A Model and a Task Scheduling Method for Double-Deep Tier-Captive SBS/RS with Alternative Elevator-Patterns

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ABSTRACT

The last decade has brought the blooming of shuttle-based automated storage and retrieval system (SBS/RS) in many industrial applications thanks to its high throughput and expandability in handling mini-load commodity. In the paper, we focus on a double-deep tier-captive SBS/RS. Specifically, we propose a simulation model to support analyzing the performance related data of a double-deep tier-captive SBS/RS with both single-elevator and double-elevator patterns. The data includes shuttle carrier waiting time, elevator idle time, and total outbound time. Then, the impact of system performance due to two different elevator-patterns has been analyzed quantitatively. Based on the model, an optimization model and an improved NSGAII algorithm for task scheduling are designed, which can make the SBS/RS achieve the shortest shuttle carrier waiting time, rearrangement time and total outbound time. Experiments are conducted considering diverse task sizes and load factors, and the two elevator-patterns in a SBS/RS. The results show that our work can optimize the performance of the SBS/RS and assist stakeholders to decide more economical elevator-pattern.

INDEX TERMS

Simulation model, Elevator-pattern, Multi-objective optimization, Shuttle-based storage and retrieval system, Task scheduling

I. INTRODUCTION

A typical shuttle-based automated storage and retrieval system (SBS/RS) consists of a certain number of storage racks, shuttle carriers, elevators, buffer areas and input/output (I/O) locations. In the last decade, SBS/RS has become a preferable solution for the automated handling of totes, cartons and bins in a lot of automated warehouses and manufacturers, because of its excellent throughput capacity and flexibility, such as Dematic, Knapp, Schaefer, Stoecklin, Savoye and so on [1].

A SBS/RS can have different configurations. Generally, from aspect of working patterns of shuttle carriers, there are two possible configurations, as tier-captive, in which, every tier has an exclusive shuttle carrier, as well as tier-to-tier, in which, a shuttle carrier can move more than one tier [2], [3]. From aspect of rack layout, a SBS/RS can be divided into single-deep system and double-deep system. In a single-deep system, there is only one rack in one side of a shuttle carrier track. Differently, in a double-deep system, there are two racks in one side of a shuttle carrier track respectively. That is to say, it is the advantage of a double-deep system that saving 50% of aisle space and effectively improving utilization rate of a SBS/RS. But, it is its disadvantage that there may be a situation that a tote in the front rack blocking a targeted tote in the behind rack, which brings the rearrangement time spent to arrange the blocking tote firstly and then return to original position for picking up the targeted tote.

Our work concerns about a double-deep tier-captive configuration, in which, the elevator is for moving totes crossing different tiers. In addition to these configurations, a SBS/RS, considering the performance and cost, can be designed with different numbers of elevators, such as a double-deep tier-captive SBS/RS with one elevator, a single-deep tier-to-tier SBS/RS with two elevators, and so on. Most stakeholders thought that more elevators mean higher performance. However, that was not the case. Actually, adding an elevator or not is a composite question about economic benefits. More elevators undoubtedly bring higher hardware investment and other costs to consider, such as equipment maintenance, equipment wastage, etc. During a particular working period...
of a SBS/RS, if the size of inbound/outbound tasks is small, redundantly running elevators will cause useless energy consumption and equipment wear, which has no contribution to critical performance of the SBS/RS, like delivery time, throughout put, and so on. Thus, for stakeholders of the SBS/RS, there exist very interesting questions about deciding the elevator pattern, for instance, how to analyze the impact of different elevator patterns on the performance of a defined SBS/RS, and whether the elevator pattern should be dynamically changed, when the SBS/RS is dealing with a different size of tasks in a certain business period.

We analyze existing researches on the SBS/RS. In summary, there are some literature of analytical and simulation models for the SBS/RS used to analyze related performance data, in which, the work about the single-deep SBS/RS is more than the double-deep SBS/RS. Additionally, for reducing delivery time, some researches concern about task scheduling optimization for a SBS/RS, and the objectives include one or more of the follows, as the shuttle carrier waiting time, the elevator waiting time and the total outbound time. Based on the analysis, it can be found that, in the double-deep SBS/RS, in order to find answers of the questions about the elevator-pattern, the further work is required to be finished. The first is to build a simulation model of the SBS/RS to analyze related performance of the SBS/RS with different elevator-patterns, like the shuttle carries waiting time, the elevator idle time and the quantization of the impact of a SBS/RS performance due to the elevator-pattern. Furthermore, it is required to be proposed that a task scheduling optimization method supporting different elevator-patterns SBS/RS. Especially, besides existing objectives of minimizing the shuttle carrier waiting time and the total outbound time, another optimized objective is considered, which is minimizing rearrangement time.

Hence, in this paper, we dedicate to these work for analyzing and optimizing the double-deep tier-captive SBS/RS with alternative elevator-patterns. Summarily, there are 3 contributions.

• Our established simulation model can support to analyze the double-deep tier-captive SBS/RS with both single-elevator pattern and double-elevator pattern, which is also the basic work to optimize task scheduling of the SBS/RS and analyze the impact of different elevator-patterns on a specific SBS/RS.

• In order to optimize performance of a SBS/RS, we build an optimization model and apply the elite strategy-based NSGAII [22] multi-objective optimization algorithm to solve the problem of task scheduling optimization, respectively on these two elevator-patterns. Three designed optimization objectives are the shortest shuttle waiting time, rearrangement time and total outbound time. Based on optimization objectives in existing work, we firstly add a new objective as the shortest rearrangement time in the double-deep SBS/RS. Comparing to only considering shuttle waiting time, our designed objectives mean higher utilization ratio of the shuttle carrier in the double-deep SBS/RS.

• We provide a series of experiments in order to verify and analyze our work in different running situations of a particular SBS/RS. Specially, the situations include the SBS/RS with incremental task sizes and changeable load factors, and the SBS/RS in single-elevator or double-elevator pattern. According to analysis of experimental results, we verify the optimized effectiveness of our proposed task scheduling method in diverse situations of the SBS/RS. Besides that, it also verifies that our work can assist stakeholders to determine the most beneficial load factor and preferred elevator-pattern to improve the performance and economic benefit of one SBS/RS.

The organization of this paper is as follows. In Section II, we state the literature contribution of the tier-captive SBS/RS system and analyze the necessity of our work. After that, main components and the structure of a specific SBS/RS are declared in Section III. Based on the system specification, in section IV, we present the proposed simulation model, including a general working procedure of a SBS/RS in subsection IV-B, the analysis of travel time in subsection IV-C and the analysis of the elevator-pattern related performance in subsection V. With the simulation model, in Section VI, we present our designed optimization model and related optimization method based upon NSGAI multi-objective optimization algorithm with the elite strategy. Moreover, in Section VII, there are related experiments to verify the effectiveness of the multi-objective method, the optimal value of the load factor and the impact of elevator-pattern on the system. Finally, we draw conclusions of the paper in Section VIII.

