1. Introduction

Modern control systems are characterized by increased complexity, flexibility, intelligence and enhanced ability to handle uncertainty. In order to incorporate the above features, the control systems need to possess significant capabilities such as perception, knowledge acquisition, learning, adaptability, and reasoning. The ability to perceive its environment forms a very important characteristic of such control systems. The revolutionary advancement in the field of sensor technology that has led to the development of superior sensing capabilities, and progress in computing and information processing, has made it possible to develop systems with enhanced perceptive abilities. Modern control systems generally employ multiple sensors to provide diverse, complementary as well as redundant information. These multiple sensor systems necessitate the development of sensor fusion algorithms that can combine information in a coherent and synergistic manner to yield a robust, accurate, and consistent description of quantities of interest in the environment.

There are several issues that arise when fusing information (Brooks & Iyengar, 1998, Hall & Llinas 2001) from multiple sources, some of which include data association, sensor uncertainty, and data management. The most fundamental of these issues arise from the inherent uncertainty in sensor measurement. The uncertainties in sensor measurement are caused not only by the device impreciseness and noise, but also manifest themselves from the ambiguities and inconsistencies present within the environment, and from an inability to distinguish between them. The strategies used to fuse data from multiple sensors should be capable of handling these uncertainties, and combining different types of information to obtain a consistent description of the environment. Some of the popular techniques for sensor fusion that are explored extensively in literature include Dempster-Shafer theory for evidential reasoning (Dempster, 1968, Shafer, 1976), fuzzy logic (Yager & Zadeh, 1991, Mahajan et. al., 2001), neural network (Garg & Kumar, 2007, Chin, 1994), Bayesian approach (Press, 1989, Berger, 1985), and statistical techniques (McKendall & Mintz, 1992) such as Kalman filter (Maybeck, 1979, Kalman, 1960, Sasiadek, 2002). All of these methods differ in the manner they attempt to model the uncertainties inherent in the sensor measurements. Another possible uncertainty that arises in the sensor measurement process occurs when the measurements become corrupted and appear spurious in nature. Such corrupted measurements are difficult to model because they are not directly attributable to the inherent noise or other sources of uncertainty mentioned above. The cause of the corruption...
may be due to events such as permanent sensor failures, short duration spike faults, or nascent (slowly developing) failures. Previous attempts at developing experimental models usually preclude the use of spurious measurements, and represent uncertainties attributable only to sensor noise and inherent limitations. Fusion techniques based on these incomplete models provide inaccurate estimation that can eventually result in potentially damaging action by the control system. Hence, a sensor validation scheme is necessary to identify spurious measurements so that they can be eliminated before the fusion process. There are several techniques reported in the literature for sensor validation and identification of inconsistent data. Many of them are limiting because they are based on specific failure models; these techniques can work well for events that occur due to known failure modes, however, they do not capture all possible failure events and often perform poorly when unmodeled failures occur. As a means to detect inconsistency, there should be either redundancy in the data, or some availability of a priori information. For example, in the case where a priori information is available, researchers have used the Nadaraya-Watson Estimator (Wellington et al., 2002) and a priori observations to validate sensor measurements. A few researchers have used a model based Kalman filter approach (Del Gobbo et al., 2001), while others have used covariance (Nicholson, 2004, Benaskeur, 2002), probability (Soika, 1997, Ibarguengoytia et al., 2001), fuzzy logic (Frolick et al., 2001), and neural network (Rizzo & Xibilia, 2002) based approaches. Some of these methods are explicit model-based, whereas others require tuning and training. In the general case, where a priori information is often not available, these approaches are typically deficient and can often lead to undesirable results.

This chapter presents a unified sensor fusion strategy based on a modified Bayesian approach that can take uncertainty of sensor data into account and automatically identify the inconsistency in sensor measurements so that the spurious measurements can be eliminated from the data fusion process. First, a novel strategy to accurately and adaptively represent uncertainty in sensor data in the form of probabilistic sensor model is developed. The strategy establishes the dependence of sensor’s uncertainties on some of the environmental parameters or parameters of any feature extraction algorithm used in estimation based on sensor’s outputs. In order to establish this dependence, the approach makes use of a neural network that is trained via an innovative technique that obtains training signal from a maximum likelihood estimator. The proposed method, then, adds a term to the commonly used Bayesian formulation. This term is an estimate of the probability that the data is not spurious, based upon the measured data and the unknown value of the true state. In fusing two measurements, it has the effect of increasing the variance of the posterior distribution when measurement from one of the sensors is inconsistent with respect to the other. The increase or decrease in variance can be estimated using the information theoretic measure “entropy”. The proposed strategy was verified with the help of extensive computations performed on simulated data from three sensors. A comparison was made between two different fusion schemes: centralized fusion in which data obtained from all sensors were fused simultaneously, and a decentralized or sequential Bayesian scheme that proved useful for identifying and eliminating spurious data from the fusion process. The simulations verified that the proposed strategy was able to identify spurious sensor measurements and eliminate them from the fusion process, thus leading to a better overall estimate of the true state. The proposed strategy was also validated with the help of experiments performed using stereo vision cameras, one infra-red proximity sensor, and one
laser proximity sensor. The information from these three sensing sources was fused to obtain an occupancy profile of the robotic workspace.

This chapter is organized as follows: First, it introduces Bayesian technique for sensor fusion in Section 2. Next, in Section 3, it presents the neural network based sensor modeling technique. The proposed strategy for inconsistency detection and data fusion in Bayesian framework is presented in Section 4. Simulation studies to verify the proposed method for inconsistency detection is presented in Section 5. Section 6 presents the experimental validation carried out in a robotic workcell using three independent sensory sources. Finally, conclusions are presented in Section 7.

