Study on abrasive particle impact modeling and cutting mechanism

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Funding information
Natural Science Foundation of Shandong Province, Grant/Award Number: ZR2020QE164

Abstract
Abrasive water jet impact is widely applied in processing, oil fracturing, coal mining, rock crushing, and other engineering fields. The influence of abrasive particles on cutting performance is one of the main characteristics of this technology, which is different from pure fluid cutting. Therefore, as for improving the cutting performance of abrasive water jet, studying the cutting mechanism of abrasive water jet, and clarifying the mapping relationship between particle parameters and cutting, it is of great theoretical and practical significance to clarify the impact failure mechanism and material removal mechanism of abrasive particles in the process of high-speed impact. In order to study the effect of abrasive particles on cutting performance, irregular abrasive particles are simplified to angular particles and circular particles in this paper. The impact shooting process of particles in abrasive jet cutting is numerically simulated by using smoothed particle hydrodynamics (SPH) method, and the target impact experiments are carried out. The fracture processes of ductile materials and brittle targets impacted by different shapes of particles are studied. The results are showed as follows. Firstly, the SPH numerical model has high accuracy and successfully reproduces the collision process of particles in the process of abrasive jet cutting, including the deformation mechanisms such as plowing, cracking and crushing of the target, and the error of the dent curve formed by the collision is less than 0.5 mm. Secondly, the impact mechanism of angular and circular particles is different for targets with different material properties. For ductile materials, the process of material accumulating to failure value is slow, mainly by the way of accumulation removal. And for brittle materials, it mainly produces plastic deformation and failure, without obvious material accumulation and extrusion area. Thirdly, different impact angles of particles have great influence on material removal.

KEYWORDS
abrasive particles, cutting mechanism, impact simulation, particle impact, Smoothed Particle Hydrodynamics, surface erosion

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1 | INTRODUCTION

The impact process of abrasive water jets can be considered a material impact problem after coupling solid particles with fluid. The solid particles erode the surface of the workpiece and cause local damage, such as plastic deformation, tiny cracks, and material falling off. The workpiece surface is destroyed and removed under continuous erosion. The difference between abrasive water jet-cutting technology and waterjet cutting is mainly the influence of the abrasive particles. The cutting performance of the jet is enhanced after adding abrasive particles, the pressure required by the jet is reduced, and the cutting efficiency is increased. Therefore, the influence of abrasive particles is very important in the process of jet cutting.

Domestic and foreign researchers have performed much research on the impact cutting of abrasive particles, including Finnie, who first proposed the theory of surround cutting and established a rigid particle impact model. Bitter further improved the cutting theory, and then proposed the plastic target cutting model. The model divides the material removal process into two parts: erosion and wear at small and large angles. Kowsari K finds that both primary particle impacts in the conical plume emanating from the nozzle, and secondary particle impacts driven by the flow. The study of Zeng and Kim is more inclined to the microscopic study of the target surface after cutting. Through a series of experimental means, such as scanning electron microscopy, material removal is caused by crystal cracking and viscous flow. The solid particle impact model is mainly used to study the erosion wear mechanism during abrasive cutting or to predict the influence of multiple particles on the surface morphology of the target workpiece. Wang suggests an analytical model of the polished surface topography (or footprint) considering the horizontally asymmetric cross-section profiles of a single-abrasive impact footprint and the abrasive-workpiece elastic deformation and further springback when the abrasive bounces back. Slikkerveer derives relations for the parameters from the basic indentation models that are important for an industrial process: erosion rate, surface roughness and strength. For the solid particle collision model, the FEM is usually used to calculate the deformation of the solid surface. Abrasive particles are modeled as rigid spheres or polyhedrons, ignoring the material properties related to abrasive deformation, and adopting the FEM is an effective method to solve the problem of solid mechanics. However, when the abrasive particles are treated, it causes large plastic deformation and even removes the material, which reduces the accuracy of the calculation. Hence, in this paper, the meshless SPH method is used in the research of abrasive particle impact. This method can avoid the drawbacks of mesh-based algorithms and has good adaptability to solve large deformation impacts. In modeling, Feng et al. visualized the abrasive particles in the jet by the SPH method, added the coupling relationship between abrasive particles and water and studied the interaction between water and abrasive particles in the process of abrasive water jet cutting. Dong et al. established a solid particle impact process model to study the removal form of ductile targets. The SPH method is superior to the traditional numerical method in simulating the moving interface and large deformation after impact. In this paper, parametric modeling of abrasive particles is carried out, and ductile and brittle target materials are selected. The abrasive particles and targets are composed of SPH particles with different properties. The impact parameters and shape parameters of abrasive particles are considered when the model is established, and the parameterized modeling of abrasive particles is carried out according to the shape and motion characteristics of the particles. At the same time, according to the mechanical behavior of the particle impact target, the motion control equation of material removal is described, and a suitable constitutive model is introduced to describe the deformation and failure behavior of the material. The deformation failure mechanism of different shapes of abrasive particles impacting targets was obtained by comparing the experimental results with the simulation results.

2 | ESTABLISHMENT OF ABRASIVE PARTICLE IMPACT MODEL

2.1 | Abrasive impact process description

The abrasive water jet process is a complex fluid-solid coupling problem involving multiphase flow, fatigue failure, chemical corrosion and a series of physical and chemical problems. However, if the motion of abrasive particles during an abrasive water jet is simplified as the impact process of a single particle, the problem becomes a problem of impact mechanics, which, in turn, greatly simplifies the research process. Therefore, a single-abrasive particle is modeled in our approach. The particle size of abrasive particles is generally 200 μm–300 μm; In the process of instantaneous impact, the state of particles is the motion state after receiving the fluid force. Therefore, the fluid can be ignored at the moment of impact, which is simplified as the interaction between particles and target. Assuming that the abrasive particle shape is irregular, we define the particle parameters, as shown in Figure 1. The incident velocity of this abrasive particle is , and the angle between the incident velocity and the horizontal
axis is $a_i$. The reflection velocity of abrasive particles after rebound is $v_r$, the angle between the reflection velocity and the horizontal direction is $\alpha_r$, and the reflection angular velocity is $\omega_r$. The geometric angle of the shape at the impact point of the abrasive particle is defined as $A$, its value is $0~180^\circ$, the angle between the center of mass and the vertical direction is defined as the impact azimuth $\theta_i$, and is defined as shown in Figure 2. The range of the azimuth angle is $-90^\circ~90^\circ$. Because the nozzle is perpendicular to the target material during the abrasive jet cutting process, according to the statistics and the experiments reported in the literature, the azimuth is mostly in the range of $-45^\circ~45^\circ$, which covers 83% of all impact particles. Therefore, the range of impact azimuths of abrasive particles is considered to be $-45^\circ~45^\circ$.

### 2.2 Establishment of abrasive particle and target models

When the abrasive particles impact the target, because the hardness of the particles is much larger than that of the target, the crushing and deformation of the abrasive particles are not considered in the modeling. When modeling abrasive particles, according to the different impact forms, they are converted into three typical particle models, as shown in Figure 3. The circle represents the smooth impact of abrasive particles on the target; the triangle represents the impact morphology when the geometric angle of abrasive particles is acute; the square represents the impact of abrasive particles whose geometric angle is greater than or equal to 90°.

The cutting mechanism for angular particles and circular particles is different when they collide with the target material. Circular particles are more prone to extrusion of the target material, which leads to the material being "squeezed out." Angular particles result in an "excavate-fracture" removal, that is, the angular abrasive particle tip is inserted into the material surface. However, under the resistance of the target, the particles cannot continue to penetrate and the material produces a back spin, so the particle tip leads to an excavation effect, and the excavated material breaks before the end of impact. Therefore, the process is directly accompanied by material removal.