II. LITERATURE REVIEW

In this paper, we focus on the tier-captive SBS/RS. Therefore, in this section, we mainly present the state of the art about the tier-captive SBS/RS. Generally, there are two aspects of existing works. One is proposing analytical models and simulation models in order to analyze and evaluate the performance of the SBS/RS. The other is designing a better storage strategy or scheduling strategy to optimization the performance and consumption of the SBS/RS. Following, we will detail the literature from these two aspects.

Both analysis models and simulation models are two ways to describe and analyze the performance of the SBS/RS. Precisely, considering the analysis model, Lerher et al designed the travel time model to calculate single and dual command cycle times (SCC and DCC), which were used to evaluate the system performance of the single-deep SBS/RS [5, 6] or the double-deep SBS/RS [1]. But, in these proposed travel time model, the travel time of the shuttle and the elevator was considered respectively, and the interaction between the shuttle and the elevator was not taken into account. Meanwhile, other analytical models based on queuing networks were proposed, which considered both the shuttle and the elevator traveling time and the time spent waiting each other [4], [7]–[10].
Compared with the analysis model, the simulation model is another effective method to analyze the SBS/RS, which is closer to real storage environment and used to directly evaluate the performance indicators of the SBS/RS (e.g., average cycle time, equipment utilization rate, throughput, etc.) and verify different design variables (e.g., rack, tier, elevator, etc.). Particularly, Lerher et al [11], [12] proposed a simulation model of the SBS/RS to evaluate the average cycle time and the throughput, which showed the superiority of the SBS/RS by comparisons it with other storage systems. Furthermore, the simulation models concerning rack configurations were established. For example, Zhao et al [13] established a simulation model for automatic remodeling of different rack configurations, which could test a large number of rack configuration schemes and effectively determine the best solution. Similarly, Ekren et al [14] proposed a simulation model to optimize rack design based on the class storage strategy, and to evaluate the system performance according to the utilization rate of devices and cycle time of storage/retrieval transactions. Additionally, Ekren et al [15] concerned other design variables and carried out simulation modeling to determine the important variables affecting the SBS/RS performance, and determined the optimal value of these important variables.

In summary, based on analyzing the state of the art, there were existing more related literature about the single-deep SBS/RS, and to the best of my knowledge as well as the presentation of a review paper about the SBS/RS [4] published in 2020, the double-deep SBS/RS work was very few, merely including [1] and [21]. Differently with these existing models, we propose a simulation model to support performance analysis of the double-deep SBS/RS, consider the interaction of the shuttle and the elevator in the situation specially appeared in the double-deep pattern, such as rearranging block totes. Meanwhile, based on our proposed simulation model, we firstly concern the elevator configuration in the double-deep SBS/RS, which is another bottleneck variable affecting the system performance.

Besides the research for analyzing the performance, there were some works for the SBS/RS optimizing, e.g. [16], [17], [19], [20] and so on. Following, we mainly describe the literature of task scheduling optimization, which is related to our work. Particularly, Zhao et al [18] worked on the scheduling problem of two non-passing elevators in common rail SBS/RS. The genetic algorithm was used to optimize storage/retrieval requests for the shortest moving time of the collision-free elevator with acceleration and deceleration. In addition, Wang et al [19] proposed a NSGAII algorithm based task scheduling method to minimize the total outbound time, elevator idle time and shuttle waiting time.

After reviewing literature about task scheduling optimization in the SBS/RS, on one side, there are single-objective optimization and multi-objective optimization. For example, [17] and [19] considered multi-objective optimization, [18] and [20] considered single-objective optimization. Specially, as [20] presenting, the total outbound time was concerned in most literature of single-objective optimization. Compared with this single objective, multi-objective optimization can improve efficiency of the SBS/RS completely. On the other side, the literature above as well as most existing literature were applied in the single-deep SBS/RS. Differently, it is very few that the work to optimize task scheduling in the double-deep SBS/RS. Specially, a noted one of them was that Wang et al [20] designed the shortest total outbound time as the optimization objective, and indicated an improved simulated annealing algorithm to solve task scheduling. However, this work simplified the optimized time in the double-deep SBS/RS, in which, the rearranging time of the shuttle to deal with the block tote, brought by the double-rack was completely ignored.

Therefore, in our paper, a task scheduling optimization method for the double-deep SBS/RS is designed. This method considers the whole working process of storage/retrieval requests in the double-deep SBS/RS, which aims at minimizing the shuttle rearrangement time, the shuttle waiting time and the total outbound time, so as to get more completely optimization. Meanwhile, based on our proposed the simulation model, we can analyze these three times affected by alternative elevator-patterns.

### III. SYSTEM SPECIFICATION

In order to analyze a SBS/RS, it is necessary to model and conceptualize the SBS/RS from different views. Hence, it is the first step of our work that we present a specified SBS/RS. As Figure 1 and Figure 2 showing, we separately present the double-deep SBS/RS with single-elevator and double elevator from the side view and the top view. Precisely, the basic components of a double-deep tier-captive SBS/RS include the storage rack with two lanes due to the double-deep configuration, the elevator with one shuttle carrier in lifting table for moving totes crossing different tiers, the shuttle carrier for moving totes in one tier, the buffer position in each tier, and the I/O (input/output) location in bottom tier. Following, we describe how to execute the storage task and the retrieve task by a SBS/RS with single-elevator pattern and double-elevator pattern separately.

As Figure 1 presenting, in a double-deep tier-captive SBS/RS with single-elevator pattern, there are one elevator with one shuttle carrier in lifting table for vertical movement and one shuttle carrier in each tier for horizontal movement. Furthermore, in the single-elevator pattern, the buffer area is in one side of the rack besides the elevator in each tier. During a whole process of a storage task executing, firstly, the shuttle carrier inside the elevator carries up the tote from I/O point to the right tier buffer position. After that, in related tier, the shuttle carrier running between two adjacent storage racks can execute moving task for carrying up the tote from the buffer area to the storage position. During a whole process of a retrieve task executing, the shuttle carrier in related tier is required to picking up targeted tote from the storage position to the tier buffer area, and then the shuttle carrier inside the elevator can move this tote from the buffer area to the I/O
point.

What calls for special attention is that, because of the double-deep storage rack, it is possible that the directed position is placed in the first lane, which is close to the shuttle truck, or in the second lane, which is behind the first lane. That is to say, on one hand, considering the process of a storage, if the selected storage location for a tote is in one position of the second lane, but the corresponding position of the first lane has a blocking tote, the blocking tote needs to be moved in a free position firstly to make the shuttle carrier normally putting the tote in the second lane. One other hand, for a retrieve process, if the targeted tote is in the second lane, and there is also a blocking tote in the related position of the first lane, the shuttle carrier requires to arrange the blocking tote in a free position firstly to ensure normal delivery of targeted tote in the second lane. Hence, in order to reducing blocks, for executing an inbound task, the tote is usually prior putted at a position in the second lane, and the first lane is selected until the second lane is full.

