Static M-horizons

Jan Gutowski and George Papadopoulos

Department of Mathematics, King’s College London, Strand, London WC2R 2LS, U.K.

E-mail: jan.gutowski@kcl.ac.uk, george.papadopoulos@kcl.ac.uk

Abstract: We determine the geometry of all static black hole horizons of M-theory preserving at least one supersymmetry. We demonstrate that all such horizons are either warped products $\mathbb{R}^{1,1} \times_w S$ or $AdS_2 \times_w S$, where $S$ admits an appropriate Spin(7) or SU(4) structure respectively; and we derive the conditions imposed by supersymmetry on these structures. We show that for electric static horizons with Spin(7) structure, the near horizon geometry is a product $\mathbb{R}^{1,1} \times S$, where $S$ is locally a compact Spin(7) holonomy manifold. For electric static solutions with SU(4) structure, we show that the horizon section $S$ is a circle fibration over an 8-dimensional Kähler manifold which satisfies an additional condition involving the Ricci scalar and the length of the Ricci tensor. Solutions include $AdS_2 \times S^3 \times CY_6$ as well as many others constructed from taking the 8-dimensional Kähler manifold to be a product of Kähler-Einstein and Calabi-Yau spaces.

Keywords: Black Holes in String Theory, Supergravity Models
1 Introduction

The classical uniqueness theorems for 4-dimensional black holes [1]–[7] do not extend to higher dimensions. In particular, in 5 dimensions, apart from black holes with spherical horizon topology [8] there are also black rings with horizon topology $S^1 \times S^2$ [9]. In more than 5 dimensions, the results of [10–15] indicate that there are many black holes with exotic horizon topologies.

The question naturally arises as to whether there are black holes with exotic horizon topologies in 10- and 11-dimensional supergravities, which are the effective theories of strings and M-theory. For this a near horizon analysis has been carried out in the heterotic [16, 17] and IIB [18, 19] supergravities. This analysis has led to the discovery of many
new black hole near horizon geometries, and so has provided some supporting evidence for the existence of exotic black holes in these theories.

In this paper, we shall investigate the static near horizon black hole geometries of 11-dimensional supergravity which preserve at least one supersymmetry. It is expected that there are many black hole solutions in M-theory. The IIA Newton constant increases quadratically with the string coupling. So as the IIA string coupling becomes large, the strength of the gravitational force increases and IIA matter collapses to black holes. But the strong coupling limit of IIA string theory is conjectured to be M-theory\(^\text{[20, 21]}\) which has as an effective theory 11-dimensional supergravity. So these black holes should be solutions of 11-dimensional supergravity\(^\text{[22]}\).

As in the case of heterotic and IIB black hole horizons the aim is to find all black hole horizons of 11-dimensional supergravity which preserve one supersymmetry. However, unlike the heterotic case, there is no complete classification of supersymmetric backgrounds in 11-dimensions. The Killing spinor equations (KSEs) of 11-dimensional supergravity for backgrounds preserving one supersymmetry have been solved in\(^\text{[23, 24]}\), and in\(^\text{[25]}\) using spinorial geometry. A systematic method for solving the KSEs of 11-dimensional supergravity for backgrounds preserving any fraction of supersymmetry has been presented in\(^\text{[26]}\). Moreover it has been shown that all backgrounds which preserve more than 29 supersymmetries are maximally supersymmetric\(^\text{[27, 28]}\), and the maximally supersymmetric backgrounds have been classified in\(^\text{[29, 30]}\). There are also conjectures on the number of supersymmetries preserved by supersymmetric M-theory backgrounds\(^\text{[31]}\) and the geometry of solutions with more than 24 supersymmetries\(^\text{[32]}\).

The focus of the work will be on the static near horizons of black holes which preserve at least one supersymmetry. The addition of rotation makes the analysis more involved and it will be reported elsewhere. Facilitated by the spinorial geometry technique for solving KSEs of\(^\text{[25, 26]}\), we show that the solution of the KSEs of 11-dimensional supergravity implies that the near horizon geometries preserving at least one supersymmetry are either warped products $AdS_2 \times_w S$, or locally warped products $\mathbb{R}^{1,1} \times_w S$, where $S$ is the near horizon section which admits either a SU(4) or a Spin(7) structure, respectively. In both cases, we present all the geometric conditions on the SU(4) and Spin(7) structures implied by the KSEs. In the former case, if $S$ admits an isometry, then it is a fibration over an almost Hermitian symplectic 8-dimensional base manifold $B$. The skew-symmetric part of the Nijenhuis tensor of $B$ vanishes but the almost complex structure is not always integrable. The conditions on the geometry we have found and the $AdS_2$ backgrounds we have considered are more general than those that have appeared so far in the literature\(^\text{[34–37]}\) in the context of $AdS_2$ solutions in 11-dimensional supergravity.

The field equations impose additional conditions. We have solved these for the electric static horizons. The electric near horizon geometries with a Spin(7) structure are products $\mathbb{R}^{1,1} \times S$, where $S$ is locally a holonomy Spin(7) manifold, and the 4-form flux vanishes. In the SU(4) structure case, $S$ admits an isometry and is a fibration over an 8-dimensional Kähler manifold $B$. In addition, the Ricci scalar and the length of the Ricci tensor of the Kähler manifold satisfy a condition (5.31). This condition has been previously found in the context of $AdS_2$/CFT\(_1\) correspondence\(^\text{[34]}\). In the special case where the solution
is a direct product $\text{AdS}_2 \times \mathcal{S}$, i.e., the warp factor is constant, the Ricci scalar is constant and so $B$ is a Kähler-Yamabe manifold. Furthermore, the length of the Ricci tensor is also pointwise constant. It turns out that $B$ is not a Kähler-Einstein space, and $\mathcal{S}$ is not Sasakian. Solutions include $\text{AdS}_2 \times S^3 \times CY_6$ for any 6-dimensional Calabi-Yau manifold $CY_6$ and others which can be constructed by taking $B$ to be a product of Kähler-Einstein and Calabi-Yau manifolds. Such solutions have also been found in [34, 36, 37] searching for backgrounds in the context of $\text{AdS}_2$ solutions in 11-dimensional supergravity.

This paper is organized as follows. In section two, we set up our notation and solve the KSEs along the lightcone directions for a class of static $M$-horizons, which preserve at least one supersymmetry. In sections three and four, we solve the KSEs for this class of static $M$-horizons, and investigate the geometry of electric static $M$-horizons. We also present several examples. In section five, we solve the KSEs for all static $M$-horizons, and the field equations in the electric case. In section six, we give our conclusions.

