Thermal Instability of Thin Accretion Disks in the Presence of Wind and a Toroidal Magnetic Field

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Abstract

We study the local thermal stability of thin accretion disks. We present a full stability analysis in the presence of a magnetic field and, more importantly, wind. We use a general model suitable for adequately describing several kinds of winds. First, we explicitly show that the magnetic field, irrespective of the type of wind, has a stabilizing effect. This is also true when there is no wind. In this case, we confirm other works presented in the literature. However, our main objective is to investigate the local thermal stability of the disk in the presence of wind. In this case, interestingly, the response of the disk is directly related to the type of wind. In other words, in some cases, the wind can stabilize the disk. On the other hand, in some cases it can destabilize the disk. We find that in some thin disk models where the magnetic pressure cannot explain the stability of the disk by including a typical contribution for magnetic pressure, wind can provide a viable explanation for the thermal stability.

Unified Astronomy Thesaurus concepts: Stellar accretion disks (1579); Magnetic fields (994); Stellar winds (1636)

1. Introduction

In a pioneering work, Shakura & Sunyaev (1973) developed the thin accretion disk model. This model is extremely successful in explaining the physical properties of black hole X-ray binaries. However, after more than four decades there are still unsolved questions and puzzles concerning the structure of thin disks. For a comprehensive review of this model, we refer the reader to Yuan & Narayan (2014). In this paper, we focus on one of these puzzles dealing with the local thermal stability of the disk. For mass accretion rates higher than a few percent of the Eddington rate, the radiation-dominated thin disks become thermally unstable. This is a well-established fact widely investigated in the literature; for example, see Shakura & Sunyaev (1976) and Piran (1978). More realistic calculations, i.e., numerical simulations, show that the final fate of unstable regions is a nonlinear oscillation between two stable phases (Li et al. 2007). According to these theoretical studies, in real observations one may expect a substantial variability in the physical properties of black hole X-ray binaries.

However, the high/soft state of X-ray binaries appears to be quite stable on observation (Gierliński & Done 2004). These observations reveal little variability with luminosities ranging from 0.01 to 0.5 $L_{\text{Edd}}$ which directly means that this thin-disk configuration is thermally stable. This conflicts with the accretion disk theory. Generally there are two processes which may likely change this inconsistency between theory and observations. The first scenario is that the viscous stress is proportional to gas pressure instead of total pressure; in this case the disk will be stable against thermal instability (Sakimoto & Coroniti 1981). The second process considers a mechanism for making the disk cooler which may eliminate instability or equivalently increase the relative importance of gas pressure over radiation pressure. To address this process several investigations have been proposed. Svensson & Zdziarski (1994) have shown that instabilities will be removed if most of the gravitational energy released in the disk is transported to the corona. Convective cooling has been suggested as a stabilizing mechanism by Goldman & Wandel (1995), although some later investigations have pointed out that it might have a minor effect on the disk instability. Zhu & Narayan (2013) considered the possible effect of turbulence on the disk instability. Cooling the disk when the magnetic pressure becomes important in hydrodynamical equilibrium (Zheng et al. 2011) or cooling via dynamically or magnetically driven wind (Li & Begelman 2014) are other possible solutions. However, it is necessary to mention that the puzzle has not yet been resolved. As one of the last suggested solutions, Zheng et al. (2011), claimed that the existence of magnetic pressure in the system may substantially stabilize the disk. However, this requires a substantial fraction of magnetic pressure to achieve stability. More specifically, for stability, magnetic pressure should contribute more than 20% of the total pressure. This fraction is too large compared with the typical values in the simulations.

Our paper is close in spirit to Zheng et al. (2011). We revisit the stability problem by generalizing the linear analysis introduced by Zheng et al. by adding a wind mechanism. The importance of wind/outflow in angular momentum removal from many accreting systems is supported by strong observational evidence, e.g., Whelan et al. (2005). On the other hand, it is long apparent that a disk wind/outflow contributes to the loss of mass, angular momentum, and thermal energy from accretion disks in theoretical modeling, e.g., Blandford & Payne (1982). To add a wind/outflow effect we use a parametric simple model presented by Knigge (1999) which derived the radial distribution of the dissipation rate and effective temperature across a Keplerian, steady-state, mass-losing accretion disk, using a simple parametric approach. This simple model is sufficiently general to be applicable to many types of wind such as radiation-driven outflow and centrifugally driven wind. Using this model, we show that mass loss via wind can stabilize the disk.

The outline of this paper is as follows: in Section 2 we write the basic equations governing the system in the presence of magnetic field and mass loss via wind. In Section 3 we investigate the local thermal stability of the disk and find a new criterion for stability. This criterion is examined in different
situations and the result is presented in Section 4. Finally, a conclusion and discussion are presented in Section 5.

