Delivery Latency Regions in Fog-RANs with Edge Caching and Cloud Processing

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Abstract—A Fog Radio Access Network (F-RAN) is a cellular wireless system that enables content delivery via edge caching, i.e., storage of popular content at the edge nodes (ENs), and cloud processing. The existing information-theoretic analyses of F-RAN systems, and special cases thereof, make the assumption that all files in the content library are equally relevant, and hence all users requests should be guaranteed the same delivery latency (i.e., transmission time). In practice, it is generally desirable to provide different latency levels as a function of the users’ requests, e.g., based on the current popularity ranking of the contents. This paper studies the problem of characterizing the region of delivery latencies for all possible users’ requests under a per-EN cache capacity constraint. For the case with two ENs and two users, the latency region is characterized in the high-SNR regime in terms of the Normalized Delivery Time (NDT) metric, introduced in prior work.

Index Terms—Edge caching, Cloud Radio Access Network, Fog Radio Access Network, Normalized Delivery Time.

I. INTRODUCTION

Fog networking is a novel paradigm that enables computing, storage and communication functions to be implemented at both cloud and edge nodes (ENs), such as base stations, of a wireless cellular system. Content delivery can benefit from fog networking via both edge caching, whereby popular content is stored at the ENs, and cloud processing, enabling delivery of content fetched from a central content library, see Fig. 1.

The information-theoretic analysis of edge caching in [1]–[4] and of more general fog-assisted wireless networks, or Fog Radio Access Networks (F-RANs), in [5], [6] focuses on the high-signal-to-noise ratio (SNR) regime and makes the important assumption that all files in the content library are equally popular. Under the latter assumption, all users’ requests should be guaranteed the same delivery latency (i.e., transmission time) and, by symmetry, all contents should be allocated the same fraction of the ENs’ caches. In practice, however, it is generally desirable to provide different latency levels as a function of the users’ requests, e.g., based on the current popularity ranking of the contents. This requires a thorough rethinking of the system operation, including cache partitions that vary depending on the file popularity.

In this paper, we study the fundamental problem of characterizing the region of delivery latencies for all possible users’ requests, while allowing for arbitrary cache partitions at the ENs across files. Knowledge of such region would provide a fine-grained understanding of the trade-off among the latencies of different requests. This would be instrumental to solve a number of problems of interest, including the minimization of the average latency over the ENs’ cache partitions across files for a given content popularity profile.

As in [5], [6], as well as in [3], [7], delivery latencies are measured here in the high-SNR regime with respect to a reference interference-free system, yielding the performance metric of Normalized Delivery Time (NDT). As a brief review, in [5], the minimum NDT was characterized for a system with two ENs and two users under the assumption that all users’ requests should be guaranteed the same latency. Reference [6] obtains upper and lower bounds that are within a multiplicative factor of 2 for any number of ENs and users.

Focusing on the special case with two ENs and two users as in [5], the main contributions of this work are as follows: (i) The performance metric of the NDT region is introduced with the aim of analyzing the set of latencies achievable for individual users’ requests under non-uniform cache partitions across files (Sec. II); (ii) Novel achievable schemes are presented that yield an inner bound to the NDT region (Sec. III); (iii) Outer bounds on the NDT region are derived that conclusively characterize the NDT region (Sec. IV); (iv) Numerical results are provided to corroborate the analysis (Sec. V).

II. SYSTEM MODEL AND PRELIMINARIES

A. Model

As illustrated in Fig. 1 we consider an F-RAN architecture with $M$ edge nodes (ENs), which serve $K$ users over a shared wireless channel. The users request files from a content library $\mathcal{F} = \{F_1, \ldots, F_J\}$ of $J \geq K$ files, where each file is of...
length $L$ bits. Each EN $m$ can cache at most $\mu J L$ bits from the library, where $0 \leq \mu \leq 1$ is referred to as the fractional cache capacity, and has a power constraint of $P$.

The channel from the ENs to the users is defined by the standard quasi-static model

$$Y_k = \sum_{m=1}^{M} H_{m,k} X_m + Z_k,$$

(1)

where $X_m \in \mathbb{C}^{n_x}$ is a codeword of length $n_x$ symbols transmitted by the EN $m$; $H_{m,k} \in \mathbb{C}$ is the channel coefficient from edge node $m$ to user $k$; $Z_k$ is complex Gaussian additive noise with unitary power, i.i.d. over time and users and also independent of the channel coefficients; and $Y_k \in \mathbb{C}^{n_x}$ is the received signal of length $n_x$ symbols by user $k$. Henceforth, we define as symbol a channel use of the wireless channel as done above, and we use the notation $[1 : K] = \{1, \ldots, K\}$. The channel coefficients are realizations of continuous random variables, and are i.i.d. over ENs and users. We let $\mathcal{H} = \{H_{m,k}\}_{m \in [1 : M], k \in [1 : K]}$ denote the channel state information (CSI), which is assumed to be known throughout the network, i.e., at the cloud, the ENs and the users.

The cloud has orthogonal fronthaul links to each of the ENs. Using the parametrization in [6], the capacity in bits per symbol of each fronthaul link is written as $C_f = r \log(P)$, where $r$ is defined as the fronthaul rate and $P$ is the (high) SNR of the wireless edge links. This parameter describes the ratio between the fronthaul capacity and the high-SNR capacity of each EN-to-user when used with no interference from other links.

