New $B^\pm \to K\pi$ data explain absence of CP violation
Tree-penguin interference canceled by Pauli effects

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Abstract

Observation of CP violation in $B^o \to K^\pm \pi^\mp$ decays and its absence in $B^+ \to K^+\pi^0$ decays are explained in new improved data analysis of more precise $B \to K\pi$ data. Success of the “Lipkin Sum Rule” indicates that four $B \to K\pi$ branching ratios are determined by three parameters, the penguin diagram $P$ and two interference terms $P \cdot T$ and $P \cdot S$ between the dominant penguin and two tree diagrams; the color-favored and color suppressed diagrams. Previous analyzes confirmed the model with errors leaving values of interference terms less than two standard deviations from zero. The observation CP violation in $B^o \to K^\pm \pi^\mp$ decays indicates a finite value for $P \cdot T$. New precise data analysis show $P \cdot T$ and $P \cdot S$ interference contributions well above errors. Their contributions to $B^\pm \to K\pi$ decays are shown to be nearly equal with opposite phase and cancel within experimental errors. This cancelation unexpected in previous analyzes explains the failure to see CP violation in $B^\pm \to K\pi$ decays. It can be due to the Pauli antisymmetry exchange neglected in previous analyzes. Two $B^\pm \to K\pi$ tree diagrams differ by interchange of two identical $u$ quarks. $B^o \to K^{\pm} \pi^{\mp}$ diagrams have no identical quark pairs. This Pauli effect explains the difference produced by changing the flavor of the spectator quark which does not participate in the weak interaction. Our analysis differs from previous analyzes which assume SU(3) flavor symmetry to use input from $B \to \pi\pi$ data and neglect Pauli effects. We use new data, include Pauli effects and strong final state interactions to all orders in QCD with no higher flavor symmetry assumed beyond isospin. We do not use $B \to \pi\pi$ data.

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I. INTRODUCTION

A general theorem from CPT invariance shows [1] that direct CP violation can occur only via the interference between two amplitudes which have different weak phases and different strong phases. This holds also for all contributions from new physics beyond the standard model which conserve CPT.

Direct CP violation has been experimentally observed [2,3] in $B_d \to K^+\pi^-$ decays.

$$A_{CP}(B_d \to K^+\pi^-) = -0.098 \pm 0.013$$  

(1.1)

This CP violation has been attributed to the interference between the large contribution from the dominant penguin diagram and smaller contributions from tree diagrams. The failure to observe CP violation in charged decays [4] is has been considered to be a puzzle [5,6] because changing the flavor of a spectator quark which does not participate in the weak decay vertex is not expected to make a difference.

$$A_{CP}(B^+ \to K^o\pi^+) = 0.009 \pm 0.029$$

$$A_{CP}(B^+ \to K^+\pi^o) = 0.051 \pm 0.025$$  

(1.2)

We shall show here that the dependence on spectator flavor arises from the Pauli blocking by the spectator quark of a quark of the same flavor participating in the weak vertex. The quark produced by a tree diagram is a $u$-quark which is Pauli blocked by the spectator $u$ quark in $B^+$ decay but is not affected by the spectator $d$ quark in neutral decays. This difference in Pauli blocking suppresses the tree contribution and CP violation in charged $B$ decays but allows tree-penguin interference and enables CP violation to be observed in neutral decays.

II. THE CRUCIAL ROLE OF PAULI BLOCKING BY THE SPECTATOR QUARK

A. The difference between charged and neutral decays

The Pauli principle can forbid the tree-penguin interference and CP violation in charged $B$ decays and allow them in neutral decays. The decay of a $\bar{b}$ antiquark to a strange charmless final state is described by the vertex having the form

$$\bar{b} \to \bar{s}n\bar{n}$$  

(2.1)

where $n\bar{n}$ denotes a nonstrange $u\bar{u}$ or $d\bar{d}$. Pauli blocking suppresses the transition for a state $n$ which has the same flavor as the spectator quark. This suppression is lost in conventional treatments which consider color-favored and color-suppressed tree amplitudes as independent without considering Pauli suppression. We show below that it is just this Pauli selection rule forbidding a $u$ quark produced by a weak interaction to enter the same state as a $u$ spectator quark which suppresses the tree contribution and CP violation in charged $B$ decay.

As a simple approximation we neglect the color and spin degrees of freedom. Corrections from color-spin effects will be considered later. Let $u^\dagger$, $d^\dagger$ and $Q^\dagger$ denote creation operators...
respectively for a $u$ quark, a $d$ quark and a spectator quark denoted by $Q$ and let $\bar{b}\, \dagger\, , \, \bar{u}\, \dagger\, , \, \bar{d}\, \dagger$ and $\bar{Q}\, \dagger$ denote creation operators respectively for a $b$ antiquark, $\bar{u}$ antiquark, a $d$ antiquark and a spectator $\bar{Q}$ antiquark.

