Kaluza-Klein Excitations and Electroweak Symmetry Breaking

B. Grzadkowski\textsuperscript{a,b} and J.F. Gunion\textsuperscript{a}

\textsuperscript{a} Davis Institute for High Energy Physics, University of California at Davis, Davis, CA 95616, USA
\textsuperscript{b} Inst. for Theoretical Physics, Warsaw University, Warsaw, Poland

Abstract

We explore the possibility that the Kaluza-Klein graviton states induce electroweak symmetry breaking. We also demonstrate that electroweak symmetry breaking could have a large impact on KK phenomenology.

Recently, it has become clear that quantum gravity as described using extra dimensions, and the associated Kaluza-Klein excitations, could have effects at scales far below the measured Planck mass \[1\]. In the most popular approach \[2\], ordinary particles are confined on a brane (having three spatial dimensions) with gravity propagating in the bulk, a situation that can be realized in several string models \[3\]. There has been an outpouring of phenomenological work based on these ideas; see \[4, 5, 6\] and references therein. Here, we point out that the KK modes could have a dramatic impact on electroweak symmetry breaking. Indeed, they could provide the EWSB mechanism. In turn, EWSB could have a large impact on KK phenomenology.

1 Electroweak Symmetry Breaking

We consider a theory with \(\delta\) extra dimensions, for which we begin with the action

\[
S = -\frac{1}{2\kappa^2} \int d^4 x \, d^\delta y \sqrt{-\hat{g}} \, R + \frac{1}{V_\delta} \int d^4 x \, d^\delta y \sqrt{-\hat{g}} \Lambda + \sum_{\text{fields}} \int d^4 x \sqrt{-\hat{g}_{\mu\nu}} L_{\text{fields}}(x) ,
\]

where \(\kappa\) is the reduced Planck mass in \(4 + \delta\) dimensions, \(V_\delta = L^\delta\) is the volume associated with the extra compactified dimensions, \(\kappa = V_\delta^{-1/2} \hat{\kappa}\) is the usual reduced Planck mass (\(\kappa = \sqrt{16\pi G_N}\)), \(\Lambda\) is a bulk cosmological constant defined so as to have the same dimensions as a vacuum energy density in \(L_{\text{fields}}\), \(\hat{g}_{\mu\nu} (\mu, \nu = 0, 1, 2, 3, \ldots, 3 + \delta)\) refers to the full metric tensor

1\(^\text{In the usual notation, the cosmological constant term would be written as } \frac{1}{\kappa^2} \int d^4 x d^\delta y \sqrt{-\hat{g}} \Lambda^* . \Lambda^* \text{ would have dimension of mass-squared. The relation is } \Lambda^* = \kappa^2 \Lambda.\)
in 4 + δ dimensions, \( \hat{g}_{\mu\nu} \) refers to the \( \mu, \nu = 0, 1, 2, 3 \) part of \( \hat{g} \), and we have restricted all matter fields to lie on a brane at \( y = 0 \) following the prescription of \[4\] and \[5\]. (We will employ notation and definitions consistent with \[4\].) In the linearized approximation, we expand the metric tensor about the flat-space limit writing \( \hat{g}_{\mu\nu} = \eta_{\mu\nu} + \hat{\kappa} h_{\mu\nu} \) with

\[
\hat{h}_{\mu\nu} = V_{\delta}^{-1/2} \left( \begin{array}{cc} h_{\mu\nu} + \eta_{\mu\nu} \phi & A_{\mu i} \\ A_{\nu i} & 2\phi_{ij} \end{array} \right),
\]

where \( \phi \equiv \sum_{i} \phi_{ij} \). Retaining the leading terms in \( \hat{\kappa} \), the resulting equations of motion take the form:

\[
R_{\mu\nu} - \frac{1}{2} \eta_{\mu\nu} R = \hat{\kappa}^{2} \eta_{\mu i} \eta_{\nu j} T_{\mu\nu} \delta^{i}(y) - \hat{\kappa}^{2} V_{\delta}^{-1} \eta_{\mu\nu}.
\]

In Eq. (3), \( T_{\mu\nu} = \left( -\hat{g}_{\mu\nu} \mathcal{L} + 2 \frac{\delta \mathcal{L}}{\delta \phi_{ij}} \right)_{\hat{g}_{\mu\nu}=\eta_{\mu\nu}} \) and \( R_{\mu\nu} - \frac{1}{2} \eta_{\mu\nu} R \equiv G_{\mu\nu} \) with

\[
G_{\mu\nu} = \frac{\hat{\kappa}}{2} \left[ \hat{\kappa}^{2} \eta_{\mu i} \eta_{\nu j} \hat{h}_{\mu\nu} \delta^{i}(y) - \hat{\kappa}^{2} \eta_{\mu\nu} \delta^{i}(y) \right] + \frac{\delta \mathcal{L}}{\delta \phi_{ij}} = \frac{\delta \mathcal{L}}{\delta \phi_{ij}} + \frac{\delta \mathcal{L}}{\delta \phi_{ij}}.
\]

The next step is to expand:

\[
h_{\mu\nu}(x, y) = \sum_{\tilde{n}} h_{\mu\nu}^{\tilde{n}}(x) \exp \left( \frac{2\pi \tilde{n} \cdot \tilde{y}}{L} \right), \quad \phi_{ij}^{\tilde{n}}(x, y) = \sum_{\tilde{n}} \phi_{ij}^{\tilde{n}}(x) \exp \left( \frac{2\pi \tilde{n} \cdot \tilde{y}}{L} \right),
\]

and similarly for \( A_{\mu i}^{\tilde{n}}(x, y) \). Here, the \( \tilde{n} \neq 0 \) modes are the KK states and we have assumed that all compactification radii are the same. The result of this expansion yields an effective Lagrangian and corresponding equations of motion for the KK modes and Standard Model fields on the brane. For \( \tilde{n} \neq 0 \), one rewrites \( h_{\mu\nu}^{\tilde{n}}, A_{\mu i}^{\tilde{n}} \) and \( \phi_{ij}^{\tilde{n}} \) in terms of the physical fields \( \tilde{h}_{\mu\nu}, \tilde{A}_{\mu i}^{\tilde{n}} \) and \( \tilde{\phi}_{ij}^{\tilde{n}} \) as in Refs. \[4\] and \[5\].

Our main focus will be on the Higgs sector of the theory. We begin with an arbitrary Higgs potential \( V \) that is a function of a certain set of real Higgs scalar fields \( \Phi \). The corresponding contribution to the energy momentum tensor is \( T_{\mu\nu} = \eta_{\mu\nu} V \). We also have the massive KK states, the Lagrangian mass terms for which are simply

\[
\mathcal{L}_{\text{mass}}^{\tilde{n}} = \frac{1}{2} m_{n}^{2} \left( -h_{\mu\nu}^{\tilde{n}} \tilde{h}_{\mu\nu} + \tilde{h}_{\mu\nu} \tilde{h}_{\mu\nu} \right) - m_{n}^{2} \sum_{i,j=1}^{\delta} \tilde{\phi}_{ij}^{\tilde{n}} \tilde{\phi}_{ij}^{\tilde{n}},
\]

where we have singled out fields associated with a single \( \tilde{n} = (n_1, n_2, \ldots, n_\delta) \) mode and the complex conjugate \(-\tilde{n}\) fields. (This means that the total \( \mathcal{L} \) is obtained by summing \( \mathcal{L}_{\tilde{n}}^{\tilde{n}} \) only

