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Is Tether a safe haven of safe haven amid COVID-19? An assessment against Bitcoin and oil using improved measures of risk
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ABSTRACT

Bitcoin is a new speculative investment with extremely volatile movement, thus possibly failing to act as a safe haven for crude oil when the price of this energy commodity plummeted following the global outbreak of COVID-19. Meanwhile, Tether is designed to behave similarly to the US dollar with stable fluctuation. In this study, we assessed their safe-haven properties in terms of risk reduction opportunities by proposing an improved version of Value-at-Risk (VaR) and Expected Shortfall (ES). Using vine copula-based AR-GJR-GARCH models, we demonstrated that Bitcoin exhibited inconsistent risk reduction capability for oil, particularly before COVID-19. When adding Tether into a portfolio containing oil and Bitcoin, the risk reduction was achieved for any portfolio allocation and was more pronounced amid the COVID-19 period. This suggests that Tether consistently served strong support for Bitcoin to protect oil investors against extreme risk and received a significant impact from the COVID-19 outbreak. However, the consistent safe-haven functionality of Tether was not as good as that of the US dollar in most cases, and this implied the vanishing of its stability. These results were robust when considering another asymmetric volatility model and another dependence model. Furthermore, the proposed improved VaR and ES forecasts outperformed their corresponding unimproved version in quantifying portfolio risk and therefore provided a more accurate assessment of safe-haven roles.

1. Introduction

Since its introduction by Nakamoto (2008), Bitcoin has become a prominent cryptocurrency and has gained a great deal of attention from academicians, financial actors, and policy designers. Initially, it functioned as a digital currency and a fully decentralized payment system providing a more effective transfer with a peer-to-peer mechanism (Wang et al., 2020). Despite its growing popularity, its identity as a currency is still questionable. Baur et al. (2018) and Baur and Dimpfl (2021) argued that Bitcoin did not exhibit fiat currency’s essential functions, particularly as a unit of account and a medium of exchange. It was viewed as a speculative investment asset, but it still played a role as a store of value. This classification was implied by a rapid appreciation in its value and market capitalization that has appeared since 2013. A vast increase in its value was observed to be about nineteen times from the beginning of 2017 (around a thousand US dollars) to December 2017 (about 19 thousand US dollars). More recently, during the global outbreak of COVID-19, this value dramatically rose to more than 60 thousand US dollars, with a market capitalization above a trillion US dollars as of March 2021 (coinmarketcap.com). Nevertheless, several drastic increases were followed by a decline and thus formed a giant bubble, as investigated by Corbet et al. (2018).

The identity of Bitcoin as an investment asset has attracted investors and led us to understand its stylized facts and its relationship with other assets in the financial industry. Numerous studies (e.g., Katsiampa, 2017; Zhang et al., 2018; Jiménez et al., 2020) have provided evidence of conditional heteroscedasticity, volatility clustering, and leptokurticity in the Bitcoin return series that are typical characteristics of equity returns. On the other hand, Bouri et al. (2017a) and Baur and Dimpfl (2018) documented that a more considerable increase in its current volatility was generated by a positive return than a negative return of the same magnitude in the past, contrary to what we typically find in equities. This inverted asymmetric reaction of Bitcoin volatility to positive and negative shocks is potentially related to the safe-haven role of Bitcoin, similar to that of gold, against other assets. We can examine this role based on its negative or zero correlation with these assets during crises (Baur and Lucey, 2010). It has been documented that Bitcoin acted as a safe haven for investors in US and non-US equity markets over particular periods of extreme global uncertainty.
In particular, Bitcoin's potential roles for crude oil were also of interest in recent years. For instance, Selmi et al. (2018) examined these roles of Bitcoin, in comparison to those of gold, under various conditions in the Bitcoin and gold markets. Over the entire sample period they considered, the correlation between Bitcoin and oil appeared to be negatively stronger than the gold–oil correlation. In times of downward movements in the oil market, the ability of Bitcoin to serve as a safe-haven tool was found to be more pronounced than that of gold. Meanwhile, Guermu et al. (2019) concluded that adding Bitcoin into a portfolio composed of oil, gold, or equities considerably reduced portfolio risk. As Dutta et al. (2020) reported, oil price exhibited decreasing and extremely volatile movements following the outbreak of COVID-19 in December 2019, suggesting that investments in the oil market could produce more substantial losses. They also found that the Bitcoin–oil relationship became significantly positive during the COVID-19 period, indicating that Bitcoin only played a role as a diversifier. In other words, Bitcoin could behave exactly as the opposite of gold; consequently, it is not the new gold, as declared by Klein et al. (2018). This declaration was supported by Bouri et al. (2017c), Smales (2019), Conlon and McGee (2020), Conlon et al. (2020), Goodell and Goutte (2021b), Long et al. (2021), Syuhada et al. (2022), and Wen et al. (2022). They recommended investors not rely on Bitcoin to protect their investments against financial market turbulence.

The disappearance of the safe-haven capabilities of Bitcoin and other traditional cryptocurrencies might be implied by their extremely volatile movements and their controversial position as potential sources of financial market instability. This is due to the absence of a central regulator controlling the cryptocurrency market, unlike fiat currencies, whose stability is maintained by central banks. The high volatility of traditional cryptocurrencies makes investors face difficulty in obtaining stable returns, and it potentially increases their risk magnitude, implying that before being regarded as safe-haven tools, they should be protected first (Wang et al., 2020). Therefore, investors require alternative cryptocurrencies, namely, stablecoins, expected to become safe havens for traditional cryptocurrencies. Stablecoins are pegged to other (relatively) stable assets, such as the US dollar (USD) or gold (Baur and Hoang, 2021). The pegging mechanism lowers and stabilizes their volatility, as their name suggests. As a result, they are possibly able to eliminate extreme losses resulting from portfolios made up of traditional cryptocurrencies or other volatile assets.

Liu (2019) observed that Tether, one of the USD-pegged stablecoins, behaved quite differently from traditional cryptocurrencies, and it was much similar to fiat currencies that exhibit stable fluctuations. This most outstanding stablecoin was found to have the lowest expected return and volatility and to exhibit either negative or nearly zero correlation with other cryptocurrencies. As documented by Wang et al. (2020) and Baur and Hoang (2021), no other stablecoins provided better performance than Tether in serving as a safe haven for traditional cryptocurrencies. Meanwhile, Goodell and Goutte (2021b) reported evidence of the dissimilarities of Tether to Bitcoin, Ethereum, and Litecoin in terms of their safe-haven roles for equity markets. More specifically, the correlation between Tether and equities was significantly negative before COVID-19 and considerably more during COVID-19. The above findings indicate the crucial role of Tether as the strongest safe-haven tool for both traditional assets and volatile cryptocurrencies, consistent with investors seeking US dollar liquidity during market stress and uncertainty (Goodell and Goutte, 2021b). In contrast to these findings, Conlon et al. (2020) revealed a warning that amid COVID-19 bear markets, large portfolio allocation to Tether could fail to protect international equity investors against extreme risk, as measured using Value-at-Risk (VaR) and Expected Shortfall (ES). This means that its peg to the USD was not successfully maintained all the time during these circumstances.

VaR is a probability-based risk measure widely utilized to quantify portfolio risk. It, however, provides no information about the potential magnitude of losses exceeding it and fails to satisfy the subadditivity axiom of risk measure coherence. ES overcomes these weaknesses of VaR by accounting for all losses beyond the VaR through the expectation operator (Syuhada et al., 2021). This magnitude-based coherent measure of risk thus leads investors and portfolio risk managers to gain diversified portfolios. Nevertheless, ES is less intuitive to interpret and is less straightforward to backtest than VaR, suggesting that both VaR and ES have their own advantages and drawbacks (Del Brio et al., 2020).

Following Bassel Committee’s recommendation, both VaR and ES have been applied in portfolio risk management, such as in cryptocurrency markets (e.g., Gkillas and Katsiampa, 2018; Jiménez et al., 2020; Conlon and McGee, 2020), including stablecoins (e.g., Conlon et al., 2020; Wang et al., 2020), as well as in energy sectors (e.g., Velásquez-Gaviria et al., 2020). More specifically, Jiménez et al. (2020) and Velásquez-Gaviria et al. (2020) focused on validating the forecasts of these risk measures computed by employing different model specifications under normality, asymmetry, and leptokurticity assumptions. The performance of each model was assessed and compared through several backtesting procedures. Meanwhile, Gkillas and Katsiampa (2018) and Wang et al. (2020) considered only relevant information about distribution tails of returns in forecasting VaR and ES through an extreme value theory approach. Gkillas and Katsiampa (2018) argued that taking extremes into consideration was crucial for better investigating which investments are more susceptible to extreme losses and better understanding potential bubbles resulting from exceedingly high returns. On the other hand, Conlon and McGee (2020) and Conlon et al. (2020) utilized a modified version of the normal distribution’s quantile to take account of higher-order moments related to skewness and kurtosis. This approach resulted in VaR and ES forecasts reflecting potential large losses when the actual distribution of returns differed from the normal distribution. Inspired by the work of Bredin et al. (2017), whose focus was on precious metals and equities, Conlon and McGee (2020) and Conlon et al. (2020) employed the above VaR and ES forecasting approach to examine the safe-haven roles of Bitcoin and other cryptocurrencies for equities by assessing risk reduction in equity portfolios after being mixed with these crypto assets.

