Design of a Narrowband Adaptable Filter in Low-Frequency Domain

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Abstract. To improve the Signal to Noise Ratio and the precision of data acquisition systems in the condition of broad bandwidth, an adaptive narrow-band filter circuit has been designed for obtaining weak signals in low-frequency domain. And the problems of classic filter circuits, such as wave form distortion and suffering from non-linearity of varactors, can be solved by the novel design in replacing varactor with voltage control capacitor. Simultaneously, by improving LRC filter circuit, our designs also can increase quality factor in traditional RC filter circuit. Therefore, this design has great advantages over classic filter circuits for its narrow frequency band, high quality factor and low signal deformation. The above properties of this circuit can make it used widely in removing noise for modulation and acquisition circuits with high precision, and can be used to obtain accurate band-pass filtering in low frequency bands, which has been verified by experiments showing superior advantages of improving the SNR and filtering effects.

1. Introduction

Higher performance has been required for data acquisition systems with the development of power electronic technology and large-scale integrated circuits [1-2]. Hence it is necessary to expand system bandwidth as far as possible, however the Signal to Noise Ratio (SNR) and precision of the system generally would be reduced with the increase of system bandwidth [3-4]. Hence, how to design a broad-band filter of high qualities for low-frequency domain has become an important topic for signal processing.

An adaptive filter has great advantages in high quality factor, and low signal deformation, and can work in broad bands. Thus, in this paper, we propose to design a new adaptive filter for realizing noise removal in low frequency domain. This filter has adaptable range of up to the maximum of 110 kHz and has many properties, such as realizing adaptable band-pass in narrow-band according to the user input, shifting the center frequency of the band-pass filter, frequency tracing in low-frequency domain, and accomplishing narrow-band filtering in broadband [5-6].

2. Design of Filter Circuit

This filter circuit adopts the design idea of combining classic filtering and adaptive filtering. The main function of the classic filtering circuit is to realize signal amplification and filtering.
2.1. Low-Pass Filter Circuit

This low-pass filter circuit mainly performs filtering signal primarily. Based on ensuring the flatness in the band, it can reduce the bandwidth, attenuate the noise, and so improve the SNR. Active power filters are used to stabilize filter characteristics due to their high input impedance and low output impedance characteristics. In the design of filter circuits, we try to achieve excellent performances such as flatness in-band and inhibition out-band. The low-pass filter circuit used in this design is shown in figure 1.

![Diagram of the low-pass filter circuit](image)

Figure 1. The low-pass filter circuit.

According to figure 1, the cut-off frequency of the filter could be adapted to the given point by setting the values of resistors and capacitors. The gain of the positive proportional amplifier can be calculated by equation (1), which is determined by resistors $R_4$ and $R_3$, as follows:

$$A_v = 1 + \frac{R_4}{R_3}$$  \hspace{1cm} (1)

For the convenience of calculation, the value of resistor $R_4$ approximately equals to 2 $\Omega$ (measured value), the value of resistor $R_3$ should be in the level of K$\Omega$, then the channel gain reaches the approximation of 1, and the filter capacitors and other resistors in the circuit have the same value, namely $R_1 = R_2 = R_3 = R$, $C_1 = C_2 = C$. Then due to the circuit connection and operational amplifier characteristics, the transfer function of the filter circuit can be expressed by equation (2):

$$A(s) = \frac{V_v(s)}{V_i(s)} = \frac{A_v}{1 + 3sRC + (sRC)^2} = \frac{1}{1 + 3sRC + (sRC)^2}$$  \hspace{1cm} (2)

To set the low-pass filter parameters, we should get the cut-off frequency $f_0$ of the filter. In order to find the cut-off frequency $f_0$ conveniently, let us make $f_0 = 1/(2\pi RC)$, then by inserting $sRC = s/(2\pi f_0)$ into equation (2), which can be converted into equation (3):

$$A(s) = \frac{1}{1 + \frac{3s}{2\pi f_0} + \left(\frac{s}{2\pi f_0}\right)^2}$$  \hspace{1cm} (3)

