Evolution of growing black holes in axisymmetric galaxy cores

J. Fiestas,1,2★ O. Porth,3 P. Berczik1,2,4 and R. Spurzem1,2,5★

1Zentrum für Astronomie der Universität Heidelberg, Astronomisches Rechen-Institut, Mönchhofstraße 12–14, D-69120 Heidelberg, Germany
2National Astronomical Observatories of China, Chinese Academy of Sciences NAOC/CAS, 20A Datun Rd., Chaoyang District, Beijing 100012, China
3Max-Planck-Institut für Astronomie, Königstuhl 17, D-69117 Heidelberg, Germany
4Main Astronomical Observatory, National Academy of Sciences of Ukraine, MAO/NASU, 27 Akademika Zabolotnogo St. 03680 Kyiv, Ukraine
5The Kavli Institute for Astronomy and Astrophysics at Peking University, Beijing 100871, China

Accepted 2011 August 19. Received 2011 August 19; in original form 2011 April 13

ABSTRACT

N-body realizations of axisymmetric collisional galaxy cores (e.g. M32, M33, NGC 205, Milky Way) with embedded growing black holes are presented. Stars which approach the disruption sphere are disrupted and accreted to the black hole. We measure the zone of influence of the black hole and disruption rates in relaxation time-scales. We show that secular gravitational instabilities dominate the initial core dynamics, while the black hole is small and growing due to the consumption of stars. Later, the black hole potential dominates the core, and the loss cone theory can be applied. Our simulations show that central rotation in galaxies cannot be neglected for relaxed systems, and compare and discuss our results with the standard theory of spherically symmetric systems.

Key words: black hole physics – gravitation – galaxies: evolution – galaxies: kinematics and dynamics – galaxies: nuclei.

1 INTRODUCTION

Galaxy cores are the hosts of supermassive black holes (SMBHs), the engines of quasars and active galactic nuclei (AGN). There is increasing evidence that SMBHs play an important role in the formation and global evolution of galaxies. They are commonly observed at the centres of many nearby galaxies (Shankar 2009), and the existence of quasars at least up to redshifts $z = 6$ (Degraf, Di Matteo & Springel 2010; Willott et al. 2010) implies that many of these SMBHs reached nearly their current mass at very early times. The evolution of galactic nuclei during and after the era of peak quasar activity therefore took place with the SMBHs already in place. The energy released by SMBHs during and after the quasar epoch must have had a major impact on how gas cooled to form galaxies and galaxy clusters (Scannapieco, Silk & Bouwens 2005). However, the detailed history of SMBH growth is still being debated. Some work has focused on the possibility that the seeds of SMBHs were black holes (BHs) of much smaller mass – either remnants of the first generation of stars, so-called ‘Population III BHs’ (Madau & Rees 2001), or the (still speculative) ‘intermediate-mass black holes’ (IMBHs), remnants of massive stars that form in dense clusters via physical collisions between stars (Portegies Zwart et al. 2004; Mapelli et al. 2010).

Observations with the Hubble Space Telescope have elucidated the run of stellar density near the centres of nearby galaxies (Ferrarese et al. 2006a; Côte et al. 2007; Glass et al. 2011). Nevertheless, in the majority of galaxies massive enough to contain SMBHs, the central relaxation time is much greater than the age of the universe, due both to the (relatively) low stellar densities and also to the presence of a SMBH, which increases $v_{\text{rms}}$ (Faber et al. 1997; Ferrarese et al. 2006b). These long relaxation times imply that nuclear structure will still reflect the details of the nuclear formation process. Beyond the Local Group, essentially all of the galaxies for which the SMBH’s influence radius is spatially resolved have ‘collisionless’ (non-relaxed) nuclei with low nuclear densities. The nuclei of these ‘core’ galaxies may have been much denser before the cores were created by probably binary SMBHs.

Only the smallest galaxies known to harbour SMBHs have nuclear relaxation times of $\sim 10$ Gyr or shorter. In such a nucleus the stellar distribution will have had time to evolve to a collisionally relaxed system. Galactic spheroids fainter than $M_V = -18$ show this property. The Milky Way nucleus is ‘collisional’. It has a half-mass relaxation time $t_{\text{bh}} \sim 5 \times 10^9$ yr but the steep density profile implies $r_{\text{bh}} \sim 6 \times 10^7$ yr at $0.2 r_{\text{bh}}$ (0.6 pc), where $r_{\text{bh}}$ is the half-mass radius, and $\sim 3.5 \times 10^8$ yr at $0.1 r_{\text{bh}}$ (0.3 pc), assuming solar-mass stars (Merritt 2006). Collisional nuclei are present in three other Local Group galaxies (M32, M33 and NGC 205) (Lauer et al. 1998; Valluri et al. 2005) although M32 is the only one of these to exhibit dynamical evidence for a SMBH (Valluri et al. 2005).

Since the 1980s, the dominant model for the formation of elliptical galaxies and bulges – the stellar systems that contain SMBHs – has been the merger model (Toomre 1977). An almost certain consequence of a merger is the in-fall of the progenitor galaxies’ SMBHs into the nucleus of the merged system, resulting in the formation of a binary SMBH (Begelman, Blandford & Rees 1980), as observations of uncoalesced dual SMBHs show (Rodriguez et al. 2006; Valtonen et al. 2008). There is no generally agreed idea on whether and how fast a binary SMBH coalesces after a galaxy merger. Earlier work (mostly numerical simulations)
in spherically symmetric nuclei discussed a stalling problem which would hang up the binary SMBH at some sub-parsec separation (last parsec problem, cf. e.g. Gould & Rix 2000; Hensendorf, Sigurdsson & Spurzem 2002; Milosavljević & Merritt 2003; Makino & Funato 2004; Berczik, Merritt & Spurzem 2005). But it turns out that any degree of more realism helps in clearing the last parsec problem, such as axisymmetry of the galaxy merger remnant (Berczik et al. 2006; Perets & Alexander 2008; Berentzen et al. 2009; Berczik et al., in preparation; Gualandris & Merritt 2011) or the presence of gaseous material in the nucleus (Callegari et al. 2009, 2011; Dotti et al. 2009; Mayer et al. 2010). A self-consistent study of the combined effect of high-accuracy high-resolution stellar dynamics with binary black holes (BBHs), together with the evolution of a central galactic nuclear disc, and its interaction with stars and BHs, is still lacking to our knowledge. Approaching this goal could be in our view the pioneering semi-analytical study of Vilokovskij & CZerny (2002) or the detailed gas and stellar dynamical of Johansson, Burkert & Naab (2009b) and Johansson, Naab & Burkert (2009a), but the latter lack the proper resolution of previously cited purely stellar dynamical work to follow the sub-parsec evolution of the binary SMBH well. One of the most detailed descriptions of physical processes between stars and gas in galactic nuclei has been presented by Ciotti, Ostriker & Proga (2009, 2010) and Shin, Ostriker & Ciotti (2010a,b). Their central resolution is much better than that of Johansson et al. (2009a,b), but their models are spherically symmetric and unable to properly resolve the collisional evolution of binary SMBH in dense gas-star systems in galactic nuclei. Another preliminary approach of Just et al. (2011) follows the star–disc interactions with direct high-accuracy stellar dynamical models [as a follow up to Vilokovskij & CZerny (2002), but still uses a stationary disc model].

In this study we investigate following mechanisms important in determining the structure and evolution of collisional galactic nuclei embedding growing SMBHs.

(i) Destruction of stars by the SMBH. An SMBH defines a ‘loss cone’ of orbits that pass within its event horizon or tidal disruption sphere, \( r \leq r_{\text{t}} \). Indeed, the existence of a capture sphere is crucial for solutions like Bahcall & Wolf’s, since it precludes the formation of an isothermal \((f \sim e^2)\) distribution of velocities which would necessarily have a very high density near the SMBH (Peebles 1972). Continued loss of stars to the SMBH also implies that no precisely steady-state equilibrium can exist (Shapiro 1977; Baumgardt et al. 2006); the nucleus will expand, on a relaxation time-scale, due to the effective heat input as stars are destroyed (Shapiro 1977).

