Reliability of a high energy one-loop expansion
of \(e^+e^- \rightarrow W^+W^-\) in the SM and in the MSSM *

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Abstract

We compare the logarithmic Sudakov expansions of the process \(e^+e^- \rightarrow W^+W^-\) in the one-loop approximation and in the resummed version, to subleading order accuracy, in the SM case and in

a light SUSY scenario for the MSSM. We show that the two expansions are essentially identical below 1 TeV, but differ drastically at higher (2,3 TeV) center of mass energies. Starting from these conclusions, we argue that a complete one-loop calculation in the energy region below 1 TeV does

not seem to need extra two-loop corrections, in spite of the relatively large size of the one-loop effects.

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I. INTRODUCTION

Historically, $WW$ production from $e^+e^-$ annihilation has been a basic process for the purposes of testing the SM and several of its extensions or alternatives \[1\]. This fact has been known for decades, and has already led to several constraints from the available measurements at LEP2\[2\]. From a practical point of view, the derivation of the constraints is relatively straightforward since in many cases it does not require, given the total absence of any signal of new physics, an extremely precise theoretical calculation. This situation should change at the foreseen future linear colliders (LC\[3\], CLIC\[4\]), where an accuracy at the level of a relative few (1-2) percent is expected and, hopefully, evidence of some kind of new physics will have already been given. In this spirit, a rigorous calculation of effects beyond the simple tree level is a theoretical must.

In the simplest case of the SM, this complete one-loop calculation has been performed almost a decade ago \[5\]. Quite recently, Hahn\[6\] has redone the SM calculation (finding complete agreement with Ref.\[5\]) and extended it to the MSSM case. His analysis has been performed in an energy range between, roughly, 200 GeV and 1 TeV and represents, to our knowledge, the only complete analysis of this process in the MSSM which is nowadays available.

A feature that one notices at first sight in various plots of the different considered cross sections is the fact that the one-loop effects become almost immediately large, reaching a relative size of fifteen percent or more, both in the SM and in the MSSM (where they are systematically larger). On one side, this has the positive important meaning that the radiative corrections do provide a real stringent test of the model. On another side, this could introduce a disturbing warning: if a final accuracy at the one percent level is aimed, a fifteen percent one-loop effect might require the hard calculation of higher order (e.g two-loop) effects, that seems, at least in the MSSM case, beyond the available technical (and human) possibilities.

The aim of this paper is that of arguing that, at least at the relative few percent level, a one-loop calculation does not seem to need extra corrections below 1 TeV, but cannot give the same reliability at higher (2,3 TeV) center of mass (c.m.) energies. With this purpose, we shall divide our discussion into three parts. In Section II we shall compute the electroweak Sudakov logarithmic expansion (for this terminology see e.g Ref.\[7\]) of the
process to subleading logarithmic accuracy, both at the one loop level and resummed to all orders. For c.m. energies beyond a few hundred GeV this asymptotic expansion should be certainly valid for the SM. In the MSSM case, a priori, it can be assumed to be reliable in the picture of a relatively light SUSY scenario, where all the relevant electroweak sparticle masses are below a few hundred GeV; our analysis will be limited therefore to this special configuration. In Section III we shall compare the effects on the different observables of the process of the two approximate expansions and show that they are "essentially" (i.e. at the assumed few percent accuracy level) identical below 1 TeV, but drastically different at higher (2,3 TeV) energies. From this evidence we shall then try to conclude in Section IV that a complete one-loop calculation does not seem to require extra two-loop corrections in the energy region below 1 TeV, the proposed range of a future LC.

II. LOGARITHMIC SUDAKOV EXPANSIONS

In all our analysis we shall assume for the purposes of our logarithmic Sudakov expansions, that are by definition asymptotic ones, that the involved c.m. energies are sufficiently higher than all the involved masses, and therefore we shall consider the behaviour of the process in the "high energy" limit. In this range, we shall now provide a brief description of the essential kinematical properties of the scattering amplitude, both at the Born level and at the considered higher orders.

