Layered system with metamaterials

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Abstract. The layered system composed of metamaterial and vacuum layers is examined. We assume the metamaterial is the isotropic, homogeneous, dispersive and non-absorptive media. The permittivity and permeability of the metamaterial are equal and described by using a single Lorentz contribution. The electric Green’s function is obtained for the case, when the permittivity and permeability of the metamaterial are equal –1. For this case we observe the no reflection effect. Also, the photonic band gap structure of the similar infinite layered system is described. For changing layers widths, the absence of the no reflection effect is shown.

1. Introduction

Metamaterials are known as negative index materials (NIMs) since for certain frequencies they have the negative refractive index [1]. By using NIMs, new covered surfaces and cloaking materials can be created [2], as well as superlenses with the resolving power, exceeding the diffraction limit [3]. Among systems with NIMs (NIM systems), layered NIM systems are widely known. The simplest two-layer NIM system is considered in [4]. The three-layer NIM system, which is the model for the superlens, is studied in [3, 5, 6]. The studies of the multilayer NIM systems (i.e., systems with the layers count greater than three) are presented in [7, 8]. A number of NIM studies are dedicated to obtaining the Green’s function which describes an electromagnetic field of a point source [4, 6, 9]. An electromagnetic field value at any point can be determined with the Green’s function. The layered systems are considered also as one-dimensional photonic crystals (1DPCs) [10, 11]. Numerous investigations of 1DPC were carried out in recent years [12, 13]. But most of these ones consider systems without the frequency dispersion. The first goal of our work is to obtain expressions for the electric Green’s function for a multilayer NIM system, composed of arbitrary finite number of layers filled with the metamaterial and vacuum. The magnetic Green’s function can as well be obtained by the same way. The second goal of our work is to describe the photonic band gap (PBG) structure for the similar but infinite NIM system (photonic crystal). We assume the metamaterial is the isotropic, homogeneous, dispersive and non-absorptive media, the permittivity and permeability stand equal and are described by using a single Lorentz contribution [4, 14]. We are interesting in the electric field in the case (NIM situation), when the permittivity and permeability of the metamaterial are equal –1 [4].

2. Green’s function

2.1. Finite NIM system

We study the NIM system composed of \((N+M+1)\) parallel layers, where \(N, M \geq 3\) are natural odd integers (figure 1). The \(\textbf{e}_1, \textbf{e}_2, \textbf{e}_3\) unit vectors of the Cartesian basis \(\{\textbf{e}_1, \textbf{e}_2, \textbf{e}_3\}\) set a plane of layer’s
surfaces. The $\mathbf{e}_3$ unit vector set the $x_3$ axis. Every layer has an index. From the right side of the zero layer, $N$ layers are located, from the left side there are $M$ layers. Thus, the $n$ layer index is $n = -M, \ldots, 0, \ldots, N$. All even layers (as well as the zero layer) are $\Delta_1$ in width, and filled with a metamaterial. All odd layers are $\Delta_2$ in width, and filled with a vacuum. The last-to-left-side (with $-M$ index) and last-to-right-side (with $N$ index) layers are the half spaces unbounded along the direction of the $x_3$ axis and are empty (vacuum). The point source is located in the zero layer and defined by coordinates of the vector $\mathbf{y} = \{y_1, y_2, y_3\}$, which is located in the Cartesian basis $\{\mathbf{e}_1, \mathbf{e}_2, \mathbf{e}_3\}$, i.e., $0 < y_3 < \Delta_1$. We assume the translation invariance along the plane of layer’s surfaces.

![Figure 1. The NIM system composed of $(N+M+1)$ parallel layers filled with the metamaterial (grey) and the vacuum (white). The point source is located in the zero layer at $y_3$ coordinate of the $x_3$ axis.](image)

