The same-sign tetralepton signature via mixing of neutral Higgs bosons and their cascade decays to charged Higgs bosons is a unique signal in the type-II seesaw model. In this paper, we study this signature at future lepton colliders, such as ILC, CLIC, and MuC. Constrained by direct search, $H^{\pm\pm} \to W^\pm W^\pm$ is the only viable decay mode for $M_{A^0} = 400$ GeV at $\sqrt{s} = 1$ TeV ILC. With an integrated luminosity of $\mathcal{L} = 8 \text{ ab}^{-1}$, the promising region with about 150 signal events corresponds to a narrow band in the range of $10^{-4}$ GeV $\lesssim v_\Delta \lesssim 10^{-2}$ GeV. For heavier triplet scalars $M_{A^0} \gtrsim 900$ GeV, although the $H^{\pm\pm} \to \ell^\pm \ell^\pm$ decay mode is allowed, the cascade decays are suppressed. A maximum event number $\sim 16$ can be obtained around $v_\Delta \sim 4 \times 10^{-4}$ GeV and $\lambda_4 \sim 0.26$ for $M_{A^0} = 1000$ GeV with $\mathcal{L} = 5 \text{ ab}^{-1}$ at $\sqrt{s} = 3$ TeV CLIC. Meanwhile, we find that this signature is not promising for $M_{A^0} = 1500$ GeV at $\sqrt{s} = 6$ TeV MuC.
I. INTRODUCTION

The discovery of neutrino oscillations [1–3] confirms that neutrinos have sub-eV masses. Meanwhile, the underlying mechanism accounting for such tiny neutrino mass is still an open question. Regarding the standard model (SM) as a low energy effective field theory, the simplest pathway to generate neutrino mass is via the Weinberg operator $LL\Phi\Phi/\Lambda$ [4]. There are three possible ways at tree level to realize this operator [5], which correspond to the canonical type-I [6, 7], type-II [8–11], and type-III [12] seesaw. To verify whether these scenarios are realized in nature, the signatures of seesaw models at colliders have been extensively studied [13–15]. Since the conventional type-I seesaw requires the right-hand neutrinos $N$ to be quite heavy ($\gtrsim 10^{14}$ GeV), it is far beyond the reach of current and future planned colliders. Therefore, we consider the type-II seesaw in this work. Other possible low scale approaches to generate tiny neutrino mass have been summarized in Ref. [16, 17].

The type-II seesaw introduces a scalar triplet $\Delta$ with hypercharge $Y = +2$, where neutrino mass is generated by the Yukawa interaction between the lepton doublets and scalar triplet. After the spontaneous symmetry breaking of SM Higgs doublet $\Phi$, the trilinear term $\mu \Phi^T i\tau_2 \Delta^\dagger \Phi$ induces a vacuum expectation value for the neutral component of scalar triplet with $v_\Delta \sim \mu v^2 / M_\Delta^2$. Since the scalar triplet $\Delta$ also carries the lepton number $+2$, the $\mu$-term breaks the lepton number by two units. In particular, this trilinear term is the only source of lepton number violation, thus it should be naturally small. Then, for $\mu \sim v_\Delta$, we can naturally have $M_\Delta \sim v$, i.e., the mass of scalar triplet at the electroweak scale [18].

A distinct feature of this model is the presence of doubly charged Higgs $H^{\pm\pm}$. Assuming degenerate mass spectrum of the scalar triplet, the typical channels to hunt for $H^{\pm\pm}$ are the same-sign dilepton channel $H^{\pm\pm} \to \ell^\pm\ell^\pm$ and the same-sign diboson channel $H^{\pm\pm} \to W^\pm W^\pm$ [19]. For non-degenerate case, cascade decay channel $H^{\pm\pm} \to H^{\pm} W^\pm$ is also possible [20–23]. Corresponding signatures have been extensively studied at LHC [24–29], HE-LHC [30–33], $e^+e^-$ collider [34, 35], and $ep$ colliders [36, 37]. When $v_\Delta < 10^{-4}$ GeV, the $H^{\pm\pm} \to \ell^\pm\ell^\pm$ is the dominant decay mode, and direct search at LHC has already excluded the region $M_{H^{\pm\pm}} < 870$ GeV [38]. In this case, the branching ratios of $H^{\pm\pm} \to \ell^\pm\ell^\pm$ are only correlated with neutrino oscillation parameters [39]. When $v_\Delta > 10^{-4}$ GeV, the $H^{\pm\pm} \to W^\pm W^\pm$ mode becomes the dominant one, and searches for pair production of $H^{\pm\pm}$ in this diboson channel have excluded $M_{H^{\pm\pm}} < 350$ GeV [40, 41].

