Towards a Naturally Small Cosmological Constant from Branes in 6D Supergravity

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ABSTRACT: We investigate the possibility of self-tuning of the effective 4D cosmological constant in 6D supergravity, to see whether it could naturally be of order $1/r^4$ when compactified on two dimensions having Kaluza-Klein masses of order $1/r$. In the models we examine supersymmetry is broken by the presence of non-supersymmetric 3-branes (on one of which we live). If $r$ were sub-millimeter in size, such a cosmological constant could describe the recently-discovered dark energy. A successful self-tuning mechanism would therefore predict a connection between the observed size of the cosmological constant, and potentially observable effects in sub-millimeter tests of gravity and at the Large Hadron Collider. We do find self tuning inasmuch as 3-branes can quite generically remain classically flat regardless of the size of their tensions, due to an automatic cancellation with the curvature and dilaton of the transverse two dimensions. We argue that in some circumstances six-dimensional supersymmetry might help suppress quantum corrections to this cancellation down to the bulk supersymmetry-breaking scale, which is of order $1/r$. We finally examine an explicit realization of the mechanism, in which 3-branes are inserted into an anomaly-free version of Salam-Sezgin gauged 6D supergravity compactified on a 2-sphere with nonzero magnetic flux. This realization is only partially successful due to a topological constraint which relates bulk couplings to the brane tension, although we give arguments why these relations may be stable against quantum corrections.

KEYWORDS: supersymmetry breaking, string moduli, cosmological constant.
1. Introduction

At present there is no understanding of the small size of the cosmological constant which does not resort to an enormous fine-tuning. (For a review of some of the main attempts, together with a no-go theorem, see [1].) The discomfort of the theoretical community with this fact was recently sharpened by the discovery of dark energy [2], which can be interpreted as being due to a very small, but nonzero, vacuum energy which is of order $\rho = v^4$, with

$$v \sim 3 \times 10^{-3} \text{ eV},$$

(1.1)
in units for which $\hbar = c = 1$.

Any fundamental solution to the cosmological constant problem must answer two questions.
1. Why is the vacuum energy so small at the microscopic scales, \( M > M_w \sim 100 \text{ GeV} \), at which the fundamental theory is couched?

2. Why does it remain small as all the scales between \( M \) and \( v \) are integrated out?

Problem 2 has proven the thornier of these two, because in the absence of a symmetry which forbids a cosmological constant, integrating out physics at scale \( M \) leads to a contribution to the vacuum energy which is of order \( M^4 \). Thus, integrating out ordinary physics which we think we understand — for instance, the electron — already leads to too large a contribution to \( \rho \). Symmetries can help somewhat, but not completely. In four dimensions there are two symmetries which, if unbroken, can forbid a vacuum energy: scale invariance and some forms of supersymmetry. However, both of these symmetries are known to be broken at scales at least as large as \( M_w \), and so do not seem to be able to explain why \( v \ll M_w \).

1.1 The Cosmological Constant Problem in 6D Supergravity

In this paper we examine how six-dimensional supergravity with two sub-millimeter extra dimensions can help with both problems 1 and 2. Higher-dimensional supergravity theories are natural to consider in this context because in higher dimensions supersymmetry can prohibit a cosmological constant, and so they provide a natural solution to problem 1 above. However higher dimensions cannot help below the compactification scale, \( M_c \sim 1/r \) where \( r \) is the largest radius of an extra dimension, since below this scale the effective theory is four-dimensional, which is no help to the extent that \( M_c \gg v \).

These considerations select out six-dimensional theories for special scrutiny, because only for these theories can the compactification scale satisfy \( M_c \sim v \) — corresponding to \( r \sim 0.1 \text{ mm} \) — without running into immediate conflict with experiment. The extra dimensions can be this large provided that all experimentally-known interactions besides gravity are confined to a \((3+1)\)-dimensional surface (3-brane) within the six dimensions, since in this case the extra dimensions are only detectable through tests of the gravitational force on distances of order \( r \). The upper limit on \( r \) provided by the absence of deviations from Newton’s Law in current experiments \([3]\) is \( r \lesssim 0.1 \text{ mm} \).

There are other well-known virtues to six-dimensional theories having \( r \) this large. Any six-dimensional physics which would stabilize \( r \) at values of order 0.1 mm would also explain the enormous hierarchy between \( M_w \) and the Planck mass \( M_p \) \([4]\). It would do so provided only that the six-dimensional Newton’s constant, \( M_6 = (8\pi G_6)^{-1/4} \), is set by the same scale as the brane physics, \( M_6 \sim M_w \) because of the prediction \( M_p \sim M_6^2 r \). A particularly appealing feature of this proposal is its prediction of detectable gravitational phenomena at upcoming accelerator energies \([10, 8, 1, 12]\).
Astrophysical constraints can also constrain sub-millimeter scale six-dimensional models, and in some circumstances can require \( r \lesssim 10^{-5} \text{ mm} \) \[13, 8\]. These bounds are somewhat more model dependent, however, and we take the point of view that they can be temporarily put aside since they are easier to circumvent through detailed model-building than is the much harder problem of the cosmological constant.

From the perspective of the cosmological constant, the decisive difference between sub-millimeter-scale 6D supergravity and other higher-dimensional proposals is that it is higher dimensional physics which applies right down to the low scale \( v \) at which the problem must be solved, as is necessary if the proposal is to help with Problem 2. This is important because in these models the vacuum energy generated by integrating out ordinary particles is not a cosmological constant in the six-dimensional sense, but is instead localized at the position of the brane on which these particles live, contributing to the relevant brane tension, \( T \). We should therefore expect these tensions to be at least of order \( T \sim M_w^4 \).

In this context Problem 1 above splits into two parts. 1A: Why doesn’t this large a brane tension unacceptably curve the space seen by a brane observer? 1B: Why is there also not an unacceptably large cosmological constant in the two-dimensional ‘bulk’ between the branes? Interestingly, 6D supergravity can help address both of these. 1B can be solved because in six dimensions supersymmetry itself can forbid a bulk cosmological constant. It can help with 1A as well to the extent that cancellations occur between the brane tensions and the contribution of these tensions make to the curvature in the extra dimensions, as has also been remarked in refs. \[4, 5, 6, 7\].

1.2 Self-Tuning and the Cosmological Constant

The cancellations we find between the brane tensions and the bulk fields fall into the category of self-tuning solutions to the cosmological constant problem, which rely on the existence of a field — usually a dilaton of some sort when seen from the 4D perspective — whose equation of motion ensures the vanishing of the effective 4D cosmological constant. Ref. \[1\] argues quite generally that past examples of these mechanisms all fail, either because the would-be dilaton does not enforce the vanishing of the vacuum energy, or because appropriate solutions to their equations cannot be found. Since the no-go theorem of ref. \[1\] is explicitly four-dimensional, one might hope it might be circumvented within a higher-dimensional context.

Self-tuning solutions have been re-examined recently within the context of 5D gravity models \[14\], motivated by the existence of solutions having flat 3-branes which at face value do not require adjustment of the brane tensions. They ultimately do not appear to circumvent the no-go theorem because they necessarily involve singularities, which either exclude them as solutions or can be interpreted as implying the existence of new, negative-tension branes whose tensions indeed were finely tuned to achieve vanishing 4D vacuum energy \[15, 16\].
We have several motivations for investigating whether a similar mechanism can work in six dimensions in this paper, despite this 5D experience. One reason is the striking coincidence of scales mentioned above, between the dark energy density, the weak scale and the 4D Planck mass. A second motivation is the possibility of having an extra-dimensional theory right down to the relevant cosmological constant scale, with the possibility that symmetries like higher-dimensional general covariance might help. Furthermore, known compactifications of 6D supergravity have flat directions for various bulk fields, and the bulk supersymmetry-breaking scale for lifting these flat directions can plausibly be small enough since it is of order $1/r$. (Flat directions provide one of the few real loopholes to the no-go arguments of ref. [1].)

Finally, 6 dimensions are similar to 5 dimensions in that it is possible to solve the back-reaction problem in a compact space to determine explicitly the bulk fields which 3-branes produce. But because 3-branes in six dimensions have co-dimension two rather than one the kinds of singularities in the fields which they generate is qualitatively different in 6D than in 5D. This can change the nature of the self-tuning solutions since their existence partly turns on the kinds of singularities which are generated [15, 16].

Our investigation takes place within the Salam-Sezgin model of gauged 6D supergravity, for which we are able to identify the couplings which 3-branes must have in order for self-tuning to occur. We are able to do so without needing to specify an explicit solution to the 6D supergravity equations.

Finally we also obtain an explicit solution to the Salam-Sezgin field equations consisting of a compactification on a two-sphere in the presence of magnetic fluxes. We introduce 3-branes into this setting and find that the flat 4D solution with constant dilaton field survives the introduction of the branes. However a topological argument imposes a constraint which relates the different gauge couplings and the 3-brane tensions. At present this represents an obstruction to regarding this solution as a realization of the more general self-tuning arguments.

