A Chiral Supersymmetric Standard Model

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Abstract

We propose a supersymmetric extension of the Standard Model with an extra $U(1)$ gauge symmetry, so that all supersymmetric mass terms, including the $\mu$-term, are forbidden by the gauge symmetries. Supersymmetry is broken dynamically which results in $U(1)$ breaking and generation of realistic $\mu$ term and soft breaking masses. The additional fields required to cancel the $U(1)$ anomalies are identified with the messengers of supersymmetry breaking. The gaugino masses arise as in the usual gauge mediated scenario, while squarks and sleptons receive their masses from both the $U(1)$ $D$-term and the two-loop gauge mediation contributions. The scale of supersymmetry breaking in this model can be below $10^6$ GeV, yielding collider signatures with decays to goldstinos inside the detector.

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1 Introduction

The main theoretical shortcoming of the Standard Model (SM) is the lack of chirality of the Higgs sector: the gauge symmetry does not prevent a large mass for the Higgs doublet, which results in the hierarchy problem. Supersymmetry (SUSY) confers chirality to scalars, and therefore offers the hope of explaining the hierarchy between the electroweak scale and the Planck scale. However, in the Minimal Supersymmetric Standard Model (MSSM), the Higgs sector is still not chiral: the gauge symmetry allows a $\mu$ term of order the Planck scale. Furthermore, most realistic models with dynamical SUSY breaking require gauge singlets and additional vector-like fields.

Given the current lack of understanding of quantum gravity, the only rigorous way of avoiding dangerously large masses for singlet or vector-like representations, is to introduce gauge symmetries which forbid any linear or quadratic term in the superpotential. Since at least one of the Higgs doublets must carry the new charges, the simplest choice for the additional gauge group is a $U(1)$.

The known examples of $U(1)$ gauge symmetries that prevent a large $\mu$ term [1] have been constructed in the framework of supergravity mediated SUSY breaking. However, in order to give large enough gaugino masses, the dynamical SUSY breaking sector has to include a gauge singlet which couples to the gauge superfields [2]. Therefore, such models are not chiral.

Thus, we are led to consider a gauge mediated SUSY breaking scenario [3]. The $U(1)$ gauge group has to be anomaly free, so that the model must include extra fields which are vector-like under the SM gauge group and charged under the $U(1)$. These fields may naturally play the role of the messenger sector. In this letter we construct a complete, renormalizable and calculable model of gauge mediation, including an explicit dynamical SUSY breaking (DSB) sector. We use the new $U(1)$ gauge symmetry both to forbid the $\mu$ term and to communicate SUSY breaking to the SM superpartners. As a result, the scale of $U(1)$ breaking and the $\mu$ parameter are related to the scale of SUSY breaking. The model has no pure gauge singlets, nor vector-like representations, i.e., it is a purely chiral supersymmetric Standard Model.
2 Model Building

Our starting point is, within the MSSM, to forbid the $H_uH_d$ term in the superpotential by charging the two Higgs superfields under a new $U(1)_\mu$ gauge group. In order to have a Higgsino mass, the usual $\mu$ term of the Higgs sector must come from an $SH_uH_d$ term in the superpotential, where $S$ (which is a $SU(3)_C \times SU(2)_W \times U(1)_Y$ singlet, but charged under $U(1)_\mu$) acquires a vacuum expectation value (vev) at or below the $U(1)_\mu$ breaking scale.

The Yukawa couplings of the Higgs doublets require the quarks and leptons to be charged under $U(1)_\mu$ too. Therefore, some extra fields charged under the $U(1)_\mu$ and SM gauge groups will in general be required to cancel the arising anomalies. These extra fields can play the role of the messenger sector used in gauge mediation \[3\]. This sector includes some chiral superfields, $q, \bar{q}, l, \bar{l}$, transforming non-trivially under the SM gauge group, and superpotential terms $X q \bar{q}$, $X l \bar{l}$, where $X$ is a SM singlet whose scalar and $F$ components acquire vevs $\langle X \rangle$ and $\langle F_X \rangle$. Note that the relevant $q \bar{q}, l \bar{l}, X$ and $X^2$ operators in the superpotential are forbidden by the $U(1)_\mu$.

