperature and the effective size of the FS is reduced, thereby reducing the scattering of quasiparticles in the magnetic channel. More importantly, since the FS is small, i.e., $\Delta k_{max}$ is very small, the scattering is anomalous around most of the FS, as can easily be verified from Eq. (13). We return to this point shortly.

The low field longitudinal and Hall conductivities are calculated according to:

$$\sigma_{xx} = -2e^2 \sum_k (v_k \cdot n)^2 \tau_k \left( \frac{\partial f_0}{\partial \epsilon} \right)$$  \hspace{1cm} (28)

$$\sigma_{xy} = -2e^3 \sum_k (v_k \cdot n_\perp \tau_k) v_k \times B \cdot \nabla (v_k \cdot n \tau_k) \left( \frac{\partial f_0}{\partial \epsilon} \right)$$  \hspace{1cm} (29)

where $n$ and $n_\perp$ are unity vectors perpendicular to the applied electrical field $E$, and $B$ is the applied transverse magnetic field.

We start with the resistivity. At relatively low temperatures, $T \ll E_f$, where $E_f$ is the Fermi energy, one can perform the change of variables (8) and integrate over $\epsilon$. Assuming that the FS is four-fold symmetric (tetragonal symmetry), the conductivity is given by:
\[
\sigma_{xx} = \frac{e^2}{4\pi^2} \int_{FS} dk \tau_k v_f \tag{30}
\]

Assuming that the Fermi velocity does not vary appreciably around the FS, we find that
the conductivity is roughly proportional to the average of the scattering time \(\tau_k\) around
the FS. From Eq. (18) it is clear that the main contribution to the conductivity comes from
the cold regions of the FS where \(\tau_k\) is the largest, while the hot spots \((\Delta k \sim 0)\) contribute
relatively little. On averaging Eq. (19) we obtain a rather cumbersome formula for the
resistivity, which, to within logarithmic corrections, can be approximated by:

\[
\rho_{xx} \approx \frac{\alpha g^2 \pi}{v_f} \frac{T^2}{T_0 + T} \frac{1}{(\Delta k_{\text{max}})^2} \tag{31}
\]

where \(T_0\) is a crossover temperature given by

\[
T_0 \approx \omega_0 (\Delta k_{\text{max}})^2 / 2\pi \equiv (\xi (\Delta k_{\text{max}})^2 \omega_{sf} / 2\pi. \tag{32}
\]

If we define cold spots as these points on the FS which are maximally distant from a hot spot,
then \(\Delta k_{\text{max}}\) is the distance between a cold spot and the nearest hot spot. As may be seen in
Fig. 1, the size of the \(\Delta k_{\text{max}}\) is related to the departure of the FS from the magnetic Brillouin
zone boundary. For this FS and any large FS, such as is found by ARPES measurements
for YBa\(_2\)Cu\(_3\)O\(_7\) and related compounds, \(\Delta k_{\text{max}} \sim 1\). On the other hand, for a profoundly
small doping level, or in the presence of substantive incommensuration (which increases the
number of hot spots and acts to reduce significantly the distance between hot and cold
spots), one finds \(\Delta k_{\text{max}} \leq 0.3\).

It is evident from Eq. (31) that in the absence of any evolution of the Fermi surface with
temperature, \(\rho_{xx}\) will be linear in temperature as long as \(T\) is appreciably greater than \(T_0\).
In the MF regime, \(T_0\) is independent of temperature, while in the PS regime it increases
as \(\xi(\sim 1/T)\) as \(T\) decreases. If \(\Delta k_{\text{max}}\) and \(\xi(T_*)\) are small enough, then \(T_0 \leq T_*\), and
the resistivity will be linear in \(T\) down to \(T_*\). The exact condition for this extended linear
in \(T\) behavior reads \((\Delta k_{\text{max}})^2 \xi(T_*) \leq (T_*/100)K(\hat{c}/53\text{meV})\), where \(\hat{c}\) is the pseudoscaling
spin velocity, \(\omega_{sf}/\xi\). Under these circumstances, any change in \(\rho_{xx}\) at \(T_{cr}\) will be marginal,
since it can only arise from changes in the hot region contribution to $\rho_{xx}$. The linearity in $T$ continues above $T_{cr}$ until $T \sim \omega_0$, an experimentally almost unreachable temperature. Moreover, we have assumed that the only relevant scattering is in the magnetic channel. Strictly speaking, in addition to the contribution proportional to $\chi$ defined above one must add the FL contribution, $\chi_{FL}$, present in all FLs, to the effective interaction $V_{eff}$. The latter is likely to dominate at these temperatures. Without this term at $T > \omega_0$ one then has $\rho_{xx} \propto T^{1/2}$, provided $\omega_{sf} \propto T$. On the other hand, from Eq. (19) it is readily shown that if $\omega_{sf} \approx const$ then $\rho_{xx} \propto T^2$. We shall return to this point when discussing overdoped samples.

What happens at $T^*$? As mentioned in Sec. II in the vicinity of $T*$ the quasiparticle band structure begins to acquire a considerable temperature dependence, implying a FS evolution as well. Recently, Chubukov, Morr and Shakhnovich studied the FS evolution as a function of the coupling strength $g$ at $T = 0$ and found that with increasing spin-fermion interaction the FS acquires features of a preformed spin-density-wave state, as depicted in Fig. 4. Alternatively one can show that a similar evolution occurs at fixed $g$, but as a function of temperature, associated with a relatively strong temperature dependence of the spin-fluctuation damping. As a result, the FS appears to get closer to the magnetic Brillouin zone boundary, i.e., the effective size of the FS decreases, with decreasing temperature. Hence when calculating transport coefficients one should assume that $\Delta k_{max}$ acquires a temperature dependence, the precise form of which is not well known at present. Below $T*$ one finds that $\omega_0$ retains its $1/T$ overall temperature dependence although it changes its slope: $\xi$ becomes constant, while $\omega_{sf}$ increases as $T^{-1}$. Whether one gets a crossover at $T_0$ in the PG regime will then depend sensitively on the interplay between $\Delta k_{max}(T)$ and $\omega_{sf}(T)$. If a crossover to $\rho_{xx} \propto T^2/T_0$ occurs, the resistivity will then decrease faster than $T^2$, due to the temperature dependence of $T_0$.

Let us examine the crossover in the vicinity of $T*$ in more detail. The above mentioned FS evolution usually closes channels for scattering as the temperature is lowered below $T*$. New hot spots are developed and a smaller portion of the FS is now cold. At the same
time $\omega_0$ is strongly temperature dependent in both PS and PG regimes. Assuming that $T \gg T_0(T)$ above $T_*$ the reduction in size of the cold region implies that a large part of the FS has a scattering rate such as those shown in Fig. 3 for small values of $\Delta k$, where $1/\tau$ is roughly independent of $T$. At a certain temperature $T^*_r$ the FS is effectively so small, that the quasiparticles around most of the FS exhibit precisely these (almost constant in $T$) scattering rates; the resistivity thus acquires a finite negative curvature, i.e., $d^2\rho_{xx}/dT^2 < 0$.

A signature of this should be seen in the $T$ variation of the scattering rate at a hot spot $\Delta k \sim 0$. In the next Section we shall see that this is indeed the case. It is important to realize that this can occur only when $\omega_0$ is strongly temperature dependent; hence $T^*_r$ is never larger than $T_{cr}$, where $\omega_0 = const$, although in principle it can be comparable to $T_{cr}$. A rapid temperature dependence of $\omega_0$ eventually leads to a Fermi liquid like behavior at $T \ll T^*_r$, regardless of the precise size of the FS. Note that perturbation theory is only marginally applicable in the PG regime, since one must include the strong vertex corrections at or near hot spots, although the fact that the conductivity is dominated by regions away from hot spots, where the above corrections are not as pronounced, allows us to make this qualitative argument. We note that the present theory is somewhat equivalent to the work of Quader et al. although in their case the scattering channels which open up at $T^*_r$ are attributed to the multiband structure in bilayer cuprates and the proximity of one of the bands to the Fermi level.

Overdoped materials exhibit only $z = 2$ scaling and in general $\omega_{sf}$ and $\xi$ show very little $T$ dependence. Quite generally in overdoped samples $\omega_{sf} = A + BT$ displays a much larger value of $A$, associated with the strong fermionic damping of the spin fluctuations, found at high doping levels. Hence, even at a hot spot ($\Delta k = 0$) the scattering rate $1/\tau_k$ is only weakly anomalous, i.e., $1/\tau_k \sim T$ or even $T^2$. Therefore in the overdoped samples one encounters a crossover from quadratic in $T$ behavior of the resistivity to linear in $T$ behavior above $T_0$, even though the amount of the anomaly in the hot regions is weak. However, there are extreme cases, such as the overdoped samples of 214 ($x \geq 0.25$), where the anomalous behavior is so weak that the resistivity is never linear in $T$. Moreover, in overdoped materials
both the large values of $\omega_{sf}$ and the small values of $\xi$ imply that the anisotropy of scattering rates in overdoped materials is seriously reduced (see Eq. (19)), producing only a weakly temperature dependent Hall coefficient.