Based on specification of a double-deep tier-captive SBS/RS with single-elevator pattern above, we further explain a double-deep tier-captive SBS/RS with double-elevator pattern from the view of their differences. As Figure 2 presenting, in a double-elevator pattern system, an elevator is added behind the rack. That is to say, there are two elevators in two sides of the rack, each of which is with one shuttle carrier in lifting table for vertical movement. Meanwhile, the related buffers of each tier and I/O point in the bottom tier are added.

Considering the storage process and the retrieve process, the basic processes is not sensitive to two elevator-patterns. Differently, in a double-elevator pattern, during a whole process of a storage task executing, if there are totes to be put in storage in two side buffer areas in certain tier, the shuttle carrier in this tier can execute moving task according to the principle of first come first served (FCFS) that means a tote arrive to the buffer area early, which is carried up early by the shuttle carrier. Meanwhile, during a whole process of a retrieve task executing, it is required to decide the side buffer area for placing delivered tote and the elevator for moving this tote from this buffer area to the I/O point.

**IV. SIMULATION MODEL**

Based on the system specification, we propose a simulation model of the double-deep tier-captive SBS/RS. With two following reasons, the proposed simulation model focuses on outbound tasks. Firstly, the outbound task is the retrieval transaction, which is the most critical activity that represents the service level of a SBS/RS system. Secondly, other tasks could be modeled similarly as the outbound task.

The retrieval transaction could be divided as following.

- The shuttle carrier in the designated tier moves to the retrieval location to pick-up the tote, and then travels to the buffer position. Especially, if one block tote is in front shelf, and the targeted tote is in the behind shelf, the shuttle carrier firstly pick-up the block tote to another place and return to the original retrieval location to execute the above picking-up process. The time spent in this process is the shuttle rearrangement time.
  - The shuttle carrier releases the tote in the buffer position.
  - The appropriate elevator (with lifting table) moves to the designated tier and picks up the tote from the buffer position.
  - The appropriate elevator (with lifting table) moves to the I/O point (first tier) with the tote and releases it.

![FIGURE 1. Side view and top view of double-deep SBS/RS with single-elevator](image1)

![FIGURE 2. Side view and top view of double-deep SBS/RS with double-elevator](image2)
A. ASSUMPTIONS AND NOTATIONS
According to practical observations as well as academic studies for SBS/RS, we make the following fundamental assumptions.

- Adopt random storage strategy. With this strategy, the probability of the tote being stored in one tier and one position is the same.
- For each tier, there is at least one empty location in the first lane all the time, in order to make sure to avoid the situation that directed tote is located in the second lane and the rack is full.
- Both the elevator and the shuttle carrier follow the pattern of Point-of-Service-Completion (POSC). That is to say, they stop at the position where they finish one task. Considering about the outbound task, the elevator always stops at the I/O point of the system, and the shuttle carrier always stops at the position next to the buffer in related tier.
- The I/O platform is located at first tier, near the lift location.
- The shuttle carrier and the elevator follow the principle of First-Come-First-Service (FCFS).
- The retrieval task obeys the single-command cycle operation.
- The length and the height of the storage rack can provide enough distance for the shuttle carrier and the elevator accelerating to the maximum speed.

Furthermore, notations that appear in this paper are listed as follows. Precisely, Table 1 lists parameters of a SBS/RS, and Table 2 provides parameters of the shuttle carrier and the elevator.

### TABLE 1. Parameters of a SBS/RS

| Parameter | Definition |
|-----------|------------|
| \( h \)   | The height of the tier |
| \( c \)   | The length (depth) of the column |
| \( N_t \) | The number of tiers |
| \( N_c \) | The number of columns |
| \( N_r \) | The number of rows |
| \( m_{\text{buffer}} \) | The maximum capacity of the buffer at the moment |

### TABLE 2. Parameters of equipments

| Parameter | Definition |
|-----------|------------|
| \( V_{\text{max}} \) | The maximum velocity of the shuttle carrier |
| \( V_{\text{max}} \) | The maximum velocity of the elevator |
| \( a_s \) | The acceleration/deceleration of the shuttle carrier |
| \( a_l \) | The acceleration/deceleration of the elevator |
| \( t_g \) | The time of a shuttle carrier picking-up and setting-down a tote |
| \( S_{\text{real}} \) | The distance from the current position to target position |
| \( S_{\text{critical}} \) | The minimum required distance for accelerating to the maximum speed and then decelerating to 0 |

Moreover, Table 3 and 4 separately state related notations about tasks and notations of time used in our analyzing and optimizing work.

### TABLE 3. Notations of tasks

| Notation | Definition |
|----------|------------|
| \( N_{\text{task}} \) | The total number of outbound tasks |
| \( N_{\text{t}} \) | The number of outbound tasks at tier \( t \) |
| \( Task_{i,t} \) | Task \( i \) at tier \( t \) |
| \( Task_i \) | The \( i \)th task in all the outbound tasks assigned to one elevator |

### TABLE 4. Notations of time

| Notation | Definition |
|----------|------------|
| \( t_i \) | For \( Task_{i,t} \), the time required for the shuttle carrier to travel \( S_{\text{real}} \) |
| \( t^\text{total}_{i,t} \) | For \( Task_{i,t} \), the total travel time required for the shuttle carrier |
| \( t_w \) | For \( Task_{i,t} \), the time of the shuttle waiting for the elevator |
| \( t_l \) | For \( Task_{i,t} \), the travel time of the elevator from I/O point to target tier |
| \( t^\text{total}_i \) | For \( Task_i \), the total running time required for the elevator |
| \( t_{l} \) | For \( Task_i \), the elevator idle time |
| \( t_r \) | For \( Task_i \), the shuttle carrier rearranging time |
| \( t_r \) | The total rearrangement time for all the shuttle carriers |
| \( t_w \) | The total shuttle carriers waiting time for the elevator |
| \( t^\text{total} \) | The total required time for all outbound tasks |
| \( t_f \) | The total idle time of one elevator |
| \( SWT \) | The shuttle carrier waiting time |
| \( RT \) | The rearrangement time |
| \( TOT \) | The total outbound time |

B. GENERAL PROCEDURE
Based on mentioned assumptions, we propose a simulation model of the SBS/RS. The Figure 3 presents a general procedure of carrying out the outbound task in the proposed simulation model. In detail, the rack is initialized firstly for ensuring that the tote to be shipped is not at an empty position within random generation strategy. After that, the outbound tasks are classified according to different tiers, and the tasks in the same tier are added to corresponding shuttle carrier queue of this tier. Specially, there is one shuttle carrier queue related to each tier, and the task in one shuttle queue is executed according to FCFS principle.

