Optimization of low-floor vehicle dynamic performance based on Kriging model and NSGA-II algorithm

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Abstract. Echoing the pressing need to improve optimization efficiency and change the method of vehicle dynamic performance optimization by varying single parameter, a new method based on the Kriging approximate model and NSGA-II genetic algorithm is proposed to obtain more accurate results and shorten the design cycle. Accordingly, taking the suspension parameters as the variables, the dynamic performance indexes as the responses to fit the Kriging approximate model, the dynamic model of the 70% low-floor vehicle has been established. Subsequently, multi-parameter, multi-objective optimization of vehicle dynamic performance is achieved by virtue of the approximate model. The results show that, compared with the original method, all indexes are improved except the vertical comfort and stability. In particular, the optimization efficiency is significantly enhanced, and the effectiveness and feasibility of applying the optimization method to vehicle dynamic performance by virtue of the Kriging model have been proved.

1. Introduction

Increasing transport congestion in urban areas due to excessive motor vehicle traffic can favor the development of passenger rail transport. The corresponding vehicle dynamics has been promoted quickly, and the vehicle-track coupling model has also been widely used. Due to the complexity of the large wheel rail system, the process of vehicle dynamics analysis is far beyond the scope of theoretical understanding. Therefore, computer simulation should be introduced. The vehicle dynamics model finally boils down to a nonlinear two order differential equation. The railway train is made up of multi-section vehicles, and the degree of freedom is about 500, so the speed of calculation has become most relevant [1]. Due to the advantages of reducing the ‘numerical noise’, improving the optimization efficiency and shortening the actual solving time by several orders of magnitude among others. Approximate optimization technique has growingly come under the spotlight in recent years. Currently there are some widely-used approximate models, such as the RSM model, RBK model, orthogonal polynomial model, and Kriging model etc.

Kriging model, also known as the space partial interpolation method, is the optimal unbiased estimator for the regional variables in the limited region based on the variation function. It is widely used in optimization design because of its superiority of fitting in high degree of nonlinear and multimodal issues [2]. Many groups have contributed to the application of Kriging model. For instance, J.Jakumeit [3] applied the Kriging model to the plates punch forming and optimized the pressure-pad-force; ShinkyuJeong [4] used the Kriging model coupled with genetic algorithm to optimize the designs of the two-dimensional and multi-element airfoil flaps, instead of the original complex CFD
analysis-solving process; Yue-hua Gao [5] introduced the Kriging model into the field of injection molding and carried out the optimization design of injection molding process; and Dawei Yin [6] established the model of low pressure compressor for turbofan engine based on the Kriging model, and drew upon it as an example of the characteristics of the engine.

The dynamic performance optimization of rolling stock is realized by parameter optimization. However, owing to the complexity of the vehicle system, as well as the highly nonlinear wheel rail and the suspension system which give rise to computational complexity, the optimization design is time consuming. Moreover, the occurrence of the optimal failure problem caused by the non-feasible solution of approximate linear programming is frequent. In this paper, the Kriging approximate model is introduced into the optimization process to solve these problems.

2. The Kriging model

The Kriging approximate model was first applied to mining and geographical statistics, and subsequently to aerospace, stamping forming, dynamic analysis and other fields. The mechanism of the Kriging approximate model is to estimate the value of some specific unknown points, by using the weighted average method according to a certain number of sample points, which have been measured in the limited regions of the unknown points. Among them, the purpose of the weights is to minimize variance, which makes the estimation of the unknown points the best linear unbiased estimation [7]. Specific methods are as follows: Let \( X^0=(x_1^0, \ldots, x_n^0) (i=1, \ldots, n) \) be the measured sample points of size \( n, d \) the dimension of the design vector, the corresponding response value is \( y(X^0) \), and the estimated value is \( f(X) \), which is defined as equation (1), then equation here.

\[
 f(X) = \sum_{i=1}^{N} \beta_i y(X_i) 
\]  

The relationship between the responses to the function to be fitted \( Y(x) \) and the independent variables can be expressed by equation (2), then equation here.

\[
 y(x) = \sum_{i=1}^{N} \beta_i y(X_i) + z(x) = f(x) + z(x) 
\]  

\( Z(x) \) is a random process function indicating the local deviation between the estimated value and the response value and boasting the following statistical characteristics, defined as equation (3), then equation here.

