A New Type of Resonant Neutrino Conversions Induced by Magnetic Fields

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Abstract

We consider resonant neutrino conversions in magnetised matter, such as a degenerate electron gas. We show how magnetisation effects caused by axial vector interactions of neutrinos with the charged leptons in the medium can induce a new type of resonant neutrino conversion which may occur even in situations where the MSW effect does not occur, such as the case of degenerate or inverted neutrino mass spectra. Our new resonance may simultaneously affect anti-neutrino $\bar{\nu}_a \leftrightarrow \bar{\nu}_b$ as well as neutrino $\nu_a \leftrightarrow \nu_b$ flavour conversions, and therefore

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it may substantially affect supernova neutrino energy spectra. Using SN1987A data we conclude that only laboratory experiments with long baseline such as ICARUS or MINOS are likely to find neutrino oscillations due to their sensitivity to small $\Delta m^2$. We also comment on the possibility of resonant conversions induced by Majorana neutrino transition moments and mention the case of sterile neutrinos $\nu_s$. 
1. Introduction

Neutrino propagation in media with random magnetic fields has attracted considerable attention recently, both from the point of view of the early universe cosmology as well as astrophysics [1]. The presence of random magnetic fields in cosmology as well as in various astrophysical objects can strongly affect neutrino conversion rates and this could have important implications.

Some recent papers have considered neutrino propagation in media with time-varying magnetic field \( B(t) \), when the magnetic field is a regular changing field like a twist (circularly polarised) one \([2,3]\), or the linear polarised Alfven wave \([1]\). In any of these regular magnetic field cases neutrino conversions occur in oscillating regime. In contrast in the case of a random magnetic field \( \langle \tilde{B}(t) \rangle = 0 \) with \( \langle \tilde{B}(t)^2 \rangle \neq 0 \), the neutrino conversions become aperiodic \([5]\).

In general the magnetic field can be separated into two parts, \( B_j(t) = B_{j0} + \tilde{B}_j(t) \), the large-scale constant field \( B_{j0} \) and a random field \( \tilde{B}_j(t) \). One can consider neutrino propagation in a medium, such as a supernova, in two simple regimes: \( \tilde{B}(t) \gg B_0 \) and \( \tilde{B}(t) \ll B_0 \). The first case has been treated in previous works, where the effect of a strong random magnetic field, \( \sqrt{\langle B^2(t) \rangle} \gg B_0 \) on active-sterile supernova neutrino conversions was discussed \([3]\). In addition, the corresponding effect of strong random magnetic field upon neutrino transitions induced by a transition magnetic moment in the early universe hot plasma or in a supernova was discussed in ref. \([7]\).

In this paper we study a new class of resonant neutrino conversions in magnetised matter, such as a degenerate electron gas in the case of strong large-scale magnetic field.

We demonstrate how the magnetisation effects caused by axial vector interactions of neutrinos with charged leptons in the medium can induce a new type of resonant neutrino conversion which may occur even in kinematical situations where the MSW effect is forbidden. We discuss as an example the case of degenerate neutrinos and the interesting case of neutrinos with a mass difference \( \Delta m^2 \sim 10^{-5} \text{eV}^2 \) relevant to the solar neutrino problem. If such neutrinos undergo MSW conversions in the sun then their supernovae anti-neutrinos certainly do not. Nevertheless our new mechanism allows both \( \bar{\nu}_a \leftrightarrow \bar{\nu}_b \) supernova anti-neutrino conversions as well as \( \nu_a \leftrightarrow \nu_b \) solar neutrino conversions to be resonantly enhanced. Similar effects may exist also for conversions involving sterile neutrinos \( \nu_s \). The effect of our new resonance in supernovae anti-neutrino energy spectra leads to constraints (from SN1987A) on neutrino conversion parameters \( \Delta m^2 \) and \( \sin^2 2\theta \). In particular we show that only \( \Delta m^2 \lesssim 10^{-3} \text{eV}^2 \) are possible, thus ruling out the possibility of finding neutrino oscillations at laboratory experiments with short baseline. Thus, if our assumptions hold and our resonance takes place in supernovae, then only the new generation of long baseline accelerator experiments such as ICARUS and MINOS or reactor experiments such as CHOOZ or San Onofre are likely to see any effect.

