OPTIMAL PRE-SALE POLICY FOR DETERIORATING ITEMS

LIANXIA ZHAO AND HUI QIAO
School of Management, Shanghai University
Shanghai 200444, China

QI AN*
School of Engineering, Open University of China
Beijing 100039, China

Abstract. Pre-sale policy is a frequently-used sales approach for deteriorating products, e.g., fruits, vegetables, seafood, etc. In this paper, we consider an EOQ inventory model under pre-sale policy for deteriorating products, in which the demand of pre-sale period depends on price and pre-sale horizon, and the demand of spot-sale period depends on the price and stock level. Optimal pricing decisions and economic order quantity are also provided. We compare pre-sale model with a benchmark inventory model in which all the products are sold in spot-sale period. Theoretical results are derived to show the existence and uniqueness of the optimal solution. Numerical experiments are carried out to illustrate the theoretical results. And sensitivity analysis is conducted to identify conditions under which the pre-sale policy is better off than the spot-sale only policy.

1. Introduction. Product deterioration is a commonly discussed topic in inventory control and marketing. Deteriorating products such as fruits, medicines, fashion apparels, hi-tech products, etc, inevitably incur loss due to decay (expiration, devaluation) during the sale period. There have been many research articles that studied the inventory policy for deteriorating products in recent years. See [2], [10] for comprehensive reviews. Pre-sale policy arises as an opportunity for retailers to avoid costs incurred by product deterioration. That is, perishable products are pre-ordered before becoming available in the market. Before product deterioration begins, products are sold when it is good quality and consumers are interested in buying, which is expected to boost retailer’s revenue to a large degree. Therefore, it is an interesting topic for researchers to deep dive on the evaluation of pre-sale policy for deteriorating products. However, very few articles considered pre-sale policy for deteriorating products.

In addition, since pre-sale policy is an effective strategy for retailers to reduce the deteriorating cost as well as holding cost there is also a potential for price discount in the pre-sale period where products are sold to customers in advance of spot sale. In reality, it is expected that price discount would stimulate the consumer’s
purchasing desire. Another benefit of pre-sale policy for the retailers is that it also reduces the demand uncertainty, as more orders are committed in advance of spot sale.

In this paper, we first built a benchmark inventory model for deteriorating products under spot-sale only policy. Then, we propose an inventory model for pre-sale policy which consists of two selling periods: pre-sale period and spot-sale period. We assume that the potential market demand is deterministic and fixed, and the basic demand for pre-sale period and spot-sale period is distributed in proportion to the time horizon in these two periods. We also assume the demand in the pre-sale period depends on price and pre-sale horizon, while the demand in the spot-sale period depends on price and the instantaneous stock level. Since the benchmark model only has spot-sale period, it can be considered as a special case of general model.

The main contributions in this paper can be summarized as follows. First, we formulate the pre-sale policy as a two-period inventory model where the pre-sale demand and spot-sale demand are constructed in a different way corresponding to reality. Second, we provide the optimal conditions of the proposed inventory model and showed the existence and uniqueness of the inventory model. Third, by comparing the pre-sale policy and spot-sale only policy, we illustrate how the optimal choice of sales policy can be affected by different factors.

The rest of the paper is organized as follows. Section 2 reviews the related literature. Section 3 provides the notations and assumptions. Section 4 formulates a two-period inventory model for the pre-sale policy. Section 5 shows the existence and uniqueness of the optimal pricing decisions, and a benchmark inventory model for spot-sale only policy is then considered for comparison. Section 6 presents some numerical examples to illustrate the theoretical results. Managerial insights and sensitivity analysis with respect to critical parameters are also provided to explain their impact on the optimal policy. Section 7 provides a brief remarking of the paper and discusses future research directions.

2. Literature review. This paper is mainly related to the literature on pre-sale, perishable inventory management, joint optimization of pricing and economic ordering quantity. Many researchers have intensively considered inventory models for perishable product in recent years.

