Singular Isothermal Disks
and the Formation of Multiple Stars

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Abstract. A crucial missing ingredient in previous theoretical studies of fragmentation is the inclusion of dynamically important levels of magnetic fields. As a minimal model for a candidate presursor to the formation of binary and multiple stars, we therefore consider the equilibrium configuration of isopedically magnetized, scale-free, singular isothermal disks, without the assumption of axial symmetry. We find that lopsided \((M = 1)\) configurations exist at any dimensionless rotation rate, including zero. Multiple-lobed \((M = 2, 3, 4, ...)\) configurations bifurcate from an underlying axisymmetric sequence at progressively higher dimensionless rates of rotation, but such nonaxisymmetric sequences always terminate in shockwaves before they have a chance to fission into separate bodies. We advance the hypothesis that binary and multiple star-formation from smooth (i.e., not highly turbulent) starting states that are supercritical but in unstable mechanical balance requires the rapid (i.e., dynamical) loss of magnetic flux at some stage of the ensuing gravitational collapse.

1. Magnetic Fields in Star Forming Clouds

On scales larger than small dense cores \((\sim 0.1 \text{ pc})\), magnetic fields are more important than thermal pressure (but perhaps not turbulence) in the support of molecular clouds against their self-gravitation (see the review of Shu, Adams, & Lizano 1987). Mestel has long emphasized that the presence of dynamically significant levels of magnetic fields changes the fragmentation problem completely (Mestel & Spitzer 1956; Mestel 1965a,b; Mestel 1985). Associated with the flux
Φ frozen into a cloud (or any piece of a cloud) is a magnetic critical mass:

$$M_{\text{cr}}(\Phi) = \frac{\Phi}{2\pi G^{1/2}}.$$  \hspace{1cm} (1)

Subcritical clouds with masses $M$ less than $M_{\text{cr}}$ have magnetic (tension) forces that are generally larger than and in opposition to self-gravitation (e.g., Shu & Li 1997) and cannot be induced to collapse by any increase of the external pressure. Supercritical clouds with $M > M_{\text{cr}}$ do have the analog of the Jeans mass – or, more properly, the Bonnor-Ebert mass – definable for them, but unless they are highly supercritical, $M \gg M_{\text{cr}}$, they do not easily fragment upon gravitational contraction. The reason is that if $M \sim M_{\text{cr}}$ for the cloud as a whole, then any piece of it is likely to be subcritical since the attached mass of the piece scales as its volume, whereas the attached flux scales as its cross-sectional area. Indeed, the piece remains subcritical for any amount of contraction of the system, as long as the assumption of field freezing applies. An exception holds if the cloud is highly flattened, in which case the enclosed mass and enclosed flux of smaller pieces both scale as the cross-sectional area. This observation led Mestel (1965a,b; 1985) to speculate that isothermal supercritical clouds, upon contraction into highly flattened objects, could and would gravitationally fragment.

Zeeman observations of numerous regions (see the summary by Crutcher 1999) indicate that molecular clouds are, at best, only marginally supercritical. The result may be easily justified after the fact as a selection bias (Shu et al. 1999). Highly supercritical clouds have evidently long ago collapsed into stars; they are not found in the Galaxy today. Highly subcritical clouds are not self-gravitating regions; they must be held in by external pressure (or by converging fluid motions); thus, they do not constitute the star-forming molecular-clouds that are candidates for the Zeeman measurements summarized by Crutcher (1999). The clouds (and cloud cores) of interest for star formation today are, by this line of reasoning, marginally supercritical almost by default.

### 1.1. Pivotal States and Self-Similarity

The stage leading up to dynamic collapse of a magnetically subcritical cloud core to a protostar or a group of protostars is believed to be largely quasi-static (e.g., Nakano 1979, Lizano & Shu 1989, Tomisaka 1991, Basu & Mouschovias 1994). To describe the transition between quasi-static evolution by ambipolar diffusion and dynamical evolution by gravitational collapse, Li & Shu (1996) introduced the idea of a **pivotal state** to indicate scale-free, magnetostatic configurations just before the onset of gravitational collapse (protostar formation and envelope infall). For these states, the density distribution approaches $\rho \propto r^{-2}$ for an isothermal equation of state when the mass-to-flux ratio has a spatially constant value, a condition that Shu & Li (1997) and Li & Shu (1997) termed *isopedic*. Numerical simulations of the contraction of magnetized clouds do show that the mass-to-flux ratio remains constant over several decades of spatial extent (see e.g. Basu & Mouschovias 1994).