In our SPH cutting model, the most frequently used abrasive particle shapes are spherical particles and angular particles. A two-dimensional (2D) simulation is used in this study; therefore, in the equivalent to the 2D plane, we use three particle shapes: a circle, an equilateral triangle, and a quadrangle. We conduct the simulations using relatively large particle size. To maintain a similar scale for the three types of abrasive particles and exclude the effect of particle size, the three particle sizes are as shown in Figure 4 ($D = h = a$). In the simulations, since the solid deformation is very sensitive to the node spacing, a small node spacing is used to ensure that the resolution is fine enough to capture the microdeformation caused by a single particle.

When the cross-section shape of angular particles remains unchanged along the thickness direction and the impact type is orthogonal impact, the impact problem of this type can be regarded as a two-dimensional plane strain problem. The motion of particles will be limited to the plane coincident with the incident direction, and particles can also be regarded as two-dimensional particles. As shown in Figure 5, two-dimensional angular particles described by corner points and surface nodes.

### 2.3 Shape modeling of abrasive particles

In this paper, triangles and squares are considered in the study of angular particles, while circular particles are used as non-angular particles. When describing the shape of the particle, the size of the particle is first determined, assuming that the height of the triangle particle is $h$ and the side length of the square is $a$; then, the center point coordinate...
position of the abrasive particles is determined. Because the particles studied in this paper are all assumed to be regularly shaped particles, the center, centroid and center of gravity of the particles are all coincident. Finally, the impact angle $\theta_i$ is determined for the particles. The SPH particle formation process is shown in Figure 6.

A is the center point of the abrasive particle, and its coordinates are $(x_0, y_0)$, then the coordinates of the impact vertex of the equilateral triangle are $a_i(x_1, y_1)$.

$$\cos\theta_i = \frac{|x_1 - x_0|}{\frac{2}{3}h}$$

(1)
\[ \sin \theta_i = \frac{y_0 - y_1}{\frac{2}{3} h} \quad (2) \]

Therefore

\[ x_1 = x_0 + \frac{2}{3} h \cdot \cos \theta_i \quad (3) \]

or

\[ x_1 = x_0 - \frac{2}{3} h \cdot \cos \theta_i \quad (4) \]

\[ y_1 = y_0 - \frac{2}{3} h \cdot \sin \theta_i \quad (5) \]

where \( x_1 \) is the horizontal coordinate value of the impact point, and \( y_1 \) is the vertical coordinate value of the impact point.

Hence, the coordinates of the impact vertex of the triangle particles incident on the left side of the center axis are \( a_1(x_0 + \frac{2}{3} h \cdot \cos \theta_i, y_0 - \frac{2}{3} h \cdot \sin \theta_i) \). A particle incident on the right side of the center axis is \( a_1(x_0 - \frac{2}{3} h \cdot \cos \theta_i, y_0 - \frac{2}{3} h \cdot \sin \theta_i) \). The other two vertices of the triangle are \( a_2 \) and \( a_3 \), and the coordinate values are \( a_2(x_2, y_2) \) and \( a_3(x_3, y_3) \).

\[ \cos (60^\circ - \theta_i) = \frac{x_1 - x_2}{\frac{2\sqrt{3}}{3} h} \quad (6) \]

\[ \sin (60^\circ - \theta_i) = \frac{y_2 - y_1}{\frac{2\sqrt{3}}{3} h} \quad (7) \]

Therefore, \( x_2 = x_1 - \left( \frac{\sqrt{3}}{3} \cos \theta_i + \sin \theta_i \right) \cdot h \quad (8) \)

\[ y_2 = y_0 - \left( \frac{2 + \sqrt{3}}{3} \sin \theta_i + \cos \theta_i \right) \cdot h \quad (9) \]

For vertex \( a_2 \),

\[ \sin (\theta_i - 30^\circ) = \frac{x_1 - x_3}{\frac{2\sqrt{3}}{3} h} \quad (10) \]

\[ \cos (\theta_i - 30^\circ) = \frac{y_3 - y_1}{\frac{2\sqrt{3}}{3} h} \quad (11) \]

Therefore,

\[ x_3 = x_1 - \left( \sin \theta_i - \frac{\sqrt{3}}{3} \cos \theta_i \right) \cdot h \quad (12) \]

\[ y_3 = y_0 + \left( \frac{\sqrt{3} - 2}{3} \sin \theta_i + \cos \theta_i \right) \cdot h \quad (13) \]

As a result, the coordinates of the other two vertices of an equilateral abrasive particle are \( a_2(x_2, y_2) \) and \( a_3(x_3, y_3) \), or

\[ a_3 \left( x_1 - \left( \frac{\sqrt{3}}{3} \cos \theta_i + \sin \theta_i \right) \cdot h, y_0 - \left( \frac{2 + \sqrt{3}}{3} \sin \theta_i + \cos \theta_i \right) \cdot h \right) \]

\[ a_2 \left( x_1 - \left( \frac{\sqrt{3}}{3} \cos \theta_i + \sin \theta_i \right) \cdot h, y_0 - \left( \frac{2 + \sqrt{3}}{3} \sin \theta_i + \cos \theta_i \right) \cdot h \right) \]

For square particles, the coordinate of the impact vertex can be obtained when the input parameters, such as side length \( a \), impact angle \( \theta_i \) and center point coordinates \( A_i(x_{00}, y_{00}) \) are known.

\[ \sin \theta_i = \frac{|x_{31} - x_{30}|}{\sqrt{2} a} \quad (14) \]

\[ \cos \theta_i = \frac{y_{30} - y_{31}}{\sqrt{2} a} \quad (15) \]

Therefore,

\[ x_{s1} = x_{30} + \frac{\sqrt{2}}{2} a \cdot \sin \theta_i \quad (16) \]

or

\[ x_{s1} = x_{30} - \frac{\sqrt{2}}{2} a \cdot \sin \theta_i \quad (17) \]

\[ y_{s1} = y_{30} - \frac{\sqrt{2}}{2} a \cdot \cos \theta_i \quad (18) \]

where \( x_{s1} \) is the horizontal coordinate value of the impact point, and \( y_{s1} \) is the vertical coordinate value of the impact point.

Hence, the coordinates of the impact vertex of the triangle particles incident on the left side of the center axis are \( a_{s1} \left( x_{30} + \frac{\sqrt{2}}{2} a \cdot \sin \theta_i, y_{30} - \frac{\sqrt{2}}{2} a \cdot \cos \theta_i \right) \). A parti-
The other three vertices of a square abrasive particle as2, as, and as4, with coordinate values a2(x2, y2), as(x3, y3) and as4(x4, y4).

