Shot noise in metallic double dot structures with a negative differential conductance

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Abstract

The shot noise of current through a metallic double quantum dot structure exhibiting negative differential conductance is studied. We can exactly solve the master equation and derive an analytical expression of the spectral density of current fluctuations as a function of frequency in the first Coulomb staircase region. For a large range of bias voltage the noise is calculated by Monte-Carlo simulation. We show that the noise is always sub-Poissonian though it is considerably enhanced in the negative differential conductance regime.

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The shot noise (SN) is a striking consequence of charge quantization and its study has become an emerging topic in physics of nano-devices because measurement of the SN can reveal more information on transport properties which are not available through the conductance measurement alone [1,2]. In the case of uncorrelated current the SN has the full (or Poissonian) value $2eI$, where $e$ is the elementary charge and $I$ is the average current. Deviations (either suppression or enhancement) from this value are due to correlations in the motion of charge carriers. The measure of these deviations is the Fano factor $F$ which is defined as the ratio of the actual noise spectral density to the full SN-value. There are mainly two kinds of correlation: the Pauli exclusion principle and Coulomb interaction. While the former correlation always causes a suppression of SN, the latter may suppress or enhance the noise depending on the conduction regime. The non-Poissonian noise has been most extensively studied in double barrier resonant tunneling diodes, where the SN is partially suppressed in the positive differential conductance (PDC) regime and becomes super-Poissonian in the negative differential conductance (NDC) regime [3]. The super-Poissonian noise accompanied by an NDC has been also predicted and observed in quantum dot devices [4,5]. For Coulomb blockade metallic structures when the Coulomb interaction is extremely important and manifested explicitly in the so-called charging energy, the SN has been studied in a number of works focussing on the single-electron transistor (SET) in PDC regime [6-8] (and refs. in [1]). It was shown that the noise is generally suppressed with a magnitude depending on device parameters and on the range of applied voltage. Recently, we were able to derive an analytical expression for current-voltage (I-V) characteristics and a condition for observing an NDC in a Metallic Double Quantum Dot Structure (MDQDS) in the first Coulomb staircase region [9]. The NDC has been analyzed in detail with respect to device parameters as well as to the temperature and the off-set charge. Then, it is natural to raise a question about the SN-behavior in this device in an NDC regime. We will show in the present letter that the noise though considerably enhanced in NDC regime seems to be always sub-Poissonian. This has been done by solving exactly the master equation in the first Coulomb staircase region and by Monte-Carlo (MC) simulation in a large range of bias voltage.

The equivalent circuit diagram of the structure studied is drawn in the inset of Fig.1a. Within the framework of the Orthodox theory the state $|i>$ of the system is described by the probability $p(i,t)$ to have $n_i$ (and $m_i$) excess electrons in the dot $D_1$ (and $D_2$). This
probability obeys the master equation which can be written in the matrix form as \(d\hat{p}(t)/dt = \hat{M}\hat{p}(t)\), where \(\hat{p}(t)\) is the matrix of elements \(p(i, t)\) and \(\hat{M}\) is the evolution matrix of elements 
\[
M_{ij} = \Gamma(j \leftarrow i) - \delta_{ij} \sum_k \Gamma(k \leftarrow i), \text{ with } \Gamma(j \leftarrow i) \text{ being the net transition rate from } |i> \text{ to } |j>. \]

The statistically averaged current across the junction \(\nu (\nu = l, m \text{ or } r)\) is \(\langle I_\nu(t) \rangle = e \sum_i [\Gamma^+_{\nu}(i) - \Gamma^-_{\nu}(i)] p(i, t), \) where \(\Gamma^+_{\nu}(i)\) is the tunneling rate through the junction \(\nu\) to the right(+)/left(-) at the state \(|i>.\) For a stationary state the current is \(t\)-independent and the total external current is \(I = \langle I_\nu \rangle\) for any of \(\nu.\) However, if the state is not stationary, the charge accumulated in the dots is time dependent and \(\langle I(t) \rangle\) is a weighted average of \(\langle I_\nu(t) \rangle\) as \(\langle I(t) \rangle = \sum_\nu g_\nu \langle I_\nu(t) \rangle, \) where \(g_l = C_l C_m/\Sigma_c, g_m = C_l C_r/\Sigma_c, g_r = C_l C_m/\Sigma_c \) and \(\Sigma_c = C_l C_m + C_m C_r + C_l C_r.\) We will be interested in the SN for \(I,\) which is the external current and experimentally measurable [10], as well as for partial currents \(I_\nu.\)

