The explanation of heavy-fermion superconductivity is a long-standing challenge to theory. It is commonly thought to be connected to non-local fluctuations of either spin or charge degrees of freedom and therefore of unconventional type. Here we present results for the Kondo-lattice model, a paradigmatic model to describe heavy-fermion compounds, obtained from dynamical mean-field theory which captures local correlation effects only. Unexpectedly, we find robust $s$-wave superconductivity in the heavy-fermion state. We argue that this novel type of pairing is tightly connected to the formation of heavy quasiparticle bands and the presence of strong local spin fluctuations.

PACS numbers: 71.27.+a,74.20.-z,74.20.Mn

Heavy-fermion (HF) 4f and 5f intermetallic compounds constitute a paradigm for strong electronic correlations. Their low-temperature behavior is affected by f-shell local moments subject to antiferromagnetic (AF) exchange coupling to the conduction electrons, resulting in Fermi-liquid (FL) phases with strongly renormalized Landau parameters, most notably huge effective masses [1–4]. HF materials often display symmetry-breaking phases which occur either within the heavy FL [1] or compete with it [5]. While magnetic order in systems containing unscreened moments appears natural, HF superconductivity is conceptually non-trivial and indeed came as an unexpected discovery more than three decades ago [6]. By now, a wide variety of f-electron superconductors are known [7, 8], many of them confirmed to be unconventional [1, 2, 9].

Superconducting (SC) transitions in HF compounds are often assumed to be driven by non-local fluctuations of the f-shell spin degrees of freedom. This idea finds support in the close connection between HF superconductivity and magnetic quantum phase transitions (QPTs) where such fluctuations are strong [3, 5, 10, 11]. Alternatively, pairing mediated by fluctuations in the charge channel (i.e., valence fluctuations) has also been discussed [12]. Given that the basic theoretical models for HF materials, the periodic Anderson model (PAM) and Kondo-lattice model (KLM), constitute complicated interacting many-body problems which cannot be solved exactly, theoretical descriptions of HF superconductivity often employ either simple static mean-field theories or effective models of fermions coupled to spin or charge fluctuations.

A rather successful approach to study the microscopic properties of correlated-electron lattice models beyond static mean-field or effective descriptions is the dynamical mean-field theory (DMFT) together with its cluster extensions [13, 14]. The lattice problem is mapped onto a self-consistent quantum impurity model at the expense of losing information on non-local correlation effects beyond the spatial size of the impurity cluster. Therefore, it is commonly assumed that a proper description of HF superconductivity within DMFT-based approaches requires either large enough clusters or the inclusion of a bath in the two-particle channel (which explicitly models a bosonic “glue” for superconductivity). In particular, within the conventional (single-site) DMFT only $s$-wave superconductivity occurs [15] and a relation to magnetic fluctuations appears highly unlikely.

In this Letter we report on the unexpected observation of a stable SC solution to the DMFT equations for the KLM without any external glue. Although the pairing symmetry is $s$-wave, the SC state is highly unconventional: Pairing is driven by local spin fluctuations; it comes with a strong frequency dependence of the gap function and requires the formation of HF bands as a prerequisite.

We note that a hint of the possible occurrence of local superconductivity was found in an earlier DMFT study to the PAM [16] which, however, did not analyze the SC phase but only normal-state instabilities. Static mean-field descriptions of the KLM or the PAM can also yield solutions with local pairing [17–19], but it is difficult to assess their validity, as fluctuations beyond mean field may destroy pairing.

Model. Within the KLM the localized f-states are described by a (pseudo-)spin degree of freedom which couples to the conduction $(c)$ electrons via an exchange interaction. We use the simplest version of the model, i.e., a nearest-neighbor tight-binding conduction band with spin degeneracy only and a $S = 1/2$ spin located at each lattice site. The Hamiltonian reads

$$\mathcal{H} = -t \sum_{\langle i,j \rangle, \sigma} \hat{c}^\dagger_{i \sigma} \hat{c}_{j \sigma} + \frac{J}{2} \sum_{i, \alpha \beta} \hat{S}_i \cdot \hat{c}^\dagger_{i \alpha} \tau_{\alpha \beta} \hat{c}_{i \beta}. \quad (1)$$

