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Prediction studies of the epidemic peak of coronavirus disease in Brazil via new generalised Caputo type fractional derivatives

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Abstract The first reported case of coronavirus disease (COVID-19) in Brazil was confirmed on 25 February 2020 and then the number of symptomatic cases produced day by day. In this manuscript, we studied the epidemic peaks of the novel coronavirus (COVID-19) in Brazil by the successful application of Predictor–Corrector (P-C) scheme. For the proposed model of COVID-19, the numerical solutions are performed by a model framework of the recent generalized Caputo type non-classical derivative. Existence of unique solution of the given non-linear problem is presented in terms of theorems. A new analysis of epidemic peaks in Brazil with the help of parameter values cited from a real data is effectuated. Graphical simulations show the obtained results to classify the importance of the classes of projected model. We observed that the proposed fractional technique is smoothly work in the coding and very easy to implement for the model of non-linear equations. By this study we tried to exemplify the roll of newly proposed fractional derivatives in mathematical epidemiology. The main purpose of this paper is to predict the epidemic peak of COVID-19 in Brazil at different transmission rates. We have also attempted to give the stability analysis of the proposed numerical technique by the reminder of some important lemmas. At last we concluded that
when the infection rate increases then the nature of the diseases changes by becoming more deadly to the population.

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1. Introduction

The term COVID-19, which indicates to Coronavirus-2019 disease, is a epidemic accounted by a virus of the Coronaviridae family, SARS-CoV-2. This contagious disease emerged in December 2019 in Wuhan City, Hubei Province, China, and hastily spread, first all over the China and then overseas causing a universal epidemic [6]. This respiratory malady that can be deadly in patients weakened by age or another chronic disease such as Diabetes, Cancer, or arterial hypertension, breadth via adjoining with infected human. The disease also transmitted by asymptomatic patients, which makes the control of the disease a difficult task for health care staff [31]. Symptoms of the COVID-19 are fever, fatigue, and a dry cough, pain, stuffy and runny nose, sore throat, and diarrhoea. Pneumonia is the most common complication of COVID-19 [36]. There is no vaccine avowed by the World Health Organization to prevent coronavirus infections. On the other hand, the fight is focused on the barrier measures decreed by the WHO (regular washing hand with soap or an alcohol-based solution, respect for distance, cover the mouth with the bend of the elbow, or a disposable tissue, during a cough or a sneeze, wearing a mask in public places, . . .)[35]. For those presenting the symptoms, the treatment although there was no consensus from the start among the virologists, is based on hydroxychloroquine combined with azithromycin [5].

The first case of COVID-19 swelled in the Brazilian city of Sao Paulo on Feb 26. It was a 61-year-old individual presenting the symptoms of Severe Acute Respiratory Syndrome, after returning from the Italian city of Turin. The disease quickly spread in the country due to the failure to take the alert level seriously by the country’s high authorities. Indeed, Brazil is one of the last countries to have set up barrier measures, which has favoured the expansion of the virus in the various cities of the country, thus leading to an upsurge in the number of deaths. The total number of confirmed cases has increased exponentially: from 1 case as of February 26, it quickly reached 28,320 confirmed cases on April 15, 2020, with nearly 1,736 deaths [30]. From the date of its origin, this virus continuously grows in the population and infected more than 17,476,105 with 676,759 deaths and 2,284,965 recoveries on July 31 all over the world. In this concern, United States which is the unduly infected nation by this epidemic by this pandemic with 4,634,985 cases, 155,285 deaths and 2,284,965 recoveries on July 31. Brazil and India are also present in this series where the COVID-19 confirmed cases are increasing sharply day by day. In Brazil, there are 2,613,789 confirmed cases with 91,377 deaths and 1,824,095 recoveries.

Hundreds of articles have been come from researchers to record the effects of coronavirus via mathematical modelling [10,15,16,19,23,24]. Non-integer order Calculus has a very long history which is more then 300 years old and current day thousands of real world problems are being solved by the application of non-integer order calculus. Atangana et al. [1] have given some analysis on the connections between non-integer order Calculus and fractal Calculus. Some credible analysis related to the consequence, importance of the Bode diagram in calculus including integer and non-integer case by using Caputo-Fabrizio derivative are given in [2]. To elevate the stability and accuracy of numerical techniques, a new method to

