AdS$_6$/CFT$_5$ correspondence for $F(4)$ Supergravity

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Abstract: $F(4)$ supergravity, the gauge theory of the exceptional six-dimensional Anti-de Sitter superalgebra, is coupled to an arbitrary number of vector multiplets whose scalar components parametrize the quaternionic manifold $SO(4,n)/SO(4) \times SO(n)$. By gauging the compact subgroup $SU(2)_d \otimes \mathcal{G}$, where $SU(2)_d$ is the diagonal subgroup of $SO(4) \simeq SU(2)_L \otimes SU(2)_R$ (the $R$-symmetry group of six-dimensional Poincaré supergravity) and $\mathcal{G}$ is a compact group such that $\text{dim} \mathcal{G} = n$. The potential admits an AdS background for $g = 3m$, as the pure $F(4)$-supergravity. The boundary $F(4)$ superconformal fields are realized in terms of a singleton superfield (hypermultiplet) in harmonic superspace with flag manifold $SU(2)/U(1) = S^2$. We analyze the spectrum of short representations in terms of superconformal primaries and predict general features of the K-K spectrum of massive type IIA supergravity compactified on warped AdS$_6 \otimes S^4$.

1 Matter coupled $F(4)$ Supergravity and its scalar potential

Let us set up a suitable framework for the discussion of the matter coupled $F(4)$ supergravity theory and its gauging. This will allow us to set up the formalism for the matter coupling in the next section. Actually we will just give the essential definitions of the Bianchi identities approach in superspace, while all the relevant results, specifically the supersymmetry transformation laws of the fields, will be given in the ordinary space-time formalism.

First of all it is useful to discuss the main results of ref. [3] by a careful study in superspace of the Poincaré and AdS supersymmetric vacua. Let us recall the content of $D = 6, N = (1,1)$ supergravity multiplet:

$$ (V^a_\mu, A_\mu^a, B_{\mu\nu}, \psi_\mu^A, \psi^{\dot{A}}_\mu, \chi^A, \chi^{\dot{A}}, e^a) \quad (1.1) $$

where $V^a_\mu$ is the six-dimensional vielbein, $\psi_\mu^A, \psi^{\dot{A}}_\mu$ are left-handed and right-handed four-component gravitino fields respectively, $A$ and $\dot{A}$ transforming under the two factors of

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the $R$-symmetry group $O(4) \simeq SU(2)_L \otimes SU(2)_R$, $B_{\mu\nu}$ is a 2-form, $A_\mu^a (\alpha = 0, 1, 2, 3)$, are vector fields, $\chi^A, \bar{\chi}^\dot{A}$ are left-handed and right-handed spin $\frac{1}{2}$ four components dilatinos, and $\epsilon^\sigma$ denotes the dilaton.

Our notations are as follows: $a, b, \ldots = 0, 1, 2, 3, 4, 5$ are Lorentz flat indices in $D = 6$ $\mu, \nu, \ldots = 0, 1, 2, 3, 4, 5$ are the corresponding world indices, $A, \dot{A} = 1, 2$. Moreover our metric is $+(+, -, -, -, -, -)$.

We recall that the description of the spinors of the multiplet in terms of left-handed and right-handed projection holds only in a Poincaré background, while in an AdS background the chiral projection cannot be defined and we are bounded to use 8-dimensional pseudo-Majorana spinors. In this case the $R$-symmetry group reduces to the $SU(2)$ subgroup of $SU(2)_L \otimes SU(2)_R$, the $R$-symmetry group of the chiral spinors. For our purposes, it is convenient to use from the very beginning 8-dimensional pseudo-Majorana spinors even in a Poincaré framework, since we are going to discuss in a unique setting both Poincaré and AdS vacua.

