Energy relaxation of an excited electron gas in quantum wires:
many-body electron LO-phonon coupling

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Abstract

We theoretically study energy relaxation via LO-phonon emission in an excited one-dimensional electron gas confined in a GaAs quantum wire structure. We find that the inclusion of phonon renormalization effects in the theory extends the LO-phonon dominated loss regime down to substantially lower temperatures. We show that a simple plasmon-pole approximation works well for this problem, and discuss implications of our results for low temperature electron heating experiments in quantum wires.

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When excess energy is supplied to an electron gas, either by a strong applied electric field or by optical excitations, the electron gas becomes “hot”, *i.e.* it goes out of equilibrium with the lattice attaining higher electron temperature than the ambient lattice temperature. A hot electron gas loses energy to its surroundings in order to achieve equilibrium. This energy loss process is usually accomplished through emission of phonons. In polar semiconductor materials such as GaAs, the electron LO-phonon Fröhlich coupling is significantly stronger than the electron acoustic-phonon coupling. The most efficient energy relaxation process for the hot-electron gas, except at very low temperatures, is therefore to emit LO phonons. The understanding of this energy relaxation process is of great technological importance since actual electronic devices work mostly under high-field hot-electron conditions. The study of this subject also constitutes a direct probe of a fundamental interaction in condensed matter physics, namely, the electron-phonon interaction. There has been considerable recent theoretical and experimental interest in the hot-electron energy relaxation problem in polar semiconductors, particularly in three-dimensional (3D) and two-dimensional (2D) GaAs structures [1–9]. More recently, one-dimensional (1D) hot-electron relaxation in quantum wire structures has been considered theoretically [10–12], motivated by the fact that there has been successful growth of one-dimensional GaAs quantum-well wires with only the lowest subband occupied [13]. In this article, we develop a many-body theory for hot-electron energy relaxation in one-dimensional quantum wires within the electron temperature model, taking full account of LO-phonon renormalization effects which have been left out of existing theories [10,11]. Our results agree with existing theoretical results at high electron temperatures \((T > 100K)\), but at lower temperatures we find significant contributions to the energy relaxation arising from renormalized LO-phonons which have been left out of existing calculations.

It has been well recognized from the study of energy relaxation in 2D and 3D systems [8] that it is important to include the effect of the phonon propagator renormalization at low temperatures. This renormalization takes into consideration phonon self-energy due to electron-phonon interaction which opens up additional channels to energy relaxation from
coupled electron-phonon excitations. These contributions become increasingly important as the electron temperature \( T \) decreases. At low enough temperatures \( k_B T \ll \hbar \omega_{LO} \), where \( \omega_{LO} \) is the frequency of the dispersionless LO phonon, the inclusion of the phonon propagator renormalization may enhance the energy relaxation rate by orders of magnitude compared with loss through bare LO-phonon (which is exponentially small for \( k_B T \ll \hbar \omega_{LO} \)). Deviation from the naive bare-phonon result, which gives an exponentially decaying energy relaxation rate as electron temperature is lowered, is a ubiquitous phenomenon in hot-electron energy loss experiments [5–9], and has usually been uncritically ascribed to acoustic-phonon contribution, in spite of the fact that merely including the acoustic-phonon emission could not account for the total observed power loss. This puzzle is resolved by including the enhancement from the renormalization of the LO-phonon propagator. Theoretical calculations including the LO-phonon renormalization effects show excellent qualitative and reasonable quantitative agreement with the existing experiments in 2D and 3D systems. The goal of the present work is to apply a similar theoretical model to study hot-electron energy relaxation via coupling to bulk LO-phonons in semiconductor quantum wire structures. Our emphasis is on the low temperature regime, where the many-body phonon propagator renormalization enhancement is important. We compare the energy relaxation rate through the LO phonon coupling with that through acoustic phonon coupling, and estimate the electron temperature for the crossover from the LO-phonon dominated energy relaxation to acoustic phonon dominated energy relaxation as the electron temperature decreases. As expected, we find that the enhancement of the energy relaxation from the LO-phonon propagator renormalization lowers the crossover temperature substantially. For commonly available GaAs quantum wire carrier density of \( n \sim 10^5 \text{cm}^{-1} \), the naive picture where the phonon renormalization is ignored would suggest a crossover temperature of 30K, where acoustic phonon emission becomes comparable to LO-phonon emission, while our estimated crossover temperature with the phonon renormalization included is well below 30K. We also show that the plasmon-pole approximation gives reasonably accurate result for this problem, which is a direct consequence of the fact that in a 1D system the plasmon-LO-phonon coupling
is strong for all carrier densities and that quasiparticle electron-hole excitations are severely suppressed. The advantage of using the plasmon-pole approximation is that the energy relaxation rate can be written as an analytic compact formula which is formally as simple as the corresponding bare phonon result and yet it gives quantitatively accurate result.