| Date       | Confirmed Cases | Deaths | Recovered |
|------------|-----------------|--------|-----------|
| 2020-01-23 | 0               | 0      | 0         |
| 2020-02-26 | 1               | 0      | 0         |
| 2020-02-27 | 1               | 0      | 0         |
| 2020-02-28 | 1               | 0      | 0         |
| 2020-02-29 | 2               | 0      | 0         |
| 2020-03-01 | 2               | 0      | 0         |
| 2020-03-02 | 2               | 0      | 0         |
| 2020-03-03 | 2               | 0      | 0         |
| 2020-03-04 | 4               | 0      | 0         |
| 2020-03-05 | 4               | 0      | 0         |
| 2020-03-06 | 13              |        |           |
| 2020-03-07 | 13              |        |           |
| 2020-03-08 | 20              |        |           |
| 2020-03-09 | 25              |        |           |
| 2020-03-10 | 31              |        |           |
| 2020-03-11 | 38              |        |           |
| 2020-03-12 | 52              |        |           |
| 2020-03-13 | 151             |        |           |
| 2020-03-14 | 151             |        |           |
| 2020-03-15 | 162             |        |           |
| 2020-03-16 | 200             |        |           |
| 2020-03-17 | 321             |        |           |
| 2020-03-18 | 372             |        |           |
| 2020-03-19 | 621             |        |           |
| 2020-03-20 | 793             |        |           |
| 2020-03-21 | 1021            |        |           |
| 2020-03-22 | 1546            |        |           |
| 2020-03-23 | 1924            |        |           |
| 2020-03-24 | 2247            |        |           |
| 2020-03-25 | 2554            |        |           |
| 2020-03-26 | 2985            |        |           |
| 2020-03-27 | 3417            |        |           |
| 2020-03-28 | 3904            |        |           |
| 2020-03-29 | 4256            |        |           |
| 2020-03-30 | 4579            |        |           |

| Table 2 Parameter values for simulations. |
| Parameter | Description | Value/range | Reference |
| β         | Infection rate | 0.3695 – 0.6043 | [4] |
| φ         | Removal rate | 0.0714 – 0.3295 | [4] |
| c         | Onset rate | 0.1 – 0.5 | [4] |
| N         | Total population in Brazil | 2.10 × 10^8 | [4] |
| p         | Identification rate | 0.01 – 0.1 | Assumed |
discretize (fractional sense) an integral or differential equation is evaluated in [3]. A concern related to hilfer non-integer neutral stochastic systems is analysed in [7]. A study on the coronavirus model by an effective fractional computational method is established in [9]. Some analysis related to existence of unique solution for non-classical non-linear hybrid impulsive
In [12], the authors organised a study on Hilfer non-integer order difference operator. Some more results on existence of Hilfer non-integer order neutral equations are given in [14]. In [17], authors solved a malaria model via CF and ABC derivatives. Kumar et al. [18] solved a time-delay coronavirus non-integer order model in Caputo sense. A model of COVID-19 is also proposed in [21] by the help of mittag–leffler kernel. Some existence results for non-classical differential equations are given in [22]. Some other results on existence of solutions for fractional Atangana-Baleanu type differential equations are established in [28] and a study on new technique of non-integer order derivative of AB type is given in [29]. Some important results on non-classical neutral differential systems are stated in [32]. A study related to solutions to non-integer order neutral delay equations is established in [33]. Mathematicians have proposed so many different kind of non-integer order derivatives like Atangana-Baleanu in Caputo sense, Caputo, Caputo and Fabrizio etc. which are the very common derivatives in the view of the implementation in real world problems. In our study, we are exemplifying the dynamics of a coronavirus non-linear model by the application of a recent non-classical derivative named generalized Caputo fractional derivatives, which nature is similar to Caputo type derivative [25]. To find the solution of the given system, we are applying the modified version of the P-C algorithm. The major difference of the P-C algorithms in the Caputo and recent Caputo derivative is the uses of a non-uniform grid in the modified version. The newly generalized fractional Caputo integral operator is mainly supported by the two parameters z and ρ, which gives a considerable equipment to control and do mathematical modelling in fractional order sense. The main aim of this paper is to give the predictions of epidemic peak of COVID-19 in Brazil using recent data of Brazil from 25 February, 2020 to 30 March, 2020 by the application of a recent generalized Caputo non-integer derivative with Predictor–Corrector method. In these predictions, we used different sets of the parameter values which make this study more credible. The article is distributed as follows. In Section 2, we given some necessary definitions related to generalisation of Caputo derivative. We described the given non-linear SEIR model in ODE and fractional sense in Section 3. We derived some theorems for supporting the existence and uniqueness solution of the model in Section 4. We also derived the solution of the given non-linear model via modified P-C method with stability analysis in Section 5. The all graphical results for the peak predictions of COVID-19 are exemplified in Section 6. A conclusion gives the mark to the paper.

2. Preliminaries

Definition 1. [26] The description of the Caputo definition of fractional derivative of Ω ∈ C^p is given as

Fig. 2 S, E, I and R profiles with β = 0.5869, φ = 0.2004 and ρ = 0.1 (set 4 of Table 2), $R_0 = 2.93$. 
where

\[ f(x) = \sum_{m=0}^{i-1} \frac{\beta^m}{m!} (x-a)^m \]

where \( \rho > 0, a \geq 0, \) and \( n \) is an integer (non-negative).

