Dust-acoustic periodic travelling waves in a magnetized dusty plasma with trapped ions and nonthermal electrons in astrophysical situations: oblique excitations

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Abstract
Obliquely propagating three-dimensional dust-acoustic periodic travelling waves (DAPTWs) in a magnetized dusty plasma composed of negatively charged inertial dust particles, trapped ions, and nonthermal fast electrons have been undertaken. In the dusty plasma model at hand, the dynamic behaviors of DAPTWs are governed by a Schamel equation. The presence of dust acoustic solitary waves (DASWs) and DAPTWs is investigated via bifurcation analysis of the Hamiltonian system. In the nonlinear regime, the Sagdeev potential and phase portrait structures indicate the presence of small-amplitude DAPTW solutions. The influences of intrinsic physical parameters include the strength of the static magnetic field, the obliqueness of propagation, the thermal pressure of charged dust grains, the electron to dust density ratio, the trapping parameter of trapped ions, and the degree of nonthermality of fast electrons on the characteristics of DAPTWs are simulated numerically. In particular, the findings illustrate that the amplitude of DAPTWs is reduced as the numerical values of the trapping parameter are decreased. Interestingly, the theoretical simulations of numerical results can significantly highlight the physical nature of DAPTWs in astrophysical situations such as Earth’s magnetosphere, auroral region, and heliospheric environments.

Keywords Dusty plasma · Schamel equation · Bifurcation theory · Dust-acoustic periodic travelling waves · Trapped ions · Nonthermal fast electrons

1 Introduction
The trapping of electrons and ions in a dusty plasma is an interesting nonlinear phenomenon. In this situation, the electrostatic wave potential confines some charged particles (i.e., electrons and ions) in a particular region in the dusty plasma model. These particles are forced to bounce back and forth. In a dusty plasma system, the inertia is well known to be provided by the mass of dust grains, whereas the restoring force comes from the pressures of inertialess ions and electrons (Mamun et al. 1996). Negative charged dusty plasmas are usually dominated due to the flow of plasma currents to their surfaces. Hence, the trapping process makes the electron and ion species deviate from Maxwellian distribution; therefore, they follow non-thermal distribution functions depending on the laboratory and space plasma environment under consideration. One of the most commonly non-thermal distribution functions which describe the trapping process of the electron and ion species in many laboratory and space plasmas is the Schamel distribution function (Schamel 1972, 1973, 1986). In general, many investigators have studied the effects of trapped ions on the non-linear waves for different plasma models (Mamun 1998; Adhikary et al. 2014, 2017; Dev et al. 2015a, 2015b; El-Hanbaly et al. 2015; Misra and Wang 2015; Sultana 2021). For example, a theoretical investigation of dust acoustic solitary waves (DASWs) in an unmagnetized three-component dusty plasma consisting of negative dust fluid, free electrons, and trapped and free ions is studied by (Mamun 1998). Due to the departure of trapped ions from the Boltzmann distribution, the small amplitude dust acoustic waves (DAWs) dynamics are governed by a Korteweg-de-Vries (KdV) type equation. Moreover, several authors have shown that the trapped ion species substantially affect the profile of dust...
acoustic shock waves (Adhikary et al. 2014, 2017; Dev et al. 2015a, 2015b; El-Hanbaly et al. 2015). In addition, many space and laboratory observations indicate the existence of electrons with high energies that do not obey Boltzmann distribution. Instead, electron energy distribution frequently has more complicated shapes with long tails, modeled by a nonthermal distribution (Cairns et al. 1995; Saha and Chatterjee 2009; Mamun and Shukla 2009; Selim 2016). The first attempt, which describes the energetic electrons observed by the FREJA satellite using the nonthermal distribution, was developed by Cairns et al. 1995. Later, the existence of nonthermal electrons in various astrophysical environments, such as solar wind, magnetosphere, interstellar medium, and auroral zone plasmas, confirmed by the Vela satellite (Lundin et al. 1989), are investigated (Cairns et al. 1995; Saha and Chatterjee 2009; Mamun and Shukla 2009; Selim 2016; Lundin et al. 1989; Futaana et al. 2003; Gill et al. 2007). Also, the ASPERA recorded the disappearance of energetic electrons from the upper ionosphere of Mars on the Phobos 2 satellite. The effects of electrons’ nonthermal distribution and the polarity of dust charge number density on nonplanar spherical and cylindrical DIASWs are investigated by Mamun and Shukla (2009) and Selim (2016). It is observed that the features of the DIASWs are significantly modified by the effects of the nonthermality of the electron distribution and the geometry factor (Mamun and Shukla 2009). Misra and Wang (2015) have investigated the nonlinear characteristics of DAWs in magnetized dusty plasma, including negative charged dust fluid, vortex-like ion distribution, and nonthermal fast electrons. They demonstrated that under a critical value of the percentage of energetic electrons, the excitation of DAWs vanishes.