As in prior works [1]-[6], the system operates in two separate phases, namely the (offline) caching phase and the (online) delivery phase. In the caching phase, for each file $j \in [1 : J]$, EN $m$ stores an arbitrary function

$$S_{m,j} = \pi_{m,j}^c(F_j)$$

(2)

of each file $F_j$. Unlike prior work, we let the entropy of the cached content for file $F_j$ at EN $m$, namely $H(S_{m,j}) = \mu_{m,j} L$, with $0 \leq \mu_{m,j} \leq 1$, to be arbitrary, as long as the cache partition $\{\mu_{m,j}\}_{j \in [1 : J]}$ satisfies the per-EN cache capacity constraint

$$\sum_{j=1}^{J} \mu_{m,j} \leq \mu J,$$

(3)

for each $m$. We will refer to $\pi^c = (\pi_{m,j}^c)_{m \in [1 : M], j \in [1 : J]}$ as the caching policy and to the matrix

$$\mu = (\mu_{m,j})_{m \in [1 : M], j \in [1 : J]}$$

(4)

as the cache partition matrix for the given caching policy.

The delivery phase consists of an arbitrary number of slots. In any slot, each user $k$ requests a file $F_{D_k}$ in $\mathcal{F}$ with index $D_k \in [1 : J]$. We let $D = \{D_1, \ldots, D_K\} \in \mathcal{D}$ denote the demand subset in any given slot. We emphasize that $D \subseteq [1 : J]$ is modeled as a subset of size $K$, and hence: (i) due to the statistical equivalence of the received signals $[1]$, of different users, we do not differentiate between permutations of the request vector $[D_1, \ldots, D_K]$; (ii) we assume that the requested files are distinct as, for instance, in [8]. As a result, the set $\mathcal{D}$ has cardinality $\binom{J}{K}$. Each slot consists of two subsequent subslots. In the first subslot, the cloud transmits information on the requested files to the ENs on the fronthaul links, while in the second subslot the ENs use the shared wireless channel to transmit to the users. To elaborate, in the first subslot, the cloud sends a message $U_m$ to the EN $m$ on the fronthaul as a function of the demand subset, the files and the channel state information, i.e.,

$$U_m = \pi_m^c(D, F, \mathcal{H})$$

(5)

The first subslot is of duration $n_f^1$ symbols, where we make explicit the dependence on the subset $D$, and the entropy of message $U_m$ must be bounded as $H(U_m) \leq C_f n_f^1$ in order to satisfy the fronthaul capacity constraints. We call $\pi^f = (\pi_{1}^f, \ldots, \pi_{M}^f)$ the fronthaul policy.

In the second subslot, the ENs transmit a codeword $X_m$, of $n_f^2$ symbols, on the wireless channel as a function of the users’ demand $D$, the cache content $S_m = \{S_{m,j}\}_{j \in [1 : J]}$ of EN $m$, the fronthaul message $U_m$ to EN $m$ and the global channel state information $\mathcal{H}$:

$$X_m = \pi_m^e(D, S_m, U_m, \mathcal{H}).$$

(6)

We call $\pi^e = (\pi_{1}^e, \ldots, \pi_{M}^e)$ the edge transmission policy. After receiving $Y_k$ in (1), user $k$ decodes the requested file as

$$\hat{F}_{D_k} = \pi_k^e(Y_k, D, \mathcal{H}),$$

(7)

and we let $\pi^d = (\pi_{1}^d, \ldots, \pi_{M}^d)$ denote the decoding policy. The error probability of a policy $\pi = (\pi^c, \pi^f, \pi^e, \pi^d)$ is defined as

$$P_e = \max_{D \in \mathcal{D}} \max_{k \in [1 : K]} P(\hat{F}_k \neq F_{D_k}).$$

(8)

A sequence of policies, parametrized by $L$ and $P$, is defined as feasible if it satisfies the limit $\lim_{P \to \infty} \lim_{L \to \infty} P_e = 0$.

B. Problem Statement

For any sequence of feasible policies $\pi$ parametrized by $L$ and $P$, we now define the high-SNR delivery time metric for each demand subset $D \subseteq \mathcal{D}$. To this end, we introduce the normalized durations of the first and second subslots in the given transmission interval as

$$\delta^x = \lim_{P \to \infty} \lim_{L \to \infty} \frac{n_f^D}{L \log(P)}$$

(9)

where $x = f$ for the first time slot and $x = e$ for the second. In [5], the subslot durations are normalized by the high-SNR delivery time of a reference system in which each user is served on an interference-free dedicated channel by an EN, namely $L / \log(P)$. We refer to $\delta_f^x$ and $\delta_e^x$ as the fronthaul and edge NDTs, respectively, for request $D$. The overall NDT for request $D$ is hence given by $\delta_D = \delta_f^D + \delta_e^D$.

We are interested in characterizing the region $\Delta^*(\mu, r)$ of all achievable NDT tuples $\delta = (\delta_D)_{D \in \mathcal{D}}$ under the per-EN capacity constraint [3], which we refer to as NDT region.
Lemma 1. The NDT regions \( \Delta^*(\mu, r) \) and \( \Delta^+(\mu, r) \) are convex.

A detailed proof of Lemma 1 is given in Appendix A.

We finally note that the minimum NDT introduced in \([6]\), corresponds to the minimum value \( \delta \) in the NDT region \( \Delta^*(\mu, r) \), with equal cache partition \( \mu_{m,j} = \mu \), such that the equality \( \delta = \delta_j \) holds for all request subsets \( D \). In the rest of this paper, we focus on the special case \( K = M = 2 \) and we write \( \delta_D = \delta_{ij} \) for any request subset \( D = \{i, j\} \).