We now turn to the Hall effect. Here the situation is, in principle, a lot trickier, since the Hall coefficient is a non-trivial function of temperature, due to the strong anisotropy of the mean free path, $\ell \equiv \tau_k v_k$, as readily observed from the definition of $\sigma_{xy}$ (Eq. (29)). However, in practice complications occur only if $\ell$ has a very peculiar periodicity as $k$ goes around the FS. For the FSs seen in ARPES measurements on a number of samples near optimal doping one can safely assume:

$$\sigma_{xy} = \frac{e^3 B}{4\pi^2} \int_{FS} dk \ell_k^2$$

(33)

where we have performed an integration over $\epsilon$ in Eq. (29) in the usual manner. Again, assuming that $v_f$ does not vary appreciably around the FS, one concludes that $\sigma_{xy}$ is roughly proportional to the average of $\tau_k^2$. Clearly, the leading contribution to this quantity comes from the regions of the FS where $\Delta k \sim \Delta k_{\text{max}}$ (cold regions); the hot regions contribute very little. On averaging $\tau_k^2$ over $\Delta k$ (Eq. (19)), we make the same approximations as before, i.e., $\omega_{sf} \ll \pi T$ and $\Delta k \xi \gg 1$ in the cold regions of interest. The result for $\sigma_{xy}$ is again cumbersome, but can be simplified by expanding $\sigma_{xy} T^4$ in $T$, leading to a relatively simple formula:

$$\sigma_{xy} \approx \frac{e^3 B v_f^2}{4\alpha^2 g^4 \pi^2} \left( \frac{\Delta k_{\text{max}}}{T^4} \right)^3 \left( 3T_0^2 + 6T_0 T + T^2 / \sqrt{2} \right)$$

(34)

where $T_0$ is defined in Eq. (32). Eq. (34) is valid over an extended temperature range. Clearly, through the variations in $T_0$ and $\Delta k_{\text{max}}$, it leads to a rather complex morphology of the Hall effect.

Our result, Eq. (34), suggests that there are three temperature regimes for the Hall effect in NAFLs: In the low temperature regime $\sigma_{xy} \propto T_0^2 / T^4$, in the high temperature regime $\sigma_{xy} \propto 1 / T^2$, while at the intermediate temperatures $\sigma_{xy} \propto T_0 / T^3$. The exact values of the crossover temperatures depend greatly on the details of the band structure. In what follows
we shall assume, for the purpose of simplicity, that the crossover between the low and inter-
mediate temperature regimes occurs at $T_0$ and that the crossover between intermediate and
high temperatures occurs at $2\pi T_0$, where $T_0$, given by Eq. (32), is the crossover temperature
observed in the resistivity; the estimate of other quantities, such as the Hall coefficient, is
then rather convenient. Our results for the transport coefficients are given in Table II. Note
that for many systems away from PG regime one finds $\cot \Theta_H \sim T^2$, due to the relatively
large value of the second crossover temperature $T_2$.

Our discussion of the resistivity in underdoped cuprates is easily extended to $\sigma_{xy}$: in the
MF regime $T_0$ is constant and hence

$$\sigma_{xy} \sim T^{-3} \quad \text{(35a)}$$

$$\cot \Theta_H \sim T^2 \quad \text{(35b)}$$

$$\rho_{xy} \sim 1/T \quad \text{(35c)}$$

for temperatures well below $\omega_0(\Delta k_{max})^2$. At somewhat higher temperatures the second
crossover in $\sigma_{xy}$ is approached so that both $\rho_{xy}$ and $\cot \Theta_H$ deviate from this behavior. In the
PS regime $\sigma_{xy} \propto T_0/T^3$, decreases faster than $1/T^{-3}$, since $T_0$ decreases with temperature.
As $T$ approaches $T_{cr}$ one finds $\sigma_{xy} \sim T^{-3}$ even in this regime. Note that in this case the Hall
coefficient varies faster than $1/T$. Finally, due to the variation of $T_0$, the second crossover
can be approached, in which case $\rho_{xy}$ becomes only weakly temperature dependent. We
emphasize once more that in both MF and PS regimes the results depend a great deal on
details of the band structure. Moreover, the presence of impurities makes a non-trivial effect
on all of the quantities of interest.

As before the PG regime is even more complex due to the very rapidly varying $\omega_0$
and the FS evolution, which alters $\Delta k_{max}$. Although it is not clear what the temperature
dependence of $T_0$ is in this case, due to the fact that at present one does not know the
temperature variation of the band structure in detail, we can still make certain arguments.
Near $T_\ast$ the value of $T_0$ changes rapidly and one can easily switch between several regimes, due to the sensitivity of the Hall effect to the variation in the band structure. For example, one can jump from the first (low $T$) regime to the third, without noticing a substantial difference in, e.g., the Hall coefficient, since in both cases the quantity is roughly constant, while $\sigma_{xx}$ and $\sigma_{xy}$ both experience non-trivial changes. Finally, at very low temperatures one finds $\rho_{xy}$ approximately constant and $\cot \Theta_H$ varying faster than $T^2$.

The NAFL model suggests a very complex morphology of the transport coefficients in HTSs. We explore the extent to which this morphology is experimentally supported in the next Section.

IV. ANALYSIS OF TRANSPORT MEASUREMENTS

In this section we review briefly the in-plane normal state transport properties of the cuprate superconductors, with particular attention to the extent to which the doping and temperature dependence of the planar resistivity, $\rho_{xx}$, and the Hall conductivity, $\sigma_{xy}$, reflect the magnetic crossovers discussed in Sec. I and the perturbation theoretic estimates of scattering rates given in Sec. II. We then use a phenomenological model which incorporates the expected highly anisotropic quasiparticle mean free path to deduce from experiments the detailed behavior of the quasiparticle scattering rates in both the hot and cold regions of the Fermi surface. We focus on the results obtained for the bilayer 123 system (of which YBa$_2$Cu$_4$O$_8$, whose behavior corresponds closely to that found in YBa$_2$Cu$_3$O$_{6.68}$, is an “honorary” member), the single layer La$_{2-x}$Sr$_x$CuO$_4$ (214) system, and as representative of overdoped materials, the 15K single layer Tl 2201 material.

It is often stated that optimally doped materials, such as La$_{1.85}$Sr$_{0.15}$CuO$_4$ and YBa$_2$Cu$_3$O$_{6.93}$ possess a resistivity which is linear in $T$ over a wide temperature region which extends down to $T_c$. However, close examination of single crystal data shows that as $T$ approaches $T_c$, departures from linearity occur in these materials, departures which are in fact characteristic of all underdoped materials, but which become more pronounced as
one reduces the doping level below the optimum level. Quite generally, $\rho_{xx}$ in underdoped cuprates exhibits a drop (below linear behavior) at a temperature, $T'_r$, which for many systems is not far from the temperature, $T_s$, at which the low frequency magnetic behavior crosses over from the PS to the PG regime. On the other hand in overdoped materials, $\rho_{xx}$ displays an upturn (from linear behavior) with decreasing $T$, an upturn which we attribute to the comparatively weak anomalous scattering in the vicinity of the hot spots we have discussed in Sec. III. Thus one encounters qualitatively different departures from linearity, depending on whether one is describing a system which is overdoped or underdoped (using the magnetic classification proposed by Barzykin and Pines). Some of the experimental results which have led us to this conclusion are given in Fig. 5. We note that the transport results presented in Fig. 5a provide additional support for the assignment, based on analysis of NMR data, of “optimally-doped” YBa$_2$Cu$_3$O$_{6.92}$ material to the underdoped sector.

To what extent does the crossover at $T'_r$ correspond to the magnetic crossover at $T_s$? As may be seen in Fig. 5a and Fig. 6 for YBa$_2$Cu$_3$O$_7$ and YBa$_2$Cu$_3$O$_{6.63}$ and 124, within the present experimental uncertainties, the two crossover temperatures agree. However, the perturbation theory analysis of Sec. III suggests that this need not be the case in general, since if the FS is sufficiently far from the magnetic Brillouin zone boundary, $T'_r$ can be considerably higher than $T_s$; this appears to be the case for the underdoped members of the 214 family (see Fig. 5b for La$_{1.9}$Sr$_{0.1}$CuO$_4$ and Table I). On the other hand, to our knowledge there are no samples for which $T'_r$ is appreciably lower than $T_s$.

