Measuremental Data: Seven Measures of Central Tendency

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Abstract

Recently, four measures of central tendency namely Arithmetic-Geometric Mean (AGM), Arithmetic-Harmonic Mean (AHM), Geometric-Harmonic Mean (GHM) and Arithmetic-Geometric-Harmonic (AGHM) have been derived from the three Pythagorean means namely Arithmetic Mean (AM), Geometric Mean (GM) and Harmonic (HM). An attempt has here been made on establishing each of these four measures of average as a measure of central tendency of measuremental data. This paper is based on a brief description on the seven measures namely AM, GM, HM, AGM, AHM, GHM & AGHM of central tendency of data.

Keywords: Central Tendency, Measure, AGM, AHM, GHM, AGHM

1 Introduction

Average [Bakker, 2003] is a concept which describes any characteristic of an aggregate / population / class of individuals overall but not of an individual in the aggregate / population / class in particular. It is used in most of the measures associated to data (or list of numerical values). Pythagoras [Riedweg, 2005], one exponent of mathematics, defined the three most common averages namely arithmetic mean (AM), geometric mean (GM) and harmonic mean (HM) which were given the name “Pythagorean Means” [Chakrabarty, 2016b; De Carvalho, 2016; Kolmogorov, 1933] as a mark of honor to him. Later on, a number of definitions/ formulations of average like Quadratic Mean , Square Root Mean , Cubic Mean , Cube Root Mean , Generalized p Mean & Generalized p th Root Mean etc. in addition to AM, GM & HM had been derived due to necessity of handling different situations [Chakrabarty, 2016a, 2018c, 2019f, 2020d; De Carvalho, 2016; Kolmogorov, 1930].

The next trend was towards the development of generalized definitions of average namely Generalized f-Mean [Chakrabarty, 2018a,f] , Generalized fH–Mean [Chakrabarty, 2018b] chakrabarty2018four and Generalized fG–Mean [Chakrabarty, 2018d, 2019d,e] and general method of defining average [Chakrabarty, 2019a,b].

In statistics, the three Pythagorean means [Chakrabarty, 2021e] are used in measuring the central tendency of numerical data [Plackett, 1958; Weisberg and Weisberg, 1992; Williams, 1984]. However, the accuracy of the value of central tendency yielded by each of the three Pythagorean means is not known. Recently, there have been a lot of studies on analysis of numerical data based on average in general and on Pythagorean means specially [Chakrabarty, 2014a,b,c, 2015a,b,c,d,e,f,g, 2016a, 2017a,b,c, 2018e, 2019c] . In the mean time, several attempts have been made on determining accurate value of central tendency of numerical data. However, still there is necessity of more accurate measure of the same. With an objective of finding out more accurate measure of central tendency of data, four measures of central tendency namely Arithmetic-Geometric Mean (AGM) [Chakrabarty, 2020f, 2021b,c,d; David,
2004; Hazewinkel, 2001; Kolmogorov, 1933], Arithmetic-Harmonic Mean (AHM) [Chakrabarty, 2020a,b, 2021b; Foster and Phillips, 1984], Geometric-Harmonic Mean (GHM) [Chakrabarty, 2020e, 2021b] and Arithmetic-Geometric-Harmonic (AGHM) [Chakrabarty, 2020c, 2021a,b,f,g,h,i] have been derived from the three Pythagorean means. An attempt has here been made on establishing each of these four measures of average as a measure of central tendency of measuremental data. This paper is based on a brief description on the seven measures namely AM, GM, HM, AGM, AHM, GHM & AGHM of central tendency of data.

2 Measurement of average

The seven measures of average considered here are Arithmetic Mean (AM), Geometric Mean (GM), Harmonic Mean (HM), Arithmetic-Geometric Mean (AGM), Arithmetic-Harmonic Mean (AHM), Geometric-Harmonic Mean (GHM) and Arithmetic-Geometric-Harmonic (AGHM).

Let
\[ x_1, x_2, \ldots, x_N \]
be N positive numbers or values or observations (not all equal or identical) which are strictly positive.

