Higher-order moments of velocity fluctuations in the wake of a short stack

MS Adaramola\textsuperscript{1,3}, DJ Bergstrom\textsuperscript{2} and D Sumner\textsuperscript{2}

\textsuperscript{1}Division of Environmental Engineering, University of Saskatchewan, Saskatoon, SK, Canada.
\textsuperscript{2}Department of Mechanical Engineering, University of Saskatchewan, Saskatoon, SK, Canada.
\textsuperscript{3}Department of Energy and Process Engineering, Norwegian University of Science and Technology, Trondheim, Norway

E-mail: muyiwa.adaramola@ntnu.no

Abstract. The effect of jet velocity relative to the crossflow velocity on the higher-order moments of velocity fluctuations characteristics in the wake of stack is reported in this study. The cross-flow Reynolds number was $Re_D = 2.3 \times 10^4$, and the jet-to-cross-flow velocity ratio was varied from $R = 0$ to 3. The stack was partially immersed in a flat-plate turbulent boundary layer, with a boundary layer thickness-to-stack-height ratio of $\delta/H = 0.5$ at the location of the stack. The skewness factor, flatness factor and triple correlations are found to be influenced by the flow regime. The deviation of skewness and flatness factors from the Gaussian fluctuation values of zero and 3 are more pronounced outside the wake centre region due to the presence of the separated shear layers and vortex structures. The high values of the skewness and flatness factors for all values of velocity ratio within the vicinity of the stack free end are related to the tip vortex structures near the stack free end which decrease in strength as the value of $R$ increases. In addition, the separated shear layers from both sides of the stack contribute to the high values observed away from the wake centreline.

1. Introduction
The complex flow field associated with a stack is encountered in many engineering applications, especially in the area of waste gas dispersion. The near-field mixing and dispersion of the jet issuing from a stack depends on the ambient conditions, the jet exit characteristics, and the geometry of the stack. Compared with a finite circular cylinder, the presence of the jet flow issuing from the stack gives rise to a more complicated flow structure, both around the stack and in its wake. For a non-buoyant jet exiting from a short stack, the extent of this complexity depends on the jet-to-cross-flow velocity ratio, $R = U_e/U_\infty$, where $U_e$ and $U_\infty$ are the mean jet exit velocity and the freestream velocity, respectively) and the ground plane boundary layer thickness. The flow regimes in the stack wake have been classified differently by various researchers. The flow topology in the vertical plane along the stack wake centreline has been used to classify the stack and jet-wake flow patterns into a number of flow regimes based on the approximate value of $R$. Using a stack of $AR = 25$, four flow regimes were identified by [1,2]: (i) downwash flow ($R \leq 0.95$), (ii) crosswind-dominated flow ($0.95 < R < 1.4$), (iii) transitional flow (1.4 < $R < 2.4$), and (iv) jet-dominated flow ($R > 2.4$). A similar study of the local flow field of a small-aspect-ratio stack of $AR = 8.3$, by [3] identified three zones within the jet: zone 1, immediately above the stack exit, where the jet dominates the flow; zone 2, where the jet begins to bend and the jet flow and crossflow have the same velocity; and zone 3, further downstream, where the crossflow dominates the flow. Depending on the value of $R$ and the corresponding flow regime, one or more of these zones may be absent.
Recent work by Adaramola et al. [4,5] identified three distinct flow regimes within the wake of a stack of AR = 9 when R was varied between 0 and 3: the downwash flow regime for $R < 0.7$, the crosswind-dominated flow regime for $0.7 \leq R < 1.5$, and the jet-dominated flow regime for $R \geq 1.5$. Each flow regime has distinct flow structures noticeable within the turbulence field and time-averaged velocity field, and up to three pairs of time-averaged streamwise vortex structures may be found within the stack wake depending on the value of R. These streamwise vortex structures influence the turbulence characteristics within the wake of the stack [5]. Similar flow regimes were also observed within the wake of the stack by other authors [e.g., 3]. In the downwash flow regime (Figure 1(a)), two pairs of counter-rotating streamwise vortex structures, each of opposite sign, are found. The characteristics of these structures are similar to those of the finite circular cylinder [6], with the tip vortex pair located closer to the free end of the stack and the base vortex pair located within the flat-plate boundary layer on the ground plane. The same vortex pairs are found in the crosswind-dominated flow regime (Figure 1(b)), but the tip vortex pair extends above the free end of the stack. In case of the jet-dominated flow regime (Figure 1(c)), a third pair of streamwise vortices, referred to as the jet-wake vortex pair, appears in the jet wake region above the free end of the stack. The jet-wake vortex structures are associated with the jet rise and a strong upwash velocity on the jet wake centreline.

