Robust multiobjective control of singularly perturbed systems using linear matrix inequality

Xigen Wang

College of Electronic and Electrical Engineering, Shanghai University of Engineering Science, Shanghai, China

Correspondence
Xigen Wang, College of Electronic and Electrical Engineering, Shanghai University of Engineering and Technology, Shanghai 201620, China.
Email: 15221619759@139.com

SUMMARY
The article investigates the issue of designing $H_\infty$ output feedback controller for a fuzzy nonlinear singularly perturbed systems (SPSs). From overall perspective, we design a series of linear matrix inequalities conditions, which provides the optional stability bounds $\epsilon_0$, verifies the existence of $\epsilon_0$ and ensures that the T–S fuzzy SPSs are asymptotic stable for all $\epsilon \in (0, \epsilon_0]$. According to the characteristics of SPSs, we design the asymmetric auxiliary matrix $P(\epsilon)$, which reduces the conservatism and expands the application scope of $\epsilon$ in contrast to the previous results. The approach gives the $\epsilon$-dependent Lyapunov function and deduces the fuzzy $\epsilon$-dependent robust controller. It is not only can be applied to the standard or nonstandard nonlinear systems but also eliminates the ill-conditioned problems. Finally, a numerical example testifies the feasibility of conclusion.

KEYWORDS
$H_\infty$ performance, LMIs, output feedback control, robust control, SPSs, T–S fuzzy models

1 | INTRODUCTION

MIMO systems, as the modern common control systems, are almost multiple time-scale dynamical system, which includes motor control systems, scheduling systems, remote robot systems and so forth. We describe these systems as singularly perturbed systems (SPSs). In order to satisfy the multitime-scale characteristics of the singularly perturbed system, the small parameter $\epsilon$ is selected to reflect the degree of separation between the “fast” and “slow” modes. Because of perturbation parameters leading to ill-conditioned numerical problems of SPSs, decomposing the origin system into slow and fast subsystems was appeared in References 1–5. Determining the upper bound $\epsilon_0$ of $\epsilon$ in the fast and slow decomposition method is the key to study the stability of SPSs.

Over the past few years, the singular perturbation approach is applied to the dimensionality reduction of the flexible manipulator model. It has been extensively appeared in SPSs. The hybrid controller is designed to realize realtime model predictive control, the tracking control and behavior analysis of multiple groups of nonholonomic robots.5–9 In article,10 the composite sliding mode control strategy is used to study the state feedback problem of a class of nonlinear SPSs. But this approach cannot be applied to nonstandard SPSs and has limitation condition of assumptions. To settle these issues, many researchers focus on studying and analyzing new measures (see References 11 and 12).

In recently years, T–S fuzzy models are also employed in nonlinear SPSs because of the accurate degree of approximating the complex nonlinear systems, such as nonlinear time-delay SPSs, Markov jump nonlinear SPSs, large-scale nonlinear networked systems and discrete-time SPSs.13–18 Reference 13 used elimination lemma and Finsler’s lemma to optimize the boundary values in Reference 19. Simultaneously, considering the actual situation, a $\epsilon$-independent fuzzy...
controller is designed by linear matrix inequality (LMI) technique. Combining the composite fuzzy control with the dither method solves the stability problem of nonlinear SPSs with time delay. However, the assumption makes the method conservative. In Reference 15, a Markov model with unknown elements was established, and the relaxation matrix was used to decouple the system to reduce the previous conservative conclusion in Reference 16.

It is well known that stabilization bound of the SPSs is the focus of attention for researchers. Solving E-bound and optimizing its value \( E_0 \) have always been a fairly vital task. For acquiring more superior parameters bound \( E_0 \), paper\(^20\) provides an approach to determine the boundary for uncertain SPSs by some algebraic inequality approach, but the value \( E_0 \) of the obtained is very small. The papers\(^{21–23} \) aiming to alleviate the numerical stiffness design a family of linear matrix inequalities (LMIs), and propose the robust \( H_\infty \) controller to achieve the close-loop system asymptotic stability, but its results show that this means is only ensured poles locations insider LMI region for sufficiently small \( E \) and no reliable upper bound of \( E \) can be offered. The new method determines the effective range of \( E \) by giving a positive value \( E_0 \) and \( \gamma \) in Reference 19 and avoids the disadvantages in Reference 21. Especially, the tactics combine this strategy of Reference 19 with sliding mode control (SMC) to solve the problem of matched or unmatched uncertainties of SPSs,\(^24\) and it deletes the strict restrictions in Reference 10. However, References 19 and 24 consider the situation where the state is directly measurable. The state variables of the actual system are often unmeasurable or difficult to measure due to the cost, operating space and nonlinear characteristics of the system. Therefore, output feedback control is more appealing for practical applications. To the best of our knowledge, the research on dynamic output feedback control (DOFC) of nonlinear SPSs still lacks a better and effective algorithm to determine the upper boundary of perturbation parameters in recent years, so it is necessary to stabilize nonlinear SPSs and achieve the high accuracy performance in novel approaches.

