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On a new method for calculation of the number of prime numbers in the given interval nariman.sabziyev@gmail.com

Abstract. Definition of the number of prime numbers in the interval.

Key words. Prime numbers.

For describing the behavior of the distribution function of prime numbers it is necessary to investigate some auxiliary classes of natural numbers. The given paper is devoted to investigation of some subclasses of natural numbers, that allows to describe the distribution function of prime numbers [1], [2].

1. Denotation and necessary facts ([3], [4]).

Denote by $\mathbb{N}$ the set of all natural numbers, by $\mathbb{N}_{\leq n}$ the set of natural numbers not exceeding $n$.

It is obvious that the set $\mathbb{N}_{\geq 5}$ may be represented in the form

$$\mathbb{N}_{\geq 5} = \{5, 6, 7, \ldots, n, \ldots\} = \{6m - 1, 6m, 6m + 1, 6m + 2, 6m + 3, 6m + 4\}_{m=1}^{\infty} = \{6m + \alpha, \alpha = -1, 0, 1, 2, 3, 4\}_{m=1}^{\infty}. \quad (1.1)$$

From (1.1) it is seen that the natural numbers of the form $6m$, $6m + 2$, $6m + 3$ and $6m + 4$ for any natural value of $m$ are composite numbers; and only natural numbers $6m - 1$ and $6m + 1$ ($m \in \mathbb{N}$) may be also prime numbers.

Therefore we have [3]

**Theorem 1.1.** If $n$ is a prime natural number, then it is necessary that $n$ should have the form $n = 6m - 1$ or $n = 6m + 1$.

If the number $6m + 1$ (or $6m - 1$) is a composite number, then at least it has two cofactors, i.e.

$$6m \pm 1 = (6i + \alpha)(6j + \beta) \quad (1.2)$$

$$\alpha \text{ and } \beta = -1, 0, 1, 2, 3, 4. \quad (1.3)$$

From (1.2) we have

$$6m \pm 1 = 6ij + \beta 6i + \alpha 6j + \alpha \beta = 6m_0 + \alpha \beta. \quad (1.4)$$

By the prime calculations from (1.4) we get

$$\alpha \beta = \pm 1, \text{ where } \alpha \text{ and } \beta \quad (1.5)$$

were determined in (1.3).

Taking into account (1.5), from (1.2) we get

$$6m + 1 = (6i \pm 1)(6j \pm 1) \quad i, j \in \mathbb{N}, \quad i \geq j, \quad (1.6)$$

$$6m - 1 = (6i \pm 1)(6j \mp 1) \quad i, j \in \mathbb{N}, \quad i \geq j. \quad (1.7)$$
From (1.6) we get
\[ m = 6ij \pm (i + j), \quad (i \geq j) \]  
(1.8)

from (1.7) we have
\[ m = 6ij \pm (i - j), \quad (i \geq j). \]  
(1.9)

Denote the set of natural numbers of the form (1.6) and (1.7) by \( M_1 \) and \( M_2 \), respectively. Then we have

**Theorem 1.2.** For the natural number of the form \( 6m + 1 \) (or \( 6m - 1 \)), \( m \in \mathbb{N} \) to be a composite natural number, it is necessary and sufficient that \( m \in M_1 \) (or \( m \in M_2 \)).

Indeed, if \( m \in M_1 \) then \( m = 6ij \pm (i + j) \). Hence \( 6m + 1 = 36ij \pm 6(i + j) + 1 = (6i \pm 1)(6j \pm 1) \) and vice versa, if \( 6m + 1 \) is a composite number, then \( 6m + 1 = (6i \pm 1)(6j \pm 1) \), hence it follows that \( m = 6ij \pm (i + j) \in M \).

Denote by \( H_1 = N \setminus M_1 \) and \( H_2 = N \setminus M_2 \), where \( H_1 \cap M_1 \neq \emptyset \) and \( H_2 \cap M_2 \neq \emptyset \).

Then we have

**Theorem 1.3.** For the natural number of the form \( 6m + 1 \) (or \( 6m - 1 \)) to be a prime natural number, it is necessary and sufficient that \( m \in H_1 \) (or \( m \in H_2 \)).

