3D modeling of an object located in the underwater robot's manipulator workspace using a point cloud

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Abstract. Operations involving direct contact of the robot's gripping device with surrounding objects are among the hardest in the field of robotics engineering. Therefore, minimizing the risk of tool damage and processing disruption is important. This can be achieved by using the information about the position of the object in question relative to the grip and its geometric parameters. In this paper, a method of describing an object located in the workspace of an underwater robot with solids of revolution is proposed. It is done using a noisy and incomplete point cloud, which is obtained with a stereoscopic system located on the terminal link of the underwater robot's manipulator.

1. Introduction

With regards to scene modeling, processing television images obtained from the stereoscopic system allows to construct a three-dimensional map of the workspace and solve the inverse kinematics problem as well as recognize the geometry of some objects present in front of the underwater robot. This approach allows us to classify objects based on configuration and understand if it is possible to carry out intended actions (e.g., information about the size of an object is needed to determine how much the grip should open to grab it).

Authors propose a 3D modeling algorithm based on depicting a point cloud with a surface by dividing it into cross line segments. In the paper, results of field experiments are presented.

2. Overview of 3D modeling method

A lot of methods of general-purpose shape reconstruction have been developed to date [1], [2], [3]. The first 3D modeling method that was common in computer graphics is polygonal modeling. Delaunay triangulation algorithm has been widely spread [3], [4], as well as the Ball-Pivoting Algorithm [5]. The accuracy of modeling from input points depends on the number of polygons. Polygonal modeling is characterized by its calculating speed. It is used in many automated designing systems and virtual area development thanks to the possibility to use many operations to define and edit objects' geometry. Nevertheless, big polygonal network development requires the processing of growing amounts of data due to the necessity of network connectivity.

Voxels-based modeling is also common. It is usually used for visualization of technical objects and medical data, terrain representation in computer games and simulations. Voxel representation takes up a lot of space due to each voxel containing a description of the whole area of space where the 3D object is located. To lessen the amounts of data, it is most common to use octrees [4].
Application of NURBS allows for the construction of smooth complex curvy surfaces with small amounts of control vertices as well as reducing the amount of memory needed for shape storage. NURBS-based modeling is widely used in CAD [5], animation [6], [7], organic modeling, medicine. Authors [8], [9] have pointed out such obvious advantages of NURBS surface as smoothness and good adaptation to complex surfaces. Despite all these advantages, NURBS-models are difficult to edit and take longer to visualize.

The aforementioned methods of polygonal modeling are successfully used to reconstruct surfaces from a dense point cloud, but the results are notably worse if the input points are partially missing. According to [10], these cases of acquiring the incomplete point cloud are common in machine-building plants due to restrictions of measuring positions.

The task of surface reconstruction with an incomplete point cloud is solved by finding solids of revolution that satisfy the equipment and engineering machinery. In most cases, their cross-sections are circles formed by intersecting a plane that is perpendicular to the central axis. Therefore, it is possible to determine their geometric dimensions and to find the central axis by dividing the surface into cross-sections. This way planes and cylinders are drawn from point clouds during pipe reconstruction, and the shapes of the pipes are estimated [10], [11].

The principle of dividing the surface into segments was also successfully used by Stefan Bojarovski. In his work [12], the author has reconstructed a coral's surface by approximating the segments with a cylindrical 3D template according to [13].

Approximate description of an object with a solid of revolution allows us to gather information about the geometric dimensions and the shape of the object using an incomplete point cloud, meaning that it is impossible to scan the object from every angle and gather a point cloud with points from the whole object's surface. That's why the method of dividing a point cloud into primitives can handle measurement errors and has a flexible shape (variability) in terms of 3D model construction. Approximate description of an object with a shape also does not require the construction of the surface, which allows us to reduce computational resources needed from the onboard control system of the underwater robot. Therefore, this approach was taken as a baseline for the 3D modeling method proposed in this paper.

