Trajectory tracking control based on disturbance observer for omnidirectional mobile robot

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Abstract. This brief focuses on the problem of trajectory tracking control of omnidirectional mobile robot with unknown external disturbances. It is assumed that the derivative of the external disturbances is bounded. The control scheme is divided into two parts. In the first part, the external disturbances are estimated by the disturbance observer. Furthermore, the feasibility of the disturbance observer is demonstrated. In the second part, we propose the trajectory tracking controller. Based up on Lyapunov function, the stability of the closed-loop system is analysed. Finally, simulations are given to show the validity of the results.

1. Introduction

In recent decade, more and more scholars have attached importance to the control problems of omnidirectional mobile robots [1-2]. The omnidirectional mobile robots can not only run flexibly in whatever direction you like, but also revolve around the center, which are quite suitable for working in narrow space.

In literature, some researchers have studied the omnidirectional mobile robot only considering kinematic model, and the dynamic model is not employed [3-4], just to name only a few. However, when the robot is moving under high payload, the control performance may not be satisfied as the result of the lack of robot dynamic parameters. Consequently, a lot of research on omnidirectional mobile robot has been investigated considering dynamical model. In [5] and [6], a robust output feedback scheme based on linear observer linear controller is proposed for output reference trajectory tracking of omnidirectional mobile robot. In [7], for an omnidirectional mobile robot, by combining kinematic controller with an integral sliding mode dynamic controller, a new tracking controller is designed to track desired trajectory at an expected speed. However, a common feature of the above two methods is that the natural dissipation of omnidirectional mobile robot dynamics is not considered. In [8], the model predictive control method was used to design the control law. However, the approaches men-
tioned above are about the problem of path following of omnidirectional mobile robot, not trajectory tracking control problem. Although the problem of trajectory tracking has been studied in [11], the external disturbances are not considered.

As is well known, there are many disturbances in the actual working environment of omnidirectional mobile robot. Recently, the use of disturbance observer has been received increasing attention [9-10]. In [9], a disturbance-observer-based sliding mode control approach has been presented to attenuate the mismatched uncertainties. In [10], the disturbance observer introduced in [9] is extended to enable the estimation of disturbances. Therefore, for the sake of improved robustness and disturbance attenuation, the disturbance observer is a good technique.

According to the previous research progress of omnidirectional mobile robot, this brief is motivated. We consider the problem of trajectory tracking control of omnidirectional mobile robot with external disturbances. We assumed that the first derivative of external disturbances is bounded. Firstly, the disturbance observer $F$ is designed, and the feasibility of which is verified. Then, a robust controller is proposed. Based up on the Lyapunov theory, the stability analysis of the controller design is given. In addition, simulation results and comparisons show that the trajectory tracking control problem of the omnidirectional mobile robot is better solved by using our proposed controller.

The rest of this brief is arranged as follows. In Section 2, the problem formulation and two assumptions are given. In Section 3, controller design and stability analysis are introduced. In Section 4, simulations are given to illustrate the validity of the proposed control scheme. In Section 5, this brief is concluded.

2. Problem formulation

As mentioned in [11], the dynamic model of three-wheeled omnidirectional mobile robot is as follows.

$$M\ddot{q} + C\dot{q} + D\dot{q} + F = \tau$$  \hspace{1cm} (1)

where $\dot{q} = [x \ y \ z]^T$ represents the position and orientation angle of robot, $\tau = Bu$ denotes the control torques, which is a virtual control input vector. And the actual control input vector $u = [u_1 \ u_2 \ u_3]^T$ denotes the input voltages of three driving motors. Here, $M$ is the inertia matrix. $C\dot{q}$ is the centripetal and Coriolis torques. $D\dot{q}$ denotes the friction force caused by viscous friction of the motors, gears and wheel shafts and so on. It refers to the force that reduces the total mechanical energy of the robot. $F$ represents the external disturbances. According to the reference [11], the expressions of $M$, $C$, $D$ and $B$ can be written in the following.

