Parallel multicomponent interferometry with a spinor Bose-Einstein Condensate

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Atomic interferometry with high-visibility is of high demand for precision measurements. Here, a parallel multi-component interferometer is achieved by preparing a spin-2 Bose-Einstein condensate of $^{87}$Rb atoms confined in a hybrid magneto-optical trap. After the preparation of a spinor Bose-Einstein condensate with spin degrees of freedom entangled, we observe four spatial interference patterns in each run of measurements corresponding to four hyperfine states we mainly populate in the experiment. The atomic populations in different Zeeman sublevels are made controllable using a magnetic-field-pulse induced Majorana transitions. The spatial separation of atoms in different hyperfine states is reached by Stern-Gerlach momentum splitting. The high-visibility of the interference fringes is reached by designing a proper overlap of the wave-packets. Due to uncontrollable phase accumulation in Majorana transitions, the phase of each individual spin is found to be subjected to unreproducible shift in multiple experimental runs. However, the relative phase across different spins is stable, paving a way towards noise-resilient multi-component parallel interferometers.

I. INTRODUCTION

Quantum mechanical particles such as photons, electrons and atoms have been used to construct two-slit interferometry, which plays important roles in studying fundamental quantum theories and enables high-sensitivity measurements, leading to important applications such as quantum precision measurement, quantum information and quantum simulation [1–4]. For example, the “smokingkun” experimental demonstration of phase coherence in a Bose-Einstein condensate (BEC) has been achieved with atomic interferometry [5]. Matter-wave interferometers with long coherence times using ultracold atomic gases can be used for high-precision measurements in a broad context, e.g., in the study of the quantum properties of atoms [6] and the quantum phase evolution of the wave function [7–9], the correlation characteristics [10], the dynamics of isolated quantum many-body systems [11–15] and gravitational effects [16–18].

Recently, there have been growing efforts in developing matter wave interferometry with unconventional approaches. Ramsey interferometers (RIs) with atomic external motion states of a BEC trapped in a harmonic potential have been demonstrated with high interferometric visibility for several cycles [19, 20]. Coherent superposition of different momentum-spin states that entangle internal and external states has been achieved [21]. The interferometer based on the spin entanglement can be used to study quantum effects of gravity [22] and construct quantum simulation platforms. Atomic clock interferometry has offered a promising high-precision tool to study the interplay of general relativity and quantum physics, which can help to formulate a modern version of the uncertainty principle in terms of entropies, and deepen our understanding of the wave-particle duality [23–26].

In the applications of atomic interferometers, it is crucial to further improve the sensitivity and reduce the apparatus complexity, which has motivated the developments of a multi-state interferometric scheme [27]. The spatial interference fringes would provide a solid experimental evidence for phase coherence [28, 29]. However, this experimental atomic spatial interference fringes have not been observed for high spin atomic systems, although this represents one promising experimental platform to implement multi-state interferometers. The contrast and phase stability are important parameters of atomic interferometers and are important for studying coherence and correlation properties [30–32].

Here we report realization of a parallel multi-component interferometer, utilizing the BEC phase coherence in a spin-2 cold atomic gas. In our experiment, interference has been observed in different spin channels. We mainly focus on two effects that affect the visibility of interference fringes—the overlap of interfering wave-packets, and fringe spacing. By properly optimizing experimental parameters, a high-visibility multi-spin parallel interferometry is achieved. By extracting the information of the interference fringes, we find that the phase of the single component is unstable, i.e., fluctuates in repeated experiments, but the phase difference of the extracted multi-components remains stable. This achieved multi-component spatial interferometer that extracts stable phase information can be used for high-precision measurement, and its integration with optical lattice interferometer that probes the spatial and temporal decoherence of cold atoms is worth further study.

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A magnetic trap, the Majorana loss is negligible in our Thomas-Fermi approximation. Two Majorana transition by controlling the fall in the second Stern-Gerlach process. After a time of flight (TOF), the interference fringes can be observed by an absorption imaging.

