Manifestation of the odd-frequency spin-triplet pairing state in diffusive ferromagnet/superconductor junctions

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Using the quasiclassical Green’s function formalism, we study the influence of the odd-frequency spin-triplet superconductivity on the local density of states (LDOS) in a diffusive ferromagnet (DF) attached to a superconductor. Various possible symmetry classes in a superconductor are considered which are consistent with the Pauli’s principle: even-frequency spin-singlet even-parity (ESE) state, even-frequency spin-triplet odd-parity (ETO) state, odd-frequency spin-triplet even-parity (OTE) state and odd-frequency spin-singlet odd-parity (OSO) state. For each of these states, the pairing state in DF is studied. Particular attention is paid to the study of spin-singlet s-wave and spin-triplet p-wave superconductors as the examples of ESE and ETO superconductors. For spin-singlet case the magnitude of the OTE component of the pair amplitude is enhanced with the increase of the exchange field in DF. When the OTE component is dominant at low energy, the resulting LDOS in DF has a zero energy peak (ZEP). On the other hand, in DF/ spin-triplet p-wave superconductor junctions LDOS has a ZEP in the absence of the exchange field, where only the OTE paring state exists. With the increase of the exchange field, the ESE component of the pair amplitude induced in DF is enhanced. Then, the resulting LDOS has a ZEP splitting. We demonstrate that the appearance of the dominant OTE component of the pair amplitude is the physical reason of the emergence of the ZEP of LDOS.

I. INTRODUCTION

Ferromagnet/superconductor structures with conventional spin-singlet s-wave superconductors have been the subject of extensive work during the past decade. An exciting manifestation of anomalous proximity effect in these structures is the existence of the so-called π-junctions in SFS Josephson junctions confirmed experimentally. Recently, diffusive ferromagnet/superconductor (DF/S) junctions have received much attention due to the possibility of generation of the odd-frequency pairing in these structures. In DF, due to the isotropization by the impurity scattering, only even-parity s-wave pairing is allowed. Besides this, the exchange field breaks the time reversal symmetry and both spin-singlet and spin-triplet Cooper pairs can coexist. In accordance with the Pauli’s principle, this spin-triplet state belongs to the odd-frequency spin-triplet even-parity (OTE) pairing. Various aspects of this state have been addressed in recent theoretical works and first experimental observation of the long-range proximity effect due to the odd-frequency pairing was reported.

Odd-frequency pairing is an unique state which was first proposed by Berezinskii as a hypothetical state of 3He. The odd-frequency superconductivity was then discussed in the context of various pairing mechanisms involving strong correlations. However, proximity effect in the presence of odd-frequency superconducting state has not been studied up to very recently.

A general theory of the proximity effect in junctions composed of diffusive normal metal (DN) and unconventional superconductor in the framework of the quasiclassical Green’s function formalism was recently presented by two of the present authors. Various possible symmetry classes in a superconductor were considered which are consistent with the Pauli’s principle: even-frequency spin-singlet even-parity (ESE) state, even-frequency spin-triplet odd-parity (ETO) state, odd-frequency spin-triplet even-parity (OTE) state and odd-frequency spin-singlet odd-parity (OSO) state. For each of the above four cases, symmetry and spectral properties of the induced pair amplitude in the DN were determined. It was shown that the pair amplitude in a DN belongs respectively to ESE, OTE, OTE and ESE pairing states.

It is remarkable that OTE state is realized without assuming magnetic ordering in DN/ETO superconductor junctions, where the mid gap Andreev resonant state formed at the interface penetrates into the DN and the resulting local density of states (LDOS) has a zero energy peak (ZEP).

On the other hand, the existence of ZEP in LDOS in the DF/ ESE s-wave superconductor junctions has been established. Although the conditions of the formation of ZEP in DF regions were formulated by the present authors, possible relation between the ZEP and the formation of OTE paring in DF has not been yet clarified. The present paper addresses this issue. We also study the proximity effect in DF/ETO p-wave superconductor junctions. It was shown in the previous papers that only the OTE pairing state is generated without exchange field h. It is an interesting question how this unusual proximity effect is influenced by the exchange field.

