PROBING THE PRESENCE OF A SINGLE OR BINARY BLACK HOLE IN THE GLOBULAR CLUSTER NGC 6752 WITH PULSAR DYNAMICS

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ABSTRACT

The five millisecond pulsars that inhabit NGC 6752 display locations or accelerations that are quite unusual compared to all other known pulsars in globular clusters. In particular, PSR A, a binary pulsar, lives in the cluster halo, while PSR B and PSR E, located in the core, show remarkably high negative spin derivatives. This suggests that some uncommon dynamical process is at play in the cluster core, which we attribute to the presence of a massive perturber. Here we investigate whether a single intermediate-mass black hole, lying on the extrapolation of the mass $\mathcal{M}_{\text{BH}}$ versus $\sigma$ relation observed in galaxy spheroids, or a less massive binary black hole could play the requested role. To this purpose we simulated binary-binary encounters involving PSR A, its companion star, and the black hole(s). A stellar-mass binary black hole of $(50 \ M_\odot, \ 10 \ M_\odot)$ can impart the right thrust to propel PSR A into the halo during a flyby. The flyby is gentle and does not alter the internal properties of the binary pulsar. An intermediate-mass binary black hole of $(200 \ M_\odot, \ 10 \ M_\odot)$ tends to impart a recoil speed larger than the escape speed: it can release PSR A on the right orbit if its separation is wide. A single intermediate-mass black hole of mass $\mathcal{M}_{\text{BH}} \gtrsim 500 \ M_\odot$ may have ejected PSR A at the periphery of NGC 6752 in a close dynamical encounter involving the binary pulsar, the black hole, and a star belonging to its cusp. The encounter gives correct speeds but alters significantly the eccentricity of the impinging binary, so that it must occur before the neutron star of PSR A is recycled via accretion torque. The influence of an intermediate-mass binary black hole on the acceleration of the two core pulsars is studied, and the ejection of stars by the binary is briefly explored. In inspecting our close four-body encounters, we have found that a single or binary black hole may attract on a long-term stable orbit a millisecond pulsar. Timing measurements on the captured “satellite” pulsar, either a member of a hierarchical triple or a member of the cusp, could unambiguously unveil the presence of a black hole(s) in the core of a globular cluster.

Subject headings: black hole physics — globular clusters: general — pulsars: general — stars: neutron

1. INTRODUCTION

Five millisecond pulsars have been recently discovered in NGC 6752 displaying unexpected characteristics (D’Amico et al. 2002). PSR A, a canonical recycled binary pulsar, holds the record of being the farthest millisecond pulsar ever observed from the gravitational center of a globular cluster, at a distance of $\approx 3.3$ half-mass radii. PSR C, an isolated pulsar, ranks second in the list of the most offset pulsars, being at a distance of $1.4$ half-mass radii from the center. PSR B, PSR E, and PSR D are located instead within the cluster core and are single; PSR B and PSR E have remarkably high negative spin derivatives while PSR D has a positive $\dot{P}$, one of the highest measured among globular cluster pulsars. If the negative spin derivatives of PSR B and PSR E are ascribed to the overall effect of the cluster gravitational potential, this would result in a central projected mass-to-light ratio $M/L_V \sim 6$–7 (Ferraro et al. 2003b). This high ratio may imply the presence of $\approx 1000$–$2000 \ M_\odot$ of under-luminous matter enclosed within the central 0.08 pc of the cluster (Ferraro et al. 2003b), perhaps in the form of a single intermediate-mass black hole and/or in the form of collapsed stellar remnants. In this scenario even the very high positive $\dot{P}$ of PSR D could be explained as due to the line-of-sight gravitational pull of this unseen matter. However, since pulsars spin down because of rotational energy losses, the period derivative of PSR D could also be intrinsic. Alternatively, one could argue that the negative values of $\dot{P}$ of PSR B and PSR E are ascribed to the overall effect of the cluster gravitational potential, while the high positive $\dot{P}$ of PSR D could be explained as due to the line-of-sight gravitational pull of a massive target, such as a nearby passing star or even a more massive exotic objects (Ferraro et al. 2003b). The millisecond pulsars in NGC 6752 are peculiar in their location or acceleration, and the combination of these facts strongly suggests the occurrence of uncommon dynamics in the core and halo of NGC 6752. It is our aim to address this issue here in detail.

In a previous paper (Colpi, Possenti, & Gualandris 2002, hereafter CPG) we explored a number of roots for the origin of PSR A: PSR A may have originated from a primordial binary, born either in the halo or in the core. But a careful analysis (based on considerations of characteristic lifetimes and neutron star natal kicks) led us to discard both these two hypotheses (see CPG). A third, involving a three-body scattering or exchange event off core stars, was also rejected given the tight constraints imposed by the binary nature of PSR A (CPG). We thus were led to conjecture that a more massive target, such as a binary of two black holes with masses in the range $\approx 10$–$100 \ M_\odot$, could have provided, in a
four-body scattering event, the necessary thrust to propel PSR A into its current halo orbit at an acceptable event rate (CPG).

PSR A may simply signal the presence of a black hole binary in NGC 6752, but this is somewhat puzzling. Black hole binaries are expected to form in rich star clusters, but along the course of evolution they are expected to exit their parent cluster (Kulkarni, Hut, & McMillan 1993; Sigurdsson & Hernquist 1993). The black holes (relics of the most massive stars) tend to pair with other black holes in binaries as soon as they segregate in the cluster core by dynamical friction (Kulkarni et al. 1993). Some binaries may rapidly merge emitting gravitational waves (GWs; Benacquista 1999; Miller 2002), creating a more massive black hole (Miller & Hamilton 2002), but most/many leave the cluster since close (three- or four-body) dynamical encounters among the black holes eject them, singly or in binaries, because of recoil (Sigurdsson & Hernquist 1993; Kulkarni et al. McMillan 1993). Current N-body simulations (Portegies Zwart & McMillan 2000) expect no black hole or one black hole binary to remain in the cluster. Thus, if our interpretation of PSR A is correct, we may be providing the first dynamical evidence of the “only” black hole binary that avoided the escape. A goal of this paper is to further explore this possibility, narrowing the range of masses of the two black holes from pulsar dynamics.

There is an alternative root for the origin of the unusual location of PSR A that we wish to explore here in connection with the high $M/L_V$ ratio and so with the potential presence of underluminous matter in the cluster core: the ejection of PSR A into the halo of NGC 6752 by a dynamical encounter with a central intermediate-mass black hole. According to an old suggestion by Frank & Rees (1976), the separation $a$ of a massive black hole can eject other stars plunging inside (Lin & Tremaine 1980). A flyby involving a central black hole, a bound star orbiting around it, and the binary pulsar may impart to PSR A the right recoil speed to climb the potential well of the cluster and reach the halo (Colpi, Mapelli, & Possenti 2003).

