Relation between pairing gaps and transition probabilities in $^{132,136}$Te

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Abstract. Nuclear masses were measured very precisely in the region around $^{132}$Sn recently. One of the new insights obtained from the measurements is the sudden decrease in the neutron pairing gap as the neutron number $N$ increases across 82. There was an indication of that decrease based on the experimental data of the anomalous first $2^+$ states of $^{132,136}$Te and the numerical results of the quasiparticle random-phase approximation (QRPA). We show a simple analytical explanation of the relation between the anomaly and the neutron pairing gap referring to the most recent experimental gaps and equations of the QRPA.

1. Introduction

One of the frontiers of nuclear physics is study of nuclear structures of very neutron-rich heavy-mass (e.g. mass number larger than 100) region. Information at this frontier is indispensable for nucleosynthesis, in particular the r-process [1]. Recently, significant progress has been made in the measurements of the nuclear masses around $^{132}$Sn [2, 3]. The neutron pairing gaps $\Delta_n$’s were deduced [2] from the measurements, using the three-point formula, and two remarkable properties were reported: one is the different $Z$-dependences of $\Delta_n$ between $N = 81$ and 83, and another is the significant decrease in $\Delta_n$ in $N > 82$ compared to $N < 82$.

The explanation of the former property remains to be an open problem, while the latter has been discussed about 10 years ago from an abnormal relation of the first $2^+$-state energies $E_{2^+_1}$ and the reduced transition probabilities $B(E2;0^+ \rightarrow 2^+_1)$ (hereafter denoted by $B(E2)\uparrow$) of $^{132,136}$Te by the calculation of the quasiparticle random-phase approximation (QRPA) [4] and independently by a shell-model study [5] but without the recent very precise experimental pairing gaps. The purpose of this study is to re-examine the relation between the anomaly and the decrease in $\Delta_n$ by considering a simplified analytic explanation, not shown in Ref. [4], with the most recent $\Delta_n$’s in Ref. [2].

2. Analytical explanation

The experimental values of $E_{2^+_1}$ and $B(E2)\uparrow$ of $^{132,136}$Te are shown in Tab. 1. Both $E_{2^+_1}$ and $B(E2)\uparrow$ decrease by adding 4 neutrons to $^{132}$Te; this relation does not obey an empirical rule [8, 9] that $E_{2^+_1}$ and $B(E2)\uparrow$ are inversely proportional in many experimental data. We explain this anomalous behavior of the $2^+_1$ states of those two nuclei based on the QRPA with several approximations for simplicity.
Table 1. Experimental $E_{21}^+$ and $B(E2)^\dagger$ of $^{132,136}\text{Te}$ [6, 7]. The $B(E2)^\dagger$ of $^{136}\text{Te}$ may be larger by 20% according to more recent data.

|              | $E_{21}^+$ (keV) | $B(E2)^\dagger$ |
|--------------|------------------|-----------------|
| $^{132}\text{Te}$ | 974              | 0.172(17)       |
| $^{136}\text{Te}$ | 606              | 0.103(35)       |

First, we assume that those states consist of only a two-neutron quasiparticle excitation and a two-proton quasiparticle excitation approximately. This noncollective assumption comes from a fact that nuclei with the proton and the neutron numbers close to the magic numbers do not have many low-energy particle-hole (p-h) excitations compared to the mid-shell nuclei. Indeed, the fact that nuclei with the proton and the neutron numbers close to the magic numbers do not change. The same approximation for the neutrons is also used, because the average angular momenta of a few neutron levels above and below the Fermi level with $\Delta n = 82$ shell gap are close; the levels above are $1h_{9/2}$, $3p_{3/2}$ and $2f_{7/2}$, and those below are $2d_{3/2}$, $1h_{11/2}$ and $3s_{1/2}$. Thus, the average node quantum numbers are also close, and it is assumed that the average matrix element of $Q$ with the relevant neutron states for $^{132}\text{Te}$ and $^{136}\text{Te}$ are close to each other. The third and fourth approximate equations (3) are based on an assumption that the single-particle levels corresponding to the $\mu_n$, $\nu_n$, $\mu_p$ and $\nu_p$ are close to the Fermi level with $\Delta_p$ being the proton pairing gap. By inserting Eq. (3) to Eq. (1) for $^{132}\text{Te}$ and Eqs. (2) and (3) to the corresponding equations for $^{136}\text{Te}$, the ratio of the proton and neutron amplitudes of the two nuclei are obtained

$$
X_{\mu\nu}^n = \frac{1}{\mathcal{N}} \langle \mu_n | Q | \nu_n \rangle, \quad X_{\mu'\nu'}^p = \frac{1}{\mathcal{N}} \langle \mu_p | Q | \nu_p \rangle,
$$

where $\mathcal{E}_n^\mu$ denotes the quasiparticle energy of the neutron state $\mu$ relevant to the $2^+_1$ state, and $\langle \mu_n | Q | \nu_n \rangle$ is the reduced matrix element of $Q$. $\mathcal{N}$ stands for the normalization factor. The backward amplitudes are ignored here.