The organization of the present paper is as follows. In section II, we formulate the proximity effect model.
in DF / S junctions within the theory applicable to unconventional superconductor junctions where the MARS are naturally taken into account in the boundary condition for the quasiclassical Green’s function. We discuss the general properties of the proximity effect by choosing ESE, ETO, OTE, and OSO superconductor junctions. It is clarified that the OTE, ESE, ESE and OTE states are, respectively, generated in the DF in the presence of exchange field $h$. In section III we calculate the pair amplitude in DF for spin-singlet $s$-wave and spin-triplet $p$-wave superconductor junctions as an example of ESE and ETO superconductor junctions. For $s$-wave junctions, it is revealed that a generation of the OTE pairing state by the exchange field $h$ causes an enhancement of the zero energy LDOS in the DF. On the other hand, for $p$-wave superconductor junctions, a generation of ESE pairing state by $h$ results in a splitting of ZEP of LDOS. We clarify the relation between the ZEP in LDOS and the function $\tilde{f}^{R,A}$. The results is given in section IV.

II. FORMULATION

Let us start with the formulation of the general symmetry properties of the quasiclassical Green’s function in the considered system following the discussion in the Ref. 24. The elements of retarded and advanced Nambu matrices $\tilde{g}^{R,A}$

\[ \tilde{g}^{R,A} = \begin{pmatrix} g^{R,A} & f^{R,A} \\ \overline{f}^{R,A} & \overline{g}^{R,A} \end{pmatrix} \]

are composed of the normal $g^{R,A}_{\alpha,\beta}(r,\varepsilon,p)$ and anomalous $f^{R,A}_{\alpha,\beta}(r,\varepsilon,p)$ components with spin indices $\alpha$ and $\beta$. Here $p = p_F / |p_F|$, $p_F$ is the Fermi momentum, $r$ and $\varepsilon$ denote coordinate and energy of a quasiparticle measured from the Fermi level respectively. The function $f^R$ and the conjugated function $f^R$ satisfy the following relations 34,35

\[ \tilde{f}^{R}_{\alpha,\beta}(r,\varepsilon,p) = -[f^{R}_{\alpha,\beta}(r,-\varepsilon,-p)]^*. \]

(2)

The Pauli’s principle is formulated in terms of the retarded and the advanced Green’s functions in the following way 34

\[ f^{A}_{\alpha,\beta}(r,\varepsilon,p) = -f^{R}_{\beta,\alpha}(r,-\varepsilon,-p). \]

(3)

By combining the above two equations, we obtain $f^{R}_{\beta,\alpha}(r,\varepsilon,p) = [f^{A}_{\alpha,\beta}(r,\varepsilon,p)]^*$. Further, the definitions of the even-frequency and the odd-frequency pairing are $f^{A}_{\alpha,\beta}(r,\varepsilon,p) = f^{R}_{\alpha,\beta}(r,-\varepsilon,p)$ and $f^{A}_{\alpha,\beta}(r,\varepsilon,p) = -f^{R}_{\alpha,\beta}(r,-\varepsilon,p)$, respectively. Finally we get

\[ \tilde{f}^{R}_{\beta,\alpha}(r,\varepsilon,p) = [f^{R}_{\alpha,\beta}(r,-\varepsilon,p)]^* \]

(4)

for the even-frequency pairing and

\[ \tilde{f}^{R}_{\beta,\alpha}(r,\varepsilon,p) = -[f^{R}_{\alpha,\beta}(r,-\varepsilon,p)]^* \]

(5)

for the odd-frequency pairing. In the following, we consider a homogeneous ferromagnet/superconductor junctions with the exchange field $h$ in a ferromagnet and focus on the Cooper pairs with $S_z = 0$. In this case, it is possible to remove the external phase of the pair potential in the superconductor. We will concentrate on the retarded part of the Green’s function.

We consider a junction consisting of a normal (N) and a superconducting reservoirs connected by a quasi-one-dimensional diffusive ferromagnet (DF) with a length $L$ much larger than the mean free path as shown in Fig. 1.

![Fig. 1: (color online) Schematic illustration of DF/S junctions where DF is connected to normal reservoirs. (a)conventional spin-singlet s-wave superconductor and (b)spin-triplet p-wave superconductor junctions.](image)

