NONLOCAL REGULARIZATION OF ABELIAN MODELS
WITH SPONTANEOUS SYMMETRY BREAKING

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Abstract. We demonstrate how nonlocal regularization is applied to
gauge invariant models with spontaneous symmetry breaking. Moti-
vated by the ability to find a nonlocal BRST invariance that leads to
the decoupling of longitudinal gauge bosons from physical amplitudes,
we show that the original formulation of the method leads to a nontrivial
relationship between the nonlocal form factors that can appear in the
model.

1. Introduction

The formalism for generating gauge-invariant, nonlocally regulated ac-
tions [1] is a gauge-invariant application of the nonlocal regularization pi-
oneered by Efimov [2, 3]. It has been extended to non-Abelian models [4],
investigated beyond one-loop order and shown to treat overlapping diver-
gences consistently [5], and has proven to be a useful tool when quantizing
field-antifield models [4, 5, 6]. It has also been applied to the investigation
of anomalies in supersymmetric models [1, 2], and to the standard model
in order to derive a limit on the nonlocal scale from measurements of \( g - 2 \)
for the muon [1]. This quantization procedure can be thought of as a non-
locally 'deformed' (or non-canonical) quantization, and it therefore seems
likely that there is a relationship between this action-level regularization
and non-commutative field theories [12, 13].

More recently in the literature there has appeared some concern as to
how the method of nonlocal regularization is to be applied to gauge theories,
specifically to models where spontaneous symmetry breaking is present [14,
15]. The issue is essentially how to fix the gauge (that is, an \( R_\xi \) gauge) of a
vector boson in a way that does not depend on the gauge parameter \( \xi \).

We will remain cautious in assessing the claims of gauge-dependence made
in [14, 15]. While it superficially appears that they have not been sufficiently
careful when simplifying the BRST invariance of the nonlocal theory (and
therefore they do not truly have a nonlocal action that is invariant under
the BRST transformation that they use when quantizing the theory), the
model that they choose to work with is also not the simplest choice in which
to investigate such matters. Nevertheless, we feel that it is worthwhile to
make available some results on the nonlocal regularization of gauge theories that we discovered some time ago.

In Section 2 we consider an abelian Higgs model in the unbroken phase, showing that there is a natural choice for smearing functions that allows a nonlocal BRST invariance with Ward-Takahashi identities that are “essentially” identical to those of the local theory. We then show in Section 3 that the same considerations lead to a fairly simple nonlocalization in the broken phase of the model with the same properties. The essence of these results lies in the treatment of the longitudinal part of the vector field propagator and its relationship with the ghost propagator.

2. Nonlocal regularization of the unbroken model

We will consider here an Abelian vector field coupled to a complex scalar Higgs field with Lagrangian (see, for example [16, 17]):

\[
L = -\frac{1}{4e^2} F^{\mu\nu} F_{\mu\nu} + (\partial_\mu \Phi^* + i A_\mu \Phi^*)(\partial^\mu \Phi - i A^\mu \Phi) - V(|\Phi|^2),
\]

which is invariant under the infinitesimal U(1) transformation \(\delta A_\mu = \partial_\mu \theta\) and \(\delta \Phi = i \theta \Phi\). We use the standard \(R_\xi\) gauge-fixing term: \((\partial_\mu A^\mu)^2/(2\xi)\), and including BRST ghost and anti-ghost fields \(C\) and \(\bar{C}\), which in the absence of symmetry breaking, contributes a kinetic term to the Lagrangian: \(-\partial^\mu \bar{C} \partial_\mu C\). The resulting local, gauge-fixed Lagrangian has kinetic terms

\[
K = \frac{1}{2e^2} A^\mu D^{-1}_{\mu\nu} A^\nu + \Phi^* D^{-1} \Phi - \bar{C} D^{-1} C,
\]

and higher-order interactions

\[
I = i A^\mu (\Phi^* \partial_\mu \Phi - \Phi \partial_\mu \Phi^*) + A^\mu A_\mu |\Phi|^2 - V(|\Phi|^2),
\]

where we have defined the inverse propagators

\[
D^{-1} = -\Box, \quad D^{-1}_{\mu\nu} = \eta_{\mu\nu} \Box - (1 - \xi^{-1}) \partial_\mu \partial_\nu.
\]