\[
 \begin{align*}
 E[z(X)] &= 0 \\
 Var[z(X)] &= \sigma^2 \\
 Cov[Z(X^{(i)}), Z(X^{(j)})] &= \sigma^2 R(X^{(i)}, X^{(j)}) 
\end{align*}
\]  

where in \( R(X^0, X^0) \) is the correlation function to the data points \( X^0, X^0 \), and Gaussian correlation function is used in this paper as equation (4), then equation here.

\[
 R(X^{(i)}, X^{(j)}) = \exp\left(-\sum_{k=1}^{d} \theta_k |x_k^{(i)} - x_k^{(j)}|^2 \right) 
\]  

In the above table \( \theta_k \) is the unknown correlation parameter employed to fit the model. Once the correlation function is selected and the best \( \theta_k \) is estimated, the Kriging model can be used to predict the response \( \hat{y}(X) \) as equation (5), then equation here.

\[
 \hat{y}(X) = \hat{\beta} + r^T(X)R^{-1}(Y - f \hat{\beta}) 
\]  

where \( \hat{y} \) is the vector of estimated response values at each sample point, \( f \) is the vector with values of the trend function evaluated at each sample point, \( \hat{\beta} \) is a constant, and \( r^T(X) \) is the vector of correlation values between the untried location \( x \) and the sample data points.
The constant $\hat{\beta}$ can be estimated by virtue of the equation (6), then equation here.

$$\hat{\beta} = (f^T R^{-1} f)^{-1} f^T R^{-1} Y$$

(6)

The estimate of the variance is equation (7), then equation here.

$$\sigma^2 = \frac{(Y - f \hat{\beta})^T R^{-1} (Y - f \hat{\beta})}{n}$$

(7)

The maximum likelihood estimate is obtained by maximizing the $\theta_k$ likelihood estimate obtained by equation (8), then equation here.

$$\frac{n \ln(\sigma^2) + \ln(R)}{2}$$

(8)

3. Establish the vehicle Kriging approximate model

In this paper, a 70% low-floor tram designed by a company is considered as the object of the research. And according to the multi-body dynamics analysis, the Kriging model is established in accordance with the vehicle indexes, including the primary and secondary stiffness and damping of three direction, and the comfort, stability, wheel rail lateral force, wheelset lateral force, derailment coefficient, and wheel load reduction.

3.1. Vehicle dynamics model

The unit form of the tram is M+T+M, which suggests ‘two motors one trail’, and sets up the top and bottom hinge joints between the vehicles to form a statically indeterminate structure. The two ends of the tram are supported by the motor bogies, while the intermediate vehicle is supported by the trailer bogie. The motor bogie adopts the structure of the external axle box with the bolster, which can adapt to the large angle between the motor bogie and the vehicle body. In comparison, the trailer bogie uses the structure of the external axle box without bolster. The primary suspension employs conical rubber spring due to its sound performance in noise reduction, while the secondary suspension uses hourglass rubber pad in order to improve the stability of the vehicle. The way of full-suspended hollow axle is applied in the driving device, which can effectively reduce the unsprung mass.

Based on multi-body dynamics theory, the vehicle dynamics model, which couples the lateral-vertical-longitudinal with 69 degrees of freedom, is established in SIMPACK. In the model, the wheel rail contact relationship is considered as nonlinear, which is the same condition as that of the vehicle suspension system. Furthermore, the wheel rail contact force is calculated by the simplified theory FASTSIM, which is proposed by Kalker. To begin with, the curve radius is set as 100m, the length of the transition curve is 20m, the super elevation is 0.075m, and the speed is 25km/h.

To evaluate the curve performance, indexes including wheel rail lateral force, wheelset lateral force, derailment coefficient, and rate of wheel load reduction are taken into account. In the meantime, comfort is evaluated by the spelling index. Stability is analyzed by a RMS-value over 100m track length and 10m step length to the lateral acceleration of the frame. In addition, the peak acceleration of RMS is taken into account in the evaluation, according to UIC518 and EN14363.

