Analysis of regularity in fuzzy soft graphs and its application in Vehicular communication on the highways

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Abstract
The concept of the fuzzy soft graph is an admirable mathematical framework to handle different real-time applications in various domains. Fuzzy soft graphs provide a more generalized notion to resolve problems associated with uncertainty and vagueness in data as related to fuzzy graphs and soft graphs. This work describes several rudimentary concepts related to fuzzy soft graphs (FSG) including regular-FSG, totally regular-FSG, partially regular-FSG and irregular-FSG. We also discuss some of their significant properties with suitable examples. Additionally, we present a highly worthwhile application of FSG for calculating traffic rates in crossings and roads to enhance the performance of Vehicular Ad hoc Networks (VANET). In this work, we consider two traffic parameters (i.e., daytime and nighttime) as attributes to predict the traffic rate. This study shows the benefits of easier enactments and minimum computational complexity, and it can adapt additional parameters for consideration if necessary. By understanding present traffic conditions, several innovative algorithms for vehicular transmission can be developed to enhance the performance of VANET in the near future.

Keywords
Degree; edges; fuzzy soft graphs; regularity; traffic rate; VANET.

AMS Subject Classification
05C72.

1. Introduction
Of late, a number of real-world applications have proliferated in various domains which involve imprecise vague, uncertain, and inconsistent knowledge. Researchers resort to new mathematical tactics that effectively catch uncertainty or imprecision in the existing information. Some of these uncertainty issues are human-centered and therefore subjective (e.g, human opinions) whereas others are objective in nature; hitherto they are strongly surrounded by uncertain circumstances. There are some concepts namely vague set, fuzzy set, intuitionistic fuzzy set, probability, and evidence-based approaches for handling data with fuzziness. These concepts have their inherent complexities due to a lack of parametrization tactics.

Zadeh firstly presented the notion of the fuzzy set to manage the problems with ambiguity and uncertainty in real-time applications which is non-probabilistic in nature [1]. Owing
to its impending nature for handling imprecise data, the fuzzy set has gained a huge sensation. Since then, the concept of the fuzzy sets has been studied by several investigators to handle real-time applications containing uncertain and ambiguous data. Atanassov improved the concept of the fuzzy set by relating an innovative idea, known as “intuitionistic fuzzy sets” (IFS) [2]. The tenet of IFS is more significant and rigorous due to the presence of membership as well as a non-membership degree.

Applications of IFS-sets have been widely examined in other points of view such as multi-criteria decision making [3], image processing [4], pattern recognition [5], and so on. Nguyen et al. [6] and Buckley [7] integrated fuzzy sets with complex numbers. Furthermore, the complex number theory is combined with IFS [8]. Alkouri and Salleh performed some operations on complex IFS [9]. Ali et al. discussed the classes of complex IFS [10]. Fuzzy models are becoming convenient tactics due to their objective in decreasing the differences among the conventional mathematical approaches exploited in science and technologies and the representative approaches exploited in expert systems.

As a simplification of Euler’s graph theory, Rosenfeld developed the concept of fuzzy graphs 1975 [11]. Rosenfeld proposed different configurations of fuzzy graphs finding analogs of a number of notions by considering the fuzzy relations among fuzzy sets. Then, Bhattacharya [12] presented some valuable comments on fuzzy graphs. After that, some significant operations on fuzzy graphs were familiarized by Mordeson and Peng [13]. Fuzzy graphs are employing several applications for modeling real-time applications in an uncertain environment.

Similar to all other concepts, the notion of the fuzzy graph has a downside to assigning a membership degree. Owing to its challenge, it cannot be just implemented to resolve some complicated issues in different real-time scenarios. To deal with these problems, an important concept is suggested by Molodtsov, called soft sets, which is an innovative mathematical framework for dealing imprecise information which is not handled by conventional scientific models [14]. Fundamentally, a soft set gives a parameterized arrangement of the entity. Molodtsov examined many enactments of soft sets in diverse fields including data assimilation, classification, decision-making, etc. Lately, studies on soft sets are emerging swiftly.

The problem of assigning the value for the degree of membership does not arise in soft set based models, which make these models easily implemented to various application domains. It has been recognized that soft sets have stimulating enactments in diverse domains such as operations research, game theory, probability theory, measurement theory, and smoothness of functions. Ali et al. discussed some essential operations in soft sets [15]. Even though the fuzzy set and soft set frameworks are handling uncertain and imprecise data existing in real-time applications these models have some difficulties in resolving the related dilemma. Hence, a hybridization model, called the fuzzy soft set (FSS), is introduced which is the amalgamation of the soft set and fuzzy sets theory. Indeed, the concept of the FSS is more widespread than that of the fuzzy set and soft set.

Maji et al. initiated the notion of the fuzzy soft sets [16]. By utilizing the description of FSSs, several stimulating enactments of the soft sets have been studied by many investigators. Many important enactments of FSS are studied by Roy and Maji [17]. Zou and Xio discussed the application of the FSSs in an imprecise scenario [18]. Lately, Akram and Nawaz have presented new ideas known as fuzzy soft graphs [19]. In this study, we discuss a new mathematical model for dealing imprecise information by integrating the concepts of fuzzy graphs and soft graphs. We analyze the regularities of FSG and some of their characteristics. Additionally, we explore a highly worthwhile application of FSG for calculating traffic rates in crossings and roads to enhance the performance of VANET. In this work, we consider two traffic parameters (i.e., daytime and nighttime) as attributes to predict the traffic rate. This study shows the benefits of easier enactments and minimum computational complexity, and FSG can adapt dynamically to consider some additional parameters if necessary. By evaluating present traffic conditions, several innovative algorithms for vehicular transmission can be developed to enhance the performance of VANET in the near future.

