Wind-ice Joint Probability Distribution Analysis based on Copula Function

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Abstract. Taking the wind speed and ice thickness field measurement data in Southwest China during November 2016 to March 2018 as analytical sample, the probability distribution of the wind speed and ice intensity were analyzed and fitted by using Gumbel distribution model, Weibull distribution model and Generalized extreme value distribution (GEV) model, respectively. The goodness of fitting results had been compared. After the determination of marginal distribution functions for wind speed and ice intensity, 500-year pseudo wind speed and ice intensity samples were generated based on the Monto Carlo method. Five Copula functions were employed in the construction of joint probability distribution function and the most suitable Copula function was chosen for building the wind-ice joint probability distribution model. On the basis of the model, the return periods for 30 years, 50 years and 100 years had been calculated, respectively. The analysis results show that the GEV model well matches the characteristics of wind speed and ice intensity. As for binary Copula function, the Frank Copula function has the best goodness for the joint probability distribution of wind speed and ice intensity. The return period calculations show that the design value of wind-ice joint return period (JRP) is larger than that of co-occurrence return period (CRP) and the value of Kendall return period (KRP) is between that of JRP and CRP. Considering that the dangerous areas of return periods are different, the return period for design should be determined by the combination analysis of different kinds of the return periods. Meanwhile, the importance and the failure results of the project should be taken into account.

1. Introduction
During the running process of power grid, the icing load and wind load are two main factors which influence the structural safety of power transmission line. To guarantee that the power transmission line in our country can run normally under severe climatic environmental conditions such as freezing, rain and snow, it is very significant in practice to research how to determine the safe and economic operation of design icing load of power transmission line to the power transmission line in a scientific way. For the power transmission line in the icing zone, the conductor icing is usually formed by a certain wind-induced effect. Both the wind speed and ice thickness are two key parameters which influence the wind load and ice load values. The statistical regression to wind speed and ice thickness in different return periods based on the probability distribution function of wind speed and ice thickness simply neglects
the authentic combination of wind speed and ice thickness actually. To research the structural power change under extreme climate, firstly, it is necessary to make a statistic and conduct analysis on the joint probability distribution of wind speed and ice thickness, while at present, there is the lack of research about this item.

There are many methods for constructing the joint probability distribution. For example, Fan Wenliang [1] have established the joint probability distribution of wind speed and wind direction based on the full probability formula, but these traditional models of multivariable frequency analysis require that various variables shall follow the marginal distribution of the same type, thus limiting their application. The Copula function is relatively flexible for the selection of marginal distribution, and it can consider the marginal distribution and correlation among variables in a separated way. Therefore, we can find out a better marginal distribution which conforms to actual situation, and can catch the non-linear correlation among variables, thus expanding the scope of correlation measurement among variables. It is a powerful tool for solving the joint distribution of multiple variables, and it is more and more widely applied into actual work. The Copula function is frequently applied into the financial and hydrological fields. Zhang Yaoting [2] firstly introduced the Copula into the financial field. Internationally, Favre [3] et al. have discussed the application of Copula function in the modeling of multi-dimensional extreme value distribution, and used it to analyze the joint distribution of flood peak and flood volume. The Journal of Hydrologic Engineering published by American Society of Civil Engineers (ASCE) [4] introduces the application of Copula function theory and method in the hydrology.

In our country, Guo Shenglian [5] has systematically introduced the basic theory and method of Copula function, analyzed and discussed the adaptability and superiority of Copula function in the multivariable hydrological calculation field. In recent years, some scholars also have applied it into the wind engineering field. Wu Zhanke[6] have described the double random variables of annual extreme value wind speed and annual extreme rainfall by utilizing the joint probability model of Gumbel Copula function and the extreme value I type; Lou Wenjuan[7] have adopted the Copula function to construct the joint probability density function of wind speed and wind direction, Chen Zishen et al.[8] have adopted Copula function to research the relationship between extreme value wave height and wind speed in the sea area of Shanwei; Wang Xiuyong[9] et al. have adopted Archimedean Copula function to establish wind-rain joint distribution targeting the wind speed and rainfall observation data at the bridge site of Qiongzhou Strait Crossing Bridge and Chengmai meteorological station, and have analyzed the wind and rain extreme value return period of Qiongzhou Strait Bridge according to the obtained optimal Copula function.

