The spectral form factor near the Ehrenfest-time

Piet W. Brouwer and Saar Rahav
Laboratory of Atomic and Solid State Physics, Cornell University, Ithaca 14853, USA

Chushum Tian
Institut für Theoretische Physik, Universität zu Köln, Köln, 50937, Germany
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We calculate the Ehrenfest-time dependence of the leading quantum correction to the spectral form factor of a ballistic chaotic cavity using periodic orbit theory. For the case of broken time-reversal symmetry, our result differs from that previously obtained using field-theoretic methods [Tian and Larkin, Phys. Rev. B 70, 035305 (2004)]. The discrepancy shows that short-time regularization procedures dramatically affect physics near the Ehrenfest-time.

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From a statistical point of view, the spectra of a quantum particle confined to a cavity with disorder or confined to a cavity with ballistic and chaotic classical dynamics are remarkably similar [1]. In both cases, the spectral statistics are independent of system specifics and follow the predictions of random matrix theory (RMT) on energy scales comparable to the mean spacing between energy levels \(\Delta\) or, equivalently, on time scales comparable to the Heisenberg time \(t_H = 2\pi\hbar/\Delta\) [2, 3], provided that the latter is much larger than the time \(\tau_{\text{erg}}\) needed for ergodic exploration of the phase space. Calculations of the spectral statistics have been the theoretical vehicle through which the profound correspondence between RMT, impurity diagrammatic perturbation theory, periodic-orbit theory, and the field-theoretic zero-dimensional \(\sigma\) model has been demonstrated [2, 4, 5, 6, 7].

In the last decade, it has been understood that chaotic quantum systems are characterized by a time scale intermediate between \(\tau_{\text{erg}}\) and \(t_H\) that does not have its counterpart in diffusive systems. This time scale is the “Ehrenfest time” \(\tau_E\) [8, 9, 10, 11, 12, 13, 14, 15]. The Ehrenfest time is the time required for two classical trajectories initially a quantum distance (wave-length) apart to diverge and reach a classical separation (system size). It is expressed in terms of the Lyapunov exponent \(\lambda\) of the corresponding classical dynamics as \(\tau_E = \lambda^{-1} \ln(c^2/\hbar)\), where \(c^2\) is a classical (action) scale. The existence of the Ehrenfest time does not affect the universality of the spectral statistics. However, it is responsible for differences between otherwise universal properties of chaotic and disordered quantum systems for energies \(\sim \hbar/\tau_E\), which is well inside the universal regime if \(\hbar/c^2 \rightarrow 0\) [8, 10, 11, 13, 14, 15, 16]. In this article we address the spectral statistics at this energy scale.

Most of the literature on spectral statistics considers the spectral form factor \(K(t)\), which is the Fourier-transform of the two-point correlation function of the level density \(\rho(\varepsilon)\),

\[
K(t) = \hbar \left( \int d\omega e^{i\omega t} \rho(\varepsilon + \hbar \omega/2)\rho(\varepsilon - \hbar \omega/2) \right).
\] (1)

In the perturbative regime, which already occurs to order \((t/t_H)^2\), \(K(t)\) is dominated by a perturbative expansion in \(t/t_H\) [1, 14, 17]

\[
K(t) = \frac{t}{\beta\pi\hbar} + \delta K_{\beta}(t),
\] (2)
where \(\beta = 1 (2)\) in the presence (absence) of time-reversal symmetry and

\[
\delta K_1(t) = -\frac{t}{\pi\hbar} \left( \frac{t}{t_H} - \frac{t^2}{t_H^2} + \ldots \right),
\]
\[
\delta K_2(t) = 0.
\] (3)

For times \(t \geq t_H\) the perturbative expansion in \(t/t_H\) breaks down, and \(K(t)\) is governed by non-perturbative contributions [10].

