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On the optimal ordering policy of a dual-sourcing system considering stochastic supply disruption together with stochastic ordering yield

Chirakiat Saithong¹ and Wipha Rukanna¹

Abstract: Due to the prevalence of supply disruptions disturbing supply chain and logistics operations, a firm should tackle such disruptions using appropriate approaches. In this research work, a dual-sourcing approach is applied by a firm. The main decision regarding the firm sourcing from two unreliable suppliers is to derive an optimal ordering policy that maximizes the firm’s expected profit. Considering the stochastic ordering yield bridges a gap in the literature. The firm’s expected profit function is analytically formulated, and the viability of the developed mathematical expression is illustrated using numerical experiments and sensitivity analyses. Through the sensitivity analyses, the importance of considering the stochastic ordering yield is confirmed, as the optimal ordering policy varies on the variance of the ordering yield. Furthermore, failing to consider the stochastic ordering yield leads to incorrect conclusions regarding the optimal ordering policy; in addition, the increase in the variance of the ordering yield prevents the firm from benefiting from incremental profit.

Subjects: Technology; Engineering & Technology; Industrial Engineering & Manufacturing; Operations Research; Manufacturing Engineering; Logistics; Supply Chain Management; Operations Management; Operations Research; Mathematics & Statistics for Engineers

Keywords: dual-sourcing; supply disruption; yield uncertainty; ordering yield; optimal ordering policy

ABOUT THE AUTHORS

Chirakiat Saithong is a lecturer in the Department of Industrial Engineering, Faculty of Engineering at Sriracha, Kasetsart University Sriracha Campus. He received a Doctor of Engineering (D.Eng) from the Asian Institute of Technology, Thailand, in 2018. His research interest is in analyzing uncertainties in supply chains.

Wipha Rukanna received a degree of Bachelor of Engineering (B.Eng) from the Kasetsart University Sriracha Campus in 2021.

PUBLIC INTEREST STATEMENT

Diversifying the risks arising from a supply disruption prevails in business practice, and sourcing from two suppliers is appropriate. Utilizing the dual-sourcing approach, the main decision of the firm is to determine the optimal ordering quantity placed to each supplier aiming at profit maximization. In addition to the optimal ordering decision, the ordering yield, which is the fraction of the amount of order that a supplier could respond to in the realization of disruption, is naturally uncertain; thus, the stochastic ordering yield is incorporated into the decision-making process. Through our sensitivity analyses, it can be confirmed that considering the stochastic ordering yield helps a firm derive a precise optimal ordering policy.
1. Introduction

The present situation of supply chain and logistics operations entails many uncertain events leading to risks and deteriorated profitability. These uncertain events include natural disasters, political instability, striking workers, fire, transportation interruptions, bankruptcy of suppliers, etc. Among various kinds of uncertainty, a focus on the supply disruption problem is appropriate due to the prevalence of supply disruption effects (Tang, 2006). Supply disruption refers to the interruption of the supply, either partially or completely, of inputs or raw materials required for the firm’s production process that results in failing to respond to customer demands as well as a loss in profit (Snyder et al., 2016). There are many practical examples of supply disruption. For example, Toyota’s manufacturing plant in Tianjin halted production due to a workers’ strike at a supplier plant, Tianjin Toyoda Gosei (Shirouzu, 2010). A production delay of electronic devices such as networking chips was caused due to a fire at Nittobo’s production facility that supplies glass fabrics to an ABF substrate supplier (MH&L Staff, 2021). TECT Aerospace, an important supplier of Boeing, filed for bankruptcy in 2021 due to the COVID-19 pandemic leading to a slowdown in production (Hayward, 2021). Thus, applying appropriate mitigation and contingency approaches helps firms not only to reduce financial loss but also to ensure continuity of business operations.

Dealing with the supply disruption problem, the acquisition of an optimal approach mainly depends on the supply disruption profile (Snyder et al., 2016; Tomlin, 2006). Even though a single approach is not viable in all situations, the dual-sourcing approach is optimal for a wide spectrum of supply disruption profiles (Tomlin, 2006). Additionally, under the condition that the firm is averse to supply disruption risk and prefers to diversify to avoid such risks, sourcing from two suppliers is therefore appropriate. The dual-sourcing strategy is the focus of this research work.

The dual-sourcing strategy is a strategy where a firm is supplied with materials or products from two suppliers, and the decision regarding the strategy is the determination of an optimal ordering policy, specifically, determination of the amount of order that should be placed with each supplier. Determining an optimal ordering policy, the firm considers the suppliers’ risk profile, the associated purchasing costs of each supplier, as well as the shortage costs and the salvage values. The suppliers’ risk profile includes the suppliers’ disruption probability and the stochastic ordering yield after a disruption occurs. Due to the short life cycle of a product, the firm has only one opportunity to order, and an emergency ordering is not possible. In normal circumstances when both suppliers function normally, the firm’s inventory is replenished at the beginning of the selling period, and the firm uses the inventory to satisfy customer demand during the selling period. The unsold units are salvaged via a secondary market, and any unsatisfied demand is lost. However, upon facing disruption by a supplier, the firm receives only a fraction of the total amount of the order that the firm wants to purchase from the disrupted supplier. We refer to this fraction as the ordering yield, and the ordering yield is stochastic. In critical situation, such as when both suppliers are disrupted, the firm may receive a partial amount of the total purchased units from the stochastic ordering yield of both suppliers. Considering the associated costs, the salvage revenue, and the suppliers’ risk profile, the question arises regarding how much the firm should order from each supplier to maximize the firm’s expected profit.

Therefore, the current research work aims to determine an optimal ordering policy of the firm to achieve the maximum profit. Based on the existing literature addressing the supply disruption problem using the dual-sourcing strategy, a few research works consider the ordering yield, namely, Iakovou et al. (2010), Xanthopoulos et al. (2012), Watanabe and Kusukawa (2015), Ledari et al. (2018), Yan et al. (2019), Gupta and Ivanov (2020), and Gupta et al. (2021). The ordering yield could represent (1) the level of suppliers’ production achievement towards the plan in the presence of a production failure and (2) the fraction of the total order quantity which could arrive on time in the presence of a distribution disruption (Iakovou et al., 2010; Xanthopoulos et al., 2012). Thus, capturing the ordering yield better explains reality. Since it spontaneously involves uncertainties, a stochastic ordering yield should be considered. However, most of the above research works consider the ordering yield as deterministic, and Yan et al. (2019) considered a stochastic ordering yield. The following discussion describes the differences between the work of Yan et al. (2019) and the current
research work. First, regarding the modeling of the disruption’s occurrence and the yield in the work of Yan et al. (2019), the disruption occurs at a time within the selling period, and the yield is dependent on the time at which the disruption occurs. However, in our research work and the existing literature, the ordering yield is considered as the suppliers’ risk profile, and it should be independent of the length of a selling period. In addition, even though the analytical expression of the expected profit function is derived in the work of Yan et al. (2019), the illustration of the viability of the proposed analytical expression assumes a uniformly distributed demand, whilst a normally distributed demand is considered in our research work. Last but not least, since the ordering yield is spontaneously stochastic, the effect of the ordering yield variance of a supplier on the optimal ordering policy is examined in our research work, and some important insights are addressed. To this end, this research work contributes to the literature by considering the stochastic ordering yield, which is independent of the length of a selling period, to derive an optimal ordering policy in a two-stage supply chain where the supplies are susceptible to disruption.

