Two-part pricing, public discriminating monopoly and redistribution: a note

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Abstract

This note analyzes some properties of optional two-part pricing in a two-type economy. First, the optimal contracts along the Pareto frontier are described. Then, the duality relation between the Rawlsian program and the discriminating monopoly is demonstrated. Last, this property is used to build a mutualist mechanism implementing the constrained Pareto optima.

1. Introduction

Optional two-part pricing, extensively used by many public utilities (electricity, water, railways etc.), gives, as shown by Sharkey and Sibley (1993), some freedom to redistribute the social surplus. These authors show in a partial equilibrium framework that, when a monopoly proposes a menu of contracts, each specifying the fee and the charge price, it is possible for a social planner controlling this monopoly to redistribute towards the weak demand consumer. Our contribution is not to extend their study to a new framework or to other pricings. Our ambition is, first, to emphasize the redistributive mechanism of optional two-part pricing, notably with the help of some graphical presentations, and, second, to propose a simple incentive mechanism implementing the more redistributive optima. As we shall see, this mechanism takes advantage of the dual relationship between the program of the discriminating monopoly and the social planner’s program.

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1 See also Roberts (1979) for the case of nonlinear pricing.
This note is organized as follows. The economy is described in section 2. The constrained Pareto optima are characterized in section 3 even though an implementing mechanism is proposed and studied in section 4. Limits and possible extensions of this work are discussed in the last section.

2. THE ECONOMY

There are two goods, the produced good and the numéraire good. Their quantities are respectively denoted $q$ and $w$. The economy is composed of two types of agents, indexed $i = 1, 2$, defined by their quasi-linear utility functions:

$$u_i(q_i, w_i) = V_i(q_i) + w_i$$

The functions $V_i$ verify the following properties.

**Assumption 1:** $V_i$ is continuously twice differentiable with

$$V_i'(q) := \frac{\partial V_i(q)}{\partial q} > 0, \quad V_i''(q) := \frac{\partial^2 V_i(q)}{\partial q^2} < 0, \quad V_i(0) = 0 \quad (1)$$

and

$$V_2'(q) > V_1'(q) \quad (2)$$

Relations (1) state that the inverse demand is positive and strictly decreasing. Relation (2) implies that type 2’s demand is higher than type 1’s; for quasi-linear utility functions this relation is also the standard single crossing assumption.

The cost function $C$ of the monopoly which produces the good $q$ verifies the following assumption.

**Assumption 2:** $C$ is a convex function on $]0, +\infty[$:

$$C'(q) \geqslant 0, \quad C''(q) \geqslant 0 \quad (3)$$

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1 Fixed costs are thus allowed.
The monopoly using this technology proposes two contracts \((p_1, E_1)\) and \((p_2, E_2)\), where \(p_1, p_2\) are the usage charges, \(E_1\) and \(E_2\) the fees. As asymmetric information prevents perfect discrimination, the contracts must be incentive-compatible. Moreover, in order to eliminate trivial cases, we will suppose that first-best optima are characterized by strictly positive consumptions.

3. CONSTRAINED PARETO OPTIMA

In this section, the constraints regimes of the Paretian program are specified. Then, the Pareto frontier is outlined and the main properties of optimal contracts are discussed. This section does not propose any new results; its aim is simply to clarify the characterization of the Pareto frontier and above all to present, with the help of a graph, a pedagogical analysis of the redistributive mechanism of optional two-part pricing.

In our economy, the constrained Pareto program \(P_p(s_2)\) is

\[
\max_{(p_i, E_i) = 1,2} S_1(p_1) - E_1
\]

subject to

- \(SC_2: \quad S_2(p_2) - E_2 \geq s_2\)
- \(IC_i: \quad S_i(p_i) - E_i \geq S_i(p_j) - E_j \quad i, j = 1, 2\)
- \(B: \quad \sum_{i=1,2} n_i [p_i D_i(p_i) + E_i] = C[D(p_1, p_2)]\)

where \(D_i(p_i) = V_i^{-1}(p_i), \quad S_i(p_i) = V_i[D_i(p_i)] - p_i D_i(p_i), \quad s_i = S_i(p_i) - E_i, \quad D(p_1, p_2) = n_1 D_1(p_1) + n_2 D_2(p_2)\) and \(n_i\) is the number of agents of type \(i\).