## 2 Solution of Killing spinor equations

### 2.1 Static near horizon geometry

To describe the near horizon geometry of 11-dimensional black holes, we shall use the Gaussian null coordinates of [38] to describe the geometry near the black hole horizons. In particular, assuming appropriate analyticity conditions as well as the existence of an extreme limit and an analysis similar to that done for 5-dimensional supergravity in [39] or for IIB supergravity in [18, 19], we find that after taking the extreme limit the metric and 4-form field strength of the near horizon geometry of 11-dimensional black holes can be written as

$$
\begin{align*}
 ds^2 &= 2e^+ e^- + \delta_{ij}e^i e^j, \\
 F &= e^+ \wedge e^- \wedge Y + re^+ \wedge d_h Y + X, \quad dX = 0,
\end{align*}
$$  \tag{2.1}

where $(u, r, y^I)$ are the coordinates of spacetime, $d_h Y = dY - h \wedge Y$ and

$$
\begin{align*}
 e^+ &= du, \\
 e^- &= dr + rh - \frac{1}{2} r^2 \Delta du, \\
 e^i &= e^i J dy^J,
\end{align*}
$$  \tag{2.2}

is a frame basis with $h = h_i(y) e^i$ a 1-form and $\Delta = \Delta(y)$ a function which depend only on the $y$ coordinates. The horizon section $\mathcal{S}$ is the 9-dimensional submanifold given by $r = u = 0$ with metric $ds^2(\mathcal{S}) = \delta_{ij} e^i e^j$. Observe that $\Delta$ and $h$ are a globally defined scalar and 1-form on $\mathcal{S}$, respectively.

Static horizons are those for which\footnote{We thank James Lucietti for a discussion on this point.}

$$
e^- \wedge de^- = 0, \tag{2.3}$$

which yields

$$
\begin{align*}
 dh &= 0, \\
 d\Delta &= \Delta h.
\end{align*}
$$  \tag{2.4}
Static horizons can be subdivided in two classes. One subclass is to take
\[ \Delta = 0, \quad dh = 0, \]  
(2.5)
and other is
\[ \Delta > 0, \quad h = d \log \Delta. \]  
(2.6)

For supersymmetric horizons \( \Delta \geq 0 \) since \( \partial_u \) is either null or time-like.

In the former case, on introducing a local co-ordinate \( x \) such that \( h = dx \) and making a change of co-ordinates \( r \rightarrow e^x r \), the metric can be rewritten as
\[ ds^2 = 2 e^{-x} dudr + ds^2(S), \]  
(2.7)
and the near horizon geometry is a warped product \( \mathbb{R}^{1,1} \times_w S \). However if \( h \) is closed but not exact, the resulting warped product is local.

For the latter case, the metric can be rewritten, after a change of coordinates \( r \rightarrow r \Delta \), as
\[ ds^2 = 2 \Delta^{-1} du \left( dr - \frac{1}{2} r^2 du \right) + ds^2(S), \]  
(2.8)
and so the near horizon geometry is a warped product \( AdS_2 \times_w S \).

In the investigation of field and KSEs, it is instructive to begin with backgrounds for which
\[ h = 0, \]  
(2.9)
and \( \Delta \) an arbitrary function of \( S \). It may seem that these horizons are not static because they do not a priori satisfy the static condition (2.4). However, as we shall show the field equations imply that \( \Delta \) is constant and so all horizons satisfying (2.9) are static. The advantage with dealing with condition (2.9) is that the solution of the KSEs is particularly simple and the geometry of the horizons can be easily described.

After understanding the geometry of the (2.9) horizons, we shall present the solution of the KSEs for all static horizons without going into details. This is because static horizons with \( h \neq 0 \) are more easily investigated in the context of rotating horizons which will be presented elsewhere.

The metric and 4-form flux of static \( h = 0 \) horizons become
\[ ds^2 = 2 e^+ e^- + \delta_{ij} e^i e^j, \quad F = e^+ \wedge e^- \wedge Y + r e^+ \wedge dY + X, \quad dX = 0, \]  
(2.10)
where now
\[ e^+ = du, \quad e^- = dr - \frac{1}{2} r^2 \Delta du, \quad e^i = e^i J dy^J. \]  
(2.11)

Observe that if \( \Delta > 0 \), the vector field \( V = \partial_u \) is time-like and Killing and becomes null at \( r = 0 \) the location of the horizon. \( V \) is identified with the stationary vector field of the black hole spacetime at the near horizon limit.
We shall consider static near horizon geometries which preserve at least one supersymmetry. For this, we shall require that (2.10) solves the Killing spinor equations
\[ \nabla_M \epsilon + \left( - \frac{1}{288} \Gamma_{M}^{L_1 L_2 L_3 L_4} F_{L_1 L_2 L_3 L_4} + \frac{1}{36} F_{M L_1 L_2 L_3} \Gamma^{L_1 L_2 L_3} \right) \epsilon = 0, \] (2.12)
of 11-dimensional supergravity. To achieve this, we shall use spinorial geometry and the techniques and notation developed in [26]. In this context, we set \( i, j = 1, 2, 3, 4, 6, 7, 8, 9, \# \), where \# is identified with the 10-th direction, and the light-cone directions \( e^+, e^- \) are spanned by the time and 5-th directions of the spacetime.

We shall also make use of the field equations
\[ R_{MN} = \frac{1}{12} F_{ML_1 L_2 L_3} F_{N}^{L_1 L_2 L_3} - \frac{1}{144} g_{MN} F_{L_1 L_2 L_3 L_4} F^{L_1 L_2 L_3 L_4}, \]
\[ d \ast F = \frac{1}{2} F \wedge F, \] (2.13)
of 11-dimensional supergravity, where the spacetime orientation is taken as
\[ d\text{vol}_{11} = e^+ \wedge e^- \wedge d\text{vol}(S), \] (2.14)
and \( d\text{vol}(S) = e^{12346789\#} \).

### 2.2 Light-cone integrability of Killing spinor equations

The KSEs of 11-dimensional supergravity for the background given in (2.10) with \( h = 0 \) and \( \Delta \) a function on \( S \) can be integrated along the light-cone directions. For this, we decompose the Killing spinor as
\[ \epsilon = \epsilon_+ + \epsilon_-, \quad \Gamma^\pm \epsilon_\pm = 0. \] (2.15)
Then a straightforward calculation reveals that
\[ \epsilon_+ = \eta_+, \quad \epsilon_- = \eta_- + r \Gamma_- \Theta_+ \eta_+ \] (2.16)
and
\[ \eta_+ = \phi_+ + u \Gamma_+ \Theta_- \phi_-, \quad \eta_- = \phi_- \] (2.17)
where now the spinors \( \phi_\pm = \phi_\pm(y) \) do not depend on \( r \) or \( u \), and we have set
\[ \Theta_\pm = \frac{1}{288} X_{\ell_1 \ell_2 \ell_3 \ell_4} \Gamma^{\ell_1 \ell_2 \ell_3} \pm \frac{1}{12} Y_{\ell_1 \ell_2} \Gamma^{\ell_1 \ell_2}. \] (2.18)
In addition, the + and - components of the KSEs impose the following algebraic conditions
\[ \left( \frac{1}{2} \Delta + \frac{1}{12} dY_{\ell_1 \ell_2 \ell_3} \Gamma^{\ell_1 \ell_2} - 2 \Theta_- \Theta_+ \right) \phi_+ = 0, \] (2.19)
\[ \left( \partial_\ell \Delta \Gamma^\ell + \frac{1}{6} dY_{\ell_1 \ell_2 \ell_3} \Gamma^{\ell_1 \ell_2} \Theta_+ \right) \phi_+ = 0, \] (2.20)
\[ \left( \frac{1}{2} \Delta - \frac{1}{12} dY_{\ell_1 \ell_2 \ell_3} \Gamma^{\ell_1 \ell_2} - 2 \Theta_- \Theta_+ \right) \Theta_- \phi_- = 0, \] (2.21)
\[ \left( \frac{1}{2} \Delta + \frac{1}{24} dY_{\ell_1 \ell_2 \ell_3} \Gamma^{\ell_1 \ell_2} + 2 \Theta_+ \Theta_- \right) \Theta_- \phi_- = 0, \] (2.22)
\[ \left( - \frac{1}{2} \Delta + \frac{1}{24} dY_{\ell_1 \ell_2 \ell_3} \Gamma^{\ell_1 \ell_2} + 2 \Theta_+ \Theta_- \right) \phi_- = 0. \] (2.23)
These further constrain both the spinors $\phi_{\pm}$, the fluxes and geometry of the horizon. We shall solve all these conditions as well as the remaining KSEs along the directions of the horizon section $S$ for one Killing spinor.

