Integral definition method to solve magnetic force of axial permanent magnetic bearing

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Abstract. In the design process of the permanent magnetic bearing, the calculation of the magnetic force between the magnetic rings is an important element. Transforming complicated physical problems into mathematical problems by establishing the mathematical model of the magnetic ring is a common method when calculating the magnetic force. In this paper, the equivalent magnetic charge model of a permanent magnet bearing is established by the former's magnetic charge viewpoint, and the quadratic integral based on this model is solved by the integral definition method. This paper compares the results of integral definition method with those obtained by Monte Carlo method and ANSYS in order to verify the correctness.

1. Introduction

Compared with sliding bearings and rolling bearings, permanent magnetic bearings (PMB) have some unique advantages, such as non-friction, non-pollution, high efficiency and low energy consumption [1]. These characteristics of PMB drew attentions of industrial engineers in recent years. China, the major country with rare earth resources, has excellent conditions to develop and research PMB. In the design process of PMB, the method of calculating the magnetic force between the magnetic rings has always been the significant part [2-5].

At present, there are four common methods to solve the magnetic force of PMB, which are the equivalent magnetic charge method, molecular current method, virtual displacement method and finite element method [6-9]. However, for one hand, the mathematical model established by the equivalent magnetic charge method and the molecular current method are both required to solve a difficult quadratic integral, and the process for calculating the quadruple integral is also complicated; for another hand, the analytical solution is hardly to obtain. Some scholars use the Monte Carlo method to solve quadruple integrals [6-7]. They obtain an approximate solution of the similarity by the method of finding a similarity of probabilistic statistics and the experimental sampling process, which means the Monte Carlo method is actually a numerical calculation method based on the probability theory [10]. In fact, the integration accuracy and numerical stability of it depend on the size of the sample. In the actual programming process, the editor needs strong probabilistic foundation, to some extent, which limits the application of this method. In this paper, the integral definition method is used to solve the quadruple integral. The main idea is to divide the integral domain of the integrand into a finite number of small units. Firstly, we calculate the magnetic force between two small units in different planes, then add the forces between the small units in the entire integral domain, and the superposition of magnetic force results is an approximate calculation of the quadruple integral [11]. The accuracy of this result depends on the number of small units. The integral definition method is simple and intuitive to solve quadruplet integration, and it is easy to use MATLAB to compile the calculation program.
This paper selects the equivalent magnetic charge method and establishes the mathematical model of the magnetic ring based on this method, then obtains the quadruple integral calculation formula of the magnetic force, and uses the integral definition method to calculate the quadruple integral formula. The calculation obtained are compared with the results obtained by ANSYS simulation and Monte Carlo method to demonstrate the feasibility and validity of the method.

2. PMB equivalent magnetic charge model

The equivalent magnetic charge method is based on the following assumption [12]: the internal magnetic field of homogeneous magnetic medium is regarded as 0, the magnetic charge exists only on the two end faces of the magnetization direction, which have positive or negative magnetic charges. The magnetic field is generated by the equivalent magnetic charge at end faces. Just as the basic law of interaction between charges is Coulomb's law, the basic law of the interaction between magnetic charges is the Coulomb’s law of magnetism, which is the origin of magnetic charge theory. Therefore, the force between magnetic charges can be calculated by Coulomb’s law.

The point magnetic charges 1 and 2 are provided, the magnetic charge amounts are \( q_1 \) and \( q_2 \), and the vector between the two magnetic charges is \( r_{12} \) as shown in Figure 1. The point magnetic charge 1 generates a magnetic field strength \( H \) at the distance \( |r_{12}| \). If the point magnetic charge is in vacuum, the value of \( H \) is:

\[
\vec{H} = \frac{1}{4\pi \mu_0} \frac{q_1}{|r_{12}|} \vec{r}_{12}
\]

In equation (1), \( \mu_0 \) is the vacuum permeability.

If the point magnetic charge 2 is in the field strength \( H \) generated by the point magnetic charge 1, the force \( \vec{F}_{12} \) is:

\[
\vec{F}_{12} = q_2 \vec{H}
\]

Substituting equation (1) into equation (2), the force between two point magnetic charges is:

\[
\vec{F}_{12} = \frac{1}{4\pi \mu_0} \frac{q_1 q_2}{|r_{12}|} \vec{r}_{12}
\]

According to the integral definition method, calculate the magnetic force of all points in two end faces of the magnetic ring acting on the other permanent magnets in magnetic field, and then the total force on other permanent magnets can be obtained by summing the force vectors.
According to the equivalent magnetic charge theory, an axial permanent-magnet bearing model is established as shown in Figure 2. Two magnetic rings are same size and both axially magnetized. The equivalent magnetic charge are concentrated on the end faces, P₁, P₂, P₃, P₄. The lower magnetic ring 1 is a static magnetic ring, the upper magnetic ring 2 is the moving one, R₁ is the inner radius of the magnetic ring, Rₑ is the outer radius of the magnetic ring, and L is the magnetic ring thickness, g is the axial gap between the two magnetic rings, which varies with the displacement of the moving magnet ring in the positive direction of the z-axis, and e is the eccentricity. The moving magnetic ring 2 is subjected to the repulsion of the magnetic ring 1, and the repulsive force can be decomposed into an axial force \( F_z \) along the z-axis and a radial force \( F_x \) along the x-axis. \( F_z \) is the bearing capacity of the permanent magnetic bearing along the z-axis direction. For the mechanical equilibrium condition in the z-direction, \( F'_z = F_z \), so the calculation of the bearing capacity \( F'_z \) of the permanent magnet bearing is transformed into the calculation of the magnetic force \( F_z \) between the magnetic rings.