2. Bayesian technique for sensor fusion

Bayesian inference (Press, 1989, Berger, 1985) is a data fusion algorithm based on Bayes’ theorem (Bayes, 1763) that calculates posterior probability distribution of \( n \)-dimensional state vector \( \mathbf{X} \), after the observation or measurement denoted by \( \mathbf{Z} \) has been made. The probabilistic information contained in \( \mathbf{Z} \) about \( \mathbf{X} \) is described by a probability density function (p.d.f.) \( p(\mathbf{Z} | \mathbf{X}) \), known as likelihood function, or the sensor model, which is a sensor dependent objective function based on observation. The likelihood function relates the extent to which the \textit{a posteriori} probability is subject to change, and is evaluated either via offline experiments or by utilizing the available information about the system. If the information about the state \( \mathbf{X} \) is made available independently before any observation is made, then the likelihood function can be improved to provide more accurate results. Such \textit{a priori} information about \( \mathbf{X} \) can be encapsulated as the prior probability and is regarded as subjective because it is not based on observed data. Bayes’ theorem provides the posterior conditional distribution of \( \mathbf{X} = \mathbf{x} \), given \( \mathbf{Z} = \mathbf{z} \), as

\[
p(\mathbf{X} = \mathbf{x} | \mathbf{Z} = \mathbf{z}) = \frac{p(\mathbf{Z} = \mathbf{z} | \mathbf{X} = \mathbf{x})p(\mathbf{X} = \mathbf{x})}{\int p(\mathbf{Z} = \mathbf{z} | \mathbf{X} = \mathbf{x})p(\mathbf{X} = \mathbf{x})d\mathbf{x}} = \frac{p(\mathbf{Z} = \mathbf{z} | \mathbf{X} = \mathbf{x})p(\mathbf{X} = \mathbf{x})}{P(\mathbf{Z} = \mathbf{z})} \tag{1}
\]

Since the denominator depends only on the measurement (the integration is carried out over all possible values of state), an intuitive estimation can be made by maximizing this posterior distribution, i.e., by maximizing the numerator of Equation (1). This is called \textit{Maximum a posteriori} (or MAP) estimate, and is given by:

\[
x_{\text{MAP}} = \arg \max_{\mathbf{x}} p(\mathbf{X} = \mathbf{x} | \mathbf{Z} = \mathbf{z}) = \arg \max_{\mathbf{x}} p(\mathbf{Z} = \mathbf{z} | \mathbf{X} = \mathbf{x})p(\mathbf{X} = \mathbf{x}) \tag{2}
\]

The data from multiple sensors can be fused simultaneously (centralized fusion scheme), or sequentially (decentralized fusion). In this chapter, we will focus on decentralized fusion scheme in which, at any given instant, only two measurements or beliefs are fused. The recent interest in sensor networks, where distributed nodes possess capability to process information, has necessitated the development of algorithms to fuse information in a decentralized manner. The decentralized approach can be easily implemented in a distributed Bayesian framework where the posterior distribution obtained from old measurements becomes the prior distribution. Hence, the addition of new sensor measurement \( \mathbf{z}_n \) to the belief obtained from \( n-1 \) sensors \( \bar{\mathbf{Z}}_{1:n-1} = \mathbf{z}_1, \mathbf{z}_2, \ldots, \mathbf{z}_{n-1} \) can be achieved in an incremental manner via Equation (3):
It may be noted that Equation (3) is valid only when measurements from different sensors are conditionally independent.

The Bayesian approach offers several advantages, including: appropriate representation of uncertainties using probability distributions; a well-defined mechanism to combine prior information with current sensor information; the existence of several machine learning algorithms to carry out the calculation of estimates and predictions; and thorough statistical characterization of the quantities of interest. Since the estimation takes into account available data from all previous as well as current experiments, the approach leads to a theoretically optimal solution. However, for most practical applications, a lack of priors or use of non-informative priors presents difficulties for Bayesian-based sensor fusion approaches. Assumptions regarding informative priors creates the possibility of unreasonable fusion between priors and likelihood functions. Moreover, most of the fusion strategies based on Bayesian approaches reported in the literature handle inconsistency in data rather poorly. In practical real-world scenarios, where data generated by sensors might be incomplete, incoherent or inconsistent, this approach might lead to erroneous results. Consequently, the inconsistency in data needs to be dealt with accordingly when Bayesian approaches are used.

3. Sensor modeling

Sensor modeling (Manyika & Durrant-Whyte, 1994, Kumar et al., 2005b, Kumar et al., 2006a) deals with developing an understanding of the nature of measurements provided by the sensor, the limitations of the sensor, and probabilistic understanding of the sensor performance in terms of the uncertainties. The information supplied by a sensor is usually modeled as a mean about a true value, with uncertainty due to noise represented by a variance that depends on both the measured quantities themselves and the operational parameters of the sensor. A probabilistic sensor model is particularly useful because it facilitates the determination of the statistical characteristics of the data obtained. This probabilistic model is usually expressed in the form of probability density function (p.d.f.) \( p(z|x) \) that captures the probability distribution of measurement by the sensor \( z \) when the state of the measured quantity \( x \) is known. This distribution is extremely sensor specific and can be experimentally determined (Durrant-Whyte, 1988).

3.1 Estimation of sensor model parameters

Maximum Likelihood (ML) method is a procedure for finding the value of one or more parameters for a given statistical data which maximizes the known likelihood distribution. If Gaussian distribution is considered, the distribution representing the sensor model is given by:

\[
p_{D}(z_{i} | \sigma, x_{i}) = \frac{1}{\sigma \sqrt{2\pi}} e^{-\frac{(z_{i} - x_{i})^{2}}{2\sigma^{2}}} \quad i=1,2,\ldots,n
\]
parameter to be estimated. The likelihood function is the joint probability of the data given by:

\[
L(\sigma) = \prod_{i=1}^{n} \frac{1}{\sigma \sqrt{2\pi}} e^{-\frac{(x_i - \mu)^2}{2\sigma^2}} = \frac{1}{\sigma^n (2\pi)^{n/2}} e^{-\frac{\sum (x_i - \mu)^2}{2\sigma^2}}
\]  

(5)

and the parameter \( \sigma \) can be estimated via ML method by maximizing \( L(\sigma) \) given by Equation (5). This results in a constant value of \( \sigma \) representing a rigid sensor model.