C. Existing F-RAN Strategies

Here we review some of the existing achievable strategies for the F-RAN model as presented in \([6]\). These strategies will be used as elements of the proposed achievable strategy. 1) Hard-transfer fronthauling (HT): Via HT, the cloud delivers a fraction of one of the requested files to an EN and a fraction of the other to the other EN on the respective fronthaul links. 2) Zero-forcing beamforming (ZF): If both ENs have both requested messages, or a fraction thereof, available in the respective caches, cooperative ZF beamforming can be carried out on this fraction, yielding parallel interference-free channels to both users. 3) Soft-transfer fronthauling with zero-forcing beamforming (ST+ZF): The cloud implements ZF beamforming and transmits the resulting baseband signals to the ENs in quantized form. The ENs simply forward the quantized signals over the shared wireless channel \([9]\). 4) X-channel interference alignment (X-IA): If ENs cache different parts of each requested file, or a fraction thereof, the resulting channel model for the delivery for this fraction is an X-channel, for which interference alignment (IA) strategies were presented in \([10]\).

Lemma 2. The following fronthaul and edge NDTs are achievable.

HT: Let \( G_1 \) and \( G_2 \) be messages of \( \nu L \) bits that are available in the cloud. HT requires the fronthaul NDT \( \delta^f = \frac{\nu}{r} \) to transmit \( G_1 \) to \( EN_1 \) and \( G_2 \) to \( EN_2 \).

ZF: Let both ENs have messages \( G_1 \) and \( G_2 \) of \( \nu L \) bits available. ZF requires the edge NDT \( \delta^e = \nu \) to transmit \( G_1 \) to user 1 and \( G_2 \) to user 2.

ST+ZF: Let \( G_1 \) and \( G_2 \) be messages of \( \nu L \) bits that are available in the cloud. ST+ZF requires the fronthaul and edge NDTs \( \delta^f = \frac{\nu}{r} \) and \( \delta^e = \nu \) to transmit \( G_1 \) to user 1 and \( G_2 \) to user 2.

X-IA: Let \( G_{i,1} \) and \( G_{i,2} \) be messages of \( \nu L \) bits that are available at \( EN_i \), \( i = 1, 2 \). X-IA requires the edge NDT \( \delta^e = 3\nu \) to transmit \( G_{i,1} \) and \( G_{2,1} \) to user 1 and \( G_{i,2} \) to user 2.

III. Achievable NDT Region

In this section, we present achievable strategies that yield an inner bound on the NDT region \( \Delta^*(\mu, r) \). To this end, we consider policies with cache partitions \( \mu \) such that the two ENs cache the same number of bits for each file, i.e., \( \mu_{1,j} = \mu_{2,j} \).

Theorem 1. An inner bound on the NDT region is given by the inclusion

\[
\Delta^*(\mu, r) \supseteq \Delta^{(0)}(\mu, r) = \bigcup_{\mu: \mu_{i,j} = \mu_{2}, \mu_{1} = \mu_{i,j}} \Delta^{(0)}(\mu, r),
\]

where the region

\[
\Delta^{(0)}(\mu, r) = \{\delta_{ij} | \delta_{ij} \geq \delta^{(0)}_{ij}(\mu, r), \forall \{i, j\} \subseteq D\}
\]

is included in \( \Delta^*(\mu, r) \), and we have

\[
\delta^{(0)}_{ij}(\mu, r) = \begin{cases} 
\delta^{(0,1)}_{ij}(\mu, r), & \text{if } r \leq 1 \text{ and } \mu_i \leq \frac{1}{2} \text{ and } \mu_j \leq \frac{1}{2}, \\
\delta^{(0,2)}_{ij}(\mu, r), & \text{if } r \leq 1 \text{ and } (\mu_i \leq \frac{1}{2} \text{ or } \mu_j \leq \frac{1}{2}), \\
\delta^{(0,3)}_{ij}(\mu, r), & \text{if } r \leq 1 \text{ and } \mu_j \leq \frac{1}{2}, \\
\delta^{(0,4)}_{ij}(\mu, r), & \text{if } r > 1,
\end{cases}
\]

with the definitions

\[
\delta^{(0,1)}(\mu, r) = 1 + \frac{1}{r} - \frac{1}{r^2} \max\{\mu_i, \mu_j\},
\]

\[
\delta^{(0,2)}(\mu, r) = \frac{3}{2} \sum_{i \neq j} \min\{\mu_i, \mu_j\},
\]

\[
\delta^{(0,3)}(\mu, r) = 2 - \min\{\mu_i, \mu_j\},
\]

\[
\delta^{(0,4)}(\mu, r) = 1 + \frac{1}{r} - \frac{1}{r^2} \min\{\mu_i, \mu_j\}.
\]

In the remainder of this section, we present the achievable strategies that yield the inner bound in the previous theorem at an intuitive level. The detailed proof of Theorem 1 is given in Appendix B. To this end, we will present two different caching policies for the cases \( r \leq 1 \) and \( r > 1 \), while the fronthaul and edge transmission policies depend on both the demand subset \( D = \{i, j\} \) and on \( r \). In the following, we set \( \mu_i \leq \mu_j \) without loss of generality.

Caching policy for \( r \leq 1 \): As seen in Figures 2, 3 and 4, each file \( F_i \) is cached so that the bits indexed by \( 1, \ldots, \mu_i L \) are stored in \( EN_1 \) and bits \( (1 - \mu_i) L, \ldots, L \) are stored in \( EN_2 \), i.e., we minimize the overlap in the cached content in \( EN_1 \).
and EN2 by storing the first part of the file in EN1 and the last part of the file in EN2. If \( \mu_i \geq 1/2 \) some overlap will occur and some bits will be stored in both ENs.