The equivalent (4) is the formula of magnetic force \( F_{14,z} \) between P₁, and P₄ that can be obtained according to the equivalent magnetic charge theory [13]. Similarly, the magnetic force equivalent (5),(6),(7) between other equivalent magnetic charge planes can be obtained. \( B_r \) is the remanence (the remanence of two ring is same), \( (r₁, α₁, r₂, α₂, r₃, β₃, r₄, β) \) are the position of any points on the magnetic surface P₁,P₂,P₃,P₄ in the polar coordinates as shown in Figure 3.

\[
F_{14,z} = \frac{B_r^2}{4\pi \mu_0} \int_0^{2\pi} \int_0^{R_1} \int_0^{R_2} \left( L_1 + g + L_2 \right) \cdot r_1 \cdot r_3 \cdot d\alpha \cdot d\beta
\]

\[
F_{13,z} = \frac{B_r^2}{4\pi \mu_0} \int_0^{2\pi} \int_0^{2\pi} \int_0^{R_1} \int_0^{R_2} \left( L_1 + g \right) \cdot r_1 \cdot r_3 \cdot d\alpha \cdot d\beta
\]

\[
F_{23,z} = \frac{B_r^2}{4\pi \mu_0} \int_0^{2\pi} \int_0^{2\pi} \int_0^{2\pi} \int_0^{R_1} \int_0^{R_2} \left( L_2 + g \right) \cdot r_1 \cdot r_3 \cdot d\alpha \cdot d\beta
\]
3. Solving quadruple integral with the integral definition method

Compared with Monte Carlo method for solving quadruple integrals, in which flow charts and knowledge of probability theory are required [6], it is more intuitive and easy to utilize the integral definition method.

In Figure 4, the two interacting equivalent magnetic charge surfaces in Figure 3 are respectively divided along their radial and circumferential directions. As shown in Figure 4, the two annular magnetic charge surfaces are respectively divided into \( N_{r1} \), \( N_{r2} \) rings radially and \( N_{d1}, N_{d2} \) sectors circumferentially. It transforms two rings into \((N_{r1} \times N_{d1})\) and \((N_{r2} \times N_{d2})\) small sectors accordingly [11]. \( P[i,j] \) and \( Q[k,l] \) represent the numbers of small sectors on the two rings, \( i \) and \( k \) denote the numbers of the rings from the inside out radially, \( j \) and \( l \) denote the sectors number divided clockwise from the \( x \) axis \((x' \text{axis})\). For example, point \( P \) in Figure 4 can be represented as \( P[4,7] \). When the amount of \( N_r \) and \( N_d \) is large enough, each small-sector magnetic charge unit can be regarded as a magnetic charge. We assume that all the sector magnetic charges on each unit are concentrated on their center points.

Then analyze the calculation process with Figure 4. \( P[i,j] \) and \( Q[k,l] \) respectively are small magnetic charge units on \( P_1 \), \( P_2 \). Their position are expressed in polar coordinates, then we get the formulas as below with geometry:

\[
r_{i}[i] = R_{r1} + i \cdot \frac{R_{d2} - R_{r1}}{N_{r1}}
\]  

(8)
The calculation program by using MATLAB ‘for’ loop structure can calculate the magnetic force of material of the magnetic ring is N35 for the axial permanent magnetic structure shown in Fig. 4.2.

According to Figure 4 and Equation (4), in equation (12), 

\[ \Delta F_{14} = \frac{1}{4\pi \mu_0} \frac{q_i q_j}{r_{14}} \]

In equation (12), \( q_i = B_i r[i] dr[i] da[j] \), \( q_j = B_j r[k] dr[k] d\beta[l] \)

\[ r_{14} = (r[k] \cos \beta[l] + e - r[i] \cos \alpha[j]) \hat{i} + (r[i] \cos \alpha[j] - r[k] \cos \beta[l]) \hat{j} + (L_1 + g + L_2) \hat{k} \]

According to Figure 4 and Equation (4), \( r[i] dr[i] da[j] \), \( r[k] dr[k] d\beta[l] \) are differential area elements, so there are:

\[ r[i] dr[i] da[j] = \Delta S_1 = \frac{\pi}{N_{z1}} \cdot (r_{o1}^2[i] - r_{o1}^2) \]

\[ r[k] dr[k] d\beta[l] = \Delta S_4 = \frac{\pi}{N_{z4}} \cdot (r_{o4}^2[k] - r_{o4}^2) \]

After the above analysis, the magnetic force between the magnetic charge surfaces P1 and P4 can be represented as the superposition of the force between the small magnetic charges on the two magnetic charge planes. Equation (4) can be transformed into equation (15). Similarly, equation (5) (6) (7) can be transformed too.

\[ F_{14} = \frac{B}{4\pi \mu_0} \sum_{i=1}^{N_{z1}} \sum_{j=1}^{N_{z4}} (L_1 + g + L_2) \cdot \Delta S_1 \cdot \Delta S_4 \]

The magnetic force of the axial permanent magnetic bearing is a superposition of the force among four magnetic charge surfaces. The force between the two magnetic charge surfaces with the same sign is positive. Otherwise, the force is negative.

