Hilbert transform based analyses on ship-rocking signals

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The ship-rocking is a crucial factor which affects the accuracy of the ocean-based flight vehicle measurement. Here we have analyzed four groups of ship-rocking time series in horizontal and vertical directions utilizing a Hilbert based method from statistical physics. Our method gives a way to construct an analytic signal on the two-dimensional plane from a one-dimensional time series. The analytic signal share the complete property of the original time series. From the analytic signal of a time series, we have found some information of the original time series which are often hidden from the view of the conventional methods. The analytic signals of interest usually evolve very smoothly on the complex plane. In addition, the phase of the analytic signal is usually moves linearly in time. From the auto-correlation and cross-correlation functions of the original signals as well as the instantaneous amplitudes and phase increments of the analytic signals we have found that the ship-rocking in horizontal direction drives the ship-rocking in vertical direction when the ship navigates freely. And when the ship keeps a fixed navigation direction such relation disappears. Based on these results we could predict certain amount of future values of the ship-rocking time series based on the current and the previous values. Our predictions are as accurate as the conventional methods from stochastic processes and provide a much wider prediction time range.

In the ocean-based flight vehicle measurement, a very important task is to characterize special properties of the objective from massive quantity of data. Based on this one may have a way to increase the accuracy of the measurement. Unfortunately, there are many factors in the measurement which could affect the accuracy of the results. And it is even worse since these factors may correlate with one and another. As a result, the time series obtained in the measurements often display the following characteristics: periodicity, may be non-stationary, noisy and may contain certain stochastic components; such as outliers which are hard to explain their origin and random jumps in the data. To increase the accuracy of the measurement, much effort has been done to study two crucial time series: the ship rocking and the ship deformation. The current strategy is to make appropriate corrections based on these two time series, and thus to reduce or even eliminate the periodicity in the signals of interest. However, after these corrections the measurement results still show considerable difference from those basement standard results obtained through other ways, such as GPS, laser, etc.

In this paper we consider the ship-rocking time series based on the approach from statistical physics. To those time series which show strong periodicity, an effective method is to investigate the instantaneous analytic signal of a time series [1]. The instantaneous analytic signal is obtained based on a Hilbert transform. The original time series as well as its Hilbert transform construct an analytic signal at each measurement time. On the complex plane one can calculate the instantaneous amplitude and the instantaneous phase of the analytic signal. Such instantaneous amplitude and phase may reveal certain intrinsic properties of systems which are not seen from the conventional methods. For example, previous research shows that, the change of the correlation in human auto-regulation may not happen in the original signal, however it does show in the correlation of its instantaneous phase increments [1]. Here, similarly we investigate the characteristics of the analytic signal. We hypothesize that one may find some properties which are hidden from the view of traditional methods.

I. DATA

From our collaborators we have obtained four groups of ship-rocking time series: GX1129JL839RW839RW840. The data starting with “GX” are those obtained when the ship navigates freely, while others are those obtained when the ship navigates while maintaining a fixed orientation. All other information is unknown to us. For each group we have the time series in three directions of the Euler angles: the navigation direction, the horizontal direction and the vertical direction. In the following we mark the navigation direction as “KC”, the horizontal direction as “OC”, and the vertical direction as “PC”. If not specially indicated, the unit of all angles is in radian. All time series are measured every 50 milliseconds.