We introduce 4 approximate equations:

$$
\langle \mu_p' | Q | \nu_p' \rangle^{^{132}\text{Te}} \simeq \langle \mu_p' | Q | \nu_p' \rangle^{^{136}\text{Te}}, \quad \langle \mu_n | Q | \nu_n \rangle^{^{132}\text{Te}} \simeq \langle \mu_n' | Q | \nu_n' \rangle^{^{136}\text{Te}},
$$

$$
\mathcal{E}_p^\mu + \mathcal{E}_p^\nu \simeq 2\Delta_p, \quad \mathcal{E}_n^\mu + \mathcal{E}_n^n \simeq 2\Delta_n,
$$

The first equation of the approximation (2) is justified, because the proton configuration does not change. The same approximation for the neutrons is also used, because the average angular momenta of a few neutron levels above and below the $N = 82$ shell gap are close; the levels above are $1h_{9/2}$, $3p_{3/2}$ and $2f_{7/2}$, and those below are $2d_{3/2}$, $1h_{11/2}$ and $3s_{1/2}$. Thus, the average node quantum numbers are also close, and it is assumed that the average matrix element of $Q$ with the relevant neutron states for $^{132}\text{Te}$ and $^{136}\text{Te}$ are close to each other. The third and fourth approximate equations (3) are based on an assumption that the single-particle levels corresponding to the $\mu_p$, $\nu_p$, $\mu_n$ and $\nu_n$ are close to the Fermi level with $\Delta_p$ being the proton pairing gap. By inserting Eq. (3) to Eq. (1) for $^{132}\text{Te}$ and Eqs. (2) and (3) to the corresponding equations for $^{136}\text{Te}$, the ratio of the proton and neutron amplitudes of the two nuclei are obtained

$$
\frac{(X_{\mu'\nu'}^p/X_{\mu\nu}^n)^{^{132}\text{Te}}}{R} = \frac{(2\Delta_n - E_{21}^+)_{^{132}\text{Te}}}{(2\Delta_p - E_{21}^+)_{^{132}\text{Te}}},
$$

$$
\frac{(X_{\mu'\nu'}^p/X_{\mu'\nu'}^n)^{^{134}\text{Te}}}{R} = \frac{(2\Delta_n - E_{21}^+)_{^{134}\text{Te}} + 2\Delta_n + \Delta E_{21}^+}{(2\Delta_p - E_{21}^+)_{^{134}\text{Te}} + \delta E_{21}^+},
$$

$$
\delta E_{21}^+ \equiv E_{21}^+ (^{132}\text{Te}) - E_{21}^+ (^{136}\text{Te}), \quad \delta \Delta_n \equiv \Delta_n (^{136}\text{Te}) - \Delta_n (^{132}\text{Te}),
$$
\[ R \equiv \langle \mu' | Q | \nu' \rangle / \langle \mu | Q | \nu \rangle. \]  
(7)

\( R \) is a constant according to one of the approximations mentioned above. The experimental data \([2, 6, 7]\) yield the values

\[ \delta E_{2^+} = 0.37\text{MeV}, \quad \delta \Delta_n = -0.21\text{MeV}. \]  
(8)

From these values and Eqs. (4) and (5), we see

\[ \left( X_{\mu' \nu'}^p / X_{\mu \nu}^p \right)^2 (^{132}\text{Te}) > \left( X_{\mu' \nu'}^n / X_{\mu \nu}^n \right)^2 (^{136}\text{Te}). \]  
(9)

With this relation and the normalization condition

\[ \left( X_{\mu' \nu'}^p \right)^2 + \left( X_{\mu \nu}^n \right)^2 = 1, \]  
(10)

it is found that

\[ \left( X_{\mu' \nu'}^p \right)^2 (^{132}\text{Te}) > \left( X_{\mu' \nu'}^p \right)^2 (^{136}\text{Te}), \]  
(11)

and this equation explains the decrease in \( B(E2) \uparrow \) with the increase in \( N \) across 82. It is emphasized, as is seen from Eqs. (4) and (5), that the negative \( \delta \Delta_n \) plays a key role in decreasing the \( B(E2) \uparrow \) in \( ^{136}\text{Te} \). Therefore, the anomaly of the \( 2^+_1 \) states of \( ^{132,136}\text{Te} \) reflects on the change in \( \Delta_n \) and is consistent with the experimental \( \Delta_n \)'s obtained by the recent very precise mass measurements.

3. Summary

We have shown the analytical explanation of the anomalous relation of the \( 2^+_1 \) states of \( ^{132,136}\text{Te} \) in terms of \( \Delta_n \) based on the QRPA and the several approximations for simplicity. The only input data for this explanation are \( E_{2^+_1} \)'s and \( \Delta_n \)'s, and how the \( \Delta_n \) affects the \( B(E2) \uparrow \) is clear; this explanation is still consistent with the most recent experimental paring gaps. This study was inspired by the recent very precise mass measurements, which confirmed the \( N \)-dependence of the \( \Delta_n \) indicated by the discussion on the \( 2^+_1 \) states. The anomaly of the \( 2^+_1 \) states is an interesting example that information on the ground states can be obtained through properties of the excited states. Finally, it is mentioned that the mixed-symmetry state is also a topic of the excited states around \( ^{132}\text{Sn} \) [7] recently.

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