We have called attention, in Sec. III, to the possibility of a crossover to almost Fermi liquid like behavior for $\rho_{xx}$. This occurs when $T < T_0(T)$ (see Eq. (32)). This crossover is found in most, if not all, underdoped materials, at sufficiently low temperatures, where $\rho_{xx}$ displays a finite positive curvature above linearity, $\rho_{xx} = \rho_{xx}(0) + \beta T^b$, with $b$ larger than 1. Indeed, as may be seen in Fig. 8, reasonable fits to the data may be obtained with $b \sim 2$. Note that here $\rho_{xx}(0)$ is the actual residual resistivity, due presumably to disorder in a sample.

In the overdoped materials one also observes a crossover from $\rho = \rho_{xx}(0) + \beta T^2$ to linear
in $T$ behavior at higher temperatures, as already illustrated by the Tl 2201 result in Fig. 5a. We emphasize once again that the physics of this crossover is different than in underdoped materials, as it arises from the fact that the the anomaly of scattering at hot spots is rather weak in these compounds, rather than in the pseudogap effects found in the underdoped cuprates. It is important to notice, however, that there is no second crossover, i.e., $T^*_r$ has not been observed. Moreover, in some overdoped materials, such as La$_{2-x}$Sr$_x$CuO$_4$ at the doping level $x = 0.3$, the resistivity is never linear in $T$, i.e., $\rho \approx T^{3/2}$ over the entire $T$ range.

The morphology of the resistivity is still more complex in materials with CuO chains. As may be seen in Fig. 7, where we show how $\rho_{aa}$ and $\rho_{bb}$ deviate from linearity in the 124 compound, chains lead to both a different magnitude and a different temperature dependence for $\rho_{aa}$ and $\rho_{bb}$ (see Ref. 37). Qualitatively similar results are found for optimally doped 123, although the level of a-b plane anisotropy is considerably less prominent. On comparing Figs. 5 and 7 it may be seen that the temperature dependence of $\rho_{aa}$ and $\rho_{bb}$ in 124 resembles that found in the underdoped and overdoped cuprates respectively. A detailed explanation of this unique behavior in terms of NAFL theory is given in Section V.

We consider next the Hall effect in a transverse magnetic field. In ordinary FLs one finds a Hall conductivity which is roughly proportional to the square of the longitudinal conductivity, $(\sigma_{xy} \sim \sigma_{xx}\sigma_{yy})$, and as a consequence the Hall resistivity, $\rho_{xy} = \sigma_{xy}/(\sigma_{xx}\sigma_{yy} - \sigma_{xy}\sigma_{yx}) \approx \sigma_{xy}/\sigma_{xx}\sigma_{yy}$, is only weakly temperature dependent. However, this is not the case in HTSs, where typically the Hall resistivity decreases sharply with increasing temperature. As discussed in the Introduction, it has been suggested (in the context of the RVB ground state) that the scattering processes which involve $\rho_{xx}$ and $\rho_{xy}$ are intrinsically different, and that the cotangent of the Hall angle, $\cot \Theta_H \equiv \rho_{xx}/\rho_{xy}$ is universal and proportional to $T^2$ (Ref. 11). Much of the experimentally available data focus on the Hall coefficient, $R_H = \rho_{xy}/B$, where $B$ is the applied magnetic field, although the above proposal has led a number of authors to plot $\cot \Theta_H$. However, we find that an examination of the behavior of $\sigma_{xy}(T)$, a quantity more directly related to theoretical calculations, provides more insight.
into the underlying physical origin of the measured anomalous Hall effect behavior.

Since $\rho_{xx}$ typically varies linearly with $T$ for $T > T_0$, $\sigma_{xy}$ must decrease faster than $T^{-2}$ in order to bring about the temperature dependent behavior of $R_H$. Examination of the experimental data presented in Figs. 9 and 10 shows that this is indeed the case at high temperatures, where $\sigma_{xy}$ is seen to vary as $T^{-3}$ for both the 123 and 214 systems. However, experiment also shows deviations from this high temperature behavior in many systems. From Eq. (34) it is clear that if $T_0$ is temperature independent, $\sigma_{xy}$ should indeed vary as $T^{-3}$. However, in PS regime this is not the case, since $T_0 \sim 1/T$ and hence the result in Fig. 9 is to some extent in contradiction with our result. However, the inset of the Figure shows clearly that in YBa$_2$Cu$_3$O$_{6.63}$ $\sigma_{xy}$ obeys $T^{-4}$ quite well up to temperatures $T \sim 200K$, where the first deviations from $T^{-4}$ behavior occur. In fact, only well above this temperature does $\sigma_{xy}$ become proportional to $T^{-3}$, in agreement with our predictions. One can observe similar deviations at other doping levels in this system. Note also that $\sigma_{xy} \sim T^{-3}$ is obtained also well above $2\pi T_0$, for $T_0 \sim 1/T$ (see Table II). We draw two important conclusions. First, $\sigma_{xy}$ has a crossover from $T^{-4}$ to $T^{-3}$ behavior, the details of which depend not only on whether one is above $T_0$, but also on the temperature regimes considered. Second, and more importantly, the “disagreement” in the experimental data in Fig. 9 and its inset show clearly that one should not use simple minded power law fits, as is often done in the literature, but more complex formulae like Eq. (34), which include all necessary crossovers. In addition, we note that one must be very careful when fitting to the Hall effect data, since impurities play an important role, and their presence can lead to large positive intercepts of $1/\sigma_{xy}$ as a function of $T^3$, such as that seen in Fig. 10. Moreover, as mentioned above, the chain contribution in various materials is non-trivial and may obscure a crossover in the in-plane Hall effect. Finally, we remind the reader that Eq. (34) is obtained in the limit $\omega_{sf} \ll T$. This limit is not well satisfied in highly overdoped materials where a somewhat different sequence of crossovers may occur.

Further insight into the role played by anisotropic quasiparticle scattering may be obtained using a phenomenological model for the variation in the mean free path as one goes
around the FS. For simplicity, we consider a representative FS and anisotropic quasiparticle behavior which exhibits a four-fold symmetry. We assume a cylindrical FS with approximately constant \( k_f \), but with an anisotropic effective mass \( m_k \). We parametrize the mean free path (MFP) by assuming that there are hot regions on the FS at \( 90^\circ \) with respect to each other, and choose as a representative FS that depicted in Fig. 1. In the extended zone scheme this FS is approximately a large distorted circle, centered around \((\pi, \pi)\), with hot spots near \( k = \pm(\pi, 0) \) and \( k = \pm(0, \pi) \). Then the MFP around the FS is given by:

\[
\ell(\theta) = \frac{1}{\ell_{\text{hot}}} \left[ 1 + a \cos(4\theta) \right] \quad (36)
\]

where the anisotropy parameter, \( a \), varies between 0 and 1 (\( a \approx 1 \) for a highly anisotropic scattering rate and \( a \approx 0 \) for weak anisotropy), and \( \ell_{\text{hot}} \) is the MFP at a hot spot on the FS. Note that, in general, both \( a \) and \( \ell_{\text{hot}} \) are functions of temperature. For simplicity we shall assume \( a = (1 - r)/(1 + r) \), where \( r = \ell_{\text{hot}}/\ell_{\text{cold}} \equiv \ell(0)/\ell(\pi/4) \). This particular choice may not be the most suitable for an effective interaction which is sharply peaked at \( Q \), since for long correlation lengths, as one moves away from a hot spot \( 1/\ell \) decays faster with \( \theta \) than is assumed here. However, if the effective interaction \( (1) \) has incommensurate peaks in momentum space, then the anomalously scattered (hot) regions are large, \( 1/\ell \) is a slower function of \( \theta \), and Eq. (36) is quite reasonable approximation. Therefore, one should consider \( \ell_{\text{hot}} \) and \( \ell_{\text{cold}} \) here in a broader sense, recognizing that under certain circumstances it may not be possible to identify them with the results quoted in the previous section.