In recent years, the bifurcation theory (Chow and Hale 1981; Saha and Banerjee 2021) has become one of the most interesting and famous approaches to study the dynamical behavior of plasma systems. Indeed, each trajectory of a phase portrait of the dynamical system indicates a nonlinear wave in the plasma model. In this sense, the bifurcation analysis of the phase portrait for the Hamiltonian of the system has been extensively employed to study the physical nature of the travelling nonlinear dust acoustic waves in plasma. Moreover, this analysis has significant applications in different plasma environments (Samanta et al. 2013; Selim et al. 2015; Kanti Das et al. 2017; Abdelwahed et al. 2017; Saha et al. 2020; El-Shamy et al. 2020; Abdikian et al. 2021; Pradhan et al. 2021; Tolba 2021; Selim et al. 2022a). For instance, Selim et al. (2015) have investigated the bifurcation analysis of nonlinear ion-acoustic waves traveling in a multicomponent magnetoplasma with superthermal electrons. The propagation of nonlinear dust ion acoustic periodic waves (DIAPWs) in a dusty plasma consisting of stationary charged dust grains, cold ions, and two temperature superthermal electrons is studied by Abdelwahed et al. (2017).

The possible range of Mach number on the existence of supernonlinear waves in a two-component Maxwellian plasma has been reported by Saha et al. (2020). Recently, the bifurcation method has been employed to analyze the nonlinear and supernonlinear ion acoustic waves in electron-ion plasmas with electrons obeying generalized \((r, q)\)-distribution (Abdikian et al. 2021). Also, El-Shamy et al. (2020) applied the bifurcation approach to study the properties of the nonlinear acoustic waves in a magnetized ultrarelativistic degenerate plasma composed of warm fluid ultrarelativistic degenerate inertial electrons and positrons and immobile heavy negative ions. Recently, Selim et al. (2022a) studied the effects of trapped ions concentration on the dust acoustic solitary and periodic travelling wave dynamics in a dusty plasma consisting of dust fluid, trapped ions, and Maxwellian electrons. However, in the light of the previous discussion, the influences of the trapped ions, nonthermal electrons, the strength of the external static magnetic field, and the obliqueness of propagation on the physical behavior of DAPTWs in dusty plasma seem to be a vital problem; thus, it will be the target of this work. This paper is organized as follows: the mathematical model for a magnetized dusty plasma composed of negatively charged inertial dust particles, trapped ions, and nonthermal fast electrons is presented in Sect. 2. Furthermore, a Schamel equation, i.e., Korteweg-de-Vries (KdV) type equation, that governs the dynamics of nonlinear waves propagating in the current plasma is derived. Sagdeev potential and bifurcation analysis are developed to examine the possibility of DAPTWs existence in Sect. 3. Numerical investigations and discussions are outlined in Sect. 4. Finally, Sect. 5 is devoted to the conclusions.

2 Model equations

In the present work, we consider a magnetized three-component plasma system consisting of collisionless, massive, micrometer-sized dust grains with negative charge, ions with trapped particles, and nonthermally distributed fast electrons. An external magnetic field is applied in the \(z\)-direction (i.e. \(B = B_0\hat{z}\)). The propagation of low frequency, compared to the dust cyclotron frequency, DAWs in this system is governed by the following set of normalized coupled nonlinear partial differential equations (Shukla and Mamun 2002; Fortov et al. 2005):

\[
\frac{\partial N_d}{\partial \tau} + \frac{\partial}{\partial X} (N_d U_{dx}) + \frac{\partial}{\partial Y} (N_d U_{dy}) + \frac{\partial}{\partial Z} (N_d U_{dz}) = 0, \tag{1}
\]

