Rotating Black Branes in the presence of nonlinear electromagnetic field

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Received: date / Revised version: date

Abstract. In this paper, we consider a class of gravity whose action represents itself as a sum of the usual Einstein-Hilbert action with cosmological constant and an U(1) gauge field for which the action is given by a power of the Maxwell invariant. We present a class of the rotating black branes with Ricci flat horizon and show that the presented solutions may be interpreted as black brane solutions with two event horizons, extreme black hole and naked singularity provided the parameters of the solutions are chosen suitably. We investigate the properties of the solutions and find that for the special values of the nonlinear parameter, the solutions are not asymptotically anti-de Sitter. At last, we obtain the conserved quantities of the rotating black branes and find that the nonlinear source effects on the electric field, the behavior of spacetime, type of singularity and other quantities.

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1 Introduction

Seventy five years ago Born and Infeld proposed a type of nonlinear electrodynamics starting from the principle of finiteness with the aim of obtaining a finite value for the self-energy of a point-like charge [1,2]. They proposed the following Lagrangian

\[ L(F) = 4\beta^2 \left( 1 - \sqrt{1 + \frac{F}{2\beta^2}} \right). \]  

(1)

where the Maxwell invariant \( F = F_{\mu\nu}F^{\mu\nu} \) in which \( F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu \) is the electromagnetic tensor field and \( A_\mu \) is the vector potential. After these significant achievements, there were not many works on nonlinear electromagnetic fields, as Plebanski mentioned in 1970 at the beginning of his monograph [3]: "If in recent times the interest in nonlinear electrodynamics cannot be said to be very popular, it is not due to the fact that one could rise some serious objections against this theory. It is simply rather difficult in its mathematical formulation, what causes that it is very unlikely to derive some concrete results in closed form". The major responsible of the actual revival, more than twenty years later of the moment of formulation of quoted statement, is the fact that the effective action for the open string ending on D-branes can be written in a nonlinear form [4,5,6]. In recent years nonlinear electrodynamics models are attracting much interest, too. The nonlinear equations of theoretical and mathematical physics attract the significant attention because of the specific properties such as the absence of the superposition principle, the nonlinear fields interactions and existence of the soliton solutions. Some of the main reasons to consider nonlinear electrodynamics theories are as follows: First point out that these theories are considerably richer than that of the classical linear electromagnetic field and in the special case they can reduce to Maxwell field. Second, it can be used to find and analyze new solutions with different behavior of the nonlinear Maxwell equations. Third, it is remarkable that all the nonlinear electrodynamics models coupled to gravity satisfy the zeroth and the first laws of black hole mechanics [7]. Fourth, there has been a significant amount of interest in cosmological models involving nonlinear electromagnetic fields [8,9,10,11,12,13,14,15,16,17,18,19,20,21,22,23,24,25,26,27]. If the early universe is dominated by radiation governed by Maxwell’s equations it is well known that there will be a spacelike initial singularity in the past. However, if Maxwell’s equations become modified in the early universe, when the electromagnetic field is large, it may be possible to avoid the initial singularity. In fact, recent results [8,9,10,11] show that a magnetic universe can avoid the initial singularity and have a period of late time acceleration if the electromagnetic Lagrangian is of the nonlinear form [28]. More recently, in the light of the AdS/CFT correspondence [29], the nonlinear electrodynamics string approach has been used to obtain solutions describing baryon configurations which are consistent with confinement [30]. Moreover, in [31,32] it was found that cosmological models with non-
linear electromagnetic fields can be compatible with the recently measured accelerated expansion of the universe.

From this point of view, the exact solutions of Einstein-nonlinear Maxwell equations are worth to study since they may indicate the physical relevance of nonlinear effects in strong gravitational and strong electromagnetic fields in cosmological and general relativity models. In nonlinear electrodynamics coupled to general relativity there exists several solutions describing electrically charged rotating black holes and black branes in Sec. 3, we present a new class of rotating black brane solutions with flat horizon in Einstein-nonlinear Maxwell gravity, investigate their properties and obtain conserved charges are finite [63]. Physically, this means that a collection of observers on the hypersurface whose metric is all observe the same value of $Q_s$ provided this hypersurface has an isometry generated by $\xi$, in the context of counterterm method, the limit in which the boundary $B$ becomes infinite $(B_{\infty})$ is taken, and the counterterm prescription ensures that the total action and conserved charges are finite [63].

