Superconducting properties of BaBi₃

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Abstract
We report the superconducting properties of single crystals of the intermetallic compound BaBi₃, whose crystal structure is perovskite related. The superconducting transition temperature \( T_{c} = 5.82 \) K was obtained from heat capacity measurements. Using the measured values for the critical fields \( H_{c1}, H_{c2} \), and the specific heat \( C \), we estimate the thermodynamic critical field \( H_{c0} \), coherence length \( \xi(0) \), Debye temperature \( \Theta \) and coupling constant \( \lambda \). The results suggest that BaBi₃ is a weakly coupled superconductor. Electronic band structure calculations show a complex Fermi surface and a moderately high DOS at the Fermi level. Further analysis of the electronic specific heat shows that the superconducting properties are dominated by \( s \)-wave gap.

Keywords: superconductor, perovskite, bismuth, physical properties, superconducting gap, alkaline, critical field

(Some figures may appear in colour only in the online journal)

1. Introduction

Perovskite structure compounds are among the most widely studied superconductors. Interest in non-oxide perovskites as superconductors was enhanced with the discovery of superconductivity in MgCNi₃, leading to experimental and theoretical studies on related systems [1–4]. Alkali-Bi based compounds have recently attracted attention because the characterization of the electronic structure of Na₃Bi has led to its proposal as a Dirac Semimetal [9–11]. Superconductors based on heavy metals such as Bi are potentially interesting because their properties may be significantly influenced by spin orbit coupling.

Here we report a detailed study of the superconducting properties of BaBi₃, supported by electronic structure calculations. BaBi₃ is a previously reported superconductor, but it has not so far been the subject of detailed experimental or theoretical studies [5, 6]. This material crystallizes in a perovskite-related crystal structure [7, 8] which consists of an array of corner sharing the Bi₆ octahedra, with no atoms inside, and Ba in the perovskite A site. The Bi₆ octahedra are considered as creating a three dimensional framework. Since the dominant element is bismuth, heavy element, our interest is to study this system as a possible candidate for unconventional superconductivity [13].

2. Experiment and calculation

BaBi₃ single crystals were prepared by a Bi-self-flux method starting from elemental Ba (99.99 %; Alfa Aesar) and Bi (99.99 %; Alfa Aesar) pieces. The starting materials (Ba:Bi ratio 1:4) were added into carbon-coated quartz tube inside an Ar-filled glove box. The sealed tubes were slowly heated up to 700°C for 6 hrs. They were then slowly cooled to 330°C over a period of 5 days. Finally, the excess Bi-flux was removed by decanting. Cubic shape single crystals (1 mm³) were observed, and were kept inside the glove box until characterization. Such handling is necessary to avoid decomposition. The purity and cell parameters of the samples were evaluated by powder X-ray diffraction (PXRD) at room temperature on a Bruker D8 FOCUS diffractometer (Cu Kα).

The electrical resistivity was measured using a standard four-probe dc technique with an excitation current of 10 mA; small diameter Pt wires were attached to the sample using a conductive epoxy (Epotek H20E). Data were collected from 300—2 K and in magnetic fields up to 3 T using a Quantum Design Physical Property Measurement System (PPMS). The specific heat was measured between 0.4 and 20 K in the PPMS equipped with a ⁴He cryostat, using a time-relaxation method, at 0 and 5 Tesla applied magnetic fields. Magnetic susceptibility was measured in a constant magnetic field of 20 Oe, the sample was cooled down to 2 K in zero-field, and then magnetic field was applied, followed by heating to 8 K [zero-field-cooled (ZFC)] and then cooled down again to 2 K.
Figure 1. (a) PXRD pattern of BaBi$_3$ single crystals. (b) 3D crystal structure of BaBi$_3$. (c) Semi log plot of resistivity as a function of temperature for BaBi$_3$. Solid symbol represents zero field data and open symbol represents resistance at 2 T field. The solid line represents the Fermi-liquid fit, with $\rho = \rho_0 + AT^n$. (d) Magnetoresistance analysis of BaBi$_3$ single crystals. The main panel shows $\mu_0 H_c^Z$ as a function of $T$, and the inset shows resistivity as a function of temperature with applied magnetic fields up to 2 Tesla.

