Atom Localization in two and three dimensions via level populations in an M-type atomic system

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Schemes for two-dimensional (2D) and three-dimensional (3D) atomic states localization in a five-level M-type system using standing-wave laser fields are presented. In the upper two levels of the system we see a ‘coupled’ localization for both 2D and 3D case. Here, the state in which majority of population will be found depends on the sign of the detunings between the upper levels and the intermediate level. The experimental implementation of the scheme using the D2 line of Rb is also proposed. The scheme may be manipulated to achieve subwavelength localization of atoms in one dimension to a spatial width, smaller by a factor of 1000 than the incident wavelength.

I. INTRODUCTION

In recent years atom localization has received considerable attention owing to its potential applications in the areas of quantum information science [1], laser cooling and trapping of neutral atoms [2], Bose-Einstein condensation [3], atom nanolithography [4, 5] and microscopy [6]. Initial proposals for localizing atoms were confined to one dimension only [7-16]. In this regard, many methods based on quantum interference [7], electromagnetically induced transparency (EIT) [1,8], coherent population trapping (CPT)[9,10] and stimulated Raman adiabatic passage (STIRAP) [11] etc. are suggested. Recently, schemes have been proposed to localize atoms even in two and three dimensions.Ivanov and Rozhdestvensky have proposed a scheme for two dimensional (2D) atom localization by laser fields in a four-level tripod system [17]. Very recently, Ivanov et al. have proposed a scheme for three dimensional atom localization in the same system [18]. It should be noted that other schemes for 2D localization have also been proposed [19-22]. On the other hand, Qi et al.[23] have proposed a scheme for 3D atom localization based on EIT. Clearly these new developments are going to open up numerous applications in many areas of science. For example, three-dimensional atomic localization is speculated to be useful in high-precision position dependent chemistry [18].Few experimental developments have already been made towards localizing atoms [4,24,25]. Johnson et al. reported localization of metastable atom beams and opened the door for nanolithography at the Heisenberg limit[4]. One notable progress in experimental realization of atom localization in one dimension is made by Yavuz’s group [25]. They have reported, using EIT, experimental realization of subwavelength localization of atoms to a spatial width smaller by a factor of eight to the incident wavelength. It is expected that many more experiments will be carried out soon and we need practical schemes to achieve the goal of atom localization in all the dimensions. In this work, we propose schemes to localize atoms both in two and three dimensions, via level populations, by using laser fields in an M-type five level atomic system. Our schemes are not only useful to localize atoms in two and three dimensions, but also enable one to achieve subwavelength localization of atoms in one dimension to a spatial width, smaller by a factor of 1000 than the incident wavelength. This article is organized as follows. In Sec. II we introduce our model and give the basic equations. Sec. III contains our simulated results and discussions followed by conclusions in Sec. IV.

II. MODEL

A standing wave pattern is created by four laser fields $\vec{E}_1$, $\vec{E}_2$, $\vec{E}_3$ and $\vec{E}_4$. The electric field here is given by[26]

$$\vec{E}(\vec{r}, t) = \vec{E}_1^+(x)e^{-i\omega_1 t} + \vec{E}_2^+(y)e^{-i\omega_2 t} + \vec{E}_3^+(y)e^{-i\omega_3 t} + \vec{E}_4^+(x)e^{-i\omega_4 t} + \text{cc.}$$

The electric fields are such that $\vec{E}_1$ is a $\sigma^-$ polarized wave with frequency $\omega_1$, $\vec{E}_2$ is a $\sigma^+$ polarized wave with frequency $\omega_2$, $\vec{E}_3$ is a $\pi$ polarized wave with frequency $\omega_3$, $\vec{E}_4$ is again a $\sigma^-$ polarized wave with frequency $\omega_4$.

As shown in the Fig.(1), the upper levels are denoted by |4⟩ and |5⟩ and three grounds levels by |1⟩, |2⟩ and |3⟩. The transitions |1⟩ ↔ |5⟩, |2⟩ ↔ |5⟩, |2⟩ ↔ |4⟩,
where $G_j = |\vec{d}_j, \vec{e}_j/\hbar|$ is coefficient of Rabi frequency and $\vec{d}_j$ is dipole moment corresponding to $j^{th}$ transition[27].

