Elliptic flow of thermal photons and dileptons

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Abstract

In this talk we describe the recently discovered rich phenomenology of elliptic flow of electromagnetic probes of the hot matter created in relativistic heavy-ion collisions. Using a hydrodynamic model for the space-time dynamics of the collision fireball created in Au+Au collisions at RHIC, we compute the transverse momentum spectra and elliptic flow of thermal photons and dileptons. These observables are shown to provide differential windows into various stages of the fireball expansion.

1 Introduction

The strong radial and elliptic flow of hadrons observed in relativistic heavy-ion collisions at RHIC have led to the important conclusion that the quark-gluon plasma (QGP) created in these collisions acts like a strongly coupled plasma with almost perfect liquid behaviour. This conclusion is based on the successful prediction of the hadron momentum distributions, in particular of their anisotropies in non-central collisions, by dynamical calculations which treat the expanding QGP as an ideal fluid. While no other equally successful model exists, one has to remain conscious of the fact that the new “perfect liquidity” paradigm is based on a model back-extrapolation of the measured data to the early stages of the collision which are not directly accessible with hadronic observables. There are strong arguments that this back-extrapolation is fairly unique [1] and hence that the above-mentioned qualitative conclusion is robust. On a quantitative level, however, the extraction from experimental data of the (small) QGP viscosity is presently hampered not only by the unavailability of consistent hydrodynamic codes for viscous relativistic fluids, but even more by uncertainties about the hydrodynamic effects of changes in the equation of state of the QGP matter [2] and about details of the initial conditions at the beginning of the hydrodynamic expansion stage [3,4].

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It would therefore be invaluable to have data on additional experimental observables which probe directly the earlier expansion stages and help to further constrain the space-time and momentum-space characteristics of the fireball expansion and the related model uncertainties. Electromagnetic probes, in particular direct photons and dileptons, provide such observables. Due to the weakness of the electromagnetic interaction, real and virtual photons are not created abundantly, making their measurement difficult, but once created they escape the fireball without reinteraction, turning them into direct probes of the conditions under which they were created. The shape of their transverse momentum and invariant mass spectra has long been advertised as a direct probe of the extremely hot temperatures during the earliest stages of the expanding fireball. We here study these spectra at the next finer level of detail, by analyzing their anisotropies, especially their elliptic flow, in non-central collisions. Our goal is to use such measurements to illuminate with better resolution the early evolution of the fireball’s spatial deformation through its imprint on the momentum anisotropies of the photons emitted during these early stages. Future studies will further complement the analysis presented here by directly measuring the space-time structure of the early collision fireball with two-photon correlations.

2 Spectra and elliptic flow

Both real and virtual photon (dilepton) momentum spectra can be written as

$$E \frac{dN}{d^3p} = \int \left[ (...) \exp \left( -p \cdot u(x)/T(x) \right) \right] d^4x,$$

where the quantity inside the square brackets indicates the thermal emission rates from the QGP or hadronic matter. The photon 4-momentum is parametrized by its rapidity $Y$, its transverse momentum $p_T = (p_x^2 + p_y^2)^{1/2}$, and its azimuthal emission angle $\phi$ as $p^\mu = (M_T \cosh Y, p_T \cos \phi, p_T \sin \phi, p_T \sinh Y)$. For real photons $M_T = p_T$; for dileptons $M_T = [M^2 + p_T^2]^{1/2}$ where $M$ is the invariant mass of the virtual photon and lepton pair. The spectral shape is dominated by the Boltzmann factor describing a flow-boosted thermal distribution. Assuming boost-invariant longitudinal expansion and using standard proper time and space-time rapidity coordinates, it is given by

$$\frac{p \cdot u(x)}{T(x)} = \frac{\gamma_T(x)}{T(x)} \left[ M_T \cosh(Y - \eta) - p_T v_T(x) \cos(\phi - \phi_v(x)) \right],$$

where $v_T$ (with $\gamma_T = (1-v_T^2)^{-1/2}$) is the magnitude and $\phi_v = \tan^{-1}(v_y/v_x)$ the azimuthal angle of the transverse flow velocity.

