Spontaneous Partial Breaking of $\mathcal{N} = 2$ Supersymmetry and the U(N) Gauge Model

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Abstract. We briefly review the properties of the $\mathcal{N} = 2$ U(N) gauge model with/without matters. On the vacua, $\mathcal{N} = 2$ supersymmetry and the gauge symmetry are spontaneously broken to $\mathcal{N} = 1$ and a product gauge group, respectively. The masses of the supermultiplets appearing on the $\mathcal{N} = 1$ vacua are given. We also discuss the relation to the matrix model.

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The supercurrent algebra plays a key role in the partial breaking of global extended supersymmetries

$$\left\{ \tilde{Q}_a^I, J^m_{\alpha\dot{\alpha}}(x) \right\} = 2(\sigma^n)_{\alpha\dot{\alpha}} \delta^I_J T^n_m(x) + (\sigma^m)_{\alpha\dot{\alpha}} C^{IJ}_n(x)$$  (1)

where $J^m_{\alpha\dot{\alpha}}$ are extended supercurrents, $T^n_m$ is the stress-energy tensor and $C^{IJ}_n$ is a field independent constant matrix, which is permitted by the constraint for the Jacobi identities [1]. The new term does not modify the supersymmetry algebra on the fields.

Besides active researches on the non-linear realization of extended supersymmetry in the partially broken phase, Antoniadis, Partouche and Taylor (APT) [2] (see also [3]) gave a model in which linearly realized $\mathcal{N} = 2$ supersymmetry is partially broken to $\mathcal{N} = 1$ spontaneously. APT model is $\mathcal{N} = 2$ supersymmetric, self-interacting $U(1)$ model composed of one (or several) abelian constrained $\mathcal{N} = 2$ vector multiplet(s) with electric & magnetic Fayet-Iliopoulos (FI) terms. In [4], we have generalized this model to the $U(N)$ gauge model and shown that the $\mathcal{N} = 2$ supersymmetry is partially broken to $\mathcal{N} = 1$ spontaneously. Further in [5], we have analyzed the vacua with broken gauge symmetry and revealed the $\mathcal{N} = 1$ supermultiplets on the vacua. The relation to the matrix model is discussed in [6]. In addition, a manifestly $\mathcal{N} = 2$ supersymmetric formulation of the model coupled with/without $\mathcal{N} = 2$ hypermultiplets was given in [5] by using unconstrained $\mathcal{N} = 2$ superfields on harmonic superspace [10]. These models contain the prepotential $F$ as a prime ingredient. So, our model should be regarded as

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1 Talks given by HI at 14th International Conference on Supersymmetry and the Unification of Fundamental Interactions (SUSY06), Irvine, California, USA, 12-17 June 2006, and given by KF at MSJ-IHES Joint Workshop on Noncommutativity, Bures-sur-Yvette, France, 15-18 November 2006.

2 This series of works [4, 5, 6] is based on $\mathcal{N} = 1$ superspace and construction of the most general $\mathcal{N} = 2$ Lagrangian based on special Kähler geometry [7] which was developed after tensor calculus [8].
a low-energy effective action for systems given by $\mathcal{N} = 2$ bare actions spontaneously broken to $\mathcal{N} = 1$. We introduce $\mathcal{N} = 1$ superfields: chiral $\Phi(x^m, \theta) = \Phi^a t_a \supset (A, \psi, F)$ and vector $V(x^m, \theta) = V^a t_a \supset (v_m, \lambda, D)$, where $t_a (a = 0, \ldots, N^2 - 1)$ generate $u(N)$ algebra, $[t_a, t_b] = if_{ab}^c t_c$ and $t_0$ generates the overall $u(1)$. The model is composed of the Kähler term

$$\mathcal{L}_K = \int d^2 \theta d^2 \bar{\theta} \mathcal{K}(\Phi^a, \Phi^* a), \quad \mathcal{K} = \frac{i}{2} (\Phi^a \bar{\mathcal{F}}_a^* - \Phi^* a \mathcal{F}_a),$$

