Abstract

This paper focuses on a solution method for unit commitment problems which are to be solved repeatedly with different demand data but with the same problem structure. We propose a heuristic based on offline training that can be used to solve new instances quickly. In the training phase, a neural network model is trained to predict for any demand data and for each binary variable how likely it is that the variable takes each of two possible values. After the training, given a test instance, the model can be used with a rounding threshold to fix some binary variables. The reduced problem can then be solved by an MILP solver quickly to find a feasible, near-optimal solution to the original instance. To provide a solution within a prescribed optimality tolerance, we combine the aforementioned heuristic with Dantzig-Wolfe decomposition. Our numerical experiments compare our methods with solving the undecomposed problem and also with a decomposition approach with a baseline primal heuristic and reveal that our approach solves test instances to high precision in shorter running times and scales to handle larger-scale instances.

1 Introduction

The unit commitment (UC) problem is a combinatorial optimization problem to find the optimal operating schedule of power plants for given demand over a fixed period. This problem has been actively studied for the past few decades, see [1] for a recent survey. The current paper focuses on UC problems which are to be solved repeatedly with different demand data but with the same problem structures. This is typically the case when UC problems are solved as day-ahead planning problems by electricity generating companies. We consider fast primal heuristics to solve new instances quickly, using a pre-trained machine learning model.

One popular solution method for a UC problem is to use Dantzig-Wolfe decomposition to decompose the problem by generators. The reformulated problem is then solved with a column generation procedure, which yields a sequence of (hopefully improving) lower bounds. This procedure is the dual of a cutting-plane approach to Lagrangian relaxation, as discussed in [9]. For problems with integer variables, the Dantzig-Wolfe reformulation is a relaxation of the original problem, however Bertsekas et al. [4] and Bard [2] reported that the optimality gap introduced by the reformulation is typically small, especially if the problem size is large. In such cases, Dantzig-Wolfe decomposition with a suitable primal heuristic is likely to solve the UC problem to high precision.
Primal heuristics based on the decomposition are commonly used alongside Dantzig-Wolfe decomposition. Such primal heuristics collect information through iterations of the column generation procedure and create primal feasible solutions based on it. However, primal heuristics based on the decomposition are not the only options in Dantzig-Wolfe decomposition. When we have any strong primal heuristic, it can be combined with Dantzig-Wolfe decomposition. Dantzig-Wolfe decomposition then provides tight lower bounds and hence bounds on the sub-optimality of the solutions provided by the primal heuristic. This may be advantageous since decomposition-based primal heuristics typically need a few iterations to collect enough data to create solutions of high quality, which requires some computational time.

In recent years, there has been a growing interest in the use of machine learning in primal heuristics. Bello et al. [3], Khalil et al. [10], Nazari et al. [14] and Kool et al. [11] applied reinforcement learning on various combinatorial problems, including the travelling salesman problems and the vehicle routine problems. Nair et al. [13] used reinforcement learning to solve two-stage stochastic programming approximately. In all of the above studies, a machine learning model was designed to create primal feasible solutions directly. Bertsimas and Stellato [5] and Bertsimas and Stellato [6] proposed a method of mixed-integer convex/quadratic optimization problems, where a machine learning model was used to simplify the optimization problems. They first trained a model to predict the tight (active) constraints and the values of the integer variables with supervised learning. Given a new instance, the model was used to delete inactive constraints and fix all the integer variables, which were then solved quickly. Xavier et al. [19] studied the use of a support vector machine on a security-constrained UC problem where the model was used to reduce the problem size by fixing a subset of binary variables. They first trained a model for each binary variable and then selected all models whose accuracies were higher than a prescribed value. The selected models were used to fix the corresponding variables and the reduced problem were then solved with a mixed-integer linear programming (MILP) solver. Wang [18] proposed a similar approach to reduce the problem size of a knapsack problem, which was based on a nearest neighbour method. Typically these approaches take a longer time compared with the methods which are purely based on machine learning models, however they are likely to provide solutions of better quality.

In our study, we extend the latter approach, which combines a pre-trained model with an MILP solver. That is, we use a model prediction to reduce problem size by fixing a subset of binary variables. We consider a method, thresholded rounding, with which we can adjust the amount of problem size reduction flexibly. This approach allows us to balance the quality of output solutions and computational time. We also demonstrate that by combining the above heuristics with Dantzig-Wolfe decomposition, we can provide a solution with guaranteed suboptimality quickly. We note that none of the aforementioned previous works provides a solution with provable suboptimality online. Our numerical experiments test the approach on large-scale UC instances.

