Superconducting Quantum Annealing Architecture with LC Resonators

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We propose a novel architecture for superconducting circuits to improve the efficiency of a quantum annealing system. To increase the capability of a circuit, it is desirable for a qubit to be coupled not only with adjacent qubits but also with other qubits located far away. We introduce a circuit that uses a lumped element resonator coupled each with one qubit. The resonator-qubit pairs are coupled by rf-superconducting quantum interference device (SQUID) based couplers. These pairs make a large quantum system for quantum annealer. This system could prepare the problem Hamiltonian and tune the parameters for quantum annealing procedure.

1. Introduction

With the present information explosion in our society, it is indispensable to realize efficient quantum information-processing systems for the coming generation. Such quantum systems are being researched and developed. Among them, superconducting quantum circuits are making remarkable progress. Quantum annealing is a class of quantum information-processing specialized for solving the optimization problems.1–3 In general, a wide range of real-world problems can be classified as optimization problems, which cover the fields of fundamental science, the improvement of productivity, and the development of infrastructure. However, it is practically impossible to solve these optimization problems with von Neumann computers when the size of the problems exceeds certain limits.4

For a quantum annealing, a problem to be solved is encoded as strengths of interactions in a “spin glass” that consists of many spins and interactions between spins.5 By suitably encoding the time evolution of the spin glass, nature itself will find the minimum energy of the whole system, giving us the solution to the optimization problem. To build the quantum annealer, we need to consider what is required for such a physical system. When the number of the interactions for each qubit increase, the encoding of optimization problems becomes more efficient, therefore it is possible to reduce the overhead of the number of physical spins.6,7 Thus, a larger problem can be solved efficiently. In general, it is indispensable to increase the number of spins as well as couplings between these spins to efficiently solve large-scale problems by the quantum annealing.

In this paper, we propose a novel architecture for scalable quantum annealing circuits with full coupling in which a spin is coupled to all other spins. The existing superconductive quantum annealing systems8,9 utilize flux qubits as spins, which are coupled with each other by an rf-superconducting quantum interference device (SQUID)-based coupler.10 On the other hand, in our proposed architecture, the coupling structure between qubits is mediated by superconducting resonators. Here, the pair of the qubit and resonator function as a very long quantum system (spin), enabling itself to be coupled to the large number of other spins. A strongly coupled qubit-resonator pair enables to make a large quantum system, comparing to a self-size of a qubit. Additionally, to increase the coupling energy between spins, deep-strong coupling11,12 between the qubit and resonator is introduced. It is also possible to introduce a dispersive readout.13,14

2. Proposed Architecture

We propose a novel architecture for superconducting circuits to realize a quantum annealing system that is consists of flux-qubits and lumped element resonators (Fig. 1).

The flux-qubit has longitudinal (Z) and transverse (X) degrees of freedoms.15,16 Their energies of Z and X are controlled by applied external magnetic flux to the main-loop and a-loop (shown in Fig. 1(b)), respectively. It is common to use flux-qubits for quantum annealing, because the quadratic structure of the energy band of the flux-qubit allow a transverse magnetic field and longitudinal magnetic field to easily and continuously increase or decrease.17 For this reason, we also employ flux-qubits for our proposed architecture.

In general, a lumped element (LC) resonator has a uniform current distribution on its inductive parts, in contrast to a distributed resonator such as those of co-planar type, with standing wave depended on resonant frequency. In our architecture, we utilize an LC resonator with a long inductive limb, which plays an important role in our architecture (see Fig. 1(a)). The long inductive limbs make it possible to couple many spins. Accordingly, an LC resonant mode with a uniform current is realized, while other non-LC resonant modes inevitably exist. However, by optimizing the parameters of the circuit, the energy of the LC resonant mode can be realized in the vicinity of the qubit energy, while making the other modes be far away from the energy. Thus, the coupling of the other resonant modes to the qubit can be ignored.