The fact that $X$ and the messengers carry $U(1)_\mu$ charges implies that this symmetry is broken at a scale of order $\langle X \rangle$ or higher, so that the $U(1)_\mu$ $D$-term is expected to be significant. As a result, the $D$-term contributions to squark and slepton masses may dominate or be comparable to the usual two loop contributions mediated by the SM gauge interactions. Since the $D$-term contributions to the squared masses are proportional to the scalar charges, we need to give same sign $U(1)_\mu$ charges to all the quarks and leptons.

Apparently, it is non-trivial to cancel the $U(1)_\mu$ anomalies in this case. The MSSM has $[SU(3)_C]^2 \times U(1)_\mu$, $[SU(2)_W]^2 \times U(1)_\mu$, $U(1)_Y^2 \times U(1)_\mu$ and $U(1)_Y \times U(1)_\mu^2$ anomalies, which should be cancelled by the anomalies of the messengers. A simple solution is to observe that $E_6 \supset SU(5)_{\text{SM}} \times U(1)_\mu$, where $SU(5)_{\text{SM}}$ includes the SM gauge group. The fundamental representation of $E_6$ decomposes as $27 = (10 + \bar{5} + 1, +1) + (\bar{5} + 5, -2) + (1, +4)$, and is anomaly free. Motivated by this, although we do not require $E_6$ gauge unification, we introduce (in addition to the MSSM fields, which include the right handed neutrinos) three $q, \bar{q} \in (3 + \overline{3})$ of $SU(3)_C$, and two $l, \bar{l} \in (2 + 2)$ of $SU(2)_W$. Then, we can assign $U(1)_\mu$ charge +1 to all the quarks and leptons; and charge $-2$ to $H_{u,d}$ as well as to all the messengers. This ensures that the $U(1)_\mu$ anomalies mentioned above cancel. In addition, the $S$ and $X$ fields can be identified with two of the three $(1, +4)$ representations of $SU(5)_{\text{SM}} \times U(1)_\mu$ required for $U(1)_\mu^3$ anomaly cancellation.
In order to allow $X$ to acquire an $F$ term, we include a new SM singlet, $N$, with $U(1)_\mu$ charge $-2$, and a $XN^2$ term in the superpotential. The complete field content of the MSSM + messenger sector is given in Table 1. One can check that indeed the gauge symmetries do not allow any supersymmetric mass term. The renormalizable superpotential that we consider includes, in addition to the usual couplings of $H_{u,d}$ to quarks and leptons, only the following terms

$$W = f_qXq_i\overline{q}_i + f_lXl_j\overline{l}_j + \frac{\lambda}{2}XN^2 - \frac{\epsilon}{2}SN^2 + \kappa SH_uH_d. \quad (2.1)$$

Apart from the MSSM + messenger sector described so far, there should be a sector that breaks SUSY dynamically. The DSB sector also contains fields which transform under $U(1)_\mu$, so that $U(1)_\mu$ can play the role of the messenger group which communicates SUSY breaking to the visible sector. The properties of this DSB sector are constrained by the low energy structure we introduced so far. The first condition is that the $U(1)_\mu$ and $U(1)_3$ anomalies of the MSSM + messenger sector, given by $(−4) + (−2)$ and $(−4)_3 + (−2)_3$, have to be cancelled by the $U(1)_\mu$ and $U(1)_3$ anomalies of the DSB sector. Another condition is that the supertrace of the $U(1)_\mu$ charged fields in the DSB sector is positive, so that the $X$ and $S$ scalars receive negative squared masses and acquire vevs. In addition, we
have to make sure that after $U(1)_\mu$ is broken by the $N$, $S$, $X$ vevs, the $U(1)_\mu$ $D$-term contributions to the squared masses of the squarks and sleptons are positive.