2. Literature review

Multiple-sourcing strategies prevail in many businesses and attract the eye of not only practitioners but also scholars. Svoboda et al. (2021) conducted a literature survey regarding the multiple-sourcing strategy, defining the strategy characteristic as a trade-off between the costs of having multiple suppliers and the inventory shortage costs of a single supplier. Three further research avenues were suggested for a more practical applicability purpose: a multi-echelon model, resilience in the presence of risks, and industrial big data. Boulaksil et al. (2021) studied a dual-sourcing system in which a buyer sourced a single product from two suppliers; one was onshore and the other was offshore. The offshore supplier was cheaper but slower; on the other hand, the onshore supplier was faster but more expensive. Studying the system, performance was compared from a supply chain perspective between the Dynamic Order Policy and the Standing Order Policy. The result revealed that the Standing Order Policy outperformed the Dynamic Order Policy in several cases from the supply chain perspective.

Focusing on the supply disruption problem, despite there is a variety of strategies available to deal with the supply disruption problem, several research works address the supply disruption problem using a dual-sourcing scheme. Iakovou et al. (2010) studied a two-echelon supply chain where a manufacturer procured products from two unreliable suppliers. Upon the disruption of a supplier, the supplier was unable to respond to the order partially, or completely as a special case. The optimal ordering policy leading to the maximum retailer’s profit was derived. Oberlaender (2011) considered a risk-aversion of a decision-maker in a dual-sourcing system incorporated with a two-ordering-opportunity policy to determine the optimal order quantities which maximized profit. The risk preference was modeled using an exponential utility function. Like Oberlaender (2011), Xanthopoulos et al. (2012) also considered the risk preference of the decision-maker to determine the optimal order quantities; however, the risk preference was modeled as a service level constraint. The results revealed a decrease in profit as the level of risk aversion of the decision-maker was very high. In addition, the work of Xanthopoulos et al. (2012) also considered the ordering yield resulting from the disruption of a supplier. Considering both the ordering yield and the players’ risk-preference, Gupta and Ivanov (2020) considered the risk aversion of suppliers where products were substitutable and developed a game-theoretical framework. The result revealed that the retailer should set a lower price for a non-disrupted product as the suppliers’ risk-aversion increased.

Taking the ordering yield into consideration, Gupta et al. (2021) investigated a system comprised of a retailer sourcing two products from two suppliers, where these two supply channels were uncertain. In this case, the retailer offered the products to the end customers, and the customers chose either of them. The pricing decisions including the wholesale price and the retail price were determined. Paying no attention to the ordering yield, Zhu and Fu (2013) derived the optimal ordering policy for a dual-sourcing system. Considering both ordering yield and a special order in the presence of supply disruption, Ledari et al. (2018) characterized an optimal ordering policy. Then, relaxing the assumption of deterministic ordering yield, Yan et al. (2019) took a stochastic ordering yield in the face of disruption into consideration for deriving an optimal ordering policy. The
stochastic yield considered in the work of Yan et al. (2019) was dependent on not only the length of a selling period but also the point in time where supply disruption takes place within a selling period. Numerical experiments were conducted based on a uniformly distributed demand. Hou and Zhao (2012) studied a system where an unreliable supplier supplied a single product to a retailer, and the retailer also had a backup source of supply. The backup source of supply built up an inventory and supplied to the retailer in need. The optimal decisions of the two players were determined using a process of sequential optimization, and numerical experiments were conducted to examine the effects of input parameters. Meena and Sarmah (2014) investigated an optimal sourcing decision from multiple suppliers, where despite providing a quantity discount scheme, all suppliers were susceptible to disruption. The optimal number of suppliers, as well as order allocation, were determined using numerical experiments. Watanabe and Kusukawa (2015) analyzed the issue of supply chain coordination in a dual-sourcing supply chain of a single type of product. Unlike previous studies, Li (2017) studied a system comprised of a firm facing deterministic demand and two suppliers; one was susceptible to disruption, and the other had a yield uncertainty problem. The optimal sourcing strategy was determined under specified conditions. Investigating a backup source for the supply system, Hou et al. (2017) studied a capacity reservation contract agreed between a buyer and a backup source of the supply. The constraint that the buyer committed to purchase at least a certain quantity was incorporated into the investigation. The optimal ordering decision, as well as reservation quantity were derived. Apart from using a capacity reservation contract to engage the player in the backup channel, Köle and Bakal (2017) utilized an option contract and analytically characterized an optimal ordering policy. Different scenarios of information availability were considered, i.e., both supply disruption and demand information, only supply disruption information, and neither of the information. Zhang and Wang (2019) aimed at determining an optimal procurement strategy in a backup sourcing system. Taking into consideration the supply information sharing, a reserved capacity, as well as an optimal emergency order, were determined. Zhang and Wang (2021) studied a system comprised of two competing suppliers and two competing manufacturers, where one of the two manufacturers sourced from a reliable supplier only, with the other utilizing a contingent sourcing strategy. The optimal order quantity of both manufacturers, as well as the optimal wholesale price of both suppliers, were derived. Gupta et al. (2015) also investigated a system with two competing manufacturers. One manufacturer sourced from an unreliable supplier in an ordinary case, while in the presence of disruption, the manufacturer utilized an emergency order placed to another reliable supplier, with the supplier also providing the components to another manufacturer. The result revealed that both the supply state realization and the time to place the order of the competitive manufacturer played important roles in the profit of the manufacturer utilizing a contingent dual-sourcing scheme. Aslani and Heydari (2019) analyzed a system comprised of one manufacturer and one retailer selling a single type of green product through two channels, namely, a retailing channel and an internet-based channel. These two channels were susceptible to disruption. Using an internet-based channel, the customers received the products from the manufacturer directly, while passing through the retailer in the retailing channel. In the face of disruption in one channel, the products were transshipped across the channel from a disrupted to another non-disrupted channel. The optimal level of products’ greenness and selling price in the channel were derived. Lin et al. (2020) characterized the retailer’s preference towards sourcing from two suppliers simultaneously or using one as a backup. Li et al. (2021) analyzed a supply chain consisting of a retailer and two unreliable suppliers where the retailer was averse to the risks. Additionally, a correlation of suppliers’ risks was considered for deriving an optimal ordering strategy.