In case of the resonant \( \bar{\nu}_\mu \leftrightarrow \bar{\nu}_e \) conversions which are suppressed in absence of magnetic field this allows us to obtain some bounds on neutrino mixing parameters from the non-observation of distortions of the SN1987A
electron anti-neutrino spectrum in the Kamiokande and IMB experiments. We also comment on the possibility of such resonant conversions induced by Majorana neutrino transition moments.

2. Neutrino conversions in magnetised electron gas

The Schrödinger evolution equation describing propagation of a system of two Majorana neutrino species in a magnetic field is given by a four dimensional evolution Hamiltonian \[ \mathcal{H} \]. Here we are interested in the situation where the neutrinos propagate in a medium. However, in contrast to ref. \[ \mathcal{H} \] we will include the magnetisation effects due to the mean axial vector neutrino interactions with the charged leptons in the medium. For definiteness we assume two neutrino species \( \nu_a \) where \( a = e, \mu, \tau \) denotes a definite neutrino flavour and \( \nu_x \) where \( x = s, b \) denotes either a sterile or an active neutrino. In the following we will specialise our attention to two simple cases:

1. neutrino transition magnetic moment is neglected, \( \mu = \mu^\nu_{ax} = 0. \) This is the generalisation of MSW theory \[ \mathcal{H} \] in the presence of magnetic field but neglecting neutrino magnetic moment

2. neutrino mixing is neglected, \( \theta_{ax} \equiv \theta = 0. \) This is a generalisation of the spin-flavour conversion theory \[ \mathcal{H} \] in the presence of non-vanishing mean axial vector neutrino interactions

In these cases the evolution Hamiltonian simplifies to a two dimensional evolution one, which may be generically written as

\[ i \frac{d}{dt} \begin{pmatrix} \nu_a \\ \nu_x \end{pmatrix} = \begin{pmatrix} H_{aa} & H_{ax} \\ H_{xa} & H_{xx} \end{pmatrix} \begin{pmatrix} \nu_a \\ \nu_x \end{pmatrix}, \]  

(2.1)

By convention we always choose \( H_{xx} = 0. \)

In case 1 (2.1) the diagonal component \( H_{aa} \) is given by

\[ H_{aa} = V_a - \Delta \cos 2\theta + f(\mu_{eff})B_\parallel(t) \]  

(2.2)

while in case 2 it will take the form

\[ H_{aa} = V_a - \Delta + f(\mu_{eff})B_\parallel(t) \]  

(2.3)

where \( \Delta = (m_2^2 - m_1^2)/2E \) vanishes for degenerate Majorana neutrinos and \( V_a \) is the appropriate difference of vector potentials describing the interactions with matter. Following ref. \[ \mathcal{H} \] we include the effect of the mean axial vector current \( V^{(axial)} \) of charged leptons in an external magnetic field \( B_\parallel = Bq/q \), denoting the relevant term by \( f(\mu_{eff})B_\parallel(t) \). This term involves the difference of axial vector potentials of the two neutrinos. Both terms depend on the channel of neutrino conversions considered. For case 1, we will consider helicity conserving \( \nu_e \leftrightarrow \nu_\mu \) and \( \nu_e \leftrightarrow \nu_s \) conversions. For case 2, we will consider helicity flipping conversions of active neutrinos to both active and sterile neutrinos.
Now we turn to the off-diagonal entry $H_{ax} = H_{xa}$. For case 1 it takes the form

\[ H_{ax} = \Delta \sin 2\theta / 2 \]

The second case corresponds to the spin-flavour transitions when we neglect mixing $s \to 0$ so that $\cos 2\theta = 1$. In this case the off-diagonal entry becomes

\[ H_{ax} = \mu B_\perp(t), \]

where $B_\perp$ is the component of the magnetic field transverse to the neutrino momentum $q$, $\mu B_\perp(t) \equiv H_\perp(t)$. For Dirac neutrinos $\Delta = 0$ and $\mu$ corresponds to the corresponding magnetic moment $\mu_B$.

