Giant Planet Formation Models with a Self-consistent Treatment of the Heavy Elements

Claudio Valletta© and Ravit Helled©
Center for Theoretical Astrophysics & Cosmology, Institute for Computational Science, University of Zurich, Zurich, Switzerland

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Abstract
We present a new numerical framework to model the formation and evolution of giant planets. The code is based on the further development of the stellar evolution toolkit Modules for Experiments in Stellar Astrophysics. The model includes the dissolution of the accreted planetesimals/pebbles, which are assumed to be made of water ice, in the planetary gaseous envelope, and the effect of envelope enrichment on the planetary growth and internal structure is computed self-consistently. We apply our simulations to Jupiter and investigate the impact of different heavy-element and gas accretion rates on its formation history. We show that the assumed runaway gas accretion rate significantly affects the planetary radius and luminosity. It is confirmed that heavy-element enrichment leads to shorter formation timescales due to more efficient gas accretion. We find that with heavy-element enrichment Jupiter’s formation timescale is compatible with typical disks’ lifetimes even when assuming a low heavy-element accretion rate (oligarchic regime). Finally, we provide an approximation for the heavy-element profile in the innermost part of the planet, providing a link between the internal structure and the planetary growth history.

Unified Astronomy Thesaurus concepts: Extrasolar gas giants (509); Planet formation (1241); Exoplanet formation (492)

1. Introduction
Understanding the origin of giant planets is a key goal in planetary science. In particular, it is of great interest to link formation models with the planetary composition and internal structure. It is therefore required that giant formation models provide predictions on the expected planetary bulk composition and the distribution of the heavy elements in the deep interior.

In the core accretion model (Pollack et al. 1996), the leading scenario for giant planet formation (Helled et al. 2013), the growth of a giant planet begins with the formation of a heavy-element core. Once the core reaches about Mars’s mass, its gravity is strong enough to accrete hydrogen and helium (hereafter H–He) gas from the protoplanetary disk. The protoplanet then continues to grow by accreting both solids (heavy elements), in the form of planetesimals (0.1–100 km-sized objects), and H–He until the so-called crossover mass is reached (Lissauer et al. 2009; Ginzburg & Chiang 2019) and runaway gas accretion takes place.

An alternative scenario to core formation via planetesimal accretion is pebble accretion. Pebbles are small solid particles with typical sizes of 1–10 cm (Ormel & Klahr 2010; Lambrechts & Johansen 2012, 2014; Bitsch et al. 2015). Due to their small sizes, pebbles experience significant gas drag that slows them down. This leads to a flux of pebbles that can be accreted very efficiently by the growing planet. The pebble accretion rate is high and is estimated to be ∼10−3 M⊙/yr (Morbidelli et al. 2015). Pebble accretion is stopped by the perturbation induced by the planet on the disk. This occurs when the protoplanet reaches the so-called pebble isolation mass. At Jupiter’s location of 5.2 au, the pebble isolation mass is ∼20 M⊕ (Lambrechts & Johansen 2014).

Independently on the size of the accreted solids, a common assumption in giant planet formation models is that the heavy elements reach the planetary center (“core”). This, however, is a strong simplification because the accreted solids can undergo ablation and fragmentation, and therefore much of their mass (if not all) is deposited in the envelope. Indeed, as shown in previous studies (Pollack et al. 1986; Alibert 2017; Brouwers et al. 2018; Lozovsky et al. 2017) when the core mass is between 1 and 3 M⊕ solids are expected to dissolve in the gaseous envelope. Therefore, only for the naïve scenario in which all the accreted heavy elements are assumed to go to the center the heavy-element mass in the planet is comparable to the core’s mass. This is clearly not a realistic scenario as shown by various previous studies (Helled & Stevenson 2017; Venturini & Helled 2017; Brouwers & Ormel 2020; Bodenheimer et al. 2018; Brouwers et al. 2018).

Previous studies including heavy-element enrichment have mainly focused on analyzing the ablation of planetesimals by the protoplanetary envelope, inferring a maximum core mass of giant planets, or computing the planetary evolution with the presence of a heavy-element gradient. In fact, already in Pollack et al. (1986) where planetesimal accretion was considered, a maximum core mass between 1 and 3 M⊕ for 100 km-sized planetesimals was derived. Lozovsky et al. (2017) investigated the heavy-element distribution in protoplanet accounting for different solid surface densities and planetesimal sizes. A maximum core mass of 2–3 M⊕ was found, with the rest of the heavy elements having a gradual distribution in the planetary deep interior. Venturini et al. (2016) presented a calculation that included the heavy-element enrichment in the planetary growth. It was assumed that the protoplanet accretes planetesimals composed of rock and water and that the rocks sink to the center (i.e., joining the core) while the water is dissolved in the envelope. The water in the envelope was assumed to be instantaneously uniformly mixed, so the envelope composition is always homogeneous but enriched in water. It was shown that the planetary formation timescale can be significantly reduced when envelope enrichment is considered. The same computational method was used in Venturini & Helled (2017) where the aim was to predict the occurrence rate of mini-Neptunes, which was found to be larger if heavy-element enrichment is considered. Finally, Bodenheimer et al. (2018) analyzed the formation of the Kepler 36 system, modeling the dissolution and fragmentation of 100 km
rocky planetesimals. They included mass loss induced by stellar X-ray and ultraviolet radiation as well as planetary migration. This simulation could reproduce the mass and radius of Kepler 36-c. It was found that, when heavy-element enrichment is included, in the region of the envelope, which is highly enriched with heavy elements (outer core in their paper), the temperature increases significantly, while the density decreases. The inferred lower densities imply that only paper is highly enriched with heavy elements of Kepler 36-c. It was found that, when heavy-element migration. This simulation could reproduce the mass and radius.

The planet is assumed to be in hydrostatic equilibrium and spherically symmetric. The original EOS present in MESA is not applicable to describe a mixture of water and H—He. Therefore use the EOS module developed by Müller et al. (2020) to properly model a mixture of water and H—He in planetary conditions. Details on the implementation of the high-metallicity EOS in MESA can be found in Müller et al. (2020) and references therein. Although it is clear that the heavy elements could consist of a mixture of elements, for simplicity in this work the accreted planetesimals and pebbles are assumed to be composed of pure water (H2O). Large fractions of water are justified by the assumed formation locations of 5.2 and 10 au, which are clearly beyond the water ice line (e.g., Sasselov & Lecar 2000). Representing heavy elements with water is a common assumption in giant planet formation and evolution simulations (Vazan et al. 2013, 2015, 2016; Venturini et al. 2016; Alibert et al. 2018). This is because the EOS of water has been widely investigated and the available tables cover a large-enough range of pressures and temperatures expected in planetary interiors (More et al. 1988; French et al. 2009; Mazevet et al. 2019). Since in this study the inclusion of heavy elements in the model corresponds to the modification of the EOS, as long as the heavy elements are represented by materials that are significantly heavier than H—He the inferred results do not change significantly (Vazan et al. 2013; Müller et al. 2020). We hope to consider various heavier elements in future research.