As shown in Fig. 1, the most apparent characteristic of the ship-rocking signal is periodicity. For all four groups of data available, the period in signals varies from around 6 seconds to more than 20 seconds. Such period also fluctuates at different measurement position of the same time series. In the KC direction, the situation becomes more complicated since the running average of the time series also varies at different time positions, e.g., as shown in Fig. 1. Such kind of signal apparently shows nonstationarity at different times. Thus many conventional methods may not apply. One possible approaches to such kind of signals is to apply the Detrended Fluctuation Analysis (DFA), which is very popular in recent years to quantify characteristics in non-stationary signals [2–4]. Another approach is to reduce such nonstationarity in the frequency domain, i.e., remove the low frequency part in the Fourier transform of the signal, then apply the reverse Fourier transform to obtain the new, stationary time series. However, the validity of these approaches to the current data has to be carefully examined before any convincing results could be accepted. This will be done in the future paper. Here we instead focus on stationary ship-rocking time series in two
other directions: OC and PC. We will investigate the original signals as well as their instantaneous amplitudes and phases. We expect that certain intrinsic properties of the current system may show in the auto-correlation functions and the cross-correlation functions of these signal.

II. THE BASIC TECHNIQUE

Our results in this paper are based on the Hilbert transform (HT) to a stationary time series. To any stationary time series $s(t)$, its HT is defined as:

$$\tilde{s}(t) = \frac{1}{\pi} P \int_{-\infty}^{\infty} \frac{s(\tau)}{t-\tau} d\tau,$$

(1)

where $P$ denotes the Cauchy principal value. In the frequency domain, the Fourier component at any frequency $f$ for the Fourier transform of $\tilde{s}(t)$ can be obtained very easily from the Fourier transform of $s(t)$, i.e., that of $s(t)$ at the same frequency $f$ rotates $90^\circ$ clockwise (for $f > 0$) or anticlockwise (for $f < 0$) in the complex plane. As a simple example, if $s(t) = \sin(\alpha t)$, then one would obtain $\tilde{s}(t) = \cos(\alpha t)$. Based on this fact, one could define an “analytic signal” for this time series:

$$S(t) = s(t) + i\tilde{s}(t) = A(t)e^{i\phi(t)},$$

(2)

where $A(t)$ and $\phi(t)$ are the instantaneous amplitude and phase of $s(t)$, respectively.

III. CHARACTERISTICS OF THE ANALYTIC SIGNALS

A. the original signal and its Hilbert transform

Following the procedure presented in the last section, one would obtain two additional time series from the original time series $s(t)$: its amplitude and its phase. In Fig.2 we show the patterns of the ship-rocking time series for the group RW839 in the OC direction. Similar behaviors are observed in signals of other groups and in other directions. In the analyses, we always remove the zero frequency part in the Fourier transform of $s(t)$ (which is related to the average of $s(t)$). Thus, in the complex plane, when the time proceeds, the analytic signal will rotate around the origin of the plane. At any time, the distance of the point on the analytic signal to the origin is the instantaneous amplitude, and the angle of the point to the positive direction of the $x$-axis is the phase.

From the lowest panel of Fig.2, one could find that the evolution of the analytic signal $S(t)$ is very slow when seen in the complex plane. It usually follows the similar behavior when rotating around the origin. (This can be further verified through the behavior of the instantaneous phase shown in Sec. III B.) However $s(t)$ could evolve very slowly at certain times, e.g., at the though and the peak of the wave. In contrast, $s(t)$ may also evolve very rapidly at the middle part between the though and the peak of the wave. Based on this characteristic, it may be possible to predict the behavior of the ship-rocking time series in certain future times from the data at the present and in previous times. We will show the details in the Sec. IV.

B. Behaviors of the instantaneous amplitude and phase

We have investigated the characteristics in the instantaneous amplitudes and instantaneous phases in all four groups of the ship-rocking time series. We find that they behave similarly between different groups and in different directions. As an example, in Fig.3 we show how the instantaneous amplitudes and the phase increments during unit time interval (50 ms) in OC and in PC directions evolve in time for the group JL839.

As shown in Fig.3, the instantaneous amplitude of the original data is non-negative and still displays periodicity in time.
However its shape is more irregular and its periods are much larger than those of the original data. In contrast, the instantaneous phase does not show any periodicity. In a large enough time window, one could find that the instantaneous phase depends approximately linearly on the time. Thus in Fig. 3 we show instead the phase increments of the original data during the unit time interval. In most times the phase increments are approximately a constant. However, we also notice that at certain times there are some sudden jumps in the value of phase increments. Such jumps could be some outliers contributed by certain stochastic noise. However it could also due to certain human or environmental activities on the ship. We are working closely with our collaborators to investigate the origin of these sudden jumps.

