Exploring the Cosmic Web in the Sloan Digital Sky Survey Data Release Six using the Local Dimension

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Abstract.

It is possible to visualize the Cosmic Web as an interconnected network of one dimensional filaments, two dimensional sheets and three dimensional volume filling structures which we refer to as clusters. We have used the Local Dimension $D$, which takes value $D = 1, 2, \text{ and } 3$ for filaments, sheets and clusters respectively; to analyse the Cosmic Web in a three dimensional volume limited galaxy sample from the Sloan Digital Sky Survey Data Release Six. The analysis was carried out separately using three different ranges of length-scales, 0.5 to $5 h^{-1}\text{Mpc}$, 1 to $10 h^{-1}\text{Mpc}$ and 5 to $50 h^{-1}\text{Mpc}$. We find that there is a progressive increase in the $D$ values as we move to larger length-scales. At the smallest length-scales the galaxies predominantly reside in filaments and sheets. There is a shift from filaments to sheets and clusters at larger scales. Filaments are completely absent at the largest length-scales ($5$ to $50 h^{-1}\text{Mpc}$). Considering the effect of the density environment on the Cosmic Web, we find that the filaments preferentially inhabit regions with a lower density environment as compared to sheets and clusters which prefer relatively higher density environments. A similar length-scale dependence and environment dependence was also found in a galaxy sample drawn from the Millennium Simulation which was analysed in exactly the same way as the actual data.

1. Introduction

Galaxy redshift surveys (like Two-Degree Field Galaxy Redshift Survey (2dFGRS), [1], Sloan Digital Sky Survey (SDSS), [2]) map the distribution of galaxies in the Universe and tell us directly how the present Universe looks like. These surveys clearly demonstrate the distribution of galaxies in an interconnected complex network of filaments, sheets and clusters, encircling the empty voids, called Cosmic Web. Quantifying the Cosmic Web and understanding it’s origin is one of the most interesting and challenging issues in cosmology.

There are wide variety of statistical measures ([3]) to quantify the Cosmic Web but each quantifies one or atmost a few aspects of the Cosmic Web. We have used the Local Dimension ([4], [5]) as a means to analyse the patterns in the galaxy distribution in the Sloan Digital Sky Survey Data Release Six (SDSS DR6). We have also carried out a similar analysis on a galaxy catalogue from the semi-analytic model of galaxy formation implemented in the Millennium Simulation [6], and compared the results from the actual data with those from the simulation. The Local Dimension classifies the different structural elements along the Cosmic Web as filaments, sheets and clusters. It is well accepted that the Cosmic Web is a complex network whose morphology and connectivity will depend on the length-scale at which the analysis is carried out. [7] shows that the SDSS DR6 is consistent with homogeneity at length-scales.
Figure 1: This shows the fraction of galaxies with a particular $D$ value for the SDSS data (left) and the Millennium data (right). The three different curves correspond to $D$ values that were determined using the length-scales $0.5 - 5 \, h^{-1}\text{Mpc}$, $1 - 10 \, h^{-1}\text{Mpc}$ and $5 - 50 \, h^{-1}\text{Mpc}$ respectively. The bins in $D$ have size $\pm 0.25$.

beyond $60 - 70 \, h^{-1}\text{Mpc}$. We have carried out the entire analysis for three different ranges of length-scales namely $0.5$ to $5 \, h^{-1}\text{Mpc}$, $1$ to $10 \, h^{-1}\text{Mpc}$ and $5$ to $50 \, h^{-1}\text{Mpc}$.

2. Data and Method of Analysis

2.1. Data

The analysis presented here is based on galaxy redshift data from the Main Galaxy Sample of SDSS DR6 [8]. The target selection algorithm of the Main Galaxy sample of SDSS DR6 is detailed in [9]. We have considered a volume-limited subsample in the Northern Galactic Cap which spans $50^\circ < \lambda < 30^\circ$ and $6^\circ < \eta < 35^\circ$, where $\lambda$ and $\eta$ are survey coordinates. The subsample contains 32740 galaxies in the $0.035 \leq z \leq 0.076$ (redshift range) and $20.5 \leq M_r \leq -19$ (absolute magnitude range). For estimating the cosmic variance of the data, we have generated ten bootstrap sample of the SDSS data.

