ELECTROWEAK THEORY CONFRONTING PRECISION DATA\[^{†}\]

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ABSTRACT
We review the empirical evidence for the validity of the Standard Electroweak Theory in nature. The experimental data are interpreted in terms of an effective Lagrangian for Z physics, allowing for potential sources of SU(2) violation and containing the predictions of the Standard Electroweak Theory as a special case. Particular emphasis is put on discriminating loop corrections due to fermion-loop vector-boson propagator corrections on the one hand, from corrections depending on the non-Abelian structure and the Higgs sector on the other hand. Results from recently obtained fits of the Higgs-boson mass are reported, yielding $M_H \lesssim 550$ GeV [800 GeV] at 95% C.L. based on the input of $\bar{s}_W^2$ (LEP + SLD) = 0.23165 ± 0.00024 [$\bar{s}_W^2$ (LEP) = 0.23200 ± 0.00027]. With respect to LEP2, it is emphasized that first direct experimental evidence for non-zero non-Abelian couplings among the vector bosons can be obtained even with restricted $e^+e^-$ luminosity.

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1 Z Physics

The spirit in which I will look at the electroweak precision data may be characterized by quoting Feynman who once said:

"In any event, it is always a good idea to try to see how much or how little of our theoretical knowledge actually goes into the analysis of those situations which have been experimentally checked."

R.P. Feynman

1.1 The \(\alpha(0)\)-Born Prediction

The quality of the data on electroweak interactions may be particularly well appreciated by starting with an analysis in terms of the Born approximation of the Standard Electroweak Theory (Standard Model, SM). From the input of

\[
\alpha(0)^{-1} = 137.0359895(61),
G_\mu = 1.16639(2) \cdot 10^{-5}\text{GeV}^{-2},
M_Z = 91.1863 \pm 0.0020\text{GeV},
\]

one may predict the partial width of the Z for decay into leptons, \(\Gamma_l\), the weak mixing angle, \(s^2_W\), and the W mass, \(M_W\). The Born approximation, more precisely the \(\alpha(0)\)-Born approximation, in distinction from the \(\alpha(M_Z^2)\)-Born approximation to be introduced below,

\[
\bar{s}_W^2 (1 - \bar{s}_W^2) = \frac{\pi \alpha(0)}{\sqrt{2} G_\mu M_Z^2},
\]

\[
\Gamma_1 = \frac{G_\mu M_Z^3}{24 \pi \sqrt{2}} \left(1 + (1 - 4\bar{s}_W^2)^2\right),
M_W^2 = M_Z^2 (1 - \bar{s}_W^2),
\]

then yields

\[
\bar{s}_W^2 = 0.2121,
\Gamma_1 = 84.85\text{ MeV},
M_W = 80.940\text{ GeV}.
\]

A comparison with the experimental data from tab. 1,

\[
\bar{s}_W^2(\text{LEP + SLD}) = 0.23165 \pm 0.00024,
\Gamma_1 = 83.91 \pm 0.11\text{ MeV},
M_W = 80.356 \pm 0.125\text{ GeV},
\]

shows discrepancies between the \(\alpha(0)\)-Born approximation and the data by many standard deviations.

\footnote{Compare the lecture by Martin Pohl, these Proceedings.}
Table 1: The 1996 precision data, consisting of the LEP data, the SLD value, and the world average for $\bar{s}_W^2$. The partial widths $\Gamma_1, \Gamma_h, \Gamma_b,$ and $\Gamma_c$ are obtained from the observables $R = \Gamma_h/\Gamma_1,$ $\sigma_h = (12\pi\Gamma_1\Gamma_h)/(M_Z^2\Gamma_T^2),$ $R_b = \Gamma_b/\Gamma_h,$ $R_c = \Gamma_c/\Gamma_h,$ and $\Gamma_T$ using the given correlation matrices. The data in the upper left-hand column will be referred to as “leptonic sector” subsequently. Inclusion of the data in the upper right-hand column will be referred to as “all data”. If not stated otherwise, the theoretical predictions will be based on the input parameters given in the lower left-hand column of the table, where $\alpha(\mu^2)$ is taken from Ref. 7, $\alpha_s(\mu^2)$ results from the event-shape analysis at LEP, and $m_t$ represents the direct Tevatron measurement.

| leptonic sector | hadronic sector |
|-----------------|-----------------|
| $\Gamma_1 = 83.91 \pm 0.11$ MeV | $R = 20.778 \pm 0.029$ |
| $\bar{s}_W^2|_{\text{LEP}} = 0.23200 \pm 0.00027$ | $\Gamma_T = 2494.6 \pm 2.7$ MeV |
| $\bar{s}_W^2|_{\text{SLD}} = 0.23061 \pm 0.00047$ | $\sigma_h = 41.508 \pm 0.056$ |
| $\bar{s}_W^2|_{\text{LEP+SLD}} = 0.23165 \pm 0.00024$ | $\Gamma_h = 1743.6 \pm 2.5$ MeV |
| $M_W = 80.356 \pm 0.125$ GeV | $R_b = 0.2179 \pm 0.0012$ |
| $M_Z = 91.1863 \pm 0.0020$ GeV | $\Gamma_b = 379.9 \pm 2.2$ MeV |
| $G_\mu = 1.16639(2) \cdot 10^{-5}$ GeV$^{-2}$ | $R_c = 0.1715 \pm 0.0056$ |
| $\alpha(M_Z^2)^{-1} = 128.89 \pm 0.09$ | $\Gamma_c = 299.0 \pm 9.8$ MeV |
| $\alpha_s(M_Z^2) = 0.123 \pm 0.006$ | |
| $m_h = 4.7$ GeV | |
| $m_t = 175 \pm 6$ GeV | |

### 1.2 The $\alpha(M_Z^2)$-Born, the Full Fermion-Loop and the Complete One-Loop Standard Model Predictions

Turning to corrections to the $\alpha(0)$-Born approximation, I follow the 1988 strategy “to isolate and to test directly the ‘new physics’ of boson loops and other new phenomena by comparing with and looking for deviations from the predictions of the dominant-fermion-loop results”. Accordingly, let us strictly discriminate vacuum-polarization contributions due to fermion loops in the photon, $Z$ and $W$ propagators from all other loop corrections, the “bosonic” loops, which contain virtual...
vector bosons within the loops. I note that this distinction between two classes of loop corrections is gauge invariant in the $SU(2)_L \times U(1)_Y$ electroweak theory. Otherwise the theory would fix the number of fermion families. The reason for systematically discriminating fermion loops in the propagators from the rest is in fact obvious. The fermion-loop effects, leading to “running” of coupling constants and to mixing among the neutral vector bosons, can be precisely predicted from the \textit{empirically known couplings} of the leptons and the (light) quarks, while other loop effects, such as vacuum polarization due to boson pairs and vertex corrections, depend on the \textit{empirically unknown couplings} among the vector bosons and the properties of the Higgs scalar. It is in fact the difference between the fermion-loop predictions and the full one-loop results which sets the scale \cite{10} for the precision required for empirical tests of the electroweak theory beyond (trivial) fermion-loop effects. One should remind oneself that the experimentally unknown bosonic interactions are right at the heart of the celebrated renormalizability properties \cite{10} of the electroweak non-Abelian gauge theory \cite{3}.

