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A constructive analysis of convex-valued demand correspondence for weakly uniformly rotund and monotonic preference

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Abstract

Bridges([4]) has constructively shown the existence of continuous demand function for consumers with continuous, uniformly rotund preference relations. We extend this result to the case of multi-valued demand correspondence. We consider a weakly uniformly rotund and monotonic preference relation, and will show the existence of convex-valued demand correspondence with closed graph for consumers with continuous, weakly uniformly rotund and monotonic preference relations. We follow the Bishop style constructive mathematics according to [1], [2] and [3].

Keywords: constructive analysis, demand correspondence, weakly uniformly rotund and monotonic preference.

1 Introduction

Bridges([4]) has constructively shown the existence of continuous demand function for consumers with continuous, uniformly rotund preference relations. We extend this result to the case of multi-valued demand correspondence. We consider a weakly uniformly rotund and monotonic preference relation, and will show the existence of convex-valued demand correspondence with closed graph.

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for consumers with continuous, weakly uniformly rotund and monotonic preference relations

In the next section we summarize some preliminary results most of which were proved in [4]. In Section 3 we will show the main result.

We follow the Bishop style constructive mathematics according to [1], [2] and [3].

2 Preliminary results

Consider a consumer who consumes \( N \) goods. \( N \) is a finite natural number larger than 1. Let \( X \subset \mathbb{R}^N \) be his consumption set. It is a compact (totally bounded and complete) and convex set. Let \( \Delta \) be an \( n-1 \)-dimensional simplex, and \( p \in \Delta \) be a normalized price vector of the goods. Let \( p_i \) be the price of the \( i \)-th good, then \( \sum_{i=1}^{N} p_i = 1 \) and \( p_i \geq 0 \) for each \( i \). For a given \( p \) the budget set of the consumer is

\[
\beta(p, w) \equiv \{ x \in X : p \cdot x \leq w \}
\]

\( w > 0 \) is his initial endowment. A preference relation of the consumer \( \succ \) is a binary relation on \( X \). Let \( x, y \in X \). If he prefers \( x \) to \( y \), we denote \( x \succ y \). A preference-indifference relation \( \succeq \) is defined as follows;

\[
x \succeq y \text{ if and only if } \neg(y \succ x)
\]

\( x \succ y \) entails \( x \succeq y \), the relations \( \succ \) and \( \succeq \) are transitive, and if either \( x \succeq y \succ z \) or \( x \succ y \succeq z \), then \( x \succ z \). Also we have

\[
x \succeq y \text{ if and only if } \forall z \in X (y \succ z \Rightarrow x \succ z).
\]

A preference relation \( \succ \) is continuous if it is open as a subset of \( X \times X \), and \( \succeq \) is a closed subset of \( X \times X \).

A preference relation \( \succ \) on \( X \) is uniformly rotund if for each \( \varepsilon \) there exists a \( \delta(\varepsilon) \) with the following property.

**Definition 1** (Uniformly rotund preference). Let \( \varepsilon > 0 \), \( x \) and \( y \) be points of \( X \) such that \( |x - y| \geq \varepsilon \), and \( z \) be a point of \( \mathbb{R}^N \) such that \( |z| \leq \delta(\varepsilon) \), then either \( \frac{1}{2}(x + y) + z \succ x \) or \( \frac{1}{2}(x + y) + z \succ y \).

Strict convexity of preference is defined as follows;

**Definition 2** (Strict convexity of preference). If \( x, y \in X \), \( x \neq y \), and \( 0 < t < 1 \), then either \( tx + (1-t)y \succ x \) or \( tx + (1-t)y \succ y \).

Bridges [5] has shown that if a preference relation is uniformly rotund, then it is strictly convex.

On the other hand convexity of preference is defined as follows;

**Definition 3** (Convexity of preference). If \( x, y \in X \), \( x \neq y \), and \( 0 < t < 1 \), then either \( tx + (1-t)y \succeq x \) or \( tx + (1-t)y \succeq y \).

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We define the following weaker version of uniform rotundity.

