Research Article

Teaching Performance Evaluation Based on the Proportional Hesitant Fuzzy Linguistic Prioritized Choquet Aggregation Operator

Lei Wang\(^1\), Lili Rong\(^2,3,7,*)\(^,\) Fei Teng\(^2\), Peide Liu\(^2,\)\(^,\)

\(^1\)School of International Education, Shandong University of Finance and Economics, Jinan Shandong, 250014, China
\(^2\)School of Management Science and Engineering, Shandong University of Finance and Economics, Jinan Shandong, 250014, China
\(^3\)School of Science and Technology, Shandong TV University, Jinan Shandong, 250014, China

\(^*\)Corresponding author. Email: ronglili@126.com

**ABSTRACT**

The quality of teaching can be improved by teaching performance evaluation from multiple experts, which is a multiple attribute group decision-making (MAGDM) problem. In this paper, a group decision-making method under proportional hesitant fuzzy linguistic environment is proposed to evaluate teaching performance. Firstly, proportional hesitant fuzzy linguistic term set (PHFLTS) is applied to express the decision makers’ (DMs) preferences for teaching performance index. Secondly, the PHFLP, CA operator is developed and its properties are discussed. Then based on the PHFLP, CA operator, a MAGDM method is formulated. Thirdly, the method is applied in teaching performance evaluation of Chinese-foreign cooperative education project. Finally, this method is proved more scientific, objective and accurate by compared with other two methods.

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1. INTRODUCTION

Chinese-foreign cooperative education project is the main form of transnational education in China. With the continuous expansion of its scale and the gradual upgrade of project-running, its sustainable development has become an inevitable requirement, which depends on the improvement of teaching quality. In the process of quality improving, teachers play an important role, such as curriculum designers and implementers. Teachers’ qualifications, teaching methods, and teaching ability have become the factors that affect the teaching quality. In order to improve the teaching quality, it is necessary to conduct the teaching performance evaluation of Chinese-foreign cooperative education project.

The teaching performance evaluation of Chinese-foreign cooperative education project can be performed by a group of decision makers (DMs) based on multiple attributes, which may be inaccuracy, ambiguity, and uncertainty. Therefore, the teaching performance evaluation of Chinese-foreign cooperative education project can be considered as a fuzzy multiple attribute group decision-making (MAGDM) problem. It plays a vital role in determining the quality of teaching performance evaluation to apply scientific methods comprehensively and effectively. In the aspect of teaching performance evaluation, some researches on this topic have been done by using MAGDM methods [1–6]. However, there are two drawbacks in the current researches. Firstly, most of the relevant studies are hard to deal with group information. Secondly, these studies assumed that the indicators were independent of each other, without considering the priority or correlation between the indicators. However, in the process of practical evaluation, there will be mutual relationship between the indicators.

In order to overcome these two drawbacks of current researches, a better method should be proposed to solve them. In the first place, the appropriate linguistic expression is chosen for teaching performance evaluation. On the one hand, appropriate linguistic expression should enable DMs to express their preferences as clearly as possible, and reduce the subjectivity and uncertainty in the decision-making process; on the other hand, the rational expression of linguistic information is the premise and basis for solving MAGDM problems effectively [7–11]. Based on the above two aspects, this paper adopts the proportional hesitant fuzzy linguistic term sets (PHFLTSS) [12–15]. PHFLTSSs are developed from traditional linguistic information, which are used to reasonably describe the subjective preference information given by DMs. Experts put forward several linguistic information representation models based on different situations, such as linguistic variables [16–18], hesitant fuzzy linguistic variables [19,20], extended hesitant fuzzy linguistic variables [21,22], linguistic distribution assessment variables [23,24], etc. By reducing the implicit constraints of the
above linguistic information representation models, a more general linguistic information representation model, i.e., proportional hesitant fuzzy linguistic elements (PHFLEs), can be proposed. PHFLEs are general forms of the above linguistic variables, which are more conducive for DMs freely expressing their subjective preferences, and more suitable for teaching performance evaluation of Chinese-foreign cooperative education project.

In the second place, after choosing the appropriate linguistic expression, another key problem is to select the information aggregation operator, that is, to find the appropriate tool for effective integration of teaching performance evaluation values. Choosing the suitable information aggregation tool is a crucial step. At present, information aggregation operators have drawn extensive attention and achieved fruitful results, especially several kinds of aggregation operators, such as Choquet integration operator [25], and prioritized integration operator [26,27]. However, the research on information aggregation operators based on PHFLEs is rare, and fails to be applied directly in teaching performance evaluation with priority and correlation.

Since Yager [26] proposed the prioritized aggregation (PrA) operator, it has been extensively improved [28–30], but the existing PrA operators cannot deal with the PHFLEs. Therefore, this paper enriches the PrA operators and proposes the proportional hesitant linguistic priority weighted average (PHFLP,WA m) operator.

Besides prioritized relation among elements, there are interdependence or interrelated features among elements, thus it is unreasonable to aggregate elements by additive measures. Sugeno [31] put forward the concept of non-additive measure (fuzzy measure), which just has monotonicity but not additive property. Choquet integral (CI) based on fuzzy measure can capture the interaction between different elements, but it hasn’t been applied into the proportional hesitant linguistic information. Therefore, it is important to propose an innovative CI operator to integrate the PHFLEs and deal with the MAGDM problems in the proportional hesitant linguistic environment.

Three innovative points in this paper are:

1. The first one is to enable DMs to express their preferences more freely and accurately. The proportional hesitant linguistic information can show the advantage of a group of DMs who have their own preferences when they make decisions.

2. The second one is to make the PHFLEs be applied in the situation with priority and correlation. The phenomenon of attributes with priority and correlation in MAGDM problems is common in real life, but it has never been extended to the PHFLEs.

3. The third one is to provide a new solution to the teaching performance evaluation of Chinese-foreign cooperative education project.

The remaining sections of this paper are outlined as follows: Section 2 introduces PHFLEs and its related concepts; Section 3 proposes proportional hesitant fuzzy linguistic Choquet aggregation (PHFLCA m) operator, PHFLP, WA m operator, and PHFLP,CA operator; Section 4 establishes a model based on PHFLP,CA operator method; Section 5 demonstrates a practical example about teaching performance evaluation of Chinese-foreign cooperative education project, then shows the advantages of the proposed method by comparing with other two methods. Finally, Section 6 comes to the overall conclusion.

2. PRELIMINARIES

This section introduces the concept of PHFLEs, which can be transformed into different linguistic information representation models according to the decrease of proportional constraints, and some related theories such as expectation function, deviation function, and their corresponding ranking methods are applied. In addition, the CI and the prioritized integration operators are reviewed.

2.1. PHFLEs and its Related Concepts

**Definition 1.** [32] Let $L = \{l_1, l_2, ..., l_t\}$ be a fully ordered finite discrete set, which $l_i$ represents a linguistic term. If $L$ meets the following conditions, $L$ is called a linguistic term set (LTS): (1) The set $L$ is ordered: if $i \geq k$, then $l_i \geq l_k$; (2) there is a negative operator: $Neg(l_i) = t - i$.

For example, when $t = 5$, a linguistic glossary is presented as follows:

$L = \{l_1 : \text{verylow}, l_2 : \text{low}, l_3 : \text{moderate}, l_4 : \text{high}, l_5 : \text{veryhigh}\}$.

**Definition 2.** [33] Suppose $X$ is a given domain $\forall x_i \in X$, and $L = \{l_i | i = 1, 2, ..., t\}$ be a LTS, then $H_L = \{(x_i, h_L(x_i)) | x_i \in X\}$ is called hesitant fuzzy LTS (HFLTS),

$h_L(x_i) = \{l_{\tau, t} | \tau = 1, 2, ..., #\}$ is a set of elements of $L$, # is the number of linguistic terms in $h_L(x_i)$. For convenience, $h_L(x_i)$ is called hesitant fuzzy linguistic element (HFL) and $H_L$ is a set of HFLs.

**Definition 3.** [34] Let $X$ be a given domain and $\forall x_i \in X$, $L$ be a given LTS and $\forall l_i \in L$, then $H_L(p) = \{(x_i, h_L^j(p)) | x_i \in X\}$ of $X$ is called probabilistic LTS (PLTS), where $h_L^j(p) = \{(\tau, t) | p(\tau, t) = 1\}$ is the $\tau$th linguistic term $P^{(\tau)}$ and its probability distribution $p^{(\tau)}$, # is the number of different linguistic terms in $h_L^j(p)$. Linguistic terms $p^{(\tau)}(\tau = 1, 2, ..., #)$ are arranged in ascending order. For convenience, $h_L^j(p)$ is called the probabilistic linguistic element (PLE).

**Definition 4.** [24] Let $X$ be a given domain and $\forall x_i \in X$, $L$ be a given LTS and $\forall l_i \geq L$, then $H_L(p) = \{(x_i, h_L^j(p)) | x_i \in X\}$ of $X$ is called a distributed LTS, where $h_L^j(p) = \{l_i(p_i') | l_i \in L, p_i' \geq 0, i = 1, 2, ..., t, \sum_{i=1}^{t} p_i' = 1\}$ is called a distributed linguistic element (DLE).

**Definition 5.** [12] Let $X$ be a given domain and $\forall x_i \in X$, $L$ be a given LTS and $\forall l_i \in L$, then $H_{PH}(p) = \{(x_i, h_{PH}^j(p)) | x_i \in X\}$ of $X$ is called PHFLTS, where $h_{PH}^j(p) = ...$
\[
\{ (l_i, p_i^j) | l_i \in L, 0 \leq p_i^j \leq 1, i = 1, 2, \ldots, t \} \] is LTS

\( l_i \) and \( p_i^j \) denotes the degree of possibility that the alternative carries an assessment value \( l_i \) provided by a group of DM. For convenience, \( \hat{h}_{PH}^j (p) \) is called PHFLE.

**Definition 6.** Let \( \hat{h}_{PH}^j (p) = \{ (l_i, p_i^j) | l_i \in L, i = 1, 2, \ldots, t \} \) be a PHFLE, then the expectation function of \( \hat{h}_{PH}^j (p) \) is

\[
E \left( \hat{h}_{PH}^j (p) \right) = \sum_{i=1}^{t} i \cdot p_i^j
\] (1)

**Definition 7.** Let \( \hat{h}_{PH}^j (p) = \{ (l_i, p_i^j) | l_i \in L, i = 1, 2, \ldots, t \} \) be a PHFLE, \( \hat{h}_{PH}^j (p) = \sum_{i=1}^{t} i \cdot p_i^j \), then the deviation function of \( \hat{h}_{PH}^j (p) \) is

\[
\sigma \left( \hat{h}_{PH}^j (p) \right) = \left( \sum_{i=1}^{t} \left( p_i^j \left( i - E \left( \hat{h}_{PH}^j (p) \right) \right) \right)^2 \right)^{1/2}
\] (2)

**Definition 8.** Let \( \hat{h}_{PH}^1 (p) \) and \( \hat{h}_{PH}^2 (p) \) be any two PHFLEs, then there are

1. if \( E \left( \hat{h}_{PH}^1 (p) \right) > E \left( \hat{h}_{PH}^2 (p) \right) \), then \( \hat{h}_{PH}^1 (p) > \hat{h}_{PH}^2 (p) \);
2. if \( E \left( \hat{h}_{PH}^1 (p) \right) < E \left( \hat{h}_{PH}^2 (p) \right) \), then \( \hat{h}_{PH}^1 (p) < \hat{h}_{PH}^2 (p) \);
3. if \( E \left( \hat{h}_{PH}^1 (p) \right) = E \left( \hat{h}_{PH}^2 (p) \right) \), then \( \hat{h}_{PH}^1 (p) = \hat{h}_{PH}^2 (p) \).