Magnetized self-gravitating equilibria tend to be somewhat flattened, unless high levels of toroidal fields are present. When the support against self-gravity is dominated by poloidal magnetic fields and/or rotation, the configuration becomes a thin disk, not necessarily axisymmetric or time independent. Also, if
turbulent support is modeled as a polytropic or logatropic scalar pressure with a relatively soft equation of state (e.g. Lizano & Shu 1989, Holliman & McKee 1993), then all magnetostatic configurations with $\lambda \leq 1$ are highly flattened in the direction perpendicular to the magnetic field.

Li & Shu (1996; see also Baureis, Ebert & Schmitz 1989) have shown that the general, axisymmetric, magnetized equilibria representing such pivotal states assume the form of singular isothermal toroids (SITs): $\rho(r, \theta, \varphi) \propto r^{-2}R(\theta)$ in spherical polar coordinates $(r, \theta, \varphi)$, where $R(\theta) = 0$ for $\theta = 0$ and $\pi$ (i.e., the density vanishes along the magnetic poles). We regard these equilibria as the isothermal (rather than incompressible) analogs of Maclaurin spheroids, but with the flattening produced by magnetic fields rather than by rotation. In the limit of vanishing magnetic support, SITs become singular isothermal spheres. In the limit where magnetic support is infinitely more important than isothermal gas pressure, SITs become singular isothermal disks (SIDs), with $\rho(\varpi, z) = \Sigma(\varpi)\delta(z)$ in cylindrical coordinates $(\varpi, \varphi, z)$, where $\delta(z)$ is the Dirac delta function, and the surface density $\Sigma(\varpi) \propto \varpi^{-1}$.

In a fashion analogous to the singular isothermal sphere (Shu 1977), the gravitational collapses of SITs have elegant self-similar properties (Allen & Shu 2000). But it should be clear that the formation of binary and multiple stars could never result from any calculation that imposes a priori an assumption of axial symmetry. In this regard, we would do well to remember the warning of Jacobi in 1834:

“One would make a grave mistake if one supposed that the axisymmetric spheroids of revolution are the only admissible figures of equilibrium.”

2. Nonaxisymmetric Equilibria and Bifurcations

Shu et al. (2000) and Galli et al. (2000) started the campaign to understand binary and multiple star-formation by considering the equilibrium and stability of nonaxisymmetric self-gravitating, magnetized, differentially-rotating, completely flattened SIDs, with critical or supercritical ratios of mass-to-flux

$$\lambda \equiv 2\pi G^{1/2} \frac{M(\Phi)}{\Phi} \geq 1,$$

(see Li & Shu 1996, Shu & Li 1997). The dimensionless mass-to-flux ratio $\lambda$ is taken to be a constant both spatially (the isopedic assumption) and temporally (the field-freezing assumption). The governing equations of our problem are the usual gas dynamical equations for a completely flattened disk (see Shu & Li 1997, Shu et al. 2000), except for two modifications introduced by the presence of magnetic fields that thread vertically through the disk, and that fan out above and below it without returning back to the disk. First, magnetic tension reduces the (horizontal) gravitational force by a multiplicative factor $\epsilon = 1 - \lambda^{-2} \leq 1$. Second, the gas pressure is augmented by the presence of magnetic pressure; this increases the square of the effective sound speed by a multiplicative factor $\Theta = (\lambda^2 + 3)/(\lambda^2 + 1) \geq 1$. In other words, the equations of motions for the isopedic, isothermal SID are identical with those of a nonmagnetized disk except for the transformations

$$G \to \epsilon G, \quad a^2 \to \Theta a^2,$$
where $G$ is the gravitational constant and $a$ the isothermal sound speed.

Under the assumption of field freezing, i.e. keeping $\lambda$ constant in time, Galli et al. (2000) found that prestellar molecular cloud cores modeled as magnetized SIDs need not be axisymmetric. The most impressive distortions are those that make slowly rotating circular cloud cores lopsided ($M = 1$ asymmetry, see also Syer & Tremaine 1996). In particular, in the absence of rotation, the system of equations of the problem has an analytical solution, where iso-surface-density contours are ellipses of eccentricity $e$,

$$\Sigma(\varpi, \varphi) = \frac{K}{\varpi(1 - e \cos \varphi)},$$

with $K$ constant and $0 < e < 1$. Notice that the limit $e \rightarrow 1$ produces a semi-infinite filament with mass per unit length $2\pi K$. For values of $e$ between these two extremes, both iso-surface-density contours and equipotentials are confocal ellipses of eccentricity $e$. Fig. 1 shows an example of a nonrotating SIDs with $e = 0.3$.