The azimuthal anisotropy ($\phi$-dependence) of the spectrum is controlled by an interplay between the collective flow anisotropy and the geometric deformation of the temperature field $T(x, y, \tau)$ at non-zero impact parameter. In the present work both are given by a hydrodynamical calculation, using the
boost invariant hydrodynamic code AZHYDRO [5] with standard [5] initial conditions for Au+Au collisions at $\sqrt{s} = 200 \text{ A GeV}$. The only adjustment we make [6] is to extrapolate the initial entropy density from the usual initial time $\tau_0 = 0.6 \, \text{fm}/c$ to a 3 times smaller value $\tau_0 = 0.2 \, \text{fm}/c$, assuming 1-dimensional boost-invariant expansion between these times, in order to account for at least a fraction of the pre-equilibrium photon production at very early times [7]. Its contribution to the photon spectrum is important at large $p_T$, and it will suppress its anisotropy there because very little transverse flow develops before 0.6 fm/c.

We concentrate on photons and dileptons emitted at midrapidity $Y = 0$ so that the spectrum has only even azimuthal Fourier components $v_n$. The elliptic flow $v_2$ is computed as the angular average of $\cos(2\phi)$ with the spectrum (1) as weight function: $v_2 = \langle \cos(2\phi) \rangle$.

### 3 Thermal photons [6]

For the photon emission rate from the QGP phase we use the complete leading-order expression from Ref. [8], while the latest results in Ref. [9] are used for photon radiation from the hadron gas phase. The hadronic emission rate sums over a large number of hadronic rescattering channels. Figure 1 shows that these cluster into two different classes: at low $p_T$, photon production is dominated by radiation accompanying vector meson production from or decay into pions, whereas at large $p_T$ collision induced conversion of vector mesons into photons dominates. Correspondingly the emitted photons trace the pion momentum distribution at low $p_T$ and the vector meson momentum distribution at higher $p_T$. The latter is flattened by the effects of radial collective flow on the heavy vector mesons, exhibiting, in fact, a weak “blast wave peak” around $p_T = 0.4 – 0.5 \text{ GeV}/c$. This also explains why in Figure 2 the photons from $\pi\pi \rightarrow \rho\gamma$ etc. scattering exhibit the standard almost linear rise of the
elliptic flow $v_2$ with $p_T$, well known from the pions themselves, whereas the vector meson conversion photons carry much less elliptic flow, even contributing negatively at low $p_T$ below the “blast wave peak” in their $p_T$ spectrum.

![Graph showing total photon elliptic flow $v_2(p_T)$](image)

**Fig. 3.** Total photon elliptic flow $v_2(p_T)$, as well as hadronic and quark matter contributions for comparison. (Figure taken from [6].)

Figure 3 shows the total thermal photon elliptic flow (solid red line) and compares it with the elliptic flow of the early quark matter photons (dotted blue line) and of the late hadronic matter photons (dashed red line). Consistent with the spectra shown in Fig. 1, the hadron gas photons are seen to track the elliptic flow of pions at low $p_T$ and that of $\rho$ mesons at higher $p_T$ (both shown by solid blue lines in Fig. 3 and exhibiting the almost linear rise of $v_2$ with $p_T$ that is characteristic of hadrons in the hydrodynamic model [10]). The cross-over between these two patterns causes a distinctive peak-valley structure in $v_2$ around $p_T = 0.4 - 0.5$ GeV/$c$. Since the hydrodynamic model is known to work quantitatively very well in this transverse momentum range [10], we expect this structure to be robust [11].