We use the standard low temperature (Freedman et al. 2014) Rosseland tables for the gas opacities by using the options in the inlist_evolute file kappa_lowT_prefix = ‘-lowT_Freedman11’ kappa_file_prefix = ‘gs98’.

The heavy-element compact core is modeled using an inner boundary condition that sets the mass, radius, and luminosity at the base of the envelope. We use the tables from Chen & Rogers (2016) to determine the core’s radius at a given mass. The infalling planetesimals typically reach the core only at the very early stages of the formation process when the envelope’s mass is negligible (VH19; Brouwers & Ormel 2020).

To simulate giant planet formation with heavy-element enrichment, we implemented three main subroutines into MESA: (1) a subroutine that computes the heavy-element accretion rate; (2) a subroutine that computes the interaction of the accreted solids with the envelope, including mass ablation and fragmentation; and (3) a subroutine that controls the gas accretion rate.

The results of each of the three subroutines affect the others: the heavy-element accretion rate drives the accretion of gas, which in turn affects the ablation. On the other hand, the mass deposition of heavy elements determines the envelope’s composition, which affects both the gas and solid accretion rate. Finally, the total envelope’s mass (which depends on the
gas accretion) determines the feeding zone of the protoplanet, and therefore the heavy-element accretion rate. The subroutines are called at each timestep by using the other_wind subroutine in MESA.

### 2.1. Heavy-element Accretion Rate

We implemented different solid (heavy-element) accretion rates: (1) the planetesimal accretion rate from Pollack et al. (1996); (2) the oligarchic accretion rate based on the N-body simulations from Fortier et al. (2007); (3) the accretion rate of Shiraishi & Ida (2008), which corresponds to a scenario with the absence of phase 2; and (4) the pebble accretion rate of Lambrechts & Johansen (2014) where we include the 3D effects as shown by Morbidelli et al. (2015) (Equation 17).

Table 1 lists the various heavy-element accretion rates used in this work. Table 2 lists the different gas accretion rates we consider. The planet’s capture radius is determined using the prescription of Ikoma et al. (2008) at the beginning of each timestep.

When the value of the heavy-element accretion rate is determined, we accordingly update the inner boundary condition for the structure equations:

```plaintext
s%M_center = s%M_center + s % dt/secyermdotcore
```

```
call CoreRadius(),
```

```
s%L_center = 3/5 * G * s%M_center/s%R_center*mdotcore, where mdotcore indicates the fraction of the accreted heavy elements that are deposited onto the core.
```

In the CoreRadius() subroutine, we compute the core’s radius using the tables of Chen & Rogers (2016) as described above.

### 2.2. Gas Accretion Rate

At each timestep we determine the gas (i.e., H–He) accretion rate. There are two different regimes for the gas accretion rate. In the first, the outer planetary radius cannot be larger than either the planet’s Hill radius $R_H$ or the Bondi radius $R_B = GM_p/c_s^2$, where $c_s$ is the disk’s sound speed at the planet’s location, whichever is smaller (see Pollack et al. 1996 for details).

In this first stage, when the protoplanet mass is low ($\lesssim 30\ M_\oplus$) the gas accretion rate $\dot{M}_g$ is obtained at each timestep through the requirement that the computed planet’s radius $R_p$ equals

$$R_A = \sqrt{\frac{GM_p}{c_s^2 + 4GM_p/R_H}}. \tag{1}$$

MESA is a Lagrangian code where the independent coordinate is the mass. As a result, it is not possible to set the radius that is iteratively found at each timestep. This is done by adding different amounts of gas until we reach convergence and $R_p$ matches $R_A$ within a given precision. This is done in the extras_check_model routine. More details on the implementation of this routine are given in Appendix B.

When the planet’s mass is large enough so that the computed $M_p$ is greater than the amount of gas that the disk can supply ($\dot{M}_{cy,\max}$), runaway gas accretion begins (detached phase). In this later stage the gas accretion rate is given by prescriptions based on hydrodynamical simulations. We implemented three different prescriptions: (1) the results of Lissauer et al. (2009), (2) the analytical result of Ginzburg & Chiang (2019), and (3) Equation (16) of Tanigawa & Ikoma (2007), where we substitute in $\Sigma_{acc}$ Equation (10) of Kanagawa et al. (2017). Table 2 lists the different gas accretion rates we consider.

During this formation stage, the outer boundary conditions of the disk fixed_T_and_P are removed and we adopt the simple_photosphere MESA boundary condition. More details on the outer boundary conditions in this case can be found in Appendix C.

### 2.3. Planetesimal-envelope Interaction

We include planetesimal’s ablation and fragmentation in the calculation. The planetesimal mass that is deposited at each layer of the envelope is computed at each timestep following Equation (13) of VH19. The heavy elements in the model are

| Heavy-element Accretion rate | Reference | Size | Density | Disk’s Viscosity |
|-----------------------------|-----------|------|---------|-----------------|
| $M_2$1                      | Pollack et al. (1996), their Equation (1) | 100 km | $\sigma = 10 \text{ g cm}^{-2}$ | … |
| $M_2$2                      | Lambrechts & Johansen (2014), their Equation (31) | $\sim 1$ cm | $\Sigma = 50 \text{ g cm}^{-3}$ | $\alpha = 10^{-3}$ |
| $M_2$3                      | Lambrechts & Johansen (2014), their Equation (31) | $\sim 1$ cm | $\Sigma = 50 \text{ g cm}^{-3}$ | $\alpha = 10^{-4}$ |
| $M_2$4                      | Fortier et al. (2007), their Equation (10) | 100 km | $\sigma = 10 \text{ g cm}^{-2}$ | … |
| $M_2$5                      | Shiraishi & Ida (2008), their Equation (24) and 25 | 100 km | $\sigma = 10 \text{ g cm}^{-2}$ | … |

| Runaway Gas Accretion Rate | Reference | $\alpha$ |
|---------------------------|-----------|----------|
| $M_{He,He}$1              | Lissauer et al. (2009), their Equation (3) | $4 \times 10^{-3}$ |
| $M_{He,He}$2              | Lissauer et al. (2009), Equation (C5) | $4 \times 10^{-4}$ |
| $M_{He,He}$3              | Ginzburg & Chiang (2019), their Equation (18) | $4 \times 10^{-3}$ |
| $M_{He,He}$4              | Ginzburg & Chiang (2019), their Equation (18) | $4 \times 10^{-4}$ |
| $M_{He,He}$5              | Kanagawa et al. (2017), their Equation (10) | $4 \times 10^{-3}$ |
| $M_{He,He}$6              | Kanagawa et al. (2017), their Equation (10) | $4 \times 10^{-4}$ |

Note.

