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Mathematical search technique for detecting moving novel coronavirus disease (COVID-19) based on minimizing the weight function

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A B S T R A C T

The study of search plans has found considerable interest between searchers due to its interesting applications in our real life like searching for located and moving targets. This paper develops a method for detecting moving targets. We propose a novel strategy based on weight function \( W(Z) \), \( W(Z) = \lambda H(Z) + (1-\lambda)L(Z) \), where \( H(Z) \), \( L(Z) \) are the total probabilities of un-detecting, and total effort respectively, is searching for moving novel coronavirus disease (COVID-19) cells among finite set of different states. The total search effort will be presented in a more flexible way, so it will be presented as a random variable with a given distribution. The objective is searching for COVID-19 which hidden in one of \( n \) cells in each fixed number of time intervals \( m \) and the detection functions are supposed to be known to the searcher or robot. We look in depth for the optimal distribution of the total effort which minimizes the probability of undetected the target over the set of possible different states. The effectiveness of this model is illustrated by presenting a numerical example.

Introduction

The COVID-19 early reported in January 2020. Since then, COVID-19 has rapidly spread around the world in just two months, as per the publicly available data sources. The COVID-19 has been the worst pandemic in last 100 years of human life. The World Health Organization (WHO) announced a pandemic emergency across the world: a review of the 2019 novel coronavirus [28]. The COVID-19 has spread to the whole world with nearly 2.4 million diagnosed cases and over 165,000 deaths up to April 20, 2020 [29]. Across the world, almost all scientists, mathematicians, biologists, environmental scientists, etc. are working to deal this outbreak pandemic.

The searching for a lost target either located or moved is often a time-critical issue where the target is very important. Search problems arise commonly in many diverse areas, like looking for a missing person, a missing black box in the sea, searching for fugitives, and prospectors explore for mineral deposits. Now systematic search is known as search theory and the object sought is called the target. More studies presented in search theory since World War II, starting with a linear search, whether this linear system is independent or intersecting [1–3]. Also, one of more interesting techniques has been studied in case of linear search is called coordinated search [4–6], the authors discussed the coordinated search technique for a located and moving target on two intersected lines and two independent lines respectively. Recently, one of an important and modern search model has been illustrated in the three dimensional space that determines a located target in a 3-D known zone by a single searcher, two searchers and 4 searchers to find a randomly located lost target [7–10].

More recently, authors illustrated a novel coordinated search algorithm for a multi searchers random walk, where the lost target is a random walker on one of \( n \) disjoint lines [11]. On the other side, the discrete search problems are not new. Whereabouts search has been studied [12–13]. The search and stop rule where all search outcomes are independent, conditional on the location of the searched object and the search policy has been studied [14–16]. The search process, where the search time of a cell depends on the number of searches so far studied [17]. The case of the target moving between two states according to a discrete parameter Markov chain has been studied [18]. The case of the target moving among a finite number of states according to a Markov chain with a discrete parameter is also studied under a variety of
conditions [19,20].

The distribution of effort which made the probability of undetection for unrestricted effort when the states are not identical and the cost of finding the target are minimum has been studied (see [21]). When the search effort is available at each fixed number of time intervals with a given distribution has been studied [22]. Recently, the searching algorithm for detecting a Markovian target based on maximizing the discount effort reward search has been introduced [23]. More recently, the authors illustrated optimal discrete search for a randomly moving COVID-19 among finite set of different states using monitoring system to search for COVID-19 which hidden in one of $n$ cells of the respiratory system in the human body in each fixed number of time intervals $m$ [24]. Also, the existence of multi generalized linear search problem to detect a lost target in one of several lines [25].

Various researchers published their work on the cure of patients of dialysis. In the field of medical science, a mathematical model for the minimization of the risk due to coronavirus is developed for patients to cure the dialysis. [26]. A study was performed to ensure that the mathematical modelling is able to solve the crisis of the current Covid-19 pandemic [27]. A mathematical model is formulated to provide a

Fig. 1. A human body divided into systems.

Fig. 2. The probability of detecting COVID-19 at time interval 1.
better understanding for economies at the outbreak of coronavirus pandemic [28]. For more different kinds of search problems [30–35].

In this paper, the main idea to distribute the effort of the single searcher in a perfect way to facilitate the detecting of Covid-19 as soon as possible by using a novel strategy based on weight function $W(Z)$, $W(Z) = \lambda H(Z) + (1 - \lambda) L(Z)$.

Materials and computational methods

Problem formulation

Discrete search problem is applied in defense problems in our life, such as searching for a hostage (target) hidden in a city or detecting improvised explosive devices, have underscored the need for efficient
and effective search methods for detecting targets of various types.