In contrast to the hadronic photons, the quark matter photons show an elliptic flow which decreases at high $p_T$. This reflects the fact that they track quark momenta, and that quark flow is small at early times when the high-$p_T$ photons are emitted. As one goes down in $p_T$ one probes later emission times and sees an increasing $v_2$ of the quark matter photons, reflecting the buildup of elliptic flow in the quark fluid. The behavior of the total photon elliptic flow, finally, can be understood by realizing that hadronic photons dominate the total photon spectrum only at low $p_T$ while quark matter radiation begins to take over around $p_T \approx 0.4$ GeV/$c$, completely outshining the hadron gas for $p_T > 1 - 2$ GeV/$c$. This cuts off the linear rise of the hadronic photon $v_2$, and the total photon elliptic flow at high transverse momenta thus reflects the small elliptic flow during the very early collision stages.

One should remember, however, that ideal fluid dynamics gradually breaks down at higher $p_T$. Data suggest [12] that near the hadronization point quark
elliptic flow begins to be seriously affected by viscous effects for $p_T > 1 \text{GeV}/c$, and this threshold may be even lower at earlier times when the longitudinal expansion rate is higher and shear viscous effects are larger. For $p_T > 1 \text{GeV}/c$ our hydrodynamic prediction of photon elliptic flow must thus be regarded as an upper limit, and its already small values at large $p_T$ will be further reduced by viscous corrections and prompt photon contributions [13,14].

4 Thermal dileptons [15]

With dileptons (virtual photons) we can probe the elliptic flow as a function of an additional variable, their invariant mass $M = M_{\ell \bar{\ell}} = M_{\gamma^*}$:

$$v_2(M, p_T; b) = \frac{\int d\phi \cos(2\phi) \frac{dN_{\ell \bar{\ell}}(b)}{dM^2 dY p_T dp_T d\phi}}{\int d\phi \frac{dN_{\ell \bar{\ell}}(b)}{dM^2 dY p_T dp_T d\phi}}. \quad (3)$$

In Figures 4 and 5 we show $p_T$-spectra and elliptic flow of thermal dileptons with invariant mass $M = m_{\phi}$. We see in Fig. 4 that for this value of $M$ the $p_T$-spectrum is completely dominated by virtual photon emission from the hadronic phase, all the way up to $p_T = 4 \text{GeV}/c$. The elliptic flow of the dileptons emitted from the hadronic phase closely tracks that of $\phi$-mesons at thermal freeze-out, as seen in Fig. 5. The elliptic flow of hadronic dileptons with $M = m_{\phi}$ (blue dotted line) is slightly larger than that $\phi$ mesons (green solid line) since radial flow (which suppresses $v_2$) continues to build up during the hadronic phase and the hadronic dileptons are on average emitted somewhat earlier than the $\phi$ mesons [16].

We have checked [15] that the calculated $\phi$ meson $p_T$-spectrum at thermal freeze-out agrees with the measured spectrum of $\phi$ mesons reconstructed from $K^+K^-$ decays by the PHENIX Collaboration [17]. The elliptic flow of quark matter dileptons with $M = m_{\phi}$ (black dashed line in Fig. 5) is much smaller
and shows the same characteristic decrease at large $p_T$ as the elliptic flow of thermal photons in Fig. 3, reflecting their early emission when the flow anisotropy of the quark fluid was still small. Due to the dominance of hadronic dileptons at this invariant mass, this decrease of $v_2$ at large $p_T$ is seen in the overall elliptic flow only at very large $p_T > 4 \text{ GeV}/c$; below $p_T = 2 \text{ GeV}/c$, the total dilepton elliptic flow follows almost perfectly the hadronic $v_2$.

Fig. 6. Same as Fig 4, but for $M = 2 \text{ GeV}$.  
Fig. 7. Same as Fig 5, but for $M = 2 \text{ GeV}$.

Qualitatively the same pattern is observed for dileptons with invariant mass $M = m_\rho$ when compared with $\rho$ mesons emitted at thermal freeze-out [15]. But things are different for dileptons with an invariant mass of $M = 2 \text{ GeV}$ (Figures 6 and 7). Now the QGP dileptons outshine those from the hadron gas for all $p_T$ by three orders of magnitude or more. While the elliptic flow of the hadronic dileptons still shows the hydrodynamic almost linear rise with $p_T$ (dotted line in Fig. 7), they are completely buried underneath the quark matter dileptons, and the total thermal dilepton spectrum at $M = 2 \text{ GeV}$ thus exhibits clearly the rise and fall of the elliptic flow with increasing $p_T$ that is characteristic of emission from the early QGP phase.

