PHOTON SPLITTING IN THE SUPERSTRONG MAGNETIC FIELDS OF PULSARS

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ABSTRACT

We discuss the polarization selection rules for the splitting of the two principal electromagnetic modes that propagate in a vacuum polarized by a superstrong magnetic field \( B > 0.1B_c \approx 4 \times 10^{12} \text{ G} \). We show that below the threshold of free pair creation, the selection rules found by Adler in the limit of weak dispersion remain unaffected by taking the resonant effects into consideration; i.e., splitting of one mode is strictly forbidden, while splitting of the other is allowed.

Subject headings: gamma rays: theory — magnetic fields — pulsars: general — stars: neutron

1. INTRODUCTION

Current models for the generation of coherent radio emission of pulsars require formation of free \( e^+e^- \) pairs in the pulsar magnetospheres (e.g., Ruderman & Sutherland 1975; Michel 1991; Melrose 1993; Usov 1996). Such pairs may be created via conversion of \( \gamma \)-rays into \( e^+e^- \) pairs in a strong magnetic field, \( \gamma + B \rightarrow e^+ + e^- + B \) (Erber 1966). Besides, some (if not all) pulsars are powerful sources of \( \gamma \)-ray emission (e.g., Hartmann 1995; Thompson et al. 1995; Daugherty & Harding 1996). Therefore, propagation of \( \gamma \)-rays in the magnetosphere is one of the central problems in the theory of pulsars (Usov & Melrose 1995, 1996; Harding, Baring, & Gonthier 1997; Baring & Harding 2001 and references therein).

In the magnetospheres of pulsars, the plasma dispersion for \( \gamma \)-rays is negligible, and to understand the process of \( \gamma \)-ray propagation in the vicinity of a pulsar it suffices to consider propagation in the vacuum polarized by a strong magnetic field (e.g., Usov & Melrose 1995; Bulik 1998). The principal modes of propagation for a photon in the magnetized vacuum are linearly polarized with the photon electric field either perpendicular (\( \perp \) mode) or parallel (\( \parallel \) mode) to the plane formed by the external magnetic field \( B \) and the photon wave vector \( K \). Both modes are generated in the magnetospheres of pulsars (Baring & Harding 2001). A single photon in either mode can decay into a free \( e^+e^- \) pair provided that a relevant threshold \( \epsilon, \sin \theta = 2mc^2 \) for \( \parallel \) mode and \( \epsilon, \sin \theta = mc^2\eta \) for \( \perp \) mode is exceeded, where \( \epsilon \) is the photon energy, \( B_c = m^2c^2\eta e/\hbar = 4.4 \times 10^{13} \text{ G} \), \( \eta = 1 + [1 + (2B/B_c)]^{1/2} \), and \( \theta \) is the angle between the photon wave vector \( K \) and the magnetic field \( B \). For typical magnetic fields of pulsars \( B \approx 10^{12} \text{ G} \), the absorption coefficient for photon pair production is at least a few orders of magnitude higher than the absorption coefficient for any other inelastic interaction of photons and magnetic fields (Adler 1971). However, for all known pulsars, \( \gamma \)-rays generated near the neutron star surface are produced in a state below the pair creation threshold (Usov & Melrose 1995; Baring & Harding 2001). In this case, the process of pair creation by single photons is kinematically forbidden, and the main (inelastic) interaction of photons and the magnetic field is a splitting into two photons: \( \gamma + B \rightarrow \gamma_1 + \gamma_2 + B \) (Adler 1971; Stoneham 1979; Melrose 1983; Baring 1991). Adler (1971) showed that in the limit of weak dispersion, where the refractive indices for both modes are very close to unity, only \( \perp \) mode may undergo the splitting process. Near the threshold of pair creation, however, the dispersion law differs considerably from the vacuum case, \( |K| = \omega c \), and the weak-dispersion limit is no longer applicable (Shabad 1975). The polarization selection rules for photon splitting, taking into account the resonant dispersion law near the threshold of free pair creation, were discussed by Usov & Shabad (1983). Later, it was shown that bound \( e^+e^- \) pairs (positronium) may be created by single photons in a strong magnetic field (Shabad & Usov 1985, 1986; Herold, Ruder, & Wunner 1985; Usov & Shabad 1985; Usov & Melrose 1995, 1996). The threshold of bound pair creation is slightly below the threshold of free pair creation, and therefore creation of bound pairs may be allowed while creation of free pairs is yet forbidden. The dispersion curves, corrected by taking into account formation of the mixed photon-positronium states, were calculated in Shabad & Usov (1986). These dispersion curves near and below the thresholds of pair creation differ significantly from the photon dispersion curves calculated by Shabad (1975) and used in Usov & Shabad (1983). In this Letter, using the corrected dispersion curves (Shabad & Usov 1986), the polarization selection rules for splitting of both modes in a superstrong \( B > 0.1B_c \) magnetic field are considered.

