Velocity analysis of a spherical parallel robot

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Abstract. A spherical mechanism of parallel structure containing three kinematic chains and performing spherical motions is presented. A distinctive feature of the architecture of the spherical mechanisms is the provision of the constancy of the position of the point – center of rotations. This characteristic causes a wide range of applications of these mechanisms. The kinematic analysis of the mechanism is based on the differentiation of the coupling equations imposed by the kinematic chains. As a result of differentiation, a matrix is obtained by means of which it is possible to solve the direct and inverse problems of kinematics. The velocity analysis is necessary to control the robot and its dynamic analysis.

1. Introduction
Spherical parallel robots are used in surgery for orientation of various devices, for example, television cameras, in measuring and technological devices [1–5]. One of the robots of this type is a device with a circular guide (figure 1), in which the drive carriages are moved along the circular guide [6–8].

![Figure 1. Schematic of a spherical parallel mechanism with a circular guide.](image-url)
This design allows, in particular, to use this robot for surgical operations, as it can move the surgical instrument, while maintaining a constant point of entry of the instrument. One of the most important tasks associated with the control of this robot is the problem of speeds [9, 10] which will be considered in this work.

2. Problem
The problem of velocity analysis is to determine the relationship between generalized and absolute velocities. The rates of changes in the orientation angles of the output link are referred as the absolute speed. Orientation angles \( \alpha, \beta \) and \( \gamma \) are the angles of the output link rotation relative to the coordinate axes \( Ox \), \( Oy \) and \( Oz \) respectively. The rates of changes in the angles of rotation (movement) of the carriages \( \phi_{11}, \phi_{21} \) and \( \phi_{31} \), which move along a circular guide, are referred as the generalized velocities. The output link oriented with respect to the coordinate axes can carry the working tool, moving relative to the output link by means of additional kinematic pairs.

3. Theory
Consider the spherical mechanism of a parallel structure with three kinematic chains (figure 1). Each of the three kinematic chains contains three rotational kinematic pairs, one of which connected with the drive, and the other two are non-driven. The values of the input generalized coordinates \( \phi_{11}, \phi_{21} \) and \( \phi_{31} \) are given by three drives associated with the input kinematic pairs of kinematic chains.

According to the solution of the positioning problem, one can write implicit function equal to zero and linking the three absolute coordinates \( \alpha, \beta \) and \( \gamma \) and the corresponding generalized coordinate for each kinematic chain. Let’s find the partial derivatives of this implicit function with respect to both the absolute and generalized coordinates for each of the kinematic chains. Since this function is zero, its full differential is also zero. On the basis of the relations for partial derivatives one can write matrix velocity equation. The resulting system of linear equations can be solved at known absolute velocities or at known generalized velocities.

Let us take the following parameters of kinematic chains. For all kinematic chains the unit vectors of the first kinematic pairs have coordinates:

\[ \mathbf{e}_{11} = \mathbf{e}_{21} = \mathbf{e}_{31} = (0 \ 0 \ 1)^T. \]

Unit vectors of the second and third kinematic pairs of the first kinematic chain in the initial position respectively:

\[ \mathbf{e}_{12} = (1 \ 0 \ 0)^T; \quad \mathbf{e}_{13} = (0 \ 0.707 \ 0.707)^T. \]

The power screw transmitted from the side of this kinematic chain to the output link in the initial position is equal to the vector product:

\[ \mathbf{r}_1 = \mathbf{e}_{12} \times \mathbf{e}_{13} = (0 \ -0.707 \ 0.707)^T. \]

Note that the power screw in this case is a pure moment.

The vector of the second kinematic pair of the second kinematic chain has coordinates:

\[ \mathbf{e}_{22} = (-0.5 \ 0.866 \ 0)^T. \]

The vector of the third kinematic pair is rotated relative to the vector of the third kinematic pair of the first kinematic chain by an angle of \( 2\pi/3 \) rad, therefore its coordinates are equal to:

\[
\mathbf{e}_{23} = \begin{pmatrix}
\cos(2\pi/3) & -\sin(2\pi/3) & 0 \\
\sin(2\pi/3) & \cos(2\pi/3) & 0 \\
0 & 0 & 1
\end{pmatrix} \cdot \mathbf{e}_{13} = (-0.612 \ -0.354 \ 0.707)^T.
\]
Power screw transmitted to the output link from the second kinematic chain:

\[ \mathbf{r}_2 = \mathbf{e}_{23} \times \mathbf{e}_{23} = (0.612 \quad 0.354 \quad 0.707)^T. \]

