Experimental Measurement of Absorption Coefficients for Effective Erbium-Doping Concentration to Optimize Few-Mode Erbium-Doped Fiber Amplifiers with Low Differential Mode Gain

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Abstract: According to the analytical expression for modal gain of few-mode erbium-doped fiber amplifiers (FM-EDFAs), we propose a method of measuring the absorption loss coefficients of few-mode signals in few-mode erbium-doped fibers (FM-EDFs) by extrapolating the mode–gain curve dependent on the average population inversion. The absorption loss coefficient of an FM-EDF was measured in our experimental platform and used to estimate the effective erbium-ion doping concentration. The feasibility of the extrapolation method was verified by simulation and comparison with the transmission method. Furthermore, the FM-EDFAs with high modal gain and low differential mode gain (DMG) could be optimized by adjusting the FM-EDF’s length and pump power. The analysis process presented here is very useful for the efficient design of FM-EDFAs from a practical point of view.

Keywords: few-mode erbium-doped fiber amplifier; erbium-ion doping concentration; mode division multiplexing; absorption loss

1. Introduction

Single-mode fiber (SMF) communication systems are facing a capacity limit resulting from fiber’s nonlinear effects [1]. The core or mode division multiplexing technology has become an effective way to increase fiber transmission capacity [2]. Just like the commercially available few-mode erbium-doped fiber amplifiers (EDFAs) used in SMF communication systems, few-mode EDFAs (FM-EDFAs) can also play an important role in mode division multiplexing (MDM) systems [3]. As a unique performance parameter for FM-EDFAs, the differential mode gain (DMG) not only affects the transmission performance of long-haul MDM systems [4], but also increases the complexity of the multiple-input multiple-output digital signal processing algorithm [5].

In 2011, Y. Jung et al. first realized the simultaneous amplification of two mode groups in FM-EDFAs and investigated the influence of the refractive index and erbium-ion distribution of the erbium-doped fiber (EDF) on the gain performance [6]. The same year, N. Bai et al. studied the gain equalization of LP_{01} and LP_{11}-mode signals under different pumping modes according to FM-EDFA theory [7]. In 2012, R. Ryf et al. proposed a modal gain control scheme for optimizing the thickness of the ring-doping layer in the active fiber [8], which was also used for two-mode gain equalization [9]. In 2017, D. Vigneswaran et al. proposed the mode-selective bidirectional pumping scheme for the uniform gain pattern for different LP modes with the minimal DMG close to 0 dB [10]. In 2021, Qayoom Taban et al. used the center-depressed erbium-ion doping profile to improve the pumping efficiency of the FM-EDFA supporting six mode groups, with the LP_{21a}-mode pump power of 55.567 mW and the minimum DMG of approximately 4.41 dB [11]. It is...
well-known that the DMG can be reduced by optimizing pump modes, refractive index profile, and erbium-doping concentration, or combining the above methods, in which the erbium-ion concentration distribution is the basis of optimizing the pumping scheme and the FM-EDF’s length in the simulation or design of FM-EDFAs. Unfortunately, the erbium-ion concentration distribution as a commercially confidential parameter is usually unavailable for the FM-EDFA’s designers. Therefore, it is necessary to characterize or measure the erbium-ion doping concentration by an effective method with application to the FM-EDFA’s theoretical model.

In the paper, we propose a measurement method for the effective erbium doping concentration in terms of modal absorption coefficients of FM-EDFs. The absorption coefficient of the fundamental mode was firstly measured by extrapolating the mode-gain curve dependent on the average population inversion. Then, the effective erbium-ion doping concentration of the EDF under test was estimated and used to calculate the absorption coefficients of high-order modes. The measured data were also used to optimally design the FM-EDFAs with high gain and low DMG under forward pumping.

The rest of the paper is organized as follows. Section 2 presents the analytical expression for modal gain of FM-EDFAs in terms of the average population inversion concentration in FM-EDFs. In Section 3, we build an experimental platform to measure the gain curve relative to the population inversion concentration by analyzing the modal splicing loss and crosstalk between the single- and few-mode fibers. Then, the absorption coefficient of LP_{01} mode and the effective erbium-ion concentration is obtained by calculation. The comparison between our measurement method and the traditional transmission method is given in Section 4. The effective erbium-ion concentration obtained above is also used to optimally design the FM-EDFAs with low DMG, and the detailed results are presented in Section 5. Section 6 discusses the repeatability of the experiment and the practicability of the extrapolation method for the absorption coefficients. Section 7 gives the conclusion.