2. Hydrodynamics Equations in the Presence of Wind and a Magnetic Field

In this section, we introduce the basic equations governing the dynamics of our accretion thin disk model. More specifically, to find a criterion for the thermal stability of the disk, we use the conservation equations of energy, mass, and angular momentum in the presence of wind and a toroidal magnetic field. For convenience, we choose the cylindrical coordinate system \((r, \phi, z)\). In our simple model, the disk consists of a differentially rotating disk around a central mass \(M\). For the sake of simplicity, we assume that the flow is static and axisymmetric, i.e., \(\frac{\partial}{\partial r} = 0\) and \(\frac{\partial}{\partial \phi} = 0\).

Assuming that the disk lies in the \(x-y\) plane, the hydrostatic condition in the vertical direction \(z\) takes the following form:

\[
\frac{\partial p}{\partial z} + \frac{\partial \psi}{\partial z} = 0
\]

in which \(p\) is the fluid pressure, \(\rho\) is the mass density, and \(\psi\) is the Newtonian gravitational potential of the central mass. It should be mentioned that we restrict ourselves to regions far enough from the center of the disk to ignore relativistic effects. Furthermore, we assume that the gravitational potential is dominated by the central mass and ignore the contribution of the disk itself. On the other hand, as we mentioned, we deal with a thin disk. In this case, one may simply assume that at every radius \(r\), the vertical thickness is very small compared to \(R\), i.e., \(R/z \ll 1\). Keeping this approximation in mind, we can write

\[
\frac{\partial p}{\partial z} \simeq -\frac{p_{\text{gas}}}{H}, \quad \frac{\partial \psi}{\partial z} \simeq \frac{GMH}{R^3}
\]

(2)

where \(p_{\text{tot}}\) is the pressure in the midplane, \(H\) is the disk scale height, and \(R\) is the radial coordinate. By total pressure we mean the combination of all pressure contributions, namely the gas pressure \(p_{\text{gas}}\), radiation pressure \(p_{\text{rad}}\), and the magnetic pressure \(p_{\text{mag}}\). Therefore \(p_{\text{tot}}\) reads

\[
p_{\text{tot}} = p_{\text{gas}} + p_{\text{rad}} + p_{\text{mag}}.
\]

(3)

Hereafter we assume that the gas content of the disk is an ideal classic gas. On the other hand, the magnetic field is assumed to be toroidal, i.e., \(B \simeq B_0 \hat{\phi}\) where \(\hat{\phi}\) is the azimuthal unit vector. The pressure contributions can be written as

\[
p_{\text{gas}} = \frac{2k_B \rho T}{m_H},
\]

(4)

\[
p_{\text{rad}} = \frac{1}{3}aT^4,
\]

(5)

\[
p_{\text{mag}} = \frac{B_0^2}{8\pi}
\]

(6)

where \(k_B\) is the Boltzmann constant, \(m_H\) is hydrogen mass, \(T\) is the temperature at midplane, and \(a\) is the radiation constant. On the other hand, in the thin disk limit the surface density is written as \(\Sigma = 2\rho H\). Therefore it is straightforward to show that Equation (2) leads to the following relation for the total pressure:

\[
p_{\text{tot}} = \frac{M G H \Sigma(R)}{2R^2}.
\]

(7)

Flow equations in the presence of wind can be easily obtained from Navier–Stokes equations; see Knigge (1999) for more details. The continuity equation reads

\[
\frac{\partial}{\partial t} (2\pi R \Sigma) - \frac{\partial M_{\text{acc}}}{\partial R} + \frac{\partial M_{\text{w}}}{\partial R} = 0
\]

(8)

where \(M_{\text{w}}\) is the mass-loss rate from the disk caused by the wind. This rate is related to the mass-loss rate per unit area, i.e., \(\dot{m}_{\text{w}}(R)\), as

\[
M_{\text{w}}(R) = 4\pi \int_{R_*}^R \dot{m}_{\text{w}}(R') R' dR'
\]

(9)

where \(R_*\) is the inner edge of the disk. In principle it cannot be smaller than the radius of the central mass. In our analysis we consider it as a few times the Schwarzschild radius \(R_s = 2M_G/c^2\). On the other hand, \(M\) is the mass accretion rate defined as \(M_{\text{acc}} = -2\pi R v_r \Sigma > 0\), where \(v_r < 0\) is the radial inflow velocity. Since we have assumed that the flow is static, one may simply infer from Equation (8) that \(-\dot{M}_{\text{acc}}(R) + M_{\text{w}}(R) = \text{const}\). The mass-loss rate at \(R = R_*\) is zero. Therefore the above-mentioned constant is \(M_{\text{acc}}(R_*\) and we have

\[
-M_{\text{acc}}(R) + M_{\text{w}}(R) = M_{\text{acc}}(R_*).
\]

(10)