When considering fermion loops, let us first of all look at the contributions of leptons and quarks to the photon propagator. Vacuum polarization due to leptons and quarks, or rather hadrons in the latter case, leads to the well-known increase (“running”) of the electromagnetic coupling as a function of the scale, at which it is measured. While the contribution of the leptons can be calculated in a straightforward manner, the contributions due to quarks are more reliably obtained from the cross section for $e^+e^-$ annihilation into hadrons via a dispersion relation \cite{7,17}. As a consequence of the experimental errors in the cross section for $e^+e^-$ annihilation, in particular in the region below about 3.5 GeV, the value of the electromagnetic fine-structure constant at the $Z$ scale, relevant for LEP1 physics, contains a non-negligible error,

$$
\alpha(M_Z^2)^{-1} = 128.89 \pm 0.09.
$$

Replacing $\alpha(0)$ in Eq. 2 by $\alpha(M_Z^2)$ implies replacing the electroweak mixing angle in Eq. 3 by $s_0^2$,

$$
s_0^2(1 - s_0^2) = \frac{\pi \alpha(M_Z^2)}{\sqrt{2}G_M M_Z^2},
$$

which may be expected to be a more appropriate parameter for electroweak physics at the $Z$-boson scale than the mixing angle from the $\alpha(0)$-Born approximation of Eq. 3. As the transition from $\alpha(0)$ to $\alpha(M_Z^2)$ is an effect purely due to the electromagnetic interactions of leptons and quarks (hadrons), even present in the absence of weak interactions, the relations in Eq. 4 with the replacement $s_W^2 \rightarrow s_0^2$ from Eq. 3 may appropriately be called the “$\alpha(M_Z^2)$-Born approximation” \cite{18} of the electroweak theory.

Numerically, one finds

$$
s_0^2 = 0.23112 \pm 0.00023,
$$
Figure 1: Three-dimensional plot of the 1σ ellipsoid of the experimental data in $(M_{W^±}/M_Z, s_W^2, \Gamma_1)$-space, using $s_W^2$ (LEP + SLD) as experimental input for $s_W^2$, in comparison with the full SM prediction (connected lines) and the pure fermion-loop prediction (single line with cubes). The full SM prediction is shown for Higgs-boson masses of $M_H = 100$ GeV (line with diamonds), 300 GeV, and 1 TeV parametrized by $m_t$ ranging from 120–220 GeV in steps of 20 GeV. In the pure fermion-loop prediction the cubes also indicate steps in $m_t$ of 20 GeV starting with $m_t = 120$ GeV. The cross outside the ellipsoid indicates the $\alpha(M_Z^2)$-Born approximation with the corresponding error bars, which also apply to all other theoretical predictions (1996 update from Ref. [5]). Note that in the projections on the planes also the 2σ contours are shown.
\[ \Gamma_l^{(0)} = 83.563 \pm 0.012 \text{ MeV}, \]
\[ M_W^{(0)} = 79.958 \pm 0.011 \text{ GeV}, \]  

(7)
i.e. a large part of the above discrepancy between the predictions in Eq. 3 and the data in Eq. 4 is due to the use of the inappropriate value of \( \alpha(0) \), instead of \( \alpha(M_Z^2) \), as appropriate for Z physics. Note that the uncertainty in \( s_W^2 \), as a consequence of the error in \( \alpha(M_Z^2) \), is as large as the error of \( s_W^2 \) from the measurements at the Z resonance (compare Eq. 4 or tab. 1).

All other fermion-loop effects are due to fermion loops in the W propagator (relevant since \( G_\mu \) enters the predictions) and in the Z propagator, and due to the important effect of \( \gamma Z \) mixing induced by fermions. Light fermions as well as the top quark accordingly yield important contributions to the “full fermion-loop prediction” which includes all fermion-loop propagator corrections.

In fig. 1, an update of a figure in Ref. 15, we show the experimental data from the “leptonic sector”, \( s_W^2, \Gamma_l, M_W \), in comparison with the \( \alpha(M_Z^2) \)-Born approximation, the full fermion-loop prediction, and the complete one-loop Standard Model results.

We conclude that:

- contributions beyond the \( \alpha(M_Z^2) \)-Born approximation are needed for agreement with the data,
- contributions beyond the full fermion-loop predictions, based on \( \alpha(M_Z^2) \), the fermion-loop contributions to the W and Z propagators and to \( \gamma Z \) mixing, and the top quark effects, are necessary, and provided
- by additional contributions involving bosonic loops, dependent on the non-Abelian couplings and the properties of the Higgs boson.

The question immediately arises of what can be said in more detail about the various contributions due to fermionic and bosonic loops, leading to the final agreement between theory and experiment.

1.3 Effective Lagrangian, \( \Delta x, \Delta y, \varepsilon, \Delta y_b \) Parameters

This question can be answered by an analysis in terms of the parameters \( \Delta x, \Delta y \) and \( \varepsilon \) which within the framework of an effective Lagrangian\[\text{[2,3,4]}\] specify potential sources of \( SU(2) \) violation. The “mass parameter” \( \Delta x \) is related to \( SU(2) \) violation by the masses of the triplet of charged and neutral (unmixed) vector boson via

\[ M_W^2 \equiv (1 + \Delta x)M_{W^0}^2 \equiv xM_{W^0}^2, \]  

(8)
while the “coupling parameter” \( \Delta y \) specifies \( SU(2) \) violation among the \( W^\pm \) and \( W^0 \) couplings to fermions,

\[ g_{W^\pm}^2(0) \equiv M_{W^\pm}^2 4\sqrt{2}G_\mu \equiv (1 + \Delta y)g_{W^0}(M_Z^2) \equiv yg_{W^0}(M_Z^2). \]  