2. Theoretical Background

This section presents the required definitions and a brief description of FSG which would be useful for subsequent discussions.

2.1 Fuzzy Set (FS)

Fuzzy sets in mathematical frameworks are pigeonholed by membership degrees. Fuzzy sets are pioneered as an advancement of the conventional set theory conferred by Zadeh [1].

Definition 2.1. If \((X, U)\) is the FS, then \(X\) is a non-empty set and \(U : X \to [0, 1], \forall x_i \in X\). The value \(U(x_i)\) is known as the membership degree of \(x_i\) in \((X, U)\). For the given set of nodes \(X = \{x_1, x_2, \ldots, x_n\}\), \((X, U)\) is denoted as \(\{U(x_1)/(x_1), U(x_2)/(x_2), \ldots, U(x_n)/(x_n)\}\). The value of the membership degree is in \([0, 1]\).

2.2 Soft Set (SS)

The concept of SS is intended to handle data with uncertainty. Owing to the deficiency of constraints on the approximate functions, the soft sets are very appropriate and convenient for implementation. In this concept, we can process different types of parameters that we desire, in terms of a real number, words or sentences, mapping, function, etc.

Definition 2.2. Consider \(X\) is an initial universe and \(P\) is the attribute set. The \((FP)\) is known as SS on \(X\) if and only if \(F\) is a mapping of \(P\) such that \(F : P \to P(X)\); here \(P(X)\) denotes the power set of \(X\).
Example 2.3. Assume $X = \{x_1, x_2, x_3, x_4, x_5\}$ is a finite and non-empty set of cars and $P$ is the attribute set and defined as below

$$P = \{a_1(\text{latest}), a_2(\text{expensive}), a_3(\text{cheap}), a_4(\text{stylish}), a_5(\text{good quality})\}$$

The following equation shows the SS ($G$)

$$F(P) = \{\text{latest} = x_1, x_3, x_4, \text{expensive} = x_1, x_2, x_3, \text{cheap} = x_4, x_5, \text{stylish} = x_3, x_4, x_5, \text{good quality} = x_1, x_3\}$$

2.3 FSS

By combining the concepts of SS and FS, Maji et al. established the theory of FSS which can be considered as a fuzzification of conventional SS [16].

Definition 2.4. Consider $P(X)$ is the power set of an initial base set $X$ and $P$ is the attribute set. A fuzzy soft set $\hat{F}$ on the base set $X$ is a set denoted by $\hat{F}$ indicating $\hat{F}_A : P \rightarrow P(X)$ such that $\hat{F}_A(a) = \emptyset$ if $a \notin A$ where $A \subseteq P$ and $\hat{F}_A$ is a FS on $X$, $\forall a \subseteq P$ Therefore, an FSS $\hat{F}$ on $X$ can be denoted as

$$\hat{F} = (a, \hat{F}_A(a)) : a \in P \hat{F}_A(a) \in P(X)$$

Indeed, an FSS is a soft set as it maps each attributes to an initial base set. The dissimilarity among the SS and the FSS is that the set of fuzzy subsets of $X$ is considered as the universe in the FSS.

Example 2.5. Let us consider the problem given in Example 2.3. Let any car has the attribute “good quality” then it will not be possible to define its value with only the two crisp values 0 or 1. Therefore, we can describe it by a membership degree rather than the crisp values related to each element in $[0, 1]$. Now the FSS for the information given in Example 2.3 can be denoted as

$$\hat{F}(a_1)\hat{F}(a_2)\hat{F}(a_3)\hat{F}(a_4)\hat{F}(a_5)$$

$$= \left\{ \begin{array}{ccccccc} x_1 & x_2 & x_3 & x_4 & x_5 & x_1 & x_2 & x_3 & x_4 & x_5 \\ 0.5 & 0.0 & 0.2 & 0.2 & 0.0 & 0.1 & 0.2 & 0.0 & 0.0 & 0.0 \\ x_1 & x_2 & x_3 & x_4 & x_5 & x_1 & x_2 & x_3 & x_4 & x_5 \\ 0.0 & 0.0 & 0.0 & 0.2 & 0.7 & 0.0 & 0.0 & 0.4 & 0.2 & 0.3 \\ x_1 & x_2 & x_3 & x_4 & x_5 & x_1 & x_2 & x_3 & x_4 & x_5 \\ 0.6 & 0.0 & 0.3 & 0.0 & 0.0 & \end{array} \right\}$$

2.4 Fuzzy Graph (FG)

A graph $G = (X, E)$ comprises of a finite set of nodes $X$ and links set $E$. The nodes $x_i$ and $x_j$ are neighboring nodes if $\{x_i, x_j\} \in E$.