The aim of this paper is to consider the nonlinear electrodynamics field coupled to Einstein gravity and introduce a class of rotating black brane solutions with flat horizon. Also, we present a study of the effects produced by a nonlinear type field on the asymptotic behavior of the solutions and conserved quantities of the $(n+1)$-dimensional black brane solutions. We finish our paper with some concluding remarks.

2 Basic Field Equations of Einstein Gravity with nonlinear Electromagnetic Source

The $(n+1)$-dimensional action in which gravity is coupled to nonlinear electromagnetic field is given by

$$I = -\frac{1}{16\pi} \int_M d^{n+1}x \sqrt{-g} \left( \mathcal{L}_1 + \mathcal{L}_2 \right),$$

where $\mathcal{L}_1 = \mathcal{R} - 2\Lambda, \mathcal{L}_2 = -\alpha F^s, \mathcal{R}$ is scalar curvature, $\Lambda$ refers to the negative cosmological constant which is in general equal to $-\pi(n-1)/2l^2$ for asymptotically AdS solutions, in which $l$ is a scale length factor, $\alpha$ denotes a coupling constant and the exponent $s$ is related to the power of nonlinearity. Varying the action with respect to the metric $g_{\mu\nu}$ and the gauge field $A_\mu$, the field equations are obtained as

$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} L_1 = T_{\mu\nu},$$

$$\partial_\nu \left( \sqrt{-g} F^{\mu\nu} F^{s-1} \right) = 0,$$

where

$$T_{\mu\nu} = 2\alpha F^{s-1} \left( s F_{\mu\rho} F^\rho_{\nu} - \frac{1}{4} g_{\mu\nu} F_{\rho\sigma} F^{\rho\sigma} \right).$$

For a well-defined variational principle [61,62], one has to supplement the action with the Gibbons-Hawking boundary term

$$I_b = \frac{1}{8\pi} \int_{\partial M} d^n x \sqrt{-\gamma} K,$$

where $\gamma$ and $K$ are the trace of the induced metric $\gamma_{ij}$ and extrinsic curvature $K_{\mu\nu}$ on the boundary $\partial M$. To compute the conserved charges of the asymptotically AdS solutions in Einstein gravity, we use the counterterm approach [63].

This technique was inspired by AdS/CFT correspondence and consists in adding suitable counterterms $I_{ct}$ to the action of the theory in order to ensure the finiteness of the boundary stress tensor derived by the quasilocal energy definition [64]. Therefore we supplement the general action (which contains the boundary terms $I_{ct}$) with the following boundary counterterm

$$I_{ct} = \frac{1}{8\pi} \int_{\partial M} d^n x \sqrt{-\gamma} \left( \frac{n-1}{l} \right).$$

Varying the total action ($I_{tot} = I + I_b + I_{ct}$) with respect to the induced metric $\gamma_{ab}$, we find the divergence-free boundary stress-tensor

$$T_{ab} = K_{ab} - \left( K + \frac{n-1}{l} \right) \gamma_{ab}.$$

Provided the boundary geometry has an isometry generated by a Killing vector $\xi^a$, a conserved charge

$$Q_\xi = \int_B d^{n-1} \varphi \sqrt{\sigma} T_{ab} n^a \xi^b,$$

can be associated with the boundary $B$ [63]. In Eq. (9), $\sigma$ is the determinant of the metric $\sigma_{ij}$, and $n^a$ is the timelike unit normal vector to the boundary $\partial M$. In the context of counterterm method, the limit in which the boundary $B$ becomes infinite $(B_{\infty})$ is taken, and the counterterm prescription ensures that the total action and conserved charges are finite [63].