(9)
Finally, the “mixing parameter” $\varepsilon$ refers to the mixing strength in the neutral vector boson sector and quantifies the deviation of $s_w^2$ from $e^2(M_Z^2)/g_W^2(M_Z^2)$,

$$s_w^2 \equiv \frac{e^2(M_Z^2)}{g_W^2(M_Z^2)}(1 - \varepsilon),$$

thus allowing for an unconstrained mixing strength in the neutral vector-boson sector. The effective Lagrangian incorporating the mentioned sources of $SU(2)$ violation for $W$ and $Z$ interactions with leptons is given by

$$\mathcal{L}_C = -\frac{1}{2} W^{\pm \mu \nu} W_{\mu \nu} + \frac{g_W}{\sqrt{2}} \left( j_\mu^+ W^{+ \mu} + h.c. \right) + M_W^2 W_{\mu}^+ W_{-\mu}^\mu$$

and

$$\mathcal{L}_N = -\frac{1}{4} Z_{\mu \nu} Z^{\mu \nu} + \frac{1}{2} \frac{M_W^2}{1 - s_w^2(1 - \varepsilon)} Z_\mu Z^{\mu} - \frac{1}{4} A_{\mu \nu} A^{\mu \nu} - e j_{em}^\mu A_{\mu} + \frac{g_W}{\sqrt{1 - s_w^2(1 - \varepsilon)}} \left( j_3^\mu - s_w^2 j_{em}^\mu \right) Z_\mu.$$ (12)

For the observables $s_w^2$, $M_W$ and $\Gamma_1$, from Eq. (11) and Eq. (12) one obtains

$$s_w^2(1 - s_w^2) = \frac{\pi \alpha(M_Z^2)}{\sqrt{2} G_\mu M_Z^2} \frac{y}{x} \left( 1 - \varepsilon \right) \frac{1}{\left( 1 + \frac{s_w^2}{1 - s_w^2} \varepsilon \right)},$$

$$\frac{M_W^2}{M_Z^2} = (1 - s_w^2) \frac{x}{1 + \frac{s_w^2}{1 - s_w^2} \varepsilon};$$

$$\Gamma_1 = \frac{G_\mu M_Z^3}{24 \pi} \frac{y}{x} \left( 1 + (1 - 4 s_w^2)^2 \right) \left( 1 - \frac{3\alpha}{4\pi} \right).$$ (13)

For $x = y = 1$ (i.e., $\Delta x = \Delta y = 0$) and $\varepsilon = 0$ one recovers the $\alpha(M_Z^2)$-Born approximation, $s_w^2 = s_0^2$, discussed previously.

The extension of the effective Lagrangian Eq. (12) to interactions of neutrinos and quarks requires the additional coupling parameters $\Delta y_{\nu}$ for the neutrino, $\Delta y_b$ for the bottom quark, and $\Delta y_h$ for the remaining light quarks. In the analysis of the data, for $\Delta y_{\nu}$ and $\Delta y_h$ which do not involve the non-Abelian structure of the theory, the SM theoretical results may be inserted without loss of generality as far as the guiding principle of separating vector-boson–fermion interactions from interactions containing non-Abelian couplings is concerned.

We note that the parameters in our effective Lagrangian are related to the parameters $\varepsilon_{1,2,3}$ and $\varepsilon_b$, introduced by isolating the quadratic $m_t$ dependence,

$$\varepsilon_1 = \Delta x - \Delta y + 0.2 \times 10^{-3}, \quad \varepsilon_2 = -\Delta y + 0.1 \times 10^{-3},$$

$$\varepsilon_3 = -\varepsilon + 0.2 \times 10^{-3}, \quad \varepsilon_b = -\Delta y_b/2 - 0.1 \times 10^{-3}.$$ (14)
Essentially the two sets of parameters only differ in $\varepsilon_1$. As $\varepsilon_1$ contains a linear combination of $\Delta x$ and $\Delta y$, the $M_H$-dependent bosonic corrections in $\Delta x$ are confused with the $M_H$-insensitive bosonic corrections in $\Delta y$. The theoretically interesting, but numerically irrelevant additive terms in Eq. [14], considerably smaller than $1 \times 10^{-3}$, originate from a refinement in the mixing involved in Lagrangian Eq. [12] and a corresponding refinement in Eq. [13]. We refer to the original paper [14] for details.

By linearizing relations Eq. [13] with respect to $\Delta x$, $\Delta y$ and $\varepsilon$ and inverting them, $\Delta x$, $\Delta y$ and $\varepsilon$ may be deduced from the experimental data on $\bar{s}_W^2$, $\Gamma_l$ and $M_W$. Inclusion of the hadronic $Z$ observables requires that $\Delta x$, $\Delta y$, $\varepsilon$ and $\varepsilon_b$ are fitted to the experimental data. Actually, one finds that the results for $\Delta x$, $\Delta y$, $\varepsilon$ are hardly affected by inclusion of the hadronic observables. On the other hand, $\Delta x$, $\Delta y$, $\varepsilon$ and $\Delta y_b$ may be theoretically determined in the standard electroweak theory at the one-loop level, strictly discriminating between pure fermion-loop predictions and the rest which contains the unknown bosonic couplings. The most recent 1996 update [21] of such an analysis [13] is shown in fig. 2.

According to fig. 2, the data in the ($\varepsilon$, $\Delta x$) plane are consistent with the theoretical predictions obtained by approximating $\Delta x$ and $\varepsilon$ by their pure fermion-loop values,

$$\Delta x = \Delta x_{\text{ferm}}(\alpha(M_Z^2), s_0^2, m_t^2 \ln m_t) + \Delta x_{\text{bos}}(\alpha(M_Z^2), s_0^2, \ln M_H^2)$$

$$\approx \Delta x_{\text{ferm}}(\alpha(M_Z^2), s_0^2, m_t^2, \ln m_t),$$

$$\varepsilon = \varepsilon_{\text{ferm}}(\alpha(M_Z^2), s_0^2, \ln m_t) + \varepsilon_{\text{bos}}(\alpha(M_Z^2), s_0^2, \ln M_H^2)$$

$$\approx \varepsilon_{\text{ferm}}(\alpha(M_Z^2), s_0^2, \ln m_t).$$

(15)

The small contributions of $\Delta x_{\text{bos}}$ and $\varepsilon_{\text{bos}}$ to $\Delta x$ and $\varepsilon$, respectively, and the logarithmic dependence on the Higgs mass, $M_H$, imply the well-known result that the data are fairly insensitive to the mass of the Higgs scalar. It is instructive to also note the numerical results for $\Delta x_{\text{ferm}}$ and $\varepsilon_{\text{ferm}}$, obtained in the Standard Model. They are given by $[13]

$$\Delta x_{\text{ferm}} = (2.61t + 1.34 \log(t) + 0.52) \times 10^{-3},$$

$$\varepsilon_{\text{ferm}} = (-0.45 \log(t) - 6.43) \times 10^{-3},$$

(16)

with $t \equiv m_t^2/M_Z^2$. The mass parameter $\Delta x$ is dominated by the $m_t^2$ term [22] due to weak isospin breaking induced by the top quark, while $\varepsilon$ is dominated by the constant term due to mixing induced by the light leptons and quarks.