Accordingly the angular momentum conservation for the thin disk can be obtained as follows (Knigge 1999):

\[
\frac{\partial}{\partial t} (\Sigma R^2 \Omega) + \frac{\partial}{\partial R} \left( \frac{\Sigma R^2 \nu \partial \Omega}{\partial R} \right) + \frac{\nu R^2 \Omega}{2\pi R} \frac{\partial M_{\text{w}}}{\partial R} = 0
\]

(11)

where \(\nu\) is the kinematic viscosity. We have assumed that at radius \(R\), the ejected matter through the wind carries away angular momentum per unit mass \(I^2 \Omega R\). This leads to a simple approach suitable for many types of wind models; for more details on this model we refer the reader to Knigge (1999). Conveniently, \(\Omega(R)\) is the angular velocity of the flow at radius \(R\) and \(l\) is a dimensionless parameter useful for characterizing the main properties of the wind. More specifically, \(l = 0\) indicates a non-rotating disk wind. On the other hand, \(l = 1\) corresponds to outflowing material that carries away the angular momentum it had at the radius of ejection (Knigge 1999). Similarly, \(l > 1\) belongs to centrifugally driven disk winds that remove a lot of angular momentum from the disk.

We reiterate that we use a Newtonian potential for the central mass and ignore the self-gravity of the disk. Consequently, the angular velocity is given by the Keplerian angular velocity \(\Omega_k = \sqrt{M_{\text{c}} G/R^3}\).

Let us first assume that there is no wind mechanism in the system. In this case, one may simply integrate Equation (11) over the radial range \((R_*, R)\) to obtain

\[
\dot{M}_{\text{acc}}(R)(\Omega_k R^2 - l_{\text{in}}) = 2\pi R^2 T_{R,\phi}
\]

(12)
in which \( R_{\text{in}} = 3R_h \) indicates a region inside which there is no stable circular orbit, and \( l_{\text{in}} = \sqrt{M_*G R_{\text{in}}} \) is the specific angular momentum of the last stable orbit. On the other hand the \( R_\phi \) component of the energy–momentum tensor when the system is axisymmetric is given by \( T_{R_\phi} = \nu \Sigma R d\Omega/dR \). Now, let us modify Equation (12) in order to include the wind contribution to the distribution of angular momentum throughout the disk. To do so we multiply Equation (12) by \( 2\pi R \) and ignore the time derivatives. Using the continuity Equation (8), and by integrating along the radial coordinate in the interval \( (R_{\text{in}}, R) \), we obtain

\[
M_{\text{acc}}(R)(\Omega_K R^2 - l_{\text{in}}) + C_w(R) = 2\pi R^2 T_{R_\phi}
\]

(13)

where \( C_w(R) \) is the correction term induced by the existence of the wind in the system. This term is written as

\[
C_w(R) = \frac{4\pi K}{R^\xi}(l^2 - l_{\text{in}})(R^{\xi+5/2} - R_{\text{in}}^{\xi+5/2}).
\]

(14)

It should be emphasized that we have taken a simple power-law model for the mass-loss rate per area as follows:

\[
m_{w}(R) = K R^{\xi}
\]

(15)

where \( K \) and \( \xi \) are two free parameters (Krigge 1999). By setting \( l = 1 \) we see that \( C_w(R) \) vanishes and Equation (14) coincides with (13). Therefore it seems that the effects of wind disappear in the system. However one should note that although \( C_w(R) = 0 \), the accretion rate \( M_{\text{acc}} \) in Equation (14) is a function of radius because of the wind. When there is no wind, the accretion rate is constant everywhere throughout the disk.

The last governing equation is the energy equation. This equation in the presence of wind can be written as

\[
Q_{\text{vis}} = Q_{\text{rad}} + Q_{\text{adv}} + Q_{\text{win}}
\]

(16)

where the plus (minus) sign stands for physical processes that produce (suppress) heating in the system. The viscous heating reads

\[
Q_{\text{vis}}^+ = -T_{R_\phi} \frac{d\Omega}{dR}
\]

(17)

Before introducing the other energy components, let us briefly explain the specific form of \( T_{R_\phi} \) widely used to study the structure of thin and thick accretion disks. Both analytic and magnetohydrodynamic (MHD) simulations show that \( T_{R_\phi} \) is the dominant component of the stress tensor and one may model it as

\[
T_{R_\phi} = 2\alpha \rho H
\]

(18)

where \( \alpha \) is the viscosity coefficient and plays a key role in the \( \alpha \) viscosity model of accretion disks. For a short review on this choice, we refer the reader to Zheng et al. (2011). One may simply write \( T_{R_\phi} \) in terms of the surface pressure \( P \) as \( T_{R_\phi} = \alpha P \). In this case, the dimension of \( P \) is Newtons per meter.

