Results on a Chromatic Number of a Bi-polar Fuzzy Complete Bipartite Graphs and Labelling of Tri-Polar Fuzzy Graphs

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Abstract: The ultimate objective of a piece of research work is to present the labelling of vertices in 3-PFG and labelling of distances in 3-PFG. Also, we characterize some of its properties. Later, we define the vertex and edge chromatic number BF-Complete Bipartite graph. Further, we illustrated an example for BFRGS which represents a Route Network system.

Keywords: 3-polar-Fuzzy Graphs, Regular Graph, Labelling Graph, Chromatic number, Complete Bipartite graph.

1. Introduction

The conventional investigation of SG starts in the mid-20\textsuperscript{th} century. SGs are fundamental logarithmic models in numerous parts of designing formal dialects, in coding, Finite State Machine, automaton. The significant part of graph theory in computer presentations is the improvement of calculations in graphs. A graph structure is a helpful tool in cracking the combinatory problems in diverse areas of computer science containing clustering, image capturing, data mining, image segmentation, networking, and computational intelligence systems. (Zadeh. L. A.,1965) presented the fuzzy theory. Further, (Rosenfeld., 1971) applied it to the classical theory of subgroups. Later, (Bhargavi. Y., 2020) and (Mordeson J. N) developed the classical theory of fuzzy graphs, SGs. (Akram. M., 2011) presented several new ideas including bipolar Fuzzy-Graphs. Furthermore, quite a few authors (Bhargavi. Y 2020; Bhargavi. Y 2020; Bhargavi. Y 2020; Loganathan, J., 2017; Murali Krishna Rao. M. (2015); Ranjeeth.S. (2020); Ragamayi.S. (2019); Ragamayi.S. (2015) done on Fuzzy and vague structures of SGs and Nearings. Later (Akram. M., 2011) proved outstanding results on applications of graph theory and Labelling. Accepting the above examination as starting point, in this Research article present the labelling of vertices in 3-PFG and labelling of distances in 3-PFG. Also, we characterize some of its properties. Later, we define the vertex and edge chromatic number BF-Complete Bipartite graph. Further, we illustrated an example for BFRGS which represents a Route Network system. Moreover, we discussed about Cartesian product of two BFRGS.

Notations

1) BF represents Bipolar Fuzzy.
2) BFG represents Bipolar Fuzzy Graph.
3) F-Graph represents Fuzzy-Graph
4) C-Graph represents Crisp-Graph
5) BFGS represents Bipolar Fuzzy Graph of Semi-group.
6) BF-IGS represents bipolar fuzzy ideal graph of Semi-group.
7) BF-IS represents bipolar fuzzy ideal of a Semi-group
8) SG represents Semi-group.
9) FS represents Fuzzy Subset.
10) BFRG represents Bipolar Fuzzy Regular Graph
11) BFRGS represents Bipolar Fuzzy Regular Graph of a Semi-group
12) 3-PFG represents 3-polar fuzzy graph or tri-polar fuzzy graph
13) 3-PFP represents 3-polar fuzzy Pathor tri-polar fuzzy path

2. Preliminaries

Definition 2.1 (Ragamayi, S. 2020) A pair (V, E) is a graph if \( V \neq \emptyset \) and E is a set of un-ordered pairs of elements of V.

Definition 2.2 (Zadeh. L. A.,1965) A non-empty set A is said to be a fuzzy subset if a mapping \( g: A \rightarrow [0,1] \).

Definition 2.3 (Akram. M., 2011) A finite fuzzy subset, V is a mapping \( \mu: V \rightarrow [0,1] \) which assigns to each element \( x \in V \) a degree of membership \( 0 \leq \mu \leq 1 \), and a fuzzy subset of V X V is a mapping \( \rho: V \times V \rightarrow [0,1] \) which assigns to each pair \( (x, y) \) a degree of membership \( 0 \leq \rho(x, y) \leq 1 \).