2.2. Cl Operator and PrA Operator

2.2.1. Fuzzy measure and Cl

**Definition 9.** [31] Let \( P(X) \) be the power set of \( X = \{ x_1, x_2, \ldots, x_n \} \), and the fuzzy measure \( \mu : P(X) \rightarrow [0, 1] \) of \( X \) satisfies the following conditions:

1. \( \mu (\emptyset) = 0, \mu (X) = 1; \)
2. if \( \forall A, B \in P(X) \) and \( A \subseteq B \), then \( \mu (A) \leq \mu (B) \).

Fuzzy measure can be regarded as monotone set function, the fuzzy measure on \( X \) has the following characteristics:

1. Additivity: \( \mu (A \cup B) = \mu (A) + \mu (B) \).
2. Sub-additivity: \( \forall A, B \in P(X), \mu (A \cup B) \leq \mu (A) + \mu (B) \).
3. Super-additivity: \( \forall A, B \in P(X), \mu (A \cup B) \geq \mu (A) + \mu (B) \).

In MAGDM, \( \mu (A) \) can be regarded as the importance of attribute subset \( A \in P(X) \). The monotony of fuzzy measure means that when new attributes are added to attribute subset, the importance of attribute subset will not decrease [35]. Nonadditivity is the main feature of fuzzy measure, which can more flexibly express the relationships between decision attributes from redundancy (negative interaction) to complementarity (positive interaction) [25,36].

Since the fuzzy measure is a function defined on the power set, it is necessary to determine \( 2^n - 2 \) parameters for calculating the fuzzy measure of \( n \) attributes, and the calculation amount for solving the fuzzy measure is large. Facing the large-scale calculation, Sugeno [31] proposed \( \lambda \) fuzzy measure to replace the general fuzzy measures, which simplified the computational complexity of fuzzy measures.

**Definition 10.** [31] Let \( P(X) \) be the power set of \( X = \{ x_1, x_2, \ldots, x_n \} \), \( \forall A, B \in P(X), A \cap B = \emptyset \), if the fuzzy measure \( g \) on \( X \) satisfies the following conditions:

\[
g(A \cup B) = g(A) + g(B) + \lambda g(A) g(B), \lambda \in (-1, \infty)
\]

then \( g \) is called \( \lambda \) fuzzy measure.

For \( \forall A, B \in P(X), A \cap B = \emptyset \): if \( \lambda = 0 \), \( g(A \cup B) = g(A) + g(B) \); attribute subsets \( A \) and \( B \) are independent; if \( -1 < \lambda < 0 \), \( g(A \cup B) < g(A) + g(B) \), attribute subset \( A \) and \( B \) are redundant; if \( 0 < \lambda < 1 \), \( g(A \cup B) > g(A) + g(B) \), attribute subsets \( A \) and \( B \) are complementary. In attribute-related multi-attribute decision-making, the role of attribute subset \( D \in P(X) \) in decision-making process is determined not only by \( g(D) \) itself, but also by other attribute subsets. If \( g(D) = 0 \), then attribute subset \( D \) is irrelevant. For attribute subset \( H \in P(X) \), \( g(H \cup D) - g(H) > 0 \) indicates that attribute subset \( H \) is important.

According to the definition of \( \lambda \) fuzzy measure \( g \), \( \forall x_j \in X, j = 1, 2, \ldots, n, j \neq k, x_j \cap x_k = \emptyset, \cup x_i = X \), then \( \lambda \) fuzzy measure is shown as follows:

\[
g(X) = g \left( \bigcup_{j=1}^{n} x_j \right) = \frac{1}{\lambda} \left( \sum_{j=1}^{n} g(x_j) \right) - \left( \prod_{j=1}^{n} (1 + \lambda g(x_j)) - 1 \right), \lambda \neq 0
\]

Since \( g(X) = 1 \), when \( \lambda \neq 0 \), the value of \( \lambda \) is determined according to the following formula:

\[
\lambda + 1 = \prod_{j=1}^{n} (1 + \lambda g(x_j))
\]

**Definition 11.** [25] Let \( X = \{ x_1, x_2, \ldots, x_n \} \) be a nonempty set, \( f \) be a nonnegative real value function defined on \( X \), \( g \) be a fuzzy measure defined on \( X \), and the CI of function \( f \) on \( g \) is defined as follows:

\[
\sum_{j=1}^{n} \left[ f(x_j) - f(x_{j-1}) \right] \cdot g(x_j)
\]
It can also be expressed as
\[
\int f dg = \sum_{j=1}^{n} \left[ g \left( X_{j} \right) - g \left( X_{j+1} \right) \right] \cdot f \left( X_{j} \right) \tag{6}
\]
where \( f \left( X_{j} \right) \leq f \left( X_{j+1} \right) \leq \ldots \leq f \left( X_{n} \right) \), \( f \left( X_{0} \right) = 0 \), \( X_{j} = \{ x_{j}, x_{j+1}, \ldots, x_{n} \} \) and \( x_{n+1} = \emptyset \).

Aggregation characteristics of CI are idempotency, compensation, monotone additivity, etc. In addition, CI is an extension of weighted average and orderly weighted average. As long as the fuzzy measure is additive, the CI degenerates into a weighted average or an ordered weighted average.

### 2.2.2. PrA operator

**Definition 12.** [26] Let \( C = \{ C_{1}, C_{2}, \ldots, C_{n} \} \) be a set of attributes, then there exists a linear ordered prioritized relationship between attributes, which can be expressed as \( C_{1} \succ C_{2} \succ \ldots \succ C_{n} \), that is, \( C_{j} \) priority ranks are higher than \( C_{k} \), \( \forall j < k \). \( C_{j} \) (x) is the evaluation value of alternative x under attribute \( C_{j} \), which satisfies \( C_{j} \) (x) \( \in [0,1] \). If
\[
P_{A} \left( C_{j} \right) (x) = \sum_{j=1}^{n} w_{j} C_{j} (x)
\]
where \( w_{j} = T_{j} / \sum_{j=1}^{n} T_{j}, T_{j} = \prod_{k=1}^{j-1} C_{k} (x) \) (\( j = 2,3, \ldots, n \)), \( T_{1} = 1 \) then \( P_{A} \) is called the PrA operator.

The PrA operator [26] based on priority measure cannot solve all of priority decision problem. To be specific, for two alternatives, this problem cannot be solved when the satisfaction of their highest priority attribute is the same and the satisfaction of these attributes is the smallest of all priority attributes. The fundamental reason is that the measurement of priority is too strict to compensate for any priority attributes. Based on this, Chen et al. [37] proposed a generalized prioritized operator to optimize the prioritized operator [9].

**Definition 13.** [37] Let \( C = \{ C_{1}, C_{2}, \ldots, C_{n} \} \) be a set of attributes, then there exists a linear ordered prioritized relationship between attributes, which can be expressed as \( C_{1} \succ C_{2} \succ \ldots \succ C_{n} \). Let \( A = \{ C_{1}, C_{2}, \ldots, C_{n} \} \) be a subset of attribute set \( C \), by reordering the elements in subset \( A \), \( A = \{ C_{1}, C_{2}, \ldots, C_{n} \} \) can be obtained, of which \( \sigma \left( 1 \right), \sigma \left( 2 \right), \ldots, \sigma \left( \ell \right) \) is an arbitrary sequence of \( \{ l_{1}, l_{2}, \ldots, l_{\ell} \} \), satisfying \( \sigma \left( 1 \right) < \sigma \left( 2 \right) < \cdots < \sigma \left( \ell \right) \). \( C_{j} \) (x) \( \in [0,1] \) (\( i = 1,2, \ldots, n \)) is the satisfaction of alternative x under attribute \( C_{i} \). A numerical value of \( b_{i} \) for each priority attribute \( C_{i} \) in attribute set \( C \), which satisfies \( b_{i} \in [0,1] \) and \( \sum_{i=1}^{n} b_{i} = 1 \), \( b_{1} \geq b_{2} \geq \ldots \geq b_{n} \). The generalized prioritized measure \( m \) : \( P \left( C \right) \rightarrow [0,1] \) is defined as follows:
\[
m \left( A \right) = \sum_{j=1}^{\ell} b_{\sigma \left( j \right)} f_{j} \left( \sigma \left( j \right) \right) \quad \text{and} \quad m \left( \emptyset \right) = 0 \tag{8}
\]
where, \( P \left( C \right) \) is the power set of attribute set \( C \), \( f \left( \cdot \right) \) is a monotone decreasing function which satisfies \( f_{j} \left( j \right) = 1, f_{j} \left( n \right) = 0 \) (\( j = 1, 2, \ldots, n \)).

If \( f_{j} \left( \sigma \left( j \right) \right) = \frac{n+1-\sigma \left( j \right)}{n+1-j} \) (\( j = 1, 2, \ldots, n \)), then the new prioritized measures are expressed as follows:
\[
m \left( A \right) = \sum_{j=1}^{\ell} b_{\sigma \left( j \right)} \frac{n+1-\sigma \left( j \right)}{n+1-j} \tag{9}
\]
Based on this, Chen et al. [37] proposed a generalized PrA operator, i.e.
\[
\chi \left( x \right) = Ch_{m} \left( c_{1} \left( x \right), c_{2} \left( x \right), \ldots, c_{n} \left( x \right) \right) = \sum_{i=1}^{n} c_{i} \left( x \right) \left( m \left( h_{i} \right) - m \left( h_{i-1} \right) \right)
\]
where \( c_{i} \left( x \right) \) is the \( i \)-th largest value in \( \{ c_{1} \left( x \right), c_{2} \left( x \right), \ldots, c_{n} \left( x \right) \} \), \( h_{i} \) is \( \{ c_{1}, c_{2}, \ldots, c_{i} \} \) and \( h_{0} = \emptyset \), \( m \left( h_{i} \right) \) (\( i = 0, 1, \ldots, n \)) is generalized prioritized measures based on \( h_{i} \).

In this section, the basic theory of MAGDM based on PHFLTS is studied. Firstly, the related linguistic information representation models are briefly reviewed. In order to overcome the shortcoming of several kinds of linguistic information representation models, the PHFLTS is proposed, and related theories, such as expectation function, deviation function, and ranking method are put forward. Secondly, two kinds of typical functional information integration operators, namely CI integration operator and PrA operator, are reviewed.

