Hadronic molecules composed of a doubly charmed tetraquark state and a charmed meson

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Abstract The three pentaquark states, \( P_c(4312) \), \( P_c(4440) \) and \( P_c(4457) \), discovered by the LHCb Collaboration in 2019, can be arranged into a complete heavy quark spin symmetry multiplet of hadronic molecules of \( \bar{D}^{(*)} \Sigma_{cB}^{(*)} \). In the heavy quark mass limit, the \( \Sigma_{cB}^{(*)} \) baryons can be related to the doubly charmed tetraquark states of isospin 1, i.e., \( T_{cB}^{(*)} \), via heavy antiquark diquark symmetry, which dictates that the \( \bar{D}^{(*)} \Sigma_{cB}^{(*)} \) interactions are the same as the \( \bar{D}^{(*)} T_{cB}^{(*)} \) interactions up to heavy antiquark diquark symmetry breakings. In this work, we employ the contact-range effective field theory to systematically study the \( \bar{D}^{(*)} T_{cB}^{(*)} \) systems, and we show the existence of a complete heavy quark spin symmetry multiplet of hadronic molecules composed of a doubly charmed tetraquark state and a charmed meson. These are a new kind of hadronic molecules and, if discovered, can lead to a better understanding of the many exotic hadrons discovered so far. In addition, we summarise the triply charmed hexaquark states formed by different combinations of hadrons. In particular, we show that \( \Omega_{c_c} \) can bind by the Coulomb force, which is analogous to a hydrogen-like atom.

1 Introduction

A hadronic molecule is a bound or resonant state composed of conventional hadrons that is held together by the residual strong interaction, which is analogous to the deuteron that is bound by the nuclear force. The studies on hadronic molecules containing charm quarks began with the proposal that \( \psi(4040) \) is a \( D^*D^* \) hadronic molecule \([1,2]\). Although nowadays \( \psi(4040) \) is widely viewed as a conventional charmonium state, the idea motivated a lot of theoretical studies on heavy hadronic molecules \([3–6]\). Such studies were revived after the discovery of \( X(3872) \) by the Belle Collaboration in 2003 \([7]\). Treated as a \( D\bar{D}^* \) bound state, the lower mass (than the conventional quark model prediction) and the isospin violation of the strong decays of \( X(3872) \) can be easily understood, which implies that \( X(3872) \) contains a large molecular component \([8–10]\). Since then a series of states beyond the naive quark model picture have been discovered and many of them can be regarded as molecular candidates. See Refs. \([11–19]\) for some recent reviews.

Symmetry is a particularly important and useful concept in particle physics. The SU(3)-flavor symmetry has already achieved great successes, e.g., in classifying the ground-state mesons and baryons. Symmetry plays an important role in studies of hadronic molecules as well, such as SU(3)-flavor and heavy quark spin symmetry (HQSS). Assuming that \( D_0(2317) \) discovered by the BarBar Collaboration \([20]\) is a \( DK \) molecule, it is natural to expect a \( D^*K \) molecule in terms of HQSS, corresponding to the exotic state \( D_{s1}(2460) \) \([21–24]\). The \( Z_c(3900) \) and \( Z_{cs}(3985) \) states, discovered by the BESIII Collaboration \([25,26]\), are often interpreted as \( \bar{D}D^* \) and \( \bar{D}D_{s1}^*/\bar{D}sD^* \) molecules, and are related to each other by SU(3)-flavor symmetry \([27–33]\). Another example is the three hidden charm pentaquark states, \( P_c(4312) \), \( P_c(4440) \) and \( P_c(4457) \), discovered by the LHCb Collaboration \([34]\). They can be arranged into a complete HQSS multiplet of \( \bar{D}^{(*)}\Sigma_{cB}^{(*)} \) hadronic molecules \([35–41]\). The existence of such multiplets of hadronic molecules not only enriches the hadron...
spectrum, but also can help verify the molecular nature of some exotic states [42,43].