According to the theory and method of Copula function, and based on the synchronously acquired wind speed and icing observation data, this paper establishes the wind-ice joint probability model, analyzes the probability distribution characteristics between wind speed and ice thickness in a deep way, and conducts research on the return period of wind speed and ice thickness.

2. Wind-ice Joint Distribution Theory

2.1. Binary Copula Function Model

Based on the Sklar theorem, the Copula can describe the statistical relationship among multi-dimensional edges exactly, and it is a bridge which connects the multivariable distribution and its unitary marginal distribution. The two-dimensional model construction steps of Copula function are shown as follow: firstly, determine the marginal distribution firstly, and then adopt suitable Copula function for connection, so as to describe the correlation among variables well, and finally obtain the two-dimensional model. Among the top three categories of Copula functions - oval Copula, Archimedean Copula and Plackett Copula, the oval Copula and Archimedean Copula are most widely applied. In this paper, the frequently used five Copula functions in the oval Copula and Archimedean Copula are adopted to construct the wind-ice joint probability distribution, and the expression and adaptability of the two-dimensional Copula function are shown as follows:

(1) Gumbel Copula
\[ C(u,v;\theta) = \left( e^{-\theta} + v^\theta - 1 \right)^{-\frac{1}{\theta}} \]  

Wherein, \( \theta \geq 1 \) refers to relevant parameter, and the relationship between it and the Kendall relevant coefficient \( \tau \) is \( \tau = 1 - \frac{1}{\theta} \). When \( \theta = 1 \), U and V are independent mutually; when \( \theta \to +\infty \), U and V tend to completely related to each other.

(2) Clayton Copula

\[ C(u,v;\theta) = \exp \left\{ - \left[ (\ln u)^\theta + (\ln v)^\theta \right]^{\frac{1}{\theta}} \right\} \]  

Where \( 0 < \theta < +\infty \) refers to relevant parameter, and the relationship between it and Kendall rank correlation coefficient \( \tau \) is \( \tau = 1 + \frac{\theta}{\theta + 2} \). When \( \theta \to 0 \), the U and V tend to independent; when \( \theta \to +\infty \), U and V tend to completely related.

(3) Frank Copula

\[ C(u,v;\theta) = -\frac{1}{\theta} \ln \left[ 1 + \frac{\left( e^{-\theta u} - 1 \right) \left( e^{-\theta v} - 1 \right)}{\left( e^{-\theta} - 1 \right)} \right] \]  

Where \( \theta \neq 0 \) refers to relevant parameter, and the relationship between it and Kendall rank correlation coefficient \( \tau \) is \( \tau = 1 + \frac{4}{\theta} \int_1^\theta \frac{\tau}{\theta} \ln \left( \frac{\tau}{\theta} - 1 \right) d\tau - 1 \); when \( \theta > 0 \), it means that the U and V are of positive correlation; when \( \theta < 0 \), it means that the U and V are negative correlation; and when \( \theta \to 0 \), U and V tend to independent.

(4) Gaussian Copula

\[ C(u,v;\theta) = \int_0^{\Phi^\theta(v)} \int_0^{\Phi^\theta(u)} \frac{1}{2\pi \sqrt{1-\theta^2}} \exp \left\{ -\frac{s^2 - 2\theta st + \tau}{2(1-\theta^2)} \right\} ds dt \]  

Where \( \Phi(\cdot) \) refers to standard normal distribution function, and \( \Phi^{-1}(\cdot) \) refers to its inverse function; \( -1 \leq \theta \leq 1 \) refers to the linear correlation coefficient between \( \Phi^{-1}(U) \) and \( \Phi^{-1}(V) \).