The transition from an initial $B$ meson state $\bar{b}\, \dagger\, Q\, |\, 0\rangle$ consisting of a $\bar{b}$ antiquark and nonstrange quark to a strange charmless two-meson final state is written

$$|B\rangle = \bar{b}\, \dagger\, Q\, |\, 0\rangle \rightarrow \bar{s}\, \dagger\, \left[ d\, \dagger\, d\, \dagger\, + \, u\, \dagger\, \bar{u}\, \dagger\, + \, \xi \cdot u\, \dagger\, \bar{u}\, \dagger \right] Q\, |\, 0\rangle \tag{2.2}$$

where the final state is written as the sum of an isoscalar $q\bar{q}$ pair and a $u\bar{u}$ pair together with a strange antiquark and a spectator quark. This is analogous to conventional description as the sum of a penguin contribution and a tree contribution with a parameter $\xi$ generally considered to be small expressing the ratio of the tree and penguin contributions. Substituting the two spectator flavors $u$ and $d$ then gives

$$\bar{b}\, \dagger\, u\, \dagger\, |\, 0\rangle \rightarrow \bar{s}\, \dagger\, \left[ d\, \dagger\, d\, \dagger\, + \, (1 + \xi)u\, \dagger\, \bar{u}\, \dagger \right] u\, \dagger\, |\, 0\rangle = \bar{s}\, \dagger\, \left[ K^o\pi^+ + \frac{K^+\pi^o}{\sqrt{2}} \right]$$

$$\bar{b}\, \dagger\, d\, \dagger\, |\, 0\rangle \rightarrow \bar{s}\, \dagger\, \left[ d\, \dagger\, d\, \dagger\, + \, (1 + \xi)u\, \dagger\, \bar{u}\, \dagger \right] d\, \dagger\, |\, 0\rangle = (1 + \xi)\bar{s}\, \dagger\, \left[ u\, \dagger\, \bar{u}\, \dagger \right] |\, 0\rangle = \left[ K^+\pi^- + \frac{K^o\pi^o}{\sqrt{2}} \right] \cdot (1 + \xi) \tag{2.3}$$

where we have noted that the products of two identical fermion creation operators $d\, \dagger\, d\, \dagger$ and $u\, \dagger\, u\, \dagger$ must vanish. The transitions for the neutral decays are seen to depend upon the parameter $\xi$ while the charged transitions are seen to be independent of the parameter $\xi$. The parameter $\xi$ is proportional to the strength of the tree amplitude. Thus tree-penguin interference which might produce CP violation is present in neutral decays and absent in charged decays. This explains how CP violation can be drastically changed by changing the spectator quark and the otherwise mysterious result (1.2).

**B. The Lipkin Sum Rule**

The transition matrices denoted by $T$ must satisfy the isospin constraints for the coupling of an isospin 1 meson to an isospin $\langle 1/2 \rangle$ kaon

$$\langle K^o\pi^o\rangle \, T \, |B^o\rangle = -\frac{1}{\sqrt{2}} \cdot \langle K^+\pi^-\rangle \, T \, |B^o\rangle \; ; \; \; \; \; 2 \cdot |\langle K^o\pi^o\rangle \, T \, |B^o\rangle|^2 = |\langle K^+\pi^-\rangle \, T \, |B^o\rangle|^2 \tag{2.4}$$

$$\langle K^+\pi^o\rangle \, T \, |B^o\rangle = -\frac{1}{\sqrt{2}} \cdot \langle K^o\pi^+\rangle \, T \, |B^o\rangle \; ; \; \; \; \; 2 \cdot |\langle K^+\pi^o\rangle \, T \, |B^o\rangle|^2 = |\langle K^o\pi^+\rangle \, T \, |B^o\rangle|^2 \tag{2.5}$$

Then

$$\frac{\langle K^+\pi^-\rangle \, T \, |B^o\rangle}{\langle K^o\pi^+\rangle \, T \, |B^o\rangle} = (1 + \xi) = \frac{\langle K^o\pi^o\rangle \, T \, |B^o\rangle}{\langle K^+\pi^o\rangle \, T \, |B^o\rangle} \tag{2.5}$$

$$2 \cdot |\langle K^+\pi^o\rangle \, T \, |B^o\rangle|^2 - |\langle K^o\pi^+\rangle \, T \, |B^o\rangle|^2 = 0 \right) = (1 + \xi)^2 \cdot \left[ |\langle K^+\pi^-\rangle \, T \, |B^o\rangle|^2 - 2 \cdot |\langle K^o\pi^o\rangle \, T \, |B^o\rangle|^2 \right] \tag{2.6}$$

this can be rewritten

$$2 \cdot \left[ |\langle K^+\pi^o\rangle \, T \, |B^o\rangle|^2 + |\langle K^o\pi^+\rangle \, T \, |B^o\rangle|^2 \right] = |\langle K^o\pi^+\rangle \, T \, |B^o\rangle|^2 + |\langle K^+\pi^-\rangle \, T \, |B^o\rangle|^2 \tag{2.7}$$
This relation (2.7) is seen to be identical to the approximate equality [7,8] called the “Lipkin sum rule” [4].

\[ R_L \equiv \frac{2\Gamma(B^+ \rightarrow K^+\pi^o) + \Gamma(B^o \rightarrow K^o\pi^o)}{\Gamma(B^+ \rightarrow K^o\pi^+) + \Gamma(B^o \rightarrow K^+\pi^-)} \approx 1 \quad (2.8) \]

III. THREE PARAMETERS DETERMINE FOUR \( B \rightarrow K\pi \) BRANCHING RATIOS

A. Two independent derivations of the sum rule

The sum rule (2.8) has been derived here using a completely different set of assumptions from the previous derivation [7–10]. Both derivations begin with the relation (2.2) The derivation of the relation (2.7) neglects the color and spin degrees of freedom but makes no further assumptions and includes Pauli blocking. The experimental observation (1.1) and the knowledge that the penguin amplitude is dominant for the decay [4] require that the decay amplitude must contain at least one additional amplitude with both weak and strong phases different from those of the penguin.

