Collider phenomenology of the E$_6$SSM

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We consider collider signatures of the exceptional supersymmetric (SUSY) standard model (E$_6$SSM). This E$_6$-inspired SUSY model is based on the SM gauge group together with an extra $U(1)$ gauge symmetry under which right-handed neutrinos have zero charge. To ensure anomaly cancellation and gauge coupling unification the low energy matter content of the E$_6$SSM involves extra exotic matter beyond the MSSM. We discuss the collider signatures associated with the production of new particles predicted by the E$_6$SSM and consider the implications of this model for dark matter and Higgs phenomenology. Since exotic quarks in the E$_6$SSM can be either diquarks or leptoquarks they may provide spectacular new physics signals at the LHC.

1. Introduction

Softly broken supersymmetry (SUSY) provides a very attractive framework for physics beyond the standard model (BSM), in which the hierarchy problem is solved and the unification of gauge couplings can be realised [1]. Despite these attractive features, the minimal supersymmetric standard model (MSSM) suffers from the problem. The superpotential of the MSSM contains the bilinear term $\mu H_d H_u$, where $H_u$ and $H_d$ are the Higgs doublet superfields. In order to get the correct pattern of electroweak (EW) symmetry breaking the parameter $\mu$ is required to be in the TeV region. At the same time the incorporation of the MSSM into supergravity (SUGRA) or Grand Unified theories (GUT) implies that $\mu$ should be of the order of GUT or Planck scales.

An elegant solution to this problem arises within E$_6$-inspired SUSY models. At high energies E$_6$ GUT symmetry can be broken to the rank–5 subgroup $SU(3)_C \times SU(2)_L \times U(1)_Y \times U(1)'$ where in general

$$U(1)' = U(1)_X \cos \theta + U(1)_Y \sin \theta$$

and the two anomaly-free $U(1)_Y$ and $U(1)_X$ symmetries originate from the breakings $E_6 \rightarrow SO(10) \times U(1)_{Y}, SO(10) \rightarrow SU(5) \times U(1)_X$. If $\theta \neq 0$ or $\pi$ the extra $U(1)'$ gauge symmetry forbids the bilinear $\mu$ term but allows interaction $\lambda S H_d H_u$ in the superpotential. At low energies ($\sim$ TeV) the scalar component of the SM singlet superfield $S$ acquires a non–zero vacuum expectation value (VEV) breaking $U(1)'$ and giving rise to an effective $\mu$ term.

Within the class of rank–5 $E_6$-inspired SUSY models with extra $U(1)'$ gauge symmetry, there is a unique choice of Abelian gauge group that allows zero charges for right-handed neutrinos. This is the $U(1)_N$ gauge symmetry given by $\theta = \arctan \sqrt{15}$. Only in this exceptional supersymmetric standard model (E$_6$SSM), which is based on the $SU(3)_C \times SU(2)_W \times U(1)_Y \times U(1)_N$ gauge group, right-handed neutrinos may be superheavy, shedding light on the origin of the mass hierarchy in the lepton sector [2, 3].

To ensure anomaly cancellation the particle content of the E$_6$SSM is extended to include three complete fundamental 27 representations of $E_6$. These multiplets decompose under the $SU(5) \times U(1)_N$ subgroup of $E_6$ as follows:

$$27_i \rightarrow \left(10, \frac{1}{\sqrt{40}}\right)_i + \left(5^*, \frac{2}{\sqrt{40}}\right)_i + \left(5^*, -\frac{3}{\sqrt{40}}\right)_i + \left(5, \frac{2}{\sqrt{40}}\right)_i + \left(1, \frac{5}{\sqrt{40}}\right)_i + (1, 0)_i.$$ (2)