(5) t Copula

\[ C(u,v;\theta,k) = \int_0^{\Phi^\theta(v)} \int_0^{\Phi^\theta(u)} \frac{1}{2\pi \sqrt{1-\theta^2}} \left[ 1 + \frac{s^2 - 2\theta st + \tau}{k(1-\theta^2)} \right]^{\frac{k+2}{2}} ds dt \]  

Wherein, \( T_k \) is the distribution function of one-dimensional student t distribution with degree of freedom of k, and \( T_k^{-1} \) is its inverse function; \( -1 < \theta < 1 \) is the linear correlation coefficient between \( T_k^{-1}(U) \) and \( T_k^{-1}(V) \).

The estimation method of Copula parameter\(^{[10]}\) can be divided into 3 types: ① correlation index method: it can be worked out indirectly according to the relationship between the Kendall rank correlation coefficient and \( \theta \); ② curve fitting method: under a certain curve fitting criterion, work out the statistical parameter of frequency curve which fits the experience point the best; ③ maximum likelihood method: the Copula parameters are divided into two parts: the parameters of marginal distribution, and the parameters of the Copula function \( \theta \); according to different estimation methods of marginal distribution, they can be divided into full-parameter maximum likelihood method, step-by-step maximum likelihood method and semi-parameter maximum likelihood method.

For different Copula functions represent different relevant structures, the selection of Copula function will directly influence some analysis and statistical inference results. The evaluation methods for goodness-of-fit of the Copula function\(^{[11]}\) include the Genest-Rivest method of Copula function
selection via intuitive schematic diagram, root mean square error (RMSE) criterion method and AIC information criterion method. In this paper, the RMSE method is adopted to evaluate the Copula function, i.e. calculate the European square distance between each Copula function and the experience Copula function; the smaller the distance is, the better the goodness-of-fit will be.

2.2. Wind-ice Joint Probability Distribution and Return Period

According to the Copula function, the joint distribution of wind speed and ice thickness is:

\[ F(w, i) = P(W \leq w, I \leq i) = C(F_w(w), F_i(i)) \]  

(6)

According to the marginal distribution of wind speed and ice thickness, it can be known that the return period of the extreme value wind speed \( W \) and the extreme value ice thickness \( I \) are:

\[ T_w(w) = \frac{1}{1 - F_w(w)}; \quad T_i(i) = \frac{1}{1 - F_i(i)} \]  

(7)

For the multivariable return period, there are three return periods which are frequently seen, i.e. joint return period, co-occurrence period and Kendall return period. When at least one between the wind speed \( W \) or the ice thickness \( I \) is exceeded, it is recorded as event \( E^\cup \), and the corresponding return period is called joint return period, and when both the wind speed and ice thickness exceed the set threshold value at the same time, it is recorded as event \( E^\cap \), and the corresponding return period is recorded as co-occurrence period\(^{[12]}\), and the calculation formulas of the joint return period and the co-occurrence period are shown as follows:

\[ T^\cup(w, i) = \frac{1}{P^\cup(w, i)} = \frac{1}{1 - C(u, v)} \]  

(8)

\[ T^\cap(w, i) = \frac{1}{P^\cap(w, i)} = \frac{1}{1 - u - v + C(u, v)} \]  

(9)

In the formula, \( u \) and \( v \) represent the marginal distribution functions of wind speed and ice thickness respectively. \( P^\cup(w, i) \) is the probability of occurrence event \( E^\cup(w, i) \), \( T^\cup(w, i) \) is the dual-variable joint return period of \( W \) and \( I \), with the unit of year.