Here $c^\dagger_{i \sigma}$ denote annihilation (creation) operators of conduction electrons with spin $\sigma$ at site $i$, and $\langle \ldots \rangle$ denotes nearest-neighbors. $\hat{S}_i$ is the operator for the localized spin, $\tau$ the vector of Pauli matrices, and the interaction between the conduction electrons and the localized spin is modeled as an isotropic exchange coupling with $J > 0$. For simplicity, we will consider nearest-neighbor hopping $t$ on the infinite-dimensional
Bethe lattice, leading to a semi-circular density of states with bandwidth $W$. We have checked that using different lattice types (for example hypercubic or square lattice tight-binding) does not change the results qualitatively.

Methods. Within standard DMFT, the conduction-electron self-energy is approximated as local in space, $\Sigma(k, \omega) \rightarrow \Sigma(\omega)$. Then, the KLM maps onto an effective single-impurity Kondo model (SIKM) [4] which needs to be solved within a self-consistency loop. In this work we treat the effective SIKM using Wilson’s numerical renormalization group (NRG) [20]. It allows one to access arbitrarily small energy scales, to calculate spectra directly on the real-frequency axis, and to work at both $T = 0$ and $T > 0$. We work with the discretization parameter $\Lambda = 2.0$, keep $N_{st} = 1000, \ldots, 2000$ states, and perform $z$-averaging with $N_z = 2$ [20].

To allow for solutions with SC order, we generalize the DMFT equations and the impurity solver to a Nambu formulation with $2 \times 2$ matrix propagators [20, 21]. This constrains our calculations to spin-singlet even-frequency $s$-wave superconductivity. The DMFT treatment of superconductivity is non-perturbative and thus goes beyond the standard Eliashberg theory [22]: it does not rely on any assumption about a separation of energy scales for the fermions and the bosonic glue responsible for the formation of superconductivity. At present, we restrict the calculations to SC order only, suppressing possible magnetic order. Our result below show that strong pairing occurs in a regime without magnetic order, justifying this neglect.

Results: Superconductivity at $T = 0$. Our numerical solution of the DMFT equations yielded, for a range of model parameters, stable SC solutions whose properties we discuss in the following. The conduction-band density of states (DOS) of both the paramagnetic normal (N) and SC solutions of the KLM for fixed conduction band filling $n = 0.9$ and different $J$ are shown in Fig. 1, left panels. The only feature of the N DOS is a hybridization pseudo-gap above the Fermi energy, signaling the formation of heavy quasiparticles, which can be rationalized within the picture of hybridized $c$ and $f$ bands. The N solution is unstable against superconductivity, where the DOS exhibits two additional features: (i) A true gap $\Delta_{sc}$ with well-developed van-Hove singularities is present around the Fermi energy; as a function of $J$, it first increases up to $J/W = 0.5$, and then slowly decreases. (ii) In addition to the SC coherence peaks, there are side resonances at positions which roughly scale with $J$. These structures are sharp for smaller $J$, but become increasingly washed out for larger $J$. These features are likely related to local spin fluctuations stabilizing the pairing, as discussed in the supplemental material.

As the appearance of a gap alone is not sufficient to identify the solution as a SC, one needs to look at the anomalous parts of the Nambu Green’s function respectively the anomalous part of the self-energy. From it, a SC gap function can be defined, as in standard Eliashberg analysis, via

$$\Delta(\omega) = \frac{\Sigma_1(\omega) + i\Sigma_2(\omega)}{1 - \Sigma(\omega)/\omega},$$

where $\Sigma_\alpha(\omega)$ denote the components of the electronic self-energy expanded into Pauli matrices, $\Sigma = \Sigma_\alpha \tau_\alpha$ ($\alpha = 0, 1, 2, 3$), in Nambu space. The resulting real parts $\text{Re}\Delta(\omega)$ are shown in the right panels of Fig. 1. As expected for even-frequency pairing, $\text{Re}\Delta(\omega)$ is symmetric. It shows a strong frequency dependence, with sharp features shifting to larger energies and broadening with increasing $J$. These structures are linked to the side resonances in the DOS: the zeroes of $\text{Re}\Delta$ coincide with the resonances in the DOS. The $\omega = 0$ limit, $\text{Re}\Delta(0)$, provides an estimate of the gap seen in the DOS; it exhibits the same non-monotonic behavior with $J$ as noted above for the gap in the DOS.