Definition 2.4 (Akram. M., 2011) Let \( A= (\mu_{AP}, \mu_{AN}) \). A BF relation on \( X \neq \emptyset \) is defined as \( A: X \times X \rightarrow [0,1] \times [-1,0] \) where \( \mu_{AP}(p,q) \in [0,1] \) and \( \mu_{AN}(p,q) \in [-1,0] \).
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Definition 2.5 (Akram. M., 2011) If A = (μAP, μAN) is a BF set on an underlying set V and B = (μBP, μBN) is a BF set in \( V \) 2 for which \( \mu BP(\{m, n\}) \leq \min \{ \mu AP(m), \mu AP(n) \}, \forall m, n \in V \) and \( \mu BN(\{m, n\}) \geq \max \{ \mu AN(m), \mu AN(n) \}, \forall m, n \in V \) and \( \mu BP(\{m, n\}) = \mu BN(\{m, n\}) = 0, \forall m, n \in (V 2 - E) \), then \( G = (V, A, B) \) is termed a BFG of the graph \( G = (V, E) \).

Definition 2.6 (Akram. M., 2011) The order of a BFG, \( G = (V, A, B) \) is represented by \( O(G) = (\mu OP, \mu ON) \) and is distinguished as \( \mu OP = \sum_{a \in V} \mu AP(a) \) and \( \mu ON = \sum_{a \in V} \mu AN(a) \). The size of a BFG, \( G = (V, A, B) \) is denoted by \( S(G) = (\mu SP, \mu SN) \) and is distinguished as \( \mu SP = \sum_{a_1, a_2 \in V^2} \mu BP(a_1, a_2) \) and \( \mu SN = \sum_{a_1, a_2 \in V^2} \mu BN(a_1, a_2) \).

Definition 2.7 (Loganathan, J., 2017) The open neighbourhood degree of a vertex ‘m’ in a BFG, \( G \) is distinguished as \( \deg OP(m) = \sum_{a \in \langle V, B \rangle} \mu AP(m, a) \) and \( \deg ON(m) = \sum_{a \in \langle V, B \rangle} \mu BN(m, a) \).

Definition 2.8 (Loganathan, J., 2017) A FS, \( \xi \) of a Semi-group, \( T \) is known as a fuzzy-sub-SG of \( T \) if \( \xi(\{m, n\}) \geq \min \{ \xi(m), \xi(n) \} \), \( \forall m, n \in T \) and \( \xi \) is a FSoT of \( S(T) \), \( T \) is known as an anti-fuzzy ideal of \( T \).

Definition 2.9 (Ragamayi, S., 2020) A relation \( \sigma : T \times T \to [0,1] \) is known as a fuzzy relation on a FS, \( \xi \) of \( T \) if \( \sigma(m, n) \leq \min \{ \xi(m), \xi(n) \} \), \( \forall m, n \in T \) and \( \xi \) is a FSoT of \( S(T) \), \( T \) is known as an anti-fuzzy ideal of \( T \).

Definition 2.10 (Loganathan, J., 2017) If \( \xi(\{m, n\}) \geq \min \{ \xi(m), \xi(n) \} \), \( \forall m, n \in T \) then \( \xi \) is a FSof \( T \) if \( \sigma(m, n) \leq \min \{ \xi(m), \xi(n) \} \), \( \forall m, n \in T \) and \( \xi \) is a FSoT of \( S(T) \), \( T \) is known as an anti-fuzzy ideal of \( T \).

Definition 2.11 (Ragamayi, S., 2020) If \( \sigma(m, n) \leq \min \{ \mu(m, n) \} \) \( \forall \{m, n\} \in T \) and \( G = (\mu, \sigma) \) is known to be a F-Graphon vertex set \( V \neq \emptyset \) where \( v \) and \( \sigma \) are FS on \( V \) and \( V \times V \) correspondingly.

