Comparing omnidirectional reflection from periodic and quasiperiodic one-dimensional photonic crystals

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We determine the range of thicknesses and refractive indices for which omnidirectional reflection from quasiperiodic multilayers occurs. By resorting to the notion of area under the transmittance curve, we assess in a systematic way the performance of the different quasiperiodic Fibonacci multilayers.

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I. INTRODUCTION

Photonic crystals are periodically structured dielectric media possessing photonic band gaps: ranges of frequency in which strong reflection occurs for all angles of incidence and all polarizations. They then behave as omnidirectional reflectors, free of dissipative losses. Since the initial predictions of Yablonovitch [1] and John [2], photonic crystals have been attracting a lot of attention and a wide variety of applications have been suggested.

In the one-dimensional case, a photonic crystal is nothing more than a periodic dielectric structure. Bragg mirrors consisting of alternating low- and high-index layers constitute, perhaps, the archetypical example [4]. In particular, quarter-wave stacks (at normal incidence) are the most thoroughly studied in connection with omnidirectional reflection (ODR) [5, 6, 7, 8, 9, 10].

The introduction of Fibonacci multilayers by Kohmoto and coworkers [11] spurred the interest for both possible optical applications [12] and theoretical aspects of light transmission in aperiodic media [13, 14, 15, 16, 17]. In fact, the possibility of obtaining ODR in quasiperiodic Fibonacci multilayers has been put forward recently [18, 19, 20, 21, 22].

Underlying all these efforts a crucial question remains concerning whether quasiperiodic Fibonacci multilayers would achieve better performance than usual periodic ones. To answer such a fundamental question one first needs to quantify the idea of ODR performance in a unique manner that permits unambiguous comparison between different structures. Only quite recently a suitable figure of merit has been introduced: the area under the transmittance curve as a function of the incidence angle [23]. In this paper, we resort to this concept of area to rank in a consistent way the ODR characteristics of these quasiperiodic systems.

II. QUASIPERIODIC FIBONACCI MULTILAYERS

A Fibonacci system is based on the recursive relation \( S_0 = \{H\}, S_1 = \{L\} \) and \( S_j = S_{j-1}S_{j-2} \) for \( j \geq 2 \). Here \( H \) and \( L \) are defined as being two dielectric layers with refractive indices \((n_H, n_L)\) and thicknesses \((d_H, d_L)\), respectively. The material \( H \) has a high refractive index while \( L \) is of low refractive index. The number of layers is given by \( F_j \), where \( F_j \) is a Fibonacci number obtained from the recursive law \( F_j = F_{j-1} + F_{j-2} \), with \( F_0 = F_1 = 1 \).

In order to properly compare the optical response of these systems we will rely on the transfer-matrix technique. The transfer matrix \( M_j \) for the Fibonacci system \( S_j \) can be computed as [11]

\[
M_0 = M_H, \quad M_1 = M_L, \\
M_j = M_{j-1}M_{j-2}, \quad j \geq 2.
\] (2.1)

The transfer matrix for the single layer \( H \) is

\[
M_H = \begin{pmatrix} \cos \beta_H & q_H \sin \beta_H \\ \frac{1}{q_H} \sin \beta_H & \cos \beta_H \end{pmatrix},
\] (2.2)

and a analogous expression for \( L \). Here \( \beta_H = (2\pi/\lambda) n_H d_H \sin \theta_H \) is the layer phase thickness, \( \theta_H \) being the angle of refraction, which is determined by Snell law. The wavelength in vacuum of the incident radiation is \( \lambda \). The parameter \( q_H \) can be written for each basic polarization \( (p \text{ or } s) \) as

\[
q_H(p) = \frac{n_H \cos \theta}{n \cos \theta_H}, \quad q_H(s) = \frac{n \cos \theta}{n_H \cos \theta_H},
\] (2.3)

where we have assumed that the layer is imbedded in a medium of refractive index \( n \). Henceforth \( \theta \) will denote the angle of incidence and, for simplicity, the surrounding medium we will supposed to be air \((n = 1)\).
Let us consider a $N$-period finite structure whose basic cell is precisely the Fibonacci multilayer $S_j$. We denote this system as $[S_j]^N$ and its overall transfer matrix is

$$M_j^{(N)} = (M_j)^N. \tag{2.4}$$

When the unit cell is $S_2$, the resulting multilayer $[S_2]^N$ is periodic. For $j > 2$, $[S_j]^N$ are quasiperiodic.

The transmittance $T_j^{(N)}$ is given in terms of the matrix $M_j^{(N)}$ as \cite{11}

$$T_j^{(N)} = \frac{4}{||M_j^{(N)}||^2 + 2}, \tag{2.5}$$

where $||M_j^{(N)}||^2$ denotes the sum of the squares of the matrix elements.