$$M = \begin{pmatrix}
\frac{3n^2I_0 + m}{2r^2} & 0 & 0 \\
0 & \frac{3n^2I_0 + m}{2r^2} & 0 \\
0 & 0 & \frac{3n^2I_0 + l_0^2}{r^2} + I_r
\end{pmatrix}, \quad C = \begin{pmatrix}
0 & \frac{3n^2I_0\theta}{2r^2} & 0 \\
\frac{3n^2I_0\theta}{2r^2} & 0 & 0 \\
\frac{3n^2I_0\theta}{2r^2} & 0 & 0
\end{pmatrix}, \quad D = \begin{pmatrix}
\frac{3n^2}{2r^2}(b_0 + k_\epsilon k_b) & 0 & 0 \\
0 & \frac{3n^2}{2r^2}(b_0 + k_\epsilon k_b) & 0 \\
0 & 0 & \frac{3n^2}{r^2}(b_0 + k_\epsilon k_b)
\end{pmatrix}, \quad B = \frac{nkr}{rR_v} \begin{pmatrix}
-\frac{1}{2} \cos \theta - \frac{\sqrt{3}}{2} \sin \theta & -\frac{1}{2} \cos \theta + \frac{\sqrt{3}}{2} \sin \theta & \cos \theta \\
-\frac{1}{2} \sin \theta - \frac{\sqrt{3}}{2} \cos \theta & -\frac{1}{2} \sin \theta + \frac{\sqrt{3}}{2} \cos \theta & \sin \theta \\
-\frac{1}{2} \sin \theta + \frac{\sqrt{3}}{2} \cos \theta & -\frac{1}{2} \sin \theta - \frac{\sqrt{3}}{2} \cos \theta & \sin \theta
\end{pmatrix}$$

where $L_0$ is the contact radius, $r$ is the wheel radius, $m$ represents the robot mass, $k_\epsilon$ is the motor
torque constant, \( k_b \) represents the motor back EMF constant, \( R_a \) means the motor armature resistance, \( n \) is the gear reduction ratio, \( I_v \) denotes robot moment of inertia, \( I_0 \) and \( b_0 \) denote the combined moment of inertia and the combined viscous friction coefficient of motor, gear train and wheel referred to the motor shaft.

For the design of the controller, the standard assumptions are introduced.

**Assumption 1:** The changing rate of the disturbances \( F \) is bounded, namely,
\[
\|F\| \leq \mu < \infty
\]  

**Assumption 2:** The desired trajectory \( q_d(t) = [x_d(t), y_d(t), \theta_d(t)]^T \) is smooth and bounded, which has bounded first and second time derivatives \( \dot{q}_d \) and \( \ddot{q}_d \).

## 3. Controller Design

For the system (1), the design of robust controller is divided into two parts. First of all, a disturbance observer is applied to estimate the disturbances \( F \) in real time and its rationality is proved. Secondly, a virtual controller is proposed to enable the omnidirectional mobile robot to track the desired trajectory accurately.

### 3.1 Disturbance Observer Design

In this part, we define state variables \( x_1 = q, \ x_2 = \dot{q} \). Then, the system (1) can be transformed into the form of state space.

\[
\begin{align*}
\dot{x}_1 &= x_2 \\
M \dot{x}_2 &= \tau - C x_2 - D x_2 - F
\end{align*}
\]

According to Equations (3)-(4) and reference [12], the disturbance observer is constructed as follows.

\[
\begin{align*}
\hat{F} &= \ddot{x} - K_0 M x_2 \\
\dot{x} &= -K_0 \ddot{x} + K_0 (\tau - C x_2 - D x_2 + K_0 M x_2)
\end{align*}
\]

where \( \hat{F} \) denotes the disturbances estimation.

Now, we define the estimation error \( \hat{\tilde{F}} \) as the following.

\[
\hat{\tilde{F}} = \hat{F} - F
\]

**Theorem 1:** Consider the dynamic model in Equation (1). Supposed that Assumption 1 is satisfied. Under the disturbance observer (5) and (6), the estimation error \( \hat{\tilde{F}} \) can be guaranteed to be exponentially converge to a ball. By properly choosing the matrix \( K_0 \), the boundedness of the estimation error \( \hat{\tilde{F}} \) can be arbitrarily small.

**Proof:** From (3), (4), (5) and (6), we have

\[
\hat{\tilde{F}} = \ddot{x} - K_0 M x_2 = -K_0 \ddot{x} + K_0 \dddot{x} + K_0 F = -K_0 \hat{\tilde{F}}
\]

Therefore, we can get the derivative of (7):

\[
\dot{\hat{\tilde{F}}} = \dot{\hat{F}} - \dot{\hat{\tilde{F}}} = -K_0 \hat{\tilde{F}} - \hat{\tilde{F}}
\]

Then, we choose the Lyapunov candidate function as:

\[
V_1 = \frac{1}{2} \hat{\tilde{F}}^T \hat{\tilde{F}}
\]

