The presence of scalar fields with color and electric charge in supersymmetric theories makes feasible the existence of dangerous charge and color breaking (CCB) minima and unbounded from below directions (UFB) in the effective potential, which would make the standard vacuum unstable. The avoidance of these occurrences imposes severe constraints on the supersymmetric parameter space. We give here a comprehensive and updated account of this topic.

1 Introduction

Experimental observation tells us that color and electric charge are gauge quantum numbers preserved in nature. From the theoretical point of view, in the Standard Model they are certainly conserved in an automatic way since the only fundamental scalar field is the Higgs boson, a colorless electroweak doublet. The Higgs potential has a continuum of degenerate minima, but these are all physically equivalent and one can always define the unbroken $U(1)$ generator to be the electric charge. In supersymmetric (SUSY) extensions of the Standard Model things become more complicated. First, the Higgs sector must contain for consistency at least two Higgs doublets $H_1, H_2$ (plus perhaps some singlets or triplets). Hence, one has to check that the minimum of the Higgs potential $V(H_1, H_2)$ still occurs for values of $H_1, H_2$ which are appropriately aligned in order to preserve the electric charge; otherwise the whole electroweak symmetry becomes spontaneously broken. Second, the supersymmetric theory has a large number of additional charged and colored scalar fields, namely all the sleptons and squarks, say $\tilde{l}_i, \tilde{q}_i$. Consequently one has to verify that the minimum of the whole potential $V(H_1, H_2, \tilde{q}_i, \tilde{l}_i)$ still occurs at a point in the field space, which we will call “realistic minimum” in what follows, where $\tilde{q}_i, \tilde{l}_i = 0$, thus preserving color and electric charge.

The generic situation is that the scalar potential does not present just a single minimum, and, besides the realistic minimum, there is a number of additional charge and color breaking (CCB) minima. Then, a reasonable requirement is that the realistic minimum is the deepest one, i.e the global minimum of the theory. This is certainly the usual constraint imposed in the literature and represents the most conservative attitude in order to be safe. Nevertheless, a situation with CCB minima deeper than the realistic minimum could still be acceptable if the cosmology leads the universe to the latter and this is stable.
enough. This issue will be discussed in the last section of this chapter.

CCB minima are not the only disease that the supersymmetric scalar potential can present. It may also happen that the field space contains directions along which the potential becomes unbounded from below (UFB), which is obviously undesirable. Both issues, CCB and UFB, are closely related, as we will see throughout the chapter.

In order to introduce some notation and to illustrate some relevant aspects and warnings concerning CCB, let us briefly review the CCB condition which has been most extensively used in the literature, namely the “traditional” bound, first studied by Frere et al. and subsequently by others. The tree-level scalar potential, $V_0$, in the minimal supersymmetric standard model (MSSM) is given by

$$V_0 = V_F + V_D + V_{\text{soft}},$$

with

$$V_F = \sum_\alpha \left| \frac{\partial W}{\partial \phi_\alpha} \right|^2,$$

$$V_D = \frac{1}{2} \sum_a g_a^2 \left( \sum_\alpha \phi_\alpha^\dagger T^a \phi_\alpha \right)^2,$$

$$V_{\text{soft}} = \sum_\alpha m_{\phi_\alpha}^2 |\phi_\alpha|^2 + \sum_{i\,\text{generations}} \{ A_u, \lambda_u, Q_i H_2 u_i + A_d, \lambda_d, Q_i H_1 d_i + A_e, \lambda_e, L_i H_1 e_i + \mu H_1 H_2 + \text{h.c.} \},$$

where $W$ is the MSSM superpotential

$$W = \sum_{i\,\text{generations}} \{ \lambda_i, Q_i H_2 u_i + \lambda_d, Q_i H_1 d_i + \lambda_e, L_i H_1 e_i \} + \mu H_1 H_2,$$

$\phi_\alpha$ runs over all the scalar components of the chiral superfields and $a, i$ are gauge group and generation indices respectively. $Q_i$ ($L_i$) are the scalar partners of the quark (lepton) $SU(2)_L$ doublets and $u_i, d_i$ ($e_i$) are the scalar partners of the quark (lepton) $SU(2)_L$ singlets. In our notation $Q_i \equiv (u_L, d_L)_i, L_i \equiv (\nu_L, e_L)_i, u_i \equiv u_{Ri}, d_i \equiv d_{Ri}, e_i \equiv e_{Ri}$. Finally, $H_{1,2}$ are the two SUSY Higgs doublets. The first observation is that the previous potential is extremely involved since it has a large number of independent fields. Furthermore, even assuming universality of the soft breaking terms at the unification scale, $M_X$, it contains a large number of independent parameters: $m, M, A, B, \mu$, i.e. the
universal scalar and gaugino masses, the universal coefficients of the trilinear and bilinear scalar terms, and the Higgs mixing mass, respectively. In addition, there are the gauge ($g$) and Yukawa ($\lambda$) couplings which are constrained by the experimental data. Notice that $M$ does not appear explicitly in $V_0$, but it does through the renormalization group equations (RGEs) of all the remaining parameters.

The complexity of $V$ has made that until recently only particular directions in the field-space have been explored. The best-known example of this is the “traditional” bound, first studied by Frere et al. and subsequently by others. These authors considered just the three fields present in a particular trilinear scalar coupling, e.g. $\lambda_u A_u Q_u H_2 u$, assuming equal vacuum expectation values (VEVs) for them:

$$|Q_u| = |H_2| = |u|,$$

where only the $u_L$-component of $Q_u$ takes a VEV in order to cancel the D-terms. The phases of the three fields are taken in such way that the trilinear scalar term in the potential gets negative sign. Then, the potential (1) gets extremely simplified and it is easy to show that a very deep CCB minimum appears unless the famous constraint

$$|A_u|^2 \leq 3 \left( m_{Q_u}^2 + m_u^2 + m_{H_2}^2 \right)$$

is satisfied. In the previous equation $m_{Q_u}^2, m_u^2, m_{H_2}^2$ are the mass parameters of $Q_u, u, H_2$. Notice from eq.(1) that $m_{H_2}^2$ is the sum of the $H_2$ squared soft mass, $m_{H_2}^2$, plus $\mu^2$. Similar constraints for the other trilinear terms can straightforwardly be written. These “traditional” bounds have extensively been used in the literature. Notice that the trilinear coefficient, $A$, plays a crucial role for the appearance of a CCB minimum. This is logical since the scalar trilinear terms are essentially negative contributions to the scalar potential (they are negative for a certain combination of the phases of the fields). However, we will see in sect.4 that they are irrelevant for UFB directions.

From the previous bound we can extract two important lessons. First, many ordinary CCB bounds (as the one of eq.(5)) come from the analysis of particular directions in the field-space, thus corresponding to necessary but not sufficient conditions to avoid dangerous CCB minima. Consequently a complete analysis requires a more exhaustive exploration of the field space. Second, the bound of eq.(5) has been obtained from the analysis of the tree-level potential $V_0$. Hence, the radiative corrections should be incorporated in some way. With regard to this point a usual practice has been to consider the tree-level scalar potential improved by one-loop RGEs, so that all the parameters appearing in it (see eq.(1)) are running with the renormalization
scale, $Q$. Then it is demanded that the previous CCB constraints, i.e. eq.(5) and others, are satisfied at any scale between $M_X$ and $M_Z$. As we will see in sect.2 this procedure is not correct and leads to an overestimate of the restrictive power of the bounds. Therefore a more careful treatment of the radiative corrections is necessary when analyzing CCB bounds.