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\[\text{We note that the expansion parameter multiplying the fields is } \hat{\kappa} V_{\delta}^{-1/2} = \kappa \equiv \sqrt{16\pi/M_{\text{Pl}}} \text{, where } M_{\text{Pl}} \text{ is the usual Planck mass for three spatial dimensions. Thus, for this expansion to be appropriate, it is important that the relevant values for the } h_{\mu\nu}, A_{\mu i} \text{ and } \phi_{ij} \text{ fields all be small relative to } M_{\text{Pl}}. \text{ This will indeed be the case so long as the string scale, } M_{S}, \text{ above which the effective theory that we will be discussing is invalid, is much smaller than } M_{\text{Pl}}. \text{ This expansion also assumes that in the absence of quantum excitations a flat metric is a consistent solution of the equations of motion, at least locally on the brane. This, in turn requires that the vacuum energy density on the brane be very small (small effective cosmological constant). As discussed later, this latter can be arranged by an appropriate choice of the bulk } A \text{ in the case that will be of interest to us where the other Lagrangian terms lead to a very substantial vacuum energy density on the brane.}\]
over \(\vec{n}\) values such that the first non-zero \(n_k\) is positive. We denote this restricted sum by \(\sum_{\vec{n}>0}\) in Eq. (6), \(m_n^2 = \frac{4\pi^2n^2}{L^2}\) with \(n^2 = \vec{n}^2\) and \(\~h = \~h_{\mu}^\mu\), which is non-zero in the general off-shell situation.\(^3\)

The next key ingredient is the coupling between the graviton KK states and the scalar field contributions to \(T_{\mu\nu}\) following from the last term in Eq. (6). To lowest order in \(\kappa = \sqrt{16\pi/M_{Pl}^2}\), and singling out fields with index \(\vec{n}\) and their complex conjugates, this takes the form\(^4\)

\[
\mathcal{L}_{\text{mix}}^{\vec{n}} = -\frac{\kappa}{2} \left[ (\~h^{\mu\nu,\vec{n}} + \~h^{\mu\nu,-\vec{n}}) T_{\mu\nu} + \omega_\delta \left( \~\phi^{\vec{n}} + \~\phi^{-\vec{n}} \right) T_\mu^\mu \right], \tag{7}
\]

where

\[
\omega_\delta = \left[ \frac{2}{3(\delta + 2)} \right]^{1/2} \tag{8}
\]

and \(\~\phi = \sum_{i} \~\phi_{ij}\).

In employing both Eq. (3) and (7), we must keep in mind the \(\delta\) constraints \(n_{ij} \~\phi_{ij} = 0\), which means that only \(\delta(\delta - 1)/2\) of the \(\~\phi_{ij}\) are independent. This is particularly crucial as we consider the effects of the mixing between the \(\~h, \~\phi\) fields and the Higgs field through \(\mathcal{L}_{\text{mix}}\).

We first wish to consider whether the mixing term can lead to non-zero vacuum expectation values for the \(\Phi\) Higgs field and the \(\~h, \~\phi\) fields. To this end, we look for an extremum of \(V_{\text{tot}} = V(\Phi_i) - \mathcal{L}_{\text{mass}} - \mathcal{L}_{\text{mix}}\), where \(V(\Phi_i)\) is the relevant part of \(-\mathcal{L}_\Phi, \mathcal{L}_\Phi\) being the Lagrangian for the Higgs field(s). We will argue later that in general \(V(\Phi_i)\) should be the full effective potential as computed for the Higgs sector. Writing \(\~h^{\mu\nu,\vec{n}} = \Re(\~h^{\mu\nu,\vec{n}}) + i\Im(\~h^{\mu\nu,\vec{n}})\) and \(\~h^{\mu\nu,-\vec{n}} = \Re(\~h^{\mu\nu,\vec{n}}) - i\Im(\~h^{\mu\nu,\vec{n}})\), and similarly for the \(\~\phi_{ij}\) fields, we find that the extremum conditions for \(\~h^{\mu\nu}\) and \(\~\phi_{ij}\) give

\[
\Re(\~h_{\mu\nu}^{\vec{n}}) = \eta_{\mu\nu} \frac{\kappa}{3m_n^2} V, \quad \Re(\~\phi_{ij}^{\vec{n}}) = -P_{ij} \frac{2\kappa\eta_{\mu\nu}\delta}{m_n^2} V, \tag{9}
\]

where \(\eta_{\mu\nu} = \text{diag}(1, -1, -1, -1)\), \(P_{ij} = \delta_{ij} - n_i n_j / n^2\) and all imaginary components are zero. Substituting into \(\mathcal{L}_{\text{mix}}\) and \(\mathcal{L}_{\text{mass}}\) gives a result for \(V_{\text{tot}}\) which we denote by \(V_{\text{tot}}\)\(^5\).

\[
V_{\text{tot}} = V + \sum_{\vec{n}>0} \frac{1}{3m_n^2} \frac{2\kappa^2}{3} V^2 - \sum_{\vec{n}>0} \frac{1}{4\kappa^2} \left( \frac{2}{3} \frac{\delta - 1}{\delta + 2} \right) V^2 = V - 2\kappa^2 V^2 \delta - 2 \delta + 2 \sum_{\vec{n}>0} \frac{1}{m_n^2}, \tag{10}
\]

\(^3\)Note that we do not agree with the notation in Eq. (24) of \(^3\) which implies that the 2nd term in Eq. (6) should only be summed over the \(\delta(\delta + 1)/2\) independent values of \(i, j\). We also see that \(\mathcal{L}_{\vec{n}}^{\vec{n}}\) of their Eq. (24) should only be summed over \(\vec{n} > 0\), as specified above.

\(^4\)Here, the full \(\mathcal{L}\) is again obtained as \(\mathcal{L} = \sum_{\vec{n}>0} \mathcal{L}_{\vec{n}}^{\vec{n}}\).

\(^5\)We note that this same result is obtained if one computes the \(V^2\) terms using virtual exchanges of the \(\~h_{\mu\nu}^{\vec{n}}\) and \(\~\phi_{ij}^{\vec{n}}\) fields at zero momentum. (We believe that the propagator for \(\~h_{\mu\nu}^{\vec{n}}\) given in \(^3\) is incorrect. One should use \(\Delta_{\~h_{\mu\nu,\vec{n}},\mu,\nu,\vec{n}} = i\delta_{\~h_{\mu\nu,\vec{n}}} B_{\mu,\nu,\vec{n}}(k^2 - m_n^2 + i\epsilon)^{-1}\), i.e. no factor of 1/2 in their normalization. This is required also for consistency with \(^3\).) Indeed, one obtains from virtual \(\~h\) and virtual \(\~\phi\) exchanges the results \(\sum_{\vec{n}>0} \frac{i\omega_{\~h_{\mu\nu,\vec{n}}}}{m_n^2} T_{\mu\nu} T_{\mu\nu} - \frac{1}{4} T_{\mu\nu} T_{\mu\nu}^{\vec{n}} \) and \(\sum_{\vec{n}>0} \frac{i\omega_{\~\phi_{ij,\vec{n}}}}{m_n^2} T_{\mu\nu} T_{\mu\nu}^{\vec{n}}\), respectively, where we will substitute \(T_{\mu\nu} = \eta_{\mu\nu} V\). To convert to an effective Lagrangian, we must remove the \(i\) and multiply by 1/2 in order to avoid over counting contractions. After changing the sign in order to convert to an effective potential, using \(\sum_{\vec{n}>0}\) and writing \(T_{\mu\nu} = \eta_{\mu\nu} V\), we reproduce the corresponding terms of Eq. (6). One can easily see that this must be the result by integrating out the \(\~h\) and \(\~\phi\) fields in the path integral formulation.
where we use the shorthand notation $V$ for $V(\Phi_i)$. It will be convenient to define

$$
\sum_{n=1}^N \frac{1}{m_n^2} = 2 \sum_{n>0} \frac{1}{m_n^2} = \frac{\mathcal{D} \delta + 2}{\kappa^2 \delta - 2},
$$

so that we have

$$
\nabla_{\text{tot}} V = V - \mathcal{D} V^2.
$$

One must next search for an extremum with respect to the $\Phi_i$. Using the result of Eq. (12), one finds

$$
\frac{\partial \nabla_{\text{tot}}}{\partial \Phi_i} = \frac{\partial V}{\partial \Phi_i} \left[ 1 - 2 \mathcal{D} \right].
$$

Requiring this to be 0, we obtain

$$
V = \frac{1}{2D}, \quad \text{or} \quad \frac{\partial V}{\partial \Phi_i} = 0, \quad \text{for all } i.
$$

The 2nd solution corresponds to the usual minimum while the first is of a very unusual nature, as we shall explore. Whichever extremum is appropriate, we denote the extremum values of $V$ and $\Phi_i$ by $V_0$ and $v_i$, respectively. We also denote the value of $V$ at the usual minimum by $V_S$.