The presence of the Ehrenfest time does not affect the leading contribution to \(K(t)\), but it does impact \(\delta K\). The leading \(\tau_E\) dependence of \(\delta K_1\) in the perturbative regime, which already occurs to order \((t/t_H)^2\), was first considered by Aleiner and Larkin [12], using a field-theoretic approach. Recently, Tian and Larkin used the field-theoretic approach to calculate the leading \(\tau_E\)-dependence of \(\delta K_2\) [8], which appears to order \((t/t_H)^3\) and causes the perturbative quantum correction \(\delta K_2\) no longer to be strictly zero. Their result is

\[
\delta K_1(t) = -\frac{t^2}{\pi\hbar^2 t_H} \theta(t - 2\tau_E) + \ldots, \tag{4}
\]
\[
\delta K_2(t) = -\frac{t^2}{2\pi\hbar^2 t_H} \left[ \Theta(t - 3\tau_E) - \Theta(t - 4\tau_E) \right] + \ldots, \tag{5}
\]
where \(\theta(x) = 1\) if \(x > 0\) and \(\theta(x) = 0\) otherwise, \(\Theta(x) = \int_{-\infty}^{x} dx' \theta(x') = x\theta(x)\), and the dots refer to terms of higher order in \(t/t_H\) that were not considered in the calculation. (Tian and Larkin also considered the \(\tau_E\)-dependence of non-perturbative contributions to the spectral form factor, but these will not be considered here.)
In a parallel development, the full perturbation expansion for \( K(t) \) was derived from periodic-orbit theory \[ \text{Ref. 5, 6, 7, 18, 19} \]. The connection between \( K(t) \), which is a quantum-mechanical object, and classical periodic orbits follows from Gutzwiller’s trace formula, which expresses \( K(t) \) as a double sum over periodic orbits \( \gamma \) and \( \gamma' \) \[17, 20\],

\[
K(t) = \left\{ \sum_{\gamma, \gamma'} A_\gamma A_{\gamma'}^* e^{i(S_\gamma - S_{\gamma'})/\hbar} \delta \left( t - \frac{T_\gamma + T_{\gamma'}}{2} \right) \right\}. \tag{6}
\]

Here \( A_\gamma, S_\gamma, \) and \( T_\gamma \) are the stability amplitude, classical action, and period of \( \gamma \), respectively. While the leading contribution to \( K(t) \) comes from the diagonal terms \( \gamma = \gamma' \) (up to time-reversal, if \( \beta = 1 \)) \[17\], off-diagonal contributions are responsible for \( \delta K(t) \). Sieber and Richter \[18\] and Heusler et al. \[6, 7, 19\] succeeded in classifying the relevant off-diagonal orbit pairs and calculated their contribution to \( \delta K(t) \) in the limit \( \tau_E/t_H \to 0 \).

Below, we show that periodic-orbit theory can also be used to calculate the Ehrenfest-time dependence of \( \delta K(t) \). Interestingly, while we confirm the field-theoretic result for the \( O(t^2) \) term in \( \delta K_1(t) \) \[6, 7, 21\], our result for the leading \( O(t^3) \) contribution to \( \delta K_2(t) \) differs from that of Ref. \[6\].

\[
\delta K_2(t) = \frac{3\mu^2}{2\pi \hbar^2 t_H} \times \left[ \Theta(t - 2\tau_E) - 2\Theta(t - 3\tau_E) + \Theta(t - 4\tau_E) \right]. \tag{7}
\]

In particular, we find that the minimum duration of off-diagonal pairs of orbits that contribute to \( \delta K_2 \) is \( 2\tau_E \), not \( 3\tau_E \). We also find that there are no \( \tau_E \)-dependent corrections for \( t > 4\tau_E \), in contrast to Ref. \[6\], where the \( \tau_E \)-dependent corrections to \( \delta K_2 \) persist up to \( t \sim t_H \). However, our leading-order perturbative calculation does not answer the question whether such \( \tau_E \)-insensitivity at long times persist to the higher order terms in \( t/t_H \).

Instead of calculating \( K(t) \) directly, it is more convenient to calculate the Laplace transform

\[
K(\alpha) = \left\{ \sum_{\gamma, \gamma'} A_\gamma A_{\gamma'}^* e^{i(S_\gamma - S_{\gamma'})/\hbar - \alpha(T_\gamma + T_{\gamma'}/2)} \right\}. \tag{8}
\]

The leading diagonal contribution to \( K(\alpha) \) can be calculated using the sum rule of Hannay and Ozorio de Almeida \[22\],

\[
\left\{ \sum_{\gamma} |A_\gamma|^2 e^{-\alpha T_\gamma} \right\} = \frac{1}{2\pi \hbar \alpha^2}. \tag{9}
\]

so that

\[
K(\alpha) = \frac{1}{\pi \hbar \alpha^2 \beta} + \delta K(\alpha). \tag{10}
\]

The inverse Laplace transform of the first term in Eq. \[10\] reproduces the leading term in Eq. \[2\] above.