**Def.

**Definition 4.** [25] The recent generalization of Caputo-type derivative of non-integer order, \( D_{\alpha,\beta}^{n}f(\cdot) \), of order \( \alpha > 0 \) is defined as:

\[
(D_{\alpha,\beta}^{n}f(\cdot))(\zeta) = \left[ \frac{\beta^{n-\alpha-1}}{\Gamma(n-\alpha)} \right] \left( \zeta-I_{\alpha}^{n} \frac{\partial^{n}}{\partial \zeta^{n}} \right) f(s)ds, \quad \zeta > a,
\]

where \( \rho > 0, a \geq 0, \) and \( n \) is an integer (non-negative), \( \beta \) is a positive integer (non-negative), and \( \alpha \) is a real number.

**Lemma 1.** [20] If \( 0 < \kappa < 1 \) and \( \beta \) is an integer (non-negative), then there exists the positive constants \( C_{\kappa,1} \) and \( C_{\kappa,2} \) only dependent on \( \kappa \), s.t

\[
(\beta + 1)^{\kappa} - \beta^{\kappa} \leq C_{\kappa,1}(\beta + 1)^{\kappa-1},
\]

and

\[
(\beta + 2)^{\kappa-1} - 2(\beta + 1)^{\kappa-1} + \beta^{\kappa-1} \leq C_{\kappa,2}(\beta + 1)^{\kappa-1}.
\]

**Lemma 2.** [20] Let \( d_{p,n} = (n-p)^{\beta-1}(p=1,2,\ldots,n-1) \) & \( d_{p,n} = 0 \) for \( p \geq n, \beta, M, h, T > 0, \) and \( i \) is a positive integer. Let \( \sum_{p=1}^{n-1} d_{p,n} e_p = 0 \) for \( k > n \geq 1 \). If

\[
|e_n| \leq M h^\gamma \sum_{p=1}^{n-1} d_{p,n} |e_p| + |\eta_0|, \quad n = 1,2,\ldots,i.
\]
then
\[ |e_i| \leq C|\eta_0|, i = 1, 2, \ldots \]
where \( C \) is a positive constant independent of \( i \) & \( \eta \).

3. Model description

There are lots of non-linear mathematical models have been developed to study the COVID-19 disease in all over the world. In this series, to study the COVID-19 epidemic peaks in Brazil, we express the model proposed by Kuniya et.al in [19]. The author formulated and defined the given non-linear model in the classical derivative sense as follows:

\[
\begin{align*}
S'(\zeta) &= -\beta S(\zeta)I(\zeta), \\
E'(\zeta) &= \beta S(\zeta)I(\zeta) - \epsilon E(\zeta), \\
I'(\zeta) &= \epsilon E(\zeta) - \phi I(\zeta), \\
R'(\zeta) &= \phi I(\zeta),
\end{align*}
\]

where \( S(\zeta), E(\zeta), I(\zeta) \) and \( R(\zeta) \) indicate the susceptible, exposed, infective, and removed classes of population at time \( \zeta \), respectively. \( \beta, \frac{1}{\epsilon} \) and \( \phi \) define the infection rate, the incubation period, and the removal rate, respectively. We set \( S + E + I + R \) to be 1 and let one infective human is recognised at time \( \zeta = 0 \) among the total population of Brazil.

We fixed the range of \( \rho \) from 0.01 to 0.1. The expected value of secondary cases organised by one infective person (known as the basic reproduction number \( R_0 \)) is formulated as the largest eigenvalue of the next generation matrix \( FV^{-1} \) [34], where

\[
F = \begin{bmatrix} 0 & \beta S(0) \\ 0 & 0 \end{bmatrix},
\]

\[
V = \begin{bmatrix} \epsilon & 0 \\ -\epsilon & \phi \end{bmatrix},
\]

Thus, we evolute

\[
R_0 = \frac{\beta S(0)}{\phi} = \beta \left( 1 - \frac{1}{\rho \times 2.10 \times 10^5} \right).
\]

Now to analyse the outcomes of the projected system (5) in the sense of fractional derivatives, here we are using a newly proposed fractional derivative called new generalised Caputo type

\[
Y(\zeta) = pI(\zeta) \times 2.10 \times 10^5 = 1,
\]

\[
Y(\zeta) = pI(\zeta) \times 2.10 \times 10^5 = 1,
\]

\[
defines the number of infective persons which are recognised at time \( \zeta \). Thus, \( I(0) = \frac{1}{\rho \times 2.10 \times 10^5} \) and we assume that at \( \zeta = 0, E(0) = R(0) = 0 \), and hence,

\[
S(0) = 1 - I(0) - E(0) - R(0) = 1 - \frac{1}{\rho \times 2.10 \times 10^5}.
\]

Fig. 4 S, E, I and R profiles with \( \beta = 0.4869, \phi = 0.1552 \) and \( \rho = 0.1 \) (set 6 of Table 2), \( R_0 = 3.14 \).
fractional derivative (4). In this regard, system (5) can be organised in the new generalised Caputo fractional derivatives sense as follows:

\[
\begin{align*}
CD^c_q f(S(f)) &= C_0 b S(f) I(f); \\
CD^c_q f(E(f)) &= b S(f) I(f); \\
CD^c_q f(I(f)) &= E(f); \\
CD^c_q f(R(f)) &= I(f);
\end{align*}
\]

where \(CD^c_q f\) denotes the new generalised Caputo fractional derivative operator.