A. High energy behaviour at the Born level

Denoting by $l_1, l_2, p_1, p_2$ the $e^-, e^+, W^-, W^+$ momenta and $e_1, e_2$ the $W^-, W^+$ polarization vectors, the invariant Born amplitude due to $\nu_e, \gamma, Z$ exchanges is:

$$A^{\text{Born}} = A^{\nu_e, \text{Born}} + A^{\gamma, \text{Born}} + A^{Z, \text{Born}}$$

(2.1)

with

$$A^{\nu_e, \text{Born}} = -\frac{e^2}{2s_W^2 t} \bar{v}(e^+) (e_2) (\hat{f}_1 - \hat{f}_1) P_L u(e^-)$$

(2.2)

$$A^{\gamma + Z, \text{Born}} = \frac{2e^2}{s} \bar{v}(e^+) \left[ (e_2 . p_1) (\hat{f}_1) - (e_1 . e_2) (\hat{f}_1) - (e_1 . p_2) (\hat{f}_2) \right] \times$$

$$\left[ (1 - \chi) \frac{2s_W^2 - 1}{2s_W^2} P_L + (1 - \chi) P_R \right] u(e^-)$$

(2.3)
where \(P_{R/L} = \frac{(1 \pm \gamma^5)}{2}, \ s = (l_1 + l_2)^2 = (p_1 + p_2)^2, \ t = (l_1 - p_1)^2 = M_W^2 - \frac{s}{2}(1 - \beta \cos \vartheta), \ u = (l_2 - p_1)^2 = M_W^2 - \frac{s}{2}(1 + \beta \cos \vartheta), \ \beta = \sqrt{1 - \frac{4M_W^2}{s}}, \ \chi = s/(s - M_Z^2).\)

**Helicity amplitudes**

From these invariant forms one obtains the helicity amplitudes denoted as \(F(\lambda, \mu_1, \mu_2).\)

In the limit of negligible electron mass, \(\lambda\) is the chirality (\(L\) when \(\lambda = -1\) and \(R\) when \(\lambda = 1\)) related to the \(e^\pm\) helicities \(\lambda \equiv 2\lambda(e^-) = -2\lambda(e^+) = \pm 1.\) The \(W^\pm\) helicities are \(\mu_1(W^-) = \pm 1, 0\) and \(\mu_2(W^+) = \pm 1, 0\) and we will denote by \(W_T^\pm\) and \(W_0^\pm\) the transverse and longitudinal cases. For details see for example Ref. [5]. We now just recall the main results that we will use to write the contributions at one loop and beyond.

It is convenient to consider separately the amplitudes corresponding to the 3 different types of final states, \(W_TW_T, W_TW_0, W_0W_0,\) whose properties are, already at Born level, very different in the high energy limit \(s \gg M_W^2:\)

\((W_TW_0):\) The \(W_TW_0\) Born amplitudes are mass suppressed (i.e. behave like \(M_W/s\)) and we shall ignore them.

\((W_TW_T):\) The only non vanishing \(W_TW_T\) Born amplitudes are given by the t-channel neutrino exchange diagram and, because of the purely Left-handed \(We\bar{\nu}\) coupling, they reduce to

\[
F^{\text{Born}}(L, \mu, -\mu) \simeq -\left(\frac{e^2}{2s_W}\right)(\frac{s}{2t}) \sin \vartheta(\mu - \cos \vartheta)
\]

At high energy these amplitudes are strongly peaked forward as

\[
\frac{2t}{s} = \beta \cos \vartheta - 1 + \frac{2M_W^2}{s} \rightarrow \cos \vartheta - 1
\]

For the purposes of our asymptotic expansions, we must assume large values of \(s\) and \(t.\)

Therefore we shall introduce an angular cut in our numerical computations as discussed in Section III.