Most of the published research on sensor fusion is based on the development of rigid sensor models. In practice, the performance of sensors or any source of information depends upon several factors. These include, for example, the environmental conditions under which the measurements were made, and the performance of estimation/calibration algorithm. Establishing dependence of a sensor’s performance on various parameters of environment and other signal/feature extraction algorithms is not a trivial task. Statistical techniques such as correlation analysis can be used to determine the manner in which these factors affect the sensor’s output. Selecting the factors that can possibly affect the sensor output is difficult, and is mostly based on heuristics. Many feature extraction algorithms include goodness-of-fit function that can be investigated to observe the correlation with uncertainties in sensor output.

After the factor which affects the sensor’s performance has been selected, the next challenge is to establish a functional correspondence between the factor and the uncertainty in the sensor’s output. Statistical system identification, regression analysis, or any mapping algorithm can be investigated to establish the correspondence. It might be difficult, if not impossible, to obtain the mathematical relation, and in the absence of such mathematical relation, model-based statistical approach would be difficult to use. In this chapter, the universal approximation capabilities of neural networks have been used to establish this correspondence.

### 3.2 Proposed neural network based sensor modeling

A neural network (NN) (Rumelhart & McClelland, 1988, Haykin, 1998) is an information-processing paradigm inspired by the way in which the heavily interconnected, parallel structure of the human brain processes information. They are often effective for solving complex problems that do not have an analytical solution or for which an analytical solution is too difficult to be found. Currently, they are being applied in many real-world problems (Garg & Kumar, 2007). Three-layered NNs (i.e., one input layer, one output layer and one hidden layer), with hidden layer having sufficient nodes and a sigmoid transfer function, and linear transfer function in the input and output layers (Hornik, 1989) are considered to be universal approximators. In this chapter, a three-layered NN has been used to obtain a correspondence between the parameters of the distribution representing the sensor model and the factors which affect sensor’s performance. The input to the neural network is the vector \( Q \) which represents vector of environmental or algorithmic factors that affect the sensor’s performance. Output of the network is the vector of parameters of the distribution representing the sensor model. Hence, if the sensor model is represented by a Gaussian distribution, the parameter \( \sigma \) is given by:
\[ \sigma = NNET(Q, W, B) \tag{6} \]

\( W \) is the weight matrix, and \( B \) is the bias matrix. Back-propagation (BP), based on gradient descent technique, is a fairly popular method for training neural networks that establishes a particular set of weights obtained by adjusting them based on the errors between the actual and target output signals. For the neural network considered for the system in this research, however, the target data for \( \sigma \) is unknown, and cannot be obtained directly from experiments. Here, the neural network is trained in a novel manner from the signals obtained from Maximum Likelihood parameter estimation approach. Likelihood function that needs to be maximized is given by Equation (5), in which parameter \( \sigma \) is represented by a neural network function given by Equation (6). Hence, the likelihood function that needs to be maximized by choosing appropriate weights and biases of the neural network is given by:

\[
L(W, B) = \frac{1}{[NNET(Q, W, B)]^n} e^{-\frac{1}{2} \sum_{i=1}^{n} (z_i - x_i)^2} \tag{7}
\]

The weights and biases can be calculated using the gradient descent method or via evolutionary strategies (Goldberg, 1989). The technique described above has been used to obtain models of infra-red proximity sensor and vision sensors in stereo configuration.

### 4. Fusion of inconsistent multi-sensor data

Sensors often provide spurious data (Kumar et al., 2006b, 2007) which can be due to sensor failure or due to some inherent limitation of the sensor and/or some ambiguity in the environment. The Bayesian approach described in Section 2 is inadequate in handling this type of spurious data. The approach does not have a mechanism to identify when data from sensors is incorrect. The following paragraphs describe the use of a Bayesian-based approach for fusion of data from multiple sensors that takes into account measurement inconsistency.

While building a stochastic sensor model, generally spurious data are identified and eliminated. Hence these experimentally developed sensor models represent uncertainties arising only from sensor noise. If the event \( s = 0 \) represents that the data obtained from a sensor is not spurious, then the sensor model developed in this manner actually represents the distribution \( p(Z = z_i | X = x, s = 0) \). From Bayes' theorem, the probability that the data \( z_i \) measured by sensor ‘\( i \)' is not spurious conditioned upon the actual state \( x \), is given by:

\[
[p(s = 0 | X = x, Z = z_i)] = \frac{[P(s = 0)] [p(Z = z_i | X = x, s = 0)]}{\sum_s [P(s)] [p(Z = z_i | X = x, s)]} \tag{8}
\]

\([P(s = 0)]\) is the sensor specific prior probability that the data provided by Sensor \( i \) is not spurious. The denominator of the right hand side of the above equation is a summation carried over all possible values of \( s \) which are 0 and 1. The above equation can be re-written as:
To combine the sensor measurement from sensor \( n \) sequentially with the current belief obtained from sensors ‘\( 1, 2...n-1 \)’, Equation (3) can be re-written as:

\[
p(X = x \mid Z_{1\ldots n} = z_1, z_2, \ldots z_n) = \frac{[P(s = 0)]_n P(Z = z_n \mid X = x, s = 0) P(X = x \mid Z_{1\ldots n-1} = z_1, z_2, \ldots z_{n-1})}{p(z_n)[P(s = 0 \mid X = x, Z = z_n)]_n}
\]

(10)

Hence, the introduction of term \([P(s = 0 \mid X = x, Z = z_n)]_n\) in the denominator has the effect of increasing the spread (variance) of the posterior if the new measurement has a greater probability of being spurious, and decreasing the spread of the posterior if the new measurement has a lower probability of being spurious. The increase or decrease in the spread of the posterior distribution can be easily ascertained by determining the information content given by the entropy of distribution obtained from the following equation:

\[
H(X) = \int -p(X = x \mid Z = z_1, z_2, \ldots z_n) \log(p(X = x \mid Z = z_1, z_2, \ldots z_n)) \text{d}x
\]

(11)

Entropy of a variable represents the uncertainty in that variable. A larger value of entropy implies more uncertainty and hence less information content. The fusion of a new measurement should always lead to a decrease in entropy, and fusion should always be done in order to reduce entropy. Based on increasing or decreasing the entropy of the posterior, this method can identify and eliminate spurious data from a sensor. It is noted that the prior probability \([P(s = 0)]_n\) has a constant value and simply acts as a constant weighting factor in Equation (10). This value does not influence the posterior distribution nor the MAP estimate of the state.

