Large Intelligent Surface-Assisted Wireless Communication Exploiting Statistical CSI

Yu Han, Student Member, IEEE, Wankai Tang, Student Member, IEEE, Shi Jin, Senior Member, IEEE, Chao-Kai Wen, Member, IEEE, and Xiaoli Ma, Fellow, IEEE

Abstract—Large intelligent surface (LIS)-assisted wireless communications have drawn attention worldwide. With the use of low-cost LIS on building walls, signals can be reflected by the LIS and sent out along desired directions by controlling its phases, thereby providing supplementary links for wireless communication systems. In this study, we evaluate the performance of an LIS-assisted large-scale antenna system by formulating a tight upper bound of the ergodic spectral efficiency and investigate the effect of the phase shifts on the ergodic spectral efficiency in different propagation scenarios. In particular, we propose an optimal phase shift design based on the upper bound of the ergodic spectral efficiency and statistical channel state information. Furthermore, we derive the requirement on the quantization bits of the LIS to promise an acceptable spectral efficiency degradation. Numerical results show that using the proposed phase shift design can achieve the maximum ergodic spectral efficiency, and a 2-bit quantizer is sufficient to ensure spectral efficiency degradation of no more than 1 bit/s/Hz.

I. INTRODUCTION

As a key feature of the fifth-generation (5G) and future mobile communications, large-scale antenna systems can produce high throughput by utilizing the spatial degrees of freedom and achieve wide cell coverage with a high-gain array. However, hindrances still occur in a large-scale antenna system due to the existence of buildings, trees, cars, and even humans. To address this problem and produce fluent user experience, a typical solution is to add new supplementary links to maintain the communication link. For example, relay nodes can be introduced in areas with poor communication signal to receive the weak signals; the signals are then amplified and retransmitted to the next relay hop or terminal [1]. However, the power consumption of the relay node is still high.

In recent years, large intelligent surface (LIS) technologies have been rapidly developed [2]–[9]. LIS is a cheap passive artificial structure that can digitally manipulate electromagnetic waves and obtain preferable electromagnetic propagation environment with limited power consumption [2], [3]. Modern LIS includes reprogrammable metasurface, which has been suggested to replace the radio frequency module and reform the wireless communication architecture [4], [5]. Modern LIS also includes reflector array, which can reflect the electromagnetic wave, offering support to traditional wireless systems [6]–[9]. Reflector array and relay work in different mechanisms to provide supplementary links. Although the incident signal at the reflector array is reflected without being scaled, the propagation environment can be improved using extremely low power consumption without introducing additional noise at the reflector array. Therefore, the reflector array is a more economical and efficient choice than the relay.

In this study, we apply the reflector-array-type LIS in large-scale antenna systems to provide supplementary links between the base station (BS) and the user. By adjusting the phases of the incident signal, LIS can reflect the signal toward the desired spatial direction. We first obtain an upper bound of the ergodic spectral efficiency of the LIS-assisted large-scale antenna system and find that the ergodic spectral efficiency is largely dependent on the phase shift amount introduced by the LIS. To maximize the ergodic spectral efficiency, we propose an optimal phase shift design by exploiting statistical channel state information (CSI). Furthermore, in view of the hardware impairments, we formulate the requirement on the quantization bits of the LIS to ensure an acceptable degradation of the spectral efficiency. Numerical results verify the tightness of the upper bound of ergodic spectral efficiency, demonstrate the effectiveness of the optimal phase shift design, and show that a 2-bit quantizer ensures a spectral efficiency degradation of less than 1 bit/s/Hz.

II. SYSTEM MODEL

We focus on a single cell of an LIS-assisted large-scale antenna system, as shown in Fig. 1. The BS is equipped with an M-element large uniform linear array (ULA), which serves a single-antenna user. The LIS is set up between the BS and the user, comprising N reflector elements arranged in a ULA. A controller manages and coordinates the BS and the LIS through separate fibers1 [8].