Among various possible collider signatures of the type-II seesaw, a unique one is the same-sign tetralepton signature [42, 43], which arises from the mixing of neutral Higgs bosons and their cascade decays to singly and doubly charged Higgs bosons. Previous studies [42, 43] focus on the hadron colliders as LHC and FCC-hh with $\sqrt{s} = 100$ TeV. In this paper, we will analyze this signature at future lepton colliders.
Considering current lower bound on doubly charged Higgs $M_{H^{\pm\pm}} > 350$ GeV, this signature is beyond the reach of CEPC \cite{44}. In order to pair produce $H^{\pm\pm}$, the collision energy should be at least higher than 700 GeV. Therefore, we take the following three benchmark scenarios to illustrate, i.e., $M_{H^{\pm\pm}} \sim 400$ GeV at $\sqrt{s} = 1$ TeV ILC \cite{45, 46}, $M_{H^{\pm\pm}} \sim 1000$ GeV at $\sqrt{s} = 3$ TeV CLIC \cite{47, 48}, and $M_{H^{\pm\pm}} \sim 1500$ GeV at $\sqrt{s} = 6$ TeV Muon Collider (MuC) \cite{49, 50}.

In our paper, the type-II seesaw model will be briefly introduced in Sec. II. The branching ratios of the scalar triplet components are also discussed in Sec. II. The same-sign tetralepton signals at ILC, CLIC, and MuC are analyzed in Sec. III. Finally, the conclusion is presented in Sec. IV.

II. THE MODEL

We concisely review the type-II seesaw in this section. Besides the SM Higgs doublet $\Phi$, a scalar triplet $\Delta$ is also employed, which can be denoted as

$$
\Phi = \begin{pmatrix} \phi^+ \\ \Phi^0 \end{pmatrix}, \quad \Delta = \begin{pmatrix} \Delta^+ \\ \Delta_0 \\ -\Delta^+ / \sqrt{2} \end{pmatrix},
$$

(1)

where after spontaneous symmetry breaking, the neutral components can be further written as $\Phi^0 = \frac{1}{\sqrt{2}} (v + \phi^0 + i\chi^0)$ and $\Delta_0 = \frac{1}{\sqrt{2}} (v_\Delta + \delta^0 + i\eta^0)$, respectively. The Yukawa interaction that generates tiny neutrino mass is given by

$$
L_Y = Y_\Delta \overline{L_L} \tau_2 \Delta L_L + \text{h.c.}
$$

(2)

The scalar potential involving $\Phi$ and $\Delta$ is

$$
V(\Phi, \Delta) = m_\Phi^2 \Phi^\dagger \Phi + M^2 \text{Tr} (\Delta^\dagger \Delta) + (\mu \Phi^T i\tau_2 \Delta^\dagger \Phi + \text{h.c.}) + \frac{\lambda_0}{4} (\Phi^\dagger \Phi)^2 + \lambda_1 (\Phi^\dagger \Phi) \text{Tr} (\Delta^\dagger \Delta) + \lambda_2 \left[ \text{Tr} (\Delta^\dagger \Delta) \right]^2 + \lambda_3 \text{Tr} \left[ (\Delta^\dagger \Delta)^2 \right] + \lambda_4 \Phi^\dagger \Delta \Delta^\dagger \Phi.
$$

(3)

Mixing between the doublet and triplet scalars leads to seven physical scalars, i.e., doubly charged Higgs $H^{\pm\pm}$, singly charged Higgs $H^\pm$, CP-even Higgs bosons $h$ and $H^0$, and CP-odd Higgs $A^0$, with the mixing angles specified by

$$
\tan \beta_\pm = \frac{\sqrt{2}v_\Delta}{v}, \quad \tan \beta_0 = \frac{2v_\Delta}{v}, \quad \tan 2\alpha = \frac{4v_\Delta}{v^2} \frac{\lambda_1 + \lambda_4 - 2M_\Delta^2}{v^2 - 2\lambda_1^2 - 4\lambda_2^2 (\lambda_2 + \lambda_3)},
$$

