Collective flow of identified hadrons at the LHC

Kazuhisa Okamoto\(^1\), Yoshifumi Omura\(^{1,*}\), and Chiho Nonaka\(^{1,2,**}\)

\(^1\)Department of Physics, Nagoya University, Nagoya 464-8602, Japan
\(^2\)Kobayashi Maskawa Institute, Nagoya University, Nagoya 464-8602, Japan

Abstract. Using our developed new relativistic viscous hydrodynamics code, we investigate the QGP bulk property from comparison with the ALICE data of Pb+Pb \(\sqrt{s_{NN}} = 2.76\) TeV collisions at the Large Hadron Collider.

1 Introduction

Since the success of production of the strongly interacting quark-gluon plasma (QGP) at the Relativistic Heavy Ion Collider (RHIC) [1], a relativistic viscous hydrodynamic model has been widely used for the description of space-time evolution of the hot and dense matter created after collisions. Now at RHIC as well as at the Large Hadron Collider (LHC) high-energy heavy-ion collisions are performed and many experimental data are reported.

Figure 1 shows space-time evolutions of high-energy heavy-ion collisions. After two heavy ions collide, in a short time, around 1 fm, thermalization is achieved and hydrodynamic expansion starts. Then the phase transition from the QGP phase to the hadronic phase occurs. Through the freezeout process where interactions among hadrons terminate, many particles jump into the detector. Here relativistic hydrodynamic models can be applied to description of the expansion of strongly interacting QGP and the phase transition. Because the relativistic viscous hydrodynamic equation has close relation to an equation of state (EoS) and transport coefficients of the QCD matter, analyses of experimental data at RHIC and the LHC based on relativistic viscous hydrodynamic model can provide an insight into the detailed information of QGP bulk property.

The recent development of lattice QCD calculations for EoS at vanishing chemical potential is remarkable. Now the phase transition from the QGP phase to the hadronic phase is considered as the crossover phase transition [2]. On the other hand, there is not conclusive understanding for quantitative information of the transport coefficients of the QCD matter. Therefore a study of shear and bulk viscosities of QGP matter from the phenomenological analyses for experimental data is important. There is a large variety of observables which are sensitive to the QGP properties; higher flow harmonics, event plane correlations, three-particle correlations and so on. Also, to analyze experimental data with high statistics and accuracy, a sophisticated numerical algorithm for solving the hydrodynamic equation is indispensable.

2 Relativistic Viscous Hydrodynamic Model

In a hydrodynamic model, we numerically solve the relativistic viscous hydrodynamic equation which is based on the conservation equations,

\[
T^{\mu\nu}\_{;\mu} = 0, \tag{1}
\]

where \(T^{\mu\nu}\) is the energy-momentum tensor. In the Landau frame, the energy-momentum tensor of the viscous fluid is decomposed as

\[
T^{\mu\nu} = e u^\mu u^\nu - (p + \Pi) \Delta^{\mu\nu} + \pi^{\mu\nu}, \tag{2}
\]

where \(\Pi\) is the bulk pressure and \(\pi^{\mu\nu}\) is the shear tensor. Here we use the relativistic viscous hydrodynamic equation derived from the Boltzmann equation based on the method of moments [3, 4]. The relaxation equations for the bulk viscous pressure \(\Pi\) and the shear-stress tensor \(\pi^{\mu\nu}\) read

\[
\tau_\Pi \Pi + \Pi = -\zeta \theta - \delta_{\Pi\Pi} \Pi \theta + \lambda_{\Pi\Pi} \pi^{\mu\nu} \sigma^{\mu\nu}, \tag{3}
\]

\[
\tau_\pi \pi^{\mu\nu} + \pi^{\mu\nu} = 2 \eta \pi^{\mu\nu} - \delta_{\pi\pi} \pi^{\mu\nu} \theta + \varphi^{\mu\nu}(h^{\mu\nu})^{\pi\alpha} - \tau_{\pi\pi} \pi^{\mu\nu} \sigma^{\mu\nu}, \tag{4}
\]

\* e-mail: okamoto@hken.phys.nagoya-u.ac.jp
\*\* e-mail: omura@hken.phys.nagoya-u.ac.jp
\*\*\* e-mail: nonaka@hken.phys.nagoya-u.ac.jp
where $\tau_H$ and $\tau_\pi$ are the relaxation times and $\delta_{\pi\pi}$, $\lambda_{1\pi}$, $\lambda_{2\pi}$, $\varphi_T$, $\tau_T$, and $\lambda_{HH}$ are the transport coefficients. To analyze high-energy heavy-ion collisions where the strong longitudinal expansion exists we perform numerical computation in the Milne coordinates. For details, see Refs. [5, 6].