To characterize the evolution of superconductivity across the phase diagram, we plot in Fig. 2 the anomalous expectation value $\Phi = \langle \hat{c}_i^\dagger \hat{c}_i \rangle$, as a function of $J$ and $n$. Superconductivity is found to be stable over wide regions of the phase diagram for $J/W > 0.1$. For a fixed $J/W = 0.2$, a finite $\Phi$ is found between 0.45 < $n < 1$. For larger $J/W$ the SC re-
gion extends to even lower fillings. A maximum of $\Phi$ appears around $J/W = 0.3$ and $n = 0.9$; the side-resonances are also the most pronounced for these parameters.

In Fig. 3 we display the evolution of $\Phi$ along two cuts along the phase diagram indicated by the white dashed lines in Fig. 2. The analysis for weak Kondo coupling, $J/W < 0.1$, is difficult as the signatures of SC become very weak and hard to distinguish from numerical noise. Thus we cannot decide whether the SC solution ceases to exist for small $J$, or whether it survives down to $J \to 0$ with an (exponentially) small pairing scale. (The latter would be expected in the weak-coupling limit of certain mean-field theories [19].) For $J \gtrsim W/2$, on the other hand, we observe a decay consistent with $\Phi(J) \propto 1/J$. We will comment on this behavior further down.

**Normal-state Fermi-liquid scale.** In the normal state, the KLM realizes a heavy FL at low temperatures for $n \neq 1$, with a FL (coherence) scale $T_0$. In a local self-energy approximation, $T_0$ can be efficiently extracted from the quasiparticle weight

$$Z^{-1} = 1 - \left. \frac{d\text{Re}\Sigma(\omega)}{d\omega} \right|_{\omega=0},$$

via $T_0 = WZ$, where $\Sigma(\omega) = \Sigma_0(\omega) + \Sigma_3(\omega)$. The evolution of $T_0$ with $J$ and $n$ is also depicted in Fig. 3(a) and (b); we recall that for $J \to 0$ the scale $T_0$ depends exponentially on both $J$ and the bare $c$ DOS, $T_0 \propto \sqrt{J/W} \exp(-\alpha(n) \cdot W/J)$ with a weakly $n$-dependent coefficient $\alpha(n)$ [23, 24]. For large $J \gtrsim W/2$ the dependence of $T_0$ on $J$ significantly deviates from this Kondo form and rather tends to saturate as $J \to \infty$. Finally, for fixed $J$ and varying $n$ we recover the known dependency $T_0 \propto n \cdot e^{-\alpha n}$ [24]. These different types of behavior for $T_0(n, J)$ can be seen from the lines superimposed to the data in Fig. 3.

Apparently, there does not exist a simple connection between $T_0$ and $\Phi$. For small $J \Phi(n, J)$ seems to scale with $T_0$. However, as noted before, the results for very small $\Phi$ become unreliable for numerical reasons, i.e. one cannot readily extract a simple relation between $\Phi$ and $T_0$ in this limit. For large $J$ at $n = 0.9$, on the other hand, we do not see a direct relation between $T_0$ and $\Phi$, but find $\Phi \propto 1/J$ instead.

**Competition with magnetism.** Within the DMFT for the KLM, one also finds magnetic phases, namely antiferromagnetism (AF) close to half filling and ferromagnetism (FM) at small filling [25-27]. Note, however, that these phases have limited extent in both $J$ and $n$, for example at $n = 0.9$ we find AF only for $J < J_c(n = 0.9) \approx 0.2 W$ [26] and FM only for $n < n_c(J/W = 0.2) \approx 0.65$ [27]. Thus, the region in parameter space where we find a strong superconducting phase in Fig. 2 seems to be complementary to the regions with magnetic phases. As the boundaries seem to overlap – in particular the SC phase at $n = 1$ lies inside the AF regime – it is surely interesting to study the competition respectively interplay of AF, FM and SC in detail. This is work in progress.

