Shrinkage vs. anti-shrinkage of the diffraction cone in the exclusive vector mesons production

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Abstract

We investigate the energy behavior of the diffraction cone in the exclusive vector meson production in diffractive DIS within the $k_t$-factorization approach. In our calculations, we make full use of fits to the unintegrated gluon structure functions extracted recently from experimental data on $F_{2p}$. Confirming early predictions, we observe that shrinkage of the diffraction cone due to the slope of the Pomeron trajectory is significantly compensated by the anti-shrinkage behavior of the $\gamma \rightarrow V$ transition. In order to match recent ZEUS data on the energy behavior of the diffraction slope, $\alpha'_{\text{eff}}(J/\psi, \text{exp.}) = 0.115 \pm 0.018 (\text{stat.})^{+0.008}_{-0.015} (\text{syst.}) \text{ GeV}^{-2}$, we had to use an input value $\alpha'_{\text{IP}} = 0.25 \text{ GeV}^{-2}$. We investigate the compensation effect in detail and give predictions for $Q^2$-dependence of the rate of cone shrinkage for different vector mesons.

1 Introduction

The exclusive production of light and heavy vector mesons in diffractive DIS

$$\gamma^{(*)} p \rightarrow V p$$

is an ideal testing ground for the color-dipole approach, and of the $k_t$-factorization approach. The two approaches are linked by the transverse Fourier-Bessel transform from the color dipole size space to the momentum space: the former is based on the color dipole cross section, while the key ingredient in the latter approach is the unintegrated gluon structure function $F(x_g, \vec{\kappa}^2)$ of the proton. Recently, the unintegrated gluon density was extracted from the experimental data on $F_{2p}$ and presented in a form of simple, ready-to-use parametrizations for $x_g < 0.01$ and throughout the entire phenomenologically relevant domain of gluon momenta $\vec{\kappa}$, [10]. These fits now allow one to put many of previous qualitative predictions of the color dipole or $k_t$-factorization approaches to quantitative grounds. For example, they have already been successfully used in calculation of (virtual) photoproduction of dijets off proton or nuclei, [11, 12]. In the vector meson production, these fits have helped understand the DGLAP factorization scale in the case of $\rho$ meson production, [9], confirming the earlier...
ideas of [3]. It seems now timely to reanalyze another issue that critically depends on
the properties of the gluon content of the proton, namely, the shrinkage of the diffraction
cone in the vector meson production.

In simple Regge-type models with an approximately linear Pomeron trajectory, the
(momentum transfer) \( t \)-dependence of the intercept

\[
\frac{d\sigma}{dt} \propto \left( \frac{W}{W_0} \right)^{4[\alpha_{\text{IP}}(t)-1]}, \quad \alpha_{\text{IP}}(t) = \alpha_{\text{IP}}(0) + \alpha'_{\text{IP}} t, \quad (2)
\]

forces growth of the slope \( b \) of the diffraction cone

\[
b(W) = b(W_0) + 4\alpha'_{\text{IP}} \log \left( \frac{W}{W_0} \right), \quad (3)
\]

the phenomenon called the shrinkage of the diffraction cone. In QCD, the Regge limit is
described by the BFKL equation [13]. However, in the fixed coupling constant regime,
the BFKL equation lacks any intrinsic dimensional scale and leads to the fixed value of
the intercept. Inclusion of the running coupling constant splits the fixed cut on the \( j \)
plane into an infinite sequence of moving Regge poles [14], each of them having its own
non-zero \( \alpha' \). Within the color dipole BFKL approach [15], the value of \( \alpha'_{\text{IP}} \) was obtained
from the solution of color dipole BFKL equation for the slope [16]; its value was shown
to be nonzero [17] and to depend both on the energy and the size of the color dipole. At
very high energies, when the BFKL asymptotics would fully develop and the saturation
effects would not yet come into play, the slope of the Pomeron trajectory would tend
to a certain constant value independent of the dipole size. This asymptotic value is
governed by the gluon screening radius and in [16], under certain initial conditions, it
was estimated to be \( \alpha'_{\text{IP}}(\text{asymp.}) = 0.072 \text{ GeV}^{-2} \). At HERA, however, we still reside in
the subasymptotic region, and the values of \( \alpha'_{\text{IP}} \) can be still \( W \) and \( Q^2 \) dependent.

If we now return to the process (1), we find that the shrinkage of the cone due to
the non-zero \( \alpha'_{\text{IP}} \) is only a part of the truth. Thanks to the factorization properties,
the diffraction slope in the vector meson production can be decomposed into the beam,
target and exchange contributions. A detailed analysis of each contribution performed
in [18] showed that the \( \gamma \to V \) transition (beam) contribution might possess substantial
subasymptotic behavior. This energy behavior was expected to be of the anti-shrinkage
type, i.e. the corresponding slope contribution was predicted to decrease with energy
growth. This partially compensates the shrinkage of the exchange contribution to the
slope, and, therefore, leads to the effective rate of the cone shrinkage observed in a vector
meson \( V \) production \( \alpha'_{\text{eff}}(V) \) smaller than \( \alpha'_{\text{IP}} \).