For comfort, the compact form of the above system of equations can be expressed as the given initial value problem (IVP)

\[
\begin{align*}
CD^c_q S(\zeta) &= -\beta S(\zeta) I(\zeta), \\
CD^c_q E(\zeta) &= \beta S(\zeta) I(\zeta) - \epsilon E(\zeta), \\
CD^c_q I(\zeta) &= \epsilon E(\zeta) - \phi I(\zeta), \\
CD^c_q R(\zeta) &= \phi I(\zeta),
\end{align*}
\]

where \(CD^c_q f\) denotes the new generalised Caputo fractional derivative operator.

For comfort, the compact form of the above system of equations can be expressed as the given initial value problem (IVP)

\[
\begin{align*}
CD^c_q S(\zeta) &= G_1(\zeta, S), \\
CD^c_q E(\zeta) &= G_2(\zeta, E), \\
CD^c_q I(\zeta) &= G_3(\zeta, I), \\
CD^c_q R(\zeta) &= G_4(\zeta, R).
\end{align*}
\]

The initial conditions are taken as \(S_0 = S(0), E_0 = E(0), I_0 = I(0)\) and \(R_0 = R(0)\).

4. Mathematical analysis of the fractional model

4.1. Existence and uniqueness analysis

Now here we do the analysis of existence of unique solution for the given non-linear COVID-19 model in terms of theorems. We show the analysis only for \(S(\zeta)\) class and for other equations of the model, analysis will be similar. Now consider the first equation of the model in the following compact form

\[
\begin{align*}
CD^c_q S(\zeta) &= G_1(\zeta, S), \quad \text{(9a)} \\
S(0) &= S_0, \quad \text{(9b)}
\end{align*}
\]

and the corresponding Volterra integral equation

\[
S(\zeta) = S(0) + \int_0^\zeta \bar{G}_1(\xi - \zeta) S(\xi) d\xi. \quad \text{(10)}
\]

Theorem 1. [8,13] (Existence). Let \(0 < \gamma \leq 1, S_0 \in \mathbb{R}, K > 0\) and \(T^* > 0\). Let \(G := \{(\zeta, S) : \zeta \in [0, T^*], |S - S_0| \leq K\}\) and let the function \(G_1 : G \to \mathbb{R}\) be continuous. Further, describe \(M := \sup_{(\zeta, S) \in G} |G_1(\zeta, S)|\) and

\[
T = \begin{cases} T^*, & \text{if } M = 0, \\ \min \left\{ T^*, \left( \frac{2G_1(S_0) + 1}{M} \right)^{-1} \right\}, & \text{otherwise.} \end{cases}
\]
Then, a function $S \in C[0, T]$ is exist that solves the IVP (9a) and (9b).

**Lemma 3.** [8,13] By the assumption of the statement of Theorem 1, the function $S \in C[0, T]$ is a solution of the IVP (9a) and (9b) if, it is a solution of the non-linear Volterra integral Eq. (10).

**Theorem 2.** [8,13] (Uniqueness). Consider $S(0) \in \mathbb{R}, K > 0 \text{and} T > 0$. Also, let $0 < \gamma \leq \text{and} m = \gamma$. For the set $G$ as in Theorem 1 and let $G_1 : G \rightarrow \mathbb{R}$ be continuous and agree a Lipschitz condition with respect to the second variable, i.e.

$$|G_1(\xi, S_1) - G_1(\xi, S_2)| \leq V |S_1 - S_2|,$$

for some constant $V > 0$ independent of $\xi, S_1, \text{and} S_2$. Then, a unique solution $S \in C[0, T]$ exists for the IVP (9a) and (9b).

### 5. Solution of the given SEIR model via modified Predictor–Corrector method

Now we derive the solution of the given SEIR model in the framework of modified version of the P-C method for the IVP (9a) and (9b) by following the procedure as given in [25] with some necessary changes. After that we will discuss the stability of the given method. To fulfill this requirement, we start from the above mentioned Volterra integral Eq. (10), which provides

$$S(\xi) = S(0) + \frac{\rho^{1-\gamma}}{\Gamma(\gamma)} \int_0^\xi \xi^{\rho - 1} (\xi^{\rho} - z)^{-1} G_1(\xi, S) d\xi,$$