3. Mathematical description of the proposed method
Stereo-reconstruction of the workspace is done using two or more pictures of the workspace taken in several positions of the manipulator with an attached camera. The result is a three-dimensional point cloud, every point of which has its metric coordinates in the coordinate system of the manipulator.

Further, a pre-selection (rejection) of the points from the 3D point cloud is performed. The criteria are reachability of the point considering the size of the manipulator, belonging of the point to the area the object of interest is in, and other spatial limitations. The result of the re-projection of the point cloud onto a television frame is illustrated in Figure 1.

Figure 1. Initial image (a) and the same image with the point cloud projected onto it - a depth map (b)

Considering the fact that an operator sets the area of interest, and the object is located in the foreground, it is more rational to perform the segmentation of the point cloud obtained from the stereo pair based on the z-axis (remoteness) of the object.

For the points of the resulting point cloud, the following tasks are solved in succession:
• forming the cross segments of the area of interest;
• segmenting the points of the area of interest based on z-axis;
• calculating the center and the radius of the circumcircle of the cross segments;
• constructing the central axis of the solid of revolution;
• calculating the radius of the solid of revolution.

3.1. Formation of the cross segments of the area of interest
For every pixel that the object’s axis consists of, a straight-line equation is calculated for the lines perpendicular to the object’s axis. The segment of the calculated line is formed using the width of the area in pixels (Figure 2).

![Figure 2](image)

**Figure 2.** Area of interest and point cloud points belonging to it

Initial data for the 3D modeling task consists of the point cloud in the selected area of interest, which contains the coordinates for observed points of the scene obtained by the stereo pair.

3.2. Object segmentation based on the z-axis
Z-axis-based segmentation of an object in the area of interest is done according to the following list of actions repeated for each cross segment of the area of interest:

1. Calculating the mean \( z \)-coordinate for every pixel in the segment:

\[
\bar{z} = \frac{1}{k+1} \sum_{i=0}^{k} p_{z,\text{i}}^{c,n},
\]

where \( k \) - number of pixels in the segment, \( p_{z,\text{i}}^{c,n} \) - \( z \)-coordinate of the point in the point cloud corresponding to pixel \( i \) of segment \( n \).

2. If \( P_{z,\text{i}}^{c,n} > \bar{z} \) is true for a pixel, it is then removed from the set \( \{P^{c,n}\} \). The value is recalculated using equation (1). The result of the calculation is illustrated in figure 3, a. The "distant" pixels are shown in navy blue, while the "close" pixels are shown in light blue.

3. Calculating the standard deviation \( \sigma \) of the \( z \)-coordinate for the "close" pixels of the searching area:

\[
\sigma = \left( \frac{\sum_{i=0}^{k} (P_{z,\text{i}}^{c,n} - \bar{z})^2}{k + 1} \right)^{1/2}
\]

4. Repeating 1-3 to guarantee that all points corresponding to both "distant" and "close" background are removed. The number of repetitions is chosen based on the fact that to pick out the closest out of \( N \) objects it is necessary to make \( N-1 \) repetitions.

5. If (3) is true for a pixel, it is removed from the set \( \{P^{c,n}\} \).

\[
|P_{z,\text{i}}^{c,n} - \bar{z}| > 3 \cdot \sigma
\]
6. Pixels are added to the set \( \{ P_{c,n} \} \) based on three-sigma criteria.

Passing from left to right:

\[ (i,j) \in \{ P_{c,n} \} \leftarrow |(i,j)_z - P_{z,i-1}^c| \leq 3 \cdot \sigma \]  

4

Passing from right to left:

\[ (i,j) \in \{ P_{c,n} \} \leftarrow |P_{z,i+1}^c - (i,j)_z| \leq 3 \cdot \sigma. \]  

5

The result of the calculations described in 3-6 is illustrated in Figure 3, b. "Distant" pixels are shown in navy blue; pixels forming the set \( \{ P_{c,n}^c \} \) (from the calculations described in 5-6) are shown in yellow.