\[
\frac{\partial U_{dx}}{\partial \tau} + U_{dx} \frac{\partial U_{dx}}{\partial X} + U_{dy} \frac{\partial U_{dx}}{\partial Y} + U_{dz} \frac{\partial U_{dx}}{\partial Z} = \frac{\partial \Phi}{\partial X} - \frac{5}{3} \sigma N_d^{-1/3} \frac{\partial N_d}{\partial X} + \omega_c U_{dy}, \tag{2}
\]
\[
\frac{\partial U_{dy}}{\partial \tau} + U_{dx} \frac{\partial U_{dy}}{\partial X} + U_{dy} \frac{\partial U_{dy}}{\partial Y} + U_{dz} \frac{\partial U_{dy}}{\partial Z} = \frac{\partial \Phi}{\partial Y} - \frac{5}{3} \sigma \frac{\partial N_d}{\partial Y} - \omega_c U_{dx},
\]
\[
\frac{\partial U_{dz}}{\partial \tau} + U_{dx} \frac{\partial U_{dz}}{\partial X} + U_{dy} \frac{\partial U_{dz}}{\partial Y} + U_{dz} \frac{\partial U_{dz}}{\partial Z} = \frac{\partial \Phi}{\partial Y} - \frac{5}{3} \sigma \frac{\partial N_d}{\partial Y},
\]
\[
\frac{\partial^2 \Phi}{\partial X^2} = N_d + \delta N_e - \mu N_i.
\]

In Eqs. (1)–(5) \(X, Y, Z,\) and \(\tau\) are the space coordinates and time, normalized by the Debye length \(\lambda_D = (k_B T_i/4\pi e^2 Z_d n_{d0})^{1/2}\), and the dust plasma period \(\omega_{pd} = (m_d/4\pi \zeta Z_d n_{d0} e^2)^{1/2}\), respectively. The electron, ion, and dust number densities, \(N_e, N_i,\) and \(N_d\) are normalized by the unperturbed dust number density, \(N_{d0}\). Also, \(U_{dx}, U_{dy}, U_{dz},\) and \(\Phi\) are the dust fluid speed in \(X, Y, Z\) directions and the electrostatic potential, respectively, which are normalized by the dust acoustic speed, \(C_d = (Z_d k_B T_i/m_d)^{1/2}\), and \(k_B T_i/e,\) respectively, with \(k_B\) denotes the Boltzmann constant and \(T_i\) is the ion thermal temperature. The ratios \(\delta = N_{d0}/Z_d n_{d0}\) and \(\mu = N_e/Z_d n_{d0}\) are the number density ratios that satisfy the neutrality condition of charge, \(\mu = \delta + 1\), at the equilibrium state. The temperature ratio, \(\tau = T_d/T_i\), is the dust to ion thermal temperature ratio, and \(\omega_c = \sqrt{2k_B T_i/m_d} \) is the dust-cyclotron frequency normalized by the dust plasma oscillation frequency, \(\omega_{pd}\). Since the thermal motion of charged dust cannot keep up with the dust acoustic wave propagation, we have considered an adiabatic compression of the dust fluid and use the pressure law
\[
P_d = N_{d0} k_B T_d \left( \frac{N_d}{N_{d0}} \right) \gamma.
\]

In Eqs. (1)–(3), \(\gamma = (N + 2)/N,\) where \(N\) is the number of degrees of freedom. In the present work, \(N = 3\) for three-dimensional configuration, and hence, \(\gamma = 5/3\). It is assumed that the DAWs propagating in this system have low phase velocity, \(\lambda = \omega/k\), compared to the thermal velocities of ions and electrons, i.e. \(\nu_{id} << \lambda << \nu_{ie}\) (Mamun 1998). Hence, \(\nu_{ij} = \sqrt{k_B T_j/m_j}\), where \(m_j\) and \(T_j\), \(J = d, i, e\), being the mass and thermal temperature of dust, ions, and electrons, respectively. In the current model, the electron temperature is much greater than the ion temperature (i.e. \(T_e >> T_i\)) and \(n_{d0} >> n_{e0}\). The thermal conductivity, viscosity, and energy transfer effects of collisions are negligible. In addition, the magnetic pressure is considered more significant than thermal plasma pressure, and the dust grains’ charging is constant. Since we are dealing with ions with trapped particles distribution functions (Cairns et al. 1995; Mamun 1998; Misra and Wang 2015), the ion number density, \(n_i,\) is given in terms of the electrostatic potential, \(\Phi,\) at the small amplitude limit, as (Mamun 1998)
\[
n_i \simeq 1 - \Phi - \frac{4}{3} \left( 1 - \beta \right) \left( \Phi \right)^{3/2} + \frac{1}{2} \Phi^2,
\]
where the term \(-4/3(1-\beta)\left(\Phi\right)^{3/2}\) in the expansion of \(n_i\) represents the contribution of trapped ions, the case \(\beta = 0\), represents the plateau or flat-topped distribution, and \(\beta = 1\) corresponds to the Boltzmann distribution of ions. On the other side, the electrons are assumed to obey non-thermal distribution. Thus, the electron number density, \(n_e,\) can be expanded as
\[
n_e = (1 - \alpha \vartheta \Phi + \alpha \vartheta^2 \Phi^2) \exp(\vartheta \Phi),
\]
where \(\vartheta = T_e/T_i\) is the electron-to-ion thermal temperature ratio, the nonthermality parameter is \(\alpha = \frac{4a}{1+3a} \) with \(a > 0\), is representing the degree of nonthermality of the charged particles or the percentage of energetic or fast electrons in the plasma, and \(\alpha < 1 (\alpha > 1)\) corresponds to \(a < 1 (a > 1)\). The value \(a = 0\) corresponds to the case of electrons’ thermal equilibrium (Boltzmann distribution).