In distinction from the results for $\Delta x$ and $\varepsilon$, where the fermion loops by themselves are consistent with the data, a striking effect appears in the plots showing $\Delta y$. The theoretical predictions are clearly inconsistent with the data, unless the fermion-loop
Figure 2: The projections of the 1σ ellipsoid of the electroweak parameters $\Delta x$, $\Delta y$, $\varepsilon$, $\Delta y_b$ obtained from the 1996 set of data in comparison with the SM predictions. Both the results obtained from using $s_W^2 (\text{LEP})$ and $s_W^2 (\text{LEP + SLD})$ as experimental input are shown. The full SM predictions correspond to Higgs-boson masses of 100 GeV (dotted with diamonds), 300 GeV (long-dashed dotted) and 1 TeV (short-dashed dotted) parametrized by the top-quark mass ranging from 120 GeV to 220 GeV in steps of 20 GeV. The pure fermion-loop prediction is also shown (short-dashed curve with squares) for the same values of $m_t$. The arrows indicate the shifts of the centres of the ellipses upon changing $\alpha(M^2_Z)^{-1}$ to $\alpha(M^2_Z)^{-1} + \delta \alpha(M^2_Z)^{-1}$ and $\alpha_s(M^2_Z)$ to $\alpha_s(M^2_Z) + \delta \alpha_s(M^2_Z)$. (From Ref. 21)
contributions to $\Delta y$ (denoted by lines with small squares) are supplemented by an additional term, which in the standard electroweak theory is due to bosonic effects,

$$\Delta y = \Delta y_{\text{term}}(\alpha(M_Z^2), s_0, \ln m_t) + \Delta y_{\text{bos}}(\alpha(M_Z^2), s_0).$$  \hspace{1cm} (17)$$

Remembering that $\Delta y$, according to Eq. 9, relates the coupling of the charged boson, $W^\pm$, to leptons as measured in $\mu^\pm$ decay, to the coupling of the neutral member, $W^0$, of the vector-boson triplet at the scale $M_Z \sim M_W$, it is not surprising that $\Delta y_{\text{bos}}$ contains vertex and box corrections originating from $\mu^\pm$ decay as well as vertex corrections at the $W^0 f \bar{f} (Z f \bar{f})$ vertex. While $\Delta y_{\text{bos}}$ obviously depends on the trilinear couplings among the vector bosons, it is insensitive to the Higgs mass, $M_H$.

The experimental data have accordingly become accurate enough to isolate loop effects which are insensitive to $M_H$, but depend on the self-interactions of the vector bosons, in particular on the trilinear non-Abelian couplings entering the $W f \bar{f}'$ and $W^0 f \bar{f} (Z f \bar{f})$ vertex corrections.

With respect to the interpretation of the coupling parameter, $\Delta y$, one further step may appropriately be taken. Introducing the coupling of the $W$ boson to leptons, $g_{W^\pm}(M_W^2)$, as defined by the leptonic $W^\pm$-boson width, in addition to the previously used low-energy coupling, $g_{W^\pm}(0)$, defined by the Fermi constant in Eq. 9,

$$\Gamma^W_l = g^2_{W^\pm}(M_W^2) \frac{M_W^2}{48\pi} \left(1 + c^2_0 \frac{3\alpha}{4\pi}\right),$$

(18)

the coupling parameter, $\Delta y$, in linear approximation may be split into two additive terms,

$$\Delta y = \Delta y^{\text{SC}} + \Delta y^{\text{IB}}.$$  \hspace{1cm} (19)$$

While $\Delta y^{\text{SC}}$ (where “SC” stands for “scale change”) furnishes the transition from $g_{W^\pm}(0)$ to $g_{W^\pm}(M_W^2)$,

$$g^2_{W^\pm}(0) = (1 + \Delta y^{\text{SC}})g^2_{W^\pm}(M_W^2),$$

(20)

the parameter $\Delta y^{\text{IB}}$ (where “IB” stands for “isospin breaking”) relates the charged-current and neutral current couplings at the high-mass scale, $M_W \sim M_Z$,

$$g^2_{W^\pm}(M_W^2) = (1 + \Delta y^{\text{IB}})g^2_{W^0}(M_Z^2),$$

(21)

to each other. Note that $\Delta y^{\text{SC}}$ according to Eq. 18 with Eq. 20 and Eq. 9 can be uniquely extracted from the observables $M_W, \Gamma^W_l$ together with $G_\mu$.

As seen in tab. 2 and fig. 3, the fermion-loop and the bosonic contributions to $\Delta y$ are opposite in sign, and both are dominated by their scale-change parts, $\Delta y^{\text{SC}}$. Once, $\Delta y^{\text{SC}}_{\text{bos}}$ is taken into account, practically no further bosonic contributions are needed.

The bosonic loops necessary for agreement with the data are accordingly recognized as charged-current corrections related to the use of the low-energy parameter
Figure 3: The one-loop SM predictions for $\Delta y$, $\Delta y_{\text{ferm}}$, $\Delta y_{\text{SC}}$, and $(\Delta y_{\text{ferm}} + \Delta y_{\text{SC}})$ as a function of $m_t$. The difference between the curves for $\Delta y$ and $(\Delta y_{\text{ferm}} + \Delta y_{\text{SC}})$ corresponds to the small contribution of $\Delta y_{\text{IB}}$. The experimental value of $\Delta y$, $\Delta y^{\exp} = (8.4 \pm 3.3) \times 10^{-3}$, is indicated by the error band (From Ref. 15, 1996 update).