Replacing $s$ and $\omega$ respectively with $i\omega$ and $2\pi f$ in equation (3), we can obtain equation (4):

$$A(f) = \frac{1}{1 - \left(\frac{f}{f_0}\right)^2 + \frac{3f}{f_0}i}$$  \hspace{1cm} (4)

where $i$ is the imaginary unit.
According to the definition of cut-off frequency, when the actual frequency equals to the cut-off frequency $f_p$, the gain of the filter circuit should be $\sqrt{2}$ times than that of low frequency. Therefore, the denominator module value in equation (4) is $\sqrt{2}$, which is shown in equation (5):

$$1 - \left(\frac{f}{f_0}\right)^2 + \frac{3f}{f_0} = \sqrt{2}$$

By squaring both of sides in equation (5), thus we obtain equation (6):

$$\left(\frac{f_p}{f_0}\right)^4 + \frac{7f_p}{f_0} + 1 = \sqrt{2}$$

By solving equation (6), the positive real solution is as following:

$$f_p = \left(\frac{\sqrt{3} - 7}{2}\right)^{\frac{1}{2}} f_0 = 0.374 f_0 = \frac{0.374}{2\pi RC}$$

Thus from equation (7), it is only need to assure $RC = 5.956\times10^{-7}$ when the cut-off frequency of second-order low-pass Butterworth filter equals to 100 kHz. If we choose 10 kΩ resistors and 60 pF capacitors, the filtering effect can approximately achieve our requirements.

2.2. Single-End Input Transform into Difference Output Circuit

Because the signal input is single-ended and the ADC used in this design requires a differential input, so we should convert the single-ended signal to a differential signal [6]. Single-end input transform into difference output circuits can be implemented by using proportional amplifier circuits. One specific structure of the circuit is shown in figure 2.

![Figure 2. Single-end input transform into difference output circuit.](image)

In figure 2, OPA1, $R_1$ and $R_2$ form a voltage follower. The circuit uses series negative feedback, and only needs to ensure that the two resistances are equal, so that its feedback coefficient is 1, which can realize the characteristic of voltage following. Meanwhile, the input impedance of the voltage follower is the impedance of the OPA itself, which can reach the level of $10^9 \Omega$, so the input loss is very small.

OPA2 and $R_3$ - $R_6$ form a positive phase proportional amplifier circuit, which can change the channel gain according to the resistance ratio of $R_3$ - $R_6$. The transfer function is presented in equation (8):

$$\frac{U_o}{U_i} = \frac{(R_3 + R_6)R_6}{(R_3 + R_5)R_4}$$
Therefore, the gain of this improved positive phase proportional amplifier circuit would be less than 1. For example, if \( R_4 = R_5 = R_6 \) and \( R_3 = 3R_4 \), the gain of the transfer function would be \( 1/2 \) in equation (8).

3. Design of Adaptive Filter Circuit

In fact, most adaptive filters have high control algorithm complexities and high hardware implementation costs. Therefore, they are mostly implemented based on digital filtering. This section mainly introduces the circuit implementation of the adaptive filter. According to the design requirements, the main purpose of the adaptive filter is to complete the matching and screening of the signal, retain the target signal as much as possible, attenuate the remaining signals, maximize the SNR, and thus realize the process of frequency selection [7-9].

3.1. Design of Frequency Selection Circuit

The adaptive filter circuit in this design is based on an adjustable filter circuit design. Commonly used adjustable filter circuits include RC and LRC resonant filter circuits. Among them, the quality factor \( Q \) of the RC filter circuit is difficult to get high value. The reason for the importance of quality factor \( Q \) for the filter system is that this parameter corresponds to measure the frequency selection characteristics of the filter circuit. The passband bandwidth of the filter system can be expressed by equation (9):

\[
B_w = \frac{f_0}{Q}
\]  

where \( f_0 \) is the center frequency of the bandpass filter. It can be seen from this formula that if the quality factor \( Q \) of the system is not high, the corresponding bandpass filter has a wide passband and shows a poor frequency selection.

