HIGHER ORDER MOMENTS OF THE ANGULAR DISTRIBUTION OF EARLY GALAXIES FROM EARLY SLOAN DIGITAL SKY SURVEY DATA

ISTVÁN SZAPUDI,1 JOSHUA A. FRIEYMAN,2,3 ROMAN SOCCIMARRO,4 ALEXANDER S. ZALAY,5 ANDREW J. CONNOLLY,6 SCOTT DODELSON,2,3 DANIEL J. EISENSTEIN,2,7 JAMES E. GUNN,8 DAVID JOHNSTON,2,3 STEPHEN KENT,3 JON LOVEDAY,9 AVERY MEIKSIN,10 ROBERT C. NICHOL,11 RYAN SCRANTON,7,12 ALBERT STEBBINS,3 MICHAEL S. VOGELEY,13 JAMES ANNIS,3 NETA A. BAHCALL,5 J. BRINKMAN,15 ISTVÁN CSABAII,14 MAMORU DOI,15 MASATAKA FUKUGITA,15 ŽELIŽEK IVEZIĆ,8 RITA S. J. KIM, GILLIAN R. KNAPP, DON Q. LAMB, BRIAN C. LEE, ROBERT H. LUPTON, TIMOTHY A. MCKAY, JEFF MUNN,17 JOHN PEOPLES,1 JEFF PIER,17 CONSTANCE ROCKOSI,2 DAVID SCHLEGEL,5 CHRISTOPHER STOUGHTON,3 DOUGLAS L. TUCKER,3 BRIAN YANNY,3 AND DONALD G. YORK6,18

Received 2001 October 17; accepted 2002 January 10

ABSTRACT

We present initial results for counts in cell statistics of the angular distribution of galaxies in early data from the Sloan Digital Sky Survey (SDSS). We analyze a rectangular stripe 2.5 wide, covering approximately 160 deg², containing over 10⁶ galaxies in the apparent magnitude range 18 < r < 22, with areas of bad seeing, contamination from bright stars, ghosts, and high galactic extinction masked out. This survey region, which forms part of the SDSS early data release, is the same as that for which two-point angular clustering statistics have recently been computed. The third and fourth moments of the cell counts, s3 (skewness) and s4 (kurtosis), constitute the most accurate measurements to date of these quantities (for r < 21) over angular scales 0.015–0.73. They display the approximate hierarchical scaling expected from nonlinear structure formation models and are in reasonable agreement with the predictions of Λ-dominated cold dark matter models with galaxy biasing that suppresses higher order correlations at small scales. The results are, in general, consistent with previous measurements in the APM, EDSGC, and Deeprange surveys. These results suggest that the SDSS imaging data are free of systematics to a high degree and will therefore enable determination of the skewness and kurtosis to 1% and less than 10%, as predicted earlier.

Subject headings: galaxies: clusters: general — large-scale structure of universe

1. INTRODUCTION

In contemporary models of structure formation, initially small density fluctuations are amplified by gravitational instability to form nonlinear structures. Within such struc-
Juszkiewicz, Bouchet & Colombi 1993; Bouchet et al. 1993; Colombi, Bouchet, & Schaeffer 1995; Szapudi & Gaztañaga 1998; Colombi et al. 2000; Szapudi et al. 2000). In particular, it has been shown that higher order moments of the galaxy distribution provide important constraints on non-Gaussianity in the initial conditions (Fry & Scherrer 1994; Jaffe 1994; Chodorowski & Bouchet 1996; Gaztañaga & Mahonén 1996; Frieman & Gaztañaga 1999; Scoccimarro 2000; Feldman et al. 2001; Durrer et al. 2000) and on the bias (Fry & Gaztañaga 1993; Frieman & Gaztañaga 1994; Gaztañaga & Frieman 1999, Fry 1994; Juszkiewicz et al. 1995; Matarrese, Verde, & Heavens 1997; Frieman & Gaztañaga 1994; Fry & Scherrer 1994; Colombi et al. 2000). Among the higher order moments, it has been shown that higher order moments of the galaxy distribution in early SDSS data; these can be viewed as normalized area averages of the projected (angular) p-point correlation functions. We use the same data set for which second-order statistics were recently reported by Connolly et al. (2001), Dodelson et al. (2001), Scranton et al. (2001), Szalay et al. (2001), and Tegmark et al. (2001).

Here we measure the s_p parameters (which characterize the connected moments of counts in cells, hereafter CIC) of the angular galaxy distribution in early SDSS data; these can be viewed as normalized area averages of the projected (angular) p-point correlation functions. We use the same data set for which second-order statistics were recently reported by Connolly et al. (2001), Dodelson et al. (2001), Scranton et al. (2001), Szalay et al. (2001), and Tegmark et al. (2001). Direct estimation of the angular and redshift-space three-point correlation functions in early SDSS data will be presented elsewhere.

Previous measurements of angular galaxy CIC include those obtained for the Automated Plate Measuring (APM: Gaztañaga 1994; Szapudi et al. 1995) and Edinburgh Durham Southern Galaxy Catalog (EDSGC: Szapudi, Meiksim, & Nichol 1996) surveys. Both have a wide area covering ~1 sr to h2 = ~20 and constructed from digitized UK Schmidt plates. Similar measurements were performed in the Deep-range catalog (see Postman et al. 1998; Szapudi et al. 2000) based on a deeper (I < 23.5), small-area (16 deg^2) CCD imaging survey. The SDSS sample considered here is intermediate between these two extremes in both area (~160 deg^2) and depth (r < 22). We note that all of these data sets contain of order 10^6 galaxies with measured angular positions and apparent magnitudes. While the SDSS data set is based on only two nights of early imaging data and constitutes less than 2% of the eventual survey area, the high quality of the data has enabled us to extract statistical measurements with smaller errors than those for the earlier catalogs; the SDSS results therefore provide an important consistency check on and interpolation between the previous measurements. According to Colombi, Szapudi, & Szalay (1998), the full SDSS will pinpoint the skewness and kurtosis with smaller then 1% and 10% errors, respectively, in a large dynamic range up to 10 h^-1 Mpc.