Note that the other set of \(W_TW_T\) amplitudes \(F^{\text{Born}}(\lambda, \mu, \mu)\) receives contributions from both \(\nu_e, \gamma\) and \(Z\) exchanges whose sum cancels in the high energy limit

\[
F^{\text{Born}}(\lambda, \mu, \mu) \simeq O(M_W^2/s)
\]

\((W_0W_0):\) The \(W_0W_0\) Born amplitudes result from the addition of \(\nu_e, \gamma\) and \(Z\) exchange
contributions leading to

\[ F^{\text{Born}}(L, 0, 0) \simeq -\frac{e^2}{4s_W^2 c_W^2} \sin \vartheta \quad F^{\text{Born}}(R, 0, 0) \simeq \frac{e^2}{2c_W^2} \sin \vartheta \quad (2.7) \]

The cancellations leading to the above results at order \(M_W^2/s\), usually dubbed as ”gauge cancellations" (resulting from the relation between the \(W e \nu\) coupling and the self-gauge boson \(\gamma W W, Z W W\) couplings), lead to results that are in agreement with the equivalence theorem \([8]\) which states the equality between the \(e^+ e^- \rightarrow W_0^+ W_0^-\) amplitudes and the amplitudes for charged Goldstone boson production \(e^+ e^- \rightarrow G^+ G^-\). This equality is true not only at Born level but also at higher orders at logarithmic accuracy \([9]\). The high energy behaviour of \(G^+ G^-\) can be found in previous papers \([10, 11]\).

**B. Amplitudes at one loop**

At one loop the amplitudes at logarithmic accuracy can be obtained either by a direct computation of the high energy limit of the Feynman diagrams like in our previous works see \(e.g.\) \([10, 11, 12]\) or by using the splitting function formalism \([13]\) and the Parameter Renormalization (PR) properties giving the leading logarithms arising from soft and collinear singularities \([14]\). The results agree and give

\[ (W_T W_T): \quad F(L, \mu, -\mu) = F^{\text{Born}}(L, \mu, -\mu) \left\{ 1 + (b^{in}_L)[n_i \log \frac{s}{M_W^2} - \log \frac{s}{M_W^2}] + b^{\text{fin}, TT}[- \log \frac{s}{M_W^2}] + b^{\text{ang}, TT}[\log \frac{s}{M_W^2}] \right\} \quad (2.8) \]

\[ (W_0 W_0): \quad F(\lambda, 0, 0) = F^{\text{Born}}(\lambda, 0, 0) \left\{ 1 + b^{in}_\lambda [n_i \log \frac{s}{M_W^2} - \log \frac{s}{M_W^2}] \right. \]

\[ + b^{\text{fin},00}[n_f \log \frac{s}{M_W^2} - \log \frac{s}{M_W^2}] - b^{Yuk,00}[\log \frac{s}{M_W^2}] + b^{\text{ang},00}[\log \frac{s}{M_W^2}] \]

\[ - \frac{\alpha}{\pi} [(\frac{c_W^2}{s_W^2} \tilde{\beta}_0 + \frac{s_W^2}{c_W^2} \tilde{\beta}_0') \delta_{\lambda,L} + \frac{\tilde{\beta}_0'}{c_W^2} \delta_{\lambda,R}][\log \frac{s}{M_W^2}] \right\} \quad (2.9) \]

where a common mass scale, chosen to be \(M_W\), has been used both in the genuine Sudakov logarithms and in the linear ones of Renormalization Group (RG) origin discussed below. With this choice, the dependence on the MSSM mass parameters has been fully shifted into
residual next-to subleading terms (either constant or vanishing with energy) that are beyond the purposes of this paper.