**Definition 4** (Weakly uniformly rotund preference). Let \( \varepsilon > 0 \), \( x \) and \( y \) be points of \( X \) such that \( |x - y| \geq \varepsilon \). Let \( z \) be a point of \( R^N \) such that \( |z| \leq \delta \) for \( \delta > 0 \) and \( z \gg 0 \) (every component of \( z \) is positive), then \( \frac{1}{2}(x + y) + z \gg x \) or \( \frac{1}{2}(x + y) + z \gg y \).

We assume also that consumers’ preferences are monotonic in the sense that if \( x' > x \) (it means that each component of \( x' \) is larger than or equal to the corresponding component of \( x \), and at least one component of \( x' \) is larger than the corresponding component of \( x \)), then \( x' \gg x \).

Now we show the following lemmas.

**Lemma 1.** If \( x, y \in X \), \( x \neq y \), then weak uniform rotundity of preferences implies that \( \frac{1}{2}(x + y) \gtrless x \) or \( \frac{1}{2}(x + y) \gtrless y \).

**Proof.** Consider a decreasing sequence \( (\delta_m) \) of \( \delta \) in Definition 4. Then, either \( \frac{1}{2}(x + y) + z_m \gg x \) or \( \frac{1}{2}(x + y) + z_m \gg y \) for \( z_m \) such that \( |z_m| < \delta_m \) and \( z_m \gg 0 \) for each \( m \). Assume that \( (\delta_m) \) converges to zero. Then, \( \frac{1}{2}(x + y) + z_m \) converges to \( \frac{1}{2}(x + y) \). Continuity of the preference (closedness of \( \gtrless \)) implies that \( \frac{1}{2}(x + y) \gtrless x \) or \( \frac{1}{2}(x + y) \gtrless y \). \( \square \)

**Lemma 2.** If a consumer’s preference is weakly uniformly rotund, then it is convex.

This is a modified version of Proposition 2.2 in [5].

**Proof.**

1. Let \( x \) and \( y \) be points in \( X \) such that \( |x - y| \geq \varepsilon \). Consider a point \( \frac{1}{2}(x + y) \). Then, \( |x - \frac{1}{2}(x + y)| \geq \frac{\varepsilon}{2} \) and \( |\frac{1}{2}(x + y) - y| \geq \frac{\varepsilon}{2} \). Thus, using Lemma 1 we can show \( \frac{1}{2}(3x + y) \gtrless x \) or \( \frac{1}{2}(3x + y) \gtrless y \), and \( \frac{1}{2}(x + 3y) \gtrless x \) or \( \frac{1}{2}(x + 3y) \gtrless y \). Inductively we can show that for \( k = 1, 2, \ldots, 2^n - 1 \), \( \frac{k}{2^n}x + \frac{2^n - k}{2^n}y \gtrless x \) or \( \frac{k}{2^n}x + \frac{2^n - k}{2^n}y \gtrless y \) for each natural number \( n \).

2. Let \( z = tx + (1 - t)y \) with a real number \( t \) such that \( 0 < t < 1 \). We can select a natural number \( k \) so that \( \frac{k}{2^n} \leq t \leq \frac{k + 1}{2^n} \) for each natural number \( n \). \( (\frac{k + 1}{2^n} - \frac{k}{2^n}) = (\frac{1}{2^n}) \) is a sequence. Since, for natural numbers \( m \) and \( n \) such that \( m > n \), \( \frac{1}{2^m} \leq t \leq \frac{1}{2^n} \) and \( \frac{k}{2^n} \leq t \leq \frac{k + 1}{2^n} \) with some natural number \( l \), we have

\[
\left| \left( \frac{l + 1}{2^m} - \frac{l}{2^m} \right) - \left( \frac{k + 1}{2^n} - \frac{k}{2^n} \right) \right| = \left| \frac{2^n - 2^m}{2^m 2^n} \right| < \frac{1}{2^n}.
\]

\( (\frac{k + 1}{2^n} - \frac{k}{2^n}) \) is a Cauchy sequence, and converges to zero. Then, \( (\frac{k + 1}{2^n}) \) and \( (\frac{k}{2^n}) \) converge to \( t \). Closedness of \( \gtrless \) implies that either \( z \gtrless x \) or \( z \gtrless y \). Therefore, the preference is convex. \( \square \)

