Study of parametric instability in gravitational wave detectors with silicon test masses

Jue Zhang, Chunnong Zhao, Li Ju and David Blair

School of Physics, The University of Western Australia, Crawley, WA 6009, Australia
E-mail: zhangjue.astro@gmail.com

Received 9 September 2016, revised 20 December 2016
Accepted for publication 10 January 2017
Published 10 February 2017

Abstract
Parametric instability is an intrinsic risk in high power laser interferometer gravitational wave detectors, in which the optical cavity modes interact with the acoustic modes of the mirrors, leading to exponential growth of the acoustic vibration. In this paper, we investigate the potential parametric instability for a proposed next generation gravitational wave detector, the LIGO Voyager blue design, with cooled silicon test masses of size 45 cm in diameter and 55 cm in thickness. It is shown that there would be about two unstable modes per test mass at an arm cavity power of 3 MW, with the highest parametric gain of $\sim 76$. While this is less than the predicted number of unstable modes for Advanced LIGO ($\sim 40$ modes with max gain of $\sim 32$ at the designed operating power of 830 kW), the importance of developing suitable instability suppression schemes is emphasized.

Keywords: gravitational waves, parametric instability, Advanced LIGO

(Some figures may appear in colour only in the online journal)

1. Introduction

Gravitational waves, predicted by Einstein a century ago, have finally been directly detected [1, 2]. These first detections enabled us to have a first glimpse of two black holes merging, opening a new window to the universe and providing strong reasons for improving detector sensitivity. At their design sensitivity, the kilometer scale interferometer detectors such as Advanced LIGO [3] and Virgo [4] are expected to be able to detect $\sim 10^3$ gravitational wave events per year [5]. This will be a critical resource for understanding the origin and distribution of stellar mass black holes in the universe.

To reach high sensitivity, advanced laser interferometer detectors must use very high laser power inside the optical cavities to reduce quantum noise. Braginsky et al [6] pointed out in
the early 2000s that with extremely high laser power in the interferometer cavity, radiation pressure coupling between the acoustic modes and cavity optical modes can cause 3-mode parametric instability (PI), resulting in the exponential growth of many acoustic mode amplitudes and disruption of the operation of the interferometer. The first detailed analysis of parametric instability of Advanced LIGO [7] using a single cavity model predicted that there would be seven unstable acoustic modes per fused silica test mass, with parametric gain $R$ up to 7, at circulating power of 800 kW. Thus a total of 28 unstable modes in four test masses were predicted. This compares with recent observations in Advanced LIGO of $\sim 10$ unstable modes, with the maximum parametric gain of the order of 10, operating at $\sim 25\%$ of designed power [8]. Zhao et al also discussed the use of small variations of the mirror radius of curvature (RoC) to tune the system to a local minimum of parametric gain. Thermal tuning was suggested and subsequently studied [9]. In the Advanced LIGO detectors, this method was successfully used to suppress instability of some acoustic modes, enabling the laser power to be increased to that used in the first detections. A more complicated simulation [10] took into consideration the recycling cavities to give deeper insight into coupled cavity effects on parametric instability. It also emphasised that any unstable modes present in the single cavity will always be potential threats in a full interferometer. Further work identified many methods of suppressing PI, such as electrostatic feedback, optical feedback and the use of passive dampers [11–18], as well as the thermal tuning method. Unfortunately, the conditions that enable high sensitivity for the detectors appear to always coincide with the conditions that enable PI. To date, no method has been demonstrated that can completely eliminate parametric instability. As advanced LIGO power is increased, it is anticipated that a combination of thermal tuning, electrostatic feedback and passive dampers will be able to eliminate all instabilities.

To increase the sensitivity of detectors beyond that achievable with the current Advanced LIGO detectors, effort is underway to design the next generation of detectors. Proposed designs include the Einstein telescope (ET) [19], several LIGO upgrades designs such as LIGO A+ [20], a 40 km interferometer [21] and an 8 km interferometer [22]. All future designs required the use of high laser power to overcome quantum shot noise for high frequency sensitivity in addition to injection of squeezed vacuum. It is apparent that each of these designs will need to consider the PI problem. In 2012, Strigin studied the PI problem of the 10 km Fabry–Parrot cavity in the ET design with sapphire or silicon test masses [23]. His result suggested that there would be $\sim 10$ unstable modes in such systems.