To the best of our knowledge, no study on the above markets relied on an improved version of VaR and ES forecasts. This improvement is crucial for determining more accurate portfolio risk forecasts and appropriate safe-haven (crypto) assets with enhanced confidence. Furthermore, the ability of stablecoins, particularly Tether, to serve as safe havens for crude oil has not been examined. To fill this research gap, this study aimed to assess Tether’s capability to support Bitcoin in acting as a safe haven for oil, particularly during COVID-19. We focused on protecting oil since it has been well documented as a leading commodity that plays a vital role and has significant implications for the rest markets in the world. Thus, it must be protected if its price dramatically declines, as observed at the onset of the COVID-19 outbreak. As argued by Ciaian et al. (2016), oil price measures global macro-financial development and becomes one of the significant determinants of Bitcoin price formation in the short run. As the most prominent cryptocurrency, Bitcoin has received much attention from numerous market participants and has been regarded as a safe haven for several markets. However, due to its excess volatility and potential positive correlation with oil, its incapability to protect oil investors against COVID-19 bear markets might be present. Tether, a USD-pegged cryptocurrency with the largest market capitalization, is thus required.
Fig. 1. Daily prices, returns, and squared returns. Note: Oil is priced in US dollars per barrel, while Bitcoin, Tether, and the US dollar index are denominated in US dollars. The shaded region represents the period of the COVID-19 outbreak from December 31, 2019, to July 22, 2022.

Table 1
Summary statistics of returns.

|                  | Min     | Max     | Mean   | Std. Dev. | Skewness | Kurtosis |
|------------------|---------|---------|--------|-----------|----------|----------|
| Before COVID-19  |         |         |        |           |          |          |
| OIL              | −6.449  | 11.070  | 0.011  | 1.943     | 0.071    | 5.576    |
| BTC              | −23.874 | 22.512  | 0.004  | 4.875     | 0.052    | 6.664    |
| USDT             | −4.820  | 5.661   | −0.000 | 0.612     | 0.724    | 24.211   |
| USD              | −1.015  | 1.125   | 0.004  | 0.331     | −0.011   | 3.183    |
| During COVID-19  |         |         |        |           |          |          |
| OIL              | −64.370 | 41.202  | 0.069  | 4.953     | −2.787   | 59.050   |
| BTC              | −46.473 | 19.153  | 0.175  | 4.869     | −1.644   | 17.654   |
| USDT             | −5.257  | 5.339   | −0.001 | 0.399     | 0.484    | 105.144  |
| USD              | −1.697  | 1.589   | 0.015  | 0.414     | 0.173    | 4.420    |

Note: JB stands for the Jarque–Bera test statistic of normality, while ADF and PP, respectively, refer to the augmented Dickey–Fuller and Phillips–Perron test statistics of a unit root. LB-$Q^{(20)}$ and LB-$Q^{2(20)}$ are the Ljung–Box statistics for testing serial correlation in the return and squared return series, respectively, while ARCH-LM(20) stands for the Lagrange multiplier statistic utilized to test conditional heteroscedasticity or ARCH effects up to lag 20. The asterisks * and *** indicate statistical significance at the 10% and 1% levels, respectively.

As Wei (2018) mentioned, Tether could be utilized for the conversion and exchange from Bitcoin to another cryptocurrency, and vice versa. They also highlighted that its issuances had a statistically significant impact on Bitcoin trading volumes. The fact that its pegging mechanism results in a stable fluctuation potentially enables it to support Bitcoin in oil protection. Since Tether is pegged to the US dollar, we also attempted to compare their safe-haven characteristics.

For the purpose of assessing and comparing the safe-haven roles of Tether and the US dollar for oil and Bitcoin, we compared risk measures of a portfolio containing oil and Bitcoin before and after being combined with Tether or the US dollar using various portfolio allocations. Motivated by the approaches of Bredin et al. (2017), Conlon and McGee (2020), and Conlon et al. (2020), we examined their risk reduction capabilities by computing the ratio of these portfolio risk measures. While they considered higher-order moments to modify VaR and ES, we employed an improved VaR forecast, proposed by Kabaila and Syuhada (2008) and Syuhada (2020), that has better accuracy in the form of conditional coverage probability. Besides, we also introduced a novel improved version of the ES forecast to overcome the shortcomings of VaR. Furthermore, we accounted for COVID-19 risk to better understand the potentially significant impact of the COVID-19 outbreak on the movements of oil, Bitcoin, Tether, and the US dollar and the assessment of safe-haven roles. We did this task by setting this COVID-19 risk as a covariate in the dynamic conditional mean equation of their return series. In addition, a GJR-GARCH model was employed to capture their asymmetric volatility, and their dependence was taken into account through an approach based on a vine copula, as in Syuhada and Hakim (2020), Chkir et al. (2020), Talbi et al. (2021), and Jin et al. (2022). To check the robustness of our findings under different model settings, we repeated our empirical study using another asymmetric volatility model and another dependence model.

In summary, this study provided the first result on the assessment of the safe-haven role of Tether for both oil and Bitcoin. Specifically,
while Bitcoin inconsistently reduced extreme risk in the oil market, Tether was found to play a consistent safe-haven role, which was more apparent in times of COVID-19 but was inferior to that of the US dollar. In addition, we contributed the first approach for accurately measuring portfolio risk and its reduction for oil, Bitcoin, and Tether by adopting an improved VaR forecast and proposing a novel improved ES forecast.

The rest of this paper is structured as follows. Section 2 reviews the literature, while Section 3 presents data sets utilized in this study and provides a preliminary analysis. Meanwhile, methodological frameworks are outlined in Section 4. We then report our main empirical results and draw conclusions in Sections 5 and 6, respectively.

2. Literature review

Since the creation of Bitcoin in 2008, the identity of cryptocurrencies has become a debated issue. By comparing the behavior of Bitcoin to that of other instruments using a variety of methods, White et al. (2020) documented that Bitcoin was far from a currency or a security. Its characteristics were similar to those of a technology-based product, an emerging asset class, or a bubble event. Meanwhile, Kwon (2020) found that Bitcoin played a role as a medium of exchange and a means of investment after comparing the tail behavior of its return to that of the US dollar, gold, and stock market index returns. In contrast, Baur et al. (2018) revealed that as a hybrid between fiat currency and commodity, Bitcoin was traded as a speculative investment asset and not as a currency nor a medium of exchange, in line with what Baur and Dimpfl (2021) concluded. Its incapability to work as a currency comes from the fact that it is unstable with excess volatility. Stablecoins, on the other hand, are designed to have low volatility by pegging them to less volatile assets, e.g., fiat currencies. More specifically, as surveyed by Mita et al. (2019), their price was formed based on stabilization mechanisms to match the price of fiat currencies with traditional cryptocurrencies. Accordingly, participants in the cryptocurrency market would prefer to hold stablecoins rather than traditional cryptocurrencies. With this, they would have the benefits of digital currencies for storing their wealth with reduced risk and fair transactions without abrupt changes in price during the transaction process (Wang et al., 2020). In particular, USD-backed stablecoins, such as Tether (anchored at 1 US dollar), are utilized by investors for the conversion and exchange from one cryptocurrency to another (Wei, 2018). As the largest stablecoin in the cryptocurrency market, Tether grew to over 80 billion US dollars in market capitalization as of March 2022 (coinmarketcap.com) and accounted for more Bitcoin transaction volumes than the US dollar (Griffin and Shams, 2020). Its issuances were found by Griffin and Shams (2020) to be timed following Bitcoin downturns and to result in a sizable appreciation in Bitcoin price. On the other hand, Wei’s (2018) study based on a Vector Autoregressive (VAR) model revealed that the impact of Tether issuances was statistically significant on Bitcoin trading volumes but not on Bitcoin returns.

Because cryptocurrencies have a special place in the financial industry, many authors were concerned with the stylized statistical properties of their returns and volatility. For instance, Katsiampa (2017) confirmed the existence of conditional heteroscedasticity or ARCH effects and leptokurtic behavior in the Bitcoin return series, leading us to apply GARCH-type models (for its conditional variance or volatility) with a heavy-tailed innovation. These stylized facts are similar to those of equities, but the possibility of extreme events in Bitcoin is higher, triggering greater financial instability (Jiménez et al., 2020). In addition to Bitcoin, Zhang et al. (2018) also studied seven other major cryptocurrencies, representing almost 70% of cryptocurrency market capitalization, and reported similar results. Meanwhile, using the GJR-GARCH model of Glosten et al. (1993), Bouri et al. (2017a) found a significant inverted relation between past shocks and the current volatility in the Bitcoin market before its price crash in December 2013. In other words, positive shocks generated the current volatility of Bitcoin more than negative shocks of the same magnitude in the past. This is contrary to what we observe in equities, but this is in the same direction as precious metals (Klein et al., 2018). Similar results were revealed by Baur and Dimpfl (2018) in 18 out of the 20 largest cryptocurrencies examined using the GJR-GARCH model and a Quantile Autoregressive (QAR)-based asymmetric volatility indicator. The inverted asymmetric reaction of most cryptocurrencies was also documented by Cheikh et al. (2020) using an asymmetric GARCH model with a smooth transition mechanism and by Fakhfekh and Jeribi (2020) using five GARCH-type models with different innovation distributions.

In addition, using an Autoregressive Distributed Lag (ARDL) model, Caiyan et al. (2018) provided evidence of interdependence among Bitcoin and altcoins, significantly stronger in the short-run than in the long-run. Meanwhile, Boako et al. (2019) and Syuhada and Hakim (2020) found their strong dependence captured by a vine copula, which allowed for different bivariate copulas for all pairs of cryptocurrencies. Through this approach, these cryptocurrencies and their dependence were, respectively, represented by nodes and edges of nested tree graphs, and Litecoin was shown to have the most connections with the others. These findings are in line with those of Ji et al. (2019) and Senssoy et al. (2021), which pointed out that Litecoin and Bitcoin were at the center of spillover networks, acting as connection hubs for connecting other cryptocurrencies. As Hoang and Baur (2021) observed, strong dependence also existed between Bitcoin and stablecoins.
More specifically, returns, volatility, and trading volumes of stablecoins were strongly correlated with returns, volatility, and trading volumes of Bitcoin, respectively, indicating that stablecoins were significantly unstable, contrary to what their name suggested.