Vector of the second kinematic pair of the third kinematic chain:

\[ \mathbf{e}_{32} = (-0.5 \quad -0.866 \quad 0)^T. \]

The vector of the third kinematic pair is rotated by an angle of \( 4\pi/3 \) rad with respect to the vector of the third kinematical pair of the first chain. Therefore, it can be found as follows:

\[ \mathbf{e}_{33} = \begin{pmatrix} \cos(4\pi/3) & -\sin(4\pi/3) & 0 \\ \sin(4\pi/3) & \cos(4\pi/3) & 0 \\ 0 & 0 & 1 \end{pmatrix} \cdot \mathbf{e}_{13} = (0.612 \quad 0.354 \quad 0.707)^T. \]

The power screw (moment) transmitted to the output link from the third kinematic chain is found as a vector product:

\[ \mathbf{r}_3 = \mathbf{e}_{32} \times \mathbf{e}_{33} = (0.612 \quad 0.354 \quad 0.707)^T. \]

To check whether this position is not singular, let’s write down the determinant of the matrix of coordinates of the power screws (moments) found above:

\[ |\mathbf{r}| = \begin{vmatrix} 0 & 0.612 & -0.612 \\ -0.707 & 0.354 & 0.354 \\ 0.707 & 0.707 & 0.707 \end{vmatrix} = 0.918. \]

The determinant is nonzero, hence the position is not singular.

The rotation of the output link around the coordinate axes \( Ox, Oy \) and \( Oz \) is described by the matrices \( \mathbf{A}_\alpha, \mathbf{A}_\beta \) and \( \mathbf{A}_\gamma \) respectively:

\[ \mathbf{A}_\alpha = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos \alpha & -\sin \alpha \\ 0 & \sin \alpha & \cos \alpha \end{pmatrix}; \quad \mathbf{A}_\beta = \begin{pmatrix} \cos \beta & 0 & \sin \beta \\ 0 & 1 & 0 \\ -\sin \beta & 0 & \cos \beta \end{pmatrix}; \quad \mathbf{A}_\gamma = \begin{pmatrix} \cos \gamma & -\sin \gamma & 0 \\ \sin \gamma & \cos \gamma & 0 \\ 0 & 0 & 1 \end{pmatrix}. \]

The position of the output link is described by the matrix of rotations \( \mathbf{A}\mathbf{A} \), obtained by the product of matrices:

\[ \mathbf{A}\mathbf{A} = \mathbf{A}_\alpha \cdot \mathbf{A}_\beta \cdot \mathbf{A}_\gamma = \begin{pmatrix} \cos \gamma \cos \beta & \cos \gamma \sin \beta \sin \alpha - \sin \gamma \cos \alpha & \cos \gamma \sin \beta \cos \alpha + \sin \gamma \sin \alpha \\ \sin \gamma \cos \beta & \sin \gamma \sin \beta \sin \alpha + \cos \gamma \cos \alpha & \sin \gamma \sin \beta \cos \alpha - \cos \gamma \sin \alpha \\ -\sin \beta & \cos \beta \sin \alpha & \cos \beta \cos \alpha \end{pmatrix}. \]

Let’s find the unit vectors coordinates of the kinematic pairs in an arbitrary position. The current position of the second kinematic pair for the first chain:

\[ \mathbf{e}_{t2} = (\cos \varphi_{t2} \quad \sin \varphi_{t2} \quad 0)^T. \]