2. The property of the set \( M_1 (\leq m) \).

By definition, if \( n \in M_1 (\leq m) \), then \( n = 6ij \pm (i + j) \leq m, \ i, j \in \mathbb{N} \).

Note that if \( m = \max(6ij - i - j) \), then there exists a natural number \( \nu \) for \( i = j = \nu, \ 6\nu^2 - 2\nu = m \), hence \( \nu = \left\lfloor \frac{1 + \sqrt{6m + 1}}{6} \right\rfloor \) i.e. \( 1 \leq i \leq \nu, \) and if \( m = \max(6ij + i + j) \), then for \( i = j = k, \ 6k^2 + 2k = m \), hence \( k = \left\lfloor -\frac{1 + \sqrt{6m + 1}}{6} \right\rfloor \) and \( 1 \leq j \leq k \).

Denote by
\[ A_i(m) = \{6ij - i - j\} = \{(6i - 1)j - i \leq m, \ i \leq j, \ i, j \in \mathbb{N}\} \]
\[ B_j(m) = \{6ij + i + j\} = \{(6j + 1)j + i \leq m, \ i \leq j, \ i, j \in \mathbb{N}\} \]
\[ 1 \leq i \leq \nu, \ 1 \leq j \leq k \]

where \( 6i - 1 \) and \( 6j + 1 \) are only prime numbers.

Call \( A_i(m) \) and \( B_j(m) \) the subclasses with prime coefficients of the set \( M_1 (\leq m) \).

It should be noted that
\[ M_1 (\leq m) = \left( \bigcup_{i=1}^{\nu} A_i(m) \right) \cup \left( \bigcup_{i=1}^{k} B_j(m) \right). \]

Denote the set of prime coefficients of the subclasses \( A_i(m) \) and \( B_j(m) \), by \( K_1(-) \) and \( K_1(+) \):
\[ K_1(-) = \{5, 11, 17, \ldots, 6\nu - 1\} = \left\{ K_1^{(1)}(-), K_1^{(2)}(-), \ldots, K_1^{(\nu)}(-) \right\}, \]
i.e.
\[ K_1^{(1)}(-) = 5, \quad K_1^{(2)}(-) = 11, \quad K_1^{(3)}(-) = 17, \ldots, K_1^{(\nu)}(-) = 6\nu - 1 \]
respectively,
\[ K_1(+) = \{7, 13, 19, \ldots, 6k + 1\} = \left\{ K_1^{(1)}(+), K_1^{(2)}(+), \ldots, K_1^{(k)}(+) \right\}, \]
i.e.
\[ K_1^{(1)}(+) = 7, \quad K_1^{(2)}(+) = 13, \ldots, \quad K_1^{(k)}(+) = 6k + 1 \]
where \(6i - 1\) and \(6j + 1\) are only prime numbers and the number of the elements of the set \(K_2(-)\) equals

\[
\nu_2(-) = C_{\nu_0}^1 \cdot C_{k_0}^1.
\]

Denote by \(K_2(-)\) the set with the elements of the form \(6\tau - 1 (\tau \in N)\) being the product of two elements of the set \(K_1(-) \cup K_1(+):\)

\[
K_2(-) = \{5 \cdot 7, 5 \cdot 13, ..., 11 \cdot 7, 11 \cdot 13, ..., (6\nu - 1) (6k + 1)\} =
= \left\{K_2^{(1)}(-), K_2^{(2)}(-), ..., K_2^{(\gamma_2(-))}(-)\right\},
\]

where

\[
K_2^{(1)}(-) = 5 \cdot 7, K_2^{(2)}(-) = 5 \cdot 13, ...
\]

Here \(\nu_0\) is the number of prime elements of the set \(K_1(-), k_0\) is the number of prime elements of the set \(K_1(+).\) Denote by \(K_2(+)\) the set with the elements of the form \(6\tau + 1 (\tau \in N)\) being the product of two elements of the set \(K_1(-) \cup K_1(+):\)

\[
K_2(+) = \{5 \cdot 11, 5 \cdot 17, ..., 7 \cdot 13, 7 \cdot 19, ...\} =
= \left\{K_2^{(1)}(+), K_2^{(2)}(+), ..., K_2^{(\gamma_2(+))}\right\},
\]

where the number of the elements of the set \(K_2(+)\) equals \(\gamma_2(+) = V_{\nu_0}^2 + C_{k_0}^2\)

\[
K_2(+) = 5 \cdot 11, K_2^{(2)}(+) = 5 \cdot 17, ..., K_2^{(3)}(+) = 7 \cdot 13, ...
\]

The set \(K_q(-)\) and \(K_q(+),\) where \(2 \leq i + j = q, 1 \leq i \leq \nu, 1 \leq j \leq k\) is determined in the same way.