For other strategies addressing the supply disruption problem, Li and Wang (2015) applied a business insurance strategy to alleviate financial loss due to a supply interruption in a system comprised of a manufacturer sourcing from two suppliers. Zeng and Xia (2015) utilized a backup supply contract to engage the supplier in a backup channel. The supply contract helped not only to protect the buyer from a supply shortage problem but also to gain the economic benefit of the backup supplier. Saithong and Luong (2019) derived an optimal inventory policy of a periodic review base stock system considering the effect of stochastic supply disruption. Switching from the periodic
Table 1. Comparison between the existing literature considering the ordering yield in a dual-sourcing system and the present research work

| Author                  | Ordering yield | Examining effect of the ordering yield variance |
|-------------------------|----------------|-----------------------------------------------|
|                         | Deterministic  | Stochastic                                    |
| Ikavou et al. (2010)    |                |                                               |
| Xanthopoulas et al. (2012) |                |                                               |
| Watanabe and Kusukawa (2015) |                |                                               |
| Ledari et al. (2018)    | ●              |                                               |
| Yan et al. (2019)       |                |                                               |
| Gupta and Ivanov (2020) | ●              |                                               |
| Gupta et al. (2021)     | ●              |                                               |
| The present research work | ●              | ●                                             |

review inventory system of Saithong and Luong (2019) to a continuous review inventory system, Saithong and Luong (2020) determined the optimal reorder point and the optimal order-up-to level that minimized the expected total costs per time unit in the face of disruption. Saithong et al. (2020) derived a closed-form expression of an optimal base-stock level in the presence of disruption. Islam et al. (2020) also utilized an inventory approach to deal with uncertainties. A three-stage supply chain comprised a supplier, who was facing stochastic capacity, a retailer, who was randomly disrupted, and a manufacturer, who was in the middle aiming at devising an optimal inventory policy leading to cost minimization, was considered. Rosyida et al. (2020) utilized a flexible shipment strategy to manage both customer disruption and link disruption problems. Kungwalsong et al. (2021) utilized a supply chain network design approach to deal with a facility disruption problem.

In the literature considering the ordering yield resulting from disruption in the derivation of an optimal ordering policy, the ordering yield was considered as deterministic in the works of Ikavou et al. (2010), Xanthopoulas et al. (2012), Watanabe and Kusukawa (2015), Ledari et al. (2018), Gupta and Ivanov (2020), and Gupta et al. (2021). Even though Yan et al. (2019) considered the stochastic ordering yield as a result of supply disruption, the disruption considered in their research work occurred within a selling period, and the yield was dependent on not only the point in time where the disruption took place within the selling period but also on the length of the selling period. The comparison between the above research works and the present research work is shown in Table 1. As far as we know, considering the stochastic ordering yield resulting from a supply disruption, there has been no previous research work on the derivation of an optimal ordering policy of a dual-sourcing system, in which each supply channel is susceptible to disruption and the disruption state is realized at the beginning of the selling period.

3. Problem description and formulation of expected profit function

Before addressing the description of the problem, Table 2 shows the notation used in this research work.

The following describes the problem considered in the present research work. A two-stage supply chain comprised a firm sourcing from two unreliable suppliers is considered. Considering all associated costs and possible revenues as well as the suppliers’ risk profile, the firm places an order $Q_i$ to supplier $i$. Taking the order lead time into consideration, the order arrives at the beginning of the selling period, and the state of disruption of each supplier is also realized. At this point in time, the firm receives an amount $Q_i$ from supplier $i$ if supplier $i$ does not encounter any problems. However, with a probability of $p_i$, supplier $i$ runs into a problem, and the firm receives only $y_iQ_i$ units, where the
minimum and the maximum values of $y_i$ are $y_{i,\min}$ and $y_{i,\max}$, respectively. It should be noted that the value of $y_{i,\min}$ cannot be less than zero, and the value of $y_{i,\max}$ cannot be greater than one. Additionally, $y_i$ is uncertain, and the probability density function of $Y_i$ is $f_{y_i}(y_i)$. In practice, the ordering yield could be uncertain in the presence of disruption; thus, they could be estimated in a range indicating the possible minimum and the maximum value of the ordering yield. The cost of purchasing from supplier $i$ is $c_i$, and the firm gains revenue from satisfying customer demand at $s$ per unit sold. Any unsold items are salvaged via a secondary market, and the salvage value per unsold unit is $r$. Unmet demand costs $k$ per unit of shortage. The firm needs to determine the optimal order quantity to place to each supplier leading to profit maximization for the firm.

In addition to the above problem description, the following assumptions are assumed.

- The demand follows a normal distribution with mean $\mu$ and variance $\sigma^2$.
- The ordering yield of supplier $i$ follows a continuous uniform distribution over the range $[y_{i,\min}, y_{i,\max}]$.
- No emergency order is allowed; thus, the firm has a single opportunity to place an order to each of the suppliers.

Regarding the formulation of the expected profit function, there are four possible suppliers' states at the beginning of the selling period: neither of the suppliers is disrupted, either of the suppliers is disrupted, or both suppliers are disrupted. The expression of the expected profit can be obtained by utilizing the following property (Ross, 2010):

$$
E[\text{Profit}] = \sum_{y_i} E[\text{Profit}| \text{Scenario}] \times P(\text{Scenario})
$$

$$
E[\text{Profit}] = E[\text{Profit}| \text{No disruption}] (1 - p_1)(1 - p_2) + E[\text{Profit}| \text{Only supplier 1 disruption}] p_1(1 - p_2) \\
+ E[\text{Profit}| \text{Only supplier 2 disruption}] (1 - p_1)p_2 + E[\text{Profit}| \text{Both suppliers disruption}] p_1p_2
$$

(1)

| Table 2. Notation |
|------------------|
| **Notation**     | **Description**                  |
| $i$              | Supplier’s index $i \in \{1, 2\}$ |
| $Q_i$            | Order quantity of supplier $i$ (units) |
| $c_i$            | Purchasing costs of supplier $i$ (monetary unit/unit) |
| $s$              | Selling price (monetary unit/unit) |
| $r$              | Salvage value (monetary unit/unit) |
| $k$              | Shortage costs (monetary unit/unit) |
| $X$              | Stochastic variable representing the customers’ demand in a selling period |
| $f_X(x)$         | Probability density function of $X$ |
| $F_X(x)$         | Cumulative distribution function of $X$ |
| $Y_i$            | Stochastic variable representing the ordering yield of supplier $i$ |
| $f_{Y_i}(y_i)$   | Probability density function of $Y_i$ |
| $y_{i,\min}$    | Lower bound of the ordering yield of supplier $i$ |
| $y_{i,\max}$    | Upper bound of the ordering yield of supplier $i$ |
| $p_i$            | Disruption probability of supplier $i$ |
From (1), there are four expressions to be determined. Regarding the $E[\text{Profit}_{\text{No disruption}}]$, the expression was determined by Xanthopoulos et al. (2012):

$$E[\text{Profit}_{\text{No disruption}}] = \int_{0}^{Q_1+Q_2} (sx - c_1Q_1 - c_2Q_2 + r(Q_1 + Q_2 - x)) f(x) dx$$