The pseudo-Majorana condition on the gravitino 1-forms is as follows:

$$(\psi_A)^{\dagger} \gamma^0 = (\bar{\psi}_A) = \epsilon^{AB} \psi_B$$

where we have chosen the charge conjugation matrix in six dimensions as the identity matrix (an analogous definition six dimensions as the identity matrix (an analogous definition holds for the dilatino fields). We use eight dimensional antisymmetric gamma matrices, with $(\gamma^\gamma)^2 = -1$. The indices $A, B, \ldots = 1, 2$, of the spinor fields $\psi_A, \gamma_A$ transform in the fundamental of the diagonal subgroup $SU(2)$ of $SU(2)_L \otimes SU(2)_R$. To study the supersymmetric vacua let us write down the Maurer-Cartan Equations (M.C.E.) dual to the $F(4)$ Superalgebra (anti)commutators:

$$DV^a - \frac{i}{2} \bar{\psi}_A \gamma_a \psi^A = 0$$

$$R^{ab} + 4m^2 \ V^a V^b + m \bar{\psi}_A \gamma_{ab} \psi^A = 0$$

$$dA^r + \frac{1}{2} \ g \ \epsilon^{rst} A_s A_t - i \ (\bar{\psi}_A \psi_B) \ \sigma^{rAB} = 0$$

$$D\psi_A - im\gamma_a \psi_A V^a = 0$$

Here $V^a, \omega^{ab}, \psi_A, A^r, (r = 1, 2, 3)$, are superfield 1-forms dual to the $F(4)$ supergenerators which at $\theta = 0$ have as $dx^\mu$ components

$$V^a_\mu = \delta^a_\mu, \ \psi_{A\mu} = A^r_\mu = 0, \ \omega^{ab} = \text{pure gauge.}$$

Furthermore $R^{ab} \equiv d\omega^{ab} - \omega^{ac} \wedge \omega^{cb}$, $D$ is the Lorentz covariant derivative, $D$ is the $SO(1, 5) \otimes SU(2)$ covariant derivative, which on spinors acts as follows:

$$D\psi_A \equiv d\psi_A - \frac{1}{4} \gamma_{ab} \omega^{ab} \psi_A - \frac{i}{2} \sigma_{AB} A_r \psi^B$$

Note that $\sigma^{rAB} = \epsilon^{BC} \sigma^A_C$, where $\sigma^{rA}_B$ $(r = 1, 2, 3)$ denote the usual Pauli matrices, are symmetric in $A, B$.

Let us point out that the $F(4)$ superalgebra, despite the presence of two different physical parameters, the $SU(2)$ gauge coupling constant $g$ and the inverse AdS radius $m$, really depends on just one parameter since the closure under $d$-differentiation of eq. (1.6) (equivalent to the implementation of Jacobi identities on the generators), implies $g = 3m$; to recover this result one has to use the following Fierz identity involving 3-$\psi$'s 1-forms:

$$\frac{1}{4} \gamma_{ab} \psi_A \bar{\psi}_B \gamma^a \psi_A \epsilon^{AC} - \frac{1}{2} \gamma_{ab} \bar{\psi}_B \gamma^a \psi_A \epsilon^{AC} + 3 \psi_C \bar{\psi}_B \psi_A \epsilon^{BC} = 0$$
The $F(4)$ superalgebra described by equations (1.3) - (1.6) fails to describe the physical vacuum because of the absence of the superfields 2-form $B$ and 1-form $A^0$ whose space-time restriction coincides with the physical fields $B_{\mu \nu}$ and $A^0_\mu$ appearing in the supergravity multiplet. The recipe to have all the fields in a single algebra is well known and consists in considering the Free Differential Algebra (F.D.A.) obtained from the $F(4)$ M.C.E.’s by adding two more equations for the 2-form $B$ and for the 1-form $A^0$ (the 0-form fields $\chi_A$ and $\sigma$ do not appear in the algebra since they are set equal to zero in the vacuum). It turns out that to have a consistent F.D.A. involving $B$ and $A^0$ one has to add to the $F(4)$ M.C.E.’s two more equations involving $dA^0$ and $dB$; in this way one obtains an extension of the M.C.E.’s to the following F.D.A:

\[
\mathcal{D}V^a - \frac{i}{2} \overline{\psi}_A \gamma_0 \psi^A = 0 \tag{1.10}
\]
\[
\mathcal{R}^{ab} + 4m^2 \ V^a V^b + m \overline{\psi}_A \gamma_{ab} \psi^A = 0 \tag{1.11}
\]
\[
dA^r \bigg[ + \frac{1}{2} g \ \epsilon^{rst} A_s A_t \bigg] - i \overline{\psi}_A \psi_B \ \sigma^{rAB} = 0 \tag{1.12}
\]
\[
dA^0 - mB - i \overline{\psi}_A \gamma_7 \psi^A = 0 \tag{1.13}
\]
\[
\overline{\psi}_A \gamma_7 \gamma_a \psi^A V^a = 0 \tag{1.14}
\]
\[
\mathcal{D}\psi_A - im \gamma_7 \psi^A = 0 \tag{1.15}
\]

