Pinched hysteresis loops in non-linear resonators

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Abstract

This study shows that pinched hysteresis can be observed in simple non-linear resonance circuits containing a single diode that behaves as a voltage-controlled switch. Mathematical models are derived and numerically validated for both series and parallel resonator circuits. The lobe area of the pinched loop is found to increase with increased frequency and multiple pinch points are possible with an odd-symmetrical non-linearity such as a cubic non-linearity. Experiments have been performed to prove the existence of pinched hysteresis with a single diode and with two anti-parallel diodes. The formation of a pinched loop in these circuits confirms the following: (1) pinched hysteresis is not a fingerprint of memristors, and (2) the existence of a non-linearity is essential for generating this behaviour. Finally, an application in a digital logic circuit is validated.

1 | INTRODUCTION

Memristors have been proposed as new electronic devices with promising analog/digital and neuromorphic applications [1–6]. Their characteristic behaviour is a pinched hysteresis loop in the current–voltage plane [7]. Many circuits (known as emulators) have been introduced to mimic the memristor’s behaviour [8–12]. Although this behaviour has been shown to exist in other non-linear devices such as a non-linear inductor or a non-linear capacitor [13–15], and that its appearance is linked to satisfying the necessary conditions of the theory of Lissajous figures [16–18], it is still mistakenly attributed to memristors only. There are actually doubts about the uniqueness of the memristor as a fundamental device which have been raised by several researchers (see [19] and the references therein) but there is no doubt that it is a dynamic and non-linear device [10,20,21]. Without being non-linear, the frequency-doubling mechanism mandated by the theory of Lissajous figures, which is essential to create a pinched loop (as explained in detail in [16] and most recently in [22]) cannot be obtained. It is important to note that even if the memristor was not a fundamental device, it may still be a useful non-linear device. Therefore, semiconductor devices labelled as memristors continue to be fabricated in different technologies and materials [23,24] and are now commercially available from some vendors (see e.g. www.knowm.com).

Any memristive system is generally defined as [25,26]:

\[
\begin{align*}
\dot{x} &= f(x, u, t) \\
\dot{u} &= g(x, u, t)
\end{align*}
\]

where, \(u\) and \(y\) are the input and output of the system and \(x\) is the state variable. To identify a device as a memristor, it has to have three fingerprints as follows [26,27]: (1) a pinched loop in current–voltage plane with (2) a loop area that decreases monotonically with increasing frequency and (3) shrink to a single-valued function when the frequency tends to infinity. This

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does not mean that pinched hysteresis loops cannot be observed in other devices or circuits (with increasing or decreasing loop area).

In this study, we provide two examples of simple fundamental circuits, namely, non-linear series and parallel resonators, that can exhibit the pinched hysteresis loop behaviour. Mathematical models are derived and numerically validated with a diode-type asymmetric switching non-linearity and also with an odd-symmetric non-linearity formed of two anti-parallel diodes. Experimental results are provided and confirm the theory. The formation of a pinched loop from these circuits confirms our previous assertions [13,16] that:

1. Pinched hysteresis is not a fingerprint of memristors only
2. Pinched loops with lobe area widening (rather than declining) with increased frequency are possible and
3. The existence of a non-linearity is essential for generating this behaviour. This is because the theory of Lissajous figures necessitates the generation of a second-harmonic frequency in the electrical current ($i(t)$) when the applied voltage on the device $v(t)$ has only one fundamental frequency (see [22] for a more detailed explanation)

We further demonstrate the use of the generated pinched loop behaviour in realizing digital logic AND/OR gates. With this, we aim to show that research in the possible applications of pinched loops [28,29] can be conducted without restricting this behaviour to memristors.

2 | NON-LINEAR RESONATORS

2.1 | Series RLC resonator

A simple non-linear resonator circuit is shown in Figure 1 composed of a series RLC resonance circuit interrupted by a voltage-controlled switching non-linearity (denoted $f[x]$) placed across the capacitor. The circuit is excited by a sinusoidal signal $v_{in}(t) = V_{DC} + V_A \sin(\omega t)$. In its simplest form, the switching device can be a single diode and in this case the circuit is described by the equation set

$$
C \frac{dv_C}{dt} = i_L - i_f, \quad (1a)
$$

$$
L \frac{di_L}{dt} = v_{in} - R i_L - v_C \quad (1b)
$$

where $i_f = i_D = \left\{ \begin{array}{ll}
(v_C - v_f)/R_D & v_C \geq v_f \\
0 & v_C < v_f
\end{array} \right. \quad (2)