To determine $E[\text{Profit}_{\text{Only supplier 1 disruption}}]$, we must determine the expected profit in this case at a fixed value of $Y_1 = y_1$ first, that is, $E[\text{Profit}_{\text{Only supplier 1 disruption}, Y_1 = y_1}]$. $E[\text{Profit}_{\text{Only supplier 1 disruption}, Y_1 = y_1}]$ was determined by Xanthopoulos et al. (2012).

$$E[\text{Profit}_{\text{Only supplier 1 disruption}, Y_1 = y_1}] = \int_{0}^{Q_1+Q_2} (sx - c_1y_1Q_1 - c_2Q_2 + r(y_1Q_1 + Q_2 - x)) f(x) dx$$

Utilizing the property $E[\text{Profit}_{\text{Only supplier 1 disruption}}] = \int_{y_1}^{y_{\text{min}}} E[\text{Profit}_{\text{Only supplier 1 disruption}, Y_1 = y_1}] f_{Y_1}(y_1) dy_1$, we have (Ross, 2010):

$$E[\text{Profit}_{\text{Only supplier 1 disruption}}] = \int_{y_1}^{y_{\text{min}}} \left( \int_{0}^{Q_1+Q_2} (sx - c_1y_1Q_1 - c_2Q_2 + r(y_1Q_1 + Q_2 - x)) f(x) dx \right) f_{Y_1}(y_1) dy_1$$

Next, the expression of $E[\text{Profit}_{\text{Only supplier 2 disruption}}]$ will be determined. Using a similar approach to the above, the expression of $E[\text{Profit}_{\text{Only supplier 1 disruption}, Y_2 = y_2}]$ must be determined first, and we will have: $E[\text{Profit}_{\text{Only supplier 2 disruption}}] = \int_{y_2}^{y_{\text{min}}} E[\text{Profit}_{\text{Only supplier 2 disruption}, Y_2 = y_2}] f_{Y_2}(y_2) dy_2$ (Ross, 2010).

Regarding the $E[\text{Profit}_{\text{Only supplier 2 disruption}, Y_1 = y_1}]$, the expression was determined by Xanthopoulos et al. (2012).

$$E[\text{Profit}_{\text{Only supplier 2 disruption}, Y_1 = y_1}] = \int_{0}^{Q_1+Q_2} (sx - c_1Q_1 - c_2y_2Q_2 + r(Q_1 + y_2Q_2 - x)) f(x) dx$$

Then,

$$E[\text{Profit}_{\text{Only supplier 2 disruption}}] = \int_{y_2}^{y_{\text{min}}} \left( \int_{0}^{Q_1+Q_2} (sx - c_1Q_1 - c_2y_2Q_2 + r(Q_1 + y_2Q_2 - x)) f(x) dx \right) f_{Y_2}(y_2) dy_2$$

Then, using the same approach as the above, $E[\text{Profit}_{\text{Both suppliers disruption}}]$ can be determined from

$$E[\text{Profit}_{\text{Both suppliers disruption}}] = \int_{y_{\text{min}}}^{y_1} \int_{y_{\text{min}}}^{y_2} E[\text{Profit}_{\text{Both suppliers disruption}, Y_1 = y_1, Y_2 = y_2}] f_{Y_1}(y_1) f_{Y_2}(y_2) dy_1 dy_2$$
(Ross, 2010). Xanthopoulos et al. (2012) derived the expression of 
\[ E\left[ \text{Profit} \middle| \text{Both suppliers disruption}, y_1, y_2 \right] \]
as:

\[
E\left[ \text{Profit} \middle| \text{Both suppliers disruption}, y_1, y_2 \right] = 
\int_0^{y_1 Q_1 + y_2 Q_2} (sx - c_1 y_1 Q_1 - c_2 y_2 Q_2 + r(y_1 Q_1 + y_2 Q_2 - x)) f_x(x) dx 
+ \int_{y_1 Q_1 + y_2 Q_2}^{\infty} (s(x - y_1 Q_1 - y_2 Q_2) - c_1 y_1 Q_1 - c_2 y_2 Q_2 - k(x - y_1 Q_1 - y_2 Q_2)) f_x(x) dx
\]

Thus,

\[
E\left[ \text{Profit} \middle| \text{Both suppliers disruption} \right] = 
\int_{y_1}^{y_2} \int_{y_1}^{y_2} \left( \int_0^{y_1 Q_1 + y_2 Q_2} (sx - c_1 y_1 Q_1 - c_2 y_2 Q_2 + r(y_1 Q_1 + y_2 Q_2 - x)) f_x(x) dx \right) f_{y_1}(y_1) f_{y_2}(y_2) dy_1 dy_2
+ \int_{y_1 Q_1 + y_2 Q_2}^{\infty} \left( \int_{y_1 Q_1 + y_2 Q_2}^{\infty} (s(x - y_1 Q_1 - y_2 Q_2) - c_1 y_1 Q_1 - c_2 y_2 Q_2 - k(x - y_1 Q_1 - y_2 Q_2)) f_x(x) dx \right) f_{y_1}(y_1) f_{y_2}(y_2) dy_1 dy_2
\]

Thus, \( E[\text{Profit}] \) of the optimal ordering policy, the optimal order quantities follow the Proposition.

**Proposition.**

The optimal order quantities, \( Q_1^* \) and \( Q_2^* \), can be determined from solving these two mathematical expressions simultaneously.

\[
\begin{align*}
(1 - p_1)(1 - p_2) F_X(Q_1^* + Q_2^*) &+ (p_1)(1 - p_2) \int_{y_1}^{\max} y_1 f_X(y_1, Q_1^* + Q_2^*) f_{Y_1}(y_1) dy_1 \\
+ (1 - p_1)(p_2) \int_{y_2}^{\max} f_X(Q_1^* + Q_2^* f_{Y_2}(y_2) dy_2 \\
+ (p_1)(p_2) \int_{y_2}^{\max} y_2 f_X(y_1 Q_1^* + Q_2^*) f_{Y_1}(y_1) dy_1 f_{Y_2}(y_2) dy_2 \] \\
&= \frac{(s + k - c_2)(1 - p_1 + p_2 E[y_1])}{(s + k - r)}
\end{align*}
\]

\[
\begin{align*}
(1 - p_1)(1 - p_2) F_X(Q_1^* + Q_2^*) &+ (p_1)(1 - p_2) \int_{y_1}^{\max} y_1 f_X(y_1, Q_1^* + Q_2^*) f_{Y_1}(y_1) dy_1 \\
+ (1 - p_1)(p_2) \int_{y_2}^{\max} y_2 f_X(Q_1^* + Q_2^* f_{Y_2}(y_2) dy_2 \\
+ (p_1)(p_2) \int_{y_2}^{\max} y_2 f_X(y_1 Q_1^* + Q_2^*) f_{Y_1}(y_1) dy_1 f_{Y_2}(y_2) dy_2 \] \\
&= \frac{(s + k - c_2)(1 - p_1 + p_2 E[y_2])}{(s + k - r)}
\end{align*}
\]

**Proof.** See Appendix B.
Regarding the proof of the concavity property of $E[\text{Profit}]$, the derivation is included in Appendix C. Next, to illustrate the applicability of the developed mathematical expression, numerical experiments and sensitivity analyses should be conducted.