where $\mathcal{F}_a = \frac{\partial \mathcal{K}}{\partial \eta^a}$, the $U(N)$ gauging counterterm $\mathcal{L}_\Gamma$, the $U(N)$ gauge kinetic term

$$\mathcal{L}_{\mathcal{W}} = -\frac{i}{4} \int d^2 \theta \mathcal{F}_{ab} \mathcal{W}^a \mathcal{W}^b + c.c. \quad \mathcal{W}_a = -\frac{1}{4} \bar{D} D e - \mathcal{V} = \mathcal{W}_a^a t_a,$$

a gauge invariant superpotential term and the FI D-term ($e$ and $m$ are real constants)

$$\mathcal{L}_W = \int d^2 \theta \ W(\Phi) + c.c., \quad \mathcal{L}_D = \xi \int d^2 \theta d^2 \bar{\theta} \mathcal{V}^0 = \sqrt{2} \xi D^0, \quad W = eA^0 + m \mathcal{F}_0.$$

In [4], it is shown that the action is invariant under $\mathfrak{R}$-action which is composed of a discrete element of the $SU(2)$ R-symmetry and a sign flip of the FI parameter

$$R : \lambda_j^0 \rightarrow \epsilon^{ij} \lambda_j^i \quad \& \quad R \xi : \xi \rightarrow -\xi,$$

where $\lambda_j^0 = (\lambda_j^a)$, so that $S^{(+\xi)} \rightarrow S^{- (\xi)} \rightarrow S^{(+\xi)}$. We have made the sign of the FI parameter manifest. This ensures the $\mathcal{N} = 2$ supersymmetry of our action. In fact, acting $\mathfrak{R}$ on the first supersymmetry transformation $\delta_{\eta_1} S^{(+\xi)} = 0$, we have, $\delta_{\eta_1} S^{- (\xi)} = 0 \rightarrow R(\delta_{\eta_1}) S^{(+\xi)} = 0 \rightarrow \mathfrak{R}(\delta_{\eta_1}) S^{(+\xi)} = 0$, which implies that the resulting $\mathfrak{R}$-invariant action is invariant under the second supersymmetry $\delta_{\eta_2} \equiv \mathfrak{R}(\delta_{\eta_1})$ as well. By applying the $\mathfrak{R}$-action on the first supersymmetry transformation, we obtain the $\mathcal{N} = 2$ supersymmetry transformation of the fermion as

$$\delta \lambda_j^a = i (\tau \cdot \bar{D}^a) F_{jK} \eta_K + \cdots, \quad \bar{D}^a = -\sqrt{2} g^{ab} \frac{\partial}{\partial \eta^b} (\mathcal{V} A^0 + \mathcal{M} \mathcal{F}_0^a)$$

where $g_{ab} = \text{Im} \mathcal{F}_{ab}$ and $\tau$ are Pauli matrices. The rigid $SU(2)$ has been fixed by making $\mathcal{E}$ and $\mathcal{M}$ point to specific directions, $\mathcal{E} = (0, -e, \xi)$ and $\mathcal{M} = (0, -m, 0)$. Under the symplectic transformation, $\Omega = \left( \begin{array}{cc} A_0^0 & \Lambda \end{array} \right) \rightarrow \Lambda \Omega$, $\Lambda \in Sp(2, \mathbb{R})$, $\left( \begin{array}{c} -e \\ \xi \end{array} \right)$ changes to $\Lambda^{-1} \left( \begin{array}{c} -e \\ \xi \end{array} \right)$. So the electric and magnetic charges are interchanged $(\mathcal{E}, \mathcal{M}) \equiv (\mathcal{M}, -\mathcal{E})$ when $\Lambda = \left( \begin{array}{cc} 0 & -1 \\ 1 & 0 \end{array} \right)$. This explains the name of the electric and magnetic FI terms.

