QUARK OFF-SHELL EFFECTS IN FLAVOUR-CHANGING DECAYS

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ABSTRACT

We discuss some flavour-changing effective Lagrangians, obtained from scales above 1 GeV, which vanish on the quark mass shell. Although the effects of such effective Lagrangians are zero in the limit of vanishing bound-state interactions, we have shown that they have a significant impact on the processes $K \rightarrow \gamma \gamma$ and $B_s \rightarrow \gamma \gamma$.

1. Introduction

Dealing with weak hadronic decays, one faces the problem of overbridging the quark world and the meson world where a physical process occurs. The analyses starting from the high-energy side evolved from the traditional Feynman diagram technique to an implementation of the operator-product expansion [1]. When studying non-leptonic decays of order $\sim G_F$, and weak radiative decays of order $\sim eG_F$ or $\sim e^2 G_F$, one normally writes down an effective Lagrangian including operators that contribute to a given process, and operators that mix with these under QCD renormalization. Within this standard procedure, one usually omits operators containing $(i\gamma \cdot D - m_q)$, by appealing to the equations of motion (EOM) for quark fields [2,3,4]:

$$(i\gamma \cdot D - m_q) \rightarrow 0,$$

where $D_\mu$ is the covariant derivative containing the gluon and the photon fields. This procedure corresponds to going on-shell with external quarks in quark operators. Certainly, quarks are not exactly on-shell in hadrons, especially not in the octet (would-be Goldstone) mesons $\pi, K, \eta$. We shall see that the naive use of [4] is not correct in general. In fact, the bound-state interactions within mesons might be understood as a change of the (perturbative) equations of motion. One expects on

*Presented by Jan O. Eeg. Dedicated to the memory of our colleague Roger Decker.
general grounds that \((i\gamma \cdot D - m_q)\), instead of having zero effect, is proportional to some binding energy or some non-perturbative parameter characterizing the problem.

We consider two cases where an effective Lagrangian containing the factor \((i\gamma \cdot D - m_q)\) has been studied 5. First, we consider the circumstances under which the renormalized \(s \rightarrow d\) self-energy transition becomes potentially relevant to \(K \rightarrow 2\pi\) decays. It was previously shown 6 that non-zero effects of \(s \rightarrow d\) transitions might persist. Then we consider Lagrangians obtained from quark diagrams for \(s \rightarrow d\gamma\) and \(s \rightarrow d\gamma\gamma\) relevant to \(K \rightarrow \gamma\gamma\), and similar Lagrangians, obtained from quark diagrams for \(b \rightarrow s\gamma\) and \(b \rightarrow s\gamma\gamma\), relevant to the \(B_s \rightarrow \gamma\gamma\) decay.

A proved example of an off-shell effect is the Lamb shift – the tiny difference between the self-energy of a free electron and the self-energy of an electron bound in the H-atom 7. One might expect more significant analogous effects for much more strongly bound quarks. Still, since one can hardly speak of the referent free-quark self-energy, one might expect that there is a better chance of finding an observable effect in the flavour-changing, non-diagonal \(s \rightarrow d\) self-energy transition.

2. Effective Lagrangians at quark level

The total effective strangeness-changing Lagrangian may be written in the form

\[
\mathcal{L}(\Delta S = 1) = \sum_i C_i Q_i ,
\]

where the \(C_i\)’s are coefficients containing the effects of short distances through electroweak loop diagrams, dressed with hard gluons. The \(Q_i\)’s are operators containing light-quark fields \((q = u, d, s)\). We consider two specific pieces of \(\mathcal{L}(\Delta S = 1)\), namely \(\mathcal{L}_{ds}\) due to the non-diagonal \(s \rightarrow d\) self-energy transition and \(\mathcal{L}(s \rightarrow d)\gamma\) due to \(s \rightarrow d\gamma\) and \(s \rightarrow d\gamma\gamma\) transitions.