On the other hand, bifurcations into sequences with $M = 2$, 3, 4, 5, and higher symmetry require non-zero rotation rates ($> 0.7$ times the magnetosonic speeds), as shown in Fig. 2. These values are considerably larger that is typically measured for observed molecular cloud cores (e.g. Goodman et al. 1993). Although seemingly more promising for binary and multiple star-formation, the models with $M = 2$, 3, 4, 5,... symmetries all terminate in shockwaves before their separate lobes can succeed in forming anything that resembles separate bodies. For these configurations to exist at all, the basic rotation rate has to be fairly close to magnetosonic. It is then not possible for the nonaxial symmetry to become sufficiently pronounced as to turn streamlines that circulate around a single center to streamlines that circulate around multiple centers (as is needed to form multiple stars), without the distortions causing supermagnetosonically flowing gas to slam into submagnetosonically flowing gas. The resultant shockwaves then transport angular momentum outward and mass inward in such a fashion as to prevent fission.

### 2.1. The Need for Magnetic Flux Loss

This negative result, combined with the analysis of the spiral instabilities that afflict the more rapidly rotating, self-gravitating, disks into which more slowly rotating, cloud cores collapse (also modeled here as SIDs), is cause for pessimism that a successful mechanism of binary and multiple star-formation can be found by either the fission or the fragmentation process acting in the aftermath of the gravitational collapse of marginally supercritical clouds during the stages when field freezing provides a good dynamical assumption.

In contrast, we know that the dimensionless mass-to-flux ratio $\lambda$ has to increase from values typically $\sim 2$ in cloud cores to values in excess of 5000 in formed stars (Li & Shu 1997). Massive loss of magnetic flux must have occurred at some stage of the gravitational collapse of molecular cloud cores to form stars. Moreover, this loss must take place at some point at a dynamical rate, or even faster, since the collapse process from pivotal molecular cloud cores is itself dynamical. It is believed that dynamical loss of magnetic fields from cosmic gases occurs only when the volume density exceeds $\sim 10^{11}$ H$_2$ molecules cm$^{-3}$.
Figure 1. (a) Iso-surface-density contours for non-rotating SIDs, seen face-on, are confocal ellipses with eccentricity $e$ (in this case, $e = 0.3$). (b) Poloidal magnetic field lines, for the same SID as in (a), seen edge-on. The magnetic field lines leave the disk at an angle of 45°.

(e.g., Nakano & Umebayashi 1986a,b; Desch & Mouschovias 2000). It might be thought that cloud cores have to collapse to fairly small linear dimensions before the volume density reaches such high values, and therefore, that only close binaries can be explained by such a process, but not wide binaries (McKee 2000, personal communication). However, this impression is gained by experience with axisymmetric collapse. Once the restrictive assumption of perfect axial symmetry is removed, we gain the possibility that some dimensions may shrink faster than others (e.g., Lin, Mestel, & Shu 1965), and densities as high as $10^{11}$ cm$^{-3}$ might be reached while only one or two dimensions are relatively small, and while the third is still large enough to accommodate the (generally eccentric) orbits of wide binaries.

The linearized stability analysis and the nonlinear simulations of Shu et al. (2000) suggests that the collapse of gravitationally unstable axisymmetric SIDs lead to configurations that are stable to further collapse but dynamically unstable to an infinity of nonaxisymmetric spiral modes that again transport angular momentum outward and mass inward in such a fashion as to prevent disk fragmentation. We suspect the same fate awaits the collapse of pivotal SIDs that are non-axisymmetric to begin with, as long as we continue with the assumption of field freezing. Thus, we speculate that rapid (i.e., dynamical rather than quasi-static) flux loss during some stage of the star formation process is an essential ingredient to the process of gravitational fragmentation to form binary and multiple stars from present-day molecular clouds.

3. A Specific Example: the Molecular Cloud Core L1544

As an example, Fig. 3 shows an overlay of one of our eccentrically displaced static models projected onto a map of thermal dust emission at 1.3 mm obtained by Ward-Thompson, Motte, & André (1999) for the prestellar molecular cloud core
Figure 2. Locus in the $D^2|-S_M|$ plane of sequences of equilibria with given $M$-fold symmetry. Here $D$ is the ratio of the rotation rate to the magnetosonic speed, and $S_M$ the amplitude of the dominant component in the Fourier expansion of the surface density. The dashed line indicates the locus of axisymmetric equilibria. Sequences of equilibria originate from axisymmetric models and terminate because of the occurrence of shocks. Isodensity contours (thick solid lines) and streamlines (thin solid lines) of terminal models are shown at the endpoints of each sequence.

