Schemes for the formation of effective resource portfolios and areas of activity of organizations based on statistical forecasts

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Abstract. The problems of efficient allocation of resources in the areas of the company activity under conditions of uncertainty are considered. The indicators of effectiveness in the areas of the company activity are of an uncertain nature and are random variables with predetermined laws of probability distribution. To form an effective distribution of the company's activities in the areas of its activities, a scheme is used to form effective portfolios based on the probabilities of priority developed at the NRNU MEPhI. As estimates of the values of the efficiency index for the directions of the company's activities, their forecasts were taken as random variables with a uniform or normal probability distribution law. Key words: resource allocation, activities of organizations, efficiency, uncertainty, portfolio, risk, effective portfolio formation schemes, probability of priority.

1. Introduction
At present the theory of portfolio investment is widely used in countries with developed market economies and is one of the main tools by which the efficiency of the use of material and financial resources is increased. The article gives an application of the scheme for the formation of effective portfolios, presented in [1], to determine the effective allocation of resources in the areas of activity of the organization. Unlike the well-known Markowitz schemes [2, 3] and the VaR schemes [4-6], the scheme presented in [1] is based on the use of priority indicators which provides the maximization of the opportunity to carry out large values of portfolio efficiency.

2. Scheme for the formation of effective portfolios of resources and company activities in accordance with Markowitz
Let’s consider the problem of allocating resources or activities of organizations in the presence of uncertainty [1-6]. Uncertainty means that the magnitude of the effectiveness of the distributed resources in the selected object or the chosen direction of the organization's activities (direction) is random.
We number the objects by the index \( i = 1, \ldots, n \). We denote by \( R_i \) the value of resource utilization efficiency in the \( i \)-th object (in the \( i \)-th direction). Mathematically, the assumption of uncertainty means that \( R_i \) are random variables.

The complete knowledge of a system of random variables \( \mathbf{R} = (R_1, \ldots, R_n)^T \) (a random vector \( \mathbf{R} \)) is determined by its distribution law. However, in order to solve the problem of the optimal allocation of the resources between the investment objects under consideration according to the Markowitz scheme, it is sufficient to know only two characteristics of the random efficiency vector - the vector of mathematical expectations \( E(\mathbf{R}) = \mathbf{m} = (m_1, \ldots, m_n)^T, m_i = E(R_i) \), the expected average value of the \( i \)-th investment object efficiency and the covariance matrix \( \mathbf{W}, W_{ij} = \text{cov}(R_i, R_j), i, j = 1, \ldots, n \). In the formulation of Markowitz, the effectiveness of the investing resources in the object under consideration is a random value, the mathematical expectation of which is taken as the expected value of efficiency from investing capital in the object under consideration. As a numerical expression of the risk value, Markowitz proposed taking a measure of the deviation from the expected value - the standard deviation of efficiency as a random variable.

Such a choice of the mathematical expression for risk made it possible to carry out the principle known in economics in the portfolio investment scheme, expressed in the phrase "do not put eggs in one basket", i.e. the diversification of resources between several objects leads to a reduction in risk compared to the risk of investing resources in different facilities.

The estimations of the vector \( \hat{\mathbf{m}} \) and matrix \( \hat{\mathbf{W}} \) can be obtained depending on the specific economic content of the resources. After having received estimations \( \hat{\mathbf{m}}_i \) and \( \hat{\mathbf{W}}_{ij} \) it is possible to set and solve the tasks of optimizing portfolios of distributed resources. Let’s denote \( x_i \) the share of the distributed resource in the object with the number \( i \). Obviously the fractions \( x_i \) satisfy the natural conditions

\[
0 \leq x_i \leq 1, \quad i = 1, \ldots, n, \quad \sum_{i=1}^{n} x_i = 1. \tag{1}
\]

Under the portfolio of distributed resources we will understand the vector of shares \( \mathbf{x} = (x_1, \ldots, x_n)^T \) of distributed resources in objects \( i = 1, \ldots, n \). The task of choosing a portfolio is equivalent to the task of selecting a vector of shares \( \mathbf{x} \). For a fixed value of the vector \( \mathbf{x} \), the value of the portfolio efficiency \( R_p \) is given by

\[
R_p = \sum_{i=1}^{n} x_i \cdot R_i = (\mathbf{x}, \mathbf{R}). \tag{2}
\]

Consequently, the expected average value \( m_p \) of efficiency and the variance of portfolio efficiency \( \sigma_p^2 \) are given by the equalities:

\[
m_p = \sum_{i=1}^{n} x_i \cdot m_i = (\mathbf{x}, \mathbf{m}), \tag{3}
\]

\[
\sigma_p^2 = \sum_{i=1}^{n} \sum_{j=1}^{n} x_i \cdot x_j \cdot W_{ij} = (\mathbf{Wx}, \mathbf{x}). \tag{4}
\]

According to Markowitz, the risk of the portfolio is equal \( \sigma_p \) (sometimes as a risk in the Markowitz scheme choose \( \sigma_p^2 \)). Thus, the portfolio is characterized by two criteria \( m_p = m_p(\mathbf{x}) \) - the expected average value of portfolio efficiency \( R_p \) and portfolio risk \( \sigma_p = \sigma_p(\mathbf{x}) \), both criteria depending on the selected portfolio composition. The larger the value of the criterion \( m_p(\mathbf{x}) \) and the smaller the criterion \( \sigma_p(\mathbf{x}) \), the better the composition of the portfolio (vector \( \mathbf{x} \)) is chosen.

