Intermediate-Mass Black Holes in Globular Clusters

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ABSTRACT

There have been reports of possible detections of intermediate-mass black holes (IMBHs) in globular clusters (GCs). Empirically, there exists a tight correlation between the central supermassive black hole (SMBH) mass and the mean velocity dispersion of elliptical galaxies, “pseudobulges” and classical bulges of spiral galaxies. We explore such a possible correlation for IMBHs in spherical GCs. In our model of self-similar general polytropic quasi-static dynamic evolution of GCs, a criterion of forming an IMBH is proposed. The key result is $M_{\text{BH}} = L\sigma^3/\epsilon\sigma_0$ where $M_{\text{BH}}$ is the IMBH mass, $\sigma$ is the GC mean stellar velocity, $L$ is a coefficient, and $2/3 < n < 1$.

Key words: accretion, accretion discs — black hole physics — galaxies: bulges — globular clusters: general — hydrodynamics — instabilities

1 INTRODUCTION

For star-forming molecular clouds, collapsed massive dense gas cores eventually lead to luminous new-born stars burning nuclear fuels. Analogously but on larger scales, we speculate that grossly spherical core-collapses of globular clusters (GCs) could also cause something singular around the dense centre; such central singularities may form IMBHs. Observations of GC cores indicate that the central concentrations of nonluminous materials are likely due to IMBHs (Bahcall & Ostriker 1975). Mass accretions onto IMBHs were proposed to power GC ULX sources (e.g. Farrell et al. 2009).

On much larger scales further, observations of galaxies reveal a strong correlation between the mass $M_{\text{BH}}$ of the central SMBHs and the mean velocity dispersion $\sigma$ of the host stellar bulge in the form of $\log(M_{\text{BH}}/M_\odot) = \epsilon + \delta \log(\sigma/\sigma_0)$ where $\epsilon$ and $\delta$ are two coefficients and $\sigma_0$ is a velocity dispersion normalization (e.g. Tremaine et al. 2002). We naturally expect a similar $M_{\text{BH}} - \sigma$ power-law relation for GCs.

Evidence for central IMBHs in GCs have been debated extensively and their possible existence bears important consequences for both the formation and evolution of GCs (e.g. Benacquista & Downing 2011 and extensive references therein). After forming such a central IMBH in the core, pertinent post-collapse mechanisms continue to operate: e.g., mass segregation maintains more massive stars or runaway collisions and coalescence or merging of stars to form supermassive stars and to trigger subsequent e\textsuperscript{\pm} pair instabilities therein). After forming such a central IMBH in the core, pertinent post-collapse mechanisms continue to operate: e.g., mass segregation maintains more massive stars around the central IMBH, while the ‘binary heating’ (e.g. Hurley et al. 2007) from primordial stellar binaries survived

al. 2008; Noyola et al. 2008; Zaharijas 2008). These empirical correlations may shed light on the origin and evolution histories of IMBHs and their host GCs. Among such observed relations, $M_{\text{BH}}$ and $\sigma$ correlate tightly (e.g. Gebhardt et al. 2002; Safonova & Shastri 2010).

Self-similar solutions for general polytropic hydrodynamics of a self-gravitating fluid with spherical symmetry were constructed recently. Asymptotic behaviours of novel quasi-static solutions in a single polytropic fluid have been revealed by Lou & Wang (2006, 2007) and was used to model rebound (MHD) shocks in SNe. Such solutions were applied to clusters of galaxies (Lou et al. 2008) for possible galaxy cluster winds. In this Letter, we invoke such quasi-static solutions to model dynamic evolution of host GCs and formation of IMBHs and to establish $M_{\text{BH}} - \sigma$ power laws.