3.2. The optimal Latin hypercube sampling

A certain number of sample points are needed to fit the response function when the approximate model is established. The optimal Latin hypercube sampling is used in this paper, which is an improved form of random Latin hypercube[8]. Its mechanism is that in a $n$-dimensional space, the coordinate interval of each dimension $[x_i^{\text{min}}, x_i^{\text{max}}], k \in [1,n]$ is evenly divided into $m$ intervals, and each subinterval is referred to as $[x_i^{k}, x_i^{k+1}]$, $i \in [1,m]$. $m$ points are randomly selected, each horizontal plane of each factor has been ensured to be considered only for once, which is a Latin hypercube sampling with $m$ samples in a $n$-dimensional space. Optimal Latin hypercube sampling method improves the
uniformity of random Latin hypercube and makes the fitting of the factor and the response more accurate, and can be robust to the changes of the model.

From past design experience, the value of the primary and secondary suspension stiffness, a total of six directions, as well as the lateral damping are determined. In order to obtain reasonable optimal results, a wider range of sampling listed in table 1 is finally used.

| Sample upper limit | Primary suspension, x&y (kN/m) | Primary suspension, z (kN/m) | Secondary suspension, x&y (kN/m) | Secondary suspension, z (kN/m) | Damper (kNs/m) |
|--------------------|-------------------------------|-----------------------------|--------------------------------|--------------------------------|----------------|
| Sample lower limit | 1500                          | 300                         | 40                            | 400                            | 40             |

3.3. Fitting Kriging approximate model

References In accordance with established ideas and methods of the Kriging model, firstly, a certain number of sample points that meet the requirements by the optimal Latin hypercube sampling should be established, and then the corresponding sample points should be adopted in the vehicle dynamics model and be solved to obtain the corresponding response of the curve performance. Subsequently, the Kriging approximate model of sample points should be established according to the response values, and an error comparison should be drawn between the results of the approximate model and the dynamic model. If the error does not meet the requirements, the number of sample points has to be raised to improve the accuracy of the approximate model.

3.4. Error analysis

The established Kriging model serves to verify the accuracy by taking other sample points in order to ensure the effectiveness of the model. And the Kriging model can be used in the approximate analysis after confirmation. When it comes to the verification of the accuracy, the optimal Latin hypercube sampling is used to take 20 sample points within the sampling range and calculate the response of those 20 sample points respectively in SIMPACK and by the approximate model. In this paper, the average relative error as equation (9) and (10) is used to analyze the accuracy of the approximate model.

\[
err = \frac{1}{m} \sum_{i=1}^{m} \left| \frac{y(x) - \hat{y}(x)}{y(x) + \varepsilon} \right| 
\]

\[
\varepsilon = \begin{cases} 
0 & |y(x)| 
eq 0 \\
0.01 & |y(x)| = 0 
\end{cases}
\]

The results are shown in table 2. The data suggest that the approximate model is highly accurate, and the maximum error is 1.30%, which falls into the acceptable range and indicates that the model meets the requirements.

| Index | Stability | Lateral comfort | Vertical comfort | Rate of wheel load reduction | Wheel rail lateral force | Wheel axle lateral force | Derailment coefficient |
|-------|-----------|-----------------|------------------|------------------------------|-------------------------|-------------------------|------------------------|
| err   | 1.30%     | 0.55%           | 0.33%            | 1.30%                        | 0.2%                    | 0.46%                   | 0.25%                  |

4. Dynamic performance optimization

The trademark characteristic of urban rail transit is full of small radius curve. For railway vehicles, sound curve performance can decrease the force between wheel and rail so as to reduce wear, and
achieve the purposes of energy-saving, noise reduction and operating cost cutting. On the contrary, poor curve performance, lack of guidance, will produce great force between wheel and rail giving rise to more wear, and even derailment or overturning or other accidents. Hence, good dynamic performance is the common goal pursued in vehicle production and operation department. For the tram used in this paper, the primary and secondary suspension parameters were optimized by the traditional method in the design process. Now, based on the Kriging approximate model, a more reasonable parameter matching relationship is obtained by genetic algorithm.

4.1. Optimization method

The NSGA-Ⅱ, which is non-dominated sorting genetic algorithm with elite strategy and also the improved version of NSGA published in 1994, is introduced to optimize the curve passing performance. It draws upon a fast non-dominated sorting algorithm to reduce complexity of the calculation. Furthermore, it employs the elite strategy, expands the sampling space, and compares the arithmetic operators between the degrees of congestion. As a result, it overcomes the defects of manually specifying the shared parameters in the NSGA.