The action is invariant under the BRST transformation:

\[
\delta A_\mu = -\partial_\mu C \zeta, \quad \delta \Phi = -i C \Phi \zeta, \quad \delta \bar{C} = \frac{1}{\xi e^2} \partial_\mu A^\mu \zeta, \quad \delta C = 0.
\]

2.1. Nonlocal Regularized Lagrangian. Nonlocal regularization of a local gauge field theory involves two ingredients: a) nonlocal form factors that act as a high frequency cutoff in the loop integrals without introducing any new degrees of freedom into the model, and b) higher-order, nonlocal interaction terms that allow an extended, nonlocal version of any gauge symmetry present in the local theory. When these two ingredients operate together we have a finite gauge field theory, with the regulator implemented directly in the Lagrangian rather than during the perturbative (loop) expansion. We will begin here by focusing primarily on the algebraic structure of the derived nonlocal action that allows a nonlocally deformed gauge symmetry to survive.
Following [3], for every physical field $\Psi$ (which we will use throughout to symbolically represent the local fields, in this case: $A_\mu$, $\Phi$, $C$ or $\bar{C}$) we introduce a shadow field $\psi$ (in this case representing one of $a_\mu$, $\phi$, $c$ or $\bar{c}$, as above), and the ‘hatted’ and ‘barred’ propagators related to $\Psi$ and $\psi$ respectively. For the scalar and ghost fields they are:

$$
\hat{D}_\Phi^{-1} = \mathcal{E}_\Phi^{-2}D^{-1}, \quad \hat{D}_\psi^{-1} = (1 - \mathcal{E}_\Phi^2)^{-1}D^{-1},
$$

(5a)

$$
\hat{D}_C^{-1} = \mathcal{E}_C^{-2}D^{-1}, \quad \hat{D}_c^{-1} = (1 - \mathcal{E}_C^2)^{-1}D^{-1},
$$

(5b)

where in order that the smearing functions $\mathcal{E}$ do not introduce any additional degrees of freedom over that of the local theory, they are taken to be entire functions of the momentum operator $\mathcal{E}$ [2, 3]. In addition, we normalize the hatted propagators to have unit residue on shell, so that since all fields are massless: $\mathcal{E}(p^2 = 0) = 1$. This condition guarantees that the barred propagators do not contain a pole, and therefore the shadow fields are not independent quantum degrees of freedom. For the vector field we introduce different smearing factors for the longitudinal and transverse propagator as:

$$
\hat{D}_{\mu\nu}^{-1} = \mathcal{E}_T^{-2}(\Box\eta_{\mu\nu} - \partial_\mu\partial_\nu) + \xi^{-1}\mathcal{E}_L^{-2}\partial_\mu\partial_\nu,
$$

(6a)

$$
\hat{D}_{\mu\nu} = (1 - \mathcal{E}_T^2)^{-1}(\Box\eta_{\mu\nu} - \partial_\mu\partial_\nu) + \xi^{-1}(1 - \mathcal{E}_L^2)^{-1}\partial_\mu\partial_\nu.
$$

(6b)

The quadratic terms of the nonlocal Lagrangian are introduced as

$$
K_{NL} = \frac{1}{2\varepsilon^2} A^\mu \hat{D}_{\mu\nu}^{-1} A_\nu + \Phi^a \hat{D}_\Phi^{-1} \Phi + \bar{C} \hat{D}_C^{-1} C
$$

$$
+ \frac{1}{2\varepsilon^2} \phi^a \hat{D}_{\mu\nu}^{-1} \phi^a + \phi^a \hat{D}_\phi^{-1} \phi + \bar{c} \hat{D}_c^{-1} c,
$$

(7)

with interaction terms identical to (2b) with the replacement $\Psi \rightarrow \Psi + \psi$:

$$
I_{NL} = I(A_\mu + a_\mu, \Phi + \phi, C + c, \bar{C} + \bar{c}).
$$

(8)

