Inflation from $R^2$ gravity: a new approach using nonlinear electrodynamics

Christian Corda and Herman J Mosquera Cuesta

December 3, 2010

Institute for Basic Research, P. O. Box 1577, Palm Harbor, FL 34682, USA

*Instituto de Cosmologia, Relatividade e Astrofísica (ICRA-BR), Centro Brasileiro de Pesquisas Fisicas, Rua Dr. Xavier Sigaud 150, CEP 22290 - 180 Urca Rio de Janeiro - RJ Brazil

E-mail addresses: +cordac.galilei@gmail.com; *herman@icra.it;

Abstract

We discuss another approach regarding the inflation from the $R^2$ theory of gravity originally proposed by Starobinski. A non-singular early cosmology is proposed, where, adding a nonlinear electrodynamics Lagrangian to the high-order action, a bouncing is present and a power-law inflation is obtained. In the model the Ricci scalar $R$ works like an inflaton field.

PACS numbers: 04.50.+h, 04.20.Jb.

Keywords: inflation; nonlinear Lagrangian.

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C. Corda is partially supported by a Research Grant of The R. M. Santilli Foundations Number RMS-TH-5735A2310
1 Introduction

The accelerated expansion of the Universe that is currently purported from observations of SNe Ia suggests that cosmological dynamics is dominated by a “new” substance of the universe constituents dubbed as Dark Energy, which is able to provide a large negative pressure to account for the late-time accelerate expansion. This is the standard picture, in which such a new ingredient is considered as a source of the right-hand-side of the field equations. It is posed that it should be some form of un-clustered non-zero vacuum energy which, together with the clustered Dark Matter, drives the global dynamics. This is the so-called “concordance model” (ΛCDM) which gives, in agreement with the data analysis of the observations of the Cosmic Microwave Background Radiation (CMBR), Lyman Limit Systems (LLS) and type Ia supernovae (SNe Ia), a good framework for understanding the currently observed Universe. However, the ΛCDM presents several shortcomings as the well known “coincidence” and “cosmological constant” problems [1].

An alternative approach to explain the purported late-time acceleration of the universe is to change the left hand side of the field equations, and to inquire whether the observed cosmic dynamics can be achieved by extending general relativity [2, 3, 4]. In this different context, it is not required to search candidates for Dark Energy and Dark Matter, which until to date, have not been found, but rather it claims that only the “observed” ingredients: curvature and baryon matter, have to be taken into account. Considering this point of view, one can posit that gravity is not scale-invariant [5]. In so doing, one allows for a room for alternative theories to be open [5, 7, 8]. In principle, interesting Dark Energy and Dark Matter models can be built by considering $f(R)$ theories of gravity [5, 9] (here $R$ is the Ricci curvature scalar).

In this perspective, even the sensitive detectors of gravitational waves like bars and interferometers (i.e., those which are currently in operation and the ones which are in a phase of planning and proposal stages [10, 11], could, in principle, test the physical consistency of general relativity or of any other theory of gravitation. This is because in the context of Extended Theories of Gravity important differences with respect to general relativity show up after studying the linearized theory [12, 13, 14, 15].

In this paper, another approach regarding the inflation from the $R^2$ theory of gravity, which is the simplest among $f(R)$ theories and was been originally proposed by Starobinski in [16], is shown. A non-singular early cosmology is proposed, where, adding a nonlinear electrodynamics Lagrangian to the high-order action, a bouncing is present and a power-law inflation is obtained. In the model the Ricci scalar $R$ works like an inflaton field.

In the general picture of high order theories of gravity, recently the $R^2$ theory has been analysed in various interesting frameworks, see [17, 18] for example.

We recall that extensions of the traditional Maxwell electromagnetic Lagrangian, which take into account high order terms of the electromagnetic scalar $F$, have been used in cosmological models [19], gravitational redshifts of neutron stars [20] and pulsars [21]. Moreover, a particular nonlinear Lagrangian
has been analysed in the context of the Pioneer 10/11 spacecraft anomaly [22].