Definition 2.6. Let $G = (X, E)$ be a FG and defined as a pair of function $U : X \rightarrow [0, 1]$ and $S : X \times X \rightarrow [0, 1]$, where $E \times (x_i, x_j) \leq U(x_i) \cup U(x_j)$ for each $(x_i, x_j) \in X \times X$ and for each $x_i, x_j \in X$ Here $X$ and $E$ are known as node and link of $G = (X, E)$ correspondingly.

2.5 Soft Graph (SG)

Definition 2.7. A soft graph is defined by 4-tuple as $G = (G^*, F_1, F_2, P)$ if it meets the following constraints:

1. $G^* = (X, E)$ is a basic graph,
2. $P$ is an attribute set with the finite elements,
3. $(F_1, P)$ is a SS over the initial node set $X$,
4. $(F_2, P)$ is a SS over the link set $E$,

For simplicity, the soft graph of $G^*$ for each attributes $a \in P$ is denoted as $H(a) = (F_1(a), F_2(a))$.

2.6 Fuzzy Soft Graph (FSG)

Definition 2.8. The FSG is defined by 4 tuple as $G = (G^*, F_1, F_2, P)$ if it meets the following constraints:

1. $G^* = (X, E)$ is a basic graph,
2. $P$ is an attribute set with the finite elements,
3. $(F_1, P)$ is an FSS over $X$,
4. $(F_2, P)$ is an FSS over $E$,

$(F_1(a), F_2(a))$ is a fuzzy graph of $G^*$, for each $a \in P$. That is, $F_2(a) \leq \min \{F_1(a)(x_1), F_1(a)(x_2)\}, \forall a \in P$ and $x_1, x_2 \in X$. Note that $F_2(a)(x_2) = 0, \forall x_1, x_2 \in E$ and $\forall a \in P$. For simplicity, the soft graph of $G^*$ for each attributes $a \in P$ is denoted as $H(a) = (F_1(a), F_2(a))$.

Example 2.9. Assume $G = (X, E)$ is the simple graph where $X = \{x_1, x_2, x_3\}$ and $E = \{x_1, x_2, x_2, x_3, x_3, x_1\}$. Consider $P = \{a_1, a_2\}$ is attribute set, and $(F_1, P)$ is a FSS on $X$ with its fuzzy mapping $F_1 : P \rightarrow P(X)$ denoted by

$$F_1(a_1) = \{x_1 | 0.3, x_2 | 0.6, x_3 | 0.7\}$$

$$F_1(a_2) = \{x_1 | 0.2, x_2 | 0.7, x_3 | 0.1\}$$

2.6 Table 1. Fuzzy soft set over $X$

| A   | $x_1$ | $x_2$ | $x_3$ |
|-----|------|------|------|
| $a_1$ | 0.3  | 0.6  | 0.7  |
| $a_2$ | 0.2  | 0.7  | 0.1  |

The FSS over $X$ is given in Table 1. Consider $(F_2, P)$ be an FSS on $E$ with its mapping $F_2 : P \rightarrow P(E)$ denoted by

$$F_2(a_1) = \{x_1, x_2 | 0.2, x_1, x_3 | 0.2, x_3, x_2 | 0.0\}$$

$$F_2(a_2) = \{x_1, x_2 | 0.1, x_1, x_3 | 0.1, x_3, x_2 | 0.1\}$$

The FSS over $E$ is given in Table 2. The representation FSGs $H(a_1)$ and $H(a_2)$ are given in Fig. 1.
Then, therefore, G is a R-FSG.

Using regular calculations, it is easy to find that FSGs, attribute set and its mapping $F$ for each attribute $a \in P$. If $H(a)$ is R-FSG of degree $d$ for each attribute $a \in P$, then G is a d-regular FSG.

Example 2.11. Let the base sets $X = \{x_1, x_2, x_3, x_4\}$ and $E = \{x_1, x_2, x_3, x_1, x_2\}$ be non-empty. Let $P = \{a_1, a_2, a_3, a_4\}$ be an attribute set and $(F_1, P)$ be a FSS on $X$ with its mapping $F_1 : P \rightarrow P(X)$ described by

$$
F_1(a_1) = \{x_1, 0.4, x_2, 0.5, x_3, 0.6, x_4, 0.3\}
$$

Consider $(F_2P)$ is an FSS on the set of links $E$ with its fuzzy mapping $F_2 : P \rightarrow P(E)$ described by

$$
F_2(a_1) = \{x_1, x_2, 0.2, x_3, 0.3, x_4, 0.2, x_1, x_4, 0.3\}
$$

Using regular calculations, it is easy to find that FSGs, $H(a_1) = (F_1(a_1), F_2(a_1))$, $H(a_2) = (F_1(a_2), F_2(a_2))$, $H(a_3) = (F_1(a_3), F_2(a_3))$, $H(a_4) = (F_1(a_4), F_2(a_4))$. The degree fuzzy soft subgraphs $d_{H(a_1)}(x_i) = 0.5$, $d_{H(a_2)}(x_i) = 0.8$, $d_{H(a_3)}(x_i) = 0.7$ and $d_{H(a_4)}(x_i) = 1.1$. The above-mentioned subgraphs are R-FSGS as given in Fig. 2. Therefore, G is a R-FSG.

