Vacuum quantum fluctuation energy in expanding universe and dark energy

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Abstract

This article is based on the Planckon densely piled vacuum model and the principle of cosmology. With Planck era as initial conditions and including early inflation, we have solved the Einstein-Friedmann equations to describe the evolution of the universe, a reasonable relation between dark energy density and vacuum quantum fluctuation energy density is obtained. The main results are: 1) the solution of Einstein-Friedmann equations has yielded the result $\frac{\rho_{\text{de}}}{\rho_{\text{vac}}} \sim \left(\frac{t}{T_0}\right)^2 \sim 10^{-122}$ (Planck time $t_P = 10^{-43}$s and universe age $T_0 = 10^{18}$s); 2) at inflation time $t_{\text{inf}} = 10^{-35}$s, the calculated universe radiation energy density is $\rho(t_{\text{inf}}) \sim 10^{-16}\rho_{\text{vac}}$ and the corresponding temperature is $E_c \sim 10^{15}$GeV consistent with GUT phase transition temperature; 3) it is showed that the expanding universe is a non-equilibrium open system constantly exchanging energy with vacuum; during its expanding, the Planckons in universe lose quantum fluctuation energy and create cosmic expansion quanta-cosmons, the energy of cosmons is the lost part of vacuum quantum fluctuation energy and contributes to the total energy of the universe with the calculated value $E_{\text{cosmos}} = 10^{22}M_\odot c^2$ ($M_\odot$ is solar mass ) agreed with astronomic data; 4) the gravity potential and gravity acceleration of cosmons are derived with the nature of repulsive force, indicating that the cosmon may be the candidate of dark energy quantum; 5) solution to three well known cosmic problems of Big Bang model is presented.
I. INTRODUCTION

Accelerating expansion of universe and dark energy are established basic facts in exact modern cosmology [1, 2]. Existence of vacuum quantum fluctuation energy is also an es-
tablished basic fact in modern accurate quantum field theory. Combining the two kinds of basic facts and explaining dark energy as vacuum quantum fluctuation energy is a big step to reach the great goal of unifying quantum theory, relativity, and cosmology [3–8]. However, this effort has met serious challenge: the ratio of the dark energy density $\rho_{de}$ to the vacuum quantum fluctuation energy density $\rho_{vac}$ shows a huge hierarchy difference of 122 order of magnitude [9]: $\frac{\rho_{de}}{\rho_{vac}} \sim 10^{-122}$. This article is based on the Planckon densely piled vacuum model [10] and the principle of cosmology. With Planck era as initial conditions and including early inflation, we have solved the Einstein-Friedmann equations to describe the evolution of the universe, a simple and reasonable relation between dark energy density and vacuum quantum fluctuation energy density is obtained. The main results are: 1) the solution of Einstein-Friedmann equations has yielded the result $\frac{\rho_{de}}{\rho_{vac}} \sim \left(\frac{t_{P}}{T_{0}}\right)^{2} \sim 10^{-122}$ (Planck time $t_{P} = 10^{-43}$ s and universe age $T_{0} = 10^{18}$ s); 2) at inflation time $t_{inf} = 10^{-35}$ s, the calculated universe radiation energy density is $\rho(t_{inf}) \sim 10^{-16} \rho_{vac}$ and the corresponding temperature is $E_{c} \sim 10^{15}$ GeV consistent with GUT phase transition temperature; 3) it is showed that the expanding universe is a non-equilibrium open system constantly exchanging energy with vacuum; during its expanding, the Planckons in universe lose quantum fluctuation energy and create cosmic expansion quanta-cosmons, the energy of cosmons—the lost part of vacuum quantum fluctuation energy contributes to the total energy of the universe with the calculated value $E_{cosmos} = 10^{22} M_{\odot} c^{2}$ ($M_{\odot}$ is solar mass) agreed with astronomical data; 4) the gravity potential and gravity acceleration of cosmons are derived and show the nature of repulsive force, indicating that the cosmon may be the candidate of dark energy quantum; 5) solution to three well known cosmic problems of Big Bang model is presented.

II. BASIC ASSUMPTIONS AND EQUATIONS

The basic assumptions of this article are: (1) the environment of the universe - the vacuum is consisted of densely piled Planckons by [10]; (2) the universe obeys the cosmological principle; (3) the evolution of the universe is conducted by Einstein-Friedmann equations with flat curvature $k = 0$ and mass contents include radiation, cold mater, dark mater, as well as dark energy; (4) the universe evolves from Planck era with the initial conditions of the energy density, radius, and time of the era; (5) the inflation occurs at the early time $\tau_{inf}$. The assumption (1) is from a study of the microscopic nature of black holes, gravity,
and vacuum [10], the assumptions (2) and (3) are based on the commonly accepted physical and cosmological principles [3, 7, 8]. Since Einstein-Friedmann equations are of nature of classical gravitation, the quantum nature of gravity and the universe should be incorporated by initial and boundary conditions, namely assumptions (4) and (5).

From the above assumptions, the following basic results can be obtained:

1) At present time, the universe energy density $\rho(T_0)$ and the vacuum quantum fluctuation energy density $\rho_{\text{vac}}$ has the ratio:

$$\frac{\rho(T_0)}{\rho_{\text{vac}}} \sim \left(\frac{t_P}{T_0}\right)^2 \sim 10^{-122}$$

2) At inflation time $\tau_{\text{inf}}$, the universe energy density $\rho(\tau_{\text{inf}})$ and the vacuum quantum fluctuation energy density $\rho_{\text{vac}}$ has the ratio:

$$\frac{\rho(\tau_{\text{inf}})}{\rho_{\text{vac}}} \sim \left(\frac{t_P}{\tau_{\text{inf}}}\right)^2 \sim 10^{-16}$$

where the Planck time $t_P \sim 10^{-43}$ s, universe age $T_0 \sim 10^{18}$ s, and inflation time $\tau_{\text{inf}} \sim 10^{-35}$ s.

First let us introduce the Planck quantum-called Planckon to describe the Planck era. Planckon is the smallest brick of vacuum, which consists of quantum radiation standing wave in the sphere with spin $1/2$, radius $r_P$, volume $v_P$, period $\tau_P$, mass $m_P$, and energy $\varepsilon_P$ as follows [10]:

$$r_P = \left(\frac{\hbar G}{c^3}\right)^{1/2} \approx c \tau_P, v_P = \frac{4\pi}{3} r_P^3, \tau_P = \left(\frac{\hbar G}{c^5}\right)^{1/2}$$

$$m_P = \frac{1}{2} \left(\frac{\hbar c}{G}\right)^{1/2} = \frac{\hbar}{2 c r_P}, \varepsilon_P = m_P c^2 = \frac{\hbar c}{2 r_P}, 2 G m_P = r_P c^2$$

$$\tau_P \sim 10^{-43} \text{s}, r_P = c \tau_P \sim 10^{-33} \text{cm}, \varepsilon_P \sim 10^{19} \text{GeV}, m_P = 10^{-5} \text{g}.$$  