Various aspects of GCs have been studied extensively (e.g. Benacquista & Downing 2011 and extensive references to excellent reviews therein). We focus on the quasi-static self-similar GC dynamic evolution (say, induced by the gravothermal instability) in the late phase of the pre-collapse regime; such asymptotic solution for GC evolution leads to diverging mass density at the center. Physically, as the inner enclosed core mass becomes sufficiently high within a radius comparable to its Schwarzschild radius, an IMBH forms inevitably (e.g. through mergers of stellar mass black holes or runaway collisions and coalescence or merging of stars to form supermassive stars and to trigger subsequent e\textsuperscript{\pm} pair instabilities therein). After forming such a central IMBH in the core, pertinent post-collapse mechanisms continue to operate: e.g., mass segregation maintains more massive stars around the central IMBH, while the ‘binary heating’ (e.g. Hurley et al. 2007) from primordial stellar binaries survived

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the IMBH formation tends to resist further core collapse or to drive post-collapse oscillations. The N-body GC simulations by Hurley et al. (2007) of up to $10^6$ stars and initial 5% binaries may eventually reach a GC core binary frequency as high as 40% at the end of the core-collapse phase. Shown in their figure 3, this core binary frequency actually fluctuates between $\sim 10\%$ to $\sim 40\%$. We would expect that an IMBH forms at the GC centre rapidly during the core-collapse phase but do not know exactly when. Physically, this IMBH would engulf the central stars and binaries at the epoch of IMBH formation. Such an explosive event may give rise to a powerful gamma-ray burst and a shock wave surrounding the GC centre. The slower relaxation, evolution and accretion then persist on a much longer time scale.

GCs are close to spherical; in their dynamic evolution, lumpiness observed could either result from merging disturbances and tidal disruptions or provide source of fluctuations that may be classified into acoustic modes, gravity modes and vortical modes on larger scales (Lou & Lian 2011). These perturbations may be unstable to trigger gravitational instability. Analogous to nuclear burnings in a star to resist stellar core collapse, the ‘binary heating’ from primordial stellar binaries may delay core collapse in GCs (e.g. Meylan & Heggie 1997). As the source of ‘binary heating’ is exhausted during a GC evolution, the inner core collapse is inevitable. This ultimately leads to the formation of IMBH which may accrete materials from immediate environs.

2 GC MODEL FOR A $M_{\text{IMBH}} - \sigma$ POWER LAW

We adopt the same hydrodynamic perspective of Lou & Jiang (2008; LJ hereafter) for the large-scale spherical GC dynamic evolution. GCs are smaller and less massive than typical galactic bulges. The random stellar velocity dispersion and the ‘binary heating’ from primordial stellar binaries in the core provide an effective pressure against the GC self-gravity. This may justify our fluid formalism for GC cores. In contrast to LJ, we emphatically focus on the reported tentative candidates of IMBHs and the properties of the host GCs. Our main goal is to extend the theory of LJ to GCs and examine whether IMBHs and properties of GCs can be sensibly fitted with data. By data comparisons, our results appear encouraging for such a physical connection. Our predictions for IMBHs and host GCs can be tested by further observations. Meanwhile, we also compare with SMBHs in their host galactic pseudobulges. This may hint at a general validity of such a self-similar dynamic evolution in spherical self-gravitating systems.

In spherical polar coordinates $(r, \theta, \phi)$, the nonlinear general polytropic hydrodynamic partial differential equations (PDEs) of spherical symmetry are PDEs (1)–(4) of LJ with the same notations. The Poisson equation is automatically satisfied. As the bulk flow of stellar fluid is slow in GCs, we invoke the quasi-static self-similar solutions of Hu & Lou (2006). We introduce the transformation in the dimensionless independent variable $x$,

$$r \equiv K^{1/2}t^{n-1}x, \quad u \equiv K^{1/2}t^{n-1}v(x), \quad \rho \equiv \frac{\alpha(x)}{4\pi G}, \quad P \equiv \frac{K^{2n-4}\beta(x)}{4\pi G}, \quad M \equiv \frac{K^{3/2}t^{3n-2}m(x)}{(3n-2)G},$$

with $K$ and $n$ being two scaling parameters; here, $u, \rho, P, M$ are radial velocity, mass density, pressure, and enclosed mass respectively, while $v(x), \alpha(x), \beta(x),$ and $m(x)$ are respectively dimensionless reduced speed, mass density, and enclosed mass of $x$ only.