Quantifying this compensation requires the knowledge of the gluon density in the
soft region\(^1\). At the time of publication [18], it was known rather poorly. Now, thanks to
the fits obtained in [10], we have a much better understanding of how the gluon density
behaves in the soft region. Therefore, it is possible now to reanalyze the anti-shrinkage
properties of \( \gamma \to V \) transition and check whether this compensation effect is important.

This is done in the present work. On the basis of the \( k_t \)-factorization approach, we
calculate the differential cross section of the exclusive vector meson production in diffrac-
tive DIS. We observe that the compensation effect is very important numerically. As we
show below, the input parameter \( \alpha'_{\text{IP}} = 0.25 \text{ GeV}^{-2} \) leads to the effective rate of the

\(^1\)To be accurate, one should understand the words “soft gluon density” only as “the dipole-proton interaction
in the soft region, parametrized in terms of gluon density”.

2
cone shrinkage in J/ψ photoproduction production $\alpha'_\text{eff} \approx 0.12 \text{ GeV}^{-2}$. This reduction implies that much care should be taken when the vector meson production results are interpreted as a direct probe of the properties of the Pomeron.

The structure of this paper is following. In Section 2 we briefly remind the basic formulae of the $k_t$-factorization approach to the calculation of the vector meson production in diffractive DIS. By performing the small-$t$ expansion of a generic helicity-conserving amplitude, we qualitatively study different contributions to the diffraction slope and discuss the origin of the anti-shrinkage properties of the $\gamma \to V$ transition. We then proceed to numerical results, which are presented in Section 3. These results are discussed in Section 4, and Section 5 contains conclusions of this work.

2 The $k_t$-factorization predictions for the diffraction slope: a qualitative analysis

2.1 The basic amplitude

The basic formulae for the vector meson production within the $k_t$-factorization approach are well known (see details in [8]). We denote the quark and gluon loop transverse momenta and the momentum transfer by $\vec{k}$, $\vec{\kappa}$, and $\vec{\Delta}$, respectively (here, vector sign denotes transverse vectors). The fraction of the photon lightcone momentum carried by the quark is denoted by $z$, while the fractions of the proton light cone momentum carried by the two gluons are $x_1$ and $x_2$. With this notation, the imaginary part of the amplitude of reaction (1) can be written in a compact form:

$$ImA = s c V \sqrt{4\pi \alpha_{em}} \frac{d^2\vec{k}}{\kappa^4} \alpha_S(q^2) F(x_1, x_2, \vec{k}_1, \vec{k}_2) \int \frac{dz d^2\vec{\kappa}}{z(1-z)} \psi^*_V(z, \vec{k}) \cdot I(\lambda_V, \lambda_\gamma).$$

(4)

where $\vec{\kappa}_{1,2} = \vec{\kappa} \pm \frac{1}{2} \vec{\Delta}$. The helicity-dependent integrands $I(\lambda_V, \lambda_\gamma)$ have form

$$I^S(L, L) = 4QM z^2(1-z)^2 \left[ 1 + \frac{(1-2z)^2}{4z(1-z)} \frac{2m}{M+2m} \right] \Phi_2;$$

$$I^S(T, T) = (\vec{e}\vec{V}^*)(m^2\Phi_2 + (\vec{k}\vec{\Phi}_1)) + (1-2z)^2(\vec{k}\vec{V}^*)(\vec{e}\vec{\Phi}_1) \frac{M}{M+2m} - (\vec{e}\vec{k})(\vec{V}^*\vec{\Phi}_1) + \frac{2m}{M+2m}(\vec{k}\vec{e})(\vec{V}^*)\Phi_2;$$

$$I^S(L, T) = 2Mz(1-z)(1-2z)(\vec{e}\vec{\Phi}_1) \left[ 1 + \frac{(1-2z)^2}{4z(1-z)} \frac{2m}{M+2m} \right]$$

$$- \frac{Mm}{M+2m}(1-2z)(\vec{e}\vec{k})\Phi_2;$$

$$I^S(T, L) = -2Qz(1-z)(1-2z)(\vec{V}^*\vec{k}) \frac{M}{M+2m}\Phi_2,$$

(5)

where

$$\Phi_2 = -\frac{1}{(\vec{r} + \vec{\kappa})^2 + \varepsilon^2} - \frac{1}{(\vec{r} - \vec{\kappa})^2 + \varepsilon^2} + \frac{1}{(\vec{r} + \vec{\Delta}/2)^2 + \varepsilon^2} + \frac{1}{(\vec{r} - \vec{\Delta}/2)^2 + \varepsilon^2};$$

3
\[ \Phi_1 = -\frac{\vec{r} + \vec{k}}{(\vec{r} + \vec{k})^2 + \varepsilon^2} - \frac{\vec{r} - \vec{k}}{(\vec{r} - \vec{k})^2 + \varepsilon^2} + \frac{\vec{r} + \vec{\Delta}/2}{(\vec{r} + \vec{\Delta}/2)^2 + \varepsilon^2} + \frac{\vec{r} - \vec{\Delta}/2}{(\vec{r} - \vec{\Delta}/2)^2 + \varepsilon^2}, \]

with \( \vec{r} \equiv \vec{k} - (1 - 2z)\vec{\Delta}/2 \) and \( \varepsilon^2 = z(1 - z)Q^2 + m_q^2 \). Finally, the strong coupling constant is taken at \( q^2 \equiv \max[\varepsilon^2 + \vec{k}^2, \vec{r}^2] \).

