Numerical analysis of blood flow in intracranial artery stenosis affected by ischemic stroke using Finite Element method

A Fatahillah1,a, S Setiawani2, A S Mandala3, S Suharto4, R P Murtikusuma5, L N Safrida6, S Hussen7 and R Adawiyah8
1,2,3,4,5,6,7,8 Mathematics Educational Study Program, University of Jember, Jember, Indonesia

E-mail : aarif.fkip@unej.ac.id

Abstract. Intracranial stenosis is a narrowing an artery inside the brain. This causes reduced blood flow to the cerebrum and damaged neurons in a short span of time due to reduced oxygenation. Intracranial stenosis can cause ischemic stroke due to a plaque occlusion. This research uses a mathematical model that expresses the velocity of arterial flow of an intracranial stenosis causing ischemic stroke that is expressed by the momentum equation and solved using the finite element method. The analysis of blood velocity is simulated using MATLAB and FLUENT software. This study looks at the effect of plaque thickness and initial speed of blood flow during intracranial stenosis. The results shows that the larger the plaque thickness, the smaller the area reduction, so the greater the increases blood velocity and the higher the initial velocity of blood that given, the greater the increase of the initial velocity of blood in the artery Intracranial stenosis. According to the MATLAB simulation, blood velocity with 80% plaque thickness increases blood velocity to 1.8665 m/sec at rest and 1.8825 m/sec at exercises. When the initial velocity of blood was 0.2 m/sec, it increases to 0.5712 m/sec at rest and 0.70058 m/sec at exercises.

1. Introduction
Stenosis (stenos) which means stricture, is common used in medical departments to explain a constriction in certain area [1]. Intracranial stenosis is an arterial constriction inside brain. This constriction may bring blood stream from the body to the brain reduced, then lead to brain cells damage due to lack of oxygen supply. Arterial intracranial stenosis may cause ischemic stroke due to embolism which is plaque formation [2].

Ischemic stroke is a medical condition that shows any dysfunctional or damage tissue in brain because of reducing blood stream into brain then disturbing blood and oxygen supply there. Ischemic stroke can be proceed from various medical condition. The most common condition is arterial stenosis in the brain. Ischemic stroke can also be proceed from various disease. However, a major cause of that is still the constriction of artery in brain or intracranial stenosis [3].

A relevant study conducted by Shi et al, [4] entitled "Suction Force-Suction Distance Relation During Aspiration Thrombectomy for Ischemic Stroke: A Computational Fluid Dynamics Study”, showed that using CFD has a good validity in giving the detail technical data including peak velocity in the tip of catheter as a noble knowledge which is useful and potential in influencing the next catheter design. Furthermore, a research conducted by Roy et al., [5] entitled “Modelling of Blood Flow in Stenosed Arteries” also showed that using CFD has a good validity in a simulation of blood streaming through
arterial stenosis in various obstruction. The result showed that the obstruction disturbed the blood stream then caused great friction force on arterial vessel. The condition of 90% thick obstruction inside arterial vessel might be very dangerous which could convert the transient stream into turbulence stream and that might be fatal to the patient.

Based on the previous research, this study modified the mathematical model of Shi and Roy. The mathematical model was finished by finite element method and quadratic approaches. The analysis and simulation of the model used MATLAB and FLUENT software. This study aimed to understand the mathematical model of blood stream velocity through arterial intracranial stenosis affected by ischemic stroke; to understand the plaque formation thickness (50 – 80% of arterial wide) and initial velocity by employing element method; and also to understand the blood stream velocity in arterial intracranial stenosis which cause ischemic stroke.