The gravity constant $G = 6.67 \times 10^{-8} \text{erg} \cdot \text{cm}^2/\text{g}^2$. Based on the Planckon densely piled vacuum model [10], the vacuum zero energy density is just the Planckon zero energy density (the Planckon energy $\varepsilon_P$ corresponds to the cutoff of vacuum quantum fluctuation energy):

$$\rho_{\text{vac}} = \rho_P = \frac{1}{2} \varepsilon_P/v_P = \frac{1}{2} m_P c^2 / \left(\frac{4\pi}{3} r_P^3\right) = \frac{3c^4}{16\pi G r_P^2} = \frac{K}{(ct_P)^2}$$

$$K = \frac{3c^4}{16\pi G}, t_P = \sqrt{\frac{3c^2}{16\pi G\rho_{\text{vac}}}}.$$  

According to the above assumptions, the time evolution of the isotropic and homogeneous universe obeys the Einstein-Friedmann equations [3]:
Expanding equation:

\[
\left( \frac{\dot{R}}{R} \right)^2 = \frac{8\pi G}{3c^2} \rho = \Lambda = \lambda^2, \Lambda = \frac{8\pi G\rho}{3c^2}, \lambda = \sqrt{\frac{8\pi G\rho}{3c^2}} \tag{8}
\]

Accelerating equation:

\[
\frac{\ddot{R}}{R} = -\frac{4\pi G}{3c^2} (\rho + 3p) \tag{9}
\]

In general, the universe energy density contains the components of radiation \(\rho_r\), cold mater \(\rho_m\), dark mater \(\rho_{dm}\), and dark energy \(\rho_{de}\). The equation of state is as follows:

\[
p_i = w_i \rho_i, \rho = \sum_i \rho_i, p = \sum_i p_i = \sum_i w_i \rho_i = \sum_i w_i x_i \rho = w \rho \tag{10}
\]

\[
w(t) = \sum_i w_i x_i(t) \frac{\rho_i(t)}{\rho(t)}, \sum_i x_i(t) = 1, i = r, m, dm, de \tag{11}
\]

\[
w_m = w_{dm} = 0, w_r = 1/3, w_{de} = -1, -1 \leq w(t) \leq 1/3 \tag{12}
\]

In the following section, we shall obtain the solutions to Einstein-Friedmann equations, which show that during the evolution of the universe, the components of \(\rho_i(t), x_i(t),\) and \(w(t)\) vary in their own intervals and make the total energy density \(\rho(t)\) vary in time continuously. Instead, the radial scale factor \(R(t)\) of the universe will have the phase transition from power law functions to exponential function or inverse. Einstein-Friedmann equations can describe both the inflation phase or power law phases for the radial scale factor function \(R(t)\).

### III. EVOLUTION OF THE UNIVERSE

#### A. Evolution of universe energy density \(\rho(t)\)

Eqs. (8) and (9) lead to

\[
d\frac{\rho}{\sqrt{\rho(\rho + p)}} = -\sqrt{\frac{24\pi G}{c^2}} dt \tag{13}
\]

By using the equation of state \(p(t) = w(t)\rho(t)\), one has

\[
d\frac{\rho}{\rho^{3/2}} = -\sqrt{\frac{24\pi G}{c^2}} [1 + w(t)] dt, d\frac{1}{\rho^{1/2}} = \sqrt{\frac{6\pi G}{c^2}} [1 + w(t)] dt \tag{14}
\]

The solution to eq. (14) is

\[
\frac{1}{\rho^{1/2}(t - t_0)} = \frac{1}{\rho^{1/2}(t_0)} \left\{ 1 + \sqrt{\frac{6\pi G\rho(t_0)}{c^2}} \int_{t_0}^{t} [1 + w(\tau)] d\tau \right\} \tag{15}
\]
or
\[
\rho(t - t_0) = \rho(t_0)/\left\{1 + \left(3/2\sqrt{2}\right)\frac{16\pi G\rho(t_0)}{3c^2}\left[(t - t_0) + \int_{t_0}^{t} w(\tau)d\tau\right]\right\}^2
\]  
(16)

In view of the interval \([-1, \frac{1}{3}]\) of \(w(t)\) and the \((t - t_0)\)-term in eq. (16), in general, \(\rho(t - t_0)\) is not sensitive to \(w(t)\) (only when \(w(t) \to -1\), \(\rho(t - t_0)\) changes from inverse quadrature functions to constant). In contrast, \(R(t - t_0)\) is very sensitive to \(w(t)\) and can change from power functions to exponential function. This behaviour leads to the problem of consistence between \(\rho(t)\) and \(R(t)\). The evolution detail of different energy density components \(\rho_i(t)\) controlling the evolution of \(w(t)\), is very important to produce a physically reasonable and realistic evolution for both \(\rho(t)\) and \(R(t)\), and establishes the consistence between \(\rho(t)\) and \(R(t)\). Since the evolution of \(\rho_i(t)\) is related to the interactions and reaction cross-sections of elementary particles under astronomic conditions, the knowledge of particle physics is needed.

If the initial conditions are \(t_0 = t_p = \sqrt{\frac{3c^2}{16\pi G\rho_{vac}}}\) and \(\rho(t_p) = \rho_{vac}\), the solution is
\[
\rho(t - t_p) = \frac{\rho_{vac}}{\left\{1 + \left(3/2\sqrt{2}\right)\frac{1}{t_p}\left[(t - t_p) + \int_{t_p}^{t} w(\tau)d\tau\right]\right\}^2}
\]
\[
= \frac{\rho_{vac}}{\left\{1 + \left(3/2\sqrt{2}\right)\frac{(t - t_p)}{t_p}\left(1 + w_{mid}(t)\right)\right\}^2}
\]  
(17)

\(w_{mid}(t)\) is the integration mid value of \(w(t)\) from \(t_p\) to \(t\):
\[
\int_{t_p}^{t} w(\tau)d\tau = w_{mid}(t)(t - t_p).
\]

For dark energy:
\[
w_{de}(t) = -1, \rho_{de}(t - t_p) = \rho_{vac};
\]  
(18)

For radiation:
\[
w_r(t) = \frac{1}{3}, \rho_r(t - t_p) = \frac{\rho_{vac}}{\left[1 + \sqrt{2}(t - t_p)/t_p\right]^2};
\]  
(19)

For cold matter:
\[
w_m(t) = 0, \rho_m(t - t_p) = \frac{\rho_{vac}}{\left[1 + \frac{3}{2\sqrt{2}}(t - t_p)/t_p\right]^2};
\]  
(20)

For global average:
\[
w_{mid}(t) = -1/3, \rho(t - t_p) = \frac{\rho_{vac}}{\left[1 + (t - t_p)/\sqrt{2}t_p\right]^2}.
\]  
(21)
Now consider the details of the evolution in three steps so that the inflation can be incorporated in early time.

1) From Planck era \((t_P, r_P, \rho_P = \rho_{vac})\) to inflation moment \([\tau_{inf}, R(\tau_{inf}), \rho(\tau_{inf})]\), the evolution is of radiation type: \(w(t) = 1/3\);

2) From \([\tau_{inf}, R(\tau_{inf}), \rho(\tau_{inf})]\) to \([t_{inf}, R(t_{inf}), \rho(t_{inf}) = \rho(\tau_{inf})]\), the evolution is inflation: \(w(t) = -1\);

3) From \([t_{inf}, R(t_{inf}), \rho(t_{inf})]\) to \([T_0, R_0, \rho(T_0)]\), the evolution is a mixture of radiation, cold matter, dark matter, and dark energy.