An example of a DSB sector satisfying our requirements is the $SU(4) \times SU(3)$ model described in Ref. [4]. The field content is shown in Table 2. One can check that all anomalies cancel in the combination of the MSSM + messenger sector and the DSB sector. After including the superpotential which lifts the flat directions, it breaks SUSY dynamically. The SUSY breaking minimum is discussed in the Appendix. For the analysis of the visible sector we only need to know that the DSB sector generates a negative squared mass for each scalar charged under $U(1)_\mu$, proportional to its $U(1)_\mu$ charge squared, and a contribution to the $U(1)_\mu$ $D$-term, $-\xi^2$, (which is unimportant as long as $\xi$ is much smaller than the $U(1)_\mu$ breaking scale). Then, the relevant part of the potential for the MSSM + messenger sector is given by

$$V = \frac{g_\mu^2}{2} \left( -\xi^2 + 4|X|^2 + 4|S|^2 - 2|N|^2 - 2|H_u|^2 - 2|H_d|^2 + \ldots \right)^2$$

$$- \tilde{m}^2 \left( 16|X|^2 + 16|S|^2 + 4|N|^2 + 4|H_u|^2 + 4|H_d|^2 + \ldots \right) + \frac{\lambda^2}{4}|N|^4$$

$$+ \frac{\xi}{2} N^2 - \kappa H_u H_d \right|^2 + |N|^2 |\lambda X - \epsilon S|^2 + \kappa^2 |S|^2 \left(|H_u|^2 + |H_u|^2\right) + \ldots, \quad (2.2)$$

where the ellipsis stand for terms involving squarks, sleptons, and messenger scalars. The values of $\xi$ and $\tilde{m}$ are given in the Appendix as functions of the parameters in the DSB sector. In Section 3 it is shown that the constraints on the gaugino masses and on the $B$ and $\mu$ parameters from the Higgs sector lead to $\lambda^{3/2} < \epsilon \ll \lambda \ll 1$. We assume that the hierarchy of these couplings, as well as the hierarchy of fermion masses come from some unknown physics at high scales\footnote{A supersymmetric model of flavor utilizing an extra gauge $U(1)$ for gauge mediation was just recently proposed in [5].}

Minimizing the potential is straightforward, and for $\kappa > \sqrt{\lambda^2 + \epsilon^2}$ we find a desired...
minimum at $\langle H_u \rangle = \langle H_d \rangle = 0$ and

$$\langle N^2 \rangle = \frac{24\tilde{m}^2}{\lambda^2 + \epsilon^2}, \quad \langle X \rangle = \frac{\epsilon}{\lambda} \langle S \rangle, \quad \langle S^2 \rangle = \frac{\lambda^2}{\lambda^2 + \epsilon^2} \left( \frac{\epsilon^2}{4} + \frac{\tilde{m}^2}{g^2_\mu} + \frac{12\tilde{m}^2}{\lambda^2 + \epsilon^2} \right).$$ (2.3)

This is only a local minimum, but we expect that its lifetime is sufficiently long. We comment more on this in Section 4. The corresponding SUSY-breaking $F$ and $D$-terms are given by

$$\langle F_N \rangle = 0, \quad \langle F_X \rangle = \frac{\lambda}{2} \langle N^2 \rangle \simeq \sqrt{6}\tilde{m} \langle N \rangle, \quad \langle F_S \rangle = -\frac{\epsilon}{2} \langle N^2 \rangle, \quad g^2_\mu \langle D \rangle = 4\tilde{m}^2.$$ (2.4)

The $\langle X \rangle$ and $\langle F_X \rangle$ vevs provide the SUSY preserving and breaking masses for the messenger fields, $q, \bar{q}, l, \bar{l}$, while $\langle S \rangle$ and $\langle F_S \rangle$ provide the $\mu$ and $B$ term for the Higgs sector. Gaugino masses come from the usual one-loop gauge mediation contribution. The scalar squared masses receive, in addition to the usual two-loop SM gauge mediation contributions, a $U(1)_\mu$ $D$-term contribution and a negative contribution from the $U(1)_\mu$ mediation. For superpartner masses near the weak scale, we require $\tilde{m} \lesssim 1$ TeV, $\sqrt{\langle F_X \rangle} \gtrsim 30$ TeV, hence the $U(1)_\mu$ breaking scale $\langle N \rangle \gtrsim 10^3$ TeV and $\lambda \lesssim 10^{-3}$. The resulting sparticle spectrum is discussed in the next Section.