### 2. Analytical Expression for FM-EDFA’s Gain

The optical amplification process in FM-EDFAs associated with population inversion can be explained by the two-level model of erbium ions as follows:

\[
\frac{\partial N_2(x, y, z, t)}{\partial t} = W_{13}(x, y, z)N_1(x, y, z, t) + W_{12}(x, y, z)N_1(x, y, z, t) - W_{21}(x, y, z)N_2(x, y, z, t) - \frac{N_2(x, y, z, t)}{T_1}
\]

(1)

where \(N_1(x, y, z, t)\) and \(N_2(x, y, z, t)\) are, respectively, the erbium-ion concentration at the ground and metastable states, with the total concentration \(N_0(x, y, z) = N_1(x, y, z) + N_2(x, y, z)\); \(T_1\) is the relaxation time of erbium ions at the metastable energy level; \(W_{13}, W_{12},\) and \(W_{21}\) represent the simulated transition rates of erbium ions between the corresponding energy levels.

Under the steady state, the erbium ion concentration on the metastable state can be given from Equation (1) as follows [12]:

\[
N_2(x, y, z) = \frac{\alpha_{ap} m}{v_p} \sum_{j=1}^{n} \left| f_j^{(p)}(x, y) \right|^2 p_j^{(p)}(z) + \frac{\alpha_{as} m}{v_s} \sum_{i=1}^{m} \left| f_i^{(s)}(x, y) \right|^2 p_i^{(s)}(z) \frac{N_0(x, y, z)}{1 + \frac{\alpha_{ap} m}{v_p} \sum_{j=1}^{n} \left| f_j^{(p)}(x, y) \right|^2 p_j^{(p)}(z) + \frac{\alpha_{as} m}{v_s} \sum_{i=1}^{m} \left| f_i^{(s)}(x, y) \right|^2 p_i^{(s)}(z)}
\]

(2)

where \(f_i^{(s)}(x, y)\) and \(f_j^{(p)}(x, y)\) are respectively the normalized mode field profiles of the signal and pump light, \(v_s\) and \(v_p\) are, respectively, the frequencies of signal and pump light, \(h\) is Planck’s constant, \(\alpha_{as}\) and \(\alpha_{es}\) are, respectively, the absorption and emission cross-sections of signals, and \(\alpha_{ap}\) is the absorption cross-section of pump light.
By integrating Equation (2) over the fiber cross-section, we can obtain the population inversion $\rho_i$ averaged over the length $\Delta L$ as follows [12]:

$$\rho_i = \frac{T_1}{\Delta L} \left( \frac{1}{h\nu_s} \sum_i P_{i,\text{in}}^{(s)} + \frac{1}{h\nu_p} \sum_j P_{j,\text{in}}^{(p)} - \frac{1}{h\nu_s} \sum_i P_{i,\text{out}}^{(s)} - \frac{1}{h\nu_p} \sum_j P_{j,\text{out}}^{(p)} \right)$$ (3)

Thus, the optical power of the signal mode $i$ output from the EDF section can be expressed by:

$$P_{i,\text{out}}^{(s)} = P_{i,\text{in}}^{(s)} \exp \left[ (B_i^{(s)} \rho_i - \alpha_i^{(s)}) \Delta L \right]$$ (4)

where $B_i^{(s)}$ and $\alpha_i^{(s)}$ are, respectively, the population inversion coefficient and the absorption coefficient related to the erbium-ion concentration, that is,

$$B_i^{(s)} = (\sigma_{as} + \sigma_{es}) \int \int \int \int N_i(x,y) \left| \vec{F}_i^{(s)}(x,y) \right|^2 \, dx \, dy$$ (5a)

$$\alpha_i^{(s)} = \sigma_{as} \int \int \int \int N_i(x,y) \left| \vec{F}_i^{(s)}(x,y) \right|^2 \, dx \, dy$$ (5b)