The nonlocal action $S_{NL} = \int dx(K_{NL} + I_{NL})$ possesses a nonlocal version of the local BRST invariance (4):

$$
\tilde{\delta}A_\mu = -\mathcal{E}_L^2\partial_\mu(C + c)\zeta, \quad \tilde{\delta}a_\mu = -(1 - \mathcal{E}_L^2)\partial_\mu(C + c)\zeta,
$$

(9a)

$$
\tilde{\delta}\Phi = -i\mathcal{E}_L^2(C + c)(\Phi + \phi)\zeta, \quad \tilde{\delta}\phi = -i(1 - \mathcal{E}_L^2)(C + c)(\Phi + \phi)\zeta,
$$

(9b)

$$
\tilde{\delta}\bar{C} = \frac{1}{\xi^2 \varepsilon^2} \mathcal{E}_c^2 \partial_\mu(a_\mu + c_\mu)\zeta, \quad \tilde{\delta}c = \frac{1}{\xi^2 \varepsilon^2} (1 - \mathcal{E}_c^2)\partial_\mu(a_\mu + c_\mu)\zeta,
$$

(9c)

$$
\tilde{\delta}C = 0, \quad \tilde{\delta}\bar{c} = 0.
$$

(9d)

By construction, these transformations satisfy $\tilde{\delta}\Psi + \tilde{\delta}\psi = \delta[\Psi + \psi]$, that is, the transformation of the field plus its shadow results in the local BRST transformation (4) with the replacement $\Psi \rightarrow \Psi + \psi$. This results in the interaction terms transforming in exactly the same way as in the local theory with this same replacement. Furthermore, in the variation of the kinetic terms the smearing operators are removed, and the variation of the kinetic
terms of the field and its shadow combine to give a “local” result, for example:

\[ A^\mu \tilde{D}^{-1}_\mu \delta A^\nu + a^\mu \tilde{D}^{-1}_\mu \delta a^\nu = -(A_\mu + a_\mu) D^{-1}_{\mu \nu} \partial^\nu (C + c) \zeta. \]  

(10)

We see that by construction, the nonlocal Lagrangian must possess the BRST invariance (9) as a consequence of the BRST invariance of the local theory.

2.2. A Modified Nonlocal BRST Invariance. As it stands we have a regulated nonlocal action which possesses the BRST invariance (9), but with a great deal of latitude on how to choose the different functions \( \mathcal{E} \). Different choices will result in distinct nonlocal theories, all of which will be equivalent in the local limit (it is to be expected that higher-order vertices are generated in these nonlocal models, but since they should vanish in the local limit the theory would be renormalizable in that limit).

In particular, we note (following \([4]\)) that the BRST invariance (9) does not guarantee decoupling of the longitudinal vector boson from \( n \)-point functions. A related BRST invariance \( \tilde{\delta} \) that does (and we show this in Section 2.3) is found by requiring that the linear parts of the BRST transformation can be made identical to those of the local theory (4). That this is possible puts nontrivial constraints on the form of the smearing operators–constraints that in hindsight are perhaps not so surprising.

We begin by choosing the transformation of the anti-ghost fields as

\[ \tilde{\delta}_1 \bar{C} = \frac{1}{\xi e^2} \partial_\mu A_\mu \zeta, \quad \tilde{\delta}_1 \bar{c} = \frac{1}{\xi e^2} \partial_\mu a_\mu \zeta, \]  

(11)

and since this means that \( \tilde{\delta}_1 \bar{C} + \tilde{\delta}_1 \bar{c} = \delta \bar{C} + \delta \bar{c} \), the interaction terms will be transformed in the same way by \( \tilde{\delta}_1 \) and \( \delta \). The kinetic terms for the ghosts transform into

\[-\delta C \tilde{D}_C^{-1} C - \delta \bar{c} \tilde{D}_C^{-1} c = \frac{1}{\xi e^2} \partial_\mu A_\mu \zeta \tilde{D}_C^{-1} C + \frac{1}{\xi e^2} \partial_\mu a_\mu \zeta \tilde{D}_C^{-1} c, \]  