First, we review the research on pre-sale policy. [21] modeled the decisions under advance sales discount and two-echelon trade credits by using EOQ. [13] also considered advance selling problems. They determined how the comparison of these price discrimination strategies depends on price commitment, and derived the profit maximizing selling strategy when aggregate demand is certain. Considering the uncertain supply and demand of the products, [5] examined three selling strategies of a manufacturer under uncertain supply and demand by modelling the first two strategies as single-period Stackelberg games, and modelling the last strategy as a two-period dynamic Stackelberg game. [27] studied an inventory model starting with shortage for Weibull-distributed deteriorating items, in which the shortage order are allowed partial backlogging. However, [28] considered an EOQ inventory model with pre-sale policy for deteriorating items, in which all the pre-sale orders are fully backlogged with waiting-time dependent rebate. [20] investigated a continuous review inventory model with order quantity, reorder point, backorder price discount, process quality, and lead time as decision variables. More literature about
advance selling strategy can be found in [17], [26] and [11], etc. However, all these above researches assumed that the demand is uncertain in supply chain environment. In this setting, in order to reduce the risk of deterioration, we consider the inventory control with pre-sale for perishable, and the actual demand depends on pre-sale time and price which is not known to the retailer in advance.

Second, we survey research involving demand patterns and inventory decisions for perishable products. [9] first proposed an inventory model with a time-sensitive demand rate pattern. There are numerous inventory control papers studied on such demand patterns. [16] developed an inventory model for deteriorating seasonal products with a ramp-type time-dependent demand rate over the season. In regards to the constant deteriorating rate of product, [24] considered a variable deteriorating rate of perishable product. He studied an EOQ model with ramp-type demand, Weibull distribution deterioration, and partial backlogging. In addition, based on product life cycle theory, [3], [4] considered an inventory model for deteriorating items with a trapezoidal-type time-dependent demand rate. From the consumer’s perspective, stock level/ shelf space of product is an important factor impact on consumer’s desire to order. [19] presented a deterministic EOQ model with stock-dependent demand rate where a supplier gives a retailer a price discount. For perishable products, consumers also consider the freshness of product before they ordered it. [8] studied the inventory management decisions of perishable goods in a deterministic setting with age-dependent demand rate. [7] applied an EOQ model in supply chains with partial downstream delayed payment and partial upstream advance payment for deteriorating items.

Third, pricing is an important factor in inventory control for perishable product due to its deterioration. [18] introduced an inventory model for a deteriorating item with the demand dependent on the price of the item. However, more researchers considered the demand rate contingent on both price and time, simultaneously. For example, [25] investigated an inventory system with the demands depending on price and waiting time. [1] formulated an inventory model for perishable items with price- and time-dependent demand to determine the optimal pricing, order quantity. They showed that the profit function is strictly pseudo-concave and provide the optimal policy. However, not every problem about inventory models has the exact optimal solution, [23] considered a production-inventory problem for a seasonal deteriorating product in which the demand is assumed to be price- and time-dependent. They proposed a simple algorithm to determine efficiently an optimal price for the non-concave profit function. [15] used a Tabu Search method to analyze an EOQ model that incorporates pricing and space-allocation decisions for perishable products in which the demand rate depends on the selling price and the displayed stock levels. Some other related literatures include [14], [22], [6] and [12], etc.

In summary, all the models cited above have considered the key factors in isolation. However, for the inventory model with deteriorating product, it is more practical to consider the pre-sale policy, demand patterns and pricing, etc, at the same time. In this paper, we construct the pricing and pre-sale model of deteriorating products, in which we the demand is assumed to be dependent on the sale time, the price and the stock level.

3. Notations and Assumptions. The following notations and assumptions are used throughout the paper.
Table 1. Summary of notations

| Symbol | Description |
|--------|-------------|
| $\epsilon$ | The market demand in pre-sale period |
| $\delta$ | The price sensitivity of the demand |
| $T$ | The fixed spot-sale planning horizon |
| $p_0$ | The selling price of a unit in pre-sale period under pre-sale policy |
| $p$ | The selling price of a unit in spot-sale period under pre-sale policy |
| $p_b$ | The selling price of a unit under spot-sale only policy |
| $w$ | The purchase price of a unit |
| $\theta$ | The deterioration rate of the item |
| $c$ | The sensitivity to the advertising policy, $0 \leq \delta \leq 1$ |
| $c_h$ | The holding cost per unit per unit time |
| $c_d$ | The deterioration cost per unit per unit time |
| $L$ | The length of the pre-sale horizon, and suppose $L = \epsilon T (0 \leq \epsilon \leq \frac{1}{2})$ |
| $I(t)$ | The instantaneous inventory level on hand at time $[0, T]$ |

3.1. Notations.

3.2. Assumptions. (1) The potential total market demand is assumed to be 1. Thus, $\epsilon$ and $1 - \epsilon$ denote the potential demand quantity in pre-sale period and spot-sale period, respectively. Without loss of generality, we assume that $0 \leq \epsilon \leq \frac{1}{2}$.