**Lemma 3.** Let \( x \) and \( y \) be points in \( X \) such that \( x \gg y \). Then, if a consumer’s preference is weakly uniformly rotund and monotonic, \( tx + (1 - t)y \gg y \) for \( 0 < t < 1 \).
Proof. By continuity of the preference (openness of $\succ$) there exists a point $x' = x - \lambda$ such that $\lambda \gg 0$ and $x' \succ y$. Then, since weak uniform rotundity implies convexity, we have $tx' + (1-t)y \geq y$ or $tx' + (1-t)y \geq x'$. If $tx' + (1-t)y \gg x'$, then by transitivity $tx' + (1-t)y = tx + (1-t) - t\lambda \gg y$. Monotonicity of the preference implies $tx + (1-t)y \succ y$. Assume $tx' + (1-t)y \gg y$. Then, again monotonicity of the preference implies $tx + (1-t)y \gg y$. 

Let $S$ be a subset of $\Delta \times R$ such that for each $(p, w) \in S$

1. $p \in \Delta$.
2. $\beta(p, w)$ is nonempty.
3. There exists $\xi \in X$ such that $\xi \succ x$ for all $x \in \beta(p, w)$.

In [4] the following lemmas were proved.

**Lemma 4** (Lemma 2.1 in [4]). If $p \in \Delta \subset R^N$, $w \in R$, and $\beta(p, w)$ is nonempty, then $\beta(p, w)$ is compact.

Lemma 4 with Proposition (4.4) in Chapter 4 of [1] or Proposition 2.2.9 of [3] implies that for each $(p, w) \in S \beta(p, w)$ is located in the sense that the distance

$$
\rho(x, \beta(p, w)) \equiv \inf \{|x - y| : y \in \beta(p, w)\}
$$

exists for each $x \in R^N$.

**Lemma 5** (Lemma 2.2 in [4]). If $(p, w) \in S$ and $\xi \succ \beta(p, w)$ (it means $\xi \succ x$ for all $x \in \beta(p, w)$), then $\rho(\xi, \beta(p, w)) > 0$ and $p \cdot \xi > w$.

**Lemma 6** (Lemma 2.3 in [4]). Let $(p, c) \in S$, $\xi \in X$ and $\xi \succ \beta(p, c)$. Let $H$ be the hyperplane with equation $p \cdot x = c$. Then, for each $x \in \beta(p, c)$, there exists a unique point $\varphi(x)$ in $H \cap [x, \xi]$. The function $\varphi$ so defined maps $\beta(p, c)$ onto $H \cap \beta(p, c)$ and is uniformly continuous on $\beta(p, c)$.

**Lemma 7** (Lemma 2.4 in [4]). Let $(p, w) \in S$, $r > 0$, $\xi \in X$, and $\xi \succ \beta(p, w)$. Then, there exists $\zeta \in X$ such that $\rho(\zeta, \beta(p, w)) < r$ and $\zeta \succ \beta(p, w)$.

Proof. See Appendix.

And the following lemma.

**Lemma 8** (Lemma 2.8 in [4]). Let $R, c$, and $t$ be positive numbers. Then there exists $r > 0$ with the following property: if $p, p'$ are elements of $R^N$ such that $|p| \geq c$ and $|p - p'| < r$, $w, w'$ are real numbers such that $|w - w'| < r$, and $y'$ is an element of $R^N$ such that $|y'| \leq R$ and $p' \cdot y' = w'$, then there exists $\zeta \in R^N$ such that $p \cdot \zeta = w$ and $|y' - \zeta| < t$.

It was proved by setting $r = \frac{ct}{R + 1}$.  

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3 Convex-valued demand correspondence with closed graph

With the preliminary results in the previous section we show the following our main result.

**Theorem 1.** Let $\succcurlyeq$ be a weakly uniformly rotund preference relation on a compact and convex subset $X$ of $\mathbb{R}^N$, $\Delta$ be a compact and convex set of normalized price vectors (an $(n-1)$-dimensional simplex), and $S$ be a subset of $\Delta \times \mathbb{R}$ such that for each $(p, w) \in S$

1. $p \in \Delta$.
2. $\beta(p, w)$ is nonempty.
3. There exists $\xi \in X$ such that $\xi \succ x$ for all $x \in \beta(p, w)$.