(12)

and in order for this to be canceled by transforming the vector field kinetic terms, we find that the vector field and its shadow must transform as

\[ \tilde{\delta}_1 A_\mu = -\frac{\mathcal{E}_L^2}{\mathcal{E}_C^2} \partial_\mu C \zeta, \quad \tilde{\delta}_1 a_\mu = -\frac{1}{\mathcal{E}_C^2} \partial_\mu c \zeta. \]  

(13)

From this we see that only if we choose \( \mathcal{E}_C = \mathcal{E}_L \) will \( \tilde{\delta}_1 A_\mu + \tilde{\delta}_1 a_\mu = \delta A_\mu + \delta a_\mu \), and therefore the interaction term again transform the same way under \( \tilde{\delta}_1 \) as they did under \( \delta \).
Provided we make this choice, the nonlocal action is invariant under the modified nonlocal BRST transformation:

\[ \tilde{\delta}_1 A_\mu = -\partial_\mu C \zeta, \quad \tilde{\delta}_1 a_\mu = -\partial_\mu c \zeta, \]  
\[ \tilde{\delta}_1 \Phi = -i \mathcal{E}_\Phi (C + c) (\Phi + \phi) \zeta, \quad \tilde{\delta}_1 \phi = -i (1 - \mathcal{E}_\Phi) (C + c) (\Phi + \phi) \zeta, \]  
\[ \tilde{\delta}_1 \bar{C} = \frac{1}{\xi e^2} \partial_\mu A^\mu \zeta, \quad \tilde{\delta}_1 \bar{c} = \frac{1}{\xi e^2} \partial_\mu a^\mu \zeta, \]  
\[ \tilde{\delta}_1 C = 0, \quad \tilde{\delta}_1 c = 0, \]  

(14a), (14b), (14c), (14d)

using which, as we shall see, decoupling is easily proven. Note that the nonlinear parts of the transformations \( \delta_1 \) and \( \tilde{\delta} \) are identical.

This is the first ingredient: the ability to make this transformation requires that the smearing for the ghosts and the longitudinal part of the vector field are identical. This is a simple matter to arrange, but there is further good reason to choose \( E_L \) so that it is related to \( E_T \).

Decomposing the vector field propagator into transverse and longitudinal pieces, we have

\[ D_{\mu\nu}^{-1} = \Box T_{\mu\nu} + \xi^{-1} \Box L_{\mu\nu}, \]  

(15a)

where the transverse and longitudinal projection operators are

\[ T_{\mu\nu} = \eta_{\mu\nu} - \Box^{-1} \partial_\mu \partial_\nu, \quad L_{\mu\nu} = \Box^{-1} \partial_\mu \partial_\nu. \]  

(15b)

If we construct the smeared nonlocal kinetic terms from an entire function of the local kinetic terms as outlined in [4], then we can write it as a power series in the local kinetic terms (the constant and first-order coefficients are constrained by the condition that there be a pole at \( p^2 = 0 \) with unit residue):

\[ \hat{D}_{\mu\nu}^{-1} = \hat{D}_{\mu\alpha} \eta^{\alpha\beta} \left( \eta_{\beta\nu} + \sum_{n=1}^{\infty} A_n (D^{-1})_{\beta\nu}^{\eta} \right), \]  

(16a)

where

\[ (D^{-1})_{\alpha\alpha_n}^{\eta} = \prod_{m=0}^{n-1} D_{\alpha_m \alpha_{m+1}}^{-1}. \]  

(16b)

With the longitudinal-transverse decomposition (14), this may be worked out explicitly:

\[ (D^{-1})_{\mu\nu}^{n} = \Box^n T_{\mu\nu} + \frac{1}{\xi^n \Box} L_{\mu\nu}, \]  

(17)

so that

\[ \hat{D}_{\mu\nu}^{-1} = \left( 1 + \sum_{n=1}^{\infty} A_n \Box^n \right) \Box T_{\mu\nu} + \left( 1 + \sum_{n=1}^{\infty} \frac{A_n}{\xi^n \Box^n} \right) \frac{1}{\xi} \Box L_{\mu\nu}, \]  

(18)

and by comparison with (13) we see that it is natural to choose

\[ E_L(\Box) = E_T \left( \frac{1}{\xi \Box} \right). \]  