In the architecture, N flux-qubits are arranged on a line and each qubit is connected to a different LC resonator via a mutual inductance. The N LC resonators are braided so that they fully interact with each other by the long inductive limb through the rf-SQUID-based couplers as shown in Fig. 1(a). Thus, the N flux-qubits effectively and fully interact with all other qubits via the network of LC resonators and couplers.

In contrast with the existing scheme, in which qubits interact with each other through rf-SQUID-based couplers, our system has the following advantages.

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In our circuit architecture, the qubits fully interact with each other. The number of interactions between the qubits is $N(N - 1)/2$ when there are $N$ qubits in the proposed circuit. On the other hand, in the existing scheme, the number of interactions is $2N$ in a unit cell.3)

To map optimization problems to interactions of a system, the larger the number of spins and interactions between spins, the more efficiently the problems are mapped. In our architecture, because of the long inductive limbs of resonators, it is possible to increase the number of spins and interactions.

Considering the actual realization of the quantum annealing circuit, as it is clear from Fig. 1(a), the resonator is interwoven in a stitch-like manner, therefore a standard multi-layered superconducting integration is required. On the other hand, the qubit, which is the part most sensitive to decoherence, can be separately fabricated by the standard double angle shadow evaporation of aluminum18) that all the good superconducting qubits are produced.

3. General Quantum Annealing

To perform the quantum annealing, the requirement is that a Hamiltonian of a physical system fit the form of the stoquastic Hamiltonian ($H_{QA}$), which is given by1)

$$H_{QA} = \Lambda(t) \sum_i \tilde{\epsilon}_i \sigma_i^z + \Lambda(t) \sum_{i<j} \tilde{J}_{ij} \sigma^z_i \sigma^z_j + \Gamma(t) \sum_i \tilde{\Lambda}_i \sigma^x_i,$$  \hspace{1cm} (1)

where $\tilde{\epsilon}_i$ and $\tilde{\Lambda}_i$ are the normalized energies of the $i$th spin corresponding the longitudinal and transverse magnetic fields, $\tilde{J}_{ij}$ is the normalized energy of the interaction between the spins ($-1 \leq \tilde{\epsilon}_i, \tilde{J}_{ij} \leq 1$), and $\Lambda$ and $\Gamma$ are time-dependently tunable values.

The optimization problem is mapped onto the $\tilde{\epsilon}_i$ and $\tilde{J}_{ij}$.

After mapping the optimization problem to the system, the quantum annealing is performed in accordance with the following procedure. Initially ($t = t_0$), all spins are facing the same direction by applying transverse magnetic fields, where $\Lambda(t_0) = 0$, and $\Gamma(t_0) = 1$. Then, the magnetic fields applying to spins are gradually changed to the longitudinal direction. Finally at $t = t_f$, $\Lambda(t_f) = 1$, $\Gamma(t_f) = 0$, states of the spins show us the solution to the problem. When the system is at the end of an annealing procedure, the energy of the spin and the strengths of the effective interactions between spins should be much larger than the transverse energy of the spin ($\tilde{\Lambda}_i \ll \tilde{\epsilon}_i, \tilde{J}_{ij}$). To satisfy these requirements, characteristics of our proposed architecture must be estimated and calculated.

4. Requirements of Proposed Architecture

The proposed architecture is described by the following Hamiltonian, which considers the qubits, resonators, the longitudinal and transverse inductive coupling between each qubit and resonator, and the interactions between resonators:19)

$$H_N = \sum_i (\epsilon_i \sigma_i^z + \Delta_i \sigma_i^+ \sigma_i^-) + \omega_i \left( a_i^+ a_i + \frac{1}{2} \right)$$

$$+ \sum_{i,j} g^c_{ij} (a_i^+ a_j^+ + a_i^- a_j^-) + g^t_{ij} (a_i^+ a_j^- + a_i^- a_j^+),$$  \hspace{1cm} (2)