(4)

where $M_\Delta^2 = \mu v^2 / (\sqrt{2}v_\Delta)$. The masses of the doubly and singly charged Higgs bosons $H^{++}$ and $H^+$ is given by

$$
M_{H^{++}}^2 = M_\Delta^2 - v_\Delta^2 \lambda_3 - \frac{\lambda_4}{2} v^2, \quad M_{H^+}^2 = \left( M_\Delta^2 - \frac{\lambda_4}{4} v^2 \right) \left( 1 + \frac{2v_\Delta^2}{v^2} \right).
$$

(5)
The masses of CP-even Higgs bosons $h$, and $H^0$ can be written as

\[ M_h^2 = T_{11}^2 \cos^2 \alpha + T_{22}^2 \sin^2 \alpha - T_{12}^2 \sin 2\alpha, \]  
\[ M_{H^0}^2 = T_{11}^2 \sin^2 \alpha + T_{22}^2 \cos^2 \alpha + T_{12}^2 \sin 2\alpha, \]  

where $T_{11}$, $T_{22}$, and $T_{12}$ are of the form

\[ T_{11}^2 = \frac{\lambda_0}{2} v^2, \quad T_{22}^2 = M_A^2 + 2v_\Delta^2 (\lambda_2 + \lambda_3), \quad T_{12}^2 = -\frac{2v_\Delta}{v} M^2_{A^0} + v_\Delta v (\lambda_1 + \lambda_4). \]  

Finally, the CP-odd Higgs $A^0$ has the following mass

\[ M_{A^0}^2 = M_A^2 \left( 1 + \frac{4v_\Delta^2}{v^2} \right). \]

Constrained by the $\rho$ parameter, $v_\Delta \lesssim 1$ GeV should be satisfied. Neglecting the contributions from $v_\Delta$, masses of triplet scalars have the relation

\[ M_{H^{++}}^2 - M_{H^+}^2 \approx M_{H^+}^2 - M_{H^0,A^0}^2 \approx -\frac{1}{4} \lambda_4 v^2. \]  

In this paper, we consider the scenario with $\lambda_4 > 0$, which leads to the mass spectrum $M_{H^{++}} < M_{H^+} < M_{H^0} \simeq M_{A^0}$. The mass difference between $H^0$ and $A^0$ plays a vital important role in the production of the same-sign trilepton signature, which is controlled by $v_\Delta$ as

\[ M_{H^0}^2 - M_{A^0}^2 \approx 2(\lambda_2 + \lambda_3)v_\Delta^2 - 4\frac{M_A^2}{v^2} v_\Delta^2. \]  