Similarly, we can also give the ASE noise power output from the length $\Delta L$ as follows:

$$P_{i,\text{out}}^{(\text{ASE})} = \left( P_{i,\text{in}}^{(s)} + \Delta P_{i,\text{ASE}}^{(s)} \right) \exp \left[ (B_i^{(s)} \rho_i - \alpha_i^{(s)}) \Delta L \right]$$ (6)

where,

$$\Delta P_{i,\text{ASE}}^{(s)} = \int_0^{\Delta L} \left( \frac{2\sigma_{es} h \nu_s \Delta \nu}{\exp \left[ (B_i^{(s)} \rho_i - \alpha_i^{(s)}) z \right]} \right) \int \int N_i(x,y,z) \left| \vec{F}_i^{(s)}(x,y) \right|^2 \, dx \, dy \, dz = \int_0^{\Delta L} \left( \frac{2h \nu_s \Delta \nu}{\exp \left[ (B_i^{(s)} \rho_i - \alpha_i^{(s)}) z \right]} \frac{\sigma_{es}}{\sigma_{as} + \sigma_{es}} B_i^{(s)} \rho_i \right) \, dz$$

is the power of the generated ASE noise, $\nu_s$ is the signal wavelength, $\Delta \nu$ is the equivalent amplifying bandwidth, and $P_{i,\text{in}}^{(s)}$ is the input ASE noise. In the paper, ASE is omitted in the calculation of population inversion, and the approximation is thought to be reasonable for the small ASE case [15]. Actually, the contribution of ASE power, $P_{i,\text{out}}^{(\text{ASE})}$, can be added to Equations (2) or (3) by substituting $P_{i,\text{out}}^{(\text{ASE})}$ with $P_{i,\text{out}}^{(s)} + P_{i,\text{out}}^{(\text{ASE})}$ for a large ASE case.

The output optical power of the pump mode is also of the form similar to that of the signal mode, by changing the superscript $(s) \rightarrow (p)$ and subscript $(as) \rightarrow (ap)$ in Equation (4) and Equation (5). Obviously, the background loss in EDFs is easily added into Equation (4) and is negligible for the short active fiber. According to Equation (4), the absorption coefficients $\alpha_i^{(s)}$ can be obtained from the gain value at $\rho_i = 0$, $G_{\rho_i=0}$ (in dB), that is,

$$\alpha_i^{(s)} = -0.23 \times G_{\rho_i=0} \Delta \nu^{-1}, \text{ (in m$^{-1}$)}$$ (7)

Furthermore, according to the overlap approximation method for FM-EDFAs [14], Equation (5) may also be rewritten as:

$$\alpha_i^{(s)} = \sigma_{as} \eta_i^{(s)} N_0$$ (8)

where $\eta_i^{(s)}$ is the overlap factor of the signal mode $i$, and $N_0$ is so-called effective erbium-ion concentration. In other words, $N_0$ can be calculated from the measured $\alpha_i^{(s)}$ for a given modal field. However, the case with $\rho_i = 0$ is not measurable in experiments due to the existence of the signal or the pump light in the EDFs. In this paper, we employed the extrapolation method to obtain the gain, $G_{\rho_i=0}$ or $\alpha_i^{(s,p)}$, according to the gain curve versus $\rho_i$ by adjusting the pump power.
3. Experimental Measurement of Modal Absorption Coefficient

3.1. Description of the Experimental Platform

In principle, to measure the absorption coefficient of each mode, it is necessary to build up a few-mode experimental platform. However, few-mode devices are still very expensive at present. In this paper, we took advantage of the fundamental mode LP$_{01}$ to calculate the effective erbium-ion concentration $N_0$ of the FM-EDF under test. The configuration and photo of the experimental setup are, respectively, shown in Figure 1a,b. As shown in Figure 1a, the signal and the pump were multiplexed into the 1:99 coupler by a wavelength division multiplexer (WDM). We measured the 1% port by an optical power meter (OPM) to calculate the powers input to the FM-EDF ($P_{in}^{(s)}$ and $P_{in}^{(p)}$). The 99% port was linked with the input G.652 SMF, and then the amplified light through the FM-EDF was coupled into the output G.652 SMF. The output powers from the SMF ($P_{out}^{(s)}$ and $P_{out}^{(p)}$) can be measured through the de-multiplexing ports. The pump power launched to the FM-EDF was controlled by a personal computer as shown in Figure 1b. After taking into account the coupling loss and excess loss, we could obtain the net input and output powers and then calculate $\rho_l$ from Equation (3).

![Figure 1](image1.png)

**Figure 1.** The configuration and photo of the experimental setup: (a) the configuration; (b) an image of the setup.

3.2. Mode Coupling and Excitation

The FM-EDF used here had a concave refractive index profile in the core, different from the G.652 SMF, as shown in Figure 2a,b. The modal excitation and conversion at 1550 nm signal and 980 nm pump are illustrated in Figure 2c,d, respectively. The mode excitation efficiency was simulated by VPI software.

