Top quark pair production and decay at linear colliders: signal vs. off resonance background

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Abstract

Standard Model predictions for the reactions with six fermions in the final state relevant for top quark pair production and decay at linear colliders are discussed. An issue of the double resonance signal versus non doubly resonant background is addressed. Effects related to the off-mass-shell production of the $t\bar{t}$-pair are discussed.

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1 MOTIVATION

As the top quark is the heaviest particle ever observed, the measurement of its physical properties may give hints towards better understanding of the electroweak (EW) symmetry breaking mechanism and observed fermion mass hierarchy. Should effects of physics beyond the Standard Model (SM) be visible at the energy scale below 1 TeV, then top couplings may show deviations from the corresponding SM values. Therefore high-precision measurements of the top quark properties and interactions are planned at TESLA [1] and will most certainly belong to the research program of any future $e^+e^-$ collider [2].

To disentangle the possible effects of physics beyond the SM, it is crucial to know the SM predictions for the top quark pair production and decay precisely, including radiative corrections.

The quantum chromodynamics (QCD) radiative corrections to the on-mass-shell top pair production

$$e^+e^- \rightarrow t\bar{t}$$

are known both in the threshold region, to the next-to-next-to-leading order including the effects of initial state radiation and beamstrahlung [3], and, in the continuum, to order $\alpha_s^2$ [4]. The electroweak radiative corrections to reaction (1) are known to one-loop order [5], [6]. They are typically of $O(10\%)$.

The corrections to the top decay into a $W$-boson and a $b$-quark, and to the $W$ decay into a fermion pair are known, too, to order $O(\alpha\alpha_s^2)$ [7] and to order $O(\alpha\alpha_s)$ [8], respectively.

As the $t$-quark ($\bar{t}$-quark) of reaction (1) almost immediately decays predominantly into a $b$-quark ($\bar{b}$-quark) and $W^+$ ($W^-$), and the latter decays into a fermion pair, the 6 fermion reactions of the form

$$e^+e^- \rightarrow bf_1f_1'\bar{b}f_2f_2'$$

where $f_1 = \nu_\mu, \nu_\tau, u, c$, $f_2 = \mu^-, \tau^-, d, s$ and $f_1', f_2'$ are the corresponding weak isospin partners, should be studied, at least in the lowest order of SM, with a complete set of the Feynman diagrams [9].

In the present lecture, the effects caused by off-shellness of the $t\bar{t}$ pair and by the off resonance background contributions to reactions (3), which may become important, especially for measurements at the c.m.s. energies much above the threshold, will be discussed. We will present results on

$$e^+e^- \rightarrow b\nu_\mu\mu^+\bar{b}\mu^-\bar{\nu}_\mu,$$

which is a pure EW reaction. In the unitary gauge, if the Higgs boson couplings to electrons and to muons are neglected, reaction (3) receives contributions from 452 Feynman diagrams. The results obtained with a full set of the diagrams will be compared with the double resonance approximation for a $t$- and $\bar{t}$-quark

$$e^+e^- \rightarrow \bar{t}^*t^* \rightarrow b\nu_\mu\mu^+\bar{b}\mu^-\bar{\nu}_\mu,$$

with only two signal diagrams contributing, and with the narrow width approximation for the $t$- and $\bar{t}$-quark.
As it is not feasible to calculate radiative corrections to the full set of Feynman diagrams, it will be argued that it is reasonable to include higher order effects, at least the leading ones, just for the two signal diagrams.

2 OUTLINE OF CALCULATION

2.1 A program eett6f

Matrix elements are calculated with the helicity amplitude method described in [10]. Phase space integrations are performed with the Monte Carlo (MC) method. The most relevant peaks of the matrix element squared, related to the Breit-Wigner shape of the $W, Z, Higgs$ and top quark resonances, and to the exchange of a massless photon or gluon, have to be mapped away.

As it is not possible to find out a single parametrization of the 14-dimensional phase space which would allow to cover the whole resonance structure of the integrand, it is necessary to utilize a multichannel MC approach.