In Eqs.\((2.8-2.9)\) one recognizes

— the universal effects for the incoming \(e^+, e^-\) with the form \([n_i \log \frac{s}{M_W^2} - \log^2 \frac{s}{M_W^2}]\), \(n_i = 3\) in SM and \(n_i = 2\) in MSSM, as established in Refs.\([10, 14]\) and the coefficients

\[
b^\text{fin.}_{L,R} = \frac{\alpha (1 + 2c_W^2)}{16\pi s_W^2 c_W^2} \quad b^\text{fin.}_{R} = \frac{\alpha}{4\pi c_W^2}
\]  

(2.10)

— the universal effects for the outgoing \(W^+_T, W^-_T\) with the form \([- \log^2 \frac{s}{M_W^2}]\), both in SM and in MSSM, \([14]\) and the coefficient

\[
b^\text{fin.}.TT = \frac{\alpha}{2\pi s_W^2}
\]  

(2.11)

— the universal effects for the outgoing \(W^+_0 \equiv G^+, W^-_0 \equiv G^-\) with the form \([n_f \log \frac{s}{M_W^2} - \log^2 \frac{s}{M_W^2}]\), \(n_f = 4\) in SM and \(n_f = 2\) in MSSM for what concerns the gauge part and the additional Yukawa term for both models, with the coefficients \([10, 11, 14]\)

\[
b^\text{fin.00} = \frac{\alpha (1 + 2c_W^2)}{16\pi s_W^2 c_W^2} \quad b^\text{Yuk.00} = \frac{3\alpha (m_t^2 + m_b^2)}{8\pi s_W^2 M_W^2}
\]  

(2.12)

— the non universal (angular dependent) contribution which only consists in residual terms arising from the quadratic logarithms \(\log^2 t, \log^2 u\) (from which the \(\log^2 s\) part has been subtracted and put in the universal contribution) generated by \(t\)-channel triangles and box diagrams containing \(W, Z, \gamma\) gauge boson internal lines, leading for both SM and MSSM cases to

\[
b^\text{ang.}.TT_{L,R} = -\frac{\alpha}{2\pi s_W^2} \left[ \log \frac{t}{u} + \left(1 - \frac{t}{u}\right) \log \frac{-t}{s} \right]
\]  

(2.13)

and

\[
b^\text{ang.00}_{L,R} = -\frac{\alpha}{4\pi} \left[ 4c_W^2 \log \frac{-t}{s} + \frac{1}{s_W c_W^2} \log \frac{t}{u} \right] \quad b^\text{ang.00}_{R} = -\frac{\alpha}{2\pi c_W^2} \left[ \log \frac{t}{u} \right]
\]  

(2.14)
In the \((W_0W_0)\) case, the last term is the one loop single log arising from the RG contribution (intermediate \(\gamma, Z\) self-energy contributions). It can be directly obtained from the corresponding Born contribution through the expression

\[
F^{\text{RG}} = -\frac{1}{4\pi^2} \left( g^4 \beta_0 \frac{dF^{\text{Born}}}{dg^2} + g'^4 \beta'_0 \frac{dF^{\text{Born}}}{dg'^2} \right) \log \frac{s}{M_W^2} \tag{2.15}
\]

where \(g_{SW} = g'c_W = e\) and

\[
\beta_0 = \frac{43}{24} - \frac{N}{3} \quad \text{(SM)}, \quad \frac{5}{4} - \frac{N}{2} \quad \text{(MSSM)}
\]

\[
\beta'_0 = -\frac{1}{24} - \frac{5N}{9} \quad \text{(SM)}, \quad -\frac{1}{4} - \frac{5N}{6} \quad \text{(MSSM)} \tag{2.16}
\]

or by taking the first order expansion of the running expressions

\[
g^2(s) = \frac{g^2(\mu^2)}{1 + \beta_0 \frac{g^2(\mu^2)}{4\pi^2} \log \frac{s}{M_W^2}}, \quad g'^2(s) = \frac{g'^2(\mu^2)}{1 + \beta'_0 \frac{g'^2(\mu^2)}{4\pi^2} \log \frac{s}{M_W^2}} \tag{2.17}
\]

In the \((W_TW_T)\) case there is no such RG term. This can be seen either in the diagrammatic way (there is no \(\gamma, Z\) self-energy contribution to \(F(L, \mu, -\mu)\)), or when using the splitting function formalism and the PR analysis by observing the cancellation of the single log due to the collinear singularities associated to the final lines with the one arising from the PR contribution [14].

