Conformal anomalies of CFT’s with boundaries

D.V. Fursaev

Dubna State University
Universitetskaya str. 19
141 980, Dubna, Moscow Region, Russia
and
the Bogoliubov Laboratory of Theoretical Physics
Joint Institute for Nuclear Research
Dubna, Russia

Abstract

The trace anomaly of conformal field theories in four dimensions is characterized by ‘a’ and ‘c’-functions. The scaling properties of the effective action of a CFT in the presence of boundaries is shown to be determined by $a$, $c$ and two new functions (charges) related to boundary effects. The boundary charges are computed for different theories and different boundary conditions. One of the boundary charges depends on the bulk $c$ charge.
1 Anomaly of the effective action

Conformal anomalies are well-known. In a conformal field theory (CFT) in four dimensions the expectation value of the trace of stress energy tensor can be written in the following universal form [1]:

\[
\langle T^\mu_\mu \rangle = -2a E - c I - \frac{c}{24\pi^2} \nabla^2 R ,
\]

where \( E \) is the volume density of the Euler characteristics of the underlying background manifold \( M \), and \( C_{\mu\nu\lambda\rho} \) is the Weyl tensor of \( M \). One can also define the integral anomaly by the variation of the effective action \( W \),

\[
A \equiv \partial_\sigma W[e^{2\sigma}g_{\mu\nu}]_{\sigma=0} = \int_M \sqrt{g}d^4x \langle T^\mu_\mu \rangle ,
\]

under scaling with a constant factor \( \sigma \). The right hand side of (1.4) relates \( A \) to the trace anomaly (1.1) and holds on a closed manifold.

By analogy with two-dimensional CFT’s constants \( a \) and \( c \) are called charges, or \( a \)-function and \( c \)-function, when they are allowed to run under renormalization. There are arguments [2] that \( a \)-function changes monotonically when going from a critical point to a critical point.

The aim of this work is to study integral anomaly of the effective action (1.4) when \( M \) has a boundary. In this case the right hand side of (1.4) cannot be written solely as the integral of the trace anomaly since new boundary terms may appear. An experience with two dimensional CFT’s shows that boundary terms in some quantum quantities, such as a \( g \)-function in the entropy, may be interesting from the point of view of the renormalization group [3], [4].

We study different CFT’s in four dimensions with the boundary conditions which do not break the conformal invariance on the classical level and show that the integral anomaly has the following universal structure:

\[
A = -2a \chi_4 - c i + q_1 j_1 + q_2 j_2 .
\]

Quantities \( \chi_4 \), \( i \), \( j_1 \) and \( j_2 \) are scale invariant functionals, \( \chi_4 \) is the Euler characteristics of the background manifold, \( i \) is the integral of \( I \) over \( M \), see (1.3). The boundaries result in two new terms with constants \( q_1 \) and \( q_2 \) (the boundary charges). The boundary functionals in (1.3) are

\[
j_1 = \frac{1}{16\pi^2} \int_{\partial M} \sqrt{H}d^3x C_{\mu\nu\lambda\rho}N^\nu N^\rho \hat{K}^{\mu\lambda} \equiv \frac{1}{16\pi^2} \int_{\partial M} \sqrt{H}d^3x G_1 ,
\]

\[
j_2 = \frac{1}{16\pi^2} \int_{\partial M} \sqrt{H}d^3x \text{Tr}(\hat{K}^3) \equiv \frac{1}{16\pi^2} \int_{\partial M} \sqrt{H}d^3x G_2 .
\]

We use the following notations: \( g_{\mu\nu} \) is the metric of the background manifold \( M \), the metric induced on the boundary \( \partial M \) of \( M \) is \( H_{\mu\nu} = g_{\mu\nu} - N_\mu N_\nu \), \( N^\nu \) is a unit outward
Table 1: Charges in the anomaly of the effective action

| Theory          | a    | c    | q₁   | q₂    | boundary condition |
|-----------------|------|------|------|-------|--------------------|
| real scalar     | 1/360| 1/120| 1/15 | 2/35  | Dirichlet          |
| real scalar     | 1/360| 1/120| 1/15 | 2/45  | Robin              |
| Dirac spinor    | 1/360| 1/20  | 2/5  | 2/7   | mixed              |
| gauge Boson     | 31/180| 1/19  | 12/15| 16/35 | absolute           |
| gauge Boson     | 31/180| 1/19  | 12/15| 16/35 | relative           |

