Landau Like States in Neutral Particles

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We show the emergence of a new type of dispersion relation for neutral atoms where we find an interesting similarity with the spectrum for 2-Dimensional electrons in an applied perpendicular constant magnetic field. In strong contrast to the equi-distant infinitely degenerate Landau levels for charged particles, the spectral gap for the 2-Dimensional neutral particle increases in a particular electric field. Moreover, the spin-orbit nature of the coupling of neutral particles to the electric field confines only one component of the spin. The idea is motivated by the developments in cold atom experiments and builds on the seminal paper of Aharonov and Casher [1]. We start with the example of 2-dimensional cold atoms subject to an external electromagnetic gauge field. This happens because of the coupling of the electromagnetic gauge field $A(x)$ to the momentum of the 2-D electrons in the Hamiltonian. Landau levels are generated due to interference of orbital motion of charged particles in the external magnetic field. The spectrum is controlled by the magnitude of the physically observed field. A similar question on the possibility of Landau levels generated by neutral particles can be asked. Following the same logic, we need to find the field that would couple to the orbital motion of neutral particles. A natural possibility would be to look at the spin-orbit coupling $((\sigma \times p) \cdot E$ with $\sigma$ being the spin of the particle (assume $s=1/2$), $E$ - electric field and $p$ -momentum. This coupling will affect the orbital motion of neutral spin 1/2 particles via the electric field. One can ask therefore if one can discern any Landau level physics for neutral atoms.

In this paper, we address the question of formation of discrete Landau like states for the neutral particles. In addition to similarities, the spin-orbit nature of the coupling indicates clear differences for these states compared to the normal electronic Landau level problem. In what follows we use the standard approach as first was laid out by Aharonov and Casher [1]. We start with the example of 2 dimensional cold atoms subject to an external electric field. For neutral atoms with magnetic moment and some electric field $E$, we get the following Hamiltonian for the neutral particles [1].

$$H = \frac{p^2}{2m} + \alpha(\sigma \times p) \cdot E$$

(1)

Here $p$ is the two dimensional momentum, $\alpha$ is some coupling constant and the electric field $E$ is applied in the $\hat{y}$ direction and given by $E = \gamma y^2 \hat{y}$ and $\sigma_x, \sigma_y$ and $\sigma_z$ are the Pauli matrices. A very particular electric field configuration has been assumed here. To get this type of electric field we need a linear charge distribution in the $\hat{y}$ direction given by $\rho = \epsilon_0 \gamma y$.

We consider the neutral atoms in optical traps with the help of lasers. Several laser beams are focused into one another. These beams interferes and creates potential profiles such that the neutral atoms can reside near the minimum of these potential. By changing the intensity of the lasers we end up with a pseudo spin-orbit like interaction as in the Hamiltonian [Eq. 1] for the neutral particles[2],[9],[8],[18]. The coupling constant $\alpha$ relates these pseudo 'spins' to the magnetic moments of the particles. One motivation to incorporate cold atoms in optical traps to see any signature of the Landau level physics is that the analogy between the pseudo 'spins' and magnetic moments for the chargeless particles then can be used. In an actual optical 2-Dimensional trap, the potential will be a function of both $x, y$ coordinates, while $p_x$ and $p_y$ are not good quantum numbers. This difficulty can be circumvented by using a very shallow trap in the $x$ direction and in that case, although not exactly a good quantum number, $p_x$ can still be assumed as a physically reasonable variable to a first order of approximation. As we proceed with the simplification of the above Hamiltonian, we use $p_x$, the $x$-component momentum, as a good quantum number which will in a way give landau likes states.