Now we can calculate the number of the elements of subclasses of the set \(M_1(\leq m):\)

\[
\text{mes}A_1(\leq m) = \text{mes}\{5t - 1 \leq m\} = \left\lfloor \frac{m + 1}{5} \right\rfloor =
= \left\lfloor \frac{m + \frac{K_1^{(1)}(-) + 1}{6}}{K_1^{(1)}(-)} \right\rfloor = \left\lfloor \frac{6m + K_1^{(1)}(-) + 1}{6K_1^{(1)}(-)} \right\rfloor \tag{2.1}
\]

\[
\text{mes}A_2(\leq m) = \text{mes}\{11t - 2 \leq m\} = \left\lfloor \frac{m + 2}{11} \right\rfloor =
= \left\lfloor \frac{m + \frac{K_1^{(2)}(-) + 1}{6}}{K_1^{(1)}(-)} \right\rfloor = \left\lfloor \frac{6m + K_1^{(2)}(-) + 1}{6K_1^{(1)}(-)} \right\rfloor \tag{2.2}
\]

\[
\text{..........................}
\]

\[
\text{mes}A_\nu(\leq m) = \text{mes}\{(6\nu - 1) t - \nu \leq m\} = \left\lfloor \frac{m + \nu}{6\nu - 1} \right\rfloor =
= \left\lfloor \frac{6m + K_1^{(\nu)}(-) + 1}{6K_1^{(\nu)}(-)} \right\rfloor \tag{2.3}
\]

where \(K_1^{(1)}(-), K_1^{(2)}(-), ..., K_1^{(\nu)}(-)\) are the elements of the set \(K_1(-);\)

\[
\text{mes}B_1(\leq m) = \text{mes}\{7t + 1 \leq m\} = \left\lfloor \frac{m - 1}{7} \right\rfloor =
= \left\lfloor \frac{6m - K_1^{(1)}(-) + 1}{6K_1^{(1)}(+)} \right\rfloor \tag{2.4}
\]
\[
mesB_2 (\leq m) = \text{mes} \{ 13t + 2 \leq m \} = \left\lceil \frac{m - 2}{13} \right\rceil = \\
\left\lceil \frac{6m - K_2^{(2)}(+) + 1}{6K_2^2(+)} \right\rceil
\]
\hspace{1cm} (2.5)

\[
mesB_k (\leq m) = \text{mes} \{ (6k + 1) t + k \} = \left\lceil \frac{m - k}{6k + 1} \right\rceil = \\
\left\lceil \frac{6m - K_k^{(k)}(+) + 1}{6K_k^k(+)} \right\rceil
\]
\hspace{1cm} (2.6)

where \( K_1^{(1)} (+), K_1^{(2)} (+), \ldots, K_1^{(k)} (+) \) are the elements of the set \( K_1 (+) \).

Let \( m \in A_1 (m) \cap A_2 (m) \). Then \( m = 5t - 1 = 11\tau - 2, \ 5t = 11\tau - 1 \),
\( t = 2\tau + \frac{\tau + 1}{5} \), \( \tau = 5\tau_1 + 1 \), \( m = 5 \cdot 11\tau_1 + 9 \) and \( m = 5 \cdot 11\tau_0 - 46 (\tau = \tau_1 - 1) \).

Hence it is seen that the coefficient \( \tau \) represents the number of the form \( 6t + 1 \),
i.e. \( 5 \cdot 11 \) is a natural number of the form \( 6t + 1 \).