### 1.3 Quantum Contributions

Unfortunately, a successful classical self-tuning solution leaves as unsolved Problem 2: the question of integrating out the scales from $M_w$ down to $v$ at the quantum level. We argue here that because self-tuning allows the effective cosmological constant to adjust to a general brane tension, quantum corrections on the brane are not likely to be troublesome. The potentially dangerous degrees of freedom to integrate out are those in the bulk, and here also 6D supergravity may help.

6D supergravity may help because six-dimensional supersymmetry relates the bulk modes to the graviton. Their couplings are therefore naturally of gravitational strength, and so if supersymmetry breaks on the branes at scale $M_w$, their supersymmetric mass splittings are naturally of order $\Delta m \sim M^2_w/M_p \sim v$ [8], with $M_p = (8\pi G_4)^{-1/2} \sim 10^{18}$ GeV denoting the 4D Planck mass, where $G_4$ is Newton’s
constant in four dimensions. To the extent that their contributions to the effective 4D cosmological constant were of order \((\Delta m)^4\), they would therefore be of precisely the required size to describe the observed Dark Energy. Although we do not completely address all of the bulk corrections here, we do argue that self-tuning requires that they must be smaller than \(O(M_w^2/r^2)\), and so can be much smaller than \(O(M_w^4)\). We also give a qualitative argument that the contributions of order \(M_w^2/r^2\) may also vanish for supersymmetric solutions, but defer a definitive treatment of these corrections to future work.

1.4 Outline

The rest of our discussion is organized as follows. The next section contains a brief review of the six-dimensional supergravity we shall use, and its supersymmetric compactification on a sphere to four dimensions. This section also discusses how the equations of motion change once branes are coupled to the bulk supergravity fields. Section 3 describes the various contributions to the effective four-dimensional vacuum energy. It first shows how the various brane tensions generally cancel the classical contributions to the bulk curvature, and also shows how supersymmetry ensures the cancellation of all other bulk contributions at the classical level. These cancellations relate the effective 4D vacuum energy to the derivatives of the dilaton at the positions of the various branes. We then estimate the size of quantum corrections to the 4D cosmological constant, and argue that these give contributions which are naturally of order \(1/r^4\). In Section 4 we construct the simplest kind of supergravity solution including branes, which corresponds to two branes located at the north and south poles of an internal two-sphere. We show that the solution has constant dilaton, and so vanishing classical effective 4D vacuum energy, if the branes do not directly couple to the dilaton. In Section 5 we finish with some conclusions as well as a discussion of some of the remaining open issues.

2. Six Dimensional Supergravity

Six-dimensional supergravities come in several flavors, and we focus here on a generalization of the Salam-Sezgin version of the gauged six-dimensional supersymmetric Einstein-Maxwell system \([17, 18, 19]\). The generalization we consider involves the addition of various matter multiplets in order to cancel the Salam-Sezgin model’s six-dimensional anomalies. We choose this model because we wish to construct an explicit brane configuration which illustrates our general mechanism, and for reasons to be made clear this 6D supergravity seems the best prospect for doing so in a simple way. Our treatment follows the recent discussion of this supergravity given in ref. \([20]\).
2.1 The Model

The field content of the model consists of a supergravity-tensor multiplet — a metric \((g_{MN})\), antisymmetric Kalb-Ramond field \((B_{MN} — \text{with field strength } G_{MNP})\), dilaton \((\varphi)\), gravitino \((\psi_i^M)\) and dilatino \((\chi_i)\) — coupled to a combination of gauge multiplets — containing gauge potentials \((A_M)\) and gauginos \((\lambda^i)\) — and \(n_H\) hyper-multiplets — with scalars \(\Phi^a\) and fermions \(\Psi^\hat{a}\). Here \(i = 1, 2\) is an \(Sp(1)\) index, \(\hat{a} = 1, \ldots, 2n_H\) and \(a = 1, \ldots, 4n_H\), and the gauge multiplets fall into the adjoint representation of a gauge group, \(G\). In the model we shall follow in detail the \(Sp(1)\) symmetry is broken explicitly to a \(U(1)\) subgroup, which is gauged.

The fermions are all real Weyl spinors — satisfying \(\Gamma_7 \psi^M = \psi^M\), \(\Gamma_7 \lambda = \lambda\) and \(\Gamma_7 \chi = -\chi\) and \(\Gamma_7 \Psi^\hat{a} = -\Psi^\hat{a}\) — and so the model is anomalous for generic gauge groups and values of \(n_H\) \cite{21}. These anomalies can sometimes be cancelled \emph{via} the Green-Schwarz mechanism \cite{22}, but only for specific gauge groups which satisfy specific conditions, such as \(n_H = \text{dim}(G) + 244 \ [24, 25]\). We need not specify these conditions in detail in what follows, but for the purposes of concreteness we imagine using the model of ref. \cite{24} for which \(G = E_6 \times E_7 \times U(1)\), having gauge couplings \(g_6, g_7\) and \(g_1\). The hyper-multiplet scalars take values in the noncompact quaternionic Kähler manifold \(M = Sp(456,1)/(Sp(456) \times Sp(1))\).

The bosonic part of the classical 6D supergravity action is:\footnote{We follow Weinberg’s metric and curvature conventions \cite{26}.}

\[
e^{-1}_6 L_B = - \frac{1}{2} R - \frac{1}{2} \partial_M \varphi \partial^M \varphi - \frac{1}{2} G_{ab}(\Phi) D_M \Phi^a D^M \Phi^b - \frac{1}{12} e^{-2\varphi} G_{MNP} G^{MNP} - \frac{1}{4} e^{-\varphi} F_{MN}^\alpha F^{MN}_\alpha - e^{\varphi} v(\Phi),
\]

where we choose units for which the 6D Planck mass is unity: \(\kappa^2_6 = 8\pi G_6 = 1\). Here the index \(\alpha = 1, \ldots, \text{dim}(G)\) runs over the gauge-group generators, \(G_{ab}(\Phi)\) is the metric on \(M\) and \(D_m\) are gauge and Kähler covariant derivatives whose details are not important for our purposes. The dependence on \(\varphi\) of the scalar potential, \(V = e^{\varphi} v(\Phi)\), is made explicit, and when \(\Phi^a = 0\) the factor \(v(\Phi)\) satisfies \(v(0) = 2 g_1^2\).

As above \(g_1\) here denotes the \(U(1)\) gauge coupling. As usual \(e_6 = |\det e_M^A| = \sqrt{-\det g_{MN}}\). The bosonic part of the basic Salam-Sezgin model is obtained from the above by setting all gauge fields to zero except for the explicit \(U(1)\) group factor, and by setting \(\Phi^a = 0\).

2.2 Compactification on a Sphere

We next briefly describe the compactification of this model to four dimensions on an internal two-sphere with magnetic monopole background, along the lines of the Randjbar-Daemi, Salam and Strathdee model \cite{23} and its supersymmetric extensions, ref. \cite{19, 24}. The equations of motion for the bosonic fields which follow from the
action, eq. (2.1), are:

\[ \Box \varphi + \frac{1}{6} e^{-2\varphi} G_{MNP} G^{MNP} + \frac{1}{4} e^{-\varphi} F^\alpha_{MN} F^\alpha_{MN} - e^\varphi v(\Phi) = 0 \]

\[ D_M \left( e^{-2\varphi} G^{MNP} \right) = 0 \]  \hspace{1cm} (2.2)

\[ D_M \left( e^{-\varphi} F^\alpha_{MN} \right) + e^{-2\varphi} G_{MNP} F^\alpha_{MP} = 0 \]

\[ D_M D^M \Phi - G^{ab}(\Phi) v_b(\Phi) e^{\varphi} = 0 \]

\[ R_{MN} + \partial_M \varphi \partial_N \varphi + G_{ab}(\Phi) D_M \Phi^a D_N \Phi^b + \frac{1}{2} e^{-2\varphi} G_{MPQ} G^P_{NQ} + e^{-\varphi} F^\alpha_{MP} F^\alpha_{NP} + \frac{1}{2} \left( \Box \varphi \right) g_{MN} = 0, \]

where \( v_b = \partial v / \partial \Phi^b \).