Starting from the expressions, Eqs. (28) and (29), for \( \sigma_{xx} \) and \( \sigma_{xy} \), at \( T \ll t \) one can perform a change of variables and integrate over \( \epsilon \). The result for \( \sigma_{xx} \) is:

\[
\sigma_{xx} = \frac{e^2 k_f}{2\pi^2} \int d\theta \ell \cos^2 \theta \quad (37)
\]

where \( \ell = \tau(\theta)k_f/m(\theta) \) and \( \theta \) is the angle between the electrical field and \( k(\theta) \). On using the parameterization (36), we find:

\[
\sigma_{xx} = \frac{n e^2 \tau_{\text{hot}}}{m_{\text{hot}} \sqrt{r}} = \frac{n e^2 \tau_{\text{cold}} \sqrt{r}}{m_{\text{cold}}} \quad (38)
\]
where $n = k_f^2/2\pi$. In similar fashion the Hall conductivity is found to be:

$$\sigma_{xy} = \left(\frac{e^3 B}{2\pi^2}\right) \int d\theta \ell \cos \theta \left(\frac{d}{d\theta} \ell \sin \theta\right)$$

(39)

which leads to:

$$\sigma_{xy} = \left(\frac{e^3 B}{2\pi}\right) \ell_{hot} \frac{1 + r}{2r^{3/2}} = \sigma_{xx} eB \ell_{hot} \frac{1 + r}{2r}.$$  

(40)

Eqs. (38) and (40) may be combined to yield simple expressions for the Hall coefficient and Hall angle:

$$R_H \approx \frac{\sigma_{xy}}{\sigma_{xx} B} = \frac{1}{en} \frac{1 + r}{2\sqrt{r}}$$

(41)

and

$$\cot \theta_H = \frac{\sigma_{xx}}{\sigma_{xy}} = \frac{m_{hot}}{eB\tau_{hot}} \frac{2r}{1 + r}$$

(42)

Note that the above expressions depend only on the two temperature dependent parameters $r$ and $\ell_{hot}$. These two quantities can be directly probed by measuring the Hall coefficient (41) and the resistivity $\rho_{xx} \approx \sigma_{xx}^{-1}$, since $R_H$ depends only on $r$ and the product $\rho_{xx}R_H$ yields $\ell_{hot}$.

In the case of high anisotropy, i.e., $r \ll 1$, the above quantities take an especially simple form:

$$\sigma_{xx} = ne^2 \sqrt{\tau_{cold}/m_{cold}}$$

(43)

$$\sigma_{xy} = \sigma_{xx} \frac{\omega_c \tau_{cold}}{2}$$

(44)

$$R_H = \frac{1}{2ne} \sqrt{\tau_{cold}/m_{cold} \tau_{hot}/m_{hot}}$$

(45)

$$\cot \theta_H = \frac{2}{\omega_c \tau_{cold}}$$

(46)

where $\omega_c$ is the cold cyclotron frequency, $\omega_c = eB/m_{cold}$. Our result, Eq. (46) for the Hall angle, provides a natural explanation of why the superconducting cuprates with a variety of different behaviors for $\rho_{xx}(T)$, often display a quite similar behavior of $\cot \Theta_H$. Note that
the Eq. (43) is exact for any \( r \). It is clearly consistent with our conclusion in Sec. III that the linear in \( T \) resistivity in NAFLs is due to a fine balance between cold and hot regions. In addition, with the help of Eq. (27) one easily verifies that Eq. (44) is consistent with Eqs. (31) and (34).

One can use Eqs. (38)-(42) to deduce the temperature dependence of \((m/\tau)_{\text{hot}}\) and \((m/\tau)_{\text{cold}}\) from the experimentally measured \(\rho_{xx}\) and \(\rho_{xy}\). Only one undetermined parameter enters into this procedure, namely \( n \), which in some cases can be obtained experimentally from the high temperature limit of \( R_H \), since in this case \( r \to 1 \), and the Hall coefficient is temperature independent. The quantity, \( R_\infty \equiv R_H(T \to \infty) \) is well defined for many overdoped materials, but has been determined for only a few underdoped materials. Whenever experimental data are not available we make the best estimate for \( R_\infty \): for example, in the 214 family we extrapolate \( R_H(T) \) to \( T = T_{cr} \) and for the 124 material we assume \( R_\infty \) comparable to the result found for \( \text{YBa}_2\text{Cu}_3\text{O}_{6.63} \) (Ref. 39), which has comparable magnetic properties. However it is important to stress that as long as one has the correct order of magnitude for \( R_\infty \), qualitatively the results are virtually the same.

We proceed to extract quasiparticle scattering rates by fitting Eqs. (38)-(41) to the transport measurements considered above. Fig. 11 shows the scattering rates for \( \text{YBa}_2\text{CuO}_{6.63} \) in the cold and hot regions of the FS (in the same dimensionless units) as a function of temperature. Clearly in the hot region the scattering rate is much larger; it is approximately linear in \( T \) for \( T \leq T^*_r \approx 200 \text{K} \), and becomes weakly \( T \) dependent at higher temperatures. As noted above, for this system \( T^*_r \approx T_{cr} \). At the same time \((m/\tau)_{\text{cold}}\) barely reflects the crossover at \( T^*_r \) as may be seen in Fig. 12, although clearly both scattering rates are affected as the system approaches the PG regime. Note that these scattering rates closely resemble those shown in Fig. 4. In particular, the crossover to a constant scattering rate of \((m/\tau)_{\text{hot}}\) is strikingly similar to the calculated one for \( \Delta k \approx 0 \), shown in Fig. 3.

The results for the 124 compound are given in Fig. 13, where we have taken \( \rho_{aa} \), which does not include chain contribution and exhibits an in-plane resistivity resembling that in
underdoped materials, and used the measured values of $R_H$. The results are similar to those found for YBa$_2$Cu$_3$O$_{6.68}$, although in the measured temperature regime ($T < 300$K) the scattering rate at the hot spot does not quite saturate. However, we find that, as $T$ increases, $(m/\tau)_{cold}$ becomes approximately proportional to $T^2$ in this material; on extrapolating this behavior of $(m/\tau)_{cold}$ and the experimentally obtained resistivity, we find the same behavior of $(m/\tau)_{hot}$, albeit with a somewhat higher value of $T^*_r \approx 250$K.

Finally in Fig. 14 we show the scattering rates in the cold and hot regions for YBa$_2$Cu$_3$O$_7$: here the scattering rate in the hot region is once more seen to saturate, but at a much lower temperature corresponding to $T^*_r \approx 120 - 130$K. Both this value and that found in 124 and YBa$_2$Cu$_3$O$_{6.68}$ are very close to $T_*$, found by Barzykin and Pines for these materials (see Table I). Although it is not easy to verify directly in NMR experiments, because both $T_{cr}$ and $T_*$ are close to $T_c$ for YBa$_2$Cu$_3$O$_7$, this result provides a further indication of the presence of the PG phase even in optimally doped 123 materials.

A similar analysis can be performed in single CuO layer materials: two panels of Fig. 15 show the scattering rates at doping levels $x = 0.10$, $x = 0.15$ in 214 compound. Much like YBa$_2$Cu$_3$O$_{6.68}$, one observes the saturation of the scattering rate in the hot region and it is only for a particular anisotropy level that the resistivity in the optimally doped 214 material is approximately linear in $T$ below $T^*_r$. In fact, as pointed out in Ref. 36, a crossover in resistivity exists even in this material, but is so minor that one usually assumes that $\rho_{xx} \propto T$ at all temperatures. As mentioned before, in the 214 family $T^*_r$ is considerably larger than $T_*$.

Finally we consider the overdoped single layer materials: Fig. 16 shows the scattering rates for the overdoped 15K Tl 2201 compound. First, one immediately notices that the anisotropy of scattering is not nearly as pronounced as in previous graphs. Note that in this case $R_\infty$ is very well known and thus one can consider this reduced anisotropy in scattering rates quantitatively as well. We remark that both scattering rates have nearly as strong $T$ dependence as in the previous cases, over the entire $T$ range. Moreover, the scattering rates shown here again closely resemble the calculated result for larger $\Delta k$ in Fig. 2.
V. NUMERICAL RESULTS

In the previous sections we have established the relationship between the crossovers in the magnetic spectrum of cuprates and their transport properties analytically. However, it is unclear whether such an analysis, based on back-of-the-envelope calculations, can account for the experimental results quantitatively. In this section we present our numerical results for the transport coefficients in NAFLs: we show that for spin-fluctuation and band parameters chosen from NMR, INS and ARPES measurements, the NAFL model indeed reproduces experimentally obtained results. We compare our results to experiments performed on several compounds of interest, examine the sensitivity of our results to the input parameters of the theory, and address several important questions regarding realistic materials, including the strong a-b plane anisotropy observed in a number of bilayer compounds.