Equations (1.13) and (1.14) were obtained by imposing that they satisfy the $d$-closure together with equations (1.10). Actually the closure of (1.14) relies on the 4-$\psi_A$’s Fierz identity

\[
\overline{\psi}_A \gamma_7 \gamma_a \psi_B \epsilon^{AB} \overline{\psi}_C \gamma^a \psi_D \epsilon^{CD} = 0 \tag{1.16}
\]

The interesting feature of the F.D.A (1.10)-(1.15) is the appearance of the combination $dA^0 - mB$ in (1.13). That means that the dynamical theory obtained by gauging the F.D.A. out of the vacuum will contain the fields $A^0_\mu$ and $B_{\mu \nu}$ always in the single combination $\partial_{[\mu} A^0_{\nu]} - mB_{\mu \nu}$. At the dynamical level this implies, as noted by Romans [2], an Higgs phenomenon where the 2-form $B$ ”eats” the 1-form $A^0$ and acquires a non vanishing mass $m$.

It is easy to see that no F.D.A exists if either $m = 0$, $g \neq 0$ or $m \neq 0$, $g = 0$, since the corresponding equations in the F.D.A. do not close anymore under $d$-differentiation. In other words the gauging of $SU(2)$, $g \neq 0$ must be necessarily accompanied by the presence of the parameter $m$ which, as we have seen, makes the closure of the supersymmetric algebra consistent for $g = 3m$.

In $D = 6$, $N = 4$ Supergravity, the only kind of matter is given by vector multiplets, namely

\[
(A_\mu, \ \lambda_A, \ \phi^\alpha)_I \tag{1.17}
\]

where $\alpha = 0, 1, 2, 3$ and the index $I$ labels an arbitrary number $n$ of such multiplets. As it is well known the $4n$ scalars parametrize the coset manifold $SO(4, n)/SO(4) \times SO(n)$. Taking into account that the pure supergravity has a non compact duality group $O(1, 1)$ parametrized by $e^\sigma$, the duality group of the matter coupled theory is

\[
G/H = \frac{SO(4, n)}{SO(4) \times SO(n)} \times O(1, 1) \tag{1.18}
\]
To perform the matter coupling we follow the geometrical procedure of introducing the coset representative $L^\Lambda_\Sigma$ of the matter coset manifold, where $\Lambda, \Sigma, \ldots = 0, \ldots, 3+n$; decomposing the $O(4,n)$ indices with respect to $H = SO(4) \times O(n)$ we have:

$$L^\Lambda_\Sigma = (L^\Lambda_\alpha, L^\Lambda_I)$$  \hspace{1cm} (1.19)

where $\alpha = 0, 1, 2, 3, I = 4, \ldots, 3+n$. Furthermore, since we are going to gauge the $SU(2)$ diagonal subgroup of $O(4)$ as in pure Supergravity, we will also decompose $L^\Lambda_\alpha$ as

$$L^\Lambda_\alpha = (L^\Lambda_0, L^\Lambda_r)$$  \hspace{1cm} (1.20)

The $4+n$ gravitational and matter vectors will now transform in the fundamental of $SO(4,n)$ so that the superspace vector curvatures will be now labeled by the index $\Lambda$: $F^\Lambda \equiv (F^0, F^r, F^I)$. Furthermore the covariant derivatives acting on the spinor fields will now contain also the composite connections of the $SO(4,n)$ duality group. Let us introduce the left-invariant 1-form of $SO(4,n)$ satisfying the Maurer-Cartan equation

$$\Omega^\Lambda_\Sigma = (L^\Lambda_\Pi)^{-1}dL^\Pi_\Sigma \Rightarrow d\Omega^\Lambda_\Sigma + \Omega^\Lambda_\Pi \wedge \Omega^\Pi_\Sigma = 0$$  \hspace{1cm} (1.21)