The heavy antiquark diquark symmetry (HADS) dictates that a pair of heavy quarks behaves like a heavy antiquark from the perspective of the color degree of freedom [44]. Therefore, the charmed antimesons are the same as the doubly charmed baryons. Via HADS, one can derive the following relation between the mass splitting of the doubly charmed baryon doublet and that of the charmed meson doublet, i.e., $m_{D^{s}c_{c}} - m_{D_{c}^{s}} = \frac{3}{2}(m_{D} - m_{c})$ [45], which has been verified by a series of lattice QCD simulations [46–49]. In addition, the coupling of a doubly charmed baryon to a pion can be related to that of a charmed meson to a pion via HADS, which is found to be consistent with the quark model prediction [50,51]. Applying HADS to the $Σ_{c}^{(s)}$ baryons, it is natural to expect the existence of a set of compact doubly charmed tetraquark states [52], which have received renewed interests since the recent discovery of the doubly charged tetraquark $T_{cc}$ by the LHCb Collaboration [53,54]. In Ref. [55], HADS has been utilized to predict the existence of a series of compact doubly charmed tetraquark states as well.

The HADS has also been utilized to study hadronic molecules. In Refs. [43,56], the authors related the $D^{(s)}Σ_{cc}^{(s)}$ and $Σ_{cc}Σ_{cc}^{(s)}$ systems with the $D^{(s)}D^{(s)}$ system via HADS, and investigated the likely existence of hadronic molecules of $D^{(s)}Σ_{cc}^{(s)}$ and $Σ_{cc}Σ_{cc}^{(s)}$. Given that there exists a complete multiplet of HQSS hadronic molecules of $D^{(s)}Σ_{cc}^{(s)}$, it is natural to expect the existence of $Σ_{cc}^{(s)}Σ_{cc}^{(s)}$ hadronic molecules by replacing $D^{(s)}$ with $Σ_{cc}^{(s)}$ via HADS [57,58]. In this work, following Ref. [58], we relate the $D^{(s)}Σ_{cc}^{(s)}$ system to the $D^{(s)}Σ_{cc}^{(s)}$ system via HADS as shown in Fig. 1, and then employ the contact-range effective field theory (EFT) to investigate the likely existence of $D^{(s)}T_{cc}^{(s)}$ hadronic molecules. Such molecules composed of a charm meson and a compact doubly charmed tetraquark state are a new kind of triply charmed hexaquark states in addition to the dibaryon states $Σ_{cc}^{(s)}Σ_{cc}^{(s)}$ [58] and three-body molecules $D^{(s)}D^{(s)}D^{(s)}$ [59,60].

This paper is organized as follows. In Sect. 2, we explain how to derive the contact potential for the $D^{(s)}T_{cc}^{(s)}$ system constrained by HQSS, solve the Lippmann-Schwinger equation, and search for poles which correspond to $D^{(s)}T_{cc}^{(s)}$ molecules. In Sect. 3, we present the pole positions obtained with both single-channel and coupled-channel potentials. Finally, this paper is ended with a short summary in Sect. 4.

2 Theoretical formalism

In this section, we explain how to derive the contact-range potential for the $D^{(s)}T_{cc}^{(s)}$ system constrained by HQSS. Here we note that in terms of HADS the isospin of the $D^{(s)}T_{cc}^{(s)}$ system is 1/2 because the isospin of the $D^{(s)}Σ_{cc}^{(s)}$ system is 1/2. The $Σ^{(s)}_{cc}$ baryons are made of a charm quark and a pair of light quarks in the conventional quark model. HADS dictates that the charm quark of a $Σ^{(s)}_{cc}$ baryon can be replaced with a pair of anti-charm quarks, and then it transforms to a doubly charmed tetraquark state as shown in Fig. 1, where the pair of anti-charm quarks is in the color 3 state of spin 1. Combining with the spin and color configurations of a pair of light quarks, the doubly charmed tetraquark states can have spin 0, 1, or 2, i.e.,

$$\begin{align*}
|T_{cc}(0^+)| &= 1^3_H \otimes 1^3_L, \\
|T_{cc}(1^+)| &= 1^3_H \otimes 1^1_L, \\
|T_{cc}(2^+)| &= 1^3_H \otimes 1^3_L,
\end{align*}$$