The chapter is organized as follows. Section 2 is devoted to analyze and give prescriptions to handle the above-mentioned issue of the radiative corrections. Section 3 deals with the Higgs part of the potential, which is a requirement for subsequent analyses. In sections 4 and 5 a complete analysis of the UFB and UFB directions of the MSSM field space is performed, giving a complete set of optimized bounds. Special attention will be paid to the most powerful one, the so-called UFB-3 bound. The effective restrictive power of these bounds is examined in section 6. Section 7 is devoted to the bounds that CCB pose on flavour mixing couplings, which turn out to be surprisingly strong. Finally, the cosmological considerations are left for section 8.

2 The role of the radiative corrections

As has been mentioned in sect.1, in the CCB analysis the scalar potential is usually considered at tree-level, improved by one-loop RGEs, so that all the parameters appearing in it (see eq.(3)) are running with the renormalization scale, $Q$. The two questions that arise are:

- What is the appropriate scale, say $Q = \hat{Q}$, to evaluate $V_0$?
- How important are the radiative corrections that are being ignored?

These two questions are intimately related. To understand this it is important to recall that the exact effective potential

$$V(Q, \lambda_\alpha(Q), m_\beta(Q), \phi(Q))$$

(in short $V(Q, \phi)$), where $\lambda_\alpha(Q), m_\beta(Q)$ are running parameters and masses and $\phi(Q)$ are the generic classical fields, is scale-independent, i.e.

$$\frac{dV}{dQ} = 0 .$$

This property allows in principle any choice of $Q$, and in particular a different one for each value of the classical fields, i.e. $Q = f(\phi)$. When analyzing CCB bounds, one is interested in possible CCB minima, so one has to minimize the scalar potential. Denoting by $\langle \phi \rangle$ the VEVs of the $\phi$–fields obtained from
the minimization of $V$, it is clear from (7) that the two following minimization conditions
\[ \frac{\partial V(Q = f(\phi), \phi)}{\partial \phi} = 0 \] (8)
\[ \frac{\partial V(Q, \phi)}{\partial \phi} \bigg|_{Q = f(\phi)} = 0 \] (9)
yield equivalent results for $\langle \phi \rangle$ (for a more detailed discussion see refs. 3, 4).

The previous results apply exactly only to the exact effective potential. In practice, however, we can only know $V$ with a certain degree of accuracy in a perturbative expansion. In particular, at one-loop level
\[ V_1 = V_0(Q, \phi) + \Delta V_1(Q, \phi) \] (10)
where $V_0$ is the (one-loop improved) tree-level potential and $\Delta V_1$ is the one-loop radiative correction to the effective potential
\[ \Delta V_1 = \sum_{\alpha} \frac{n_\alpha}{64\pi^2} M_\alpha^4 \left[ \log \frac{M_\alpha^2}{Q^2} - \frac{3}{2} \right]. \] (11)

Here $M_\alpha^2(Q)$ are the improved tree-level squared mass eigenstates and $n_\alpha = (-1)^{2s_\alpha} (2s_\alpha + 1)$, where $s_\alpha$ is the spin of the corresponding particle. It is important to notice that $M_\alpha^2(Q)$ are in general field–dependent quantities since they are the eigenvalues of the $(\partial^2 V_0/\partial \phi_i \partial \phi_j)$ matrix. Hence, the values of $M_\alpha^2(Q)$ depend on the values of the fields and thus on which direction in the field space is being analyzed. $V_1(Q, \phi)$ does not obey eq.(7) for all values of $Q$. However, in the region of $Q$ of the order of the most significant masses appearing in (11), the logarithms involved in the radiative corrections, and the radiative corrections themselves (i.e. $\Delta V_1$), are minimized, thus improving the perturbative expansion. So we expect $V$ to be well approximated by $V_1$ and it is not surprising that in that region of $Q$, $V_1$ is approximately scale-independent, i.e. eq.(7) is nearly satisfied. On the other hand, due to the smallness of $\Delta V_1$, $V_1$ and $V_0$ are, in this region, very similar. Consequently (always in this region of $Q$) we can safely approximate $V$ by $V_1$ or even $V_0$, and minimize by using either eq.(8) or eq.(9), although of course eq.(9) is much easier to handle. This statement can be numerically confirmed, see e.g. refs. 4, 7.

In conclusion, the radiative corrections are reasonably well incorporated by using the tree-level potential $V_0(\phi, \hat{Q})$, where the renormalization scale $\hat{Q}$

\[ ^{a}\text{More precisely, for a choice of } Q \text{ such that } \partial \Delta V_1/\partial \phi = 0 \text{ the results from } V_0 \text{ and } V_1 \text{ are the same. In practice this precise condition is quite involved and such a degree of precision is not necessary.} \]
is of the order of the most significant mass, normally $\hat{Q} \sim \phi$. The application of these recipes to our task of determining the CCB minima and extract the corresponding CCB bounds will be shown in sects.4,5.

3 The Higgs potential and the realistic minimum

The Higgs part of the MSSM potential can be extracted (at tree level) from eq.(1). It reads

$$V_{\text{Higgs}} = m_1^2|H_1|^2 + m_2^2|H_2|^2 - m_3^2\left(\epsilon_{ij}H_1^iH_2^j + \text{h.c.}\right) - \frac{1}{2}g_2^2\left|\epsilon_{ij}H_1^iH_2^j\right|^2 + \frac{1}{8}(g_2^2 + g'^2)(|H_2|^4 + |H_1|^4) + \frac{1}{8}(g_2^2 - g'^2)|H_2|^2|H_1|^2,$$

(12)

where $H_1 = (H_1^0, H_{1}^{-})$, $H_2 = (H_2^0, H_{2}^{+})$, $m_1^2 \equiv m_{H_1}^2 + \mu^2$, $m_2^2 \equiv m_{H_2}^2 + \mu^2$, $m_3^2 \equiv -\mu B$ and $g_2$, $g'$ are the gauge couplings of $SU(2) \times SU(1)_Y$. All these parameters are understood to be running parameters evaluated at some renormalization scale $Q$.

Our first interest in $V_{\text{Higgs}}$ comes from the fact that $V_{\text{Higgs}}$ depends not only on the neutral components of $H_1$, $H_2$, but also on the charged ones, i.e. $H_1^\pm$, $H_2^\pm$. Hence, one should check that $\langle H_{1}^{-}\rangle$, $\langle H_{2}^{+}\rangle$ remain vanishing when $V_{\text{Higgs}}$ is minimized (one of them, say $\langle H_{2}^{+}\rangle$, can always be chosen as vanishing through an $SU(2)$ rotation). Fortunately, it is easy to show from (12) that the minimum of $V_{\text{Higgs}}$ always lies at $H_{2}^{+} = H_{1}^{-} = 0$. So the MSSM is safe from this point of view. It is worth remarking that non-minimal supersymmetric extensions of the standard model do not have this nice property, at least in such an automatic way. (This is e.g. the case of the so-called next-to-minimal supersymmetric standard model (NMSSM), which contains an extra singlet in the Higgs sector.) Therefore we can set $H_{2}^{+} = H_{1}^{-} = 0$ and focus our attention on the neutral part of $V_{\text{Higgs}}$, which reads

$$V_{\text{Higgs}} = m_1^2|H_1|^2 + m_2^2|H_2|^2 - 2|m_3^2||H_1||H_2| + \frac{1}{8}(g'^2 + g_2^2)(|H_2|^2 - |H_1|^2)^2.$$

(13)

Notice that, since we are interested in the minimization of the potential, we have implicitly chosen in (13) a phase of $H_1$, $H_2$ such that the mixing term $\propto (\epsilon_{ij}H_1^iH_2^j + \text{h.c.})$ gets negative.