The specific evolutions of three steps are in the following:

1) From \(t_P = 10^{-43}s\) to inflation moment \(t = \tau_{inf} \sim 10^{-35}s (w_{mid} = 1/3)\) The solution is

\[
\rho(\tau_{inf} - t_P) = \rho_{vac}/\{1 + \frac{3}{2\sqrt{2}t_P}[(\tau_{inf} - t_P) + \int_{t_P}^{\tau_{inf}} w(\tau)d\tau]\}^2 \approx \frac{8}{9} \rho_{vac}\left(\frac{t_P}{(\tau_{inf} - t_P)(1 + w_{mid})}\right)^2
\]

\[
= \frac{1}{2} \rho_{vac}\left(\frac{t_P}{\tau_{inf} - t_P}\right)^2
\]

(22)

The energy density before inflation at \(\tau_{inf}\) is

\[
\frac{\rho(\tau_{inf} - t_P)}{\rho_{vac}} = \frac{1}{2}\left[\frac{t_P}{(\tau_{inf} - t_P)}\right]^2 \approx 10^{-16}
\]

(23)

The radiation quantum energy \(E_c\) before inflation and Planckon energy \(\varepsilon_P\) has the relation:

\[
\rho(\tau_{inf} - t_P)/\rho_{vac} = \left(\frac{\lambda_P}{\lambda_c}\right)^4 = \left(\frac{r_P}{r_c}\right)^4 = \left(\frac{E_c}{\varepsilon_P}\right)^4
\]

(24)

\((\lambda_c \sim r_c\ \text{and}\ \lambda_P \sim r_P\ \text{are corresponding wave lengths})\)

From \(\varepsilon_P = 10^{19} GeV\), one obtains

\[
\rho(\tau_{inf} - t_P)/\rho_{vac} = \left(\frac{E_c}{\varepsilon_P}\right)^4 = \left(\frac{E_c}{10^{19} GeV}\right)^4 \sim 10^{-16}, E_c \sim 10^{15} GeV
\]

(25)

which is the temperature of GUT at the moment of phase transition and indicates that the first step evolution from \(t_P\) to \(\tau_{inf}\) is reasonable and yields correct result.

2) From \([\tau_{inf}, R(\tau_{inf}), \rho(\tau_{inf})]\) to \([t_{inf} = 10^{-33}s, R(t_{inf}), \rho(t_{inf}) = \rho(\tau_{inf})]\): \(w_{mid} = -1\) leads to \(\rho(t_{inf}) = \rho(\tau_{inf})\).

3) From \([t_{inf} = 10^{-33}s, \rho(t_{inf}) = \rho(\tau_{inf})]\) to present time \(T_0 = 10^{18}s\), the solution is

\[
\rho(T_0 - t_{inf}) = \rho(t_{inf})/\{1 + (3/2\sqrt{2})\frac{1}{t_{inf}}[(T_0 - t_{inf}) + \int_{t_{inf}}^{T_0} w(\tau)d\tau]\}^2 \approx \frac{8}{9} \rho_{inf}\left(\frac{T_0^2}{(T_0 - t_{inf}) + \int_{t_{inf}}^{T_0} w(\tau)d\tau}\right)^2
\]
\[
\rho(T_0 - t_{\text{inf}}) = \frac{8}{9}\rho(t_{\text{inf}})\left[\frac{t_{\text{inf}}}{(T_0 - t_{\text{inf}})(1 + w_{\text{mid}})}\right]^2 \approx 10^{-106}, \quad t_{\text{inf}} = \sqrt{\frac{3c^2}{16\pi G \rho(t_{\text{inf}})}} \approx 10^{-35}\text{s}
\] (26)

where \(w_{\text{mid}}\) is the mid value of \(w(t)\) from \(t_{\text{inf}} = 10^{-33}\text{s}\) to \(T_0 = 10^{18}\text{s}\). In the following we will know that \(w_{\text{mid}} = -1/3\). From \(\rho(\tau_{\text{inf}}) = \rho(t_{\text{inf}})\) and \(\rho(\tau_{\text{inf}})/\rho_{\text{vac}} \sim 10^{-16}\), one gets

\[
\rho(T_0 - t_{\text{inf}}) = \rho(t_{\text{inf}})10^{-106} = \rho_{\text{vac}}\frac{\rho(t_{\text{inf}})}{\rho_{\text{vac}}}10^{-106} = \rho_{\text{vac}}10^{-122}
\] (27)

and \(\frac{\rho(T_0 - t_{\text{inf}})}{\rho_{\text{vac}}} \approx 10^{-122}\).

Now consider one step evolution from \(t_P\) directly to \(T_0 = 10^{18}\text{s}\): the solution is

\[
\rho(T_0 - t_P) = \rho_{\text{vac}}/\{1 + (3/2\sqrt{2})\frac{1}{t_P}[\frac{T_0}{(T_0 - t_P) + \int_{t_P}^{T_0} w(\tau)d\tau}]^2 \approx 8\frac{\rho_{\text{vac}}}{9\rho_{\text{vac}}} t_P^2 \left[\frac{T_0}{(T_0 - t_P) + \int_{t_P}^{T_0} w(\tau)d\tau}\right]^2 = 8\frac{\rho_{\text{vac}}}{9\rho_{\text{vac}}} t_P^2 \left[\frac{T_0}{(T_0 - t_P)(1 + w_{\text{mid}})}\right]^2
\] (28)

The ratio of universe energy density to vacuum energy density is

\[
\frac{\rho(T_0 - t_P)}{\rho_{\text{vac}}} = 8\left[\frac{t_P}{(T_0 - t_P)(1 + w_{\text{mid}})}\right]^2 \approx 10^{-122}
\] (29)

where \(w_{\text{mid}}\) is the mid value between \(t_P\) and \(T_0\). Since one step evolution\((t_P \to T_0)\) and three step evolution\((t_P \to \tau_{\text{inf}} \to t_{\text{inf}} \to T_0)\) yield the same result, the time duration of phase transition thus has no effect on energy density evolution of the universe. It should be pointed out the initial condition of Planck era is a key point to obtain the correct results.