In the absence of \( \Delta - \vec{k} \) correlations, and for a very asymmetric gluon pair, the off-forward gluon structure function \( F(x_1, x_2, \vec{k}_1, \vec{k}_2) \) that enters (4) can be approximately related to the forward gluon density \( F(x_g, \vec{k}) \) via

\[ F(x_1, x_2 \ll x_1, \vec{k}_1, \vec{k}_2) \approx F(x_g, \vec{k}) \cdot \exp \left( -\frac{1}{2}b_{\text{IP}} \vec{\Delta}^2 \right). \]

Here \( x_g \approx 0.41x_1 \); the coefficient 0.41 is just a convenient representation of the off-forward to forward gluon structure function relation found in [19]. Numerical parametrizations of the forward unintegrated gluon density \( F(x_g, \vec{k}) \) for any practical values of \( x_g \) and \( \vec{k}^2 \) can be found in [10]. The slope \( b_{\text{IP}} \) contains contributions from the proton impact factor as well as from the Pomeron exchange; experimentally, it can be accessed in the high-mass elastic diffraction.

The vector meson wave function \( \psi_V(z, \vec{k}) \) describes the projection of the \( q\bar{q} \) pair onto the physical vector meson. It is normalized so that the forward value of the vector meson formfactor is unity, and the free parameters are chosen to reproduce the experimentally observed value of the vector meson electronic decay width \( \Gamma(V \rightarrow e^+e^-) \). In what concerns the shape of the radial wave function, we followed a pragmatic strategy. We took a simple Ansatz for the wave function, namely, the oscillator type wave function and performed all calculations with it. In order to control the level of uncertainty, introduced by the particular choice of the wave function, we redid the calculations with another wave function Ansatz, namely, the Coulomb wave function, and compared the results. Since these two wave functions represent the two extremes (very compact and very broad wave functions that still lead to the same value of the electronic decay width), the difference observed should give a reliable estimate of the uncertainty. This difference is typically given by factor of 1.5 for absolute values of the cross sections, while in the observables that involve ratios of the cross sections (such as slopes, intercepts, etc.) this uncertainty is reduced. Details can be found in [8] [20].

Note also that when deriving (5), we treated the vector mesons as \( 1S \) wave states and used the pure \( S \)-wave spinorial structure \( S^\mu \) instead of \( \gamma^\mu \), [21].

### 2.2 The contributions to the diffraction slope

The main feature of the \(|t| \equiv \vec{\Delta}^2\)-dependence of the cross sections is a pronounced forward diffraction cone, which can be, at very small \( \vec{\Delta}^2 \), parametrized by a single slope parameter \( b \) (see Section 3.1 for a detailed discussion on the definition of the slope). Let us understand the various contributions to the slope by studying the small-\( t \) expansion of a typical amplitude. Due to factorization properties, one can approximately decompose the slope to the target, exchange and the projectile contribution, [18]:

\[ b = b_{\text{p} \rightarrow \text{p}} + b_{\text{IP}} + b_{\gamma^* \rightarrow V}. \]  

In principle, the presence of the helicity-flip amplitudes represents yet another source of the \( t \)-dependence. Since the single helicity-flip amplitudes are proportional to \( \sqrt{|t|} \), one can introduce correction to the slope as

\[ d\sigma/dt_{\text{non-flip}} + d\sigma/dt_{\text{flip}} = A(t) + B \cdot |t| \rightarrow A(t) \cdot e^{-b_{\text{flip}}|t|}. \]
However, since the helicity-flip amplitudes are small, this correction never exceeds 2% of the value of the slope.

The first term in (6) is an intrinsically soft quantity and is related to the wave function of the proton. It is not calculable within pQCD, but its magnitude can be estimated from the proton charge radius. In our calculations, we introduced the following elastic formfactor to the amplitudes

$$F(\Delta^2) = \frac{1}{(1 + \Delta^2/\Lambda^2)^2}; \quad \text{with } \Lambda = 1 \text{ GeV},$$

which leads to the proton impact factor contribution to the slope

$$b_{p\rightarrow p} = 4 \text{ GeV}^{-2}. \quad (8)$$

The second contribution in (6) arises from the $t$-dependence of the Pomeron intercept. In our calculations, it was parametrized as

$$b_{\text{IP}} = \alpha'_{\text{IP}} \log \left( \frac{x_0}{x_g} \right); \quad \alpha'_{\text{IP}} = 0.25 \text{ GeV}^{-2}, \quad x_0 = 3.4 \cdot 10^{-4}. \quad (9)$$

This dependence was ascribed both to hard and soft components of the unintegrated gluon structure function, see [10]; more sophisticated parametrizations can be put forth as well. This parametrization was obtained by the requirement that we describe well the recent ZEUS data on the shrinkage of $J/\psi$ photoproduction [22]. We underline that, due to the compensation effect to be explained in a minute, this $\alpha'_{\text{IP}} = 0.25 \text{ GeV}^{-2}$ reduced to $\alpha'_{\text{eff}} \approx 0.12 \div 0.13 \text{ GeV}^{-2}$ in the differential cross section, in agreement with the experimental data.