Definition 2.12 (Ragamayi, S., 2020) If \( \sigma(m, n) \leq \min \{ \mu(m, n) \} \) \( \forall \{m, n\} \in T \) and \( G = (\mu, \sigma) \) is known to be a F-Graphon vertex set \( V \neq \emptyset \) where \( v \) and \( \sigma \) are FS on \( V \) and \( V \times V \) correspondingly.

Definition 2.13 The bipolar fuzzy vertex chromatic number of complete bipolar fuzzy graph \( G = (A, B) \) is\( n(n, n) \), where \( n \) is the number of vertices of \( G \).

Definition 2.14 The edge chromatic number of complete bipolar fuzzy graph \( G = (A, B) \) on \( n \) vertices is \( (n, n) \), if \( n \) is odd and is \( (n-1, n-1) \), if \( n \) is even.

3. Vertex and Edge Chromatic Number of a Bipolar Fuzzy of a Complete Bipartite Graph

In this segment, we propose the concept of BFG signifying a Route network representing a regular Graph of a SG as a generalization of Regular Graph, and Bipartite Graph and C-Graph. Here, we work on simple graphs having limited number of routes (Edges), Nodes(vertices). Also, we describe Vertex and Edge Chromatic Number of a Bipolar Fuzzy of a Complete Bipartite Graph, and Bipolar Fuzzy of a Complete Bipartite Graph through examples. Moreover, we discussed about Cartesian product of two BFGRS.

Definition 3.1 Let \( Gr(V, A, B) \) be a Regular graph signifying a Route network system. Let \( (V) \) be a commutative SG with finite vertices. If \( A = (\mu AP, \mu AN) \) is a BF set on \( V \), \( B = (\sigma BP, \sigma BN) \) is a BF set in \( B \) where \( \mu AP : V \to [0,1], \mu AN : V \to [-1,0], \sigma BP : V \times V \to V \) and \( \sigma BN : V \times V \to V \) for which \( \sigma BP(x,y) \leq \min(\mu AP(x), \mu AP(y)), V \times V = V^2 \) and \( \sigma BN(x,y) \leq \max(\mu AN(x), \mu AN(y)), V \times V = V^2 \) and \( \sigma BP(x,y) = \sigma BN(x,y) = 0, V \times V = V^2 - E \), then \( G = (V, A, B) \) is called a BFGRS and is symbolized by \( Gr = (V, A, B, \mu, \sigma) \).

Definition 3.2 Let \( Gr(V_1, A, B, \mu, \sigma) \) be a BFGRS.

(1) The order of a BFGRS, \( Gr(V_1, A, B, \mu, \sigma) \) is designated by \( O(G) = (\mu OP, \mu ON) \) and is distinguished as \( \mu OP = \sum_{a \in V} \mu AP(a) \) and \( \mu ON = \sum_{a \in V} \mu AN(a) \).

(2) The size of a BFGRS, \( Gr(V_1, A, B, \mu, \sigma) \) is designated by \( S(G) = (\mu SP, \mu SN) \) and is distinguished as \( \mu SP = \sum_{a_1, a_2 \in V^2} \sigma BP(a_1, a_2) \) and \( \mu SN = \sum_{a_1, a_2 \in V^2} \sigma BN(a_1, a_2) \).

(3) The open neighbourhood degree of a vertex \( V \) of BFGRS \( (V_1, A, B, \mu, \sigma) \) is defined as \( D(a) = (D^P(a), D^N(a)) \), where

\[
D^P(a) = \sum_{ab \in V^2} \sigma_B^P(ab) \text{ and } D^N(a) = \sum_{ab \in V^2} \sigma_B^N(ab)
\] (1)