2. POLARIZATION SELECTION RULES FOR SPLITTING

In a time-independent, spatially uniform external magnetic field \( B \), the dispersion curve for the principal mode of polarization \( a (a = \{ \parallel, \perp \}) \) may be written in the following general form (e.g., Shabad 1975; Usov & Shabad 1983):

\[
\left( \frac{\omega^2}{\epsilon} - K_a^2 \right) = f_a(K_a^2),
\]

where \( \omega = \epsilon/\hbar \) is the frequency of state (photon, positronium atom, or their mixture), \( K_a \) and \( K_a^2 \) are the components of the wave vector along and across the magnetic field \( B \), and \( f_a(K_a^2) \) is a certain function of \( K_a^2 \). In a superstrong magnetic field for the \( \parallel \) mode below the threshold of free pair creation, \( (\omega c)^2 - K_a^2 < (2mc^2/\hbar)^2 \), we have (Usov & Shabad 1985; Shabad & Usov...
that is given by equations (1)–(5) is photon-like for Fig. 1). In the process of propagation, both modes to shift along the relevant dispersion curves (see case, the curvature of the magnetic field lines causes same points ( ),

\[ n / H_{11001} \]

is at the first excited Landau level (Shabad & Usov 1986).

\[ \frac{1}{H_{11001}} \]

n

\[ \frac{2mc^2}{H_{11001}} \]

is the rest energy of positronium like for \( \gamma > (2mc^2) \) and positronium-like for \( K_2^x > (2mc^2) \) (see Fig. 1). At \( K_2^x \approx (2mc^2) \), this curve describes the mixed photon-positronium state.

The positronium states of the lowest series (\( n^+ = n^- = 0 \) and \( n^+ \geq 0 \), including the boundary of its continuum (\( \omega_c^2 - K_2^x = (2mc^2) \)), do not contribute to the polarization operator of a \( \perp \)-polarized state (Shabad & Usov 1986). Therefore, the dispersion curve (\( \omega_c^2 - K_2^x = f_j(K_j^x) \)) for the \( \perp \) mode crosses the positronium spectra of the lowest series without interfering with them (see Fig. 1). As \( K_2^x \) grows further, this curve eventually approaches the dispersion curve of positronium at the state with \( n^+ + n^- = 1 \) and \( n^+ = 0 \) when either the electron or the positron is at the first excited Landau level (Shabad & Usov 1986).

At \( B > 0.2B_\gamma \), the geometrical optics or adiabatic approximation is valid for propagation of \( \gamma \)-rays in the pulsar magnetospheres (Usos & Melrose 1995 and references therein). In this case, the curvature of the magnetic field lines causes \( \gamma \)-rays of both modes to shift along the relevant dispersion curves (see Fig. 1). In the process of propagation, \( \gamma \)-rays that are created significantly below the threshold of pair creation can pass through the mixed photon-positronium states and turn into positronium at \( K_j > 2mc^2 \). In this section we discuss the polarization selection rules for splitting of both polarization modes irrespective of whether the initial state is a photon or a positron atom. We consider the case when the initial state is below the threshold of free pair creation, (\( \omega_c^2 - K_2^x < (2mc^2) \)).

The splitting reactions that may be, in principle, in a super-strong magnetic field are \( \perp \rightarrow \perp + \perp \), \( \perp \rightarrow \perp + \perp \), \( \perp \rightarrow \perp + \perp \), \( \perp \rightarrow \perp + \perp \), \( \perp \rightarrow \perp + \perp \), and \( \perp \rightarrow + + \).

\[ f_j(K_j^x) = \frac{1}{2} \left[ \left( \frac{\epsilon_{\omega}(0, K_j^x)}{\epsilon_{\omega}(0, K_j^x)} \right)^2 + K_j^x \right] - \left( \left( \frac{\epsilon_{\omega}(0, K_j^x)}{\epsilon_{\omega}(0, K_j^x)} \right)^2 + 4A(K_j^x) \right)^{1/2}, \]

where

\[ A(K_j^x) = \frac{4\alpha eB}{c^2\hbar^2} \epsilon_{\omega}(0, K_j^x) |\psi(0)|^2 \exp \left( -\frac{chK_j^x}{2eB} \right), \]