The second aspect of $V_{\text{Higgs}}$ which interests us is that $V_{\text{Higgs}}$ should develop a minimum at $|H_1^0| = v_1$, $|H_2^0| = v_2$, such that $SU(2) \times U(1)_Y$ is broken in the correct way, i.e. $v_1^2 + v_2^2 = 2M_W^2/g_2^2 \simeq (175 \text{ rmGeV})^2$. This is the realistic minimum that corresponds to the standard vacuum. This requirement fixes one of the five independent parameters $(m, M, A, B, \mu)$ of the MSSM, say

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\(\mu\), in terms of the others. Actually, for some choices of the four remaining parameters \((m, M, A, B)\), there is no value of \(\mu\) capable of producing the correct electroweak breaking. Therefore, this requirement restricts the parameter space further, as is illustrated in Fig.1 (darked region) with a representative example (which will be discussed in detail in sect.6). In addition, the actual value of the potential at the realistic minimum, say \(V_{\text{real min}}\), is important for the CCB analysis since the possible CCB vacua are dangerous as long as they deeper than \(V_{\text{real min}}\). From (13) it is straightforward to get 

\[
V_{\text{real min}} = -\frac{1}{8}(g'^2 + g^2)(v_2^2 - v_1^2)^2 - \left\{ \frac{[(m_1^2 + m_2^2)^2 - 4|m_3|^4]^{1/2} - m_1^2 + m_2^2}{2(g'^2 + g^2)} \right\}^2
\]

Note that this is the result obtained by minimizing just the tree-level part of (13). As explained in sect.2 this procedure is correct if the minimization is performed at some sensible scale \(Q\), which should be of the order of the most relevant mass entering \(\Delta V_1\), see eq.(11). Since we are dealing here with the Higgs-dependent part of the potential, that mass is necessarily of the order of the largest Higgs-dependent mass, namely the largest stop mass. From now on we will denote this scale by \(M_{\text{Sb}}\).

Finally, to be considered as realistic, the previous minimum must be really a minimum in the whole field-space. This simply implies that all the scalar squared mass eigenvalues (squarks and sleptons) must be positive. Actually, we should go further and demand that all the not yet observed particles, i.e. charginos, squarks, etc., have masses compatible with the experimental bounds.

4 Unbounded from below (UFB) constraints

In this section we analyze the constraints that arise from directions in the field-space along which the (tree-level) potential can become unbounded from below (UFB). It is in fact possible to give a complete classification of the potentially dangerous UFB directions and the corresponding constraints in the MSSM. In order to understand what are the dangerous directions and the form of the corresponding bounds it is useful to notice the following two general properties about UFB in the MSSM:

1 Contrary to what happens to the CCB minima (see sect.1), the trilinear scalar terms cannot play a significant role along an UFB direction.

\(^{6}\)A more precise estimate of \(M_S\) was given in \(\text{[1]}\), but for our purposes this is accurate enough.
since for large enough values of the fields the corresponding quartic (and positive) F–terms become unavoidably larger.

2 Since all the physical masses must be positive at $Q = M_S$, the only negative terms in the (tree-level) potential that can play a relevant role along an UFB direction are

$$m_2^2 |H_2|^2 , \quad -2 |m_3|^2 |H_1| |H_2| .$$

Therefore, any UFB direction must involve, $H_2$ and, perhaps, $H_1$. Furthermore, since the previous terms are quadratic, all the quartic (positive) terms coming from F– and D–terms must be vanishing or kept under control along an UFB direction. This means that, in any case, besides $H_2$ some additional field(s) are required for that purpose. In all the instances, the preferred additional fields are $H_1$ and/or sleptons since they normally have smaller soft masses and therefore amount to a less positive contribution to the potential.

Using the previous general properties we can completely classify the possible UFB directions in the MSSM. Special attention should be paid to the UFB–3 bound, which is the strongest one:

**UFB-1**

The first possibility is to play just with $H_1$ and $H_2$. Then, the relevant terms of the potential are those written in eq.(13). Obviously, the only possible UFB direction corresponds to choose $H_1 = H_2$ (up to $O(m_i)$ differences which are negligible for large enough values of the fields), so that the quartic D–term is cancelled. Thus, the (tree-level) potential along the UFB-1 direction is

$$V_{UFB-1} = (m_1^2 + m_2^2 - 2 |m_3|^2) |H_2|^2 .$$

The constraint to be imposed is that, for any value of $|H_2| < M_X$,

$$V_{UFB-1}(Q = Q) > V_{real \min}(Q = M_S) ,$$

where $V_{real \min}$ is the value of the realistic minimum, given by eq.(14), and $V_{UFB-1}$ is evaluated at an appropriate scale $Q$ (see sect.2). $Q$ must be of the same order as the most significant mass along this UFB-1 direction,

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$^a$The only possible exception are the stop soft mass terms $m_{Q_t}^2 |Q_t|^2 + m_t^2 |t|^2$ since the stop masses are given by $\sim (m_{Q_t}^2 + M_{stop}^2 \pm mixing)$, but this possibility is barely consistent with the present bounds on squark masses.
which is obviously of order $H_2$. More precisely $\hat{Q} \sim \max(g_2|H_2|, \lambda_{\text{top}}|H_2|, M_S)$. Consequently, from (10) the bound (17) is accurately equivalent to the well-known condition

$$m_1^2 + m_2^2 \geq 2|m_3^2|.$$  \hspace{1cm} (18)

From the previous discussion, it is clear that the bound (18) must be satisfied at any $Q > M_S$ and, in particular, at $Q = M_X$.

**UFB-2**

If, besides $H_2, H_1$, we consider additional fields in the game, it is easy to check by simple inspection (see property 2 above) that the best possible choice is a slepton $L_i$ (along the $\nu_L$ direction), since it has the lightest mass without contributing to further quartic terms in $V$. Consequently, from eq. (1), the relevant potential reads

$$V = m_1^2|H_1|^2 + m_2^2|H_2|^2 - 2|m_3^2||H_1||H_2| + m_{L_i}^2|L_i|^2$$

$$+ \frac{1}{8}(g'^2 + g_2^2)(|H_2|^2 - |H_1|^2 - |L_i|^2)^2. \hspace{1cm} (19)$$

By minimizing $V$ with respect to $H_1, L_i$, it is possible to write these two fields in terms of $H_2$. This step leads to non-trivial results provided that $|m_3^2| < \mu^2, |H_2|^2 > 4m_{L_i}^2/(g'^2 + g_2^2) \left[1 - \frac{|m_3^2|^4}{\mu^4}\right]$; otherwise the optimum value for $L_i$ is $L_i = 0$, and we come back to the direction UFB-1. Then, the potential along the UFB-2 direction reads

$$V_{\text{UFB-2}} = \left[m_2^2 + m_{L_i}^2 - \frac{|m_3|^4}{\mu^2}\right]|H_2|^2 - \frac{2m_{L_i}^4}{g'^2 + g_2^2}. \hspace{1cm} (20)$$

From (20) it might seem that the potential is unbounded from below unless $m_2^2 + m_{L_i}^2 - \frac{|m_3|^4}{\mu^2} \geq 0$. However, strictly, the UFB-2 constraint reads

$$V_{\text{UFB-2}}(Q = \hat{Q}) > V_{\text{real min}}(Q = M_S), \hspace{1cm} (21)$$

where $V_{\text{real min}}$ is the value of the realistic minimum, given by eq. (14), and $V_{\text{UFB-2}}$ is evaluated at an appropriate scale $\hat{Q}$. Again $\hat{Q} \sim \max(g_2|H_2|, \lambda_{\text{top}}|H_2|, M_S)$.

