Abstract

The concept of neutrosophic continuous function was very first introduced by A.A. Salama et al. The main aim of this paper is to introduce a new concept of Neutrosophic continuous function namely Strongly Neutrosophic gsα* - continuous functions, Perfectly Neutrosophic gsα* - continuous functions and Totally Neutrosophic gsα* - continuous functions in Neutrosophic topological spaces. These concepts are derived from strongly generalized neutrosophic continuous function and perfectly generalized neutrosophic continuous function. Several interesting properties and characterizations are derived and compared with already existing neutrosophic functions.

Keywords: Neutrosophic gsα* - closed set, Neutrosophic gsα* - open set, Strongly Neutrosophic gsα* - continuous function, Perfectly Neutrosophic gsα* - continuous function, Totally Neutrosophic gsα* - continuous function

1. Introduction

The concept of Neutrosophic set theory was introduced by F. Smarandache [1] and it comes from two concepts, one is intuitionistic fuzzy sets introduced by K. Atanassov’s [2] and the other is fuzzy sets introduced by L.A. Zadeh’s [3]. It includes three components, truth, indeterminacy and false membership function. R. Dhavaseelan and S. Jafari [4] has discussed about the concept of strongly generalized neutrosophic continuous function. Further he also introduced the topic of perfectly generalized neutrosophic continuous function. The real life application of neutrosophic topology is applied in Information Systems, Applied Mathematics etc.

In this paper, we introduce some new concepts related to Neutrosophic gsα* - continuous function namely Strongly Neutrosophic gsα* - continuous function, Perfectly Neutrosophic gsα* - continuous function, Totally Neutrosophic gsα* - continuous function.

2. Preliminaries

Definition 2.1: [5] Let \( \mathbb{P} \) be a non-empty fixed set. A Neutrosophic set \( \mathbb{H} \) on the universe \( \mathbb{P} \) is defined as \( \mathbb{H} = \{(p, t_\mathbb{H}(p), i_\mathbb{H}(p), f_\mathbb{H}(p)) : p \in \mathbb{P}\} \) where \( t_\mathbb{H}(p), i_\mathbb{H}(p), f_\mathbb{H}(p) \) represent the degree of membership function \( t_\mathbb{H}(p) \), the degree of indeterminacy \( i_\mathbb{H}(p) \) and the degree of non-membership function \( f_\mathbb{H}(p) \) respectively for each element \( p \in \mathbb{P} \) to the set \( \mathbb{H} \). Also, \( t_\mathbb{H}, i_\mathbb{H}, f_\mathbb{H} : \mathbb{P} \rightarrow [0, 1] \) and \( 0 \).
\[ \leq t_H(p) + i_H(p) + f_H(p) \leq 3^+. \] Set of all Neutrosophic set over \( \mathbb{P} \) is denoted by \( \text{Neu}(\mathbb{P}) \).

**Definition 2.2:** [8] Let \( \mathbb{P} \) be a non-empty set.
\[
A = \{ (p, (t_A(p), i_A(p), f_A(p))) : p \in \mathbb{P} \} \text{ and } B = \{ (p, (t_B(p), i_B(p), f_B(p))) : p \in \mathbb{P} \}
\] are neutrosophic sets, then

i. \( A \subseteq B \) if \( t_A(p) \leq t_B(p), i_A(p) \leq i_B(p), f_A(p) \geq f_B(p) \) for all \( p \in \mathbb{P} \).

ii. Intersection of two neutrosophic set \( A \) and \( B \) is defined as \( A \cap B = \{ (p, (\min(t_A(p), t_B(p)), \min(i_A(p), i_B(p))), \max(f_A(p), f_B(p))) : p \in \mathbb{P} \} \).

iii. Union of two neutrosophic set \( A \) and \( B \) is defined as \( A \cup B = \{ (p, (\max(t_A(p), t_B(p)), \max(i_A(p), i_B(p))), \min(f_A(p), f_B(p))) : p \in \mathbb{P} \} \).

iv. \( A^c = \{ (p, (f_A(p), 1 - i_A(p), t_A(p))) : p \in \mathbb{P} \} \).

v. \( 0_{\text{Neu}} = \{ (p, (0, 0, 1)) : p \in \mathbb{P} \} \) and \( 1_{\text{Neu}} = \{ (p, (1, 1, 0)) : p \in \mathbb{P} \} \).

**Definition 2.3:** [5] A neutrosophic topology \( (\text{Neu}, T) \) on a non-empty set \( \mathbb{P} \) is a family \( \tau_{\text{Neu}} \) of neutrosophic sets in \( \mathbb{P} \) satisfying the following axioms,

i. \( 0_{\text{Neu}}, 1_{\text{Neu}} \in \tau_{\text{Neu}} \).

ii. \( A_1 \cap A_2 \in \tau_{\text{Neu}} \) for any \( A_1, A_2 \in \tau_{\text{Neu}} \).

iii. \( \bigcup A_i \in \tau_{\text{Neu}} \) for every family \( \{ A_i / i \in \Omega \} \subseteq \tau_{\text{Neu}} \).

In this case, the ordered pair \( (\mathbb{P}, \tau_{\text{Neu}}) \) or simply \( \mathbb{P} \) is called a neutrosophic topological space \( (\text{Neu}, \mathbb{P}) \). The elements of \( \tau_{\text{Neu}} \) are neutrosophic open set \( (\text{Neu} \text{OS}) \) and \( \tau_{\text{Neu}}^c \) is neutrosophic closed set \( (\text{Neu} \text{CS}) \).

**Definition 2.4:** [6] A neutrosophic set \( A \) in a \( \text{Neu} \text{TS} \) \( (\mathbb{P}, \tau_{\text{Neu}}) \) is called a neutrosophic generalized semi alpha closed set \( (\text{Neu}g\text{sa}^+ \text{CS}) \) if \( \text{Neu}g\text{sa}^+ = \text{int}(\text{Neu}g\text{sa}^+ \text{CS}) \) whenever \( \text{Neu}g\text{sa}^+ = \text{int}(\text{Neu}g\text{sa}^+ \text{CS}) \).

**Definition 2.5:** [7] A neutrosophic topological space \( (\mathbb{P}, \tau_{\text{Neu}}) \) is called a \( \text{Neu}g\text{sa}^+ \text{CS} \) space if every \( \text{Neu}g\text{sa}^+ \text{CS} \) in \( (\mathbb{P}, \tau_{\text{Neu}}) \) is a \( \text{Neu}g\text{sa}^+ \text{CS} \) in \( (\mathbb{P}, \tau_{\text{Neu}}) \).

**Definition 2.6:** A neutrosophic function \( f : (\mathbb{P}, \tau_{\text{Neu}}) \rightarrow (\mathbb{Q}, \sigma_{\text{Neu}}) \) is said to be

1. neutrosophic continuous [8] if the inverse image of each \( \text{Neu}g\text{sa}^+ \text{CS} \) in \( (\mathbb{Q}, \sigma_{\text{Neu}}) \) is \( \text{Neu}g\text{sa}^+ \text{CS} \) in \( (\mathbb{P}, \tau_{\text{Neu}}) \).

2. \( \text{Neu}g\text{sa}^+ \text{CS} \) – continuous [7] if the inverse image of each \( \text{Neu}g\text{sa}^+ \text{CS} \) in \( (\mathbb{Q}, \sigma_{\text{Neu}}) \) is \( \text{Neu}g\text{sa}^+ \text{CS} \) – closed set in \( (\mathbb{P}, \tau_{\text{Neu}}) \).