\[ \epsilon_{\omega}(0, K_j^x) = 2mc^2 - \Delta \epsilon_{\omega}(0, K_j^x) \]

is the binding energy, \( \psi(0) \) is the longitudinal wave function of the positronium with the constituent particles taken at the same points (\( z^+ = z^- \)),

\[ |\psi(0)|^2 = \frac{1}{a} \ln \frac{a}{r_B(1 + r_B^2 K_j^x)^{1/2}}, \]

\[ a = \hbar mc^2 \]

is the Bohr radius, \( r_B = (\hbar/eB)^{1/2} \) is the radius of the electron orbit in the magnetic field \( B \), and \( \alpha = e^2/\hbar c^2 \) is the fine-structure constant. The dispersion curve of the \( \| \) mode that is given by equations (1)–(5) is photon-like for \( K_2^x < (2mc^2) \) and positronium-like for \( K_2^x > (2mc^2) \) (see Fig. 1). At \( K_2^x \approx (2mc^2) \), this curve describes the mixed photon-positronium state.

In our case, when \( K_1 = 0 \), we have \( K_{1,1} = -K_{2,1} \).

Using equations (2)–(5), we can solve the dispersion relation (\( \omega_c^2 - K_2^x = f_j(K_j^x) \)) and find \( |K_{1,1}| \) as a function of \( \omega \) (Shabad & Usov 1986). From equations (2)–(5), it follows that the deviation of \( |K_{1,1}| \) from the vacuum dispersion curve \( |K_{1,1}| = \omega/c \) to the side of large values of \( |K_{1,1}| \) increases with increasing \( \omega \) throughout the interval \( 0 < \omega < 2mc^2/\hbar \) (see Fig. 1). In other words, we have \( \varphi(\omega_1, \omega_2) > \varphi(\omega_1) + \varphi(\omega_2) \) for \( \omega_1, \omega_2 > \omega \), and \( \varphi(\omega_1, \omega_2) > \varphi(\omega_1) + \varphi(\omega_2) \) for any values of \( \omega_1 \) and \( \omega_2 \) at \( \omega_1 + \omega_2 < 2mc^2/\hbar \). These properties of \( \varphi(\omega) \) and equation (6) yield

\[ |K_{1,1}| = \varphi(\omega) = \varphi(\omega_1, \omega_2) > \varphi(\omega_1) + \varphi(\omega_2) \]

\[ \geq \varphi(\omega_1) + \varphi(\omega_2) = |K_{1,1} + |K_{2,1}|. \]

Therefore, the conservation of \( K_{1,1} \) cannot be satisfied for the splitting reaction \( \perp \rightarrow + + \) below the threshold of free pair creation, and this reaction is kinematically forbidden. This is
valid irrespective of the fact that the initial state is either a photon or positronium atom.

The splitting reactions \( || \rightarrow || + \perp, \perp \rightarrow || + ||, \) and \( \perp \rightarrow || + \perp \) are also kinematically forbidden below the threshold of free pair creation. Indeed, if we rewrite the dispersion relation \( (\omega E)^2 - K_f^2 = f_s^2(K_f^2) \) for the \( \perp \) mode in the form \( |K_f| = \varphi_3(\omega) \), the inequality \( \varphi_3(\omega) > \varphi_3(\omega) \) holds throughout the interval \( 0 < \omega < 2mc^2/h \) (Shabad 1972, 1975; Shabad & Usov 1986). Using this inequality, the chain of inequalities (7) may be continued for splitting reactions when the initial state is the \( || \) mode and at least one final state is the \( \perp \) mode. Finally, we have \( |K_f| > |K_{\perp,1}^2 + K_{\perp,1}^2| \) for all these reactions. Hence, splitting of the \( || \) mode is strictly forbidden below the threshold of free pair creation; i.e., Adler’s conclusion remains unaffected by taking the resonant effects into consideration.