If \( a_0, g_0 \) and \( h_0 \) are respectively the AM , the GM & the HM of these N numbers (or values or observations) then

\[ a_0 = AM(x_1, x_2, \ldots, x_N) = \frac{1}{N} \sum_{i=1}^{N} x_i \] (1)

\[ g_0 = GM(x_1, x_2, \ldots, x_N) = \left( \prod_{i=1}^{N} x_i \right)^{\frac{1}{N}} \] (2)

\[ h_0 = HM(x_1, x_2, \ldots, x_N) = \left( \frac{1}{N} \sum_{i=1}^{N} x_i^{-1} \right)^{-1} \] (3)

Which satisfies the inequality

\[ \text{Largest}(x_1, x_2, \ldots, x_N) > a_0 > h_0 > g_0 > \text{Smallest}(x_1, x_2, \ldots, x_N) \] [4, 16, 49, 51] (4)

2.1 AGM

Let the two sequences \( \{a_n\} \) and \( \{g_n\} \) be defined by

\[ a_{n+1} = \frac{1}{2} (a_n + g_n) \] (5)

and

\[ g_{n+1} = (a_n \times g_n)^{\frac{1}{2}} \] (6)

respectively where the square root takes the principal value. Then, the two sequences \( \{a_n\} \) and \( \{g_n\} \) converge to a common point \( M_{AB} \). This common point of convergence can be termed as the Arithmetic-Geometric Mean of \((x_1, x_2, \ldots, x_N)\).

Accordingly AGM can be defined as follows:

**Definition of AGM:**

The Arithmetic-Geometric Mean of the N positive real numbers \((x_1, x_2, \ldots, x_N)\) denoted by \( AGM(x_1, x_2, \ldots, x_N) \) is the common limit (or equivalently the common point of convergence) \( M_{AG} \) of the two common sequences \( \{a_n\} \) and \( \{g_n\} \) defined by Eq. 5 and Eq. 6 respectively where the square root takes the principal value and \( a_0 \) & \( g_0 \) are defined by Eq. 1 and Eq. 2 respectively.
2.2 AHM
Let \( \{a_n^A\} \) and \( \{b_n^A\} \) be two sequences defined by
\[
a_{n+1}^A = \frac{1}{2}(a_n^A + b_n^A) \tag{7}
\]
\[
b_{n+1}^A = \left[\frac{1}{2}(a_n^A)^{-1} + (b_n^A)^{-1}\right]^{-1} \tag{8}
\]
respectively. Then, the two sequences \( \{a_n^A\} \) and \( \{b_n^A\} \) converges to the same point \( M_{AH} \). This common point of convergence \( M_{AH} \) can be termed as the Arithmetic-Harmonic Mean of \( (x_1, x_2, ..., x_N) \).
Accordingly, AHM can be defined as follows:

**Definition of AHM:**
The Arithmetic-Harmonic Mean of the \( N \) positive real numbers \( (x_1, x_2, ..., x_N) \) denoted by \( AHM(x_1, x_2, ..., x_N) \) is the common limit (or equivalently the common point of convergence \( M_{AH} \) of two sequences \( \{a_n^A\} \) and \( \{b_n^A\} \) defined respectively by Eq. 7 and Eq. 8 where \( a_0 \) and \( b_0 \) are defined by Eq. 1 and Eq. 3 respectively.

2.3 GHM
Let \( \{g^H\} \) and \( \{h^H\} \) be two sequences defined respectively by
\[
g_{n+1}^H = \left(\frac{g_n^H}{h_n^H}\right)^\frac{1}{3} \tag{9}
\]
\[
h_{n+1}^H = \left[\frac{1}{2}(g_n^H)^{-1} + (h_n^H)^{-1}\right]^{-1} \tag{10}
\]
where the square cube takes the principal value.
The, the two sequences \( \{g^H\} \) and \( \{h^H\} \) converges to the same point \( M_{GH} \). This common point of convergence \( M_{GH} \) can be termed as the Geometric-Harmonic Mean of \( (x_1, x_2, ..., x_N) \).
Accordingly, GHM can be defined as follows:

**Definition of GHM:**
The Geometric-Harmonic Mean (GHM) of the \( N \) positive real numbers \( (x_1, x_2, ..., x_N) \) denoted by \( GHM(x_1, x_2, ..., x_N) \) is the common limit (or equivalently the common point of convergence \( M_{GH} \) of two the two sequences \( \{g^H\} \) and \( \{h^H\} \) defined by Eq. 9 and Eq. 10 respectively where the square root takes the principal value and \( g_0 \) and \( h_0 \) are defined by Eq. 2 and Eq. 3 respectively.

