Designing hypothetical learning trajectory for learning the importance of hypothesis testing

S Syafriandi¹*, A Fauzan¹, L Lufri² and A Armiati¹

¹Mathematics Department, Universitas Negeri Padang, Padang, Indonesia
²Biology Department, Universitas Negeri Padang, Padang, Indonesia

*Corresponding author: syafriandi_math@fmipa.unp.ac.id

Abstract. This study aims to design student learning trajectories in learning the concept of the importance of hypothesis testing in inferential statistics. This study used design research. Design research consists of three stages, i.e. (1) preparing for the experiment; (2) teaching experiment; and (3) retrospective analysis. This study focused on the first stage, because the main purpose of this research was to develop a series of learning activities to learn the importance of hypothesis testing in inferential statistics. The result of this study is an hypothetical learning trajectory (HLT) to find the concept of the importance of hypothesis testing in inferential statistics. HLT was designed using a realistic mathematics education (RME) approach. The context used in designing HLT is to tossed a coin to determine which football team will kick off. Using iceberg, a series of learning trajectories are designed so that students are able to find out by themselves that testing hypotheses is an important concept in proving the assumption that coin is used in balance.

1. Introduction

The concept of hypothesis testing has been widely used in research in various fields, such as research in science, business, agriculture, medicine, social, psychology, and education. As users, researchers only utilize standard procedures in testing hypotheses that were written in many books about Introduction to Statistics. In fact, statistical software has provided a lot of hypothesis testing facilities that make it easier for users to test. Unlike the students of mathematics department, learning the concept of hypothesis testing is not only learning how to use standard procedures in testing hypotheses, but students are expected to be able to master and understand the theoretical concepts of testing the hypothesis. Students are expected to be able to retrace the concept of testing hypotheses as before the experts found them.

Many studies have found that students often see hypothesis testing as only a set of calculation procedures. Without this procedure, hypothesis testing is meaningless to them [1], [2]. In addition, the students does not understand the role of hypothesis testing in making conclusions, the students have difficulty identifying null and alternative hypotheses, interpreting significance levels and p-values, and failing to understand the role of sampling distribution in testing hypotheses [3], [4], [5], and [6].
The cases as stated above, were also found in mathematics students, Universitas Negeri Padang. For example, when given a case about information that nationally the average national examination score of mathematics in 2019 is 7.4. If a school in West Sumatra is taken and it turns out that their national math test average scores are 8.2. Can it be concluded that the average score of students in the school is higher? Most students answer “yes”, since 8.2 is higher than 7.4. Most students don’t realize that the means of samples varies about the population mean when samples are selected from a specific population. So the question arises, Is there a real difference in the means, or is the difference simply due to chance (i.e., sampling error)? This case shows that the concept of testing hypotheses is important in making conclusions.

To teach the students to be able to understand the importance of the concept of testing hypotheses, learning trajectory need to be designed. In the early stages, hypothetical learning trajectory (HLT) was designed. HLT consists of learning goals, learning activities, and hypothetical learning process [7]. Using research design will be obtained by local instructional theory (LIT). In order to support students development in understanding the importance testing hypothesis concept, researcher apply the Realistic Mathematics Education (RME) approach. The choice of RME as an underlying theory for this study relates to its potential to address issues of instructional design [8]. With the RME approach students will learn mathematical concepts using real problems that are close to their daily lives. Through doing mathematics activities, students are stimulated to democratize the real problem using their informal knowledge (horizontal mathematization). Furthermore, gradually the students will be facilitated to rediscover formal mathematical concepts through vertical mathematization.

2. Method and Design
This research is the first phase of design research, that is preparing for the experiment. In this phase, researchers designed preliminary HLT consisting of: learning goal, learning activities: planned instructional activities and the tools that will be used, and the hypothetical learning process or conjectured of learning processes. The RME approach is used to design HLT.