(2) We assume that the pre-sale total quantity is $Q_0 = \epsilon - \delta \epsilon p_0 - L$, where $0 < \delta \leq 1$.

(3) During the spot-sale period $[0, T]$, the demand rate depends on the selling price and the instantaneous inventory level, that is, $D(t, p) = (1 - \epsilon T - \delta p) + \alpha I(t), 0 \leq t \leq T$, where $\alpha$ is the inventory elasticity of product and $1 - \epsilon T$ represents the mean demand rate.

4. Model Formulation. With the above assumptions, we consider an inventory model for deteriorating product in which the planning horizon is divided into pre-sale period and spot-sale period. The inventory level over time is depicted in Figure 1.

![Figure 1. The graphic of the sale model during a cycle](image)
4.1. **Pre-sale period.** For deteriorating products, retailer can reduce the loss incurred by product deterioration through adopting pre-sale policy before products become available in the market. Customers are allowed to order the product before the start of spot-sale period. And the retailer’s profit in the pre-sale period is

\[ \pi_1(p_0) = (p_0 - w)(\epsilon - \delta p_0 - L) \]  \hspace{1cm} (1)

4.2. **Spot-sale period.** In spot-sale period, deteriorating products are usually more popular when there are large stock on the shelf for its freshness. Meanwhile, price is also an essential factor that impacts the demand in a competitive market. Thus, we suppose that the demand in spot-sale period depends on both price and inventory level. Figure 1 depicts the inventory level \( I(t) \) over the spot-sale period \([0,T]\). The instantaneous inventory level is given by

\[ \frac{dI(t)}{dt} = -\theta I(t) - (1 - \epsilon T) - \delta p - \alpha I(t), \]  \hspace{1cm} (2)

with the boundary condition \( I(T) = 0 \).

Then, we have

\[ I(t) = \frac{1}{\alpha + \theta} \left( \frac{1 - \epsilon}{T} - \delta p \right) e^{(\theta + \alpha)(T-t)} - 1. \]  \hspace{1cm} (3)

The total holding cost is obtained as

\[ HC = c_h \int_0^T I(t) dt = c_h \frac{\epsilon}{(\alpha + \theta)^2} \left[ \frac{1 - \epsilon}{T} - \delta p \right] e^{(\theta + \alpha)T} - (\alpha + \theta)T - 1. \]  \hspace{1cm} (4)

In spot-sale period, the deteriorating cost is obtained as

\[ DC = c_d \theta \int_0^T I(t) dt = c_d \theta \frac{\epsilon}{(\alpha + \theta)^2} \left[ \frac{1 - \epsilon}{T} - \delta p \right] e^{(\theta + \alpha)T} - (\alpha + \theta)T - 1. \]  \hspace{1cm} (5)

Thereby, we obtain the revenue in the spot-sale period as

\[ TR = p \int_0^T D(t,p) dt = p \left[ \frac{1 - \epsilon}{T} - \delta p \right] T + \alpha \int_0^T I(t) dt. \]  \hspace{1cm} (6)

Therefore, the retailer’s profit over spot-sale period is given as

\[ \pi_2(p) = (p - w) \left( \frac{1 - \epsilon}{T} - \delta p \right) T + \left[ (\alpha p - (\alpha + \theta)w - c_h - \theta c_d) \int_0^T I(t) dt. \] \hspace{1cm} (7)

To determine the optimal optimal pricing decisions to maximize the total profit over the selling period, we need to solve

\[ Max \ \pi(p_0, p) = \pi_1(p_0) + \pi_2(p) \]

\[ = (p_0 - w)(\epsilon - \delta p_0 - \epsilon T) \]

\[ + \left[ (\alpha p - (\alpha + \theta)w - c_h - \theta c_d) + (p - \epsilon T) \right] \left( \frac{1 - \epsilon}{T} - \delta p \right) \]

\[ s.t. \ w \leq p \leq \frac{1 - \epsilon}{\delta T}, \ w \leq p_0 \leq \frac{1 - T}{\delta}, \]  \hspace{1cm} (8)

where \( \phi = \frac{e^{(\theta + \alpha)T} - (\alpha + \theta)T - 1}{(\alpha + \theta)^2} \).