The interface between the DF and the superconductor (S) at $x = L$ has a resistance $R_b$ and the N/DF interface at $x = 0$ has a resistance $R'_b$. The Green’s function in the superconductor can be parameterized as $g_{\pm}(\varepsilon)\tilde{F}_3 + f_{\pm}(\varepsilon)\tilde{F}_2$ using Pauli’s matrices, where the subscript $+$($-$) denotes the right (left) going quasiparticles. $g_{\pm}(\varepsilon)$ and $f_{\pm}(\varepsilon)$ are given by $g_{\pm}(\varepsilon) = g_{\pm}^{R}(r,\varepsilon,p) = g_{\pm}^{\overline{R}}(r,\varepsilon,p)$, $g_{-}(\varepsilon) = g_{-}^{R}(r,\varepsilon,p) = g_{-}^{\overline{R}}(r,\varepsilon,p)$, $f_{\pm}(\varepsilon) = f_{\pm}^{R}(r,\varepsilon,p)$, and $f_{-}(\varepsilon) = f_{-}^{R}(r,\varepsilon,p)$, respectively, with $\tilde{p} = p_F / |p_F|$ and $\tilde{p}_F = (-p_{Fx},p_{Fy})$. Using the relations (4) and (5), we obtain that $f_{\pm}(\varepsilon) = [f_{\pm}(-\varepsilon)]^*$ for the even-frequency pairing and $f_{\pm}(\varepsilon) = -[f_{\pm}(-\varepsilon)]^*$ for the odd-frequency pairing, respectively, while $g_{\pm}(\varepsilon) = [g_{\pm}(-\varepsilon)]^*$ in both cases.

In the DF region, only the $s$-wave even-parity pairing state is allowed due to isotropization by impurity scattering. The resulting Green’s function with majority and minority spin in the DF can be parameterized by $\cos \theta \tilde{F}_3 + \sin \theta \tilde{F}_2$ and $\cos \theta \tilde{F}_3 - \sin \theta \tilde{F}_2$ in a junction with
an even-parity superconductor respectively. On the other hand, for odd-parity superconductor, the corresponding quantities for majority spin and minority spin are expressed by \( \cos \theta \tau_3 + \sin \theta \tau_1 \) and \( \cos \bar{\theta} \tau_3 + \sin \bar{\theta} \tau_1 \) respectively.

The function \( \theta \) satisfies the Usadel equation

\[
D \frac{\partial^2 \theta}{\partial x^2} + 2i(\varepsilon + h) \sin \theta = 0
\]

with the boundary conditions at the DF/S interface

\[
\frac{L}{R_d} \left( \frac{\partial \theta}{\partial x} \right) |_{x=L} = \frac{\langle F_1 \rangle}{R_b},
\]

and at the N/DF interface

\[
\frac{L}{R_d} \left( \frac{\partial \theta}{\partial x} \right) |_{x=0} = -\frac{\langle F_2 \rangle}{R_b},
\]

\[
F_1 = \frac{2T_1(f_s \cos \theta_L - g_s \sin \theta_L)}{2 - T_1 + T_1(\cos \theta_L g_S + \sin \theta_L f_S)}
\]

\[
F_2 = \frac{2T_2 \sin \theta_0}{2 - T_2 + T_2 \cos \theta_0},
\]

respectively, with \( \theta_L = \theta |_{x=L} \) and \( \theta_0 = \theta |_{x=0} \). Here, \( R_d \) and \( D \) are the resistance and the diffusion constant in the DF, respectively. Function \( g_S \) is given by \( g_S = (g_+ + g_-)/(1 + g_+ g_- + f_+ f_-) \) and \( f_S = (f_+ + f_-)/(1 + g_+ g_- + f_+ f_-) \) for the even-parity pairing and \( f_S = (f_+ + f_-)/(1 + g_+ g_- + f_+ f_-) \) for the odd-parity pairing, respectively, with \( g_\pm = \varepsilon / \sqrt{\varepsilon^2 - \Delta_\pm^2} \)

\( \Delta_\pm = \sqrt{\Delta_\pm^2 - \varepsilon^2} \). 

The brackets \( \langle \ldots \rangle \) denote averaging over the injection angle \( \phi \):

\[
\langle F_{1(2)}(\phi) \rangle = \frac{1}{\pi/2} \int_{-\pi/2}^{\pi/2} d\phi \cos \phi F_{1(2)}(\phi) / \int_{-\pi/2}^{\pi/2} d\phi T_{1(2)}(\phi) \cos \phi
\]

\[
T_1 = \frac{4 \cos^2 \phi}{Z^2 + 4 \cos^2 \phi}, \quad T_2 = \frac{4 \cos^2 \phi}{Z^2 + 4 \cos^2 \phi},
\]

where \( T_{1,2} \) are the transmission probabilities, \( Z \) and \( Z' \) are the barrier parameters for two interfaces.

The resistance at the interface \( R_b^{(i)} \) is given by

\[
R_b^{(i)} = \frac{2R_0^{(i)}}{\int_{-\pi/2}^{\pi/2} d\phi T_{1(2)}(\phi) \cos \phi}
\]

Here, \( R_b^{(i)} \) denotes \( R_0 \) or \( R_b' \), and \( R_0^{(i)} \) is Sharvin resistance, which in three-dimensional case is given by \( R_0^{(i)} = 4\pi^2/((\varepsilon^2 k_F^2 s_c^{(i)})^2) \), where \( k_F \) is the Fermi wavevector and \( s_c^{(i)} \) is the constriction area.