Single black holes of intermediate mass ($M_{BH} \lesssim 500 M_\odot$) may inhabit the center of globular clusters; their formation root can involve the runaway growth of a supermassive star through collisions of heavy stars in the young cluster (Portegies Zwart & McMillan 2002) or the occurrence of repeated mergers among compact objects (Miller & Hamilton 2002; see van der Marel 2003 and Miller 2003 for a review). Recently, Hubble Space Telescope observations of the globular cluster G1 in M31 (Gebhardt, Rich, & Ho 2002) and of M15 in the Milky Way (Gerssen et al. 2002) have provided first clues for the presence of a single central black hole. Surprisingly, the two postulated black holes, of masses $2.0^{+0.4}_{-0.8} \times 10^4 M_\odot$ in G1 and $1.7^{+0.7}_{-1.7} \times 10^5 M_\odot$ in M15, seem to lie only along the extrapolation of the black hole mass versus one-dimensional dispersion velocity relation ($M_{BH}$-$\sigma$) obeyed by the supermassive black holes in galaxy spheroids (Ferrarese & Merritt 2000; Gebhardt et al. 2000; Tremaine et al. 2002). Here we assume that the hypothethical black hole in NGC 6752 lies on the $M_{BH}$-$\sigma$ relation. (Note that current evidence of a large black hole in G1 and M15 is not yet compelling: Baumgardt et al. 2003a, 2003b provide an alternative to the black hole hypothesis, i.e., a cluster of lower mass collapsed objects.)

In this paper we simulate a series of binary-binary encounters to study (1) the scattering of the binary pulsar PSR A off a stellar-mass or intermediate-mass black hole binary and (2) the plunge-in of PSR A inside the cusp of a single intermediate-mass black hole. In § 2 we select the mass spectrum of the two black holes in the binary and choose the mass of the single intermediate-mass black hole. In § 3 we describe properties of the binaries and identify the possible state of binary-binary encounters (flyby, ionization, and exchange). The binary hosting the neutron star is described in its post-recycling phase as well as in its prerecycling state. In § 4 we extract from a series of binary-binary simulations postencounter distributions of recoil velocities, eccentricities, and binary separations, running more than 20,000 encounters. In § 5 we examine the statistics of the encounters, while in § 6 we briefly explore exotic end states. In § 7 we model the random walk (moderated by dynamical friction) of the black hole binary induced by stellar collisions and try to evaluate the local gravitational pull that a black hole binary may exert on the core pulsars inside NGC 6752. In § 8 we explore the survival of a black hole binary in NGC 6752, mimicking mass growth and hardening by stellar encounters. In § 9 we outline the key findings and present our conclusions.

2. BLACK HOLE MASSES

The masses of the two black holes in the binary, denoted $M_{BH}$ (for the heavier) and $m_{BH}$ (for the lighter), are free parameters (let $M_T = M_{BH} + m_{BH}$ be the total mass of the binary). Constraints on their values derive mainly from limits on their ejection due to gravitational encounters and survival against coalescence by emission of GWs.

In general, survival inside the cluster (followed by coalescence due to GWs) would win over ejection if the binary, at the separation $a_{eq}$ where the timescale for coalescence

$$
\tau_{GW} = \frac{5c^5 a_{eq}^4 (1 - e^2)^{7/2}}{256 G^3 M_{BH} m_{BH} M_T}
$$

becomes comparable to the collision time $\tau_{BH} \sim \sigma_{BH}/(m_{BH} G M_T \pi a_{eq})$, has a binding energy $E_{bin} = G M_{BH} m_{BH}/2a_{eq}$ smaller than the minimum binding energy for expulsion, $E_\xi(V_{es})$, given an encounter with a field black hole (here $n_{BH}$ and $\sigma_{BH}$ are the density and one-dimensional dispersion velocity of the field black holes interacting with the binary and $V_{es}$ is the escape speed from the core). If $m_{BH}$ is the mass of the field black hole impinging on the binary, momentum conservation implies a minimum energy for escape equal to

$$
E_\xi(V_{es}) = \left(\frac{1}{2 \xi_{BH}}\right) \left(\frac{M_T}{m_{BH,f}}\right)^2 V_{es}^2 ,
$$

where $\xi_{BH}$ controls the relative energy exchange per scattering (see Tables 1 and 3 for the values of $\xi_{BH}$). If $E_\xi > E_{bin}$ at $a_{eq}$, the binary avoids ejection during its entire lifetime (when its separation $a > a_{eq}$) and may eventually merge, producing a more massive black hole when $a$ drops below $a_{eq}$.

The condition $E_\xi(V_{es}) = E_{bin}(a_{eq})$ selects a critical mass $M_{BH, crit}$ for the heavier black hole in the binary, for fixed $m_{BH}$, above which ejection by recoil off a field black hole is avoided. In Figure 1 we plot $M_{BH, crit}$ against $m_{BH}$, considering a close dynamical encounter with a field black hole of mass $m_{BH,f}$. The black strip refers to a mass $m_{BH,f}$ set equal to $m_{BH}$, while the gray strip refers to scattering with a
The escape speed is \( V_{\text{enc}} \) for a close dynamical encounter with a field black hole of mass \( m_{\text{BH, f}} = 10 M_\odot \) (dark gray strip) and \( m_{\text{BH, f}} = 10 M_\odot \) (black strip). The binary has eccentricity \( e = 0.7 \). The escape speed is \( V_{\text{esc}} = 40 \) km s\(^{-1}\), and \( \sigma_{\text{BH}} = 0.5 \) (Miller & Hamilton 2002). The background black hole density and dispersion velocity are set equal to some scatter in \( m_\odot \), and density to bracket uncertainties. The shadowed areas in light gray above each strip indicate the permitted values of \( m_\odot \) consistent with binary black hole retention in the cluster.

Figure 1 shows clearly that black holes with larger masses (\( \gtrsim 30 M_\odot \)) and small mass ratios \( m_{\text{BH, f}}/m_{\text{BH}} \) are preferentially retained. We have considered values of \( (m_{\text{BH, f}}/m_{\text{BH}}, m_{\text{BH, f}}/m_{\text{BH}}) \) equal to \((10, 10), (30, 3), (50, 10)\), and \((200, 10)\) for the binary black holes in our simulations of binary-black hole encounters; in one series of runs we considered a \((50, 1.4)\) binary comprising a black hole and a canonical neutron star.\(^5\) The \((10, 10)\) binary is in the forbidden region; it is set to study comparisons with heavier binaries and because its mass is in the range of observed black hole masses (Bailyn et al. 1998). Binaries with the heaviest black hole below \( 100 M_\odot \) will be considered as stellar-mass black hole binaries. Those with \( m_{\text{BH}} > 100 M_\odot \) will be referred to as intermediate-mass black hole binaries.