### 3. PROPORTIONAL HESITANT FUZZY LINGUISTIC PRIORITIZED CHOQUET AGGREGATION OPERATOR

This section focuses on the aggregation of PHFLTs, and proposes the PHFLPCA operator to deal with the MAGDM problems with both priority and correlation.

#### 3.1. PHFLPCA\(_{\mu}\) Operator Based on Fuzzy Measure

**Definition 14.** Let \( \hat{H}_{PH} \left( \hat{p} \right) = \left\{ ^{1}_{n} \hat{h}_{PH} \left( \hat{p} \right), ^{2}_{n} \hat{h}_{PH} \left( \hat{p} \right), \ldots, ^{n}_{n} \hat{h}_{PH} \left( \hat{p} \right) \right\} \) be the PHFLTS, where \( \hat{h}_{PH}^{i} \left( \hat{p} \right) = \left\{ \left( i_{1}, p_{1}^{i} \right), \left( i_{2}, p_{2}^{i} \right), \ldots, \left( i_{t}, p_{t}^{i} \right) \right\}, i = 1, 2, \ldots, t, \mu \) is fuzzy measure based on \( \hat{H}_{PH} \left( \hat{p} \right) \). Then the \( \text{PHFLPCA}_{\mu} \) operator is defined as follows:
\[
\text{PHFLPCA}_{\mu} \left( \hat{H}_{PH} \left( \hat{p} \right), \ldots, \hat{H}_{PH} \left( \hat{p} \right) \right) = \left( \mu \left( H_{PH}^{1} \left( \hat{p} \right) \right), \ldots, \mu \left( H_{PH}^{t} \left( \hat{p} \right) \right) \right) \hat{h}_{PH}^{(\mu)} \left( \hat{p} \right)
\]
where \( \hat{h}_{PH}^{(1)} \left( \hat{p} \right) \leq \hat{h}_{PH}^{(2)} \left( \hat{p} \right) \leq \cdots \leq \hat{h}_{PH}^{(n)} \left( \hat{p} \right) \) and \( \hat{H}_{PH}^{(\mu)} \left( \hat{p} \right) = \left\{ \hat{h}_{PH}^{(1)} \left( \hat{p} \right), \ldots, \hat{h}_{PH}^{(n)} \left( \hat{p} \right) \right\} \).

**Theorem 1.** Let \( \hat{H}_{PH} \left( \hat{p} \right) = \left\{ ^{1}_{n} \hat{h}_{PH} \left( \hat{p} \right), ^{2}_{n} \hat{h}_{PH} \left( \hat{p} \right), \ldots, ^{n}_{n} \hat{h}_{PH} \left( \hat{p} \right) \right\} \) be the PHFLTS, where \( \hat{h}_{PH}^{i} \left( \hat{p} \right) = \left\{ \left( i_{1}, p_{1}^{i} \right), \left( i_{2}, p_{2}^{i} \right), \ldots, \left( i_{t}, p_{t}^{i} \right) \right\} \) for \( i = 1, 2, \ldots, t, p_{i}^{i} = 1 \),
\( i = 1, 2, ..., t \), \( \mu \) be fuzzy measure based on \( \hat{H}_{pfi} (\hat{p}) \), then the result of PHFLCA\( _{\mu} \) operator is as follows:

\[
\text{PHFLCA}_{\mu}(\hat{h}_{pfi}^{1}(\hat{p}), \hat{h}_{pfi}^{2}(\hat{p}), ..., \hat{h}_{pfi}^{n}(\hat{p})) = \left\{ \left( \left( \sum_{i=1}^{n} \mu \left( \hat{H}_{pfi}^{j}(\hat{p}) \right) \right) \right) \hat{p}_{i} \right\}_{i \in L,}
\]

(11)

Next, the properties of PHFLCA\( _{\mu} \) operator are idempotency, permutation invariance, monotonicity, and boundedness.

Property 1. (Idempotency) Let \( \hat{h}_{pfi}^{j}(\hat{p}) = \{\hat{h}_{pfi}^{j}(\hat{p}), \hat{h}_{pfi}^{2}(\hat{p}), ..., \hat{h}_{pfi}^{n}(\hat{p})\} \) be the PHFLTS, where \( \hat{h}_{pfi}^{j}(\hat{p}) = \hat{h}_{pfi}(\hat{p}) = \{l_{i}, \hat{p}_{i} \}_{i \in L,} \sum_{i=1}^{t} p_{i} = 1, i = 1, 2, ..., t \}, j = 1, 2, ..., n, then

\[
\text{PHFLCA}_{\mu}(\hat{h}_{pfi}^{1}(\hat{p}), \hat{h}_{pfi}^{2}(\hat{p}), ..., \hat{h}_{pfi}^{n}(\hat{p})) = \hat{h}_{pfi}(\hat{p})
\]

Proof: Because of \( \hat{h}_{pfi}^{j}(\hat{p}) = \hat{h}_{pfi}(\hat{p}), \mu \left( \hat{H}_{pfi}^{j}(\hat{p}) \right) > \mu \left( \hat{H}_{pfi}^{j+1}(\hat{p}) \right) \geq 0 \) and \( n \sum_{j=1}^{n} \mu \left( \hat{H}_{pfi}^{j}(\hat{p}) \right) = 1 \), it is available that

\[
\text{PHFLCA}_{\mu}(\hat{h}_{pfi}^{1}(\hat{p}), \hat{h}_{pfi}^{2}(\hat{p}), ..., \hat{h}_{pfi}^{n}(\hat{p})) = \left\{ \left( \sum_{i=1}^{n} \mu \left( \hat{H}_{pfi}^{j}(\hat{p}) \right) \right) \hat{p}_{i} \right\}_{i \in L,}
\]

Property 2. (Commutativity) Let \( \{\hat{h}_{pfi}^{1}(\hat{p}), \hat{h}_{pfi}^{2}(\hat{p}), ..., \hat{h}_{pfi}^{n}(\hat{p})\} \) be any PHFLTS, and \( \{\hat{h}_{pfi}^{1}(\hat{p}), \hat{h}_{pfi}^{2}(\hat{p}), ..., \hat{h}_{pfi}^{n}(\hat{p})\} \) is any sequence of \( \{\hat{h}_{pfi}^{1}(\hat{p}), \hat{h}_{pfi}^{2}(\hat{p}), ..., \hat{h}_{pfi}^{n}(\hat{p})\} \), then

\[
\text{PHFLCA}_{\mu}(\hat{h}_{pfi}^{1}(\hat{p}), \hat{h}_{pfi}^{2}(\hat{p}), ..., \hat{h}_{pfi}^{n}(\hat{p})) = \text{PHFLCA}_{\mu}(\hat{h}_{pfi}^{1}(\hat{p}), \hat{h}_{pfi}^{2}(\hat{p}), ..., \hat{h}_{pfi}^{n}(\hat{p}))
\]

Proof: Since \( \{\hat{h}_{pfi}^{1}(\hat{p}), \hat{h}_{pfi}^{2}(\hat{p}), ..., \hat{h}_{pfi}^{n}(\hat{p})\} \) is an arbitrary sequence of \( \{\hat{h}_{pfi}^{1}(\hat{p}), \hat{h}_{pfi}^{2}(\hat{p}), ..., \hat{h}_{pfi}^{n}(\hat{p})\} \), CI operator can be regarded as an extended ordered weighted average operator. Based on the above formula, we can find that PHFLCA\( _{\mu} \) operators have permutation invariance.

Property 3. (Monotonicity) Let \( \{\hat{h}_{pfi}^{1}(\hat{p}), \hat{h}_{pfi}^{2}(\hat{p}), ..., \hat{h}_{pfi}^{n}(\hat{p})\} \) and \( \{\hat{h}_{pfi}^{1}(\hat{q}), \hat{h}_{pfi}^{2}(\hat{q}), ..., \hat{h}_{pfi}^{n}(\hat{q})\} \) be any two PHFLTSs, of which

\[
\{l_{i}, \hat{p}_{i} \}_{i \in L,} \sum_{i=1}^{t} p_{i} = 1, i = 1, 2, ..., t \}
\]

and \( \{l_{i}, \hat{q}_{i} \}_{i \in L,} \sum_{i=1}^{t} q_{i} = 1, i = 1, 2, ..., t \}

\( \hat{h}_{pfi}^{j}(\hat{p}) = \{l_{i}, \hat{p}_{i} \}_{i \in L,} \sum_{i=1}^{t} p_{i} = 1, i = 1, 2, ..., t \}

(12)

and \( \hat{h}_{pfi}^{j}(\hat{q}) = \{l_{i}, \hat{q}_{i} \}_{i \in L,} \sum_{i=1}^{t} q_{i} = 1, i = 1, 2, ..., t \}

(13)

and \( E \left( \hat{h}_{pfi}^{j}(\hat{p}) \right) \leq E \left( \hat{h}_{pfi}^{j}(\hat{q}) \right) \)

(14)

\[
E \left( \hat{h}_{pfi}^{j+1}(\hat{p}) \right) \leq E \left( \hat{h}_{pfi}^{j+1}(\hat{q}) \right)
\]

(15)

\[
E \left( \hat{h}_{pfi}^{j}(\hat{p}) \right) \leq E \left( \hat{h}_{pfi}^{j}(\hat{q}) \right)
\]

Then,

\[
\text{PHFLCA}_{\mu}(\hat{h}_{pfi}^{1}(\hat{p}), \hat{h}_{pfi}^{2}(\hat{p}), ..., \hat{h}_{pfi}^{n}(\hat{p})) \leq \text{PHFLCA}_{\mu}(\hat{h}_{pfi}^{1}(\hat{q}), \hat{h}_{pfi}^{2}(\hat{q}), ..., \hat{h}_{pfi}^{n}(\hat{q}))
\]

Proof: According to Definition 6, we get \( E \left( \hat{h}_{pfi}^{j}(\hat{p}) \right) = 1 \), \( E \left( \hat{h}_{pfi}^{j}(\hat{p}) \right) = t \).