Since at present time \(T_0\), the dark energy density \(\rho_{\text{de}}\) is 73% of the universe total energy density, so we have

\[
\rho(T_0) \approx \rho_{\text{de}}(T_0), \quad \frac{\rho_{\text{de}}}{\rho_{\text{vac}}} = 10^{-122}
\] (30)

To study the consistence between \(\rho(t)\) and \(R(t)\), we should investigate radial scale factor \(R(t)\) evolution.
B. Evolution of $R(t)$

Combing the expansion equation with the accelerating equation, we have the equation for $R(t)$:
\[
\frac{dR}{R} = \sqrt{\frac{8\pi G \rho(t)}{3c^2}} dt = \lambda(t) dt, \quad \sqrt{\frac{8\pi G \rho(t)}{3c^2}} = \lambda(t)
\] (31)

The universe energy density $\rho(t)$ has been obtained before as follows
\[
\rho(t - t_0) = \rho(t_0) / \left\{ 1 + (3/2) \sqrt{\frac{8\pi G \rho(t_0)}{3c^2}} [(t - t_0) + \int_{t_0}^{t} \omega(\tau) d\tau] \right\}^2
\] (32)

The exact solution of $R(t)$ is
\[
R(t) = R(t_0) \exp\left[ \int_{t_0}^{t} \lambda(\tau) d\tau \right]
\] (33)

The integration in the exponential can be calculated by the substitution of $\rho(t)$,
\[
\int_{t_0}^{t} \lambda(\tau) d\tau = \sqrt{\frac{8\pi G \rho(t_0)}{3c^2}} \int_{t_0}^{t} \frac{d\tau}{1 + 3/2 \sqrt{\frac{8\pi G \rho(t_0)}{3c^2}} [(\tau - t_0) + \int_{t_0}^{\tau} \omega(\tau') d\tau']} \] (34)

For dark energy dominant $w(t) = -1$, it follows
\[
\int_{t_0}^{t} \lambda(\tau) d\tau = \sqrt{\frac{8\pi G \rho(t_0)}{3c^2}} (t - t_0)
\] (35)

\[
R(t) = R(t_0) \exp\left[ \sqrt{\frac{8\pi G \rho(t_0)}{3c^2}} (t - t_0) \right]
\] (36)

which is radial inflation.

With mid value theorem $\int_{t_0}^{t} \omega(\tau) d\tau = w_{mid}(t)(t - t_0)$, one has
\[
\int_{t_0}^{t} \lambda(\tau) d\tau = \ln \left\{ 1 + \frac{3(1 + w_{mid})}{2} [\alpha(t) - \alpha(t_0)] \right\}^{\frac{2}{3(1 + w_{mid})}}, \quad \alpha(t) = \sqrt{\frac{8\pi G \rho(t_0)}{3c^2}} t
\] (37)

The solution in terms of $w_{mid}$ is
\[
R(t) = R(t_0) \left[ 1 + \frac{1}{\frac{3(1 + w_{mid})}{2}} \sqrt{\frac{8\pi G \rho(t_0)}{3c^2}} (t - t_0) \right]^{\frac{2}{3(1 + w_{mid})}}
\] (38)
which is very sensitive to $w_{\text{mid}}$.

1) As $\dot{w}_{\text{mid}} \rightarrow -1$ (dark energy dominant),

$$
\frac{2}{3(1 + w_{\text{mid}})} \rightarrow \infty,
$$

$$
R(t) = R(t_0)[1 + \frac{1}{3(1 + w_{\text{mid}})}\sqrt{\frac{8\pi G \rho(t_0)}{3c^2}}(t - t_0)] \rightarrow
$$

$$
R(t) = R(t_0) \exp[\sqrt{\frac{8\pi G \rho(t_0)}{3c^2}}(t - t_0)]
$$

(39)

Eq. (39) is same as that obtained before. The velocity after inflation:
from $\tau_{\text{inf}} = \sqrt{\frac{3c^2}{8\pi G \rho(\tau_{\text{inf}})}} = 10^{-35}s$ and $t_{\text{inf}} = 10^{-33}s$, one gets

$$
\dot{R} \quad (t_{\text{inf}} = 10^{-33}s) = (R(\tau_{\text{inf}} = 10^{-35}s)/\tau_{\text{inf}}) \times e^{(t_{\text{inf}} - \tau_{\text{inf}})/\tau_{\text{inf}}} =
$$

$$
R \quad (\tau_{\text{inf}} = 10^{-35}s)/\tau_{\text{inf}} \times 10^{33} = (10^{-29}/10^{-35}) 10^{43} \text{cm/s} \sim 10^{49} \text{cm/s} \quad (e^{100} \approx 10^{43})(40)
$$

The effect of curvature $k$ term approaches zero: $\frac{\rho - \rho_c}{\rho_c} = \frac{c^2 k}{H^2 R^2} = \frac{c^2 k}{R^2} \sim 10^{-78} \rightarrow 0$, so after inflation the space becomes flat. As $w_{\text{mid}} \rightarrow -1/3$,

$$
R(t) = R(t_0)[1 + \frac{8\pi G \rho(t_0)}{3c^2}(t - t_0)]
$$

(41)

The evolution is linear.

2) As $w_{\text{mid}} \rightarrow 1/3$ (radiation dominant)

$$
R(t) = R(t_0)[1 + 2\sqrt{\frac{8\pi G \rho(t_0)}{3c^2}}(t - t_0)]^{1/2}
$$

(42)

Radial velocity at $t_P$ and $\tau_{\text{inf}}$:

$$
\dot{R}(t) = \frac{r_P}{\sqrt{2t_P}}[1 + \sqrt{2(t - t_P)/t_P}]^{-1/2} \approx 10^{10} \times [1 + \sqrt{2(t - t_P)/t_P}]^{-1/2} \text{cm/s},
$$

(43)

Velocity at $t \approx t_P$ : $\dot{R}(t \sim t_P) \sim 10^{10} \text{cm/s} \sim c$.

Velocity before inflation at $t = \tau_{\text{inf}} = 10^{-35}s$: $\dot{R}(\tau_{\text{inf}}) \sim 10^6 \text{cm/s}$.

The curvature effect before inflation is very lager:

$$
\frac{\rho - \rho_c}{\rho_c} = \frac{c^2 k}{H^2 R^2} = \frac{c^2 k}{R^2} \approx 10^8 >> 1$, the space is not flat.

3) As $w_{\text{mid}} \rightarrow 0$(cold mater dominant)

$$
R(t) = R(t_0)[1 + \frac{3}{2}\sqrt{\frac{8\pi G \rho(t_0)}{3c^2}}(t - t_0)]^\frac{3}{2}
$$

(44)

4) Global average evolution from $t_P$ to $T_0$: we shall show later that Nature requires $w_{\text{mid}} = -\frac{1}{3}$ and

$$
R(t) = R(t_0 = t_P)[1 + \sqrt{\frac{8\pi G \rho(t_0)}{3c^2}}(t - t_0)] = r_P[1 + (t - t_P)/\sqrt{2t_P}]
$$

(45)
The radial scales of the universe at \( \tau_{\text{inf}} = 10^{-35} \) s and \( t_{\text{inf}} = 10^{-33} \) s:

1. Radiation dominant \((w_{\text{mid}} = 1/3)\), from \((t_P, r_P, \rho_{\text{vac}})\) to \((\tau_{\text{inf}} = 10^{-35} \) s, \( R(\tau_{\text{inf}}), \rho(\tau_{\text{inf}}))\):

\[
R(\tau_{\text{inf}}) = R(t_P)[1 + 2\sqrt{\frac{8\pi G \rho(t_P)}{3c^2}}(t - t_P)]^{1/2} = 2r_P(t_{\text{inf}} - t_P)^{1/2} \approx 10^{-29} \text{ cm.} \tag{46}
\]