On the other hand the cooling via radiation is given by

\[
Q_{\text{rad}} = \frac{32\pi T^4}{3\tau}
\]

(19)

where \( \tau = \kappa \Sigma/2 \) is the optical depth and \( \kappa \approx 0.4 \text{ cm}^2\text{ g}^{-1} \) if the opacity in the inner parts of the disk is mostly due to electron scattering. Furthermore, \( \sigma \) is the Stefan–Boltzmann constant and \( T \) is the temperature of the disk.

The advection cooling in the disk is related to accretion rate as follows (Abramowicz et al. 1995):

\[
Q_{\text{adv}} = \mu^2 \frac{M_{\text{acc}}(R)\Omega_K H^2}{2\pi R^3}
\]

(20)

where we use the typical value \( \mu = 1.5 \) already employed in Zheng et al. (2011). As already mentioned, for the wind we use the model described in Krigge (1999). In this case the wind cooling is written as

\[
Q_{\text{win}} = \frac{1}{2}(\eta_b + \eta_k f^2) K R^{\xi+2} \Omega_K^2
\]

(21)

where \( f \) is a velocity parameter defined as the ratio of the Keplerian velocity over the escape velocity. Therefore we will set it as \( f = \sqrt{2} \). Moreover, \( \eta_b \) and \( \eta_k \) are efficiency parameters: for \( l < \sqrt{3/2} : \eta_b = 3 - 2l^2 \) and \( \eta_k = 1 \); and for \( l > \sqrt{3/2} : \eta_b = 0 \) and \( \eta_k = 1 - 2f^{-2}(l^2 - 3/2) \); see Krigge (1999) for details.

Now we have completed the main equations necessary to describe the thermal stability of the flow. Before moving on to close this section and start the stability analysis, let us introduce some useful relations. In what follows we assume that the radius of the disk is \( R_h \). In this case the total mass-loss rate, i.e., \( M_w(R_d) \) can be obtained from Equation (9) as

\[
M_w(R_d) = \frac{4\pi K}{s}(R^d_d - R^d_{*d})
\]

(22)

where the new parameter \( s \) is defined as \( s = 2 + \xi \). One may use this equation to find \( K \) and normalize \( m_w(R) \) as follows:

\[
m_w(R) = \frac{sM_w(R_d)}{4\pi} \frac{R^{s-2}}{R^2_d - R^2_{*d}}
\]

(23)

As another useful equation, we rewrite the continuity Equation (10) as follows:

\[
M_{\text{acc}}(R) = M_{\text{acc}}(R_*) + \frac{M_w(R_d)}{R_d^2 - R_{*d}^2}(R^d - R_{*d})
\]

(24)

It should be mentioned that magnetic field is present in our model. Therefore in order to construct a complete set of equations to describe the unknown functions, we need one more constraint on the magnetic field. To do so, we follow the description introduced in Zheng et al. (2011). Based on MHD simulations (Machida et al. 2006), one may assume that the strength of the magnetic field decreases with vertical height from the disk midplane. Therefore we simply assume that

\[
B_\phi H^\gamma = \text{constant} = \Phi_0
\]

(25)

where \( \gamma \) is a constant parameter. This equation combined with the contribution of the magnetic field in the pressure content of the flow characterizes the impact of the magnetic field on the evolution of the thermal instability.

### 3. Thermal Instability

In this section we use the main equations presented in the previous section and find a general criterion for the thermal stability of the disk in the presence of wind. We generalize the analysis of Zheng et al. (2011) to include wind cooling. To do
so let us assume that the thermal instability does not change the surface density at the given radius \( R \). Note that we are interested in finding the stability criterion at an arbitrary radius \( R \). Consequently, using Equation (4) we find

\[
d \ln p_{\text{tot}} \simeq d \ln H = \beta_{\text{gas}}(d \ln T - d \ln H) + 4(1 - \beta_{\text{gas}} - \beta_{\text{mag}})d \ln T + 2\beta_{\text{mag}}d \ln B_\phi
\]

where \( \beta_{\text{mag}} = p_{\text{mag}}/p_{\text{gas}}, \beta_{\text{gas}} = p_{\text{gas}}/p_{\text{tot}} \) and \( \beta_{\text{rad}} = p_{\text{rad}}/p_{\text{tot}} \) are dimensionless quantities. It is clear that

\[
\beta_{\text{mag}} + \beta_{\text{gas}} + \beta_{\text{rad}} = 1.
\]

