Constraining the early-Universe baryon density and expansion rate

Vimal Simha\textsuperscript{1} and Gary Steigman\textsuperscript{2,3}

\textsuperscript{1} Department of Astronomy, The Ohio State University, 140 West 18th Avenue, Columbus, OH 43210, USA
\textsuperscript{2} Departments of Physics and Astronomy, The Ohio State University, 191 West Woodruff Avenue, Columbus, OH 43210, USA
\textsuperscript{3} Center for Cosmology and Astro-Particle Physics, The Ohio State University, 191 West Woodruff Avenue, Columbus, OH 43210, USA
E-mail: vsimha@astronomy.ohio-state.edu and steigman@mps.ohio-state.edu

Received 16 April 2008
Accepted 16 May 2008
Published 18 June 2008

Online at stacks.iop.org/JCAP/2008/i=06/a=016
doi:10.1088/1475-7516/2008/06/016

Abstract. We explore the constraints on those extensions to the standard models of cosmology and particle physics which modify the early-Universe, radiation dominated, expansion rate $S \equiv H'/H$ (parameterized by the effective number of neutrinos $N_\nu$). The constraints on $S(N_\nu)$ and the baryon density parameter $\eta_B \equiv (n_B/n_\gamma) = 10^{-10} \eta_{10}$, derived from big bang nucleosynthesis (BBN, $t \sim 20$ min), are compared with those inferred from the cosmic microwave background anisotropy spectrum (CMB, $t \sim 400$ kyr) and large scale structure (LSS, $t \sim 14$ Gyr). At present, BBN provides the strongest constraint on $N_\nu$ ($N_\nu = 2.4 \pm 0.4$ at 68% confidence), but a weaker constraint on the baryon density. In contrast, while the CMB/LSS data best constrain the baryon density ($\eta_{10} = 6.1^{+0.2}_{-0.1}$ at 68% confidence), independent of $N_\nu$, at present they provide a relatively weak constraint on $N_\nu$ which is, however, consistent with the standard value of $N_\nu = 3$. When the best fit values and the allowed ranges of these CMB/LSS-derived parameters are used to calculate the BBN-predicted primordial abundances, there is excellent agreement with the observationally inferred abundance of deuterium and good agreement with $^4$He, confirming the consistency between the BBN and CMB/LSS results. However, the BBN-predicted abundance of $^7$Li is high, by a factor of 3 or more, if its observed value is uncorrected for possible dilution, depletion, or gravitational settling. We comment on the relation between the value of $N_\nu$ and a possible anomaly in the matter power spectrum inferred from observations of the Ly-$\alpha$ forest. Comparing our BBN and CMB/LSS results permits us to constrain any post-BBN entropy changes.

©2008 IOP Publishing Ltd and SISSA
Constraining the early-Universe baryon density and expansion rate

production as well as the production of any non-thermalized relativistic particles. The good agreement between our BBN and CMB/LSS results for $N_{\nu}$ and $\eta_B$ permits us to combine our constraints finding, at 95% confidence, $1.8 < N_{\nu} < 3.2$ and $5.9 < \eta_B < 6.4$.

**Keywords:** cosmological neutrinos, big bang nucleosynthesis, baryon asymmetry

**ArXiv ePrint:** 0803.3465

## Contents

1. Introduction 2
2. Non-standard early-Universe expansion rate or radiation density 3
3. $N_{\nu}$ and $\eta_B$ from BBN
   3.1. Observationally inferred primordial abundances 6
   3.2. BBN constraints on $N_{\nu}$ and $\eta_B$ 7
4. $N_{\nu}$ and $\eta_B$ from the CMB and LSS
   4.1. Analysis and data sets 10
   4.2. $N_{\nu}$ and the Lyman-$\alpha$ forest 13
   4.3. CMB and LSS constraints on $N_{\nu}$ and $\eta_B$ 13
5. Comparing the BBN and CMB/LSS constraints 16
6. Conclusions 18
    **Acknowledgments** 20
    **References** 21

### 1. Introduction

The standard models of particle physics and of cosmology with dark energy, baryonic matter, radiation (including three species of light neutrinos), and dark matter is consistent with cosmological data from several widely separated epochs. However, there is room to accommodate some models of non-standard physics within the context of this well tested, $\Lambda$CDM cosmology. One possibility, explored here, is that of a non-standard expansion rate ($S \equiv H'/H$, where $H$ is the Hubble parameter) during the early, radiation dominated evolution of the Universe driven perhaps, but not necessarily, by a non-standard content of relativistic particles ($\rho'_R \neq \rho_R$), parameterized by the equivalent number of additional neutrinos ($\rho'_R \equiv \rho_R + \Delta N_{\nu} \rho_{\nu}$, where $\Delta N_{\nu} \equiv N_{\nu} - 3$ prior to the epoch of $e^\pm$ annihilation). In the epoch just prior to $e^\pm$ annihilation, which is best probed by big bang nucleosynthesis (BBN), $\rho_R = \rho_\gamma + \rho_e + 3\rho_{\nu} = 43 \rho_{\gamma}/8$, so that

\[
S^2 \equiv \left( \frac{H'}{H} \right)^2 \equiv \frac{\rho'_R}{\rho_R} \equiv 1 + \frac{7 \Delta N_{\nu}}{43}. 
\]  

In the epoch after the completion of $e^\pm$ annihilation, best probed by the cosmic microwave background (CMB) and by large scale structure (LSS), the relations between $\rho_R$ and $\rho_\gamma$...
and between $S$ and $\Delta N_{\nu}$ differ from those in equation (1), as described below in some detail in section 2.

We use the comparison between the predicted and observed abundances of the light elements produced during BBN, and between the predicted and observed CMB anisotropy spectra, along with data from LSS observed in the present/recent Universe, to constrain new physics which leads to a non-standard, early-Universe expansion rate ($S$) or, equivalently, to place bounds on the effective number of neutrinos ($N_{\nu}$); for related earlier work, see e.g. [4,13]. In addition, the baryon density and any variation in it over widely separated epochs in the evolution of the Universe are constrained simultaneously with $N_{\nu}$, thereby testing the standard-model expectation that the ratio (by number) of baryons to CMB photons ($\eta_B \equiv n_B/n_\gamma$) should be unchanged from $e^\pm$ annihilation ($T \lesssim 1/2$ MeV; $t \gtrsim 3$ s) until the present ($T = 2.725$ K $\approx 2 \times 10^{-10}$ MeV; $t \approx 14$ Gyr).

In section 2 a non-standard expansion rate ($S$) is related to a non-standard radiation density, parameterized by an effective number of neutrinos $N_{\nu}$. In section 3, the observationally inferred primordial light element (D, $^4$He, $^7$Li) abundances are used to constrain the radiation density (expansion rate) and the baryon density. In section 4, we assume a flat, $\Lambda$CDM cosmology and use data from the CMB, along with a prior on the Hubble parameter from the HST key project, luminosity distances of type Ia supernovae, and the matter power spectrum to provide independent constraints on the radiation density and the baryon density. The results from these two epochs, very widely separated in time, are compared constraining any post-BBN entropy production. The good agreement between them permits us to combine them to obtain a joint constraint on the baryon density parameter ($\eta_B$) and the effective number of neutrinos ($N_{\nu}$).