1Note that even though both the LIS-assisted system and the distributed antenna system have controllers, the two systems work in different ways and have completely different signal models and system designs.
The channel between the BS and the user is in Rayleigh fading condition and is denoted by $g \in \mathbb{C}^{1 \times M}$, where the elements of $g$ are i.i.d. in the complex Gaussian distribution with zero mean and unit variance. For the channel between the BS and the LIS and that between the LIS and the user, line-of-sight (LoS) paths exist in practical implementation. Therefore, these two channels are modeled in Rician fading. We denote $\tilde{H}$ as the channel between the BS and the LIS as

$$
\tilde{H} = \begin{bmatrix} K_1^{-1} \tilde{H}_1 + \sqrt{\frac{1}{K_1+1}} \tilde{H}_1 \\
\end{bmatrix},
$$

where $K_1$ is the Rician $K$-factor of $H_1$; $\tilde{H}_1 \in \mathbb{C}^{N \times M}$ is the LoS component, which remains unchanged within the channel coherence time; and $\tilde{H}_1 \in \mathbb{C}^{N \times M}$ is the non-LoS (NLoS) component. The elements of $\tilde{H}_1$ are i.i.d. complex Gaussian distributed, and each element has zero mean and unit variance. Similarly, the channel between the LIS and the user is expressed as

$$
h_2 = \begin{bmatrix} K_2^{-1} \tilde{h}_2 + \sqrt{\frac{1}{K_2+1}} \tilde{h}_2 \\
\end{bmatrix},
$$

where $K_2$ is the Rician $K$-factor of $H_2$; $\tilde{h}_2 \in \mathbb{C}^{1 \times N}$ is the LoS component, and $\tilde{h}_2 \in \mathbb{C}^{1 \times N}$ is the NLoS component. Each element of $\tilde{h}_2$ is i.i.d. complex Gaussian distributed with zero mean and unit variance.

The LoS components are expressed by the responses of the ULA. The array response of an $N$-element ULA is

$$
a_N(\theta) = \begin{bmatrix} e^{j2\pi \frac{\pi}{d} \sin \theta}, \ldots, e^{j2\pi \frac{\pi}{d} (N-1) \sin \theta} \end{bmatrix},
$$

where $\theta$ is the angle of departure (AoD) or angle of arrival (AoA) of a signal. Under this condition, the LoS components are expressed as $H_1 = a_N^{H}(\theta_{AoA,1}) a_M(\theta_{AoD,1})$ and $H_2 = a_N^{H}(\theta_{AoA,2}) a_M(\theta_{AoD,2})$, respectively, where $\theta_{AoD,1}$ is the AoD from the ULA at the BS, $\theta_{AoA,1}$ is the AoA to the ULA at the LIS, and $\theta_{AoD,2}$ is the AoD from the LIS to the BS.

We focus on the downlink of the LIS-assisted system. The signal can travel along $g$ directly, or be reflected by the LIS. When reflected by the LIS, the phases of the signals are changed. The signal received by the user side is

$$
r = \sqrt{P} (h_2 \Phi H_1 + g)f^H s + w,
$$

where $P$ is the transmit power, $\Phi = \text{diag}\{e^{j \phi_1}, \ldots, e^{j \phi_N}\}$, $\phi_n \in [0, 2\pi)$ is the phase shift introduced by the $n$th element of the LIS, $f \in \mathbb{C}^{1 \times M}$ is the beamforming vector satisfying $\|f\|^2 = 1$, $s$ is original signal satisfying $\mathbb{E}\{|s|^2\} = 1$, $\mathbb{E}\{\cdot\}$ represents taking expectation, and $w$ is the complex Gaussian noise with zero mean and unit variance. We denote $h = h_2 \Phi H_1$ as the assistant channel. We also view $\Phi$ as the beamforming weight introduced by the LIS. Maximum ratio transmission (MRT) is adopted at BS. Then, assuming $h + g$ is known at BS, the beamforming vector $f$ is defined as

$$
f = \frac{h + g}{\|h + g\|}.
$$

Note that $h$ and $g$ can be estimated by the BS using the pilots sent from the user. By applying (5), we obtain the ergodic spectral efficiency of the LIS-assisted large-scale antenna system as

$$
C = \mathbb{E}\left\{\log_2 \left(1 + \frac{P}{\sigma_w^2} \|h_2 \Phi H_1 + g\|^2\right)\right\},
$$

where $\sigma_w^2 = 1$ is the noise variance.