Then, for each $(p, w) \in S$ there exists a subset $F(p, w)$ of $\beta(p, w)$ such that $F(p, w) \succcurlyeq x$ (it means $y \succ x$ for all $y \in F(p, w)$) for all $x \in \beta(p, w)$, $\Delta \cdot F(p, w) = w$ ($p \cdot y = w$ for all $y \in F(p, w)$), and the multi-valued correspondence $F(p, w)$ is convex-valued and has a closed graph.

A graph of a correspondence $F(p, w)$ is

$$G(F) = \cup_{(p, w) \in S} (p, w) \times F(p, w).$$

If $G(F)$ is a closed set, we say that $F$ has a closed graph.

**Proof.**

1. Let $(p, w) \in S$, and choose $\xi \in X$ such that $\xi \succ \beta(p, w)$. By Lemma 7 construct a sequence $(\zeta_m)$ in $X$ such that $\zeta_m \succ \beta(p, w)$ and $\rho(\zeta_m, \beta(p, w)) < \frac{1}{m}$ with $r > 0$ for each natural number $m$. By convexity and transitivity of the preference $t\zeta_m + (1 - t)\zeta_{m+1} \succ \beta(p, w)$ for $0 < t < 1$ and each $m$. Thus, we can construct a sequence $(\zeta_n)$ such that $|\zeta_n - \zeta_{n+1}| < \varepsilon^n$, $\rho(\zeta_n, \beta(p, w)) < \delta^n$ and $\zeta_n \succ \beta(p, w)$ for some $0 < \varepsilon < 1$ and $0 < \delta < 1$, and so $(\zeta_n)$ is a Cauchy sequence in $X$. It converges to a limit $\zeta^* \in X$. By continuity of the preference (closedness of $\succcurlyeq$), $\zeta^* \succcurlyeq \beta(p, w)$, and $\rho(\zeta^*, \beta(p, w)) = 0$. Since $\beta(p, w)$ is closed, $\zeta^* \in \beta(p, w)$. By Lemma 5 $p \cdot \zeta_n > w$ for all $n$. Thus, we have $p \cdot \zeta^* = w$. Convexity of the preference implies that $\zeta^*$ may not be unique, that is, there may be multiple elements $\zeta'$ of $\beta(p, w)$ such that $p \cdot \zeta' = w$ and $\zeta' \succ \beta(p, w)$. Therefore, $F(p, w)$ is a set and we get a demand correspondence. Let $\zeta \in F(p, w)$ and $\zeta' \in F(p, w)$. Then, $\zeta \succcurlyeq \beta(p, w)$, $\zeta' \succcurlyeq \beta(p, w)$, and convexity of the preference implies $t\zeta + (1 - t)\zeta' \succcurlyeq \beta(p, w)$. Thus, $F(p, w)$ is convex.