The standard treatment assumes that four \( B \rightarrow K\pi \) branching ratios are determined by three parameters, the penguin diagram \( P \) shown in Fig. 1 and two interference terms \( P \cdot T \) and \( P \cdot S \) between the dominant penguin and the color-favored and color suppressed tree diagrams shown in Figs. 2 and 3. This derivation assumes the difference between the two three contributions shown in Figs. 2 and 3 are independent and neglects Pauli blocking. It also assumes that the two tree amplitudes are sufficiently small to be treated in first order. Second order terms \( T \cdot T \), \( S \cdot T \) and \( S \cdot S \) are shown to be negligible. The agreement [4] with experiment [2,3] confirms either these assumptions [7–10] or the neglect of spin and color while including Pauli blocking.

B. The Difference rule

We now investigate what is observable in the experimental data, how to separate the signal from the noise, how to find the other amplitude and examine what can we learn about it from experiment. The sum rule (2.8) has been rearranged [10] to obtain a “difference rule”

\[ 2B(B^+ \rightarrow K^+\pi^o) - B(B^+ \rightarrow K^o\pi^+) \approx B(B^o \rightarrow K^+\pi^-) - 2B(B^o \rightarrow K^o\pi^o) \quad (3.1) \]

where for simplicity the result was expressed in terms of branching ratios, denote by \( B() \). Corrections for the difference between the \( B^+ \) and \( B^o \) lifetime ratio are included in the subsequent analysis

The difference rule (3.1) has real significance only if there is interference between the dominant penguin and another amplitude leading to an I=3/2 final state. It is trivially satisfied if the decays are described entirely by a pure penguin or other I=1/2 contribution where the final state is an isospin eigenstate with I=1/2. For a pure penguin transition both sides of the difference rule (3.1) vanish and the relation reduces to the trivial 0=0.

Four experimental branching ratios for \( B \rightarrow K\pi \) are available [2,3]. Three different independent differences between these branching ratios can be defined which eliminate the
penguin contribution. These overdetermine the two remaining free parameters in the theory, the interference contributions between the penguin amplitude and the two tree amplitudes. The next step in the analysis would have been to solve the equations and obtain the values of the two interference terms $P \cdot T$ and $P \cdot S$ from the experimental data. However the large experimental errors at the time left these values less that two standard deviations from zero [7,8].

C. New experimental data show statistically significant penguin-tree interference

New data now show that the $B \to K\pi$ transition is not a pure penguin. They give a finite experimental value for an expression which vanishes in a pure penguin transition.

$$\frac{\tau^o}{\tau^+} \cdot 2B(B^+ \to K^+\pi^o) - B(B^o \to K^+\pi^-) = 4.7 \pm 0.82$$

(3.2)

where $\tau^o/\tau^+$ denotes the ratio of the $B^o$ and $B^+$ lifetimes and we have used the experimental values

$$B(B^o \to K^+\pi^-) = 19.4 \pm 0.6$$

$$\frac{\tau^o}{\tau^+} \cdot B(B^+ \to K^o\pi^+) = \frac{(23.1 \pm 1.0)}{1.07} = 21.6 \pm 0.93$$

(3.3)

$$\frac{\tau^o}{\tau^+} \cdot 2B(B^+ \to K^+\pi^o) = \frac{2 \cdot (12.9 \pm 0.6)}{1.07} = 24.1 \pm 0.56$$

$$B(B^o \to K^o\pi^o) = (9.4 \pm 0.6)$$

D. A surprising cancelation suggests Pauli effects $P \cdot (T + S) \approx 0$

We now note that the right hand side of eq.(3.1) gives

$$B(B^o \to K^+\pi^-) - 2B(B^o \to K^o\pi^o) = (19.4 \pm 0.6) - 2 \cdot (9.4 \pm 0.6) = 0.6 \pm 1.3$$

(3.4)

The expression (3.4) also vanishes in the case of a pure penguin transition. The significant difference between the experimental values of expressions (3.4) and (3.2) which both vanish in the case of a pure penguin transition seems to indicate a surprising cancelation and motivates a search for a theoretical explanation.

We first note that the two transitions (3.4) which have a d-quark spectator are seen to be described respectively by the color-favored and color-suppressed tree diagrams shown respectively in figs. 2 and 3 in addition to the dominant common penguin diagram shown in fig. 1. This cancelation between the contributions of the two tree diagrams is surprising because the standard treatments assume that the these two tree contributions are completely independent and are not expected to cancel. A further analysis of the new data [2,3] isolates the color-favored and color-suppressed contributions, and minimizes the importance of experimental errors.
\[ \vec{P} \cdot (\vec{T} + \vec{S}) = \frac{2B(B^o \to K^o\pi^o) - B(B^o \to K^+\pi^-)}{\vec{P} \cdot (\vec{T} - \vec{S})} = [\tau^o/\tau^+] \cdot [B(B^+ \to K^o\pi^+) + 2B(B^+ \to K^+\pi^0)] - 2B(B^o \to K^+\pi^-) = 0.09 \pm 0.1 \]  

(3.5)

We now note that the Pauli principle neglected in conventional treatments can produce this cancelation. The amplitudes \( T \) and \( S \) go into one another under the interchange of the two identical \( u \) quarks in \( A[K^+\pi^o] \). A full examination of Pauli effects requires antisymmetrization of the \( uu \) wave function including the color and spin correlations. As a first approximation we neglect color and spin. Then Pauli antisymmetry requires \( T \) and \( S \) amplitudes to be equal and opposite and explains the cancelation (3.5). Both \( B^+ \) decays are then pure penguin decays to the \( I = 1/2 \) \( K\pi \) state. Experiment [3] shows agreement with this prediction to between one and two standard deviations.

\[ 2B(B^+ \to K^+\pi^o) = 25.8 \pm 1.2 \approx B(B^+ \to K^o\pi^+) = 23.1 \pm 1.0 \]  

(3.6)

where \( B \) denotes the branching ratio in units of \( 10^{-6} \).