Following, we explain how an outbound task could be finished with two key steps in the procedure of simulation model, as the shuttle carrier processing flow and the elevator processing flow. Firstly, we regard the workflow of the shuttle carrier, which is presented in blue in Figure 3. In detail, it is the first step that the shuttle carrier moves from on call position (the buffer position) to targeted position. There are two situations. One is that, there is nothing blocks. The shuttle carrier can move the tote directly. Contrarily, there is a blocking tote at corresponding position in the first lane. In this situation, the shuttle carrier needs to move the blocking tote to the nearest empty position firstly, in order to create a passable route. In the meantime, related information of this tote is updated. For example, if the blocking tote is in one shuttle task queue, the targeted position of this tote is updated. Dealing with these two situations, the next step is that the shuttle carrier transports the tote to the buffer area. In the event of the buffer area being full, the shuttle requires to wait for the elevator taking out certain tote from the buffer...
Initialize the rack and classify shuttle carrier

Start of a task (Task 1)

According to our analysis of the general procedure, we are minimizing travel time of shuttle carriers. The three rules are a more appropriate elevator in the step of the shuttle carrier. Therefore, it is another question to decide operating in parallel, and two buffer areas beside each elevator is a double-elevator system means that there are two elevators in the double-elevator procedure. As the system specification, it is the I/O point. After that, the elevator finishes all processes of executing one task and turns to being on call.

The above is the general procedure that is not sensitive to elevator-patterns. Based on that, we further analyze the double-elevator procedure. As the system specification, double-elevator system means that there are two elevators operating in parallel, and two buffer areas beside each elevator in one tier. Therefore, it is another question to decide a more appropriate elevator in the step of the shuttle carrier transporting the tote to related buffer area. At great length, we design a balancing strategy with three rules, which aims at balancing assigned tasks of two elevators as well as minimizing travel time of shuttle carriers. The three rules are as follows.

- When both buffer areas of one tier are not full, the elevator near targeted position is preferential.
- If only one buffer area is not full, the elevator beside free buffer area is preferential.
- When both buffer areas is full, the elevator with fewer assigned tasks is preferential.

C. TRAVEL TIME ANALYSIS

According to our analysis of the general procedure, we further analyze the core time consumption during the process of a task (Task k_i) executing. There are five crucial actions and the moment (timing) of each action happening, as listed in Table 5.

**Table 5.** The action and the moment

| Time | Corresponding action |
|------|-----------------------|
| T1_i | The shuttle carrier starting to process a tote |
| T2_i | The shuttle carrier arriving at the position of the buffer area |
| T3_i | The tote being placed at the position of the buffer area |
| T4_i | The elevator starting to process a tote |
| T5_i | The tote being placed at the I/O point |

1) The shuttle carrier waiting time

In the tier-captive SBS/RS, shuttle carriers in particular tier are independent and parallel. That is to say, we analyze the waiting time of one shuttle carrier due to the task queue of one tier. Precisely, T3_i is the same as T1_i, because the task Taskk_i^t is will be processed as soon as the shuttle carrier unloads the tote corresponding to Taskk_i^t−1. However, there is a waiting period of the shuttle carrier (t_w^i) between T1_i and T3_i, as Equation (1) presenting.

\[ t_w^i = T3_i - T2_i \] (1)

In detail, in Equation (1), T3_i = T4_i−n_buffer, the tote of Taskk_i^t placed in the buffer must wait for the elevator take the earlier tote out, so Equation (1) can be transformed into Equation (2).

\[ t_w^i = T4_i - n_buffer - T2_i \] (2)

We assume that \( \theta_i \) representing the state of the buffer area at the time of T2_i, then \( \theta_i \) could be stated as Equation (3).

\[ \theta_i = \begin{cases} 0, & T4_i - n_buffer \leq T2_i \\ 1, & T4_i - n_buffer > T2_i \end{cases} \] (3)

Particularly, if there is enough free capacity of the buffer area, there will be no waiting time of the shuttle carrier. In other words, if the number of totes in Taskk_i^t is less than current free capacity of the buffer area (n_buffer), there will be no waiting time of the shuttle carrier. Combining Equation (2) and (3), we can infer Equation (4).

\[ t_w^i = \begin{cases} 0, & \theta_i \times (T4_i - n_buffer - T2_i), \quad Taskk_i^t < n_buffer \\ \theta_i \times (T4_i - n_buffer - T2_i), \quad Taskk_i^t > n_buffer \end{cases} \] (4)

According to Table 3, Taskk_i^t represents that the i task in the t tier, and \( N_{task}^t \) represents that the number of outbound tasks at tier t. Hence, we can infer the total number of outbound tasks, \( N_{task} \), by Equation (5).

\[ N_{task} = \sum_{t=0}^{N_t-1} N_{task}^t \] (5)

Combining Equation (4) and (5), \( t_w^i \) can be inferred as Equation (6).

\[ t_w^i = \sum_{t=0}^{N_t-1} \sum_{i=0}^{N_{task}^t-1} t_w^i \] (6)
2) The elevator idle time

Considering about the elevator idle time, it is related with the whole process of performing an outbound task. We use $Task_{i}$ to represent the $i$-th task in all the outbound tasks. Correspondingly, for the double-elevator configuration, $Task_{i}$ represents the $i$-th task in the tasks assigned to one elevator. As soon as the elevator placing the previous tote (the tote in $Task_{i-1}$) at the I/O point, the elevator turns to idle state, until next tote (the tote in $Task_{i}$) being placed in the buffer area. That is to say, the idle time of the elevator is the period between $T_{i-1}^{5}$ and $T_{i}^{3}$, as presented in Table 5.

$$t_{i}^{f} = T_{i}^{3} - T_{i-1}^{5}$$  \hspace{1cm} (7)

It should be considered that the time from the shuttle carrier starting to handle the first outbound task to placing the tote in the buffer area cannot be avoided. Assuming that $\rho_{i}$ represents the state of the elevator at the time of $T_{i-1}^{5}$, $\rho_{i}$ could be modeled as Equation (8), in which, case 0 means the elevator is free, and case 1 means the elevator is executing one task.

$$\rho_{i} = \begin{cases} 
0, & T_{i}^{3} \leq T_{i-1}^{5} \\
1, & T_{i}^{3} > T_{i-1}^{5} 
\end{cases}$$  \hspace{1cm} (8)

Finally, combining Equation (7) and Equation (8), we can obtain the elevator idle time during the process of performing $Task_{i}$ is as Equation (9).

$$t_{i}^{f} = \rho_{i} \times (T_{i}^{3} - T_{i-1}^{5})$$  \hspace{1cm} (9)

Combining Equation (9), the total idle time of the one elevator is calculated as Equation (10).

$$t_{i}^{f} = \sum_{i=0}^{N_{task}-1} t_{i}^{f}$$  \hspace{1cm} (10)

3) Running time of the shuttle carrier and the elevator

As Table 5 showing, the shuttle travel time is the period between $T_{i}^{1}$ and $T_{i}^{2}$, and the elevator travel time is the period between $T_{i}^{4}$ and $T_{i}^{5}$. Additionally, the travel time of the shuttle carrier and the elevator is affected by their characteristics. In general speaking, there are two cases:

- When $S_{real}$ is not more than $S_{critical}$, the running process of the equipment mainly includes two processes:
  1. The process of accelerating from 0 to $v_{0}$;
  2. The process of decelerating from $v_{0}$ to 0.