Table 2. Fuzzy soft set over $E$.

| $F_1$ | $x_1$, $x_2$, $x_3$, $x_4$ |
|-------|-----------------------------|
| $a_1$ | $0.2$, $0.2$, $0.0$         |
| $a_2$ | $0.1$, $0.1$, $0.1$         |

Figure 1. Fuzzy graphs $H(a_1)$ and $H(a_2)$.

2.7 Regular-FSG (R-FSG)

Definition 2.12. Consider G be an FSG over the set $X$. Then G is known as R-FSG if $H(a)$ is an R-FSG for each attributes $a \in P$. If $H(a)$ is R-FSG of degree $d$ for each attribute $a \in P$, then G is a d-regular FSG.

Example 2.13. Let the set of nodes $X = \{x_1, x_2, x_3, x_4\}$ and set of links $E = \{x_1, x_2, x_3, x_4\}$ be non-empty sets. Consider $P = \{a_1, a_2\}$ is an attribute set and $(F_1, P)$ be a FSS on base set $X$ with its mapping $F_1 : P \rightarrow P(X)$ described by

$$
F_1(a_1) = \{x_1, 0.4, x_2, 0.2, x_3, 0.2, x_4, 0.4\}
$$

Consider $(F_2P)$ be an FSS on $E$ with its mapping $F_2 : P \rightarrow P(E)$ described by

$$
F_2(a_1) = \{x_1, x_2, 0.2, x_3, x_4, 0.2\}
$$

The FSGs $H(a_1) = (F_1(a_1), F_2(a_1))$ and $H(a_2) = (F_1(a_2), F_2(a_2))$ are shown in Fig. 3. Using regular calculations, the total degrees can be calculated as $\Gamma(x_i) = 0.6$, for $i = 1, 2, 3, 4$ in fuzzy subgraph $H(a_1)$. Since $\Gamma(x_1) = \Gamma(x_2) = \Gamma(x_3) = \Gamma(x_4)$, it is said to be a tR-FSG. Similarly $\Gamma(x_i) = 0.9$, for $i = 1, 2, 3, 4$ in $H(a_2)$, and it is also a tR-FSG. Therefore G is tR-FSG. However $d(x_1) = 0.2$ and $d(x_2) = 0.4$ in $H(a_1)$. Since $d(x_1) \neq d(x_2)$, $H(a_1)$ is not R-FSG. Therefore G is not R-FSG.

Example 2.14. Let $X = \{x_1, x_2, x_3, x_4\}$ and $E = \{x_1, x_2, x_3, x_1, x_3\}$ are non-empty sets. Consider $P = \{a_1, a_2\}$ be an attribute set and $(F_1, P)$ be a FSS on $X$ with its mapping $F_1 : P \rightarrow P(X)$ described by

$$
F_1(a_1) = \{x_1, 0.4, x_2, 0.5, x_3, 0.1\}
$$

Consider $(F_2P)$ be an FSS on $E$ with its mapping $F_2 : P \rightarrow P(E)$ described by

$$
F_2(a_1) = \{x_1, x_2, 0.1, x_3, x_4, 0.1\}
$$

The above-mentioned subgraphs are R-FSGS as given in Fig. 2. Therefore, G is a R-FSG.
Using regular calculations, it is easy to prove that fuzzy subgraphs $H(a_1) = (F_1(a_1), F_2(a_1))$, $H(a_2) = (F_1(a_2), F_2(a_2))$ and $H(a_3) = (F_1(a_3), F_2(a_3))$ are fuzzy graphs. However, $H(a)$ is not a tR-FSG for each $a \in P$. Therefore, $G$ is not a tR-FSG.

**Example 2.15.** Let $X = \{x_1, x_2, x_3, x_4, x_5\}$ and $E = \{x_1 x_2, x_2 x_3, x_3 x_4, x_4 x_5, x_5 x_1\}$ are non-empty sets. Consider $P = \{a_1, a_2\}$ be an attribute set and $(F_1, P)$ is a FSS over $X$ with its mapping $F_1 : P \rightarrow P(X)$ described by

$$F_1(a_1) = \{x_1 | 0.4, x_2 | 0.4, x_3 | 0.1, x_4 | 0.4, x_5 | 0.4\}$$

$$F_1(a_2) = \{x_1 | 0.2, x_2 | 0.2, x_3 | 0.2, x_4 | 0.2, x_5 | 0.2\}$$

Consider $(F_2, P)$ be an FSS on $E$ with its mapping $F_2 : P \rightarrow P(E)$ described by

$$F_2(a_1) = \{x_1, x_2 | 0.3, x_3, x_4 | 0.3, x_5 | 0.3, x_1 | 0.3\}$$

$$F_2(a_2) = \{x_1, x_2 | 0.1, x_3, x_4 | 0.1, x_5 | 0.1, x_1 | 0.1\}$$

Obviously $d(x_1) = 0.6$ in $H(a_1)$ and $d(x_1) = 0.2$ in $H(a_1)$ for $i = 1, 2, 3, 4, 5$. Hence $G$ is a FSG. Similarly, $\Gamma(x_1) = 1.0$ in $H(a_1)$ and $\Gamma(x_1) = 0.4$ in $H(a_2)$ for $i = 1, 2, 3, 4, 5$. Therefore $G$ is a tR-FSG. From this example, it is observed that there is no correlation between R-FSG and tR-FSG. Subsequently, we state and prove some theorems as below.

