The black hole and FRW geometries of non-relativistic gravity

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Abstract

We consider the recently proposed non-relativistic Hořava-Lifshitz four-dimensional theory of gravity. We study a particular limit of the theory which admits flat Minkowski vacuum and we discuss thoroughly the quadratic fluctuations around it. We find that there are two propagating polarizations of the metric. We then explicitly construct a spherically symmetric, asymptotically flat, black hole solution that represents the analog of the Schwarzschild solution of GR. We show that this theory has the same Newtonian and post-Newtonian limits as GR and thus, it passes the classical tests. We also consider homogeneous and isotropic cosmological solutions and we show that although the equations are identical with GR cosmology, the couplings are constrained by the observed primordial abundance of $^4\text{He}$.
1 Introduction

A UV completion of gravity has recently been proposed [1, 2] and various aspects of it have been discussed [3]-[7]. This proposal is quite heretic as it introduces back non-equality of space and time. Indeed, space and time in this approach, exhibit Lifshitz scale invariance $t \to \ell^z t$ and $x^i \to \ell x^i$ with $z \geq 1$ (actually, $z = 3$ for the case at hand). Moreover, the theory is not invariant under the full diffeomorphism group of GR, but rather under a subgroup of it, manifest in the standard ADM splitting.

The breaking of the 4D diffeomorphism invariance, allows for a different treatment of the kinetic and potential terms for the metric. Thus, although the kinetic term is quadratic in time derivatives of the metric, the potential has higher-order space derivatives. In particular, the UV behavior of the potential is determined by the square of the Cotton tensor of the 3D geometry, which also appeared previously in topological massive extensions of 3D GR [8]. As the Cotton tensor contains third derivatives of the 3D metric, there is a contribution of order $k^6$ ($k$ is the 3-momentum) to the propagator, dominating at UV and renders the theory renormalizable power-counting. This is similar in spirit with many previous attempts where higher derivative terms has been added in the theory. However, in all these cases, the full 4D diffeomorphism invariance introduces higher-order time derivatives as well, leading to the appearance of ghost and various instabilities.

At large distances, higher derivative terms do not contribute and the theory runs to standard
GR if a particular coupling \( \lambda \), which controls the contribution of the trace of the extrinsic curvature has a specific value. Indeed, \( \lambda \) is running and if \( \lambda = 1 \) is an IR fixed point, standard GR is recovered.\(^1\) However, for generic values of \( \lambda \), the theory does not exhibit the full 4D diffeomorphism invariance at large distances and deviations from GR are possible. As there are severe restrictions on the possible deviations of GR, it is crucial to confront this type of non-relativistic theories with experimental and observational data. The basic issue is the Newtonian and post-Newtonian limits of this theory, which are crucial for the classical tests of GR. Moreover, the dynamics of cosmological solutions may also provide an interesting place for confronting the theory with observations, which is the main task of this work.

We should also mention here, that the generic IR vacuum of this theory is anti-de Sitter. In particular, the Newton constant and the speed of light are related to the cosmological constant (\( \sim \Lambda_W^2 \)). Thus, it is important to look for limits of the theory which eventually lead to a Minkowski vacuum in the IR. For this we deform the theory with a relevant operator proportional to the Ricci scalar of the three-geometry, \( \mu^4 R^{(3)} \) and then take the \( \Lambda_W \to 0 \) limit. This does not modify the UV properties of the theory but it does the IR ones. Namely, there exists a Minkowski vacuum and one may start discussing possible deviations from GR \cite{11}. The far more important solution for such a discussion is the Schwarzschild analog in this theory, which we will describe below in section 3.