The number of the elements of the set \( A_1 (m) \cap A_2 (m) \) equals
\[
\left\lceil \frac{m + 46}{5 \cdot 11} \right\rceil = \left\lceil \frac{m + 5 \cdot 5 \cdot 11 + 1}{5 \cdot 11} \right\rceil = \left\lceil \frac{6m + 5 \cdot 5 \cdot 11 + 1}{6 \cdot 5 \cdot 11} \right\rceil.
\]
\hspace{1cm} (2.7)

And if \( m \in A_1 (m) \cap B_1 (m) \), then
\( 5t - 1 = 7\tau + 1, \ 5t = 7\tau + 2, \ t = \tau + 2 \frac{\tau + 1}{5}, \ \tau = 5\tau_1 - 1, \ m = 5 \cdot 7\tau_1 - 6 \),
hence it is seen that the coefficient \( \tau_1 \) represents the number of the form \( 6t - 1 \),
i.e. \( 5.7 \) is a natural number of the form \( 6t - 1 \); the number of the elements of the set \( A_1 (m) \cap B_1 (m) \) equals
\[
\left\lceil \frac{m + 6}{5 \cdot 7} \right\rceil = \left\lceil \frac{m + 5 \cdot 7 + 1}{5 \cdot 7} \right\rceil = \left\lceil \frac{6m + 5 \cdot 7 + 1}{6 \cdot 5 \cdot 7} \right\rceil.
\]

Similarly continuing (by induction), we find that the number of the elements of the set
\[
s (m) = \left( \bigcap_{s_1=0}^s A_{i_{s_1}} (m) \right) \cap \left( \bigcap_{r_1=0}^r B_{j_{r_1}} (m) \right), \ 2 \leq s_1 + r_1 \leq q, \ 2 \leq q \leq \nu + k
\]
we get
\[
mesS (m) = \left\lceil \frac{6m + \prod_{s_1=0}^s (6s_1 - 1) \prod_{r_1=0}^r (6r_1 + 1) + 1}{6 \prod_{s_1=0}^s (6s_1 - 1) \prod_{r_1=0}^r (6r_1 + 1)} \right\rceil,
\]
\hspace{1cm} (2.8)

where
\[
a = \begin{cases} 
1, & \text{if } s \text{ is an odd number}; \\
5, & \text{if } s \text{ is an even number},
\end{cases}
\]
\[
a \prod_{s_1=0}^0 (-1) = 1, \ 1 \leq i_{s_1} \leq \nu, \ 1 \leq j_{r_1} \leq k.
\]

If we denote the number of the composite numbers of the form \( 6t + 1 (t \in \mathbb{N}) \)
by \( P_2^{(+)} (6m + 1) \), then from (2.1)-(2.8) we have
Theorem 2.1. For the given $m \in \mathbb{N}$ the number of the elements of the set $M_1 (\leq m)$ (i.e. the number of the composite numbers of the form $6\tau+1 (\tau \in \mathbb{N})$ doesn’t exceed $6m+1$) equals

$$P^+ (6m+1) = \sum_{i=1}^{\nu} \left\lfloor \frac{6m + K_1^{(i)} (-) + 1}{6K_1^{(i)} (-)} \right\rfloor + \sum_{j=1}^{k} \left\lfloor \frac{6m - K_1^{(j)} (+) + 1}{6K_1^{(j)} (+)} \right\rfloor + \sum_{q=2}^{\nu+k} (-1)^{q-1} \left( \sum_{i=1}^{\nu} \left\lfloor \frac{6m + K_q^{(i)} (-) + 1}{6K_q^{(i)} (-)} \right\rfloor + \sum_{j=1}^{k} \left\lfloor \frac{6m + 5K_q^{(j)} (+) + 1}{6K_q^{(j)} (+)} \right\rfloor \right)$$

where

$$\gamma_2 (-) = C_1^1, C_1^2, \gamma_3 (-) = C_1^3C_2^3 + C_3^3, \ldots$$

$$\gamma_2 (+) = C_1^2 + C_1^3, \gamma_3 (+) = C_1^2C_2^3 + C_3^3, \ldots$$

Denote by $\pi^+ (6m+1)$ the number of prime numbers of the form $6\tau+1 (\tau \in \mathbb{N})$, then by $H_1 (\leq m) = N (\leq m) \setminus M_1 (\leq m)$ it holds

