Measurement of branching fraction and direct $CP$ asymmetry in charmless $B^+ \rightarrow K^+K^-\pi^+$ decays at Belle

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We report a study of the charmless hadronic decay of the charged $B$ meson to the three-body final state $K^+K^−\pi^+$. The results are based on a data sample that contains $772 \times 10^6 \bar{B}B$ pairs collected at the $\Upsilon(4S)$ resonance with the Belle detector at the KEKB asymmetric-energy $e^+e^−$ collider. The measured inclusive branching fraction and direct $CP$ asymmetry are $(5.38 \pm 0.40 \pm 0.35) \times 10^{−6}$ and $−0.170 \pm 0.073 \pm 0.017$, respectively, where the first uncertainties are statistical and the second are systematic. The $K^+K^−$ invariant mass distribution of the signal candidates shows an excess in the region below 1.5 GeV/$c^2$, which is consistent with the previous studies from BABAR and LHCb. In addition, strong evidence of a large direct $CP$ asymmetry is found in the low $K^+K^−$ invariant-mass region.

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Charmless decays of $B$ mesons to three charged hadrons are suppressed in the standard model (SM), and thus provide an opportunity to search for physics beyond the SM through branching fraction enhancements. Large $CP$ asymmetries can occur in these decays, due to interference of tree and loop diagrams with similar amplitudes. Beyond-the-SM particles could also contribute in the loops. Figure 1 shows some of the SM Feynman di-
where $\text{suppressed}$). (a) Tree diagram, (b) $W$-exchange diagram leading to $K^+K^-\pi^+$ decay. The LHCb studies revealed a nonzero inclusive $CP$ asymmetry of $-0.123 \pm 0.017 \pm 0.012 \pm 0.007$ and a large unquantified local $CP$ asymmetry in the same mass region. These results suggest that final-state interactions may contribute to $CP$ violation \[4, 5\]. This study attempts to quantify the $CP$ asymmetry and branching fraction as a function of the $K^+K^-$ invariant mass.

$$A_{CP} = \frac{N(B^+ \to K^+K^-\pi^-) - N(B^+ \to K^+K^+\pi^+)}{N(B^+ \to K^+K^+\pi^-) + N(B^+ \to K^+K^+\pi^+)},$$

(1)

where $N$ denotes the yield obtained for the corresponding mode. The data sample consists of $772 \times 10^6 B\bar{B}$ pairs accumulated at the $\Upsilon(4S)$ resonance, corresponding to an integrated luminosity of 711 fb$^{-1}$, and an additional 89.4 fb$^{-1}$ of off resonance data recorded at a center-of-mass (c.m.) energy about 60 MeV below the $\Upsilon(4S)$ resonance.

The Belle detector consists of a silicon vertex detector (SVD), a 50-layer central drift chamber (CDC), time-of-flight scintillation counters (TOF) an array of aerogel threshold Cherenkov counters (ACC), and a CsI(Tl) electromagnetic calorimeter located inside a superconducting solenoid coil that provides a 1.5 T magnetic field. Outside the coil, the $K^0_L$ and muon detector, composed of resistive plate counters, detects $K^0_L$ mesons and identifies muons. The detector is described in detail elsewhere \[7\]. The data set used in this analysis was collected with two different inner detector configurations. A data sample corresponding to 140 fb$^{-1}$ was collected with a beam pipe of radius 2.0 cm and with three layers of SVD, while the rest of the data were collected with a beam pipe of radius 1.5 cm and four layers of SVD \[8\]. A GEANT3-based \[9\] Monte Carlo (MC) simulation of the Belle detector is used to optimize event selection and to estimate the signal efficiency. The signal MC sample is generated with the EvtGen package \[10\], assuming a three-body phase space combined with an intermediate resonance decaying to two kaons as observed by BABAR and LHCb \[1, 3\].

To reconstruct $B^+ \to K^+K^-\pi^+$, we combine two oppositely charged kaons with a charged pion. Charged tracks originating from a $B$-meson decay are required to have a distance of closest approach with respect to the interaction point of less than 5.0 cm along the $z$ axis (opposite to the positron beam direction) and less than 0.2 cm in the $r$-$\phi$ transverse plane, and a transverse momentum of at least 100 MeV/c.