We are interested in a compactified solution to these equations which distinguishes four of the dimensions – \( x^\mu, \mu = 0, 1, 2, 3 \) – from the other two – \( y^m, m = 4, 5 \). A convenient compactification proceeds by choosing \( \varphi = \text{constant} \), \( g_{MN} = \left( \begin{array}{cc} g_{\mu\nu}(x) & 0 \\ 0 & g_{mn}(y) \end{array} \right) \) and \( T^a F^\alpha_{MN} = Q \left( \begin{array}{cc} 0 & 0 \\ 0 & F^\alpha_{MN} \end{array} \right), \) (2.3)

where \( g_{\mu\nu} \) is a maximally-symmetric Lorentzian metric (\( i.e. \) de Sitter, anti-de Sitter or flat space) and \( g_{mn} \) is the standard metric on the two-sphere, \( S_2: g_{mn} dy^m dy^n = r^2 (d\theta^2 + \sin^2 \theta d\phi^2) \). The only nonzero Maxwell field corresponds to a \( U(1) \), whose generator \( Q \) we take to lie anywhere amongst the generators \( T^a \) of the gauge group. Maximal symmetry on the 2-sphere requires we choose the Maxwell field to be \( F_{mn} = f \epsilon_{mn}(y) \) where \( \epsilon_{mn} \) is the sphere’s volume form.\(^2\) All other fields vanish.

As is easily verified, the above ansatz solves the field equations provided that \( r, f \) and \( \varphi \) are constants and the following three conditions are satisfied: \( R_{\mu\nu} = 0, F_{mn} F^{mn} = 8 g_1^2 e^{2\varphi} \) and \( R_{mn} = -e^{-\varphi} F_{mp} F^{p}_{n} = -f^2 e^{-\varphi} g_{mn} \).\(^3\) These imply the following conditions:

1. Four dimensional spacetime is flat;

2. The magnetic flux of the electromagnetic field through the sphere is given – with an appropriate normalization for \( Q \) – by \( f = n/(2 g_1 r^2) \), where \( r \) is the radius of the sphere, \( g_1 \) is the \( U(1) \) gauge coupling which appears in the scalar potential when we use \( v(0) = 2 g_1^2 \), and the monopole number is \( n = \pm 1 \);

3. The sphere’s radius is related to \( \varphi \) by \( e^\varphi r^2 = 1/(4 g_1^2) \). Otherwise \( \varphi \) and \( r \) are unconstrained.

\(^2\)In our conventions \( \epsilon_{\theta\phi} = e_2 = \sqrt{\det g_{mn}} \).

\(^3\)Notice that the symmetries of the 2D sphere actually imply that \( \varphi \) should be constant, and select flat Minkowski space over dS or AdS as the only maximally symmetric solution of the field equations, given this ansatz.
What is noteworthy here is that the four dimensions are flat even though the internal two dimensions are curved. In detail this arises because of a cancellation between the contributions of the two-dimensional curvature, $\mathcal{R}$, the dilaton potential and the Maxwell action. This cancellation is not fine-tuned, since it follows as an automatic consequence of the field equations given only the choice of a discrete variable: the magnetic flux quantum, $n = \pm 1$. It is important to notice that the requirement $n = \pm 1$ is required both in Einstein’s equation to obtain flat 4D space, and in the dilaton equation to obtain a constant dilaton. By contrast, in a non-supersymmetric theory like the pure Einstein-Maxwell system the absence of the dilaton equation allows one to always have flat 4D spacetime for any monopole number by appropriately tuning the 6D cosmological constant.

It is instructive to ask what happens in supergravity if another choice for $n$ were made. In this case the cancellation in the first of eqs. (2.2) no longer goes through, with the implication that $\Box \varphi \neq 0$. It follows that with this choice $\varphi$ cannot remain constant and so the theory spontaneously breaks the $SO(3)$ invariance of the two-sphere in addition to curving the noncompact four dimensions.$^4$

The above compactification reduces to that of the basic Salam-Sezgin model if $Q$ is taken to be the generator of the explicit $U(1)$ gauge factor. In this case the compactification also preserves $N = 1$ supersymmetry in four dimensions [19]. Consequently in this case the flatness of the noncompact four dimensions for the choice $n = \pm 1$ is stable against perturbative quantum corrections, because it is protected by the perturbative non-renormalization theorems of the unbroken four-dimensional $N = 1$ supersymmetry. (See ref. [20] for a more detailed discussion of the resulting 4D supergravity which results.)

2.3 Including Branes: Field Equations

We next describe the couplings of this supergravity to branes that we use in later sections. We take the coupling of a 3-brane to the bulk fields discussed above to be given by the brane action

$$S_b = -T \int d^4 x \ e^{\lambda \varphi} (- \det \gamma_{\mu\nu})^{1/2},$$

(2.4)

where $\gamma_{\mu\nu} = g_{MN} \partial_\mu x^M \partial_\nu x^N$ is the induced metric on the brane. For simplicity we do not consider any direct couplings to the bulk Maxwell field, such as is possible

$$S_b' = -q \int \ast F,$$

(2.5)

where $q$ is a coupling constant and $\ast F$ denotes the (pull back of the) Hodge dual (4-form) of the Maxwell field strength. We also do not write a brane coupling to

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$^4$Note added: These solutions have since been found, in ref. [48], with the required 4D curvature coming from a warping of the extra dimensions.
of Dirac-Born-Infeld form, although this last coupling could be included without
substantially changing our later conclusions.

The constant $\lambda$ controls the brane-dilaton coupling and depends on the kind of
brane under consideration. For instance consider a D$p$-brane, for which the coupling
to the ten-dimensional dilaton is known to have the following form in the string
frame:

$$S_{\text{SF}} = -T \int d^n x \ e^{-\varphi} (- \det \hat{\gamma}_{\mu \nu})^{1/2} ,$$

where $n = p + 1$ is the dimension of the brane world-sheet. This leads in the Einstein
frame, $\hat{\gamma}_{\mu \nu} = e^{\phi/2} \gamma_{\mu \nu}$, to the result $\lambda = \frac{1}{4} n - 1$. We see that $\lambda = 0$ for a 3-brane
while $\lambda = \frac{1}{2}$ for a 5-brane. Our interest is in 3-branes, and we shall choose $\lambda = 0$ in
what follows.

The choice $\lambda = 0$ is not quite so innocent as it appears, however, since the ten-
dimensional dilaton need not be the dilaton which appears in a lower-dimension
compactification. More typically the lower-dimensional dilaton is a combina-
tion of the 10D dilaton with various radions which arise during the compactif-
cation. Since it has not yet been possible to obtain the Salam-Sezgin 6D supergravity,
or its anomaly free extensions, from a ten-dimensional theory, it is not yet possible to
precisely identify the value of $\lambda$ which should be chosen for a particular kind of brane.
The choice $\lambda = 0$ is nonetheless very convenient since it implies the branes do not
source the dilaton, which proves important for our later arguments.

Imagine now that a collection of plane parallel 3-branes are placed at various
positions $y_i^a$ in the internal 2-sphere. Here $i = 1, \ldots, N$ labels the branes, whose
tensions we denote by $T_i$. We use a gauge for which $\partial_\mu x^\nu_i = \delta^\nu_\mu$. Adding the brane
actions, (2.4), to the bulk action, (2.1), adds delta-function sources to the right-
hand-side of the Einstein equation (and the dilaton equation if $\lambda \neq 0$), giving

$$e_6 \left[ \Box \varphi + \frac{1}{6} e^{-2\varphi} G^2 + \frac{1}{4} e^{-\varphi} F^2 - v(\Phi) e^\varphi \right] = \lambda e^{\lambda \varphi} e_4 \sum_i T_i \delta^2(y - y_i) ,$$

and

$$e_6 \left[ R_{MN} + \partial_M \varphi \partial_N \varphi + G_{ab}(\Phi) D_M \Phi^a D_N \Phi^b + \frac{1}{2} e^{-2\varphi} G_{MPQ} G_{NPQ}^{} \right.\left. + e^{-\varphi} F^\alpha_{MP} F^\alpha_{NP} - \left( \frac{1}{12} e^{-2\varphi} G^2 + \frac{1}{8} e^{-\varphi} F^2 - \frac{1}{2} v(\Phi) e^\varphi \right) g_{MN} \right]$$

$$= e^{\lambda \varphi} e_4 \left( g_{\mu \nu} \delta_M^\mu \delta_N^\nu - g_{MN} \right) \sum_i T_i \delta^2(y - y_i) .$$

In these expressions $F^2 = F^\alpha_{MN} F^\alpha_{MN}$, $G^2 = G_{MN} G^{MN}$, $e_6 = \sqrt{-\det g_{MN}}$ in the
bulk and $e_4 = \sqrt{-\det g_{\mu \nu}}$ on the brane. The other field equations remain unchanged
by the presence of the branes.
2.4 Supersymmetry Breaking

Notice that we do not require the brane actions, eq. (2.4), to be supersymmetric. This is a virtue for any phenomenological brane-world applications, wherein we assume ourselves to be confined on one of them. In this picture, assuming that the original compactification preserves supersymmetry, the supersymmetry-breaking scale for brane-bound particles is effectively of order the brane scale (which in our case must be of order $M_w$ if we are to obtain the correct value for Newton’s constant given two extra dimensions which are sub-millimeter in size). Since superpartners for brane particles are not required having masses smaller than $O(M_w)$ they need not yet have appeared in current accelerator experiments. Supersymmetry breaking on a brane is also not hard to arrange since explicit brane constructions typically do break some or all of a theory’s supersymmetry.