## 3 Sparticle spectrum

The communication of SUSY breaking from the DSB sector to the visible sector proceeds in two steps. First, at the scale

$$M_\mu \equiv g_\mu \langle N \rangle \simeq 2\sqrt{3} \frac{g_\mu}{\lambda} \tilde{m} \quad (\gg \tilde{m})$$ (3.5)

of $U(1)_\mu$ breaking, each scalar with $U(1)_\mu$ charge $Q^f_\mu$ which does not acquire a vev receives a soft mass contribution

$$m^2_f(M_\mu) = Q^f_\mu (4 - Q^f_\mu) \tilde{m}^2.$$ (3.6)

In particular, all squarks and sleptons get a positive squared mass $+3\tilde{m}^2$, while the two Higgs doublets get a negative soft squared mass of $-12\tilde{m}^2$. The $\mu$ and $B$ terms are also generated at this scale:

$$\mu(M_\mu) = \kappa \langle S \rangle \simeq 2\sqrt{3} \frac{\kappa}{\lambda} \tilde{m} \quad (\gtrsim \tilde{m}),$$ (3.7)

$$B(M_\mu) = \frac{\langle F_S \rangle}{\langle S \rangle} \simeq -2\sqrt{3} \frac{\epsilon}{\lambda} \tilde{m} \quad (|B| \ll \tilde{m}).$$ (3.8)
One may check that the condition $\kappa > \sqrt{\lambda^2 + \epsilon^2}$ ensures that the tree-level squared masses for the Higgs doublets are positive and thus electroweak symmetry is unbroken at this stage.

The messenger fermions and scalars also become massive at the scale $M\mu$. The masses of the $d_i, \bar{d}_i$ and $l_i, \bar{l}_i$ fermion messengers are

$$M_{q,l} \equiv f_{q,l}(X) \simeq 2\sqrt{3}f_{q,l}\tilde{m}/\lambda \quad (\gg \tilde{m}).$$

The messenger scalars, on the other hand, have the following mass matrices:

$$M_{\tilde{q},\tilde{l}}^2 \approx M_{q,l}^2 \left[1 + \frac{\lambda^3}{\epsilon^2f_{q,l}} \begin{pmatrix} -\lambda/f_{q,l} & 1 \\ 1 & -\lambda/f_{q,l} \end{pmatrix} \right].$$

The eigenvalues are positive if $\epsilon > \mathcal{O}(\lambda^{3/2})$, assuming $f_{q,l} \sim \mathcal{O}(1)$.

Finally, the gauge singlets $X, S$ and $N$ also get masses. Their fermionic components mix among themselves, and with the $U(1)_\mu$ gaugino. As a result, we find two Dirac fermions, with masses of order $24(g_\mu/\lambda)\tilde{m}$ and $4\tilde{m}$, respectively. The scalar components of the singlets also mix, and the resulting mass spectrum is as follows. There is a massless Nambu-Goldstone boson, which is eaten by the $U(1)_\mu$ gauge boson; and there is a scalar of mass $24(g_\mu/\lambda)\tilde{m}$, which becomes a member of the heavy gauge supermultiplet. The rest of the scalars are light, with masses $2\sqrt{6}\tilde{m}, 2\sqrt{6}\tilde{m}, 2\sqrt{3}\tilde{m}$ and $2\sqrt{2}\tilde{m}$, correspondingly.

Below the messenger scale, $M \simeq M_q \simeq M_l$, the messengers $d_i, \bar{d}_i$ and $l_i, \bar{l}_i$ are integrated out inducing gaugino masses

$$M_n(M) = c_n\frac{\alpha_n}{4\pi}\Lambda g(\Lambda/M),$$

where $n = 1, 2, 3$ corresponds to $U(1)_Y$, $SU(2)_W$ and $SU(3)_C$, $c_1 = 4, c_2 = 2, c_3 = 3$ (we do not use the $SU(5)$ normalization for $\alpha_1$), and

$$\Lambda \equiv \frac{\langle F_X \rangle}{\langle X \rangle} \simeq 2\sqrt{3}\frac{\lambda}{\epsilon}\tilde{m}.$$
in Ref. [6]. In order to get large enough gaugino masses, we need \( \Lambda \sim 30 \text{ TeV} \gg \tilde{m} \), hence \( \epsilon \ll \lambda \). In eq. (3.13) we neglect small contributions of order \( [\alpha/(4\pi)^2] \tilde{m}^2 \ln(M_\mu/M) \), arising due to the nonvanishing messenger supertrace [4].