[field-cooled (FC)] in the PPMS. The electronic structure calculations were performed by density functional theory (DFT) using the WIEN2K code with a full-potential linearized augmented plane-wave and local orbitals [FP-LAPW + lo] basis [14–17] together with the PBE parameterization [18] of the GGA, including spin orbit coupling (SOC). The plane-wave cutoff parameter $R_{\text{MT}}K_{\text{MAX}}$ was set to 8 and the Brillouin zone was sampled by 20,000 k points. Convergence was confirmed by increasing both $R_{\text{MT}}K_{\text{MAX}}$ and the number of k points until no change in the total energy was observed. The Fermi surface was plotted with the program Crysdan.

3. Results and discussion

Figure 1 shows the resistivity, PXRD analysis of BaBi$_3$ single crystals and the 3D crystal structure. BaBi$_3$ has tetragonal symmetry. (P4/$\text{mmm}$, space group number 123) figure 1(a) shows that the PXRD pattern of the crystals employed (crushed for the PXRD pattern) matches the standard pattern in the ICSD database (code number 58634) [7]. (The hump in the low 20 range of the PXRD pattern is due to the paratone-oil that covers the sample to prevent it from decomposing during the pattern acquisition.) A schematic view of crystal structure of BaBi$_3$ is shown in figure 1(b). The corner sharing Bi$_6$-octahedra that make the 3D-bismuth network are easily discerned.

Figure 1(c) shows the temperature dependent resistivity from 300 K to 2 K. Metallic behavior ($\frac{d\rho}{dT} > 0$) can be observed in the normal state resistance of the BaBi$_3$ single crystals. An $S$-like inflection point can be observed around 40 K, with room temperature $\rho = 160 \mu \Omega \, \text{cm}$ and residual resistivity ratio $\left(\frac{\rho_{\text{RRR}}}{\rho_{\text{RRR}}(0)} = 10\right)$. Similar behavior is observed in other Bi-based superconductors [19] and the perovskite-type intermetallic system MgCNi$_3$ [1]. The $\rho(T)$ (inset in figure 1(c)) shows a tendency to saturate at high temperature with convex curve above 50 K. The behavior could be related to the Ioffe-Regel limit, [26] when the charge carrier mean free path is comparable to the interatomic spacing and/or to the two-band conductivity [27]. The low-temperature resistivity data can be described by the power law $\rho = \rho_0 + AT^n$ with $n = 2$, the residual resistivity $\rho_0 = 12 \mu \Omega \, \text{cm}$, and the coefficient $A = 0.014 \frac{\mu \Omega \text{cm}}{K}$. The Fermi-liquid fit is shown as solid lines in figure 1(a). The value of $A$ is often taken as a measure of the degree of electron correlations. The value found here suggests that BaBi$_3$ is a weakly correlated electron system; the variation of electron correlations. The value found here suggests that BaBi$_3$ is a weakly correlated electron system; the variation of electron correlations.

Figure 1(d) shows an analysis of the magnetoresistance data for a BaBi$_3$ single crystal. The width of the superconducting transition increases slightly with increasing magnetic field. Selecting the 50% normal state resistivity drop point as the transition temperature, we estimate the orbital upper critical field, $\mu_0 H_{c2}(0)$, from the Whamper-Helfand-Hohenberg (WHH) expression, $\mu_0 H_{c2}(0) = -0.693 T_c \frac{dT_c}{dT} \frac{dH_{c2}}{dT}$. A nearly linear relationship is observed in figure 1(d) between $\mu_0 H_{c2}$ and $T_c$. The slope is used to calculate $\mu_0 H_{c2}(0) = 2.82$ T. The value of $\mu_0 H_{c2}(0)$ is smaller than the weak coupling Pauli paramagnetic limit $\mu_0 H_{c2}^\text{Pauli} = 1.82 T_c = 10.9$ T for BaBi$_3$. We also used the empirical formula $H_{c2}(T) = H_{c2}(0) \left[1 - (\frac{T}{T_c})^2\right]$ to calculate orbital upper critical field ($\mu_0 H_{c2}(0) = 2.91$ T), which yields a value that agrees well with the calculated value using the WHH method. The WHH expression and the empirical formula are widely used to calculate the $\mu_0 H_{c2}(0)$ for a variety of intermetallics and oxide superconductors [20, 28, 29, 31–33, 36]. Also, for a one-band superconductor, the orbital upper critical field derived from the slope $k = \frac{dH_{c2}}{dT}$ of the $H$-$T$ phase boundary at $T_c$ is an indication of clean limit ($\frac{\mu_0 H_{c2}}{k T_c} = -0.73$) or dirty limit ($\frac{\mu_0 H_{c2}}{k T_c} = -0.69$) behavior. BaBi$_3$ has the $\left(\frac{\mu_0 H_{c2}}{k T_c} = -0.74\right)$
value, therefore BaBi$_3$ is a type II BCS-superconductor that is close to the clean limit [42, 43]. The upper critical field value $H_{c2}(0)$ of the BaBi$_3$ superconductor can be used to estimate the Ginzburg-Landau coherence length $\xi(0) = \sqrt{\Phi_0/2\pi H_{c2}(0)} = 110.45$ Å, where $\Phi_0 = hc/2\pi$ is the magnetic flux quantum [37, 38]. This value is comparable to that for the alkali-Bi superconductor NaBi and larger than that of MgCNi$_3$ (see table 1) [1, 12].