Charged kaons and pions are identified using specific ionization in the CDC, time-of-flight information from the TOF, and Cherenkov light yield in the ACC. This information is combined to form a $K$-$\pi$ likelihood ratio $R_{K/\pi} = \mathcal{L}_K/(\mathcal{L}_K + \mathcal{L}_\pi)$, where $\mathcal{L}_K$ and $\mathcal{L}_\pi$ are the likelihoods for the kaon and pion hypothesis, respectively. Tracks with $R_{K/\pi} > 0.6$ are regarded as kaons and those with $R_{K/\pi} < 0.4$ as pions. With these requirements, the identification efficiencies for 1 GeV/c kaons and pions are 83% and 90%, respectively; 6% of the pions are misidentified as kaons and 12% of the kaons as pions.

Candidate $B$ mesons are identified using two kinematic variables: the beam-energy constrained mass, $M_{bc} = \sqrt{E_{\text{beam}}^2 - \frac{|p_B|^2}{c^2}}$, and the energy difference, $\Delta E = E_B - E_{\text{beam}}$, where $E_B$ and $p_B$ are the reconstructed energy and momentum of $B$-meson candidates in the c.m. frame, respectively, and $E_{\text{beam}}$ is the run-dependent beam energy. The fit region is defined as $M_{bc} > 5.24$ GeV/c$^2$ and $|\Delta E| < 0.3$ GeV, while the signal-enhanced region is given by $5.27$ GeV/c$^2 < M_{bc} < 5.29$ GeV/c$^2$ and $|\Delta E| < 0.05$ GeV. For 19% of the events, there is more than one $B$-meson candidate; we choose the one with the best fit quality from the $B$ vertex fit. The $B$ vertex fit uses the three charged tracks. This criterion selects the correct $B$-meson candidate in 92% of MC events.

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1 The inclusion of charge-conjugate modes is implied throughout this paper, unless explicitly stated otherwise.
The dominant background is from continuum $e^+e^- \rightarrow q\bar{q}$ ($q = u, d, s, c$) processes. A neural network [11] is used to distinguish the spherical $B\bar{B}$ signal from the jetlike continuum background. It combines the following observables based on the event topology: a Fisher discriminant formed from 17 modified Fox-Wolfram moments [12], the cosine of the angle between the $B$-meson candidate direction and the beam axis, the cosine of the angle between the thrust axis [13] of the $B$-meson candidate and that of the rest of event (all of these quantities being calculated in the c.m. frame), the separation along the $z$ axis between the vertex of the $B$-meson candidate and that of the remaining tracks, and the tagging quality variable from a $B$-meson flavor-tagging algorithm [14].

The training and optimization of the neural network are performed with signal and continuum MC samples. The neural network output ($C_{NN}$) selection requirement is optimized by maximizing a figure of merit defined as $N_{S}/\sqrt{N_{S}+N_{B}}$ in the signal-enhanced region, where $N_{S}$ denotes the expected number of signal events based on MC simulation for a branching fraction of $5 \times 10^{-6}$ and $N_{B}$ denotes the expected number of background events.

The requirement on $C_{NN}$ removes 99% of the continuum events while retaining 48% of the signal.

Background contributions from $B$-meson decays via the dominant $b \rightarrow c$ transition (generic $B$ decays) are investigated with an MC sample of such decays. The resulting $\Delta E$ distribution is found to peak strongly in the signal region. Peaks are observed in the $K^+K^-$ and $K^+\pi^-$ invariant-mass spectra, arising from $b \rightarrow c$ decays.

These contributions are from $D^0 \rightarrow K^+K^-$ or $K^+\pi^+$ peaking at the nominal $D^0$ mass, and $D^0 \rightarrow K^-\pi^+$ with the peak slightly shifted from the $D^0$ mass in the $M_{K^+K^-}$ spectrum owing to $K^-\pi$ misidentification. To suppress these backgrounds, the candidates for which the invariant mass of the $K^+K^-$ or $K^+\pi^-$ system lies in range of 1850-1880 MeV/c$^2$ are removed. The selection window corresponds to $\pm 3.75\sigma$ around the nominal $D^0$ mass, where $\sigma$ is the mass resolution. In the case of $K^-\pi$ misidentification, we use the pion hypothesis for one of the kaons. To suppress the possible charmonium backgrounds from $J/\psi \rightarrow \ell^+\ell^-(\ell = e, \mu)$ decays, we apply the lepton hypothesis for both charged daughters and exclude candidates that lie in the range of 3060-3140 MeV/c$^2$, which corresponds to $\pm 4\sigma$ around the nominal $J/\psi$ mass. Since no significant resonance is found in the $\psi(2S)$ mass region, we do not apply a veto selection for it.

