Fermion masses and mixings and some phenomenological aspects of a 3-3-1 model with linear seesaw mechanism

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We propose a viable model based on the $SU(3)_C \times SU(3)_L \times U(1)_X$ gauge group supplemented by the $S_4$ family symmetry and other auxiliary cyclic symmetries, whose spontaneous breaking gives rise to the observed pattern of SM fermion masses and mixing angles. In the proposed model the small light active neutrino masses are generated from a linear seesaw mechanism mediated by three Majorana neutrinos. The model is capable of reproducing the experimental values of the physical observables of both quark and lepton sectors. Our model is predictive in the quark sector having 9 effective parameters that allow to successfully reproduce the four CKM parameters and the six Standard Model (SM) quark masses. In the SM quark sector, there is particular scenario, motivated by naturalness arguments, which allows a a good fit for its ten observables, with only six effective parameters. We also study the single heavy scalar production via gluon fusion mechanism at proton-proton collider. Our model is also consistent with the experimental constraints arising from the Higgs diphoton decay rate.

Keywords: Extensions of electroweak gauge sector, Extensions of electroweak Higgs sector, Electroweak radiative corrections, Neutrino mass and mixing

1. INTRODUCTION

Although the Standard Model (SM) is a very well established quantum field theory highly consistent with the experimental data, it has several unexplained issues. For instance, the current pattern of SM fermion masses and mixing angles, the number of SM fermion families, the tiny values of active neutrino masses are some of the issues that do not find an explanation within the context of the SM. The pattern SM fermion mass hierarchy is spanned over a range of 13 orders of magnitude from the light active neutrino mass scale up to the top quark mass. In addition, the experimental data shows that the quark mixing pattern is significantly different from the leptonic mixing one. In the quark sector, the mixing angles are small, thus implying that the Cabibbo-Kobayashi-Maskawa (CKM) quark mixing matrix close to the identity matrix. On the other hand, two of the leptonic mixing angles are large and one is small, of the order of the Cabbibo angle, thus implying a Pontecorvo-Maki-Nakagawa-Sakata (PMNS) leptonic mixing matrix very different from the identity matrix. This is the so called flavor puzzle, which is not addressed by the SM and motivates to consider theories with augmented particle content and extended symmetries introduced to explain the current SM fermion mass and mixing pattern.

Theories with an extended $SU(3)_C \times SU(3)_L \times U(1)_X$ gauge symmetry [1–48] (3-3-1 models) are used to explain the origin of the three family structure in the fermion sector, which is left unexplained in the SM. In these models, the chiral anomaly cancellation condition is fulfilled when there are equal number of $SU(3)_L$ fermionic triplets and antitriplets, which occurs when the number of fermion families is a multiple of three. In addition, when the chiral anomaly cancellation condition is combined with the asymptotic freedom in QCD, models based on the $SU(3)_C \times SU(3)_L \times U(1)_X$ gauge symmetry predict the existence of three fermion families. Furthermore, the large mass difference between the heaviest quark and the two lighter ones can be explained in 3-3-1 models due to the fact that the third family is treated under a different representation than the first and second ones. Furthermore, the 3-3-1 models explain the quantization of the electric charge [49, 50], have sources of CP violation [51, 52], have a natural Peccei-Quinn symmetry, which solves the strong-CP problem [53–56], predict the limit $\sin^2 \theta_W < \frac{1}{4}$, for the weak mixing parameter. Besides that, if one includes heavy sterile neutrinos in the fermionic spectrum of the 3-3-1 models, such theories will have cold dark matter candidates as weakly interacting massive particles (WIMPs) [57–60]. A concise review of WIMPs in 3-3-1 Electroweak Gauge Models is provided in Ref. [61]. Finally, if one considers 3-3-1 Electroweak Gauge Models with three right handed Majorana neutrinos and without exotic charges, one can implement a low scale linear or inverse seesaw mechanism, useful for generating the tiny active neutrinos masses.

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In this work, motivated by the aforementioned considerations, we propose an extension of the 3-3-1 model with right handed Majorana neutrinos, where the scalar spectrum is enlarged by the inclusion of several gauge singlet scalars. Our model successfully explains current SM fermion mass spectrum and fermionic mixing parameters. In the proposed model, the $SU(3)_C \times SU(3)_L \times U(1)_X$ gauge symmetry is supplemented by the $S_4$ family symmetry and other auxiliary cyclic symmetries, whose spontaneous breaking produces the current pattern of SM fermion masses and mixings. In the model under consideration, the masses for the Standard Model charged fermions lighter than the top quark are produced by a Froggatt-Nielsen mechanism and the tiny masses for the light active neutrinos are generated by a linear seesaw mechanism. We use the $S_4$ family symmetry since it is the smallest non abelian group having a doublet, triplet and singlet irreducible representations, thus allowing to naturally accommodate the three fermion families of the SM. It is worth mentioning that the $S_4$ discrete group [10, 62–83] has been shown to provide a nice description for the observed pattern of SM fermion masses and mixing angles.

The layout of the remainder of the paper is as follows. In section 2 A we describe the proposed model, its symmetries, particle content and Yukawa interactions. The gauge sector of the model is described in section 2 B, whereas its low energy scalar potential is presented in section 2 C. In section 3 we discuss the implications of our model in the quark mass and mixing pattern. In Section 4, we present our results on lepton masses and mixing. The implications of our model in the Higgs diphoton decay rate are discussed in section 5. The production of the heavy $H_1$ scalar at proton-proton collider is discussed in Section 6. We conclude in section 7. Appendix A provides a description of the $S_4$ discrete group. Appendices B and C present a discussion of the scalar potentials for a $S_4$ scalar doublet and $S_4$ triplet, respectively.

2. THE MODEL.

A. Particle content

We consider an extension of the 3-3-1 model with right handed Majorana neutrinos, where the $SU(3)_C \times SU(3)_L \times U(1)_X$ gauge symmetry is supplemented by the $S_4 \times Z_3 \times Z_{12} \times Z_{16}$ discrete group and the scalar sector is enlarged by the inclusion of several gauge singlet scalars, which are introduced to generate viable textures for the fermion sector that successfully explain the current pattern of SM fermion masses and mixing angles. The scalar and fermionic content with their assignments under the $SU(3)_C \times SU(3)_L \times U(1)_X \times S_4 \times Z_3 \times Z_{12} \times Z_{16}$ group are shown in Tables I and II, respectively. The dimensions of the $SU(3)_C$, $SU(3)_L$ and $S_4$ representations shown in Tables I and II are specified by the numbers in boldface and the different $U(1)_X$ and $Z_N$ charges are written in additive notation. It is worth mentioning that a field $\psi$ transforms under the $Z_N$ symmetry with a corresponding $q_n$ charge as: $\psi \rightarrow e^{2\pi i q_n} \psi$, $n = 0, 1, 2, 3 \cdots N - 1$. We choose the $S_4$ symmetry since it is the smallest non abelian group having doublet, triplet and singlet irreducible representations, thus allowing us to naturally accommodate the three families of the SM left handed leptonic fields into a $S_4$ triplet, the three gauge singlet right handed Majorana neutrinos into one $S_4$ singlet and one $S_4$ doublet, the three right handed SM down type quarks into a $S_4$ singlet and a $S_4$ doublet and the remaining fermionic fields as $S_4$ singlets. In addition, the $S_4$, $Z_3$ and $Z_{12}$ symmetries shape the textures of the SM fermion mass matrices thus yielding a reduction of the model parameters, especially in the SM quark sector. In addition, the $Z_3$ symmetry separates the $S_4$ scalar triplets $(\Phi, \Xi, \Omega)$ participating in the charged lepton Yukawa interactions, from the ones $(\zeta, \Sigma, \Sigma)$ appearing in the neutrino Yukawa terms. The $Z_{16}$ symmetry shapes the hierarchical structure of the SM fermion mass matrices crucial to yield the observed SM fermion mass and mixing pattern. In our model, the masses of the Standard Model charged fermions lighter than the top quark arise from a Froggatt-Nielsen mechanism [84], triggered by non renormalizable Yukawa interactions invariant under the different discrete group factors. Thus, the current pattern of SM fermion masses and mixing angles arises from the spontaneous breaking of the $S_4 \times Z_3 \times Z_{12} \times Z_{16}$ discrete group. The masses of the light active neutrinos are generated from a linear seesaw mechanism, which can be implemented in our model because the third component of the $SU(3)_L$ leptonic triplet is electrically neutral and the fermionic spectrum includes three right handed Majorana neutrinos. In addition, the SM fermions in our model do not have exotic electric charges. Consequently, the electric charge is defined as:

$$Q = T_3 + \beta T_8 + XI = T_3 - \frac{1}{\sqrt{3}} T_8 + XI,$$

with $I = \text{diag}(1, 1, 1)$, $T_3 = \frac{1}{2} \text{diag}(1, -1, 0)$ and $T_8 = (\frac{1}{2\sqrt{3}}) \text{diag}(1, 1, -2)$ for a $SU(3)_L$ triplet.
In our model the full symmetry $G$ experiences the following three-step spontaneous breaking:

$$G = SU(3)_C \times SU(3)_L \times U(1)_X \times S_4 \times Z_3 \times Z_{12} \times Z_{16} \xrightarrow{\Lambda_{int}}$$

$$SU(3)_C \times SU(3)_L \times U(1)_X \xrightarrow{\nu}$$

$$SU(3)_C \times SU(2)_L \times U(1)_Y \xrightarrow{\nu_\nu}$$

$$SU(3)_C \times U(1)_Q,$$  

(2.2)

where the different symmetry breaking scales satisfy the following hierarchy $\Lambda_{int} \gg v_\chi \gg v_\eta, v_\rho$, with $v_\eta^2 + v_\rho^2 = 246$ GeV. The first step of spontaneous symmetry breaking is triggered by all gauge singlet scalars (excepting $\zeta$), charged under the discrete symmetries, assumed to acquire vacuum expectation values (VEVs) at a very large energy scale $\Lambda_{int} \gg v_\chi \sim \mathcal{O}(10) \text{ TeV}$. The second step of spontaneous symmetry breaking is caused by the $SU(3)_{int}$ scalar triplets $\chi$, whose third component acquires a 10 TeV scale vacuum expectation value (VEV) that breaks the $SU(3)_L \times U(1)_X$ gauge symmetry, thus providing masses for the exotic fermions, non Standard Model gauge bosons and the heavy CP even neutral scalar state of $\chi$. We further assume that the $S_4$ triplet gauge singlet scalar $\zeta$ acquires a VEVs at the same scale of $v_\eta$. Finally, the remaining two $SU(3)_L$ scalar triplets $\eta$ and $\rho$, whose first and second components, respectively, get VEVs at the Fermi scale, thus producing the masses for the SM particles and for the physical neutral scalar states arising from those scalar triplets. Here we are considering that the $SU(3)_L \times U(1)_X$ gauge symmetry is spontaneously broken at a scale of about 10 TeV in order to comply with collider constraints [85] as well as with the constraints arising from the experimental data on $K$, $D$ and $B$ meson mixings [86] and from the $B_{s,d} \to \mu^+\mu^-$ and $B_d \to K^*(K)\mu^+\mu^-$ decays [9, 87–90].