(19)
Accepting this condition, we see that we only have the freedom to choose two smearing operators: $E_T$ and $E_\Phi$—the smearing function for the ghost is constrained by requiring the existence of the BRST symmetry \cite{14}, and the smearing functions for the longitudinal and transverse related by \cite{13}. In fact, if we follow the method of nonlocal regularization as formulated in \cite{1} to the letter, we should really be considering entire functions of the operator appearing in the quadratic part of the local action as a whole. That is, writing \cite{24} as $K = \Psi D^{-1} \Psi$, then we should write $\hat{D}^{-1} = \sum_n A_n D^{-1}$. This leads to $E_\Phi = E_T$, and we have a nonlocalization that depends on a single nonlocal form factor function only.

2.3. Quantization and Ward-Takahashi Identities. Quantization of the nonlocal theory proceeds in two steps. First, because the propagators for the shadow fields do not possess a pole in the propagator, the shadow field equations:

$$\psi = -\hat{D}\frac{\delta I_{NL}}{\delta \psi},$$

constitute in implicitly-defined, but local relationship between the shadow fields and physical fields $\Psi$. The quantum nonlocal BRST action is generated by iteratively replacing the shadow fields in $S_{NL}$ using \cite{20}, which generates the nonlocal Lagrangian as a series which is straightforward to evaluate to any order. Similarly, the resulting action will be invariant under the nonlocal BRST transformations of the local fields $\Psi$ as given by $\hat{\delta}$ or $\hat{\delta}_1$, with the same replacement of the shadow fields using \cite{20}.

Once this procedure is completed, path integral quantization is completed through the prescription to compute the vacuum expectation of any operator $O$ via \cite{1} for a discussion of the $T^*$ ordered product

$$\langle T^*[O] \rangle = \int d\mu_{inv} O \exp(iS_{NL}).$$

The measure $d\mu_{inv}$ is an invariant measure that can be written as

$$d\mu_{inv} = d\Psi \exp(iS_{meas}),$$

where $S_{meas}$ may be determined from the BRST transformations (up to BRST-invariant contributions) via the condition

$$\delta S_{meas} = i\text{Tr} \left[ \frac{\partial}{\partial \Psi} \hat{\delta}_1 \Psi \right];$$

see \cite{1, 4} for a more complete discussion on this matter.

We have explicitly used the modified nonlocal BRST transformation $\hat{\delta}_1$ in \cite{23}, since generating the measure from this transformation will lead to a nonlocal quantum theory will be invariant under this transformation. Note though, that beginning from different nonlocal gauge symmetries we would generate a different invariant measure factor, and therefore a different nonlocal quantum theory and different Ward-Takahashi identities. Also,
since $E_L$ is $\xi$-dependent, we have to expect that the measure will also have nontrivial dependence on the gauge parameter.

To generate the Ward-Takahashi identities for the nonlocal quantum theory, we introduce the generating functional

$$Z_J = \int d\mu_{\text{inv}} \exp(iS_{NL} + iJ \cdot \Psi),$$

(24)

where

$$J \cdot \Psi = \int dx \left[ J^\mu A_\mu + J\Phi + \bar{C}D + \bar{D}C + \bar{i}U \mathcal{E}_\Phi^2(C + c)(\Phi + \phi) \right],$$

(25)

and we have added a complex source $U$ for the BRST invariant field combination: $\bar{i}E_2^2(\Phi + \phi)$ (to see this, note that $(C + c)^2 = 0$). Following [16], in order for the generating functional to result in a BRST invariant perturbation theory, it must itself be gauge invariant, and transforming $Z_J$ using (14) we end up with the condition

$$\int dx \left[ J^\mu \partial_\mu \frac{\delta Z_J}{\delta D} + \frac{1}{\xi e^2} \partial_\mu \frac{\delta Z_J}{\delta J^\mu} D + J^\mu \frac{\delta Z_J}{\delta U} \right] = 0.$$  

(26)