(1)

where $1^3_H$ and $1^3_L$ represent an anti-charm quark pair and a light quark pair, respectively. In the following, we denote $T_{cc}(0^+)$, $T_{cc}(1^+)$, and $T_{cc}(2^+)$ as $T_{cc}^0$, $T_{cc}^1$, and $T_{cc}^2$, respectively. The existence of isovector tetraquark states $T_{cc}^{(s)}$ not only is allowed by Quantum Chromodynamics (QCD) but also were predicted by a series of theoretical studies [52,61–65]. In this work, we take the HADS to argue for their existence. The recent theoretical studies have shown that the masses of such isovector $T_{cc}^{(s)}$ states are above the mass thresholds of a pair of relevant charmed mesons [55,66–69], which implies that such states would decay into $DD\pi$ via an on-shell $D^*$ meson, leading to larger widths than that of $T_{cc}^0$ (3875) [70]. As a result, it will be necessary for the LHCb Collaboration to update their data sample and enlarge the energy region studied to search for these isovector $T_{cc}^{(s)}$ states. Since these states have not been discovered experimentally, we turn to the theoretical works [55,68,69,71] for their masses, which are given in Table 1. In the following numerical study, we use their averages given in the last column of Table 1.

The HQSS dictates that the interactions between light quarks are independent of the spin of the heavy quark(s). In the following, we apply HQSS to derive the $D^{(s)}T_{cc}^{(s)}$ contact-range potentials. First, we express the spin wave functions of the $D^{(s)}T_{cc}^{(s)}$ pairs in terms of the spins of the heavy quarks $s_{1H}$ and $s_{2H}$ and those of the light quarks(s) (often referred to as brown muck [72,73]) $s_{1L}$ and $s_{2L}$, where 1 and 2 denote...
Table 1: Masses (in units of MeV) of $T_{cc}^1$, $T_{cc}^2$, and $T_{cc}^3$ in several models and their averages.

| Tetraquark      | [55] | [69] | [71] | [68] | A.V |
|-----------------|------|------|------|------|-----|
| $T_{cc}^0$      | 3999.8 | 4132 | 4032 | 3969.2 | 4033.3 |
| $T_{cc}^1$      | 4124.0 | 4151 | 4117 | 4053.2 | 4111.3 |
| $T_{cc}^2$      | 4194.9 | 4185 | 4179 | 4123.8 | 4170.7 |

For the $T_{cc}^0$ single channel, the contact-range potential is

$$V_{J=0}^{T_{cc}^0} = \begin{pmatrix} C_a & 1/2 C_b & -3/2 C_b \\ \sqrt{3} C_b & C_a + \frac{1}{2} C_b & \frac{3}{2} C_b \\ \frac{3}{2} C_b & \frac{3}{2} C_b & C_a - \frac{1}{2} C_b \end{pmatrix}.$$  \(5\)

For the $T_{cc}^2$ single channel, the contact-range potential is $V_{J=0}^{T_{cc}^2} = C_a + C_b$. In the present work, we denote the diagonal and off-diagonal elements of the above matrices as elastic potentials and inelastic potentials of the $D^{(*)} T_{cc}^{(*)}$ system, respectively.

With the above potentials, we can solve the Lippmann-Schwinger equation

$$T = (1 - V G)^{-1} V,$$  \(6\)

where $V$ is the coupled-channel potential determined by the contact-range EFT approach described above, and $G$ is the two-body propagator. In evaluating the loop function $G$, we introduce a regulator of Gaussian form $e^{-2q^2/\Lambda^2}$ in the integral

$$G(s) = \int \frac{d^3 q}{(2\pi)^3} \frac{e^{-2q^2/\Lambda^2}}{\sqrt{s} - m_1 - m_2 - q^2/(2\mu_{12}) + i\epsilon},$$  \(7\)

where $\sqrt{s}$ is the total energy in the center-of-mass (c.m.) frame of $m_1$ and $m_2$, $\mu_{12} = \sqrt{m_1 m_2}$ is the reduced mass, and $\Lambda$ is the momentum cutoff. Following our previous works [74,75], we take $\Lambda = 0.75$ GeV and $\Lambda = 1.5$ GeV in the present work, which reflects the internal structure of the involved hadrons and provides an estimate of the uncertainties of the numerical results. The dynamically generated states correspond to poles in the unphysical sheet. In this sheet, the loop function of Eq. (7) becomes

$$G_{IJ}(s, m_1, m_2) = G(s, m_1, m_2) + i \mu_{12} \frac{p}{2\pi} e^{-2p^2/\Lambda^2},$$  \(8\)

where the c.m. momentum $p$ is

$$p = \sqrt{2\mu_{12}(\sqrt{s} - m_1 - m_2)}.$$  \(9\)