The wind impact does not appear directly in Equation (26). On the other hand, using the energy Equation (16) we find

\[
d \ln Q_{\text{vis}}^+ - d \ln (Q_{\text{rad}}^+ + Q_{\text{adv}}^- + Q_{\text{win}}^-) = -4(1 - f_{\text{adv}} - f_{\text{win}})d \ln T + d \ln T_{R\Phi} - f_{\text{adv}}(d \ln M_{\text{acc}} + 2d \ln H)
\]

where \( f_{\text{adv}} \) and \( f_{\text{win}} \) are defined as

\[
f_{\text{adv}} = \frac{Q_{\text{adv}}^-}{Q_{\text{vis}}}, \quad f_{\text{win}} = \frac{Q_{\text{win}}^-}{Q_{\text{vis}}}
\]

it is clear that wind effects directly appear in Equation (28). Moreover, using Equations (13) and (25) we find

\[
d \ln M_{\text{acc}} = d \ln T_{R\Phi} = d \ln p_{\text{tot}} + d \ln H \quad (30)
\]

\[
d \ln B_\phi = -\gamma d \ln H \quad (31)
\]

and using Equations (26) and (31) we eliminate \( B_\phi \) to obtain

\[
d \ln p_{\text{tot}} = d \ln H = \frac{4 - 3\beta_{\text{gas}} - 4\beta_{\text{mag}}}{1 + 2\gamma\beta_{\text{mag}} + \beta_{\text{gas}}}d \ln T.
\]

Now in order to find the thermal stability criterion in the presence of wind, we substitute Equations (16), (30), and (32) into Equation (28). The result is

\[
\left[ \frac{\partial Q_{\text{vis}}^+ - Q_{\text{rad}}^+ - Q_{\text{adv}}^- - Q_{\text{win}}^-}{\partial T} \right]_\Sigma Q_{\text{vis}}^+ = \frac{\psi}{1 + 2\gamma\beta_{\text{mag}} + \beta_{\text{gas}}} \quad (33)
\]

where \( \psi \) is defined as

\[
\psi = 4 - 10\beta_{\text{gas}} - 8(1 - \gamma)\beta_{\text{mag}} - 12f_{\text{adv}} + 16f_{\text{adv}}\beta_{\text{gas}} + (16 + 8\gamma)\beta_{\text{mag}}f_{\text{adv}} + 4f_{\text{win}}(1 + 2\gamma\beta_{\text{mag}} + \beta_{\text{gas}}).
\]

As we know, thermal instability occurs when

\[
\left[ \frac{\partial Q_{\text{vis}}^+ - Q_{\text{rad}}^+ - Q_{\text{adv}}^- - Q_{\text{win}}^-}{\partial T} \right]_\Sigma > 0. \quad (35)
\]

On the other hand the denominator in the right-hand side of (33) is positive. Therefore for thermal instability, the numerator should be positive, namely \( \psi > 0 \). In order to study this criterion in a quantitative way for the given parameters \( s, M, R, l, \gamma, M_{\text{acc}}(R_\ast) \), and \( M_w(R_\ast) \), we solve the governing Equations (7), (13), (16), (24), and (25) for six unknowns: \( \Sigma, H, T, B_\phi, M_{\text{acc}}(R) \), and \( \Phi \). It is clear that we need one more algebraic equation to construct a complete set of equations. To do so we use \( \beta_{\text{mag}} = \text{const} \). Finally it is straightforward to plot \( \psi \) for the given physical quantities. We have followed the method introduced in Zheng et al. (2011).

4. Results

Now let us investigate the thermal stability of the system taking account of changes in the relevant physical quantities. There are several parameters that can influence stability. For example, it turns out that the accretion rates, magnetic pressure, and wind parameters \( l \) and \( s \) play a significant role in the local stability of the disk.

Interestingly, the response of the system is sensitive to the magnitude of \( l \). More specifically, \( l^2 = 5/2 \) is a threshold and the system behaves completely different for \( l^2 > 5/2 \) and \( l^2 < 5/2 \). There is a simple interpretation of the existence of this threshold. We already mentioned that when \( l^2 > 3/2 \) then \( \eta_l = 0 \) and \( \eta_l = 5/2 - l^2 \) (note that \( f = \sqrt{2} \)). On the other hand, \( \eta_l \) directly controls the sign of wind cooling \( Q_{\text{win}} \). When \( l^2 < 5/2 \) we have \( Q_{\text{win}} > 0 \). This means that this type of wind heats the disk. Consequently, in this case, we expect destabilizing behavior due to wind. In contrast, when \( l^2 > 5/2 \) the wind cooling is positive and stabilizes the disk.

4.1. Stability Function \( \psi(M_{\text{acc}}, \beta_{\text{mag}}) \) for Different \( M_w \)

In Figure 1, we have plotted \( \psi \) as a function of \( M_{\text{acc}} \) and \( \beta_{\text{mag}} \). In the top and bottom panels, we have set \( l^2 = 2 \) and \( l^2 = 3 \) respectively. In all figures we have \( \gamma = 1 \) and \( R = 10MG/c^2 \). Contours indicate curves with constant \( \psi \). The solid curve is the stability boundary \( \psi = 0 \). In both rows \( M_w \) increases from left to right. It is clear that in the top row, by increasing the wind accretion rate, the stability region gets smaller. In other words, when \( l^2 < 5/2 \), the existence of wind destabilizes the disk in the sense that higher values of \( \beta_{\text{mag}} \) are required to stabilize the disk. One should note that this value for \( l \) corresponds to a specific form of wind.