The resulting set \( \{ P_{c,n} \} \) is filtered using two thresholds:

- The required density \( (T_d) \) of the pixels that passed the filtration according to 1-6 compared to all points of the point cloud belonging to the longitudinal segment \( n \) of the object \( P_{L,n}^{cl} \);

- The required density \( (T_{de}) \) of the points of the point cloud belonging to the longitudinal segment \( n \) of the object \( P_{L,n}^{sg} \) compared to the whole length of the longitudinal segment.

\[
\{ P_{L,n}^{cl} \} \rightarrow \{ P \} \leftarrow \left( \frac{\sum_1 P_{i}^{L,n}}{P_{L,n}^{cl}} < T_d \cap \frac{P_{L,n}^{L,n}}{P_{L,n}^{sg}} < T_{de} \right) \]  

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The result of the filtration is illustrated in Figure 3, c. "Distant" pixels are shown in navy blue, and the segmented object is shown in red.

![Figure 3. Segmented object](image)

3.3. Calculating the center and the radius of the circumcircle of the cross segments

Each cross-segment \( n \) of the object on the screen \( (P_{c,n}^c) \) corresponds to a plane in the space.

For each cross-segment \( P_{c,n}^c (n = 0..P_{c,n}^c) \) the following list of actions is performed:

1. Defining the base points for the circle construction:

   - finding the first segmented point of the segment \( S(S_0, S_0, S_0) = P_{0}^{c,n} \);
   - finding the last point of the segment \( F(F_0, F_0, F_0) = P_{k}^{c,n} \), where \( k \) - number of segmented pixels in the segment \( P_{c,n}^c \);
   - finding the middle point of the segment \( C(C_0, C_0, C_0) = P_{k/2}^{c,n} \).

2. Calculating the directional vectors of lines \( SC \) and \( CF \).

3. Calculating the coefficients for the equation of plain \( SCF \):
\[ A_{SCF} \cdot x + B_{SCF} \cdot y + C_{SCF} \cdot z + D_{SCF} = 0, \]  

which is determined by points S, C and F.

4. Calculating the coefficients for the equations of the plains \( \alpha \) and \( \beta \), which are perpendicular to segments \( SC \) and \( CF \) and intersect them in their middle.

5. Calculating the coordinates of the intersection point \( (O) \) of the plains \( SCF \), \( \alpha \), and \( \beta \). The point \( O \) is the sought-out center of the circle and is found by solving the following system of linear equations:

\[
\begin{align*}
A_{SCF} \cdot x + B_{SCF} \cdot y + C_{SCF} \cdot z + D_{SCF} &= 0, \\
A_{\alpha} \cdot x + B_{\alpha} \cdot y + C_{\alpha} \cdot z + D_{\alpha} &= 0, \\
A_{\beta} \cdot x + B_{\beta} \cdot y + C_{\beta} \cdot z + D_{\beta} &= 0,
\end{align*}
\]

The radius of the circle corresponding to the cross segment \( n \) of the object is calculated using the following equation:

\[ R^n = \left( (O_x - S_x)^2 + (O_y - S_y)^2 + (O_z - S_z)^2 \right)^{1/2}, \]

3.4. **Circumscribing the segmented object with a cylinder**

Let \{O\} be the set of points corresponding to the centers of the circles built from the set of points \{P\}.

To get straight-line equations \( y = ax + b \) for all the points from the set \{O\}, the method of least squares is used. Then the coefficients \( a \) and \( b \) of the equation can be found from the following equations:

\[
\begin{align*}
a &= \left( \frac{\sum_{i=0}^{N-1} O_{x,i} \cdot O_{y,i}}{N} - \frac{\sum_{i=0}^{N-1} O_{x,i} \cdot \sum_{i=0}^{N-1} O_{y,i}}{N} \right) \left( \frac{\sum_{i=0}^{N-1} (O_{x,i})^2}{N} - \frac{\sum_{i=0}^{N-1} O_{x,i}^2}{N} \right)^{1/2}, \\
b &= \frac{\sum_{i=0}^{N-1} O_{y,i}}{N} - a \cdot \frac{\sum_{i=0}^{N-1} O_{x,i}}{N}
\end{align*}
\]

where \((O_{x,i}, O_{y,i})\) - the coordinate of the point \( i \) from the set \{O\}, \( N \) - number of points in the set \{O\}.