pointing normal vector to ∂M, \( K_{\mu\nu} = H_\mu^\lambda H_\nu^\rho N_{\lambda\rho} \) is the extrinsic curvature tensor of ∂M, \( \hat{K}_{\mu\nu} = K_{\mu\nu} - H_{\mu\nu} K/3 \) is a traceless part of \( K_{\mu\nu} \). Conformal invariance of \( j_1, j_2 \) follows from the fact that \( \hat{K}_{\mu\nu} \) transforms homogeneously under conformal transformations. Definitions of the Riemann tensor, the Ricci tensor, the scalar curvature, respectively, are \( R_{\lambda\mu\nu\rho} = -\Gamma_{\mu\nu,\rho}^\lambda + ... \), \( R_{\mu\nu} = R_{\mu\lambda\nu}^\lambda, R = R_{\mu\nu}^\mu \). Our definitions of \( R \) and \( K_{\mu\nu} \) coincide with those used by Dowker and Schofield in [5] and by Vassilevich in [6]. The only difference with those works is in the sign of \( R_{\mu\nu\rho}^\lambda \) and direction of \( N^\mu \). In our paper, in [5], and in [6] the scalar curvature on \( S^n \) is positive. The curvatures constructed with the metric of \( \partial M \) will be denoted as \( \hat{R}_{\mu\nu\rho}^\lambda, \hat{R}_{\mu\nu}, \hat{R} \).

Our results for the boundary charges in (1.5) are listed in Table 1, \( q_2 \) can depend on the type of the boundary conditions.

It should be noted that (1.5) for a scalar field with the Dirichlet boundary condition follows from results by Dowker and Schofield [5]. Table 1 agrees with [5] for this case. Anomalous rescalings of the effective action of Dirac fields and gauge bosons have been also studied by Moss and Poletti [7] for Einstein spaces with boundaries.

To our knowledge the model independent nature of Eq. (1.5) has not been emphasized so far.

In Sec. 2 we relate the anomaly to known computations of boundary terms in the corresponding heat kernel coefficients. Anomalies in the models presented in Table 1 are discussed in Sec. 3. Concluding remarks are given in Sec. 4. We point out a universal relation between \( c \) and \( q_1 \) (valid at least for all above models). It hints that \( q_1 \) is not an independent new charge.

## 2 The heat coefficient

We use the relation between the anomaly and the heat coefficient of a Laplace operator \( \Delta = -\nabla^2 + X \) for the corresponding conformal theory

\[
A = \eta A_4,
\]  
(2.1)
where $\eta = +1$ for Bosons and $\eta = -1$. We ignore in (3.4) a possible contribution of zero modes of $\Delta$. The heat coefficients for the asymptotic expansion of the heat kernel of $\Delta$ are defined as

$$K(\Delta; t) = \text{Tr} e^{-t \Delta} \simeq \sum_{p=0} A_p(\Delta) t^{(p-4)/2}, \quad t \to 0.$$  \hspace{1cm} (2.2)

If the classical theory is scale invariant the heat coefficient $A_4$ is a conformal invariant, see e.g. [8]. Therefore $A_4$ can be represented as a linear combination of conformal invariants constructed of the geometrical characteristics of $M$, $\partial M$ and embedding of $\partial M$ in $M$. On dimensional grounds, these invariants (in four dimensions) are $\chi_4$, $i$, $j_1$ and $j_2$. The bulk part of $A_4$ is determined by $\chi_4$ and $i$, and it is well known.

We are interested in boundary terms in $A_4$. To find them one should take into account the boundary term in $\chi_4$. The definition of the Euler number for a four-dimensional manifold with a boundary is as follows:

$$\chi_4 = B_4[\mathcal{M}] + S_4[\partial \mathcal{M}] + S_4[\partial \mathcal{M}].$$  \hspace{1cm} (2.3)

$$B_4[\mathcal{M}] = \int_{\mathcal{M}} \sqrt{g} d^4 x \ E,$$  \hspace{1cm} (2.4)

$$S_4[\partial \mathcal{M}] = \frac{1}{32 \pi^2} \int_{\partial \mathcal{M}} \sqrt{H} d^3 x Q,$$  \hspace{1cm} (2.5)

$$Q = -8 \left[ \det K_{\mu\nu} + \hat{G}_{\mu\nu} K_{\mu\nu} \right], \quad \hat{G}_{\mu\nu} = \hat{R}_{\mu\nu} - \frac{1}{2} H_{\mu\nu} \hat{R}.$$  \hspace{1cm} (2.6)