3 The $(n+1)$-dimensional Charged Rotating Black Branes with Flat Horizon

Static black hole solutions of nonlinear Maxwell field with spherical horizon have been investigated in [46,47,48,49], and here we want to generalize it to the rotating solutions.
Consider the \((n+1)\)-dimensional static metric with flat horizon with the following form
\[
ds^2 = -F(r)dt^2 + \frac{dr^2}{F(r)} + r^2 \sum_{i=1}^{\frac{n-1}{2}} d\phi_i^2,
\] (11)
we want to generalize this metric \([13]\) to the rotating metric with a global rotation. These kinds of rotating solutions in Einstein gravity have been introduced in Ref. \([65,66]\). In order to add angular momentum to the spacetime, we perform the following rotation boost in the \(t - \phi_i\) planes
\[
t \mapsto \Xi t - a_i \phi_i, \quad \phi_i \mapsto \Xi \phi_i - \frac{a_i}{\Xi^2} t,
\] (12)
for \(i = 1...[n/2]\), where \([x]\) is the integer part of \(x\). The maximum number of rotation parameters is due to the fact that the rotation group in \((n+1)\)-dimensions is \(SO(n)\) and therefore the number of independent rotation parameters is \([n/2]\). Thus the \((n+1)\)-dimensional metric of rotating solutions with \(\kappa \leq [n/2]\) rotation parameters for flat horizon can be written as
\[
ds^2 = -F(r) \left( \Xi dt - \sum_{i=1}^{\kappa} a_i d\phi_i \right)^2 + \frac{dr^2}{F(r)} + \sum_{i=1}^{\frac{n-1}{2}} (a_i dt - \Xi^2 t d\phi_i)^2
\]
\[- r^2 \sum_{i \neq j} (a_i d\phi_j - a_j d\phi_i)^2 + r^2 dX^2,
\] (13)
where \(\Xi = \sqrt{1 + \sum a_i^2 / l^2}\) and \(dX^2\) is the Euclidean metric on the \((n-\kappa-1)\)-dimensional submanifold. Because of the periodic nature of \(\phi_i\), the transformation \([12]\) is not a proper coordinate transformation on the entire manifold. Therefore, the metrics \([11]\) with \(k = 0\) and \([13]\) can be locally mapped into each other but not globally, and so they are distinct \([67]\). If we consider Eq. \([11]\), the gauge potential \(A_\mu\) and the metric function of the rotating metric \([14]\) can be written as
\[
A_\mu = h(r) \left( \Xi \delta^0_\mu - \delta^0_\mu t \right) \quad \text{(no sum on } i),
\] (14)
\[
F(r) = - \frac{2\Lambda r^2}{n(n-1)} - \frac{m}{r^{n-2}} + H_1,
\] (15)
\[
H_1 = \begin{cases} 
\frac{(2s-1)^2 (2(2s-1)^2 - s)}{(n-1)(2s-n + 1)(2s-1)} & 0 < s < \frac{1}{2}, s < 0 \\
\frac{1}{r - \frac{s}{2}} & s = \frac{1}{2} \\
\frac{-(2s-1)^2 (2(2s-1)^2 - s)}{(n-1)(2s-n + 1)(2s-1)} & \text{Otherwise}
\end{cases}
\] (16)
where \(m\) is the integration constant which is related to mass parameter. and \(h(r) = -q \ln r\) for \(s = \frac{1}{2}\) and \(h(r) = -q r^{(2s-n)/(2s-1)}\) otherwise and then for \(s = (n+1)/4\), the electric field \(F_{tr}\), is proportional to \(r^{-2}\) in arbitrary dimension \([68,69]\). In the last relations \(q\) is an integration constant which is related to the electric charge parameter. It is clear that for \(s = 0\) and \(s = 1/2\), the function \(h(r)\) is constant and we do not have any electromagnetic field and the field equations reduce to uncharged solutions which discussed in many literatures and we are not interested in it. In the rest of the paper we assume that \(s \neq 0, 1/2\). The metric function \(F(r)\), presented here, differ from the linear higher dimensional Reissner-Nordström black hole solutions; it is notable that the electric charge term in the linear case is proportional to \(r^{-2n-2}\), but in the presented metric function, nonlinear case, this term also depends on the exponent parameter \(s\). But, in the linear limit \((s = 1)\), these solutions reduce to the higher dimensional Reissner-Nordström solutions, as they should be. In addition, it is easy to show that these solutions reduce to the solutions obtained in Ref. \([68]\) for \(s = (n+1)/4\). Also, it is notable that the solutions of the rotating metric has the same as static one. But in the next sections we show that the rotating metric has different conserved and thermodynamics quantities, and therefore the new spacetime is different from static one.