3. \( \text{Neu}g\text{sa}^+ \text{CS} \) – irresolute map [7] if the inverse image of each \( \text{Neu}g\text{sa}^+ \text{CS} \) – closed set in \( (\mathbb{Q}, \sigma_{\text{Neu}}) \) is \( \text{Neu}g\text{sa}^+ \text{CS} \) – closed set in \( (\mathbb{P}, \tau_{\text{Neu}}) \).

4. strongly neutrosophic continuous [4] if the inverse image of each neutrosophic set in \( (\mathbb{Q}, \sigma_{\text{Neu}}) \) is both \( \text{Neu}g\text{sa}^+ \text{OS} \) and \( \text{Neu}g\text{sa}^+ \text{CS} \) in \( (\mathbb{P}, \tau_{\text{Neu}}) \).

5. perfectly neutrosophic continuous [4] if the inverse image of each \( \text{Neu}g\text{sa}^+ \text{CS} \) in \( (\mathbb{Q}, \sigma_{\text{Neu}}) \) is both \( \text{Neu}g\text{sa}^+ \text{OS} \) and \( \text{Neu}g\text{sa}^+ \text{CS} \) in \( (\mathbb{P}, \tau_{\text{Neu}}) \).
Definition 2.7: [9] Let τ_N_eu = {0_N_eu, 1_N_eu} is a neutrosophic topological space over P. Then (P, τ_N_eu) is called neutrosophic discrete topological space.

Definition 2.8: A neutrosophic topological space (P, τ_N_eu) is called a neutrosophic clopen set (N_eu - clopen set) if it is both N_eu - OS and N_eu - CS in (P, τ_N_eu).

3. Strongly neutrosophic gsa*-continuous function

Definition 3.1: A neutrosophic function f : (P, τ_N_eu) → (Q, σ_N_eu) is said to be strongly N_eu gsa*-continuous if the inverse image of every N_eu gsa* - CS in (Q, σ_N_eu) is a N_eu - CS in (P, τ_N_eu). (ie) f^{-1}(A) is a N_eu - CS in (P, τ_N_eu) for every N_eu gsa* - CS in (Q, σ_N_eu).

Theorem 3.2: Every strongly N_eu gsa* - continuous is neutrosophic continuous, but not conversely.

Proof:
Let f : (P, τ_N_eu) → (Q, σ_N_eu) be any neutrosophic function. Let A be any N_eu - CS in (Q, σ_N_eu). Since every N_eu - CS is N_eu gsa* - CS, then A is N_eu gsa* - CS in (Q, σ_N_eu). Therefore, f is neutrosophic continuous.

Example 3.3: Let P = {p} and Q = {q}. τ_N_eu = {0_N_eu, 1_N_eu, A} and σ_N_eu = {0_N_eu, 1_N_eu, B} are N_eu TS on (P, τ_N_eu) and (Q, σ_N_eu) respectively. Also A = { (p, 0.6, 0.4, 0.4) } and B = { (q, 0.4, 0.6, 0.2) } are N_eu(P) and N_eu(Q).

Define a map f : (P, τ_N_eu) → (Q, σ_N_eu) by f(p) = q + 0.2. Let B' = { (q, 0.2, 0.4, 0.4) } be a N_eu - CS in (Q, σ_N_eu). Then f^{-1}(B') = { (p, 0.4, 0.6, 0.6) } . Now, N_eu - cl(f^{-1}(B')) = A^c ∩ 1_{N_eu} = A^c = f^{-1}(B') ⇒ f^{-1}(B') is N_eu - CS in (P, τ_N_eu). Therefore, f is neutrosophic continuous, but f is not strongly N_eu gsa* - continuous.

Theorem 3.4: Let f : (P, τ_N_eu) → (Q, σ_N_eu) be strongly N_eu gsa* - continuous if the inverse image of every N_eu gsa* - OS in (Q, σ_N_eu) is N_eu - OS in (P, τ_N_eu).

Proof:
Assume that f is strongly N_eu gsa* - continuous. Let A be any N_eu gsa* - OS in (Q, σ_N_eu). Then A^c is N_eu gsa* - CS in (Q, σ_N_eu). Since f is strongly N_eu gsa* - continuous, then f^{-1}(A^c) is N_eu - CS in (P, τ_N_eu) ⇒ (f^{-1}(A))^c is N_eu - CS in (Q, σ_N_eu). Conversely, Let A be any N_eu gsa* - CS in (Q, σ_N_eu). Then A^c is N_eu gsa* - OS in (Q, σ_N_eu). By hypothesis, f^{-1}(A^c) is N_eu - OS in (P, τ_N_eu) ⇒ (f^{-1}(A))^c is N_eu - OS in (P, τ_N_eu) ⇒ f^{-1}(A) is N_eu - CS in (P, τ_N_eu). Therefore, f is strongly N_eu gsa* - continuous.

Theorem 3.5: Every strongly N_eu gsa* - continuous is N_eu gsa* - continuous, but not conversely.

Proof:
Let f : (P, τ_N_eu) → (Q, σ_N_eu) be any neutrosophic function. Let A be any N_eu - CS in (Q, σ_N_eu). Then A is N_eu gsa* - CS in (Q, σ_N_eu). Since f is strongly N_eu gsa* - continuous, then f^{-1}(A) is a N_eu - CS in (P, τ_N_eu) ⇒ f^{-1}(A) is N_eu gsa* - CS in (P, τ_N_eu). Therefore, f is N_eu gsa* - continuous.
Define a map \( f : (\mathbb{P}, \tau_{\mathbb{N}}) \rightarrow (\mathbb{Q}, \sigma_{\mathbb{N}}) \) by \( f(\mathbf{p}) = \mathbf{q} \). Let \( \mathbf{B}^c = \{ (\mathbf{q}, (0.4, 0.2, 0.6)) \} \) be a \( N_{\mathbb{N}} \) - CS in \((\mathbb{Q}, \sigma_{\mathbb{N}})\). Then \( f^{-1}(\mathbf{B}^c) = \{(\mathbf{p}, (0.4, 0.2, 0.6)) \} \). \( N_{\mathbb{N}} \alpha \rightarrow \mathbb{OS} = N_{\mathbb{N}} \alpha - \mathbb{OS} = \{ 0_{\mathbb{N}}, 1_{\mathbb{N}}, A \} \) and \( N_{\mathbb{N}} \alpha - \mathbb{CS} = \{ 0_{\mathbb{N}}, 1_{\mathbb{N}}, A \} \). Now, \( N_{\mathbb{N}} \alpha - int( N_{\mathbb{N}} \alpha - cl( f^{-1}(\mathbf{B}^c)) = \mathbb{A} \subseteq N_{\mathbb{N}} - int(1_{\mathbb{N}}) = 1_{\mathbb{N}} \), whenever \( f^{-1}(\mathbf{B}^c) \subseteq 1_{\mathbb{N}} \Rightarrow f^{-1}(\mathbf{B}^c) \) is \( N_{\mathbb{N}} \alpha \rightarrow \mathbb{CS} \) in \((\mathbb{P}, \tau_{\mathbb{N}})\). Therefore, \( f \) is \( N_{\mathbb{N}} \alpha \rightarrow \mathbb{CS} \) continuous. But \( f \) is not strongly \( N_{\mathbb{N}} \alpha \rightarrow \mathbb{CS} \) continuous. Let \( \mathbb{C} = \{ (\mathbf{q}, (0.3, 0.1, 0.7)) \} \) be a \( N_{\mathbb{N}} \alpha \rightarrow \mathbb{CS} \) in \((\mathbb{Q}, \sigma_{\mathbb{N}})\). Then \( f^{-1}(\mathbb{C}) = \{(\mathbf{p}, (0.3, 0.1, 0.7)) \} \). Now \( N_{\mathbb{N}} - cl( f^{-1}(\mathbb{C})) = \mathbb{A} \cap 1_{\mathbb{N}} = \mathbb{A} \neq f^{-1}(\mathbb{C}) \) is not \( N_{\mathbb{N}} \alpha \rightarrow \mathbb{CS} \) in \((\mathbb{P}, \tau_{\mathbb{N}})\).

