Finite formation time effects in inclusive and semi-inclusive electro-disintegration of few-body nuclei

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Finite Formation Time (FFT) effects in the exclusive reaction $^4\text{He}(e,e'p)^3\text{H}$ at high values of $Q^2$ are introduced and discussed. It is shown that the minimum in the momentum distributions predicted by the Plane Wave Impulse Approximation (PWIA), which is filled by the Glauber-type Final State Interaction (FSI), is completely recovered when FFT effects are taken into account. The semi-inclusive process $^4\text{He}(e,e'p)X$ is also investigated.

1. Introduction

Recently \cite{1} the Glauber approach to Final State Interaction (FSI) in the inclusive quasi-elastic $A(e,e'p)X$ process, has been extended by taking into account the virtuality of the hit nucleon after $\gamma^* \gamma$ absorption. It has been found that at large $Q^2$, due to the fact that the hit nucleon need a Finite Formation Time (FFT) to reach its asymptotic form, the interaction with the remainder of the target nucleus becomes very weak and vanishes in the asymptotic limit. In this contribution, we present the results of a calculation based upon the extension of the method of \cite{1} to the exclusive, $^4\text{He}(e,e'p)^3\text{H}$, and semi-inclusive, $^4\text{He}(e,e'p)X$, processes.

2. The Distorted momentum distributions

Under certain approximations whose validity will not be discussed here (see e.g. \cite{2}), the cross section for the process $A(e,e'p)X$, can be shown to have the following form

$$\frac{d^5 \sigma}{d\vec{k}_e d\vec{k}_p d\Omega_{\vec{k}_m}} = \mathcal{K} n_D(\vec{k}_m), \quad \vec{k}_m = \vec{q} - \vec{k}_p,$$

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where $\mathcal{K}$ is a kinematical factor and $n_D(\vec{k}_m)$ the nucleon distorted momentum distributions, which in the case of the semi-inclusive process $^4\text{He}(e,e'p)X$, are defined as follows

$$
n_D(\vec{k}_m) = (2\pi)^{-3} \int d\vec{r} d\vec{r}^\prime \exp(i\vec{k}_m \cdot (\vec{r} - \vec{r}^\prime)) \rho_D(\vec{r}, \vec{r}^\prime)
$$

with

$$
\rho_D(\vec{r}, \vec{r}^\prime) = \int d\vec{R}_1 d\vec{R}_2 \psi_\alpha^*(\vec{R}_1, \vec{R}_2, \vec{R}_3 = \vec{r}) S_G^{1,1} S_G^\prime \psi_\alpha(\vec{R}_1, \vec{R}_2, \vec{R}_3 = \vec{r}').
$$

being the one-body mixed density matrix. In the above equation $\psi_\alpha$ denotes the $^4\text{He}$ wave function, $\vec{R}_i$s are usual Jacobi coordinates $\vec{R}_1 = \vec{r}_2 - \vec{r}_1$, $\vec{R}_2 = \vec{r}_3 - (\vec{r}_1 + \vec{r}_2)/2$, $\vec{R}_3 = \vec{r}_4 - (\vec{r}_1 + \vec{r}_2 + \vec{r}_3)/3$, and $S_G$ is the Glauber operator.

In the exclusive process $^4\text{He}(e,e'p)^3\text{H}$, $n_D(\vec{k}_m)$ is given by

$$
n_D(\vec{k}_m) = |w(\vec{k}_m)|^2
$$

where

$$
w(\vec{k}_m) = (2\pi)^{-3/2} \int d\vec{r} \exp(-i\vec{k}_m \cdot \vec{r}) A(\vec{r})
$$

and

$$
A(\vec{r}) = \sqrt{4} \int d\vec{R}_1 d\vec{R}_2 d\vec{R}_3 \psi_1^*(\vec{R}_1, \vec{R}_2) S_G^{1,1} \psi_\alpha(\vec{R}_1, \vec{R}_2, \vec{R}_3 = \vec{r}),
$$

with $\psi_1$ denoting the $^3\text{H}$ wave function. In our calculations both the $^4\text{He}$ and $^3\text{H}$ wave functions, which correspond to the Reid Soft Core $V_8$ interaction, have been taken from $[3]$.