Korotkov [7] suggested in detail the procedure of calculating the noise spectrum of a correlated tunneling current in a SET. Extending this procedure for the MDQDS of interest we have

\[
S_{\nu\nu}(\omega) = 2A_\nu + 4e^2 \sum_{ij} \left[ \Gamma^+_{\nu}(i) - \Gamma^-_{\nu}(i) \right] B_{ij} \left[ \Gamma^+_{\nu}(j |\nu^-) p_{st}(j |\nu^-) - \Gamma^-_{\nu}(j |\nu^+) p_{st}(j |\nu^+) \right], \\
S_{II}(\omega) = 2 \sum_\nu g_\nu^2 A_\nu + 4e^2 \sum_{\nu\mu} \sum_{ij} g_\nu g_\mu \left[ \Gamma^+_{\nu}(i) - \Gamma^-_{\nu}(i) \right] B_{ij} \left[ \Gamma^+_{\mu}(j |\mu^-) p_{st}(j |\mu^-) - \Gamma^-_{\mu}(j |\mu^+) p_{st}(j |\mu^+) \right].
\]

Here \(S_{\nu\nu}\) and \(S_{II}\) are the spectral densities of current fluctuations (SDCF) for currents \(I_\nu\) and \(I,\) respectively; \(A_\nu = e (I^+_\nu + I^-_\nu)\) with \(I^\pm_\nu = e \sum_i p_{st}(i) \Gamma^{\pm}_{\nu} ;\) the conditional probability \(p(i \leftarrow j |\tau)\) for having state \(|i>\) at the time \(t = \tau > 0\) under the condition that the state was \(|j>\) at an earlier time \(t = 0\) obeys the same master equation as for the probability \(p(i, t);\) the stationary probability \(p_{st}(i)\) is defined as \(p(i \leftarrow j |\tau \rightarrow \infty) = p_{st}(i) \delta_{ij}; \hat{B} = Re\{i\omega \hat{1} - \hat{M}\}^{-1}\); \(\langle j |\nu^\pm \rangle\) is the state obtained from the state \(|j>\) by transferring an electron across the \(\nu\)-junction to the right(+)/left(-).

For the structure under study, using the well-known expression of the tunneling rate across a junction (see, eqs.(4.5) in [9]), in principle, one can calculate the SDCFs (1). In practice, however, one can not solve the master equation exactly with all possible states. Recently [9], we have shown that at zero temperature and in the first Coulomb staircase region, \(V_{s1} \leq V \leq V_{s2},\) where \(V_{s2} = e/2C_r\) and \(V_{s1}\) is the maximum from \(e/2C_l\) and \(e|C_l - C_m|/2C_r(C_l + C_m)\) (assuming \(C_l \geq C_r\)), the master equation can be exactly solved.
and therefore the I-V characteristics can be derived for the range of parameters as

\[ C_r \leq C_l \leq 3C_r \quad \text{and} \quad C_m \leq C_r(3C_r - C_l)/(C_l - C_r). \]  

(2)

Under this condition all probabilities \( p_{st}(i) \) are equal to zero except those for three states \(|1\rangle = (0,0), |2\rangle = (1,0)\) and \(|3\rangle = (1,-1)\): \( p_{st}(1) = bc/\Sigma \Gamma \); \( p_{st}(2) = ca/\Sigma \Gamma \) and \( p_{st}(3) = ab/\Sigma \Gamma \), where

\[
a \equiv \Gamma (2 \leftarrow 1) = [C_l (C_m + C_r) / e\Sigma_c R_l] (V - e/2C_l),
\]
\[
b \equiv \Gamma (3 \leftarrow 2) = [C_r (C_m + C_l) / e\Sigma_c R_r] (V - e (C_l - C_m) / 2C_r (C_l + C_m)),
\]
\[
c \equiv \Gamma (1 \leftarrow 3) = [C_l C_r / e\Sigma_c R_m] (e/2C_l + e/2C_r - V),
\]