From Figure 2, the SMF supports the LP$_{01}$ mode at 1550 nm and the LP$_{01}$, LP$_{11a}$, and LP$_{11b}$ modes at 980 nm. In contrast, the FM-EDF supports three modes (i.e., LP$_{01}$, LP$_{11a}$, and LP$_{11b}$) and six modes (i.e., LP$_{01}$, LP$_{11a}$, LP$_{11b}$, LP$_{21a}$, LP$_{21b}$, and LP$_{02}$) for the signal and pump light, respectively. For the fundamental-mode signal, the coupling or
excitation efficiency between the SMF and the EDF was 69.7% (−1.6 dB) with no crosstalk. Under the pump case, the LP\textsubscript{01} mode was converted to LP\textsubscript{01} and LP\textsubscript{02} modes in the FM-EDF, corresponding to the excitation efficiencies of 63.2% and 30.1%, respectively. The pump mode conversion had a great influence on the FM-EDFA’s gain or DMG, which was dependent on the average population inversion. A similar analysis was also applied to the EDF-to-SMF coupling. It should be pointed out that, for the LP\textsubscript{01}-mode pumping case, the pump coupling loss from the SMF to FM-EDF was 0.3 dB [15], different from the case of the FM-EDF-to-SMF coupling with the loss of 2.8 dB, since the two modes (i.e., LP\textsubscript{01} and LP\textsubscript{02}) in the FM-EDF were simultaneously excited in different efficiencies. When the signal and pump powers’ input and output from the FM-EDFA were measured for the total population inversion, the coupling loss between the SMF and the FM-EDF was taken into account.

Figure 2. Spatial modes supported in the SMF and FM-EDF: (a) connection of single-mode pigtail and few-mode erbium-doped fiber; (b) the refractive index of the two fibers; (c) mode excitation process at a 1550 nm wavelength; (d) mode excitation process at a 980 nm wavelength.
3.3. Extrapolation Method for Measurement of the Absorption Coefficient

By virtue of the experimental platform, as shown in Figure 1, we measured the gain values of the LP\textsubscript{01}-mode signal dependent on the average population inversion in the 1.9 m-long FM-EDF as shown in Figure 3, the intercept point of the fitting curve at \( \rho_l = 0 \) corresponding to \( G_{\rho_l=0} = -19.71 \) dB. From Equation (7), the absorption coefficient of the FM-EDF for LP\textsubscript{01}-mode signal was \( \alpha_{01} = 2.38 \) m\textsuperscript{−1}, corresponding to the effective erbium-ion concentration \( N_0 = 9.82 \times 10^{24} \) m\textsuperscript{−3}. In our calculation, the signal absorption and emission cross-sections and the pump absorption cross-section were used here as follows: \( \sigma_{as} = 2.49 \times 10^{-25} \) m\textsuperscript{2}, \( \sigma_{es} = 3.89 \times 10^{-25} \) m\textsuperscript{2}, and \( \sigma_{ap} = 2.86 \times 10^{-25} \) m\textsuperscript{2}. Further, according to the normalized modal field distribution of the LP\textsubscript{11}-mode signal, we could also obtain the absorption coefficient of the LP\textsubscript{11} mode from Equation (8), \( \alpha_{11} = 2.03 \) m\textsuperscript{−1}.

![Figure 3](image_url). The gain curve of the LP01-mode signal with the population inversion concentration.

4. Comparison of Experimental and Simulated Results

According to the effective erbium-ion concentration obtained previously, along with the erbium-doped fiber refractive index distribution and other experimental parameters, we simulated the gain characteristic of the setup shown in Figure 1 by VPI software. From Figure 3, the simulated results are basically identical to the experimental data. It is shown that the extrapolation method for the modal absorption coefficients is feasible, and then the effective erbium-ion concentration can also be estimated.