The SM part of the above results agrees completely with a previous analysis [15]. We have also checked that these high energy \(W_TW_T\) properties are in agreement with those of the Wino components in chargino pair production [16], due to supersymmetry.

C. Resummed Amplitudes

The general procedure for writing the so-called resummed amplitude, \(i.e.\) the exponential form containing all orders at subleading logarithmic accuracy has been described in several papers [9, 17] and applied to specific cases [10, 16]. It has also been very recently successfully checked by a non trivial comparison with a partial two-loop calculation to leading order [18] and with the angular dependent subleading contribution for arbitrary processes [19]. So we will not repeat it here, but will immediately write the result in a rather transparent form.
For $W_TW_T$ amplitudes the expression is:

$$F(L, \mu, -\mu) = F^{Born}(L, \mu, -\mu) \exp \left\{ \left[ \tilde{b}_L^{in} + \tilde{b}^{fin, TT} \right] \frac{1}{3} \log^3 \left( \frac{s}{M_W^2} \right) \right\}$$

$$+ \left( \tilde{b}_L^{in} \right) \left[ n_i \log \frac{s}{M_W^2} - \frac{1}{2} \log^2 \frac{s}{M_W^2} + \tilde{b}^{fin, TT} \right] \left[ - \log^2 \frac{s}{M_W^2} \right]$$

$$+ b^{WPR} \log \frac{s}{M_W^2} + b^{\text{ang}, TT} \left[ \log \frac{s}{M_W^2} \right]$$

$$+ F^{RG}(L, \mu, -\mu) \quad (2.18)$$

and for $W_0W_0 \simeq G^+G^-$ amplitudes:

$$F(\lambda, 0, 0) = F^{Born}(\lambda, 0, 0) \exp \left\{ \left[ \tilde{b}^{in}_\lambda + \tilde{b}^{fin, LL} \right] \frac{1}{3} \log^3 \left( \frac{s}{M_W^2} \right) \right\}$$

$$+ b^{in}_\lambda \left[ n_i \log \frac{s}{M_W^2} - \frac{1}{2} \log^2 \frac{s}{M_W^2} \right] + b^{fin,00} \left[ n_f \log \frac{s}{M_W^2} - \frac{1}{2} \log^2 \frac{s}{M_W^2} \right]$$

$$- b^{Yuk,00} \left[ \log \frac{s}{M_W^2} \right] + b^{\text{ang},00} \left[ \log \frac{s}{M_W^2} \right]$$

$$+ F^{RG}(\lambda, 0, 0) \quad (2.19)$$

with $n_i = 3$ or 2, $n_f = 4$ or 2, as in the one loop case. The new quantities arising from Parameter Renormalization of high order diagrams, not defined in the one loop expression, are:

$$\tilde{b}^{fin, TT} = \frac{\alpha^2 \beta_0}{2 \pi^2 s_w^4} \quad b^{WPR} = \frac{\alpha \beta_0}{\pi s_w^2} \quad (2.20)$$

$$\tilde{b}^{fin, LL} = \frac{3 \alpha^2 \beta_0}{16 \pi^2 s_w^4} + \frac{\alpha^2 \beta'_0}{16 \pi^2 c_w^4} \quad \tilde{b}^{in}_R = \frac{\alpha^2 \beta'_0}{4 \pi^2 c_w^4} \quad (2.21)$$

