Confinement Driven by Scalar Field in 4d Non Abelian Gauge Theories

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Abstract. We review some of the most recent work on confinement in 4d gauge theories with a massive scalar field (dilaton). Emphasis is put on the derivation of confining analytical solutions to the Coulomb problem versus dilaton effective couplings to gauge terms. It is shown that these effective theories can be relevant to model quark confinement and may shed some light on confinement mechanism. Moreover, the study of an interquark potential derived from Dick Model, in the heavy meson sector, proves that phenomenological investigation of this mechanism is more than justified and deserves more efforts.

INTRODUCTION

Full Understanding of the QCD vacuum structure and color confinement mechanism are still lacking. Despite enormous amount of work performed over more than thirty years, particularly in lattice simulations of QCD, direct derivation of confinement from first principles remain still elusive, and there is no totally convincing proposal about its generating mechanism. On the other hand, it is known that the vacuum topological structure of theories with dilaton fields is drastically changed compared to the non dilatonic ones [1]. Therefore much about confinement might be learned from such theories, particularly string inspired ones. Indeed the appearance of fundamental scalars with direct coupling to gauge curvature terms in string theories offers a challenge with attractive implications in four-dimensional gauge theories.\footnote{The dilaton is an hypothetical scalar particle predicted by string theory and Kaluza-Klein type theories. In string theory, its expectation value probes the strength of the gauge coupling [2].} Besides, color confinement can be signaled through the behavior of the interaction potential at large distances. In this context, it was suggested in [3] that an effective coupling of a massive dilaton to the 4-dimensional gauge fields may provide an interesting mechanism which accommodate both the Coulomb and confining phases. The derivation performed in [3, 4] suggest a new scenario to generate color confinement which may be considered as a challenge to the mechanism based on monopole condensation.

The outline of this contribution is as follows. In the next section, We describe the influence of the dilaton on a low energy gauge theory and look into the problem how dilatonic degrees of freedom modifies Coulomb potential and how transition to
a confining phase occurs. Then, we review several recent work by presenting the corresponding effective coupling functions used. We briefly comment on the analytic solutions of the field equations and their confinement features. Also, it seems to us more than justified to dedicate some efforts to phenomenological investigations. We summarize the results obtained from study of Dick interaction potential in the heavy quarkonium systems.

THE MODEL

The imprint of dilaton on a 4d effective nonabelian gauge theory is described by a Lagrangian density:

\[ L(\phi; A) = \frac{1}{4F(\phi)} G^a_{\mu\nu} G^{\mu\nu}_a + \frac{1}{2} \partial_\mu \phi \partial^\mu \phi \quad V(\phi) + J^\mu A^a_\mu \]  

(1)

where \( V(\phi) \) denotes the non perturbative dilaton potential and \( G^{\mu\nu} \) is the standard field strength tensor of the theory. \( F(\phi) \) is the coupling function depending on the dilaton field. Several forms of \( F(\phi) \) have been proposed in literature. The most popular one \( F(\phi) = e^{k\phi} \) occurred in string theory and Kaluza-Klein theories[2].

The problem of the Coulomb gauge theory augmented with dilatonic degrees of freedom in (1) is analyzed as follows:

First, we consider a point like static Coulomb source which is defined in the rest frame by the current:

\[ J^\mu_a = g \delta(\rho) C^a_\nu \phi_0^\mu = \rho_a \eta^\mu_0 \]  

(2)

where \( C^a_\nu \) is the expectation value of \( SU(N_c) \) generator.

The field equations emerging from the static configuration (2) are given by:

\[ D_\mu \, F^1(\phi) G^{\mu\nu} = J^\nu \]  

(3)

and

\[ \partial_\mu \partial^\mu \phi = \frac{\partial V(\phi)}{\partial \phi} \quad \frac{1}{4} \frac{\partial F^1(\phi)}{\partial \phi} G^a_{\mu\nu} G^{\mu\nu}_a \]  

(4)

At this stage, by setting \( G^{0i}_a = E^i \gamma_a = \nabla^i \Phi_a \), after some algebra, we derive the chromo-electric field:

\[ E_a = \frac{Q^{a}_{\text{eff}}(\rho)}{r^2} \]  

(5)

where the effective charge is defined by

\[ Q^{a}_{\text{eff}}(\rho) = g \frac{C_a}{4\pi} F(\phi(\rho)) \]
From Eq(5), we learn that it is the running of the effective charge that makes the potential stronger than the Coulomb potential. In other words, Coulomb spectrum is recovered if the effective charge did not run.