Derivation of these formulae can be found in [5]. Some values of $\chi_4$ are: $\chi_4 = 1$, if $\mathcal{M}$ is a domain in $\mathbb{R}^3$ with the spherical boundary $\partial \mathcal{M} = S^3$, $\chi_4 = 0$, if $\mathcal{M}$ is a domain of a torus $S^1 \times \mathbb{R}^3$ with a boundary $\partial \mathcal{M} = S^1 \times S^2$.

The boundary part of $A_4$, therefore, is

$$A_4^{bd} = \eta(q_1 j_1 + q_2 j_2 - 2a S_4).$$  \hspace{1cm} (2.7)

It is the aim of computations to check that the coefficient at $S_4$ does equal $-2\eta a$, where $a$ is the same constant which appears in the trace anomaly (1.1).

Appearance of $S_4$ among counter terms in a one-loop effective action dates back to works in 1980’s, see [10]. If $\mathcal{M}$ is a domain of a flat Euclidean background and $\partial \mathcal{M} = S^3$, one easily finds for (1.5) that $A = -2a$. In this case the anomaly is solely determined by $S_4$. Mode-by-mode computations of the anomaly for the spinor and gauge fields have been done in [11],[12].

Our starting point is formula (5.33) from Vassilevich’s review [6] for $A_4$ for mixed boundary conditions. We put there $f = 1, \ n = 4$. The boundary conditions are

$$(\nabla_N - S)\Pi_+ \phi = 0 \quad , \quad \Pi_- \phi = 0,$$  \hspace{1cm} (2.8)

where $\nabla_N = N^\mu \nabla_\mu$, $\Pi_\pm$ are corresponding projectors, $\Pi_+ + \Pi_- = 1$, definition of $S$ coincides with [6].

After converting total derivatives in the bulk into surface terms and some algebra the boundary part $A_4^{bd}$ of $A_4$ can be written as

$$A_4^{bd} = \frac{1}{(4\pi)^2} \int_{\partial \mathcal{M}} \sqrt{H} d^3 x \ Tr C_4.$$  \hspace{1cm} (2.9)
\[ C_4 = \Pi_+ C_4^+ + \Pi_- C_4^- + C_4^{++} , \quad (2.10) \]

\[ C_4^+ = -\frac{1}{360} Q + \frac{1}{15} G_1 + \frac{2}{45} G_2 - \frac{1}{3} \left( X - \frac{1}{6} R \right) K + \frac{1}{2} \nabla_N \left( X - \frac{1}{6} R \right) + \]

\[ \frac{4}{3} \left( S \Pi_+ + \frac{1}{3} K \right)^3 - 2 \left( X - \frac{1}{6} R \right) S + \left( S + \frac{1}{3} K \right) \left( \frac{2}{15} \text{Tr} K^2 - \frac{2}{45} K^2 \right) , \quad (2.11) \]

\[ C_4^- = -\frac{1}{360} Q + \frac{1}{15} G_1 + \frac{2}{35} G_2 - \frac{1}{3} \left( X - \frac{1}{6} R \right) K + \frac{1}{2} \nabla_N \left( X - \frac{1}{6} R \right) , \quad (2.12) \]

\[ C_4^{++} = -\frac{1}{3} (\Pi_+ - \Pi_-) \Pi_{+;a} \Omega_{a\mu} N_{\mu} - \]

\[ \frac{2}{15} \Pi_{+_a} \Pi_{+_a} K - \frac{4}{15} \Pi_{+_a} \Pi_{++b} K^{ab} - \frac{4}{3} \Pi_{+_a} \Pi_{+_a} \Pi^{+} S . \quad (2.13) \]

Here we use 'flat' indices \( a, b \) in the tangent space to the boundary, see [6]. \( \Omega_{\mu\nu} \) is the field strength of the connection defined by (2.10) in [6]. The notations are \( \text{Tr} K^m = K_{\mu_1}^{\mu_2}...K_{\mu_p}^{\mu_p} \), \( K = \text{Tr} K \).

