CONICS, $(q + 1)$-ARCS, PENCIL CONCEPT OF TIME AND PSYCHOPATHOLOGY

Metod Saniga
Astronomical Institute of the Slovak Academy of Sciences,
05960 Tatranská Lomnica, Slovak Republic
(E-mail: msaniga@astro.sk)

Abstract: – It is demonstrated that in the (projective plane over) Galois fields $\text{GF}(q)$ with $q = 2^n$ and $n \geq 3$ ($n$ being a positive integer) we can define, in addition to the temporal dimensions generated by pencils of conics, also time coordinates represented by aggregates of $(q + 1)$-arcs that are not conics. The case is illustrated by a (self-dual) pencil of conics endowed with two singular conics of which one represents a double real line and the other is a real line pair. Although this pencil does not generate the ordinary (i.e., featuring the past, present and future) arrow of time over $\text{GF}(2^n)$, there does exist a pencil-related family of $(q+1)$-arcs, not conics, that closely resembles such an arrow. Some psycho(patho)logical justifications of this finding are presented, based on the ‘peculiar/anomalous’ experiences of time by a couple of schizophrenic patients.

Keywords: – pencils of conics, $(q + 1)$-arcs, Galois fields, psychopathology of time

1. Introduction

In one of our debut papers devoted to the theory of pencil-generated temporal dimensions [1] we discussed basic properties of the structure of time over a Galois field of even order, $\text{GF}(2^n)$. Our attention was focused exclusively on a specific pencil of conics featuring two singular conics and two distinct base points. Although this pencil has been found to reproduce quite well the qualitative properties of the physical world when considered over the ground field of the real numbers [2], it leads to a very peculiar arrow of time over $\text{GF}(2^n)$, the one lacking (the moment of) the present [1]. The aim of this short contribution is to show that there exists an interesting way of ‘recovering/restoring’ the familiar arrow of time also in the latter case.

2. Conics and $(q + 1)$-Arcs

To this end in view, let us consider, following (the symbols and notation of) [1,2], a conic, i.e., the curve of second order

$$\mathcal{Q}_{\vec{x}\vec{x}} \equiv \sum_{i \leq j} c_{ij} \vec{x}_i \vec{x}_j = 0, \quad i, j = 1, 2, 3; \quad (1)$$

here $c_{ij}$ are regarded as fixed quantities, while $\vec{x}_i$ are viewed as variables (the so-called homogeneous coordinates of a projective plane). The conic is degenerate (or singular) if there exists a change of coordinate system reducing Eq. (1) into a form in fewer variables; otherwise, the conic is non-degenerate (or proper). It is well known (see, e.g., [3]) that the equation of any proper conic $\mathcal{Q}$ in a projective plane over $\text{GF}(q)$ (the latter being henceforth denoted as $\text{PG}(2,q)$) can be brought into the canonical form

$$\tilde{\mathcal{Q}}_{\vec{x}\vec{x}} = \vec{x}_1 \vec{x}_2 - \vec{x}_3^2 = 0. \quad (2)$$
From the last equation it follows that the points of \( \bar{Q} \) can be parametrized as \( \varrho \bar{x}_i = (\sigma^2, 1, \sigma) \), \( \varrho \neq 0 \), and this implies that a proper conic in \( \text{PG}(2, q) \) contains \( q + 1 \) points; the point \((1, 0, 0)\) and \( q \) other points specified by the sequences \((\sigma^2, 1, \sigma)\) as the parameter \( \sigma \) runs through the \( q \) elements of \( \text{GF}(q) \). Moreover, it can easily be verified that any triple of distinct points of \( \bar{Q} \) are linearly independent, for \[ \begin{vmatrix} 1 & 0 & 0 \\ \sigma_1^2 & 1 & \sigma_1 \\ \sigma_2^2 & 1 & \sigma_2 \end{vmatrix} = \sigma_2 - \sigma_1 \neq 0 \] (3)

and

\[ \begin{vmatrix} \sigma_1^2 & 1 & \sigma_1 \\ \sigma_2^2 & 1 & \sigma_2 \\ \sigma_3^2 & 1 & \sigma_3 \end{vmatrix} = (\sigma_1 - \sigma_2)(\sigma_2 - \sigma_3)(\sigma_3 - \sigma_1) \neq 0. \] (4)

Hence, a proper conic of \( \text{PG}(2, q) \) is an example of a \((q + 1)\)-arc – a set of \( q + 1 \) points no three of which are collinear. Although every non-degenerate conic of \( \text{PG}(2, q) \) is a \((q + 1)\)-arc, the converse is true only for \( q \) odd; for \( q \) even and greater than four there also exist \((q + 1)\)-arcs that are not conics [3–5].