In this picture the effective 6D theory at scales below $M_w$ is unusual in that it consists of a supersymmetric bulk sector coupled to various non-supersymmetric branes. Because the bulk and brane fields interact with one another we expect supersymmetry breaking also to feed down into the bulk, once the back-reaction onto it of the branes is included. In this section we provide simple estimates of some of the aspects of this bulk supersymmetry breaking which are relevant for the cosmological constant problem.

There are several ways to see the order of magnitude of the supersymmetry breaking which is thereby obtained in the bulk. The most direct approach starts from the observation that bulk fields only see that supersymmetry breaks through the influence of the branes, and that the branes only enter into the definition of the bulk mass eigenvalues through the boundary conditions which they impose there. (In this sense this kind of brane breaking can be thought to be a generalization to the sphere of the Scherk-Schwarz [28] mechanism.) Since boundary conditions can only affect the eigenvalues of $\Box$ by amounts of order $1/r^2$, one expects in this way supersymmetric mass-splittings between bosons and fermions which are of order

$$\Delta m \sim \frac{h(T)}{r}. \quad (2.9)$$

We include here an unknown function, $h(T)$, which must vanish for $T \to 0$ since this limit reduces to the supersymmetric spherical compactification of the Salam-Sezgin model. (For our later purposes we need not take $h$ to be terribly small and so we need not carefully keep track of this factor.)

An alternative road to the same conclusion starts from the observation that supersymmetry breaks on the brane with breaking scale $M_w$. In a globally supersymmetric model, the underlying supersymmetries of the theory would still be manifest on the brane because they would imply the existence of one or more massless goldstone fermions, $\xi_i$, localized on each brane. All of the couplings of these goldstone fermions are determined by the condition that the theory realizes supersymmetry
nonlinearly \[32\], with the goldstone fermions transforming as \( \delta \xi_i = \epsilon_i + \ldots \), where \( \epsilon_i \) is the corresponding supersymmetry parameter and the ellipses denote the more complicated terms involving both the parameters and fields. Because of its inhomogeneous character, this transformation law implies that the goldstino appears linearly in the corresponding supercurrent: \( U_\mu^i = a_i \gamma^\mu \xi_i + \ldots \), and so can contribute to its vacuum-to-single-particle matrix elements. Here the \( a_i = O(M^2_w) \) are nonzero constants whose values give the supersymmetry-breaking scale on the brane.

For local supersymmetry the gravitino coupling \( \kappa \bar{\psi}^i U_\mu^i + \ldots \) implies that the goldstone fermions mix with the bulk gravitini at the positions of the brane \[32\], and so can be gauged away in the usual super-Higgs mechanism. By supersymmetry the coupling, \( \kappa = O(M^{-2}_w) \), is of order the six-dimensional gravitational coupling. Because these couplings are localized onto the branes we expect the resulting gravitino modes to generically acquire a singular dependence near the branes, much as does the metric.

Since all of the bulk states are related to the graviton by 6D supersymmetry, we expect the size of their supersymmetry-breaking mass splittings, \( \Delta m \), to be of order of the splitting in the gravitino multiplet. An estimate for this is given by the mass of the lightest gravitino state, which from the above arguments is of order

\[
\Delta m \sim \frac{\kappa a_i}{r} \sim \frac{1}{r} \sim \frac{M^2_w}{M_p},
\]

in agreement with our earlier estimate. Here the factor of \( 1/r \) comes from canonically normalizing the gravitino kinetic term, which is proportional to the extra-dimensional volume, \( e_2 \sim r^2 \). (Of course, precisely the same argument applied to the graviton kinetic terms is what identifies the 4D Planck mass, \( M^2_p \sim M^4_w r^2 \).)

The factor of \( 1/M_p \) in the couplings is generic for the couplings of each individual bulk KK mode to the brane. These couplings must be of gravitational strength because the bulk fields are all related to the 4D metric by supersymmetry and extradimensional general covariance. (As usual, for scattering processes at energies \( E \sim M_w \) the effective strength of the interactions is instead suppressed only by \( 1/M_w \), as is appropriate for six-dimensional fields, because the contributions of a great many KK modes are summed \[32\].)

3. The 4D Vacuum Energy

We now return to the main story and address the size that is expected for the effective 4D vacuum energy as seen by an observer on one of the parallel 3-branes positioned about the extra two dimensions. This is obtained as a cosmological constant term within the effective action obtained by integrating out all of the unobserved bulk fields as well as fields on other branes.
In this section we do this in several steps. First we imagine *exactly* integrating out all of the brane fields, including the electron and all other known elementary particles. In so doing we acquire a net contribution to the brane tension which is of order $T \sim M_4^w$. Provided this process does not also introduce an effective coupling to the dilaton or Maxwell fields, this leads us to an effective brane action of precisely the form used in the earlier sections.

The second step is to integrate out the bulk fields to obtain the effective four-dimensional bulk theory at energies below the compactification scale, $M_c \sim 1/r$, and in so doing we focus only on the effective 4D cosmological constant. The bulk integration can be performed explicitly at the classical level, which we do here to show how the large brane tensions automatically cancel the 2D curvature in the effective 4D vacuum energy. We then estimate the size of the quantum corrections to this classical result.

### 3.1 Classical Self-Tuning

We start by integrating out the bulk massive KK modes, which at the classical level amounts to eliminating them from the action using their classical equations of motion. If, in particular, our interest is in the vacuum energy it also suffices for us to set to zero all massless KK modes which are not 4D scalars or the 4D metric. The scalars can also be chosen to be constants and the 4D metric can be chosen to be flat. We may do so because the only effective interaction which survives in this limit is the vacuum energy. With these choices, Lorentz invariance ensures that the relevant solution for any KK mode which is not a 4D Lorentz scalar is the zero solution, corresponding to the truncation of this mode from the action. In particular this allows us to set all of the fermions in the bulk to zero.

For parallel 3-branes positioned about the internal dimensions we therefore have

$$
\rho_{\text{eff}} = \sum_i T_i + \int_M d^2 y \ e^2 \left[ \frac{1}{2} R_6 + \frac{1}{2} (\partial \varphi)^2 + \frac{1}{2} G_{ab} (D \Phi^a) (D \Phi^b) 
+ \frac{1}{12} e^{-2 \varphi} G^2 + \frac{1}{4} e^{-\varphi} F^2 + v(\Phi) e^\varphi \right] \bigg|_{g_{\mu \nu} = \eta_{\mu \nu}} \tag{3.1}
$$

where $M$ denotes the internal two-dimensional bulk manifold and the subscript ‘$\text{cl}$’ indicates the evaluation of the result at the solution to the classical equations of motion. Eliminating the metric using the Einstein equation, (2.8), allows the 6D curvature scalar to be replaced by

$$
R_6 = - (\partial \varphi)^2 - G_{ab} D \Phi^a D \Phi^b - 3v(\Phi) e^\varphi - \frac{1}{4} e^{-\varphi} F^2 - \frac{2}{e^2} \sum_i T_i \delta^2 (y-y_i) \tag{3.2}
$$

Substituting this into eq. (3.1) we find

$$
\rho_{\text{eff}} = \int_M d^2 y \ e^2 \left[ \frac{1}{12} e^{-2 \varphi} G^2 + \frac{1}{8} e^{-\varphi} F^2 - \frac{1}{2} v(\Phi) e^\varphi \right] \bigg|_{g_{\mu \nu} = \eta_{\mu \nu}} \tag{3.3}
$$
Notice that the Einstein, dilaton-kinetic and brane-tension terms all cancel once the extra-dimensional metric is eliminated. In particular, it is this cancellation — which is special to six dimensions — between the singular part of the two-dimensional Ricci scalar, \( R_2 \), and the brane terms (first remarked in ref. [3]) which protects the low-energy effective cosmological constant from the high-energy, \( O(M_4^4) \), contributions to the effective brane tensions. We note in passing that this cancellation is special to the brane tension, and does not apply for more complicated metric dependence of the brane action (such as to renormalizations of Newton’s constant).