2 Quadratic fluctuations

To proceed, let us consider the ADM decomposition of the metric in standard GR

\[
ds^2 = -N^2 dt^2 + g_{ij} \left( dx^i + N^i dt \right) \left( dx^j + N^j dt \right),
\]

where \( g_{ij}, N \) and \( N^i \) are the dynamical fields of scaling mass dimensions 0, 0, 2, respectively. The action for the fields of the theory is

\[
S = \int dtd^3x \sqrt{g} N \left\{ \frac{2}{\kappa^2} \left( K_{ij} R^{ij} - \lambda K^2 \right) - \frac{\kappa^2}{2w^2} C_{ij} C^{ij} + \frac{\kappa^2 \mu}{2w^2} \epsilon^{ijk} R^{(3)}_{i\ell} \nabla_j R^{(3)}_{k\ell} \right. \\
- \frac{\kappa^2 \mu^2}{8} R^{(3)}_{ij} R^{(3)ij} + \frac{\kappa^2 \mu^2}{8(1 - 3\lambda)} \left( 1 - \frac{4\lambda}{4} \left( R^{(3)} \right)^2 + \Lambda_W R^{(3)} - 3\Lambda_W^2 \right) + \mu^4 R^{(3)} \right\} \quad (2.2)
\]

\(^1\)The idea that Lorentz symmetry arises as an IR fixed point dates back to \cite{9}. The broader related idea that symmetries arise as IR attractive fixed points has been also explored, notably in \cite{10}.
We should note that

\[ K_{ij} = \frac{1}{2N} (\dot{g}_{ij} - \nabla_i N_j - \nabla_j N_i) \]  

is the second fundamental form,

\[ C^{ij} = \epsilon^{ik\ell} \nabla_k \left( R^{(3)j\ell} - \frac{1}{4} R^{(3)} \delta^j_{\ell} \right) \]  

is the Cotton tensor, \( \kappa, \lambda, w \) are dimensionless coupling constants, whereas \( \mu, \Lambda_W \) are dimensionful of mass dimensions \([\mu] = 1, [\Lambda_W] = 2\). The action (2.2) is the action in [2] where we have added the last term, which represents a soft violation of the detailed balance condition.

We will now consider the limit of this theory such that

\[ \Lambda_W \to 0 \]  

In this particular limit, the theory turns out to be

\[
S = \int dt d^3x \sqrt{g} N \left\{ \frac{2}{\kappa^2} (K_{ij} K^{ij} - \lambda K^2) - \frac{\kappa^2}{2w^4} C_{ij} C^{ij} + \frac{\kappa^2 \mu^2}{2w^2} \epsilon^{ijk} R^{(3)}_{\ell k} \nabla_j R^{(3)\ell} 
- \frac{\kappa^2 \mu^2}{8} R^{(3)}_{ij} R^{(3)ij} \right. 
+ \left. \frac{\kappa^2 \mu^2}{8(1 - 3\lambda)} \frac{1 - 4\lambda}{4} (R^{(3)})^2 + \mu^4 R^{(3)} \right\} .
\]

Introducing the coordinate \( x^0 = ct \), we may write the action (2.6) in the IR limit as the standard Einstein-Hilbert action (up to surface terms)

\[
S_{EH} = \frac{1}{16\pi G_N} \int d^4x \sqrt{g} N (K_{ij} K^{ij} - K^2 + R^{(3)})
\]

provided

\[
\lambda = 1, \quad c^2 = \frac{\kappa^2 \mu^4}{2}, \quad G_N = \frac{\kappa^2}{32\pi c}.
\]

For a general \( \lambda \) we get

\[
S_{EH\lambda} = \int dt d^3x \sqrt{g} N \left( \frac{2}{\kappa^2} (K_{ij} K^{ij} - \lambda K^2) + \mu^4 R^{(3)} \right).
\]

