Magnetic properties of spin-1/2 Fermi gases with ferromagnetic couplings

Baobao Wang\textsuperscript{1}, Jihong Qin\textsuperscript{1} and Huaiming Guo\textsuperscript{2}
\textsuperscript{1}Department of Physics, University of Science and Technology Beijing, Beijing 100083, China and \textsuperscript{2}Department of Physics, Beihang University, Beijing 100191, China

Within the mean-field theory, the magnetic properties of spin-1/2 charged Fermi gases with ferromagnetic interactions are investigated. A competition among the paramagnetism, diamagnetism and ferromagnetism are found in this system, and there exists a spontaneous magnetization for weak magnetic field. The intensity of the paramagnetic effect which comes from the spin degree of freedom is described by the Landé factor $g$. When the magnetic field intensity increases, both the paramagnetism and diamagnetism increase. The critical value of reduced ferromagnetic coupling constant $I_c$ increases with the increase of the reduced temperature $t$. The reduced magnetization density $M$ increases firstly as the temperature increases, then reaches a maximum, finally decreases smoothly in the high temperature region. The domed shape of the magnetization density $M$ is different from the behavior of Bose gas. We also calculate the susceptibility. We find the curve of susceptibility follows the Curie-Weiss law, and for a given temperature the susceptibility increases with the increase of the Landé factor $g$.

Keywords: Charged Fermi gases, Ferromagnetic coupling, Curie-Weiss law

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I. INTRODUCTION

In recent years the interests in the magnetism of materials, especially the magnetism of quantum Fermi gases have increased. It is well known that at low temperatures, the Fermi particles crowd into the lowest energy configuration, subject to the Pauli exclusion principle, which is different from the Bose statistics. Since the first observation of Bose-Einstein condensation in 1995, the field of trapped ultracold Fermi gases have received a lot of attention in recent years.\textsuperscript{1–4} In ultracold Fermi gases, after considering finite-temperature effects, the measured magnetic susceptibility of ideal gas is in quantitative accordance with the Pauli paramagnetic prediction.\textsuperscript{5}

The research on Pauli paramagnetism, Landau diamagnetism, and de Haas-van Alphen effect in an ideal Fermi gases goes beyond the result of Pauli-Landau on the magnetism of the three dimensional ideal gas of electrons, and is capable to describe the crossover of the de Haas-van Alphen oscillation to the saturation of magnetization.\textsuperscript{6} For the zero-field and magnetized pair-fermion gases in an external magnetic field, the intrinsic spin has an important contribution to the relativistic paramagnetism or diamagnetism.\textsuperscript{7}

The strong interacting fermions are ubiquitous in nature and play an important role in magnetism and superconductivity. In any confining geometry, by introducing a new technique that combines topological classification and the variational principle, the one-dimensional strongly-interacting fermionic systems with two or more internal spin states are exactly solvable in the hard-core limit.\textsuperscript{8} Many complex magnetic structures in solids can be understood within the Heisenberg model. In one and two dimensions, the low-temperature properties of quantum Heisenberg models, both ferromagnetic (FM) and anti-ferromagnetic (AFM) situation, have been investigated with two different large-$N$ formulations.\textsuperscript{9} The high temperature expansions of the Heisenberg model have been discussed for any lattice in relation to the Curie temperature in FM materials, and the spin susceptibility of the nearly FM Fermi systems is large and varies strongly with the change of temperature below the Fermi temperature $T_F$.\textsuperscript{10,11} Due to the magnetic dipole-dipole interaction, an ensemble of spinor Bose-Einstein condensates confined in a one-dimensional optical lattice can undergo a FM phase transition and spontaneous magnetization.\textsuperscript{12}

The present status of the theory of itinerant electron magnetism has been outlined with particular emphasis on the developments, which are characterized by the substantial advances in the theory of spin fluctuations.\textsuperscript{13} Itinerant ferromagnetism of a trapped two dimensional atomic gas has been studied, results show that the effective interaction strength has no relation to the particle number density, besides the characteristics of the FM phase are reinforced.\textsuperscript{14} The FM phase transition of a two dimensional itinerant electrons Stoner Hamiltonian has been studied with Quantum Monte Carlo calculations. With a screened Coulomb interaction, a first-order ferromagnetic transition for short screening lengths has been observed.\textsuperscript{15} Different from the conclusion of Bose gas that an infinitesimal value of the coupling can induce a FM phase transition,\textsuperscript{16} the Stoner coupling of Fermi gases $I_s$ cannot lead to a FM phase transition unless it is larger than a threshold $I_0$. Using the $1/d$ expansion theory, the two mechanisms of the Curie-Weiss law for the itinerant electron FM material have been investigated.\textsuperscript{17}