The bare unrenormalized self-energy transition (see Fig. 1) is divergent and non-vanishing on the mass shell. Since there are no direct \(s \rightarrow d\) transitions in the original Lagrangian, the renormalization is carried out so that \(s \rightarrow d\) transitions are absent for on-shell \(s\)- and \(d\)-quarks. This requirement defines the physical \(s,d\) fields in the presence of weak interactions, and the effective Lagrangian corresponding to the renormalized self-energy transition takes the form

\[
\mathcal{L}_{ds}^R = -A \bar{d}(i\gamma \cdot D - m_d)(i\gamma \cdot DR + M_R R + M_L L)(i\gamma \cdot D - m_s)s + h.c. ,
\]
Figure 2: Quark diagrams for $s \rightarrow d\gamma$ and $s \rightarrow d\gamma\gamma$ corresponding to $\mathcal{L}(s \rightarrow d)_\gamma$, $\mathcal{L}_F$ and $\mathcal{L}_\sigma$ in eqs. (4) - (7).

where $M_L, M_R$ are constants depending on quark masses, and $A$ contains the result of the loop integration.

In the pure electroweak case, the CP-conserving part of $A$ is of order $G_F m_c^2/M_W^2$. However, the CP-violating part of $A$ has no such suppression for a $t$-quark with a mass of the same order as the $W$-boson or heavier. Moreover, adding perturbative QCD to lowest order, in any case one obtains an unsuppressed contribution $\sim G_F\alpha_s(\log m^2)^2$, where $m = m_c$ and $m \sim M_W$ in the CP-conserving and CP-violating cases, respectively.

If one applies the EOM as in (1), then $\mathcal{L}_{ds}^R \rightarrow 0$. According to the standard procedure, one would discard contributions from $\mathcal{L}_{ds}^R$ to physical amplitudes, such as $K \rightarrow 2\pi$. However, if (1) is violated for off-shell bound quarks in $\pi$ and $K$, physical effects could be obtained, and one should explore possible consequences for the $\Delta I = 1/2$ rule and for $\epsilon'/\epsilon$ in $K \rightarrow 2\pi$ decays.

The evaluation of the loop diagrams for $s \rightarrow d\gamma$ and $s \rightarrow d\gamma\gamma$ transitions (for real photons; see Fig. 2) without going to the mass shell, results in an effective Lagrangian

$$\mathcal{L}(s \rightarrow d)_\gamma = B \epsilon^{\mu\nu\lambda\rho} F_{\mu\nu} \left( \bar{d}_L i \overset{\leftrightarrow}{D}_{\lambda} \gamma_\rho s_L \right) + h.c., \quad (4)$$

where $F$ is the electromagnetic field tensor, and $B \sim eG_F\lambda_{KM}$ depends on the loop integration ($\lambda_{KM}$ is the relevant KM parameter). In order to follow the fate of the off-shell contribution, it is convenient to rewrite (4) in the form

$$\mathcal{L}(s \rightarrow d)_\gamma = \mathcal{L}_F + \mathcal{L}_\sigma, \quad (5)$$

where

$$\mathcal{L}_F = B_F \bar{d}[(i\gamma \cdot D - m_d) \sigma_{\mu\nu} F^{\mu\nu} L + \sigma_{\mu\nu} F^{\mu\nu} R(i\gamma \cdot D - m_s)] s + h.c., \quad (6)$$
and $\mathcal{L}_\sigma$ is the well-known magnetic-moment term,

$$\mathcal{L}_\sigma = B_\sigma \bar{d} \left( m_s \sigma_{\mu\nu} F^{\mu\nu} R + m_d \sigma_{\mu\nu} F^{\mu\nu} L \right) s + h.c. .$$

(7)

Here we have anticipated that the coefficients $B_F$ and $B_\sigma$, being equal at the $W$-scale, evolve differently down to the scale of $\sim 1$ GeV. The anomalous dimension of $\mathcal{L}_F$ is zero \[15\], while $\mathcal{L}_\sigma$ is known to have a nonzero anomalous dimension \[16\].