The $SU(3)_L$ triplet scalars $\chi, \eta$ and $\rho$ can be expanded around the minimum as follows:

$$\chi = \left( \begin{array}{c} \chi_0^0 \\ \chi_1^0 \\ \frac{1}{\sqrt{2}}(v_\chi + \xi_\chi \pm i\zeta_\chi) \end{array} \right), \quad \eta = \left( \begin{array}{c} v_\eta + \xi_\eta \pm i\zeta_\eta \\ \eta_3^0 \\ \eta_3^0 \end{array} \right), \quad \rho = \left( \begin{array}{c} v_\rho + \xi_\rho \pm i\zeta_\rho \\ \rho_3^0 \\ \rho_3^0 \end{array} \right).$$

(2.3)

The $SU(3)_L$ fermionic antitriplets and triplets are represented as:

$$Q_{nL} = \left( \begin{array}{c} D_n \\ -U_n^* \\ J_n \end{array} \right)_L, \quad Q_{3L} = \left( \begin{array}{c} U_3 \\ D_3 \\ T \end{array} \right)_L, \quad L_{iL} = \left( \begin{array}{c} \nu_i \\ \bar{c}_i \\ \bar{e}_i \end{array} \right)_L, \quad n = 1, 2, \quad i = 1, 2, 3.$$  

(2.4)

|       | $\chi$ | $\eta$ | $\rho$ | $\sigma_1$ | $\sigma_2$ | $\xi$ | $\Delta$ | $\Theta$ | $\Phi$ | $\Xi$ | $\Omega$ | $\zeta$ | $\Sigma$ | $S$ |
|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|
| $SU(3)_C$ | 1     | 1     | 1     | 1     | 1     | 1     | 1     | 1     | 1     | 1     | 1     | 1     | 1     | 1     |
| $SU(3)_L$ | 3     | 3     | 3     | 1     | 1     | 1     | 1     | 1     | 1     | 1     | 1     | 1     | 1     | 1     |
| $U(1)_X$ | $-\frac{1}{8}$ | $-\frac{1}{8}$ | 0     | 0     | 0     | 0     | 0     | 0     | 0     | 0     | 0     | 0     | 0     | 0     |
| $S_4$ | 1     | 1     | 1     | 1     | $1'$ | 2     | 2     | 2     | 3     | 3     | 3     | 3' | 3' | 3' |
| $Z_3$ | 0     | 0     | 0     | 0     | 0     | 0     | 0     | 0     | 1     | 1     | 1     | 0     | 0     | 0     |
| $Z_{12}$ | 0     | 0     | 0     | 0     | 0     | 0     | 0     | 0     | 1     | 1     | 1     | 1     | 1     | 0     |
| $Z_{16}$ | 0     | 0     | 0     | 0     | 0     | 0     | 0     | 0     | 0     | 0     | 0     | 0     | 0     | 0     |

Table I: Scalar assignments under $SU(3)_C \times SU(3)_L \times U(1)_X \times S_4 \times Z_3 \times Z_{12} \times Z_{16}$.

With the particle content shown in Tables I and II, the following relevant Yukawa interactions for both quark and lepton sectors arise:

$$-\mathcal{L}_Y^{(q)} = y_3^{(U)} \mathcal{Q}_{3L} \chi T_R + y_3^{(U)} \mathcal{Q}_{3L} \eta U_{3R} + y_3^{(U)} \mathcal{Q}_{3L} \rho^* U_{3R} \frac{\sigma_1^4}{A_1^4} + y_4^{(U)} \mathcal{Q}_{2L} \rho^* U_{2R} \frac{\sigma_1^4}{A_1^4} + y_4^{(U)} \mathcal{Q}_{1L} \rho^* U_{1R} \frac{\sigma_2^8}{A_8^8} + y_1^{(D)} \mathcal{Q}_{1L} \chi^* J_{1R} + y_2^{(D)} \mathcal{Q}_{2L} \chi^* J_{2R} + y_3^{(D)} \mathcal{Q}_{3L} \rho (\Theta D_R)_1 \frac{\sigma_2^8}{A_8^8} + y_4^{(D)} \mathcal{Q}_{2L} \eta^* (\Delta D_R)_1 \frac{\sigma_2^8}{A_8^8} + y_4^{(D)} \mathcal{Q}_{1L} \eta^* (\xi D_R)_1 \frac{\sigma_2^8}{A_8^8}$$

(2.5)
$-\mathcal{L}^{(l)}_Y = x_1^{(L)} (\bar{T}_L \rho \Phi)_1 e_1 R \frac{\sigma_2^2}{\Lambda^2} + x_2^{(L)} (\bar{T}_L \rho \Xi)_1 \xi_1 R \frac{\sigma_2^8}{\Lambda^9} + x_3^{(L)} (\bar{T}_L \rho \Omega)_1 \xi_1 R \frac{\sigma_2^8}{\Lambda^9}$

$+ y_1^{(L)} (\bar{T}_L \rho \Phi)_1 e_2 R \frac{\sigma_2^2}{\Lambda^2} + y_2^{(L)} (\bar{T}_L \rho \Xi)_1 \xi_2 R \frac{\sigma_2^8}{\Lambda^9} + y_3^{(L)} (\bar{T}_L \rho \Omega)_1 \xi_2 R \frac{\sigma_2^8}{\Lambda^9}$

$+ z_1^{(L)} (\bar{T}_L \rho \Phi)_1 e_3 R \frac{\sigma_2^2}{\Lambda^2} + z_2^{(L)} (\bar{T}_L \rho \Xi)_1 \xi_3 R \frac{\sigma_2^8}{\Lambda^9} + z_3^{(L)} (\bar{T}_L \rho \Omega)_1 \xi_3 R \frac{\sigma_2^8}{\Lambda^9}$

$+ y_1^{(L)} (\bar{T}_L \sigma)_1 \chi \eta_1 R \frac{1}{\Lambda} + y_2^{(L)} (\bar{T}_L \chi)_1 \lambda \eta_1 R \frac{1}{\Lambda} + y_3^{(L)} (\bar{T}_L \sigma)_1 \chi \eta_1 R \frac{1}{\Lambda} + y_4^{(L)} (\bar{T}_L \sigma)_1 \chi \eta_1 R \frac{1}{\Lambda} + H.c.$

(2.6)

As shown in detail in Appendices B and C, the following VEV configurations for the $S_4$ scalar doublets and $S_4$ scalar triplets are consistent with the scalar potential minimization equations for a large region of parameter space:

$\langle \xi \rangle = v_\xi (0, -1), \quad \langle \Delta \rangle = \frac{v_\Delta}{\sqrt{3}} (2, 1), \quad \langle \Theta \rangle = v_\Theta (1, 0), \quad \langle \Phi \rangle = v_\Phi (-1, 1, 1), \quad \langle \Xi \rangle = \frac{v_\Xi}{\sqrt{3}} (1, -1, 1), \quad \langle \Omega \rangle = \frac{v_\Omega}{\sqrt{2}} (1, 1, -1), \quad \langle \zeta \rangle = \frac{v_\zeta}{\sqrt{1 + c^2}} (1, 0, c), \quad \langle \Sigma \rangle = \frac{v_\Sigma}{\sqrt{3}} (1, -1, 1), \quad \langle S \rangle = \frac{v_S}{\sqrt{3}} (1, 1, 1).$  (2.7)

Furthermore, since the breaking of the $S_4 \times Z_3 \times Z_{12} \times Z_{16}$ discrete group gives rise to the charged fermion mass and quark mixing pattern, we set the VEVs of the $SU(3)_L$ singlet scalar fields with respect to the Wolfenstein parameter $\lambda = 0.225$ and the model cutoff $\Lambda$, as follows:

$v_\eta \sim v_\rho \ll v_\xi \sim v_\chi \ll v_\Delta \sim v_\Phi \sim v_\Xi \sim v_\Omega \sim v_\Theta \sim v_\Sigma \sim v_\Omega \sim \Lambda_{int} = \lambda \Lambda, \quad n = 1, 2$.  (2.8)

where the model cutoff $\Lambda$ can be interpreted as the scale of the UV completion of the model, e.g. the masses of Froggatt-Nielsen messenger fields.

### B. The gauge sector

The gauge bosons associated with the group $SU(3)_L$ with $\beta = -1/\sqrt{3}$ are given by:

$W_\mu = W_\mu^a G_a$

$= \frac{1}{2} \begin{pmatrix}
W_\mu^3 + \frac{1}{\sqrt{3}} W_\mu^8 & \sqrt{2} W_\mu^+ & \sqrt{2} K_\mu^0 \\
\sqrt{2} W_\mu^- & -W_\mu^3 + W_\mu^8 & \sqrt{2} K_\mu^- \\
\sqrt{2} K_\mu^+ & \sqrt{2} K_\mu^- & -\frac{2}{\sqrt{3}} W_\mu^8
\end{pmatrix}$  (2.9)

where $G_a$ ($a = 1, 2, 3$) are the Gell-Mann matrices. In addition, for $\beta = -\frac{1}{\sqrt{3}}$, the charges $Q$ for each field are:

$Q_W = \begin{pmatrix}
0 & 1 & 0 \\
-1 & 0 & -1 \\
0 & 1 & 0
\end{pmatrix}$  (2.10)

The gauge field associated with the $U(1)_X$ gauge symmetry has $Q_B = 0$ charge and is represented by:

$B_\mu = I_{3 \times 3} B_\mu$  (2.11)
In 331 models, three gauge fields have $Q = 0$ and combine to form the photon as well as the $Z$ and $Z'$ gauge bosons. Furthermore, there are two gauge fields with $Q = \pm 1 \ (W^\pm)$ and four exotic fields $W^{\prime \pm}, \ K^0, \ K^0$.

The covariant derivative in 331 models reads:

$$D_\mu = \partial_\mu + igW_\mu^\alpha G_\alpha + ig'X_\mu B_\mu$$  

(2.12)

Replacing (2.12) in the scalar kinetic interactions give rise to the gauge boson mass terms as well as to the interactions between the scalar and gauge bosons [91]:

$$\mathcal{L}_K = \sum_{\Phi = n, \rho, \chi} (D^\mu \Phi)\partial_\mu \Phi$$

$$= \sum_{\Phi = n, \rho, \chi} \left[ (\partial^\mu \Phi)\partial_\mu \Phi + (D^\mu \Phi)\partial_\mu \Phi - (\partial^\mu \Phi)\partial_\mu \Phi + \Phi (gW^\mu + g'X_\mu B_\mu) + (gW^\mu + g'X_\mu B_\mu)\Phi \right]$$  

(2.13)

This expression can be useful to get from (1) the couplings among gauge fields and the derivatives of the scalar fields, from which one can get information about each would-be Goldstone boson coupling to the corresponding massive gauge boson. Furthermore, from (2) one can get the gauge boson masses and its couplings with the physical Higgs fields.