2.4 AGHM
Let us define the three sequences \( \{a_n^{AH}\} \), \( \{g_n^{AH}\} \), \( \{h_n^{AH}\} \) defined by
\[
a_{n+1}^{AH} = \frac{1}{3}\left(a_n^{AH} + g_n^{AH} + h_n^{AH}\right) \tag{11}
\]
\[
g_{n+1}^{AH} = \left(\frac{a_n^{AH}}{g_n^{AH}} \times h_n^{AH}\right)^\frac{1}{3} \tag{12}
\]
\[
a_{n+1}^{AH} = \left[\frac{1}{3}\left(\left(a_n^{AH}\right)^{-1} + \left(g_n^{AH}\right)^{-1} + \left(h_n^{AH}\right)^{-1}\right)\right]^{-1} \tag{13}
\]
respectively where the cube root takes the principal value.
Then, the three sequences \( \{a_n^{AH}\} \), \( \{g_n^{AH}\} \), \( \{h_n^{AH}\} \) converge to a common point \( M_{AGH} \). This common point of convergence can be termed as the Arithmetic-Geometric-Harmonic Mean of \( (x_1, x_2, ..., x_N) \).
Accordingly, AGHM can be defined as follows:

**Definition of AGHM:**
The Arithmetic-Geometric-Harmonic Mean (AGHM) of the \( N \) positive real numbers \( (x_1, x_2, ..., x_N) \) denoted by \( AGHM((x_1, x_2, ..., x_N) \) is the common limit (or equivalently the common point of convergence \( M_{AGH} \) of the three sequences \( \{a_n^{AH}\} \), \( \{g_n^{AH}\} \), \( \{h_n^{AH}\} \) defined by Eq. 11, Eq. 12, Eq. 13 respectively where the cube root takes the principal value and \( a_0 \), \( g_0 \), \( h_0 \) are defined by Eq. 1, Eq. 2, Eq. 3.
3 Measure of Central Tendency of Data

The seven measures of central tendency of data, measuremental in nature, discussed here are AM, GM, HM, AGM, AHM, GMH & AGHM. Among these seven, the first three are already established and widely used ones while the other four have been attempted here. The logical background of the seven measures has been explained below:

3.1 AM as a Measure of Central Tendency

If \( \mu \) is the central tendency of the observations of \((x_1, x_2, \ldots, x_N)\) then the observations are composed of \( \mu \) and random errors. In other words, these can be described by or expressed as

\[ x_i = \mu + \epsilon_i, (i = 1, 2, \ldots, N) \quad (14) \]

where \( \epsilon_1, \epsilon_2, \ldots, \epsilon_N \) are the random errors, which assume positive and negative values in random order, associated to \((x_1, x_2, \ldots, x_N)\) respectively. In this case, \( AM(x_1, x_2, \ldots, x_N) \to \mu \) as \( N \to \infty \) where

\[ AM(x_1, x_2, \ldots, x_N) = \frac{1}{N} \sum_{i=1}^{N} x_i \quad (15) \]

This implies that \((x_1, x_2, \ldots, x_N)\) can be regarded as a measure of the value of \( \mu \) or equivalently the central tendency of the observations \((x_1, x_2, \ldots, x_N)\).