**Theorem 3.7:** Every strongly neutrosophic continuous is strongly \( N_{\mathbb{N}} \alpha \rightarrow \mathbb{CS} \) continuous, but not conversely.

**Proof:**
Let \( f : (\mathbb{P}, \tau_{\mathbb{N}}) \rightarrow (\mathbb{Q}, \sigma_{\mathbb{N}}) \) be any neutrosophic function. Let \( \mathbb{A} \) be any \( N_{\mathbb{N}} \alpha \rightarrow \mathbb{CS} \) in \((\mathbb{Q}, \sigma_{\mathbb{N}})\). Since \( f \) is strongly neutrosophic continuous, then \( f^{-1}(\mathbb{A}) \) is both \( N_{\mathbb{N}} \alpha \rightarrow \mathbb{CS} \) and \( N_{\mathbb{N}} \alpha \rightarrow \mathbb{OS} \) in \((\mathbb{P}, \tau_{\mathbb{N}})\). Hence, \( f \) is strongly \( N_{\mathbb{N}} \alpha \rightarrow \mathbb{CS} \) continuous.

**Example 3.8:** Let \( \mathbb{P} = \{ \mathbf{p} \} \) and \( \mathbb{Q} = \{ \mathbf{q} \} \). \( \tau_{\mathbb{N}} = \{ 0_{\mathbb{N}}, 1_{\mathbb{N}}, A, \mathbb{C} \} \) and \( \sigma_{\mathbb{N}} = \{ \mathbb{B} \} \) be neutrosophic functions. Let \( \mathbb{B} = \{ \mathbf{p}, (0.4, 0.6, 0.2) \} \). \( \mathbb{C} = \{ \mathbf{q}, (0.4, 1, [0.6, 1], [0.0, 2]) \} \) and \( \mathbb{A} = \{ \mathbf{q}, (0.4, 0.6, 0.2) \} \) are \( N_{\mathbb{N}} \alpha \rightarrow \mathbb{OS} \) and \( N_{\mathbb{N}} \alpha \rightarrow \mathbb{CS} \) in \((\mathbb{P}, \tau_{\mathbb{N}})\). Define a map \( f : (\mathbb{P}, \tau_{\mathbb{N}}) \rightarrow (\mathbb{Q}, \sigma_{\mathbb{N}}) \) by \( f(\mathbf{p}) = \mathbf{q} \). Let \( T = \{ (\mathbf{q}, (0.0, 0.2), (0.4, 0.4), (0.4, 1)) \} \) be a \( N_{\mathbb{N}} \alpha \rightarrow \mathbb{CS} \) in \((\mathbb{Q}, \sigma_{\mathbb{N}})\). Then \( f^{-1}(T) = \{ (\mathbf{p}, (0.0, 0.2), (0.4, 0.4), (0.4, 1)) \} \). Now \( N_{\mathbb{N}} - cl( f^{-1}(T)) = \mathbb{A} \cap \mathbb{C} \subseteq 1_{\mathbb{N}} = \mathbb{C} \neq f^{-1}(T) \). Therefore, \( f \) is strongly \( N_{\mathbb{N}} \alpha \rightarrow \mathbb{CS} \) continuous. But \( f \) is not strongly neutrosophic continuous. Let \( \mathbb{E} = \{ (\mathbf{q}, (0.4, 0.6, 0.2)) \} \) be a neutrosophic set in \((\mathbb{Q}, \sigma_{\mathbb{N}})\). Then \( f^{-1}(E) = \{ (\mathbf{p}, (0.4, 0.6, 0.2)) \} \). Now \( N_{\mathbb{N}} - int( f^{-1}(E)) = 0_{\mathbb{N}} \cup \mathbb{A} = \mathbb{A} \neq f^{-1}(E) \Rightarrow f^{-1}(E) \) is not \( N_{\mathbb{N}} \alpha \rightarrow \mathbb{CS} \) in \((\mathbb{P}, \tau_{\mathbb{N}})\). Also \( N_{\mathbb{N}} - cl( f^{-1}(E)) = 1_{\mathbb{N}} \neq f^{-1}(E) \Rightarrow f^{-1}(E) \) is not \( N_{\mathbb{N}} \alpha \rightarrow \mathbb{CS} \) in \((\mathbb{P}, \tau_{\mathbb{N}})\). Therefore, \( f^{-1}(E) \) is not both \( N_{\mathbb{N}} \alpha \rightarrow \mathbb{OS} \) and \( N_{\mathbb{N}} \alpha \rightarrow \mathbb{CS} \) in \((\mathbb{P}, \tau_{\mathbb{N}})\).

**Remark 3.9:** Every strongly neutrosophic continuous is \( N_{\mathbb{N}} \alpha \rightarrow \mathbb{CS} \) continuous, but not conversely. (by Theorem 3.5 & 3.7).

**Theorem 3.10:** Let \( f : (\mathbb{P}, \tau_{\mathbb{N}}) \rightarrow (\mathbb{Q}, \sigma_{\mathbb{N}}) \) be neutrosophic function and \( (\mathbb{Q}, \sigma_{\mathbb{N}}) \) be \( N_{\mathbb{N}} \alpha \rightarrow \mathbb{CS} \) - \( T_{1/2} \) space. Then the following are equivalent.

1. \( f \) is strongly \( N_{\mathbb{N}} \alpha \rightarrow \mathbb{CS} \) continuous.
2. \( f \) is neutrosophic continuous.

**Proof:**
1. \( \Rightarrow (2) \). Proof follows from theorem 3.2.

2. \( \Rightarrow (1) \). Let \( \mathbb{A} \) be any \( N_{\mathbb{N}} \alpha \rightarrow \mathbb{CS} \) in \((\mathbb{Q}, \sigma_{\mathbb{N}})\). Since \( (\mathbb{Q}, \sigma_{\mathbb{N}}) \) is \( N_{\mathbb{N}} \alpha \rightarrow \mathbb{CS} \) - \( T_{1/2} \) space, then \( \mathbb{A} \) is \( N_{\mathbb{N}} \alpha \rightarrow \mathbb{CS} \) in \((\mathbb{Q}, \sigma_{\mathbb{N}})\). Since \( f \) is neutrosophic continuous, then \( f^{-1}(\mathbb{A}) \) is \( N_{\mathbb{N}} \alpha \rightarrow \mathbb{CS} \) in \((\mathbb{P}, \tau_{\mathbb{N}})\). Therefore, \( f \) is strongly \( N_{\mathbb{N}} \alpha \rightarrow \mathbb{CS} \) continuous.

**Theorem 3.11:** Let \( f : (\mathbb{P}, \tau_{\mathbb{N}}) \rightarrow (\mathbb{Q}, \sigma_{\mathbb{N}}) \) be \( N_{\mathbb{N}} \alpha \rightarrow \mathbb{CS} \) continuous. Both \( (\mathbb{P}, \tau_{\mathbb{N}}) \) and \( (\mathbb{Q}, \sigma_{\mathbb{N}}) \) are \( N_{\mathbb{N}} \alpha \rightarrow \mathbb{CS} \) - \( T_{1/2} \) space, then \( f \) is strongly \( N_{\mathbb{N}} \alpha \rightarrow \mathbb{CS} \) continuous.
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Proof:
Let A be any $\text{Neugsa}^*$ - CS in $(Q, \sigma_{N_\alpha})$. Since $(Q, \sigma_{N_\alpha})$ is $\text{Neugsa}^*$ - $1_{12}$ space, then A is $\text{Neu}^*$ - CS in $(Q, \sigma_{N_\alpha})$. Since $f$ is $\text{Neugsa}^*$ - continuous, then $f^{-1}(A)$ is $\text{Neugsa}^*$ - CS in $(P, \tau_{N_\alpha})$. Since $(P, \tau_{N_\alpha})$ is $\text{Neugsa}^*$ - $1_{12}$ space, then $f^{-1}(A)$ is $\text{Neu}^*$ - CS in $(P, \tau_{N_\alpha})$. Therefore, $f$ is strongly $\text{Neugsa}^*$ - continuous.