Mathematical modelling, a branch of mathematical field, can represent and explain the physical systems or the problem in the real life into mathematical expression. The result of this mathematical representation is called “Mathematical Model”[6]. The general form of mathematical model of blood stream velocity in intracranial stenosis affected by ischemic stroke uses momentum equation which is explained below:

\[
\frac{\partial \rho \phi_0}{\partial t} + \frac{\partial \rho u \phi_x}{\partial x} = -\frac{\partial P}{\partial x} + \frac{\partial}{\partial x} \left( \mu \frac{\partial u}{\partial x} \right)
\]

where,

\[u = 2 \bar{u} \left( 1 - \frac{r^2}{R^2} \right) \]  

\( \rho \) = blood specific mass \((1.06 \times 10^3)\), \( u \) = blood stream velocity, \( P \) = pressure, \( P_{\text{rest}}(t) \) = Fourier series from pressure in rest condition equation, \( P_{\text{exercise}}(t) \) = Fourier series from pressure in exercise condition equation, \( \bar{u} \) = average velocity, \( \bar{u}_{\text{rest}}(t) \) = Fourier series from velocity wave in rest condition equation, \( \bar{u}_{\text{exercise}}(t) \) = Fourier series from velocity wave in exercise condition equation, \( R \) = constriction radius and \( R \) = arterial radius. After that, the equation would be finished using finite element method to quadratic approach.

Once arterial vessel experienced stenosis, there will be a potential to patient having high blood pressure. Therefore, the high blood pressure wave profile was used in this analysis. For analyzing the physiological impact due to arterial stenosis, the two common condition which is rest and exercises in high blood pressure was used as limitation [5].

a. Rest Condition
\[
\bar{u}_{\text{rest}}(t) = 0.378 + 0.0893 \sin(2\pi \omega_1 t) - 0.10679 \cos(2\pi \omega_1 t) + 0.0418 \sin(4\pi \omega_1 t) - 0.0223 \cos(4\pi \omega_1 t) + 0.0167 \sin(6\pi \omega_1 t) - 0.0406 \cos(6\pi \omega_1 t) - 0.0086 \sin(8\pi \omega_1 t) - 0.0243 \cos(8\pi \omega_1 t) - 0.0118 \sin(10\pi \omega_1 t) - 0.0167 \cos(10\pi \omega_1 t)
\]
\[
P_{\text{rest}}(t) = 117.98 + 17.11 \sin(2\pi \omega_1 t) + 9.22 \cos(2\pi \omega_1 t) + 12.7 \sin(4\pi \omega_1 t) + 3.1 \cos(4\pi \omega_1 t) + 9.78 \sin(6\pi \omega_1 t) + 7.31 \cos(6\pi \omega_1 t) + 1.45 \sin(8\pi \omega_1 t) - 3.01 \cos(8\pi \omega_1 t) + 3.82 \sin(10\pi \omega_1 t) - 2.35 \cos(2\pi \omega_1 t)
\]

b. Exercise Condition
\[
\bar{u}_{\text{exercise}}(t) = 0.416 + 0.0168 \sin(2\pi \omega_1 t) - 0.0551 \cos(2\pi \omega_1 t) + 0.00225 \sin(4\pi \omega_1 t) - 0.0868 \cos(4\pi \omega_1 t) + 0.0236 \sin(6\pi \omega_1 t) - 0.0456 \cos(6\pi \omega_1 t) - 0.0335 \sin(8\pi \omega_1 t) - 0.0094 \cos(8\pi \omega_1 t) - 0.0883 \sin(10\pi \omega_1 t) - 0.0114 \cos(10\pi \omega_1 t)
\]
\[
P_{\text{exercise}}(t) = 117.98 + 17.11 \sin(2\pi \omega_1 t) + 9.22 \cos(2\pi \omega_1 t) + 12.7 \sin(4\pi \omega_1 t) + 3.1 \cos(4\pi \omega_1 t) + 9.78 \sin(6\pi \omega_1 t) + 7.31 \cos(6\pi \omega_1 t) + 1.45 \sin(8\pi \omega_1 t) - 3.01 \cos(8\pi \omega_1 t) + 3.82 \sin(10\pi \omega_1 t) - 2.35 \cos(2\pi \omega_1 t)
\]
2. Research method
Finite element method is kind of numeric approach method which is based on the problem of each element called finite element [7][8]. Therefore, finite element method is a numeric approach method used to solve continuum mechanics problems which based on each elements called finite element [9]. The method employed in this study is simulation. A study needs a procedure which contains a series of event until the result gained or the data was analyzed to reach the conclusion related to the aim of the study. The research concept is used as the basis of the steps of this study. The procedure employed in this study is doing literature review in term of blood stream velocity through arterial intracranial stenosis which cause ischemic stroke, then making blood stream through arterial intracranial stenosis model, and examining plaque formation thickness and the velocity of blood stream through arterial intracranial stenosis.