We also made use of the transmission method to measure the absorption coefficient of the LP\textsubscript{01} mode by the single-mode experiment platform in which only 1550 nm signal light was injected into the SMF and FM-EDF. The transmittance, defined by the ratio of output to input powers of signal modes, was obtained by adjusting the optical power input to the FM-EDF. Under the case with no pump light, both optical amplification and ASE noise did not occur. In the experimental test, the 3.2 dB coupling loss and 0.4 dB excess loss between the SMF and the FM-EDF were introduced. The measured transmission of the LP\textsubscript{01} mode is shown in Figure 4. For comparison, the simulated transmission curves of the LP\textsubscript{01} and LP\textsubscript{11} modes were also plotted in Figure 4, and the VPI simulation parameters were identical with those used in Figure 3. From Figure 4, the measured data were basically consistent with the simulated results for the LP\textsubscript{01} mode. It was reconfirmed that the effective erbium-ion concentration calculated from the absorption coefficient was credible. From Figure 4, the transmission depends on the input optical powers and applied to the case with low input power. The absorption coefficient of the LP\textsubscript{01} mode at the input power of \(-10\) dBm could be
calculated to be $\alpha_{01} = 2.40 \text{ m}^{-1}$, which was close to the value of $2.38 \text{ m}^{-1}$ obtained by our extrapolation method. Similarly, the simulated transmission coefficient was $\alpha_{11} = 2.08 \text{ m}^{-1}$ for the LP$_{11}$-mode signal. In contrast, our extrapolation method was especially applied to the erbium-doped fiber under the operating states of pumping and amplification.

**Figure 4.** Measured and simulated transmission curves.

5. Optimal Design of the 3M-EDFAs with forward Pumping LP$_{21}$-Mode

The parameters of modal gain and DMG were often used to characterize the amplification performance of the FM-EDFAs. The FM-EDFAs with high gain and low DMG are especially desirable for MDM transmission systems. In what follows, according to the effective erbium-ion concentration calculated from the measured absorption coefficient, we built a VPI simulation system to optimally design 3M-EDFAs under forward pumping as shown in Figure 5. The pump and three mode signals (i.e., LP$_{01}$, LP$_{11a}$ and LP$_{11b}$) were multiplexed into the FM-EDF and demultiplexed after amplification. The output powers were measured by the optical power meters to further calculate the modal gain and the DMG. Three signal modes had the same input power of $-10 \text{ dBm}$. The LP$_{21a}$-mode pumping was employed for a large overlap integral with LP$_{01}$ and LP$_{11}$ modes, which is also helpful for low DMG [7]. In the VPI simulation, the pump power and the length of FM-EDF were simultaneously swept by the step sizes of 100 mW and 0.1 m, respectively. On the 2D plane of the pump power and FM-EDF’s length, we plotted the contour lines of 20 dB modal gain, and the color values represent the corresponding DMG as shown in Figure 6. The region above the contour lines of $G_{01}$ and $G_{11} = 20$ dB had a gain of more than 20 dB. Similarly, the DMG lower than 2 dB was also bounded by the contour line of DMG = 2 dB. It should be pointed out that, the mode conversion between LP$_{11a}$ and LP$_{11b}$ modes in the short FM-EDFs was neglected in our simulation for simplicity; that is, the transverse profile of the signal or pump modal field was kept fixed or with no azimuthal rotation [16].

From Figure 6, if the modal gain $G \geq 20 \text{ dB}$ for all three mode signals was required for the 3M-EDFAs, it should be satisfied that the FM-EDF’s length $L \geq 1.8$ m and the pump power $P_p \geq 240$ mW. For example, at the lowest pump power of 240 mW, the optimal length of the FM-EDF was 3.8 m, with a DMG of 1.8 dB. As a reference, two contour lines of DMG = 0.1 dB were also plotted in Figure 6 for further optimizing 3M-EDFAs. Clearly, to design the 3M-EDFAs with a $G \geq 20 \text{ dB}$ and $\text{DMG} \leq 0.1 \text{ dB}$, the FM-EDF’s length and
pump power should be no less than 2 m and 650 mW, respectively. This optimization process was also applied to the case with backward or bidirectional pumping.

![Figure 5. The structure of the VPI simulation system.](image)

![Figure 6. The contour lines of modal gain and DMG to optimize the pump power and the FM-EDF’s length.](image)

6. Discussion

Here, we discuss the repeatability of the experiment and the practicability of the extrapolation method for the absorption coefficients of few-mode fibers.