To discuss the constraints regimes of \(P_p(s_2)\), it is useful to consider the first-best optima which verify the incentive constraints. Actually, as for every first-best optimum, prices are equal to the marginal cost, and incentive constraints require equality of fees. Hence, there is a unique first-best optimum which verifies the incentive constraints, the so-called Coase two-part pricing. The types surpluses at the Coase solution are noted \(s_1^{co}\) and \(s_2^{co}\).

\(\text{\textsuperscript{3}}\) Of course, the Coase solution only exists if the fixed cost is not too big with respect to the demand.
In the following, only the domain $s_2 < s_2^{co}$ is studied. In this domain the binding incentive constraint is IC$_2$. To know if SC$_2$ binds, it is useful to introduce the Rawlsian solution defined by the maximization of the type 1 surplus subject to incentive and budget constraints. As these constraints always bind, the Rawlsian objective function, after some substitutions, can be rewritten

$$W_R(p_1, p_2) := \frac{1}{n_1 + n_2} \{S_2(p_1, p_2) - n_2[S_2(p_1) - S_1(p_1)]\}$$

(4)

where the social surplus $S_2(p_1, p_2) = \sum_i n_i V_i[D_i(p_i)] - C[D(p_1, p_2)]$.

As usual in this literature, we assume the concavity of this function, and hence the unicity of the Rawlsian solution $(p_1^R, p_2^R)$. If $s_i^R$ is the surplus of type $i$ at this Rawlsian optimum, two cases must be distinguished depending on whether $s_2$ is above or below $s_2^R$. For $s_2^R \leq s_2 < s_2^{co}$, the constrained Pareto program is equivalent to maximizing $W_R(p_1, p_2)$ subject to SC$_2$. The (assumed) strict concavity of $W_R$ implies two results. First, SC$_2$ is binding; second, the second-best frontier, in the surplus space $(s_1, s_2)$, is continuous and strictly monotonic (see figure 1).

The remaining question is what the properties of the optimal contracts are along the second-best frontier. As only incentive and participation constraints of type 2 bind for $s_2^R \leq s_2 < s_2^{co}$, the program $P_p(s_2)$ is reduced to the following one:

$$\max_{p_1, p_2} \frac{1}{n_1 + n_2} \{S_2(p_1, p_2) - n_2[S_2(p_1) - S_1(p_1)]\}$$

subject to

$$SC_2: \sum_{i=1,2} n_i V_i[D_i(p_i)] = C[D(p_1, p_2)]$$

$$- n_1[S_2(p_1) - S_1(p_1)] + (n_1 + n_2)s_2$$

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4 The other case is symmetrical. To extend our results to the domain $s_2 > s_2^{co}$, we only need to rewrite program $P_p(s_2)$ by permuting indices, i.e. we need to maximize $s_2$ subject to the participation constraint of type 1.

5 The space being limited here and the proof being classical, it is not reproduced in this note.

6 As by assumption $s_i^{co}$ is strictly positive, $s_i^R > 0$. Then, from IC$_2$ and equation (2), it can be shown that $s_2^R > s_1^R > 0$.

7 Of course, for $s_2 < s_2^R$, as $s_2^R$ is the lower value that the Paretian social planner can actually assign to type 2, SC$_2$ of program $P_p(s_2)$ is always released.
For each $s_2$, the first-order conditions give optimal prices:

\[
p_2 = C'[D(p_1, p_2)]
\]  

\[
p_1 = (1 + \lambda)C'[D(p_1, p_2)] - \lambda Rm_1[D_1(p_1)] + \frac{\lambda}{D_1(p_1)} D_2(p_1)
\]  

\[
+ \frac{n_2}{n_1 D_1(p_1)} [D_2(p_1) - D_1(p_1)]
\]  

where $Rm_1(q_1) = V'_1(q_1) + q_1 V''_1(q_1)$ is the marginal revenue upon type 1 and $\lambda$ is the Lagrangian multiplier.  

Equation (5) reflects the fact that $p_2$ is not an incentive tool when one tries to increase the type 1 surplus above this Coasian level. Second, we

\[
p_i = C'[D(p_1, p_2)] + \frac{n_2 - \lambda n_1}{1 + \lambda} \frac{D_i(p_1) - D_2(p_1)}{n_1 D_i(p_1)}
\]

We note that for $\lambda = n_2/n_1$ we obtain the Coasian prices: $p_i = p_2 = C'[D(p_1, p_2)]$. Otherwise, one could show that for $\lambda = 0$, $p_2 = p_2^R$. So, intuitively, we could interpret $\lambda$ as a relative weight of type 2 in a linear social welfare function; but this interpretation assumes the concavity of the Pareto frontier (in the surplus space).

