Modeling of the influence of non-stationary waves in three-component medium in the formation of a plasma jet

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Abstract. This article discusses the mathematical description of the physical processes of transformation of composite substances of a plasma jet. A mathematical model describing the propagation of elastic waves in a three-component medium was developed for the prediction and planning of the use of plasma deposition technology, in particular in the study of the kinematic characteristics of the plasma flow. Also in the course of mathematical modeling the table of coefficients of compressibility of liquid (plasma jet) and gas and velocities of longitudinal and transverse elastic waves is obtained. Graphs of the velocity of propagation of longitudinal and transverse waves in the plasma coating material on its physical and mechanical characteristics-compressibility coefficients are plotted.

1. Introduction.

To date, one of the promising methods of restoration of worn surfaces is plasma spraying on the working surface of parts of protective coatings. This process ensures the application of the required composite composition in local areas of the working surface of the part, which allows to further reduce the cost of repair [1].

Mathematically, it is quite difficult to describe the physical real processes of transformation of a substance under the influence of the applied pressure pulse, so models are built that can adequately reflect the behavior of the composite material under operating conditions [2].

This work is devoted to the development of a mathematical model, in particular, to the assessment of the influence of the velocity of propagation of unsteady waves in a three-component medium on the coefficients of compressibility of the plasma jet and plasma gas.

2. Methods of conducting research.

In this paper, we take the moment of formation of a plasma jet as the basis of the research base (Fig. 1).

Namely, when a composite powder is supplied, three components come into contact: a plasma-forming gas, a solid element (a powder particle) and a liquid (a plasma jet). When predicting plasma spraying and mathematical modeling of this process, it was assumed that the plasma jet acts as a liquid.
Since real substances react to loads and the spectrum of different factors in operational tests is very difficult and ambiguous, rheological relations reflecting its thermomechanical properties should be set for each medium. Depending on the temperature and pressure, the same material can be in both solid and liquid phase. The concept of idealized medium is introduced to describe the characteristic properties of materials [3].

**Figure 1.** Diagram of plasma spraying: 1 – plasma gas; 2 – place of introduction of the sprayed material; 3 – power supply; 4 – cathode; 5 – anode; 6 – composite powder, 7 – plasma gas; 8 – plasma jet

On the basis of the idealized medium, we write down a system of equations that determine the dynamic behavior of an elastic, saturated liquid and gas three-component medium in the displacements of the components, expressing the coefficients Lame $\lambda$, $\mu$ through young modulus $E$ and Poisson's ratio $\nu$ by formula [4]:

$$\lambda = \frac{E\nu}{(1 + \nu)(1 - 2\nu)} , \quad \mu = \frac{E}{2(1 + \nu)}$$  \hspace{1cm} (1)

Based on the work [5,6], we derive a cubic equation for determining the velocity of propagation of nonstationary longitudinal waves:

$$kz^3 + bz^2 + dz + f = 0$$

$$b = \sigma_{22}\gamma_{13} + \sigma_{33}\gamma_{11} - \sigma_{13}\gamma_{12} - \sigma_{12}\gamma_{13} - \sigma_{23}\gamma_{23} - \sigma_{33}\gamma_{22} + \sigma_{13}\gamma_{12} + \sigma_{12}\gamma_{13} + \sigma_{13}\gamma_{23} + \sigma_{12}\gamma_{23} + \sigma_{13}\gamma_{33}$$

$$d = -\sigma_{1}^{12}\gamma_{13}^{3} + \sigma_{1}^{13}\gamma_{12}^{3} + \sigma_{1}^{13}\gamma_{23}^{3} - \sigma_{1}^{12}\gamma_{12}^{3} + \sigma_{1}^{13}\gamma_{33}^{3} + \sigma_{1}^{12}\gamma_{23}^{3} + \sigma_{1}^{12}\gamma_{33}^{3} - \sigma_{1}^{13}\gamma_{13}^{3}$$

$$f = \sigma_{1}^{12}\gamma_{12}^{2} - \sigma_{1}^{13}\gamma_{13}^{2} + \sigma_{1}^{12}\gamma_{23}^{2} + \sigma_{1}^{13}\gamma_{33}^{2} - \sigma_{1}^{13}\gamma_{12}^{2}$$  \hspace{1cm} (2)