2 Action and Lagrangian

Let us consider the high order action [16, 17, 18]

\[ S = \int d^4x \sqrt{-g}(R + \alpha R^2 + \mathcal{L}_m). \] (1)

Such an equation (1) is a particular choice in respect to the well known canonical one of General Relativity (the Einstein - Hilbert action [23]) which is

\[ S = \int d^4x \sqrt{-g}(R + \mathcal{L}_m). \] (2)

We are going to show that the action (1), applied to the Friedman-Robertson-Walker Cosmology, generates a non singular inflationary phase of the Universe where the Ricci scalar acts like inflaton, and a bouncing is present, if \( \mathcal{L}_m \) is the non linear electrodynamics Lagrangian. Note that in this letter we work with \( 8\pi G = 1, c = 1 \) and \( \hbar = 1 \).

Inflationary models of the early Universe were analysed in the early and middles 1980's (see [24] for a review), starting from an idea of Starobinski [15] and Guth [25]. These are cosmological models in which the Universe undergoes a brief phase of a very rapid expansion in early times. In this context the expansion could be power-law or exponential in time. Inflationary models provide solutions to the horizon and flatness problems and contain a mechanism which creates perturbations in all fields [24].

In Cosmology, the Universe is seen like a dynamic and thermodynamic system in which test masses (i.e. the “particles”) are the galaxies that are stellar systems with a number of the order of \( 10^9 - 10^{11} \) stars [23]. Galaxies are located in clusters and super clusters, and observations show that, on cosmological scales, their distribution is uniform. This is also confirmed by the WMAP data on the Cosmic Background Radiation [26, 27]. These assumption can be summarized in the so called Cosmological Principle: the Universe is homogeneous everywhere and isotropic around every point. Cosmological Principle simplifies the analysis of the large scale structure, because it implies that the proper distances between any two galaxies is given by an universal scale factor which is the same for any couple of galaxies [23].

In this framework, the cosmological line - element is the well known Friedman-Robertson-Walker one, and for a sake of simplicity we will consider the flat case, because the WMAP data are in agreement with it [26, 27]:

\[ ds^2 = -dt^2 + a^2(dz^2 + dx^2 + dy^2). \] (3)
Following \cite{23} we also get
\[
g_{\mu\nu} = \begin{pmatrix}
-1 & 0 & 0 & 0 \\
0 & +a^2 & 0 & 0 \\
0 & 0 & +a^2 & 0 \\
0 & 0 & 0 & +a^2
\end{pmatrix},
\]
and
\[
\sqrt{-g} = a^3
\]
(4)

and
\[
R = -6\left[\frac{1}{a} \frac{da}{dt} + \left(\frac{\dot{a}}{a}\right)^2\right].
\]
(5)

One can use the Lagrange multipliers putting
\[
S = 2\pi^2 \int dt \{a^3(R + \alpha R^2) - \beta[R + 6\frac{\dot{a}}{a} + 6(\frac{\dot{a}}{a})^2 + a^3L_m]\}.
\]
(6)

\(\beta\) can be obtained by varing the action in respect to \(R\). It is
\[
a^3 \frac{\partial (R + \alpha R^2)}{\partial R} \delta R - \beta \delta R,
\]
(7)

which gives
\[
\beta = a^3 \frac{\partial (R + \alpha R^2)}{\partial R} = a^3(2\alpha R + 1).
\]
(8)

Thus, substituting in eq. (7) one obtains
\[
S = 2\pi^2 \int dt \{-2a^3\alpha R^2 - 6a^2\dddot{a}(2\alpha R + 1) - 6a(\dddot{a})^2(2\alpha R + 1) + a^3L_m\}.
\]
(9)

The term \(-6a^2\dddot{a}(2\alpha R + 1)\) is critical as it contains a second derivative of \(a\).