The current position of the third kinematic pair unit vector of the first chain is determined as:

\[ \mathbf{e}_{t3} = \mathbf{A}\mathbf{A} \cdot \mathbf{e}_{13} = \begin{pmatrix} 0.707 \left[ \sin \gamma \left( \sin \alpha - \cos \alpha \right) + \cos \gamma \sin \beta \left( \sin \alpha + \cos \alpha \right) \right] \\ 0.707 \left[ \cos \gamma \left( \cos \alpha - \sin \alpha \right) + \sin \gamma \sin \beta \left( \sin \alpha + \cos \alpha \right) \right] \\ 0.707 \cos \beta \left( \sin \alpha + \cos \alpha \right) \end{pmatrix}. \]
The scalar product of vectors \( \mathbf{e}_{t12} \) and \( \mathbf{e}_{t13} \) is zero:

\[
\mathbf{e}_{t12} \cdot \mathbf{e}_{t13} = 0.707 \cos \varphi_{11} \left[ \sin \gamma (\sin \alpha - \cos \alpha) + \cos \gamma \sin \beta (\sin \alpha + \cos \alpha) \right] \\
+ 0.707 \sin \varphi_{11} \left[ \cos \gamma (\cos \alpha - \sin \alpha) + \sin \gamma \sin \beta (\sin \alpha + \cos \alpha) \right] = 0.
\]

After the transformation we obtain the value of the first generalized coordinate as a ratio:

\[
\varphi_{11} = \arctg \left( \frac{-\sin \gamma (\sin \alpha - \cos \alpha) + \cos \gamma \sin \beta (\sin \alpha + \cos \alpha)}{\cos \gamma (\cos \alpha - \sin \alpha) + \sin \gamma \sin \beta (\sin \alpha + \cos \alpha)} \right).
\]

The following notation is introduced for the scalar product of vectors of kinematic pairs:

\[
\mathbf{F}_i = \mathbf{e}_{t12} \cdot \mathbf{e}_{t13}.
\]

Next, we can find the partial derivatives:

\[
\frac{d \mathbf{F}_i}{d \varphi_{11}} = 0.707 \cos \varphi_{11} \left[ \cos \gamma (\cos \alpha - \sin \alpha) + \sin \gamma \sin \beta (\sin \alpha + \cos \alpha) \right] \\
- 0.707 \sin \varphi_{11} \left[ \sin \gamma (\sin \alpha - \cos \alpha) + \cos \gamma \sin \beta (\sin \alpha + \cos \alpha) \right];
\]

\[
\frac{d \mathbf{F}_i}{d \alpha} = 0.707 \cos \varphi_{11} \left[ \sin \gamma (\cos \alpha + \sin \alpha) + \cos \gamma \sin \beta (\cos \alpha - \sin \alpha) \right] \\
- 0.707 \sin \varphi_{11} \left[ \cos \gamma (\sin \alpha + \cos \alpha) + \sin \gamma \sin \beta (\sin \alpha - \cos \alpha) \right];
\]

\[
\frac{d \mathbf{F}_i}{d \beta} = 0.707 \cos \varphi_{11} \cos \gamma \cos \beta (\sin \alpha + \cos \alpha) + 0.707 \sin \varphi_{11} \sin \gamma \cos \beta (\sin \alpha + \cos \alpha);
\]

\[
\frac{d \mathbf{F}_i}{d \gamma} = 0.707 \cos \varphi_{11} \left[ \cos \gamma (\sin \alpha - \cos \alpha) - \sin \gamma \sin \beta (\sin \alpha + \cos \alpha) \right] \\
- 0.707 \sin \varphi_{11} \left[ \sin \gamma (\cos \alpha - \sin \alpha) - \cos \gamma \sin \beta (\sin \alpha + \cos \alpha) \right].
\]

Using a similar approach, we can write scalar product of vectors \( \mathbf{e}_{t22} \) and \( \mathbf{e}_{t23} \), that is equal to zero, and find the value of the generalized coordinate of the second kinematic chain:

\[
\mathbf{e}_{t22} \cdot \mathbf{e}_{t23} = \mathbf{F}_2;
\]

\[
= 0.354 \cos \varphi_{21} \left[ \sin \gamma (2 \sin \alpha + \cos \alpha) + \cos \gamma \sin \beta (2 \cos \alpha - \sin \alpha) - 1.732 \cos \gamma \cos \beta \right] \\
+ 0.354 \sin \varphi_{21} \left[ \cos \gamma (\cos \alpha - 2 \sin \alpha) + \sin \gamma \sin \beta (\sin \alpha + 2 \cos \alpha) - 1.732 \sin \gamma \cos \beta \right] = 0;
\]

\[
\varphi_{21} = \arctg \left( \frac{-\sin \gamma (2 \sin \alpha + \cos \alpha) + \cos \gamma \sin \beta (2 \cos \alpha - \sin \alpha) - 1.732 \cos \gamma \cos \beta}{\cos \gamma (\cos \alpha - 2 \sin \alpha) + \sin \gamma \sin \beta (\sin \alpha + 2 \cos \alpha) - 1.732 \sin \gamma \cos \beta} \right) + \pi.
\]

In the ratio above the term \( \pi \) appeared due to the fact, that the second drive operates in the corresponding sector of the circular guide.