The next section briefly describes the SDSS data set used, while § 3 presents an outline of the method used for analysis. Section 4 contains the measurements and various tests for systematic effects, while § 5 is devoted to comparisons with measurements in other surveys. We present our conclusions in § 6.

2. THE SLOAN DIGITAL SKY SURVEY EARLY DATA SET

The Sloan Digital Sky Survey is a wide-field photometric and spectroscopic survey being undertaken by the Astrophysical Research Consortium at the Apache Point Observatory in New Mexico (York et al. 2000). The completed survey will cover approximately 10,000 deg^2. CCD imaging with the SDSS camera (Gunn et al. 1998) will include 10^8 galaxies in five passbands (u, g, r, i, and z; see Fukugita et al. 1996) to an approximate detection limit of r = 23 at S/N = 5 (strictly true for point sources).

In this paper, we focus on a small section of imaging data that was taken on the nights of 1999 March 20 and 21, during commissioning of the survey telescope; these data are designated Runs 752 and 756 and are in the northern equatorial stripe of the survey. These interleaved scans are centered on the celestial equator, covering a stripe 2°52 wide, with declination |b| < 1°26, and approximately 90° long, with right ascension ranging from h^40°40^m48^s to 15^h45^m12^s (J2000). These data, designated EDR-P and discussed extensively by Scranton et al. (2001), constitute a high-quality subset of the data made publicly available as part of the SDSS early data release (Stoughton et al. 2001).

The imaging data were reduced by the SDSS photometric image processing software (PHOTO), which detects objects and measures their apparent magnitudes based on the best-fit point-spread function-convolved de Vaucouleurs or exponential model, including an arbitrary scale size and axis ratio. All magnitudes were corrected for Galactic extinction using the reddening maps of Schlegel, Finkbeiner, & Davis (1998). For this data set, two methods of star-galaxy separation were available. The version of the photometric pipeline used to reduce these data makes a binary decision about whether a given detected object is stellar or galactic. For most of the results shown below, we selected all objects flagged as galaxies in this way by PHOTO. While the PHOT0 separation works well at bright magnitudes, a Bayesian star-galaxy separation method (Scranton et al. 2001), which assigns a probability to each object of being a star or a galaxy, provides a more precise separation at faint magnitudes. Below, we also show results for a sample of objects which the Bayesian method assigns a very high probability (p > 0.99) of being galaxies. We find that these two samples yield consistent results for the higher order moments.

Observing conditions, particularly the seeing, varied considerably over the course of these two nights of data taking. Scranton et al. (2001) carried out extensive tests of the effects of seeing, galactic extinction, variable stellar density, etc., on the integrity of the data. These tests showed that the data were essentially free of systematic contamination (as demonstrated by cross-correlation analysis), provided certain regions were excluded, and we apply the same masks here. The resulting masks depend on apparent magnitude: a “bright” seeing mask (used for r < 21) excludes regions where the seeing disk is larger than 1.75, while the “faint” seeing mask (used for 21 < r < 22) excludes regions for which the seeing exceeds 1.6. These masks exclude roughly 30% of the data area. In addition, regions where the reddening is greater than 0.2 mag in r, small rectangles around saturated stars, and ghost images were also masked out. The resulting catalog contains 1.46 × 10^6 galaxies in the apparent magnitude range 18 < r < 22 and covers an area of 160 (140) deg^2 for the bright (faint) samples. The resulting data region has a complex geometry (see Figs. 13–14 in Scranton et al. 2001), complicating the CIC analysis, as described below. Since the data tests described in Scranton et al. (2001) were based solely on two-point analyses, the results shown below provide additional constraints on the data quality: the consistency of the s_p measurements with both previous results and theoretical predictions indicates that stellar and other contamination of the sample is small.
As in Connolly et al. (2001), Dodelson et al. (2001), Scranton et al. (2001), Szalay et al. (2001), and Tegmark et al. (2001), we divide the sample into four (dereddened) apparent magnitude slices: \( r = 18-19, 19-20, 20-21, 21-22 \). We also consider a “pseudo APM” slice, with \( r = 15.9-18.9 \), in order to compare with the results of Gaztañaga (2001a), as well as a series of slices with \( r = 17.9-18.9, 18.9-19.9, 19.9-20.9, 20.9-21.9 \) for direct comparison with the results from the Deeprange survey (Postman et al. 1998; Szapudi et al. 2001).