![Figure 1: Mean streamwise vorticity field (iso-vorticity contours of $\omega_z = \omega_z D/U_a$) downstream of the stack at $x/D = 6$, from Ref. [3], for (a) $R = 0.5$; (b) $R = 1$; and (c) $R = 2.5$. Solid lines represent positive (CCW) vorticity; dashed contour lines represent negative (CW) vorticity. Minimum vorticity contour of $\omega_z = \pm 0.04$, and contour increment of $\Delta \omega_z = 0.04$.](image)

Specific information about the high-order moments of the velocity fluctuations in the wake of a stack is scarce, or even absent in the open literature. The higher-order moments contain useful information, such as that related to coherent structures [7]. The current study presents part of a comprehensive set of measurements taken recently to investigate the effect of the jet flow on the turbulent wake of a stack. This information is important for validation of computational studies, and also supports a better understanding of the overall wake structure of a stack or elevated jet in crossflow. Therefore, the focus of this paper is to report the turbulence characteristics in the wake of a stack with a specific emphasis on the higher-order moments of the turbulence parameters.
2. **Experimental Approach**

The present experiments were conducted in a low-speed, closed-return wind tunnel with a test section of 0.91 m (height) × 1.13 m (width) × 1.96 m (length). The streamwise freestream turbulence intensity was less than 0.6% and the velocity non-uniformity outside the test section wall boundary layers was less than 0.5%. The test section floor was fitted with a ground plane. A roughness strip located about 200 mm from the leading edge of the ground plane was used to enhance the development of a turbulent boundary layer. At the location of the stack (with the stack removed), the boundary layer provided a thickness-to-height ratio of \( \delta/H \approx 0.5 \), and the Reynolds number based on momentum thickness, \( \theta \), was \( \text{Re}_\theta = 8 \times 10^3 \).

2.1 **Experimental Apparatus**

A cylindrical stack of \( H = 171.5 \text{ mm}, \) \( D = 19.1 \text{ mm}, \) \( d/D = 0.67, \) and \( \text{AR} = 9 \), was used in the present study. The stack was located 700 mm downstream of the roughness strip on the ground plane. The experiments were conducted at a single stack Reynolds number (based on \( D \) and \( U_\infty \)) of \( \text{Re}_D = 2.3 \times 10^4 \). The exhaust velocity of the non-buoyant stack jet was varied with two MKS 1559A-200L mass flow controllers arranged in parallel. The jet-to-crossflow velocity ratio was varied from \( R = 0 \) (no jet exiting the stack) to \( R = 3 \), corresponding to momentum flux ratios of \( \text{Re}_\text{m} (R^2) = 0 \) to 9. The jet Reynolds number, \( \text{Re}_\text{e} \) (based on \( d \) and \( U_e \)) ranged from \( 7.6 \times 10^3 \) to \( 4.7 \times 10^4 \). The experimental set-up for the study is shown schematically in Fig. 2, and was similar to that adopted by [4,5].

