The path planning method for anthropomorphic manipulator for avoidance an obstacle approximated by a parallelepiped

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Abstract. There are various methods of planning paths for anthropomorphic manipulators that allow to bypass the obstacle and to work in real time. One of these methods was developed by the authors for circumventing an obstacle, which was approximated by a sphere. The approximation by a sphere of elongated obstacles overstates their size that leads to an excessive length of the path of avoiding the obstacle. The developed method combines analytical and numerical solutions with relatively low computational complexity, allowing it to work in real time. The effectiveness of the method is illustrated by a numerical example. The trajectory of the anthropomorphic manipulator required for its movement can be obtained using the existing methods of generating a trajectory based on the path of movement calculated using the proposed method. The developed method can be implemented in the systems of intellectual or supervisory control by anthropomorphic manipulators.

1. Introduction
The modern period of scientific and technological development in the world is characterized by the continuous growth of demand for robotic technologies. These trends in the development of robotics are due to the need to replace people when performing routine operations and working in potentially dangerous areas of activity [1]. Depending on the conditions, various methods of controlling robotic products can be applied [2].

The main area of implementation of routine operations is the industry. The implementation of routine operations by industrial robots leads to an increase in labor productivity and quality of the manufactured products. Routine labor involves performing repetitive actions in a controlled, deterministic environment. Software control of industrial robots is used for these conditions, which consists in performing actions on a predetermined program and moving along predetermined trajectories. In the case of the possibility of minor changes in the environment within the specified limits, adaptive control is applied. Adaptive-controlled robots are equipped with sensing tools that allow to analyze the external environment and adjust the characteristics of the operation being performed accordingly.

A high level of development of robotic systems including both hardware and software components of the robot is necessary for a partial or complete replacement of a person when performing complex tasks in life-threatening conditions.

The systems of supervisory and intellectual management possess the highest level of complexity. Supervisory control systems imply a statement of the task by the operator and its independent execution by the robot. In intelligent control systems, the task is set up by the robot independently. The task in this case refers to the sequence of operations leading to the finished result. The systems of intelligent and
supervisory control are similar at the execution level of individual operations they operate in automatic mode.

The use of copy control systems that implement the virtual presence of the operator in the body of the robot is an alternative technical solution, that allows to replace a person to perform complex tasks in a hazardous environment [3].

In systems with software control, the trajectory of the manipulator for repetitive actions can be calculated in advance. In adaptive control systems, all trajectories have a common pattern and change only quantitatively. In copy control systems, the trajectory of the manipulator is formed by the hand of the operator. In supervisory and intelligent control systems, the trajectory of an anthropomorphic manipulator should be generated in real time. The trajectory should be formed taking into account the need of circumventing the obstacle and optimality by any criterion. The calculation of such paths is based on the results of the work of the path planning methods. The problem is that many existing path planning methods do not work in real time, which does not allow them to be used for intelligent control. This work is aimed at the development of one of the existing methods of planning a path operating in real time.

The introduction of the developed method in intelligent control systems allows us to achieve a greater degree, which is the most important development direction for automatic control systems.

2. Review of research on the subject of work
There are different methods of the path planning. Relevant to the systems of intelligent and supervisory control are methods that work in real time.

The study [4] proposes an approach for planning paths based on affine transformations. It is based on the concept of trajectory invariance, which avoids obstacles, but this requires considering a certain excessive number of possible ways to circumvent obstacles.

The article [5] presents several practical algorithms for planning the movement of mobile robot manipulators designed with a common trajectory generation base. Methods are focused on the idea of generating a motion path graph. The article [6] describes a method for planning a path and generating a trajectory based on a probabilistic approach. The work of [7] proposes a combined method using a genetic algorithm for finding a path. The solution obtained using the methods [5-7] is admissible, but not necessarily optimal according to such important criteria as the length of the trajectory and the time of movement.