Here, we briefly discuss the decay properties of triplet scalars with the mass spectrum $M_{H^{++}} < M_{H^+} < M_{H^0} \simeq M_{A^0}$. Expressions of partial decay widths of triplet scalars can be found in Ref. [21]. In this scenario, the doubly charged Higgs $H^{\pm\pm}$ is the lightest. The possible decay channels are same-sign dilepton $H^{\pm\pm} \rightarrow \ell^\pm \ell^\pm$ and same-sign diboson $H^{\pm\pm} \rightarrow W^\pm W^\pm$. The branching ratios are plotted in Fig. 1 for three benchmark cases with $M_{A^0} = 400, 1000,$ and $1500$ GeV. The decay widths of dilepton $H^{\pm\pm} \rightarrow \ell^\pm \ell^\pm$ channel is proportional to $1/v_\Delta^2$, while that of diboson $H^{\pm\pm} \rightarrow W^\pm W^\pm$ is proportional to $v_\Delta^2$. Therefore, we have $\text{BR}(H^{\pm\pm} \rightarrow \ell^\pm \ell^\pm) \sim 1$ for $v_\Delta \lesssim 10^{-5}$ GeV, $\text{BR}(H^{\pm\pm} \rightarrow H^{\pm\pm} W^*) \sim 1$ for $v_\Delta \gtrsim 10^{-3}$ GeV. Increasing the mass of $H^{\pm\pm}$ do not have a large impact on the results of $\text{BR}(H^{\pm\pm})$. As for the singly charged Higgs $H^\pm$, possible decay channels are leptonic $H^\pm \rightarrow \ell^\pm \nu$, bosonic $H^\pm \rightarrow W^\pm Z/W^\pm h$, quarks $H^\pm \rightarrow tb/CS$, and cascade $H^\pm \rightarrow H^{\pm\pm} W^*$. Here, we focus on the same-sign tetralepton signature related channel, i.e., the cascade decay $H^\pm \rightarrow H^{\pm\pm} W^*$. This channel is the dominant one in the range of $10^{-6} \text{ GeV} \lesssim v_\Delta \lesssim 10^{-3}$ GeV when $M_{A^0} = 400$ GeV. As the mass of triplet scalars increase to about 1000 GeV, the dominant range of this channel shrinks to $v_\Delta \sim 5 \times 10^{-5}$ GeV, and the corresponding branching ratio never reaches one. Meanwhile, this channel can not become the dominant one when $M_{A^0} = 1500$ GeV.
III. SAME-SIGN TETALEPTON SIGNATURE

In this section, we explore the same-sign tetralepton signature resulting from the neutral Higgs decay. First, let’s consider the production cross section of $H^0 A^0$. The results are shown in Fig. 2 where the cross section $\sigma(H^0 A^0)$ at 14 TeV LHC and 100 TeV FCC-hh are also illustrated for comparison. All the results are computed by using Madgraph5_aMC@NLO [51]. For lepton colliders, the neutral Higgs pair $H^0 A^0$ can be produced when $M_{A^0} < \sqrt{s}/2$. At the 1 TeV ILC, the cross section $\sigma(H^0 A^0)$ is larger than at 14 TeV LHC in the range of $300 \text{ GeV} \lesssim M_{A^0} \lesssim 500 \text{ GeV}$. For $500 \text{ GeV} \lesssim M_{A^0} \lesssim 1300 \text{ GeV}$, the 3 TeV CLIC generates the largest cross section among lepton colliders. Notably, $\sigma(H^0 A^0)$ at 3 TeV CLIC can be two
FIG. 2. Production cross section of $H^0 A^0$ at various colliders. The solid red, green, and blue lines are the results at 1 TeV ILC, 3 TeV CLIC, and 6 TeV MuC, respectively. The dashed cyan and pink lines are the results at 14 TeV LHC and 100 TeV FCC-hh.

orders of magnitudes larger than at LHC for $M_{A^0} \sim 1000$ GeV. When $M_{A^0} \gtrsim 1300$ GeV, the 6 TeV MuC becomes one of the best options. Especially, in the range of 1700 GeV $\lesssim M_{A^0} \lesssim 2700$ GeV, $\sigma(H^0 A^0)$ at 6 TeV MuC is even larger than at 100 TeV FCC-hh.

At the 1 TeV ILC with $M_{A^0} = 400$ GeV, this signal is generated via the tetraboson process

$$e^+ e^- \rightarrow H^0 A^0 \rightarrow H^\pm W^* H^\pm W^* \rightarrow H^{\pm \pm} W^* W^* + W^* W^* \rightarrow 4W^\pm + X,$$

with the leptonic decay $W^\pm \rightarrow \ell^\pm \nu (\ell = e, \mu)$. Note that the dilepton decay $H^{\pm \pm} \rightarrow \ell^\pm \ell^\pm$ has already been excluded by direct search at LHC. Since the typical mass splitting between triplet scalars for the same-sign tetralepton signature is at the order of $O(\text{GeV})$, the final states from off-shell $W$ decay are hard to be detected. Such signature occurs due to the interference effect between $H^0$ and $A^0$, which is sizable when $\delta M = M_{H^0} - M_{A^0} \sim \Gamma_{H^0/A^0}$. The cross section for this signal is calculated as