DOFC, as also a common control method, is used to study stability of many kinds of SPSs. However, this controller contains weighting constants needed to be chosen subjectively, leading to conservatism and inconvenience in practice to same extent. Summarize the results of DOFC in recent years.\(^{20,22,25–28} \) Reference 25 compared with Reference 20, the former gives an inequality algorithm to determine the boundary and avoids the trouble caused by the trial value in the latter. Reference 26 considers the robust control for the linear singularly perturbed system by the approach in Reference 19, but there is no in-depth research on nonlinear SPSs. Reference 29 researches the DOFC of the reduced-order nuclear reactor model. However, four weighting constants require to be chosen subjectively in the design procedures. As we all know, few works on the DOFC of nonlinear SPSs has been reported in the existing literature, so DOFC is still an open research.

The contribution of the article can be summarized as follows: (1) based on the characteristics of SPSs, design asymmetric matrix \( P(E) \) to exclude the conservatism of symmetry \( P(E) \) in Reference 30 and the inverse operation of \( P(E) \). (2) A strategy that can obtain the dynamic output feedback controller by selecting \( E_0 \) eliminates the limitation that \( E \) is sufficiently small in the previous papers. (3) Compared with the papers\(^{20,21} \) this method makes the past result less conservative and gives a series of general conclusions. (4) By combining the lemma in Reference 19 with the elimination method, the \( E \)-independent sufficient conditions of multi-objective control are given and the conditions can determine \( E_0 \) more flexible. (5) The technique effectively enlarges stability bound \( E_0 \) and ensures that the \( E \)-dependent controller makes fuzzy SPSs work well for all \( E \in (0, E_0) \). (6) This way is independent of perturbation parameters and the \( E \)-dependent Lyapunov function avoids the fast and slow decomposition of states, so it avoids the numerical stiffness problem and can also be applied to nonstandard SPSs.

The rest of this article contains mainly the following contents. In Section 2, the description of singular perturbed systems is presented. In Section 3, \( H_\infty \) output feedback controller design for the system in Section 2, and the close-loop system. Section 4 certifies the Theorem 1. In Section 5, an illustrative instance demonstrates the validity of the methods. Finally, conclusions about this article are given.

2 SYSTEM DESCRIPTION

On the ground of a T–S fuzzy SPS, the ith rule is formulate as follows:

Model rule i:

If \( v_1(t) \) is \( M_{11} \), \( v_2(t) \) is \( M_{12} \), ..., \( v_6(t) \) is \( M_{18} \) then

\[
E(\varepsilon)\dot{x}(t) = A_1x(t) + B_1\alpha(t) + B_2u(t),
\]

\[
z(t) = C_1z(t) + D_{11}u(t),
\]

\[
y(t) = C_2x(t) + D_{21}\alpha(t),
\]

where

- \( E(\varepsilon) \) is the \( \varepsilon \) dependent positive definite matrix,
- \( x(t) \) is the state vector,
- \( \alpha(t) \) is the sliding mode control signal,
- \( u(t) \) is the control input.

The \( \varepsilon \) dependent matrix \( E(\varepsilon) \) is bounded by a variable \( E_0 \), which is determined by the given upper bound \( E_0 \).

The \( \varepsilon \) dependent controller is designed to achieve the system asymptotic stability when \( E \leq E_0 \).
where $E(E) = \text{diag}[I, EI]$, $E > 0$ is the singular perturbation parameter. $M_{ij}$ ($i = 1, 2, \ldots, p$, $j = 1, 2, \ldots, \delta$) are fuzzy sets. $v(t) = [v_1(t) \cdots v_\delta(t)]^T$ is the premise vector that may depend on states in many cases, $\delta$ is the number of premise variables. $p$ is the number of fuzzy rules. $x(t) \in \mathbb{R}^n$ is the state vector. $u(t) \in \mathbb{R}^m$ is the input. $\omega(t) \in \mathbb{R}^n$ is the disturbance which belongs to $L_2 [0, \infty)$. $y(t) \in \mathbb{R}^l$ is the measurement. $z(t) \in \mathbb{R}^k$ is the controlled output. $A_i, B_{1i}, B_{2i}, C_{1i}, D_{1i}, C_{2i}, D_{2i}$ are constant matrices of appropriate dimensions.