For vertex as2:

\[
\cos(45^\circ - \theta_i) = \frac{x_{s1} - x_{s2}}{a} \quad (19)
\]

\[
\sin(45^\circ - \theta_i) = \frac{y_{s2} - y_{s1}}{a} \quad (20)
\]

Therefore,

\[
x_{s2} = x_{s1} - \left(\frac{\sqrt{2}}{2}\sin \theta_i + \frac{\sqrt{2}}{2}\cos \theta_i\right) \cdot a \quad (21)
\]

\[
y_{s2} = y_{s0} - \frac{\sqrt{2}}{2} a \cdot \sin \theta_i \quad (22)
\]

Similarly, the coordinates of the other vertices can be obtained as follows:

\[
x_{s3} = x_{s1} - \sqrt{2} a \cdot \sin \theta_i \quad (23)
\]
\[ y_{s3} = y_{s0} + \frac{\sqrt{2}}{2} a \cdot \cos \theta_i \]  
(24)

\[ x_{s4} = x_{s1} + \left( \frac{\sqrt{2}}{2} \cos \theta_i - \frac{\sqrt{2}}{2} \sin \theta_i \right) \cdot a \]  
(25)

\[ y_{s4} = y_{s0} + \frac{\sqrt{2}}{2} a \cdot \sin \theta_i \]  
(26)

As a result, the coordinates are

\[ a_{s2} \left( x_{s1} - \left( \frac{\sqrt{2}}{2} \sin \theta_i + \frac{\sqrt{2}}{2} \cos \theta_i \right) \cdot a, y_{s0} - \frac{\sqrt{2}}{2} a \cdot \sin \theta_i \right) \]  

\[ a_{s3} \left( x_{s1} - \frac{2a}{3} \cdot \sin \theta_i, y_{s0} + \frac{\sqrt{2}}{2} a \cdot \cos \theta_i \right) \]  

\[ a_{s4} \left( x_{s1} + \frac{\sqrt{2}}{2} \cos \theta_i - \frac{\sqrt{2}}{2} \sin \theta_i \right) \cdot a, y_{s0} + \frac{\sqrt{2}}{2} a \cdot \sin \theta_i \right) \]

Therefore, the mass and moment of inertia of angular abrasive particles are defined as follows:

\[ n_{k+1} = \left[ \begin{array}{l} y_{k+2} - y_k \\ x_{k+2} - x_k \end{array} \right], \frac{x_{k+2} - x_k}{x_{k+2} - x_k} \]  
(27)

where the nodes adjacent to \( k + 1 \) are \( k \) and \( k + 2 \), \( \{x_{k+2}, y_{k+2}\} \), and \( \{x_k, y_k\} \).

Triangular and square abrasive particles can be integrated into angular particles, which can be triangulated when calculating the moment of inertia of the angular particles. The coordinates of the center of mass are obtained after triangle dissection of the angular particles. The coordinate of the center of mass is \( x_c^- \)

\[ x^-_c = \frac{\sum A_n \cdot x^-_{nc}}{\sum A_n} \]  
(28)

where \( \sum A_n \) is the total area of angular particles, and \( |x^-_{nc} \) is the centroid coordinate of subtriangle \( n \) after the split.

Therefore, the mass and moment of inertia of angular abrasive particles are defined as follows:

\[ \left\{ \begin{array}{l} M = \sum m_n \\ I_z = \sum \left( i_{zn} + m_n \left( x^-_{nc} - x^-_c \right)^2 \right) \end{array} \]  
(29)

\[ i_{zn} = \frac{m_n}{36} \left( a^2_n + b^2_n + c^2_n \right) \]  
(30)

where \( i_{zn} \) is the moment of inertia of the \( n \)th subtriangle around its centroid rotation axis, \( m_n \) is the mass of the \( n \)th subtriangle, and the three subtriangle sides are, \( a_n, b_n, \) and \( c_n \).

The moment of inertia of the circular abrasive particles can be obtained by the following formula:

\[ I_z = \frac{m_c \cdot d^2}{8} \]  
(31)

where \( m_c \) is the mass of the circular abrasive particle, and \( d \) is the diameter.

3 | BRIEF INTRODUCTION OF THE SPH METHOD

3.1 | Implementation of SPH method process

The SPH method is a meshless algorithm that does not need to use a mesh when describing the problem domain.\(^{12}\) Instead, it uses a series of SPH particles instead of meshing. The particles have their own physical properties, including mass, density, velocity, acceleration, and energy, and move according to the conservation of the control equation. Compared with the meshless nodes in other meshless methods that only act as interpolation points, SPH particles also have material properties and motion based on internal interactions and external forces. Because SPH particles act as approximate points and have material properties, real-world conditions can be accurately represented.\(^{13}\) In the process of calculation, it takes two basic steps to convert the partial differential equation into the SPH equation, namely, kernel approximation and particle approximation. The kernel approximation approximates the field variables and their derivatives by using the information in the nearby regions \( \Omega \),\(^{14-16}\) while the particle approximation discretizes the integral function into a sum of finite particles in the \( \Omega \) domain. The problem domain is shown in Figure 8.

For continuous smooth functions, any point \( m \) in the domain can be expressed as follows\(^{17}\):

\[ f(r) = \int_{\Omega} f(r') \delta(r - r') \, dr' \]  
(32)

where \( r \) is the space position vector, and \( \delta(r - r') \) is the Dirac function.

\[ \delta(r - r') = \begin{cases} \infty, & r = r' \\ 0, & r \neq r' \end{cases} \]  
(33)

When solving this problem, it is difficult to solve the integral by a numerical method. Therefore, the \( f(r) \) function ...
is approximated by the finite integral form given by formula (34) on the domain \( \Omega \), that is, the kernel approximation of the function:\(^18\)

\[
 f(\mathbf{r}) = \int_{\Omega} f(\mathbf{r'}) W(\mathbf{r} - \mathbf{r'}, h) \, d\mathbf{r'}
\]  
(34)

where \( h \) is the smoothing length.

A kernel function is compact, and the value of the function is 0 beyond \( 2h \) of the definition domain.

\[
 W(\mathbf{r} - \mathbf{r'}, h) = 0
\]  
(35)

where \(|\mathbf{r} - \mathbf{r'}| \geq 2h\).

The kernel function is normalized as follows:

\[
 \int W(\mathbf{r} - \mathbf{r'}, h) \, d\mathbf{r'} = 1
\]  
(36)

According to formula (34), the spatial derivative of function \( f(\mathbf{r}) \) can be expressed as \( \nabla \cdot f(\mathbf{r}) \).

\[
 \nabla \cdot f(\mathbf{r}) = \int_{\Omega} [\nabla \cdot f(\mathbf{r'})] W(\mathbf{r} - \mathbf{r'}, h) \, d\mathbf{r'} = - \int_{\Omega} f(\mathbf{r'}) \cdot \nabla W(\mathbf{r} - \mathbf{r'}, h) \, d\mathbf{r'}
\]  
(37)

The particle approximation can be expressed as follows:

\[
 f(\mathbf{r}) = \int f(\mathbf{r'}) W(\mathbf{r} - \mathbf{r'}, h) \, d\mathbf{r'} \approx \sum_{j=1}^{N} \frac{m_j}{\rho_j} \mathbf{f}(\mathbf{r}_j) W(\mathbf{r} - \mathbf{r}_j, h_i)
\]  
(38)

Therefore,

\[
 f(\mathbf{r}_i) = \sum_{j=1}^{N} \frac{m_j}{\rho_j} \mathbf{f}(\mathbf{r}_j) W(\mathbf{r} - \mathbf{r}_j, h_i)
\]  
(39)

where \( m_j \) is the mass of particle \( j \), \( \rho_j \) is the density of particle \( j \), and \( \frac{m_j}{\rho_j} \) is the volume of particle \( j \).

The spatial derivative of the field function at particle \( i \) is approximately

\[
 \nabla \cdot f(\mathbf{r}_i) = \rho_i \sum_{j=1}^{N} \frac{m_j}{\rho_j^2} \left[ \frac{f(\mathbf{r}_j)}{\rho_j} + \frac{f(\mathbf{r}_i)}{\rho_i^2} \right] \cdot \nabla_i W_{ij}
\]  
(40)

### 3.2 Smooth length

In large deformation impact problems particles can be separated. When the smoothing length is constant, it is possible to increase the distance between particles. When the smoothing length reaches a certain value, the interaction between particles disappears. If the smoothing length is compressed, the particles may enter the adjacent computational domain, which leads to a significant reduction in computational speed. To solve this problem, according to the average density, the smoothing length is modified as follows:\(^19\):

\[
 h = h_0 \left( \frac{\rho_0}{\rho} \right)^{\frac{2}{3}}
\]  
(41)

where \( h_0 \) is the initial smoothing length, \( \rho_0 \) is the initial density, \( \rho \) is the density, and \( d \) is the dimension of the computational problem.