**Theorem 3.12:** Let $f : (P, \tau_{N_\alpha}) \rightarrow (Q, \sigma_{N_\alpha})$ be strongly $\text{Neugsa}^*$ - continuous, then $f$ is $\text{Neugsa}^*$ - irresolute.

**Proof:**
Let A be any $\text{Neugsa}^*$ - CS in $(Q, \sigma_{N_\alpha})$. Since $f$ is strongly $\text{Neugsa}^*$ - continuous, then $f^{-1}(A)$ is $\text{Neu}^*$ - CS in $(P, \tau_{N_\alpha})$, implying $f^{-1}(A)$ is $\text{Neugsa}^*$ - CS in $(P, \tau_{N_\alpha})$. Hence, $f$ is strongly $\text{Neugsa}^*$ - irresolute.

**Theorem 3.13:** Let $f : (P, \tau_{N_\alpha}) \rightarrow (Q, \sigma_{N_\alpha})$ be $\text{Neugsa}^*$ - irresolute and $(P, \tau_{N_\alpha})$ be $\text{Neugsa}^*$ - $1_{12}$ space, then $f$ is strongly $\text{Neugsa}^*$ - continuous.

**Proof:**
Let A be any $\text{Neugsa}^*$ - CS in $(Q, \sigma_{N_\alpha})$. Since $f$ is $\text{Neugsa}^*$ - irresolute, then $f^{-1}(A)$ is $\text{Neu}^*$ - CS in $(P, \tau_{N_\alpha})$. Since $(P, \tau_{N_\alpha})$ is $\text{Neugsa}^*$ - $1_{12}$ space, then $f^{-1}(A)$ is $\text{Neu}^*$ - CS in $(P, \tau_{N_\alpha})$. Therefore, $f$ is strongly $\text{Neugsa}^*$ - continuous.

**Theorem 3.14:** Let $f : (P, \tau_{N_\alpha}) \rightarrow (Q, \sigma_{N_\alpha})$ and $g : (Q, \sigma_{N_\alpha}) \rightarrow (R, \gamma_{N_\alpha})$ be strongly $\text{Neugsa}^*$ - continuous, then $gof : (P, \tau_{N_\alpha}) \rightarrow (R, \gamma_{N_\alpha})$ is strongly $\text{Neugsa}^*$ - continuous.

**Proof:**
Let A be any $\text{Neugsa}^*$ - CS in $(R, \gamma_{N_\alpha})$. Since $g$ is strongly $\text{Neugsa}^*$ - continuous, then $g^{-1}(A)$ is $\text{Neu}^*$ - CS in $(Q, \sigma_{N_\alpha})$. Since $f$ is $\text{Neugsa}^*$ - continuous, then $f^{-1}(g^{-1}(A)) = (gof)^{-1}(A)$ is $\text{Neu}^*$ - CS in $(P, \tau_{N_\alpha})$. Therefore, $gof$ is strongly $\text{Neugsa}^*$ - continuous.

**Theorem 3.15:** Let $f : (P, \tau_{N_\alpha}) \rightarrow (Q, \sigma_{N_\alpha})$ be strongly $\text{Neugsa}^*$ - continuous and $g : (Q, \sigma_{N_\alpha}) \rightarrow (R, \gamma_{N_\alpha})$ be $\text{Neugsa}^*$ - continuous, then $gof : (P, \tau_{N_\alpha}) \rightarrow (R, \gamma_{N_\alpha})$ is neutrosophic continuous.

**Proof:**
Let A be any $\text{Neugsa}^*$ - CS in $(R, \gamma_{N_\alpha})$. Since $g$ is $\text{Neugsa}^*$ - continuous, then $g^{-1}(A)$ is $\text{Neu}^*$ - CS in $(Q, \sigma_{N_\alpha})$. Since $f$ is strongly $\text{Neugsa}^*$ - continuous, then $f^{-1}(g^{-1}(A)) = (gof)^{-1}(A)$ is $\text{Neu}^*$ - CS in $(P, \tau_{N_\alpha})$. Therefore, $gof$ is neutrosophic continuous.

**Theorem 3.16:** Let $f : (P, \tau_{N_\alpha}) \rightarrow (Q, \sigma_{N_\alpha})$ be strongly $\text{Neugsa}^*$ - continuous and $g : (Q, \sigma_{N_\alpha}) \rightarrow (R, \gamma_{N_\alpha})$ be $\text{Neugsa}^*$ - irresolute, then $gof : (P, \tau_{N_\alpha}) \rightarrow (R, \gamma_{N_\alpha})$ is strongly $\text{Neugsa}^*$ - continuous.

**Proof:**
Let A be any $\text{Neugsa}^*$ - CS in $(R, \gamma_{N_\alpha})$. Since $g$ is $\text{Neugsa}^*$ - irresolute, then $g^{-1}(A)$ is $\text{Neu}^*$ - CS in $(Q, \sigma_{N_\alpha})$. Since $f$ is strongly $\text{Neugsa}^*$ - continuous, then $f^{-1}(g^{-1}(A)) = (gof)^{-1}(A)$ is $\text{Neu}^*$ - CS in $(P, \tau_{N_\alpha})$. Therefore, $gof$ is strongly $\text{Neugsa}^*$ - continuous.

**Theorem 3.17:** Let $f : (P, \tau_{N_\alpha}) \rightarrow (Q, \sigma_{N_\alpha})$ be $\text{Neugsa}^*$ - continuous and $g : (Q, \sigma_{N_\alpha}) \rightarrow (R, \gamma_{N_\alpha})$ be strongly $\text{Neugsa}^*$ - continuous, then $gof : (P, \tau_{N_\alpha}) \rightarrow (R, \gamma_{N_\alpha})$ is $\text{Neugsa}^*$ - irresolute.

**Proof:**
Let A be any $\text{Neugsa}^*$ - CS in $(R, \gamma_{N_\alpha})$. Since $g$ is strongly $\text{Neugsa}^*$ - continuous, then $g^{-1}(A)$ is $\text{Neu}^*$ - CS in $(Q, \sigma_{N_\alpha})$. Since $f$ is $\text{Neugsa}^*$ - continuous, then $f^{-1}(g^{-1}(A)) = (gof)^{-1}(A)$ is $\text{Neu}^*$ - CS in $(P, \tau_{N_\alpha})$. Hence, $gof$ is $\text{Neugsa}^*$ - irresolute.
Theorem 3.18: Let \( f : (\mathbb{P}, \tau_{N_{\alpha}}) \rightarrow (\mathbb{Q}, \sigma_{N_{\alpha}}) \) be neutrosophic continuous and \( g : (\mathbb{Q}, \sigma_{N_{\alpha}}) \rightarrow (\mathbb{R}, \gamma_{N_{\alpha}}) \) be strongly \( N_{eu}gsa^* \) continuous, then \( g \circ f : (\mathbb{P}, \tau_{N_{\alpha}}) \rightarrow (\mathbb{R}, \gamma_{N_{\alpha}}) \) is strongly \( N_{eu}gsa^* \) continuous.