In both cases $V_a$ is the difference of the active neutrino vector interaction potentials, $V_a = V_{\nu a} - V_{\nu a}$, (for $\nu_a$, $a \neq b$; $a = e, \mu, \tau$). For the sterile neutrino case $V_{\nu s} = 0$. For a left-handed electron neutrino in a supernova with core density $\rho = \rho_{14} \times 10^{14}$ g/cm$^3$ the vector potential is

\[ V_{\nu e} \simeq 3.8 \times 10^{-6} \rho_{14} f^{(e)}(Y_e) \text{MeV}, \quad (2.4) \]

where $f^{(e)}(Y_e) = 3Y_e - 1$ is a function of the abundances for our probe electron neutrino. Here $Y_e = n_e/n_B$ is the electron abundance, $n_B$ is the baryon density, and we neglect neutrino background contribution, $Y_\nu = 0$.

For the electron anti-neutrino case one would have $V_{\bar{\nu} e} = -V_{\nu e}$.

For a left-handed muon neutrino the potential is

\[ V_{\nu \mu} \simeq 3.8 \times 10^{-6} \rho_{14} 2Y_e \text{MeV}, \quad (2.5) \]

with abundance function $f^{(\mu)}(Y) = Y_e - 1$. Similarly for the muon anti-neutrino the potential $V_{\bar{\nu} \mu} = -V_{\nu \mu}$.

Using these neutrinos and anti-neutrino potentials we can easily calculate the resulting vector potential ($V_a$) for different flavour and (or) spin-flavour conversions. For $\nu_e \leftrightarrow \nu_\mu$ flavour conversion the potential is

\[ V_a \simeq 3.8 \times 10^{-6} \rho_{14} 2Y_e \text{MeV} \quad (2.6) \]

and for spin-flavour conversions $\bar{\nu}_e \leftrightarrow \nu_\mu$, the potential is

\[ V_a \simeq 3.8 \times 10^{-6} \rho_{14} 2(1 - 2Y_e) \text{MeV}. \quad (2.7) \]

For the active-sterile neutrino conversions $\nu_e \leftrightarrow \bar{\nu}_s$ one gets

\[ V_a \simeq 3.8 \times 10^{-6} \rho_{14}(3Y_e - 1) \text{MeV}. \quad (2.8) \]

The averaged matrix element for the axial current denoted by the symbol $< ... >_0$ is given by

\[ V_{\bar{\nu} s}^{(axial)} = \frac{G_F}{\sqrt{2}} \sum_{a=e,\mu,\tau} (-2c_A^{ba}) < \bar{\psi}_a \gamma_z \gamma_5 \psi_a >_0 \]

\[ = \frac{G_F}{\sqrt{2}} \sum_{a=e,\mu,\tau} (-2c_A^{ba}) \mu_B \frac{2eB_\parallel}{(2\pi)^2} \times \int_0^\infty dp_z \left[ \frac{1}{exp((\sqrt{p_z^2 + m_a^2} - \zeta)/T) + 1} + \frac{1}{exp((\sqrt{p_z^2 + m_a^2} + \zeta)/T) + 1} \right], \quad (2.9) \]

\[ ^3 \text{From the two-component point of view the Dirac neutrino magnetic moment is a particular case of transition moment, involving an active to a sterile neutrino.} \]
where \( \mu_B = e/2m_e \) is the Bohr magneton and \( c_A^2 = \mp 0.5 \) is the axial coupling constant (upper sign for \( b = a \) and lower one for \( b \neq a \)). Comparing (2.9) with \( \mu_{\text{eff}}B_\parallel \) and neglecting the contribution of positrons, muons and taus in a supernova degenerate electron gas we can define

\[
\mu_{\text{eff}} = -9c_A \times 10^{-13} \mu_B(p_F/eV)\quad (2.10)
\]

Note that the quantity \( \mu_{\text{eff}} \) has no relation to a real magnetic moment since it does not lead to a change of helicity.

In the diagonal entry of \( H_{aa} \) the function \( f(\mu_{\text{eff}}) \) is the difference in \( \mu_{\text{eff}} \) for the two species of neutrinos considered, which is determined by the value of \( c_A \).

