Detection of supernova neutrinos with neutrino-iron scattering

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The $\nu_e$-$^{56}$Fe cross section is evaluated in the projected quasiparticle random phase approximation (PQRPA). This model solves the puzzle observed in RPA for nuclei with mass around $^{12}$C, because it is the only RPA model that treats the Pauli Principle correctly. The cross sections as a function of the incident neutrino energy are compared with recent theoretical calculations of similar models. The average cross section weighted with the flux spectrum yields a good agreement with the experimental data. The expected number of events in the detection of supernova neutrinos is calculated for the LVD detector, leading to an upper limit for the electron neutrino energy of particular importance in this experiment.

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I. INTRODUCTION

A careful knowledge of the semileptonic weak interactions in nuclei allows the possibility of testing implications of physics beyond the standard model, such as exotic properties of neutrino oscillations and massiveness. The dynamics of supernova collapse and explosions as well as the synthesis of heavy nuclei are strongly dominated by neutrinos. For example, neutrinos carry away about 99% of gravitational binding energy in the core collapse of a massive star, and only a small fraction ($\sim$1%) is transferred to the stalled shock front, creating ejected neutrino fluxes observed in supernova remnants [1].

It was shown in Ref. [2] that accurate nuclear structure calculations are essential to constrain the neutrino oscillation parameters of the LSND experiment [3]. This was also noted in previous works, e.g., Hayes and Towner [4]. This work, based on a shell-model, which includes configuration mixing, is also supported by the results of a similar calculation described in Ref. [5]. This shows the importance of including configuration mixing (as done in both references) for this nucleus. The importance of configuration mixing in $^{12}$C is known since the very first work in p-shell nuclei by Cohen and Kurath in '65 [6]. Nevertheless the quasiparticle random phase approximation (QRPA) predictions of Ref. [5] do not yield good results for this nucleus because the configuration mixing is not properly accounted for and the projection procedure (as done in Ref. [7]) is not included. In particular, the employment of the projected quasiparticle random phase approximation (PQRPA) for the inclusive $^{12}$C($\nu_e, e^-$)$^{12}$N cross section, instead of the continuum RPA (CRPA) used by the LSND collaboration in the analysis of $\nu_\mu \rightarrow \nu_e$ oscillations of the 1993–1995 data sample, leads to an increased oscillation probability. Then, the previously found consistency between the $(\sin^2 2\theta, \Delta m^2)$ confidence level regions for the $\nu_\mu \rightarrow \nu_e$ and the $\bar{\nu}_\mu \rightarrow \bar{\nu}_e$ oscillations is decreased [2].

The measured observables are flux-averaged cross sections. The KARMEN Collaboration measured charged and neutral cross sections induced on $^{12}$C [8]. They also measured (the only experimental data for a medium-heavy nucleus) the neutrino reaction $^{56}$Fe($\nu_e, e^-$)$^{56}$Co from $e^-$-bremsstrahlung with the detector surrounding shield [8]. This cross section is important to test the ability of nuclear models in explaining reactions on nuclei with masses around iron, which play an important role in supernova collapse [9]. Experiments on neutrino oscillations such as MINOS [10] use iron as material detector, and future experiments, such as SNS at ORNL [11], plan to use the same material.

In a recent work, Agafonova et al. [12] studied the effect of neutrino oscillations on the supernova neutrino signal with the LVD detector. This detector studies supernova neutrinos through their interactions with protons and carbon nuclei in a liquid scintillator and with iron nuclei in the support structure. Several estimates on deviations of the detected signal arising from different constraints on the astrophysical parameters, the oscillation parameters, and the nonthermal nature of the neutrino fluxes were studied before [12]. Nevertheless, in all their estimates the corresponding $\nu$-nucleus cross sections were kept within strict limits.

In this work, we calculate the $\nu_e$-$^{56}$Fe cross sections using QRPA and PQRPA models to account for allowed and forbidden transitions. The present calculations are the first within the PQRPA framework for this purpose. In Ref. [13] it was established that PQRPA is the proper theory to treat both short-range pairing and long-range random phase (RPA) correlations. When QRPA was implemented for the triad $^{12}$B, $^{12}$C, $^{12}$N there were difficulties in choosing the ground state of $^{12}$N because the lowest state is not the most collective [5]. PQRPA solves this puzzle because it treats correctly the Pauli Principle, yielding better results for the distribution of the Gamow-Teller (GT) strength. This problem does not exist in heavier nuclei, where the neutron excess allows QRPA to account for pairing and RPA correlations [14]. In the case of medium-heavy nuclei, such as $^{56}$Fe, the consequences of the projection technique procedure can be manifest.