The rest of the paper is structured as follows. Section 2 briefly introduces Dantzig-Wolfe decomposition, the column generation procedure and decomposition-based primal heuristics. Section 3 presents primal heuristics based on pre-trained models and considers an approach that combines them with Dantzig-Wolfe decomposition. The proposed approach is tested on large scale UC problems in Section 4. Finally, in Section 5 conclusions of this work are discussed.
2 Dantzig-Wolfe decomposition

In this section, we briefly review Dantzig-Wolfe decomposition and the column generation procedure. For further background, see [17].

Consider the following family of mixed-integer programmes parametrized by \( \omega \):

\[
\begin{align*}
  z(\omega) = \min_{(x_s)_{s \in S}} & \sum_{s \in S} c_s^T x_s \\
  \text{s.t.} & \sum_{s \in S} A_s x_s = a(\omega), \\
                 & x_s \in X_s(\omega) \quad \forall s \in S,
\end{align*}
\]

where \( x_s \) is a vector of decision variables for each \( s \) and \( X_s(\omega) \) is the set of feasible points given by

\[
X_s(\omega) = \{ x_s = (x^I_s, x^C_s) \in \mathbb{Z}^n \times \mathbb{R}^m \mid Dx_s \leq d(\omega) \} \quad s \in S.
\]

For simplicity, we assume that \( X_s(\omega) \) is non-empty and bounded for every \( \omega \) and \( s \in S \), and that the problem (2.1) has an optimal solution for every \( \omega \). To reduce clutter in what follows we drop the dependence on \( \omega \) unless it causes confusion.

In Dantzig-Wolfe decomposition, we consider a relaxation of (2.1), referred to as the master problem (MP), by replacing \( X_s \) with \( \operatorname{conv}(X_s) \) for every \( s \in S \). Let \( \{ x_{si} : i \in I_s \} \) denote the extreme points of \( X_s \). Given the boundedness assumption on \( X_s \), the MP is written as

\[
\begin{align*}
  \min_{p} & \sum_{s \in S, i \in I_s} c_s^T x_{si} p_{si} \\
  \text{s.t.} & \sum_{s \in S, i \in I_s} A_s x_{si} p_{si} = a, \\
                 & \sum_{i \in I_s} p_{si} = 1 \quad \forall s \in S, \\
                 & p_{si} \geq 0 \quad \forall s \in S, i \in I_s.
\end{align*}
\]

Since the MP is too large to formulate and solve explicitly, a column generation procedure is used. The restricted master problem (RMP) is defined by replacing \( I_s \) in the MP with a subset \( \tilde{I}_s \subset I_s \) for every \( s \in S \). Suppose that the RMP is feasible (the \( \tilde{I}_s \)'s must have enough elements so that the first constraint in (2.2) can be satisfied) and let \( y, \sigma_s \) for \( s \in S \) be the optimal dual solution to the RMP corresponding to the first and second constraint respectively. To find columns to be added to the RMP, the following problems, known as the pricing subproblems, are solved:

\[
r_s(y) = \min_{x_s} \{(c_s - y^T A_s)x_s : x_s \in X_s\} \quad s \in S.
\]

If \( r_s(y) \geq \sigma_s \) for all \( s \in S \), the RMP has found the optimal solution to the MP. Otherwise, the solutions to the pricing subproblems are added to the RMP and the above process is repeated. It follows from linear programming (LP) duality that given a dual value \( y \)

\[
l(y) = a^T y + \sum_{s \in S} r_s(y)
\]

is a lower bound to the MP, which also bounds (2.1).
It is known that the naive column generation approach described above suffers from instability. To mitigate the issue, quadratic regularization on the dual variable may be added to the RMP, as described by Briant et al. [9]. It has been numerically observed that by initializing the regularization center to values close to the optimal one we can warmstart the algorithm and reduce the computational time. Schulze et al. [15] solved the linear programming relaxation (LPR) of the original problem (2.1) and used the optimal dual values to the LPR to initialize the column generation procedure. We follow their approach in this study.