Since the dispersion curve \( |K_f| = \varphi_3(\omega) \) for the \( \perp \) mode crosses the positronium spectra of the lowest series \( (n^2 = n^2 = 0) \) without interfering with them, the resonant effects are unessential for the splitting of the \( \perp \) mode below the threshold of free pair creation, \( \omega \leq 2mc^2/h \) (see Fig. 1). In this case, the splitting reaction \( \perp \rightarrow || + \perp \) is kinematically forbidden, while the splitting reactions \( \perp \rightarrow || + ||, || \rightarrow || + \perp, \) and \( \perp \rightarrow || + || \) are kinematically allowed (Adler 1971). Anyone can also come to this conclusion from the following properties of the functions \( \varphi_3(\omega) \) and \( \varphi_3(\omega) \) below the threshold of free pair creation: \( \varphi_3(\omega) > \varphi_3(\omega) \) for \( \omega > \omega_0, \varphi_3(\omega + \omega_0) > \varphi_3(\omega) + \varphi_3(\omega_0), \) and \( \varphi_3(\omega) > \varphi_3(\omega) \) (Shabad & Usov 1986). The splitting reactions \( || \rightarrow || + \perp \) and \( \perp \rightarrow || + || \) that involve an odd number of \( \perp \) polarized photons are forbidden by CP (charge conjugation and parity) invariance in the limit of zero dispersion (Adler 1971). In the magnetized vacuum, dispersive effects guarantee a nonzero probability for these reactions. Therefore, the channels \( || \rightarrow || + \perp \) and \( \perp \rightarrow || + || \) are strongly suppressed in comparison with \( || \rightarrow || + || \) when the refraction index of the magnetized vacuum is close to unity, \( n - 1 = 0.1a(B/B_c)^2 \ll 1 \) (Adler 1971). However, this suppression may not hold in extremely strong magnetic fields \( B \approx (10/\alpha)^{1/2} B_c, \sim 10^{15} \) G.

3. DISCUSSION

In this Letter we have discussed splitting of the principal modes that propagate in the vacuum polarized by a superstrong \( (B > 0.1B_c) \) magnetic field. We have shown that splitting of the \( || \) mode is strictly forbidden below the threshold of free pair creation. This is valid irrespective of the fact that the initial state is either a photon \( (|K_f| < 2mc/h) \) or a positronium atom \( (|K_f| > 2mc/h) \). Splitting of the \( \perp \) mode is allowed, and only the one channel \( || \rightarrow || + || \) of splitting operates if the magnetic field is not extremely high (Adler 1971).

In the study of the polarization selection rules for splitting, Adler (1971) used the refractive indices

\[
n_{||,1} = \frac{c|K_f|}{\omega} = \left[ \frac{\varphi_3^2(\omega) + K_f^2}{(\omega/c)^2 + K_f^2} \right]^{1/2}, (8)
\]

while we use the dispersion curves \( |K_f| = \varphi_3(\omega) \). From equation (8), the properties of \( n_{||,1} \) used by Adler (1971) follow immediately from the properties of \( \varphi_3(\omega) \) used in our study, and therefore both these methods are equivalent.

The fact that the resonant effects do not change the polarization selection rules found by Adler for photon splitting in the limit of weak dispersion may be easily explained. Indeed, to formulate the selection rules for the case when photons are significantly below the threshold of pair creation, the following properties of functions \( \varphi_3(\omega) \) and \( \varphi_3(\omega) \) were used, in fact, by Adler (1971), \( \varphi_3(\omega) > \varphi_3(\omega) \), and for both principal modes the deviation of \( |K_f| \) from the vacuum dispersion curve \( |K_f| = \omega/c \) to the side of large values of \( |K_f| \) increases with the increase of \( \omega \). These properties are only strengthened by the resonant effects, especially when the dispersion curve of the \( \perp \) mode approaches the positronium state at \( |K_f| \gg 2mc/h \) (see Fig. 1). Therefore, for a positronium atom at ground state \( (n' = n = n = 0) \), with \( |K_f| \gg 2mc/h \), its stability against spontaneous decay into photons is evident and has been mentioned long ago (Herold et al. 1985).

The dispersion curves calculated in Shabad & Usov (1986) and used in this Letter are exact relativistic consequences of the adiabatic approximation, and therefore they are valid for an arbitrary strong magnetic field. (This relates to eqs. [1], [2], and [3]) The nonrelativity is introduced only when the binding energy of the positronium (eq. [4]) and the longitudinal wave function (eq. [5]) that are the solutions of the nonrelativistic Schrödinger equation are substituted into equation (3). The relativistic corrections to both the binding energy and the longitudinal wave function are small if the positron and the electron remain nonrelativistic along the field direction, i.e., if \( \Delta E_{cm}(0, 0) \ll mc^2 \) (Angelie & Deutsch 1978; Lai & Salpeter 1995). From equation (4) it follows that the dispersion curves calculated in Shabad & Usov (1986), and therefore their considerations, are valid even if the magnetic field is as high as the virial magnetic field \( (\sim 10^{11}-10^{14} \) G) for neutron stars.