Substituting self-similar transformation (1) into nonlinear PDEs (1)–(4) of LJ and defining $q \equiv 2(n+\gamma - 2)/(3n - 2)$, we derive two coupled nonlinear ordinary differential equations (ODEs) for $\alpha'$ and $v'$

$$D(x, \alpha, v)\alpha' = N_1(x, \alpha, v), \quad D(x, \alpha, v)v' = N_2(x, \alpha, v),$$

where three functionals $D, N_1$ and $N_2$ are defined explicitly in Hu & Lou (2009) with zero magnetic field. An exact global static solution of eq (2) in physical dimensions, known as the singular general polytropic sphere, has $u = 0$,

$$\rho = \frac{A}{4\pi G}K^{1/n}r^{-2/n}, \quad M = \frac{nAK^{1/n}}{(3n - 2)G}(3n - 2)/n,$$

where coefficient $A \equiv \left[\frac{\nu^2-q}{2(2n-3n^2)}\right]^{1/(n(3n^2-2))}$. This solution serves as an asymptotic ‘quasi-static’ solution for small $x$; i.e. they are leading terms of $v(x)$ and $\alpha(x)$ and there exist higher order terms for a self-similar asymptotic evolution.

For such quasi-static self-similar hydrodynamic asymptotic solutions at small $x$, we consistently presume

$$\alpha = Ax^{-2/n} + Jx^{1-2/n}S, \quad v = Lx^S,$$

two coupled nonlinear ODEs (2), and derive two nonlinear algebraic equations for the coefficients $J, S$ and $L$,

$$n(S - 1)J = (S + 2 - 2/n)AL,$$

$$\left[\frac{n^2}{2(3n - 2)} + \frac{(3n - 2)}{2}W\right]S^2 + \frac{(3n - 4)}{n}S = 0,$$

where $W \equiv n^q/[2(2n-3n^2)]$. Once the proper roots of $S$ are known, coefficients $J$ and $L$ are related by eq (3); only one is free to choose. The existence of SPS solution (3) and the requirement of $\Re(S) > 1$ constrain the parameter regime of such quasi-static solution. Oscillatory behaviours can also emerge (Lou & Wang 2006), which may be relevant to the interesting controversy of gravitational and post-collapse oscillations in GCs. For a sufficiently small $W \neq 0$, we can have two roots $S > 1$ from quadratic eq (3). It also happens for one root $S > 1$ and the other root $S < 1$.

As a physical requirement for sensible similarity solutions of a general polytropic flow, both $v(x)$ and $\alpha(x)$ approach zero at large $x$. Thus for either $x \to 0^+$ or $x \to +\infty$, the reduced velocity $v \to 0$, which means at time $t$, for either $r \to 0^+$ or $r \to +\infty$ the flow speed $u \to 0$, or at a radius $r$, when $t$ is either short or long enough, the radial flow speed $u \to 0$. This model describes a self-similar dynamic GC evolution towards a quasi-static configuration a long time later and may grossly fit relaxed spherical GCs.

From the general polytropic EoS (LJ), the pressure is

$$P = (4\pi)^{\gamma-1}G^{q+\gamma-1}(3n-2)^6K^{1-3q/2}\rho^\gamma M^q,$$

and $\sigma_L(r, t) = (\gamma P/\rho)^{1/2}$ is the local stellar velocity dispersion in a GC. Asymptotically as $t \to +\infty$, $\sigma_L(r, t)$ becomes

$$\sigma_L(r) = \gamma^{1/2}K^{1/(2n)}t^{n/2}A^{(q+\gamma-1)/2}(r^{(n-1)/n}).$$
To check against data, we derive the spatial average of velocity dispersion $\sigma$ in a GC. The GC boundary is taken as either radius $r_e$ where mass density $\rho_e$ is indistinguishable from the surrounding or the tidal radius. Within $r_e$ of a GC, the spatial average of stellar velocity dispersion $\sigma_L(r)$ is

$$\sigma = \frac{3}{4\pi r_e^2} \int_0^{r_e} \sigma_L(r) 4\pi r^2 dr = QK^{1/2}$$

$$\equiv \left[3n^{1+q/2}/(4\pi - 1)\right]\{4\pi G\rho_c\}^{(1-n)/2} A^{3n/4} K^{1/2}.$$  