Second-best frontier
Coasian solution
Rawlsian solution
First-best frontier

Figure 1. The frontier of the constrained Pareto optima with optional two-part pricing.

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can easily prove that \( p_1 > C' \). Indeed, either \( Rm_1 > C' \) or \( Rm_1 \leq C' \); in the first case, we have of course \( p_1 > C' \), and, if \( Rm_1 \leq C' \), equation (6) implies that \( p_1 > C' \). Furthermore, after some manipulation, the differentiation of IC\(_2\) gives

\[
\frac{ds_1}{ds_2} - 1 = \left[ D_2(p_1) - D_1(p_1) \right] \frac{dp_1}{ds_2}
\]

(7)

Along the constrained Pareto frontier (for \( s_2^R < s_2 < s_2^{co} \), \( ds_1/ds_2 < 0 \), we get from equation (7) \( dp_1/ds_2 < 0 \). Hence, to raise \( s_1 \), the social planner must decrease \( s_2 \), i.e. increase \( p_1 \). The intuition of this result can be grasped graphically.

In figure 2, the Coasian equilibrium is depicted by points A and B in the space \((q, T)\) where \( T \) is the total spending of each type. The upward line passing through these points is the nil profit line; it is also the spending line of each type when the fee is \( E \) and the price is equal to the marginal cost \( c \).\(^{10}\) The curves passing through A and B are the iso-surplus curves corresponding respectively to types 1 and 2.

At the Coase optimum, and in fact at each constrained Pareto optimum, the only way to increase \( s_1 \) is obviously to decrease \( s_2 \). Nevertheless, for this, one needs to release the incentive constraint of type 2, i.e. to decrease the surplus of the dishonest type 2. Starting from the Coasian point A, the only way to proceed is to raise \( p_1 \) (with an appropriate adjustment of \( E_1 \) leaving \( s_1 \) constant).\(^{11}\) As the surplus of the dishonest type 2, reached at point F, is now only \( s_2' \) (< \( s_2 \)), the social planner can extract at most \( d\tau_2 \) with the new contract \((E', c)\). As we can see in figure 2, the increase of budget surplus \( d\tau_2 \) over type 2 exceeds the budget loss \( d\tau_1 \) over type 1.\(^{12}\) So starting from the Coase equilibrium, such an adjustment leaves a positive net budget surplus which, equally redistributed to check incentive constraints, increases type 1’s surplus.\(^{13}\)

Since it permits more redistributive surplus allocation than the Coase solution, optional two-part pricing is a useful tool for the social planner. But, if he does not directly control the monopoly, implementation of the

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10 For simplicity, we supposed in this graph that the marginal cost is constant.
11 Indeed, it is easy to see that a decrease of \( p_1 \) (leaving \( s_1 \) constant) incites type 2 to lie, increases \( s_2 \), and breaks the budget constraint.
12 In fact, to first order, \( d\tau_1 \) is negligible, which is not the case for \( d\tau_2 \).
13 Those adjustments can be reproduced for all constrained Pareto optima but the Rawlsian one.
constrained optimum is questionable: how can he induce the monopoly to select the right two-part pricing?

4. IMPLEMENTATION BY DISCRIMINATING MONOPOLY

In this section we build mechanisms which implement the more redistributive optima. To reach this aim we study a regulated monopoly, the so-called monopoly \_à la Edgeworth\_.\textsuperscript{14} This monopoly is supposed to use optional two-part pricing and is subject to an additional constraint to leave a minimal surplus to type 1. The implementing mechanisms are then deduced from the duality relation between the discriminating program of this monopoly and the Rawlsian program; this duality relation was incidentally noticed by Roberts (1979, pp. 80–81) in a continuous types economy for nonlinear pricing.

\textsuperscript{14} This solution deserves to be called monopoly \_à la Edgeworth\_ with reference to Edgeworth’s contributions to the regulated monopoly literature (e.g Edgeworth (1910)).

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\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{figure2}
\caption{Starting from the Coasian equilibrium (A and B), a rise in \( p_1 \) permits a decrease in \( s_2 \) (with \( s_1 \) constant) in the space \((q, T)\) where \( T \) is total spending.}
\end{figure}
Before introducing the monopoly à la Edgeworth, let us first introduce the program \( P_m \) of the simple discriminating monopoly using optional two-part pricing

$$\max_{(p_i, E_i)_{i=1,2}} \sum_{i=1,2} n_i[p_i D_i(p_i) + E_i] - C[D(p_1, p_2)]$$

subject to

- **PC\(_i\):** \( S_i(p_i) - E_i \geq 0 \quad i = 1, 2 \)
- **IC\(_i\):** \( S_i(p_i) - E_i \geq S_i(p_j) - E_j \quad i, j = 1, 2 \)

and begin to show that the Rawlsian prices are also the monopolistic ones.