3 N=1 supersymmetry

3.1 Killing vector bilinear

Let us assume that (2.10) with $h = 0$ and $\Delta$ a function of $S$ admits one Killing spinor. The existence of a Killing spinor implies that the spacetime admits a Killing vector field $W$, which is constructed as a spinor bilinear, that is either time-like or null. The analysis of the KSEs proceeds by identifying the Killing vector bilinear $W$ with the Killing vector field of the black hole horizon $V$. The associated 1-form of the latter is given by

$$V = e^{-} - \frac{1}{2} \bar{r}^2 \Delta e^{+}$$

(3.1)

while the 1-form associated with the former can be computed from the expression

$$W = (B \epsilon^*, \Gamma_M \epsilon) e^M,$$

(3.2)

for the 1-form spinor bilinear.

To compute $W$ and compare it to $V$, we have to evaluate the expression for $W$. For this we shall use the residual Spin(9) gauge symmetry of KSEs which fixes the two light-cone directions that we have integrated over. To proceed the 32-dimensional Majorana representation of Spin(10,1) decomposes under Spin(9) into two 16-dimensional Majorana representations. This decomposition has already been given in (2.15), where the Killing spinor was written as a sum of two spinors with opposite chirality along the light-cone directions. In addition, Spin(9) acts transitively on the $S^{15}$ sphere in the 16-dimensional Majorana representation with isotropy group Spin(7), Spin(9)/Spin(7) = $S^{15}$. Using this, the spinor $\phi_{-}$ can be chosen to lie in any direction and in particular one can set

$$\phi_{-} = w(e_5 + \epsilon_{12345}),$$

(3.3)

for some real function $w$. Next, on comparing $W$ and $V$ in the basis (2.11) one finds that

$$W_{+}|_{r=0} = 0.$$

(3.4)

The $W_{+}$ component can be computed using (3.2) and $\phi_{-} = w(e_5 + \epsilon_{12345})$ to reveal that $w = 0$. Thus, we find that

$$\phi_{-} = 0.$$

(3.5)

To continue, one can again use $r,u$-independent Spin(9) transformations to set, without loss of generality,

$$\phi_{+} = z(1 + \epsilon_{1234}).$$

(3.6)
for some real \((r, u\text{-independent})\) function \(z\). Using this and (3.2), one finds that

\[
W_- = -2\sqrt{2}z^2.
\]  

(3.7)

But \(V_- = 1\), and so \(z\) is constant. For convenience, we set \(z = 1\), and so

\[
\phi_+ = 1 + e_{1234}.
\]  

(3.8)

To summarize the results so far, substituting (3.5) and (3.8) into the expression for the Killing spinor \(\epsilon\), (2.16) and (2.17), we find that

\[
\epsilon = (1 + e_{1234}) + r\Gamma_-\Theta_+(1 + e_{1234}) .
\]  

(3.9)

The Killing spinor \(\epsilon\) can be further simplified. As \(\Gamma_+\Theta_+(1 + e_{1234}) = 0\), \(\Theta_+(1 + e_{1234})\) is also a Spin(9) Majorana spinor and so it can be expanded in the basis \(1, e_{1234}, e_i, e_{ij}, e_{ijk}\) for \(i, j, k = 1, \ldots, 4\). Using this and the above expression for the Killing spinor \(\epsilon\), it is straightforward to evaluate the remaining components of the spinor bilinear \(W\) in the directions transverse to the light cone directions, and determine the resulting constraints imposed on the components of \(\Theta_+(1 + e_{1234})\). In particular, requiring that \(W^\sharp = 0\) implies that the component of \(\Theta_+(1 + e_{1234})\) in the \(1 + e_{1234}\) direction vanishes.

Furthermore, requiring that \(W^\alpha = 0\) forces the components of \(\Theta_+(1 + e_{1234})\) in the \(e_i\) and \(e_{ijk}\) directions to vanish as well. As a result \(\Theta_+(1 + e_{1234})\) must be a linear combination of \(i(1 - e_{1234})\) and \(e_{ij}\) and so must lie in the vector representation of Spin(7) the isotropy group of \(1 + e_{1234}\). On the other hand Spin(7) acts transitively on the \(S^6\) sphere in the 7-dimensional vector representation with isotropy group SU(4). As a result \(\Theta_+(1 + e_{1234})\) can be chosen to lie in any direction and in particular one can then without loss of generality take

\[
\Theta_+(1 + e_{1234}) = i\Phi(1 - e_{1234}),
\]  

(3.10)

for some real function \(\Phi = \Phi(y)\). On examining the component \(W_+\) of the Killing spinor bilinear, one finds that

\[
\Delta = 4\Phi^2.
\]  

(3.11)

This concludes all the conditions on the Killing spinor which arise from the identification of \(W\) with the Killing vector field of the black hole horizon.

After considering the KSEs along the directions transverse to the light-cone, the independent conditions which have to be solved so that the near horizon geometry (2.10) admits at least one supersymmetry are

\[
\Theta_+(1 + e_{1234}) = i\Phi(1 - e_{1234}),
\]  

(3.12)

\[
\nabla_i(1 + e_{1234}) + \left(\frac{1}{24}X_{i\ell_1}\ell_2\ell_3\Gamma^{\ell_1\ell_2\ell_3} + \frac{1}{8}\Gamma_i\ell_1\ell_2Y_{\ell_1\ell_2}\right)(1 + e_{1234})
\]

\[
- i\Phi\Gamma_i(1 - e_{1234}) = 0 ,
\]  

(3.13)

\[
\nabla_i(i\Phi(1 - e_{1234})) + i\Phi\left(-\frac{1}{24}X_{i\ell_1}\ell_2\ell_3\Gamma^{\ell_1\ell_2\ell_3} + \frac{1}{8}\Gamma_i\ell_1\ell_2Y_{\ell_1\ell_2}\right)(1 - e_{1234})
\]

\[
+ \left(\frac{1}{4}\Delta\Gamma_i - \frac{1}{48}dY_{\ell_1\ell_2\ell_3}\Gamma^{\ell_1\ell_2\ell_3}\Gamma_i\right)(1 + e_{1234}) = 0 ,
\]  

(3.14)
where $\Delta$ and $\Phi$ are related as in (3.11), and $\nabla$ is the Levi-Civita connection on the horizon section $S$. We remark that conditions (2.19) and (2.20) have been omitted from this list, because they are implied by (3.12), (3.13), (3.14) and (3.11).