The effect of the renormalization group equations (RGEs) on the sparticle spectrum between the \( M_\mu \) and \( M \) scales is peculiar: due to their larger Yukawa couplings, the third generation sfermions are driven heavier than those of the first two generations. The reason is that in our model the soft squared masses for the Higgs doublets are negative due to the \( U(1)_\mu \)\( D \)-term, and as a result, the scalar mass combinations

\[
M^2_{Q_3} + M^2_{U_3} + M^2_{H_u} \simeq M^2_{Q_3} + M^2_{D_3} + M^2_{H_d} \simeq M^2_{L_3} + M^2_{E_3} + M^2_{H_d} \simeq -6\tilde{m}^2 ,
\]

(3.14)

which appear in the RGEs, are also negative. Of course, below the messenger scale the usual gauge mediated contributions are included, and their Yukawa RGE effect on the third generation soft masses is just the opposite. The net effect depends on the values of the model parameters. Note that the charge assignments are such that \( \text{Tr} Q_Y \cdot Q_\mu = 0 \), so that a hypercharge \( D \)-term is not induced through RGE renormalization.

The supersymmetric mass spectrum at the electroweak scale is determined as a function of the following parameters: \( \{\tilde{m}, \kappa/\lambda, \epsilon/\lambda, f_{q,l}/\lambda, g_\mu/\lambda\} \). Using eqns. (3.5), (3.12), (3.7)-(3.9), and the requirement for radiative electroweak symmetry breaking, they can be exchanged for \( \{\Lambda, M, M_\mu, \tan \beta, \text{sign}(\mu)\} \). Note the presence of the extra parameter \( M_\mu \) as compared to the minimal gauge mediated models. The only constraint on the parameter space is \( M_\mu \gg M > \Lambda \). We would also expect to be in the large \( \tan \beta \) region, since our model predicts low values of \( B \). In Fig. [1] we show the RGE evolution of representative soft masses for \( \Lambda = 40 \text{ TeV}, M = 10^2 \text{ TeV}, M_\mu = 10^4 \text{ TeV}, \tan \beta = 40 \) and \( \mu > 0 \). We find that these values correspond to \( \tilde{m} = 80 \text{ GeV}, \kappa/\lambda = 1.5, \epsilon/\lambda = 7 \times 10^{-3}, \)

\( f_{q,l}/\lambda = 5.1 \times 10^4 \) and \( g_\mu/\lambda = 3.6 \times 10^4 \).

Qualitatively, the superpartner spectrum looks similar to the pure gauge mediation scenario, if \( \tilde{m} \) is smaller than the SM gauge mediated soft masses. Then, as \( \tilde{m} \) gets larger than the SM gauge mediation contributions, the squarks and sleptons become heavier in comparison to the gauginos, and increasingly degenerate, since they carry the same \( U(1)_\mu \) charge.

### 4 Discussion

Several features of the model presented here warrant further comments.
1. It is well known that the “µ problem” is more difficult to solve in gauge mediation models. Besides the general problem of a potentially large µ-term induced by Planck scale physics, a totally separate sector is often required solely for generating the µ-term\[3, 8\]. If one tries to generate the µ term directly from the messenger sector by a small coupling or a loop contribution, the resulting ratio of $Bµ$ and µ is too big$^{2}$, $Bµ/µ \sim F_X/X \sim 10^4 - 10^5$ GeV. In our model, the supersymmetric Higgs mass term is forbidden by the $U(1)_µ$ gauge symmetry, and the µ term is generated only after $U(1)_µ$ is broken. The $S$ field which gives rise the µ-term is an integral part of this model and is required for anomaly cancellation. The $F_S/S$ ratio is small, allowing acceptable values for $Bµ/µ$. However, without understanding the small Yukawa couplings ($\lesssim 10^{-3}$) needed in this model, we can not claim that the scale of the µ term is completely natural.