**Proof:**

Let \( A \) be any \( N_{eu}gsa^* - \text{CS} \) in \((\mathbb{R}, \gamma_{N_{\alpha}})\). Since \( g \) is strongly \( N_{eu}gsa^* \) continuous, then \( g^{-1}(A) \) is \( N_{eu} - \text{CS} \) in \((\mathbb{Q}, \sigma_{N_{\alpha}})\). Since \( f \) is neutrosophic continuous, then \( f^{-1}(g^{-1}(A)) = (g \circ f)^{-1}(A) \) is \( N_{eu} - \text{CS} \) in \((\mathbb{P}, \tau_{N_{\alpha}})\). Hence, \( g \circ f \) is strongly \( N_{eu}gsa^* \) continuous.

**Inter-relationship 3.19:**

| Strongly neutrosophic continuous | Neutrosophic continuous |
|----------------------------------|------------------------|
| \( N_{eu}gsa^* \) - continuous  | \( N_{eu}gsa^* \) - continuous |
| \( N_{eu}gsa^* \) - irresolute  | \( N_{eu}gsa^* \) - continuous |

4. Perfectly neutrosophic \( gs^* \) -continuous function

**Definition 4.1:** A neutrosophic function \( f : (\mathbb{P}, \tau_{N_{\alpha}}) \rightarrow (\mathbb{Q}, \sigma_{N_{\alpha}}) \) is said to be perfectly \( N_{eu}gsa^* \) continuous if the inverse image of every \( N_{eu}gsa^* \) - CS in \((\mathbb{Q}, \sigma_{N_{\alpha}})\) is both \( N_{eu} - \text{OS} \) and \( N_{eu} - \text{CS} \) (ie, \( N_{eu} - \text{clopen set} \)) in \((\mathbb{P}, \tau_{N_{\alpha}})\).

**Theorem 4.2:** Every perfectly \( N_{eu}gsa^* \) - continuous is strongly \( N_{eu}gsa^* \) continuous, but not conversely.

**Proof:**

Let \( f : (\mathbb{P}, \tau_{N_{\alpha}}) \rightarrow (\mathbb{Q}, \sigma_{N_{\alpha}}) \) be any neutrosophic function. Let \( A \) be any \( N_{eu}gsa^* - \text{CS} \) in \((\mathbb{Q}, \sigma_{N_{\alpha}})\). Since \( f \) is perfectly \( N_{eu}gsa^* \) continuous, then \( f^{-1}(A) \) is both \( N_{eu} - \text{OS} \) and \( N_{eu} - \text{CS} \) in \((\mathbb{P}, \tau_{N_{\alpha}})\) \( \Rightarrow f^{-1}(A) \) is \( N_{eu} - \text{CS} \) in \((\mathbb{P}, \tau_{N_{\alpha}})\).

Example 4.3: Let \( \mathbb{P} = \{p\} \) and \( \mathbb{Q} = \{q\} \), \( \tau_{N_{\alpha}} = \{0_{N_{\alpha}}, 1_{N_{\alpha}}, \text{A}, \text{C}\} \) and \( \sigma_{N_{\alpha}} = \{0_{N_{\alpha}}, 1_{N_{\alpha}}, \text{B}\} \) are \( N_{eu} - \text{TS} \) on \((\mathbb{P}, \tau_{N_{\alpha}})\) and \((\mathbb{Q}, \sigma_{N_{\alpha}})\) respectively. Also \( A = \{(p, (0.7, 0.8, 0.3))\}, \text{C} = \{(p, (0.7, 1), (0.8, 1), (0, 0.3))\}\) and \( B = \{(q, (0.7, 0.8, 0.3))\} \) are \( N_{eu}(\mathbb{P}) \) and \( N_{eu}(\mathbb{Q}) \). Define a map \( f : (\mathbb{P}, \tau_{N_{\alpha}}) \rightarrow (\mathbb{Q}, \sigma_{N_{\alpha}}) \) by \( f(p) = q \). Let \( T = \{(q, (0.3, 0.2, 0.7, 1))\} \) be a \( N_{eu}gsa^* - \text{CS} \) in \((\mathbb{Q}, \sigma_{N_{\alpha}})\).

Then \( f^{-1}(T) = \{(p, (0, 0.3, 0, 0.2, 0.7, 1))\} \). Now \( N_{eu} - cl(f^{-1}(T)) = A^c \cap C^c \cap 1_{N_{\alpha}} = C^c = f^{-1}(T) \). Therefore, \( f \) is strongly \( N_{eu}gsa^* \) continuous. But \( f \) is not perfectly \( N_{eu}gsa^* \) continuous, because \( f^{-1}(T) \) is not both \( N_{eu} - \text{OS} \) and \( N_{eu} - \text{CS} \) in \((\mathbb{P}, \tau_{N_{\alpha}})\). Since, \( N_{eu} - int(f^{-1}(T)) = 0_{N_{\alpha}} \neq f^{-1}(T) \Rightarrow f^{-1}(T) \) is not \( N_{eu} \) - \( \text{CS} \) in \((\mathbb{P}, \tau_{N_{\alpha}})\). Therefore, \( f^{-1}(T) \) is not both \( N_{eu} - \text{OS} \) and \( N_{eu} - \text{CS} \) in \((\mathbb{P}, \tau_{N_{\alpha}})\).

**Theorem 4.4:** Every perfectly \( N_{eu}gsa^* \) continuous is perfectly neutrosophic continuous, but not conversely.
Proof:
Let $f : (\mathbb{P}, \tau_{N_u}) \rightarrow (\mathbb{Q}, \sigma_{N_u})$ be any neutrosophic function. Let $A$ be any $N_u - CS$ in $(\mathbb{Q}, \sigma_{N_u})$. Then $A$ is $N_{eqgsa^*} - CS$ in $(\mathbb{Q}, \sigma_{N_u})$. Since $f$ is perfectly $N_{eqgsa^*}$ - continuous, then $f^{-1}(A)$ is both $N_u - OS$ and $N_u - CS$ in $(\mathbb{P}, \tau_{N_u})$. Therefore, $f$ is perfectly neutrosophic continuous.

Example 4.5: Let $\mathbb{P} = \{p\}$ and $\mathbb{Q} = \{q\}$. $\tau_{N_u} = \{0_{N_u}, 1_{N_u}, A, C, E\}$ and $\sigma_{N_u} = \{0_{N_u}, 1_{N_u}, B\}$ are $N_u$-TS on $(\mathbb{P}, \tau_{N_u})$ and $(\mathbb{Q}, \sigma_{N_u})$ respectively. Also $A = \{(p, (0.4, 0.2, 0.6))\}, C = \{(p, (0.6, 0.8, 0.4))\}, E = \{(p, (0.6, 0.2, 0.6))\}$ and $B = \{(q, (0.6, 0.8, 0.4))\}$ are $N_u(\mathbb{P})$ and $N_u(\mathbb{Q})$. Define a map $f : (\mathbb{P}, \tau_{N_u}) \rightarrow (\mathbb{Q}, \sigma_{N_u})$ by $f(p) = q$. Let $B' = \{(q, (0.4, 0.2, 0.6))\}$ be a $N_u - CS$ in $(\mathbb{Q}, \sigma_{N_u})$. Then $f^{-1}(B') = \{(p, (0.4, 0.2, 0.6))\}$. Now $N_u - cl\left(f^{-1}(B')\right) = A^c \cap C^c \cap E^c \cap 1_{N_u} = C^c = f^{-1}(B')$. Also, $N_u - int\left(f^{-1}(B')\right) = A \cup C \cup 0_{N_u} = A = f^{-1}(B') \Rightarrow f^{-1}(B')$ is both $N_u - OS$ and $N_u - CS$ in $(\mathbb{P}, \tau_{N_u})$. Therefore, $f$ is perfectly neutrosophic continuous. But $f$ is not perfectly $N_{eqgsa^*}$ - continuous. Let $T = \{(q, (0.4, 0.2, 0.6))\}$ be $N_{eqgsa^*} - CS$ in $(\mathbb{Q}, \sigma_{N_u})$. Then $f^{-1}(T) = \{(p, (0.4, 0.2, 0.6))\}$. Since, $N_u - int\left(f^{-1}(T)\right) = E \cup 0_{N_u} = E = f^{-1}(T) \Rightarrow f^{-1}(T)$ is $N_u - OS$ in $(\mathbb{P}, \tau_{N_u})$. Also, $N_u - cl\left(f^{-1}(T)\right) = A^c \cap C^c \cap E^c \cap 1_{N_u} = C^c \neq f^{-1}(T) \Rightarrow f^{-1}(T)$ is not $N_u - CS$ in $(\mathbb{P}, \tau_{N_u})$. Therefore, $f^{-1}(T)$ is not both $N_u - OS$ and $N_u - CS$ in $(\mathbb{P}, \tau_{N_u})$.