Thereby the interquark potential reads as \[4\],

\[
U(r) = 2\alpha_s Z F(\phi(r)) \frac{1}{r^2} dr
\]

(6)

with \(\alpha_s = \frac{g^2}{4\pi}\) and \(\alpha = \frac{\alpha_s}{8\pi N_c} \frac{1}{2N_c}\).

The formula in is remarkable since it provides a direct relation between the interquark potential and the coupling function \(F(\phi(r))\). Moreover, it shows that existence of a confining phase in the theory in (1) is subject to the following requirement,

\[
\lim_{r \to \infty} \frac{1}{r} F(\phi(r)) = finite
\]

(7)

The main objective is to solve the field equations of motion (3) and (4) and determine analytically \(\phi(r)\) and \(\Phi_a(r)\). For this, \(F(\phi)\) and \(V(\phi)\) have to be fixed. In the sequel the dilaton potential is set to \(V(\phi) = \frac{1}{2} m^2 \phi\). Below, we will briefly describe the main features of three recent models and present their solutions.

1. Dick Model

In this effective theory, Dick used the form:

\[
\frac{1}{r(\phi)} = \frac{\phi^2}{f^2 + \beta \phi^2}
\]

where \(f\) represents a coupling scale characterising the strength of the scalar-gluon coupling. \(\beta\) is a parameter. Then he found for the radial dependance of the dilaton field and the interquark potential (up to a color factor) \[3\]:

\[
\phi(r) = \frac{1}{r} \frac{k}{m} \left( \frac{k^2}{m} \frac{k}{2} \exp(2mr) \right)
\]

\[
V(r) = \left( \frac{\beta g^2}{4\pi r} \right) g f \left( \frac{N_c}{2} (\frac{1}{N_c} ln(2mr) + \frac{m}{k} \phi^2) \right)
\]

With the abbreviation:

\[
k^2 = \frac{\alpha_s f^2}{8\pi N_c} \frac{1}{N_c}
\]

Note that the potential \(V(r)\) comes with the required behavior: a first term which accomodates the Coulomb interaction at short distances, and a second term linearly rising in the asymptotic regime with a string tension \(^2\).

\(^2\) In the massless case, \(V(\phi) = 0\), solutions of the field equations reduced to:

\[
\frac{\phi^2}{2} + \frac{N_c}{N_c} \frac{1}{N_c} \phi^2
\]

\[
V(r) = \left( \frac{\beta g^2}{8\pi r N_c} \right) g f \left( \frac{N_c}{2} \frac{1}{N_c} \right) r
\]
\[ \sigma \ gmf \] which depends on the dilatonic degrees of freedom \( m, f \).

2. Cornwall-Soni Model

In this model, the glueballs are represented by a massive scalar field \( \phi \), and couple in a non minimal way to gluons, through \( \frac{1}{F(\phi)} = \frac{\phi}{f} \) \[ 6 \].

Analytical Solutions were found for \( r \to \infty \) \[ 7 \],

\[ \phi (r) = \frac{\hbar \alpha_s f (N_c - 1)}{16 \pi m^2 N_c} r^{\frac{1}{4}} \]

\[ V (r) = 3 g N_c \frac{\hbar g f^2 N_c m^2}{2 \pi} (N_c - 1) r^{\frac{1}{4}} \]

These formulas show that their model provides confinement of quarks detected through an interaction potential proportional to \( r^{1/3} \) at large distances and considered by the authors as non perturbative correction to the Coulomb potential.

3. Chabab-Sanhaji Model

The main aim in this work was to construct a low energy effective field theory from which some of the popular phenomenological potentials may emerge. For this, we used the following coupling function \( F(\phi) = 1 + \beta \frac{\phi^2}{f^2} \) \[ 8 \].