In deriving (2.10)-(2.13) we used Gauss-Codazzi identities and the relations:

\[ G_1 = R_{\mu\rho\lambda\nu} K^{\mu\lambda} N^{\nu} N^\rho - \frac{1}{2} R_{\mu\nu} (N^{\mu} N^\nu K + K^{\mu\nu}) + \frac{1}{6} K R \ , \quad (2.14) \]

\[ G_2 = \text{Tr} K^3 - K \text{Tr} K^2 + \frac{2}{9} K^3 \ , \quad (2.15) \]

\[ Q = 8 R_{\mu\nu\lambda\rho} K^{\mu\lambda} N^{\nu} N^\rho - 8 R_{\mu\nu} (N^{\mu} N^\nu K + K^{\mu\nu}) + 4 K R + \]

\[ \frac{8}{3} K^3 + \frac{16}{3} \text{Tr} K^3 - 8 K \text{Tr} K^2 \ , \quad (2.16) \]

which can be easily obtained from definitions (1.6), (1.7), (2.6).

Representation of the boundary terms in form (2.10)-(2.13) follows Moss and Poletti [7]. Calculations of \( A_4 \) in case of boundaries have been done by several authors. The key paper is by Branson and Gilkey [9]. A complete list of references can be found in [6].

### 3 Computations

#### 3.1 Conformal scalar field

In this case \( X = 1/6 \). For the Dirichlet condition \( \Pi_+ = 0 \), \( C_4^{++} = 0 \), and the boundary charges are determined by \( C_4^- \), (2.12).

Conformally invariant Robin condition requires \( S = -K/3 \), \( \Pi_+ = 1 \). Then again \( C_4^{++} = 0 \), and boundary charges follow from (2.11).

#### 3.2 Massless Dirac spinor

In case of a massless Dirac field \( \psi \) the operator is \( \Delta^{(1/2)} = (i\gamma^\mu \nabla_\mu)^2 \). The boundary conditions are mixed ones,

\[ \Pi_- \psi \mid_{\partial M} = 0 \ , \ (\nabla_N + K/2) \Pi_+ \psi \mid_{\partial M} = 0 \ , \quad (3.1) \]
where $\Pi_{\pm} = \frac{1}{2}(1 \pm i\gamma_s N^{\mu}\gamma_{\mu})$, and $\gamma_s$ is a chirality gamma matrix. Therefore, $X = R/4$, $S = -\Pi_{\pm} K/2$.

The physical meaning of (3.1) is that the normal component of the spinor current vanishes on the boundary. Condition (3.1) does not break conformal invariance. The strength of the spin connection is

$$\Omega_{\mu\nu} = \frac{1}{4} R_{\mu\nu\rho\sigma} \gamma^\sigma \gamma^\rho .$$

The rest computation is straightforward. One finds

$$\text{Tr}(\Pi_{\pm} C^+_4 + \Pi_{-} C^-_4) =$$

$$r \left( -\frac{1}{360} Q + \frac{1}{15} G_1 + \frac{1}{315} G_2 + \frac{1}{72} KR + \frac{1}{1620} K^3 - \frac{1}{90} K \text{Tr}(K^2) \right) ,$$

(3.2)

$$\text{Tr}(C^+_4) = r \left( -\frac{1}{12} R_{\mu\nu\lambda\rho} K^{\mu\lambda} N^\nu N^\rho - \frac{1}{15} \text{Tr}(K^3) + \frac{1}{20} K \text{Tr}(K^2) \right) ,$$

(3.3)

where $r = 4$ is the number of components of the Dirac spinor in four dimensions. In computing $C^+_4$ one uses the relation

$$\Pi_{\pm,\alpha} = \frac{i}{2} \gamma^b \gamma_a K^b .$$

The data of Table 1 follow from the sum of (3.2) and (3.3) and relations (2.14)-(2.16).

### 3.3 Gauge boson

By following [6] we consider the quantization of an Abelian gauge field $V_{\mu}$ in the Lorentz gauge $\nabla V = 0$. The results of Table 1 are valid for a gauge invariant combination

$$A = A_4(\Delta^{(1)}) - 2A_4(\Delta^{(ab)}) ,$$

(3.4)

where $(\Delta^{(1)})^\nu_{\mu} = -\nabla^2 \delta^\nu_{\mu} + R^\nu_{\mu}$ is the vector Laplacian, and $\Delta^{(ab)} = -\nabla^2$ is the Laplacian for ghosts.