Our aim is to gauge a compact subgroup of $O(4,n)$. Since in any case we may gauge only the diagonal subgroup $SU(2) \subset O(4) \subset H$, the maximal gauging is given by $SU(2) \otimes \mathcal{G}$ where $\mathcal{G}$ is a $n$-dimensional subgroup of $O(n)$. According to a well known procedure, we modify the definition of the left invariant 1-form $L^{-1}dL$ by replacing the ordinary differential with the $SU(2) \otimes \mathcal{G}$ covariant differential as follows:

$$\nabla L^\Lambda_\Sigma = dL^\Lambda_\Sigma - f^\Lambda_{\Pi\Gamma} A^\Gamma L^\Pi_\Sigma$$  \hspace{1cm} (1.22)

where $f^\Lambda_{\Pi\Gamma}$ are the structure constants of $SU(2) \otimes \mathcal{G}$. More explicitly, denoting with $\epsilon^{rst}$ and $\Omega^{IJK}$ the structure constants of the two factors $SU(2)$ and $\mathcal{G}$, equation (1.22) splits as follows:

$$\nabla L^0_\Sigma = dL^\Lambda_\Sigma$$  \hspace{1cm} (1.23)

$$\nabla L^r_\Sigma = dL^r_\Sigma - g\epsilon^r_{ts} A^t L^s_\Sigma$$  \hspace{1cm} (1.24)

$$\nabla L^I_\Sigma = dL^I_\Sigma - g\Omega^I_{KJ} A^K L^J_\Sigma$$  \hspace{1cm} (1.25)

Setting $\hat{\Omega} = L^{-1}\nabla L$, one easily obtains the gauged Maurer-Cartan equations:

$$d\hat{\Omega}^\Lambda_\Sigma + \hat{\Omega}^\Lambda_\Pi \wedge \hat{\Omega}^\Pi_\Sigma = (L^{-1} \mathcal{F} L)^\Lambda_\Sigma$$  \hspace{1cm} (1.26)

where $\mathcal{F} \equiv F^\Lambda T_\Lambda$, $T_\Lambda$ being the generators of $SU(2) \otimes \mathcal{G}$.

After gauging, by appropriate decomposition of the indices, we find from (1.26):

$$R^r_s = -P^r_I \wedge P^I_s + (L^{-1} \mathcal{F} L)^r_s$$  \hspace{1cm} (1.27)

$$R^r_0 = -P^r_I \wedge P^I_0 + (L^{-1} \mathcal{F} L)^r_0$$  \hspace{1cm} (1.28)

$$R^I_J = -P^I_r \wedge P^r_J - P^I_0 \wedge P^0_J + (L^{-1} \mathcal{F} L)^I_J$$  \hspace{1cm} (1.29)

$$\nabla P^I_r = (L^{-1} \mathcal{F} L)^I_r$$  \hspace{1cm} (1.30)

$$\nabla P^I_0 = (L^{-1} \mathcal{F} L)^I_0$$  \hspace{1cm} (1.31)

Where we have set

$$P^I_\alpha = \begin{cases} P^I_0 & \equiv \Omega^I_0 \\ P^I_r & \equiv \Omega^I_r \end{cases}$$
Note that $P^I_r$, $P^I_t$ are the vielbeins of the coset, while $(\Omega^r, \Omega^0)$, $(R^r, R^0)$ are respectively the connections and the curvatures of $SO(4)$ decomposed with respect to the diagonal subgroup $SU(2) \subset SO(4)$.

Starting from the F.D.A. discussed before, one can now define suitable gauged curvatures in superspace, and apply the Bianchi identities technique in order to retrieve the space-time supersymmetry transformation laws. One finds:

\[
\delta V^\alpha = -i\bar{\psi} A^\alpha \gamma^\alpha \varepsilon^A \tag{1.32}
\]
\[
\delta B_{\mu \nu} = 2e^{-2\phi} \bar{\chi} A\gamma^\mu \gamma^\nu \varepsilon^A - 4e^{-2\phi} A\gamma^\mu \gamma^\nu \psi_{[\mu}]^A \tag{1.33}
\]
\[
\delta A^\mu = 2e^{\phi} \bar{\psi} A\gamma^\mu \chi^B L_{0}^A \varepsilon_{AB} + 2e^\phi \bar{A}\gamma^\mu \chi^B L^A \sigma_{rAB} - e^\phi L^A \gamma^\mu \gamma^\nu \varepsilon_{AB} + \\
+ 2ie^\phi L^A \gamma^\mu \gamma^\nu \psi_{rAB} + 2ie^\phi L^A \sigma_{rAB} \varepsilon_{AB} \psi_B \tag{1.34}
\]
\[
\delta \psi_{A\mu} = D_{\mu} \varepsilon_{A} + \frac{1}{16} e^{-\sigma}[T_{[AB] \mu \nu} \gamma_7 - T_{(AB) \mu \nu}] \gamma^\mu \gamma^\nu \delta \varepsilon + \\
+ \frac{i}{32} e^{\phi} H_{\mu \lambda \rho} \gamma^\lambda (\gamma^\mu \gamma^\lambda \lambda - 3 \delta^\gamma \gamma^\lambda \lambda) \varepsilon_{A} + \frac{1}{2} e^{\phi} A \gamma^\alpha \psi_{C \mu} + \\
+ \frac{1}{2} e^{\phi} \gamma_7 \bar{\psi} \gamma^\alpha \gamma^\alpha \psi_{C \mu} + \gamma_7 \gamma \gamma^\mu \varepsilon_{A} - 4 \gamma_7 \gamma \gamma^\mu \varepsilon_{C \mu} + \\
- \frac{1}{4} \gamma_7 \gamma \gamma^\mu \varepsilon_{A} - \frac{1}{4} \gamma_7 \gamma \gamma^\nu \varepsilon_{C \mu} + S_{AB}^{(g,g^\prime,m)} \gamma_{\varepsilon} \varepsilon_{B} \tag{1.35}
\]
\[
\delta \gamma = \frac{2}{3} \gamma \partial_\mu \varepsilon_{A} + \frac{1}{16} e^{-\sigma}[T_{[AB] \mu \nu} \gamma^\nu + T_{(AB) \mu \nu}] \gamma^\mu \gamma^\nu \varepsilon_{B} + \frac{1}{32} e^{\phi} H_{\mu \nu \lambda} \gamma_7 \gamma^\mu \gamma^\nu \varepsilon_{A} + \\
+ A_{\varepsilon}^{(g^\prime,m)} \gamma_{\varepsilon} \varepsilon_{B} \tag{1.36}
\]
\[
\delta \chi = \frac{1}{2} \chi_7 \gamma \gamma^\mu \varepsilon_{A} \tag{1.37}
\]
\[
\delta \lambda^{IA} = -iP^I_{r} \sigma^{rAB} \partial_\mu \phi^A \gamma_7 \gamma^\mu \varepsilon_{B} + iP^I_{0} \epsilon^{AB} \partial_\mu \phi^B \gamma_7 \gamma^\mu \varepsilon_{B} + i \frac{1}{2} e^{\phi} T^I_{\mu \nu} \gamma^\mu \gamma^\nu \varepsilon_{A} + \\
+ M_{AB}^{(g,g^\prime,m)} \gamma_{\varepsilon} \varepsilon_{B} \tag{1.38}
\]
\[
P^I_{0} \delta \phi^i = \frac{1}{2} \chi_7 \gamma \varepsilon \tag{1.39}
\]
\[
P^I_{r} \delta \phi^i = \frac{1}{2} \chi_7 \gamma \bar{\sigma} \tag{1.40}
\]

where we have introduced the "dressed" vector field strengths

\[
T_{[AB] \mu \nu} \equiv \varepsilon_{AB} L_{0 \lambda}^{-1} F^\lambda_{\mu \nu} \tag{1.41}
\]
\[
T_{(AB) \mu \nu} \equiv \varepsilon_{AB} L_{r \lambda}^{-1} F^\lambda_{\mu \nu} \tag{1.42}
\]
\[
T^I_{\mu \nu} \equiv L_{I \lambda}^{-1} F^\lambda_{\mu \nu} \tag{1.43}
\]

and we have omitted in the transformation laws of the fermions the three-fermions terms of the form $(\chi \chi \varepsilon)$, $(\lambda \lambda \varepsilon)$.