Let us integrate it. It is
\[
-6 \int dt a^2\dddot{a}(2\alpha R + 1) = -6a^2\dot{a}(2\alpha R + 1) + 6 \int dt [2a a^2 \dddot{a} R + 2a(\dddot{a})^2(2\alpha R + 1)] =
\]
\[
= 6 \int dt [2a a^2 \dddot{a} R + 2a(\dddot{a})^2(2\alpha R + 1)],
\]
(10)

where we have taken into account that the term outside the integral is equal to zero as it is a pure divergence.

Substituting in eq. (10), one gets
\[
S = 2\pi^2 \int dt \{-a^3\alpha R^2 + 12a a^2 \dddot{a} R + 6a(\dddot{a})^2(2\alpha R + 1) + a^3L_m\}.
\]
(11)

Then, the Lagrangian is
\[
\mathcal{L} = -a^3\alpha R^2 + 12a a^2 \dddot{a} R + 6a(\dddot{a})^2(2\alpha R + 1) + a^3L_m.
\]
(12)
The energy function associated to the Lagrangian is [23]

\[ E_L = \frac{\partial L}{\partial \dot{a}} \dot{a} + \frac{\partial L}{\partial \dot{R}} \dot{R} - L. \]  

(14)

Combining eq. (13) with eq. (14), the condition

\[ E_L = 0 \]  

(15)

together with the definition of the Hubble constant, i.e. \( H = \frac{1}{a} \frac{da}{dt} \), and with a little algebra gives

\[ H^2 = \frac{\mathcal{L}_m}{3\alpha R} - H \frac{\dot{R}}{R}. \]  

(16)

From the Euler-Lagrange equation for \( a \) and \( \dot{a} \), i.e. [23]

\[ \frac{\partial L}{\partial a} = \frac{d}{dt} \left( \frac{\partial L}{\partial \dot{a}} \right) \]  

(17)

one gets

\[ \ddot{R} + 3H \dot{R} = \frac{2\mathcal{L}_m}{3\alpha}. \]  

(18)

An important question is where Eq. (15) comes from [29]. In general relativity, due to the reparametrization invariance of the time coordinate, the total energy (including the contribution from the gravity sector) vanishes [29]. In the action (12), however, there is not the reparametrization invariance because the total derivative terms are dropped [29]. Then, one can think that the total energy does not always vanish [29]. We clarify this point as it follows. Let us start by the original action [1] from which the action (12) arises. Let us consider the conformal transformation [30]

\[ \tilde{g}_{\alpha\beta} = e^{2\Phi} g_{\alpha\beta} \]  

(19)

where the conformal rescaling

\[ e^{2\Phi} = 2\alpha R + 1 \]  

(20)

has been chosen. By applying the conformal transformation [19] to the action [1] the conformal equivalent Hilbert-Einstein action

\[ A = d^4x \sqrt{-\tilde{g}[\tilde{R} + \mathcal{L}(\Phi, \Phi; \alpha) + \mathcal{L}_m]}, \]  

(21)

is obtained. \( \mathcal{L}(\Phi, \Phi; \alpha) \) is the conformal scalar field contribution derived from

\[ \tilde{R}_{\alpha\beta} = R_{\alpha\beta} + 2(\Phi_{,\alpha} \Phi_{,\beta} - g_{\alpha\beta} \Phi_{,\delta} \Phi^{,\delta} - \frac{1}{2} g_{\alpha\beta} \Phi_{,\delta} \Phi_{,\delta}) \]  

(22)

and

\[ \tilde{R} = e^{-2\Phi} + (R - 6\square \Phi - 6\Phi_{,\delta} \Phi^{,\delta}). \]  

(23)
Clearly, the reparametrization invariance of the time coordinate is consistent with the new action \(21\) in the conformal Einstein frame and the total energy (including the contribution from the gravity sector) vanishes in this case too. One could object that the energy in the conformal Einstein frame is different with respect to the energy in the original Jordan frame, but in ref. \[31\] it has been shown that the two conformal frames are energetically equivalent if, together with the conformal rescaling \[19\], times and lengths are rescaled as \(e^\Phi\), while the mass-energy is rescaled as \(e^{-\Phi}\). This analysis permits to enable the condition of Eq. (15) in the present discussion too.