II. EXPERIMENTAL DEMONSTRATION AND DATA ANALYSIS

A. Experimental implementation

In the experiment, we prepare a BEC of $^{87}$Rb in an atomic hyperfine state $|F, m_F\rangle = |2, 2\rangle$ in a hybrid trap, combining an optical dipole trap formed by a laser beam with a wavelength of 1064nm and a quadruple magnetic trap with a field gradient $B_y = 12.4\, \text{G/cm}$. The atom number is about $2 \times 10^5$ atoms and the temperature 70nK. The trapping frequencies in three directions are $(\omega_x, \omega_y, \omega_z) = (2\pi\times(28\, \text{Hz}, 55\, \text{Hz}, 65\, \text{Hz})$, corresponding to the diameter of atom cloud 25µm, 8.6µm, and 6.6µm in three directions, respectively, under the Thomas-Fermi approximation.

After the preparation of BEC, we turn off the optical trap in about 200µs and let the atom cloud expand 3.8ms to make a bigger size, so let have a better wave function overlap in the latter process. Then two non-adiabatic Majorana transitions are applied to induce population in five hyperfine states utilized to make a parallel multicomponent interferometry. Given that the time between the shut off of the optical trap and the first magnetic field pulse is short around 3.8 ms, and that the atom cloud is confined about 158µm below the center of the quadruple magnetic trap, the Majorana loss is negligible in our experiment.

Although Majorana transition is a very old problems, the technical difficulty of using Majorana transition in constructing the multicomponent interferometer lies in the precise control of spin flips at the time of applying magnetic pulses so that the atoms in different submagnetic states are finely tunable. Here the spin projection in the Majorana transition is controlled by changing the off-time of the magnetic field pulse with a signal generator. This magnetic field pulse is applied with an added Helmholtz coil with a current 1.1A during the holding time, which provides a magnetic field of a few hundred milligauss along $z$ direction. The Larmor frequency, when the magnetic field pulse is off, is about 274kHz with the weak magnetic field along the $y$ direction. The rising time of the magnetic field pulse which is about 40µs is rather slow comparable to the Larmor frequency, so the turn-on process is adiabatic. The switch-off time can be as short as 5µs, which is comparable to the Larmor precession and is thus nonadiabatic leading to Majorana transitions. In between the turning-on and -off of the pulse is kept by 20µs to balance the magnetic field along the $z$ axis.

In the first magnetic field pulse (MP1), the magnetic field in the $z$ direction $B_z$ is adiabatically turned on and then switched off in 16µs, where the falling curve consists of two straight lines with different slopes. This ramp time is chosen to project the system into nearly two hyperfine states of approximately equal population, our measurement shows the resultant populations for the hyperfine states $|m_F\rangle = |2\rangle$, $|m_F\rangle = |1\rangle$, and $|m_F\rangle = |0\rangle$ are 50%, 45%, and 5%, respectively, and the populations for other states are essentially negligible.

In the interval $t_1$ and $t_2$, there is a field gradient $B_y$ supplied by the quadrupole magnetic trap. Due to the difference of the magnetic forces, atoms in different hyperfine states will be spatially separated. Taking into account the gravity acceleration $g$ along the $y$ direction, the resultant netforce on the $|m_F\rangle$ is $f_{m_F} \approx (2-m_F)mg$ in our experiment. The measured relative acceleration between $|1\rangle$ and $|2\rangle$ states is consistent with this. The accumulated velocity difference of two species in the time interval of $t_1$ is $\delta v = \frac{gt_1}{2}$ and a spatial separation is estimated to be on the order of submicron.

The second magnetic field pulse (MP2), chosen to be identical to the first one except that the ramp-off time is 8.5 µs, i.e., roughly two times faster than the first pulse, in which the action of the MP2 can be almost seen as a nonadiabatic transitions, its detail calculation is given in Section V. After this pulse, atoms populate five hyperfine states. Following the MP2, we hold the system for a time duration $t_2$, from which different atomic species acquire a velocity difference given by $\delta v_{m_F} = (2-m_F)gt_2/2$. Meanwhile, the separation of atoms cloud of the same hyperfine state coming from the two copies made by the MP1 keeps increasing owing to their different velocity accumulated during the $t_1$ interval. The eventual separation of two copies of each hyperfine state at time $T_2$ is then $d \approx gt_1t_2/2$, which is independent of $m_F$.