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$^d$Eq. (20) relies on the equality $m_1^2 - m_{L_i}^2 = \mu^2$, which only holds under the assumption of degenerate soft scalar masses for $H_1$ and $L_i$ at $M_X$ and in the approximation of neglecting the bottom and tau Yukawa couplings in the RGEs. Otherwise, one simply must replace $\mu^2$ by $m_1^2 - m_{L_i}^2$ in eq. (20).
The only remaining possibility is to take $H_1 = 0$. Then, the $H_1$ F–term can be cancelled with the help of the VEVs of sleptons of a particular generation, say $e_{L_j}, e_{R_j}$, without contributing to further quartic terms. More precisely

$$\left| \frac{\partial W}{\partial H_1} \right|^2 = |\mu H_2 + \lambda_{e_j} e_{L_j} e_{R_j}|^2 = 0,$$

(22)

where $\lambda_{e_j}$ is the corresponding Yukawa coupling. It is important to note that this trick is not useful if $H_1 \neq 0$, as it happens in the UFB–2 direction, since then the $e_{L_j}, e_{R_j}$ F–terms would eventually dominate.

Now, in order to cancel (or keep under control) the $SU(2)_L$ and $U(1)_Y$ D–terms we need the VEV of some additional field, which cannot be $H_1$ for the above mentioned reason. Once again the optimum choice is a slepton $L_i$ (with $i \neq j$) along the $\nu_L$ direction, as in the UFB–2 case.

Denoting $|e_{L_j}| = |e_{R_j}| \equiv |e| = \sqrt{\frac{\mu}{\lambda_{e_j}}} |H_2|$, the relevant potential reads

$$V = (m_2^2 - \mu^2)|H_2|^2 + (m_{L_j}^2 + m_{e_j}^2)|e|^2 + m_{L_i}^2 |L_i|^2$$

+ $\frac{1}{8}(g^2 + g_2^2) (|H_2|^2 + |e|^2 - |L_i|^2)^2$.

(23)

Now, the value of $L_i$ can be written, by simple minimization, in terms of $H_2$, namely $|L_i|^2 = -\frac{4m_{L_i}^2}{g^2 + g_2^2} + (|H_2|^2 + |e|^2)$. It turns out that for any value of $|H_2| < M_X$ satisfying

$$|H_2| > \sqrt{\frac{\mu^2}{4\lambda_{e_j}^2} + \frac{4m_{L_i}^2}{g^2 + g_2^2} - \frac{|\mu|}{2\lambda_{e_j}}},$$

(24)

the value of the potential along the UFB–3 direction is simply given by

$$V_{\text{UFB–}3} = \left[ m_2^2 - \mu^2 + m_{L_i}^2 \right] |H_2|^2 + \frac{|\mu|}{\lambda_{e_j}} \left[ m_{L_j}^2 + m_{e_j}^2 + m_{L_i}^2 \right] |H_2|^2 + \frac{2m_{L_i}^4}{g^2 + g_2^2},$$

(25)

Otherwise

$$V_{\text{UFB–}3} = \left[ m_2^2 - \mu^2 \right] |H_2|^2 + \frac{|\mu|}{\lambda_{e_j}} \left[ m_{L_j}^2 + m_{e_j}^2 \right] |H_2|$$

+ $\frac{1}{8}(g^2 + g_2^2) \left[ |H_2|^2 + \frac{|\mu|}{\lambda_{e_j}} |H_2|^2 \right]^2$.

(26)
Then, the UFB-3 condition reads

\[ V_{\text{UFB-3}}(Q = \tilde{Q}) > V_{\text{real min}}(Q = M_S), \]

where \( V_{\text{real min}} \) is given by eq.(14), \( \tilde{Q} \sim \text{Max}(g_2|e|, \lambda_{\text{top}}|H_2|, g_2|L_i|, M_S) \).

It is interesting to mention that the previous constraint (27) with the replacements \( e \to d \), \( \lambda_{e_i} \to \lambda_{d_j} \), \( L_j \to Q_j \), must also be imposed. Now \( i \) may be equal to \( j \) (the optimum choice is \( d_j = \text{sbottom} \)) and \( \tilde{Q} \sim \text{Max}(\lambda_{\text{top}}|H_2|, g_3|d|, \lambda_{u_j}|d|, g_2|L_i|, M_S) \).

Anyway, the optimum condition is the one with the sleptons (note e.g. that the second term in eqs.(25, 26) is proportional to the slepton masses and thus smaller) and indeed represents, as we will see in sect.6, the strongest one of all the UFB and CCB constraints in the parameter space of the MSSM.

5 Charge and color breaking (CCB) constraints

These constraints arise from the existence of CCB minima in the potential deeper than the realistic minimum. We have already mentioned the "traditional" CCB constraint (2) of eq.(2). Other particular CCB constraints have been explored in the literature \cite{9,10,11}. In this section we will perform a complete analysis of the CCB minima, obtaining a set of analytic constraints that represent the necessary and sufficient conditions to avoid the dangerous ones. As we will see, for certain values of the initial parameters, the CCB constraints "degenerate" into the previously found UFB constraints since the minima become unbounded from below directions. In this sense, the following CCB constraints comprise the UFB bounds of the previous section, which can be considered as special (but extremely important) limits of the former.

In order to gain intuition about CCB, let us enumerate a number of general properties which are relevant when one is looking for CCB constraints in the MSSM. (Formal proofs of the following statements can be found in ref.\cite{7}).

1. The most dangerous, i.e. the deepest, CCB directions in the MSSM potential involve only one particular trilinear soft term of one generation (see eq.(2)). This can be either of the leptonic type (i.e. \( A_{e_i} \lambda_{e_i} L_i H_1 e_i \)) or the hadronic type (i.e. \( A_{u_i} \lambda_{u_i} Q_i H_2 u_i \) or \( A_d \lambda_{d_i} Q_i H_1 d_i \)). Along one of these CCB directions the remaining trilinear terms are vanishing or negligible. This is because the presence of a non-vanishing trilinear term in the potential gives a net negative contribution only in a region of the field space where the relevant fields are of order \( A/\lambda \) with \( \lambda \) and \( A \) the corresponding Yukawa coupling and soft trilinear coefficient; otherwise either
the (positive) mass terms or the (positive) quartic F–terms associated with these fields dominate the potential. In consequence two trilinear couplings with different values of \( \lambda \) cannot efficiently “cooperate” in any region of the field space to deepen the potential. Accordingly, to any optimized CCB constraint there corresponds a unique relevant trilinear coupling, which makes the analysis much easier.

2 If the trilinear term under consideration has a Yukawa coupling \( \lambda^2 \ll g^2 \), where \( g \) represents a generic gauge coupling constant, then along the corresponding deepest CCB direction the D-term must be vanishing or negligible. This occurs essentially in all the cases except for the top, and simplifies enormously the analysis.