![Figure 2: Schematic of a cylindrical stack mounted normal to a ground plane and partially immersed in a plane boundary layer.](image)

2.2 **Measurement Instrumentation**

The wind tunnel data were acquired by a computer with a 1.8-GHz Intel Pentium 4 processor, a National Instruments PCI-6031E 16-bit data acquisition board, and LabVIEW software. The freestream conditions were obtained with a Pitot-static probe (United Sensor, 3.2-mm diameter), Datametrics Barocell absolute and differential pressure transducers, and an Analog Devices AD590 integrated circuit temperature transducer. Turbulence measurements were made with a TSI model 1243-20 boundary layer X-probe and a TSI IFA-100 constant-temperature anemometer. The probe was manoeuvred to the measuring points using the wind tunnel’s three-axis computer-controlled traversing system. At each measurement point, 100,000 instantaneous velocity data per channel were acquired at a sampling rate of 10 kHz per channel after low-pass filtering at 5 kHz. The uncertainties in the streamwise and wall-normal fluctuating velocity components were estimated to be ±4% and ±7%, respectively, while the uncertainty in the Reynolds shear stress was estimated to be ±9%. The X-probe was oriented to
measure the streamwise, $u$, and wall-normal, $w$, velocity components. For a given value of the velocity ratio, $R$, the profile measurements were made in the $y$-$z$ plane at $x/D = 10$. The measurement plane extended to $z/D = 14$ in the wall-normal direction.

3 Results and Discussion
The higher-order moments contain valuable statistical information that relates to the turbulent diffusion of the Reynolds stresses. The transport equation for the Reynolds stresses, $\langle u_i u_j \rangle$, can be expressed as:

$$\frac{\partial}{\partial t} \rho \langle u_i u_j \rangle + \frac{\partial}{\partial x_k} \rho U_k \langle u_i u_j \rangle = - \frac{\partial}{\partial x_k} \left[ \rho \langle u_i u_j u_k \rangle + \langle p u_i \delta_{jk} + p u_j \delta_{ik} \rangle \right] + \frac{\partial}{\partial x_k} \left[ \frac{\partial}{\partial x_i} \langle u_i u_j \rangle \right] - \left( \rho \langle u_i u_k \rangle \frac{\partial U_j}{\partial x_k} + \rho \langle u_j u_k \rangle \frac{\partial U_i}{\partial x_k} \right) - p \left( \frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i} \right) - 2 \mu \left( \frac{\partial u_i}{\partial x_k} \frac{\partial u_j}{\partial x_k} \right)$$

or in symbolic terms,

$$\text{Local Time Derivative} + C_{ij} = D_{ij}^P + D_{ij}^P + D_{kij} + P_{ij} + \varphi_{ij} + \varepsilon_{ij}$$

In the above equation, $C_{ij}$ is the convection term representing the transport of the turbulent flux by the mean velocity, $D_{ij}^P$ is the turbulent diffusion due to interaction between the fluctuating velocity components or triple velocity correlation, $D_{ij}^P$ is the pressure diffusion due to the interaction between the fluctuating pressure and fluctuating velocity fields, $D_{kij}$ is the viscous diffusion of the turbulent fluxes $P_{ij}$ is the production of the turbulent fluxes arising from the interaction between the fluctuating mean velocity gradient, $\varphi_{ij}$ is the pressure-strain correlation caused by the interaction between the fluctuating pressure and the fluctuating strain rates, and $\varepsilon_{ij}$ is the dissipation rate of the turbulent fluxes. For numerical calculations, $D_{ij}^P, D_{ij}^P, \varphi_{ij}$ and $\varepsilon_{ij}$ need to be modeled to close the transport equation. In this study, experimental data obtained for the triple velocity correlations, as well as skewness and flatness factors, obtained in the wake of a stack are presented.

3.1 Skewness and flatness factors
This third-order moment provides statistical information on the temporal distribution of the fluctuations about the mean velocity in the probability density function of turbulent fluctuations. For a symmetric Gaussian distribution, $S_n = 0$, and a non-zero value shows the degree of temporal irregularity in the fluctuation. The streamwise and wall-normal skewness factors for select values of the velocity ratio $R$, at $z/H = 0.5$ and 0.75 and $x/H = 10$ are presented in Figures 3 and 4, respectively.