The charmless (i.e., “rare”) $B$-meson decays are studied with a large MC sample where one of the $B$ mesons decays via a process with known or estimated branching fraction. There are a few modes that contribute in the $M_{K\bar{K}}$ signal region with a corresponding $\Delta E$ peak, denoted collectively as the “rare peaking” background.

These peaking backgrounds are due to $K^-\pi$ misidentification, including $B^+ \rightarrow K^+K^-K^+$, $B^+ \rightarrow K^+\pi^-\pi^+$, and their intermediate resonant modes. The events that remain after removing the peaking components are called the “rare combinatorial” background.

The signal yield and direct $CP$ asymmetry are extracted by performing a two-dimensional extended unbinned maximum likelihood fit to $M_{K\bar{K}}$ and $\Delta E$ distributions in bins of $M_{K^+K^-}$. In order to study the finer structure in the enhancement region, the $M_{K^+K^-}$ region is divided into five nonuniform bins. The first two bins are chosen to cover the interesting enhancement and $A_{CP}$ signal found by LHCb and BABAR. The remaining bin ranges are chosen in order to have an approximately equal number of signal events in each bin. The likelihood is defined as

$$L = \frac{e^{-\sum_i N_i} \prod_i^N \left( \sum_j N_j \mathcal{P}_j \right)}{N!},$$

where

$$\mathcal{P}_j = \frac{1}{2} \left( 1 - q^j \cdot A_{CP,j} \right) \times \mathcal{P}_j(M_{bc}, \Delta E^i).$$

Here, $N$ is the total number of candidate events, $i$ is the event index, and $N_j$ is the yield of events for category $j$, which indexes the signal, continuum, generic $B$, and rare $B$ components. $\mathcal{P}_j(M_{bc}, \Delta E)$ is the probability density function (PDF) in $M_{bc}$ and $\Delta E$ for the $j$th category. The electric charge of the $B$-meson candidate in event $i$ is denoted $q^i$ and $A_{CP,j}$ is the direct $CP$ asymmetry for category $j$. In the signal $B$ decays, there are two cases: all final state particles are correctly combined (“true” signal), or one of the daughter particles is a product of the other $B$-meson decay (“self-cross-feed” [SCF] background). We prepare the corresponding PDFs, $\mathcal{P}_{sig}$ and $\mathcal{P}_{SCF}$. The SCF background is described by $(N_{sig} \cdot f) \times \mathcal{P}_{SCF}$, where $N_{sig}$ is the signal yield and $f$ is the fraction of SCF component, which is fixed to the MC expectation. The signal PDF is represented by the product of a double Gaussian in $M_{bc}$ and a triple Gaussian in $\Delta E$, where the shape parameters are determined from the signal MC sample and are calibrated by a control sample of $B^+ \rightarrow \bar{D}^0(K^+K^-)\pi^+$. The PDF that describes the continuum background is the product of an ARGUS function [15] in $M_{bc}$ and a second-order polynomial in $\Delta E$. The parameters of the continuum PDF are derived from MC simulation, which agree with the off resonance data. In contrast, the distributions for $\Delta E$ and $M_{bc}$ are strongly correlated in the $BB$ background samples, including generic $B$, rare combinatorial, rare peaking, and SCF components. Therefore, they are modeled with two-dimensional smoothed histograms from MC simulation. The free parameters in the fit are the signal yield, the signal $A_{CP}$, the generic $B$ yield, the rare peaking yields, and the continuum yield. The yields of rare combinatorial backgrounds are also derived from the MC study. The $A_{CP}$ of all backgrounds is fixed to zero in the fit. The stability and bias of the
two-dimensional fit is checked by large ensemble tests using both toy and simulated MC events. The validity of the fit and branching fraction extraction method is checked using data in a high-statistics control sample of the $B^+ \to \bar{D}^0(K^+K^-)\pi^+$ decays. The measured branching fraction for the control sample is consistent with the world-average value \cite{pdg}.