It is known that the optimal solution of multicriteria problems must be searched among the set of Pareto solutions possessing the property that the improvement of any criterion for the Pareto solution leads to the deterioration of other criteria. A set of Pareto solutions for a two-criteria problem can be found by fixing one of the criteria and optimizing the other.

According to the Markowitz scheme, in order to find the Pareto solutions of the two-criteria portfolio optimization problem, portfolio risk \( \sigma_p(\mathbf{x}) \) is minimized for fixed values of the expected mean value \( m_p(\mathbf{x}) \).

Thus, the mathematical model of portfolio optimization according to the Markowitz scheme has the form [2, 3]:

\[
\sigma_p^2 = (\mathbf{Wx}, \mathbf{x}) - \min \tag{5}
\]
\[ 0 \leq x_i \leq 1, \ i = 1, ..., n, \ \sum_{i=1}^{n} x_i = 1, \ \sum_{i=1}^{n} x_i \cdot m_i = m_p. \]

Obviously, \( m_p \) belongs to the interval \([m_{\text{min}}, m_{\text{max}}]\), where \( m_{\text{min}} = \min\{m_1, ..., m_n\} \), \( m_{\text{max}} = \max\{m_1, ..., m_n\} \).

3. Formation of effective resource portfolios or company activities according to the VaR scheme

In the Markowitz scheme, the uncertainty expressed by the variance of efficiency is two-sided since it determines the possibility of deviating efficiency both at a lower side of the average expected value (which is undesirable and determines essentially the risk), and in the larger side (which, on the contrary, is desirable, the opposite of understanding the risk). Therefore, the concept of uncertainty used in the Markowitz scheme leads to the fact that a decrease in uncertainty \( \sigma_p^2 \) in the allocation of resources is possible only due to a large degree of diversification of resources. And the greatest decrease in uncertainty is achieved, as a rule, with a significant degree of uniform distribution of resources between the objects. Thus, the Markowitz scheme implements the rule that a greater decrease in uncertainty is due to a greater diversification of resources.

At first glance, this property of uncertainty in the Markowitz scheme does not raise objections. However, the consideration of specific practical problems of resource allocation in conditions of uncertainty shows that the rule of "large diversification of resources - a lower risk value" is not universal. Often, a greater degree of resource concentration and even a complete lack of diversification (when the entire resource is given to a single facility) can lead to a significant increase in the efficiency of its use while significantly reducing the risk of loss of efficiency.

Unfortunately, the concept of uncertainty concluded in the Markowitz scheme does not take into account the last circumstance. In fact, the main reason for the last lack of the concept of uncertainty in the Markowitz scheme is the inconsistency of this concept of uncertainty for the effectiveness of an individual object and the risk of a decrease for the total effectiveness of the entire distributed resource.

When setting and solving the problem of optimal allocation of resources under uncertainty, the risk should reflect the possibility of a loss of resource use efficiency due to its improper diversification. Thus, in order that the used concept of risk in setting up problems of optimizing resource allocation in conditions of uncertainty does not lead to undesirable results, the concept of risk should include the possibility of loss of efficiency due to improper diversification of resources. Unfortunately, the transfer of the concept of uncertainty in the Markowitz scheme does not provide the above account for the loss of fulfilled efficiency.

As mentioned above, the use of dispersion as a measure of risk includes a double focus of possible changes in efficiency, both to a smaller and larger extent. At the same time, in many applied fields of science and technology, a unidirectional risk concept is traditionally used associated with the probability of implementing unfavorable random events (probability of a catastrophe, probability of an accident, probability of failure of equipment, etc.). Therefore, in recent years, the economy has also begun to use the concept of risk based on the counting of adverse events of a kind \( p(R < -R_{\nu}) \), where \( R \) is efficiency and \( R_{\nu} > 0 \) determines the level of efficiency losses and is called Value-at-Risk (VaR) [5, 6].

Instead of posing Markowitz (5) for the problem of optimal allocation of resources under conditions of uncertainty, we propose the formulation of this problem on the basis of VaR:

\[
\begin{align*}
    p^* & = \min_{R^*} \left( \sum_{i=1}^{n} x_i \right), \\
    R^* & = \max_{R_i \text{ efficiency of resource using in the } i\text{-th facility}} \left( R_i \right), \\
    0 & \leq x_i \leq 1, \ i = 1, ..., n, \ \sum_{i=1}^{n} x_i = 1,
\end{align*}
\]

where \( p^* = p \left( R_p(x) \right) \), \( R_p(x) = \sum_{i=1}^{n} x_i \cdot R_i \), \( R_i \) is level of efficiency; \( p^* \) is the probability level of the adverse event that the total resource allocation efficiency will be less than the level \( R^* \). By solving the problem of optimal allocation of resources in a VaR setting, we mean the Pareto solutions of the two-criteria problem (6) [4].
4. Algorithm for the numerical calculation of the composition of effective portfolios for the areas of company activity based on the probability of priority

Let’s consider an algorithm for solving the problem of forming effective portfolios with the help of priority probabilities [1].