We consider the Maxwell’s equations in a differential form

$$\frac{d\mathbf{D}(x,t)}{dt} = \nabla \times \mathbf{H}(x,t), \quad \frac{d\mathbf{B}(x,t)}{dt} = -\nabla \times \mathbf{E}(x,t), \quad \nabla \cdot \mathbf{D}(x,t) = 0, \quad \nabla \cdot \mathbf{B}(x,t) = 0,$$

where $\mathbf{x} = \{x_1, x_2, x_3\}$ is the vector located in the Cartesian basis $\{\mathbf{e}_1, \mathbf{e}_2, \mathbf{e}_3\}$, $\nabla$ is the Hamilton operator, $\times$ is a cross product symbol as well as a numerical product symbol, $\cdot$ is an inner product symbol as well as a matrix product symbol. Also, we consider the auxiliary field equations

$$\mathbf{D}(x,t) = \varepsilon_0 \mathbf{E}(x,t) + \mathbf{P}(x,t), \quad \mathbf{B}(x,t) = \mu_0 \left[\mathbf{H}(x,t) + \mathbf{M}(x,t)\right],$$

$$\mathbf{P}(x,t) = \varepsilon_0 \int_0^t \mathbf{\chi}_e(x,s) \cdot \mathbf{E}(x,s) \, ds, \quad \mathbf{M}(x,t) = \int_0^t \mathbf{\chi}_m(x,s) \cdot \mathbf{H}(x,s) \, ds,$$

where $\varepsilon_0$ and $\mu_0$ are the electric and magnetic constants, which we set $\varepsilon_0 = \mu_0 = 1$ for brevity [4], $\mathbf{\chi}_e(x,t)$ and $\mathbf{\chi}_m(x,t)$ are the electric and magnetic susceptibility tensors. We use the causality condition $\mathbf{\chi}_e(x,t) = \mathbf{\chi}_m(x,t) = 0$ for $t < t_0$ (we assume $t_0 = 0$) and the passivity condition [4].

The considered system composed of layers divided by plane unbounded surfaces. The boundary conditions for each one are presented in the general form as follows:

$$(\mathbf{E}_1 - \mathbf{E}_2) \cdot \mathbf{e}_3 = 0, \quad (\mathbf{H}_1 - \mathbf{H}_2) \cdot \mathbf{e}_3 = 0, \quad (\mathbf{D}_1 - \mathbf{D}_2) \cdot \mathbf{e}_3 = 0, \quad (\mathbf{B}_1 - \mathbf{B}_2) \cdot \mathbf{e}_3 = 0.$$

$E_j = E_j(\tilde{x}, t)$, $H_j = H_j(\tilde{x}, t)$, $D_j = D_j(\tilde{x}, t)$, and $B_j = B_j(\tilde{x}, t)$ stand for the limits with $\mathbf{x} \rightarrow \tilde{\mathbf{x}}$, where $\tilde{x} = x^+ + x^- \mathbf{e}_3$, $\mathbf{x} = x^+ + x^- \mathbf{e}_3$, $x^+ = \{x_1, x_2, 0\}$ and $x_3 \rightarrow \tilde{x}_3$, $j$ is the index distinguishing limits calculated on different surface sides ($j = 1, 2$). We use the Laplace transform with $t$ time

$$\hat{f}(z) = \int_0^\infty \exp[izt]f(t)\,dt, \quad f(t) = \frac{1}{2\pi i} \int_A \exp[-iz\tau] \hat{f}(z)\,dz,$$

and the Fourier transform with $x_1$ and $x_2$ coordinates

$$f_k(x_1) = \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} \exp[i(k_1x_1 + k_2x_2)]f(x)\,dx_1\,dx_2 = \int_{k_2} \exp[i k_1 \cdot x^+ ] f(x)\,dx^+,$$
and can be are the coordinates of the two-dimensional wave vector with the

\[ \mathbf{k}^\perp = \{k_x, k_y, 0\} \]

and the unit vector \( \mathbf{e}_k \), i.e., \( \mathbf{k}^\perp = \mathbf{k} \mathbf{e}_k \). The electric and magnetic permeabilities do not depend on coordinates of the vector, i.e., \( \mathbf{e}(x, z) = \mathbf{e}(z) \mathbf{U} \), \( \mathbf{\mu}(x, z) = \mu(z) \mathbf{U} \), where \( \mathbf{U} \) is the 3x3 unit matrix. Also, we assume that the metamaterial is the dispersive non-absorptive media. In that case, the susceptibilities consist of a sum of Lorentz contributions [14]. We deal with a single dispersive Lorentz contribution [4]. We assume that the permittivity and permeability of the metamaterial stand equal and

\[ \varepsilon(z) = \mu(z) = \frac{1 - \Omega_0^2}{z^2 - \omega_0^2}, \]

where \( \Omega \), \( \omega_0 \) are constants, \( z = \omega + i\alpha \), \( \alpha \to 0 \), \( \alpha > 0 \), and \( \varepsilon(z) = \mu(z) = 1 \) in vacuum layers. Note that the frequency (NIM frequency [4]) leads to the NIM situation, i.e., \( \varepsilon(\omega) = \mu(\omega) = 1 \).