3.2 GM as a Measure of Central Tendency

Again since the observations \((x_1, x_2, \ldots, x_N)\) consists of \( \mu \) and random errors, these can be described by or expressed as

\[ x_i = \mu \epsilon_i^l, (i = 1, 2, 3, \ldots, N) \quad (16) \]

where \( \epsilon_1^l, \epsilon_2^l, \ldots, \epsilon_N^l \) are the random errors, which assume values in \((0,1)\) and in \((1, \infty)\) in random order associate to \((x_1, x_2, \ldots, x_N)\) respectively. In this case, \( GM(x_1, x_2, \ldots, x_N) \to \mu \) as \( N \to \infty \) where

\[ GM(x_1, x_2, \ldots, x_N) = \left[ \sum_{i=1}^{N} x_i \right]^{\frac{1}{N}} \quad (17) \]

This implies that \( GM(x_1, x_2, \ldots, x_N) \) can also be regarded as a measure of the value of \( \mu \) or equivalently the central tendency of the observations \((x_1, x_2, \ldots, x_N)\).

3.3 HM as a Measure of Central Tendency

Again since the observations \((x_1, x_2, \ldots, x_N)\) consist of \( \mu \) and random errors, therefore, the reciprocals \((x_1^{-1}, x_2^{-1}, \ldots, x_N^{-1})\) are composed of \( \mu^{-1} \) and random errors different from the respective random errors \( \epsilon_1, \epsilon_2, \ldots, \epsilon_N \) provided \((x_1, x_2, \ldots, x_N)\) are different from zero.

In this case thus,

\[ x_i^{-1} = \mu^{-1} + \epsilon_i^l, (i = 1, 2, \ldots, N) \quad (18) \]

where, \( \epsilon_1^l, \epsilon_2^l, \ldots, \epsilon_N^l \) are the random errors, which assume positive and negative values in random order, associated to are the random errors associated to \((x_1^{-1}, x_2^{-1}, \ldots, x_N^{-1})\) respectively. In this case, \( HM(x_1, x_2, \ldots, x_N) \to \mu \) as \( N \to \infty \) where

\[ HM(x_1, x_2, \ldots, x_N) = \left[ \frac{1}{N} \sum_{i=1}^{N} x_i \right]^{-1} \quad (19) \]

This implies that \( HM(x_1, x_2, \ldots, x_N) \) can also be regarded as a measure of the value of \( \mu \) or equivalently the central tendency of the observations \((x_1, x_2, \ldots, x_N)\).
3.4 AGM as a Measure of Central Tendency

Since each of $a_n$ & $g_n$ is approximate value of $\mu$,

$$a_0 = \mu + \delta_0$$

and

$$g_0 = \mu + \xi_0$$

for real numbers $\delta_0$ and $\xi_0$.

This implies, $\delta_0 > \xi_0$ since $a_0 > g_0$.

By the same logic,

$$a_{n+1} = \mu + \delta_{n+1} + 1$$

and

$$g_{n+1} = \mu + \xi_{n+1} + 1$$

for real numbers $\delta_{n+1}$ and $\xi_{n+1}$.

Since $a_{n+1}$ is the AM of $a_n$ and $g_n$ Therefore, $a_n > a_{n+1} > g_n$

Which implies, $\delta_n > \delta_{n+1}$, i.e., sequence $\{\delta_n\}$ is decreasing.

This implies, $\delta_n < \delta_0$, i.e, $\delta_n < a_0 - \mu$

Also $\delta_0 > \xi_0 \implies \delta_0 > g_n - \mu$

Hence, $g_0 - \mu < \delta_n < a_0 - \mu$, i.e., the sequence $\{\delta_n\}$ is bounded.

Hence, the sequence $\{\delta_n\}$ is convergent and converges to a point $\delta_{AG}$ in $(g_0 - \mu, a_0 - \mu)$.

Accordingly, $\{a_n\}$ & $\{g_n\}$ and hence $AGM(x_1, x_2, \ldots, x_N)$ converges to a point $\mu + \delta_{AG}$.

Therefore, $AGM(x_1, x_2, \ldots, x_N)$ can be regarded as a measure of the value of $\mu$ and consequently the central tendency of the values $(x_1, x_2, \ldots, x_N)$ with deviation $\delta_{AG}$ in $(g_0 - \mu, a_0 - \mu)$.