The method used to obtain the results presented here has already been discussed elsewhere and hence we only review it briefly. We study the transport in NAFLs by solving the Boltzmann equation (BE) numerically. We obtain the displacement of the quasiparticle (fermionic) distribution function,

\[ f_0(k) - f(k) = \Phi_k \left( \frac{\partial f_0(k)}{\partial \epsilon} \right) \]  

(47)

as a function of momentum \( k \), temperature and frequency on a fine mesh of points in the Brillouin zone, using a standard relaxation method. The transport coefficients are then obtained from

\[ j = e \sum_k \Phi_k v_k \left( \frac{\partial f_0}{\partial \epsilon} \right) \]  

(48)

where \( v = \nabla \epsilon_k \). We assume that the interaction present in the collision integral in the BE, is given by Eq. (I). The numerical method can be described as follows: we start from a reasonable choice of \( \Phi_k \), which we then use to calculate the collision integral, which in turn provides for a new value of \( \Phi_k \). The iteration procedure is stopped when the difference between two subsequent values of \( \Phi_k \) is smaller than a given tolerance.
In our earlier numerical work\cite{23,31}, we demonstrated the viability of a NAFL description of transport in the cuprates. By using spin-fluctuation spectra seen in NMR experiments, we were able to obtain both qualitative and quantitative agreement with the analysis presented in Sec. III and Sec. IV. However, for the band parameters, doping level, and spin-fluctuation spectrum we assumed for optimally-doped YBa$_2$Cu$_3$O$_7$, the crossover temperature, $T_o$, was high compared to that seen experimentally, while $\rho_{xx}$ exhibited FL behavior up to temperatures $\sim 200K$. Because $T_o$ was large, on extrapolating our linear in T results for $\rho_{xx}$ to zero temperature, we found a large negative intercept. Moreover, although the calculated Hall coefficient decreased with increasing temperature, it displayed a far weaker temperature dependence than that seen experimentally. Our results thus resembled those found for overdoped cuprates, rather than an optimally doped sample.

On the basis of the analysis presented in \cite{III}, we can identify several potential causes for these discrepancies. First, the actual value of $\omega_0 \equiv \omega_{SF}\xi^2$ in optimally doped 123 is $\lesssim 60$meV\cite{32} rather than the larger value, $\omega_0 = 76$meV, adopted in these calculations. Second, the spin fluctuation spectrum was assumed to possess a commensurate peak, rather than the four incommensurate peaks which a recent analysis of NMR and INS results suggests might provide a better fit to experiments on the system\cite{32}. Third, the doping level was taken to be $n \sim 0.25\%$, while the LDA calculations of bonding and antibonding in bilayer materials discussed below suggest a considerably lower doping level might be more appropriate. Each of these effects acts to bring about either a lower value of $T_o$, an increase in the linear in T regime of $\rho_{xx}$, and/or an increased temperature dependence of $R_H$. To explore their combined influence, we carry out a numerical solution of the BE for a representative “standard” 123 material for which $n = 20\%$, $\omega_0 = 60$meV, and $\omega_{SF}(K) = 60 + 0.6T(K)$; the band spectrum was assumed to be $(t = 0.2eV; t' = -0.35t)$, which is slightly different from the unrenormalized LDA spectrum\cite{40} while the incommensurate peaks in the spin spectrum are assumed to lie along the diagonals, at $Q_i = Q + (\pm 0.1\pi, \pm 0.1\pi)$, as proposed by Zha et al\cite{32}.

In Fig. 17 we compare our calculated resistivity as a function of temperature with the experimental results obtained for optimally-doped YBa$_2$Cu$_3$O$_7$. The coupling constant $g$,
which sets the scale for the magnitude of the resistivity, has been adjusted to $g = 0.48$ eV, in order to obtain the best fit. Clearly, the calculated result agrees very well with the data down to temperatures only slightly above $T_c$. Note that the extrapolated value of $\rho_{xx}$ at $T = 0$ is now quite small, as is seen in the experiment. On the other hand, the Hall coefficient, shown in the inset of the figure, is strongly temperature dependent, with a relative decrease which is approximately the same as that seen experimentally in the temperature interval, 150-400K, although the exact shape of $R_H$ is not the same. However, as mentioned above, a better knowledge of the band structure, including the chain contribution, and of the effective interaction $V_{eff}$ for $q$ away from $Q_i$ might well remove this problem. Moreover, a quick look at Eq. (45), as well as Eqs. (31) and (34) indicate that the FS reduction (i.e., decreasing $\Delta k_{max}$) with the approaching PG phase at low temperatures, should provide for an additional temperature dependence of $R_H$. Our calculated $R_H$ saturates at higher $T$, to a somewhat higher value than that seen experimentally, implying that a different value of the doping level should be used; nevertheless, our choice of parameters would seem quite reasonable (see Eq. (41)). Another reason for a discrepancy in the shape of $R_H$ lies in the fact that we have calculated this quantity assuming a tetragonal lattice, rather than the orthorhombic one found in YBa$_2$Cu$_3$O$_{7}$. Recent measurements on twinned and detwinned crystals of this material show a considerable difference in $R_H$, indicating the importance of including the a-b plane anisotropy in the calculation. We return to this point at the end of this section.

The doping level used in this calculation is somewhat low. If one assumes that every Oxygen atom doped into the 123 system extracts a single hole from CuO planes, the doping level of the optimally doped 123O$_{6.93}$ should be somewhat higher, $n \approx 23\%$. On the other hand, simple arguments, based on LDA calculations, suggest that in bilayer materials, such as 123 compounds, there are two bands, bonding and antibonding, separated by twice the hopping matrix element between the CuO layers, $t_\perp$ (see Refs. [40][42]). If one assumes that $t_\perp$ is weakly momentum dependent, one finds the top (antibonding) band is more heavily doped, while the bottom (bonding) band should be close to half filling, regardless of the Oxygen
Since the interaction between quasiparticles is much weaker in the metallic state of cuprates than it is in the insulating state, the nearly half filled bonding band has a larger value of $v_f$ and should provide the dominant contribution to the conductivity in the bilayers (see Eq. (31)). Hence the doping level chosen here, $n = 20\%$, may be quite reasonable for 123 and 124 compounds.

To study the effect of doping on the resistivity at fixed spin fluctuation spectra we have calculated $\rho_{xx}(T)$ for the same values of the spin and band parameters as in our “standard model,” but with the chemical potential adjusted to yield an $n = 30\%$ doping level, and compared it to the above case in Fig. 18. Clearly, $\rho_{xx}$ ceases to be linear in $T$ at lower temperatures, while continuing to exhibit typical FL behavior as $T \to 0$. In the same figure we also show the influence of incommensuration, by plotting $\rho_{xx}(T)$ for the same representative set of parameters as above, but with a commensurate spin-fluctuation spectrum. Although the commensurate result has a slope at higher temperatures which is remarkably similar to the incommensurate (representative) one (due to the fact that $\rho_{xx}$ is independent of $\omega_0$ above $T_0$ (see Eq. (31))), we again observe FL-like behavior at lower temperatures. In both cases the explanation is simple: enlarging $\Delta k_{max}$, leads to an increase in $T_0$; this can be achieved either by increasing the doping level, which increases the size of the FS, or by removing the extra hot spots brought about by incommensuration. Fig. 18 demonstrates quite clearly that the behavior of the resistivity is sensitive to the choice of band parameters and leads us to conclude that self-consistent calculations which take into account changes in the quasiparticle interaction should be used in comparing with experiment. On the other hand, assuming that the magnetic (band) parameters are well known, our model can be used to put constraints on the allowed band (magnetic) parameter values.

Next we examine the dependence of $\rho_{xx}$ and $\sigma_{xy}$ on $\omega_{sf} = A + BT$. In Fig. 19 we plot the resistivity as a function of temperature for four different values of $A$ and $B$: we show the cases where $A = 20K$ and $B = 0.2$, $A = 0.6$ and $B$ is the same, the case where $B$ is 0.6 and $A$ is the same, and the case where $\omega_{sf} = 60 + 0.6T$. The band parameters are set to
t = 250meV, \( t' = -0.45t \) and the doping level is \( n = 15\% \); the coupling constant is set to \( g = 1eV \). In agreement with our analysis in Sec. III, a large value of \( A \), corresponding to an enhanced damping of spin-fluctuations due to spin-fermion interactions, shifts the resistivity down; it leads to a higher value of \( T_0 \) in Eq. (31) and only marginally anomalous scattering rates even in hot regions of the FS, in agreement with the result presented in Fig. 16.