3 Results and Discussions

Following our previous works [58,74], the parameters $C_a$ and $C_b$ of the contact-range potentials are determined by reproducing the masses of $P_c(4440)$ and $P_c(4457)$ in two scenarios. In Scenario A, $P_c(4440)$ and $P_c(4457)$ have $J^P = 1/2^-$ and $J^P = 3/2^-$, while in Scenario B the assignment interchanges. In Table 2, we present the values of $C_a$ and $C_b$ obtained in Scenario A and B with $\Lambda = 1.5$ GeV and $\Lambda = 0.75$ GeV. One should note that the uncertainty of HADS is expected to be of the order of $\Lambda_{QCD}/(m_{QCD})$ [44].
where \( v \) is the velocity of the heavy quark pair. In general, the QCD scale is taken to be \( \Lambda_{QCD} \sim 200–300 \text{ MeV} \) and \( m_Q v \) is estimated to be 800 MeV for charm quarks. The breaking of HADS is then estimated to be 25–40\% [58]. The mass splitting between the \( J = 1/2 \) and \( J = 3/2 \) doubly charmed baryons obtained in lattice QCD simulations are 20–25\% smaller than the HADS prediction [46–49]. Therefore, we settle on a 25\% uncertainty for the \( \bar{D}^{(*)} T_{cc}^{(*)} \) potentials.

First, we search for poles of \( \bar{D}^{(*)} T_{cc}^{(*)} \) with single-channel potentials. In Table 3, we present the poles found in Scenario A and Scenario B, where the values inside and outside the brackets correspond to the cutoff \( \Lambda = 0.75 \text{ GeV} \) and \( \Lambda = 1.5 \text{ GeV} \), respectively. One can see that our results are weakly dependent on the cutoff. Even taking into account the 25\% uncertainty of HADS, all of these states are below the \( \bar{D}^{(*)} T_{cc}^{(*)} \) mass thresholds. Obviously, we obtain a complete HQSS multiplet of hadronic molecules composed of a \( \bar{D}^{(*)} \) meson and a \( T_{cc}^{(*)} \) tetraquark state. In other words, the existence of \( \bar{D}^{(*)} \Sigma_{cc}^{(*)} \) hadronic molecules implies the existence of \( \bar{D}^{(*)} T_{cc}^{(*)} \) molecules, if HQSS is approximately satisfied. We note in passing that the existence of \( \bar{D}^{(*)} \Sigma_{cc}^{(*)} \) molecules also implies the existence of a complete HQSS multiplet of \( \Sigma_{cc}^{(*)} \Sigma_{cc}^{(*)} \) hadronic molecules in terms of HADS [58], which agrees with the lattice QCD simulations [76].

Next we take into account the inelastic potentials, and search for poles with coupled-channel potentials. We present the results in Table 4, where some poles are located in the complex plane instead of in the real axis due to the contribution of the inelastic channels. We find that the resulting widths are rather small, similar to the \( \bar{D}^{(*)} \Sigma_{cc}^{(*)} \) molecules [36]. We stress that the number of \( \bar{D}^{(*)} T_{cc}^{(*)} \) molecules remain unchanged after including the inelastic potentials, and the masses of the \( \bar{D}^{(*)} T_{cc}^{(*)} \) molecules change little compared with the results shown in Table 3. That is to say, even with coupled-channel potentials, we still obtain a complete HQSS multiplet of \( \bar{D}^{(*)} T_{cc}^{(*)} \) hadronic molecules. One should note that as we reduce the strength of the \( \bar{D}^{(*)} T_{cc}^{(*)} \) potential, all the poles are still located in the same Riemann sheet. However, if we increase the strength of the potentials, some poles move to different Riemann sheets, leading to two poles located in the same Riemann sheet.