We will consider perturbations of the metric around Minkowski space-time, which is a solution of the full theory (2.6)

\[
g_{ij} \approx \delta_{ij} + wh_{ij}, \quad N \approx 1 + wn, \quad N_i \approx wn_i .
\]
At quadratic order the action turns out to be

\[ S_2 = w^2 \int dtd^3x \frac{1}{\kappa^2} \left[ \frac{1}{2} \dot{h}_{ij}^2 - \frac{\lambda}{2} \dot{h}^2 + (\partial_i n_j)^2 + (1 - 2\lambda)(\partial \cdot n)^2 - 2\partial_i n_j (\dot{h}_{ij} - \lambda \dot{h} \delta_{ij}) \right] \]

\[ + \frac{\mu^4}{2} \left[ -\frac{1}{2}(\partial_k h_{ij})^2 + \frac{1}{2}(\partial_i h)^2 + (\partial_i h_{ij})^2 - \partial_i h_{ij} \partial_j h + 2n(\partial_i \partial_j h_{ij} - \partial^2 h) \right]. \tag{2.11} \]

This theory is invariant under

\[ \delta x^i = \xi^i(x, t), \quad \delta t = f(t), \tag{2.12} \]

which induce

\[ \delta h_{ij} = \partial_i \xi_j + \partial_j \xi_i + \xi_k \partial_k h_{ij} + f \dot{h}_{ij}, \]

\[ \delta n_i = \dot{\xi}_i, \quad \delta n = \dot{f}. \tag{2.13} \]

As usual, we fix the invariance of the theory by imposing the gauge condition

\[ n_i = 0, \tag{2.14} \]

which from the corresponding eq. of motion gives the momentum constraint

\[ \partial_i \dot{h}_{ij} - \lambda \partial_j \dot{h} = 0. \tag{2.15} \]

The above gauge fixing leaves time-independent spatial diffeomorphisms unfixed. We choose the gauge fixing condition of the latter to be

\[ \partial_i h_{ij} - \lambda \partial_j h = 0, \tag{2.16} \]

which remains invariant in time thanks to the above constraint. Varying \( n \) we get the hamiltonian constraint

\[ \partial_i \partial_j h_{ji} - \partial^2 h = 0, \tag{2.17} \]

which, combined with (2.16) leads to

\[ (\lambda - 1) \partial^2 h = 0. \tag{2.18} \]

Thus, for \( \lambda \neq 1 \) we get that

\[ \partial^2 h = 0. \tag{2.19} \]
We may define the transverse field

\[ H_{ij} = h_{ij} - \lambda \delta_{ij} h , \quad \partial_t H_{ij} = 0 \]  
(2.20)

and the transverse traceless part of \( \tilde{H}_{ij} \) of \( H_{ij} \) by

\[ H_{ij} = \tilde{H}_{ij} + \frac{1}{2} \left( \delta_{ij} - \frac{\partial_i \partial_j}{\partial^2} \right) H . \]  
(2.21)

From these we obtain

\[ h_{ij} = \tilde{H}_{ij} + \frac{1}{2(1-3\lambda)} \delta_{ij} H - \frac{1}{2} \frac{\partial_i \partial_j}{\partial^2} H , \quad h = \frac{H}{1-3\lambda} . \]  
(2.22)

Then the quadratic part of the action (2.9) turns out to be

\[ S_2 = \int dt d^3 x \left\{ \frac{w^2}{2\kappa^2} (\partial_0 \tilde{H}_{ij})^2 - \frac{\mu^4 w^2}{4} \left( \partial_k \tilde{H}_{ij} \right)^2 + \frac{w^2(1-\lambda)}{4\kappa^2(1-3\lambda)} \dot{H}^2 \right\} . \]  
(2.23)