The first and second quantities in brackets are the $SU(5)$ representation and extra $U(1)_N$ charge respectively, while $i$ is a family index that runs from 1 to 3. An ordinary SM family, which contains the doublets of left–handed quarks $Q_i$ and leptons $L_i$, right–handed up– and down–quarks ($u^c_i$ and $d^c_i$) as well as right–handed
charged leptons, is assigned to \( \left( 10, \ \frac{1}{\sqrt{40}} \right)_i + \left( 5^*, \ \frac{2}{\sqrt{40}} \right)_i \). Right-handed neutrinos \( N^c_i \) should be associated with the last term in Eq. (2), (1, 0). The next-to-last term, \( \left( 1, \ \frac{5}{\sqrt{40}} \right)_i \), represents SM-singlet fields \( S_i \), which carry non-zero \( U(1)_N \) charges and therefore survive down to the EW scale. The pair of \( SU(2)_W \)-doublets \((H^d_i, H^u_i)\) that are contained in \( (5^*, -\frac{3}{\sqrt{40}})_i \) and \( (5, -\frac{2}{\sqrt{40}})_i \) have the quantum numbers of Higgs doublets. They form either Higgs or inert Higgs \( SU(2)_W \) multiplets. Other components of these \( SU(5) \) multiplets form colour triplets of exotic quarks \( D_i \) and \( \bar{D}_i \) with electric charges \(-1/3\) and \(+1/3\), respectively. These exotic quark states carry a \( B - L \) charge \( \pm 2/3 \), twice that of ordinary ones.

In addition to the complete 27, multiplets the low energy matter content of the EqSSM can be supplemented by an \( SU(2)_W \) doublet \( L_4 \) and anti-doublet \( \bar{L}_4 \) from the extra 27' and \( 27'' \) to preserve gauge coupling unification. These components of the \( E_6 \) fundamental representation originate from \( (5^*, \ \frac{2}{\sqrt{40}})_i \) of 27' and \( (5, -\frac{2}{\sqrt{40}})_i \) of 27'' by construction. Anomaly cancellation is still guaranteed since \( L_4 \) and \( \bar{L}_4 \) originate from the 27' and 27'' supermultiplets. The analysis performed in [4] shows that the unification of gauge couplings in the E6SSM can be achieved for any phenomenologically acceptable value of \( \alpha_3(M_Z) \) consistent with the measured low energy central value.

The successful leptogenesis in the early epoch of the Universe is the distinctive feature of the EqSSM. Indeed, the heavy Majorana right-handed neutrinos may decay unequally into final states with lepton number \( L = \pm 1 \), thereby creating a lepton asymmetry in the early Universe. Because in the EqSSM the Yukawa couplings of the new exotic particles are not constrained by the neutrino oscillation data, substantial values of CP-violating \( S \) may be achieved for any phenomenologically acceptable value of \( \alpha_3(M_Z) \) consistent with the measured low energy central value.

As in the MSSM the gauge symmetry in the EqSSM does not forbid lepton and baryon number violating operators that result in rapid proton decay. Moreover, exotic particles in EqSSM inspired SUSY models give rise to new Yukawa interactions that in general induce unacceptably large non–diagonal flavour transitions. To suppress these effects in the EqSSM an approximate \( Z_2^H \) symmetry is imposed. Under this symmetry all superfields except one pair of \( H^d_i \) and \( H^u_i \) (say \( H_d \equiv H^d_1 \) and \( H_u \equiv H^u_1 \)) and one SM-type singlet field \((S \equiv S_3)\) are odd. The \( Z_2^H \) symmetry reduces the structure of the Yukawa interactions to

\[
W_{\text{EqSSM}} \simeq \lambda \hat{S}(\hat{H}_u \hat{H}_d) + \lambda_{ab} \hat{S}(\hat{H}^d_a \hat{H}^u_b) + f_{ab} \hat{S}_a(\hat{H}^d_b \hat{H}_d) + f_{ab} \hat{S}_a(\hat{H}^u_b \hat{H}_d) + \kappa \hat{S}(\hat{D}_i \hat{\bar{D}}_i)
\]

where \( \alpha, \beta = 1, 2 \) and \( i, j = 1, 2, 3 \). The \( SU(2)_W \) doublets \( \hat{H}_u \) and \( \hat{H}_d \) and SM-type singlet field \( \hat{S} \), that are even under the \( Z_2^H \) symmetry, play the role of Higgs fields. At the physical vacuum they develop vacuum expectation values (VEVs)

\[
\langle H_d \rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} v_1 \\ 0 \end{pmatrix}, \quad \langle H_u \rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v_2 \end{pmatrix}, \quad \langle S \rangle = \frac{s}{\sqrt{2}}
\]