After that, we turn off the quadrupole magnetic trap, and atoms will expand. Atoms of the same hyperfine state would interfere in the ballistic expansion. A typical picture in the plane $y, z$ after 26 ms time of flight (TOF) is shown in Fig. 2(a), where multiple interference fringes...
spatially separated corresponding to the five hyperfine components are observed with the chosen $t_1 = 158\mu s$ and $t_2 = 1300\mu s$. To show it more clearly, we integrate over the $z$ direction, and the interference distribution in $y$ axis are given as the points in Fig. 2(b1-b5), corresponding to the different state $|2\rangle$, $|1\rangle$, $|-1\rangle$, $|-2\rangle$, $|0\rangle$, respectively. The interference pattern is clearly revealed (see Fig. 2).

B. Theory for the multi-component interferometer

The process of designing the parallel multi-component atomic interferometry with the different sub-magnetic level $m_F$ and atomic spin $F$, as shown in Fig. 1. Firstly, the initial prepared condensate with the wave-packet $\psi = |F, m_F\rangle$ is projected into two submagnetic states $\psi_+ = |F, m_F\rangle$ and $\psi_- = |F, m_F + 1\rangle$ by an optimized state-selective operation at the gravity $y$ direction. These two states evolve a period of time $t_1$ in a gradient magnetic field and obtain a speed difference $\delta v$, while the wave-packets are still overlap. Each state is separated into multi-magnetic state ($n = 2F + 1$) by another state-selective operation, which varies from $|F, m_F\rangle$ to $|F, m_F + n\rangle$. Then the different submagnetic states evolve a period of time $t_2$, and the different components will be separated in space for the gradient of the magnetic field. Finally, after the time-of-flight with time $t_3$ with switching off all the trap, the same submagnetic state with the separation $D$ will give an interference pattern, and a parallel multi-component interferometer is achieved in camera with the absorb detect method. From these pictures, the interference information including the fringes spacing and the visibility can be gotten.

In order to understand the processes, the time-dependent condensate wave-function after the time of flight takes an approximate form with the width of the wave-packet $\sigma_{m_F}$ as $[7, 21]$

$$\psi_{q, m_F} = \Gamma_{q, m_F} \exp[-\alpha_{m_F}(y + qD/2)^2 + iv_{q, m_F}(y + qD/2) + i\theta_{q, m_F}],$$

which is associated with the hyperfine $|m_F\rangle$ component generated from the $q$th copy—$q = \pm$ labels the two copies of atom cloud before applying the MP2 (see Fig. 1). Here $\alpha_{m_F} = \frac{1}{2\sigma_{m_F}} - \frac{1}{2}i\frac{\gamma_{m_F}}{2\sigma_{m_F}}$ is a complex number, and becomes purely imaginary in the long TOF limit. The linear term $v_{p, m_F}$ is the average propagation speed of the wave-packet, and $\Gamma_\pm$ represents the amplitude of the wavepacket.

The coherent superposition $|\psi_{+, m_F}(y) + \psi_{-, m_F}(y)|^2$ describes the interference patterns formed by the two copies of atom cloud populating the $|m_F\rangle$ state. In the long TOF limit where the interfering wavepackets are well-overlapped, $\alpha_{m_F}$ is approximately given by $\alpha_{m_F} \approx M/(2i\hbar\lambda)$, where $M$ is the atomic mass, which leads to a simplified form of interference pattern,

$$|\psi_{+, m_F}(y) + \psi_{-, m_F}(y)|^2 = A_{m_F, const} + A_{m_F, osc} \cos[ky + \phi_{m_F}].$$

Here we have a constant part $A_{m_F, const} = \Gamma_{+, m_F}^2 + \Gamma_{-, m_F}^2$, and an oscillating part $A_{osc} = 2\Gamma_{+, m_F}\Gamma_{-, m_F}$ that leads to the interference fringes, which are characterized by the wavenumber $k = D/t_3$ and the phase $\phi_{m_F}$. In the experiment, $k$ is controllable by varying the distance $d$. Although $\phi_{m_F}$ fluctuates in different experimental runs due to the uncontrollable phase accumulation in the MP2. The phase differences among the five spin components are stable, as they are determined by the coherent processes controllable in the experiment.