Partial derivative with respect to the generalized coordinate of the second kinematic chain:

\[
\frac{d \mathbf{F}_2}{d \varphi_{21}} = 0.354 \cos \varphi_{21} \left[ \cos \gamma (\cos \alpha - 2 \sin \alpha) + \sin \gamma \sin \beta (\sin \alpha + 2 \cos \alpha) - 1.732 \sin \gamma \cos \beta \right] \\
- 0.354 \sin \varphi_{21} \left[ \sin \gamma (2 \sin \alpha + \cos \alpha) + \cos \gamma \sin \beta (2 \cos \alpha - \sin \alpha) - 1.732 \cos \gamma \cos \beta \right].
\]

Partial derivatives of the second kinematic chain with respect to the absolute coordinates:
\[
\frac{d\mathbf{F}_1}{d\alpha} = 0.354 \cos \phi_{31} \left[ \sin \gamma (2 \cos \alpha - \sin \alpha) - \cos \gamma \sin \beta (2 \sin \alpha + \cos \alpha) \right] \\
-0.354 \sin \phi_{31} \left[ \cos \gamma (\sin \alpha + 2 \cos \alpha) + \sin \gamma \sin \beta (2 \sin \alpha - \cos \alpha) \right];
\]
\[
\frac{d\mathbf{F}_2}{d\beta} = 0.354 \cos \phi_{31} \left[ \cos \gamma \cos \beta (2 \cos \alpha - \sin \alpha) + 1.732 \cos \gamma \sin \beta \right] \\
+0.354 \sin \phi_{31} \left[ \sin \gamma \cos \beta (\sin \alpha + 2 \cos \alpha) + 1.732 \sin \gamma \sin \beta \right];
\]
\[
\frac{d\mathbf{F}_3}{d\gamma} = 0.354 \cos \phi_{31} \left[ \cos \gamma (2 \sin \alpha + \cos \alpha) - \sin \gamma \sin \beta (2 \cos \alpha - \sin \alpha) + 1.732 \sin \gamma \cos \beta \right] \\
-0.354 \sin \phi_{31} \left[ \sin \gamma (\cos \alpha - 2 \sin \alpha) - \cos \gamma \sin \beta (\sin \alpha + 2 \cos \alpha) + 1.732 \cos \gamma \cos \beta \right].
\]

With a similar approach for the third kinematic chain, we can write scalar product of vectors \( \mathbf{e}_{32} \) and \( \mathbf{e}_{33} \), that is equal to zero, and find the value of the generalized coordinate \( \phi_{31} \):

\[
\mathbf{e}_{32} \cdot \mathbf{e}_{33} = \mathbf{F}_i
\]

\[
= 0.354 \cos \phi_{31} \left[ \sin \gamma (2 \sin \alpha + \cos \alpha) + \cos \gamma \sin \beta (2 \cos \alpha - \sin \alpha) + 1.732 \cos \gamma \cos \beta \right] \\
+0.354 \sin \phi_{31} \left[ -\cos \gamma (2 \sin \alpha + \cos \alpha) + \sin \gamma \sin \beta (2 \cos \alpha - \sin \alpha) + 1.732 \sin \gamma \cos \beta \right] = 0;
\]

\[
\phi_{31} = \arctg \left[ \frac{-\sin \gamma (2 \sin \alpha + \cos \alpha) + \cos \gamma \sin \beta (2 \cos \alpha - \sin \alpha) + 1.732 \cos \gamma \cos \beta}{-\cos \gamma (2 \sin \alpha + \cos \alpha) + \sin \gamma \sin \beta (2 \cos \alpha - \sin \alpha) + 1.732 \sin \gamma \cos \beta} \right] + \pi.
\]