4. Numerical experiments and sensitivity analyses
The viability of the developed mathematical expression is illustrated in this section. The section is divided into two parts. The first part is the numerical experiments; several experiments are conducted in this part. The latter part is the sensitivity analysis that aims to analyze the effects of changing the values of parameters on the optimal firm’s decision.

4.1. Numerical experiments
There are three sets of parameters used to examine the objective of the research work.

Set 1: The following values of parameters are assumed. $s = $80 per unit, $c_1 = $20 per unit, $c_2 = $30 per unit, $r = $10 per unit, $k = $200 per unit, $\mu = 400$ units, $\sigma = 6$ units, $p_1 = 0.3$, $p_2 = 0.3$, $y_{1,\min} = 0$, $y_{1,\max} = 1$, $y_{2,\min} = 0$, $y_{2,\max} = 1$. According to this set of parameters, the optimal ordering policy, as well as the corresponding firm’s expected profit, are shown in Table 3.

| $Q_1$ (units) | $Q_2$ (units) | Firm’s expected profit ($) |
|---------------|---------------|----------------------------|
| 388           | 324           | 16,705                     |

It can be seen that the optimal ordering policy, which maximizes the firm’s expected profit can be derived. It can be further noticed that these two suppliers have the same disruption probability, while one supplier is cheaper than the other one. The cheaper supplier receives a greater order quantity than the more expensive supplier.

Set 2: The following values of parameters are assumed. $s = $45 per unit, $c_1 = $27 per unit, $c_2 = $27 per unit, $r = $10 per unit, $k = $150 per unit, $\mu = 400$ units, $\sigma = 6$ units, $p_1 = 0.1$, $p_2 = 0.1$, $y_{1,\min} = 0$, $y_{1,\max} = 1$, $y_{2,\min} = 0$, $y_{2,\max} = 1$. According to this set of parameters, the optimal ordering policy, as well as the corresponding firm’s expected profit, are shown in Table 4.

| $Q_1$ (units) | $Q_2$ (units) | Firm’s expected profit ($) |
|---------------|---------------|----------------------------|
| 237           | 237           | 4,172                      |

It can be seen that the optimal ordering policy, which maximizes the firm’s expected profit can be determined. It can be further noticed that these two suppliers share the same disruption profile with equal purchasing costs. The numerical result reveals the equal optimal ordering quantity.

Set 3: The following values of parameters are assumed. $s = $70 per unit, $c_1 = $30 per unit, $c_2 = $30 per unit, $r = $10 per unit, $k = $250 per unit, $\mu = 400$ units, $\sigma = 6$ units, $p_1 = 0.7$, $p_2 = 0.3$.
According to this set of parameters, the optimal ordering policy, as well as the corresponding firm’s expected profit, are shown in Table 5.

It can be seen that the optimal ordering policy which maximizes the firm’s expected profit can be derived. It can be further noticed that these two suppliers have the same purchasing costs; however, one supplier is more susceptible to disruption than the other supplier. The numerical result reveals that the more reliable supplier receives more order quantity.

### 4.2. Sensitivity Analyses

Sensitivity analyses are conducted to examine the effect of changing parameter values on the optimal order quantity of both suppliers as well as the firm’s expected profit. In this subsection, the effect of disruption probability, yield variance, purchasing costs, demand-mean, demand-standard deviation, selling price, salvage value, and shortage costs are examined. The following values of parameters are used at the base-case: \( s = 580 \text{ per unit}, c_1 = 25 \text{ per unit}, c_2 = 25 \text{ per unit}, r = 10 \text{ per unit}, k = 200 \text{ per unit}, \mu = 400 \text{ units}, \sigma = 6 \text{ units}, p_1 = 0.1, p_2 = 0.1, y_{1,\text{min}} = 0, y_{1,\text{max}} = 1, y_{2,\text{min}} = 0, y_{2,\text{max}} = 1 \). For the base-case parameter values, both suppliers share the same purchasing costs as well as disruption profile. According to this set of parameters, the firm’s optimal decision can be determined as \( Q_1 = 277 \text{ units}, Q_2 = 277 \text{ units} \), with the firm’s expected profit of $18,422.

The organization of this subsection is as follows. First, the effect of disruption probability will be examined. Since stochastic yield is an important feature of this research work, the effect of changing yield variance will be investigated. Then, the effect of changing purchasing costs, demand-mean, demand-standard deviation, selling price, salvage value, and shortage costs will be respectively examined.

#### 4.2.1. Effect of changing disruption probability

Regarding the examination of the effect of disruption probability, the value of \( p_2 \) is varied while other parameter values are unchanged. The firm’s expected profit and the optimal order quantity of each supplier are illustrated in Figure 1. It can be seen that the increase in disruption probability causes a decrease in the firm’s expected profit. Clearly, the disruption is unfavorable to the firm. Furthermore, the increase in the disruption probability of a supplier tends to increase the optimal order quantity of the other supplier.

#### 4.2.2. Effect of changing yield variance

Because the stochastic ordering yield is considered, the effect of yield variance is examined. According to the base case parameters, \( E[Y_1]=E[Y_2]=0.5 \). We keep the expected value of the ordering yield unchanged, while the value difference between \( y_{2,\text{min}} \) and \( y_{2,\text{max}} \), which represents the ordering yield variance, is varied. The values of \( y_{2,\text{min}} \) and \( y_{2,\text{max}} \) used in analyzing the effect are shown in Table 6; the optimal firm’s decision, as well as the firm’s expected profit, are illustrated in Figure 2.

According to Figure 2, the increase in ordering yield variance of supplier-2 leads to not only a decrease in the optimal order quantity of supplier-2 but also an increase in the optimal order quantity of the other supplier. This implies that the greater the ordering yield variance of a supplier, the lesser the order quantity the supplier receives. Thus, the ordering yield variance is important in

| Table 5. Optimal ordering policy and the firm’s expected profit for Set 3 of the numerical experiment |
|------------------------------------------|----------|------------------|
| \( Q_1 \) (units) | \( Q_2 \) (units) | Firm’s expected profit ($) |
| 340 | 383 | 8,970 |

0.2, \( y_{1,\text{min}} = 0, y_{1,\text{max}} = 1, y_{2,\text{min}} = 0, y_{2,\text{max}} = 1 \).
deriving an optimal ordering policy. Furthermore, it can be seen that the increase in ordering yield variance causes a decrease in the firm’s expected profit; thus, the firm also benefits from a reduction in the supplier’s yield variance. By the same token, for a low ordering yield variance, the firm’s expected profit is higher. In this meaning, ignoring the variance of the ordering yield and considering it as deterministic, not only the firm’s expected profit but also the optimal ordering quantity will be distorted.