2. Non-standard early-Universe expansion rate or radiation density

In the radiation dominated early Universe ($\rho_{\text{TOT}} \to \rho_R$) the expansion rate ($H$) is related to the radiation density through the Friedman equation,

$$H^2 = \frac{8\pi}{3}G\rho_R.$$  \hspace{1cm} (2)

Any modification to the radiation density or to the Friedman equation by a term which evolves like the radiation density (as the inverse fourth power of the scale factor) can be parameterized by an equivalent number of additional neutrinos $\Delta N_{\nu}$ where, prior to $e^\pm$ annihilation, $\Delta N_{\nu} \equiv N_{\nu} - 3$. For the standard models of particle physics and cosmology, in the epoch after muon annihilation ($T \lesssim 100$ MeV) and prior to $e^\pm$ annihilation ($T \gtrsim 0.5$ MeV), the radiation consists of an equilibrium mixture of photons, relativistic $e^\pm$ pairs, and three flavors of extremely relativistic, left-handed neutrinos (and their right-handed, antineutrino counterparts). In this case, the total radiation density may be written in terms of the photon density as

$$\rho_R = \frac{43}{8} \rho_\gamma = 5.375 \rho_\gamma.$$  \hspace{1cm} (3)

In this same epoch, prior to $e^\pm$ annihilation, a modified radiation density can be written as

$$\rho'_R = \rho_R \left(1 + \frac{7\Delta N_{\nu}}{43}\right) = \rho_R (1 + 0.163\Delta N_{\nu}),$$  \hspace{1cm} (4)
where $\rho_R$ is the standard-model radiation energy density and $\rho'_R$ is the modified, non-standard-model radiation energy density. In a sense, this modified energy density is simply a proxy for a non-standard expansion rate during the radiation dominated epoch relevant for comparison with BBN,

$$S \equiv \frac{H'}{H} = \left( \frac{\rho'_R}{\rho_R} \right)^{1/2} = \left( 1 + \frac{7\Delta N_\nu}{43} \right)^{1/2}. \quad (5)$$

After $e^\pm$ annihilation the surviving relativistic particles are the photons (which will redshift to the currently observed CMB) and the now decoupled, relic neutrinos. In the approximation that the neutrinos are fully decoupled at $e^\pm$ annihilation, the post-$e^\pm$ annihilation photons are hotter than the neutrinos by a factor of

$$T_\gamma / T_\nu = \left( \frac{4}{11} \right)^{1/3} \frac{1}{3} + 0.227 \Delta N_\nu,$$

so that

$$\rho'_R = \rho_\gamma (1 + 0.135\Delta N_\nu), \quad (6)$$

where, as before, $\Delta N_\nu \equiv N_\nu - 3$.

However, it is well known that the standard-model neutrinos are not fully decoupled at $e^\pm$ annihilation. As a result, the relic neutrinos share some of the energy/entropy released by $e^\pm$ annihilation and they are warmer than in the fully decoupled approximation, increasing the ratio of the post-$e^\pm$ annihilation radiation density to the photon energy density. While the post-$e^\pm$ annihilation phase space distribution of the decoupled neutrinos is no longer that of a relativistic, Fermi–Dirac gas, according to the additional contribution to the total energy density can be accounted for by replacing $N_\nu = 3$ with $3.046$, so that

$$\rho_R \to (1 + 3 \times 0.227)\rho_\gamma = 1.692\rho_\gamma. \quad (8)$$

For deviations from the standard model that can be treated as equivalent to contributions from fully decoupled neutrinos,

$$\rho'_R = \rho_R (1 + 0.134\Delta N_\nu), \quad (9)$$

where $\Delta N_\nu \equiv N'_\nu - 3.046$ in the post-$e^\pm$ annihilation Universe relevant for comparison with the CMB and LSS. Note that in the standard model, where $N_\nu = 3$ prior to $e^\pm$ annihilation, the neutrino contribution to the post-$e^\pm$ annihilation radiation energy density is equivalent to $N'_\nu = 3.046$, so that for the standard models of cosmology (standard expansion rate) and of particle physics (standard radiation energy density), $N_\nu = 3$, $N'_\nu = 3.046$ and $\Delta N_\nu = 0$.

We emphasize that although the non-standard radiation density (expansion rate) has been parameterized as if it were due to additional species of neutrinos, this parametrization accounts for any term in the Friedman equation whose energy density varies as $a^{-4}$, where $a$ is the scale factor. From this perspective, $\Delta N_\nu$ could either be positive or negative; the latter does not necessarily imply fewer than the standard-model number of neutrinos but could, for example, be a sign that the three standard-model neutrinos failed to be fully populated in the early Universe, or could reflect modifications to the 3 + 1-dimensional Friedman equations arising from higher dimensional extensions of the standard model of particle physics.
3. $N_\nu$ and $\eta_B$ from BBN

The stage is being set for BBN when the Universe is about a tenth of a second old and the temperature is a few mega-electron volts. At this time the energy density of the Universe is dominated by relativistic particles. When the temperature drops below $\sim 2$ MeV, the neutrinos begin to decouple from the photon–$e^\pm$ plasma. However, they do continue to interact with the neutrons and protons through the charged-current weak interactions ($n + \nu_e \leftrightarrow p + e^-, p + \bar{\nu}_e \leftrightarrow n + e^+$, $n \leftrightarrow p + e^- + \bar{\nu}_e$), maintaining the neutron-to-proton ratio at its equilibrium value of $n/p = \exp(-\Delta m/kT)$, where $\Delta m$ is the neutron–proton mass difference. When the temperature drops below $\sim 0.8$ MeV, and the Universe is $\sim 1$ second old, the reactions regulating the neutron–proton ratio become slower than the Universal expansion rate ($\Gamma_{wk} < H$). As a result, the neutron–proton ratio deviates from (exceeds) its equilibrium value, so that $n/p > \exp(-\Delta m/kT)$, and the actual $n/p$ ratio depends on the competition between the expansion rate ($H$) and the charged-current weak-interaction rate ($\Gamma_{wk}$), as well as on the neutron decay rate, $1/\tau_n$, where $\tau_n$ is the neutron lifetime.

Although nuclear reactions such as $n + p \leftrightarrow D + \gamma$ proceed rapidly during these epochs, the large $\gamma$-ray background (the blue-shifted CMB) ensures that the deuterium (D) abundance is very small, inhibiting the formation of more complex nuclei. The more complex nuclei begin to form only when $T \lesssim 0.08$ MeV, after $e^\pm$ annihilation, when the Universe is about 3 min old. At this time the number density of photons with sufficient energy to photodissociate deuterium is comparable to the baryon number density, and various two-body nuclear reactions can begin to build more complex nuclei. Note that the neutron-to-proton ratio has decreased slightly since ‘freeze-out’ (at $T \lesssim 0.8$ MeV) through the residual two-body reactions as well as via beta decay. Once BBN begins, neutrons and protons combine to form D, $^3$He, and $^4$He. The absence of a stable mass-5 nuclide presents a road-block to the synthesis of heavier elements in the expanding, cooling Universe, ensuring that the abundances of heavier nuclides are severely depressed below those of the lighter nuclei. In standard BBN (SBBN) only D, $^3$He, $^4$He, and $^7$Li are produced in astrophysically interesting abundances (for a recent review see [61]). While the BBN-predicted abundances of D, $^3$He, and $^7$Li are most sensitive to the baryon density, that of $^4$He is very sensitive to the neutron abundance when BBN begins and, therefore, to the competition between the weak-interaction rate and the Universal expansion rate. The primordial abundances of D, $^3$He, or $^7$Li are baryometers, constraining $\eta_B$, while the $^4$He mass fraction ($Y_P$) is a chronometer, depending mainly on $S$ or $N_\nu$. In the standard model of particle physics and cosmology with three species of neutrinos and their respective antineutrinos, the primordial element abundances depend on only one free parameter, the baryon density parameter, the post-$e^\pm$ ratio (by number) of baryons to photons, $\eta_B = n_B/n_\gamma$. This parameter may be related to $\Omega_B$, the present-Universe ratio of the baryon mass density to the critical mass–energy density (see [59])

$$\eta_{10} = 10^{10} \frac{n_B}{n_\gamma} = 273.9 \Omega_B h^2. \tag{10}$$

The abundance of $^4$He is very sensitive to the early expansion rate. Since a non-standard expansion rate ($S \neq 1$) would result in fewer or more neutrons at BBN and since most neutrons are incorporated into $^4$He, the predicted $^4$He abundance differs from that in SBBN ($S = 1; \Delta N_\nu = 0$). In contrast, the abundance of $^4$He is not very sensitive to the baryon density since, to first order, all the neutrons available at BBN are rapidly
converted to \(^4\)He. For \(\eta_{10} \approx 6\), \(N_\nu \approx 3\) and for a primordial \(^4\)He mass fraction in the range \(0.23 \lesssim Y_P \lesssim 0.27\), to a very good approximation \[32,61\],

\[Y_P = 0.2485 \pm 0.0006 + 0.0016[(\eta_{10} - 6) + 100(S - 1)].\]  

\(11\)

In equation (11), the effect of incomplete neutrino decoupling on the \(^4\)He mass fraction is accounted for according to \[39\] and \(S\) is related to \(\Delta N_\nu\) \((\Delta N_\nu \equiv N_\nu - 3.0)\) by equation (5). As a result, for a fixed \(^4\)He abundance, a variation in \(\eta_{10}\) of \(\sim \pm 0.2\) (corresponding to a \(\sim 3\%\) uncertainty in the baryon density) is equivalent to an uncertainty in \(\Delta N_\nu\) of \(\sim \pm 0.02\).