where $i$ and $j$ are integers from 1 to $N$, which is the total number of qubits (resonators), $\sigma_i^+$ and $\sigma_i^-$ are the $i$th spin operators of the longitudinal and transverse degrees of freedom, $\epsilon_i = \epsilon_i(f_i^c)$ and $\Delta_i = \Delta_i(f_i^t)$ are the energies of each degree
of freedom of \(i\)th qubit, \(f^i = \Phi^i/\Phi_0\) and \(f_j^i = \Phi^i_j/\Phi_0\). \(\Phi^i\) and \(\Phi^i_j\) are the fluxes of the \(i\)th qubit in the \(\alpha\)-loop and main-loop, \(\Phi_0\) is flux quantum, \(a_i^\dagger\) and \(a_i\) are the bosonic creation and annihilation operators of the \(i\)th resonator, \(\omega'_j\) is the energy of the \(i\)th resonator, \(g^i_j\) and \(g^i\) are the longitudinal and transverse coupling constants between the \(i\)th qubit and resonator, and \(g^i_{ij}\) is the coupling constant between the \(i\)th and \(j\)th resonators, respectively.

When the applied flux of main-loop changes away from a half-integer multiple of the flux quantum, the \(Z\) and \(X\) energies of the \(i\)th qubit \((\varepsilon_i, \Delta_i)\) become \(\Delta_i = 0\) and \(\varepsilon_i = \varepsilon'_i\), and the transverse coupling \(g^i_{ij}\) is neglected. The third term of the Hamiltonian \(\mathcal{H}_N\), which describes the qubit-resonator interactions, is exactly diagonalized (following and expanding Billangeon’s method in Refs. 20 and 21 by the unitary operator given by

\[
\mathcal{U}_N = \exp \left[ \sum_{i=1}^{N} \sum_{j=1}^{N} -\theta_{ij} \sigma_i^z (a_i^\dagger - a_i) \right].
\]  

(3)

Here, \(\theta_{ij}\) are set to satisfy the following simultaneous equations for all \(i\) and \(j\):

\[
\forall i, j \quad g^i_{ij} \delta_{ij} - \sum_{k=1}^{N}(2g^i_{kj} + \omega'_k \delta_{kj}) \theta_{ik} = 0,
\]  

(4)

where we impose \(g^i_{ij} = g^i_{ji}\) because the coupling strength of the resonators is symmetric, and \(g_{ii} = 0\) because the self-coupling refers to the self-energy of the resonator \(\omega'_i\), which has already been included. Under this constraint, using the Baker–Campbell–Hausdorff (BCH) formulation, we have:

\[
\mathcal{H}'_N = \mathcal{U}_N^\dagger \mathcal{H}_N \mathcal{U}_N
\]

\[
= \sum_i g^i \sigma_i^x + \sum_{i,j} J_{ij} \sigma_i^x \sigma_j^x + \sum_{i,j} \omega_j (a_i^\dagger a_i + \frac{1}{2}) + \sum_{i,j} g^i_{ij} (a_i^\dagger + a_i)(a_j^\dagger + a_j),
\]  

(5)

where \(J_{ij} = -g^i_{ij} \theta_{ji}\).

\(J_{ij}\) is the strength of the effective interaction between the \(i\)th and \(j\)th qubits. We are able to obtain \(J_{ij}\) by applying Cramer’s rule to Eq. (4). \(\theta_{ij}\) denotes the route from the \(i\)th qubit to the \(j\)th qubit through the network of resonators. In proposed architecture, the full interactions between qubits are effectively realized. In Hamiltonian (5), qubit-resonator interaction term have already been unitary transformed. The remained terms of resonator couplings change eigenenergies of resonators only and not other terms. Therefore, to evaluate energies of qubits and effective couplings between qubits for quantum annealing Hamiltonian (1), it is not necessary to consider the 3rd and the 4th terms of Hamiltonian (5) that depends only on resonators themselves.