Continued supply of stars to the SMBH requires some mechanism for loss cone re-population. The most widely discussed mechanism is gravitational encounters, which drive a diffusion in energy \((E)\) and angular momentum \((J)\). The latter dominates the loss rate (Frank & Rees 1976; Lightman & Shapiro 1977). Roughly speaking, many of the stars dominated by the BH potential (i.e. within its influence sphere) will be deflected into \( r_{\text{t}} \) in one relaxation time, i.e. the loss rate is roughly \((M_{\ast} m / \dot{m} (r_{\text{t}}))^{-1}\). In a collisional nucleus with \( M_{\ast} \approx 10^6 M_{\odot}\), this is \(10^{10} / (10^{10} \text{yr}) \approx 10^{-4} \text{yr}^{-1}\).

Stellar disruption has direct observational consequences. Tidally disrupted stars produce X- and UV radiation with luminosities of \(10^{44} \text{erg s}^{-1}\), potentially outshining their host galaxies for a period of days or weeks (Khokhlov & Melia 1996; Kobayashi et al. 2004). A handful of X-ray flaring events have been observed that have the expected signature (Komossa et al. 2004), and the number of detections is crudely consistent with theoretical estimates of the event rate (Wang & Merritt 2004). Tidal flaring events may dominate the X-ray luminosity function of AGN at \( L_x \lesssim 10^{44} \text{erg s}^{-1}\) (Rees 1988; Milosavljević, Merritt & Ho 2006). Compact objects (neutron stars or stellar-mass black holes) can remain intact at much smaller distances from the SMBH; these objects would emit gravitational waves at potentially observable amplitudes before spiralling in and may dominate the event rate for low-frequency gravitational wave interferometers like Laser Interferometer Space Antenna (LISA) (Hopman & Alexander 2006; Eilon, Kupi & Alexander 2009).

That loss cones can be much more quickly refilled in even only slightly axisymmetric or triaxial nuclei had been realized much earlier in the context of tidal accretion of stars on to single BHs (Norman & Silk 1983; Yu & Tremaine 2002; Merritt & Poon 2004). It is quite natural after a galaxy merger that the merger remnant is not spherically symmetric and is not completely void of gas, so one would expect frequent mergers of SMBHs as well. It seems this is consistent with the cosmological evolution of galaxy and BH populations (Hirschmann et al. 2010), and also the LISA gravitational wave community expects frequent coalescences of binary SMBH in the Universe (Sesana 2010). Tidal disruptions of stars by BHs have recently only been studied by Chen, Liu & Magorrian (2008), Chen et al. (2009, 2011) and Liu, Li & Chen (2009) in the context of X-ray flares. They find, while the total amount of X-ray flares related to binary SMBH may be small (of the order of 10 per cent), they could show a special behaviour in the form of bursts and interruptions of tidal disruptions.

(ii) The Bahcall–Wolf mechanism.

In a collisional nucleus exchange of energy between stars drives the system towards an approximately steady-state distribution of stars around the SMBH in a relaxation time. For a single stellar mass, this is \( f(E) \sim |E|^{1/4}, \rho \sim r^{-7/4} \) (Bahcall & Wolf 1977). Since galaxies with collisional nuclei probably always have \( M \lesssim 10^8 M_{\odot}\) the tidal disruption sphere is more relevant than the Schwarzschild radius. Another condition for the Bahcall–Wolf solution is that \( r_{\text{t}} \) is much smaller than \( r_{\text{m}} \) (\( |E| \geq GM/r_{\text{m}} \)), which is the case in real nuclei. It represents a ‘zero-flux’ solution. Nevertheless, in the numerical solutions, the steady-state flux is found to be small but non-zero. The flux is determined by the rate at which stars can diffuse into the disruption sphere at \( r_{\text{t}} \).

The Bahcall–Wolf solution has been verified in a number of other studies based on fluid (Amaro-Seoane, Freitag & Spurzem 2004) or Monte Carlo (Marchant & Shapiro 1980; Freitag & Benz 2002) approximations to the Fokker–Planck equation. It has been tested via direct \( N \)-body integrations, avoiding the approximations of the Fokker–Planck formalism (Preto, Merritt & Spurzem 2004; Baumgardt et al. 2004).

Observational measurements of the galactic centre reveal a stellar cusp, which appears to be flatter than expected in a collisionally relaxed population around an SMBH (Schödel et al. 2007; Do et al. 2009; Merritt 2010). A possible explanation for the absence of a cusp in the observed stars around Sgr A* includes mass segregation (which leads to the formation of flatter cusps by lighter stars and steeper cusps by massive central stars, by relaxation), or the destruction of the envelopes of giant stars in the densest parts of the cluster (Dale et al. 2009). It is not clear that Bahcall–Wolf cusps are present in any other galaxy however, both because relaxation times are generally \( \gg 10^{10} \text{yr} \) and also because they are difficult to be observationally resolved.

Moreover, if binary SMBHs formed during galaxy mergers, they can destroy dense nuclei, as has been observed in the central density profiles of ‘mass deficits’ of bright elliptical galaxies (Merritt & Szell 2006). An important question is whether the existence of dense cusps at the centres of galaxies such as the Milky Way and M32 implies that no binary SMBH was ever present, or whether a
collisional cusp could have spontaneously regenerated after being destroyed.

The relative importance of these and other various mechanisms, like physical collisions between stars, gas inflow to the SMBH and the nature of the dark matter that permeate galaxies and their possible interactions, is still not well understood. One of the most advanced physical descriptions including a detailed multi-phase treatment of interstellar matter and mechanical and thermal feedback due to stars and central BHs (AGN) has been presented by Ciotti et al. (2009, 2010) and Shin et al. (2010a). Their approach, however, still lacks spatial and physical resolution compared to isolated galactic models.

2 THE METHOD

We define stellar accretion via the loss cone, which in a spherical galaxy is given by orbits, for which specific energy \( \epsilon \) and angular momentum \( J \) lie within

\[
J^2 \leq J_\theta^2(\epsilon) \equiv 2r_\epsilon^2[\phi(r_\epsilon) - \epsilon] \simeq 2GM_r r_\epsilon,
\]

(1)

where \( G \) is the gravitational constant, \( \phi \) is the BH potential and \( J_\theta \) denotes the loss cone size. Stars of mass \( m_\star \) and radius \( r_\star \) that come within a distance

\[
r_\epsilon = \alpha r_\star \left( \frac{2M_\star}{m_\star} \right)^{1/3}
\]

(2)

of the central BH with mass \( M_\star \) will be tidally disrupted and accreted. 100 per cent accretion efficiency is assumed. We include in this definition a free parameter \( \alpha \), for scaling of the tidal radius in our simulations (see description in Section 3). For \( M_\star \lesssim 10^8 M_\odot \), the tidal radius does not fall below the Schwarzschild radius, if we assume solar-type stars (Frank & Rees 1976).

Time-scales considered here are of the order of the relaxation time, or

\[
t_r \approx 0.065 \sigma^3/(G^2m_\rho \rho \ln \Lambda),
\]

(3)

where \( \rho \) is the mass density, \( \sigma \) is the 3D velocity dispersion and \( \ln \Lambda \) is the Coulomb logarithm (Spitzer 1987). We adopt \( \Lambda = 0.11N \) (Giersz & Heggie 1994). The relation between the dynamical or crossing time \( t_{\text{dyn}} \) and \( t_r \) is given by

\[
t_{\text{dyn}} \sim \frac{\ln \Lambda}{N} t_r,
\]

(4)

where \( t_{\text{dyn}} = \sigma \rho /r \) is a characteristic radius of the system, usually \( r_{\text{in}} \). Here we point out the strong dependence of \( t_r \) on \( N \). We will specially scale our models in Section 3.2.