Theorem 4.6: Let $f : (\mathbb{P}, \tau_{N_u}) \rightarrow (\mathbb{Q}, \sigma_{N_u})$ be perfectly $N_{eqgsa^*}$ continuous if and only if the inverse image of every $N_{eqgsa^*} - OS$ in $(\mathbb{Q}, \sigma_{N_u})$ is both $N_u - OS$ and $N_u - CS$ in $(\mathbb{P}, \tau_{N_u})$.

Proof:
Assume that $f$ is perfectly $N_{eqgsa^*}$ - continuous function. Let $A$ be any $N_{eqgsa^*} - OS$ in $(\mathbb{Q}, \sigma_{N_u})$. Then $A^c$ is $N_{eqgsa^*} - CS$ in $(\mathbb{Q}, \sigma_{N_u})$. Since $f$ is perfectly $N_{eqgsa^*}$ - continuous, then $f^{-1}(A^c)$ is both $N_u - OS$ and $N_u - CS$ in $(\mathbb{P}, \tau_{N_u}) \Rightarrow f^{-1}(A^c)$ is both $N_u - OS$ and $N_u - CS$ in $(\mathbb{P}, \tau_{N_u})$. Conversely, let $A$ be any $N_{eqgsa^*} - CS$ in $(\mathbb{Q}, \sigma_{N_u})$. Then $A^c$ is $N_{eqgsa^*}$ - OS in $(\mathbb{Q}, \sigma_{N_u})$. By hypothesis, $f^{-1}(A^c)$ is both $N_u - OS$ and $N_u - CS$ in $(\mathbb{P}, \tau_{N_u}) \Rightarrow f^{-1}(A^c)$ is both $N_u - OS$ and $N_u - CS$ in $(\mathbb{P}, \tau_{N_u}) \Rightarrow f^{-1}(A)$ is both $N_u - OS$ and $N_u - CS$ in $(\mathbb{P}, \tau_{N_u})$. Therefore, $f$ is perfectly $N_{eqgsa^*}$ - continuous.

Theorem 4.7: Let $(\mathbb{P}, \tau_{N_u})$ be a neutrosophic discrete topological space and $(\mathbb{Q}, \sigma_{N_u})$ be any neutrosophic topological space. Let $f : (\mathbb{P}, \tau_{N_u}) \rightarrow (\mathbb{Q}, \sigma_{N_u})$ be a neutrosophic function, then the following statements are true.

1. $f$ is strongly $N_{eqgsa^*}$ - continuous.
2. $f$ is perfectly $N_{eqgsa^*}$ - continuous.

Proof:
1. $\Rightarrow (2)$, Let $A$ be any $N_{eqgsa^*} - CS$ in $(\mathbb{Q}, \sigma_{N_u})$. Since $f$ is strongly $N_{eqgsa^*}$ - continuous, then $f^{-1}(A)$ is $N_u - CS$ in $(\mathbb{P}, \tau_{N_u})$. Since $(\mathbb{P}, \tau_{N_u})$ is neutrosophic discrete topological space, then $f^{-1}(A)$ is $N_u - OS$ in $(\mathbb{P}, \tau_{N_u}) \Rightarrow f^{-1}(A)$ is both $N_u - OS$ and $N_u - CS$ in $(\mathbb{P}, \tau_{N_u})$. Therefore, $f$ is perfectly $N_{eqgsa^*}$ - continuous.

2. $\Rightarrow (1)$, Let $A$ be any $N_{eqgsa^*} - CS$ in $(\mathbb{Q}, \sigma_{N_u})$. Since $f$ is perfectly $N_{eqgsa^*}$ - continuous, then $f^{-1}(A)$ is both $N_u - OS$ and $N_u - CS$ in $(\mathbb{P}, \tau_{N_u}) \Rightarrow f^{-1}(A)$ is $N_u - CS$ in $(\mathbb{P}, \tau_{N_u})$. Therefore, $f$ is strongly $N_{eqgsa^*}$ - continuous.
Theorem 4.8: Let $f : (\mathbb{P}, \tau_\mathbb{P}) \rightarrow (\mathbb{Q}, \sigma_\mathbb{Q})$ be perfectly neutrosophic continuous and $(\mathbb{Q}, \sigma_\mathbb{Q})$ be $N_{e_u}gsa^*$ – $T_{1/2}$ space, then $f$ is perfectly $N_{e_u}gsa^*$ – continuous.

Proof:
Let $A$ be any $N_{e_u}gsa^*$ – CS in $(\mathbb{Q}, \sigma_\mathbb{Q})$. Since $(\mathbb{Q}, \sigma_\mathbb{Q})$ is $N_{e_u}gsa^*$ – $T_{1/2}$ space, then $A$ is $N_{e_u} – CS$ in $(\mathbb{Q}, \sigma_\mathbb{Q})$. Since $f$ is perfectly neutrosophic continuous, then $f^{-1}(A)$ is both $N_{e_u} – OS$ and $N_{e_u} – CS$ in $(\mathbb{P}, \tau_\mathbb{P})$. Therefore, $f$ is perfectly $N_{e_u}gsa^*$ – continuous.

Theorem 4.9: Let $f : (\mathbb{P}, \tau_\mathbb{P}) \rightarrow (\mathbb{Q}, \sigma_\mathbb{Q})$ and $g : (\mathbb{Q}, \sigma_\mathbb{Q}) \rightarrow (\mathbb{R}, \gamma_\mathbb{R})$ be perfectly $N_{e_u}gsa^*$ – continuous, then $gof : (\mathbb{P}, \tau_\mathbb{P}) \rightarrow (\mathbb{R}, \gamma_\mathbb{R})$ is perfectly $N_{e_u}gsa^*$ – continuous.

Proof:
Let $A$ be any $N_{e_u}gsa^*$ – CS in $(\mathbb{R}, \gamma_\mathbb{R})$. Since $g$ is perfectly $N_{e_u}gsa^*$ – continuous, then $g^{-1}(A)$ is both $N_{e_u} – OS$ and $N_{e_u} – CS$ in $(\mathbb{Q}, \sigma_\mathbb{Q})$. Since $g$ is perfectly neutrosophic continuous, then $f^{-1}(g^{-1}(A)) = (gof)^{-1}(A)$ is both $N_{e_u} – OS$ and $N_{e_u} – CS$ in $(\mathbb{P}, \tau_\mathbb{P})$. Therefore, $gof$ is perfectly $N_{e_u}gsa^*$ – continuous.