**Proposition 1:** The Rawlsian prices \((p_1^R, p_2^R)\) are the solutions of the monopoly program.

**Proof:** Under assumption 1, IC\(_2\) and PC\(_1\) are the only active constraints and we get

\[ E_1 = S_1(p_1) \quad E_2 = S_1(p_1) + [S_2(p_2) - S_2(p_1)] \]

Hence, after substitutions, the objective function becomes

\[ \Pi(p_1, p_2) = \sum_{i=1,2} n_i V_i[D_i(p_i)] - C[D(p_1, p_2)] - n_2[S_2(p_1) - S_1(p_1)] \]
\[ = (n_1 + n_2) W_R(p_1, p_2) \]

The end of the proof is now obvious. \( \blacksquare \)

Consequently, the monopoly equilibrium and the Rawlsian solution differ only by \( E_1 \) and \( E_2 \). In fact, this result hides a fundamental link between them: they are the two polar solutions of the monopoly à la Edgeworth. The latter is a discriminating monopoly with an additional surplus constraint for type 1:
Two-Part Pricing

\[
\max_{(p_{i, E_i})_{i=1,2}} \sum_{i=1,2} n_i[p_i D_i(p_i) + E_i] - C[D(p_1, p_2)]
\]

subject to

\[
\text{SC}_1: \quad S_1(p_1) - E_1 \geq s_1
\]

\[
\text{PC}_2: \quad S_2(p_2) - E_2 \geq 0
\]

\[
\text{IC}_{ij}: \quad S_i(p_i) - E_i \geq S_i(p_j) - E_j \quad i, j = 1, 2
\]

As one can easily demonstrate using classical arguments, equation (2) of assumption 1 implies that SC_1 and IC_2 are the only binding constraints. After manipulations, the previous program is reduced to the following free maximization:

\[
\max_{p_1, p_2} (n_1 + n_2) [W_R(p_1, p_2) - \bar{s}_1]
\]

Optimal prices and quantities are independent of the \( s_1 \) level and equal to the monopoly ones. By raising \( s_1 \) (from 0 to \( s_1^R \)), all surplus distributions between the monopoly equilibrium and the Rawlsian solution can be achieved. Actually, for \( s_1 = s_1^R \), the program of the monopoly \( \text{à la} \) Edgeworth is the dual of the Rawlsian program. So, naturally, it gives not only the same prices but also the same fees. Of course, the monopoly \( \text{à la} \) Edgeworth is an abstract mechanism since \( s_1 \) is exogenous.

A way to make the mechanism more realistic is to consider a mutualist mechanism, i.e. a profit-sharing device. In our framework, one can view a mutualist monopoly as a firm which redistributes all its profit to its members\(^{15} \) according to a sharing key. If this key is contingent upon the chosen contracts, membership guarantees a part of the profit even if the member does not consume. Furthermore, we will suppose that this sharing key is fixed \( \text{ex ante} \) and the profits are redistributed \( \text{ex post} \). The mutualist monopoly’s customers are thus considered just as shareholders. Therefore, it is natural to suppose that the aim of the mutualist monopoly is to maximize profit. Finally, for a sharing key \((\theta_1, \theta_2, \bar{\theta})\) the program \( P_{mu}(\theta_1, \theta_2, \bar{\theta}) \) of this monopoly is then

\(^{15}\text{If the good produced is a public utility, all agents are potential consumers and can be viewed as members of the mutualist monopoly.}\)
\[
\max_{(p_i, E_i)_{i=1,2}} \sum_{i=1,2} n_i [p_i D_i(p_i) + E_i] - C[D(p_1, p_2)]
\]

subject to

\[
\begin{align*}
PC_1: \quad & S_1(p_1) - E_1 + \theta_1 \Pi \geq \bar{\Pi} \\
PC_2: \quad & S_2(p_2) - E_2 + \theta_2 \Pi \geq \bar{\Pi} \\
IC_i: \quad & S_i(p_i) - E_i + \theta_i \Pi \geq S_i(p_j) - E_j + \theta_j \Pi \\
& i, j = 1, 2
\end{align*}
\]

where \(\bar{\Pi}\) is the guaranteed share profit and \(\theta_i \Pi\) is the profit share of type \(i\).