L1544. Apart from relatively minor fluctuations due to the cloud turbulence, the solid curves depicting the iso-surface-density contours of the theoretical model match well both the observed shapes and grey-scale of the dust isophotes.

Zeeman measurements of the magnetic-field component parallel to our line of sight toward L1544 have been made by Crutcher & Troland (2000), who obtain $B_\parallel = 11 \pm 2 \mu G$. For a highly flattened disk, which is reflection symmetric about the plane $z = 0$, integration along the line of sight yields cancelling contributions of $B_\varphi$ and $B_\varphi$ to $B_\parallel$. The $z$-component of the magnetic field of our model core is given by

$$B_z = \frac{2\pi G^{1/2}}{\lambda} \Sigma. \quad (5)$$

We may now calculate the average value of $\Sigma$ within a radius $R$ as

$$\langle \Sigma \rangle = \frac{1}{\pi R^2} \int d\varphi \int_0^R \Sigma \varpi d\varpi = \frac{\lambda^2(\lambda^2 + 3)}{\lambda^4 - 1} \frac{a^2}{\pi G R}. \quad (6)$$
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\[ \langle \Sigma \rangle = \frac{\lambda}{\langle \Sigma \rangle} = \frac{\lambda(\lambda^2 + 3)(\lambda^4 - 1)}{2a^2 G^{1/2} R} \quad (7) \]

Since we model L1544 as a thin disk with elliptical iso-surface-density contours, its orientation in space is defined by three angles, two specifying the orientation of the disk plane, the third giving the position of the elliptical contours in this plane. We fix the first angle by assuming for simplicity that the major axis of the elliptical contours lies in the plane of the sky. The second angle \( i \) is the inclination of the minor axis with respect to the plane of the sky \( (i = 0 \text{ for a face-on disk}) \) and can be adjusted to fit the observations. The third angle, specifying the ellipse’s orientation in the disk plane, is given as 38° north through east by Ward-Thompson et al. (2000).

We choose the eccentricity \( e \) and inclination \( i \) by the following procedure. From Fig. 3, we can estimate that a typical dust contour has a ratio of distances closest and farthest from the core center given in a model of nested confocal ellipses by \( (1 - e^2)/(1 + e) \approx 0.30 \), which implies \( e \approx 0.54 \). Similarly, we may estimate that these ellipses have an apparent minor-to-major axis-ratio of \( (1 - e^2)^{1/2} \cos i \approx 0.54 \), which implies \( \cos i \approx 0.64 \). The resulting ellipses for three

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\text{Figure 3. Iso-surface-brightness contours (thick solid lines) from a theoretically computed, lopsided, magnetized, self-gravitating figure of equilibrium compared with isophotal measurements of Ward-Thompson et al. (1999) of the submillimeter emission from heated dust grains in L1544. The short solid line and dashed line show the directions of predicted and measured field inferred from submillimeter-wave polarization observations (Ward-Thompson et al. 2000).}
\]
iso-surface-density contours, spaced in a geometric progression 1:2:4, are shown as solid curves in Fig. 3.

Determination of $\cos i$ allows us to compute an expected $\langle B_{\parallel} \rangle = \langle B_z \rangle \cos i$. Similarly, we obtain the expected hydrogen column density by multiplying $\langle \Sigma \rangle$ by $(\cos i)^{-1}$ for a slant path through an inclined sheet and by 0.7 for the mass fraction of H nuclei of mass $m_H$: $N_H = 0.7 \langle \Sigma \rangle / (m_H \cos i)$.

The sound speed for the 10 K gas in L1544 is $a = 0.19$ km s$^{-1}$ (Tafalla et al. 1998). These authors give $\Delta V = 0.22$ km s$^{-1}$ as the typical linewidth for their observations of $^{34}$S in this region. For such a heavy molecule, turbulence is the main contributor to the linewidth, which allows us to estimate the mean square turbulent velocity along a typical direction (e.g., the line of sight) as $v_t^2 = \Delta V^2 / 8 \ln 2$. We easily compute that $v_t^2$ has only 24% the value of $a^2$. Assuming that it is possible to account for the "pressure" effects of such weak turbulence by adding the associated velocities in quadrature, $a^2 + v_t^2$, we adopt an effective isothermal sound speed of $a = 0.21$ km s$^{-1}$ for L1544.