For our purposes, the most interesting quantities are the fermionic shifts denoted in boldface in the previous equations, since they are the objects in terms of which one can retrieve the scalar potential. Their explicit form is the following:

\[
S_{AB}^{(g,g^\prime,m)} = \frac{i}{24} [A \epsilon^A + 6me^{-3\phi} (L^{-1})_{00}] \varepsilon_{AB} - \frac{i}{8} [B \epsilon^A - 2me^{-3\phi} (L^{-1})_{00}] \gamma_7 \sigma^I_{AB} \tag{1.44}
\]
\[
N_{AB}^{(g,g^\prime,m)} = \frac{1}{24} [A \epsilon^A - 18me^{-3\phi} (L^{-1})_{00}] \varepsilon_{AB} + \frac{1}{8} [B \epsilon^A + 6me^{-3\phi} (L^{-1})_{00}] \gamma_7 \sigma^I_{AB} \tag{1.45}
\]
\[ M_{AB}^{I(g,g',m)} = (-C_i^I + 2i\gamma^7 D_i^I)\epsilon^\sigma \sigma_{AB}^{I} - 2m e^{-3\sigma} (L^{-1})^{I}_0 \gamma^7 \epsilon_{AB} \] (1.46)

where
\[ A = \epsilon^{rst} K_{rst}; \quad B^i = \epsilon^{ijk} K_{jk0}; \quad C_I^i = \epsilon^{trs} K_{It}; \quad D_{It} = K_{0It} \] (1.47)

and the threefold completely antisymmetric tensors \( K' \)s are the so-called ”boosted structure constants” given explicitly by:
\[ K_{rst} = g e_{0mn} L^I_r (L^{-1})^s_t m^n t + g' \mathcal{C}_{IJK} L^I_r (L^{-1})^s_t J L^K_t \] (1.48)
\[ K_{r0} = g e_{0mn} L^I_r (L^{-1})^s_t m^n t + g' \mathcal{C}_{IJK} L^I_r (L^{-1})^s_t J L^K_t \] (1.49)
\[ K_{rIt} = g e_{0mn} L^I_r (L^{-1})^s_t m^n t + g' \mathcal{C}_{IJK} L^I_r (L^{-1})^s_t J L^K_t \] (1.50)
\[ K_{0It} = g e_{0mn} L^I_0 (L^{-1})^s_t m^n t + g' \mathcal{C}_{IJK} L^I_0 (L^{-1})^s_t J L^K_t \] (1.51)

By performing the supersymmetry variation of the Lagrangian, indeed one finds the following Ward identity [10]:
\[ \delta^A \mathcal{W} = -20 g^{BA} S_{BC} - 4 N^{BA} N_{BC} + \frac{1}{4} M_I^{BA} M_J^{BC} \] (1.52)

One can verify that the r.h.s. of (1.52) is indeed proportional to a Kronecker delta, by using the explicit form of the shifts. The resulting potential turns out to be:
\[ \mathcal{W}(\phi) = 5 \left \{ \left [ \frac{1}{12} (A e^\sigma + 6 m e^{-3\sigma} L_{00}) \right ]^2 + \frac{1}{4} (e^\sigma B_i - 2 m e^{-3\sigma} L_{0i})^2 \right \} + \]
\[ - \left \{ \left [ \frac{1}{12} (A e^\sigma - 18 m e^{-3\sigma} L_{00}) \right ]^2 + \frac{1}{4} (e^\sigma B_i + 6 m e^{-3\sigma} L_{0i})^2 \right \} + \]
\[ - \frac{1}{4} \left \{ C'_i C_{It} + 4 D'_i D_{It} \right \} e^{2\sigma} - m^2 e^{-6\sigma} L_{0f} L^0_i \] (1.53)

A further issue related to the scalar potential, which is an important check of all our calculation, is the possibility of computing the masses of the scalar fields by varying the linearized kinetic terms of the Lagrangian and the potential \( \mathcal{W} \), after power expansion of \( \mathcal{W} \) up to the second order in the scalar fields \( q^I \). If we use as mass unity the inverse \( AdS \) radius, which in our conventions is \( R_{AdS}^{-2} = 4m^2 \) we get:
\[ m^2_{\sigma} = -6; \quad m^2_{q^0} = -4; \quad m^2_{q^r} = -6 \] (1.54)

These values should be compared with the results obtained in reference [3] where the supergravity and matter multiplets of the \( AdS_6 \) \( F(4) \) theory were constructed in terms of the singleton fields of the 5-dimensional conformal field theory, the singleton being given by hypermultiplets transforming in the fundamental of \( G \equiv \mathcal{E} \). It is amusing to see that the values of the masses of the scalars computed in terms of the conformal dimensions are exactly the same as those given in equation (1.54). This coincidence can be considered as a non-trivial check of the \( AdS/CFT \) correspondence in six versus five dimensions.