To derive the nonlinear KdV type equation for the small amplitude DAWs from Eqs. (1) to (5), we have to find an appropriate coordinate frame where the wave can be described smoothly. For this purpose, the reductive perturbation technique (RPT) (Washimi and Taniuti 1966) is applied as a powerful tool to derive the KdV type equation for the propagation of DAWs in the plasma system. According to the RPT, the independent variables in Eqs. (1) to (5) are stretched as
\[
\lambda = \varepsilon^{1/4} \left( L_x X + L_y Y + L_z Z - \lambda T \right) = \varepsilon^{1/4} (\eta - \lambda T),
\]
\[
T = \varepsilon^{3/4} \tau,
\]
where \(\varepsilon\) is a small parameter that measures the weakness of the wave amplitude, and \(\lambda\) is the wave phase velocity normalized by the dust acoustic velocity, \(C_d\). The direction cosines, \(L_x, L_y,\) and \(L_z\) of the wave vector along the axes, \(X, Y,\) and \(Z\) satisfy the equality; \(L_x^2 + L_y^2 + L_z^2 = 1\). The dependent variables \(N_d, U_{dx,y,z}\) and \(\Phi,\) in Eqs. (1) to (8), can be expanded as
\[
N_d = 1 + \varepsilon N_{d1} + \varepsilon^2 N_{d2}^2 + \varepsilon^3 N_{d3}^3 + \ldots,
\]
\[
U_{dx} = \varepsilon^{5/4} U_{dx1}^{(1)} + \varepsilon^{3/2} U_{dx2}^{(2)} + \varepsilon^2 U_{dx3}^{(3)} + \ldots,
\]
\[
U_{dy} = \varepsilon^{5/4} U_{dy1}^{(1)} + \varepsilon^{3/2} U_{dy2}^{(2)} + \varepsilon^2 U_{dy3}^{(3)} + \ldots,
\]
\[
U_{dz} = \varepsilon^{5/4} U_{dz1}^{(1)} + \varepsilon^{3/2} U_{dz2}^{(2)} + \varepsilon^2 U_{dz3}^{(3)} + \ldots,
\]
\[
\Phi = \varepsilon^{5/4} \Phi^{(1)} + \varepsilon^{3/2} \Phi^{(2)} + \varepsilon^2 \Phi^{(3)} + \ldots.
\]

Now, substituting these expansions into Eqs. (1) to (5), and collecting the terms of different powers of \(\varepsilon,\) in the lowest
order, we obtain
\[
\frac{\partial N_d^{(1)}}{\partial \eta} = \frac{L_x \partial U_d^{(1)}}{\lambda}, \quad \frac{\partial U_d^{(1)}}{\partial \eta} = \frac{5 \sigma L_x \partial N_d^{(1)}}{3 \lambda \partial \eta} - \frac{L_x \partial \Phi^{(1)}}{\lambda \partial \eta},
\]
\[
N_d^{(1)} = -[\mu + \Delta \theta (1 - \alpha)] \Phi^{(1)}.
\]
From these equations, one can derive the phase velocity as
\[
\lambda = L_x \sqrt{\left(\frac{1}{\mu + \Delta \theta (1 - \alpha)}\right) + \frac{5 \sigma}{3}}.
\]

