Fast calibration method of alignment prism based on image transformation relation

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Abstract: The alignment assembly based on the principle of coaxial imaging has high accuracy, and the accuracy of coaxial imaging is the key to it. Manually calibrating the pose of the prism to ensure the accuracy of coaxial imaging, which has disadvantages such as difficult operation and long time consuming. This paper analyzes the image relation between the upper and lower parts of the prism based on the optical imaging theory, and solves the image transformation relation between the target part and the assembly part with an experimental method, and proposes a rapid calibration method for the alignment prism based on the image transformation relation. The tedious steps of manually adjusting the prism are eliminated. The experimental results show that this method realizes the high-precision assembly of parts by the micro-assembly system.

1. Introduction
The micro-assembly system is mainly composed of a camera module, an illumination module, an optical alignment module, an image processing unit module and a mechanical motion actuator module. The orientation adjustment accuracy of the prism is a key factor affecting the alignment accuracy of the assembly[1]. Right-angle prism is a key component of optical inspection, and its processing and installation accuracy will seriously affect the alignment and assembly accuracy. The installation and fixation accuracy must be ensured by the prism fixture and subsequent adjustments[2]. At present, the image alignment accuracy is calibrated by manually adjusting the prism pose, but there is a lack of analysis of the transformation relation between the part images when the prism pose is deflected. According to the principle of optical imaging, this paper analyzes the light path of the parts with the law of refraction and reflection in vector form, demonstrates the image transformation relation, and is applied in the alignment assembly experiment.

2. Alignment assembly based on the principle of coaxial imaging
The key to coaxial alignment assembly is coaxial imaging, and the realization method of coaxial imaging is to place a prism that deflects the light direction between the target part and the part to be assembled. The image information of the two parts can be located on the prism. The camera on the side acquires it at the same time, and obtains the pose deviation between the two parts in the image, converts it into pulse information and transmits it to the micro-motion platform, and adjusts the pose of the parts to be assembled.
The cube beam splitter prism is composed of two triangular prisms. The bonding surface of the two is coated with a semi-reflective semi-transparent film, and the working surface far away from the lens is coated with a total reflection film. Its main function is to change the reflection of the measured target through total reflection light\(^3\). It can be seen from Figure 1 that there is an alignment optical path difference between the target part and the part to be assembled. In order to obtain a clear image at the same time, a preliminary initial calibration must be performed before the alignment starts to eliminate the optical path difference.

3. Imaging relation analysis

3.1. Prism pose representation
The optical axis of the reflecting prism becomes a broken line due to the effect of the reflecting surface, and the optical axis sometimes even turns in space. Therefore, the concept of space must be used to study the conjugate relationship between the object and the image. For this purpose, a right-hand rectangular object coordinate needs to be established in the object space\(^4\). The ideal position of the prism is: the center line of the prism is perpendicular to the reference plane of the part to be assembled and the target part\(^5\), and the upper and lower incident working surfaces are parallel to the plane, and the exit surface is parallel to the plane, and the side parallel to the plane. In this way, a pair of object points with a coaxial relationship on the top and bottom of the prism are coincident in the image, which is used to establish the image reference for coaxial alignment assembly.

When the spatial position of the prism is shifted, the field of view of the visual system will shift accordingly, causing the position of the part in the image to change, but there is no imaging error between the upper and lower images of the prism. Therefore, when the prism is not calibrated, only the influence caused by the rotation of the prism is considered when analyzing the deviation of the upper and lower points of the same projection on the plane on the image.

As shown in Figure 2, the apex angle A of the prism coincides with the origin O of the coordinate system. When the prism rotates, the position of this point remains unchanged. When the prism is at rest, the direction of each working surface is known. Let the upper incident surface be \(\pi_1\), the lower incident surface be \(\pi_2\), the total reflection surface be \(\pi_3\), the semi-reflective semi-transparent film plane be \(\pi_4\), the exit surface be \(\pi_5\), and the normal vectors of the plane are respectively \(t_1\), \(t_2\), \(t_3\), \(t_4\) and \(t_5\).
3.2. Light direction analysis

The light $L_p$ emitted from the object point $P_0(x_0, y_0, h + l)$ of the upper part passes through $L_{p1} - L_{p2} - L_{p3} - L_{p4}$, and finally enters from the lens objective lens to participate in imaging. Since the lens is a telecentric lens, the imaging principle is shown in Figure 3 below, so it is only parallel to the lens. Only the light emitted from the optical axis can participate in imaging, and the direction of $L_{p4}$ is (0,1,0).