3.5 AHM as a Measure of Central Tendency

Since each of $a'_n$ & $h'_n$ is approximate value of $\mu$,

$$a'_0 = \mu + \delta'_0$$

and

$$h'_0 = \mu + e'_0$$

for real numbers $\delta'_0$ and $e'_0$.

This implies, $\delta'_0 > e'_0$ since $a'_0 > h'_0$.

By the same logic,

$$a'_{n+1} = \mu + \delta'_{n+1} + 1$$

and

$$h'_{n+1} = \mu + e'_{n+1} + 1$$

for real numbers $\delta'_{n+1}$ and $e'_{n+1}$.

Since $a'_{n+1}$ is the AM of $a'_n$ and $h'_n$ Therefore, $a'_n > a'_{n+1} > h'_n$

Which implies, $\delta'_n > \delta'_{n+1}$, i.e., sequence $\{\delta'_n\}$ is decreasing.

Moreover, $h_0 - \mu < \delta'_n < a_0 - \mu$

i.e, the sequence $\{\delta'_n\}$ is bounded.

Hence, the sequence $\{\delta'_n\}$ is convergent and converges to a point $\delta_{AH}$ in $(h_0 - \mu, a_0 - \mu)$.

Accordingly, $\{a'_n\}$ & $\{h'_n\}$ and hence $AHM(x_1, x_2, \ldots, x_N)$ converges to a point $\mu + \delta_{AH}$.

Therefore, $AHM(x_1, x_2, \ldots, x_N)$ can be regarded as a measure of the value of $\mu$ and consequently the central tendency of the values $(x_1, x_2, \ldots, x_N)$ with deviation $\delta_{AH}$ in $(h_0 - \mu, a_0 - \mu)$.

3.6 GHM as a Measure of Central Tendency

Since each of $g''_0 = g_0$ & $h''_0 = h_0$ is approximate value of $\mu$,

$$g''_0 = \mu + \xi_0$$

and

$$h''_0 = \mu + e_0$$

for real numbers $\xi_0$ and $e_0$.

This implies, $\xi_0 > e_0$ since $g''_0 > h''_0$.

By the same logic,

$$g''_{n+1} = \mu + \xi''_{n+1} + 1$$

and

$$h''_{n+1} = \mu + e''_{n+1} + 1$$

for real numbers $\xi''_{n+1}$ and $e''_{n+1}$.

Since $g''_{n+1}$ is the GM of $g'_n$ and $h'_n$ Therefore, $g''_n > g''_{n+1} > h''_n$

Which implies, $\xi''_n > \xi''_{n+1}$, i.e., sequence $\{\xi''_n\}$ is decreasing.

Moreover, $h_0 - \mu < \xi''_n < g_0 - \mu$

i.e, the sequence $\{\xi''_n\}$ is bounded.

Hence, the sequence $\{\xi''_n\}$ is convergent and converges to a point $\xi_{GH}$ in $(h_0 - \mu, g_0 - \mu)$.

Accordingly, $\{g''_n\}$ & $\{h''_n\}$ and hence $GHM(x_1, x_2, \ldots, x_N)$ converges to a point $\mu + \xi_{GH}$.

Therefore, $GHM(x_1, x_2, \ldots, x_N)$ can be regarded as a measure of the value of $\mu$ and consequently the central tendency of the values $(x_1, x_2, \ldots, x_N)$ with deviation $\xi_{GH}$ in $(h_0 - \mu, g_0 - \mu)$.  

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3.7 AGHM as a Measure of Central Tendency

In this case as earlier,
\[
a_0'' = \mu + \delta_0, \quad g_0'' = \mu + \xi_0 \quad \text{and} \quad h_0'' = \mu + e_0
\]
is approximate value of \(\mu\), \(g_0'' = \mu + \xi_0\) and \(h_0'' = \mu + e_0\), for real numbers \(\delta_0, \xi_0\) and \(e_0\).

This implies, \(\delta_0 > \xi_0 > e_0\) since \(a_0'' > g_0'' > h_0''\).