On the other hand, we see in the bottom row that by increasing the wind accretion rate the stability zone gets wider. Strictly speaking, in this case we need lower \( \beta_{\text{mag}} \) to stabilize all the accretion rates. One should note that, for intermediate values for the wind accretion rate, disks with low accretion rates that are already stable become thermally unstable. It is important to mention that, for both rows, by increasing \( \beta_{\text{mag}} \) the disk gets stabilized. This result is in agreement with those obtained by Zheng et al. (2011). In other words, we confirm that, regardless of the nature of the wind, the magnetic field induces stabilizing effects. Of course one should note that we have considered the magnetic field and the wind to be totally independent. This is a restriction for the parametric model adopted here. More specifically, even the power-law mass-loss rate used here may not explain the real winds. This means that a full dynamical model would be needed to obtain more reliable results. In this regard, this parametric model should be considered as an approximative method which reveals some important features for the system.

Another important feature is that, except for the right panels in both rows, when \( \beta_{\text{mag}} \) is small the line \( \beta_{\text{mag}} = \text{const} \) intersect the stability boundary curve \( \psi = 0 \) at two points. This means that in this interval of mass accretion rate the disk is thermally unstable, while for values of mass accretion rate outside this interval the disk is stable. This behavior for the effect of the total
Figure 1. Top three panels: $l^2 = 2$; bottom panels: $l^2 = 3$. In both rows from left to right the dimensionless wind accretion rate is chosen as $M_{\infty}/M_{\text{Edd}} = 0.001, 0.1, 0.5$ respectively. In all panels the curves indicate contours of $\psi = 1, 0.5, 0, -0.5, -1, -1.5$ from bottom to top. The solid curve shows the stability boundary $\psi = 0; s = 0.3$ in all panels.

Figure 2. Top three panels: $l^2 = 2$; bottom panels: $l^2 = 3$. In both rows from left to right $\beta_{\infty}$ is chosen as $0.1, 0.125, 0.15$ respectively. In the top row the contours indicate $\psi = 1, 0.5, 0, -0.5, -1$ curves, while for the bottom panel we have $\psi = 0.2, 0.1, 0, -0.1, -0.25, -0.5, -1$. The solid curve shows the stability boundary $\psi = 0; s = 0.3$ in all panels.
accretion rate is also reported in Zheng et al. (2011). However, we mention that for one of the points we have $\dot{M}_{\text{acc}} > \dot{M}_{\text{Edd}}$, which refers to slim disk solutions. However, thin disk solutions are of interest in this study. Therefore we have truncated most of the figures at $\dot{M}_{\text{acc}} = \dot{M}_{\text{Edd}}$.

### 4.2. Stability Function $\psi(M_{\text{acc}}, M_\nu)$ for Different $\beta_{\text{mag}}$

In a similar way, it is helpful to plot the stability function $\psi$ as a function of $M_{\text{acc}}$ and $M_\nu$; see Figure 2. In both rows, from left to right we vary $\beta_{\text{mag}}$ as $\beta_{\text{mag}} = 0.1, 0.125,$ and 0.15. As before, the top row refers to $l^2 = 2$ and in the bottom panel we have $l^2 = 3$. The solid curves indicate the stability boundary, i.e., $\psi = 0$.

Let us start with the top panel. It is clear that by increasing the magnetic pressure contribution the stability region gets wider. Also if we keep the mass accretion rate constant and increase the wind accretion rate, we see that the disk eventually becomes unstable. Furthermore, if we keep the wind accretion rate constant and move along the mass accretion rate axis, at least for relatively small $\dot{M}_w$, we see that the line $\dot{M}_w$ constant intersects the stability boundary at two different values of $M_{\text{acc}}$ (as mentioned above, we discard high mass accretion rates). This directly means that the disk is stable for small and large mass accretion rates, while it is unstable for intermediate values of $\dot{M}_{\text{acc}}$.

Now let us discuss the bottom row in Figure 2. We recall that in this figure $l^2 = 3$. Completely in agreement with our discussion for Figure 1, we see that if we keep the mass accretion rate constant and move along the wind accretion rate, the disk becomes stable. As already mentioned, for $l^2 > 5/2$, we expect that the existence of wind stabilizes the disk. On the other hand, as expected, magnetic pressure has a stabilizing effect and its overall role does not depend on $l$. In other words, we see that the instability region, which covers large mass accretion rates, gets smaller and smaller. This means that the magnetic pressure stabilizes the disks with high mass accretion rates and low $\dot{M}_w$. As we see, there is a region at low $\dot{M}_w$ and $\dot{M}_{\text{acc}}$ where our parametric model cannot produce physically accreting disk solutions. This region is not affected by magnetic pressure.