Table 2: The different contributions (according to Eq. 19) to the coupling parameter $\Delta y$ (from Ref. 15).

|        | $\Delta y_{\text{ferm}} \times 10^3$ | $\Delta y_{\text{bos}} \times 10^3$ | $\Delta y \times 10^3$ |
|--------|--------------------------------------|------------------------------------|------------------------|
| SC     | $-7.8$                               | $12.4$                             | $4.6$                  |
| IB ($m_t = 175$ GeV) | $1.5$                                 | $1.2$                              | $2.7$                  |
| SC + IB | $-6.3$                               | $13.6$                             | $7.3$                  |
$G_\mu$ in the analysis of the data at the Z scale. Their contribution, due to a gauge-invariant combination of vertex, box and vacuum-polarization, is opposite in sign and somewhat larger than the contribution due to fermion-loop vacuum polarization.

Once the input parameters at the Z scale, $M_Z$ and $\alpha(M_Z^2)$, are supplemented by the coupling $g_{W\pm}(M_W^2)$, also defined at the scale of $M_W \sim M_Z$ and replacing $G_\mu$, all relevant radiative corrections are contained in $\Delta x_{\text{ferm}}$, $\varepsilon_{\text{ferm}}$, and $\Delta y_b$, and are either related to weak isospin breaking by the top quark or due to mixing effects induced by the light leptons and quarks and the top quark. Compare the numerical results for $\Delta x_{\text{ferm}}$ and $\varepsilon_{\text{ferm}}$ in Eq. 16. In addition to $\Delta x_{\text{ferm}}$ and $\varepsilon_{\text{ferm}}$, there is a (small) $\log(m_t)$ isospin-breaking contribution to $\Delta y$ as shown in tab. 2, and even an smaller bosonic isospin-breaking contribution.

In fig. 2, we also show the result for $\Delta y_b$ in the $(\Delta y_b, \varepsilon)$ plane. The theoretical prediction for $\Delta y_b$, as a consequence of a quadratic dependence on $m_t$, is similar in magnitude to the one for $\Delta x$. The experimental result for $\Delta y_b$ at the 1$\sigma$ level almost includes the theoretical expectation implied by the Tevatron measurement of $m_t^{\text{exp}} = 175 \pm 6$ GeV. This reflects the fact that the 1996 value of $R_b$ from tab. 1 is approximately consistent with theory, since the $R_b$ enhancement, present in the 1995 data, has practically gone away. I will come back to this point when discussing the bounds on $M_H$ implied by the data.

1.4 Empirical Evidence for the Higgs Mechanism?

As the experimental results for $\Delta x$ and $\varepsilon$ are well represented by neglecting all effects with the exception of fermion loops, and as the bosonic contribution to $\Delta y$, which is seen in the data, is independent of $M_H$, the question as to the role of the Higgs mass and the concept of the Higgs mechanism with respect to precision tests immediately arises.

More specifically, one may ask the question whether the experimental results, i.e. $\Delta x$, $\Delta y$, $\epsilon$, and $\Delta y_b$, can be predicted even without the very concept of the Higgs mechanism.

In Ref. 25 we start from the well-known fact that the standard electroweak theory without Higgs particle may credibly be reconstructed within the framework of a massive vector-boson theory (MVB) with the most general mass-mixing term which preserves electromagnetic gauge invariance. This theory is then cast into a form which is invariant under local $SU(2) \times U(1)$ transformations by introducing three auxiliary scalar fields á la Stueckelberg. As a consequence, loop calculations may be carried out in an arbitrary $R_\xi$ gauge in close analogy to the SM, even though the non-linear realization of the $SU(2) \times U(1)$, obviously, does not imply renormalizability of the theory.
Explicit loop calculations show that indeed the Higgs-less observable $\Delta y$, evaluated in the MVB, coincides with $\Delta y$ evaluated in the standard electroweak theory, i.e. in particular for the bosonic part, we have\footnote{Actually, in the SM there is an additional contribution of $\mathcal{O}(1/M_H^2)$ which is irrelevant numerically for $M_H \gtrsim 100$ GeV. Note that the $M_H$-dependent contributions to interactions violating custodial SU(2) symmetry turn out to be suppressed by a power of $1/M_H^2$ in the SM relative to the expectation from dimensional analysis. The absence of a $\log M_H$ term in $\Delta y$ and the absence of a $M_H^2 \log M_H$ term in $\Delta x$ in the SM thus appear on equal footing from the point of view of custodial SU(2) symmetry. In contrast, no suppression relative to dimensional analysis is present in the mixing parameter $\varepsilon$, which does not violate custodial SU(2) symmetry.}

$$\Delta y_{bos}^{MVB} \equiv \Delta y_{bos}^{SM}. \tag{22}$$

In the case of $\Delta x_{bos}$ and $\varepsilon_{bos}$, one finds that the MVB and the SM differ by the replacement $\ln M_H \leftrightarrow \ln \Lambda$, where $\Lambda$ denotes an ultraviolet cut-off. For $\Lambda \lesssim 1$ TeV, accordingly,

$$\Delta x_{MVB} \approx \Delta x_{SM}, \quad \varepsilon_{MVB} \approx \varepsilon_{SM}. \tag{23}$$

In conclusion, the MVB can indeed be evaluated at one-loop level at the expense of introducing a logarithmic cut-off, $\Lambda$. This cut-off only affects $\Delta x$ and $\varepsilon$, whose bosonic contributions cannot be well resolved experimentally anyway.

The quantity $\Delta y$, whose bosonic contributions are essential for agreement with experiment, is independent of the Higgs mechanism, i.e. it is convergent for $\Lambda \to \infty$ in the MVB theory. It depends on the non-Abelian couplings of the vector bosons among each other, which enter the vertex corrections at the $W$ and $Z$ vertices. Even though the data cannot discriminate between the MVB and the standard model with Higgs scalar, the Higgs mechanism nevertheless yields the only known simple physical realization of the cut-off $\Lambda$ (by $M_H$) which guarantees renormalizability.

1.5 Bounds on the Higgs-Boson Mass

We return to the description of the data in the SM, and in particular discuss the question, in how far the mass of the Higgs boson, $M_H$, can be deduced from the precision data.