Theorem 4.10: Let $f : (\mathbb{P}, \tau_\mathbb{P}) \rightarrow (\mathbb{Q}, \sigma_\mathbb{Q})$ and $g : (\mathbb{Q}, \sigma_\mathbb{Q}) \rightarrow (\mathbb{R}, \gamma_\mathbb{R})$ be neutrosophic continuous and $g$ : $(\mathbb{Q}, \sigma_\mathbb{Q}) \rightarrow (\mathbb{R}, \gamma_\mathbb{R})$ be strongly $N_{e_u}gsa^*$ – continuous, then $gof : (\mathbb{P}, \tau_\mathbb{P}) \rightarrow (\mathbb{R}, \gamma_\mathbb{R})$ is strongly $N_{e_u}gsa^*$ – continuous.

Proof:
Let $A$ be any $N_{e_u}gsa^*$ – CS in $(\mathbb{R}, \gamma_\mathbb{R})$. Since $g$ is perfectly $N_{e_u}gsa^*$ – continuous, then $g^{-1}(A)$ is both $N_{e_u} – OS$ and $N_{e_u} – CS$ in $(\mathbb{Q}, \sigma_\mathbb{Q})$. Since $f$ is neutrosophic continuous, then $f^{-1}(g^{-1}(A)) = (gof)^{-1}(A)$ is both $N_{e_u} – OS$ and $N_{e_u} – CS$ in $(\mathbb{P}, \tau_\mathbb{P})$. Therefore, $gof$ is strongly $N_{e_u}gsa^*$ – continuous.

Theorem 4.11: Let $f : (\mathbb{P}, \tau_\mathbb{P}) \rightarrow (\mathbb{Q}, \sigma_\mathbb{Q})$ be perfectly $N_{e_u}gsa^*$ – continuous and $g : (\mathbb{Q}, \sigma_\mathbb{Q}) \rightarrow (\mathbb{R}, \gamma_\mathbb{R})$ be strongly $N_{e_u}gsa^*$ – continuous, then $gof : (\mathbb{P}, \tau_\mathbb{P}) \rightarrow (\mathbb{R}, \gamma_\mathbb{R})$ is perfectly $N_{e_u}gsa^*$ – continuous.

Proof:
Let $A$ be any $N_{e_u}gsa^*$ – CS in $(\mathbb{R}, \gamma_\mathbb{R})$. Since $g$ is strongly $N_{e_u}gsa^*$ – continuous, then $g^{-1}(A)$ is both $N_{e_u} – CS$ in $(\mathbb{Q}, \sigma_\mathbb{Q})$. Since $f$ is perfectly $N_{e_u}gsa^*$ – continuous, then $f^{-1}(g^{-1}(A)) = (gof)^{-1}(A)$ is both $N_{e_u} – OS$ and $N_{e_u} – CS$ in $(\mathbb{P}, \tau_\mathbb{P})$. Therefore, $gof$ is perfectly $N_{e_u}gsa^*$ – continuous.

5. Totally neutrosophic $gsa^*$ – continuous function

Definition 5.1: A neutrosophic function $f : (\mathbb{P}, \tau_\mathbb{P}) \rightarrow (\mathbb{Q}, \sigma_\mathbb{Q})$ is said to be totally $N_{e_u}gsa^*$ – continuous if the inverse image of every $N_{e_u} – CS$ in $(\mathbb{Q}, \sigma_\mathbb{Q})$ is both $N_{e_u}gsa^*$ – OS and $N_{e_u}gsa^*$ – CS (i.e., $N_{e_u}gsa^*$ – clopen set) in $(\mathbb{P}, \tau_\mathbb{P})$.

Definition 5.2: A neutrosophic topological space $(\mathbb{P}, \tau_\mathbb{P})$ is called a $N_{e_u}gsa^*$ – clopen set $(N_{e_u}gsa^* – clopen set)$ if it is both $N_{e_u}gsa^*$ – OS and $N_{e_u}gsa^*$ – CS in $(\mathbb{P}, \tau_\mathbb{P})$.
Also, $N_{eu\alpha-int}(\mathcal{f}^{-1}(B')) = 0_{N_{eu}}$. Now, $N_{eu\alpha-cl}(N_{eu\alpha-int}(\mathcal{f}^{-1}(B'))) = 0_{N_{eu}} \subseteq N_{eu} - cl(0_{N_{eu}}) = 0_{N_{eu}}$, whenever $\mathcal{f}^{-1}(B') \supseteq 0_{N_{eu}} \Rightarrow \mathcal{f}^{-1}(B')$ is $N_{eu}^{gsa^*} - OS$ in $(\mathbb{P}, \tau_{N_{eu}})$. Therefore, $f$ is totally $N_{eu}^{gsa^*} - continuous$.

**Theorem 5.4:** Every perfectly $N_{eu}^{gsa^*} - continuous$ is totally $N_{eu}^{gsa^*} - continuous$, but not conversely.

**Proof:**

Let $f : (\mathbb{P}, \tau_{N_{eu}}) \rightarrow (\mathbb{Q}, \sigma_{N_{eu}})$ be any neutrosophic function. Let $A$ be any $N_{eu} - CS$ in $(\mathbb{Q}, \sigma_{N_{eu}})$. Then $A$ is $N_{eu}^{gsa^*} - CS$ in $(\mathbb{Q}, \sigma_{N_{eu}})$. Since $f$ is perfectly $N_{eu}^{gsa^*} - continuous$, then $\mathcal{f}^{-1}(A)$ is both $N_{eu} - OS$ and $N_{eu} - CS$ in $(\mathbb{P}, \tau_{N_{eu}}) \Rightarrow \mathcal{f}^{-1}(A)$ is both $N_{eu}^{gsa^*} - OS$ and $N_{eu}^{gsa^*} - CS$ in $(\mathbb{P}, \tau_{N_{eu}})$. Therefore, $f$ is totally $N_{eu}^{gsa^*} - continuous$.

**Example 5.5:** Let $\mathbb{P} = \{p\}$ and $\mathbb{Q} = \{q\}$. $\tau_{N_{eu}} = \{0_{N_{eu}}, 1_{N_{eu}}, A\}$ and $\sigma_{N_{eu}} = \{0_{N_{eu}}, 1_{N_{eu}}, B\}$ are $N_{eu}$-TS on $(\mathbb{P}, \tau_{N_{eu}})$ and $(\mathbb{Q}, \sigma_{N_{eu}})$ respectively. Also $A = \{(p, (0.2, 0.4, 0.6))\}$ and $B = \{(q, (0.6, 0.8, 0.4))\}$ are $N_{eu}(\mathbb{P})$ and $N_{eu}(\mathbb{Q})$. Define a map $f : (\mathbb{P}, \tau_{N_{eu}}) \rightarrow (\mathbb{Q}, \sigma_{N_{eu}})$ by $f(p) = q$. Let $\mathcal{B} = \{(q, (0.4, 0.2, 0.6))\}$ be a $N_{eu} - CS$ in $(\mathbb{Q}, \sigma_{N_{eu}})$. Then $\mathcal{f}^{-1}(\mathcal{B}) = \{(p, (0.4, 0.2, 0.6))\}$. $N_{eu}\alpha - OS = N_{eu} - OS = \{0_{N_{eu}}, 1_{N_{eu}}, A\}$ and $N_{eu} - CS = \{0_{N_{eu}}, 1_{N_{eu}}, A\}$. $N_{eu}\alpha-cl\{(f^{-1}(B'))\} = A' \cap \mathcal{N}_{eu} = A'$. Now, $N_{eu}\alpha - int\{N_{eu}\alpha - cl\{(f^{-1}(B'))\}\} = A' \cup 0_{N_{eu}} = A \subseteq N_{eu} - int\{1_{N_{eu}} = 1_{N_{eu}} \Rightarrow f^{-1}(B')$ is $N_{eu}^{gsa^*} - CS$ in $(\mathbb{P}, \tau_{N_{eu}})$. Therefore, $f$ is totally $N_{eu}^{gsa^*} - continuous$. But $f$ is not perfectly $N_{eu}^{gsa^*} - continuous$. Let $T = \{(q, (0.3, 0.1, 0.8))\}$ be $N_{eu}^{gsa^*} - CS$ in $(\mathbb{Q}, \sigma_{N_{eu}})$. Then $\mathcal{f}^{-1}(T) = \{(p, (0.3, 0.1, 0.8))\}$. Now, $N_{eu} - int\{f^{-1}(T)\} = 0_{N_{eu}} \neq f^{-1}(T) \Rightarrow f^{-1}(T)$ is not $N_{eu} - OS$ in $(\mathbb{P}, \tau_{N_{eu}})$. Also, $N_{eu}\alpha-cl\{f^{-1}(T)\} = A' \neq f^{-1}(T) \Rightarrow f^{-1}(T)$ is not $N_{eu} - CS$ in $(\mathbb{P}, \tau_{N_{eu}})$. Therefore, $f^{-1}(T)$ is not both $N_{eu} - OS$ and $N_{eu} - CS$ in $(\mathbb{P}, \tau_{N_{eu}})$.