### 4.3. Stability Function $\psi(s, \dot{M}_w)$ for Different Mass Accretion Rates

$s$ is another important parameter directly related to the properties of wind in the system, therefore it is useful to investigate the response of the system to changes in this parameter. To do so, we have plotted the stability function $\psi(s, \dot{M}_w)$ as a density plot in Figure 3.

From top to bottom we increase the wind mass accretion rate as $\dot{M}_w / \dot{M}_{\text{Edd}} = 0.01, 0.05,$ and 0.07. In this case, for larger values of the wind accretion rates the disk is completely unstable and the $s$ parameter is not helpful. On the other hand, it is important to mention that, in this case, the behavior of the system in $s-M_\nu$ is not significantly sensitive to the magnitude of $l$. Therefore we have illustrated only the $l^2 = 2$ case. Furthermore the magnetic pressure contribution is fixed as $\beta_{\text{mag}} = 0.1$. The solid black curves separate the stability and instability regions. As already mentioned for this case, by increasing the wind mass accretion rate, the stability region gets smaller. It is clear from the top panel that when the wind accretion rate is small, the $s$ parameter does not have any impact on the local stability of the system. We see that the mass accretion rate plays a key role. However, there is a narrow region for low mass accretion rates which becomes unstable when $s \to 1$.

Figure 3. Stability function $\psi$ with respect to $s$ and $\dot{M}_w$ when $l^2 = 2$. From top to bottom the dimensionless wind accretion rate is chosen as $\dot{M}_w / \dot{M}_{\text{Edd}} = 0.01,$ 0.05, and 0.07 respectively. In all panels the solid curves indicates the stability contour $\psi = 0$. 
We see that there is a region around $0.1 \dot{M}_{\text{Edd}} < \dot{M}_{\text{acc}} < 0.2 \dot{M}_{\text{Edd}}$ which does not lead to physical solutions with $s > 0.8$. From the middle and bottom panels one may deduce that the parameter $s$ has a stabilizing effect. More specifically, although $s$ cannot stabilize the disks with large mass accretion rates $\dot{M}_{\text{acc}}$, it can stabilize disks with low mass accretion rates.

### 4.4. Local Thermal Equilibria: $\dot{M}_{\text{acc}}-\Sigma$ Diagram

It is also convenient and instructive to plot the $\dot{M}_{\text{acc}}-\Sigma$ diagram for the local stability of the system. In this diagram the negative slop shows the thermally unstable solution. We have plotted this diagram in Figure 4. It is crucial to mention again that only accretion rates smaller than the Eddington critical rate, i.e., $\dot{M}_{\text{acc}} \lesssim \dot{M}_{\text{Edd}}$, are of interest to us. However, for the sake of clarity, we have included high accretion rates in Figure 4. In this case we can see the well-known $S$-shaped stability curve.

Let us first discuss the top row. In the left panel, we have $l^2 = 2$ and in the right panel $l^2 = 3$. To see the response of the system to changes in $l$ and wind accretion rate, in both panels we keep $s$ constant ($s = 0.3$). Other relevant parameters are $\gamma = 1$ and $\Phi = 3 \times 10^{13}$. In this row, the dashed, solid, and dotted curves indicate $M_{w}/M_{\text{Edd}} = 0.05$, 0.1, and 0.15 respectively. In the bottom panels the dashed, thick, dotted, and dotted-dashed curves indicate $s = 0.2, 0.4, 0.6,$ and 10 respectively. The wind accretion rate is constant: $M_{w} = 0.1$.

![Figure 4](image_url)

**Figure 4.** Left panels: $l^2 = 2$; right panels: $l^2 = 3$. In the top panels $s$ is fixed as $s = 0.3$ and $\gamma = 1$ and $\Phi = 3 \times 10^{13}$; the dashed, solid, and dotted curves indicate $M_{w}/M_{\text{Edd}} = 0.05$, 0.1, and 0.15 respectively. In the bottom panels the dashed, thick, dotted, and dotted-dashed curves indicate $s = 0.2, 0.4, 0.6,$ and 10 respectively. The wind accretion rate is constant: $M_{w} = 0.1$.