The repeatability of the experiment mainly relies on the acquisition for the signal and pump powers’ input and output from the active erbium-doping fiber. However, the measurement of these parameters is usually taken in an indirect way due to the influence of insert loss, coupling loss, mode crosstalk, and other factors. In this paper, we calculated the coupling loss between the different fibers by the VPI software, and the insert loss was measured by experiment. The resulting coupling loss and excess loss introduced in our experiment were used to estimate the net powers for the experimental repeatability.
Before taking analysis on the practicability of the extrapolation method, we described several techniques for deriving the absorption coefficients as follows: (1) the indirect method, in which the absorption coefficients are indirectly deduced from the gain coefficients according to their relationship in terms of the known absorption and emission cross sections [17]; (2) the direct method, in which the absorption coefficients are directly calculated from the absorption cross-section and the uniformly distributed erbium-doping concentration, which can, respectively, be determined by the gain saturation measurement and the chemistry methods [18]; (3) the transmission method, which is equivalent to the case with no pump power. In Section 4, it was shown that the absorption coefficients given by the extrapolation method proposed in our paper were basically identical with the transmission method [19]. In other words, we also presented an alternative method, that is, the absorption coefficients can be estimated by the extrapolation of the gain curve dependent on the pump power in which the value of $G_{\rho=0}$ may approximately be replaced by the case with no pump power. By comparison, our method is especially applied to the erbium-doped fiber under the operating states of pumping and amplification. Most of the relevant references reported the gain curves dependent on pump power, but the data are not applied to the extrapolation method for the accurate absorption coefficient due to the inadequate fitting points [20,21]. In other words, it is difficult to compare our simulation results with the data presented in the literature.

7. Conclusions

In the paper, we put forward the extrapolation method of measuring the modal absorption coefficients $\alpha_i^{(s)}$ from the gain curve with $\rho_l$ and then estimate the effective erbium-ion concentration $N_0$ of the EDF under test. By building a single-mode experimental platform, the absorption coefficient of the LP_{31}-mode was measured, and it was basically consistent with that of the transmission method. The mode coupling and crosstalk between the SMF and the FM-EDF for the signal and pump light were also analyzed by the VPI simulation. Our simulation reconfirms the reliability of the effective erbium-ion concentration estimated by the extrapolation method. Finally, a universal process of optimally designing the 3M-EDFAs by the contour lines of modal gain and DMG on the 2D plane of the pump power and FM-EDF’s length was described in detail. It was shown by the VPI simulation that when the FM-EDF’s length and LP_{21a}-mode pump power are respectively more than 2 m and 650 mW, the 3M-EDFAs under forward pumping way can possess the desirable gain performance of $G \geq 20$ dB and DMG $\leq 0.1$ dB.

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References

1. Essiambre, R.-J.; Kramer, G.; Winzer, P.J.; Foschini, G.J.; Goebel, B. Capacity Limits of Optical Fiber Networks. *Lightwave Technol.* 2010, 28, 662–701. [CrossRef]

2. Guan, P.; Tang, M.; Cao, M.; Mi, Y.; Liu, M.; Ren, W.; Ren, G. Transverse Asymmetry of the Index Modulation Profile in Few-Mode Fiber Bragg Grating. *Photonics* 2021, 8, 87. [CrossRef]
3. Sillard, P.; Bigot-Astruc, M.; Molin, D. Few-Mode Fibers for Mode-Division-Multiplexed Systems. *J. Lightwave Technol.* **A Jt. IEEE/OSA Publ.** **2014**, *32*, 2824–2829. [CrossRef]

4. Ono, H.; Hosokawa, T.; Ichii, K.; Matsuo, S.; Yamada, M. Improvement of differential modal gain in few-mode fiber amplifier by employing ring-core erbium-doped fiber. *Electron. Lett.* **2015**, *51*, 172–173. [CrossRef]

5. Ryf, R.; Randel, S.; Grauch, A.H.; Bolle, C.; Essiambre, R.J.; Winzer, P.J.; Peckham, D.W.; McCurdy, A.; Lingle, R. Space-division multiplexing over 10 km of three-mode fiber using coherent $6 \times 6$ MIMO processing. In *National Fiber Optic Engineers Conference*; Optical Society of America: Los Angeles, CA, USA, 2011; p. PDPB10.