### 3.1 Properties of the solutions

In order to study the general structure of these spacetime, we first investigate the effects of the nonlinearity on the asymptotic behavior of the solutions. It is worthwhile to mention that for \(0 < s < \frac{1}{2}\), the asymptotic dominant term of Eq. \([15]\) is third term and the presented solutions are not asymptotically AdS, but for the cases \(s < 0\) or \(s > \frac{1}{2}\) (include of \(s = \frac{1}{2}\)), the asymptotic behavior of rotating Einstein-nonlinear Maxwell field solutions are the same as linear AdS case. It is easy to show that the electromagnetic field is zero for the cases \(s = 0, \frac{1}{2}\), and the metric function \([16]\) does not possess a charge term.

Fig. 1. \(F(r)\) versus \(r\) for \(n = 4, q = 1, s = 1, A = -1, m < 0, 0 < m < m_{\text{ext}}, m = m_{\text{ext}}\) and \(m > m_{\text{ext}}\) from up to down, respectively.
Fig. 2. $F(r)$ versus $r$ for $n = 4$, $q = 1$, $s = 3$, $\Lambda = -1$, $m < m_{\text{ext}} < 0$, $m = m_{\text{ext}} < 0$, $m_{\text{ext}} < m < 0$ and $m > 0$ from up to down, respectively.

Fig. 3. $F(r)$ versus $r$ for $n = 4$, $q = 1$, $s = 3/4$, $\Lambda = -1$, $m < 0$, $0 < m < m_{\text{ext}}$, $m = m_{\text{ext}}$ and $m > m_{\text{ext}}$ from up to down, respectively.

Fig. 4. $F(r)$ versus $r$ for $n = 4$, $q = 1$, $s = -3$, $\Lambda = -1$, $m < m_{\text{ext}} < 0$, $m = m_{\text{ext}} < 0$, $m_{\text{ext}} < m < 0$ and $m > 0$ from up to down, respectively.

Fig. 5. $F(r)$ versus $r$ for $n = 4$, $q = 0.5$, $s = n/2$, $\Lambda = -1$, $m < 0$, $0 < m < m_{\text{ext}}$, $m = m_{\text{ext}}$ and $m > m_{\text{ext}}$ from up to down, respectively.

and it corresponds to uncharged asymptotically AdS one.

Second, we look for the essential singularities. One can show that all curvature invariants (such as Kretschmann scalar, Ricci square, Weyl square and so on) are functions of $F''$, $F'/r$ and $F/r^2$ and therefore it is sufficient to study the Kretschmann scalar for the investigation of the spacetime curvature. After some algebraic manipulation, we can show that the Kretschmann scalar diverges at $r = 0$ and is finite for $r \neq 0$. Thus, there is a curvature singularity located at $r = 0$. The metric (13) has two types of horizons: a Killing horizon and an event horizon. The Killing horizon is a null surface whose null generators are tangent to a Killing field. It is proved that a stationary black hole event horizon should be a Killing horizon in four-dimensional Einstein gravity [70,71]. In general this proof is not valid for higher dimensional black holes, but the result is true for all the known static solutions. The event horizon is the hypersurface in which light can no longer escape from the gravitational pull of a black hole. For calculation of the event horizon, one can use the time dilation interpretation (gravitational red shift) [71,72,73].

It is straightforward to show that the event horizon of the presented rotating solutions are located at the root(s) of $F(r) = 0$. Numerical calculations (see for e.g. Figs. 1, 3 and 5) show that for $s = 1$ (linear case) and $1/2 < s \leq n/2$, the metric of Eqs. (13) and (15) has two inner and outer event horizons located at $r_-$ and $r_+$, provided the mass parameter $m$ is greater than $m_{\text{ext}}$ given as

$$m_{\text{ext}} = \frac{4A_{\text{ext}}}{n(n-1)(n-2)} + \frac{\Pi_2}{(n-2)}$$

(17)
\[ I_2 = \begin{cases} 2^{n/2} r_{\text{ext}}^{n} \left[ 1 - (n - 2) \ln r_{\text{ext}} \right], & s = \frac{n}{2} \\ \frac{2(2s - 1)(ns - 3s + 1)}{n(n - 1)(n - 2)(n - 2s)} \left( \frac{2r_{\text{ext}}^2(n - 2s)^2}{(2s - 1)(2s - 1)2^s} \right)^s r_{\text{ext}}^{2s - n} & \frac{1}{2} < s < \frac{3}{2} \end{cases} \]