We study two sorts of boundary conditions. The absolute (or electric [7]) boundary condition:

$$N^\mu F_{\mu\nu} |_{\partial M} = 0 ,$$

(3.5)

and relative (or magnetic [7]) boundary condition:

$$N^\mu \tilde{F}_{\mu\nu} |_{\partial M} = 0 ,$$

(3.6)

where $F_{\mu\nu} = \nabla_{\mu} V_{\nu} - \nabla_{\nu} V_{\mu}$, and $\tilde{F}_{\mu\nu}$ is the Hodge dual of $F_{\mu\nu}$.

The both conditions are manifestly gauge and conformally invariant.

Condition (3.5) requires that components of an electric field normal to $\partial M$ and components of the magnetic field which are tangential to $\partial M$ vanish on the boundary. This means that the boundary is a perfect conductor. In condition (3.6) the roles of electric and magnetic fields are interchanged.

In the Lorentz gauge the absolute boundary condition is reduced to the following conditions on the vector field and ghosts [6]:

$$(\delta^\nu_{\mu} \nabla N + K^\nu_{\mu}) V^+_{\nu} |_{\partial M} = 0 , \quad V^-_{\mu} |_{\partial M} = 0 ,$$

(3.7)
where $V^\pm = \Pi^\pm V$, and

$$(\Pi_+)^\nu_\mu = \delta^\nu_\mu - N^\nu_\mu$$, $(\Pi_-)^\nu_\mu = N^\nu_\mu$.

(3.8)

The corresponding boundary condition for a ghost field $c$ is

$$\partial N^c \mid_{\partial M} = 0.$$ (3.9)

The relative condition in the same gauge is

$$(\nabla N + K)V^\pm_\mu \mid_{\partial M} = 0$$, $V^\mu_- \mid_{\partial M} = 0$$,

$$(\Pi_+)^\nu_\mu = N^\nu_\mu$$, $(\Pi_-)^\nu_\mu = \delta^\nu_\mu - N^\nu_\mu$.

(3.10)

By taking into account that $X^\nu_\mu = R^\nu_\mu$, $(\Omega^\lambda_\rho)^\nu_\mu = R^\rho_\mu \delta^\lambda_\nu$, for the vector component with the absolute condition (3.7), (3.8) ($S^\mu_\nu = -K^\nu_\mu$) we find

$$\text{Tr}(\Pi^+ C^+_4 + \Pi^- C^-_4) =$$

$$\left(\frac{N^\mu R^a_\mu}{6} - \frac{4}{360} Q + \frac{4}{15} G_1 - \frac{8}{15} G_2 - \frac{4}{9} KR + R^\mu_\mu (N^\nu_\mu N^\nu K + K^\mu_\nu)\right),$$

(3.13)

$$\text{Tr}(C^+_4) = \frac{2}{3} R^\mu_\nu \lambda \rho K^\lambda \rho N^\nu N^\rho + \frac{4}{5} \text{Tr}(K^3) - \frac{4}{15} K \text{Tr}(K^2).$$

(3.14)

For the ghost part with condition (3.9) $S = 0$ and

$$C_4 = C^+_4 = \frac{1}{12} R_N - \frac{1}{360} Q + \frac{4}{15} G_1 + \frac{2}{45} G_2 + \frac{1}{18} KR + \frac{14}{405} K^3 + \frac{2}{45} K \text{Tr}(K^2),$$

(3.15)

For the vector component with relative conditions (3.10), (3.11) ($S^\mu_\nu = -K^\nu_\mu$)

$$\text{Tr}(\Pi^+ C^+_4 + \Pi^- C^-_4) = -(N^\mu R^a_\mu)^a + \frac{1}{6} R_N$$

$$- \frac{4}{360} Q + \frac{4}{15} G_1 + \frac{8}{45} G_2 - \frac{4}{9} KR + R^\mu_\mu (N^\nu_\mu N^\nu K + K^\mu_\nu) - \frac{34}{405} K^3 - \frac{4}{45} K \text{Tr}(K^2),$$

(3.16)

$$\text{Tr}(C^+_4) = -\frac{2}{3} R^\mu_\nu \lambda \rho K^\mu \lambda N^\nu N^\rho - \frac{8}{15} \text{Tr}(K^3) + \frac{16}{15} K \text{Tr}(K^2).$$