We also see an analogue of the gauge field coupling to the momentum of the neutral particles compared to the electromagnetic gauge coupling to the momentum of the electrons. These gauge fields are not Maxwell electromagnetic gauges, but are called synthetic gauges [2]. We write the Hamiltonian [Eq. 1] in the following manner to explore the emergence of the electric field and 'spin' dependent gauges. The components of these gauge fields
also do not commute with each other. Therefore, we see that these gauge fields are quite different from the U(1) electromagnetic gauge fields. Mathematically, they represent SU(2) gauge fields. The particular $y^2$ dependent gauge fields can be contrasted with the electronic landau level problem. If we recollect the problem of Landau quantization in electrons, we use the gauge choice as $A_x = By$; similarly here we have a $y^2$ dependent gauge which origins from the particular choice of the electric field. Actually, in this case we need both the neutral particles with magnetic moments and a charge distribution to achieve level quantization.

\[
H = \frac{p_x^2}{2m} + \alpha (\sigma \times p) \cdot E
= \frac{p_x^2}{2m} + \alpha p_x A_{\text{eff}}
A_{\text{eff}} = E \times \sigma
\]

Coupling $\alpha$ in the above equation plays same role as $e/c$ coupling in the case of the U(1) gauge potential. The above Hamiltonian [Eq. 1] can be further simplified (APPENDIX: A). Because of the particular form of the electric field in Eq. 1, it appears that there is an effective $y$-dependent ‘spin-orbit’ coupling.

\[
H = \frac{p_x^2}{2m} + \frac{p_y^2}{2m} + \alpha \gamma y^2 p_x \sigma_z
\]

The last Hamiltonian contains $\sigma_z$. Therefore, it separates into two parts. [$\sigma_z$ has two eigenvalues $\pm 1$].

\[
H = \frac{p_x^2}{2m} + \frac{p_y^2}{2m} \pm \alpha \gamma y^2 p_x
\]

The Hamiltonian does not depend explicitly on $x$. So, $p_x$ is a good quantum number as explained in the beginning of the paper and we can express the full wave function as $\psi (x, y) = e^{i k_x x} \phi (y)$. Inserting $\psi (x, y)$ in Eq. 3, we get an effective Hamiltonian which acts on $\phi (y)$.

\[
H_{\text{eff}} = -\frac{\hbar^2}{2m} \nabla_y^2 \pm \alpha \gamma \hbar k_x y^2
= -\frac{\hbar^2}{2m} \nabla_y^2 \pm \frac{1}{2} \frac{m \omega_c^2 y^2}{\alpha \gamma}.
\]

Here the cyclotron frequency is $\omega_c^2 = \frac{\alpha \gamma \hbar k_x}{m}$. It depends on the mass of the neutral atoms as well as the on the $x$ component momentum $k_x$. Only one part of the above hamiltonian gives us bound states for particles with spin $+1$ while the particles with the opposite spin experience an unbounded potential $-\frac{1}{4} m \omega_c^2 y^2$. As a result this trap will be confining only for particles with one spin projection and the other particles will scatter off.

The wave functions and the dispersion relation are given below,

\[
\psi_{n, +} (x, y) = e^{i k_x x} \frac{1}{\sqrt{2^n n!}} \left( \frac{m \omega_c}{\hbar} \right)^{1/4} e^{-\frac{m \omega_c}{2} y^2} \times
E_{n, +} = \hbar \omega_c \left( n + \frac{1}{2} \right) + \frac{\hbar^2 k_x^2}{2m}
\]

The harmonic oscillator frequency $\omega_c$, depending both on the mass of the atom and the $k_x$, gives extra freedom to control the spectrum in optical traps. If we assume that highest wave vector that can be achieved within the trap is $k_{\text{max}}$, then the corresponding cyclotron frequency is $\omega_{\text{max}} = \sqrt{\frac{\alpha \gamma \hbar k_{\text{max}}}{2m}}$. A plot of $E_n/\hbar \omega_{\text{max}}$ as a function of $k_x/k_{\text{max}}$ is given in Fig 1.

![Figure 1: Energy as a function of x-Component momentum $k_x$. Different from standard Landau level dispersion](image)

We also plot the probabilities for the ground and first few excited states, in the unit of $\frac{\hbar}{m \omega_c} = 1$, in Fig. 2.