The additional RG contribution to all orders are obtained explicitly using Eq.(2.17) in the SM or MSSM cases:

$$F^{RG}(L, \mu, -\mu) = -\left( \frac{s}{4t} \right) \sin \vartheta (\mu - \cos \vartheta) \left\{ g^2(s) - [g^2]_{\text{Born}} \right\} \quad (2.22)$$

$$F^{RG}(L, 0, 0) = - \left( \frac{\sin \vartheta}{4} \right) \left\{ g^2(s) + g'^2(s) - [g^2 + g'^2]_{\text{Born}} \right\} \quad (2.23)$$

$$F^{RG}(R, 0, 0) = \frac{\sin \vartheta}{2} \left\{ g^2(s) - [g^2]_{\text{Born}} \right\} \quad (2.24)$$

One can check that a first order expansion of Eq.(2.18,2.19) reproduces the one loop results. In particular for the $W_TW_T$ amplitudes one observes the cancellation of the single
log associated to the outgoing $W^+, W^-$ when adding the part coming from the exponential and the RG part.

**D. Observables**

In this Section we summarize the definition of the various observables that will be considered in the analysis.

The unpolarized angular distribution is given by

$$\frac{d\sigma}{d\cos \theta} = \frac{\beta}{128\pi s} \sum_{\lambda, \mu, \mu'} |F(\lambda, \mu, \mu')|^2$$ (2.25)

The angular distribution for longitudinally polarized W is obtained by reducing the sum over $\mu, \mu'$ to the case $\mu = \mu' = 0$.

The left-right angular distribution is given by

$$\frac{d\sigma_{LR}}{d\cos \theta} = \frac{1}{128\pi s} \sum_{\mu, \mu'} \left[ |F(L, \mu, \mu')|^2 - |F(R, \mu, \mu')|^2 \right]$$ (2.26)

The differential cross sections can be integrated with an angular cut-off $|\cos \theta| \leq \cos \theta_{cut}$ in the forward or backward cones:

$$\int_F \equiv \int_0^{\cos \theta_{cut}} d\cos \theta, \quad \int_B \equiv \int_{-\cos \theta_{cut}}^0 d\cos \theta$$ (2.27)

This leads to the cross sections

$$\sigma = (\int_F + \int_B) \frac{d\sigma}{d\cos \theta}$$ (2.28)

$$\sigma_{LR} = (\int_F + \int_B) \frac{d\sigma_{LR}}{d\cos \theta}$$ (2.29)

$$\sigma_{FB} = (\int_F - \int_B) \frac{d\sigma}{d\cos \theta}$$ (2.30)

and the related asymmetries

$$A_{FB} = \frac{\sigma_{FB}}{\sigma}, \quad A_{LR} = \frac{\sigma_{LR}}{\sigma}$$ (2.31)

The relative effect due to radiative corrections (treated in the one-loop approximation or by the resummation formula) is defined to be

$$\text{cross-sections} : \Delta \sigma / \sigma \equiv \frac{\sigma - \sigma^{\text{Born}}}{\sigma^{\text{Born}}} , \quad \text{asymmetries} : \Delta A \equiv A - A^{\text{Born}}$$ (2.32)
In the high-energy limit \((s \to \infty)\) the Born values of the above quantities are \((\delta \equiv \cos \vartheta_{cut})\)

\[
\sigma = \frac{\pi \alpha^2}{192 \ s s_w^4 \ c_w^4} \left[ -3(23 - 48 s_w^2 + 20 s_w^4) \delta + (-9 + 16 s_w^2 - 12 s_w^4) \delta^3 + 48 c_w^4 \log \frac{1 + \delta}{1 - \delta} \right]
\]

\[
\sigma_{FB} = \frac{\pi \alpha^2}{8 \ s s_w^4 c_w^4} \left[ -\delta^2 - 2 \log(1 - \delta^2) \right]
\]

\[
\sigma_{LR} = \frac{\pi \alpha^2}{192 \ s s_w^4 c_w^4} \left[ -3(23 - 48 s_w^2 + 28 s_w^4) \delta + (-9 + 16 s_w^2 - 4 s_w^4) \delta^3 + 48 c_w^4 \log \frac{1 + \delta}{1 - \delta} \right]
\]

\[
\sigma_{\text{long}, W} = \frac{\pi \alpha^2 (1 - 4 s_w^4)}{96 \ s s_w^4 c_w^4}
\]

After this, we hope not too long, technical presentation we are now ready to move to the explicit comparison of the relative effects in the two expansions. This will be done in the forthcoming Section III.