With regard to the above two categories of return periods, the variable value is adopted to divide the dangerous zone directly, and there is a certain irrationality. To solve above problems, Salvadori et al.\(^{[13]}\) have proposed the return period calculation method of defining dangerous zone according to the joint probability. The probability that an event occurs within the subcritical zone can be described by adopting the Kendall measurement. Assume that \( t \in [0,1] \), then the measurement \( K_c \) of Kendall is:

\[ K_c = P\{C(u, v) < t\} \]  

(10)

The formula above which represents the relation of binary quantile is the probability distribution function of \( C_u(u, v) \). \( E^K(w, i) = \{F(w, i) = C_u(u, v) > t\} \), and the analytical expression of \( K_c \) cannot be worked out easily. To facilitate the calculation, this paper adopts the numerical method proposed by Salvadori and De Michele\(^{[14]}\) to calculate \( K_c \). The return period formula of Kendall is shown as follows:

\[ T_c = \frac{1}{1 - P\{C(u, v) < t\}} = \frac{1}{1 - K_c(t)} \]  

(11)

For the given dual-variable return period, there are infinitely many combinations of wind speed and ice thickness which meet design standard. They constitute a piece of isoline, and there will be many combination values of wind speed and ice thickness. In this paper, the adopted most widely applied and the most possible combination method is to select the point with the maximum joint probability density on the isoline as the most possible design point, and then work out the design value through the inverse function of the distribution function:
\[
(u,v) = \text{arg max} \ f_w(f_w^{-1}(w), f_i^{-1}(i)) \quad (12)
\]
\[
w = f_w^{-1}(u), i = f_i^{-1}(v) \quad (13)
\]

Next, we will calculate the wind-ice joint design value combinations with the joint return period, Kendall return period and co-occurrence period as calculation standard respectively, thus providing reference for the engineering design.

3. Research of Wind-ice Joint Probability for Southwest China

3.1. Field Measurement Data
The basic data adopts the wind speed and icing data observed at the ice observation stations of power grids in three countries in the power transmission line zone of southwest power grid from November 2016 to March 2018, and the basic statistical characteristic value of sample shows that the maximum wind speed is 9.8m/s, and the maximum ice thickness is 23.72mm.

3.2. Wind-ice Joint Probability Distribution Model

3.2.1. Marginal Distribution Function. In case of determining the conductor icing thickness under some return period, the results obtained by adopting different probability statistic models may be very different; meanwhile, due to difference of terrain and climate in different regions, it is very difficult to determine a general probability model for extreme value analysis. In meteorology, it is necessary to fit the extreme values of climatic factors, and usually, the generalized extreme value distribution function is adopted for fitting. In this paper, we adopt the Gumbel distribution, Weibull distribution and generalized extreme value distribution function to conduct fitting and inspection of wind speed and ice thickness.

Assume that the average wind speed and ice thickness near the ground surface refer to a stable random process, are adopt MATLAB to fit the three probability models. The expressions of the three probability distribution functions are shown as follows:

1. Generalized extreme value distribution (GEV):
\[
F(x) = \exp \left[ \left( 1 + k \frac{x - \mu}{\sigma} \right)^{-1/k} \right] \quad (14)
\]

In the formula, \( \mu \) and \( \sigma \) are position diameter and scale parameter respectively, and \( k \) refers to shape parameter, \(-\infty < k < +\infty \).

2. Gumbel distribution
\[
F(x) = \exp \left[ -\exp \left( -\frac{x - \mu}{\sigma} \right) \right] \quad (15)
\]

In the formula, \( \mu \) and \( \sigma \) are position parameter and scale parameter respectively.

3. Weibull distribution (extreme value III type distribution):
\[
F(x) = 1 - \exp \left[ -\left( \frac{x}{a} \right)^{b} \right] \quad (16)
\]

In the formula, \( a > 0 \), it refers to shape parameter, \( b > 0 \), it refers to scale parameter.

Fit the daily extreme values of wind speed and ice thickness in original data, and the parameters are to be estimated by adopting the maximum likelihood method with relatively strong adaptability and relatively high precision. The fitting inspection adopts the Kolmogorov - Smirnov test (K-S) method for inspection, and the obtained parameters and inspection values are as shown in Table 1. From the Table, it can be seen that no matter the wind speed or the ice thickness, the KS statistical amount of generalized extreme value distribution is the minimum, and the fitting is good. The fitting results of actually
measured value and each theoretical distribution are as shown in Figure 1 and Figure 2, and the result shows that the fitting effect of generalized extreme value distribution is relatively good.