Similarly, the light $L_Q$ emitted by the object point $Q_0(x_0, y_0, -h)$ of the lower part passes through $L_{Q1} - L_{Q2} - L_{Q3} - L_{Q4} - L_{Q5}$, and the final $L_{Q5}$ direction incident from the lens is also (0,1,0).

![Figure 3 Imaging principle of telecentric lens](image)

According to the reversible characteristics of the light path, combined with the vector form of the law of refraction and the law of reflection, the direction of each segment of light is deduced. The law of refraction in vector form is as follows:

$$e = \frac{n_1}{n_2}$$  \hspace{1cm} (1)

$$\cos \theta_1 = -\vec{L} \cdot \vec{N}$$  \hspace{1cm} (2)

$$\cos \theta_2 = \sqrt{1 - \frac{1 - \cos^2 \theta_1}{e^2}}$$  \hspace{1cm} (3)

$$\vec{T} = \frac{\vec{L}}{e} + \vec{N} \cdot \left( \frac{\cos \theta_1}{e} - \cos \theta_2 \right)$$  \hspace{1cm} (4)

Among them, $\vec{T}$ is the exit vector after refraction, $\vec{L}$ is the incident vector, and $\vec{N}$ is the normal vector of the refracting surface. Obviously, when the direction of the incident light and the normal vector of the refraction surface are known, the direction of the refracted light is also determined.

The reflection law in vector form is as follows:

$$\vec{T} = \vec{L} - 2(\vec{L} \cdot \vec{N}) \cdot \vec{N}$$  \hspace{1cm} (5)

Among them, $\vec{T}$ is the exit vector after reflection, $\vec{L}$ is the incident vector, and $\vec{N}$ is the normal vector of the reflecting surface. Similarly, when the direction of the incident light and the normal vector of the reflecting surface are known, the direction of the reflected light is also determined.

Since the normal vector of each working surface of the prism and the final direction of the emitted light are both known quantities, the direction vector of each segment of light can be reversibly derived, that is, the direction vector of each segment of light is related to the normal vector of each working surface of the prism. In a certain pose of the prism, both are constants.

3.3. Derivation of the coordinate relation between the deflection points of the optical path

From the above analysis, when the prism is at rest, regardless of the prism's pose, the upper light is the same as the lower light, and the direction and the direction of each segment of light and the equations of each plane passing by are known. And the coordinate points of the light rays starting from the part plane are known numbers, and the coordinate transformation relation between the turning points of the light rays can be derived for the general situation.

Suppose a certain piece of light $L_i$ is emitted from the point $P_{i-1}(x_{i-1}, y_{i-1}, z_{i-1})$, and its direction vector is $(m_i, n_i, r_i)$, then the linear equation expression is:

$$\frac{x - x_{i-1}}{m_i} = \frac{y - y_{i-1}}{n_i} = \frac{z - z_{i-1}}{r_i}$$  \hspace{1cm} (6)
The ray $L_i$ intersects the plane $\pi_i$ at $P_i(x_i, y_i, z_i)$, and the expression of the plane is:

$$A_i x + B_i y + C_i z + D_i = 0$$  \hspace{1cm} (7)

Simultaneously the expression equations of the straight line and the plane, the coordinates of $P_i(x_i, y_i, z_i)$ can be obtained, expressed in matrix form as:

$$\begin{bmatrix}
x_i
y_i
z_i
\end{bmatrix} = \frac{1}{A m_i + B n_i + C r_i} \begin{bmatrix}
B r_i + C r_i & -B m_i & -C m_i & -D m_i
-A r_i & B r_i + C r_i & -C n_i & -D n_i
0 & 0 & A m_i + B n_i & -D r_i
\end{bmatrix} \begin{bmatrix}
x_{i-1}
y_{i-1}
z_{i-1}
\end{bmatrix}$$  \hspace{1cm} (8)

Since $A_i, B_i, C_i, D_i, m_i, n_i, r_i$ are all constants in a specific pose of the prism, the coordinate value $(x_i, y_i, z_i)$ of $P_i$ is the coordinate value $(x_{i-1}, y_{i-1}, z_{i-1})$ of $P_{i-1}$ function, let:

$$\begin{bmatrix}
x_{i-1}
y_{i-1}
z_{i-1}
\end{bmatrix} = T_i \cdot \begin{bmatrix}
x_{i-1}
y_{i-1}
z_{i-1}
\end{bmatrix}$$  \hspace{1cm} (9)