The data are now sufficiently precise to show that the interference terms between the dominant penguin amplitude and the two tree amplitudes are both individually finite and one is well above the experimental errors. The sum rule is satisfied and is now nontrivial. But the interference term \( \vec{P} \cdot (\vec{T} + \vec{S}) \) is now equal to zero well within the experimental errors (3.5). This confirms the Pauli symmetry prediction (3.6) with smaller experimental errors.

Thus tree-penguin interference with normally ignored Pauli effects can explain the observed CP violation in charged B-decays and its absence in neutral decays.

This shows how a nontrivial change in the weak decay amplitude can arise from a change of the flavor of the spectator quark.

E. A new analysis of the data pinpointing tree-penguin interference

We now show how a detailed conventional analysis of new experimental data with no new theory leads to the result (3.5). We later show how this cancelation can result from the Pauli principle.

The conventional analysis expresses the four \( B \to K\pi \) amplitudes in terms of the three amplitudes \( P \), \( T \), and \( S \) denoting respectively the penguin, color favored tree and color suppressed tree amplitudes while neglecting other contributions at this stage [7–10]

\[
A[K^o\pi^+] = P; \quad A[K^+\pi^-] = T + P \\
A[K^o\pi^o] = \frac{1}{\sqrt{2}}[S - P]; \quad A[K^+\pi^o] = \frac{1}{\sqrt{2}}[T + S + P] 
\]  

(3.7)

When the interference terms are taken only to first order,

\[
|A[K^o\pi^+]|^2 = |\vec{P}|^2; \quad |A[K^+\pi^-]|^2 = |\vec{P}|^2 + 2\vec{P} \cdot \vec{T} \\
2 \cdot |A[K^o\pi^o]|^2 = |\vec{P}|^2 - \vec{P} \cdot \vec{S}; \quad 2 \cdot |A[K^+\pi^o]|^2 = |\vec{P}|^2 + 2\vec{P} \cdot (\vec{T} + \vec{S}) 
\]  

(3.8)
where the approximate equalities hold to first order in the $T$ and $S$ amplitudes.

We now get a more sensitive experimental test by using all the $B \to K \pi$ data.

We use new data and define new differences which optimize the signal to noise ratio. Noting that the branching ratio $B(B^o \to K^+\pi^-)$ has the smallest experimental error, we define three independent differences which vanish for a pure penguin transition and are chosen to have the smallest experimental errors.

$$\Delta(K^o\pi^+) \equiv |A[K^o\pi^+]|^2 - |A[K^+\pi^-]|^2 \approx -2 \vec{P} \cdot \vec{T}$$

$$\Delta(K^+\pi^o) \equiv 2|A[K^+\pi^o]|^2 - |A[K^+\pi^-]|^2 \approx 2 \vec{P} \cdot \vec{S}$$

$$\Delta(K^o\pi^o) \equiv 2|A[K^o\pi^o]|^2 - |A[K^+\pi^-]|^2 \approx -2 \vec{P} \cdot (\vec{T} + \vec{S})$$

where the approximate equalities hold to first order in the $T$ and $S$ amplitudes. The isospin sum rule [7,8] is easily expressed in terms of these differences.

$$\Delta(K^o\pi^o) + \Delta(K^+\pi^o) - \Delta(K^o\pi^+) \approx 0$$

Since each of the three terms in eq. (3.10) vanish for a pure penguin transition, the sum rule is trivially satisfied in this case. We improve the previous analysis [10] that only showed the sum rule trivially satisfied with real data and all terms proportional to tree-penguin interference were still statistically consistent with zero.

These individual differences are now sufficiently different from zero with available experimental branching ratio data corrected for the lifetime ratio [2,3]

$$\frac{\tau^o}{\tau^+} \cdot B(B^+ \to K^o\pi^+) - B(B^o \to K^+\pi^-) = 2.2 \pm 1.1 \propto -\vec{P} \cdot \vec{T}$$

$$\frac{\tau^o}{\tau^+} \cdot 2B(B^+ \to K^+\pi^o) - B(B^o \to K^+\pi^-) = 4.7 \pm 0.82 \propto \vec{P} \cdot \vec{S}$$

$$2B(B^o \to K^o\pi^o) - B(B^o \to K^+\pi^-) = -0.6 \pm 1.3 \propto -\vec{P} \cdot (\vec{T} + \vec{S})$$

Combining these equations gives the relation (3.5).

There is no new theory here. Choosing three independent differences in a way to minimize experimental errors shows two significant signals well above the noise of experimental errors that still fit an overdetermination of the two parameters and lead to the result (3.5). These show two finite tree-penguin interference contributions that can produce the observed direct CP violation in neutral B-decays. However the third difference is consistent with zero well below the noise and below the other two contributions. The absence of tree-penguin contributions in this difference is completely unpredicted in the standard treatments.