In this model, we have only one searcher, the main task of him is to distribute his effort in a perfect way to find the lost target as soon as possible.

The allocation of search effort is \( Z_{ij} \) where \( i = 1, 2, ..., n \) and \( j = 1, 2, ..., m \), which gives the effort to be put into state \( j \) at time \( i \), we call \( Z_{ij} \) a search plan, the conditional probability of finding the target at time given that it is located in state \( j \), is given by the detection function \[ b(i, j, Z_{ij}) \]. Our main purpose is to minimize the weight function \( W(Z) \).

**Design and Procedure:**

Fig. 1 shows a human body divided into systems (states), the lost COVID-19 which moved from one state to another in each new time interval \( i = 1, 2, 3 \). Here, each state indicates the expected system of the lost COVID-19.

Our main purpose to minimize the weight function \( W(Z) \) where,

\[
W(Z) = \lambda H(Z) + (1 - \lambda) L(Z) \quad 0 \leq \lambda \leq 1
\]

Subject to

Fig. 5. The probability of detecting COVID-19 at time interval 4.

Fig. 6. The probability of detecting COVID-19 at time interval 5.
\[ H(Z) = \prod_{i=1}^{n} \sum_{j=1}^{m} P_{ij} e^{T_{ij} (Z_{ij})} \]  

Here, \( H(Z) \) is the total probability of un-detecting the COVID-19 over the whole time (see [21]). It is known that the minimizing of probability of un-detecting COVID-19 means the maximizing probability of detecting our lost COVID-19.

Total effort

\[ L(Z) = \sum_{i=1}^{n} \sum_{j=1}^{m} Z_{ij} \]  

So

\[ W(Z) = \lambda \prod_{i=1}^{n} \sum_{j=1}^{m} P_{ij} e^{T_{ij} (Z_{ij})} + (1-\lambda) \sum_{j=1}^{m} \sum_{i=1}^{n} Z_{ij} \]

Here,

\[ \sum_{j=1}^{m} Z_{ij} = L_i(Z), Z_{ij} \geq 0, \quad i = 1, 2, ..., n, \quad j = 1, 2, ..., m, \quad \sum_{j=1}^{m} P_{ij} = 1 \]

\( T_{ij} \) is the mean effort of detection in state \( j \).

By assuming,

\[ Z(v_i) = \{ Z \in \mathbb{R}^{n \times m} \mid g(Z) = L_i(Z) - E(v_i) - k_p \sqrt{\text{var}(v_i)} \leq 0 \} \]

where \( E(v) \) and \( \text{var}(v) \) denote the mean and variance of the normally distributed random variable \( v \) when the target moves among \( m \) states.

By using Kuhn-Tucker conditions, we have

\[ \frac{\partial W(Z)}{\partial Z_{ij}} + \sum_{i=1}^{n} U_{ij} \frac{\partial W(Z)}{\partial Z_{ij}} = 0, \]

\[ Z(V_i) \leq 0, U_i Z(V_i) = 0, U_i \geq 0. \]

This leads to

\[ \frac{p_{0j}}{T_j} e^{\frac{Z_{0j}}{T_j}} \prod_{i=1}^{n} \sum_{j=1}^{m} p_{ij} e^{\frac{Z_{ij}}{T_j}} + nm(1-\lambda) + u_i = 0 \]

\[ L_i(Z) = E(v_i) - k_p \sqrt{\text{var}(v_i)} \leq 0 \]

\[ U_i(L_i(Z) - E(v_i) - k_p \sqrt{\text{var}(v_i)}) = 0 \]

Since \( L_i(Z) - E(v_i) - k_p \sqrt{\text{var}(v_i)} \leq 0 \) and \( U_i \geq 0 \) then from the above Kuhn-Tucker conditions, we get

\[ \frac{p_{0j}}{T_j} e^{\frac{Z_{0j}}{T_j}} \prod_{i=1}^{n} \sum_{j=1}^{m} p_{ij} e^{\frac{Z_{ij}}{T_j}} + nm(1-\lambda) = 0 \]

Where, \( U_i(U_i L_i(Z) - E(v_i) - k_p \sqrt{\text{var}(v_i)}) = 0 \) this leads to \( U_i = 1 \)

If the target was detected in the cell \( j \) at time \( i \), then logically the total non-detection probability in all cells until the cell \( j \) can be given from

\[ \frac{p_{0j}}{T_j} e^{\frac{Z_{0j}}{T_j}} \prod_{i=1}^{n} \sum_{j=1}^{m} p_{ij} e^{\frac{Z_{ij}}{T_j}} + nm(1-\lambda) + 1 = 0 \]

Let \( \tau_i = E(v_i) - k_p \sqrt{\text{var}(v_i)} \) be the total effort on the time interval \( i \).