In this paper, by using the mean-field theory, the magnetic properties of charged spin-1/2 Fermi gases with FM interactions are studied. Our results uncover a competition among paramagnetism, diamagnetism and ferromagnetism, which is in contrast to the magnetic properties of a charged spin-1 Bose gas with FM interactions.\textsuperscript{18} We find that there is a spontaneous magnetization for weak magnetic field. As the increase of the reduced tem-
perature, there is not a pseudo-critical temperature for the charged spin-1/2 Fermi gases with FM interaction, which is different from the case of Bose gas. The relationship of susceptibility and the reduced temperature obey the Curie-Weiss law. The following structure of this paper is as follows: in section II a model containing Landau diamagnetism, Pauli paramagnetism and the FM effect is constructed. Then the total magnetization density and susceptibility are obtained through the analytical derivation. In section III, the results are obtained and discussed. A summary is given in section IV.

II. THE MODEL

The effective Hamiltonian of spin-1/2 Fermi gases with FM couplings is

\[ \hat{H} - \mu \hat{N} = D_L \sum_{j,k_z,\sigma} (\epsilon^j_{k_z} + \epsilon^e_{\sigma} + \epsilon^m_{\sigma} - \mu) \hat{n}_{j,k_z,\sigma}, \]

(1)

Then the grand thermodynamic potential is expressed as

\[ \Omega_{T\neq 0} = -\frac{1}{\beta} \ln \text{Tr} e^{-\beta(\hat{H} - \mu \hat{N})} \]

\[ = -\frac{1}{\beta} D_L \sum_{j,k_z,\sigma} \ln \left[ 1 + e^{-\beta(\epsilon^j_{k_z} + \epsilon^e_{\sigma} + \epsilon^m_{\sigma} - \mu)} \right] \]

(2)

where \( \beta = \frac{1}{k_B T} \), \( \mu \) is the chemical potential of the system.

The quantized Landau levels of the charged fermions are

\[ \epsilon^j_{k_z} = (j + \frac{1}{2}) \hbar \omega + \frac{\hbar^2 k_z^2}{2m^*}, \]

(3)

\( j = 0, 1, 2, \ldots \) labels different Landau levels, \( \omega = qB/m^*c \) is the gyromagnetic frequency, with charge \( q \), effective mass \( m^* \) and the magnetic induction intensity \( B \).

The degeneracy of the Landau levels is

\[ D_L = \frac{qBS}{2\pi \hbar c}. \]

(4)

where \( S \) is the section area of x-y plane of the system.

The Zeeman energy levels associated with the spin degree of freedom,

\[ \epsilon^e_{\sigma} = -\frac{q}{\hbar} \omega \hbar (qB/m^*c)\sigma = -g_\sigma \hbar \omega, \]

(5)

where \( g \) is the Landé factor, and \( \sigma \) denotes the spin-z index of Zeeman state \( |F = 1/2, m_F = \sigma \rangle \) \( (\sigma = 1/2, -1/2) \).

The contribution to the effective Hamiltonian from the FM couplings is

\[ \epsilon^m_{\sigma} = -4I\sigma (m + 2\sigma n_\sigma), \]

(6)

where \( I \) denotes FM coupling constant and relative particle number density \( m = n_{\frac{3}{2}} - n_{-\frac{3}{2}} \). Performing Taylor expansions and integrating out \( k_z \), then Eq. (2) becomes

\[ \Omega_{T\neq 0} = -\frac{\omega V}{\hbar^2} \left( \frac{m^*}{2\beta} \right)^\frac{3}{2} \sum_{l=1}^{\infty} \sum_{\sigma} \left[-1\right]^{l+1} \frac{\frac{3}{2} l\hbar \omega - g_\sigma \hbar \omega - 4I\sigma (m + 2\sigma n_\sigma) - \mu}{1 - e^{-\beta l\hbar \omega}} \]

(7)

where \( V \) is the volume of the system.