It has been shown that $\mathcal{L}_F$ does not contribute to $s \rightarrow d \gamma \gamma$ when the external quarks are on-shell: The irreducible $s \rightarrow d \gamma \gamma$ part, with $iD_\mu \rightarrow e_{s(d)} A_\mu$, is exactly cancelled by reducible diagrams \[13\], \[14\], i.e. with one photon on an external line of the $s \rightarrow d \gamma$ vertex, with $D_\mu \rightarrow \partial_\mu$. Thus, for on-shell quarks, the remaining contribution from $\mathcal{L}(s \rightarrow d)_\gamma$ to $s \rightarrow d \gamma \gamma$ is due to the reducible diagrams, where the effective flavour-changing vertex corresponds to $\mathcal{L}_\sigma$ alone. Moreover, this remaining contribution vanishes in the chiral limit $m_{s,d} \rightarrow 0$, as seen from (7). In the pure electroweak case, the CP-conserving part of the quantity $B$ is very small, $\sim e G_F m_s^2/M_W^2$, owing to an effective GIM cancellation between $u$- and $c$- quarks, whereas the CP-violating part is substantial ($\sim e G_F$) owing to the heavy $t$-quark \[12\]. In the CP-conserving case, a significant amplitude $\sim e G_F \alpha_s \log(m_c^2)$ is induced by perturbative QCD \[16\].

3. Bosonization and the chiral quark model

One possibility of including non-perturbative confining and chiral-symmetry aspects of QCD is to use some version of the chiral quark model, an effective low-energy QCD model advocated by many authors \[17\], \[18\], \[19\]. To quote Weinberg \[17\], such a framework will introduce “fictitious elementary particles into the theory, in rough correspondence with the bound states” – pseudoscalar Goldstone mesons among the degrees of freedom of the constituent quark model. The chiral quark model includes the ordinary QCD Lagrangian and adds a term $\mathcal{L}_\chi$ that takes care of chiral-symmetry breaking,

$$\mathcal{L}_\chi = -M(\bar{q}_R U q_L + \bar{q}_L U^\dagger q_R) ,$$

(8)

where $\bar{q} = (\bar{u}, \bar{d}, \bar{s})$ and the $3 \times 3$ matrix $U \equiv \exp \left( 2i \Pi / f \right)$ contains the pseudoscalar octet mesons $\Pi = \sum_a \pi^a \lambda^a / 2$ ($a = 1, \ldots, 8$), and $f$ can be identified with the pion decay constant, $f = f_\pi = (92.4 \pm 0.2)$ MeV ($= f_K$, in the chiral limit). This term, proportional to the constituent quark mass $M \sim 300$ MeV, includes the Goldstone meson octet in a chiral-invariant way, and provides a meson-quark coupling that makes it possible to calculate matrix elements of quark operators as loop diagrams. In this effective field theory it is of course no problem to handle off-shell quarks. It should be noted that a calculation based on (8) reproduces the amplitude for $\pi^0 \rightarrow 2\gamma$, governed by the triangle anomaly.

The term $\mathcal{L}_\chi$ in (8) can be transformed into a pure mass term $-M \bar{Q} Q$ for rotated “constituent quark” fields $Q_{L,R}$:

$$q_L \rightarrow Q_L = \xi q_L ; \quad q_R \rightarrow Q_R = \xi^\dagger q_R ; \quad \xi \cdot \xi = U .$$

(9)
Figure 3: The total $s \rightarrow d\gamma\gamma$ amplitude due to $\mathcal{L}_F$ (represented by a cross within a circle): The irreducible contribution (left) is cancelled by the reducible contribution (middle) for on-shell quarks. The last diagram (right) originates from $\mathcal{L}_{ds}^R$.

Then the meson-quark couplings in this “rotated” ($R$) picture are transformed into the kinetic (Dirac) part of the “constituent quark” Lagrangian. These interactions can be described in terms of vector and axial vector fields coupled to constituent quark fields $Q$. The rotated picture is of course equivalent to the unrotated picture defined by (8). However, an explicit diagrammatic evaluation gives zero result for $\pi^0 \rightarrow 2\gamma$ in the rotated picture. The explanation is that the anomaly term is contained in the Jacobian of the quark field rotation in eq. (9).

Although the $\pi^0$ axial anomaly is not conventionally termed the off-shell effect, it might be viewed in this way, because the divergence of the axial current cannot be reproduced by the classical equations of motion.