We aim to identify the optimal phase shift design $\Phi$ at the LIS to maximize the ergodic spectral efficiency. This low-cost long-term design utilizes the statistical CSI and can remain unchanged within the channel coherence time.

## III. Ergodic Spectral Efficiency Analysis

Before designing the phase shift, we theoretically analyze the ergodic spectral efficiency of the LIS-assisted large-scale antenna system and then evaluate the effects of using different phase shift amounts under different propagation scenarios.

### A. Upper Bound of Ergodic Spectral Efficiency

To have a direct cognition on the ergodic spectral efficiency, we provide an upper bound in the following proposition.

**Proposition 1:** The ergodic spectral efficiency of the LIS-assisted large-scale antenna system is upper bounded by

$$
C \leq C_{\text{ub}} = \log_2 \left(1 + P \left(\gamma_1 \|H_2 \Phi H_1\|^2 + \gamma_2 MN + M\right)\right),
$$

where

$$
\gamma_1 = \frac{K_1 K_2}{(K_1+1)(K_2+1)}, \quad \gamma_2 = \frac{K_1 + K_2 + 1}{(K_1+1)(K_2+1)}.
$$

**Proof:** See Appendix A.

We focus on high signal-to-noise ratio region, where the upper bound is tight and can be viewed as a good approximation. According to **Proposition 1**, when $P$, $H_1$, and $H_2$ remain unchanged, the ergodic spectral efficiency is determined by $\gamma_1$, $\gamma_2$, and $\|H_2 \Phi H_1\|^2$, which are further determined by the Rician-$K$ factors and the phase shift amounts at the LIS.

### B. Effects of Rician-$K$ Factors and Phase Shifts

To acquire deep insights on the effects of the Rician-$K$ factors and the phase shift amounts on the ergodic spectral efficiency, we investigate the following cases.

**Case 1:** If $K_1 = 0$ or $K_2 = 0$, then the ergodic spectral efficiency of the LIS-assisted system is upper bounded by

$$
C_{\text{ub}} = \log_2 \left(1 + PM(N + 1)\right).
$$
We observe that when either $\mathbf{h}_1$ or $\mathbf{h}_2$ is under Rayleigh fading condition, $C_{\text{ub}}$ is proportional to $M$ and $N$, but is independent of $\Phi$. This phenomenon is caused by the spatial isotropy that holds upon the assistant channel, which is insensitive to the beamforming between $\mathbf{h}_1$ and $\mathbf{h}_2$. Under this condition, even 0-bit phase-shifting is sufficient, and the phase shift amount can be set arbitrarily.

Case 2: If $K_1, K_2 \to \infty$, then the upper bound of the ergodic spectral efficiency of the LIS-assisted system approaches

$$C_{\text{ub}} \to \log_2 \left(1 + P(\|\mathbf{h}_2 \mathbf{H}_1\|^2 + M)\right).$$

(10)

In the extreme Rician fading condition, only LoS components exist, and the assistant channel remains unchanged. The ergodic spectral efficiency increases proportional to $\|\mathbf{h}_2 \mathbf{H}_1\|^2$. That is, in spatially directional propagation environment, the ergodic spectral efficiency of the LIS-assisted system is sensitive to the beamforming weights at the LIS.

If the amount of phase shifts is properly set, then the wireless signal can be beamformed on the main lobe of the assistant channel. Then, $\|\mathbf{h}_2 \mathbf{H}_1\|^2 \gg N$, and the ergodic spectral efficiency under extreme Rician fading condition is considerably higher than that of Rayleigh fading comparing (9) with (10). Otherwise, when $\|\mathbf{h}_2 \mathbf{H}_1\|^2 < N$, the ergodic spectral efficiency under extreme Rician fading condition is even inferior to that of Rayleigh fading. Therefore, in Rician fading condition, $\Phi$ should be carefully designed to fully utilize the LoS components of $\mathbf{h}$.

IV. PHASE SHIFT DESIGN OF REFLECTOR ARRAY

In this section, we propose an optimal design of $\Phi$ to maximize the ergodic spectral efficiency exploiting the statistical CSI and provide a criterion of the quantization bit to ensure an acceptable ergodic spectral efficiency degradation of the LIS-assisted large-scale antenna system.