The smoothing length can also be adjusted by using the time derivative of the smoothing function.

\[
 \frac{dh}{dt} = - \frac{1}{d} \frac{h \, d^2 \rho}{d \rho \, dt}
\]  
(42)

### 3.3 SPH equations for solids

By using the mass conservation equation, the Navier-Stokes equation of solid media can be expressed in tensor form:

\[
 \frac{d\rho}{dt} = - \rho \frac{\partial \mathbf{v}^\theta}{\partial x^\theta}
\]  
(43)

The momentum conservation equation is defined as follows:

\[
 \frac{dv^\theta}{dt} = \frac{1}{\rho} \frac{\partial \rho a^\theta}{\partial x^\theta} + f^\theta
\]  
(44)

where \( \rho \) is the solid medium density, \( v^\theta \) is the velocity vector, \( x^\theta \) is the position vector, and \( t \) is the time.

The SPH equations for solids can be expressed as:\(^20\)

\[
 \frac{d\rho_i}{dt} = \rho_i \sum_{j=1}^{N} \frac{m_j}{\rho_j} v^\theta_j \frac{\partial W_{ij}}{\partial x^\theta_i}
\]  
(45)

\[
 \frac{dv^\theta_i}{dt} = - \sum_{j=1}^{N} m_j \left[ \frac{p_i + p_j}{\rho_i \rho_j} - \frac{\mu_i \varepsilon_i^{\alpha \beta} + \mu_j \varepsilon_j^{\alpha \beta}}{\rho_i \rho_j} \right] + \Pi_{ij} \right] \frac{\partial W_{ij}}{\partial x^\theta_i}
\]  
(46)

where \( W \) is the smooth kernel function, \( \mu \) is Poisson’s ratio, \( \varepsilon \) is the strain, and \( \Pi_{ij} \) is the artificial viscosity term.
In solid media, stress can be expressed as the sum of hydrostatic pressure and stress deviation, \[\sigma^{\alpha\beta} = -p\delta^{\alpha\beta} + S^{\alpha\beta}\] (46)

where \(S^{\alpha\beta}\) is the stress bias. The Jaumann rate is substituted into the stress bias, which can be expressed as follows:

\[S^{\alpha\beta} = 2G\left(\dot{e}^{\alpha\beta} - \frac{1}{3}\delta^{\alpha\beta}\dot{\varepsilon}^{rr}\right) + S^{\alpha\gamma}R^{\beta\gamma} + S^{\beta\gamma}R^{\alpha\gamma}\] (47)

where \(G\) is the shear modulus, and \(\dot{\varepsilon}\) is the strain rate tensor, which can be expressed as follows:

\[\dot{\varepsilon}^{\alpha\beta} = \frac{1}{2}\left(\frac{\partial v^{\alpha}}{\partial x^{\beta}} + \frac{\partial v^{\beta}}{\partial x^{\alpha}}\right)\] (48)

where above, \(R^{\alpha\beta}\) is the rotation rate tensor, which can be expressed as follows:

\[R^{\alpha\beta} = \frac{1}{2}\left(\frac{\partial v^{\alpha}}{\partial x^{\beta}} - \frac{\partial v^{\beta}}{\partial x^{\alpha}}\right)\] (49)

Assume that the material obeys the von Mises yield criterion. When the equivalent stress \(J_2\) is greater than the yield stress \(Y_{JC}\) of the material, the stress deviation is corrected to

\[S^{\alpha\beta} = S^{\alpha\beta}Y_{JC}/J_2\] (50)

where the equivalent stress can be expressed as follows:

\[J_2 = \sqrt{3/2S^{\alpha\beta}S^{\alpha\beta}}\] (51)

\subsection*{3.4 Abrasive particle movement}

When the abrasive particles impact the target material, the velocity can be decomposed into the translational velocity and the rotational velocity, and the conservation equation of the abrasive particles can be expressed as follows:

\[
\begin{align*}
\frac{dV_j^\alpha}{dt} &= F_j^\alpha \\
\frac{dW_j^\alpha}{dt} &= T_j^\alpha \\
\frac{dX_j^\alpha}{dt} &= I_j \\
\frac{dx_j^\alpha}{dt} &= V_j^\alpha
\end{align*}
\] (52)

where \(j\) is the retrieved \(j\)th abrasive particle, \(V_j^\alpha\) is the translational velocity, \(W_j^\alpha\) is the rotational velocity, \(X_j^\alpha\) is the central position of the particle, \(M_j\) is the mass of the \(j\)th particle, \(I_j\) is the moment of inertia of the \(j\)th particle, \(F_j^\alpha\) is the resultant force on the \(j\)th particle, and \(T_j^\alpha\) is the action time.

In this model, each abrasive particle is associated with several discrete SPH nodes, and each SPH node carries field variables, including mass and velocity. The equations of motion of these nodes are defined as follows:

\[v_{j-1}^\alpha = v_{j}^\alpha + W_{j}^\alpha \times (X_{j-1}^\alpha - X_{j}^\alpha)\] (53)

where \(j\) is the \(j\)th node in the \(i\)th particle, and \(X_{j-1}^\alpha\) is the position vector of the node.

For the nodes close to the interface, two or three particles of the material can be included. The common method to address this situation is to solve all particles in the region by control. The material properties of these SPH particles are not considered. Accordingly, the total force \(F_j^\alpha\) and moment \(T_j^\alpha\) acting on the \(j\)th rigid body can be expressed as follows:

\[
\begin{align*}
F_j^\alpha &= \sum_{i=1}^{N} m_{i-j} \frac{dv_{i-j}^\alpha}{dr} \\
T_j^\alpha &= \sum_{i=1}^{N} m_{i-j} \frac{dv_{i-j}^\alpha}{dr} \times (X_{i-j}^\alpha - X_j^\alpha)
\end{align*}
\] (54)

where \(i\) and \(j\) are the labels of SPH nodes and particles, \(m_{i-j}\) is the mass of the \(i\)th node from the \(j\)th particle, and \(\frac{dv_{i-j}^\alpha}{dr}\) is the acceleration caused by the surrounding continuous phase in the \(i\)th node circle.

\subsection*{3.5 Constitutive model of target material}

When modeling, the accuracy of the description of the plastic behavior of the target material directly affects the accuracy of the calculation. Target materials mainly consider ductile materials and brittle materials, and in the study of this paper these two kinds of materials are considered elastoplastic materials.

Ductile materials and brittle materials do not have the same impact erosion mechanism. The ductile material is dominated by plastic deformation, and the surface of materials is cut and peeled after the high-speed jet impact, while the fracture of brittle materials is through cracks and crack propagation.