In order to determine which extremum corresponds to the smallest $\nabla_{\text{tot}}$, we compute

$$
\nabla_{\text{tot}} \left( V = \frac{1}{2D} \right) - \nabla_{\text{tot}}(V = V_S) = \frac{1}{4D} - \left( V_S - \mathcal{D} V_S^2 \right) = \mathcal{D} \left( V_S - \frac{1}{2D} \right)^2.
$$

We see that the $V = \frac{1}{2D}$ extremum is preferred if $\mathcal{D} < 0$, whereas the standard extremum is preferred for $\mathcal{D} > 0$ unless $V_S = \frac{1}{2D}$. The procedure of Ref. [4] yields $\mathcal{D} = \frac{2}{M_S^2(\delta - 2)}$, where $M_S$ is an ultraviolet cutoff. This suggests that $\mathcal{D} > 0$. However, the ultraviolet cutoff is the point at which the physics of the string enters. The exact manner in which the divergent sum is regularized is thus uncertain and either sign for $\mathcal{D}$ is possible, as considered, for example, in Ref. [4]. Thus, we consider $\mathcal{D}$ to simply be a parameter determined by the detailed physics at the string scale.

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6We note that we end up with exactly the same conditions whether we substitute using Eq. (4) and then minimize (as we have done) or take the derivatives of $V(\Phi) - L_{\text{mass}} - L_{\text{mix}}$ with respect to the $\tilde{h}_{ij}$, $\tilde{\phi}$ and $\Phi$ fields independently and then afterwards substitute. Technically speaking, the former is appropriate if we are integrating out the $\tilde{h}$ and $\tilde{\phi}$ fields, whereas the latter is appropriate if we retain them as physical degrees of freedom. One can equally well mix the two approaches and obtain the same conditions. This ‘matching’ is important as it guarantees that there is no sensitivity to exactly where we place the boundary between heavy and light fields.

7A simple example is $\zeta$ regularization. Defining $\zeta(x) \equiv \sum_{n=1}^\infty n^{-x}$, $\zeta(x)$ is easily computed and is positive for $\Re(x) > 1$. But, using analytical extension to define the divergent summation for negative $x$ gives $\zeta(-1) = -1/12, \zeta(-5) = -1/152, \ldots$. In general, $\zeta(1 - 2m)$ has sign $(-1)^m$, i.e. a negative value results for odd integer $m$ even though $\zeta(1 - 2n)$ is formally a sum of positive numbers. In fact, the large $n$ terms in the summation $\sum_{n} \frac{1}{m_n^2}$ behave like $\sum_{n} c_d n^{d-3}$ with $c_d = \frac{L_{\text{x}}^{2+2/\delta - 2}}{2\Gamma(\delta/2)}$. Thus, we could write $\sum_{n} \frac{1}{m_n^2} = c_d \zeta(\delta - 3) + d_d$, where $d_d$ is a finite correction. However, since $\zeta$ regularization is not the only possibility, results obtained in this specific manner would probably be misleading. The only firm conclusion is that string physics could act to regulate apparently divergent sums in a manner such that $\mathcal{D} < 0$. 
Given a definite minimum, we must consider an appropriate quantum state expansion. To this end, we first note that for a set of scalar fields \( \Phi_i \) one has \( T_{\mu}^\nu = -2\mathcal{L}_\Phi + 2V(\Phi_i) \), where \( \mathcal{L}_\Phi = \sum_i \mathcal{L}_{\text{kin},i} - V(\Phi_i) \), where \( \mathcal{L}_{\text{kin},i} = \frac{1}{2}(\partial_{\mu}\Phi_i)(\partial^\mu\Phi_i) \). By substituting the vacuum field values of Eq. (10) into Eq. (7) and summing over all \( \vec{n} > 0 \), we find

\[
\mathcal{L}_{\text{mix}} \sim \kappa^2 V_0 \tilde{\partial}^2 \mathcal{V} \frac{\delta - 2}{\delta + 2} \sum_{\vec{n} > 0} \frac{1}{m_n^2} T_{\mu}^\mu = \frac{1}{2} \partial\mathcal{V} \partial_{\mu} T_{\mu}^\mu = -\partial\mathcal{V} \mathcal{L}_\Phi - V(\Phi_i) \frac{1}{2} \partial\mathcal{V} \mathcal{L}_\Phi - V(\Phi_i) \ .
\]

(16)

To determine the appropriate quantum expansion for the scalar fields, we focus on the derivative terms

\[
\mathcal{L}_\Phi + \mathcal{L}_{\text{mix}} \equiv \left(1 - \partial\mathcal{V}_0\right) \frac{1}{2} \sum_i \partial^\mu \Phi_i \partial_\mu \Phi_i .
\]

(17)

(For later reference, we note that \( \mathcal{L}_\Phi + \mathcal{L}_{\text{mix}} \) also contains non-derivative terms of the form \( [2\partial\mathcal{V}_0 - 1]V(\Phi_i) \). From Eq. (17) we see that, for the \( V_0 = \frac{1}{2m^2} \) minimum, half of each usual \( \mathcal{L}_{\text{kin},i} \) derivative term is canceled by \( \mathcal{L}_{\text{mix}} \); whatever the value of \( V_0 \), we must rescale the \( \Phi_i \) in order to have canonical normalization for their kinetic energy terms. We write

\[
\Phi_i = \tilde{\Phi}_i \left(1 - \partial\mathcal{V}_0\right)^{-1/2} .
\]

(18)

Note that rescaling is not necessary for the KK \( \vec{n} \) and \( \vec{\phi} \) fields since \( \mathcal{L}_{\text{mix}} \) does not involve their derivatives.

The next step is to expand \( V_{\text{tot}} = V(\Phi_i) - \mathcal{L}_{\text{mass}} - \mathcal{L}_{\text{mix}} \) about the extremum. We write

\[
\tilde{\Phi}_i = \tilde{v}_i + s_i , \quad \Re(\tilde{\phi}^i_{\vec{n}}) = -P_{ij} \frac{2\kappa \omega_0}{m_n^2} V_0 + \sum_s \epsilon_{ij} s^s \frac{\delta_{k}\sqrt{2}}{2} .
\]

(19)

It will not be necessary to expand \( h_{\mu\nu}^i \) about its minimum since \( T^\mu_{\nu} \propto \eta^\mu_{\nu} \) and the spin-2 quantum states have polarizations \( \epsilon_{\vec{n}}^i_{\mu\nu} \) such that \( \eta^\mu_{\nu} \epsilon_{\vec{n}}^i_{\mu\nu} = 0 \). We may also choose to define the \( s^s_n \) states so that only \( \epsilon^s_{ii} \neq 0 ; \epsilon^s_{ii} = (\delta - 1)^{1/2} \) is required for correct normalization (summation over \( i \) is implied in both formulae). Only \( s^1_n \) has the potential for mixing with the \( s_i \) states. The resulting form for \( V_{\text{tot}} \) is:

\[
V_{\text{tot}} = V_0 - \partial V_0 + \frac{1}{2} \sum_{\vec{n} > 0} m_n^2 (s^1_n)^2 + 2\sqrt{2}\kappa \omega_0 (\delta - 1)^{1/2} \sum_{\vec{n} > 0} s^1_n \sum_j s_j \left( \frac{\partial V}{\partial \Phi_j} \right) \tilde{\Phi}_k = \tilde{v}_k , \text{ all } k
\]

\[
+ (1 - 2\partial V_0) \sum_i s_i s_j \left( \frac{1}{2} \frac{\partial^2 V}{\partial \Phi_i \partial \Phi_j} \right) \tilde{\Phi}_k = \tilde{v}_k , \text{ all } k + \ldots ,
\]