At the quark level, the \( \bar{D}^{(*)} T_{cc}^{(*)} \) molecules can be viewed as triply charmed hexaquark states. We note that in the molecular picture, such triply charmed hexaquark states can be formed by different combinations of hadrons.\(^1\) In the following, we investigate all the possibilities in detail. In terms of baryon number, the triply charged hexaquark states can be classified into two groups: one with baryon number \( B = 0 \) and the other with \( B = 2 \). In the \( B = 2 \) case, the triply charged hexaquark states can be formed by either the \( \Sigma_{cc}^{(*)} \Sigma_{cc}^{(*)} \) system or the \( \Omega_{cc} \Sigma_N \) system, where \( N \) represents the proton or neutron. In Ref. [58], utilizing HADS we have predicted a series of \( \Sigma_{cc}^{(*)} \Sigma_{cc}^{(*)} \) hadronic molecules from the

1 Of course, triply charged hexaquark states can also be compact multiquark states.
Table 4: Pole positions relative to the corresponding two-body thresholds (in units of MeV), $M - i\Gamma/2$, for the $D_s\Sigma_c^+$ molecules in Scenario A and Scenario B obtained with coupled-channel potentials.

| Molecule | $J^P$ | Threshold | $M_{\text{threshold}} - M$ (Scenario A) | $M_{\text{threshold}} - M$ (Scenario B) |
|----------|-------|-----------|----------------------------------------|----------------------------------------|
| $\bar{D}^* T^0_{cc}$ | $0^-$ | 5900.3 | 26.3±4.2$^e$14.2$^{+17.4}_{-13.0}$ | 34.2±4.2$^{+17.4}_{-13.0}$ |
| $\bar{D}^* T^1_{cc}$ | $1^-$ | 5978.3 | 29.0±4.2$^{+14.0}_{-14.0}$ | 34.2±4.2$^{+17.4}_{-13.0}$ |
| $\bar{D}^* T^2_{cc}$ | $2^-$ | 6037.7 | 29.2±4.2$^{+13.9}_{-13.9}$ | 34.2±4.2$^{+17.4}_{-13.0}$ |

The numbers inside and outside the brackets correspond to $\Lambda = 0.75$ GeV and $\Lambda = 1.5$ GeV. The superscripts and subscripts represent corrections induced by a 25% breaking of HADS.

existence of $\bar{D}^*(s)\Sigma_c^+(s)$ hadronic molecules. These predictions are based on the assumed molecular nature of the three pentaquark states, $P_c(4312)$, $P_c(4440)$, and $P_c(4457)$, discovered by the LHCb Collaboration in 2019. As for the $\Omega_{c\bar{c}}N$ system, we find that it can not bind because of the repulsive Coulomb force and the Okubo–Zweig–Iizuka (OZI) rule.

The triply charmed hexaquark states with baryon number $B = 0$ exist either in the form of two-body systems $\bar{D}^*\Sigma_c$ and $\Omega_{c\bar{c}}p$, or the three-body system $\bar{D}^*(s)\bar{D}^*(s)\bar{D}^*(s)$ as shown in Fig. 2. Among the three possible configurations, the existence of $\bar{D}^*\Sigma_c$ molecules have been studied in terms of HADS in the present work, which are based on the existence of $\bar{D}^*(s)\Sigma_c^+(s)$ molecules, similar to the existence of $\Sigma_c^+(s)\Sigma_c^-(s)$ molecules. In Refs. [59,60], we have confirmed the existence of three-body molecules $\bar{D}^*(s)\bar{D}^*(s)\bar{D}^*(s)$, which are related to the molecular nature of $T^0_{cc}$ (3875) recently discovered by the LHCb Collaboration. For the $\Omega_{c\bar{c}}p$ system, it is quite difficult to bind by the strong interaction due to the OZI rule, while it can bind by the Coulomb force as shown below.