The first two terms describe the usual (transverse traceless) graviton whereas, for \( \lambda \neq 1 \) there is another mode \( H \). This mode is physical, but non-propagating nevertheless, as in empty space its equation is simply \( \ddot{H} = 0 \). So a natural question is if the higher derivative terms in the full action (2.2) may provide spatial derivatives of \( H \) turning the latter into a true propagating mode. It is not difficult to see that taking into account the higher derivative terms in (2.2) does not change the Hamiltonian and momentum constraints. As a result, taking again the gauge condition (2.16), \( h \) still satisfies (2.18) and the quadratic part of the perturbed action is

\[ S_2 = \int dt d^3 x \left\{ -\frac{w^2}{2\kappa^2} \tilde{H}_{ij} \partial_t^2 \tilde{H}_{ij} + \frac{\mu^4 w^2}{4} \tilde{H}_{ij} \partial^2 \tilde{H}_{ij} + \frac{\kappa^2 \mu^2 w^2}{32} \tilde{H}_{ij} (\partial^2)^2 \tilde{H}_{ij} \\
+ \frac{\kappa^2}{8w^2} \tilde{H}_{ij} (\partial^2)^3 \tilde{H}_{ij} + \frac{\kappa^2 \mu}{8} \varepsilon^{ijk} \tilde{H}_{im} (\partial^2)^2 \partial_j \tilde{H}_{mk} \\
+ \frac{w^2(1-\lambda)}{4\kappa^2(1-3\lambda)} \dot{H}^2 \right\} . \]  
(2.24)

We see that \( H \) still has no spatial derivatives and although physical, is not propagating. Thus, there are two physical degrees of freedom (transverse and traceless \( \tilde{H}_{ij} \)) which corresponds to the physical graviton.

Returning to the quadratic action (2.23) in the infrared, we may determine the speed of the gravitational interaction after introducing \( x^0 \)

\[ S_2 = \int dx^0 d^3 x \left\{ \frac{w^2}{2\kappa^2} \left( \partial_0 \tilde{H}_{ij} \right)^2 - \frac{\mu^4 \kappa^2}{2c^2} \left( \partial_k \tilde{H}_{ij} \right)^2 + \frac{w^2 c^2(1-\lambda)}{4\kappa^2(1-3\lambda)} (\partial_0 H)^2 \right\} . \]  
(2.25)
Note that for $1/3 < \lambda < 1$ the kinetic term of $H$ becomes negative indicating a ghost instability. Thus, either $\lambda$ runs to $1^+$ from above in the IR or $H$ does not couple at all to matter.

We also see from (2.25) that the speed of gravitational interaction is

$$c_g^2 = \frac{\mu^4 \kappa^2}{2c_0^2 c_0^2},$$

(2.26)

where $c_0^2$ is the speed of light. The stability of pulsar clocks has allowed to measure very small orbital period decay of binary systems and thereby a direct experimental confirmation, namely that, the propagation of gravity interaction equals the velocity of light to better than $1:1000$ [12]. Hence, we get that

$$c^2 = \frac{\mu^4 \kappa^2}{2},$$

(2.27)

with the above accuracy, independent of the value of the couplings $\lambda, \omega$.

### 3 The black hole solution

Let us consider now a static, spherically symmetric background

$$ds^2 = -N(r)^2 dt^2 + \frac{dr^2}{f(r)} + r^2 (d\theta^2 + \sin^2 \theta d\phi^2).$$

(3.1)

Using that

$$R^{(3)}_{rr} = -\frac{f'}{rf}, \quad R^{(3)}_{\theta\theta} = 1 - f - \frac{r}{2} f', \quad R^{(3)}_{\phi\phi} = \sin^2 \theta (1 - f - \frac{r}{2} f'),$$

(3.2)

we find that the Lagrangian (2.6) after the angular integration reduces to

$$\mathcal{L} = \frac{\kappa^2 \mu^2}{8(1 - 3\lambda) \sqrt{f}} \left( (2\lambda - 1) \frac{(f - 1)^2}{r^2} - 2\lambda \frac{f - 1}{r} f' + \frac{\lambda - 1}{2} f'^2 - 2\omega (1 - f - rf') \right),$$

(3.3)

where $\omega = 8\mu^2(3\lambda - 1)/\kappa^2$ and has dimension $[\omega] = 2$. Note that, due to the fact that the ansatz for the spatial part is conformally flat, the Cotton tensor does not contribute. Also, since the Ricci tensor is diagonal, the last term in the first line of (2.6) does not contribute, as well.