**Caching policy for** \( r > 1 \): As seen in Figure 5, each file \( F_i \) is cached so that bits \( 1, \ldots, \mu_i L \) in both EN1 and EN2, i.e., we cache only the first part of the file and we maximize the overlap between the content that is cached in the ENs.

**Delivery strategy for** \( \mu_j < 1/2 \) and \( r \leq 1 \): This case is illustrated in Figure 2 and achieves \( \delta_{i,j}^{(\text{in},1)}(\mu, r) \). The delivery proceeds in three phases: a) we use HT to deliver bits \( \mu_i L, \ldots, (1 - \mu_j)L \) of file \( F_i \) to EN1 and EN2, respectively; b) we use X-IA to deliver complete files from the ENs to the users. We remark that this scenario is not relevant for the special case considered in [6]. Furthermore, we emphasize the important role played by HT, which is used here but not in [6], to derive an achievable strategy.

**Delivery strategy for** \( \mu_i > 1/2 \) and \( r \leq 1 \): This case is illustrated in Figure 4 and achieves \( \delta_{i,j}^{(\text{in},3)}(\mu, r) \). The delivery proceeds in two phases: a) we use X-IA to deliver bits \( 1, \ldots, (1 - \mu_i)L \) and \( \mu_i L, \ldots, L \) of file \( F_i \) to EN1 and EN2, respectively; and b) we use ZF to transmit bits \( (1 - \mu_i)L, \ldots, \mu_i L \) from the ENs to the users. Note that this strategy does not make use of the front haul during the delivery.

**Delivery strategy for** \( r > 1 \): The first \( \min\{\mu_i, \mu_j\} \) bits of both files, which are stored at both ENs, are delivered using ZF. The remaining \( (1 - \min\{\mu_i, \mu_j\}) \) bits are delivered using ST+ZF. The strategy is illustrated in Figure 5 and achieves \( \delta_{i,j}^{(\text{in},4)}(\mu, r) \).

### IV. Characterization of the NDT Region

In this section, we present an outer bound on the NDT region \( \Delta^*(\mu, r) \) and we demonstrate that the inner bound from the previous section is in fact tight, hence characterizing the NDT region. The first result of this section provides an outer bound on the achievable NDT tuple region for a fixed, and generic, cache partition \( \mu \).

**Theorem 2.** For any cache partition \( \mu \), we have the outer bound \( \Delta^*(\mu, r) \leq \Delta^{(\text{out})}(\mu, r) \), where

\[
\Delta^{(\text{out})}(\mu, r) = \left\{ \delta \mid \delta_{i,j} \geq \delta_{i,j}^{(\text{out})}(\mu, r), \forall \{i,j\} \in D \right\}, \quad (18)
\]

with

\[
\delta_{i,j}^{(\text{out})}(\mu, r) = \begin{cases} \max_{\ell=1,\ldots,3} \left\{ \delta_{i,j}^{(\text{out}, \ell)}(\mu, r) \right\}, & \text{if } r \leq 1, \\ \left\{ \delta_{i,j}^{(\text{out}, 4)}(\mu, r) \right\}, & \text{if } r > 1, \end{cases} \quad (19)
\]

and the definitions

\[
\delta_{i,j}^{(\text{out}, 1)}(\mu, r) = 1 + \frac{1}{r} - \min \{\mu_{1,i}, \mu_{2,i}, \mu_{1,j}, \mu_{2,j}\} - \frac{1}{2}\left(\frac{1}{r} - 1\right)(\mu_{1,i} + \mu_{2,i} + \mu_{1,j} + \mu_{2,j}), \quad (20)
\]
\[ \delta_{i,j}^{(1)}(\mu, r) = \frac{3}{2} + \frac{1}{2r} - \min \{ \mu_{1,i}, \mu_{2,i}, \mu_{1,j}, \mu_{2,j} \} \]
\[ - \frac{1}{2} \left( \frac{1}{r} - 1 \right) \min \{ \mu_{1,i} + \mu_{2,i}, \mu_{1,j}, \mu_{2,j} \} \] (21)
\[ \delta_{i,j}^{(2)}(\mu, r) = 2 - \min \{ \mu_{1,i}, \mu_{2,i}, \mu_{1,j}, \mu_{2,j} \} \] (22)
\[ \delta_{i,j}^{(3)}(\mu, r) = 1 + \frac{1}{r} - \frac{1}{r} \min \{ \mu_{1,i}, \mu_{2,i}, \mu_{1,j}, \mu_{2,j} \} \] (23)

The proof of Theorem 3 is given in Appendix C. Using the outer bound in the previous theorem, we now show that the inner bound from the previous section is tight. This result implies that, in order to exhaust the NDT region, it is sufficient to consider cache partitions in which \( \mu_{1,j} = \mu_{2,j} \) for all files \( j \in [1 : J] \).

**Theorem 3.** The NDT region is given as \( \Delta^*(\mu, r) = \Delta^{(in)}(\mu, r) \).

The proof of Theorem 3 is given in Appendix D.