[8] uses in his work a bidirectional fast search algorithm. Its performance for the mobile platform in two-dimensional space is demonstrated. This algorithm has a computational complexity, growing exponentially; therefore it is not applicable to manipulators with a large number of degrees of mobility, working in three-dimensional space.

[9–12] describe in their works various methods for finding a path. These methods are reliable, leading to a valid solution, if one exists. However, the path of movement for different configurations of obstacles may be far from optimal.

In the work of [13] Bezier curves are used to plan the path of a robotic ball. This method requires serious adaptation for use in manipulators, since various design constraints are imposed on the joints of the manipulator.

In the work of [14] a method for planning a path and trajectory based on Bezier curves for a manipulator was proposed. However, this method takes into account only the movements of the gripper and is used to bypass relatively small obstacles. For relatively large obstacles, the lack of consideration for the movement of the entire manipulator can lead to unacceptable collisions.

In the last article of the [15] the authors proposed a numerical-analytical method for planning a path for an obstacle of arbitrary size approximated by a sphere. This article is a development of this method. The approximation of the elongated objects in the form of a sphere leads to an unnecessary lengthening of the path around the obstacle. This article describes the adaptation of the developed method to circum-
vent an obstacle approximated by a parallelepiped. The required trajectory for the movement of an anthropomorphic manipulator can be generated using various methods [16, 17] based on the obtained obstacle avoidance path.

3. Methods
The algorithm of the method described in [15] is universal for the obstacles of any form and includes the following steps:

- The solution of the inverse problem of kinematics for calculating the final position of the manipulator.
- Construction of the trajectory of direct movement of the manipulator from the initial position to the final one.
- Analysis of the possibility of moving the manipulator along a straight path to bypass an obstacle.
- In case of impossibility of direct movement search for the “worst” position in the trajectory of movement at which the manipulator touches the obstacle most of all.
- Offset the worst position to the outside of the obstacle by entering an additional intermediate position outside the obstacle.
- Repeat steps 2-5 for moving the manipulator from the initial position to the intermediate position and from the intermediate position to the final position.

Adaptation of the method to a specific obstacle implies the implementation of the following steps of the method:

- Analysis of the possibility of moving the manipulator along a straight path to bypass an obstacle.
- Search for the “worst” position in the trajectory of movement at which the manipulator touches the obstacle most of all.
- Offset the worst position to the outside of the obstacle by entering an additional intermediate position outside the obstacle.

Task setting and implementation of the corresponding steps are described in the subsequent subsections of the article.

![Figure 1. AM kinematic scheme with 7 degrees of mobility.](image)
The kinematic scheme of the considered anthropomorphic manipulator (AM) with 7 degrees of mobility is shown in figure 1. A similar kinematic scheme is used in Russian anthropomorphic robots of the AR-600, SAR-400, FEDOR series, the manufacturer of which is the JSC «Scientific Production Association "Android Technology"». In this figure, the following notation is entered: \(B_1 - B_4\) – shoulder, elbow, wrist nodes and working end, respectively; \(A_1 - A_7\) – 7 rotational degrees of mobility of the manipulator AP; \(A_8\) – its working ending.

Known values for the manipulator are the lengths of the links \(l_{B_1-B_2}, l_{B_1-B_3}, l_{B_1-B_4}\), corresponding to the shoulder, elbow, and wrist sections \(B_1\) and \(B_2\), \(B_3\) and \(B_4\), respectively.

Known data are:

1. the initial position of AM in the space of joint coordinates;
2. the final position of AM in the space of joint coordinates;
3. the data about the parallelepiped, including the coordinates of its center \(O\), and coordinates of all its vertices \(C_i, i = 1,8\) (see figure 2).

![Figure 2. Obstacle approximated by parallelepiped.](image)

The following subsections describe the implementation of the necessary steps of the method described in [15], necessary for its adaptation to the circumvention of an obstacle approximated by a parallelepiped.