\subsection*{3.5.1 Model of ductile materials}

For engineering, the most commonly used material constitutive model is the Johnson-Cook model based on experimental data. The Johnson-Cook constitutive model has a good predictive power when describing the
mechanical behavior of metal materials in the process of large deformation and high strain rate cutting.\textsuperscript{21} As a result, the Johnson-Cook viscoplastic model is used, which can be expressed as follows:

\[
\sigma_y = \left( A + B \left( \overline{\varepsilon}^p \right)^n \right) \left( 1 + C \ln \dot{\varepsilon}^p \right) \left( 1 - \left( \frac{T}{T_{\text{m}} - T_{\text{r}}} \right) \right)
\]

(55)

\[
\overline{\varepsilon}^p = \frac{\varepsilon^p}{\varepsilon_0}
\]

(56)

\[
T^* = \frac{T - T_{\text{r}}}{T_{\text{m}} - T_{\text{r}}}
\]

(57)

where \(\sigma_y\) is the yield stress of the material, \(A, B, C, m, n\) are all the relevant parameters of the material, \(\overline{\varepsilon}^p\) is the equivalent plastic strain, \(\dot{\varepsilon}_\text{eq}^p\) is the dimensionless plastic strain rate, \(T^*\) is the homologous temperature, \(\varepsilon_0\) is the plastic strain, \(\varepsilon_0\) is the reference strain rate, the \(T\) is the actual temperature, \(T_r\) is the room temperature, and \(T_m\) is the melting point of the material.

The Johnson-Cook fracture criterion is adopted for the failure fracture behavior of the target material.\textsuperscript{20} In the Johnson-Cook failure model, the failure strain \(\varepsilon_f\) is expressed as follows:

\[
\varepsilon_f = \varphi \left( \sigma^*, T^*, \dot{\varepsilon}_\text{eq}^p \right) = \left[ D_1 + D_2 \exp \left( D_3 \sigma^* \right) \right] \left[ 1 + D_4 \ln \dot{\varepsilon}_\text{eq}^p \right] \left[ 1 + D_4 T^* \right]
\]

(58)

\[
\sigma^* = \frac{\sigma_m}{\sigma_{\text{eq}}}
\]

(59)

where \(D_1-D_3\) is the material performance parameter, \(\sigma_m\) is the average value of the main stress, and \(\sigma_{\text{eq}}\) is the equal effect force.

For the high-speed impact of abrasive particles on ductile material, to calculate the isotropic pressure of the material, the Mie-Gruneisen equation is used to simulate the impact effect of high-speed abrasive particles on the target material. The state equation is expressed as follows:\textsuperscript{23}

\[
p = \frac{\rho_0 C_0^2 \eta \left( 1 + \left( \frac{T_0}{\eta} \right) \right) \eta}{\left( 1 - (S_a - 1) \eta \right)} + \rho_0 \Gamma_0 \varepsilon
\]

(60)

where \(\rho_0\) is the reference density (kg/m\(^3\)), \(C_0\) is the Mie-Gruneisen equation constant, \(S_a\) is the linear Hugoniot coefficient, and \(\varepsilon\) is the internal energy per unit mass.

In the selection of ductile target material, a titanium alloy (TC4) is selected, and its material parameters are shown in Table 1.

\[
\text{FIGURE 8 Approximation of SPH particles in the two-dimensional problem domain}
\]

### 3.5.2 Model of brittle materials

For brittle materials, the most commonly used model is the Johnson-Holmquist constitutive model.\textsuperscript{24} Model failure is defined as follows:

\[
\sigma^* = \left[ A (1 - D) + B P^s N \right] \left( 1 + C \ln \dot{\varepsilon}^p \right)
\]

(61)

where \(\sigma^*\) is the equivalent strength (MPa), \(A\) is the cohesive strength (MPa), \(D\) is the damage factor, \(B\) is the pressure hardening factor, \(P^s\) is the dimensionless HEL pressure, \(P^s = P/P_{\text{HEL}}\), \(N\) is the stress hardening index, \(C\) is the strain rate coefficient, and \(\dot{\varepsilon}^p\) is the equal effect rate (s\(^{-1}\)).

The damage factor \(D\) represents the amount of damage of the target material. When \(D\) equals 0, it means that the target material is not damaged, and when \(D\) equals 1, it means that the target material begins to break. The expression is defined as follows:

\[
D = \sum \frac{\Delta \varepsilon_p + \Delta \mu_p}{\varepsilon^p + \mu^p}
\]

(62)

where \(\Delta \varepsilon_p\) is the increment of equal plastic deflection strain, \(\Delta \mu_p\) is the increment of plastic volume strain, \(\varepsilon^p\) is the equivalent plastic deflection strain, and \(\mu^p\) is the plastic volume strain. In the damage relationship of the JH\(_2\) model, \(T\) represents the maximum tensile strength, \(D_1\) and \(D_2\) are the damage constants, and \(E_{\text{lin}}\) represents the minimum plastic strain variable before fracture. where \(f_c\) is the static uniaxial compressive strength, parameter \(A\) represents the normalized cohesive strength, \(B\) is the normalized pressure hardening coefficient, \(C\) is the strain rate coefficient, \(\sigma^*\) is the normalized equivalent strength, and \(P\) is the static pressure.
In the JH2 model, impacted by the abrasive particles, the target material is first compressed, and then it is stretched and peeled off when it reaches the limit state. The two processes are analyzed independently. In the compression stage, it is divided into an elastic zone, plastic transition zone and compact compression material zone, while in the process of tensile spalling, there is only the damage factor elastic zone.

In the elastic compression stage, the volume change of the brittle material is linear with the pressure, namely,

\[ P = K \mu \]  \hspace{1cm} (63)

where \( K \) is the bulk modulus (MPa), \( \mu \) is the volume strain, \( \mu = \rho / \rho_0 - 1 \), \( \rho \) is the density of brittle materials (kg/m\(^3\)), and \( \rho_0 \) is the initial density of the brittle material.

In the plastic transition stage, some parts of the brittle materials have already produced non-recoverable damage, that is, there are microcracks, and the pressure can be expressed as follows:

\[ P = P_c + K_c (\mu - \mu_c) \]  \hspace{1cm} (64)

\[ K_c = \frac{P_t - P_c}{\mu_t - \mu_c} \]  \hspace{1cm} (65)

where \( P_c \) is the pressure at the impact point (MPa), \( P_t \) is the pressure at the material compaction point (MPa), \( \mu_c \) is the volume strain at the impact point, and \( \mu_t \) is the volume strain at the material compaction point.

The time of the material compaction stage is very short. After the compaction stage is complete, the microcracks begin to expand, and then staggered complex cracks are formed, which leads to spalling of the materials. The expression for pressure is defined as follows:

\[ P = K_1 \bar{\mu} + K_2 \bar{\mu}^2 + K_3 \bar{\mu}^3 \]  \hspace{1cm} (66)

\[ \bar{\mu} = \frac{\mu - \mu_t}{\mu_t} \]  \hspace{1cm} (67)

where \( K_1, K_2 \) and \( K_3 \) are material constants.

The equation of state for brittle materials can be expressed as follows:

\[ \begin{align*}
P &= K_1 \mu + K_2 \mu^2 + K_3 \mu^3 \quad \mu > 0 \\
&= K_1 \mu \mu \leq 0
\end{align*} \]  \hspace{1cm} (68)

where \( K_1, K_2 \) and \( K_3 \) are the correlation constants of the material and their values are measured by the plate impact experiment. \( \mu = \rho / \rho_0 - 1 \), where \( \rho \) is the current density and \( \rho_0 \) is the initial density.

Glass was selected as the brittle target, and its specific material properties are shown in Table 2.