Example 3.3 Let \( V_1 = \{K, L, M, N\} \). The ‘·’ is a binary operation on \( V_1 \) is defined by

|  · | (4) K | (5) L | (6) M | (7) N |
|---|---|---|---|---|
| K | 8 K | 9 L | 10 M | 11 N |
A Route Network, $Gr(V_1, E_1)$ is taken with route set, $E_1 = \{(K, L), (K, M), (K, N), (L, N), (L, M), (M, N)\}$ where $(V_1, \cdot)$ is a finite junctions of SG. Then

\[
\begin{align*}
\mu_A^P(p) &= \begin{cases} 
0.7 & \text{if } p = K \\
0.6 & \text{if } p = L \\
0.4 & \text{if } p = M \\
0.1 & \text{if } p = N 
\end{cases} \\
\sigma_B^P(pq) &= \begin{cases} 
0.9 & \text{if } pq = KL \\
0.7 & \text{if } pq = KM \\
0.6 & \text{if } pq = KN \\
0.5 & \text{if } pq = LN \\
0.4 & \text{if } pq = LM \\
0.2 & \text{if } pq = MN \\
-0.5 & \text{if } pq = KL \\
-0.5 & \text{if } pq = KM \\
-0.4 & \text{if } pq = KN \\
-0.3 & \text{if } pq = LN \\
-0.3 & \text{if } pq = LM \\
-0.3 & \text{if } pq = MN 
\end{cases}
\end{align*}
\]

Then from Definition 3.1, $Gr(V_1, A, B, \mu, \sigma)$ is a BFRGS of $V_1$.

![Figure 1. A Regular Network graph (k_4)](image)

**Figure 1. A Regular Network graph (k₄)**

Let a FS, $\mu_A^P: V_1 \rightarrow [0,1]$ be a Positive membership degree of $A$, which is distinguished for every $p \in V_1$ and $\{p, q\} \in E_1$.

Then from Definition 3.1, $Gr(V_1, A, B, \mu, \sigma)$ is a BFRGS of $V_1$.

![Figure 2. A Bi-polar fuzzy Route Network of a Regular graph (K4)](image)

**Figure 2. A Bi-polar fuzzy Route Network of a Regular graph (K4)**

Since $Gr(V_1, E_1) = k_4$ which is a complete Regular graph, and from definition 3.2, we can also say that the BFRGS, $Gr(V_1, A, B, \mu, \sigma)$ is an anti- BF-IGS,$V_1$.

1. The order of a BFRGS, $Gr(V, A, B, \mu, \sigma)$ is designated by $O(G) = (O(P(G), ON(G))$ which is distinguished as

$$OP(G) = \sum_{a \in V} \mu_P(a) = \mu_P(K) + \mu_P(L) + \mu_P(M) + \mu_P(N)$$

$$= 0.7 + 0.6 + 0.4 + 0.1 = 1.8$$ and

| m | L | L | M | N | K | L |
|---|---|---|---|---|---|---|
| s | M | N | K | L | L | N |
| 1 | L | M | N | K | L | L |
| 2 | L | M | N | K | L | L |
| 3 | L | M | N | K | L | L |
ON(G) = \sum_{x \in V} \mu N (a) \\
= \mu N^1(K) + \mu N(L) + \mu N(M) + \mu N(N) \\
= -0.4 - 0.3 - 0.2 - 0.2 \\
= -1.1

2. The size of a BFRGS, Gr(V, A, B, µ, σ) is designated by S(G) = (SP(G), SN(G)) which is distinguished as

\[ SP(G) = \sum_{a1, a2 \in V} \sigma BP(a1a2) = \sigma B^P(KL) + \sigma B^P(KM) + \sigma B^P(KN) + \sigma B^P(LN) + \sigma B^P(LM) + \sigma B^P(MN) \]
\[ = 0.9 + 0.7 + 0.6 + 0.5 + 0.4 + 0.2 \]
\[ = 3.3 \]

\[ SN(G) = \sum_{a1, a2 \in V} \sigma BN(a1a2) = \sigma B^N(KL) + \sigma B^N(KM) + \sigma B^N(KN) + \sigma B^N(LN) + \sigma B^N(LM) + \sigma B^N(MN) \]
\[ = -0.5 - 0.5 - 0.4 - 0.3 - 0.3 - 0.3 \]
\[ = -2.3 \]