It is assumed that the center of mass of the atom is at rest, the interaction only affects the internal states and hence the Raman-Nath approximation[28] is valid. So in the interaction picture and rotating wave approximation (RWA)[29], the Liouville equation:

$$i\hbar \dot{\rho} = [\mathcal{H}, \rho] - i\gamma \rho$$

becomes

$$i\dot{\rho}_{11} = g_1(\rho_{15} - \rho_{31}) + i\gamma_1 \rho_{55}, \quad (4a)$$

$$i\dot{\rho}_{22} = g_3(\rho_{24} - \rho_{42}) + g_2(\rho_{25} - \rho_{52}) + i\gamma_{54} \rho_{55} + i\gamma_{42} \rho_{44}, \quad (4b)$$

$$i\dot{\rho}_{33} = g_4(\rho_{44} - \rho_{43}) + i\gamma_3 \rho_{44}, \quad (4c)$$

$$i\dot{\rho}_{44} = g_3(\rho_{42} - \rho_{24}) + g_4(\rho_{43} - \rho_{34}) - i\gamma_4 \rho_{44}, \quad (4d)$$

$$i\dot{\rho}_{55} = g_1(\rho_{51} - \rho_{15}) + g_2(\rho_{52} - \rho_{25}) - i\gamma_5 \rho_{55}, \quad (4e)$$

$$i\dot{\rho}_{12} = \rho_{12}(\Delta_{12} - i\Gamma_{23}) + g_3 \rho_{14} + g_3 \rho_{15} - g_1 \rho_{52}, \quad (4f)$$

$$i\dot{\rho}_{13} = g_4 \rho_{14} - g_1 \rho_{53} + (\Delta_{1234} - i\Gamma_{13}) \rho_{13}, \quad (4g)$$

$$i\dot{\rho}_{14} = -g_1 \rho_{54} + g_3 \rho_{12} + g_4 \rho_{13} + (\Delta_{123} - i\Gamma_{14}) \rho_{14}, \quad (4h)$$

$$i\dot{\rho}_{15} = g_1(\rho_{11} - \rho_{55}) + g_2 \rho_{12} + (\Delta_1 - i\Gamma_{15}) \rho_{15}, \quad (4i)$$

$$i\dot{\rho}_{23} = g_4 \rho_{24} - g_3 \rho_{43} - g_2 \rho_{53} + (\Delta_{43} - i\Gamma_{23}) \rho_{23}, \quad (4j)$$

$$i\dot{\rho}_{24} = g_3(\rho_{22} - \rho_{44}) + g_4 \rho_{23} - g_2 \rho_{54} + (\Delta_4 - i\Gamma_{24}) \rho_{24}, \quad (4k)$$

$$i\dot{\rho}_{25} = g_2(\rho_{22} - \rho_{55}) + g_1 \rho_{21} - g_3 \rho_{45} + (\Delta_2 - i\Gamma_{25}) \rho_{25}, \quad (4l)$$

$$i\dot{\rho}_{34} = g_4(\rho_{33} - \rho_{44}) + g_3 \rho_{32} + (\Delta_4 - i\Gamma_{34}) \rho_{34}, \quad (4m)$$

$$i\dot{\rho}_{35} = g_1 \rho_{31} + g_2 \rho_{32} - g_4 \rho_{45} + (\Delta_{234} - i\Gamma_{35}) \rho_{35}, \quad (4n)$$

$$i\dot{\rho}_{45} = g_1 \rho_{41} + g_2 \rho_{42} - g_3 \rho_{25} - g_4 \rho_{45} + (\Delta_{23} - i\Gamma_{45}) \rho_{45}, \quad (4o)$$

Since $\rho$ is a density matrix, we have $\rho_{ij} = \rho_{ji}^*$. $\sum \rho_{ii} = 1$. For the $\Delta$s we have used following notation: $\Delta_{ijkl} = \Delta_1 - \Delta_2 - \Delta_3 - \Delta_4$, $\Delta_{xyz} = \Delta_x - \Delta_y + \Delta_z$ and $\Delta_{mn} = \Delta_m - \Delta_n$. $\gamma_1$ and $\gamma_3$ are the decay rates which correspond to the relaxation ‘into’ ground states $|1\rangle$ and $|3\rangle$ respectively, $\gamma_{52}$ is the decay rate that corresponds to the relaxation ‘from’ excited state $|5\rangle$ ‘into’ ground state $|2\rangle$, similarly other $\gamma$s are defined. $\Gamma_{34}$ corresponds to the decay rate between states $|3\rangle$ and $|4\rangle$, similarly other $\Gamma$s are defined. The decay rate between states $|1\rangle$ and $|2\rangle$, $\Gamma_{12}$, can safely be neglected because there is no field driving the transition and it is usually much smaller than the decay rates corresponding to driven transitions. Similarly other decay rates can be neglected. In effect we have

$$\Gamma_{12} \approx \Gamma_{13} \approx \Gamma_{14} \approx \Gamma_{23} \approx \Gamma_{35} \approx \Gamma_{45} \approx 0. \quad (5)$$

### III. RESULTS AND DISCUSSION

#### 2D Localization

As addressed by Ivanov[17], we take the two probe fields $g_2$ and $g_3$ to be weak so that in the long time we have $g_2$ and $g_3$ to be much smaller than $g_1$, $g_4$, $\Delta_2$, $\Delta_3$, $\Gamma_{24}$, $\Gamma_{25}$ and other quantities of interest.