Parametric instability arises through the coincidental matching of test mass acoustic mode frequencies and mode shapes with optical cavity transverse modes for which the frequency difference from the main TEM$_{00}$ pump laser mode is in the 1–100kHz range. Detailed predictions must take into account the details of the test mass shapes and optical cavity design. However, in general, the risk of modal coincidences depends on the optical mode density and the acoustic mode density. The optical mode density and the mode gap depends on the cavity free spectral range, determined by the cavity length, and the inverse cosine of the cavity g-factor. The acoustic mode density depends on test mass sizes and the sound velocity of the material. The particular design analyzed here replaces the 40kg fused silica test masses of Advanced LIGO with $\sim 200$kg cooled silicon test masses. Silicon was chosen as the candidate for the mirror substrate due to its low mechanical loss, which reduces the thermal noise according to the fluctuation dissipation theorem [24–26]. Another advantage of silicon is the fact that its thermal expansion coefficient falls to zero at 123 K. If operated at this temperature there is a small reduction of thermal noise, and more importantly, a large advantage in that thermal stress associated with residual absorbed light power falls to zero. This greatly suppresses the problem of thermal lensing or wavefront distortions in the test masses.
Silicon also has higher sound velocity than fused silica. Higher sound velocity of the test mass material means higher mechanical mode frequency. In general, this leads to less acoustic modes at the frequencies where parametric interaction occurs, thereby reducing the risk of PI. However, the increased mass leads to lower acoustic modes frequencies and acts to increase the risk of PI.

This paper investigates this issue in detail for the specific design called the LIGO Voyager blue design. While the results are specific to this design, they give an indication of the problems that could be faced by many future 3rd generation gravitational wave detectors. For this analysis we use a single cavity model for PI modeling because of its general agreement with observations. Any more complex models depend on details of the power and signal recycling cavities, and do not change the general pattern of instability. Hence this model provides a good indication of the level of the PI.

2. Parametric instability

The three-mode parametric interaction involves two optical modes (the carrier fundamental mode TEM\(_{00}\) and a higher order optical mode TEM\(_{mn}\)) and an acoustic mode. If the transverse modes and acoustic modes have a similar spatial distribution and appropriate frequency relations, three mode interactions can be strong. The parametric gain \(R\) is used to describe the three-mode interaction in the cavity, which can be described as follows in the case of a single optical cavity [6, 27]:

\[
R = \frac{4PQ_nQ_m}{MC\omega_m^2} \left( \sum_{i}^{\text{TEM}} \frac{\Lambda_a}{1 + \frac{\omega_i^2}{\delta^2}} - \sum_{j}^{\text{TEM}} \frac{\Lambda_a}{1 + \frac{\omega_j^2}{\delta^2}} \right),
\]

where \(P\) is the circulating power in the cavity arm, \(Q_n\) and \(Q_m\) are the quality factors of the acoustic and the optical modes (which specify their respective modal linewidths), \(M\) is the mass of the test mass, \(c\) is the speed of light, \(L\) is the length of the cavity, \(\omega_m\) is the frequency of acoustic mode, \(\delta\) is the half line-width of the optical mode, and \(\Lambda\) is the overlapping factor discussed below. The frequency detuning parameter is given by

\[
\Delta\omega_{sa} = |\omega_{00} - \omega_{mn}|_{30} - \omega_{mn},
\]

with \(\omega_{00}\) and \(\omega_{mn}\) being the cavity carrier frequency and the cavity higher order transverse mode frequency, respectively. Subscripts \(s\) and \(a\) indicate Stokes and anti-Stokes processes, which describe either excitation or damping of the acoustic modes. The overlapping factor is given as [6]

\[
\Lambda = \frac{V(\int E^{00}(\vec{r})E^{\text{hom}}(\vec{r})\mu_0(\vec{r})d\vec{r}_\perp)^2}{\int |\mu_0(\vec{r})|^2 d\vec{r}_\perp \int E^{00}(\vec{r})^2 d\vec{r}_\perp \int E^{\text{hom}}(\vec{r})^2 d\vec{r}_\perp},
\]

where \(V\) is the volume of the test mass. Mathematically, the spatial match between the carrier mode \(E^{00}\), transverse mode \(E^{\text{hom}}\), and the acoustic mode \(\mu\) is defined by the integral \(\int E^{00}(\vec{r})E^{\text{hom}}(\vec{r})\mu_0(\vec{r})d\vec{r}_\perp\), where \(\mu_0\) is the displacement vector of the acoustic mode normal to the test mass surface. Both \(\mu\) and \(\mu_0\) are generated by finite element analysis. The larger the value of the integral, the greater is the overlap between the three modes. The integral \(\int |\mu_0(\vec{r})|^2 d\vec{r}_\perp\) represents the effective volume of the acoustic mode.