The entries of the squared mass matrices for gauge bosons arise from the relation:

$$M^2_{V_i V_j} = \frac{\partial^2 \mathcal{L}_K}{\partial V_i \partial V_j},$$

(2.14)

where for the charged gauge bosons $V_i = W^\pm, W'^\pm$, whereas for the neutral ones $V_i = W^3, W^8, B, K^0, K^0$. The squared gauge bosons mass matrices are:

$$M^2_{\text{charged}} = \begin{pmatrix}
\frac{1}{4}g^2(\nu_\eta^2 + \nu_\rho^2) & \frac{1}{2}g^2\mu_\eta^2 - \frac{1}{2}g'\nu_\eta^2 - \frac{1}{4}g^2\mu_\rho^2 & 0 \\
\frac{1}{2}g^2\mu_\eta^2 - \frac{1}{2}g'\nu_\eta^2 - \frac{1}{4}g^2\mu_\rho^2 & \frac{1}{2}g^2\nu_\eta^2 + \frac{1}{2}g^2\nu_\rho^2 + \frac{1}{2}g^2\nu_\chi^2 & -\frac{1}{2}g'\nu_\eta^2 - \frac{1}{4}g'\nu_\rho^2 \\
0 & -\frac{1}{2}g'\nu_\eta^2 - \frac{1}{4}g'\nu_\rho^2 & \frac{1}{2}g^2\nu_\eta^2 + \frac{1}{2}g^2\nu_\rho^2 + \frac{1}{2}g^2\nu_\chi^2
\end{pmatrix}$$

(2.15)

$$M^2_{\text{neutral}} = \frac{1}{2}g^2\nu_\eta^2 + \frac{1}{2}g^2\nu_\rho^2 \left( \begin{array}{ccc}
\frac{1}{4}g^2(\nu_\eta^2 + \nu_\rho^2) & \frac{1}{2}g^2\nu_\eta^2 + \frac{1}{2}g^2\nu_\rho^2 + \frac{1}{2}g^2\nu_\chi^2 & 0 \\
\frac{1}{2}g^2\nu_\eta^2 + \frac{1}{2}g^2\nu_\rho^2 + \frac{1}{2}g^2\nu_\chi^2 & \frac{1}{2}g^2\nu_\eta^2 + \frac{1}{2}g^2\nu_\rho^2 + \frac{1}{2}g^2\nu_\chi^2 & 0 \\
0 & 0 & \frac{1}{2}g^2(\nu_\eta^2 + \nu_\rho^2)
\end{array} \right)$$

(2.16)

After the diagonalization, the gauge bosons mass spectrum is summarized in Table (III)

| Gauge Boson | Square Mass |
|-------------|-------------|
| $W^\pm$     | $\frac{1}{4}g^2(\nu_\eta^2 + \nu_\rho^2)$ |
| $W'^\pm$    | $\frac{1}{4}g^2(\nu_\eta^2 + \nu_\rho^2)$ |
| $\gamma$    | 0                                                      |
| $Z$         | $\frac{1}{8}(\Xi_1 - \Xi_2)$ |
| $Z'$        | $\frac{1}{8}(\Xi_1 + \Xi_2)$ |
| $K^0, \bar{K}^0$ | $\frac{g^2}{2}(\nu_\eta^2 + \nu_\rho^2)$ |

Table III: Physical gauge bosons mass spectrum.

with $\nu_\eta \simeq 173.948$ GeV, $\nu_\rho \simeq 173.948$ GeV and $\nu_\chi \simeq 10$ TeV. Consequently, for these values we find that the heavy gauge bosons are $M_{W^\prime} \simeq 3.2$ TeV, $M_{Z'} \simeq 6.3$ TeV. Notice that the obtained value of $M_{Z'} \approx 6.3$ TeV is consistent with the lower bound of 4 TeV on the $Z'$ gauge boson mass imposed by the experimental data on the $K, D$ and $B$ meson mixings [86].

In what follows we briefly comment about the LHC signals of a $Z'$ gauge boson. The heavy $Z'$ gauge boson is mainly produced via Drell-Yan mechanism and its corresponding production cross section has been found to range from 85
fb up to 10 fb for Z’ gauge boson masses between 4 TeV and 5 TeV and LHC center of mass energy $\sqrt{S} = 13$ TeV [92]. Such Z’ gauge boson after being produced will decay into pair of SM particles, with dominant decay mode into quark-antiquark pairs as shown in detail in Refs. [9, 93]. A comprehensive study of the two body decays of the Z’ gauge boson in 3-3-1 models is performed in Refs. [93], where it has been shown that the branching ratios of the of the Z’ decays into a lepton pair are of the order of $10^{-2}$, thus yielding a total LHC cross section of about 1 fb for the $pp \rightarrow Z' \rightarrow l^+l^-$ resonant production at $\sqrt{S} = 13$ TeV and $M_{Z'} = 4$ TeV, which is below its corresponding lower experimental bound resulting from LHC searches [94]. Finally, as pointed out in Ref. [92], at the proposed energy upgrade of the LHC with $\sqrt{S} = 28$ TeV, the LHC production cross section for the $pp \rightarrow Z' \rightarrow l^+l^-$ resonant production will be of the order of $10^{-2}$, at $M_{Z'} = 4$ TeV, which falls in the order of magnitude of its corresponding experimental lower bound resulting from LHC searches.

C. The low energy scalar potential and scalar mass spectrum

The renormalizable low energy scalar potential of the model under consideration is given by:

$$V = -\mu_1^2(\chi^\dagger \chi) - \mu_2^2(\eta^\dagger \eta) - \mu_3^2(\rho^\dagger \rho) + \frac{1}{2} (\eta_1 \chi_j \rho_k \varepsilon^{ijk} + H.c.) + \lambda_1(\chi^\dagger \chi)(\chi^\dagger \chi) + \lambda_2(\rho^\dagger \rho)(\rho^\dagger \rho) + \lambda_3(\eta^\dagger \eta)(\eta^\dagger \eta) + \lambda_4(\chi^\dagger \chi)(\rho^\dagger \rho) + \lambda_5(\chi^\dagger \chi)(\eta^\dagger \eta) + \lambda_6(\rho^\dagger \rho)(\eta^\dagger \eta) + \lambda_7(\chi^\dagger \eta)(\chi^\dagger \eta) + \lambda_8(\chi^\dagger \rho)(\rho^\dagger \chi) + \lambda_9(\rho^\dagger \eta)(\eta^\dagger \rho)$$

(2.17)

where $\chi$, $\rho$ and $\eta$ are the scalar triplets with VEVs in the third, second and first components, respectively. The minimization equations of the low energy scalar potential yield the following relations:

$$\mu_1^2 = \frac{-f_{\nu \nu \rho}}{\sqrt{2}v_\chi} + \frac{1}{2} \lambda_5 v_\eta^2 + \frac{1}{2} \lambda_4 v_\rho^2 + \lambda_1 v_\chi^2$$

(2.18)

$$\mu_2^2 = \frac{-f_{\nu \nu \chi}}{\sqrt{2}v_\rho} + \frac{1}{2} \lambda_6 v_\sigma^2 + \frac{1}{2} \lambda_2 v_\rho^2 + \frac{1}{2} \lambda_4 v_\chi^2$$

(2.19)

$$\mu_3^2 = \frac{-f_{\nu \nu \eta}}{\sqrt{2}v_\chi} + \frac{1}{2} \lambda_3 v_\eta^2 + \frac{1}{2} \lambda_6 v_\rho^2 + \frac{1}{2} \lambda_5 v_\rho^2$$

(2.20)

Replacing these mass parameters in the Higgs potential, the neutral and charged mass spectrum can be obtained doing the respective derivatives:

$$M_{\Phi_i}^2 = \frac{\partial^2 V}{\partial \Phi_i \partial \Phi_j} \bigg|_{\Phi_i=0}$$

$$M_{\Phi^c_i}^2 = \frac{\partial^2 V}{\partial \Phi_i^c \partial \Phi_j} \bigg|_{\Phi_i=0}$$

(2.21)

for the neutral scalar masses $\Phi_i = \xi, \epsilon, \chi, \xi, \chi, \chi, \epsilon, \eta_1, \eta_2$ and charged scalar masses $\Phi_i = \eta_2, \rho_1, \rho_2, \rho_3$ respectively. The scalar mass matrices are shown below:

$$M_{\xi, \epsilon, \chi}^2 = \begin{pmatrix}
\frac{f_{\nu \nu \eta}^{2}}{\sqrt{2}v_\chi} & \frac{f_{\nu \nu \rho}}{\sqrt{2}v_\rho} & \frac{f_{\nu \nu \chi}}{\sqrt{2}v_\chi} \\
\frac{f_{\nu \nu \rho}}{\sqrt{2}v_\rho} & \frac{f_{\nu \nu \chi}}{\sqrt{2}v_\chi} & \frac{f_{\nu \nu \eta}}{\sqrt{2}v_\chi} \\
\frac{f_{\nu \nu \chi}}{\sqrt{2}v_\chi} & \frac{f_{\nu \nu \eta}}{\sqrt{2}v_\chi} & \frac{f_{\nu \nu \rho}}{\sqrt{2}v_\rho}
\end{pmatrix}$$

$$M_{\xi, \epsilon, \chi}^2 = \begin{pmatrix}
2\lambda_1 v_\chi^2 + f_{\nu \nu \eta}^{2} v_\chi & \lambda_4 v_\rho v_\chi - f_{\nu \nu \rho} v_\chi & \lambda_5 v_\nu v_\chi - f_{\nu \nu \eta} v_\chi \\
\lambda_4 v_\rho v_\chi - f_{\nu \nu \rho} v_\chi & 2\lambda_2 v_\rho^2 + f_{\nu \nu \chi} v_\chi & \lambda_6 v_\nu v_\rho - f_{\nu \nu \rho} v_\chi \\
\lambda_5 v_\nu v_\chi - f_{\nu \nu \eta} v_\chi & \lambda_6 v_\nu v_\rho - f_{\nu \nu \rho} v_\chi & 2\lambda_3 v_\eta^2 + f_{\nu \nu \rho}^2 v_\eta
\end{pmatrix}$$