3.1. Analysis of the possibility of direct movement

Consider the movement of an arbitrary link \(B_iB_{i+1}\) of the anthropomorphic manipulator from the initial position to the final one. The direct movement is understood as the movement in which all the joint coordinates depend on time linearly, and reach the final value at the same time. Such a time dependence can be described with the following function:

\[
\theta(t) = \theta_S + t(\theta_F - \theta_S),
\]

where \(t\) – normalized time, changing from 0 to 1;
\(\theta(t)\) – the law of motion of AM along joint coordinates;
\(\theta_S\) – the joint coordinates of AM in the initial state;
\(\theta_F\) – the joint coordinates of AM in the final position.

Suppose that in the process of movement between the local initial and local final positions, link \(B_iB_{i+1}\) crosses an obstacle-parallelepiped. In this case, the set of link positions is a certain unclosed curvilinear surface. The selected metric for finding the worst position depends heavily on the eject method used below. Unlike a sphere, for a given obstacle one cannot use as a metric the distance to the
center of the obstacle, together with the pushing in the direction from the center of the obstacle to its surface. The problem case for using this approach, when considering the movement of a link from the position $H$ to the position $K$ is shown in the figure 3.

![Figure 3. Problem case of pushing an arbitrary link.](image)

As can be seen from the figure, in addition to unnecessary movement along the surface of the obstacle, the link comes to a standstill when trying to solve the task of planning a path from position 3 to position $K$, when pushing the link from the center of the obstacle. The link motion should initially be oriented to bypassing the top of the obstacle located near position 3.

To solve this problem, it is necessary to consider the distance not to the center of the obstacle as the metric of the worst position, but to the point which is the most serious obstacle. The method of finding the worst position based on such a metric is as follows.

Consider the motion of an arbitrary link $B_iB_{i+1}$ beyond a certain point in time $t$. Consider the tangent plane to the instantaneous movement of a link. The normal vector to this plane can be found as follows. By solving the direct kinematics problem, it is necessary to find the values of the vector $B_iB_{i+1}$ at times $t - dt$ and $t + dt$, where $dt$ is a certain small time interval. Then the normal vector of the tangent plane can be found as the vector product of the given position vector of the link $B_iB_{i+1}$:

$$n_t(t) = B_iB_{i+1}(t - dt) \times B_iB_{i+1}(t + dt),$$  \hspace{1cm} (2)

where $n_t(t)$ – the normal vector of the tangent plane at the time $t$; $B_iB_{i+1}(t - dt)$ – vector of the link $B_iB_{i+1}$ at the moment $t - dt$; $B_iB_{i+1}(t + dt)$ – the vector of the link $B_iB_{i+1}$ at the moment $t - dt$.

The deviation of the link is performed in a plane perpendicular to the plane of tangency. The normal vector of this plane forms an orthogonal triple with vectors $B_iB_{i+1}$ and $n_t$:

$$n_n(t) = B_iB_{i+1}(t) \times n_t(t),$$  \hspace{1cm} (3)

where $n_n(t)$ – the normal vector of the plane of deflection at the moment of time $t$.

Let the deflection plane be described by the following canonical equation:

$$A_nx + B_ny + C_nz + D_n = 0,$$  \hspace{1cm} (4)

then the coefficients of this equation can be found as follows:

$$\begin{pmatrix} A_n \\ B_n \\ C_n \end{pmatrix} = n_n, \hspace{1cm} (5)$$

$$D_n = -n_n \cdot B_i.$$

Next, it is necessary to analyze the section of the parallelepiped by the plane of deflection for the magnitude of the required deflection.
Consider an arbitrary section of the parallelepiped formed by the deflection plane. This section can be three-, four-, five- and hexagonal. In the case of a collision of a link with an obstacle, this section intersects with a circle, radius $B_iB_{i+1}$ and center at a point $B_i$ (figure 4).