The open neighborhood degree of a vertex V of BFRGS (V, A, B, µ, σ) is distinguished as D(a) = (D^P(a), D^N(a)), ∀ ‘a’ in V, where

\[ D^P(K) = \sigma B^P(KL) + \sigma B^P(KM) + \sigma B^P(KN) = 0.9 + 0.7 + 0.6 = 2.2 \]
\[ D^P(L) = \sigma B^P(LK) + \sigma B^P(LN) + \sigma B^P(LM) = 0.9 + 0.5 + 0.4 = 1.8 \]
\[ D^P(M) = \sigma B^P(MK) + \sigma B^P(ML) + \sigma B^P(MN) = 0.7 + 0.4 + 0.2 = 1.3 \]
\[ D^P(N) = \sigma B^P(NK) + \sigma B^P(NM) + \sigma B^P(NL) = 0.6 + 0.2 + 0.5 = 1.3 \]
\[ D^N(K) = \sigma B^N(KL) + \sigma B^N(KM) + \sigma B^N(KN) = -0.5 - 0.5 - 0.4 = -1.4 \]
\[ D^N(L) = \sigma B^N(LK) + \sigma B^N(LN) + \sigma B^N(LM) = -0.5 - 0.3 - 0.3 = -1.1 \]
\[ D^N(M) = \sigma B^N(MK) + \sigma B^N(ML) + \sigma B^N(MN) = -0.5 - 0.3 - 0.3 = -1.1 \]
\[ D^N(N) = \sigma B^N(NK) + \sigma B^N(NM) + \sigma B^N(NL) = -0.4 - 0.3 - 0.3 = -1.0 \]

The neighborhood of each vertex is 3 {i.e., N(x) =3\forall x \in V}. Since G = (V, E) = K4, which is a complete regular graph.

**Definition 3.4:** A BFRGS of Gr(V, A, B, µ, σ) is stated to be semi strong if \( \mu B^P_1 (kp) = \min \{ \mu A_1(k), \mu A_1(p) \} \) or \( \mu B^N_1 (kp) = \max \{ \mu A_1(k), \mu A_1(p) \} \) ∀k, p ∈ E.

**Theorem 3.5:** If Gr1 × Gr2 is strong BFRGS, then at least Gr1 or Gr2 must be semi-strong.

**Proof.** Suppose that Gr1 and Gr2 are not semi-strong BFRGS.

Then \( \exists k1p1 \in E1, k2p2 \in E2 \exists \)
\( \mu B^P_1 (k1p1) < \min \{ \mu A_1(k1), \mu A_1(p1) \} \) or
\( \mu B^N_1 (k1p1) > \max \{ \mu A_1(k1), \mu A_1(p1) \} \) and
\( \mu B^P_1 (k2p2) < \min \{ \mu A_1(k2), \mu A_1(p2) \} \) or
\( \mu B^N_1 (k2p2) > \max \{ \mu A_1(k2), \mu A_1(p2) \} \)

Let E = {(x, k2)( x, p2): x ∈ V1, k2p2 ∈ E2} ∪ {(k1, z)( p1, z): k1p1 ∈ E1, x ∈ V2}.
Let \((x, k_2)(x, p_2))\in E.