This leads to the Ward-Takahashi identities on the vertex functional $\Gamma[\psi]$, which is related to $Z_J$ by $Z_J = \exp(i\Gamma + iJ \cdot \psi)$:

$$\int dx \left[ \frac{\delta \Gamma}{\delta A^\mu} \partial_\mu C + \frac{1}{\xi e^2} \frac{\delta \Gamma}{\delta C} \partial_\mu A^\mu + i \frac{\delta \Gamma}{\delta \Phi} \mathcal{E}_\Phi^2(C + c)(\Phi + \phi) \right] = 0.$$  

(27)

These conditions are identical to those of the local theory except for the nonlocal vertices appearing in the final term. In particular, taking functional derivatives with respect to $C(y)$ and $A_\nu(z)$ leads to the relationship between the ghost two-point function $\Pi_C$ and the longitudinal projection of the vector field two-point function: $\xi e^2 \partial_\mu \Pi_\mu^\nu + \partial^\nu \Pi_C = 0$, which, since no ghosts can exist on external legs, shows that the physical vector field propagator is transverse. Further functional derivatives with respect to the vector field guarantee that the longitudinal vector field decouples from all physical pure vector field $n$-point functions.

The first manifestation of nonlocality in the Ward-Takahashi identities is in the relationship between the scalar field vacuum expectation value $\langle \Phi \rangle$ and the longitudinal part of the two-point mixing of the vector field and the scalar field $\Pi_\mu^\nu$: $\partial_\mu \Pi^\mu_\nu = i\mathcal{E}_\Phi^2(\Phi)$. In the local theory this relation indicates how a real vacuum expectation value for the scalar field will result in a mixing between the Goldstone boson and the longitudinal part of the vector field, resulting in a mass for the vector field. That this relation involves the scalar field smearing function $\mathcal{E}_\Phi^2$, foreshadows that in the spontaneously broken theory we will be forced to choose the smearing function for the scalar field to be related to that of the vector field if we want the modified BRST invariance to exist for the nonlocal theory.
3. Broken symmetry phase

It is noteworthy that spontaneous symmetry breaking necessarily mixes the quadratic and interacting terms—the masses are ‘fed’ down from the interaction terms to the quadratic terms by the shift to the potential minimum. Since the nonlocal regularization treats the kinetic and interaction terms differently, it is not a straightforward matter to go from the nonlocally-regularized model in the previous section to a regularized model in the broken phase. Instead we will begin with the BRST invariant local theory in the broken phase, and apply the nonlocal regularization method as outlined in Section 2.

Assuming a form of the scalar field potential that leads to spontaneous symmetry breaking:

\[ V(|\phi|^2) = \lambda \left( |\phi|^2 - \frac{1}{2} v^2 \right)^2, \]

we redefine the scalar field fluctuations about the minimum of this potential as

\[ \phi = 2^{-\frac{1}{2}} (v + H + iF), \]

and at the same time introduce the gauge-fixing and ghost contributions:

\[ L_{\text{gf}} = -\frac{1}{2}\xi e^2 (\partial_\mu A^\mu + \xi e^2 v F)^2, \quad L_{\text{ghost}} = -\partial_\mu \bar{C} \partial_\mu C + \xi e^2 v (v + H) \bar{C} C. \]

The resulting BRST Lagrangian has quadratic terms

\[ K = \frac{1}{2e^2} A^\mu D_{A,\mu\nu}^{-1} A_\nu + \frac{1}{2} H D_H^{-1} H + \frac{1}{2} F D_F^{-1} F - \bar{C} D_C^{-1} C, \]

and higher-order interactions

\[ I = A^\mu (F \partial_\mu H - H \partial_\mu F) + v H (A^\mu A_\mu - \lambda F^2 - \lambda H^2) + \frac{1}{2} A^\mu A_\mu (H^2 + F^2) - \frac{1}{4} \lambda (H^2 + F^2)^2 + \xi e^2 v H \bar{C} C. \]