### 3.2 Solution to the Killing spinor equations

The KSEs (3.12)–(3.14) can be easily solved using the spinorial geometry techniques of [25, 26] and the general results of [26]. In particular, the differential and algebraic conditions turn into a linear system for the geometry as expressed in terms of the spin connection and the components of the fluxes. This system can be solved to express some of the components of the fluxes in terms of the geometry and find the conditions on the spacetime geometry imposed by supersymmetry.

Before we proceed with the solution to the linear system, the spacetime admits an 1-form, 2-form and 5-form bilinear. As a consequence of the assumption that the Killing spinors are globally defined, all these three bilinears are also globally defined on the spacetime. We have already stated the 1-form bilinear. The remaining two are

\[
\alpha = 2 \left( e^{- \frac{1}{2} \Delta e^+} \right) \wedge e^\sharp - 4r \Phi \omega, \quad (3.15)
\]

and

\[
\sigma = - \left\{ \left( e^{- \frac{1}{2} \Delta r^2 e^+} + 2ir \Phi e^\sharp \right) \wedge \chi + \text{c.c.} \right\} + \left( e^{- \frac{1}{2} \Delta r^2 e^+} \right) \wedge \omega \wedge \omega, \quad (3.16)
\]

where $\omega = -i \delta_{\alpha \beta} e^\alpha \wedge e^\beta$ is almost Hermitian 2-form and $\chi$ is a (4,0)-form on the directions transverse to the light-cone and $e^\sharp$. Taking the light-cone directions as globally defined, $e^\sharp$, $\omega$ and $\chi$ are also globally defined. Note that the index $i$ transverse to the light-cone directions decomposes as $i = \alpha, \bar{\alpha}, \sharp, \bar{\sharp}$, where $\alpha = 1, 2, 3, 4$. As a result both the near horizon geometry and the horizon section $S$ admit an SU(4) structure.

We shall not give the linear system as it is easily derived from the KSEs. The solution of the linear system expresses the flux $Y$ in terms of the geometry as

\[
Y = -de^\sharp - 2\Phi \omega, \quad (3.17)
\]

and $\Phi$ as

\[
\Phi = -i \left( \Omega_{\xi,\lambda} - \Omega_{\lambda,\xi} \right), \quad (3.18)
\]

respectively. Also, $\Phi$ satisfies

\[
\partial_\alpha \Phi = \Phi \left( -2\Omega_{\lambda,\bar{\lambda} \alpha} + 2\Omega_{\alpha,\lambda} \right), \quad (3.19)
\]
\[
\partial_\sigma \Phi = -\frac{1}{4} \Phi X_{\lambda, \sigma} \omega. \quad (3.20)
\]
The 4-form \( X \) is expressed in terms of the geometry as

\[
X_{\mu \lambda_1 \lambda_2 \lambda_3} = \left( -\Omega_{\mu, \sigma} + \Omega_{\sigma, \mu} - 2\Omega_{[\mu][\sigma]} \right) \epsilon_{\lambda_1 \lambda_2 \lambda_3},
\]

(3.21)

\[
X_{\beta \alpha \lambda}^\lambda + \frac{1}{4} X_{\lambda}^\sigma \sigma_{\beta \alpha} = \Omega_{\beta, \lambda} + \Omega_{\lambda, \beta},
\]

(3.22)

\[
\frac{2}{3} \Omega_{\lambda}^\lambda + \frac{2}{3} \Omega_{\lambda}^\lambda + \frac{1}{6} X_{\lambda_1 \lambda_2 \lambda_3} \epsilon_{\lambda_1 \lambda_2 \lambda_3} = 0,
\]

(3.23)

\[
X_{\beta \lambda_2 \lambda_3} = \left( \Omega_{\beta, \lambda}^\lambda - \frac{1}{2} \Omega_{[\beta]}^\lambda \right) \epsilon_{\lambda_1 \lambda_2 \lambda_3},
\]

(3.24)

\[
X_{\beta \sigma_1 \sigma_2} = \frac{2}{3} (\Omega_{\beta, \mu_1 \mu_2} + \Omega_{\mu_1, \beta \mu_2}) \epsilon_{\mu_1 \mu_2 \sigma_1 \sigma_2} - 2\Omega_{\beta, \sigma_1} \sigma_2
\]

\[
+ \left( -\frac{4}{3} \Omega_{\lambda}^{\lambda} \sigma_1 + \frac{4}{3} \Omega_{[\sigma_1, \lambda]}^{\lambda} \lambda + \frac{2}{3} \Omega_{\mu_1 \mu_2}^{\mu_1 \mu_2} \right) \delta_{\lambda_{\sigma_2}} \beta.
\]

(3.25)

The conditions on the geometry are

\[
\Omega_{\lambda}^\lambda + \Omega_{\lambda}^\lambda = 0,
\]

\[
-2\Omega_{\lambda_1 \lambda_2 \lambda_3} \epsilon_{\lambda_1 \lambda_2 \lambda_3} + 4\Omega_{\lambda_1}^{\lambda_1} - 2\Omega_{\alpha, \lambda}^{\lambda} + \Omega_{[\alpha]}^{\lambda} = 0,
\]

\[
\Omega_{[\mu_1, [\sigma_2]} - \Omega_{\beta, \mu_1} \mu_2 - \frac{1}{2} (\Omega_{\sigma_1, \beta \mu_2} - \Omega_{\beta, \sigma_1 \mu_2}) \epsilon_{\sigma_1 \sigma_2} \mu_1 \mu_2 = 0.
\]

(3.26)

Observe that the \((2,2)\) and traceless part of \( X \) is not determined by the KSEs in terms of the geometry.

### 3.3 Field equations

In addition to the KSEs the near horizon geometries must satisfy the field equations (2.13). In particular, \( M = +, N = + \) component of the Einstein equation gives

\[
\frac{1}{2} \nabla^2 \Delta = \frac{1}{12} (dY)(\ell_1, \ell_2, \ell_3)(dY)(\ell_1, \ell_2, \ell_3).
\]

(3.27)

As \( S \) is assumed to be compact, this implies that \( \Delta \) is constant, and

\[
dY = 0.
\]

(3.28)

Also note that the \( M = +, N = - \) component of the Einstein equation gives

\[
\Delta = \frac{1}{6} Y_{\ell_1, \ell_2} Y_{\ell_1, \ell_2} + \frac{1}{144} X_{\ell_1, \ell_2, \ell_3, \ell_4} X_{\ell_1, \ell_2, \ell_3, \ell_4}.
\]

(3.29)

If \( \Delta = 0 \) then this condition implies that \( Y = 0 \) and \( X = 0 \), and hence the 4-form also vanishes; in this case the spacetime is \( \mathbb{R}^{1,1} \times S \), where \( S \) is a compact Spin(7) holonomy manifold.