These solutions present naked singularity for \( m < m_{\text{ext}} \) and when \( m = m_{\text{ext}} \), we have an extreme black brane with horizon radius

\[ r_{\text{ext}} = \begin{cases} \left( \frac{2r_{\text{ext}}^2(n - 2s)^2}{(2s - 1)(2s - 1)2^s} \right)^s r_{\text{ext}}^{(2s-n)/(2s-1)}, & s = \frac{n}{2} \\ \left[ \frac{2r_{\text{ext}}^2(n - 2s)^2}{(2s - 1)(2s - 1)2^s} \right]^{s} r_{\text{ext}}^{(2s-n)/(2s-1)}, & \frac{1}{2} < s < \frac{3}{2} \end{cases} \]

For \( s = 0, 1/2 \), the solutions do not have any charge term and they are uncharged black holes, then these cases are excluded from the discussion of \( m_{\text{ext}} \). Also for \( 0 < s < 1/2 \), the metric function does not have a good behavior asymptotically and we are not interested in this case. Unlike the Reissner-Nordström solutions (\( s = 1 \)) or \( 1/2 < s \leq n/2 \), for \( s > n/2 \) and \( s < 0 \), the singularity at \( r = 0 \) for the solutions with non-negative mass is spacelike, and therefore it is unavoidable. This is due to the fact that \( f(r) \) approaches to \(-\infty\) as \( r \) goes to zero and goes to \(+\infty\) at large \( r \). These solutions with positive mass present black branes with one horizon and this spacetime with negative mass presents a naked singularity if \( m < m_{\text{ext}} \), an extreme black brane for \( m = m_{\text{ext}} \) and a black brane with inner and outer horizons provided \( m_{\text{ext}} < m < 0 \), where \( m_{\text{ext}} \) is

\[ m_{\text{ext}} = \frac{4\pi r_{\text{ext}}^n}{n(n - 1)(n - 2)} + \frac{2(2s - 1)(ns - 3s + 1)}{n(n - 1)(n - 2)(n - 2s)} \times \left( \frac{2r_{\text{ext}}^2(n - 2s)^2}{(2s - 1)(2s - 1)2^s} \right)^s r_{\text{ext}}^{(2s-n)/(2s-1)}, \]

where

\[ r_{\text{ext}} = \begin{cases} \left( \frac{2r_{\text{ext}}^2(n - 2s)^2}{(2s - 1)(2s - 1)2^s} \right)^s r_{\text{ext}}^{(2s-n)/(2s-1)}, & \frac{1}{2} < s \leq \frac{3}{2} \\ \left[ \frac{2r_{\text{ext}}^2(n - 2s)^2}{(2s - 1)(2s - 1)2^s} \right]^{s} r_{\text{ext}}^{(2s-n)/(2s-1)}, & -1 < s < \frac{1}{2} \end{cases} \]

and \( - \) or \( + \) correspond to \( s > n/2 \) or \( s < 0 \), respectively (see figures [2] and [1] for clarity).

In some of the classic solutions of the general relativity equations (e.g. Schwarzschild and Robertson-Walker) no such closed causal curves occur, although several examples of spacetimes which do admit closed timelike curves are known [71, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90]. The possibilities of formation of closed causal curves in spacetime, within the framework of general relativity have been discussed in many literatures [22, 83, 84, 85, 86, 87]. Also, the rotating black objects have a net angular momentum, in which its value may approach a critical value. In this “under-rotating” case (under critical value) the closed timelike curves are enclosed inside a velocity of light surface which lies inside the horizon. In the over-rotating case there are closed timelike curves outside what superficially appears to be a horizon [88, 89, 90]. If we consider the rotating \((n + 1)\)-dimensional metric in the following form