C. Fitting the interference patterns

The interference pattern reached in Eq. (2) relies on the long TOF limit and perfect experimental realization with single magnetic state condensate $[7, 38]$. The actual interference pattern in the experiment follows an empirical fit having the same form $[21]$ as in Eq. (2)

$$\Phi(y) = A_y G(y)[1 + V \cos(\frac{2\pi}{\lambda} y + \phi)],$$

where $A_y$ is an overall amplitude, $G(y)$ is a Gaussian, $\lambda$ is the spacing of the fringes, and $V$ is the visibility. Due to experimental imperfection we expect the

FIG. 2. (Color online) (a) The typical interference picture in the $(y, z)$ plane. (b1)-(b5) After integrating along $z$ axis in (a), the density distribution along $y$ direction for the five component condensates. The points are the experiment data and the curves are fitting results by Eq. (3). (c) The average of consecutive experimental measurements with a contrast reduction of 0 for the chosen state $m_F = | -1 \rangle$. 


visibility to be smaller than the theoretical value of
\[ 2\Gamma_{+mF}\Gamma_{-mF}/[\Gamma_{+mF}^2 + \Gamma_{-mF}^2] \] (Eq. (2)). We use the expression Eq. (3) to fit the experimental data, as shown by the solid curves in Fig. 2(b1-65). From the fitting, we get that the visibility for the state \(|1\rangle, |−1\rangle, |2\rangle, \) and \(|−2\rangle\) are about 0.6±0.1, where the experiment parameter \(t_1 = 158\mu s\) and \(t_2 = 1300\mu s\) are chosen. Meanwhile the fringes spacing are about 26±1\mu m. We observe almost no interference fringes in the \(|0\rangle\) species for that the fraction of \(|0\rangle\) component generated from the MP2 is tiny, consistent with the theoretical analysis in Section V. Hence in the following we mainly study the interference pattern for other four components.

III. TOWARDS OPTIMAL VISIBILITY WITH TIME SEQUENCE DESIGN

A. Visibility of interference fringes

We find the interference fringes are highly dependent on the distance between wave-packet centers for every hyperfine state. In the experiment the time \(t_1\) in Fig. 1 can be changed from 80\mu s to 210\mu s, so the separation \(D\) can increase from 20\mu m to 44\mu m with \(D = d + \sqrt{\lambda t_2 t_3}\). The experimental interference is shown in Fig. 3, where we observe a nonmonotonic dependence of visibility on \(D\). The visibility firstly improves with increasing \(D\), and then decreases, with an optimal separation at \(D_{\text{optimal}} \approx 35\mu m\). The optimal interference fringes are obtained for different hyperfine states at roughly the same \(D\). The initial improvement of the visibility upon increasing \(D\) can be attributed to the increasing of the wavenumber \(k\), which leads to more interference fringes accessible to experimental measurements. The eventual decrease in the visibility at a larger \(D\) is because TOF allowed in the experiment is not long enough to make a good overlap of the wave-packets, which causes a smaller number of atoms in the interference regime. This understanding is further confirmed with our simulations provided below.

B. Simulating interference fringes by changing the overlapping of wave-packets

To understand the results in Fig. 3 more clearly, we simulate the fringes by Eq. (2) with different \(D/\sigma\) with the same wave-packets width \(\sigma = 20\mu m\), as is shown in Fig. 4. In characterizing the quality of the observed interference, there are three important length scales to keep in mind—the interference fringe spacing \(\lambda\), the distance \(D\), and the width of each interfering wave-packet \(\sigma\). When \(D\) is significantly smaller than \(\sigma\), the interference quality is mainly affected by the ratio of \(\sigma/\lambda\). The larger it is, the more interference fringes would fall into the region we have large number of atoms (see Fig. 4(a,b)), which would lead to a better visibility in measurements. This explains the initial improvement of visibility by increasing \(D\) (Fig. 3). On the other hand, when the distance \(D\) becomes larger than the wave-packet width \(\sigma\), the number of atoms that contribute to the interference becomes smaller (see Fig. 4(c)). This causes a decrease in the visibility because of the practical limitation in the atom number resolution in experiments. In Fig. 4(d), by extracting the contrast of the simulated fringes, we show that keeping \(\sigma/\lambda = 0.8\) unchanged, changing the ratio \(D/\sigma\), and obtaining the contrast change curve, which is consistent with the theoretical analysis.