**4.1 Bayesian fusion without consideration of spuriousness in data (method 1)**

If the spurious nature of the sensor data is not considered, and the models of the ‘\( n \)’ sensors are given by the following Gaussian likelihood function:

\[
p(Z = z_k \mid X = x) = \frac{1}{\sigma_k \sqrt{2\pi}} e^{-\frac{(x - z_k)^2}{2\sigma_k^2}}\quad k = 1,2,\ldots n
\]

(12)

then, from Bayes’ Theorem the fused MAP estimate is given by:

\[
\hat{x}_{MAP} = \arg \max_x [p(Z = z_1 \mid X = x)p(Z = z_2 \mid X = x)\ldots p(Z = z_n \mid X = x)]
\]

(13)

**4.2 Bayesian fusion with consideration of spuriousness in data (method 2)**

If the spurious nature of the sensor data is considered, then the Gaussian sensor model represented by distribution \( p(Z = z \mid X = x, s = 0) \) is given by:
\[
[p(Z = z | X = x, s = 0)]_k = \frac{1}{\sigma_k \sqrt{2\pi}} e^{-\frac{(x-z_k)^2}{2\sigma_k^2}} k=1,2,...,n
\] (14)

The probability that the measurement from Sensor ‘\(k\)’ is not spurious given the true state ‘\(x\)’ and measurement ‘\(z_k\)’, is assumed to be represented by the following equation:

\[
\left[p(s = 0 | X = x, Z = z_k)\right]_k = e^{-\frac{-(x-z_k)^2}{2\sigma_k^2}} 
\] (15)

An advantage of choosing the above formulation for representing the probability is that the probability is 1 when measurement ‘\(z_k\)’ is equal to the true state ‘\(x\)’, and decreases when the measured value moves away from the true state. The rate at which the probability decreases when the measured value moves away from the true estimate depends upon the parameter ‘\(a_k\)’. The value of the parameter is dependent on the variances of the sensor models and the distance between the output of sensor \(k\) with respect to other sensors.

In the decentralized or sequential fusion scheme, measurements from only two sources are fused at once. The belief resulting from the fusion of two sensors is then fused with the next sensor, and the process continues henceforth. Fusion of two sensors ‘\(k\)’ and ‘\(k+1\)’ using Equation (10) yields:

\[
p(X = x | Z = z_k, z_{k+1}) = \frac{p(s = 0)}{p(s = 0 | X = x, Z = z_k)} \times \frac{p(Z = z_k | X = s = 0)}{p(Z = z_k | X = x, Z = z_{k+1})} \times \frac{p(X = x)}{p(X = x)} \times \frac{p(Z = z_{k+1})}{p(Z = z_{k+1})}
\] (16)

The value of parameter ‘\(a_k\)’ in Equation (15) is assumed to be given by:

\[
a_k^2 = \frac{b_k^2}{(z_k - z_{k+1})^2}
\] (17)

which leads to

\[
p(X = x | Z = z_k, z_{k+1}) = \frac{p(X = x)}{p(Z = z_k, z_{k+1})} \times \frac{1}{\sigma_k \sqrt{2\pi}} e^{-\frac{(x-z_k)^2}{2\sigma_k^2}} \times \frac{1}{\sigma_{k+1} \sqrt{2\pi}} e^{-\frac{(x-z_{k+1})^2}{2\sigma_{k+1}^2}}
\] (18)

The value of parameter ‘\(b_k\)’ is chosen to satisfy the following inequality:

\[
b_k^2 \geq 2\sigma_k^2 (z_k - z_{k+1})^2
\] (19)

Satisfaction of this inequality ensures that the posterior distribution in Equation (18) remains Gaussian and hence has a single peak. The parameter value should be chosen based on
maximum expected difference (represented by ‘\( m \)’) between the sensor readings so that inequality (19) is always satisfied. Hence,

\[ b_i^2 = 2\sigma_i^2m^2 \]  \hspace{1cm} (20)

Substituting Equation (20) in Equation (18) gives:

\[
p(X = x \mid Z = z_k, z_{k+1}) = \frac{P(X = x)}{P(Z = z_k, z_{k+1})} \times \left[ P(s = 0) \right]_k \frac{1}{\sigma_k \sqrt{2\pi}} e^{-\frac{(x-z_k)^2}{m^2}} \times \left[ P(s = 0) \right]_{k+1} \frac{1}{\sigma_{k+1} \sqrt{2\pi}} e^{-\frac{(x-z_{k+1})^2}{m^2}} \times \left[ P(s = 0) \right]_k \frac{1}{\sigma_k \sqrt{2\pi}} e^{-\frac{(x-z_k)^2}{m^2}} \times \left[ P(s = 0) \right]_{k+1} \frac{1}{\sigma_{k+1} \sqrt{2\pi}} e^{-\frac{(x-z_{k+1})^2}{m^2}}
\]  \hspace{1cm} (21)

It is apparent that the entire process has the effect of increasing the value of the variance of individual distribution by a factor of \( \frac{m^2}{m^2-(z_k-z_{k+1})^2} \). Larger differences in sensor measurement imply that the variance increases by a bigger factor. Depending on the squared difference in measurements from the two sensors, the variance of the posterior distribution may increase or decrease as compared to the variance of individual Gaussian distributions representing the sensor models. Therefore, the strategy is capable of determining if fusion of the two measurements would lead to an increase or decrease of the variance of the posterior distribution. In information theoretic terms, the strategy is capable of determining if the fusion leads to an increase in information content (or entropy given by Equation (11)) or not. Based on increasing or decreasing of entropy in the posterior, a decision can be made whether to fuse those two sensors or not. This approach provides an opportunity to eliminate sensor measurements that are spurious and fuse measurements from only those sensors that are consistent, ensuring an increase in information content after fusion.