On the contrary, increasing the value of \( B \) only changes the slope of the resistivity curves shown in Fig. 19, but does not much affect the behavior with respect to the origin. This is to be expected, since according to Eq. (19) the term \( BT \) in \( \omega_{sf} \) acts in conjunction with the \( \pi T \) term in \( \sqrt{\pi T + \omega_{sf}(1 + (\Delta k)^2\xi^2)} \) to bring about linear in \( T \) resistivity. However, since in all experimentally studied samples \( B \ll \pi \), the effective value of \( T_0 \sim \omega_0(\Delta k_{max})^2/2(\pi + B) \) differs only slightly from that quoted in Eq. (32). As noted in Sec. III, for practically all cuprates \( BT \) is never large enough to produce the \( T^{1/2} \) behavior of the resistivity discussed there, which is found at \( T \sim \omega_0(\Delta k_{max})^2 \), provided the FS is large enough. Hence, the crossover at \( T^* \), where a curvature resembling \( T^{1/2} \) is found in many underdoped samples at \( T \geq T_* \), has to be attributed to a small value of \( \Delta k_{max} \) resulting from the FS evolution. Finally we note that the Hall conductivities for all of the above cases obey a \( T^{-3} \) law quite well, as shown in the Inset of the figure, although due to the large value of \( T_0 \) the Hall resistivity may be only weakly temperature dependent.

We turn now to the \( z = 1 \) PS regime. As indicated in Sec. III, the resistivity is linear in \( T \) for \( T > T_0 \) regardless of whether or not \( T_0 \) is temperature dependent. However, since \( \xi \sim 1/T \) in this regime, the condition \( \xi \Delta k < 1 \) is satisfied over most of the FS at relatively low temperatures. In this case the resistivity due to spin-fluctuation scattering tends to a constant, since \( T \gg 2\pi T_0 \). To further explore the PS regime, we solve the B.E. numerically for a representative member of the 2-1-4 system, \( [\hat{c} = \omega_{sf}\xi = 50meV, \xi^{-1} = 0.1 + T/1000K, g = 1eV, n = 15\%, t = 250meV] \) and several values of \( t' \) (\( -t'/t = 0.2, 0.3 \), and 0.4 respectively). Our results are shown in Fig. 20a. In all of the cases displayed there we have assumed that the effective interaction has incommensurate peaks at \( Q_i = Q + (\pm \Delta Q, 0) \) and \( Q + (0, \pm \Delta Q) \), with \( \Delta Q = 0.25\pi \), in order to be consistent with neutron
scattering experiments performed in 214 materials at sufficiently high doping levels. For \( t' = -0.4t \) the FS is relatively large, as is the corresponding \( T_0(T) \). As a result one observes a crossover to linear in \( T \) resistivity at \( T \sim 200K \) from the low \( T \) Fermi liquid like behavior. For \( t' = -0.2t \) the FS is always close to the Brillouin zone boundary and hence the effective \( \Delta k_{max} \) is small, \( \Delta k_{max} \sim 0.1 \). Hence the condition \( T = 2\pi T_0(T) \) is satisfied for \( T \sim 1000K \), and the resistivity deviates from linearity already at \( T = 150 \). Obviously, in the intermediate case \( \rho_{xx} \propto T \) over an extended temperature range. We conclude that the resistivity is very sensitive to changes in band parameters, especially when \( T_0 \) is a strong function of temperature.

The parameters used in Fig. 20a are close to those found for \( \text{La}_{1.85}\text{Sr}_{0.15}\text{CuO}_4 \) material in the PS regime. In Fig. 20b we compare \( \rho_{xx}(T) \), obtained for \( t' = -0.33t \) (solid line), to experiment. The coupling constant has been adjusted to yield the same slope as the experimental curve. Obviously even in this case the calculated resistivity exhibits the features shown in Fig. 20a, but the fit is quite good, especially keeping in mind the fact that the experimental result is subject to disorder. On the other hand, when one compares \( \cot \Theta_H \) in these two cases (solid line in Fig. 21), the fit, although remaining reasonable, is not as good. The curvature of the calculated result (and lack thereof in the experimental result) implies that the cold regions of the FS are too small in the calculation, whence \( \cot \Theta_H \sim T^{1.6} \), rather than the \( T^2 \), behavior frequently seen, although some samples of this material have the smaller exponent in \( \cot \Theta_H \) found here. However, even for a small value of \( t' \), the addition of a Fermi liquid term, \( \chi_{FL} = \chi_0(T)/(1 - i\pi \omega/\Gamma(T)) \), to the effective interaction, makes the resistivity appear linear in \( T \) up to higher temperatures and the agreement with experiment for the Hall effect much better, as shown by the dashed line in Figs. 20 and 21. In both cases we have used \( \Gamma(T) = (0.5 - T/2500)\text{eV} \) and \( \chi_0(T) = 1 + 0.4T/100\text{states/eV} \), the values obtained from the fits to NMR measurements. This is as expected, since adding a Fermi liquid term affects predominantly the cold regions of the FS and its effect is similar to that of the strong band renormalization present in the PS regime.

We next consider the issue of the a-b plane anisotropy. Although found in practically
all cuprates, this anisotropy is especially pronounced in materials with CuO chains, leading to quite different transport results along different crystallographic directions. In principle, these are two reasons for an anisotropic resistivity: the chains can form a conducting band, or the chain bands hybridize with Oxygen bands in CuO plane, producing an anisotropic quasiparticle effective mass. Since the chains in, e.g., YBa$_2$Cu$_3$O$_7$ are very close to the CuO planes, and the Oxygen p-orbitals are rather large, we believe that the latter effect must dominate. In order to simulate its effect on the in-plane resistivity we consider a 2D band with anisotropic hopping integrals: we alter the first term in Eq. (2) to read

$$-2t(\cos k_x + a \cos k_y)$$  \hfill (49)

In Eq. (49) the anisotropy parameter $a$, lies between 0 and 1; obviously, $a = 1$ in tetragonal systems. A FS for an anisotropic system with $a = 0.55$ and the doping level $n = 10\%$ is shown in Fig. 22. Although this shape of the FS is somewhat incompatible with that observed in, e.g., the 124 material, one must keep in mind that strictly speaking a single band model, such as that considered here, is an oversimplification; we only use the present model in order to qualitatively explain the physics in the above material.

In Fig. 23 we show the resistivities for the current running along $x$ and $y$ directions for the band depicted by the FS in Fig. 22. We assume that $\omega_{sf}(K) = 20 + 0.2T$, which is close to $\omega_{sf}$ found in 124 materials. However, we retain the $z = 2$ scaling regime in order to make the comparison with previous results easier. Clearly, not only are the magnitudes of the two resistivities different, but their temperature dependence is also different; indeed our results for $\rho_{xx}$ and $\rho_{yy}$ resemble those found in the 124 material. The lower curve ($\rho_{xx}(T)$) appears to have a considerable negative intercept, indicating a large value of $T_0$, as in overdoped materials. The upper curve, $\rho_{yy}(T)$, exhibits a crossover at $T \sim 300K$, similar to that observed experimentally at $T^*$ in underdoped cuprates. This quite different behavior arises from the modified positions of the hot spots: as may be seen on examining the FS shown in fig. 22, the hot spots are now located at points $(1, \pm \pi)$ and $(\pi - 1, 0)$ and symmetry related points on the FS, i.e., they are asymmetric with respect to the inversion of $x$ and $y$.  

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axis and cannot be obtained by drawing the usual magnetic Brillouin zone boundary. For current running along the $x$ direction, a large segment of the FS, roughly along the $(1, y)$ direction, can be displaced in transport (the vertex function $v_x$ is large at these $k$ vectors) and more importantly, these segments of the FS are far from the hot spot at $(1, \pm \pi)$, so that it is necessary to balance the hot and cold regions, to arrive at a $\rho_{xx}$ which is linear in $T$. For current running along the $y$ direction, the vertex $v_y$ is small almost everywhere on the FS (see Eq. (48) and the hot spots sit in the middle of the region of the FS where $v_y$ is the largest, further reducing the conductivity in this case. Effectively the FS appears to be small, producing the anomalous (non-linear with negative curvature) $\rho_{yy}(T)$ seen in the figure. Note that the Hall conductivity is proportional to $T^{-3}$, as seen in Fig. 24. We also note that our result for $\sigma_{xy}$ is the same to within a few percent for both current directions, which serves as a nice check of the quality of numerics in this case.

We make another important comment regarding this result: unlike the case of 124 material, where $T^*_r$ is very close to $T_*$, so that the anomalous behavior of the resistivity occurs at least in the pseudoscaling ($z = 1$) regime if not in the pseudogap one, here we have similar behavior with $z = 2$. We again conclude that the crossover at $T^*_r$ is closely related to the effective size of the FS and is less closely related to a particular magnetic regime. However, we must keep in mind that the FS here is reduced artificially (by introducing a very large anisotropy). In reality, magnetic fluctuations alter the shape of the FS as the system enters into the pseudogap regime. Moreover, one cannot obtain a similar anomalous behavior of $\rho_{yy}$ using a large Fermi surface and the same spin-fluctuation parameters as those utilized here; and hence this result serves as a further indication that the pseudogap regime involves an evolution of the FS.