C. Auto-correlation functions for the original data and their instantaneous amplitudes and phase increments

Here we investigate the auto-correlation functions for the original data and their instantaneous amplitudes and phases in OC and PC directions. The results in most groups can be summarized in the upper panel of Fig. 4 where we mark the results for the original signal as “sig”, the results for the instantaneous amplitude as “amp”, and the results for the instantaneous phase increments as “pha”. Note that in conventional methods only the correlation of the original signal was considered.

As shown in the upper panel of Fig. 4 for most groups of data we find that the periodicity is presented in the auto-correlation functions $C(\tau)$ of the original signal. The period shown in the correlation function is approximately the averaged period of the original signal. Thus when the time lag $\tau$ increase from zero, $C(\tau)$ will decrease rapidly from 1 to a small value. Different from that of the original signal, auto-correlations of both corresponding instantaneous amplitude and phase increment do not show apparent periodicity as a function of the time lag $\tau$. $C(\tau)$ will normally decrease as the time lag $\tau$ increases, and finally $C(\tau)$ would fluctuate around zero. Very interestingly the correlation length (in unit of measurement time) for the instantaneous amplitude is around one period of the original signal. However the situation is different for the instantaneous phase increment. As shown in Fig. 4 such correlation length is only 1/10 of one period of the original signal. Thus the correlation in the instantaneous phase increment may be lost for even two patches in the original signal which are in the same period in time and are not very far away. In contrast, the correlation in the instantaneous amplitude is usually kept for two patches in the original signal which are in the same period.
It is very striking that the data in the group GX1129 display two different properties from those shown above. We find that in horizontal (OC) direction the original signal presents very strong correlations even when the time lag $\tau$ is huge. In vertical (PC) direction, the correlation length of the instantaneous amplitude series becomes much larger than one period of the original signal. These two results both imply that the ship is not interrupted by any considerable actions when the measurement is processing, thus the characteristics of the system could be maintained in time. This clarification is verified through the conversations with our collaborators: this group of data are actually obtained when the ship navigates freely.

**D. Cross-correlations for the original data and their instantaneous amplitudes and phase increments**

Similar to those we present in the last section, the cross-correlations for the original data and their instantaneous amplitudes and phase increments also present some distinct properties. The typical case for most groups is shown in the upper panel of Fig. 5. The OC-PC cross-correlation is calculated utilizing the following formula:

$$C_{\text{cross}}(\tau) = \langle (s_{OC}(t) - \langle s_{OC} \rangle)(s_{PC}(t + \tau) - \langle s_{PC} \rangle) \rangle,$$

where $\tau$ is the time lag.

As shown in Fig. 5, the group GX1129 continue presenting its special properties in cross-correlations of signals in different directions. For the cross-correlations of the original signals, it is apparent that $C_{\text{cross}}(\tau)$ behave differently for positive and negative time lag $\tau$. For positive $\tau$ we observe much stronger correlations, which implies that the ship-rocking in the horizontal (OC) direction at the present time will affect more effectively to the ship-rocking in the vertical (PC) direction at the future time. Thus the ship-rocking in the horizontal direction is the one which provides the “power” to ship-rocking in the vertical direction. Such result explains why in the last section the increase in the correlation length of the instantaneous amplitudes happens in the vertical direction when the ship navigates freely, while in the horizontal direction the correlation length is almost unchanged.