With the Laplace transform (5) we obtain the following Helmholtz equation for the electric field function \( \mathbf{E}(x, \omega) \):

\[ \mathbf{L}'(x, z) \cdot \hat{\mathbf{E}}(x, z) = \mathbf{g}'(x, z), \]

\[ \mathbf{L}'(x, z) \cdot \hat{\mathbf{E}}(x, z) = \nabla \times [\mathbf{e}(x, z) \hat{\mathbf{E}}(x, z) - \nabla \times [\mathbf{\mu}^{-1}(x, z) \cdot \nabla \times \hat{\mathbf{E}}(x, z)]], \]

\[ \mathbf{g}'(x, z) = iz\mathbf{E}(x, 0) - \nabla \times [\mathbf{\mu}^{-1}(x, z) \cdot \mathbf{H}(x, 0)], \]

where \( \mathbf{L}'(x, z) \) is the electric Helmholtz operator, \( \mathbf{g}'(x, z) \) is the function of the initial electric field configuration. Let us introduce the \( \mathbf{G}'(x, y, z) \) electric Green’s function [4] that satisfies

\[ \mathbf{L}'(x, z) \cdot \mathbf{G}'(x, y, z) = \delta(x - y) \mathbf{U}, \]

where \( \delta(x - y) \) is the Dirac delta function. Then, the \( \mathbf{E}(x, \omega) \) electric field function is given by the inverse Laplace transform (5) of

\[ \hat{\mathbf{E}}(x, z) = \int \mathbf{G}'(x, y, z) \cdot \mathbf{g}'(y, z) dy. \]

Note that the magnetic Green’s function \( \mathbf{G}''(x, y, z) \) and the magnetic field function \( \mathbf{H}(x, \omega) \) can be obtained in a similar way. Therefore, below we consider only electric Green’s function.

2.2. Electric Green’s function

By using the Laplace (5) and Fourier (6) transforms for the Helmholtz equation (8) and the boundary conditions (4) we obtain the system of equations for the electric Green’s function in every layer with transverse electric (TE) and transverse magnetic (TM) modes. We solve this system by using the recurrence relation approach, and obtain results in the NIM situation with \( z \to \pm \omega_0 \). The electric Green’s function in the NIM situation is expressed in the following way: for \( \omega > \kappa \) in the layers filled with the metamaterial, i.e., with \( n = -(M - 1), -2, 0, 2, \ldots, (N - 1), \)

\[ \mathbf{G}'_{k^\perp}(x_\perp, y_\perp, \pm \omega) = \pm \frac{1}{2i\rho(\omega)} \exp\left[ \pm i\rho(\omega) \left| n|A_x| - |x_\perp - y_\perp| \right| \right] \mathbf{e}_k \otimes (\mathbf{e}_3 \times \mathbf{e}_{k^\perp}). \]
\[ + \frac{\rho^2(\omega)}{\omega^2} \left\{ e_k^+ \otimes e_k^+ + \pm \text{sign}(x_1 - y_1) \left\{ \frac{\kappa}{\rho(\omega)} \left( e_k^+ \otimes e_k^+ + e_k^- \otimes e_k^- \right) + \frac{\kappa^2}{\rho^2(\omega)} e_k^+ \otimes e_k^- \right\} \right\}, \]

in the layers filled with a vacuum, i.e., with \( n = -M, \ldots, -3, -1, 1, \ldots, N \),

\[ G_k'(x_1, y_1, \pm \omega) = \frac{1}{2\rho(\omega)} \exp \left[ -\rho(\omega) \left( |n|\Delta_1 + |x_1 - \Delta_1 + y_1| \right) \right] \left\{ (e_j \times e_k^+) \otimes (e_j \times e_k^-) + \right. \]

\[ + \frac{\rho^2(\omega)}{\omega^2} \left\{ e_k^+ \otimes e_k^- \pm \text{sign}(x_1 - \Delta_1 + y_1) \left\{ \frac{\kappa}{\rho(\omega)} \left( e_k^+ \otimes e_j + e_k^- \otimes e_j \right) - \frac{\kappa^2}{\rho^2(\omega)} e_k^+ \otimes e_j \right\} \right\}, \]