From equation (1), it can be seen that the parametric gain \(R\) depends on the overlap \(\Lambda_{sa}\) between an acoustic mode and the optical cavity modes, as well as the frequency match.
between the acoustic mode frequency and the cavity mode frequency $\Delta \omega_{\text{ac}}$. If the overlap is large, when $\Delta \omega < \delta$, where $\delta$ is the line-width of the transverse mode, strong interaction will occur. Figure 1 shows a schematic diagram of the mode structure of a cavity, up to the 10th optical mode order. When (a) the frequency of an acoustic mode is close to the frequency difference between a higher order mode and a fundamental mode, shown as the distance between the vertical lines, and (b) the mode shapes match, then the opto-acoustic interaction can be large.

In the Stokes process ($\omega_{mn} \sim \omega_{00} - \omega_{\text{trans}}$, $\omega_{mn} > \omega_{00}$), optical power is transferred to an acoustic mode. This corresponds to a parametric gain $R > 1$. When $0 < R < 1$, the acoustic mode amplitude will be amplified by a factor $\sim (1 - R)^{-1}$ but the system will remain stable. If the transferred optical power exceeds the energy loss rate due to acoustic dissipation ($R > 1$), the mode amplitude will grow exponentially. In the anti-Stokes process ($\omega_{\text{trans}} > \omega_{00}$), the parametric gain $R < 0$. The acoustic mode transfers energy to the optical mode, and the acoustic mode is damped. Stokes and anti-Stokes processes both occur in optical cavities, but due to the asymmetrical mode structure, the Stokes and anti-Stokes processes do not cancel [6]. Moreover, multiple optical modes can interact with one acoustic mode simultaneously [27]. This is reflected by the summation in equation (1). These issues will be addressed in detail below for a particular interferometer design.

3. Parametric instability for LIGO Voyager design

3.1. LIGO Voyager design parameters

The Voyager conceptual design was proposed as a major upgrade of Advanced LIGO. It was first proposed in 2012 and subsequent studies led to three ‘straw-man’ Voyager designs, known as Red, Green and Blue. Here the Blue design is used for our simulation. The parameters used for calculating the parametric gain for the Voyager Blue design are listed in table 1.

The Voyager design was proposed as a dual recycling interferometer similar to Advanced LIGO [28]. As stated above, our analysis in this paper is for a single arm cavity. According to Gras [10], any unstable mode revealed in the single cavity is a potential threat that can cause instability in the more complex dual recycling interferometer. Although simplified, the study for single cavity is a good indicator of the scale of the instability problem.

3.2. Overlap

We used the finite element analysis software COMSOL to simulate the acoustic modes of the test mass with the properties shown in table 1. We considered a total of 2194 acoustic modes from $\sim 5$ kHz up to $\sim 74$ kHz. Compared with an aLIGO test mass, the mode density is of comparable magnitude.

The frequencies and shapes of the high order optical transverse modes are calculated using the eigenvalue method [29, 30].

Figure 2 shows an example of the spatial match between the transverse optical mode, and the acoustic mode at 6197 Hz on the surface of the test mass. It is clear that they correspond to a large overlap in our calculation.

In our analysis, we considered the worst case scenario by rotating the displacement of each acoustic mode every $\frac{\pi}{18}$ from 0 to $\pi$ to locate the best match orientation with the optical modes.
to obtain the maximal overlapping parameter $\Lambda$. The overlap of each acoustic mode was calculated for all 21 optical modes up to 10th order.