* In the case of $M_{He,He}$2, we fit the data derived by Lissauer et al. (2009) (blue line in Figure 3 of their paper) with a high order polynomial. The coefficients of the fit can be found in Equation (C5) of Appendix C.
represented by one substance, which is taken to be water, and therefore we cannot properly model different types of heavy elements in the envelope in a self-consistent manner (e.g., 50\% rock, 50\% water). As a result, for the sake of simplicity, in this work both planetesimals and pebbles are assumed to be composed of pure water. Clearly it is desirable to include mixtures of heavy elements (e.g., water+rock) in the EOS calculation, providing the possibility to include solids with various compositions and we hope to address this in future work.

The heavy elements are added to the envelope by modifying the composition of the layers. We calculate the amount of heavy elements deposited at each layer and increase the metallicity accordingly, \( Z(i) = Z(i) + \text{deposited mass} / \text{layer mass} \), where \( Z(i) \) indicates the metallicity of the shell \( i \). The increased metallicity of the layer implies a decrease in the amount of H–He, which is then added at the top of the envelope using the mass_change MESA variable. This could result in an artificial inflation of the radius and a decrease in the gas accretion rate. In Appendix B we show that our results are insensitive to the metallicity of the outermost layer.

The change in the envelope’s composition has two main effects:

1. A change in the EOS. The fraction of water determines the EOS at a given layer. This in turn changes the density, temperature, and pressure profile in the envelope.
2. A change in the opacity. The presence of a heavy element affects the gas opacity, which strongly affects the planetary growth (Ikoma et al. 2008; Mordasini 2014). The opacity changes due to direct and indirect effects. A different metallicity leads to a direct change in the gas opacity. In addition, a different amount of heavy elements changes the temperature and pressure in the envelope, which in turn affects the opacity.

Fragmentation is the main mechanism for the mass deposition of large planetesimal \((r \geq 1 \text{ km})\) (Pollack et al. 1996), and is assumed to occur when the pressure gradient of the surrounding gas across the planetesimal exceeds the material strength (Equation (1) of VH19). We do not include Rayleigh–Taylor instabilities (Korycansky et al. 2000; Mordasini et al. 2006) to determine the fragmentation of the planetesimal. In Appendix D we investigate the sensitivity of the results when changing the fragmentation strength and the latent heat of vaporization of the high-Z material.

We include heavy-element settling following Iaroslavitz & Podolak (2007). In this calculation, the heavy-element mass that settles to the underlying layers is determined by comparing the pressure of the heavy elements present in the layer to the water vapor pressure (the pressure above which water condenses). The vapor pressure of water is given by \( P_{\text{vap}} = e^{-5640.34/T - 28367} \) (Rumble 2012), where the temperature is in Kelvin and the pressure is in dyne cm\(^{-2}\). If the partial pressure exceeds the vapor pressure, the heavy elements are assumed to settle to the layer below. Energy is also added in each layer due to the conversion of the planetesimal’s energy into heat. The amount of energy \( \Delta E_i \) released into a given atmospheric layer \( i \) is given by (Pollack et al. 1986)

\[
\Delta E_i = F_d d_i + \frac{1}{2} \Delta m_i v_p^2,
\]

where \( F_d \) is the drag force exerted on the planetesimal at layer \( i \), \( d_i \) is the width of the layer \( i \), \( \Delta m_i \) is the mass of the planetesimal ablated in shell \( i \), and \( v_p \) is the planetesimal’s local velocity. The first term on the right-hand side of Equation (2) represents the energy deposited in a layer due to gas drag that slows the planetesimal, while the second term represents the deposition of the kinetic energy of the ablated material. The energy provided by the planetesimals is included in MESA into the extra_energy subroutines. If a fraction of the planetesimal’s mass reaches the core, its energy is added onto the core’s luminosity as

\[
L_{\text{core}} = \frac{GM_{\text{core}}}{R_{\text{core}}} M_{\text{core}},
\]

where \( M_{\text{core}} \) is the total heavy-element mass that reaches the core.

2.4. Heavy-element Mixing

After the heavy elements are deposited in the envelope we investigate whether they mix and redistribute by convection. The heavy-element distribution is affected by the mixing mechanism, which is determined by the heat transport mechanism and can be radiation, conduction, or convection. To determine whether a region with composition gradients is stable against convection we use the standard Ledoux criterion \( \nabla_T < \nabla_{\text{ad}} + (\varphi/\delta) \nabla_{\mu} \), where \( \nabla_T = d \ln T / d \ln \rho \), \( \nabla_{\text{ad}} \), and \( \nabla_{\mu} = d \ln \mu / d \ln \rho \) are the temperature, adiabatic, and the mean molecular weight gradient, respectively, while \( \varphi = (\partial \ln \rho / \partial \ln \mu)_{T} \) and \( \delta = (\partial \ln \rho / \partial \ln T)_{\mu} \) are material derivatives. In a homogeneous region, \( \nabla_{\phi} = 0 \) and the Ledoux criterion reduces to the Schwarzschild criterion.

If the mean molecular weight increases toward the planetary center the composition gradient acts against convection. Due to planetesimal fragmentation it can happen during the planetary growth in which the mean molecular weight decreases toward the planetary center, and the composition gradient leads to convective mixing. The mixing length theory used in MESA requires the knowledge of a mixing length \( \ell_m = \alpha_{\text{mllt}} H_P \), where \( H_P \) is the pressure scale height and \( \alpha_{\text{mllt}} \) is a dimensionless parameter. For stars the value of \( \alpha_{\text{mllt}} \) is of the order of unity (Sonoi et al. 2019). The value of \( \alpha_{\text{mllt}} \) for planets is poorly constrained, but convincing arguments point toward values significantly lower than for stars (Leconte & Chabrier 2012). Following previous studies on Jupiter’s evolution with convective mixing (Vazan et al. 2015; Müller et al. 2020) we adopt \( \alpha_{\text{mllt}} = 0.1 \). Since the efficiency of mixing does not play a big role in our study the exact value of \( \alpha_{\text{mllt}} \) is of secondary importance. Nevertheless, we emphasize the importance of determining its value in the planetary regime.

2.5. Putting the Pieces Together

At time \( t \) the planet has a total heavy-element mass \( M_z \) and a total H–He mass \( M_{\text{xy}} \). The outer radius is equal to \( K_A \) given by Equation (1).

During a timestep \( dt \) of the planet’s growth we apply the following:

1. We compute the capture radius and the disk’s solid surface density. We then compute the solid accretion rate.
2. Using the heavy-element accretion rate, we compute the fraction that reaches the core and the fraction that is
dissolved in the envelope. The heavy-element mass is deposited (nonuniformly) in each layer following the description in Section 2.3.