To map a particular optimization problem that we wish to solve, it is necessary that the set of interactions \((J_{ij})\) is encoded to the set of coupling constants of the system \((g^i_{ij})\). If the encoding time is not polynomial when using a classical computer, it makes no sense to build the system. In our proposed circuit, once the set of interactions of the problem is fixed, it can be converted to the set of \(g^i_{ij}\) by a common matrix method using a classical computer in polynomial time with Eq. (4).

We define the coupling matrix \(G\) with non-diagonal elements \(2g^i_{ij}\) and diagonal elements \(\omega'_i\), then, using Cramer’s rule, we obtain \(J_{ij} = -g^i_{ij} (\text{det} G^i_{ij}/\text{det} G)\), where \(G^i_{ij}\) is \(G\) with \(i\)th columns replaced with the vector \((0, \cdots, 0, g^i_{ij}, 0, \cdots, 0)^T\), which has the \(j\)th qubit-resonator coupling strength at the \(j\)th element with other elements equal to zero. The highest orders of \(\text{det} G\) and \(\text{det} G^i_{ij}\) for the resonator energy are

\[
\text{det} G \propto \prod_{k=1}^{N} \omega'_{k}, \quad \text{det} G^i_{ij} \propto 2g^i_{ij} g^j_{ij} \prod_{k=1, k \neq i, j}^{N} \omega'_{k}.
\]

(6)

Therefore, we can estimate the strength of the effective interaction through just two \((i, j)\) resonators,

\[
|J_{ij}| \propto (g^i_{ij}/\omega'_i) (g^j_{ij}/\omega'_j) g^i_{ij},
\]

(7)

where the lower-order terms of \(\omega'\) are ignored, which is much smaller than the highest-order term because the lower-order terms correspond to coupling through more than two resonators.

In the strong-coupling regime, which is usually used in the field of superconducting circuits, the resonator energy \(\omega'_i\) is larger than the coupling constant \(g\) between a qubit and resonator \((\kappa, \gamma \ll g \ll \omega'_i)\), where \(\kappa\) and \(\gamma\) are the photon leak rate from the resonator and the relaxation rate from the qubit, respectively. In this regime, the value of \(|J_{ij}|\) is much smaller than the sufficient strength of the interactions: \(|J_{ij}| \ll 1\).

For example, when \(N = 2\), \(J_{ij}\) is given by

\[
J_{12} = \frac{4g^i_{12} g^j_{12} g^i_{1j}}{\omega'_i \omega'_j - (2g^i_{1j})^2}.
\]

(8)

The value of \(J_{12}\) is lower than the order of MHz when common values of the qubit-resonator coupling strength \((\sim 100 \text{ MHz})\) are used in the strong-coupling regime. To satisfy the requirement of the final procedure (see Sec. 3) of the quantum annealing, \(g\) must also be lower than the order of MHz. However, the thermal fluctuation of a quantum circuit in a 10 mK environment is equivalent to a frequency fluctuation of around 200 MHz. Thus, this system cannot give the correct solution to problems because the final state of qubit is easy to be buried in thermal noise.

The ultrastrong-coupling regime \((\sim 0.1 \text{ MHz})\) is stronger than the strong coupling regime, however, the coupling strength is still smaller than the energy of resonator by one order of magnitude \((g \sim 0.1 \omega'_i\)), so the strength of the effective interactions is also insufficient in this regime. To resolve this problem of the strength, we adopt the deep-strong-coupling regime, which was recently realized in experiments.\(^{11,12}\)

In the deep-strong-coupling regime, the coupling strength between the qubits and resonators is similar to the energy of resonator \((g \sim \omega'_i)\). To increase the coupling strength, the deep-strong-coupling can be achieved by the qubit and resonator of the pare sharing a line with a Josephson junction (called \(\beta\)-junction) that generates a large inductance (Fig. 1(b)). In this regime, the strength of the effective interaction is larger and Eq. (7) becomes

\[
|J_{ij}| \propto g^i_{ij}.
\]

(9)
Here, the order of strength of \( J_{ij} \) depend the set of \( (q_i^j) \).