2. Inflation \((w_{\text{mid}} = -1)\), from \((\tau_{\text{inf}}, R(\tau_{\text{inf}}), \rho(\tau_{\text{inf}}))\) to \((t_{\text{inf}} = 10^{-33} \) s, \( R(t_{\text{inf}}), \rho(t_{\text{inf}}))\):

\[
t_0 = \tau_{\text{inf}}, t = t_{\text{inf}} = 10^{-33} \) s, \( \rho(t_0) = \rho(\tau_{\text{inf}}), \sqrt{\frac{8\pi \rho(\tau_{\text{inf}})/3c^2}{\tau_{\text{inf}}}} = 1/\tau_{\text{inf}}, \tau_{\text{inf}} = 10^{-35} \) s, \( R(\tau_{\text{inf}}) = 10^{-29} \) cm,
\]

\[
R(t_{\text{inf}}) = R(\tau_{\text{inf}}) \exp[\sqrt{\frac{8\pi G \rho(\tau_{\text{inf}})}{3c^2}}(t_{\text{inf}} - \tau_{\text{inf}})] = 10^{-29} \times e^{t_{\text{inf}} \tau_{\text{inf}} cm} = 10^{-29} \times 10^4 e^{100} cm
\]

\[(\text{if } t_{\text{inf}} = 10^{-34.633} \) s, \( \tau_{\text{inf}} = 10^{-35} \) s, then \( R(t_{\text{inf}}) = 10 cm). \tag{47}\]

C. \( \rho(R) - R \) relation

The evolution equation for \( R(t) \) is related to energy conservation as follows:

\[
\frac{d(\rho R^3)}{dt} = -p \frac{d(R^3)}{dt}. \tag{48}\]

Physical implication of eq.(48): as \( p > 0 \) and \( \frac{d(R^3)}{dt} > 0 \), \( \frac{d(\rho R^3)}{dt} < 0 \) and the universe loses energy to vacuum; as \( p < 0 \) and \( \frac{d(R^3)}{dt} > 0 \), \( \frac{d(\rho R^3)}{dt} > 0 \) and the universe acquires energy from vacuum.

Total energy \( E \) of the universe: From \( p = w\rho, E = \rho V \), and \( V = \frac{4\pi}{3} R^3 \), one obtains

\[
d \ln[E] = -wd \ln V \tag{49}\]

If \( w \) assumes the constant mid value \( w = w_{\text{mid}} = \text{const} \), then

\[
d \ln[EV^{w_{\text{mid}}}] = 0, E = C/V^{w_{\text{mid}}} \tag{50}\]

The total energy evolution with \( R \) are as follows:

Radiation dominant \((w_{\text{mid}} = 1/3)\):

\[
E_r = C/R \tag{51}\]

Cold mater dominant \((w_{\text{mid}} = 0)\):
\[ E_m = C \]  

Dark energy dominant \((w_{\text{mid}} = -1)\):

\[ E_{de} = CR^3 \]  

Global average evolution \((w_{\text{mid}} = -1/3)\):

\[ \tilde{E} = CR \]  

The above evolutions of energy density and total energy of the universe indicate that the expanding universe is a non-equilibrium open system constantly exchanging energy with vacuum. The universe had expanded from the Planck era and the Planckon constitutes the initial condition. During its expansion, more and more Planckons are involved in the universe evolution and all the involved Planckons in the universe lose their quantum fluctuation energy and the lost irregular energy of Planckons converts into cosmons and contributes to the universe. Thus the expanding universe acquires energy from vacuum in the form of cosmons which are created and made of the lost quantum fluctuation energy of Planckons. The cosmons are the candidates of dark energy quanta with the nature of repulsive force as shown later. The cold matter has no net energy exchange with vacuum on average.

Combining the energy conservation equation and radial evolution equation, one has

\[
\frac{d\rho}{\rho^{3/2}} = -\sqrt{\frac{24\pi G}{c^2}}[1 + w(t)]dt, \quad \frac{dR}{R} = \sqrt{\frac{8\pi G\rho(t)}{3c^2}}dt = \lambda(t)dt
\]  

and

\[
\frac{dR}{R} = -\frac{d\rho}{3[1 + w(\rho)]\rho}, \quad \ln \frac{R}{r_P} = \int_{\rho}^{\rho_P} \frac{d\rho}{3[1 + w(\rho)]\rho} = \frac{1}{3[1 + w(\rho_{\text{mid}})]} \ln \frac{\rho_P}{\rho}
\]

Their solutions yield the \(\rho = \rho(R)\)-\(R\) relation:

\[ \rho(R) = \rho_{\text{vac}} \left( \frac{r_P}{R} \right)^{3(1+w_{\text{mid}})} \quad (\rho_P = \rho_{\text{vac}}) \]  

Different \(w_{\text{mid}}\) lead to different power laws:

Radiation dominant:

\[ w_{\text{mid}} = 1/3, \rho = \rho_{\text{vac}} \left( \frac{r_P}{R} \right)^4 \]
Matter dominant:

\[ w_{\text{mid}} = 0, \rho = \rho_{\text{vac}} \left( \frac{r_P}{R} \right)^3, \]  

(59)

Dark energy dominant:

\[ w_{\text{mid}} = -1, \rho = \rho_{\text{vac}} \]  

(60)

Global average:

\[ w_{\text{mid}} = -1/3, \rho = \rho_{\text{vac}} \left( \frac{r_P}{R} \right)^2 \]  

(61)

Summary of the evolution phase transitions:

\( \rho(t) - t \) function: insensitive to \( w(t) \), transitions within power laws;
\( R(t) - t \) function: very sensitive to \( w(t) \), transitions between power law and exponential;
\( \rho(R) - R \) function: sensitive to \( w(R) \), transitions among different power laws.

More information from \( \rho(t) \) and \( R(t) \). From the evolution of \( \rho(t) \), one can obtain more information: The value \( w_{\text{mid}} > -1 \) definitely leads to \( \rho(T_0) / \rho_{\text{vac}} \sim 10^{-122} \) (see eq. (28) and eq. (29)). From \( R(t) - t_P \) relation:

\[ R(t) = R(t_P)[1 + \frac{1}{2} \frac{\sqrt{8\pi G \rho(t_P)}}{3c^2} (t - t_P)]^{\frac{2}{3(1 + w_{\text{mid}})}} \]  

(62)

\[ R(T_0) = r_P, \rho(T_0) = \rho_{\text{vac}}, \sqrt{8\pi G \rho_{\text{vac}}/3c^2} = 1/\sqrt{2} t_P \]  

(63)

One can derive that if \( R(T_0) = 10^{28} \text{cm} \) and \( \ddot{R} > 0 \) today, then there must have \( w_{\text{mid}} \approx -1/3 \), namely: \( R(T_0) = 10^{28} \text{cm} \to w_{\text{mid}} \sim -1/3, \ddot{R} > 0 \).

In fact, as \( w_{\text{mid}} \approx -1/3, \frac{2}{3(1 + w_{\text{mid}})} \approx 1 \),

\[ R(T_0 - t_P) = r_P[1 + (T_0 - t_P)/\sqrt{2} t_P] \approx 10^{-33} \times 10^{61} \text{cm} = 10^{28} \text{cm} \]

\( (r_P = 10^{-33} \text{cm}, t_P = 10^{-43} \text{s}, T_0 = 10^{18} \text{s}) \)

The above \( R(T_0 - t_P) \) also yields the acceleration \( \ddot{R}(T_0) \sim 10^{-7} \text{cm/s}^2 \) and velocity \( \dot{R}(T_0) \sim 10^{10} \text{cm/s} \approx c \) at present time.