**Results for $T > 0$.** Figure 4 displays finite-temperature results for the DOS. With increasing temperature $T$ the gap shrinks and the spectral side resonances are depleted. The reduced pair correlations are also reflected in a decrease of $\Phi$ (see inset to Fig. 4). Close to $T_c$ the gap is progressively filled, and both the hybridization gap and the side resonances move towards the Fermi level. Finally, in the normal-state solution for $T > T_c$ only the hybridization pseudo-gap is visible.

Estimating $T_c$ from the numerical data at finite $T$ is hard, as close to the critical temperature the signatures of SC become very small and also the convergence of the DMFT rather slow. From the data in Fig. 4 we extract $T_c \approx 0.0036 W$ at $n = 0.9$ and $J = 0.25 W$, which is well below $T_0$. In Tab. I we collect the resulting estimates for $T_c$ for fixed $n = 0.9$ and several values of $J$ in the region where according to Fig. 3 we have optimal conditions for pairing. As a rule we observe that always $T_c < T_0$, i.e. the HF state seems to be a necessary ingredient for the appearance of the SC phase. For small $J$ this obviously leads to a strong suppression of $T_c$. On the other hand, the gap $\text{Re}\Delta(0)$ appears to be much less sensitive to $J$ respectively $T_0$ in this regime. An interesting characteristic quantity is the ratio between the gap and $T_c$. The results are given in the last row of Tab. I. Obviously, the ratio exceeds the
| $J/W$  | 0.2  | 0.25 | 0.3  | 0.4  | 0.5  |
|-------|------|------|------|------|------|
| $T_0/W$ | 0.0200 | 0.0418 | 0.0658 | 0.1139 | 0.1548 |
| $T_c/W$ | 0.0027 | 0.0036 | 0.0054 | 0.0058 | 0.0054 |
| $\Phi(T = 0)$ | 0.0160 | 0.0163 | 0.0174 | 0.0153 | 0.0140 |
| $\text{Re}\Delta(0)/W$ | 0.0138 | 0.0165 | 0.0180 | 0.0193 | 0.0195 |
| $\text{Re}\Delta(0)/T_c$ | 5.169 | 4.583 | 3.321 | 3.305 | 3.585 |

TABLE I: Quantities characterizing the superconducting solution for $n = 0.9$ as a function of $J$ in the region of optimal conditions for pairing.

BCS value $\Delta/T_c \approx 1.74$ by a sizeable factor between 2 and 3. Such values are actually observed in HF superconductors [11], although the interpretation there is usually given in terms of a weak-coupling theory for a $d$-wave state.

**Pairing mechanism at strong coupling.** In the strong-coupling limit, $J > W$, the pairing mechanism and behavior of $\Phi(J)$ can be understood perturbatively, Fig. 5. We consider a conduction-band filling of $n = N_{el}/N_s \lesssim 1$ on $N_s$ sites. For $J/W \rightarrow \infty$ there are $N_s$ Kondo singlets and $(N_s - N_{el})$ uncompensated local moments. The $c$ electrons are mobile, such that the uncompensated moments can alternatively be interpreted as spinful $c$ holes with density $(1 - n)$ and a hard-core repulsion, forming the Fermi liquid with a coherence scale $T_0 \propto W$ (whereas the impurity Kondo scale simply diverges $\propto J$).

For large but finite $J/W$ additional excitations out of this manifold are allowed. The lowest one consists of converting a singlet into a triplet and can be created by $c$ (hole) hopping. Interestingly, the excited triplet may decay via a different neighboring hole provided that its spin is opposite to the first one. Together, this second-order virtual process leads to correlated hopping, with an energy gain $\propto W^2/J$, which binds two holes into a singlet state, Fig. 5. The pairing is local – it occurs on the site of the virtual triplet – but the holes share this site only in the virtual triplet state, so that the pairing is strongly retarded.

It is plausible that this pairing mechanism continues to operate at smaller $J$. Importantly, the existence of the virtual state, whose energy may now be approximated by the Kondo binding energy $T_0$, requires Kondo screening to be intact – this naturally explains the limitation $T_c < T_0$. The picture also makes clear that superconductivity is more favorable close to half filling: Here, Kondo screening is done by $c$ electrons whereas the paired carriers are $c$ holes (for $n < 1$). In contrast, in the opposite (exhaustion) limit of small $n$, Kondo screening becomes strongly non-local, and it is the same $c$ electrons doing the screening that need to be paired, thus the pairing is weak.