3. The planet is growing in mass and is heavy-element dominated. This has two effects: on the one hand, it enlarges the accretion radius $R_A$ given by Equation (1), on the other hand, the planet’s radius shrinks due to the increased gravitational mass and mean molecular weight, where $R_p \leq R_A$.

4. We compute the gas accretion rate in an iterative way. We solve the structure equations at each iteration assuming different H–He masses (i.e., H–He accretion rate) until we reach radius convergence, where $R_p \sim R_A$.

### 3. Results

#### 3.1. Formation without Heavy-element Enrichment

For simplicity, in this section, we neglect the effect of heavy-element enrichment, and all the heavy elements are assumed to join the core. This assumption is relaxed in the following sections. We present Jupiter’s formation history using different heavy-element accretion rates as well as various gas accretion rates for the runaway gas accretion phase.

First, in order to check the validity of our numerical methods, we present in Figure 1 a comparison between our work and Figure 12 of Lissauer et al. (2009). We simulate the planetary formation assuming the solid accretion rate $M_f$ of Table 1 and the runaway gas accretion rate $M_{\text{H–He}}$ of Table 2. We call this run $Z_1\text{HHE}_2$ (see Table 3 for details), and in Figure 1 we compare it with a simulation of $3R_{\text{J}}$ of Lissauer et al. (2009).

As can be seen from the figure there is a very good agreement. The small differences between the two curves are probably caused by different assumed dust opacities.

Figure 1 shows clearly the three phases of giant planet formation. Initially, the heavy-element accretion rate is very high, and the protoplanet rapidly reach the planetesimal isolation mass (<1 Myr). When the planet has accreted almost all of the solid mass present in its feeding zone (end of phase 1), the heavy-element accretion rate decreases and the protoplanet starts to accrete gas. This is known as phase 2 or the attached phase and its duration determines the formation timescale. Finally, in phase 3, gas is accreted rapidly, the mass of the planet increases dramatically, and the planet opens a gap and detaches from the disk.

We next compare Jupiter’s growth assuming different accretion rates. The planet is assumed to be formed in situ, i.e., at 5.2 au. The results are summarized in Figure 2. The left panel shows the growth of the planetary mass as a function of time assuming different solid (heavy-element) accretion rates. The blue curve represents the solid accretion rate $M_f$ (Pollack et al. 1996). Initially, the planetesimals accretion rate is very high, and the isolation mass ($\sim0.1 M_{\oplus}$) is reached rapidly. Then, the solid accretion rate decreases and gas starts to be accreted efficiently. Finally, runaway gas accretion occurs and the planet detaches from the disk. The solid accretion rate $M_f$ (Fortier et al. 2007) is indicated by the black line. This accretion rate results in the longest formation timescale, which is substantially larger than typical lifetimes of protoplanetary disks (Mamajek 2009; Pfalzner et al. 2014; Ribas et al. 2015). Note that in the oligarchic growth scenario, the planet does not grow via the standard three phases. The oligarchic regime rate predicts a low accretion rate at the beginning of the simulation, which is slowly growing with time, and as a result, the feeding zone is never depleted (no phase 2). Therefore, in this scenario there is no isolation mass. The green line corresponds to the solid accretion rate $M_f$ (Shiraishi & Ida 2008). In this case, the gas accretion rate is already very high, after that the planet reaches $\sim10 M_{\oplus}$. Before the planet reaches $\sim10 M_{\oplus}$, we use the solid accretion rate of $M_f$. In this scenario, a giant planet is formed very rapidly, in a timescale shorter than a million years.

Finally, the yellow line corresponds to the pebble accretion rate $M_{\text{pf}}$ (Lambrechts & Johansen 2014). At Jupiter’s location the pebble isolation mass is $\sim20 M_{\oplus}$. In this scenario, the formation of a giant planet is much shorter than in the case of planetesimal accretion, as already indicated by previous studies (Lambrechts & Johansen 2014; Venturini & Helled 2017; Johansen & Lambrechts 2017).

The right panel of Figure 2 shows the planetary bulk metallicity as a function of the total planetary mass, which is defined as

$$Z_{\text{bulk}} = \frac{M_Z}{M_Z + M_{\text{H–He}}}.$$  \hspace{1cm} (4)

where $M_Z$ is the total heavy element (solid mass) and $M_{\text{H–He}}$ is the H–He mass. In the standard oligarchic planetary growth $M_f$, the planet’s metallicity decreases almost linearly with planetary mass. The prescription for $M_f$ (Shiraishi & Ida 2008) results in very efficient H–He accretion leading to a low bulk metallicity during all the stages of the planetary

### Table 3

| Run $Z_1\text{HHE}_2$ | $M_f$ | $M_{\text{H–He}}$ |
|------------------------|-------|-------------------|
| Run $3R_{\text{J}}$    |       |                   |
formation. For the case of pebble accretion with $\dot{M}_2$, it is found that gas accretion is inefficient and the bulk planetary metallicity remains very high. Finally, assuming the standard solid accretion rate of Pollack et al. (1996) ($\dot{M}_2$) leads to a bulk metallicity close to 1 (i.e., mostly heavy elements) until the feeding zone is depleted and gas is accreted. At this point, the planetary metallicity decreases rapidly.

In each case the gas accretion is computed via the requirement that the planet’s radius $R_p$ matches $R_A$ introduced in Equation (1). During this stage, the protoplanet accretes H–He gas due to cooling. At each timestep, the planet radiates energy, cools down, and therefore can accrete more gas. It is clear from Figure 2 that H–He accretion is driven by the heavy-element accretion rate. Different solid accretion rates result in different H–He accretion rates.

At some point the disk is unable to provide gas at a sufficient rate, and the computed $\dot{M}_{\text{gas}}$ exceeds the maximum amount of gas that the disk can supply. The gas accretion is then controlled by hydrodynamics rather than thermodynamics. During this phase the planet detaches from the disk. The outer boundary conditions of the disk fixed_T_and_F are removed and the simple_photosphere option of MESA is adopted. Therefore, the surface temperature, pressure, and density are changed and evolve with time. More details can be found in Appendix C.

We investigate the differences in the inferred planetary radius, luminosity, and mass when we use different H–He accretion rates (as listed in Table 2).

Figure 3 shows Jupiter’s growth when we use the different runaway gas accretion formulae assuming two different disk $\alpha$ viscosity parameters of $3 \times 10^{-3}$ and $3 \times 10^{-4}$, indicated by solid and dashed lines, respectively. For the purpose of comparison, in all the cases the initial mass is set to 0.5 $M_J$.

All the accretion rates are multiplied by a cap function equal to

$$f_{\text{cap}} = 1 - \frac{M_p}{M_f},$$

which has the role to terminate the planet’s growth smoothly at Jupiter’s mass.