3. The Glauber operator and the Final State Interaction

The Glauber operator $S_G$ is explicitly given by $[2]$,

$$
S_G = \prod_{i=1}^3 G(4i), \quad G(4i) = 1 - \theta(z_i - z_4) \Gamma(\vec{b}_4 - \vec{b}_i),
$$

where the knocked out nucleon is denoted by "4". Here the $z$-axis is oriented along the direction of the motion of the knocked out proton, $\vec{b}$ is the component of the nucleon coordinate in the $xy$ plane, and $\Gamma$ stands for the usual Glauber profile function

$$
\Gamma(\vec{b}) = \frac{\sigma_{tot}(1 - i\alpha)}{4\pi b_0^2} e^{-\vec{b}/2b_0^2},
$$

with $\sigma_{tot}$ denoting the total proton-nucleon cross section, and $\alpha$ the ratio of the real to imaginary parts of the forward elastic $pN$ scattering amplitude. The value of $\sigma_{tot}$ and $\alpha$ were chosen at the proper values of the invariant mass of the process, and the numerical values were taken from ref. $[4]$, whereas the value of $b_0$ has been determined by using the relation $\sigma_{el} = \sigma_{tot}^2 (1 + \alpha^2)/16\pi b_0^2$.

When FFT effects are considered, the $G(4i)$ in eq. (7) is replaced by $[4]$

$$
G(4i) = 1 - J(z_i - z_4) \Gamma(\vec{b}_4 - \vec{b}_i), \quad J(z) = \theta(z)(1 - \exp(\frac{z x m M^2}{Q^2})),
$$

where $x$ is the Bjorken scaling variable, $m$ the nucleon mass, and $M$ represents the average virtuality defined by $M^2 = (m_{Av}^*)^2 - m^2$. Eq. (9) shows that at high values of $Q^2$ FFT effects reduce the Glauber-type FSI, depending on the value of $M$. In our calculations the value of the average excitation mass $m_{Av}^*$ was taken to be $1.8(GeV/c)$ $[4]$. 

4. Results of calculations

The distorted momentum distributions $n_D(\vec{k}_m)$ for the exclusive process $^4\text{He}(e,e'p)^3\text{H}$ reaction are shown in Fig. 1 versus the missing momentum $\vec{k}_m$, in correspondence of the parallel kinematics, i.e. when the missing momentum $\vec{k}_m$ is oriented along the virtual photon momentum $\vec{q}$.

Concerning the results presented in Fig. 1, the following remarks are in order. The PWIA predicts a minimum at $k \sim 2.2 \text{ (fm}^{-1})$, which is completely filled up by the Glauber-type FSI. The latter exhibits a very small $Q^2$ dependence, unlike FFT effects which depend on $Q^2$ in such a way that at $Q^2 = 20 \text{ (GeV}/c)^2$ they completely kill the FSI, so that the minimum is fully recovered.

The FFT effects on the semi-inclusive reaction $^4\text{He}(e,e'p)X$ at parallel kinematics are shown in Fig. 2 and it can be seen that here the effects from Glauber-type FSI and FFT, are less relevant.
Figure 2. The $Q^2$ dependence of $n_D(\vec{k})$ in the semi-inclusive $^4$He(e,e’p)X process. Dotted curve: undistorted $n(k_m)$; dashed (full) curve: Glauber-type FSI+FFT at $Q^2 = 2(20)(GeV/c)^2$.  

5. Summary

The results of the calculations that we have exhibited, show that the exclusive process $^4$He(e,e’p)$^3$H at high values of $Q^2$ could provide a clear cut check of various models which go beyond the treatment of FSI effects in terms of Glauber-type rescattering. In particular, the FFT approach of Ref. [1] predicts a clean and regular $Q^2$ behaviour leading to the vanishing of FSI effects at moderately large values of $Q^2$; such a prediction would be validated by the experimental observation of a dip in the cross section at $k_m \simeq 2.2 fm^{-1}$. Recently, Benhar et al [5] have analyzed the same process we have considered, viz. the $^4$He(e,e’p)$^3$H reaction, using a colour transparency model. At variance with our results, their model does not lead to the vanishing of FSI at $Q^2 \simeq 20(GeV/c)^2$. We conclude, therefore, that exclusive electron scattering off $^4He$ would really represent a powerful tool to discriminate various models of hadronic final state rescattering.

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