3.3. The $Q$-factors

For the Voyager blue design, it is proposed [31] to have black coating on the test mass barrel for radiative cooling. It is clear that the lossy black coating on the barrel will reduce the $Q$-factor of acoustic modes and increase the thermal noise [14], depending on their strain energy distribution on the barrel. We assume the acoustic mode $Q$-factors are reduced by an order of magnitude from the silicon intrinsic $Q$-factor ($Q_m$) of $10^8$ to $10^7$ due to the combined effects of both black coating on the barrel and the optical coating on the surfaces.
The strength of the parametric interaction is governed by the three-mode detuning parameter $\delta_\omega$ (equation (2)) and the ratio of $\delta_\omega / \delta$. The half line-width (also called the relaxation rate) of the transverse mode $\delta$ is defined as $\delta = \frac{\text{FSR}}{2F}$, where $F$ is the finesse of the cavity and FSR is the free spectral range. These are assumed to be the same as Advanced LIGO as indicated in table 1. Narrower linewidths represent smaller cavity loss and higher cavity Q-factor. For laser frequency $\omega$, the quality factor is defined as $Q = \frac{\omega}{\delta}$.

3.4. Parametric gain in Voyager Blue design

We calculated the parametric gain for acoustic modes up to 74kHz and optical modes up to 10th order. The results are plotted in figure 3. It can be seen that, at cavity power of 3 MW, there are 1161 modes with $R > 0$, of which two modes have $R > 1$, four modes with $R > 0.1$, and 121 modes with $R > 10^{-3}$. The maximum gain is 76.

It can also be seen from figure 3 that the frequency condition $\left(1 + \frac{\omega^2}{\delta^2}\right)^{-1}$ plays a big role in the parametric gain. Those acoustic modes with frequencies near the optical mode gaps between the TEM$_{00}$ and TEM$_{mn}$ modes tend to have higher parametric gain.

![Figure 2](image1.png)

**Figure 2.** Mode shapes of one of the acoustic mode and optical mode showing similarity in spatial distribution. (a) The acoustic mode shape of the 6197 Hz mode of the test mass. (b) The product of TEM$_{00}$ and TEM$_{01}$ optical modes.

![Figure 3](image2.png)

**Figure 3.** Parametric gain for different acoustic modes taking into account one acoustic mode interacting with up to 10th order optical modes, using parameters of table 1. The vertical lines represent the mode gap $\omega_{mn} - \omega_{mn0}$.
Thermal tuning of the RoC of the cavity mirrors for changing the mode gaps of the optical cavity ($\omega_{00} - \omega_{\text{mI}}$) was proposed [7] to suppress parametric instability. However, if the acoustic mode density is high, tuning the cavity mode gaps away from one acoustic mode frequency to suppress the instability may result in increasing the parametric gain for other acoustic modes.

3.5. Effect on parametric gain by changing RoC of test masses

For cryogenic detectors, thermal tuning of the RoC of the mirrors is not possible. However, from a previous study [10] for aLIGO, the number of unstable modes are smaller for some RoC-values for the cavity mirrors. Hence we investigate the effect of RoC on the parametric gain with Voyager Blue design.

Figures 4 and 5 show the maximum parametric gain and the number of unstable modes as a function of the RoC of the input test mass (ITM, also known as input mirror).

It can be seen that although there is no ROC-value for which all parametric gains are below unity, there are some ‘sweet regions’ where both the number of unstable modes and the maximum parametric gain are smaller. For example, at RoC $\approx 1790$ m, there would be less than three unstable modes with maximum R less than 4. Even with the uncertainties in finite element modeling for acoustic mode frequencies (usually a few hundred Hz to one kHz error...
from measured values) [32], figures 4 and 5 provide useful information for choosing favorable RoC-values in regards to PI. The analysis also shows that the maximum $R$ in figure 4 comes from the same acoustic mode in the middle part of the ROC range ($RoC \in [1790, 1815]$) and then changes to another acoustic mode. The dominating acoustic mode is one at 12 880 Hz, interacting with a second-order optical mode. If we could damp this acoustic mode, the maximum parametric gain would be reduced dramatically.

Figures 4 and 5 can be used in optimising the interferometer engineering design. From the optical mode gap point of view, RoC tuning is equivalent to tuning the cavity length. To operate the interferometer close to one of the ‘sweet regions’, the RoC needs to be specified within a few meters for the low finesse designs such as indicated in table 1. However, for a high finesse design considered below, cm-level errors in RoC would be significant. In this case tuning could be achieved by varying the interferometer length by up to 50 cm.
3.6. Influence on parametric gain by changing the finesse

For silicon test masses, the optical absorption of large samples is still uncertain. It may be possible to avoid high absorbed power in the input test mass and beamsplitter by using reduced circulating power in the power recycling cavity, compensated for by using higher arm cavity finesse to maintain the same arm cavity circulating power (i.e. 3 MW). Here we use a much higher finesse ($F = 3000$) than that given in table 1 to investigate the effect of parametric instability with high cavity finesse. The results are shown in figures 6–8.