4. Application to $K \rightarrow \pi\pi$ and $K \rightarrow \gamma\gamma$

Using the meson-quark couplings $\sim M_{\gamma\pi}/f_\pi$ obtained from (8), and effective electroweak transition vertices obtained from $\mathcal{L}_{ds}^R$, $\mathcal{L}_F$ and $\mathcal{L}_\sigma$, the calculation of weak transitions for mesons can be performed in terms of quark loops. An explicit calculation of the contribution to $K \rightarrow 2\pi$ from $\mathcal{L}_{ds}^R$ gives a non-zero but negligible contribution in the CP-conserving case. For the CP-violating case, one might obtain a significant contribution. Anyway, it should be noted that the $K_L \rightarrow \pi\pi$ amplitude obtained from $\mathcal{L}_{ds}^R$ is suppressed by $M^2/\Lambda^2$, where $\Lambda = 2\pi f_\pi \sqrt{6/N_c}$ is the chiral symmetry-breaking scale. Thus, this contribution disappears in the limit $M/f_\pi \rightarrow 0$, when the meson-quark interactions from (8) are switched off.

In our previous work we found a substantial CP-violating $K_S \rightarrow \gamma\gamma$ amplitude from irreducible diagrams for $s \rightarrow d\gamma\gamma$. Owing to Ward identities between $s \rightarrow d\gamma\gamma$ and $s \rightarrow d\gamma$ transitions, there is a cancellation between 1PI diagrams for $s \rightarrow d\gamma\gamma$ and reducible diagrams for the two-photon emission (where the 1PI transition $s \rightarrow d\gamma$ is a building block). However, one cannot expect this free-quark cancellation to persist in the real world: the hadronic matrix elements of the reducible graphs (see Fig. 3, middle) are of highly non-local character, whereas the matrix elements of the irreducible graphs (see Fig. 3, left) are proportional to a quark current, having a well-known matrix element.

There have been considerable efforts devoted to the study of the direct
$K_{L,S} \to \gamma\gamma$ amplitudes induced by the operators (4)–(7). By explicit calculation within the chiral quark model, we have found a non-zero contribution to $K \to \gamma\gamma$ from $\mathcal{L}_F$. The result can be written as an effective interaction

$$\mathcal{L}(K \to 2\gamma) = G_{K2\gamma} F \cdot \tilde{F} \Phi_K; \quad G_{K2\gamma} \sim e B_F f_\pi \frac{M^2}{\Lambda^2},$$  \hspace{1cm} (10)

where $\Phi_K$ is the K-meson field. The result of the irreducible part of $\mathcal{L}_F$ (Fig. 3, left), being proportional to $f_K (= f_\pi$ in the chiral limit) is cancelled by the leading part of the reducible graphs (Fig. 3, middle), the net result being the formally non-leading part of the reducible graphs. Strictly speaking, (10) is the result in the chiral limit, i.e. the contribution from $\mathcal{L}_F( m_{s,d} \to 0) = \mathcal{L}_\gamma (s \to d)$. It should also be noted that $K \to \gamma\gamma$ receives a contribution from $\mathcal{L}_\sigma$ (Fig. 3, middle) and $\mathcal{L}_{\Delta s}^R$ (Fig. 3, all diagrams). However, these contributions are less important numerically.

Although $G_{K2\gamma}$ in (10) is formally suppressed by $M^2/\Lambda^2$, its coefficient is sizeable, yielding a significant amplitude, both in the CP-conserving $K_L \to \gamma\gamma$ case \cite{22}, and in the CP-violating $K_S \to \gamma\gamma$ case \cite{5,12}. Thus we disagree with some authors \cite{14,24} who claim that this effect is unimportant.

An important property of the $K \to \gamma\gamma$ amplitude obtained from $\mathcal{L}_F$, is that it is zero when diagramatically calculated within the rotated basis \cite{22}. Thus, this amplitude has a similar anomalous nature as $\pi^0 \to \gamma\gamma$! One should, however, note that in contrast to $\pi^0 \to \gamma\gamma$, the contribution to $K \to \gamma\gamma$ from $\mathcal{L}_F$ (and similarly from $\mathcal{L}_{\Delta s}^R$) is mass-dependent through the factor $M^2$.