For the case of \( \nu_e \leftrightarrow \nu_\mu \) flavour conversions the axial contribution doubles in the difference of \( 2\mu_{\text{eff}} \), so that we obtain

\[
H_{aa} = 3.8 \times 10^{-6} \rho_{14} 2Y_e - \Delta \cos 2\theta + 2\mu_{\text{eff}}B_\parallel. \quad (2.11)
\]

In contrast, for spin-flavour transitions (\( \bar{\nu}_e \leftrightarrow \nu_\mu \)) the axial contribution cancels in the difference of \( \mu_{\text{eff}} \), so that the entry \( H_{aa} \) is given by the corresponding neutrino vector potential (2.7),

\[
H_{aa} = 3.8 \times 10^{-6} \rho_{14} 2(1 - 2Y_e)MeV - \Delta. \quad (2.12)
\]

As a result of this cancellation the effects of magnetisation are absent for spin-flip flavour conversions, and will not consider in what follows. Finally for the active-sterile neutrino conversions \( \nu_e \leftrightarrow \nu_s \) we obtain for the upper diagonal entry

\[
H_{aa} = 3.8 \times 10^{-6} \rho_{14} (3Y_e - 1) - \Delta \cos 2\theta + \mu_{\text{eff}}B_\parallel, \quad (2.13)
\]

for \( s \neq 0, \mu = 0 \) \( (H_{ax} = \Delta \sin 2\theta/2) \) and

\[
H_{aa} = 3.8 \times 10^{-6} \rho_{14} (3Y_e - 1) - \Delta + \mu_{\text{eff}}B_\parallel, \quad (2.14)
\]

for \( s = 0, \mu \neq 0 \) \( (H_{ax} = \mu B_\perp) \).

With the help of the equations derived in this section many new types of neutrino conversions can be described. In the next session we will focus in the case of resonant neutrino flavour conversions in supernovae, neglecting neutrino transition moments.

### 3. Resonant neutrino conversions in magnetised electron gas with constant magnetic field

Consider a strong large-scale magnetic field \( B_{0j}(t) \) and let us neglect random components \( \tilde{B}_j(t) \), i.e. \( B_{0j}(t) \gg \tilde{B}_j(t) \). Expressing the magnetic field in terms of its transverse and longitudinal components we have

\[
B^2(t) = B^2(t) \cos^2 \alpha + B^2(t) \sin^2 \alpha = B^2_\parallel(t) + B^2_\perp(t), \quad (3.1)
\]

\(^4c_A\) changes sign for corresponding anti-neutrinos.
where \( \alpha \) is the angle between neutrino momentum \( \mathbf{q} \) and magnetic field \( \mathbf{B} \). In this case one can write the differential equation describing the evolution of the neutrino flavour conversion probability \( \mathcal{P} \) as

\[
\dot{\mathcal{P}} + \omega_0^2 \mathcal{P} = \frac{\varepsilon \sin \theta \times \bar{\varepsilon}}{\varepsilon}.
\] (3.2)

In (3.2) the square of the conversion frequency is

\[
\omega_0^2 = (V - \Delta \cos 2\theta + f(\mu_{eff})B_{\parallel})^2 + \Delta^2 \sin^2 2\theta.
\] (3.3)

where \( f(\mu_{eff}) = 2\mu_{eff} \) for \( \nu_e \leftrightarrow \nu_\mu \) transitions, \( f(\mu_{eff}) = 0 \) for \( \bar{\nu}_e \leftrightarrow \nu_\mu \) transitions and \( f(\mu_{eff}) = \mu_{eff} \) for \( \nu_e \leftrightarrow \nu_s \) conversions.

Let us consider a supernova with a strong constant magnetic field \( B \) \((\bar{B}_j(t) = 0)\). In this case the solution of (3.2) for the case of constant density reduces to

\[
\mathcal{P}(\bar{\varepsilon}) = \frac{\varepsilon \sin \theta \times \bar{\varepsilon}}{\varepsilon} \left( \frac{\omega_0 \bar{\varepsilon}}{\bar{\varepsilon}} \right) \sin \left( \frac{\omega_0 \bar{\varepsilon}}{\bar{\varepsilon}} \right).
\] (3.4)

where the frequency \( \omega_0 \) is given (3.3).