$$\sigma_W(4\ell^\pm + X) = \sigma(e^+ e^- \rightarrow H^0 A^0) \times \left(\frac{2 + x^2}{1 + x^2} \frac{x^2}{1 + x^2}\right) \times \text{BR}(H^0/A^0 \rightarrow H^{\pm W^*})^2 \times \text{BR}(H^{\pm} \rightarrow H^{\pm\pm} W^*)^2 \times \text{BR}(W^\pm \rightarrow \ell^\pm \nu)^4,$$

where $x = \delta M/\Gamma_{H^0/A^0}$. The initial cross section $\sigma(e^+ e^- \rightarrow H^0 A^0)$ is about 10 fb at the 1 TeV ILC with $M_{A^0} = 400$ GeV. In the left panel of Fig. 3, we show the product of BRs in the above process. As shown in Fig. 1, BR($H^{\pm \pm} \rightarrow W^\pm W^\pm$) is quickly suppressed for $v_\Delta < 10^{-4}$ GeV, which corresponds to the left boundary. While the right one is determined by the cascade decay branching ratios as BR($H^\pm \rightarrow H^{\pm W^*}$). In this way, a larger $\lambda_4$ leads to a larger mass splitting, hence a wider range of $v_\Delta$. In the right panel of Fig. 3, we show the expected event number for the same-sign tetralepton signature at the 1 TeV ILC with an integrated luminosity of $\mathcal{L} = 8 \text{ ab}^{-1}$. A detector level simulation with Delphes
FIG. 3. Left panel: Product of branching ratios \( \text{BR}(H^0/A^0 \to H^0W^-)^2 \times \text{BR}(H^\pm \to H^\pm W^-)^2 \times \text{BR}(H^\pm \to W^\pm W^\pm)^2 \times \text{BR}(W^\pm \to \ell \nu)^4 \) for the process \( e^+e^- \to H^0A^0 \) with mass of \( A^0 \) being fixed as \( M_{A^0} = 400 \text{ GeV} \). Right panel: Event number of the same-sign tetralepton signature \( 4\ell^\pm + X \) for the mass \( M_{A^0} = 400 \text{ GeV} \) from \( e^+e^- \to H^0A^0 \) and subsequent decays at the \( \sqrt{s} = 1 \text{ TeV} \) ILC with \( \mathcal{L} = 8 \text{ ab}^{-1} \).

\[ \text{[52]} \] is also performed, where we only require \( p_T(\ell^\pm) > 10 \text{ GeV} \) and \( |\eta(\ell^\pm)| < 2.5 \). The total cut efficiency we applied is \( c_{eff} = 0.6 \) for \( M_{A^0} = 400 \text{ GeV} \). The promising region in the \( \lambda_4 - v_\Delta \) plane fills a narrow band, where the maximum event number can reach about 160. Such a narrow band is formed mainly due to the interference effect between \( H^0 \) and \( A^0 \). For fixed value of \( v_\Delta \), the mass splitting \( \delta M \) is then determined. A certain value \( \lambda_4^M \) resulting suitable cascade decay width, i.e., \( x = \delta M/\Gamma_{H^0/A^0} \sim 1 \), leads to the maximum event number. If \( \lambda_4 > \lambda_4^M \), then \( \Gamma_{H^0/A^0} \) will increase, thus \( x \) will decrease, and the final event number also will decrease. Considering the fact that for a small mass splitting of triplet scalars \( \Delta M \sim \lambda_4 v_\Delta^2/(8M_{A^0}) \), the cascade decay dominant width \( \Gamma_{H^0/A^0} \sim \Delta M^5 \), and \( \delta M \sim v_\Delta^2 \), it is easy to derive the relation \( \lambda_4 \propto v_\Delta^{2/5} \) by taking \( \delta M \sim \Gamma_{H^0/A^0} \).