Now, the total order quantity over the selling period becomes

\[ Q = \epsilon (1 - \delta p_0 - T) + \frac{1}{\alpha + \theta} \left( \frac{1 - \epsilon}{T} - \delta p \right) e^{(\theta + \alpha)T} - 1. \]  \hspace{1cm} (9)
5. Model analysis.

5.1. Spot-sale only policy. We consider a benchmark inventory model where products are only sold in spot-sale period. With market size normalized to 1, the demand rate is modified as $D(t, p_b) = (\frac{1}{T} - \delta p_b) + \alpha I(t)$. Thus, the instantaneous inventory level is characterized by

$$\frac{dI(t)}{dt} = -\theta I(t) - \left(\frac{1}{T} - \delta p_b\right) - \alpha I(t),$$

with the boundary condition $I(T) = 0$. Thereby, we have

$$I(t) = \frac{1}{\alpha + \theta} \left(\frac{1}{T} - \delta p_b\right)(e^{(\theta + \alpha)(T-t)} - 1).$$

Then the total order quantity becomes $Q_0 = \frac{1}{\alpha + \theta} \left(\frac{1}{T} - \delta p_b\right)(e^{(\theta + \alpha)T} - 1)$.

Similar to (7), we can obtain the total profit function over the entire selling period

$$\pi(p_b) = \frac{(1 - \delta p_b T)\left[(\alpha p_b - (\alpha + \theta)w - c_d \theta - c_h) \phi \right]}{T} + (1 - \delta p_b T)(p_b - w).$$

**Proposition 1.** $\pi(p_b)$ is a concave function with respect to $p_b$ and the optimal price $p^*_b = \frac{T + \alpha \phi + \delta T[wT + (\theta c_d + c_h + \alpha w) \phi]}{2 \delta T (\alpha \phi + T)}$.

**Proof.** The optimal price $p^*_b$ has to satisfy

$$\frac{d\pi(p_b)}{dp_b} = \frac{\alpha \phi + T}{T} + \delta [c_d \theta + c_h + (\alpha + \theta)w] \phi - 2p_b (\alpha \phi + T) + Tw],$$

where $\phi = \frac{e^{(\alpha + \theta)T} - (\alpha + \theta)T - 1}{(\alpha + \theta)^2}$.

Taking the second derivative of $\pi(p_b)$ with respect to $p_b$, we have $\frac{d^2\pi(p_b)}{dp_b^2} = -2(\alpha \phi + T) \delta < 0$, which means that $\pi(p_b)$ is a concave function with respect to $p_b$. Let

$$f(p_b) = \alpha \phi + T + \delta T[(c_d \theta + c_h + (\alpha + \theta)w) \phi - 2p_b (\alpha \phi + T) + Tw].$$

We have

$$\frac{df(p_b)}{dp_b} = -2\delta T (\alpha \phi + T) < 0,$$

and

$$f(w) = (1 - \delta Tw) (\alpha \phi + T) + \delta \phi T (c_d \theta + c_h + (\theta + \theta)w) > 0.$$

Thus, from $\frac{d\pi(p_b)}{dp_b} = 0$, we have

$$p^*_b = \frac{T + \alpha \phi + \delta T [wT + (\theta c_d + c_h + (\alpha + \theta)w) \phi]}{2 \delta T (\alpha \phi + T)}.$$

Substituting the optimal price $p^*_b$ into (13), the total profit of the retailer can be obtained as

$$\pi p^*_b = \frac{[\alpha \phi + T - \delta wT^2 - \delta T (c_d \theta + c_h + (\alpha + \theta)w) \phi]^2}{4 \delta T^2 (\alpha \phi + T)}.$$
5.2. Pre-sale policy. We propose a more general model that incorporates pre-sale strategy to reduce deteriorating cost of the product. We prove that the total profit is strictly concave with respect to price, and then obtain the optimal pricing decision.