In (31a) we have integrated by parts and defined the inverse propagators:

\[ D_H^{-1} = -(\Box + 2\lambda v^2), \quad D_F^{-1} = D_C^{-1} = -(\Box + \xi e^2 v^2), \]

and action is invariant under the local BRST transformations

\[ \delta A_\mu = -\partial_\mu C \zeta, \quad \delta F = -C (v + H) \zeta, \quad \delta H = C F \zeta, \]

\[ \delta C = 0, \quad \delta \bar{C} = \frac{1}{\xi e^2} (\partial_\mu A^\mu + \xi e^2 v F) \zeta. \]
3.1. Nonlocal Regularized Lagrangian. The construction goes through as described in Section 2.1: we introduce shadow fields for all local fields, make the \( \Psi \rightarrow \Psi + \psi \) replacement in the interaction terms, and introduce the hatted and barred propagators for the scalar and ghost fields of the same form as (3):

\[
\hat{D}_\Phi^{-1} = \mathcal{E}_\Phi^{-2}D_\Phi^{-1}, \quad \bar{D}_\Phi^{-1} = (1 - \mathcal{E}_\Phi^2)^{-1}D_\Phi^{-1}.
\]

(35)

Once again we allow the possibility that the smearing functions are different for the different fields, except that now the propagator for the Higgs field \( H \) will now have a pole at \( p^2 = 2\lambda v^2 \), and so it is natural to choose \( \mathcal{E}_H = \mathcal{E}_L(\square + 2\lambda v^2) \).

Noting that the local propagator for the vector field can be written as

\[
D_{A,\mu\nu} = (\square + e^2v^2)T_{\mu\nu} + \xi^{-1}(\square + \xi e^2v^2)L_{\mu\nu},
\]

(36)

then the same argument that led to (19) leads us to choose

\[
\mathcal{E}_T = \mathcal{E}_T(\square + e^2v^2), \quad \mathcal{E}_L = \mathcal{E}_T(\xi^{-1}\square + e^2v^2),
\]

(37)

and the propagators for the vector field and its shadow are:

\[
\hat{D}_{A,\mu\nu}^{-1} = \mathcal{E}_T^{-2}(\square + e^2v^2)T_{\mu\nu} + \mathcal{E}_L^{-2}\xi^{-1}(\square + \xi e^2v^2)L_{\mu\nu},
\]

(38a)

\[
\bar{D}_{A,\mu\nu}^{-1} = (1 - \mathcal{E}_T^2)^{-1}(\square + e^2v^2)T_{\mu\nu} + (1 - \mathcal{E}_L^2)^{-1}\xi^{-1}(\square + \xi e^2v^2)L_{\mu\nu}.
\]

(38b)

Note that the vector field propagator has a physical pole at \( p^2 = e^2v^2 \) as well as the gauge-dependent pole at \( p^2 = \xi e^2v^2 \), both of which are reflected in the smearing functions.

The quadratic terms in the nonlocal Lagrangian are:

\[
K_{NL} = \frac{1}{2e^2}A^\mu \hat{D}_{A,\mu\nu}^{-1}A_\nu + \frac{1}{2}H\hat{D}_H^{-1}H + \frac{1}{2}F\hat{D}_F^{-1}F + \bar{C}\bar{D}_C^{-1}C
\]

(39)

with higher-order interaction terms determined as before from the local interaction terms (31b): \( I_{NL} = I(\Psi + \psi) \), and the nonlocal BRST action is invariant under the nonlocal version of the local BRST symmetry (34):

\[
\delta A_\mu = -\mathcal{E}_L^2(\partial_\mu(C + c))\zeta,
\]

(40a)

\[
\delta C = 0, \quad \delta \bar{C} = \mathcal{E}_L^2\frac{1}{\xi e^2}(\partial_\mu(A^\mu + a^\mu) + \xi e^2v(F + f))\zeta,
\]

(40b)

\[
\delta F = -\mathcal{E}_L^2(C + c)(v + H + h)\zeta, \quad \delta H = \mathcal{E}_L^2(C + c)(F + f)\zeta,
\]

(40c)

with shadow field transformations that follow the pattern given in (3).