for \( \omega < \kappa \) in the layers filled with the metamaterial, i.e., with \( n = -(M-1), \ldots, -2, 0, 2, \ldots, (N-1) \),

\[ G_k'(x_1, y_1, \pm \omega) = \frac{1}{2\rho(\omega)} \exp \left[ -\rho(\omega) \left( |n|\Delta_2 + |x_1 - \Delta_2 + y_1| \right) \right] \left\{ (e_j \times e_k^+) \otimes (e_j \times e_k^-) + \right. \]

\[ + \frac{\rho^2(\omega)}{\omega^2} \left\{ e_k^+ \otimes e_k^- \pm \text{sign}(x_1 - \Delta_2 + y_1) \left\{ \frac{\kappa}{\rho(\omega)} \left( e_k^+ \otimes e_j + e_k^- \otimes e_j \right) - \frac{\kappa^2}{\rho^2(\omega)} e_k^+ \otimes e_j \right\} \right\}, \]

in the layers filled with a vacuum, i.e., with \( n = -M, \ldots, -3, -1, 1, \ldots, N \),

\[ G_k'(x_1, y_1, \pm \omega) = \frac{1}{2\rho(\omega)} \exp \left[ -\rho(\omega) \left( |n|\Delta_3 + |x_1 - \Delta_3 + y_1| \right) \right] \left\{ (e_j \times e_k^+) \otimes (e_j \times e_k^-) + \right. \]

\[ + \frac{\rho^2(\omega)}{\omega^2} \left\{ e_k^+ \otimes e_k^- \pm \text{sign}(x_1 - \Delta_3 + y_1) \left\{ \frac{\kappa}{\rho(\omega)} \left( e_k^+ \otimes e_j + e_k^- \otimes e_j \right) - \frac{\kappa^2}{\rho^2(\omega)} e_k^+ \otimes e_j \right\} \right\}, \]

where the sign function equals +1 for \( x \geq 0 \) and −1 for \( x < 0 \), and the \( \rho(\omega) \) function is defined as

\[ \rho(\omega) = \sqrt{|k|^2 - |\omega|^2} = \sqrt{\omega^2 \left( 1 - \frac{\Omega^2}{\omega^2 - \omega_0^2} \right)^2 - \kappa^2}, \]

where \( \rho(\hat{\omega}) = \sqrt{\hat{\omega}^2 - \kappa^2} \). \( \Omega \) denotes the speed of light in vacuum. We use the similar to (5) Fourier transform with \( \tau \) time

\[ \hat{f}(\omega) = \int_{-\infty}^{\infty} \exp[-i\omega \tau] f(\tau) d\tau, \quad f(\tau) = \frac{1}{2\pi} \int_{-\infty}^{\infty} \exp[i\omega \tau] \hat{f}(\omega) d\omega, \]
and consider the permittivity and permeability of the metamaterial as in (7) with \( z = \omega \). They are negative and the metamaterial behaves like the NIM when the \( \omega \) frequency is inside the \((\omega_0, \omega_\infty)\) interval (NIM interval), where \( \omega_\infty = \sqrt{\omega_0^2 + \omega^2} \), \( \varepsilon(\omega) = \mu(\omega) = 0 \), and \( \varepsilon(\omega_0 + 0) = \mu(\omega_0 + 0) = -\infty \).

With the Fourier transform (10) we obtain the following Helmholtz equation for the electric field function \( \mathbf{E}(x, \omega) \):

\[
\nabla \times \nabla \times \mathbf{E}(x, \omega) = (\omega / c)^2 \varepsilon(\omega) \mu(\omega) \mathbf{E}(x, \omega).
\]

By using the Fourier transforms (10) and (6) for the Helmholtz equation (11), the boundary conditions (4) and the Floquet-Bloch theorem [15, 16] we obtain the following dispersion equation:

\[
\left( e^{-i\theta(\omega, k_1)} \right)^2 - \left[ \frac{\sigma_{1, 2}^2}{4} \left( 1 + e^{i2\mu(\omega)} \right) e^{i2\mu(\omega)} + \frac{\sigma_{1, 2}^2}{4} \left( e^{i2\mu(\omega)} - e^{-i2\mu(\omega)} \right) + 1 = 0, \right.
\]

where \( \theta \) is a yet undefined wave vector, called the Bloch wave vector, \( \sigma_{m, p} = (\varepsilon_p k_1^m, \mu_p k_3^m) / \varepsilon_m k_3^m \) with indices \( m = 1 \) and \( p = 2 \), or \( m = 2 \) and \( p = 1 \), and \( \varepsilon_m \) and \( k_3^m \) are calculated for the metamaterial if \( m = 1 \), and vacuum if \( m = 2 \). The dispersion equation is the same for the TE and TM modes. It follows from the equality of the permittivity and permeability (7). Therefore, we have the identical PBG structure for TE and TM modes.