Next, we focus on the Green’s function of minority spin. The function \( \bar{\theta} \) satisfies the following equation

\[
D \frac{\partial^2 \bar{\theta}}{\partial x^2} + 2i(\varepsilon - h) \sin \bar{\theta} = 0
\]

with the boundary condition at the DF/S interface

\[
\frac{L}{R_d} \left( \frac{\partial \bar{\theta}}{\partial x} \right) |_{x=L} = \frac{\langle \bar{F}_1 \rangle}{R_b}.
\]

Here, \( \bar{F}_1 \) is given by

\[
\bar{F}_1 = \frac{2T_1(f_s \cos \theta_L - g_s \sin \bar{\theta}_L)}{2 - T_1 + T_1(\cos \theta_L g_S + \sin \theta_L f_S)}
\]

for spin-triplet superconductor and

\[
\bar{F}_1 = \frac{2T_1(-f_s \cos \theta_L - g_s \sin \bar{\theta}_L)}{2 - T_1 + T_1(\cos \theta_L g_S - \sin \theta_L f_S)}
\]

for spin-singlet superconductor respectively. At the N/DF interface, the boundary condition reads

\[
\frac{L}{R_d} \left( \frac{\partial \bar{\theta}}{\partial x} \right) |_{x=0} = -\frac{\langle \bar{F}_2 \rangle}{R_b}, \quad \bar{F}_2 = \frac{2T_2 \sin \bar{\theta}_0}{2 - T_2 + T_2 \cos \bar{\theta}_0}.
\]

The pair amplitude is defined as

\[
f_3(\varepsilon) = (\sin \theta - \sin \bar{\theta})/2
\]

in the spin-singlet case and as

\[
f_0(\varepsilon) = (\sin \theta + \sin \bar{\theta})/2
\]

in the spin-triplet case.

Since only an even-parity \( s \)-wave pairing can exist in the DF due to the impurity scattering, \( f_3 \) and \( f_0 \) belong to the ESE and OTE state, respectively.

In the following, we will consider four possible symmetry classes of superconductivity in the junction, consistent with the Pauli’s principle: ESE, ETO, OTE and OSO pairing states.

(1) Junction with ESE superconductor
Even-frequency
Odd-frequency
OTE +
Odd-frequency

Similar to the case of ETO junctions, OTE pairing is induced in the presence of DF. Similar to the case of ETO junctions, ESE pairing is obtained by the fact that these equations are consistent with each other only when \( \sin \theta (\varepsilon) = -\sin \theta (\varepsilon) \) and \( \cos \theta (\varepsilon) = -\cos \theta (\varepsilon) \). After simple calculation, we can show that \( f_3(\varepsilon) = f_3^* (-\varepsilon) \) and \( f_0(\varepsilon) = -f_0^* (-\varepsilon) \). This relation is consistent with the fact that \( f_3 \) and \( f_0 \) are the even-frequency and odd-frequency pairing state, respectively. When \( h=0 \), since \( \sin \theta (\varepsilon) = -\sin \theta (\varepsilon) \) is satisfied, the resulting \( f_0 \) is vanishing and only the ESE state exist. For \( h \neq 0 \), \( f_0 \) becomes nonzero and the OTE state is generated in DF.

(2) Junction with ETO superconductor

Now we have \( f_{\pm}(\varepsilon) = f_{\pm}^* (-\varepsilon) \) and \( g_{\pm}(\varepsilon) = g_{\pm}^* (-\varepsilon) \). Then, \( f_S(-\varepsilon) = -f_S^* (-\varepsilon) \) and \( g_S(-\varepsilon) = g_S^* (-\varepsilon) \). As a result, \( F_1^*(-\varepsilon) \) is given by

\[
F_1^*(-\varepsilon) = \frac{2T_1 [-f_S \cos \theta_1^* (-\varepsilon) - g_S \sin \theta_1^* (-\varepsilon)]}{2 - T_1 + T_1 \cos \theta_1^* (-\varepsilon) g_S - \sin \theta_1^* (-\varepsilon) f_S}.
\]

Eqs. [4,9] and Eqs. [17,19] are consistent if \( \sin \theta^*(-\varepsilon) = -\sin \theta (\varepsilon) \) and \( \cos \theta^*(-\varepsilon) = -\cos \theta (\varepsilon) \). As in the case of ESE pairing, we can show that \( f_3(\varepsilon) = f_3^* (-\varepsilon) \) and \( f_0(\varepsilon) = -f_0^* (-\varepsilon) \).