Many calculations of the $^{56}$Fe($\nu_e, e^-$)$^{56}$Co cross sections with microscopic and global models have been reported previously. The first were shell model calculations developed by Bugaev et al. [15]. They obtained the $\nu$-nucleus cross sections as a function of the incident neutrino energy. A second estimate was obtained by Kolbe et al. [16] using a nuclear Hybrid model: a shell model for the GT and Fermi (F) transitions and a continuum RPA (CRPA) for forbidden
transitions. This cross section was employed to estimate the number of events from \( v_e \)-\(^{56}\)Fe reactions in the LVD detector [12]. Lazauskas and Volpe [17] used QRPA with the Skyrme force to explore the possibility of performing nuclear structure studies using neutrinos from low energy \( \beta \) beams [18]. Several \( v_\nu \)-nucleus cross sections for different nuclei were also obtained recently with the relativistic QRPA (RQRPA) [19]. The \( v_\nu ^{-}\)\(^{56}\)Fe cross sections were also described with the gross theory of \( \beta \) decay (GTBD) [20].

II. NEUTRINO-NUCLEUS SCATTERING

The cross section for \( v_\nu + (Z, A) \rightarrow (Z + 1, A) + e^- \) is given by

\[
\sigma(E_\nu, J_f) = \frac{|p_\nu|E_\nu}{2\pi} F(Z + 1, E_\nu) \int_{-1}^{1} d(\cos \theta) T_\nu(|k|, J_f),
\]

where \( F(Z + 1, E_\nu) \) is the usual scattering Fermi function, \( k = p_\nu - q_\nu \) is the momentum transfer, \( p_\nu \) and \( q_\nu \) are the corresponding electron and neutrino momenta, and \( \theta \equiv \cos^{-1} \hat{q}_\nu \cdot \hat{p}_\nu \) is the angle between the incident neutrino and the emerging electron. The \( \sigma(E_\nu, J_f) \) cross section is obtained within first-order perturbation theory according to Ref. [7], where velocity-dependent terms are included in the weak effective Hamiltonian. The transition amplitude \( T_\nu(|k|, J_f) \) depends on the neutrino leptonic traces and on the nuclear matrix elements (NME), as explained in Ref. [7]. Here, the NME are evaluated in QRPA and in PQRPA. We employ the \( \delta \) interaction (in MeV fm\(^3\))

\[
V = -4\pi (v_s P_s + v_l P_l) \delta(r),
\]

with different coupling constants \( v_s \) and \( v_l \) for the particle-hole, particle-particle, and pairing channels. This interaction was used in Refs. [21–24], leading to a good description of single and double \( \beta \) decays.

For \(^{56}\)Fe we work within a configuration space of 12 single-particle levels, including the oscillator shells \( 2h_\omega, 3h_\omega, \) and \( 4h_\omega \). The single-particle energies of the active \( 3h_\omega \) shell correspond to the experimental energies of \(^{56}\)Ni. For the other \( 2h_\omega \) and \( 4h_\omega \) shells we used the harmonic oscillator energies with \( \hbar \omega / \text{MeV} = 45A^{1/3} - 25A^{2/3} \). The parameters \( v_s^{\text{par}}(p) \) and \( v_s^{\text{par}}(n) \) were obtained with the procedure of Ref. [25], i.e., by fitting the experimental gap pairing energies of protons and neutrons, \( \Delta_{n,p}(N, Z) \) (Eq. (2.96) of Ref. [26]), to \( \Delta_{n,p} \) defined by the usual BCS equations. The BCS or PBCS equations were solved in the full space of three oscillator shells. For the particle-hole channel we used \( v_s^{\text{ph}} = 27 \) and \( v_s^{\text{ph}} = 64 \) (in MeV fm\(^3\)). These values were fitted to \(^{48}\)Ca from a systematic study of the GT resonances [24] and shown to yield a good description of double \( \beta \) decay. For the particle-particle channel, it is convenient to define the parameters

\[
s = \frac{2v_p^{pp}}{v_s^{\text{par}}(p) + v_s^{\text{par}}(n)}
\]

and

\[
t = \frac{2v_p^{pp}}{v_s^{\text{par}}(p) + v_s^{\text{par}}(n)},
\]