Figure 1. Effect of disruption probability

Table 6. Variation in ordering yield

| $y_2$ | $y_{2,\text{min}}$ | $y_{2,\text{max}} - y_{2,\text{min}}$ | Var($y_2$) |
|------|----------------|-----------------------------------|-----------|
| 0.5000001 | 0.4999999 | 2E-07 | 3.3333E-15 |
| 0.500001 | 0.499999 | 2E-06 | 3.3333E-13 |
| 0.5001 | 0.4999 | 2E-05 | 3.3333E-11 |
| 0.501 | 0.499 | 0.002 | 3.3333E-09 |
| 0.51 | 0.49 | 0.02 | 3.3333E-07 |
| 0.60 | 0.40 | 0.20 | 3.3333E-05 |
| 0.65 | 0.35 | 0.30 | 3.3333E-03 |
| 0.70 | 0.30 | 0.40 | 3.3333E-02 |
| 0.75 | 0.25 | 0.50 | 3.3333E-01 |
| 0.80 | 0.20 | 0.60 | 3.3333E+00 |
| 0.85 | 0.15 | 0.70 | 3.3333E+01 |
| 0.90 | 0.10 | 0.80 | 3.3333E+02 |
| 0.95 | 0.05 | 0.90 | 3.3333E+03 |
| 1.00 | 0.00 | 1.00 | 3.3333E+04 |

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4.2.3. Effect of changing cost of purchasing

Next, the effect of changes in purchasing costs is examined. In Figure 3, the value of $c_2$ is varied, while the values of the other parameters remain unchanged. As expected, the increase in the purchasing costs of supplier-2 leads to a decrease in the optimal order quantity from supplier-2; on the other hand, the optimal order quantity of the other supplier increases. The increase in purchasing costs does not benefit the firm; thus, the firm’s expected profit decreases.

4.2.4. Effect of changing $\mu$ and $\sigma$

The effects of changing mean and standard variation of demand are examined. The value of both demand-mean and demand-standard deviation are varied, and other parameter values are kept unchanged. Regarding the effect of changing $\mu$, the increase in demand-mean causes an increase in both the optimal order quantities and the maximum profit, as illustrated in Figure 4. Thus, the firm gains profitability as the demand-mean increases.
Figure 4. Effect of demand-mean.

Figure 5. Effect of demand-standard deviation.
Next, regarding the effect of changing demand-standard deviation, the value of demand-standard deviation is varied, and the other parameter values are kept unchanged. From Figure 5, it can be observed that the increase in demand-standard deviation tends to decrease the firm’s expected profit; on the other hand, the optimal order quantities increase. Thus, it can be confirmed that the increase in demand variability deteriorates the firm’s profit.

4.2.5 Effect of changing selling price
Despite the selling price is exogenously determined, the effect of changing selling price is examined in this part. Apart from the selling price, which is varied, all parameter values are kept unchanged. From Figure 6, it is clear that the increase in the selling price increases the profit for the firm, and the optimal order quantities increase as well.

![Figure 6. Effect of selling price.](image)

4.2.6. Effect of changing salvage value
Next, as the leftover items are salvaged via a secondary market, the effect of changing salvage value is examined. From Figure 7, it is clear that the increase in salvage value encourages the firm to order greater quantities, and the firm gains extra profit as well. Therefore, the increase in salvage value helps the firm increase profit.

4.2.7. Effect of changing shortage costs
A penalty for unsatisfied demand is examined. From Figure 8, the increase in the shortage costs leads to an increase in order quantities; however, the firm still loses profit. Thus, the increase in shortage costs harms the firm’s expected profit.

5. Conclusions
A dual-sourcing strategy is applied to deal with a supply disruption problem in this research work. Employing the dual-sourcing scheme, an optimal ordering policy is determined for a firm sourcing from two unreliable suppliers so that the expected profit of the firm is maximized. A mathematical expression of the firm’s expected profit is analytically formulated. Because the ordering yield is
Figure 7. Effect of salvage value.

Figure 8. Effect of shortage costs.
spontaneously stochastic, the stochastic ordering yield is incorporated into the formulation of the firm's expected profit function, which bridges a gap in the literature. The viability of the developed mathematical expression is illustrated using numerical experiments and sensitivity analyses. From a managerial perspective, the stochastic ordering yield plays a role in deriving an optimal ordering policy, and it should be taken into consideration. Ignoring the variance of ordering yield leads to an incorrect conclusion regarding the optimal ordering policy. In addition, the higher variance of the ordering yield, the lesser profit the firm receives.

For further research avenue, since there is an improvement in the firm’s expected profit as the yield variance decreases, it would be interesting to examine an incentive mechanism by which the firm incentivizes the supplier to improve the ordering yield variance. The firm may benefit via incremental profit, while the supplier may benefit from an increased order quantity as well as the associated profitability. Thus, a joint benefit of the adoption of this incentive mechanism could be further investigated. In addition to this point, the developed mathematical model is built on a premise that the order lead time between these two suppliers must be equal. Therefore, a further study could be conducted by considering unequal lead time for orders to determine an optimal ordering policy.

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Author details
Chirakiat Saithong
E-mail: chirakiat@eng.srku.ac.th
ORCID ID: http://orcid.org/0000-0002-6615-9861
Wipha Rukanna
E-mail: wipha@srku.th
1 Department of Industrial Engineering, Faculty of Engineering at Siracha, Kasetsart University Siracha Campus, 199, Sukhumvit Rd., Siracha, Chonburi 20230, Thailand.

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Appendix A. Mathematical expression of the firm’s expected profit

\[
E[\text{Profit}] = \left[ \int_{0}^{Q_1+Q_2} \left[ sx - c_1Q_1 - c_2Q_2 + r(Q_1 + Q_2 - x) \right] f_x(x) \, dx \right. \\
+ \left. \int_{Q_1+Q_2}^{\infty} \left[ s(Q_1 + Q_2) - c_1Q_1 - c_2Q_2 - k(x - Q_1 - Q_2) \right] f_x(x) \, dx \right] (1 - p_1)(1 - p_2)
\]

\[
+ \left[ \int_{y_1,\text{min}}^{y_1,\text{max}} \left[ \int_{0}^{Q_1+Q_2} \left[ sx - c_1y_1Q_1 - c_2Q_2 + r(y_1Q_1 + Q_2 - x) \right] f_x(x) \, dx \right. \\
+ \left. \int_{Q_1+Q_2}^{\infty} \left[ s(y_1Q_1 + Q_2) - c_1y_1Q_1 - c_2Q_2 - k(x - y_1Q_1 - Q_2) \right] f_x(x) \, dx \right] f_{y_1}(y_1) \, dy_1 \right]
\]

\[
+ \left[ \int_{y_2,\text{min}}^{y_2,\text{max}} \left[ \int_{0}^{Q_1+Q_2} \left[ sx - c_1Q_1 - c_2y_2Q_2 + r(Q_1 + y_2Q_2 - x) \right] f_x(x) \, dx \right. \\
+ \left. \int_{Q_1+Q_2}^{\infty} \left[ s(Q_1 + y_2Q_2) - c_1Q_1 - c_2y_2Q_2 - k(x - Q_1 - y_2Q_2) \right] f_x(x) \, dx \right] f_{y_2}(y_2) \, dy_2 \right]
\]

\[
\frac{(p_1)(1 - p_2)}{1 - p_1} \left( \frac{y_1,\text{min}}{y_1,\text{max}} \right) \left( \frac{y_2,\text{min}}{y_2,\text{max}} \right) (p_1)(p_2)
\]