3.2. A Modified Nonlocal BRST Invariance. As described in Section 2.3, we want to find a modified nonlocal BRST transformation \( \delta_1 \) in which the linear part of the transformation is identical to the linear part of the local BRST transformation (34), and the nonlinear part identical to the nonlinear part of (34). Requiring that the ghosts transform as:

\[
\delta_1 \bar{C} = (\frac{1}{\xi e^2}\partial_\mu A^\mu + vF)\zeta,
\]

then transforming the quadratic terms for the
vector field we find that we have to choose $\mathcal{E}_C = \mathcal{E}_L$, and the vector field would transform as (14). Also transforming the quadratic terms of the Goldstone boson we find that we have to choose $\mathcal{E}_F = \mathcal{E}_C$, and the linear part of the BRST transformation of the Goldstone boson $F$ is also ‘localized’. The result of this is the modified nonlocal BRST invariance:

$$\tilde{\delta} F = -vC\zeta - \mathcal{E}_2^L(C + c)(H + h)\zeta,$$

$$\tilde{\delta} H = \mathcal{E}_2^H(C + c)(F + f)\zeta, \quad \tilde{\delta} A_\mu = -\partial_\mu C\zeta,$$

$$\tilde{\delta} \bar{C} = \left(\frac{1}{\xi e^2}\partial_\mu A^\mu + vF\right)\zeta, \quad \tilde{\delta} C = 0,$$

with the ghosts transforming following the pattern in (14).

Note that choosing $\mathcal{E}_L = \mathcal{E}_C = \mathcal{E}_F$ does not interfere with the shadow fields $f, c$ and $\bar{c}$ being removable at the classical level, since their ‘barred’ propagators still do not contain a pole. This would not necessarily been the case with an arbitrary $\mathcal{E}_L$, but would be possible with any $\mathcal{E}_L = \mathcal{E}_L(\Box + \xi e^2 v^2)$ that satisfies $\mathcal{E}_L(0) = 1$. Nevertheless, the relation (37) is well-motivated, and leaves only the freedom to choose $\mathcal{E}_T$ and $\mathcal{E}_H$. If we further require the theory to generated from a single entire function of the kinetic terms of the local theory, as described at the end of Section 2.2, the we would also be forced to choose $\mathcal{E}_H(x) = \mathcal{E}_T(x)$, that is, $\mathcal{E}_H$ is the same function of $\Box + 2\lambda v^2$ that $\mathcal{E}_T$ is of $\Box + e^2 v^2$.

3.3. Quantization and Ward-Takahashi Identities. Path integral quantization proceeds as before, except that in the generating functional (24) we will write the source terms as

$$J \cdot \Psi = \int dx \left[ J^\mu A_\mu + JF + KH + \bar{C}D + \bar{D}C + U\mathcal{E}_2^H(C + c)(F + f) - V\mathcal{E}_2^L(C + c)(H + h)\right],$$

and proceeding as before, we find that the vertex functions will satisfy:

$$\int dx \left[ \frac{\delta \Gamma}{\delta A^\mu} \partial_\mu C + vC\frac{\delta \Gamma}{\delta F} + \left(\frac{1}{\xi e^2}\partial_\mu A^\mu + vF\right)\frac{\delta \Gamma}{\delta C} + \frac{\delta \Gamma}{\delta F}\mathcal{E}_2^H(C + c)(F + f) - \frac{\delta \Gamma}{\delta H}\mathcal{E}_2^L(C + c)(H + h)\right] = 0.$$

As before, these lead to the decoupling of the longitudinal vector boson from $n$-point functions that involve external vector fields.

4. Discussion

We have presented a simple prescription for nonlocal regularization of field theory models with spontaneous symmetry breaking in an arbitrary $R_\xi$ gauge. The smearing functions depend on the gauge parameter in a nontrivial way, and we therefore expect that the path integral measure factor will also depend on $\xi$. Nevertheless, requiring that the nonlocal theory possess a nonlocal BRST invariance with Ward-Takahashi identities that imply the
decoupling of longitudinal gauge bosons, leads to a nontrivial relationship
between the nonlocal form factors for different fields. This relationship was
shown to follow naturally from the formalism presented in [4], which results
in a nonlocal theory that depends on a single nonlocal form factor.

Although in hindsight these relationships are not surprising, recent related
constructions have appeared in the literature [14, 15] do not impose them.
While one cannot say that alternate constructions are incorrect, one can
make the case that the method presented herein is preferred on the grounds
of simplicity of implementation and interpretation.

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