3. Results and discussion
The study of mathematical model of blood stream velocity through arterial intracranial stenosis affected by ischemic stroke is an equation which is expressed in momentum equation. Then, the equation would be finished by finite element method with quadratic approach and simulated by MATLAB and FLUENT software.

This blood stream simulation used MATLAB software which can help the researcher describe the result in each domain. The first simulation is the effect of plaque formation thickness with 50 – 80% thick of arterial wide in rest and exercise condition.

Figure 1. The graph of the blood stream velocity which is influenced by the thickness of the plaque in rest condition

Figure 2. The graph of the blood stream velocity which is influenced by the thickness of the plaque in exercise condition

Figure 1 and figure 2 shows that, blood stream velocity of 50% thick plaque was 0.2848 m/s in rest condition and 0.28725 m/s in exercise condition. At 55% thick plaque, the velocity was 0.36869 m/s in rest condition and 0.37185 m/s in exercise condition. At 60% thick plaque, the velocity was 0.48586 m/s in rest condition and 0.49003 m/s in exercise condition. Furthermore, at 65% thick plaque, the velocity began to rise significantly up to 0.65351 m/s in rest condition and 0.65911 m/s in exercise condition. At 70% thick plaque, the velocity was increasing until 0.90012 m/s in rest condition and 0.90784 m/s in exercise condition. At 75% thick plaque, the velocity was 1.2748 m/s in rest condition and 1.2858 m/s in exercise condition. The highest velocity was gained at 80% thick plaque. It gained 1.8665 m/s in rest condition and 1.8825 m/s in exercise condition. Based on the result, we can state that the bigger the size of plaque, the higher the velocity of the bloodstream was.
Table 1. Comparison of plaque formation thickness for rest and exercise conditions.

| Thickness of the plaque | 50%    | 55%    | 60%    | 65%    | 70%    | 75%    |
|-------------------------|--------|--------|--------|--------|--------|--------|
| Blood stream velocity   | Rest   | 0.2848 | 0.3687 | 0.4851 | 0.6535 | 0.9001 | 1.2748 |
|                         | Exercise| 0.2872 | 0.3715 | 0.4906 | 0.6592 | 0.9078 | 1.2858 |

The second simulation is about the effect of initial velocity to the blood stream velocity through arterial intracranial stenosis which occurred in both rest and exercise condition.

Figure 3 and figure 4 shows that, blood stream with initial velocity about 0.2 m/s, had gained blood stream velocity about 0.5712 m/s in rest condition and 0.70058 m/s in exercise condition. At 0.4 m/s initial velocity, the blood stream velocity was 2.2178 m/s in rest and 2.6248 m/s in exercise condition. At 0.6 m/s initial velocity, the blood stream velocity increased to 4.2432 m/s in rest and 4.7701 m/s in exercise condition. This case state that the higher the initial velocity, the higher the blood stream velocity in constriction area. Then, when the patients do some physical activities, the heart will beat faster to make sure the oxygen supply is available. Therefore, there will be an increasing velocity of blood stream in exercise condition rather than in rest condition.

Table 2. Comparison of initial velocity to the blood stream velocity for rest and exercise conditions.

| Initial velocity (m/s) | 0.2 | 0.4 | 0.6 |
|------------------------|-----|-----|-----|
| Blood stream velocity  | Rest|     |     |
|                         | Exercise| 0.70058 | 2.6248 | 4.7701 |

The result of simulation with FLUENT will present the figure of simulation from blood stream velocity through arterial intracranial stenosis which cause ischemic stroke. The model that will be simulated with FLUENT previously will be designed used GAMBIT program in 50%, 65% and 80% thick plaque formation. These various 50% and 80% thick plaque was selected as limitation thickness of the plaque while 65% thick plaque was selected due to its ability to express significant increasing velocity of blood stream which recorded by MATLAB.
The first simulation was aimed to understand the bloodstream velocity which is influenced by plaque thickness through arterial intracranial stenosis which cause ischemic stroke. The simulation was conducted using three GAMBIT with various thickness plaque about 50%, 65% and 80% of normal radius with initial velocity about 0.4 m/s.