The equations of motions are

$$(2\lambda - 1) \frac{(f - 1)^2}{r^2} - 2\lambda \frac{f - 1}{r} f' + \frac{\lambda - 1}{2} f'^2 - 2\omega (1 - f - rf') = 0,$$

$$\left( \log \frac{N}{\sqrt{f}} \right)' \left\{ (\lambda - 1) f'' - 2\lambda \frac{f - 1}{r} + 2\omega r \right\} + (\lambda - 1) \left( f'' - \frac{2(f - 1)}{r^2} \right) = 0.$$

(3.4)
For the $\lambda = 1$ ($\omega = 16\mu^2/\kappa^2$) case, we get for asymptotically flat space-time

$$N^2 = f = 1 + \omega r^2 - \sqrt{r(\omega^2 r^3 + 4\omega M)} , \quad (3.5)$$

where $M$ an integration constant, with dimension $[M] = -1$. The static, spherically symmetric solution is in this case

$$ds^2 = -f dt^2 + \frac{dr^2}{f} + r^2 (d\theta^2 + \sin^2 \theta d\phi^2) . \quad (3.6)$$

For $r \gg (M/\omega)^{1/3}$ we get the usual behavior of a Schwarzschild black hole

$$f \approx 1 - \frac{2M}{r} + \mathcal{O}(r^{-4}) . \quad (3.7)$$

The Ricci scalar diverges as $1/r^{3/2}$ and therefore the metric is singular at $r = 0$. There are two event horizon at

$$r_\pm = M \left(1 \pm \sqrt{1 - \frac{1}{2\omega M^2}} \right) . \quad (3.8)$$

For the singularity at $r = 0$ not to be naked the inequality

$$\omega M^2 \geq \frac{1}{2} , \quad (3.9)$$

has to be satisfied. When it is saturated the two horizons coincide. The conventional GR arises when $\omega M^2 \gg 1$. Then, the outer horizon approaches the usual Schwarzschild horizon $r_+ \approx 2M$, whereas the inner one approaches the singularity $r_- \approx 0$.

For $\lambda$ near the GR value $\lambda = 1$ we set $\lambda = 1 + \delta \lambda$, $f \rightarrow f + \delta f$ and obtain

$$\delta f = \frac{3}{4} \frac{\delta \lambda}{M} \frac{r}{r^3} \left(1 + \frac{4M}{\omega r^3}\right)^{-1/2} \ln \left(1 + \frac{4M}{\omega r^3}\right) . \quad (3.10)$$

At large distances

$$\delta f = 3 \frac{\delta \lambda}{M^2} \frac{M^2}{\omega r^4} + \mathcal{O} \left(\frac{1}{r^7}\right) . \quad (3.11)$$

In addition, letting $N^2 \rightarrow f + \delta f + \delta \lambda f A$, we find

$$A = -\frac{3M}{\omega r^3} \left(1 + \frac{4M}{\omega r^3}\right)^{-1} + \frac{3}{4} \frac{\ln \left(1 + \frac{4M}{\omega r^3}\right)}{r^6} = 6 \frac{\delta \lambda}{\omega^2 r^6} + \mathcal{O} \left(\frac{1}{r^9}\right) . \quad (3.12)$$

Hence, for large distances $N^2(r)$ and $f(r)$ remain equal to first order in the deviation from $\lambda = 1$.