Finally, the third term in (6) originates from the photon to vector meson transition amplitude. This contribution possesses both $Q^2$ and energy dependence, which can be understood as follows.

We start with the amplitude $L \rightarrow L$ and consider function $\Phi_2$ at large enough values of $\varepsilon^2$ so that $\tilde{\kappa}^2$ can be safely neglected. Expand it at small $\tilde{\Delta}^2$ and average over all possible directions of $\tilde{\Delta}$. The result reads

$$\Phi_2 \approx \frac{2\tilde{\kappa}^2}{\varepsilon^2(\tilde{\kappa}^2 + \varepsilon^2)} - \frac{\tilde{\Delta}^2}{(\tilde{\kappa}^2 + \varepsilon^2)^3} \left[ \frac{1}{2}(\varepsilon^2 - \tilde{\kappa}^2) + 4[z^2 + (1 - z)^2]\tilde{\Delta}^2 \left( 1 + \frac{3\varepsilon^2}{4\varepsilon^2} + \frac{\tilde{\kappa}^2}{4\varepsilon^4} \right) \right].$$

To the leading log $Q^2$, $\tilde{\kappa}^2 \ll \varepsilon^2$, and one has

$$\Phi_2 \approx \frac{2\varepsilon^2}{\varepsilon^4} - 2(1 - 2z)^2 \frac{\tilde{\Delta}^2\varepsilon^2}{\varepsilon^6} - \frac{\tilde{\Delta}^2}{2\varepsilon^4}.$$

After performing the relevant integration, one obtains that the amplitude $L \rightarrow L$ is proportional to

$$\frac{2}{Q_L} G(x_g, Q_L^2) - \frac{2}{Q_L} G(x_g, Q_L^2) \cdot \eta_L \frac{\tilde{\Delta}^2}{Q_L^2} - \frac{\tilde{\Delta}^2}{2Q_L^2} \frac{F(x_g, \mu^2)}{\mu^2}.$$

Here $G(x_g, Q_L^2)$ is the conventional gluon density, $\eta_L = \langle (1 - 2z)^2 \rangle_L$ and $\mu^2$ is an appropriately defined soft scale. The relevant hard scale $Q_L^2$, which is linked to the scanning
radius of the color dipole approach \( r_s^2 \leftrightarrow 1/\sqrt{Q_s^2} \), was investigated in detail in [9]. Thus, one obtains the following contributions to the slope

\[
 b_{\gamma \to V} = \frac{2\eta}{Q_L} + \frac{1}{\mu^2} \frac{\mathcal{F}(x_g, \mu^2)}{G(x_g, \sqrt{Q_s^2} L)}.
\]

(11)

The first term in (11) is a perturbative contribution. At \( Q^2 = 0 \), it should be of the order of several \( \text{GeV}^{-2} \), but it quickly falls off with the \( Q^2 \) growth. The second contribution is a predominantly soft quantity, its dependence on \( Q^2 \) is weak.

The \( T \to T \) amplitude can be analyzed in a similar way, and the qualitative results of this analysis remain the same as for the longitudinal case. This is why we use the above qualitative results even when discussing the properties of the transverse amplitudes.

2.3 The compensation effect: sources of the cone anti-shrinkage

The total slope of the diffraction cone is given by the sum of all three contributions, (6). Since the target contribution to the slope is taken constant, the energy behavior of the slope originates only from \( b_{\text{IP}} \) and \( b_{\gamma \to V} \).

The exchange contribution to the diffraction slope, \( b_{\text{IP}} \), logarithmically grows with energy and leads to the well-known shrinkage of the diffraction cone. In a simple Regge-type models, the properties of the Pomeron coupling to hadrons are assumed to be energy independent, and this contribution is the only source of the energy dependence of the diffraction cone. In a more elaborate models, such as the \( k_t \)-factorization approach, the beam contribution to the diffraction slope, \( b_{\gamma \to V} \), also possesses the energy dependence.

On the basis of the above qualitative analysis, one can identify two distinct sources of this energy behavior. First, even within the leading log \( Q^2 \), the second term in (11) depends on energy. Indeed, both \( \mathcal{F}(x_g, \mu^2) \) and \( G(x_g, \sqrt{Q_s^2} L) \) rise with energy, but the exponents of their rise are different, see [10]. For \( \sqrt{Q_s^2} > 1 \text{ GeV} \), the integrated glue taken at \( \sqrt{Q_s^2} \) rises with energy faster than the differential glue at the soft scale \( \mu^2 \), and this contribution to the slope will decrease with energy rise. This source of the energy behavior of \( b_{\gamma \to V} \) was also discussed in [18].