Denote

$$q_i(v(t)) = \prod_{k=1}^{\delta} M_{ik}(v_k(t)), \quad i = 1, 2, \ldots, p,$$

where $M_{ik}(v_k(t))$ is the grade of membership of $v_k(t)$ in $M_{ik}$.

$$q_i(v(t)) \geq 0, \quad \sum_{i=1}^{p} q_i(v(t)) > 0, \quad \forall t \geq 0,$$

$$\mu_i = \frac{q_i(v(t))}{\sum_{i=1}^{p} q_i(v(t))}, \quad \sum_{i=1}^{p} \mu_i = 1.$$

The SPSs is described by the following T–S fuzzy model:

$$E(\varepsilon)\ddot{x}(t) = \sum_{i=1}^{p} \mu_i [A_i x(t) + B_{1i} \omega(t) + B_{2i} u(t)],$$

$$z(t) = \sum_{i=1}^{p} \mu_i [C_{1i} x(t) + D_{1i} u(t)],$$

$$y(t) = \sum_{i=1}^{p} \mu_i [C_{2i} x(t) + D_{2i} \omega(t)].$$

Simplify the above formula:

$$E(\varepsilon)\ddot{x}(t) = A(\mu)x(t) + B_1(\mu)\omega(t) + B_2(\mu)u(t),$$

$$z(t) = C_1(\mu)x(t) + D_1(\mu)u(t),$$

$$y(t) = C_2(\mu)x(t) + D_2(\mu)\omega(t), \quad (1)$$

where

$$A(\mu) = \sum_{i=1}^{p} \mu_i A_i, \quad B_1(\mu) = \sum_{i=1}^{p} \mu_i B_{1i}, \quad B_2(\mu) = \sum_{i=1}^{p} \mu_i B_{2i}, \quad C_1(\mu) = \sum_{i=1}^{p} \mu_i C_{1i},$$

$$D_1(\mu) = \sum_{i=1}^{p} \mu_i D_{1i}, \quad C_2(\mu) = \sum_{i=1}^{p} \mu_i C_{2i}, \quad D_2(\mu) = \sum_{i=1}^{p} \mu_i D_{2i}.$$
Simplify the above formula:

\[ E(\varepsilon) \dot{\tilde{x}}(t) = A_c(\mu)\tilde{x}(t) + B_c(\mu)\omega(t), \]
\[ u(t) = C_c(\mu)\tilde{x}(t), \] (2)

where

\[ A_c(\mu) = \sum_{i=1}^{p} \mu_i \tilde{A}_i, B_c(\mu) = \sum_{i=1}^{p} \mu_i \tilde{B}_i, C_c(\mu) = \sum_{i=1}^{p} \mu_i \tilde{C}_i. \]

\( \tilde{x}(t) \) is the feedback controller’s state set, \( \tilde{A}_i, \tilde{B}_i, \) and \( \tilde{C}_i \) are the controller’s parameters need to be determined. Substituting (2) into (1) yields the close-loop system

\[ E_{cl}(\varepsilon) \dot{\hat{x}}(t) = A_{cl}(\mu)\hat{x}(t) + B_{cl}(\mu)\omega(t), \]
\[ z(t) = C_{cl}(\mu)\hat{x}(t) + D_{cl}(\mu)\omega(t), \] (3)

where

\[ E_{cl}(\varepsilon) = \begin{bmatrix} E(\varepsilon) & 0 \\ 0 & E(\varepsilon) \end{bmatrix}, \hat{x}(t) = \begin{bmatrix} x(t) \\ \tilde{x}(t) \end{bmatrix}, \]
\[ A_{cl}(\mu) = \begin{bmatrix} A(\mu) & B_2(\mu)C_c(\mu) \\ B_c(\mu)C_2(\mu) & A_2(\mu) \end{bmatrix}, \]
\[ B_{cl}(\mu) = \begin{bmatrix} B_1(\mu) \\ B_c(\mu)D_2(\mu) \end{bmatrix}, C_{cl}^T(\mu) = \begin{bmatrix} C_1(\mu) \\ D_1(\mu)\tilde{C}_2(\mu) \end{bmatrix}. \]

The definitions about the problem which we study is described:

**Definition 1.** Given an \( H_\infty \) performance \( \gamma > 0 \), design an output feedback controller for the SPSs (1), so that \( \forall T_f \geq 0 \) and \( \forall \omega(t) \in L_2(0, T_f] \) the following inequality holds (see Reference 25):

\[ \int_0^{T_f} z(t)z^T(t)dt \leq \gamma^2 \int_0^{T_f} \omega(t)\omega^T(t)dt \]

In this article, the problem is formulated as follows:

**Problem.** Given an \( H_\infty \) performance bound \( \gamma > 0 \) and an upper bound \( \varepsilon_0 \) for \( \varepsilon \), design an output feedback controller of the form (2), so that the close-loop system (3) is asymptotically stable for all \( \varepsilon \in (0, \varepsilon_0] \) and an \( H_\infty \)-norm less than or equal to \( \gamma \) (see Reference 19).

