Optimization of DTC algorithm based on distributed parameter modelling of PMSM

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Abstract. Accurate mathematical model is the basis and precondition of electric motor control. The existing motor model under concentrated parameters is difficult to describe the harmonic and nonlinear characteristics of the motor, while the motor model under distributed parameters can accurately describe the torque and flux characteristics of the motor precisely at all working points, which provides the foundation for model-based motor control. The stator flux and torque observer have an essential influence on the performance of direct torque control (DTC) for PMSM. The concentrated parameter model of PMSM considering only the fundamental wave vector of the magnetic field for DTC will have obvious deficiencies in the performance such as torque ripple. This paper therefore uses the reconstructed magnetic co-energy (MCE) model to establish a torque and flux observer based on distributed parameters, which is introduced into the DTC algorithm of PMSM as an observer. The optimized DTC algorithm makes full use of the flux and torque harmonic description capabilities of the distributed parameter model, which improves control performance. The superiority of the distributed parameter observer is verified by simulation and bench test.

1. Introduction

Compared with electric excitation synchronous motors and induction motors, permanent magnet synchronous motors (PMSM) are characterized by high reliability, high efficiency, high energy density, wide speed range and low vibration noise, making them widely used in the field of electric vehicle drives[1]. Compared with industrial motor control, higher requirements are placed on motor control performance under vehicle operating conditions. The accurate mathematical model is the basis and precondition of the electric motor control. The common mathematical model of PMSM today is represented by concentrated parameters such as d-q axis inductance \(L_d, L_q\) and rotor flux \(\psi_f\), while the concentrated parameter model is mainly based on assumptions such as the sinusoidal distribution of the air-gap magnetic field. However, the air-gap magnetic field often does not guarantee a good sinusoidal distribution for actual motors, especially for automotive motors. Figure 1 shows the air-gap magnetic field distribution of PMSM in an electrical cycle under no-load and load conditions. It can be seen from the waveforms of the air-gap magnetic field distribution that the spatial distribution of the air-gap magnetic field is non-sinusoidal. As the magnetic saturation of the iron core is further deepened in case of load conditions, the harmonic content in the air-gap magnetic field is further increased.
Due to the non-sinusoidal characteristics and magnetic saturation of the air-gap magnetic field of the concentrated parameter model, it does not meet the preconditions for its establishment. Therefore, the concentrated parameter model often cannot fully describe the operating characteristics of PMSM. In order to fully describe the harmonic and nonlinear characteristics of PMSM, the distributed parameter modelling of PMSM based on magnetic co-energy (MCE) reconstruction is proposed [2]. The distributed parameter modelling is based on the periodicity of the magnetic ensemble in the two dimensions of the rotor position $\theta_r$ and the torque angle $\beta$. A two-dimensional Fourier series expansion and a univariate polynomial fitting are used to obtain a distributed parameter matrix $C^h$ representation. The magnetic compositing model is then based on the reconstructed MCE model to establish the corresponding analytical models of torque, flux and stator voltage. The distributed parameter modelling of PMSM has been successfully applied to active torque ripple suppression [2], but the current application of PMSM distributed parameter model is mainly based on Field oriented control (FOC), which has not been applied to direct torque control (DTC). The research on DTC, therefore, combines the many advantages of dynamic performance with distributed parameter modelling of PMSM, and optimizes the DTC algorithm based on PMSM distributed parameter model to extend the application range of the distributed parameter model and improve the control performance of DTC.

In the 1980s, the DTC algorithm for asynchronous motors was proposed [3]. From that time to the present, the connotation of DTC has been continuously developed and enriched. The conventional DTC (CDTC) refers to the closed loop control system, which through two hysteresis regulators respectively for torque and flux control. It is now generally accepted that direct control of torque and flux can be classified as DTC. In order to distinguish it from the traditional CDTC, the improved control method is called IDTC (Improved Direct Torque Control). For example, some researchers have developed an IDTC algorithm called SVPWM-DTC, which introduces Space Vector Pulse Width Modulation (SVPWM) into DTC to replace the conventional switch table of CDTC. Comparing with CDTC, which uses only a single basic voltage vector of the inverter in a control period, SVPWM-DTC decreases the amplitude of torque ripple by using SVPWM to produce the desired voltage vector [4][5]. However, the current research on DTC algorithm for PMSM is based on the concentrated parameter model. Therefore, this paper will further optimize the CDTC and IDTC control algorithms by combining the distributed parameter model of PMSM.