2.1 Influence and wandering radius

Motion of stars surrounding the BH in a certain region is directly influenced by its gravitational field. This region is given by the influence radius of the BH, defined as the radius, where the mass in stars is of the order of \( M_\star \) [for an isothermal sphere \( M(<r_\star) = 2M_\star; \) Merritt & Szell 2006]. Frank & Rees (1976) define it as

\[
r_\star = \frac{GM_\star}{\sigma^2}
\]

(5)

where \( \sigma \) is here the 1D velocity dispersion. Although both definitions do not always agree well, if the mass distribution is known, the first one is easy to determine.

The motion of a heavy particle in a sea of lighter particles can be treated as the Brownian motion. As Chatterjee, Hernquist & Loeb (2002) point out, for King models with \( W_\odot \geq 3 \) (as used in the present study) equipartition of the BH with its surrounding core is a valid assumption (Merritt, Berczik & Laun 2007). The wandering radius can be define as

\[
r_{\text{walk}} = 0.5 \left( \frac{m_\star}{M_\star} \right)^{1/2} r_{\text{in}}.
\]

(6)

where \( r_{\text{in}} \) is the N-body unit of length. This empirical relation provides a decent fit for the measured BH wandering during the whole simulated time in all considered models. This random walk might hinder the formation of a (7/4) cusp, as it stirs up the central region and smears out the eventual overdensities. Superposed on the random walk, it might have a non-vanishing mean velocity varying on much larger time-scales. The BH exerts high-frequency oscillations, and has an additional drift, which is observed in the centre of mass vector as well (Makino & Sugimoto 1987). We correct this effect by subtracting every time-step of the centre of mass from the BH position.

2.2 Loss cone flux

In a time \( t_r \), gravitational encounters between stars can exchange orbital energy and angular momentum. Core collapse (shrinking of the core to zero size and infinite density) does not happen in galactic nuclei embedding a SMBH, since the central potential triggers core expansion and the central density drops.

The concept of a loss cone as introduced by Frank & Rees (1976) can be used to identify stars on an orbit penetrating the BHs Roche lobe. As these loss cone orbits are disrupted within an orbital period, the stellar mass supply must run out immediately if they are not replenished by relaxation. The loss cone can be defined as an angle-like variable giving the half aperture of the BH as seen from the stars’ distance. Trajectories with a smaller aperture have a peribothron \( \leq r_\star \) and will be lost.

In a potential dominated by the BH, taking advantage of the Keplerian velocity profile,

\[
\sigma = \sqrt{\frac{GM_\star}{r}}
\]

(7)

and using equation (1) for \( \epsilon < \epsilon_1 \) (Frank & Rees 1976), the loss cone angle \( \theta_{\text{lc}} = v_\phi /\sigma \) evaluates to

\[
\theta_{\text{lc}}(r) = \left( \frac{2 r_\epsilon}{\sqrt{3} r} \right)^{1/2}.
\]

(8)

In a steady-state spherical system, the only process refilling the loss cone is scattering of stars due to distant gravitational encounters. For a general \( r^{-\eta} \) density cusp and \( \sigma \) given by equation (7), the relaxation time within the BH sphere of influence becomes

\[
t_r = 0.338 \frac{M_\star^{3/2}}{G^{1/2}m_\rho \rho \ln \Lambda} r^{\eta-3/2},
\]

(9)

where \( r_\rho \) and \( \rho_\rho \) are radius and mass density at the influence radius, and the angular diffusion per orbital period \( \theta_\rho = (t_{\text{dyn}}/t_r)^{1/2} \) turns out to be

\[
\theta_\rho = 2.960 r^{3-\eta} r_\rho^3 M_\star^{-2} m_\rho \rho \ln \Lambda.
\]

(10)

Comparing the two angles, one customarily defines two regimes: the empty loss cone or diffusive regime for small \( r \) where \( \theta_{\text{lc}} > \theta_\rho \) and the full loss cone or pinhole regime, where stars can move through the loss cone within one orbital period \( (\theta_{\text{lc}} < \theta_\rho) \). Lightman

© 2011 The Authors, MNRAS 419, 57–69

Monthly Notices of the Royal Astronomical Society © 2011 RAS
& Shapiro (1977) showed that the flux into the loss cone peaks at \( \theta_D = \theta_k \), which defines the critical radius

\[
r_{\text{crit}} = \left( \frac{0.225 M^2}{m_\ast \rho_\ast \ln \Lambda} \right)^{\frac{1}{3}} r_{\text{crit}}
\]

(11)

for the assumption of a power-law density profile within the BHs radius of influence. Hence, the last relation is valid for \( r_{\text{crit}} < r_{\text{in}} \), which holds for most of the 51 elliptical galaxies in the sample of Wang & Merritt (2004).

Lightman & Shapiro (1977) showed that the integrated number flux per orbital period can be approximated to match its value at the critical energy,

\[
F(E) \sim F(E_{\text{crit}})|E_{\text{crit}}|
\]

(12)

where \( F \) gives the flux of stars at \( E = E_{\text{crit}} \). A simplified expression of the expected disruption rate for the steady-state solution (\( \eta = 1.75 \)) can be obtained by applying the scaling of \( N(r) \propto r^{5/4} \), and \( t_c \propto \alpha^{-1/2} r(r) \propto r^{3/2} \ln \Lambda \propto N(r)/r(r) \), and \( E \propto r^{-1} \), and the scaling of \( r(t) \propto t^{2/3} \) during self-similar expansion (Shapiro 1977). It results in

\[
N_{\text{crit}} \propto t^{-1.25}
\]

We use an equivalent expression (Frank & Rees 1976)

\[
N_{\text{crit}} \propto \frac{4 \pi^2 \theta_{\text{crit}}^2(r) n(r)}{3 \xi(r)} \left| r = r_{\text{crit}} \right|
\]

for a \( \eta = 7/4 \) power-law cusp and \( t_{\text{dyn}} \propto r_{\text{in}}^{3/2} \) to derive

\[
N_{7/4} \propto 6.39 \sqrt{\frac{\pi}{G}} \ln \Lambda \frac{5/8}{1^{11/8}} \frac{10^9}{1^{4/9}} \frac{10^{14}}{r_{\text{in}}^{14/9}} \frac{10^{19}}{r_0^{49/18}}.
\]

Moreover in physical units,

\[
N_{7/4} \propto \frac{6.89 \times 10^{-6}}{\text{Myr}} \ln \left( \frac{M_*}{2 \times 10^9 \text{M}_\odot} \right)^{5/8} \left( \frac{r_*}{R_\odot} \right)^{4/9} \left( \frac{m_*}{\text{M}_\odot} \right)^{26/27} \left( \frac{1000 \text{M}_\odot}{1 \text{pc}^3} \right)^{-25/24} \left( \frac{n_0}{\text{pc}^{-3}} \right)^{14/9} \left( \frac{r_0}{\text{pc}} \right)^{49/18}.
\]

This equation will help us to properly scale our simulations, as described in the next section.

