I describe a turbulence-inspired model for the stellar initial mass function which includes feedback and self-regulation via protostellar outflows. A new aspect of the model provides predictions of the star formation rate in molecular clouds and gas complexes. A similar approach is discussed for self-regulation on kiloparsec scales via supernova input, and an expression is presented for the global star formation rate that depends on the turbulent pressure of the interstellar medium.

Keywords: star formation-galaxies-turbulence

Introduction

The turbulence paradigm seems to be increasingly accepted in star formation theory. The dissipation time of the observed supersonic turbulence especially in giant molecular clouds (GMCs) is less than the cloud lifetime, demonstrating that cloud support is by turbulent pressure. Implications include the consequences that molecular clouds on all scales are short-lived and hence that large-scale star formation is relatively inefficient. This corresponds to what is observed: about 0.1-1 percent of a GMC is in stars, and the dynamical time-scale of a GMC is $10^6 - 10^7$ yr, giving consistency with the global star formation rate of the Milky Way galaxy, that requires about 10 percent efficiency of conversion of gas to stars over a galactic rotation time. On small scales, such as cloud cores, supersonic turbulence is still present, although less dominant, and one generally observes a much higher efficiency of star formation. The efficiency for star clusters embedded in molecular clouds, which have ages less than $5 \times 10^6$ yr, is 10-30 % (Lada and Lada 2003). Presumably an important difference is that self-gravity plays a prominent role on these scales. The turbulence drivers are not well understood. Possible sources include external drivers (galactic rotation, Parker instability, OB wind/SN-driven super-bubbles) as well as internal sources (protostellar jets and outflows, gravitational collapse), and the turbulence is likely to be multi-scale, controlled both by cascades and inverse cascades.

I will focus here mostly on protostellar outflows on small scales, although I will also discuss supernova input on larger scales, as sources of interstellar turbulence. The aim will be to demonstrate that outflow-generated turbulence allows self-regulation of star formation via control of the accretion rate. A GMC is viewed as a network of overlapping, interacting protostellar-driven outflows. This leads to simple derivations of the IMF and of the star formation rate.

Similar ideas are implicit in models of competitive accretion by turbulent fragments (Bonnell, Vine and Bate 2004). The idea of a network of interacting wind-driven shells is a manifestation of the fragmentation models developed by Klessen and collaborators, in which colliding, supersonic
flows shock, dissipate and produce dense gravitationally unstable cores (Klessen, Heitsch and MacLow 2000; Burkert 2003).

1 Observational issues

Substantial churning of molecular clouds by protostellar outflows is a common phenomenon. Observations of some cloud complexes such as Circinus, Orion and NGC 1333 support the idea that protostellar outflows drive the observed turbulence (Yu, Bally and Devine 1997; Bally et al. 1999; Bally and Reipurth 2001; Sandell and Knee 2001). Quantitatively, such flows seem to be an important source of turbulence. One may estimate the outflow strength as (e.g., Konigl 2003)

\[ \dot{M}_{\text{wind}} \sim \left( \frac{r_d}{r_A} \right)^2 \dot{M}_{\text{acc}} \sim 0.1 \dot{M}_{\text{acc}} \sim 0.1 v_{\text{amb}}^3 / G, \]

where the wind velocity is of order 100 km/s, and \( v_{\text{amb}} \sim 0.3 - 3 \) km s\(^{-1}\). In general, one expects that \( v_{\text{wind}} > v_{\text{amb}} \). Moreover, outflow momenta accumulate since typical outflow durations are \( t_{\text{wind}} \sim 10^5 \) yr, so that the cloud lifetime \( t_{\text{dyn}} \gg t_{\text{wind}} \). Hence one wind outflow event per forming star ejecting 10 percent of its mass can, on the average, balance the accretion momentum in a typical cloud core that forms stars at \( \sim 10\% \) efficiency.

An important issue is the extent to which the outflows are localised. Observed jets suggest that some outflows deposit energy far from the dense cores, but this inference cannot easily be generalised since it is likely to be highly biased by extinction. Jets may be ubiquitous, but could still be unstable enough to drive bipolar outflows and mostly deposit their energy in and near dense cores. Protostellar outflows are a possible source of the observed molecular cloud turbulence, and the only one that is actually observed in situ. The relevant driving scales cover a wide range, and may well yield the observed, apparently scale-free cloud turbulence.