- When $S_{real}$ is more than $S_{critical}$, the running process of the equipment mainly includes three processes:
  1. The process of accelerating from 0 to $v_{max}$;
  2. The process of running at a constant speed of $v_{max}$;
  3. The process of decelerating from $v_{max}$ to 0.

a) The shuttle carrier:

We consider about the distance of the shuttle carrier running. For $Task_{i}$, it is assumed that the directed tote is at the tier $t_{i}$, the column $x$, and the row $y$. We set the shuttle carrier starting to execute the outbound task from the position $(0,0,0)$.

Therefore, the actual distance of the shuttle carrier running to $x$, $S_{real}$, can be derived as Equation (11), in which, it is case 1 that the elevator is in front of the rack, and it is case 2 that the elevator is behind the rack.

$$S_{real} = \begin{cases}
 x \times c, & \text{case 1} \\
 (N_{c} - x - 1) \times c, & \text{case 2}
\end{cases}$$  \hspace{1cm} (11)

Meanwhile, the shuttle carrier critical distance $S_{critical}$ is calculated as Equation (12).

$$S_{critical} = \frac{V_{max}^{2}}{a_{s}}$$  \hspace{1cm} (12)

With Equation(11) and (12), the travel time of the shuttle carrier from the buffer area to column $x$ is as Equation (13).

$$t_{i}^{s} = \begin{cases}
 2 \times \frac{S_{real}}{V_{max}}, & S_{real} \leq S_{critical} \\
 2 \times \frac{S_{real}-S_{critical}}{V_{max}} + 2 \times \frac{V_{max}}{a_{s}}, & S_{real} > S_{critical}
\end{cases}$$  \hspace{1cm} (13)

And, the total time of the shuttle carrier to transport $Task_{i}$ is as Equation (14). Exactly, there are two cases, in which, case 1 is that departure point is the same as the unloading point, and case 2 is that departure point is different with the unloading point. Moreover, the single-elevator pattern is exclusively possible in case 1, and the double-elevator pattern is possibly in case 1 or case 2. Precisely, when the departure point is the same as the unloading point, the return time of the shuttle carrier is equal to the departure time as $t_{i}^{s}$, as shown in Figure 4 case 1. Oppositely, when the departure point and the unloading point are different, the shuttle carrier need drive to the position of targeted tote and select the other buffer area, as shown in Figure 4 case 2. Hence, $t_{i}^{s_{next}}$ is the return time of the shuttle carrier, $t_{g}$ is time picking-up and setting-down the tote by the shuttle carrier, and $t_{i}^{f}$ is the rearrangement time of the blocking tote, which will be detailed in Section VI.

$$t_{i}^{s_{total}} = T_{i}^{2} - T_{i}^{1}$$
$$= \begin{cases}
 t_{i}^{s} + t_{g} + t_{i}^{s_{next}} + t_{i}^{f}, & \text{case 1} \\
 t_{i}^{s} + t_{g} + t_{i}^{s_{next}} + t_{i}^{f}, & \text{case 2}
\end{cases}$$  \hspace{1cm} (14)

b) The elevator:

The actual distance of the elevator, $S_{real}$, can be derived as Equation (15), and the elevator critical distance $S_{critical}$ is as Equation (16).

$$S_{real} = t \times h$$  \hspace{1cm} (15)

$$S_{critical} = \frac{V_{max}^{2}}{a_{l}}$$  \hspace{1cm} (16)

With Equation (15) and (16), the travel time of the elevator from the I/O point to the targeted tier $t_{i}$, $t_{i}^{e}$, is calculated as

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Equation (17), and the total time of the elevator to transport Task_i is as Equation (18).

\[ t_i = \begin{cases} 
2 \times \sqrt{\frac{S_{\text{real}}}{v_{\text{max}}}}, & S_{\text{real}} \leq S_{\text{critical}} \\
S_{\text{real}} - S_{\text{critical}} + 2 \times v_{\text{max}} \frac{1}{a_i}, & S_{\text{real}} > S_{\text{critical}} 
\end{cases} \]

(17)

\[ t_i^\text{total} = T_i^5 - T_i^4 = t_i^l + t_g + t_i^l \]

(18)

V. ELEVATOR RELATED ANALYSIS

Many scholars [1], [5], [8], [11]–[14], [17], [18] have mentioned that the elevator is the bottleneck of improving system performance. Therefore, the one contribution of our paper is that analyzing the impact of single-elevator pattern and double-elevator pattern on a specific SBS/RS. Theoretically, the double-elevator system means double buffer areas and greater operating efficiency of the shuttle carrier. In addition, the pattern of double-elevator means processing outbound tasks in parallel, and reduces the total time by half. However, it is only a matter of theoretical analysis. The actual impact is much more complicated. The paper will provide deeper insight.

Consequently, in order to analyze the efficiency of the elevator-pattern, we state two quantifiable values, as \( \eta_{\text{TOT}} \) and \( \eta_{\text{SWT}} \). Perspectively, \( \eta_{\text{TOT}} \) is the change rate of TOT (the total outbound time) in the single-elevator pattern relative to the double-pattern, which is calculated as Equation (19). Moreover, \( \eta_{\text{SWT}} \) is the change rate of SWT (the shuttle waiting time) in the single-elevator pattern relative to the double-pattern, which is calculated as Equation (20). With these two values, we can measure the impact of the elevator-pattern and gain greater insight into it.

\[ \eta_{\text{TOT}} = \frac{\text{SingleElevator} t^{\text{total}} - \text{DoubleElevator} t^{\text{total}}}{\text{SingleElevator} t^{\text{total}}} \]

(19)

\[ \eta_{\text{SWT}} = \frac{\text{SingleElevator} t^{w} - \text{DoubleElevator} t^{w}}{\text{SingleElevator} t^{w}} \]

(20)

VI. TASK SCHEDULING MODEL AND OPTIMIZATION METHOD

The simulation model above provides the solution to analysis related performance value of the SBS/RS with alternative elevator-pattern. Based on that, we work on the optimization of the SBS/RS and the analysis of optimized results. Specially, we design a task scheduling method based on NSGAI to optimize the task scheduling with three goals, as minimizing the shuttle waiting time, the rearrangement time, and the total outbound time. Especially, we firstly consider the rearrangement time as one of multi purposes.

A. THE OPTIMIZATION MODEL

In the research of traditional automation distribution systems, the task scheduling problem is mostly converted into a trav-
eling salesman problem (TSP), in order to reduce the travel distance. However, the SBS/RS is not a simple question to only get the shortest sum of paths. Precisely, when a task is handed over from the shuttle carrier to the elevator, there are the shuttle carrier waiting time and the elevator idle time. Meanwhile, these two kinds of waiting time affect each other. Additionally, the load factor is the ratio of occupied storage locations to the total storage locations. In the double-deep SBS/RS, different load factors have a greater impact on the rearrangement time of blocking totes and cause the movement of unrelated goods, which can increase the shuttle travel time and reduce the utilization rate of the shuttle carrier. That is to say, we must reduce idle time of equipments and the rearrangement time of blocking totes. The most desired result is that the time interval between two tasks reaching the buffer area is exactly the time required for the elevator to execute the first task.