In Section 1.3 we noted that the full (logarithmic) dependence on $M_H$ is contained in the mass parameter, $\Delta x$, and in the mixing parameter, $\varepsilon$. The experimental restrictions on $M_H$ may accordingly be visualized by showing the contour of the data in the $(\Delta x, \varepsilon)$ plane for the fixed (theoretical) value of $\Delta y \approx 7 \times 10^{-3}$ (corresponding to $m_t = 175 \pm 6$ GeV) in comparison with the $M_H$-dependent theoretical predictions for $\Delta x$ and $\varepsilon$. Fig. 4 illustrates the delicate dependence of bounds for $M_H$ on the experimental input for $s_{W,0}(M_H^2)$ and $m_t^{exp}$. The bounds on $M_H$, one can read off from fig. 4, are qualitatively in agreement of the results of fits to be discussed next.
Figure 4: The $1\sigma$ contour of the experimental data in the $(\Delta x, \varepsilon)$ plane defined by 
$\Delta y \simeq 7 \times 10^{-3}$ (corresponding to $m_t = 175 \pm 6$ GeV). The cut of the contour with the theoretical predictions for $m_t = 175 \pm 6$ GeV yields the experimental bounds on $M_H$. The projection of the data ellipsoid on the $(\Delta x, \varepsilon)$ plane, also shown, differs slightly from the one in fig. 2, since the data from the leptonic sector only were used for the present figure.

Precise bounds on $M_H$ require a fit to the experimental data. In order to account for the experimental uncertainties in the input parameters of $\alpha(M_Z^2), \alpha_s(M_Z^2)$ and $m_t$, four-parameter ($m_t, M_H, \alpha(M_Z^2), \alpha_s(M_Z^2)$) fits to various sets of observables from tab. 1 were actually performed in Ref. 29, 21. The Higgs-boson mass, $M_H$, and $\alpha_s(M_Z^2)$ were treated as free fit parameters, while for $\alpha(M_Z^2)$ and $m_t$ the experimental constraints from tab. 1 were used.

The results of the 1996 update (taken from Ref. 21) of the fits29 21 are presented in the plots of $\Delta \chi^2 \equiv \chi^2 - \chi^2_{\text{min}}$ against $M_H$ of fig. 5.

As $\chi^2_{\text{min}}$ is smallest for the fit to the “leptonic sector” of $s_W^2, M_W, \Gamma_1$ together with $m_t^{\text{exp}}$, and $\alpha(M_Z^2)$, while the $1\sigma$ errors are approximately the same in the three fits shown in fig. 5, we quote the result from the leptonic sector as the most reliable one,

$M_H = 190^{+174}_{-102}$ GeV, using $s_W^2(\text{LEP + SLD})_{96} = 0.23165 \pm 0.00024,$

$M_H = 296^{+243}_{-143}$ GeV, using $s_W^2(\text{LEP})_{96} = 0.23200 \pm 0.00027$  (24)

\[\text{ref 4, 30 for } M_H\text{-fits to the 1996 electroweak data, and Ref. 31, 32 for } M_H\text{ fits to previous sets of data.}\]
Figure 5: $\Delta \chi^2 = \chi^2 - \chi_{\text{min}}^2$ is plotted against $M_H$ for the $(m_t, M_H, \alpha(M_Z^2), \alpha_s(M_Z^2))$ fit to various sets of physical observables. For a chosen input for $s_W^2$, as indicated, we show the result of a fit to (i) the full set of 1996 data, $s_W^2$, $M_W$, $\Gamma_T$, $\sigma_h$, $R$, $R_b$, $R_c$, together with $m_t^{\text{exp}}$, $\alpha(M_Z^2)$, (ii) the 1996 set of (i) upon exclusion of $R_b$, (iii) the 1996 “leptonic sector” of $s_W^2$, $M_W$, $\Gamma_l$, together with $m_t^{\text{exp}}$, $\alpha(M_Z^2)$. (From Ref. [21])
Figure 6:
Figure 6: The results of the two-parameter \((m_t, M_H)\) fits within the SM are displayed in the \((m_t, M_H)\) plane. The different columns refer to the sets of experimental data used in the corresponding fits,

(i) “all data \(\setminus R_b\)”: \(\hat{s}_W^2(\text{LEP} + \text{SLD}), M_W, \Gamma_T, \sigma_b, R, R_c\),

(ii) “all data”: \(R_b\) is added to set (i),

(iii) “all data + \(m_t^{\text{exp}}\)”: \(R_b, m_t^{\text{exp}}\) are added to the set (i).

The second and third row shows the shift resulting from changing \(\alpha_s(M_Z^2)^{-1}\) and \(\alpha_s(M_Z^2)\), respectively, by one standard deviation in the SM prediction. The fourth row shows the effect of replacing \(\hat{s}_W^2(\text{LEP} + \text{SLD})\) by \(\bar{s}_W^2(\text{LEP})\) and \(\bar{s}_W^2(\text{SLD})\) in the fits. Note that the 1\(\sigma\) boundaries given in the first row are repeated identically in each row, in order to facilitate comparison with other boundaries. The value of \(\chi^2_{\text{min}}/\text{d.o.f.}\) given in the plots refers to the central values of \(\alpha_s(M_Z^2)^{-1}\) and \(\alpha_s(M_Z^2)\). In all plots the empirical value of \(m_t^{\text{exp}} = 175 \pm 6\) GeV is also indicated. (From Ref. 21)

based on the 1996 set of data. It implies the 1\(\sigma\) bounds of \(M_H \lesssim 360\) GeV and \(M_H \lesssim 540\) GeV, using \(\bar{s}_W^2(\text{LEP} + \text{SLD})\) and \(\bar{s}_W^2(\text{LEP})\), respectively, and

\[
\begin{align*}
M_H & \lesssim 550\text{GeV (95\%C.L.) using } \bar{s}_W^2(\text{SEP} + \text{SLD})_{96}, \\
M_H & \lesssim 800\text{GeV (95\%C.L.) using } \bar{s}_W^2(\text{LEP})_{96}.
\end{align*}
\]

(25)

The fact that the results Eq. 24 and Eq. 25 do not require \(\alpha_s(M_Z^2)\) as input parameter (apart from two-loop effects), and accordingly are independent of the uncertainties in \(\alpha_s(M_Z^2)\), provides an additional reason for the restriction to the leptonic sector when deriving bounds for \(M_H\). Moreover, we note that according to fig. 5 the results for \(M_H\) given by Eq. 24 and Eq. 25 practically do not change if the \(\alpha_s(M_Z^2)\)-dependent observables, \(\Gamma_T\) and \(\Gamma_h\), the total and hadronic \(Z\) widths, are included in the fit. Inclusion of \(\Gamma_T\) and \(\Gamma_h\) provides important information on \(\alpha_s(M_Z^2)\), however. One obtains \(\alpha_s(M_Z^2) = 0.121 \pm 0.003\) and \(\alpha_s(M_Z^2) = 0.123 \pm 0.003\) depending on whether \(\bar{s}_W^2(\text{LEP} + \text{SLD})\) or \(\bar{s}_W^2(\text{LEP})\) was used in the fit. Both values are consistent with the event-shape result given in tab. 1. The impact of also including \(R_b\) in the fit, also shown in fig. 5, will be commented upon below. Inclusion or exclusion of \(R_c\) is unimportant, as the error in \(R_c\) is considerable.