3. THE COUNTS IN CELLS METHOD

The probability distribution of CIC, \( P_N(\theta) \), is the probability that an angular cell of (linear) dimension \( \theta \) contains \( N \) galaxies.\(^{19}\) The factorial moments of this distribution are defined by \( F_k \equiv \sum_N P_N(N)k^\theta \), where \( (N)_k = N(N-1)\ldots(N-k+1) \) is the \( k \)th falling factorial of \( N \). The factorial moments are closely related to the moments of the underlying continuum random field (which is assumed Poisson-sampled by the galaxies), \( \rho = \langle N \rangle(1+\delta) \), through \( \langle (1+\delta)^k \rangle = F_k/\langle N \rangle^k \) (Szapudi & Szalay 1993), where angle brackets in the last relation denote an area average over cells of size \( \theta \). The factorial moments therefore provide a convenient way to estimate the angular connected moments, \( \sigma_p \equiv \langle \delta \rangle_c/\langle \delta^2 \rangle_c^{-1} \), where the subscript \( c \) denotes the connected contribution, and \( \langle \delta \rangle_c \) denotes the area average (over scale \( \theta \)) of the \( p \)-point angular correlation function. The moments \( s_3 \) (skewness) and \( s_4 \) (kurtosis) quantify the lowest order deviations of the angular distribution from a Gaussian.

To measure the \( \sigma_p \) amplitudes from CIC via factorial moments, we must estimate the distribution of CIC in the survey. We use the infinite oversampling algorithm of Szapudi (1997), which eliminates measurement errors due to the finite number of sampling cells. From the factorial moments, the recursion relation of Szapudi & Szalay (1993), where angle brackets in the last relation denote an area average over cells of size \( \theta \). The factorial moments therefore provide a convenient way to estimate the angular connected moments, \( \sigma_p \equiv \langle \delta \rangle_c/\langle \delta^2 \rangle_c^{-1} \), where the subscript \( c \) denotes the connected contribution, and \( \langle \delta \rangle_c \) denotes the area average (over scale \( \theta \)) of the \( p \)-point angular correlation function. The moments \( s_3 \) (skewness) and \( s_4 \) (kurtosis) quantify the lowest order deviations of the angular distribution from a Gaussian.

To measure the \( \sigma_p \) amplitudes from CIC via factorial moments, we must estimate the distribution of CIC in the survey. We use the infinite oversampling algorithm of Szapudi (1997), which eliminates measurement errors due to the finite number of sampling cells. From the factorial moments, the recursion relation of Szapudi & Szalay (1993), where angle brackets in the last relation denote an area average over cells of size \( \theta \). The factorial moments therefore provide a convenient way to estimate the angular connected moments, \( \sigma_p \equiv \langle \delta \rangle_c/\langle \delta^2 \rangle_c^{-1} \), where the subscript \( c \) denotes the connected contribution, and \( \langle \delta \rangle_c \) denotes the area average (over scale \( \theta \)) of the \( p \)-point angular correlation function. The moments \( s_3 \) (skewness) and \( s_4 \) (kurtosis) quantify the lowest order deviations of the angular distribution from a Gaussian.

The masks described in \( \S \, 2 \) represent the most serious practical problem for CIC estimation, since they generate a complex geometry for the survey area. In addition to the seeing masks, there are a large number of small cutout holes around bright stars, ghosts, etc., yielding about 20,000 separate mask regions. Since the cutout holes are distributed across the entire survey area, a randomly placed cell larger than about 0'1 on a side has a high probability of intersecting a mask. In the standard CIC technique, cells that overlap mask regions are discarded, since one has no information about the galaxy field in the masked portion of the cell. In principle, this would limit our analysis to cells smaller than \( \sim 0'1 \).

To remedy this situation, we follow Szapudi et al. (2001) and carry out the CIC analysis by ignoring all masks with area smaller than 0.0021 \( \text{deg}^2 \); this leaves only about 300 of the largest masks. Using only the large masks allows us to extend the measurements to cell sizes of order 1'. We test the validity of this prescription by comparing measurements of the \( s_p \) with all the masks and with only the large masks for cells smaller than 0'1. We find that we can reliably use just the large masks to make measurements on larger scales, since the discarded small masks occupy a falling fraction of the cell area as the cell size increases.

The errors on the \( s_p \) measurements shown in the figures were estimated by using the FORCE (Fortran for Cosmic Errors) package (Szapudi & Colombi 1996; Colombi, Szapudi, & Szalay 1998; Szapudi, Colombi, & Bernardeau 1999). This method provides the most accurate estimation of the errors from the data itself and takes into account the contributions from the six-point (eight-point) correlations to the errors on \( s_3 \) (\( s_4 \)). It includes error contributions from finite-volume effects, edge effects, and discreteness. The FORCE-estimated errors depend on a number of parameters, several of which can be accurately estimated from the data itself (e.g., the mean cell occupation number, the angular two-point function averaged over the area of a cell, and the effective area of the survey). With one exception, the remaining parameters are obtained from models; e.g., extended perturbation theory (Colombi et al. 1997) is used to estimate \( s_6 \) and \( s_8 \). As shown by Szapudi, Colombi, & Bernardeau (1999), the \( s_p \) errors are relatively insensitive to variations of these model parameters within their presently acceptable ranges. The final parameter needed is the estimate for the area-average of the two-point correlation function over the full survey region, which we have measured from 100 \( \Lambda \text{CDM} \) (cold dark matter) realizations (including galaxy bias) obtained with the PTHalos code (Scoccimarro & Sheth 2001). These simulations of the galaxy distribution are generated by using second-order Lagrangian perturbation theory to follow clustering at large scales and to identify dark matter halos, which are then replaced by nonlinear dark matter profiles to build the small-scale correlations. Galaxy distributions are obtained by sampling the dark matter profiles with a number of galaxies that depend on the halo mass \( m \), according to \( N_{\text{gal}}(m) \sim m^{0.8-0.9} \). This galaxy-halo scaling leads to an approximate match between the measured angular two-point function in early SDSS data and the \( \Lambda \text{CDM} \) model; see Scranton et al. (2001) for a detailed description of this scaling relation and comparison of the PTHalos angular two-point correlation with measurements in the data.