\[ ds^2 = -V dt^2 + 2W dt d\phi + X d\phi^2 + g_{ab} dx^a dx^b, \]

where \( a, b = 1, 2, ..., n - 1 \), and \( V, X \) and \( W \) depend on \( x^n \). Ergospheres are given by \( V = 0 \) and the velocity of light surface is given by \( X = 0 \). Comparing our rotating spacetime [13] with the former metric, one can show that \( X = 0 \) leads to

\[ F(r) = -\frac{\Xi^2 - 2r^2}{\Xi^2 - r^2}, \]  

where for sufficiently large rotation parameter(s) \( \Xi \rightarrow \infty \), reduces to \( F(r) = -\frac{r^2}{\Xi} \). This result is overlap to our solution for sufficiently large distance from the black hole and positive cosmological constant \((\Lambda = n(n - 1)/2l^2)\). Thus in the presented spacetime [13] with negative \( \Lambda \), there is not closed timelike curves.

Although our solution is not static, the Killing vector

\[ \chi = \partial_t + \sum_{i=1}^{n} \Omega_i \partial_{\phi_i}, \]

is the null generator of the event horizon where \( \Omega_i \) is the \( i \)-th component of angular velocity of the outer horizon. The angular velocities \( \Omega_i \)'s may be obtained by analytic continuation of the metric. Setting \( a_i \rightarrow ia_i \) yields the Euclidean section of (13), whose regularity at \( r = r_{+} \) requires that we should identify \( \phi_i \sim \phi_i + \beta \Omega_i \). One obtains

\[ \Omega_i = \frac{a_i}{2l^2} \]  

The temperature may be obtained through the use of the definition of surface gravity,

\[ T_{+} = \beta^{-1} = \frac{1}{2}\sqrt{-\frac{1}{2} (\nabla_{\mu} \chi_{\nu}) (\nabla^{\mu} \chi^{\nu})} \]

where \( \chi \) is the Killing vector [22]. One obtains

\[ T_{+} = \frac{A_{r_{+}}}{2\pi \Xi(n - 1)} + \frac{\Pi_{3}}{4n \Xi} \]

\[ \frac{(2s-1)^2 - 4s^2}{(2s-1)(2s-1)^2 r_{+}} \]  

\[ 0 < s < \frac{1}{2}, \frac{1}{2} < s < 0 \]

\[ \Pi_{3} = \frac{-\frac{2r_{+}^n}{n^2}}{r_{+}} \]  

\[ \frac{(2s-1)^2 - 4s^2}{(2s-1)(2s-1)^2 r_{+}} \]  

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3.2 Conserved quantities

More than thirty years ago, Bekenstein argued that the entropy of a black hole is a linear function of the area of its event horizon, which so-called area law [91, 92]. He also proposed a value for the proportionality constant, deduced from a semiclassical calculation of the minimum increase in the area of a black hole when it absorbs a particle. Bekenstein’s statements can in fact be generalized by considering the quantum nature of the particle
and taking then into account the uncertainty principle and the energy-momentum dispersion relation \[93\]. This generalized argument leads essentially to the same conclusion about the linearity of the entropy with respect to the black hole area. Since the area law of the entropy is universal, and applies to all kinds of black holes/branes in Einstein gravity \[91,92\], therefore the entropy per unit volume \( V_{n-1} \) of the presented black branes is equal to one-quarter of the area of the horizon, i.e.,

\[
S = \frac{\pi}{4} r^{n-1}.
\]

The electric charge per unit volume \( V_{n-1} \) of the black branes, \( Q \), can be found by calculating the flux of the electromagnetic field at infinity, yielding

\[
Q = \begin{cases} 
\frac{(-1)^{(n+2)/2} n^{n/2} \Xi n^{-1}}{8\pi}, & s = \frac{n}{2} \\
\frac{(-1)^{n+1} 2^n \Xi (2^n - 1)}{8\pi} 2^{n-1}, & \text{otherwise}
\end{cases}
\]