The electromagnetic interaction of the $\Omega_{c\bar{c}}p$ system reads

$$V_{\Omega_{c\bar{c}}p} = -\frac{2}{\alpha} e^2,$$

(10)

where $\alpha = 1/137$ is the fine structure constant. Since the $\Omega_{c\bar{c}}$ baryon has not been observed so far, we take the mass $m_{\Omega_{c\bar{c}}} = 4795.6$ MeV predicted by lattice QCD [77], solving the Schrödinger equation via the Gaussian expansion approach, we find that there indeed exists a shallow $\Omega_{c\bar{c}}p$ bound state with a binding energy of 83.6 keV for the orbital angular momentum $l = 0$. Recall that for a Hydrogenlike atom, the energy level is given by

$$E_n = -\frac{\mu e^4}{2\hbar^2 n^2},$$

(11)

where $\mu$ is the reduced mass of the hydrogenlike atom, $e$ is the electron charge, and $Z$ is the charge number. Applying the above formula to the $\Omega_{c\bar{c}}p$ system, one obtains the binding energy 83.6 keV for the ground state, in agreement with our numerical result. In Fig. 3, we present the energy spectrum of the $\Omega_{c\bar{c}}p$ system, which is typical of a two-body system bound by the electromagnetic force. We note that the $\Omega_{c\bar{c}}p$ is similar to the $D^\pm D^{\ast\mp}$ hadronic atom of Ref. [78] but differs in that the strong interaction between $\Omega_{c\bar{c}}p$ and $p$ is more reduced because of the OZI rule.

It should be noted that in the molecule picture the triply charmed hexaquark states with $B = 0$ can have the form of either $\bar{D}^*(s)T^0_{cc}$, $\bar{D}^*(s)\bar{D}^*(s)$, or $\Omega_{c\bar{c}}p$ as shown in Fig. 2. We find that the $\bar{D}^*(s)\bar{D}^*(s)$ and $\bar{D}^*(s)T^0_{cc}$ molecules are significantly different from the $\Omega_{c\bar{c}}p$ atom due to their different binding mechanisms. As for the $\bar{D}^*(s)\bar{D}^*(s)$ and $\bar{D}^*(s)T^0_{cc}$ molecules, the number of $\bar{D}^*(s)\bar{D}^*(s)$ and $\bar{D}^*(s)T^0_{cc}$ molecules are less than that of $\bar{D}^*(s)T^0_{cc}$ molecules due to the constraint of Bose–Einstein statistics. In addition, the $\bar{D}^*(s)\Sigma_c$ molecules can decay into three charmed mesons [52], while the $\bar{D}^*(s)\bar{D}^*(s)$ and $\bar{D}^*(s)T^0_{cc}$ molecules are more likely to decay into three charmed mesons together with a $\pi$ or a $\gamma$ [59], which can be used to discriminate the $\bar{D}^*(s)T^0_{cc}$ and $\bar{D}^*(s)\Sigma_c^+(s)$ molecules.

4 Summary and conclusion

The pentaquark states, $P_c(4312)$, $P_c(4440)$, and $P_c(4457)$, discovered by the LHCb Collaboration, have been suggested to be $D^*(s)\Sigma_c$ hadronic molecules. Together with the other four states dictated by HQSS, a complete HQSS multiplet of hadronic molecules may exist. HADS allows one to relate the $\bar{D}^*(s)T^0_{cc}$ system to the $\bar{D}^*(s)\Sigma_c^+(s)$ system. The $T^0_{cc}$ are
Fig. 2 Three possible configurations of triply charmed hexaquark states with the same quark contents: three charmed mesons $D^{(*)} D^{(*)} D^{(*)}$ (a), a charmed meson and a doubly charmed tetraquark $D^{(*)} T_{cc}$ (b), and a triply charmed anti-baryon and a proton $\Omega_{ccc} P$ (c).

\[ n \quad E / \text{keV} \\
\infty \quad 0 \\
4 \quad -3.77 \\
3 \quad -3.29 \\
2 \quad -20.90 \\
1 \quad -83.59 \]

Fig. 3 Energy spectrum of the $\Omega_{ccc} P$ molecule for the orbital angular momentum $l = 0$.

isovector doubly charmed compact tetraquark states, which have been extensively studied after the discovery of $T_{cc}^+$ by the LHCb Collaboration. Since the masses of the isovector tetraquark states $T_{cc}^{(*)}$ are unknown, we took the average masses estimated by several theoretical models, which are the main uncertainty of our present work.