For solutions with \( \Delta \neq 0 \), as \( \Delta \) is constant, the near horizon geometry is a product \( AdS_2 \times S \). Since \( \Phi \) is constant, (3.17) and \( dY = 0 \) imply that

\[
d\omega = 0.
\]

(3.30)

Hence one finds the following additional conditions on the spin connection

\[
\Omega_{[a_1, a_2, a_3]} = 0, \quad \Omega_{a_1, b_1, b_2} = 0, \quad -\Omega_{[a, \mu_1 \mu_2} + \Omega_{[\mu_1, [\mu_2]} = 0, \quad \Omega_{(\alpha, \beta)} = 0.
\]

(3.31)
Comparing these to the geometric conditions derived from the KSEs (3.26) and (3.19), one finds the additional remaining geometric condition

$$2\Omega_{\alpha,\beta}^\beta - \Omega_{\sharp,\sharp}^\alpha = 0 .$$  \hspace{1cm} (3.32)

Implementing all the geometric conditions (3.31) and (3.32) on the fluxes, we find

$$X_{\alpha_{1}\alpha_{2}\alpha_{3}} = -\Omega_{\sharp,\sharp}^\beta \epsilon_{\alpha_{1}\alpha_{2}\alpha_{3}} ; \quad X_{\alpha_{1}\beta_{1}\beta_{2}} = \Omega_{\alpha,\gamma_{1}\gamma_{2}} \epsilon_{\gamma_{1}\gamma_{2}\beta_{1}\beta_{2}} ; \quad X_{\alpha,\gamma} = 0 ,$$

$$\Phi = -i \Omega_{\sharp,\alpha}^\alpha = i \Omega_{\alpha,\sharp}^\alpha , \quad \Omega_{\alpha,\beta}^\beta = 0 .$$  \hspace{1cm} (4.1)

Moreover, $Y = -de^\sharp - 2\Phi \omega$ and it is constrained as

$$\Delta = \frac{1}{6} Y_{\ell_{1}\ell_{2}} Y_{\ell_{1}\ell_{2}} .$$  \hspace{1cm} (4.2)

In addition, the 3-form flux field equations imply that $Y$ is co-closed on $\mathcal{S}$. As $dY = 0$, $Y$ is harmonic. We remark that these conditions are sufficient to ensure that the solution
preserves (at least) $N = 2$ supersymmetry. To see this, note that (3.12), (3.13) and (3.14) are also satisfied if one replaces the Majorana spinors $1 + e_{1234}$ and $i(1 - e_{1234})$ by $i(1 - e_{1234})$ and $-(1 + e_{1234})$ respectively throughout.

First observe that the geometric conditions (4.1) imply that the vector field associated to $e^\sharp$ is Killing on the horizon section $S$ and of constant length. Therefore the metric on $S$ can be written as

$$ds^2(S) = (d\tau + \lambda)^2 + ds^2(B)$$

where $\tau$ is the coordinate along the Killing vector field, $\lambda$ is a 1-form on the base space $B$. Thus $S$ can be thought of as a U(1) fibration over a 8-dimensional manifold $B$. Furthermore, $i_\omega \omega = 0$ and $\omega$ is invariant under the action of the $e^\sharp$ vector field, and so it descends to a closed (almost) Hermitian form on $B$. Since in addition $B$ is complex, one concludes that $B$ is Kähler. The geometric conditions also imply that curvature of the fibration $de^\sharp$ is $(1,1)$ and its trace is constant. As a result $e^\sharp$ is a Hermitian-Einstein connection with a non-vanishing cosmological constant $\Phi$.

These restrictions on the fibration solve all the conditions in (4.1) apart from

$$\Omega_{\alpha,\beta} = 0, \quad \Phi = -i \Omega_{\sharp, \lambda}, \quad \Delta = \frac{1}{6} Y_{\ell_1 \ell_2} Y^{\ell_1 \ell_2}. \quad (4.4)$$

It is clear that the first two conditions can be expressed in terms of components of $d\chi$. In particular the first condition imposes a certain restriction on the canonical class of $B$. However, the identification of the precise condition is not apparent as $\Omega_{\sharp, \lambda} \neq 0$ which indicates that the chosen frame $e^\alpha$ depends on the coordinate along $e^\sharp$, even though the metric and $\omega$ do not, and so it is not adapted to the fibration. We shall illustrate this with an example.

4.2 Example

We shall demonstrate that $AdS_2 \times S^3 \times CY_6$ is a solution, where $CY_6$ is any Calabi-Yau 6-dimensional manifold. In such case $S = S^3 \times CY_6$. To see this, parameterize $S^3$ in terms of Euler angles as

$$\sigma^1 = \sin \frac{\psi}{2} \sin \theta d\phi + \cos \frac{\psi}{2} d\theta, \quad \sigma^2 = -\cos \frac{\psi}{2} \sin \theta d\phi + \sin \frac{\psi}{2} d\theta, \quad \sigma^3 = \frac{1}{2} d\psi + \cos \theta d\phi. \quad (4.5)$$

Then write the metric on $S$ as

$$ds^2(S) = (e^\sharp)^2 + 2e^1 e^{\bar{1}} \sum_{\alpha, \beta > 1} \delta_{\alpha \beta} e^\alpha e^{\bar{\beta}}, \quad (4.6)$$

where

$$e^\sharp = \sigma^3, \quad e^1 = \frac{\sigma^1 + i \sigma^2}{\sqrt{2}} = \frac{1}{\sqrt{2}} e^{\frac{i}{2} \psi} (-i \sin \theta d\phi + d\theta), \quad (4.7)$$
and \( e^\alpha, \alpha > 1 \), a frame on \( CY_6 \) which is independent from the coordinates of \( S^3 \). Observe that \( e^1 \) depends explicitly on the coordinate \( \psi \) of the isometry but neither the metric nor the Kähler form

\[
\omega = -i e^1 \wedge \bar{e}^1 + \omega_{(6)}, \quad \omega_{(6)} = -i \sum_{\alpha, \beta > 1} \delta_{\alpha \bar{\beta}} e^\alpha \wedge \bar{e}^\beta,
\]

(4.8)
on the base space \( B = S^2 \times CY_6 \), depend on \( \psi \), where \( \omega_{(6)} \) is the Kähler form on \( CY_6 \). This gives

\[
\Omega_{1,1} = \frac{i}{2}, \quad \Omega_{\sharp, \alpha}^{\alpha} = 0 \quad \text{for} \quad \alpha > 1,
\]

(4.9)
and so \( \Phi = \frac{1}{2} \).
Moreover
\[
\Omega_{1,11} = 0,
\]
(4.10)
and so
\[
\Omega_{\alpha, \beta}^{\beta} = 0.
\]
(4.11)
Observe also that
\[
Y = -\omega_{(6)}
\]
(4.12)
and so \( \Delta = \frac{1}{6} Y_{\ell_1 \ell_2} Y_{\ell_1 \ell_2} \).

### 4.3 General electric static horizons

Since the metric and Kähler form \( \omega \) are independent from the coordinate \( \tau \) along the isometry, there is a \( \tau \)-independent frame \( \hat{e}^\alpha \) and a unitary transformation \( U \), which may depend on all coordinates of \( S \), such that

\[
e^\alpha = U^\alpha_{\beta} \hat{e}^\beta.
\]
(4.13)
However \( e^\alpha \) is defined up to a local SU(4) transformation which preserves the Killing spinor \( \epsilon \) and so all the conditions we have derived from the field and KSEs. As a result, such a transformation can be used to specify \( U \) up to a phase. Thus we can write

\[
e^\alpha = e^{i \xi} \hat{e}^\alpha,
\]
(4.14)
where \( \xi \) can depend on all coordinates of \( S \). As a result

\[
\Omega_{I, \alpha}^{\beta} = -e^{-i \xi} \partial_I e^{i \xi} \delta_{\alpha \beta} + \tilde{\Omega}_{I, \alpha}^{\beta},
\]
(4.15)
where \( \tilde{\Omega} \) is the spin connection of the frame \((\hat{e}^\alpha, \hat{e}^\beta)\), with \((\hat{e}^\alpha, \hat{e}^\bar{\beta})\) adapted to \( B \). To specify the geometry of \( B \), we have to determine the restrictions on \( \Omega \) implied by (4.4). In particular

\[
\Omega_{\sharp, \alpha}^{\alpha} = 4 e^{-i \xi} \partial e^{i \xi} + \tilde{\Omega}_{I, \alpha}^{\alpha} = i \Phi, \quad \Omega_{\sharp, \alpha}^{\alpha} = -i \Phi
\]
(4.16)
and so
\[ \partial_{\tau} \xi = \frac{1}{2} \Phi \]
leading to \( \xi = \frac{1}{2} \Phi \tau + \beta \), where \( \beta \) does not depend on \( \tau \). The gauge transformation generated by \( \beta \) is inconsequential as it can be absorbed in the definition of the frame \( \hat{e}^\alpha \). So without loss of generality, we can set
\[ \xi = \frac{1}{2} \Phi \tau . \]  
(4.18)