(3) Junction with OTE superconductor

In this case \( f_\pm(\varepsilon) = -f_\pm^* (-\varepsilon) \) and \( g_\pm(\varepsilon) = g_\pm^* (-\varepsilon) \). Then \( f_S(-\varepsilon) = -f_S^* (-\varepsilon) \) and \( g_S(-\varepsilon) = g_S^* (-\varepsilon) \) and one can show that \( F_1^*(-\varepsilon) \) has the same form as in the case of ESE and ETO superconductor junctions. Then, we obtain \( \sin \theta^*(-\varepsilon) = -\sin \theta (\varepsilon) \) and \( \cos \theta^*(-\varepsilon) = -\cos \theta (\varepsilon) \). Also \( f_3(\varepsilon) = f_3^* (-\varepsilon) \) and \( f_0(\varepsilon) = -f_0^* (-\varepsilon) \) are satisfied. For \( h=0 \), only the OTE pairing state is generated in DF. Similar to the case of OTO junctions, OTE pairing is induced in the presence of \( h \).

(4) Junction with OSO superconductor

We have \( f_\pm(\varepsilon) = f_\pm^* (-\varepsilon) \), \( g_\pm(\varepsilon) = g_\pm^* (-\varepsilon) \), \( f_S(-\varepsilon) = f_S^* (-\varepsilon) \), and \( g_S(-\varepsilon) = g_S^* (-\varepsilon) \). One can show that \( F_1^*(-\varepsilon) \) takes the same form as in the cases of ESE, ETO, OTE superconductor junctions. Then, we obtain \( \sin \theta^*(-\varepsilon) = -\sin \theta (\varepsilon) \) and \( \cos \theta^*(-\varepsilon) = -\cos \theta (\varepsilon) \). Also \( f_3(\varepsilon) = f_3^* (-\varepsilon) \) and \( f_0(\varepsilon) = -f_0^* (-\varepsilon) \) are satisfied. For \( h=0 \), only the ESE pairing state is generated in DF. Similar to the case of ETO junctions, OTE pairing is induced in the presence of \( h \).

We can now summarize the above results in the table below. As seen from the above discussion, \( \sin \theta^*(-\varepsilon) = -\sin \theta (\varepsilon) \), \( \cos \theta^*(-\varepsilon) = -\cos \theta (\varepsilon) \), \( f_3(\varepsilon) = f_3^* (-\varepsilon) \) and \( f_0(\varepsilon) = -f_0^* (-\varepsilon) \) are satisfied for all cases. The real part of \( f_3 \) is an even function of \( \varepsilon \) while the imaginary part of it is an odd function of \( \varepsilon \) consistent with even-frequency pairing. On the other hand, the real part of \( f_0 \) is an odd function of \( \varepsilon \) while its imaginary part is an even function of \( \varepsilon \) consistent with odd-frequency pairing.

| Symmetry of the pairing in superconductors | Symmetry of the pairing in DF without exchange field | Symmetry of the pairing in DF |
|------------------------------------------|-----------------------------------------------|-------------------------------|
| (1) Even-frequency spin-singlet even-parity (ESE) | ESE | ESE + OTE |
| (2) Even-frequency spin-triplet odd-parity (ETO) | OTE | OTE + ESE |
| (3) Odd-frequency spin-triplet even-parity (OTE) | OTE | OTE + ESE |
| (4) Odd-frequency spin-singlet odd-parity (OSO) | ESE | ESE + OTE |

Within this formulation, the LDOS in the DF layer is given by

\[
N/N_0 = \frac{1}{2} (\text{Re } \theta + \text{Re } \cos \theta) \tag{22}
\]

where \( N_0 \) denotes the LDOS in the normal state. Below we will calculate \( f_3 \), \( f_0 \) and LDOS at zero temperature. For this purpose, we will use the following parameter set

\[
Z = 3, \quad Z' = 3, \quad E_{Trh} \equiv D/L^2 = 0.1 \Delta \quad \text{and} \quad R_d/R_0' = 0.1,
\]

which represents a typical DF/S junction. Our qualitative conclusions are not sensitive to the parameter choice.