The Hamiltonian [Eq. 2] contains only a $\sigma_z$ dependent term. This particular form can be attained in optical traps by tuning an equal mixture of Rashba and Dresselhaus interactions. Tuning the optical coupling we write effective hamiltonian with either Rashba $\alpha (\sigma_x k_y - \sigma_y k_x)$ or Dresselhaus $\beta (\sigma_x k_y + \sigma_y k_x)$ interaction. In this case, the particular form of the Hamiltonian is very similar to an equal weight of the Rashba and Dresselhaus interactions\[2, 5, 8, 10\].

**Estimate:**

Now let us try to give some estimate of the energy gap in the Landau like dispersion relation. A typical energy gap is of the order of $\hbar \omega_c$. Typically, this is $\hbar \sqrt{\alpha \gamma v_x}$.
Taking some realistic value for the $\gamma = 10^{10} \text{V/m}^3$ and $v_x = 10^3 \text{m/s}$ and $\alpha = 10 \text{m}^2 \text{s}^{-1} / \text{V}$ we get a typical energy gap of the order of $6.5 \times 10^{-9} \text{eV}$. Therefore, the typical energy gap is of the order of few microKelvin $10^{-9} \text{eV} \sim 10^{-5} \text{K}$.

In optical traps one can attain low temperature as this. Therefore, one can observe Landau levels in neutral atoms with suitable magneto-optical trapping with microKelvin temperature[15], [16], [17], [18].

**Conclusion:**

In this paper we demonstrated that upon the application of quadratic electric field and induced spin-orbit coupling in optically trapped cold atoms, one can induce nontrivial spin dependent levels in the spectrum of the atoms. In contrast to Landau levels where the energy spectrum is equispaced, we find here a different energy spectrum with continuously increasing levels. We note that for the neutral particles with magnetic moments in the presence of specific charge distribution, a new type of gauge fields arise and couple to the momentum in the Hamiltonian. This is to be contrasted with the Landau level physics of electrons in applied magnetic fields. Due the presence of $\sigma_z$ in [Eq. 3] only half of the trapped neutral particles show the Landau like behaviour and the other half just "flies away" due to the unbounded potential.

We have given an order of magnitude estimate of the gaps in the spectrum with some reasonable values of the parameters. An important point to note is that the cyclotron frequency depends on $k_x$. For big wave number one can therefore tune the approximate energy gap to any value that is practical. At same time there are experimental limitations for observing a very large $k_x$. The traps which can be created by the laser beams are of finite strength. The maximum value of $k_x$ depends of the potential depth as otherwise the atoms will fly off from the traps and therefore it will not be possible to do the experiment.

We addressed the problem in optical traps. This set up gives one a reasonable freedom in choosing a large $k_x$, but if some other setup can be arranged where we can have an electric field with $\gamma$ as strong as $10^{20} \text{V/m}^3$ and $v_x \approx 10^6 \text{m/s}$ we can create the gap in the meV scale. Ultimately the proposed mechanism to control neutral particles might be useful for optical applications.

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**APPENDIX: A Spin-Orbit Hamiltonian**

The simplification of the Hamiltonian in Equation 1 is given by,

$$H = \frac{p^2}{2m} + \alpha (\sigma \times p).E$$

$$= \frac{p^2}{2m} + \alpha E \cdot y^2 \hat{y} \cdot (\sigma \times p)$$

$$= \frac{p^2}{2m} + \alpha E \cdot y^2 p \cdot (\hat{y} \times \sigma)$$

$$= \frac{p^2}{2m} + \alpha E \cdot y^2 p \cdot (-\sigma_x \hat{z} + \sigma_z \hat{x})$$

$$= \frac{p^2}{2m} + \alpha E \cdot y^2 p \cdot \sigma_z$$

$$= \frac{p^2}{2m} + \frac{p_y^2}{2m} + \alpha E \cdot y^2 p \cdot \sigma_z$$ (5)

[1] Y. Aharonov, A. Casher, "Topological quantum effects for neutral particles" Phys. Rev. Lett. 53(4), pp. 319-321, 1984.