The story is further clarified by studying the invariant mass dependence of the $p_T$-integrated elliptic spectrum (the “dilepton mass spectrum”, Figure 8) and of the corresponding $p_T$-integrated elliptic flow $v_2(M)$ (Figure 9). We note that these spectra are preliminary and do not properly account for the $\omega$ meson; complete mass spectra of the dilepton elliptic flow which include the $\omega$ contribution as well as dileptons from vector meson decays after thermal freeze-out will be presented in [15]. Figure 8 shows that near the vector mesons peaks ($\rho$, $\omega$, $\phi$), dilepton emission from the late hadronic stage dominates by at least an order of magnitude over QGP radiation. Correspondingly, the total dilepton elliptic flow approaches in these mass regions the value of the hadronic matter dileptons (dotted line in Fig. 9). If one adds the post-freeze-out vector meson decays, the total elliptic flow around $M = m_\omega$ and $M = m_\omega$ even approaches the elliptic flow of the corresponding hadrons, $\omega$ and $\phi$, emitted at kinetic freeze-out. This last observation is explained by the relatively long lifetimes of the $\omega$ and $\phi$ mesons (23 and 45 fm/c, respectively), which signifi-
Fig. 8. QGP and hadron gas contributions to the dilepton invariant mass spectrum for 200 $A$ GeV Au+Au collisions at $b = 7$ fm. The solid line is the total dilepton mass spectrum.  

Fig. 9. The invariant mass dependence of the total $p_T$-integrated dilepton elliptic flow, as well as that of its hadron and quark matter contributions. Shown for comparison are also the $p_T$-integrated elliptic flows for a number of hadrons emitted at thermal freeze-out.

Note that the elliptic flow of the hadronic dileptons (dotted line in Fig. 9) remains significantly below that of the hadrons emitted at thermal freeze-out at $T_{dec} = 130$ MeV (crossed circles in Fig. 9). In view of the similarity of the $p_T$-dependences of their elliptic flows (see, e.g., the blue dotted and solid green lines in Fig. 5) this looks surprising. The puzzle is resolved by noting that, at a fixed invariant mass, the hadronic dilepton $p_T$-spectrum is steeper than the thermal freeze-out hadron spectrum for a hadron with the same mass [15]. The difference is likely due to additional radial flow built up between the average time of hadronic photon emission and final hadron freeze-out.

5 Conclusions

Elliptic flow of thermal photons and dileptons was shown to be a versatile and potentially powerful probe of the fireball dynamics at RHIC and LHC, complementary to the already well-studied flow anisotropies in the hadronic sector. Contrary to hadron elliptic flow, which in a hydrodynamic picture rises monotonically with increasing $p_T$ and in real life, due to viscous effects, saturates at high $p_T$, photon and dilepton elliptic flow decrease to zero at large $p_T$ and large dilepton mass, reflecting direct emission from the early QGP at a stage when flow anisotropies have not yet had time to grow strong. $v_2^\gamma(p_T)$ and $v_2^\ell(M)$ exhibit rich structures which reflect the interplay of different emission processes, opening a window on detailed and differential information from a variety of different stages of the fireball expansion. The elliptic flow of photons and dileptons emitted from the late hadronic stage was seen to track the
$v_2$ of the emitting hadrons, which suggests the possibility of subtracting the hadronic photon contributions from the total (virtual) photon signal in order to isolate and study in greater detail the elliptic flow of early QGP photons.

Obviously, the measurements will be difficult and the theoretical treatment can use further refinement, but this first glimpse suggests that photon and dilepton elliptic flow have the potential of turning into profitable gold mines.

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