IV. SYMMETRY ARGUMENTS SUPPORTING THE VANISHING OF $\vec{P} \cdot (\vec{T} + \vec{S})$

A. The $u\bar{u}s$ tree diagrams for $B \to K \pi$ decays

In tree diagrams for charmless strange $B$ decays, the $\bar{b} \to u\bar{u}s$ transition produces a four-quark state $u\bar{u}s$ where $n$ denotes the nonstrange spectator quark.
The $K^+\pi^-$ final state $(us)(n\bar{u})$ can be produced by a $B^o$ tree diagram in which the spectator quark $n$ is a $d$ quark and combines with the $\bar{u}$ in a color-favored transition shown in Fig. 2. The CP violation observed in this state indicates that is produced by appreciable $P \cdot T$ interference.

The $K^+\pi^o$ final state $(u\bar{s})(n\bar{u})$ or $(n\bar{s})(u\bar{u})$ can be produced by a $B^+$ tree diagram in which the spectator quark $n$ is a $u$ quark and combines with either the $\bar{u}$ in a color-favored transition shown in Fig. 2 or the $\bar{s}$ in a color-suppressed transition shown in Fig. 3. The failure to observe CP violation in this state while CP violation is observed in the $K^+\pi^-$ final state indicates that both $P \cdot T$ and $P \cdot S$ interference contributions are appreciable and their interference contributions have opposite phase and tend to cancel any CP violation.

The experimental data for these two transitions thus present predictions for the following two final states.

The $K^0\pi^0$ final state $(n\bar{s})(n\bar{u})$ can be produced by a $B^o$ tree diagram in which the spectator quark $n$ is a $d$ quark and combines with the $\bar{s}$ in a color-suppressed transition shown in Fig. 3. This transition is produced by $P \cdot S$ interference which is expected to be similar to the $P \cdot T$ interference contribution and produce a similar CP violation to that observed in the $K^+\pi^-$ final state.

The $K^0\pi^+$ final state $(d\bar{s})(u\bar{d})$ contains a $\bar{d}$ antiquark and cannot be produced by a tree diagram leading to a $unu\bar{s}$ state. The prediction for the transition to this final state is that it has no tree contribution, no penguin-tree interference and no CP violation.

B. Experimental results suggest underlying symmetry

Experimental results for $B^o$ decays now show finite penguin-tree interference and a possibility of CP violation. Results for $B^{\pm}$ decays show negligible penguin-tree interference and little possibility of CP violation.

We now show how these results follow from symmetry conditions.

In charged $B$ decays the spectator quark is a $u$ quark and the $un$ pair has s unique isospin $I = 1$. The two identical fermions must obey Pauli antisymmetry. In neutral $B$ decays the spectator quark is a $d$ quark, the $un$ pair is a mixture of two isospins $I = 1$ and $I = 0$ and has no Pauli constraints.

A full analysis of symmetry and Pauli effects must include color-spin correlations. The state of two $u$ quarks in a relative $S$ wave which are symmetric in space and flavor must be antisymmetric in color and spin. A group-theoretical treatment of this symmetry involves the color-spin SU(6) group in which all pseudoscalar mesons are color-spin singlets, while the $uu$ diquark in a relative $s$-wave is classified in the antisymmetric 15 dimensional representation.

C. Isospin Analysis

We first examine Pauli effects in the present data using only isospin and see how these produce a selection rule that cancels the tree contribution to $B^+ \to K^+\pi^o$.

The tree diagram for $B^+ \to K^+\pi^o$ has a four-body $u\bar{s}u\bar{u}$ state containing a $u$ spectator quark and the $\bar{u}\bar{s}$ produced by the $\bar{b}$ antiquark weak decay. This state contains two $u$ quarks with isospin $I = 1, I_z = +1$. The $\bar{u}$ antiquark is in a well defined isospin state with
\( I = 1/2, I_z = -(1/2) \) and the strange antiquark has isospin zero. The total four-body state is thus a state with well defined isospin. It is a definite mixture of two eigenstates of the total four-body isospin with \( I = 1/2 \) and \( I = 3/2 \) with unique relative magnitudes and phases with determined by isospin Clebsch-Gordan coefficients for coupling two states with \( I = 1 \) and \( I = 1/2 \) to \( I = 1/2 \) and \( I = 3/2 \).

\[
|i; u\bar{s}u\bar{u}\rangle \propto \left| \frac{1}{2}; \frac{1}{2} \right\rangle \left\langle \frac{1}{2}| \frac{1}{2}(-\frac{1}{2}) \right| \frac{1}{2}; \frac{1}{2}; \frac{1}{2} + \frac{3}{2}; \frac{1}{2} \right\rangle \left\langle \frac{1}{2}| \frac{1}{2}(-\frac{1}{2}) \right| \frac{1}{2}; \frac{1}{2}; \frac{1}{2} \right\rangle \tag{4.1}
\]

where \( \langle j_1 j_2 m_1 m_2 | j_1 j_2 JM \rangle \) denotes a Clebsch-Gordan coefficient.