The non-zero contribution from $\mathcal{L}_F$ to $K \to \gamma\gamma$ was recently confirmed in a bound-state calculation \cite{23}. To evaluate the hadronic matrix elements in the bound-state approach, the variant of an effective meson bilocal theory was used. The bound-state calculation in essence confirms the previous chiral-quark results: our off-shell contribution is, within the language of chiral perturbation theory, an entirely new $\mathcal{O}(p^4)$ direct-decay piece \cite{22} not contained in previous analysis \cite{23}, whereas the reducible pole contributions \cite{26} are numerically uncertain, and the non-diagonal magnetic-moment term belongs to the $\mathcal{O}(p^6)$ terms.

5. The quark-loop $B_s \to \gamma\gamma$ amplitude

As in the $\Delta S = 1$ case \cite{22}, the flavour-changing radiative vertices have to be supplemented by the quark-meson vertex in order to perform the full quark-loop evaluation. In contradistinction to the analogous $K \to \gamma\gamma$ decay \cite{24}, the heavy $B$ meson cannot be treated as a Goldstone boson of the chiral quark model adopted earlier. However, the pseudoscalar character of $B$ mesons allows us to write down a simple $\bar{b}s B_s$ interaction, replacing the $\mathcal{L}_X$ term in (8) by

$$iG_B \bar{s}\gamma_5 b B_s.$$

This interaction may in general be non-local, i.e. $G_B$ might be momentum-dependent \cite{27}. Thereby, as usually done \cite{1}, we trade the meson-quark coupling $G_B$
in favour of the meson-decay constant $f_B$. In calculating the contributions from $\mathcal{L}_F$ and $\mathcal{L}_\sigma$ in (12) and (13), respectively (with the obvious replacements $s \to b$ and $d \to s$), we obtained an amplitude of the following form

$$
M(B_s \to \gamma\gamma) = e_D f_B [A(+) F_\mu\nu F^{\mu\nu} + i A(-) F_\mu\nu \tilde{F}^{\mu\nu}],
$$

(12)

$$
A(\pm) = \tau^{(\pm)}_F B_F + B_\sigma \tau^{(\pm)}_\sigma,
$$

(13)

where the quantities $\tau^{(\pm)}_{F,\sigma}$ are dimensionless and depend on the bound-state dynamics. Numerically, they turn out to be of order one. The coefficients $B_F$ of $\mathcal{L}_F$ and $B_\sigma$ of $\mathcal{L}_\sigma$ now contain the KM factors relevant to the $b \to s$ transition and are renormalized at the scale $\mu = m_b$.

In the formal limit where the current quark masses $m_{b,s} \to 0$, the $F_\mu\nu \tilde{F}^{\mu\nu}$ term of (12) should reduce to the anomalous $K \to 2\gamma$ amplitude described in the preceding section. However, in the real world, $M_b \gg M_s \gg M \sim 300$ MeV, and the result for $\text{Br}(B_s \to 2\gamma)$ will be rather different from $K \to 2\gamma$. In order to estimate the model-dependent quantities $\tau^{(\pm)}_{F,\sigma}$, one might consider several assumptions. In a previous paper \[28\] we found that, with reasonable assumptions, $\tau^{(\pm)}_\sigma \sim 2$ to 3, whereas $\tau^{(\pm)}_F$ was formally suppressed by $\tilde{\Lambda}/M_b$ with respect to $\tau^{(\pm)}_\sigma$. (The parameter $\tilde{\Lambda}$ is a few hundred MeV). This is in agreement with intuitive expectations. We also found that the genuine off-shell term $\mathcal{L}_F$ increased the rate by a factor of $\sim 1.5$ to 3. We conclude that the branching ratios of the order $10^{-8}$ to $10^{-7}$ are realistic. This result is not far from that given in Ref.\[29\]. Our prediction is still two orders of magnitude above the LD estimates based on the vector-meson dominance \[28\].

In a recent paper \[31\], it was reported that there are off-shell bound state effects in the process $B \to K^*\gamma$. However, only the off-shellness of the $b$-quark was taken into account, whereas we have found that for $B_s \to 2\gamma$ the off-shellness of the $s$-quark cannot be neglected. In our recent study \[28\] of $B_s \to 2\gamma$ we have shown that the contribution from the two-photon piece of $\mathcal{L}_F$ is exactly cancelled by parts of its contribution from the one-photon piece. The remaining contribution from the off-shell operator $\mathcal{L}_F$ corresponds to loop diagrams containing effective $B_s \bar{b}b\gamma$ and $B_s \bar{s}s\gamma$ vertices. This result is equivalent to that presented in Ref.\[3\]: $\mathcal{L}_F$ may be transformed into the wave function, but then it reappears in the bound-state dynamics.