VI. CONCLUSIONS

We have performed an analysis of the in-plane longitudinal and Hall conductivities (resistivities), in terms of the NAFL model, using both perturbation theory and numerical
methods. From our perturbation theory results we obtained an analytical expression for the scattering rate as a function of temperature for an arbitrary point on the FS, as well as empirical expressions for $\rho_{xx}$ and $\rho_{xy}$. The results are then used to study analytically the crossovers in transport in various temperature regimes. Assuming a particular geometry of the FS, we obtained phenomenological expressions for both $\rho_{xx}$ and $\rho_{xy}$ which we used to analyze the experimental results and gain further insight into the scattering rates. Finally, we performed a set of numerical calculations, verifying for a realistic set of input parameters that NAFL model yields results in agreement with experiment.

Our analytical results display a relatively strong anisotropy of scattering rates around the FS, leading to a complex morphology of both $\rho_{xx}$ and $\rho_{xy}$, in contrast to the common belief that in all cuprates $\rho_{xx} \propto T$ and $\rho_{xy} \propto 1/T$. A close examination of the available experimental data shows that the universality of the temperature behavior of both $\rho_{xx}$ and $\rho_{xy}$ is quite marginal over larger temperature regions, i.e., only extremely clean, optimally doped samples of 123 show $\rho \sim \rho_0 + \alpha T$ with $\rho_0 \sim 0$ and $\cot \Theta_H \sim T^2$. Even optimally doped 214 compounds show considerable discrepancy from this behavior in that the intercept of $\cot \Theta_H$ at $T = 0$ is non-negligible. All other doping levels show more or less pronounced deviations from the above optimal behavior at low temperatures, in agreement with the NAFL model. We therefore conclude that the optimally doped compounds are rather unique and the temperature variation of their transport coefficients is somewhat accidental. The calculated results for the scattering rates are found to be in detailed qualitative agreement with those coming from the fits to the experimental data. It is remarkable how systematically the scattering rates vary with temperature and doping and how precisely they reflect the magnetic properties in cuprates.

Finally our numerical results show reasonable agreement with experiments for a realistic set of spin-fluctuation and band parameters in both $z = 1$ and $z = 2$ scaling regimes. We also verified that improved quantitative agreement can be achieved through the inclusion of strong coupling effects, which bring about a temperature dependent quasiparticle band structure. In addition, we showed that the NAFL model provides for a natural explanation
for the unusual temperature dependence of the resistivities along different directions in cuprate material. Both the numerical results and the available experimental data provide yet another indication that NAFL is a strong candidate for the explanation of the anomalous transport in cuprates.

It is straightforward to extend the NAFL model to optical frequencies and to take into account the influence of impurities and we will report on these results in future publications.

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TABLES

TABLE I. Crossovers temperatures and spin fluctuation parameters for representative cuprates

|                | $T_c$ | $T_*$ | $T_{cr}$(K) |
|----------------|-------|-------|-------------|
| La$_{1.9}$Sr$_{0.1}$CuO$_4$ | 33    | 50    | $\geq$500  |
| La$_{1.85}$Sr$_{0.15}$CuO$_4$ | 39    | 85    | 325         |
| La$_{1.8}$Sr$_{0.2}$CuO$_4$   | 30    | 75    | 120         |
| La$_{1.76}$Sr$_{0.24}$CuO$_4$ | 25    | 35    | 100         |
| YBa$_2$Cu$_3$O$_{6.63}$       | 60    | 180   | $\geq$600  |
| YBa$_2$Cu$_4$O$_8$            | 80    | 210   | 475         |
| YBa$_2$Cu$_3$O$_7$            | 90    | 110   | 150         |

TABLE II. The transport coefficients in various temperature regimes.

|                | $\sigma_{xy}$ | $\rho_{xx}$ | $\rho_{xy}$ | $\cot\Theta_H$ |
|----------------|----------------|--------------|--------------|-----------------|
| $T < T_0$     | $T_0/T^4$      | $T^2/T_0$    | const        | $T^2/T_0$       |
| $T_0 < T < 2\pi T_0$ | $T_0/T^3$    | $T$          | $T_0/T$      | $T^2/T_0$       |
| $T > 2\pi T_0$ | $1/T^2$       | $T$          | const        | $T$             |
FIGURES

FIG. 1. A model of a Fermi surface in cuprates (solid line) and the magnetic Brillouin zone boundary (dashed line). The intercepts of two lines marks the regions of the FS near $(\pi, 0)$, which can be connected by the wavevector $Q_i$. These regions are most strongly scattered into each other (see Eq. (1)).

FIG. 2. Scattering rates, in arbitrary units, as a function of $T$ in the mean field $z = 2$ regime, for several values of $\Delta k$, the displacement of the wavevector $k$ from a hot spot. In both plots we have assumed $\omega_{sf} = 0.2T$, $\omega_0 = 600K$. The curves correspond to (top to bottom) $\Delta k = 0, 0.1, 0.25, 0.5, 1$ and 2. Panel (a) shows the case of a quite large FS, $\Delta k_{max} = 2$, and panel (b) that of a typical small one, $\Delta k_{max} = 0.5$.

FIG. 3. Scattering rates, in arbitrary units, as a function of $T$ in the pseudoscaling $z = 1$ regime, for several values of $\Delta k$, the displacement of the wavevector $k$ from a hot spot. In both plots we have assumed $\omega_{sf} = 0.2T$, $\omega_0 = 1000K/T$ and the curves correspond to (top to bottom) $\Delta k = 0, 0.1, 0.25, 0.5, 1$ and 2. Panel (a) shows the case of a large FS, $\Delta k_{max} = 2$, and panel (b) that of a small one $\Delta k_{max} = 0.5$.

FIG. 4. The FS as a function of coupling constant $g$ in a NAFL. The figure shows only one quarter of the Brillouin zone. The curves are obtained using the same formalism and the same values of spin fluctuation and band parameters as in Ref. 35. The dashed-dotted, solid and dashed lines correspond to increasing value of $g$. As shown in Ref. 29, these curves also correspond to high, intermediate and low temperatures respectively. Note that the effect of the interaction is to move the FS towards the magnetic Brillouin zone boundary (here depicted by the dotted line). Hence the interaction effectively reduces $\Delta k_{max}$. 
FIG. 5. The measured resistivity as a function of temperature, showing the deviation from linearity in $T$ in underdoped, overdoped and optimally doped cuprates. Panel (a) shows the results from Refs. 44, 45 and 46, obtained in (top to bottom) 15K Tl 2201, YBa$_2$Cu$_3$O$_7$, and YBa$_2$Cu$_3$O$_{6.63}$ compounds. The dashed line is a guide to the eye and the arrow marks the crossover from pseudoscaling to pseudogap behavior at $T_*$ in the underdoped and optimally doped material. Panel (b) shows La$_{2-x}$Sr$_x$CuO$_4$ at three doping levels (top to bottom) $x = 0.22$, 0.15 and 0.10 (Ref. 36). The quantity plotted is $(\rho_{xx}(T) - \rho_0)/\alpha T$, where $\rho_0$ and $\alpha$ are obtained by fitting the high $T$, linear part of the resistivity.

FIG. 6. The resistivity $\rho_{aa}$, for current running along the $a$ crystallographic axis, as a function of $T$ in YBa$_2$Cu$_4$O$_8$ material (Ref. 37). The quantity plotted is $(\rho_{aa}(T) - \rho_{PS})/\alpha_{PS}T$, where $\rho_{PS}$ and $\alpha_{PS}$ are obtained by fitting $\rho_{PS} + \alpha_{PS}T$ in the PS temperature regime (200K $< T < 500$K; see Table I). The arrows mark the crossovers at $T_*$ and $T_{cr}$. Note that $\rho_{aa}$ retains its linearity in $T$ above $T_{cr}$ with only minor change in slope.

FIG. 7. The reduced resistivity, $(\rho_{aa}(T) - \rho_0)/\alpha T$, in YBa$_2$Cu$_4$O$_8$ (Ref. 37) $\alpha$ and $\rho_0$ are determined as in Fig. 5. The bottom (top) set of data shows $\rho_{aa}$ ($\rho_{bb}$), where $\rho_{aa}$ ($\rho_{bb}$) is obtained with current running along $a$ ($b$) crystallographic directions. Inset: $\rho_{aa}$ and $\rho_{bb}$ in YBa$_2$Cu$_4$O$_8$ material. Note that at higher $T$ $\rho_{aa}$ $\sim$ $3\rho_{bb}$.