It can be seen that for the particular RoCs in table 1, the number of unstable modes for the high finesse case is the same as for the lower finesse case, but with a smaller maximum gain $R = 21$. From figures 7 and 8, it is clear that due to the higher Q-factor and narrower linewidths of optical modes for the high finesse cavity, the worst parametric gain is much higher than that of the lower finesse case, but with fewer unstable modes. It can also be seen that for the high finesse case, there would be several windows without instability, such as those near RoC $\sim 1785$ m, RoC $\sim 1790$ m. Moreover, there are more windows with only one unstable mode, which will be easier to suppress.

4. Conclusion

The result presented here is the first step in analyzing parametric instability in laser interferometer gravitational wave detectors with silicon test masses. A single cavity model, used because of its proven ability to predict PI in Advanced LIGO, shows that interferometers with 200 kg silicon test masses are likely to experience parametric instability comparable, but somewhat reduced, to instabilities in Advanced LIGO. In practice the power and signal recycling cavities will change the parametric gain depending on their detailed design.

Assuming a Q-factor of $10^7$ for test mass acoustic modes, we estimated that there would be $\sim 2$ unstable modes per test mass with maximum parametric gain of 76 for the Voyager Blue design. Higher acoustic Q-values would increase this number. We note that this design includes a very ambitious optical power level of 3 MW in the arm cavities. Overall it is clear that the potential parametric instability risk is comparable to that of the advanced LIGO detectors. The choice of a high sound velocity test mass material is advantageous in regards to parametric instabilities because it acts to reduce the density of ultrasonic acoustic modes. This compensates for the increased mass and optical power. A significant disadvantage of the silicon design, which is cooled to achieve zero thermal expansion coefficient, is the inability

![Figure 8. Number of unstable modes as a function of the ITM RoC ($F = 3000$).](image)
to undertake thermal tuning. We pointed out that length variations could be used to tune out mirror radius of curvature errors, especially if a high finesse design was chosen. Alternatively, an effective damping scheme will need to be implemented, that suppresses the Q-factor of ultrasonic modes without degrading the thermal noise of the detector.

As designs are further developed, it will be necessary to model PI for the full recycling cavity designs as well as details of test mass structures such as flats on the barrel for suspensions, the effects of manufacturing tolerances, and the diffraction losses of high order modes.

Acknowledgments

The authors would like to thank the the LIGO Scientific Collaboration for all the benefits of our close collaborations, and Peter Fritschel and Matt Evans. We also thank Jiayi Qin, Xu Chen and Carl Blair for many useful discussions. This work was supported by the Australian Research Council and is part of the Australian Consortium for Interferometric Gravitational wave Astronomy.