\(\Sigma \Gamma = ab + bc + ca\). With (3) the SDCFs (1) become

\[
S_{ll} = 2eI (1 + 2aB_{12}), \quad S_{mm} = 2eI (1 + 2aB_{31}), \quad S_{rr} = 2eI (1 + 2aB_{23}),
\]
\[
S_{ll} = 2eI \sum \nu g_{\nu}^2 + 4eI [g_l a (g_m B_{11} + g_l B_{12} + g_r B_{13}) + g_r b (g_m B_{21} + g_l B_{22} + g_r B_{23}) + g_m c (g_m B_{31} + g_l B_{32} + g_r B_{33})].
\]

(4)

The matrix \((i\omega \hat{J} - \hat{M})\) has then a simple form giving the matrix \(\hat{B}\) with elements

\[
B_{ij} = Re \{D_{ij} [i\omega (ab + bc + ca - \omega^2) - \omega^2 (a + b + c)]^{-1}\},
\]

(5)

where \(D_{11} = bc - \omega^2 + i\omega (b + c); \quad D_{12} = bc; \quad D_{13} = bc + i\omega c; \quad D_{21} = ac + i\omega a; \quad D_{22} = ac - \omega^2 + i\omega (a + c); \quad D_{23} = ac; \quad D_{31} = ab; \quad D_{32} = ab + i\omega b; \quad D_{33} = ab - \omega^2 + i\omega (a + b)\). The expression (4) (with (3) and (5)) is our main analytical result obtained for the first Coulomb staircase region in the condition (2). Substituting (3) and (5) into (4) we see that for the model under study all \(S_{\nu\nu}(\omega) (\nu = l, m, r)\) are identical

\[
S_{\nu\nu}(\omega) = 1 - \frac{2abc (a + b + c)}{(ab + bc + ca - \omega^2)^2 + \omega^2 (a + b + c)^2}.
\]

(6)

In the limit of zero frequency the noises \(S_{\nu\nu}(0)\) and \(S_{ll}(0)\) are coincident with a single Fano factor

\[
F = [1/a^2 + 1/b^2 + 1/c^2][1/a + 1/b + 1/c]^{-2}.
\]

(7)

In the opposite limit of large frequency \(F_{\nu\nu} \to 1\) (the Poissonian value), whereas \(F_{ll} \to \sum_{\nu} g_{\nu}^2 < 1\).

For given values of device parameters (capacitances and tunneling resistances) satisfying the condition (2) it is easy to calculate the SDCFs (4) in a large range of frequency. To this
end we use the zero temperature tunneling rate across a junction \( \Gamma = \Theta(-\Delta F) |\Delta F| / e^2 R_t \), where \( R_t \) is the junction tunneling resistance, \( \Delta F \) is the change in the free energy \( F \) of the system after the tunneling event has occurred. For the model under study \( F(i) = (e n_i - C_i V/2)^2 / 2 C^*_i + (e m_i + C_r V/2)^2 / 2 C^*_r + (e n_i - C_i V/2)(e m_i + C_r V/2) / 2 C^*_m + e V(n_l - n_r) / 2 - (C_i + C_r) V^2 / 8 \), where \( C^*_i = \Sigma C_i / (C_i + C_m) \), \( C^*_r = \Sigma C_r / (C_i + C_m) \), \( C^*_m = \Sigma C / C_m \); \( n_l(n_r) \) is the number of electrons that have entered the structure from the left (right) [9]. In calculations as well as in MC-simulations below the elementary charge \( e \), the capacitance \( C_r \) and the tunneling resistance \( R_r \) are chosen as the basic units. The voltage, the current and the frequency are then measured in the units of \( e / C_r \), \( e / C_r R_r \) and \( (C_r R_r)^{-1} \), respectively.