In contrast to the case for \(^4\)He, since the primordial abundances of \(^3\)He and \(^7\)Li are set by the competition between two-body production and destruction rates, they are more sensitive to the baryon density than to the expansion rate. For example, for \(\eta_{10} \approx 6\) and for a primordial ratio of \(D\) to \(H\) by number \(y_D \equiv 10^5(D/H)_P\) in the range \(2 \lesssim y_D \lesssim 4\), to a very good approximation \[32,61\],

\[y_D = 2.64(1 \pm 0.03) \left[\frac{6}{\eta_{10} - 6(S - 1)}\right]^{1.6} .\]  

\(12\)

The effect of incomplete neutrino decoupling on this prediction is at the \(~0.3\%\) level [39], about ten times smaller than the overall error estimate above.

### 3.1. Observationally inferred primordial abundances

Given the monotonic post-BBN evolution of deuterium (as gas is cycled through stars, deuterium is destroyed) and the significant dependence of its predicted BBN abundance on the baryon density \((y_{DP} \propto \eta_{10}^{-1.8})\), deuterium is the baryometer of choice among the light nuclides produced during primordial nucleosynthesis. While observations of \(D/H\) in the solar system and the local interstellar medium provide a lower limit to the relic deuterium abundance, it is the \(D/H\) ratio (by number) measured from observations of high redshift, low metallicity QSO absorption line systems which provide an estimate of its primordial abundance. The weighted mean of the six high redshift, low metallicity \(D/H\) ratios from \[31,47\] is [61]

\[y_{DP} = 2.68^{+0.27}_{-0.25}\]  

\(13\)

Since the post-BBN evolution of \(^3\)He is more complex and model dependent than that of deuterium and since \(^3\)He is only observed in chemically evolved H\(\text{II}\) regions in the Galaxy and since the \(^3\)He primordial abundance is only weakly dependent on the baryon density \((10^5(^3\text{He}/H) \equiv y_{3P} \propto \eta_{10}^{-0.6})\), its role as a baryometer is limited.

As for deuterium, the post-BBN evolution of \(^4\)He is monotonic, with \(Y_P\) increasing along with increasing metallicity. At low metallicity, the \(^4\)He abundance should approach its primordial value. As a result, it is the observations of helium and hydrogen recombination lines from low metallicity, extragalactic H\(\text{II}\) regions which are most useful in determining \(Y_P\). At present, corrections for systematic uncertainties (and their uncertainties) dominate estimates of the observationally inferred \(^4\)He primordial mass fraction and, especially, of its error. Following [61], we adopt for our estimate here

\[Y_P = 0.240 \pm 0.006 .\]  

\(14\)

While the central value of \(Y_P\) adopted here is low, the conservatively estimated uncertainty is relatively large (some ten times larger than the uncertainty in the BBN-predicted
abundance for a fixed baryon density). In this context, it should be noted that although very careful studies of the systematic errors in very limited samples of H II regions provide poor estimators of \( Y_p \) as a result of their uncertain and/or model dependent extrapolation to zero metallicity, they are of value in providing a robust upper bound to \( Y_p \). Using the results of [46, 21, 48], we follow [61] in adopting

\[
Y_p < 0.251 \pm 0.002. \tag{15}
\]

Although the BBN-predicted \( ^7 \)Li relic abundance provides a potentially sensitive baryometer ((Li/H) \( \propto \eta_{10}^2 \) for \( \eta_{10} \gtrsim 3 \)), its post-BBN evolution is complicated and model dependent. For these reasons, it is the observations of lithium in the oldest, most metal-poor stars in galactic globular clusters and in the halo of the Galaxy which have the potential to provide the best estimate of the primordial abundance of \( ^7 \)Li. The complication associated with this approach is that these oldest galactic stars have had the most time to dilute or deplete their lithium surface abundances, leading to the possibility that the observed abundances require large, uncertain, and model dependent corrections in order to infer the primordial abundance of \( ^7 \)Li. In the absence of corrections for depletion, dilution, or gravitational settling, the data of [56, 2] suggest

\[
\[\text{[Li]}\text{P} \equiv 12 + \log(\text{Li/H}) = 2.1 \pm 0.1. \tag{16}\]
\]

In contrast, in an attempt to correct for evolution of the surface lithium abundances, [34] use their observations of a small, selected sample of stars in the globular cluster NGC6397, along with stellar evolution models which include the effect of gravitational settling, to infer

\[
\[\text{[Li]}\text{P} = 2.54 \pm 0.1. \tag{17}\]
\]

In the following analysis, the inferred primordial abundances of D and \( ^4 \)He adopted here are used to estimate \( \eta_{10} \) and \( \Delta N_\nu \). Given the inferred best values and uncertainties in these two parameters, the corresponding BBN-predicted abundance of \( ^7 \)Li can be derived and compared to its observationally inferred abundance.

**3.2. BBN constraints on \( N_\nu \) and \( \eta_B \)**

Since the primordial abundance of deuterium is most sensitive to \( \eta_{10} \), while that of \( ^4 \)He is most sensitive to \( N_\nu \), isoabundance contours of D/H and \( Y_p \) in the \( \{\eta_{10}, \Delta N_\nu\} \) plane are very nearly orthogonal; see [32]. The analytic fits to BBN from [32], updated by [61], are used in concert with the primordial abundances of D and \( ^4 \)He adopted here to infer the best values, and to constrain the ranges of \( \eta_{10} \) and \( \Delta N_\nu \). While these fits do have a limited range of applicability, they are, in fact, accurate within their quoted uncertainties for the range of parameter values and observed abundances considered here.

In SBBN, with three species of neutrinos (\( \Delta N_\nu = 0 \)), the primordial abundances are only functions of the baryon density, \( \eta_B \). For SBBN, the primordial deuterium abundance adopted in section 3.1, \( y_{DP} = 2.68^{+0.27}_{-0.25} \), implies

\[
\eta_{10}(\text{SBBN}) = 6.0 \pm 0.4. \tag{18}\]

This result is in excellent agreement with the independent estimate of \( \eta_{10} = 6.1_{-0.1}^{+0.2} \) from the CMB and LSS (discussed below in section 4). The probability distributions of \( \eta_{10} \) inferred from SBBN and from the CMB and LSS are shown in figure 1.
Constraining the early-Universe baryon density and expansion rate

Figure 1. The probability distribution of the baryon density parameter, $\eta_{10}$. The dashed line shows the probability distribution inferred from SBBN ($N_\nu = 3$) and the adopted primordial abundance of deuterium (see section 3). The solid line is the probability distribution of $\eta_{10}$ inferred for $N_\nu = 3$ from the combination of the WMAP five-year data, small scale CMB data, matter power spectrum data from 2dFGRS and SDSS LRG, SNIa, and the HST Key Project (see section 4).

For non-standard BBN, with $\Delta N_\nu \neq 0$ ($S \neq 1$), there is a second free parameter, $N_\nu$ (or $S$). In this case, in addition to $y_{DP}$, the $^4$He abundance $Y_P$ is used to constrain the $\{\eta_{10}, N_\nu\}$ pair. Adopting the D and $^4$He abundances from section 3.1 ($Y_P = 0.240 \pm 0.006$), along with the analytic fits in equations (11) and (12), leads to

$$\eta_{10} = 5.7 \pm 0.4, \quad N_\nu = 2.4 \pm 0.4.$$ (19)

In figure 2 are shown the 68% and 95% contours in the $N_\nu$–$\eta_{10}$ plane which follow from a comparison of the BBN predictions with the observationally inferred primordial abundances of D and $^4$He. Notice that while the best fit value of $N_\nu$ is less than the standard-model value of $N_\nu = 3$, the standard-model value is consistent with the relic abundances at the $\sim$68% level.