Figure 2 shows the analysis of the DC-magnetization of BaBi$_3$ single crystals. The main panel shows $H_{c1}$ as a function of $T_c$. The upper right inset shows the DC-magnetization as a function of applied magnetic field at different temperatures below the superconducting $T_c$. The lower left inset shows the ZFC and FC data through $T_c$.

Figure 3. Analysis of the heat capacity of a BaBi$_3$ single crystal. (a) The main panel shows the heat capacity in 0 T and 5 T fields. The upper left inset shows the heat capacity data fit with the equation $C = \gamma T + \beta_1 T^3 + \beta_2 T^4$. The lower right inset shows the heat capacity jump at $T_c$. (b) The heat capacity data below $T_c$ down to 0.4 K. Analysis suggests that BCS type s-wave pairing symmetry dominates the superconducting state.
Figure 3 shows the characterization of the superconducting transition by specific heat measurements. Figure 3(a) shows $\frac{C}{T}$ as a function of $T$, characterizing the specific heat jump at the thermodynamic transition. This jump is completely suppressed under a 5 T applied magnetic field. The superconducting transition temperature $T_c = 5.8$ K is shown in the lower left inset of figure 3(a), as extracted by the standard equal area construction method. The low temperature normal state specific heat is non-Debye-like. Non-Debye behavior has often been reported in superconductors and is ascribed either to a large Einstein contribution or a low Debye temperature $\theta_D$. Because an Einstein phonon contribution is negligible below 20 K, a second term in the harmonic-lattice approximation is needed to improve the fit to the specific heat data [22, 24, 25]. We find that the low temperature normal state specific heat can be well fitted with

$$\frac{C}{T} = \gamma_n + \beta_1 T^2 + \beta_2 T^4, \text{ where } \gamma_n T \text{ represents the electronic contribution in normal state and } \beta_1 T^3 \text{ and } \beta_2 T^5 \text{ describe the lattice-phonon contributions to the specific heat.}$$

The solid line in figure 3(a) shows the fitting; the electronic specific heat coefficient $\gamma_n = 41.6 \text{ mJ mol}^{-1} \text{ K}^{-2}$ and the phonon/lattice contributions $\beta_1 = 16.05 \text{ mJ mol}^{-1} \text{ K}^{-2}$ and $\beta_2 = -0.055 \text{ mJ mol}^{-1} \text{ K}^{-2}$ are extracted from the fit. The value of $\gamma$ obtained is relatively larger than that of MgCNi$_3$ and NaBi (see Table 1) [1, 12]. The $\gamma$ is an indication of a moderately high density of states near the Fermi energy.

The ratio $\frac{\Delta C}{\gamma T_c}$ can be used to measure the strength of the electron-phonon coupling [39]. The specific heat jump $\frac{\Delta C}{\gamma T_c}$ for the sample is about $42 \text{ mJ mol}^{-1} \text{ K}^{-2}$, setting the value of $\frac{\Delta C}{\gamma T_c}$ to 1.02. This is smaller than the weak-coupling limit of 1.43 for a conventional BCS superconductor. The results suggest that BaBi$_3$ is a weakly electron-phonon coupled superconductor.