generating the masses of the quarks and leptons. Instead of \( v_1 \) and \( v_2 \) it is more convenient to use \( \tan \beta = v_2/v_1 \) and \( v = \sqrt{v_1^2 + v_2^2} = 246 \text{ GeV} \). The VEV of the SM-type singlet field, \( s \), breaks the extra \( U(1)_N \) symmetry generating exotic fermion masses and also inducing that of the \( Z' \) boson. Therefore the singlet field \( S \) must acquire a large VEV in order to avoid conflict with direct particle searches at present and past accelerators. This also requires the Yukawa couplings \( \lambda_{ab} \) and \( \kappa \) to be reasonably large. If \( \lambda_{ab} \) or \( \kappa \) are large enough at the GUT scale they affect the evolution of the soft scalar mass \( m^2_S \) of the singlet field \( S \) rather strongly resulting in a negative value of \( m^2_S \) at low energies which triggers the breakdown of the \( U(1)_N \) symmetry.
2. $Z'$ and Exotica phenomenology

Although $Z^H_2$ eliminates any problems related with baryon number violation and non-diagonal flavour transitions it also forbids all Yukawa interactions that would allow the exotic quarks to decay. Since models with stable charged exotic particles are ruled out by various experiments the $Z^H_2$ symmetry must be broken. At the same time, the breakdown of $Z^H_2$ should not give rise to operators that would lead to rapid proton decay. There are two ways to overcome this problem: the Lagrangian must be invariant with respect to either a $Z^L_2$ symmetry, under which all superfields except leptons are even (Model I), or a $Z^B_2$ discrete symmetry, which implies that exotic quark and lepton superfields are odd whereas the others remain even (Model II). If the Lagrangian is invariant under the $Z^L_2$ symmetry, then the terms in the superpotential which permit exotic quarks to decay and are allowed by the $E_6$ symmetry can be written in the form

$$W_1 = g^{Q}_{ijk} \hat{D}_i (\hat{Q}_j \hat{Q}_k) + g^{q}_{ijk} \hat{D}_i \hat{u}_j \hat{c}_k,$$

that implies that exotic quarks are diquarks. If $Z^B_2$ is imposed then the following couplings are allowed:

$$W_2 = g^{E}_{ijk} \hat{e}_i \hat{D}_j \hat{u}_k + g^{D}_{ijk} (\hat{Q}_i \hat{L}_j) \hat{D}_k.$$  

(6)

In this case baryon number conservation requires the exotic quarks to be leptoquarks.

In the $E_6$ SSM some of the exotic quarks can be relatively light. Then from Fig. 1 one can see that the exotic quark production cross section at the LHC can be comparable with the cross section of $t\bar{t}$ production \cite{2}. In the $E_6$ SSM, the $D_i$ and $\bar{D}_i$ fermions are SUSY particles with negative $R$–parity so they must be pair produced and decay into quark–squark (if diquarks) or quark–slepton, squark–lepton (if leptoquarks), leading to final states containing missing energy from the lightest SUSY particle (LSP).

![Figure 1](image.png)

Figure 1: (Left) Differential cross section at the 14 TeV LHC for pair production of three families of exotic $D$–quarks with masses $\mu_{D_i} = 300$ GeV in comparison to top–quark pair production. (Right) Cross section at the 14 TeV LHC for pair production of exotic $D$–quarks as a function of their (common) mass $\mu_{D_i} = M_F$.  

The lifetime and decay modes of the exotic coloured fermions are determined by the $Z^H_2$ violating couplings. Assuming that $D_i$ and $\bar{D}_i$ fermions couple most strongly to the third family (s)quarks and (s)leptons, the lightest exotic $D_i$ and $\bar{D}_i$ fermions decay into $tb$, $tb$, $\bar{t}b$, $\bar{t}b$ (if they are diquarks) or $\bar{t}\tau$, $t\bar{\tau}$, $\bar{b}\nu_\tau$, $b\nu_\tau$ (if they are leptoquarks). This can lead to a substantial enhancement of the cross section of either

$$pp \rightarrow t\bar{t}bb + E_T^{\text{miss}} + X$$

if exotic quarks are diquarks or

$$pp \rightarrow t\bar{t}\tau\bar{\tau} + E_T^{\text{miss}} + X, \quad pp \rightarrow b\bar{b} + E_T^{\text{miss}} + X$$

if exotic quarks are leptoquarks. Here it is worth to point out that the SM production of $t\bar{t}\tau^+\tau^-$ is $(\alpha_W/\pi)^2$ suppressed in comparison to the $t\bar{t}$ production cross section. Therefore light leptoquarks are expected to lead
to the strong signal with low SM background at the LHC. The results presented in Fig. 1 suggest that the observation of the D fermions might be possible if they have masses below about 1.5–2 TeV [2].