5. Simulation results

A simulation study was carried out to validate the effectiveness of the proposed strategy in identifying inconsistent data while fusing data from three sensors. A comparative analysis was performed to study the efficiency with which the two methods (described in Section 4) were able to handle inconsistency in data. The following parameters were assumed in the simulation:

Sensor 1: \( P(s = 0) = 0.90 \) and \( \sigma_1 = 3 \)
Sensor 2: \( P(s = 0) = 0.98 \) and \( \sigma_2 = 2 \)
Sensor 3: \( P(s = 0) = 0.94 \) and \( \sigma_3 = 2.5 \)
True value of state: \( x = 20 \)

Simulation data was generated so that Sensor 1 provided 90% of the time normally distributed random data with a mean value of 20 and variance 9. It provided incorrect data
10% of the time which was uniformly distributed outside the Gaussian distribution. Sensor 2 provided 98% of the time normally distributed random data with a mean value of 20 and variance 4, and 2% of the time it provided incorrect data. Similarly, Sensor 3 provided 94% of the time normally distributed random data with a mean value of 20 and variance 6.25, and 6% of the time it provided incorrect data. It may be noted here that the values for \([P(x = 0)]_k\) have been assumed simply for the purpose of generating simulated data. These are not used in the fusion algorithm. Since these values are constants, they do not have any effect on the posterior distribution or the MAP estimate.

Figure 1.(a) illustrates a case when all of the three sensors are in agreement, and measurement from none of the sensors is inconsistent with the rest. It can be seen that posterior distributions obtained from both methods coincide resulting in the same value of MAP estimate. In Figure 1.(b), measurement from Sensor 1 is in disagreement from the other two sensors. Method 1, which is a simple Bayesian fusion and does not take into account inconsistency of data, results in the weighted average of the three measurements. Method 2 identifies the sensor which provides spurious measurements and eliminates that from the fusion process. Hence, it simply considers measurements from Sensors 2 and 3, and fuses

![Diagram](image)

Fig. 1. Fusion of Three Sensors 1.a: All Sensors in Agreement, 1.b: Sensor 1 in Disagreement, 1.c: Sensor 2 in Disagreement, 1.d: Sensor 3 in Disagreement
them appropriately using Equation (21). In a similar manner, Figure 1.(c) and Figure 1.(d) respectively show that measurements from Sensor 2 and Sensor 3 are spurious. The figures show the efficiency with which Method 2 identifies and eliminates spurious measurements, and results in better estimates (closer to the true value) of the variable. Ten thousand (10,000) data points were generated in the manner described above and fusion was carried out using both methods. The mean value of the sum of squared error (MSE) between the fused value and true value for all ten thousand data points was computed. The values of MSE were found as 6.94 for Method 1 and 5.50 for Method 2. Hence, Method 2 was able to reduce the mean square error by approximately 21% when compared to Method 1.

6. Experimental results

The theories developed in Sections 3 and 4 were validated with the help of experiments performed in the Robotics and Manufacturing Automation (RAMA) Laboratory at Duke University. The objective of the experiment was to obtain a three-dimensional occupancy profile of the robotic workspace using three independent sensory sources: stereo vision, an infra-red proximity sensor, and a laser proximity sensor. This section provides in detail first the sensor modeling process for the three sensory sources, and then the experiments for fusing data from them.

6.1 Sensor modeling

**Stereo Vision**: One of the most important components of stereo vision algorithm is stereo matching (Garg & Kumar, 2003) which involves finding out the location of the point in right image plane corresponding to a point in the left image plane. The relative displacement of these two points, called disparity, is used to estimate the three-dimensional position of the point. The accuracy with which stereo vision sensors are able to specify three-dimensional positional information about a point depends on how precisely the stereo vision algorithm is able to find the match of the point. The correlation score (Zhang et al, 1995) of the matched points, which measures the correlation between two template windows from left and right images, is a measure of “goodness-of-match” of the two points. The score ranges from -1 to +1, -1 representing not similar at all, and +1 representing most similar. The sensor modeling technique formulated in Section 3 has been used to develop a model for the stereo vision sensors that could take into account the effect of performance of the stereo matching algorithm on the uncertainty in sensor’s output.

An experiment was carried out in the RAMA Laboratory, wherein a set of fifty data points was obtained. The data set consisted of 3-D location of point in world coordinate system obtained via stereo vision sensors (via transformation as discussed in reference (Kumar & Garg, 2004)), correlation score for that point, and the actual 3-D location of the point in world coordinate frame. The strategy presented in Section 3.2 was used to develop a Gaussian model of the sensor. In this model the standard deviation of the distribution, which represents the uncertainty of the data, is dependent on the correlation score for the specific point. This dependence was modeled with the help of a neural network with five nodes in the hidden layer. This neural network takes correlation score as input, and outputs the value of standard deviation (sigma σ) for that particular correlation score. The neural network was trained via the process based on the Maximum Likelihood technique as presented in Section 3.2. Genetic Algorithm (GA) was used to maximize the likelihood function given by Equation (7). Though
computationally intensive as compared to back-propagation based methods, GAs provide globally optimum results. The likelihood function given by Equation (7) was calculated by obtaining the data set representing the actual 3D location of points and the corresponding measurements from the stereo vision. Figure 2 shows the graph of standard deviation $\sigma$ of the probability distribution function representing the model of stereo vision sensor as obtained by the neural network plotted against the correlation score of stereo matched points. As illustrated in Figure 2, the sensor model obtained from this approach separately for X, Y, and Z directions showed the intuitive trend that as the correlation score increases, i.e., as the stereo match gets better, the standard deviation decreases. Smaller value of standard deviations implies that the positional information obtained from stereo vision is less uncertain, and hence the degree of belief in the sensor output is more.

![Fig. 2. Stereo Vision Sensor Model: Variation of Standard Deviation of Sensor Model in X, Y, Z Directions with respect to the Correlation Score of Stereo Matched Points](image)

**Infra-Red (IR) Proximity Sensor:** The output of the IR proximity sensor is an analog voltage which is indicative of the distance of the object detected by the sensor. From the test data obtained from experiments, it was seen that the uncertainty in data increases when the distance to the object increases. Since the output of the sensor is indicative of the distance, sensor modeling process tries to capture the relationship between sensor’s uncertainties and sensor output.