Although in this regime, the approximation of the Jaynes–Cummings model fails, the second-order term of the resonator \( g_i^z (a_i^j + a_i^j)^2 \) appears in the system Hamiltonian without approximation of the Rabi model. Fortunately, following the method of Ref. 11, this second-order term can be transformed into first-order term and eliminated to obtain the form of the Hamiltonian in Eq. (2).

Next we calculate the energy levels and the coupling strengths of the qubit in the deep-strong-coupling regime. From the Fig 1(b) the Hamiltonian of the \( i \)th qubit is given by

\[
\mathcal{H}_i^0 = \frac{1}{2(\alpha + \beta + 2\alpha \beta)} \left[ (\alpha + \beta + \alpha \beta) \left( q_i^2 + q_i^2 \right) + (1 + 2\alpha \beta) q_i^2 a_i a_i - 2\alpha \beta q_i a_i b_i - 2\alpha q_i a_i + q_i b_i \right] - E_J \left( \cos \varphi_a + \cos \varphi_b + \cos \varphi_c \right)
\]

where \( g_i \) is the conjugate momentum of \( \phi_i = \varphi_i / 2\pi, j \in \{a, b, \beta\} \), \( C \) is the capacitance of the Josephson junction, \( E_J \) is the Josephson energy of the junction, and \( 0.5\alpha E_J \) and \( \beta E_J \) are the energies of the \( \alpha \)-junction and \( \beta \)-junction, respectively. To derive the energy levels, we calculate the Schrödinger equation of the Hamiltonian (\( \mathcal{H}_i^0 \)) using the wave function \( |\Psi_i^j\rangle = \sum_{k,l,m} C_{k,l,m}^j |\psi_k^j\rangle |\psi_l^j\rangle |\psi_m^j\rangle \), where \( |\psi_k^j\rangle = (2\pi)^{-1/2} \exp(-i\varphi_j) \), and \( C_{k,l,m}^j \) is an arbitrary complex number for \( \eta \in \{k, l, m\} \) and \( \xi \) is the number of the energy states. The energy band structure is shown in Fig. 2.

The coupling constant between the qubit and resonator via the \( \beta \)-junction are also calculated\(^{25} \) (shown in Fig. 3) as

\[
g_i^z = \frac{1}{2} t \times \frac{1}{2} \Phi_0 \left( |\Psi_{\beta}^j\rangle \langle \varphi_{\beta} |\Psi_{0}\rangle - |\Psi_{0}\rangle \langle \varphi_{\beta} |\Psi_{\beta}\rangle \right), \tag{11}
\]

\[
g_i^\parallel = \frac{1}{2} t \times \frac{1}{2} \Phi_0 \left( |\Psi_{0}\rangle \langle \varphi_{\beta} |\Psi_{\beta}\rangle + |\Psi_{\beta}\rangle \langle \varphi_{\beta} |\Psi_{0}\rangle \right), \tag{12}
\]

where \( \varphi_{\beta} \) is the phase difference at the \( \beta \)-junction.

As shown in Fig. 2, a flux-qubit can be well approximated as a two-level system around the optimal point \( (f_z^0 \sim 0.5) \), with its large anharmonicity.\(^{26} \) Using the Hamiltonian (10) and both of the coupling constants Eq. (11) and Eq. (12), the Hamiltonian of the resonator-qubit pair is given by

\[
\mathcal{H}_i^0 = \omega_i \left( a_i^\dagger a_i + \frac{1}{2} \right) + \omega_d \sigma_i^0 + \left( g_i^\parallel \sigma_i^0 + g_i^\perp \sigma_i^1 \right) (a_i^\dagger + a_i), \tag{13}
\]