These results are sensitive to the choices of the relic abundances of D and $^4$He. We note that if deuterium is ignored and the robust upper bound to the $^4$He mass fraction, $Y_P < 0.255$ at 95% confidence (equation (15)), is adopted, then equation (11) provides an upper limit to $S(N_\nu)$ as a function of the baryon density,

$$S < 1.10 - 0.01\eta_{10}. $$ (20)

For $\eta_{10}$ in the range $5 \leq \eta_{10} \leq 7$ (see section 4), this corresponds to a robust upper bound to $N_\nu$ ranging from 3.6 to 3.4.

4. $N_\nu$ and $\eta_B$ from the CMB and LSS

The pattern of temperature fluctuations in the cosmic microwave background contain information about the baryon density and the radiation density and thus serve as complementary probes of $\eta_B$ and $N_\nu$ some $\sim 10^5$ years after BBN.
Constraining the early-Universe baryon density and expansion rate

Figure 2. The 68% and 95% contours in the $N_\nu$-$\eta_{10}$ plane derived from a comparison of the observationally inferred and the BBN-predicted primordial abundances of D and $^4$He. The shaded region is excluded by the 95% upper bound to the helium abundance in equation (15) (see equation (20)).

The baryon density, parameterized by $\eta_B$ or $\Omega_B h^2$, affects the relative amplitudes of the peaks in the CMB temperature power spectrum. The ratios of the amplitudes of the odd peaks to the even peaks provide a determination of $\eta_B$ that is largely uncorrelated with $N_\nu$.

The radiation density, parameterized by an effective number of neutrino species $N_\nu$, affects the CMB power spectrum primarily through its effect on the epoch of matter–radiation equality. There are substantial differences in amplitudes between those scales that enter the horizon during the radiation dominated era and those that enter the horizon later, in the matter dominated era. Increasing the radiation content delays matter–radiation equality, bringing it closer to the epoch of recombination, suppressing the growth of perturbations. As a result, the redshift of the epoch of matter–radiation equality, $z_{eq}$, is a fundamental observable that can be extracted from the CMB power spectrum. $z_{eq}$ is related to the matter and radiation densities by

$$1 + z_{eq} = \frac{\rho_M}{\rho_R}. \quad (21)$$

Since $\rho_R$ depends on $N_\nu$, $z_{eq}$ is a function of both $N_\nu$ and $\Omega_M h^2$, leading to a degeneracy between these two parameters. For a flat universe, preserving the fit to the CMB power spectrum when $N_\nu$ increases requires that $\Omega_M$ and/or $H_0$ increase. As a result of this degeneracy, the CMB power spectrum alone imposes only a very weak constraint on $N_\nu$ [12, 52, 4, 26, 28]. Inclusion of additional, independent constraints on these parameters is needed to break the degeneracy between $N_\nu$ and $\Omega_M h^2$.

Besides affecting the epoch of matter–radiation equality, relativistic neutrinos leave a distinctive signature on the CMB power spectrum due to their free streaming at speeds exceeding the sound speed of the photon–baryon fluid. This free streaming creates neutrino anisotropic stresses generating a phase shift of the CMB acoustic oscillations.
in both temperature and polarization. This phase shift is unique and, for adiabatic initial conditions, cannot be generated by non-relativistic matter. In principle, this effect can be used to break the degeneracy between $N_\nu$ and $\Omega_M h^2$, leading to tighter constraints on $N_\nu$ [5].

Alternatively, since the luminosity distances of type Ia supernovae (SNIa) provide a constraint on a combination of $\Omega_M$ and $\Omega_\Lambda$ complementary to that from the assumption of flatness, they are of value in restricting the allowed values of $\Omega_M$. In concert with a bound on $H_0$, this, too, helps to break the degeneracy between $N_\nu$ and $\Omega_M h^2$.

Another way to break the degeneracy between $N_\nu$ and $\Omega_M h^2$ is to use measurements of the matter power spectrum in combination with the CMB power spectrum. To preserve a fit to the CMB power spectrum, an increase in $N_\nu$ requires that $\Omega_M h^2$ increase in order that the redshift of matter–radiation equality remain unchanged. The turnover scale in the matter power spectrum is set by this connection between $N_\nu$ and $\Omega_M h^2$. Since the baryon density is constrained by the CMB power spectrum, independently of $N_\nu$, increasing the radiation density ($N_\nu > 3$) requires a higher dark matter density in order to preserve $z_{\text{eq}}$ (in a flat universe, $\Omega_M + \Omega_\Lambda = 1$). Between the epoch of matter–radiation equality and recombination, the density contrast in the cold dark matter grows unimpeded, while the baryon density contrast cannot grow. Consequently, increasing $N_\nu$ and $\Omega_M h^2$, increases the amplitude of the matter power spectrum on scales smaller than the turnover scale corresponding to the size of the horizon at $z_{\text{eq}}$. Data from galaxy redshift surveys can be used to infer the matter power spectrum, thereby constraining $\Omega_M h^2$ and $N_\nu$. This effect may be seen in figure 3 which shows that for nearly indistinguishable CMB power spectra, different values of $N_\nu$ yield distinguishable matter power spectra. The upper panel of figure 3 shows that by making suitable adjustments to the other cosmological parameters, specifically the matter density and the spectral index, CMB power spectra which are nearly degenerate up to the third peak of the power spectrum can be produced using very different values of $N_\nu$. However, as the bottom panel of figure 3 illustrates, these models produce matter power spectra with different shapes, demonstrating that the matter power spectrum can be used to help break the degeneracy between $N_\nu$ and $\Omega_M h^2$.

4.1. Analysis and data sets

Before presenting our results, we describe the analysis and the data sets employed.

For our analysis we assume a flat, CDM cosmology with a cosmological constant, $\Lambda$, and three flavors of active neutrinos with negligible masses. Our cosmological model is parameterized by seven parameters:

$$p = \{\Omega_B h^2, \Omega_M h^2, h, \tau, n_S, A_S, N_\nu\}. \tag{22}$$

The contents of the Universe are described by the baryon density, $\Omega_B h^2$, and the matter (baryonic plus cold dark matter) density, $\Omega_M h^2$. Since a flat cosmology is assumed, the dark energy density and the matter density are related by $\Omega_\Lambda = 1 - \Omega_M$. The expansion rate of the Universe is described by the reduced Hubble parameter, $h$ ($H_0 = 100h$ km s$^{-1}$ Mpc$^{-1}$). Instantaneous reionization of the Universe is assumed with optical depth to last scattering, $\tau$. $A_S$ is the amplitude of scalar perturbations and $n_S$ is the scalar spectral index.

The code for anisotropies in the microwave background (CAMB) [36] is used to compute the CMB power spectrum for a fixed set of cosmological parameters. For a
Figure 3. The top panel shows the CMB power spectrum for the best fit models with $N_\nu$ fixed at $N_\nu = 1$ (solid black), $N_\nu = 3$ (dashed red), and $N_\nu = 5$ (dot-dashed blue) illustrating its insensitivity to $N_\nu$ in the absence of an independent constraint on $\Omega_M h^2$. The bottom panel shows the matter power spectra for the same set of parameter values, illustrating its sensitivity to $N_\nu$.

Given data set, our degree of belief in a set of cosmological parameters $\{p\}$ is quantified by the posterior probability distribution,

$$P(p|\text{data}) \propto L(\text{data}|p)\Pi(p).$$

The likelihood $L(\text{data}|p)$ quantifies the agreement between the data and the set of parameter values $\{p\}$. $\Pi(p)$ represents the prior on cosmological parameter values before the data are considered.

Markov chain Monte Carlo methods are used to explore the multi-dimensional likelihood surface. We use the publicly available COSMOMC code for our analysis [37]. Flat priors are adopted for all parameters, along with a prior on the age of the Universe, $t_0$, of $t_0 > 10$ Gyr.

The mode of the marginalized posterior probability distribution is used as a point estimate and the minimum credible interval as an estimate of the uncertainty. The
minimum credible interval selects the region of the parameter space around the mode, \( \hat{\theta} \), that contains the appropriate fraction of the volume (e.g., 68%, 95%) of the posterior probability distribution, while minimizing \( \hat{\theta} - \theta \). The minimum credible interval selects the region of the parameter space with the highest probability densities\(^4\).