Theorem 2.2. For the given $m$, the number of prime numbers of the form $6\tau+1 (\tau \in \mathbb{N})$ not exceeding $6m+1$ (i.e. the number of the elements of the set $H_1 (\leq m)$) equals

$$\pi^+ (6m+1) = m - P^+$$

(2.10)

where $P^+ (6m+1)$ was determined in equality (2.9),

$$\nu = \left\lfloor \frac{1 + \sqrt{6m+1}}{6} \right\rfloor, \quad k = \left\lfloor \frac{-1 + \sqrt{6m+1}}{6} \right\rfloor.$$

Example 1. Let $m = 50$, then $\nu = 3$, $k = 2$

$$K_1 (-) = \{5, 11, 17\} = \left\{ K_1^{(1)} (-), K_1^{(2)} (-), K_1^{(3)} (-) \right\}, \quad i.e.$$

$$K_1^{(1)} (-) = 5, \quad K_1^{(2)} (-) = 11, \quad K_1^{(3)} (-) = 17.$$

$$K_1 (+) = \{7, 13\} = \left\{ K_1^{(1)} (+), K_1^{(2)} (+) \right\}, \quad i.e.$$

$$K_1^{(1)} (+) = 7, \quad K_1^{(2)} (-) = 13.$$

Then

$$\sum_{i=1}^{3} \left\lfloor \frac{m + K_i^{(i)} (-) + 1}{6K_i^{(i)} (-)} \right\rfloor + \sum_{j=1}^{2} \left\lfloor \frac{m - K_i^{(j)} (+) - 1}{6K_i^{(j)} (+)} \right\rfloor$$

$$= \left\lfloor \frac{m+1}{5} \right\rfloor + \left\lfloor \frac{m+2}{11} \right\rfloor + \left\lfloor \frac{m+3}{17} \right\rfloor + \left\lfloor \frac{m-1}{7} \right\rfloor + \left\lfloor \frac{m-2}{13} \right\rfloor. \quad (2.11)$$

Since

$$K_2 (-) = \{5 \cdot 7, 5 \cdot 13, 11 \cdot 7, 11 \cdot 13, 17 \cdot 7, 17 \cdot 13\}.$$

The elements $K_2 (-)$ have the form $6\tau-1$, and the number of the elements of the set $K_2 (-)$ equals $\gamma_2 (-) = C_2^3 \cdot C_2^3 = 6$, then

$$\sum_{i=1}^{\nu} \left\lfloor \frac{m + K_2^{(i)} (-) + 1}{6K_2^{(i)} (-)} \right\rfloor = \left\lfloor \frac{m+6}{5 \cdot 7} \right\rfloor + \left\lfloor \frac{m+11}{5 \cdot 13} \right\rfloor + \left\lfloor \frac{m+13}{11 \cdot 7} \right\rfloor + \left\lfloor \frac{m+24}{11 \cdot 13} \right\rfloor + \left\lfloor \frac{m+20}{7 \cdot 11} \right\rfloor + \left\lfloor \frac{m+37}{13 \cdot 17} \right\rfloor. \quad (2.12)$$

and

$$K_2 (+) = \{5 \cdot 11, 5 \cdot 17, 11 \cdot 17, 7 \cdot 13\}.$$
the elements $K_2^+ (\pm)$ have the form $6\tau + 1$, and the number of the elements of the set $K_2^+ (\pm)$ equals $\gamma_2 (\pm) = C_p^2 + C_p^3 = 4$

\[
\sum_{i=1}^{\gamma_2(\pm)} \left[ m + \frac{6K_2^{(i)}(\pm)+1}{K_2^{(i)}(\pm)} \right] = \left[ m + 46 \right] + \left[ m + 71 \right] + \left[ m + 156 \right] + \left[ m + 76 \right] + \left[ \frac{m + 258}{7 \cdot 11 \cdot 13} \right]. \tag{2.13}
\]

Further,

$$K_3 (-) = \{5 \cdot 11 \cdot 17, 5 \cdot 7 \cdot 13, 11 \cdot 7 \cdot 13, 17 \cdot 7 \cdot 13\},$$

whose elements have the form $6\tau - 1$, and the number of the elements $K_3^+ (\pm)$ equals $\gamma_3 (\pm) = C_v^3 + C_v^1 \cdot C_k = 4$

\[
\sum_{i=1}^{\gamma_3(\pm)} \left[ m + \frac{6K_3^{(i)}(\pm)+1}{K_3^{(i)}(\pm)} \right] = \left[ m + 321 \right] + \left[ m + 496 \right] + \left[ m + 921 \right] + \left[ \frac{m + 2026}{11 \cdot 13 \cdot 17} \right]. \tag{2.14}
\]