The \( K\pi \) final states are also linear combinations of states with isospin \( (1/2) \) and isospin \( (3/2) \) with relative amplitudes and phases determined completely by the requirement that the pion has isospin one and the kaon has isospin \( (1/2) \).

\[
|f; \pi^o K^+ \rangle \propto \left| \frac{1}{2}; \frac{1}{2} \right\rangle \left\langle \frac{1}{2}| \frac{1}{2}(\frac{1}{2}) \right| \frac{1}{2}; \frac{1}{2}; \frac{1}{2} + \frac{3}{2}; \frac{1}{2} \right\rangle \left\langle \frac{1}{2}| \frac{1}{2}(\frac{1}{2}) \right| \frac{1}{2}; \frac{1}{2}; \frac{1}{2} \right\rangle \tag{4.2}
\]

From the orthogonality relation for Clebsch-Gordan coefficients

\[
\langle f; \pi^o K^+ | i; u\bar{s}u\bar{u} \rangle = 0. \tag{4.3}
\]

There is therefore no overlap between the initial state \( |i; u\bar{s}u\bar{u}\rangle \) produced by the weak interaction tree diagram and the \( K^+\pi^o \) final state. The relevant Clebsch-Gordan coefficients for the \( K^+\pi^o \) state are seen to be just those to make this linear combination exactly orthogonal to the combination in the \( u\bar{s}u\bar{u} \) state produced in the tree diagram by the \( \bar{b} \) decay. The overlap between the \( K^+\pi^o \) state and the initial state thus vanishes. In the “fall-apart” [13] model commonly used in tetraquark decays this vanishing overlap indicates that the tree diagram contribution to \( B^+ \rightarrow K^+\pi^o \) transition vanishes.

The transition is therefore forbidden if the four-body state that fragments into a kaon and a pion \( |u\bar{s}u\bar{u}\rangle \) retains the \( u\bar{s}u\bar{u} \) constituents with the \( uu \) pair coupled to \( I=1 \). This is the same as the original four-body state created in the tree diagram by the \( \bar{b} \) antiquark decay. This is true in the simple “fall-apart” [13] model. The experimental data seem to indicate that the tree diagram for this transition is indeed forbidden here.

The only way that the initial \( |u\bar{s}u\bar{u}\rangle \) state can change by a strong interaction conserving isospin is annihilating the \( u\bar{u} \) pair and creating a \( d\bar{d} \) pair. This adds a \( |u\bar{s}d\bar{d}\rangle \) component to the final state. This interaction is included below in a general treatment including color-spin correlations.

We now note that changing the flavor of the spectator quark makes a crucial difference in the isospin analysis. The isospin of the two quarks \( (u, d) \) in the corresponding tree diagram for the neutral \( B \) decays is not unique, it is a combination of \( I = 0 \) and \( I = 1 \). Thus there is no isospin nor Pauli constraint here and no selection rule forbidding the tree contribution.

D. Detailed symmetry and Pauli analysis

We now examine the effect of symmetry restrictions from the Pauli principle on the fragmentation of a \( uu\bar{s}\bar{u} \) state into a \( K^+\pi^o \) color singlet state with spin zero and no orbital angular momentum. Explicitly writing wave functions and imposing Pauli antisymmetry correlation.
requires the full color-spin-flavor SU(12) group. Since the $uu$ state is flavor symmetric it must be antisymmetric in color and spin if it is in a spatially symmetric S-wave. The fragmentation process is a strong interaction which conserves flavor SU(3) and charge conjugation. Since the initial state contains no $d$ quarks we can simplify the symmetry restriction by considering the $V$-spin subgroup of SU(3) which acts in the $u - s$ flavor space. This state contains a $uu$ pair required by the Pauli principle to be antisymmetric in color and spin. It is either in a color antitriplet with spin $S = 1$ or in a color sextet with spin $S = 0$. The $\bar{u}s$ pair must also be either in a color antitriplet with spin $S = 1$ or in a color sextet with spin $S = 0$. Although no Pauli principle forbids it from being in a color antitriplet with spin $S = 0$ or in a color sextet with spin $S = 1$ these states cannot combine with the $uu$ pair to make the spin-zero color singlet final state.

Both the $uu$ diquark and the $\bar{u}s$ antidiquark are thus antisymmetric in color and spin. The generalized Pauli principle requires each to be symmetric in flavor SU(3) and its SU(2) subgroup $V$-spin. Each is therefore in the symmetric $V$-spin state with $V = 1$. A final state must be even under general charge conjugation to decay into two pseudoscalar mesons in an orbital S wave. Thus the $(V = 1, V_z = +1)$ diquark and the $(V = 1, V_z = 0)$ antidiquark must be coupled symmetrically to $(V = 2, V_z = +1)$. This state is in the 27-dimensional representation of flavor SU(3). The $V$ spin analysis of the initial state $|\bar{i}; uud\bar{s}\rangle$ is

$$\langle i; uud\bar{s}| V = 2; V_z = 1 \rangle = \frac{1}{2}$$

A final $K^0\pi^+$ state $|f; \pi^0K^+\rangle$ has no $V = 2$ component, since both the $K^0$ and $\pi^+$ have $V = 1/2$. Thus the tree diagram for the $K^0\pi^+$ decay must vanish and this decay is pure penguin, in agreement with the isospin analysis (3.7).