3 Simulations and Results

The evolution of dense stellar systems harbouring growing BHs is studied using direct N-body methods with an implemented tidal disruption procedure. The BH is treated as a heavy particle with an initial mass of \( M/M_{\odot} = 0.01 \). Particle numbers are \( N = 10 \) thousand to 100,000. Simulations were run in parallel with up to 128 processors on the Rechenzentrum Garching (RZG) Power 6 Machine in Garching (Munich), which is a facility of the Distributed European Infrastructure for Supercomputing Applications (DEISA) project; the 40 nodes Kolob GPU-Cluster (Mannheim, Germany), the 85 nodes Laohu cluster (Beijing, China) and the GRAPE cluster Titan at Zentrum für Astronomie Heidelberg (ZAH) (Heidelberg, Germany), which has been recently upgraded with GPU cores. In our implementation, by using the message passing interface (MPI) parallel n-body6++ (Spurzem 1999) and the parallel GPU code \( \varphi \)-GPU (Hafst et al. 2007), we let the star proceed to its peribothron where the accretion then takes place. In the \( n \)-body6++ code, we take advantage of the neighbour scheme (Ahmad & Cohen 1973), and find candidates for the disruption by only searching the BHs neighbours – thus reducing the computational overhead. These simulations incorporate zero softening (Hafst et al. 2007) and have an energy conservation \( \Delta E/E_0 \sim 10^{-4} \), even after \( 10^4 \) N-body time units. For the high \( N \) (up to 100 K) simulations we use the parallel \( \varphi \)-GPU code, ready for use with GPU clusters which includes a softening parameter of \( 10^{-5} \). The energy conservation is of the same order as in the \( n \)-body6++ code.

In order to challenge the analytic theory of loss cone diffusion by direct N-body simulations, we have to ensure that the simulations probe the dynamics of interest for real galaxies – the empty loss cone regime. It is convenient to identify the physics at work by its dominating time-scale, which in our case demands a separation of the loss cone depletion time \( t_{\text{out}} = t_{\text{dyn}} \) from the loss cone refilling time \( t_{\text{in}} \simeq \theta_k^2 t_c \). Hence the condition to have an empty loss cone is

\[
t_{\text{dyn}} \ll \frac{r_*}{r_{\text{in}}} \frac{1}{t_c} < t_{\text{in}}.
\]

For an in-depth discussion of \( t_{\text{in}} \) we would like to refer to the gas model studies by Amaro-Seoane et al. (2004). A straightforward way to satisfy the above relation is by increasing the particle number \( N \) following equation (4). However, as the \( O(N^3) \) scaling of direct algorithms transforms to \( O(N^2) \) for relaxation processes, it was a challenge already for our \( N = 100 \) K runs, especially because of the long integration time required for our purposes. Additionally, we highlight that the inclusion of a heavy BH particle leads to a widespread time-step distribution with few very short stepped particles close to the BH. This hampers scalability and significantly increases the overall integration time compared to the standard benchmark cases. Thus, it is still very difficult to obtain models of \( N \sim 10^6 \) on the general-purpose high-performance computers used nowadays, but we expect in the near future to be able to perform such runs in our new GPU clusters.

Given the limitations on the particle number and to further separate the time-scales, we introduce the magnification factor \( \alpha \) (equation 2) to vary the tidal radius in our N-body simulations with this free parameter and in order to determine the scaling behaviour of the results as a function of \( \alpha \), which ranges between 1 and 1000 (or \( \sim 10^{-5} \) for \( 1000 \) or \( \sim 10^{-5} \)).

It allows us to improve the performance and undertake a deeper study of time-dependent stellar accretion. Our model N-body units (Heggie & Mathieu 1986) are scaled to \( G = 1 \), \( r_{\text{in}} = 1 \) pc and \( m_* = 1 \) M\odot. With this scaling the stellar radius for the single-mass runs turns out to be \( r_\ast = 2.52 \times 10^{-8} r_{\text{in}} \) and the tidal radius then turns out as \( r(0) = 2.52 \times 10^{-8} r_{\text{in}} \) for a mass ratio of 1000:1. We are neglecting in this study of single-mass systems the stellar evolution during the whole simulated time. We will present the evolution of multi-mass systems in a forthcoming publication, where we will fully incorporate the stellar evolution time-scale, which becomes short enough to play a significant role.

The BH is located initially at the centre with zero velocities and the mass of tidally disrupted stars is added completely to the BH mass every accretion event. In order to provide an actual sink in phase space to drive the diffusion, particles need to be removed from the simulation when entering the tidal radius. The subsequent accretion is bluntly modelled as a perfect inelastic collision with the BH particle, where the star is fully accreted and linear momentum is conserved. The ‘equations of motion’ reads as

\[
M_\ast = M_\ast + m_\ast
\]

(17)

\[
\dot{r}_\ast = \frac{1}{M_\ast + m_\ast} (M_r \dot{r}_\ast + m_r \dot{v}_\ast)
\]

(18)

\[
\dot{v}_\ast = \frac{1}{M_\ast + m_\ast} (M_v \dot{v}_\ast + m_v \dot{v}_\ast).
\]

As initial galaxy models, we use rotating King models (Einsel & Spurzem 1999), where we added the BH particle of mass \( M_\ast/M_{\text{tot}} = \ldots \)

© 2011 The Authors, MNRAS 419, 57–69
Monthly Notices of the Royal Astronomical Society © 2011 RAS
0.01 in the centre. For completeness we employ $W_0 = (3, 6)$ and $\omega_0 = (0.0, 0.6, 0.9)$ axisymmetric King models. $W_0 = 3$ King models have larger and denser cores than $W_0 = 6$ King models.

In the text, we refer to non-axisymmetric models ($\omega_0 = 0.0$) as an approach to spherically symmetric systems, since the first do not have any flattening due to rotation. We use, complementary to our $N$-body models, Fokker–Planck approximations to our problem, with a seed BH $M_\bullet = 10^{-5}$ and $N = 10^6$ to properly define $r_c$, as in equation (1). Here is the stellar radius $r_* = 2.52 \times 10^{-3} r_c$, and thus $r_c(0) = 2.52 \times 10^{-7} r$, where $r$ is the scaling radius in our King Models, equal to the core radius. The value of the initial $r_c$ corresponds to a mass ratio $M_\bullet/m_\star = 1000$. Table 1 summarizes the $\alpha$ models. This effect is induced by the (0.0, 0.6, 0.9) axisymmetric King models.

Table 1. Overview of the performed $N$-body runs.

| Run identity | $W_0$ | $\omega_0$ | $N$   | $\alpha$ |
|--------------|--------|------------|-------|----------|
| 10KR1a       | 3      | 0.0        | 10000 | 10       |
| 10KR1b       | 3      | 0.0        | 10000 | 100      |
| 10KR1c       | 3.0    | 10000      | 1000  |
| 10KR2a       | 3      | 0.6        | 10000 | 1000     |
| 10KR3b       | 3      | 0.9        | 10000 | 1000     |
| 10KR3c       | 3.9    | 10000      | 1000  |
| 16KR1a       | 3      | 0.0        | 16000 | 10       |
| 16KR1b       | 3      | 0.0        | 16000 | 100      |
| 16KR1c       | 3      | 0.0        | 16000 | 1000     |
| 16KR3c       | 3.9    | 16000      | 1000  |
| 16KR4c       | 6      | 0.0        | 16000 | 1000     |
| 16KR5a       | 6      | 0.6        | 16000 | 10       |
| 16KR5c       | 6      | 0.6        | 16000 | 1000     |
| 16KR6a       | 6.9    | 16000      | 10    |
| 16KR6c       | 6.9    | 16000      | 1000  |
| 32KR1b       | 3      | 0.0        | 32000 | 100      |
| 32KR1c       | 3.0    | 32000      | 1000  |
| 32KR3c       | 3.9    | 32000      | 1000  |
| 32KR4c       | 6      | 0.0        | 32000 | 1000     |
| 32KR5c       | 6      | 0.6        | 32000 | 1000     |
| 32KR6c       | 6      | 0.9        | 32000 | 1000     |
| 64KR1c       | 3      | 0.0        | 64000 | 100      |
| 64KR1c       | 3      | 0.0        | 64000 | 1000     |
| 64KR3a       | 3      | 0.9        | 64000 | 10       |
| 64KR3c       | 3.9    | 64000      | 1000  |
| 64KR4c       | 6      | 0.0        | 64000 | 1000     |
| 64KR5c       | 6      | 0.6        | 64000 | 1000     |
| 64KR6c       | 6.9    | 64000      | 1000  |
| 100KR1b      | 3      | 0.0        | 100000| 100      |
| 100KR1c      | 3      | 0.0        | 100000| 1000     |
| 100KR2c      | 3      | 0.3        | 100000| 1000     |
| 100KR3c      | 3      | 0.6        | 100000| 1000     |
| 100KR4c      | 6      | 0.0        | 100000| 1000     |
| 100KR5c      | 6      | 0.6        | 100000| 1000     |
| 100KR6a      | 6.9    | 100000     | 10    |
| 100KR6c      | 6.9    | 100000     | 1000  |
| FPKR1        | 3.0    | 1 $10^8$   | 1     |
| FPKR3        | 3.9    | 1 $10^8$   | 1     |
| FPKR4        | 6      | 1 $10^8$   | 1     |
| FPKR6        | 6.9    | 1 $10^8$   | 1     |

Notes. Column 1 is an identifier for the run; Column 2 is the King parameter of the initial model $W_0$; Column 3 is the rotation parameter in the King model $\omega_0$, Column 4 is the initial number of particles and Column 5 is tidal radius magnification factor $\alpha$.