The vacua are specified by the scalar potential given by

$$\mathcal{V} = \frac{1}{8} g_{ab} \mathcal{F}^a \mathcal{F}^b + \frac{1}{2} g_{ab} \bar{D}^a \cdot \bar{D}^b, \quad \mathcal{F}^a \mathcal{F}_b = g^{ab} \mathcal{F}_b = -if_{bc}^a A^{+b} A^c.$$ Coordinates $z^\ell = \mathcal{F}_\ell^a t_a$ or $z^\ell = \mathcal{F}_\ell^a t_a$

$$\langle (\mathcal{F}_\ell) \rangle = -2 \left( \frac{e}{m} \pm \frac{\xi}{m} \right).$$
The vev of $\mathcal{V}$ is given by $\langle \langle \mathcal{V} \rangle \rangle = 2 |m| \xi$. On the other hand, by applying supersymmetry transformation twice on the $U(1)_R$ charge conservation law \([10]\), we can read off the constant matrix $C_I^J$ in (10) as $C_I^J = +4m^2 \xi^2 (\tau_1)^J$. Thus half of supercurrents annihilates the vacuum.

Let $\langle \langle A \rangle \rangle$ be $\langle \langle A \rangle \rangle = \text{diag}(\lambda^{(1)}, \ldots, \lambda^{(1)}, \lambda^{(2)}, \ldots, \lambda^{(2)}, \ldots)$ with $\sum_i N_i = N$ where $\lambda^{(k)}$ are complex roots of (8), then $U(N)$ is broken to $\prod_i U(N_i)$. Unbroken $\Pi_i U(N_i)$ is generated by $t_\alpha \in \{ t_\alpha | t_\alpha, \langle \langle A \rangle \rangle = 0 \}$, while broken ones by $t_\mu \in \{ t_\mu | t_\mu, \langle \langle A \rangle \rangle \neq 0 \}$. The $\mathcal{N} = 2$ supersymmetry is partially broken on the vacua since

$$\langle \langle \frac{1}{\sqrt{2}} \delta (\lambda^L \mp \psi^L) \rangle \rangle = \pm im \sqrt{\frac{2}{N}} (\eta_1 \mp \eta_2), \quad \langle \langle \frac{1}{\sqrt{2}} \delta (\lambda^L \mp \psi^L) \rangle \rangle = 0, \quad \langle \langle \delta \lambda^L \rangle \rangle = 0.$$  

The mass spectrum is summarized as (\(d_u \equiv \dim \prod_i U(N_i)\))

| field | mass | label | # of polarization states |
|-------|------|-------|-------------------------|
| $v_m^\alpha$, $\frac{1}{\sqrt{2}} (\lambda^L \mp \psi^L)$ | 0 | A | 2$d_u$ |
| $A^\alpha$, $\frac{1}{\sqrt{2}} (\lambda^L \mp \psi^L)$ | $|m| \langle g^{\alpha \alpha} \mathcal{F}_{0 \alpha \alpha} \rangle$ | B | 2$d_u$ |
| $v_m^\alpha + (TA^\alpha)_2$, $\lambda^L_I$ | $\frac{1}{\sqrt{2}} |m| \mathcal{F}_{0 \alpha \alpha}^\alpha$ | C | $4(N^2 - d_u)$ |

$\frac{1}{\sqrt{2}} (\lambda^0 \mp \psi^0)$ is massless and thus is the Nambu-Goldstone (NG) fermion for the partial breaking of $\mathcal{N} = 2$ supersymmetry to $\mathcal{N} = 1$. Due to the $\mathcal{N} = 1$ supersymmetry on the vacua, the modes in (10) form $\mathcal{N} = 1$ multiplets: (A) massless $\mathcal{N} = 1$ vector multiples of spin$(0, \frac{1}{2})$, (B) massive $\mathcal{N} = 1$ chiral multiplets of spin$(0, \frac{1}{2})$ and (C) two massive $\mathcal{N} = 1$ vector multiplets of spin$(0, \frac{1}{2}, 1)$.

