Imaging of dielectric objects buried under a rough surface via distorted born iterative method

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Abstract. A method is given for the shape, permittivity and conductivity reconstruction of lossy dielectric objects buried under rough surfaces using the Distorted Born Iterative Method (DBIM). The method is based on the refreshing of the Green’s function of the two-part space media with rough interface by updating the complex permittivity of the reconstruction domain at each iteration step. The scattered field data are measured at multiple locations for multiple transmitters operating at a single frequency where both transmitters and receivers are located above the rough surface interface. The Green’s function of the problem is obtained by using the buried object approach (BOA) method where the fluctuations of the rough surface from the flat one are assumed to be buried objects in a two-part space with planar interface. The performance of the method is tested by some numerical applications and satisfactory results are obtained.

1. Introduction

Inverse scattering problems related to the objects buried in a layered media are of importance due to the practical applications in electromagnetic theory such as detection and location of dielectric mines, medical imaging, nondestructive testing and geophysical probing etc. Although during the last two decades several techniques have been developed, most of them deal with the layered backgrounds with planar boundaries [1-3]. However in general practical applications, the bodies are buried in layered media having rough interfaces and the roughness has a strong effect on the scattering phenomena as well as inversion algorithms. Therefore realistic conditions and environments should be taken into consideration for the reconstruction of dielectric object buried under rough surfaces.

The main objective of this paper is to give a method to solve the inverse scattering problem related to the inhomogeneous cylindrical bodies buried in a half-space medium with rough surface. The scattered field measurements are assumed to be performed through a line over the rough surface. For the sake of simplicity, only the surfaces having one-dimensional profiles are considered. The materials of the bodies are assumed to be inhomogeneous (i.e., their dielectric permittivities and conductivities are functions of the location). The Distorted-Born Iterative (DBI) algorithm is used to obtain the both shape and the electromagnetic parameters of the buried objects. The method is based on updating the Green’s function of the layered media with rough interface at each iteration step. The detail of the DBI method is given in [1] for the object buried under the planar interfaces. The computation of the Green’s function of the layered media with rough interface is based on the buried object approach
(BOA) given in [4], where the perturbations of the rough surface from the flat one are assumed to be buried objects in a two-part space with planar interface.

2. Formulation of the Problem

The geometry of the problem is illustrated in Figure 1 where an interface denoted by $\Gamma$ separates the whole space into two half-spaces. $\Gamma$ can be defined by either a deterministic function $f(x_1)$ or statistically. The roughness of the interface $\Gamma$ is local in a certain region $L_R$. In other words the variation of the $\Gamma$ is equal to zero out of this bounded region $L_R$. The constitutive parameters of the upper half-space above $\Gamma$ and lower half space below $\Gamma$ are denoted by $\varepsilon_1, \sigma_1$ and $\varepsilon_2, \sigma_2$, respectively. An inhomogeneous cylindrical object of arbitrary cross-section $B$ in the $O_{x_1x_2}$ plane with an unknown relative permittivity $\varepsilon^{\text{obj}}$ and the conductivity $\sigma^{\text{obj}}$ is embedded in the lower half-space as parallel to the $Ox_3$-axis. Multiple buried objects are also possible. The magnetic permeability is supposed to be $\mu = \mu_0$ in both half spaces and time dependency is assumed as $e^{-i\omega t}$ and suppressed throughout the paper. Multiple transmitters and receivers are located above the rough surface.

![Figure 1. Geometry of the problem](image)

The inverse scattering problem considered here is to determine the location and the shape as well as the electromagnetic constitutive parameters of the buried cylinder by the use of scattered field data measured along a line above the rough interface for multiple transmitter and receiver positions at a single frequency. The reconstruction domain denoted by $D$ is chosen as large enough to contain unknown body. The relative permittivity and the conductivity inside the reconstruction domain can be defined as

$$
\varepsilon(y) = \begin{cases} 
\varepsilon^{\text{obj}}(y), & y \in B \\
\varepsilon_2, & y \notin B, y \in D 
\end{cases}
$$

and

$$
\sigma(y) = \begin{cases} 
\sigma^{\text{obj}}(y), & y \in B \\
\sigma_2, & y \notin B, y \in D 
\end{cases}
$$
The transmitters are line sources, which are directed to the Ox3-axis, therefore the incident field is given by

\[ u^i(x, s) = ik_1\eta_1\overline{G}(x; s) \]  