(2.22)

$$M_{\xi, \epsilon, \chi}^2 = \begin{pmatrix}
\lambda_7 v_\chi^2 + \frac{f_{\nu \nu \rho} v_\chi}{\sqrt{2}v_\rho} & \sqrt{2} f v_\rho + \lambda_7 v_\eta v_\chi \\
\sqrt{2} f v_\rho + \lambda_7 v_\eta v_\chi & \lambda_7 v_\chi^2 + \frac{f_{\nu \nu \rho} v_\chi}{\sqrt{2}v_\rho}
\end{pmatrix}$$

$$M_{\eta_1, \eta_2}^2 = \begin{pmatrix}
\lambda_8 v_\rho^2 + \frac{f_{\nu \nu \rho} v_\rho}{\sqrt{2}v_\rho} & \sqrt{2} f v_\rho + \lambda_8 v_\rho v_\chi \\
\sqrt{2} f v_\rho + \lambda_8 v_\rho v_\chi & \lambda_8 v_\rho^2 + \frac{f_{\nu \nu \rho} v_\rho}{\sqrt{2}v_\rho}
\end{pmatrix}$$

Finally, the scalar mass spectrum is summarized in Table IV where the physical mass spectrum is shown.
The 125 GeV mass value for the SM-like Higgs boson can be reproduced for the following benchmark point: 

The low energy physical scalar spectrum of our model is composed of the following fields: 2 CP-even Higgs bosons. The experimental values of the physical quark mass spectrum [95, 96], mixing angles and Jarlskog invariant [97] are taken to be complex. From the quark Yukawa interactions given by Eq. (2.5) we find that the SM mass matrices for quarks take the form:

$$M_U = \frac{v}{\sqrt{2}} \begin{pmatrix} c_1 \lambda^8 & 0 & a_1 \lambda^4 \\ 0 & b_2 \lambda^4 & 0 \\ 0 & 0 & a_2 \end{pmatrix}, \quad M_D = \frac{v}{\sqrt{2}} \begin{pmatrix} g_1 \lambda^7 & g_4 \lambda^6 & 0 \\ 0 & g_2 \lambda^5 & 2g_2 \lambda^5 \\ 0 & 0 & g_3 \lambda^3 \end{pmatrix},$$

where $\lambda = 0.225$ and $v = 246$ GeV. In order to get quark mixing angles and a CP violating phase consistent with the experimental data, we assume that all dimensionless parameters given in Eqs. (3.1) are real, except for $a_1$, taken to be complex. The exotic quark masses are:

$$m_T = y^{(T)} \frac{\nu_X}{\sqrt{2}}, \quad m_{J_1} = y^{(J)} \frac{\nu_X}{\sqrt{2}} = \frac{y^{(J)}_{1}}{y^{(T)}} m_T, \quad m_{J_2} = y^{(J)} \frac{\nu_X}{\sqrt{2}} = \frac{y^{(J)}_{2}}{y^{(T)}} m_T.$$

The experimental values of the physical quark mass spectrum [95, 96], mixing angles and Jarlskog invariant [97] are consistent with their experimental data, as shown in Table (V), starting from the following benchmark point:

$$c_1 \simeq 1.2525, \quad |a_1| \simeq 1.48406, \quad \arg (a_1) \simeq 68^\circ, \quad a_2 \simeq 0.989375, \quad b_2 \simeq 1.41504, \quad g_1 \simeq 0.579397, \quad g_2 \simeq 0.57, \quad g_3 \simeq 1.40209, \quad g_4 \simeq 0.583.$$

### Table IV: Physical scalar mass spectrum.

| Scalars | Masses |
|---------|--------|
| $G_2^0 = -S_\alpha \xi_X + C_\alpha \xi_S$ | $M_{G_2^0} = 0$ |
| $A^0 = C_\beta \xi_S + S_\beta \xi_S$ | $M_{A^0} = \frac{f v}{\sqrt{2}} \nu_S (\nu_S + \nu_T)$ |
| $G_2^0 = -C_\gamma \xi_X + S_\gamma \xi_S$ | $M_{G_2^0} = 0$ |
| $H_0^0 = \xi_X$ | $M_{H_0^0} = \lambda_1 \nu_\chi^2$ |
| $h^0 = C_\delta \xi_S + S_\delta \xi_S$ | $M_{h^0} = \lambda_2 \nu_\chi^2 + \lambda_3 \nu_\eta^2$ |
| $H_2^0 = S_\delta \xi_S + C_\delta \xi_S$ | $M_{H_2^0} = \lambda_\theta \nu_\chi^2 (\nu_\chi^2 + \nu_\eta^2)$ |
| $G_3^0 = -S_\alpha \lambda_1^0 + C_\alpha \lambda_3^0$ | $M_{G_3^0} = 0$ |
| $H_1^0 = S_\alpha \lambda_1^0 + C_\alpha \lambda_3^0$ | $M_{H_1^0} = (\sqrt{2} f \nu_\eta + \lambda_5 \nu_\eta \nu_\chi) (\nu_\eta^2 + \nu_\chi^2)$ |
| $G_4^0 = -C_\alpha \chi^0_1 + C_\alpha \chi_3^0$ | $M_{G_4^0} = 0$ |
| $H_4^0 = S_\alpha \chi^0_1 + C_\alpha \chi_3^0$ | $M_{H_4^0} = 0$ |

$$\Delta_1 = \lambda_2 \nu_\chi^2 + \lambda_1 \nu_\eta^2,$$

$$\Delta_2 = \sqrt{\lambda_3^2 \nu_\eta^2 \nu_\chi^2 - 2 \lambda_1 \lambda_2 \nu_\eta^2 \nu_\chi^2 + \lambda_3^2 \nu_\eta^2 + \lambda_1^2 \nu_\chi^2}.$$
The result given in Eq. (3.3) motivates to consider the simplified benchmark scenario:

\[ c_1 \simeq 1.2525, \quad |a_1| \simeq 1.48406, \quad \arg (a_1) \simeq 68^\circ, \quad a_2 \simeq 0.989375, \]
\[ g_1 \simeq g_2 \simeq g_4 \simeq 0.579397, \quad b_2 \simeq g_3 \simeq 1.41504. \]  

(3.4)

Notice that a successful fit of the ten physical observables in the quark sector can be obtained in the above described scenarios where the first (Eq. 3.3) and the second one (Eq. 3.4) only have 9 and 6 effective free parameters, respectively. Thus, the symmetries of our model give rise to quark mass matrix textures that successfully explain the SM quark mass spectrum and mixing parameters, with quark sector effective free parameters of order unity.

| Observable | Model value with (3.3) | Model value with (3.4) | Experimental value |
|------------|-----------------------|-----------------------|-------------------|
| \( m_u(\text{MeV}) \) | 1.44999 | 1.44999 | 1.45^{+0.56}_{-0.45} |
| \( m_c(\text{MeV}) \) | 635 | 635 | 635 \pm 86 |
| \( m_t(\text{GeV}) \) | 172.101 | 172.101 | 172.1 \pm 0.6 \pm 0.9 |
| \( m_d(\text{MeV}) \) | 2.89988 | 2.90313 | 2.9^{+0.5}_{-0.4} |
| \( m_s(\text{MeV}) \) | 59.1145 | 60.021 | 57.7^{+16.8}_{-15.7} |
| \( m_b(\text{GeV}) \) | 2.79418 | 2.82003 | 2.82^{+0.09}_{-0.04} |
| \( \sin \theta_{12} \) | 0.225402 | 0.220611 | 0.225 |
| \( \sin \theta_{23} \) | 0.0412799 | 0.0415761 | 0.0412 |
| \( \sin \theta_{13} \) | 0.00386484 | 0.0038648 | 0.00351 |
| \( \delta \) | 68.021^\circ | 68.0198^\circ | 68^\circ |

Table V: Model and experimental values of the quark masses and CKM parameters.

Finally to close this section we briefly comment about the LHC signatures of exotic \( T, \ J^1 \) or \( J^2 \) quarks in our model. Such exotic quarks will mainly decay into a top quark and either neutral or charged scalar and can be pair produced at the LHC via Drell-Yan and gluon fusion processes mediated by charged gauge bosons and gluons, respectively. A detailed study of the collider phenomenology of the model is beyond the scope of this paper and is left for future studies.

4. LEPTON MASSES AND MIXINGS.

From Eq. (2.6), and using the product rules of the \( S_4 \) group given in Appendix (A) we find that the charged lepton mass matrix is given by:

\[
M_l = \frac{\nu}{\sqrt{2}} \begin{pmatrix}
    f_{11} \lambda^0 & f_{12} \lambda^5 & f_{13} \lambda^3 \\
f_{21} \lambda^0 & f_{22} \lambda^5 & f_{23} \lambda^3 \\
f_{31} \lambda^0 & f_{32} \lambda^5 & f_{33} \lambda^3
\end{pmatrix}.
\]  

(4.1)

Regarding the neutrino sector, from the Eq. (2.6), we find the following neutrino mass terms:

\[
-L_{\text{mass}}^{(\nu)} = \frac{1}{2} \left( \nu_L^T \bar{\nu_R} \bar{N_R} \right) M_\nu \left( \nu_L \right) + H.c.,
\]  

(4.2)

where the neutrino mass matrix is given by:

\[
M_\nu = \begin{pmatrix}
    0_{3 \times 3} & M_1 & M_2 \\
    M_1^T & 0_{3 \times 3} & M_3 \\
    M_2^T & M_3^T & 0_{3 \times 3}
\end{pmatrix},
\]  

(4.3)
and the submatrices are given by:

\[
M_1 = \frac{h_\nu v_\nu v_\nu}{2\Lambda} \begin{pmatrix}
0 & a & 0 \\
a & 0 & b \\
0 & -b & 0
\end{pmatrix}, \quad M_2 = \frac{h_\eta v_\eta v_\eta}{\sqrt{6}\Lambda} \begin{pmatrix}
x & y & -y \\
-x & \omega^2 y & -\omega y \\
x & \omega y & -\omega^2 y
\end{pmatrix},
\]

\[
M_3 = \frac{h_\chi v_\chi v_\chi}{\sqrt{6}\Lambda} \begin{pmatrix}
r & z & -z \\
r & \omega^2 z & -\omega z \\
r & \omega z & -\omega^2 z
\end{pmatrix}, \quad \omega = e^{\frac{2\pi i}{3}}.
\] (4.4)

The light active neutrino masses arise from a linear seesaw mechanism and the physical neutrino mass matrices are:

\[
M_\nu^{(1)} = -\left[ M_2 M_3^{-1} M_1^T + M_1 (M_3^T)^{-1} M_2^T \right],
\]

\[
M_\nu^{(2)} = -\frac{1}{2} (M_3 + M_3^T) - \frac{1}{2} \left[ M_1^T M_1 (M_3^T)^{-1} + (M_3)^{-1} M_1^T M_1 \right],
\]

\[
M_\nu^{(3)} = \frac{1}{2} (M_3 + M_3^T) + \frac{1}{2} \left[ M_1^T M_1 (M_3^T)^{-1} + (M_3)^{-1} M_1^T M_1 \right],
\] (4.5–7)

where \(M_\nu^{(1)}\) corresponds to the active neutrino mass matrix whereas \(M_\nu^{(2)}\) and \(M_\nu^{(3)}\) are the sterile neutrino mass matrices. The physical neutrino spectrum is composed of 3 light active neutrinos and 6 nearly degenerate sterile exotic pseudo-Dirac neutrinos. Furthermore, from Eqs. (2.6) and (2.8) and considering \(v_\nu \sim O(10)\) TeV, \(v_\eta \sim v_\chi \sim O(100)\) GeV and the Yukawa couplings of order unity, we find that the light active neutrino mass scale \(\sim 50\) meV is estimated as \(m_\nu \sim \frac{v_\nu v_\nu v_\nu}{v_\chi A} \sim \frac{v_\nu v_\eta}{\Lambda}\), which implies for the model cutoff the estimate \(\Lambda \sim O(10^{16})\) GeV.