Some compact notations for the class of sums are introduced for the sake of convenience,

\[ F^*_T [\alpha, \delta] = \sum_{l=1}^{\infty} \left[-1\right]^{l+1} \frac{\frac{3}{2} l\hbar \omega - g_\sigma \hbar \omega - 4I\sigma (m + 2\sigma n_\sigma) - \mu + \delta}{1 - e^{-\beta l\hbar \omega}} \]

(8)

With this notation, equation (7) can be rewritten as

\[ \Omega_{T\neq 0} = -\frac{\omega V}{\hbar^2} \left( \frac{m^*}{2\beta} \right)^\frac{3}{2} \sum_{\sigma} F^*_T [-1,0] \]

(9)

where \( D = 3 \) is the space dimensionality.

The particle number density \( n = N/V \) can be obtained through the grand thermodynamic potential,

\[ n = -\frac{1}{V} \left( \frac{\partial \Omega_{T\neq 0}}{\partial \mu} \right)_{T,V} \]

\[ = \left( \frac{m^*}{2\beta \hbar^2} \right) \sum_{\sigma} F^*_T [-1,0] \]

(10)

where \( x = \beta \hbar \omega \).

Taking the grand thermodynamic potential derivative with respect to the magnetic induction intensity \( B \), the total magnetization density can be obtained

\[ M_{T\neq 0} = -\frac{1}{V} \left( \frac{\partial \Omega_{T\neq 0}}{\partial B} \right)_{T,V} \]

\[ = \frac{q \hbar}{m^* c} \left( \frac{m^*}{2\beta \hbar^2} \right)^\frac{3}{2} \sum_{\sigma} \left[ F^*_T [-3,0] + x \left[ \left( g_\sigma - \frac{1}{2} \right) F^*_T [-1,0] - F^*_T [-1,1] \right] \right] \]

(11)

The relation among the magnetic induction intensity \( B \), external magnetic field \( H \) and total magnetization density \( M \) is formally expressed as

\[ B = H + 4\pi M \]

(12)

We introduce some dimensionless parameters for computational convenience, such as

\[ t = \frac{T}{T^*}, \tilde{M} = \frac{m^* c M}{\hbar q}, \tilde{\omega} = \frac{\hbar \omega}{k_B T^*}, \tilde{I} = \frac{I n}{k_B T^*}, \tilde{\mu} = \frac{\mu}{k_B T^*}, \tilde{n}_\sigma = \frac{n_\sigma}{n}, \tilde{h} = \frac{\hbar H}{m^* c k_B T^*}, \tilde{x} = \frac{\tilde{\omega}}{t} \]

The characteristic temperature of the system \( T^* \) is given by \( k_B T^* = 2\pi \hbar^2 n^*/m^* \).
The mean-field self-consistent equation are derived as follows:

\begin{equation}
1 = \bar{\omega} t^2 \sum_{\sigma} \bar{F}_{1}^{\sigma} [-1, 0] \tag{13a}
\end{equation}

\begin{equation}
\bar{M}_{T \neq 0} = t^2 \sum_{\sigma} \left\{ \bar{F}_{1}^{\sigma} [-3, 0] + \frac{\bar{\omega}}{t} \left[ (g\sigma - \frac{1}{2}) \bar{F}_{1}^{\sigma} [-1, 0] - \bar{F}_{2}^{\sigma} [-1, 1] \right] \right\} \tag{13b}
\end{equation}

\begin{equation}
\bar{\omega} = h + 4\pi \gamma \bar{M} \tag{13c}
\end{equation}

where \( \bar{\omega} = (q^2 n^{1/3})/(2\pi m^* c^2) \), and

\begin{equation}
\bar{F}_{\sigma}^{\sigma} [\alpha, \delta] = \sum_{l=1}^{\infty} (-1)^{l+1} l^{\alpha/2} e^{-l \bar{\omega}} \left[ \frac{\delta - g\sigma - \frac{1}{2}(m + 2\pi l + \bar{\rho})}{1 - e^{-l \bar{\omega}}} \right] \tag{14}
\end{equation}

where \( \bar{\mu} \) is the dimensionless parameter of the chemical potential, which can be determined from the mean-field self-consistent calculations.