### III. COMPARISON OF THE TWO EXPANSIONS

Starting from the formulae given in the previous Section II, we have now computed the various electroweak logarithmic effects on the considered observables in the two approximate expansions, to subleading logarithmic accuracy, in an energy region between 500 GeV and 3 TeV, that should include the foreseen LC and CLIC c.m. energy domains. To simplify the presentation of our results, we have considered the known fact that the number of final longitudinal \(WW\) pairs is much smaller than that of the transverse ones, that completely dominate the available statistics (leaving aside the technical difficulties of analyzing the final polarization). In this spirit, we shall present a detailed analysis for the unpolarized final \(WW\) state, showing all the considered observables in this case in Fig. (1a, c,d). For sake of comparison with previous papers, we also draw in Fig. (1b) the plot representing the cross section for final longitudinal \(WW\) pairs.

The results are given both for the SM and for the MSSM. From a glance to the different graphs, a number of features emerge. To proceed with order, it is more convenient to give a separate discussion of the three considered observables.

a) Cross sections. One sees from Fig. (1a) that the two approximate expansions for unpolarized \(WW\) pairs are "essentially" \((i.e. \ \text{within the assumed 1-2 percent accuracy})\) identical for c.m. energies below, roughly, 1 TeV. When the energy increases beyond this
value, the difference between the two approximations becomes larger, reaching a dramatic (in our opinion) value of ten percent at about 3 TeV. We observe also quite a small difference between the SM and the MSSM effects in our considered logarithmic expansions, in agreement with the observation of Ref. [6] for what concerns the one-loop approximation.

As one sees from inspection of Fig. (1b) identical remarks strictly apply for the longitudinal $WW$ cross section. We can therefore conclude that for what concerns cross sections, at the subleading electroweak logarithmic accuracy, a one-loop expansion does not require extra corrections below one TeV, but appears to be drastically inadequate in the higher (2,3 TeV) c.m. energy domain.

b) Forward-backward asymmetry. As one immediately sees from Fig. (1c), the same conclusions given for the cross sections case are valid. In particular, at 3 TeV, the difference between the two approximations reaches a value of five percent, to be compared with a Born value of approximately 0.77 (with our choice of cut at 30 degrees). Again, no appreciable difference exists between the SM and the MSSM results.

c) Electron-positron longitudinal polarization asymmetry. This is the only case where, between the one-loop expansion and the resummed one, no appreciable difference exists in the full considered energy range (see Fig. (1d)). This is relatively simple to understand since this process is dominated by left-handed electron contributions, given the nature of the final state. In fact, not much information seems to be provided by a measurement of this observable, at least in the framework of a "conventional" model like the MSSM.