The leading \( O(t^2) \) quantum correction for \( K(\alpha) \) exists in the presence of time-reversal symmetry only. The relevant pairs of periodic orbits \( \gamma \) and \( \gamma' \) are shown in Fig. \[11\]. The existence of such pairs was pointed out by Sieber and Richter \[18\]; an equivalent configuration of classical trajectories appears in the field-theoretic formulation \[6, 12, 21\] and in the diagrammatic calculation of the form factor for disordered cavities \[6\]. The periodic orbit \( \gamma \) in Fig. \[11\] has a small-angle self-intersection. There are two loops of duration \( T_1 \) and \( T_2 \) through which \( \gamma \) returns to the self-intersection. The trajectory \( \gamma' \) is equal to \( \gamma \) in one of these loops, whereas \( \gamma' \) is the time-reversed of \( \gamma \) in the other loop. Following Ref. \[6, 7, 19\], we perform the sum over such periodic orbits with the help of a Poincaré surface of section taken at an arbitrary point during the self-intersection. The Poincaré surface of section is parameterized using stable and unstable phase space coordinates \( s \) and \( u \), normalized such that \( ds du \) is the cross-sectional area element. Denoting the coordinate differences between the two points where \( \gamma \) pierces the Poincaré surface of section by \( s \) and \( u \), the action difference \( S_\gamma - S_{\gamma'} = su \) \[23, 24\]. The duration \( t_{\text{enc}} \) of the self-encounter is defined as the time during which the two stretches of \( \gamma \) are within a phase space distance \( c \), where \( c \) is a classical scale below which the classical dynamics can be linearized. The periodic-orbit sum is then expressed...
in terms of an integral over \(s, u, T_1\) and \(T_2\),
\[
\delta K_1(\alpha) = |dT_1dT_2| \int_{-c}^{c} ds du \frac{(T_1 + T_2 + 2t_{enc})^2}{(2\pi h)^2 t_{H_{\text{enc}}}} \times \cos(su/h) e^{-\alpha(T_1 + T_2 + 2t_{enc})} \\
= \frac{\partial^2}{\partial \alpha^2} \frac{1}{\alpha^2} \int_{-c}^{c} ds du \frac{\cos(su/h) e^{-2\alpha t_{enc}}}{(2\pi h)^2 t_{H_{\text{enc}}}},
\]
(11)
where
\[
t_{\text{enc}} = (1/\lambda) \ln(c^2/|su|),
\]
(12)
\(\lambda\) being the Lyapunov exponent of the classical dynamics in the cavity. The factor \(t_{\text{enc}}\) in the denominator cancels an unwanted contribution from the freedom to choose the Poincaré surface of section anywhere inside the encounter region. Whereas Refs. [6, 19] calculate the remaining integral in the limit \(\alpha \tau_E \to 0\), we need to consider the effect of a finite Ehrenfest time. These integrals have been considered in the context of quantum transport and we will be able to obtain the remaining integral in Eq. (11) as well as all other necessary integrals from the literature. The integral needed here has been calculated in Ref. [25] and gives
\[
\delta K_1(\alpha) = -\frac{1}{\pi h t_H} \frac{\partial^2}{\partial \alpha^2} \frac{e^{-2\alpha \tau_E}}{\alpha}.
\]
(13)
where
\[
\tau_E = \frac{1}{\lambda} \ln\frac{c^2}{h}.
\]
(14)
This result is in agreement with the \(\tau_E\) dependence of \(\delta K_1\) calculated by Tian and Larkin [6]. Its inverse Laplace transform is Eq. (4) above.

We now consider the Ehrenfest-time dependent correction \(\delta K_2(t)\) to the spectral form factor in the absence of time-reversal symmetry. There are three classes of periodic orbits that give a contribution to \(\delta K_2(t)\) to order \(t^3\). These are shown in Fig. 4c–d. Now, the classical orbit \(\gamma\) has two small-angle self-encounters. Its partner orbit \(\gamma'\) follows \(\gamma\) between the encounters, but connects the ends of the encounters in a different way. Figure 4f shows two separate encounters. Figure 4b shows a “three-encounter”, which arises when the two small-angle encounters of Fig. 4b are merged along one of the connecting stretches (while keeping the duration of the other stretches finite). The orbit \(\gamma\) of Fig. 4c goes through three loops between returns to the encounter region. Finally, Fig. 4h shows a periodic orbit for which the self-encounter fully extends along one of these loops [26]. Note that the encounter region can not simultaneously extend along two or more of the loops, because then these loops, and, hence, the trajectories \(\gamma\) and \(\gamma'\) would be equal.