We invoke a heuristic criterion of forming an IMBH in a GC (LJ). An IMBH mass $M_{\BH}$ is given by $M_{\BH} = r_e\sigma^2/(2G)$ where $r_e$ is its Schwarzschild radius and $c$ is the speed of light. By solution 5 and when

$$\frac{nAK^{1/n}}{(3n-2)G}^{(3n-2)/n} = r_e^2/(2G),$$

an IMBH forms with the Schwarzschild radius

$$r = r_e = [(3n - 2)c^2/(2nAK^{1/n})]^{n/(2n-2)}.$$  

Only those asymptotic quasi-static GCs with $n < 1$ can thus form central IMBHs (see fig. 1 of LJ). Consequently,

$$M_{\BH} = \left[c^2/(2G)\right][(3n - 2)c^2/(2nA)]^{n/(2n-2)} K^{1/2}.$$  

or equivalently, the explicitly $M_{\BH} - \sigma$ power law

$$M_{\BH} = \frac{c^2}{2G} \left[\frac{2nA}{(3n-2)c^2}\right]^{n/(2n-2)} \left(\frac{\sigma}{c}\right)^{1/(1-n)} \equiv \mathcal{C} \sigma^{1/(1-n)},$$

where the exponent $1/(1-n) > 3$ since $2/3 < n < 1$ (LJ).

To validate our quasi-static model for the nine GCs with available observational data and references summarized in Tables 1 and 3, we fit a $M_{\BH} - \sigma$ power law in Fig. 1. Applying the least-square criterion to the data of Table 1 (e.g. Meylan & Mayor 1991; Sañonova & Shastri 2010), we obtain $M_{\BH} = 4.102 \times 10^7 M_\odot (\sigma/200 \text{ km s}^{-1})^{3.6251}$ with parameters $\{n, \gamma, \rho_c\}$ being $(0.7241, 1.99, 4.40 M_\odot \text{ pc}^{-3})$. This $\rho_c$ value appears somewhat larger but grossly consistent with the data in rough orders of magnitude (e.g. Meylan 1987). Specifically, $\rho_c \sim 0.87 M_\odot \text{ pc}^{-3}$ for GC 47Tuc (Meylan 1988), $\rho_c \sim 0.091 M_\odot \text{ pc}^{-3}$ for GC NGC6397 (Meylan & Mayor 1991) and $\rho_c \sim 0.42 M_\odot \text{ pc}^{-3}$ for GC G1 (Meylan et al. 2001). No published $\rho_c$ values for NGC2808, M80, M62, NGC6388 and M15 are available. The nine GCs 47Tuc, NGC2808, ω Cen, M80, M62, NGC6388, NGC6397, M15, and G1 correspond to $K = \{1.2, 2.0, 7.7, 1.5, 2.1, 3.1, 0.22, 1.9, 5.8\} \times 10^{13} \text{ cgs}$ unit and $r_e = 13, 16, 32, 14, 17, 20, 5, 16, 28 \text{ pc}$, respectively in rough agreement with the estimated data in orders of magnitude. Harris (1996) reported $r_e$ of 47Tuc, NGC2808, ω Cen, M62, NGC6388, NGC6397, and M15 to be 56, 43, 88, 18, 18, 11, and 64 pc, while Bahcall & Hausman (1976) reported $r_e$ of M80 to be $\sim 18 \text{ pc}$ and Ma et al. (2007) estimated $r_e$ of G1 to be $\sim 81 \text{ pc}$.