In the laboratory experiments, the Infra-Red sensor was mounted on the wrist of the robot so that it looked vertically down (negative Z direction in world coordinate frame). The IR sensor provided the information about the distance to the nearest object detected directly in front of the object. Information about the position of end effector was obtained from the encoders of the robot. Hence, IR sensor can be effectively used in conjunction with robot encoders to provide 3-D information about any object. Similar to vision sensor, in this research, the model of the IR sensor was obtained separately for all three X, Y, and Z directions based on the method described in Section 3.2. The variation of standard deviation of the Gaussian sensor model obtained from this approach, as illustrated by Figure 3, showed a decrease when the sensor output increased which means that when the distance to
the object decreases (i.e. sensor’s output is larger) the standard deviation becomes smaller, and the sensor’s measurement becomes less uncertain.

![Graph showing variation of standard deviation with sensor output](image1)

**Fig. 3.** Infra-Red Proximity Sensor Model: Variation of Standard Deviation of Sensor Model in X, Y, and Z Directions with respect to the Sensor Output

**Laser Proximity Sensor:** Similar to IR proximity sensor, the output of laser sensor is indicative distance to the detected object. Sensor modeling for laser proximity sensor was done in a similar manner as the IR proximity sensor. The variation of standard deviation of the Gaussian sensor model obtained from this approach, as illustrated by Figure 4, showed a flat curve which means that the uncertainty in sensor measurement remained indifferent to distance to the detected object. In practice, the laser proximity sensor was very accurate, and the uncertainty in sensor measurement was not dependent on the distance.

![Graph showing variation of standard deviation with sensor output](image2)

**Fig. 4.** Laser Proximity Sensor Model: Variation of Standard Deviation of Sensor Model in X, Y, and Z Directions with respect to the Laser Sensor Output
6.2 Sensor fusion for 3D modeling of workspace

The model of workspace was obtained in an occupancy grid framework. The occupancy grid (Elfes, 1992, Kumar et al, 2005a) is a multi-dimensional field (usually of dimension two or three) where each cell (or unit of the grid) stores or represents the probabilistic estimate of the state of spatial occupancy. Occupancy grids are one of the most common low-level models of an environment, which provide an excellent framework for robust fusion of uncertain and noisy data. If the state variable (occupancy, in this case) associated with a cell, \( C_i \), is denoted by \( s(C_i) \), then the occupancy probability \( P[s(C_i)] \) represents the probabilistic estimate of occupancy of that particular cell. If \( P[s(C_i) = \text{occ}] \approx 0 \), then the cell is assumed to be empty, while, if \( P[s(C_i) = \text{occ}] \approx 1 \), then the cell is assumed to be occupied. If a single sensor is used to obtain the occupancy grid, Bayes’ Theorem can be used in the following manner to determine the state of the cell:

\[
P[s(C_i) = \text{occ} \mid z] = \frac{p[z \mid s(C_i) = \text{occ}]P[s(C_i) = \text{occ}]}{\sum_{s(C_i)} p[z \mid s(C_i)]P[s(C_i)]}
\]

where \( z \) is the sensor measurement. The probability density function (p.d.f.) \( p[z \mid s(C_i) = \text{occ}] \) is dependent on the sensor characteristics and is called the sensor model. The probability \( P[s(C_i) = \text{occ}] \) is called prior probability mass function and specifies the information made available prior to any observation.

Occupancy grids were obtained individually for stereo vision, infra-red, and laser proximity sensors, and then the individual grids were fused using two techniques: i) Simple Bayesian Fusion, and ii) Sequential Bayesian Fusion with Proposed Inconsistency Detection and Elimination Strategy. The details of the process for obtaining occupancy grids and sensor fusion are explained in reference (Kumar et al, 2005a).

In the experiment a cylindrical object was placed on the robot’s work-table. Figure 5 shows the images of the work-table obtained from the stereo cameras. Figure 6.a shows the actual occupancy grid of the workspace. This was obtained based on the geometric dimensions of the object and its location in the workspace. For the occupancy grid developed in this research, each grid is of size 5mm X 5mm X 5mm. Figures 6.b, 6.c, and 6.d show the occupancy grids independently obtained from stereo vision, IR proximity sensor, and laser proximity sensor respectively. Figure 6.e shows the occupancy grid obtained from simple Bayesian approach, and Figure 6.f shows the occupancy grid obtained from the Bayesian approach that utilizes the inconsistency detection and elimination technique proposed earlier.

Fig. 5. Images of the Worktable Obtained from the Left and the Right Cameras
Multi-Sensor Data Fusion in Presence of Uncertainty and Inconsistency in Data

| Stereo Vision | IR Proximity | Laser Proximity | Simple Bayesian | Bayesian with Proposed Inconsistency Detection |
|---------------|--------------|-----------------|----------------|-----------------------------------------------|
| 1279          | 1062         | 399             | 459            | 384                                           |

Table 1. Error Associated with Occupancy Grids Obtained from Fusion Process

Fig. 6. Occupancy Grids a) Actual Grid, b) Grid Obtained from Stereo Vision, c) Grid Obtained from IR Proximity Sensor, d) Grid Obtained from Laser Proximity Sensor, e) Fused Grid (Simple Bayesian Approach, Method 1), and f) Fused Grid (Proposed Bayesian Fusion with Inconsistency Detection and Elimination, Method 2)
To facilitate a comparison of the performance of the fusion process via different algorithms, a measure of error was formulated which is given by the following equation:

$$ Error = \sum_{C_i} \left[ |s(C_i)_{\text{actual}}| - |s(C_i)_{\text{sensor}}| \right]^2 $$  

(23)

where $|s(C_i)_{\text{actual}}|$ is the actual state of the cell, and $|s(C_i)_{\text{sensor}}|$ is the state of the cell obtained from the sensor and/or fusion process. The state of the cell is either 1 (for occupied) or 0 (for empty). Table I provides the error value associated with the occupancy grid obtained from the fusion process described above. The table compares the error value obtained via the two approaches. The first approach is based on the simple Bayesian fusion scheme, and the second approach is based on the proposed Bayesian fusion scheme embedded with the mechanism for inconsistency detection and elimination.