Equation (18) represents the phase velocity of the DAWs propagating with velocity, $\lambda$, which must be real-valued. For $\lambda$ to be real, the expression in the square brackets of Eq. (18) must be positive, which gives $\alpha > \alpha_c = 1 + \frac{1}{\Delta \theta} (\mu + \frac{2}{\sigma})$. Therefore, DAWs propagate in the current plasma system when the percentage of nonthermal energetic electrons exceeds some critical value. For the following order of $\varepsilon$, we obtain
\[
\frac{\partial N_d^{(2)}}{\partial \eta} = \frac{1}{\lambda} \frac{\partial N_d^{(1)}}{\partial \eta} + \frac{L_x}{\lambda} \frac{\partial U_d^{(2)}}{\partial \eta} + \frac{L_x}{\lambda} \frac{\partial U_d^{(2)}}{\partial \eta} + \frac{L_x}{\lambda} \frac{\partial U_d^{(2)}}{\partial \eta},
\]
\[
\frac{\partial U_d^{(2)}}{\partial \eta} = \frac{1}{\lambda} \frac{\partial U_d^{(1)}}{\partial \eta} - \frac{L_x}{\lambda} \frac{\partial \Phi^{(2)}}{\partial \eta} + \frac{5 \sigma L_x}{3 \lambda} \frac{\partial N_d^{(2)}}{\partial \eta},
\]
\[
N_d^{(2)} = \frac{\partial^2 \Phi^{(2)}}{\partial \eta^2} - \frac{4 \mu (1 - \beta)}{3 \sqrt{\pi}} (-\Phi^{(1)})^{3/2} - c \Phi^{(2)},
\]
\[
U_d^{(2)} = \frac{\lambda L_x}{\omega_x^2} \left(1 + \frac{5 \sigma}{3} (\mu + \Delta \theta (1 - \alpha))\right) \frac{\partial^2 \Phi^{(1)}}{\partial \eta^2},
\]
\[
U_d^{(2)} = \frac{\lambda L_x}{\omega_x^2} \left(1 + \frac{5 \sigma}{3} (\mu + \Delta \theta (1 - \alpha))\right) \frac{\partial^2 \Phi^{(1)}}{\partial \eta^2}.
\]

Now, by eliminating the second-order perturbed terms and their derivatives from Eqs. (19)–(23) and substituting the first-order perturbed quantities from Eqs. (15)–(17), we obtain the following Schamel (KdV type) equation:
\[
\frac{\partial \Phi^{(1)}}{\partial \tau} + A \sqrt{-\Phi^{(1)}} \frac{\partial \Phi^{(1)}}{\partial \xi} + B \frac{\partial^3 \Phi^{(1)}}{\partial \xi^3} = 0,
\]
where
\[
A = \frac{1 - \beta}{\sqrt{\pi \lambda}} \frac{\mu L_x^2}{(\mu + \Delta \theta (1 - \alpha))^2},
\]
\[
B = \frac{\lambda (3 + (\mu + \Delta \theta (1 - \alpha))) (5 \sigma - 3 \lambda^2)}{6 \omega_x^2 (\mu + \Delta \theta (1 - \alpha))} + \frac{L_x^2}{2 \lambda (\mu + \Delta \theta (1 - \alpha))^2}.
\]

3 DAW and DAPTW bifurcation points

We now apply the bifurcation theory (Chow and Hale 1981; Selim et al. 2015; Abdelwahed et al. 2017; El-Shamy et al. 2020; Abdikian et al. 2021; Selim et al. 2022b; Samanta et al. 2013; Pradhan et al. 2021; Tolba 2021) to study the presence of DAW and DAPTW solutions of Eq. (24). Our starting point here is the analysis of Tolba 2021 that establishes the existence of DAW and DAPTW for a Schamel equation. We assume that the DASWs and DAPTWs move with a constant wave speed $v > 0$ and introduce the change of coordinates $\xi = \zeta - v \tau$. Accordingly, Schamel Eq. (24) can be transformed, using $\Phi^{(1)}(\zeta, \tau) = \Phi(\xi)$, into
\[
\frac{d^2 \Phi}{d \xi^2} = \frac{v}{B} \Phi + \frac{2A}{3B} (-\Phi)^{3/2}.
\]