This has clearly a resonant form. The most interesting case from the point of view of observation is the case of anti-neutrino flavour conversions. The corresponding resonant condition can be written as

\[
V + \Delta \cos 2\theta + 2\mu_{eff}B \cos \alpha = 0.
\] (3.5)

The above resonance condition can be fulfilled for the outer layers of a supernova, where the density reduces to \( \rho \simeq 10^5 \text{ gcm}^{-3} \) and the Fermi momentum \( p_F/M_eV \simeq (\rho Y_e/10^7 \text{ gcm}^{-3})^{1/3} \).

Note that this condition can always be fulfilled for some point along the neutrino trajectory, since the last term can take on any values for \(-1 < \cos \alpha < 1\) \(^5\).

It is well-known that anti-neutrino flavour transitions are suppressed in the absence of a magnetic field for \( \Delta = (m_2^2 - m_1^2)/2E > 0 \). In contrast the third term in (3.3) allows us to obtain a resonance.

Note also that our new resonance condition can be fulfilled even for the case of degenerate neutrinos \( m_1 = m_2 \) when \( \Delta = 0 \) and the MSW resonance \(^[9]\) is absent. This is similar to the mechanism described in ref. \(^[11]\). For this \( \Delta = 0 \) case (3.5) simplifies to

\[
\left( \frac{|B \cos \alpha|}{10^{12}G} \right)^{3/2} \simeq 17 Y_e \rho_5.
\] (3.6)

The role of the non-universal interaction in ref. \(^[11]\) is played here by the axial vector interactions of neutrinos with the charged leptons of the medium. If (3.6) is fulfilled, analogously to ref. \(^[11]\), the resonance will take place simultaneously for both anti-neutrino as well as neutrino channels.

\(^5\)Note that for isotropic emission, neutrinos always cross magnetic field force lines. As a result it seems unlikely to expect \( B_\parallel = 0 \), i.e. that neutrinos will propagate strictly perpendicular to the magnetic field.
Now we discuss the issue of adiabaticity of our resonant conversion. We can define an adiabaticity parameter at resonance as follows:

$$\kappa = \frac{2(\Delta \sin 2\theta)^2}{\left|dV/dr + 2d(\mu_{eff}B) /dr \right|^2},$$ (3.7)

for $\nu_e \leftrightarrow \nu_\mu$ and $\bar{\nu}_e \leftrightarrow \bar{\nu}_\mu$. Adiabatic neutrino conversions would require the adiabaticity parameter (3.7) to be very large at resonance, $\kappa_R \gg 1$. As we show below this condition is not fulfilled in our case. Indeed, note that for a fixed electron abundance $Y_e \approx constant$, the profile of the electron density is the same as the matter density profile, which is assumed to be $\rho \sim r^{-3}$, so that $p_{Fe} \sim r^{-1}$. Thus for negligible $\Delta \simeq 0$ and at resonance the adiabaticity parameter (3.7) becomes infinity $\kappa_R = \infty$ if we have conservation of the magnetic field flux $\Phi_B$ so that $B \sim r^{-2}$ and $\rho \sim r^{-3}$. This would hold irrespective of the magnetic field and density profiles. Similarly, this will be the case for dipole type regular magnetic field $B \sim r^{-3}$ if $\rho \sim r^{-4.5}$.

In reality, during the first seconds of the main neutrino burst the density drops more steeply $\rho = \rho_0 \left(\frac{30 km}{r} \right)^5$. (3.8)

where $\rho_0 = 10^{12} g/cm^3$. We may describe the magnetic field profile via the scaling index $m$ defined as

$$B = B_0 \left(\frac{10 km}{r} \right)^m.$$ (3.9)

Let us note that the magnetic field profile (3.9) is more speculative than of (3.8). For example there has been so far no clear X-ray observations for SN1987A. Below we assume different scaling indexes $m = 1/2, 1, 3/2$ for the mean random magnetic field corresponding to different large scale ($r \sim R_{res} \sim 700 km$) field structure. For instance, $m = 3/2$ is appropriate for 3-d domains like dipole magnetic fields with size of order $L_0 \sim 1$ km randomly oriented in the supernova [13].

Unfortunately this profile is compatible with our resonance condition (3.6) only for very large field values at the supernova core, $B_0 \sim 3 \times 10^{16} G$ [13].