FIG. 8. $\rho_{xx}(T)$ in underdoped cuprates at low temperatures. The three sets of data are shifted by 1 for clarity and correspond to (top to bottom) YBa$_2$Cu$_3$O$_{6.68}$, YBa$_2$Cu$_4$O$_8$ and La$_{1.9}$Sr$_{0.1}$CuO$_4$ (Refs. 36, 37 and 46). The dashed lines are guides to the eye. The quantity plotted is $\rho_{xx}(T) - \rho_{xx}(0))/\beta T^2$, where $\rho_{xx}(0)$ and $\beta$ are obtained by fitting $\rho_{xx} = \rho_{xx}(0) + \beta T^2$ at low $T$. 

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FIG. 9. The inverse Hall conductivity as a function of temperature in 123 and YBa$_2$Cu$_4$O$_8$ compounds. The data correspond to the measured results obtained in (top to bottom) YBa$_2$Cu$_4$O$_8$, YBa$_2$Cu$_3$O$_{6.63}$ and YBa$_2$Cu$_3$O$_7$ (Refs. 37, 45 and 46). Note that at sufficiently high temperature all curves satisfy $T^3$ quite well. Inset: the result for YBa$_2$Cu$_3$O$_{6.63}$, showing that $\sigma_{xy} \propto T^{-4}$ at low $T$.

FIG. 10. The inverse Hall conductivity for the 214 compound. The data are taken from Ref. 36 and correspond to doping levels (top to the bottom) $x = 0.10, 0.15, \text{ and } 0.22$. The curves show large positive intercepts, suggesting a considerable amount of disorder and/or a smaller FS in this system.

FIG. 11. The hot (top) and cold (bottom) scattering rates in YBa$_2$Cu$_3$O$_{6.68}$ in dimensionless units, obtained by fitting the results of Ito et al.$^{46}$ for $\rho_{xx}$ and $\rho_{xy}$, shown in the previous figures, to the phenomenological formulae (38) and (41) (see text). The rates closely resemble the calculated scattering rates shown in Fig. 3. The arrow marks the crossover at $T^*$. 

FIG. 12. The scattering rate in the cold regions of YBa$_2$Cu$_3$O$_{6.68}$ obtained from experimental results of Ito et al.$^{46}$ Note that, to within an additive constant, this scattering rate is the same as that found in our perturbation theoretic calculations. The arrow marks the crossover at $T^*$, discussed in the text.

FIG. 13. The hot (top) and cold (bottom) scattering rates for YBa$_2$Cu$_4$O$_8$ obtained from the experimental results of Bucher et al.$^{37}$ There scattering rates resemble closely those found in YBa$_2$Cu$_3$O$_{6.68}$, as might be expected from the similarities in their magnetic behavior. The arrow marks the crossover at $T^*$. 

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FIG. 14. The hot (top) and cold (bottom) scattering rates for YBa$_{2}$Cu$_{3}$O$_{7}$ obtained from the experimental results of Ginsberg et al.$^{45}$ Note that $1/\tau_{\text{hot}}$ shows a crossover from the $\Delta k = 0$ result in Fig. 3 to the same result in Fig. 2, which should occur near $T_{cr}$, marked in the figure by an arrow. $1/\tau_{\text{hot}}$ weakly increases with temperature for $T \ll T_{cr}$ (see Fig. 2). $T_{*}$ is the crossover temperature equivalent to those shown in Figs. 11 and 13. Note that $1/\tau_{\text{cold}}$ is approximately quadratic in temperature, in agreement with our perturbation theory calculations.

FIG. 15. The hot (top) and cold (bottom) scattering rates for two 214 compounds obtained from the experimental results of Hwang et al.$^{36}$ The top (bottom) panel shows the results for $x = 0.10$ ($x = 0.15$) doping level. In the bottom panel the crossover temperature $T_{*}$ is clearly observed in $1/\tau_{\text{hot}}$, as expected in the NAFL model.

FIG. 16. The hot (top) and cold (bottom) scattering rates for a 15K 2201 Tl compound obtained from the experimental results of Mackenzie et al.$^{44}$ Note that the anisotropy of scattering is considerably reduced in overdoped materials, due to the smaller correlation length and the strong fermionic damping, manifested in a larger value of $\omega_{sf}$ (see text).

FIG. 17. A comparison of our calculated result for $\rho_{xx}(T)$ with the experimental result of Carrington et al.$^{20}$ The coupling constant $g \approx 0.48\text{eV}$ has been adjusted to yield the best fit. Note that in this case $T_{0}$, Eq. (32), is of order 20K and hence one observes a very small negative intercept of the resistivity, $\sim 20\mu\Omega\text{cm}$, as seen in experiment. Inset: the Hall resistivity as a function of temperature in comparison with experiment for the same system.

FIG. 18. The sensitivity of $\rho_{xx}(T)$ with respect to the band and incommensuration parameters. The solid line shows the same $\rho_{xx}(T)$ as in Fig. 17 with $g = 1\text{eV}$. The dashed and dotted lines show $\rho_{xx}(T)$ at a $n = 30\%$ doping level and zero incommensuration ($\Delta Q = 0$) respectively.
FIG. 19. The sensitivity of $\rho_{xx}(T)$ with respect to the changes in spin-fluctuation parameters. The curves correspond to (top to bottom) $\omega_{sf} = 20 + 0.2T(K)$, $60 + 0.2T(K)$, $20 + 0.6T(K)$ and $60 + 0.6T(K)$ respectively. All other parameters are assumed the same as in Fig. 18. Inset: $\sigma_{xy}(T)$ for the input parameters defined in the figure.

FIG. 20. (a) $\rho_{xx}(T)$ in the PG regime: the parameters used are: $\xi^{-1} = 0.1 + T/1000(K)$, $\omega_{sf}^\xi = 50\text{meV}$, $g = 1\text{eV}$ and $n = 15\%$. The solid, dashed and dotted lines correspond to $-t'/t = 0.2$, $0.3$ and $0.4$ respectively. The effective interaction is assumed to have incommensurate peaks at $Q + (\pm \Delta Q, 0)$ and $Q + (0, \pm \Delta Q)$, where $\Delta Q = 0.25\pi$. The three curves show a vast difference in the crossover temperature $T_0(T)$. (b) $\rho_{xx}(T)$ in comparison with experiment (Ref. 36). The solid line corresponds to $t' = -0.3t$, with $g$ adjusted to yield the same slope as seen experimentally. The dashed line corresponds to $t' = -0.25t$, with the FL scattering included (see text).

FIG. 21. Comparison of the calculated value of cot $\Theta_H$ with experiment (Ref. 36). The solid (dashed) line corresponds to the result depicted by the solid (dashed) line in Fig. 20b.

FIG. 22. A model FS for a system with strong a-b plane anisotropy. The anisotropy parameter is assumed to be $a = 0.5$ (see Eq. (49)), the doping level is $n \approx 15\%$.

FIG. 23. The resistivities along $a$ and $b$ crystallographic directions as a function of temperature for the system with the FS depicted in Fig. 22, using the spin fluctuation parameters appropriate to YBa$_2$Cu$_4$O$_8$ system. Note that there is not only a large quantitative difference, but a qualitative difference as well, as is seen in experiment. The anisotropy of the distribution of the hot regions leads to the anisotropy of the resistivities.

FIG. 24. The Hall conductivity as a function of temperature for the system depicted in Fig. 23. Note that, as usual, $\sigma_{xy}$ reflects an average contribution of the hot and cold regions and hence is proportional to $T^{-3}$ as in the isotropic case (see Fig. 17 and its inset).
\[
\left( \rho_{xx} - \rho_0 \right) / \alpha T
\]

(a)

(b)
\[ \frac{\rho_{aa} - \rho_{PS}}{\alpha_{PS}} \]
\( \frac{\rho - \rho_0}{\alpha T} \) vs. \( T \) (K)
\[ \frac{(\rho_{xx} - \rho_{xx}(0))}{\beta T^2} \]
$\frac{1}{\sigma_{xy}} (\mu\Omega\text{cm})$

$T^3 \left(10^6 K^3\right)$
$1/\sigma_{xy} \times (10 \mu\Omega \mathrm{cm})$

$T^3 \times (10^6 \mathrm{K}^3)$
\[ \frac{1}{\tau_{\text{cold}}} \]

\[ T^2 \left(10^4 \text{ K}^2\right) \]
$\sigma_{xy} \quad (\mu\Omega\text{cm})^{-1}$

$T \quad (\text{K})$