The correlation or dependence analysis was also adopted by numerous studies that focused on assessing the roles of Bitcoin and other forms of cryptocurrencies for traditional assets, particularly in times of financial market turbulence. According to Baur and Lucey (2010), an asset is said to serve as a safe haven for another if they are uncorrelated or negatively correlated during such turbulence. When considering entire periods on average, the former asset is regarded as a hedge for the latter. In particular, a zero (negative) correlation indicates weak (strong) safe-haven or hedging roles (Baur and McDermott, 2010).³

³ Baur and Lucey (2010) also defined a diversifier, i.e., an asset exhibiting a positive (but not perfectly strong) correlation with another on average.

Fig. 3. Scatter plot and Pearson’s correlation matrices for pairs of (squared) returns. Note: For readability, the plot range of oil, Bitcoin, Tether, and USD returns is restricted to [−7.5, 7.5], [−15, 15], [−1.5, 1.5] (or [−0.75, 0.75]), and [−1, 1], respectively, while their squared-return range is limited to [0, 24], [0, 120], [0, 0.24] (or [0.00, 0.24]), and [0, 1], respectively. The asterisks ** and *** indicate statistical significance at the 5% and 1% levels, respectively.

Fig. 4. Numbers of daily COVID-19 confirmed new cases worldwide and their rates defined as COVID-19 risks. Note: The data period is from December 31, 2019, to July 22, 2022.
Table 2
Estimated parameters of marginal AR(1)-GJR-GARCH(1,1) models with a normal or Student’s $t$ innovation.

|               | $\omega$  | $\psi_1$ | $\psi_2$ | $\eta_1$ | $\eta_2$ | $\delta$ | $\gamma$ | $\nu$ | $\lambda + \gamma$ | LL   | AIC | LR   |
|---------------|-----------|-----------|-----------|----------|----------|----------|----------|------|------------------|------|-----|------|
| **Before COVID-19** |           |           |           |          |          |          |          |      |                  |      |     |      |
| OIL $N$       | 0.025     | -0.004    | 0.137***  | 0.013    | 0.908*** | 0.083**  | 0.962    | -1105.683 | 2223.366 | 45.328*** |
| $t$           | 0.072     | -0.010    | 0.121*    | 0.011    | 0.917*** | 0.076**  | 6.257*** | 0.966  | -1099.964 | 2203.992 | 26.126*** |
| BTC $N$       | -0.104    | 0.074     | 1.367**   | 0.089**  | 0.852*** | 0.003    | 0.942    | -1588.363 | 3188.726 | 77.108*** |
| $t$           | -0.008    | 0.028     | 0.143     | 0.095**  | 0.901    | 0.004    | 3.190*** | 0.999  | -1520.213 | 3054.426 | 84.939*** |
| USDT $N$      | -0.004    | -0.414*** | 0.017***  | 0.278*** | 0.680    | 0.059    | 0.988    | -337.770 | 687.541  | 269.089*** |
| $t$           | -0.012    | -0.368*** | 0.028**   | 0.281**  | 0.580    | 0.244    | 4.013*** | 0.983  | -293.861 | 601.721  | 107.295*** |
| USD $N$       | 0.000     | -0.029    | 0.000     | 0.003    | 0.988*** | 0.016    | 0.999    | -160.254 | 332.509  | 18.751*** |
| $t$           | 0.000     | -0.029    | 0.000     | 0.003    | 0.988*** | 0.016    | 62.458   | 0.999  | -160.183 | 334.366  | 17.886*** |
| **During COVID-19** |           |           |           |          |          |          |          |      |                  |      |     |      |
| OIL $N$       | 0.167*    | 0.024     | 0.001     | 0.339*** | 0.097*** | 0.797*** | 0.188**  | 0.988  | -1606.069 | 3226.138 | 703.530*** |
| $t$           | 0.300***  | -0.008    | 0.004     | 0.366*** | 0.106*** | 0.807*** | 0.125*   | 4.003*** | 0.976  | -1555.447 | 3126.894 | 171.194*** |
| BTC $N$       | 0.188     | -0.016    | 0.015     | 0.578**  | 0.000    | 0.665*** | 0.187**  | 0.759  | -1930.915 | 3875.830 | 30.297*** |
| $t$           | 0.255***  | -0.065*** | 0.003     | 0.450    | 0.085*** | 0.941*** | 0.056*   | 3.004*** | 0.998  | -1850.745 | 3717.490 | 19.234*** |
| USDT $N$      | 0.002     | -0.242*** | 0.000     | 0.318*** | 0.829*** | 0.296*** | 0.999    | 432.076 | -850.152 | 1394.280*** |
| $t$           | 0.000     | -0.368*** | 0.000     | 0.000    | 0.356*** | 0.687*** | 0.088    | 3.856*** | 0.999  | 742.415   | -1468.830 | 588.191*** |
| USD $N$       | 0.010     | 0.028     | 0.000     | 0.011**  | 0.105*** | 0.829*** | 0.000    | 0.933  | -303.450 | 620.900  | 85.734*** |
| $t$           | 0.010     | 0.027     | 0.000     | 0.011**  | 0.106**  | 0.829*** | 0.002    | 78.507 | 0.933   | -303.421 | 622.842  | 62.051*** |

Note: LL and AIC refer to the maximized conditional log-likelihood and the corresponding Akaike information criterion, respectively. Meanwhile, LR is the likelihood ratio test statistic utilized to test the null hypothesis of homoscedasticity, i.e., $H_0: \eta = \beta = \gamma = 0$, as in Hoang and Baur (2021). This statistic is asymptotically chi-square distributed with three degrees of freedom. Each number in parentheses is the standard error of the estimated parameter. The asterisks *, **, and *** indicate statistical significance at the 10%, 5%, and 1% levels, respectively.

Based on these definitions, examining safe-haven (and hedging) properties has been carried out using not only correlation coefficients and regression models (e.g., Bouri et al., 2017b, c; Stensæs et al., 2019; Marianna et al., 2021) but also tail dependence coefficients and copulas (e.g., Rebrokere, 2013; Wen and Cheng, 2018; Elie et al., 2019; Talbi et al., 2021), cross-quantilogram (e.g., Baumöl and Ljócsa, 2017; Shahzad et al., 2019; Bouri et al., 2020a), and wavelet analysis (e.g., Bredin et al., 2015; Bouri et al., 2020b; Goodell and Gouette, 2021a, b; Ali et al., 2022; Kumar and Padakandla, 2022).

Particularly from oil investors’ viewpoint, Selmi et al. (2018) highlighted that Bitcoin was able to be a strong safe-haven asset in the face of various circumstances in the oil market. More specifically, Selmi et al. (2018) utilized a quantile-on-quantile regression model by taking economic, monetary policy, financial, and political uncertainty indicators into account and showed that the safe-haven role of Bitcoin was stronger than that of gold. Their findings were robust in terms of portfolio diversification and risk reduction opportunities. Using several multivariate GARCH models, Guesmi et al. (2019) demonstrated significant return and volatility spillovers between Bitcoin and other financial assets. They found that a short position in the Bitcoin market could hedge the risk investment for oil, gold, and stocks from emerging markets. However, Dutta et al. (2020), Syuhada et al. (2022), and Wen et al. (2022) highlighted that this safe-haven functionality of Bitcoin disappeared when a sharp decline in oil price occurred at the onset of the global outbreak of COVID-19. Similar disappearance of the safe-haven properties of Bitcoin and other traditional cryptocurrencies for other markets was also observed by, e.g., Bouri et al. (2017c), Smale (2019), Conlon and McGee (2020), Conlon et al. (2020), Goodell and Gouette (2021b), and Long et al. (2021). Since these traditional cryptocurrencies were statistically unstable, and stablecoins were created to overcome this problem, the examination of the safe-haven roles of the latter for the former has also been a crucial and appealing study.

Baur and Hoang (2021) considered the six largest stablecoins based on their market capitalization in protecting five traditional cryptocurrencies (in particular, Bitcoin) against extreme conditions. They employed a regression model with dummy variables and showed that many stablecoins’ returns were negatively correlated with Bitcoin’s extreme negative returns, indicating that they were strong safe-haven tools for Bitcoin. Baur and Hoang (2021) argued that this evidence, however, implied that these safe-haven stablecoins did not exactly do what their name suggested, i.e., being stable, which was then confirmed by Hoang and Baur (2021). Using a DCC-GARCH approach combined with dummy variable regression, Wang et al. (2020) demonstrated that USD-pegged and gold-pegged stablecoins functioned as safe havens for traditional cryptocurrencies better than the respective underlying assets (i.e., the US dollar and gold). Furthermore, USD-pegged stablecoins were found to outperform gold-pegged ones, in line with the fact that the US dollar is more stable than gold. This is also in line with the finding of Wen and Cheng’s (2018) study that highlighted the US dollar is more stable than gold. This is also in line with the finding of Wen and Cheng’s (2018) study that highlighted the US dollar is more stable than gold.
on improved measures of portfolio risk with the purpose of assessing and comparing Tether and the US dollar’s safe-haven roles for both oil and Bitcoin. To the best of our knowledge, this purpose has not been considered in previous studies.