(20)

where \( \frac{\partial V}{\partial \Phi_i} = (1 - \partial V_0)^{-1/2} \frac{\partial V}{\partial \Phi_i} \). The physics at the two different extrema of Eq. (14) is quite different. If \( \frac{\partial V}{\partial \Phi_i} = 0 \) for all \( i \), then Eq. (20) shows that there is no tree-level mixing between \( s^1_n \) and any of the \( s_i, i \). The Higgs and KK modes remain in separate sectors. The case of \( V_0 = \frac{1}{2m^2} \)

\[\text{As we shall show shortly, the same is true at the one-loop level.}\]
is more subtle. To illustrate, we assume that there is only one $\Phi_i$. In this case, the mass terms for the quantum fluctuations read

$$V_{\text{tot}} \rightarrow \frac{1}{2} \sum_{\vec{n} > 0} \left[ m_n^2 (s_n^1)^2 + 2 \epsilon s_n^1 s_n^2 \right], \quad \text{with} \quad \epsilon \equiv 2 \sqrt{2} \kappa \omega \lambda (\delta - 1)^{1/2} \left( \frac{\partial V}{\partial \Phi} \right)^{-1/2}_{\Phi = \hat{v}}, \quad (21)$$

where Eq. (18) implies that $\frac{\partial V}{\partial \Phi} = 2 \sqrt{2} \kappa \omega \lambda$ when $V_0 = \frac{1}{2} \hat{v}$. In order to determine whether or not we are in a local minimum, we must diagonalize $V_{\text{tot}}$. The mass matrix takes the form

$$\mathcal{M}^2 = \begin{pmatrix} 0 & \rho_1 \epsilon & \rho_2 \epsilon & \rho_3 \epsilon & \ldots \\ \rho_1 \epsilon & \rho_1 m_1^2 & 0 & 0 & \ldots \\ \rho_2 \epsilon & 0 & \rho_2 m_2^2 & 0 & \ldots \\ \rho_3 \epsilon & 0 & 0 & \rho_3 m_3^2 & \ldots \\ \ldots & 0 & 0 & 0 & \ldots \end{pmatrix} \quad (22)$$

where the notation $\rho_n \epsilon$ and $\rho_n m_n^2$ means that there are actually $\rho_n = \sum_{\vec{n} > 0, \vec{n}^2 = n^2}$ identical entries in the matrix. The precise values of $\rho_{1,2,3,\ldots}$ depend upon $\delta$. For example, $\rho_1 = \delta$. (The $\vec{n} > 0$ solutions of $1^2 = n_1^2 + n_2^2 + \ldots + n_\delta^2$ are $n_1 = 1$, $n_{k \neq i} = 0$ for any $i = 1, \ldots, \delta$.) A typical value of $m_n^2 = \frac{4 \pi^2 L^2}{\vec{n}^2}$ is set by the relation $4 \pi^2 L^2 = \left( \frac{M_{\text{Pl}}}{\sqrt{8 \pi}} \right)^4 M_S^{2+4/\delta}$ from which one obtains $\frac{4 \pi^2 L^2}{\vec{n}^2} = 1.7 \cdot 10^{-7} \text{eV}^2, 3 \cdot 10^3 \text{eV}^2, 4 \cdot 10^8 \text{eV}^2, 5 \cdot 10^{11} \text{eV}^2, 6 \cdot 10^{13} \text{eV}^2, 2 \cdot 10^{16} \text{eV}^2$ for $\delta = 2, 3, 4, 5, 6, 8$ for a $4 + \delta$ dimension Planck mass $M_S$ of order 1 TeV. This is to be compared to the off-diagonal entries characterized by $\epsilon \sim m^2 v / M_{\text{Pl}} \leq 10^7 \text{eV}^2$ for $m < 1 \text{ TeV}$. Thus, for $\delta \geq 4$ the off-diagonal entries are always small compared to the diagonal entries. Even more importantly, the upper cutoff in $m_n^2$ to which we sum and which dominates the relevant summations is of order $M_S^2 \sim 10^{24} \text{eV}^2$ (for $M_S \sim 1 \text{ TeV}$) or larger, which is much larger than $\epsilon$. Thus, for the relevant matrix entries, the $s_n^1$ KK states mix slightly with $s$ with a mixing angle $\theta_{s1} \sim - \sqrt{\frac{\epsilon}{m_n^2}}$. The physical eigenstate corresponding to the original $s$ is rotated to

$$s_{\text{phys}} \sim \begin{pmatrix} \Pi_n \theta_{s1} \\ s_{\theta_1} \\ s_{\theta_2} \\ \ldots \end{pmatrix} \cong \begin{pmatrix} 1 \\ \theta_1 \\ \theta_2 \\ \ldots \end{pmatrix} \cong \begin{pmatrix} 1 \\ - \rho_1 \epsilon / m_1^2 \\ - \rho_2 \epsilon / m_2^2 \\ \ldots \end{pmatrix}, \quad (23)$$

where we use the short-hand notation of lumping all $\rho_n$ of the states of a given $m_n$ together. The mass of $s_{\text{phys}}$ is given by

$$m_{s_{\text{phys}}}^2 = - \epsilon^2 \sum_n \frac{\rho_n}{m_n^2} \rightarrow - \epsilon^2 \sum_{\vec{n} > 0} \frac{1}{m_n^2} \sim - \frac{8}{3} \frac{\partial V}{\partial \Phi} \delta - 1 \left( \frac{\partial V}{\partial \Phi} \right)^2_{\Phi = \hat{v}}, \quad (24)$$

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\*It is remarkable that this result is obtained whether we treat the $\tilde{\phi}_D^2$ fields as being light, i.e. without integrating them out, (as we have done) or first integrate them out. Just as for the minimization condition of Eq. (14), the final expression for $m_{s_{\text{phys}}}^2$ does not depend upon where the dividing line between ‘heavy’ and ‘light’ fields is placed.
where we have used Eqs. (11) and (8). We note that the $\mathcal{D} < 0$ requirement, needed to ensure that we are expanding about a local minimum that is deeper than the standard minimum, is also that which implies a positive mass-squared for $s_{\text{phys}}$. Mixing of $s$ with the full tower of KK states and whatever physics is present at the string scale to cutoff the ultraviolet divergence of $\sum \vec{n}_{m_{n}}$ is critical to obtaining $m_{s_{\text{phys}}}^2 > 0$.

Let us now make a few comments before turning to phenomenology. First, it is very amusing to note that the $\mathcal{D} < 0$ minimum yields nonzero $v$ at the minimum even if $V(\Phi)$ itself does not have a minimum with $\Phi \neq 0$. In particular, $V(\Phi) = \frac{1}{2}m^2\Phi^2 + \Xi$ (25) is entirely satisfactory, provided $V_0 = \frac{1}{2}m^2v^2 + \Xi = \frac{1}{2D}\delta < 0$, where $v = \sqrt{2}\hat{v}$. For this form of $V(\Phi)$ we have

$$m_{s_{\text{phys}}}^2 = -\frac{32}{3}\mathcal{D}\delta - \frac{1}{\delta - 2}m^4\hat{v}^2,$$

which would be of order $\hat{v}^2$ if $m \sim |\mathcal{D}|^{-1/4} \sim M_S$, as might possibly be natural. We note that if $m$ is of this size, then to achieve $V_0 = \frac{1}{2D}$ it is necessary that $\Xi$ be negative with absolute magnitude of order $M_S^4$. This is all that is required in order for the interactions of the tower of KK states with the Higgs field to generate electroweak symmetry breaking.

Second, we argue that in our approach it is incorrect to include in $V(\Phi)$ tree-level diagrams containing virtual exchanges of the $s_{\vec{n}}$ fields. (These would, in particular, create effective $\Phi^4$ and higher interactions for $V(\Phi) = \frac{1}{2}m^2\Phi^2 + \Xi$). This is because the graphs required are one-particle reducible (in the $s_{\vec{n}}$ fields). However, true one-loop corrections to $V(\Phi)$ should be included beyond the tree level. The usual Higgs, fermion and vector loops will be discussed later. In addition, there are one-loop corrections that first appear at $\mathcal{O}(\kappa^2)$. For example, the sea-gull $\tilde{\phi}\tilde{\phi}\Phi\Phi$ vertex of order $\kappa^2$ gives rise to a one-loop diagram coupling two pairs of $\Phi$’s together via a loop containing two (independent) $\tilde{\phi}$ exchanges. Summing over all the independent $\vec{n}$ states for each $\tilde{\phi}$ and integrating over the loop momentum, one obtains a one-loop $\Phi^4$ interaction with coefficient of order $[\kappa^2][\mathcal{D}/\kappa^2]^2/\mathcal{D} \sim \mathcal{D}$. This is of similar form to the $\mathcal{D}V^2$ term in Eq. (12), but suppressed by a one-loop numerical factor. We will consistently neglect all diagrams containing KK loops.