**Lemma 1** (19). For \( \varepsilon_0 > 0 \) and the symmetric matrices \( R_1, R_2, \) and \( R_3 \) of compatible dimensions, if the inequalities

\[ R_1 \geq 0 \]
\[ R_1 + \varepsilon_0 R_2 > 0 \]
\[ R_1 + \varepsilon_0 R_2 + \varepsilon_0^2 R_3 > 0 \]

hold, then

\[ R_1 + \varepsilon R_2 + \varepsilon^2 R_3 > 0 \quad \forall \varepsilon \in (0, \varepsilon_0]. \] (4)

**4 | MAIN RESULT**

In this section, we will put forward the following theorems to solve the problem.
Theorem 1. Given an $H_\infty$ performance $\gamma$ and a positive upper bound $\varepsilon_0$, if there exist the appropriate dimensional matrices $Z_k(k = 1, 2, \cdots, 10)$ with $Z_k = Z_k^T(k = 1, \cdots, 4, 6, \cdots, 9)$ and $\bar{A}_i, \bar{B}_i, \bar{C}_i$ satisfying the following LMIs

$$
\begin{bmatrix}
Z_1 & I \\
I & Z_6
\end{bmatrix} > 0,
$$

$$
\begin{bmatrix}
Z_1 + \varepsilon_0 Z_3 & \varepsilon_0 Z_5^T & I & 0 \\
\varepsilon_0 Z_5 & \varepsilon_0 Z_2 & 0 & \varepsilon_0 I \\
I & 0 & Z_6 + \varepsilon_0 Z_8 & \varepsilon_0 Z_{10}^T \\
0 & \varepsilon_0 I & \varepsilon_0 Z_{10} & \varepsilon_0 Z_7
\end{bmatrix} > 0,
$$

$$
\begin{bmatrix}
Z_1 + \varepsilon_0 Z_3 & \varepsilon_0 Z_5^T & I & 0 \\
\varepsilon_0 Z_5 & \varepsilon_0 Z_2 + \varepsilon_0^2 Z_4 & 0 & \varepsilon_0 I \\
I & 0 & Z_6 + \varepsilon_0 Z_8 & \varepsilon_0 Z_{10}^T \\
0 & \varepsilon_0 I & \varepsilon_0 Z_{10} & \varepsilon_0 Z_7 + \varepsilon_0^2 Z_9
\end{bmatrix} > 0,
$$

$$
\Psi_{1ii} < 0 \quad \text{for} \quad i = 1, 2, \cdots, p,
$$

$$
\Psi_{1ij} + \varepsilon_0 \Psi_{1ji} < 0 \quad 1 \leq i < j \leq p,
$$

$$
\Psi_{1ii} + \varepsilon_0 \Psi_{2ii} < 0 \quad \text{for} \quad i = 1, 2, \cdots, p,
$$

$$
\Psi_{1ij} + \varepsilon_0 (\Psi_{2i} + \Psi_{2j}) < 0 \quad 1 \leq i < j \leq p,
$$

where

$$
\Psi_{1ij} = \begin{bmatrix}
\varphi_{1ij} + \varphi_{1ji}^T & * & * & * \\
A_1^T + \bar{A}_j & \varphi_{2ij} + \varphi_{2ji} & * & * \\
B_1^{T_B} U_1 + D_{ii} \bar{C}_j & C_{ii} & 0 & -\gamma I \\
C_{ii} U_3 + D_{ii} \bar{C}_j & C_{ii} & 0 & -\gamma I
\end{bmatrix},
\Psi_{2ij} = \begin{bmatrix}
U_4^T A_1^T + A_1 U_4 & 0 & 0 & * \\
0 & U_2^T A_1 + A_1^T U_2 & 0 & 0 \\
0 & B_1^T U_2 & 0 & 0 \\
C_{ii} U_4 & 0 & 0 & 0
\end{bmatrix},
$$

where

$$
\varphi_{1ij} = A_i U_3 + B_2 \bar{C}_j, \quad \varphi_{2ij} = A_i^T U_1 + \bar{B}_j C_{ii},
$$

$$
\bar{A}_j = X^T(\varepsilon) A_i Y(\varepsilon) + \bar{B}_j^T C_{ii} Y(\varepsilon) + X^T(\varepsilon) B_2 \bar{C}_j + (Y(\varepsilon)^{-1} - X(\varepsilon))^T \bar{A}_j Y(\varepsilon),
$$

$$
\bar{B}_j = (Y(\varepsilon)^{-1} - X(\varepsilon))^T \bar{B}_j,
$$

$$
\bar{C}_j = \bar{C}_j Y(\varepsilon).
$$

$$
X(\varepsilon) = U_1 + \varepsilon U_2 = \begin{bmatrix}
Z_1 + \varepsilon Z_3 & \varepsilon Z_5^T \\
Z_5 & Z_2 + \varepsilon Z_4
\end{bmatrix}, \quad Y(\varepsilon) = U_3 + \varepsilon U_4 = \begin{bmatrix}
Z_6 + \varepsilon Z_8 & \varepsilon Z_{10}^T \\
Z_{10} & Z_7 + \varepsilon Z_9
\end{bmatrix}.
$$