Now, let’s consider the same-sign tetralepton signature at the 3 TeV CLIC. In this scenario, we set \( M_{A^0} = 1000 \text{ GeV} \), and the same-sign dilepton decay \( H^\pm \to \ell^\pm \ell^\pm \) is still allowed. Therefore, in addition to the tetraboson process in Eqn. \[12\], we also have the direct tetralepton channel

\[ e^+e^- \to H^0A^0 \to H^\pm W^*H^\pm W^* \to H^\pm W^*H^\pm W^* + W^*W^* \to 4\ell^\pm + X. \quad (14) \]

The corresponding cross section is then calculated as

\[
\sigma(4\ell^\pm + X) = \sigma(e^+e^- \to H^0A^0) \times \left( \frac{2 + x^2}{1 + x^2} \right) \times \text{BR}(H^0/A^0 \to H^\pm W^*)^2 \times \text{BR}(H^\pm \to W^*W^*)^2 \times \text{BR}(H^\pm \to \ell^\pm \ell^\pm)^2.
\]

In the left panel of Fig. 4, we show the product of BRs in the direct tetralepton decay process. As shown in Fig. 1, the cascade decays are suppressed for \( v_\Delta \lesssim 10^{-5} \text{ GeV} \) with \( M_{A^0} = 1000 \text{ GeV} \), so we do not show the region \( v_\Delta < 10^{-5} \text{ GeV} \). The right boundary corresponds to the area where \( \text{BR}(H^\pm \to \ell^\pm \ell^\pm) \) is
FIG. 4. Left panel: Product of branching ratios $\text{BR}(H^0/A^0 \to H^\pm W^-)^2 \times \text{BR}(H^\pm \to H^{\pm\pm} W^-)^2 \times \text{BR}(H^{\pm\pm} \to \ell^\pm \ell^\pm)^2$ for the process $e^+e^- \to H^0A^0$ with $M_{A^0} = 1000\text{GeV}$. Middle panel: Product of branching ratios $\text{BR}(H^0/A^0 \to H^\pm W^-)^2 \times \text{BR}(H^\pm \to H^{\pm\pm} W^-)^2 \times \text{BR}(H^{\pm\pm} \to W^\pm W^\pm)^2 \times \text{BR}(W^\pm \to \ell \nu)^4$. Right panel: Event number of the same-sign tetralepton signature $4\ell^\pm + X$ for the mass $M_{H^0} \sim M_{A^0} = 1000\text{GeV}$ from $e^+e^- \to H^0A^0$ and subsequent decays at the $\sqrt{s} = 3\text{ TeV}$ CLIC with luminosity $\mathcal{L} = 5\text{ ab}^{-1}$.

suppressed. For $\lambda_4 > 0.5$, there are large parameter space where the product of BRs reaches the maximum, i.e., 0.25. In the middle panel of Fig. 4 the product of BRs in the diboson process is also shown. Comparing with the region of $M_{A^0} = 400\text{GeV}$ in Fig. 3, the region of $M_{A^0} = 1000\text{GeV}$ is much smaller. For instance, when product of BRs is larger than 0.002, one needs $\lambda_4 \gtrsim 0.5$ and $10^{-4}\text{GeV} \lesssim v_\Delta \lesssim 10^{-3}\text{GeV}$. This is because for heavier scalar triplet, the branching ratios of cascade decays are suppressed.

In the right panel of Fig. 4 we show the expected event number for the same-sign tetralepton signature at the 3 TeV CLIC with an integrated luminosity of $\mathcal{L} = 5\text{ ab}^{-1}$. Here, the expected event number is the sum of both diboson decay process in Eqn. (13) and the dilepton decay process in Eqn. 15. In a small area around $v_\Delta \sim 4 \times 10^{-4}\text{GeV}$ and $\lambda_4 \sim 0.26$, we have the maximum number $\sim 16$, where the dominant contribution is from $H^{\pm\pm} \to \ell^\pm \ell^\pm$. Meanwhile, the $H^{\pm\pm} \to W^\pm W^\pm$ dominant tail region with $10^{-4}\text{GeV} \lesssim v_\Delta \lesssim 10^{-3}\text{GeV}$ only predicts a total event number less than three, thus this long tail region is not promising.