Next, we have
\[ \Omega_{\beta, \alpha}^\alpha = 4e^{-i\xi} \partial_{\beta} e^{i\xi} + \hat{\Omega}_{\beta, \alpha}^\alpha = 0 . \]
(4.19)

Thus, we find
\[ \hat{\Omega}_{\beta, \alpha}^\alpha = 2i \Phi \lambda_\beta . \]
(4.20)

So the Ricci form of \( B \) is
\[ \hat{\rho} = -i\hat{\Theta}_{\beta; \alpha}^\alpha \hat{e}^\beta \wedge \hat{e}^\gamma = 2\Phi d e^\delta . \]
(4.21)

Clearly the curvature of the canonical bundle of \( B \) is proportional to that of the fibration of \( S \) over \( B \). In terms of the Ricci form, the last condition in (4.4) can be rewritten as
\[ \frac{1}{6} \hat{\rho}_{ij} \hat{\rho}^{ij} = 4\Phi^4 , \]
(4.22)

where
\[ \hat{\rho}_{ij} = \hat{\rho}_{ij} + \Phi^2 \omega_{ij} , \quad \hat{\rho}_\alpha^\alpha = 4i \Phi^2 \]
(4.23)
is the (1,1) and traceless component of the curvature of the canonical bundle. Thus \( B \) is a Kähler manifold for which the canonical bundle is equipped with a Hermitian-Einstein connection, the Ricci curvature has point-wise constant length but it is not Einstein, and the Ricci scalar is constant. A consequence of the Hermitian-Einstein condition for the connection of the canonical bundle is that the Kähler metric on \( B \) is Yamabe, ie the Ricci scalar is constant.

### 4.4 More examples

To classify the electric static horizons, one has to find the 8-dimensional Kähler-Yamabe manifolds which admit a Kähler metric such that the Ricci tensor has point-wise constant length. To find examples, we shall take \( B \) to be a product of Kähler-Einstein and Calabi-Yau manifolds \( N_p \) of real dimension \( 2n_p \). Such examples have also been constructed in [34, 36, 37] in the context of AdS\(_2\)/CFT\(_1\) correspondence. So the Ricci forms are
\[ \rho_p = \Phi \ell_p \omega_p , \quad p \leq 4 , \quad \sum_p n_p = 4 , \]
(4.24)
with
\[ \hat{\rho} = \sum_p \rho_p, \quad \omega = \sum_p \omega_p, \]  
(4.25)

where \( \omega_p \) is the Kähler form of \( N_p \). The geometric conditions imply that
\[ \sum_p n_p \ell_p = -4\Phi, \quad \sum_p n_p \ell_p^2 = 16\Phi^2. \]  
(4.26)

Solutions to these equations will give examples of near horizon geometries. For the explicit example given above \( B = S^2 \times CY_6 \) and so \( n_1 = 1, n_2 = 3 \) and \( \ell_1 = -4\Phi, \ell_2 = 0 \). For more examples, take \( B = N_1 \times N_2 \), where \( N_1, N_2 \) are 4-dimensional Kähler Einstein spaces and so \( n_1 = n_2 = 2 \). The conditions on the geometry imply that
\[ \ell_1 = (-1 \pm \sqrt{3})\Phi, \quad \ell_2 = (-1 \mp \sqrt{3})\Phi. \]  
(4.27)

In either case, one of the spaces has negative Ricci curvature. Since all Kähler manifolds with negative first Chern class admit Einstein metrics and there are 4-dimensional Kähler manifolds with positive first Chern class admitting Einstein metrics, there are many examples of electric static horizons.

5 Static horizons

Now we shall turn to the investigation of static horizons with \( h \neq 0 \). There are two classes described by the conditions (2.5) and (2.6), respectively. The \( M = +, N = + \) component of the Einstein equation can be written as
\[ \frac{1}{2} \tilde{\nabla}^2 \Delta - \frac{3}{2} h \tilde{\nabla}_i \Delta - \frac{1}{2} \Delta \tilde{\nabla}^i h_i + \Delta h^2 = \frac{1}{12} (dY - h \wedge Y)_{ijk} (dY - h \wedge Y)^{ijk} \]  
(5.1)

For both (2.5) and (2.6) cases, the l.h.s. of this equation vanishes identically, and we therefore find that
\[ dY - h \wedge Y = 0. \]  
(5.2)

From now on, we shall investigate the two classes separately.

5.1 Static \( dh = \Delta = 0 \) horizons

5.1.1 Solution of KSEs

As we have already mentioned, the solution of the KSEs for static horizons with \( h \neq 0 \) are a special case of that of rotating horizons. Because of this, we shall not explain the solution of the KSEs in detail. Instead, we shall simply state the solution. It turns out that since \( \Delta = 0 \), the Killing spinor is
\[ \epsilon = 1 + \epsilon_{1234}. \]  
(5.3)
Substituting this and (2.5) into the KSEs, one finds the spacetime geometry is restricted as

\begin{align}
2\Omega_{\mu,\nu}^{\lambda} - \Omega_{\lambda,\nu}^{\mu} + \Omega_{\lambda,\mu}^{\nu} &= 0 , \\
\Omega_{\mu_1,\mu_2} - \Omega_{\mu_1,\mu_2} - \frac{1}{2}(\Omega_{\lambda_1,\lambda_2} - \Omega_{\lambda_1,\lambda_2})\varepsilon_{\lambda_1,\lambda_2}^{\mu_1,\mu_2} &= 0 , \\
h_{\gamma} &= \frac{1}{2}(\Omega_{\lambda,\nu}^{\lambda} + \Omega_{\lambda,\nu}^{\lambda}) , \\
h_{\alpha} &= \frac{1}{2}(\Omega_{\lambda_1,\lambda_2,\lambda_3} - \delta_{\lambda_1,\lambda_2,\lambda_3}^{\mu_1,\mu_2} + \frac{4}{3}\Omega_{\beta,\beta}^\alpha - \frac{2}{3}\Omega_{\alpha,\beta}^\beta + \frac{1}{3}\Omega_{\gamma,\delta}^\gamma .
\end{align}

Observe that \( h \) is specified in terms of the Levi-Civita connection along the horizon section directions. In addition, one has to impose

\[ dh = 0 . \]