The Hamiltonian system for Eq. (27) can be obtained by setting $\frac{d \Phi}{d \xi} = \varphi$, resulting in first-order differential equations that can be written as:
\[
\frac{d \Phi}{d \xi} = \psi,
\]
\[
\frac{d \psi}{d \xi} = \frac{v}{B} \Phi + \frac{2A}{3B} (-\Phi)^{3/2}.
\]

We are interested in numerically finding the DAW and DAPTW solutions to the Hamiltonian system (28). Therefore, one can express the Hamiltonian system in terms of Hamiltonian function $H(\Phi, \varphi)$ as follows:
\[
H(\Phi, \varphi) = \frac{\Psi^2}{2} + \frac{4A}{15B} (-\Phi)^{5/2} - \frac{v}{2B} \Phi^2 = h,
\]
where $h$ is a constant that determines the value of the energy. In addition, one can adapt Eq. (27) to express the potential, $V(\Phi)$, as
\[
V(\Phi) = \frac{4A}{15B} (-\Phi)^{5/2} - \frac{v}{2B} \Phi^2.
\]

Hamiltonian system (28) admits the following two equilibrium points on the axis $\varphi = 0$ and $F(\Phi) = 0$: $E_1(\Phi_1, 0)$, where $\Phi_0 = 0, \Phi_1 = -\left(\frac{3v}{2A}\right)^2$ and $i = 0$ and 1, respectively. Furthermore, the determinant of the Jacobi matrix corresponding to Eq. (28) admits the following expression
\[
\det |J(\Phi_1, 0)| = \begin{vmatrix} \frac{v}{B} + A & 1 \\ \frac{A}{B} & \Phi_1^{1/2} \end{vmatrix} = -\frac{v}{B} + \frac{A}{B} (-\Phi_1^{1/2}).
\]

Based on the mathematical analysis of the planar dynamic system, the equilibrium points $E_1(\Phi_1, 0)$ of the Hamiltonian system (28) are either saddle point if $\det |J(\Phi_1, 0)| < 0$ or
center point if \( \det |J(\Phi_i, 0)| > 0 \) or cusp if \( \det |J(\Phi_i, 0)| = 0 \). Now let us determine the energy values, \( h \), at the equilibrium points \( E_i(\Phi_i, 0) \). Hence, at the equilibrium point \( E_0(0, 0) \), the energy value \( h = H(0, 0) \geq 0 \). Moreover, the energy value \( h = H\left(-\left(\frac{3\nu}{2A}\right)^2, 0\right) \equiv -\frac{\nu^5}{2BA^2} \) at the equilibrium point \( E_1\left(-\left(\frac{3\nu}{2A}\right)^2, 0\right) \). We now identify different curves in the phase portraits depending on the equilibrium points \( E_0(\Phi_i, 0) \). As displayed in Fig. 1, the saddle point of the solid curve is at \( h \gg 0 \), where \( \det |J(0, 0)| = -\frac{\nu^2}{B} > 0 \), ensuring DASW solutions. These types of curves are called nonlinear homoclinic trajectories. In addition, the center point of the closed dashed curve with no separatrix is at \( h < 0 \), where \( \det |J(0, 0)| = -\frac{\nu^2}{B} > 0 \) guarantees DAPTW solutions. These types of closed curves are named nonlinear heteroclinic cycles. This paper will focus on the center point of DAPTW solutions.