The sterile neutrinos can be produced in pairs at the LHC, via quark-antiquark annihilation mediated by a heavy \(Z'\) gauge boson. They can decay into SM particles giving rise to a SM charged lepton and a \(W\) gauge boson in the final state. Thus, observing an excess of events with respect to the SM background in the opposite sign dileptons final states can be a signal in support of this model at the LHC. Studies of inverse seesaw neutrino signatures at the colliders as well as the production of heavy neutrinos at the LHC are carried out in Refs. [98–112]. A detailed study of the implications of our model at colliders goes beyond the scope of this paper and is deferred for a future work.

The light active neutrino mass matrix is given by:

\[
M_\nu^{(1)} = \begin{pmatrix}
2A & B - 2A & A - B \\
B - 2A & 2(A - B) & 2B - A \\
A - B & 2B - A & -2B
\end{pmatrix},
\] (4.8)

and the light active neutrino masses are:

\[
m_1 = 0,
\]

\[
m_2 = \sqrt{(10A - 7B)A^* + (10B - 7A)B^* - 4\sqrt{3}\sqrt{(A - B)}(A^* - B^*)((2A - B)A^* - (A - 2B)B^*)},
\]

\[
m_3 = \sqrt{(10A - 7B)A^* + (10B - 7A)B^* + 4\sqrt{3}\sqrt{(A - B)}(A^* - B^*)((2A - B)A^* - (A - 2B)B^*)}.
\] (4.9–11)

then, the best fit results adjusting these parameters are:

\[
|A| \simeq 0.0131338\text{eV}, \quad \arg(A) \simeq 45^\circ,
\]

\[
|B| \simeq 0.0028061\text{eV}, \quad \arg(B) \simeq 45^\circ.
\]

The corresponding PMNS leptonic mixing matrix is defined as \(U = R^\dagger L\nu\), and from the standard parametrization of \(U\), it follows that the lepton mixing parameters are given by:

\[
\sin^2(\theta_{13}) = |U_{13}|^2, \quad \sin^2(\theta_{12}) = \frac{|U_{12}|^2}{1 - |U_{13}|^2}, \quad \sin^2(\theta_{23}) = \frac{|U_{23}|^2}{1 - |U_{13}|^2}.
\]

It is worth mentioning that due to the complexity of the expression for the PMNS matrix, the analytic form cannot be shown.
Furthermore, the Jarlskog invariant $J_{CP}$ is determined from the relation:

$$J_{CP} = \text{Im}(U_{11}^* U_{22} U_{13} U_{21})$$

(4.12)

whereas the leptonic Dirac CP violating phase $\delta_{CP}$ can be extracted from the equivalent definition of $J_{CP}$ [113] in the standard parametrization

$$J_{CP} = \frac{1}{8} \sin(2\theta_{12}) \sin(2\theta_{23}) \sin(2\theta_{13}) \cos(\theta_{13}) \delta_{CP}$$

(4.13)

The charged lepton masses, leptonic mixing parameters and CP-phase can be very well reproduced for the scenario of normal neutrino mass ordering in terms of natural parameters of order one, as shown in Table VI, starting from the following benchmark point:

- $|f_{11}| \approx 1.4634$,  \hspace{0.5cm} \text{arg} (f_{11}) \approx -8.241^\circ$,
- $|f_{12}| \approx 0.4893$,  \hspace{0.5cm} \text{arg} (f_{12}) \approx 135.146^\circ$,
- $|f_{13}| \approx 0.3698$,  \hspace{0.5cm} \text{arg} (f_{13}) \approx 53.5383^\circ$,
- $|f_{21}| \approx 1.0$,  \hspace{0.5cm} \text{arg} (f_{21}) \approx 80.9978^\circ$,
- $|f_{22}| \approx 0.6153$,  \hspace{0.5cm} \text{arg} (f_{22}) \approx 80.9978^\circ$,
- $|f_{23}| \approx 0.5303$,  \hspace{0.5cm} \text{arg} (f_{23}) \approx 80.9978^\circ$,
- $|f_{31}| \approx 0.668$,  \hspace{0.5cm} \text{arg} (f_{31}) \approx 99.0022^\circ$,
- $|f_{32}| \approx 0.6574$,  \hspace{0.5cm} \text{arg} (f_{32}) \approx -164.714^\circ$,
- $|f_{33}| \approx 0.6017$,  \hspace{0.5cm} \text{arg} (f_{33}) \approx -11.618^\circ$.

As indicated by Table VI, our model is consistent with the experimental data on lepton masses and mixings. Notice that the ranges for the experimental values in Table (VI) were taken from [114] for the case of normal hierarchy. Note that we only consider the case of normal hierarchy since it is favored over more than $3\sigma$ than the inverted neutrino mass ordering.

| Observable | Model Value | Experimental value |
|------------|-------------|--------------------|
| $m_e$ [MeV] | 0.487       | 0.487              | 0.487 |
| $m_\mu$ [MeV] | 102.8       | 102.8 ± 0.0003     | 102.8 ± 0.0006 | 102.8 ± 0.0009 |
| $m_\tau$ [GeV] | 1.75        | 1.75 ± 0.0003      | 1.75 ± 0.0006  | 1.75 ± 0.0009  |
| $\Delta m^2_{21} [10^{-5} eV^2]$ | 7.54999    | $7.55^{+0.20}_{-0.16}$ | 7.20 – 7.94 | 7.05 – 8.14 |
| $\Delta m^2_{31} [10^{-3} eV^2]$ | 2.50        | 2.50 ± 0.03        | 2.44 – 2.57  | 2.41 – 2.60  |
| $\sin^2(\theta_{12})/10^{-1}$ | 3.20664    | $3.20^{+0.20}_{-0.16}$ | 2.89 – 3.59  | 2.73 – 3.79  |
| $\sin^2(\theta_{23})/10^{-1}$ | 4.82618    | $5.47^{+0.20}_{-0.30}$ | 4.67 – 5.83  | 4.45 – 5.99  |
| $\sin^2(\theta_{13})/10^{-2}$ | 2.1773      | $2.166^{+0.08}_{-0.09}$ | 2.03 – 2.34  | 1.96 – 2.41  |
| $\delta_{CP}$ | 161.327°     | $218^{+38}_{-27}$°   | 182° – 315°  | 157° – 349° |

Table VI: Model values are the best fit results for the neutrino mass squared differences, mixing angles and the CP-violating phase for the case of normal hierarchy.

5. **HIGGS DIPHOTON DECAY RATE CONSTRAINTS.**

The explicit form of the $h \to \gamma \gamma$ decay rate is [115–122]

$$\Gamma(h \to \gamma \gamma) = \frac{\alpha^2 m_h^3}{256\pi^3 v^2} \left| \sum_f a_{hff} N_C Q^2_f F_{1/2}(\rho_f) + a_{hWW} F_1(\rho_W) + a_{hW'W'} F_1(\rho_{W'}) + \frac{\lambda_{hH^\mp H^\mp v}}{2m_{H^\mp}^2} F_0(\rho_{H^\mp}) \right|^2$$

(5.1)

where

$$a_{hWW} = \sin(\beta - \alpha)$$

(5.2)

$$a_{hW'W'} = \cos \alpha \sin \gamma$$

(5.3)

$$a_{hH^\mp} = \frac{\cos \alpha}{\sin \beta}$$

(5.4)

$$\lambda_{hH^\mp H^\mp v} = 2 \left( -\lambda_5 \sin(\alpha) \sin^2(\gamma) \nu_\eta + \lambda_6 (\sin(\alpha)) \cos^2(\gamma) \nu_\eta + \cos(\alpha) \nu_\rho \right.$$  

$$+ \left( (\lambda_4 + \lambda_5) \sin^2(\gamma) + 2\lambda_2 \cos^2(\gamma) \right) \lambda_8 \sin(\gamma) \cos(\gamma) \nu_\chi \right)$$

(5.6)
Here $\rho_i$ are the mass ratios $\rho_i = \frac{m_i^2}{4M_i^2}$ with $M_i = m_f, M_W, M_W'$; $\alpha_{em}$ is the fine structure constant; $N_C$ is the color factor ($N_C = 1$ for leptons and $N_C = 3$ for quarks); and $Q_f$ is the electric charge of the fermion in the loop. From the fermion-loop contributions we consider only the dominant top quark term.

The dimensionless loop factor $F_{1/2}(\rho)$ and $F_1(\rho)$ (for spin-$1/2$ and spin-1 particles in the loop, respectively are:

\[
F_{1/2}(\rho) = 2(\rho + (\rho - 1)f(\rho))\rho^{-2}, \quad (5.7)
\]
\[
F_1(\rho) = -2(2\rho^2 + 3\rho + 3(2\rho - 1)f(\rho))\rho^{-2}, \quad (5.8)
\]
\[
F_0 = -(\rho - f(\rho))\rho^{-2}, \quad (5.9)
\]

with

\[
f(\rho) = \begin{cases} 
\arcsin^2 \sqrt{2} & \text{for } \rho \leq 1 \\
-\frac{1}{4} \left( \ln \left( \frac{1+\sqrt{1-\rho^{-1}}}{1-\sqrt{1-\rho^{-1}}-i\pi} \right)^2 \right) & \text{for } \rho > 1
\end{cases} \quad (5.10)
\]

In what follows we determine the constraints that the Higgs diphoton signal strength imposes on our model. To this end, we introduce the ratio $R_{\gamma\gamma}$, which normalizes the $\gamma\gamma$ signal predicted by our model relative to that of the SM:

\[
R_{\gamma\gamma} = \frac{\sigma(pp \to h)\Gamma(h \to \gamma\gamma)}{\sigma(pp \to h)_{SM}\Gamma(h \to \gamma\gamma)_{SM}} \approx a^2_{htt} \frac{\Gamma(h \to \gamma\gamma)}{\Gamma(h \to \gamma\gamma)_{SM}} \quad (5.11)
\]

The normalization given by (5.11) for $h \to \gamma\gamma$ was also used in [76, 120, 123–128].