Figure 5. Stream velocity vector for arterial intracranial stenosis with 50% thick plaque.

Figure 6. Stream velocity vector for arterial intracranial stenosis with 60% thick plaque.

Figure 7. Stream velocity vector for arterial intracranial stenosis with 80% thick plaque.

Figure 5 shows that bloodstream velocity through arterial Intracranial stenosis with 50% thick plaque was the lowest velocity. In this case, we can see the identical color which is blue in bloodstream into and out from the constriction area. Figure 6 shows that bloodstream velocity through arterial intracranial stenosis at 65% thick plaque was faster than at 50% thick plaque. In this case, we can see the green light-dominant color where bloodstream come out from the constriction area. Figure 7 shows that bloodstream velocity through arterial intracranial stenosis with 80% thick plaque was the highest velocity. In this case, we can see through the green dominant color where bloodstream come out from the constriction area.

The second simulation was about bloodstream influenced by initial velocity through arterial intracranial stenosis affected by ischemic stroke. This simulation used one GAMBIT which has 50% thick plaque and three various which are 0.2 m/s, 0.4 m/s and 0.6 m/s. The result of simulation can be seen at figure 8, 9 and 10.
Based on Figure 8, 9 and 10 from the three figures above, the velocity increased when bloodstream came into the constriction area. Figure 8 was when bloodstream entering the constriction area, the light became blue and turned into green then orange when it stayed inside the stricture, then it came out from the stricture bringing the blue light back. This case stated that the lowest velocity was occurred when it influenced by initial velocity about 0.2 m/s. Figure 9 revealed faster result rather than in figure 8. We can see it from the alteration of light-blue colour gradation which came out from the stricture. Moreover, figure 10 shows the simulation result had light-blue alteration dominantly in the area where bloodstream came out from the stricture. Therefore, the highest velocity was which influenced by initial velocity about 0.6 m/s. The velocity alteration can be seen on the number in the gradation indicator located in the left side of the figure.

Based on this research, the shape of the plaque on CFD is described into 3 models using different thick plaque to produce a more specific and detailed design than before. The relevant study conducted by Roy et al., [5] entitled “Modelling of Blood Flow in Stenosed Arteries”, stated that 90% thick plaque might cause serious problem. It could be seen at their study when it was about 90% thick plaque, the FLUENT could not figure any parabolic vector line formation from it which was quite different with in 50% and 75% thick plaque. However, in this study, when it was about 65% thick plaque, the significantly increased bloodstream was occurred as shown at figure 1. At Figure 3 (c), 80% thick plaque eventually had velocity vector line formation which was different with in 50% and 65%, then 80% thick plaque had a very high interruption intensity. These provide an explanation that before the plaque thickness increased up to 90%, in 80% level inside arterial vessel, it might be very dangerous condition and also had high risk on the damage of blood vessel due to the high bloodstream occurred.

4. Conclusion and suggestion
Based on the study about the bloodstream velocity through arterial intracranial stenosis affected by ischemic stroke with the thickness of plaque about 50 – 80 % of the sectional area, we conclude that the
thicker the plaque formation, the smaller the sectional area and the higher the velocity of the bloodstream was. Bloodstream velocity with initial rate about 0.2 m/s, 0.4 m/s and 0.6 m/s had increased airflow velocity respectively about 0.5712 m/s, 2.2178 m/s and 4.2432 m/s in the rest condition while respectively about 0.70058 m/s, 2.6248 m/s and 4.7701 m/s in exercise condition. The higher amount of initial velocity given, the higher the bloodstream velocity through arterial intracranial stenosis was.

After conducting a study about mathematical modelling on the bloodstream velocity through arterial intracranial stenosis affected by ischemic stroke, the researcher may give a suggestion about improving the study result by using real geometrically figure-based of the patient’s blockaded vessel. In the future, the authors will extend this work on patient-specific image-based real geometries of stenosis, extracted from Magnetic Resonance Imaging (MRI) on the patients and then the analyzing of another factor of blood pressure and plaque formation could be conducted.

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