\text{with } v_C \text{ and } i_L \text{ are, respectively, the voltage and current across the capacitor and in the inductor while } i_f \text{ is the non-linear device current given bywhere, } R_D \text{ is the diode switch-on resistance while } v_f \text{ is the switch-on voltage (approximately 0.6 V for silicon devices). Introducing the dimensionless variables: } x = \frac{v_C}{v_f}, \gamma = \frac{R i_L}{v_f}, a = \frac{R}{R_D}, b = \frac{R' C L}{v_f}, A = \frac{V_{DC}}{v_f}, B = \frac{V_A}{v_f} \text{ a normalized frequency } \omega_n = \alpha RC \text{ and a normalized time } t_n = t/RC, \text{ the above equations transform into}

$$
\frac{dx}{dt} = y - f(x), \quad (3a)
$$

$$
\frac{dy}{dt} = b(g(t_n) - y - x) \quad (3b)
$$

with $g(t_n) = A + B \sin(\omega_n t_n)$ being the sinusoidal excitation and $f(x)$ being the non-linearity. In the case of the switching-diode, $f(x)$ is asymmetric and given by the following:

$$
f(x) = \left\{ \begin{array}{ll}
a(x - 1) & x \geq 1 \\
0 & x < 1
\end{array} \right. \quad (4)
$$

Numerical simulations of this model were performed using the ode45 solver in MATLAB with $A = 1, b = 0.2, B = 2$ for different values of $a$ and $\omega_n$ as shown in Figures 2a and 2b which plot the observed pinched loop in the $g(t_n) - y(t_n)$ plane; that is, in the $v_i - i_L$ plane of the circuit. Therefore, this pinched loop represents an impedance

![Figure 1](image1.png)

**Figure 1** Non-linear series resonator circuit with diode-based non-linear switches

![Figure 2](image2.png)

**Figure 2** Numerical simulation results of the non-linear model of the series resonator circuit with (a), (b) a single diode-non-linearity and (c), (d) with two anti-parallel diodes. The effect of different values of parameter $a$ at $\omega_n = 0.1$ are shown in (a) and (c) and the effect of different values of $\omega_n$ at $a = 1$ are shown in (b) and (d)
analogous to the ‘memristance’. It is noted from these simulations that the lobe area increases with increasing the value of the parameter $a$ and also with increasing the frequency $\omega_n$. Note that $\omega_n$ can also be written as $\omega_n = \omega \cdot BW / \omega_r$, where $\omega_r = 1 / \sqrt{LC}$ is the resonance frequency and $BW = R / L$ is the resonance bandwidth. Hence, satisfying a certain value of $\omega_n$ is achieved not only by changing the applied excitation signal frequency $\omega$ but also by adjusting the resonance bandwidth and resonance frequency.

Based on the detailed analysis and explanation of [16], the appearance of the pinched loop in this non-linear resonator is a result of satisfying the conditions of the theory of Lissajous figures. In particular, the non-linearity $f(x)$ is responsible for generating a strong second-harmonic in the current signal $i(t)$ with a normalized frequency $2\omega_n$. The existence of this harmonic can easily be verified from the FFT of $y(t)$. So long as the phase shift between this harmonic and the fundamental frequency component remains less than $\pi/2$, the pinched loop will be observable [16]. The location of the pinch point can be found using the general equations derived in [16]. When the diode-type non-linearity is replaced with an odd-symmetric one, such as a cubic-type non-linearity or the non-linearity resulting from two anti-parallel diodes given by (5) (see Supplementary Appendix), two pinched points are obtained in this case as shown in Figure 2c and 2d for the parameter values $A = 0$, $b = 0.2$ and $B = 2$. A pure sinusoid is used in this case for excitation ($A = 0$).

$$f(x) = \begin{cases} a(x-1) & x \geq 1 \\ 0 & -1 < x < 1 \\ a(x+1) & x \leq -1 \end{cases}$$

(5)

Shaping of the pinched loop is thus clearly related to the choice of the function $f(x)$. In particular, the odd-symmetric non-linearity generates a third-order harmonic from the applied fundamental frequency of the excitation voltage. The total number of pinch points in a Lissajous figure formed from two sinusoids with frequencies $p \cdot \omega$ and $q \cdot \omega$ is known to equal $q (p-1) + p (q-1)$ which is equal to 2 when $p = 1$ and $q = 3$.

