Branching instability in the flux creep regime of type-II superconductors

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Abstract

We study theoretically the space-time evolution of the thermal and electromagnetic perturbation in a superconductor with a nonlinear current-voltage characteristics in the flux creep regime. On the basis of a linear analysis of a set of differential equations describing small perturbations of temperature and electromagnetic field, it is found that under some conditions a branching instability may occur in a superconductor sample.

Key words: thermal and electromagnetic perturbations, critical state, flux creep.

The dynamics of thermomagnetic instabilities of the critical state and flux jumps in hard superconductors with high values of the critical current density and the critical magnetic fields have been earlier investigated by many authors [1-5]. The general concept of thermomagnetic instabilities in type-II superconductors was developed in literature [5]. The dynamics of small thermal and electromagnetic perturbations, whose development leads to the flux jump, have been investigated theoretically by Mints and Rakhamnov [4, 5]. The authors have found the stability criterion for the flux jumps in the framework of adiabatic and dynamic approximations in the viscous flux flow regime of type-II superconductors. The detailed theoretical analyze of the flux jumping in the flux creep regime, where the current-voltage characteristics of a sample is a nonlinear have been carried out recently by Mints [6] and by Mints and Brandlt [7]. Nonuniform magnetic flux penetration in type II superconductors, creating finger and dendritic patterns, has recently attracted considerable interest. Such patterns have been directly observed in a large number of superconducting films employing magneto-optical imaging techniques [8-15]. The existing experimental data [8-15], and the recently developed theoretical models [16, 17], suggest that the origin of these patterns is thermomagnetic instability of the vortex matter in the superconducting films. The instability arises from local temperature increase due to flux motion, which, in turn, decreases flux pinning and hence facilitates further flux motion. Linear stability analysis, based on the coupled nonlinear Maxwell and thermal diffusion equations, showed that instability in the form of narrow fingers perpendicular to the background electric field occurs when this field exceeds of its threshold value [16, 17].

In the present work, we study the spatial and temporal evolution of small thermal and electromagnetic perturbation in type-II superconductor sample in the flux creep regime with a nonlinear voltage-current characteristics, assuming that an applied field parallel to the surface of the sample. On the basis of a linear analysis of a set of differential equations describing small perturbations of temperature and electromagnetic field we found that under some conditions a branching instability may occur in the sample.

Mathematical problem of theoretical study the dynamics of thermal and electromagnetic perturbations in a superconductor sample in the flux creep regime can be formulated on the basis of a system nonlinear diffusion-like equations for the thermal and electromagnetic field perturbations with account nonlinear relationship between the field and current in superconductor sample. The distribution of the magnetic flux density \(\mathbf{B}\) and the transport current density \(\mathbf{j}\) inside a superconductor is described by the equation

\[
\text{rot}\mathbf{B} = \mu_0 \mathbf{j},
\]

When the penetrated magnetic flux changes with time, an electric field \(\mathbf{E}\) is generated inside the sample according to Faraday’s law

\[
\text{rot}\mathbf{E} = \frac{d\mathbf{B}}{dt}.
\]

The temperature distribution in superconductor is governed by the heat conduction diffusion equation

\[
\nu(T) \frac{dT}{dt} = \nabla [\kappa(T) \nabla T] + \mathbf{j} \cdot \mathbf{E},
\]

where \(\nu\) and \(\kappa\) are the specific heat and thermal conductivity, respectively. The above equations should be supplemented by a current-voltage characteristics of superconductors, which has the form

\[
j = j_c(T, B) + j(E).
\]

In order to obtain analytical results of a set Eqs. (1)-(3), we suggest that \(j_c\) is independent on magnetic field induction and \(B\) and use the Bean critical state model \(j_c = j_c(B_\phi, T) = j_0 - \alpha(T - T_0)\) [1, 18], where \(B_\phi\) is the external applied magnetic field induction; \(\alpha = j_0/(T_c - T_0)\); \(j_0\) is the equilibrium current density, \(T_0\) and \(T_c\) are the equilibrium and critical temperatures of the sample, respectively, [5]. For the sake of simplifying of the calculations, we perform our calculations on the assumption of negligibly small heating \((T - T_0 \ll T_c - T_0)\) and assume that the temperature profile is a constant within the across sample and thermal conductivity \(\kappa\) and heat capacity \(\nu\) are independent on the temperature profile. We shall study the problem in the framework of a macroscopic approach, in which all lengths scales are larger than the flux-line spacing; thus, the superconductor is considered as an uniform medium. The system of differential equations (1)-(3) should be supplemented by a current-voltage curve \(j = j(E)\). In the flux creep regime the current-voltage characteristics of type II conventional superconductors is highly nonlinear due to thermally activated dissipative flux motion [19, 20]. For the logarithmic current dependence of the potential barrier \(U_j\), proposed by [21] the dependence \(j(E)\) has the form