When the probe fields are weak, $g_2, g_3 \to 0$, we have most of the population in the state $|2\rangle$, hence $\rho_{22} \approx 1$ during the interaction of atom with the field. In the long time limit, we have $\rho_{ij} = 0$. From these considerations we can obtain $\rho_{44}$, $\rho_{55}$ from Eq.(4).

Considering Eq.(4f) in the long time limit and with weak probe fields $g_2$ and $g_3$, we get

$$\rho_{21} \approx g_1 \frac{\Delta_{12}}{\Delta_{12}} \rho_{25}. \quad (6)$$

Now we consider Eq.(4o). Near the nodes of $g_2$, we get

$$\rho_{45} \approx g_4 \frac{\Delta_{34}}{\Delta_{34}} \rho_{25}. \quad (7)$$

Putting Eq.(6) and Eq.(7) in Eq.(4l) along with $(\rho_{22} - \rho_{55}) \approx 1$, we have

$$\rho_{25} \approx \frac{g_2}{i\Gamma_{25} - (\frac{g_1^2}{\Delta_{12}} - \frac{g_4^2}{\Delta_{34}} + \Delta_2)} . \quad (8)$$
From Eq.(4a) and Eq.(4e), we get
\[
\rho_{55} \approx \frac{i g_2 (\rho_{25} - \rho_{52})}{\gamma_5 - \gamma_1}.
\]  
(9)

From Eq.(8) and Eq.(9), we get
\[
\rho_{55} \approx \frac{2 g_2^2 \Gamma_{25}}{(\gamma_5 - \gamma_1) (\Gamma_{25}^2 + \frac{g_7^2}{\Delta_{12}^2} - \frac{g_7^2}{\Delta_{23}^2} + \Delta_2^2)}.
\]  
(10)

This expression gives the upper level population in state \( |5\rangle \) in terms of Rabi frequencies \( g_1, g_2, g_3 \). It is valid near nodes of \( g_2 \) and away from nodes of \( g_3 \).

To obtain population in state \( |4\rangle \), we consider Eq.(4j) in the long time limit and for weak \( g_2 \) and \( g_3 \). We obtain
\[
\rho_{23} \approx \frac{g_1}{\Delta_{34}} \rho_{24}.
\]  
(11)

Considering Eq.(4o) again. Near nodes of \( g_3 \), we obtain
\[
\rho_{54} \approx -\frac{g_2}{\Delta_{23}} \rho_{24}.
\]  
(12)

Putting \( (\rho_{22} - \rho_{44} \approx 1) \) in Eq.(4k) and from Eq.(11) and Eq.(12), we get
\[
\rho_{24} \approx \frac{i g_3}{\Gamma_{24} - \frac{g_7^2}{\Delta_{34}^2} + \frac{g_7^2}{\Delta_{23}^2} + \Delta_3}.
\]  
(13)

From Eq.(4c) and Eq.(4d), we get
\[
\rho_{44} \approx \frac{i g_3 (\rho_{24} - \rho_{42})}{\gamma_4 - \gamma_3}.
\]  
(14)

From Eq.(13) and Eq.(14), we have
\[
\rho_{44} \approx \frac{2 g_2^2 \Gamma_{24}}{(\gamma_4 - \gamma_3) (\Gamma_{24}^2 + \frac{g_7^2}{\Delta_{23}^2} + \Delta_3^2)}.
\]  
(15)

This expression gives the upper level population in state \( |4\rangle \) in terms of Rabi frequencies \( g_2, g_3 \) and \( g_4 \). It is valid near the nodes of \( g_1 \) and away from nodes of \( g_2 \).