Now we assume that a unique solution of the given problem exists for the function $G_1$ on some interval $[0, T]$ and divided the interval $[0, T]$ into $N$ unequal subintervals $[\xi_k, \xi_{k+1}], k = 0, 1, \ldots, N - 1$ using the mesh points

$$\{ \xi_0 = 0, \quad \xi_{k+1} = (\xi_k + h)^{1/\rho}, k = 0, 1, \ldots, N - 1, \}$$

where $h = \frac{T}{N}$. Now, we are trying to analyse the approximations $S_k, k = 0, 1, \ldots, N$, to solve numerically the given IVP. The basic step, supposing that we have already estimated the approximations $S_k \approx S(\xi_k)(j = 1, 2, \ldots, k)$, is that we want to estimate the approximation $S_{k+1} \approx S(\xi_{k+1})$ by means of the integral equation

$$S(\xi_{k+1}) = S(0) + \frac{\rho^{1-\gamma}}{\Gamma(\gamma)} \int_0^\xi \xi^{\rho - 1} (\xi^{\rho} - z)^{-1} G_1(\xi, S) d\xi,$$

doing the substitution $z = \xi^\rho$, we get

$$S(\xi_{k+1}) = S(0) + \frac{\rho^{1-\gamma}}{\Gamma(\gamma)} \int_0^\xi (\xi_k^{\rho} - z)^{-1} G_1(z^{1/\rho}, S(z^{1/\rho})) dz.$$
Published the corrector formula for Eq. (16), if we use the trapezoidal quadrature rule with respect to the weight function \( (\zeta^j_{k+1} - z)^{-1} \) then shifting the function \( G_1(z^{1/\rho}, S(z^{1/\rho})) \) by its piecewise linear interpolant with nodes chosen at the \( \zeta^j_k \) \((j = 0, 1, \ldots, k + 1)\), then we get

\[
\int_{\zeta^j_k}^{\zeta^j_{k+1}} (\zeta^j_{k+1} - z)^{-1} G_1(z^{1/\rho}, S(z^{1/\rho})) dz \approx \frac{k}{b_{k+1}} \left[ \left( (k-j)^{\gamma+1} - (k-j-\gamma)(k-j+1)^{\gamma+1} \right) \times G_1(\zeta^j_k, S(\zeta^j_k)) + \left( (k-j+1)^{\gamma+1} - (k-j+\gamma+1)(k-j+\gamma)^{\gamma+1} \right) G_1(\zeta^j_{k+1}, S(\zeta^j_{k+1})) \right].
\] (17)

Thus, putting the above approximations into Eq. (16), we established the corrector formula for \( S(\zeta^j_{k+1}), k = 0, 1, \ldots, N - 1 \),

\[
S(\zeta^j_{k+1}) \approx S(0) + \frac{\rho^{-1}h^\gamma}{\Gamma(\gamma + 2)} \sum_{j=0}^{k} [a_j h_j + G_1(\zeta^j_k, S(\zeta^j_k))]
\]

\[
+ \frac{\rho^{-1}h^\gamma}{\Gamma(\gamma + 2)} G_1(\zeta^j_{k+1}, S(\zeta^j_{k+1})),
\] (18)

where

\[
a_{k,j+1} = \begin{cases} 
  k^{\gamma+1} - (k-j)(k+1)^{\gamma} & \text{if } j = 0, \\
  (k-j+2)^{\gamma+1} + (k-j)^{\gamma+1} - 2(k-j+1)^{\gamma+1} & \text{if } 1 \leq j \leq k.
\end{cases}
\] (19)

The final attempt of our method is to change the quantity \( S(\zeta^j_{k+1}) \) appeared on the right-hand side of the formula (18) with the predictor value \( S^p(\zeta^j_{k+1}) \) that can be calculated by using the one-step Adams–Bashforth technique to the integral Eq. (15).

In this case, by changing the mapping \( G_1(z^{1/\rho}, S(z^{1/\rho})) \) by the quantity \( G_1(\zeta^j_k, S(\zeta^j_k)) \) at each integral in Eq. (16), we get

\[
S^p(\zeta^j_{k+1}) \approx S(0) + \frac{\rho^{-1}h^\gamma}{\Gamma(\gamma + 2)} \sum_{j=0}^{k} [a_j h_j + G_1(\zeta^j_k, S(\zeta^j_k))] dz
\]

\[
= S(0) + \frac{\rho^{-1}h^\gamma}{\Gamma(\gamma + 2)} \sum_{j=0}^{k} [(k+1-j)^{\gamma} - (k-j)^{\gamma}] G_1(\zeta^j_k, S(\zeta^j_k)).
\] (20)

Therefore, our P-C scheme, for finding the approximation \( S_{k+1} \approx S(\zeta^j_{k+1}) \), is totally evaluated by the formula

\[
S_{k+1} \approx S(0) + \frac{\rho^{-1}h^\gamma}{\Gamma(\gamma + 2)} \sum_{j=0}^{k} a_{j,k+1} G_1(\zeta^j_k, S_j)
\]

\[
+ \frac{\rho^{-1}h^\gamma}{\Gamma(\gamma + 2)} G_1(\zeta^j_{k+1}, S^p_{k+1}),
\] (21)