Similar considerations apply to the case of exotic \( \tilde{D}_i \) and \( \tilde{D}_i \) scalars except that they are non-SUSY particles so they may be produced singly and decay into quark-quark (if diquarks) or quark-lepton (if leptoquarks) without missing energy from the LSP. It is possible to have relatively light exotic coloured scalars due to mixing effects. The Tevatron and LHC searches for dijet resonances ruled out the presence of light scalar diquarks. However, scalar leptoquarks may be as light as 300 GeV since at hadron colliders they are pair produced through gluon fusion. Scalar leptoquarks decay into quark–lepton final states through small \( Z^0 \) violating terms, for example \( \tilde{D} \rightarrow t\tau \), and pair production leads to an enhancement of \( pp \rightarrow tt + \tau\tau \) (without missing energy) at the LHC.

Other possible manifestations of the EqSSM at the LHC are related to the presence of \( Z' \) boson. The production of a TeV scale \( Z' \) will provide an unmistakable signal leading to enhanced production of \( l^+ l^- \) pairs \( (l = e, \mu) \) [2]. The differential distribution in invariant mass of the lepton pair \( l^+ l^- \) in Drell–Yan production is expected to be measurable at the CERN collider with a high resolution and would enable one to not only confirm the existence of a \( Z' \) state but also establish the possible presence of additional exotic matter, by fitting to the data the width of the \( Z' \) resonance. At the LHC, the \( Z' \) boson that appears in the EqSSM can be discovered if it has a mass below 4–4.5 TeV [7]–[8]. The determination of its couplings should be possible if \( M_{Z'} \lesssim 2–2.5 \) TeV [8]. The new physics signals associated with the presence of \( Z' \) boson and exotic particles predicted by the EqSSM were discussed in [2]–[3], [10]–[11]. Recently the particle spectrum and collider signatures associated with it were studied within the constrained version of this model [12]–[13].

### 3. Higgs phenomenology

Although the \( Z^0 \) symmetry can only be an approximate one from here on we assume that \( Z^0 \) symmetry violating couplings are small and can be neglected in our analysis. This assumption can be justified if we take into account that the \( Z^0 \) symmetry violating operators may give an appreciable contribution to the amplitude of \( K^0 - \bar{K}^0 \) oscillations and give rise to new muon decay channels like \( \mu \rightarrow e^- e^+ e^- \). In order to suppress processes with non–diagonal flavour transitions the Yukawa couplings of the exotic particles to the quarks and leptons of the first two generations should be smaller than \( 10^{-3} - 10^{-4} \). Such small \( Z^0 \) symmetry violating couplings can be ignored in the first approximation.

When \( Z^0 \) symmetry violating couplings tend to zero only \( H_u \), \( H_d \) and \( S \) acquire non-zero VEVs. The Higgs effective potential can be written in the following form:

\[
V = V_F + V_D + V_{soft} + \Delta V, \\
V_F = \lambda^2 |S|^2 (|H_d|^2 + |H_u|^2) + \lambda^2 (H_d H_u)^2, \\
V_D = g_2^2 \left( H_d^\dagger \sigma_2 H_d + H_u^\dagger \sigma_2 H_u \right) + g_2^2 \left( |H_d|^2 - |H_u|^2 \right)^2 + \frac{g_1^2}{2} \left( Q_1 |H_d|^2 + Q_2 |H_u|^2 + \hat{Q}_S |S|^2 \right)^2, \\
V_{soft} = m_3^2 |S|^2 + m_1^2 |H_d|^2 + m_2^2 |H_u|^2 + \left( \lambda \hat{A}_1 S (H_d H_d) + h.c. \right),
\]

where \( g_2, g' = \sqrt{3/5} g_1 \) and \( g'_i \) are the low energy \( SU(2)_W \), \( U(1)_Y \) and \( U(1)_N \) gauge couplings while \( \hat{Q}_1, \hat{Q}_2 \) and \( \hat{Q}_S \) are the \( U(1)_N \) charges of \( H_d \) and \( S \). The term \( \Delta V \) represents the contribution from loop corrections to the Higgs effective potential. Here \( H_d^\dagger = (H_d^0, H_d^+) \), \( H_u^\dagger = (H_u^0, H_u^+) \) and \( (H_d H_u) = H_d^+ H_u^0 - H_u^+ H_d^0 \). The couplings \( g_2 \) and \( g' \) are known precisely. Assuming gauge coupling unification one can determine the value of extra \( U(1)_N \) gauge coupling. It turns out that \( g'_1(Q) \equiv g_1(Q) \) for any renormalization scale \( Q \lesssim M_X \) [2].