Now we consider the case where all of the four fields are standing wave fields. We take \( g_1, g_2, g_3, g_4 \) according to Eq.(2). For simplicity, consider \( \gamma_5 - \gamma_1 = \gamma_4 - \gamma_3 = \gamma_0, \Delta_{12} = \Delta_{34} = \Delta_0, g_1 = g_4 = g_2, g_2 = g_3 = g_9 \). We get
\[
\rho_{44} \approx \frac{2 g_3^2 \Gamma_{24}}{\gamma_0 (\Gamma_{24}^2 + \frac{g_7^2}{\Delta_{23}^2} + \Delta_3^2)},
\]  
(16)

\[
\rho_{55} \approx \frac{2 g_2^2 \Gamma_{25}}{\gamma_0 (\Gamma_{25}^2 + \frac{g_7^2}{\Delta_{23}^2} + \Delta_2^2)}.
\]

The population obtained from these expressions is shown in Fig.(2). For plotting purpose we take \( \Gamma_{24} = 1.6 \gamma_0, \Gamma_{25} = 1.4 \gamma_0, g_2 = 4 \gamma_0 \sin(k_2 y), g_3 = 4 \gamma_0 \sin(k_3 y), g_1 = 6 \gamma_0 \sin(k_1 x), g_4 = 6 \gamma_0 \sin(k_4 x), \Delta = -6 \gamma, \Delta_{23} = -2 \gamma, \Delta_2 = 15 \gamma \) and \( \Delta_3 = 17 \gamma \).

FIG. 2. (Color online) Population in states \( |4\rangle \) and \( |5\rangle \) as a function of \((x,y)\).

(363x145 to 516x256)

(a) Population in level \(|4\rangle\) for \( g_2 \) as a cosine wave

(b) Population in level \(|5\rangle\) for \( g_2 \) as a cosine wave

FIG. 3. (Color online) Phase shifted Population in states \(|4\rangle\) and \(|5\rangle\) as a function of \((x,y)\).
When fields are of the form Eq. (2), the localization patterns are according to Fig. (2) where most of the excited state population is localized in a single excited state depending on the sign of the detuning $\Delta_{23}$. Instead if $g_2$ is of the form $g_2 = G_2 \cos(k_2 y)$ while other three fields being of the form Eq. (2), we get a localization pattern where both the excited states are almost equally populated with different localization structures as shown in Fig. (3).

From Eq. (10) and Eq. (15) we see that the state where majority of population is localized depends crucially on the sign of the difference between detunings $\Delta_2$ and $\Delta_3$ i.e. on the sign of $\Delta_{23}$. For a positive value of $\Delta_{23}$, the excited state population localized in state $|4\rangle$ is much higher and is localized very tightly whereas for state $|5\rangle$, the population is low and area of localization is considerably larger. For $-\Delta_{23}$, excited state population is tightly and highly localized in state $|5\rangle$ whereas for state $|4\rangle$ the population is low and localized over a greater area.

One can estimate from the contour plots in Fig. 4 that a greater population could be very tightly localized up to a subwavelength region having an area on the order of $(\frac{\lambda}{300\pi})^2$ in the state $|4\rangle$. On the other hand the population in the state $|5\rangle$ can be localized from a region having area $(\frac{2\lambda}{3\pi})^2$ to a region with any desired area. Both these observations are expressed by Eq. (17) below:

$$A_{\text{max}}^{(4)} \approx \left(\frac{\lambda}{300\pi}\right)^2 \quad \text{and} \quad A_{\text{min}}^{(5)} \approx \left(\frac{\lambda}{3\pi}\right)^2.$$  \hspace{1cm} (17)

where $A_{\text{max}}^{(4)}$ represents the minimum area up to which atomic population can be localized in state $|4\rangle$ while $A_{\text{max}}^{(5)}$ represents the maximum area to which atoms can be localized in the state $|5\rangle$. It is interesting to note that we may achieve subwavelength localization of atoms in 1D to a spatial width which is a factor of 1000 smaller than the wavelength of the laser beam, via level population in the state $|4\rangle$ through judicious manipulation of the proposed scheme.

3D localization

The scheme for localization of atoms in three dimensions is also presented below. The atomic states arrangement is same as Eq. (21) except that here two addition fields are required whose $\hat{k}$ vectors are along $\hat{z}$ and satisfy the selection rule Eq. (22). For the electric field configuration of the form

$$\vec{E}(\vec{r},t) = \vec{E}_1^+ (x)e^{-i\omega_1 t} + \vec{E}_2^+ (y)e^{-i\omega_{21} t} + \vec{E}_{21}^-(y)e^{-i\omega_{21} t} + \vec{E}_{31}^-(y)e^{-i\omega_{31} t} + \vec{E}_{32}^+(y)e^{-i\omega_{32} t} + \vec{E}_4^- (x)e^{-i\omega_4 t} + \text{cc.}$$  \hspace{1cm} (18)