In terms of risk reduction opportunities, an asset is said to be a safe haven for another if combining the former with the latter into a portfolio results in a reduction in portfolio risk compared to holding the latter only. In other words, the relative ratio of the risk of this portfolio and that of the latter is below one. Bredin et al. (2017) introduced this relative ratio concept to examine precious metals’ safe-haven roles for equities, where the risk was measured using Value-at-Risk (VaR) with the inclusion of higher-order moments. It was then adopted by Conlon and McGee (2020) and Conlon et al. (2020), whose focus was on the safe-haven roles of cryptocurrencies for equities. In addition to VaR, they also relied on ES, which accounted for all losses beyond the VaR. On the other hand, Wang et al. (2020) computed the difference between the VaR/ES of a single asset and the VaR/ES of a portfolio made up of this asset and a safe-haven candidate, where the VaR and ES were determined through an extreme value theory approach, more specifically through a generalized Pareto distribution. This study preferred to consider improved risk measure forecasts that were formulated so that their accuracy increased. An improved VaR forecast was first proposed by Barndorff-Nielsen and Cox (1994). It originally referred to an improved version of the estimative upper forecasting limit of future observation since VaR was basically the (conditional) quantile of future random loss at a given level of confidence. While an estimative VaR forecast possesses first-order accuracy, its improved version can have better accuracy up to the third order, where their accuracy is assessed using their (conditional) coverage probability. Other studies, e.g., Vidoni (2004), Ueki and Fueda (2007) and Kabaila and Syuhada (2008) proposed different approaches to carry out this improvement. Recently, Syuhada (2020) adopted Kabaila and Syuhada’s (2008) simulation-based approach to compare the improved VaR forecast of an ARCH model and that of a Stochastic Volatility (SV) model. In this study, we contributed to the literature by formulating an improved ES forecast as an alternative to the improved VaR forecast.

3. Data and preliminary analysis

This study paid particular attention to (1) Brent crude oil, (2) Bitcoin (BTC), (3a) Tether (USDT), and (3b) the US dollar index (USD).

![Fig. 5. Scatter plot and Kendall's τ matrices for pairs of uniformly distributed pseudo observations. Note: The asterisks *, **, and *** indicate statistical significance at the 10%, 5%, and 1% levels, respectively.](image-url)
Fig. 6. Vine copula-based dependence structures among individual returns following AR(1)-GJR-GARCH(1,1) models with a normal or Student’s $t$ innovation. Note: C, G, and F stand for the best-fitting bivariate Clayton, Gumbel, and Frank copulas, respectively. Meanwhile, C90/C180/C270 and G90/G180/G270 refer to the rotated versions of Clayton and Gumbel copulas, respectively, based on clockwise rotations by 90/180/270 degrees. The estimates for their parameters are provided in Table A.2 in Appendix. The shortest and thickest edge represents the strongest dependence between two instruments linked.

The former’s daily spot prices were sourced from the Energy Information Administration’s website (eia.gov), while the daily closing price data for the two cryptocurrencies and USD were downloaded from coinmarketcap.com and investing.com, respectively. The data period spanned from November 9, 2017, to July 22, 2022. We transformed these price data into the daily return data $\{R_{i,t}\}$ with $R_{i,t} = 100(\ln P_{i,t} - \ln P_{i,t-1})$, where $P_{i,t}$ and $P_{i,t-1}$ denote the prices of the $i$th instrument on day $t$ and day $t-1$, respectively. These price and (squared) return data are plotted in Fig. 1.

We found that oil price shrank from around 70 US dollars per barrel at the beginning of 2020 to the lowest value below 10 US dollars per barrel in the second quarter of 2020. This phenomenon indicates that oil demand dramatically declined since restricted travel and business activities and even lockdown policies were enforced by governments worldwide in response to the outbreak of COVID-19. During this period, oil return also exhibited extreme volatility, as measured using its squared return, suggesting high uncertainty in the oil market. From several statistics reported in Table 1, this extremely volatile movement of oil over the COVID-19 period led to a rise in its standard deviation by more than two and a half times. Amid this period, its kurtosis also appeared to become huge, implying that its return followed a heavy-tailed distribution, as confirmed by the Jarque–Bera test rejecting the null hypothesis of normality. On the other hand, Bitcoin price experienced a drastic growth following the COVID-19 outbreak, with a value from about 10 thousand US dollars on the first day of 2020 to above 60 thousand US dollars in March 2021. In COVID-19 times, Bitcoin return possessed a positive mean far from zero as well as from the mean of oil return. Nevertheless, its standard deviation and kurtosis values also
increased, but they were lower than those of oil. These empirical facts imply that investors investing in Bitcoin gained less acute losses than those holding oil.

In contrast, Tether had the lowest magnitude of expected return before and during COVID-19 and the lowest volatility during COVID-19. This quite different feature of Tether is related to its peg to the USD. It is well known that this kind of stablecoin is designed to remain stable at around 1 USD dollar, making it behave more similar to the USD rather than to a speculative financial asset. However, we can observe from Fig. 1 that Tether exhibited downturns and upturns in value; it fluctuated from around 0.96 to 1.08 USD dollars in the times under consideration. Fluctuation in its return series also occurred with a range between a minimum of −5.257 and a maximum of 5.661, being much wider than the range of USD return data. Besides, the magnitude of the skewness and kurtosis of Tether return was found to be much larger than those of USD return data before and during the COVID-19 outbreak.

The dissimilarity between Tether and USD was further supported by the test of Engle’s (1982) ARCH effects using the Lagrange multiplier (LM) statistic. More specifically, this test revealed evidence of conditional heteroscedasticity, being significant in the Tether return series but insignificant in the USD return series. As depicted in Fig. 2, significant autocorrelations at several lags in the squared-return series of Tether demonstrated strong proof for the rejection of the null hypothesis of homoscedasticity using the LB-Q\(^2\) test statistic. The above results indicate the instability of Tether, contrary to what its name suggests. Its stability should require homoscedastic or constant volatility over time, as argued by Hoang and Baur (2021). According to the value of the ARCH-LM test statistic, Tether was more stable than Bitcoin, but it was not as stable as its underlying asset (i.e., the USD).

Evidence of Tether’s instability can also be suggested from its reactions to the volatile movement of Bitcoin (and oil), related to safehaven, hedging, and diversification properties. Following the demonstration of Baur and Hoang (2021), if Tether does not co-move with Bitcoin, it is exactly stable and therefore is a weak safe-haven asset. Surprisingly, Fig. 3 reveals that Tether and Bitcoin returns were significantly correlated at the 1% level with a positive direction (0.133) before the COVID-19 period. As the COVID-19 outbreak progressed, the Tether–Bitcoin correlation became significantly negative (−0.218) at the 1% level and was stronger than the Tether–oil correlation (−0.083). This result indicates Tether’s ability to act as a strong safe-haven tool for both oil and Bitcoin. Consequently, investors can add Tether into their portfolio consisting of oil and Bitcoin to help them reduce their portfolio risk instead of holding the two only. The involvement of Tether is required in this case since Bitcoin exhibited significantly positive co-movement with oil and therefore failed to protect oil against market turbulence. Amid the COVID-19 outbreak, we can observe that the safe-haven property of Tether was better than that of the US dollar, although the former was less stable than the latter. In addition, Tether’s instability was possibly sourced from Bitcoin. The reason is that a highly significant correlation between squared returns or realized volatilities of these two cryptocurrencies was present with a value of 0.117 (before COVID-19) and 0.651 (during COVID-19).

From the risk management viewpoint, it is crucial to assess how large the magnitude of extreme risks of oil, Bitcoin, Tether, and the US dollar before and after they are mixed. Technically, if Bitcoin is a safe haven for oil, investors combining them would be exposed to reduced extreme risk compared to those holding oil only. Similarly, risk reduction achieved after Tether (or the US dollar) is mixed with oil and Bitcoin suggests that Tether plays a safe-haven role against extreme risks in the oil and Bitcoin markets. In the next section, we assessed these extreme risks after setting up stochastic models describing the evolution of oil, Bitcoin, Tether, and the US dollar returns during the pre-COVID-19 and COVID-19 periods. More specifically, we adopted stationary and dependent GARCH-type models with the additional capability to handle asymmetric volatility. In addition to the presence of conditional heteroscedasticity and pairwise correlation, another reason for considering this approach is that each return series was not a unit-root process based on augmented Dickey–Fuller and Phillips–Perron test results, as previously summarized in Table 1. Besides, there existed asymmetric responses of tomorrow’s volatility to the current positive and negative shocks, as displayed in Fig. 2. Due to the fact that the outbreak of COVID-19 has shaken the aforementioned markets and has affected the empirical behaviors of their returns, we also involved changes in the numbers of daily confirmed new cases of COVID-19 worldwide in constructing the marginal return models. More specifically, using the data (Case\(_t\)) collected from the website covid19.who.int of the World Health Organization (WHO), we defined the following COVID-19 risk denoting the rate of daily addition in COVID-19 confirmed cases:

\[
R_{COVID} = -100(\ln \text{Case}_t - \ln \text{Case}_{t-1}).
\]  

(1)

It might influence the magnitude of each return series in times of COVID-19. This definition led to a simple understanding of the interpretation of COVID-19 risk analogous to that of the return series, such as (i) being dimensionless and (ii) riskiness indication from its negative level. See Fig. 4 to investigate the movements of the data \{Case\(_t\)\} and the COVID-19 risk data \(R_{COVID}\).
the COVID-19 period, we inserted such a first-order Autoregressive or AR(1) model. To investigate the possible error or innovation independent of previous information was proposed, where $\varepsilon_{i,t}$ is a stochastic term denoting the standardized Student's $t$ distribution with a distribution function $F_{\nu}$ equal to $\Phi$ or $\tau_{i}$, respectively, where $\nu_{i}$ denotes the degrees of freedom. To ensure that the return process $(R_{i,t})$ is stationary under the above model settings, the value of several components of the parameter vector $\theta = (\phi_{i}, \psi_{i}, \xi_{i}, \omega_{i}, \eta_{i}, \beta_{i}, \gamma_{i}, \nu_{i})^T$ must be restricted as follows: $-1 < \psi_{i} < 1$, $\eta_{i} + \beta_{i} + \gamma_{i} < 1$, and $\nu_{i} > 2$. In particular, $\psi_{i}$ and $\eta_{i} + \beta_{i} + \gamma_{i}$, respectively, measure the persistence of the conditional mean and volatility of $(R_{i,t})$.

