A violation of the factorization theorem for "hard" inclusive production

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Abstract. This talk is a digest of Ref. [1], where a new mechanism for hard inclusive production, which leads to a violation of the factorization theorem (FT), is suggested. Numerical estimates for the effect are given for high energy hadron (nucleus) scattering.

INTRODUCTION

In this talk we consider a new mechanism for hard inclusive production, which violates the factorization theorem [2]. The suggested mechanism is present in any central hard inclusive production, such as heavy Higgs meson production and/or the high $p_T$ inclusive production of mini jets and jets. We illustrate this mechanism for the case of inclusive heavy Higgs production in nucleon - deutron high energy scattering.

The usual description of inclusive production of a Higgs meson with mass $M$ in a nucleon - deuteron interaction, is based on the factorization theorem [2], using which the cross section of interest is given by

$$\sigma(Higgs) = \int dx_1 dx_2 F_N^p(x_1, M^2) F_D^p(x_2, M^2) \sigma(hard),$$

(1)

where $F_D^p(x_2, M^2)$ denotes the parton distribution within the deuteron, and in the impulse approximation $F_D^p = 2 F_N^p$. $\sigma(hard)$ denotes the cross section for Higgs meson production in the parton - parton collision.
FACTORIZATION THEOREM AND AGK CUTTING RULES.

The factorization theorem (FT) can be proven for the nucleon - deuteron scattering using the AGK cutting rules [3]( see Ref. [4] and references therein for detail discussion of the AGK cutting rules in QCD). Indeed, let us assume that the FT holds for nucleon - nucleon scattering. For nucleon - deuteron scattering we have two contributions to the total cross section: the projectile nucleon interacts with one nucleon in the deuteron ( impulse approximation ) and it interacts with both nucleons in the deuteron ( Glauber corrections). The first process obviously leads to the FT of Eq.(1), while for the second one we can use the AGK cutting rules for the decomposition of the total cross section with respect to the multiplicity of produced particles. The relation between different contributions is:

\[ \sigma^{(0)}_{\text{tot}} : \sigma^{(1)}_{\text{tot}} : \sigma^{(2)}_{\text{tot}} = (1 + \rho^2) : -4 : 2 , \]

where \( \rho = \frac{\text{Re}A_N}{\text{Im}A_N} \) and \( A_N \) is the amplitude of the nucleon - nucleon interaction. \( \sigma^{(0)}_{\text{tot}} \) is the cross section of all processes where no particles were produced in the central rapidity region ( elastic scattering and single and double diffractive dissociation); \( \sigma^{(1)}_{\text{tot}} \) is the contribution to the total inelastic cross section, which has the same multiplicity distribution as in nucleon - nucleon inelastic cross section, and \( \sigma^{(2)}_{\text{tot}} \) is part of the total inelastic cross section for the nucleon - deuteron scattering which describes the inelastic interaction of the projectile nucleon with two nucleons in the deuteron.

One can see that Eq.(2) leads to the cancellation of the interaction with the two nucleon in the deuteron for the inclusive cross section of the particle in the central rapidity region. Indeed, \( \sigma^{(0)}_{\text{tot}} \) does not contribute to this production while two other contributions cancels since in the subprocess described by \( \sigma^{(2)}_{\text{tot}} \) there is two ways of the production of the particle of the interest. Therefore, the inclusive production can be described in the impulse approximation for which the FT is valid by our assumption. The natural question arises: what is wrong in our above discussion.

THE AGK CUTTING RULES AND THE INTERFERENCE DIAGRAMS.

Let us start with the short answer to the formulated question: we missed the interference diagrams given in Fig.1 and these diagrams give the new mechanism for centrally produced Higgs meson ( or for any other “hard” process).

Trying to describe what kind of processes we picture in Fig.1, one can see that Fig.1a shows the process where the Higgs, which is produced centrally, is separated by two large rapidity gaps (LRG) from the small final state multiplicities which occur on the edges of the rapidity plot. We define this process as a double Pomeron
exchange reaction, and denote its contribution as $\sigma_2^{(0)}$. The contribution of Fig.1b is denoted by $\sigma_2^{(1)}$. This is a mixed diagram, where the Higgs which is produced in a double Pomeron process, is superimposed on a normal uniform rapidity distribution typical of an inelastic nucleon-nucleon reaction. Finally, Fig.1c describes Higgs production as part of the nucleon-nucleon background to the rapidity distribution. We denote this contribution by $\sigma_2^{(2)}$. To obtain our final result we need to sum over all three of the above contributions, noting that these are not necessarily positive.

The nucleon-nucleon (NN) amplitude for Higgs production via double Pomeron exchange, has been calculated \cite{5}, to be

$$A_H = A(NN \rightarrow N + (LRG) + H + (LRG) + N) = 2g_H A_P , \tag{3}$$

$g_H$ is the vertex of the hard parton - parton $\rightarrow$ Higgs process and $A_P$ is the Pomeron exchange amplitude.

The second ingredient in our calculation is the amplitude shown in Fig.1c (see also one of the amplitude in Fig.1b). This amplitude has no analog in the case of a single nucleon-nucleon interaction, and it depicts the cut in the diagram shown in Fig.1a. We note that this diagram is equal to $Im A_H$, unlike the case for inelastic nucleon-nucleon cross section where $\sigma_{in} = 2Im A_P$. The above result follows from the unitarity constraint \cite{3}.