To find the boundaries of this section and determine the magnitude of the required deviation, it is proposed to use the following sequence of steps:

- Find the top of the section.
- Find the equation of the edges of the section.
- Find the characteristic points of the section.
- Find the value of the minimum required deviation.

Figure 4. Section of an obstacle in the plane of deflection.

At the first step, the vertices of the section can be found as a solution of eight systems of equations of the form:

\[
\begin{align*}
\frac{x - C_{xi}}{C_{xi} - C_{xj}} &= \frac{y - C_{yi}}{C_{yi} - C_{yj}} = \frac{z - C_{zi}}{C_{zi} - C_{zj}}, \\
A_n x + B_n y + C_n z + D_n &= 0,
\end{align*}
\]

(6)

where $C_{xj}, C_{yj}, C_{zj}$ – coordinates $x, y, z$ of point $C_i$ of parallelepiped; $C_{xj}, C_{yj}, C_{zj}$ – coordinates $x, y, z$ of point $C_j$ of parallelepiped.

The first equation of the system (6) describes a straight line passing through two adjacent vertices of the parallelepiped, the second one - the plane of deviation of the link, the last three strict inequalities impose a limit on the intersection of the parallelepiped on the point of intersection of the straight line and the plane.

As a result, the solution of eight equations can be obtained from 0 to 6 vertices of the section, we denote them as $D_j$. Zero of vertices corresponds to the absence of intersection between the plane of deflection and the obstacle, one vertex - the intersection at a point, two - the intersection is a straight line, three or more - the section is a convex polygon.

If there is no intersection in the considered position of the link, there is no intersection with the obstacle, therefore such a case can be discarded.

In case the cross section is a point, it is characteristic according to the definition given below and the transition can be made immediately to the third step.

In the second step, it is necessary to find the equations for the edges of the section. To do this, find the neighboring vertices of the polygon they form. The neighborhood of the vertices can be determined

\[
\begin{align*}
\frac{x - C_{xi}}{C_{xi} - C_{xj}} &= \frac{y - C_{yi}}{C_{yi} - C_{yj}} = \frac{z - C_{zi}}{C_{zi} - C_{zj}}, \\
A_n x + B_n y + C_n z + D_n &= 0,
\end{align*}
\]

(6)
in the first step. Adjacent are vertices of a section lying on intersecting edges or lying on parallel edges belonging to the same face.

Then the edge equations can be written in canonical form:

\[
\frac{x - D_{xj}}{D_{xj} - D_{xk}} = \frac{y - D_{yj}}{D_{yj} - D_{yk}} = \frac{z - D_{zj}}{D_{zj} - D_{zk}},
\]

(7)

where \(D_{xj}, D_{yj}, D_{zj}\) – coordinates \(x, y, z\) of point \(D_j\) of the section; \(D_{xk}, D_{yk}, D_{zk}\) – coordinates \(x, y, z\) of point \(D_k\) of the section; \(D_j\) and \(D_k\) – adjacent section points.

Let us call vertex of a section as characteristic points of a section, lying inside a circle formed by rotating a point \(B_{i+1}\) around a point \(B_i\), as well as points lying at the intersection of a given circle and section edges. Let’s designate them as \(E_j\). Characteristic points for the section in figure 4 are shown in figure 5.

![Figure 5. Characteristic points of the section.](image)

The vertices of the section are characteristic points in the case of the following condition:

\[B_i D_j \leq B_i B_{i+1}.\]

(8)

Characteristic points lying at the intersection of the edges of the section and the circle can be found by solving the systems of equations composed of equation (6) and the following equation:

\[(x - B_{ix})^2 + (y - B_{iy})^2 + (z - B_{iz})^2 = B_i B_{i+1}^2.\]

(9)

The deviation of the link \(B_i B_{i+1}\) can be performed in one direction or another. The required angle of deflection in each direction is determined by the most distant characteristic point. Thus, it is necessary to perform the separation of characteristic points that require a clockwise or counterclockwise deflection for the deflection plane in question. Separation can be performed by the sign of the scalar product of vectors \(B_i B_{i+1}\) and \(B_i E_j\).