Our primary data set is the CMB data from the Wilkinson microwave anisotropy probe (WMAP) accumulated from five years of observations [27, 45]. For the WMAP data, likelihoods are computed using the code provided on the LAMBDA webpage\(^5\). There are a number of ground-and balloon-based CMB experiments whose high angular resolutions probe scales smaller than those probed by WMAP. These are more sensitive to the higher order acoustic oscillations beyond the third peak in the CMB power spectrum. In particular, the 2008 results from the Arcminute Cosmology Bolometer Array Receiver (ACBAR) [55] impose the strongest constraints at present on the CMB power spectrum at small angular scales. In our analysis, data from the following CMB experiments are used: BOOMERANG [51, 44], ACBAR [55], CBI [54, 41], VSA [18], MAXIMA [24] and DASI [22].

To help break the degeneracy between \( N_\nu \) and \( \Omega_M h^2 \), we adopt a Gaussian prior on the Hubble parameter, \( H_0 = 72 \pm 8 \) km s\(^{-1}\) Mpc\(^{-1}\) from the Hubble Space Telescope Key Project [20].

Since the luminosity distances of type Ia supernovae (SNIa) provide a constraint on a combination of \( \Omega_M \) and \( \Omega_\Lambda \), they are of value in restricting the allowed values of \( \Omega_M \), helping to break the degeneracy between \( N_\nu \) and \( \Omega_M h^2 \). We use the luminosity distances for 115 type Ia supernovae measured by the Supernova Legacy Survey (SNLS) [3]. For each supernova, the observed luminosity distance is compared to that predicted for a given set of cosmological parameters.

We use the matter power spectrum inferred from galaxy redshift surveys such as the Sloan Digital Sky Survey (SDSS) [62, 63] and the 2dFGRS [10]. In using the matter power spectrum to infer cosmological parameters, it has become clear that the constraint on \( \Omega_M \) is sensitive to the choice of length scales on which the power spectrum is measured. The power spectrum on smaller scales favors higher values of \( \Omega_M \), provided that those scales are correctly described by linear perturbation growth and scale independent galaxy bias [10, 49]. For example, using the SDSS LRG power spectrum [63] in combination with the three-year WMAP data set [58], Dunkley \textit{et al} [19] find a disagreement between the matter density inferred on scales with \( k \leq 0.1h \) Mpc\(^{-1}\) and \( k \leq 0.2h \) Mpc\(^{-1}\). Hamann \textit{et al} [23] have shown that the constraint on \( N_\nu \) obtained from the matter power spectrum is affected similarly, resulting from the correlation between \( \Omega_M \) and \( N_\nu \). Therefore, here we only use matter power spectrum data on scales that are likely to be safely linear. We truncate the matter power spectrum at \( k = 0.07h \) Mpc\(^{-1}\), keeping data only on scales with \( k \leq 0.07h \) Mpc\(^{-1}\). We assume that galaxy bias is constant and scale independent for these scales.

\(^4\) The commonly used GETDIST analysis package uses the mean of the marginalized posterior probability distribution as a point estimate and gives uncertainty estimates based on the central credible interval [23]. These estimates are identical for Gaussian probability distributions but differ significantly for non-Gaussian distributions, particularly for asymmetric probability distributions.

\(^5\) http://lambda.gsfc.nasa.gov.
4.2. $N_{\nu}$ and the Lyman-\(\alpha\) forest

Measurements of the flux power spectrum of the Lyman-\(\alpha\) forest in QSO absorption spectra can be used to reconstruct the matter power spectrum on small scales \([11,42]\). Observations of the SDSS Ly-\(\alpha\) flux power spectrum have been used to constrain the linear matter power spectrum at \(z \sim 3\) \([43]\). When these constraints are combined with CMB power spectrum data, they favor values of \(N_{\nu}\) considerably higher than those we find here (section 5). For example, \([57]\) found \(N_{\nu} = 5.2\) and a 95\% range \(3.4 \leq N_{\nu} \leq 7.3\) and \([23]\) found \(N_{\nu} = 6.4\) and a 95\% range \(3.2 \leq N_{\nu} \leq 11\), the difference being accounted for by their different combinations of data sets. Each of these excludes the standard-model value of \(N_{\nu} = 3\) at more than 95\% confidence.

As discussed earlier and shown in the lower panel of figure 3, the principal effect of an increase in \(N_{\nu}\) is to increase the amplitude of the matter power spectrum on scales smaller than those corresponding to the horizon at matter–radiation equality, \(z_{\text{eq}}\). The \(\Lambda\) CDM fits to the Lyman-\(\alpha\) forest data favor higher amplitudes of density fluctuations on small scales compared to those expected from measurements of the WMAP power spectrum \([64]\), favoring higher values of \(N_{\nu}\). This effect is seen in figure 3 which shows that for nearly indistinguishable CMB power spectra, different values of \(N_{\nu}\) yield distinguishable matter power spectra. To preserve \(z_{\text{eq}}\) and the fit to the CMB, the model with \(N_{\nu} = 1\) has a lower value of \(\Omega_{M} h^2\) and, therefore, the matter power spectrum for that model has a lower amplitude on scales smaller than the scale corresponding to the horizon at the epoch of matter–radiation equality. Conversely, the model with \(N_{\nu} = 5\) has a higher value of \(\Omega_{M} h^2\) and the matter power spectrum for that model has a higher amplitude on scales smaller than the scale corresponding to the horizon at \(z_{\text{eq}}\).

Before reaching any conclusions about \(N_{\nu}\) based on the Lyman-\(\alpha\) forest data, it is worth noting that assumptions about the thermal state of the IGM play an important role in reconstructing the matter power spectrum from the Lyman-\(\alpha\) forest flux power spectrum. Bolton \textit{et al} \([8]\) compare measurements of the Lyman-\(\alpha\) forest flux probability distribution by Kim \textit{et al} \([30]\) to hydrodynamic simulations of the Lyman-\(\alpha\) forest, finding evidence for an \textit{inverted} temperature–density relation for the low density intergalactic medium. Bolton \textit{et al} \([8]\) suggest that He\II reionization could be a possible physical mechanism for achieving an inverted temperature–density relation. Such an inversion would result in a smaller amplitude of the matter power spectrum for a given observed flux power spectrum, thereby alleviating the tension with the other data sets which tended to drive \(N_{\nu}\) to high values. However, in their power spectrum fits, \([43]\) marginalize over equation of state parameters, so this explanation of the tension may not be entirely satisfactory. Future studies of the Lyman-\(\alpha\) forest may weaken or strengthen the evidence for a discrepancy. For these reasons, we do not use data from the Lyman-\(\alpha\) forest in our analysis.

4.3. CMB and LSS constraints on $N_{\nu}$ and $\eta_B$

According to \([19]\) the WMAP five-year data only impose a lower limit on \(N_{\nu}\) of \(N_{\nu} > 2.3\) but, according to \([33]\), do not lead to an upper limit due to the degeneracy between \(N_{\nu}\) and \(\Omega_{M} h^2\). Inclusion of data from small scale CMB experiments do not break this degeneracy. Figure 4 illustrates this degeneracy, as well as how it may be broken by non-CMB constraints on \(\Omega_{M} h^2\). The dashed line in figure 4 is the locus of points with constant...
Constraining the early-Universe baryon density and expansion rate

\[ \Omega_M h^2 \]

\[ N_{\nu} \]

\[ \eta_{10} \]

\[ z_{eq} = 3144 \]

Figure 4. The 68% and 95% contours in the \( N_{\nu} - \Omega_M h^2 \) plane inferred from the combination of the WMAP five-year data, small scale CMB data, luminosity distances of SNIa and the HST Key Project prior on \( H_0 \). The dashed line shows the locus of points corresponding to the same value of \( z_{eq} (=3144) \), illustrating the degeneracy between these two parameters. As the contours reveal, this degeneracy may be broken if complementary data are used to constrain \( \Omega_M h^2 \).

\[ z_{eq} = 3144 \]

Figure 4 shows how the joint probability distributions of \( N_{\nu} \) and \( \Omega_M h^2 \) inferred from the WMAP five-year data and small scale CMB experiments, supplemented by independent data from measurements of SNIa luminosity distances and the HST Key Project prior on \( H_0 \), are used to bound \( \Omega_M h^2 \). The range of \( N_{\nu} \) is now limited.

Figure 4 shows how the joint probability distributions of \( N_{\nu} \) and \( \Omega_M h^2 \) inferred from the WMAP five-year data and small scale CMB experiments, supplemented by independent data from measurements of SNIa luminosity distances and the HST Key Project prior on \( H_0 \), are used to bound \( \Omega_M h^2 \). The range of \( N_{\nu} \) is now limited.