The second source of the \( b_{\gamma \to V} \) energy behavior appears, if we consider (10) beyond the leading log \( Q^2 \). In this case denominators will contain \( \sqrt{Q_s^2} + \bar{\kappa}^2 \) instead of just \( \sqrt{Q_s^2} \).

Due to specific properties of the unintegrated gluon density, the typical values of \( \bar{\kappa}^2 \) grow with energy even at fixed \( \sqrt{Q_s^2} \). This is clear from Fig. 1, taken from [10], where we showed, in a single plot, how the unintegrated gluon density \( \mathcal{F}(x_g, \bar{\kappa}^2) \) changes with energy (or \( 1/x_g \)) growth. One sees that the relative weight of the large \( \bar{\kappa}^2 \) region increases as we move from \( x_g = 10^{-2} \) to \( x_g = 10^{-4} \). This proves that \( \sqrt{Q_s^2} + \bar{\kappa}^2 \) increases — and the contribution (11) to the slope again decreases — with the energy rise. This effect mostly relies on specific, yet unavoidable, properties of the gluon density.

Both sources of the energy behavior of the beam contribution to the slope are of the \textit{anti-shrinkage type}. They lead to the partial compensation of the diffraction cone shrinkage, and eventually result in \( \alpha'_{\text{eff}} < \alpha'_{\text{IP}} \).

It is interesting to note that one can, in principle, disentangle these two sources of the compensation effect. What one needs is the study of the \( k_t \)-factorization prediction for the ultrahigh energy behavior of \( \alpha'_{\text{eff}} \). Note that the second contribution to the compensation effect works at full strength only at \( x_g > 10^{-4} \). At smaller \( x_g \), i.e. at higher
energies, the $\vec{\kappa}$-shape of the unintegrated gluon density practically does not change, and the growth of average values of $\kappa^2$ with energy stops. Thus, in this region, the compensation effect should be only due to the first mechanism.

3 Numerical results

3.1 Definitions of the slope

Before comparing the results of the $k_t$-factorization predictions of the diffraction slope with the experimental data, we would like to discuss the definition of the slope itself. The literal definition of the local slope of the diffraction cone as a logarithmic derivative of the differential cross section is

$$b(t) = -\frac{d}{d|t|} \log \left( \frac{d\sigma}{d|t|} \right).$$  (12)

Since the differential cross section flattens as $|t|$ increases, the value of the local slope will decrease with $|t|$ growth. In Fig. 2 we show the $k_t$-factorization calculation of the local slope for the $J/\psi$ photoproduction within the region $|t| \leq 1$ GeV$^2$. 

Figure 1: The $\vec{\kappa}$-dependence of the unintegrated gluon density $F(x_g, \kappa^2)$ (fit D-GRV) for several values of $x_g$. The solid and dashed curves correspond, respectively, to the full gluon density and to the soft contribution only.
Figure 2: The momentum transfer dependence of the local slope \((1 2)\) for the photoproduction of \(J/\psi\) meson. The solid and dashed curves correspond to total energies of the \(\gamma p\) collision equal to 50 GeV and 200 GeV, respectively.

The values of the diffraction slope obtained in the experiment are often the results of exponential law fits to the measured differential cross section performed within a certain \(t\) interval. This procedure approximately corresponds to a finite-difference slope defined as

\[
b(t_1, t_2) = \frac{1}{|t_2| - |t_1|} \left( \log \left| \frac{d\sigma}{dt} \right| |t_1| - \log \left| \frac{d\sigma}{dt} \right| |t_2| \right),
\]

(13)

where \(t_1\) and \(t_2\) define the region of the experimental fit. When comparing our predictions with such data, we will use precisely this definition of the slope.

In literature, other definitions of the slope can be encountered, such as \(\frac{1}{\sigma} \frac{d\sigma}{dt}|_{t=0}\) or \(1/\langle |t| \rangle\). If the differential cross section were a pure exponential law, all these definitions would lead to the same values. However, in a realistic situation, the offset among them can reach \(\sim 1 \pm 2 \text{ GeV}^{-2}\). This should be kept in mind when comparing different results of the slope.

3.2 The energy dependence of the slope parameter

We start our analysis with the \(J/\psi\) photoproduction. The local slope parameter, predicted from the \(k_t\)-factorization approach, has already been shown in Fig. 2 within the region \(|t| \leq 1 \text{ GeV}^2\) for two values of the \(W_{\gamma p}\). One observes a steady growth of the slope as the energy increases, which leads to the shrinkage of the diffraction cone. Motivated by the Regge-model considerations, this growth is usually described by the following law:

\[
b(W) = b(W_0) + 4\alpha_{eff}' \log \left( \frac{W}{W_0} \right).
\]

(14)

The quantity \(\alpha_{eff}'\), which we will call the rate of the diffraction cone shrinkage, is often assumed to be equal to the slope of the Pomeron trajectory \(\alpha_{IP}'\). However, as we argued
in the previous section, there are grounds to expect that these two will differ from each other.