We now apply the same procedure to integrating out the dilaton, which amounts to imposing its equation of motion: \( v(\Phi) e^{\phi} - \frac{1}{4} e^{-\phi} F^2 - \frac{1}{6} e^{-2\phi} G^2 = \Box \phi \). The result, when inserted into eq. (3.3), gives

\[
\rho_{\text{eff}} = -\frac{1}{2} \int_M d^2 y \, e_2 \Box \phi \bigg|_{g_{\mu\nu} = \eta_{\mu\nu}} = -\frac{1}{2} \sum_i \int_{\partial M_i} d\Sigma_m \partial^m \phi \bigg|_{y = y_i},
\]

(3.4)

where the final sum is over all brane positions, which we imagine having excised from \( M \) and replaced with infinitesimal boundary surfaces \( \partial M_i \). Here \( d\Sigma_m = ds \, e_2 \, n_m \), where \( n_m \) is the outward-pointing unit normal and \( ds \) is a parameter along \( \partial M_i \). We see that, provided these surface terms sum to zero, all bulk \textit{and} brane contributions to the effective 4D action cancel once the bulk fields are integrated out. At the weak scale the problem of obtaining a small 4D vacuum energy is equivalent to finding brane configurations for which eq. (3.4) vanishes. In particular \( \rho_{\text{eff}} = 0 \) if \( \phi \) is smooth at the 3-brane positions (and so, in particular, if \( \phi \) is constant).

We have the remarkable result that at the classical level the low-energy observer sees no effective cosmological constant despite there being an enormous tension situated at each brane. There are two components to this cancellation. First, the singular part of the internal 2D metric precisely cancels the brane tensions, as is generic to gravity in 6 dimensions. Second, the supersymmetry of the bulk theory ensures the cancellation of all of the smooth bulk contributions. Neither of these requires the fine-tuning of properties on any of the branes.

Furthermore, this cancellation is quite robust inasmuch as it does not rely on any of the details of the classical solution involved so long as its boundary conditions near the branes ensure the vanishing of eq. (3.4). In particular, its validity does not require \( \phi \) to be constant throughout \( M \) or the tensions to be equal. On the other hand, the cancellation \textit{does} assume some properties for the branes, such as requiring the absence of direct dilaton-brane couplings (\( \lambda = 0 \)).

### 3.2 Quantum Corrections

Although not trivial, the classical cancellation of the effective 4D vacuum energy is

\[5\text{Note added: The new warped solutions of ref. [15] strikingly illustrate this self-tuning, where they find that the absence of a singular dilaton assures the flatness of the 4D surfaces having fixed positions in the 2 compact dimensions.}\]
only the first step towards a solution to the cosmological constant problem. We must also ask that quantum corrections to this result not ruin the cancellation if we are to properly understand why the observed vacuum energy is so small.

Since we have already integrated out all of the brane modes to produce the effective brane tensions, the only modes left for which quantum corrections are required are those of the bulk. A complete discussion of the quantum corrections goes beyond the scope of this paper, so our purpose in this section is simply to classify the kinds of contributions which can arise, and to argue that they are smaller than $O(M_w^4)$.

For these purposes we divide the quantum contributions into four categories: those that are of order $m_{KK}^4$ and so are not ultraviolet (UV) sensitive; and three types of UV-sensitive contributions.

**UV-Insensitive Contributions**

Before addressing the UV-sensitive contributions we wish to re-emphasize that the ‘generic’ contributions to the 4D cosmological constant are of the right order of magnitude to describe naturally the observed dark energy density. To this end it is useful to think of the bulk theory in four-dimensional terms, even though this is the hard way to actually perform calculations. From the 4D perspective the bulk theory consists of a collection of KK modes all of whom are related to the ordinary 4D graviton by supersymmetry and/or extra-dimensional Lorentz transformations. The theory therefore consists of a few massless fields, plus KK towers of states whose masses are all set by the Kaluza-Klein scale, $m_{KK} \sim 1/r$.

An important feature of this complicated KK spectrum is its approximate supersymmetry (we are assuming here that supersymmetry breaking is only due to the presence of the branes). As we have seen, due to their connection with the graviton the individual KK modes only couple to one another and to brane modes with 4D gravitational strength, proportional to $1/M_p$. As a result the typical supersymmetry-breaking mass splitting within any particular bulk supersymmetry multiplet is quite small, being of order

$$\Delta m \sim \frac{1}{r} \sim \frac{M_w^2}{M_p},$$

(3.5)

Supersymmetric cancellations within a supermultiplet therefore fail by this amount. Remarkably, to the extent that the residual contribution to the 4D vacuum energy has the generic size

$$\rho_{\text{eff}} \sim (\Delta m)^4 \sim 1/r^4$$

(3.6)

it is precisely the correct order of magnitude to account for the dark energy density which now appears to be dominating the observable universe’s energy density.

**UV-Sensitive Contributions**

There are several kinds of contributions to the effective 4D cosmological constant
which are undesirable because they would swamp the above UV-insensitive result. One might worry, in particular, that the sum over the very large number of KK modes might introduce more divergent contributions to the vacuum energy than normally arise within 4D supersymmetric theories having only a small number of fields.

For these purposes it is less useful to think of the theory in 4D terms. After all, since terms of the form $\rho_{\text{eff}} \sim M_w^4$ or $M_w^2/r^2$ can only be generated by integrating out modes whose energies are much larger than $1/r$, they arise within a regime where the effective theory is six-dimensional. UV-sensitive contributions therefore must be describable by local effective interactions within the six-dimensional theory, and must respect all of the microscopic six-dimensional symmetries including in particular 6D general covariance and supersymmetry. Since the 2D curvatures are very small compared to $M_w$ they may be examined in a small-curvature expansion, and within this context powers of $1/r$ emerge when these local interactions are evaluated at the metric of interest. The terms in the effective 6D theory which depend most strongly on the 6D ultraviolet scale, $M_w$, then have the schematic form

$$\mathcal{L}_{\text{eff}} = -e^6 \left[ c_0 M_w^6 + c_1 M_w^4 R_6 + c_2 M_w^2 R_6^2 + c_3 R_6^3 + \cdots \right],$$  \hspace{1cm} (3.7)$$

plus all of their supersymmetric extensions. The $R_6^2$ term here in general could include the square of the Riemann and Ricci tensors in addition to the Ricci scalar, and the curvature-cubed terms can also be more complicated than simply the Ricci-scalar cubed. The powers of $M_w$ appearing here are set by dimensional analysis, so the constants $c_i$ are dimensionless. These effective interactions might also include terms having no more derivatives than appear in the classical action, but higher powers of the dilaton such as can arise in string theory through higher string loops [34, 35, 36, 37].

Imagine now evaluating the above effective interactions at a typical background configuration, for which $e^6 \sim r^2$ and $R_6 \sim R_2 \sim 1/r^2$. This gives contributions to the effective 4D vacuum energy which are of order

$$\rho_{\text{eff}} \sim c_0 M_w^6 r^2 + c_1 M_w^4 + c_2 M_w^2/r^2 + c_3/r^4 + \cdots.$$  \hspace{1cm} (3.8)$$

Successful suppression of the quantum contributions to the effective 4D vacuum energy clearly require the vanishing of the terms involving $c_0$, $c_1$ and $c_2$. Let us consider these in more detail.

$c_0$ and $c_1$ Terms:

Quantum contributions to $c_0$ and $c_1$ can be nonzero and can come in two forms. The simplest of these simply provides an overall renormalization of the classical action, which contains both a scalar potential and two-derivative terms. As such they are covered by any successful self-tuning solution where the self-tuning solution is regarded as applying directly to the renormalized, quantum-corrected action rather
than to the bare classical action. This argument relies on the cancellation of the 4D vacuum energy not requiring any special adjustment of the bulk couplings which might be disturbed by renormalization.

More complicated are contributions to terms having only two derivatives but more complicated dilaton dependence, such as can arise from higher string loops. In principle, these might be dangerous to the extent that they are nonzero once evaluated at the self-tuning solution of interest. In practice they are unlikely to be so, for two reasons. First, since the flat directions of Salam-Sezgin supergravity are supersymmetric, they are protected by nonrenormalization theorems so long as supersymmetry is unbroken. To the extent that supersymmetry breaking only arises through the brane couplings, this ensures that pure bulk stringy radiative corrections cannot lift the lowest-order flat directions. Second, since the classical vacua satisfies $1/r^2 \sim e^{\phi}$ along the flat direction (see, e.g. ref. [20]), any additional powers of the string coupling, $e^{\phi}$, which do arise once supersymmetry breaks are just as small as are the additional powers of $1/r^2$ which extra derivatives would imply.

On this basis we see that any successful self-tuning solution to 6D supergravity automatically also removes the danger of obtaining $O(M_4^4)$ and larger quantum contributions to the effective 4D vacuum energy once the bulk modes are integrated out. In this sense these particular bulk contributions are similar to brane modes, whose quantum contributions can be absorbed into renormalizations of the brane tensions when evaluated at $g_{\mu \nu} = \eta_{\mu \nu}$ (as is sufficient for determining the contribution to the 4D cosmological constant).

c_2 Terms:

More difficult to analyze are the curvature-squared contributions, which can in principle contribute $\rho_{\text{eff}} \sim M_6^2/r^2 \sim M_6^6/M_\mu^2 \sim (10 \text{ keV})^4$. It is worth remarking that even such terms do dominate $\rho_{\text{eff}}$, they are much smaller than the $O(M_4^4)$ contributions which make up the usual cosmological-constant problem. Furthermore, these kinds of terms do not appear to arise within the few extant explicit one-loop calculations which exist [27] of Casimir energies in higher-dimensional supersymmetric models, with supersymmetry broken by brane-dependent or Scherk-Schwarz-type [28] mechanisms.