A. Optimal Phase Shift Design

Based on (7), when $P$, $K_1$, $K_2$, $M$, and $N$ are fixed, then maximizing the tight upper bound $C_{\text{ub}}$ is equivalent to maximizing $\|\mathbf{h}_2 \mathbf{H}_1\|^2$. Thus, the optimal $\Phi$ satisfies

$$\Phi_{\text{opt}} = \max_\Phi \|\mathbf{h}_2 \mathbf{H}_1\|^2.$$  

(11)

Denoting $z = a_N(\theta_{\text{AoA},2}) \Phi a_N^{-1}(\theta_{\text{AoA},1})$, we rewrite (11) as

$$\Phi_{\text{opt}} = \max_\Phi \|z a_M(\theta_{\text{AoD},1})\|^2 = \max_\Phi |z|^2 \|a_M(\theta_{\text{AoD},1})\|^2,$$

(12)

where $\|a_M(\theta_{\text{AoD},1})\|^2 = M$ is a constant. Hence, the optimal $\Phi$ can also maximize $|z|^2$. We derive that

$$z = \sum_{n=1}^N e^{j2\pi \frac{\Phi}{M}(n-1)(\sin \theta_{\text{AoD},2} - \sin \theta_{\text{AoA},1}) + j\phi_n}.$$  

(13)

The ergodic spectral efficiency depends on $\theta_{\text{AoA},1}$ and $\theta_{\text{AoD},2}$ but is independent of $\theta_{\text{AoD},1}$. Hence, the phase shift can only affect the links that are directly connected by the LIS. Meanwhile, $0 \leq |z| \leq N$. If $|z| = 0$, then the LoS component of the assistant channel is completely blocked due to the inadequate setting of $\Phi$, that is, the power transmitted along the LoS component is wasted. This phenomenon further demonstrates the significance of a proper phase shift design.

Therefore, in order to maximize $|z|^2$, the optimal phase shift on the $n$th reflector element of the LIS should be

$$\phi_{\text{opt},n} = 2\pi \frac{d}{\lambda}(n-1)(\sin \theta_{\text{AoA},1} - \sin \theta_{\text{AoD},2}).$$

(14)

That is, the wireless signal is reflected and sent out along the LoS component of the assistant channel. When adopting the optimal phase shift design, $z = N$ holds, and

$$C_{\text{ub}}^\text{opt} = \log_2 \left(1 + PM(\gamma_1 N^2 + \gamma_2 N + 1)\right).$$

(15)

The comparison of (15) and (9) indicates that the existence of LoS component is beneficial at all times if the phase shift amount is well-designed. Moreover, the ergodic spectral efficiency is proportional to $N$. Increasing $N$ helps enhancing the receiving power at the user side, thereby further improving the ergodic spectral efficiency of the LIS-assisted system.

B. Influence of Bit Quantization

In practical systems, the phase shift amount is constrained by the quantization bits of the LIS. We denote the number of quantization bits as $B$. Then, each theoretical value $\phi_n$ is quantized to its nearest value in

$$\left\{0, \frac{2\pi}{2^B}, \ldots, \frac{2\pi(2^B - 1)}{2^B}\right\}.$$  

(16)

The following proposition provides guidance for the selection of LISs with different phase bit precisions.

**Proposition 2:** To promise an acceptable ergodic spectral efficiency degradation of $\xi$ bits/s/Hz compared with using optimal full-resolution phase shift amounts, the number of quantization bits of the LIS should satisfy

$$B \geq \log_2 \pi - \log_2 \arccos \left(1 + \alpha N^{-2}\right) - \alpha N^{-2},$$

(17)

where

$$\alpha = \frac{1}{\gamma_1} \left(\gamma_2 N + 1 + \frac{1}{PM}\right).$$

(18)

**Proof:** See Appendix B.

**Proposition 2** indicates that if $\alpha = 0$, that is, the assistant channel is under Rayleigh fading condition, then $B \geq 0$. This conclusion is in accordance with the analytical results in Case 1 of Section III. Moreover, when $\alpha > 0$ remains unchanged, if $\xi = 0$, then $B \geq \infty$. This crucial requirement is released gradually as $\xi$ increases. These reasonable findings demonstrate the correctness of **Proposition 2**.