Furthermore some of the components of the flux can be expressed in terms of the geometry. In particular one finds that

\[ Y = -e^t \wedge h - de^t , \]

and

\[ \frac{1}{3}X_{\mu_1,\mu_2,\mu_3,\mu_4}e^{\mu_1,\mu_2,\mu_3,\mu_4} + X_{\sigma,\rho}^{\sigma,\rho} = \Omega_{\lambda,\nu}^{\lambda} - 7\Omega_{\lambda,4}^{\lambda} , \]

\[ X_{\mu_1,\mu_2,\mu_3} = -2\Omega_{\mu_1,\mu_2,\mu_3} - \frac{2}{3}(\Omega_{\nu,\nu}^{\sigma,\nu} - \Omega_{\gamma,\mu}^{\sigma,\nu} + \Omega_{\mu,\tau}^{\sigma,\nu})\varepsilon_{\mu_1,\mu_2,\mu_3}^\nu , \]

\[ X_{\beta,\beta,\beta,\beta} = \frac{2}{3}(\Omega_{\beta,\mu_1,\mu_2} + \Omega_{\mu_1,\beta,\mu_2} + \Omega_{\mu_1,\beta,\mu_2})\varepsilon_{\mu_1,\mu_2}^{\beta,\beta} - 2\Omega_{\beta,\beta,\beta,\beta} \]

\[ + \left( -\frac{4}{3}\Omega_{\nu,\nu}^{\sigma,\nu} + \frac{4}{3}\Omega_{\gamma,\mu,\mu}^{\sigma,\nu} + \frac{2}{3}\Omega_{\mu,\nu}^{\sigma,\nu} \right) \delta_{\beta,\beta,\beta,\beta} , \]

\[ X_{\alpha,\beta}^{\lambda} = \frac{1}{4}\delta_{\alpha,\beta}X_{\sigma,\rho}^{\sigma,\rho} = -2\Omega_{\alpha,\beta}^{\gamma,\beta} + \frac{1}{4}(\Omega_{\lambda,4}^{\lambda} + \Omega_{\lambda,4}^{\lambda})\delta_{\alpha,\beta} , \]

\[ X_{\mu_1,\beta,\beta,\beta} - \frac{1}{2}X_{\mu_\alpha,\lambda}^{\lambda}e^{\sigma,\beta,\beta,\beta} = -\Omega_{\mu_1,\gamma} + \Omega_{\mu_1,\gamma,\beta} - \Omega_{\lambda_1,\lambda_2,\gamma}^{\mu_1,\mu_2}e^{\lambda_1,\lambda_2,\gamma}e^{\sigma,\beta,\beta,\beta} . \]

Observe again that the (2,2) and traceless component of the magnetic flux \( X \) is not constrained by the KSEs.

The isotropy subgroup of the Killing spinor in \( \text{Spin}(10, 1) \) is \( \text{Spin}(7) \rtimes \mathbb{R}^0 \). Therefore, the horizon section \( \mathcal{S} \) admits a \( \text{Spin}(7) \) structure. In particular although we have decomposed the conditions that arise from the KSEs in \( \text{SU}(4) \) representations, they can be rewritten in terms of \( \text{Spin}(7) \) representations. Further investigation of the geometry of \( \mathcal{S} \) requires the solution of the field equations. As in the previous case, this is rather involved in the presence of magnetic fluxes \( X \). So we shall set \( X = 0 \) and explore the geometry of electric horizons.

### 5.1.2 Electric static \( dh = \Delta = 0 \) horizons

The \( Y \) flux of electric, \( X = 0 \), static horizons can be rewritten as

\[ Y = -\frac{3}{2}e^t \wedge h + Z , \quad i_{e^t}Z = 0 . \]
The additional conditions on the geometry obtained by setting \( X = 0 \) in the expressions for the fluxes in the previous section imply

\[
h_x = 0 . \tag{5.15}
\]

Then the \( M = +, N = - \) component of the Einstein equations can be rewritten as

\[
\tilde{\nabla}^i h_i = -\frac{1}{4} h^2 - \frac{1}{6} Z_{ij} Z^{ij} . \tag{5.16}
\]

On integrating both sides of this condition over \( \mathcal{S} \), one finds that \( h = 0 \) and \( Z = 0 \). So \( Y = 0 \) and since \( X = 0 \), the 4-form flux \( F \) vanishes. The spacetime is a product \( \mathbb{R}^1 \times \mathcal{S} \), where \( \mathcal{S} \) is a compact Spin(7) holonomy manifold. The Berger classification in turn implies that locally \( \mathcal{S} = S^1 \times N \), where \( N \) is an 8-dimensional holonomy Spin(7) manifold.

5.2 Static \( h = d \log \Delta \) horizons

5.2.1 Solution of KSEs

Next let us turn to the solution of the KSEs for static horizons satisfying \((2.6)\). The Killing spinor in this case can be chosen as

\[
\epsilon = 1 + e_{1234} + i r \Phi \Gamma - (1 - e_{1234}) , \tag{5.17}
\]

where \( \Delta = 4 \Phi^2 \). In fact \( \Phi \) can be chosen to be a positive function\(^2\) up to a Spin(7) gauge transformation. Substituting this and \((2.6)\) into the KSEs, one finds the conditions

\[
d \left( \Delta^{\frac{1}{2}} \omega \right) = 0 , \tag{5.18}
\]

\[
-2 \Omega_{\alpha, \beta} + \Omega_{\xi, \alpha} = 0 , \tag{5.19}
\]

\[
\Delta^{\frac{1}{2}} = - i \left( \Omega_{\xi, \lambda} + \frac{1}{2} \Omega_{\lambda, \xi} - \frac{1}{2} \Omega_{\xi, \lambda} \right) , \tag{5.20}
\]

on the geometry of spacetime, where \( \omega = - i \delta_{\alpha \beta} e^\alpha \wedge e^\beta \).

In addition, the KSEs express some of the fluxes in terms of the geometry as

\[
Y = - d e^i - \Delta^{\frac{1}{2}} \omega - \Delta^{-1} e^i \wedge d \Delta \tag{5.21}
\]

and

\[
X_{\alpha_1 \alpha_2 \alpha_3} = \left( - \Omega_{\alpha_\beta \gamma} + \frac{1}{2} \Delta^{\alpha} \partial_{\beta} \Delta \right) e^{\alpha_1 \alpha_2 \alpha_3 ,} , \quad X_{\alpha_1 \beta_1 \beta_2} = \Omega_{\alpha, \gamma_1 \gamma_2} e^{\gamma_1 \gamma_2} \beta_1 \beta_2 , \tag{5.22}
\]

Observe again that the \((2,2)\) and traceless part of the \( X \) is not determined in terms of the geometry.

\(^2\)Changing the sign of \( \Phi \) corresponds to a sign choice for the almost complex structure on the horizon section.
The spacetime is a warped product $\text{AdS}_2 \times_w S$, where $S$ admits a SU(4) structure. The SU(4) structure is further restricted by the geometric conditions $(5.18)$–$(5.20)$. Although $S$ admits a preferred direction $e^\sharp$, generically this direction is not an isometry. Moreover, the almost complex structure in the 8-dimensions transverse to $e^\sharp$ is not integrable. However, $S$ admits a conformally symplectic form in the directions transverse to $e^\sharp$.