The projected density profiles of some dense cores are well fit by a pressure-confined self-gravitating isothermal sphere (Alves, Lada and Lada 2002). Others require supercritical cores undergoing subsonic collapse (Harvey et al. 2003). Magnetic support offers one explanation of the initial conditions, with magnetically supercritical cores surrounded by subcritical envelopes. However the core profiles can also, more debatably, be reproduced as supersonic turbulence-compressed eddies (Ballesteros-Paredes, Klessen and Vazquez-Semadini 2003).

2 IMF Theory

There are many explanations that yield a power-law IMF, \( m dN/dm = A m^{-x} \), with \( x \approx 4/3 \), the Salpeter value. However it is harder to account for the interesting physics embedded in \( A(p,t...) \), which determines the star formation rate and efficiency. It is likely that molecular clouds are controlled by feedback. Protostellar outflows feed the turbulence which controls ambient pressure, which in turn regulates core formation. Pressure support includes magnetic fields. There are different views about the details of self-regulation, depending on the relative roles of magnetic and gravitational support versus turbulent compression. Collapse of the supercritical cores into protostellar disks might drive outflows that levitate the envelopes, thereby limiting core masses (Shu et al 2003). Ambipolar diffusion and magnetic reconnection would be controlled by the turbulence, thereby setting the mass scale of cores (Basu & Ciolek 2004). The outflows should cumulatively set the ambient pressure that in turn controls both core masses and the accretion rate at which cores can grow (Silk 1995).

Consider a simple model for a spherically symmetric outflow into a uniform medium that drives a shock-compressed shell until either the shell encounters another expanding shell or until pressure balance with the ambient medium is achieved. At this point, the shell breaks up into blobs confined by ram or by turbulent pressure, with the relevant turbulent velocity being that of the shell at break-up.
Now the shell radius is \( R = \left( \frac{\dot{M}_{\text{wind}} v_w}{\rho_a} \right)^{1/4} t^{1/2} \Rightarrow R \propto v^{-1}, \quad t \propto v^{-2} \). Here \( v = dR/dt \) is the shell velocity. The protostellar outflows generate a network of interacting shells that form clumps with a velocity distribution:

\[
N(>v) = 4\pi R^3 \dot{N}_s = \left( \frac{\dot{M}_{\text{wind}} v_w}{\rho_a} \right)^2 v^{-5} \dot{N}_s.
\]

To convert to a mass function, I combine this expression for the velocity distribution of clumps with the relation between mass \( m \) and turbulent velocity (identified with \( v \)) for a clump: \( m = v^3 G^{-3/2}(v/B) \) for magnetically supercritical cores or \( m = v^3 G^{-3/2}(vp^{-1/2}) \) for gravitationally supercritical Bonnor-Ebert cores. Now \( B \) scaling suggests \( B \propto v \Rightarrow \epsilon = 3 \) (c.f. Shu et al. 2003). Alternatively, scaling from Larson’s laws yields \( \rho \propto v^{-2} \Rightarrow \epsilon = 4 \). If I instead generalise the wind-driven, approximately momentum-conserving, shell evolution to \( R \propto t^\delta \), the IMF can now be written in the form \( \dot{m} N/dm \propto m^{-x} \) with \( x = \frac{3b+1}{\epsilon(1-\delta)} \) where \( R \propto t^\delta \) and \( m \propto v^\epsilon \). One finally obtains the following values for \( x \): \( x = 5/3 \) if \( \delta = 1/2 \); \( x = 4/3 \) if \( \delta = 3/7 \) (\( \epsilon = 3 \)), and \( x = 5/4 \) if \( \delta = 1/2 \); \( x = 4/3 \) if \( \delta = 13/25 \) (\( \epsilon = 4 \)). There seems to be little difficulty in obtaining a Salpeter-like IMF.