**V. NUMERICAL EXAMPLE**

Consider a set-up in which the set of popular files is partitioned into two disjoint classes as \( \mathcal{F} = \mathcal{F}(1) \cup \mathcal{F}(2) \), where class \( \mathcal{F}(i) \) has \( J_i \) files. In this section, we illustrate and discuss a slice of the NDT region in which we impose that the same NDT \( \Delta(i,j) \) be achieved for all subsets \( D \) for which one file is in the class \( \mathcal{F}(i) \) and the other in \( \mathcal{F}(j) \). Note that this slice is three-dimensional, where given by \( \delta(1), \delta(2), \delta(1,2) \). To further reduce the dimensionality, we let \( \delta(2,2) \) be arbitrary, so as to focus only on the plane \( \delta(1),(2), \delta(2),(2) \). In order to evaluate the boundary of this slice of the NDT region, it can be argued that it is sufficient to consider cache partitions such as files within the same class, which we denote as \( \mu(1) \) and \( \mu(2) \) for the files of class 1 and 2, respectively. With this choice, the cache capacity constraint (3) reduces to \( J_1 \mu(1) + J_2 \mu(2) \leq \mu(J_1 + J_2) \).

The slice of the NDT region at hand is illustrated in Figure 6 for \( r = 1/5, J_1 = J_2 \) and various values of \( \mu \). The figure also indicates the values of the cache allocations \( (\mu(1), \mu(2)) \) that are required to obtain various points on the boundary of the region as well as the delivery strategy that should be used at various segments of the boundary. As it can be seen, the slice of the NDT region is a polyhedron and each linear portion of the boundary corresponds to a different delivery strategy as indicated in the figure (see Sec. III for a correspondence between strategies and NDT tuples \( \delta^{(in, t)} \)). For instance, it is seen that, for \( \mu = 3/8 \), one has to use a different strategy depending on the operating point: as \( \delta(1,2) \) increases, one needs to switch between the strategies that achieve the NDTs \( \delta^{(1)}, \delta^{(2)} \)

**VI. CONCLUSIONS**

This work characterized the set of delivery latencies supported by an F-RAN with two ENs and two users in the high SNR regime, when allowing for any cache partition across the files in a set of popular contents. Various aspects call for further investigation, including the explicit minimization of the average delivery latency as a function of the content popularity profile, the extension of the main results to any number of ENs and users (see [6] for the case of uniform file popularity) and the derivation of an extended NDT region in which the same contents may be requested by multiple users.

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**APPENDIX A**

**PROOF OF LEMMA 1**

The proof is based on cache sharing and file splitting, similar to [6].
Let \( \delta \) and \( \delta' \) be NDT tuples that are achievable with policies \( \pi \) and \( \pi' \) and cache partitions \( \mu \) and \( \mu' \), respectively, that satisfy (3). Moreover, let \( 0 < \alpha < 1 \) be arbitrary. We will prove that the region \( \Delta^\pi(\mu, r) \) is convex by demonstrating that the NDT tuple \( \alpha \delta + (1 - \alpha) \delta' \) (with entry-wise sum) is achievable for a cache partition \( \alpha \mu + (1 - \alpha) \mu' \), which also satisfies (5).

We start by splitting each of the files in \( F \) in two parts of sizes \( \alpha L \) and \( (1 - \alpha) L \) bits. Moreover, we split the cache at each EN in two parts of sizes \( \alpha \mu_j L \) bits and \( (1 - \alpha) \mu_j L \) bits. Now, to achieve the NDT tuple \( \alpha \delta + (1 - \alpha) \delta' \) we transmit the first fraction of the pair of requested files using policy \( \pi \) and the first part of the caches and then transmit the second part of the files using \( \pi' \) and the second part of the caches. This policy uses cache partition \( \alpha \mu + (1 - \alpha) \mu' \) and hence satisfies the cache capacity constraints (3). Furthermore, it achieves the desired NDT tuple \( \alpha \delta + (1 - \alpha) \delta' \), since the NDT is proportional to the file size.

It remains to be shown that for any \( \mu \), \( \Delta^\pi(\mu, r) \) is convex. This follows directly by taking in the above argument \( \mu' = \mu \) and observing that the overall cache allocation in the two phase policy achieving \( \alpha \delta + (1 - \alpha) \delta' \) is again equal to \( \mu \).

**APPENDIX B**

**Proof of Theorem 1**

We analyze the fronthaul and edge NDTs using Lemma 2. W.l.o.g. we consider \( \mu_i \leq \mu_j \). We focus on the case \( r = 1 \) since the result for the case \( r > 1 \) is an immediate consequence of Lemma 2.

First, for \( \mu_i < \frac{1}{2} \) and \( \mu_j < \frac{1}{2} \), the HT delivery of bits \( \mu_i L, \ldots, \mu_j L \) and \( (1 - \mu_i) L, \ldots, (1 - \mu_j) L \) of file \( F_i \) to EN1 and EN2, respectively, is done over parallel channels to EN1 and EN2 and takes a fronthaul NDT \( \delta_f' = \frac{1}{r}(\mu_j - \mu_i) \) by Lemma 2. In the X-I A phase we deliver four parts, two of each file, of \( \mu_j L \) bits, and hence the edge NDT is \( \delta_e = 3\mu_j \). The fronthaul and edge NDTs to deliver the remaining \( (1 - 2\mu_j) L \) bits of both files through ST+ZF are \( \delta_f' = \frac{1}{r}(1 - 2\mu_j) \) and \( \delta_e = 1 - 2\mu_j \), respectively. Summing all the NDT terms gives the result.

Second, for \( \mu_i \leq \frac{1}{2} \) and \( \mu_j \geq \frac{1}{2} \), the HT transmission of \( (1/2 - \mu_i) L \) bits to each EN requires a fronthaul NDT \( \delta_f' = \frac{1}{r}(1/2 - \mu_i) \). The X-I A phase, in which messages of \( L/2 \) bits are transmitted, instead takes an edge NDT of \( \delta_e = 3/2 \).