C. Analysis of the visibility for the different components

To characterize the property of the multicomponent interferometer, it is important to know how the interference fringe spacing \(\lambda_{mF}\) depends on the initial separation of the wave-packets \(d\) before TOF. In the long TOF limit, we have \(\lambda_{mF} = 2\pi t_3/D\). We find the experimental observation does not agree with this quantitatively, which
can be attributed to finite time effects. We then fit experimental results with an empirical form [7, 21, 39]

\[ \lambda_{m_F} = 2\pi/(\beta_{m_F}d + \zeta_{m_F}), \] (4)

which is confirmed with numerical simulations of the interference. The hyperfine state \( m_F \) dependence is from their different magnetic forces.

We vary the distance \( d \) by choosing the pulse interval \( t_2 \) from 110\( \mu s \) to 190\( \mu s \). The experimental results are shown in Fig. 5. For different hyperfine states, we obtain \( \beta_2 = 0.027, \beta_1 = 0.022, \beta_{-2} = 0.011, \beta_{-2} = -0.029 \) by fitting with Eq. (4). The intercepts are found to be \( \zeta_2 = 0.20, \zeta_1 = 0.20, \zeta_{-1} = 0.21, \zeta_{-2} = 0.28 \), which exhibits a weaker \( m_F \) dependence compared to the slope term \( \beta_{m_F} \).

IV. THE PHASE STABILITY

By fitting the experimental data as shown in Fig. 2(b) using Eq. (3), we also can get the phase information \( \phi \) of wave function from the fringes and the center of Gaussian wave-packet. In repeating the experiment, if we take an average of these pictures, the interference pattern actually disappears—the averaging washes out the interference fringes, as shown in Fig. 2(c). This means that the phase repeatability is poor and the phase for each component in every experimental run is evenly distributed. At the same time, phase information is very important in the study of quantum coherence and also for precision measurements. To further study it, we give the phase distribution for the continuously measured more than 61 experimental run in Fig. 6(a). The phase for each component \( \phi_{m_F} \) \( (m_F = 2, 1, -1, -2) \) almost is random, this means we can not control the phase of the wave function. This is due to the fluctuations in the magnetic field [21] during the adiabatic procedure of the MP2.

However, we found the phase difference between different hyperfine states is stable. This is because the relative phase across different states is determined by the coherent process following the MP2. As shown in Fig. 6(b), the distribution of the relative phase \( \Delta \phi \) for \( \phi_{2} - \phi_{1}, \phi_{-2} - \phi_{-1}, \phi_{2} - \phi_{-2}, \phi_{1} - \phi_{-1} \) are given in (b1) to (b4), respectively. The main values for the \( \phi_{2} - \phi_{1} \) and \( \phi_{-2} - \phi_{-1} \) are concentrated at about zero degrees, while the latter two have the distribution about 180 degrees, where the \( t_1 = 158\mu s \) and \( t_2 = 1300\mu s \), and the optical trap is turned off at \( t_0 = 3.8ms \) in advance.

This distribution of the relative phase difference, can be controlled by the position of the atomic ensemble in gradient magnetic field, according to change time \( t_0 \) that defined as the time difference between switching off the optical trap and the MP1. If we change the above value \( t_0 = 3.8ms \) in Fig. 6 to \( t_0 = 3.6ms \), the phase distribution for \( \phi_{2} - \phi_{1}, \phi_{1} - \phi_{-1} \) are given in (a1) and (a2) of Fig. 7, respectively. Its value of the phase difference is about 180 degrees and 350 degrees. If \( t_0 = 3.5ms \), these distribution changes to about 90 degrees and 60 degrees, as shown (b1) and (b2) of Fig. 7. This means we can obtain the steady phase difference for the different patterns by adjusting the time of the optical trap turned off in advance of MP1, it supplies the possible precise measurements by this multicomponent interferometry.
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$$F.\text{ DISCUSSION}$$