Then we have,
\[
\mu_{B_1}^P \times \mu_{B_2}^P (x, k_2)(x, p_2) = \min \{\mu_{A_1}^P(x), \mu_{A_2}^P(k_2), \mu_{A_2}^P(p_2)\} < \min \{\mu_{A_1}^P(x), \mu_{A_2}^P(k_2), \mu_{A_2}^P(p_2)\} = \min \{\mu_{A_1}^P(x), \mu_{A_2}^P(k_2), \mu_{A_2}^P(p_2)\} = \min \{\mu_{A_1}^P(x), \mu_{A_2}^P(k_2), \mu_{A_2}^P(p_2)\}
\]

**Definition 3.6** The vertex chromatic number of BFGS, \(\text{Gr}(V_1, A_1, B_1, \mu, \sigma)\) is \((n, n)\) and is denoted by \(\chi(\text{Gr}) = (\chi^P(\text{Gr}), \chi^N(\text{Gr})) = (n, n)\) where \(n\) is the number of vertices of \(\text{Gr}\). And The fuzzy value of colouring of a vertex in BFGS is \((\mathcal{C}^P, \mathcal{C}^N)\) where \(\mathcal{C}^P\) and \(\mathcal{C}^N\) are the fuzzy values of providing and not providing certain color to the vertex.

**Definition 3.7** The edge chromatic number of BFGS, \(\text{Gr}(V_1, A_1, B_1, \mu, \sigma)\) on \(n\) vertices is \((n, n)\), if \(n\) is odd and is \((n-1, n-1)\), if \(n\) is even and is denoted by
\[
|\tau(\text{Gr})| = \left\{\begin{array}{ll}
(n, n) & \text{if } n \text{ is odd} \\
(n-1, n-1) & \text{if } n \text{ is even}
\end{array}\right.
\]

And The fuzzy value of colouring of an edge in BFGS is \((\mathcal{D}^P, \mathcal{D}^N)\) where \(\mathcal{D}^P\) and \(\mathcal{D}^N\) are the fuzzy values of providing and not providing certain color to the edge.

**Example 3.8** The vertex chromatic number of a BFGS of a Complete Bipartite Graph is at most \((2, 2)\). Consider a complete bipartite graph \(K_{2,3}\).

\[
\mu^P(V_i) = \begin{cases} 
0.2 & \text{if } V_i \text{ = Blue} \\
0.1 & \text{if } V_i \text{ = Red}
\end{cases}
\]
\[
\mu^N(V_i) = \begin{cases} 
-0.4 & \text{if } V_i \text{ = Blue} \\
-0.3 & \text{if } V_i \text{ = Red}
\end{cases}
\]

Also, For a complete bipartite graph \(K_{1,3}\)

\[
\mu^P(V_i) = \begin{cases} 
0.2 & \text{if } V_i \text{ = Blue} \\
0.1 & \text{if } V_i \text{ = Red}
\end{cases}
\]
\[
\mu^N(V_i) = \begin{cases} 
-0.4 & \text{if } V_i \text{ = Blue} \\
-0.3 & \text{if } V_i \text{ = Red}
\end{cases}
\]

Since no two adjacent colours should fill with same color, Hence, the vertex chromatic number of a BFGS of a Complete Bipartite Graph is \((2, 2)\).

**4. Labelling of Tri-polar Fuzzy of Graph**

**Definition 4.1** An edge AB is called a 3–polar fuzzy bridge of G if its removal reduces the strength of connectedness between some other pair of nodes in G.

**Definition 4.2** In an edge AB, the node B is stated to be3–polar fuzzy cut node of G if its removal reduces the strength of connectedness between some other pair of nodes in G.

**Definition 4.3** In an edge AB, the node A is stated to be 3–polar fuzzy end node of G if it has exactly one strong neighbour in G.

**Definition 4.4** In an edge AB of \(A3\)-PFPG is called strong edge if its weight is as great as the strength of connectedness of its 3–polar fuzzy end nodes.

**Definition 4.5** An 3-PFP, \(P = U - V\) is a sequence of distinct vertices \(U = u_1, u_2, ..., u_n = V, V j; \exists \) at least one \(i \in \Omega, Pi = (xj . xj+1) > 0\).
Definition 4.6 A 3-PFP is strong if all its arcs are strong. A 3-PFP x \( \rightarrow y \) is said to be strongest 3-PFP if its strength equals to its connectedness.