Calculating the distance \( O_{d,n} \) from each point \( O_n \) from the set \{O\} to the line using the resulting straight-line equation:

\[ O_{d,n} = \frac{|a \cdot O_{x,n} - O_{y,n} + b|}{(a^2 + 1)^{1/2}}. \]

The mean distance \((O_d)\) from each point \{O\} to the line is calculated as follows:

\[ O_d = \frac{1}{N} \sum_{i=0}^{N-1} O_{d,i}. \]

If \( O_{d,n} \geq O_d \) is true for a point, it is then removed from the set \{O\}. The same operations are then repeated for the projections of the points onto the plane \( O_{xz} \).

Calculating the radius of the cylinder \( R \) is done according to the following list of actions:

1. Calculating the mean radius of the circles constructed from \{P\}:
3.5. Circumscribing the segmented object with a sphere
There are two approaches to circumscribing an arbitrary object with a sphere. The first one consists of constructing the central axis (for a sphere that would be the one corresponding to the diameter of the cylinder) and calculating the radiuses $R_n$ for each resulting cross section of the object, which was illustrated in the previous sections of the paper. The resulting radius values are then circumscribed into the circle using the method of least squares.

The second approach is universal and can be used for objects of any shape. It is, in essence, an addition to the calculations from the previous sections. In place of the equation (15) the sought-out sphere radius is calculated as half of its axis' length:

$$ R = \frac{1}{2} \left( (O_{x,0} - O_{x,N-1})^2 + (O_{y,0} - O_{y,N-1})^2 + (O_{z,0} - O_{z,N-1})^2 \right)^{1/2} , $$

(16)

where $N$ - size of $\{O\}$.

4. Researching the chosen method using a digital model
A mathematical model described in section 3 was realized using the C++ programming language. The initial data consists of an image from the stereoscopic system and a previously filtered point cloud. To visualize the resulting mathematical model, a re-projection of a spatial shape onto the initial image was done.

The re-projection of the bases of the constructed cylinder and a diametric cross section of the constructed sphere are illustrated in Figure 4.

![Re-projection of bases and cross section](image)
Figure 4. Re-projection of the bases of the constructed cylinder (a) and a diametric cross section of the constructed sphere (a) onto the initial image

The experiment was repeated for wires illustrated in Figure 5.

Figure 5. Initial image of the wires (a), the initial image with a point cloud projection (b), with a re-projection (c)

Table 1. Comparison of the actual dimensions of the objects with the ones obtained experimentally

| Object in question     | Actual diameter, mm | Diameter obtained experimentally, mm | Error, % |
|------------------------|----------------------|--------------------------------------|----------|
| Cylindrical chair leg  | 36                   | 32                                   | 11       |
| Soccer ball            | 21,6                 | 19,2                                 | 11       |
| Street wire            | 25                   | 22                                   | 12       |

5. Conclusion

In this paper, an algorithm for an approximate description of a point cloud collected from the stereoscopic system with a 3D surface using the method of dividing into cross segments was proposed. An approximate description of an object with a solid of revolution using a point cloud allows us to get the information about the geometric dimensions and the shape of an object with a noisy and incomplete point cloud and also doesn’t require the construction of the surface, which allows for less computational power needed from the onboard control system of the underwater robot. A mathematical description for the method is provided and tested on a digital model. The dimensions of some objects located in the workspace of the manipulator were obtained from the experiment. The deviation of those dimensions from the real ones is 11-12%, which could be connected to the inaccuracy of the stereoscopic system calibration and the algorithm used for the calculation of the center and the radius of the circle in section 3.3. The error could be reduced if the calculations are done using methods of estimating the parameters of the model using a sample (least-squares method, RANSAC method). For the following research, an underwater experiment is suggested, as well as the expansion of the group of the figures that are described.

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