Regarding the case of the single intermediate-mass black hole \( \mathcal{M}_{\text{BH}} \) in NGC 6752, we imposed a mass of \( \sim 500 M_\odot \). This value is derived according to the relation

\[
\log(\mathcal{M}_{\text{BH}}/M_\odot) = \alpha + \beta \log(\sigma/200 \text{ km s}^{-1}),
\]

with \( \alpha = 8.13 \pm 0.06 \) and \( \beta = 4.02 \pm 0.32 \) (Tremaine et al. 2002) extrapolated down to \( \sigma \approx 10 \) km s\(^{-1}\). The central value of the line-of-sight dispersion velocity \( \sigma \) in NGC 6752 has been estimated to be between 2.1 and 9.7 km s\(^{-1}\) (Dubath, Meylan, & Mayor 1997). Recent Fabry-Perot spectroscopy of single stars in NGC 6752 has shown a flat profile with typical dispersion of \( \sim 7 \) km s\(^{-1}\) within the central 1’ (Xie et al. 2002), while proper-motion measurements of stars in the central part of the cluster suggest a much higher value of \( \sim 9–15 \) km s\(^{-1}\) (Drukier et al. 2003). In Figure 2 we have drawn the \( \mathcal{M}_{\text{BH}} \) versus \( \sigma \) relation and indicated the upper limit on the central underluminous mass implied by the pulsar acceleration measurements, together with the upper limit on the black hole mass \( \mathcal{M}_{\text{BH}} \lesssim 1,000 M_\odot \) imposed by the lack of a rise in the stellar density inside the core resolved down to a scale of 0.08 pc (Ferraro et al. 2003b).

3. Binary-Binary Encounters

3.1. The Binary Pulsar

Here we study the four-body dynamics of two binaries interacting under Newtonian gravity. We explore four cases resulting from the combination of “projectiles” occurring in two flavors and “targets” of two types. The projectile is labelled as [PSR A, CO] when it coincides with the observed binary pulsar (in its postrecycling phase). It is labeled as [NS, CO] when the neutron star, not yet recycled, is orbiting around a more massive main-sequence star (in its prerecycling phase).

The binary pulsar [PSR A, CO] is described as in D’Amico et al. (2002). PSR A has likely experienced a phase of recycling and of orbital circularization (Bhattacharya & van den Heuvel 1991) that has driven the neutron star to spin at the observed period of 3.27 ms (CPG). PSR A orbits

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\(^5\) Three solar masses is the most general upper limit on the mass of the neutron star, which we considered as the lower limit for the mass of the lighter black hole.
around a companion star of mass $m_{c0} = 0.2 M_\odot$ (a value derived adopting a mass of 1.4 $M_\odot$ for the pulsar and an orbit inclination in the range of 60°–90°). The binary separation and eccentricity are $a_{\text{PSR A}} = 0.0223$ AU ($P_{\text{orb,PSR A}} = 0.86$ days) and $e_{\text{PSR A}} \leq 10^{-5}$, respectively (D'Amico et al. 2002).

In the prerecycling phase, the neutron star has a mass of 1.4 $M_\odot$ and the companion a mass $m_{c0} = 0.8 M_\odot$, consistent with evolutionary scenarios of low-mass binary pulsars (Tauris & Savonije 1999). The initial semimajor axis of the binary [NS, CO] is 0.03 AU, corresponding to an orbital period $P_{\text{orb}} = 1.3$ days, below the bifurcation point (CPG). The initial eccentricity is 0.7. From here on, the total mass of the binary will be denoted as $m_B$ (1.6 $M_\odot$ for [PSR A, CO], 2.2 for [NS, CO]).

3.2. The [BH, BH] or [BH, *] Binary as Target

The target is either a binary composed of two black holes [BH, BH] (in one case a black hole–neutron star binary [BH, NS]) or an intermediate-mass black hole with a star [BH, *] orbiting around.

In the first hypothesis, the initial semimajor axis $a_{\text{BH}}$ of all our black hole binaries [BH, BH] is of 1 AU, unless specified otherwise, and the initial eccentricity is $e_{\text{BH}} = 0.7$. The black hole binary is always sufficiently hard to provide the right recoil speed $V_{\text{p,PSR A}}$ to [PSR A, CO] (or [NS, CO]). Since this speed is of the order of 30–40 km s$^{-1}$, the separation $a_{\text{BH}}$ fulfills the inequality

$$a_{\text{BH}} \leq 6 \xi_{\text{BH}} (\mu_{\text{BH}}/10 M_\odot)(40 \text{ km s}^{-1}/V_{\text{p,PSR A}})^2 \text{ AU}$$

(see eq. [3] and CPG for details; $\mu_{\text{BH}}$ is the reduced mass of the black hole binary). The binary is also wide enough to have a coalescence time $\tau_{\text{GW}}$ longer than $\sim 10$ Gyr, which implies

$$a_{\text{BH}} \geq 0.4[(\mu_{\text{BH}}/10 M_\odot)(M_{\odot}/100 M_\odot)]^2 \times (\tau_{\text{GW}}/10 \text{ Gyr})^{1/4} \text{ AU}$$

for an eccentricity of 0.7. This is a necessary condition since the characteristic timescale for dynamical friction $\tau_{\text{DF}}$ to drive [PSR A, CO] from the current position to the core has been estimated as $\sim 1$ Gyr (see CPG; Sigurdsson 2003). This timescale imposes a lifetime for the black hole binary comparable to the age of the cluster itself, unless the binary formed recently. As an example, for a target binary with (50 $M_\odot$, 10 $M_\odot$) and $e_{\text{BH}} = 0.7$, the suitable interval for $a_{\text{BH}}$ is (0.3, 6) AU.

In the second hypothesis, the target is the central intermediate-mass black hole of $M_{\text{BH}} \sim 500 M_\odot$ surrounded by a swarm of bound stars belonging to the cusp, with the region of influence of the black hole extending up to a distance $r_{\text{BH}} \approx G M_{\text{BH}}/\sigma^2 \approx 0.02 M_{\text{BH}}/\sigma_{10} \text{ pc}$, where $\sigma_{10}$ is the stellar central line-of-sight velocity dispersion in units of 10 km s$^{-1}$. (Note that even in the absence of a stellar cusp, a large central black hole $M_{\text{BH}}$ can easily capture a star or exchange with a binary star in the cluster core, likely a neutron star or a stellar-mass black hole.) We selected a cusp star (or companion star) of 1 $M_\odot$ tightly bound to the large black hole, moving on a Keplerian orbit with semimajor axis $a_{\text{BH},*}$ of 1 AU and eccentricity 0.7 ($P_{\text{orb, BH}} = 16$ days). The cusp star is well inside the so-called critical radius $r_{\text{crit}}$ where the flow of bound stars percolating across the loss cone peaks. At distances $r < r_{\text{crit}}$ relaxation leads to relatively small changes in the integrals of motion of the stars over an orbital period (Lightman & Shapiro 1977), the stars avoiding prompt capture by the hole and the loss cone remaining empty. Thus, conditions $a_{\text{BH},*} < r_{\text{crit}}$ and $e \leq 0.7$ for a cusp star guarantee stability of the orbit during the characteristic time of the dynamical interaction $\tau_{\text{enc}} \sim (a_{\text{BH},*} b)^{1/2} / \sigma$ with [PSR A, CO] (or [NS, CO], where $b$ is the impact parameter of the encounter. In general, the following inequalities hold: $P_{\text{orb,PSR A}} < P_{\text{orb, BH}} \leq \tau_{\text{enc}} < \tau_{\text{rel}}$, where $\tau_{\text{rel}}$ is the relaxation time inside the cusp.