**Summary.** We have identified a novel mechanism for superconductivity in heavy-fermion materials: Local spin fluctuations due to the Kondo exchange coupling can act as a retarded pairing interaction and drive $s$-wave superconductivity in the heavy FL. A particularly interesting feature is the appearance of structures in the tunneling DOS at scales related to the spin fluctuation spectrum, i.e. well separated from the coherence peaks at the gap edges. Such structures have been observed for example in recent STS measurements on iron pnictides [28]. It would be interesting, to extend such STS experiments to systematically study HF superconductors.

For model parameters relevant to HF systems, $T_c$ can be as large as several Kelvins, a typical $T_c$ scale for existing HF superconductors. However, given that pairing in our theory is $s$-wave, it can be ruled out for those materials where the existence of gap nodes have been established experimentally. More generally, it is an interesting question to what extent this mechanism can cooperate or will actually compete with SC driven by e.g. non-local magnetic fluctuations. To address this point numerical studies beyond DMFT will be required. Work along these lines is in progress.

This work was supported by the DFG through PR293/13-1 (OB,TP), the BMBF through IND 10/067 (TP) as well as FOR 960 and GRK 1621 (MV). RŽ acknowledges the support of ARRS through Program P1-0044. NRG calculations have been done using a modification of the NRG _Ljubljana_ package. Computer support was provided by the Jožef Stefan Institute Ljubljana and the Gesellschaft für wissenschaftliche Datenverarbeitung Göttingen (GWDG).

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LOCAL PAIRING IN MEAN-FIELD THEORY

Here we summarize how local pairing emerges in the Kondo lattice model within mean-field theory [1, 2]. One starts by representing the f-shell local moments in terms of auxiliary fermions:

\[ S_i = \frac{1}{2} \sum_{\sigma\sigma'} f_i^{\dagger} \tau_{\sigma\sigma'} f_{i\sigma'} \]  

(S1)

where \( f_i^{\dagger} \) describes a spinful (but charge-neutral) fermion destruction operator at site \( i \). This representation requires the constraint \( \sum_{\sigma} f_i^{\dagger} f_{i\sigma} = 1 \) and displays a local U(1) gauge invariance.

The standard decoupling of the Kondo interaction in the KLM [3] is done via a bosonic field \( b_i \) in the particle–hole channel, resulting in an interaction term of the form

\[ \mathcal{H}_b = \sum_{i\sigma} (b_i^{\dagger} c_{i\sigma} f_{i\sigma} + h.c.) . \]  

(S2)

In the particle–particle channel, the interaction can be decoupled in the particle–hole channel, resulting in an interaction term of the form

\[ \mathcal{H}_p = \sum_{i\sigma} (\bar{b}_i^{\dagger} c_{i\sigma} f_{i\sigma} + h.c.) . \]  

(S3)

Bose condensation of the \( \bar{b}_i \) alone yields the same metallic HF state, due to the local U(1) gauge invariance. Superconductivity is generated by a simultaneous condensation of \( b_i \) and \( \bar{b}_i \) which induces anomalous expectation values among the physical c electrons, \( \langle c_{i\uparrow} c_{i\downarrow} \rangle \neq 0 \), and this breaks the physical U(1) electromagnetic symmetry [4]. The resulting state displays local s-wave pairing with a full gap.

This mean-field description of Kondo-lattice superconductivity via two decoupling channels of the Kondo interaction has, however, obvious deficiencies. In particular, in the limit \( J/W \to \infty \) it yields strong pairing on the scale \( J \); this result is incorrect as the Kondo interaction only causes spin singlet formation on this scale (but no Cooper pairing). In the opposite limit of small \( J \), pairing in this mean-field theory becomes exponentially weak, with \( T_c \) scaling as the single-impurity Kondo temperature \( T_K \).

As an aside, we note that if one starts from an Anderson (instead of Kondo) lattice Hamiltonian, the \( f \) particles always carry a physical charge, such that only a mean-field decoupling in the particle–particle channel is required (in addition to the c–f hybridization) to induce superconductivity. Then, the resulting pairing may be understood as local c–f pairing [1, 2].