associated with the coupling constant of the \( T = 1, S = 0 \) (singlet) and \( T = 0, S = 1 \) (triplet) channels, respectively [24]. We adopt \( s \approx 1 \), which restores the isospin symmetry in QRPA for \( N > Z \). As the experimental errors in the averaged cross sections are very large, the agreement of the theoretical cross section is not sufficient to select the best nuclear structure calculation and other observables must be found. We use the behavior of the \( B(GT^\pm) \) strength as a function of the parameter \( t \) to conclude that better results could be obtained when the particle-particle channel is off, \( t = 0 \). With this value the theoretical value \( B(GT^-) = 17.7 \) overestimates the experimental value \( (9.9 \pm 2.4 \) [28]) similarly to previous and more sophisticated QRPA calculations for \(^{56}\)Fe \( B(GT^-) = 18.68 \) [29]) with the Skyrme force.

III. NUMERICAL RESULTS

The flux averaged inclusive cross section reads

\[
\langle \sigma \rangle = \int dE_\nu \sigma(E_\nu)n(E_\nu),
\]

where \( \sigma(E_\nu) = \sum_f \sigma_s(E_\nu, J_f^+) \) is the inclusive cross section as a function of the neutrino energy and \( n(E_\nu) \) is the neutrino normalized flux. As a first test, we fold the \( \sigma_e(E_\nu) \) with the Michel energy spectrum [16],

\[
n(E_\nu) = \frac{96E_\nu^5(M_\mu - 2E_\nu)}{M_\mu^4},
\]

where \( M_\mu \) is the muon mass. In Table I we compare our \(^{56}\)Fe \( (v_\nu, e^-) \) \(^{56}\)Co cross section \( \langle \sigma \rangle \) in QRPA and PQRPA with other nuclear models for the energy window of \( \mu\)-Decay-At-Rest (DAR) neutrinos that the KARMEN experiment observed.

From Table I we note that our results for \( \langle \sigma \rangle \) are \( 264.6 \times 10^{-42} \) cm\(^2\) (QRPA) and \( 197.3 \times 10^{-42} \) cm\(^2\) (PQRPA) are

| Model      | \( \langle \sigma \rangle \) |
|------------|-----------------|
| QRPA       | 264.6           |
| PQRPA      | 197.3           |
| Hybrid\(^a\) [16] | 228.9          |
| Hybrid\(^a\) [16] | 238.1          |
| TM [30]    | 214             |
| RPA [31]   | 277             |
| QRPA\(^a\) [17] | 352            |
| RQRPA [19] | 140             |
| Exp [8]    | 256 \pm 108 \pm 43 |

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\(^a\) superscript “a” denotes partial occupation in the ground state and a superscript “b” denotes no occupation.
in agreement with the experimental value $256 \pm 83 \text{(stat)} \pm 42 \text{(syst)} \times 10^{-42} \text{ cm}^2$ [8]. The main difference between QRPA and PQRPA, both solved consistently with the same interaction, shows that the projection procedure is important in a medium mass nucleus such as $^{56}\text{Fe}$.

In Fig. 1 we plot the inclusive $^{56}\text{Fe}(\nu, e^-)^{56}\text{Co}$ cross sections (in $10^{-42} \text{ cm}^2$) evaluated in QRPA (dashed line) and PQRPA (solid line) in the DAR region. A comparison is shown with other nuclear structure models: GTBD (dashed-dot-dot-dot line) [20], Hybrid (dashed-dot line) [27], QRPA$_{s}$ (dashed-dot dot line) [17], and RQRPA(dashed line) [19].

The number of events detected for supernova explosions is calculated as

$$N_\alpha = N_i \int_0^\infty \mathcal{F}_\alpha(E_\nu) \cdot \sigma(E_\nu) \cdot \epsilon(E_\nu) \, dE_\nu,$$

where the index $\alpha = \nu_e, \bar{\nu}_e, \nu_x$ and $(\nu_x = \nu_\tau, \nu_\mu, \bar{\nu}_\mu, \bar{\nu}_\tau)$ indicates the neutrino or antineutrino type, $N_i$ is the number of target nuclei, $\mathcal{F}_\alpha(E_\nu)$ is the neutrino flux, $\sigma(E_\nu)$ is the neutrino–nucleus cross section, $\epsilon(E_\nu)$ is the detection efficiency, and $E_\nu$ is the neutrino energy. Recent calculations by the LVD group [12] estimate that the $(\nu_\mu + \bar{\nu}_\mu)$ interactions on $^{56}\text{Fe}$ are almost 17% of the total detected signal.

