Radiative decays of radially excited mesons $\pi^0', \rho^0', \omega'$ in NJL model

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Abstract

Radiative decays $\pi^0(\pi^0') \rightarrow \gamma + \gamma$, $\pi^0' \rightarrow \rho^0(\omega) + \gamma$, $\rho^0'(\omega') \rightarrow \pi^0 + \gamma$, and $\rho^0'(\omega') \rightarrow \pi^0 + \gamma$ are considered in the framework of the non-local SU(2)$\times$SU(2) NJL model. Radially excited mesons are described with the help of polynomial form factor $f(k_{\perp}^2)$, where $k_{\perp}$ is the quark momentum transverse to the external meson momentum. In spite of mixing of the ground and excited meson states in this model, the decay widths of $\pi^0 \rightarrow \gamma + \gamma$ and $\rho^0(\omega) \rightarrow \pi^0 + \gamma$ are found to be in good agreement with experimental data as in the standard local NJL model. Our predictions for decay widths of radially excited mesons can be verified in future experiments.

Keywords: Nambu-Jona-Lasinio model, radially excited meson, radiative decays, SU(2)$\times$SU(2) NJL model

PACS: 12.39.Fe 13.20.Jf 13.40.Hq

High luminosity modern electron-positron accelerators with CMS energy of about several GeV (e.g. BEPC-II (Beijing), VEPP-2000 (Novosibirsk), DAΦNE (Frascati)) allow studying properties of different mesons with masses up to 3 GeV, including radially excited meson states. Let us note that the standard local quark Nambu-Jona-Lasinio (NJL) model allows to describe the low energy meson physics in a satisfactory agreement with the experiment \cite{1, 2, 3, 4, 5, 6, 7, 8, 9}. In these papers, the properties of the ground meson states were considered. A non-local version of the NJL model with a polynomial form factor was used in order to describe also the first radial excitations of scalar pseudoscalar and vector mesons \cite{9, 10, 11, 12, 13}. The mass spectrum and strong interactions of mesons were studied there. Here we In this paper we consider radiative decays of $\pi^0(1300)$, $\rho^0(1450)$ and $\omega'(1420)$ which can measured in the current and future experiments \cite{14}.

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\textsuperscript{1} Recently the two-photon decay widths of $\pi^0$ and $\pi^0'$ and the cross sections of these meson production in $e^+e^-$ collisions were calculated in Ref. \cite{14}. Unfortunately, some technical mistakes were made there. They will be corrected in the present paper (see also v.2 of arXiv:0908.1628).
The form factor of the most simple polynomial type is chosen:

\[
\begin{align*}
    f_{\pi,\rho}(k^{\perp 2}) & = c_{\pi,\rho} f(k^{\perp 2}), \\
    f(k^{\perp 2}) & = (1 - d |k^{\perp 2}|) \Theta(\Lambda^2 - |k^{\perp 2}|),
\end{align*}
\]

where \( k \) and \( p \) are the quark and meson four-momenta, respectively. In the rest frame of the external meson \( k^{\perp} = 0, k^{\perp 2} = -k^2 \). The parameter \( c_{\pi,\rho} \) defines only the meson masses and can be omitted in the description of meson interactions. The cut-off parameter is taken to be \( \Lambda = 1.03 \) GeV.

In our model the slope parameter \( d \) is chosen from the condition so that the excited scalar meson states do not influence the value of the quark condensate, i.e. do not change the constituent quark mass \( m_u = m_d = 280 \) MeV (see Refs. [10, 11, 9]). This condition can be written in the form

\[
I_1^f = -i \frac{N_C}{(2\pi)^4} \int \frac{d^4k}{m_\pi^2 - k^2} = 0, 
\]

where \( N_C = 3 \) is the number of colors. This gives \( d \approx 1.78 \) GeV\(^{-2}\). This means that the quark tadpole connected with excited scalar mesons vanishes. Therefore the quark condensate acquires only a contribution from the quark tadpole connected with the ground scalar state \( \bar{q}q \).