Based on (9), we take the derivative of (10), which yields
\[
\dot{V}_1 = \ddot{\hat{F}}^T \hat{F} = -K_0 \ddot{\hat{F}}^T \hat{F} - \ddot{\hat{F}}^T \hat{F} \leq -K_0 \ddot{\hat{F}}^T \hat{F} + \varepsilon_1 \ddot{\hat{F}}^T \hat{F} + \frac{1}{4\varepsilon_1} \hat{F}^T \hat{F}
\]

\[
\leq 2(\lambda_{\min}(K_0) - \varepsilon_1) \frac{1}{2} \ddot{\hat{F}}^T \hat{F} + \frac{\mu^2}{4\varepsilon_1} \leq -\alpha \dot{V}_1 + \delta
\]

where \( \alpha = 2(\lambda_{\min}(K_0) - \varepsilon_1) \), \( \delta = \frac{\mu^2}{4\varepsilon_1} \), \( \lambda_{\min}(K_0) \geq 0 \), and \( \lambda_{\min}(K_0) \) denotes the smallest eigenvalue of matrix \( K_0 \). For completeness, we solve the inequality (11). Multiplying (11) by \( e^{\alpha t} \) yields

\[
\frac{d}{dt}(V_1 e^{\alpha t}) \leq \dot{\alpha} e^{\alpha t}
\]

Simultaneous integration on both sides of inequality, we can obtain

\[
V_1 \leq \frac{\delta}{\alpha} + (V_1(0) - \frac{\delta}{\alpha}) e^{-\alpha t}
\]

Therefore, from (13), it is easy to show that \( V_I \) is bounded and exponentially converges in a sphere. Consequently, from (10), the boundedness of the estimation error \( \hat{F} \) can be guaranteed and the estimation error exponentially converges in a sphere, which is centered at the origin with the radius \( R_F = \frac{1}{4 \varepsilon_1 (\lambda_{\min}(K_0) - \varepsilon_1)} \). Furthermore, it can be seen that increasing the value of \( \lambda_{\min}(K_0) \) can reduce the radius of \( R_F \).

Thus, we complete the proof of Theorem 1.

**Remark 1:** Specially, if \( \mu = 0 \), namely, the disturbances \( F \) are an unknown constant vector. It is obvious that the proposed disturbance observer can make the estimation error exponentially converge to zero.

### 3.2 The Controller Design

In this subsection, we concentrate on solving the trajectory tracking control problem of omnidirectional mobile robot with unknown external disturbances \( F \). As mentioned before, the disturbances \( F \) can be estimated by the disturbance observer \( \hat{F} \). Hence, we should find a control input which can make the actual output \( q = [x \ y \ \theta]^T \) track the desired output \( q_d = [x_d \ y_d \ \theta_d]^T \).

Firstly, we define the tracking error and the auxiliary variable as:

\[
e = q - q_d = \begin{bmatrix} x - x_d \\ y - y_d \\ \theta - \theta_d \end{bmatrix}
\]

\[
r = \dot{e} + \Lambda e
\]

where \( \Lambda \) is a positive definite matrix.

Then, from (1), (14) and (15), it is derived that

\[
Mr = \tau - C\ddot{q} - D\dot{q} - F + M\dot{\dot{e}} - M\dot{\dot{q}}_d
\]

Therefore, the controller is proposed as follows.

\[
\tau = C\ddot{q} + D\dot{q} + \hat{F} + M\dot{\dot{q}}_d - M\dot{\dot{e}} - K_1 e - K_2 r
\]

As mentioned before, \( \tau \) is virtual control input. Thus, the actual control input \( u \) can be expressed below.

\[
u = B^{-1} \tau = B^{-1} (C\ddot{q} + D\dot{q} + \hat{F} + M\dot{\dot{q}}_d - M\dot{\dot{e}} - K_1 e - K_2 r)
\]

where \( K_1 \) and \( K_2 \) are positive definite constant matrices.

**Theorem 2:** Consider the dynamic model in Equation (1). Supposed that Assumptions 1-2 are satisfied. Under the proposed actual tracking controller \( u \), described by (18), the tracking error is bounded and exponentially converges to a ball. Furthermore, by appropriately choosing the relative parameters, the boundedness of the tracking error \( e \) can be arbitrarily small.
Proof: The Lyapunov candidate function is considered as follows.

\[ V_2 = V_1 + \frac{1}{2} e^T K_1 e \] (19)

Then, in terms of (16) and (17), we can obtain the time derivative of the function \( V_2 \).

\[ \dot{V}_2 = \dot{V}_1 + \frac{1}{2} e^T K_1 \dot{e} = \dot{V}_1 + r^T \dot{e} F - r^T K_1 e - r^T K_2 r + e^T K_1 \dot{e} \] (20)

From the complete square inequality and (11), it is hold that

\[ \delta \rho \leq \lambda_{\min}(K_0) - e_1 - e_2, -2(\lambda_{\min}(K_2) - \frac{1}{4\epsilon_2}) \frac{1}{2} r^T r - 2\lambda_{\min}(\lambda^T K_1) \frac{1}{2} e^T e \] (21)

where

\[ \rho = \min \left\{ \frac{2(\lambda_{\min}(K_0) - e_1 - e_2)}{\lambda_{\max}(M)}, \frac{2\lambda_{\min}(\lambda^T K_1)}{\lambda_{\max}(K_1)} \right\} \] (22)

here, \( \lambda_{\min}(K_0) \), \( \lambda_{\min}(K_2) \), and \( \lambda_{\min}(\lambda^T K_1) \) represent the smallest eigenvalues of \( K_0, K_2 \) and \( \lambda^T K_1 \), respectively. \( \lambda_{\max}(M) \) denotes the largest eigenvalue of \( M \).