In contrast, for other groups we find that $C_{\text{cross}}(\tau)$ is symmetric for positive and negative small time lag $\tau$, as shown in Fig. 5. Thus there is no priority in vertical or horizontal direction for the ship-rocking. As a result at $\tau = 0$ we find that $C_{\text{cross}}(\tau)$ achieves absolute maximum for the instantaneous amplitudes. However for the group GX1129, at $\tau = 0$ the cross-correlation $C_{\text{cross}}(\tau)$ is close to zero. The conclusion is similar for the cross-correlation of the instantaneous phase increments, though when $\tau = 0$ we observe that $C_{\text{cross}}(\tau)$ only achieves local maximum.

**IV. PREDICTION FOR THE SHIP-ROCKING AT THE FUTURE TIME**

In reality, the ability of the prediction to the ship-rocking signal at the future time is very crucial in the ocean-based flight vehicle measurement. However, with the accuracy of 20 arc seconds, the current available methods based on the stochastic processes could predict only the behavior of the ship-rocking signal in the very near future, i.e., less than 200 ms [5]. In addition, the possibility to improve this kind of methods is very limited since such methods contain many adjusting parameters. In Sec. III A we find that the evolution of the analytic signal $S(t)$ is very slow when seen in the complex plane. Based on this fact and results in Sec. III B here we set up a new method to make predictions to behaviors of the ship-rocking signal at certain future times.

In an initial setup we provide here a simple model. In two-dimensional plane one can use two polar-coordinate parameters $\rho$ and $\theta$ to quantify the behavior of a time series. For the data of interest, $\rho$ corresponds to the instantaneous amplitude, and $\theta$ corresponds to the instantaneous phase. From Sec. III A and Sec. III B we have learned that the analytic signal usually evolve very smoothly in time on the complex plane. On
FIG. 6: The absolute errors and the standard deviations of the prediction from the Hilbert transform based method as a function of the prediction time interval.

the other side, the ability of our prediction is also limited by the correlation in signals. From Sec. III C and Sec. III D we estimate that the maximal time range in which we could obtain convincing prediction is around 1/10 of the periods in the original signal. In such a short time range we utilize the simplest model in the polar coordinate: the helix model. In this model $\rho$ evolve linearly with the change in $\theta$. Based on the results in Sec. III D we assume that the instantaneous phase also evolves linearly in time. In reality this assumption is invalid only at the points where we observe outliers, as shown in in Fig. 3.

From this model we propose to utilize 10 known values in the original time series to predict values of the original time series at one to ten following measurement time. (Note that the data are measured every 50 milliseconds). The data measured before these ten known values are also considered known values. From these known values we first construct its corresponding analytic signal. We next extend the track of the analytic signal on the complex plane from the helix model. The ten predicted values are then compared to the actual values in the original time series. The results of comparison are shown in Fig. 6. From the results of both the averaged absolute error and the standard deviation we find that our predicted values are very close to that of the original signals within a much wider range of the prediction time window than that of current methods. The results seem better in the averaged absolute error than that in the standard deviation. We attribute this to certain outliers in the instantaneous phase increments. Some tuning at positions near the outliers should be able to further improve the current accuracy.

V. SUMMARY

In summary, we have analyzed four groups of ship-rocking signals in horizontal and vertical directions utilizing a Hilbert based method from statistical physics. Our method gives a way to construct an analytic signal on the two-dimensional plane from a time series. The analytic signal share the complete property of the original time series. From the analytic signal of a time series, we have found some information of the original time series which are often hidden from the view of the conventional methods. For the current data, their analytic signals usually evolve very smoothly on the complex plane. In addition, the phase of the analytic signal is usually moves linearly in time. From the auto-correlation and cross-correlation functions of the original signals as well as the instantaneous amplitudes and phase increments of the analytic signals we have found that the ship-rocking in horizontal direction drives the ship-rocking in vertical direction when the ship navigates freely. And when the ship keeps a fixed navigation direction such relation disappears. Based on these results we could predict certain amount of future values of the ship-rocking time series based on the current and the previous values. Our predictions are as accurate as the conventional methods from stochastic processes and provide a much wider prediction time range.

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