In [4], it was shown that the fermionic shift symmetry arises in a certain limit of our model, due to second supersymmetry, which is spontaneously broken. More precisely in [6], we study the decoupling limit of the NG fermion and the fermionic shift symmetry in $\mathcal{N} = 1$ $U(N)$ gauge model. The fermionic shift symmetry plays a key role in the proof of the Dijkgraaf-Vafa conjecture which asserts that non-perturbative quantities in $\mathcal{N} = 1$ supersymmetric gauge theory can be computed by a matrix model. The $\mathcal{N} = 1$ action was obtained by “softly" breaking of $\mathcal{N} = 2$ supersymmetry by adding the tree-level superpotential $\int d^2 \theta \text{Tr} W(\Phi)$. Thanks to the fermionic shift symmetry, effective superpotential is written as $W_{\text{eff}} = \int d^2 \chi \mathcal{F}_p$ with some function $\mathcal{F}_p$, which is related to the free energy of the matrix model [11].

We consider the $\mathcal{N} = 1$ $U(N)$ action realized on the vacua, in which the discrete $SU(2)$ R-symmetry is broken by the superpotential. How can we realize the situation in which the NG fermion is decoupled while the superpotential remains non-trivial? The answer is to consider the large limit of the electric and magnetic FI parameters $(e, m, \xi)$. Parametrizing

$$(e, m, \xi) = (\Lambda e', \Lambda m', \Lambda \xi'),$$  

$\mathcal{F} = \text{tr} \left( c_0 1 + c_1 \Phi + \frac{c_2}{2} \Phi^2 \right) + \frac{1}{\Lambda} \sum_{\ell = 3}^n \frac{c_{\ell}}{\ell!} \Phi^\ell \quad (11)$$

and taking the limit $\Lambda \to \infty$, we derived the general $\mathcal{N} = 1$ action discussed in [11], in which the NG fermion is decoupled while partial breaking of $\mathcal{N} = 2$ supersymmetry
is realized as before. It shows that the fermionic shift symmetry is due to the free NG fermion.

In [8], we provide a manifestly $\mathcal{N} = 2$ supersymmetric formulation of the $\mathcal{N} = 2$ $U(N)$ gauge model with/without $\mathcal{N} = 2$ hypermultiplets. It is convenient to use $\mathcal{N} = 2$ superfields in harmonic superspace: vector multiplet $V^{++a}(x^i, \theta^+, \bar{\theta}^+) \equiv (A^a, \nu_i^a, \lambda^{\alpha i}, D^{aIJ})$ and hypermultiplet $q^{-a}$ in the adjoint representation. The action for the gauge field and the $U(N)$ gauged matter is

\[ S_{V+q} = -\frac{i}{4} \int d^4x \left[ (D)^4 \mathcal{F}(W) - (\bar{D})^4 \mathcal{F}(\bar{W}) \right] - \int dud \zeta^{(-4)}_{a} \bar{q}^{a} D^{++} q^{a} \tag{12} \]

where $W$ is the curvature of $V^{++}$ and $D^{++} q^{a} = D^{++} q^{a} + iV^{++c} f_{cb}^{a} q^{b}$. The electric FI term, $S_e = \int dud \zeta^{(-4)}_{a} \text{tr}(\bar{\zeta}^{++} V^{++}) + c.c.$, shifts the dual auxiliary field $D^{aIJ}_{\bar{b}}$ in $W^a_{\bar{b}} \equiv \mathcal{F}_{a}$ by an imaginary constant. So we introduce the magnetic FI term $S_m$ so as to shift the auxiliary field $D^{aIJ}$ as $D^{aIJ} \rightarrow D^{aIJ} = D^{aIJ} + 4i\xi^{IJ} \delta^a_{\bar{a}}$. By this, the $\mathcal{N} = 2$ supersymmetry transformation law changes to $\delta_{\eta} \lambda^{aI} = (D^{aI})_{\eta} \eta^{I} + \cdots$, under which the total action $S = S_V + S^{\text{gauged}}_q + S_e + S_m$ is invariant. We find that on the Coulomb phase the $\mathcal{N} = 2$ supersymmetry is partially broken to $\mathcal{N} = 1$ spontaneously, and (8) is reproduced by fixing $SU(2)$ appropriately. A generalization to the case with $\mathcal{N} = 2$ local supersymmetry is discussed in [13].

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