By substituting \( F(\phi) \) in the field equations, they were found too complicated to integrate analytically. However, as in Cornwall-Soni Model, since the focus is on the long range behavior of the dilaton field and on how it modifies the Coulomb phase, the analysis is restricted to the infrared region. Thus, the asymptotic solutions are found to be,

\[ \phi = \frac{\hbar f^2}{\beta} \frac{\beta}{f^2} \frac{2n \alpha_s}{m^2} \frac{1}{r^{\frac{1}{4}}} \]

and the chromo-electric potential:

\[ \Phi_a (r) = \frac{g C_a}{4 \pi} 2n \alpha_s \frac{m^2 f^2}{3n} \frac{n + 1}{r^{\frac{3n - 1}{n + 1}}} \]

We see that the occurrence of confinement depends on the parameter \( n \) and our effective theory can serve to model quark quark confinement when \( n \geq \frac{1}{3} \)\).

On the other hand, we attained the above mentioned objective: by selecting specific values of \( n \), we reproduced the following known interquark potentials

- \( n = 1 \) linear term of Cornwall potential.
• $n = 11 = 29$ ) Martin’s potential [13].
• $n = 3 = 5$ ) Song-Lin, or Motyka-Zalewski’ potential [14].
• $n = 5 = 9$ ) Turin potential [15].

These quark potentials, which gained credibility only through their confrontation to the hadron spectrum, are now supplied with a theoretical framework since they can be derived from a low energy effective theory.

**PHENOMENOLOGICAL ANALYSIS: RESULTS AND DISCUSSION**

The interquark potential resulting from Dick model is quit attractive and deserves phenomenological investigations. A first study has been performed in [26] in the heavy mesons sector. Therein, the semi-relativistic wave equation has been solved using Dick potential. This problem was addressed as in [11] where the shifted-$l$ expansion technique is used (SLET), $l$ is the angular momentum. This method provides a powerful analytic technique for determining the bound states of the semi-relativistic wave equation consisting of two quarks of masses $m_1$, $m_2$ and total binding meson energy $M$ in any spherically symmetric potential. It is rapidly converging and handles highly excited states which pose problems for variational methods [12]. Moreover, relativistic corrections are included in a consistent way.

Dick interquark potential reads,

$$V_D (r) = \frac{4}{3} \frac{\alpha_s}{r} + \frac{4}{3} \frac{g f}{N_c} \frac{N_c}{2} \ln \left[ \exp \left( \frac{2mr}{\lambda} \right) \right]$$

The SLET technique used to obtain results from the theory requires us to specify several inputs, namely, $m_c$, $m_b$, $m$, $f$ and $\alpha_s$. In our numerical analysis, we set the charm and bottom quark masses to the values $m_c = 1.89$ GeV and $m_b = 5.19$ GeV. For the QCD coupling constant, in contrast to the Lattice potentials which use the same effective coupling in the description of heavy quarkonium, we take into account the running of $\alpha_s$,

$$\alpha_s (\lambda) = \frac{\alpha_s (m_z)}{1 + \left(1 - \frac{1}{2} n_f \right) \alpha_s (m_c) \frac{2 \pi \gamma_n (m_c = \lambda)}{1}}$$

where the renormalization scale is fixed to $\lambda = 2\mu$, with $\mu$ is the reduced mass,

$$\mu = \frac{m_1 m_2}{m_1 + m_2}$$

Thus, combination of the leading order formula (9) and the world experimental value $\alpha_s (m_z) = 0.118$ yields,

$$\alpha_s (charmonium) = 0.31; \quad \alpha_s (bottomonium) = 0.20;$$
while \( \alpha_s = 0.22 \) for the \( b\bar{c} \) quarkonia. On the other hand, the interquark potential parameters \( m \) and \( f \) are treated as being free in our analysis and are obtained by fitting the spin-averaged \( c\bar{c} \) and \( b\bar{b} \) boundstates. An excellent fit with the available experimental data can be seen to emerge when the following values are assigned,

\[
m = 57 \text{ MeV} \quad g f \left( \frac{N_c}{2 N_c} \right) = 430 \text{ MeV}^2
\]  

\( \text{(12)} \)