(3)

where \( k_1 \) is the wave number of upper half-space and defined by \( k_1^2 = \omega^2\varepsilon_1\mu_0 + i\omega\sigma_1\mu_0 \). \( \overline{G}(x; s) \) is the Green’s function of the two-half space media with rough interface [4], \( I \) is the electric current and “\( s=(s_1,s_2) \)” is the position vector of the transmitters. The problem can be reduced to a 2D one since the geometry is uniform in the Ox3 direction and line source is also parallel to this direction. Then \( x_3 \) component can be omitted in the subsequent analysis. The scattered field \( u' \) at the receiver points satisfies the following integral

\[ u'(x, s) = k_2^2 \int_D \overline{G}(x, y)v(y)u'(y, s)dy \]

(4)

where \( k_2 \) is the wave number of lower half-space defined by \( k_2^2 = \omega^2\varepsilon_2\mu_0 + i\omega\sigma_2\mu_0 \). \( u' \) is the total field (i.e., the summation of the incident field \( u^i \) and the scattered field \( u' \)) and \( \nu(y) \) is the actual object function, which is defined by

\[ \nu(y) = \begin{cases} 
\frac{k_2^2(y)}{k^2(y)} - 1, & y \in B \\
0, & y \notin B, y \in D \end{cases} \]  

(5)

where \( k_2^{obj} = \omega^2\varepsilon^{obj}\mu_0 + i\omega\sigma^{obj}\mu_0 \). The Green’s function \( \overline{G}(x; s) \) is obtained by using the buried object approach in which the roughness of the surfaces are assumed to be buried bodies to the two half-space media with planar interface. By using the Green’s function of the two half-space media with planar interface, \( \overline{G}(x; s) \) can be formulated as

\[ \overline{G}(x; s) = G(x, s) + k^2(x_3) \int_R G(x, z)v_g(z)\overline{G}(z, s)dz \]

(5)

where \( G(x, s) \) is the Green’s function of the two half-space media with planar interface, \( R \) is two dimensional area bounded by \( \Gamma \) and \( x_3 = 0 \) plane, which corresponds to the cross-section of the roughness and \( z = (z_1, z_2) \) is the position vector in \( R \). In (5) \( k^2(x_3) \) is defined as

\[ k^2(x_3) = \begin{cases} 
k_1^2, & x_3 > 0 \\
k_2^2, & x_3 < 0 \end{cases} \]  

(6)

and

\[ v_g(z) = \begin{cases} 
k_2^2/k_1^2 - 1, & z_2 > 0 \\
k_1^2/k_2^2 - 1, & z_2 < 0 \end{cases} \]  

(7)

is the object function of the roughness.
Unlike the equation (4) an inhomogeneous Green’s function, which is updated at each iteration step, is used in the Distorted Born Iterative inversion instead of the Green’s function of the half-space media with rough interface. By using the object function reconstructed in the previous step \( (v^{\varepsilon}) \), the scattered field at receiver points can be written as

\[
 u_s(x, s; v) - u_s(x, s; v^{\varepsilon}) = k^2 \int_D \overline{G}(x, y; \varepsilon^2) \delta v(y) u^\prime(y, s; v^{\varepsilon}) dy
\]  

(8)

where \( u_s(x, s; v) \) and \( u^\prime(x, s; v^{\varepsilon}) \) are measured and computed scattered fields at the receiver points, respectively. \( u^\prime(x, s; v^{\varepsilon}) \) is calculated by using the object function \( v^{\varepsilon}(y) \) obtained in the previous iteration step. The inhomogeneous Green’s function of the two-part space media with rough interface, \( \overline{G}(x, y; \varepsilon^2) \), is updated using the inhomogeneous complex permittivity of the reconstruction domain, which is obtained at each iteration step. In (8) \( \delta v(y) = v(y) - v^{\varepsilon}(y) \) is the difference of the actual object function and the computed object function calculated in the previous step. Since \( u^\prime(y, s; v) \) is the total field in the reconstruction domain in terms of the actual object function \( v(y) \) which is unknown, (8) is a nonlinear integral equation that needs to be linearized to get \( \delta v(y) \). The distorted born approximation is applied to provide the linearity by using \( u^\prime(y, s; v^{\varepsilon}) \) in (8) as follows

\[
 u_s(x, s; v) - u_s(x, s; v^{\varepsilon}) = k^2 \int_D \overline{G}(x, y; \varepsilon^2) \delta v(y) u^\prime(y, s; v^{\varepsilon}) dy
\]  

(9)

where \( u^\prime(y, s; v^{\varepsilon}) \) is the total field in the reconstruction domain in terms of the updated object function \( v^{\varepsilon}(y) \) and can be obtained from

\[
 u^\prime(y, s; v^{\varepsilon}) = k^2 \int_D \overline{G}(x, y; v^{\varepsilon}) (y) u^\prime(y, s; v^{\varepsilon}) dy = u^\prime(y, s; v^{\varepsilon}).
\]  