In the theory of periodic systems it is well established that band gaps appear whenever the trace of the basic period satisfies \cite{4}

$$|\text{Tr}(M_j)| \geq 2. \tag{2.6}$$

This should be worked out for both basic polarizations. The trace map is a powerful tool to investigate this condition, especially when the index $j$ is high \cite{24}. In our context, it reads as

$$\text{Tr}(M_{j+1}) = \text{Tr}(M_j) \text{Tr}(M_{j-1}) - \text{Tr}(M_{j-2}). \tag{2.7}$$

This simple recurrence relation allows us to compute easily the band gaps. We quote here the first nontrivial cases, namely, the pure periodic system $S_2 = \{LH\}$ and the first quasiperiodic one $S_3 = \{LHL\}$, respectively:

$$|\cos\beta_L \cos\beta_H - \Lambda_{LH} \sin\beta_L \sin\beta_H| \geq 1,$$

$$|\cos(2\beta_L) \cos\beta_H - \Lambda_{LH} \sin(2\beta_L) \sin\beta_H| \geq 1. \tag{2.8}$$

The function $\Lambda_{LH}$ is

$$\Lambda_{LH} = \frac{1}{2} \left( \frac{q_L}{q_H} + \frac{q_H}{q_L} \right), \tag{2.9}$$

which is frequency independent but takes different values for $p$ and $s$ polarizations. However, one can check that, irrespective of the angle of incidence, the following relation for both basic polarizations holds:

$$\frac{\Lambda_{LH}(p)}{\Lambda_{LH}(s)} \leq 1. \tag{2.10}$$

Due to the restriction \cite{2.10}, whenever Eqs. \cite{2.8} are fulfilled for $p$ polarization, they are always true also for $s$ polarization. In consequence, the $p$-polarization bands are more stringent than the corresponding $s$-polarization ones \cite{22}.

III. ASSESSING ODR FROM QUASIPERIODIC MULTILAYERS

We first investigate the range of layer thicknesses for which ODR exists; that is, when condition \cite{2.6} holds true for all the incidence angles. Although for the simple periodic system $S_2$ analytic approximations are at hand, the general problem seems to be very involved and we content ourselves with a numerical exploration.

For definiteness, we fix the refractive indices to the values $n_L = 1.75$ and $n_H = 3.35$ at $\lambda = 10 \, \mu m$. In Fig. 1 we have plotted the zones of ODR for the basic periods $S_j$ (with $j = 2, 3, 4, 5$) in terms of the adimensional thicknesses $n_L d_L / \lambda$ and $n_H d_H / \lambda$. Note that the use of these adimensional variables not only simplifies the presentation of the results, but, as dispersion can be neglected, the results apply to more general situations.

![FIG. 1: Regions where ODR (for $p$ polarization) occurs for the Fibonacci systems $S_2 = \{LH\}$, $S_3 = \{LHL\}$, $S_4 = \{LHLLH\}$, and $S_5 = \{LHLLLHL\}$. We have taken $n_L = 1.75$ and $n_H = 3.35$ at $\lambda = 10 \, \mu m$. The inset identifies the filled ellipses. The marked points correspond to the minimum area for each one of the systems.](image-url)
These regions of ODR are not enough to fully quantify the performance of the multilayer. In Ref. [23] we have proposed that, once the materials and the wavelength are fixed, the area under the transmittance as a function of the angle of incidence \( \theta \)

\[
A_j^{(N)} = \int_0^{\pi/2} T_j^{(N)}(\theta) \, d\theta,
\]

is an appropriate figure of merit for the structure: the smaller this area, the better the performance as ODR. In Fig. 2 we have plotted this area as a function of \( n_L d_L / \lambda \) and \( n_H d_H / \lambda \) for \( S_2 \). The area has been computed solely for the points fulfilling the ODR condition, so the abrupt steps give the boundaries of ODR plotted in Fig. 1. However, this function varies significantly in the ODR region.

In fact, for the present case the minimum of this area is reached at the point

\[
n_L d_L / \lambda = 0.34305, \quad n_H d_H / \lambda = 0.25416. \tag{3.3}
\]

While the value of \( n_H d_H / \lambda \) essentially coincide with the quarter wavelength solution (3.1), \( n_L d_L / \lambda \) differs more than 30 % of that solution.

In Fig. 1 we have marked the points of minimum area for each one of the Fibonacci systems \( S_j \). We see the strong difference for the periodic and the quasiperiodic systems. In fact, for the latter (\( S_j \) with \( j \geq 3 \)) we can summarize the results saying that the optimum area is reached approximately at the values of the parameters

\[
n_L d_L / \lambda = 1/8, \quad n_H d_H / \lambda = 1/4. \tag{3.4}
\]

In our view, this is a remarkable result: from the principle of minimum area, we have consistently derived optimum parameters for ODR, which differ a lot from the usual solutions found in the literature.