From \( w_{\text{mid}} \approx -1/3 \), one gets the average weights for matter and dark energy (neglecting the small radiation weight \( \bar{x}_r \sim 0 \)): \( \bar{x}_m = 0.67, \bar{x}_{\text{de}} = 0.33 \). The following calculations of \( R(T_0) \) indicates that any single component can not yield the observed value of \( R(T_0) \):

Radiation dominant: \( w_{\text{mid}} \to 1/3 \)
\[ R(t) = R(t_0)[1 + 2 \sqrt{\frac{8\pi G \rho(t_0)}{3c^2}} (t - t_0)]^{1/2}, R(T_0) \sim 10^{-3} \text{cm} \ll 10^{28} \text{cm} = R_0, \text{ too small;} \]
Matter dominant: $w_{mid} \to 0$

$$R(t) = R(t_0)[1 + \frac{3}{2} \sqrt{\frac{8\pi G \rho(t_0)}{3c^2}}(t - t_0)]^{2/3}, \quad R(T_0) \sim 10^7 \text{cm} \ll 10^{28} \text{cm} = R_0, \text{ too small;}$$

Dark energy dominant: $w_{mid} \to -1$

$$R(t) = R(t_0)[1 + \frac{1}{3(1 + w_{mid})} \sqrt{\frac{8\pi G \rho(t_0)}{3c^2}}(t - t_0)]^{2/3(1 + w_{mid})} \to$$

$$R(t) = R(t_0) \exp[\sqrt{\frac{8\pi G \rho(t_0)}{3c^2}}(t - t_0)]$$

$$R(T_0) = r_P e^{T_0/t_P} \approx 10^{-33+0.43 \times 10^{61}} \text{ cm} \gg 10^{28} \text{ cm} = R_0,$$

too large.

**IV. UNIVERSE MASS AND BACKGROUND MICROWAVE TEMPERATURE**

**A. Cosmic expansion quantum-cosmon and universe mass**

Now let us calculate the universe total energy and mass from the cosmic expansion quantum-cosmons, which are excitations of vacuum and their energies come from the lost energies of Planckons during the expansion of universe. Since the energy $e_{\text{cosmos}}$ of the cosmon contributed to the universe is the lost energy of Planckon and vacuum consists of densely piled Planckons, the lost energy density of Planckon at present time is just the universe energy density $\rho(T_0)$. From the Planckon volume $v_P$ and its lost energy density $\rho(T_0)$, one obtains the cosmon energy $e_{\text{cosmos}}$ as follows: $e_{\text{cosmos}} = \rho(T_0) \times v_P$. From $\rho(T_0) = \rho_{\text{vac}}(\frac{r_P}{T_0})^2$, $\rho_{\text{vac}} = \rho_P$, and $\varepsilon_P = \rho_{\text{vac}} \times v_P$, one obtains $e_{\text{cosmos}} = \varepsilon_P(\frac{r_P}{T_0})^2 = \varepsilon_P(\frac{r_P}{R_0})^2 \approx 10^{-122}\varepsilon_P \sim 10^{-94}\text{eV}$, which is extremely small. Since during the expansion of universe, each Planckon loses its energy and creates a cosmon, the number $N_{\text{cosmon}}$ of cosmons is just the number $N_{\text{Planckon}}$ of Planckons in the universe. According to the Planckon densely piled vacuum model \[10\],

$$N_{\text{cosmon}} = N_{\text{Planckon}} = \frac{4\pi R_0^3}{3} = \left(\frac{R_0}{r_P}\right)^3 \approx 10^{183}, \quad \left(\frac{R_0}{r_P} = \frac{10^{28} \text{ cm}}{10^{-33} \text{ cm}} = 10^{61}\right) \quad (64)$$

The universe total energy and mass are

$$E_{\text{cosmos}} = e_{\text{cosmon}} N_{\text{cosmon}} = \varepsilon_P\left(\frac{R_0}{r_P}\right) = 10^{80}\text{GeV}, \quad (m_P = 10^{-5} \text{ g}, \varepsilon_P = 10^{19}\text{GeV}) \quad (65)$$

$$M_{\text{cosmos}} = m_P\left(\frac{R_0}{r_P}\right) = 10^{-5} \times 10^{61} \text{ g} = 10^{66} \text{ g} = 5 \times 10^{22} M_{\odot}, \quad (\text{solar mass} \quad M_{\odot} = 2 \times 10^{33} \text{ g}) \quad (66)$$
which is just the universe astronomic mass. Thus the universe acquires energy and mass in the form of cosmons which are created with the lost energy of Planckons (namely the decreased part of vacuum quantum fluctuation energy) during the expansion of the universe.

B. Temperature of background microwave radiation

At the decouple time $t_{\text{decouple}} = 10^{11} s$ the photon energy can be calculated as follows:

$$\rho(t_{\text{decouple}} = 10^{11} s) / \rho_{\text{vac}} = \left( \frac{t_P}{t_{\text{decouple}}} \right)^2 = \left( \frac{10^{-43}}{10^{11}} \right)^2 = 10^{-108} = \left( \frac{E_{\text{decouple}}}{\varepsilon_P} \right)^4 \quad (67)$$

$$E_{\text{decouple}} = 10^{-8} \text{GeV} = 10eV \sim 10^5 K^0. \quad (68)$$

The radius of the universe at $t_{\text{decouple}} = 10^{11} s$ is

$$R(t_{\text{decouple}}) = r_P \left( \frac{t_{\text{decouple}}}{t_P} \right) \sim 10^{22} \text{cm} \sim R_0 10^{-6} \quad (69)$$

Due to expansion stretching of wave length of photon, the photon energy today is,

$$E_0 = \frac{R(t_{\text{decouple}})}{R_0} E(t_{\text{decouple}}) \sim 10^{-1} K^0 \quad (70)$$

which is near the astronomic value $3K^0$ (the error maybe due to the decouple time).

V. DIFFERENT EFFECTS OF TWO KINDS OF VACUUM EXCITAIONS BASED ON PLANCKON DENSELY PILED VACUUM MODEL

Now we discuss two different kinds of energy losses of Planckons and their different gravity effects. In the Planckon densely piled vacuum model [10], the gravity of black holes is generated by Casimir effect due to the cutoff condition of black hole horizon singular surface. Suppose (not really happens) that the universe $R_0$-spherical surface provides a cutoff for radial wave modes of radiation excitations. Inside the universe, each Planckon will lose energy from radial modes and the vacuum has less zero quantum fluctuation energy density than that in flat space and the effective temperature of vacuum energy quantum fluctuation will decrease, which is the microscopic origin of gravity (equilibrium space Casimir effect). According to [10], the inside Planckon has the zero quantum energy loss from $\varepsilon_P$,

$$\Delta \varepsilon_P = \varepsilon_H = \varepsilon_P \left( \frac{r_P}{R_0} \right) = \frac{\hbar c}{2R_0} \quad (71)$$
\[ \Delta \varepsilon_p \] will produce radial outward gravity acceleration and driving particles to horizon. The corresponding excitation quanta of vacuum will be accumulated in horizon in the form of radiation quantum with energy \( e_H \) and the number of the radiation quanta in horizon is just the number \( N_H = 4(R_0/r_P)^2 \) of Planckons in the spherical horizon Planckon layer. The total mass \( M_H \) is the excitation quantum mass \( e_H/c^2 \) times its number \( N_H = 4(R_0/r_P)^2 \),