The configurations of Figs. 4b and c were also considered by Heusler et al. [6, 19]. These two contributions cancel in the limit \(\alpha \tau_E \to 0\), so that one finds \(\delta K_2 = 0\) in that limit. The contribution of the trajectories shown

Fig. 4i vanishes in the limit \(\alpha \tau_E \to 0\), which is why it was not considered in Refs. [6, 19]. However, as we will show below, it is needed to be taken into account when calculating \(\tau_E\)-dependent corrections. The field-theoretic calculation of Ref. [6] also has two contributions to \(\delta K_2\) only, one with two two-encounters and the other with a single three-encounter.

The contribution \(\delta K_{2\gamma}\) of the two separate encounters for the periodic-orbit pair in Fig. 4f factorizes. Taking a Poincaré surface of section at each of the encounters and proceeding as in the calculation of \(\delta K_1\), one finds
\[
\delta K_{2\gamma}(\alpha) = \frac{\partial^2}{\partial \alpha^2} \frac{1}{\alpha^4} \left( \int_{-c}^{c} \cos(su/h) e^{-2\alpha t_{enc}} \right)^2
\]
(15)
where we included a combinatorial factor \(1/4\) to account for permutations of the \(2 \times 2\) passages of \(\gamma\) through the two encounters [6, 19]. This result is in agreement with that obtained by calculating the two two-encounter diagram in the field-theoretic approach [27].

For the calculation of the contribution of the trajectory of Fig. 4i one needs only one Poincaré surface of section, taken at a point where all three stretches of \(\gamma\) are within a phase space distance \(c\). Labeling the phase space coordinates of the three piercings of \(\gamma\) through the surface of section as \((s_i, u_i), i = 1, 2, 3\), the action difference is
\[
\delta S_{\gamma} - \delta S_{\gamma'} = su + s'u',
\]
(16)
where \(s = s_1 - s_3, s' = s_1 - s_5, u = u_1 - u_3\), and \(u' = u_1 - u_5\). Integrating over the durations of the three stretches of \(\gamma\) that connect the three-encounter to itself, we find
\[
\delta K_{2\gamma}(\alpha) = \frac{1}{3} \frac{\partial^2}{\partial \alpha^2} \frac{1}{\alpha^4} \int ds'dsu' \cos((su + s'u')/h) e^{-\alpha(2t_{\text{enc}} + t'_{\text{enc}})}
\]
\[
\times \frac{e^{-\alpha(2t_{\text{enc}} + t'_{\text{enc}})}}{(2\pi h)^2 t_{H_{\text{enc}}}} t_{\text{enc}}' \cos(su'/h) e^{-\alpha t_{\text{enc}}'},
\]
(17)
where \(t_{\text{enc}}\) is the duration of the encounter,
\[
t_{\text{enc}}' = \frac{1}{\lambda} \ln\frac{c^2}{\min(|s|, |s'|, |s - s'|) \min(|u|, |u'|, |u + u'|)}
\]
and \(t'_{\text{enc}}\) is the time that all three trajectories involved in the encounter are within a phase space distance \(c\),
\[
t_{\text{enc}}' = \frac{1}{\lambda} \ln\frac{c^2}{\max(|s|, |s'|, |s - s'|) \max(|u|, |u'|, |u + u'|)}.
\]
(18)
The prefactor \(1/3\) in Eq. (17) accounts for permutations of the three passages of \(\gamma\) through the three encounters. The integration domain in Eq. (17) is \(\max(|u|, |u'|, |u + u'|)\), \(\max(|s|, |s'|, |s - s'|) < c\). The exponential factor contains the total time \(2t_{\text{enc}} + t'_{\text{enc}}\) that the orbit \(\gamma\) spends
in the encounter region. Taking the remaining integral over $s$, $s'$, $u$, and $u'$ from Ref. 28, we find

$$\delta K_{2c}(\alpha) = \frac{1}{2\pi\hbar^2_H} \frac{\partial^2}{\partial\alpha^2} \left[ 3e^{-3\alpha\tau_E} - 4e^{-4\alpha\tau_E} \right].$$