We now consider a sample of SMBHs in galactic pseudobulges (e.g. Kormendy & Kennicutt 2004) summarized in Table 2. For the reasons in Hu (2008), we do not regard galaxy NGC3227 as containing a pseudobulge. In Table 2, $\sigma_e$ is defined by Gebhardt et al. (2000) and Tremaine et al. (2002) as the luminosity-weighted rms velocity dispersion within a slit aperture of length 2$R_e$, where $R_e$ is the effective or half-light radius of a galactic bulge. Parameter $\sigma_e$ is the rms velocity dispersion within a circular aperture of radius $r_e/8$. Ferrarese & Merritt (2000) used “central stellar velocity dispersion” $\sigma_e$. There is a relation $\mathcal{C}_{\sigma_e} = \sigma_e (S R_{ap}/R_e)^{0.04}$ (Jorgensen et al. 1995), where $R_{ap} \simeq 2.''$ (e.g. Davies et al. 1987). Actually, the difference between $\sigma_e$ and $\sigma_e$ are much smaller than their errors (e.g. Hu 2008).

A pseudobulge border is at radius $r_e$ where $\rho$ reaches a value $\rho_c$ indistinguishable from the environs. With the least-square fit to Table 2 data, we obtain $M_{\BH} = 2.4350 \times 10^7 M_\odot (\sigma/200 \text{ km s}^{-1})^{3.7680}$ with parameters $\{n, \gamma, \rho_c\}$ being $(0.7346, 1.999, 1.8 M_\odot \text{ pc}^{-3})$, respectively. This estimated $\rho_c$ appears grossly consistent with the data in the order of magnitudes, e.g. $\rho_c \sim 0.1 M_\odot \text{ pc}^{-3}$ for the pseudobulge in the Milky Way (Lopez-Corredoira et al. 2005). No published $\rho_c$ values for other pseudobulge are available. The eight pseudobulges NGC1068, NGC2787, NGC3079, NGC3384, NGC3393, Circinus, IC2560, Milky Way correspond to values of $K = \{3.4, 5.8, 1.3, 3.5, 5.0, 0.85, 1.4, 2.6\} \times 10^{21}$ cgs unit and $r_e = 1.3, 1.7, 0.79, 1.3, 1.5, 0.64, 0.82, 1.1 \text{ kpc}$, respectively in rough agreement with the data in orders of mag-

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\[ \text{(1)} \]

The prime distinguishes this approximation for $\sigma_e$ from the actual value of $\sigma_e$ and the ratio $\sigma_e'/\sigma_e$ may depend systematically on the velocity dispersion of a galaxy.
As in Hu (2008), NGC3227 may not possess a pseudobulge.\footnote{The SMBH mass has been updated by Das et al. (2007).}

The stellar velocity dispersion is taken from Sarzi et al. (2001).\footnote{The stellar velocity dispersion is from Busarello et al. (1996).}

As in Hu (2008), NGC3227 may not possess a pseudobulge.\footnote{The SMBH mass has been updated by Falcke et al. (2009) and the stellar velocity dispersions are from Walter et al. (2006).}

\begin{table}[h]
\centering
\caption{A sample of SMBHs in pseudobulges of galaxies}
\begin{tabular}{lccc}
\hline
\textbf{galaxy} & \textbf{$M_{\text{BH}}(10^8 M_\odot)$} & \textbf{$\sigma_{\text{c}}$ (km s$^{-1}$)} & \textbf{$\sigma_{\text{r}}$ (km s$^{-1}$)} \\
\hline
NGC1068 & 0.15 & 165 ± 17 & 165 ± 17 \\
NGC2787 & 0.41 ±0.04 & 210 & 210 \\
NGC3079 & 0.025 ±0.025 & 146 ± 15 & 146 ± 15 \\
NGC3384 & 0.16 ±0.01 & 160 & 160 \\
NGC3393 & 0.31 ±0.02 & 184 ± 18 & 184 ± 18 \\
Circinus & 0.011 ± 0.002 & 75 ± 20 & 75 ± 20 \\
IC 5260 & 0.029 ± 0.006 & 137 ± 14 & 137 ± 14 \\
Milky Way & 0.036 ± 0.003 & 132.5 & 132.5 \\
\hline
\end{tabular}
\end{table}