**Theorem 2.16.** Consider $G$ is an FSG over the set of nodes $X$. If $G$ is an R-FSG and $F_1$ is a constant function in $H(a_i)$, $\forall a_i \in P$ where $i = 1, 2, \ldots, n$ then $G$ is a tR-FSG.

**Proof.** Consider $G$ is an R-FSG and $F_1$ be a constant function. Then $F_1(a_i)(x) = c_i$, where $c_i$ is an invariant and $c_i \in [0, 1]$, $\forall x \in X$, $\forall a_i \in P$ for $i = 1, 2, \ldots, n$. Now, the degree $d(x) = d_i$ in fuzzy subgraphs $H(a_i), \forall x \in X, \forall a_i \in P$ for $i = 1, 2, \ldots, n$. Then $\Gamma(x) = d(x) + F_1(a_1)(x)$. This indicates $d(x) = d_i + c_i$ in $H(a_i), \forall a_i \in P_i = 1, 2, \ldots, n$ and $\forall x \in X$. Therefore $G$ is a tR-FSG.

**Theorem 2.17.** Consider $G$ is an FSS on $X$. If $G$ is a tR-FSG and $F_1$ is a constant function in $H(a_i)$, $\forall a_i \in P$ where $i = 1, 2, \ldots, n$, then the given $G$ is an R-FSG.

**Proof.** Assume $G$ is a tR-FSG and $F_1$ is a constant function. Then $F_1(a_i)(x) = c_i$, where $c_i$ is an invariant, $c_i \in [0, 1]$, $\forall x \in X, \forall a_i \in P$ for $i = 1, 2, \ldots, n$ and $\Gamma(x) = d_i$ in $H(a_i), \forall a_i \in P$ for $i = 1, 2, \ldots, n$ and $\forall x \in X$. As $\Gamma(x) = d(x) + F_1(a_1)(x)$ in $H(a_i), \forall a_i \in P$ for $i = 1, 2, \ldots, n$ and $\forall x \in X$. This denotes $d(x) = \Gamma(x) - F_1(a_1)(x)$ in fuzzy graphs $H(a_i), \forall a_i \in P$ for $i = 1, 2, \ldots, n$ and $\forall x \in X$. That is $d(x) = d_i - c_i$ in $H(a_i), \forall a_i \in P$ for $i = 1, 2, \ldots, n$ and $\forall x \in X$. Therefore, $G$ is an R-FSG.

**Theorem 2.18.** If $G$ is both R-FSG and tR-FSG, then $F_1$ is a constant function in fuzzy graphs $H(a_i)$, $\forall a \in P$ where $i = 1, 2, \ldots, n$.

**Proof.** Suppose that $G$ is both R-FSG and tR-FSG. Then $d(x) = d_i$ and $\Gamma(x) = e_i$ in $H(a_i), \forall a \in P$ for $i = 1, 2, \ldots, n$ and $\forall x \in X$. This denotes $e_i = d(x) + F_1(a_1)(x)$ in $H(a_i), \forall a \in P$ for $i = 1, 2, \ldots, n$ and $\forall x \in X$. That is $F_1(a_1)(x) = e_i - d_i$ in $H(a_i), \forall a \in P$ for $i = 1, 2, \ldots, n$ and $\forall x \in X$. Therefore, $F_1$ is a constant function in $H(a_i), \forall a \in P$ for $i = 1, 2, \ldots, n$.

**Example 2.19.** Let $X = \{x_1, x_2, x_3, x_4\}$ and $E = \{x_1 x_2, x_2 x_3, x_3 x_4, x_4 x_1\}$ be non-empty sets. Consider $P = \{a_1, a_2\}$ is a set of attributes and $(F_1, P)$ is a FSS over $X$ with mapping $F_1 : P \rightarrow P(X)$ described by

$$F_1(a_1) = \{x_1 | 0.5, x_2 | 0.5, x_3 | 0.5, x_4 | 0.6\}$$

$$F_1(a_2) = \{x_1 | 0.3, x_2 | 0.3, x_3 | 0.3, x_4 | 0.3\}$$

Consider $(F_2, P)$ be an FSS on $E$ with its mapping $F_2 : P \rightarrow P(E)$ described by

$$F_2(a_1) = \{x_1, x_2 | 0.2, x_3 | 0.3, x_4 | 0.5, x_4 | 0.4\}$$

$$F_2(a_2) = \{x_1, x_2 | 0.3, x_3 | 0.3, x_4 | 0.4, x_4 | 0.3\}$$

$F_1(a_i)$ is constant in $H(a_i)$ for $i = 1, 2, 3$ and $G$ is neither R-FSG nor tR-FSG.

**Theorem 2.20.** An R-FSG on the set $X$ with $|X| = 3$ and $H(a)$ is R-FSG of degree $e_i > 0$, for $i = 1, 2, \ldots, n$ has no end node.

**Proof.** As $H(a_i)$ is R-FSG of degree $e_i, d_{H(a_i)}(x) = e_i, \forall a_i \in P$ for $i = 1, 2, \ldots, n$ and $\forall x \in X$. Since $e_i > 0$, $d_{H(a_i)}(x) > 0, \forall x \in X$. This implies that each node is neighboring to at least one other node. Assume that $y$ is an end node, then $d_{H(a_i)}(x) = e_i = d_{H(a_i)}(y)$. Since $H(a_i)$ is R-FSG with $|X| \geq 3$ for $i = 1, 2, \ldots, n$ then the node $x$ must be a neighboring node to another node $z = y$. Now, $d_{H(a_i)}(x) = d_{H(a_i)}(y)$.