The final $K^+\pi^0$ state contains a $\pi^0$ which is a linear combination of a $u\bar{u}$ pair and a $d\bar{d}$ pair. Since the $d$ quark has $V = 0$ the $d\bar{d}$ pair has $V = 0$ and cannot combine with a $V = 1$ $K^+$ to make $V = 2$. The $u\bar{u}$ pair is an equal mixture of $V = 0$ and $V = 1$. Only the $V = 1$ component can combine with a $V = 1$ to make $V = 2$. Thus the $V$-spin analysis of the final $K^0\pi^+$ state $|f; \pi^0K^+\rangle$ and its overlap with the initial state are

$$\langle K^+\pi^0| V = 2; V_z = 1 \rangle = \frac{1}{4}$$

$$\langle K^+\pi^0| i; uud\bar{s} \rangle = \frac{1}{8}$$

We thus see that the tree diagram for transition $B^+ \rightarrow K^+\pi^0$ state vanishes in the “fall-apart” model where the initial $u\bar{u}$ pair does not have enough time to annihilate and create a $d\bar{d}$ pair. When there is time for the $u\bar{u}$ pair annihilation and the creation a $d\bar{d}$ pair, the tree diagram does not vanish but is suppressed by a significant factor. Present data are consistent with complete suppression but evidence for a partial suppression is still down in the noise.

The $u\bar{d}s\bar{s}$ state created in the tree diagram for $B_d$ decay has no such restrictions. It can be in a flavor SU(3) octet as well as a 27. Its “diquark-antidiquark” configuration includes the
flavor-SU(3) octet constructed from the spin-zero color-antitriplet flavor-antitriplet “good” diquark found in the Λ baryon and its conjugate “good” antidiquark. These “good diquarks” do not exist in the corresponding $uu\bar{s}$ configuration.

We again see that the Pauli effects produce a drastic dependence on spectator quark flavor in the tree diagrams for $B \to K\pi$ decays.

Thus tree-penguin interference can explain both the presence of CP violation in neutral decays and its absence in charged decays.

V. A FLAVOR TOPOLOGY ANALYSIS WHICH INCLUDES FINAL STATE INTERACTIONS

The unique flavor topology of the charmless strange quasi-two-body weak B decays enables the results (3.11) to be obtained in a more general analysis of these decays including almost all possible diagrams including final state interactions and complicated multiparticle intermediate states.

Consider diagrams for a charmless $B(\bar{b}qs)$ decay into one strange and one nonstrange meson, where $q_s$ denotes either a $u$ or $d$. The allowed final states must have the quark constituents $\bar{s}n\bar{q}_s$ where $n$ denotes a $u$ or $d$ nonstrange quark. We consider the topologies of all possible diagrams in which a $\bar{b}$ antiquark and a nonstrange quark enter a black box from which two final $q\bar{q}$ pairs emerge. We follow the quark lines of the four final state particles through the diagram going backward and forward in time until they reach either the initial state or a vertex where they are created. There are only two possible quark-line topologies for these diagrams:

1. We call a generalized penguin diagram, shown in Fig. 1, the sum of all possible diagrams in which a $q\bar{q}$ pair appearing in the final state is created by a gluon somewhere in the diagram. The quark lines for the remaining pair must go back to the weak vertex or the initial state. This diagram includes not only the normally called penguin diagram but all other diagrams in which the final pair is created by gluons somewhere in the diagram. This includes for example all diagrams normally called “tree diagrams” in which an outgoing $u\bar{u}$ or $c\bar{c}$ pair is annihilated into gluons in a final state interaction and a new isoscalar $q\bar{q}$ pair is created by the gluons. There are two topologies for penguin diagrams:
   - A normal penguin diagram has a the spectator quark line continuing unbroken from the initial state to the final state. This penguin contribution is described by a single parameter, denoted by $P$ which is independent of the spectator quark flavor and contributes equally to the $\bar{s}u\bar{u}q_s$ and $\bar{s}d\bar{d}q_s$ states.
   - A diagram which we call here an “anomalous penguin” has the spectator “$u$” quark in a $B^+$ decay annihilated in a final state interaction against the $\bar{u}$ antiquark produced in a tree diagram. This diagram also contributes equally to the $\bar{s}u\bar{u}q_s$ and $\bar{s}d\bar{d}q_s$ states. But this diagram denoted by $P_u$ is present only in charged decays.

2. We call the “tree diagram” the sum of all possible diagrams in which all of the four quark lines leading to the final state go back to a initial $\bar{s}u\bar{u}$ state created by the weak
decay of the $b$ quark and the $q_s$ spectator whose line goes back to the initial state. There are two possible couplings of the pairs to create final two meson states from this diagram

- The $\bar{s}u$ pair is coupled to make a strange meson; the $\bar{u}q_s$ pair is coupled to make a nonstrange meson as shown in Fig.2. This is conventionally called the color-favored coupling. The contribution of this coupling is described by a single parameter, denoted by $T$.
- The $\bar{s}q_s$ pair is coupled to make a strange meson; the $u\bar{u}$ is coupled to make a nonstrange meson as shown in Fig. 3. This is conventionally called the color-suppressed coupling. The contribution of this coupling is described by a single parameter, denoted by $S$.

All the results (3.11) obtained with the conventional definitions of $P$, $T$ and $S$ are seen to hold here with the new definitions of $P$, $T$ and $S$. They now include contributions from all final state interactions which conserve isospin and do not change quark flavor. The one final state interaction not included is the $P_u$ diagram occurring in $B^+$ decays. The flavor topology of this diagram creates an additional $I = 1/2$ state which is neglected in the derivation of the results (3.11). These results hold as long as the contribution of this $P_u$ diagram by final state interactions to the observed final states is negligible. That this additional $I = 1/2$ contribution does not affect the sum rule (2.8) is easily seen by its representation as a “difference rule” (3.1) which considers only the $I = 3/2$ contributions.