In a simple Debye model for the phonon contribution to the specific heat, the $\beta$ coefficient is related to the Debye temperature $\theta_D$ through $\beta = n N_A \frac{1}{5} \pi^4 \rho \theta_D^3$, where $R = 8.314 \text{ J mol}^{-1} \text{ K}^{-1}$, $n$ is the number of atoms per formula unit and $N_A$ is Avogadro’s number. The calculated Debye temperature for BaBi$_3$ is thus 142 K. This value of the Debye temperature is comparable to that of elemental Bi and Bi-based superconductors such as NaBi and NiBi$_3$, but is slightly smaller than that of the MgCNi$_3$ [3, 12, 22, 23]. An estimation of the strength of the electron-phonon coupling can be derived from the McMillan formula

$$\lambda_{ep} = \frac{1.04 + \rho^* \ln \frac{\mu_n}{\mu}}{(1 - 0.62 \rho^*) \ln \frac{\mu_n}{\mu} - 1.04}$$

[40, 41]. McMillan’s model contains the dimensionless electron-phonon coupling constant $\lambda_{ep}$. Which, in the Eliashberg theory, is related to the phonon spectrum and the density of states. This parameter $\lambda_{ep}$ represents the attractive interaction, while the second parameter $\rho^*$ accounts for the screened Coulomb repulsion. Using the Debye temperature $\theta_D$, critical temperature $T_c$, and making the common assumption that $\mu^* = 0.15$, [40] the electron-phonon coupling constant ($\lambda_{ep}$) obtained for BaBi$_3$ is 0.82, which suggests weak electron-phonon coupling behavior and agrees well with $\frac{\Delta C}{\gamma T_c} = 1.02$.

The electronic specific heat ($C_e$) is further analyzed to study the superconducting gap function. $\Delta C_s$ was fitted to the forms $e^{-\Delta/T}$, $T^2$, and $T^4$, which are the expected temperature dependencies for gaps that are isotropic, contain line nodes, or linear point nodes, respectively. The data agrees well with the exponential fit $\frac{\Delta C}{\gamma T_c}$ below $T/T_c = 0.8$, while the other two fits do not describe the data. $C_e(T)$ can also be well described with the semiempirical approximation, the so-called $\alpha$ model, $C_e(T) = A \exp\left(\frac{-\Delta(0)}{k_B T}\right)$, where $k_B$ is the Boltzmann constant and $\Delta(0)$ is the superconducting gap at 0 K [39]. This equation allows for varying the coupling strength $\alpha = \frac{\Delta(0)}{k_B T}$, instead of fixing it at the BCS weak-coupling limit, $\alpha = 1.76$. The obtained coupling strength $\alpha = 1.46$ (2 $\Delta(0) = 2.93 k_B T_c$) gives an excellent fit, which agrees well with the BCS-weak coupling limit. Therefore, the above analysis suggests that BaBi$_3$ is a BCS-type isotropic-gapped superconductor. This further confirms that the superconducting properties of this system are dominated by $s$-wave pairing symmetry [28, 30, 31].

Figure 4 shows the analysis of the electronic density of states for BaBi$_3$ based on the electronic structure calculations. Figure 4(a) shows the band structure in the vicinity of Fermi energy $E_F$. According to the calculated band structure, BaBi$_3$
is a three-dimensional metal; 4 bands with large dispersion cross the Fermi level. The bands at the Fermi level are all derived from Bi $p$-orbitals. The total DOS (see figure 4(b)) shows that the Fermi level is located near the edge of a local maximum. The value of the DOS at $E_F$ is in the range of 2.1–4.0 eV/f.u. (f.u. = formula unit), which agrees well with the moderately high $\gamma$ value observed from the heat capacity data. The partial DOS shows that the total DOS is dominated by the contributions from the two types of Bi atoms in the tetragonal symmetry structure and that the contribution from the Ba atom near the Fermi level is almost negligible. Figure 4(c) shows the calculated Fermi surfaces of the four bands that crosses the Fermi level in BaBi$_3$. This suggests that the combination of all four bands creates a very complex Fermi surface in this compound.

The superconducting parameters are presented in table 1. Comparison with MgCNi$_3$ is given because it shares perovskite-type structure with BaBi$_3$ and comparison to NaBi is given because both compounds are alkaline-bismuth based superconducting compounds. The superconducting parameters of BaBi$_3$ are more close to the perovskite-MgCNi$_3$ superconducting compound. It is interesting to see that electron-phonon coupling constants of BaBi$_3$ and MgCNi$_3$ are very similar.