The rate of the shrinkage $\alpha'_{\text{eff}}$ can be analyzed quantitatively on a plot of $b$ as a function of energy $W$. As can be seen from Fig. 2, the rate of the cone shrinkage is roughly independent of the value of $|t|$, therefore one can expect that different definitions of the slope will still produce similar $\alpha'_{\text{eff}}$. In Fig. 3 we plotted the $k_t$-factorization calculations of the energy dependence of the diffraction slope $b(W)$ in the $J/\psi$ photoproduction. In order to have control on the level of uncertainty introduced by the wave function Ansatz, we calculated $b(W)$ for both the oscillator and the Coulomb wave functions.

![Effective slope, b(W)](image)

**Figure 3:** The energy dependence of the diffraction slope in $J/\psi$ photoproduction. Experimental data points are from ZEUS [22].

The same Figure contains also the recent experimental data from ZEUS [22]. Keeping in mind the discussion of the previous subsection, we attempted in our calculations to match the experimental procedure of the diffraction slope evaluation. The $k_t$-factorization values of the slopes were calculated according to (13) with $|t_1| = 0.1$ GeV$^2$ and $|t_2| = 0.7$ GeV$^2$. As we mentioned above, it is precisely these $J/\psi$ photoproduction experimental data that we use to fix the two free parameters of the diffraction slope (9). Therefore the agreement between the curves and the data points is nothing else but just the double-check of the consistency of our calculations.

As can be seen from this Figure, the $k_t$-factorization calculations do not predict $b(W)$ to be strictly linear function of log($W$), as at the low energy end of the plot the curves flatten out. This is a consequence of the fact that the compensation effect discussed...
above can be well energy dependent (see also next subsection). However, at \( W \gtrsim 70 \) GeV, the approximate linearity is indeed observed. Note also that \( J/\psi \) photoproduction at low energies, \( W \lesssim 25 \) GeV, corresponds to \( x_g \gtrsim 0.01 \). This is the very edge of the \( x_g \) region the gluon density fits were devised for. We prefer to eliminate potential problems with applicability of the gluon density parametrizations and focus on region \( W \geq 50 \) GeV. If we now evaluate the shrinkage of the cone between \( W = 50 \) GeV and \( W = 250 \) GeV, we find the \( k_t \)-factorization values

\[
\alpha'_{\text{eff}} \approx \begin{cases} 
0.127 \text{ GeV}^{-2} & \text{for the oscillator wave function}, \\
0.121 \text{ GeV}^{-2} & \text{for the Coulomb wave function},
\end{cases}
\tag{15}
\]

which are, of course, in agreement with the experimentally measured value \[\alpha'_{\text{eff}}(\text{exp.}) = 0.115 \pm 0.018(\text{stat.})^{+0.008}_{-0.015}(\text{syst.}) \text{ GeV}^{-2}.\] \( \tag{16} \)

The key observation here is that the \( \alpha'_{\text{eff}} \) value predicted by the \( k_t \)-factorization is significantly less than the input parameter \( \alpha'_{\text{IP}} = 0.25 \) GeV\(^{-2}.\)

![Effective Interception vs. Energy Growth](image)

**Figure 4**: The \( t \)-dependence of the effective intercept \( \alpha_{\text{eff}}(t) \) calculated from the energy growth of the differential cross section of the \( J/\psi \) photoproduction between points \( W = 50 \) GeV and \( W = 250 \) GeV. Experimental data points are from ZEUS,[22].

Another look at how the diffraction cone behaves with the energy growth is given by the energy rise of the differential cross section itself. Introducing the effective intercept
for the differential cross section

\[
\frac{d\sigma}{dt}|_W = \frac{d\sigma}{dt}|_{W_0} \cdot \left( \frac{W}{W_0} \right)^{4[\alpha'_{\text{eff}}(t) - 1]},
\]

one can study how this intercept changes with \( t \). The results of this study, together with ZEUS experimental data [22], are shown in Fig. 4. Although the experimental data were available down to \( |t| = 1.34 \text{ GeV}^2 \), we limited ourselves only to \( |t| < 1 \text{ GeV}^2 \) region. Within this region, the \( k_t \)-factorization calculations based on either of the two wave functions are in a qualitative agreement with the data. Besides, one sees that the shape of the vector meson wave function has rather minor effect on the diffraction cone shrinkage, and, in the subsequent plots we will give only results based on the oscillator wave function.

3.3 The \( Q^2 \) and \( W \) behavior of the compensation effect

It is known experimentally, and it follows from our analysis in Section 2 as well, that the values of the diffraction slope \( b \) are sensitive to the virtuality \( Q^2 \) and to the mass of the vector meson produced \( m_V \). It is, therefore, interesting to check how the values of \( Q^2 \) and \( m_V \) will affect the rate of the diffraction cone shrinkage \( \alpha'_{\text{eff}} \).