the Rabi frequencies are given by

$$g_1 (x) = G_1 \sin(k_1 x),$$

$$g_2 (y) = G_2 \sin(k_2 y) + iG_{22} \sin(k_{22} z),$$

$$g_3 (y) = G_{31} \sin(k_{31} y) + iG_{32} \sin(k_{32} z),$$

$$g_4 (x) = G_4 \sin(k_4 x).$$

where $G_1$, $G_4$ are of the form Eq. (2). The Rabi frequency $G_{21}$ is given by $G_{21} = |\vec{d}_2, \vec{d}_{21}/\hbar|$. $G_{22}$, $G_{31}$, $G_{32}$ are defined in the same way. Then the solutions of the Liouville equation, Eq. (3), for the upper state population $\rho_{44}$ and $\rho_{55}$ are given by, in the long time limit, with RWA and in interaction picture,

$$\rho_{44} \approx \frac{2|g_2|^2 \Gamma_{25}}{(\gamma_4 - \gamma_3)(\Gamma_{24}^2 + (\frac{g_2^2}{\Delta_{23}} - \frac{|g_3|^2}{\Delta_{23}} + \Delta_3)^2)}.$$  \hspace{1cm} (19)

$$\rho_{55} \approx \frac{2|g_2|^2 \Gamma_{25}}{(\gamma_5 - \gamma_1)(\Gamma_{25}^2 + (\frac{g_2^2}{\Delta_{23}} - \frac{|g_3|^2}{\Delta_{23}} + \Delta_2)^2)}.$$  \hspace{1cm} (20)

where $|.|$ is the magnitude of the quantity. All the notations and approximations are same as that of Sec(II).

A simple plot for 3D localization of the population $\rho_{44}$ is shown in Fig. (5). Here we take $g_3$ to be a constant/ running-wave pulse of the form $g_3 = G_3$, $g_1$ and $g_4$ to be sine waves along $\hat{x}$ and $g_2$ to be along $\hat{y}$ and $\hat{z}$. For plotting, we take $g_3 = 0.3\gamma$, $g_2^2 = 16\gamma^2$ ($\sin^2(k_{21} y) + \sin^2(k_{22} z)$), $g_4 = 6\gamma \sin(k_{4} x)$, $g_1 = 6\gamma \sin(k_{1} x)$, $\Delta_{31} = 9\gamma$, $\Delta_{23} = 4\gamma$, $\Delta_{3} = -12\gamma$, $\Gamma_{24} = 1.5\gamma$ and $\gamma_4 - \gamma_3 = \gamma_0$.
FIG. 5. Plot for 3D localization of the population $\rho_{44}$

Implementation of 2D and 3D localization schemes using Rubidium

The model proposed in Section (II) can be practically implemented using the $D_2$ line of Rubidium atom where two upper levels are from $5^2P_\frac{3}{2}$ and three grounds levels are from $5^2S_\frac{1}{2}$ [30].

\[ |1\rangle = |5^2S_\frac{1}{2}, F = 1, m = +1\rangle, \]
\[ |2\rangle = |5^2S_\frac{1}{2}, F = 1, m = -1\rangle, \]
\[ |3\rangle = |5^2S_\frac{1}{2}, F = 1, m = 0\rangle, \]
\[ |4\rangle = |5^2P_\frac{3}{2}, F' = 1, m = -1\rangle, \]
\[ |5\rangle = |5^2P_\frac{3}{2}, F' = 0, m = 0\rangle. \]  (21)

The polarization of the laser fields driving this transition should be in accordance with the selection rule. If

\[ m_f - m_i = \begin{cases} +1 & \text{then } \sigma^+ \\ 0 & \text{then } \pi \\ -1 & \text{then } \sigma^- \end{cases} \]  (22)

The polarization of electric fields for the implementation of 2D localization scheme is also shown in Fig.(1). The 3D localization scheme can be implemented using Eq.(21) as well with addition of two extra laser fields. The setup is shown in Fig.(6).

IV. CONCLUSIONS

In conclusion, we have shown how to localize atoms in a M-type system in two dimensions as well as three dimensions. It is also pointed out that for 2D localization, the state in which majority of the population resides depends crucially on the sign of detuning $\Delta_{23}$. Same conclusion is valid for three dimensions as can be easily seen from Eq.(20). We also estimated the range of localization numerically, Eq.(17). Further, we estimate that by judicious manipulation of the scheme one could achieve atom localization in 1D to a spatial width smaller by a factor of 1000 to the incident wavelength. The effect of a phase shift of $\pi/2$ in making both the states almost equally populated with different localization structure is also studied. In the end, we also proposed a practical implementation of the two and three dimensional localization schemes using Rubidium atom.

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