### 3.6 Fracture and contact algorithms

The contact effects between SPH particles with different material properties can be achieved by applying contact
forces to the SPH particles. In Figure 3, when the distance between the injected SPH particles (water jet particles or abrasive particles) and the coal particles reaches twice the smoothing length, contact forces are generated on the relevant SPH particles. The radius of the SPH particle support area $\Omega$ is twice the smoothing length. The contact function is defined as follows:

$$
\phi(x_i) = \sum_{j}^{N} \frac{m_j}{\rho_j} K \left( \frac{W(r_{ij})}{W(\Delta d_{ij}^{avg})} \right)^n
$$

(69)

where $N$ is the number of particles of different materials in the supporting domain $\Omega$ of particle $i$, as shown in Figure 9, and the value of $N$ is 3. When calculating the contact force of the particle, only $N_1$, $N_2$, and $N_3$ are included as SPH particles in the support domain $\Omega$. When $x_a$ and $x_b$ belong to the same kind of material, the value of the kernel function is 0. The values of $K$ and $n$ should be determined according to the working conditions. The $K$ value is similar to the penalty stiffness values in finite element contact and is related to the material properties and impact velocity.

The potential function is similar to the repulsive force to avoid the tension instability. It has the following characteristics: (a) the value of the function inside the object is 0; (b) the value is usually positive; (c) the value of the potential function increases with decreasing particle spacing.

The SPH scheme is used to discretize the gradient of the contact potential function. The contact forces acting on the particles are obtained as follows:

$$
f_c(x_i) = \sum_{j}^{N} \frac{m_j}{\rho_j} m_i K_n \left( \frac{W(r_{ij})}{W(\Delta d_{ij}^{avg})} \right)^{n-1} \nabla x_i W(r_{ij})
$$

(70)

After introducing the contact force into the SPH momentum equation, the results are obtained as follows:

$$
\frac{dv_i^a}{dt} = \sum_{j=1}^{N} m_j \left( \sigma_{ij}^{a\beta} + \sigma_{ij}^{a\beta} - \Pi_{ij} + R_{ij}\delta^a_{ij} \right) \frac{\partial W_{ij}}{\partial x_i^a} - \frac{f_c(x_i)}{m_i}
$$

(71)

The smoothing length of the SPH particles at the fracture point decreases when calculating the fracture zone. This method reduces the interaction between SPH particles of the fracture zone and other SPH particles. However, reducing the smoothing length of invalid particles leads to a decrease in the time step and an increase in the calculation time. Therefore, when the critical value is reached, the particles are considered invalid, and fracture is assumed to occur at the position related to the particles. Then, the particles at that location are removed from the SPH calculation. However, to ensure the mass and momentum conservation of the whole jet system, the mass and momentum of the invalid particles that are not involved in the calculation are still retained.

In Figure 10, the particles $j$ identified by red dots are invalid particles and are removed from the list of adjacent particles associated with particle $i$. Therefore, although the mass and momentum are still constantly updated, the SPH calculations for particles $j$ are not performed. When the target is in a compression state, the target can still bear the load. Therefore, although the particles associated with the damage variables have reached the fracture critical value, these particles are still used for SPH calculations under compression conditions.
3.7 | Artificial viscous forces

When solving the problem of high-speed impact, a large amount of energy accumulation is generated in the impact region, which result in nonphysical oscillation in the calculation. The shock wave discontinuities in the impact region do not occur in real physics, but in a very small transition region, the material is very thin, the number of molecules is very small, and the size of the region is generally the average free path of several molecules. If there is no artificial viscosity term in the governing equation, the SPH solution presents significant nonphysical oscillations or fluctuations. This is because impact and discontinuity always exist under initial conditions, and the viscous force is not eliminated without artificial viscosity. Therefore, if there is no damping force in the momentum equation represented by equation (45), it causes unreasonable numerical oscillation or fluctuation problems.

In this paper, Monaghan’s artificial viscosity form is used, and this artificial viscosity is the most widely used impact problem. It can not only convert kinetic energy into heat energy and provide essential dissipation of the collision surface but also prevent the nonphysical penetration of particles when they are close to each other. The specific expression is defined as follows:

\[
\Pi_{ij} = \begin{cases} 
-\alpha_{II} \vec{c}_{ij} \phi_{ij} + \beta_{II} \phi_{ij}^2 & \text{if } \mathbf{v}_i \cdot \mathbf{x}_{ij} < 0, \\
0 & \text{if } \mathbf{v}_i \cdot \mathbf{x}_{ij} \geq 0
\end{cases}
\]  

(72)

where

\[
\phi_{ij} = \frac{h_{ij} \mathbf{v}_i \cdot \mathbf{x}_{ij}}{||\mathbf{x}_{ij}||^2 + \varphi^2}
\]  

(73)

\[
\vec{c}_{ij} = \frac{1}{2} (c_i + c_j)
\]  

(74)

\[
\bar{\rho}_{ij} = \frac{1}{2} (\rho_i + \rho_j)
\]  

(75)

\[
h_{ij} = \frac{1}{2} (h_i + h_j)
\]  

(76)

where \(\alpha_{II}\) and \(\beta_{II}\) are the Monaghan-type artificial viscosity constants, and \(\varphi = 0.1h_{ij}\) is used to prevent numerical divergence when particles are close to each other. That is, when the \(x_{ij}\) term in the denominator is zero, the calculation is unstable. The following ranges are used for the values of \(\alpha_{II}\) and \(\beta_{II}\). When simulating free surface flow, \(\alpha_{II}\) is taken as 0.01; when dealing with solid mechanics problems, \(\alpha_{II}\) is taken as 2.5; and when dealing with impact problems, \(\alpha_{II}\) is taken as 1. The parameter \(\beta_{II}\) is used to address the particle penetration problem at a high Mach number, while in the process of abrasive water jet impact, the value of the jet velocity is 100 m/s–300 m/s, which corresponds to a low Mach number, so the value is 1.

3.8 | Time step

In this paper, the frog leaping method with low storage is used for time integration. When a time step is completed, the density, velocity and internal energy are advanced by half a step from the initial state, and the particle position is advanced by one step:

\[
\begin{align*}
\rho_i^0 &= \rho_i^0 + \frac{\Delta t^1}{2} \frac{d\rho_i^0}{dt} \\
\mathbf{v}_i^0 &= \mathbf{v}_i^0 + \frac{\Delta t^1}{2} \frac{d\mathbf{v}_i^0}{dt} \\
\mathbf{x}_i^0 &= \mathbf{x}_i^0 + \frac{\Delta t^1}{2} \mathbf{v}_i^0
\end{align*}
\]  

(77)

(78)

(79)

where \(\Delta t\) is the time step, \(\rho\) is the density of the particles, \(\mathbf{v}\) is the velocity vector of the particle motion, \(\mathbf{x}\) is the position vector of the particles, and the upper corner of each parameter represents the running time step.

In the overall calculation, to ensure that the particle density, velocity and internal energy of the half time step running parameters can be consistent with the position of the particle, these parameters advance forward half a time step at the beginning of each operation step to obtain the value of the integer step.

\[
\rho_i^n = \rho_i^{n-\frac{1}{2}} + \frac{\Delta t^n}{2} \frac{d\rho_i^{n-\frac{1}{2}}}{dt}
\]  

(80)
\[ \nu_i^n = \nu_i^{n-\frac{1}{2}} + \frac{\Delta t^n \, d\nu_i^{n-1}}{2} \]

where \( n \) is the time step label.