2. Distributed parameter flux and torque observer

2.1. Establishment of PMSM reconstructed magnetic co-energy model

Magnetic co-energy (MCE) is a physical quantity that characterizes magnetic field energy storage as opposed to magnetic energy (ME). It uses current as an independent variable to integrate the flux. In PMSM, the generalized coordinates $[i_A, i_B, i_C, \theta_r]$ are used to represent the magnetic co-energy $W_c$ by selecting the appropriate integral path, which can be expressed as
\[ W_e(i_A, i_B, i_C, \theta_r) = \int_0^{i_A} \psi_A(i_A', 0,0,0) di_A' + \int_0^{i_B} \psi_B(i_B', 0,0,0) di_B' + \int_0^{i_C} \psi_C(i_C', 0,0,0) di_C' \]  

(1)

According to the principle of virtual displacement, the flux of each phase and the electromagnetic torque of the motor are the partial derivatives of MCE to each generalized coordinate, as shown in equations (2) and (3).

\[ \psi_X = \frac{\partial W_c(i_A, i_B, i_C, \theta_r)}{\partial i_X} (X = A, B, C) \]  

(2)

\[ t_e = p \frac{\partial W_c(i_A, i_B, i_C, \theta_r)}{\partial \theta_r} \]  

(3)

In the equations, \( \psi_X \) is the flux of each phase \( (X=A, B, C) \), \( p \) is the motor pole pair number, \( t_e \) is the output electromagnetic torque.

Finite Element Numerical Analysis (FEA) is widely used in motor design, which accurately considers the harmonic and nonlinear characteristics of PMSM. It is based on Maxwell's differential equations to obtain accurate simulation results from PMSM model, which are solved by discrete finite element and electromagnetic field matrix. In this paper, the accurate MCE values of PMSM under different operating conditions are obtained by FEA simulation, which are used for reconstruction of MCE modelling. Besides, the electromagnetic torque, stator flux and voltage values obtained by FEA simulation are used for verifying the accuracy of the reconstructed mathematical model of PMSM.

The specific MCE reconstruction process is established to obtain the MCE model of PMSM with the rotor position \( \theta_r \) and the torque angle \( \beta \) under all current operating points, as shown in Figure 2. Firstly, the FEA simulation is used to obtain the exact numerical solution of the magnetic co-energy with an electric cycle of rotor position under each operating point of PMSM. Then, the MCE numerical solutions of PMSM with each single stator current vector \( I_s \) are obtained, which are fitted to a mathematical model by using two-dimensional Fourier series expansion according to the periodicity of rotor position \( \theta_r \) and the torque angle \( \beta \). The Fourier series coefficients with different \( I_s \) are characterized as the function of \( I_s \) \( (C(I_s)) \) by polynomial fitting. Finally, the MCE analytical equation is collated to obtain the reconstructed MCE model as shown in equation (4).

\[ W_c(I_s, \beta, \theta_r) = V(\theta_r)C(I_s)U(\beta) \]  

(4)

In the equation, \( C(I_s) = \sum_{i=1}^{N_3} C_i I_s^i \)

![Figure 2. The specific progress of magnetic co-energy reconstruction.](image)

2.2. The analytical model of electromagnetic torque

Substituting the reconstructed magnetic co-energy expression (4) into equation (3) gives the analytical expression of electromagnetic torque (5).
The stator current $I_s$ can be decomposed under the rotor magnetic field oriented synchronous coordinate system, d-q axis system, to obtain the equation (6).

$$I_s = \sqrt{i_d^2 + i_q^2}$$  \hspace{1cm} (6)

The equations (7) and (8) can be obtained from the equation (6).

$$\frac{\partial i_s}{\partial \theta_r} = \frac{\partial}{\partial \theta_r} \sqrt{i_d^2 + i_q^2} = \frac{1}{2} (i_d^2 + i_q^2)^{-\frac{1}{2}} \left( 2i_d \frac{\partial i_d}{\partial \theta_r} + 2i_q \frac{\partial i_q}{\partial \theta_r} \right) = 0$$  \hspace{1cm} (7)

$$\frac{\partial \beta}{\partial \theta_r} = \frac{\partial}{\partial \theta_r} \arctan(i_q/i_d) = \frac{1}{1 + (i_q/i_d)^2} \frac{i_q}{i_d^2} = -1$$  \hspace{1cm} (8)

The equation (9) can be obtained by substituting equations (7) and (8) into equation (5).