Then, according to **Proposition 2**, the minimum value of $B$ is inversely proportional to either $P$, $M$, or $N$. Thus, the LIS-assisted large-scale antenna system becomes less sensitive to the bit quantization constraint if the transmit power is enhanced or either the antenna array or the LIS is enlarged. We take $M = N = 64$, $K_1 = K_2 = 10$, and $P = 0$ dB as an example. When $\xi = 1$, $B \geq 2$ holds. Consequently, the ergodic spectral efficiency degradation does not exceed $1$ bit/s/Hz as well, if we remain $B = 2$ and set $P = 10$ dB,

2Note that the acquisition of $\theta_{\text{AoA},1}$ and $\theta_{\text{AoD},2}$ at the LIS is left for future work. One possible solution is to measure $\theta_{\text{AoA},1}$ when initially setting up the LIS, and then $\theta_{\text{AoD},2}$ using pilots sent from the user.
ergodic spectral efficiency degradation of less than 1 bit/s/Hz. We set phase shift design for the LIS-assisted large-scale antenna system. We rewrite (19) as

\[ r = g f_1^H s + h_2 f_2^H y + w_2, \]  

(19)

where \( y = \sqrt{P} u^H H_1 f_1^H s + u^H w_1 \) is the signal received by the AF relay node; \( f_1 = (u^H H_1 + g) / \| u^H H_1 + g \| \) and \( f_2 = h_2 / \| h_2 \| \) are the MRT precoding vectors at the BS and the relay node, respectively; \( u \in \mathbb{C}^{N \times 1} \) is the first column of the left singular matrix of \( H_1 \); and \( w_1 \in \mathbb{C}^{N \times 1} \) and \( w_2 \) are additive noises at the relay node and at the user, respectively. We rewrite (19) as

\[ r = \sqrt{P} (h_2 f_2^H u^H H_1 + g) f_1^H s + h_2 f_2^H u^H w_1 + w_2. \]  

(20)

Considering half duplex, the ergodic spectral efficiency of the AF relay system is

\[ C = \frac{1}{2} \mathbb{E} \left\{ \log_2 \left( 1 + \frac{P \| h_2 f_2^H u^H H_1 + g \| ^2}{\sigma_w^2 \| h_2 f_2^H u^H H_1 + g \|^2 + 1} \right) \right\}. \]  

(21)

Fig. 2. Comparison of the Monte Carlo results and the upper bounds of the ergodic spectral efficiency.

We also compare the ergodic spectral efficiency of the amplify-and-forward (AF) relay system. To make a fair comparison with the LIS-assisted system, the AF relay node does not amplify the forwarded signal. We apply MRT and singular value decomposition-based receiving at each hop. The signal received by the user is

\[ r = g f_1^H s + h_2 f_2^H y + w_2, \]

(19)

where \( y = \sqrt{P} u^H H_1 f_1^H s + u^H w_1 \) is the signal received by the AF relay node; \( f_1 = (u^H H_1 + g) / \| u^H H_1 + g \| \) and \( f_2 = h_2 / \| h_2 \| \) are the MRT precoding vectors at the BS and the relay node, respectively; \( u \in \mathbb{C}^{N \times 1} \) is the first column of the left singular matrix of \( H_1 \); and \( w_1 \in \mathbb{C}^{N \times 1} \) and \( w_2 \) are additive noises at the relay node and at the user, respectively. We rewrite (19) as

\[ r = \sqrt{P} (h_2 f_2^H u^H H_1 + g) f_1^H s + h_2 f_2^H u^H w_1 + w_2. \]  

(20)

Considering half duplex, the ergodic spectral efficiency of the AF relay system is

\[ C = \frac{1}{2} \mathbb{E} \left\{ \log_2 \left( 1 + \frac{P \| h_2 f_2^H u^H H_1 + g \| ^2}{\sigma_w^2 \| h_2 f_2^H u^H H_1 + g \|^2 + 1} \right) \right\}. \]  

(21)

We first test the tightness of the upper bound of ergodic spectral efficiency in Proposition 1. The optimal phase shift is adopted. As shown in Fig. 2, the upper bound is tightly closed with the Monte Carlo results. With the increase of the Rician K-factor, the gap between the Monte Carlo results and the upper bound diminishes. When the Rician K-factor grows to infinity, the ergodic spectral efficiency approaches a constant, as discussed in Case 2. These results demonstrate the correctness of Proposition 1. Hence, designing the phase shift amount based on the upper bound is reliable. Besides, given the half-duplex mode and the existence of noise at the relay node, the AF relay system has lower ergodic spectral efficiency than the LIS-assisted system.