(App. 1)

Appendix B. Proof of Proposition

The optimal order quantities, \(Q_1^*\) and \(Q_2^*\), can be determined from solving \(\frac{\partial E[\text{Profit}]}{\partial Q_1} = 0\) and \(\frac{\partial E[\text{Profit}]}{\partial Q_2} = 0\) simultaneously. Thus, we need to determine both \(\frac{\partial E[\text{Profit}]}{\partial Q_1}\) and \(\frac{\partial E[\text{Profit}]}{\partial Q_2}\). For readability purpose, the mathematical expression of (A-1) should be broken into 4 main components, namely, \(G_0(Q_1, Q_2)\), \(G_1(Q_1, Q_2)\), \(G_2(Q_1, Q_2)\), and \(G_3(Q_1, Q_2)\), by which

\[
G_0(Q_1, Q_2) = \left[ \int_{0}^{Q_1+Q_2} \left[ sx - c_1Q_1 - c_2Q_2 + r(Q_1 + Q_2 - x) \right] f_x(x) \, dx \right. \\
+ \left. \int_{Q_1+Q_2}^{\infty} \left[ s(Q_1 + Q_2) - c_1Q_1 - c_2Q_2 - k(x - Q_1 - Q_2) \right] f_x(x) \, dx \right] f_{y_1}(y_1) \, dy_1
\]

\[
G_1(Q_1, Q_2) = \left[ \int_{y_1,\text{min}}^{y_1,\text{max}} \left[ \int_{0}^{Q_1+Q_2} \left[ sx - c_1y_1Q_1 - c_2Q_2 + r(y_1Q_1 + Q_2 - x) \right] f_x(x) \, dx \right. \\
+ \left. \int_{Q_1+Q_2}^{\infty} \left[ s(y_1Q_1 + Q_2) - c_1y_1Q_1 - c_2Q_2 - k(x - y_1Q_1 - Q_2) \right] f_x(x) \, dx \right] f_{y_1}(y_1) \, dy_1 \right]
\]
\[ G_2(Q_1, Q_2) = \left( \int_{y_2 = 0}^{y_2 = \infty} \left( \int_{x = 0}^{Q_1 + y_1 Q_2} (s x - c_1 Q_1 - c_2 y_2 Q_2 + r(Q_1 + y_2 Q_2 - x)) f(x) dx \right) f_2(y_2) dy_2 \right) \]

\[ G_{12}(Q_1, Q_2) = \left( \int_{y_1 = 0}^{y_1 = \infty} \left( \int_{x = 0}^{Q_1 + y_1 Q_2} (s x - c_1 y_1 Q_1 - c_2 y_2 Q_2 + r(y_1 Q_1 + y_2 Q_2 - x)) f(x) dx \right) f_1(y_1) dy_1 \right) f_2(y_2) \]

Thus, (A-1) becomes

\[ E[\text{Profit}] = (G_0(Q_1, Q_2))(1 - p_1)(1 - p_2) + (G_1(Q_1, Q_2))(p_1)(1 - p_2) + (G_2(Q_1, Q_2))(1 - p_1)(p_2) + (G_{12}(Q_1, Q_2))(p_1)(p_2) \]

Taking the derivative of \( E[\text{Profit}] \) with respect to \( Q_1 \), we have

\[ \frac{\partial E[\text{Profit}]}{\partial Q_1} = \left( \frac{\partial G_0(Q_1, Q_2)}{\partial Q_1} \right)(1 - p_1)(1 - p_2) + \left( \frac{\partial G_1(Q_1, Q_2)}{\partial Q_1} \right)(p_1)(1 - p_2) + \left( \frac{\partial G_2(Q_1, Q_2)}{\partial Q_1} \right)(1 - p_1)(p_2) + \left( \frac{\partial G_{12}(Q_1, Q_2)}{\partial Q_1} \right)(p_1)(p_2) \] (A-2)

Taking the derivative of \( E[\text{Profit}] \) with respect to \( Q_2 \), we have

\[ \frac{\partial E[\text{Profit}]}{\partial Q_2} = \left( \frac{\partial G_0(Q_1, Q_2)}{\partial Q_2} \right)(1 - p_1)(1 - p_2) + \left( \frac{\partial G_1(Q_1, Q_2)}{\partial Q_2} \right)(p_1)(1 - p_2) + \left( \frac{\partial G_2(Q_1, Q_2)}{\partial Q_2} \right)(1 - p_1)(p_2) + \left( \frac{\partial G_{12}(Q_1, Q_2)}{\partial Q_2} \right)(p_1)(p_2) \] (A-3)

With the aim of replacing all corresponding components in (A-2) and (A-3), taking the derivative of \( G_0(Q_1, Q_2) \) with respect to \( Q_1 \) and \( Q_2 \), respectively, we have

\[ \frac{\partial G_0(Q_1, Q_2)}{\partial Q_1} = (s - c_1 + k) + (r - s - k) F_x(Q_1 + Q_2) \] (A-4)

\[ \frac{\partial G_0(Q_1, Q_2)}{\partial Q_2} = (s - c_2 + k) + (r - s - k) F_x(Q_1 + Q_2) \] (A-5)
After that, taking the derivative of $G_1(Q_1, Q_2)$ with respect to $Q_1$ and $Q_2$, respectively, we have

$$\frac{\partial G_1(Q_1, Q_2)}{\partial Q_1} = (s - c_1 + k)E[Y_1] + (r - s - k)\int_{y_{1,\min}}^{y_{1,\max}} y_1 F_X(y_1 Q_1 + Q_2) f_Y(y_1) dy_1$$  \hspace{1cm} (A - 6)$$

$$\frac{\partial G_1(Q_1, Q_2)}{\partial Q_2} = (s - c_2 + k)E[Y_2] + (r - s - k)\int_{y_{2,\min}}^{y_{2,\max}} F_X(y_1 Q_1 + Q_2) f_Y(y_1) dy_1$$  \hspace{1cm} (A - 7)$$