LOCAL SPIN-FLUCTUATION THEORY FOR PAIRING

Let us try to give an argument why for the KLM in DMFT the spin fluctuations can be a driving glue for superconductivity. To this end we use the approximate form of the pairing equations as given in [5], and employ Hewson’s renormalized perturbation theory [6], i.e. we assume that we have a local Fermi liquid with correspondingly renormalized parameters. Note that this also means that the Kondo spin is now a dynamic quantity with a fluctuation spectrum living on the order of the Kondo scale. We do not use the usual approximation in the Eliashberg equations that all fermionic energies can be put to the Fermi energy, because all relevant energy scales are of the order of \( T_0 \), including the spin fluctuation scale.

We are working within DMFT, i.e. all momentum dependencies can be ignored and the \( \bar{K} \) sums for the Green’s functions be performed using the non-interacting DOS. The equation for the gap can then be written as

\[ \Sigma^{(12)}(\omega+i0^+) = \int_{-\infty}^{\infty} d\omega' K(\omega, \omega') \left[ -\frac{1}{\pi} \text{Im} G^{(12)}(\omega' + i0^+) \right] \]

where the kernel is given by

\[ K(\omega, \omega') = \frac{1}{2} \int_{-\infty}^{\infty} d\Omega \frac{g^2}{g^2 + (\Omega - \omega - \omega')^2} \left[ \tanh \frac{\omega'}{2T} + \coth \frac{\Omega}{2T} \right] . \]

For the integral the principal value has to be taken and \( \chi_S^2(\omega) \) is the imaginary part of the dynamical susceptibility of the Kondo spin. The constant \( g^2 \) collects the (unknown) coupling matrix element and some constant numerical factors.

At this point one usually assumes that \( \omega_0 \approx \omega' \approx 0 \), which then leads to

\[ K(\omega, \omega') \approx -\frac{\pi g^2}{2} \tanh \frac{\omega'}{2T} \chi(0) . \]

As \( \chi(0) > 0 \), this kernel is repulsive and hence will not generate s-wave even-frequency superconductivity. As already
mentioned, we here cannot safely assume $\omega, \omega' \ll \mu$, since for our Fermi liquid $\mu \to \tilde{\mu} \sim T_0$. We therefore must try to analyze the equation somewhat more carefully. To this end we nevertheless will assume that we are deep enough in the Fermi liquid phase so that temperature dependencies of the Fermi liquid parameters are negligible, and that – at least for a determination of $T_c$ – we can approximate $\Sigma^{(12)}(\omega + i0^+) \approx -\Delta \Theta(J/2 - \omega) \in \mathbb{R}$. The estimate for the cut-off is taken from the evolution of the gap function in Fig. 1. Since the gap is frequency independent, we can at least put $\omega = 0$ and – ignoring the term with $\text{coth}[\epsilon_\omega]$ – find a kernel

\[
K(0, \omega') \approx \frac{\tilde{\gamma}^2}{2} \frac{\omega'}{2T} \int_{-\infty}^{\infty} d\Omega \frac{\chi'(\Omega)}{-\omega' - \Omega}
\]

\[
= -\frac{\pi \tilde{\gamma}^2}{2} \tanh \frac{\omega'}{2T} \chi'(0)
\]

As final input we need the anomalous Green’s function, which is given by

\[
G^{(12)}_{k^*}(z) = \frac{\Sigma^{(12)}(z)}{(z + \epsilon_k - \Sigma^{(12)}(z))(z - \epsilon_k - \Sigma^{(11)}(z)) - \Sigma^{(12)}(z)^2}
\]

using the independence of the self-energy on momentum in DMFT. We replace $\Sigma^{(12)}(z)$ by our approximation and also set it to zero in the denominator, as we are interested in finding $T_c$. Furthermore, the diagonal self-energies are to be replaced by the expressions for the local Fermi liquid, i.e.