3.3. End States

Sampling of the initial conditions is carried on using the prescriptions outlined in Hut & Bahcall (1983) and Sigurdsson & Phinney (1993). The integration method, a fourth-order Runge-Kutta scheme with adaptive step size and quality control, maintains the accuracy on total energy and angular momentum conservation up to $\Delta E/E_0 \sim 10^{-3}$ to $10^{-5}$ and $\Delta J/J_0 \sim 10^{-11}$ to $10^{-10}$, respectively. After a close dynamical interaction, typical values of the fractional change of the binding energy of the black hole binary ($\Delta E_{\text{BH}}/E_0\text{BH}$) are of the order of $\sim 10^{-2}$ to $10^{-3}$ (here $E_0\text{BH} = G M_{\text{BH}} m_{\text{BH}}/2 a_{\text{BH}}$). During integration, a four-body to two-body switch is performed in the asymptotic regions, both for the incoming and outgoing states. The code has been tested against Sigurdsson & Phinney (1993) in the case of three-body scattering. A trial four-body experiment for a reference case (the 50 $M_\odot$, 10 $M_\odot$ binary) has been compared with the output of STARLAB thanks to the cooperation of A. Gualandris and S. Portegies Zwart. We found agreement between the two outputs within the statistical uncertainties involved in the simulations. 5

Impact parameters and relative velocities of the two interacting binaries have been selected to ensure an effective energy exchange between the two binaries leading to postencounter states that can be classified into five main groups: pure flybys (FBs), ionizations (IONs), quartets (Qs), unresolved encounters (UEs), and exchanges (EXs) (the last two being very rare). Ionizations can further be sampled in (1) resonant ionizations in which [PSR A, CO] is dissociated (their relative separation $\geq 20 a_{\text{PSR A}}$) but the two stars are bound to [BH, BH] (or [BH, *]) (within a separation relative to the center of mass of black hole binary smaller than $30 a_{\text{BH}}$), (2) ionizations with the escape of the two stars, or (3) ionizations with formation of a hierarchical triple in which PSR A (or CO) escapes to infinity, leaving CO (or PSR A) bound to the [BH, BH] (or [BH, *]) binary. These triplets may end in either stable or unstable states (Hut 1993; Mandling & Aarseth 2001; we return to these systems in § 6). Qs correspond to cases where the total energy relative to the centers of mass of the binaries is negative. UEs are very tight Qs occurring when the separation of [PSR A, CO] relative to the black hole(s) never exceeds $30 a_{\text{BH}}$ after $10^8$ time steps. Integration is aborted when Qs or UEs appear. On the contrary, IONs and FBs are integrated

5 A. Gualandris and S. Portegies Zwart kindly agreed to run a comparison model. A detailed comparison between the outcomes of the two codes will be presented elsewhere.

6 Qs can be stable (Hut 1993) but are likely to be perturbed by flying-by stars in the dense star cluster (Bacon et al. 1996).
until the outgoing star(s) reach the asymptotic state. As an illustration, Figure 3 shows an FB for [PSR A, CO] of interest to our studies.

All end states, i.e., FBs, IONs, UEs, and Qs, are recorded to calculate relative probabilities, and for the case of FBs we produce postencounter distributions for the recoil velocity and eccentricity of [PSR A, CO] (or [NS, CO]). For [BH, BH] the distributions include IONs, UEs, and Qs. All distributions are normalized to unity.

### 3.4. Energy Exchange and Recoil Velocities

As in the case of three-body encounters (Hills 1983a, 1983b; Quinlan 1996), and given the large mismatch between the mass of the binary pulsar [PSR A, CO] (or [NS, CO]) and the mass of the black hole binary [BH, BH] (or [BH, *]), we are led to quantify the mean energy exchange per scattering of the target binary ([BH, BH] or [BH, *]) as

$$\frac{\Delta E_{BH}}{E_{0, BH}} = \xi_{BH} \frac{m_B}{M_T},$$  \hspace{1cm} (2)

where $M_T = M_{BH} + m_B$ or $M_T = M_{BH} + m_*$ for the two cases and $\xi_{BH}$ is derived from our four-body simulations. When the black hole binary hardens, most of its energy change will go into kinetic energy of the light projectile, giving a postencounter recoil speed

$$V_{PSR-A}^2 \sim \xi_{BH} \frac{G \mu_{BH}}{a_{BH}},$$ \hspace{1cm} (3)

where again $\mu_{BH}$ is the reduced mass of either [BH, BH] or [BH, *].

The cross section, enhanced by gravitational focusing, is defined as

$$\Sigma_{BH} = \pi b_{\text{max}}^2 = \Sigma \pi (a_{BH}^2 + a_B^2) V_{ion}^2 / V_{\infty}^2 \approx \Sigma a_{BH} GM_T / V_{\infty}^2,$$ \hspace{1cm} (4)

where $b_{\text{max}}$ is the maximum impact parameter for which the fractional energy exchange per scattering $\Delta E_{BH} / E_{0, BH}$ is $10^{-5}$; $b_{\text{max}}$ is estimated at the start from the analytical expression of Sigurdsson & Phinney (1993) and later is determined more accurately from the numerical scattering experiments. Here $V_{\infty}$ denotes the preencounter relative velocity between the two centers of mass of the binaries; it is taken as close to 7–10 km s$^{-1}$, while

$$V_{ion}^2 = 2(m_B + M_T)(E_{0, BH} + E_{0, B}) / (m_B M_T)$$

is the velocity necessary to dissolve the four-body system (Bacon, Sigurdsson, & Davies 1996).

### 4. RESULTS

#### 4.1. Postrecycling Binary-Binary Encounters

As an example, we first compare the end states between [PSR A, CO] and the stellar-mass black hole binary [BH, BH] of $(50 \, M_{\odot}, 10 \, M_{\odot})$ with those resulting from the interaction of [PSR A, CO] off the black hole plus cusp star system [BH, *] of $(500 \, M_{\odot}, 1 \, M_{\odot})$. We recall that the initial semimajor axis of the binaries hosting the black holes (or the hole and the cusp star) is of 1 AU, corresponding to a binding energy of $4.5 \times 10^{48}$ ergs (note that [BH, BH] and [BH, *] have equal energy in this case). [PSR A, CO] at the observed separation of 0.0223 AU has a binding energy of $10^{47}$ ergs. Figure 4 shows the postencounter distribution of