Thirdly, we wish to note that the actual size of $\mathcal{D}$ is extremely uncertain. For example, the recent work of Ref. [10] suggests that the brane recoil effects will provide an effective cutoff for the $\sum \frac{1}{m_{\vec{n}}}$ that can reduce the size of $\mathcal{D}$: $\mathcal{D} \rightarrow \mathcal{D} \times \left(\frac{f^2}{M_S}\right)^{\delta - 2}$ for $\delta > 2$, where $f$ is the brane tension. However, if $\mathcal{D} < 0$ and $V_0 = 1/(2\mathcal{D})$, this does not affect the size of $\mathcal{D}V_0$. As a result, if $\mathcal{D} < 0$ its exact magnitude is not critical to the resulting phenomenology.

Finally, we note that the result of Eq. (24) for the physical Higgs mass squared will receive loop corrections from a number of sources. For example, there are the corrections to the Higgs mass coming from the virtual KK corrections as computed in Ref. [1]. These are proportional to the $m^2$ parameter appearing in $V$ times functions of $m^2/M_S^2$. Most naturally, all mass parameters are of order $|\mathcal{D}|^{-1/4} \sim M_S$, in which case the virtual KK correction would be of the order of $M_S^4$, and therefore possibly somewhat larger than $m_{s_{\text{phys}}}^2$ (which might be of order
v^2). More importantly, there are one-loop contributions from vector boson and fermion loops. As usual, these are quadratically divergent unless we introduce the usual supersymmetric partners. In this paper, we do not consider the supersymmetric extension. Quadratically divergent contributions are then absorbed in the usual renormalization procedure.

2 Electroweak Phenomenology

Let us first focus on vector boson mass generation. There are two contributions; one coming from $\mathcal{L}_{\text{mix}}$ and the other from the standard $\mathcal{L}_{\text{kin}}$ kinetic energy portion of the scalar Lagrangian. After substituting the vacuum values for $\Re(\tilde{h}_{\mu\nu}^{\bar{n}})$ and $\Re(\tilde{\phi}_{ij}^{\bar{n}})$ given in Eq. (3) into Eq. (4), we find the result of Eq (17) for the pure derivative terms. After rescaling to $\hat{\Phi}$, one obtains the usual canonically normalized derivative form $\mathcal{L}_{\text{mix}} + \mathcal{L}_{\text{kin}} \ni \frac{1}{2} (D^\mu H)^\dagger (D_\mu H)$, where for the simple one-doublet model we employ $H = (0, \hat{\Phi})$ and $D_\mu = \partial_\mu + igA^{a}_\mu T^a$ with $T^a = \tau^a/2$. Keeping only the $W$ and $Z$ boson portions of Eq. (27) and writing $\hat{\Phi} = \hat{v} + s$ as before, we find

$$\mathcal{L}_{\text{mix}} + \mathcal{L}_{\text{kin}} \ni \frac{1}{4} g^2 (\hat{v} + s)^2 W_\mu W^{\mu} - \frac{1}{8} (g^2 + g'^2) (\hat{v} + s)^2 Z_\mu Z^\mu.$$  

(28)

However, before proceeding further, we must demonstrate that the $W$ and $Z$ fields themselves do not need to be rescaled in order to have canonical normalization. The crucial result is that following from Eq. (16):

$$\mathcal{L}_{\text{mix}} = \frac{1}{2} D^\nu T^\mu_\nu$$  

(29)

after substituting the vacuum values of the $\tilde{h}$ and $\tilde{\phi}$ fields. From the form of $T^{\nu}_{\mu}$ for a vector field given in Eq. (37) of [4], one finds that the $F^{\mu\nu} F^{\mu\nu}$ kinetic energy part of $T^\nu_{\mu}$ is zero.

Thus, we can proceed to read off masses and couplings from Eq. (28). One finds $m^2_W = \frac{1}{4} g^2 \hat{v}^2$ and $m^2_Z = \frac{1}{4} (g^2 + g'^2) \hat{v}^2$, which are the usual expressions but in terms of the vev of the rescaled field, $\hat{\Phi}$. (Note that the standard result for the ratio $m_W/m_Z$ is preserved.)

We must now compute the coupling of $WW$ and $ZZ$ to $s_{\text{phys}}$. We focus on the $WW$ coupling, that for $ZZ$ being entirely analogous. In principle, both $WW$s and $WW^s_{\bar{n}}$ couplings contribute after mixing. The former coupling is easily read off from Eq. (28). The latter coupling is obtained by substituting the kinetic part of the Higgs $T_{\mu\nu}$ into Eq. (7), yielding a $\mathcal{L}_{\text{mix}}$ term of the form $\sqrt{2} \kappa \omega (\hat{v} + s)^2 s_{\text{phys}}^{1/2} L_{\text{kin}}$. The net result for the $WW s_{\text{phys}}$ coupling is

$$\frac{g^2 \hat{v}}{2} + \sum_{\bar{n} > 0} \kappa(\hat{v} + s)^2 \hat{\theta}_{\bar{n}} = \frac{1}{2} g m_W - \frac{1}{3} \frac{\hat{D}}{\hat{D}} \frac{\hat{v}}{\delta - 2} g^2 \hat{\theta}^2 \left( \frac{\partial V}{\partial \hat{\Phi}} \right)_{\hat{\Phi} = \hat{v}}.$$  

(30)
The 2nd term only contributes for the \( D < 0 \) minimum where the mixing angles, proportional to \( \partial V / \partial \Phi \) at the minimum, are non zero. In this case, assuming the simple \( V(\Phi) \) form of Eq. (25), this 2nd term is of order \( g m_W (\hat{v}^2 m_2 / M_S^4) \) and will yield a small correction to the usual \( g m_W \) strength for the \( WWs_{\text{phys}} \) coupling. To the extent that it can be neglected, \( WW \) scattering (for example) will not violate unitarity bounds if \( m_{s_{\text{phys}}} \lesssim 1 \) TeV. The unitarity problems associated with the correction term in Eq. (30) would, in fact, generally be smaller than those arising from virtual \( \tilde{h} \) and \( \tilde{\phi} \) contributions to \( WW \) scattering. These latter are summarized in the amplitude form

\[
A(s) = -\frac{\kappa^2}{2} \sum_{\text{all } \vec{n}} \frac{1}{s - m_\vec{n}^2} T,
\]

where \( T = T_{\mu\nu} T^{\mu\nu} - \frac{1}{s + 2} T_\mu T_\mu \), where for \( WW \) scattering the \( T_{\mu\nu} \) would be that for the \( W \)'s. As usual, for \( s < M_S^2 \), \( \sum_{\vec{n} > 0} \frac{1}{s - m_\vec{n}^2} \) would be dominated by contributions at the \( M_S \) scale yielding \( -\kappa^2 \sum_{\vec{n} > 0} \frac{1}{s - m_\vec{n}^2} \sim D \sim 1/M_S^4 \), implying an effective contact interaction form. The \( WW \) unitarity problems deriving from the correction term of Eq. (30) and from the effective contact interaction of Eq. (31) can both be suppressed simply by taking \( M_S \) to be large. Still, it would be interesting to try to probe the correction term of Eq. (30) by a high precision measurement of the Higgs-\( WW \) coupling once the Higgs boson has been discovered.