The three different formulae predict that the gas accretion rate decreases with increasing viscosity. The formulation of Kanagawa et al. (2017) ($M_{\text{He}}$) and $M_{\text{He}}$) predicts the fastest planetary growth. The planet’s growth based on the runaway gas accretion rate $M_{\text{He}}$ (Lissauer et al. 2009) in excellent agreement with the $M_{\text{He}}$ case Kanagawa et al. (2017), but there is a clear difference when we compare $M_{\text{He}}$ to $M_{\text{He}}$. The analytical result of Ginzburg & Chiang (2019) (runs $M_{\text{He}}$ and $M_{\text{He}}$) leads to slow growth of the planet, and when coupled with a low value of viscosity, it results in a rather long formation timescale (>10 Myr). The middle panel of the figure shows the planet’s radius as a function of planetary mass for the different formulae. It is clear from the figure that the exact value of the planetary radius changes significantly, depending on the assumed accretion rate. The differences can be up to several Jupiter radii. The cases associated with faster planetary growth result in a larger radius. This happens because for these cases the growing planet has less time to cool down. Finally, the right panel presents the planetary luminosity as a function of mass. As with the radius, here there is also a substantial difference up to several orders of magnitude when the different prescriptions are used. In this phase, the planet’s luminosity is driven by the gas accretion and the planetary contraction, which cause a change in the gravitational energy. Therefore, different gas accretion formulae result in different luminosities. Determining the planetary luminosity of a young giant planet is of significant importance for determining the masses of planets detected by direct imaging, especially of young planetary candidates that are still accreting gas from their protoplanetary disk (Sallum et al. 2015; Guidi et al. 2018; Wagner et al. 2018). The
luminosity of such planets is a key observable; it provides hints about the thermodynamic state of the planet (Berardo & Cumming 2017) and it can be used to constrain planet formation theories (e.g., Marleau & Cumming 2013). This highlights the importance of accurately modeling of the terminal phase of nebular accretion, and we suggest that this topic should be investigated in more detail in future studies.

3.2. Planetesimal Accretion

In this section, we include heavy-element enrichment in the planetary envelope when modeling Jupiter’s formation. We use the numerical tools described in Section 2 to include the deposition of mass and energy of 100 km-sized planetesimals.

As already shown in VH19 the main mass deposition mechanism of a large planetesimal is fragmentation. The planetesimal’s velocity increases as it travels toward the planetary center until the pressure acting upon its surface is greater than the material strength, leading to fragmentation. The details of the fragmentation process and the distribution of the material are important (Hills & Goda 1993; Revelle 2005; Register et al. 2017; VH19). Different fragmentation models result in different distributions of the heavy elements which in turn also affect the inferred core mass and further growth.

Figure 4 shows the total planet’s mass versus time when using different fragmentation models (see VH19 for details). It can be seen that instantaneous, common bowl and halved models lead to a similar planetary growth history, while the pancake model results in a slower formation time. The yellow line corresponds to a case where all the heavy elements are assumed to join the core. In this case the envelope’s composition does not change and is set to 0.02. The different derived formation timescales when using the various fragmentation models is affected by the mass of the inner compact core. As shown analytically by Brouwers et al. (2020) the planetary growth timescale when heavy-element enrichment is considered depends on the heavy-element fraction that is deposited into the envelope relative to the amount that is added to the core. The smaller the inner core is, the faster is the growth of the planet. This relation can be seen clearly from Figure 7 of VH19. The instantaneous, common bowl, and halved fragmentation models lead to very similar (compact) core masses, while the pancake model results in a more massive core.

Figure 5 shows the effect of including heavy-element enrichment assuming the standard core accretion rate of Pollack et al. (1996).
We use the solid accretion rate $\dot{M}_s$, but investigate how the planetary growth depends on the assumed solid surface density. The assumed initial solid surface density $\sigma_{\text{init}}$ in the planet’s feeding zone is a key property that influences the formation timescale and growth history. The dependence of the planet’s growth on the initial solid surface density is consistent with the results of Pollack et al. (1996), where it was shown that the formation timescale goes as $\sim \sigma_{\text{init}}^{-3.5}$. We assume different solid surface densities ranging between 7 and 10 g cm$^{-2}$ as indicated by the different colors. For in situ formation of Jupiter the value of 10 g cm$^{-2}$ corresponds to 3 times the density of the minimum mass solar nebula model. The solid curves in Figure 5 correspond to the case where heavy-element enrichment is included, while the dashed lines represent cases where all the heavy elements are assumed to have joined the core. We hope to address this in future research.

3.3. Oligarchic Regime

In this section, we investigate the effect of heavy-element enrichment when assuming that the planet growth follows the oligarchic regime ($M_s < 4$). Figure 6 is similar to Figure 5 but using the heavy-element accretion rate derived by Fortier et al. (2007). We indicate a maximum disk’s lifetime by a red line, which is placed at 10 Myr. If all the solids join the core and heavy-element enrichment is neglected, the oligarchic regime is incompatible with formation timescales of a few million years (Fortier et al. 2007), as can be seen from the figure. For both assumed locations and for all the assumed solid surface densities, the dashed lines are beyond the red line, suggesting that the disk dissipates before the planet reaches runaway gas accretion. Furthermore, in this case the planetary growth is less sensitive to the initial solid surface density. As already seen in Figure 5 including heavy-element enrichment significantly shortens the formation timescale. When including heavy-element enrichment, even the low solid accretion rate of the oligarchic regime leads to a formation timescale that is compatible with the disk’s lifetime. In the left panel which corresponds to a formation location of 5.2 au, all the solid lines reach $40 M_{\oplus}$ within 3–4 Myr. We conclude that the oligarchic accretion rate, coupled with heavy-element enrichment, can realistically be applied for giant planet formation at Jupiter’s location. For a formation location of 10 au, including heavy-element enrichment leads to formation timescales between 7 and 10 Myr (see right panel). Overall, as expected, the planetary growth is faster at 5.2 au than at 10 au, where in the latter formation location the growth timescale is comparable to the disk’s lifetime. This difference in the formation timescale could explain the different masses of Jupiter and Saturn. We suggest that this topic should be investigated in more detail and we hope to address this in future research.

3.4. Pebble Accretion

In this section, we investigate the effect of heavy-element enrichment on Jupiter’s formation assuming that the planet is
growing via pebble accretion. We use the solid accretion rate $M_z$ but we perform a parameter study on the gas surface density. Figure 7 shows the planetary growth as a function of time at two different locations, 5.2 and 10 au, for various gas surface densities, with and without heavy-element enrichment.

As expected, the planetary formation with pebble accretion is faster and more efficient compared to the planetesimal case. There are no runs where the formation timescale exceeds the expected disk’s lifetime. When heavy-element enrichment is included, for a large set of parameters, the predicted timescale is less than 1 Myr, which is too fast and is in conflict with the occurrence rates of giant exoplanets (Venturini et al. 2016; Venturini & Helled 2017). However, as shown by various studies (Ormel et al. 2015; Alibert 2017) a significant fraction of the accreted pebbles can be recycled back onto the disk. This can lead to a slower growth of the planet, reducing the efficiency of giant planet formation in the pebble accretion scenario. It is therefore desirable to investigate heavy-element enrichment in the pebble accretion model where pebble recycling is included.