We have also considered a semi analytic galaxy catalogue ([10]) from Millennium Run Simulation ([6]). The catalog contains around 9 million galaxies in a $(500 \, h^{-1}\text{Mpc})^3$ box. Using the peculiar velocities, we map the galaxies to redshift space and then identify a region having the same geometry as our actual data. We have extracted three different sample from the Millennium Simulation.

2.2. Method of analysis

We choose a particular galaxy as center and determine the no of other galaxies $N(<R)$ inside a sphere of comoving radius $R$. This counting process is repeated by varying $R$. We assigned Poisson error $\Delta N(<R) = \sqrt{N(<R)}$ to each measured value of $N(<R)$. The Local Dimension $D$ in the neighbourhood of the center is then determined by using a power-law fit of the equation $N(<R) \propto R^D$ over some length-scales $R_1 \leq R \leq R_2$, iff $\chi^2/\nu \leq 1$. We have carried out the analysis for three different ranges of length-scales, each covering a decade. Values of $D$ were determined separately across the length-scales $0.5$ to $5 \, h^{-1}\text{Mpc}$, $1$ to $10 \, h^{-1}\text{Mpc}$ and $5$ to $50 \, h^{-1}\text{Mpc}$.

3. Results and Conclusions

We first analyse the fraction of centers with different $D$ values, shown in Figure 1. The solid curves in the Figure 1a, corresponds to the results for $0.5$ to $5 \, h^{-1}\text{Mpc}$ for the SDSS data, shows a broad peak with a maxima at $D = 1.5$. This bin together with bin centered at $D = 2$...
Figure 2: The three curves which correspond to $D = 1, 2$ and $3$ respectively show the fraction of galaxies as a function of $\nu$. Figure 2a and 2b are for the SDSS and Millennium data respectively using $R_1 = 0.5 \, h^{-1}\text{Mpc}$, $R_1 = 5 \, h^{-1}\text{Mpc}$. Figure 2c and 2d shows the same for $R_1 = 1 \, h^{-1}\text{Mpc}$, $R_1 = 10 \, h^{-1}\text{Mpc}$ for SDSS and Millennium data respectively.

contains more than 50% of the centers. The dash curve in the Figure 1a, which corresponds to length-scales 1 to 10 $h^{-1}\text{Mpc}$, shows a sharp peak at $D = 2$. The three bins at $D = 1.5$, 2 and 2.5 together contains more than 80% of the centers. The dot-dashed curve in Figure 1, which corresponds to the length-scales 5 to 50 $h^{-1}\text{Mpc}$, shows a broader peak with maxima at $D = 2.5$. The two bins $D = 2$ and $D = 2.5$ contains more than 70% of the centers.

The above results implies that the majority of galaxies are predominantly contained in sheets and filaments at length-scales 0.5 to 5 $h^{-1}\text{Mpc}$, while at length-scale 1 to 10 $h^{-1}\text{Mpc}$ Cosmic Web are predominantly in sheets. At length-scale 5 to 50 $h^{-1}\text{Mpc}$, the galaxies in Cosmic Web is predominantly in sheets and clusters and interestingly we don’t find any centers with $D = 1$ or 1.5. The three curves shows that there is a shift to larger $D$ values in Figure 1a as we progressively consider larger length-scales. It is quite evident from this that the nature of the structural elements that make up the Cosmic Web differs depending on the length-scale at which we view the Cosmic Web. For comparison, we have also applied the Local Dimension to analyse the galaxy distribution in the Millennium Simulation (Figure 1b). We find that the fraction of centers with different $D$ values have very similar distributions in the actual SDSS data and the Millennium Simulation.