2. The intrinsic SUSY breaking scale in this model can be as low as $10^5 - 10^6$ GeV, lower than that in the original models of gauge mediation [3]. To see this, we assume that the $U(1)_µ$ breaking scale ($\langle N \rangle$, determined by $\tilde{m}$ and $\lambda$) is about the same as the scale

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$^2$However, there are ways to cure this problem [8].
of heavy fields in the DSB sector, $\langle \mathcal{R} \rangle$, so that the formulae (A.8), (A.9) in the appendix can still apply. Then we have

$$F_X \approx \sqrt{6\tilde{m}}\langle N \rangle \sim c\sqrt{480} \left( \frac{g_{\mu}^2}{16\pi^2} \right) m_b \langle \mathcal{R} \rangle \sim c'\sqrt{480} \left( \frac{g_{\mu}^2}{16\pi^2} \right) E_{\text{vac}}^2,$$

(4.15)

where $c$, $c'$ are $\mathcal{O}(1)$ constants, $E_{\text{vac}}^4$ is the vacuum energy density, and $m_b$ is defined in (A.7). We can see that the intrinsic SUSY breaking scale $E_{\text{vac}}$ has to be only about less than an order of magnitude higher than the messenger scale. For $\sqrt{F_X} \sim 10^4 - 10^5$ GeV, $E_{\text{vac}}$ can be lower than $10^6$ GeV. In that case, the next to lightest supersymmetric particle (NLSP) can decay inside the detector, yielding interesting collider signals [10]. Note that most gauge mediation models have the SUSY breaking scale so high that NLSP will escape the detector, giving similar signals as in the traditional supergravity mediation scenario. All previously known models with the SUSY breaking scale below $10^6$ GeV so far involve some assumptions about noncalculable strong dynamics [11], therefore it is not certain that they are viable.

3. The vev of the $N$ field can give Majorana masses to the right-handed neutrinos on the order of the $U(1)_\mu$ breaking scale via superpotential interactions $N\nu_i\nu_j$. However, without knowing the Yukawa couplings of the neutrinos, we are not able to predict the neutrino masses and mixing patterns.

4. We did not include all possible terms consistent with the gauge symmetry in the superpotential (2.1). Since we allow small Yukawa couplings, most of the missing couplings could be sufficiently small, so that they do not have significant effects on our model. Some of them may change the low energy parameters. For example, a small coupling between $X$ and $H_u, H_d$ can give extra contribution to the $B\mu$ parameter, and hence affect $\tan\beta$. The only dangerous terms are those matter-messenger couplings which induce proton decays, so we may need an extra symmetry to forbid them (the flavor changing constraints are not very severe [12]). A more attractive solution in this framework would be to have different messenger fields or different charge assignments so that the messenger-matter couplings which allow rapid proton decays can not exist.

5. The $U(1)_\mu$ gauge symmetry forbids $R$ parity violating operators. Moreover, the $U(1)_\mu$ is broken only by fields with even charges, such that a $Z_2$ symmetry, identified as the $R$ parity, is automatically conserved.

6. As in the original models of gauge mediation [3], the minimum we considered is a local minimum [13, 14]. There exist lower minima and even runaway directions with $\langle X \rangle = 0$, $\langle \bar{q}q \rangle$ and/or $\langle \bar{l}l \rangle \neq 0$. Ref. [14] estimates the vacuum tunneling rate. It is
found that in order for the lifetime of the local minimum to be longer than the age of the universe, some Yukawa couplings have to be small ($\lambda < 0.1$), which is easily satisfied in our case.

To our knowledge, the model we have presented here is the first example of a purely chiral supersymmetric Standard Model. This is a viable model and yields interesting superpartner spectrum and phenomenology, which may be tested in future experiments.

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A The DSB Sector

An interesting set of DSB models is the $SU(N) \times SU(N-1)$ models of Poppitz, Shadmi and Trivedi [1]. One can require that the superpotential preserves an $SU(N-2)$ global symmetry. If we add a fundamental representation of the $SU(N-2)$ (singlet under $SU(N) \times SU(N-1)$), it is anomaly free [15]. In addition, there is a non-anomalous $U(1)$, so that we can gauge a $U(1)$ subgroup of $SU(N-2) \times U(1)$ and use it as the messenger to transmit SUSY breaking to the visible sector.