**Proposition 2:** If the monopoly profit is uniformly distributed, \(\theta_1 = \theta_2 = \bar{\theta}\), the mutualist monopoly equilibrium gives the Rawlsian surplus to each type.

**Proof:** With the uniform sharing key, the program is reduced to the program \(P_m\). So the optimal quantities and fees are the Rawlsian ones.

The intuition of the previous proposition can easily be grasped graphically (see figure 3). Points A and B correspond to the private monopoly equilibrium (where \(s_1 = 0\)). Because of the quasi-linearity of preferences, a uniform monetary transfer (to both types) implies a vertical translation of the Edgeworthian monopoly equilibrium: when a surplus \(s_1\) is granted to type 1, the private equilibrium is translated to the new equilibrium represented by \(A'\) and \(B'\).

Furthermore, the previous mechanism suggests its extension to the set of constrained Pareto optima with \(s^R_2 \leq s_2^c\).

**Proposition 3:** For every \(\bar{s}_2\), with \(s^R_2 < \bar{s}_2 \leq s^c_2\), there exists a price cap \(\bar{p}\) such that the mutualist discriminating monopoly mechanism implements quantities and prices of the constrained Pareto optimum corresponding to \(\bar{s}_2\).

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16 As it is depicted, a further rise of \(p_1\) does not increase the net profit \((0 < d\pi_2 = -d\pi_1)\).
Proof: With the price cap \( \bar{p} \), the monopoly program becomes

\[
P_{\text{mpc}}(\bar{p}) = \max_{(p_i, E_i)_{i=1,2}} \sum_{i=1,2} n_i[p_iD_i(p_i) + E_i] - C[D(p_1, p_2)]
\]

subject to

PC\(_i\):
\[ S_i(p_i) - E_i \geq 0 \quad i = 1, 2 \]

IC\(_i\):
\[ S_i(p_i) - E_i \geq S_i(p_j) - E_j \quad i, j = 1, 2 \]

PCC\(_i\):
\[ p_i \leq p \quad i, j = 1, 2 \]

Assumption 1 always implies that IC\(_1\) and IC\(_2\) cannot both be binding. As PC\(_2\) is released, IC\(_2\) must bind and IC\(_1\) is then loosened. So, PC\(_1\) is active and the previous program is reduced to
max \, S_s(p_1, p_2) - n_2[S_2(p_1) - S_1(p_1)]

subject to

PCC_i: \quad p_i \leq p \quad i = 1, 2

For right values of \( p \) (\( p_1^{co} \leq p \leq p_1^R \)), this program implies, for every value of \( p_1, p_2 = C' \). And by strict concavity, \( p_1 = p \). To implement constrained Pareto optima (in quantities and prices) for \( s_2^R < s_2 \leq s_2^{co} \), it is sufficient to set \( \bar{p} = p_1^d(s_2) \), where \( p_1^d(s_2) \) is the optimal price \( p_1 \) of program \( P_1(s_2) \).\(^{17}\)

5. CONCLUSION

This note explores the redistributive properties of optional two-part pricing in a two-type economy. It shows that a monopolistic structure market augmented by a uniform profit sharing allows one to implement the most redistributive optimum, the Rawlsian solution. If a price cap is added, this mutualist mechanism allows one to achieve less redistributive constrained optima. However, there are three caveats to bear in mind.

First, in a pure mutualist mechanism, only customers share profit, even though, in the proposed mechanism, each agent receives profit independently of his consumption decision. However, as here each agent is a customer, the difference is blurred. So, this mechanism can only be applied to a subset of quasi-universally consumed goods, such as electricity, water, public transport.

Second, the efficiency of this mechanism requires of course the social planner to have such precise information as to prevent managers and employees from capturing profits. So, the mechanism supposes a strict monitoring of the managers.

Last, a strong implicit assumption of this paper is the fact that the social planner has a unique redistribution tool: public pricing. Of course, in a more general framework he can also use income taxation. So, a natural extension would be to study the complementarity between discriminating public pricing and income taxation.\(^{18}\)

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\(^{17}\) To understand that the optimal price \( p_1 \) of program \( P_1(s_2) \) is a function of \( s_2 \), it is useful to notice that equation (6) implicitly depends on \( s_2 \) (through \( \lambda \)).

\(^{18}\) See for example Boadway and Marchand (1995).

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