The present spacetime \[13\], have boundaries with timelike \((\xi = \partial/\partial t)\) and rotational \((\varsigma = \partial/\partial \varphi)\) Killing vector fields. From Eq. \[9\], one obtains the quasilocal mass and angular momentum

\[
M = \int_B d^{n-1} \varphi \sqrt{\tilde{g}} \tilde{T}_{\alpha\beta} \tilde{h}^{\alpha \beta} = \frac{V_{n-1}}{16\pi} (n\Xi^2 - 1) m
\]

\[
J = \int_B d^{n-1} \varphi \sqrt{\tilde{g}} \tilde{T}_{\alpha\beta} \tilde{h}^{\alpha \beta} = \frac{V_{n-1}}{16\pi} n\Xi m a_i
\]

provided the hypersurface \( B \) contains the orbits of \( \varsigma \). Eqs. \[27\]–\[29\] show that, like the Born-Infeld gravity \[30\,50\,51\,52\,53\,57\,63\,65\,66\], although, the presence of the nonlinear Maxwell field has no effects on the mass and angular momentum of the black brane, the nonlinear parameter, \( s \), change the electrical charge of black brane. It is a matter of straightforward calculation to show that the conserved and thermodynamics quantities satisfy the first law of thermodynamics

\[
dM = TdS + \sum_{i=1}^{k} \Omega_i dJ_i + \Phi dQ.
\]

It is notable that, in general, the temperature and the other conserved quantities are independent of the choice of coordinates. Eqs. \[27\]–\[29\] show that the thermodynamics and conserved quantities are different for static and rotating spacetimes, and consequently, confirm that the static and rotating metrics are distinct and they describe two different spacetimes.

**4 CLOSING REMARKS**

In this paper we generalized the solutions of Ref. \[68\] for arbitrary values of nonlinear parameter \( s \). At first, we considered Einstein-Hilbert action with cosmological constant coupled to an abelian gauge field for which the Lagrangian is given by a power of the Maxwell invariant. The presented nonlinear parameter effected on the electromagnetic field and charge term of metric functions. One can show that these solutions reduce to the solutions of linear and conformally invariant Maxwell field provided the nonlinear parameter \( s = 1 \) and \( s = (n+1)/4 \), respectively.

Then, by a suitable rotation boost, we presented the horizon flat rotating spacetime with \( \kappa \leq [n/2] \) rotation parameters. We investigated the asymptotic behavior of these solutions and found that for the special values of the nonlinear parameter \( s \), the solutions are not asymptotically AdS. Also, we found the essential singularity located at \( r = 0 \) and showed that for a suitable values of \( s \), the presented solutions may be interpreted as black brane solutions with two event horizons, extreme black hole or naked singularity provided the mass parameter, \( m \), of the solutions is more than, less than or equal to the extreme value \( m_{ext} \).

In addition, it is worth to mention that for \( s > n/2 \) and \( s < 0 \), the singularity at \( r = 0 \) for the solutions with non-negative mass is spacelike. This is due to the fact that \( f(r) \) approaches to \( -\infty \) as \( r \) goes to zero and goes to \( +\infty \) at large \( r \). These solutions with positive mass present black holes with one horizon. Also, we investigated the existence of closed timelike curves and showed that they can exist for sufficiently large distance from the black hole and sufficiently large rotation parameter, only for positive cosmological constant.

At last, using the counterterm approach, area and Gauss laws, we computed the quasi local mass and angular momentum, entropy and electrical charge of the rotating black branes. These calculations showed that only the electrical charge depend on the nonlinear parameter. In fact the nonlinearity effects on the electric field, the behavior of spacetime, temperature, type of singularity, electrical charge and all the expressions include of the charge parameter. Finally, it is worthwhile to examine the thermodynamical stability as well as dynamical (gravitational) stability of these black branes.

This work has been supported financially by Research Institute for Astronomy and Astrophysics of Maragha.