We also use the PTHalos realizations to independently estimate \( s_p \) errors and their covariance matrix by averaging over the Monte Carlo pool. Due to the significant computational cost required to measure higher order moments, however, we have only used this approach for the \( r = 18-19 \) and \( 19-20 \) magnitude slices, which have the fewest galaxies. Directly comparing the FORCE and PTHalos errors in this case for both \( s_3 \) and \( s_4 \), we found that they agree to better than 30\%, with the FORCE errors being larger (smaller) than the PTHalos errors for angular scales larger (smaller) than \( 0'05 \). Given the different assumptions that go into the two methods, this consistency is quite encouraging; while the FORCE uses as input the measured higher order correlations and assumes the hierarchical model (\( s_p = \text{const.} \) independent of \( \theta \)), the PTHalos estimate is based on correlations built from \( \Lambda \text{CDM} \) models plus a bias prescription which does not necessarily lead to \( s_p = \text{const.} \) (see, e.g., the solid curves in Fig. 2 below). In addition, the elongated geometry of the survey region and the large number of masks present an extra challenge for the FORCE method; the agreement between the PTHalos estimates for the two bright slices indicates that the FORCE maintains accuracy under these conditions.

\(^{19}\) We are using square-shaped cells, which yield virtually indistinguishable results from circular cells of the same area.
The FORCE error estimation is based on a series expansion: when the relative errors approach unity, it breaks down. When this occurs, the FORCE correctly indicates that the errors are qualitatively large, but the actual size of the error bars has no precise statistical meaning in this regime. Finally, while the FORCE method is able to calculate the cosmic bias of the estimators of $s_p$ (Hui & Gaztañaga 1999; Szapudi, Colombi, & Bernardeau 1999), its effects are always negligible in the regime where the FORCE errors are accurate (i.e., smaller than $\sim 100\%$); therefore we do not include a cosmic bias estimate.

It should be borne in mind that error estimation using the FORCE or the PTHalos mock catalogs each represent model-dependent statistical error calculations. This is unavoidable, since the errors are determined in part by galaxy clustering over scales larger than the survey region, which we cannot directly measure. The accuracy of the error estimates is determined by the difference between the assumed model and the actual distribution. In addition, these estimates do not include possible systematic errors in the data; however, the tests described in Scranton et al. (2001), although only performed for two-point statistics, suggest the latter are not dominant over the scales we consider. The FORCE error estimates are easily obtained for faint magnitudes where the PTHalos estimates require costly computational resources due to the relatively large number of galaxies. On the other hand, the PTHalos method can deal trivially with complex masks and edge effects; in addition, it provides the covariance matrix of the errors, essential for testing models since measurements at different scales are correlated.

4. MEASUREMENTS OF COUNTS IN CELLS STATISTICS

We have carried out a series of measurements of CIC statistics in the early SDSS angular clustering data. We first present our principal results for the normalized connected moments $s_3$ and $s_4$ and their estimated statistical errors. To ascertain the reliability of these results, we have performed a number of auxiliary measurements aimed at quantifying the possible level of systematic errors. The most important of these tests are discussed in § 4.2.

4.1. Principal Measurements

Figure 1 shows the main results of this paper, the measurement of the angular amplitudes $s_3$ (triangles) and $s_4$ (squares) in the early SDSS data in four apparent magnitude bins. The error estimates were made using the FORCE method, as described in § 3. The open symbols show the results using all the masks of Scranton et al. (2001); as the figure shows and as described in § 3, these measurements cannot be extended to cells much larger than 0.1$''$. The solid symbols show the results when only the large masks are used; on scales smaller than 0.1$''$, these estimates are essentially identical to those obtained with all the masks. As argued in § 3, this agreement indicates that the measure-

![Fig. 1.](image-url)
ments on larger angular scales using only the large masks should be reliable; as a result, the measurements can be extended to scales as large as $\approx 1.5$, about half the width of the data stripe. The exception to this argument is the measurement of $s_3$ in the faintest bin, $21 < r < 22$: in this case, there appears to be a systematic discrepancy between the all-mask and large-mask measurements, even on small scales at approximately $2\sigma$ level. We have checked that this is not due to objects masked out by the full mask, i.e., the results are not influenced by keeping or omitting objects masked out by the full mask. All calculations were repeated with a catalog which was first filtered through the full bright and faint mask, respectively: partial masks on the filtered and unfiltered catalogs produce identical results.

To check the accuracy of the FORCE error estimates (in the presence of the complex masked geometry) and to compare the SDSS results with the predictions of structure formation models, we show in Figure 2 the results for $s_3$ and $s_4$ for the brightest flux slice, $18 < r < 19$, in 11 logarithmically spaced angular bins in the range $0.01468$–$176813$ (filled symbols, using the large masks). We also show the predictions based on 100 mock catalogs with a radial selection function chosen to match that of the data (Dodelson et al. 2001), based on the PTHalos code for a $\Lambda$CDM model (with $\sigma_8 = 0.84$, $\Omega_m = 0.3$, $\Omega_\Lambda = 0.7$, $\Gamma = 0.21$ at $z = 0$) with galaxies generated as briefly described in § 3 (see Scranton et al. 2001 for details). The parameters of this model were chosen in order to reproduce the angular two-point function $w(\theta)$ measured in the same SDSS data by Scranton et al. (2001) and Connolly et al. (2001). The solid curves show the mean of the 100 realizations, and the dotted curves show the $\pm 1\sigma$ deviations. The agreement between the variance estimates on $s_3$ and $s_4$ from FORCE and PTHalos is evident on scales $0.03 < \theta < 0.3$. As noted above, on scales larger than $\sim 0.3$,