Pebble accretion is characterized by the so-called pebble isolation mass. This is the mass at which the perturbation of the planet on the disk causes the pebble accretion rate to stop. It is typically assumed (Lambrechts & Johansen 2014; Bitsch et al.

---

**Figure 6.** Oligarchic growth case: total planetary mass vs. time for different assumed values of the initial solid surface density $\sigma_{\text{init}}$ represented by different colors. The assumed solid accretion rate is taken from Fortier et al. (2007). The solid lines represent cases where the heavy-element enrichment is included, while the dashed ones correspond to cases where all the heavy elements are assumed to join the core. The right and left panels correspond to formation locations of 5.2 and 10 au, respectively. The red line represents the typical maximum disk lifetime, which is assumed to be 10 Myr.

**Figure 7.** Pebble accretion case: total planetary mass vs. time for growth via pebble accretion assuming different values of the gas disk’s density $\Sigma$ in the feeding zone represented by different colors. The solid lines represent cases where the heavy-element enrichment is included, while the dashed ones correspond to cases where all the heavy elements are assumed to join the core. The right and left panels correspond to formation locations of 5.2 and 10 au, respectively.
Figure 8. H–He mass fraction for different planetary locations and disk viscosity. Each column corresponds to a different formation location, while each line indicates a different disk viscosity. Solid lines indicate runs where heavy-element enrichment is included, while the dashed lines correspond to cases where solids are assumed to join the core. The red line indicates the assumption of Johansen & Lambrechts (2017) and Bitsch et al. (2015) where the H–He mass fraction is 10% of the total mass until pebble isolation mass is reached. All the plots end at the pebble isolation mass.

2015; Johansen & Lambrechts 2017) that the protoplanet begins to accretes substantial amounts of gas only after pebble isolation mass is reached, while before that it is assumed that the gaseous (H–He) mass is 10% of the total planetary mass.

Figure 8 shows the inferred H–He fraction for the different cases, including (solid line) and neglecting (dashed line) heavy-element enrichment. The different columns represent different formation locations, while different line styles indicate different disks’ $\alpha$ viscosity. The red line indicates a H–He mass fraction of 10% as assumed in the pebble accretion simulations. We find that the fraction of H–He is increasing as pebbles are accreted. Therefore, assuming that the H–He mass fraction is constant until pebble isolation mass is reached is an oversimplification. Nevertheless, in some cases the fraction of H–He is indeed around 10% when pebble isolation mass ($\sim 20\ M_\oplus$) is reached. We suggest that this approximation is appropriate for formation location of 5.2 au and for small values for the disk’s viscosity. However, at 10 and 20 au the pebble isolation mass is significantly higher, being 35 and 60 $M_\oplus$, respectively. In these cases we find that the growing protoplanet accretes large amounts of H–He of the order of several tens of percents. Again, when heavy-element enrichment is included gas can be accreted more efficiently.

4. The Connection between the Accretion Rates and the Planetary Internal Structure

The heavy-element distribution within the planetary envelope has an important impact on the thermal evolution and on the final planet’s structure (Vazan et al. 2015, 2018). In this section, we provide an approximation for the heavy-element distribution in the planet’s envelope at the end of phase 2, i.e., before runaway gas accretion begins. This approximation can be used to infer the heavy-element profile of the deep interiors of gas giants or the structure of intermediate-mass planets ($\sim 10–50\ M_\oplus$) that never reached runaway gas accretion, such as Uranus and Neptune. We show that the heavy-element profile of such planets mainly depends on the ratio between the heavy-element and gas accretion rates.

At the beginning of the planetary formation, the envelope’s mass is negligible and the heavy elements deposit their mass in the deep interior. Once the envelope’s mass becomes a significant fraction of the total mass, ablation becomes important and the heavy elements are deposited far from the planetary center. For simplicity, we assume that heavy elements are deposited in the outermost layer. This is a reasonable approximation in particular for the case of water (ice) planetesimals, which are easily fragmented or vaporized in the upper envelope, and therefore typically deposit their mass in the outer regions. Rocky planetesimals can penetrate deeper into the envelope and the approximation becomes less
mass coordinate

If the heavy elements remain where they are deposited (i.e., no convective mixing), then the metallicity $Z$ of a particular shell $j$ reflects the solid (heavy-element) and gas accretion rates at the time when that shell was the outermost layer. In this case the heavy-element profile as a function of the mass coordinate $z (m)$ can be approximated by (Helled & Stevenson 2017)

$$Z(m) \sim \frac{M_{Z,\text{env}}(M)}{M_{\chi}(M) + M_{Z,\text{env}}(M)}, \quad (6)$$

where $M_{Z,\text{env}}$ is the heavy-element mass dissolved in the envelope, which can be computed following the semi-analytical formula presented in VH19, and $M_{\chi}$ is the H–He mass. On the left-hand side of Equation (6) $m$ represents the mass coordinate in the profile, while on the right-hand side $M$ is the total planetary mass.

We test this approximation for the cases of both planetesimal and pebble accretion assuming different heavy-element accretion rates. The results are presented in Figures 9 and 10, respectively. The orange line corresponds to Equation (6), while the black one indicates the inferred heavy-element profile in the planet’s envelope. The calculation includes convection (using the Ledoux criterion) and the sinking of heavy-element as described in Section 2. Therefore, Equation (6) does not always perfectly reproduce the $Z$ profile, since convective mixing redistributes the heavy elements. However, the orange line follows qualitatively the behavior of the black one.

Figure 9 corresponds to the case of planetesimal accretion using $M_{Z,1}$. The solid surface density and the planet’s location are specified in the figure. As discussed above, large planetesimals deposit mass in the envelope mainly with fragmentation. Fragmentation creates jumps in the planet’s heavy-element profile due to the deposition of all the planetesimal’s mass in a small location in the envelope. As a result, there is a difference between the approximation and the actual profile of the planet, but still the orange line qualitatively follows the the black one for all the assumed locations solid surface densities. The blue dashed line represents the planetesimal isolation mass. For $M \lesssim M_{\text{iso}}$, the heavy-element fraction is high and $Z \sim 1$. Once the planetesimal isolation mass is reached, the heavy-element fraction decreases rapidly.

The case of pebble accretion ($M_{Z,2}$) is presented in Figure 10. The location of the planet and the gas surface density are specified in the figure. It can be seen that in all plots, there is a convective inner region in the envelope, where the red and black lines do not overlap. In the outer, radiative part, the agreement between the planetary profile and Equation (6) is excellent.