In terms of HADS, the $D^{(*)} T_{cc}^{(*)}$ potentials are the same as the $D_1^{(*)} \Sigma_{1S}^{(*)}$ potentials. We determined the unknown parameters of the $D^{(*)} T_{cc}^{(*)}$ potentials by reproducing the masses of $P_c$ (4440) and $P_c$ (4457) with two spin assignments. We then employed the Lippmann-Schwinger equation to search for poles near the mass thresholds of $D^{(*)} T_{cc}^{(*)}$ with single-channel and coupled-channel potentials, respectively. We find that, even taking into account a 25% breaking of HADS, there exists a complete HQSS multiplet of $D^{(*)} T_{cc}^{(*)}$ hadronic molecules composed of a charmed meson and a doubly charmed compact tetraquark state. Such states are a new kind of hadronic molecules, which, if discovered, will further enlarge the family of hadronic molecules. At last, we summarised the triply charmed hexaquark states formed by two kind of combinations of hadrons, $D^{(*)} T_{cc}^{(*)}$ two-body hadronic molecules, $\bar{D}^{(*)} \bar{D}^{(*)} \bar{D}^{(*)}$ three-body hadronic molecules, or $\Omega_{ccc} P$ hadronic atom with baryon number 0 and $\Xi_{cc} \Sigma_{1S}^{(*)}$ two-body hadronic molecules with baryon number $B = 2$. In particular, we predicted the existence of a different kind of triply charmed hadronic atom of $\Omega_{ccc} P$ held together solely by the Coulomb force. It is interesting to note that the same mechanism could also lead to the existence of other hadronic atoms as well, such as $\Omega_{ccc} \Omega_{ssS}$, $\Omega_{bbp}$, and $\Omega_{bbS} \Omega_{ccc}$.

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Appendix A: Contact-range potentials for the $T_{cc}^{(*)} \bar{D}^{(*)}$ system

The $T_{cc}^{(*)} \bar{D}^{(*)}$ states with spin $J = 0, 1, 2, 3$ are expressed in terms of the total spin of the light quark and heavy quark pairs as follows:

\[
|T_{cc}^{0} \bar{D}(0^-)\rangle = \frac{1}{\sqrt{3}} \frac{1}{2} H \otimes 1/2_L + \sqrt{\frac{2}{3}} \frac{3}{2} H \otimes 3/2_L,
\]

\[
|T_{cc}^{1} \bar{D}(1^-)\rangle = \frac{\sqrt{3}}{3} \frac{1}{2} H \otimes 1/2_L - \frac{1}{3} \frac{1}{2} H \otimes 3/2_L + \frac{1}{3} \frac{3}{2} H \otimes 1/2_L + \sqrt{\frac{2}{3}} \frac{3}{2} H \otimes 3/2_L,
\]
\[ |T_{ee}^2 \tilde{D}(2^-)\rangle = \frac{1}{\sqrt{3}} 3/2_H \otimes 1/2_L - \frac{1}{\sqrt{3}} 1/2_H \otimes 3/2_L + \frac{1}{\sqrt{3}} 3/2_H \otimes 3/2_L, \]
\[ |T_{ee}^0 \tilde{D}^*(1^-)\rangle = \frac{1}{3\sqrt{3}} 1/2_H \otimes 1/2_L + \frac{2}{3 \sqrt{3}} 1/2_H \otimes 3/2_L - \frac{2}{3 \sqrt{3}} 3/2_H \otimes 1/2_L + \frac{1}{3 \sqrt{3}} 3/2_H \otimes 3/2_L, \]
\[ |T_{ee}^1 \tilde{D}^*(0^-)\rangle = \frac{1}{\sqrt{3}} 3/2_H \otimes 1/2_L - \frac{1}{\sqrt{3}} 3/2_H \otimes 3/2_L, \]
\[ |T_{ee}^1 \tilde{D}^*(1^-)\rangle = \frac{1}{\sqrt{2}} 1/2_H \otimes 3/2_L + \frac{1}{\sqrt{2}} 3/2_H \otimes 1/2_L, \]
\[ |T_{ee}^1 \tilde{D}^*(2^-)\rangle = \frac{1}{\sqrt{2}} 3/2_H \otimes 1/2_L + \frac{1}{\sqrt{2}} 1/2_H \otimes 3/2_L + \frac{2}{\sqrt{2}} 3/2_H \otimes 3/2_L, \]
\[ |T_{ee}^2 \tilde{D}^*(1^-)\rangle = \frac{2}{3 \sqrt{3}} 1/2_H \otimes 1/2_L + \frac{1}{3 \sqrt{3}} 1/2_H \otimes 3/2_L + \frac{1}{3 \sqrt{3}} 3/2_H \otimes 3/2_L. \]