From the previous properties it can be checked that for a given trilinear coupling under consideration there are \( \text{two} \) different relevant directions to explore. Next, we illustrate them taking the trilinear coupling of the first generation, \( A_u \lambda_u Q_u H_2 u_R \), as a guiding example.

**Direction (a)**

It exploits the trick expounded in the direction UFB-3. Namely, if \( H_1 = 0 \), then one can take two \( d \)-type squarks \( d_{L_j}, d_{R_j} \) (or sleptons \( e_{L_j}, e_{R_j} \)) such that \( \lambda_{d_j} \gg \lambda_u \) (or \( \lambda_{e_j} \gg \lambda_u \)), so that their VEVs cancel the \( H_1 \) F–term, i.e.

\[
\left| \frac{\partial W}{\partial H_1} \right|^2 = \left| \mu H_2 + \lambda d_j d_L j d_R j \right|^2 = 0 .
\]

Notice that \( H_1 \) must be very small or vanishing, otherwise the (positive) \( d_{L_j}, d_{R_j} \) F–terms, \( \lambda_{d_j}^2 \left\{ |H_1 d_{R_j}|^2 + |d_{L_j} H_1|^2 \right\} \), would clearly dominate the potential (this is also in agreement with the property above).

Since \( |d_{L_j}|^2, |d_{R_j}|^2 \ll |H_2|^2, |Q_u|^2, |u_R|^2 \), the \( d_{L_j}, d_{R_j} \) mass terms are negligible and the net effect of eq. (28) is to decrease the \( H_2 \) squared mass from \( m_{2}^2 \) to \( m_{2}^2 - \mu^2 \). Furthermore, in addition to \( H_2, Q_u, u_R, d_{L_j}, d_{R_j} \), other fields could take extra non-vanishing VEVs. As in the above-explained UFB-2 direction (see sect.4) and for similar reasons, it turns out that the optimum choice is \( L_i \neq 0 \), with the VEV along the \( \nu_L \) direction. Therefore, along the direction (a)

\[
H_2, Q_u, u_R \neq 0 , \quad \text{Possibly} \quad L_i \neq 0 , \quad (29a)
\]

\[
|d_{L_j}|^2 = |d_{R_j}|^2 \quad ; \quad d_{L_j} d_{R_j} = - \frac{\mu}{\lambda_{d_j}} H_2 \quad (29b)
\]

\[\text{Recall that } m_{2}^2 - \mu^2 = m_{H_2}^2, \text{ i.e. the } H_2 \text{ soft mass, see sect.3.}\]
**Direction (b)**

If we allow for $H_1 \neq 0$, then we cannot play the trick of eq. (28) to cancel the $H_1$ F-term. Therefore, along this alternative direction $H_2, Q_u, u_R, H_1 \neq 0$, Possibly $L_i \neq 0$.

Let us now write the potential along the directions $(a), (b)$. It is useful for this task to express the various VEVs in terms of the $H_2$ one, using the following notation

$$|Q_u| = \alpha |H_2|, \quad |u_R| = \beta |H_2|,$$

$$|H_1| = \gamma |H_2|, \quad |L_i| = \gamma_L |H_2|.$$  

(30)

E.g. the “traditional” direction, eq. (4), is recovered for the particular values $\alpha = \beta = 1$, $\gamma = \gamma_L = 0$. Now, the basic expression for the scalar potential (see eq. (1)) is

$$V = \lambda^2 u F^2(\alpha, \beta, \gamma, \gamma_L) \alpha^2 \beta^2 |H_2|^4 - 2\lambda_u \hat{A}(\gamma) |H_2|^3 + \hat{m}^2(\alpha, \beta, \gamma, \gamma_L) |H_2|^2,$$

(32)

where

$$F(\alpha, \beta, \gamma, \gamma_L) = 1 + \frac{1}{\alpha^2} + \frac{1}{\beta^2} + \frac{f(\alpha, \beta, \gamma, \gamma_L)}{\alpha^2 \beta^2},$$

$$f(\alpha, \beta, \gamma, \gamma_L) = \frac{1}{\lambda^2 u} \left\{ \frac{1}{8} g_2^2 (1 - \alpha^2 - \gamma^2 - \gamma_L^2)^2 + \frac{1}{8} g_2^2 \left( \frac{1}{3} \alpha^2 - \frac{4}{3} \beta^2 - \gamma^2 - \gamma_L^2 \right)^2 + \frac{1}{6} g_2^2 (\alpha^2 - \beta^2)^2 \right\},$$

$$\hat{A}(\gamma) = |A_u| + |\mu| \gamma,$$

$$\hat{m}^2(\alpha, \beta, \gamma, \gamma_L) = m_2^2 + m_2^2 Q_u \alpha^2 + m_2^2 \beta^2 + m_1^2 \gamma^2 + m_2^2 \gamma_L^2 - 2 |m_3^2| \gamma.$$  

(33)

For the $(a)$–direction eqs. (32,33) hold replacing $\gamma = 0$, $m_2^2 \rightarrow m_2^2 - \mu^2$ in eq. (33). For the $(b)$–direction, when $\text{sign}(A_u) = \text{sign}(B)$, it is not possible to choose the phases of the fields in such a way that the trilineal scalar coupling ($\propto A_u \lambda_u Q_u H_2 u_R$), the cross term in the $H_2$ F–term ($\propto \mu \lambda_u Q_u H_1 u_R$) and the Higgs mixing term ($\propto \mu B H_1 H_2$) become negative at the same time.

Correspondingly, one (any) of the three terms $\{|A_u|, |\mu| |\gamma|, -2 |m_3^2| \gamma\}$ in eq. (33) must flip the sign.

Minimizing $V$ with respect to $|H_2|$ for fixed values of $\alpha, \beta, \gamma, \gamma_L$, we find, besides the $|H_2| = 0$ extremal (all VEVs vanishing), the following CCB solution

$$|H_2|_{\text{ext}} = |H_2(\alpha, \beta, \gamma, \gamma_L)|_{\text{ext}} = \frac{3 \hat{A}}{4 \lambda_u \alpha \beta F} \left\{ 1 + \sqrt{1 - \frac{8 \hat{m}^2 F}{9 A^2}} \right\}.$$  

(34)
\[ V_{\text{CCB min}} = -\frac{1}{2} \alpha \beta |H_2|^2_{\text{ext}} \left( \hat{A}\lambda_u |H_2|_{\text{ext}} - \frac{\hat{m}^2}{\alpha \beta} \right). \] (35)

Notice that, as was stated above (see property 1), the typical VEVs at a CCB minimum are indeed of order \( A/\lambda \). The previous CCB minimum will be negative (and much deeper than the realistic minimum) unless

\[ \hat{A}^2 \leq F\hat{m}^2 \] (36)

This is in fact the most general form of a CCB constraint.

The previous CCB bound takes a more handy form if we realize that since \( \lambda^2 \ll 1 \) (see property 3 above) the D–terms should vanish. This implies \( \alpha^2 - \beta^2 = 0 \), \( 1 - \alpha^2 - \gamma^2 - \gamma^2_{L} = 0 \). As a consequence \( f(\alpha, \beta, \gamma, \gamma_{L}) \) becomes vanishing and \( F = 1 + \frac{2}{\alpha^2} \).