In this case, you need to find the maximum required deviation in each direction, and then choose from the two obtained values the smallest one, which will be the metric of the worst position.

![Figure 6. Bypassing of the link \(B_i B_{i+1}\) characteristic point \(E_j\).](image)
Consider the deviation of the link $B_iB_{i+1}$ to bypass an arbitrary characteristic point $E_j$ (figure 6). In figure 6 $h$ – certain margin of bypass, depending on the accuracy of following the manipulator along a given trajectory.

As follows from the figure, the deviation angle can be found by the following formula:

$$\alpha = \left| \acos \left( \frac{B_iB_{i+1} \cdot B_iE_j}{|B_iB_{i+1}| \cdot |B_iE_j|} \right) \right| + \asin \left( \frac{h}{2 \cdot |B_iE_j|} \right), \quad (10)$$

where $\alpha$ – the angle of the required deflection.

3.2. Finding the worst position
The angle of deviation $\alpha$ is determined by the movement of the link $B_iB_{i+1}$. In this case, the search for the worst situation is reduced to solving the problem of one-dimensional optimization:

$$\alpha(t) \rightarrow \max, \quad (11)$$

and the obstacle avoidance condition is:

$$\max_{t \in [0,1]} \alpha(t) = 0. \quad (12)$$

3.3. Entering an additional state
In case the condition (12) is not fulfilled in the worst position, it is necessary to introduce an additional intermediate position.

To do this, sequentially, starting from the shoulder link, the manipulator is “pushed out” in the direction from the center of the obstacle with some margin $h$.

For a parallelepiped-shaped obstacle, pushing can be carried out by rotating the link $B_iB_{i+1}$ by an angle $\alpha$, found in formula (9) around the axis found by formula (2).

4. Results
To illustrate the effectiveness of the proposed method, a simulation was performed. During the simulation, the oblong obstacle was approximated by a sphere and a parallelepiped. For both cases, path planning was performed using the developed methods. The visualization of the results is shown in figure 7.

![Figure 7. Bypass of the oblong obstacle: a) approximated by a sphere; b) approximated by a parallelepiped.](image)
5. Discussion
As follows from figure 7, approximation of the oblong obstacle by the parallelepiped and the use of the corresponding method for path planning can significantly reduce the length of the path of movement of the anthropomorphic manipulator. The movement of AM passes near the parallelepiped, which indicates the efficiency of the calculated path of movement along the path. Thus, the development goal can be considered achieved. An unsolved problem is the path planning of movement in an environment with several obstacles or a complex obstacle approximated by several primitives. The solution to this problem is the goal of subsequent work.

6. Conclusion
In developing the method proposed in the article, the method of path planning of movement of an anthropomorphic manipulator to circumvent an obstacle approximated by a sphere was taken as the basis. The disadvantage of this method is the limited functionality associated with the possibility of bypassing only spherical obstacles. In the case of an oblong obstacle, approximation by a sphere leads to an over-estimation of the overall dimensions of the obstacle, which leads to an increase in the length of the circumvention path of the obstacle. To eliminate this drawback, an adaptation of the base method was performed to circumvent an obstacle approximated by a parallelepiped. To solve the problem, method steps were identified that require a different implementation. For the new implementation of the identified steps, numerical and analytical solutions have been developed that have a relatively low computational complexity. Low computational complexity allows the new method to work in real time. To assess the effectiveness of the method, a comparative analysis was made of the results of path planning for an oblong obstacle approximated by a sphere and a parallelepiped. For the conditions of the case under consideration, the new method showed significantly better results. Thus, the goal can be considered achieved. An unsolved problem is the path planning of movement in an environment with several obstacles or a complex obstacle approximated by several primitives. The solution to this problem is the goal of subsequent work.

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