Coefficients $\sigma_{11}, ..., \sigma_{33}$ and $\gamma_{11}, ..., \gamma_{33}$ are on formulas:
\[ y_{11} = \frac{\rho_{11}}{\rho}, y_{12} = \frac{\rho_{12}}{\rho}, y_{22} = \frac{\rho_{22}}{\rho}, y_{13} = \frac{\rho_{13}}{\rho}, y_{23} = \frac{\rho_{23}}{\rho}, \]

\[ y_{33} = \frac{\rho_{33}}{\rho} - \rho = \rho_{11} + 2\rho_{12} + \rho_{22} + 2\rho_{13} + 2\rho_{23} + \rho_{33} \]

\[ \sigma_{11} = \frac{\lambda}{M}, \sigma_{12} = \frac{(1-m)R_{0}^{(2)}}{M}, \sigma_{22} = \frac{mR_{0}^{(2)}}{M}, \]

\[ \sigma_{33} = \sigma_{23} = \frac{(1-m)R_{0}^{(3)}}{M}, \sigma_{33} = \frac{mR_{0}^{(3)}}{M} \]  

where \( \rho_{11}, \rho_{22}, \rho_{33} \) – effective densities of powder mechanical mixture for plasma spraying, plasma and plasma-forming gas, respectively; \( \rho_{12}<0, \rho_{13}<0, \rho_{23}<0 \) – dynamic coupling coefficients of the skeleton; \( R_{0}^{(2)}, R_{0}^{(3)} \) – compressibility factors for components filled with liquid and gas; \( m \) – the porosity of the medium; \( \sigma_{11}, \sigma_{33} \) – stress tensor; \( y_{11}, y_{33} \) – weighting factor; \( M \) – the force acting on the liquid and gas related to the unit cross-sectional area of the porous medium. In parentheses, the numbers at the top indicate: 1 – solid component, 2 – liquid, 3 – gas.

Solving the cubic equation (2) find the formulas of Cardan [7].

Also, taking as a basis the work [5,6], we obtain an expression to determine the velocity of the transverse wave, distributed in a three-component environment:

\[ a_{\gamma} = \sqrt{\frac{(y_{22}y_{33} - y_{23}^2)E}{2(1+\nu)M'(y_{12}y_{33} - y_{13}^2y_{23} - y_{12}y_{23})y_{11}^2y_{22}y_{33} + y_{12}y_{13}y_{23} + y_{12}y_{23} - y_{12}^2y_{33})}} \]  

(4)

3. The results of the research and their discussion.

According to the data entered in the idealized environment, as well as the results of calculations of equations (1-4), we obtain a summary table of values.

| The coefficients of compressibility of the plasma jet Co(2) and To a plasma-forming gas(3) | The speed of longitudinal waves | The speed of transverse waves |
|---------------------------------------------|-------------------------------|-------------------------------|
| \( R_{0}^{(2)} \) | \( z \) | \( a_{\gamma} \) |
| 0.5 | 2.89994E-07 | 530516509.1 |
| 0.47619048 | 2.90765E-07 | 531222969.1 |
| 0.45454545 | 2.91538E-07 | 531929376.7 |
| 0.43478261 | 2.92311E-07 | 532635731.7 |
| 0.41666667 | 2.93085E-07 | 533342034.3 |
| \( R_{0}^{(3)} \) | \( z \) | \( a_{\gamma} \) |
| 0.83333333 | 2.8999420857E-07 | 530516509.1 |
| 0.76923077 | 2.9030803724E-07 | 530804106.2 |
| 0.71428571 | 2.9062201203E-07 | 531091681.6 |
| 0.66666667 | 2.9093613288E-07 | 531379235.5 |
| 0.625 | 2.9125039977E-07 | 531666767.7 |

On the basis of the table of values of coefficients of compressibility and velocities of waves we will make graphs of dependences. Figures 2 and 3 show the dependence of the velocity of propagation of longitudinal and transverse elastic waves in a three-component medium on its physical and mechanical characteristics – the compressibility coefficients of the liquid and gas.
Figure 2. The graph of $R_0^{(2)}$ from the velocity of longitudinal and transverse wave propagation

Figure 3. The graph of $R_0^{(3)}$ from the velocity of longitudinal and transverse wave propagation
4. Conclusions.

The influence of the propagation velocity of unsteady waves in a three-component medium on the compressibility coefficients of the plasma jet and plasma gas is estimated. Graphs of the velocity of propagation of longitudinal and transverse waves in the material of the plasma coating on its physical and mechanical characteristics—the coefficients of compressibility of the liquid (plasma jet) and gas are plotted.

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