In our algorithm [6], we split the conservation equation into two parts, an ideal part and a viscous part using the Strang splitting method. It is also applied to evaluate the constitutive equations of the viscous tensors. We decompose them into the following three parts, the convection equations, the relaxation equations, and the equations with source terms. In numerical simulation of relativistic hydrodynamic equation, a time-step size $\Delta \tau$ is usually determined by the Courant-Friedrichs-Lewy (CFL) condition. However in the relativistic dissipative hydrodynamics, one needs to determine the value of $\Delta \tau$ carefully. To save computational cost, we use the Piecewise Exact Solution (PES) method [7], instead of using a simple explicit scheme. If, however, the relaxation times are larger than $\Delta \tau$ determined by the CFL condition, the PES method is not applied. We have checked the energy and momentum conservation in one-dimensional expansion of high-energy heavy ion collisions [5] and the correctness of our code in the following test problems; the viscous Bjorken flow for one-dimensional expansion and the Israel-Stewart theory in Gubser flow regime for the three-dimensional calculation [6].

For an initial condition of our hydrodynamic model, we use the TRENTo [8, 9] which is applied for Bayesian analysis [10]. We use the parameters in the initial condition with the same values as in Ref.[11]. Also, we set initial flows and initial values of viscous tensors to be vanishing at $t_0 = 0.6$ fm. We use a realistic parametrized EoS [2] based on continuum-extrapolated lattice QCD results in the physical quark mass limit, which is also combined with a hadron resonance gas model at low temperature. In the parametrization, the sound velocity takes the minimum value at $T_c \sim 167$ MeV [2]. At the switching temperature $T_{SW} = 150$ MeV the hydrodynamic expansion terminates and the UrQMD which is a hadron based event generator [12, 13] starts for the description of space-time evolution. We sample produced particles from the fluid, using the Cooper-Frye formula [14]

$$E \frac{dN}{dp^3} = \frac{g_i}{2\pi} \int_{\Sigma} f_i(x, p)p^\mu d^3\sigma_{\mu}, \label{eq:cooper_frye}$$

where $i$ is an index over particle species, $f_i$ is the distribution function, and $d^3\sigma_{\mu}$ is a volume element of the isothermal hypersurface $\Sigma$ defined by $T_{SW}$. We introduce only the shear viscous correction to the distribution function $f_i$.

### 3 Calculated Results

Using our relativistic viscous hydrodynamic model, we analyze Pb-Pb $\sqrt{s_{NN}} = 2.76$ TeV collisions in 0-5% and 30-40% centralities. First we show pseudorapidity distribution for charged hadrons in Fig. 2. Here for all calculations we only include the constant ratio of shear viscosity to entropy density $\eta/s = 0.17$, though we already have performed investigation with finite bulk viscosity and temperature dependence of transport coefficients [11]. From comparison with the ALICE data for central collision, we fix the parameters in the initial condition TRENTo. Using the same parameter, we can also reproduce the pseudorapidity distribution for charged hadrons in 30-40% centrality.

In Fig. 3 the $p_T$ distributions for pions in 0-5% and 30-40% centralities are shown, together with the ALICE data [18]. Within the error bars our calculated results are consistent with the ALICE data. However, from $p_T > 1$ GeV, our $p_T$ spectra are slightly larger and the slope of them is flatter than the experimental data. It indicates that in our calculation radial flow grows stronger, compared with the ALICE data. If we include finite bulk viscosity which causes reduction of radial flow, our $p_T$ spectra may show good agreement with the experimental result [11].

In Figs. 4, 5 and 6, we show preliminary results of elliptic flow as a function of transverse momentum $p_T$ for pions, kaons and protons in 30-40% centrality, together with the ALICE data [17]. The elliptic flow $v_2$ is the second coefficient of Fourier expansion of the particle yield as a function of the azimuthal angle on the transverse plane. Recently not only the elliptic flow but also triangular flow $v_3$ and higher harmonics $v_n$ ($n > 4$) are measured at the
LHC. From the detailed analyses of the collective flows, we understand the QGP bulk property in detail. The difficulty in measurement of collective flow is correctly to evaluate the non-flow effect. To distinguish between flow and non-flow effect, the rapidity gap measurement is introduced. However, here we evaluate elliptic flow for pions, kaons and protons only around mid rapidity |η| < 0.5, which is different from the way in experimental measurement. Also, we mention that statistics of the calculation is much smaller than experimental data.

Figure 4 shows the elliptic flow of pion as a function of $p_T$. Though error bars are still large, our results show good agreement with experimental data.