Let us now analyze the fermion sector. We again use the general result of Eq. (29). The rescaling of the fermion fields is determined by noting that for the fermionic \( T_F^{\mu\nu} \) of Eq. (42) of Ref. [4] we have

\[
T_F^{\mu\nu} \ni -3 \bar{\psi} i \gamma^\rho D_\rho \psi,
\]

where we have dropped total derivative terms. Thus, we have

\[
\mathcal{L}_\psi + \mathcal{L}_{\text{mix}} \ni \left( 1 - \frac{3}{2} D V_0 \right) \bar{\psi} i \gamma^\rho D_\rho \psi,
\]

implying that we should rescale to

\[
\hat{\psi} = \left( 1 - \frac{3}{2} D V_0 \right)^{1/2} \psi.
\]

Considering next the Yukawa coupling, for which we use the notation \( \mathcal{L}_\psi \ni -f_Y \bar{\psi} \psi \Phi \), implying \( T_{\mu\nu} = \eta_{\mu\nu} f_Y \bar{\psi} \psi \Phi \), we find that

\[
\mathcal{L}_\psi + \mathcal{L}_{\text{mix}} \ni - \left( 1 - 2 D V_0 \right) f_Y \bar{\psi} \psi \Phi = - \frac{\left( 1 - 2 D V_0 \right)}{\left( 1 - \frac{3}{2} D V_0 \right) \left( 1 - D V_0 \right)}^{1/2} f_Y \bar{\psi} \psi \Phi.
\]

If \( 1 - 2 D V_0 \neq 0 \), then this simply amounts to a redefinition of the Yukawa coupling strength \( f_Y \), which does not affect the standard relation between the \( s \bar{\psi} \psi \) coupling and the mass \( m_{\bar{\psi}} \) induced by \( \hat{v} \). Any Yukawa coupling term involving the real doublet \( H \) will always contain the combination \( \hat{v} + s \). However, for the \( D < 0 \) minimum, \( 1 - 2 D V_0 = 0 \) and it appears that the Yukawa interaction is automatically zeroed.
As remarked earlier, if \( \mathcal{D} > 0 \) then \( \partial V / \partial \Phi = 0 \) is required at the minimum, in which case Eq. (20) implies that there is no mixing between \( s_{\tilde{t}}^1 \) and the Higgs fields. As we have stressed, since \( V(\Phi) \) is the full effective potential, this is not just a tree-level result. Even though there are diagrams involving Higgs boson, fermion and vector boson loops that appear to mix \( s_{\tilde{t}}^1 \) with \( s \), the full calculation is such that the sum of all such diagrams simply serves to modify \( \mathcal{L}_{\text{mix}} \) in such a way that it is the full effective potential \( V \) that should be written in \( T^\mu_{\nu} \), and not just the tree-level potential. We will explicitly demonstrate this for one-loop. We believe the result to be general. The one-loop contribution to the effective potential \( V \) is conveniently summarized as

\[
\delta V(\Phi) = \frac{1}{64\pi^2} \left[ 3m_4^4(\Phi) + m_4^4(\Phi) - 4m_4^2(\Phi) \right] \ln \frac{\Phi^2}{M^2} \equiv \delta V_1(\Phi) + \delta V_2(\Phi) + \delta V_3(\Phi),
\]

where \( M = \langle \Phi \rangle \). In the above, \( m_V(\Phi) = e \Phi \) (where \( e \) is a generic gauge coupling), \( m_4^2(\Phi) = m_4^2 + \frac{1}{4} \partial^2 \Phi \) (where we use the Higgs Lagrangian form \( \mathcal{L}_{\Phi} = \frac{1}{2} (\partial_\mu \Phi)^2 - V_{\text{tree}}(\Phi) \) with \( V_{\text{tree}}(\Phi) = \frac{1}{2} m_4^2 \Phi^2 + \frac{1}{2} \Phi^4 \) and \( m_4(\Phi) = f_X \Phi \) (\( f_X \) being the Yukawa coupling appearing in \( \mathcal{L}_\psi = \bar{\psi} \gamma^\mu D_\mu \psi - f_X \bar{\psi} \psi \Phi \)). We wish to demonstrate that the \( \tilde{h} \) and \( \phi \) fields interact with \( \delta V(\Phi) \) just as they do with the tree-level \( V(\Phi) \), see Eq. (36). To do so, consider the Lagrangian constructed by adding \( \mathcal{L}_{\text{mix}}^{\bar{n}} \) to \( \mathcal{L}_{\phi} \). (For convenience, we focus on a single value of \( \bar{n} \).) Since we are interested in expanding about the potential minimum for which \( \tilde{h}_{\mu \nu} \propto \eta_{\mu \nu} \), we may write \( \mathcal{L}_{\text{mix}} = \alpha T^\mu_{\nu} \), where \( \alpha \equiv - \frac{2}{3} \left( \frac{1}{4} \eta^{\mu \nu} [\tilde{h}_{\mu \nu} + \tilde{h}_{\mu \nu}] + \omega_\delta [\tilde{\phi}^2 + \tilde{\phi}^2] \right) \). Let us first focus on the scalar sector contribution, for which \( T^\mu_{\nu} = - \partial_\rho \Phi \partial^\rho \Phi + 4V_{\text{tree}}(\Phi) \). We now recast \( \alpha T^\mu_{\nu} + \mathcal{L}_{\phi} \) in the form of the original \( \mathcal{L}_{\phi} \) by using appropriate rescalings of fields and couplings. First, for a canonical kinetic energy normalization we must define \( (1 - 2\alpha) \frac{1}{2} \partial_\mu \Phi \partial^\mu \Phi = \frac{1}{2} \partial_\rho \Phi' \partial^\rho \Phi', \) implying \( \Phi' = (1 - 2\alpha)^{1/2} \Phi \sim (1 - \alpha) \Phi \). (Since we are considering the linearized expansion in powers of \( \kappa \), we need only keep \( \mathcal{O}(\alpha) \) terms.) One then finds that the net coefficient of \( \Phi'^2 \) is \( \sim - \frac{1}{2} (1 - 2\alpha) m_4^2 \), implying that we should define \( m_4^{\phi'} \sim (1 - 2\alpha) m_4^2 \). In contrast, we find that rescaling of \( \lambda \) is not necessary; \( \lambda' = \lambda \). We now compute the contribution to the one-loop potential coming from the scalar sector using the \( \Phi' \) Lagrangian. The result is analogous to that contained in Eq. (25), namely \( \frac{1}{64\pi^2} m_4^4(\Phi) \ln \frac{\Phi^2}{M^2} \), with \( m_4(\Phi') = (m_4^2 + \frac{3}{2} \lambda \Phi'^2)^2 \sim (1 - 4\alpha) m_4^2(\Phi) \) (both terms in \( m_4^2(\Phi') \) scale in the same way) and \( \ln \frac{\Phi^2}{M^2} = \ln \frac{\Phi^2}{M^2} \). The net result is \( \mathcal{L}_{\phi} + \mathcal{L}_{\text{mix}} \supset -(1 - 4\alpha) V_{\text{tree}}(\Phi) - (1 - 4\alpha) \delta V_{\phi}(\Phi) \), which is equivalent to including \( \delta V_{\phi}(\Phi) \) in both \( \mathcal{L}_{\phi} \supset -V(\Phi) \) and in \( \mathcal{L}_{\text{mix}} = \alpha T^\mu_{\nu} \supset 4\alpha V(\Phi) \). We turn next to the vector boson loop. After rescaling, \( m_4^2(\Phi') = e^4 \Phi'^2 - (1 - 4\alpha) e^4 \Phi'^4 \), which is equivalent to including \( \delta V_{\psi}(\Phi) \) in the full \( V(\Phi) \) in both \( \mathcal{L}_{\phi} \) and \( \mathcal{L}_{\text{mix}} \). Finally, consider the fermionic sector. Since \( \mathcal{L}_{\psi} + \alpha T^\mu_{\nu} \supset (1 - 3\alpha) \bar{\psi} \gamma^\rho D_\rho \psi \), canonical normalization for the net kinetic terms requires rescaling \( \psi \sim (1 - \frac{3}{2} \alpha) \psi \). The scaling for the Yukawa constant is determined by requiring \( \mathcal{L}_{\psi} + \alpha T^\mu_{\nu} \supset -(1 - 4\alpha) f_X \bar{\psi} \psi \Phi = - f_X \bar{\psi} \psi \Phi' - f_X (1 - 3\alpha) \bar{\psi} \psi \Phi \), implying \( f_X' \sim - f_X \). We then have \( m_4^2(\Phi') = (f_X')^4 \sim (1 - 4\alpha) f_X^4, \) equivalent to including \( \delta V_{\phi}(\Phi) \) in \( V(\Phi) \) in both \( \mathcal{L}_{\phi} \) and \( \mathcal{L}_{\text{mix}} \).