**References**

1. M. Born and L. Infeld, Proc. R. Soc. London A 143, 410 (1934)
2. M. Born and L. Infeld, Proc. R. Soc. London A 144, 425 (1934)
3. J. Plebanski, *Lectures on Non–Linear Electrodynamics* (Nordita, 1968)
4. E. S. Fradkin and A. Tseytlin, Phys. Lett. B, 163, 123 (1985)
5. A. Tseytlin, Nucl. Phys B 276, 391 (1986)
6. N. Seiberg and E. Witten, JHEP 09, 032 (1999)
7. D. A. Rasheed, [arXiv:9702087]
8. M. Novello and S. E. Perez Bergliaffa, Phys. Rept. 463, 127 (2008)
9. M. Novello, E. Goulart, J. M. Salim and S. E. Perez Bergliaffa, Class. Quant. Grav. 24, 3021 (2007)
37. N. Breton, S. E. Perez Bergliaffa and J. Salim, Phys. Rev. D 69, 127301 (2004)
38. R. G. Cai, D. W. Pang and A. Wang, Phys. Rev. D 75, 044030 (2007)
39. R. G. Cai and Y. W. Sun, JHEP 09, 124018 (2006)
40. S. H. Mazharimousavi and M. Halilsoy, Phys. Lett. B 681, 190 (2009)
41. M. Dehghani and H. R. Rastegar Sedehi, Phys. Rev. D 74, 124018 (2006)
42. M. Dehghani, S. H. Hendi, A. Sheykhi and H. Rastegar Sedehi, JCAP 02, 020 (2007)
43. S. H. Hendi, J. Math. Phys. 49, 082501 (2008)
44. K. A. Bronnikov, Phys. Rev. D 63, 044005 (2001)
45. M. Dehghani, N. Bostani and S. H. Hendi, Phys. Rev. D 78, 064031 (2008)
46. M. Dehghani, A. Sheykhi and S. H. Hendi, Phys. Lett. B 659, 476 (2008)
47. M. Dehghani and S. H. Hendi, Int. J. Mod. Phys. D 16, 1829 (2007)
48. S. Mukherji and S. Pal, [arXiv:08062507]
49. M. Dehghani and S. H. Hendi, Gen. Rel. Grav. 41, 1853 (2009)
50. M. Aiello, R. Ferraro and G. Giribet, Phys. Rev. D 70, 104014 (2004)
51. M. Dehghani, N. Alinejadi and S. H. Hendi, Phys. Rev. D 77, 104025 (2008)
52. R. G. Cai, S. Wang, J. He and Y. W. Sun, JHEP 09, 124034 (2006)
53. R. G. Cai and Y. W. Sun, JHEP 09, 124018 (2006)
54. S. H. Mazharimousavi and M. Halilsoy, Phys. Lett. B 681, 190 (2009)
55. M. Dehghani and H. R. Rastegar Sedehi, Phys. Rev. D 74, 124018 (2006)
56. M. Dehghani, S. H. Hendi, A. Sheykhi and H. Rastegar Sedehi, JCAP 02, 020 (2007)
57. S. H. Hendi, J. Math. Phys. 49, 082501 (2008)
58. K. A. Bronnikov, Phys. Rev. D 63, 044005 (2001)
59. M. Dehghani, N. Bostani and S. H. Hendi, Phys. Rev. D 78, 064031 (2008)
60. M. Dehghani, A. Sheykhi and S. H. Hendi, Phys. Lett. B 659, 476 (2008)
61. M. Dehghani and S. H. Hendi, Int. J. Mod. Phys. D 16, 1829 (2007)
62. S. Mukherji and S. Pal, [arXiv:08062507]
63. M. Dehghani and S. H. Hendi, Gen. Rel. Grav. 41, 1853 (2009)
64. M. Aiello, R. Ferraro and G. Giribet, Phys. Rev. D 70, 104014 (2004)
65. M. Dehghani, N. Alinejadi and S. H. Hendi, Phys. Rev. D 77, 104025 (2008)
66. R. C. Myers, Phys. Rev. D 36, 392 (1987)
67. S. C. Davis, Phys. Rev. D 67, 024030 (2003)
68. V. Balasubramanian and P. Kraus, Commun. Math. Phys. 208 (1999) 413
94. S. W. Hawking and C. J. Hunter, Phys. Rev. D 59, 044025 (1999)
95. S. W. Hawking, C. J. Hunter and D. N. Page, Phys. Rev. D 59, 044033 (1999)
96. R. B. Mann, Phys. Rev. D 60, 104047 (1999)
97. R. B. Mann, Phys. Rev. D 61, 084013 (2000)
98. C. J. Hunter, Phys. Rev. D 59, 024009 (1999)