(3.17)

For the ghost part with condition (3.12) $S = 0$ and

$$C_4 = C^-_4 = \frac{1}{12} R_N - \frac{1}{360} Q + \frac{1}{15} G_1 + \frac{2}{35} G_2 + \frac{1}{18} KR.$$

(3.18)

In deriving (3.13), (3.16) we used the identity

$$\frac{1}{2} R_N = (N^\mu R^a_\mu)^a + KR^\mu_\nu N^\nu N^\rho - R^\mu_\mu K^\mu_\nu + N^\mu N^\nu N^\lambda R^\mu_\nu \lambda.$$
4 Concluding remarks

The aim of this paper was to demonstrate a model independent form of the integral anomaly (1.5) in the presence of boundaries and obtain specific values of the boundary charges for some typical CFT’s. It would be important now to study evolution of the boundary charges under the renormalization group.

Computations of boundary charges for other models can be continued along the lines of the present paper. Extensions of (1.5) to higher dimensional CFT’s, say to 6 dimensions, are possible. Since the number of scale invariant structures is increasing we expect more boundary charges. A principal challenge here is the knowledge of boundary terms in the heat coefficient \( A_6 \).

An interesting issue is a possible relation between bulk charges \( a, c \) and boundary charges \( q_1, q_2 \). A conjecture of [13] is that \( q_1 \) and \( c \) may be related. Indeed, all models presented in Table 1 satisfy a universal relation

\[
q_1 = 8c.
\] (4.1)

We leave a general proof and implications of (4.1) for a future analysis.

It should be noted that formula (1.5) was also discussed in a recent work [14] which appeared several days earlier of the present publication.

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References

[1] M.J. Duff, *Observations on Conformal Anomalies*, Nucl. Phys. **B125** (1977) 334.

[2] Z. Komargodski, A. Schwimmer, *On Renormalization Group Flows in Four Dimensions*, JHEP **1112** (2011) 099, e-Print: arXiv:1107.3987 [hep-th].

[3] I. Affleck and A. Ludwig, *Universal Noninteger 'Ground State Degeneracy’ in Critical Quantum Systems*, Phys. Rev. Lett. **67** (1991) 161-164.

[4] D. Friedan and A. Konechny, *On the Boundary Entropy of One-dimensional Quantum Systems at Low Temperature*, Phys. Rev. Lett. **93** (2004) 030402, e-Print: hep-th/0312197.

[5] J.S. Dowker, J.P. Schofield, *Conformal Transformations And The Effective Action In The Presence Of Boundaries*, J. Math. Phys. **31** (1990) 808.

[6] D.V. Vassilevich, *Heat Kernel Expansion: User’s Manual*, Phys. Rept. **388** (2003) 279-360, e-Print: hep-th/0306138.

[7] I.G. Moss, S.J. Poletti, *Conformal anomalies on Einstein spaces with boundary*, Phys. Lett. **B333** (1994) 326-330, gr-qc/9405044.

[8] D. Fursaev and D. Vassilevich, *Operators, Geometry and Quanta: Methods of Spectral Geometry in Quantum Field Theory*, Springer Series 'Theoretical and Mathematical Physics', Springer, 2011.

[9] T.B. Branson and P. B. Gilkey, Commun. Part. Diff. Eqs. **15** (1990) 245.

[10] E.S. Fradkin and A.A. Tseytlin, *Renormalizable asymptotically free quantum theory of gravity*, Nucl. Phys. **B201** (1982) 469-491.

[11] P.D. D'Eath and G. Esposito, *Local boundary conditions for the Dirac operator and one loop quantum cosmology*, Phys. Rev. **D43** (1991) 3234-3248, gr-qc/9507004.

[12] G. Esposito, A.Yu. Kamenshchik, I.V. Mishakov, G. Pollifrone, *Euclidean Maxwell theory in the presence of boundaries. Part 2*, Class. Quant. Grav. **11** (1994) 2939-2950, gr-qc/9506061.

[13] S.N. Solodukhin, a private communication.

[14] C.P. Herzog, Kuo-Wei Huang, K. Jensen, *Universal Entanglement and Boundary Geometry in Conformal Field Theory*, e-Print: arXiv:1510.00021 [hep-th].