The time spectra can be approximated by the zero-pinched Fermi–Dirac distribution. For the neutrino of flavor $\alpha$, it is

$$\mathcal{F}_\alpha^0(E_\nu, T_\alpha) = \frac{L_\alpha}{4\pi D^2 T_\alpha F_\alpha(0) \exp\left(E_\nu / T_\alpha\right) + 1},$$

where $D$ is the distance to the supernova, $E_\nu$ is the neutrino energy, $L_\alpha$ is the time-integrated energy of flavor $\nu_\alpha, T_\alpha$ is the neutrino effective temperature, and $F_\alpha(0) \equiv \int_0^\infty d^3 \mathbf{x} \mathcal{F}_\alpha(E_\nu) / (e^{E_\nu / T_\alpha} + 1)$ is the normalization factor. For a galactic supernova explosion at a typical distance $D = 10 \text{ kpc}$, it was assumed that the total binding energy for each flavor is $L_\alpha = f_\nu E_b$, with $E_b = 3 \times 10^{53} \text{ erg}$ and a perfect energy equipartition between the neutrino flavors, $f_\nu = f_{\bar{\nu}} = f_\nu = 1 / 6$. Hence, it is possible to assume that the fluxes $(\nu_\mu, \nu_\tau, \bar{\nu}_\mu, \bar{\nu}_\tau)$ are identical. Because the pinched factor was assumed to be zero for all neutrino flavors, we can fix the effective neutrino temperature as $T_\nu = 1.5 T_\nu$ and $T_\nu = 0.8 T_\nu$, leaving $T_\nu$ as a variable parameter in the interval studied in Ref. [12].

When the neutrinos escape from the star, they cross regions of different densities where a flavor transition could happen. Usually one assumes two resonance layers, which we call Mikheyev-Smirnov-Wolfenstein (MSW) resonances throughout the text (see, for example, Ref. [32]). According to the mass scheme shown in Ref. [12], the observed electron neutrino fluxes ($\mathcal{F}_{\nu_e}, \mathcal{F}_{\bar{\nu}_e}$) originating from MSW resonances are linear combinations of the original neutrinos fluxes in the star, $\mathcal{F}_{\nu_\alpha}$ and $\mathcal{F}_{\bar{\nu}_\alpha}$, with coefficients governed by the crossing probability in the high density resonance layer, $P_R(\Delta m^2, \theta_{13})$. For simplicity, we only show differences that appear in the number of events calculated from the convolution of cross sections obtained with different nuclear structure models with the original supernova fluxes, i.e.,

$$N_\alpha \equiv N_\alpha(T_\alpha) = N_i \int_0^\infty \mathcal{F}_{\nu_\alpha}^0(E_\nu, T_\alpha) \cdot \sigma_\alpha(E_\nu) \cdot \epsilon_\alpha(E_\nu) \, dE_\nu,$$

for the “direct” electron neutrino event, and

$$\bar{N}_\alpha \equiv \bar{N}_\alpha(T_\alpha) = N_i \int_0^\infty \mathcal{F}_{\bar{\nu}_\alpha}^0(E_\nu, T_\alpha) \cdot \sigma_\alpha(E_\nu) \cdot \epsilon_\alpha(E_\nu) \, dE_\nu,$$

for the “direct” antineutrino event.
for the “indirect” number of events for electron neutrinos associated with the total $\nu_x$-flux coming from the contribution of $F^{0}_{\nu_e}$. Due to the MSW effect, electron neutrino fluxes mix with non-electron neutrino fluxes (i.e., $\nu_x \equiv \nu_\mu, \nu_\tau$), and therefore with the MSW resonance the $\nu_x$-s might get a “hot” contribution to their flux. Another important issue, not considered for simplicity in the present work, is the spectral swapping of the neutrino flux (Ref. [33]). Duan et al. have shown that certain numerical results in the simulation of neutrino and antineutrino flavor evolution in the region above the post-supernova explosion proto-neutron star cannot be easily explained with the conventional MSW mechanism Ref. [1].