As mentioned, the above results on \(M_H\) are based on the 1996 set of data which was presented at the Warsaw International Conference on High Energy Physics which took place towards the end of July, two weeks after the International School of Subnuclear Physics in Erice. Two results presented in Warsaw are of particular importance with respect to the bounds on \(M_H\).

First of all, the value of \(m_t = 175 \pm 6\) GeV reported in Warsaw and given in tab. 1 is significantly more precise than the 1995 result\(^23\) of \(m_t = 180 \pm 12\) GeV. The decrease in the error on \(m_t\), due to the \((m_t, M_H)\) correlation in the theoretical predictions for the observables, clearly visible in fig. 4, led to a substantially narrower
$\Delta \chi^2$ distribution in fig. 5 compared with the results based on the 1995 set of data. Indeed, the 1995 leptonic set of data had implied

\[ M_H = 152^{+282}_{-106} \text{ GeV} \quad \text{using} \quad \bar{s}_W^2 \ (\text{LEP + SLD})_{95} = 0.23143 \pm 0.00028, \]
\[ M_H = 353^{+540}_{-224} \text{ GeV} \quad \text{using} \quad \bar{s}_W^2 \ (\text{LEP})_{95} = 0.23186 \pm 0.0034, \]

(26)
i.e., central values similar to the ones in Eq. [24], but with substantially larger errors.

The second and most pronounced change occurred in the result for $R_b \equiv \Gamma_b/\Gamma_h$. The enhancement in the 1995 value [29] of $R_b = 0.2219 \pm 0.0017$ of almost four standard deviations with respect to the theoretical prediction, according to the 1996 result of $R_b = 0.2179 \pm 0.0012$ presented in Warsaw, has reduced to less than two standard deviations. In order to discuss the impact of $R_b$ on the results for $M_H$, if $R_b$ is included in the fits, we recall that the theoretical prediction for $R_b$ is (practically) independent of the Higgs mass, but significantly dependent on $m_t$. As the theoretical prediction for $R_b$ increases with decreasing mass of the top quark, $m_t$, an experimental enhancement of $R_b$ effectively amounts [29] to imposing a low top-quark mass in fits of $m_t$ and $M_H$, as soon as $R_b$ is included in the fits. Lowering the top-quark mass in turn implies a lowering of $M_H$ as a consequence of the $(m_t, M_H)$ correlation present in the theoretical values of the other observables. Looking at fig. 5, we see that this effect of lowering $M_H$ is not very significant with the 1996 value of $R_b$ and the 1996 error in $m_t$. The “$R_b$-crisis” in the 1995 data, in contrast, led to a substantial decrease in the deduced value of $M_H$ to e.g. $M_H = 81^{+144}_{-52}$ GeV with $\bar{s}_W^2$ (LEP + SLD). As stressed in Ref. [29], this low value of $M_H$ had to be rejected, however, as the effective top-quark mass induced by including $R_b$ was substantially below the result from the direct measurements at the Tevatron. Other consequences from the “$R_b$-crisis”, such as an exceedingly low value of $\alpha_s \approx 0.100$ required upon allowing for a necessary non-standard $Zb\bar{b}$ vertex, as discussed during my lecture in Erice, have also gone away, and a very satisfactory and consistent overall picture of agreement with Standard Model predictions has emerged. Speculations on the existence of a “leptophobic” or a “hadrophilic” extra boson, offered as potential solutions to the “$R_b$-crisis”, do not seem to be realized in nature.

The delicate interplay of the experimental results for $\bar{s}_W^2$, $R_b$ and $m_t$ in constraining $M_H$ and the dependence of $M_H$ on $\alpha(M_Z^2)$ and $\alpha_s(M_Z^2)$ is visualized in the two-parameter ($m_t, M_H$) fits shown in fig. 6. With its caption, fig. 6 is fairly self-explanatory. For a detailed discussion we refer to the original papers [29,21]. We only note the considerable dependence of the bounds resulting for $M_H$ on whether the experimental value for $m_t$ is included in the fit and the strong dependence of $M_H$ on a $1\sigma$ variation of $\alpha(M_Z^2)$ and $\alpha_s$. Fig. 6 also shows that the SLD value of $\bar{s}_W^2$, when taken by itself, would rule out an interpretation of the data in terms of the standard Higgs mechanism, since the resulting Higgs mass, $M_H$, is much below the lower bound of $M_H \geq 65$ GeV following from the direct Higgs-boson search at LEP.

**Compare the Erice lectures by Paul Frampton and Dimitri Nanopoulos, these Proceedings.**
2 Production of $W^+W^-$ at LEP2

At this meeting we learnt that LEP2 has successfully started running at an energy above the $W^+W^-$ production threshold, and the first $W^+W^-$ events were presented. It is appropriate to add a remark on what we can learn on the couplings of the vector bosons among each other, even with the restricted luminosity to be accumulated in a few weeks or in a few months of running of LEP 2.

I start by quoting my distinguished late friend J.J. Sakurai, who was a frequent lecturer at the International School of Subnuclear Physics here in Erice. In his characteristic way of looking at physics, he said \[36\]:

“To quote Weinberg [Rev. Mod. Phys. 46 (1974) 255]

‘Indeed, the best way to convince oneself that gauge theories may have something to do with nature is to carry out some specific calculation and watch the cancellations before one’s very eyes’.

Does all this sound convincing? In any case it would be fantastic to see how the predicted cancellations take place experimentally at colliding beam facilities - LEPII? - in the 200 to 300 GeV range.”

Unfortunately, J.J. was overly optimistic concerning the energy range of LEP2.