The context used in designing HLT is to tossed a coin to determine which football team will kick off. Using iceberg, a series of learning trajectories are designed so that students are expected to find themselves that testing hypotheses is an important concept in proving the assumption that coin was used in balance.

3. Findings and Discussion
The design of the HLT begins with developing Iceberg using the principles of the RME. Iceberg describes three main principles in RME, namely: (1) guided reinvention and progressive mathematizing, (2) didactical phenomenology, and (3) self developed models [9].

The problem to be solved is to prove the assumption that any coin is balanced, through an experiment tossing a coin n times. The real problem used are as follows " In order to determine which team will get the ball for kickoff in a soccer match, the referee draws a coin which is witnessed by the two team captains. Before the toss of a coin, the referee makes an agreement with the two captains, ie if the result of the toss is head, then Team A gets the ball for kickoff, otherwise if the result of the toss is tail, then Team B gets the ball for kickoff. Raihan and his friends who came to watch the match wondered, why did the referee have to use coins? Has the referee done anything fair? Rafi Raihan's friend explained that the referee assumed that the coins used were balanced, that a coin to land with Heads up half the time and with Tails up half the time. Rafi added, this is what makes the referee believes that he has done fair in determining the team that gets the ball for kickoff. However, Raihan and his friends are not convinced by Rafi's statement. How do you convince Raihan and her friends that any coin is balanced?". The following is the result of the Iceberg design on the importance of the hypothesis testing.
Based on Iceberg, HLT was prepared. The results are as follows.

Title of lesson: The importance of the hypothesis testing
Activity: Toss a coin n times (n = 10 to n = 30)

3.1. Rationale for this activity
This activity uses the context of the use of coin by soccer referee who will kick off. The problem is make sure that the coins used are balanced. Through this activity students are invited to discover for themselves that to prove the assumption that the coins used are balanced, the concept of hypothesis testing is needed. Drawing inferences from data is an important part in a research. The concept of hypothesis testing was used to draw a conclusion about a particular population using data-based evidence provided by a sample.

3.2. Learning goal
Analyze and make sure that the concept of hypothesis testing is needed to prove the assumptions about a parameter based on statistic.

3.3. Description of the lesson
The first experiment conducted was that each student tossed a coin several times (10 - 30 times). Students make a conjecture about what would happen. Next, the students discuss the results of their experiments, to conclude whether the coin tossed is balanced. They are asked how to determine if a balanced coin is just as likely to land Heads up as it is to land Heads down. If this is done many times, would it also land Heads and Tails in fairly equal numbers? It is assumed that the experimental results obtained will vary. When the experiment produces the same number of heads as the number of tails, it is concluded that the coin is balanced. However, when the amount is not the same, it is concluded that the coins are not balanced. Then, what if the number of throws is odd?. They might argue that because coins are tossed differently, the results are also different. There are some balanced coins and some others are not balanced.

To build students' knowledge in obtaining formal concepts from the importance of hypothesis testing, the students re-experiment with tossing a coin in turn with the same number of tosses in each
The conclusion obtained remains the same as in the first experiment, namely a balanced coin, if just as likely to land Heads up as it is to land Heads down. Students are confused, why can happen on the same coin to produce two conclusions, i.e some experiments concluded that the coin is balanced, and several other experiments concluded that unbalanced. Students discuss this case, so they can find that the results of the experiments carried out are statistic, and the concept of hypothesis testing is needed to deduce the parameters.