Third, for \( \mu_i > \frac{1}{2} \) and \( \mu_j > \frac{1}{2} \), the X-I A phase delivers four messages of \( (1 - \mu_i) L \) bits using an edge NDT \( \delta_e = 3(1 - \mu_i) \). The ZF phase instead delivers \( (2\mu_i - 1) L \) bits of each file and requires an edge NDT \( \delta_e = 2\mu_i - 1 \).

**APPENDIX C**

**Proof of Theorem 2**

In this appendix, we let \( \epsilon_P \) denote any function such that \( \epsilon_P / \log(P) \to 0 \) as \( P \to \infty \) and let \( \epsilon_L \) denote any function such that \( \epsilon_L \to 0 \) as \( L \to \infty \). Furthermore, we drop the dependence of \( n_{EL}^i \) and \( n_{EL}^P \) on \( D \) in order to streamline the notation.

We start with two technical results that appear in [6].

**Lemma 3** ([6], Lemma 6). For \( k = 1, 2 \) we have

\[
H(F_i,F_j | Y_1,S_k,F_{[1:j]\setminus(j)} ) \leq L \epsilon_L + n^c e_P .
\]

**Lemma 4** ([6], Lemma 5). For \( k = 1, 2 \) we have

\[
I(F_i,F_j ; Y_k | F_{[1:j]\setminus(j)}) \leq n^c \log P + n^c e_P .
\]

Note that Lemma 4 does not appear in this form in [6], but it follows directly from Lemma 5 in [6].

We will now develop several bounds on linear combinations of the edge NDT \( \delta_e^i \) and fronthaul NDT \( \delta_f^i \) for any sequence of feasible schemes. These bounds will be used to construct a bound on the NDT \( \delta_e^i + \delta_f^i \). The following three lemmas provide such bounds.

**Lemma 5.** Let \( i,j \in [1:J] \). Then, any sequence of achievable strategies satisfies the inequality

\[
\delta_e^i + \epsilon_f^i \geq 2 \min\{\mu_{1,i}, \mu_{2,i}, \mu_{1,j}, \mu_{2,j}\}.
\]

Proof: W.l.o.g. we prove the inequality \( \delta_e^i + \epsilon_f^i \geq 2 - \mu_{m,j} \), \( m = 1, 2 \). First, we can write

\[
2L = H(F_i,F_j | F_{[1:j]\setminus(i,j)}) + I(F_i,F_j ; Y_1,S_m,U_m | F_{[1:j]\setminus(i,j)}) + H(S_m | F_{[1:j]\setminus(i,j)}) \leq I(F_i,F_j ; Y_1,S_m,U_m | F_{[1:j]\setminus(i,j)}) + H(S_m | F_{[1:j]\setminus(i,j)}) \leq n^c \log P + n^c \epsilon_P + L \epsilon_L .
\]

where the equality follows from the independence of files and the inequality follows from Lemma 3 and the fact that conditioning on \( U_m \) reduces entropy. Next, we write

\[
I(F_i,F_j ; Y_1,S_m,U_m | F_{[1:j]\setminus(i,j)}) \leq I(F_i,F_j ; Y_1,S_m,U_m | F_{[1:j]\setminus(i,j)}) + I(F_i,F_j ; F_{[1:j]\setminus(i,j)}, Y_1) + H(S_m | F_{[1:j]\setminus(i,j)}) \leq n^c \log P + n^c \epsilon_P + L \epsilon_L
\]

where: (32) follows by bounding the first term in (31) using Lemma 4 and the second term in (31) using Fano’s inequality; (32) follows from \( H(U_m) \leq \frac{n^c}{4} \log P \) by the fronthaul capacity constraints; and (33) follows from the definition of \( \mu_{m,j} \).

Combining (32) and (34) and rewriting in terms of \( \delta_e^i \) and \( \delta_f^i \) gives

\[
\delta_e^i \left( 1 + \frac{\epsilon_P}{\log(P)} \right) + \delta_f^i \leq 2 - \mu_{m,j} - \epsilon_L .
\]

and the result follows by taking \( L \to \infty \) and \( P \to \infty \).
Lemma 6. Let $i,j \in [1:J]$. Then, any sequence of achievable strategy satisfies the inequality

$$2\delta^f_{i,j} \geq 1 - \min\{\mu_{1,i} + \mu_{2,i}, \mu_{1,j} + \mu_{2,j}\}. \quad (36)$$

Proof: W.l.o.g. we prove the inequality $2\delta^f_{i,j} \geq 1 - \mu_{1,i} - \mu_{2,j}$. We have

$$L = I(F_i; S_1, U_1, S_2, U_2 | F_{[1:J] \setminus \{i,j\}}) + H(F_i | S_1, U_1, S_2, U_2, F_{[1:J] \setminus \{i,j\}}) \leq H(S_1, S_2 | F_{[1:J] \setminus \{i,j\}}) + H(U_1 | F_{[1:J] \setminus \{i,j\}}) + H(U_2 | F_{[1:J] \setminus \{i,j\}}) + L \epsilon_L \leq \mu_{1,i} + \mu_{2,i} + 2r n^f \log P + L \epsilon_L, \quad (37)$$

where the first inequality follows from the equality $H(S_1, U_1^T, S_2, U_2^T | F_{[1:J]}) = 0$ and from Fano’s inequality, since file $F_i$ can be recovered from $S_1, U_1^T, S_2, U_2^T$ given that the input signals are functions of these variables. The second inequality follows from the frontier rate constraint. The result follows by taking the limits $L \to \infty$ and $P \to \infty$.

