The $s\bar{s}s\bar{s}$ tetraquark states and the newly observed structure $X(2239)$ by BESIII Collaboration

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We investigate the mass spectrum of the $s\bar{s}s\bar{s}$ tetraquark states within the relativized quark model. By solving the Schrödinger-like equation with the relativized potential, the masses of the $S^−$ and $P^−$ wave $s\bar{s}s\bar{s}$ tetraquarks are obtained. The screening effects are also taken into account. It is found that the newly observed resonant structure $X(2239)$ in the $e^+e^− \to K^+K^−$ process by BESIII Collaboration can be assigned as a $P^−$ wave $1^−−\ s\bar{s}s\bar{s}$ tetraquark state. Furthermore, the radiative transition of this structure is also estimated, and the 27 keV radiative decay width can provide helpful information for future experimental searches.

PACS numbers:
Keywords: Tetraquark; Spectrum; Radiative decay; Relativized quark model

I. INTRODUCTION

Recently, the BESIII Collaboration analyzed the cross section of the $e^+e^− \to K^+K^−$ process at center-of-mass energies varying from 2.00 to 3.08 GeV. A resonant structure is observed in the line shape, which has a mass of $2239.2 ± 7.1 ± 11.3$ MeV and a width of $139.8 ± 12.3 ± 20.6$ MeV [1]. Given its production process, the quantum number of this resonant structure can be assigned as $J^{PC} = 1^{−−}$.

From the Review of Particle Physics, there exist four $J^{PC} = 1^{−−}$ observed states around 2.2 GeV, such as $φ(2170), ρ(2150), ω(2205)$ and $ρ(2270)$ [2]. The $φ(2170)$ state with $I^G(J^{PC}) = 0^−(1^{−−})$, labeled previously as $Y(2175)$, has been investigated within many theoretical interpretations, which include conventional $s\bar{s}$ state [3–7], hybrid state [3–8], tetraquark state [9–14], $ΛΛ(3S_1)$ bound state or hexaquark state [15–19], and $φKK$ resonance state [20–21]. The $ρ(2150), ω(2205)$ and $ρ(2270)$ are also studied as conventional radial or orbital excited mesons in the consistent quark model [22–24]. As mentioned by BESIII Collaboration, the newly observed resonant structure, denoted as $X(2239)$ in present work, differs from the masses and widths of the $φ(2170)$ and $ρ(2150)$, which seems to be a new resonance [1]. The $ω(2205)$ and $ρ(2270)$ listed in the further states are both broad states, which can not be the same structure as this newly observed one [2].

In the conventional quark model, several highly excited $1^{−−}$ $ρ, ω$, and $φ$ states are predicted in this energy region, and their strong decay behaviors have been investigated in the quark pair creation model. Due to the large phase space, the predicted total decay width of these states are rather broad, which suggests that the newly observed state with about 140 MeV width may not be a conventional excited meson. More exotic interpretations, such as the $s\bar{s}s\bar{s}$ tetraquark state, are needed to be considered to clarify its nature. In the literature, the $P^−$ wave $s\bar{s}s\bar{s}$ system was mostly investigated by the QCD sum rule method [25–27] or the simple quark models [13–14], and their results are inconsistent with each other. Hence, it is essential to study this system in a more realistic potential model.