**Proposition 2.** For the inventory model with pre-sale stage, we have (i) the total profit $\pi(p_0, p)$ is concave in $p_0$ and $p$, and (ii) there exist optimal prices as follows:

$\pi_0 = \frac{1 - T + \delta w}{2\delta}$ and $p^* = \frac{(1 - \epsilon)(T + \alpha \phi) + \delta T[wT + (c_d \theta + c_h + (\alpha + \theta)w)\phi]}{2\delta T(T + \alpha \phi)}$.

**Proof.** (i) Taking the first-order partial derivative of $\pi(p_0, p)$ with respect to $p_0$ and $p$, respectively, we have

$$\frac{\partial \pi(p_0, p)}{\partial p_0} = (1 - T)\epsilon - 2\delta p_0 + \delta w,$$

$$\frac{\partial \pi(p_0, p)}{\partial p} = \delta [\phi(c_d \theta + c_h + \alpha w) - 2p(\alpha \phi + T) + Tw] + \frac{(1 - \epsilon)(\alpha \phi + T)}{T}. \quad (15)$$

Taking the second-order partial derivative of $\pi(p_0, p)$ with respect to $p_0$ and $p$, respectively, we have

$$\frac{\partial^2 \pi(p_0, p)}{\partial p_0^2} = -2\delta(T + \alpha \phi) < 0,$$

$$\frac{\partial^2 \pi(p_0, p)}{\partial p^2} = -2\delta \epsilon < 0,$$

$$\frac{\partial^2 \pi(p_0, p)}{\partial p_0 \partial p} = 0.$$

Thus, the Hessian matrix of $\pi(p_0, p)$ is

$$H_\pi = \begin{bmatrix} -2\delta(T + \alpha \phi) & 0 \\ 0 & -2\delta \epsilon \end{bmatrix}.$$

Obviously, $H_\pi$ is negative definite and $\pi(p_0, p)$ is concave in $p_0$ and $p$.

From $\frac{\partial \pi(p_0, p)}{\partial p_0} = 0$ and $\frac{\partial \pi(p_0, p)}{\partial p} = 0$, we have

$$p_0^* = \frac{1 - T + \delta w}{2\delta},$$

$$p^* = \frac{(1 - \epsilon)(T + \alpha \phi) + \delta T[wT + (c_d \theta + c_h + (\alpha + \theta)w)\phi]}{2\delta T(T + \alpha \phi)}.$$

Based on Proposition 2, the total profit of the inventory system is given as

$$\pi(p_0^*, p^*) = \epsilon[T - 1 + \delta w]^2$$

$$+ \frac{[(\alpha \phi + T)(1 - \epsilon) - \delta \phi T(c_d \theta + c_h + \alpha w + \theta w) - \delta T^2w^2]}{4\delta T^2(\alpha \phi + T)}. \quad (16)$$

5.3. Policy comparison. We compare the pre-sale and spot-sale only policies and discuss the conditions under which retailer should choose pre-sale policy over spot-sale only policy.

**Proposition 3.** When the retailer adopts pre-sale strategy, the optimal pricing decisions satisfy $p_0^* < p^* < p_b^*$. 
Therefore, if $\theta > \theta_S$, solving the equation $\pi_0 = \frac{1 - \epsilon - \epsilon T + \epsilon T^2}{2\delta T} + \frac{\phi(c_d \theta + c_h + \theta w)}{2(\alpha \phi + T)} > 0$, and

$$p^* - p_0^* = -\frac{\epsilon}{2\delta T} < 0.$$  

Thus, we have $p_0^* < p^* < p_b^*$. \hfill \qed

**Proposition 4.** The retailer’s optimal strategy depends on market sale $\epsilon$: i) When $\epsilon > \epsilon_0$, it is more profitable for the retailer to choose pre-sale policy; ii) When $\epsilon \leq \epsilon_0$, it is more profitable for the retailer to choose spot-sale only policy, where $\epsilon_0 = 2 - \frac{2\delta T(\theta c_d + c_h + \alpha w + \theta w) - 2\delta w T^2 + T^2(1 + \delta w)^2}{\alpha \phi + T}$. 