In order to see this explicitly, we first recall that all the tangents to a proper conic \( Q \) of \( \text{PG}(2, q = 2^n) \) are concurrent, i.e. they pass via one and the same point, called the nucleus [1,3–5]. Hence, the conic \( Q \) together with its nucleus form a \((q + 2)\)-arc. Now, let us delete from this \((q + 2)\)-arc a point belonging to \( Q \); we will obtain a \((q + 1)\)-arc \( K \) which shares \( q = 2^n \) points with \( Q \). Taking into account the fact that a proper conic is uniquely specified by five of its points, no three collinear, it then follows that the above described \((q + 1)\)-arc \( K \) cannot be a conic for \( n \geq 3 \); for, indeed, if it were then it would have with \( Q \) more than five points in common and would thus coincide with the latter, a contradiction.

3. Pencil-Time Comprising \((q + 1)\)-Arches

By the very definition, a straight-line (henceforth only line) of \( \text{PG}(2, q) \) can have at most two points in common with a \((q + 1)\)-arc \( K \); if it has just two, it is called – following the terminology used for conics – a secant of \( K \), if one, a tangent to \( K \), and if none, a line external to (or, skew with) \( K \). So, a \((q + 1)\)-arc can be regarded as a natural and straightforward generalization of the concept of conic for \( \text{PG}(2, 2^n) \), \( n \geq 3 \). As a consequence, instead of viewing the time dimension as being generated by a pencil of conics, we can introduce its generalized concept based on a one-parametric family of \((q + 1)\)-arcs. Moreover, after affinizing \( \text{PG}(2, q) \) we define, in a completely analogous way to what we did in the case of pencils of conics [2], the domain of the past/future to be represented by those \((q + 1)\)-arcs (of a given family) to which the ideal line is secant/external, while the \((q + 1)\)-arc(s) having this line as a tangent stand(s) for the moment(s) of the present.

In order to see an explicit realization of this idea, we will again consider our favoured pencil of conics [1,2]

\[ Q^\vartheta_{\bar{x}} = \vartheta_1 \bar{x}_1 \bar{x}_2 + \vartheta_2 \bar{x}_2^2 = 0. \] (5)
The pencil features two distinct base points, \( B_1 : \varrho \bar{x}_i = (0, 1, 0) \) and \( B_2 : \varrho \bar{x}_i = (1, 0, 0) \), each of multiplicity two, and two degenerate conics: \( \vartheta (\equiv \vartheta_2 / \vartheta_1) = \pm \infty \) (i.e., the double real line \( \bar{x}_3^2 = 0 \)) and \( \vartheta = 0 \) (i.e., a pair of real lines \( \bar{x}_1 = 0 \) and \( \bar{x}_2 = 0 \) concurring at the point \( N : \varrho \bar{x}_i = (0, 0, 1) \)). As already mentioned, this pencil, when affinized in such a way that the ideal line \( \mathcal{L}^\infty \) meets neither \( B_1, B_2 \) nor \( N \), reproduces nicely the ordinary arrow of time if considered over the field of the reals \([2]\), but leads to a very peculiar arrow, the one lacking the present, when we switch to \( \text{GF}(2^n) \) \([1]\); this happens because the point \( N \) is the common nucleus for all the proper conics of pencil (5) and as \( \mathcal{L}^\infty \) is not incident with \( N \) it cannot be a tangent to any of them. Let us select one line, \( \mathcal{L}^* \), from the pencil of lines carried by \( N \) such that \( \mathcal{L}^* \neq NB_1, NB_2 \). It is obvious that the point \( A \) at which \( \mathcal{L}^\infty \) and \( \mathcal{L}^* \) meet each other lies on just one (proper) conic \( Q^* \) of pencil (5), to which \( \mathcal{L}^\infty \) must clearly be a secant. Now, let us create a family of \((q + 1)\)-arcs in such a way that we delete from each proper conic the point at which \( \mathcal{L}^* \) touches the latter, and add to such a \( q \)-arc the nucleus \( N \) (recalling once again that \( N \) is the nucleus for all the proper conics of (5)). The family of \((q + 1)\)-arcs created this way thus contains not only \((q + 1)\)-arcs to which the ideal line \( \mathcal{L}^\infty \) is a secant (the past) and/or an external line (the future), as in the case of conics \([1]\), but also a unique \((q + 1)\)-arc, that composed of \( Q^* \setminus \{A\} \) and the point \( N \), having with \( \mathcal{L}^\infty \) just one point in common (and standing thus for the present): this aggregate is thus qualitatively identical with a geometrical structure that was in \([2]\) recognized and demonstrated to reproduce remarkably well our ordinary/normal perception of time.