Similar to (12) and (13), we can obtain

\[ V_2 \leq \frac{\delta}{\rho} + (V_2(0) - \frac{\delta}{\rho}) e^{-\rho t} \] (23)

Therefore, from (23), it is easy to know that \( V_2 \) is bounded and exponentially converges to a ball, which is centered at the origin with radius \( R_{V_2} = \frac{\delta}{\rho} \). Furthermore, it is easy to hold that the tracking error \( e \) is also bounded and exponentially converges to a sphere. Moreover, we can reduce the convergence domain of the tracking error by increasing the values of \( \lambda_{\min}(K_2) \) or \( \lambda_{\min}(\lambda^T K_1) \).

Thus, we complete the proof of Theorem 2.

4. Simulation results

In this section, simulations are presented to illustrate the effectiveness of the proposed method. Here, comparing with active disturbance rejection controller in reference [13], the proposed controller is applied to make the omnidirectional mobile robot to track a given circle trajectory.

Similar to reference [11], the robot physical parameters are chosen as follows:

\[
\begin{align*}
    n &= 185.7 \quad m = 33kg \quad L_0 = 0.1915m \quad I_1 = 1.35kgm^2 \quad I_0 = 3.15 \times 10^{-5} kgm^2 \quad k_r = 0.0292N.m/A \\
    k_b &= 0.00304V.s/rad \quad R_a = 2.53\Omega \quad r = 0.06m \quad b_0 = 1 \times 10^{-4} Nms/rad
\end{align*}
\]

In the simulation, we take a test that the robot with uniform velocity tracks a circle trajectory with a radius of 2m as an example.

The desired circle trajectory and external disturbances are given as follows:

\[
\begin{align*}
    x &= 2 \cos \left( \frac{\pi t}{10} \right), \quad y = 2 \sin \left( \frac{\pi t}{10} \right), \quad 0 \leq t \leq 20s. \\
    \theta &= \frac{\pi t}{10}
\end{align*}
\]

\[ F = \begin{bmatrix} 0.5 \cos \left( \frac{\pi}{10} t \right) + 0.5 \sin \left( \frac{\pi}{10} t \right) \\ 0.5 \cos \left( \frac{\pi}{20} t \right) + 0.5 \sin \left( \frac{\pi}{20} t \right) \end{bmatrix} \]

We set the initial position and orientation angle of robot as \( \begin{bmatrix} x(0) \ y(0) \ \theta(0) \end{bmatrix}^T = \begin{bmatrix} 2 \ 0 \ 0 \end{bmatrix}^T \).

In the case of tracking the circle trajectory, the control parameters are chosen as
Choose the control parameters of active disturbance rejection controller as

\[
\begin{align*}
K_0 &= \begin{pmatrix} 9 & 0 & 0 \\ 0 & 9 & 0 \\ 0 & 0 & 9 \end{pmatrix}, &
K_1 &= \begin{pmatrix} 24 & 0 & 0 \\ 0 & 24 & 0 \\ 0 & 0 & 24 \end{pmatrix}, &
K_2 &= \begin{pmatrix} 72 & 0 & 0 \\ 0 & 72 & 0 \\ 0 & 0 & 72 \end{pmatrix}, &
\Lambda &= \begin{pmatrix} 24 & 0 & 0 \\ 0 & 24 & 0 \\ 0 & 0 & 24 \end{pmatrix}
\end{align*}
\]

Comparing with the active disturbance rejection controller, the tracking performance of our proposed control scheme is better. Also, the tracking error in each direction of our proposed controller is smaller in Fig.1. In addition, the curves of the actual control inputs of two controllers are shown in Fig.2. It is obvious that the disturbances estimation is better in Fig.3.

**Fig.1** The trajectory tracking performance

**Fig.2** The trajectory of control inputs. **Fig.3** The disturbances estimation.

5. Conclusions

In this brief, the trajectory tracking control problem of the omnidirectional mobile robot with disturbances has been considered. Firstly, to estimate the unknown disturbances \( F \), the disturbance observer \( \hat{F} \) has been designed. Secondly, the virtual controller \( \tau \) has been proposed. By doing this, the expression for the actual controller \( u \) has been provided. Based up on the Lyapunov theory, the stability of the control system has been analyzed. Finally, simulation results show that the problem of trajectory tracking for the robot has been solved by the proposed controller.

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