With these analysis, our general idea is that scheduling tasks in a time window, in order to achieve these three purposes, including effectively increasing the utilization rate of shuttle carriers and elevators, reducing waiting time of shuttle carriers and idle time of elevators, and avoiding rearrangements as many as possible.

Generally, the objective function for the task scheduling optimization is as Equation 21 presenting, which explains the design of optimization purpose $F$. Following, we will detail the three elements in the objective function $F$, as the shuttle waiting time $t_w$, the total outbound time $t_{total}$ and the rearrangement time $t_r$.

$$F = f_1 + f_2 + f_3$$
$$t_w = \min t_w + \min t_{total} + \min t_r$$  \hspace{1cm} (21)

1) The shuttle waiting time

In Equation 21, the shuttle waiting time, $t_w$, has been analyzed in section IV-C1, which can be obtained by Equation (6).

2) The total outbound time

From the aspect of the shuttle carrier, outbound tasks are executed in parallel. From the aspect of the elevator, outbound tasks are executed sequentially. Therefore, we calculate the total outbound time from the aspect of the elevator. Especially, in the double-elevator system, as traveling tasks assigned to both two elevators being finished separately, all the outbound tasks are finished. In detail, the total outbound time is composed of the shuttle carrier travel time of the first task, the elevator travel time of all tasks and the elevator idle time of all tasks, which is as Equation (22) showing. Specially, $t_i^{total}$ is calculated based on Equation (14), and $t_i^{total}$ is calculated based on Equation (18), $t_i^{r}$ is calculated based on Equation (9).

$$t_{total} = t_1^{total} + \sum_{i=0}^{N_{task}-1} t_i^{total} + t_i^r$$
$$t_1^{total} + \sum_{i=0}^{N_{task}-1} t_i^{total} + \sum_{i=0}^{N_{task}-1} t_i^r$$  \hspace{1cm} (22)

3) Rearrangement time

Rearrangement time is the time for a shuttle carrier rearranging the blocking tote and returning the position of targeted tote. The rearrangement time is generated, because that the position of targeted tote of one task is in the second lane, and the first lane is blocked by another tote. Hence, as Equation (23) presenting, the rearrangement time is composed of four parts, including the time picking blocking tote up, the time of the shuttle carrier running from the original position to the nearest empty position, the time setting blocking tote down, and the time of the shuttle carrier returning to the original position of the targeted tote. Especially, in the double-elevator system, the nearest empty position is determined relative to the assigned elevator. In Equation (23), $t_i^r$ represents the time required for the shuttle carrier to run from the original position to the nearest empty position, which is the same as the calculation method of Equation (13).

$$t_r = \sum_{i=0}^{N_{task}-1} t_i^{r}$$
$$= \sum_{i=0}^{N_{task}-1} (t_g + t_i^s + t_g + t_i^r)$$  \hspace{1cm} (23)

B. TASK SCHEDULING OPTIMIZATION METHOD

Based on this optimization model, a task scheduling optimization method is designed. We apply a non-dominated sorting genetic algorithm (NSGAI) with elite strategy used for multi-objective solution, which is described as Algorithm 1.

As Algorithm 1 presenting, we firstly read out the outbound list to generate targeted tasks, initialize the storage rack according to the load factor, and randomly generate initial population $P0$ in the size of $N$. The following step is that calculating fitness value of each individual according to our proposed simulation model. Based on obtained fitness values, we use non-dominated sorting to classify $P0$ and calculate the shared fitness value of each individual. With the binary tournament method, individuals of $P0$ are selected. Then, we perform crossover and mutation of these individuals, in order to produce new generation (offspring population) $Qt$. Furthermore, from the second generation, the parent population $Pt$ and the offspring population $Qt$ are merged to perform rapid non-dominated sorting. Meanwhile, it is carried out that the crowding degree calculation of the individuals in each non-dominated layer, and then $N$ individuals are selected to
Algorithm 1 NSGAII algorithm based task scheduling method

input: Related data of outbound tasks
output: Optimal tasks order and objective function value

1: function NSGAII(Tasks order, operating parameters)
2:   Data preprocessing
3:   Randomly generate initial population P0 in size of N
4:   Put P0 into proposed simulation model to calculate the fitness value and the shared fitness value
5:   Non-dominated sorting of the initial population P0
6:   Binary tournament method selection, order crossover, gene exchange mutation
7:   Generate offspring population Qt
8:   for gen = 2 → Number of iterations – 1 do
9:     Merge parent Pt and offspring population Qt
10:    Calculate fitness value of merge population in proposed simulation model
11:   Fast non-dominated sorting
12:   Crowding distance calculation
13:   Select N individuals through crowding distance and elite retention strategy to form a new parent population Pt
14:   Selection, Crossover, Mutation for new offspring population Qt
15: end for
16: return Optimal tasks order and Corresponding function value
17: end function

generate a new parent population Pt, according to crowding distance and elite retention strategy. Finally, new offspring populations Qt are generated following basic operations of genetic algorithms. We repeat the above steps until predefined conditions are satisfied. At this time, the Pareto optimal solution and the corresponding objective function value are obtained. Related key steps in the proposed task scheduling method with NSGAII are explained as follows.

- Code: A complete order of outbound tasks is used as a chromosome, which is formed as \((task_{t1}, task_{t2}, ..., task_{tN})\), where \(task_{ti}\) is a gene on one chromosome, which is determined by the position of the good \((t, r, c)\).
- Fast non-dominated sorting method: Fast non-dominated sorting is mainly to classify the population, which is divided into two parts.
  - Group the individuals that can be dominated by the individual into a set \(S_{p}\), and record the number of individuals dominated \(n_p\).
  - Make the individuals with \(n_p == 0\) the first front face \(F_1\) and assign the individual \(rank\) to 1. Remove each individual from \(F_1\), decrease \(n_p\) by 1 for each individual in the individual set \(S_{p}\), and add to the collection \(F_{i+1}\) if \(n_p == 0\). Repeat the above operation until the entire population is classified.
  - Crowding distance calculation: The crowding distance is equal to the sum of the distance between two solutions in the direction of each objective function. It could be divided into two steps as follows.
    - Step 1: \(n_d = 0, n = 1, 2, \ldots, N;\)
    - Step 2: For each objective function, first sort the population based on the objective function, and then make the crowding degree of the two individuals at the boundary infinite, which is \(L_d = N_d = \infty\), then \(n_d = n_d + (f_m(i + 1) - f_m(i - 1)), n = 2, 3, \ldots, N - 1\).
  - Elite strategy: After fast non-dominated sorting and crowding degree calculation, the front face \(F_i\) formed by the individual \(rank\) is added to the next generation population in order from low to high, when \(F_i\) is added to make the population size exceed the limit, individuals are added to the population according to the crowding distance from large to small.
  - Genetic algorithm operation: In our proposed method, the binary operator is used for selection, the order crossover operator is used for crossover, and the method of exchanging two random gene is used for mutation.