To proceed, it is convenient to introduce a new frame $\hat{e}$ on $S$ as

$$e^\sharp = 2\Delta^{-\frac{1}{2}} \hat{e}^\sharp, \quad e^a = \frac{1}{\sqrt{2}} \Delta^\frac{1}{2} \hat{e}^a.$$  \hspace{1cm} (5.23)

In particular the metric on $S$ written in terms of the new frame is

$$ds^2(S) = 4\Delta^{-1} (\hat{e}^\sharp)^2 + \Delta^\frac{1}{2} \delta_{ab} \hat{e}^a \hat{e}^b.$$  \hspace{1cm} (5.24)

The geometric conditions $(5.18)$–$(5.20)$ on $S$ can now be rewritten as

$$d\hat{\omega} = 0,$$  \hspace{1cm} (5.25)

$$-2\hat{\Omega}_{\alpha,\beta} + \hat{\Omega}_{\alpha,\beta} = 0,$$  \hspace{1cm} (5.26)

and

$$-\frac{i}{2} \left( \hat{\Omega}_{\alpha,\beta} + \frac{1}{2} \hat{\Omega}_{\alpha,\beta} - \frac{1}{2} \hat{\Omega}_{\alpha,\beta} \right) = 1,$$  \hspace{1cm} (5.27)

where $\hat{\omega} = -i\delta_{ab} \hat{e}^a \hat{e}^b$, and $\hat{\Omega}$ is the spin connection computed in the $\hat{e}$ frame.

### 5.2.2 Electric static $h = d \log \Delta$ horizons

As for the $h = 0$ solutions, if one furthermore imposes $X = 0$, then additional conditions on the geometry are obtained. In particular, on taking the vector field dual to $\hat{e}^\sharp$ to be $\frac{\partial}{\partial \tau}$, one finds that $\frac{\partial}{\partial \tau}$ is an isometry of $S$, and $\Delta$ does not depend on $\tau$. Furthermore, on making an appropriate U(4) transformation on the holomorphic basis elements $\hat{e}^a$, one can without loss of generality work with a $\tau$-independent basis

$$\hat{e}^a = e^{-\frac{i}{2} \tau} \hat{e}^a \quad \hat{e}^\sharp = \hat{e}^\sharp.$$  \hspace{1cm} (5.28)

After some analysis of the KSEs, one finds that $S$ is a U(1) fibration over an 8-dimensional compact Kähler base manifold $B$ and the metric can be written as

$$ds^2(S) = 4\Delta^{-1}(d\tau + \lambda)^2 + \frac{1}{2}\Delta^{-\frac{1}{2}} ds^2(B),$$  \hspace{1cm} (5.29)

where the $\tau$-independent Kähler form$^3$ is $\hat{\omega}$. The Ricci scalar and Ricci form of $B$ satisfy

$$\hat{R} = \Delta^{-\frac{3}{2}}, \quad \hat{\rho} = 2d\lambda,$$  \hspace{1cm} (5.30)

and

$$\nabla^2 \hat{R} = \frac{1}{2} \hat{R}^2 - \hat{R}_{ij} \hat{R}^{ij}.$$  \hspace{1cm} (5.31)

$^3$Integrability of the almost complex structure depends on the additional conditions obtained from setting $X = 0$. 


The 2-form $Y$ is

$$Y = -3\Delta^{-\frac{3}{2}}(d\tau + \lambda) \wedge d\Delta - \Delta^{-\frac{1}{2}}\left(\rho + \frac{1}{2}R\omega\right).$$

So to construct such near horizon geometries, one has to find a 8-dimensional Kähler manifold such that the Ricci scalar and Ricci tensor satisfy (5.31) with $R > 0$. This equation has been obtained [34] before in the search for gravitational duals in AdS$_2$/CFT$_1$ correspondence.

6 Conclusions

We have demonstrated that all static M-horizons are (local) warped products $\mathbb{R}^{1,1} \times_w S$ or $AdS_2 \times_w S$, where $S$ is a 9-dimensional manifold which admits either a Spin(7) or SU(4) structure respectively, and the conditions on these structures imposed by supersymmetry have been determined. If the M-horizons are electric and $S$ has a Spin(7) structure, the near horizon geometry is $\mathbb{R}^{1,1} \times S$, $S$ is locally a Spin(7) holonomy manifold, and the 4-form flux vanishes. However, for electric M-horizons such that $S$ admits a SU(4) structure, we have shown that $S$ is a fibration over a 8-dimensional Kähler manifold $B$ whose Ricci scalar and Ricci tensor must satisfy (5.31). This condition has also been found in [34] in the context of AdS$_2$ solutions in 11-dimensional supergravity.

It is remarkable that the classification of supersymmetric black hole horizons is closely related to that of Riemannian manifolds with special geometry. In the heterotic case, the understanding of horizons leads to a Calabi type of differential system on conformally balanced Calabi-Yau manifolds with torsion [16, 17]. In the IIB case, the horizons have sections which admit 2-strong Calabi-Yau structure with torsion [18, 19]. Furthermore as we have seen in M-theory, the existence of electric static horizons with SU(4) structure leads to a condition on the curvature of 8-dimensional Kähler manifolds (5.31). We remark that a special case of this condition arises when we take the Ricci scalar of the 8-manifold to be constant. Then the 8-manifold is Kähler-Yamabe, with the additional requirement that the point-wise length of the Ricci tensor is constant. More generally, it remains to determine the types of 8-dimensional compact Kähler manifolds, with positive Ricci scalar, satisfying (5.31). It is well-known that a Kähler metric $g$ can be deformed within its Kähler class and the deformation is determined by a single real function $f$ as $g \rightarrow g + i\partial\bar{\partial}f$. So starting from an arbitrary Kähler metric $g$, it may be possible to deform it such that the deformed metric satisfies

$$(\nabla^2 R)_{g + i\partial\bar{\partial}f} = \left(\frac{1}{2}R^2 - q(n)R_{ij}R^{ij}\right)_{g + i\partial\bar{\partial}f}$$

for a unknown function $f$, where we have allowed Kähler manifolds of any dimension and so we have modified (5.31) with a constant $q(n)$ which depends on the dimension $n$ of the Kähler manifold. The subscript indicates that the Ricci scalar $R$, Ricci tensor $R_{ij}$, covariant derivative $\nabla$ and all the inner products are taken with respect to the deformed metric $g + i\partial\bar{\partial}f$. There are many solutions to this equation but it is not apparent that the general problem always has a solution. We remark that the r.h.s. of (6.1) is non-negative if
$q \leq 0$ and non-positive if $q \geq \frac{n}{2}$; in both cases compactness implies that the Ricci scalar is constant. In the case $q < 0$, compactness implies the Kähler manifold is Ricci flat, and if $q = 0$ the Ricci scalar vanishes. If $q > \frac{n}{2}$ then compactness also implies that the manifold is Ricci flat. Kähler-Einstein manifolds satisfy (6.1) for $q = \frac{n}{2}$, and Riemann surfaces satisfy (6.1) with $q = 1$. For the case of interest for horizons with $n = 8$ and $q = 1$, the r.h.s. of (5.31) is of indeterminate sign.

Thus, for the systematic understanding of all horizons, natural non-linear differential systems have to be solved on compact manifolds with a special structure. The systematic investigation of solutions to such differential systems is an interesting problem in geometry which will have widespread applications in physics.

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