We now give a qualitative argument as to why self-tuning might help ensure these curvature-squared corrections do not contribute dangerously to $\rho_{\text{eff}}$, even if they are generated by bulk quantum effects. To this end it is instructive to first consider how things work in a simple toy model. Consider therefore the following toy lagrangian

$$ S = - \int d^n y \left[ \frac{1}{2} (\partial \phi)^2 + \frac{\lambda}{2} \phi \Box^2 \phi + \phi J \right], \quad (3.9) $$

where $J(x) = \sum_i Q_i \delta(x - x_i)$ is the sum of localized sources. In this model the scalar field $\phi$ represents a generic bulk field, $J$ represents its brane sources and the second
term is a representative four-derivative effective term. The argument of the previous section as applied to this model amounts to eliminating \( \varphi \) using its classical equation of motion, and we wish to follow how the four-derivative term alters the result to linear order in the effective coupling \( \lambda \).

For \( \lambda = 0 \) the classical solution is

\[
\varphi_0 = \sum_i Q_i G(x, x_i),
\]

(3.10)

where \( \Box G(x, x') = \delta(x-x') \). To linear order in \( \lambda \) the classical solution is \( \varphi_c = \varphi_0 + \delta \varphi \), where

\[
\Box \delta \varphi = \lambda \Box^2 \varphi_0 = \lambda \Box J,
\]

(3.11)

and so \( \delta \varphi(x) = \lambda J(x) = \lambda \sum_i Q_i \delta(x-x_i) \).

Our evaluation of \( \rho_{\text{eff}} \) amounts in this model to evaluating the action at \( \varphi_c \). A straightforward calculation gives

\[
S[\varphi_c] - S[\varphi_0] = -\frac{\lambda}{2} \int d^n y J^2(y) = -\frac{\lambda}{2} \sum_i \int d^n y Q_i^2 \delta^2(y - y_i).
\]

(3.12)

Although \( S[\varphi_0] \neq 0 \) for this model, the important point for our purposes is that the \( O(\lambda) \) contribution, \( S[\varphi_c] - S[\varphi_0] \), is purely localized at the positions of the sources (or branes).

If the same were true for the effective corrections to 6D supergravity, for the purposes of their contributions to \( \rho_{\text{eff}} \) they would again amount to renormalizations of the brane tensions, and so would be cancelled by the mechanism described previously. Should we expect this for the influence of these 6D effective corrections? We now argue that this should be so, provided that the two dimensions are compactified in a way which preserves an unbroken supersymmetry (for instance, as does the Salam-Sezgin compactification on a sphere described in section 2).

The basic reason why the toy model generates only source terms in the action is that the effective interaction \( \lambda \varphi \Box^2 \varphi \) has the property that it vanishes if it is evaluated at the solution \( \varphi_0 \) in the absence of sources (which would then satisfy \( \Box \varphi_0 = 0 \)). But supersymmetry also ensures that this is also true for the supersymmetric higher-derivative (and other) corrections in six dimensions. To see this imagine removing the various 3 branes and asking how these effective terms contribute to \( \rho_{\text{eff}} \). In this case we know that their contribution is zero because the low-energy theory has unbroken supersymmetry in a flat 4D space, and \( \rho_{\text{eff}} \) is in this case protected by a non-renormalization theorem. This is perhaps most easily seen by considering the effective 4D supergravity which describes this theory [20].

Once branes are re-introduced, we expect the contributions to \( \rho_{\text{eff}} \) to no longer vanish, just as for the toy model, but just as for the toy model their effects should be localized at the positions of the branes. As such, for the purposes of contributing
to $\rho_{\text{eff}}$ they amount to renormalizations of the brane tensions and so are cancelled according to the mechanism of the previous section. In this sense none of these 6D effective terms may be dangerous, because their effects may correspond to renormalizations of brane properties whose values are not important for obtaining the conclusion that $\rho_{\text{eff}}$ is small.

A more detailed explicit calculation of the one-loop contributions to the effective 4D vacuum energy using six-dimensional supergravity is clearly of great interest, along the lines begun in ref. [27]. Besides its utility in clarifying the nature of the mechanism described above, such a calculation would be invaluable for determining the nature of the dynamics which is associated with the dark energy.

4. An Explicit Brane Model

The previous sections outline a mechanism which relates a small 4D vacuum energy to brane properties at higher energies $E \sim M_w$, and can explain why this vacuum energy remains small as the modes between $M_w$ and $1/r$ are integrated out. It remains to see if an explicit brane configuration can be constructed which takes advantage of this mechanism to really give such a small cosmological constant.

In this section we take the first steps in this direction, by constructing a simple two-brane configuration within the 2-sphere compactification of the Salam-Sezgin model described earlier, taking into account the back-reaction of the branes. Since our construction also has a constant dilaton field, it furnishes an explicit example of a model for which the classical contributions to $\rho_{\text{eff}}$ precisely cancel.

Our attempt is not completely successful in one sense, however, because our construction is built using a non-supersymmetric compactification of 6D supergravity. As such, our general arguments as to the absence of quantum corrections may not apply, perhaps leading to corrections which are larger than $1/r^4$. The model has the great virtue that it is sufficiently simple to explicitly calculate quantum corrections, and so to check the general arguments, and such calculations are now in progress. Readers in a hurry can skip this section as being outside our main line of argument.

4.1 Branes on the Sphere

The great utility of the spherical compactification of Salam-Sezgin supergravity is the simplicity with which branes can be embedded into it, including their back-reaction onto the bulk gravitational, dilaton and Maxwell fields. Because the solution we find has a constant dilaton, our construction of these brane solutions turns out to closely resemble the analysis of the Maxwell-Einstein equations given in ref. [6].

The field equations of 6D supergravity have a remarkably simple solution (when $\lambda = 0$) for the special case of two branes having equal tension, $T$, located at opposite poles of the two-sphere. In this case the solution is precisely the same as obtained before in the absence of any branes, but with the two-dimensional curvature now
required to include a delta-function singularity at the position of each of the branes. More precisely, the only change implied for the solution by the brane sources comes from the two-dimensional components of the Einstein equation, which now requires that the two-dimensional Ricci scalar can be written \( R_2 = R_2^{\text{smth}} + R_2^{\text{sing}} \), where \( R_2^{\text{smth}} \) satisfies precisely the same equations as in the absence of any branes, and the singular part is given by

\[
R_2^{\text{sing}} = -\frac{2T}{e_2} \sum_i \delta^2(y - y_i),
\]  

(4.1)

where as before \( e_2 = \sqrt{\det g_{mn}} \).

The resulting solution therefore involves precisely the same field configurations as before: \( \varphi = \text{(constant)}, g_{\mu\nu} = \eta_{\mu\nu}, g_{mn} \, dy^m \, dy^n = r^2 \left( d\theta^2 + \sin^2 \theta \, d\phi^2 \right) \) and \( T_\alpha F^\alpha_{mn} = Q f \epsilon_{mn} \), for a \( U(1) \) generator, \( Q \), embedded within the gauge group. As before the parameters of the solution are related by \( r^2 e^\varphi = 1/(4g_1^2) \) and \( f = n/(2g_1 \, r^2) \) where \( n = \pm 1 \). The singular curvature is then ensured by simply making the coordinate \( \phi \) periodic with period \( 2\pi (1 - \varepsilon) \) rather than period \( 2\pi \) — thereby introducing a conical singularity at the branes’ positions at the north and south poles. The curvature condition, eq. (4.1), is satisfied provided that the deficit \( \varepsilon \) is related to the brane tension by \( \varepsilon = 4 G_6 T \).

---

**Figure 1:** The effect of two 3-branes at the antipodal points in a 2-sphere. The wedge of angular width \( 2\pi \varepsilon \) is removed from the sphere and the two edges are identified giving rise to the rugby-ball-shaped figure. The deficit angle is related to the branes tensions (assumed equal) by \( \varepsilon = 4G_6 T \).