### 4 Numerical investigations

In this part, we will test the DASW and DAPTW bifurcation points by numerical simulations to verify the predictions given by bifurcation analysis. Accordingly, we have carried the numerical simulations over various plasma parameters. In this study, the numerical values of the parameters were frequently used in literature (Mamun 1998; Misra and Wang 2015), for example, in space plasmas (such as Earth’s magnetosphere, auroral region, heliospheric environments) and laboratory dusty plasma situations. For this purpose, we choose some of the normalized physical parameters in plasma, such as (Mamun 1998; Misra and Wang 2015) \( L_z \approx 0.1 - 0.2 \), \( \delta \approx 0.4 - 2.5 \), \( \sigma \approx 0.10 - 0.12 \), \( \beta \approx -1.5 - -3 \), \( \alpha \approx 0.01 - 1.5 \) and \( \omega_c \approx 0.3 - 0.5 \). It should be mentioned here that we complemented the work done in (Mamun 1998; Misra and Wang 2015), which concentrated on the behavior of DASWs. This work will focus on the physical and numerical simulation of the Sagdeev potential, \( V(\Phi) \), and DAPTWs. So, we did not include DASWs figures here. Now we present the numerical representation of Eq. (30) (i.e., the Sagdeev potential, \( V(\Phi) \)) by changing the values of the obliqueness of wave propagation, \( L_z \), the ratio of unperturbed electron density to the unperturbed dust density, \( \delta \), the ratio of dust to electron temperature, \( \sigma \), free ion-trapped ion temperature ratio, \( \beta \), the nonthermal parameter, \( \alpha \), the dust-cyclotron frequency, \( \omega_c \). As shown in Figs. 2-7, the deep negative values of \( V(\Phi) \) are dependent on \( L_z, \delta, \sigma, \beta, \alpha, \) and \( \omega_c \). Obviously, there are two values given for \( \Phi \) (i.e., \( \Phi_0 = 0 \) and \( \Phi_1 = -\left(3\nu/2A\right)^2 \)), where \( V(\Phi) \) becomes zero. Clearly, \( V(\Phi) \) is defined physically as a pseudoparticle that periodically oscillates back and forth in the potential well between two zeroes points. Figure 2 illustrates that the depth of \( V(\Phi) \) and the amplitude of the DAPTW (i.e., the magnitude of the amplitude of the DAPTW, which is rarefaicative in nature) increase with the decrease in the obliqueness of propagation, \( L_z \). Furthermore, increasing \( \delta (\sigma) \) leads...
Fig. 4 The variation of the Sagdeev pseudopotential $V(\phi)$ versus $\phi$, at different values of $\sigma$ for $L_z = 0.18$, $\delta = 1.5$, $\beta = -2.5$, $\alpha = 0.1$ and $\omega_c = 0.3$

Fig. 5 The variation of the Sagdeev pseudopotential $V(\phi)$ versus $\phi$, at different values of $\beta$ for $L_z = 0.18$, $\delta = 1.5$, $\sigma = 0.1$, $\alpha = 0.1$ and $\omega_c = 0.3$

to an increase (slightly increase) of the $V(\Phi)$ depth and the amplitude of the DAPTW, as shown in Fig. 3 (4), respectively. For $\beta < 0$ (i.e., a vortex-like excavated trapped ion distribution), Fig. 5 demonstrates that the depth of $V(\Phi)$ and the amplitude of the DAPTW decrease with the decrease in the absolute numerical values of $\beta$ (i.e., $|\beta|$). Figure 6 shows that $V(\Phi)$ depth and the amplitude of the DAPTW are slightly enhanced with decreasing $\alpha$. As shown in Fig. 7, an increase in the dust-cyclotron frequency, $\omega_c$, leads to an increase in $V(\Phi)$ depth, but the amplitude of the DAPTW remains constant. It is interesting to note that enhancing the depth of $V(\Phi)$ makes DAPTWs narrower. Therefore, we can expect physically, as will be discussed in the following paragraphs, that the growth in $\delta$, $\sigma$, $|\beta|$, and $\omega_c$, makes DAPTWs more spiky, but the increase in $L_z$ and $\alpha$ makes the DAPTWs broader and shorter.

To complete the picture, we must also include the influences of the plasma parameters $L_z$, $\delta$, $\sigma$, $\beta$, $\alpha$, and $\omega_c$ on the structures of DAPTW solutions. First, we can numerically find the DAPTW solutions via Mathematica software. Figures 8-13 show the profiles of the DAPTWs as a function in the space coordinate, $\xi$, at various values of $L_z$, $\delta$, $\sigma$, $\beta$, $\alpha$, and $\omega_c$, respectively. Figure 8 shows that the DAPTWs become less deep and more expansive as $L_z$ increases. Figure 9 (10) displays that the depth and the width of DAPTWs
Fig. 9 The evolution of the DAPTW, $\phi$, for different values of $\delta$ at $L_z = 0.18$ for $\beta = -2.5$, $\sigma = 0.1$, $\alpha = 0.1$ and $\omega_c = 0.3$

Fig. 10 The evolution of the DAPTWs, $\phi$, for different values of $\sigma$ at $L_z = 0.18$, $\delta = 1.5$, $\beta = -2.5$, $\alpha = 0.1$ and $\omega_c = 0.3$

Fig. 11 The evolution of the DAPTW, $\phi$, for different values of $\beta$ at $L_z = 0.18$, $\delta = 1.5$, $\sigma = 0.1$, $\alpha = 0.1$ and $\omega_c = 0.3$