\[
E \left( \text{PHFLCA}_{\mu}(\hat{h}_{pfi}^{1}(\hat{p}), \hat{h}_{pfi}^{2}(\hat{p}), ..., \hat{h}_{pfi}^{n}(\hat{p})) \right) = \sum_{i=1}^{t} \left( \sum_{j=1}^{n} \mu \left( \hat{H}_{pfi}^{j}(\hat{p}) \right) - \mu \left( \hat{H}_{pfi}^{j+1}(\hat{p}) \right) \right) \hat{p}_{i} \leq t \sum_{j=1}^{n} \left( \sum_{i=1}^{t} \mu \left( \hat{H}_{pfi}^{j}(\hat{p}) \right) - \mu \left( \hat{H}_{pfi}^{j+1}(\hat{p}) \right) \right) \hat{p}_{i} = t
\]
The same, we get
\[
E \left( PHFLCA_\mu \left( \hat{h}^1_{PH} (\hat{p}), \hat{h}^2_{PH} (\hat{p}), ..., \hat{h}^n_{PH} (\hat{p}) \right) \right) \\
= \frac{\hat{h}^+_{PH} (\hat{p})}{\hat{h}^-_{PH} (\hat{p})} \\
\geq \frac{\hat{h}^+_{PH} (\hat{p})}{\hat{h}^-_{PH} (\hat{p})} = 1
\]
so,
\[
E \left( \hat{h}^-_{PH} (\hat{p}) \right) \leq E \left( PHFLCA_\mu \left( \hat{h}^1_{PH} (\hat{p}), \hat{h}^2_{PH} (\hat{p}), ..., \hat{h}^n_{PH} (\hat{p}) \right) \right) \\
E \left( \hat{h}^+_{PH} (\hat{p}) \right)
\]
and
\[
\hat{h}^-_{PH} (\hat{p}) \leq PHFLCA_\mu \left( \hat{h}^1_{PH} (\hat{p}), \hat{h}^2_{PH} (\hat{p}), ..., \hat{h}^n_{PH} (\hat{p}) \right) \leq \hat{h}^+_{PH} (\hat{p}).
\]
Noted: When the PHFLEs \( \hat{h}^j_{PH} (\hat{p}) (j = 1, 2, ..., n) \) are independent of each other, the fuzzy measure \( \mu \) degenerates into an additive measure, i.e.,
\[
\mu \left( \hat{h}^j_{PH} (\hat{p}) \right) = \sum_{\hat{h}^j_{PH}(\hat{p}) \in \hat{h}^j_{PH}(\hat{p})} \mu \left( \hat{h}^j_{PH} (\hat{p}) \right).
\]
∀ \( \hat{h}^j_{PH} (\hat{p}) \in \hat{H}_{PH} (\hat{p}) \). The PHFLCA_\mu operator degenerates to the proportional hesitant linguistic orderly weighted average operator.

3.2. PHFLP_{WA_m} Operator Based on Generalized Prioritized Measure

The generalized prioritized measure proposed by Chen et al. [37], can solve the following problems when the satisfaction degree of the highest priority attribute is same and the satisfaction degree is the lowest among all priority attributes. This is because the prioritized measure is too strict to compensate for any priority attributes. Therefore, this section proposes PHFLP_{WA_m} operator based on generalized prioritized measure.

**Definition 15.** Let \( C = \{C_1, C_2, ..., C_n\} \) be a set of attributes, then there exists a linear ordered priority relationship between attributes, which can be expressed as \( C_1 > C_2 > ... > C_n \), i.e., \( C_j \) priority ranks are higher than \( C_k \), \( \forall j < k \). \( \hat{h}^j_{PH} (\hat{p}) \) is the evaluation result of alternative \( x \) under attribute \( C_j \), i.e.,
\[
\hat{h}^j_{PH} (\hat{p}) = \left\{ \left( l_i, \hat{p}^i_j \right) \mid l_i \in L, \sum_{i=1}^t \hat{p}^i_j = 1, i = 1, 2, ..., t \right\}, \hat{H}_{PH} (\hat{p}) \in \hat{H}_{PH} (\hat{p}), \hat{H}_{PH} (\hat{p}) = \left\{ \hat{h}^1_{PH} (\hat{p}), \hat{h}^2_{PH} (\hat{p}), ..., \hat{h}^n_{PH} (\hat{p}) \right\} \) is the PHFLTS.

The PHFLP_{WA_m} operator is defined as follows:
\[
PHFLP_{WA_m} \left( \hat{h}^1_{PH} (\hat{p}), \hat{h}^2_{PH} (\hat{p}), ..., \hat{h}^n_{PH} (\hat{p}) \right) \\
= \sum_{j=1}^n \hat{h}^j_{PH} (\hat{p}) \left( m \left( H_j \right) - m \left( H_{j-1} \right) \right)
\]
where \( \hat{h}^j_{PH} (\hat{p}) \) is the \( j \)th largest element in \( \hat{h}^1_{PH} (\hat{p}), \hat{h}^2_{PH} (\hat{p}), ..., \hat{h}^n_{PH} (\hat{p}) \), \( H_T = \{C_{\sigma(1)}, C_{\sigma(2)}, ..., C_{\sigma(\tau)}\}, \{\sigma (1), \sigma (2), ..., \sigma (\tau)\} \) is the sequence of \( \{1, 2, ..., \tau\} \), satisfying if \( \varepsilon < k, C_{\sigma(\varepsilon)} > C_{\sigma(k)} \) and \( H_{(0)} = \emptyset, m \left( H_j \right) (j = 1, 2, ..., n) \) is generalized prioritized measure.
\[
m \left( H_T \right) = \sum_{i=1}^t b_{\varepsilon(e)} \frac{n + 1 - \varepsilon \left( \varepsilon \right)}{n + 1 - \varepsilon}
\]

**Theorem 2.** Let \( C = \{C_1, C_2, ..., C_n\} \) be a set of attributes, then there exists a linear ordered priority relationship between attributes, which can be expressed as \( C_1 > C_2 > ... > C_n \), i.e., \( C_j \) priority ranks are higher than \( C_k \), \( \forall j < k \). \( \hat{h}^j_{PH} (\hat{p}) \) is the evaluation result of alternative \( x \) under attribute \( C_j \), i.e.,
\[
\hat{h}^j_{PH} (\hat{p}) = \left\{ \left( l_i, \hat{p}^i_j \right) \mid l_i \in L, \sum_{i=1}^t \hat{p}^i_j = 1, i = 1, 2, ..., t \right\}, \hat{H}_{PH} (\hat{p}) = \left\{ \hat{h}^1_{PH} (\hat{p}), \hat{h}^2_{PH} (\hat{p}), ..., \hat{h}^n_{PH} (\hat{p}) \right\} \) is the PHFLTS, then the integration result of PHFLP_{WA_m} operator is as follows:
\[
PHFLP_{WA_m} \left( \hat{h}^1_{PH} (\hat{p}), \hat{h}^2_{PH} (\hat{p}), ..., \hat{h}^n_{PH} (\hat{p}) \right) \\
= \left\{ \left( l_i, \sum_{j=1}^n \hat{h}^j_{PH} \left( m \left( H_j \right) - m \left( H_{j-1} \right) \right) \right) \mid l_i \in L \right\}
\]
Through the above theorem, we can easily find that PHFLP_{WA_m} operator has the following properties:

**Property 5. (Idempotency)** Let \( \hat{H}_{PH} (\hat{p}) = \left\{ \hat{h}^1_{PH} (\hat{p}), \hat{h}^2_{PH} (\hat{p}), ..., \hat{h}^n_{PH} (\hat{p}) \right\} \) be the PHFLTS, of which \( \hat{h}^j_{PH} (\hat{p}) = \left\{ \left( l_i, \hat{p}^j_i \right) \mid l_i \in L, \sum_{i=1}^t \hat{p}^j_i = 1, i = 1, 2, ..., t \right\} \). If \( \hat{h}^j_{PH} (\hat{p}) = \hat{h}^j_{PH} (\hat{p}) = \left\{ \left( l_i, \hat{p}^j_i \right) \mid l_i \in L, \sum_{i=1}^t \hat{p}^j_i = 1, i = 1, 2, ..., t \right\} \) and \( \hat{h}^j_{PH} (\hat{q}) \geq \hat{h}^j_{PH} (\hat{q}), \) i.e., the proportional distribution of corresponding linguistic term is \( \hat{p}^j_i \geq \hat{q}^j_i \) (i = 1, 2, ..., t), then
\[
PHFLP_{WA_m} \left( \hat{h}^1_{PH} (\hat{p}), \hat{h}^2_{PH} (\hat{p}), ..., \hat{h}^n_{PH} (\hat{p}) \right) \\
\geq PHFLP_{WA_m} \left( \hat{h}^1_{PH} (\hat{q}), \hat{h}^2_{PH} (\hat{q}), ..., \hat{h}^n_{PH} (\hat{q}) \right)
\]
Proof: Based on Theorem 2, we get
\[
\text{PHFLP,WA}_m\left(h^1_{PH} (\hat{p}), h^2_{PH} (\hat{p}), ..., h^n_{PH} (\hat{p})\right) = \left\{ l, \sum_{j=1}^n \hat{p}_j \left( m(H_j) - m(H_{j-1}) \right) \right\},
\]
\[
\text{PHFLP,WA}_m\left(h^1_{PH} (\hat{q}), h^2_{PH} (\hat{q}), ..., h^n_{PH} (\hat{q})\right) = \left\{ l, \sum_{j=1}^n \hat{q}_j \left( m(H_j) - m(H_{j-1}) \right) \right\}.
\]
According to the Definition 8, the expectation values are
\[
E\left( \text{PHFLP,WA}_m\left(h^1_{PH} (\hat{p}), h^2_{PH} (\hat{p}), ..., h^n_{PH} (\hat{p})\right) \right) = \sum_{i=1}^t i \cdot \left( \sum_{j=1}^n \hat{p}_j \left( m(H_j) - m(H_{j-1}) \right) \right),
\]
\[
E\left( \text{PHFLP,WA}_m\left(h^1_{PH} (\hat{q}), h^2_{PH} (\hat{q}), ..., h^n_{PH} (\hat{q})\right) \right) = \sum_{i=1}^t i \cdot \left( \sum_{j=1}^n \hat{q}_j \left( m(H_j) - m(H_{j-1}) \right) \right).
\]
Because \( \hat{p}_i \geq \hat{q}_i \), we get \( \hat{p}_j \left( m(H_j) - m(H_{j-1}) \right) \geq \hat{q}_j \left( m(H_j) - m(H_{j-1}) \right) \).
Then \( \sum_{j=1}^n \hat{p}_j \left( m(H_j) - m(H_{j-1}) \right) \geq \sum_{j=1}^n \hat{q}_j \left( m(H_j) - m(H_{j-1}) \right) \), we can get
\[
E\left( \text{PHFLP,WA}_m\left(h^1_{PH} (\hat{p}), h^2_{PH} (\hat{p}), ..., h^n_{PH} (\hat{p})\right) \right) \geq E\left( \text{PHFLP,WA}_m\left(h^1_{PH} (\hat{q}), h^2_{PH} (\hat{q}), ..., h^n_{PH} (\hat{q})\right) \right).
\]
Therefore, \( \text{PHFLP,WA}_m\left(h^1_{PH} (\hat{p}), h^2_{PH} (\hat{p}), ..., h^n_{PH} (\hat{p})\right) \geq \text{PHFLP,WA}_m\left(h^1_{PH} (\hat{q}), h^2_{PH} (\hat{q}), ..., h^n_{PH} (\hat{q})\right) \).