where \( \omega_i = \sqrt{\Delta_i^2 + \epsilon_i^2} \) and the Pauli matrix \( \sigma_i^j \) basis are \( |\Psi_0\rangle \) and \( |\Psi_\beta\rangle \). \( g_i^\parallel \) and \( g_i^\perp \) in the coordinate \( \sigma_i^j \) of Hamiltonian (2) can be calculated by \( g_i^\parallel, \beta, e_i \) and \( \Delta_i \) (Fig.3). Thereby, \( g_i^z \) is negligible because it is much smaller than \( g_i^\parallel \).

In the deep-strong-coupling regime, the state of the pair of the qubit and resonator are displaced. When the qubit energy is sufficiently smaller than the resonator energy, the state of the pair at the transverse magnetic field can be approximated\(^{27} \) to \(|\langle \cdots |\rangle \equiv |\langle +\rangle \rangle \otimes \exp \left[ -i g_i^z / \omega_i (a_i^\dagger - a_i) \right] |n\rangle \rangle \). Where \(|\langle +\rangle \rangle \equiv |(\langle 0 | \rangle \rangle + |\langle 2 | \rangle \rangle)/\sqrt{2}, |\langle 0 | \rangle \rangle \) and \(|\langle 2 | \rangle \rangle \) are the basis of the \( i \)th qubit and correspond the current directions, and \( n_i \) is a photon number of the Fock state in the \( i \)th resonator.

Next, we describe a procedure to perform quantum annealing using the parameters in the proposed circuit \( (\Delta_i, e_i, g_i^\parallel, g_i^\perp, g_i^z) \). An example of the procedure of each parameter during annealing is shown in Fig. 4. This graph is based on the assumption that the flux biases are linearly changed at each loop.

In the beginning of the quantum annealing procedure, the state of the all qubits are set to the \( |\langle \cdots |\rangle \rangle = |\langle -\rangle \rangle \rangle \). Then, to fit the Hamiltonian of the proposed architecture Eq. (2) to the form of Eq. (1), \( J_{ij} = \Delta_i, e_i \) must be controlled with time. To control the parameters, they are time-dependently tuned by external flux biases. Because \( J_{ij} \) depends on \( g_i^\parallel, g_i^\perp, g_i^z \), and the set of \( g_i^\parallel, g_i^\perp, g_i^z \) depend on flux biases of the main-loop and the \( \alpha \)-loop. Such standard annealing path is shown in Fig. 4.

In proposed system, we can freely choose \( e_i \) and \( |f_z^0| \) in range 0 to around 2 GHz. At the end of the path, \( \Delta_i \) should

\[\text{Fig. 2. Calculated energy levels of a flux qubit with a } \beta \text{-junction as a function of main-loop flux bias. The energy gap between } E_1 \text{ and } E_2 \text{ is } 2 \sqrt{\Delta_i^2 + \epsilon_i^2}. |\Psi_1\rangle \text{ is the } i \text{-th eigenstate given } E_i. \text{ We take the calculation space maximum values of } k, l, m \text{ of 7 for a good approximation. And the parameters are } E_i/h = 5 \text{ GHz, } E_j/h = 250 \text{ GHz, } \alpha = 0.7, \beta = 4.\]

\[\text{Fig. 3. Calculated coupling constant of each degree of freedom as a function of the main-loop flux bias. A solid and dashed curves represent } g_i^\parallel \text{ and } g_i^z \text{, respectively. Those constants are calculated from } g_i^\parallel, g_i^z, e_i \text{ and } \Delta_i. \text{ The other parameters are shown in Table. I.}\]
be set much smaller than $\varepsilon_i$ and $|J_{ij}|^{29}$ Fortunately, $\Delta$ is reduced by a factor $\exp[-2(\varepsilon_i^2/\omega_i^2)^2]$ in deep-strong-coupling regime. $^{27}$ Therefore the final state of the system corresponds to the solution to an optimization problem. After the annealing, the flux-qubit can be measured by dispersive readout with high accuracy.