Initially the EWSB sector involves ten degrees of freedom. However four of them are massless Goldstone modes which are swallowed by the \( W^\pm \), \( Z \) and \( Z' \) gauge bosons that gain non-zero masses when Higgs fields acquire VEVs given by Eq. 4. In the limit where \( s \gg v \) the masses of the \( W^\pm \), \( Z \) and \( Z' \) gauge bosons are

\[
M_W = \frac{g_2}{2} v, \quad M_Z \simeq \frac{\bar{g}}{2} v, \quad M_{Z'} \simeq g'_1 \hat{Q}_S s,
\]

where \( \bar{g} = \sqrt{g_2^2 + g'^2} \). When CP–invariance is preserved the other degrees of freedom form two charged, one CP–odd and three CP–even Higgs states. The masses of the charged and CP-odd Higgs bosons are

\[
m_{H^\pm}^2 = \frac{\sqrt{2} \lambda_1 A_1}{\sin2\beta} s - \frac{\lambda_2}{2} v^2 + M_W^2 + \Delta_\pm, \\
m_{A}^2 = \frac{\sqrt{2} \lambda A_1}{\sin2\beta} s + \Delta_A,
\]

where \( \lambda_1 = \sqrt{\lambda^2 - (\lambda_2 v)^2} \).
where $\Delta_\perp$ and $\Delta_A$ are the loop corrections. If all Higgs states except the lightest one are considerably heavier than the EW scale the mass matrix of the CP-even Higgs sector can be diagonalised using the perturbation theory \[16\]-\[20\]. Then the masses of two heaviest CP-even Higgs states are set by $M_{Z'}$, and $M_A$, i.e.

$$m^2_{h_3} \approx m^2_A + O(M^2_Z), \quad m^2_{h_4} \approx M^2_{Z'} + O(M^2_Z). \quad (9)$$

The lightest CP–even Higgs state remains light, i.e. $m^2_{h_1} \sim O(M^2_Z)$, even when $m_A$ and $M_{Z'} \gtrsim 1$ TeV.

At least one CP–even Higgs boson is always heavy preventing the distinction between the E6SSM and MSSM Higgs sectors. Indeed, the mass of the singlet dominated Higgs scalar particle $m_{h_2}$ is always close to the mass of the $Z'$ boson that has to be considerably heavier than $800 - 900$ GeV. When $\lambda \gtrsim g'_1$, vacuum stability requires $m_A$ to be considerably larger than $M_{Z'}$ and the EW scale so that the qualitative pattern of the Higgs spectrum is rather similar to the one which arises in the PQ symmetric NMSSM \[19\]-\[21\]. In the considered limit the heaviest CP–even, CP–odd and charged states are almost degenerate around $m_A$ and lie below the TeV range \[2\]. If $\lambda \lesssim g'_1$ the charged, CP–odd and second lightest CP–even Higgs states may have masses in the $200 - 300$ GeV range. However in this case we get a MSSM–type Higgs spectrum with the lightest SM–like Higgs boson below $130 - 135$ GeV and with the heaviest scalar above $800 - 900$ GeV being singlet dominated and irrelevant.

SUSY models predict that the mass of the lightest Higgs particle is limited from above. The E6SSM is not an exception. When the soft masses of the superpartners of the top-quark are equal, i.e. $m^2_{h_3} = m^2_{h_4} = M^2_Z$, the two-loop upper bound on the lightest CP-even Higgs boson mass $m_{h_1}$ in the E6SSM can be written in the following form:

$$m^2_{h_1} \lesssim \left(\frac{\lambda}{2}\right)^2 \sin^2 \beta + \cos^2 \beta + g'^2 v^2 \left(\frac{\tilde{Q}_1 \cos^2 \beta + \tilde{Q}_2 \sin^2 \beta}{8\pi^2}\right) \left(1 - \frac{3h^2_{l}}{8\pi^2}\right) + \frac{3h^4_{l} \sin^4 \beta}{8\pi^2} \left(\frac{X_1^2}{M^2_Z} \left(1 - \frac{1}{12} \frac{X_2^2}{M^2_Z}\right) + l + \frac{1}{16\pi^2} \left(\frac{3}{2} h^2_{l} - 8g'^2_3\right) \left(\frac{2}{3} \frac{X_1^2}{M^2_Z} - \frac{1}{6} \frac{X_2^2}{M^2_Z}\right) + l\right), \quad (10)$$