Property 7. (Boundedness) Let \( \left\{h^1_{PH} (\hat{p}), h^2_{PH} (\hat{p}), ..., h^n_{PH} (\hat{p})\right\} \) be a PHFLTS, where \( h^i_{PH} (\hat{p}) = \left\{ l, \hat{p}_i \right\} \), \( \left\{ l, \hat{p}_i \right\} \in L \), \( \sum_{i=1}^t \hat{p}_i = 1, i = 1, 2, ..., t \), then
\[
h^-_{PH} (\hat{p}) = \{ (l_1, 0), (l_2, 0), ..., (l_1, 0) \} \text{ and } h^+_{PH} (\hat{p}) = \{ (l_1, 1), (l_2, 0), ..., (l_1, 0) \},
\]
then
\[
h^-_{PH} (\hat{p}) \leq \text{PHFLP,WA}_m\left(h^1_{PH} (\hat{p}), h^2_{PH} (\hat{p}), ..., h^n_{PH} (\hat{p})\right) \leq h^+_{PH} (\hat{p}).
\]
Proof: Based on the Definition 8, we get that
\[
E\left( h^+_{PH} (\hat{p}) \right) = t, E\left( h^-_{PH} (\hat{p}) \right) = 1,
\]
\[
E\left( \text{PHFLP,WA}_m\left(h^1_{PH} (\hat{p}), h^2_{PH} (\hat{p}), ..., h^n_{PH} (\hat{p})\right) \right) \leq \sum_{i=1}^t \sum_{j=1}^n \hat{p}_j \left( m(H_j) - m(H_{j-1}) \right).
\]
Because
\[
E\left( \text{PHFLP,WA}_m\left(h^1_{PH} (\hat{p}), h^2_{PH} (\hat{p}), ..., h^n_{PH} (\hat{p})\right) \right) = \sum_{i=1}^t \sum_{j=1}^n \hat{p}_j \left( m(H_j) - m(H_{j-1}) \right) \leq \sum_{i=1}^t \sum_{j=1}^n \hat{p}_j \left( m(H_j) - m(H_{j+1}) \right) = t.
\]
Similarly,
\[
E\left( \text{PHFLP,WA}_m\left(h^1_{PH} (\hat{q}), h^2_{PH} (\hat{q}), ..., h^n_{PH} (\hat{q})\right) \right) = \sum_{i=1}^t \sum_{j=1}^n \hat{q}_j \left( m(H_j) - m(H_{j-1}) \right) \leq \sum_{i=1}^t \sum_{j=1}^n \hat{q}_j \left( m(H_j) - m(H_{j+1}) \right) = 1.
\]
So, we get \( h^-_{PH} (\hat{p}) \leq \text{PHFLP,WA}_m\left(h^1_{PH} (\hat{p}), h^2_{PH} (\hat{p}), ..., h^n_{PH} (\hat{p})\right) \leq h^+_{PH} (\hat{p}).
\]

3.3. PHFLP,CA Operator

Yager [26] pointed out that prioritized decision problems can be categorized in two forms: (1) one form is strictly ordered priority, i.e., each priority corresponds to one attribute; (2) the other form is weakly ordered priority, i.e., each priority corresponds to one or more attributes. PHFLP,WA_m operator can only deal with the first kind of priority problems, but it cannot deal with the second kind of MAGDM problems. However, the second kind of MAGDM problems often occur in real life, thus a new integration operator needs proposing to handle the problems of weakly ordered priority, i.e., PHFLP,CA operator. The operation of PHFLP,CA operator can be described in two steps. Firstly, the PHFLCA operator is based on λ fuzzy measure to get the satisfaction, which is displayed in each priority level. Secondly, the PHFLP,WA_m operator based on prioritized measure is to get the overall satisfaction, which is displayed in each alternative. Thus the PHFLP,CA operator proposed in this section considers both the priority relationship among priority levels and the correlation among attributes.

In the MAGDM problems with weakly ordered priority, attribute set \( C = \{ C_1, C_2, ..., C_n \} \) is divided into \( q \) independent priority levels \( H = \{ H_1, H_2, ..., H_q \} \), \( H_{\varepsilon} = \{ C'_1, C'_2, ..., C'_{n_{\varepsilon}} \} \) of which \( n_{\varepsilon} \) is the number of attributes contained in priority level \( H_{\varepsilon} \). Assuming that there is a priority relation \( H_1 > H_2 > ... > H_k \) in the independent priority level \( \{ H_1, H_2, ..., H_k \} \), when the attribute priority of \( k > \varepsilon \) in \( H_k \) is higher than that in \( H_{\varepsilon} \). Attribute set \( C = \bigcup_{\varepsilon=1}^{q} H_{\varepsilon} \), numbers
of attributes \( n = \sum_{\tau=1}^{q} n_{\tau} \), evaluation value of alternative \( x \) under attribute \( C_{\tau} \in H_{\tau} \) is PHFLE, expressed as \( h_{\tau}^{\tau}(p) \).

PHFLCA\(_{\tau}\) operator is based on \( \lambda \) fuzzy measure to calculate the satisfaction \( h^{\tau}(p) \), which is displayed in each priority level:

\[
PHFLCA_{\tau}\left( \sum_{j=1}^{n_{\tau}} h_{\tau}^{(j)}(p) \right) = \bigoplus_{h_{\tau}^{(j)}(p)} (g(H_{\tau}^{(j)}) - g(H_{\tau}^{(j-1)}))
\]

where \( h_{\tau}^{(j)}(p) \) is the \( j \)th largest value in \( \{h_{1\tau}(p), h_{2\tau}(p), ..., h_{n\tau}(p)\} \), and \( H_{\tau}^{(j)} = \{C_{1\tau}^{(j)}, C_{2\tau}^{(j)}, ..., C_{m\tau}^{(j)}\} \).\( H_{\tau}^{(0)} = \emptyset \), \( g(H_{\tau}^{(j)}) \) is a \( \lambda \) fuzzy measure of \( H_{\tau}^{(j)} \).

According to the above formulas, the ensemble results of PHFLCA\(_{\tau}\) operators based on \( \lambda \) fuzzy measure is as follows:

\[
\hat{h}_{\tau}(p) = PHFLCA_{\tau}\left( \sum_{j=1}^{n_{\tau}} h_{\tau}^{(j)}(p) \right) = \left\{ \left( \sum_{j=1}^{n_{\tau}} p_{\tau}^{(j)}(g(H_{\tau}^{(j)}) - g(H_{\tau}^{(j-1)})) \right) \mid l = 1, 2, ..., l \right\}
\]

The PHFLP\(_{m}\) operator is based on prioritized measure to calculate the overall satisfaction \( h(p) \), which is displayed in each alternative,

\[
PHFLP_{m}\left( \hat{h}_{1}(p), \hat{h}_{2}(p), ..., \hat{h}_{q}(p) \right) = \bigoplus_{\tau=1}^{q} \hat{h}_{\tau}(p) (m(h^{(\tau)}) - m(h^{(\tau-1)}))
\]

where \( \hat{h}_{\tau}(p) \) is the \( \tau \)th largest value in \( \{\hat{h}_{1\tau}(p), \hat{h}_{2\tau}(p), ..., \hat{h}_{q\tau}(p)\} \), \( h^{(\tau)} = \{C^{(1)}, C^{(2)}, ..., C^{(q)}\} \), \( H^{(0)} = \emptyset \), \( m(h^{(\tau)}) \) is a generalized prioritized measure.

According to the above formula, the integration result of PHFLP\(_{m}\) operator based on prioritized measure is as follows:

\[
PHFLP_{m}\left( \hat{h}_{1}(p), \hat{h}_{2}(p), ..., \hat{h}_{q}(p) \right) = \left\{ \left( \sum_{\tau=1}^{q} \hat{h}_{\tau}(p) (m(h^{(\tau)}) - m(h^{(\tau-1)})) \right) \right\}
\]

Through the above formulas, it is easy to prove that the operators proposed in this section are idempotent, monotonic and bounded.

4. MAGDM METHOD BASED ON PHFLPrCA OPERATOR

4.1. MAGDM Problems with Priority Relations

In a MAGDM problem with both priority and correlation, it is assumed that there are \( m \) alternatives, i.e., \( \{A_{1}, A_{2}, ..., A_{m}\} \). \( \{G\} \) DMs evaluate the alternative according to \( n \) attributes, i.e., \( \{C_{1}, C_{2}, ..., C_{n}\} \). \( n \) attributes are divided into \( q \) independent priority levels, i.e., \( H = \{H_{1}, H_{2}, ..., H_{q}\} \). Priority level \( H_{\tau} \) contains \( n_{\tau} \) attributes, i.e., \( H_{\tau} = \{C_{\tau}^{1}, C_{\tau}^{2}, ..., C_{\tau}^{n_{\tau}}\} \). Assuming that the attributes in the same priority level are interactive, the priority relationship among the \( q \) independent priority levels is \( H_{1} > H_{2} > ... > H_{q} \).

The attributes in all priority levels constitute the whole set of attributes, \( C = \bigcup_{\tau=1}^{q} H_{\tau} \). \( G \) DMs evaluated alternative \( A_{\tau} \) under attribute \( C_{\tau} \). The evaluation information was hesitant fuzzy linguistic \( \hat{h}_{\tau}^{\tau}(p) \) \( (\tau = 1, 2, ..., q; k_{\tau} = 1, 2, ..., n_{\tau}; \zeta = 1, 2, ..., m; \epsilon = 1, 2, ..., |G|) \). The evaluation information of \( |G| \) DMs could be transformed into PHFLEs \( \hat{h}_{\tau}^{\tau}(p) \).

4.2. Model of Priority Level in Generalized Prioritized Measure

In order to determine the priority weight, it is necessary to determine the value \( b_{\tau} (\tau = 1, 2, ..., q) \) in the generalized prioritized measure. Based on O’Hagan’s maximum entropy method, a mathematical programming model is established, with predefined priority attributes as constraints, and with entropy as objective function to determine \( b_{\tau} (\tau = 1, 2, ..., q) \). Entropy is measured by the degree of discreteness. The greater the degree of discreteness, the more optimistic the DM will be. Entropy is measured by the degree of discreteness. The greater the degree of discreteness, the more information will be involved in the process of information integration. Specific models are as follows:

\[
\max = \sum_{\tau=1}^{q} b_{\tau} \ln b_{\tau}
\]

s.t.

\[
\sum_{\tau=1}^{q} b_{\tau} = \Omega, \quad 0 \leq \Omega \leq 1
\]

\[
\sum_{\tau=1}^{q} b_{\tau} - 1 = 0, \quad b_{\tau} \in [0, 1]
\]

where \( \Omega \) represents the eigenvalue of preferential attitude. Generally speaking, attributes with higher priority are more important, so the value of \( \Omega \) is between 0.5 and 1. \( \sum_{\tau=1}^{q} b_{\tau} \ln b_{\tau} \) is about the dispersion of \( b_{\tau} (\tau = 1, 2, ..., q) \).

