Is it Possible to Find a Probable Solution for the Problem of Divergence in QFT?

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Abstract. There are some fundamental questions in quantum physics and quantum field theory which cannot be answered considering the positive energy states, alone. Negative energy states show up in many places in physics and could help us to provide an answer to some ‘awkward’ questions of physics [1, 2]. In recent years much research has been done on extending classical mechanics, quantum mechanics and quantum field theory into the complex domain [3, 4, 5]. On the other hand, applying negative energy states in QFT and performing the field quantization results in a theory without ultraviolet and infrared divergences and it seems as if evaluating divergent integrals of QED space leads to convergent values.

Due to Dirac, “negative energies and probabilities should not be considered as nonsense...”, so that comparison of different approaches and reviving quantum theories that were thought to be dead may be helpful in leading us toward a general formalism and finding an appropriate answer to the problem of divergence in QFT.

1. Introduction
Quantum field theory is plagued by infinities, and it is therefore considered ugly [6]. The singular behavior of Green function at short/large relative distances leads to main divergences in quantum field theory. Entering the negative frequency states, which are also solutions of the field equation, leads to Krein space quantization. As a result, the vacuum energy becomes automatically zero and all ultraviolet divergences of QFT except the light-cone singularity are removed. The latter is removed with the consideration of quantum metric fluctuations in the linear approximation [7, 8]. Krein space method of quantization could be extended to free boson and spinor fields in Minkowski spacetime [9, 10], Wick’s theorem and Feynman rules could be rewritten and the basic structure of QFT could be reformulated [11]. It is remarkable that the auxiliary negative frequency states play the role of regularization of the theory.

2. Origin of Divergence in QFT and a Probable Solution
The origin of divergences in standard quantum field theory lies in the singularity of the Green’s function. The divergence appears in the imaginary part of the Feynman propagator, and the real part is convergent [12]:

\[ G_F(x, x') = -\frac{1}{8\pi} \delta(\sigma_0) + \frac{m^2}{8\pi} \theta(\sigma_0) \left[ J_1(\sqrt{2m^2}\sigma_0) - iN_1(\sqrt{2m^2}\sigma_0) \right] \frac{1}{\sqrt{2m^2\sigma_0}} - \frac{im^2}{4\pi^2} \theta(-\sigma_0) \frac{K_1(\sqrt{2m^2(-\sigma_0)})}{\sqrt{2m^2(-\sigma_0)}} \]

Consideration of negative frequency states removes singularity of the Green function with exception of delta function singularity:
\[ G_T(x, x') = -\frac{1}{8\pi} \delta(\sigma_0) + \frac{m^2}{8\pi} \theta(\sigma_0) \frac{J_1(\sqrt{2m^2\sigma_0})}{\sqrt{2m^2\sigma_0}}, \quad \sigma_0 \geq 0. \]

However, considering the quantum metric fluctuations removes the latter singularity:

\[ < G_T(x, x') > = -\frac{1}{8\pi} \sqrt{\frac{\pi}{2}} \frac{\sigma_0^2}{\sigma_1^2} \exp\left(-\frac{\sigma_0^2}{2\sigma_1^2}\right) + \frac{m^2}{8\pi} \theta(\sigma_0) \frac{J_1(\sqrt{2m^2\sigma_0})}{\sqrt{2m^2\sigma_0}}, \]

where \( < \sigma_1^2 > \) is related to the density of gravitons. When \( \sigma_0 = 0 \), due to the metric quantum fluctuation \( < \sigma_1^2 > \neq 0 \), and we have

\[ < G_T(0) > = -\frac{1}{8\pi} \sqrt{\frac{\pi}{2}} + \frac{m^2}{16\pi}. \]

By using the Fourier transformation, we obtain [13]

\[ < \tilde{G}_T(p) > = \tilde{G}_T(p) + PP \frac{m^2}{p^2(p^2 - m_0^2)}. \]

However, in the one-loop approximation, the contribution of delta function is negligible and the Green function in Krein space quantization appearing in the transition amplitude is

\[ < \tilde{G}_T(p) > |_{\text{one-loop}} = \tilde{G}_T(p) |_{\text{one-loop}} \equiv PP \frac{m^2}{p^2(p^2 - m_0^2)}. \]

In the standard QED the divergent quantities are found in the self-energy graph, the vacuum polarization graph and the vertex graph. Applying the above resulted Green function can be regarded as a new method for regularization and considering the renormalization point at \( p^2 = m_0^2 \), the previous results for physical quantities in standard QED are obtained.

Since applying Krein method to QFT leads to the standard results, so it is safer to regard this procedure as a convenient way or another viewpoint of performing standard procedure of introducing counter-terms, specially the wave function renormalization term. The Krein space method takes care of divergence on the propagator level, while the standard method takes care of it on the interaction level by introducing counter-terms.

The role of ‘unphysical’ states appears in the method as a natural tool for removing the divergences of the theory. The crucial point is that while imposing the physical boundary conditions on the field operator, the positive frequency states are only affected. The negative frequency states do not interact with the physical states or real physical world. So that, they can not be affected by the physical boundary conditions as well indicating that the former are technical artifacts rather than real objects of deeper meaning. The situation is quite similar to the ‘method of images’ for finding an electromagnetic field in electrostatics. As the simplest example, suppose one tries to find out the electromagnetic field produced by a positively charged point-particle, placed in front of an infinite flat conductor plate. One can find out the field configuration either by a system of a point charge and an infinite flat conductor or a virtual system of a point charge and a negatively charged virtual point-particle. In this case, the ‘positive charge’, the conductor plate and the virtual charge correspond to, respectively, positive frequency modes, counter-terms and negative frequency modes. One can replace one object (the conductor plate or counter-terms) with another object (the virtual charge or negative frequency modes) to get the same result. However, no new aspects are added to the situation, so that nobody claims that the virtual negative charge is a real object. In the same manner, we may introduce unphysical modes if convenient as far as they do not contradict with the correct results. It does
not mean, however at a first glance, that they are real objects of physical meaning. A question arises here: Can this method provide an answer when asked for what he had won the Nobel prize, Feynman replied, “For sweeping the infinities under the rug”? However, at a second glance it is not bad to mention that negative energy states show up in other places and help to explain some fundamental behaviors and provide an answer to some ‘awkward’ questions of physics, such as Klein paradox, original version of EPR paradox [2], the miracle of creation, etc. As stated by Dirac, “negative energies and probabilities should not be considered as nonsense. They are well-defined concepts mathematically, like a negative sum of money, since the equations which express the important properties of energies and probabilities can still be used when they are negative. Thus, negative energies and probabilities should be considered simply as things which do not appear in experimental results. The physical interpretation of relativistic theory involves these things and is thus in contradiction with experiment. We, therefore, have to consider ways of modifying or supplementing this interpretation.” [14, 15].

3. Conclusion
In standard quantum field theory the procedures of normal ordering and renormalization have been adopted for removing the divergences that appear in the physical quantities for free and interacting fields. Relaxation of the requirement for the positivity of norm and energy, results in the disappearance of the divergences. Addition of the new unphysical states leads us to the Krein space quantization. It is remarkable that they play no role in the physical world but an element for removing the divergences of the theory. It could be shown that the divergent integrals of QED in Krein space quantization leads to convergent values.

Negative energy states are entered in different approaches to quantum mechanics and quantum field theory, so that the comparison of them will be helpful, as stated by Dirac, “negative energies and probabilities should not be considered as nonsense...”.

Reviving quantum theories that were thought to be dead, is it possible to find an appropriate answer to the problem of divergence in QFT?

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