On the other hand, elliptic flows of kaons and protons as a function of $p_T$ of our results have large error bars, though the rough behavior of them is not so far from the experimental data. Still further analyses with more statistics and improvement of evaluation of elliptic flow are needed to extract the detailed information of the QGP bulk property.

4 Summary

Using our developed relativistic viscous hydrodynamic model, we have investigated the QGP bulk property from comparison with experimental data at the LHC. First we fixed the parameters of the initial condition, TRENTo from pseudorapidity distribution of charged hadrons in central collision. Then we calculated the $p_T$ spectra of pions in 0-5 % and 30-40 % centralities. Furthermore we showed the preliminary results of elliptic flow of pions, kaons and protons. Though further investigation with high statistics is needed to reach confirmed results, our preliminary calculations are promising compared with the ALICE data.

Acknowledgments

The work of C.N. is supported by the JSPS Grant-in-Aid for Scientific Research (S) No. 26220707 and the JSPS Grant-in-Aid for Scientific Research (C) No. 17K05438.

References

[1] T-D. Lee, Nucl. Phys. A750(2005)1.
[2] M. Bluhm, P. Alba, W. Alberico, A. Beraudo and C. Ratti, Nucl. Phys. A 929 (2014) 157 doi:10.1016/j.nuclphysa.2014.06.013 [arXiv:1306.6188 [hep-ph]].
[3] G.S. Denicol, H. Niemi, E. Molnar, D.H. Rischke, Phys. Rev. D85(2012) 114047, doi:10.1103/PhysRevD.85.114047, 10.1103/PhysRevD.91.039902, [arXiv:1202.4551 [nucl-th]].
[4] G.S. Denicol, S. Jeon, and C.Gale, Phys. Rev. C90(2014) 024912, doi:10.1103/PhysRevC.90.024912, [arXiv:1403.0962 [nucl-th]].
[5] K. Okamoto, Y. Akamatsu and C. Nonaka, Eur. Phys. J. C 76 (2016) no.10, 579 doi:10.1140/epjc/s10052-016-4433-x [arXiv:1607.03630 [nucl-th]].
[6] K. Okamoto and C. Nonaka, Eur. Phys. J. C 77 (2017) no.6, 383 doi:10.1140/epjc/s10052-017-4944-0 [arXiv:1703.01473 [nucl-th]].
[7] Y. Akamatsu, S. Inutsuka, C. Nonaka and M. Takamoto, J. Comput. Phys. 256 (2014) 34 doi:10.1016/j.jcp.2013.08.047 [arXiv:1302.1665 [nucl-th]].

[8] W. Ke, J. Moreland, J. E. Bernhard and S. A. Bass, Phys. Rev. C 96 (2017) 044912 doi:10.1103/PhysRevC.96.044912 [arXiv:1610.08490 [nucl-th]].

[9] J. S. Moreland, J. E. Bernhard and S. A. Bass, Phys. Rev. C 92 (2015) no.1, 011901 doi:10.1103/PhysRevC.92.011901 [arXiv:1412.4708 [nucl-th]].

[10] J. E. Bernhard, J. S. Moreland, S. A. Bass, J. Liu and U. Heinz, Phys. Rev. C 94 (2016) no.2, 024907 doi:10.1103/PhysRevC.94.024907 [arXiv:1605.03954 [nucl-th]].

[11] K. Okamoto and C. Nonaka, [arXiv:1712.00923 [nucl-th]].

[12] S. A. Bass et al., Prog. Part. Nucl. Phys. 41 (1998) 255 [Prog. Part. Nucl. Phys. 41 (1998) 225] doi:10.1016/S0146-6410(98)00058-1 [nucl-th/9803035].

[13] M. Bleicher et al., J. Phys. G 25 (1999) 1859 doi:10.1088/0954-3899/25/9/308 [hep-ph/9909407].

[14] F. Cooper and G. Frye, Phys. Rev. D 10 (1974) 186. doi:10.1103/PhysRevD.10.186

[15] E. Abbas et al. [ALICE Collaboration], Phys. Lett. B 726 (2013) 610 doi:10.1016/j.physletb.2013.09.022 [arXiv:1304.0347 [nucl-ex]].

[16] J. Adam et al. [ALICE Collaboration], Phys. Lett. B 754 (2016) 373 doi:10.1016/j.physletb.2015.12.082 [arXiv:1509.07299 [nucl-ex]].

[17] B. B. Abelev et al. [ALICE Collaboration], JHEP 1506 (2015) 190 doi:10.1007/JHEP06(2015)190 [arXiv:1405.4632 [nucl-ex]].

[18] B. Abelev et al. [ALICE Collaboration], Phys. Rev. C 88 (2013) 044910 doi:10.1103/PhysRevC.88.044910 [arXiv:1303.0737 [hep-ex]].