The ‘rugby-ball’ geometry\(^6\) so described corresponds to removing from the 2-sphere a wedge of angular width \( 2\pi \varepsilon \), which is bounded by two lines of longitude

---

\(^6\)We use the name rugby-ball to resolve the cultural ambiguity in the shape meant by ‘football’, which was used previously in the literature \[\text{[reference]}\]. The name ‘periodic lune’ has also been used \[\text{[reference]}\].
running between the branes at the north and south poles, and then identifying the edges on either side of the wedge \[30, 29, 4, 6\]. The delta-function contributions to \(R_2\) are then just what is required to keep the Euler characteristic unchanged, since

\[
\chi = -\frac{1}{2\pi} \int d^2y \left( R_{2}^{\text{smth}} + R_{2}^{\text{sing}} \right) = 2. \tag{4.2}
\]

The singular contribution precisely compensates the reduction in the contribution of the smooth curvature, \(R_{2}^{\text{smth}}\), due to the reduced volume of the rugby-ball relative to the sphere.

Finally, the above configuration also satisfies the equations of motion for the branes, which state (for constant \(\varphi\) or vanishing \(\lambda\)) that they move along a geodesic according to

\[
\ddot{y}^m + \Gamma^m_{pq} \dot{y}^p \dot{y}^q = 0, \tag{4.3}
\]

where \(\Gamma^m_{pq}\) is the Christoffel symbol constructed from the 2D metric, \(g_{mn}\). Consequently branes placed precisely at rest anywhere in the two dimensions will remain there, and this configuration is likely to be marginally stable due to the absence of local gravitational forces in two spatial dimensions.

4.2 Topological Constraint

We now show that the above solution is further restricted by a topological argument. This will exclude for instance the possibility of the supersymmetric Salam-Sezgin compactification in which the monopole background is fully embedded into the explicit \(U(1)\) gauge group factor. But it allows other embeddings, in particular the \(E_6\) embedding of \[25\] that is non-supersymmetric.

In order to make this argument we write the electromagnetic field strength obtained from the field equations as

\[
F = \frac{n}{2 g_1} \sin \theta \, d\theta \wedge d\phi, \tag{4.4}
\]

where \(n = \pm 1\). The gauge potential corresponding to this field strength can be chosen in the usual way to be

\[
A_\pm = \frac{n}{2 g_1} \left[ \pm 1 - \cos \theta \right] d\phi, \tag{4.5}
\]

where the subscript ‘\(\pm\)’ denotes that the configuration is designed to be nonsingular on a patch which respectively covers the northern or southern hemisphere of the rugby-ball.

Now comes the main point. \(A_+\) and \(A_-\) must differ by a gauge transformation on the overlap of the two patches along the equator, and this — with the periodicity condition \(\phi \approx \phi + 2\pi (1 - \varepsilon)\) — implies \(A_\pm\) must satisfy \(gA_+ - gA_- = N \, d\phi / (1 - \varepsilon)\), where \(N\) is any integer and \(g\) denotes the gauge coupling constant which is
appropriate to the generator $Q$. In particular $g = g_6$ if $Q$ lies within the $E_6$ subgroup, as is in ref. [24], or $g = g_1$ if $Q$ corresponds to the explicit $U(1)$ gauge factor, as in ref. [19]. Notice that this is only consistent with eq. (4.4) if $g$ and $g_1$ are related by

$$\frac{g}{g_1} = \frac{N}{n(1-\epsilon)}.$$  \hspace{1cm} (4.6)

In particular, $g$ cannot equal $g_1$ if $\epsilon \neq 0$, and so we cannot choose $Q$ to lie in the explicit $U(1)$ gauge factor, as for the supersymmetric Salam-Sezgin compactification.

A deeper understanding of this last condition can be had if the 3-brane action is generalized to include the coupling, eq. (2.3), to the background Maxwell field since in this case the 3-brane acquires a delta-function contribution to the magnetic flux of size $Q \propto q$. Denoting the flux at the position of each brane by $Q_{\pm}$, eq. (4.3) generalizes to

$$A_{\pm} = \left[ \frac{Q_{\pm}}{2\pi} + \frac{n}{2g_1}(\pm 1 - \cos \theta) \right] \text{d}\phi.$$  \hspace{1cm} (4.7)

The same arguments as above then lead to the following generalization of formula (4.6)

$$\frac{Q_+ - Q_-}{2\pi} + \frac{n}{g_1} = \frac{N}{g(1-\epsilon)},$$  \hspace{1cm} (4.8)

which relates the difference, $Q_+ - Q_-$, to the integers $n$ and $N$. This shows that the constraint we are obtaining is best interpreted as a topological condition on the kinds of magnetic fluxes which are topologically allowed in order for a solution to exist (much like the condition that the tensions on each to the two 3-branes must be equal or, in another context, to the Gauss’ Law requirement that the net charge must vanish for a system of charges distributed within a compact space). Within this context eq. (4.4) expresses the conditions which are required in order to have a solution with $Q_+ = Q_-$. Given its topological (long-distance) character, such a condition is very likely to be preserved under short-distance corrections, and so be stable under renormalization.\footnote{Note added: This stability is easier to see for the single-brane solutions of ref. [48], where it is very much like the usual quantization of monopole charge.}

Although the choice $Q_+ = Q_-$ precludes a solution with $g = g_1$, it does allow solutions where $Q$ lies elsewhere in the full gauge group, such as the $E_6$ embedding above. This model has the great virtue of simplicity, largely due to the constancy of both the dilaton and the magnetic flux over the two-sphere. It has the drawback that this simple embedding of the monopole gauge group breaks supersymmetry, and so may allow larger quantum corrections than would be allowed by the general arguments of the previous sections. On the other hand, the choice $g = g_1$ may be possible if $Q_\pm$ are not equal, and if so would allow a solution with unbroken bulk supersymmetry as in the original Salam-Sezgin model.
It clearly would be of great interest to find an anomaly-free embedding that also preserves some of the supersymmetry, since any such embedding would completely achieve precisely the scenario we are proposing with a naturally small cosmological constant. However, although supersymmetry was required to eliminate the contributions of curvature squared terms, which contribute to \( \rho_{\text{eff}} \) an amount of order \( M_w^2/r^2 \), we see that even without supersymmetry this model achieves a great reduction in the cosmological constant relative to the mass-splittings, \( M_w \), between observable particles and any of their superpartners. A full study of monopole solutions and their quantum fluctuations is presently being investigated.

5. Conclusions

In this paper we present arguments that supersymmetric six-dimensional theories with 3-branes can go a long way towards solving the cosmological constant problem. Unlike most approaches, the arguments we present address (at least partially) both the high-energy and the low-energy part of the cosmological constant problem: i.e. why is the cosmological constant so small at high energies and why does it remain small after integrating out comparatively light degrees of freedom (like the electron) whose physics we think we understand.

We are motivated to examine six-dimensional theories because of the remarkable fact that the cosmological constant scale \( v \) coincides in these theories with the compactification scale \( 1/r \) and the gravitino mass \( M_w^2/M_p \). This potentially makes the nonvanishing of the cosmological constant less of a mystery since it becomes related to the relevant scales of the theory, providing an explanation for the phenomenological relationship \( v \sim M_w^2/M_p \) which relates the cosmological constant to the hierarchy problem. Notice that this relationship would also account for the ‘Why Now?’ problem — which asks why the Dark Energy should be just beginning to dominate the Universe at the present epoch — provided the cold dark matter consists of elementary particles having weak-interaction cross sections \cite{38}.

A satisfying consequence of this kind of six-dimensional solution to the cosmological constant problem is that it shares the many experimental implications of the sub-millimeter 6D brane scenarios. These are very likely to be testable within the near future in two distinct ways. First, the scenario predicts violations to Newton’s gravitational force law at distances below \( \sim 0.1 \) mm, which is close to the edge of what can be detected. It also predicts the existence of a 6D fundamental scale just above the TeV scale, and so predicts many forms of extra-dimensional particle emission and gravitational effects for high-energy colliders. Both predictions provide a fascinating and unexpected connection between laboratory physics and the cosmological constant.

We find that self-tuning in 6D supergravity potentially provides a new twist to the connection between supersymmetry and the cosmological constant. Usually
supersymmetry is thought not to be useful for solving the low-energy part of the cosmological constant problem since it can at best suppress it to be of order \( (\Delta m)^4 \). The low-energy problem is then how to reconcile this with the absence of observed superpartners, which requires \( \Delta m \gtrsim M_w \). We overcome this problem by separating the two scales. No unacceptable superpartners arise for ordinary particles because particle supermultiplets on the brane are split by \( O(M_w) \). Although their contribution to the vacuum energy is therefore \( O(M_w^4) \), this is not directly a contribution to the observed 4D cosmological constant because it is localized on the branes and is cancelled by the contribution of the bulk curvature.