6. Results and conclusions

We have demonstrated the quark off-shell effects in flavour-changing two-photon decays, such as $s \to d\gamma\gamma$ ($b \to s\gamma\gamma$) and its hadronic $\bar{K}^0 \to \gamma\gamma$ ($B_s \to \gamma\gamma$) counterparts. Thus, the same basic off-shell effect seems to take place in processes belonging to such different calculating environments as the chiral perturbation theory and the one accounting for the heavy-light bound-states. Thus the naive use of the (perturbative) EOM \[1\] is not applicable in general. The bound-state dynamics changes the (perturbative) equations of motion \[1\].
The genuine off-shell effects are formally suppressed in a certain limit by \( \tilde{\Lambda}/M_b \) for \( B \to \gamma\gamma \) and by \((M/\Lambda)^2\) for \( K \to \gamma\gamma \) (and \( K \to \pi\pi \)). Numerically, the suppression is not equally pronounced in these cases. For \( K \to \gamma\gamma \), the effect of \( \mathcal{L}_F \) is bigger than that of \( \mathcal{L}_\sigma \). Indeed, the latter effect is chirally suppressed and of order \( \mathcal{O}(p^6) \).

We have assumed that chiral-symmetry breaking only affects the strong sector, and does not induce any new terms in the electroweak sector. It might be argued that this is not obvious.\(^3\)\(^2\)\(^3\) It is of course possible to write down effective Lagrangian terms containing meson fields in the combined strong and electroweak sector which contribute to our processes. However, we cannot see how such terms should be generated. One way of addressing this issue could be within Nambu-type models. Within such models, one (or more) gluon exchanges generate four quark operators in the strong sector which are supposed to be responsible for the term \( \mathcal{L}_F \). Apriori, \( \mathcal{L}_F \) could generate a new relevant operator if another quark line is attached through gluon exchange. However, the sum of such contributions vanishes on the mass shell, and for off-shell quarks, will correspond to a complicated higher dimensional operator which, for instance, will not cancel our \( K \to \gamma\gamma \) amplitude.

The quark off-shellness represents a link that brings the electroweak \( K \to \gamma\gamma \) decays close to the electromagnetic \( \pi^0 \to \gamma\gamma \) decay.\(^3\)^\(^4\) Although the \( \pi^0 \) axial anomaly is not conventionally termed the off-shell effect, it might be viewed in this way, because the divergence of the axial current cannot be reproduced by the classical equations of motion. It should be emphasized that \( K \to \gamma\gamma \) can be treated as textbook approaches for \( \pi^0 \to \gamma\gamma \), with one of the electromagnetic vertices replaced by that obtained from \( \mathcal{L}_F \). Strictly speaking, all that is assumed through our treatment of \( K \to 2\gamma \) is the PCAC! \( K \)-field is replaced by the divergence of the axial quark current (corresponding to the rotated picture), or by the pseudoscalar quark density (unrotated picture).

However, the direct amplitude originating in the quark off-shellness in the kaon is only a fraction of the total \( K_L \to \gamma\gamma \) amplitude, and it is model-dependent through the constituent quark mass \( M \). The various LD aspects, including the reducible pole contributions, seem to play a dominant role in this case.\(^2\)\(^6\) However, the CP-violating \( K_S \to \gamma\gamma \) amplitude receives its main contribution from \( \mathcal{L}_F \).\(^4\)

For \( B_s \to \gamma\gamma \), we have also found a non-zero genuine off-shell contribution. Although the hadronic matrix element is model dependent, there are substantial off-shell contributions that increase the rate by a factor of \( \sim 1.5 \) to \( 3 \). It is hoped that some of the uncertainties in calculating the effects of \( \mathcal{L}_F \) could be resolved within some variant of a QCD sum-rule\(^2\)\(^8\) calculation.

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