Since

$$K_3^+ (\pm) = \{5 \cdot 11 \cdot 7, 5 \cdot 17 \cdot 7, 11 \cdot 17 \cdot 7, 5 \cdot 11 \cdot 13, 5 \cdot 17 \cdot 13, 11 \cdot 17 \cdot 13\}.$$

The elements $K_3^+ (\pm)$ have the form $6\tau + 1$, and the number of the elements equals $\gamma_3^+ (\pm) = C_v^2 \cdot C_k^2 = 6$

\[
\sum_{j=1}^{\gamma_3^+} \left[ m + \frac{5K_3^{(j)}(\pm)+1}{K_3^{(j)}(\pm)} \right] = \left[ m + 321 \right] + \left[ m + 496 \right] + \left[ m + 921 \right] + \left[ \frac{m + 2026}{11 \cdot 13 \cdot 17} \right]. \tag{2.15}
\]

For

$$K_4^+ (-) = \{5 \cdot 11 \cdot 17 \cdot 7, 5 \cdot 11 \cdot 17 \cdot 13\},$$

the elements $K_4^+ (\pm)$ have the form $6\tau - 1$, and the number of the elements equals $\gamma_4 (\pm) = C_v^1 \cdot C_k^2 = 2$

\[
\sum_{i=1}^{\gamma_4(\pm)} \left[ m + \frac{6K_4^{(i)}(\pm)+1}{K_4^{(i)}(\pm)} \right] = \left[ m + 1091 \right] + \left[ m + 921 \right] + \left[ \frac{m + 2026}{11 \cdot 13 \cdot 17} \right]. \tag{2.16}
\]

For

$$K_4^+ (\pm) = \{5 \cdot 11 \cdot 7 \cdot 13, 5 \cdot 17 \cdot 7 \cdot 13, 11 \cdot 17 \cdot 7 \cdot 13\},$$

the elements $K_4^+ (\pm)$ are of the form $6\tau + 1$, and the number of the elements equals $\gamma_4 (\pm) = C_v^2 \cdot C_k^3 = 3$

\[
\sum_{j=1}^{\gamma_4^+} \left[ m + \frac{5K_4^{(j)}(\pm)+1}{K_4^{(j)}(\pm)} \right] = \left[ m + 4171 \right] + \left[ \frac{m + 6446}{5 \cdot 7 \cdot 11 \cdot 13} \right] + \left[ \frac{m + 1418}{7 \cdot 11 \cdot 13 \cdot 17} \right]. \tag{2.17}
\]

$$K_5^+ (-) = \{5 \cdot 11 \cdot 17 \cdot 7 \cdot 13\}, \; \gamma_5^+ (\pm) = C_v \cdot C_k^3 = 1.$$
and the number of the elements equals
\[
\left\lfloor \frac{m + 14181}{5 \cdot 7 \cdot 11 \cdot 13 \cdot 17} \right\rfloor
\]
(2.18)
and
\[
K_5(+) \equiv \emptyset.
\]
Thus, taking into account equalities (2.11)-(2.18), we get
\[
P^{(+)}(301) = 22
\]
(2.19)
and from (2.10) we have
\[
\pi^{(+)}(6 \cdot 50 + 1) = \pi^{(+)}(306) = 50 - 22 = 28.
\]
(2.20)