A computer program eett6f for calculating cross sections of reactions (2) at c.m.s. energies typical for linear colliders has been written. Version 1.0 of eett6f [11] allows for calculating both total and differential cross sections at tree level of SM. The program can be used as the Monte Carlo generator of unweighted events as well.

eett6f is a package written in FORTRAN 90. It consists of 50 files including a makefile, all stored in one working directory. The user should specify the physical input parameters in module inprms.f and select a number of options in the main program csee6f.f. The options allow, among other, for calculation of the cross sections while switching on and off different subsets of the Feynman diagrams. It is also possible to calculate cross sections in two different narrow width approximations, for $t\bar{t}$-quarks, or $W^\pm$-bosons. The program allows for taking into account both the electroweak and QCD lowest order contributions.

Constant widths of unstable particles, the massive electroweak vector bosons, the Higgs boson and the top quark, are introduced through the complex mass parameters

$$
M_V^2 = m_V^2 - i m_V \Gamma_V, \quad V = W, Z,
$$

$$
M_H^2 = m_H^2 - i m_H \Gamma_H, \quad M_t = m_t - i \Gamma_t/2,
$$

which replace masses in the corresponding propagators, both in the s- and t-channel Feynman diagrams,

$$
\Delta_F^{\mu\nu}(q) = - g^{\mu\nu} + q^\mu q^\nu / M_V^2, \quad S_F(q) = \frac{q + M_t}{q^2 - M_t^2}.
$$

Propagators of a photon and a gluon are taken in the Feynman gauge.
Table 1: Lowest order total cross sections for 3 different channels of reaction (2) with cuts of [15] and zero external fermion masses

| Reaction                        | eett6f   | LUSIFER  |
|---------------------------------|----------|----------|
| $e^+e^- \rightarrow b\nu_\mu + \bar{b}\mu - \bar{\nu}_\mu$ | 5.8065(33) | 5.8091(49) |
| $b\nu_\mu + \bar{b}\tau - \bar{\nu}_\tau$ | 5.8196(32) | 5.7998(36) |
| $b\nu_\mu + \bar{b}d\bar{u}$ | 17.275(28) | 17.171(24) |

The EW mixing parameter may be defined either real or complex,

$$\sin^2 \theta_W = 1 - \frac{m_W^2}{m_Z^2}, \quad \text{or} \quad \sin^2 \theta_W = 1 - \frac{M_W^2}{M_Z^2}. \tag{7}$$

As light fermion masses are not neglected, cross sections can be calculated without any kinematical cuts.

2.2 Checks

A number of checks of a program eett6f have been performed.

Matrix elements of different ‘subprocesses’ of (2): $e^+e^- \rightarrow t\bar{t}$, $bW^+\bar{b}W^-$, $bW^+\bar{b}f\bar{f}Z$ and $f_1f_1'f_2f_2'Z$ have been checked against MADGRAPH [12]. Moreover, matrix elements of the ‘subprocesses’ with the single on-mass-shell top quark have been calculated in an arbitrary linear gauge [13]. Finally, matrix elements of reaction (3) have been programmed independently by each of the two authors.

The multichannel phase space generation routine has been checked by comparing normalization of different channels against each other and testing energy-momentum conservation and on-mass-shell relations. Moreover, results obtained with the MC integration routine VEGAS [14] have been reproduced with a more efficient own made MC integration routine carlos.

Results for different channels of reaction (2) have been checked against those published in the literature. A comparison against LUSIFER [15] is shown in Table 1. The discrepancy of a few standard deviations between the results can most probably be explained by slightly different implementation of the top quark width. The invariant mass and angular distributions of the $bud$-quark triple of $e^+e^- \rightarrow bud\bar{b}\nu_\mu$ at $\sqrt{s} = 500$ GeV of [15] are nicely reproduced by eett6f within accuracy of plots.

Lowest order SM total cross sections of $e^+e^- \rightarrow b\nu_\mu + \bar{b}d\bar{u}$ agree nicely with those of [16]. Moreover, a qualitative agreement with the results of [17] have been found, see [9] for details.
3 NUMERICAL RESULTS

Lowest order SM total cross sections of reaction (3) at 3 different c.m.s. energies typical for a linear collider are collected in Table 2. The physical input parameters which are used are the same as those of [9]. The cross section $\sigma$ obtained with the full set of the Feynman diagrams is shown in column 1, the cross section $\sigma_{t^*-t^*}$ obtained with the two $t\bar{t}$ signal diagrams only, in column 2, and the cross section in the narrow top width approximation, $\sigma_{tt}$, in column 3. The results differ between each other by a few per cent at $s^{1/2} = 360$ GeV and 500 GeV, while at $s^{1/2} = 800$ GeV the differences become even larger, of more than 10%.