### III. RESULTS

In the following, we will study two typical cases. As an example of ESE superconductor, the conventional spin-singlet \( s \)-wave pairing will be considered. We will clarify the generation of OTE pairing in DF by the exchange field \( h \) consistent with preexisting results.\(^2\)\(^13\) We will also study spin-triplet \( p \)-wave superconductor as a typical example of ETO superconductor. In this case, ESE pairing state is induced by \( h \). It should be remarked again that \( f_3 \) and \( f_0 \) denote the ESE and OTE pairing amplitudes, respectively.

### A. Spin singlet \( s \)-wave superconductor junctions

Let us first study DF/spin-singlet \( s \)-wave superconductor junctions where we choose \( R_d/R_0 = 1 \) and the form factor \( \Psi_{\pm} \) is given by \( \Psi_{\pm} = 1 \). Real and imaginary parts of \( f_3 \) and \( f_0 \) at \( x = 0 \) for various \( h/\Delta \) are shown in Fig. 2. Without exchange field, i.e., \( h = 0 \), only the \( f_3 \) is
nonzero, consistent with conventional theory of proximity effect.\textsuperscript{37,38,39} By introducing the exchange field $h$, the magnitude of $f_3$ is suppressed for small $\varepsilon$ while it is enhanced for large $\varepsilon$ as shown in Figs. 2(a) and 2(b). On the other hand, the imaginary part of $f_0$ is enhanced for small magnitude of $\varepsilon$. The corresponding LDOS at N/DF interface normalized by its value in the normal state is plotted as a function of $\varepsilon$ in Fig. 3. The LDOS has a minigap at $h = 0$.\textsuperscript{38,39} As shown in Fig. 3 the LDOS is influenced crucially by $h$. A peak appears at zero energy with $h/\Delta = 0.05$. In this case $\text{Im}f_0$ has a large value at zero energy as shown in Fig. 2(d). Thus large magnitude of $\text{Im}f_0$ at $\varepsilon = 0$ is responsible for the peak of the LDOS.

It was shown in our previous work\textsuperscript{33} that the condition for the formation of ZEP in the LDOS is given by $E_{T\theta} \sim 2hR_0/R_d$. This condition is consistent with the results shown in Fig. 3. As shown in Fig. 2 when this condition is satisfied, $\text{Im}f_0$ has a large value at the zero energy. Thus it corresponds to the generation of the odd-frequency pairing amplitude $f_0$ at low energy. The spatial dependences of the pair amplitudes $f_3$ and $f_0$ at $\varepsilon = 0$ are shown in Fig. 4. The amplitude of $f_3$ is dominant near the DF/S interface while the magnitude of $f_0$ is enhanced at the N/DF interface.

Let us study the crossover between singlet and triplet pairing states. We show $f_3$ and $f_0$ as a function of $h$ for $\varepsilon = 0$ at (a) $x = 0$, (b) $x = L/2$ and (c) $x = L$ in Fig. 5. $f_0$ increases from zero with $h$. At a certain value of $h$, $f_0$ has a maximum. If the value of $h$ is larger than this value, the triplet component becomes dominant as shown in Fig. 5(a) and Fig. 5(b). The value of $h$ at the crossover regime is given by the minigap in DN/S junctions. Let us discuss this regime in more detail. As shown in section II, $\sin \theta(\varepsilon) = -\sin \theta^* (-\varepsilon)$ and $\cos \theta(\varepsilon) = \cos \theta^* (-\varepsilon)$ are satisfied for any case. Then the ESE and OTE pair wave functions in the DF are given by

$$f_3(\varepsilon) = \frac{\sin \theta(\varepsilon) + \sin \theta^* (-\varepsilon)}{2}, \quad \text{and} \quad f_0(\varepsilon) = \frac{\sin \theta(\varepsilon) - \sin \theta^* (-\varepsilon)}{2}.$$ \hfill (23) \hfill (24)

At $\varepsilon = 0$, we denote $\theta(0) = \text{Re} \theta(0) + i \text{Im} \theta(0)$, where $\text{Re} \theta(0)$ and $\text{Im} \theta(0)$ are the real and imaginary part of $\theta(0)$. Then $f_3(0)$ and $f_0(0)$ are given by $\cosh |\text{Im} \theta(0)| \sin |\text{Re} \theta(0)|$ and $i \sin |\text{Im} \theta(0)| \cos |\text{Re} \theta(0)|$. Thus the following equation is satisfied:

$$\frac{f_3(0)}{f_0(0)} = \frac{\tanh |\text{Re} \theta(0)|}{i \tanh |\text{Im} \theta(0)|}.$$ \hfill (25)

It is easy to show that $|\text{Re} \theta(0)| < |\text{Im} \theta(0)|$ is satisfied when the crossover occurs, i.e., $\tan \text{Re} \theta(0) = \tanh |\text{Im} \theta(0)|$.