Fig. 7  S, E, I and R profiles with \( \beta = 0.4869, \phi = 0.1552 \) and \( \rho = 0.01 \) (set 6 of Table 2), \( R_0 = 3.14 \).
SEIR COVID-19 system (8) can be established as:

\[ S_j(t) = S(0) + \frac{\rho \gamma}{\Gamma(\beta + 2)} \sum_{j=0}^{k} a_{j,k+1} G_1(\zeta_j, S_j(t)) + \frac{\rho \gamma}{\Gamma(\beta + 2)} G_1(\zeta_{k+1}, S_{k+1}), \]

\[ E_j(t) = E(0) + \frac{\rho \gamma}{\Gamma(\beta + 2)} \sum_{j=0}^{k} a_{j,k+1} G_2(\zeta_j, E_j(t)) + \frac{\rho \gamma}{\Gamma(\beta + 2)} G_2(\zeta_{k+1}, E_{k+1}), \]

\[ I_j(t) = I(0) + \frac{\rho \gamma}{\Gamma(\beta + 2)} \sum_{j=0}^{k} a_{j,k+1} G_3(\zeta_j, I_j(t)) + \frac{\rho \gamma}{\Gamma(\beta + 2)} G_3(\zeta_{k+1}, I_{k+1}), \]

\[ R_j(t) = R(0) + \frac{\rho \gamma}{\Gamma(\beta + 2)} \sum_{j=0}^{k} a_{j,k+1} G_4(\zeta_j, R_j(t)) + \frac{\rho \gamma}{\Gamma(\beta + 2)} G_4(\zeta_{k+1}, R_{k+1}). \]

where \(S_j(t)\), \(E_j(t)\), \(I_j(t)\), and \(R_j(t)\) are the solutions of given SEIR COVID-19 system (8). The predicted value \(S_{k+1}\) can be evaluated as given in Eq. (20) with the weights \(a_{j,k+1}\) being calculated according to (19).

So by the above manner, we can derive the approximation formulae for the whole system (7).

So, the approximation solution formulae for the given SEIR COVID-19 system (8) can be established as:

\[ S_j(t) = S(0) + \frac{\rho \gamma}{\Gamma(\beta + 2)} \sum_{j=0}^{k} a_{j,k+1} G_1(\zeta_j, S_j(t)) + \frac{\rho \gamma}{\Gamma(\beta + 2)} G_1(\zeta_{k+1}, S_{k+1}), \]

\[ E_j(t) = E(0) + \frac{\rho \gamma}{\Gamma(\beta + 2)} \sum_{j=0}^{k} a_{j,k+1} G_2(\zeta_j, E_j(t)) + \frac{\rho \gamma}{\Gamma(\beta + 2)} G_2(\zeta_{k+1}, E_{k+1}), \]

\[ I_j(t) = I(0) + \frac{\rho \gamma}{\Gamma(\beta + 2)} \sum_{j=0}^{k} a_{j,k+1} G_3(\zeta_j, I_j(t)) + \frac{\rho \gamma}{\Gamma(\beta + 2)} G_3(\zeta_{k+1}, I_{k+1}), \]

\[ R_j(t) = R(0) + \frac{\rho \gamma}{\Gamma(\beta + 2)} \sum_{j=0}^{k} a_{j,k+1} G_4(\zeta_j, R_j(t)) + \frac{\rho \gamma}{\Gamma(\beta + 2)} G_4(\zeta_{k+1}, R_{k+1}). \]

Theorem 3. Let the Lipschitz condition satisfied by \(G_1(\zeta, S)\) in (9a) and \(S_j(\zeta_j)\), \(E_j(\zeta_j)\), \(I_j(\zeta_j)\), and \(R_j(\zeta_j)\) are the solutions of given Predictor–Corrector algorithm (22) and (23). Then, the given fractional numerical scheme (22) and (23) is conditionally stable.
Using the Lipschitz condition, we obtain

\[ |S_{k+1}^-| \leq \zeta_0 + \frac{\rho^{-\gamma} m_1}{(\gamma + 1)} \left( |\tilde{S}_{k+1}^p| + \sum_{j=0}^{k} \alpha_{j,k+1} |\tilde{S}_j| \right), \]  

where \( \zeta_0 = \max_{0 \leq k \leq N} \{ |\tilde{S}_0| + \frac{\rho^{-\gamma} m_1}{(\gamma + 2)} |\tilde{S}_0| \} \). Also, from Eq. (3.18) in [20] we derive

\[ |\tilde{S}_{k+1}^p| \leq \eta_0 + \frac{\rho^{-\gamma} m_1}{(\gamma + 1)} \sum_{j=0}^{k} \alpha_{j,k+1} |\tilde{S}_j|, \]

where \( \eta_0 = \max_{0 \leq k \leq N} \{ |\tilde{S}_0| + \frac{\rho^{-\gamma} m_1 a_{k+1}}{(\gamma + 2)} |\tilde{S}_0| \} \). Substituting \( |\tilde{S}_{k+1}^p| \) from Eq. (27) into Eq. (26) results

\[ |S_{k+1}^-| \leq \gamma_0 + \frac{\rho^{-\gamma} m_1}{(\gamma + 2)} \left( \frac{\rho^{-\gamma} m_1}{(\gamma + 1)} \sum_{j=1}^{k} \alpha_{j,k+1} |\tilde{S}_j| + \sum_{j=1}^{k} \alpha_{j,k+1} |\tilde{S}_j| \right), \]  