Lemma 7. Let $i,j \in [1:J]$. Then, any sequence of achievable strategy satisfies the inequality

$$2\delta^f \geq 2 - \mu_{1,i} - \mu_{2,i} - \mu_{1,j} - \mu_{2,j}. \quad (38)$$

Proof: We have

$$2L = I(F_i, F_j; S_1, U_1, S_2, U_2 | F_{[1:J] \setminus \{i,j\}}) + H(F_i, F_j | S_1, U_1, S_2, U_2, F_{[1:J] \setminus \{i,j\}}) \leq H(S_1, S_2 | F_{[1:J] \setminus \{i,j\}}) + H(U_1 | F_{[1:J] \setminus \{i,j\}}) + H(U_2 | F_{[1:J] \setminus \{i,j\}}) + L \epsilon_L \leq \mu_{1,i} + \mu_{2,i} + \mu_{1,j} + \mu_{2,j} + 2r n^f \log P + L \epsilon_L, \quad (39)$$

where the first inequality follows from the equality $H(S_1, U_1^T, S_2, U_2^T | F_{[1:J]}) = 0$ and from Fano’s inequality, since the files $F_i$ and $F_j$ can be recovered from the variables $S_1, U_1, S_2, U_2$ as discussed above.

We now summarize the constraints of Lemmas 5, 6 as

$$\delta^e_{i,j} + r \delta^f_{i,j} \geq 2 - \min\{\mu_{1,i} + \mu_{2,i}, \mu_{1,j} + \mu_{2,j}\}, \quad (40)$$

and we add

$$\delta^e_{i,j} \geq 1, \quad \delta^f_{i,j} \geq 0, \quad (41)$$

with weights 1 and $(1/r - 1)$, respectively, we obtain

$$[\delta^e_{i,j} + r \delta^f_{i,j}] + \left(\frac{1}{r} - 1\right) \left[r \delta^f_{i,j}\right] \geq [2 - \min\{\mu_{1,i} + \mu_{2,i}, \mu_{1,j} + \mu_{2,j}\}] + \left(\frac{1}{r} - 1\right) \left[1 - \frac{\mu_{1,i} + \mu_{2,i} + \mu_{1,j} + \mu_{2,j}}{2}\right], \quad (42)$$

from which it follows that $\delta_{i,j} \geq \delta_{i,j}(\text{out}^1)$. Next, for $r \leq 1$, taking $[44]$ and $[45]$ with weights 1 and $(1/r - 1)$, respectively, gives

$$[\delta^e_{i,j} + r \delta^f_{i,j}] + \left(\frac{1}{r} - 1\right) \left[r \delta^f_{i,j}\right] \geq [2 - \min\{\mu_{1,i} + \mu_{2,i}, \mu_{1,j} + \mu_{2,j}\}] + \left(\frac{1}{r} - 1\right) \left[1 - \frac{\mu_{1,i} + \mu_{2,i}}{2}\right], \quad (43)$$

which is equivalent to $\delta_{i,j} \geq \delta_{i,j}(\text{out}^2)$. Continuing with $r \leq 1$, the inequality $\delta_{i,j} \geq \delta_{i,j}(\text{out}^3)$ follows from

$$[\delta^e_{i,j} + r \delta^f_{i,j}] + \left(\frac{1}{r} - 1\right) \left[r \delta^f_{i,j}\right] \geq [2 - \min\{\mu_{1,i} + \mu_{2,i}, \mu_{1,j} + \mu_{2,j}\}] + \left(\frac{1}{r} - 1\right) \cdot 0, \quad (44)$$

which is obtained by taking [44] and [46] with weights 1 and $(1/r - 1)$, respectively.

Finally, for $r > 1$, by taking [44] and [47] with weights 1 and $(1 - 1/r)$, respectively, we obtain

$$\frac{1}{r} \left[\delta^e_{i,j} + r \delta^f_{i,j}\right] + \left(1 - \frac{1}{r}\right) \left[\delta^e_{i,j}\right] \geq \frac{1}{r} [2 - \min\{\mu_{1,i} + \mu_{2,i}, \mu_{1,j} + \mu_{2,j}\}] + \left(1 - \frac{1}{r}\right) \cdot 1, \quad (45)$$

from which it follows that $\delta_{i,j} \geq \delta_{i,j}(\text{out}^4)$.

Appendix D

Proof of Theorem 3

To prove Theorem 3, we need to show that the NDT region $\Delta^*(\mu, r)$ is given as

$$\Delta^*(\mu, r) = \bigcup_{\mu \in \mathcal{U}} \Delta^\text{(in)}(\mu, r), \quad (46)$$

where we have introduced the set

$$\mathcal{U} = \left\{ \mu \mid \forall i \in [1:J] : \mu_{1,i} = \mu_{2,i} = \mu_i, \text{ and } \sum_{j=1}^J \mu_j \leq \mu \right\} \quad (47)$$

for convenience of notation. The proof consists of two parts. 1) We demonstrate that for any $\tilde{\mu} \notin \mathcal{U}$ such that [43] holds, there exists a $\tilde{\mu} \in \mathcal{U}$ for which

$$\Delta^*(\tilde{\mu}, r) \subset \Delta^*(\tilde{\mu}, r) \quad (48)$$
and (3) hold. This allows us to restrict the union in (10) with no loss of optimality to set $\mathcal{U}$ as

$$\Delta^*(\mu, r) = \bigcup_{\mu \in \mathcal{U}} \Delta^*(\mu, r).$$

(56)

2) We show that, for any $\mu \in \mathcal{U}$, we have

$$\Delta^*(\mu, r) = \Delta^{(\text{in})}(\mu, r) = \Delta^{(\text{out})}(\mu, r),$$

(57)

which reduces (56) to (53), hence concluding the proof. Details are provided next.