Our constraints on \( N_{\nu} \) and \( \eta_{10} \) from various CMB and LSS data sets and combinations of them are summarized in table 1 and in figures 5–7.

Using the WMAP five-year data in combination with data from other CMB experiments (ACBAR, BOOMERANG, CBI, DASI, MAXIMA and VSA), along with the HST Key Project prior on \( H_0 \) and luminosity distance measurements of type Ia supernovae, we find central values \( N_{\nu} = 2.9 \) and \( \eta_{10} = 6.2 \), along with 68% (95%) ranges of \( 2.0 < N_{\nu} < 4.1 \) (1.3 < \( N_{\nu} \) < 5.4) and \( 6.0 < \eta_{10} < 6.3 \) (5.9 < \( \eta_{10} \) < 6.4). Figure 5 shows the joint probability distribution of \( N_{\nu} \) and \( \eta_{10} \) for this combination of CMB data sets.

Adding the 2dFGRS power spectrum [10] to the CMB power spectrum from the WMAP five-year data set, along with ground-based CMB experiments (see above), the HST prior on \( H_0 \) and luminosity distance measurements from SNIa, we find central values \( N_{\nu} = 3.0 \) and \( \eta_{10} = 6.1 \), along with 68% (95%) ranges \( 2.1 < N_{\nu} < 4.2 \) (1.3 < \( N_{\nu} \) < 5.2) and \( 6.0 < \eta_{10} < 6.3 \) (5.9 < \( \eta_{10} \) < 6.4) respectively; see table 1. Replacing the 2dFGRS power spectrum with the SDSS LRG power spectrum [63], we obtain very similar results: \( N_{\nu} = 2.8, 2.1 < N_{\nu} < 3.9 \) (1.5 < \( N_{\nu} \) < 5.2) and \( \eta_{10} = 6.2, 6.1 < \eta_{10} < 6.3 \) (5.9 < \( \eta_{10} \) < 6.5). In contrast, the SDSS DR2 [62] power spectrum favors considerably higher values of \( N_{\nu} \) compared to those inferred from the 2dFGRS and the SDSS (LRG) power spectra.
Constraining the early-Universe baryon density and expansion rate

Figure 5. The 68% and 95% contours in the $N_\nu$–$\eta_{10}$ plane inferred from the combination of the WMAP five-year data, small scale CMB data, SNIa luminosity distances, and the HST Key Project prior on $H_0$ (see the text and table 1).

Table 1. $N_\nu$ and $\eta_{10}$ from different data sets. (Note: best fits and 68% and 95% confidence intervals for $N_\nu$ and $\eta_{10}$ from our principal data sets. WMAP refers to the CMB power spectrum data from the WMAP experiment and CMB to the data from ACBAR + BOOM + CBI + VSA + MAXIMA + DASI. CMB(07) uses the 2007 ACBAR data set [35] while CMB uses the 2008 ACBAR data set [55]. HST refers to the prior on $H_0$ from the HST Key Project. SN stands for the SNIa luminosity distance measurements. SDSS(LRG) and 2dF refer to the respective LSS matter power spectra, truncated at $k = 0.07h$ Mpc$^{-1}$.)

| Data set                          | $N_\nu$      | $\eta_{10}$      |
|----------------------------------|--------------|-------------------|
| BBN ($Y_P$ and $y_{DP}$)         | $2.4^{+0.4+0.9}_{-0.4-0.8}$ | $5.7^{+0.4+0.8}_{-0.4-0.8}$ |
| WMAP(1 yr) + HST                 | $2.8^{+4.5+5.5}_{-0.4-0.8}$ | $6.1^{+0.5+1.8}_{-0.5-0.7}$ |
| WMAP(3 yr) + HST + SN            | $2.9^{+1.2+1.7}_{-1.2-1.7}$ | $6.06^{+0.23+0.42}_{-0.20-0.39}$ |
| WMAP(3 yr) + CMB(07) + HST + SN  | $2.5^{+1.7+3.8}_{-1.2-2.3}$ | $6.13^{+0.23+0.39}_{-0.18-0.36}$ |
| 2dFGRS + WMAP(3 yr) + CMB(07) + HST + SN | $2.9^{+1.4+2.3}_{-1.2-2.3}$ | $6.11^{+0.20+0.38}_{-0.15-0.32}$ |
| SDSS(DR2) + WMAP(3 yr) + CMB(07) + HST + SN | $3.7^{+1.6+3.3}_{-1.4-2.4}$ | $6.15^{+0.15+0.35}_{-0.20-0.39}$ |
| SDSS(LRG) + WMAP(3 yr) + CMB(07) + HST + SN | $2.0^{+1.2+2.5}_{-1.2-2.3}$ | $6.12^{+0.19+0.36}_{-0.16-0.34}$ |
| WMAP(5 yr) + HST + SN            | $3.9^{+2.0+4.5}_{-1.2-2.4}$ | $6.11^{+0.16+0.33}_{-0.16-0.33}$ |
| WMAP(5 yr) + CMB + HST + SN      | $2.9^{+1.4+2.5}_{-0.9-1.6}$ | $6.16^{+0.14+0.25}_{-0.16-0.30}$ |
| 2dFGRS + WMAP(5 yr) + CMB + HST + SN | $3.0^{+1.2+2.2}_{-0.9-1.7}$ | $6.14^{+0.16+0.27}_{-0.14-0.25}$ |
| SDSS(LRG) + WMAP(5 yr) + CMB + HST + SN | $2.8^{+1.1+2.4}_{-0.7-1.3}$ | $6.16^{+0.14+0.27}_{-0.14-0.27}$ |
| SDSS(LRG) + 2dFGRS + WMAP(5 yr) + CMB + HST + SN | $2.9^{+1.0+2.0}_{-0.8-1.4}$ | $6.14^{+0.16+0.30}_{-0.11-0.25}$ |
| BBN + SDSS(LRG) + 2dFGRS + WMAP(5 yr) + CMB + HST + SN | $2.5^{+0.4+0.7}_{-0.4-0.7}$ | $6.11^{+0.12+0.26}_{-0.13-0.27}$ |
Constraining the early-Universe baryon density and expansion rate

\[ \eta_{10} = 6.1^{+0.3}_{-0.2}, \]
\[ N_\nu = 2.9^{+2.0}_{-1.4}. \]  

At present the combined CMB and LSS data provide the best baryometer, determining the baryon density to better than 3%, but only a relatively weak chronometer, still allowing a large range in \( S \) (0.87 \( \leq S \leq 1.14 \) at 95% confidence). Within their uncertainties, the CMB/LSS data, which probe the Universe at \( \gtrsim 400,000 \) years, are consistent with BBN, which provides a window on the Universe at \( \lesssim 20 \) min.

5. Comparing the BBN and CMB/LSS constraints

Using the ranges of \( \eta_{10} \) and \( N_\nu \) allowed by the CMB and LSS data, the BBN-predicted primordial abundances of \(^4\text{He}, \(^D\), and \(^7\text{Li}\) may be inferred. Figure 8 compares these constraints to the observationally inferred primordial abundances adopted in section 3.1. The CMB/LSS-inferred BBN abundances of D and \(^4\text{He}\) are in excellent agreement, within the errors, with the observationally inferred relic abundances. For
Constraining the early-Universe baryon density and expansion rate

Figure 7. On the left (red, dashed), the probability distribution of $N_\nu$ inferred from the combination of the WMAP five-year data, small scale CMB data, SNIa and the HST Key Project prior on $H_0$ and matter power spectrum data from 2dFGRS and SDSS LRG. The solid blue curve is the BBN (D plus $^4$He) distribution. On the right (same line types and colors) are the probability distributions of $\eta_{10}$ using the same data sets.