Now, we can write the explicit form of the bounds for the directions \((a, b)\):

**CCB-1**

This bound arises by considering the direction \((a)\) and thus the general condition (36) takes the form

\[ |A_u|^2 \leq \left( 1 + \frac{2}{\alpha^2} \right) \left[ m_2^2 - \mu^2 + (m_0^2_{Q_u} + m_0^2_u) \alpha^2 + m_{L_i}^2 \gamma^2_{L} \right], \] (37)

where \( \alpha^2 \) is arbitrary and \( \gamma^2_{L} \) is given by \( \gamma^2_{L} = 1 - \alpha^2 \). More precisely

1. If \( m_2^2 - \mu^2 + m_{L_i}^2 > 0 \) and \( 3m_{L_i}^2 - (m_0^2_{Q_u} + m_0^2_u) + 2(m_2^2 - \mu^2) > 0 \), then the optimized CCB-1 occurs for \( \alpha = 1 \), i.e.

\[ |A_u|^2 \leq 3 \left[ m_2^2 - \mu^2 + m_{Q_u}^2 + m_u^2 \right. \] (38)

2. If \( m_2^2 - \mu^2 + m_{L_i}^2 > 0 \) and \( 3m_{L_i}^2 - (m_0^2_{Q_u} + m_0^2_u) + 2(m_2^2 - \mu^2) < 0 \), then the optimized CCB-1 bound is

\[ |A_u|^2 \leq \left( 1 + \frac{2}{\alpha^2} \right) \left[ m_2^2 - \mu^2 + (m_0^2_{Q_u} + m_0^2_u) \alpha^2 + m_{L_i}^2 (1 - \alpha^2) \right. \] (39)

with \( \alpha^2 = \sqrt{\frac{2(m_{L_i}^2 + m_0^2 - \mu^2)}{m_0_{Q_u}^2 + m_0^2 - m_{L_i}^2}} \).

\[ \text{The mere existence of a CCB minimum is discarded by demanding } \hat{A}^2 < (8/9)F\hat{m}^2, \text{ see eq. (34).} \]
3. If \( m_2^2 - \mu^2 + m_{L_i}^2 < 0 \), then the CCB-1 bound is automatically violated. In fact the minimization of the potential in this case gives \( \alpha^2 \to 0 \), and we are exactly led to the UFB-3 direction shown above, which represents the correct analysis in this instance.

**CCB-2**

This bound arises by considering the direction \((b)\). Then the general condition (36) takes the form

\[
(|A_u| + |\mu|\gamma)^2 \leq \left( 1 + \frac{2}{\alpha^2} \right) \left[ m_2^2 + (m_{Q_u}^2 + m_{L_i}^2)\alpha^2 \right. \\
+ \left. m_1^2\gamma^2 + m_{L_i}^2\gamma_L^2 - 2|m_3^2|\gamma \right]
\]

(40)

where \( \alpha^2, \gamma^2 \) are arbitrary and \( \gamma_L^2 = 1 - \alpha^2 - \gamma^2 \). Rules to handle this bound in an efficient way (i.e. to take the values of \( \alpha^2, \gamma^2 \) that make the bound as strong as possible) can be found in ref.\(^7\).

If \( \text{sign}(A_u) = \text{sign}(B) \), the sign of one of the three terms \( \{ |A_u|, |\mu|\gamma, -2|m_3^2|\gamma \} \) in (40) must be flipped (see comments after eq. (33)). Notice that, due to the form of (40) flipping the sign of \( |A_u| \) or the sign of \( |\mu|\gamma \) leads to the same result. Therefore, there are only two choices to examine.

Concerning the renormalization scale at which the previous CCB-1, CCB-2 constraints must be evaluated, a sensible choice is \( Q \sim \text{Max}(M_S, g_3 A_u^4, \lambda_t A_u^4) \), since \( H_2 \sim A_u^4 \), see eq. (34).

The previous CCB-1, CCB-2 bounds are straightforwardly generalized to all the couplings with coupling constant \( \lambda \ll 1 \). This essentially includes all the couplings apart from the top. The generalization to the top Yukawa coupling case is more involved since \( \lambda_{top} = O(1) \), so the D-terms should not be assumed to vanish anymore. Furthermore, the CCB-1 bounds are no longer applicable due to the absence of \( d \)-type squarks such that \( \lambda_d >> \lambda_{top} \). Finally, the associated CCB minima have in many cases a similar size to the realistic one. So, it is important to examine explicitly the condition \( V_{CCB\min} > V_{real\min} \).

For more details, the interested reader is referred to ref.\(^7\).

6 Constraints on the SUSY parameter space

In sections 4–6 a complete analysis of all the potentially dangerous unbounded from below (UFB) and charge and color breaking (CCB) directions has been carried out. Now, we wish to show explicitly, through a numerical analysis,
the restrictive power of the constraints on the MSSM parameter space. We will see that this is certainly remarkable.

We will consider the whole parameter space of the MSSM, \( m, M, A, B, \mu \), with the only assumption of universality. Actually, universality of the soft SUSY-breaking terms at \( M_X \) is a desirable property not only to reduce the number of independent parameters, but also for phenomenological reasons, particularly to avoid flavour-changing neutral currents (see, e.g., ref.\(^9\)). As discussed in sect.3, the requirement of correct electroweak breaking fixes one of the five independent parameters of the MSSM, say \( \mu \), so we are left with only four parameters \((m, M, A, B)\). In order to present the results in a clear way we will start by considering the particular case \( m = 100 \text{ GeV} \) and \( B = A - m \) (i.e. the well-known minimal SUGRA relation), and later we will let \( B \) to vary freely.

Fig.1a shows the region excluded by the “traditional” CCB bounds of the type of eq.\(^\text{(5)}\), evaluated at an appropriate scale (see sect.2). Clearly, the “traditional” bounds, when correctly evaluated, turn out to be very weak. In fact, only the leptonic (circles) and the \( d \)-type (diamonds) terms do restrict, very modestly, the parameter space. Let us recall here that it has been a common (incorrect) practice in the literature to evaluate these traditional bounds at all the scales between \( M_X \) and \( M_W \), thus obtaining very important (and of course overestimated) restrictions in the parameter space. Fig.1b shows the region excluded by the “improved” CCB constraints obtained in sect.5. Clearly, the excluded region becomes dramatically increased. Notice also that all the trilinear couplings (except the top one in this case) give restrictions, producing areas constrained by different types of bounds simultaneously. The restrictions coming from the UFB constraints, obtained in sect.4, are shown in Fig.1c. By far, the most restrictive bound is the UFB–3 one (small filled squares). Indeed, the UFB–3 constraint is the strongest one of all the UFB and CCB constraints, excluding extensive areas of the parameter space. This is a most remarkable result. Finally, in Fig.1d we summarize all the constraints plotting also the excluded region due to the experimental bounds on SUSY particle masses (filled diamonds). The finally allowed region (white) is quite small.

How do these results evolve when we vary the values of \( m \) and \( B \)? The results indicate that the smaller the value of \( m \) the more restrictive the bounds become (an explanation of this behavior will be given below). More precisely for \( m < 50 \text{ GeV} \) the whole parameter space becomes forbidden (for any value of the remaining parameters). So, from UFB and CCB constraints we can

\(^9\)Let us remark, however, that the constraints found in previous sections are general and they can also be applied to the non-universal case.
conclude

\[ m \geq 50 \text{ GeV} \]  \hspace{1cm} (41)

Obviously, the limiting case \( m = 0 \) is excluded. This is very relevant for no-scale models, since \( m = 0 \) is a typical prediction in that kind of scenarios. Concerning the remaining parameter, \( B \), the results indicate that the larger the value of \( B \), the more restrictive the bounds. In general, for \( m \lesssim 500 \text{ GeV} \), \( B \) has to satisfy the bound

\[ |B| \lesssim 3.5 \, m \]  \hspace{1cm} (42)

Figures illustrating eqs.(41,42) can be found in refs. [7,15].