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**FIG. 2:** (color online) Real (a) and imaginary (b) parts of $f_3$, and real (c) and imaginary (d) parts of $f_0$ in spin-singlet $s$-wave superconductor junctions. We choose $R_d/R_0 = 1$.

**FIG. 3:** (color online) Normalized LDOS as a function of $\varepsilon$ for $R_d/R_0 = 1$ with various $h/\Delta$ in spin-singlet $s$-wave superconductor junctions.

**FIG. 4:** (color online) Spatial dependence of the pair amplitudes $f_3$ and $f_0$ in DF for $\varepsilon = 0$ in spin-singlet $s$-wave superconductor junctions. For $\varepsilon = 0$, $\text{Im}f_3 = 0$ and $\text{Re}f_0 = 0$ are satisfied.\hfill (a) \hfill (b)
tanh Imθ(0). As shown in our previous work, this inequality is satisfied when the exchange field is of the order of the minigap energy in DN/S junctions, i.e., $h \sim (R_d/R_b)(E_{TB}/2)$. Therefore the crossover occurs around this value of the exchange field.

![Graph](image1)

**FIG. 5:** (color online) The pair amplitudes $f_3$ and $f_0$ as a function of $h$ in DF for $\varepsilon = 0$ in spin-singlet s-wave superconductor junctions. (a) $x = 0$. (b) $x = L/2$. (c) $x = L$.

**B. Spin-triplet p-wave superconductor junctions**

Next we focus on the DF / spin-triplet p-wave superconductor junctions, where we choose $R_d/R_b = 0.1$ and the form factor $\Psi_{\pm}$ is given by $\Psi_{\pm} = \pm \cos \phi$ corresponding to the case of $\alpha = 0$ (see Fig. 4). In order to make numerical calculations stable, we introduce small imaginary number in the quasiparticle energy: $\varepsilon \to \varepsilon + i\gamma$, with $\gamma = 0.01\Delta$. The real and imaginary parts of $f_3$ and $f_0$ at $x = 0$ are plotted in Fig. 6 for various $h/\Delta$. Similar to the case of DN/s-wave superconductor junctions, the imaginary part of $f_3$ and the real part of $f_0$ vanish at $\varepsilon = 0$. For $h=0$, $f_3 = 0$ and only $f_0$ is nonzero as shown in Fig. 6. The feature of this unusual proximity effect was already discussed in our previous paper, where OTE pairing state is generated in the DN of DN/ETO superconductor junctions. In this case, the LDOS has a ZEP and odd-frequency component $f_0$ becomes a purely imaginary number at $\varepsilon = 0$. With increasing $h$, the amplitude of $f_3$ is enhanced as shown in Figs. 6(a) and 6(b), in contrast to the case of DN/spin-singlet s-wave superconductor junctions. At the same time, the magnitude of $f_0$ near the zero energy is suppressed. Then the features of the proximity effect in DF are the same as in conventional superconductor junctions. The corresponding LDOS normalized by its value in the normal state is plotted as a function of $\varepsilon$ in Fig. 7. With the increase of $h$, the magnitude of LDOS at $\varepsilon = 0$ is suppressed and the LDOS peak is splitted. The magnitude of the splitting increases with the increase of $h$. Note that the peak positions in Im$f_0$ and LDOS coincide with each other.

![Graph](image2)

**FIG. 6:** (color online) Pair amplitudes for DF/ spin-triplet p-wave superconductor junctions. Real (a) and imaginary (b) parts of $f_3$. Real (c) and imaginary (d) parts of $f_0$. Here we choose $R_d/R_b = 0.1$.

![Graph](image3)

**FIG. 7:** (color online) Normalized LDOS as a function of $\varepsilon$ for $R_d/R_b = 0.1$ and various $h/\Delta$ in p-wave superconductor junctions.
The spatial dependences of the real part of \( f_3 \) and the imaginary part of \( f_0 \) at \( \varepsilon = 0 \) are shown in Fig. 8. For \( h = 0 \), \( f_3 \) is absent and the magnitude of the imaginary part of \( f_0 \) reaches its maximum at the DF/S interface. With the increase of \( h \), the amplitude of \( f_0 \) is drastically reduced. The spatial dependence of \( f_3 \) is rather weak and its amplitude is most strongly enhanced for \( h = 0.05\Delta \). At the same time, the magnitude of LDOS at \( \varepsilon = 0 \) is most strongly suppressed (see Fig. 7).