From Eq 8, the coupling strength ($J_{12}$) is expressed in terms of the circuit parameters as

$$J_{12} \approx \frac{M^2}{L^2} M_r I_{1}^r I_{2}^r, \quad (14)$$

where for $M$ is the mutual inductance between a qubit and a resonator, $M_r$ is the effective mutual inductance between resonators (1 and 2) through the coupler. $I_{i}^r$ is the screening current of the $i$th qubit (i = 1, 2). $L_r$ is the effective inductance of resonators.

To realize anti-ferromagnetic and ferromagnetic interactions between qubits for mapping of problems in Eq. (8), the rf-SQUID-based coupler connecting the resonators require that the coupling must be able to take positive and negative value by external biases. $^{10,29}$ To meet the requirement from a coupler, the circuit parameters are chosen to obtain the coupling strength ($J_{12}$) of the order of GHz. The parameters are listed in Table I.

Parasitic direct couplings exist between resonators because of geometric mutual inductance at their limbs. However, in a case that two resonators with parameters given in Table I are positioned 100 $\mu$m apart, the parasitic direct couplings should be lower than the order of MHz. The simulation showed that such small parasitic coupling could be ignored. When the length of the resonator limb is elongated to the order of cm, the parasitic coupling probably need to be suppressed with a superconducting ground plane.

An N-qubit system can clearly be realized in the same way. In this N pairs circuit, we are able to show that the order of coupling strengths are not reduced by increase of N. We deal with this N-qubit system as a unit cell because the number of qubits in the unit cell is limited by the length of the long inductive limb of the LC resonators. From Eq. (14), to make $J_{ij}$ as large as possible, the inductance of the resonator cannot be made too large. It is necessary to suppress $L_r$ to nH order or below to construct dozens qubits full coupled circuit. For this reason, the length of the long inductive limb is limited to cm order.

Table I. Parameters for Annealing calculation.$I_c$ is the root-mean-square current of the resonator, $I_c$ and $I_s$ is critical current of Josephson junction in a rf-SQUID-based coupler and a qubit, respectively. $E_1 = I_c \Phi_0/2\pi, E_c = \varepsilon^2/2C$. The resonator with below parameters has 2 mm length of inductor limb.

| Parameter | Value | Unit |
|-----------|-------|------|
| $E_i/h$   | 5     | GHz  |
| $E_f/h$   | 250   | GHz  |
| $\omega$ | 7.2   | GHz  |
| $I_c$     | 41    | nA   |
| $I_s$     | 1.4   | nH   |
| $M_s$     | 154   | pH   |
| $I_f$     | 10    | $\mu$A |
| $\alpha$ | 0.8   | -    |
| $\beta$  | 1.1   | -    |

Fig. 4. Calculated circuit parameter dynamics for a typical annealing path. This graph is based on the assumption that $g_{ci}$ is linearly increased from 0 to 400 MHz. $f_c$ and $f_0$ are also linearly changed from 0.5 to 0.4997 and from 0.21 to 0, respectively. During annealing path $J_{ij}^c$ (solid curve) and $J_{ij}^s$ (dashed curve) are calculated from $g_{ci}$ and $g_{si}$ in Eq. (2). The other parameters are shown in Table I.

5. Conclusion

We have described the architecture of the quantum annealing circuit with lumped element resonators to dramatically increase the number of coupled qubits, which is important for an efficient quantum annealing system. Although the total number of fully coupled qubits would be limited to around 100 in a unit at present, this unit can be scaled up by combining with other units via other couplers.

Quantum annealing machines with fully coupled dozens qubit unit cells should have an obvious advantage in mapping problems such as social networks, economics and categorized advertisements. Those problems can be decomposed into many subsets, where tight relationships exist within the subset, while only shallow relationships are required among subsets.

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