\[
\Sigma^{(\alpha\alpha)}(z) = \frac{V^2}{z + \tilde{\mu}}
\]

where the effective chemical potential $\tilde{\mu}$ and hybridization $\tilde{V}$ are both $O(T_0)$. Using this self-energy expression in the normal phase leads to the well-known formation of heavy quasiparticle bands [6, 7]. In the following we further assume $\tilde{\mu} = 0$. With these simplifications we get

\[
G^{(12)}_{k^*}(z) = -\Delta \left\{ \frac{1}{z - |\epsilon_k| - \tilde{V}^2/z} - \frac{1}{z + |\epsilon_k| - \tilde{V}^2/z} \right\}
\]

We can now perform the $k^*$-sum. To this end we assume a featureless DOS of the band states with value $N_F$ and use the wide-band limit to obtain

\[
G^{(12)}_{k^*}(z) = i\pi \Delta N_F \frac{1}{z - \tilde{V}^2/z}
\]

Putting everything together into the gap equation we arrive at

\[
1 = N_F \int_0^{J/2} d\omega K(0, \omega) \left[ \frac{1}{\omega - V} + \frac{1}{\omega + V} \right] = 0
\]

as equation to determine $T_c$, where the integral has to be evaluated as principal value integral.

The kernel contains $\chi'(\omega)$, which is shown in Fig. S1. Although the kernel is negative for small $\omega$ and would be considered repulsive in a standard calculation, the contribution from the anomalous Green’s function within the approximation of a local Fermi liquid due to the Kondo effect is also negative for energies less than some constant times $T_0$, hence the total contribution from the region $\omega \in [0, T_0]$ is positive. There is of course the pole, which has to be treated in the principal value sense, leading to a sign change for $\omega > \tilde{V}$. Here it becomes important that $\chi'(\omega)$ decays rapidly on a scale $T_0$, too.

As very simple approximation let us assume that $\chi'(\omega) \approx \chi'(0) \Theta(\omega_0 - |\omega|)$, where $\omega_0$ is some cut-off for the local spin fluctuations. According to Fig. S1, a suitable cut-off would be again $O(T_0)$. We restrict the discussion to $T = 0$, because if we cannot find a solution there, we will surely not find it for any $T > 0$. Under these conditions the integral on the right hand side of (S4) then evaluates to

![FIG. S1: $\chi'(\omega)$ for $J = 0.2W$ and $n = 0.9$. The dashed line marks the zero. In the inset the result for the integral on the right hand side of Eq. (S4) as a function of $V$, evaluated for $T = 0$, is shown.](image-url)
\[
-\frac{\pi \bar{g}^2 \chi_S(0)}{2} \int_0^{\omega_0} d\omega \left[ \frac{1}{\omega - V} + \frac{1}{\omega + V} \right] = -\frac{\pi \bar{g}^2 \chi_S(0)}{2} \ln \left| \frac{\omega_0^2}{V^2} - 1 \right|.
\]

As long as \( \omega_0 < \sqrt{2} \bar{V} \), the logarithm on the right hand side is negative, i.e. the total expression positive and hence one has a non-trivial solution to equation (S4).

Let us now use the numerical data for \( \chi'_S(\omega) \) and see, if the above rough arguments are compatible with them. As parameter we vary the effective hybridization between some small value \( \bar{V} \ll T_0 \) to some large value \( \bar{V} \gg T_0 \). The result is shown in the inset to Fig. S1. In both limits, the value of the integral is negative, hence no pairing is supported. However, in a reasonably large regime with \( \bar{V} = O(T_0) \) one indeed obtains a positive value and hence the tendency to form Cooper pairs.

One thus needs two absolutely necessary ingredients – hybridized bands with a small energy scale, and a spin-fluctuation spectrum that decays on the same energy scale. Note that this result also predicts that one has to expect a strongly reduced \( T_c \) for a phonon-mediated superconducting heavy-fermion state, as here the kernel enters with a negative sign from the outset (see e.g. [5]), and hence the low-energy part of the integral now rather tends to suppress superconductivity.

We note that the above discussion – considering the various uncontrolled approximations – can neither be considered a proof of the existence of \( s \)-wave superconductivity, nor does it provide a reliable formula for estimating \( T_c \). For instance, we have assumed a frequency-independent gap, obviously inconsistent with the full numerical results in Fig. 1. The discussion nevertheless gives a qualitative foundation for our results, namely that, contrary to common wisdom, spin fluctuations can lead to \( s \)-wave superconductivity, at least under the very special conditions met in heavy-fermion systems.

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