4.3. The Steps of MAGDM Method

Priority relation among attributes is a common phenomenon in decision-making problems, as well as, interaction between attributes, i.e., correlation, is another common phenomenon in decision-making process. Therefore, this section proposes a PHFLP\(_{m}\)CA operator-based decision-making method to solve the multi-attribute decision-making problem with weak ordered priority and association. The framework is shown in Figure 1. The specific steps are as follows:

Step 1: DMs evaluate the alternatives under each attribute to give HFLEs \( \hat{h}_{\tau}^{\tau}(p) \), convert HFLEs \( \hat{h}_{\tau}^{\tau}(p) \) into PHFLEs \( \hat{h}_{\tau}^{\tau}(p) \), thus forming the PHFLEs decision matrix \( Y = \left[ \hat{h}_{\tau}^{\tau}(p) \right] \).

Step 2: Use formula (4) to determine \( \lambda_{\tau} \) \((\tau = 1, 2, ..., q)\) in each priority level \( H_{\tau} \).

Step 3: Use formula (3) to determine the optimal fuzzy measure \( g_{\lambda_{\tau}}(S_{\tau}) \) of attribute set \( S_{\tau} = \{C_{\tau}^{1}, C_{\tau}^{2}, ..., C_{\tau}^{n_{\tau}}\} \) in each priority level \( H_{\tau} \).

Step 4: Use formula (16) to calculate the satisfaction \( h\tau^{\tau}(p) \) of alternative \( A_{\tau} \) under each priority level \( H_{\tau} (\tau = 1, 2, ..., q) \).
Step 5: Use formula (19) to determine \( b_\tau \) (\( \tau = 1, 2, ..., q \)) of each priority level in the generalized prioritized measure.

Step 6: Use formula (18) to calculate the overall satisfaction \( \hat{h}_\zeta (\hat{p}) \) of alternative \( A_\zeta \) (\( \zeta = 1, 2, ..., m \)).

Step 7: Use formula (1) and formula (2) to calculate the expectation \( E(\hat{h}_\zeta (\hat{p})) \) and deviation \( \sigma (\hat{h}_\zeta (\hat{p})) \) of the overall satisfaction of each alternative.

Step 8: Rank the alternatives according to the expectations and deviations.

5. CASE STUDY: TEACHING PERFORMANCE EVALUATION OF CHINESE-FOREIGN COOPERATIVE EDUCATION PROJECT OF S COLLEGE

Chinese-foreign cooperative education project is an educational model with the background of globalization. It usually refers to the educational projects that foreign educational institutions and domestic educational institutions have set up in China. Under such background S college carries out many projects with several foreign universities.

However, with the expansion of the projects’ scale and the increasing number of students enrolled, the problems of resource integration, personnel training, and scientific research within the S college has gradually emerged. In order to solve the hidden problems within the college, improve the efficiency of teachers’ performance and promote the realization of project objectives, S college has the demand that carries out a reasonable evaluation index system. According to the requirement of trinity (teaching ability, scientific research ability, and other ability), the college evaluation committee has established an evaluation index system to combine the characteristics and development goals of Chinese-foreign cooperative education project in S college, as shown in Table 1.

The focus of the evaluation is teachers’ teaching ability, so the priority of three elements is \( Teaching ability (H_1) \) \( Scientific research ability (H_2) \) \( Other abilities (H_3) \). It shows that the lack of teaching ability cannot be compensated by the increase of the corresponding scientific research ability. In addition, some correlations are displayed between the specific indicators contained in each element, so the sum of the importance of elements may not be 1. In the process of formulating the evaluation index system, the importance of each indicator is obtained from expert interviews, i.e., in the teaching ability \( H_1 \), the importance of three specific indicators are \( g_{\lambda_1}(\{C_{11}^1\}) = g_{\lambda_1}(\{C_{11}^2\}) = 2/3, g_{\lambda_1}(\{C_{11}^3\}) = 1/3 \), respectively; in the scientific research ability \( H_2 \), the importance of three specific indicators are \( g_{\lambda_2}(\{C_{21}^1\}) = g_{\lambda_2}(\{C_{21}^2\}) = 1/3, g_{\lambda_2}(\{C_{21}^3\}) = 2/3 \), respectively; in the other abilities \( H_3 \), the importance of two specific indicators are \( g_{\lambda_3}(\{C_{31}^1\}) = g_{\lambda_3}(\{C_{31}^2\}) = 2/5 \), respectively.
Table 1  Teaching performance evaluation of Chinese-foreign cooperative education project of S college.

| Serial Number | Elements                  | Specific Indicators                                      |
|---------------|---------------------------|----------------------------------------------------------|
| 1             | Teaching ability $H_1$    | Bilingual teaching ability $C_1^1$                      |
|               |                           | Developing international course ability $C_1^2$        |
|               |                           | Cooperative teaching ability $C_1^3$                    |
| 2             | Scientific researchability $H_2$ | The level of scientific research projects $C_2^1$   |
|               |                           | The level of academic papers $C_2^2$                   |
|               |                           | International research cooperation ability $C_2^3$     |
| 3             | Other abilities $H_3$     | Project management ability $C_3^1$                      |
|               |                           | Cross-cultural ability $C_3^2$                         |

Table 2  Proportional hesitant fuzzy linguistic elements (PHFLEs) decision matrix.

|       | $A_1$     | $A_2$     | $A_3$     | $A_4$     |
|-------|-----------|-----------|-----------|-----------|
| $C_1^1$ | $\{(l_3, 0.230), (l_4, 0.385)\}$, $(l_5, 0.385)$ | $\{(l_4, 0.462), (l_5, 0.538)\}$ | $\{(l_2, 0.385), (l_3, 0.538), (l_4, 0.077)\}$ | $\{(l_2, 0.250), (l_3, 0.583), (l_4, 0.417)\}$ |
| $C_1^2$ | $\{(l_3, 0.357), (l_4, 0.429)\}$, $(l_5, 0.214)$ | $\{(l_4, 0.571), (l_5, 0.429)\}$ | $\{(l_2, 0.308), (l_3, 0.538), (l_4, 0.154)\}$ | $\{(l_2, 0.214), (l_3, 0.571), (l_4, 0.214)\}$ |
| $C_1^3$ | $\{(l_3, 0.286), (l_4, 0.500)\}$, $(l_5, 0.214)$ | $\{(l_4, 0.538), (l_5, 0.462)\}$ | $\{(l_2, 0.357), (l_3, 0.571), (l_4, 0.072)\}$ | $\{(l_2, 0.333), (l_3, 0.500), (l_4, 0.167)\}$ |
| $C_2^1$ | $\{(l_3, 0.357), (l_4, 0.500)\}$, $(l_5, 0.143)$ | $\{(l_4, 0.500), (l_5, 0.500)\}$ | $\{(l_2, 0.231), (l_3, 0.538), (l_4, 0.231)\}$ | $\{(l_2, 0.308), (l_3, 0.385), (l_4, 0.308)\}$ |
| $C_2^2$ | $\{(l_3, 0.072), (l_4, 0.571)\}$, $(l_5, 0.357)$ | $\{(l_4, 0.538), (l_5, 0.462)\}$ | $\{(l_2, 0.214), (l_3, 0.572), (l_4, 0.214)\}$ | $\{(l_2, 0.333), (l_3, 0.250), (l_4, 0.417)\}$ |
| $C_2^3$ | $\{(l_3, 0.214), (l_4, 0.429)\}$, $(l_5, 0.357)$ | $\{(l_3, 0.076), (l_4, 0.462)\}$, $(l_5, 0.462)$ | $\{(l_2, 0.231), (l_3, 0.461), (l_4, 0.308)\}$ | $\{(l_2, 0.417), (l_3, 0.250), (l_4, 0.333)\}$ |
| $C_3^1$ | $\{(l_3, 0.076), (l_4, 0.462)\}$, $(l_5, 0.357)$ | $\{(l_3, 0.071), (l_4, 0.571), (l_5, 0.357)\}$ | $\{(l_2, 0.154), (l_3, 0.538), (l_4, 0.308)\}$ | $\{(l_2, 0.417), (l_3, 0.417), (l_4, 0.167)\}$ |
| $C_3^2$ | $\{(l_3, 0.286), (l_4, 0.428)\}$, $(l_5, 0.286)$ | $\{(l_4, 0.429), (l_5, 0.571)\}$ | $\{(l_2, 0.214), (l_3, 0.572), (l_4, 0.214)\}$ | $\{(l_2, 0.333), (l_3, 0.417), (l_4, 0.250)\}$ |

Evaluation team is composed of 11 experts. Experts express evaluation values in the form of linguistic words, because the evaluation values are highly uncertain and difficult to express in precise numbers. Experts may be more accustomed to using qualitative words such as “very low,” “low,” “medium,” “high,” and “very high.” Experts can choose one or more linguistic words from $S = \{s_1: \text{very low}, s_2: \text{low}, s_3: \text{moderate}, s_4: \text{high}, s_5: \text{very high}\}$ to evaluate according to their own understanding. When they know nothing about it, they can also give no evaluation value. Therefore, the evaluation results of 11 experts are summarized in Appendix (Table A1).

5.1. Decision-Making Steps

According to the MAGDM method described in the previous section, the teaching performance in S college is evaluated. The specific steps are as follows:

Step 1: The evaluation information given by experts is transformed into PHFLEs, and the PHFLE matrix is constructed, as shown in Table 2.

Step 3: Determine the $\lambda$ fuzzy measure of each attribute subset under each priority level $\lambda_\tau (\tau = 1, 2, 3)$ according to formula (3) and formula (4).

1. For priority level $H_1$, $\lambda_{A_1} \left( \{C_1^1\} \right) = \lambda_{A_1} \left( \{C_1^3\} \right) = 2/3$, $\lambda_{A_1} \left( \{C_2^3\} \right) = 1/3$, according to formula (4), $\lambda_1 = -0.879$ can be obtained.

According to formula (3), $\lambda_{A_1} \left( \{C_1^1, C_1^3\} \right) = 0.943$, $\lambda_{A_1} \left( \{C_2^2, C_3^1\} \right) = 0.805$, and $\lambda_{A_1} \left( \{C_1^1, C_1^3, C_2^2, C_3^1\} \right) = 1$ can be obtained.

2. For priority level $H_2$, $\lambda_{A_2} \left( \{C_2^2\} \right) = 2/3$, according to formula (4), $\lambda_2 = -0.658$ can be obtained.

According to formula (3), $\lambda_{A_2} \left( \{C_2^2, C_3^1\} \right) = 0.594$, $\lambda_{A_2} \left( \{C_2^2, C_3^1\} \right) = 0.854$, and $\lambda_{A_2} \left( \{C_2^2, C_3^1\} \right) = 1$ can be obtained.
3. For priority level $\mathcal{H}_3$, $g_{\lambda_3} (\{C_{13}^1\}) = g_{\lambda_3} (\{C_{13}^2\}) = 2/5$, according to formula (4), $\lambda_3 = 1.25$ can be obtained. According to formula (3), $g_{\lambda_3} (\{C_{13}^1, C_{23}^2\}) = 1$ can be obtained.