In the non-dominated sorting genetic algorithm with elite strategy, the degree of congestion calculation is significant in ensuring the diversity of the population. Its calculation steps are as follows:

- Each degree of congestion \( i_d \) is set as 0;
- For each objective, non-dominated sorting is conducted on the population, the degree of congestion of the two individuals on the boundary is infinite, which is \( o_d=\text{inf} \).
- Calculate the degree of congestion to other individuals as equation (11), then equation here.

\[
i_d = \sum_{j=1}^{n} \left( \left| f_j^{i+1} - f_j^{i-1} \right| \right)
\]

where in, \( i_d \) represents the degree of congestion of the \( i \) point, \( f_j^{i+1} \) represents the \( j \)-th objective function value of the \( i+1 \) point, and \( f_j^{i-1} \) represents the \( j \)-th objective function value of the \( i-1 \) point.

When it comes to multi-objective optimization, due to the contradiction between the objective functions, it is difficult to find the solution to make each objective function reach their optimal value. At present, the common solution would be the Pareto solution, which indicates the value of any objective function cannot be further improved, otherwise, it would worsen the value of other objective function. The Pareto optimal solutions are continuous and the number of the solutions is usually infinite, which constitutes the Pareto front. The final solution of multi-objective optimization is to choose the best compromised solution from all the Pareto optimal solutions.

4.2. Optimization results

The vehicle dynamic performance is directly affected by the matching relationship of the stiffness of the vehicle suspension system. In this paper, the Kriging approximate model is employed to optimize the vehicle curve performance based on the NSGA-Ⅱ algorithm, which considers the stiffness of primary and secondary with six directions in total and the damping of shock absorber as the design variables. Moreover, it takes indexes including the wheel rail lateral force, wheelset lateral force, derailment coefficient, wheel load reduction, stability index, lateral stability, and vertical stability as the response values. Table 3 shows the optimized suspension parameters, and table 4 shows the comparison between the calculated results and the original parameters. In the comparison of the results before and after optimization, it shows that the wheel rail lateral force is reduced by 767.4N, the wheelset lateral force is reduced by 581.5N, and all the performance indexes are improved except the vertical comfort and stability. Compared with the traditional optimization method of changing single variable, genetic algorithm has greatly increased the optimization speed For instance, in this case using a genetic algorithm takes only 18 hours instead of approximately 200 hours by optimizing in the original design process, which serves to significantly improve efficiency and shorten the design cycle. Thus, it indicates smaller human consumption, and the workload is greatly reduced.
| Table 3. Optimization results. |
|--------------------------------|
| | Stabi lity | Lateral comfort | Vertical comfort | Wheel rail lateral force (N) | Wheel axle lateral force (N) | Derailment coefficient | Rate of wheel load reduction |
| Optimized | 0.84 | 1.82 | 2.23 | 35214 | 17854 | 0.616 | 0.246 |
| Original | 0.78 | 2.25 | 2.22 | 35981.4 | 18435.5 | 0.624 | 0.257 |
| Difference | -0.06 | 0.43 | -0.01 | 767.4 | 581.5 | 0.008 | 0.011 |

| Table 4. Optimized suspension parameters. |
|------------------------------------------|
| | x stiffness (kN/m) | y stiffness (kN/m) | z stiffness (kN/m) | Damper (kNs/m) |
| Optimized | Primary | 3395.587 | 2376.82 | 457.50 | 13.11 |
| | Secondary | 72.28 | 47.77 | 613.55 | |
| Original | Primary | 3000 | 3000 | 500 | |
| | Secondary | 120 | 120 | 600 | 20 |

5. Conclusion

In this paper, a new 70% low-floor tram is selected as the study object. According to its structure, a multi-body dynamics model with 69 degrees of freedom has been established, and the Kriging approximate model is applied to optimize the vehicle dynamic performance based on the NSGA-II algorithm. It takes the primary and secondary suspension parameters with seven in total as the design variables, and takes the vehicle dynamic performance indexes as the response values.

The results show that, NSGA-II optimization algorithm serves to deliver good results, moreover, compared with the original design parameters, all the performance indicators are improved except the vertical comfort and the stability, so the optimization method can be used directly in the design of the vehicle suspension parameters. The method based on the Kriging model can increase the optimization efficiency without significant sacrifice of accuracy. Meanwhile, the optimization target has been remarkably improved, an indicator of its effectiveness and feasibility applied to the optimization of rolling stock dynamic performance.

6. Reference

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