### 4. Methodology

#### 4.1. Marginal return models

We assumed the return series of each instrument $i$ to have a dynamic non-zero conditional mean $\mu_{i,t}$ and a time-varying volatility $\sigma_{i,t}^2$. Therefore, a marginal stochastic model

$$R_{i,t} = \mu_{i,t} + \sigma_{i,t} \varepsilon_{i,t}$$

(2)

was proposed, where $\varepsilon_{i,t}$ is a stochastic term denoting the standardized error or innovation independent of previous information $\xi_{i,t-1}$ available at time $t-1$. More specifically, we modeled the conditional mean using a first-order Autoregressive or AR(1) model. To investigate the possible impact of the COVID-19 risk level $V_{COVID}$ on each return series during the COVID-19 period, we inserted such $R_{COVID,i}$ in the AR(1) equation as follows:

$$\mu_{i,t} = \phi_{i} + \psi_{i} R_{i,t-1} + \xi_{i} R_{COVID,i}$$

(3)

where $(\phi_{i}, \psi_{i}, \xi_{i}) \in \mathbb{R}$. Meanwhile, the GJR-GARCH(1,1) model of Glosten et al. (1993) was applied for the volatility $\sigma_{i,t}^2$ as follows:

$$\sigma_{i,t}^2 = \omega_{i} + \eta_{i} (R_{i,t-1} - \mu_{i,t-1})^2 + \beta_{i} \sigma_{i,t-1}^2 + \gamma_{i} (R_{i,t-1} - \mu_{i,t-1})^2 \mathbb{I}(R_{i,t-1} - \mu_{i,t-1} < 0),$$

(4)

with $(\omega_{i}, \eta_{i}, \beta_{i}, \gamma_{i})^T$ belonging to $(0, \infty) \times (0, \infty)^2 \times \mathbb{R}$. The nonzero leverage parameter $\gamma_{i}$ allows us to accommodate the asymmetric impact of previous negative and positive shocks on the current volatility. This impact is distinguished by the indicator function $\mathbb{I}(R_{i,t-1} - \mu_{i,t-1} < 0)$ having a value of one if the event $(R_{i,t-1} - \mu_{i,t-1} < 0)$ occurs and zero otherwise. In addition, we supposed the innovation $\varepsilon_{i,t}$ to follow a standard normal distribution $N(0,1)$ (as a benchmark) or a standard Student's $t$ distribution $t(0,1,\nu_{i})$ with a distribution function $F_{\nu}$ equal to $\Phi$ or $\tau_{i}$, respectively, where $\nu_{i}$ denotes the degrees of freedom.

### 4.2. Structure of return dependence

The classical Pearson’s $r$ and Kendall’s $\tau$ are the simplest tools we can employ when assuming dependence between different random variables. However, these measures are insufficient to capture the complex features of dependence, such as tail dependence and asymmetric dependence, which are typical features of high-dimensional financial asset returns (Breckmann and Czado, 2013; Syuhada and Hakim, 2020). We, therefore, adopted the copula method, which provides a flexible way to link their marginal distributions. This method is required to construct the joint distribution of a pairwise innovation $(\varepsilon_{i,j}, \varepsilon_{j,i})^T$ determining the return models for given instruments $i$ and $j$. More specifically, we utilized a bivariate copula $C_{U_{i}}$ and the corresponding copula density $c_{U_{i}}(u_{i,j}) = \partial^2 C_{U_{i}}(u_{i,j})/\partial u_{i} \partial u_{j}$ as the joint distribution function and joint probability function of $(U_{i}, U_{j})$, respectively, with $U_{i} = F_{i}(\varepsilon_{i,j})$ and $U_{j} = F_{j}(\varepsilon_{j,i})$ uniformly distributed over a unit interval $[0,1]$. Accordingly, based on Sklar’s theorem given in Joe and Smith (1987), it follows

$$c_{U_{i}}(u_{i,j}) = c_{C_{U_{i}}}(F_{i}(u_{i,j}), F_{j}(u_{i,j})).$$

A set $(U_{i,j})$ of pseudo observations derived from the estimated standardized errors through the probability integral transformation was employed in the estimation of the copula’s parameter, where $U_{ij} = F_{i}(\varepsilon_{ij})$ with $\varepsilon_{ij} = (R_{i,j} - \mu_{ij})/\hat{\sigma}_{ij}$; the terms $\hat{\mu}_{ij}$ and $\hat{\sigma}_{ij}$ are the fitted conditional mean and volatility, respectively.
Fig. 7. VaR and ES forecasts at the 99% level of confidence for a portfolio containing various allocations of $w_B$ to Bitcoin and the remaining $1 - w_B$ to oil, whose marginal returns follow AR(1)-GJR-GARCH(1,1) models with a normal or Student’s $t$ innovation. Note: $w_B$ varies between 0% and 100%.

(2015), the joint distribution function of $(\varepsilon_i, \varepsilon_j)^T$ can be defined as follows:

$$F_{(\varepsilon_i, \varepsilon_j)} (y_i, y_j) = C_{ij} (F_{\varepsilon_i}(y_i), F_{\varepsilon_j}(y_j)) = C_{ij} (u_i, u_j),$$

for all $(y_i, y_j)^T \in \mathbb{R}^2$. An appropriate bivariate copula employed in this study was selected from the Archimedean copula family, including (rotated) Clayton, (rotated) Gumbel, and Frank copulas, as in Syuhada et al. (2022); see Table A.1 in Appendix.

There are extended versions of the above copula family for higher dimensions. Nevertheless, all possible pairs of random variables linked by a multivariate Archimedean copula exhibit a similar dependence structure since it generally has only one or two parameters controlling such dependence (Brechmann and Czado, 2013). The pair-copula construction method can be employed to overcome this limitation by constructing copula densities for three or more random variables from (conditional) bivariate copulas, which are not necessarily identical. In
K. Syuhada et al.

Panel A: OIL–BTC–USDT Portfolio

Panel B: OIL–BTC–USDT Portfolio Relative to OIL–BTC Portfolio

Fig. 8. VaR and ES forecasts at the 99% level of confidence for a portfolio consisting of various allocations of \( w_B \) to Bitcoin, \( w_T \) to Tether, and the remaining \( 1 - w_B - w_T \) to oil, whose marginal returns follow AR(1)-GJR-GARCH(1,1) models with a normal or Student’s \( t \) innovation. Note: Both \( w_B \) and \( w_T \) vary between 0% and 100%, but \( w_B + w_T \leq 100\% \).

our context, we assumed that \( (\epsilon_{1,t}, \epsilon_{2,t}, \epsilon_{3,t})^T \) required to determine oil \((R_{1,t})\), Bitcoin \((R_{2,t})\), and Tether or the US dollar \((R_{3,t})\) return models has a trivariate copula \( C_{123} \), whose density is decomposed into

\[
c_{123}(u_1,u_2,u_3) = c_{12}(u_1,u_2) \times c_{13}(u_1,u_3) \times c_{23}(u_2,u_3)
\]

or other permutations of \( \{1,2,3\} \), where \( c_{23}(u_2|u_1) = \partial c_{23}(u_2,u_3)/\partial u_1 \) and \( c_{31}(u_3|u_1) = \partial c_{31}(u_3,u_2)/\partial u_1 \) (Aas et al., 2009). A nested set \( \{T_1, T_2\} \) of tree graphs, called regular vine, is also required to represent the marginal distributions and the bivariate copulas using nodes and edges, respectively. In particular, \( T_1 \) is a maximum spanning tree that maximizes absolute measures of pairwise dependence attached to its edges; see, e.g., Brechmann and Czado (2013) and Dißmann et al. (2013).

4.3. Improved measures of risk

Given available information \( \hat{R}_{i,n} \) from each market on day \( n \), Value-at-Risk (VaR) has widely been used to forecast future risk or return \( \hat{R}_{i,n+1} \) on day \( n + 1 \). For a specified \( \alpha \in (0,1) \), we computed this risk measure at the \( 1 - \alpha \) level of confidence by solving a value \( \text{VaR}^{1-\alpha}_{\hat{R}_{i,n+1}, \hat{\theta}} \).
Fig. 9. VaR and ES forecasts at the 99% level of confidence for a portfolio consisting of various allocations of \( w_B \) to Bitcoin, \( w_U \) to USD, and the remaining \( 1 - w_B - w_U \) to oil, whose marginal returns follow AR(1)-GJR-GARCH(1,1) models with a normal or Student’s \( t \) innovation. Note: Both \( w_B \) and \( w_U \) vary between 0% and 100%, but \( w_B + w_U \leq 100\% \).