At last, we calculated the susceptibility of charged Fermi gases with FM couplings. From the formula \( \chi_M = \left( \partial M / \partial H \right)_{T, V} \), the derivation of equation (11) for the magnetic field \( H \), the expression of susceptibility can be obtained as follows,

\begin{equation}
\chi_M = \frac{\gamma b}{\bar{\omega} - 4\pi \gamma b} \tag{15}
\end{equation}

with

\begin{equation}
b = 2\bar{\omega} t^{1/2} \sum_{\sigma} (g\sigma - \frac{1}{2}) \bar{F}_{1}^{\sigma} [-1, 0] - 2\bar{\omega}^2 t^{-1/2} \tag{16}
\end{equation}

\begin{equation}
\sum_{\sigma} (g\sigma - 1) \bar{F}_{2}^{\sigma} [1, 1] - 2\bar{\omega} t^{1/2} \sum_{\sigma} \bar{F}_{2}^{\sigma} [-1, 1] + \bar{\omega}^2 t^{-1/2} \tag{16}
\end{equation}

\begin{equation}
\sum_{\sigma} (g\sigma - 1) \bar{F}_{2}^{\sigma} [1, 0] + 2\bar{\omega}^2 t^{-1/2} \sum_{\sigma} \bar{F}_{2}^{\sigma} [1, 2] \tag{16}
\end{equation}

III. RESULTS AND DISCUSSIONS

In the following calculations from figures 1~5 and figure 7, the characteristic parameter \( \gamma \) has been set as \( 10^{-6} \), which is estimated for a system with the charge and mass of nearly thin electron gas, and the particle density being set as \( 8/nm^3 \). Figure 1 is plotted in a weak reduced magnetic field \( h = 0.005 \). As shown in figure 1(a), when the reduced FM coupling constant \( \bar{I} \) is smaller than a certain critical value of reduced FM coupling constant \( \bar{I}_c \), the reduced total magnetic density \( \bar{M} \) is always equal to zero. Only when \( \bar{I} \) is equal to \( \bar{I}_c \), \( \bar{M} \) begins to increase from zero. It suggests that with the increase of \( \bar{I} \), there exists a spontaneous magnetization when the reduced magnetic field \( h \) is very small. In figure 1(b), we can find that although the value of Landé factor \( g \) changes, the curve of the reduced relative particle number density \( \bar{m} = n_{1/2} - \bar{n}_{-1/2} \) versus \( \bar{I} \) entirely overlaps. It suggests that \( \bar{m} \) is independent of the Landé factor \( g \). And for the given reduced temperature \( t \) the value of \( \bar{I}_c \) is identical, although the values of landé factor are different.

It is shown that the critical value of FM coupling constant \( \bar{I}_c \) approximates to 0.5 in figure 1. When \( \bar{I} < \bar{I}_c \), \( \bar{m} \) is equal to zero, and the value of \( \bar{m} \) increases with the increase of \( \bar{I} \) from \( \bar{I} > \bar{I}_c \) until saturation. The reduced total magnetization density \( \bar{M} \) increases with the Landé factor when the value of \( \bar{I} \) is fixed in the region of \( \bar{I} > \bar{I}_c \), which owes to the contribution of paramagnetism. It shows that there exists a competition among diamagnetism, paramagnetism and ferromagnetism in such a system. The diamagnetism caused by spontaneous magnetization cannot overcome ferromagnetism in a very weak magnetic field. While the competition between
paramagnetism and diamagnetism has been discussed in Ref.\textsuperscript{12}.

Figure 2 plots the threshold of reduced FM coupling constant vs reduced temperature at reduced magnetic field $h = 0.005$. The region above $I_c$ is the FM phase, while the region below $I_c$ is the paramagnetic phase. The value of $I_c$ increases monotonically with the increase of reduced temperature. The spontaneous magnetization is difficult to occur at high temperature, when the Fermi statistics reduces to Boltzmann statistics. The results is similar to the Bose gas with FM coupling\textsuperscript{12}, although they submit to different statistical rules, respectively.

Figure 1 and 2 have discussed the spontaneous magnetization in the weak reduced magnetic field. Figure 3 is plotted in order to understand the influence of Landé factor $g$. Comparing to the figure 1 and figure 2, the magnetic field is chosen relatively strong as $h = 0.8$ and the reduced temperature $t = 1.5$ in figure 3. In figure 3(a), the horizontal line $M = 0$ is used to distinguish the region of paramagnetism and diamagnetism. Merely for the fixed small value of $g$, the value of $M$ changes slowly when the value of FM coupling constant $I$ varies. This is similar to the result of the charged Bose gas with FM coupling\textsuperscript{12}. When $g$ is small $M$ is negative, which mainly derives from the diamagnetic contribution. The following figure 3(b) shows that, with the increase of Landé factor $g$, $\bar{m}$ increase until approximates saturation at higher $I$. We can find that the larger $I$ the larger $\bar{m}$ for the identical value of Landé factor $g$, where the Landé factor denotes the intensity of the paramagnetism\textsuperscript{19,20}.