Then

\[ L_i(Z) = \frac{Z_{0j}}{T_j}, \text{where the target detected on the cell } j. \text{ Thus} \]

\[ \lambda \prod_{i=1}^{n} \sum_{j=1}^{m} P_{ij} e^{-(\tau_{ij}/T_j)} + (1-\lambda) \sum_{j=1}^{m} \sum_{i=1}^{n} Z_{ij} \]
The values of probability of undetected the lost moving COVID-19 in case of two time intervals and a random variable effort. Figs. 2-6 show the probability of detecting moving COVID-19 or cancer cells at time intervals 1, 2, 3, 4 and 5 respectively.

| Time interval | $p_1$ | $p_2$ | $E(v_i)$ | $\sigma(v_i)$ | $k_i$ | $r_i$ | $Z_{1i}$ | $Z_{2i}$ | $H(Z)$ |
|---------------|-------|-------|-----------|----------------|--------|--------|----------|----------|---------|
| 1             | 0.64  | 0.36  | 0.82      | 0.04           | 3      | 1.42   | 1.3900   | 0.1009   | 9.66E-3 |
| 2             | 0.656 | 0.344 | 0.38      | 0.09           | 3      | 1.73   | 1.4691   | 0.2608   |         |
| 3             | 0.6624| 0.3376| 0.76      | 0.16           | 3      | 1.96   | 1.5647   | 0.3952   |         |

Table 2
The values of probability of undetected the lost moving COVID-19 in case of three time intervals and a random variable effort.

| Time interval | $p_1$ | $p_2$ | $p_3$ | $E(v_i)$ | $\sigma(v_i)$ | $k_i$ | $r_i$ | $Z_{1i}$ | $Z_{2i}$ | $Z_{3i}$ | $W(Z)$ |
|---------------|-------|-------|-------|-----------|----------------|--------|--------|----------|----------|----------|---------|
| 1             | 0.12  | 0.21  | 0.67   | 0.2       | 0.36           | 3      | 2      | 0.20029  | 0.26705  | 1.53264  | 5.92E-3 |
| 2             | 0.114 | 0.227 | 0.649  | 0.7       | 0.01           | 3      | 1      | 5.87E-5  | 0.03883  | 0.96110  |         |
| 3             | 0.1158| 0.2289| 0.6553 | 0.4       | 0.09           | 3      | 1.3    | 0.05896  | 0.10820  | 1.13182  |         |
| 4             | 0.1153| 0.2313| 0.6534 | 0.1       | 0.25           | 3      | 1.6    | 0.10715  | 0.21733  | 1.27551  |         |
| 5             | 0.1154| 0.2306| 0.6540 | 0.6       | 0.16           | 3      | 1.8    | 0.14112  | 0.28137  | 1.37750  |         |

Application and statistical Analysis:

By supposing, $$s = \prod_{i=1}^{\infty} \left( \sum_{j=1}^{\infty} T_j \right) \left( \prod_{k=1}^{\infty} \left( T_k \right)^{\lambda} \right) \left( \prod_{k=1}^{\infty} \left( T_k \right)^{-\sigma} \right)$$

$$W(Z) = \lambda \prod_{i=1}^{\infty} \sum_{j=1}^{\infty} P_{ij} e^{-i}$$

Conclusion
In this paper, we illustrated a new search technique that using minimizing both of probability of un-detecting the lost COVID-19 and the total effort in a new function which calls the weight function. The probabilities of detecting and un-detecting COVID-19 have been calculated, and illustrated with the help of various graphs. In fighting with such an outbreak, mathematical modeling and optimization play a crucial role in appreciating how an infectious disease outbreak is occurring and where it may transmit.

We hope that the proposed work can produce more understanding and awareness for fighting the COVID-19 pandemic. We conclude that our study also provides an understanding how fast this outbreak is spreading. The proposed work may also be very helpful for the control of this outbreak, proper utilization of available resources.

CRediT authorship contribution statement

Saad J. Almalki: Formal analysis, Funding acquisition, Investigation, Methodology, Project administration, Supervision, Validation, Visualization, Writing - review & editing. W.A. Affif: Data curation, Software, Validation, Writing - original draft. Abd Al-Aziz Hosni El-Bagouy: Data curation, Investigation, Resources, Software, Writing - original draft, Writing - review & editing. Gamal A. Abd-Elmougd: Formal analysis, Methodology, Project administration, Resources, Supervision, Writing, Review & editing.

Declaration of Competing Interest
The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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