However in practice, the IMF is not a simple power-law in mass. It may be described by a combination of three different power-laws: \( x = -2/3 \) over 0.01 to 0.1 \( M_\odot \); \( x = 1/3 \) over 0.1 to 1 \( M_\odot \); \( x = 4/3 \) over 1 to 100 \( M_\odot \) (Kroupa 2002). A scale of around 0.3 \( M_\odot \) must therefore be built into the theory. One clue may come from the fact that the observed IMF is similar to the mass function of dense clumps in cold clouds, at least on scales above \( \sim 0.3M_\odot \) (Motte et al. 2001).

### 3 Feedback

I now describe an approach that yields the normalisation of the IMF, and in particular its time-dependence. The idea is that the network of interacting shells must self-regulate, in that star formation provides both the source of momentum that drives the shells, and is itself controlled by the cumulative pressure that enhances clump collapse. I introduce porosity as the parameter that controls self-regulation, via the overlap of outflows. I define porosity as \( Q = 4\pi R_{\text{max}}^3 \dot{N}_s t_{\text{max}} \).

Now for self-regulation, I expect that \( Q \sim 1 \). One may rewrite the IMF as

\[
\dot{m} N/dm = Q(m_a/m)^x
\]

where \( m_a = v_a^3 G^{-3/2}(v_a/B_a) \) or \( m_a = v_a^3 G^{-3/2} \rho_a^{-1/2} \). Now with \( Q \sim 1 \), one can expect self-regulation. However in addition, one must require self-gravity to avoid clump disruption. This allows the possibility of either negative or positive feedback.

The predicted IMF is \( \dot{m} N/dm = Am^{-x} \). The preceding argument yields \( A \). More generally with regard to \( x \), if \( R_{\text{wind}} \propto t^\delta \) and \( m_{\text{clump}} \propto v^\gamma \), we obtain \( \delta = 2/5, 3/7, 1/2 \) \( \rightarrow x = 2/3, 4/3, 5/3 \).

The principal new result is the self-regulation ansatz that yields \( A \). We infer that \( A \propto Q \) where porosity \( Q \) can be written as \( f_{\text{low density phase}} = 1 - e^{-Q} \). The IMF slope is in accordance with observations for plausible choices of parameters. Of course numerical simulations in 3-D are needed to make a more definitive calculation of the IMF in the context of the present model.

Nevertheless, there is one encouraging outcome. Turbulent feedback seems to be significant for stars of mass \( \gtrsim 0.3M_\odot \). This is an observed fact, and is attributed to detailed models that generally invoke magnetically-driven accretion disk outflows and jets. Of course even sub-stellar objects display outflows but the outflow rates are dynamically unimportant for the parent cloud (e.g., Barrado et al. 2004). Such a hypothesis could help explain why a feedback explanation of the IMF naturally selects a characteristic mass of \( \sim 0.3M_\odot \). As the stellar mass increases, negative feedback mediates the numbers of more massive clumps and stars. The numbers of increasingly massive stars fall off according to the power-law derived here.
4 Summary of IMF results

There are 3 crucial components to star formation phenomenology. These are the initial stellar mass function or IMF, the star formation efficiency or SFE, and the star formation rate or SFR. The outflow-driven turbulence model predicts these quantities, provided we can identify the mass of a star with 

\[ m = \mu v^3 G^{-3/2} \rho^{-1/2} \]

where 

\[ \mu = p^{1/2}/B \text{ or } 1 \]

for either magnetically or pressure-supported clouds. One then finds that, above the feedback scale, the IMF is

\[ mdN/dm \propto Q(v_a/v) = f \mu \rho_a Q m_a^{2/3} m^{-5/3}. \]

Here \( f \) is a constant of order unity.

The SFR is

\[ \dot{n}_s = \frac{Q}{R_a} = Q v_a (\frac{\rho_a}{M_{\text{wind}} v_w})^{1/3} \propto Q \rho_a^2 v_a. \]

In the final expression, I set \( \delta = 1/2 \) and \( \dot{M}_{\text{wind}} = 0.1 v_a^3 G \). The SFR \( \propto \rho_a^2 \). This means that the SFR accelerates as the cloud evolves and contracts. The enhanced dissipation from outflows most likely results in the increase of turbulent density in the cloud, at least until sufficiently massive stars form whose energetic outflows, winds and eventual explosions blow the cloud apart. Evidence for accelerating star formation in many nearby star-forming regions, based on pre-main-sequence evolutionary tracks, is presented by Palla and Stahler (2000). This suggests that a ministarburst is a common phenomenon.