(10)

Inhomogeneous Green’s function of the background medium \( \overline{G}(x, y; \varepsilon^2) \) is equal to the electric field at the receiver points generated by the point sources at the reconstruction domain. Through the reciprocity theorem, the Green’s function shown in (9) can be expressed as

\[
 \overline{G}(x, y; \varepsilon^2) = \frac{1}{ik\varepsilon^2} u^\prime(y, x; v^{\varepsilon})
\]  

(11)

and it can be calculated by using (10). Equation (9) is solved iteratively until a desired level of accuracy is achieved. In other words, the iteration is continued up to the following inequality is acquired

\[
 \left| \frac{u_s(x, s; v) - u_s(x, s; v^{\varepsilon})}{|u_s(x, s; v) - u_s(x, s; v^{\varepsilon})|^2} - \text{tolerance} \right| < 0
\]  

(12)

3. Numerical Results

In this section, some numerical examples are given to show the validity and the effectiveness of the method. In all examples the upper half-space is assumed to be free space while the relative permittivity and conductivity of the lower half space are taken as \( \varepsilon_2 = 2 \) and \( \sigma_2 = 10^{-8} \text{ S/m} \), respectively. In order to avoid non-physical solutions the negative values of the reconstructed conductivity are also forced to zero in the algorithm.
In the first example, a kite shaped object with relative permittivity $\varepsilon_r^{\text{obj}} = 3$ and conductivity $\sigma^{\text{obj}} = 10^{-3} \, S/m$ is considered. 16 transmitters and 40 receivers are located above the rough surface along the line $x_2 = 0.1m$ with length $16m$ at the frequency $100 \, MHz$. The surface of the lower half space has random variation with rms height $0.15m$, correlation length $0.7m$ and the roughness length $L_r = 6m$. The reconstruction domain with dimensions $3m \times 2m$ divided into $30 \times 20 = 600$ pixels. Figures 2a and 2b display the reconstructed relative permittivities and the conductivities of the buried objects respectively. The results are achieved after 5 iterations.

Figure 2.a. Reconstructed relative permittivity of the kite shaped object
Figure 2.b. Reconstructed conductivity of the kite shaped object

In the second example, two square shaped objects with relative permittivity $\varepsilon_r^{\text{obj}} = 3$ and conductivity $\sigma^{\text{obj}} = 10^{-4} \, S/m$ are considered to be reconstructed and all other parameters used for reconstruction algorithm are taken as in the previous application. In Figures 3a and 3b, the reconstructed relative permittivities and the conductivities of the buried objects are plotted, respectively, where it is clearly observed that the resolution of the method is satisfactory. Although the reconstructed objects in Figure 3b are sliding a bit up, it is still clear that there are two distinct objects nearby the exact locations.

Figure 3.a. Reconstructed relative permittivity of two square shaped objects
Figure 3.b. Reconstructed conductivity of two square shaped objects

In order to examine the effect of the noisy data on the reconstruction algorithm presented here, random noise is added to the measurement data which is generated synthetically by solving forward problem
for the same configuration and buried bodies given in previous example. In Figures 4a and 4b the reconstructed relative permittivity of two square shaped objects, obtained using 2% and 3% noisy data are plotted, respectively. As it is observed from the figures, it is still clear that there are two distinct objects and they are reconstructed at real locations, but the value of permittivity diverges and resolution of the reconstruction deteriorates as the noise increases.

The last example is devoted to show the effect of the roughness on the reconstruction. To this aim two square shaped objects with parameters $\varepsilon_{r}^{\text{obj}} = 3$ and $\sigma_{\text{obj}} = 10^{-8} S/m$ are considered as buried objects. In Figure 5a the interface of the two half spaces is flat while in Figure 5b it is a random rough surface with rms height 0.15m, correlation length 0.7m and the roughness length $L_R = 6m$. A $1.6m \times 1.6m$ square reconstruction domain is divided into $30 \times 30 = 900$ pixels. The transmitters and receivers are located at $x_1 = 0.42m$ and $x_2 = 0.4m$ respectively. It is observed from the figures that the roughness has no significant effect on the quality of reconstruction for this example.

4. Conclusions

In this work a numerical solution for imaging of the cylindrical bodies buried under rough surfaces is introduced. The reconstruction algorithm presented here is based on the distorted born iterative
method. The method requires the Green’s function of the two half space media with rough interface which is obtained by the buried object approach as explained in [4]. It has been shown through the numerical examples that the DBI method, which is introduced for the reconstruction of the lossy dielectric bodies buried underground with planar interface, can also be applied to the reconstruction of the objects buried under rough surfaces.

References

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