In connection with the discussion of the coupling parameter $\Delta y$ in sect. 3, we stressed that the agreement with the LEP1 data at the $Z$ provides convincing indirect experimental evidence for the non-Abelian couplings of the Standard Model. More direct, quantitative information can be deduced from future data on $e^+e^- \to W^+W^-$. My remark will be brief, and essentially consists of showing two figures on the accuracy which we may expect, when extracting trilinear vector-boson couplings from measurements of the reaction $e^+e^- \to W^+W^-$ at LEP2. Restricting ourselves to dimension-four, P- and C-conserving interactions, the general phenomenological Lagrangian for trilinear vector boson couplings $^{37}$

$$
\mathcal{L}_{int} = -ie[A_\mu (W^{-\mu\nu}W_\nu^+ - W^{+\mu\nu}W_\nu^-) + F_{\mu\nu}W^{+\mu}W^{-\nu}]
-ie\gamma F_{\mu\nu}W^{+\mu}W^{-\nu}
-ie\left(\frac{C_W}{s_W} + \delta_Z\right)[Z_\mu (W^{-\mu\nu}W_\nu^+ - W^{+\mu\nu}W_\nu^-) + Z_{\mu\nu}W^{+\mu}W^{-\nu}]
-ixZ\mu\nu W^{+\mu}W^{-\nu}
$$

is obtained by supplementing the trilinear interactions of the SM with an additional anomalous magnetic-moment coupling of strength $x_\gamma$, by allowing for arbitrary normalization of the Z coupling via $\delta_Z$, and by adding an additional anomalous weak
magnetic dipole coupling of the $Z$ of strength $x_Z$. Compare Ref. 38 for a representation of the effective Lagrangian Eq. 27 in an $SU(2) \times U(1)$ gauge-invariant form. The SM corresponds to $x_\gamma = \delta_Z = x_Z = 0$.

Non-vanishing values of $x_\gamma$ parametrize deviations of the magnetic dipole moment, $\kappa_\gamma$, from its SM value of $\kappa_\gamma = 1$, as according to Eq. 27,

$$x_\gamma \equiv \kappa_\gamma - 1. \quad (28)$$

We note that $\kappa_\gamma = 1$ corresponds to a gyromagnetic ratio, $g$, of the $W$ of magnitude $g = 2$ in units of the particle’s Bohr-magneton $e/2M_W$, while $\kappa_\gamma = 0$ corresponds to $g = 1$ as obtained for a classical rotating charge distribution. The weak dipole coupling, $x_Z$, may be related to $x_\gamma$ by imposing “custodial” $SU(2)$ symmetry via

$$x_Z = -\frac{s_W}{c_W}x_\gamma, \quad (29)$$

thus reducing the number of free parameters to two independent ones in Eq. 27. Relation Eq. 29 follows from requiring the absence of an $SU(2)$-violating interaction term solely among the members of the $SU(2)$ triplet, $W^3_{\mu\nu}W^{\mu\nu}$, when rewriting the Lagrangian in the $BW^3$ base (or the $\gamma W^3$ base). This requirement is motivated by the validity of $SU(2)$ symmetry for the vector-boson mass term, i.e. from the observation that the deviation of the experimental value for $\Delta x$ from $\Delta x = 0$ in sect. 1.3 is fully explainable by radiative corrections, thus ruling out a violation of “custodial” $SU(2)$ symmetry by the vector boson masses at a high level of accuracy.

We also note the relation of $\delta_Z$ to the gauge coupling $\hat{g}$ describing the trilinear coupling between $W^0$ and $W^\pm$ in the $BW^0$ (or $\gamma W^3$) base,

$$e\delta_Z \equiv g_{ZW^0} - e\frac{c_W}{s_W} = \hat{g} - \frac{e}{s_Wc_W}. \quad (30)$$

The SM corresponds to $\hat{g} = e/s_W$.

Figs. 7a and 7b from Ref. 40 are based on the assumption that future data on $e^+e^- \rightarrow W^+W^-$ at an energy of 175 GeV will agree with SM predictions within errors. Under this assumption, fig. 7a shows that an integrated luminosity of $8pb^{-1}$, corresponding to a few weeks of running at 175 GeV will be sufficient to provide direct experimental evidence for the existence of a non-vanishing coupling of the non-Abelian type, $\hat{g} \neq 0$, among the members of the vector-boson triplet (at 95% C.L.). Likewise, according to fig. 7b, an integrated luminosity of $100pb^{-1}$, corresponding to about seven months of running at LEP2, will provide direct experimental evidence for a non-vanishing anomalous magnetic moment of the $W$ boson (at 95% C.L.), $\kappa_\gamma \neq 0$.

3 Conclusions

Let me conclude as follows:
Figure 7: a: Detecting the existence of a non-Abelian vector-boson coupling, \( \hat{g} \neq 0 \), at LEP 2. b: Detecting a non-zero anomalous magnetic dipole moment, \( \kappa_\gamma \neq 0 \), of the \( W^\pm \) at LEP 2.

- The Z data and the W-mass measurements require electroweak corrections beyond fermion-loop contributions to the vector-boson propagators.

- In the Standard Model such corrections are provided by bosonic loops. The dominant bosonic correction, needed for agreement with the data can be traced back to the difference in scale between \( \mu \) decay, entering via \( G_\mu \), and W or Z decay. While not being sensitive to the Higgs mechanism, these bosonic corrections depend on the non-Abelian couplings among the vector bosons. The data accordingly “see” the non-Abelian structure of the Standard Model.

- The bounds on the mass, \( M_H \), of the Higgs scalar are most reliably derived from the reduced set of data containing \( s_W^2, M_W, \Gamma_1, m_t^{\text{exp}} \) and \( \alpha(M_Z^2) \) besides \( M_Z \) and \( G_\mu \). At 95% C.L. the 1996 set of data implies \( M_H \lesssim 550 \text{ GeV} \) and \( M_H \lesssim 800 \text{ GeV} \), depending on whether \( s_W^2 \) (LEP+SLD) or \( s_W^2 \) (LEP) is used as
input. These bounds are quite remarkable, as for the first time they seem to fairly reliably predict a Higgs mass in the perturbative region of the SM.

- Since the “$R_b$-crisis” has meanwhile been resolved by our experimental colleges a short time after my talk in Erice, there is now perfect overall agreement with the predictions of the SM, even upon including hadronic $Z$ decays in the analysis. The strong coupling, $\alpha_s(M^2_Z)$, obtainable from the hadronic $Z$-decay modes, comes out consistently with the event-shape analysis. Various speculations on “hadrophilic” or “leptophobic” bosons do not seem to be realized in nature.

- Forthcoming experiments at LEP2 on $e^+e^- \rightarrow W^+W^-$ will allow one to find first direct experimental evidence for the existence of non-vanishing couplings of non-Abelian type among the vector bosons.

- The available data by themselves do not discriminate a MVB from the Standard Theory based on the Higgs mechanism. The issue of mass generation will remain open until the Higgs scalar will be found - or something else?

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