There is now compelling evidence for the existence of “magnetars”—neutron stars with magnetic field strengths in excess of \( B_c \). The evidence comes primarily from observations of rapid spin down in the pulsations observed from soft \( \gamma \) -ray repeaters (e.g., Kouveliotou et al. 1998, 1999). Recently, radio surveys have discovered a few pulsars with surface magnetic fields approaching \( 10^{14} \) (Camilo et al. 2000). Kulkarni & Frail (1993) and Kouveliotou et al. (1994) estimate that the birthrate of magnetars is about 10% that of ordinary pulsars. The processes of photon splitting and bound pair creation may be very important for magnetars (Thompson & Duncan 1995; Usov & Melrose 1995; Baring & Harding 2001 and references therein). We briefly discuss these processes in the magnetospheres of pulsars with very strong magnetic fields.

For pulsars with surface magnetic field \( B \sim 10^{15} \) G, \( \perp \) -polarized \( \gamma \)-rays generated near the neutron star surface split into \( \parallel \) -polarized \( \gamma \)-rays before they reach the pair creation threshold (e.g., Baring & Harding 2001). In turn, in such a strong magnetic field, \( \parallel \) -polarized \( \gamma \)-rays create bound pairs that are stable, in the absence of such external factors as electric fields and ionizing radiation (Usov & Melrose 1995 and references therein). The bound pairs form a gas of electroneutrality particles. Such a gas does not undergo plasma processes, such as plasma instabilities, which are responsible for the generation of coherent radio emission of pulsars. Maybe the suppression of free pair creation in superstrong magnetic fields results in a death line of pulsars at \( B \sim B_c \) (e.g., Baring & Harding 1998; Heyl & Kulkarni 1998). For this suppression it is very important that splitting of \( \perp \) -polarized \( \gamma \)-rays into \( \parallel \) -polarized \( \gamma \)-rays prevents from formation of bound pairs at the state with \( n' + n = 1 \) when either the electron or the positron is in an excited state with the spin quantum number, \( s = 1 \), opposite to that in the ground state. Otherwise, bound pairs in the excited state may be ionized by the recoil from photons radiated in a spin-flip transition, and free pairs may form in the magnetospheres of pulsars.

Since bound pairs do not screen the electric field \( E_i \), the formation of bound rather than free pairs may cause increases in the height of the polar gap and the total power carried away by
relativistic particles from the polar gap into the pulsar magnetosphere by large factors (e.g., Shabad & Usov 1985). For the case when there is no emission of particles from the surface of a strongly magnetized neutron star, these increases were considered by Usov & Melrose (1995, 1996) in detail. Recently, the polar gap model, which is rather close to reality for typical pulsars, was developed for the case when particles flow freely from the stellar surface (Muslimov & Harding 1997; Harding & Muslimov 1998; Zhang & Harding 2000; Hibschman & Arons 2001). In this model, the \( E_1 \)-field in the polar gap is due to the effect of general relativistic frame dragging discovered by Muslimov & Tsygan (1992). For the pulsar period \( P \approx 0.1 \) s and \( 0.1 B_z \approx B_z \approx 0.1 B_\star \), this field (e.g., Harding & Muslimov 1998) is significantly smaller than the value \( E_1 \approx (1-2) \times 10^{10} \) V cm\(^{-1}\), at which field ionization of bound pairs becomes important (Usov & Melrose 1996). The mean free path of bound pair photoionization is \( \lambda_\text{ph} \approx 10^3 \) cm (Usov & Melrose 1995), where \( \Gamma \) is the Lorentz factor of bound pairs and \( T_\star \) is the surface temperature. For high-energy bound pairs \( (\Gamma > 10^3-10^4) \) that might be responsible for the \( E_1 \)-field screening after their ionization, we have that \( l_\text{ph} \) is larger than the polar gap height even if \( T_\star \) is as high as a few times \( 10^6 \) K. Therefore, the polar gap model (developed in Muslimov & Harding 1997; Harding & Muslimov 1998; Zhang & Harding 2000; Hibschman & Arons 2001) is probably accurate for magnetic fields \( B_z \approx (0.1-0.2)B_\star \), while for \( B_z > 0.2B_\star \) it is necessary to modify this model by taking into account photon splitting and bound pair creation.

In the vicinity of pulsars with \( B_z \approx 10^{13} \) G, splitting of \( \perp \)-polarized \( \gamma \)-rays into \( \parallel \)-polarized \( \gamma \)-rays provides a mechanism for the production of linearly polarized \( \gamma \)-rays. At energies \( \epsilon_\gamma \approx 10^2 \) MeV, the \( \gamma \)-ray polarization may be up to 100% (Shabad & Usov 1983). By observing the polarization of the \( \gamma \)-ray emission of pulsars it would be possible to estimate the magnetic field near the pulsar surface.

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