After bosonization of the SU(2)\( \times \)SU(2) chiral symmetric four-quark Lagrangian and renormalization of the meson fields, we get the following form of the quark–meson interaction:

\[
\mathcal{L}^{\text{int}} = \bar{q}(k) \left\{ \epsilon Q \gamma_\mu A^\mu(p) + \tau^3 \gamma_5 \left[ g_{\pi_1}(p) + g_{\pi_2}(p) f(k^{\perp 2}) \right] \right\} q(k'),
\]

\[
+ \frac{1}{2} \gamma_\mu \left[ g_{\rho_1}(\rho_1^\mu(p)\tau^3 + \omega_1^\mu(p)) + g_{\rho_2}(k^{\perp 2})(\rho_2^\mu(p)\tau^3 + \omega_2^\mu(p)) \right] q(k').
\]

Here \( q \) are the \( u \) and \( d \) quark fields; \( Q = \text{diag}(\frac{2}{3}, -\frac{1}{3}) \) is the quark charge matrix; \( A^\mu \) is the electromagnetic field; \( \pi_1, \rho_1, \) and \( \omega_1 \) are non-physical pseudoscalar and vector meson fields. The coupling constants have the form [11, 12]:

\[
\begin{align*}
    g_{\pi_1} & = \left[ 4 I_2 \left( 1 - \frac{6m_u^2}{M_{\pi_1}^2} \right) \right]^{-1/2}, \\
    g_{\pi_2} & = \left[ 4 I_2^{(2)} \right]^{-1/2}, \\
    g_{\rho_1} & = \left[ \frac{2}{3} I_2 \right]^{-1/2}, \\
    g_{\rho_2} & = \left[ \frac{2}{3} I_2^{(2)} \right]^{-1/2},
\end{align*}
\]

where

\[
\begin{align*}
    I_2 & = -i N_C \int \frac{d^4k}{(2\pi)^4} \frac{\Theta(\Lambda^2 - k^2)}{m_\pi^2 - k^2}, \\
    I_m^n & = -i N_C \int \frac{d^4k}{(2\pi)^4} \frac{f(k^{\perp 2})^n}{m_m^2 - k^2}^m, \quad n = 1, 2, \quad m = 1, 2.
\end{align*}
\]

Note that in \( g_{\pi_1} \) we take into account \( \pi - a_1 \) transitions [4]. \( M_{a_1} = 1.23 \) GeV is the \( a_1 \) meson mass. In the constant \( g_{\pi_2} \) these transitions can be neglected, see Refs. [10, 11].
The free part of the Lagrangian for the pion fields contains non-diagonal kinetic terms:

\[ \mathcal{L}^\text{free}_\pi = \frac{\mu^2}{2} \left( \pi_1^2 + 2\Gamma_{\pi_1\pi_2} + \pi_2^2 \right) - \frac{M_{\pi_1}^2}{2} \pi_1^2 - \frac{M_{\pi_2}^2}{2} \pi_2^2. \]  

(6)

The mass terms have a diagonal form because of the condition (2), and

\[ \Gamma_{\pi} = \frac{I^2_\pi}{\sqrt{I^2_\pi I^2_\rho}}, \]

\[ M_{\pi_1}^2 = g^2_{\pi_1} \left[ \frac{1}{G_1} - 8I_1 \right], \quad M_{\pi_2}^2 = g^2_{\pi_2} \left[ \frac{1}{G_1 c_\pi} - 8I_1^2 \right], \]

(7)

where \( G_1 = 3.47 \text{ GeV}^{-2} \) is the interaction constant of scalar and pseudoscalar quark currents in the initial NJL model [11, 12].

The following transformation allows us to get a diagonal form of the free part of the Lagrangian for the pion fields with quarks [11, 12].

\[ \pi^0 = \pi_1 \cos(\alpha - \alpha_0) - \pi_2 \cos(\alpha + \alpha_0), \]

\[ \pi^{0'} = \pi_1 \sin(\alpha - \alpha_0) - \pi_2 \sin(\alpha + \alpha_0), \]

(8)

where

\[ \sin \alpha_0 = \sqrt{1 + \Gamma_{\pi}}, \quad \tan(2\alpha - \pi) = \sqrt{1 + \frac{1}{\Gamma_{\pi}} - 1 \left[ \frac{M_{\pi_1}^2 - M_{\pi_2}^2}{M_{\pi_1}^2 + M_{\pi_2}^2} \right]}. \]

(9)

For the angles we obtained the following values \( \alpha_0 = 59.06^\circ \) and \( \alpha = 59.38^\circ \).

The free pion Lagrangian takes the standard form

\[ \mathcal{L}^\text{free}_\pi = \frac{\mu^2}{2} \left( \pi^0^2 + \pi^{0'}^2 \right) - \frac{M_{\pi_1}^2}{2} \pi^0^2 - \frac{M_{\pi_2}^2}{2} \pi^{0'}^2. \]

(10)

For \( c_{\pi} = 1.36 \) the values \( M_{\pi} \approx 134.8 \text{ MeV} \) and \( M_{\pi'} \approx 1308 \text{ MeV} \) were received, in agreement with the experimental ones 134.9766 ± 0.0006 MeV and 1300 ± 100 MeV, respectively [10].