The approximation given by Equation (6) can be used to investigate the primordial heavy-element profile in the deep interior of giant planets or the overall profile of intermediate-mass gas-rich planets, ($\sim 10–50 \ M_{\oplus}$). It should be noted, however, that this calculation corresponds to the planetary primordial internal structure, which could change during the planetary long-term evolution due to due to convective mixing. Nevertheless, our calculation implies that the deep interiors of Uranus and Neptune shortly after their formation are expected to have composition gradients.

5. Summary and Conclusions

We developed a new numerical framework for giant planet formation using MESA. We include the dissolution of the accreted heavy elements self-consistently, where the high-Z material is included in the EOS and opacity calculations. Our formation model can be used to model the deposition of the high-Z material in the envelope of growing planets and to follow the planetary long-term evolution.

We investigated Jupiter’s growth history assuming different solid (heavy-element) and runaway gas accretion rates, different sizes of the accreted solids, and the fragmentation model. We find that the planetary growth strongly depends on the assumed solid and runaway gas accretion rates. Very different formation timescales are inferred when assuming different solid accretion rates. For example, when heavy-element enrichment is neglected, the oligarchic regime leads to formation timescales longer than 10 Myr (the maximum expected disk’s lifetime). This is not the case when assuming the solid accretion rate from Pollack et al. (1996).

In addition, the planetary luminosity shortly after formation depends on the assumed runaway gas accretion rate. Different runaway gas accretion rates can result in a difference of three order of magnitude in the planetary luminosity. This should be taken into account when determining the masses of giant planets detected by direct imaging, since a given planetary mass can have different luminosity depending on its growth history (runaway accretion). This is particularly relevant for young planetary candidates that are still accreting gas from their protoplanetary disk (Sallum et al. 2015; Guidi et al. 2018; Wagner et al. 2018). As a result, the terminal phase of gas accretion should be investigated in more detail.

In all the simulations we performed including heavy-element enrichment lead to shorter formation timescales. This has been already shown by different authors (e.g., Stevenson 1982; Venturini et al. 2016) and is a consequence of the higher mean molecular weight of the envelope, which results in a more efficient accretion of gas. For the case of pebble accretion, we find that the fraction of H–He in the planet can significantly exceed 10% even before pebble isolation mass is reached. We also show that when we assume the oligarchic accretion rate from Fortier et al. (2007) including heavy-element enrichment, the formation timescale is reduced to a few million years, a timescale that is consistent with the estimated lifetimes of protoplanetary disks.

Finally, we also show that the heavy-element profile within the planet’s deep interior (before runaway gas accretion) reflects its accretion history, i.e., the ratio between the heavy-element and gas accretion rates. This, however, assumes that no significant mixing has occurred during the planetary evolution. Indeed, recent structural models of Jupiter suggest that the planet has a fuzzy core (Wahl et al. 2017; Debras & Chabrier 2019). Such a structure could be a result of primordial composition gradients that do not mix during the planetary evolution (Vazan et al. 2018). However, it was recently shown by Müller et al. (2020) that composition gradients associated with the formation process are unlikely to retain their primordial shape. It was shown that Jupiter’s envelope after runaway gas accretion is hot enough for the majority of the envelope to become convective, despite the stabilizing
composition. Therefore, the origin of Jupiter’s extended diluted core is still unknown and should be investigated further.

Our study represents an additional step forward in giant planet studies, but clearly much more work is needed. In this work, the high-Z material was represented by water. Although the results are insensitive to the assumed material strength and density of the accreted planetesimals as shown in Appendix D, assuming a different material (e.g., rock) would affect the EOS and opacity calculations, and therefore the planetary growth. As a consequence, including the presence of other elements in the EOS and opacity calculations is desirable as it can impact the growth, internal structure, and long-term evolution of the planet (Vazan et al. 2013). Furthermore, a more realistic treatment for the dust opacity with a proper modeling of the grain microphysics is desirable. We hope to address these topics in future research. Future investigations should also include a hybrid pebble-planetesimal formation path (Alibert et al. 2018; Venturini & Helled 2020). Finally, using this new numerical framework, future studies could focus on the formation history, evolution, and internal structures of Saturn, Uranus, and Neptune, as well as gaseous-rich exoplanets.

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Appendix A
Comparison with Standard Assumptions

The stellar structure equations of mass conservation, hydrostatic balance, thermal gradients, and energy conservation...
are described by

\begin{align}
\frac{dm}{dr} &= 4\pi r^2 \rho \\
\frac{dP}{dr} &= -G \frac{m}{r^2} \rho \\
\frac{dT}{dr} &= \nabla_T \frac{dP}{P \, dr} \\
\frac{dL}{dr} &= 4\pi r^2 \rho \left( -T \frac{dS}{dt} \right)
\end{align}

where \( r \) is the radial coordinate, \( L \) is the luminosity, \( T, P, \rho \), and \( S \) are the temperature, pressure, density, and entropy, respectively. \( \nabla_T \) is the temperature gradient, which depends on the heat transport mechanism within the envelope. In many planetary formation/evolution codes (Piso & Youdin 2014; Piso et al. 2015; Venturini et al. 2016; Venturini & Helled 2017), Equation (A4), which describes the local conservation of energy, is replaced by a global energy balance. The set of time-dependent partial differential equations describing the evolution of the planet is transformed into a set of time independent ordinary differential equations, which is more easy to be solved numerically. However, this transformation has two main limitations. First, these codes cannot be used to properly model the planetary long-term evolution of the planet because the time is not present anymore in the equations. Second, Equation (A4) is no longer satisfied, and the planetary luminosity is assumed to be constant within the envelope.

Below we explore the limitations of this approach.

We compare our code, which solves all the structure equations with the work of Venturini et al. (2016), where the term A4 is neglected. For this comparison, we assume a constant solid accretion rate. The results are shown in Figure A1. As can be seen from the left panel there is a good agreement between the two codes regarding the growth of the planet. In the right panel of Figure A1 we perform the same

Figure 10. Growth via pebble accretion: comparison between the accretion ratio (Equation (6)) and the heavy-element profile for nine cases, each with a different location and initial solid surface density. Each line represents a different location in the disk of the protoplanet.
simulation with using an ideal-gas EOS and a constant opacity of 0.01 g cm\(^{-2}\) in order to ensure that the only difference between the two cases lies in the treatment of luminosity. The agreement between our simulations and this of Venturini et al. (2016) provides an additional benchmark to our code.

Finally, in Figure A2 we show the luminosity profile within the planet’s envelope at four different times. The dashed lines represent results from Venturini et al. (2016), while the solid lines indicate the luminosity profile derived with our code where the luminosity is not constant. It is clear from the figure that the constant luminosity approximation is inapplicable when the envelope’s mass reaches ~2–3 \(M_\oplus\). We therefore suggest that in order to properly model the formation and evolution of gaseous-rich planets assuming a constant luminosity should be avoided.