3.2. Solutions

The Helmholtz equation has non-trivial solutions when the dispersion equation (12) holds true. By using the Fourier transforms (10) and (6), the boundary conditions (4) and the Floquet-Bloch theorem [16, 17], we obtain the following solutions in metamaterial layers (subscript 1) and vacuum layers (subscript 2) for TM mode:

\[
E_1(x_3, \omega, \kappa) = C(\omega, \kappa) \left( \frac{\sigma_{1, 2}^2}{4} \left( e^{i2\theta(\omega, k_1)} - e^{-i2\theta(\omega, k_1)} \right) e^{-i2\theta(\omega, k_1)} e^{i2\mu(\omega, k_1)} - \left[ \left( \frac{\sigma_{1, 2}^2}{4} \right)^2 - \left( \frac{\sigma_{1, 2}^2}{4} \right) \left( e^{i2\theta(\omega, k_1)} - e^{-i2\theta(\omega, k_1)} \right) \right] e^{i2\mu(\omega, k_1)} \right),
\]

\[
E_2(x_3, \omega, \kappa) = 2C(\omega, \kappa) \left[ \frac{\sigma_{1, 2}^2}{4} \left( e^{-i2\theta(\omega, k_1)} - e^{i2\theta(\omega, k_1)} e^{i2\mu(\omega, k_1)} \right) e^{i2\mu(\omega, k_1)} \right) - \sigma_{1, 2}^2 \left( e^{-i2\theta(\omega, k_1)} - e^{i2\theta(\omega, k_1)} e^{i2\mu(\omega, k_1)} \right) e^{-i2\theta(\omega, k_1)} e^{i2\mu(\omega, k_1)} \right),
\]

where \( C(\omega, \kappa) \) is an unknown function, which can be obtained from some initial conditions. For the TE mode we obtain the same expressions (13)-(14) but with swapped \( \sigma_{1, 2}^2 \) for \( \sigma_{2, 1}^2 \). The solutions (13)-(14) in the NIM situation for TE and TM modes are expressed for \( \omega > \kappa \) as follows:

\[
E_1(x_3, \omega, \kappa) = -4C(\omega, \kappa) \left( e^{i\rho_\kappa} - e^{i\rho_\kappa} e^{i\omega(k_1 + \kappa)} \right) e^{-i\rho_\kappa},
\]

\[
E_2(x_3, \omega, \kappa) = -4C(\omega, \kappa) \left( e^{i\rho_\kappa} - e^{i\rho_\kappa} e^{i\omega(k_1 + \kappa)} \right) e^{-i\rho_\kappa},
\]

and for \( \omega < \kappa \) as follows:

\[
E_1(x_3, \omega) = 4C(\omega, \kappa) \left( e^{i\rho_\kappa} - e^{i\rho_\kappa} e^{i\omega(k_1 + \kappa)} \right) e^{-i\rho_\kappa},
\]

\[
E_2(x_3, \omega) = 4C(\omega, \kappa) \left( e^{i\rho_\kappa} - e^{i\rho_\kappa} e^{i\omega(k_1 + \kappa)} \right) e^{-i\rho_\kappa},
\]

where \( \rho = \rho(\omega) \) is given by equation (9). Note that the solution (15), as well as the solutions (16)-(18), has only one term responsible for propagation and no terms responsible for reflection, i.e., there is no reflection at the NIM frequency \( \omega \). But this effect is observed only when the dispersion equation (12) holds true, i.e., only for permitted bands (at the spectrum).
3.3. Numerical results