On the other hand the case $m = 1/2$ corresponds to $B_0 \sim 3 \times 10^{13}$ Gauss at the core, which is acceptable for a magnetised neutron star. While we lack a compelling physical motivation for such an exotic random magnetic field profile, it is not in conflict with any observational fact. It would correspond to filaments like super-conducting needles aligned along the neutrino trajectory [7].

As we will show below, we find that our neutrino parameter bounds are not sensitive to the magnetic field structure (3.9) which is crucial for resonant condition (3.6) itself only.

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6From Gauss theorem $div \textbf{B} = 0$, we expect the magnetic field profile to be $B \simeq 10^{12} G/(10 km/r)^2$.

7Note that the large-scale form (3.9) of the instantaneous mean field consisting of such dipoles has a completely different profile than the individual dipoles. As shown in ref. [14] it is characterised by the scaling index $m = 3/2$

8D. D. Sokoloff, private communication
In the crucial region \( r = R_{\text{res}} \sim 700 \text{ km} \) obeying the resonance condition (3.6) the index \( m \) is never \( 10/3 \), which would be necessary for adiabatic neutrino conversions, \( \kappa \gg 1 \) in (3.7). Thus, for our profiles (3.8) and (3.9) we need to consider neutrino conversions in the non-adiabatic regime. For the neutrino transition probability describing non-adiabatic \( \bar{\nu}_e \leftrightarrow \bar{\nu}_\mu,\tau \) flavour conversions we use the same as for the corresponding \( \nu_e \leftrightarrow \nu_\mu,\tau \) MSW conversions [15],

\[
P_{\bar{\nu}_e \leftrightarrow \bar{\nu}_\mu} = \frac{1}{2} \left[ 1 - (1 - 2P_{LZ}) \cos 2\theta \cos 2\theta_m \right],
\]

(3.10)

where the Landau-Zener probability

\[
P_{LZ} = \exp \left( -\frac{\pi}{4} \kappa F(\tan^2 \theta) \right)
\]

(3.11)

in the case of the density profile (3.8) differs from the linear Landau-Zener potential by the correction factor

\[
F(\tan^2 \theta) \simeq (1 - \tan^2 \theta) \left[ 1 + \frac{1}{5} \left( \ln(1 - \tan^2 \theta) + 1 - \frac{1 + \tan^2 \theta}{\tan^2 \theta} \ln(1 + \tan^2 \theta) \right) + \ldots \right].
\]

(3.12)

due to the fact that in our case \( V \sim \rho \sim r^{-5} \) instead of \( V_{LZ} \sim r \). We now turn to our main application of (3.10). It has been recently argued [17] that the non-observation of a hard energy tail in the electron anti-neutrino spectrum from SN1987A in the Underground Detectors at the Kamiokande and IMB experiments may place stringent limits in neutrino oscillation parameters. In ref. [17] this argument was used in order to severely constrain the possibility of large mixing MSW solutions to the solar neutrino problem. We now proceed to the implications of their argument to our case. Using (3.7) to (3.12) and adapting the results of ref. [17] to our resonant \( \bar{\nu}_e \leftrightarrow \bar{\nu}_\mu \) flavour conversions we obtain the following constraint on the active-active light neutrino mixing parameters,

\[
\exp \left( -\frac{\pi}{4} \kappa F(\tan^2 \theta) \right) \gtrsim \frac{0.3}{2 \cos 2\theta | \cos 2\theta_m |} + 0.5,
\]

(3.13)

which is shown in Fig. 1. For the case of small mixing, \( \theta \ll 1 \), this bound reduces to \( \exp(-\pi \kappa/4) \gtrsim 0.65 \). We see from Fig. 1 that assuming strong random magnetic field generation so as to realize our new resonant anti-neutrino conversion mechanism in SN1987A we can exclude here the possibility of observing neutrino oscillations in all previous searches at accelerators as well as reactors.