VII. EXPERIMENT ANALYSIS

In this section, we design and analyze a series of experiments, in order to verify our work in different running situations of the specific SBS/RS. Specially, the first experiment is about testing task scheduling method in the single-pattern SBS/RS and the double-pattern SBS/RS with incremental task sizes. The second experiment is dedicated to analyze the impact of two elevator-patterns on the SBS/RS system. Lastly, we change initial load factor of the SBS/RS to verify our work in different filled-grade SBS/RS and find the best load factor.

A. EXPERIMENT DATA

Our experiment data is from an actual SBS/RS based automated warehousing in the city of Kunshan in China. We use all the outbound tasks in three days to randomly generate 200 outbound tasks. Table 6 is listing parameters of related equipments and the warehouse. In the experiments, each task is carrying out a particular tote, the position of which is represented by three-dimensional coordinate of the SRS/BS, as the format of \((tier, row, column)\). Among them, the value of \(tier\) is an integer ranging from 0 to 5, the value of \(row\) is an integer ranging from 0 to 3, and the value of \(column\) is an integer ranging from 0 to 59. Specially, it is the tote in the second lane that \(row\) value is 0 or 3. Contrarily, \(row\) value being 1 or 2 means in the first lane.

B. EXPERIMENTS OF TASK SCHEDULING METHOD

Firstly, we take a series of experiments, in order to analyze the effectiveness of our proposed task scheduling method.
TABLE 6. Related parameters in experiments

| Parameter                        | Value         |
|----------------------------------|---------------|
| Number of tiers                  | 6             |
| Height of tier                   | 1.5m          |
| Number of rows                   | 4             |
| Number of columns                | 60            |
| Column width                     | 0.6m          |
| Maximum speed of the shuttle carrier | 2m/s         |
| Acceleration of the shuttle carrier | 1.2m/s²      |
| Maximum speed of the elevator    | 2m/s          |
| Acceleration of the elevator     | 1.2m/s²       |
| The time for the shuttle carrier picking-up a tote | 2s           |
| The time for the shuttle carrier setting-down a tote | 2s           |
| Maximum number of totes in the buffer | 2             |

Precisely, these experiments includes executing different number of tasks in SBS/RS with single-elevator pattern and double-elevator pattern respectively.

Besides, from aspect of NSGAII algorithm, all the results of the experiments are calculated with parameters as follows. The number of iterations is 2000, the number of population is 100, the crossover rate is 0.9, and the mutation rate is 0.02.

The results of experiments are listed in Table 7, in which, we collect related experimental data during executing different numbers of tasks in the SBS/RS with single-elevator pattern and double-elevator pattern, including the waiting time of the shuttle carrier (SWT), the rearrangement time (RT) and the total outbound time (TOT).

Based on the data in Table 7, we analyze our work for optimizing the performance of the SBS/RS. Firstly, we analyze separately two patterns of the SBS/RS. According to original unoptimized result and our optimized results, we draw Figure 5 and Figure 6 separately presenting value curves of SWT (in red), RT (in green) and TOT (in black) in the single-elevator pattern and the double-elevator pattern. In these two figures, the horizontal ordinate is the number of tasks, and the vertical coordinate is the time in second. Moreover, three dotted curves present original unoptimized results and the other three solid curves declare the optimized solution with our proposed task scheduling method. As these two figures presenting, it could be found that all solid curves are below corresponding dotted curves. That is to say, compared with the original results, the optimized values of three key time have become smaller, and the proposed task scheduling method obviously improves the system performance.

C. EXPERIMENTS OF THE IMPACT OF THE ELEVATOR-PATTERN

With our proposed simulation model with the single-elevator and double-elevator pattern, we research the impact of the elevator on the system performance, which can provide a lot of useful suggestions to stakeholders. For example, in design phase of the SBS/RS, it can assist to decide the configuration of the SBS/RS with related requirements of the performance and the budget. In operation phase, as the size of tasks changing in different period, it can suggest stakeholder to alter current running elevator-pattern dynamically and get more economical configuration. Therefore, based on the experiments above, we calculate the change rate of the total outbound time and the shuttle waiting time due to the contribution of the elevator-pattern under different task sizes. As Table 8 presenting, the number of tasks ranges from 10 to 150. And $\eta_{\text{SWT}}$ represents the change rate of the shuttle waiting time (SWT) in the single-elevator pattern relative to the double-pattern, which is calculated according to Equation 20 explained in Section V. Similarly, $\eta_{\text{TOT}}$ represents the change rate of the total outbound time (TOT) as Equation 19 in Section V. Specially, due to the divisor $\text{SingleElevator } t^w$ is 0, the value of SWT is an invalid value.

From the Table 8, it can be concluded that as the size of tasks increasing, the improvement rate for the total outbound time is getting higher and higher to approach 50% , but the improvement rate of the shuttle waiting time is decreasing from 100% to 96.12%. The improvement curve of the total outbound time and the shuttle waiting time are shown in Figure 7. We noticed that the curve is not smooth. This is because the outbound tasks are randomly generated, so that the degree of task scheduling optimization is different. But in
TABLE 7. Experimental results of SWT, RT and TOT in the single-elevator SBS/RS and the double-elevator SBS/RS

| Task number | Optimized results | Original results |
|-------------|-------------------|-----------------|
|             | Single-elevator pattern | Double-elevator pattern | Single-elevator pattern | Double-elevator pattern |
|             | SWT | RT | TOT | SWT | RT | TOT | SWT | RT | TOT |
| 10          | 0   | 39 | 124 | 0   | 39 | 98  | 0   | 42 | 101 |
| 20          | 11  | 47 | 229 | 0   | 44 | 165 | 38  | 58 | 175 |
| 30          | 92  | 84 | 312 | 0   | 92 | 195 | 193 | 96 | 235 |
| 40          | 229 | 118| 420 | 0   | 79 | 254 | 493 | 121| 278 |
| 50          | 506 | 162| 521 | 0   | 148| 303 | 759 | 173| 323 |
| 60          | 868 | 183| 634 | 0   | 171| 351 | 1129| 199| 387 |
| 70          | 1296| 211| 748 | 0   | 191| 402 | 1634| 216| 431 |
| 80          | 1641| 258| 854 | 0   | 272| 450 | 1892| 281| 475 |
| 90          | 1921| 333| 974 | 9   | 304| 511 | 2055| 344| 537 |
| 100         | 2391| 325| 1070| 12  | 323| 558 | 3176| 360| 577 |
| 110         | 2728| 390| 1219| 17  | 388| 645 | 2982| 402| 670 |
| 120         | 2930| 405| 1323| 26  | 399| 698 | 3390| 423| 743 |
| 130         | 3277| 440| 1457| 52  | 438| 769 | 3686| 458| 821 |
| 140         | 3505| 497| 1569| 108 | 476| 825 | 4083| 511| 891 |
| 150         | 4204| 577| 1678| 163 | 557| 883 | 4461| 597| 986 |