From the figures as well as from the table of results, it is evident that the proposed fusion scheme based on Bayesian approach with inbuilt mechanism to identify and eliminate spurious/inconsistent measurement presented in this chapter has been able to reduce the uncertainty inherent in individual sensors. The proposed method has been able to reduce the error by approximately 70% as compared to stereo vision, 64% as compared to IR proximity sensor, and 4% as compared to laser proximity sensor. On the other hand, simple Bayesian technique was able to reduce the error by approximately 64% as compared to stereo vision and by 56% as compared to IR proximity sensor. The technique based on simple Bayesian approach led to an increase in error by approximately 15% as compared to laser proximity sensor. The increase in error demonstrates the fact that it is not necessary that incorporation of additional sensor data will lead to improved accuracy of estimation. This is particularly more evident in cases when the accuracy of measurements from sensors differs by a large amount. In this case, the measurements from laser proximity are far more accurate (as seen from sensor models) than measurements from the stereo vision or IR proximity sensor, and fusion of measurements from the laser with stereo vision and IR proximity leads to an increase in error. However, the proposed technique has an inbuilt mechanism to determine if the fusion process leads to an increase in the information content, and, in this way was able to eliminate inconsistent data and improve the overall accuracy of the fusion process. Of the 24000 points (or cells) where the fusion of data from three sensors occurred (fusion occurred at 30x40x20 cells of the occupancy grid), the proposed technique detected 393 points where data from IR sensor were inconsistent and 1028 points where data from stereo vision were inconsistent. None of the data from laser sensor were detected to be inconsistent. This observation is consistent with the fact that laser sensor was far more accurate than the other two sensors.

One of the limitations of the proposed technique is that when there is a large number of sensors supporting an inconsistent measurement, then, based on the beliefs of the individual measurements, the technique may consider inconsistent measurement to be the correct one, and might disregard the correct measurements obtained by fewer numbers of sensors. In psychology, this kind of problem is termed as group conformity. For example, when an individual’s opinion differs significantly from that of others in a group, the individual is likely to feel extensive pressure to align his or her opinion with others. In the case of sensor
systems, this kind of condition is more likely to occur in adversarial situations, such as the battlefield, where events are prone to be camouflaged to escape detection. Hence, a formal criterion to establish the difference between spuriousness and opinion difference must be developed for the sensor fusion process to be accurately carried out in such adversarial situations. For example, in these situations, the technique proposed in this chapter could be applied if sensor models could be developed that represent the possibility/likelihood of events being camouflaged. Real time implementation and scalability aspects of the proposed sequential scheme have to be considered. To improve real time applicability of decentralized sensor fusion approaches, concepts from parallelization of processing can be incorporated. The recent interest in distributed sensing can incorporate such parallel/distributed framework of processing and sensor fusion.

7. Conclusions

Sensors measurements are inherently uncertain and often inconsistent. Appropriate consideration of uncertainty and identification/elimination of inconsistent measurements are essential for carrying out accurate estimation. The research reported in this chapter proposes a unified and formalized approach to fuse data from multiple sources which can take uncertainty of sensor data into account and automatically identify inconsistency in sensor data. Appropriate modeling of uncertainties in sensor measurement is necessary. This chapter presents an innovative neural network based method to model sensor’s uncertainties. Further, the chapter presents a strategy that adds a term to the popular Bayesian approach corresponding to a belief that the sensor data is not spurious conditioned upon the data and true state. An information theoretic measure is utilized to observe the information content of the posterior distribution to identify and eliminate inconsistent data. An extensive simulation study was performed where data from three sensors was fused. It was observed that the presented method was very effective in identifying spurious data, and, elimination of spurious data ensured more accurate results. Finally, the effectiveness of the proposed technique to identify and eliminate inconsistent sensor data in sequential Bayesian fusion was demonstrated with the help of an experiment performed in a robotic workcell where measurements from stereo vision, infra-red proximity, and laser proximity sensor were fused to obtain three-dimensional occupancy profile of robotic workspace.

8. References

Bayes T. (1763). An essay towards solving a problem in doctrine of chances, *Philosophical Transactions*, Vol. 53, 1763, pp. 370 – 418.

Benaskeur, A. R. (2002). Consistent fusion of correlated data sources, *Proceedings of the IEEE Annual Conference of the Industrial Electronics Society*, Vol. 4, 5-8 Nov. 2002, pp. 2652 – 2656.

Berger, J. O. (1985). *Statistical Decision Theory and Bayesian Analysis*, Springer-Verlag, Berlin.

Brooks, R. R. and Iyengar, S. S. (1998). *Multi-Sensor Fusion: Fundamentals and Applications with Software*, Prentice-Hall, Inc., Upper Saddle River, NJ, USA.
Chin, L. (1994). Application of neural networks in target tracking data fusion", IEEE Transactions on Aerospace and Electronic Systems, Vol. 30, No. 1, January 1994, pp. 281 – 287.

Del Gobbo, D., Napolitano, M., Famouri, P., and Innocenti, M. (2001). Experimental application of extended Kalman filtering for sensor validation, IEEE Transactions on Control Systems Technology, Vol. 9, No. 2, March 2001, pp. 376 – 380.

Demspter, A. P. (1968). A generalization of Bayesian inference, Journal of Royal Statistical Society, B, Vol. 30, No. 2, 1968, pp. 205 – 247.

Durrant-Whyte, H.F. (1988). Integration, Coordination and Control of Multi-Sensor Robot Systems, Kluwer Academic Publishers, Norwell, MA.

Elfes, A. (1992). Multi-source spatial data fusion using Bayesian reasoning, Data Fusion in Robotics and Machine Intelligence, Abidi, M. A. and Gonzalez, R. A. (eds.), Academic Press.

Frolík, J., Abdelrahman, M., and Kandasamy, P. (2001). A confidence-based approach to the self-validation, fusion and reconstruction of quasi-redundant sensor data", IEEE Transactions on Instrumentation and Measurement, Vol. 50, No. 6, Dec. 2001, pp. 1761 – 1769.