The SFE is

\[ m_{\text{char}} \frac{\dot{n}_s t_{\text{dyn}}}{\rho_a} \propto m_{\text{char}} \frac{\dot{n}_s}{G^{1/2} \rho_a^{3/2}} \propto Q \rho_a^{1/2} v_a. \]

This scaling gives an SFE that is \( \sim 10 - 100 \) times larger in cores than in a GMC, more or less as is observed.

There are a number of unresolved issues. The scale-free nature of the observed turbulence in molecular clouds is suggestive of a cascade. Normally these proceed from large to small scales. With internal protostellar sources, an inverse cascade must be invoked, such as could arise via injection of turbulence associated with jet-driven helical magnetic fields. Alternatively, the wide range of jet and outflow scales suggests that the driving scale may largely be erased.

Efficient thermal accretion onto low protostellar mass cores coupled with protostellar outflows and turbulent fragmentation, for which \( M_{\text{j}}^{\text{turb}} \sim M^2 M_{\text{therm}}^2 \) (Padoan and Nordlund 2002), will help to imprint the characteristic stellar mass scale. This suggests that magnetic fields, insofar as they regulate and drive outflows, are likely to play an important role in setting the characteristic stellar mass scale. Moreover, regions of enhanced turbulence, such as would be associated with star formation induced by merging galaxies, could plausibly have a increased feedback scale and hence a top-heavy IMF.

5 A theory for kiloparsec-scale outflows

I now show that a porosity formulation of outflows can also lead to a large-scale burst of star formation. Rather than consider molecular cloud regions, where the physics is more complex, I discuss a more global environment where the physics can be simplified but the essential ingredient of interacting outflows remains. Consider a larger-scale version of self-regulated feedback. I model a cubic kiloparsec of the interstellar medium, which contains atomic and molecular gas clouds and ongoing star formation. I assume that the dominant energy and momentum to the multiphase interstellar medium is via supernovae. One expects self-regulation to lead to a situation in which the porosity \( Q \sim 1 \). The porosity is initially small, but increases as outflows and bubbles develop. If it is too large, I argue that molecular clouds are disrupted and the
galaxy blows much of the gas out of the disk, e.g. via fountains into the halo. Star formation is quenched until the gas cools and resupplies the cold gas reservoir in the disk. Whether the gas leaves in a wind is not clear; this may occur for dwarf galaxy starbursts, but cannot happen for Milky-Way type galaxies as long as the supernova rates are those assumed to apply in the recent past. I further speculate that the feedback is initially positive, in a normal galaxy. The outflows drive up the pressure of the ambient gas which enhances the star formation rate by accelerating collapse of molecular clouds. The feedback eventually is negative in dwarf galaxies, once a wind develops. In more massive galaxies, the ensuing starburst is only limited by the gas supply.

To develop a simple model, I make the following ansatz. The porosity may be defined by

$$ Q \sim (SN \text{ bubble rate}) \times (\text{maximum bubble 4-volume}) $$

$$ \propto (\text{star formation rate}) \times (\text{turbulent pressure}^{-1.4}). $$

Expansion of a supernova remnant is limited by the ambient pressure, when it can be described as a radiation pressure-driven snowplow with $R_{\text{clt}}^{1.4} \propto p_{\text{turb}}^{1.4}$ (Cioffi et al. 1988). Hence the star formation rate may be taken to be $\propto Q_{\text{P,1.4}}$, and by introducing a new parameter $\epsilon$ may also be written as $\epsilon \times \text{rotation rate} \times \text{gas density}$. What is in effect the global star formation efficiency is now given by $\epsilon \equiv (\frac{\sigma_{\text{gas}}}{\sigma_f})^{2.7}$, where $\sigma_f \approx 20\text{km}s^{-1} (E_{\text{SN}}/10^{51})^{1.27} (200M_\odot/m_{\text{SN}})$. We expect positive feedback at high gas turbulent velocities. High resolution numerical simulations of a multiphase medium demonstrate that a starburst is generated, and that the porosity formalism describes the star formation rate (Slyz et al. 2004). The porosity formulation yields a star formation rate that gives a remarkably good fit to the numerical results. The positive feedback arises from the implementation of the derived star formation law with star formation rate proportional to turbulent gas pressure. Pressure enhancements are mostly due to shocked gas. It is interesting to note that a star formation law which favours shock dissipation can more readily account for the spatial extent of star formation as modelled for interacting galaxies (Barnes 2004) than can an expression in which the star formation rate is only a function of gas density (as in the Schmidt-Kennicutt law).