![Figure 4](image-url)
statistics of the outgoing states in the post-recycling scenario

| State          | (500 $M_\odot$, 1.0 $M_\odot$) | (500 $M_\odot$, 1.4 $M_\odot$) | (30 $M_\odot$, 3 $M_\odot$) | (10 $M_\odot$, 10 $M_\odot$) | (50 $M_\odot$, 10 $M_\odot$) | (200 $M_\odot$, 10 $M_\odot$) |
|----------------|--------------------------------|--------------------------------|----------------------------|--------------------------------|--------------------------------|--------------------------------|
| FBs            | 0.452                         | 0.623                         | 0.645                      | 0.867                         | 0.657                         | 0.594                         |
| EXs            | 0.0                            | 0.001                         | 0.001                      | 0.0                            | 0.0                            | 0.0                            |
| IONs (triplets)| 0.273 (0.267)                 | 0.052 (0.043)                 | 0.003 (0.072)              | 0.059 (0.048)                 | 0.107 (0.092)                 | 0.121 (0.103)                 |
| Qs             | 0.027                         | 0.013                         | 0.024                      | 0.027                         | 0.033                         | 0.027                         |
| UEs            | 0.003                         | 0.011                         | 0.025                      | 0.002                         | 0.003                         | 0.008                         |

Notes.—The end-state distributions of the physical quantities are highly asymmetric. To quantify skewness we have introduced asymmetric values for their dispersions. Dispersions around the mean values are calculated considering those values which contain 34% of the total area in the left and right wings, respectively. Dispersion around the peak values are calculated considering those values which contain 50% of the total area descending from the peak. The 12,000 binary-binary encounters simulated have been subdivided as follows: 1000 (for the 500 $M_\odot$, 1 $M_\odot$ binary), 1000 (500, 1.4), 1000 (500, 3), 1000 (10, 10), 1000 (40, 40), and 7000 (200, 200).

$V_{p,PSR A}$ obtained for the two cases, collecting only FBs (the relevant end states for the description of $[PSR A, CO]$). The distributions peak at $V_{p,PSR A} \sim 30$ km s$^{-1}$ (for $[BH, BH]$) and 45 km s$^{-1}$ (for $[BH, BH]$), just around the value of the recoil speed necessary to propel the binary pulsar in the halo NGC 6752. The distributions are remarkably asymmetric, and we quantify their dispersion around the peak value in Table 1. Table 1 collects postencounter physical quantities, while Table 2 collects data on relative occurrence probabilities for the different end states. In general, FBs are associated to less-vigorous/weak-recoil dynamical encounters compared to IONs. For $[BH, BH]$, FBs amount to 66% of all events, while only 11% of the outcomes end in IONs (see Table 2). For $[BH, BH]$, IONs amount to 27% of the events, against 45% of FBs. The single massive black hole, having a stronger tidal field and a stronger gravitational focusing (the overall field being closer to a monopole), imposes a higher frequency of IONs ending in a narrower velocity distribution for the FBs.

The postencounter eccentricity of $[PSR A, CO]$ is remarkably different in the two cases. Figure 5 shows the two distributions. In the case of interaction of $[PSR A, CO]$ with a [BH, BH] binary, the end state eccentricities peak around $\sim 10^{-5}$, and so $[PSR A, CO]$ preserves the very low initial eccentricity. The interactions off $[BH, BH]$ are instead quite damaging, yielding a significant change in $E_{p,PSR A}$. The difference can be ascribed, again, to gravitational focusing. In fact, in the interaction of $[PSR A, CO]$ off $[BH, BH]$, gravitational stresses onto the incoming binary pulsar are reduced (because of the smaller total mass of the system) and softened (because of the dipolar nature of the interaction with the two BHs), thus causing less damage to the internal orbits of $[PSR A, CO]$.

Table 1 surveys results from a series of runs that considered stellar-mass and intermediate-mass binary black holes. The recoil speed of $[PSR A, CO]$ increases with increasing $\mu_{BH}$, but for the most massive binary, i.e., the (200 $M_\odot$, 10 $M_\odot$) case, the rise of $V_{p,PSR A}$ is faster than that implied by the linear scaling with $\mu_{BH}$ (eq. [2]), suggesting a dependence of $\mu_{BH}$ on the total mass of the binary. For the (200 $M_\odot$, 10 $M_\odot$) binary, we find that the peak value of the recoil speed of $[PSR A, CO]$ largely exceeds the escape speed. It varies with $a_{BH}$ as

$$V_{p,PSR A} = V_{p,PSR A}(1 AU) \left( \frac{1 AU}{a_{BH}} \right)^{0.44},$$

(5)
so we can reduce the magnitude of the recoil considering FBs off a much wider [BH, BH]; at a separation \(a_{BH} \approx 7 \text{ AU} \), [PSR A, CO] would receive the right pull. Note that this intermediate-mass black hole binary does not damage significantly the eccentricity of [PSR A, CO], despite the fact that the mass of the heaviest hole (200 \(M_\odot\)) is not so far from the mass \(M_\text{BH}\) of the single hypothetical black hole.

The close encounters with the binary pulsar exert random impulses on the [BH, BH] binary. Linear momentum conservation then imposes a recoil velocity to the center of mass of the [BH, BH] binary. Figure 6 shows the skewed distribution of the recoil velocity for the (50 \(M_\odot\), 10 \(M_\odot\)) black hole binary scattering off the recycled [PSR A, CO]. Table 1 collects characteristic values of the recoil speed computed averaging over all FBs and IONs. The rms recoil velocity of the binary black holes is clearly in excess to its equipartition value (as expected for a point mass in a star background). It will be moderated by frictional drag, and we defer to a discussion (see Merritt 2002 for a study of the Brownian motion of a massive binary near equipartition).

As described by equation (2), the target black hole binary transfers its gravitational binding energy to the binary pulsar (mainly in the form of kinetic energy of the center of mass of the projectile binary). In the last row of Table 1 we collect the values of \(\xi_{BH}\) obtained averaging over all encounters, and in parentheses we show the values obtained selecting only FBs. The significantly lower value of \(\xi_{BH}\) when inclusive of all events is related to the formation of triple systems: the formation of a hierarchical triple requires loss in binding energy of the target binary that widens to incorporate the third star.

The black hole binary transfers its internal orbital angular momentum \(J_{0,BH}\) to the orbit of the outgoing binary pulsar (which in this context can be treated as a point mass, having negligible internal angular momentum). The initial Keplerian angular momentum of the binary black holes,

\[
J_{0,BH} \sim 10^{55}(M_{BH,200})(m_{BH,10})(M_{T,210})^{-1/2} \times (a_{BH,1AU})^{1/2} \text{ g cm}^2 \text{ s}^{-1} \]

[scripts refer to units consistent with a (200 \(M_\odot\), 10 \(M_\odot\)) binary] exceeds by a factor of \(\sim 10\) the orbital angular momentum of the pulsar

\[
J_{0,B} \sim 10^{44}m_{B,1.6}(b_\infty/30 \text{ AU})(V_\infty/10 \text{ km s}^{-1}) \text{ g cm}^2 \text{ s}^{-1} ,
\]

where the impact parameter \(b_\infty \sim (0.5a_{BH}GM_T/V_\infty^{1/2})\) and \(V_\infty\) define the initial unperturbed hyperbolic orbit of the projectile. We find that the fractional angular momentum change of the black hole binary \(\Delta J_{BH}/J_{0,BH}\) amounts to \(\sim 0.01–0.1\) (see Table 1). This is comparable to or less than \(J_{0,B}/J_{0,BH}\), so the binary pulsar can only marginally modify its postencounter orbital angular momentum vector. A more massive binary black hole (with mass \(\gtrsim 1000 \ M_\odot\)) would be necessary to produce a sizeable change in the angular momentum vector of the binary pulsar. Stars of mass near turnoff \((m \sim 0.5 \ M_\odot)\) could instead be affected (see Miller 2003 for a discussion on the possible origin of rotation observed in the core of globular clusters and attributed to a binary black hole).