3. Property of the set \( M_2(\leq m) \).

By definition, if \( n \in M_2(\leq m) \) then
\[
n = 6it - i + t \leq m, \quad i \leq t, \quad i, t \in \mathbb{N}
\]
or
\[
n = 6jt + j - t \leq m, \quad j \leq t, \quad j, t \in \mathbb{N}.
\]
where \( m = \max \{6it - i + t\} \) or \( m = \max \{6jt + j - t\} \). Then there exists a natural number \( r \), for \( i = j = t = r \) we have \( 6r^2 = m \), hence we get
\[
r = \left\lfloor \sqrt{\frac{6m}{6}} \right\rfloor,
\]
where \( 1 \leq i \leq r, \ 1 \leq j \leq r \).
Denote by
\[
C_i(m) = \{(6i - 1)t + i \leq m, \quad i \leq t, \quad i, t \in \mathbb{N}, \ 1 \leq i \leq r\}
\]
and
\[
D_j(m) = \{(6j - 1)t - i \leq m, \quad j \leq t, \quad j, t \in \mathbb{N}, \ 1 \leq j \leq r\}
\]
where \( 6i - 1 \) and \( 6j + 1 \) are only prime numbers.
Call \( C_i(m) \) and \( D_j(m) \) the subclasses with prime coefficients of the set \( M_2(\leq m) \). Obviously,
\[
M_2(\leq m) = \left( \bigcup_{i=1}^{r} C_i(m) \right) \cup \left( \bigcup_{j=1}^{r} D_j(m) \right).
\]

Denote by \( K_1^{(1)}(-) \) and \( K_2^{(1)}(+) \) the set of prime coefficients of the subclasses \( C_i(m) \) and \( D_j(m) \) and the set \( M_2(\leq m) \):
\[
K_1(-) = \{5, 11, 17, ..., 6r - 1\},
\]
\[
K_1(+) = \{7, 13, 19, ..., 6r + 1\},
\]
where the elements of the set \( K_1(-) \cup K_1(+) \) are only prime numbers.
As in the set \( M_1(\leq m) \), here we also determine the set
\[
K_2(-), \ K_2(+), \ K_3(-), \ K_3(+), ..., K_q(-) \text{ and } K_q(+),
\]
and calculate the number of the elements of the subclasses \( C_i(m) \) and \( D_j(m) \)
\[
mes(C_1(\leq m)) = mes(5t + 1 \leq m) = \left\lfloor \frac{m - 1}{5} \right\rfloor = \left\lfloor \frac{6m - K_1^{(1)}(-) - 1}{6K_1^{(1)}(-)} \right\rfloor,
\]
\[
\begin{align*}
\text{mes}(C_2 \leq m) &= \text{mes}(11t + 2 \leq m) = \left[ \frac{m - 2}{11} \right] = \left[ \frac{6m - K_1^{(2)} (-) - 1}{6K_1^{(2)} (-)} \right], \\
\text{mes}(D_1 \leq m) &= \text{mes}(7t - 1 \leq m) = \left[ \frac{m + 1}{7} \right] = \left[ \frac{6m + K_1^{(1)} (+) - 1}{6m} \right], \\
\text{mes}(D_2 \leq m) &= \text{mes}(13t - 2 \leq m) = \left[ \frac{m + 2}{13} \right] = \left[ \frac{6m + K_1^{(2)} (+) - 1}{6m} \right],
\end{align*}
\]

and etc.

If
\[
R(m) = \left( \bigcap_{s_1=0}^{s} C_{i,s_1}(m) \right) \cap \left( \bigcap_{r_1=0}^{r} D_{j,r_1} \right),
\]

\[
t = s_1 + r_1 = q \leq s + r
\]

then
\[
\text{mes} R(m) = \left[ \frac{6m + b \prod_{s_1=0}^{s} (6s_1 - 1) \prod_{r_1=0}^{r} (6r_1 + 1)}{6 \prod_{s_1=0}^{s} (6s_1 - 1) \prod_{r_1=0}^{r} (6r_1 + 1)} \right]
\]

where
\[
b = \begin{cases} 
1, & \text{if } s \text{ is an even number} \\
5, & \text{if } s \text{ is an odd number} 
\end{cases}
\]

\[
0 \prod_{i=1}^{1} (-1) = 1, \quad 1 \leq i, \quad j \leq r
\]

where \( K_1^{(1)} (+), \ K_1^{(1)} (-), \ K_1^{(2)} (+), \ K_1^{(2)} (-) \ldots \) is determined as in calculating the number of the elements of subclasses of the set \( M_1 (\leq m) \).