$$t_e = p \frac{d}{d\theta_r} V(\theta_r) C(I_s) U(\beta) - V(\theta_r) C(I_s) \frac{d}{d\beta} U(\beta)$$  \hspace{1cm} (9)

The derivative of $V(\theta_r)$ and $U(\beta)$ can be expressed as a form of itself multiplied by a diagonal matrix, as shown in equation (10).

$$\frac{d}{d\theta_r} V(\theta_r) = V(\theta_r) \cdot P, \frac{d}{d\beta} U(\beta) = M \cdot U(\beta)$$  \hspace{1cm} (10)

Where

$$P = \begin{bmatrix} -jN_z\omega_{\theta_r} & \ldots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \ldots & jN_z\omega_{\theta_r} \end{bmatrix}, \quad M = \begin{bmatrix} -jN_z\omega_{\theta_r} & \ldots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \ldots & jN_z\omega_{\beta} \end{bmatrix}$$  \hspace{1cm} (11)

In summary, the analytical model of electromagnetic torque can be expressed as the equation (12).

$$t_e = \frac{3}{2} p V(\theta_r) \{ PC(I_s) - C(I_s) M \} U(\beta)$$  \hspace{1cm} (12)

2.3. The analytical model of stator flux

It can be seen from equation (2) that the partial derivatives of the magnetic co-energy to the d-q axis current is the flux of the d-q axis equivalent phase. Similar to the torque model, since the analytical formula of the magnetic co-energy is a function with $I_s$ and $\beta$ as independent variables, the partial derivatives of the $I_s$ and $\beta$ to the d-q axis current will appear in the process of using the magnetic co-energy to derive the d-q axis current. The d-q axis stator flux can be expressed as the equations (13) and (14).

$$\psi_d = \frac{\partial W_e(I_s, \beta, \theta_r)}{\partial i_s} \cdot i_s + \frac{\partial W_e(I_s, \beta, \theta_r)}{\partial \beta} \cdot \beta + \frac{\partial W_e(I_s, \beta, \theta_r)}{\partial \theta_r} \cdot \theta_r$$  \hspace{1cm} (13)

$$\psi_q = \frac{\partial W_e(I_s, \beta, \theta_r)}{\partial i_s} \cdot i_q + \frac{\partial W_e(I_s, \beta, \theta_r)}{\partial \beta} \cdot \beta + \frac{\partial W_e(I_s, \beta, \theta_r)}{\partial \theta_r} \cdot \theta_r$$  \hspace{1cm} (14)

According to the equation (6), the equations (15) and (16) can be obtained.

$$\frac{\partial i_s}{\partial i_d} = \frac{\partial i_q}{\partial i_d} = \cos \beta, \quad \frac{\partial i_s}{\partial \beta} = -\sin \beta \frac{i_s}{i_d}, \quad \frac{\partial i_q}{\partial \beta} = 0$$  \hspace{1cm} (15)

$$\frac{\partial i_s}{\partial i_q} = \sin \beta \frac{i_s}{i_q}, \quad \frac{\partial i_q}{\partial i_q} = \cos \beta, \quad \frac{\partial \beta}{\partial i_d} = 0, \quad \frac{\partial \beta}{\partial i_q} = 0$$  \hspace{1cm} (16)

Further the equations (17) and (18) can be obtained.

$$\psi_d = \cos \beta \times V(\theta_r) \frac{dC(I_s)}{d\beta} U(\beta) - \frac{\sin \beta}{i_s} \times V(\theta_r) C(I_s) \frac{dU(\beta)}{d\beta}$$  \hspace{1cm} (17)

$$\psi_q = \sin \beta \times V(\theta_r) \frac{dC(I_s)}{d\beta} U(\beta) + \frac{\cos \beta}{i_s} \times V(\theta_r) C(I_s) \frac{dU(\beta)}{d\beta}$$  \hspace{1cm} (18)

Define

$$D(I_s) = \frac{C(I_s)}{l_s} = \sum_{i=1}^{N_s} C_i l_i^{i-1}$$  \hspace{1cm} (19)
In summary, the analytical model expression of the stator d-q axis flux is obtained as the formula (20).

\[
\begin{bmatrix}
\psi_d \\
\psi_q
\end{bmatrix} =
\begin{bmatrix}
\cos\beta & -\sin\beta \\
\sin\beta & \cos\beta
\end{bmatrix}
\begin{bmatrix}
V(\theta_r) \frac{dC(I_s)}{dt} U(\beta) \\
V(\theta_r) D(I_s) M U(\beta)
\end{bmatrix}
\]