\[
M_H = 4(e_H/c^2)(R_0/r_P)^2 = 4m_P(R_0/r_P) \sim M_{\text{cosmos}} \tag{72}
\]

which is in the same order of magnitude to the universe mass \( M_{\text{cosmos}} \). On the other hand, it is really happens that the expansion of universe to the radius \( R_0 \) leads to the quantum fluctuation energy loss of Planckons in the universe in quite a different way which can be called as "non-equilibrium time Casimir effect". Since the lost energy of every Planckon is fed to the universe with the energy density \( \rho(T_0) \) in the form of cosmons created by expanding universe, the cosmon energy should be

\[
e_{\text{cosmon}} = \rho(T_0)v_P = \varepsilon_P \left( \frac{r_P}{R_0} \right)^2 = \frac{\hbar c}{2r_P} \left( \frac{r_P}{R_0} \right)^2 = \frac{\hbar c}{2R_0} \left( \frac{r_P}{R_0} \right) = \frac{\hbar c}{2R_\kappa}, R_\kappa = R_0 \left( \frac{R_0}{r_P} \right) \tag{73}
\]

They contribute to the universe a total mass \( M_{\text{cosmos}} \) as calculated before \( M_{\text{cosmos}} \sim 10^{22}M_\oplus \). Now let us investigate the gravity effect of the cosmon with energy \( e_{\text{cosmon}} = \frac{\hbar c}{2R_\kappa} = \frac{\hbar c}{2c} \) and \( \kappa = \frac{c^2}{2R_\kappa} \). From the period condition of temperature Green’s function for fermion, we have \( e_{\text{cosmos}}/k_B T_{\text{cosmos}} = \pi \), and the Hawking-Unruh formulae naturally follows \( \pi k_B T_{\text{cosmos}} = e_{\text{cosmon}} = \frac{\hbar c}{2c} = \frac{\hbar c}{2R_\kappa} \tag{10} \). The gravity acceleration (or gravity strength) of the cosmons \( \kappa = \frac{d\varphi(R)}{dR} = \frac{c^2}{2R_\kappa} \) leads to the Einstein gravity potential \( \varphi(R) = -\frac{c^2}{2R_\kappa}R \). Since the gravity acceleration of the cosmoms is in the radial direction and outward, the gravity effect of cosmoms is of the nature of a repulsive force. The cosmon is a standing quantum radiation wave with wave length \( \lambda_{\text{cosmon}} = 4\pi R_\kappa \) and spin \( 1/2 \) \( \tag{10} \). Thus the cosmon is a radiation quantum of fermion-type, a candidate of dark energy quantum. It is dual to the radiation quantum for Schwarzschild black hole \( \tag{10} \) as follows: The cosmon of the expanding universe:

\[
e_{\text{cosmon}} = \frac{\hbar c}{2R_\kappa}, \quad R_\kappa = R_0 \left( \frac{R_0}{r_P} \right) \tag{74}
\]

The radiation quantum of the Schwarzschild black hole:

\[
e_H = \frac{\hbar c}{2r_H}, \quad (r_H - \text{black hole radius}) \tag{75}
\]

It should be noted that although both the universe surface cutoff and the universe expansion lead to quantum fluctuation energy loss of all its Planckons which are basic bricks of
vacuum, the mechanism is quite different. The universe surface cutoff leads to the quantum fluctuation energy loss of Planckon with the energy decrease of \( \Delta \varepsilon_P(\text{space}) = \varepsilon_H = \varepsilon_P \left( \frac{r_P}{R_0} \right) \), which is a violin cord effect (equilibrium space Casimir effect); while the universe expansion leads to the quantum fluctuation energy loss of Planckon with the energy decrease of \( \Delta \varepsilon_P(\text{time}) = \varepsilon_{\text{cosmon}} = \varepsilon_P \left( \frac{r_P}{R_0} \right)^2 = \varepsilon_H \left( \frac{r_P}{R_0} \right) \), which is \( r_P/R_0 \) time smaller than \( \Delta \varepsilon_P(\text{space}) \) and shows the holographic nature of outward radiation (non-equilibrium time casimir effect). The former effect moves the quantum energy \( \varepsilon_H \) to the horizon surface, the later effect keeps the cosmon quantum energy \( \varepsilon_{\text{cosmon}} \) in universe uniformly. Both are excitation effects of vacuum which is densely piled by Planckons.

VI. PHYSICAL EXPLANATION OF THE RESULTS

The present study tells us that our universe started from an explosion in the Planckon densely piled vacuum. The initial condition is the Planck era with the Planckon space-time scales \((r_P, t_P)\) and energy density \( \rho_{\text{vac}} \). The evolution of the universe is controlled by the Einstein-Fridmann equations with different energy density components. During the evolution more and more Planckons are involved, lose their quantum fluctuation energy, and create cosmons which change the Planckon lost irregular energies into the universe regular energies of different components. The gravity effect of the newly created cosmon quanta is to produce repulsive force outward in radial direction, and it may thus be the candidate of dark energy quantum.

The evolution took three steps: (1) explosion from Planck era to the inflation moment, (2) inflation within a very short period, (3) evolution from the end of inflation to present time. Evidently, the evolution of our universe is controlled by the basic principles of general relativity, cosmology, and quantum theory. Of course, the information of particle physics is need to achieve a complete and detailed description. From our model, the energy density of dark energy and cold matter, inflation phase transition temperature, background microwave temperature, and the universe total mass can be calculated in agreement with the observational data.

An intuitively physical picture can be presented for the time evolution of the universe energy density and the creation of cosmons as follows. In the Planckon densely piled vacuum model\[10\], the universe and vacuum constitute a compound system, each of them are open
subsystems. Planckons in vacuum as radiation sources can radiate and absorb radiation quantum energy. In the stationary universe, the radiation and absorption processes are in equilibrium, no net effect can be observed physically in the universe subsystem. As the universe subsystem undergoes expansion, the stationary equilibrium of the two subsystem is broken, every Planckon in the expanding universe is now in non-equilibrium and radiates energy quantum as a cosmon. Therefore a net effect of cosmons in the universe subsystem can be observed physically as a radial outward gravity force. Let the pressure of Planckons at their spherical surface is $p(r_P)$. As the universe has expanded to $R_0$, the radiation pressure of the Planckons at $R_0$ sphere is $p(R_0)$. Since the spherical pressure is proportional to the energy quanta number per unit time and per unit area—the radiation energy flux, according to energy conservation, $p(R_0)$ is inversely proportional to the spherical surface area $(4\pi R_0^2)$. Hence

$$\frac{p(R_0)}{p(r_P)} \sim \left(\frac{r_P}{R_0}\right)^2, \quad p(R_0) \sim \left(\frac{r_P}{R_0}\right)^2 p(r_P), \quad (76)$$

Using the the energy density and pressure relation $p(r_P) \sim \rho_{vac}(r_P)$ and $p(R_0) \sim \rho(R_0)$, finally we have

$$\rho(R_0) \sim \rho_{vac}\left(\frac{r_P}{R_0}\right)^2 \sim \rho_{vac}\left(\frac{t_P}{T_0}\right)^2 \quad (77)$$

This is exactly the results obtained by directly solving Einstein-Friedmann equations.