We adopt the standard electron-temperature model which assumes that the electron gas and the substrate lattice are separately in equilibrium at different temperature $T$ and $T_L$ with $T > T_L$. We also assume that the electrons in the quantum wire structure occupy only the lowest subband, which is a valid assumption for carrier density $n \leq 10^6 \text{cm}^{-1}$ and wire width $a \leq 300 \text{Å}$. For lattice temperature $T_L = 0K$, the energy relaxation rate is

$$P = \sum_q \int \frac{d\omega}{\pi} \omega n_T(\omega)|M_q|^2 \text{Im} \chi^{\text{ret}}(q, \omega) \text{Im} D^{\text{ret}}(q, \omega),$$  \hspace{1cm} (1)$$

where $n_T(\omega)$ is Bose distribution factor at temperature $T$. $\chi^{\text{ret}}(q, \omega)$ is the retarded density-density response function for an uncoupled interacting 1D electron gas. In the random-phase approximation (RPA), $\chi(q, \omega) = \chi_o(q, \omega)/[1 - v_q \chi_o(q, \omega)]$, with the finite temperature $\chi_o(q, \omega)$ of a 1D free electron gas obtained from its zero-temperature counterpart by an integration over chemical potential. For a quantum wire with finite wire widths, the Coulomb interaction potential is $v_q = (e^2/\epsilon_\infty) \int d\eta |I(\eta)|^2 H(q, \eta)/((q^2 + \eta^2)^{1/2}$. The form factor $I(\eta)$ and $H(q, \eta)$ used in our calculation are taken from the infinite well confinement model. The Fröhlich coupling matrix is $|M_q|^2 = v_q(\omega_{\text{LO}}/2)[1 - (\epsilon_\infty/\epsilon_0)]$, with $\epsilon_0(\epsilon_\infty)$ as the static (high frequency) dielectric constant. The phonon propagator in Eq. (1) is

$$D(q, \omega) = \frac{2\omega_{\text{LO}}}{\omega^2 - \omega_{\text{LO}}^2 - 2\omega_{\text{LO}}|M_q|^2 \chi(q, \omega)}.$$  \hspace{1cm} (2)$$

The last term in the denominator is the phonon self-energy. When this phonon self-energy is ignored, one has $\text{Im} D^{\text{ret}}(q, \omega) = \text{Im} D_0^{\text{ret}}(q, \omega) = \pi[\delta(\omega + \omega_{\text{LO}}) - \delta(\omega - \omega_{\text{LO}})]$. Inserting this into Eq. (1), one obtains the energy relaxation rate for bare phonon emission as

$$P_0 = \omega_{\text{LO}} n_T(\omega_{\text{LO}}) \sum_q (-2)|M_q|^2 \chi(q, \omega_{\text{LO}}).$$  \hspace{1cm} (3)$$
The characteristic of the bare phonon result is an approximate exponential temperature dependence $P_0 \propto \exp(-\omega_{LO}/k_BT)$, which comes from the Bose factor $n_T(\omega_{LO})$.