For the neutrino reactions $^{56}\text{Fe}(\nu_e, e^-)^{56}\text{Co}$, we calculate $N_e$ and $\tilde{N}_e$ as a function of the neutrino temperatures $T_{\nu_e}$ and $T_{\nu_x}$, folding $\sigma_\nu(E_\nu)$ from different nuclear structure models with the neutrino fluxes $F_{\nu_e}^{0}(E_{\nu_e}, T_{\nu_e})$ and $F_{\nu_x}^{0}(E_{\nu_x}, T_{\nu_x})$, respectively. The limits for the temperatures, $T_{\nu_e}$ and $T_{\nu_x}$, were obtained from the interval $T_{\nu} \in [4, 7]$ MeV and the relations $T_{\nu_e} = 1.5T_{\nu}$ and $T_{\nu_x} = 0.8T_{\nu}$, employed by the LVD group [12]. The $\epsilon(E_{\nu_e})$ efficiency is taken from Fig. 1b of Ref. [12]. The results obtained are shown in Fig. 3. The left panel shows the number of events $N_e$, with different $\sigma_\nu(E_\nu)$, our QRPA and PQRPA, QRPA$_S$ [17], RQRPA [19], and the Hybrid model [16] employed by the LVD detector. In the right panel we show the number of events $\tilde{N}_e$. Although one knows that $\nu_x$ neutrinos at supernova energies can only induce neutral current events, we evaluate this quantity because it will be modified by MSW oscillations according to the scheme presented in Eqs. (2) and (4) of Ref. [12] or in Eqs. (10) and (12) of Ref. [34]. Despite the fact that $\tilde{N}_e$ can be obtained from the expression of $N_e$ by extending its $T_{\nu_e}$-dependence to 10.5 MeV, this lacks physical meaning. The $T_{\nu_e}$ region of electron-neutrino temperatures of interest for supernovae explosions is $T_{\nu_e} < 6$ MeV, based on supernovae modeling with neutrino transport [35].

We note that $N_e$ and $\tilde{N}_e$ increase with the temperatures $T_{\nu_e}$ and $T_{\nu_x}$. The increase for each $N_e$ follows the increase of the different $\sigma_\nu$. The contribution of the neutrino flux, $F_{\nu_e}^{0}$, in $N_e$ is strongly concentrated in the region below 60 MeV. This is because (i) the mean neutrino energy $\langle E_{\nu_e} \rangle$ of the flux varies from 10.1 to 17.6 MeV approximately [36] and (ii) the contribution of the product of $\sigma_\nu$ with the flux tail is not important. Notice that the ordering of the $N_e$ in Fig. 3 is the same as the ordering of $\sigma_\nu$ shown in Fig. 1. For example, the crossing between $N_e$’s of our QRPA and Hybrid model at $T_{\nu_e} \sim 4.8$ MeV originates from the crossing of the corresponding $\sigma_\nu$ at $E_{\nu} \sim 32$ MeV as Fig. 1 shows.

The above behavior also applies to $\tilde{N}_e$, but they are shifted according to the shift that $F_{\nu_e}^{0}$ has with respect to $F_{\nu_e}^{0}$. This means that the main contribution to $\tilde{N}_e$ comes from the convolution of $F_{\nu_e}^{0}$ with $\sigma_\nu$ in the energy interval [18.9, 33.1] MeV where the larger energy flux of $\nu_e$ is concentrated. The right panel of Fig. 3 shows additional crossings at $T_{\nu_e} \sim 10.5$ MeV, which are the result of the corresponding crossings of $\sigma_\nu$(QRPA-QRPA$_S$) at $E_{\nu} \sim 56$ MeV and $\sigma_\nu$(PQRPA-RQRPA).
at $E_{\nu} \sim 60$ MeV, shown in Fig. 1. We conclude that the relevant energy interval for the $\nu_e$-$^{56}$Fe reaction is $E_{\nu} \lesssim 60$ MeV for the astrophysical parameters adopted in LVD.

**IV. CONCLUSIONS**

In summary, we employed the projected QRPA to calculate the $^{56}$Fe($\nu_e$, $e^-$)$^{56}$Co cross section. The calculated cross section was compared with a QRPA calculation with the same interaction showing that the projection procedure is important for medium mass nuclei. The cross section was also compared with other RPA and Hybrid models. The PQRPA yields a cross section smaller than those of almost all RPA models with the exception of relativistic QRPA [19] for $E_{\nu} \lesssim 60$ MeV. Above this energy and up to $E_{\nu} = 100$ MeV, the PQRPA leads to the smallest cross section. Therefore, we feel that a more detailed study of allowed and forbidden transitions in the region below $E_{\nu} = 100$ MeV is imperative, both experimentally and theoretically. In particular, the region with $E_{\nu} \lesssim 60$ MeV is the most important for the LVD detector [12]. In a future work we plan to include the MSW effect in the same way as was done by Agafonova et al. [12] and an explicit account of the uncertainties in the supernova neutrino flux will be considered.