Table 2: Lowest order SM total cross sections of (3)

| $\sqrt{s}$ (GeV) | $\sigma$ (fb) | $\sigma_{t^*-t^*}$ (fb) | $\sigma_{tt}$ (fb) |
|------------------|---------------|-------------------------|-------------------|
| 360              | 4.416(6)      | 4.262(1)                | 4.624(2)          |
| 500              | 6.705(6)      | 6.354(2)                | 6.400(7)          |
| 800              | 3.538(29)     | 3.058(2)                | 2.973(4)          |

Dependence of the total cross section of (3) on a width of the top quark, $\Gamma_t$, treated as a free parameter, is shown in Table 3. It is amazing, that despite the contribution of 450 off resonance Feynman diagrams, we see substantial dependence on the top width, $\sim 1/\Gamma_t^2$, even at c.m.s. energies much above threshold, a behaviour typical for the resonance $t\bar{t}$-pair production which proceeds through the two signal diagrams only. This may be helpful in determining $\Gamma_t$ from measurements of the total top pair production cross section in the continuum, complementary to the measurement of the top quark width at the $t\bar{t}$ threshold.

Table 3: Lowest order SM total cross sections of (3) in fb for different values of the top quark width $\Gamma_t$

| $\sqrt{s}$ (GeV) | $\Gamma_t$ (GeV) 1.5 | 1.6 | 1.7 |
|------------------|-----------------------|-----|-----|
| 360              | 4.416(6)              | 3.862(7) | 3.423(7) |
| 500              | 6.705(6)              | 5.942(9) | 5.296(8) |
| 800              | 3.538(29)             | 3.130(14) | 2.785(11) |
Figure 1. Total cross sections of $e^+e^- \rightarrow b\nu_\mu \mu^+ \bar{b}\mu^- \bar{\nu}_\mu$ as functions of the c.m.s. energy.

Figure 2. Angular distributions of a $\mu^+$ at $\sqrt{s} = 500$ GeV.
The energy dependence of the total cross sections of reaction (3) is shown in Figure 1. The full lowest order cross section $\sigma$ is plotted with the solid line, the signal cross section $\sigma_{t\bar{t}}$, with the dotted line, and the cross section in the narrow top width approximation, $\sigma_{tt}$, with the dashed line. A comparison of the solid and dotted lines shows the effects of the off resonance background contributions to reaction (3), and a comparison of the dotted and dashed lines shows the pure effect of the off-mass-shell production of the $t\bar{t}$-pair.

How the off-resonance effects may distort the shape of differential cross sections is shown in Figs 2, 3 and 4, where the angular distribution of a $\mu^+$ and the energy distributions of a $b$-quark and $\mu^+$ of reaction (3) at $\sqrt{s} = 500$ GeV are plotted, respectively.

![Graph showing energy distributions](image)

Figure 3. Energy distributions of a $\mu^+$ at $\sqrt{s} = 500$ GeV.

4 SUMMARY AND OUTLOOK

Top quark pair production and decay into 6 fermions in $e^+e^-$ annihilation at c.m.s. energies typical for linear colliders has been studied with a program eett6f and some sample results on reaction (3) have been presented. It has been shown that, although the two $t\bar{t}$ signal diagrams dominate total cross sections even at c.m.s. energies much above the $t\bar{t}$ threshold, the effects related to off-mass-shell production of the $t\bar{t}$-pairs and the off resonance background may be relevant for the analyses of future precision data.
All reactions, except for those that include $e^\pm$ in the final state, have been implemented in \texttt{eett6f v 1.0}. However, the $t$-channel contributions to those reactions that are missing can be suppressed by even a small angular cut in the direction of initial beams, see \textit{e.g.} \cite{13}. Some gluon contributions to $e^+e^- \rightarrow b\bar{b}d\bar{d}$ are missing too, but their implementation into the program is underway. Implementation of anomalous couplings is being done as well.

The fact that the total cross section of reactions (2) is dominated by the doubly resonant signal justifies inclusion of leading radiative effects solely to the two signal diagrams. Work in this direction is planned in collaboration with the Zeuthen–Bielefeld group \cite{9}.

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