Optimization Analysis of Vibration Characteristics for Precision Positioning Stage

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Abstract: As the core of semiconductor equipment, the vibration of precision positioning stage has great influence on its comprehensive performance. The research object of this paper is a high-precision positioning stage for chip detection. The precision positioning stage is driven by linear motors and supported by rolling linear guides. During the movement, vibration will be generated due to the influence of cable disturbance force and motor thrust harmonics. Based on Newton mechanics, the dynamic formula group of the motion table is established, and the reasons for the vibration of each degree are analyzed. Based on the geometric nonlinear theory, the cable model is established with the help of finite element software, and the influence of installation dimensions on the dynamic change of the cable assembly is analyzed. The motor thrust formula is established based on the equivalent magnetizing current method, the characteristics and influencing factors of the thrust harmonics are analyzed, and the method to suppress the thrust harmonics from the source is sought. In this paper, the vibration source characteristics of the stage are obtained through theory and simulation, which lays a foundation for the subsequent use of control strategy compensation.

1. Introduction
The cleanliness of the silicon wafer surface is very important for the lithography process. With the continuous narrowing of the line width of the lithography process, the silicon wafer surface detection equipment plays an increasingly important role in today's IC manufacturing industry [1]. In order to achieve micron level tracking accuracy in the high speed and high acceleration of the large stroke trajectory, linear motor + rolling guide platform structure is mainly used at home and abroad [2]. The stage of this structure type will be affected by the drag line disturbance power and the harmonic action of the motor thrust during the movement process, which will cause the vibration in the movement direction. The same processing and installation errors will also cause the vertical direction and the rotation vibration around each axis [3]. These vibrations have great influence on the comprehensive performance of the precision motion table. It is of great significance to analyze and weaken their influence on improving the performance of the worktable.

As one of the key factors of the comprehensive performance of the precision positioning stage, the vibration characteristics directly affect the motion accuracy of the system. Many scholars at home and abroad have done relevant research on weakening its vibration. Cheng Rong et al carried out
optimization analysis on the disturbance force fluctuation of the cable assembly [4]; Liu Xiao uses finite element method and analytical method to optimize the structure of linear motor [5]. Based on previous studies, this paper analyzes the vibration characteristics of the high-precision mobile platform for chip testing equipment. In Section 2, the dynamic equation of the motion stage is established based on Newton mechanics, and the simulation model is built in Simulink to analyze the vibration causes of the motion stage. In Section 3, the motor thrust is modeled based on the equivalent magnetization current method to suppress the motor thrust wave from the source. Section 4 is based on the geometric nonlinear theory, using the finite element analysis software to optimize the analysis of the cable disturbance force. The optimization analysis is carried out. Finally, the research results are summarized.

2. Vibration analysis and model construction of positioning stage

Through the electromechanical coupling analysis and modeling, we can know the influence law of internal vibration of precision positioning stage, which is the basis for suppressing vibration. As shown in the figure 1, it is the lower shaft of the precision positioning stage. The stage is fixedly connected with the motor mover and driven by the linear motor to move back and forth along the guide rail. Under the action of the motor and the support force of the guide rail, in addition to the vibration along each axis, there are also pitching (\(\varphi\)) around the X axis, rolling (\(\theta\)) around the Y axis, and yaw (\(\gamma\)) around the Z axis. Finally, the dynamic equations of the Y-direction motion platform can be obtained[6,7]:

![Figure 1 Structural dynamics analysis](image)
\[
\begin{align*}
M_y \ddot{Y} &= F_e - \mu \left( \sum_{i=1}^{4} |N_{ih}| + \sum_{i=1}^{4} |N_{iv}| \right) - F_{\text{cable}} \\
M_y \ddot{Z} &= \sum_{i=1}^{4} N_{iz} - M_y g \\
I_x \ddot{\varphi} &= -(N_{1z} + N_{2z})L_1 + (N_{3z} + N_{4z})L_2 - F_e H + (f_1 + f_2 + f_3 + f_4)h + F_{\text{cable}} h_1 \\
I_y \ddot{\theta} &= (N_{1z} + N_{2z})d_2 - (N_{2z} + N_{4z})d_1 + (N_{2x0y} + N_{4x0y} - N_{1x0y} - N_{3x0y})h \\
I_z \ddot{y} &= (N_{2x0y} - N_{1x0y})L_1 + (N_{3x0y} - N_{4x0y})L_2 + (f_1 + f_3)d_2 - (f_2 + f_4)d_1
\end{align*}
\]

Where \( N_{z} \) is the vertical force of guide slider, \( N_{x0y} \) is the horizontal force, \( f \) is the friction, \( F_e \) is motor thrust, and \( F_{\text{cable}} \) is the disturbing force of cable. The simulation and experiment show that the vibration in the direction of motion is very obvious, and the vibration amplitude in other directions is very small, so it can be ignored. The vibration in the direction of motion is mainly caused by the traction disturbance and the motor thrust harmonic, so it is necessary to analyze its mechanism and find a way to weaken its influence from the source.