3 Nonlinear electrodynamics Lagrangian and Inflation

In order to show that our model admits a power law inflationary phase, we need to postulate some matter Lagrangian \(\mathcal{L}_m\) which can perform the condition of inflation \(P < -\rho\) \[24\]. We will use the non linear electrodynamics Lagrangian of \[19\], which is

\[
\mathcal{L}_m \equiv \frac{1}{4} F + c_1 F^2 + c_2 G^2, \tag{24}
\]

where \(F\) is the electromagnetic scalar, \(c_1, c_2\) are two constants and, considering the electromagnetic field tensor \(F^{\alpha\beta}\) (see \[23\] the definition of this object), \(G\) is defined like \[19\] \(G \equiv \frac{1}{2} \eta^{\alpha\beta\mu\nu} F^{\alpha\beta} F^{\mu\nu}\).

The Lagrangian \(24\), differently from the one of the singular Einstein-Maxwell Universe, performs a non singular Universe with \textit{bouncing} \[19\]. This is because the energy condition of singularity theorems \[28\] is not satisfied in the case of the non linear electrodynamics Lagrangian (see \[19\] for details).

In fact, following \[19\], one uses the equation of state

\[
p = \frac{1}{3} \rho - \rho_*, \tag{25}
\]

where

\[
\rho_* \equiv \frac{16}{3} c_1 B^4 \tag{26}
\]

(see eq. 15, 16 and 25 of \[19\]) and \(B\) is the magnetic field associated to \(F\). This equation of state is no longer given by the Maxwellian value, thus, using eq. \(24\), from eqs. \[16\] and \[18\] one gets

\[
B = \frac{B_0}{2a^2}, \tag{27}
\]

where \(B_0\) is a constant \[19\], and

\[
\dot{a}^2 = \frac{B_0^2}{12 a^4 R} (1 - \frac{8c_1 B_0^2}{a^4}) - 2 H a^2 \frac{\dot{R}}{R}, \tag{28}
\]
which can be solved by suitably choosing the origin of time. One gets

\[ a^2 = \frac{B_0}{\sqrt{\alpha}} \sqrt{\frac{2}{3} (t^2 + 12c_1)}. \] (29)

This expression is not singular for \( c_1 > 0 \). In this case we see that at the instant \( t = 0 \) a minimum value of the scale factor is present:

\[ a^2_{\text{min}} = \frac{B_0}{\sqrt{\alpha}} \sqrt{8c_1}. \] (30)

This also implies that, for a value \( t = \sqrt{12c_1} \), the energy density \(-\rho\) reaches a maximum value \( \rho_{\text{max}} = 1/64c_1 \). For smaller values of \( t \) the energy density decreases, vanishing at \( t = 0 \), while the pressure becomes negative [19].

In this way, the condition of inflation \( P - \rho < 0 \) gives the inflationary solutions for equations (16) and (18), if one assumes that the Ricci scalar \( R \) acts like inflaton:

\[ R(t) \simeq (1 + Ht/\beta)^2 \]
\[ a_{\text{inf}}(t) \simeq (1 + Ht/\beta)^{w+1/2}, \]

with \( \beta \simeq w \) and

\[ H_{\text{inf}} \simeq \sqrt{\mathcal{L}_m^*}, \] (31)

where \( \mathcal{L}_m^* \) is the right hand side of equation (10) which is constant during the inflationary phase. The idea of considering the Ricci scalar as an effective scalar field (scalaron) arises from Starobinski [16].

4 Conclusion remarks

Another approach regarding the inflation from the \( R^2 \) theory of gravity, which was originally proposed by Starobinski, has been analysed. A non-singular early cosmology has been proposed, where, adding a nonlinear electrodynamics Lagrangian to the high-order action, a bouncing is present and a power-law inflation is obtained. In the model which has been discussed, the Ricci scalar \( R \) works like an inflaton field.

Acknowledgements

We would like to thank Professor Mario Novello for useful discussions on the topics of this paper. We also thank an unknown referee for precious advices and suggestions which permitted to improve this paper.
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