Similarly, let us discuss the bottom row in Figure 4. In the left and right panels, we have $l^2 = 2$ and $l^2 = 3$ respectively. In this row, we are interested in checking the response of the disks to $s$. To do so, we keep the wind accretion rate as $M_{w} = 0.1$ and
vary the parameter $s$. We recall that other parameters are chosen as $\gamma = 1$ and $\Theta = 3 \times 10^{13}$. The dashed, thick, dotted, and dotted-dashed curves indicate $s = 0.2$, 0.4, 0.6, and 10. We see that when $s$ is small the stability of the system is not significantly sensitive to $s$ in both cases, i.e., $l^2 < 5/2$ and $l^2 > 5/2$. On the other hand, when $s$ is relatively large, we see in the bottom left panel that the parameter $s$ causes stabilizing effects. For example $s = 10$ stabilizes all the equilibrium solutions with $M_{\text{acq}} \lesssim 2M_{\text{edd}}$. Accordingly, one may conclude from the bottom right panel that $s$ has destabilizing effects. One should note that by increasing this parameter, the slope of the diagram becomes negative for a wider interval of $\Sigma$. This behavior was not clear in our previous figures dealing with the stability function $\psi$. We recall that $s$ is not completely independent of $M_{\text{acq}}$. However, its behavior is opposite to that of wind accretion rate.

There is another interesting feature in all the panels of Figure 4. It is clear that wind cannot influence the thermal stability of disks with high mass accretion rates. We see that the upper stable branch, i.e., $M_{\text{acq}} \gtrsim 10 M_{\text{edd}}$, in the panels does not change by varying the wind parameters. In this branch of solutions, advective cooling plays a key role and the radiation pressure gets very large. Both of these quantities have a stabilizing nature dominating the dynamics of the system. Therefore it is natural that in this case neither magnetic pressure nor wind has a serious impact on the system. On the other hand, as we already discussed, wind effects significantly influence the lower stable branch and the unstable interval.

As a final remark in this subsection, we have also explored the behavior of the special case $l = 1$. As already mentioned, in this case, $C_w = 0$. We found that a thin disk with $l = 1$ does not show any different behavior compared to $1 < l^2 < 5/2$. In other words, this type of wind with $l = 1$ destabilizes the disk.

5. Summary and Discussion

In this paper, we have studied the thermal stability of thin accretion disks. We present a full local stability analysis when the disk is magnetized and there is a wind mechanism in the system. We recall that the thermal stability of radiation-dominated thin disks has revealed some puzzles. For example, in the standard picture, a radiation-dominated thin disk is thermally unstable when the mass accretion rate is higher than a few percent of the Eddington rate. However, observations of X-ray binaries do not confirm this prediction. We have revised this issue by including the role of wind in the system. We found a criterion for the stability of the disk when both the magnetic field and wind exist in the disk; see Equation (34) ($\psi > 0$). We use the key assumption already used in Zheng et al. (2011), namely the magnetic field will become weaker with an increase of the scale height $H$ or temperature $T$. However, our focus is on the wind’s effects and we used a wind model already introduced in Knigge (1999). We showed that, depending on the type of wind, the disk can be stabilized or destabilized. In other words, when the wind parameter $l^2 < 5/2$ then the presence of wind makes the disk unstable. On the other hand, for $l^2 > 5/2$, the wind significantly stabilizes the disk. Interestingly, this stabilizing effect is completely in agreement with that presented in Li & Begelman (2014). In fact, Li & Begelman show that the critical accretion rate, corresponding to the thermal instability threshold, is significantly increased in the presence of magnetically driven disk winds. On the other hand, when $l > 1$ in the parametric wind model presented in Knigge (1999), then the model is suitable for describing magnetically driven disk winds. This means that our analysis also confirms the stabilizing role of the magnetically driven wind reported by Li & Begelman. In some sense, this consistency shows that Knigge’s parametric wind model is a viable one at least for the magnetically driven disk case. However, we should mention that the parametric approach presented here is an approximative procedure. Strictly speaking, real wind mechanisms may not follow a simple model as presented by Knigge (1999). As discussed by Knigge, a full dynamical model is required to obtain an accurate description. However, this simple approximative model is useful for investigating some main features of thin accretion disks. In fact, we have shown in this paper that Knigge’s parametric model is useful for investigating the thermal stability of disks. Another restriction is that we have ignored the direct effects of the outflow on the magnetic filed, e.g., via field line stretching. A full analysis removing the above-mentioned restrictions is beyond the scope of this paper and we leave it for future studies.

It is important to mention that, when there is no wind to stabilize the disk, a large contribution from the magnetic pressure compared to the total pressure is required ($\beta_{\text{mag}} > 0.2$). This contribution is larger than the typical value $\beta_{\text{mag}} \approx 1$ widely used in relevant simulations. Therefore, to stabilize the disk with normal values of magnetic pressure, one needs to assume $\gamma > 3$; see Zheng et al. (2011) for more details. This value gets even larger when the stability of the radiation-dominated thin disk simulated in Hirose et al. (2009) is considered. To explain the stability of this thin disk with magnetic field one needs $\gamma > 7$. These values for $\gamma$ seem uncomfortably large, as reported in Zheng et al. (2011).

However, when wind exists in the disk, we can achieve stability for any value of the mass accretion rate. In this case it is not required to assume a large value for $\beta_{\text{mag}}$ or $\gamma$. Of course, in this case, one needs to justify the existence of wind in thin disks.

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