Before ending this subsection, we investigate the crossover between singlet and triplet pairing states. Let us plot \( f_3 \) and \( f_0 \) for \( \varepsilon = 0 \) as a function of \( h \) at (a) \( x = 0 \), (b) \( x = L/2 \) and (c) \( x = L \) in Fig. 9. \( f_3 \) has a maximum at a certain value of \( h \). When \( h \) exceeds this value, the singlet component becomes dominant as shown in Fig. 9. The value of \( h \) at the crossover increases with the increase of \( Z, R_d/R_B \) and \( E_{TR} \), i.e., with the enhancement of the proximity effect.

C. Relevance of the odd-frequency component to ZEP of LDOS

Let us discuss the relation between the generation of the odd-frequency pairing and ZEP in LDOS, using general properties of solutions of the proximity effect problem. Since \( \cos \theta(\varepsilon) = \cos \theta^*(-\varepsilon) \) are satisfied, the LDOS normalized by its value in the normal state is given by

\[
N/N_0 = |\cos \theta(\varepsilon) + \cos \theta^*(-\varepsilon)|/2.
\]

For \( \varepsilon = 0 \), the normalized LDOS reads \( \cosh[\text{Im} \theta(0)] \cos[\text{Re} \theta(0)] \), while \( f_3(0) \) and \( f_0(0) \) are given by \( \cosh[\text{Im} \theta(0)] \sin[\text{Re} \theta(0)] \) and \( i \sinh[\text{Im} \theta(0)] \cos[\text{Re} \theta(0)] \) respectively. As seen from these relations, \( f_0 \) becomes zero when the LDOS is zero. In addition, whether the spin-singlet component \( f_3 \) dominates the spin-triplet component \( f_0 \) or not crucially depends on the value of \( \text{Re} \theta(0) \). The most favorable condition where \( N/N_0 \) is enhanced is the large magnitude of \( \text{Im} \theta(0) \) and the absence of \( \text{Re} \theta(0) \), where \( f_0 \) dominates \( f_3 \). For the sufficiently large magnitude of \( \text{Im} \theta(0) \) and small magnitude of \( \text{Re} \theta(0) \), \( N/N_0 \sim \exp[\text{Im} \theta(0)]/2 \sim \exp[\text{Im} \theta(0)]/2 \) and \( f_0(0) \sim i \cos[\text{Re} \theta(0)] \exp[\text{Im} \theta(0)]/2 \sim i \exp[\text{Im} \theta(0)]/2 \) are satisfied. Then we obtain \( N/N_0 \sim -i f_0(0) \). This means that the generation of the odd-frequency pair amplitude \( f_0(0) \) leads to the enhancement of the density of states at zero energy.
IV. CONCLUSIONS

We have studied the proximity effect in diffusive ferromagnet (DF) / superconductor (S) junctions. Various possible symmetry classes in a superconductor were considered which are consistent with the Pauli’s principle: even-frequency spin-singlet even-parity (ESE) state, even-frequency spin-triplet odd-parity (ETO) state, odd-frequency spin-triplet even-parity (OTE) state and odd-frequency spin-singlet odd-parity (OSO) state. As was established in the previous work, in the absence of the exchange field the induced pair amplitude in a DF belongs respectively to ESE, OTE, OTE and ESE pairing states. It is shown in the present paper that, in addition to these states, the OTE, ESE, ESE and OTE pairing states are generated in DF in the presence of the exchange field $h$.

As a typical example of ESE superconductor, we have chosen spin-singlet $s$-wave state. We have clarified that when the OTE state dominates the ESE state in the DF, the resulting LDOS has a zero energy peak. At the same time, the amplitude of the OTE pair wave function near the N/DF interface is enhanced at zero energy. As suggested by our findings, the odd-frequency pairing state was possibly realized in the experiment by Kontos et al., where the ZEP was observed in ferromagnet / $s$-wave superconductor junctions.

We have also studied spin-triplet $p$-wave superconductor junctions. In this case, the ZEP in the LDOS splits into two peaks due to the generation of the ESE pairing state by the exchange field. The features of proximity effect specific to spin-triplet $p$-wave superconductor junctions can be studied in experiments with $\text{Sr}_2\text{RuO}_4-\text{Sr}_3\text{Ru}_2\text{O}_7$ eutectic system. Based on general properties of solutions of the proximity effect problem, we have demonstrated that the generation of the odd-frequency pairing state at zero energy leads to the ZEP in LDOS.

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