Let the possible values of the efficiency \( R_i \) belong to the interval \([r_{i\min}, r_{i\max}], i = 1, \ldots, n\). This, in particular, means that the probability density values \( q_i(x), i = 1, \ldots, n \) are zero outside the interval \([r_{i\min}, r_{i\max}], i = 1, \ldots, n\). If among the intervals \([r_{i\min}, r_{i\max}], i = 1, \ldots, n\) there are those for which the inequalities \( r_{i\max} < r_{j\min} \) are satisfied for some \( j = 1, \ldots, n \), then objects with such numbers \( i \) are excluded from the composition of the portfolio.

Let the remaining \( l \) objects be renumbered in descending order \( r_{i\max} \):

\[
r_{1\max} > r_{2\max} > \cdots > r_{l\max}.
\]

We introduce the probability of priority of efficiency for objects according to equation

\[
p(R_i > R_{i+1}) = \int_{r_{i\max}}^{r_{i+1\max}} q_i(x)dx, \quad i = 1, \ldots, l - 1.
\]

The formation of an effective portfolio is carried out according to the scheme of step-by-step increase of the shares of the distributed resource into objects, starting with the object with the number 1 with the largest value \( r_{1\max} \).

At the first stage, the proportion of the first object is assigned the value

\[
x_1 = p(R_1 > R_2).
\]

In the second stage, the percentage of the first object is assigned with the value

\[
x_1 = x_1 + \left(\int_{r_{3\max}}^{r_{2\max}} q_3(x)dx\right) \cdot (1 - x_1),
\]

where \( x_1 \) in the right-hand side of (10) is taken from equation (9), and the fraction of the second object is assigned with the value

\[
x_2 = p(R_2 > R_3) \cdot (1 - x_1),
\]

where \( x_1 \) in the right-hand side of (11) is taken from (10).

Similarly, the assignment of shares for objects up to the object with the number \( l \) [1].

Table 1 presents the results of calculations according to scheme (7) - (11) of the composition of effective portfolios for the company activity based on the data of the forecast of performance indicators (the numerical values of the shares are given in percentages).

| Area | 2018 | 2019 | 2020 | 2021 | 2022 | 2023 | 2024 | 2025 | 2026 | 2027 | 2028 | 2029 | 2030 |
|------|------|------|------|------|------|------|------|------|------|------|------|------|------|
| Area 1 | 50.0 | 50.0 | 50.0 | 50.0 | 50.0 | 50.0 | 49.9 | 46.3 | 43.3 | 38.7 | 35.6 | 33.0 |
| Area 2 | 50.0 | 50.0 | 50.0 | 50.0 | 50.0 | 50.0 | 49.9 | 46.3 | 43.3 | 38.7 | 35.6 | 33.0 |
| Area 3 | 0.0  | 0.0  | 0.0  | 0.0  | 0.0  | 0.0  | 0.0  | 0.0  | 0.0  | 6.4  | 7.3  | 8.7  |
| Area 4 | 0.0  | 0.0  | 0.0  | 0.0  | 0.0  | 0.0  | 0.0  | 0.0  | 0.2  | 7.4  | 13.4 | 16.2 | 19.3 |
| Area 5 | 0.0  | 0.0  | 0.0  | 0.0  | 0.0  | 0.0  | 0.0  | 0.0  | 0.0  | 0.0  | 0.0  | 0.0  |
| Area 6 | 0.0  | 0.0  | 0.0  | 0.0  | 0.0  | 0.0  | 0.0  | 0.0  | 0.0  | 2.2  | 2.3  |

5. Conclusion
The article gives an application of schemes for the formation of effective portfolios, resource allocation, including a scheme based on the probabilities of priority. The presented scheme allows to determine the optimal allocation of resources or company activities from the use of these forecasts for indicators of the efficiency of resource development or the implementation of directions. The results of numerical calculations of effective portfolios for selected areas of the company are presented.
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References
[1] Kryanev A V., Sliva D E and Udumyan D K 2018 Methods for the formation of effective portfolios with the constraints and algorithms for their implementation. Bulletin of the National Research Nuclear University “MEPhI” 7 №3 pp 289-293 (in Russian)

[2] Markowitz H M 1990 Mean Variance Analysis in Portfolio Choice and Capital Markets Basil Blackwell

[3] Sharpe U F, Alexander G Dg and Baily D V 2001 Investment Moscow: Infra-M

[4] Kryanev A V and Lukin G V 2001 On the formulation and solution of problems of optimization of investment portfolios Moscow (Preprint MEPhI 006-2001) (in Russian)

[5] Rockafellar R T and Uryasev S 2000 Optimization of conditional value-at-risk J. Risk 2(3)

[6] Lim Churlzu, Sherali Hanif D and Uryasev S 2010 Portfolio optimization by minimizing conditional value-at-risk via non differentiable optimization Computational Optimization and Applications 46(3)