In this work, we firstly employ a relativized quark model to estimate the masses of $s\bar{s}s\bar{s}$ tetraquark states. The relativized quark model, proposed by Godfrey, Capstick, and Isgur, has been widely used to study the properties of the conventional hadrons and gives a unified description of the traditional hadron spectra [24–26]. Also, it has been extended to investigate various tetraquark systems, such as $ΩqΩ̅q$, $Ωqq$ and so on [37–39]. Moreover, the relativistic effects are involved in this model, which may be essential for the up, down and strange quarks. Therefore, it is suitable to deal with the $s\bar{s}s\bar{s}$ tetraquark states, where strange quarks and antiquarks are included. To calculate the tetraquark masses, we restrict our works in the diquark-antidiquark picture [37–38]. We first calculate the masses and wave functions of the axial-vector and vector $s\bar{s}$ diquarks, and then obtain the mass spectra and diquark-antidiquark wave functions by solving the Schrödinger-type equation between the diquark and antidiquark. The total wave functions can be expressed as the multiplication of the diquark, antidiquark, and diquark-antidiquark wave functions. The predicted mass of the lowest $1^{−−}$ $s\bar{s}s\bar{s}$ tetraquark is 2227 MeV, which is consistent with the experimental data $2239.2 ± 7.1 ± 11.3$ MeV by BESIII Collaboration. It suggests that the newly observed resonant structure $X(2239)$ can be assigned as the lowest $J^{PC} = 1^{−−}$ $s\bar{s}s\bar{s}$ tetraquark state. Furthermore, using the wave functions obtained from the relativized quark model and the electromagnetic transition operator, we estimate the radiative decays of the $s\bar{s}s\bar{s}$ tetraquarks. It is found that the radiative decay width for the lowest $1^{−−}$ $s\bar{s}s\bar{s}$ tetraquark state is 27 keV, which is significant and useful for future experimental searches.

This paper is organized as follows. In Sec. II the relativized quark model is briefly introduced, and the masses of $s\bar{s}s\bar{s}$ tetraquark states are calculated. In Sec. IV the radiative transitions of $s\bar{s}s\bar{s}$ tetraquark states are numerically estimated.
Finally, we give a short summary in the last section.

II. MASS SPECTRUM

The Hamiltonian between the quark and antiquark in the relativized quark model can be expressed as

\[ \tilde{H} = H_0 + \tilde{V}(p, r), \]

with

\[ H_0 = (p^2 + m_f^2)^{1/2} + (p^2 + m_i^2)^{1/2}, \]

\[ \tilde{V}(p, r) = \tilde{H}_{ij}^{conf} + \tilde{H}_{ij}^{cont} + \tilde{H}_{ij}^{ten} + \tilde{H}_{ij}^{so}, \]

where the \( \tilde{H}_{ij}^{conf} \) includes the spin-independent linear confinement and Coulomb-like interaction, the \( \tilde{H}_{ij}^{cont} \), \( \tilde{H}_{ij}^{ten} \), and \( \tilde{H}_{ij}^{so} \) are the color contact term, the color tensor interaction, and the spin-orbit term, respectively. The \( \tilde{H} \) represents these interactions and the details of this relativization scheme via the relativized procedure. The explicit forms of these interactions and the details of this relativization scheme can be found in Ref. [26, 27]. In the original GI model, the coupled channel or screening effects are ignored, which may influence on the properties of the excited mesons and tetraquarks [33, 37, 43, 50]. The modified procedure \( br \rightarrow b(1 - e^{-\mu r})/\mu \) with a new screening parameter \( \mu \) performs a better description of meson and tetraquark spectra, especially for the strange quark systems [33, 57]. Hence, we take the screening effects into account for completeness.

In present work, only the antitriplet diquark \([\bar{3}, 1]_{ss}\) are considered, while the \([6, 1]_{ss}\) type diquarks cannot be formed in the GI quark model. For the quark-quark interaction in the antitriplet diquark and triplet antidiquark systems, the relation \( \tilde{V}_{ss}(p, r) = \tilde{V}_{\bar{3}3}(p, r) = \tilde{V}_{\bar{3}3}(p, r)/2 \) is employed. The parameters used in our calculations are the same as the ones in the original work [28]. The structures of the \( ss\bar{s}\bar{s} \) tetraquarks are illustrated in Fig. 1. The interaction between diquark and antidiquark \( \tilde{V}_{ss, \bar{3}3}(p, r) \) equals to the quark-antiquark interaction \( \tilde{V}_{ss}(p, r) \). The ground \( ss \) diquark lies in \( S \)-wave, and has the spin-parity \( J^P = 1^+ \) named as axial-vector diquark. For the excited ones, we only consider the \( P \)-wave \( ss \) diquark with spin-parity \( J^P = 1^- \), which is denoted as vector diquark.