### Appendix B

**Attached Phase—Gas Accretion Rate**

When the protoplanet mass is low (\(\lesssim 30 \ M_\oplus\)) the gas accretion rate \(\dot{M}_g\) is obtained at each timestep through the requirement that the computed planet’s radius \(R_p\) equals the accretion radius \(R_A\) as shown in Equation (1). This is done iteratively, adding different amounts of gas until convergence is reached where and \(R_p\) equals \(R_A\) within a given precision. The gas accretion rate in this phase is determined using the extras_check_model subroutine. At the beginning of the timestep the heavy-element accretion rate is computed and the mass of the core is updated accordingly. At this point the radius is smaller than the one at the beginning of the timestep, we call it \(R_{\text{inter}}\), representing an intermediate radius. The first guess for the gas accretion rate is done is the following:

\[
\text{s\% mass_change = out_rho *4. / 3.*pi*( -R_inter**3+R_acc**3)/(MSun*s%dt/secyer),}
\]

where \(\text{out_rho}\) is the density of the outermost layer. At this point MESA solves the structure equations with the updated mass and composition of the envelope. We then check whether the radius matches (within the precision given) the accretion radius. If the two radii match, we proceed to the next timestep, otherwise the calculation is repeated assuming a different amount of gas, which depends on whether the radius is smaller or larger than the accretion radius. The extras_check_model subroutine is given by

\[
\text{if(abs(R_acc/R_inter -1) .ge.tolerance then}
\]

\[
\quad \text{if(R_acc/R_inter .ge. 0) then}
\]

\[
\quad \quad \text{s% mass_change = s% mass_change +2}
\]

\[
\quad \quad \text{extras_check_model = retry}
\]

\[
\quad \text{else}
\]

\[
\quad \quad \text{s% mass_change = s% mass_change /1.5}
\]

\[
\quad \quad \text{extras_check_model = retry}
\]

\[
\text{else}
\]

\[
\quad \quad \text{extras_check_model = keep_going.}
\]

The left panel of Figure B1 shows the planet’s radius and the accretion radius (Equation (1)) as a function of time. In this simulation, heavy-element enrichment is neglected and all the planetesimals are assumed to join the core. We use the solid accretion rate from Pollack et al. (1996), where the initial solid surface density is 10 g cm\(^{-2}\) and the protoplanet is located at 5.2 au. The tolerance for satisfying \(R_A = R_p\) is set to 5%. In the right panel of Figure B1 we show the ratio \(R_A/R_p\), i.e., the
accretion radius (Equation (1)) divided by the computed planet’s radius. The red lines indicate the limit of 5%. As can be seen from the figure, the ratio is always within the used tolerance.

In principle, a change of the metallicity of the outermost layer could result in an artificial inflation of the radius, which in turn can affect the gas accretion rate. In Figure B2 we show that imposing a fixed metallicity to the outermost layer does not significantly affect our result. The orange line represents the case where the metallicity of the outermost layer is fixed to $Z = 0.01$, while the blue line indicate the case where no fixed metallicity is imposed on the outermost layer. We therefore conclude that our results for the planetary growth are robust.

### Appendix C

#### Detached Phase—Outer Boundary Conditions

During the first stage of the core accretion model, the total planetary mass is small ($\lesssim 50 M_\oplus$) and it is attached to the disk. The outer envelope’s temperature and pressure are fixed (therefore also the density) and are equal to the disk’s temperature and pressure at the planet’s location. When the computed gas accretion rate exceeds the limit that can be supplied by the disk, the planet detaches from the disk, and the gas accretion rate is significantly higher and is obtained via hydrodynamic formulae. The outer boundary conditions used in this case is the simple photosphere MESA option. The surface temperature, pressure, and density are obtained by solving the stellar structure equations using the Eddington $T(\tau)$ relation. Therefore, in this phase, the planetary surface temperature, pressure, and density vary with time. In addition, their actual values depend on the runaway gas accretion formula that is used and also whether the accretion energy can be deposited or is radiated away (Berardo & Cumming 2017; Cumming et al. 2018).

We implemented three different prescriptions for the H–He accretion rate during runaway as listed in Table 2. We fit with high order polynomials the $\alpha = 4 \times 10^{-5}$ case of Lissauer et al. (2009). The fit is given by

$$\log \left( \frac{\dot{M}_g}{\Sigma \gamma F_R / P} \right) = \sum_{i=0}^{6} c_i \log^i \left( \frac{M_p}{M_\star} \right),$$

where the coefficients are: $C_0 = 259.26$, $C_1 = 584.32$, $C_2 = 478.51$, $C_3 = 195.81$, $C_4 = 43.25$, $C_5 = 4.94$, $C_6 = 0.229$.

The left panel of Figure C1 shows the planet’s surface temperature during the runaway accretion phase when assuming different formulae for the gas accretion rate as indicated by the different colors. The changes in the surface temperature are due to two reasons. On the one hand, the gravitational energy provided by the accreted gas increases the surface temperature. On the other hand, the cooling of planet result in a loss of energy and therefore a decrease in its surface temperature. Therefore, the actual planetary temperature depend on the efficiency of these two processes.

It is clear that the planet’s surface temperature does not remain constant also if the planet is not fully convective. This is
different from the hypothesis in Ginzburg & Chiang (2019), who assumed that the temperature changes from the initial value only when the planet’s envelope becomes fully convective. The middle and right panels of the plot show the pressure and density of the planet, respectively.

Appendix D
Dependence on Material Strength

In this section, we investigate the sensitivity of our results on the assumed material strength and latent heat of vaporization of the accreted planetesimals. We assume the material strengths and latent heat of vaporization for an icy (water) planetesimal to be equal to $10^9$ dyne cm$^{-2}$ and $2.2 \times 10^{10}$ erg g$^{-1}$, respectively. A rocky planetesimal has a higher material strength, which we set to $10^8$ dyne cm$^{-2}$ and latent heat of vaporization equal to $8 \times 10^{10}$ erg g$^{-1}$. However, in both cases the heavy elements are represented by water for the EOS calculation.

Figure D1 shows the planetary growth at two different planet’s location assuming the Pollack et al. (1996) solid accretion rate. The blue and brown lines correspond to the planetesimal’s composition of water and rock, respectively. The red lines represent the case where all the solids are assumed to reach the core. We find that the growth history is rather similar for the two assumed planetesimals’ compositions, where the planet’s growth is slightly faster assuming rocky planetesimals. As expected, neglecting heavy-element enrichment results in a larger formation timescale. Future studies
should perform a detailed comparison when the different assumed heavy elements are also considered for the EOS and opacity calculation and we hope to address this in future work.

**ORCID iDs**

Claudio Valletta © https://orcid.org/0000-0001-5172-7929

Ravit Helled © https://orcid.org/0000-0001-5555-2652

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