In order to understand why the $|0\rangle$ state is basically free of fringes. Here we given the calculation for the concrete form is

$$\hat{U}_{m_F', m_F}(a, b) = \sum_n (-1)^n \xi \chi$$

where $\xi = \sqrt{(F-m_F)(F+m_F)}$ and $\chi = a^{F+m_F'-n}(a^*)^{F-m_F-n}b^n(b^*)^{n+m_F-m_F'}$, where the value $n$ in the summation contains all the integers making the four factorials in the denominator reasonable, and $|a|^2 + |b|^2 = 1$. For convenience, by taking $a = b = \frac{1}{\sqrt{2}}$, the concrete form is

$$\hat{U} \left( \frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}} \right) = \frac{1}{4} \begin{pmatrix} 1 & -2 & \sqrt{6} & -2 & 1 \\ 2 & -2 & 0 & 2 & -2 \\ \sqrt{6} & 0 & -2 & 0 & \sqrt{6} \\ 2 & 2 & 0 & -2 & -2 \\ 1 & 2 & \sqrt{6} & 0 & 2 & 1 \end{pmatrix}$$

Before action of $\hat{U}$, the state is denoted as

$$|\psi\rangle = \sum_{l=1}^5 \sqrt{n_{m_F}} e^{i\varphi_{m_F}} |m_F\rangle \otimes |p_{m_F}\rangle.$$
[1] A. D. Cronin, J. Schmiedmayer, and D. E. Pritchard, Rev. Mod. Phys. 81, 1051 (2009).
[2] D. M. Stamper-Kurn and M. Ueda, Rev. Mod. Phys. 85, 1191 (2013).
[3] A. Negretti, P. Treutlein, and T. Calarco, Quant. Inf. Proc. 10, 721 (2011).
[4] M. Han, P. Ge, Y. Shao, Q. Gong, and Y. Liu, Phys. Rev. Lett. 120, 073202 (2018).
[5] M. R. Andrews, C. G. Townsend, H.-J. Miesner, D. S. Durfee, D. M. Kurn, and W. Ketterle, Science 275, 637 (1997).
[6] K. Bongs and K. Sengstock, Rep. Prog. Phys. 67, 907 (2004).
[7] J. Simsarian, J. Denschlag, M. Edwards, C. W. Clark, L. Deng, E. W. Hagley, K. Helmerson, S. Rolston, and W. D. Phillips, Phys. Rev. Lett. 85, 2040 (2000).
[8] M. H. Wheeler, K. M. Mertes, J. D. Erwin, and D. S. Hall, Phys. Rev. Lett. 93, 170402 (2004).
[9] T. Schumm, S. Hofferberth, L. M. Andersson, S. Widermuth, S. Groth, I. Bar-Joseph, J. Schmiedmayer, and P. Krger, Nat. Phys. 1, 57 (2005).
[10] A. Polkovnikov, E. Altman, and E. Demler, P. Natl. Acad. Sci. 103, 6125 (2006).
[11] M. Gring, M. Kuhnert, T. Langen, T. Kitagawa, B. Rauer, M. Schreitl, I. Mazets, D. A. Smith, E. Demler, and J. Schmiedmayer, Science 337, 1318 (2012).
[12] T. Langen, R. Geiger, M. Kuhnert, B. Rauer, and J. Schmiedmayer, Nat. Phys. 9, 640 (2013).
[13] T. Langen, S. Erne, R. Geiger, B. Rauer, T. Schweigler, M. Kuhnert, W. Rohringer, I. E. Mazets, T. Gasenzer, and J. Schmiedmayer, Science 348, 207 (2015).
[14] A. Polkovnikov, K. Sengupta, A. Silva, and M. Vengalattore, Rev. Mod. Phys. 83, 863 (2011).
[15] M. Rigol, V. Dunjko, and M. Ishihani, Nature 452, 854 (2008).