In the heavy quark limit, the interactions are only dependent on the degree of freedom of light quarks. As a result, we can write the matrix elements of the potentials as \(F_{1/2} = (1/2_L|V|1/2_L)\) and \(F_{3/2} = (3/2_L|V|3/2_L)\), then the potentials for the \(T_{ee}^{(s)} \tilde{D}^{(s)}\) system can be expressed in terms of two parameters, e.g., \(F_{1/2}\) and \(F_{3/2}\). For the \(T_{ee} \tilde{D}^* - T_{ee} \tilde{D}^*\) coupled channels, the contact-range potentials \(V\) in matrix form read

\[ V^{J=0}_{T_{ee}^0 \tilde{D}^* - T_{ee}^0 \tilde{D}^*} = \begin{pmatrix} \frac{1}{4} F_{1/2} + \frac{2}{3} F_{3/2} & -\frac{1}{3 \sqrt{3}} (F_{1/2} - F_{3/2}) & \frac{1}{\sqrt{3}} F_{1/2} - F_{3/2} \\ -\frac{1}{3 \sqrt{3}} (F_{1/2} - F_{3/2}) & \frac{1}{4} F_{1/2} + \frac{2}{3} F_{3/2} & -\frac{2}{3 \sqrt{3}} (F_{1/2} - F_{3/2}) \\ \frac{1}{3 \sqrt{3}} (F_{1/2} - F_{3/2}) & -\frac{2}{3 \sqrt{3}} (F_{1/2} - F_{3/2}) & \frac{1}{4} F_{1/2} + \frac{2}{3} F_{3/2} - \frac{1}{6 \sqrt{3}} (F_{1/2} - F_{3/2}) \end{pmatrix}. \]  

For the \(T_{ee}^1 \tilde{D}^* - T_{ee}^0 \tilde{D}^* - T_{ee}^1 \tilde{D}^* - T_{ee}^2 \tilde{D}^*\) coupled channels, the contact-range potentials \(V\) in matrix form read

\[ V^{J=0}_{T_{ee}^1 \tilde{D}^* - T_{ee}^0 \tilde{D}^* - T_{ee}^1 \tilde{D}^* - T_{ee}^2 \tilde{D}^*} = \begin{pmatrix} \frac{1}{4} F_{1/2} + \frac{2}{3} F_{3/2} & -\frac{1}{3 \sqrt{3}} (F_{1/2} - F_{3/2}) & \frac{1}{\sqrt{3}} F_{1/2} - F_{3/2} \\ -\frac{1}{3 \sqrt{3}} (F_{1/2} - F_{3/2}) & \frac{1}{4} F_{1/2} + \frac{2}{3} F_{3/2} & -\frac{2}{3 \sqrt{3}} (F_{1/2} - F_{3/2}) \\ \frac{1}{3 \sqrt{3}} (F_{1/2} - F_{3/2}) & -\frac{2}{3 \sqrt{3}} (F_{1/2} - F_{3/2}) & \frac{1}{4} F_{1/2} + \frac{2}{3} F_{3/2} - \frac{1}{6 \sqrt{3}} (F_{1/2} - F_{3/2}) \end{pmatrix}. \]  

For the \(T_{ee}^1 \tilde{D}^*\) single channel, the contact-range potential is

\[ V^{J=3}_{T_{ee}^0 \tilde{D}^*} = F_{3/2}. \]
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