1) To prove (55), we start by constructing the mentioned cache partition $\tilde{\mu}$ from $\tilde{\mu} \not\in \mathcal{U}$ as

$$\tilde{\mu}_i = \hat{\mu}_{1,i} = \hat{\mu}_{2,i} = \frac{\hat{\mu}_{1,i} + \hat{\mu}_{2,i}}{2}.$$  

(58)

The choice $\tilde{\mu}$ satisfies the capacity constraint (3), since we have

$$\sum_{i=1}^{J} \tilde{\mu}_{m,i} = \frac{1}{2} \left( \sum_{i=1}^{J} \hat{\mu}_{1,i} + \sum_{i=1}^{J} \hat{\mu}_{2,i} \right) \leq J \mu,$$

(59)

where the inequality holds because of the constraints $\sum_{i=1}^{J} \tilde{\mu}_{1,i} \leq J \mu$ and $\sum_{i=1}^{J} \tilde{\mu}_{2,i} \leq J \mu$. We now argue that, for an arbitrary $(i,j) \in \mathcal{D}$, the achievable NDT under $\tilde{\mu}$ by Theorem 1 is strictly smaller than the lower bound on the NDT under $\hat{\mu}$ obtained in Theorem 2, i.e.,

$$\delta^{(\text{in})}_{i,j}(\hat{\mu}, r) < \delta^{(\text{out})}_{i,j}(\tilde{\mu}, r).$$

(60)

This would conclude the proof of part 1). To this end, we leverage the following inequalities: for all $\ell \in [1:4]$, we have

$$\delta^{(\text{out}, \ell)}_{i,j}(\tilde{\mu}, r) \leq \delta^{(\text{out}, \ell)}_{i,j}(\hat{\mu}, r),$$

(61)

with equality if and only if $\tilde{\mu}_{1,i} = \tilde{\mu}_{2,i} = \hat{\mu}_i$ and $\tilde{\mu}_{1,j} = \tilde{\mu}_{2,j} = \hat{\mu}_j$; and

$$\delta^{(\text{in}, \ell)}_{i,j}(\mu, r) = \delta^{(\text{out}, \ell)}_{i,j}(\mu, r),$$

(62)

for all $\mu \in \mathcal{U}$. The proofs of (61) and (62) are deferred to the end of this appendix. Now, w.l.o.g. assume that $\delta^{(\text{in})}_{i,j}(\hat{\mu}, r) = \delta^{(\text{in})}_{i,j}(\hat{\tilde{\mu}}, r)$. Then, we can write

$$\delta^{(\text{in})}_{i,j}(\hat{\mu}, r) = \delta^{(\text{in}, \ell)}_{i,j}(\hat{\mu}, r) = \delta^{(\text{out}, \ell)}_{i,j}(\hat{\mu}, r) < \delta^{(\text{out}, \ell)}_{i,j}(\hat{\tilde{\mu}}, r) \leq \delta^{(\text{out})}_{i,j}(\hat{\tilde{\mu}}, r),$$

(63)

where the second equality follows from (62), the first inequality from (62) and the second inequality from the definition of $\delta^{(\text{out})}_{i,j}(\hat{\tilde{\mu}}, r)$. This proves (55).

2) Equation (57) follows directly from (62), since the latter equality shows that any point on the boundary of the region $\Delta^{(\text{out})}(\hat{\mu}, r)$ can be achieved with the scheme proposed in Theorem 1.

Proof of (61) and (62): Observe that, for all $\ell \in [1 : 4]$, we have

$$\delta^{(\text{out}, \ell)}_{i,j}(\hat{\mu}, r) - \delta^{(\text{out}, \ell)}_{i,j}(\tilde{\mu}, r)$$

$$= \min \{ \hat{\mu}_{1,i} + \hat{\mu}_{2,i} \} - \min \{ \tilde{\mu}_{1,i} + \tilde{\mu}_{2,i} \}$$

$$= \min \left\{ \frac{\hat{\mu}_{1,i} + \hat{\mu}_{2,i}}{2}, \frac{\tilde{\mu}_{1,i} + \tilde{\mu}_{2,i}}{2} \right\}$$

$$- \min \{ \min \{ \tilde{\mu}_{1,i}, \tilde{\mu}_{2,i} \}, \min \{ \hat{\mu}_{1,i}, \hat{\mu}_{2,i} \} \}.$$  

(64)

Now, (61) follows from the inequality

$$\frac{\hat{\mu}_{1,k} + \hat{\mu}_{2,k}}{2} \geq \min \{ \tilde{\mu}_{1,k}, \tilde{\mu}_{2,k} \},$$

(65)

for $k = 1, 2$, which holds with equality if and only if $\hat{\mu}_{1,k} = \hat{\mu}_{2,k}$.

Finally, for (62), with $\mu \in \mathcal{C}$, we have

$$\delta^{(\text{in}, \ell)}_{i,j}(\mu, r) = 1 + \frac{1}{r} - \min \{ \hat{\mu}_i, \hat{\mu}_j \}$$

$$- \left( \frac{1}{r} - 1 \right) \left( \min \{ \hat{\mu}_i, \hat{\mu}_j \} + \max \{ \hat{\mu}_i, \hat{\mu}_j \} \right) = \delta^{(\text{in}, \ell)}_{i,j}(\mu, r).$$

(66)

In a similar fashion, it follows that $\delta^{(\text{in}, \ell)}_{i,j}(\mu, r) = \delta^{(\text{out}, \ell)}_{i,j}(\mu, r)$ for all $\ell \in [1 : 4]$. 