[16] A. D. Cronin, J. Schmiedmayer, and D. E. Pritchard, Rev. Mod. Phys. 81, 1051 (2009).
[17] S. Dimopoulos, P. W. Graham, J. M. Hogan, and M. A. Kasevich, Phys. Rev. Lett. 98, 111102 (2007).
[18] H. H. Muntinga, H. Ahlers, M. Krutzik, A. Wenzlawski, S. Arnold, D. Becker, K. Bongs, H. Dittus, H. Duncker, N. Gaaloul, et al., Phys. Rev. Lett. 110, 093602 (2013).
[19] S. van Frank, A. Negretti, T. Berrada, R. Bcker, S. Montangero, J.-F. Schaff, T. Schumm, T. Calarco, and J. Schmiedmayer, Nat. Commun. 5, 4009 (2014).
[20] D. Hu, L. Niu, S. Jin, X. Chen, G. Dong, J. Schmiedmayer, and X. Zhou, Commun. Phys. 1, 29 (2018).
[21] S. Machluf, Y. Japha, and R. Folman, Nat. Commun. 4, 2424 (2013).
[22] S. Bose, A. Mazumdar, G. W. Morley, H. Ulbricht, M. Toro, M. Paternostro, A. A. Geraci, P. F. Barker, M. S. Kim, and G. Milburn, Phys. Rev. Lett. 119, 240401 (2017).
[23] M. Zych, F. Costa, I. Pikovski, and Č. Brukner, Nat. Commun. 2, 505 (2011).
[24] Y. Margalit, Z. Zhou, S. Machluf, D. Rohrlich, Y. Japha, and R. Folman, Science 349, 1205 (2015).
[25] Z. Zhou, Y. Margalit, D. Rohrlich, Y. Japha, and R. Folman, Classical Quant. Grav. 35, 185003 (2018).
[26] I. Pikovski, M. Zych, F. Costa, and Č. Brukner, New J. Phys. 19, 025011 (2017).
[27] J. Petrovic, I. Herrera, P. Lombardi, F. Schaefer, and F. S. Cataliotti, New J. Phys. 15, 043002 (2013).
[28] D. E. Miller, J. R. Anglin, J. R. Abo-Shaeer, K. Xu, J. K. Chin, and W. Ketterle, Phys. Rev. A 71, 043615 (2005).
[29] Y. Yang and W. Wang, Phys. Rev. A 91, 013623 (2015).
[30] S. P. Rath and W. Zwerger, Phys. Rev. A 82, 053622 (2010).
[31] S. Watanabe, S. Aizawa, and T. Yamakoshi, Phys. Rev. A 85, 043621 (2012).
[32] K. von Prillwitz, L. Rudnicki, and F. Mintert, Phys. Rev. A 92, 052114 (2015).
[33] B. Yang, S. Jin, X. Dong, Z. Liu, L. Yin, and X. Zhou, Phys. Rev. A 94, 043607 (2016).
[34] Z. Wang, B. Yang, D. Hu, X. Chen, H. Xiong, B. Wu, and X. Zhou, Phys. Rev. A 94, 033624 (2016).
[35] X. Zhou, S. Jin, and J. Schmiedmayer, New J. Phys. 20, 055005 (2018).
[36] F. Dalfovo, S. Giorgini, L. P. Pitaevskii, and S. Stringari, Rev. Mod. Phys. 71, 463 (1999).
[37] E. Majorana and N. Cimento, 9, 43 (1932).
[38] C. Fort, P. Maddaloni, F. Minardi, M. Modugno, and M. Inguscio, Opt. Lett. 26, 1039 (2001).
[39] Y. Castin and R. Dum, Phys. Rev. Lett. 77, 5315 (1996).
[40] X. Ma, L. Xia, F. Yang, X. Zhou, Y. Wang, H. Guo, and X. Chen, Phys. Rev. A 73, 013624 (2006).
[41] L. Xia, X. Xu, R. Guo, F. Yang, W. Xiong, J. Li, Q. Ma, X. Zhou, H. Guo, and X. Chen, Phys. Rev. A 77, 043622 (2008).