Above we have discussed the relationship between magnetization density $\bar{M}$ and Landé factor $g$. Now we turn to investigated the influence of magnetic field. The gas show diamagnetism when the Landé factor $g$ is small in figure 4. What is more, when the Landé factor $0.3 < g < 0.6$, there exists a threshold $g_c$ of Landé factor $g$ making $\bar{M} < 0$ when $g < g_c$, which is independent of the magnetic field. When the Landé factor $g = 0.6$, the reduced magnetization density is close to a small positive value which approximates zero at the beginning. As the increase of the magnetic field, the reduced magnetization density increases at first and then descends when the Landé factor $g \geq 0.6$. When the magnetic field continues to increase, diamagnetism increases and exceeds paramagnetism at higher magnetic field.

To further understand the paramagnetism and diamagnetism in detail. The dependence of the reduced paramagnetism density $M_p = g \bar{m}$ and the reduced diamagnetism density $M_d = M - M_p$ versus the reduced magnetic field $h$, at reduced temperature $t = 1.5$ and $\bar{t} = 0.1$, are studied in figure 5. $M_p$ tends to saturate when the magnetic field intensity is strong. While the absolute...
FIG. 4: (a) The reduced magnetization density $\bar{M}$, (b) the reduced relative particle number density $\bar{m} = \bar{n}_{1/2} - \bar{n}_{-1/2}$, as a function of reduced magnetic field $h$ at $t = 1.5$ and $I = 0.1$, where Landé factor $g = 0.1$ (solid line), 0.3 (dashed line), 0.6 (dotted line), 0.8 (dash dotted line), and 1.0 (dash dot dotted line).

The relationship between the reciprocal of susceptibility and the reduced temperature $t$ are investigated in figure 7. It can be seen from the diagram that the curves

a maximum, and ultimately decreases at high temperature region. The upward trend at low temperature region reflects that, with the increase of temperature, the diamagnetic contribution to the total magnetization density weakens. While a flat decline can be seen when $\bar{M}$ is close to zero, which can be seen clearly from the inset of figure 6(a). This is obviously different from the Bose gas. In our present study on Bose gas with FM coupling, a sharp decline can be seen when $\bar{M}$ is close to zero, which suggests that there is a pseudo-condensate temperature in the transition from ferromagnetism to paramagnetism. However, for the charged Fermi gases with FM interaction, we find when the temperature is high, the reduced total magnetization density $\bar{M}$ and the reduced relative particle number density $\bar{m}$ are asymptotic with respect to the zero point for different values of $\bar{I}$. It suggests that there does not exist the pseudo-critical temperature for Fermi gases. This may ascribe to that they obey different statistical distribution, respectively.

In order to investigate the FM phase transition of the charged spin-1/2 Fermi gases, we assume $\gamma = 10$. The evolution of the reduced magnetization density $\bar{M}$ and the reduced relative particle number density $\bar{m} = \bar{n}_{1/2} - \bar{n}_{-1/2}$ versus reduced temperature $t$ at $h = 0.005$ and $g = 0.8$ is shown in figure 6. It is shown that $\bar{M}$ increases with increasing temperature, then reaches

value of diamagnetism density $|\bar{M}_d|$ increases with the increase of magnetic field intensity. This is in conformity with the result of figure 4 in qualitatively. With the increase of magnetic field, the contribution come from ferromagnetism, paramagnetism and diamagnetism reach to maximum, so the peak of total magnetization density appears when Landé factor $g$ is 0.8 or 1.0. But the diamagnetism increase faster than paramagnetism as the magnetic field continues to increase. This accounts for the reduced total magnetization decreases when the magnetic field is strong in figure 4. However, this will not change the diamagnetism of this system when the magnetic field is high, which is different from the conclusion of Bose gas with FM coupling.

FIG. 5: (a) The reduced paramagnetism density $\bar{M}_p$, (b) the reduced diamagnetism density $\bar{M}_d$ as a function of reduced magnetic field $h$ at $t = 1.5$ and $I = 0.1$, where Landé factor $g = 0.1$ (solid line), 0.3 (dashed line), 0.6 (dotted line), 0.8 (dash dotted line), and 1.0 (dash dot dotted line).
FIG. 6: (a) The reduced magnetization density $\bar{M}$, (b) the reduced relative particle number density $\bar{m}$ versus the reduced temperature $\bar{t}$ at reduced magnetic field $\bar{h} = 0.005$ and Landé factor $g = 0.8$, where $\bar{I} = 0.1$ (solid line), 0.5 (dashed line), 0.6 (dotted line), 0.8 (dash dotted line), and 1.0 (dash dot dotted line). The inset of figure (a) depicts the smooth decline of reduced magnetization density $\bar{M}$ when the reduced temperature lies between 10 and 15.6.