### 3. Analysis of thrust fluctuation of linear motor

#### 3.1. Linear motor thrust modeling

![Figure 2 Linear motor mover coil](image)

The precision positioning stage is driven by a linear motor without iron core. There is no iron core in the motor, so there is no cogging effect. The thrust fluctuation is mainly caused by the end effect and the thrust harmonic caused by the non ideal sinusoidal magnetic field [6,7]. As shown in Figure 2, it is the schematic diagram of linear motor coil.

This paper analyzes the non stack winding and considers the ideal situation of zero coil spacing. At this time, the coils are closely arranged and the length of the mover can be shortened. For linear motor, the formula is as follows:

\[
\omega = \frac{\pi n}{L} v = \frac{\pi v}{\tau}
\]  

(2)

For non stacked windings, \( \theta_m = 4\pi/3 \). It can be seen from the figure, the flux linkage of the single turn coil at \( \omega t + \Delta \) is as follows:

\[
\varphi = \int_{\omega t + \Delta}^{\omega t + \theta_m - \Delta} B_y dx = \int_{\omega t + \Delta}^{\omega t + \theta_m - \Delta} B_y l \frac{L}{\pi} d\theta
\]  

(3)

Where \( B_y \) can be calculated by the equivalent magnetizing current method:
\[ B_y = \sum_{k=1}^{\infty} \frac{\mu_0 M_k \text{sh}(m_k h_m)}{\text{sh} \left( \frac{m_k \delta}{2} \right)} \text{ch}(m_k y) \sin(m_k x) \]  

The results show that the flux linkage of single turn coil is as follows:

\[ \varphi = \frac{2lL}{\pi p} \sum_{k=0}^{\infty} B_k \frac{B_k}{(2k-1)} \sin \left( (2k-1)(0.5\theta_m - \Delta) \right) \text{ch}(m_k y) \sin \left( (2k-1)(wt + 0.5\theta_m) \right) \]  

Assuming that a coil group has \( n \) turns of coils, we can get the flux linkage of the coil group as follows:

\[ \psi = \frac{2LN}{\pi p} \sum_{k=0}^{\infty} B_k \frac{k_{pk} \sin \left( (2k-1)(wt + 0.5\theta_m) \right)}{(2k-1)} \]  

Where \( k_{pk} \) can be calculated as:

\[ k_{pk} = \frac{2}{(2k-1)h_c m_k \text{sh} \left( \frac{h_c m_k}{2} \right)} \sin \left( (2k-1)(0.5\theta_m(1-\eta)) \sin \left( (2k-1)0.5\eta\theta_m \right) \right) \]  

Where \( h_c \) is the actual height of the coil, \( \eta = \frac{\theta_r}{\theta_m} \).

\( e \) is the no-load induced electromotive force. Among them, \( a \) is the winding parallel coefficient and \( k_d \) is the winding distribution coefficient.

\[ e = \sum_{k=1}^{\infty} E_k \cos \left( (2k-1)(wt + 0.5\theta_m) \right) \]  

Where \( E_k \) can be calculated as:

\[ E_k = \frac{2\omega q B_k LN k_{pk} k_d}{\pi p a} \]  

According to the above formula, the induced electromotive force only contains odd harmonics such as 3, 5, 7,9 and fundamental waves. Then by vector control and the law of conservation of energy, the motor thrust formula can be obtained:

\[ F = \frac{P}{v} = \frac{3}{2v} E_1 l + \frac{3}{2v} \sum_{k=1}^{\infty} (E_{6k-1} + E_{6k+1}) l \cos(6kw + 0.5\theta_m) \]  

In the motor thrust, there are constant thrust and six-fold harmonics of 6, 12, 18, etc.