![Graph showing the rate of cone shrinkage for \( \gamma p \rightarrow \rho p \) and \( \gamma p \rightarrow J/\psi p \) as functions of \( Q^2 \).](image)

Figure 5: The effective rate of the shrinkage of the diffractive cone \( \alpha'_{\text{eff}} \) for \( \rho \) meson and \( J/\psi \) meson production as functions of \( Q^2 \). The \( k_t \)-factorization results are calculated between \( W = 50 \text{ GeV} \) and \( W = 250 \text{ GeV} \), and for the oscillator wave function only. The photoproduction data points are from ZEUS, [23, 22].

In Fig. 5 we show the \( k_t \)-factorization predictions of the \( Q^2 \) behavior of \( \alpha'_{\text{eff}}(\rho) \) and \( \alpha'_{\text{eff}}(J/\psi) \). We also show the two ZEUS data points available [23, 22], both of
which correspond to the photoproduction limit. What one sees on both plots is a slight decrease of $\alpha'_\text{eff}$ from $\approx 0.13$ GeV$^{-2}$ down to $\approx 0.10$ GeV$^{-2}$, which takes place at typical $Q^2 \sim m_V^2$. This rather weak sensitivity of $\alpha'_\text{eff}$ to the values of $Q^2 + m_V^2$ is in accordance with [10], where, too, the value of $\alpha'_\text{eff}$ was found to depend mostly on energy but not on the scanning radius, which is related to the virtuality, or to be more precise, to the hard scale $Q^2$.

As mentioned in the previous section, the ultrahigh energy behavior of $\alpha'_\text{eff}$ can help disentangle the two sources of the compensation effect. If $\gamma p$ collision energy grows to high enough values, such that $x_g \lesssim 10^{-4}$ (which corresponds, for $J/\psi$ photoproduction, to $W > 300$ GeV), then the second source of the anti-shrinkage behavior discussed in Sect. 2.3 should get suppressed. As a result, what one probes in this region is only the first contribution to the compensation effect. The overall compensation should, therefore, weaken.

Numerical study shows that at $W \gg 100$ GeV the effective rate of the diffraction cone shrinkage $\alpha'_\text{eff}$ significantly rises, in accordance with expectations. At $W = 1000$ GeV and $W = 4000$ GeV, $\alpha'_\text{eff}$ rises up to 0.19 GeV$^{-2}$ and 0.22 GeV$^{-2}$, respectively, which should be compared to [15] at HERA energies. This is, first, in agreement with our expectation that the compensation effect should decrease with energy growth. Second, this results can be treated as an evidence that, at HERA energies, both sources of the anti-shrinkage effect are of comparable importance.

We would like to stress that this analysis is just a self-contained investigation of the energy behavior of the compensation effect, which is the property the $k_t$-factorization approach. We do not attempt to predict the energy behavior of $\alpha'_\text{eff}$. Such a prediction might be feasible only with a reliable information on the energy behavior of the input parameter $\alpha'_\text{IP}$. In our calculations, we used constant value of $\alpha'_\text{IP} = 0.25$ GeV$^{-2}$ for all energies, which might be very inaccurate beyond the HERA energy range.

4 Discussion

4.1 Is this the Pomeron trajectory?

The experimental results presented in Fig. 4 were explicitly interpreted in [22] as a measurement of the Pomeron trajectory. Here, we argue that this interpretation is, at least, not that straightforward.

The most direct argument comes from the value of $\alpha'_\text{eff}$. The numerical calculations show that the rate of the diffraction cone shrinkage $\alpha'_\text{eff}$ predicted by the $k_t$-factorization approach is significantly less than the input parameter $\alpha'_\text{IP}$, which quantifies the $t$-dependence of the intercept of the underlying Pomeron exchange. Therefore, there exists a mechanism of compensation of the cone shrinkage. The qualitative analysis of Section 2 suggests that the energy decreasing contribution [1] of the $\gamma \to V$ transition to the overall diffraction slope is in charge of this compensation effect. In Sect. 2.3 we pointed out two sources of this anti-shrinkage behavior, and the subsequent analysis gave evidences that both of them were equally important.

In order to make sure that there is no other source for anti-shrinkage but the $\gamma \to V$ transition, we made a double-check and switched off the $\Delta$-dependence of integrands $I^S(\lambda_V, \lambda_\gamma)$ in [5]. This made $\alpha'_\text{eff}$ jump up to $\approx 0.24$ GeV$^{-2}$, which is very close to the input value $\alpha'_\text{IP} = 0.25$ GeV$^{-2}$. This proves that it is precisely the $\gamma \to V$ transition vertex that causes such a strong reduction of the rate of the diffraction cone shrinkage.
in vector meson production.

Thus, we are led to a conclusion that, first, the experimentally measured values of $\alpha'_{\text{eff}}$ should not be interpreted directly as the slope of the Pomeron trajectory. Second, as we checked, $\alpha'_{\text{eff}} - \alpha'_{\text{IP}}$ relation is rather robust within the $k_t$-factorization scheme,

$$\alpha'_{\text{eff}} \approx \alpha'_{\text{IP}} - 0.13 \text{ GeV}^{-2},$$

and the experimental data shown in Fig. 4 might be in fact an evidence that the true value of the slope of the Pomeron trajectory, as measured in the $J/\psi$ photoproduction, is around $0.25 \text{ GeV}^{-2}$.