4.2. Prerecycling Binary-Binary Encounters

In this section we explore binary-binary encounters before the neutron star is recycled, studying two cases: a (50 \(M_\odot\), 10 \(M_\odot\)) binary and the single black hole [BH, *].

As shown in Table 3, the thrust received by [NS, CO] is within the correct range for expulsion of the binary (progenitor of PSR A) in the halo of NGC 6752 in either case ([BH, BH] or [BH, *]). Despite the large difference in mass between
5. THE STATISTICS OF THE ENCOUNTERS

In the postrecycling scenario, FBs are statistically more frequent. In general, with increasing mass of the black hole binary and decreasing mass ratio $m_{BH}/M_{BH}$, the frequency of IONs increases from 8%–12% if the target is a [BH, BH] binary and up to ~30% in the [BH, *] case. Triple systems have a high probability of formation among IONs, and they are described in § 6. Qs account for ~20%–30% of the end states; many of these systems are wide and loose, such that the interaction with the gravitational potential of the cluster causes their fission into two separate binaries. In the prerecycling case the statistics reverse, causing IONs to become more important than FBs, with a frequency as high as 85% for the [BH, *] case. FBs comprise only 7% of the events in the black hole–cusp star hypothesis and up to 23% in the [BH, BH] case.

Having computed the frequency of FBs relative to all possible end states, we can estimate the collision rate of the binary off the [BH, BH] or [BH, *]:

$$\dot{Q}_{coll} = \frac{1}{\tau_{coll}} \sim 2\pi a_{BH} G M T \frac{f_{INS}}{V_{\infty}},$$

where $n_{NS}$ is the number density of neutron stars impinging on the target black hole(s) and $f$ is the fraction in binaries with a star. The density of neutron stars is largely unknown given the uncertainties in their retention fraction at birth, but its value is expected to be larger than the density $n$ of stars (Sigurdsson & Hernquist 1993). The value of $f$ is even more uncertain. To circumvent the problem of estimating $n_{NS}$ and $f$, we list in Table 5 the minimum density of recycled neutron stars $f_{INS}$ necessary to have an encounter probability comparable to that of detecting [PSR A, CO] at the current position. Since the estimated time of dynamical friction $\tau_{DF}$ is ~1 Gyr, this minimum density is computed imposing a rate $\dot{Q}_{coll} \sim \tau_{DF}^{-1}$ and a black hole binary separation $a_{BH}$ equal to the maximum necessary to acquire a recoil velocity of the order of ~35 km s$^{-1}$. These minimum densities cluster around 50 to 10$^3$ pc$^{-3}$, implying a ratio $f_{INS}/n$ that is less than that expected from dynamical arguments (Sigurdsson & Hernquist 1993). Note that in the case of prerecycling, $f_{INS}$ can be much larger, since there are no constraints on the spin history of the neutron star.

6. EXOTIC END STATES

A black hole binary in a cluster is a catalyst for the formation of exotic triple systems composed of the two black holes and a star. This is a consequence of the high frequency of ionization events in binary-binary encounters ending with the capture of one of the two binary stars. Tables 2 and 4 give in parentheses the fraction of triple systems that form out of the total. These comprise from $\leq 5\%$ up to 30% of all events in [BH, BH] binaries. The question is now whether the triplet that forms is long-term stable or short-term stable, i.e., destined to lose one of its components after a few outer-orbital times (Mardling & Aarseth 2001).

In our experiments we have focused attention on binary-binary encounters leading to the formation of triplets with an active pulsar (in the postrecycling case) orbiting around one of the two black holes. We then extract from the sample those triplets that fulfill the long-term stability criterion following Mardling & Aarseth (2001; their eq. [90]). The stable triplets are more frequently composed by the millisecond pulsar orbiting the heavier black hole (forming the

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**Table 3**

| Quantity | (500 $M_{\odot}$, 1.0 $M_{\odot}$) | (50 $M_{\odot}$, 10 $M_{\odot}$) |
|----------|----------------------------------|-------------------------------|
| $V_{PSR A}$ (km s$^{-1}$)          | 45.4 (0.10)                    | 25.0 (0.01)                   |
| $\langle V_{PSR A} \rangle$ (km s$^{-1}$) | 62.1 (0.13)                    | 67.3 (0.08)                   |
| $V_{BH}$ (km s$^{-1}$)             | 0.4 (0.01)                     | 3.2 (0.02)                    |
| $V_{INS}$ (km s$^{-1}$)            | 0.5 (0.02)                     | 3.2 (0.02)                    |
| $\eta_{PSR A}$                    | 0.9 (0.04)                     | 0.4 (0.01)                    |
| $\langle \Delta E/E_{PSR A} \rangle (\times 10^{-5})$ | -1.6 (0.13)                    | -3.5 (0.24)                   |
| $\langle \Delta E/E \rangle$ (\times 10^{-5}) | 0.1 (0.26)                     | 1.3 (0.23)                    |
| $\langle \Delta f_{BH} \rangle (\times 10^{-5})$ | -0.01 (0.30)                   | -1.0 (0.31)                   |

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**Table 4**

| State               | (500 $M_{\odot}$, 1.0 $M_{\odot}$) | (50 $M_{\odot}$, 10 $M_{\odot}$) |
|---------------------|----------------------------------|-------------------------------|
| FBs                 | 0.068                            | 0.232                         |
| EXs                 | 0.0                              | 0.0                           |
| IONs (triplets)     | 0.844 (0.669)                    | 0.686 (0.341)                 |
| Qs                  | 0.087                            | 0.079                         |
| UEs                 | 0.0003                           | 0.003                         |

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*Notes.*—Tabulated values are as in Table 1. We have simulated 4000 binary-binary encounters for two types of binary black holes ($M_{BH}$, $m_{BH}$ in solar masses): 3000 for (500, 1) binaries and 1000 for (50, 10) binaries.
inner binary), with the lighter black hole (the outer binary) orbiting the inner pair. The typical parameters of the inner binaries found in our simulations have orbital periods ranging between 10 and 100 days, corresponding to orbital separations of 200–10,000 lt-s. The orbital acceleration imparted to the millisecond pulsar by the black hole would not hamper its detection by current deep globular cluster surveys (Possenti et al. 2003). Figure 7 shows the formation of a triplet for the (50 $M_\odot$, 10 $M_\odot$) binary. Triplets with the millisecond pulsar and the light black hole as inner binary are long-term stable only if their orbit is very tight. The stronger acceleration suffered by the millisecond pulsar due to its proximity to the hole would induce very rapid changes in the apparent spin period along the orbital motion implying a strong bias against the discovery of such pulsars.