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{figure2.png}
\caption{Mass $M_{\text{BH}}/M_\odot$ versus the mean velocity dispersion $\sigma/\sigma_0$ for galactic pseudobulges and nine GCs. The line is the least-square fit to the combined data of galactic pseudobulges and nine GCs with $\log(M_{\text{BH}}/M_\odot) = 7.3529 + 3.4125 \log(\sigma/\sigma_0)$ with $n = 0.7070$.}
\end{figure}

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{figure3.png}
\caption{Masses of central IMBH $M_{\text{BH}}$ (in $M_\odot$) versus GC masses (in $M_\odot$) of the nine GCs respectively. The straight line is the least-square fit to the data in a log-log display by $M_{\text{BH}}/M_\odot = 1.25 \times 10^{-4}(M_{\text{GC}}/M_\odot)^{1.2128}$ with $n = 0.7252$.}
\end{figure}

\begin{table}[h]
\centering
\caption{Nine GCs with the reported total GC masses $M_{\text{GC}}$}
\begin{tabular}{lll}
\hline
\textbf{GC Name} & $M_{\text{GC}}(10^6 M_\odot)$ & \textbf{Major Relevant References} \\
\hline
47 Tuc & 1.26 & Pryor & Meylan (1993) \\
NGC2808 & 1.46 & Servillat et al. (2008) \\
\omega Cen & 3.1 & Miocchi (2010) \\
M80 & 1.0 & Pryor & Meylan (1993) \\
M62 & 0.63 & Pryor & Meylan (1993) \\
NGC6388 & 2.6 & Lanzoni et al. (2007) \\
NGC6397 & 0.062 & Heggie & Giersz (2009) \\
M15 & 0.44 & van den Bosch et al. (2006) \\
G1 (M31) & 7.37 ± 2.15 & Ma et al. (2009) \\
\hline
\end{tabular}
\end{table}

\section{Relation for $M_{\text{BH}} - M_{\text{GC}}$ Power Law}

By our model analysis, the total mass $M_{\text{GC}}$ of a GC is

$$M_{\text{GC}} = \frac{n(4\pi\rho_c)^{2(3n-3)}/(3n-2)}{3n-2} \left(\frac{A}{G}\right)^{3n/2} K^{3/2}.$$

By eq. 5, a smaller $\rho_c$ corresponds to a larger $r_c$ and thus a larger $M_{\text{GC}}$ with $2/3 < n < 1$. By relation (7), $M_{\text{BH}}$ and $M_{\text{GC}}$ are related by the power law

$$M_{\text{BH}} = \left[\frac{4\pi\rho_c}{3n-2}\right]^{1/3} \left(\frac{2G}{c^2}\right)^{(3n-2)/(2-2n)} M_{\text{GC}}^{1/(3-3n)}.$$ 

In Fig. 3 we show the IMBH mass $M_{\text{BH}}$ vs the GC mass $M_{\text{GC}}$. By the least-square fit to these data, we obtain

$$M_{\text{BH}}/M_\odot = 1.25 \times 10^{-4}(M_{\text{GC}}/M_\odot)^{1.2128}$$

with $n = 0.7252$. The $\rho_c$ value is $4.93 M_\odot$ pc$^{-3}$, grossly consistent with the data in orders of magnitude. Note that the $n$ value of $M_{\text{BH}} - \sigma$ power law for the nine GCs is 0.7241. These two $n$ values are fairly close, indicating that our model may consistently explain both the $M_{\text{BH}} - \sigma$ and $M_{\text{BH}} - M_{\text{GC}}$ power-laws for the nine GCs.

In Table 4, we list the central SMBH masses and the stellar masses of galactic pseudobulges (Hu 2009). As noted, NGC3227 is not taken as a galaxy having a pseudobulge. Moreover, NGC2787 and NGC3384 are two galaxies with composite structures consisting of both pseudobulges and small inner classical bulges (e.g. Erwin 2008); we do not treat them as pseudobulges. We include the Milky Way as having a pseudobulge with $M_{\text{BH}} = (3.6 \pm 0.3) \times 10^6 M_\odot$ (e.g. Falcke et al. 2009) and stellar bulge mass $M_s = (1.3 \pm 0.5) \times 10^9 M_\odot$ (e.g. de Zeeuw et al. 1995). In Table 4, $M_{4.36 - V}$ is...
Table 4. A sample of SMBHs in galactic pseudobulges