Next, taking the derivative of $G_2(Q_1, Q_2)$ with respect to $Q_1$ and $Q_2$, respectively, we have

$$\frac{\partial G_2(Q_1, Q_2)}{\partial Q_1} = (s - c_1 + k)E[Y_2] + (r - s - k)\int_{y_{2,\min}}^{y_{2,\max}} F_X(Q_1 + y_2 Q_2) f_Y(y_2) dy_2$$  \hspace{1cm} (A - 8)$$

$$\frac{\partial G_2(Q_1, Q_2)}{\partial Q_2} = (s - c_2 + k)E[Y_2] + (r - s - k)\int_{y_{1,\min}}^{y_{1,\max}} y_2 F_X(Q_1 + y_2 Q_2) f_Y(y_2) dy_2$$  \hspace{1cm} (A - 9)$$

Then, taking the derivative of $G_{12}(Q_1, Q_2)$ with respect to $Q_1$ and $Q_2$, respectively, we have

$$\frac{\partial G_{12}(Q_1, Q_2)}{\partial Q_1} = (s - c_1 + k)E[Y_1]$$

$$+ (r - s - k)\int_{y_{2,\min}}^{y_{2,\max}} \left( \int_{y_{1,\min}}^{y_{1,\max}} y_1 F_X(y_1 Q_1 + y_2 Q_2) f_Y(y_1) dy_1 \right) f_Y(y_2) dy_2$$  \hspace{1cm} (A - 10)$$

$$\frac{\partial G_{12}(Q_1, Q_2)}{\partial Q_2} = (s - c_2 + k)E[Y_2]$$

$$+ (r - s - k)\int_{y_{1,\min}}^{y_{1,\max}} \left( \int_{y_{2,\min}}^{y_{2,\max}} y_2 F_X(Q_1 + y_2 Q_2) f_Y(y_1) dy_2 \right) f_Y(y_2) dy_2$$  \hspace{1cm} (A - 11)$$

Replacing (A-4), (A-6), (A-8), (A-10) in (A-2) and equating to zero, we have
\[
\left(1 - p_1\right)\left(1 - p_2\right) F_X(Q_1 + Q_2) + (p_1)(1 - p_2) \int_{y_1, \min}^{y_1, \max} y_1 F_X(y_1 Q_1 + Q_2) f_{r_1}(y_1) dy_1
\]
\[
+ \left(1 - p_1\right)(p_2) \int_{y_2, \min}^{y_2, \max} y_2 F_X(Q_1 + y_2 Q_2) f_{r_2}(y_2) dy_2
\]
\[
+ \left(p_1\right)(p_2) \left( \int_{y_1, \min}^{y_1, \max} y_1 F_X(y_1 Q_1 + y_2 Q_2) f_{r_1}(y_1) dy_1 \right) f_{r_2}(y_2) dy_2
\]
\[
= \frac{(s + k - c_1)((1 - p_1) + p_2 \mathbb{E}[Y_1])}{(s + k - r)}
\]

Similarly, replacing (A-5), (A-7), (A-9), (A-11) in (A-3) and equating to zero, we have

\[
\left(1 - p_1\right)\left(1 - p_2\right) F_X(Q_1 + Q_2) + (p_1)(1 - p_2) \int_{y_1, \min}^{y_1, \max} y_1 F_X(y_1 Q_1 + Q_2) f_{r_1}(y_1) dy_1
\]
\[
+ \left(1 - p_1\right)(p_2) \int_{y_2, \min}^{y_2, \max} y_2 F_X(Q_1 + y_2 Q_2) f_{r_2}(y_2) dy_2
\]
\[
+ \left(p_1\right)(p_2) \left( \int_{y_1, \min}^{y_1, \max} y_1 F_X(y_1 Q_1 + y_2 Q_2) f_{r_1}(y_1) dy_1 \right) f_{r_2}(y_2) dy_2
\]
\[
= \frac{(s + k - c_2)((1 - p_2) + p_2 \mathbb{E}[Y_2])}{(s + k - r)}
\]

**Appendix C. Concavity property of \( \mathbb{E}[\text{Profit}] \)**

The second order partial derivatives of (A-1) with respect to \( Q_1 \) and \( Q_2 \) are:

\[
\frac{\partial^2 \mathbb{E}[\text{Profit}]}{\partial Q_1^2} =
\]
\[
\frac{\partial^2 \mathbb{E}[\text{Profit}]}{\partial Q_2^2} =
\]

(A-12)

(A-13)
\[
\frac{\partial^2 E[\text{Profit}]}{\partial Q_2 \partial Q_1} = \\
\left((1 - p_1)(1 - p_2)f_X(Q_1 + Q_2) + \left(p_1(1 - p_2)\int_{y_1,\min}^{y_1,\max} y_1 f_X(y_1, Q_1 + Q_2) f_{Y_1}(y_1) dy_1\right)ight) \\
(r - s - k) + \left((1 - p_1)p_2\int_{y_2,\min}^{y_2,\max} y_2 f_X(Q_1 + Q_2) f_{Y_2}(y_2) dy_2\right) \\
+ \left(p_1p_2\int_{y_2,\min}^{y_2,\max} \int_{y_1,\min}^{y_1,\max} y_1 y_2 f_X(y_1, Q_1 + Q_2) f_{Y_1}(y_1) f_{Y_2}(y_2) dy_1 dy_2\right)
\]
(A - 14)

\[
\frac{\partial^2 E[\text{Profit}]}{\partial Q_2^2} = \\
\left((1 - p_1)(1 - p_2)f_X(Q_1 + Q_2) + \left(p_1(1 - p_2)\int_{y_1,\min}^{y_1,\max} y_1 f_X(y_1, Q_1 + Q_2) f_{Y_1}(y_1) dy_1\right)\right) \\
(r - s - k) + \left((1 - p_1)p_2\int_{y_2,\min}^{y_2,\max} y_2 f_X(Q_1 + Q_2) f_{Y_2}(y_2) dy_2\right) \\
+ \left(p_1p_2\int_{y_2,\min}^{y_2,\max} \int_{y_1,\min}^{y_1,\max} y_1 y_2 f_X(y_1, Q_1 + Q_2) f_{Y_1}(y_1) f_{Y_2}(y_2) dy_1 dy_2\right)
\]

Due to the fact that \(s > r\), (A-12), (A-13), and (A-14) expressing \(\frac{\partial^2 E[\text{Profit}]}{\partial Q_2 \partial Q_1}\), \(\frac{\partial^2 E[\text{Profit}]}{\partial Q_2^2}\), and \(\frac{\partial^2 E[\text{Profit}]}{\partial Q_1^2}\), respectively, will be less than or equal to zero. Regarding (A-15) expressing \(\frac{\partial^2 E[\text{Profit}]}{\partial Q_2 \partial Q_1} - \left(\frac{\partial^2 E[\text{Profit}]}{\partial Q_2^2}\right)^2\), it is difficult to justify its non-negativity. Thus, several numerical instances (more than 24,400 instances) are used in testing, and the negative value of (A-15) cannot be found. Therefore, \(Q_1^*\) and \(Q_2^*\) result in the maximization of \(E[\text{Profit}]\).