where $l = \ln\left(\frac{M^2_Z}{m^2_{l}}\right)$ and $X_l$ is the usual stop mixing parameter. At tree level the upper limit on the mass of the lightest Higgs particle is described by the sum of the three terms in the square brackets. One-loop corrections from the top-quark and its superpartners increase the bound on the lightest CP-even Higgs boson mass substantially while the inclusion of leading two-loop corrections reduces the upper limit on $m_{h_1}$. In order to enhance the contribution of loop effects we assume maximal mixing in the stop sector (i.e. $X_l = \sqrt{6}M_S$).

We also adopt $M_S = 700$ GeV. Then since $g'_1(M_Z)$ is determined uniquely if we require the unification of gauge couplings the theoretical restriction on the lightest Higgs mass \[10\] depends on $\lambda$ and $\tan \beta$ only. The requirement of validity of perturbation theory up to the GUT scale constrains the parameter space further setting a limit on the Yukawa coupling $\lambda$ for each value of $\tan \beta$. Relying on the results of the analysis of the renormalisation group (RG) flow in the E6SSM presented in \[2\] one can obtain the maximum possible value of the lightest Higgs boson mass for each particular choice of $\tan \beta$.

The dependence of the tree level and two-loop upper bounds on the mass of the lightest Higgs state on $\tan \beta$ is examined in Fig. 2 where these bounds are compared with the corresponding limits in the MSSM and NMSSM. One can see that in the interval of $\tan \beta$ from 1.2 to 3.4 the maximum value of the mass of the lightest Higgs boson in the E6SSM is larger than the experimental lower limit on the SM–like Higgs boson even at tree–level. At moderate values of $\tan \beta$ (tan $\beta = 1.6 - 3.5$) the two–loop upper limit on $m_{h_1}$ in the E6SSM is also considerably higher than in the MSSM and NMSSM. It reaches the maximum value $\sim 150 - 155$ GeV at $\tan \beta = 1.5 - 2$. In the considered part of the parameter space the theoretical restriction on the mass of the lightest CP-even Higgs boson in the NMSSM exceeds the corresponding limit in the MSSM because of the extra contribution to $m^2_{h_1}$ induced by the additional $F$-term in the Higgs scalar potential of the NMSSM. The size of this contribution, which is described by the first term in the square brackets of Eq. (10), is determined by the Yukawa coupling $\lambda$. The upper limit on the coupling $\lambda$ caused by the validity of perturbation theory in the NMSSM is more stringent than in the E6SSM due to the presence of exotic matter. As a result the upper limit on $m_{h_1}$ in the NMSSM is considerably less than in the E6SSM at moderate values of $\tan \beta$.

At large $\tan \beta \gtrsim 10$ the contribution of the $F$-term of the SM-type singlet field to $m^2_{h_1}$ vanishes. Therefore with increasing $\tan \beta$ the upper bound on the lightest Higgs boson mass in the NMSSM approaches the corresponding limit in the MSSM. In the E6SSM the theoretical restriction on the mass of the lightest Higgs scalar also diminishes when $\tan \beta$ rises. But even at very large values of $\tan \beta$ the upper limit on $m_{h_1}$ in the E6SSM is still $4 - 5$ GeV larger than the ones in the MSSM and NMSSM because of the $U(1)_X$ D-term contribution to $m_h$ (last term in the square brackets of Eq. (10)).
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that the masses of all inert chargino states, which are formed by the charged components of the inert Higgsinos $\chi^\pm$, and second lightest inert neutralinos $\chi_2$, in the validity of perturbation theory up to the GUT scale. The restrictions specified above set very strong limits on the masses of the lightest inert neutralinos. In particular, our numerical analysis indicates that the lightest inert neutralino tends to be the lightest SUSY particle in the spectrum and can play the role of dark matter. The exact $Z^H$ symmetry limit the scalar components of the corresponding superfields do not acquire VEVs and inert neutralino states do not mix with the ordinary neutralinos. In the field basis $(\tilde{H}_2^0, \tilde{H}_2^0, \tilde{S}_2, \tilde{H}_1^0, \tilde{H}_1^0, \tilde{S}_1)$ the mass matrix of the inert neutralinos takes a form