The following conditional expectation:

\[
\text{ES}_{R_{i,n+1}|\theta_i}^{1-\alpha} = \mathbb{E}\left( -R_{i,n+1} \mid -R_{i,n+1} > \text{VaR}_{R_{i,n+1}|\theta_i}^{1-\alpha} \right) = -\mu_{i,n+1} - \sigma_{i,n+1} \mathbb{E}\left[ \epsilon_{i,n+1} \mid \epsilon_{i,n+1} < F_{\epsilon_i}^{-1}(\alpha) \right].
\]  

If \( \epsilon_{i,n+1} \sim N(0,1) \) with \( F_{\epsilon_i} = \Phi \) and \( f_{\epsilon_i} = \phi \), we can obtain

\[
\mathbb{E}\left[ \epsilon_{i,n+1} \mid \epsilon_{i,n+1} < F_{\epsilon_i}^{-1}(\alpha) \right] = -\frac{\phi(\Phi^{-1}(\alpha))}{\alpha}.
\]
Meanwhile, if a standard Student’s $t$ distributed innovation with $v_i$ degrees of freedom is taken into consideration such that $F_{t_i} = t_{v_i}$ and $f_{t_i} = t'_{v_i}$, we can derive

$$E \left[ \varepsilon_i, s_{i+1} \mid F_{t_i}^{-1}(a) \right] = \frac{-t'_{v_i} \left( t_{v_i}(a) \right)}{a} \left( 1 + \left( t_{v_i}(a) \right)^2 - 1 \right).$$

When the parameter $\theta_i$ is estimated by $\hat{\Theta}_i$, one can obtain an estimative VaR$^{1-a}_{R_{t_i}, \hat{\Theta}_i}$ forecast by simply replacing $\theta_i$ with $\hat{\Theta}_i$. The conditional coverage probability of this forecast can be shown to differ from $1 - \alpha$ by $O(n^{-1}),$ that is

$$P \left( -R_{t_i, s_{i+1}} \leq \text{VaR}^{1-a}_{R_{t_i}, \hat{\Theta}_i} \mid \hat{\Theta}_s \right) = E \left[ F_{-R_{t_i,s_{i+1}}} \left| \hat{\Theta}_s \right. \left\{ \text{VaR}^{1-a}_{R_{t_i}, \hat{\Theta}_i} \right\} \right] \mid \hat{\Theta}_s
$$

where $1 - \alpha$ is negative (positive) because the conditional coverage probability of VaR$^{1-a}_{R_{t_i}, \hat{\Theta}_i}$ is lower (higher) than $1 - \alpha$, the corresponding improved VaR forecast is larger (smaller) than this estimative VaR forecast. In this study, a novel improved ES forecast was also introduced with the following formulation:

$$\hat{\text{ES}}^{1-a}_{R_{t_i}, \hat{\Theta}_i} = E \left[ -R_{t_i, s_{i+1}} \mid -R_{t_i, s_{i+1}} > \text{VaR}^{1-a}_{R_{t_i}, \hat{\Theta}_i} \right] \mid \hat{\Theta}_s
$$

where $\text{VaR}^{1-a}_{R_{t_i}, \hat{\Theta}_i} = \frac{-t'_{v_i} \left( t_{v_i}(a) \right)}{a} \left( 1 + \left( t_{v_i}(a) \right)^2 - 1 \right).$ Compared to the previously mentioned estimative form, this improved VaR forecast has a better conditional coverage probability equal to $1 - \alpha + O(n^{-1/2}).$ We can observe from Eq. (8) that when the term $\text{VaR}^{1-a}_{R_{t_i}, \hat{\Theta}_i}$ is negative (positive) because the conditional coverage probability of VaR$^{1-a}_{R_{t_i}, \hat{\Theta}_i}$ is lower (higher) than $1 - \alpha$, the corresponding improved VaR forecast is larger (smaller) than this estimative VaR forecast. In this study, a novel improved ES forecast was also introduced with the following formulation:

$$\hat{\text{ES}}^{1-a}_{R_{t_i}, \hat{\Theta}_i} = E \left[ -R_{t_i, s_{i+1}} \mid -R_{t_i, s_{i+1}} > \text{VaR}^{1-a}_{R_{t_i}, \hat{\Theta}_i} \right] \mid \hat{\Theta}_s
$$

where $\text{VaR}^{1-a}_{R_{t_i}, \hat{\Theta}_i} = \frac{-t'_{v_i} \left( t_{v_i}(a) \right)}{a} \left( 1 + \left( t_{v_i}(a) \right)^2 - 1 \right).$

### 4.4 Portfolio risk reduction

Suppose that $R_i = (R_{i, j}, R_{i, 2}, R_{i, 3})^\top$ is a vector consisting of individual returns of oil, Bitcoin, and Tether (or the US dollar). From all these returns, we constructed a portfolio $S_i = w^\top R_i$ with a specified weighting vector $w = (1 - w_B - w_{U/T}, w_B, w_{U/T})^\top \in [0, 1]^3$, where $w_{U/T} \leq w_B \leq (1 - w_B - w_{U/T})$. In other words, we allocated $w_B$ to Bitcoin and $w_{U/T}$ to Tether (or the US dollar); the remaining $1 - w_B - w_{U/T}$ is allocated to oil. If $w_{U/T} = 0$, we have a portfolio consisting of oil and Bitcoin only. Meanwhile, when both $w_B$ and $w_{U/T}$ are equal to zero, the portfolio consists of oil only. Using a variance–covariance approach, the VaR and ES of future portfolio return $S_{t_i, s_{i+1}}$ are, respectively, formulated as follows:

$$\text{VaR}^{1-a}_{S_{t_i, s_{i+1}}, \hat{w}(w)} = -w^\top \mu_{t_i+1} - \sqrt{w^\top \Sigma_{t_i+1} w} \cdot \phi^{-1}(a)$$

where $\mu_{t_i+1}$ and $\Sigma_{t_i+1}$ represent, respectively, the conditional mean vector and conditional covariance matrix of $R_{t_i, s_{i+1}}$. Following Eqs. (8) and (9), their improved versions can also be found.

Using the above measures of portfolio risk, we assessed risk reduction in a portfolio after and before being mixed with Tether (or the US dollar). More specifically, we computed the following relative ratio:

$$RR_p = \frac{\text{VaR}^{1-a}_{R_{t_i}, \hat{\Theta}_s} (\hat{w}(w))}{\text{VaR}^{1-a}_{R_{t_i}, \hat{\Theta}_s}}$$

where $\phi$ refers to either VaR, ES, or their improved version, while $O, B, T,$ and $U$ stand for oil, Bitcoin, Tether, and the US dollar, respectively. When the value of $RR_p$ is below one, the risk reduction is achieved. The smaller this $RR_p$ is, the stronger the risk reduction capability is. This approach has been implemented by, e.g., Bredin et al. (2017), Conlon and McGee (2020), Conlon et al. (2020), and Syuhada et al. (2022) to examine the safe-haven properties of precious metals and cryptocurrencies. In this study, we performed our assessment for several cases detailed as follows.

- **Case 1:** $RR_p = \frac{\text{VaR}^{1-a}_{R_{t_i}, \hat{\Theta}_s} (\hat{w}(w))}{\text{VaR}^{1-a}_{R_{t_i}, \hat{\Theta}_s}}$, where the portfolios are equally weighted.
- **Case 2:** $RR_p = \frac{\text{VaR}^{1-a}_{R_{t_i}, \hat{\Theta}_s} (\hat{w}(w))}{\text{VaR}^{1-a}_{R_{t_i}, \hat{\Theta}_s}}$, where the portfolios are optimally weighted with $w_{opt} = \arg \min \{ w^\top \Sigma_{t_i+1} w : w = (1 - w_B - w_{U/T}, w_B, w_{U/T})^\top \in [0, 1]^3 \}$ and $w_{opt} = \arg \min \{ w^\top \Sigma_{t_i+1} w + \phi^{-1}(a) \cdot \Sigma_{t_i+1} w : w = (1 - w_B - w_{U/T}, w_B, w_{U/T})^\top \in [0, 1]^3 \}$.
- **Case 3:** $RR_p = \frac{\text{VaR}^{1-a}_{R_{t_i, s_{i+1}}, \hat{w}(w)} - \text{VaR}^{1-a}_{R_{t_i}, \hat{\Theta}_s}}{\text{VaR}^{1-a}_{R_{t_i}, \hat{\Theta}_s}}$, for various allocation weights $w_B, w_{U/T} \in [0, 1]$ with $w_B + w_{U/T} \in [0, 1]$.

This purpose of assessing risk reduction in portfolios with any weight-

### 5. Empirical findings

#### 5.1 Return models and their dependence structure

We estimated the parameters of the marginal AR(1)-GJR-GARCH(1,1) model previously defined in Eqs. (2)–(4) for oil, Bitcoin, Tether, and USD return series. More specifically, we maximized its conditional likelihood function under normality and Student’s $t$ assumptions. The estimation results are detailed in Table 2. We can observe from this table that before COVID-19, oil and Bitcoin returns possessed an insignificant conditional mean equation. As the COVID-19 outbreak progressed, their conditional mean became significant, particularly when assuming Student’s $t$ distribution for its innovation term. Meanwhile, the conditional mean of USD return was not found to be significant and persistent during the pre-COVID-19 and COVID-19 periods. In contrast, the persistence of the conditional mean of Tether return was consistently present during the two periods with negative effects, indicating that yesterday’s Tether return of a certain sign tended to be followed by Tether return of the opposite sign today. In addition, we found that the COVID-19 risk level tended to influence the return of each instrument positively. However, the parameter $\xi$ driving this effect was statistically significant at the 5% level only for the Tether return model conditionally normally distributed. Based on the definition of COVID-19 risk in Eq. (1), this result indicates that all the instruments examined became riskier (safer) in response to large (small) increments in the daily COVID-19 risk levels, in line with what Iqbal et al. (2021) documented but contrary to the finding of Goodell and Goutte (2021a).
from a large and significant value of $\eta$ value of their persistence parameter volatility for all the considered instruments. In particular, a huge existence of conditional heteroscedasticity effects and highly persistent market turbulence due to COVID-19. This inverted leverage effect was

Table A.2
Estimated parameters of the best-fitting bivariate copulas of the regular vines, whose margins are AR(1)-GJR-GARCH(1,1) models with a normal or Student’s $t$ innovation.