From this point of view the important modes to whose quantum fluctuations the 4D vacuum energy is sensitive are those in the bulk. But this sector is only gravitationally coupled, and so in it supersymmetry breaking can really be of order the cosmological constant scale, \( \Delta m \sim v \sim 10^{-3} \text{ eV} \) without being immediately inconsistent with observations. We argue here that self-tuning precludes bulk quantum corrections to the 4D cosmological constant from being larger than \( \rho_{\text{eff}} \sim M_w^2/r^2 \sim M_w^6/M_p^2 \), making them much smaller than the \( O(M_w^4) \) contributions of most 4D supersymmetric theories. Explicit one-loop calculations \[27\] seem to indicate that the \( O(M_w^2/r^2) \) are also not present for theories where supersymmetry is broken by boundary conditions, giving a generic zero-point energy \( (\Delta m)^4 \), and we argue qualitatively why this might also be ensured by the self-tuning mechanism. In this sense our proposal goes beyond explaining why the cosmological constant is zero, by also explaining why supersymmetry breaking at scale \( M_w \) requires it to be nonzero and of the observed size.

Our proposal shares some features with other brane-based mechanisms which have been proposed to suppress the vacuum energy after supersymmetry breaking. For instance, the special role played by supersymmetry in two transverse dimensions echoes earlier ideas \[39\] based on (2+1)-dimensional supersymmetry. We regard the present proposal to be an improvement on the brane-based mechanism of suppressing the 4D vacuum energy relative to the splitting of masses within supermultiplets proposed in ref. \[40\]. This earlier proposal was difficult to embed into an explicit string model, and required an appeal to negative-tension objects, such as orientifolds, in order to obtain a small \( \rho_{\text{eff}} \) at high energies. Furthermore, the low-energy part of the cosmological constant problem was not fully addressed. Our mechanism also shares some of the features of the self-tuning proposals of \[14\] in the sense that flat spacetime is a natural solution of the field equations. But we do not share the difficulties of that mechanism, such as the unavoidable presence of singularities or the need for negative tension branes \[13, 16\].

Our mechanism is most closely related to recent attempts to obtain a small cosmological constant from branes in non-supersymmetric 6D theories \[5, 6\]. In particular we use the special role of 6D to cancel the brane tensions from the bulk curvature, independent of the value of the tensions. However, our framework goes
beyond theirs in several ways. In the scenarios of [3], the singular part of the Ricci scalar cancels the contribution from the brane tensions, but the smooth part does not cancel the other contributions to the cosmological constant, such as a bulk cosmological constant. The explicit compactifications considered there, including the presence of 4-branes, either do not achieve the natural cancellation of the cosmological constant or have naked singularities. In [3], the same rugby-ball geometry that we consider was studied in detail. Because of the lack of supersymmetry it was necessary to tune the value of the bulk cosmological constant to obtain a cancellation with the monopole flux and obtain flat 4D spacetime. Furthermore none of these proposals address question 2 of our introduction. Our proposal, being based on supersymmetry, avoids those problems and addresses both questions 1 and 2 of the introduction.

5.1 Open Questions

Even though our scenario has a number of attractive features, it leaves a great many questions unanswered.

First, our attempt to realize the self-tuning in an explicit solution to the 6D equations led to a topological constraint that appears to require a relationship between the brane tension and other (gauge) couplings in the bulk action. It remains to be seen whether this condition is an artifact of the simplicity of our solution (such as being due to our requiring the dilaton and Maxwell fields to be nonsingular at the brane positions) or if it is actually unavoidably required in order to obtain flat 3-branes. In particular, we argue that the topological relation is better interpreted as a constraint on what magnetic fluxes which may be carried by the branes given the topology of the internal space. As such it might be expected to be stable under renormalization, in much the same way as is the condition that the net electric charge vanish for a configuration of charged particles in a compact space.

Since the scale of the cosmological constant (and the electroweak/gravitational hierarchy) is set by $r$ in this picture, it becomes all the more urgent to understand how the radion can be stabilized at such large values. Six dimensions are promising in this regard, since they allow several mechanisms for generating potentials which depend only logarithmically on $r$ [42, 43]. (See ref. [20] for a discussion of stabilization issues within the 6D Salam-Sezgin model.)

More generally, it is crucial to understand the dynamics of the radion near its minimum within any such stabilization mechanism, since this can mean that the radion is even now cosmologically evolving, with correspondingly different implications for the Dark Energy’s equation of state. Indeed, it has recently been observed that viable cosmologies based on a sub-millimeter scale radion can be built along these lines [44].

More precise calculations of the quantum corrections within these geometries is clearly required in order to sharpen the general order-of-magnitude arguments
presented here. This involves a detailed examination of the full classical solution to the Einstein-Maxwell-dilaton system in the presence of the branes, as well as the explicit integration over their quantum fluctuations. Such calculations as presently exist (both in string theory and field theory [27]) support the claim that the net 4D vacuum energy density after supersymmetry breaking can be \( O(1/r^4) \).

At a more microscopic level, it would be very interesting to be able to make contact with string theory. This requires both a derivation of the effective 6D supergravity theory as a low-energy limit of a consistent string theory (or any other alternative fundamental theory which may emerge), as well as a way of obtaining the required types and distributions of branes from a consistent compactification. In particular it is crucial to check if we can derive the absence of a dilaton coupling to the branes directly within a stringy context.

One approach is to try to obtain Salam-Sezgin supergravity from within string theory. As mentioned in [20], the possibility of compactifications based on spheres [45] in string theory, or fluxes in toroidal or related models [46], could be relevant to this end. Remembering that the Salam-Sezgin model has a potential which is positive definite, this may actually require non-compact gaugings and/or duality twists, such as those recently studied in [47]. Ideally one would like a fully realistic string model that addresses all of these issues. An alternative approach is to see if our mechanism generalizes to other 6D supergravities, whose string-theoretic pedigree is better understood. Work along these lines is also in progress.

A virtue of identifying a low-energy mechanism for controlling the vacuum energy is that obtaining its realization may be used as a guideline in the search for realistic string models. This may suggest considering anisotropic string compactifications with four small dimensions (of order the string scale \( \sim M_w \)) and two large dimensions \((r \sim 0.1 \text{ mm})\) giving rise to a large Planck scale \( M_p \sim M_w^2 r \) and a small cosmological constant \( \Lambda \sim 1/r^4 \sim (M_w^2/M_p)^4 \).

All in all, we believe our proposal to be progress in understanding the dark energy, inasmuch as it allows an understanding of the low-energy — and so also the most puzzling — part of the problem. We believe these ideas considerably increase the motivation for studying the other phenomenological implications of sub-millimeter scale extra dimensions [8], and in particular to the consequences of supersymmetry in these models [8]. We believe the potential connection between laboratory observations and the cosmological constant makes the motivation for a more detailed study of the phenomenology of these models particularly compelling.

Note Added: There have been several interesting developments since this paper appeared on the arXiv, which we briefly summarize here.

Ref. [41] provide an interesting analysis of the Salam-Sezgin model without branes, in which they verify the topological condition, eq. (4.6) (as also did ref. [49]), and show that if the Kaluza-Klein scale is of order \( 10^{-3} \text{ eV} \), then the 4D gauge cou-
pling of the bulk gauge fields must be $g_4 \sim 10^{-31}$ (as opposed to the value of $10^{-15}$ which is obtained in the absence of a dilaton [3]). Since this follows directly from the large size of the extra dimensions, its explanation rests with whatever physics stabilizes the size of the extra dimensions, and does not represent an additional fine tuning beyond this. The physics of radius stabilization at such a large value remains of course an open question.\footnote{Notational point: We adopt in this paper a slightly different metric convention than we did in ref. [24] since we here do not work in the 4D Einstein frame. Consequently in this paper KK masses are of order $1/r$ instead of being of order $ge^{\phi/2}/r \sim 1/r^2$, as they are in ref. [24], and as is shown explicitly in ref. [41].}

Ref. [18] finds the general nonsingular solution to the Salam-Sezgin equations having maximal symmetry in the noncompact 4 dimensions, for arbitrary monopole number. These solutions nicely illustrate many of the arguments made here, since the noncompact 4 dimensions are always flat, as our general self-tuning arguments predict. The 4D curvature which the field equations require for non-constant dilaton is in this case provided by warping in the extra dimensions. Furthermore, the solutions with monopole number greater than 1 provide examples whose topological constraints are very plausibly stable against renormalization inasmuch as they closely resemble the standard monopole quantization condition.

Progress towards embedding our picture into string theory has also been made. Ref. [50] finds a higher-dimensional derivation of a new supergravity which shares the bosonic part of the Salam-Sezgin theory. Ref. [51] obtains exactly the Salam-Sezgin supergravity, by consistently reducing type I/heterotic supergravity on the non-compact hyperboloid $H^{2,2}$ times $S^1$.

Finally, ref. [52] provides an explicit recent one-loop string calculation of the vacuum energy within a supersymmetry-breaking framework similar to that considered here. They find a result which is of order $1/r^4$, in agreement with our arguments and with previous calculations [27].

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