3.2. Thrust harmonic optimization analysis

According to the formula (10), it can be known that reducing the harmonic amplitude of the induced electromotive force can reduce the thrust harmonic amplitude. The amplitude of the induced electromotive force is related to the harmonic amplitude of the magnetic field:

\[ B_{yk} = \frac{\mu_0 M_k \text{sh}(m_k h_m)}{\text{sh} \left( \frac{m_k \delta}{2} \right)} \]  

The harmonic wave of the air gap density is related to the structure parameters of the permanent magnet, mainly including the permanent magnet width (\( \tau_m \)), the permanent magnet height (\( h_m \)). And the total height (\( g \)), which is shown in Fig. 3.
Figure 3 Structure diagram of permanent magnet

Definition $\alpha = \frac{\tau_m}{\tau}, \beta = \frac{h_m}{\tau}, \gamma = \frac{\delta}{\tau}$, the relationship between the harmonic of magnetic field and motor parameters can be obtained by formula (10). By changing the values of $\alpha$, $\beta$, and $\gamma$, the relationship between the ratio of the thrust harmonic assignment to the fundamental assignment and the motor parameters $\alpha$, $\beta$, and $\gamma$ can be obtained, as shown in Figure 4-6.

Figure 4 Relationship between $F_{XV}/F_{X1}$ and $\alpha$

Figure 5 Relationship between $F_{XV}/F_{X1}$ and $\beta$

Figure 6 Relationship between $F_{XV}/F_{X1}$ and $\gamma$

It can be seen from the figure that with the increase of order, the harmonic amplitude of magnetic field decreases gradually; choosing appropriate permanent magnet width ($\alpha$) can reduce the harmonic amplitude of magnetic field; the harmonic amplitude of magnetic field increases gradually with the
increase of permanent magnet height ($\beta$); the force harmonic proportion decreases gradually with the increase of total height ($\gamma$).

4. Dynamic analysis of towing line disturbance

The physical and schematic diagram of the line pipe assembly is shown in Figure 7. It is mainly composed of hoses for conveying coolant and gas, and cables for transmitting power and signals. The moving end will produce disturbing force along the movement direction with the movement of the platform, which will deteriorate the movement performance of the worktable.

The cable assembly is simplified to flexible beam structure, and the material parameters are simplified to isotropic linear elastic materials for analysis. There are three parameters for the installation of towline, $d$ is the distance between the midpoint of the stroke and the fixed end, $s$ is the total stroke, $l$ is the length of the cable, and $h$ is the installation height. This kind of problem is geometric nonlinear problem, which can be analyzed with the help of the finite element analysis software Workbench. Different load steps are set in the static module to extract the force on the surface of the mobile end, and then optimize the installation parameters. The indicator function in the optimization process is as follows:

$$F_{op} = \sqrt{\frac{1}{s} \int_{-s/2}^{s/2} (F_L(x) - \bar{F}_L)^2 dx}$$

(12)

Where, $F_L(x)$ is the static disturbing force of the towline at a certain point, $\bar{F}_L$ is the average value of the moving end disturbance in the stroke. Change the installation structure parameters for simulation, and the results are shown in Figure 8.
It can be seen from the above figure that the thrust disturbance force fluctuation can be reduced by selecting a larger height \((h)\) value, a larger absolute value of the midpoint \((d)\) of the stroke, and a larger cable length \((l)\).

5. Conclusions
This paper takes the high-precision mobile platform for chip detection equipment as the research object, analyzes its vibration characteristics, and obtains its vibration causes and optimization direction.

1. Based on Newton mechanics, the dynamic equation of the motion table is established, and the vibration types of the motion table are analyzed. There are mainly multi degree of freedom vibration caused by machining and installation error, vibration of moving direction caused by line dragging disturbance force and thrust harmonic of linear motor. Among them, the vibration of motion direction is more significant, and the vibration of other degrees of freedom can be ignored.

2. Based on the equivalent magnetization current method, the thrust formula of coreless linear motor is established, and the relationship between thrust harmonic and motor parameters is obtained, which provides the direction for motor optimization. In uniform motion, the thrust harmonic is mainly the sixth harmonic, and with the increase of order, the thrust harmonic amplitude gradually decreases, which lays the foundation for the subsequent compensation.

3. With the help of the finite element software, the change of the disturbing force of the towing line in the process of the towing line movement is analyzed, and the larger height \((h)\) value, the larger stroke midpoint \((d)\) value and the larger cable length \((l)\) can reduce the thrust fluctuation.

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