The above picture implies that: the evolution of the universe energy density, the creation of cosmons and dark energy, are a problem of a non-equilibrium and open system, and the non-equilibrium process is more important and essential. Only in its non-equilibrium expansion, the universe can exchange energy with vacuum, the Planckons in the universe can lose energy and change their lost quantum fluctuation energy into cosmon’s regular energy, finally the dark energy quanta-cosmons are created. Therefore, the dark energy is not the vacuum quantum fluctuation energy itself, it is the lost energy of Planckons under the condition of the universe non-equilibrium expansion. The cosmons are made of the lost energy of Planckons and act as dark energy quanta with the repulsive nature of gravity effect.

VII. SUMMARY OF THE RESULTS

The results of this paper are summarized as as follows.

1) Based on the Planckon densely piled model, vacuum energy density is that of Planckons
\( \rho_{\text{vac}} = \rho_P \).

2) By solving Einstein-Fridmann equations with Planck era as initial conditions and in three steps, we have obtained \( \rho(T_0)/\rho(t_P) \sim \left( \frac{t_P}{T_0} \right)^2 \sim 10^{-122} \). Since the dark energy is 73% of the present universe energy density, so \( \rho_{\text{de}}(T_0) \sim \rho(T_0) \sim \rho_{\text{vac}} 10^{-122} \).

3) The relation between dark energy and vacuum quantum fluctuation energy is explained and the cosmon with energy \( e_{\text{cosmos}} = \varepsilon_P \left( \frac{r_P}{R_0} \right)^2 = 10^{-122} \varepsilon_P \) is introduced as the candidate of dark energy quantum.

4) The calculated inflation phase transition temperature is \( E_c \sim 10^{15} \text{GeV} \).

5) The universe mass calculated is \( M_{\text{cosmos}} \approx 10^{22} M_\odot \) and the calculated background microwave temperature is \( T \sim 0.1 K^0 \).

6) Two different excitation quanta of vacuum-the radiation quantum \( e_H \sim \varepsilon_P \left( \frac{r_P}{R_0} \right)^2 \) due to space Casimir effect and the cosmon \( e_{\text{cosmos}} \sim \varepsilon_P \left( \frac{r_P}{R_0} \right)^2 \) due to universe expansion are compared and discussed.

7) The gravity potential and gravity acceleration of cosmons are derived and the nature of repulsive force has been unveiled.

8) The scaling law of the universe energy density evolution is explored.

\[
\rho(t - t_P) = \rho_{\text{vac}} / \left\{ 1 + \left( \frac{3}{2} \sqrt{2} \right) \frac{1}{t_P} \left[ (t - t_P) + \int_{t_P}^{t} w(\tau) d\tau \right] \right\}^2
\]  \hspace{1cm} (78)

Evolution period: Planck period \( t_P \) inflation period \( t_{\text{inf}} \) present \( T_0 \)

Energy density:
\[
\rho_{\text{vac}} \sim \left( \frac{\rho_{\text{vac}}}{(t_P/t_P)^2} \right) \rho \sim \left( \frac{\rho_{\text{vac}}}{[(t_{\text{inf}} - t_P)/t_P]^2} \right) \rho \sim \left( \frac{\rho_{\text{vac}}}{(T_0/t_P)^2} \right)
\]

Time scaling:
\[
t_P \rightarrow s t_P, \quad t_{\text{inf}} \rightarrow s t_{\text{inf}}, \quad T_0 \rightarrow s T_0
\]

Energy density scaling:
\[
\rho_{\text{vac}} \rightarrow \rho_{\text{vac}}/s^2, \quad \rho(t_{\text{inf}}) \rightarrow \rho(t_{\text{inf}})/s^2, \quad \rho(T_0) \rightarrow \rho(T_0)/s^2
\]

The above scaling law is invalid for the dark energy density since \( \rho(t) = \rho_{\text{de}} = \text{const.} \)

VIII. RELATION WITH OTHER THEORIES

A. Solution to three puzzles of Big Bang model

Solutions to three puzzles of Big Bang model are as follows:

(1) The initial singular problem is solved by the initial conditions of Planck era.
(2) The problem of flatness of space is solved by following calculations:
before inflation, $\dot{R} = 10^6 \text{cm/s}$, $|\rho - \rho_c| \approx \frac{c^2|k|}{R^2} \gg 1$, the curvature $k$ term is very large and the space is not flat; after inflation, $\dot{R} = 10^{39} \text{cm/s}$, $|\rho - \rho_c| \approx \frac{c^2|k|}{R^2} = \frac{c^2|k|}{R^2} \sim 10^{-58} \sim 0$, the curvature $k$ term is negligible and the space becomes flat.

(3) The horizon problem is solved by the following calculated results: before inflation the expansion velocity $\dot{R}(\tau_{inf}) \sim 10^6 \text{cm/s} \ll c$, the causal connection of the whole universe can be established; after inflation the average radial velocity $\bar{\dot{R}} \approx c$, the average horizon distance $D$ is approximately equal to the average expansion distance $R$ so that $\frac{D}{R} \approx 1$.

B. Comparing with Guth theory

To solve the above difficulties of Big Bang model, instead of using thermodynamic formulas for energy density, entropy, and temperature within the radiation gas model by Guth [7, 8], our model is based on (1) Einstein-Friedmann equation, (2) the initial conditions ($t_P, r_P, \rho_P = \rho_{vac}$) of Planck era, (3) the state of equation with $w(\rho) = w[\rho(t)]$, and (4) including a short period of inflation.

C. Comparing with holographic models

As discussed in sections IV and V, dark energy as the gravity effect is related to cosmons which are created from the lost quantum fluctuation energies of Planckons during the expansion of the universe. Its energy density has the relation $\rho(R) = \rho_{vac}(\frac{r_P}{R})^2$, just like photons radiated from the source, propagated outward and left their information at different passed spherical surfaces. Thus the dark energy with above property possesses the nature of holography, consistent with the holographic models of dark energy [11, 12].

D. Different Casimir effects

In section IV, we have pointed out that the gravity and temperature of black holes are stemmed from the equilibrium and stationary space Casimir effect [10] and the dark energy is from the non-equilibrium and time-dependent casimir effect. The Radiation quantum $e_H$ and cosmon $e_{cosmon}$ are two kinds of basic excitations of vacuum which is consisted of
densely piled Planckons. Both $e_H$ and $e_{\cos\text{mon}}$ are closely related to quantum fluctuation energy loss of Planckons in vacuum under different conditions: the space boundary cutoff of black holes causes the space Casimir effect and induces the radiation quantum $e_H$, while the expansion of universe causes the time Casimir effect and induces the cosmon $e_{\cos\text{mon}}$. Since these two kinds of basic effects are related to fundamental and important physics, the issue of different Casimir effects deserves further study.

Acknowledgements

This work was supported in part by the National Natural Science Foundation of China under the grant No. 10974137 and 10775100, the Doctoral Education Fund of the Education Ministry of China, and by the Research Fund of the Nuclear Theory Center of HIRFL of China.

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