With a standard plasmon-pole approximation for $\chi(q, \omega)$, the spectral function of the phonon propagator in Eq. (2) becomes a pair of $\delta-$functions at the coupled plasmon-phonon excitation frequencies. Inserting this spectral function into Eq. (1), a compact expression for energy relaxation rate is obtained as

$$P_{PP} = P_+ + P_-$$
$$P_\pm = \sum_q \omega_{\pm} n_T(\omega_{\pm}) \frac{\omega_{LO}|\omega_{\pm}^2 - \omega_P^2|}{\omega_{\pm}(\omega_{\pm}^2 - \omega_{\mp}^2)} |M_q|^2 (-2) \text{Im}\chi(q, \omega_{\pm}),$$

(4)

where $\omega_{\pm}$ are the coupled plasmon-phonon excitation energies, and $\omega_P$ is the uncoupled plasmon excitation for a 1D electron gas. Note that we use the plasmon-pole approximated form of $\chi(q, \omega)$ only in the phonon self-energy in Eq. (2), not in the energy loss rate of Eq. (1). Most of our calculations are concerned with comparing the energy relaxation rates from the different approximations expressed in Eq. (1), (3), and (4).

In our calculation, the hot phonon “bottleneck” effect is ignored. This is equivalent to setting the lifetimes of the emitted LO-phonons to zero. The reasons for doing this are that the effect from the finite lifetime of the emitted phonon is less important at low temperatures and the values of the lifetimes for the electron-phonon coupled excitations are unknown. It is easy to include the hot phonon bottleneck effect in the theory if the phonon lifetime is known.

The results of our calculation are shown in Fig. 1 to 4, where we take the parameters which are appropriate for GaAs materials: $\epsilon_0 = 12.9$, $\epsilon_\infty = 10.9$, and $\omega_{LO} = 36.8$ meV. In Fig. 4, energy relaxation rates with bare phonons, renormalized phonons, and plasmon-pole approximation are shown as functions of electron temperature. The bare phonon result shows an approximate exponential temperature dependence mainly due to the Bose factor $n_T(\omega_{LO})$. In addition to phonon-like excitation, the renormalized phonon propagator also contains coupled plasmon- and quasiparticle-like excitations. The phonon-like excitation has large spectral weight and high energy ($\hbar\omega_{LO} = 427K$), while the plasmon- and quasiparticle-
like excitations have small spectral weights, but with arbitrarily low energies. At high
temperatures \((k_B T \sim h\omega_{\text{LO}})\), the phonon-like excitation dominates because of its large
spectral weight. The energy relaxation rate of the renormalized phonon is essentially the
same as that of bare phonon. As the temperature is lowered, the plasmon- and quasiparticle-
like excitations begin to dominate because of their low energies. The energy relaxation rate of
the renormalized phonon starts to deviate from that of the bare phonon. As shown in Fig. 1,
the low temperature enhancement to the energy relaxation from the phonon renormalization
at high density \((n = 10^6 \text{cm}^{-1})\) is weaker than that at low density \((n = 10^5 \text{cm}^{-1})\), but it is
still significant. The temperature where the deviation from the bare phonon result begins
is about 100 K (compare to \(h\omega_{\text{LO}} = 427 K\)). Due to strong plasmon LO-phonon coupling
and phase space restriction on quasiparticle excitation in 1D systems, plasmon-pole
approximation is expected to give reasonably accurate result. This is basically true in Fig.
1, especially for the case of low density \((n = 10^5 \text{cm}^{-1})\).

In a polar semiconductor material such as GaAs, electron LO-phonon coupling is much
stronger than electron-acoustic phonon coupling. Hot-electron energy relaxation through the
LO-phonon channel dominates over that through the acoustic phonon channel, except at low
temperatures where emission of the high energy LO-phonon is effectively frozen by energy
conservation. The temperature for the crossover from the LO-phonon dominated energy
relaxation to the acoustic phonon dominated energy relaxation should be significantly shifted
by the enhancement to the energy relaxation from LO-phonon renormalization. In Fig. 2 the
energy relaxation rates for bare LO-phonons, renormalized LO-phonons and deformation-
potential acoustic phonons are shown as functions of electron temperature \(T\). The
crossover temperature without considering the LO-phonon renormalization is about 30K
for both densities. When the LO-phonon renormalization effect is included, the crossover
temperature is shifted down to about 10K for electron density of \(n = 10^6 \text{cm}^{-1}\) and well
below 1K for \(n = 10^5 \text{cm}^{-1}\). The basic conclusion from our calculation is that the crossover
temperature can be shifted down substantially by the phonon renormalization effect.