References

[1] Abbott B P et al 2016 Observation of gravitational waves from a binary black hole merger Phys. Rev. Lett. 116 061102
[2] Abbott B P et al 2016 Gw151226: Observation of gravitational waves from a 22-solar-mass binary black hole coalescence Phys. Rev. Lett. 116 241103
[3] Aasi J et al 2015 Advanced LIGO Class. Quantum Grav. 32 074001
[4] Accadia T et al 2012 Virgo: a laser interferometer to detect gravitational waves J. Instrum. 7 P03012
[5] Abadie J et al 2010 Predictions for the rates of compact binary coalescences observable by ground-based gravitational-wave detectors Class. Quantum Grav. 27 173001
[6] Braginsky V B, Strigin S E and Vyatchanin S P 2001 Parametric oscillatory instability in Fabry–Perot interferometer Phys. Lett. A 287 331–8
[7] Zhao C, Ju L, Degallaix J, Gras S and Blair D G 2005 Parametric instabilities and their control in advanced interferometer gravitational-wave detectors Phys. Rev. Lett. 94 121102
[8] Evans M et al 2015 Observation of parametric instability in advanced LIGO Phys. Rev. Lett. 114 161102
[9] Susmithan S, Zhao C, Qi F, Ju L and Blair D 2012 Thermal tuning the optical cavity for 3 mode interaction studies using a Co2 laser J. Phys.: Conf. Ser. 363 012018
[10] Gras S, Zhao C, Blair D G and Ju L 2010 Parametric instabilities in advanced gravitational wave detectors Class. Quantum Grav. 27 205019
[11] Degallaix J, Zhao C, Ju L and Blair D 2007 Thermal tuning of optical cavities for parametric instability control J. Opt. Soc. Am. B 24 1336–43
[12] Ju L, Blair D G, Zhao C, Gras S, Zhang Z, Barriga P, Miao H, Fan Y and Merrill L 2008 Strategies for the control of parametric instability in advanced gravitational wave detectors Class. Quantum Grav. 26 015002
[13] Gras S, Blair D G and Ju L 2008 Test mass ring dampers with minimum thermal noise Phys. Lett. A 372 1348–56
[14] Gras S, Blair D G and Zhao C 2009 Suppression of parametric instabilities in future gravitational wave detectors using damping rings Class. Quantum Grav. 26 135012
[15] Fan Y-H, Merrill L, Zhao C-N, Ju L, Blair D, Slagmolen B, Hosken D, Brooks A, Veitch P and Munch J 2010 Testing the suppression of opto-acoustic parametric interactions using optical feedback control Class. Quantum Grav. 27 084028
[16] Miller J, Evans M, Barsotti L, Fritschel P, MacInnis M, Mittleman R, Shapiro B, Soto J and Torrie C 2011 Damping parametric instabilities in future gravitational wave detectors by means of electrostatic actuators Phys. Lett. A 375 788–94
Zhao C, Ju L, Fang Q, Blair C, Qin J, Blair D, Degallaix J and Yamamoto H 2015 Parametric instability in long optical cavities and suppression by dynamic transverse mode frequency modulation Phys. Rev. D 91 092001

Gras S, Fritschel P, Barsotti L and Evans M 2015 Resonant dampers for parametric instabilities in gravitational wave detectors Phys. Rev. D 92 082001

Punturo M et al 2010 The Einstein telescope: a third-generation gravitational wave observatory Class. Quantum Grav. 27 194002

LIGO Scientific Collaboration 2015 Instrument Science White Paper https://dcc.ligo.org/LIGO-T1400316/public

Dwyer S, Sigg D, Ballmer S W, Barsotti L, Mavalvala N and Evans M 2015 Gravitational wave detector with cosmological reach Phys. Rev. D 91 082001

Blair D et al 2015 The next detectors for gravitational wave astronomy Sci. China Phys. Mech. Astron. 58 1–34

Strigin S E 2012 The effect of parametric oscillatory instability in a Fabry–Perot cavity of the Einstein telescope Opt. Spectrosc. 112 373–6

Liu X, Queen D R, Metcalf T H, Karel J E and Hellman F 2014 Hydrogen-free amorphous silicon with no tunneling states Phys. Rev. Lett. 113 025503

Murray P G, Martin I W, Craig K, Hough J, Robie R, Rowan S, Abernathy M R, Pershing T and Penn S 2015 Ion-beam sputtered amorphous silicon films for cryogenic precision measurement systems Phys. Rev. D 92 062001

Pohl R O, Liu X and Thompson E J 2002 Low-temperature thermal conductivity and acoustic attenuation in amorphous solids Rev. Mod. Phys. 74 991–1013

Ju L, Gras S, Zhao C, Degallaix J and Blair D G 2006 Multiple modes contributions to parametric instabilities in advanced laser interferometer gravitational wave detectors Phys. Lett. A 354 360–5

Harry G M et al, LIGO Scientific Collaboration 2010 Advanced LIGO: the next generation of gravitational wave detectors Class. Quantum Grav. 27 084006

Gras S, Blair D G and Ju L 2010 Opto-acoustic interactions in gravitational wave detectors: Comparing flat-top beams with Gaussian beams Phys. Rev. D 81 042001

Siegman A E 1986 Lasers (Washington DC: OSA Publishing)

Brooks A et al 2015 LIGO Voyager Upgrade Conceptual Design https://dcc.ligo.org/DocDB/0112/T1400226/007/VoyagerConcept-v7.pdf

Strigin S E, Blair D G, Gras S and Vyatchanin S P 2008 Numerical calculations of elastic modes frequencies for parametric oscillatory instability in advanced ligo interferometer Phys. Lett. A 372 5727–31