Garg, D. and Kumar, M. (2003). Object classification via stereo vision in a flexible manufacturing work cell, Proceedings of the 10th International Conference on Mechatronics and Machine Vision in Practice, Perth, Western Australia, December 9-11, 2003.

Garg, D. and Kumar, M. (2007). Neural controllers, In Wiley Encyclopedia of Computer Science and Engineering (Benjamin Wah, ed.), Hoboken: John Wiley & Sons, Inc.

Goldberg, D. (1989). Genetic Algorithms in Search, Optimization & Machine Learning. Addison Wesley Longman, Inc..

Hall, D. L. and Llinas, J. (2001). Handbook of Multisensor Data Fusion, CRC Press, Boca Raton, FL.

Haykin, S. S. (1998). Neural Networks: A Comprehensive Foundation, Prentice Hall Press.

Hornik, K., Stinchcombe, M., and White, H. (1989). Multilayer feedforward network are universal approximators, Neural Networks, Vol. 2, 1989, pp. 359 – 366.

Ibarguengoytia, P. H., Sucar, L. E., and Vadera, V. (2001). Real time intelligent sensor validation, IEEE Transactions on Power Systems, Vol. 16, No. 4, Nov. 2001, pp. 770 – 775.

Kalman, R. E. (1960). A new approach to linear filtering and prediction problems, Transactions of the ASME-Journal of Basic Engineering, Vol. 82, Ser. D, 1960, pp. 35 – 45.

Kumar, M. and Garg, D. (2004). Intelligent multi sensor fusion techniques in flexible manufacturing workcells, Proceedings of American Control Conference, 2004, Boston, MA, pp. 5375 – 5380.

Kumar, M., Garg, D., and Zachery, R. (2005a). Multi-sensor fusion strategy to obtain 3-D occupancy profile", Proceedings of the 31st Annual Conference of the IEEE Industrial Electronics Society (IECON), Raleigh, NC, November 2005, pp. 2083 – 2088.
Kumar, M., Garg, D., and Zachery, R. (2005b). Intelligent sensor modeling and data fusion via neural network and maximum likelihood estimation”, Paper No. IMECE2005-80972, Proceedings of the ASME International Mechanical Engineering Congress and Exposition, November 2005, Orlando, FL.

Kumar, M., Garg, D., and Zachery, R. (2006a). Stochastic adaptive sensor modeling and data fusion, Proceedings of SPIE Conference on Sensors and Smart Structures Technologies for Civil, Mechanical, and Aerospace Systems, San Diego, CA, Feb-March 2006, pp. 61740C-1 to 61740C-11.

Kumar, M., Garg, D., and Zachery, R. (2006b). A generalized approach for inconsistency detection in data fusion from multiple sensors, Proceedings of the American Control Conference, Minneapolis, MN, 2006, pp. 2078 – 2083.

Kumar, M., Garg, D., and Zachery, R. (2007). A method for judicious fusion of inconsistent multiple sensor data, IEEE Sensors Journal, Vol. 7, No. 5, May 2007, pp. 723-733.

Mahajan, A., Wang, K., and Ray, P. K. (2001). Multisensor integration and fusion model that uses a fuzzy inference system, IEEE/ASME Transactions on Mechatronics, Vol. 6, Issue 2, June 2001, pp. 188 – 196.

Manyika, J. and Durrant-Whyte, H. (1994). Data Fusion and Sensor Management: a Decentralized Information-Theoretic Approach, Ellis Howard Limited.

McKendall, R. and Mintz, M. (1992). Data fusion techniques using robust statistics, In Data Fusion in Robotics and Machine Intelligence, Abidi, M. A. and Gonzalez, R. A. (eds.), Academic Press.

Maybeck, P. S. (1979). Stochastic Models, Estimation and Control, Vol. 1, Academic Press, Inc.

Nicholson, D. (2004). An automatic method for eliminating spurious data from sensor networks, Proceedings of the IEE Conference on Target Tracking: Algorithms and Applications, 23-24 March 2004, pp. 57 – 61.

Press, S. J. (1989). Bayesian Statistics: Principles, Models and Applications, John Wiley and Sons.

Rizzo, A. and Xibilia, M.G. (2002). An innovative intelligent system for sensor validation in Tokamak machines, IEEE Transactions on Control Systems Technology, Vol. 10, No. 3, May 2002, pp. 421 – 431.

Rumelhart, D. E. and McClelland, J. L. (1988). Explorations in Parallel Distributed Processing: a Handbook of Models, Programs, and Exercises, MIT Press.

Sasiadek, J. Z. (2002). Sensor fusion, Annual Reviews in Control, Vol. 26, 2002, pp. 203-228.

Shafer, G. (1976). A Mathematical Theory of Evidence, Princeton University Press, Princeton, NJ.

Soika, M. (1997). A sensor failure detection framework for autonomous mobile robots, Proceedings of the IEEE/RSJ International Conference on Intelligent Robots and Systems, Vol. 3, 7-11 Sept. 1997, pp. 1735 – 1740.

Wellington, S. J., Atkinson, J. K., and Sion, R. P. (2002). Sensor validation and fusion using the Nadaraya-Watson statistical estimator, Proceedings of the IEEE International Conference on Information Fusion, Vol. 1, July 2002, pp. 321 – 326.

Yager, R. R. and Zadeh, L. A., (Eds.) (1991). An Introduction to Fuzzy Logic Applications in Intelligent Systems, Kluwer Academic Publishers.
Zhang, Z., Deriche, R., Faugeras, O., and Luong, Q.-T. (1995). A robust technique for matching two uncalibrated images through the recovery of the unknown epipolar geometry, *Artificial Intelligence Journal*, Vol.78, October 1995, pp. 87 – 119.
Data fusion is a research area that is growing rapidly due to the fact that it provides means for combining pieces of information coming from different sources/sensors, resulting in ameliorated overall system performance (improved decision making, increased detection capabilities, diminished number of false alarms, improved reliability in various situations at hand) with respect to separate sensors/sources. Different data fusion methods have been developed in order to optimize the overall system output in a variety of applications for which data fusion might be useful: security (humanitarian, military), medical diagnosis, environmental monitoring, remote sensing, robotics, etc.

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