In Fig. 3 energy relaxation rates as functions of electron density and wire widths are
shown respectively. As the density increases, among other things, the screening is increased and the energies of the coupled electron-phonon excitation are raised, so the energy relaxation rate for bare phonons declines slightly while the energy relaxation rate for renormalized phonons decreases more noticeably. The dependence of energy relaxation rate on wire widths is shown to be weak.

Finally, the energy resolved contribution to the relaxation rate as functions of renormalized phonon frequency is shown in Fig. 4, where $I(\omega)$ is defined by rewriting Eq. (11) as $P = \int_0^\infty I(\omega)d\omega$. It is clearly seen that at high temperature ($T = 250K$) the energy relaxation is dominated by emission of bare LO-phonons with $\omega \sim \omega_{LO}$, while at low temperature ($T = 36K$), energy relaxation comes mainly from emission of renormalized phonons with $\omega \ll \omega_{LO}$. The significant contribution to energy relaxation from low energy phonon emission at $T = 36K$ suggests that it is necessary to incorporate many-body phonon renormalization into energy relaxation theories in order to obtain meaningful results at low temperatures.

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FIGURES

FIG. 1. Energy relaxation rate per electron as functions of electron temperature $T$. The solid-line, dot-line, and dot-dashed-line are respectively the results with renormalized phonon propagator, bare phonon propagator, and plasmon-pole approximated phonon propagator. The widths of the quantum wire are $L_y = L_z = 200 \ang$. (a) electron density $n = 10^5 \text{cm}^{-1}$. (b) electron density $n = 10^6 \text{cm}^{-1}$.

FIG. 2. Energy relaxation rate per electron as functions of electron temperature $T$. The solid-line and dot-line are respectively the results with renormalized LO phonon propagator and bare LO phonon propagator. The dot-dashed-line is the energy relaxation rate through acoustic phonon coupling (see Ref. 12). The widths of the quantum wire are $L_y = L_z = 200 \ang$. (a) electron density $n = 10^5 \text{cm}^{-1}$. (b) electron density $n = 10^6 \text{cm}^{-1}$.

FIG. 3. (a) Energy relaxation rate per electron as functions of electron density $n$. The widths of the quantum wire are $L_y = L_z = 50 \ang$. (b) Energy relaxation rate per electron as functions of quantum wire widths. The electron density is $n = 10^5 \text{cm}^{-1}$. In both (a) and (b), the solid-line and dot-line are respectively the results of renormalized phonon and bare phonon, and the electron temperature is $T = 35 K$.

FIG. 4. Energy relaxation rate as functions of renormalized phonon frequency $\omega$ at high temperature (circle-marked line) and low temperature (diamond-marked line), where $I(\omega)$ is defined in the text. The diamond-marked line represents the low temperature result multiplied by a factor of $10^4$. The widths of the quantum wire are $L_y = L_z = 50 \ang$. The electron density is $n = 10^6 \text{cm}^{-1}$.
Fig. 1(a) Zheng & Das Sarma

![Graph showing the relationship between log(P/N) (Watts/carrier) and 1/T (K⁻¹). The graph includes three curves labeled P, P_{pp}, and P_{0}.]
Fig. 1(b) Zheng & Das Sarma

\[ \log(P/N) \text{ (Watts/carrier)} \]

\[ \frac{1}{T} \text{ (K}^{-1}) \]

- Graph showing \( \log(P/N) \) vs. \( 1/T \) for different states or processes labeled as \( P \), \( P_{PP} \), and \( P_0 \).
Fig. 3(a) Zheng & Das Sarma

\[ \log(P/N) \text{ (Watts/carrier)} \]

\[ \log(n \times 10^5 \text{cm}^{-1}) \]

- \( P \)
- \( P_0 \)