Here, we use the Gaussian expansion method to solve the Hamiltonian [1] with \( \tilde{V}_{ss}(p, r) \) potential [51]. The obtained masses of the axial-vector and vector \( ss \) diquarks are presented in Table. I. Since the \( \mu = 0.02 \text{ GeV} \) case can give a rather better description of the strange quark systems [33, 37], we would like to use the diquark masses at this value to calculate the masses and wave functions of \( ss\bar{s}\bar{s} \) tetraquarks.

With the diquarks listed in Table. I, one can calculate the masses of the \( ss\bar{s}\bar{s} \) tetraquarks and the wave functions between diquarks and anti-diquarks. Then, the total wave function of the \( ss\bar{s}\bar{s} \) tetraquark can be expressed as the multiplication of the diquark, anti-diquark and relative wave functions. The masses of \( ss\bar{s}\bar{s} \) tetraquark states composed of the \( A \) and \( V \) diquarks and antidiquarks are presented in Tab. II and Fig. 2.

The predicted mass of the lowest \( 0^{++} \) state is 1716 MeV, which is consistent with the \( f_0(1710) \) state. For the \( 1^{+} \) \( ss\bar{s}\bar{s} \) state, only \( h_1(1965) \) state listed in the PDG book lies in this energy region [2]. Since the \( h_1(1965) \) was observed in \( \omega\eta \) and \( \omega\pi\pi \) final states, which disfavors its assignment as \( 1^{+} ss\bar{s}\bar{s} \) tetraquark. In Ref. [12, 25], the authors suggest that the new structure \( X(2063) \) observed in the \( J/\psi \rightarrow \phi\eta' \) by BESIII Collaboration [52] is a \( 1^{+} ss\bar{s}\bar{s} \) tetraquark candidate within the QCD sum rule method. However, our calculated mass is 100 MeV lower than the experimental mass, which does not support this interpretation. Considering the mass, spin, parity, and \( \phi\phi \) decay mode of the \( f_2(2300) \), we may assign it as a \( 2^{++} ss\bar{s}\bar{s} \) tetraquark state.

For the \( P \)-wave \( ss\bar{s}\bar{s} \) tetraquarks, we predict three \( 1^{--} \) states. The lowest one has the internal excitation in the diquark or antidiquark, while the others have the relative excitations between diquarks and antidiquarks. The three \( 1^{--} \) states together with other theoretical works are listed in Tab. III for comparisons. It can be seen that our quark model classification is significantly different with the QCD sum rule works [9–11, 25], and the authors did not consider the internal excitation of the diquark or anti-diquark within the simple quark model [13]. The predicted lowest one has a mass of 2227 MeV, which agrees well with the \( X(2239) \) observed by BESIII Collaboration [11]. The experimental mass of the \( \phi(2170) \) is about 50 MeV lower than our calculation, which can not be excluded as the \( ss\bar{s}\bar{s} \) tetraquarks. The evidences of these other
TABLE II: Masses of the $ss\bar{s}\bar{s}$ tetraquark states composed of the $A$ and $V$ diquarks and antidiquarks. For the $V$ diquark and $A$ antidiquark case, the linear combinations together with $V$ diquark and $A$ antidiquark are understood to form the eigenstates of charge conjugation $^{35\,38}$. The units are in MeV.

| $J^{PC}$ | Diquark | Anti-diquark | $S$ | $L$ | Mass | Candidate |
|----------|---------|-------------|-----|-----|------|-----------|
| $0^{++}$ | $A$     | $\bar{A}$  | 0   | 0   | 1716 | $f_0(1710)$ |
| $1^{++}$ | $A$     | $\bar{A}$  | 1   | 0   | 1960 |           |
| $2^{++}$ | $A$     | $\bar{A}$  | 2   | 0   | 2255 | $f_2(2300)$ |