**Definition 2.21.** Consider $G$ is an FSG on the set of nodes $X$. The given FSG $G$ is said to be a partially regular-FSG (pR-FSG) if $H(a)$ is pR-FSG for each attributes $a \in P$.

**Definition 2.22.** If the FSG $G$ is both R-FSG and pR-FSG, then $G$ is said to be a full regular-FSG (fR-FSG).

**Example 2.23.** Let $X = \{x_1, x_2, x_3, x_4\}$ and $E = \{x_1 x_2, x_2 x_3, x_3 x_4, x_4 x_1\}$ be non-empty sets. Consider $P = \{a_1, a_2\}$ is an attribute set and $(F_1, P)$ is a FSS on $X$ with its mapping $F_1 : P \rightarrow P(X)$ described by

$$F_1(a_1) = \{x_1 | 0.4, x_2 | 0.5, x_3 | 0.7, x_4 | 0.3\}$$

$$F_1(a_2) = \{x_1 | 0.9, x_2 | 0.6, x_3 | 0.8, x_4 | 0.4\}$$

Consider $(F_2, P)$ be an FSS on $E$ with its mapping $F_2 : P \rightarrow P(E)$ described by

$$F_2(a_1) = \{x_1, x_2 | 0.3, x_3 | 0.4, x_4 | 0.1, x_4 | 0.2\}$$

$$F_2(a_2) = \{x_1, x_2 | 0.5, x_3 | 0.4, x_4 | 0.2, x_4 | 0.3\}$$
The fuzzy subgraphs for the given problem are defined as $H(a_1) = (F_1(a_1), F_2(a_1))$ and $H(a_2) = (F_1(a_2), F_2(a_2))$. $H(a_1)$ and $H(a_2)$ are pR-FSG as shown in Fig. 4. Therefore $G$ is a pR-FSG.

**Remark 2.24.** Every R-FSG need not be a pR-FSG. Similarly, each pR-FSG need not be an R-FSG.

**Theorem 2.25.** Consider $G$ is an FSG such that $F_2$ is a constant function in fuzzy subgraph $H(a_i)$, $\forall a_i \in P$, for $i = 1, 2, \ldots, n$. Now $G$ is an R-FSG if and only if $G$ is a pR-FSG.

**Proof.** Suppose that $F_2(a_i)(xy) = c_i$, a constant for all $x, y \in E$ and $\forall a_i \in P$ for $i = 1, 2, \ldots, n$. Then, $d_{H(a_i)}(x) = \sum_{y \in E} F_2(a_i)(xy) = \sum_{y \in E} c_i d_{H^*(a_i)} (x)$, for each $x \in X$, $\forall a_i \in P$ for $i = 1, 2, \ldots, n$. Consider $G$ be an R-FSG. Then, $d_{H(a_i)} (x) = c_i d_{H^*(a_i)} (x) = g_i$, for each $x \in X$ and $\forall a_i \in P$ for $i = 1, 2, \ldots, n$. Consider $G$ be an R-FSG.

**Definition 2.27.** An FSG $G = (G^*, F_1, F_2, P)$ is said to be an irregular-FSG (IR-FSG) if $H(a) = (F_1(a), F_2(a))$ is IR-FSG for each $a \in P$. Similarly, an FSG $G$ is said to be an IR-FSG if there is a node which is neighboring to the nodes with distinct degrees in $H(a)$, for each $a \in P$.

**Definition 2.28.** An FSG $G = (G^*, F_1, F_2, P)$ is said to be a neighbourly irregular-FSG (NI-FSG) if $H(a) = (F_1(a), F_2(a))$ is NI-FSG for each attributes $a \in P$. Similarly, an FSG $G$ is said to be a NI-FSG if each two neighboring nodes have distinct degrees in $H(a), \forall a \in P$.

**Definition 2.29.** An FSG $G = (G^*, F_1, F_2, P)$ is said to be a highly irregular-FSG (HIR-FSG) if $H(a) = (F_1(a), F_2(a))$ is HIR-FSG for each attributes $a \in P$. Similarly, an FSG $G$ is said to be a HIR-FSG if each node is adjacent to the nodes of distinct degrees in $H(a), \forall a \in P$.

**Remark 2.30.** An HIR-FSG need not be a NI-FSG. Similarly, a NI-FSG need not be an HIR-FSG.

**Definition 2.31.** A FSG $G = (G^*, F_1, F_2, P)$ is said to be a totally irregular-FSG (TIR-FSG) if $H(a) = (F_1(a), F_2(a))$ is a TIR-FSG for each attributes $a \in P$.

**Definition 2.32.** A FSG $G = (G^*, F_1, F_2, P)$ is called a neighbourly totally irregular-FSG (NTIR-FSG) if $H(a) = (F_1(a), F_2(a))$ is NTIR-FSG for each attributes $a \in P$.

**Theorem 2.33.** Let $G$ be an FSG. If $G$ is a NI-FSG and $F_1$ is a constant function then the given graph $G$ is a NI-FSG.