\(^{10}\)There is no rescaling of the gauge fields since the gauge kinetic terms do not appear in \( T^\mu_{\nu} \) for the vector theory. As a result, the gauge coupling to the \( \Phi \) is also not rescaled since the gauge interactions arise from the covariant derivative \( D_\rho \supset i e A_\rho \).
We reemphasize that we have consistently neglected all terms that arise from expanding the metric tensor to higher order in powers of $\kappa(h_{\mu\nu} + \eta_{\mu\nu} \phi_{\mu\nu})$. These would lead (being very schematic) to corrections of order $\kappa^2 \tilde{h} \tilde{h} T$ and $\kappa^2 \tilde{\phi} \tilde{\phi} T$ to $L_{\text{mix}}$ of Eq. (1). The extremum solution will be shifted by such terms. The higher order expansion terms also lead to new contributions to $m_W$, for instance through the quartic $W W \Re(\tilde{\phi} \tilde{\phi} \vec{n}) \Re(\tilde{\phi} \tilde{\phi} \vec{m})$ interactions that will contribute to $m_W^2$ when the (shifted) vacuum expectation values for the $\Re(\tilde{\phi} \tilde{\phi})$ are substituted.

3 Basic Issues and Phenomenology for $V_0 \neq 0$

We first wish to point out that the basic gauge-theory interaction strengths are not altered by the rescaling required when $V_0 \neq 0$. For example, consider the interaction of the fermionic $\psi$ field with a vector field. After the rescaling, $L_\psi + L_{\text{mix}} \ni \tilde{\psi} \gamma^\rho D_\rho \tilde{\psi}$ is of canonical form. This, in combination with the fact that there is no rescaling for the vector fields contained in $D_\rho$ implies that the $W \tilde{\psi} \tilde{\psi}$ and $Z \tilde{\psi} \tilde{\psi}$ couplings are the same as always. The same remarks apply also to the interactions of the Higgs fields with the vector fields. Indeed, after rescaling we have already noted that the Higgs kinetic energy terms have a canonical normalization; i.e. after rescaling and making the derivatives covariant $L_\Phi + L_{\text{mix}} \ni \sum_i \frac{1}{2} (D_\rho \Phi_i)(D^\rho \Phi_i)$. This structure guarantees standard gauge couplings for the Higgs bosons. Clearly, the gauge structure of the theory is being preserved precisely because the vector fields do not require rescaling (due to the fact, noted earlier, that their contribution to the trace of the energy momentum tensor vanishes at tree-level).

If $\overline{D} > 0$, then electroweak symmetry breaking is only possible if $V$ itself has a minimum for non-zero $\Phi$. A typical form is $V = \lambda(\Phi^2 - v^2)^2 + \Xi$, leading to $V_0 = \Xi$. The Higgs self interactions induced by such a potential will not be related in the usual way to the Higgs mass if $V_0 \neq 0$ and the Higgs fields are rescaled.

Another issue regarding the $\overline{D} > 0$ case is the following. The contribution of $L_{\text{mix}}$ to $V_{\text{tot}}$ means that when the KK modes take their appropriate values at the minimum of $V_{\text{tot}}$, $L_{\text{mix}}$ yields a negative contribution to $V_{\text{tot}}$ as reflected in the form $\nabla V_{\text{tot}} = V - \overline{D} V^2$ of Eq. (12). Then, for large enough $V(\Phi)$ (large $\Phi$) $\nabla V_{\text{tot}}$ is unbounded from below. Thus, the standard electroweak symmetry breaking minimum is intrinsically unstable. However, if early universe evolution is such that we enter the $\Phi = v$ minimum, the height of the potential barrier that must be overcome to reach large $\Phi$ is given by the maximum of $\nabla V_{\text{tot}}$, i.e. $(\nabla V_{\text{tot}})_{\text{max}} = 1/(4\overline{D})$, which is most naturally of order $\nabla V_{\text{tot}} \sim M_S^4$. For large enough $M_S$, the tunneling probability will be very small. From another perspective, the model we are discussing is only an effective theory valid below a certain cutoff scale (the string scale $M_S$). In this context, we need only require that $V_{\text{tot}}(\Phi_i) > V_0 - \overline{D} V_0^2$ for field strengths $\Phi_i$ smaller than $M_S$, as is the case.

Finally, there is the question of how to resolve $V_0 \neq 0$ (required to be of order $M_S^4$ in magnitude if $\overline{D} < 0$ and most naturally of this magnitude even if $\overline{D} > 0$) with the known fact that the vacuum energy on the brane (i.e in our world of three spatial dimensions) is very

\[1^1\text{However, the anomaly in the trace of the energy momentum tensor would modify this conclusion; see Ref. }\]
small. We first note that adding an explicit cosmological constant that exists only on the brane by including a term in Eq. (1) of the form \( \int d^4x \sqrt{-g_4} \Lambda \) is simply equivalent to shifting the value of the constant \( \Xi \) as already included in the Higgs potential. Indeed, after expanding \( \sqrt{-g_4} \sim 1 + \frac{1}{2} h + 2\kappa \phi \) (where \( h = h_\mu^\nu \) and \( \phi = \phi_{ii} \), i.e. not \( h \) and \( \phi \)), we find that the effect is to alter Eq. (12) to \( \nabla_{\text{tot}}^2 = (V - \Lambda_4) - \overline{D}(V - \Lambda_4)^2 \). Clearly this amounts to a redefinition of \( \Xi \rightarrow \Xi - \Lambda_4 \). Further, the effective vacuum energy seen by the four-dimensional subset of the basic equations of motion, Eq. (4), including the massless gravitational (\( \tilde{n} = 0 \)) modes, would be \( V_0 - \Lambda_4 \). Thus, if the cosmological constant only resides on the brane, it is equivalent to shifting \( \Xi \rightarrow \Xi - \Lambda_4 \). As a result, the vacuum energy at the minimum will be exactly as before. For instance, the \( \overline{D} < 0 \) minimum would correspond to an effective vacuum energy of \( V_0 - \Lambda_4 = \frac{1}{2\overline{D}} \), which is not only non-zero but has a very large magnitude of order \( M_5^4 \).

At least one solution to this potential problem, which would also seem to be quite natural in the string theory context, is to introduce a cosmological constant throughout the bulk in the manner of the \( \Lambda \) terms of Eqs. (3) and (4). Using Eq. (3), and multiplying the resulting equation by \( \exp \left( -i \frac{2m \tilde{n} \tilde{\eta}}{L} \right) \) and integrating over the \( y \) coordinates projects out the equations of motion for the various \( \tilde{n} \) components of the fields. The crucial point is that the \( \Lambda \) term in Eq. (3) contributes only for \( \tilde{n} = 0 \). The \( \tilde{n} \neq 0 \) considerations we have been discussing are unaltered. For \( \tilde{n} = 0 \), after integration over \( y \), in the 4-dimensional space the right hand side of Eq. (3) takes the form \( \tilde{\kappa}^2 (T_{\mu\nu} - \Lambda \eta_{\mu\nu}) \). A vacuum energy density \( T_{\mu\nu} = \eta_{\mu\nu} V_0 \) can be canceled by choosing \( \Lambda = V_0 \). In the \( \overline{D} < 0 \) case, with \( V_0 \sim -M_5^4 \), one requires \( \Lambda < 0 \) and of order \( M_5^4 \).

A final point is that if \( \Lambda \neq 0 \) then one should in principle account for the fact that the background metric in the bulk in the absence of matter is not exactly flat, implying that we should not expand about \( \eta_{\mu\nu} \) except on the brane where we have fine tuned \( \Lambda = V_0 \). Indeed, one could ask if it is consistent to presume the existence of a brane at \( y = 0 \) in the presence of a non-trivial background metric. However, it can be shown that for \( \Lambda < 0 \) (as required for \( \overline{D} < 0 \)) the metric is conformally equivalent to a flat metric for a slice of constant \( y \), which we have chosen to be at \( y = 0 \). In addition, there could be non-trivial dynamics in the bulk (see for example [3]) that could stabilize the brane in the required manner.