By the same logic,
\[
a_{n+1}'' = \mu + \delta_{n+1}, \quad g_{n+1}'' = \mu + \xi_{n+1}, \quad h_{n+1}'' = \mu + e_{n+1}
\]
is the AM of \(a_n'' \), \(g_n'' \) and \(h_n'' \).

Which implies, \(\delta_n'' > \xi_n'' > e_n''\), i.e., sequence \(\{\delta_n''\}\) is decreasing.

Moreover, \(h_0 - \mu < \delta_n'' < a_0 - \mu\)
i.e, the sequence \(\{\delta_n''\}\) is bounded

Hence, the sequence \(\{\delta_n''\}\) is convergent and converges to a point \(\delta_{AGH}\) in \((h_0 - \mu, a_0 - \mu)\). Accordingly, the sequences \(\{a_n''\}, \{g_n''\} \) & \(\{h_n''\}\) and hence AGHM\((x_1, x_2, \ldots, x_N)\) converges to a point \(\mu + \xi_{GH}\).

Therefore, GHM\((x_1, x_2, \ldots, x_N)\) can be regarded as a measure of the value of \(\mu\) and consequently the central tendency of the values \((x_1, x_2, \ldots, x_N)\) with deviation \(\delta_{AGH}\) in \((h_0 - \mu, a_0 - \mu)\).

4 Numerical Example – Application to Numerical data

Observed data considered here are the data on each of annual maximum & annual minimum of surface air temperature, occurred in temperature periodic year (TPR), at Guwahati during the period from 1969 to 2013 [Chakrabarty (2014c, 2015a, 2015c, 2015d, 2015e, 2015f)]. The objective here is to evaluate the value of central tendency of each of annual maximum & annual minimum of surface air temperature at Guwahati.

4.1 Central Tendency of Annual Minimum of Surface Air Temperature at Guwahati

From the observed data on annual minimum of surface air temperature, occurred in temperature periodic year (TPR), at Guwahati during the period from 1969 to 2013, values (in Degree Celsius) of the seven measures of central tendency have been found as follows:

\[
AM = 37.2093023255814, \\
GM = 37.1922871485760, \\
HM = 37.17539890356262, \\
AGM = 37.20079425067069371656824015813, \\
AHM = 37.18811147922283218438295127449, \\
GHM = 37.183841587880081504883830979786, \\
AGHM = 37.192326883785690452815011297444
\]

4.2 Central Tendency of Annual Minimum of Surface Air Temperature at Guwahati

From the observed data on annual minimum of surface air temperature, occurred in temperature periodic year (TPR), at Guwahati during the period from 1969 to 2013, values (in Degree Celsius) of the seven
measures of central tendency have been found as follows:

\[ AM = 7.3634146341463414634146341463415, \]

\[ GM = 7.2597176194576185608709616351297, \]

\[ HM = 7.154393380282352520984974707569, \]

\[ AGM = 7.3114742070301664641236221835825, \]

\[ AHM = 7.258151618339946610217427950892, \]

\[ GHM = 7.206766895137370073793727700802, \]

\[ AGHM = 7.2586735571288657555393158774538 \]

5 Conclusion

From the description, presented above, one can arrive at the following conclusions:

1. In addition to AM, GM, & HM, each of the four formulations namely AGM, AHM, GHM & AGHM can be regarded as a measure of average of a set of numbers.

2. AGM, AHM, GHM & AGHM are defined only when the associated numbers are strictly positive. For numbers other than strictly positive, there is necessity of searching for technique of finding these types of average.

3. Each of AGM, AHM, GHM & AGHM can be accepted as measure of central tendency of data in addition to its usual measures namely AM, GM, & HM.

4. Each of AM, GM, & HM becomes closer and closer to the value of the central tendency of data if the data size becomes larger and larger. However, for data of small size, if is the difference among AM, GM, & HM is found to be unacceptable then AGM, AHM, GHM & AGHM may be preferred in finding the value of central tendency of data.

From the meaning of research [Chakrabarty, 2018g,h, 2019f], it can be concluded that the derivation of AGM, AHM, GHM & AGHM as measures of central tendency of data can be regarded as research findings carrying fundamental importance and high significance in the theory of analysis of data specially of measure of central tendency of data.

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