It is interesting to note that this value is close to what is usually believed to be the soft $^2\alpha'$, see [16]. One can also quote a similar value measured in the elastic hadronic interaction at high energies [27, 28], but, in its own turn, the single Pomeron exchange in these reactions receive sizable absorption corrections [28] and, therefore, these results should not be interpreted straightforwardly as well. In any case, due to the strong compensation effect, we think that it would be premature to conclude that the measured value of $\alpha'_{\text{eff}}$ in $J/\psi$ photoproduction is inconsistent with the soft Pomeron.

4.2 Comments on other works

We would like to mention that our approach to $\alpha'_{\text{eff}}$ is different from approaches of [25] and [26]. In [25] the differential cross section of vector meson production was fitted to the experimental data with no attempt to disentangle the real physics that leads to this form of the cross section. An explicitly non-linear Pomeron trajectory was introduced, which, in constrast to our predictions, leads to a strong $t$-dependence of $\alpha'$. Although in this model $\alpha'(t = 0) = 0.25 \text{ GeV}^{-2}$, it becomes twice smaller at $|t| = 12m^2_\pi \approx 0.24 \text{ GeV}^2$, which allows the authors of [25] to describe reasonably well the ZEUS data on shrinkage in $J/\psi$ photoproduction, [22]. It must be pointed out that this agreement is not surprising, since the fits to the cross section were derived precisely from the $J/\psi$ photoproduction data. By the construction of their model, the authors of [25] directly relate the observed shrinkage in vector mesons photoproduction to the Pomeron properties, $\alpha'_{\text{eff}} = \alpha'_{\text{IP}}$. At the end, authors state that they “have reached a deeper understanding of the properties of dipole Pomeron”, however, in the light of the present paper’s results, such statement looks rather questionable.

The authors of [26] worked in the color dipole formalism and, as can be expected from the early work [18], they should have observed the compensation effect studied in the present paper. When discussing $\alpha'$, they indeed mention a similar effect, but they estimate it to be very inessential numerically, contrary to the claim of our work. The origin of this discrepancy lies in the fact that the authors of [26] overlooked the main source of the energy dependence of the slope of $\gamma \to V$ transition and took only inessential part into account. Namely, in their phenomenological parametrization of the slope, the authors of [26] included only the first term in [11] (this is the $\langle b^2 \rangle$ term in the notation of [26]). The authors of [26] neither took into account the second term in [11], nor went beyond leading log $Q^2$ approximation by accounting for $\vec{r}^2$ in this term, and, as a result, missed the sizable anti-shrinkage effect discussed here. Therefore, the conclusion $\alpha'_{\text{eff}} \approx \alpha'_{\text{IP}}$ of [26] was misleading.

Ironically, several competing theoretical approaches agree on this, see also [24, 25, 26].
5 Conclusions

In this work we investigated, at the quantitative level, the energy behavior of the diffraction cone in the exclusive production of vector mesons. The work was conducted in the $k_t$-factorization scheme, closely related to the familiar color dipole formalism, and was based on recent fits to the unintegrated gluon density obtained in [10]. The fact that we do not devise models for the gluon content but instead heavily rely on the high-precision experimental data lends certain credence to the whole calculation.

Confirming the results of early work [18], we observed that the shrinkage of the diffraction cone, that could be expected from the Pomeron properties, is partially compensated by the anti-shrinkage behavior of the $\gamma \to V$ transition vertex. We pointed out two sources of this compensation effects, both of equal importance, and observed that this reduction of the cone shrinkage is very significant numerically. In order to reproduce the experimentally observed value of $\alpha'_\text{eff}(J/\psi) \approx 0.115 \text{ GeV}^{-2}$, we had to take $\alpha'_{\text{IP}} = 0.25 \text{ GeV}^{-2}$ as an input parameter. This value turned out to be intriguingly close to what is usually believed to be $\alpha'_{\text{IP}}$ of the soft Pomeron.

This observation casts doubts on too straightforward interpretations of the experimental data on diffraction cone shrinkage as a direct measurement of the slope of the Pomeron trajectory.

When studying the $Q^2$-dependence of $\alpha'_\text{eff}$ within the $k_t$-factorization approach, we observed some decrease of $\alpha'_\text{eff}$ as we shifted from photoproduction limit to DIS. The exact numerical properties of this decrease depend on the particular choice of the definition of the diffraction slope. This should not be forgotten when one compares the results of the theoretical predictions with the data. We also analyzed the energy behavior of the compensation effect and observed that it weakens with energy growth, which agree with expectations based on the qualitative analysis. Unfortunately, at this stage we cannot predict the energy behavior of the $\alpha'_\text{eff}$ beyond the HERA energy range, since such a prediction requires understanding of the energy behavior of the input parameter $\alpha'_{\text{IP}}$.

In any case, the mere presence of the dependence of the compensation effect on kinematics proves that the anti-shrinkage effect is not universal. This, in turn, supports the understanding that the whole underlying picture of the vector meson production (the “real” Pomeron) does not correspond to any simple singularity on $j$ plane.

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