**Step 4:** Use formula (16) to calculate the satisfaction $h^r_{\zeta} (\hat{p})$ of $A_1^r (\zeta = 1, 2, 3, 4)$ under priority $C_2 (\tau = 1, 2, 3)$.

Computing the satisfaction of $A_1$ at a priority level:

$$h^1_{11} (\hat{p}) = \left( g_{\lambda_1} \left( \{C_{11}^1\} \right) - g_{\lambda_1} (\emptyset) \right) h^1_{13} (\hat{p})$$

$$\bigoplus g_{\lambda_1} \left( \{C_{21}^1, C_{31}^2\} \right) - g_{\lambda_1} (\{C_{31}^1\}) )$$

$$h^1_{12} (\hat{p}) \bigoplus g_{\lambda_1} \left( \{C_{11}^1, C_{21}^2, C_{31}^3\} \right) - g_{\lambda_1} (\{C_{11}^1, C_{21}^2\}) )$$

$$h^1_{11} (\hat{p}) = \{(l_1, 0), (l_2, 0), (l_3, 0.263), (l_4, 0.409), (l_5, 0.328)\}$$

$$h^2_{11} (\hat{p}) = (g_{\lambda_2} (\{C_{22}^2\}) - g_{\lambda_2} (\emptyset)) h^2_{12} (\hat{p})$$

$$\bigoplus g_{\lambda_2} \left( \{C_{21}^2, C_{31}^3\} \right) - g_{\lambda_2} (\{C_{21}^2\}) )$$

$$h^2_{13} (\hat{p}) \bigoplus g_{\lambda_2} \left( \{C_{11}^1, C_{21}^2, C_{31}^3\} \right) - g_{\lambda_2} (\{C_{11}^1, C_{21}^2\}) )$$

$$h^2_{11} (\hat{p}) = \{(l_1, 0), (l_2, 0), (l_3, 0.147), (l_4, 0.538), (l_5, 0.315)\}$$

$$h^3_{11} (\hat{p}) = \left( g_{\lambda_3} \left( \{C_{23}^1\} \right) - g_{\lambda_3} (\emptyset) \right) h^3_{11} (\hat{p})$$

$$\bigoplus g_{\lambda_3} \left( \{C_{11}^1, C_{21}^2\} \right) - g_{\lambda_3} (\{C_{21}^2\}) )$$

$$h^3_{11} (\hat{p}) = \{(l_1, 0), (l_2, 0), (l_3, 0.130), (l_4, 0.426), (l_5, 0.387)\}$$

In the same way, the satisfaction of $A_2, A_3, A_4$ at each priority level can be obtained as follows:

$$h^1_{12} (\hat{p}) = \{(l_1, 0), (l_2, 0), (l_3, 0), (l_4, 0.539), (l_5, 0.461)\};$$

$$h^2_{12} (\hat{p}) = \{(l_1, 0), (l_2, 0), (l_3, 0.004), (l_4, 0.508), (l_5, 0.487)\};$$

$$h^3_{12} (\hat{p}) = \{(l_1, 0), (l_2, 0.048), (l_3, 0.499), (l_5, 0.396)\};$$

$$h^1_{13} (\hat{p}) = \{(l_1, 0), (l_2, 0.329), (l_3, 0.543), (l_4, 0.127), (l_5, 0)\};$$

$$h^2_{13} (\hat{p}) = \{(l_1, 0), (l_2, 0.228), (l_3, 0.519), (l_4, 0.253), (l_5, 0)\};$$

$$h^3_{13} (\hat{p}) = \{(l_1, 0), (l_2, 0.162), (l_3, 0.517), (l_4, 0.264), (l_5, 0)\};$$

$$h^1_{14} (\hat{p}) = \{(l_1, 0), (l_2, 296), (l_3, 0.524), (l_4, 0.180), (l_5, 0)\};$$

$$h^2_{14} (\hat{p}) = \{(l_1, 0), (l_2, 0.387), (l_3, 0.258), (l_4, 0.355), (l_5, 0)\};$$

$$h^3_{14} (\hat{p}) = \{(l_1, 0), (l_2, 0.370), (l_3, 0.393), (l_4, 0.180), (l_5, 0)\}.$$

**Step 5:** Use formula (19) to calculate the values $b_\tau (\tau = 1, 2, 3)$ in the generalized prioritized measure.

$$\max \{-b_1 \ln b_1 - b_2 \ln b_2 - b_3 \ln b_3\}$$

$$\begin{cases} b_1 + b_2 + b_3 = 0.75 \\
 b_1, b_2, b_3 \geq 0 \end{cases}$$

$$b_1 = 0.612, b_2 = 0.276, b_3 = 0.112.$$

**Step 6:** Use Formula (18) to calculate the overall satisfaction of each alternative at all priority levels.

$$h_{11} (\hat{p}) = \{(l_1, 0), (l_2, 0), (l_3, 0.227), (l_4, 0.435), (l_5, 0.332)\};$$

$$h_{12} (\hat{p}) = \{(l_1, 0), (l_2, 0), (l_3, 0.008), (l_4, 0.513), (l_5, 0.472)\};$$

$$h_{13} (\hat{p}) = \{(l_1, 0), (l_2, 0.301), (l_3, 0.537), (l_4, 0.158), (l_5, 0)\};$$

$$h_{14} (\hat{p}) = \{(l_1, 0), (l_2, 0.321), (l_3, 0.461), (l_4, 0.212), (l_5, 0)\}. $$

**Step 7:** Calculating expectation $E \left( h_{\zeta} (\hat{p}) \right) (\zeta = 1, 2, 3, 4)$

$$E \left( h_{11} (\hat{p}) \right) = 4.079, E \left( h_{12} (\hat{p}) \right) = 4.437, E \left( h_{13} (\hat{p}) \right) = 2.847, E \left( h_{14} (\hat{p}) \right) = 2.871.$$

**Step 8:** Rank the alternatives according to the expectation value.

$$E \left( h_{12} (\hat{p}) \right) > E \left( h_{11} (\hat{p}) \right) > E \left( h_{14} (\hat{p}) \right) > E \left( h_{13} (\hat{p}) \right), \text{so } A_2 > A_1 > A_4 > A_3.$$ That is the performance of the four teachers in S College, A2 is the best and A3 is the worst.

### 5.2. The Impact of Prioritized Attitudinal Character on Decision-Making Results

The influence of prioritized attitudinal character $\Omega$ on priority decision results is further analyzed. Firstly, O’Hagan’s maximum entropy method is used to calculate the corresponding $b_\tau (\tau = 1, 2, 3)$ based on different $\Omega$ values. As can be seen from Table 3, $\Omega$ can be used to adjust the priority of attributes. As far as $\Omega \in \{0.6, 0.7, 0.75, 0.8, 0.9\}$ is concerned, it can be found from Table 3 and Figure 2 that the $b_\tau (\tau = 1, 2, 3)$ value forms a nonsubtractive sequence, i.e., $b_1 \geq b_2 \geq b_3$. Then, the PHFLP,CA operator based on the corresponding $b_\tau (\tau = 1, 2, 3)$ value is used to rank alternatives. From the ranking results listed in Table 3 and Figure 3, no matter how $\Omega$ changes, $A_2$ is always the best solution. The prioritized attitudinal character $\Omega$ can be used to describe the DM’s psychology. The greater the value of $\Omega$ is, the more optimistic the DM is. On the contrary, the smaller the $\Omega$ is, the more pessimistic the DM is. Therefore, DMs can choose an appropriate $\Omega$ values according to their own preference and practical problems.
Table 3  Decision-making results corresponding to prioritized attitudinal character.

| Ω  | \( b_\tau (\tau=1,2,3) \) | \( E\left(\tilde{h}_\tau (\tilde{p})\right) \) | Ranking |
|----|---------------------|---------------------------------|---------|
| 0.6 | \( b_1 = 0.440, b_2 = 0.320, b_3 = 0.240 \) | \( E\left(\tilde{h}_1 (\tilde{p})\right) = 4.077, E\left(\tilde{h}_2 (\tilde{p})\right) = 4.390, E\left(\tilde{h}_3 (\tilde{p})\right) = 2.862, E\left(\tilde{h}_4 (\tilde{p})\right) = 2.842. \) | \( A_2 \succ A_1 \succ A_3 \succ A_4 \) |
| 0.7 | \( b_1 = 0.550, b_2 = 0.300, b_3 = 0.150 \) | \( E\left(\tilde{h}_1 (\tilde{p})\right) = 4.080, E\left(\tilde{h}_2 (\tilde{p})\right) = 4.425, E\left(\tilde{h}_3 (\tilde{p})\right) = 2.854, E\left(\tilde{h}_4 (\tilde{p})\right) = 2.865. \) | \( A_2 \succ A_1 \succ A_4 \succ A_3 \) |
| 0.75 | \( b_1 = 0.612, b_2 = 0.276, b_3 = 0.112 \) | \( E\left(\tilde{h}_1 (\tilde{p})\right) = 4.079, E\left(\tilde{h}_2 (\tilde{p})\right) = 4.437 \) | \( A_2 \succ A_1 \succ A_4 \succ A_3 \) |
| 0.8 | \( b_1 = 0.684, b_2 = 0.232, b_3 = 0.084 \) | \( E\left(\tilde{h}_1 (\tilde{p})\right) = 4.077, E\left(\tilde{h}_2 (\tilde{p})\right) = 4.448 \) | \( A_2 \succ A_1 \succ A_4 \succ A_3 \) |
| 0.9 | \( b_1 = 0.846, b_2 = 0.108, b_3 = 0.046 \) | \( E\left(\tilde{h}_1 (\tilde{p})\right) = 4.070, E\left(\tilde{h}_2 (\tilde{p})\right) = 4.464 \) | \( A_2 \succ A_1 \succ A_4 \succ A_3 \) |

Figure 2  Trend chart of changes.

Figure 3  Trend chart of expectation value changes relative to value changes.
5.3. Validity Test of the Method

This subsection focuses on the reliability and validity of the created method by the test criteria [38]. This is because different MAGDM method may lead to different ranking results for the same decision-making problem.

Test criterion 1. Under the condition that attributes’ weights keep unchanged, the optimal alternative still maintain its first place when any nonoptimal alternative are substituted by another non-optimal alternative. Thus, if the result meets the above conditions, this MAGDM method is effective.