IV. CONCLUSIONS

The main motivation of our work was that of examining the reliability of a one-loop expansion for the process $e^+e^-\rightarrow W^+W^-$ in the MSSM (and also in the SM). The practical reason is that we do not see personally many chances of performing a two-loop calculation in this model (unless a dedicated effort is motivated and supported). Naively, one reason of worry would be the realization that the one-loop effect is "large", say beyond the qualitative ten percent threshold, that might induce the feeling of a two-loop effect possibly beyond the
one-two percent limit. In this respect we must make a preliminary distinction between two different types of one-loop effects. The first ones are the "classical" QED ones. In the case of $WW$ production, they are known to be large and in some cases dominating the overall correction. However, they are supposedly under control, and thus the largeness of their size does not generate particular worries since an established resummation procedure exists. In addition they are purely standard, i.e. the same in SM and in MSSM. A quite different situation characterizes the "genuine" electroweak effects. For the latter a complete resummation procedure of the one-loop contribution to all orders is not known at the moment. Thus, the existence of large one-loop corrections might lead to the feeling that unknown higher order effects might be relevant. In the case of $WW$ cross sections, the Sudakov effects at one loop become indeed, rather quickly, "large". As one sees, they reach already the fifteen percent level at 1 TeV. The technical reason of this fact, which is not shared by other (fermion, scalar) final states, is not difficult to understand from our formulae. In fact, for final $W$ pairs, the coefficient $b^{fin,T\bar{T}}$ only contains a (negative) squared logarithm that is not reduced by a corresponding linear logarithm like for other final state processes. This explains the quick rise of the negative effect, a feature that would reappear for final chargino production. In particular, at 3 TeV, the size of the one-loop term reaches the $\simeq 50$ percent value, that seems, honestly, disturbingly large. To a minor level, an analogous feature characterizes the considered forward-backward asymmetry.

In such a situation, the only (to our knowledge) control is offered by the existing partial resummation procedures. For the MSSM case, we have used the only one that we are aware of (therefore we cannot compare it with other proposals). We remind the reader that both the one-loop Sudakov expansion and the fully resummed one are valid to subleading logarithmic accuracy. Within this limitation, we have verified that, below 1 TeV, no "dangerous" (i.e. at the one-two percent level) difference exists between the two approximations, while large discrepancies arise at higher energies. We can conclude that, to subleading electroweak logarithmic accuracy, a one-loop calculation is fully adequate below one TeV, and certainly unsatisfactory at higher energies. The next remaining step is to enlarge this statement to include a complete one-loop calculation. In our opinion, this conclusion is, at least, rather reasonable, given the fact that the difference between the two approaches (modulo the known QED corrections) is only due to next-to subleading (e.g. 12
constant) terms. From our previous experience in the case of final charged Higgs production [11], we expect that such terms are reasonably small and sufficient to provide an adequate description of the process, when added to the logarithmic ones. Certainly, this statement needs a professional support. In practice, this would be provided by the comparison of a complete one-loop program with a logarithmic expansion that includes e.g. an extra constant term, as we did thoroughly in Ref.[11]. This is not beyond a ‘reasonable’ effort. We consider this suggestion as a much simpler possibility compared to the (tough!) alternative of performing a two-loop calculation, and look forward to its completion in a reasonably near future.

For what concerns the sensitivity of $WW$ production to the genuine SUSY component of the MSSM, our conclusion is that, at least to subleading logarithmic accuracy, this appears to be rather small (both at the one-loop and also at the resummed level) in the full energy range that we have considered. In fact, at this level, the only difference between the SM and the MSSM is due to the change of the coefficients $n_{i,f}$ of the linear logarithms, as explained in Section 2. Although we cannot exclude a higher sensitivity in the next-to subleading terms where the parameters of the model will actually enter, it appears from the previous Hahn analysis [6] that, at least until 1 TeV, the complete calculation retains this property. This could indicate, in case of SUSY discovery, that $WW$ production might be more relevant for detecting possible signals of different types of sophisticated new physics models.

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FIG. 1: Effect of electroweak Sudakov radiative corrections in (a) cross-section for production of unpolarized \( W \), (b) cross-section for production of longitudinally polarized \( W \), (c) forward-backward asymmetry (unpolarized \( W \)), and (d) left-right asymmetry (unpolarized \( W \)). The angular integrations are performed with the cut \( \vartheta_{\text{cut}} = 30^\circ \). For each observable we show four lines corresponding to the one-loop/resummed expansions in the subleading logarithmic accuracy both in the Standard Model and in the MSSM.