### TABLE 1. Calculated mass spectra (in units of GeV) \( M_n' \) of \( c\bar{c} \) boundstates from Dick interquark potential \[26\]

| State, \( n' \) | \( M_n', \) SLET | \( M_n', \) Exp. | State, \( n' \) | \( M_n', \) SLET | \( M_n', \) Exp. |
|-----------------|------------------|-----------------|-----------------|------------------|-----------------|
| 1S              | 3.073            | 3.068           | 1P              | 3.546            | 3.525           |
| 2S              | 3.662            | 3.663           | 2P              | 3.871            | -               |
| 3S              | 4.027            | 4.028           | 1D              | 3.787            | 3.788           |

### TABLE 2. Calculated mass spectra (in units of GeV) \( M_n' \) of \( b\bar{b} \) from Dick interquark potential \[26\]

| State, \( n' \) | \( M_n', \) SLET | \( M_n', \) Exp. | State, \( n' \) | \( M_n', \) SLET | \( M_n', \) Exp. |
|-----------------|------------------|-----------------|-----------------|------------------|-----------------|
| 1S              | 9.450            | 9.446           | 1P              | 9.903            | 9.900           |
| 2S              | 10.014           | 10.013          | 2P              | 10.227           | 10.260          |
| 3S              | 10.299           | 10.348          | 1D              | 10.129           | -               |

Tables (1,2) list the results of the analysis for the spin-averaged energy levels of interest. In all cases, where comparison with experiment is possible, agreement is generally very good. Next step, to check the consistency of our predictions, we estimate the bound states energies of the \( b\bar{c} \) quarkonia. These states are expected to be produced at LHC and Tevatron. Moreover, they should provide an excellent test to discriminate between various techniques used to probe nonperturbative properties of hadrons. In table 3 we show our calculated spectrum. The estimate of the mass of the lowest pseudoscalar S-state of the \( B_c \) spectra is close to the experimental value reported by CDF collaboration \[16\]. As to the higher states masses, they compare favorably with other predictions based on QCD sum-rules \[17,18\] or potential models \[19\]-\[25\]. In conclusion, Dick interquark potential (08) is tested successfully to fit the spin-averaged quarkonium spectrum. In view of these results, it is quite encouraging to pursue phenomenological application of \( V_D (\rho) \) and other quark potentials emerging from such low effective gauge theory with dilaton.

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3 if we adopt the usual number 0.18 GeV\(^2\) for the string tension, the dilaton mass will be shifted to a value about 158 MeV.
TABLE 3. Calculated mass spectra (in units of GeV) $M_n$, of $b\bar{c}$ boundstates from Dick potential \cite{26}

| State, $n$ | $M_n$, SLET | $M_n$, Exp. | State, $n$ | $M_n$, SLET | $M_n$, Exp. |
|-----------|-------------|-------------|-----------|-------------|-------------|
| 1S        | 6.322       | 6.40 0.39 0.43 | 1P        | 6.767       | -           |
| 2S        | 6.876       | -           | 2P        | 7.072       | -           |
| 3S        | 7.181       | -           | 1D        | 6.994       | -           |

GENERAL CONCLUSION

In summary, We reviewed some of the most recent work on confinement in 4d non abelian gauge theories with a massive scalar field (dilaton) and effective coupling functions to gauge fields. Analytical solutions have been found with confinement feature in the asymptotic regime. Thus, These low energy effective theories can serve well to model quark confinement. Moreover, by using Dick interquark potential in the heavy quarkonium sector, we showed that phenomenological investigation of the confinement generating mechanism suggested by these models is more than justified. Indeed, the obtained results for charmonium and bottomonium fit well experimental data when the dilaton mass is given a value about 57 MeV. Also, for $B_c$ system, we found that the S-state energy level is close to the value reported by CDF collaboration, while those of excited states agree favorably with predictions of other theoretical works. On the other hand, This analysis allows a test to the physics beyond the standard model in relation to hadron spectroscopy. Indeed, as a by-product, the estimate of the dilaton mass lies in the range of values proposed in \cite{27,28}. This determination may shed some light on the search of the dilaton since, as suggested in \cite{29,30}, the possibility to identify this hypothetical particle to a fundamental scalar invisible to present day experiments should not be excluded.

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