Millisecond pulsar–black hole binaries have long been searched for in globular clusters since they can form via three-body exchange interactions between single black holes and binary pulsars (Sigurdsson 2003). We here foresee the possibility of detecting a hierarchical triple millisecond pulsar–black hole–black hole system that would provide a direct measure of the mass of the heaviest black hole.

Considering the case of a single intermediate-mass black hole in the cluster core, binary-binary encounters of the type studied here can lead to the capture of a millisecond pulsar thanks to the interaction of the binary pulsar with the star belonging to the cusp. This is a possibility that will be studied in more detail, introducing a model for the cusp.

7. LINKING PULSAR ACCELERATION TO THE INTERMEDIATE-MASS binary BLACK HOLE HYPOTHESIS

The single intermediate-mass black hole $M_{BH} \lesssim 500$ can account for a portion of the unseen matter required for explaining the spin derivatives of PSR B and PSR E (and perhaps PSR D).

Alternatively, one may wonder whether a suitably located perturber such as a black hole binary can accelerate the two pulsars. (This would reduce the demand of a significant amount of underluminous matter in the core of NGC 6752.) The perturber should impart a line-of-sight acceleration $GM_{F}/P^2 \sim c|\mathbf{P}|$. Considering the value of $|\mathbf{P}| = (9.6 \pm 0.1) \times 10^{-17}$ s$^{-1}$ for the two pulsars (PSR B and PSR E) and a separation $l \sim 0.03$ pc (D’Amico et al. 2002), comparable to the projected distance between the two pulsars, we can infer the minimum mass that the [BH, BH] binary should have to impart the observed acceleration: $M_{F} \gtrsim 180$ $M_\odot$. Thus, the (200 $M_\odot$, 10 $M_\odot$) binary has the right total mass. Assuming a harmonic potential for the central region of the cluster with uniform stellar mass density ($\rho$) of $\sim 10^5$ $M_\odot$ pc$^{-3}$ (where $\langle m \rangle$ is the mean stellar mass), the minimum recoil velocity for moving the black hole binary from the center of the potential well to the pulsar projected locations (at $r \sim 0.08$ pc) is

$$\langle V_{BH, min}^2 \rangle^{1/2} \sim 4 \left( \frac{r}{0.08 \text{ pc}} \right) \left( \frac{\langle m \rangle}{10^5 \text{ $M_\odot$ pc}^{-3}} \right)^{1/2} \text{km s}^{-1}. \tag{7}$$

Tables 1 and 3 show that a [BH, BH] binary of the required mass has typical $V_{BH} \sim 1$ km s$^{-1}$ (2 km s$^{-1}$) in the post-(pre-) recycling scenario, lower than the value required. However the black hole binary is subject to repeated close encounters with cluster stars before dynamical friction drives it toward the center of gravity of the cluster (Merritt 2002). This depends on the comparison between the dynamical friction timescale $\tau_{DF} \sim \sigma^3/(1.89 G^2 \langle m \rangle n M_F)$ and the collision time off stars $\tau_{coll} \sim \sigma/(2\pi GM_{BH} n_{BH})$. Their ratio

$$\frac{\tau_{DF}}{\tau_{coll}} \sim 5 \left( \frac{d_{max} \text{ BH}}{7 \text{ AU}} \right) \left( \frac{\sigma}{10 \text{ km s}^{-1}} \right)^2 \left( \frac{0.5 \text{ $M_\odot$}}{\langle m \rangle} \right) \tag{8}$$

is above unity but uncomfortably close. Because of this non-equilibrium dynamics, an intermediate-mass black hole binary of (200 $M_\odot$, 10 $M_\odot$) may random-walk up to the current position of PSR B and PSR E. This scenario is possible but needs to be fine-tuned.

8. BLACK HOLE BINARY EVOLUTION

In this section we address the issue of the survival of our stellar and intermediate-mass black hole binaries against hardening by stellar collisions.

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Footnote: See also the respective papers by B. A. Jacoby and S. M. Ransom in Globular Clusters: Formation, Evolution, and the Role of Compact Objects, ed. L. Bildsten, A. Cool, F. Rasio, & S. Zepf (available at http://online.itp.ucsb.edu/online/clusters_c03/jacoby and http://online.itp.ucsb.edu/online/clusters_c03/ransom).
8.1. Binary Hardening

The last black hole binary that remains in the cluster core (Portegies Zwart & McMillan 2000) has a large cross section and interacts with the cluster stars. Thus, we expect it to harden while progressively ejecting stars and emitting GWs; at the same time, stars may be tidally captured.

It is the heaviest black hole in the binary that preferentially eats (unbound) stars because of its higher gravitational focusing. This leads to a secular change in the mass and orbital parameters. We mimic evolution, solving the equations for the binary separation $d_{BH}$ and mass $M_{BH}$ as a function of time:

$$\frac{dM_{BH}}{dt} = 2\pi GM_{BH} \sum_{i} \frac{m_i n_i}{\sigma_i}$$

$$\frac{d\sigma}{dt} = -2\pi \xi_{BH} G a_{BH}^2 \sum_{i} m_i n_i - 64G^3 M_{BH} m_{BH} M_{T} \frac{e^2 a^3(1-e^2)^{7/2}}{c^2}$$

where the sum over index $i$ is inclusive of all stars, i.e., main-sequence stars, white dwarfs, and neutron stars, distributed according to the Salpeter initial mass function and in thermal equilibrium. The capture rate on $M_{BH}$ refers to a cross section equal to $2\pi GM_{BH} r_{\text{in}}/\sigma_i^2$, where $r_{\text{in}} = r_{i}[M_{BH}/m_i]^{1/3}$ is the tidal radius of the star considered. We assume that the entire star is eaten. Figure 8 shows $M_{BH}$ for two values of the stellar density $\eta$ (treated as a constant in time). We found that a secular increase in the mass of the binary black hole becomes appreciable only if the initial mass $M_{BH}(0) > 20 M_{\odot}$, under the background conditions imposed. A $(50 M_{\odot}, 10 M_{\odot})$ or $(30 M_{\odot}, 10 M_{\odot})$ binary avoids coalescence, and its separation remains $\lesssim 1$ AU for an appreciably long time. An intermediate-mass black hole binary (such as that considered in § 7 with $M_{BH} \gtrsim 200 M_{\odot}$ at the start) should have already terminated its life or be on the verge of coalescing, ending in a single black hole, and curiously should have grown to a mass close to $M_{BH}$. Clearly, binary evolution is rather sensitive to the background density, which is itself evolving with time (a fact that we did not include in this simplified treatment). Thus, the dynamical evolution of an intermediate-mass black hole binary should thus be addressed using a full N-body code. Lighter stellar-mass black hole binaries are still in the hardening phase, and the mass increase is not significant. If our binaries coalesce by GW emission, they remain always far from the critical regime (corresponding to mass ratios $m_{BH}/M_{BH}$ around 0.385) at which the gravitational rocket effect comes into play (Fitchett & Detweiler 1984) and avoid ejection from the cluster because of release of nonzero net linear momentum.