![Figure 2](image-url)  

**Fig. 2.**—Results for $s_3$ (triangles) and $s_4$ (squares) for $18 < r < 19$, with FORCE error bars. Solid curves show the mean results from 100 PTHalo simulations for biased $\Lambda$CDM with galaxy biasing chosen to match the angular two-point correlation function (see § 3). We have used the same survey geometry and selection function as the SDSS data. Dotted curves indicate the $1\sigma$ deviations among the simulations.

where the relative errors are of order 100% of the value or larger, the FORCE error estimates are not expected to be numerically accurate. On scales smaller than 0.03, the mock catalogs predict somewhat higher variance than the FORCE. In any case, the overall agreement between these error estimates indicates the robustness of the results.

Figure 2 also indicates that the measured $s_3$ and $s_4$ amplitudes are in reasonably good agreement with predictions for galaxies in the biased $\Lambda$CDM model described above. To be more specific, we computed the goodness of fit of the $\Lambda$CDM model to the data using the full covariance matrix from the simulations; the resulting reduced $\chi^2$ values are given in the second and third columns of Table 1. Given that this model was not explicitly constructed to give a good fit to $s_3$ and $s_4$, it is in very good agreement with the data. The amplitude of higher order correlations $s_3$ and $s_4$ are inconsistent with galaxies tracing dark matter at small scales, as this will predict much higher amplitudes than observed. We verified using mock catalogs that the Gaussian assumption for the likelihood function is a very good approximation on small scales, where most of the information is coming from.

In addition, we performed a fit of the SDSS results to a hierarchical model (HM), in which $s_3$ and $s_4$ are constrained to be constants, independent of cell size, again using the covariance matrix with galaxies tracing dark matter at small scales, as this will predict much higher amplitudes than observed. We verified using mock catalogs that the Gaussian assumption for the likelihood function is a very good approximation on small scales, where most of the information is coming from.

![Table 1](image-url)

**Table 1**

| $r$     | $\chi^2_{\Lambda CDM}(s_3)$ | $\chi^2_{\Lambda CDM}(s_4)$ | $\chi^2_{HM}(s_3)$ | $\chi^2_{HM}(s_4)$ | $\chi^2_{HM}(s_3)$ | $\chi^2_{HM}(s_4)$ |
|--------|-----------------------------|-----------------------------|--------------------|--------------------|--------------------|--------------------|
| 18–19  | 1.4                         | 1.4                         | 4.9 $\pm$ 0.4      | 0.7                | 42 $\pm$ 11        | 1.2                |
| 19–20  | 1.8                         | 1.6                         | 4.1 $\pm$ 0.4      | 0.8                | 38 $\pm$ 9         | 1.1                |
| 20–21  | ...                         | ...                         | 4.2 $\pm$ 0.4      | 0.7                | 41 $\pm$ 10        | 0.7                |
| 21–22  | ...                         | ...                         | 3.6 $\pm$ 0.3      | 0.8                | 41 $\pm$ 10        | 0.7                |

Note.—The first two columns give the $\chi^2$ per degree of freedom (d.o.f.) for the comparison of the SDSS data with the $\Lambda$CDM model. The remaining columns give the best-fitted constant values for $s_3$ and $s_4$ (hierarchical model, HM) and their goodness of fit. All fits use the covariance of the measurements between different angular scales estimated from the mock catalogs.

$^{20}$ Typically the cross-correlation coefficient between neighboring bins is larger than 0.7 and drops to 0.5 for angular scales separated by a factor of 4.

$^{21}$ For the faint magnitude slices, $r = 20–21, 21–22$, for which no explicit covariance matrix from PTHalos was available due to CPU limitations, we used the cross-correlation coefficient $(r_{ij} = C_{ij}/(C_{ii}C_{jj})^{1/2})$ from the $r = 19–20$ catalogs and scaled up this matrix by the ratio of the diagonal $(C_{ii})$ FORCE errors for the different simulation slices. The Ansatz was tested by using it to scale from the 18–19 to the 19–20 slices, in which case it accurately reproduced the measured covariance matrix for the fainter slice. Here $C_{ij}$ is the covariance matrix of the $s_p$ estimator for cell sizes $\theta_i$ and $\theta_j$.
a first exercise at this, we have crudely divided the sample described above by morphological type: “ellipticals” designate objects for which the de Vaucouleurs profile fit is of higher likelihood than the exponential profile according to the PHOTO pipeline, and “spirals” constitute the remainder. We have made no attempt to assess the reliability of this classification scheme as a function of magnitude or to correlate it with other photometric properties. The resulting $s_p$ measurements for ellipticals (open symbols) and spirals (closed symbols) are shown in Figure 3. These results are suggestive: for example, the tendency for higher clustering amplitudes for the ellipticals, particularly on small scales, is at least qualitatively consistent with the well-known morphology-density relation. However, there is no clearly discernible trend in the results through our magnitude slices, and the differences are, for the most part, not statistically significant, given the errors on this relatively small sample. While we cannot draw firm conclusions from this first assay, it is evident that the full SDSS data set should provide an excellent sample for studying higher order clustering as a function of galaxy type, especially if colors and spectral types are used in the selection (e.g., Iveic et al. 2001).

4.2. Auxiliary Measurements: Checks for Systematic Errors

We now describe a number of tests carried out in order to gauge possible systematic effects on the results. While the tests described by Scranton et al. (2000), although performed for two-point statistics, should imply the reliability of higher order statistics measurements, some extra checks are warranted.