4. Some Intriguing Psychopathology of Time

We have thus arrived at a principally new type of temporal arrow that cannot be reproduced by (any pencil of) conics whatever field we would take as the ground field of the projective plane. And because this kind of the temporal emerges only over fields of characteristic two which, we conjecture(d), represent the ‘working regime’ of the deepest parts of our subconscious \([1]\), the corresponding mental states will be extremely difficult (and, so, very rare) to attain and be fully experienced. Nevertheless, after looking carefully through a large number of references dealing with so-called ‘altered’ states of consciousness [see the exhaustive bibliography in Ref. 6], we succeeded in finding a very interesting old paper \([7]\) that seems to contain descriptions of such mental states by schizophrenic patients. \(^1\) Below are the excerpts from the narratives given by a couple of psychotics where there is a/n direct/explicit reference to a ‘strange,’ or ‘new,’ mental temporal dimension; in particular, a patient, aliased ‘Sche,’ describes their ‘weird’ time fabric as follows \([7, \text{p.} 556]\):

\[\ldots \text{and then came a feeling of horrible expectation that I could be sucked up into the past or that the past would overcome me and flow over me. It was disquieting that someone could play with time like that, somewhat daemonic. This would be perverse for humanity. What could time be for the orderlies? Did they still have ordinary time? Then everything seemed to be absolutely of no consequence, and in spite of that I was very uneasy. A foreign time sprang up. Everything was}\]

\(^1\) The article is written in German; the English translation of both the excerpts quoted was borrowed from \([8]\) (first excerpt) and \([9]\) (second one). The italics, however, are supplied by the present author.
confused, pell-mell, and I felt contracted in myself: I wanted to hold everything back, but I had to let everything go... I wanted this false time to disappear in me again...”

Another patient (‘Ge’) gives even a more ‘physically attractive’ piece of information [7, p. 567]:

“One evening during a walk in a busy street, I had a sudden feeling of nausea... Afterwards a small patch appeared before my eyes... The patch glimmered inwardly and there was a to and fro of dark threads... The web grew more pronounced... I felt drawn into it. It was really an interplay of movements which had replaced my own person. Time had failed and stood still – no, it was rather that time re-appeared just as it disappeared. This new time was infinitely manifold and intricate and could hardly be compared with what we ordinarily call time. Suddenly the idea shot through my head that time lies not only before and after me, but in every direction...”

5. Conclusion

We have outlined a conceptually very important extension of our pencil concept of the time dimension in the case of Galois fields of characteristic two and order greater than four. It has been shown that such a generalization of the model may not be a mere academic issue. On the contrary, it seems to possess a serious ‘observational/experimental’ counterpart in the domain of the psychopathology of time. The issue obviously asks for and deserves further effort and ingenuity to be properly explored and examined.

Acknowledgement

The work was partially supported by the 2001–2003 joint research project of the Italian Research Council and the Slovak Academy of Sciences entitled ‘The Subjective Time and its Underlying Mathematical Structure.’

References

[1] Saniga, M. (1998). Time dimension over Galois fields of characteristic two. Chaos, Solitons & Fractals, 9, 1095–1104.
[2] Saniga, M. (1998). Pencils of conics: a means towards a deeper understanding of the arrow of time?. Chaos, Solitons & Fractals, 9, 1071–1086.
[3] Hirschfeld, J. W. P. (1979). Projective Geometries over Finite Fields. Oxford: Clarendon Press.
[4] Kárteszi, F. (1976). Introduction to Finite Geometries. Amsterdam: North-Holland Publishing Company.
[5] Segre, B. (1961). Lectures on Modern Geometry. Rome: Cremonese.
[6] Saniga, M. (2000). Algebraic geometry: a tool for resolving the enigma of time?. In R. Bucher, V. Di Gesù and M. Saniga (eds.), Studies on the Structure of Time: from Physics to Psycho(path)ology, New York: Kluwer Academic/Plenum Publishers, pp. 137–166.
[7] Fischer, F. (1929). Zeitstruktur und Schizophrenie. Zeitschrift für die gesamte Neurologie und Psychiatrie, 121, 544–574.
[8] Minkowski, E. (1970). Lived Time: Phenomenological and Psychopathological Studies. (Translated from French by N. Metzel.) Evanston: Northwestern University Press, p. 285.
[9] Jaspers, K. (1968). General Psychopathology. (Translated from German by J. Hoenig and M. W. Hamilton.) Chicago: The University of Chicago Press, p. 87.