The kinetic of the true stress at low-cycle loading

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Abstract. Features of grows on true tension at elasto-plastic strains at static and cyclic loading of structural metal materials in connection with their structural state (hardening, softening) are considered in this article. The features of the use of strains and kinetic criterion connected with the bearing capacity of material estimated by extreme plasticity of plastic strains at the time of formation of a neck are shown. It is established that growth of true tension at quasistatic destruction is connected first of all with active accumulation of the residual strains. Methods of strains of nature of behavior of material are described (hardening, softening, stabilization) at cyclic elasto-plastic strains. When loading with the set scope of elasto-plastic strains (tough loading) significant change of tension is observed only in the first (hardening) and the last (softening) cycles. Significant changes in true tension it is observed and directly in loading cycles. It is important to consider the specified changes of true tension when modeling processes of elasto-plastic strains of structural materials and also at the solution of a question of extension of a resource of the designs which fulfilled the resource.

It is accepted that the fatigue durability of structural materials, both in low-cyclic, and in multi-cycle areas is described as dependence of conditional tension (division of the operating amplitude of load of the initial cross-sectional area of a sample) on durability (numbers of cycles before fracture) (figure 1).

![Figure 1](image-url)  
**Figure 1.** A curve of fatigue of steel of the TC (a) and 22k (b) at the room temperature

And if in the field of multi-cycle fatigue such description can be considered justified, then at low-cyclic soft loading, in particular in the field of quasistatic type of fracture when the plasticity of material of the destroyed sample is equal or comparable to plasticity at single static fracture as it shows an experiment, true tension can significantly increase in a cycle in comparison with the set initial tension.
At tough loading (with the set scope of elastoplastic strains) tension changes (increases) at an initial stage of loading, and a big share of durability (until formation of a macrocrack) practically remains invariable.

It is accepted that as at static loading, and cyclic strength characteristics of structural materials are presented in conventional units (strength, a fluidity limit).

In fact at static loading at stretching owing to reduction of section of the sample caused by plastic strains, true tension significantly differs from conditional.

In figure 2 curves of static fracture are presented to the TC and 22k.

![Figure 2. Static tension curves in conditional (σ) and true (Δ) stresses of steel TC (a, c) and steel 22k (b, d).](image)

Increases in true tension depend on a structural condition of material [1, 2]. It is accepted to characterize at cyclic elastoplastic strains as cyclically softening (hysteresis loop width with growth of number of cycles of loading increases, figure 3,a), cyclically strengthened (width of a loop of a hysteresis decreases in the course of strains, figure 3,b) and cyclically stable (width of a loop of a hysteresis remains almost invariable) a big share of durability.

The nature of behavior of material can be determined by conditional curves of static fracture: for softening material the size of the uniform strains determined by strains on strength, less than 0.5 from the general strains (figure 2,c), and for strengthened this size more than 0.5, and the more strains at the level of strength, the stronger is strengthened material and vice versa, the less this size, the more actively softening material with growth of number of cycles of loading.

Steel with perlite structure of the TC is a typical example softening material (figure 2): hysteresis loop width at increase in number of cycles of loading continuously grows (figure 3,a). For this material the value of uniform strain is about 0.3, and for the strengthened AD-33 aluminum alloy this value equally about 0.7 [3]. Steel 22k is cyclically stable material: the value of uniform strain is equal about
0.5 (figure 2, b, d). Structural and unstable materials under the influence of cyclic elastoplastic strains can increase the plasticity [4].

For softening heatproof steel of the CU cyclic strains with increase in number of cycles of loading increases (figure 3, a), and for AD-33 aluminum alloy deformation (hysteresis loop width) decreases with growth of number of cycles (figure 3, b).

The plasticity softening material at static deformation is also characterized by the high size and active growth of true tension by the time of final fracture (figure 2, and c).

At cyclic loading, in particular in the conditions of quasistatic fracture when residual strains by the time of fracture is equal or commensurable with strains of single fracture, the true tension (owing to active reduction of section of a sample from plastic strains considerably decreases), true tension increase (figure 4).

\[ \eta = \int_{0}^{N} \epsilon_p^2 \frac{dN}{\epsilon_{st}^2} + \int_{0}^{N} \Delta \epsilon \frac{dN}{\epsilon} \]  

quasistatic fracture is described by the formula:

\[ \int_{0}^{N_f} \epsilon_p^2 \frac{dN}{\epsilon_{st}^2} + \int_{0}^{N_f} \Delta \epsilon \frac{dN}{\epsilon_{st}} = 1 \]  

Figure 3. Change of variable plastic strains (hysteresis loops) of steel of the TC (a) and AD-33 (b) alloy with growth of numbers of cycles of loading.

As appears from figure 4, with increase in number of cycles of loading and growth of the accumulating of strains connected with reduction of operating section of a sample, true tension increases and becomes maximum in a limit case (at the time of loss of stability of plastic strains and the beginning of formation of a neck). At the same time the more loading amplitude means strains accumulation (reduction of diameter of section) and the above true tension increases. The loadings measured in process cross strains in this case describe real process as they correspond to true tension. In this regard more reliable in the field of low-cyclic loading are offered to carry out calculation of durability (resource) in strains characteristics on the basis of kinetics of accumulation of damages to a look [1, 2]:
Figure 4. Kinetics of true tension of steel of the TC (a) and steel 22k (b) at low-cyclic loading from the loading set by amplitude.