For $z/H = 0.5$ (Figure 3a), the effect of $R$ on $S_n$ is in general insignificant and the profiles are almost self-similar especially near the wake centreline. This may be related to the presence of shed Karman vortex structures from the sides of the stack that are similar irrespective of the flow regime [4]. In addition, for all the flow regimes, the skewness factors are either zero or have negative values. In contrast, changes in the sign of the skewness are present at $z/H = 0.75$ (Figure 3b). This change in sign is likely related to the pair of tip vortex structures that create regions of strong turbulence on either side of the stack, with a region of relatively weak turbulence between them. The location of the change in sign depends on the value of $R$, and increases with increasing value of $R$. Note that the tip vortices reduce in strength as the flow regime changes from downwash flow ($R = 0.5$) to jet-dominated flow regime ($R = 2.5$). For all values of $R$, the value of $S_n$ is non-zero with a negative peak magnitude located at about $\pm 2D$ from the wake centreline for $R = 0$ and 0.5, and at about $\pm 3D$ for $R = 2.5$. For the case of $R = 1$, no dominant negative peak value is observed. The magnitude of the peak value is also
observed to depend on the value of $R$ and hence tip vortex strength. The skewness factor is close to the Gaussian value at the wake centreline for $R \geq 0.5$, but takes a value of approximately -1 for $R = 0$. This may be due to the increase in the jet momentum resulting in less downwash flow from the stack free end along the wake centre region. The observed pattern of the skewness factor profiles in this study is similar to that reported by [8] for an infinite circular cylinder.

The wall-normal skewness factor $S_w$ follows a similar trend with respect to the effect of flow regime except that the profiles are opposite in shape to that of the streamwise skewness factor. In addition, unlike the case of the streamwise skewness factor, the wall-normal skewness factor is typically zero or positive in value, and no reversal in sign is observed. Within the vicinity of the wake centreline, the values of $S_w$ are close to zero for all values of $R$ and at both vertical locations (Figure 4a,b). At $z/H = 0.75$, there is a positive peak at approximately $\pm 2D$ from the wake centreline for $R = 0$ and 0.5, and $\pm 3D$ for $R = 1$ and 2.5, (Figure 4b).

The streamwise flatness factor ($F_u$) and wall-normal flatness factor ($F_w$) profiles are shown in Figures 5 and 6, respectively. This fourth-order moment illustrates the intermittency of turbulence in the flow. For a Gaussian distribution, its value is 3, and a flatness factor more than 3 is attributed to a "peaky signal [due to] intermittent turbulent events" [7]. Similar to the skewness factor, for the streamwise flatness factor shown in Figure 5, a flatness factor larger than 3 indicating non-Gaussian behaviour is typically observed away from the wake centreline. The largest values (>3) of the flatness factor occur in a region located between $\pm 2D$ and $\pm 3.5D$ from the centreline, depending on the value.
of $R$. In addition, it was observed that the peak of the flatness factor occurs where the skewness factor also has a peak value, on either side of the wake centreline. This is in agreement with observations made by [9] in the wake of a side mirror of a passenger car. For larger values $R$, the flatness factor approaches the Gaussian value along the centreline. The level of intermittency of turbulence in the flow as indicated by the value of the flatness is strongly dependent on vertical location. For example, it is observed to be higher at $z/H = 0.75$ (Figure 5b) than at $z/H = 0.5$ (Figure 5a) because of the presence of the tip vortices that create more turbulence in the flow in this region.

![Figure 5: The streamwise flatness factor at $x/D = 10$: (a) $z/H = 0.5$ and (b) $z/H = 0.75$](image)

The wall-normal flatness factor profiles (Figure 6) follow the same trend as for $F_u$, both in terms of pattern and magnitude at corresponding locations along the stack height. However, the peak values of $F_w$ for $R = 0$ seem to be somewhat lower than that of $F_u$. Both the streamwise and wall-normal flatness profiles reported in this study follow a similar pattern to that reported by [8] for an infinite circular cylinder. The behaviour of $F_u$ and $F_w$ indicates that there are strong intermittent turbulent events within the stack and jet wake especially around the stack free end.