It is instructive to compare our results with those of ref. [17]. In our case one can write

\[
\sin^2 2\theta_m = \frac{\tan^2 2\theta}{\tan^2 2\theta + \left[ 1 + (V + 2\mu_{eff}B_||)/(\Delta \cos 2\theta) \right]^2}
\]

(3.14)

as the generalization of Wolfenstein’s formula describing anti-neutrino mixing in a magnetized degenerate electron gas. Clearly, outside the resonance region \( \sin^2 2\theta_m \ll 1 \) for neutrino masses in the range of interest for the explanation of the solar neutrino deficit. In this case we can safely neglect anti-neutrino conversions outside the resonance region. The situation in quite opposite in ref. [17], where the anti-neutrino conversion happen in vacuo outside the star. As a result Bahcall, Spergel and Smirnov can exclude only large vacuum mixings, in contrast to our formula (3.13).
4. Discussion and conclusions

We have found new resonant mechanism of neutrino conversion induced by the presence of magnetisation effects caused by axial vector interactions of neutrinos with the charged leptons in the degenerate electron gas in a large-scale supernova magnetic field. We gave an explicit solution of the corresponding evolution equation for the case of two neutrino species. For the conversion probabilities describing $\bar{\nu}_e \leftrightarrow \bar{\nu}_{\mu,\tau}$ flavour conversions, we use the same results obtained in ref. [15] as for the corresponding $\nu_e \leftrightarrow \nu_\mu$ MSW conversions.

Our new resonance may affect both anti-neutrino as well as neutrino flavour conversions, and also those involving sterile neutrinos $\nu_s$. They may occur even in situations where the MSW effect can not occur. In particular, they may convert supernovae anti-neutrinos $\bar{\nu}_e \leftrightarrow \bar{\nu}_b$ where $b = \mu, \tau$ at the same time that solar neutrinos are converted through the usual $\nu_e \leftrightarrow \nu_b$ MSW conversions and for the same choice of parameters.

Supernova neutrino energy spectra may be substantially affected by our new resonance. Using SN1987A data we conclude that only laboratory experiments with long baseline such as ICARUS or MINOS or the new generation of reactor long baseline experiments Chooz and San Onofre are likely to observe neutrino oscillations due to their sensitivity to small $\Delta m^2$. Our result is totally complementary to the one found by Bahcall, Smirnov and Spergel. In their case the solar neutrino conversions occur in the resonant regime but the supernova antineutrino ones do not. As a result they are only able to exclude large mixing angles but with a better sensitivity to small $\Delta m^2$. On the other hand, if our new resonance takes place, we can have both solar neutrinos and supernova anti-neutrinos resonantly converted, the first by the MSW effect and the second by our new effect involving the magnetisation. Correspondingly we can exclude a much larger region, including small and intermediate mixings.

Finally, let us comment on the possibility of resonant conversions induced by Majorana neutrino transition moments. This case is analogous to the generalisation of the Aneziris-Schechter twisting magnetic field ($B_\perp = B_0 \perp \exp(i\Phi)$) result [4] by Smirnov [3], who included matter effects. For definiteness let us focus here on transitions to a sterile neutrino $\nu_s$. The frequency for $\nu_e \leftrightarrow \nu_s$ transitions

$$\omega_0 = \sqrt{(V - \Delta + \mu_{\text{eff}} B_\parallel)^2 + (2\mu_\perp B)^2}$$

contains the magnetisation term $\mu_{\text{eff}} B_\parallel$ instead of $\Phi$. The corresponding resonance condition is

$$V - \Delta + \mu_{\text{eff}} B \cos \alpha = 0.$$  \hspace{1cm} (4.2)

Notice that, in contrast to ref. [3], in order to obtain the resonance condition (4.2), all we need is a restriction on the value of magnetic field $B$, which enters through the term $\mu_{\text{eff}} B_\parallel$. Our effect would seem more physical, as it is generated by the charged leptonic axial interaction in the medium and does not depend on special field geometry details. Note also that our axial term cures the suppression effect for $\Delta = 0$ found in ref. [12] for Dirac neutrino spin flip in an external magnetic field $\nu_{eL} \leftrightarrow \nu_{eR}$.  


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Fig.1.
Constraints on neutrino parameters $\Delta m^2$ and $\sin^22\theta$ for typical average supernova neutrino energies $< E_\nu >$ and different magnetic field profiles. The curves correspond to the magnetic field profile $B = B_0(10 km/r)^m$. There are three pairs of almost identical curves, with the lower curve in each pair corresponding to $m = 1/2$ and the upper one to $m = 3/2$, illustrating the insensitivity of our conversion rates to the field profile. The three sets of curves correspond to $< E_\nu > = 11$ MeV, 16 MeV and 25 MeV.
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