\[
j = j_c \left[\frac{E}{E_0}\right]^{1/n},
\]

where \(E_0\) is the voltage criterion at which the critical current density \(j_c\) is determined [5]; a constant parameter \(n\) depends
on the pinning regimes and can vary widely for various types of superconductors. In the case \( n = 1 \) the power-law relation (4) reduces to Ohm’s law, describing the normal or flux-flow regime [18]. For infinitely large \( n \), the equation describes the Bean critical state model \( j = j_c \) [1]. When \( 1 < n < \infty \), the equation (4) describes nonlinear flux creep [22]. In this case the differential conductivity \( \sigma \) is determined by the following expression

\[
\sigma = \frac{dj}{dE} = \frac{j_c}{\pi E_b} \quad (5)
\]

According to relation (5) the differential conductivity decreases with the increasing of the background electric field \( E_b \), and strongly depends on the external magnetic field sweep rate \( E_b \sim B_c x \). Therefore the stability criterion also strongly depends on the differential conductivity \( \sigma \). For the typical values of \( j_1 = 10^9 A/cm^2, E_b = 10^{-7} V/cm \) we obtain \( \sigma = 10^{10} 1/\Omega cm \). It follows from this estimation [6, 7] that the differential conductivity \( \sigma \) which determines the dynamics of thermomagnetic instability for is high enough. We assume, for simplicity, that the value of \( n \) temperature and magnetic-field independent.

Let us formulate a differential equations governing the dynamics of small temperature and electromagnetic field perturbation in a superconductor sample. We study the evolution of the thermal and electromagnetic penetration process in a simple geometry - superconducting semi-infinite sample \( x \geq 0 \). We assume that the external magnetic field induction \( B_c \) is parallel to the \( z \)-axis and the magnetic field sweep rate \( B_c \) is constant. When the magnetic field with the flux density \( B_c \) is applied in the direction of the \( z \)-axis, the transport current \( j(x,t) \) and the electric field \( E(x,t) \) are induced inside the slab along the \( y \)-axis. For this geometry the spatial and temporal evolution of small thermal \( T(x,t) \) and electromagnetic field \( E(x,t) \) perturbations are described by the thermal diffusion equation coupled to Maxwell’s equations

\[
\nu \frac{dT}{dt} = k \frac{d^2 T}{dx^2} + j_c E, \quad (6)
\]

\[
\frac{d^2 E}{dx^2} = \mu \left( \frac{j_c}{n E_b} \frac{dE}{dt} - \frac{dj_c}{dt} \frac{dT}{dt} \right). \quad (7)
\]

It should be noted that the nonlinear diffusion-type equations (6) and (7), totally determine the problem of the space-time distribution of the temperature and electromagnetic field profiles in the flux creep regime with a nonlinear current-voltage characteristics in the semi-infinite sample.

Let us specify the thermal and electrodynamic boundary and initial conditions to the last system of equations. The thermal boundary conditions are

\[
\frac{dT(0, t)}{dx} = 0, \quad T(L,t) = T_0.
\]

We assume that the magnetic field perturbation is equal to zero at the sample surface and according to relation (2), we obtain the first electrodynamic boundary condition

\[
\frac{dE(0, t)}{dx} = 0.
\]

The second boundary condition for the electric field \( E(x, t) \) at the flux front \( x = L \) can be presented as

\[
E(L, t) = 0,
\]

The boundary conditions for the magnetic induction are

\[
dB(0, t) = B_c, \quad B(L, t) = 0,
\]

where \( L = \frac{c B_c}{4 \pi j_c} \) is the London penetration depth. For initial conditions we assume that the electric field is uniform within the cross-section of the sample \( E = E_0 \) at \( t=0 \).