Partial derivative with respect to the generalized coordinate of the third kinematic chain:

\[
\frac{d\mathbf{F}_1}{d\phi_{31}} = 0.354 \cos \phi_{31} \left[ -\cos \gamma (2 \sin \alpha + \cos \alpha) + \sin \gamma \sin \beta (2 \cos \alpha - \sin \alpha) + 1.732 \sin \gamma \cos \beta \right] \\
-0.354 \sin \phi_{31} \left[ \sin \gamma (2 \sin \alpha + \cos \alpha) + \cos \gamma \sin \beta (2 \cos \alpha - \sin \alpha) + 1.732 \cos \gamma \cos \beta \right].
\]

Partial derivatives of the second kinematic chain with respect to the absolute coordinates:

\[
\frac{d\mathbf{F}_2}{d\alpha} = 0.354 \cos \phi_{31} \left[ \sin \gamma (2 \cos \alpha - \sin \alpha) - \cos \gamma \sin \beta (2 \sin \alpha + \cos \alpha) \right] \\
+0.354 \sin \phi_{31} \left[ \cos \gamma (\sin \alpha + 2 \cos \alpha) - \sin \gamma \sin \beta (2 \sin \alpha - \cos \alpha) \right];
\]
\[
\frac{d\mathbf{F}_3}{d\beta} = 0.354 \cos \phi_{31} \left[ \cos \gamma \cos \beta (2 \cos \alpha - \sin \alpha) - 1.732 \cos \gamma \sin \beta \right] \\
+0.354 \sin \phi_{31} \left[ \sin \gamma \cos \beta (2 \cos \alpha - \sin \alpha) - 1.732 \sin \gamma \sin \beta \right];
\]
\[
\frac{d\mathbf{F}_3}{d\gamma} = 0.354 \cos \phi_{31} \left[ \cos \gamma (2 \sin \alpha + \cos \alpha) - \sin \gamma \sin \beta (2 \cos \alpha - \sin \alpha) - 1.732 \sin \gamma \cos \beta \right] \\
+0.354 \sin \phi_{31} \left[ \sin \gamma (2 \sin \alpha + \cos \alpha) + \cos \gamma \sin \beta (2 \cos \alpha - \sin \alpha) + 1.732 \cos \gamma \cos \beta \right].
\]

Partial derivative is the implicit function linking three absolute coordinates and the generalized one. It is always zero, so its total differential is also zero.

Let’s consider an example: we will set the values \( \alpha = \pi/3, \beta = \pi/6 \) and \( \gamma = \pi/4 \) and use the above relations to find the values of the generalized coordinates and partial derivatives for kinematic chains. For the first kinematic chain we obtain:

\[
\phi_{11} = -1.277 \text{ rad}; \quad \frac{d\mathbf{F}_1}{d\phi_{11}} = 0.548; \quad \frac{d\mathbf{F}_1}{d\alpha} = 0.913; \quad \frac{d\mathbf{F}_1}{d\beta} = -0.395; \quad \frac{d\mathbf{F}_1}{d\gamma} = -0.548.
\]
For the second kinematic chain we will have the following values:

\[
\varphi_{21} = 3.42 \text{ rad}; \quad \frac{d \mathbf{F}_2}{d \varphi_{21}} = 0.936; \quad \frac{d \mathbf{F}_2}{d \alpha} = 0.322; \quad \frac{d \mathbf{F}_2}{d \beta} = -0.303; \quad \frac{d \mathbf{F}_2}{d \gamma} = -0.936.
\]

Values of the generalized coordinate and partial derivatives for the third kinematic chain:

\[
\varphi_{31} = 4.539 \text{ rad}; \quad \frac{d \mathbf{F}_3}{d \varphi_{31}} = 0.964; \quad \frac{d \mathbf{F}_3}{d \alpha} = 0.35; \quad \frac{d \mathbf{F}_3}{d \beta} = 0.217; \quad \frac{d \mathbf{F}_3}{d \gamma} = -0.964.
\]