$$M_{1N} = \begin{pmatrix} A_{22} & A_{21} \\ A_{12} & A_{11} \end{pmatrix}, \quad A_{\alpha\beta} = -\frac{1}{\sqrt{2}} \begin{pmatrix} 0 & \lambda_{\alpha\beta} s & \tilde{f}_{\alpha\beta} v \sin \beta \\ \lambda_{\beta\alpha} s & 0 & \tilde{f}_{\beta\alpha} v \cos \beta \\ \tilde{f}_{\alpha\beta} v \sin \beta & \tilde{f}_{\beta\alpha} v \cos \beta & 0 \end{pmatrix}, \quad (11)$$

so that $A_{12} = A_{21}^T$. As before we choose the VEV of the SM singlet field $s$ to be large enough ($s > 2400$ GeV) so that the masses of all inert chargino states, which are formed by the charged components of the inert Higgsinos $(\tilde{H}_2^{\pm}, \tilde{H}_1^{\pm}, \tilde{H}_2^{0}, \tilde{H}_1^{0})$, are larger than 100 GeV and $Z'$ boson is relatively heavy. In addition, we also require the validity of perturbation theory up to the GUT scale. The restrictions specified above set very strong limits on the masses of the lightest inert neutralinos. In particular, our numerical analysis indicates that the lightest and second lightest inert neutralinos ($\chi_1^0$ and $\chi_2^0$) are typically lighter than $60 - 65$ GeV [22, 24]. Therefore the lightest inert neutralino tends to be the lightest SUSY particle in the spectrum and can play the role of dark matter. The neutralinos $\chi_1^0$ and $\chi_2^0$ are predominantly inert singlinos. Their couplings to the $Z$-boson can be rather small so that such inert neutralinos would remain undetected at LEP.

In order to clarify the results of our numerical analysis, it is useful to consider a simple scenario when $\alpha = \frac{\lambda_{\alpha\beta}}{\lambda_{\beta\alpha}}, f_{\alpha\beta} = f_{\alpha} \delta_{\alpha\beta}$ and $\tilde{f}_{\alpha\beta} = \tilde{f}_{\alpha} \delta_{\alpha\beta}$. In the limit where off–diagonal Yukawa couplings vanish and $\lambda_{\alpha}s \gg f_{\alpha}v$, the eigenvalues of the inert neutralino mass matrix can be easily calculated (see [25]). In particular the masses of two lightest inert neutralino states ($\chi_1^0$ and $\chi_2^0$) are given by

$$m_{\chi_0^0} \simeq \frac{\tilde{f}_{\alpha} f_{\alpha} v^2 \sin 2\beta}{2 m_{\tilde{\chi}_0^0}}. \quad (12)$$

where $m_{\tilde{\chi}_0^0} = \lambda_{\alpha}s/\sqrt{2}$ are masses of the inert charginos. From Eq. (12) one can see that the masses of $\chi_1^0$ and $\chi_2^0$ are determined by the values of the Yukawa couplings $\tilde{f}_{\alpha}$ and $f_{\alpha}$. They decrease with increasing $\tan \beta$ and chargino masses. In this approximation the part of the Lagrangian, that describes interactions of $Z$ with $\chi_1^0$...
and $\chi^0_2$, can be presented in the following form:

$$\mathcal{L}_{ZXX} = \sum_{\alpha,\beta} \frac{M_Z}{2v} Z_\mu \left( \bar{\chi}^0_\alpha \gamma_\mu \gamma_5 \chi^0_\beta \right) R_{Z\alpha\beta},$$

Eqs. (13) demonstrates that the couplings of $\chi^0_1$ and $\chi^0_2$ to the Z-boson can be very strongly suppressed or even tend to zero. This happens when $|f_\alpha| \cos \beta \approx |f_\alpha| \sin \beta$.