TABLE III: The predicted three $1^{--}$ tetraquarks states together with other theoretical works. The units are in MeV.

| $J^{PC}$ | Ours | SQM $^{13}$ | QSR $^{9}$ | QSR $^{10}$ | QSR $^{11}$ | QSR $^{25}$ |
|----------|------|-------------|-------------|-------------|-------------|-------------|
| $1^{--}$ | 2227 | 2210 $^{\pm}$ 90 | 2300 $^{\pm}$ 400 | 2340 $^{\pm}$ 170 | 3080 $^{\pm}$ 110 |
| $1^{--}$ | 2468 | 2243         | 2410 $^{\pm}$ 250 |
| $1^{--}$ | 2574 | 2333         |

FIG. 2: Masses of the $ss\bar{s}\bar{s}$ tetraquark states.

two higher $1^{--}$ states may have been observed in the previous experiments $^{4\,11\,53\,55}$, or are waiting to be discovered in future searches.

TABLE III: The predicted three $1^{--}$ $ss\bar{s}\bar{s}$ tetraquarks states together with other theoretical works. The units are in MeV.

| $J^{PC}$ | Ours | SQM $^{13}$ | QSR $^{9}$ | QSR $^{10}$ | QSR $^{11}$ | QSR $^{25}$ |
|----------|------|-------------|-------------|-------------|-------------|-------------|
| $1^{--}$ | 2227 | 2210 $^{\pm}$ 90 | 2300 $^{\pm}$ 400 | 2340 $^{\pm}$ 170 | 3080 $^{\pm}$ 110 |
| $1^{--}$ | 2468 | 2243         | 2410 $^{\pm}$ 250 |
| $1^{--}$ | 2574 | 2333         |

Furthermore, we predict several higher $ss\bar{s}\bar{s}$ tetraquarks around 2.5 GeV. For the higher $0^{--}$ state, there exists a candidate $X(2500)$ with mass of $2470^{+150}_{-190}$ MeV observed in the $J/\psi \rightarrow \gamma \phi \phi$ process by BESIII Collaboration $^{56}$. In the conventional quark model, the $X(2500)$ was assigned as the $\phi(5170)$ state given its mass and total width $^{57\,58}$, but with a tiny $\phi \phi$ partial decay width. The $ss\bar{s}\bar{s}$ tetraquark interpretation of the $X(2500)$ may avoid this defect due to its falling apart mechanism into the $\phi \phi$ final state. Other predictions can provide helpful information for future experimental searches.

III. RADIA TIVE TRANSITIONS

Besides the mass spectrum, the decay behaviors are also needed to clarify these tetraquark states in future experiments. The strong decays can occur if the tetraquarks lie above the meson-meson threshold via falling apart mechanism, and the detailed discussions can be found in Refs. $^{13\,14}$. To treat the radiative transitions between these $ss\bar{s}\bar{s}$ tetraquarks, one can adopt an EM transition operator which has been successfully applied to study the radiative decays of quarkonium and baryons $^{59\,60}$. In this model, the quark-photon EM coupling at the tree level is taken as

$$H_e = -\sum_j e_j \psi_j^\dagger A^\mu(k, r_j)\psi_j,$$

where $\psi_j$ stands for the $j$th quark field with coordinate $r_j$ and $A^\mu$ is the photon field with three-momentum $k$. To match the wave functions obtained by the Schrödinger-like equation, we adopt this quark-photon EM coupling in a nonrelativistic form. In the initial-hadron-rest system, the approximate form can be written as $^{59\,67}$

$$h_e \cong \sum_j \left[ e_j r_j \cdot e - \frac{e_j}{2m_j} \sigma_j \cdot (e \times \dot{k}) \right] e^{-ikr_j},$$

where $e_j$, $m_j$, and $\sigma_j$ stand for the charge, consistent mass, Pauli spin vector for the $j$th quark, respectively. The $e$ is the polarization vector of the final photon.