Then, the gains of suitable controller (3) can be get

$$
\hat{A}_i = (Y^{-1}(\varepsilon) - X(\varepsilon))^{-T} \left(\bar{A}_i Y^{-1}(\varepsilon) - X^T(\varepsilon) A_i - \bar{B}_i^T C_{ii} - X^T(\varepsilon) B_2 \bar{C}_i Y^{-1}(\varepsilon)\right),
$$

$$
\hat{B}_i = (Y(\varepsilon)^{-1} - X(\varepsilon))^{-T} \bar{B}_i,
$$

$$
\hat{C}_i = \bar{C}_i Y^{-1}(\varepsilon).
$$

Proof. Suppose that LMIs (5)–(11) hold water.
By using Lemma 1, the LMIs (5)–(7) imply
\[
E^T(\epsilon)X(\epsilon) = X^T(\epsilon)E(\epsilon) \geq 0 \forall \epsilon \in (0, \epsilon_0],
\] (13)
\[
E^T(\epsilon)Y(\epsilon) = Y^T(\epsilon)E(\epsilon) \geq 0 \forall \epsilon \in (0, \epsilon_0],
\] (14)
\[
\begin{bmatrix}
E^T(\epsilon) & 0 \\
0 & E^T(\epsilon)
\end{bmatrix}
\begin{bmatrix}
Y(\epsilon) & I \\
I & X(\epsilon)
\end{bmatrix}
= \begin{bmatrix}
Y^T(\epsilon) & I \\
I & X^T(\epsilon)
\end{bmatrix}
\begin{bmatrix}
E(\epsilon) & 0 \\
0 & E(\epsilon)
\end{bmatrix} > 0 \forall \epsilon \in (0, \epsilon_0].
\] (15)

Pre and postmultiplying (14) by $Y^{-T}(\epsilon)$ and its transpose, yield
\[
Y^{-T}(\epsilon)E^T(\epsilon) = E(\epsilon)Y^{-1}(\epsilon) \geq 0,
\] (16)
which shows that
\[
E^T(\epsilon)Y^{-1}(\epsilon) = Y^{-T}(\epsilon)E(\epsilon) \geq 0.
\] (17)

Based on the Schur complement, (15) can show
\[
\begin{aligned}
E^T(\epsilon)X(\epsilon) - E(\epsilon)(E^T(\epsilon)Y(\epsilon))^{-1}E^T(\epsilon) \\
= E^T(\epsilon)X(\epsilon) - E(\epsilon)Y^{-1}(\epsilon)E^{-T}(\epsilon)E(\epsilon)Y(\epsilon)Y^{-1}(\epsilon) \\
= E^T(\epsilon)X(\epsilon) - E(\epsilon)Y^{-1}(\epsilon) \\
= E^T(\epsilon)(X(\epsilon) - Y^{-1}(\epsilon)) \geq 0.
\end{aligned}
\] (18)

By using Lemma 1, the LMIs (8)–(11) imply
\[
\Psi_{111} + \epsilon \Psi_{211} < 0 \quad \forall \epsilon \in (0, \epsilon_0],
\] (19)
\[
\Psi_{11j} + \epsilon \Psi_{21j} < 0 \quad i \neq j, \forall \epsilon \in (0, \epsilon_0].
\] (20)

Choose
\[
P(\epsilon) = \begin{bmatrix}
X(\epsilon) & Y^{-1}(\epsilon) - X(\epsilon) \\
Y^{-1}(\epsilon) - X(\epsilon) & X(\epsilon) - Y^{-1}(\epsilon)
\end{bmatrix}.
\] (21)

Using the LMIs (13), (17), and (21), we obtain
\[
\begin{aligned}
E^T(\epsilon)X(\epsilon) - E^T(\epsilon)(Y^{-1}(\epsilon) - X(\epsilon))(E^T(\epsilon)(X(\epsilon) - Y^{-1}(\epsilon)))^{-1}E^T(\epsilon)(Y^{-1}(\epsilon) - X(\epsilon)) \\
= E^T(\epsilon)X(\epsilon) - (Y^{-1}(\epsilon) - X(\epsilon))^T E(\epsilon) (X(\epsilon) - Y^{-1}(\epsilon))^{-1} E^{-T}(\epsilon) E^T(\epsilon) (Y^{-1}(\epsilon) - X(\epsilon)) \\
= E^T(\epsilon)X(\epsilon) + (Y^{-1}(\epsilon) - X(\epsilon))^T E(\epsilon) \\
= E^T(\epsilon)Y^{-1}(\epsilon) \geq 0.
\end{aligned}
\] (22)

From (22) and Schur complement, one implies
\[
E^T_{cl}(\epsilon)P(\epsilon) = P^T(\epsilon)E_{cl}(\epsilon) \geq 0.
\] (23)