The ratio $R_{\gamma\gamma}$ has been measured by CMS and ATLAS collaborations with the best fit signals [129, 130]:

\[
R_{\gamma\gamma}^{CMS} = 1.14^{+0.26}_{-0.23} \quad \text{and} \quad R_{\gamma\gamma}^{ATLAS} = 1.17 \pm 0.27 \quad (5.12)
\]

With the best fit results shown in Table VII the $R_{\gamma\gamma}$ parameter has been calculated as:

\[
R_{\gamma\gamma} = 1.0267 \quad (5.13)
\]

Consequently, our model successfully accommodates the current Higgs diphoton decay rate constraints.

| Parameters | Model value |
|------------|-------------|
| $M_{h^0}$  | 125.09 GeV  |
| $M_{H^0}$  | 5319.77 GeV |
| $M_{A^0}$  | 5318.3 GeV  |
| $M_{h\pm}$ | 5503.95 GeV |
| $a_{hW^-W^+}$ | 1.0 |
| $a_{hW^+W^-}$ | 0.0122981 |
| $a_{h\ell\ell}$ | 1.0 |
| $\lambda_{hH^0H^\pm}$ | 2525.45 GeV |

Table VII: Parameters with $\nu_\eta = 173.948$ GeV, $\nu_\rho = 173.948$ GeV and $\nu_\chi = 10$ TeV.

Correlations plots have been obtained to observe the behavior of the $R_{\gamma\gamma}$ parameter as function of scalar masses and $W'$ gauge boson mass. They are shown in Figure 1.
These plots were generated using random points in a space in the neighborhood of the best fit values for $f$, $v_\chi$ and $\lambda_8$. Figure 1a shows that the parameter $R_{\gamma\gamma}$ is strongly restricted by the CP-odd Higgs mass $M_{A^0}$, since the range of allowed values for $R_{\gamma\gamma}$ decreases when the CP odd scalar mass $M_{A^0}$ is increased. The Higgs diphoton signal strength $R_{\gamma\gamma}$ features a similar behavior with the charged scalar mass $M_{H^\pm}$, as indicated by Figure 1b. Notice that despite the CP odd neutral scalar $A^0$ does not contribute to the Higgs diphoton decay rate, the Higgs diphoton signal strength indirectly depends on $M_{A^0}$ since the parameters $\alpha$, $\gamma$ and $\lambda_{H^\pm H^\pm}$ (that enter in the Higgs diphoton decay rate) as well as the CP-odd Higgs mass $M_{A^0}$ are functions of $v_\chi$. In addition, we have found that the Higgs diphoton signal strength decreases when the $W'$ mass is increased, approaching to 1 when $M_{W'} \gtrsim 10$ TeV, as indicated by Figure 1c. Furthermore, Figures 1a, 1b and 1c show that our model favors values for the Higgs diphoton decay rate larger than the SM expectation. In addition, Figure 2 shows that the Higgs diphoton decay rate constraints are fulfilled when $M_{H^\pm} \gtrsim M_{W'}$. Finally, our obtained results for the Higgs diphoton signal strength indicate that the Higgs diphoton decay is a smoking gun signature of our model, whose more precise measurement will be crucial to assess its viability.
6. HEAVY SCALAR PRODUCTION AT PROTON-PROTON COLLIDER

In this section we discuss the singly heavy scalar $H_1$ production at proton-proton collider. It is worth mentioning that the production mechanism at the LHC of the heavy scalar $H_1$ is via gluon fusion, which is a one loop process mediated by the heavy exotic $T$, $J_1$ and $J_2$ quarks. Consequently, the total $H_1$ production cross section in proton proton collisions with center of mass energy $\sqrt{S}$ is given by:

$$
\sigma_{pp \rightarrow gg \rightarrow H_1} (S) = \frac{\alpha^2 S m_{H_1}^2}{64 \pi v^2 S} \left[ I \left( \frac{m_{H_1}^2}{m_T^2} \right) + I \left( \frac{m_{H_1}^2}{m_{J_1}^2} \right) + I \left( \frac{m_{H_1}^2}{m_{J_2}^2} \right) \right]
\times \int_{\ln \sqrt{\frac{m_{H_1}^2}{S}}}^{-\ln \sqrt{\frac{m_{H_1}^2}{S}}} \int_{\ln \sqrt{\frac{m_{H_1}^2}{S}}}^{1} f_{p/g} \left( \sqrt{\frac{m_{H_1}^2 e^y}{S}}, \mu^2 \right) f_{p/g} \left( \sqrt{\frac{m_{H_1}^2 e^{-y}}{S}}, \mu^2 \right) dy
$$  

where $f_{p/g} (x_1, \mu^2)$ and $f_{p/g} (x_2, \mu^2)$ are the distributions of gluons in the proton which carry momentum fractions $x_1$ and $x_2$ of the proton, respectively. Furthermore $\mu = m_{H_1}$ is the factorization scale and $I(z)$ is given by:

$$
I(z) = \int_0^1 dx \int_0^{1-x} dy \frac{1-4xy}{1-zy}
$$  

Figure 2: Correlation plot of the CP-odd Higgs mass and the charged Higgs mass.
Figure 3: Total cross section for the $H_1$ production via gluon fusion mechanism at the LHC for $\sqrt{S} = 13$ TeV and as a function of the $SU(3)_L \times U(1)_X$ symmetry breaking scale $v_\chi$ for the scenario described in Eq. (2.23).

Figure 3 displays the $H_1$ total production cross section at the LHC via gluon fusion mechanism for $\sqrt{S} = 13$ TeV, as a function of the $SU(3)_L \times U(1)_X$ symmetry breaking scale $v_\chi$, which is taken to range from 10 TeV up to 15 TeV, which corresponds to a heavy scalar mass $m_{H_1}$ varying between 1.3 TeV and 1.9 TeV. In addition, the exotic quark Yukawa couplings have been taken equal to unity and the scenario described by Eq. (2.23) has been considered in our numerical analysis. Notice that the $SU(3)_L \times U(1)_X$ symmetry breaking scale has been taken larger than 10 TeV, which corresponds to a $Z'$ gauge boson heavier than 4 TeV, in order to comply with the experimental data on $K$, $D$ and $B$ meson mixings [86]. For such region of $H_1$ masses, we find that the total production cross section is found to be $0.28 - 0.02$ fb. However, at the proposed energy upgrade of the LHC with $\sqrt{S} = 28$ TeV, the $H_1$ production cross section is enlarged, reaching values of $2.9 - 0.4$ fb in the same mass region as indicated by Figure 4. Such small values for the $H_1$ production cross section at a proton-proton collider with $\sqrt{S} = 13$ TeV and $\sqrt{S} = 28$ TeV are small to give rise to a signal for the relevant region of parameter space. However at a $\sqrt{S} = 100$ TeV proton-proton collider, there is a significant enhancement of the $H_1$ production cross section, which takes values of $51 - 10$ fb for $1.3$ TeV $\lesssim m_{H_1} \lesssim 1.9$ TeV, as shown in Figure 5. Finally, it is worth mentioning that one can safely assume that the heavy $H_1$ scalar after being produced will mainly decay into a pair of SM Higgs bosons, since it is the lightest non SM scalar, as follows from Eq. (2.23) and Table IV. Consequently, an enhancement of the SM Higgs pair production with respect to the SM expectation, will be a smoking gun signature of this model, whose observation will be crucial to assess its viability.
Figure 4: Total cross section for the $H_1$ production via gluon fusion mechanism at the proposed energy upgrade of the LHC with $\sqrt{S} = 28$ TeV as a function of the $SU(3)_L \times U(1)_X$ symmetry breaking scale $v_\chi$ for the scenario described in Eq. (2.23).

Figure 5: Total cross section for the $H_1$ production via gluon fusion mechanism at a $\sqrt{S} = 100$ TeV proton-proton collider as a function of the $SU(3)_L \times U(1)_X$ symmetry breaking scale $v_\chi$ for the scenario described in Eq. (2.23).

7. CONCLUSIONS

We have constructed a multiscalar singlet extension of the 3-3-1 model with three right handed Majorana neutrinos, consistent with the observed SM fermion mass and mixing pattern. The model incorporates the $S_4$ family symmetry, which is combined with other auxiliary symmetries, thus allowing a viable description of the current SM fermion mass and mixing pattern, which is generated by the spontaneous breaking of the discrete group. The small masses of the light active neutrinos are produced by a linear linear seesaw mechanism mediated by three Majorana neutrinos. The model provides a successful fit of the physical observables of both quark and lepton sectors. Our model is predictive in the SM quark sector, since it only has 9 effective parameters that allow a successful fit of its 10 observables, i.e.,
the 6 SM quark masses, the 3 quark mixing parameters and the CP violating phase. In addition, we have found that the SM quark sector of our model has a particular scenario, which is inspired by naturalness arguments and has only 6 effective parameters that allows to successfully reproduce the experimental values of the ten SM quark sector observables. Furthermore, we have also shown that the proposed model successfully accommodates the current Higgs diphoton decay rate constraints provided that the charged Higgs bosons are a bit heavier, than the $W$ gauge bosons. In addition, we have found that it favors a Higgs diphoton decay rate larger than the SM expectation. Finally, we have also discussed the single production of the heavy scalar $H_1$, associated with the spontaneous breaking of the $SU(3)_C \times U(1)_X$ symmetry, at a proton-proton collider, via gluon fusion mechanism. We have considered the cases where the center of mass energy takes the values of $\sqrt{S} = 13$ TeV, $\sqrt{S} = 28$ TeV and $\sqrt{S} = 100$ TeV. For the first two cases corresponding to the current LHC center of mass energy and the proposed energy upgrade of the LHC, respectively, we have found the $H_1$ production cross sections are small to give rise to a signal for the relevant region of parameter space. However, in a future $\sqrt{S} = 100$ TeV proton-proton collider, the $H_1$ production cross section is significantly enhanced reaching values between 51 fb and 10 fb, for the mass range $1.3$ TeV $\lesssim m_{H_1} \lesssim 1.9$ TeV.