Since the column generation procedure does not provide primal feasible solutions, we need to run a primal heuristic to get feasible solutions. Primal heuristics based on the decomposition are commonly used. For example, Merlin and Sandrin [12] and Zhuang and Galiana [20] used the pricing subproblem solutions to construct a solution to the original problem. In their approaches, the values of the binary variables in the original problem were fixed to the values of the pricing subproblem solutions. Let \( x^s \) be the optimal solution to pricing subproblem \( s \). Then we add the following constraint to the original problem (2.1)

\[
    x^I_s = (x^I_s)^*, \quad \forall s.
\]

The resulting problem is an LP and its optimal objective value gives an upper bound of the original problem. The problem is not necessarily feasible and heuristics to recover feasibility were used in their studies. In this paper, we refer to this approach as the column evaluation heuristic.

Another heuristic based on the decomposition is a column combination heuristic. In this heuristic, the solutions of the pricing subproblems are stored in a pool. Then a combinatorial problem is formulated where the pattern of the solution is restricted to those in the pool. Let \( \bar{X}_s = \{ \bar{x}_{si} \}_{i \in J_s} \) be the pool of solutions of pricing subproblem \( s \) whose the index set is denoted by \( J_s \). Then, binary variables \( w_{si} (s \in si \in J_s) \) are added to the original problem (2.1) together with the following constraints

\[
    x^I_s = \sum_{i \in J_s} w_{si} \bar{x}^I_{si}, \quad \forall s,
\]

\[
    \sum_{i \in J_s} w_{si} = 1, \quad \forall s.
\]

This is referred to as the restricted master IP by Vanderbeck [16]. This is still MILP but the solution space is much smaller than the original problem. This method was used to solve a stochastic version of UC problems by Schulze et al. [15].

The above discussion on Dantzig-Wolfe decomposition, the column generation procedure and primal heuristics is summarized in Algorithm 1. In each iteration, the pricing subproblems are solved and a lower bound is evaluated. It is followed by RMP update, its solution and a primal heuristic. We note that given multiple processes many steps in the above algorithm can be parallelized. LPR and RMP are solved with an optimization solver, which typically supports parallelization over multiple processes. This also applies to primal heuristics which solve some optimization problems, such as the column evaluation heuristic and the column combination heuristic. The pricing subproblems are \( m \) independent optimization problems and can be distributed across multiple processes. A more detailed explanation of the algorithms can be found in [15].
**Algorithm 1** Dantzig-Wolfe decomposition

```plaintext
select problem parameter ω.
Initialize the lower bound $l = -∞$ and the upper bound $u = ∞$.
Solve the LPR and use the solution as the initial values of $y$.
for $i$ in \{1, 2, \ldots\} do
  Solve the pricing subproblems.
  Compute the lower bound $l(y)$ and update $l$.
  Update and solve the regularized RMP and set $y$ to the solution.
  Run primal heuristics and update $u$.
end for
```

3 Primal heuristic with pre-trained model

In this section, we consider a parametrized UC problem which is to be solved repeatedly with different demand data. For example, a UC problem is solved repeatedly on a daily basis by electricity generating companies. In such a case, typically the structure of the problem is the same and only some of the problem data such as demand is modified. We further assume that we have access to historical data of the parameter values, or that we can sample simulated parameter values which are likely to be close to those we will observe in the future. Under this setup, we design a primal heuristic based on off-line training that can be used to solve new instances quickly.

Recall that the problem is parameterized with $ω$. In the training phase, we sample $ω$ and solve as many training instances as possible. In this way, we obtain the dataset of problem parameter $ω$ and corresponding optimal values of the binary variables. Then, we train a prediction model which takes $ω$ as input and predicts the value of the binary variables. We consider two alternatives: a neural network model and a nearest neighbour model.

The neural network model we consider in this study is a feed-forward neural network [7]. The model takes $ω$ as input and outputs a vector each of whose elements is a predicted probability of the corresponding binary variables to be 1. The neural network model is trained as a standard binary classification problem.

The nearest neighbour model is also considered as an alternative model to predict the values of binary variables. When solving a test instance the nearest neighbour model compares the problem parameter $ω$ with those in the training dataset. A prescribed number of the closest neighbours are selected and the average of the values of the binary variables are computed and used as the prediction.

Once a prediction model is trained, it can be used to find a feasible solution to solve a new instance. The simplest approach is to round the prediction to the nearest integer. However, such a solution is typically infeasible when the problem is highly constrained. Instead, as described below, we use the prediction to fix a subset of binary variables so that the problem size is reduced. Similar ideas have been explored by Xavier et al. [19] and Wang [18].