In order to obtain a higher quality factor, this design uses LRC resonant filter circuit. Its working principle is based on the impedance characteristics of the resonance circuit at the resonance point: The series resonant circuit has a low reactance near the resonant frequency, which is similar to a short circuit. The network impedance is close to the resonant resistance value, and shows a higher impedance at other frequencies. The parallel resonant circuit is just the opposite. The resonance frequency of the resonance circuit can be expressed and calculated by equation (10):

\[
f_0 = \frac{1}{2\pi(LC)^{1/2}}
\]  

From the above equation (10), we can find that in the LRC resonant filter circuit, the adjustable capacitor is used as the adjustable device to construct the adjustable circuit. The adjustable capacitor is essentially a varactor diode, which is made by using the feature that the junction capacitance decreases with the increases of the applied voltage when the PN junction is reverse-biased, and can be continuously adjusted.

However, the capacitance of varactor diode is usually so small that in the level of pF. To solve this problem, this design uses Murata’s new variable capacitors, i.e., LXRW series chip. According to the input control voltage, it is able to continuously adjust capacitance of the variable capacitor and promote the adjusting capacitance range up to 1.5 times to its utmost, which shows a good comprehensive performance. Based on this variable capacitor, the LRC resonant filter circuit designed in this subject is shown in figure 3.

In figure 3, \( R_1 - L_1 - C_1 \) is a series resonant circuit, \( C_2 - L_2 - R_2 \) is a parallel resonant circuit, \( R_3 \) is a compensation resistor, \( C_1 \) and \( C_2 \) are variable capacitors, and other devices are general components. The OPAU8 is connected in the form of an inverting proportional amplification circuit, which inverts the signal phase and superimposes the phase inversion caused by the subsequent single-ended-to-differential circuit to form an integer multiple of phase shifts to eliminate phase effects.
To achieve the purpose of frequency selection, we should set the resonance frequency of the resonance circuit to the signal frequency. From equation (10), if we want to set the resonance frequency to 100 kHz, we can take $k_{Hz} = 100$ into this equation (10) and acquire that:

$$12.5 \times 10^{-5} = \frac{1}{LC}.$$ 

That is to say, as long as the inductor and the variable capacitor are appropriately set, e.g. $L = 25\text{mH}$ and $C = 100\text{pF}$, then the narrow band-pass filter with a central frequency of 100 kHz will be obtained.

For the first-order series resonant circuit, the quality factor $Q$ of the circuit is as shown in equation (11):

$$Q = \frac{1}{R} \left( \frac{L}{C} \right)^{1/2}$$  \hspace{1cm} (11)

We can see from the equation (11), under the conditions that both $L$ and $C$ have been determined, reducing the resistance value of the current limiting resistor in the resonant circuit can further improve the quality factor, which can reduce the passband bandwidth of the filter (equation (9)) and improve the system selectivity.

### 3.2. Design of Adaptive Filter

The realization of the adaptive filter in this topic can be subdivided into two parts, frequency measurement and frequency approximation.

#### 3.2.1. Frequency Measurement

Since the frequency of the input signal is unknown, it is necessary to measure the frequency of the target signal as the reference input of the adaptive filter, and then we can adjust the filter parameters, shift the central frequency of the band-pass filter, finally realize the narrow-band filtering. At present, the widely used spectrum analysis method is to transform the signal from the time domain to the frequency domain by Fourier transform, which can obtain the relationships among the amplitude, the phase and the frequency. Basing on the correspondence between amplitude and frequency, this design is designed to determine the signal frequency.

#### 3.2.2. Frequency Approximation

Due to the non-ideality of the device and the non-linearity of the voltage-capacitance curve of the variable capacitor, after receiving the feedback from the host computer, the central frequency set by the filter will deviate from the actual frequency. In actual testing, the maximum deviation between the theoretical estimated value and the actual value of the capacitor can reach 2.1 pF, the corresponding frequency deviation is close to 1 kHz, but the narrowband filter bandwidth is only 635 kHz, which is obviously unacceptable for design. And this is exactly the task to be completed in the approach link.