6. Jung, Y.; Alam, S.; Li, Z.; Dhar, A.; Giles, D.; Giles, I.P.; Sahu, J.K.; Poletti, F.; Grüner-Nielsen, L.; Richardson, D.J. First demonstration and detailed characterization of a multimode amplifier for Space Division Multiplexed transmission systems. *Opt. Express* **2011**, *19*, B952–B957. [CrossRef] [PubMed]

7. Bai, N.; Ip, E.; Wang, T.; Li, G. Multimode fiber amplifier with tunable modal gain using a reconfigurable multimode pump. *Opt. Express* **2011**, *19*, 16601–16611. [CrossRef][PubMed]

8. Kang, Q.; Lim, E.L.; Jung, Y.; Sahu, J.K.; Poletti, F.; Baskiotis, C.; Alam, S.; Richardson, D.J. Accurate modal gain control in a multimode erbium doped fiber amplifier incorporating ring doping and a simple $L P_{01}$ pump configuration. *Opt. Express* **2012**, *20*, 20835–20843. [CrossRef][PubMed]

9. Jung, Y.; Kang, Q.; Sleiffer, V.A.J.M.; Inan, B.; Kuschnerov, M.; Veljanovski, V.; Corbett, B.; Winfield, R.; Li, Z.; Teh, P.S.; et al. Three mode $Er^{3+}$ ring-doped fiber amplifier for mode-division multiplexed transmission. *Opt. Express* **2013**, *21*, 10383–10392. [CrossRef][PubMed]

10. Vigneswaran, D.; Ayyanar, N.; Sumathi, M.; Rajan, M.M. Tunable differential modal gain in FMEDFA system using dual pumping scheme at 100 Gbps system capacity. *Photonic Netw. Commun.* **2017**, *34*, 3. [CrossRef]

11. Qayoom, T.; Qazi, G. A comparative perspective on Differential Modal Gain reduction techniques for optimized few mode EDFA systems. *Optik* **2021**, *230*, 166285. [CrossRef] [PubMed]

12. Jiang, X.; Wu, B.; Xie, Y.; Wen, F.; Qiu, K. A semi-analytic method for FM-EDFA intensity model. *Opt. Fiber Technol.* **2021**, *64*, 102546. [CrossRef]

13. Lim, E.L.; Kang, Q.; Gecevicius, M.; Poletti, F.; Alam, S.U.; Richardson, D.J. Vector Mode Effects in Few Mode Erbium Doped Fiber Amplifiers. In Proceedings of the 2013 Optical Fiber Communication Conference and Exposition and the National Fiber optic Engineers Conference (OFC/NFOEC), Anaheim, CA, USA, 17–21 March 2013; pp. 1–3.

14. Chen, X.; Wu, B.; Xie, Y.; Wen, F.; Qiu, K. Analytical method for few-mode erbium doped fiber amplifiers. *Laser Phys. Lett.* **2020**, *17*, 035102. [CrossRef]

15. Noordegraaf, D.; Skovgaard, P.M.; Nielsen, M.D.; Bland-Hawthorn, J. Efficient multi-mode to single-mode coupling in a photonic lantern. *Opt. Express* **2009**, *17*, 1988–1994. [CrossRef] [PubMed]

16. Ho, K.P.; Kahn, J.M. Mode Coupling and Its Impact on Spatially Multiplexed Systems. In *Optical Fiber Telecommunications VI*; Kaminow, I.P., Li, T., Willner, A.E., Eds.; Academic Press: Los Angeles, CA, USA, 2013; pp. 491–568.

17. Giles, C.R.; Desurvire, E. Modeling Erbium-Doped Fiber Amplifiers. *J. Lightwave Technol.* **1991**, *9*, 271–283. [CrossRef]

18. Barnes, W.L.; Laming, R.I.; Morkel, P.R.; Tarbox, E.J. Absorption and Emission Cross Section of Er$_3^+$ Doped Silica Fibers. *Quantum Electron.* **1991**, *27*, 1004–1010. [CrossRef]

19. Jopson, R.M.; Saleh, A.A. Modeling of gain and noise in erbium-doped fiber amplifiers. *IEEE Photonics Technol. Lett.* **1990**, *2*, 714–717.

20. Lavrinovica, I.; Supe, A.; Porins, J. Experimental Measurement of Erbium-Doped Optical Fibre Characteristics for Edfa Performance Optimization. *Lett. J. Phys. Tech. Sci.* **2019**, *56*, 53–41. [CrossRef]

21. Le Cocq, G.; Bigot, L.; Le Rouge, A.; Bigot-Astruc, M.; Sillard, P.; Koebele, C.; Salsi, M.; Quiquempois, Y. Modeling and Characterization of a Few-Mode EDFA Supporting Four Mode Groups for Mode Division Multiplexing. *Opt. Express* **2012**, *20*, 27051–27061. [CrossRef] [PubMed]