Next, we evaluate the optimal phase shift design under \( K_1 = K_2 = 10 \) dB. For comparison, we examine the case when the phase shift amount is randomly set, and the results of 10,000 types of random amount are collected. The results are shown in Fig. 3. The ergodic spectral efficiency of the optimal phase shift design is considerably higher than that of using random phase shift amount. When the number of reflectors increases, this advantage is further enlarged. While using random phase shifts under Rician fading condition achieves even lower ergodic spectral efficiency than under Rayleigh fading condition because using inadequate phase shift amount may damage the assistant channel. Therefore, proper design of phase shift amount is essential in the LIS-assisted system.

Finally, we test the performance degradation when bit quantization is considered. As shown in Fig. 4, the ergodic spectral efficiency decreases more than 1 bit/s/Hz if 1-bit quantization is adopted. The performance gap between the ergodic spectral efficiency of using perfect phase shifts and that of using quantized phase shifts decreases with the increase of the phase shift amount.
quantiﬁcation bit. The performance degradation is below 1 bit/s/Hz when using 2-bit quantization, which is in accordance with the theoretical results derived from Proposition 2.

VI. CONCLUSION

In this study, we evaluated the ergodic spectral efﬁciency of the LIS-assisted large-scale antenna system by formulating an upper bound and discussed the signiﬁcance of using a proper phase shift design based on this expression. Particularly, we proposed an optimal phase shift design to maximize the ergodic spectral efﬁciency and obtained the requirement on the quantization bits to ensure an acceptable spectral efﬁciency degradation. Tightness of the upper bound was veriﬁed through Monte Carlo simulations, and the proposed phase shift design proved to considerably enhance the spectral efﬁciency performance than using random phase shifts. Moreover, the numerical results showed that 2-bit quantization can sufﬁciently guarantee high spectral efﬁciency, which is in accordance with the analytical results.

VII. APPENDIX A

We aim to prove Proposition 1. According to the Jensen’s inequality, it holds that
\[
\mathbb{E}\{\log_2(1 + x)\} \leq \log_2(1 + \mathbb{E}\{x\}).
\] (22)
Besides, \(\sigma_w^2 = 1\). Hence, (6) satisﬁes
\[
C \leq \log_2\left(1 + P\mathbb{E}\left\{||h_2^*\Phi H_1 + g||^2\right\}\right).
\] (23)
Then, we focus on the derivation of \(\mathbb{E}\{||h_2^*\Phi H_1 + g||^2\}\). We decompose \(||h_2^*\Phi H_1 + g||^2\) by (24) at the bottom of this page. The ﬁrst item \(x_1\) is constant, and \(\mathbb{E}\{x_1\} = 0\) holds for \(i = 2, 3, 4\). Since \(H_1, h_2\) and \(g\) have zero means and are independent with each other, we can derive that
\[
\mathbb{E}\left\{||h_2^*\Phi H_1 + g||^2\right\} = \mathbb{E}\{||g||^2\} + \frac{\mathbb{E}\{||x_1||^2\}}{(K_1 + 1)(K_2 + 1)} + \frac{\mathbb{E}\{||x_2||^2 + ||x_3||^2 + ||x_4||^2\}}{(K_1 + 1)(K_2 + 1)^2}.
\] (25)
For the channel between the BS and the user, it holds that
\[
\mathbb{E}\{||g||^2\} = M.
\] (26)
For the assistant channel, we derive that
\[
\begin{align*}
\mathbb{E}\{||x_2||^2\} &= K_1||H_1||_F^2 = K_1MN, \\
\mathbb{E}\{||x_3||^2\} &= K_2M||h_2||^2 = K_2MN, \\
\mathbb{E}\{||x_4||^2\} &= M\mathbb{E}\{||h_2||^2\} = MN.
\end{align*}
\] (27)
By applying (26) and (27) into (25), we obtain
\[
\mathbb{E}\left\{||h_2^*\Phi H_1 + g||^2\right\} = \gamma_1||h_2^*\Phi H_1||^2 + \gamma_2MN + M.
\] (28)
Finally, (7) is obtained.