Let us derive, for this geometry a differential equations, describing the spatial and temporal evolution of thermal \( T(x,t) \) and electromagnetic field \( E(x,t) \) perturbations. We present the small thermal and electromagnetic perturbations in the form

\[
\Theta(x, t) = T_0(x) + (T_c - T_0) \exp \left[ \gamma t/t_0 + i q z \right],
\]

\[
e(x, t) = E_0(x) + E_b \exp \left[ \gamma t/t_0 + i q z \right].
\]

where \( T_0(x) \) and \( E_0(x) \) are solutions to the unperturbed equations obtained in the quasi-stationary approximation describing the background distributions of temperature and electric field in the sample. Here \( \gamma \) is the eigenvalue of the problem to be determined and \( q = 2 \pi d / L \) is the wave-number of the perturbation. From solutions (8), one can see that the characteristic time of thermal and electromagnetic perturbations \( t \) is of the order of \( t_0 / \gamma \). Where, we have introduced the following dimensionless parameters and variables

\[
t_0 = \frac{\sigma \nu a}{j_c}, \quad z = \frac{x}{d}, \quad d = \frac{\nu a}{\mu_0 j_c}, \quad q = \frac{\pi d}{2 L}.
\]

As we mentioned above, the background temperature \( T_0(x) \) is practically uniform over the cross-section of the sample and under this approximation we ignore its coordinate dependence. We note that the background electric field may be created by ramping the external magnetic field, and for simplicity we assume it to be coordinate independent. It turns out that these simplifications have no qualitative influence on the results but make it possible to perform analytical calculations completely.

Substituting the expression (8) into the system equations (6), (7) one can get the following linearized system equations for \( \Theta \) and \( \epsilon \)

\[
\tau q^2 \Theta + \gamma \Theta + \frac{1}{n} \Theta - 2 \left( \frac{1}{n} \right)^2 \epsilon = 0,
\]

\[
q^2 \epsilon + \gamma \left[ k - n \Theta \right] = 0.
\]

Solving the above system equations (9) we obtain the following dispersion relation to determine an eigenvalues of the problem

\[
\gamma^2 + \left( \tau + 1 \right) q^2 - \frac{1}{n} \gamma + \left[ \tau q^2 + \frac{1}{n} \right] q^2 = 0 \quad (10)
\]

where \( \tau \) is the ratio between the characteristic time of magnetic flux diffusion and the characteristic time of heat flux diffusion [4]. The instability of the flux front is defined by the positive values of the growth rate Re \( \gamma > 0 \). It can be seen that there is a critical wave number

\[
q_c = \frac{1}{\sqrt{\gamma}} \quad (11)
\]

below which the system is always unstable at \( n = 1 \). This instability appears first at \( q = 0 \). In this case the small perturbations grow with the maximal possible rate Re \( \gamma = 1 \). The growth rate dependence on the wave number for different values of \( \gamma \) is illustrated in Figs. (1-3) at \( n = 1 \). For high enough values of
\[ \tau \text{ the system is stable. As the } \tau \text{ decreases, the growth rate } \gamma \text{ increases. The branching instability will gradually appear for relatively small values of } \tau = 0.05. \]

Thus, according to (11), the branching instability occurs at the threshold electric field \( E = E_c \)
\[ E = E_c = \frac{\pi^2 \kappa(T_e - T_0)}{4 \sigma L^2}, \]
Taking into account an expression for penetration depth \( L \), the threshold field can be written at \( B = B_{th} \) as
\[ B_{th} = \frac{\pi}{2} \sqrt{\frac{\kappa(T_e - T_0) j_c}{E_b}}. \]
The threshold field for branching instability, as can be seen from the last expression is highly sensitive to the critical current density and the shape of the background electric field \( E_b \) generated by the varying magnetic field. The threshold field \( B_{th} \) decreases monotonously with increasing the background electric field \( E_b \). If we assume that the background electric field \( E_b \) generated by a varying magnetic field as \( E_b \approx \dot{B} e \), for the considered simple geometry, then we can easily obtain the expression for the sweep rate dependence of the threshold field of branching instability.

Let us assume that the thermal diffusion is slower than the magnetic diffusion \( \tau \ll 1 \). In this adiabatic limiting case, the instability criterion is determined as \( q = q_c = 1 \) for \( n=1 \), so the threshold field can be presented as
\[ B_{th} = \frac{\pi}{2} \sqrt{\frac{\nu \mu_0 (T_e - T_0)}{E_b}}. \]
This is a well known adiabatic criteria \([2]\), which assumes that the heat transport from the sample surface to the environment can be neglected.

**Conclusion**

In conclusion, on the basis of a linear analysis of a set of differential equations describing small perturbations of temperature and electromagnetic field we found that under some conditions a branching instability may occur in the sample.

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Fig.1-3. The dependence of the growth rate on the wave number for $\tau = 0.5, 0.05, 0.08$ and $n=1$. 

Growth rate $\gamma$ vs. Wavenumber $q$