The value of $\gamma$ extracted from the measured specific heat data corresponds to an electronic density of states at the Fermi energy $N(E_F)\exp \mu \Delta (mol^{-1})$. As estimated from the relation $\gamma = \frac{\pi^2 k_B^2}{6} N(E_F)(1 + \lambda_{ep})$. This value is comparable to the value obtained from our band structure calculation $N(E_F)\exp \mu \Delta (mol^{-1})$. A straightforward calculation of the condensation energy from the relation $U(0) = \frac{1}{2} \Delta^2(0)N(E_F)$ produces a value of $U(0) = 1190 \Delta_0^2/mol$ [20, 21]. Using the upper and lower critical fields and the relation $H_{c2}(0)H_{c1}(0) = H_{c1}(0)^2[\ln \kappa(0) + 0.08]$, the thermodynamic critical field $H_c(0)$ was found to be 0.15 T. This value is comparable to that of MgCNi$_3$. Assuming $g = 2$ for conduction electrons, one can estimate the Pauli limiting field for BaBi$_3$ from the relation of $\mu_B H_{Pulim} = \frac{\Delta(0)}{\mu_B \Delta_{c1}^{41}} = 11.7$ T.

This is very close to the value obtained from the orbital upper critical field. The actual $H_{c2}$ of real materials is generally influenced by the both orbital and spin-paramagnetic effects. The relative importance of the orbital and spin-paramagnetic effects can be described by the Maki parameter, [44] which can be calculated by $\mu_B H_{c2}(0) = a \gamma_{41}^{41} = 0.3$ [34, 43, 45]. Even though BaBi$_3$ shows moderately high density of states at the Fermi level $\alpha < 1$ indicates that this system dose not represent heavy electron mass or multiple small Fermi pockets, which is also consistent with our calculation.

4. Conclusion

We have grown single crystals and characterized the superconducting properties of perovskite-like BaBi$_3$. A bulk superconducting transition was confirmed and characterized through magnetization and heat capacity measurements on the single crystals. The electronic contribution to the specific heat is relatively large, $\gamma = \frac{m_L}{\Delta_0^2}$ and the electron-phonon coupling constant is $\lambda_{ep} = 0.82$. BCS-type $s$-wave pairing symmetry is inferred from the behavior of the electronic heat capacity in the superconducting state. The electronic density of states calculation suggests that BaBi$_3$ is a good 3D-metal and the Bi $p$-bands are dominant through the Fermi level. Many bands can be found at energies through the Fermi level, which creates a complex Fermi surface.

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References

[1] He T et al 2001 Nature 411 54
[2] Nagamatsu J, Nakagawa N, Muranaka T, Zenaitani Y and Akimitsu J 2001 Nature 410 63
[3] Mollah S 2004 J. Phys. Cond. Matt. 16 R1237R1276
[4] Cava R J, Batlogg B, Krajewski J J, Farrow R, Rupp L W Jr, White A E, Short K, Peck W F and Kometani T 1988 Nature 332 814–6
[5] Matthias B T and Corenzwi K 1957 Phys. Rev. 107 06
[6] Matthias B T and Hulm J K 1952 Phys. Rev. 87 05
[7] Zhuravlev N N and Melik-Adyman V P 1961 Kristallografiya 06 121–4
[8] Hawinga E E, Dansma H and van Maarem M H 1970 J. Phys. Chem. Solids 03 2653–62
[9] Wang Zhijun, Sun Yan, Chen Xing-Qiu, Franchini Cesare, Xu Gang, Weng Hongming, Dai Xi and Fang Zhong 2012 Phys. Rev. B 85 195320
[10] Liu Z K et al 2014 Science 343 864
[11] Su-Yang Xu et al 2014 arXiv:1312.7624
[12] Kushwaha S K, Krizan J W, Xiong Jun, Klimczuk T, Gibson Q D, Liang Tian, Ong N P and Cava R J 2014 J. Phys. Cond. Matters 26 122201
[13] Bauer E, Hilscher G, Michor H, Paul Ch, Scheidt E W, Gribanov A, Seropogin Yu, Noel H, Sigrist M and Rogl P 2004 Phys. Rev. Lett. 92 027003
[14] Blaha P, Schwarz K, Madsen G, Kvasnicka D and Luitz J 2001 WIEN2K, An Augmented Plane Wave + Local Orbitals Program for Calculating Crystal Properties (Austria: Technische Universitat Wien)
[15] Singh D J and Nordstrom L 1996 Planewaves, Pseudopotentials, and the LAPW Method 2nd edn (New York: Springer)
[16] Madsen G K H, Blaha P, Schwarz K, Sjostedt E and Nordstrom L 2001 Efficient linearization of the augmented plane-wave method Phys. Rev. B 19 195134
[17] Sjostedt E, Nordstrom L and Singh D J 2000 An alternative way of linearizing the augmented plane-wave method Solid State Communications 114 15–20
[18] Perdew J P, Burke K and Ernzerhof M 1996 Phys Rev Lett. 77 3865
[19] Xiao Zhang Hechang Lei and Petrovic C 2012 Phys. Rev. B 86 054502
[20] Karki A B, Xiong Y M, Vekhter I, Browne D, Adams P W, Thomas K R, Kim H, Prozorov R, Chan Julia Y and Young D P 2010 Phys. Rev. B 82 064512
[21] Tinkham M 1975 *Introduction to Superconductivity* 2nd edn (New York: McGraw Hill)