\[ \leq \gamma_0 + \frac{\rho^{-\gamma} m_1}{(\gamma + 2)} \sum_{j=1}^{k} \left( \frac{\rho^{-\gamma} m_1}{(\gamma + 1)} \alpha_{j,k+1} + \alpha_{j,k+1} \right) |\tilde{S}_j|, \]  

\[ \leq \gamma_0 + \frac{\rho^{-\gamma} m_1 C_{\gamma,2}}{(\gamma + 2)} \sum_{j=1}^{k} (k + 1 - j)^{-1} |\tilde{S}_j|, \]

where \( \gamma_0 = \max(\zeta_0 + \frac{\rho^{-\gamma} m_1 a_{k+1}}{(\gamma + 2)} \eta_0) \). \( C_{\gamma,2} \) is a positive constant only depends on \( \gamma \) (Lemma 1) and \( h \) is supposed to be small enough. Applying Lemma 2 concludes \( |\tilde{S}_{k+1}^-| \leq C_{\gamma,0} \), which ends the proof. (see Table 1)

6. Simulation results

To establish graphical simulations, we use parameter values cited from a real data, summarized in Table 2. We also use the following initial conditions \( S(0) = 1 - \frac{1}{p \times 10^{10}}, I(0) = \frac{1}{p \times 10^{10}}, E(0) = 0, \) and \( R(0) = 0 \). These parameter values were
estimated using COVID-19 real data in Brazil, between 25 February ($\zeta = 0$) to 30 March 2020. For the model (5), the basic reproductive number is given by

$$R_0 = \frac{\beta}{\phi} \left( 1 - \frac{1}{p \times 2.10 \times 10^3} \right),$$

where the identification rate $p$ ranges from 0.01 to 0.1.

We estimated basic reproduction number $R_0$ for different values of infection rate $\beta$ and removal rate $\phi$ given as follows: where the identification rate $p$ ranges from 0.01 to 0.1. (see Fig. 1).

We denote the coronavirus epidemic peak $\zeta^*$ for the exposed and infected individuals attains its maximum in 1 year. We first set $p = 0.1$. In this case, we obtained the following figures (Figs. 2–4) on the long time behaviour of all given classes for $\beta = 0.5869 \& \phi = 0.2004, \beta = 0.6043 \& \phi = 0.2004$ and $\beta = 0.4869 \& \phi = 0.1552$, respectively. We can see from Fig. 2 that the estimated epidemic peaks for exposed and infected individuals are $\zeta^* = 145$ near about $\gamma = 1$. That is, starting from 25 February ($\zeta = 0$), the estimated epidemic peaks are $19$ July ($\zeta = 145$). We can also observe that in Fig. 2 if we shifted to $\gamma = 0.95$, the peak shifted to $t \approx 162$ (5 August). We observe from Fig. 3 that the estimated epidemic peaks are $\zeta^* = 145$ for both exposed and infective individuals near about $\gamma = 1$. That is, starting from 25 February ($\zeta = 0$), the estimated coronavirus peak is 19 July ($\zeta = 145$). We can also observe that in Fig. 3 if we shifted to $\gamma = 0.95$, the peak shifted to $\zeta \approx 152$ (26 July) and $\zeta \approx 160$ (3 August) respectively. From Fig. 4, the estimated coronavirus peak is $\zeta^* = 152$ for exposed and $\zeta^* = 162$ for infected near about $\gamma = 1$, respectively. That is, starting from 25 February ($\zeta = 0$), the estimated epidemic peak are 26 July ($\zeta = 152$) and 5 August ($\zeta = 162$), respectively. When we shifted to $\gamma = 0.95$, the peak shifted to $\zeta \approx 165$ (8 August) and $\zeta \approx 172$ (15 August), respectively. We next set $p = 0.01$. In this case, we obtain the following figures (Figs. 5–7) on the long time behaviour of all given classes for $\beta = 0.5869 \& \phi = 0.2004, \beta = 0.6043 \& \phi = 0.2004$ and $\beta = 0.4869 \& \phi = 0.1552$, respectively. We can see from Fig. 5 that the estimated epidemic peaks for exposed and infected individuals are $\zeta^* = 130$ (4 July) and $\zeta^* = 132$ (6 July) respectively near about $\gamma = 1$. We can also observe that in Fig. 5 if we shifted to $\gamma = 0.95$, the peak shifted to $\zeta \approx 138$ (12 July) and $\zeta \approx 145$ (19 July). We observe from Fig. 6 that the estimated epidemic peaks are $\zeta^* = 122$ (26 June) for exposed and $\zeta^* = 125$ (29 June) near about $\gamma = 1$. We also observe that in Fig. 6 if we shifted to $\gamma = 0.95$, the peak shifted to $\zeta \approx 130$.