As an example, Fig. 2 shows the fit results in $B^+$ and $B^-$ samples of the first $M_{K^+K^-}$ bin in a signal-enhanced region. We use the efficiency and fitted yield in each bin to calculate the branching fraction:

$$B = \frac{N_{\text{sig}}}{\epsilon \times C_{\text{PID}} \times N_{BB}},$$

where $N_{BB}$, $\epsilon$, and $C_{\text{PID}}$, respectively, are the number of $BB$ pairs $(772 \times 10^6)$, the reconstruction efficiency, and the correction factor for particle identification (94.2%) that accounts for possible data-MC difference. We assume that charged and neutral $BB$ pairs are produced equally at the $\Upsilon(4S)$ resonance. Table I lists the fitted yields, efficiencies, and measured $A_{CP}$ in all such bins. To determine the significance of our measurements, we use the convolution of the likelihood function with a Gaussian function of width equal to the additive systematic uncertainties that only affect the signal yield and $A_{CP}$. The corresponding significance is given by $\sqrt{-2\ln(L_0/L_{\text{max}})}$, where $L_{\text{max}}$ and $L_0$ are the likelihood values with and without the signal component, respectively. Figure 3 shows the results, where an excess and a large $A_{CP}$ are seen in $M_{K^+K^-} < 1.5$ GeV/$c^2$, confirming the observations by BABAR and LHCb. We find strong evidence of a large $CP$ asymmetry of $-0.90 \pm 0.17 \pm 0.03$ with 4.8$\sigma$ significance for $M_{K^+K^-} < 1.1$ GeV/$c^2$. We integrate the differential branching fractions over the entire mass range to obtain an inclusive branching fraction:

$$B(B^+ \to K^+K^-\pi^+) = (5.38 \pm 0.40 \pm 0.35) \times 10^{-6},$$

where the quoted uncertainties are statistical and systematic, respectively. The weighted average $A_{CP}$ over the entire $M_{K^+K^-}$ region is

$$A_{CP} = -0.170 \pm 0.073 \pm 0.017,$$

where the $A_{CP}$ value in each bin is weighted by the fitted yield divided by the detection efficiency in that bin. The statistical uncertainties are independent among bins; thus, the term is a quadratic sum. For the systematic uncertainties, the contribution from the bin-correlated sources is a linear sum while the contribution from the bin-uncorrelated sources is a quadratic sum.

Systematic uncertainties in the branching fraction are itemized in Table II. The uncertainty due to the total number of $BB$ pairs is 1.37%. The uncertainty due to the charged-track reconstruction efficiency is estimated to be 0.35% per track by using the partially reconstructed $D^{*+} \to D^0\pi^+$, $D^0 \to \pi^+\pi^-K_S^0$ events. The uncertainty due to the $R_{K/\pi}$ requirements is determined by a control sample study of $D^{*+} \to D^0\pi^+$ with $D^0 \to K^+\pi^-$. The uncertainties due to the continuum suppression selection criteria and the signal PDF shape are estimated using a control sample of $B^+ \to D^0(K^+K^-)\pi^+$ decays. The
TABLE I. Signal yield, efficiency, differential branching fraction, and $A_{CP}$ for individual $M_{K^+K^-}$ bins. The first uncertainties are statistical and the second systematic.

| $M_{K^+K^-}$ (GeV/c^2) | $N_{sig}$ | Eff. (%) | $d\mathcal{B}/dM$ ($\times 10^{-7}$) | $A_{CP}$ |
|-------------------------|-----------|----------|----------------------------------|----------|
| 0.8 − 1.1 | 59.8 ± 11.4 ± 2.6 | 19.7 | 14.0 ± 2.7 ± 0.8 | −0.90 ± 0.17 ± 0.04 |
| 1.1 − 1.5 | 212.4 ± 21.3 ± 6.7 | 19.3 | 37.8 ± 3.8 ± 1.9 | −0.16 ± 0.10 ± 0.01 |
| 1.5 − 2.5 | 113.5 ± 26.7 ± 18.6 | 15.6 | 10.0 ± 2.3 ± 1.7 | −0.15 ± 0.23 ± 0.03 |
| 2.5 − 3.5 | 110.1 ± 17.6 ± 4.9 | 15.1 | 10.0 ± 1.6 ± 0.6 | −0.09 ± 0.16 ± 0.01 |
| 3.5 − 5.3 | 172.6 ± 25.7 ± 7.4 | 16.3 | 8.1 ± 1.2 ± 0.5 | −0.05 ± 0.15 ± 0.01 |

potential fit bias is evaluated by performing an ensemble test comprising 1000 pseudoexperiments, where the signal component is taken from the signal MC sample, and the PDF shapes are used to generate the data for the other event categories. The observed 2.3% bias is included in the systematic uncertainty calculation. The uncertainty due to the continuum background PDF modeling is evaluated by varying the PDF parameters by ±1σ of their statistical errors. The uncertainty due to the data-MC difference is taken into account by using the fit model from the off resonance data, which is included in the background PDF modeling in Table II. For the $BB$ background PDFs that are modeled by a two-dimensional smoothed histogram PDFs, the associated uncertainty is evaluated by changing the bin sizes. The uncertainty due to the fixed yields of rare combinatorial backgrounds is also evaluated by varying each fixed yield up or down by its statistical error. The uncertainty due to nonzero $A_{CP}$ of rare peaking backgrounds is estimated by assuming the $A_{CP}$ values to be higher or lower than the LHCb measured values by 1σ of the LHCb measurement uncertainty [3]. In the absence of knowledge of the distribution of the SCF background in $M_{K^+K^-}$, we use a conservative approach to evaluate the uncertainty by varying the fraction by ±50%; the resulting deviation from the nominal value is included in the fixed yields in Table II.