In the results shown above, we used the SDSS image processing pipeline to perform star-galaxy separation: the resulting catalog contained all objects selected as galaxies by PHOTO. As noted in §2, the Bayesian star-galaxy separation described in Scranton et al. (2001) provides a more accurate method at faint magnitudes by assigning stellar and galactic probabilities to each object. To test the effects of possible stellar contamination of the PHOTO-selected sample, we have repeated the $s_p$ measurements using the Bayesian probabilities by constructing a catalog containing all objects with galactic probability $p_{gal} \geq 0.99$. The results, shown in Figure 4, are almost indistinguishable from the PHOTO-selected results of Figure 1. In the faintest magnitude slice, there appears to be a small difference in the results, but it is not statistically significant. This test indicates that stellar contamination of the sample is not a significant source of systematic error in the measurement of $s_3$ and $s_4$.

Another potential source of systematic error is incomplete masking of areas of poor-quality data, particularly bad seeing. The results shown above used the masks recommended by Scranton et al. (2001): the bright mask (used for $18 < r < 21$) excludes regions with seeing >1.75, and the more conservative faint mask (used for $21 < r < 22$) excludes regions where the seeing >1.8. Scranton et al. (2001) showed that these cuts yield a galaxy data set with negligible cross-correlations with systematic effects such as.

![Figure 3](image_url)

**Fig. 3.**—Measurements of $s_3$ and $s_4$ for “elliptical” (open symbols) and “spiral” galaxies (closed symbols) in early SDSS data. Only measurements with the large masks are shown, and error bars are omitted for clarity. Objects in this sample were selected to have $p_{gal} \geq 0.99$, according to the Bayesian star-galaxy separator (see §4.2).
stellar density, seeing, and dust extinction. As a further test, we have repeated the \( s_p \) estimates by excluding data with seeing \( > 1.6 \) in all four magnitude bins, i.e., applying the more stringent faint seeing mask to the brighter \( (18 < r < 21) \) data as well. The results are shown in Figure 5: solid symbols indicate the bright seeing mask of \( 1'75 \), and open symbols indicate results for the more conservative faint seeing mask of \( 1'6 \). In both cases, we show results using only the large masks. There is evidence for small systematic shifts in the \( s_p \) amplitudes between the two masks, particularly for the two brightest slices; we speculate that the reduced effective survey area for the data with the faint mask could lead to a small cosmic bias (a systematic underestimate) in the estimate of the \( s_p \). However, the more important conclusion from this comparison is that almost all the changes are well within the 1 \( \sigma \) errors, indicating that systematic errors due to incomplete masking are well within the statistical error bars. (Note that the \( s_p \) estimates at different \( \theta \) are correlated, so they cannot simply be combined to increase the significance of any differences.) To further test the effect of masks, we also computed the higher order moments without using any mask at all. As shown in Figure 6, this results in an underestimate of \( s_3 \) and \( s_4 \) comparable to the differences seen in Figure 5. This should represent a conservative upper limit on possible systematic errors due to incorrect masking.

As a further check, we have used the 100 PTHalos simulations of the \( 18 < r < 19 \) and \( 19 < r < 20 \) magnitude slices to check if the masks introduce any systematic errors into our measurements. We have performed three measurements in each mock SDSS catalog: (1) with no mask, (2) with the bright (large) masks, and (3) with the faint (large) masks, using the same CIC method as applied to the data. As expected, the error bars were larger for the samples with more masked regions (smaller effective survey area), with the effect being more significant on large angular scales. On most scales, there is no systematic shift in the mean values of \( s_p \) (no bias) between the three cases, although there is a slight bias toward lower \( s_p \) values on the very largest scales for the samples with more masked regions. As speculated above, it is possible that an effective cosmic bias due to the skewness of the distributions of \( s_3 \) and \( s_4 \) values leads to a systematic underestimate of these amplitudes at large scales when the masked area is increased. We have attempted to quantify this effect from the mock catalogs, but the relatively small number of realizations (100) precludes us from reaching a definite conclusion. In any case, the amplitude of the bias of the \( s_p \) estimates is negligible compared to the measured variance. We conclude that no appreciable bias in the estimates of \( s_3 \) and \( s_4 \) can be attributed to the complicated geometry of the survey region.

5. COMPARISON WITH PREVIOUS MEASUREMENTS

It is instructive to compare the early SDSS results for \( s_3 \) and \( s_4 \) with previous measurements. To compare with results from the APM and EDSGC surveys as well as with the early SDSS results reported by Gaztanaga (2001a), we have measured CIC statistics for SDSS galaxies in the 752/756 data set with 15.9 < \( r < 18.9 \). This choice is based on that of Gaztanaga (2001a), who demonstrated that the
apparent magnitude range $15.9 < g < 18.9$ in the SDSS yields a galaxy surface density comparable to the $b_J < 20$ samples of the APM and EDSGC surveys. The $r$-selected sample used here will have a slightly different effective depth than his $g$-selected sample, but $s_3$ and $s_4$ should be relatively insensitive to this. A more substantive difference likely arises from the fact that samples selected in different passbands will contain different mixes of galaxy types and therefore yield different clustering amplitudes (see Fig. 3); as a result, we do not expect these samples to yield identical higher order moments, but they should be qualitatively similar.

Figure 6 shows the results of this comparison for $s_3$ (triangles and lower curves) and $s_4$ (squares and upper curves) for the SDSS data with large masks (filled symbols with error bars) compared with results from the APM (dotted curves) and EDSGC (solid curves) surveys. Open symbols without error bars show the SDSS results when the masks are not used.