of $1/\chi \sim t$ conforms to the Curie-Weiss law, meaning the susceptibility is inversely proportional to the reduced temperature $t$. The susceptibility of Fermi gases with FM coupling is subject to Curie-Weiss law, which is similar to the conclusion of the FM material. It can be found that when the reduced temperature is fixed, the susceptibility increases with the increase of the Landé factor $g$. The Landé factor $g$ denotes the strength of paramagnetism, so it shows that the increase of susceptibility is mainly attributed to the contribution of paramagnetism at the fixed reduced temperature.

IV. SUMMARY

In summary, we study the interplay among paramagnetism, diamagnetism and ferromagnetism of the charged spin-$1/2$ Fermi gases with FM interaction in the framework of the mean-field theory. In a very weak magnetic field, it is shown that the ferromagnetism is stronger than the diamagnetism when $I > I_c$. The critical value of the reduced FM coupling constant $\bar{I}_c$, which denotes the paramagnetic phase to the FM phase transition, increases with the increase of the reduced temperature $\bar{t}$. The Landé factor $g$ is served as a variable to evaluate the strength of the paramagnetic effect. In a strong magnetic field, Fermi gases exhibits a shift from diamagnetism to paramagnetism when $g$ increases. The reduced magnetization density $\bar{M}$ shows negative values for the high intensity magnetic field, which shows that the diamagnetism dominates in the strong magnetic field. As the increase of reduced temperature, the total reduced magnetization density asymptotically approximates to zero, which is different from the case of Bose gas with FM coupling. It shows that there is not a pseudo-critical temperature for charged Fermi gases with FM interaction. The diagram of susceptibility is in accordance with the Curie-Weiss law. The values of susceptibility increase as the increase of Landé factor $g$.

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* Corresponding author. Tel.&fax: +86 10 62316177. E-mail address: jhqin@sas.ustb.edu.cn

1. J. Noronha and D. J. Toms, Phys. Lett. A 267 (2000) 276.
2. S. Giorgini, L. P. Pitaevskii and S. Stringari, Rev. Mod. Phys. 80 (2008) 1215.
3. I. Bloch, Nature Physics 1 (2005) 23.
4. D. Greif, T. Uehlinger, G. Jotzu, L. Tarruell, and T. Esslinger, Science 340 (2013) 1307.
5. Y. R. Lee, T. T. Wang, T. M. Rvachov, J. H. Choi, W. Ketterle, and M. S. Heo, Phys. Rev. A 87 (2013) 043629.
6. S. Biswas, S. Sen, and D. Jana, Phys. of. Plasm. 20 (2013) 052503.
7. J. Daicic, N. E. Frankel, R. M. Gailis and V. Kowalenko, Physics reports 237 (1994) 63.
8. A. G. Volosniev, D. V. Fedorov, A. S. Jensen, M. Valiente, and N. T. Zinner, arXiv 1306.4610.
9. D. P. Arovas and A. Auerbach A, Phys. Rev. B 38 (1988) 316.
10. G. S. Rushbrooke and P. J. Wood, Proceedings of the Physical Society. Section A 68 (1955) 1161.
11. S. G. Mishra and T. V. Ramakrishnan, Phys. Rev. B 18 (1978) 2308.
12. H. Pu, W. Zhang, and P. Meystre, Phys. Rev. Lett. 87 (2001) 140405.
13. T. Moriya, Journal of magnetism and magnetic materials 100 (1991) 261.
14. G. J. Conduit, Phys. Rev. A 82 (2010) 043604.
15. G. J. Conduit, Phys. Rev. B 87 (2013) 184414.
16. Q. Gu and R. A. Klemm, Phys. Rev. A 68 (2003) 031604.
17. E. Miyai and F. J. Okawa, Phys. Rev. B 61 (2000) 1357.
18. J. H. Qin, X. L. Jian, and Q. Gu, J. Phys.: Condensed Matter 24 (2012) 366007.
19. X. L. Jian, J. H. Qin, and Q. Gu, Phys. Lett. A 374 (2010) 2580.
20. X. L. Jian, J. H. Qin, and Q. Gu, J. Phys.: Condensed Matter 23 (2011) 026003.