2.2 | Parallel RLC resonator

Another non-linear resonator circuit is shown in Figure 3 composed of a parallel RLC resonant circuit interrupted by non-linearity placed in series with the inductor $C_p$ is a parasitic capacitor that must be included in the analysis since the non-linearity is voltage-controlled. In this case the circuit is described by the equation set

$$RC \frac{dv_C}{dt} = v_m - v_C - i_L R,$$

$$L \frac{di_L}{dt} = v_C - v_c,$$

(6a) (6b)

**FIGURE 3** Non-linear parallel resonator circuit

**FIGURE 4** Numerical simulation results of the non-linear model of the parallel resonator circuit with a single diode non-linearity and with two anti-parallel diodes: (a), (c) at $\omega_n = 0.1$ for different values of $a$ and (b), (d) at $a = 1$ for different values of $\omega_n$

$$C_p \frac{q_{c}}{dt} = i_L - i_f$$

(6c)

With the same dimensionless variables introduced in the previous subsection, the above equations can be written as the following:

$$\frac{dx}{dt_n} = g(t_n) - x - y,$$

$$\frac{dy}{dt_n} = b(x - z)$$

(7a) (7b)

$$\epsilon \frac{dz}{dt_n} = y - f(z)$$

(7c)

where $z = i_R R / v_C$, $\epsilon = C_p / C$ and $f(z)$ is the switching non-linearity function given by (4) or (5). When $\epsilon$ tends to zero, (7c) yields $y = f(z)$. 

Figure 4 shows the numerical simulation of (7) with \( A = 1 \), \( b = 0.2 \), \( B = 2 \) and \( \epsilon = 0.01 \). The area enclosed inside the lobes increases with increasing the frequency \( a_\pi \) but decreases with increasing the value of parameter \( a \).

## 3 | EXPERIMENTAL RESULTS

The non-linear series resonator circuit was experimentally tested with discrete components \( L = 1 \) mH, \( C = 5.6 \) nF, \( R = 10 \) k\( \Omega \) and using a 1N914 diode. Figure 5 shows the observed pinched loop when the excitation voltage has an offset voltage \( V_{DC} = 1 \) V, an amplitude of \( V_A = 2 \) V and at different frequencies. The current in the resonator was measured using a differential probe.

At 100 Hz (see Figure 5a) it is seen that the \( v_i - i_L \) portrait resembles the diode characteristics as expected. As frequency increases, the pinched loop formation is observed in Figure 5b, 5c and 5d. The voltage and current waveforms corresponding to the portrait of Figure 5d are shown in Figure 6. Adding another diode with opposite polarity in parallel to the diode in the circuit, a loop with two pinch points is seen in Figure 7a as predicted numerically. The applied voltage in this case had no offset value \( (V_{DC} = 0 \) V), an amplitude of \( V_A = 7 \) V and a 1 kHz frequency. The corresponding power spectrum of the current signal \( i_L \) is shown in Figure 7b from which it is visible that strong odd harmonics have been generated. However, the two pinched points are created because the phase angle between the fundamental signal at 1 kHz and the generated third-order harmonic at 3 kHz is less than \( \pi/2 \) satisfying the Lissajous conditions [16].

The remaining harmonics contribute to the shape of the loop but not to the creation of the pinch points.

## 4 | APPLICATION AND COMPARISON WITH A COMMERCIAL MEMRISTOR Device

Digital application of AND/OR gates are used to compare their functionality with pinched loops created (i) from the series resonator described above and (ii) from a KNOWM commercial memristor device using their discovery kit [30]. The gates are built using two memristor devices, as shown in Figure 8, driving a capacitive load (\( C = 10 \) nF). Figure 9 shows the waveform on the capacitor for both the AND/OR gates after using the experimental memristance data of the pinched loop obtained from the non-linear resonator circuit and that obtained from a commercial memristor device [30]. It is clear from the figure that similar responses are obtained in both cases. Therefore, from an application point of view, what matters is the existence of the pinched loop rather than its origin. On the other hand,
resonators are easily realizable on the micro- and nano-scales [31,32], which paves the way for possibly easier implementations of memristors based on the technique presented here. In terms of the non-volatility of these circuits, a procedure to dis-connect the ground terminal connection should be devised in order to maintain the states of the state variables (inductor current and capacitor voltage) un-changed. It is important to mention, that the non-volatility of memristors is not related to whether they are fundamental devices or not.

5 | CONCLUSION

We have verified numerically and experimentally the existence of a dynamic pinched loop behaviour in a simple non-linear resonator circuits. The key behind generating such a loop is to produce in the electrical current a second-order harmonic (for a single pinch point case) or third-order harmonic (for a two pinch point case) from the applied fundamental frequency of the excitation voltage. When these harmonics have proper phase and amplitude relations with respect to the fundamental [16], pinched loops appear. The circuits presented here rely on the diode non-linearity as opposed to MOS transistor quadratic non-linearity [17,23] or multiplier-type non-linearity [10,33]. The proposed systems represent second-order and third-order dynamical systems which is not the case of memristive systems as defined in [25].

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How to cite this article: Elwakil AS, Fouda ME, Majzoub S, Radwan AG. Pinched hysteresis loops in non-linear resonators. IET Circuits Devices Syst. 2021;15:88–93. https://doi.org/10.1049/cds.12003