For the thicknesses giving minimum area, we have calculated the region in the \((n_L, n_H)\) plane for which ODR occurs. In Fig. 3 we have plotted the boundary of such a region for the same Fibonacci multilayers as before: above such curves we have the ODR region. It is again the periodic system the first in fulfilling ODR: the onset of the ODR curve is at \( n_H \approx 2.5 \), in agreement with previous estimations [10].

Of course, the optimum parameters for the system \( S_j \) do not need to be optimum for \([S_j]^N\). To elucidate this question, we have computed numerically the values of \( n_L d_L / \lambda \) and \( n_H d_H / \lambda \) for different systems containing up to 42 layers and for the same indices as before. In Table 1 we have summarized the corresponding data. We have included only results for the five first periods \( N = 1, \ldots, 5 \), since from \([S_j]^5\) onwards, all the thicknesses are fairly stable, while the area tends rapidly to 0, as one would expect from a band gap. We can conclude that the optimum parameters do not depend strongly on the number of layers.

In Fig. 4 we have plotted the logarithm of the area computed for the systems \([S_j]^N\) as a function of the number of layers.
Basic Bandwidth # Periods $n_d d_L / \lambda$ $n_d h_{dH} / \lambda$ Area

| $S_2$ | 0.217 |
|-------|--------|
| 1     | 0.34305 0.25416 1.01660 |
| 2     | 0.33807 0.22187 0.47147 |
| 3     | 0.30817 0.23429 0.17894 |
| 4     | 0.29281 0.23926 0.06294 |
| 5     | 0.29572 0.24422 0.02171 |
| $S_3$ | 0.233 |
| 1     | 0.11978 0.26906 1.07725 |
| 2     | 0.12917 0.26657 0.47955 |
| 3     | 0.13319 0.26409 0.17693 |
| 4     | 0.13587 0.26409 0.06902 |
| 5     | 0.13722 0.26409 0.02055 |
| $S_4$ | 0.195 |
| 1     | 0.14795 0.25912 0.48632 |
| 2     | 0.16538 0.25912 0.10423 |
| 3     | 0.16002 0.29389 0.01647 |
| 4     | 0.16136 0.29389 0.00251 |
| 5     | 0.16270 0.29389 0.00038 |
| $S_5$ | 0.198 |
| 1     | 0.14929 0.28396 0.24841 |
| 2     | 0.15063 0.28396 0.01365 |
| 3     | 0.15331 0.28396 0.00073 |
| 4     | 0.15197 0.28644 0.00004 |
| 5     | 0.15331 0.28644 0.00002 |

From previous results for the case of Bragg mirrors, it is reasonable to assume that the transmittance of $[S_j]^N$ tends to zero exponentially with the number of layers. To test such an ansatz, we have plotted the area (in a logarithmic scale) for all these systems. The results are presented in Fig. 4. We think that a simple glance at this figure is enough to decide on the performance of the quasiperiodic systems as omnidirectional reflectors.

It is quite clear that all the quasiperiodic systems, irrespective of the index $j$, behave essentially in the same way as far as ODR is concerned. All the points for these systems fit into a straight line. On the other hand, the periodic system $S_2$ lies on another straight line, but with a better slope. That is, for a given number of layers of the system (and under the hypothesis of optimum thicknesses), the system $[LH]^N$ offers better performance than any other.

Of course, one may think that the bandwidth of these systems is different. Sometimes the bandwidth is defined at normal incidence, and then it has been argued that quasiperiodic systems offer fundamental advantages. If we denote by $\lambda_{\text{short}}$ and $\lambda_{\text{long}}$ the longer- and shorter-wavelength edges for given ODR bands (of the basic period), it seems more appropriate to define the ODR bandwidth as

$$B = \frac{\lambda_{\text{long}} - \lambda_{\text{short}}}{4(\lambda_{\text{long}} + \lambda_{\text{short}})}.$$  (3.5)

Note that this is the appropriate definition in our case. Obviously, the parameters chosen for the purpose of comparison must be the ones giving minimum area; i.e., optimum ODR behavior. In fact, we have numerically checked that the parameters giving optimum area offer also a good bandwidth. In Table 1 we have indicated this parameter, confirming again that with the proper definition the periodic system is the best.

IV. CONCLUDING REMARKS

In summary, we have exploited the idea of minimum area to fully assess in a systematic way the performance of omnidirectional reflectors. Although quasiperiodic systems has attracted a lot of interest due to their unusual physical properties, Bragg reflectors offer the best performance, although not at a quarter-wavelength thick at normal incidence. We believe that the best feature of our approach is that it provides a very clear thread to deal with omnidirectional reflection properties in a systematic way. Our method is general and can be applied to any spectral region.

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