As the first correction to the $1/r$ law behaviour for large distances is of order $r^{-4}$, the Eddington–Robertson–Schiff Post-Newtonian parameters are identical to that of GR, i.e.,

$$\beta = \gamma = 1 . \quad (3.13)$$
3.1 The minimal theory with Minkowski vacuum

We stress, that the theory (2.2) is not the minimal theory with a Minkowski vacuum. Indeed, the minimal theory is described by the action

\[ S_{\text{min}} = \int dt d^3x \sqrt{g_N} \left\{ \frac{2}{\kappa^2} (K_{ij} K^{ij} - \lambda K^2) - \frac{\kappa^2}{2w^4} C_{ij} C^{ij} + \mu^4 R^{(3)} \right\} \]  

(3.14)

This theory has for generic values of \( \lambda \) the standard Schwarzschild GR solution

\[ N^2 = f = 1 - \frac{M}{r} , \]  

(3.15)

as in the static case, there is no contribution from the \( K^2 \) part of action and the Cotton tensor is exactly zero for spherical symmetry.

4 The cosmological solution

We now consider a homogeneous and isotropic cosmological solution to the theory (2.6) with the standard FRW geometry

\[ ds^2 = -c^2 dt^2 + a^2(t) \left[ \frac{dr^2}{1 - kr^2} + r^2 (d\theta^2 + \sin^2 \theta d\phi^2) \right] . \]  

(4.1)

As usual, \( k = 0, -1, 1 \) corresponds to a flat, open and closed universe, respectively. Assuming the matter contribution to be of the form of a perfect fluid, the Friedmann equation turns out to be

\[ H^2 = \frac{\kappa^2}{6(3\lambda - 1)} \left( \rho - \frac{6k\mu^4}{a^2} - \frac{3k\kappa^2\mu^2}{8(3\lambda - 1)a^4} \right) , \]  

(4.2)

where \( H = N^{-1} \dot{a}/a \). The conservation of energy-momentum tensor gives as usual

\[ \dot{\rho} + 3H(\rho + p) = 0 . \]  

(4.3)

We may read-off from the Friedmann equation the “cosmological” Newton constant \( G_{\text{cosmo}} \)

\[ G_{\text{cosmo}} = \frac{2}{3\lambda - 1} G_N . \]  

(4.4)

Current observational bounds on the observed primordial helium \( ^4He \) abundance require [13, 14]

\[ \left| \frac{G_{\text{cosmo}}}{G_N} - 1 \right| < \frac{1}{8} . \]  

(4.5)
In our case, we get

$$\left| \frac{\lambda - 1}{3\lambda - 1} \right| < \frac{1}{24} ,$$

(4.6)

or

$$0.926 \lesssim \frac{25}{27} < \lambda < \frac{23}{21} \approx 1.095 .$$

(4.7)

Thus, the IR value of the coupling constant $\lambda$ is restricted to be within the above range.

## 5 Conclusions

We have studied here a specific limit of Hořava-Lifshitz gravity. This is the vanishing $\Lambda_W$ limit after introducing a relevant operator proportional to the Ricci scalar of the 3D geometry. In this limit, the theory is close to GR in the IR depending on the value of the coupling $\lambda$, which controls the contribution of the trace of the extrinsic curvature. Moreover, it admits a Minkowski vacuum. We studied perturbations around this vacuum and found that there exists a massless excitation with two propagating polarizations described by the transverse traceless part of the perturbation.

We also found a static spherically symmetric black hole solution of this theory, which is the analog of the Schwarzschild one of GR. We discussed the geometry of the solution and its Newtonian and post-Newtonian limits. We have not checked explicitly, but it is very likely that an analog of Birkhoff’s theorem for GR applies to the $\lambda = 1$ case as well. Namely that, a spherically symmetric gravitational field in empty space must be static, with metric given by (3.6). For generic values of $\lambda$, it is not at all obvious that all spherically symmetric vacuum solutions are static.

We also discussed homogeneous and isotropic cosmological solutions and we presented a constrained on $\lambda$ from Big Bang nucleosynthesis (BBN).

**Acknowledgment:** AK wishes to thank support from the PEVE-NTUA-2007/10079 programme and the European Research and Training Network MRTPN-CT-2006 035863-1.
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