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Appendix A: The $S_4$ discrete group

The $S_4$ is the smallest non abelian group having doublet, triplet and singlet irreducible representations. $S_4$ is the group of permutations of four objects, which includes five irreducible representations, i.e., $1, 1', 2, 3, 3'$ fulfilling the following tensor product rules [131]

$$
3 \otimes 3 = 1 \oplus 2 \oplus 3 \oplus 3', \quad 3' \otimes 3' = 1 \oplus 2 \oplus 3 \oplus 3', \quad 3 \otimes 3' = 1' \oplus 2 \oplus 3 \oplus 3',
$$

(A.1)

$$
2 \otimes 2 = 1 \oplus 1' \oplus 2, \quad 2 \otimes 3 = 3 \oplus 3', \quad 2 \otimes 3' = 3' \oplus 3.
$$

(A.2)

$$
3 \otimes 1' = 3', \quad 3' \otimes 1' = 3, \quad 2 \otimes 1' = 2.
$$

(A.3)

Explicitly, the basis used in this paper corresponds to Ref. [131] and results in

$$
(A)_3 \times (B)_3 = (A \cdot B)_1 + \left( A \cdot \Sigma \cdot B \right)_2 + \left( A \cdot \Sigma^* \cdot B \right)_3 + \left( \begin{array}{c} A_y B_z \\ A_x B_x \\ A_x B_y \end{array} \right)_3 + \left( \begin{array}{c} A_y B_z \\ A_x B_x \\ A_x B_y \end{array} \right)_3',
$$

(A.4)

$$
(A)_{3'} \times (B)_{3'} = (A \cdot B)_1 + \left( A \cdot \Sigma \cdot B \right)_2 + \left( A \cdot \Sigma^* \cdot B \right)_3 + \left( \begin{array}{c} A_y B_z \\ A_x B_x \\ A_x B_y \end{array} \right)_3 + \left( \begin{array}{c} A_y B_z \\ A_x B_x \\ A_x B_y \end{array} \right)_3',
$$

(A.5)

$$
(A)_3 \times (B)_{3'} = (A \cdot B)_1 + \left( A \cdot \Sigma \cdot B \right)_2 + \left( A \cdot \Sigma^* \cdot B \right)_3 + \left( \begin{array}{c} A_y B_z \\ A_x B_x \\ A_x B_y \end{array} \right)_{3'} ,
$$

(A.6)

$$
(A)_{2} \times (B)_{2} = (A_x B_y)_1 + (A_x B_y)_{1'} + \left( \begin{array}{c} A_y B_y \\ A_x B_x \end{array} \right),
$$

(A.7)

$$
\left( \begin{array}{c} A_x \\ A_y \end{array} \right)_2 \times \left( \begin{array}{c} B_x \\ B_y \\ B_z \end{array} \right)_3 = \left( \begin{array}{c} (A_x + A_y) B_z \\ (\omega^2 A_x + \omega A_y) B_y \\ (\omega A_x + \omega^2 A_y) B_z \end{array} \right)_3 + \left( \begin{array}{c} (A_x - A_y) B_x \\ (\omega^2 A_x - \omega A_y) B_y \\ (\omega A_x - \omega^2 A_y) B_z \end{array} \right)_{3'},
$$

(A.8)
Here we consider the phase $S$. This result indicates that the VEV pattern of the scalar potential (B.1) of this model for a large region of parameter space. The previously described procedure can be used to show that the VEV patterns of the remaining doublets of the model are also consistent with the minimization conditions of the scalar potential.

\( V = -\mu_\Delta (\Delta \Delta^*)_1 + \kappa_1 (\Delta \Delta^*)_1 (\Delta \Delta^*)_1 + \kappa_2 (\Delta \Delta^*)_1 (\Delta \Delta^*)_1 + \kappa_3 (\Delta \Delta^*)_2 (\Delta \Delta^*)_2 + h.c. \)  

This scalar potential has six free parameters: one bilinear and four quartic couplings. The $\mu_\Delta$ parameter can be written as a function of the other five parameters by the scalar potential minimization condition:

\[ \frac{\partial [V(\Delta)]}{\partial \nu_\Delta} = 16\kappa_1 \nu_\Delta^3 + 8\kappa_3 \nu_\Delta^2 - \mu_\Delta^2 \nu_\Delta^2 \]  

\[ = 0 \]  

Solving the leading equation for $\mu_\Delta^2$:

\[ \mu_\Delta^2 = 16(2\kappa_1 + \kappa_3)^2 \nu_\Delta^4 \]  

This result indicates that the VEV pattern of the $S_4$ doublet $\Delta$ in (2.7) is consistent with a global minimum of the scalar potential (B.1) of this model for a large region of parameter space. The previously described procedure can be used to show that the VEV patterns of the remaining $S_4$ doublets of the model are also consistent with the minimization conditions of the scalar potential.

Appendix C: The scalar potential for a $S_4$ triplet

The scalar potential for a $S_4$ triplet $\Phi$ takes the form:

\[ V = -\mu_\rho (\Phi \Phi^*)_1 + \kappa_1 (\Phi \Phi^*)_1 (\Phi \Phi^*)_1 + \kappa_2 (\Phi \Phi^*)_1 (\Phi \Phi^*)_3 + \kappa_3 (\Phi \Phi^*)_3 (\Phi \Phi^*)_3 + \kappa_4 (\Phi \Phi^*)_2 (\Phi \Phi^*)_2 + h.c. \]  

This scalar potential has six free parameters: one bilinear and four quartic couplings. The $\mu_\rho$ parameter can be written as a function of the other four parameters by the scalar potential minimization condition:

\[ \frac{\partial [V(\Phi)]}{\partial \nu_\rho} = 36\kappa_1 \nu_\rho^3 + 48\kappa_2 \nu_\rho^3 + 4\kappa_4 \left( 2e^{2\pi i/3} \nu_\rho + 2e^{-2\pi i/3} \nu_\rho + 2\nu_\rho \right) \left( e^{2\pi i/3} \nu_\rho^2 + e^{-2\pi i/3} \nu_\rho^2 + \nu_\rho^2 \right) - 6\mu_\rho \nu_\rho \]  

\[ = 0 \]  

Here we consider the phase $\omega = e^{2\pi i/3}$ in the multiplications rules for tensor product of the scalar triplets of $S_4$. Solving the leading equation for $\mu_\rho^2$:

\[ \mu_\rho^2 = 4(3\kappa_1 + 4\kappa_2)^2 \nu_\rho^4 \]
This result indicates that the VEV pattern of the $S_4$ triplet $\rho$ in (2.7) is consistent with a global minimum of the scalar potential (C.1) of this model for a large region of parameter space. Following the same procedure previously described, one can also show that the VEV patterns of the remaining $S_4$ triplets of the model are also consistent with the minimization conditions of the scalar potential.

[1] H. Georgi and A. Pais, Phys. Rev. D 19, 2746 (1979). doi:10.1103/PhysRevD.19.2746
[2] J. W. F. Valle and M. Singer, Phys. Rev. D 28, 540 (1983). doi:10.1103/PhysRevD.28.540
[3] F. Pisano and V. Pleitez, Phys. Rev. D 46, 410 (1992) doi:10.1103/PhysRevD.46.410 [hep-ph/9206242].
[4] R. Foot, O. F. Hernandez, F. Pisano and V. Pleitez, Phys. Rev. D 47, 4158 (1993) doi:10.1103/PhysRevD.47.4158 [hep-ph/9207624].
[5] P. H. Frampton, Phys. Rev. Lett. 69, 2889 (1992). doi:10.1103/PhysRevLett.69.2889
[6] H. N. Long, Phys. Rev. D 54, 4691 (1996) doi:10.1103/PhysRevD.54.4691 [hep-ph/9607439].
[7] H. N. Long, Phys. Rev. D 53, 437 (1996) doi:10.1103/PhysRevD.53.437 [hep-ph/9504274].
[8] R. Foot, H. N. Long and T. A. Tran, Phys. Rev. D 50, no. 1, R34 (1994) doi:10.1103/PhysRevD.50.34 [hep-ph/9402243].
[9] A. E. Carcamo Hernandez, R. Martinez and F. Ochoa, Phys. Rev. D 73, 035007 (2006) doi:10.1103/PhysRevD.73.035007 [hep-ph/0510421].
[10] P. V. Dong, H. N. Long, D. V. Soa and V. V. Vien, Eur. Phys. J. C 71, 1544 (2011) doi:10.1140/epjc/s10052-011-1544-2 [arXiv:1009.2328 [hep-ph]].
[11] P. V. Dong, H. N. Long and D. V. Soa, Phys. Rev. D 81, 053004 (2010) doi:10.1103/PhysRevD.81.053004 [arXiv:1001.4625 [hep-ph]].
[12] P. V. Dong, H. N. Long, C. H. Nam and V. V. Vien, Phys. Rev. D 85, 053001 (2012) doi:10.1103/PhysRevD.85.053001 [arXiv:1111.3630 [hep-ph]].
[13] R. H. Benavides, W. A. Ponce and Y. Giraldo, Phys. Rev. D 82, 013004 (2010) doi:10.1103/PhysRevD.82.013004 [arXiv:1006.3248 [hep-ph]].
[14] P. V. Dong, H. N. Long and H. T. Hung, Phys. Rev. D 86, 033002 (2012) doi:10.1103/PhysRevD.86.033002 [arXiv:1205.5648 [hep-ph]].
[15] D. T. Huong, L. T. Hue, M. C. Rodriguez and H. N. Long, Nucl. Phys. B 870, 293 (2013) doi:10.1016/j.nuclphysb.2013.01.016 [arXiv:1210.6776 [hep-ph]].
[16] P. T. Giang, L. T. Hue, D. T. Huong and H. N. Long, Nucl. Phys. B 864, 85 (2012) doi:10.1016/j.nuclphysb.2012.06.008 [arXiv:1204.2902 [hep-ph]].
[17] D. T. Binh, L. T. Hue, D. T. Huong and H. N. Long, Eur. Phys. J. C 74, no. 5, 2851 (2014) doi:10.1140/epjc/s10052-014-2851-1 [arXiv:1308.3085 [hep-ph]].
[18] A. E. Carcamo Hernandez, R. Martinez and F. Ochoa, Phys. Rev. D 87, no. 7, 075009 (2013) doi:10.1103/PhysRevD.87.075009 [arXiv:1302.1757 [hep-ph]].
[19] A. E. Cárcamo Hernández, R. Martínez and F. Ochoa, Eur. Phys. J. C 76, no. 11, 634 (2016) doi:10.1140/epjc/s10052-016-4480-3 [arXiv:1309.6567 [hep-ph]].
[20] A. E. Cárcamo Hernández, R. Martínez and J. Nisperuza, Eur. Phys. J. C 75, no. 2, 72 (2015) doi:10.1140/epjc/s10052-015-3278-z [arXiv:1401.0937 [hep-ph]].
[21] A. E. Cárcamo Hernández, E. Cataño Mur and R. Martínez, Phys. Rev. D 90, no. 7, 073001 (2014) doi:10.1103/PhysRevD.90.073001 [arXiv:1407.5217 [hep-ph]].
[22] C. Kelso, H. N. Long, R. Martínez and F. S. Queiroz, Phys. Rev. D 90, no. 11, 113011 (2014) doi:10.1103/PhysRevD.90.113011 [arXiv:1408.6203 [hep-ph]].
[23] V. V. Vien and H. N. Long, JHEP 1404, 133 (2014) doi:10.1007/JHEP04(2014)133 [arXiv:1402.1256 [hep-ph]].
[24] V. Q. Phong, H. N. Long, V. T. Van and L. H. Minh, Eur. Phys. J. C 75, no. 7, 342 (2015) doi:10.1140/epjc/s10052-015-3550-2 [arXiv:1409.0750 [hep-ph]].
[25] V. Q. Phong, H. N. Long, V. T. Van and N. C. Thanh, Phys. Rev. D 90, no. 8, 085019 (2014) doi:10.1103/PhysRevD.90.085019 [arXiv:1408.5657 [hep-ph]].
[26] S. M. Boucenna, S. Morisi and J. W. F. Valle, Phys. Rev. D 90, no. 1, 013005 (2014) doi:10.1103/PhysRevD.90.013005 [arXiv:1405.2332 [hep-ph]].
[27] G. De Conto, A. C. B. Machado and V. Pleitez, Phys. Rev. D 92, no. 7, 075031 (2015) doi:10.1103/PhysRevD.92.075031 [arXiv:1505.01343 [hep-ph]].
[28] S. M. Boucenna, J. W. F. Valle and A. Vicente, Phys. Rev. D 92, no. 5, 053001 (2015) doi:10.1103/PhysRevD.92.053001 [arXiv:1502.07546 [hep-ph]].
[29] S. M. Boucenna, S. Morisi and A. Vicente, Phys. Rev. D 93, no. 11, 115008 (2016) doi:10.1103/PhysRevD.93.115008 [arXiv:1512.06878 [hep-ph]].
[30] R. H. Benavides, L. N. Epele, H. Fanchiotti, C. G. Canal and W. A. Ponce, Adv. High Energy Phys. 2015, 813129 (2015) doi:10.1155/2015/813129 [arXiv:1503.01686 [hep-ph]].
[31] A. E. Cárcamo Hernández and R. Martínez, Nucl. Phys. B 905, 337 (2016) doi:10.1016/j.nuclphysb.2016.02.025 [arXiv:1501.05937 [hep-ph]].
[32] L. T. Hue, H. N. Long, T. T. Thuc and T. Phong Nguyen, Nucl. Phys. B 907, 37 (2016).
A. Das, P. S. B. Dev and C. S. Kim, Phys. Rev. D 86, 073007 (2012) doi:10.1103/PhysRevD.86.073007 [arXiv:1208.2875 [hep-ph]].