### 2 \( F(4) \otimes G \) Superconformal Field Theory

Here we describe the basics of the \( F(4) \) highest weight unitary irreducible representations “UIR’s” and exhibit two towers of short representations which are relevant for a K-K
analysis of type IIA theory on (warped) $AdS_6 \otimes S^4$ \[8, 9\].

We will not consider here the $G$ representation properties but we will only concentrate on the supersymmetric structure.

Recalling that the even part of the $F(4)$ superalgebra is $SO(2, 5) \otimes SU(2)$, from a general result on Harish-Chandra modules \[11, 12\] of $SO(2, 2n + 1)$ we know that there are only a spin 0 and a spin 1/2 singleton unitary irreducible representations \[13\], which, therefore, merge into a unique supersingleton representation of the $F(4)$ superalgebra: the hypermultiplet \[5\].

To describe shortening is useful to use a harmonic superfield language \[8\]. The harmonic space is in this case the 2-sphere $SU(2)/U(1)$, as in $N = 2$, $d = 4$ and $N = 1$, $d = 6$. A highest weight UIR of $SO(2, 5)$ is determined by $E_0$ and a UIR of $SO(5) \simeq Usp(4)$, with Dynkin labels $(a_1, a_2)$. We will denote such representations by $D(E_0, a_1, a_2)$. The two singletons correspond to $E_0 = 3/2$, $a_1 = a_2 = 0$ and $E_0 = 2$, $a_1 = 1$, $a_2 = 0$.

In the $AdS/CFT$ correspondence \[3, 4\] ($E_0$, $a_1$, $a_2$) become the conformal dimension and the Dynkin labels of $SO(1, 4) \simeq Usp(2, 2)$.

The highest weight UIR of the $F(4)$ superalgebra will be denoted by $D(E_0, a_1, a_2; I)$ where $I$ is the $SU(2)$ $R$-symmetry quantum number (integer or half integer). The basic superfield is the supersingleton hypermultiplet $W^A(x, \theta)$, which satisfies the constraint

$$D_\alpha^A W^B(x, \theta) = 0$$

(2.1)

corresponding to the irrep. $D(E_0 = \frac{3}{2}, 0, 0; I = \frac{1}{2})$ \[8\].

By using harmonic superspace, $(x, \theta_I, u_I)$, where $\theta_I = \theta_i u_i$, $u_i$ is the coset representative of $SU(2)/U(1)$ and $I$ is the charge $U(1)$-label, from the covariant derivative algebra

$$\{D^A_\alpha, D^B_\beta\} = i\epsilon^{AB} \partial_{\alpha\beta}$$

(2.2)

we have

$$\{D^I_\alpha, D^J_\beta\} = 0 \quad D^I_\alpha = D^i_\alpha u_i^I$$

(2.3)

Therefore from eq. (2.1) it follows the $G$-analytic constraint:

$$D^I_\alpha W^1 = 0$$

(2.4)

which implies

$$W^1(x, \theta) = \varphi^1 + \theta_2^3 \zeta_\alpha + d.t.$$  

(2.5)

(d.t. means “derivative terms”).

Note that $W^1$ also satisfies

$$D^2_\alpha D^{2\alpha} W^1 = 0$$

(2.6)

because there is no such scalar component\[10\] in $W^1$.

$W^1$ is a Grassman analytic superfield, which is also harmonic (that is $D^2_\alpha W^1 = 0$ where, using notations of reference \[14\], $D^2_\alpha$ is the step-up operator of the $SU(2)$ algebra acting on harmonic superspace).