![Figure 5](image1.png)

*Fig. 5.* — Skewness ($s_3$) and kurtosis ($s_4$) measured with the Scranton et al. (2001) masks (filled symbols) compared with results using the more stringent 1.76 faint seeing mask for the three brightest flux slices (open symbols). The differences are well within the statistical errors. For clarity, error bars are only plotted for the principal measurements.

![Figure 6](image2.png)

*Fig. 6.* — Skewness ($s_3$) and kurtosis ($s_4$) measurements in the SDSS for $15.9 < r < 18.9$ (filled symbols with FORCE error bars) compared with results from the APM (dotted curves) and EDSGC (solid curves) surveys. Open symbols without error bars show the SDSS results when the masks are not used.
plates, the differences between their results on small scales must be largely due to systematic effects in one or both of these catalogs. As noted by Szapudi & Gaztañaga (1998), the APM (EDSGC) amplitudes are likely biased low (high) by their different methods of deblending objects in crowded fields. For the kurtosis, $s_4$, the SDSS results on small scales are also intermediate between the other two surveys, although somewhat closer to the APM values. Overall, the consistency between the measurements of the higher order moments in these different data sets is encouraging.

The results for $s_3$ in the SDSS shown in Figure 6 appear to be in qualitative agreement with those obtained by Gaztañaga (2001a) on scales smaller than 0.1, but they are systematically higher than his results on larger scales. There are several possible reasons for this discrepancy, and they may act in combination. The data in the survey region used in his analysis (runs 94/125, southern equatorial stripe), which does not overlap the region analyzed here, was taken for the most part in relatively poor seeing conditions early in the commissioning phase of the project. Application of the bright seeing cut (1.75) recommended by Scranton et al. (2001) for the removal of systematic effects would mask out roughly 37% of the 94/125 area. Along with the use of an undersampled CIC method, not including the masks could lead to an underestimate of $s_3$ (Szapudi, Meiksin, & Nichol 1996; Szapudi & Gaztañaga 1998). As an example, the open symbols in Figure 6 show the results for $s_3$ and $s_4$ for the 752/756 data set when the bright mask, which covers about 20% of this data region, is not used. In addition to the mask, cosmic variance could also contribute to the difference in $s_3$ results seen on large scales. Other small differences in the data selection between Gaztañaga (2001a) and this paper ($g$ vs. $r$ selection, use of uncorrected versus extinction-corrected magnitudes, use of Petrosian versus model magnitudes) are probably less significant and unlikely to account for the difference in results. In fact, as we were about to submit this paper for publication, Gaztañaga (2001b) appeared, which also included an analysis of $s_3$ for the early data release north slice, which we analyze in this paper. His results for this data slice are remarkably consistent with the open triangles in Figure 6, in which the mask is not used.

We turn next to comparison of the SDSS results with measurements in the Deeprange survey (Szapudi et al. 2000). The Deeprange catalog was selected in $I$; for a typical galaxy spectral energy distribution, and assuming a mixture of early and late types, the approximate correspondence between SDSS $r$ and $I$ band is $r_{sdss} \approx I_{Deeprange} + 0.9$ (M. Postman 2001, private communication). We therefore constructed new SDSS samples in the $r$ ranges 17.9–18.9, 18.9–19.9... to compare with the Deeprange samples of $I = 17–18$, 18–19... These SDSS samples are shifted by just 0.1 mag from those analyzed in § 4. We note that the Deeprange magnitudes were not corrected for extinction, which should be negligible over this small, high-latitude field. The results of this comparison for $s_3$ and $s_4$ are shown in Figure 7, with solid symbols corresponding to the SDSS

![Figure 7](https://example.com/fig7.png)

**Fig. 7.**—Skewness ($s_3$) and kurtosis ($s_4$) for the Deeprange survey (open symbols), in $I$-band magnitude slices 17–18...20–21, compared with SDSS results in the approximate corresponding $r$ slices 17.9–18.9...20.9–21.9 (solid symbols). For each survey, the appropriate large masks were used, and the error bars were calculated by the FORCE package.
data and open symbols to the Deeprange. The figure panels are labeled by the Deeprange J-band range, e.g., 1 19–20 is compared with \( r = 19.9–20.9 \) in the SDSS.

As expected, comparison of Figure 7 with Figure 1 shows that the 0.1 mag shift in the SDSS magnitude ranges causes very little change in the measured \( \sigma_t \) s. The SDSS results for \( \sigma_t \) are in excellent agreement with the Deeprange values in the three brightest slices, while the values are \( \sim 1–2 \sigma \) discrepant in the faintest SDSS slice. This suggests that the early SDSS data represents the most accurate measurement to date of the angular skewness for magnitudes \( r < 21 \). For \( \sigma_t \), there is also qualitatively good agreement between the surveys for the three brighter magnitude slices, with a few discrepant points; this is not surprising, since the kurtosis is more sensitive to both systematic errors and cosmic variance than the skewness.

The comparison for the faintest SDSS slice \( r = 21–22 \) is qualitatively different from the others: while the Deeprange data indicate a systematic falling off of the amplitudes with increasing sample depth, this trend is either absent or not significant in the SDSS data (see also Table 1). Given possible differences due to cosmic variance or undetected systematic errors, the current data cannot decide between these two trends. The answer, which has important implications for the evolution of galaxy bias, should come when a larger SDSS sample is available.