One can obtain the helicity amplitude $\mathcal{A}$ of the radiative transition $^{59\,60}$

$$\mathcal{A} = -i \left[ \frac{\omega_e}{2} \sum_j |f|h_e|i\right].$$

Then, we can estimate the radiative transitions straightforward $^{59\,60}$

$$\Gamma = \frac{|k|^2}{\pi} \frac{2}{2J_f + 1} \frac{M_f}{J_f} \sum_{j_1, j_2} |\mathcal{A}|^2,$$

where $J_i$ is the total angular momentum of the initial tetraquarks, and $J_{f_1}$ and $J_{f_2}$ are the components of the total angular momenta along the $z$ axis of the initial and final tetraquarks, respectively. In present work, the masses and wave functions of the $ss\bar{s}\bar{s}$ tetraquarks are adopted from our theoretical predictions.

The radiative transitions of the $ss\bar{s}\bar{s}$ tetraquarks are estimated and listed in Tab.$^{19\,23}$. Here, we eliminate the notation
AA of the three ground states without causing misunderstanding. The predicted radiative transitions of the three ground states 0++, 1−++, and 2++ are

\[ \Gamma[^{1+}\,\rightarrow\, ^{0+}\,\gamma] = 157 \text{ keV}, \]
\[ \Gamma[^{2+}\,\rightarrow\, ^{1+}\,\gamma] = 175 \text{ keV}, \]

respectively, which are significant large. As we assign the \( f_0(1710) \) and \( f_2(2300) \) as the 0++ and 2++ states respectively, the rather large radiative decay rates are useful to searching for the missing \( 1^- \) \( ss\bar{s}\bar{s} \) tetraquark. Since the 0++ state has large branching ratios of \( K\bar{K} \) and \( \eta\eta \), more studies of the \( ss\bar{s}\bar{s}(1^-) \rightarrow ss\bar{s}(0^+)\gamma \rightarrow K\bar{K}\gamma \) and \( ss\bar{s}\bar{s}(1^-) \rightarrow ss\bar{s}(0^+)\gamma \rightarrow \eta\eta\gamma \) decay processes are suggested in future experiments.

| TABLE IV : Radiative Transitions. |
|----------------------------------|
| Decay mode | \( M_f(\text{MeV}) \) | \( M_i(\text{MeV}) \) | Width(keV) |
|--------------------------------------------------|
| \( ^^1\gamma \rightarrow \, ^{0+}\gamma \) | 1960 | 1716 | 157 |
| \( ^^2\gamma \rightarrow \, ^{1+}\gamma \) | 2255 | 1960 | 175 |
| \( \,VA, \, ^{0+}\gamma \rightarrow \, ^{1+}\gamma \) | 2004 | 1960 | 0.001 |
| \( \,VA, \, ^{1-}\gamma \rightarrow \, ^{0+}\gamma \) | 2227 | 1716 | 26.6 |
| \( \,VA, \, ^{1-}\gamma \rightarrow \, ^{1+}\gamma \) | 2227 | 1960 | 3.1 |
| \( \,VA, \, ^{2-}\gamma \rightarrow \, ^{0+}\gamma \) | 2497 | 1716 | 49.4 |
| \( \,VA, \, ^{2-}\gamma \rightarrow \, ^{2+}\gamma \) | 2497 | 2255 | 4.6 |
| \( \,VA, \, ^{2-}\gamma \rightarrow \, ^{1+}\gamma \) | 2497 | 1960 | 65.4 |
| \( \,A\bar{A}, \, ^{0+}\gamma \rightarrow \, ^{1+}\gamma \) | 2450 | 1960 | 1345 |
| \( \,A\bar{A}, \, ^{1-}\gamma \rightarrow \, ^{1+}\gamma \) | 2581 | 1960 | 1444 |
| \( \,A\bar{A}, \, ^{1-}\gamma \rightarrow \, ^{0+}\gamma \) | 2574 | 1716 | 1137 |
| \( \,A\bar{A}, \, ^{1-}\gamma \rightarrow \, ^{2+}\gamma \) | 2574 | 2255 | 0.0 |
| \( \,A\bar{A}, \, ^{1-}\gamma \rightarrow \, ^{1+}\gamma \) | 2468 | 1716 | 0.0 |
| \( \,A\bar{A}, \, ^{1-}\gamma \rightarrow \, ^{2+}\gamma \) | 2468 | 2255 | 119 |
| \( \,A\bar{A}, \, ^{1-}\gamma \rightarrow \, ^{1+}\gamma \) | 2619 | 1960 | 954 |
| \( \,A\bar{A}, \, ^{2-}\gamma \rightarrow \, ^{1+}\gamma \) | 2622 | 2255 | 809 |
| \( \,A\bar{A}, \, ^{3-}\gamma \rightarrow \, ^{2+}\gamma \) | 2660 | 2255 | 606 |