Choose a Lyapunov function as
\[
V(\hat{x}(t)) = \hat{x}^T(t)E^T_{cl}(\epsilon)P(\epsilon)\hat{x}(t).
\] (24)
Design inequality to asymptotically stable the system (3) with Definition 1:

\[ V(\hat{x}(t)) + \gamma^{-1}z^T(t)z(t) - \gamma \omega^T(t)\omega(t) < 0 \]

\[ = \left[ A_{cl}(\mu)\hat{x}(t) + B_{cl}(\mu)\omega(t) \right]^T P(\epsilon)\hat{x}(t) + \hat{x}^T(t)P^T(\epsilon) \left[ A_{cl}(\mu)\hat{x}(t) + B_{cl}(\mu)\omega(t) \right] + \gamma^{-1}z^T(t)z(t) - \gamma \omega^T(t)\omega(t) \]

\[ = \hat{x}^T(t)A_{cl}^T(\mu)P(\epsilon)\hat{x}(t) + \hat{x}^T(t)P^T(\epsilon)A_{cl}(\mu)\hat{x}(t) + \omega^T(t)B_{cl}^T(\mu)P(\epsilon)\hat{x}(t) + \hat{x}^T(t)P^T(\epsilon)B_{cl}(\mu)\omega(t) \]

\[ + \gamma^{-1}z^T(t)z(t) - \gamma \omega^T(t)\omega(t) < 0 \]

Applying the Schur complement to the formula (25), one requires

\[
\begin{bmatrix}
A_{cl}^T(\mu)P(\epsilon) + P^T(\epsilon)A_{cl}(\mu) & \ast & \ast \\
B_{cl}^T(\mu)P(\epsilon) & -\gamma I & \ast \\
C_{cl}(\mu) & 0 & -\gamma I
\end{bmatrix} < 0.
\]

Substituting \( A_{cl}(\mu), B_{cl}(\mu), C_{cl}(\mu), P(\epsilon) \) into (26) and pre and post multiplying (26) by \( \text{diag}[I, I] \) and its transpose with \( \Pi = \begin{bmatrix} Y(\epsilon) & I \\ Y(\epsilon) & 0 \end{bmatrix} \), yield

\[ \sum_{i=1}^{p} \mu_i^2 (\Psi_{1ii} + \epsilon \Psi_{2ii}) + \sum_{\substack{i,j=1 \\ i \neq j}}^{p} \mu_i \mu_j (\Psi_{1ij} + \epsilon \Psi_{2ij}) < 0. \]

Based on LMIs (25) and (27), the finally result of the Lyapunov function can be shown:

\[ \gamma V(\hat{x}(t)) \leq -z^T(t)z(t) + \gamma^2 \omega^T(t)\omega(t). \]

Integrating both sides of (28) from 0 to \( T_f \), with \( \hat{x}(0) = 0 \), yields

\[ \gamma V(\hat{x}(t)) \leq -\int_0^{T_f} z^T(t)z(t)dt + \gamma^2 \int_0^{T_f} \omega^T(t)\omega(t). \]

Because \( V(\hat{x}(t)) \geq 0 \), the following formula can be hold:

\[ \int_0^{T_f} z^T(t)z(t)dt \leq \gamma^2 \int_0^{T_f} \omega^T(t)\omega(t). \]

The inequalities (5)–(7) imply the hold of condition (23). The LMIs (8)–(11) and (23) testify that \( \dot{V}(t) < 0 \). So, the close-loop system is asymptotically stable. Lemma 1 guarantees the following content: when \( \epsilon_0 \) exists, conditions (5)–(11) are all true for optional \( \epsilon \in (0, \epsilon_0) \). This completes the proof.

5 | ILLUSTRATIVE EXAMPLES

**Example 1.** An illustrative instance is chosen to explain the validity of the above theory.\(^{22}\)

\[
\begin{align*}
\dot{x}_1(t) &= 2x_1(t) + 0.1x_1^3 + 10x_2(t) + 0.1\omega(t), \\
\dot{x}_2(t) &= -x_1(t) - x_2(t) + u(t), \\
\dot{\omega}(t) &= 0.1x_1(t) + 0.1\omega(t), \\
z(t) &= x_1(t) + 0.1u(t).
\end{align*}
\]

\[
(29)
\]
Choose the membership functions of the fuzzy sets as follows:
\[ M_1(x_1(t)) = \frac{1}{9}x_1^2(t), M_2(x_1(t)) = 1 - \mu_1(t). \]

If \( x_1(t) \) is \( M_1(x_1(t)) \), then
\begin{align*}
E(\epsilon)x(t) &= A_1x(t) + B_{11}\omega(t) + B_{21}u(t), \\
z(t) &= C_{11}x(t) + D_{11}u(t), \\
y(t) &= C_{21}x(t) + D_{21}\omega(t).
\end{align*}
(30)

If \( x_1(t) \) is \( M_2(x_1(t)) \), then
\begin{align*}
E(\epsilon)x(t) &= A_2x(t) + B_{12}\omega(t) + B_{22}u(t), \\
z(t) &= C_{12}x(t) + D_{12}u(t), \\
y(t) &= C_{22}x(t) + D_{22}\omega(t),
\end{align*}
(31)

where
\[
A_1 = \begin{bmatrix} 2 & 10 \\ -1 & -1 \end{bmatrix}, B_{11} = B_{12} = \begin{bmatrix} 0.1 \\ 0 \end{bmatrix}, A_2 = \begin{bmatrix} 2.9 & 10 \\ -1 & -1 \end{bmatrix}, B_{21} = B_{22} = \begin{bmatrix} 0 \\ 1 \end{bmatrix},
C_{11} = C_{12} = \begin{bmatrix} 1 & 0 \end{bmatrix}, D_{11} = D_{12} = 0.1, C_{21} = C_{22} = \begin{bmatrix} 1 & 0 \end{bmatrix}, D_{21} = D_{22} = 0.1.
\]