Pick a threshold value $α \in (0.5, 1]$. If the prediction is larger than $α$ (smaller than $1 - α$), we fix the corresponding binary variables to 1 (0). Namely, we leave variables unfixed if the predictions are between $1 - α$ and $α$ and fix other binary variables to the predicted values. Then the resulting partially-fixed MILP is solved with a MILP solver. We refer to this procedure as thresholded rounding.
Choosing the threshold value is a subtle task. If we fix many binary variables the problem becomes small and can be solved quickly. However, fixing too many variables may result in infeasibility or unacceptably large suboptimality. On the other hand, fixing fewer variables results in a harder problem which takes longer to solve. We explore this point more in the section on numerical experiments.

Instead of fixing the threshold value to a single value a priori, we try various values adaptively. Namely, we first try a small threshold value and solve the partially-fixed MILP. If the resulting problem is infeasible, or if the resulting problem is solved, we try a larger threshold value. This process is described in Algorithm 2.

Algorithm 2 Thresholded rounding primal heuristic

\[
\begin{align*}
\text{select} & \quad \text{problem parameter } \omega, \text{ rounding thresholds } \{\alpha_i\}. \\
\text{Initialize} & \quad \text{the upper bound } u = \infty. \\
\text{Compute} & \quad \text{the prediction using a model.} \\
\text{for } \alpha & \quad \text{in } \{\alpha_i\} \quad \text{do} \\
& \quad \text{Apply threshold } \alpha \text{ and fix binary variables.} \\
& \quad \text{Solve the partially-fixed MILP. Update } u \text{ when an incumbent is found.} \\
\text{end for}
\end{align*}
\]

The above procedure provides a series of feasible solutions and upper bounds. To compute the suboptimality, we need to compute a lower bound. In this manuscript, we use Dantzig-Wolfe decomposition to this end. For simplicity, we run the aforementioned primal heuristic and Dantzig-Wolfe decomposition independently in parallel, as shown in the upper part of Figure 1. One process is assigned to run the primal heuristic as described in Algorithm 2 and solves partially-fixed MILP successively, while another process is assigned to run Dantzig-Wolfe decomposition as described in Algorithm 1. In this setup, we may skip running primal heuristics in Dantzig-Wolfe decomposition, at least for the first few iterations. If Dantzig-Wolfe decomposition finds a tight lower bound but the primal heuristic fails to provide a primal feasible solution within the tolerance, it may be helpful to run decomposition-based primal heuristics in Dantzig-Wolfe decomposition. As soon as the gap between the best lower and upper bound found by Dantzig-Wolfe decomposition and the thresholded rounding primal heuristic gets smaller than a prescribed tolerance, both processes are terminated. For comparison, an implementation of the standard Dantzig-Wolfe decomposition is sketched on the lower part of the figure.

4 Numerical experiments

In this section, the proposed methods are compared with standard Dantzig-Wolfe decomposition with decomposition-based primal heuristics and with an MILP solver. All methods are implemented with Python. IBM ILOG CPLEX\(^1\) is used as the optimization solver and PyTorch to implement the neural networks.

4.1 Problem

In the experiments, we consider a setup where UC problems are solved repeatedly with a fixed set of generators but different demand forecasts which are assumed to be accurate. To assess the

\(^1\)https://www.ibm.com/products/ilog-cplex-optimization-studio
Thresholded rounding primal heuristic

| 1 process | Thresholded rounding primal heuristic |
|-----------|----------------------------------------|
| LPR       | Pricing                                |
| RMP       | Pricing                                |
| RMP       | Pricing                                |

| 1 process | Thresholded rounding primal heuristic |
|-----------|----------------------------------------|
| LPR       | Pricing                                |
| RMP       | Pricing                                |
| RMP       | Pricing                                |

Decomposition-based primal heuristic

| 2 processes | Decomposition-based primal heuristic |
|-------------|--------------------------------------|
| LPR         | Pricing                              |
| RMP         | Pricing                              |
| Decom. PH   | Pricing                              |
| RMP         | Pricing                              |
| Decom. PH   | Pricing                              |

Figure 1: Sketches of implementations of the primal heuristics in 2-process environment

Table 1: Statistics on the training of neural network models

|                | 200  | 600  | 1000 |
|----------------|------|------|------|
| number of training instance | 11,438 | 5,437 | 3,949 |
| time to train a model (s)    | 631  | 1,168 | 1,523 |

Scalability, we consider 3 different problem sizes; problems with 200, 600 and 1,000 generators. In all cases, the length of the planning horizon is 48 hours with a time resolution of 1 hour. The generator data is based on Borghetti et al. [8]. Since their sets of generators contain 200 generators at most, we combine multiple sets to create larger ones. Each generator is distinct and combining the sets of generators does not introduce symmetry. The demand data is based on the historical demand data in the UK published by National Grid ESO. For each set of generators, the demand is scaled so that the median of the daily peaks becomes 50% of the total generation capacity. A detailed description of the problem formulation is found in the appendix.