Among four alternatives, A3 is a nonoptimal alternative. C3 can be taken place by C1 = {(l2, 0.385), (l3, 0.538), (l4, 0.077)}, C2 = {(l2, 0.357), (l3, 0.643)}, by using test criterion 1. The overall satisfaction values of each alternative are obtained:

\[ \hat{h}_1(\hat{p}) = \{(l_1, 0), (l_2, 0), (l_3, 0.227), (l_4, 0.435), (l_5, 0.332)\} ; \]
\[ \hat{h}_2(\hat{p}) = \{(l_1, 0), (l_2, 0), (l_3, 0.008), (l_4, 0.513), (l_5, 0.472)\} ; \]
\[ \hat{h}_3(\hat{p}) = \{(l_1, 0), (l_2, 0.329), (l_3, 0.534), (l_4, 0.097), (l_5, 0)\} ; \]
\[ \hat{h}_4(\hat{p}) = \{(l_1, 0), (l_2, 0.321), (l_3, 0.461), (l_4, 0.212), (l_5, 0)\} . \]

The expectations of the alternatives are

\[ E(\hat{h}_1(\hat{p})) = 4.079, E(\hat{h}_2(\hat{p})) = 4.437, E(\hat{h}_3(\hat{p})) = 2.647, \]
\[ E(\hat{h}_4(\hat{p})) = 2.871. \]

Thus, according to the ranking results A2 > A1 > A4 > A3, A2 is still the best teacher in S College. Therefore, the created MAGDM method is verified effectively by test criterion 1.

Test criterion 2. An effective MAGDM method should be proved transitive property, which will be demonstrated in test criterion 3.

Test criterion 3. If the same method is used to solve sub-problems, which are divided from the original MAGDM problem, the ranking result should be the same as the original MAGDM problem.

According to criteria 2 and 3 the initial MAGDM problem is divided to two sub-problems, \{A1, A2, A4\} and \{A2, A3, A4\}. These two sub-problems are solved by the created MAGDM method, so the ranking results of them are A2 > A1 > A4 and A2 > A4 > A3. The integration of the above ranking results is A2 > A1 > A4 > A3, that is the same ranking result of original MAGDM problem, which proved transitive property. Therefore, the created MAGDM method is verified effectively by test criterion 2 and 3.

5.4. Advantages of the Method

To further illustrate the advantages of the proposed method, the PHFLP, CA operator-based MAGDM method proposed in this paper is compared with other two decision-making methods [12], which are PHFLWA operator and PHFLOWA operator. The two methods mentioned above are used to rank the alternatives of the cases in this paper. The ranking results are shown in the following table:

Compared with the other two methods, this method has the following advantages:

1. The created method is flexible. By changing the \( \Omega \) value, the result of decision-making problem can be solved more flexibly. When \( \Omega = 0.6 \), the ranking results of the proposed method are slight differences with those of other two methods (shown in Table 4); when \( \Omega = 0.8 \), the ranking results of the proposed method are consistent with them. This shows that the proposed method is more flexible than the other two methods in dealing with decision-making problems.

2. The created method can solve the decision-making problems with both priority and correlation. The existing two methods in this section have the same shortcomings, i.e., they do not consider the relationship between attributes, and cannot deal with the priority relationship between different levels. However, according to the actual situation, this paper can solve the decision-making problems of priority and correlation (complementary relationship, redundant relationship, independent relationship) among attributes, whereas the above two methods are not suitable for dealing with such problems.

3. The created method owns a simple calculation process. Both methods 1 and 2 aggregate indicators by means of two times. If there are more than two indicators involved, they have to aggregate them one by one, which results in a complicated calculation process. On the contrary, the created method can aggregate indicators by means of only one time, no matter how many indicators are. Fewer steps make the created method simpler than methods 1 and 2.

In this section, seven steps are applied in teaching performance evaluation to obtain the best alternative. And the best alternative won’t change by adjusting \( \Omega \), thus DMs can choose an appropriate \( \Omega \) values according to their own preference and practical problems. Moreover, this method passed validity test and is proved to own many advantages by compared with other two methods.

6. CONCLUSION

In order to solve teaching performance evaluation of Chinese-foreign cooperative education project, this paper mainly studies the PHFLE information aggregation operator with priority and correlation function and its application. Firstly, the PHFLECA operator is proposed for MAGDM problems with correlations among attributes (redundancy, complementarity, and independence). Secondly, the PHFLP, CA operator is proposed for the case that attributes have both priority and correlation. In this paper, the properties of correlation operators are briefly introduced while corresponding operators are proposed. Thirdly, based on the PHFLP, CA operator, a new MAGDM method is proposed. In order to verify the effectiveness of the proposed method, this paper chooses the typical MAGDM problem on teaching performance evaluation of Chinese-foreign cooperative education project in S College, which has priority and correlation among its attributes. Then this paper illustrates the role of the proposed method in solving the problem. Finally, the advantages of this method are illustrated by comparing with other two methods.
The limitations of this paper are shown in following two aspects, which will be optimized in our future research. The first limitation is the LTS supposed in balanced and symmetric environment, however, in real life the LTS is unbalanced and asymmetric sometimes. The second limitation is that the DMs adopt the same LTS, however, DMs often choose different LTSs by their experience and preference. In the future PHFLE will be applied into unbalanced and multi-granular environment.

CONFLICTS OF INTEREST

The authors declare no conflicts of interest.

AUTHORS’ CONTRIBUTIONS

Conceptualization, Lei Wang and Peide Liu; methodology, Lili Rong and Fei Teng; formal analysis, Lei Wang and Lili Rong; writing—original draft preparation, Lei Wang and Fei Teng; writing—review and editing, Lei Wang and Peide Liu; supervision, Peide Liu; funding acquisition, Lei Wang, Lili Rong, Fei Teng and Peide Liu.

ACKNOWLEDGMENTS

This work is supported by the National Natural Science Foundation of China (Nos. 71471172 and 71801142), the Special Funds of Taishan Scholars Project of Shandong Province (No. ts201511045), the Natural Science planning project of Shandong Province (No. 20CSDJ23), the Scientific research project of Shandong TV University (Nos. 2019KQ002 and 2020XY001Z), Key projects of Art Science in Shandong Province (Nos. ZD202008366 and ZD202008492).

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Table 4  Ranking results of different methods.

| Methods               | Ranking Values (R) | Ranking Results          |
|-----------------------|--------------------|--------------------------|
| Method 1 (PHFLEWA)    | R (A1) = 2.568, R (A2) = 2.944, R (A3) = 1.084, R (A4) = 1.404. | A2 ≻ A1 ≻ A4 ≻ A3       |
| Method 2 (PHFLOWA)    | R (A1) = 2.751, R (A2) = 3.234, R (A3) = 1.006, R (A4) = 1.009. | A2 ≻ A1 ≻ A4 ≻ A3       |
| Our method            | Ω = 0.6            | A2 ≻ A1 ≻ A4 ≻ A3       |
|                       | Ω = 0.8            | A2 ≻ A1 ≻ A4 ≻ A3       |


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## APPENDIX

Table A1 | Information on performance evaluation of four teachers in S college.

|       | A1     | A2     | A3     | A4     |
|-------|--------|--------|--------|--------|
| C1    | \{1\}, \{1\}, \{1\}, \{1\}, \{1\}, \{1\} | \{1\}, \{1\}, \{1\}, \{1\}, \{1\}, \{1\} | \{1\}, \{1\}, \{1\}, \{1\}, \{1\}, \{1\} | \{1\}, \{1\}, \{1\}, \{1\}, \{1\}, \{1\} |
|       | \{1\}, \{1\} | \{1\}, \{1\}, \{1\}, \{1\}, \{1\}, \{1\} | \{1\}, \{1\}, \{1\}, \{1\}, \{1\}, \{1\} | \{1\}, \{1\}, \{1\}, \{1\}, \{1\}, \{1\} |
| C2    | \{1\}, \{1\}, \{1\}, \{1\}, \{1\}, \{1\} | \{1\}, \{1\}, \{1\}, \{1\}, \{1\}, \{1\} | \{1\}, \{1\}, \{1\}, \{1\}, \{1\}, \{1\} | \{1\}, \{1\}, \{1\}, \{1\}, \{1\}, \{1\} |
|       | \{1\}, \{1\}, \{1\}, \{1\}, \{1\}, \{1\} | \{1\}, \{1\}, \{1\}, \{1\}, \{1\}, \{1\} | \{1\}, \{1\}, \{1\}, \{1\}, \{1\}, \{1\} | \{1\}, \{1\}, \{1\}, \{1\}, \{1\}, \{1\} |
| C3    | \{1\}, \{1\}, \{1\}, \{1\}, \{1\}, \{1\} | \{1\}, \{1\}, \{1\}, \{1\}, \{1\}, \{1\} | \{1\}, \{1\}, \{1\}, \{1\}, \{1\}, \{1\} | \{1\}, \{1\}, \{1\}, \{1\}, \{1\}, \{1\} |
|       | \{1\}, \{1\}, \{1\}, \{1\}, \{1\}, \{1\} | \{1\}, \{1\}, \{1\}, \{1\}, \{1\}, \{1\} | \{1\}, \{1\}, \{1\}, \{1\}, \{1\}, \{1\} | \{1\}, \{1\}, \{1\}, \{1\}, \{1\}, \{1\} |
| C4    | \{1\}, \{1\}, \{1\}, \{1\}, \{1\}, \{1\} | \{1\}, \{1\}, \{1\}, \{1\}, \{1\}, \{1\} | \{1\}, \{1\}, \{1\}, \{1\}, \{1\}, \{1\} | \{1\}, \{1\}, \{1\}, \{1\}, \{1\}, \{1\} |
|       | \{1\}, \{1\}, \{1\}, \{1\}, \{1\}, \{1\} | \{1\}, \{1\}, \{1\}, \{1\}, \{1\}, \{1\} | \{1\}, \{1\}, \{1\}, \{1\}, \{1\}, \{1\} | \{1\}, \{1\}, \{1\}, \{1\}, \{1\}, \{1\} |
| C5    | \{1\}, \{1\}, \{1\}, \{1\}, \{1\}, \{1\} | \{1\}, \{1\}, \{1\}, \{1\}, \{1\}, \{1\} | \{1\}, \{1\}, \{1\}, \{1\}, \{1\}, \{1\} | \{1\}, \{1\}, \{1\}, \{1\}, \{1\}, \{1\} |
|       | \{1\}, \{1\}, \{1\}, \{1\}, \{1\}, \{1\} | \{1\}, \{1\}, \{1\}, \{1\}, \{1\}, \{1\} | \{1\}, \{1\}, \{1\}, \{1\}, \{1\}, \{1\} | \{1\}, \{1\}, \{1\}, \{1\}, \{1\}, \{1\} |
| C6    | \{1\}, \{1\}, \{1\}, \{1\}, \{1\}, \{1\} | \{1\}, \{1\}, \{1\}, \{1\}, \{1\}, \{1\} | \{1\}, \{1\}, \{1\}, \{1\}, \{1\}, \{1\} | \{1\}, \{1\}, \{1\}, \{1\}, \{1\}, \{1\} |
|       | \{1\}, \{1\}, \{1\}, \{1\}, \{1\}, \{1\} | \{1\}, \{1\}, \{1\}, \{1\}, \{1\}, \{1\} | \{1\}, \{1\}, \{1\}, \{1\}, \{1\}, \{1\} | \{1\}, \{1\}, \{1\}, \{1\}, \{1\}, \{1\} |