VIII. APPENDIX B

We aim to prove Proposition 2. Based on (12) and (15), we write the upper bound of the ergodic spectral efﬁciency under bit quantization constraint as
\[
C_{\text{ub}BQ} = \log_2\left(1 + P M(\gamma_1||z||^2 + \gamma_2N + 1)\right).
\] (29)
For the \(n\)th element of the LIS, we denote the quantization error of the phase shift amount as \(\delta_n\), which satisﬁes
\[
\frac{2\pi}{2B + 1} \leq \delta_n \leq \frac{2\pi}{2B + 1}, n = 1, \ldots, N.
\] (30)
When the optimal phase shift amounts are employed, it holds that \(z = \sum_{n=1}^N e^{j\delta_n}\). Suppose that \(N\) is even. Since \(B \geq 1\), according to (30), we obtain
\[
|z|^2 \geq \left|\frac{N}{2}(e^{j\pi B + \pi} + e^{-j\pi B + \pi})\right|^2 = N^2\cos^2\left(\frac{\pi}{2B}\right).
\] (31)
It is obvious that degradation occurs under bit quantization constraint. We discuss how many bits are sufﬁcient to promise the acceptable spectral efﬁciency degradation of \(\xi\), i.e., \(C_{\text{ub}BQ} \leq \xi\). Applying (15) and (29), we derive that
\[
|z|^2 \geq (N^2 + \alpha)2^{-\xi} - \alpha,
\] (32)
Recalling (31), we obtain the requirement on bit quantization
\[
N^2\cos^2\left(\frac{\pi}{2B}\right) \geq (N^2 + \alpha)2^{-\xi} - \alpha,
\] (33)
which is further translated to (17).

REFERENCES

[1] B. Rankov, and A. Wittneben, “Spectral efﬁcient protocols for half-duplex fading relay channels,” IEEE J. Sel. Areas Commun., vol. 25, no. 2, pp. 379-389, Feb. 2007.
[2] T. J. Cui, M. Q. Qi, X. Wan, J. Zhao, and Q. Cheng, “Coding metamaterials, digital metamaterials and programmable metamaterials,” Light Sci. Appl., vol. 3, pp. 1-9, Oct. 2014.
[3] L. Zhang, X. Q. Chen, S. Liu, Q. Zhang, J. Zhao, Y. Dai, G. B. Bai, X. Wan, Q. Cheng, G. Castaldi, V. Galdi, and T. J. Cui, “Space-time-coding digital metasurfaces,” Nat. Commun., vol. 9, pp. 1-11, Oct. 2018.
[4] J. Zhao, X. Yang, J. Y. Dai, Q. Cheng, X. Li, N. H. Qi, J. C. Ke, G. D. Bai, S. Liu, S. Jin, and T. J. Cui, “Controlling spectral energies of all harmonics in programmable way using time-domain digital coding metasurface,” arXiv:1806.04414, Jun. 2018.
[5] W. Tang, J. Dai, M. Chen, X. Li, Q. Cheng, S. Jin, K. K. Wong, and T. J. Cui, “A programmable metasurface based RF chain-free 8PSK wireless transmitters,” IET Electronics Letters, 2019.
[6] C. Huang, G. C. Alexandropoulos, A. Zappone, M. Debbah, and C. Yuen, “Large intelligent surfaces for energy efﬁciency in wireless communication,” arXiv:1810.06934, Oct. 2018.
[7] C. Huang, A. Zappone, M. Debbah, and C. Yuen, “Achievable rate maximization by passive intelligent mirrors”, in Proc. IEEE ICASSP, 2018, pp. 3714-3718.
[8] Q. Wu, and R. Zhang, “Intelligent reﬂecting surface enhanced wireless network: Joint active and passive beamforming design,” in Proc. IEEE GLOBECOM, 2018, pp. 1-6.
[9] Q. Wu, and R. Zhang, “Beamforming optimization for intelligent reﬂecting surface with discrete phase shifts,” in Proc. IEEE ICASSP, 2019, pp. 7830-7833.