(18)

Although this result agrees with the field-theoretic calculation in the limit $\alpha\tau_E \to 0$ [4, 5], the appearance of a term proportional to $\exp(-4\alpha\tau_E)$-oscillation differs from the field-theoretic approach [5], in which only one term proportional to $\exp(-3\alpha\tau_E)$ appears with prefactor $-1$ ($=-3\alpha\tau_E$).

Finally, we have to calculate the contribution from trajectories with a three-encounter where the encounter region fully wraps around one of the loops. In order to make optimal use of the literature on the Ehrenfest-time dependence of quantum transport, we calculate this contribution in an indirect way: We again consider the case of two two-encounters, but now allow the encounters to approach each other at a pre-assigned stretches of $\gamma$. This situation shown in Fig. 11 (again), and allow the encounters to approach each other along the central loop in the figure. We take a Poincaré surface of section at each encounter, and measure the durations of between the two surfaces of section along the central loop by $T'_1$ and $T'_2$. Along the remaining stretches (the outer loop in Fig. 11), we still require non-overlapping encounters in order to enforce $\gamma \neq \gamma'$. Hence, we parameterize their duration using $T_1$ and $T_4$ measured between the ends of the encounters. Finally, $t_{enc,1}$ and $t_{enc,2}$ denote time that the inner and outer loops are within a phase space distance $c$, and $s$ and $t_u$ are the durations of eventual stretches that the two segments of the outer loop are within a phase space distance $c$ from each other but not from the inner loop [28]. The times $t_s$ and $t_u$ are zero except in the case of overlapping encounters. With this parameterization, the total duration of $\gamma$ is

$$T_\gamma = T'_1 + T'_2 + T_3 + T_4 + t_{enc,1} + t_{enc,2} + 2t_s + 2t_u.$$  

(19)

We now consider the integral

$$I = \int dT'_1 dT'_2 dT_3 dT_4 \int_{-c}^c ds_1 ds_2 du_1 du_2 \times \frac{T'_1^2 \cos[(u_1 s_1 + u_2 s_2)/\hbar]e^{-\alpha T'_1}}{(2\pi\hbar)^3 T'_1^2 t_{enc,1} t_{enc,2}},$$

(20)

where $s_i$ and $u_i$ are phase space coordinates at the two Poincaré surfaces of section, $i = 1, 2$. This integral contains both the case that the two encounters are separate and the case that the two encounters overlap. If the two encounters are separate, which requires $T'_1 > \lambda^{-1} \ln(|u_1 s_1|/c^2)$ and $T'_2 > \lambda^{-1} \ln(|u_2 s_2|/c^2)$, the two stretches in the outer loop are never close to each other, hence $t_s = t_u = 0$. One then recovers the expression for $\delta K_{2b}$, multiplied by four because of the combinatorial factor $1/3$ which is present in Eq. (17) but not in Eq. (20). (Note that in this case the times $t_s$ and $t_u$ need not be zero.) If the encounters overlap at two ends, they span the reference loop. This scenario is neither contained in $\delta K_{2b}$ nor in $\delta K_{2c}$. Since there is no combinatorial factor in this case, this is precisely the contribution $\delta K_{2d}$ corresponding to the trajectories of the type shown in Fig. 11. Hence

$$\delta K_{2d}(\alpha) = I - 4\delta K_{2b}(\alpha) - 3\delta K_{2c}(\alpha).$$

(21)

From Sec. IV of Ref. 25, where an integral similar to $I$ was calculated, we find

$$I = \frac{1}{2\pi\hbar^2_H} \frac{\partial^2}{\partial\alpha^2} \left[ 3e^{-2\alpha\tau_E} - 2e^{-4\alpha\tau_E} \right].$$

(22)

Combining everything, we arrive at

$$\delta K_2(\alpha) = \delta K_{2b}(\alpha) + \delta K_{2c}(\alpha) + \delta K_{2d}(\alpha) = \frac{3}{2\pi\hbar^2_H} \frac{\partial^2}{\partial\alpha^2} \left[ 1 - e^{-\alpha\tau_E} \right]^2.$$  

(23)

The inverse Laplace transform of this result is Eq. 17 above.