The approximation process means that the central frequency of the filter is moved to the midpoint of the two ends, and the resulting data is subjected to Fast Fourier Transform (FFT) to obtain the amplitude of its maximum energy point. Comparing with the value of the two ends, we can use the midpoint value to replace the point with smaller peak amplitude. Repeat this process until the peak
amplitude measured by FFT is the largest, then this point is the corresponding point of the actual frequency. The execution process is shown in figure 4.

![Flowchart of adaptive adjustment in approximation process](image)

**Figure 4.** Flowchart of adaptive adjustment in approximation process.

4. **The Experiments of Adaptive Filter**

In order to test the actual performance of the adaptive filter circuit, we can use a signal source with a maximum deviation of 1 mV to simulate a noisy input, a sinusoidal signal with an output frequency of 50 kHz and an amplitude value of 1 V to test the actual filtering effect of the adaptive filter. This design separately passes the signal through the all-pass channel and the adaptive filtering channel, and performs FFT on the data obtained by the host computer, and compares the two graphs to reflect the effect of adaptive filtering. It is shown in figure 5.

![FFT comparison](image)

**Figure 5.** FFT comparison (a) all-pass channel (b) adaptive filter channel.

It can be seen from the comparison of the two figures in figure 5 that after adaptive filtering, the overall amplitude of the noise in the input signal is reduced by about 40 dB, while the target signal amplitude of 50 kHz is basically unchanged. So, after adaptive filtering, the SNR is improved obviously and the filtering effect is good. It is verified that the design in this paper has superior advantages of improving the SNR and filtering effect.

5. **Conclusion**

This paper designed an adaptive filter circuit with a maximum adjustable frequency range from 0 Hz to 110 kHz. According to requirements, it can change the central frequency of the band-pass filter so that it can realize adaptive narrow-band filtering. It has been shown that this filter has a narrow frequency band, a high quality factor, and can avoid a signal distortion effectively. Hence, it can be used to obtain accurate band-pass filtering in low frequency bands.
References

[1] Takizawa M and Yukawa M 2016 Efficient dictionary-refining kernel adaptive filter with fundamental insights IEEE Transactions on Signal Processing 64 (16) 4337-4350.

[2] Yu Y, Zhao H and Chen B 2016 Steady-state mean-square-deviation analysis of the sign subband adaptive filter algorithm Signal Processing 120 36-42.

[3] Quadri A, Manesh M R and Kaabouch N 2016 Denoising signals in cognitive radio systems using an evolutionary algorithm based adaptive filter IEEE 7th Annual Ubiquitous Computing, Electronics & Mobile Communication Conference (UEMCON) pp 1-6.

[4] Safarian C, Ogunfunmi T and Kozacky W J 2015 FPGA implementation of LMS-based FIR adaptive filter for real time digital signal processing applications IEEE International Conference on Digital Signal Processing (DSP) pp 1251-1255.

[5] Yoon J M, Bahn W and Kim T I 2017 Discrete derivative method for adaptive notch filter-based frequency estimators International Journal of Control, Automation and Systems 15 (2) 668-679.

[6] Venkatesan C, Karthigaikumar P and Varatharajan R 2019 FPGA implementation of modified error normalized LMS adaptive filter for ECG noise removal Cluster Computing 22 (5) 12233-12241.

[7] Quan X, Dou X and Wu Z 2017 A concise discrete adaptive filter for frequency estimation under distorted three-phase voltage IEEE Transactions on Power Electronics 32 (12) 9400-9412.

[8] Cho J, Baek H J and Park B Y 2018 Variable step-size sign subband adaptive filter with subband filter selection Signal Processing 152 141-147.

[9] Yu Y and Zhao H 2017 Novel combination schemes of individual weighting factors sign subband adaptive filter algorithm International Journal of Adaptive Control and Signal Processing 31 (8) 1193-1204.