2. Next we prove that the demand correspondence has a closed graph. Consider $(p, w)$ and $(p', w')$ such that $|p - p'| < r$ and $|w - w'| < r$ with $r > 0$. Let $F(p, w)$ and $F(p', w')$ be demand sets. Let $y' \in F(p', w')$,
$$c = \rho(0, \Delta) > 0$$ and $$R > 0$$ such that $$X \subset \bar{B}(0, R)$$. Given $$\varepsilon > 0$$, $$t = \delta > 0$$ such that $$\delta < \varepsilon$$, and choose $$r$$ as in Lemma 8. By that lemma we can choose $$\zeta \in R^N$$ such that $$p \cdot \zeta = w$$ and $$|y' - \zeta| < \delta$$. Similarly, we can choose $$\zeta'(y) \in R^N$$ such that $$p' \cdot \zeta'(y) = w'$$ and $$|y - \zeta'(y)| < \delta$$ for each $$y \in F(p, w)$$. $$y' \in F'(p', w')$$ means $$y' \succeq \zeta'(y)$$. Either $$|y' - y| > \frac{\delta}{2}$$ for all $$y \in F(p, w)$$ or $$|y' - y| < \varepsilon$$ for some $$y \in F(p, w)$$. Assume that $$|y' - y| > \frac{\delta}{2}$$ for all $$y \in F(p, w)$$ and $$y \succ \zeta$$. If $$\delta$$ is sufficiently small, $$|y' - y| > \frac{\delta}{2}$$ means $$|y - \zeta| > \frac{\delta}{2}$$ and $$|y' - \zeta'(y)| > \frac{\delta}{2}$$ for some finite natural number $$k$$. Then, by weak uniform rotundity there exist $$z_n$$ and $$z'_n$$ such that $$|z_n| < \tau_n$$, $$|z'_n| < \tau_n$$ with $$\tau_n > 0$$, $$z_n \gg 0$$ and $$z'_n \gg 0$$, $$\frac{\delta}{2}(y + \zeta) + z_n \gg \zeta$$ and $$\frac{\delta}{2}(y' + \zeta'(y)) + z'_n \gg \zeta'(y)$$ for $$n = 1, 2, \ldots$$ Again if $$\delta$$ is sufficiently small, $$|y - \zeta(y)| < \delta$$ and $$|y - \zeta| < \delta$$ imply $$\frac{\delta}{2}(y + \zeta) + z_n \gg y$$ and $$\frac{\delta}{2}(y' + \zeta'(y)) + z'_n \gg y$$. And it follows that $$\frac{\delta}{2}(y + \zeta) - \frac{\delta}{2}(y' + \zeta'(y)) < \delta$$. By continuity of the preference (openness of $$\succ$$) $$\frac{\delta}{2}(y + \zeta) + z'_n \gg y$$. Let $$y_1 = \frac{\delta}{2}(y + \zeta)$$. Consider a sequence $$(\tau_n)$$ converging to zero. By continuity of the preference (closedness of $$\succ$$) $$y_1 \gg y'$$ and $$y_1 \gg y$$. Note that $$p \cdot y_1 = w$$. Thus, $$y_1 \in \beta(p, w)$$. Since $$y \in F(p, w)$$, we have $$y_1 \in F(p, w)$$. Replacing $$y$$ with $$y_1$$, we can show that $$\frac{\delta}{2}(y + \zeta) + z'_n \gg y$$ for each natural number $$n$$. Then, we have $$|y - \zeta| < \eta$$ for some $$y \in F(p, w)$$ for any $$\eta > 0$$. It contradicts $$|y - \zeta| > \frac{\delta}{2}$$. Therefore, we have $$|y' - y| < \varepsilon$$ or $$\zeta \succeq y$$ (it means $$|y' - \zeta| < \delta$$ and $$\zeta \in F(p, w)$$), and so $$F(p, w)$$ has a closed graph.

□

Appendix: Proof of Lemma 7

This proof is almost identical to the proof of Lemma 2.4 in Bridges [4]. They are different in a few points.

Let $$H$$ be the hyperplane with equation $$p \cdot x = w$$ and $$\xi$$ the projection of $$\zeta$$ on $$H$$. Assume $$|\xi - \xi'| > 3r$$. Choose $$R$$ such that $$H \cap \beta(p, w)$$ is contained in the closed ball $$\bar{B}(\xi', R)$$ around $$\xi'$$, and let

$$c = \sqrt{1 + \left(\frac{R}{|\xi - \xi'|}\right)^2}.$$  

Let $$H'$$ be the hyperplane parallel to $$H$$, between $$H$$ and $$\xi$$ and a distance $$\frac{c}{\sqrt{}}$$ from $$H$$; and $$H''$$ the hyperplane parallel to $$H$$, between $$H$$ and $$\xi$$ and a distance $$\frac{c}{\sqrt{}}$$ from $$H$$. For each $$x \in \beta(p, w)$$ let $$\varphi(x)$$ be the unique element of $$H \cap [x, \xi]$$, $$\varphi'(x)$$ be the unique element of $$H' \cap [x, \xi]$$, and $$\varphi''(x)$$ be the unique element of $$H'' \cap [x, \xi]$$. Since $$\xi \succ \beta(p, w)$$, we have $$\varphi''(x) \succ \varphi(x) \succeq x$$ by convexity and continuity of the preference. $$\varphi'(x)$$ is uniformly continuous, so