The output feedback controller can be described as follows:
If \( x_1(t) \) is \( M_1(x_1(t)) \), then
\[ E(\epsilon)\hat{x}(t) = \hat{A}_1\hat{x}(t) + \hat{B}_1y(t), \]
\[ u(t) = \hat{C}_1x(t). \]

If \( x_1(t) \) is \( M_2(x_1(t)) \), then
\[ E(\epsilon)\hat{x}(t) = \hat{A}_2\hat{x}(t) + \hat{B}_2y(t), \]
\[ u(t) = \hat{C}_2x(t), \]
then, the fuzzy controller is expressed as
\[ E(\epsilon)\hat{x}(t) = \sum_{i=1}^{2} \mu_i \left[ \hat{A}_i\hat{x}(t) + \hat{B}_iy(t) \right], \]
\[ u(t) = \sum_{i=1}^{2} \mu_i \hat{C}_i\hat{x}(t), \]
(32)

where \( \mu_1(\nu(t)) = M_1(x_1(t)), \mu_2(\nu(t)) = M_2(x_1(t)). \)

On account of Theorem 1, the upper bounds of \( E \) are subject to different \( H_\infty \) performance index, which is shown in Tables 1 and 2.

By the LMIs in Theorem 1 with \( \varepsilon_0 = 0.6 \) and \( \gamma = 0.5 \) acquires
\[
U_1 = \begin{bmatrix} 0.9917 & 0 \\ -0.0852 & 3.3775 \end{bmatrix}, U_2 = \begin{bmatrix} 0.0450 & -0.0852 \\ 0 & -0.4891 \end{bmatrix}, U_3 = \begin{bmatrix} 3.5842 & 0 \\ -1.1487 & 3.8767 \end{bmatrix}, U_4 = \begin{bmatrix} 0.0448 & -1.1487 \\ 0 & 0.3004 \end{bmatrix},
\]
\[
\hat{A}_1 = \begin{bmatrix} -1.2483 & 1.3654 \\ -9.8627 & 1.0037 \end{bmatrix}, \hat{A}_2 = \begin{bmatrix} -2.7169 & 1.3741 \\ -9.9189 & 1.0025 \end{bmatrix},
\]
Table 1 Upper bound $\varepsilon_0$ for $\varepsilon$ for different index $\gamma$

| The performance index $\gamma$ | $\gamma = 0.15$ | $\gamma = 0.2$ | $\gamma = 0.3$ |
|--------------------------------|-----------------|-----------------|-----------------|
| Theorem 3.3 of Reference 22   | no reliable upper bound |
| Theorem 1 of Reference 28     | 0.5372          | 0.7379          | 1.2989          |
| Theorem 1                     | $> 1.29$        | $> 1.44$        | $> 1.51$        |

Table 2 Upper bound $\varepsilon_0$ for $\varepsilon$ for different index $\gamma$

| The performance index $\gamma$ | $\gamma = 0.15$ | $\gamma = 0.2$ | $\gamma = 0.3$ |
|--------------------------------|-----------------|-----------------|-----------------|
| Theorem 1                       | 0.0017          | 0.0602          | 0.1369          |

$$\bar{B}_1 = \begin{bmatrix} -4.7355 & -6.8611 \end{bmatrix}, \bar{B}_2 = \begin{bmatrix} -5.6142 & -6.8275 \end{bmatrix}, \bar{C}_1 = \begin{bmatrix} -36.4909 & 1.1842 \end{bmatrix}, \bar{C}_2 = \begin{bmatrix} -36.1924 & 1.1829 \end{bmatrix}.$$  

Apply (32) to (30) and (31) with $x(0) = 0$ and $\omega(t) = \sin(\pi t) \cdot e^{-0.6t}$. The simulation result shows the square root of the ratio of the output energy to the disturbance input energy is almost settled in a constant value, which is $\sqrt{7.556 \times 10^{-3}} = 0.0849$ and $\sqrt{5.5552 \times 10^{-3}} = 0.0745$ after 2 s in Figure 1. The values are less than the prescribed value 0.5. Figures 2 and 3, respectively, show the response of each state quantity of the closed-loop system when $\varepsilon = 0.4$ and $\varepsilon = 0.04$. The changing trend reflects the asymptotic stability of the system. In Table 1, the $\varepsilon_0$ corresponding to different $\gamma$ values of the $H_\infty$-performance is optimized in Reference 28, and its value is not limited to sufficiently small constraints in Reference 22.