So far, we have just presented the numerical results in the figs.1a–d and eqs.(41,42) with no attempt to explain the physical reasons underlying them. It is, however, very instructive to examine this question. The first thing to note is that, due to their structure, the CCB bounds on \( A/m \) (see eqs.(37,40)) are essentially \( m \)–invariant and \( B \)–invariant. The numerical analysis confirms this fact [7,15]. On the other hand, the UFB–3, which is the strongest (CCB and UFB) bound, becomes more stringent as \( m^2_{H_2} = m^2 - \mu^2 \) (i.e. the \( H_2 \) soft mass) becomes more negative. This is clear from eqs.(25–27). The precise value of \( m^2_{H_2} \) at low energy depends on its initial value at \( M_X \), i.e. \( m \), and on the RG running that, due to the effect of \( \lambda_{top} \), brings \( m^2_{H_2} \) to negative values. Consequently, the smaller \( m \) and the larger \( \lambda_{top} \), the stronger the UFB–3 bound becomes. Concerning \( m \) this result is certainly well reflected in eq.(41). Concerning \( \lambda_{top} \), since \( m_{top} \sim \lambda_{top} \langle H_2 \rangle \), where \( \langle H_2 \rangle = 2M^2_W \sin \beta/g^2 \), it is clear that the smaller \( \tan \beta \), the larger \( \lambda_{top} \) and therefore the stronger the UFB–3 bound. But \( \tan \beta \) decreases as \( B \) increases, thus the form of eq.(42). On the other hand, values of \( \tan \beta \) too close to 1 demand a value of \( \lambda_{top} \) at low energy higher than the infrared fixed point value, which is impossible to get from the running from high energies. This fact also contributes to the upper bound on \( |B| \), eq.(42).

\[ ^{17} \text{Larger values of } m \text{ start to conflict clearly the naturality bounds for electroweak breaking}\]

\[ ^{18} \text{so they are not realistic.} \]
Figure 1: Excluded regions in the parameter space of the MSSM, with $B = A - m$, $m = 100$ GeV and $M_{\text{phys}}^{\text{top}} = 174$ GeV. The darked region is excluded because there is no solution for $\mu$ capable of producing the correct electroweak breaking. (a) The circles and diamonds indicate regions excluded by the “traditional” CCB constraints associated with the $e$ and $d$-type trilinear terms respectively. (b) The same as (a) but using the “improved” CCB constraints. The triangles correspond to the $u$-type trilinear terms. (c) The crosses, squares and small filled squares indicate regions excluded by the UFB-1,2,3 constraints respectively. (d) The previous excluded regions together with the one arising from the experimental lower bounds on supersymmetric particle masses (filled diamonds).
To summarize, the UFB and CCB bounds, specially the UFB-3 bound, put important constraints on the MSSM parameter space. Contrary to a common believe, the bounds affect not only the trilinear parameter, $A$, but also the values of the universal scalar mass, $m$, the bilinear term parameter, $B$, and the universal gaugino mass, $M$. This can be noted from the figures 1a–d and eqs. (41, 42). Also, the frequently used constraint $|A| \leq 3m$ is not in general a good approximation. The actual bounds on $A$ depend on the values of the other SUSY parameters $(m, M, B)$.

The application of the UFB and CCB bounds to particular SUSY scenarios has been considered in some works. It is worth–mentioning that the string-inspired dilaton-dominated scenario is completely excluded on these grounds (as the above-mentioned no-scale scenarios). The infrared fixed point scenario
is also severely constrained\footnote{We work in a basis for the superfields where the Yukawa coupling matrices are diagonal.} (in particular it requires $M < 1.1 \, m$).

7 CCB constraints on flavor-mixing couplings

Supersymmetry has sources of flavor violation which are not present in the Standard Model.\footnote{This is why, contrary to eq.\eqref{eq:standardmodel}, we have not factorized the Yukawa couplings, $\lambda$, in the trilinear terms in eq.\eqref{eq:susyflavor}} These arise from the possible presence of non-diagonal terms in the squark and slepton mass matrices, coming from the soft-breaking potential (see eq.\eqref{eq:Vsoft})

\begin{align}
V_{\text{soft}} &= (m_{L}^2)_{ij} \bar{L}_i L_j + (m_{e_R}^2)_{ij} \bar{e}_{R_i} e_{R_j} \\
&+ (m_{Q}^2)_{ij} \bar{Q}_i Q_j + (m_{u_R}^2)_{ij} \bar{u}_{R_i} u_{R_j} + (m_{d_R}^2)_{ij} \bar{d}_{R_i} d_{R_j} \\
&+ \left[A_{ij}^L \bar{L}_i H_{1} e_{R_j} + A_{ij}^u \bar{Q}_i H_{1} u_{R_j} + A_{ij}^d \bar{Q}_i H_{1} d_{R_j} + \text{h.c.} \right] + \ldots\, (43)
\end{align}

where $i, j = 1, 2, 3$ are generation indices. A usual simplifying assumption of the MSSM is that $m_{L}^2_{ij}$ is diagonal and universal and $A_{ij}$ is proportional to the corresponding Yukawa matrix. Actually, we have implicitly used this assumption in all the previous sections. However, there is no compelling theoretical argument for these hypotheses\footnote{\textsuperscript{1}}.

The size of the off-diagonal entries in $m_{L}^2_{ij}$ and $A_{ij}$ is strongly restricted by FCNC experimental data. Here, we will focus our attention on the $A_{ij}^{\left(f\right)}$ terms; a summary of the corresponding FCNC bounds is given in the second column of Table\,\ref{table:FCNC}. The $\left(\delta_{LR}^{\left(f\right)}\right)_{ij}$ parameters used in the table are defined as

\begin{equation}
\left(\delta_{LR}^{\left(f\right)}\right)_{ij} \equiv \frac{\Delta M_{L}^2 \left(f\right)_{ij}}{M_{\text{av}}^2 \left(f\right)}
\end{equation}

where $f = u, d, l$; $M_{L}^2 \left(f\right)$ is the average of the squared sfermion ($\tilde{f}_L$ and $\tilde{f}_R$) masses and $\left(\Delta M_{L}^2 \left(f\right)\right)_{ij} = A_{ij}^{\left(f\right)} \langle H_{f}^0 \rangle$, with $H_{0}^u \equiv H_{2}^0, H_{d,l}^0 \equiv H_{1}^0$, are the off-diagonal entries in the sfermion mass matrices. It is remarkable that the $A_{ij}^{\left(f\right)}$ terms are also restricted on completely different grounds, namely from the requirement of the absence of dangerous charge and color breaking (CCB) minima or unbounded from below (UFB) directions. These bounds are in general stronger than the FCNC ones. Other properties of these bounds are the following:
i) Some of the bounds, particularly the UFB ones, are genuine effects of the non-diagonal $A^{(f)}_{ij}$ structure, i.e. they do not have a “diagonal counterpart”.

ii) Contrary to the FCNC bounds, the strength of the CCB and UFB bounds does not decrease as the scale of supersymmetry breaking increases.