8.2. Star Ejection

Since black hole hardening (occurring at a rate $da_{BH}/dt = -2\pi \xi_{BH} G (m/\sigma^2)$) is followed by stellar ejection, a correlation exists between the mass lost in stars

$$M_{\text{deficit}} \sim \frac{M_{T}}{\xi_{BH}} \ln \left[ \frac{a_{BH}(0)}{a_{BH}(t)} \right]$$

and the total mass $M_{T}$ of the binary black hole (treated as a constant). In equation (11), $a_{BH}(0)$ refers to the maximum semimajor axis at which the interactions are strong enough to eject stars. When $a_{BH}(t) \sim a_{GW}$, $M_{\text{deficit}}$ is the mass that needs to be ejected in order to drive the binary toward coalescence by GWs. If NGC 6752 still hosts a binary black hole, one may ask whether the “deficit” in stars created by their ejection from the cluster would alter appreciably the stellar density profile or velocity field in the central regions.

If, similarly to what is claimed for bright “core ellipticals,” a binary black hole turns an otherwise power-law density profile into a core (or shallower power law) because of the ejection of stars (see Ferrarese et al. 1994; Milosavljevic & Merritt 2001; Milosavljevic et al. 2002 for details), then one can verify whether the “mass deficit” (defined as the mass in stars that would need to be removed from an initially power law profile in order to produce the observed core) correlates with the mass of the binary black hole according to equation (11). Deprojecting the power-law+core profile of Ferraro et al. (2003b; Fig. 6), we infer for NGC 6752 a deficit of 300 “missing” stars. This is close to the mass that would be ejected by one of our binary black holes (eq. [11]). Since the central relaxation time is of the order of the ejection time, a flow of stars can refill an underlying power law, but gravitational heating (Spitzer 1987) induced by star loss can counteract this process. Only a detailed cluster model with a core binary black hole can address this question in detail, and one should be designed specifically for NGC 6752.

9. CONCLUSIONS

What have we learned from this analysis?

1. A stellar-mass binary black hole of $(50 M_{\odot}, 10 M_{\odot})$ can impart the right thrust to PSR A in a gentle flyby, leaving its eccentricity almost unperturbed. Flybys off this binary can occur at an acceptable rate without imposing
unrealistic conditions on the neutron star density inside the central 0.1 pc. This binary black hole lives long in the cluster. Lighter black hole binaries or binaries with a black hole and a neutron star have interaction rates smaller by a factor \(\sim 10\) and can be excluded (see Table 5).

2. An intermediate-mass binary black hole of \((200 M_{\odot}, 10 M_{\odot})\) is more aggressive on the binary pulsar, since it tends to impart large recoil speeds to any incoming particle. Thus, it has to be sufficiently wide \(a_{\text{BH}} \sim 7–10\) AU to propel [PSR A, CO] (or [NS, CO]) into the halo of NGC 6752 at the speed required. Such a binary demands the lowest neutron star density to allow one ejection every billion years, i.e., over a time comparable to the lifetime of PSR A in the halo. The values of \(f_{\text{NS}}\) refer to the pre-recycling scenario for a \((500 M_{\odot}, 1 M_{\odot})\) binary and to the post-recycling scenario for all remaining binaries.

### Table 5: Minimum Neutron Star Density

| \(M_{\odot}\) (M.) | \(a_{\text{max,BH}}\) (AU) | \(f_{\text{NS}}\) (pc\(^{-3}\)) |
|-------------------|-------------------|------------------|
| 500, 1.0...........| 1                 | 2.9 \times 10^2  |
| 50, 1.4............| 1                 | 1.5 \times 10^1  |
| 30, 3..............| 2                 | 1.1 \times 10^1  |
| 10, 10............| 3.6               | 7.6 \times 10^2  |
| 50, 10............| 6                 | 2.0 \times 10^2  |
| 200, 10...........| 7                 | 5.5 \times 10^1  |

**Notes.** Minimum neutron star density \(f_{\text{NS}}\) in the cluster core to ensure an interaction every billion years, i.e., over a time comparable to the lifetime of PSR A in the halo. The values of \(f_{\text{NS}}\) refer to the pre-recycling scenario for a \((500 M_{\odot}, 1 M_{\odot})\) binary and to the post-recycling scenario for all remaining binaries.

3. A single intermediate-mass black hole of \(M_{\text{BH}} \gtrsim 500 M_{\odot}\) is a possibility. The recoil speed of PSR A falls in the correct interval, and considering its weak dependence on \(a_{\text{BH}}\) (recoil is more sensitive to the reduced mass of the system), even a 1000 \(M_{\odot}\) one would fit in this picture. The gravitational encounter with PSR A must occur before recycling since the binary hosting the neutron star suffers a considerable change in its eccentricity. Circularization can be achieved later, during the phase of Roche lobe mass transfer; we find that the binary is left in an outgoing state such that scattering-induced recycling is favored.

Considerations 1, 2, and 3 do not address the problem of the acceleration of PSR B and PSR E. Can we provide a self-consistent picture? If there is a stellar-mass binary black hole in the cluster propelling PSR A, this binary cannot act as a local perturber on the two pulsars. It is too light. The best candidate for the local perturber hypothesis is our intermediate-mass binary black hole (the \(200 M_{\odot}, 10 M_{\odot}\) binary). The binary is sufficiently light that it can still random-walk across the core to produce the local acceleration of the two pulsars. If instead the acceleration of PSR E (and perhaps PSR D) is caused by the overall effect of the cluster potential well, the single intermediate-mass black hole plays a role but some additional mass should be present in the form of collapsed remnants such as white dwarfs and neutron stars.

Black hole binaries are special catalysts for the formation of “stable” triplets. The interaction of a binary pulsar with an intermediate-mass binary black hole can create an extraordinary system: a millisecond pulsar–black hole–black hole hierarchical triple (the companion star leaving the system). The single intermediate-mass black hole may, on the other hand, host in its cusp a millisecond pulsar. In this perspective, timing measurements on this “planetary-like pulsar,” either a member of a triplet or a member of the cusp, would give the unique possibility of discriminating the nature, either single or binary, of the black hole(s), providing also a reliable estimate of their mass. This opportunity exists in globular clusters contrary to the case of the Galactic center, where the signal from a millisecond pulsar belonging to the cusp of our central black hole would be completely smeared out by scattering in the interstellar medium.

A binary black hole may steadily perturb its environment (§ 8.2). The distinction between a stellar-mass and an intermediate-mass binary black hole is subtle and difficult to make. Accurate evolutionary models of clusters hosting a binary black hole of various masses need to be developed, since they can indicate the fingerprints left by such exotic binaries. They also may provide clues for discriminating between the single versus binary nature of the central black hole(s) that may inhabit NGC 6752.

**Note added in manuscript.**—During the revision of this manuscript, Ferraro et al. (2003a) reported the optical identification of the companion star to PSR A, a helium white dwarf. The derived cooling age (\(\sim 1.2–2.8\) Gyr) suggests that the dynamical encounter responsible for the ejection of PSR A into the cluster halo occurred after the neutron star was recycled or was triggered at the time of ejection. In the latter case the binary would have had to have suffered very rapid evolution (see Bassa et al. 2003).

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