**Proof.** Consider $G$ is a NI-FSG. Therefore, all the neighboring nodes have unique degrees. Assume $x_1$ and $x_2$ be two neighboring nodes with distinct degrees $e_i$ and $g_i$ correspondingly in $H(a_i), \forall a_i \in P$ for $i = 1, 2, \ldots, n$. That is, $d(x_1) = e_i$ and $d(x_2) = g_i$ in $H(a_i)$, where $e_i \neq g_i$, for $i = 1, 2, \ldots, n$. Since $F_1$ is a constant function $F_1(a_1)(x_1) = c_i = F_1(a_1)(x_2)$, where $c_i$ is a constant function and $c_i \in \{0, 1\}$, $\forall x \in X$, $\forall a_i \in P$ for $i = 1, 2, \ldots, n$. Now, $\Gamma(x_1) = d(x_1) + c_i = e_i + c_i$, $\Gamma(x_2) = d(x_2) + c_i = g_i + c_i$, in $H(a_i), \forall a_i \in P$ for $i = 1, 2, \ldots, n$. Therefore, the given graph is a NI-FSG.

**Theorem 2.34.** Consider $G$ is an FSG. If it is a TIR-FSG and $F_1$ is a constant function then $G$ is a TIR-FSG.

**Proof.** Consider $G$ is a TIR-FSG. Therefore, each neighboring nodes have unique total degrees. Assume $x_1$ and $x_2$ be two neighboring nodes with unique total degrees $\tau_i$ and $\sigma_i$ correspondingly in $H(a_i), \forall a_i \in P$ for $i = 1, 2, \ldots, n$. Viz., $\Gamma(x_1) = \tau_i$ and $\Gamma(x_2) = \sigma_i$ in $H(a_i)$, where $\tau_i \neq \sigma_i$, for $i = 1, 2, \ldots, n$. Since $F_1$ is a constant function, hence $F_1(a_1)(x_1) = \phi_i = F_1(a_1)(x_2)$, where $\phi_i$ is a constant function, $\phi_i \in [0, 1], \forall x \in X, \forall a_i \in P$ for $i = 1, 2, \ldots, n$. Then, $d(x_1) = \Gamma(x_1) - \phi_i = \tau_i - \phi_i, d(x_2) = \Gamma(x_2) - \phi_i = \sigma_i - \phi_i$. If $d(x_1) \neq d(x_2)$, Then

\[ \begin{align*} \Gamma(x_1) - \phi_i & = \Gamma(x_2) - \phi_i \\
\Rightarrow \tau_i - \phi_i & = \sigma_i - \phi_i \\
\Rightarrow \tau_i - \sigma_i & = 0 \end{align*} \]

\[ \Rightarrow \tau_i = \sigma_i \]

which is a paradox to the statement that $\tau_i \neq \sigma_i$.

\[ \Rightarrow d(x_1) \neq d(x_2) \]

\[ \Rightarrow \text{no two neighboring nodes have equal degrees.} \]

\[ \Rightarrow H(a_i) \text{ is NIR-FSG for } i = 1, 2, \ldots, n. \]

Therefore $G$ is a NI-FSG.
Consider \((F_2P)\) be an FSS on \(E\) with its mapping \(F_2 : P \rightarrow P(E)\) described by
\[
F_2(a_1) = \{x_1x_2|0.3, x_2x_3|0.2, x_3x_4|0.1, x_4x_5|0.2, x_5x_6|0.1, x_6x_1|0.1, x_3x_6|0.1\}
\]
\[
F_2(a_2) = \{x_1x_2|0.3, x_2x_3|0.4, x_3x_4|0.5, x_4x_5|0.5, x_5x_6|0.1, x_6x_1|0.2, x_3x_6|0.1\}
\]

All the neighboring nodes have unique total degrees. Hence the subgraphs \(H(a_1)\) and \(H(a_2)\) are \(t\text{NIR-FSG}\). Therefore, \(G\) is \(t\text{NIR-FSG}\). However, \(d(x_4) = d(x_3) = d(x_6) = 0.5\) in \(H(a_1)\) and \(d(x_3) = d(x_4) = 1\) in \(H(a_2)\). Therefore, \(H(a_1)\) and \(H(a_2)\) are not \(\text{NIR-FSG}\). Therefore \(G\) is not \(\text{NIR-FSG}\).

### 3. Applications of Fuzzy Soft Graphs in Vehicular Communication on Highways

The proliferation of vehicle population in the recent past has paved the way towards Vehicular Ad hoc Network (VANET). They are the special form of mobile ad hoc networks, roaming on the road that is providing multi-hop wireless ubiquitous connectivity among vehicles-to-vehicles and vehicles-to-infrastructure units. They exploit different valuable data in an effective manner. The vehicles participating in VANETs are fortified with onboard units, roadside infrastructures, etc., to develop brilliant applications with the intelligent transport system.

VANETs increase the coziness of driving by propagating the data such as current traffic rate, speed of the vehicle, location of vehicle, direction and driver’s performance. These data will support to improve traffic efficiency and road safety for the drivers and passengers. For instance, using VANET transmission, they can assist drivers to circumvent accidents and direct driving at the highway entrance, crossing, and other hazardous zones. This is the reason for several car manufacturing industries and government organizations are conducting extensive study in this domain. We trust such research will modernize our nomadic behaviors by improving road safety while entertaining our feelings simultaneously.