For the transitions between \( V\bar{A} \) type and ground states, the partial radiative decay widths range from 1 eV to tens keV. The \( \,VA, \, ^{1-}\gamma \rightarrow \, ^{0+}\gamma \) process is 26.6 keV, which shows that the newly observed \( X(2239) \) state has a significant radiative decay width. The other two \( 1^- \) states with relative excitations between diquarks and anti-diquarks can decay into 0++ and 2++ ground states, respectively,

\[ \Gamma[^{A\bar{A}, \, ^{1-}\gamma \rightarrow \, ^{0+}\gamma}] = 1137 \text{ keV}, \]
\[ \Gamma[^{A\bar{A}, \, ^{1-}\gamma \rightarrow \, ^{2+}\gamma}] = 119 \text{ keV}, \]

where the ten times divergence of the partial widths derives from the different phase spaces. The radiative decay of the \( S = 0 \) state to 2++ final state is highly suppressed, and also for the \( S = 2 \) state to 0++ final state. These predictions may be helpful for searching and distinguishing the two higher \( 1^- \) \( ss\bar{s}\bar{s} \) tetraquark states. About these radiative transitions of the excited \( ss\bar{s}\bar{s} \) tetraquarks, few discussions are found in the literature, thus, more theoretical and experimental studies are expected to be carried out in future.

IV. SUMMARY

In this work, we investigate the masses of \( ss\bar{s}\bar{s} \) tetraquark states using the relativized quark model proposed by Godfrey and Isgur. Here, only the antitriplet diquark \([\bar{3}, \bar{c}]_c \) is considered. The masses of \( ss\bar{s}\bar{s} \) tetraquark states are obtained by solving the Schrödinger-like equation between diquark and antidiquark. The color screening effects are also added in present calculations. It is found that the newly observed resonant structure \( X(2239) \) in the \( e^+e^- \rightarrow K^+K^- \) process by BESIII Collaboration can be assigned as a \( P^- \) wave \( 1^- \) \( ss\bar{s}\bar{s} \) tetraquark state.

Besides the mass spectrum, the wave functions of the \( ss\bar{s}\bar{s} \) tetraquark states are obtained simultaneously. With the wave functions, the radiative transitions between these tetraquarks are also estimated. The \( P^- \) wave \( 1^- \) \( ss\bar{s}\bar{s} \) tetraquark state radiate to the ground state is 27 keV, which can provide helpful information for future experimental searches. Moreover, other \( ss\bar{s}\bar{s} \) tetraquark candidates \( f_0(1710), f_2(2300), \) and \( X(2500) \) are also discussed here. We hope our assignments can be tested by future experiments.

ACKNOWLEDGEMENTS

We would like to thank Wen-Biao Yan, Xian-Hui Zhong and Dian-Yong Chen for valuable discussions. This project is supported by the National Natural Science Foundation of China under Grants No. 11705056, No. 11475192 and No. U1832173. This work is also supported by the Sino-German CRC 110 "Symmetries and the Emergence of Structure in QCD" project by NSFC under the Grant No. 11621131001, and the Key Research Program of Frontier Sciences, CAS, under the Grant No. Y7292610K1.

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