Let's write down the Angeles-Gosselin matrix for the absolute coordinates:

\[
\begin{pmatrix}
\frac{d \mathbf{F}_1}{d \alpha} & \frac{d \mathbf{F}_1}{d \beta} & \frac{d \mathbf{F}_1}{d \gamma} \\
\frac{d \mathbf{F}_2}{d \alpha} & \frac{d \mathbf{F}_2}{d \beta} & \frac{d \mathbf{F}_2}{d \gamma} \\
\frac{d \mathbf{F}_3}{d \alpha} & \frac{d \mathbf{F}_3}{d \beta} & \frac{d \mathbf{F}_3}{d \gamma}
\end{pmatrix}
= 
\begin{pmatrix}
0.913 & -0.395 & -0.548 \\
0.322 & -0.303 & -0.936 \\
0.35 & 0.217 & -0.964
\end{pmatrix}
\]

And Angeles-Gosselin matrix for the generalized coordinates:

\[
\begin{pmatrix}
\frac{d \mathbf{F}_1}{d \varphi_{11}} & 0 & 0 \\
0 & \frac{d \mathbf{F}_2}{d \varphi_{21}} & 0 \\
0 & 0 & \frac{d \mathbf{F}_3}{d \varphi_{31}}
\end{pmatrix}
= 
\begin{pmatrix}
-0.548 & 0 & 0 \\
0 & -0.936 & 0 \\
0 & 0 & -0.964
\end{pmatrix}
\]

The inverse Jacobian matrix will be:

\[
\text{Job} = \text{AAbCo} \cdot \text{AObCo}^{-1} = \begin{pmatrix}
-1.665 & 0.422 & 0.568 \\
-0.587 & 0.324 & 0.971 \\
-0.639 & -0.232 & 1
\end{pmatrix}
\]

And the Jacobian direct matrix:

\[
\text{Jpr} = \text{Job}^{-1} = \begin{pmatrix}
-0.749 & 0.755 & -0.307 \\
0.045 & 1.775 & -1.749 \\
-0.468 & 0.894 & 0.398
\end{pmatrix}
\]

Let the absolute velocity be equal to one, then the generalized velocities will be:

\[
\begin{pmatrix}
\dot{\varphi}_{11} \\
\dot{\varphi}_{21} \\
\dot{\varphi}_{31}
\end{pmatrix} = \text{Job} \cdot \begin{pmatrix}
\dot{\alpha} \\
\dot{\beta} \\
\dot{\gamma}
\end{pmatrix} = \begin{pmatrix}
-1.665 & 0.422 & 0.568 \\
-0.587 & 0.324 & 0.971 \\
-0.639 & -0.232 & 1
\end{pmatrix} \cdot \begin{pmatrix}
1 \\
1 \\
1
\end{pmatrix} = \begin{pmatrix}
-0.675 \\
0.708 \\
0.129
\end{pmatrix}
\]

Let the generalized velocity be equal to one, then the absolute velocities will be:
4. Experimental result

The considered mechanism was manufactured using gears between the drive carriages and the circular guide (figure 2). The experiment confirmed the correctness of the approach to structural synthesis: the output link moves while maintaining the constant center of rotation. The third kinematic pair of each kinematic chain is made as spherical, which is quite acceptable, since one of the rotations according to this pair passes through the center \( O \) in any case.

\[
\begin{pmatrix}
\alpha \\
\beta \\
\gamma
\end{pmatrix} = J_{pr} \begin{pmatrix}
\dot{\phi}_{11} \\
\dot{\phi}_{21} \\
\dot{\phi}_{31}
\end{pmatrix} = \begin{pmatrix}
-0.749 & 0.755 & -0.307 \\
0.045 & 1.775 & -1.749 \\
0.468 & 0.894 & 0.398
\end{pmatrix} \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} \begin{pmatrix}
1 \\
0.301 \\
0.071
\end{pmatrix}.
\]

5. Discussion of results

Thus, this paper presents a solution to the problem of velocities for a spherical robot of a parallel structure. The solution is based on the differentiation of the coupling equations (the Angeles- Gosselin method). This task is important, in particular, in the control of the considered robot. In addition, it will be further used in the dynamic analysis of the given device.

6. Conclusion

In conclusion, we note that the considered parallel structure spherical robot is an effective mean of solving a number of technical problems, in particular those related to the manipulation of surgical instruments. At the same time, the input point of the mentioned tool remains constant. The solution of the velocity problem required to control this robot can be quite effectively implemented on the basis of coupling equations differentiation.

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