3. DTC optimization based on distributed parameter observer

3.1. CDTC optimization based on distributed parameter observer

The specific form of the flux and torque observer of the PMSM distributed parameter model is shown in equations (12) and (20). Considering the complexity and accuracy of the model, the order of MCE model is selected as \(N_1 = N_2 = N_3 = 5\). By using the MCE reconstruction modelling method mentioned in the section 2.1, the expression of the corresponding coefficients in reconstructed MCE model can be obtained. According to the mentioned models in section 2.2 and 2.3, the flux and torque observer based on the distributed parameter model are obtained, as shown in Figure 3. For the convenience of discussion, the stator flux and torque observer with concentrated parameters are called concentrated parameter observer, and the stator flux linkage and torque observer with distributed parameters are called distributed parameter observer.

![Figure 3. PMSM flux and torque observer based on the distributed parameter model.](image)

The distributed parameter observer shown in Figure 3 is used as the flux and torque observer of the CDTC system. The PMSM control framework shown in Figure 4 is established. The data exchange and co-simulation are performed by using Simulink and the motor FEA model established by Maxwell. The relevant structural parameters of PMSM are shown in Table 1 and Table 2.

| Parameters                  | Value |
|-----------------------------|-------|
| Stator outer diameter (mm)  | 180   |
| Stator inner diameter (mm)  | 105   |
| Stator core length (mm)     | 80    |
| Stator slots                | 36    |
| Slot width (mm)             | 2.2   |
| Slot length (mm)            | 1     |
| Connection height (mm)      | 1     |
| Slot depth (mm)             | 26    |
| Poles                       | 6     |

Table 1. The stator’s main parameters of PMSM
Table 2. The rotor’s main parameters of PMSM

| Parameters                        | Value  |
|----------------------------------|--------|
| Rotor outer diameter (mm)        | 103.6  |
| Rotor inner diameter (mm)        | 50     |
| Rotor core length (mm)           | 80     |
| Bridge width (mm)                | 2.5    |
| Permanent magnet width (mm)      | 35     |
| Permanent magnet thickness (mm)  | 10     |

Based on the motor parameters in Table 1 and Table 2, the CDTC based on the concentrated parameter observer and the CDTC based on the distributed parameter observer were simulated according to the operating conditions in Table 3. The results are shown in Figure 5 ~ Figure 16.

Figure 4. CDTC control block diagram of PMSM.

Table 3. Operating conditions setting

| Parameters | Speed (rpm) | Control Period (s) | Reference Torque (Nm) |
|------------|-------------|--------------------|-----------------------|
| Value      | 500         | 5e-5               | 5                     |
Figure 5. Torque response with concentrated parameters.

Figure 6. FFT analysis of torque response with concentrated parameters.

Figure 7. Torque response with distributed parameters.

Figure 8. FFT analysis of torque response with distributed parameters.

Figure 9. Phase A current response with concentrated parameters.

Figure 10. FFT analysis of Phase A current with concentrated parameters.
Figure 11. Phase A current response with distributed parameters.

Figure 12. FFT Analysis of Phase A current with distributed parameters.

Figure 13. Phase A instantaneous switching frequency with concentrated parameters.

Figure 14. Phase A average switching frequency with concentrated parameters.

Figure 15. Phase A instantaneous switching frequency with distributed parameters.

Figure 16. Phase A average switching frequency with distributed parameters.

Compared with Figure 5 using the concentrated parameter observer, Figure 7 shows the torque response of a CDTC using the distributed parameter observer with the amplitude of torque ripple reduced by approximately 68.4%. Figure 8 is the FFT analysis of the torque response with distributed parameters. Compared with the CDTC simulation results in Figure 6 under concentrated parameters, the 12th-time torque ripple in Figure 8 is significantly reduced. Figure 11 shows the phase A current response based on the distributed parameter observer, and Figure 12 shows the FFT analysis of the current response. Compared with the results in Figure 10, the 6k-1 and 6k+1 (k=1, 2, 3, ...) harmonics of the current in Figure 12 are obviously increased. Figure 15 and Figure 16 show the instantaneous switching frequency and average switching frequency of Phase A based on the distributed parameter observer. The average switching frequency is about 5 kHz in Figure 16. Compared with Figure 14, the average switching frequency in Figure 16 is significantly reduced. The main reason is that the torque ripple with distributed parameter observer is weaker and the switching rate of basic voltage vectors is
reduced due to the use of the distributed parameter observer. The joint simulation results show that the distributed parameter observer significantly improves the torque control effect of CDTC.