The model which we use is the $SU(4) \times SU(3)$ model. The field content of the DSB sector and their charges under the “messenger” $U(1)_\mu$ are shown in Table 2. The $U(1)_\mu^3$ and $U(1)_\mu$ anomalies (which would be cancelled by adding two fields with charges $-4$, $-2$) are cancelled by the combination of the MSSM fields and the messenger sector.

The superpotential of the DSB sector is given by

$$W_{DSB} = \lambda_1 L_1 Q R_1 + \lambda_2 L_2 Q R_2 + \lambda_3 L_3 Q R_3 + \frac{\alpha}{3!} R_1 R_2 R_3. \quad (A.1)$$

For this model to be calculable, we assume that $\alpha \ll \lambda_1, \lambda_2, \lambda_3 \sim 1$, so that the vacuum lies in the weakly coupled regime. The detailed analysis of this family of models can be found in Ref. [15]. Here we just sketch the result. The $R_i$ fields develop large vevs and give large masses to the $L_i$ and $Q$ fields. After integrating out the heavy fields, the low energy nonlinear SUSY sigma model is described by the baryons $b_i$, where

$$b_i = \frac{1}{3!} \epsilon_{ijkl} R_j R_k R_l. \quad (A.2)$$
with a superpotential
\[ W_{\text{eff}} = (\Lambda_D^9 b_4)^{\frac{1}{4}} + \alpha b_3. \] (A.3)

The first term comes from the gaugino condensation of the \( SU(4) \) gauge group (with scale \( \Lambda_4 \)). We have absorbed the \( \lambda_i \) couplings into \( \Lambda_D, \Lambda_9^D \sim \lambda_1 \lambda_2 \lambda_3 \Lambda_4^9 \). Combining it with the Kähler potential,
\[ K_{\text{eff}} = 3(b_1^b b_i)^{1/3}, \] (A.4)
we find that the minimum occurs at
\[ b_3 = -0.075(\alpha^{-\frac{3}{4}} \Lambda_D)^3, \quad b_4 = 0.102(\alpha^{-\frac{3}{4}} \Lambda_D)^3. \] (A.5)

The energy density at the minimum and the masses of the scalar components of \( b_1, b_2 \) are
\[ E_{\text{vac}}^4 = 0.220 \alpha^{\frac{3}{4}} \Lambda_D^4, \] (A.6)
\[ m_{b_{1,2}}^2 \equiv m_b^2 = 0.445 \alpha^{\frac{3}{2}} \Lambda_D^2. \] (A.7)

One can easily see that \( E_{\text{vac}}^2 \sim m_b \langle \mathcal{R} \rangle \). Because the light fields \( b_1 \) and \( b_2 \) have \( U(1)_\mu \) charges +4 and +2 respectively, they will generate the Fayet-Illiopoulos D term for the \( U(1)_\mu \) gauge group as discussed in Ref. [3],
\[ -\xi^2 = -\sum_j \frac{g_\mu^2}{16\pi^2} Q_\mu^b m_{b_j}^2 \ln \frac{M_V^2}{p^2} = -6 \left( \frac{g_\mu^2}{16\pi^2} \right) m_b^2 \ln \frac{M_V^2}{p^2}, \] (A.8)
where \( M_V \) represents the mass scale of the heavy fields in the DSB sector, and \( p^2 \) is the larger scale between the \( U(1)_\mu \) breaking scale, \( M_\mu \), and \( m_b^2 \). They also generate a negative contribution to the mass squared of each scalar field charged under \( U(1)_\mu \) at two-loop, proportional to the field’s charge squared,
\[ \frac{m_i^2}{q_i^2} \equiv -\bar{m}^2 = -\sum_j 4 \left( \frac{g_\mu^2}{16\pi^2} \right)^2 \left( Q_\mu^b \right)^2 m_{b_j}^2 \ln \frac{M_V^2}{p^2} = -80 \left( \frac{g_\mu^2}{16\pi^2} \right)^2 m_b^2 \ln \frac{M_V^2}{p^2}. \] (A.9)

Note that the formulae (A.8), (A.9) only apply when \( p^2 < M_V^2 \). If the \( U(1)_\mu \) breaking scale \( (p^2 = M_\mu^2) \) is higher than \( M_V^2 \), the results will be suppressed by a factor \( M_V^2/M_\mu^2 \).

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