Although $\chi^0_1$ and $\chi^0_2$ might have extremely small couplings to Z, their couplings to the lightest CP–even Higgs boson $h_1$ cannot be negligibly small if $\chi^0_1$ and $\chi^0_2$ have appreciable masses. When the SUSY breaking scale $M_S$ and the VEV $s$ of the singlet field are considerably larger than the EW scale, the lightest CP–even Higgs state is the analogue of the SM Higgs field and is responsible for all light fermion masses in the E6SSM. Therefore it is not so surprising that in the limit when $\lambda_s \gg f_{\alpha}v$, $f_{\alpha}v$ the part of the Lagrangian that describes the interactions of $\chi^0_1$ and $\chi^0_2$ with $h_1$ takes a form

$$\mathcal{L}_{HXX} = \sum_{\alpha,\beta} (-1)^{\theta_\alpha + \theta_\beta} X_h^{h_1} \left( \bar{\psi}_\beta (e^{i\gamma_5})^{\theta_\alpha \theta_\beta} \psi_\alpha \right) X_h^{h_1} \approx \frac{|m_{\chi^0_1}|}{v} \delta_{\sigma \sigma},$$

i.e. the couplings of $h_1$ to $\chi^0_1$ and $\chi^0_2$ are proportional to the mass/VEV. In Eq. (15) $\psi_0^0 = (e^{i\gamma_5})^{\theta_\alpha \chi^0_0}$ is the set of inert neutralino eigenstates with positive eigenvalues, while $\theta_\alpha$ equals 0 (1) if the eigenvalue corresponding to $\chi^0_\alpha$ is positive (negative).

In our analysis we require that the lightest inert neutralino account for all or some of the observed dark matter relic density. This sets another stringent constraint on the masses and couplings of $\chi^0_1$. Indeed, because the lightest inert neutralino states are almost inert singlinos, their couplings to the gauge bosons, Higgs states, quarks (squarks) and leptons (sleptons) are rather small resulting in a relatively small annihilation cross section of $\chi^0_1 \chi^0_1 \rightarrow$ SM particles and the possibility of an unacceptably large dark matter density. Thus the bulk of the E6SSM parameter space, that leads to small masses of $\chi^0_1$, is almost ruled out$^1$.

A reasonable density of dark matter can be obtained for $|m_{\chi^0_1}| \sim M_Z/2$ when the lightest inert neutralino states annihilate mainly through an s–channel Z-boson, via its inert Higgsino doublet components which couple to the Z–boson. If $\chi^0_1$ annihilation proceeds through the Z–boson resonance, i.e. $2|m_{\chi^0_1}| \approx M_Z$, then an appropriate value of dark matter density can be achieved even for a relatively small coupling of $\chi^0_1$ to Z. Since the masses of $\chi^0_1$ and $\chi^0_2$ are much larger than the b–quark mass and the decays of $h_1$ into these neutralinos are kinematically allowed, the SM–like Higgs boson decays predominantly into the lightest inert neutralino states and has very small branching ratios (2%–4%) for decays into SM particles$^{22,23}$.

The lightest inert neutralino states can get appreciable masses $\sim M_Z/2$ only if at least one light inert chargino state and two inert neutralinos states, which are predominantly components of the SU(2)$_W$ doublet, have masses below 200 GeV. The inert chargino and neutralinos states that are mainly inert Higgsinos couple rather strongly to W and Z–bosons and therefore can be efficiently produced at the LHC and then decay into the LSP and pairs of leptons and quarks giving rise to remarkable signatures which can be observed in the near future.

If the masses of $\chi^0_1$ and $\chi^0_2$ are very close then the decays of $h_1$ into $\chi_0 \chi_1$ will give rise to a large invisible branching ratio of the SM–like Higgs boson. When the mass difference between the second lightest and the lightest inert neutralinos is larger than 10 GeV the invisible branching ratio remains dominant but some of the decay products of $\chi_2$ might be observed at the LHC. In particular, there is a chance that soft $\mu^+ \mu^-$ pairs may be detected. Since the branching ratios of $h_1$ into SM particles are extremely suppressed, the decays of the SM–like Higgs boson into $l^+ l^- + X$ could be important for Higgs searches$^{22}$.

$^1$When $f_{\alpha\beta}, f_{\alpha\beta} \rightarrow 0$ the masses of $\chi^0_1$ and $\chi^0_2$ tend to zero and inert singlino states essentially decouple from the rest of the spectrum. In this limit the lightest non-decoupled neutralino may be rather stable and can play the role of dark matter$^{22}$. The presence of very light neutral fermions in the particle spectrum might have interesting implications for the neutrino physics (see, for example$^{23}$).
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