Figure 8. The solid black curves show the probability distributions for the primordial abundances of D, $^4$He and $^7$Li derived from the values of $\eta_{10}$ and $N_\nu$ inferred from the CMB, LSS, SNIa and the HST prior and matter power spectrum data from 2dFGRS and SDSS LRG. The blue dashed curves show the probability distributions for the observationally inferred primordial abundances of D, $^4$He and $^7$Li; see section 3.1. The red, dot–dashed curve in the far right panel is the $^7$Li abundance from [34].

the central values of $\eta_{10} = 6.14$ and $N_\nu = 2.9$, the BBN-predicted deuterium abundance of $y_{DP} = 2.54$ is, within the errors, in agreement with its observationally inferred primordial value of $y_{DP} = 2.68$. For $^4$He, the BBN-predicted mass fraction is $Y_P = 0.247$, slightly high compared to the central value of the primordial abundance adopted in section 3.1, $Y_P = 0.240$, but within $1.2\sigma$ of it, and completely
consistent with the evolution model independent upper bound presented in equation (15), $Y_P < 0.251 \pm 0.002$.

For $^7$Li the BBN-predicted best fit from the CMB/LSS data is $[\text{Li}]_P = 2.66$, considerably higher than the value ($[\text{Li}]_P = 2.1 \pm 0.1$) determined from observations of metal-poor halo stars [56, 2] without any correction for depletion, destruction, or gravitational settling. If, however, the correction proposed by [34] is applied, the predicted and observed $^7$Li abundances may, perhaps, be reconciled, as may be seen from the lower panel of figure 8. It remains an open question whether this lithium problem is best resolved by a better understanding of stellar physics or whether it is providing a hint of new physics beyond the standard model.

6. Conclusions

While our CMB +LSS constraints on $\eta_B$ and $N_\nu$ are consistent with most previous analyses [12, 52, 26, 4, 57, 28, 58, 40, 23, 19, 33], they are tighter because we have used more and/or more recent data. However, as discussed above in section 4.2, analyses that include the Lyman-$\alpha$ forest data generally find higher values of $N_\nu$. Until the advent of WMAP and the other ground- and balloon-based CMB experiments, BBN provided the best baryometer (mainly from deuterium) and chronometer (mainly from helium-4). As may be seen from table 1, while the WMAP first-year data set provided a competitive baryometer, it offered a relatively poor chronometer. This improves with the WMAP three-year and five-year data, especially when they are combined with the other CMB and LSS data. These now lead to a determination of the baryon density at the $\sim 2$–3% level, a factor of $\gtrsim 2$ better than that from BBN. However, although the CMB/LSS constraint on $N_\nu$ has improved significantly and is consistent with that from BBN, it remains weaker than the BBN constraint by a factor of $\sim 2$. For some time now BBN has clearly established at high confidence that $N_\nu > 1$ when the Universe was $\sim 20$ min old. For example, using a slightly different estimate of the primordial helium mass fraction, [4] found $N_\nu > 1.7$ at 95% confidence (see also [60]). The more recent WMAP five-year data, combined with the recent ACBAR and other CMB and LSS data sets, the CMB/LSS now confirm that $N_\nu > 1$ (or $N_\nu > 2$ [19]) when the Universe was $\gtrsim 400$ kyr old.

As may be seen from table 1 and from figures 7 and 8, and from the left-hand panel of figure 9, BBN and the CMB, which probe physics at widely separated epochs in the evolution of the Universe, are in excellent agreement. This permits constraints on any differences in physics between BBN and recombination and/or the present epoch. For example, since baryons are conserved, $\eta_B$ relates the number of thermalized black body photons in a comoving volume at different epochs (see figure 7), constraining any post-BBN entropy production. Our results from the CMB/LSS and from BBN imply

$$\frac{N_{\gamma}^{\text{CMB}}}{N_{\gamma}^{\text{BBN}}} = 0.92 \pm 0.07. \quad (26)$$

This ratio is consistent with 1 at $\sim 1\sigma$ and places an interesting upper bound on any post-BBN entropy production.

Alternatively, late decaying particles could produce relativistic particles (radiation), but not necessarily thermalized black body photons (see for example [29]). Deviations
Constraining the early-Universe baryon density and expansion rate

Figure 9. Left. In blue (solid), the 68% and 95% contours in the $N_\nu - \eta_{10}$ plane derived from a comparison of the observationally inferred and BBN-predicted primordial abundances of D and $^4$He (see figure 2). In red (dashed), the 68% and 95% contours derived from the combined WMAP five-year data, small scale CMB data, SNIa, and the HST Key Project prior on $H_0$ along with matter power spectrum data from 2dFGRS and SDSS LRG (see the text and table 1). Right. The 68% and 95% joint BBN–CMB–LSS contours in the $N_\nu - \eta_{10}$ plane.

from the standard-model radiation density can be parameterized by the ratio of the radiation density, $\rho'_R$, to the standard-model radiation density, $\rho_R$. In the post-$e^\pm$ annihilation Universe (see equation (9)),

$$ R = S^2 = \frac{\rho'_R}{\rho_R} = 1 + 0.134\Delta N_\nu. \tag{27} $$

Comparing this ratio at BBN and at recombination (see figure 7), any post-BBN production of relativistic particles can be constrained:

$$ \frac{R_{\text{CMB}}}{R_{\text{BBN}}} = 1.07^{+0.16}_{-0.13}. \tag{28} $$

This ratio, too, is consistent with 1 within 1σ, placing an upper bound on post-BBN production of relativistic particles.

Although the non-standard expansion rate has been parameterized in terms of an equivalent number of additional species of neutrinos, we have emphasized that a non-standard expansion rate need not be related to extra (or fewer) neutrinos. For example, deviations from the standard expansion rate could occur if the value of the early-Universe gravitational constant, $G_N$, were different from its present, locally measured value [65, 7, 1, 14]. For the standard radiation density with three species of light, active neutrinos, the constraint on the expansion rate can be used to constrain variations in the gravitational constant, $G_N$. From BBN

$$ S^2 = \frac{G_N^{\text{BBN}}}{G_N} = 0.91 \pm 0.07, \tag{29} $$
and at the epoch of the recombination

\[ S^2 = \frac{G_{\text{CMB}}}{G_N} = 0.99^{+0.13}_{-0.11}, \]

consistent with no variation in \( G \) at the \( \sim 1\sigma \) level.

The agreement between \( \eta_B \) and \( N_\nu \) evaluated from BBN (\( \sim 20 \) min) and from the CMB/LSS (\( \gtrsim 400 \) kyr) is shown in the left-hand panel of figure 9. As figure 9 illustrates, BBN and the CMB/LSS, which probe the Universe at widely separated epochs in its evolution, are completely consistent. As already noted, at present the CMB is a better baryometer while BBN remains a better chronometer. Since these independent constraints from the CMB/LSS and BBN are in very good agreement, we may combine them to obtain the joint fit in table 1 and shown in the right-hand panel of figure 9. We note that while the best fit value of \( N_\nu \) is less than 3, this is not statistically significant since the results are consistent with the standard model of three species of active neutrinos at the \( \sim 1\sigma \) level.

Of course, our BBN results are sensitive to the relic abundances that we have adopted. For comparison, we have repeated our analysis for an alternative set of primordial abundances. For deuterium, we adopted the [47] results based on the weighted mean of the \( \log(y_D) \) values, \( y_D = 2.84^{+0.27}_{-0.25} \), and for \( ^4\text{He} \) we chose, somewhat arbitrarily, \( Y_P = 0.247 \pm 0.004 \). For this alternative abundance set the BBN-predicted value of \( \eta_B \) is virtually unchanged from our previous result, \( \eta_{10} = 5.7 \pm 0.3 \), while \( N_\nu = 2.9 \pm 0.3 \) is much closer to the standard-model expectation. As a result, while the constraint on entropy production (equation (26)) is unchanged, \( R_{\text{CMB}}/R_{\text{BBN}} = 1.00^{+0.16}_{-0.13} \) and \( G_{\text{BBN}}^{N}/G_N = 0.99^{+0.13}_{-0.11} \). When the alternative BBN constraints are combined with those from the CMB and LSS, we find \( \eta_{10} = 6.09^{+0.12}_{-0.13} \) and \( N_\nu = 2.9 \pm 0.3 \).

Very recent observations of deuterium in a high redshift, low metallicity damped Lyman-\( \alpha \) absorber by [50] lead to a deuterium abundance very close to the mean of the previous six abundances used in this paper. As a result, the change in \( y_D \) is very small (\( y_D = 2.70 \) rather than \( y_D = 2.68 \) adopted in this paper). Consequently, the change in the parameters inferred from it (\( N_\nu \) and \( \eta_{10} \)) is also very small.