At the end of a time step operation, the density, velocity, internal energy, and position of the particles advance one time step:

\[ \rho_i^{n+\frac{1}{2}} = \rho_i^{n-\frac{1}{2}} + \frac{\Delta t^n + \Delta t^{n+1}}{2} \, d\rho_i^n \]  
\[ \nu_i^{n+\frac{1}{2}} = \nu_i^{n-\frac{1}{2}} + \frac{\Delta t^n + \Delta t^{n+1}}{2} \, d\nu_i^n \]
\[ x_i^{n+1} = x_i^n + \Delta t^{n+1} \nu_i^{n+\frac{1}{2}} \]

Considering the stability of the numerical calculation, each time step is limited according to the time step of viscous dissipation and external force. The expression is defined as follows:

\[ \Delta t_{cv} = \min \{ \frac{h_i}{c_i + 0.6 \left[ \alpha_{II} c_i + \beta_{II} \max (\phi_j) \right]} \} \]  
\[ \Delta t_f = \min \left( \frac{h_i}{f_i} \right)^{\frac{1}{2}} \]

where \( c_i \) is the sound velocity of particle \( i \) in the material, \( h_i \) is the smoothing length of SPH particle \( i \), \( \alpha_{II} \) and \( \beta_{II} \) are artificial viscosity constants, \( f_i \) is the unit mass force acting on particle \( i \), and the final time step is the minimum value in equations (85) and (86). The total number of steps of the impact process is determined by the actual process time of the impact phenomenon, and the total number of steps includes a complete process of particles impacting the target.

4 | NUMERICAL RESULTS AND ANALYSIS

4.1 | Numerical simulation

In this paper, the model parameters of angular particles (square and regular triangles) and circular particles are set to study the impact of different shapes of abrasive particles on the surface of ductile and brittle materials. The two-dimensional model is established by using the SPH code written in Fortran according to the basic principle outlined above. The target material is set as a rectangular target block with material properties, and the impact effect of different forms of abrasive particles is compared under the same conditions. The initial velocity of particle impact is 100 m/s, the initial azimuth angles are 30°, 45° and 0°.

4.1.1 | Comparison of the results of the impact ductility material (TC4)

As shown in Figure 11, a V-shaped pit is formed when the abrasive particles (the impact azimuth angle is 0°) impact the target surface, and material accumulates on both sides of the impact pit. The simple vertical impact is not very effective for material removal because in the continuous impact process, the process of material accumulation to a failure value is slow. When the triangular abrasive particles impact the target with a certain impact azimuth angle, for example, when \( \theta_i \) is equal to 30°, at the simulation time of 0–45 μs, the target material accumulates on one side. When the abrasive particles cannot continue to penetrate the target body, the direction of the particle motion is deflected, which produces a certain angular velocity and it breaks away from the target; however, with increasing impact azimuth angle, when \( \theta_i \) is equal to 45°, a single impact of an abrasive particle results in material surface removal due to the shallow impact extrusion area.

Taking square abrasive particles as the impact body, the impact azimuth angle \( \theta_i \) is 0° (vertical impact target) and 30°, and the impact speed is 100 m/s. At different times, the extracted collision effect is shown in Figure 12. For vertical incidence, at the same calculation time point, the pits generated by the impact on the target surface are shallow relative to the pits generated by the regular triangle abrasive particles, and the impact of the particles does not cause material removal and shedding. When the square abrasive particles impact the target at an azimuth angle of 30°, an area of unilateral accumulation is formed. When the target material in the accumulation area reaches the fracture limit of the material, the material is removed.

The impact mode of the circular abrasive particles is relatively simple compared with that of the angular particles, and the simulation results of the vertical impact on target material are shown in Figure 13. The target material begins to form a stress concentration. With the increase in the calculation step, plastic deformation gradually occurs, and material accumulation is formed. Circular abrasive particles cannot effectively remove the surface material when impacting the target.

4.1.2 | Comparison of the results of impact on brittle materials (glass)

Glass was selected as the brittle target material in the simulation of the impact process, abrasive particles of different shapes are used to impact the target with different...
initial impact azimuth angles, and the impact velocity is set to 100 m/s. Figure 14 shows that the removal method of the target material is different from that of the ductile material after being impacted. The brittle material mainly produces plastic deformation and breakage, and there is no obvious material accumulation or extrusion area. For the three shapes of abrasive particles, the cracks produced by the impact of regular triangular abrasive particles are the most obvious, and the longitudinal cracks are the deepest, followed by square abrasive particles and circular abrasive particles. For abrasive particles of the same shape, with increasing impact azimuth angle $\theta_i$, the longitudinal depth of the stress concentration area becomes shallower, which is not conducive to material removal. For circular
abrasive particles, the longitudinal depth of the stress concentration region produced by impacting brittle materials is deeper than that of impacting ductile materials.

5 | EXPERIMENTAL RESULTS

In this section, aiming at the impact erosion process of a single particle, an ejection launcher for the study of the erosion wear mechanism of small particles is designed, which can launch a single solid particle to impact the target material, and capture the impact dynamic process of particles through a high-speed camera. The device can effectively study the influence of single factors (impact
velocity, impact angle, azimuth angle, etc.) on the erosion process, and realize the controllable research variables.

The design objectives of the ejection experiment system are,

1. The impact erosion process of a single particle with arbitrary shape can be studied, and the particle size is about 5 mm;
2. The micro-mechanical behavior of single particle impact of different target materials was studied;
3. The impact angle and impact azimuth of particles are adjustable, and the impact speed is limited.

As shown in Figure 15, the ejection launching device adopts the lever principle to convert the elastic potential energy of the spring into the kinetic energy of the ejection rod, so as to give the initial kinetic energy to the particles. The specific operation process is as follows: the spring is connected to one end of the ejection rod, the other end of the ejection rod is equipped with a particle holder (as shown in the upper right figure of Figure 15), and the particles to be studied are placed on the particle holder; Press down one end of the ejection rod with particles to the locking position, and the spring at the other end reaches the maximum tensile position; Operate the release device, and the spring drives the ejection rod to rotate; The ejection rod rotates to a certain position and stops after hitting the stagnation block (hard rubber block). At this time, the particles placed by the particle holder still have kinetic energy, fly out by inertia, and finally hit the target material at a certain speed, causing plastic deformation to the target material. The target fixing device is provided with a rotation degree of freedom, which can adjust the incidence angle of erosion particles by rotation; The particle azimuth can be adjusted by rotating the position of the particle holder. The particle holder can be made into different shapes and sizes to meet the requirements of emitting particles with different shapes and sizes, so as to study the influence of particle shape and size on erosion; The target holder can load and unload targets of different materials to realize the research of different target materials.

The high-speed imaging system includes high-speed camera, light source, camera support and supporting software. The maximum frame rate of the high-speed camera is 200,000 frames/s. Other performance parameters of the camera are as follows: full resolution (pixels): 2,560 × 1,920 (5 megapixels); Exposure time: minimum to 1.1 ms; Ultra high sensitivity: ISO black and white up to 80,000, color up to 16,000; In the impact test, the frame rate is 5,000 frames/s (resolution 1,280 × 720), the particle shape is shown in Figure 16.

The main work flow is as follows: (1) the target material block is installed on the target fixing mechanism, and according to the requirement of initial azimuth, the angle of the fixing mechanism of the target body is adjusted before the impact test, so as to meet the experimental requirements. (2) Install the particles to be studied to the particle fixing mechanism and lock the ejection switch; (3) During the experimental operation, the ejection switch

---

**FIGURE 15** Particle impact test bench. 1-test bench base, 2-spring, 3-connecting rod, 4-stop, 5-protective baffle, 6-target fixing mechanism, 7-particle fixing mechanism, 8-ejection switch

**FIGURE 16** Diagram of impact angle coordinates. (A) Impact test bench, (B) The coordinate system of impact angle
is released, the spring shrinks and pulls the connecting rod to rotate. When it reaches the stop position, it stops. At this time, the particles continue to move according to the inertia and hit the target material at a certain speed.