We now focus on the distribution of centers with a particular $D$ values at different density environments. To determine the density environment, we consider the entire galaxy distribution to a density field on a grid of spacing $[0.5 \, h^{-1}\text{Mpc}]^3$ using Cloud-in-Cell method. This density field is then smoothed with a Gaussian kernel having a smoothing length $R_S$. The density field at any grid point was quantified using the dimensionless ratio $\nu = \delta/\sigma$ where $\delta = \delta\rho/\bar{\rho}$.

\[ \int_0^\infty \rho(x) dx = \int_0^\infty \frac{1}{\sqrt{2\pi}} e^{-x^2/2} dx = 1 \]

\[ z = \frac{\rho}{\bar{\rho}} = \frac{\rho}{\rho_T} \]

\[ \phi(x) = \int_0^\infty \rho(x) dx = \int_0^\infty \frac{1}{\sqrt{2\pi}} e^{-x^2/2} dx = 1 \]

\[ \nu = \frac{\delta}{\sigma} \]

\[ \delta = \frac{\rho}{\bar{\rho}} \]

\[ \sigma = \sqrt{\frac{1}{2\pi} \int_0^\infty x^2 e^{-x^2/2} dx} = 1 \]

\[ \phi(x) = \frac{1}{\sqrt{2\pi}} e^{-x^2/2} \]

\[ \rho_0 = \bar{\rho} = \frac{1}{\sqrt{2\pi}} \]

\[ \nu = \frac{\delta}{\sigma} \]

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\[ \phi(x) = \frac{1}{\sqrt{2\pi}} e^{-x^2/2} \]

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is the density contrast of the smoothed density field at the particular grid point and \( \sigma \) is the standard deviation of the density contrast of the smoothed density field evaluated using all the grid points that lie within the survey volume. The value of \( \nu \) associated with any of the galaxies gives an estimate of the density environment in the vicinity of the structural element centered on that galaxy. Figure 2a shows the results for the range of length-scale \( R_1 = 0.5h^{-1}\)Mpc and \( R_2 = 5h^{-1}\)Mpc. For this scale \( R_\sigma = 1.58h^{-1}\)Mpc and \( \sigma = 5.78 \). Figure 2c shows the same quantities as Figure 2a with the difference that the range of length-scales now corresponds to \( R_1 = 1 \) and \( R_2 = 10h^{-1}\)Mpc for which \( R_\sigma = 3.16h^{-1}\)Mpc and \( \sigma = 3.53 \). The correspond curves for Millennium Simulations was shown in Figure 2b and 2d respectively.

We may interpret the different curves in Figure 2 as representing the probability of finding a particular kind of structural element in the density environment corresponding to \( \nu \). For example, the curve for \( D=1 \) in Figure 2a gives the probability of finding a filament in the interval \( \nu - 0.25 \) to \( \nu + 0.25 \) for different values of \( \nu \). Similarly, the curves for \( D = 2 \) and \( D = 3 \) show the probability of finding a sheet and a cluster respectively. Our analysis shows that the filaments, sheets and clusters have different probability distributions. The filaments are preferentially distributed in low density environments relative to the distribution of sheets and clusters. We have a cross-over from this behavior at intermediate densities. The sheets are preferentially distributed relative to filaments and clusters in the high density environments. These findings indicate that the way in which different structural elements are distributed along the Cosmic Web depends jointly on two factors (a) the local density environment, and (b) the length-scale at which we view the Cosmic Web.

In summary the analysis of both the SDSS DR6 and the Millennium Simulation exhibit similar trends. The Local Dimensions were estimated separately in three different ranges of length-scales, 0.5 to 5 \( h^{-1}\)Mpc, 1 to 10 \( h^{-1}\)Mpc and 5 to 50 \( h^{-1}\)Mpc. We find that there is a progressive shift in the \( D \) values as we move to larger length-scales. At small length-scales there is a mixtures of sheets and filaments, and the fraction of sheets increases as we move to larger length-scales (1 to 10 \( h^{-1}\)Mpc). We find that sheets and clusters are the predominant structures at the largest length-scales (5 to 50 \( h^{-1}\)Mpc). Filaments are absent at thesis length-scales. It is interesting to note that [11] find that the mass fraction of filaments decreases while that of sheet increases with an increase in the smoothing length-scale, which is consistent with the results of this paper.

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