Since $W^1$ satisfies $D^1 W^1 = 0$, any $p$-order polynomial

$$I_p(W^1) = (W^1)^p$$

(2.7)

\[2\] Note that the $Usp(4)$ Young labels $h_1, h_2$ are related to $a_1, a_2$ by $a_1 = 2h_2; a_2 = h_1 - h_2$.

\[3\] This is rather similar to the treatment of the $(1,0)$ hypermultiplet in $D = 6$ \[15\].
will also have the same property, so these operators form a ring under multiplication \[14\],
they are the 1/2 BPS states of the \( F(4) \) superalgebra and represent massive vector multiplets \((p > 2)\), and massless bulk gauge fields for \( p = 2 \).
The above multiplets correspond to the \( D(E_0 = 3I, 0, 0; I = \frac{q}{2}) \) h.w. U.I.R.'s of the \( F(4) \) superalgebra.

The AdS squared mass for scalars is
\[
 m_s^2 = E_0(E_0 - 5) \tag{2.8}
\]
so there are three families of scalar states with
\[
 m_1^2 = \frac{3}{4}p(3p - 10) \quad p \geq 2
\]
\[
 m_2^2 = \frac{1}{4}(3p + 2)(3p - 8) \quad p \geq 2
\]
\[
 m_3^2 = \frac{1}{4}(3p + 4)(3p - 6) \quad p \geq 4
\]
The only scalars states with \( m^2 < 0 \) are the scalar in the massless vector multiplet \((p = 2)\) with \( m_1^2 = -6, m_2^2 = -4 \) (no states with \( m^2 = 0 \)) exist and in the \( p = 3 \) multiplet with \( m^2 = -\frac{9}{4} \).

We now consider the second ”short” tower containing the graviton supermultiplet and its recurrences.
The graviton multiplet is given by \( W^1 \bar{W}^1 \). Note that such superfield is not \( G \)-analytic, but it satisfies
\[
 D^1_\alpha D^{1\alpha}(W^1 \bar{W}^1) = D^2_\alpha D^{2\alpha}(W^1 \bar{W}^1) = 0 \tag{2.9}
\]
this multiplet is the \( F(4) \) supergravity multiplet. Its lowest component, corresponding to the dilaton in \( AdS_6 \) supergravity multiplet, is a scalar with \( E_0 = 3 \) \( (m^2 = -6) \) and \( I = 0 \).
The tower is obtained as follows
\[
 G_{q+2}(W) = W^1 \bar{W}^1 (W^1)^q \tag{2.10}
\]
where the massive graviton, described in eq. (2.10) has \( E_0 = 5 + \frac{3}{2}q \) and \( I = \frac{q}{2} \).
Note that the \( G_{q+2} \) polynomial, although not \( G \)-analytic, satisfies the constraint
\[
 D^1_\alpha D^{1\alpha} G_{q+2}(W) = 0 \tag{2.11}
\]
so that it corresponds to a short representation with quantized dimensions and highest weight given by \( D(E_0 = 3 + 3I, 0, 0; I = \frac{q}{2}) \).
We call these multiplets, following \[13\], ”intermediate short” because, although they have some missing states, they are not BPS in the sense of supersymmetry. In fact they do not form a ring under multiplication.

There are also long spin 2 multiplets containing \( 2^8 \) state where \( E_0 \) is not quantized and satisfies the bound \( E_0 \geq 6 \).
Finally let us make some comments on the role played by the flavour symmetry \( \mathcal{G} \).
It is clear that, since the supersingleton \( W^1 \) is in a representation of \( \mathcal{G} \) (other than the gauge group of the world-volume theory), the \( I_p \) and \( G_{q+2} \) polynomials will appear in the \( p \)-fold and \((q + 2)\)-fold tensor product representations of the \( \mathcal{G} \) group. This representation is in general reducible, however the 1/2 BPS states must have a first component totally
symmetric in the $SU(2)$ indices and, therefore, only certains $\mathcal{G}$ representations survive. Moreover in the $(W^1)^2$ multiplet, corresponding to the massless $\mathcal{G}$-gauge vector multiplets in $AdS_6$, we must pick up the adjoint representation $\text{Adj}\mathcal{G}$ and in $W^1 W^1$, corresponding to the graviton multiplet, we must pick up the $\mathcal{G}$ singlet representation. However in principle there can be representations in the higher symmetric and antisymmetric products, and the conformal field theory should tell us which products remains, since the flavor symmetry depends on the specific dynamical model.

The states discussed in this paper are expected to appear $[6], [7]$ in the K-K analysis of IIA massive supergravity on warped $AdS_6 \otimes S^4$.

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