At last, we consider the same-sign tetralepton signature at the 6 TeV MuC. The corresponding production processes at muon collider are

\begin{align*}
\mu^+\mu^- &\to H^0A^0 \to H^\pm W^* H^\pm W^* \to H^{\pm\pm} W^* H^{\pm\pm} W^* \to 4W^\pm (\to \ell^\pm \nu) + X, \quad (16) \\
\mu^+\mu^- &\to H^0A^0 \to H^\pm W^* H^\pm W^* \to H^{\pm\pm} W^* H^{\pm\pm} W^* \to 4\ell^\pm + X. \quad (17)
\end{align*}

The production cross section is then obtained by simply replace $\sigma(e^+e^- \to H^0A^0)$ in Eqn. (13) and Eqn. (15) with $\sigma(\mu^+\mu^- \to H^0A^0)$. In the left and middle panel of Fig. 5 we show the product of BRs in the direct tetralepton and tetraboson decay process with $M_{A^0} = 1500\text{GeV}$. To realize a relatively large value of BRs, $\lambda_4$ has to be larger than 0.8. However, such large $\lambda_4$ leads to too large mass splitting of triplet scalars that
FIG. 5. Same as Fig. 4 but for $M_{{H^0}} \sim M_{{A^0}} = 1500\text{GeV}$ from the process $\mu^+\mu^- \rightarrow H^0 A^0$ at the $\sqrt{s} = 6\text{ TeV}$ MuC with luminosity $\mathcal{L} = 10\text{ ab}^{-1}$.

the interference factor $x$ is suppressed. In the right panel of Fig. 5, we show the total event number for the same-sign tetralepton signature at the 6 TeV MuC with an integrated luminosity of $\mathcal{L} = 10\text{ ab}^{-1}$. It is obvious that the event number is always smaller than three. Therefore, the same-sign tetralepton signature is not promising at the MuC for $M_{{A^0}} = 1500\text{ GeV}$.

IV. CONCLUSION

In this paper, we study the novel same-sign tetra-lepton signature in type-II seesaw at the future lepton colliders (including 1 TeV ILC, 3 TeV CLIC, and 6 TeV MuC). The signature arises from the mixing of associated production of Higgs fields $H^0 A^0$ followed by the cascade decays $H^0/A^0 \rightarrow H^\pm W^*$, $H^\pm \rightarrow H^{\pm\pm} W^*$, and $H^{\pm\pm} \rightarrow \ell^\pm \ell^\pm / W^\pm W^\pm$ with $W^\pm \rightarrow \ell^\pm \nu$. There are two important parameters $\lambda_4$ and $v_\Delta$ closely related to this signature, where $\lambda_4$ controls the mass splitting of triplet scalars and $v_\Delta$ determines the decay mode of $H^{\pm\pm}$.

We first consider a low mass benchmark scenario with $M_{{A^0}} = 400\text{ GeV}$ at 1 TeV ILC. In this scenario, $H^{\pm\pm} \rightarrow W^\pm W^\pm$ is the only viable decay mode. The production cross section of the process $e^+ e^- \rightarrow H^0 A^0$ varies around 10 fb. The promising region corresponds to a narrow band in the range of $10^{-4} \text{ GeV} \lesssim v_\Delta \lesssim 10^{-2} \text{ GeV}$. With an integrated luminosity of $\mathcal{L} = 8\text{ ab}^{-1}$, we find that a neutral Higgs of mass around 400 GeV can lead to around 150 events at ILC. For heavier triplet scalars, we then consider $M_{{A^0}} = 1000\text{ GeV}$ at 3 TeV CLIC, where the cross section $\sigma(e^+ e^- \rightarrow H^0 A^0)$ is about 2 fb. Although this value is about two orders of magnitudes larger than at 14 TeV LHC, the cascade decay branching ratios are suppressed for small $\lambda_4$. This leads to a mismatch between cascade decays and the interference effect. A maximum event number $\sim 16$ can be obtained around $v_\Delta \sim 4 \times 10^{-4} \text{ GeV}$ and $\lambda_4 \sim 0.26$ with an integrated luminosity of $\mathcal{L} = 5\text{ ab}^{-1}$ at CLIC. In this high mass scenario, the $H^{\pm\pm} \rightarrow \ell^\pm \ell^\pm$ decay mode is the dominant contribution.
to the same-sign tetralepton signature. If the triplet scalars are even heavier than 1 TeV, e.g., $M_{A^0} = 1500$ GeV, the cascade decays will be heavily suppressed. With an integrated luminosity of $\mathcal{L} = 10 \text{ ab}^{-1}$ at 6 TeV MuC, there are at best have 3 signal events. Therefore, this signature is not promising at MuC.

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