3.4. The resulting HLT design

Table 1 is a HLT that has been designed, which contains learning goals, learning activities, and hypothetical learning process. Learning begins with understanding the context of the coin toss by the soccer referee in determining which team will kick off.

| Name of activity          | Learning goal                                                   | Learning activities                                                                 | Hypothetical learning process                                                                 |
|---------------------------|-----------------------------------------------------------------|-------------------------------------------------------------------------------------|------------------------------------------------------------------------------------------------|
| Activity 1: Let's tossing a different coin | The students make sure that a coin is balanced | • Students are divided into small groups of 3-5 people  
• Each person in the group tosses a coin with a specified number of tosses and records the results  
• Each of them draws a conclusion as to whether the a coin being toss is balanced  
• Discuss experimental results in groups  | • Some students tossed an even number of coins, others tossed an odd number of coins  
• Some students record the results of the experiment using a table, and some others without using a table.  
• There are three possible experimental results obtained by students, namely (1) balanced coin, if the number of heads that appear is equal to the number of tails, (2) coin is not balanced, if the number of heads that appear is not the same as the number of tails, and (3) can not be concluded, because the number of tossed is odd |

| Activity 2: Let's tossing a coin in turn | The students make sure that to prove the assumption of a balanced coin requires the concept of hypothesis testing | • Everyone in the group tosses the same coin, and with the same number of tosses then records the result  
• compare the results obtained by each group member.  
• discuss the results of an experiment to make a conclusion | • It is possible that the experimental results obtained are the same as in activity 1.  
• Students are confused, because they find unreasonable conclusions. Why for a coin, sometimes it's balanced and other trials aren't balanced  
• Students discuss, why such things happen.  
• Some students relate to the concept of population and sample, and some others relate to the concept of parameters and statistic. |

3.5. The results of one-to-one phase

HLT that has been designed, was tested on three students (one high ability, one medium ability, and one low ability). The trial results show that the HLT that has been designed can be implemented well.
All the allegations that are estimated, appear in the learning process. The following are the results of student work.

![Figure 2. The results of student work in activity 1](image)

Next, the lecturer conducts an interview with a student A regarding the results of the experiment.

*Lecturer*: Is a coin that you toss balanced?

*Student A*: Yes

*Lecturer*: Why?

*Student A*: Because, the number of heads is equal to the number of tails obtained

*Lecturer*: If you repeat tossing the coin, are you still sure that the coin is balanced?

*Student A*: I am not sure.

*Lecturer*: Why

*Student A*: Because, the results obtained from the experiment were just a coincidence.

*Lecturer*: Then, how to conclude that your coin is balanced or not?, please discuss in your group.

Next, the lecturer interview C student

*Lecturer*: What if the number of toss is odd?

*Student C*: I can’t conclude, because I can’t compare the number of heads and the number of tails.

*Lecturer*: Then, how do you know if the coin you use is balanced?, please discuss in your group.

Students are confused, why are their coins different?. Is the condition of the coin really like that?. Is it possible that the coin used by the referee is also unbalance? The lecturer asks students to check the coins they have. Next, the lecturer instructs students to do the activity 2. In activity 2, students choose a coin that has been used in activity 1, and toss the coin alternately with the same number of tosses. Figure 3 is the result of an experiment from one of the groups.

![Figure 3. The results of student work in activity 2](image)
Of the three students who carried out activity 2, two students concluded that the coin tossed was unbalance, while one other student concluded that the coin was balanced. There is a cognitive conflict in student thinking. Why is a coin sometimes balanced and on other occasions unbalanced.

The lecturer asked several questions (1) Is it enough to compare the proportions of many heads and the number of tails obtained from coin toss activities to conclude that the coins are balanced?. (2) If this activity is repeated many times or the number of toss increased, what will happen?. Students discuss in their groups, and one of the groups selected presents their results in front of the class. Utilizing the results of class discussions, students return to discuss in their groups. Some groups find that the experimental results obtained from activities 1 and 2 are sample data, so we need tools to generalize the population, and the lecturer informed that the tool was "hypothesis testing".

4. Conclusion

Based on the results of one to one phase, it was seen that HLT was designed to be able to facilitate students to achieve learning goals. Gradually students are facilitated to rediscover formal mathematical concepts through vertical mathematics. The learning trajectory that is compiled based on the principle of RME has been able to facilitate students to make sure that the concept of hypothesis testing is needed to prove the assumptions about a parameter based on statistic.

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