P. S. Bhupal Dev, B. Dutta, R. N. Mohapatra and M. Severson, Phys. Rev. D 86, 035002 (2012) doi:10.1103/PhysRevD.86.035002 [arXiv:1202.4012 [hep-ph]].

I. de Medeiros Varzielas and L. Lavoura, J. Phys. G 40, 085002 (2013) doi:10.1088/0954-3899/40/8/085002 [arXiv:1212.3247 [hep-ph]].

G. J. Ding, S. F. King, C. Luhn and A. J. Stuart, JHEP 1305, 084 (2013) doi:10.1007/JHEP05(2013)084 [arXiv:1303.6180 [hep-ph]].

H. Ishimori, Y. Shimizu, M. Tanimoto and A. Watanabe, Phys. Rev. D 83, 033004 (2011) doi:10.1103/PhysRevD.83.033004 [hep-ph].

G. J. Ding and Y. L. Zhou, Nucl. Phys. B 876, 418 (2013) doi:10.1016/j.nuclphysb.2013.08.011 [arXiv:1304.2645 [hep-ph]].

C. Hagedorn and M. Serone, JHEP 1110, 083 (2011) doi:10.1007/JHEP10(2011)083 [arXiv:1106.4021 [hep-ph]].

M. D. Campos, A. E. Cárcamo Hernández, H. Pás and E. Schumacher, Phys. Rev. D 91, no. 11, 116011 (2015) doi:10.1103/PhysRevD.91.116011 [arXiv:1408.1652 [hep-ph]].

V. V. Vien, H. N. Long and D. P. Khoi, Int. J. Mod. Phys. A 30, no. 17, 1550102 (2015) doi:10.1142/S0217751X1550102X [arXiv:1506.06063 [hep-ph]].

F. J. de Anda, S. F. King and E. Perdomo, JHEP 1712, 075 (2017) Erratum: [JHEP 1904, 069 (2019)] doi:10.1007/JHEP12(2017)075, 10.1007/JHEP04(2019)069 [arXiv:1710.03229 [hep-ph]].

F. J. de Anda and S. F. King, JHEP 1807, 057 (2018) doi:10.1007/JHEP07(2018)057 [arXiv:1803.04978 [hep-ph]].

A. E. Cárcamo Hernández and S. F. King, arXiv:1903.02565 [hep-ph].

P. T. Chen, G. J. Ding, S. F. King and C. C. Li, arXiv:1906.11414 [hep-ph].

I. De Medeiros Varzielas, S. F. King and Y. L. Zhou, arXiv:1906.02208 [hep-ph].

I. De Medeiros Varzielas, M. Levy and Y. L. Zhou, arXiv:1903.15006 [hep-ph].

C. D. Froggatt and H. B. Nielsen, Nucl. Phys. B 147, 277 (1979). doi:10.1016/0550-3213(79)90316-X

S. F. King, B. Dutta, R. N. Mohapatra and J. T. Piran, Phys. Rev. D 86, 035002 (2012) doi:10.1103/PhysRevD.86.035002 [arXiv:1202.4012 [hep-ph]].
[108] A. Das, S. Jana, S. Mandal and S. Nandi, Phys. Rev. D 99, no. 5, 055030 (2019) doi:10.1103/PhysRevD.99.055030 [arXiv:1811.04291 [hep-ph]].

[109] A. Das, Adv. High Energy Phys. 2018, 9785318 (2018) doi:10.1155/2018/9785318 [arXiv:1803.10940 [hep-ph]].

[110] A. Bhardwaj, A. Das, P. Konar and A. Thalapillil, arXiv:1801.00797 [hep-ph].

[111] J. C. Helo, H. Li, N. A. Neill, M. Ramsey-Musolf and J. C. Vasquez, Phys. Rev. D 99, no. 5, 055042 (2019) doi:10.1103/PhysRevD.99.055042 [arXiv:1812.01630 [hep-ph]].

[112] P. I. Krastev and S. T. Petcov, Phys. Lett. B 205, 84 (1988). doi:10.1016/0370-2693(88)90404-2

[113] S. Pascoli, R. Ruiz and C. Weiland, JHEP 1906, 049 (2019) doi:10.1007/JHEP06(2019)049 [arXiv:1812.08750 [hep-ph]].

[114] M. A. Shifman, A. I. Vainshtein, M. B. Voloshin and V. I. Zakharov, Sov. J. Nucl. Phys. 30, 711 (1979) [Yad. Fiz. 30, 1368 (1979)].

[115] P. F. de Salas, D. V. Forero, C. A. Ternes, M. Tortola and J. W. F. Valle, Phys. Lett. B 782, 633 (2018) doi:10.1016/j.physletb.2018.06.019 [arXiv:1708.01186 [hep-ph]].

[116] M. B. Gavela, G. Girardi, C. Malleville and P. Sorba, Nucl. Phys. B 193, 257 (1981). doi:10.1016/0550-3213(81)90529-0

[117] M. Spira, Fortsch. Phys. 46, 203 (1998) doi:10.1002/(SICI)1521-3978(199804)46:3<203::AID-PROP203>3.0.CO;2-4 [hep-ph/9705337].

[118] A. Djouadi, Phys. Rept. 459, 1 (2008) doi:10.1016/j.physrep.2007.10.005 [hep-ph/0503173].

[119] W. J. Marciano, C. Zhang and S. Willenbrock, Phys. Rev. D 85, 013002 (2012) doi:10.1103/PhysRevD.85.013002 [arXiv:1109.5304 [hep-ph]].

[120] A. E. Carcamo Hernandez, C. O. Dib and A. R. Zerwekh, Eur. Phys. J. C 74, 2822 (2014) doi:10.1140/epjc/s10052-014-2822-6 [arXiv:1304.0286 [hep-ph]].

[121] A. E. Carcamo Hernandez, C. O. Dib and A. R. Zerwekh, Nucl. Phys. B 193, 257 (1981). doi:10.1016/0550-3213(81)90529-0

[122] M. Spira, Fortsch. Phys. 46, 203 (1998) doi:10.1002/(SICI)1521-3978(199804)46:3<203::AID-PROP203>3.0.CO;2-4 [hep-ph/9705337].

[123] A. E. Carcamo Hernandez, C. O. Dib and A. R. Zerwekh, Eur. Phys. J. C 74, 2822 (2014) doi:10.1140/epjc/s10052-014-2822-6 [arXiv:1304.0286 [hep-ph]].

[124] G. Bhattacharyya and D. Das, Phys. Rev. D 91, 015005 (2015) doi:10.1103/PhysRevD.91.015005 [arXiv:1408.6133 [hep-ph]].

[125] E. C. F. S. Fortes, A. C. B. Machado, J. Montaño and V. Pleitez, J. Phys. G 42, no. 11, 115001 (2015) doi:10.1088/0954-3899/42/11/115001 [arXiv:1408.0780 [hep-ph]].

[126] A. E. Carcamo Hernandez, C. O. Dib and A. R. Zerwekh, Nucl. Part. Phys. Proc. 267-269, 35 (2015) doi:10.1016/j.nulphysbps.2015.10.079 [arXiv:1503.08472 [hep-ph]].

[127] A. E. Carcamo Hernández, I. de Medeiros Varzielas and E. Schumacher, Phys. Rev. D 93, no. 1, 016003 (2016) doi:10.1103/PhysRevD.93.016003 [arXiv:1509.02083 [hep-ph]].

[128] A. E. Carcamo Hernandez, B. Díaz Sáez, C. O. Dib and A. Zerwekh, Phys. Rev. D 96, no. 11, 115027 (2017) doi:10.1103/PhysRevD.96.115027 [arXiv:1707.05195 [hep-ph]].

[129] V. Khachatryan et al. [CMS Collaboration], Eur. Phys. J. C 74, no. 10, 3076 (2014) doi:10.1140/epjc/s10052-014-3076-z [arXiv:1407.0558 [hep-ex]].

[130] G. Aad et al. [ATLAS Collaboration], Phys. Rev. D 90, no. 11, 112015 (2014) doi:10.1103/PhysRevD.90.112015 [arXiv:1408.7084 [hep-ex]].

[131] H. Ishimori, T. Kobayashi, H. Okki, Y. Shimizu, H. Okada and M. Tanimoto, Prog. Theor. Phys. Suppl. 183, 1 (2010) doi:10.1143/PTPS.183.1 [arXiv:1003.3552 [hep-th]].