According to the order of light incidence, apply the above process to each working surface of the prism one by one to complete the actual optical path tracing of various prism systems [4]. For the upper light, recursively starting from the known point $P_0$, the exit point $P_3$ and $P_0$ have the following relation:

$$\begin{bmatrix}
x_{P_3}
y_{P_3}
z_{P_3}
\end{bmatrix} = T_{P_3} \cdot T_{P_2} \cdot T_{P_1} \cdot \begin{bmatrix}
P_0
1
\end{bmatrix} = T_3 \cdot \begin{bmatrix}
P_0
1
\end{bmatrix} = \begin{bmatrix}
a_{l_1} & a_{l_2} & a_{l_3} & a_{l_4}
b_{l_1} & b_{l_2} & b_{l_3} & b_{l_4}
c_{l_1} & c_{l_2} & c_{l_3} & c_{l_4}
0 & 0 & 0 & 1
\end{bmatrix} \begin{bmatrix}
x_0
y_0
z_0
1
\end{bmatrix}$$  \hspace{1cm} (10)

Similarly, for the light below:

$$\begin{bmatrix}
x_{Q_4}
y_{Q_4}
z_{Q_4}
\end{bmatrix} = T_{Q_4} \cdot T_{Q_3} \cdot T_{Q_2} \cdot T_{Q_1} \cdot \begin{bmatrix}
Q_0
1
\end{bmatrix} = T_4 \cdot \begin{bmatrix}
Q_0
1
\end{bmatrix} = \begin{bmatrix}
a_{r_1} & a_{r_2} & a_{r_3} & a_{r_4}
b_{r_1} & b_{r_2} & b_{r_3} & b_{r_4}
c_{r_1} & c_{r_2} & c_{r_3} & c_{r_4}
0 & 0 & 0 & 1
\end{bmatrix} \begin{bmatrix}
x_0
y_0
z_0
1
\end{bmatrix}$$  \hspace{1cm} (11)

### 3.4. Image transformation relation derivation

Since the rays finally entering the lens are parallel to the optical axis, the pixel coordinates in the image reflect the projection coordinates of $P_3(x_{P_3}, y_{P_3}, z_{P_3})$ and $Q_4(x_{Q_4}, y_{Q_4}, z_{Q_4})$ on the xoz plane. Let $P_3'(x_{P_3}, z_{P_3})$ and $Q_4'(x_{Q_4}, z_{Q_4})$ be the projection coordinates of $P_3$ and $Q_4$ respectively on the plane:

$$\begin{bmatrix}
x_{P_3}
y_{P_3}
z_{P_3}
\end{bmatrix} = \begin{bmatrix}
a_{l_1} & a_{l_2} & a_{l_3}
c_{l_1} & c_{l_2} & c_{l_3}
0 & 0 & 1
\end{bmatrix} \begin{bmatrix}
x_0
y_0
z_0
\end{bmatrix} = \begin{bmatrix}
a_{l_1} & a_{l_2} & a_{l_3}
c_{l_1} & c_{l_2} & c_{l_3}
0 & 0 & 1
\end{bmatrix} \begin{bmatrix}
h+l
1
\end{bmatrix}$$  \hspace{1cm} (12)

$$\begin{bmatrix}
x_{Q_4}
y_{Q_4}
z_{Q_4}
\end{bmatrix} = \begin{bmatrix}
a_{r_1} & a_{r_2} & a_{r_3}
c_{r_1} & c_{r_2} & c_{r_3}
0 & 0 & 1
\end{bmatrix} \begin{bmatrix}
x_0
y_0
z_0
\end{bmatrix} = \begin{bmatrix}
a_{r_1} & a_{r_2} & a_{r_3}
c_{r_1} & c_{r_2} & c_{r_3}
0 & 0 & 1
\end{bmatrix} \begin{bmatrix}
h
1
\end{bmatrix}$$  \hspace{1cm} (13)

Where $T_3'$ and $T_4'$ are matrices, and $h+l$ and $-h$ are constants, the above equation can be rewritten as:

$$T_3'^{-1}(x_{P_3}, z_{P_3}) = T_3'^{-1}(x_{Q_4}, z_{Q_4})$$  \hspace{1cm} (14)
Further establish the relation between the image of the upper part and the image of the lower part, let 
\
$$T_{o5} = T'_o \cdot T^{-1}_o$$
\
then for the points on the \(xoy\) plane with the same projection coordinates, but respectively above and below the prism, the transformation relation in the image is:

$$T_o \cdot (x_{p3}, z_{p3}, 1)^T = (x_{q4}, z_{q4}, 1)^T$$

Since the above equations hold true for any values of \(x_{p3}\) and \(z_{p3}\), the third row element of \(T_{o5}\) is \((0,0,1)\), so the transformation relation between these two points in the image is an affine transformation relation.