The main difference between our result and that previously obtained by the field-theoretic approach is that, in contrast to Ref. 5, in Eq. 17 universal quantum corrections already appear after a time $2\tau_E$ [29]. This shortest-duration contribution to $\delta K_2$ stems from periodic orbit pairs which (for a part of their duration) wind around another, shorter, periodic orbit. Such orbits explained [29] the numerically observed $\tau_E$-independence of conductance fluctuations in chaotic cavities [30, 31]. Another difference between our result and that previously obtained by the field-theoretic approach is the appearance of a discontinuity at $4\tau_E$ for the contribution $\delta K_{2c}$ of a single three encounter. A feature at $4\tau_E$, which is absent in the field-theoretical calculation, is essential for the validity of the ‘effective random matrix theory’ [17] of the Ehrenfest-time dependence of the spectral gap in a chaotic cavity coupled to a superconductor [25].

The calculation of the $\tau_E$-dependence of the perturbative correction $\delta K_2$ is, to the best of our knowledge, the first example in which the periodic orbit and the field-theoretical approaches lead to different results in the universal range. It is also the first example in which the two approaches are compared for a quantity that involves more than a single two-encounter. We believe this discrepancy can be traced to a difference in the way both approaches handle the encounter regions, in particular to differences implicitly or explicitly imposed by the short-time regularization procedures.

In the periodic-orbit approach, a generic phase-space density function is described in reference to a classical trajectory. It expands and contracts along the unstable and stable directions in phase space, but with a uniform density function at each point during the encounter. The total duration of the encounter region is
\[ \sim \tau_E, \] beyond which the density functions are projected onto the low-lying sectors of the Perron-Frobenius modes and evolve irreversibly \[ \delta K, \] despite the deterministic nature of the classical dynamics. On the other hand, in the field-theoretic approach of Ref. 5, each ‘orbit’ represents the evolution from a singular density field concentrated at a point in phase space. Under the action of elliptical regulators, such singular fields first become locally smooth, spreading over the scale determined by the regulator strength. This scale is adjusted to be \( \sim h \) in order to mimic the evolution of quantum wave packets with minimal variance \[ \sim \tau E. \] At the time \( \tau E/2 \) away from the origin (both forward and backward in time), the (now) locally smooth fields are projected onto the low-lying sectors of the Perron-Frobenius modes \[ \delta K. \] The two approaches give the same result for a single two-encounter: in both cases the encounter region is a generic classical trajectory of duration \( \tau E. \) However, whether encounters are built from a reference trajectory or a reference ‘point’ matters for encounters of more than two trajectories. In the periodic-orbit approach, a reference orbit can be a periodic orbit with an arbitrarily short period. These reference orbits then give the universal contribution to \( \delta K \) at the time \( 2\tau E \) that was calculated above. The reference orbit can also be a trajectory of length \( \lesssim 2\tau E, \) which is involved in two almost non-overlapping two-encounters. This case gives the feature at time \( 4\tau E \) for the three-encounter contribution to the form factor. In the field-theoretic approach of Ref. 6, the evolution of the phase space density field is always altered irreversibly a time \( \tau E/2 \) away from the reference point, thus, in the case of a three-encounter, leaving no room for a quantum correction at time \( 2\tau E, \) nor for a feature at \( 4\tau E. \)

Summarizing, we used periodic orbit theory to calculate the leading Ehrenfest time dependent corrections to the spectral form factor \( K(t). \) We find that, in the absence of time-reversal symmetry, the corrections to the leading RMT prediction do not strictly vanish for \( 2\tau E < t < 4\tau E \) if the Ehrenfest time \( \tau E \) is finite. The fact that finite-\( \tau E \) corrections give a nonzero correction to \( K(t) \) is in agreement with a previous field-theoretic calculation \[ \delta K, \] although our detailed expression for \( K(t) \) is not. We attribute the difference to the different short-time regularization used in Ref. 2.

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[33] Perhaps unexpectedly, with an appropriately coarse grained density function this principle also appears in the ‘Moyal quantization’ — much resembling peri-
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[34] For a review, see, e.g., C. Zachos, Int. J. Mod. Phys. A 17, 297 (2002) and references therein.