Computer cluster where simulations were performed: (a) n-body6++ in RZG, (b) $\psi$ GPU in Titan, (c) $\psi$ GPU in Kolob, (d) $\psi$ GPU in Loaobu (see the text for a brief description of the hardware).

3.1 Cluster evolution

The negative heat capacity of self-gravitating systems leads to the process of core collapse. In the standard picture proposed by Hénon (1965) and Aarseth (1973), the singularity is avoided due to the formation and subsequent hardening of tight central binaries. The presence of a star-disrupting BH can act as an energy source just like binary hardening. On the event of star disruption, a bound object is removed from the system, which loses (negative) binding-energy, (energy conservation is established, if the inner energy of the BH is considered), therefore the system gains energy and expands. Shapiro (1977) studied the combined effect of star disruption and the change of density in a homological model. He found a self-similar expansion of the core radius ($r_\text{c}$) according to

$$r_\text{c}(t) \propto [1 + g(M_\bullet) t]^{2/3},$$

where $t_\text{c}$ is the collapse time, $^1$ and $g(M_\bullet)$ also depends on the initial — and on the ‘minimum’ — core radius and respective density, as well as on the relaxation time. Given the similar underlying physics it is unsurprising that the time-dependence is of the same type ($\alpha^2/3$) as in the binary hardening case considered by Hénon (1965) and Goodman (1984).

We show the Lagrange radii (radii containing the given percentage of the initial total stellar mass) for the runs 16KR3c and 16KR6c in Fig. 1. We are using a small particle number, which permits us to obtain longer evolutionary times, in relaxation time-scales.

After a less pronounced collapse compared to the case without BH (Kim et al. 2008), the Lagrange radii expand self-similarly according to the $r \propto t^{2/3}$ law. This behaviour, which has also been seen in gas models (Amaro-Seoane et al. 2004) and axisymmetric Fokker–Planck models (Fiestas & Spurzem 2010), is here verified in a self-consistent direct $N$-body simulation for axisymmetric systems.

As we can see, Lagrangian radii give us a qualitative description of the interaction of a growing BH and the stellar mass shells. Initially, Lagrangian radii are dominated by core contraction and the BH mass growth is slow due to the low central density. Later, density grows due to gravitational instabilities (the Lagrangian radii shrink) and the collapse is halted and reversed (mass shells re-expand), while the growing BH potential dominates the system. The axisymmetric models of Fig. 1 show a similar evolution as the well-studied spherically symmetric case.

In Fig. 1 we observe that $t_{\text{c}}$ is shorter than the expected value for systems without BH, which are $t_{\text{c}}/t_{\text{bh}} \sim 16$ for $W_0 = 3$ models, and $t_{\text{c}}/t_{\text{bh}} \sim 10$ for $W_0 = 6$ models. This effect is induced by the magnification of the tidal radius and the high initial $M_\bullet$, which enhance stellar accretion, accelerating the reverse of collapse and further expansion. Nonetheless, it does not affect the obtained self-similar expansion. Note that the influence radius approaches the initial $r_{\text{bh}}$ during the post-collapse phase, $r_{\text{crit}}$ (equation 11) is also shown in this figure, as well as $r_{\text{walk}}$ (equation 6), which decreases with increasing $M_\bullet$, and becomes more than an order of magnitude smaller than $r_{\text{crit}}$, and more than two orders of magnitude smaller than $r_\text{c}$.

Spherically symmetric systems with BH are known to develop steady-state solutions in relaxation time-scales. These solutions have characteristic density and velocity dispersion profiles. The

$^1$ The collapse time corresponds to the time at which central density grows to infinity, and core radius shrinks to zero. In systems with an energy source, like here, it is the time at which the contraction phase is halted and reversed.
density distribution scales theoretically with radius as $r^{-7/4}$ and the velocity dispersion as $r^{-1/2}$. We follow the evolution of our models in this time-scale and compare them with the standard theory.

Fig. 2 shows the evolution of the density profile for the models 100K1c and 100K4c (non-rotating), and 100K3c and 100K6c (rotating). We use here the highest particle number in order to obtain a better resolution of the cusp. Note that the cores are initially flat and the time needed to build the cusp is of the order of $r_0$. The influence radius (green diamonds) moves outwards while BH mass grows, and the critical radius (orange triangles) appears in our resolution range before the cusp is formed. $r_{\text{cusp}}$ is smaller than the minimum radius in this figure. In post-collapse, these radii are well separated and the profile approaches the expected zero flux solution between $r_{\text{cusp}}$ and $r_\star$. Systems with larger cores ($W_0 = 3.0$) evolve slower than $W_0 = 6.0$ models, reaching the latter their final density cusps in shorter times.

During post-collapse the system expands and the cut-off radius in our models extends up to $\sim 10 r_{\text{hm}}$. Closer to the centre, one can see that at later times, the critical radius grows and the cusp is shallower inside this radius, due to the effective stellar accretion around the BH and the growing tidal radius, which perturbs the formation of a cusp at the very centre. Although the critical radius is resolved in our simulations, it contains only a few dozens of stars. We are performing higher $N$ ($\geq 256$ K) simulations, which will provide a more accurate measurement of the central cusp and especially of the empty loss cone region.

### 3.2 Disruption rates

In our models, stellar accretion is driven by small angle, two-body encounters, which under the influence of the BH gravitational potential causes some stars to lose energy and move closer to the BH before being eventually consumed. During the contraction phase, high central densities are expected to trigger higher BH mass growth rates. Thus, maximal rates occur close to $r_{\text{cusp}}$, when angular momentum and energy diffusion are most effective.

As already discussed, in our $N$-body models, stars in orbits of $J < J_{\text{cusp}}$, which reach their apocentres inside the tidal radius, define an accretion event. In our axisymmetric Fokker–Planck models, stars in orbits of $J < J_{\text{cusp}}$ are accreted (Fiestas & Spurzem 2010), being only energy and $J_x$ conserved quantities. Since the other angular momentum components are not conserved, accretion can be artificially enhanced in these models, especially during the initial evolution, before self-similar expansion sets on. Regarding initial conditions, our Fokker–Planck models use comparatively smaller BH seeds ($M_*/M_{\text{tot}} \sim 10^{-5}$), while we fix this value in the $N$-body models to 0.01.