![Figure 6: The wall-normal flatness factor at $x/D = 10$: (a) $z/H = 0.5$ and (b) $z/H = 0.75$](image)
addition, the wall-normal components of the skewness and flatness factors (Figures 7b, 8b) are generally higher than those of the streamwise components (Figures 7a, 8a).

![Figure 7: The skewness factor iso-contour plots for $R = 2.5$ at $x/D = 10$: (a) streamwise component $S_u$ and (b) wall-normal component $S_w$.](image)

![Figure 8: The flatness factor iso-contour plots for $R = 2.5$ at $x/D = 10$: (a) streamwise component $F_u$ and (b) wall-normal component $F_w$.](image)

3.2 Triple Correlation

It was shown above that due to the presence of the tip vortex structures, the deviation of both skewness and flatness factors from their respective Gaussian values is more pronounced at $z/H = 0.75$ than at $z/H = 0.5$. Therefore, the fluctuation triple correlation will only be considered at $z/H = 0.75$. It was previously shown that the Reynolds stress within the wake of a stack is strongly dependent on the flow regime and the location along the stack height [5]. Additional information about the changes in turbulence structure can be provided by the velocity triple correlation. The profiles of $\langle u^3 \rangle / U_\infty^3$, which relate to the transport of $u'$ by the turbulent motion in the wall-normal direction, are shown in Figure 9, for different values of $R$. Alternatively, the gradient $\partial < u^3 w > / \partial z$ represents the turbulent diffusion of $u'$ in the Reynolds stress transport equation for $u'$. The profiles shown in Figure 9 behave opposite to the Reynolds shear stress profiles; they also have a lower absolute magnitude (see ref. [5]), which depends on the value of the velocity ratio. Note that the gradients associated with rapid spatial changes in the correlation profile will result in significant turbulent diffusion of $u'$ within the stack wake.

The profiles of $\langle w^3 \rangle / U_\infty^3$, which represents the transport of the wall-normal Reynolds stress $w^3$ by the turbulent motion in the streamwise direction, are shown in Figure 10 for different values of $R$. Two peaks (one on either side of the wake centre line) are observed for $R = 0, 0.5$ and $1.0$, while only a single peak close to the centreline is observed in the case of $R = 2.5$. The magnitudes of these profiles, unlike the case of $\langle u^3 w \rangle / U_\infty^3$, are generally positive.

4. Conclusions

In this study, the effect of the jet-to-crossflow velocity ratio on the characteristics of the higher-order moments within the wake of a stack was investigated experimentally. Two-component thermal anemometry was used to measure the turbulence parameters at different locations along the stack height and at different downstream locations. The probe was oriented to measure the streamwise, $u$, and wall-normal, $w$, velocity components. The results presented in this study show that the skewness and flatness factors, as well as the triple correlation of the velocity fluctuations, are dependent on the velocity ratio and the location along the stack height.
The high values observed in the skewness and flatness factors for all values of $R$ near the stack free end region ($z/H = 0.75$) are most likely related to the tip vortex structures. The strength of the tip vortex structures decreases as the value of $R$ increases, and the fluctuations near the centreline were found to approach their Gaussian values with increasing value of $R$. The separated shear layers from both sides of the stack also contribute to the high values observed in the flatness and skewness away from the wake centreline. However, at the mid-height of the stack, the degree of deviation of the skewness and flatness factors from the Gaussian values is considerably lower outside wake centre region compared to a section near the free end of the stack. This suggests that the tip vortex structures generate more turbulence in the flow than the Karman vortex structures, which are the dominant structures near the stack mid-height. The triple velocity correlations near the free end of the stack also exhibit a strong dependence on the jet-to-crossflow velocity ratio, also demonstrating the influence of the tip vortices on the turbulence structure.

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