4.2 Methods

Neural network This is an implementation of the method described in Section 3 with a neural network model. In the training phase, as many training instances as possible are solved to 0.25% optimality. The training budget is 24 hours on 8 CPU cores. To solve each training instance we use the baseline Dantzig-Wolfe decomposition which is described below. The number of solved instances are reported in Table 1. Once the dataset is constructed, a neural network model is trained to predict the values of the binary variables. We use a feed-forward neural network with 2 hidden-layers of 400 units per layer with ReLU activation function. For simplicity, the time to train a neural network model, which is shown in Table 1, is not included in the training budget. In Dantzig-Wolfe decomposition, we do not run any primal heuristics for the first 10 iterations. After the 10th iteration, we run the column combination heuristic using the columns generated in the 2 last iterations.

2https://www.nationalgrideso.com/
Table 2: Computational time and required number of iterations

| size | method          | tol: 0.5% |            |               |            |               |               |
|------|----------------|-----------|------------|---------------|------------|---------------|---------------|
|      |                | solved    | time       | iter          | solved     | time       | iter          |
|      |                |           |            |               |            |            |               |
| 200  | network        | 40        | 7.5        | 1.0           | 40         | 9.3        | 1.6           | 40            | 27.8        | 6.4           |
|      | nearest        | 40        | 7.6        | 1.1           | 40         | 9.8        | 1.8           | 40            | 38.2        | 8.2           |
|      | baseline       | 40        | 12.1       | 3.2           | 40         | 39.4       | 9.2           | 40            | 108.6       | 13.7          |
|      | CPLEX          | 40        | 228.5      | -             | 40         | 281.3      | -             | 30            | 810.3       | -             |
| 600  | network        | 40        | 27.5       | 1.2           | 40         | 30.3       | 1.4           | 40            | 64.6        | 4.7           |
|      | nearest        | 40        | 32.0       | 1.5           | 40         | 35.0       | 1.8           | 40            | 76.1        | 5.6           |
|      | baseline       | 40        | 31.1       | 2.0           | 40         | 59.8       | 5.4           | 40            | 124.0       | 10.4          |
|      | CPLEX          | 38        | 771.2      | -             | 38         | 777.4      | -             | 21            | 970.5       | -             |
| 1000 | network        | 40        | 44.6       | 1.2           | 40         | 45.1       | 1.2           | 40            | 89.7        | 4.2           |
|      | nearest        | 40        | 62.5       | 2.1           | 40         | 69.5       | 2.5           | 40            | 132.6       | 6.0           |
|      | baseline       | 40        | 48.4       | 1.8           | 40         | 81.3       | 4.1           | 40            | 145.2       | 8.6           |
|      | CPLEX          | 10        | 1164.1     | -             | 9          | 1165.3     | -             | 7             | 1175.8      | -             |

**Nearest neighbour** This is an alternative approach that uses a nearest neighbour model. It uses the same dataset as the neural network model. When solving a test instance, the parameter of the instance is compared with those of training instances, and the 50 closest instances are chosen to compute the average values of the binary variables. Dantzig-Wolfe decomposition is configured in the same way as the method with the neural network model.

**Baseline Dantzig-Wolfe decomposition** This is an implementation of the standard method based on Dantzig-Wolfe decomposition as described in Algorithm 1 with 2 processes. For the first 10 iterations, the column evaluation primal heuristic is run [20]. If the resulting LP is infeasible, we commit the cheapest available generators to make it feasible. After 10 iterations, the column combination heuristic is run additionally to the column evaluation heuristic [15].

**CPLEX** For comparison, we run CPLEX on the original MILP problem without decomposition. CPLEX is given 2 processes.

**4.3 Evaluation**

To evaluate the performance of the methods described in the previous section, 40 test instances are sampled and solved. The demand data to construct the test instances are sampled from a different year than those of the training instances.