To study the PBG structure of the considered 1DPC we use a numerical approach. We consider the equality (12) and seek for which $\omega$ and $\kappa$ values it holds true with any $\theta$ value, which belongs to the $(0, 2\pi/\Delta)$ interval. We fix the constants $\omega_0 = 30$ THz, $\Omega = 90$ THz. Then, the NIM frequency is $\hat{\omega} = 70.36$ THz and the NIM interval is $(30$ THz, $94.87$ THz). We consider the intervals for $\omega$ values from 42 till 96 THz (then the $\omega/c$ normalized frequency has values from $1.4 \times 10^6$ till $3.2 \times 10^5$ m$^{-1}$) and for $\kappa$ values from 0 till 0.8 $\eta$m$^{-1}$ (i.e., from 0 till $8 \times 10^5$ m$^{-1}$). We use four following cases of the $\Delta_1$ and $\Delta_2$ parameters: 1. $\Delta_1 = \Delta_2 = 10$ $\eta$m (see (a) in figure 2); 2. $\Delta_1 = 20$ $\eta$m and $\Delta_2 = 10$ $\eta$m (see (b) in figure 2); 3. $\Delta_1 = 10$ $\eta$m and $\Delta_2 = 20$ $\eta$m (see (c) in figure 2); 4. $\Delta_1 = 10$ $\eta$m and $\Delta_2 = 100$ $\eta$m (see (d) in figure 2).

Figure 2. Dependences of PBG structure on the $\omega$ frequency and $\kappa$ values for TE and TM modes simultaneously. Permitted bands are grey, forbidden bands are white. The vertical straight dot line shows the NIM frequency 70.36 THz. The dot curves divide the space on four areas. The bottom and top areas correspond to the radiative and evanescent regime, respectively, in the metamaterial and vacuum simultaneously. The right area corresponds to the evanescent regime in the metamaterial and the radiative regime in the vacuum. The left area corresponds to the radiative regime in the metamaterial and the evanescent regime in the vacuum. The 1, 2, 3, and 4 cases are presented in (a), (b), (c), and (d), respectively.

In the case 1, we observe the NIM frequency belonging to a permitted band for all $\kappa$ values. Therefore, there is no reflection for any $\kappa$ value in the NIM situation. With changing the $\Delta_1$ or $\Delta_2$ parameters, i.e., in the cases 2, 3, and 4, we observe the faster narrowing of permitted bands with the increasing of $\kappa$ value, than it is in the case 1. In cases 3 and 4 for small $\kappa$ value permitted bands shift to the zero $\omega$ value. The contained the NIM frequency permitted band splits into two bands. As in the case 2, one observes the absence of reflection in the NIM situation only for a part of $\kappa$ values set.

4. Conclusion

In this paper, we solved the problem of obtaining the electric Green’s function for the layered NIM system with the point source was located inside the certain metamaterial layer. We assumed that the permittivity and permeability of the metamaterial stand equal and are defined with the single Lorentz contribution. In the NIM situation, we obtained relations for the Fourier transformed electric Green’s
function. The magnetic Green’s function can as well be obtained with reasoning similar to the presented above. The obtained formulas are symmetric, relative to the position of the point source. This fact shows the correlation with the physical conception of the electromagnetic field propagated into the system composed of isotropic homogeneous layers. The system can be composed of arbitrary finite number of layers. This fact allows us to use the considered system as a model for simulation or engineering of the real objects, such as superlens systems and multilayer NIM coverings. Besides, the reflection in the NIM situation is absent. This fact was discussed earlier [4, 6] and holds true only if the NIM frequency does not belong to the spectrum of the electric Helmholtz operator. For a limiting case, when we deal with infinite periodic NIM system, the obtained formulas correspond to gaps.

Also, we solved the problem of describing the photonic band gap structure of the similar infinite layered NIM system (1DPC). In the permitted bands we obtained expressions for the scalar field functions, which contained only one term responsible for propagation and no terms responsible for reflection, i.e., there was no reflection at the NIM frequency in permitted bands. By the numerical approach, we observed the NIM frequency belonging to a permitted band, i.e., in the NIM situation there was no reflection for any direction. With changing the layers widths the permitted band contained the NIM frequency was splitted into two bands, i.e., for the NIM frequency there was the absence of reflection not for any direction.

5. Acknowledgements
The work was partially financially supported by the Government of the Russian Federation (grant 074-U01), by the Ministry of Education and Science of the Russian Federation (GOSZADANIE 2014/190, Projects No 14.Z50.31.0031 and No. 1.754.2014/K), by grant MK-5001.2015.1 of the President of the Russian Federation.

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