Since we expect a self-similar evolution during core expansion, independent of initial conditions, as shown in Figs 1 and 2, at this stage both methods should be comparable, if radial and time units are properly re-scaled. We check this by scaling rates, correcting the changes induced in equation (14) by $M_*, m_\star = 1/10$ and $r_\star$ in each model. First, we bring rates to common dimensionless units $[\Delta N/N]/[\Delta t t_{\text{ref}}]$, and we calculate the factor $N_{\text{modelA}}/N_{\text{modelB}}$ by replacing the correspondent values of $M_*, m_\star$ and $r_\star$ of each model. $r_\star$ enhances the disruption rates by a factor of $\sim 1$ to 22 (for $\alpha = 1$ to 1000). A variation of $N$ between 10 and 100 K modifies the stellar mass $m_\star$ as $1 - 6 \times 10^{-5}$, and $M_*/m_\star$ changes like $\sim 100 - 1000$. A (−7/4) cusp in the density profile is assumed. We finally scale the results obtained by model B to model A by multiplying the first with the previous factor.

On the other side, as previously discussed, the larger seed BH in the $N$-body models triggers core heating and enhances accretion events, with the consequence of an earlier bounce of core density. It modifies the time at which density reaches its maximum, and maximal disruption rates appear. We can observe this effect in the evolution of Lagrangian radii (Fig. 1). Thus, self-similar expansion during post-collapse can be compared by bringing the collapse times of the compared models together.

Fig. 3 (left-hand panel) shows disruption rates before scaling. Note that peak rates are higher for higher $\alpha$ and $N$-body models approach Fokker–Planck models by decreasing this parameter (the ‘real’ tidal radius is given by $\alpha = 1$). Since maximal disruption rates dominate the stellar accretion events over time, they can give us the extent of the influence of the different initial parameters and kinematics (rotation) used in our models. Fig. 3 (right-hand panel) shows maximal disruption rates ($N_{\text{modelA}}^{\text{max}}$) obtained for all models in dimensionless units $(dM/dt)_\text{ref}/(dM/dt_{\text{ref}})$ against the parameter $\alpha$, as used in the simulations. This plot gives only approximately the $N(\alpha)$-dependence, since the $m_\star$- and $M_*$-dependence are also present. Additionally, the different peak rates at constant $\alpha$ are a consequence of the better resolution reached in high-$N$ $N$-body runs, which show higher peaks. Moreover, $N_{\text{modelA}}^{\text{max}}$ for smaller values of $\alpha$ converges to rates corresponding to $\alpha = 1$ (actual $r_\star$), as we
Black holes in axisymmetric galaxy cores

Figure 2. Evolution of the density profile of non-rotating models 100KR1c and 100KR4c (top), and rotating models 100KR3c and 100KR6 (bottom). Symbols are: influence radius $\circ$ and critical radius $\triangle$. In post-collapse, the radii are well separated and the profile approaches the expected zero flux solution between $r_{\text{crit}}$ and $r_n$.

Figure 3. Left-hand panel: non-scaled disruption rates for N-body (16KR1a,b,c) and Fokker–Planck models (FKKR1). The influence of the magnified $r_t$ in the evolution is clearly seen. The solutions approach the Fokker–Planck results for $\alpha = 1$. Right-hand panel: maximal disruption rates versus parameter $\alpha$ for all models. Filled symbols are $W_0 = 6$ models, and empty symbols correspond to $W_0 = 3$ models. The results converge to the ideal case ($\alpha = 1$) as in the Fokker–Planck approximation.

Show by plotting the maximal rates of our Fokker–Planck models (black dots in Fig. 3, right-hand panel). The measured values of $N_{\text{max}}$ are shown in Table 2. Mainly models using $\alpha = 1000$ and $\omega_0 = 0.0, 0.9$ were here selected for comparison. Table 2 shows as well $\dot{N}_{\text{scaled}}$ scaled to $\alpha = 1$, as in our Fokker–Planck models. We correct for the $N \propto r_t^{-1}$-dependence, as in equation (14).

Fig. 4 shows the disruption rates for models of the King parameter $W_0 = 3.0$ (left-hand panel) and $W_0 = 6.0$ (right-hand panel). In each
By comparing non-rotating with rotating models in Table 2 (Column 4), we can observe that the latter show slightly higher peak rates, for constant $N$. Maximal rates versus the rotational parameter $\omega_0$ for all models are shown in Fig. 5 (top). Low $N$ runs show roughly constant peaks, while the better spatial resolution reached by high $N$ runs, especially at $r_{\rm crit}$ (as seen in Fig. 2), shows higher disruption rates for higher $\omega_0$. The higher peaks reached by rotating systems lead to the BH masses shown in Table 2. Final BH masses ($M_{\ast, f}$) are measured at times, which are kept the same for constant $N$ and $W_0$ to facilitate the comparison between the models. We observe in the $N = 100$ K runs that non-rotating models reach a similar final mass, since rotating models converge to a $\sim 20\%$ larger mass, independent of $W_0$. It implies a direct influence of rotation in the process of stellar accretion.

In order to understand how this effect is triggered, Fig. 5 (middle) shows the distribution of $J_x$, $J_y$, $J_z$ of accreted stars for the model 100KR6 (rotating). Values are fractions of the total number of accreted stars. All distributions peak at $J = 0$, as expected. The contribution of $J_x$, $J_y$ to accretion is symmetric with respect to $J = 0$, while a higher fraction of prograde rotating stars ($J_z > 0$) is consumed by the BH. This was expected, since our initial models rotate in the positive direction of $J_z$. Fig. 5 (bottom) shows the distribution of $J_z$ of accreted stars for the model 100KR6 (rotating) in comparison to the non-rotating model (100KR4). We observe that rotating models show an excess of accreted prograde rotating stars.

We show in Fig. 6 the radial distribution of the semimajor axis, normalized by $r_{\rm crit}$, of accreted stars in models 100KR4c (non-rotating) and 100KR6c (rotating). A larger fraction of accreted stars in the rotating models have semimajor axes a few times larger than $r_{\rm crit}$. Distribution of accreted stars in the non-rotating model peaks at $r_{\rm crit} \sim 1$ as expected from the loss cone theory (Lightman & Shapiro 1977). The distribution of eccentricities is similar in both models (Fig. 6, bottom). It shows a steep maximum above $e = 0.995$.

From Figs 5 and 6 we can conclude that the excess of accreted rotating stars originate mainly from regions outside $r_{\rm crit}$. According to the loss cone theory, diffusion of energy and angular momentum is higher than the loss cone size in this region (full loss cone regime), and orbits in this region are able to escape the loss cone in a dynamical time. In order to obtain a larger $r_{\rm crit}$, as in our rotating models, $t_{\rm in} \sim \theta_{25}^2 t_{\rm h}$ should be larger, or $t_{\rm in} \sim \theta_{25}^4 t_{\rm h}$ shorter in these region.

This hints to a breakdown of the classical theory which depends on the conservation of angular momentum that is not given in the rotating models. An enhanced relaxation in a system that is supported by bulk rotation, as compared to a non-rotating one which is pressure supported, could explain this effect (Athanassoula, Vojtkov & Lambert 2001). A decrease in the velocity dispersion for $r < r_{\ast}$, which could substantially modify $t_{\ast CR}t_{\ast}$ within the BH influence zone, was nevertheless not detected in our models. It is thus tempting to interpret this behaviour in terms of the additional presence of orbits with non-conserved $J_x, J_y$ angular momentum (e.g. box orbits). To further investigate this effect, an orbit study of the accreted stars should be performed, which we must leave open for future research.