In neutral $B_d$ decays there is no $P_u$ diagram. Thus the simple relations (3.11) between the $P$, $T$ and $S$ amplitudes hold for neutral decays are not changed by isospin conserving final state interactions.

Further analysis of the contribution of this additional $I = 1/2$ contribution to final state interactions is needed to obtain definite values for CP violation in charged $B$ decay.

VI. COMPARISON WITH OTHER APPROACHES

Previous analyses [5,6] were performed at a time when experimental values for $B \rightarrow K\pi$ branching ratios were not sufficiently precise to enable a significant test of the sum rule (3.10). Values of each of the three interference terms in (3.11) were statistically consistent with zero. The full analysis required the use of data from $B \rightarrow \pi\pi$ decays and the assumption of $SU(3)_{\text{flavor}}$ symmetry. Contributions of the electromagnetic penguin diagram were included and the relevant CKM matrix elements were included. But there was no inclusion of constraints from the Pauli principle nor contributions from final state interactions.

The present analysis uses new experimental data which enable a statistically significant evaluation of the interference terms (3.11) without additional information from $B \rightarrow \pi\pi$ decays or the assumption of $SU(3)_{\text{flavor}}$ symmetry. Contributions from all isospin invariant finite state interactions are included as well as constraints from the Pauli principle. The flavor topology definition of the interference parameters includes contributions from the electromagnetic penguin diagram since the quark states in final state of a photon can be rewritten as the sum of an isoscalar and a $u\bar{u}$ state. However the flavor topology parameters
are no longer simply related to the CKM matrix elements. Additional assumptions and information are necessary to determine the CKM matrix elements and explain CP violation. The main advantage of this approach is that it gives a simple explanation for the experimental value (3.5) and the vanishing of the experimental value

$$\frac{2B(B^o \to K^o\pi^o) - B(B^o \to K^+\pi^-)}{[\tau^o/\tau^+] \cdot [B(B^+ \to K^o\pi^+) + 2B(B^+ \to K^+\pi^0)] - 2B(B^o \to K^+\pi^-)} = 0.09 \pm 0.1$$ (6.1)

This vanishing of tree-penguin interference $B^+$ decays is explained by a symmetry analysis including the constraints of the Pauli principle on states containing a pair of identical $u$ quarks.

**VII. CONCLUSION**

Experiment has shown that the penguin-tree interference contribution in $B^+ \to K^+\pi^o$ decay is very small and may even vanish. The corresponding interference contributions to neutral $B \to K\pi$ decays have been shown experimentally to be finite. In charged decays the previously neglected Pauli antisymmetrization produces a cancelation between color-favored and color-suppressed tree diagrams which differ by the exchange of identical $u$ quarks. This explains the smallness of penguin-tree interference and small CP violation in charged $B$ decays. Pauli cancelation does not occur in neutral decay diagrams which have no pair of identical quarks. This can explain why CP violation has been observed in neutral $B \to K\pi$ decays and not in charged decays.
FIGURE 1.
“Gluonic penguin” (P) diagram. \( G \) denotes any number of gluons. \( n \) denotes \( u \) or \( d \) quark.

FIGURE 2.
Color favored tree (T) diagram.

FIGURE 3.
Color suppressed tree (S) diagram.

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REFERENCES

[1] Harry J. Lipkin, in Proceedings of the International Workshop on B-Factories; Accelerators and Experiments, BFWS92, KEK, Tsukuba, Japan November 17-20, 1992, edited by E. Kikutani and T. Matsuda, Published as KEK Proceedings 93-7, June 1993, p.8; Phys. Lett. B357, (1995) 404
[2] K. Nakamura et al. (Particle Data Group), J. Phys. G 37, 075021 (2010).
[3] Heavy Flavor Averaging Group http://www.slac.stanford.edu/xorg/hfag/rare/
[4] Ahmed Ali, Invited Plenary Talk at the 32nd International Conference on High Energy Physics (ICHEP’04), Beijing, China, Aug. 16-22, 2004, hep-ph/0412128
[5] Michael Gronau and Jonathan L. Rosner, hep-ph/0608040, Phys.Rev.D74:057503,2006
[6] Michael Gronau, Jonathan L. Rosner and David London, Phys. Rev. Lett. 73 (1994) 21; Michael Gronau, Oscar F. Hernandez, David London and Jonathan L. Rosner, Phys. Rev. D52 (1995) 6356 and 6374
[7] Harry J. Lipkin, hep-ph/9810351, Physics Letters B445 (1999) 403
[8] Michael Gronau and Jonathan L. Rosner, hep-ph/9809384, Phys. Rev. D 59 (1999) 113002
[9] Michael Gronau, hep-ph/0008292, Phys. Lett. B492, (2000) 297
[10] Harry J. Lipkin, hep-ph/0507225
[11] Michael Gronau and Jonathan L. Rosner, hep-ph/0003119, Phys. Lett. B482 (2000) 71
[12] Harry J. Lipkin, hep-ph/9710342, Phys. Lett. B415 (1997) 186
[13] T. Barnes, hep-ph/0608175
[14] Marcello A. Giorgi, From BABAR to the future, lectures at the 44th Course, International School of Subnuclear Physics 2006, “Ettore Majorana” Centre for Scientific Culture, Erice, Italy
[15] A. Buras, R. Fleischer et al, Phys J C (2006) 701