TABLE 8. Experimental results of changeable rate of TOT and SWT

| Task Number | \(\eta_{TOT}\) | \(\eta_{SWT}\) |
|-------------|---------------|---------------|
| 10          | 20.97%        | Invalid value |
| 20          | 27.95%        | 100.00%       |
| 30          | 37.50%        | 100.00%       |
| 40          | 39.52%        | 100.00%       |
| 50          | 41.84%        | 100.00%       |
| 60          | 44.64%        | 100.00%       |
| 70          | 46.26%        | 100.00%       |
| 80          | 47.31%        | 100.00%       |
| 90          | 47.54%        | 99.53%        |
| 100         | 47.85%        | 99.50%        |
| 110         | 47.09%        | 99.38%        |
| 120         | 47.24%        | 99.11%        |
| 130         | 47.22%        | 98.41%        |
| 140         | 47.42%        | 96.92%        |
| 150         | 47.38%        | 96.12%        |

general, it conforms to the trend we analyzed.

We can conclude that when the number of tasks is small, the contribution of the elevator is very small. That is to say, for the SBS/RS with low frequency outbound tasks, the double-elevator configuration is useless and unnecessary. However, when the number of tasks is relatively large, the double-elevator configuration becomes more and more useful, and the total outbound time is greatly improved as well as the elevator utilization is very high. The shuttle carrier waiting time greatly decreases due to the number of the elevator and the buffer increasing. Moreover, the double-elevator pattern equalizes the number of tasks on both sides and converts a part of the waiting time of the shuttle carrier into the running time.

In particularly, considering about the special size of tasks in experiments, we can make an analysis about the critical point. When the number of tasks is below 70, a single-elevator pattern is preferred, and as the number of tasks increasing, positive effect of applying a double-elevator pattern will be more obvious. When the number of tasks is larger than 100, a double-elevator pattern is totally preferred.
D. EXPERIMENTS OF THE LOAD FACTOR

The load factor is the ratio of occupied storage locations to the total storage locations. Considering about a SBS/RS system, it is the most expected situation that the outbound efficiency and the load factor are maximized. Therefore, we design interrelated experiments of load factors aims at verifying our work in different fill-graded SBS/RS.

In particular, we test the shuttle waiting time, the rearrangement time and the total outbound time in the SBS/RS with diverse load factors, including 0.5, 0.55, 0.6, 0.65, 0.7, 0.75, 0.8, 0.85, 0.9, 0.95 and the limit situation (0.996). Precisely, it is the limit situation that there are the minimize number of 6 free positions in the whole system to support the rearrangement of the blocking tote, in which, only one free position in each tier. Based on a number of experiments, we selected the Pareto optimal solution with the shortest rearrangement time and the shortest total outbound time. The results of SWT, RT and TOT are listed in Table 9.

In order to analyze the trend and the correlation, we draw Figure 8 and Figure 9 with obtained results. As these two figures representing, it could be found that as the load factor decreasing, the rearrangement time is decreasing, the shuttle waiting time is increasing, and the total outbound time is almost unchanged.

Following, we further analyze experiments results. Firstly, the results present that shuttle waiting time is inversely proportional to the rearrangement time. Furthermore, we insist on the total outbound time, TOT. From the aspect of the elevator, in the single-elevator pattern system (as the Figure 8 showing), if the transport capacity of the elevator can not satisfy the transport capacity of the shuttle carrier, the idle time of the elevator will be close to zero, and the essential time will be exactly used to transport totes. Hence, the minimum total outbound time is basically equal to the essential time, which is almost unchanged. On the other side, in the double-elevator pattern system (as the Figure 9 showing), if the transport capacity of the elevator covers the transport capacity of the shuttle carrier, so that the elevator should have idle time. However, the elevator idle time is basically within a small range after our optimization. As our analysis above, the total outbound time is equal to the sum of unchanged elevator essential time and sight fluctuated elevator idle time. Hence, there is a sight fluctuation existing in curve of the total outbound time (TOT). Meanwhile, as the load factor decreasing, the rearrangement time is decreasing, and the utilization rate of the shuttle carrier is gradually increasing, but the elevator can still completely cover the transport capacity of the shuttle carrier, so there is no waiting time for the shuttle carrier. Combining the two figures, we can conclude that the rearrangement time drops the fastest in the range of Group 1 to Group 2. That is to say, in our current specified SBS/RS, with the purpose of minimizing the total outbound time and the shuttle carrier energy consumption, the best load factor is about 0.9.

Lastly, as Figure 8 presenting, in the single-pattern system, the reduction of the load factor increases the waiting time of shuttle carriers, but the total outbound time is hardly changed. The reasons is as following. As the load factor decreasing, free positions of racks increases, which means that the rearrangement time of blocking totes could be reduced, and shuttle carriers can put totes into the buffer areas more quickly. Therefore, if the rate that the elevator moving totes out from the buffer areas is lower than the rate that totes arriving at the buffer areas, the buffer areas will be full, and shuttle carriers will require to wait the buffer areas have free space. It is the reason that the waiting time of shuttle carriers increases as the load factor reducing within a certain range. From the aspect of system optimization, the experiment results also show that it is another interesting topic that optimizing configuration and layout of the buffer area, the elevator and the shuttle carrier.

### VIII. CONCLUSION

In this paper, we proposed our work dedicated to analyze and optimize the double-deep tier-captive SBS/RS and concerned the impact of the elevator-pattern on the SBS/RS. Precisely, we built the simulation model to support analyze the workflow as well as diverse travel time of the shuttle carrier and the elevator in the SBS/RS with different elevator-patterns, and produced the simulation results to show the best elevator-pattern.
patterns. Based on this simulation model, in order to optimize the performance of the SBS/RS, we designed the optimization model and NSGAII based multi-objective optimization method, which can be used in the double-deep tier-captive SBS/RS with two elevator-patterns. In the our work of the optimization, we firstly added the minimum of the rearrangement time as one of optimization goals. The three optimization objectives are the shuttle waiting time, the rearrangement time and the total outbound time. Furthermore, we executed three kinds of experiments. With analysis of experimental results, we verified the optimized effectiveness of the proposed model and method in diverse situation of the SBS/RS. Meanwhile, we also verified that our work can assist stakeholders to determine the most beneficial load factor and preferred elevator-pattern to improve the performance and economic benefit of one SBS/RS.

In the future, we will continue our work from these aspects. One is considering more storage strategies, such as class-based storage policy. The other is analyzing a double-deep tier-captive SBS/RS with more changeable configurations, such as the number of shuttle carriers in one tier, the different elevator positions, etc. Moreover, we will attempt to let the elevator can decide and run in advance to the related tier, while the shuttle carrier is conducting the retrieval task, in order to make the system more intelligent. These works must be a very useful assistant for stakeholders of the SBS/RS.

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