![Figure 1](image1.png)

**Figure 1** Fitting of daily extreme value function of sample wind speed

![Figure 2](image2.png)

**Figure 2** Fitting of daily extreme value function of sample ice thickness

| Distribution Type | Sample Type | $\mu$ | $\sigma$ | k | KS Statistical Amount |
|-------------------|-------------|-------|-------|---|-----------------------|
| Gumbel            | WS (m/s)    | 5.57  | 3.02  | - | 0.30                  |
|                   | IT (mm)     | 14.18 | 6.66  | - | 0.37                  |
| Weibull           | WS (m/s)    | 7.50  | 3.12  | - | 0.32                  |
|                   | IT (mm)     | 18.71 | 4.23  | - | 0.30                  |
| GEV               | WS (m/s)    | 6.64  | 2.49  | -0.76 | 0.20                  |
|                   | IT (mm)     | 16.93 | 5.69  | 1.05 | 0.23                  |

Note: WS: wind speed; IT: Ice thickness

For the measured data of conductor icing is relatively small, the relatively shorter measured sample sequence brings error for the evaluation of probability of icing event inevitably. Therefore, in this paper, the Monte Carlo method is adopted to generate 500-year pseudo wind speed and pseudo ice thickness samples with the generalized extreme value distribution as master die proof distribution by referring to the fitting result of actually measured daily extreme value wind-ice data. Through adopting the annual extreme value sampling method for the pseudo wind speed and ice thickness sample, we will obtain the annual maximum value samples of wind speed and ice thickness. After conducting parameter estimation by adopting generalized extreme value distribution, the obtained annual extreme value parameter values of wind speed and ice thickness are as shown in the Figure 3, Figure 4 and Table 2.

![Figure 3](image3.png)

**Figure 3** Comparison between Daily Extreme Value & Annual Extreme Value of Simulated Wind Speed and Daily Extreme Value of Actually Measured Data
Figure 4 Comparison between daily extreme value & annual extreme value of simulated ice thickness and daily extreme value of measured data

Table 2 Distribution parameters of annual extreme values of wind speed and ice thickness simulation

| Samples            | Distribution Type | µ     | σ     | k      |
|--------------------|-------------------|-------|-------|--------|
| Wind speed (m/s)   | GEV               | 8.97  | 0.78  | -0.83  |
| Ice thickness (mm) | GEV               | 21.38 | 1.05  | -1.08  |

\[
F(w) = \exp\left[\frac{-\left(1-0.83w-8.97\right)^{\frac{1}{0.83}}}{0.78}\right]^{17}
\]

\[
F(i) = \exp\left[\frac{-\left(1-1.08i-21.38\right)^{\frac{1}{1.05}}}{1.05}\right]^{17}
\]

3.2.2. Determination of Copula Function for Joint Probability Model. The joint probability distribution selects the relatively extensive Gumbel Copula, Clayton Copula, Frank Copula, Gaussian Copula and t Copula, and the parameter estimation adopts the maximum likelihood estimation, while the goodness-of-fit evaluation adopts the AIC criterion. The AIC value of each Copula model is as shown in Table 3, and it is known that the AIC value of the joint distribution Frank Copula is the minimum, with the best fitting effect. The Frank Copula probability density function diagram, distribution diagram and probability density contour map are as shown in Figure 5-Figure 7.