Table 2 shows the number of instances solved within the time limit of 20 minutes, the average computational time and the average number of column generation iterations to solve the instances or to reach the time limit. For the instances that are not solved within the time limit, no penalty is added on the computation of the average and the time is set to be 20 minutes, and the number of iterations reached by the 20-minute time limit is used.
If the target tolerance is tight (0.25% or 0.1%), the methods based on the pre-trained models outperform the baselines in all cases. It is noteworthy that the prediction models are trained with a dataset generated with tolerance of 0.25%. However, they are helpful to solve UC instances to tighter tolerance such as 0.1%.

The result with the loose tolerance (0.5%) has a different trend. The method based on a prediction model still outperforms in the 200-generator case, but not in the larger cases. In the 600- and 1000-generator cases, all of the methods except CPLEX often find a solution within 1 or 2 iterations of the column generation procedure. If the required number of iterations is the same, the baseline with the standard Dantzig-Wolfe decomposition is likely to be faster. This is because the baseline is given 2 processes to run the column generation procedure while the others are given 1 process.

In all cases, the method based on a neural network model outperforms the one based on a nearest neighbour model. This indicates that the prediction ability of the underlying model used in the thresholded rounding primal heuristic affects the overall performance (e.g., computational time).

To understand the effect of the threshold value, upper bounds found by CPLEX applied to partially-fixed MILP with different threshold values are plotted in Figure 2. This result is based on a single test instance with 200 generators. For each threshold value, the partially-fixed MILP is formulated and CPLEX is used to solve for the remaining (integer and continuous) variables. The upper bounds found by CPLEX are plotted in the figure against the time. With a small threshold, we fix more binary variables. Thus the partially-fixed MILP can be solved quickly, but the optimal value suffers large suboptimality. Indeed the partially-fixed MILP is solved (to the default tolerance of CPLEX) in approximately 30 seconds. If we use a large threshold, we obtain better upper bounds, but the required computational time grows quickly.

### 4.4 Analysis on the training budget

In this section, we study the effect of the training budget. We consider cases where the training budget is 6, 12, 36 or 48 hours instead of 24 hours and observe how the performance of the methods are affected. To this end, the neural network model and the nearest neighbour model are trained using the dataset generated within each of these training budgets. The performance
of the models are then evaluated as before and the results are reported in Table 3. In all cases, all of the test instances are solved within 0.1% tolerance.

As expected, in many cases, both the neural network model and the nearest neighbour model tend to perform better with a larger dataset. Comparing the models with 6-hour training, those with 48-hour training tend to be faster. However, the improvement of the performance after 24 hours of training is unclear, especially for the neural network model. The room for additional performance gain seems limited if further training budget is given.

4.5 Analysis on neural network model architecture

In the following, the effect of the model architecture is studied. In the previous experiments, small feed-forward neural network models were considered. Here, we additionally train deeper neural network models and measure their performances. A deeper model consists of 4 hidden layers of 1000 units per layer. We use the same training dataset which is generated with the training budget of 24 hours. The performance of the models are evaluated similarly and the results are reported in Table 4. The difference in performance is relatively small. Although we observed a notable difference between the performances of the neural network model and the nearest neighbour model, the gain of using the deeper, more expressive model is unclear.

5 Conclusion

We considered primal heuristics which use a machine learning model to solve parametrized UC problems quickly. Given a pre-trained neural network model to predict the values of binary variables, we consider an approach to reduce the problem size by fixing a subset of the binary variables. The MILP solver could quickly solve the resulting partially-fixed MILP and provide feasible solutions with small suboptimality. Furthermore, combined with Dantzig-Wolfe decomposition, we were able to provide a solution within prescribed suboptimality, such as 0.1% in a short time. The numerical experiments also show the scalability of our approach for solving large-scale instances.