Equation (16) can be obtained:

\[ F_z = F_{14,z} - F_{15,z} + F_{23,z} - F_{24,z} \]

4. Example and verification

4.1. Related parameters of PMB
For the axial permanent magnetic structure shown in Figure 2, the dimensions and physical parameters of the magnetic ring are determined as follows: \( R_{i1} = R_{o2} = 38 \text{mm} \), \( R_{o1} = R_{i2} = 50 \text{mm} \), \( L_1 = L_2 = 12 \text{mm} \); the material of the magnetic ring is N35 NdFeB, remanence \( B_r = 1.2106 \text{T} \), coercive force \( H_c = 918266 \text{A/m} \).

4.2. Calculation accuracy with integral definition method
The calculation program by using MATLAB ‘for’ loop structure can calculate the magnetic force of the axial PMB. According to the above, the accuracy of the integral definition method depends on the
accuracy of the division of the magnetic charge surface, which is the number of \((N_{r1} \times N_{d1})\) and \((N_{r2} \times N_{d2})\). Low precision will result in a large error in the calculation result, thus losing the actual value, while excessive calculation precision will greatly increase the calculation time. How to choose the appropriate accuracy is also a problem that needs to be considered in the calculation.

The calculations of 4 different accuracy are shown in the Figure 5. It is obvious the magnetic force \(F_z\) obtained by the four kinds of calculation accuracy decreases as the gap \(g\) increases, which is in accordance with reality. And it is obvious when the calculation accuracy is \(N_{r1} \times N_{d1} \times N_{r1} \times N_{d1}=10 \times 90 \times 10 \times 90\), the result is different from other precisions; the second accuracy is \(N_{r1} \times N_{d1} \times N_{r1} \times N_{d1}=20 \times 90 \times 20 \times 90\), and the result is basically consistent with the other two high-precision calculations, but there are small discrepancies when the gap \(g\) is less than 3mm. It means the bigger the gap of the magnetic ring is, the lower the precision is required for calculation. Comparing the results of the two higher accuracy, \(N_{r1} \times N_{d1} \times N_{r1} \times N_{d1}=20 \times 180 \times 20 \times 180\) and \(N_{r1} \times N_{d1} \times N_{r1} \times N_{d1}=30 \times 180 \times 30 \times 180\), the curves tend to be consistent. When the calculation accuracy is \(N_{r1} \times N_{d1} \times N_{r1} \times N_{d1}=20 \times 180 \times 20 \times 180\), it is more appropriate due to reducing the time of calculation and the correctness of the result.

4.3. Comparison of integral definition method and other methods
In order to verify the results obtained by using the integral definition method, the calculation are compared with the results by using the Monte Carlo method and ANSYS, the results of comparison are shown in Figure 6.
Figure 6. Comparison of calculation results of different methods.

From Figure 6, it can be seen that the magnetic force calculated using the three methods has the same trend. On one hand, comparing the calculation results of the integral definition method and the Monte Carlo method, there is 5% deviation when gap $g$ is 1mm. As the gap $g$ increases, the deviation of the calculations of the two methods decreases. Because these two methods are based on the equivalent magnetic charge method, the Monte Carlo method has been proved the feasibility in engineering [6-7]. This shows that the quadratic integral based on the equivalent magnetic charge model can be solved by the integral definition method. This also shows that it is feasible to use the integral definition method to solve the quadruple integral based on the equivalent magnetic charge model. On the other hand, the results of the integral definition method are compared with the results of the ANSYS simulation. As gap $g$ is 1 mm, the deviation is the largest, which is 9%. During the most commonly used gap $g = 2 \sim 3$mm, the deviation is still less than 10%, which shows that this method can be used to solve magnetic forces in engineering.

5. Conclusions

Compared to using the Monte Carlo method to solve magnetic forces, the integration definition method is easy to program, and does not require a probabilistic knowledge. The results are the same as those obtained by the Monte Carlo method on the whole.

The results based on the integral definition method are compared with the results of ANSYS simulation. As the gap of the magnetic ring decreases, the deviation between the two results increases, but the maximum deviation is still less than 10%. This shows that the integral definition method can be applied in engineering.

The calculation accuracy of the integral definition method should be selected according to the gap of the magnetic ring. When the gap $g$ is less than 3mm, a higher calculation accuracy should be selected to ensure the precision of the result; in other cases, the accuracy can be appropriately reduced to save time of calculation.

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