Predicting traffic rate on a highway is imperative as it can aid to deliver dynamic navigation and enhance the performance of VANETs. The prevailing methods for calculating the traffic rate frequently exhibit the downsides of hard implementations and large computation. This work implements FSG to calculate the traffic rate for VANET transmissions. The FSG-based algorithms can envisage the rate of traffic flow timely. We demonstrate the benefits of simple enactments and less complexity, and it can adapt dynamically to include more traffic parameters if required (e.g., speed, lane occupancy, weather condition, etc.). By understanding present traffic conditions, several innovative algorithms for vehicular transmission can be developed to enhance the performance of VANET and assure consistent communication in various environments.

Several investigators are recognizing that the overall performance of vehicular communications can be upgraded using traffic conditions. For this, the present traffic conditions should be assimilated in real-time, which reveal the variations in network conditions. Then, an appropriate forwarding technique might be embraced. We now discuss an FSG-based framework for calculating the traffic rate in the road network. A distributed vehicle traffic rate prediction method is implemented, where a road section is subdivided into various fixed-size cells (zones) [20]. The zone traffic rate is evaluated by calculating the number of automobiles in the zone through a probe car. By transmitting the information of the traffic rates between the selected automobiles, the mean traffic rate on the highway is also gathered. We take 10 cells in the vicinity with crossings and roads as shown in Figure 5.

Consider \(X\) is the set of nodes (cells) and it is defined as \(X = \{x_1, x_2, x_3, x_4, x_5, x_6, x_7, x_8, x_9, x_{10}\}\) and the attribute set \(P = \{a_1, a_2\}\), where \(a_1 = \text{daytime}\) and \(a_2 = \text{nighttime}\). An \(\text{FSGH}(a) = (F_1(a), F_2(a))\), \(\forall a \in P\) is given in Table 3 The objective of this study is to find the traffic rate in road network during day time as well as night time. The FSG corresponding to attribute \(a_1\) (i.e., daytime), \(H(a_1) = (F_1(a_1), F_2(a_1))\), as given in Fig. 6. The FSG corresponding to attribute \(a_2\) (i.e., nighttime), \(H(a_2) = (F_1(a_2), F_2(a_2))\), is given in Fig. 7

| \(F_1(a)\) | \(x_1\) | \(x_2\) | \(x_3\) | \(x_4\) | \(x_5\) | \(x_6\) | \(x_7\) | \(x_8\) | \(x_9\) | \(x_{10}\) |
|-----------|--------|--------|--------|--------|--------|--------|--------|--------|--------|--------|
| \(a_1\)   | 0.9    | 0.6    | 0.7    | 0.5    | 0.7    | 0.8    | 0.5    | 0.6    | 0.7    |        |
| \(a_2\)   | 0.8    | 0.5    | 0.6    | 0.4    | 0.6    | 0.4    | 0.5    | 0.5    | 0.6    |        |

| \(F_2(a)\) | \(x_1x_2\) | \(x_2x_3\) | \(x_3x_4\) | \(x_4x_5\) | \(x_5x_6\) | \(x_6x_7\) | \(x_7x_8\) | \(x_8x_9\) | \(x_9x_{10}\) |
|-----------|--------|--------|--------|--------|--------|--------|--------|--------|--------|
| \(a_1\)   | 0.6    | 0.7    | 0.5    | 0.4    | 0.6    | 0.5    | 0.4    | 0.6    | 0.7    |
| \(a_2\)   | 0.5    | 0.5    | 0.3    | 0.6    | 0.6    | 0.4    | 0.5    | 0.6    | 0.4    |

Figure 5. An example of VANET Scenario

Table 3. Fuzzy soft subgraph \(H(a_1)\)
about flow of traffic on crossings and roads corresponding to various attributes. In this work, we consider only two traffic parameters (i.e., daytime and nighttime) as attributes to predict the traffic rate. This study shows the benefits of easier enactments and minimum computational complexity, and it can consider additional traffic parameters if necessary. By understanding the present traffic condition on the road, several innovative algorithms for vehicular transmission can be developed to enhance the performance of VANET.

Figure 6. FSG corresponding to attribute $a_1$.

Figure 7. FSG corresponding to attribute $a_2$.

4. Conclusion

The concept of the fuzzy soft graph is an admirable mathematical framework to handle different real-time applications in various domains. Fuzzy soft graphs provide a more generalized notion to resolve problems associated with uncertainty and vagueness in information as compared with fuzzy graphs and soft graphs. In this paper, we describe several rudimentary concepts related to fuzzy soft graphs (FSG) including R-FSG, tR-FSG, pR-FSG and IR-FSG. We also discuss some of their significant properties with suitable examples. Additionally, we present a highly worthwhile application of FSG for calculating traffic rates in crossings and roads to enhance the performance of Vehicular Ad hoc Networks (VANET). In this work, we consider two traffic parameters (i.e., daytime and nighttime) as attributes to predict the traffic rate. This study shows the benefits of easier enactments and minimum computational complexity, and it can consider additional traffic parameters if necessary. By understanding the present traffic condition on the road, several innovative algorithms for vehicular transmission can be developed to enhance the performance of VANET in the near future.

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