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| size | method   | budget | tol: 0.5% | tol: 0.25% | tol: 0.1% |
|------|----------|--------|-----------|-----------|-----------|
|      |          |        | solved    | time      | iter      | solved    | time      | iter      | solved    | time      | iter      |
| 200  | network  | 6      | 40        | 7.7       | 1.1       | 40        | 9.4       | 1.7       | 40        | 30.5      | 6.8       |
|      |          | 12     | 40        | 7.8       | 1.1       | 40        | 8.9       | 1.5       | 40        | 29.5      | 6.6       |
|      |          | 24     | 40        | 7.5       | 1.1       | 40        | 9.3       | 1.6       | 40        | 27.8      | 6.3       |
|      |          | 36     | 40        | 7.5       | 1.1       | 40        | 8.6       | 1.4       | 40        | 25.4      | 6.0       |
|      |          | 48     | 40        | 7.3       | 1.0       | 40        | 8.4       | 1.4       | 40        | 26.3      | 6.3       |
|      | neighbour| 6      | 40        | 8.0       | 1.2       | 40        | 10.2      | 1.9       | 40        | 41.9      | 8.6       |
|      |          | 12     | 40        | 7.7       | 1.1       | 40        | 10.7      | 2.1       | 40        | 40.8      | 8.6       |
|      |          | 24     | 40        | 7.6       | 1.1       | 40        | 9.8       | 1.8       | 40        | 38.2      | 8.2       |
|      |          | 36     | 40        | 7.6       | 1.1       | 40        | 9.3       | 1.6       | 40        | 34.0      | 7.6       |
|      |          | 48     | 40        | 7.9       | 1.2       | 40        | 10.2      | 1.9       | 40        | 31.4      | 7.3       |
| 600  | network  | 6      | 40        | 31.2      | 1.5       | 40        | 33.7      | 1.8       | 40        | 71.2      | 5.3       |
|      |          | 12     | 40        | 27.8      | 1.2       | 40        | 32.1      | 1.5       | 40        | 65.8      | 4.8       |
|      |          | 24     | 40        | 27.5      | 1.1       | 40        | 30.3      | 1.4       | 40        | 64.6      | 4.7       |
|      |          | 36     | 40        | 29.7      | 1.4       | 40        | 30.6      | 1.4       | 40        | 64.0      | 4.7       |
|      |          | 48     | 40        | 30.2      | 1.4       | 40        | 30.2      | 1.4       | 40        | 64.5      | 4.7       |
|      | neighbour| 6      | 40        | 33.6      | 1.8       | 40        | 37.4      | 2.1       | 40        | 83.7      | 6.0       |
|      |          | 12     | 40        | 32.7      | 1.6       | 40        | 36.8      | 2.0       | 40        | 75.4      | 5.6       |
|      |          | 24     | 40        | 32.0      | 1.5       | 40        | 35.0      | 1.9       | 40        | 76.1      | 5.5       |
|      |          | 36     | 40        | 32.4      | 1.6       | 40        | 35.3      | 1.9       | 40        | 71.7      | 5.4       |
|      |          | 48     | 40        | 34.2      | 1.8       | 40        | 37.3      | 2.1       | 40        | 73.8      | 5.6       |
| 1000 | network  | 6      | 40        | 50.3      | 1.5       | 40        | 53.9      | 1.7       | 40        | 105.7     | 5.0       |
|      |          | 12     | 40        | 50.5      | 1.4       | 40        | 53.2      | 1.6       | 40        | 97.9      | 4.5       |
|      |          | 24     | 40        | 44.6      | 1.2       | 40        | 45.1      | 1.2       | 40        | 89.7      | 4.2       |
|      |          | 36     | 40        | 45.5      | 1.2       | 40        | 45.9      | 1.3       | 40        | 91.3      | 4.3       |
|      |          | 48     | 40        | 47.2      | 1.4       | 40        | 48.4      | 1.4       | 40        | 89.5      | 4.2       |
|      | neighbour| 6      | 40        | 56.8      | 1.9       | 40        | 63.3      | 2.4       | 40        | 143.5     | 6.6       |
|      |          | 12     | 40        | 55.2      | 1.9       | 40        | 59.7      | 2.1       | 40        | 122.4     | 5.7       |
|      |          | 24     | 40        | 62.5      | 2.1       | 40        | 69.5      | 2.5       | 40        | 132.6     | 6.0       |
|      |          | 36     | 40        | 47.8      | 1.4       | 40        | 52.4      | 1.6       | 40        | 110.7     | 5.1       |
|      |          | 48     | 40        | 52.1      | 1.6       | 40        | 57.3      | 1.9       | 40        | 112.9     | 5.2       |
| size | model  | tol: 0.5% | 0.25% | 0.1% |
|------|--------|-----------|-------|------|
|      |        | solved    | time  | iter | solved    | time  | iter | solved    | time  | iter |
| 200  | original | 40 | 7.5 | 1.0 | 40 | 9.3 | 1.6 | 40 | 27.8 | 6.4 |
|      | deeper  | 40 | 7.4 | 1.0 | 40 | 8.6 | 1.4 | 40 | 28.5 | 6.6 |
| 600  | original | 40 | 27.5 | 1.2 | 40 | 30.3 | 1.4 | 40 | 64.6 | 4.7 |
|      | deeper  | 40 | 28.6 | 1.2 | 40 | 33.3 | 1.6 | 40 | 63.1 | 4.6 |
| 1000 | original | 40 | 44.6 | 1.2 | 40 | 45.1 | 1.2 | 40 | 89.7 | 4.2 |
|      | deeper  | 40 | 43.9 | 1.1 | 40 | 44.3 | 1.2 | 40 | 97.5 | 4.6 |