4. Prism image transformation relation calibration

4.1. Calibration images acquisition

The visual inspection module built by this coaxial alignment assembly system is shown in the figure 4 above. It uses Basler's acA4024-8gc camera with a resolution of 4024*3036 and a pixel size of 1.85 microns. It has the characteristics of high pixels and small unit pixel size. The lens adopts VST's VS-TCH-08-108 telecentric lens, which can eliminate the phenomenon of near large and small objects when shooting objects. The light source adopts two RIH square light sources, which are respectively installed on the upper and lower incident surfaces of the prism.

In the experiment, a pair of standard shaft holes are used, the shaft is clamped on the fixture in the alignment assembly station, and the hole is placed on the micro-motion platform below. By operating the fixture and the micro-motion platform, the shaft and the hole can be coaxially assembled. Then the Z axis of the operating fixture is moved up vertically, which is convenient for moving in the coaxial alignment detection module for visual inspection of alignment.

Because the coaxial alignment detection module has freedom of movement in the x and y directions, the prism can be extended between the shaft and the hole, and the coaxial alignment detection module can be moved while ensuring that the shaft hole appears in the camera's field of view at the same time. So that the prism can be moved to different horizontal positions between the shaft holes, and in this way, multiple sets of images of different positions of the shaft holes in the prism imaging field of view are obtained.
4.2. Axle and hole position recognition

After the image is thresholded, morphologically processed, and edge detected, the CircleFinder function in the image processing software MIL10 is used to detect the position of the shaft hole in the image, and the results are as follows:

| Axis coordinates | Hole coordinates | Axis coordinates | Hole coordinates |
|------------------|-----------------|-----------------|-----------------|
| (693.37,980.06)  | (1164.44,750.46) | (2340.26,1925.64) | (2801.23, 1700.87) |
| (1015.92,995)    | (1477.89,760.6)  | (2076.31,1920.11) | (2533.48, 1689.51) |
| (1366.57,1005.27) | (1828.74,769.75) | (1690.51, 1910.24) | (2150.89, 1682.47) |
| (1726.25,1014.37) | (2189.44,780.52) | (1416.35, 1900.61) | (1872.73,1677.05) |
| (2075.6,1021.75) | (2539.12,789.04) | (1099.69, 1894.69) | (1557.14, 1661.04) |
| (2363.97,1030.06) | (2826.35,796.85) | (765.3, 1885.08)  | (1218.03,1651.86) |
| (2357.93,1175.58) | (2822.14,942.66) | (748.14, 2167.54) | (1206.1, 1940.99) |
| (2095.92,1170.87) | (2557.87,936.741) | (1042.04, 2176.14) | (1500.23, 1946.84) |
| (1720.6,1161.4)  | (2183.24,926.692) | (1276.81, 2179.4) | (1734.2,1951.91) |
| (1232.98,1146.88) | (1695.84,912.903) | (1703.05, 2191.95) | (2159.67,1963.35) |
| (983.6,1139.98)  | (1441.91,907.536) | (2016.68, 2197.75) | (2474.96,1970.71) |
| (658.3,1129.65)  | (1118.68,897.578) | (2392.71,2210.1)  | (2849.29,1981.11) |
| (647.02,1485.01) | (1108.76,1255.17) | (2289.3,2372.57)  | (2744.09,2142.35) |
| (1159.83,1501.4) | (1617.78,1267.37) | (1885.18,2359.35) | (2340.85,2132.41) |
| (1538.59,1509.57) | (2000.11,1278.34) | (1617.99,2351.23) | (2071.57,2125.8)  |
| (1893.52,1519.06) | (2351.88,1290.07) | (1308.84,2343.19) | (1763.48,2118.27) |
| (2126.21,1523.32) | (2586.53,1295.8)  | (1004.03,2334.42) | (1462.1, 2108.49) |
| (2360.4,1528.4)  | (2815.9,1301.04) | (713.15, 2327.1) | (1171.17,2100.64) |