As the result the interaction Lagrangian for physical pion fields with quarks takes the form

\[ \mathcal{L}^\text{int}_\pi = g(k) \tau^3 \gamma_5 \left\{ \left[ g_{\pi_1} \frac{\sin(\alpha + \alpha_0)}{\sin(2\alpha_0)} + g_{\pi_2} f(k^{-2}) \frac{\sin(\alpha - \alpha_0)}{\sin(2\alpha_0)} \right] \pi^0(p) \right. \]

\[- \left. \left[ g_{\pi_1} \frac{\cos(\alpha + \alpha_0)}{\sin(2\alpha_0)} + g_{\pi_2} f(k^{-2}) \frac{\cos(\alpha - \alpha_0)}{\sin(2\alpha_0)} \right] \pi^{0'}(p) \right\} q(k'). \]

(11)

An analogous procedure for the vector mesons leads to

\[ \mathcal{L}^\text{int}_{\rho,\omega} = g(k) \frac{\gamma_5}{2} \left\{ \left[ g_{\rho_1} \frac{\sin(\beta + \beta_0)}{\sin(2\beta_0)} + g_{\rho_2} f(k^{-2}) \frac{\sin(\beta - \beta_0)}{\sin(2\beta_0)} \right] (\tau^3 \rho_{\mu}^0(p) + \omega_{\mu}(p)) \right. \]

\[- \left. \left[ g_{\rho_1} \frac{\cos(\beta + \beta_0)}{\sin(2\beta_0)} + g_{\rho_2} f(k^{-2}) \frac{\cos(\beta - \beta_0)}{\sin(2\beta_0)} \right] (\tau^3 \rho_{\mu}^{0'}(p) + \omega_{\mu}'(p)) \right\} q(k'), \]

(12)
where the angles $\beta_0 = 61.53^\circ$ and $\beta = 76.78^\circ$ are defined analogously to Eqs. (9), using

$$M_{\rho_1}^2 = \frac{3}{8G_2l^2}, \quad M_{\rho_2}^2 = \frac{3}{8c_\rho G_2l^2}.$$  \hspace{1cm} (13)

For $c_\rho = 1.15$ and $G_2 = 13.1$ GeV$^{-2}$ we get $M_\rho = M_\omega \approx 783$ MeV and $M_{\rho'} = M_{\rho'} \approx 1450$ MeV. The corresponding experimental values are $M_\rho = 775.49 \pm 0.34$ MeV, $M_\omega = 782.65 \pm 0.12$ MeV, $M_{\rho'} = 1465 \pm 25$ MeV, and $M_{\rho'} = 1425 \pm 25$ MeV.

Let us consider now the standard two-photon decay, described by the triangle quark diagram of the anomalous type. The decay amplitude has the form

$$A_{\mu\nu\to\gamma\gamma} = 8m_u\varepsilon_{\mu\nu\gamma\sigma}q_1^\gamma q_2^\gamma e^2(Q_u^2 - Q_d^2)$$

$$\times \frac{(-1)N_C}{(2\pi)^4} \int \frac{d^4k}{(k^2 - m_\pi^2 + i0)((k - q_1)^2 - m_\pi^2 + i0)((k + q_2)^2 - m_\pi^2 + i0)}$$

$$\times \left\{ g_\pi \frac{\sin(\alpha + \alpha_0)}{\sin(2\alpha_0)} + g_{\pi_2} f(k^2) \frac{\sin(\alpha - \alpha_0)}{\sin(2\alpha_0)} \right\},$$  \hspace{1cm} (14)

where $Q_{u,d}$ are the $u$ and $d$ quark charges; $q_{1,2}$ are the photon momenta. The amplitude contains two types of one-loop integrals: with and without form factor in the quark-pion vertex. The expression for the $\pi^0 \to \gamma\gamma$ amplitude differs from the above one only by the coupling constant of pion and quarks, see Eq. (11), and by the mass of the decaying particle. In the calculation we take into account only the real part of the loop integrals. This ansatz corresponds to the naïve confinement definition [15], which was used in some of our recent works [16, 17, 18].