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A Problem formulation

We follow one of the standard formulations in literature and formulate the following constraints:

- **Load balance**: Generators have to meet the all demand in each time period.
- **Reserve**: To deal with contingencies, it is required to keep a sufficient amount of back up in each time period, which can be activated quickly.
- **Power output bounds**: Each generator’s power output has to be within its limit.
- **Ramp rate**: Generators can only change their outputs within the ramp rates.
- **Minimum up/downtime**: If switched on (off), each generator has to stay on (off) for a given minimum period. This is to avoid thermal stress in the generators which may cause wear and tear of the turbines.

To formulate the model, we use the following notation.

- **Sets**
  - $G = \{1, 2, \ldots, n_G\}$: set of generators
  - $T = \{1, 2, \ldots, n_T\}$: set of time indices where decisions are taken

- **Parameters**
  - $C^\text{nl}_g$: no-load cost of generator $g$
  - $C^\text{mr}_g$: marginal cost of generator $g$
  - $C^\text{up}_g$: startup cost of generator $g$
  - $P^\text{max/\min}_g$: maximum/minimum generation limit of generator $g$
  - $P^\text{ru/rd}_g$: operating ramp up/down limits of generator $g$
  - $P^\text{su/sd}_g$: startup/shutdown ramp limits of generator $g$
- $T^{u/d}_g$: minimum uptime/downtime of generator $g$
- $P^d_t$: power demand at time $t$
- $P^r_t$: reserve requirement at time $t$

• Variables
  - $\alpha_{gt} \in \{0, 1\}$: 1 if generator $g$ is on in period $t$, and 0 otherwise
  - $\gamma_{gt} \in \{0, 1\}$: 1 if generator $g$ starts up in period $t$, and 0 otherwise
  - $\eta_{gt} \in \{0, 1\}$: 1 if generator $g$ shut down in period $t$, and 0 otherwise
  - $p_{gt} \geq 0$: power output of generator $g$ in period $t$

The objective is the total cost

$$\min \sum_{t \in T} \sum_{g \in G} \left( C_{nl}^{\text{nl}} g \alpha_{gt} + C_{mr}^{\text{mr}} p_{gt} + C_{up}^{\text{up}} g \gamma_{gt} \right).$$

This is to be minimized subject to the following constraints.

• Load balance
  $$\sum_{g \in G} p_{gt} \geq P^d_t \quad t \in T$$

• Reserve
  $$\sum_{g \in G} (P_{g}^{\text{max}} \alpha_{gt} - p_{gt}) \geq P^r_t \quad t \in T$$

• Power output bounds
  $$P_{g}^{\min} \alpha_{gt} \leq p_{gt} \leq P_{g}^{\max} \alpha_{gt} \quad g \in G, t \in T$$

• Ramp rate
  $$p_{gt} - p_{gt-1} \leq P_{g}^{\text{ru}} \alpha_{gt-1} + P_{g}^{\text{su}} \gamma_{gt} \quad g \in G, t \in T \setminus \{1\}$$
  $$p_{gt-1} - p_{gt} \leq P_{g}^{\text{rd}} \alpha_{gt} + P_{g}^{\text{sd}} \eta_{gt} \quad g \in G, t \in T \setminus \{1\}$$

• Minimum up/downtime
  $$\sum_{i = \max\{t-T_g^u+1,1\}}^{t} \gamma_{gi} \leq \alpha_{gt} \quad g \in G, t \in T$$
  $$\sum_{i = \max\{t-T_g^u+1,1\}}^{t} \eta_{gi} \leq 1 - \alpha_{gt} \quad g \in G, t \in T$$

• Polyhedral/Switching constraints (to enforce binaries to work as we expect)
  $$\alpha_{gt} - \alpha_{gt-1} = \gamma_{gt} - \eta_{gt} \quad g \in G, t \in T$$
  $$1 \geq \gamma_{gt} + \eta_{gt} \quad g \in G, t \in T$$