**Example 2.** Consider the illustrative instance of dc motor in Reference 21.

\[
\begin{align*}
\dot{x}_1(t) &= x_2(t) + 0.1 \omega(t), \\
\dot{x}_2(t) &= 9.8 \sin x_1(t) + x_3(t), \\
\varepsilon \dot{x}_3(t) &= -x_2(t) - x_3(t) + u(t), \\
y(t) &= x_1(t) + 0.1 \omega(t), \\
z(t) &= 0.1 x_1(t) + 0.1 u(t).
\end{align*}
\] (33)

Describe the system (33) as the following T–S fuzzy SPS:

\[
E(\varepsilon) \dot{x}(t) = \sum_{i=1}^{2} A_i x(t) + B_{1i} \omega(t) + B_{2i} u(t).
\]

**Figure 1** Simulation result for the ratio of the output energy to the disturbance input energy with $\varepsilon = 0.4$ and $\varepsilon = 0.04$.  

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\end{align*}
\] (33)

Describe the system (33) as the following T–S fuzzy SPS:

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E(\varepsilon) \dot{x}(t) = \sum_{i=1}^{2} A_i x(t) + B_{1i} \omega(t) + B_{2i} u(t).
\]
\[ z(t) = \sum_{i=1}^{2} C_{1i} x(t) + D_{1i} u(t), \]
\[ y(t) = \sum_{i=1}^{2} C_{2i} x(t) + D_{2i} \omega(t), \]  \hspace{1cm} (34)

where

\[
A_1 = \begin{bmatrix} 0 & 1 & 0 \\ 9.8 & 0 & 1 \\ 0 & -1 & -1 \end{bmatrix}, \quad A_2 = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & -1 & -1 \end{bmatrix},
\]

\[
B_{1i} = B_{12} = \begin{bmatrix} 0 & 0 & 1 \end{bmatrix}^T, \quad B_{21} = B_{22} = \begin{bmatrix} 0.1 & 0 & 1 \end{bmatrix}^T.
\]

\[
C_{1i} = C_{12} = \begin{bmatrix} 0.1 & 0 & 0 \end{bmatrix}, \quad C_{21} = C_{22} = \begin{bmatrix} 1 & 0 & 0 \end{bmatrix},
\]

\[
D_{1i} = D_{12} = 0.1, \quad D_{21} = D_{22} = 0.1.
\]

Solving the LMIs in Theorem 1 with \( \epsilon_0 = 0.2 \) and \( \gamma = 0.5 \) gives
Choose the conditions that include \( x(0) = 0 \) and \( \omega(t) = \sin(\pi t) \cdot e^{-0.6t} \). In Figure 4, the ratio is asymptotically stable at the fixed values \( \sqrt{3.8927 \times 10^{-2}} = 0.1973 \) and \( \sqrt{3.2041 \times 10^{-2}} = 0.1790 \). By choosing the upper bounds of different disturbance parameters \( \mathcal{E}_0 \), Theorem 1 can be used to obtain different solutions to the variables \( U_1, U_2, U_3, U_4, \bar{A}_1, \bar{B}_1, C_1, \bar{A}_2, \bar{B}_2, C_2 \). Finally, the disturbance parameter \( \mathcal{E} \) (\( 0 < \mathcal{E} \ll 1 \)) that satisfies the condition is selected and substituted into Equation (12),
we can get the effective solution to controller, which is proved by the simulation examples. Figures 5 and 6, respectively, show the state response when $\varepsilon = 0.1$ and $\varepsilon = 0.01$.

6 | CONCLUSION

Design the $H_\infty$ output feedback controller to make the fuzzy SPSs stable. According to LMI-based means, we apply the $\varepsilon$-independent conditions to establish an $\varepsilon$-dependent dynamic output feedback controller. The fuzzy $H_\infty$ controller makes the close-loop system asymptotically stable for the value of any singular perturbation parameter in the stabilization bound. Simultaneously, the $H_\infty$ performance is less or equal to a prescribed value. Compared with Reference 22, the proposed method eliminates the limitation of $\varepsilon$ sufficient small and gives the optimized boundary value. This method is suitable for standard and nonstandard SPSs. In future, we will optimize the theorem for enlarging $\varepsilon_0$ and consider the SMC, uncertainties and the unmeasurability of singular perturbation parameters.

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ENDNOTE
Singularly perturbed systems: $\dot{x}_1(t) = A_{c11}x_1(t) + A_{c12}x_2(t) + B_{c1}u(t)$, $\varepsilon\dot{x}_2(t) = A_{c21}x_1(t) + A_{c22}x_2(t) + B_{c2}u(t)$. $x_1(t), x_2(t)$ are the state vector. $u(t)$ is the input. $A_{c11}, A_{c12}, A_{c21}, A_{c22}, B_{c1}, B_{c2}$ are constant matrices. When $A_{c22}$ is singular, the SPSs are nonstandard SPSs. When $A_{c22}$ is nonsingular, the SPSs are standard SPSs.

DATA AVAILABILITY STATEMENT
Data sharing is not applicable as no new data generated, or the article describes entirely theoretical research.

ORCID
Xigen Wang © https://orcid.org/0000-0002-9663-9938

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