3.2. SVPWM-DTC optimization based on distributed parameter observer

The CDTC is controlled only by the eight basic voltage vectors of the inverter, where the amplitude of non-zero voltage vectors is the same as the bus voltage. But the bus voltage amplitude is large. Large normal voltage vector causes a rapid change in the magnitude of the flux, causing it to exceed the hysteresis range. Besides, large tangential voltage vector causes the absolute value of the stator current angular frequency $\omega_s$ to be too large, resulting in the rotation speed of $\psi_s$ is too fast. It will easily cause a large change in the torque angle $\beta$, resulting in overshoot of the torque response. In addition, the non-zero voltage vectors have only six basic directions, but these directions are often not the optimal directions.

It can be seen that a single non-zero voltage vector with a large amplitude and direction is the main cause of awful CDTC flux and torque control performance. At present, the SVPWM method widely used in FOC control can modulate the voltage vector of any amplitude and direction within the allowable range of the bus voltage. And its switching frequency is fixed, so it is introduced into DTC to improve the control performance. It is called SVPWM-DTC, and its system block diagram is shown in Figure 17.

![Figure 17. SVPWM-DTC control block diagram.](image)

The torque and flux observer in Figure 17 is selected as the distributed parameter observer shown in Figure 3. The joint simulation results are obtained according to the working conditions in Table 3, as shown in Figure 18 ~ Figure 21.
Figure 18 shows the torque response of SVPWM-DTC with distributed parameters. Figure 19 shows the FFT analysis of the torque response. Compared with the CDTC under the distributed parameters in Figure 7, the torque ripple amplitude in Figure 18 is reduced by 51.6%, which proves that the SVPWM-DTC has better torque control performance than the CDTC.

Figure 20 shows the phase A current response with distributed parameters, and Figure 21 shows the FFT analysis of the current response. Compared with the CDTC under the distributed parameters in Figure 12, the 13th harmonic of SVPWM-DTC in Figure 21 increases, and the other harmonics decrease.

4. Experimental Verification of DTC Optimization Strategy for PMSM
The optimization algorithm is tested and verified by using the PMSM motor controller on the test bench. The test bench is shown in Figure 22, which is mainly composed of the dynamometer and the motor to be tested. The main test equipment and its parameters are shown in Table 4. The electrical parameters of the tested motor are shown in Table 1 and Table 2.
The test conditions are set as shown in Table 3. The SVPWM-DTC control algorithm is tested by using the concentrated parameter observer and the distributed parameter observer respectively. The torque response obtained by the torque sensor and its FFT analysis are shown in Figure 23 ~ Figure 26.
As can be seen from the comparison of Figure 23 and Figure 24, the torque ripple amplitude of the motor is significantly reduced after using the distributed parameter observer instead of concentrated parameter observer. By comparing the FFT analysis results in Figure 25 and Figure 26, the reduced 12th harmonic is the main cause of the torque ripple reduction. The 12th harmonic amplitude of the torque response with the concentrated parameter observer is 2.62 Nm, and with the distributed parameter observer is 1.01 Nm. It can be seen that the 12th torque ripple amplitude is reduced by 61.30% after using distributed parameter observer.

Figure 27 ~ Figure 30 show the phase A current response and its FFT analysis results measured by the current sensor under the concentrated parameter observer and the distributed parameter observer.

It can be seen that the harmonic content in the current response is significantly increased after using the distributed parameter observer by comparing Figure 27 with Figure 28, which is consistent with the results of the joint simulation. The significant increase of the 13th harmonic is the main reason for the enhancement of the current fluctuation.

According to the test result and its analysis, the DTC performance with the distributed parameter observer is significantly better than that with the concentrated parameter observer. When the distributed parameter observer is introduced into the SVPWM-DTC, the torque control performance is significantly improved.

5. Conclusion
In this paper, the FEA model of PMSM is constructed in Maxwell. Through the joint simulation of Maxwell, Simplorer and Simulink, it is found that the amplitude of torque ripple generated by CDTC under the concentrated parameter observer is large, which exposes the disadvantages of the concentrated parameter observer. The flux and torque observer of the distributed parameter were introduced into CDTC and SVPWM-DTC, which improved the torque control performance of both algorithms.
According to the analysis of the simulation, the distributed parameter observer can more accurately describe the changes in torque and stator flux. Finally, through the bench test, the torque ripple amplitude under the distributed parameter observer is reduced by about 50% compared with the concentrated parameter observer, which verifies the superiority of the distributed parameter observer.

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