### 3.3 Rotational velocity

In Fig. 7 (top) we show the initial distribution of the velocity component corresponding to $J_z$ of stars for the model 100KR6c. The crossing point in fig. 13 of Amaro-Seoane et al. (2004) is shifted to the right in our rotating models, when compared to non-rotating models.
Black holes in axisymmetric galaxy cores

Figure 4. Disruption rates in units of fractional mass per relaxation time for King models 16KR1c, 16KR4 (non-rotating, top) and 16KR3, 16KR6c (rotating, bottom). N-body results are black lines, green lines are Fokker–Planck models and the expected time dependence for a Bahcall–Wolf cusp ($\propto t^{-1.25}$) is shown by the red line. The predicted rate following equation (14) is shown as a blue line. The rate peaks after a few $t_{\text{rh}}$ and decreases afterwards. The models follow the analytical predictions in the post-collapse phase.

initial rotating King models show a maximum of rotation ($v_{\text{rot, max}}$) at around $r_{\text{hm}}$ and central rigid body rotation. We plot velocities of bins of five stars in order to get a more detailed stellar distribution, and average over five N-body time units. Vertical dashed lines mark the radius of influence of the BH and $r_{\text{crit}}$ in units of $r_{\text{hm}}$ at the given time, as indicated. The rotation profile from bins of 50 stars is overplotted (red dots). The evolved distribution of rotational velocities for the system after relaxation ($\sim 3 t_{\text{rh}}$) is shown in Fig. 7 (middle). As a consequence of angular momentum transport through gravitational scatterings, the original $v_{\text{rot, max}}$ decreases during the evolution. In the model $v_{\text{rot, max}}(t_{\text{cc}})/v_{\text{rot, max}}(0) \sim 0.85$. Additionally, one can observe that $v_{\text{rot, max}}$ moves inwards, in a region close and inside the influence radius, leading to an increasing of rotation in this region, with respect to the initial configurations. With constant $\omega = 0.9$, as an initial rotational parameter, this effect is more pronounced in concentrated models (100KR6c, $W_0 = 0.6$) than in models with larger cores (100KR3c, $W_0 = 0.3$), because the first contain initially more amount of rotational energy with respect of the kinetic energy ($T_{\text{rot}}/T_{\text{kin}} \sim 0.3$) than the second ones ($T_{\text{rot}}/T_{\text{kin}} \sim 0.1$). After the system relaxes, central stars rotate with velocities in average larger than the original maximum, initially located in the outer parts of the system.

Consider the region $r_{\text{crit}} < r < r_{\text{hm}}$, where stars are dominated by the BH central potential. Stars populate always more this region in time (Fig. 7, middle), while they interact dynamically with other stars. A fraction of them will be disrupted, when their orbits reach $r < r_{\text{crit}}$. As we discussed, the excess of disrupted stars in rotating models comes mainly from this region, and is dominated by stars in orbits with positive $J_z$. It is not surprising, since the concentration of rotating stars in the BH zone of influence is triggered by the initial configuration in our axisymmetric systems. Moreover, the growing central density, caused by gravitational and gravogyro instabilities (Einsel & Spurzem 1999), together with the angular momentum transport, enhances this effect, especially before expansion sets in.

The few counter-rotating stars observed in Fig. 7 (middle) are remaining stars from the initial distribution, which together with pro-rotating stars lead to no rotation in the very centre. The enhancement of rotation inside the influence radius builds a wide maximum of rotating stars inside $r_{\text{hm}}$, which is now close to $r_{\text{crit}}$. At a radius inside $\sim r_{\text{crit}}$ one finds only a few dozens of stars (in our N=100 K runs), and it is difficult to talk about rotation in this region. Moreover, in the region $r_{\text{crit}} \lesssim r \lesssim r_{\text{hm}}$ a rotating core can be detected. By comparison with Fig. 2, we observe that this is the region where the cusp forms, and the BH potential dominates the stellar environment.

In order to investigate how strong rotation dominates the dynamics at this evolutionary stage, we show the rate of rotational velocity versus 1D velocity dispersion ($v_{\text{rot}}/\sigma$) for the same relaxed
Figure 5. Top: maximal disruption rates versus rotational parameter $\omega_0$ for all models. Symbols are as in Fig. 3 (right-hand side). Especially high $N$ runs show how rotation influences the disruption rates. We keep a constant parameter $\alpha = 1000$ for comparison. Middle: distribution of $J_x, J_y, J_z$ of accreted stars for the rotating model 100KR6. A higher fraction of prograde rotating stars is accreted. Bottom: distribution of $J_z$ of accreted stars for the models 100KR4 (non-rotating) and 100KR6 (rotating). The excess of accreted prograde rotating stars leads to the obtained higher masses. The maximum $J_{z,c}$, for the largest energy of the stellar orbits, is indicated in the middle and bottom panels with a vertical dashed line.

axisymmetric model (Fig. 7, bottom) corresponding to the previous figures. This parameter shows the relative importance of rotational versus pressure-supported kinematics as used in observational studies of ellipticals and galaxy bulges of spirals. Here, we used velocity bins of 50 stars and averaged over 50 time-steps in order to get a better defined profile. In relaxed systems, the initial peak at $\sim r_{hm}$ still dominates and becomes wider. Although the profile continuously decreases towards the centre, an enhanced $v_{\text{rot}}/\sigma$ inside $r_h$ with respect to the initial configuration can be observed. It is especially interesting since the BH gravitational potential builds a velocity dispersion cusp ($\propto r^{-1/2}$), which requires a strong increase of $v_{\text{rot}}$ in order to be detected.

4 CONCLUSIONS

The loss cone theory as developed in the classic papers of Frank & Rees (1976), Lightman & Shapiro (1977) and Cohn & Kulsrud (1978) can be used to estimate feeding rates for SMBHs in galactic nuclei. None the less, total consumption occurs from orbits that could extend beyond the BH influence radius, hence the contribution of the stars to the gravitational potential cannot be ignored. These orbits interact with central stars and are able to interchange energy and angular momentum in relaxation time-scales. In this work,
We investigate accretion rates in axisymmetric systems by using direct $N$-body and Fokker–Planck simulations, harbouring a star accreting growing BH, which evolves dynamically in time-scales of relaxation.

Our main results are as follows.

(i) The stellar distribution is strongly influenced by the interplay between diffusion of energy and angular momentum (gravitational instabilities) and the feeding of the BH influence zone by stars in high-energy, low-$J$ (eccentric) orbits. These systems undergo core collapse (reach a central density maximum at $r_\text{cc}$) in the presence of a star-accreting BH. The growing central BH potential dominates always larger zones of the system, with a growing influence radius, which reaches almost the half-mass radius during the post-collapse phase.

Axisymmetric (rotating) systems, like the spherically symmetric, reach in our simulations steady-state solutions during the post-collapse phase. Systems with smaller cores ($W_0 = 6.0$) reach self-similar expansion in shorter times than systems with larger cores ($W_0 = 3.0$). For relaxed systems, the BH $r_\text{walk}$ is about one order of magnitude smaller than $r_\text{crit}$, which is well resolved in our simulations, and itself about two orders of magnitude smaller than $r_0$. Thus, these systems fulfil the conditions for the loss cone theory.

Furthermore, we are currently investigating the consequences for relaxation times and especially for the loss cone theory in multi-mass models. Mass segregation time-scales in real systems are short enough to lead to a faster relaxation in the central parts, where the more massive stars concentrate, and consequently, to an earlier formation of cusps by these central stars (a work to be presented in a forthcoming publication). This is especially important, since galactic nuclei are often less than one relaxation time old, and stellar density near the SMBH seems not to have the Bahcall–Wolf form, although observations need to resolve the influence radius to be able to detect them. Moreover, relaxation times can be themselves much shorter, the smaller the radius of the system, and the higher the central densities.

(ii) We measured tidal disruption rates in axisymmetric $N$-body models and compared them to 2D Fokker–Planck realizations. Stellar accretion acts as an indirect heating source reversing core collapse, like in isolated star clusters it is the role of hard binaries. In our low $N$, $N$-body runs with high initial seed BH masses the indirect heating is stronger, so the density maximum and thus the peak rates are wide. For high $N$ or small seed BH masses the heating is small, a very peaked central density and disruption rates occur (like in the FP models), as shown in Fig. 4.

In order to investigate the self-similar evolution during the post-collapse phase, we apply a scaling procedure to our models, which use partly different initial conditions, and we obtain a non-trivial result for the rates through all the post-collapse phase where $N$-body and Fokker–Planck models agree with each other. This means that in this phase we can make from these data predictions for stellar disruption rates and other kinematical parameters of dry galactic nuclei, which are independent of the previous history [as also shown in Fiestas & Spurzem (2010)], even independent of whether the BH has grown earlier by gas or star accretion, a statement, which is confirmed here by using direct $N$-body realizations.