6. CONCLUSION

We have measured the moments of angular counts in cells (CIC) in early SDSS data, using the northern equatorial stripe. Although the results presented here focus on the higher order moments, we have also checked that the measured second order moments from CIC are consistent with the direct measurements of the angular two-point correlation function presented in Connolly et al. (2001) and Scarron et al. (2001). To control statistical and systematic errors, we have used state-of-the-art measurement techniques and the highest quality segment of the available data. This resulted in a set of complicated masks, which probably does influence the accuracy of our measurement via edge effects. We have shown with auxiliary measurements and simulations that this should not produce any significant bias, only extra variance. According to Figure 4, star-galaxy separation has insignificant contaminating effects on the \( \sigma_t \) measurements, except perhaps for the faintest magnitude slice. These results suggest that the SDSS imaging data are free of systematics to a high degree and will therefore enable determination of the skewness and kurtosis to 1% and less than 10%, as predicted by Colombi, Szapudi, & Szalay (1998).

Figures 1–2 and Table 1 show that the \( \sigma_t \) values behave qualitatively as expected from theories of hierarchical structure formation. The \( \sigma_t \) and \( \sigma_s \) amplitudes display an approximate hierarchical scaling, with deviations consistent with cosmic (co)variance. As quantified in Table 1, the measurements of \( \sigma_t \) and \( \sigma_s \) are consistent with the predictions of non-linear CDM models, in which the galaxy bias is determined by the fact that the number of galaxies in halos of mass \( m \) scales as \( N_{gal} \propto m^{0.8–0.9} \). While this conclusion should be regarded as preliminary given the limitations of the current data set and the restricted number of models considered, it is worth noting that a similar model matches the APM power spectrum and \( \sigma_t \) values on small scales (Scoccimarro et al. 2001).

We have also presented preliminary results for \( \sigma_t \) and \( \sigma_s \) for galaxies separated by morphological type into ellipticals and spirals based on profile fits to the images by the PHOTO pipeline. As expected from the morphology-density relation, it appears that ellipticals are more strongly clustered than spirals; the robustness of this result remains to be demonstrated with a larger sample and a more refined method of morphological classification.

The Sloan Digital Sky Survey (SDSS)\(^22\) is a joint project of The University of Chicago, Fermilab, the Institute for Advanced Study, the Japan Participation Group, The Johns Hopkins University, the Max-Planck-Institute for Astronomy (MPIA), the Max-Planck-Institute for Astrophysics (MPA), New Mexico State University, Princeton University, the United States Naval Observatory, and the University of Washington. Apache Point Observatory, site of the SDSS telescopes, is operated by the Astrophysical Research Consortium (ARC).\(^23\)

Funding for the project has been provided by the Alfred P. Sloan Foundation, the SDSS member institutions, the National Aeronautics and Space Administration, the National Science Foundation, the US Department of Energy, the Japanese Monbukagakusho, and the Max Planck Society. I. S. thanks Stephane Colombi for useful discussions and was partially supported by a NASA AISR, NAG-10750.

\(^{22}\) The SDSS Web site is http://www.sdss.org/.

\(^{23}\) We also acknowledge the use of the FORCE (FORtran for Cosmic Errors) package by S. Colombi and I. S., available from http://www.ifa.hawaii.edu/szapudi/istvan.html.

REFERENCES

Bardeen, J. R., Bond, J. R., Kaiser, N. & Szalay, A. S. 1986, ApJ, 304, 15

Bernardeau, F. 1992, ApJ, 292, 1

Bouchet, F. R., Strauss, M. A., Davis, M., Fisher, K. B., Yahil, A., & Huchra, J. P. 1993, ApJ, 417, 36

Chodorowski, M., & Bouchet, F. R. 1996, MNRAS, 279, 557

Colombi, S., Bernardae, F., Bouchet, F. R., & Hernquist, L. 1997, MNRAS, 287, 241

Colombi, S., Bouchet, F. R., & Schaeffer, R. 1995, A&A, 281, 301

Colombi, S., Szapudi, I., Jenkins, A., & Colberg, J. 2000, MNRAS, 313, 711

Colombi, S., Szapudi, I., & Szalay, A. S. 1998, MNRAS, 296, 253

Connolly, A. J., et al. 2001, ApJ, submitted (astro-ph/0107417)

Davis, M., Efstathiou, G., Frenk, C. S., & White, S. D. M. 1985, ApJ, 292, 371

Dodelson, S., et al. 2001, ApJ, submitted (astro-ph/0107418) 23508

Durrer, R., Juszkiewicz, R., Kunz, M., & Uzan, J-P. 2000, Phys. Rev. D, 62, 021301

Feldman, H. A., Fry, J. N., Frieman, J., & Scoccimarro, R. 2001, Phys. Rev. Lett., 86, 1434

Frieman, J. A., & Gaztanaga, E. 1994, ApJ, 425, 392

———. 1999, ApJ, 521, L83

Fry, J. N. 1984, ApJ, 279, 499

———. 1999, ApJ, 521, L83

Fry, J. N., & Scherrer, R. J. 1994, ApJ, 429, 36

Fukugita, M., et al. 1996, AJ, 112, 3010

Gaztanaga, E. 1993, ApJ, 417, 36

Gaztanaga, E., & Mäho¨nen, P. 1996, ApJ, 462, L1

Gunn, J. E. 1998, AJ, 111, 1718

Hui, L., & Gaztanaga, E. 1999, ApJ, 519, 622

Ivezic, Z. et al. 2001, AJ, 122, 2749

Jaffe, A. H. 1994, Phys. Rev. D, 49, 3893