4.3. Image transformation matrix calculation

The coordinates of the 36 sets of corresponding points in the above content can be used as overdetermined equations to calculate the coefficients of the affine transformation matrix. After calculation, the transformation matrix $R$ between the point in the image above the prism and the corresponding point in the lower image is:

$$
\begin{bmatrix}
  a_0 & a_1 & a_2 \\
  b_0 & b_1 & b_2 \\
  0 & 0 & 1
\end{bmatrix} = \begin{bmatrix}
  9.99761 \times 10^{-1} & -5.35360 \times 10^{-3} & 4.68693 \times 10^2 \\
  2.76521 \times 10^4 & 1.00492 & -2.38719 \times 10^2 \\
  0 & 0 & 1
\end{bmatrix}
$$

5. Coaxial alignment assembly experiment

5.1. Image affine transformation processing

In the coaxial alignment assembly, the upper part is used as an assembly reference and remains stationary, and the lower part is placed on the micro-motion platform. The vision module recognizes the posture deviation of the upper and lower parts, and outputs the adjustment amount to the micro-motion platform to adjust the lower part. The pose of the part allows it to be coaxially aligned and assembled with the target part above.
The adjustment amount of the micro-motion stage is converted according to the relation between the motion coordinate system of the micro-motion stage and the image coordinate system of the lower part. Therefore, as shown in Figure 6 above, the image of the upper part needs to be affine transformed, that is, normalized to After the image coordinate system of the parts below, the pose is compared to facilitate the conversion of the adjustment amount of the micro-motion stage.

5.2. Pose recognition of parts
According to the assembly process of the part, draw a template with both assembly relation and shape characteristics, and use the GeometricModelFinder module in the MIL10 software to identify the target part and the image of the part to be assembled to detect the position and pose of the part. The results are as follows:

According to the angle and position information of the part, the angle and position deviation between the target part and the part to be assembled are obtained and output to the control system.

5.3. Assembly adjustment
After the part pose feature recognition, the control system converts the pose deviation amount in the image into the pulse value adjustment amount of the adjustment motor, and outputs it to the micro-motion stage motor. The following formula is the calculation formula for converting the pixel value deviation amount to the pulse value adjustment amount of the micro-motion stage.

\[
\begin{pmatrix}
p_x \\
p_y \\
\end{pmatrix} = \begin{pmatrix}
-23.218199 & -0.614941 \\
-0.602567 & 23.155790
\end{pmatrix} \begin{pmatrix}
\Delta u \\
\Delta v
\end{pmatrix}
\]

In the assembly adjustment, the micro-motion stage is first rotated to complete the angle alignment; then the parts to be assembled are recognized for the second time, and the translational adjustment of the micro-motion stage is performed according to the coordinate difference with the target part. Finally, the fixture carries the target part and moves down vertically, and stops when it reaches the assembly position to complete the assembly of the two parts.

6. Conclusion
This paper proposes a rapid calibration method for alignment prism based on image transformation relation. This method solves the affine transformation relation matrix between the upper and lower images of the prism. During the assembly alignment detection process, the image of the target part above the prism is simulated. The shooting transformation is compared with the parts to be assembled below. Compared with the previous alignment assembly process, manual calibration of the prism is omitted.
And through experiments, the image transformation matrix was solved to be used for part pose recognition and assembly adjustment. Finally, the assembly was successfully realized, and the feasibility of the calibration method was verified.

References
[1] Yonglong Tang, Zhijing Zhang, Xiaofeng Zhang, Yuan Sun. Design of Orthogonal Accurate Alignment System for Micro-assembly[J]Optical Precision Engineering, 2012,07:1542-1550.
[2] Qiang Wang, Zhijing Zhang, Xin Ye, Yuan Sun, Xiaofeng Zhang, Meixia Bu. Orientation adjustment and error analysis of micro-assembly inspection prism mechanism[J]Journal of Beijing Institute of Technology, 2011,08:896-900.
[3] Xuegen Zhang, Yu Sun, Yongxi He. The influence of the position change of the beam splitter prism on the angle measurement error of the self-collimating goniometer tube[J]Missile and Space Launch Technology, 2017,01:22-25.
[4] Zhijian Wang. Mathematical analysis of prism rotation theorem[J] Optical technology, 1982,03:14-21.
[5] Yongtian Wang. Actual optical path tracing of complex prisms[J]Acta Optics, 1991,07:640-650.