Consider now two particles decay modes of pseudoscalar and vector mesons with a single photon. The amplitude of $\rho^0 \to \pi^0\gamma$ takes the form

$$A_{\mu\nu\to\pi^0\gamma} = 4m_u\varepsilon_{\mu\nu\gamma\sigma}q_1^\gamma q_2^\gamma e^2(Q_u + Q_d)$$

$$\times \frac{(-1)N_C}{(2\pi)^4} \int \frac{d^4k}{(k^2 - m_\pi^2 + i0)((k - q_1)^2 - m_\pi^2 + i0)((k + q_2)^2 - m_\pi^2 + i0)}$$

$$\times \left\{ \frac{g_\rho}{2} \frac{\sin(\beta + \beta_0)}{\sin(2\beta_0)} + \frac{g_{\pi_2}}{2} f(k^2) \frac{\sin(\beta - \beta_0)}{\sin(2\beta_0)} \right\}$$

$$\times \left\{ \frac{g_\pi}{2} \frac{\sin(\alpha + \alpha_0)}{\sin(2\alpha_0)} + g_{\pi_2} f(k^2) \frac{\sin(\alpha - \alpha_0)}{\sin(2\alpha_0)} \right\},$$  \hspace{1cm} (15)

where $q_1$ and $q_2$ are the vector meson and photon momenta, respectively.

First we recalculate the width of radiative decays of the ground meson states, see Table [19]. The results are in a satisfactory agreement with the experimental data [19]. Note that similar strong decays of considered here radial-excited mesons $\rho \to \omega\pi$ and $\omega' \to \rho\pi$ within the same non-local model were earlier found in Ref. [12] to be also in a satisfactory agreement with observations [19, 21]:

$$\Gamma_{\rho' \to \omega\pi}^{\text{theor.}} \approx 75 \text{ MeV}, \quad \Gamma_{\rho' \to \omega\pi}^{\text{exper.}} = 65.1 \pm 12.6 \text{ MeV},$$

$$\Gamma_{\omega' \to \rho\pi}^{\text{theor.}} \approx 225 \text{ MeV}, \quad \Gamma_{\omega' \to \rho\pi}^{\text{exper.}} = 174 \pm 60 \text{ MeV}.$$  \hspace{1cm} (16)
We remind that the decay width of the ground states have been obtained within the local NJL model in 1986 [4] in a good agreement with the experimental data. In the considered here non-local version of the NJL model radial–excited meson states are mixed with the ground ones. However this mixing does not lead to distortion of the description of the ground meson states interaction among each other, received in the local model.

Table 1: Ground state meson radiative decays widths.

| Decay         | Theory | Exper.  |
|---------------|--------|---------|
| \( \pi^0 \to \gamma \gamma \) | 7.7 eV | 7.5 ± 1.1 eV |
| \( \rho^0 \to \pi^0 \gamma \) | 77 keV | 88 ± 12 keV |
| \( \omega^0 \to \pi^0 \gamma \) | 710 keV | 700 ± 30 keV |

Table 2: Radiative decay widths of \( \pi^0' \) and \( \rho^0' \)

| Decay         | Theory | Theory | Theory |
|---------------|--------|--------|--------|
| \( \pi^0' \to \gamma \gamma \) | 3.2 keV | 1.8 keV | 450 keV |
| \( \rho^0' \to \rho^0 \gamma \) | 450 keV | 450 keV |
| \( \rho^0' \to \pi^0 \gamma \) | 450 keV | 450 keV |
| \( \rho^0' \to \pi^0' \gamma \) | 24 keV | 24 keV |

Table 3: Decay widths of radiative processes with \( \omega \) and \( \omega' \) mesons.

| Decay         | Theory | Theory | Theory |
|---------------|--------|--------|--------|
| \( \pi^0' \to \omega \gamma \) | 17 keV | 17 keV |
| \( \omega' \to \pi^0 \gamma \) | 3.7 MeV | 3.7 MeV |
| \( \omega' \to \pi^0' \gamma \) | 93 keV | 93 keV |

Tables 2 and 3 contain our results of theoretical calculations of radiative decays widths of the radial–excited states of \( \pi^0, \rho^0 \) and \( \omega \) mesons calculated in frames of the NJL model. For the calculations of the phase space we used the present experimental values for the excited meson masses. The widths of these decay channels are not yet measured experimentally. So we made predictions which can be relevant for future experiments.

We plan to perform similar calculations to describe radiative decay channels of radial–excited of \( \eta, \eta' \), and \( \phi \) mesons in non-local U(3)×U(3) NJL model and, besides, to investigate production of the radial–excited mesons at modern \( e^+e^- \) colliders.

**Acknowledgments**

The authors are grateful to L.I. Volkova for the help in manuscript preparation. We also are grateful to A. Akhmedov for discussions. The work was supported by the RFBR grant 10-02-01295-a.

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