The radius of the Arecibo telescope beam at the distance of L1544 is $R = 0.06$ pc (Crutcher & Troland 2000). Ambipolar diffusion calculations by Nakano (1979), Lizano & Shu (1989), Basu & Mouschovias (1994) suggest that $\lambda \approx 2$ when the pivotal state is approached (see the summary of Li & Shu 1996). Putting together the numbers, $\cos i = 0.64$, $R = 0.06$ pc, $a = 0.21$ km s$^{-1}$, and $\lambda = 2$, we get $\langle B_{\parallel} \rangle = 11 \; \mu G$, in excellent agreement with the Zeeman measurement of Crutcher & Troland (2000). These authors also deduce $N_H = 1.8 \times 10^{22}$ cm$^{-2}$ from their OH measurements, whereas we compute a hydrogen column density within the Arecibo beam of $N_H = 1.4 \times 10^{22}$ cm$^{-2}$. The slight level of disagreement is probably within the uncertainties in the calibration or calculation of the fractional abundance of OH in dark clouds (cf. Crutcher 1979, van Dishoeck & Black 1986, Heiles et al. 1993).

Our ability to obtain good fits of much of the observational data concerning the prestellar core L1544 with a simple analytical model should be contrasted with other, more elaborate, efforts. Consider, for example, the axisymmetric numerical simulation of Ciolek & Basu (2000), who were forced to assume a disk close to being edge-on ($\cos i \approx 0.3$ when $e$ is assumed to be 0) to reproduce the observed elongation, but who left unexplained the eccentric displacement of the cloud core’s center (very substantial for ellipses of eccentricity $e \approx 0.54$).

The adoption of axisymmetric cores leads to another problem: Ciolek & Basu’s deprojected magnetic field is on average 3-4 times stronger than ours, values never seen directly in Zeeman measurements of low-mass cloud cores. [See the comments of Crutcher & Troland (2000) concerning the need for magnetic fields in Taurus to be all nearly in the plane of the sky if conventional models are correct.] Natural elongation plus projection effects, as anticipated in the comments of Shu et al. (1999), allow us to model L1544 as a moderately supercritical cloud, with $\lambda \approx 2$, fully consistent with the theoretical expectations from ambipolar diffusion calculations, and in contrast with the value $\lambda \approx 8$ estimated by Crutcher & Troland (2000) from the measured values of $B_{\parallel}$ and $N_H$. In addition, if L1544 is a thin, intrinsically eccentric, disk seen moderately face-on, as implied by our model, then the extended inward motions observed by Tafalla et al. (1998; see also Williams et al. 1999) may be attributable to a (relatively fast) core-amplification mechanism that gathers gas (neutral and ionized) dy-
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namically but subsonically along magnetic field lines on both sides of the cloud toward the disk’s midplane.

Finally, we show in Fig. 3 the direction of the average magnetic field projected in the plane of the sky predicted by our model (thin solid line) and derived from submillimeter polarization observations of Ward-Thompson et al. (2000) (thin dashed line). Since we have assumed in our model that the major axis of iso-surface-density contours is in the plane of the sky, the predicted projection of the magnetic field is parallel to the cloud’s minor axis. The offset between the measured position angle of the magnetic field and the cloud’s minor axis might indicate some inclination of the cloud’s major axis with respect to the plane of the sky. The turbulent component of the magnetic field, not included in our model, may also contribute to the observed deviation.

4. Conclusions

We close with the following analogies. The basic problem with trapped magnetic fields is that they compress like relativistic gases (i.e., their stresses accumulate as the 4/3 power increase of the density in 3-D compression). Such gases have critical masses [e.g., the Chandrasekhar limit in the theory of white dwarfs, or the magnetic critical mass of equation (1)] which prevent their self-gravitating collections from suffering indefinite compression, no matter how high is the surface pressure, if the object masses lie below the critical values. Moreover, while marginally supercritical objects might collapse to more compact objects (e.g., white dwarfs into neutron stars, or cloud cores into stars), a single such object cannot be expected to naturally fragment into multiple bodies (e.g., a single white dwarf with mass slightly bigger than the Chandrasekhar limit into a pair of neutron stars).

In order for fragmentation to occur, it might be necessary for the fluid to decouple rapidly from its source of relativistic stress. For example, the universe as a whole always has many thermal Jeans masses. Yet in conventional big-bang theory, this attribute did not do the universe any good in the problem of making gravitationally bound subunits, as long as the universe was tightly coupled to a relativistic (photon) field. Only after the matter field had decoupled from the radiation field in the recombination era, did the many fluctuations above the Jeans scale have a chance to produce gravitational “fragments.” It is our contention that this second analogy points toward where one should search for a viable theory of the origin of binary and multiple stars from the gravitational collapse of magnetized molecular cloud cores.

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