Fig.1b shows the normalized SDCFs \( S_{\nu \nu}(\omega) / 2 e I \) (two upper lines) and \( S_{II}(\omega) / 2 e I \) (two lower lines) calculated at the same bias voltage \( V = 0.5 \) for two cases of I-V characteristics with an PDC (dashed line) and an NDC (solid line) as shown correspondingly in Fig.1a. This result should be in comparison with that for the SET shown in Fig.3 of [7]. For the more interesting case of NDC the Fano factor is plotted versus the bias voltage in the inset of this figure. Clearly, the noise is considerably greater in the NDC regime, the factor \( F \) is however still limited by the Poissonian value. An enhancement of the SN in an NDC regime is observed in various structures due to a strong fluctuation of current. The present result of \( F \leq 1 \) is different from that observed in resonant tunneling diode devices where the electrostatic potential fluctuation of the band bottom leads to a super-Poissonian noise [1,4]. It should be emphasized that the origin of the super-Poissonian noise is related to the nature of potential fluctuations producing a positive feedback of charge which is absent in the MDQDS under study. On the other hand, the sub-Poissonian noise accompanied by an NDC has been observed in \( GaAs-AlAs-GaAs \) tunneling structure with embedded self-assembled \( InAs \) quantum dots in the single-electron tunneling regime [11] and also suggested in strongly correlated double quantum dot systems in the Kondo regime [12].

For a large range of bias voltage we simulate the noise using basically the MC-program [9] which was shown to give I-V curves in good agreement with analytical calculations (see Fig.2 in [9]). However, it is impossible to simulate the spectral density in the limit of zero frequency and we have to consider the low-frequency limit \( \omega_c = 10^{-3} \) (indicated by the arrow in Fig.1b). For all frequencies \( \omega \geq \omega_c \) the simulation noises are practically coincident with corresponding analytical curves in Fig.1b. As an additional test, our noise program has well reproduced the qualitative behavior of experimental data in Fig.1 of [2] for a single
dot structure. Fig.2 presents the voltage dependence of normalized simulation SDCFs for sample with parameters given in the figure. The calculation has been performed at $\omega = \omega_c$ in a range of $V$ where the I-V curve (dashed line) exhibits several peaks and valleys. Clearly, two solid curves describing $S_{\nu\nu}(V)$ and $S_{II}(V)$ have practically the same form, though at the chosen finite frequency there is still a considerable separation between them. At each bias voltage the Fano factor $F$ is somewhere between these normalized noises and we can guess that the $F(V)$-dependence should have the same saw-tooth behavior like solid curves in Fig.2. The most interesting feature observed in this figure is the modulation of the noise amplitude as a function of $V$ with peaks at the points where the NDC reaches the highest magnitude and moreover the noise seems to be always sub-Poissonian even in NDC regimes. Such a suppression of noise is due to the strong Coulomb interaction as generally suggested in ref.[8]. We like to mention that a similar voltage-dependent behavior of the Fano factor has been experimentally observed in [11] for a single quantum dot structure. The gradual decrease of noise peaks at NDC regimes as the voltage increases is closely related to the corresponding decrease of the peak-to-valley ratio of current as can be seen in Fig.2. Note that qualitatively all the simulation results discussed are not particular for the chosen frequency $\omega_c$. Thus, both the analytical results of eqs.(6) and (7) and the MC-simulation data suggest that the SN in the MDQDS under study is always sub-Poissonian though it is considerably enhanced in NDC regimes. This is the main conclusion of the present work which, as shown by additional simulation data (should be published elsewhere), is well preserved even in the case gates are included.

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FIGURE CAPTIONS

FIG.1 The I-V characteristics (a) and corresponding normalized SDCFs calculated at $V = 0.5$ (b) for $R_l = 1.1$ (PDC - dashed lines) and $R_l = 0.2$ (NDC - solid lines), everywhere $C_l = 1.5, C_m = 2.0, R_m = 2.0$. Insets: in (a): circuit diagram of the model, in (b): the Fano factor as a function of the bias for the NDC case.

FIG.2 The voltage dependence of normalized SDCFs: solid curves (upper for $S_{vv}$ and lower for $S_{II}$) at the frequency $\omega_c = 10^{-3}$. The corresponding I-V characteristics is shown by the dashed curve. Simulation parameters: $C_l = 1.0, C_m = 1.0, R_l = 1.0, R_m = 10.0$, zero temperature.