[2] Y.J. Lin, K. Jimenez-Garca and I. B. Spielman, "Spin-orbit-coupled Bose-Einstein condensates" Nature Vol. 471, pp. 8386 March 2011.
[3] A. Jacob, P. Ohberg, G. Juzeliunas and L. Santos, "Landau levels of cold atoms in non-Abelian gauge fields" Arxiv 0801.2935v1, January 2008.
[4] K. Bakke, C. Furtado "Relativistic Landau quantization for a neutral particle" ArXiv 0902.1474, February 2009.
[5] D. L. Campbell, G. Juzeliunas, and I. B. Spielman, "Realistic Rashba and Dresselhaus spin-orbit coupling for neutral atoms" Phys. Rev. A 84, 025602 August 2011.
[6] C. Furtado, J.R. Nascimento, L.R. Riberiron, "Landau quantization of Neutral Particles in an External Fields" Phys. Lett. A 358, pp. 336-338, 2006.
[7] L. R. Ribeiro, Claudio Furtado, J. R. Nascimento, "Landau Levels Analog to Electric Dipole" ArXiv 0502129, December 2005.
[8] J. Larson, J.P. Martikainen, A. Collin and Erik Sjovqvist, "Spin-orbit coupled Bose-Einstein condensate in a tilted optical lattice" Arxiv 1001.2527v2, January 2011.
[9] V. Galitski and T. B. Spielman, "Spin-orbit coupling in quantum gases" Arxiv 1312.3292, December 2013.
[10] G. Dresselhaus, "Spin-Orbit Coupling Effects in Zinc Blende Structures" Phys. Rev. Lett. 100, 580 October, 1955.
[11] L. R. Ribeiro, Claudio Furtado, J. R. Nascimento, "Landau Levels Analog to Electric Dipole" ArXiv 0502129, December 2005.
[12] A. Stern, B. I. Halperin, F. von Oppen, S. H. Simon, "Half-filled Landau level as a Fermi liquid of dipolar quasiparticles", ArXiv 9812135, December 1998.
[13] J.K. Jain, R.K. Kamilla, "Composite Fermions in the Hilbert Space of the Lowest Electronic Landau Level" Arxiv 9704031, April 1997.
[14] Gediminas Juzelinas, Julius Ruseckas, and Jean Dalibard, "Generalized Rashba-Dresselhaus spin-orbit coupling for cold atoms" Phys. Rev. A Vol. 81, 053403, May 2010.
[15] D. J. Wineland and Wayne M. Itano, "Laser cooling of atoms" Phys. Rev. A Vol. 20, 1521, October, 1979.
[16] D. C. McKay, D. Jervis, D. J. Fine, J. W. Simpson-Porco, G. J. A. Edge, and J. H. Thywissen, "Low-temperature high-density magneto-optical trapping of potassium using the open 4S5P transition at 405 nm" Phys.Rev. A Vol.84, 063420,December 2011.
[17] P. M. Duarte, R. A. Hart, J. M. Hitchcock, T. A. Corcovilos, T.-L. Yang, A. Reed, R. G. Hulet, "All-Optical Production of a Lithium Quantum Gas Using Narrow-Line Laser Cooling" ArXiv 1109.6635, January 2012.
[18] Keir C. Neuman and Steven M. Block, "Optical trapping" Review of Scientific Instruments Vol. 75, No. 9, September 2004.
[19] Armin Ridinger, Saptarishi Chaudhuri, Thomas Salez, Ulrich Eismann, Diogo Rio Fernandes, David Willkowski, Frederic Chevy, Christophe Salomon, "Large atom number dual-species magneto-optical trap for fermionic 6Li and 40K atoms" arXiv 1103.0637 March 2011.