Figure 5. Frank Copula probability density diagram

Figure 6. Frank Copula probability distribution diagram
The joint probability distribution of wind speed and ice thickness obtained from Frank Copula is:

\[
C(u,v) = -\frac{1}{0.38} \ln \left[ 1 + \frac{(e^{-0.38u} - 1)(e^{-0.38v} - 1)}{(e^{-0.38} - 1)} \right] \quad (19)
\]

\[
c(u,v) = -0.38(e^{-0.38} - 1)e^{-0.38(u+v)} \left[ (e^{-0.38} - 1) + (e^{-0.38} - 1) \right]^{2} \quad (20)
\]

| Copula Distribution Type | AIC value |
|--------------------------|-----------|
| Gaussian copula          | 2.08      |
| t copula                 | 2.08      |
| Gumbel Copula            | 2.00      |
| Clayton Copula           | 2.00      |
| Frank Copula             | 0.76      |

4. Joint Probability and Return Period

The isoline of wind-ice joint return period (JRP), contour line of co-occurrence return period (CRP) and Kendall return period (KRP) are as shown in [Figure 8-10] respectively, and it can be known that the wind speed and ice thickness joint return period is slightly shorter than the return period, and the Kendall return period is within both of them.
The most possible design values of the 30-year, 50-year and 100-year return periods which correspond to the joint return period, co-occurrence period and Kendall return period are listed in Table 4. It can be known that the design value of joint return period is greater than the design value of the co-occurrence return period, and the design value of Kendall return period is within both of them.

Table 4 Results of different return periods

| Design Standards (years) | CRP WS (m/s) | CRP IT (mm) | JRP WS (m/s) | JRP IT (mm) | KRP WS (m/s) | KRP IT (mm) |
|-------------------------|--------------|-------------|--------------|-------------|--------------|-------------|
| 30                      | 9.56         | 23.36       | 11.65        | 24.31       | 10.02        | 24.00       |
| 50                      | 9.75         | 23.64       | 11.85        | 24.53       | 10.06        | 24.01       |
| 100                     | 9.84         | 23.94       | 11.95        | 24.68       | 10.07        | 24.02       |

Considering from the definition of return period, the dangerous zones of the joint return period and the co-occurrence return period are directly divided according to value of each variable. Although this will cause expansion or shrinkage of dangerous zone, its definition for the dangerous event is clear, and this can meet some specific application demands, for example, in case that two variables are required to exceed the critical value at the same time, the co-occurrence return period can be used as the calculation standard. However, the dangerous zone of Kendall return period divided according to joint probability value avoids too high or too low estimation to the dangerous zone, and it can be used to describe the dual-variable dangerous event under general situations reasonably. Therefore, under the general situation when the dangerous event is not defined in advance, the Kendall return period is recommended to be used as the calculation standard of dual-variable return period.

5. Conclusions

Based on the joint probability model of binary Copula function, this paper makes a statistic and analyzes the observation data of wind speed and ice thickness obtained by the observation station of the southwest region from November 2016 to March 2018. Then, it obtains the annual extreme value simulation sample by adopting the Monte Carlo method according to the statistical characteristics of actually measured data, constructs the wind-ice joint probability distribution with Copula function, and conducts research about the return periods of wind speed and icing thickness; finally, it obtains the following conclusions:

(1) Through the fitting of wind speed and ice thickness data measured in the power transmission line zone of southwest power grid, it is found that both wind speed and ice thickness conform to the generalized extreme value distribution. For the icing thickness is influenced by tiny terrain and weather, under different micro-meteorological conditions, we shall adopt different probability statistic models according to measured data, so as to obtain a safe and economic design result.
(2) This paper adopts binary Copula function to construct the wind-ice joint probability distribution, selects five Copula functions to construct the joint probability distribution of wind speed and ice thickness. The result shows that the AIC value of Frank Copula is the minimum, with the best fitting effect.

(3) By adopting the joint probability distribution model, this paper can forecast the joint return period, co-occurrence return period and Kendall return period of wind speed and ice thickness, thus providing parameters for the engineering design. This Project infers the combination of design values of three kinds of return periods from the perspective of statistic, and it is suggested that in the engineering practice, the design combination of several kinds of return periods should be determined comprehensively according to engineering importance and engineering failure result.

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