Fig. 10 Simulations of Exposed class v/s susceptible, infective and removed for $\beta = 0.4869, \phi = 0.1552$ and $p = 0.1$ (set 6 of Table 2).
and $\zeta \approx 135$. From Fig. 7, the estimated epidemic peaks are $\zeta^* = 132$ for exposed and $\zeta = 140$ for infected near about $\gamma = 1$ respectively. That is, starting from 25 February ($\zeta = 0$), the estimated COVID-19 peak are 6 July ($\zeta = 132$) and 14 July ($\zeta = 140$) respectively. When we sifted to $\gamma = 0.95$, the peak shifted to $\zeta \approx 142$ (16 July) and $\zeta \approx 150$ (24 July), respectively.

After the successful study of epidemic peaks, here we do some graphical simulations of the nature of the given classes respect to each other. In Fig. 8, we show the nature of susceptible, infected and removed classes versus exposed individuals for $\beta = 0.5869, \phi = 0.2004$ and $p = 0.1$ at different fractional order $\gamma$. In Fig. 9, we show the nature of susceptible, infected and removed classes versus exposed individuals for $\beta = 0.6043, \phi = 0.2004$ and $p = 0.1$ at different fractional order $\gamma$. In Fig. 10, we show the nature of susceptible, infected and removed classes versus exposed individuals for $\beta = 0.4869, \phi = 0.1552$ and $p = 0.1$ at different fractional order $\gamma$. Similarly, we do the graphical simulations of the nature of the given classes respect to each other for $p = 0.01$. In Fig. 11, we show the nature of susceptible, infected and removed classes versus exposed individuals for $\beta = 0.5869, \phi = 0.2004$ and $p = 0.01$ at different fractional order $\gamma$. In Fig. 12, we show the nature of susceptible, infected and removed classes versus exposed individuals for $\beta = 0.6043, \phi = 0.2004$ and $p = 0.01$ at different fractional order $\gamma$. In Fig. 13, we show the nature of susceptible, infected and removed classes versus exposed individuals for $\beta = 0.4869, \phi = 0.1552$ and $p = 0.01$ at different fractional order $\gamma$. From the all above simulations, we can say that there are so much uncertainties of epidemic peaks for the given real data range. Given new generalised Caputo derivative works well to study these different natures. All simulations are done by the use of Mathematica software. Also, we have fixed the parameter $p$ equal to 1.2 for all simulations, which can make more varieties in the simulations for different numerical values. The calculative value of the basic reproduction number $R_0$ in our study is different for specific data. In Table 2, we gave some estimated values of basic reproductive number but for graphical simulations we studied only three sets of values for $R_0 = 2.93, 3.02$ and 3.14. In this study, the estimated values of the basic reproduction number $R_0$ are not so different from some estimations given in [4] (see Table 3).
7. Conclusions

In this paper, by using a compartmental mathematical non-linear model to the reported cases of Coronavirus in Brazil from 25 February to 30 March, we have evaluated that the basic reproduction number $R_0$ has so many possible values for the specific range of infection rate and removal rate when identification rate assumed between 0.01 to 0.1. Of course, such type of long range peak prognostication would ally the necessary suspicion due to the event of some large transformations in the social and natural conditions. We have observed that the basic reproductive number $R_0$ (essential epidemic size) would not be dominance by the identification rate $\rho$ in a parameter range 0.01 – 0.1. After giving the theory of necessary definitions and the description of the non-linear model, the proposed time-fractional non-linear model is studied by the applications of modified version of Corrector-Predictor algorithm in the sense of recent generalised version of Caputo non-integer derivative. The given method stability is also concerned by the application of some specific well known lemmas. The framework of existence of unique solution is also given. Our results are rich for making a full idea of epidemic peaks of coronavirus cases in Brazil. All necessary calculations for plots of the data are given to specify the nature of the given solutions at various fractional order values. We concluded that there are a lot of uncertainties in the given peaks for the mentioned data range. The given method is strong and highly reliable in finding the solution to non-integer order models of physical, biological, and medical point of view. The given numerical approach exemplifies the uses of the given scheme and considered non-integer order operator while analysing real world problems.

Fig. 12 Simulations of Exposed class v/s susceptible, infective and removed for $\beta = 0.6043$, $\phi = 0.2004$ and $\rho = 0.01$ (set 5 of Table 2).
Availability of data and material

Not applicable.

Declaration of Competing Interest

This work does not have any conflicts of interest.

Authors contribution

All authors equally contributed to this work.

Acknowledgement

Not applicable.

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Fig. 13 Simulations of Exposed class v/s susceptible, infective and removed for $\beta = 0.4869$, $\phi = 0.1552$ and $p = 0.01$ (set 6 of Table 2).

Table 3 Estimation of basic reproduction number $R_0$.

| Sets | $\beta$   | $\phi$   | $R_0$   |
|------|-----------|----------|--------|
| 1    | 0.6043    | 0.0714   | 8.46   |
| 2    | 0.3695    | 0.3295   | 1.12   |
| 3    | 0.4869    | 0.2004   | 2.43   |
| 4    | 0.5869    | 0.2004   | 2.93   |
| 5    | 0.6043    | 0.2004   | 3.02   |
| 6    | 0.4869    | 0.1552   | 3.14   |
| 7    | 0.4869    | 0.1142   | 4.26   |