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[1] H. Duan, G. M. Fuller, J. Carlson, and Y.-Z. Qian, Phys. Rev. D 74, 105014 (2006).
[2] A. Samana, F. Krmpotić, A. Mariano, and R. Zukananovich Funchal, Phys. Lett. B642, 100 (2006).
[3] C. Athanassopoulous et al. (LSND Collaboration), Phys. Rev. C 58, 2489 (1998); Phys. Rev. Lett. 81, 1774 (1998).
[4] A. C. Hayes and I. S. Towner, Phys. Rev. C 61, 044603 (2000).
[5] C. Volpe, N. Auerbach, G. Colò, T. Suzuki, and N. Van Giai, Phys. Rev. C 62, 015501 (2000).
[6] S. Cohen and D. Kurath, Nucl. Phys. 73, 1 (1965).
[7] F. Krmpotić, A. Samana, and A. Mariano, Phys. Rev. C 71, 044319 (2005).
[8] R. Maschuw et al. (KARMEN Collaboration), Prog. Part. Nucl. Phys. 40, 183 (1998).
[9] S. E. Woosley, D. Hartmann, R. D. Hoffman, and W. C. Haxton, Astrophys. J. 356, 272 (1990).
[10] P. Adamson et al. (MINOS Collaboration), Phys. Rev. D 76, 072005 (2007).
[11] Yu. Efremenko, Nucl. Phys. B138 (Proc. Suppl), 343 (2005); F. T. Avignone III and Yuri V. Efremenko, J. Phys. G 29, 2615 (2003).
[12] N. Yu. Agafonova et al., Astroparticle Phys. 27, 254 (2007).
[13] F. Krmpotić, A. Mariano, and A. Samana, Phys. Lett. B541, 298 (2002).
[14] F. Krmpotić, A. Mariano, T. T. S. Kuo, and K. Nakayama, Phys. Lett. B319, 393 (1993).
[15] E. V. Bugaev, G. S. Bisnovaty-Kogan, M. A. Rudzsky, and Z. F. Seidov, Nucl. Phys. A324, 350 (1979).
[16] E. Kolbe, K. Langanke, and G. Martínez-Pinedo, Phys. Rev. C 60, 052801(R) (1999).
[17] R. Lazauskas and C. Volpe, Nucl. Phys. A792, 219 (2007).
[18] Cristina Volpe, J. Phys. G 30, L1 (2004).
[19] N. Paar, D. Vretenar, T. Marketin, and P. Ring, Phys. Rev. C 77, 024608 (2008).
[20] N. Itoh and Y. Kohyama, Nucl. Phys. A306, 527 (1978).
[21] J. Hirsch and F. Krmpotić, Phys. Rev. C 41, 792 (1990).
[22] J. Hirsch and F. Krmpotić, Phys. Lett. B246, 5 (1990).
[23] F. Krmpotić, J. Hirsch, and H. Dias, Nucl. Phys. A542, 85 (1992).
[24] F. Krmpotić and Shelly Sharma, Nucl. Phys. A572, 329 (1994).
[25] J. Hirsch and F. Krmpotić, Phys. Rev. C 41, 792 (1990).
[26] A. Bohr and B. R. Mottelson, Nuclear Structure (Benjamin, New York/Amsterdam, 1969), Vol. I.
[27] E. Kolbe and K. Langanke, Phys. Rev. C 63, 025802 (2001).
[28] J. Rappaport et al., Nucl. Phys. A410, 371 (1983).
[29] P. Sarriuguren, E. Moya de Guerra, and R. Álvarez-Rodríguez, Nucl. Phys. A716, 230 (2003).
[30] S. L. Mintz, J. Phys. G 28, 451 (2002).
[31] M. S. Athar, A. Ahmad, and S. K. Singh, Nucl. Phys. A764, 551 (2006).
[32] E. Kh. Akhmedov, Lectures given at Trieste Summer School in Particle Physics, June 7–9, 1999, arXiv:hep-ph/0001264v2.
[33] H. Duan, G. M. Fuller, J. Carlson, and Y.-Z. Qian, Phys. Rev. Lett. 99, 241802 (2007).
[34] G. L. Fogli, E. Lisi, A. Mirizzi, and D. Montanino, J. Cosmol. Astropart. Phys. 4, 2 (2005).
[35] M. Th. Keil, G. G. Raffelt, and H.-T. Janka, Astrophys. J. 590, 971 (2003).
[36] The mean neutrino energy results $\langle E_{\nu} \rangle \approx 3.15T_{\nu}$ with the pinching parameter $\eta = 0$. 

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