Recalling that the integration over the longitudinal components of the momentum carried by the Pomeron in Fig.1b results in a negative sign for the interference diagram (see Ref. \cite{3} for details), we can easily calculate all diagrams contributing to the inclusive Higgs meson production. Indeed, calculating $\sigma_2^{(0)}$, the sum of the diagrams in Fig.1a, and using Eq.(3), one has
\[\sigma_2^{(0)} = 4g_H^2 \left\{ (ReA_P)^2 + (ImA_P)^2 \right\}. \quad (4)\]

For the sum of diagrams in Fig.1b we have
\[\sigma_2^{(1)} = -8g_H^2 (ImA_P)^2. \quad (5)\]

Finally, for \(\sigma_2^{(2)}\), shown in Fig.1c, we obtain
\[\sigma_2^{(2)} = 4g_H^2 (ImA_P)^2. \quad (6)\]

The relation between the different contributions is
\[\sigma_2^{(0)} : \sigma_2^{(1)} : \sigma_2^{(2)} = (1 + \rho^2) : -2 : 1, \quad (7)\]
where \(\rho = \frac{ReA_P}{ImA_P}\).

Summing all contributions we obtain an additional term in the cross section compare to the one given in Eq.(1)
\[\sigma(N + D \rightarrow H + X) = 4g_H^2 (ReA_P)^2. \quad (8)\]

We would like to draw attention to the novel fact, that the contribution of the new mechanism is proportional to the real part of the amplitude. For a soft Pomeron with \(\alpha_P(0) = 1\), the contribution of Eq.(8) vanishes, and we recover the factorization theorem [2], i.e. an exact cancellation of the diagrams shown in Fig.1c. However, for the hard processes (“hard” Pomeron) both theory and experiment lead to a steep behaviour of the cross section as a function of energy (see Ref. [1]) which results in considerable violation of the factorization theorem for totally inclusive production. Indeed, at high energy (low \(x\)) \(\rho\) is equal to
\[\rho = \frac{ReA_P}{ImA_P} = \frac{\pi}{2} \frac{dImA_P}{dln(1/x)} = <\omega>. \quad (9)\]

In all available parametrization for the deep inelastic structure function \(<\omega> \approx 0.3 - 0.5\) at \(10^{-2} < x < 10^{-5}\).

**NUMERICS.**

To give a quantative measure of the effect of the nonfactorization we plot in Fig.2 the ratio of the nonfactorized contribution (see Ref. [1] for the formula of this contribution) to the cross section from the factorization theorem of Eq.(1), namely
\[R = \frac{2\pi^2}{\int_{Q_0^2}^\infty dk_1^2 \frac{d\phi_1(x_1,k^2)}{dln(1/x_1)} \phi_2(x_2,k^2)} \left(\frac{R_1}{R_2}\right)^2 \frac{x_1G_1(x_1,M^2/4)x_2G_2(x_2,M^2/4)}{x_1G_1(x_1,M^2/4)x_2G_2(x_2,M^2/4)}, \quad (10)\]
where \( x_i G_i(x_i, M^2) \) is the gluon density of the \( i \)-th hadron and \( \phi \) is the unintegrated gluon density, which closely related to the gluon structure function, namely, \( \alpha_S(Q^2) x G(x, Q^2) = \int Q^2 dk^2 \alpha_S(k^2) \phi(x, k^2) \). In Fig. 2 we took \( y = 0 \), where \( y \) is rapidity of produced Higgs meson. Notice, that this ratio does not depend on the hard cross section, and as such is also applicable to any central production process, in particular, hard minijet and jet production. In evaluating Eq.(10) we use [1] the GRV parametrization [6] for the gluon density. We assume that \( R_1^2 = R_2^2 = 5 \text{ GeV}^{-2} \) (see Ref. [7]). For the initial virtuality we take \( Q_0^2 = 1 \text{ GeV}^2 \), as the GRV parametrization is in agreement with the HERA data on \( F_2(x, Q^2) \) [8] for all \( Q^2 \geq 1 \text{ GeV}^2 \).

One can see, that \( R \) is quite big for low masses and decreases when \( M \) increases. For example, \( R = 1.2 \) for \( M = 10 \text{ GeV} \). We do not expect the Higgs meson mass to be that small, but our result is also applicable in the case of jet production where \( M \approx 2p_\perp \). Accordingly, with such a value of \( R \) we can expect minijet production (jets with \( p_\perp \approx 5\text{ GeV} \)) which can be responsible for the structure of the minimum bias events at the Tevatron energy.

The value of \( R \) is bigger for nucleus - nucleus interaction. To estimate this value we need to multiply Eq.(10) by a factor of \( A_{1\text{eff}}^\frac{1}{2} A_{2\text{eff}}^\frac{1}{2} \). Using the simple relation \( R_A^2 = r_0^2 A_{1\text{eff}}^\frac{1}{2} A_{2\text{eff}}^\frac{1}{2} \), one has \( A_{\text{eff}} = \frac{R_A^2}{r_0^2} (A_{1\text{eff}}^{\frac{1}{2}} - 1) + 1 \). Therefore, for a gold - gold interaction we expect that \( R \) is enhanced by factor 4 - 9.

**CONCLUSIONS.**

Our conclusion is very simple: the factorization theorem [2] is only approximate and the violation of the FT can be rather big especially for heavy ion collisions and/or for hadron ones at the LHC energies. We firmly believe that the formal
proof of the FT given in Ref. [2] should be reconsidered to find out the correct region of the applicability of this theorem.

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