**Proof.** From Propositions 1 and 2 and the assumption $0 \leq \epsilon \leq \frac{1}{2}$, we have

$$p^* - p_0^* = \frac{\epsilon(\alpha \phi + T)}{4\delta T^2} \left(2\delta w T + \epsilon - 2\right)$$

$$+ \frac{\epsilon[2\delta \phi(c_d \theta + c_h + \theta w) + T(T + \delta w - 1)^2]}{4\delta T^2}.$$ 

Solving the equation $\pi_{p_0^*, p^*} - \pi_{p_b^*} = 0$ with respect to $\epsilon$, we have

$$\epsilon_0 = 2 - \frac{2\delta T(\theta c_d + c_h + \alpha w + \theta w) - 2\delta w T^2 + T^2(1 + \delta w)^2}{\alpha \phi + T}.$$ 

Obviously, if $\epsilon > \epsilon_0$, then $\pi_{p_0^*, p^*} > \pi_{p_b^*}$ which means that the retailer should choose pre-sale policy, and if $\epsilon \leq \epsilon_0$, then $\pi_{p_0^*, p^*} \leq \pi_{p_b^*}$ which means that the retailer should choose spot-sale only policy. \hfill \qed

**Proposition 5.** The retailer’s optimal strategy depends on the deteriorating rate $\theta$: i) when $\theta > \theta_0$, it is more profitable for the retailer to choose pre-sale policy; ii) when $\theta \leq \theta_0$, it is more profitable for the retailer to choose spot-sale only policy, where $\theta_0 = \frac{(2 + \alpha T)(2 - \epsilon - 2\delta w T) - 2T(T + \delta w - 1)^2 - \delta T^2 c_h}{2\delta T^2(c_d + w)}$. 

**Proof.** Since $\phi = \frac{e^{(\theta + \alpha)T} - (\alpha + \theta)T - 1}{(\alpha + \theta)^2}$, we use $T^2$ to approximately instead of $\phi$. From above analysis, we have

$$\pi_{p_0^*, p^*} - \pi_{p_b^*} = \frac{\epsilon(\alpha T + 2)(2\delta w T + \epsilon - 2\phi)}{4\delta T^2} + \frac{\epsilon[\delta T(c_d \theta + c_h + \theta w) + (T + \delta w - 1)^2]}{4\delta T^2(c_d + w)}.$$ 

Solving the equation $\pi_{p_0^*, p^*} - \pi_{p_b^*} = 0$, we obtain

$$\theta_0 = \frac{(2 + \alpha T)(2 - \epsilon - 2\delta w T) - 2T(T + \delta w - 1)^2 - \delta T^2 c_h}{2\delta T^2(c_d + w)}.$$ 

Therefore, if $\theta > \theta_0$, then $\pi_{p_0^*, p^*} > \pi_{p_b^*}$ which means that the retailer should choose pre-sale policy; and if $\theta \leq \theta_0$, then $\pi_{p_0^*, p^*} \leq \pi_{p_b^*}$ which means that the retailer should choose spot-sale only policy. \hfill \qed

From Propositions 4 and 5, we know that the retailer’s sale strategy depends on the pre-sale market size and the deteriorating rate. If the pre-sale market size $\epsilon$ (the deteriorating rate, $\theta$) goes beyond than a threshold, the retailer should adopt the pre-sale policy; Otherwise, the retailer should adopt spot-sale only policy.
6. Numerical study. We illustrate the theoretical results and provide some managerial insights on main parameters involving the inventory model. We also will examine the impact of the coefficients $\epsilon$, $\theta$, $\delta$ and $T$ on the retailer’s optimal decision. We control for the following parameters: $w = 3.0$, $\alpha = 0.3$, $c_d = 0.4$, $c_h = 0.25$, $\theta = 0.04$, $\delta = 0.32$, $T = 0.6$, and vary different parameter to conduct sensitivity analysis.