There is no room here to review in detail how these bounds arise, although the philosophy is similar to that explained in sects.4, 5 (for further details see ref.21). Let us write however the final form of the constraints

**CCB bounds**

\[
\left| A^{(u)}_{ij} \right|^2 \leq \lambda^2_{uk} \left( m_{u_L}^2 + m_{u_R}^2 + m_\tau^2 \right), \quad k = \text{Max} (i, j)
\]

\[
\left| A^{(d)}_{ij} \right|^2 \leq \lambda^2_{dk} \left( m_{d_L}^2 + m_{d_R}^2 + m_{\nu_m}^2 \right), \quad k = \text{Max} (i, j)
\]

\[
\left| A^{(l)}_{ij} \right|^2 \leq \lambda^2_{ek} \left( m_{e_L}^2 + m_{e_R}^2 + m_{\nu_m}^2 \right), \quad k = \text{Max} (i, j)
\] (45)

**UFB bounds**

\[
\left| A^{(u)}_{ij} \right|^2 \leq \lambda^2_{uk} \left( m_{u_L}^2 + m_{u_R}^2 + m_{\tau_L}^2 + m_{\tau_R}^2 \right), \quad k = \text{Max} (i, j), \quad p \neq q.
\]

\[
\left| A^{(d)}_{ij} \right|^2 \leq \lambda^2_{dk} \left( m_{d_L}^2 + m_{d_R}^2 + m_{\nu_m}^2 \right), \quad k = \text{Max} (i, j)
\]

\[
\left| A^{(l)}_{ij} \right|^2 \leq \lambda^2_{ek} \left( m_{e_L}^2 + m_{e_R}^2 + m_{e_m}^2 \right), \quad k = \text{Max} (i, j), \quad m \neq i, j.
\] (46)

The CCB bounds must be evaluated at a renormalization scale $Q \sim 2 A^{(f)}_{ij} / \lambda^2_{f_k}$, while the UFB bounds must be imposed at any $Q^2 \gg (m/\lambda_{f_k})^2$. This can be relevant in many instances. For example, for universal gaugino and scalar masses ($M_1/2$ and $m$ respectively) satisfying $M_1/2 \gtrsim m$, the UFB bounds are more restrictive at $M_X$ than at low energies (especially the hadronic ones). This trend gets stronger as the ratio $M_{1/2}/m$ increases.

The previous CCB and UFB bounds can be expressed in terms of the $(\delta^{(f)}_{LR})_{ij}$ parameters defined in eq.(44) and compared with the corresponding FCNC
bounds. It turns out that the former are almost always stronger. This is illustrated in Table 1 for the particular case \( M_{av}^{(f)} = 500 \text{ GeV} \). The only exception is \((\delta_{LR}^{(f)})_{12}\), which is experimentally constrained by the \( \mu \rightarrow e, \gamma \) process. As the scale of supersymmetry breaking increases the FCNC bounds are easily satisfied whereas the CCB and UFB bounds continue to strongly constrain the theory.

Another case in which the FCNC constraints are satisfied is when approximate “infrared universality” emerges from the RG equations.\(^{22,18,19}\) Again, the CCB and UFB bounds continue to impose strong constraints on such theories. This is because, as argued before, these bounds have to be evaluated at different large scales and do not benefit from RG running.

Table 1: FCNC bounds versus CCB and UFB bounds on \((\delta_{LR}^{(f)})_{ij}\) for \( M_{av}^{(f)} = 500 \text{ GeV} \). The bounds have been obtained from ref.[20] taking \( x = (m_{\text{gaugeo}}/M_{av}^{(f)})^2 = 1.\)

| \( \delta_{LR}^{(f)} \) | FCNC       | CCB and UFB |
|------------------|------------|-------------|
| \( (\delta_{LR}^{(d)})_{12} \) | \( 4.4 \times 10^{-3} \) | \( 2.9 \times 10^{-4} \) |
| \( (\delta_{LR}^{(d)})_{13} \) | \( 3.3 \times 10^{-2} \) | \( 10^{-2} \) |
| \( (\delta_{LR}^{(d)})_{23} \) | \( 1.6 \times 10^{-2} \) | \( 10^{-2} \) |
| \( (\delta_{LR}^{(u)})_{12} \) | \( 3.1 \times 10^{-2} \) | \( 2.3 \times 10^{-3} \) |
| \( (\delta_{LR}^{(l)})_{12} \) | \( 8.5 \times 10^{-6} \) | \( 3.6 \times 10^{-4} \) |
| \( (\delta_{LR}^{(l)})_{13} \) | \( 5.5 \times 10^{-1} \) | \( 6.1 \times 10^{-3} \) |
| \( (\delta_{LR}^{(l)})_{23} \) | \( 10^{-1} \) | \( 6.1 \times 10^{-3} \) |

8 Cosmological considerations and final comments

As has been mentioned in sect.1, the CCB and UFB bounds presented here are conservative; they correspond to sufficient, but not necessary, conditions for the viability of the standard vacuum. It is possible that we live in a metastable vacuum,\(^2 \), whose lifetime is longer than the age of the universe. This certainly softens the constraints obtained here.

The first study on CCB-metastability bounds was performed by Claudson et al.\(^2 \). They showed that only the top-Yukawa CCB bounds are dangerous
from this point of view. In other words, among the various CCB minima (see sect.5), the one associated with the top-Yukawa coupling is the only one to which the realistic minimum has a substantial probability to decay during the universe life-time. The remaining CCB minima, although deeper, present too high barriers for an efficient tunnelling. That analysis has been re-done by Kusenko et al.\textsuperscript{23} taking into account some subtleties when analyzing the transition probabilities. Their results are qualitatively similar to those of ref.\textsuperscript{4}. Quantitatively, they obtain a bound similar to the CCB-1 bound (see eq.(38)), empirically modified as $|A_t|^2 \lesssim 3 \left[ m_i^2 - \mu^2 + 2.5(m_{Q_t}^2 + m_t^2) \right]$, which of course is weaker than the pure stability bound. On the other hand, the UFB bounds (in particular the UFB-3 bound, which is the strongest of all the CCB and UFB bounds) have not been analyzed yet from this point of view.

It is important to keep in mind that the metastability bounds represent necessary but perhaps not sufficient conditions to be safe (in the same sense that the stability bounds presented in sects.4–5 represent sufficient but perhaps not necessary conditions). The reason is that for the applicability of the metastability bounds, the universe should be driven by some mechanism into the realistic (but local and metastable) minimum. This problem has been treated in several papers\textsuperscript{23,24}. Of course, a definite answer (not based in an anthropic principle) requires the consideration of a particular cosmological scenario in order to determine the initial values of the relevant fields at early times. Apparently, the realistic minimum is indeed favoured in many cosmological scenarios. Namely, if the initial conditions are dictated by thermal effects, the universe tends to fall into the realistic minimum since it is the closest one to the origin. This can also be the case in some inflationary scenarios. However, a more systematic analysis of these issues would be welcome.

Finally, from a more philosophic point of view, it is conceptually difficult to understand how the cosmological constant is vanishing precisely in a local “interim” vacuum (especially from an inflationary point of view). It is also interesting that many of the (tree-level) UFB directions presented here, particularly the ones associated with flavour violating couplings, are really unbounded from below (after radiative corrections) and, if present, make the theory ill-defined, at least until Planckian physics comes to the rescue. These issues, however, enter the realm of still unknown pieces of fundamental physics.

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