To verify the accuracy of the numerical model and improve the SPH model, numerical simulations and experiments are carried out for the process of single-abrasive particles impacting the target. The single particle impact
The experiment is realized by using the particle impact test bench. For different shapes of particles impacting on target material with different properties, the high-speed camera is used to capture and track the process dynamically. The main parameters in the experiment are as follows: (1) The size of square, regular triangle and circular particle is $a = h = d = 5\, \text{mm}$, respectively; (2) The particle impact velocity is $100\, \text{m/s}$; (3) Figure 3 shows that when abrasive particles impact the target material in water, the main impact azimuth angle ranges from $-45^\circ$ to $45^\circ$ (angular symmetry). Therefore, the impact azimuth angles are taken as $0^\circ$, $30^\circ$ and $45^\circ$ in the experimental process.

The initial impact velocity of the particle is perpendicular to the connecting rod. In order to simplify the research process, the initial velocity angle is set as $\alpha$, and impact azimuth is $\theta_i$. The relationship of them is: $\theta_i = 90^\circ - \alpha$. The angle of the target fixing mechanism is adjustable to ensure that the initial impact azimuth meets the requirements of different experimental angles. The coordinate system of impact angle is shown in Figure 16(B). The coordinate system is established by taking the connecting rod as the $x$-axis direction and perpendicular to the connecting rod as the $y$-axis, $\alpha$ is the angle with the $x$-axis, $\theta_i$ is the angle with the $y$-axis.

In the experimental process, the particle impact experimental device is shown in Figure 15. The particles adopt a high-speed steel blade with high hardness. The dynamic process of the particle impact on the target is captured by a high-speed camera, as shown in Figure 16(A).

During the impact process, the particles will rebound after impacting the target. As shown in Figure 17, the impact process of the regular triangle (the processing side is diamond) and the motion trajectory of the rebound process are obtained through capture and post-processing. The incident angle is equal to the rebound angle.
After impacting the target, impact dents will be generated on the surface of the target due to the soft target material relative to the particle material. The impact contour of the target surface is drawn. After the vertical impact, the impact dent on the target surface caused by the regular triangular particles is obtained by the point tracing method, as shown in Figure 18. After the vertical impact TC4, the impact trace is approximately symmetrically distributed. Because the target is a ductile material, there will be accumulation deformation at the dent boundary, making the height of both sides slightly higher than the plane.

To verify the accuracy of the SPH model, the calculation results of the single-abrasive particle impacting the target are compared with the experimental results, as shown in Figure 19. The SPH models were used to simulate regular triangle, square particles, and then the titanium alloy (TC4) materials was impacted at 100 m/s, and the glass material was impacted by regular triangle, square and circular particles at the same speed. When the incident angle is 30° and 45°, the insertion depth is almost the same, but the simulation results show that the horizontal cutting amount is larger than the measured value, and the interpolation becomes larger as the angle increases. For the plastic deformation-dominated impact processes, Johnson-Cook parameters are based on a certain range of strain rates (less than $10^5$). However, in the process of abrasive particle impact, the strain rate can reach $10^5$. Therefore, the yield limit obtained by numerical simulation is lower than the actual situation under this strain rate, which results in the calculated results being larger than the experimental results. When calculating the impact dependence on azimuth, the direction of stress concentration changes, so the displacement in the direction of stress concentration is greater than in the actual situation. As a result, the impact dependence on azimuth angle is larger than the measured value for the horizontal displacement, and the difference is more obvious with increasing azimuth angle. A comparison of the simulation and experimental results of the impact deformation of ductile materials shows that the maximum error in the vertical direction is approximately 0.2 mm, the maximum error in the
horizontal direction is approximately 0.5 mm, and the degree of coincidence between the simulation results curve and the experimental curve is high.

In the impact experiment of brittle materials, glass is selected as the target material. A comparison of the impact process is shown in Figures 20 and 21. The transparency of glass is good, and the morphology caused by some cracks and impact can be observed. When abrasive water jet cutting is carried out later, the contour of cutting can be clearly seen. It can be clearly seen that a large number of debris will be generated at the moment of impact on the target, which is also an obvious feature different from ductile materials, and the rebound trajectory is also obvious compared with impact ductile materials. Therefore, the motion trajectory of particles is also overlapped in the figure.

Figure 22 shows a comparison between the numerical simulation results and the experimental results for brittle materials. The experimental results in the figure are the dent profile of Section 3 when the experimental drawing points are taken. By comparison, it can be found that the dent trajectory similarity is higher when the brittle material is impacted, but more broken points in the measured curves of angular particles lead to uneven curves, and the numerical simulation results are smoother. Because the behavior of the material in the impact process is mainly dynamic failure, and there are many factors affecting the fracture of brittle materials, only the fracture trend and morphology can be predicted. The impact form of circular particles is relatively simple, so the simulation results curves are more consistent with the measured curves. Generally, the error of the numerical simulation results for impacting brittle materials is larger than that for ductile materials, but it also has an allowable range of errors. Therefore, it can be concluded that the impact model of a single-abrasive particle can simulate the impact on ductile and brittle materials well, and the simulation results are more accurate than those of other methods.

6 | CONCLUSIONS

In this paper, the SPH method is used to construct the process model of abrasive particles impacting targets, and the model is verified by experiments. The conclusions are as follows:
1. The SPH method provides a new numerical model for simulating the impact process of abrasive particles. The target is modeled as an elastic-plastic material, and the abrasive particles are modeled as rigid bodies with irregular shapes. The model can be used to study the impact process of microparticles and provide a new numerical model for the mechanism of abrasive water jets.

2. Through the experiment of particle impact, the numerical simulation results are verified, and it is found that the accuracy of the numerical model is high, the impact process of a single particle can be simulated well, and the error of the dent curve formed by impact is less than 0.5 mm.

3. The target materials with different material properties are impacted by circular, regular triangle and square abrasive particles. For ductile materials, a V-shaped pit is produced when the angular particles are vertically incident on the target surface, the materials accumulate on both sides of the impact pit, and the process of material accumulation to the failure value is slow. When the triangular abrasive particles impact the target at an azimuth angle of 30°, the target material accumulates on one side. When the abrasive particles cannot continue to penetrate the target, the movement direction of the particles is deflected, which produces a certain angular velocity and the particles detach from the target; however, with increasing impact azimuth angle, a single impact of an abrasive particle leads to the removal of surface material. When the square abrasive particles are incident vertically, at the same calculation time point, the pits produced by the impact on the target
surface are shallower than those produced by the regular triangle, and the impact of the particles does not cause the material to be removed and shed. When the square particles impact the target at an azimuth angle of 30°, a single side accumulation is formed. When the target material in the accumulation area reaches the fracture limit of the material, material is removed. The impact mode of circular abrasive particles is simpler than those of the angular particles. When the circular abrasive particles impact the target material vertically, an initial stress concentration of the target material is formed. With increasing the calculation step, plastic deformation is gradually produced, and the accumulation of materials is formed. Circular abrasive particles cannot effectively remove the surface material when impacting the target.

Brittle materials differ from ductile materials in the way of removing material after being impacted; brittle materials mainly produce plastic deformation and breakage, and there is no obvious material accumulation and extrusion area. For the three shapes of abrasive particles, the cracks produced by the impact of regular triangular abrasive particles are the most obvious, and the longitudinal cracks are the deepest, followed by square abrasive particles and circular abrasive particles. For abrasive particles of the same shape, with increasing impact azimuth angle θ, the longitudinal depth of the stress concentration area becomes shallow, which is not conducive to material removal. For the circular abrasive particles, the longitudinal depth of the stress concentration region produced by the impact on brittle material is deeper than that of the impact on ductile material.

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**How to cite this article:** Mu M, Feng L, Zhang Q, Zang W, Wang H. Study on abrasive particle impact modeling and cutting mechanism. *Energy Sci Eng*. 2022;10:96–119. doi:10.1002/ese3.1012