**Figure 2.** Comparison of the profits and prices at different pre-sale market potential

Figure 2 compares the pre-sale policy and spot-sale only policy at different pre-sale market potential from the profit point of view, and also provides the corresponding optimal pricing decisions. We observe that there exists a threshold of pre-sale market potential $\epsilon_0$ such that if $\epsilon \leq \epsilon_0$, the retailer should adopt spot-sale only policy and if $\epsilon > \epsilon_0$, the retailer should adopt pre-sale policy for better profitability. When $\epsilon > \epsilon_0$, the retailer’s profit increases with $\epsilon$, implying if there is enough potential for advance order, the retailer can benefit more from adopting pre-sale policy for deteriorating product, even with price ($p$) in normal sale stage decreasing with $\epsilon$.

**Figure 3.** Comparison of the profits and prices at different deteriorating rate

Figure 3 compares the pre-sale policy and spot-sale only policy at different deteriorating rate from the profit point of view, and also provides the corresponding optimal pricing decisions. We observe that there exists a threshold of deteriorating rate $\theta_0$ such that if $\theta \leq \theta_0$, the retailer should adopt spot-sale only policy and if $\theta > \theta_0$, the retailer should adopt pre-sale policy so as to maximize the profit. The retailer’s profit from spot-sale only policy is more sensitive to $\theta$. This implies the pre-sale policy provides more benefits to retailer in case of highly deteriorating
products. For either sale policy, the retailer’s profit decreases with $\theta$. We also see that the optimal prices $p$ and $p_b$ increase with $\theta$ either pre-sale policy or spot-sale only policy.

Figure 4. Comparison of the profits and prices at different price sensitivity

Figure 4 compares the pre-sale policy and spot-sale only policy at different price sensitivity from the profit point of view, and also provides the corresponding optimal pricing decisions. We observe that there exists a threshold of deteriorating rate $\delta_0$ such that if $\delta \leq \delta_0$, the retailer should adopt spot-sale only policy and if $\delta > \delta_0$, the retailer should adopt pre-sale policy so as to maximize his system profit. When $\delta > \delta_0$, the retailer’s profit increase with $\delta$ for either sale policy. However, the retailer’s profit from pre-sale policy is more sensitive to $\delta$. This implies that the retailer should adopt pre-sale policy to obtain more profit for demand highly sensitive to price. We also find that the optimal prices $p_0$, $p$, and $p_b$ decrease with $\delta$, and the price $p_0$ is more sensitive to $\delta$ for low $\delta$.

Figure 5. Comparison of the profits and prices at different planning horizon

Figure 4 compares the pre-sale policy and spot-sale only policy at different planning horizon rate from the profit point of view, and also provides the corresponding optimal pricing decision. We observe that there exists a threshold of planning horizon $T_0$ such that $T \leq T_0$, the retailer should adopt spot-sale only policy and if $T > T_0$, the retailer should adopt pre-sale policy so as to maximize his system profit. For both sale policies, when $T$ increases, the retailer’s profit decreases. However, the retailer’s profit is more sensitive to the change of $T$ under spot-sale only policy and the profit increasingly trends for large $T$. We also find that the optimal prices $p_0$, $p$, and $p_b$ decrease with $T$. Moreover, $p$ and $p_b$ are more sensitive to $T$. It means
that the retailer should adopt spot-sale only policy for small $T$, and adopt pre-sale policy for more profit for large $T$.

7. Conclusion. This paper proposed pre-sale policy for an EOQ inventory model with deteriorating products in which the inventory system is composed of a pre-sale period and a spot-sale period. We assume that the total demand quantity depends on product’s price and pre-sale time in pre-sale period, while the demand rate of spot-sale period depends on product’s price and stock level. For comparison, we propose a benchmark inventory model which only includes spot-sale period. We theoretically derived to demonstrate the existence and uniqueness of the optimal pricing decision for both sale policies. We also obtain the retailer’s optimal choice of sale policy considering pre-sale market potential, deteriorating rate, demand’s price sensitivity, planning horizon, different parameters. Numerical experiments of the main parameters are also carried out to illustrate the theoretical findings.

In this setting, we considered an inventory model with a deteriorating product, single cycle and determined demand rate. However, there are more research opportunities to be considered in the future, such as multiple products, multiple cycle and stochastic demand in inventory setting, etc

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E-mail address: zhaolianxia@staff.shu.edu.cn
E-mail address: 18752090828@163.com
E-mail address: anq@mail.ouchn.edu.cn