[22] Fujimori Yasunobu, Kan Shin-ichi, Shinozaki Bunjyu and Kawaguti Takasi 2000 *Journal of the Physical Society of Japan* 69 3017–26

[23] Robie R A and Edwards J L 1966 *J. Appl. Phys.* 37 2659

[24] Lin X, Budko S L, Samolyuk G D, Torikachvili M S and Canfield P C 2011 *J. Phys. Condens. Matter* 23 455703

[25] Imai Y, Nabheshima F, Yoshinaka T, Miyatani K, Kondo R, Komiya S, Tsukada I and Maeda A 2012 *J. Phys. Soc. Jpn.* 81 113708

[26] Ioffe A F and Regel A R 1961 *Prog. Semicond* 4 237

[27] Zverev V N, Korobenko A V, Sun G L, Sun D L, Lin C T and Boris A V 2009 *JETP Lett.* 90 130

[28] Karki A B, Xiong Y M, Haldolaarachchige N, Phelan W A, Chan Julia Y, Stadler S, Vekhter I, Adams P W and Young D P 2011 *Phys. Rev. B* 83 144525

[29] Haldolaarachchige N, Gibson Q, Krizan J and Cava R J 2014 *Phys. Rev. B* 89 104520

[30] Klimczuk T, Ronning F, Sidorov V, Cava R J and Thompson J D 2007 *Phys. Rev. Lett.* 99 257004

[31] Hirai Daigorou, Ali Mazhar N and Cava Robert J 2013 *Journal of the Physical Society of Japan* 82 124701

[32] Ali Mazhar N, Gibson Quinn D, Klimczuk T and Cava R J 2014 *Phys. Rev. B* 89 020505

[33] Luo H, Klimczuk T, Muchler L, Schoop L, Hirai D, Fuccillo M K, Felser C and Cava R J 2013 *Phys. Rev. B* 87 214510

[34] Carbotte J P 1990 *Rev. Mod. Phys.* 62 1027

[35] Kittel Charles 1996 *Introduction to Solid State Physics* 7th edn (New York: John Wiley and Sons, Inc.)

[36] Lan M D, Chang J C, Lu K T, Lee C Y, Shih H Y and Jeng G Y 2001 *IEEE Transaction on Applied Superconductivity* 11 3607–10

[37] Werthamer N R, Helfand E and Hohenberg P C 1966 *Phys. Rev.* 147 295

[38] Clogston A M 1962 *Phys. Rev. Lett.* 9 266

[39] Padamsee H, Neighbor J E and Shiffman C A 1973 *J. Low Temp. Phys.* 12 387

[40] McMillan W L 1968 *Phys. Rev.* 167 331

[41] *Handbook of Superconductivity* Poole C P Jr ed 2009 19 (New York, 1999: Academic Press) p 478.422 Chap. 9, section G

[42] Helfand E and Werthamer R 1966 *Phys. Rev.* 147 288–94

[43] Zehetmayer M, Eisterer M, Jun J, Kazakov S M, Karpinski J, Wisniewski A and Weber H W 2002 *Phys. Rev. B* 66 252505

[44] Maki Kazumi 1966 *Phys. Rev.* 142 362–9

[45] Junod A 1996 Studies of high temperature superconductors (Nova Science 19) ed A Norliker (New York)