Definition 4.7 A 3–polar fuzzy weakest arc is an arc having least degree of membership.

Example 4.8 Consider a 3–polar fuzzy labelling graph as shown in Figure 3, it is familiar to see
1. \( x_2x_5, x_1x_2, x_2x_4 \) are 3–polar fuzzy bridges.
2. \( x_2 \) is 3–polar fuzzy cut node.
3. \( x_1, x_5, x_4 \) are 3–polar fuzzy end nodes of \( G \).
4. \( x_1x_2, x_2x_5, x_2x_4 \) are 3–polar fuzzy strong arcs.
5. \( x_1 \rightarrow x_2 \rightarrow x_5, x_1 \rightarrow x_2 \rightarrow x_4 \) are 3–PFP.
6. \( x_1 \rightarrow x_2 \rightarrow x_5, x_1 \rightarrow x_2 \rightarrow x_4, x_4 \rightarrow x_2 \rightarrow x_5 \) are strongest 3–PFP.

In Figure 4, The 3–polar fuzzy weakest arc is \( x_1x_2 \).

Definition 4.9 A graph \( G_p^w = (C_p^w, \mathcal{D}_p^w) \) is stated to be a 3–polar fuzzy labelling graph, if \( C_p^w : V \rightarrow [0, 1]^3 \) and \( \mathcal{D}_p^w : V \times V \rightarrow [0, 1]^3 \) are bijective such that the membership values of vertices and edges are distinct and also \( \Pi \circ \mathcal{D}_p^w (ab) < \Pi \circ C_p^w (a) \land \Pi \circ C_p^w (b) \land a, b \in V, 1 \leq i \leq 3 \).

Example 4.10 Consider a 3–polar fuzzy labelling graph as shown in Figure 3, Labelling of 3–polar fuzzy set \( C_p^w \) is

| \( C_p^w \) | \( x_1 \) | \( x_2 \) | \( x_3 \) | \( x_4 \) | \( x_5 \) |
|---|---|---|---|---|---|
| \( P_1 \circ C_p^w \) | 0.5 | 0.5 | 0.6 | 0.7 | 0.8 |
| \( P_2 \circ C_p^w \) | 0.4 | 0.6 | 0.4 | 0.5 | 0.7 |
| \( P_3 \circ C_p^w \) | 0.8 | 0.7 | 0.3 | 0.3 | 0.9 |

Labelling of 3–polar fuzzy relation \( \mathcal{D}_p^w \) is,

| \( \mathcal{D}_p^w \) | \( x_1x_2 \) | \( x_1x_5 \) | \( x_2x_5 \) | \( x_2x_3 \) | \( x_2x_2 \) | \( x_2x_4 \) |
|---|---|---|---|---|---|---|
| \( P_1 \circ \mathcal{D}_p^w \) | 0.4 | 0.8 | 0.4 | 0.3 | 0.4 | 0.5 |
| \( P_2 \circ \mathcal{D}_p^w \) | 0.4 | 0.7 | 0.6 | 0.4 | 0 | 0 |
| \( P_3 \circ \mathcal{D}_p^w \) | 0.6 | 0.9 | 0.6 | 0.2 | 0 | 0.2 |

Definition 4.11 A star in 3-PFG can be defined as having two 3–polar fuzzy node sets V and E with |V | = 1 and |E | > 1, \( \exists \), \( \Pi \circ \mathcal{D}_p^w (abj) > 0 \) and \( \Pi \circ \mathcal{D}_p^w (bjb + 1) = 0, 1 \leq j \leq n \). It is denoted by \( S_p(1, n) \).

Example 4.12 A 3–polar fuzzy star given below
5. Conclusion

In this article, we presented the concept of BFRG symptomatic of a Route network system on SG. Also, the notion of BFRG and the concept of 3-PFP and 3-PFG is characterized through some examples.

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