We found that disruption rates and BH masses are influenced by axisymmetry/rotation, in the way that rotation leads to higher peak rates and higher $M_\ast$. The excess of accreted stars, origins mainly from prograde rotating stars, is located in regions outside $r_\text{crit}$. This hints to a breakdown of the classical theory, given by the rotating models, which could be interpreted as a consequence of the presence of box orbits with non-conserved $J_x/J_y$ angular momentum. To further investigate this effect, an orbit study of the accreted stars is necessary. This is of importance, since galactic nuclei need not be even axisymmetric. In a triaxial nucleus containing centrophilic orbits, the mass in stars on orbits that intersect the SMBH’s capture...
sphere can be enormous, much greater than $M_\star$, so that the loss cone is never fully depleted. Galactic nuclei sometimes undergo catastrophic changes, due to galaxy mergers, in-fall of star clusters or BHs, star formation, etc., all of which can substantially affect the feeding rate on both the short and long terms.

We apply, for illustration, disruption rates given by equation (15) to the galactic centre, by using $M_\star = 3.3 \times 10^6$, $r_\text{in} = 1.65$, $\rho_0 = 2.8 \times 10^6 M_\odot pc^{-3}$, $r_0 = 0.22$ pc and $\eta = 1.75$ (Schödel et al. 2007), and obtain stellar disruption rates of $\sim 1.2 \times 10^{-4} M_\odot yr^{-1}$. Our results presented in Table 2 give around 50 per cent higher rates in axisymmetric models, which lead to final BH masses in average 20 per cent higher with respect to spherically symmetric systems. This factor would change the rates to $\sim 1.8 \times 10^{-4} M_\odot yr^{-1}$. Integrated over the age of the universe, the mass gain is of the order of the BH mass. It means that relaxation processes might play an important role in the growth of the galactic centre BH. Moreover, for a galaxy core of $M \approx 10^7 M_\odot$ and a relaxation time of 10 Gyr, our models give us peak rates of the order of $\sim 10^{-4} M_\odot yr^{-1}$. These rates are comparable to the accretion rates of some power-law galaxies found by Wang & Merritt (2004).

(iii) The evolution of initially rotating systems with BHs substantially affects the orbital distribution of stars, especially in the regions inside the BH influence radius. We have found that the rotation in relaxation time-scales cannot be neglected, and it triggers higher disruption rates with an excess of rotating stars. Central rotation has been detected in relaxation time-scales in the zone of influence of the BH ($r_{\text{core}} < r < r_\text{in}$), being at this time $r_{\text{rot}} \sim r_{\text{in}}(0)$. The original $v_{\text{rot,max}}$ moves inwards, in a region where the BH potential dominates the stellar environment. For comparison, in systems without BH, thus without post-collapse evolution, dynamical instabilities cause $v_{\text{rot,max}}$ to move outwards from the centre (Kim et al. 2008).

In the central profile of the parameter $v_{\text{rel}}/\sigma$ this maximum is lowered, mainly due to the cusp in the central velocity dispersion, but is not negligible.

The presence of different stellar populations is expected to enhance this effect in the central regions, as shown by Kim, Lee & Spurzem (2004). More massive stars segregate to the centre in time-scales shorter than a relaxation time, and they can rotate faster. Our current investigations of multi-mass axisymmetric cores with the stellar evolution and BHs aim to obtain more detailed measurements of different stellar populations in the centre, which could be detectable by observations. Another task would be the treatment of a BBH, which can in a similar way lead to a more efficient support in the development of rotation in its zone of influence (Berczik et al. 2006; Berentzen et al. 2009).

More realistic N-body simulations by using $\alpha = 1$ and higher particle numbers up to $N \sim 10^6$ or more are nevertheless necessary and still challenging to perform, especially in relaxation time-scales, but will be possible in the near future. The advantage of using direct N-body models, together with computationally faster Fokker–Planck realizations, makes it possible to study the evolution of kinematical and structural parameters in more detail, which can complement and test observational measurements. Observational studies of ‘collisional’ galactic nuclei embedding massive BHs can be compared to evolutionary models to elucidate theoretical predictions and have a better understanding of galaxy evolution.

ACKNOWLEDGMENTS

We acknowledge support by the Chinese Academy of Sciences Visiting Professorship for Senior International Scientists, Grant Number 2009S1-5 (The Silk Road Project) (RS, PB and JF partly). The special supercomputer Laohu at the High Performance Computing Center at National Astronomical Observatories, funded by Ministry of Finance under the grant ZDYZ2008-2, has been used. Simulations were performed on the GRACE supercomputer (grants 1/80 041-043 of the Volkswagen Foundation and 823.219-439/30 and 823.219-439/36 of the Ministry of Science, Research and the Arts of Baden-Württemberg). The Kolob cluster is funded by the excellence funds of the University of Heidelberg in the Frontier scheme. We thank the SPP 1177 (SP 345/17-2) for the financial support of this project. PB acknowledges the special support by the NAS Ukraine under the Main Astronomical Observatory GRAPE/GRID computing cluster project. PB’s studies are also partially supported by the program Cosmicmophysics of NAS Ukraine. We thank the DEISA Consortium (http://www.deisa.eu), co-funded through EU FP6 projects RI-508830 and RI-031513, for support within the DEISA Extreme Computing Initiative. We thank the referee for fruitful comments, which helped in improving the quality of the present publication.

REFERENCES

Aarseth S. J., 1973, Vistas Astron., 15, 13
Ahmad A., Cohen L., 1973, J. Computational Phys., 12, 389
Amaro-Seoane P., Freitag M., Spurzem R., 2004, MNRAS, 352, 655
Ferrarese L., Ford H., 2005, Space Sci. Rev., 116, 523
Callegari S., Kazantzidis S., Colpi M., Governato F., Quinn T., Wadsley J., 2009, ApJ, 696, L89
Callegari S., Kazantzidis S., Mayer L., Colpi M., Bellonvary J. M., Quinn T., Wadsley J., 2011, ApJ, 729, 125
Cappellari M., Cappellari M., Gerssen J., 2006, MNRAS, 366, 1553
Cappellari M., Cappellari M., van de Ven G., 2009, MNRAS, 396, 1640
Córd O. et al., 2007, ApJ, 671, 1456
Degruf C., Di Matteo C., Springel V., 2010, MNRAS, 402, 1927
Dale J. E., Davies M. B., Church R. P., Freitag M., 2009, MNRAS, 393, 1016
Do T., Ghez A. M., Morris M. R., Yelda S., Meyer L., Lu J. R., Hornstein S. D., Matthews K., 2009, ApJ, 691, 1021
Dotti M., Ruszkowski M., Paredi L., Colpi M., Volonteri M., Haardt F., 2009, MNRAS, 396, 1640
Eilon E., Kupi G., Alexander T., 2009, ApJ, 698, 641
Einsel C., Spurzem R., 1999, MNRAS, 302, 81
Faber S. M. et al., 1997, AJ, 114, 1771
Ferrarese L., Ford H., 2005, Space Sci. Rev., 116, 523
Ferrarese L. et al., 2006a, ApJ, 644, L21
Ferrarese L. et al., 2006b, ApJS, 164, 334
Fiestas J., Spurzem R., 2010, MNRAS, 405, 194
Frank J., Rees M., 1976, MNRAS, 176, 633
Freitag M., Benz W., 2002, A&A, 394, 345
Glass L. et al., 2011, ApJ, 726, 31

© 2011 The Authors, MNRAS 419, 57–69
Monthly Notices of the Royal Astronomical Society © 2011 RAS
This paper has been typeset from a TeX/LaTeX file prepared by the author.