$$T \equiv \{\varphi'(x) : x \in \beta(p, w)\}$$

is totally bounded by Lemma 4 and Proposition (4.2) in Chapter 4 of [1].
Since \( \varphi''(x) \succ \varphi(x) \) and \( \varphi'(x) = \frac{1}{2}\varphi''(x) + \frac{1}{2}\varphi(x) \) we have \( \varphi'(x) \succ x \), and so continuity of the preference (openness of \( \succ \)) means that there exists \( \delta > 0 \) such that \( \varphi'(x_i) \succ x \) when \( |\varphi'(x_i) - \varphi'(x)| < \delta \). Let \((x_1, \ldots, x_n)\) be points of \( \beta(p, w) \) such that \( (\varphi'(x_1), \ldots, \varphi'(x_n)) \) is a \( \delta \)-approximation to \( T \). Given \( x \) in \( \beta(p, w) \) choose \( i \) such that \( |\varphi'(x_i) - \varphi'(x)| < \delta \). Then, \( \varphi'(x_i) \succ x \).

Now from our choice of \( c \) we have \( |\varphi(x) - \varphi'(x)| < \frac{\epsilon}{2} \) for each \( x \in \beta(p, w) \).

It is proved as follows. Since by the assumption \( |\varphi(x) - \xi'| < R \), \( |\varphi(x) - \xi| < \sqrt{R^2 + |\xi - \xi'|^2} \). Thus, we have

\[
|\varphi(x) - \varphi'(x)| < \frac{r}{2c} \times \sqrt{R^2 + |\xi - \xi'|^2} = \frac{r}{2c} \sqrt{1 + \left( \frac{R}{|\xi - \xi'|} \right)^2} = \frac{r}{2}.
\]

See Figure 1.

Let

\[
t_1 = 1 - \frac{r}{2n|\varphi'(x_1) - \xi'|},
\]

and

\[
\eta_1 = t_1 \varphi'(x_1) + (1 - t_1)\xi.
\]

Then, \( |\eta_1 - \varphi'(x_1)| = \frac{r}{2n} \rho(\eta_1, \beta(p, w)) < \frac{r(n+1)}{2n} \) (because \( |\varphi(x_1) - \varphi'(x_1)| < \frac{\epsilon}{2} \) and \( \varphi(x_1) \in \beta(p, w) \)), and by convexity of the preference \( \eta_1 \succeq \xi \) or \( \eta_1 \succeq \varphi'(x_1) \).

In the first case we complete the proof by taking \( \zeta = \eta_1 \). In the second, assume that, for some \( k (1 \leq k \leq n - 1) \), we have constructed \( \eta_1, \ldots, \eta_k \) in \( X \) such that

\[
\eta_k \succeq \varphi'(x_i) (1 \leq i \leq k),
\]

and

\[
\rho(\eta_k, \beta(p, w)) < \frac{r(n + k)}{2n}.
\]

As \( |\xi - \eta_k| > r \) (because \( |\xi - \xi'| > 3r \)), we can choose \( y \in [\eta_k, \xi] \) such that \( |y - \eta_k| = \frac{r}{2n} \). Then \( \rho(y, \beta(p, w)) < \frac{r(n+k+1)}{n} \) and either \( y \succeq \xi \) or \( y \succeq \eta_k \). In the

Figure 1: Calculation of \( |\varphi(x) - \varphi'(x)| \)
former case, the proof is completed by taking $\zeta = y$. If $y \succsim \eta_k$, $y + \frac{\lambda}{2} \succsim \eta_k - \frac{\lambda}{2}$ for all $\lambda$ such that $\lambda \gg 0$. Then, either $y + \frac{\lambda}{2} \succsim \varphi'(x_{k+1})$ for all $\lambda$ and so $y \succsim \varphi'(x_{k+1})$, in which case we set $\eta_{k+1} = y$; or else $\varphi'(x_{k+1}) \succsim \eta_k$ and so $\varphi'(x_{k+1}) \succsim \eta_k$, then we set $\eta_{k+1} = \varphi'(x_{k+1})$.

If this process proceeds as far as the construction of $\eta_n$, then, setting $\zeta = \eta_n$, we see that $\rho(\zeta, \beta(p, w)) < r$ and that $\zeta \succsim \varphi'(x_i)$ for each $i$; so $\zeta \succ x$ for each $x \in \beta(p, w)$.

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