In transitional area (between low-cyclic and multi-cycle fatigue) and in the field of multi-cycle fatigue (as well as in all range of destruction: low-cyclic and multi-cycle fatigue) accumulation of damages can be described by the formula:

$$\eta = \int_0^{N_f} \frac{\varepsilon_p}{\varepsilon_{st}} dN + \int_0^{N_f} \frac{\Delta \varepsilon}{\varepsilon_{st}} dN$$

(3)

and a limit case

$$\int_0^{N_f} \frac{\varepsilon_p}{\varepsilon_{st}^2} dN + \int_0^{N_f} \frac{\Delta \varepsilon}{\varepsilon_{st}} dN = 1$$

(4)

where $\varepsilon_p$ – plastic strain, $\varepsilon_{ep}$ – elastoplastic strain in a stretching half-cycle respectively, $\Delta \varepsilon$ – unilaterally the saved-up plastic strain in a cycle, $N$ – current and $N_f$ – the destroying number of cycles, $\varepsilon_{st}$ – the strain at static fracture corresponding to the true strength (resistance to a separation).

At tough loading (loading with the set scope of deformation) and at multi-cycle fatigue the second member – (4) is equal in dependences (1) to zero.

It is important to note that in dependences (1) – (4) it is necessary to use when calculating limit strains $\varepsilon_{st}$ corresponding to the extreme true tension (true strength). At the same time the true tension and the strains corresponding to it are defined by the moment of loss of stability of plastic strains (the beginning of the accelerated fracture and the beginning of formation of a neck) in figure 2.a and b this moment corresponds to strains $\varepsilon$ equal 43% ($S_k = 950$ MPa) and 40% ($S_k = 834$ MPa) respectively, and strains in a neck after destruction of samples were for steel of the TC 55%, for steel 22k – 45%.

Apparently from figure 5, for steel of the TC at soft loading there is a continuous increase in true tension in cycles (in a hysteresis loop). And at identical levels of plastic strains in a cycle understress is observed. For these conditions of loading ($\sigma_a = 660$ MPa) already in the 20th half-cycle ($k = 20$) true strains increase approximately by 1.4 times in comparison with a zero half-cycle ($k = 0$); at $k = 48$ and $k = 76$ they increase respectively in 2.6 and 8 times while width of a loop changes for the specified half-cycles respectively in 1.1; 2.4 and by 7 times. At the same time gain of true maximum tension for these half-cycles of loading was: from 670 MPa (at $k = 0$) up to 680 MPa (at $k = 20$), 708 MPa (at $k = 48$) and 820 MPa (at $k = 76$), i.e. the maximum increase in tension was about 21.5% (or 23% in relation to the conditional tension used for creation of curve fatigue and in calculations of durability).

At tough loading for softening steel of the TC increase in true tension in the several first cycles as it is shown for the second half-cycle in figure 5, with their further falling is characteristic ($k = 400$). Tough loading always comes to an end with fracture of fatigue type, (without formation of a neck with emergence and development at a certain stage of loading of a crack of fatigue). In figure 5, the given
curve changes of tension of tough loading for all materials belong by the time of before formation of a crack.

In this case for tough loading of steel of the TC the maximum change (reduction) of true tension from 2nd to the 400th half-cycles (about a half of durability, $N_f = 360$ cycles) for 22% was observed, i.e. changes of tension at tough loading before formation of a crack are commensurable with changes of true tension at soft loading in the range of quasistatic type of fracture.

At the same time taking into account true strains conditional strength for steel of the TC increases from 69.7 MPa up to 775 MPa, and for steel 22k from 63 MPa to 775 MPa.

Continuous reduction of the saved-up strains and width of a loop with growth of number of cycles of soft loading is characteristic of the strengthened AD-33 aluminum alloy. The sample which durability was 30 cycles had increase in true tension from zero to the 44th half-cycle only for 7.6% (from 340 to 360 MPa). At the same time loop width in the specified half-cycle decreased in 5.9, and true deformation – by 5.6 times.

So small difference in comparison with softening materials is explained first of all by the fact that AD-33 materials possess a small stock of plasticity and their destruction even at small durability, as a rule, has fragile character. The neck which is formed under existing conditions tests is small and defines only insignificant increases in true tension in comparison with conditional.

At tough loading the strengthened materials with small initial plasticity show also insignificant increases in true tension.

Figure 5. Kinetics of true deformations and tension in stretching half-cycles at cyclic loading steel of the TC and – soft loading: 1 – $k = 0$, 2 – $k = 2$, 3 – $k = 20$, 4 – $k = 48$, 5 – $k = 76$; – tough loading: 1 – $k = 0$, 2 – $k = 2$, 3 – $k = 400$
Thus it is important when modeling processes of deformation and fracture at low-cyclic loading it is necessary to consider the nature of behavior of material (hardening, softening, stabilization) and to accept in calculations of durability and levels of the saved-up damages according to dependences (1) – (4) plasticity of material defining it bearing capacity and corresponding to true strength (to tension, the corresponding beginning of loss of stability of plastic deformation at single fracture), but not residual plasticity $\psi$. In structural materials in most cases the extreme plasticity coincides with true strength, only for materials with high plasticity ($\psi \geq 50\%$) this coincidence is not observed.

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