The $A_{CP}$ systematic errors due to the fixed yields, background $A_{CP}$ and the background PDF modeling are estimated with the same procedure as applied for the branching fraction. A possible detector bias due to tracking acceptance and $R_{KK}$ is evaluated using the measured $A_{CP}$ value from the off resonance data. We apply the same criteria as those for the signal except for the continuum-suppression requirement and calculate $A_{CP}$ as in Eq. (1). The $A_{CP}$ value from off resonance data is 0.0024 ± 0.0014. The final $A_{CP}$ result can be either corrected with this detector bias, or it can be applied as a systematic uncertainty. For this result the central shift plus 1σ statistical error is included in the total systematic uncertainty for the $A_{CP}$. A full list of systematic uncertainties in $A_{CP}$ is shown in Table II.

In conclusion, we have reported the measured branching fraction and direct $CP$ asymmetry for the suppressed decay $B^+ \rightarrow K^+K^−\pi^+$ using the full $\Upsilon(4S)$ data sample collected with the Belle detector. We employ a two-dimensional fit to determine the signal yield and $A_{CP}$ as a function of $M_{K^+K^-}$. We confirm the excess and local $A_{CP}$ in the low $M_{K^+K^-} < 1.1$ GeV/c^2. Our measured inclusive branching fraction and direct $CP$ asymmetry are $\mathcal{B}(B^+ \rightarrow K^+K^−\pi^+) = (5.38 ± 0.40 ± 0.35) \times 10^{-6}$ and $A_{CP} = −0.170 ± 0.073 ± 0.017$, respectively. The measurement challenges the conventional description of direct $CP$ violation since it requires large contributions to separate weak tree and strong penguin amplitudes in the same small region of phase space in order to simultaneously enhance both the yield and provide the cancellation required for such a large $CP$ effect. So, for example, if the enhancement were due to a large final state resonance in a strong penguin diagram, there would have to be an accompanying tree-level process of the same magnitude and opposite phase to provide the almost complete cancellation observed in the measurement. To understand the origin of the low-mass dynamics, a full Dalitz analysis from experiments with a sizeable data set, such as LHCb and Belle II, will be needed in the future.

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TABLE II. Systematic uncertainties in the measured branching fraction and $A_{CP}$ in the individual bins. The dagger (†) indicates the $M_{K^+ K^-}$ dependence of the uncertainty. The center dots (· · ·) indicate a value below 0.05% (0.001) in $B (A_{CP})$.

| Source | Number of $BB$ pairs | Relative uncertainties in $B$ (%) |
|--------|----------------------|----------------------------------|
|        | $M_{K^+ K^-}$ (GeV/c²) | 0.8 − 1.1 1.1 − 1.5 1.5 − 2.5 2.5 − 3.5 3.5 − 5.3 |
|        |                      | 1.37                                      |
|        | Tracking             | 1.05                                      |
|        | Particle identification | 1.44                                      |
|        | Continuum suppression | 1.33                                      |
|        | Signal PDF           | 1.77                                      |
|        | Fit bias             | 2.30                                      |
|        | Background PDF†      | 3.65 2.15 16.16 3.77 3.59                |
|        | Fixed yields†        | · · · · · · · · · ·                      |
|        | Background $A_{CP}$† | 0.23 0.28 1.46 0.80 0.36                |

| Source | Absolute uncertainties in $A_{CP}$ |
|--------|------------------------------------|
|        | $M_{K^+ K^-}$ (GeV/c²) | 0.8 − 1.1 1.1 − 1.5 1.5 − 2.5 2.5 − 3.5 3.5 − 5.3 |
|        |                      |                                           |
|        | Background PDF†      | 0.036 0.005 0.028 0.006 0.003            |
|        | Fixed yields†        | · · · · · · · · · ·                      |
|        | Background $A_{CP}$† | 0.015 0.004 0.009 0.005 0.002            |
|        | Detector bias        | 0.004                                      |

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