Fig. 12 The evolution of the DAPTW, $\phi$, for different values of $\alpha$ at $L_z = 0.18$, $\delta = 1.5$, $\beta = -2.5$, $\sigma = 0.1$ and $\omega_c = 0.3$

Fig. 13 The evolution of the DAPTW, $\phi$, for different values of $\omega_c$ at $L_z = 0.18$, $\delta = 1.5$, $\beta = -2.5$, $\sigma = 0.1$ and $\alpha = 0.1$

displayed in Fig. 12. Also, increasing $\omega_c$ leads to a reduction in the width of DAPTWs, while the amplitude of DAPTWs will remain fixed, as shown in Fig. 13. It is worth noting that all DAPTW solutions (i.e., Figs. 8-13) have a negative amplitude based on Fig. 1.

From a physical point of view, decreasing $L_z$ (i.e., increasing $\theta$, where $\theta$ is the angle that the propagation vector of DAPTWs makes with the magnetic field) leads to growth (reduction) in the amplitude (width) of DAPTWs. Therefore, one can expect that as the DAPTWs approach the direction normal to the magnetic field (i.e., $L_z \to 0$), the amplitude of DAPTW increase to a maximum value, and the width decrease to a minimum value. A reduction in the electron concentration gives an increase in the dust concentration to keep the quasi neutrality in the dusty plasma model (where $\delta = n_e^{(0)}/Z_d n_d^{(0)}$, and $n_i^{(0)} = n_e^{(0)} + Z_d n_d^{(0)}$). Thus, the driving force provided by dust inertia in DAPTWs increases; hence, the amplitude of DAPTWs is enhanced. The slight growth in the amplitude is a result of the rise in the fraction of thermal dust grains, $\sigma$, which is one of the energy sources for DAPTWs. Growth in the numerical values of $\beta$ decreases
the nonlinear coefficient, $A$, which causes an increase in the amplitude, which means an increase in the energy of the DAPTWs. An increase in the degree of nonthermality of electrons, $\alpha$ (i.e., the percentage of fast electrons in the plasma), leads to a decrease in the phase velocity, $\lambda$, which, in turn, leads to an enhancement of the nonlinear coefficient leading to a reduction in the amplitude of DAPTWs. Interestingly, the degree of nonthermality of electrons has a weak effect on the physical nature of DAPTWs. An increase in the numerical values of the external uniform magnetic field, $B_0$, increases the ion cyclotron frequency, $\omega_c$, and decreases the dispersion of model plasma at hand. Thus, the static constant magnetic field constrains the charged particles of plasma, a condition referred to as magnetic confinement. Therefore, the uniform external magnetic field makes DAPTWs structures more spiky.

5 Conclusions

A plasma model of warm negative dusty fluid, including trapped positive ions and nonthermal fast electrons, has been examined. The physical nature of the small but finite amplitude of DAPTWs has been addressed. The essential features of DAPTWs are changed due to nonthermal electrons and positive trapped ions. The plasma model at hand gives only rarefactive DAPTWs (i.e., all DAPTW solutions are negative amplitudes) with characteristics that depend strongly on the obliqueness of propagation, $L_2$, the ratio of unperturbed electron density to the unperturbed dust density, $\delta$, the dust-cyclotron frequency, $\omega_c$. The degree of nonthermality of electrons, $\alpha$, has a weak effect on the behavior of DAPTWs. We can summarize the interesting physical nature of DAPTWs in the following number of points:

- Dusty fluid, including trapped positive ions and nonthermal fast electrons, supports only DAPTWs of rarefactive nature.
- The depth of DAPTWs is amplified as the physical parameters $\delta$, $\sigma$, and $|\beta|$ are increased. Conversely, the depth of DAPTWs is reduced as $L_2$ and $\alpha$ are enhanced.

Finally, we consider that the simulation model at hand may aid in understanding the salient features of the DAPTWs in astrophysical environments such as Earth’s magnetosphere, auroral region, and heliospheric environments.

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Author contributions The author E.F. El-Shamy suggested the plasma model M.M. Selim derived the equations and prepared the results. E.F. El-Shamy discussed the results and prepared the introduction. Both authors participated writing of the paper in the final form.

Data Availability The data that supports the findings of this study are available within the article.

Declarations

Competing interests The authors declare no competing interests.

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