Porosity may therefore regulate star formation, on the physical grounds that porosity can be neither too large nor too small. If it is too small, the rate of massive star formation (and death) accelerates until the porosity increases. If the porosity is too large the cloud is blown apart via a wind and loses its gas reservoir. To make the concept of a porosity-driven wind more precise, I write the disk outflow rate as the product of the star formation rate, the hot gas volume filling factor, and the cold gas mass loading factor. If the hot gas filling factor $1 - e^{-Q}$ (Q is porosity) is of order 50%, then this suggests that the outflow rate is of order the star formation rate. I emphasize that such a result is plausible but only qualitative: it has yet to be numerically simulated in a sufficiently large box.

One infers that the metal-enriched mass ejected in a wind is generically of order the mass in stars formed. This is similar to what is observed for nearby starbursting dwarf galaxies (e.g. NGC 1569: Martin, Kobulnicky and Heckman 2002). Observations suggest that massive galaxies should have had massive winds in the past, in order both to account for the observed baryonic mass and the galaxy luminosity function (Benson et al. 2003), although theory has difficulty in rising to this challenge. It is clear that, energetically, with conventional supernova rates, one cannot drive winds from massive or even Milky Way-like galaxies (Springel and Hernquist 2003). The situation is very different for dwarfs, where supernova input suffices to drive vigorous winds, although even in these cases geometric considerations are important (MacLow and Ferrara 1999).

Simulations in a multiphase medium currently lack sufficient resolution to adequately treat such instabilities as Rayleigh-Taylor and Kelvin-Helmholtz, that will respectively enhance the porosity and the wind loading. The highest resolution simulations to date (Slyz et al. 2004) of a multiphase medium already show that SN energy input efficiency is considerably underestimated.
by failure to have adequate resolution to track the motions of OB stars from their birth sites in dense clouds before they explode.

It is likely therefore that feedback may occur considerably beyond the scales hitherto estimated (Dekel and Silk 1986), possibly extending to the galactic (stellar) mass scale of about $3 \times 10^{10} M_\odot$, only above which the star formation efficiency is inferred to be approximately constant (Kaufmann et al. 2003).

Whether even more refined and detailed hydrodynamical simulations can be consistent with the requirement of substantial early gas loss from massive galaxies is uncertain (Silk 2003). One simply lacks the energy input. Instead, recourse must be made either to an early top-heavy IMF or outflows from a quasar phase that coincided with the epoch of bulge formation. A top-heavy IMF is motivated by the earlier derivation of an IMF driven by turbulent feedback. The case for an early quasar phase during galaxy bulge formation is motivated by the empirical bulge-supermassive black hole correlation, high quasar metallicities and SMBH growth times (Dietrich and Hamann 2004). Yet another option is an enhanced early rate of hypernovae in starbursts, as suggested by the interpretation of the peculiar abundances found in the starburst galaxy M82 (Umeda et al. 2002).

While all of these enhanced sources of energy and momentum are likely to play some role in forming galaxies, it is intriguing to note that early reionisation, in concordance with requirements from CMB measurements by the WMAP satellite, can also be accomplished by the first of these hypotheses which can simultaneously account for chemical evolution of the metal-poor IGM and the abundance ratios observed in extreme metal-poor halo stars (Daigne et al. 2004). Moreover, a top-heavy IMF, if identified with luminous starbursts, can also account for the faint submillimetre galaxy counts (Baugh et al. 2004) and the chemical abundances in the enriched intracluster medium (Nagashima et al. 2004).

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