To summarize, a bulk cosmological constant does not affect the \( \tilde{n} \neq 0 \) KK mode mixing and minimization process but does result in the vacuum energy seen by gravity being given by \( V_0 - \Lambda \). For the natural magnitude of \( |V_0| \sim M_5^4 \) (as certainly required for \( \overline{D} < 0 \)), the cancellation between \( V_0 \) and \( \Lambda \) must be essentially exact. However, we do not regard this as being unreasonable given that all these quantities will be determined by the ultimate string theory which might well have such an exact cancellation built in by means of symmetry or dynamics.

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12 A complete expansion of the Lagrangian, see Eq. (4), to higher order in \( \tilde{\kappa} \) would be required to consistently assess the impact of the \( \Lambda \) term at higher orders in \( \tilde{\kappa} \). All we can say at the moment is that, since \( \sqrt{-g_4} \) and \( \sqrt{-g_4} \) both involve the field \( h_\mu^\nu \) in the same way, terms involving only \( h_\mu^\nu \) will cancel for \( V_0 - \Lambda = 0 \). However, terms that mix \( h_\mu^\nu \) and \( \phi \), as well as pure \( \phi \) terms will survive. These deserve further analysis.

13 We are grateful to S. Carlip for pointing this out.
4 Implications of $V_0 \neq 0$ for KK Phenomenology

We have argued that $V_0 \neq 0$ is possible even if $\overline{D} > 0$, and obviously it is required if $\overline{D} < 0$. If $V_0 \neq 0$, then the KK phenomenology given in the literature will be altered. The crucial point is that if $V_0 \neq 0$ then the Higgs scalar fields and the fermionic fields must be rescaled to achieve canonical normalization. In addition, in the $V_0 = \frac{1}{2\overline{D}}$ minimum with $\partial V/\partial \Phi \neq 0$ there will be Higgs-KK mixing. Both effects will modify the KK couplings to the physical states.

We first consider whether the KK-mode induced mixing of $s_{\vec{n}}$ with $s$, present at tree-level if $\overline{D} < 0$, could create experimentally significant modifications. We argue that this is not the case since the mixing angles $\theta_{\vec{n}} \sim -\frac{1}{m_{\vec{n}}^2}$ are very small by virtue of $\epsilon \propto \kappa$ [see Eq. (21)]. It is only if one performs experiments at energies of order the cutoff scale $M_S$ that the cumulative effects of these small mixings might become significant.

As regards rescaling, we have already noted that the vector fields are not rescaled. However, this is not the case for fermionic and Higgs fields. In the fermion case, the Feynman rules read off from the $\mathcal{L}_{\vec{F}}$ of Eq. (44) of [4] will be modified by a factor of $(1 - \frac{3}{2}DV_0)^{-1}$. For the $V_0 = \frac{1}{2\overline{D}}$ minimum, the coupling strengths of the KK modes to $\hat{\psi}\psi$ are obtained by multiplying the Feynman rules of [4] by a factor of 4. In the Higgs field case, the Feynman rules of [4] for KK mode coupling to two Higgs fields must be multiplied by $(1 - \overline{D}V_0)^{-1}$, which is a factor of 2 for the $V_0 = \frac{1}{2\overline{D}}$ minimum. A sampling of the consequences are the following:

- The effective contact interaction of Eq. (31) is multiplied by a factor of 16 for 4-fermion interactions and by a factor of 4 in the case of vector-vector-fermion-fermion interactions. This means that the experimental constraints on $M_S$ will be increased by a factor of 2 ($\sqrt{2}$) in the respective cases.

- The amplitude for radiating a KK excitation from a fermion (vector boson) is increased by a factor of 4 (2). This means that the upper bound on $M_S$ extracted from experimental limits must be re-evaluated. For example, in $e^+e^- \rightarrow \gamma + \text{KK mode}$, the two contributing diagrams in which the KK excitation is radiated from the fermion must be multiplied by a factor of 4, the diagram involving $e^+e^- \rightarrow \gamma^* \rightarrow \gamma + \text{KK}$ is unchanged while the $e^+e^-\gamma\text{KK}$ contact term is multiplied by a factor 4.

5 Discussion and Conclusions

To summarize, we have found that mixing between the Higgs sector and the KK modes could provide a source for electroweak symmetry breaking even in the absence of tree-level Higgs self interactions. The proposed mechanism arises automatically if the KK mode sum $\sum_{\vec{n}} \frac{1}{m_{\vec{n}}^2} \propto \overline{D}$ is cutoff at the string scale in such a way that $\overline{D} < 0$. We note that electroweak symmetry breaking occurs when $\overline{D} < 0$ whatever the actual string cutoff scale, $M_S \sim |\overline{D}|^{-1/4}$, so long as $M_S$ is sufficiently below $M_{Pl}$ that the effective theory we employ can be defined. Even if the KK modes are not responsible for electroweak symmetry breaking, the phenomenology of the contact interactions and missing energy processes which they mediate could be substantially
modified if the Higgs potential vacuum expectation value is of order $M_S^4$, as is entirely possible and perhaps even natural in the string theory context. We have noted that such a vacuum expectation value does not necessarily lead to an unacceptably large cosmological constant; it can be canceled by introducing a cosmological constant in the bulk. As usual, the cancellation must be fine-tuned. However, such precise cancellation could be an automatic result of the string theory dynamics or symmetries.

Undoubtedly, the $\mathcal{D} < 0$ case is somewhat unusual. The two major issues that remain and that we cannot at the moment resolve are the following. First, we have not explicitly constructed a string theory for which $\mathcal{D} < 0$ is predicted. We can only argue that it appears to be an entirely possible result. Even though the KK states below $M_S$ give a positive contribution to $\mathcal{D}$, the way in which KK modes above $M_S$ couple to the hidden string physics and the divergence of the $\sum_n \frac{1}{m_n^*}$ summation is regulated is highly uncertain. At this time, one can only say that the result is an effective operator for which one could have $\mathcal{D} < 0$. Second, since the gravitational corrections are, in the end, of order 1 it is possible that results obtained without expanding to all orders in $\kappa$ are misleading. We have not attempted to go beyond the linear expansion, but it is clear that this issue deserves further investigation. We do wish to note that $V_0 \neq 0$ does not force us to violate the idea that we are dealing with an effective theory below the scale $M_S$. In particular, the Higgs mass and the value of $\Phi$ at the minimum can easily be below $M_S$. Indeed, as shown earlier, $\hat{\Phi} = \hat{\nu} = 246$ GeV is required for the correct value of $m_W$, and parameters can be chosen so that the (tree-level) Higgs mass is also of this same order. For instance, in the $\mathcal{D} < 0$ scenario the Higgs mass is of order $\hat{\nu}$ so long as $m$ of Eq. (25) is of order $M_S^4$. The only requirement for this to be possible is that $\Xi$ be negative with absolute magnitude of order $M_S^4$. This, we argue, would not be unnatural in a string theory context.

As regards the $\mathcal{D} > 0$ case, aside from the $V_0 \neq 0$ issues already noted above, an interesting new point is the potential instability at large $\Phi$ implied by Eq. (12). If the $\lambda$ coefficient of the quartic interaction is significantly larger than 1, $\nabla_{\text{tot}}^2$ becomes negative for $\Phi$ values significantly below $M_S$ (i.e. such that the effective theory is likely to still be valid). We have argued that if one enters the normal $\Phi = v$ minimum early in the evolution of the universe, then the barrier to penetrating to the large $\Phi$ instability will be very substantial (of order $M_S^2$). However, a more detailed investigation is probably called for.

Note added: Ideas concerning possible relations between electroweak symmetry breaking and gravity have also been considered in the past[10].

Acknowledgements

We thank S. Carlip, Z. Lalak, K. Meissner, J. Lykken, M. Olechowski, J. Pawelczyk, M. Schmaltz and J. Wells for helpful conversations. This work was supported in part by the U.S. Department of Energy, the U.C. Davis Institute for High Energy Physics, the State Committee for Scientific Research (Poland) under grant No. 2 P03B 014 14 and by Maria Sklodowska-Curie Joint Fund II (Poland-USA) under grant No. MEN/NSF-96-252. One of the authors (BG) is indebted to the U.C. Davis Institute for High Energy Physics for the great hospitality.
extended to him while this work was being performed.

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