| Tree | Edge | Copula | $\theta$ | LL | AIC | Kendall’s $\tau$ |
|------|------|--------|---------|----|-----|----------------|
| **Panel A1: OIL, BTC, and USDT with Normal Margins** |
| Before COVID-19 | OIL,USDT | Clayton | -0.517 | 1.453 | -0.907 | -0.057 |
| | USD,BTC | Clayton | 1.074 | 7.183 | -12.366 | 0.069 |
| | OIL,BTC/USDT | Frank | 0.226 | 0.254 | 1.491 | 0.025 |
| During COVID-19 | OIL,BTC | Gumbel | 1.081 | 3.436 | -4.872 | 0.075 |
| | BTC,USDT | Gumbel | 1.015 | 11.692 | -21.385 | -0.015 |
| | OIL,BTC/USDT | Clayton | 0.032 | 0.573 | 0.855 | 0.016 |
| **Panel A2: OIL, BTC, and USD with Student’s $t$ Margins** |
| Before COVID-19 | OIL,USDT | Frank | -0.441 | 1.446 | -0.892 | -0.049 |
| | USD,BTC | Gumbel | 1.075 | 5.988 | -9.977 | 0.070 |
| | OIL,BTC/USDT | Clayton | 0.037 | 0.460 | 1.081 | 0.018 |
| During COVID-19 | OIL,BTC | Clayton | 0.114 | 5.043 | -8.085 | 0.054 |
| | BTC,USDT | Clayton | 1.021 | 4.578 | -7.155 | -0.021 |
| | OIL,BTC/USDT | Clayton | 0.037 | 0.475 | 1.050 | 0.018 |

| **Panel B1: OIL, BTC, and USD with Normal Margins** |
| Tree | Edge | Copula | $\theta$ | LL | AIC | Kendall’s $\tau$ |
|------|------|--------|---------|----|-----|----------------|
| Before COVID-19 | OIL,USD | Frank | -0.087 | 0.049 | 1.901 | -0.010 |
| | USD,BTC | Frank | -0.393 | 0.869 | 0.261 | -0.044 |
| | OIL,BTC/USD | Clayton | 0.029 | 0.312 | 1.377 | 0.014 |
| During COVID-19 | OIL,USD | Frank | -0.994 | 5.105 | -8.209 | -0.109 |
| | USD,BTC | Frank | -0.956 | 4.768 | -7.535 | -0.105 |
| | OIL,BTC/USD | Clayton | 1.016 | 3.301 | -4.602 | 0.016 |

| **Panel B2: OIL, BTC, and USD with Student’s $t$ Margins** |
| Tree | Edge | Copula | $\theta$ | LL | AIC | Kendall’s $\tau$ |
|------|------|--------|---------|----|-----|----------------|
| Before COVID-19 | OIL,USD | Frank | -0.110 | 0.088 | 1.825 | -0.012 |
| | USD,BTC | Frank | -0.431 | 1.410 | -0.820 | -0.048 |
| | OIL,BTC/USD | Clayton | 0.040 | 0.532 | 0.936 | 0.019 |
| During COVID-19 | OIL,USD | Clayton | 0.178 | 7.125 | -12.249 | -0.082 |
| | USD,BTC | Frank | -0.928 | 5.912 | -9.824 | -0.103 |
| | OIL,BTC/USD | Clayton | 0.106 | 3.712 | -5.424 | 0.050 |

Note: LL and AIC refer to the maximized log-likelihood and the corresponding Akaike information criterion, respectively.

Although the conditional mean equation for several instruments is insignificant, we can observe from Table 2 that volatility modeling through GJR-GARCH specifications provided strong support for the existence of conditional heteroscedasticity effects and highly persistent volatility for all the considered instruments. In particular, a huge value of their persistence parameter $\eta + \beta + \frac{1}{2} \gamma$ was mainly sourced from a large and significant value of $\beta$ that controlled the impact of the previous volatility on the current one. Interestingly, Bitcoin and Tether were found to exhibit significantly asymmetric volatility with a negative direction in times of COVID-19. This means that investors in these markets overreacted to good news rather than bad news amid market turbulence due to COVID-19. This inverted leverage effect was significantly more pronounced in Tether than Bitcoin, and it was also reported in the USD with an insignificant leverage parameter $\gamma$. In contrast to these results, the leverage parameter of oil was significantly positive before and during COVID-19. Another interesting result is that according to the maximized conditional log-likelihood and Akaike information criterion (AIC), the AR(1)-GJR-GARCH(1,1) model under normality assumption was able to model USD return and volatility better than that with Student’s $t$ innovation. Meanwhile, the superiority of this model under Student’s $t$ assumption was present when utilized to model the return and volatility of oil, Bitcoin, and Tether. Its low and significant parameter $\nu$, denoting the degrees of freedom, supported...
empirical evidence of the heavy-tailedness of these instruments previously indicated in Table 1. In addition, the likelihood ratio test did not fail to reject the null hypothesis of homoscedasticity ($H_0: \eta = \beta = \gamma = 0$) at the 1% level of significance. From the particular point of view of Tether investment, this result significantly confirmed no evidence of absolute stability of Tether, in line with that of Hoang and Baur (2021).

After filtering each return process using the aforementioned marginal model, the resulting standardized errors were transformed into pseudo observations uniformly distributed over $[0,1]$ through the probability integral transformation. All possible pairs of these pseudo observations of oil, Bitcoin, Tether, and the US dollar are plotted in Fig. 5. Among oil, Bitcoin, and Tether, we can observe that Tether (Bitcoin) was an instrument exhibiting the strongest correlation with the others before (during) COVID-19. Meanwhile, when taking oil, Bitcoin, and the US dollar into consideration, the latter was highly correlated with the former two instruments before and during the COVID-19 outbreak. This result is consistent with what we have observed in Fig. 3, although their return data have experienced filtration and transformation. In this step, we employed the correlation coefficient of Kendall’s $\tau$ rather than Pearson’s $\rho$ due to the former’s invariance property under any increasing and nonlinear transformation. In the next step, we constructed a maximum spanning tree for oil, Bitcoin, and Tether, where a node representing Tether (Bitcoin) was at the center before (during) COVID-19. Another maximum spanning tree was also constructed for oil, Bitcoin, and the US dollar, where the latter was represented by a central node amidst the pre-COVID-19 and COVID-19 periods. Each maximum spanning tree formed a vine copula, whose graphical representation is depicted in Fig. 6 and whose estimated parameters are provided in Table A.2 in Appendix. We found that the vine copula method provided a sophisticated and flexible dependence model through different bivariate copulas attached to the edges connecting all pairs of instruments.

When adopting the methods of Reboredo (2013) and Talbi et al. (2021), where safe-haven properties were examined using copulas, we can observe from the first tree graphs in Fig. 6 that before COVID-19, Tether was capable of serving as a safe haven for oil but not for Bitcoin. This is because Tether and oil exhibited tail independence captured by a Frank copula, while Tether and Bitcoin co-moved together with evidence of lower-tail dependence accommodated by a 180°-rotated version of a Gumbel copula. Amid the COVID-19 outbreak, Tether presented its safe-haven capability for Bitcoin while Bitcoin failed to protect oil investors against extreme negative returns. Meanwhile, the USD was able to act as a safe-haven tool for oil and Bitcoin before and during COVID-19 because of the oil–USD and Bitcoin–USD tail independence. In this study, we were concerned with examining their safe-haven characteristics in terms of portfolio risk reduction opportunities discussed in the following subsection.

### 5.2 Risk quantification and portfolio risk reduction assessment

The main discussion of this paper is the assessment of risk reduction when combining oil, Bitcoin, and Tether (or the US dollar) into a portfolio. We carried out this task by computing the relative ratio ($RR$) in boldface is below one, indicating portfolio risk reduction. Meanwhile, the improvement was made by accounting for the estimated conditional coverage probability of the estimative VaR.
Table 3 summarizes the VaR and ES forecasts of oil, Bitcoin, Tether, and the US dollar before and during COVID-19 under normality and Student’s $t$ assumptions. From this table, we can observe various values of the estimative VaR forecasts ranging between 0.591 (0.058) and 8.573 (12.646) before (during) COVID-19. The US dollar and Tether possessed the lowest estimative VaR forecast before and during COVID-19, respectively. Each instrument’s estimative VaR forecast was found to increase as the COVID-19 outbreak progressed, except for that of Bitcoin.
Tether. Similar results can be observed from the columns reporting the improved VaR forecasts and the estimative and improved ES forecasts. These results provided a preliminary indication of the risk reduction abilities of these two less risky financial instruments, where Tether’s capability was more apparent over the COVID-19 period. In general, the improved VaR forecast appeared to be smaller in value than the estimative one for most instruments. Notable exceptions were oil and the US dollar, whose VaR forecast increased after being improved in COVID-19 times. The reason is that the conditional coverage probability of their estimative VaR forecast was below the 99% level of confidence.

In summary, our proposed improvement framework resulted in a VaR forecast with higher accuracy for all the instruments examined before and during COVID-19. Compared to the estimative version, this improved VaR forecast was found to have a conditional coverage probability closer to 99%. The closest distance between the conditional coverage probability and the 99% confidence level was reported in the US dollar before and during COVID-19. This means that the US dollar’s improved VaR exhibited the best forecasting performance. Similarly, improving the ES forecasting method resulted in a decrease in the ES forecast value, except for oil and the US dollar over the COVID-19 period.

Fig. A.2. VaR and ES forecasts at the 99% level of confidence for a portfolio consisting of various allocations of $w_B$ to Bitcoin, $w_T$ to Tether, and the remaining $1 - w_B - w_T$ to oil, whose marginal returns follow AR(1)-APARCH(1,1) models with a normal or Student’s $t$ innovation. Note: Both $w_B$ and $w_T$ vary between 0% and 100%, but $w_B + w_T \leq 100%$. 
period. Furthermore, the improved ES forecast appeared to outperform its estimative version for most instruments since the resulting $p$-value of ES backtesting increased after the improvement.

The estimative risk measures and their improved version were then applied to quantify portfolio risk (reduction). More specifically, we made a comparison between the risk measure forecasts for (i) a portfolio of oil before and after being combined with Bitcoin and (ii) a portfolio made up of oil and Bitcoin before and after being mixed with Tether (or the US dollar). Through the first approach, we attempted to examine the safe-haven role of Bitcoin against oil in terms of its risk reduction ability. Meanwhile, the capability of Tether to act as a safe haven for both oil and Bitcoin was assessed and then compared with that of the US dollar through the second approach. The abovementioned portfolios were equally (optimally) weighted, as presented in the upper (lower) panel of Table 4, where the dependence among their components was taken into account using the (vine) copula model.