| Galaxies       | \( \log M_{BH}(+, -) \) | \( \log M_{*, B-V} \) |
|---------------|-------------------------|----------------------|
| NGC1068       | 7.18                    | 10.36                |
| NGC3079       | 6.40 (0.30, 0.30)       | 10.19                |
| NGC3393       | 7.49 (0.03, 0.03)       | 10.57                |
| Circinus      | 6.04 (0.07, 0.09)       | 9.57                 |
| IC 2560       | 6.46 (0.08, 0.10)       | 10.26                |
| Milky Way     | 6.56                    | 10.11                |

Here, \( \log M_{BH}(+, -) \) is the logarithm of the mass and 1σ error of the SMBH and \( \log M_{*, B-V} \) is the logarithm of the host pseudobulge stellar mass inferred from the K-band mass-to-light ratio \( M/L \) derived from \( B-V \) colours. The SMBH mass and the bulge mass of NGC 1068 are from Das et al. (2007). The SMBH mass of Milky Way is from Falcke et al. (2009).

We conclude that GCs with IMBHs and pseudobulges with SMBHs might share qualitatively similar \( M_{BH} - \sigma \) and \( M_{BH} - M_{GC} \) power-law relations in general. These results would bear significance for our theoretical understanding of the dynamic evolution of GCs, the formation of IMBHs, the connection between GC and pseudobulge formation.

REFERENCES

Bahcall N. A., Hausman M. A., 1977, ApJ, 213, 93
Bahcall J. N., Ostriker J. P., 1975, Nature, 256, 23
Bash F. N., et al., 2008, AJ, 135, 182
Benaquista M. J., Downing J. M. B., 2011, arXiv:1119.4423v1
Busarello G., et al., 1996, A&A, 314, 32
Cavichia O., et al., 2011, RMDXXA, 47, 49
Das V., et al., 2007, ApJ, 656, 699
Davies R. I., et al., 2006, ApJ, 646, 754
Dehnen W., et al., 2006, MNRAS, 369, 1688
Djorgovski S. G., King I. R., 1986, ApJ, 305, L61
Dwek E., et al., 1995, ApJ, 445, 716
Erwin P., 2008, in Bureau M., Athanassoula A., Barbuy B., eds, Proc. IAU Symp. 245, Cambridge U. Press, Cambridge, p.113
Falcke H., Markoff S., Bower G. C., 2009, A&A, 496, 77
Farrell S. A., et al., 2009, Nature, 460, 73
Ferrarese L., Merritt D., 2000, ApJ, 539, L9
Freire P. C. J., et al., 2001, ApJ, 557, L105
Gradotti D. A., 2008, MNRAS, 384, 420
Gebhardt K., Rich R. M., Ho L. C., 2002, ApJ, 578, L41
Gnedin O. Y., et al., 2009, ApJ, 705, L168
Grindlay J., Gursky H., 1976, ApJ, 205, L131
Harris W. E., 1996, AJ, 112, 1487
Heggie D. C., Giersz M., 2009, MNRAS, 397, L46
Hu J., 2008, MNRAS, 386, 2242
Hu R.Y., Lou Y.-Q., 2009, MNRAS, 396, 878
Hurley J. R., Aarseth S. J., Shara M. M. 2007, ApJ, 665, 707
Jørgensen I., Franx M., Kjaergaard P., 1995, MNRAS, 276, 1341
Kormendy J., Kennicutt R. C., 2004, ARA&A, 42, 603
Lanzoni B., et al., 2007, ApJ, 668, L139
Lopez-Corredoira M., et al., 2005, A&A, 439, 107
Lou Y.-Q., Jiang Y. F., 2008, MNRAS, 391, L44
Lou Y.-Q., Lian B., 2011, MNRAS, in press, arXiv:1111.2951L
Lou Y.-Q., Wang W. G., 2006, MNRAS, 372, 885
Lou Y.-Q., Wang W. G., 2007, MNRAS, 378, L54

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