Future CMB experiments will improve the constraint on \( N_\nu \) by measuring the neutrino anisotropic stress more accurately. According to [5], PLANCK should determine \( N_\nu \) to an accuracy of \( \sigma(N_\nu) \sim 0.24 \) and CMBPOL, a satellite-based polarization experiment, might improve it further to \( \sigma(N_\nu) \sim 0.09 \), independently of the BBN constraints. In this context, we note that such tight constraints on \( N_\nu \) will be sensitive to the value of the \(^4\text{He}\) abundance adopted in the CMB analysis. To achieve these projected accuracies, it will no longer be sufficient to fix \( Y_P \) in advance. Rather, \( Y_P \) should be solved for in concert with the other cosmological parameters. To this end, we point out that for \( \eta_{10} \approx 6 \), \( N_\nu \approx 3 \) (suggested by our results), a very good, simple approximation to \( Y_P \) is provided by equation (11) [32, 61].

**Acknowledgments**

This research was supported at The Ohio State University by a grant (DE-FG02-91ER40690) from the US Department of Energy. We thank D Weinberg for a careful reading of the manuscript and for suggestions which led to an improved manuscript. We also thank R Cyburt, S Dong, J Dunkley, J Hamann, S Hannestad, A Lewis,
P McDonald, M Pettini, G Raffelt, and Y Wong for useful discussions. We thank the Ohio Supercomputer Center for allowing the use of a Cluster Ohio Beowulf cluster in this research.

References

[1] Accetta F S, Krauss L M and Romanelli P, 1990 Phys. Lett. B 248 146 [SPIRES]
[2] Asplund M et al, 2006 Astrophys. J. 644 229 [SPIRES]
[3] Astier P et al, 2006 Astron. Astrophys. 447 31 [SPIRES]
[4] Barger V, Kneller J P, Lee H-S, Marfatia D and Steigman G, 2003 Phys. Lett. B 569 123 [SPIRES]
[5] Bashinsky S and Seljak U, 2004 Phys. Rev. D 69 083002 [SPIRES]
[6] Binetruy P, Deffayet C, Ellwanger U and Langlois D, 2000 Phys. Lett. B 477 285 [SPIRES]
[7] Boesgaard A M and Steigman G, 1985 Ann. Rev. Astron. Astrophys. 23 319 [SPIRES]
[8] Bolton J S et al, 2007 Preprint 0711.2064v1
[9] Cline J M, Grojean C and Servant G, 2000 Phys. Rev. Lett. 83 4245 [SPIRES]
[10] Cole S et al, 2005 Mon. Not. R. Astron. Soc. 362 505
[11] Croft R A C et al, 1998 Astrophys. J. 495 44 [SPIRES]
[12] Crotty P, Lesgourges J and Pastor S, 2003 Phys. Rev. D 67 123005 [SPIRES]
[13] Cyburt R H, 2004 Phys. Rev. D 70 023505 [SPIRES]
[14] Cyburt R H et al, 2005 Astropart. Phys. 23 313 [SPIRES]
[15] Dicus D A, 1997 Phys. Rev. D 47 4325 [SPIRES]
[16] Dodelson S and Turner M S, 1992 Phys. Rev. D 45 1372 [SPIRES]
[17] Dunkley J et al, 2005 Astrophys. J. 340 617 [SPIRES]
[18] Dolgov A D, 2002 Phys. Rep. 370 333 [SPIRES]
[19] Dickinson C et al, 2004 Mon. Not. R. Astron. Soc. 353 732
[20] Dunkley J et al, 2006 Preprint 0803.0586 [SPIRES]
[21] Freedman W L et al, 2001 Astrophys. J. 553 47 [SPIRES]
[22] Fukugida M and Kawasaki M, 2006 Astrophys. J. 646 691 [SPIRES]
[23] Halverson N W et al, 2001 Astrophys. J. 562 38 [SPIRES]
[24] Halverson N W et al, 2001 Astrophys. J. 568 38 [SPIRES]
[25] Hamann J et al, 2007 J. Cosmol. Astropart. Phys. JCAP08(2007)021 [SPIRES]
[26] Hanany S et al, 2007 J. Cosmol. Astropart. Phys. JCAP09(2007)007 [SPIRES]
[27] Harari J et al, 2007 J. Cosmol. Astropart. Phys. JCAP07(2007)008 [SPIRES]
[28] Hishida S et al, 2007 J. Cosmol. Astropart. Phys. JCAP05(2007)004 [SPIRES]
[29] Huang J et al, 2007 J. Cosmol. Astropart. Phys. JCAP05(2007)004 [SPIRES]
[30] Johnston S and Turner M S, 1992 Phys. Rev. D 46 3337 [SPIRES]
[31] Kirkman D et al, 2007 Preprint 0711.1862 [SPIRES]
[32] Kneller J P and Steigman G, 2004 New J. Phys. 6 117 [SPIRES]
[33] Komatsu E et al, 2008 Preprint 0803.0547 [SPIRES]
[34] Korn A J et al, 2006 Nature 442 657 [SPIRES]
[35] Kuo C L et al, 2007 Astrophys. J. 660 32 [SPIRES]
[36] Lewis A et al, 1999 Astrophys. J. 538 473 [SPIRES]
[37] Lewis A and Bridle S, 2002 Phys. Rev. D 66 103511 [SPIRES]
[38] Lopez R E, Dodelson S, Heckler A and Turner M S, 1999 Phys. Rev. Lett. 82 3952 [SPIRES]
[39] Mangano G et al, 2005 Nucl. Phys. B 729 221 [SPIRES]
[40] Mangano G et al, 2007 J. Cosmol. Astropart. Phys. JCAP03(2007)006 [SPIRES]
[41] Mason B S et al, 2003 Astrophys. J. 591 540 [SPIRES]
[42] McDonald P et al, 2000 Astrophys. J. 543 1 [SPIRES]
[43] McDonald P et al, 2006 Astrophys. J. 635 761 [SPIRES]
[44] Montroy T E et al, 2006 Astrophys. J. 647 813 [SPIRES]
[45] Nolta M R et al, 2008 Preprint 0803.0593 [SPIRES]
[46] Olive K A and Skillman E D, 2004 Astrophys. J. 617 29 [SPIRES]
[47] O’Meara J M et al, 2006 Astrophys. J. 649 L61 [SPIRES]
[48] Peimbert M, Luridiana V and Peimbert A, 2007 Astrophys. J. 666 636 [SPIRES]
[49] Percival W J et al, 2007 Astrophys. J. 645 663 [SPIRES]
[50] Pettini M et al, 2008 Preprint 0805.0594 [SPIRES]
[51] Piacentini F et al, 2006 Astrophys. J. 647 833 [SPIRES]
[52] Pierpaoli E, 2003 Mon. Not. R. Astron. Soc. 342 L63
Constraining the early-Universe baryon density and expansion rate

[53] Randall L and Sundrum R, 1999a Phys. Rev. Lett. 83 3370 [SPIRES]
Randall L and Sundrum R, 1999b Phys. Rev. Lett. 83 4690 [SPIRES]
[54] Readhead A C S et al., 2004 Astrophys. J. 609 498 [SPIRES]
[55] Reichardt C L et al., 2008 Preprint 0801.1491
[56] Ryan S G et al., 2000 Astrophys. J. 530 L57 [SPIRES]
[57] Seljak U, Slosar A and McDonald P, 2006 J. Cosmol. Astropart. Phys. JCAP10(2006)014 [SPIRES]
[58] Spergel D N et al., 2007 Astrophys. J. 170 377 [SPIRES]
[59] Steigman G, 2006 J. Cosmol. Astropart. Phys. JCAP10(2006)016 [SPIRES]
[60] Steigman G, 2006 Int. J. Mod. Phys. E 15 1
[61] Steigman G, 2007 Ann. Rev. Nucl. Part. Sci. 57 463 [SPIRES]
[62] Tegmark M et al., 2004 Astrophys. J. 606 702 [SPIRES]
[63] Tegmark M et al., 2006 Phys. Rev. D 74 123507 [SPIRES]
[64] Viel M and Haehnelt M G, 2006 Mon. Not. R. Astron. Soc. 365 231
[65] Yang J M, Schramm D N, Steigman G and Rood R T, 1979 Astrophys. J. 227 697 [SPIRES]