We can observe from Table 4 that the risk measure forecasts of portfolios with equal weights in times of COVID-19 were higher than those in pre-COVID-19 times. The equally weighted portfolio of oil and Bitcoin was found to experience the highest increase in its risk; when Tether or the US dollar was included, the increase became much lower. Similar to the previous results of individual risk quantification, the risk of each portfolio was measured using the improved version of the VaR and ES forecasts more accurately than their corresponding estimative version. Relative to holding oil only, combining an allocation of 50% to oil and the remaining 50% to Bitcoin before COVID-19 resulted in an increase in portfolio risk, as quantified using the relative ratio (RR) of most of the risk measures. Meanwhile, designing an optimally weighted portfolio containing oil and Bitcoin was found to substantially reduce the extreme risk faced during the pre-COVID-19 and COVID-19 periods. The reason is that the resulting relative ratio assessed using any risk measure under any model setting was below one. The ability of Bitcoin to reduce portfolio risk was more pronounced amid the COVID-19 outbreak, which was considerably more when the improved VaR and ES forecasts were taken into consideration. These findings provided an initial indication of the inconsistency of Bitcoin’s safe-haven role for oil.

To help Bitcoin protect the oil commodity against extreme risk, Tether was suitable to combine with them. Table 4 demonstrates that the inclusion of Tether allowed us to make portfolio risk significantly reduced under any assumption, which was considerably more apparent when the portfolio was optimally designed. More specifically, relative to holding the oil–Bitcoin portfolio only, creating an equally weighted portfolio of oil, Bitcoin, and Tether before (during) COVID-19 resulted in a relative ratio of portfolio risk measure forecasts of about 67.6% (66.6%). Meanwhile, when considering an optimal portfolio weight, a reduction in portfolio risk was achieved up to 27.9% (0.6%) before (during) the COVID-19 outbreak. The drastic risk reduction was sourced from a large portfolio allocation above 90% to Tether. These findings indicate that this most outstanding stablecoin significantly supported Bitcoin to provide portfolio risk reduction even when Bitcoin alone failed to serve as a single safe-haven tool for oil. On the other hand, the utilization of the US dollar before (during) COVID-19 resulted in a portfolio risk reduction of about 66.5% (66.0%) when the portfolio was equally weighted. This reduction decreased to 17.6% (12.8%) if the portfolio was designed using an optimal weight. This means that the risk reduction capability of the US dollar was better than that of Tether in the majority of cases. Tether was able to be a stronger safe-haven than the US dollar only when combining Tether with oil and Bitcoin using an optimal allocation in COVID-19 times.

For confirmation, we repeated portfolio risk quantification and risk reduction assessment for various Bitcoin, Tether, and the US dollar’s allocation weights ranging between 0% and 100%. Taking a portfolio of oil and Bitcoin into consideration first, we found from Fig. 7 that the risk of this mixed portfolio designed before COVID-19 using Bitcoin’s small allocation was lower than oil risk. The relative ratio of their risk measures quadratically went up as the allocation weight given to Bitcoin increased. This evidence highlighted the inconsistency of Bitcoin in acting as a safe-haven tool against oil, in line with what Syuhada et al. (2022) revealed from their study on the protection of several energy commodities against extreme risk in these markets. Surprisingly, combining oil and Bitcoin amid the COVID-19 period using any portfolio allocation tended to result in consistent portfolio risk reduction, as assessed using improved risk measure forecasts. The latter result is contrary to that of Syuhada et al. (2022), who considered a shorter COVID-19 period and an unimproved measure of portfolio risk.

Finally, when adding Tether into the portfolio of oil and Bitcoin, we conducted a similar empirical study by permitting both Bitcoin and Tether’s allocation weights to vary, as depicted in Fig. 8. We found that for any allocation given to Bitcoin, the risk measure forecasts of the oil–Bitcoin–Tether portfolio appeared to decrease as the weight allocated to Tether went up. The decrease occurred more rapidly during the COVID-19 outbreak, which was more apparent when employing improved measures of portfolio risk, similar to what we observe from the surface plot of the corresponding relative ratio. These findings confirmed that Tether consistently functioned as a safe haven for both oil and Bitcoin mixed into a portfolio with any allocation. This functionality of Tether was stronger amid extreme market turbulence in COVID-19 times. In addition, we can also draw a similar conclusion when replacing Tether with the US dollar; see Fig. 9.

5.3. Robustness check

To verify whether the findings of this study are robust under different model settings, we employed another asymmetric model for conditional heteroscedastic volatility, namely, Asymmetric Power ARCH (APARCH), and another dependence model based on a truncated vine copula approach. These models were implemented to reassess portfolio risk reduction with equal weight, optimal weight, and various weights. We can observe from Table A.3 in Appendix that Bitcoin was not found to reduce portfolio risk before COVID-19 when the oil–Bitcoin portfolio was equally weighted. After adding Tether or the US dollar into this portfolio, the risk reduction was achieved before and during COVID-19. However, the capability of Tether was weaker than that of the US dollar, except for the case when optimal weight above 99% was given to Tether during COVID-19. From Fig. A.1 in Appendix, we found that the inconsistency of Bitcoin in reducing portfolio risk was present, particularly before COVID-19. Nevertheless, extreme portfolio risk was able to be reduced consistently by Tether and the US dollar before and during the COVID-19 outbreak; see Figs. A.2 and A.3 in Appendix. In summary, these results derived using truncated vine copula-based AR(1)-APARCH(1,1) models were consistent with those previously obtained using vine copula-based AR(1)-GJR-GARCH(1,1) models.

6. Conclusions

Global oil demand experienced a dramatic decline in response to the COVID-19 outbreak. This situation made oil price plummet and thus led oil investors to seek alternative investments to protect them against extreme losses. This study has attempted to examine the potential ability of Bitcoin to act as a safe haven for this commodity. Due to its highly volatile movements, Tether, designed to have a stable fluctuation, was also involved together. From the viewpoint of risk management, we assessed their safe-haven characteristics in terms of portfolio risk reduction opportunities and compared these with the US dollar’s capability. The assessment was performed using the VaR and ES risk measures and their improved version with corrected forecast accuracy.
Fig. A.3. VaR and ES forecasts at the 99% level of confidence for a portfolio consisting of various allocations of $w_B$ to Bitcoin, $w_U$ to USD, and the remaining $1 - w_B - w_U$ to oil, whose marginal returns follow AR(1)-APARCH(1,1) models with a normal or Student’s $t$ innovation. Note: Both $w_B$ and $w_U$ vary between 0% and 100%, but $w_B + w_U \leq 100\%$.

By adopting vine copula-based AR(1)-GJR-GARCH(1,1) models, the empirical results revealed that combining oil and Bitcoin into a portfolio with an optimal design resulted in a reduction in portfolio risk compared to holding oil only before and during COVID-19. However, an allocation weight of 50% or larger given to Bitcoin failed to produce risk reduction opportunities before COVID-19. This indicates the inconsistency of Bitcoin’s safe-haven capability, in line with the finding of Syuhada et al. (2022). In times of COVID-19, oil risk was able to be reduced consistently by Bitcoin when assessed using improved measures of risk. Meanwhile, the addition of Tether into the oil–Bitcoin portfolio with any allocation highlighted its consistent ability to eliminate extreme portfolio risk faced before and during the COVID-19 period. This is contrary to a conclusion drawn by Conlon et al. (2020) that revealed that Tether was an inconsistent safe-haven tool for equities. Interestingly, this capability of Tether was more apparent amid extreme bear markets due to COVID-19, suggesting that the COVID-19 outbreak impacted not only market behaviors but also the performance of safe-haven instruments. These impacts perhaps are related to the influence of the COVID-19 risk level on the return values.
and consequently on the portfolio risk forecasts, as empirically revealed in this study.

Tether’s risk reduction ability was, however, not found to be as strong as that of its peg (i.e., the US dollar) in most cases based on both of our copula and risk reduction analyses. This is in line with the result of the risk reduction analysis of Wang et al. (2020) that did not involve oil but is contrary to the result of their dummy variable regression analysis. Furthermore, the capability of this USD-pegged stablecoin to consistently play a safe-haven role against extreme downward markets suggested its unstable movement with persistent volatility, in line with its inverted leverage effect more pronounced than that of Bitcoin. This evidence is exactly contrary to what its name implies and to the initial purpose of its creation, i.e., being a stable cryptocurrency, as argued by Hoang and Baur (2021) and Baur and Hoang (2021). Nevertheless, its position in the financial industry enriches alternative classes of consistent safe-haven instruments. Investors can choose it to help them avoid more acute losses resulting from their investments, particularly in the face of crises or pandemics that can inflict an economic impact on financial markets (Goodell, 2020). Furthermore, our proposed improvement framework can lead them to design improved investment strategies to mitigate the risk they are exposed to and to prepare the reasonably accurate amount of risk-based capital requirements.

Based on our findings above, investors and policymakers should carefully select a safe-haven (crypto) asset for maintaining and protecting oil commodity values against extreme bear markets due to the ongoing COVID-19 outbreak. If Bitcoin is taken into consideration, it is worth noting that an excessive allocation given to Bitcoin can result in an increase in oil portfolio risk. Accordingly, they need to optimally design a portfolio consisting of oil and Bitcoin to achieve portfolio risk reduction. To obtain better protection against extreme risk in the oil market, Tether can be added into the oil–Bitcoin portfolio using any allocation. Although portfolio risk is able to be reduced consistently under this scheme, investors and policymakers would be exposed to a higher possibility of financial instability sourced from not only Bitcoin but also Tether. The latter, which belongs to the class of stablecoins, was not found to be as stable as its name. This means that there is a trade-off between consistent risk reduction and financial stability when involving Tether. As an alternative, we recommended replacing this stablecoin with its underlying asset, i.e., the US dollar, which exhibited higher possibility of financial instability sourced from not only Bitcoin but also Tether. The latter, which belongs to the class of stablecoins, such as Tether.

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