Denote by \( P^{(-)} (6m - 1) \) the number of composite numbers of the form \( 6\tau - 1 \) \( (\tau \in N) \) not exceeding \( 6m - 1 \), then we have

**Theorem 3.1.** For the given \( m \in N \), the number of the elements of the set \( M_2 (\leq m) \) (i.e. the number of composite numbers of the form \( 6\tau - 1 \) \( (\tau \in N) \) not exceeding \( 6m - 1 \)) equals

\[
P^{(-)} (6m - 1) = \sum_{i=1}^{r} \left[ \frac{6m + K_1^{(i)} (-) + 1}{6K_1^{(i)} (-)} \right] + \sum_{j=1}^{r} \left[ \frac{6m - K_1^{(j)} (+) + 1}{6K_1^{(j)} (+)} \right] + \sum_{q=2}^{r+r} (-1)^{q-1} \left( \gamma_q^{(-)} \left[ \frac{6m + 5K_1^{(i)} (-) + 1}{6K_1^{(i)} (-)} \right] \right) \gamma_q^{(+)} \left[ \frac{6m + K_1^{(j)} (+) - 1}{6K_1^{(j)} (+)} \right]
\]

where \( \gamma_q^{(-)} \) and \( \gamma_q^{(+)} \) is determined as in theorem 2.1.

Denote by \( \pi^{(-)} (6m - 1) \) the number of prime numbers of the form \( 6\tau - 1 \) \( (\tau \in N) \) not exceeding \( 6m - 1 \), then by

\[
H_2 (\leq m) = N (\leq m) \setminus M_2 (\leq m)
\]

it holds

**Theorem 3.2.** For the given \( m \in N \) the number of prime numbers of the form \( 6\tau - 1 \) \( (\tau \in N) \) not exceeding \( 6m - 1 \) (i.e. the number of the elements of the set \( H_2 (\leq m) \)) equals

\[
\pi^{(-)} (6m - 1) = m - P^{(-)} (6m - 1), \quad (3.2)
\]
where $P^{(-)}(6m - 1)$ is determined as equality (3.1), $r = \left\lfloor \sqrt{6m} \right\rfloor$.

**Example 2.** Let $m = 50$, then $r = 2$ and

$$K(-) = \{5, 11\}, \quad K(+) = \{7, 13\}$$

i.e.

$$K_1^1(-) = 5, \quad K_1^{(2)}(-) = 11, \quad K_1^{(1)}(+) = 7, \quad K_1^{(2)}(+) = 13.$$ 

Then from (3.1) we have

$$P^{(-)}(301) = \left[ \frac{50 - 1}{5} \right] + \left[ \frac{50 - 2}{11} \right] + \left[ \frac{50 + 1}{7} \right] + \left[ \frac{50 + 2}{13} \right] -$$

$$\left( - \left[ \frac{50 + 29}{5 \cdot 7} \right] + \left[ \frac{50 + 54}{5 \cdot 13} \right] + \left[ \frac{50 + 64}{7 \cdot 11} \right] + \left[ \frac{50 + 119}{11 \cdot 13} \right] + \left[ \frac{50 + 9}{5 \cdot 11} \right] + \left[ \frac{50 + 15}{7 \cdot 13} \right] \right) + \left( \left[ \frac{50 + 327}{5 \cdot 7 \cdot 13} \right] + \left[ \frac{50 + 834}{11 \cdot 7 \cdot 13} \right] \right) +$$

$$\left[ \frac{50 + 64}{5 \cdot 7 \cdot 11} \right] + \left[ \frac{50 + 119}{5 \cdot 11 \cdot 13} \right] - \left[ \frac{50 + 834}{5 \cdot 7 \cdot 11 \cdot 13} \right] =$$

$$= (9 + 4 + 7 + 4) - (2 + 1 + 1 + 1 + 1) = 24 - 6 = 18$$

$$P^{(-)}(6m - 1) = P^{(-)}(301) = 18$$

i.e.

$$\pi^{(-)}(6m - 1) = \pi^{(-)}(299) = 50 - 18 = 32.$$

4. Calculation of the number of prime numbers not exceeding $6m + 1$.

Denote by $\pi(6m + 1)$ the number of prime numbers not exceeding $6m + 1$.

Then from theorems 2.2 and 3.2 we have

**Theorem 4.** The number of prime numbers not exceeding $6m + 1$ (except 2 and 3) equals

$$\pi(6m + 1) = \pi^{(+)}(6m + 1) + \pi^{(-)}(6m + 1) =$$

$$= 2m - \left( P^{(+)}(6m + 1) + P^{(-)}(6m - 1) \right).$$

**References**

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