**Theorem 5.6:** Every totally $N_{eu}^{gsa^*} - continuous$ is $N_{eu}^{gsa^*} - continuous$.

**Proof:**

Let $f : (\mathbb{P}, \tau_{N_{eu}}) \rightarrow (\mathbb{Q}, \sigma_{N_{eu}})$ be any neutrosophic function. Let $A$ be any $N_{eu} - CS$ in $(\mathbb{Q}, \sigma_{N_{eu}})$. Since $f$ is totally $N_{eu}^{gsa^*} - continuous$, then $f^{-1}(A)$ is both $N_{eu}^{gsa^*} - OS$ and $N_{eu}^{gsa^*} - CS$ in $(\mathbb{P}, \tau_{N_{eu}}) \Rightarrow f^{-1}(A)$ is $N_{eu}^{gsa^*} - CS$ in $(\mathbb{P}, \tau_{N_{eu}})$. Therefore, $f$ is $N_{eu}^{gsa^*} - continuous$.

**Example 5.7:** Let $\mathbb{P} = \{p\}$ and $\mathbb{Q} = \{q\}$. $\tau_{N_{eu}} = \{0_{N_{eu}}, 1_{N_{eu}}, A\}$ and $\sigma_{N_{eu}} = \{0_{N_{eu}}, 1_{N_{eu}}, B\}$ are $N_{eu}$-TS on $(\mathbb{P}, \tau_{N_{eu}})$ and $(\mathbb{Q}, \sigma_{N_{eu}})$ respectively. Also $A = \{(p, (0.7, 0.6, 0.5))\}$ and $B = \{(q, (0.7, 0.8, 0.3))\}$ are $N_{eu}(\mathbb{P})$ and $N_{eu}(\mathbb{Q})$. Define a map $f : (\mathbb{P}, \tau_{N_{eu}}) \rightarrow (\mathbb{Q}, \sigma_{N_{eu}})$ by $f(p) = q$. Let $\mathcal{B} = \{(q, (0.3, 0.2, 0.7))\}$ be a $N_{eu} - CS$ in $(\mathbb{Q}, \sigma_{N_{eu}})$. Then $f^{-1}(B') = \{(p, (0.3, 0.2, 0.7))\}$. $N_{eu}^{gsa^*} - OS = N_{eu}^{gsa^*} - CS = \{0_{N_{eu}}, 1_{N_{eu}}, A, D\}$, and $N_{eu} - CS = \{0_{N_{eu}}, 1_{N_{eu}}, A', E\}$, where $D = \{(p, (0.7, 1), [0.6, 1], [0.5], 0), E = \{(p, ([0, 0.5], [0, 0.4], [0, 0.1]), 0\}$. $N_{eu} - cl\{f^{-1}(B')\} = A' \cap F \cap 1_{N_{eu}} = F$, where $J = \{(p, (0.3, 0.5), [0.2, 0.4], 0.7)\}$. Now, $N_{eu} - int\{N_{eu} - cl\{f^{-1}(B')\}\} = 0_{N_{eu}} \subseteq N_{eu} - int\{A, 1_{N_{eu}}, D\} - N_{eu} - int\{1_{N_{eu}} = A, 1_{N_{eu}}, D\} - N_{eu} - int\{1_{N_{eu}} = A, 1_{N_{eu}}, D\} - N_{eu} - int\{1_{N_{eu}} = A, 1_{N_{eu}}, D\} - N_{eu} - int\{1_{N_{eu}} = A, 1_{N_{eu}}, D\}$. $N_{eu} - cl\{f^{-1}(B')\} = A' \cap J$, whenever $f^{-1}(B') \subseteq A, 1_{N_{eu}} \Rightarrow f^{-1}(B')$ is $N_{eu}^{gsa^*} - CS$ in $(\mathbb{P}, \tau_{N_{eu}})$. Therefore, $f$ is $N_{eu}^{gsa^*} - continuous$. But $f$ is not totally $N_{eu}^{gsa^*} - continuous$, because $f^{-1}(B')$ is not $N_{eu}^{gsa^*} - OS$ in $(\mathbb{P}, \tau_{N_{eu}})$. Since $N_{eu} - cl\{N_{eu} - int\{f^{-1}(B')\}\} = 0_{N_{eu}} \not\subseteq N_{eu} - cl\{f_{p}\} = A'$, whenever $f^{-1}(B') \supseteq J$, where $J = \{(p, ([0, 0.3], [0, 0.2], [0, 0.7], 1))\}$ implies $f^{-1}(B')$ is not $N_{eu}^{gsa^*} - OS$ in $(\mathbb{P}, \tau_{N_{eu}})$. 
Inter-relationship 5.8:

\[ \text{Perfectly Perfectly} \]
\[ N_{eu gsa^*} - \text{continuous} \quad N_{eu gsa^*} - \text{continuous} \]
\[ \quad \downarrow \]
\[ N_{eu gsa^*} - \text{continuous} \]

**Theorem 5.9:** Let \( f : (\mathbb{P}, \tau_{N_m}) \to (\mathbb{Q}, \sigma_{N_m}) \) be totally \( N_{eu gsa^*} \) - continuous and \((\mathbb{Q}, \sigma_{N_m}) \) be \( N_{eu gsa^*} - T_{1\frac{1}{2}} \) space, then \( f \) is \( N_{eu gsa^*} \) - irresolute.

**Proof:**

Let \( A \) be any \( N_{eu gsa^*} - \text{CS in } (\mathbb{Q}, \sigma_{N_m}) \). Since \((\mathbb{Q}, \sigma_{N_m}) \) is \( N_{eu gsa^*} - T_{1\frac{1}{2}} \) space, then \( A \) is \( N_{eu} - \text{CS in } (\mathbb{Q}, \sigma_{N_m}) \). Since \( f \) is totally \( N_{eu gsa^*} \) - continuous, then \( f^{-1}(A) \) is both \( N_{eu gsa^*} - \text{OS and } N_{eu gsa^*} - \text{CS in } (\mathbb{P}, \tau_{N_m}) \) \( f^{-1}(A) \) is \( N_{eu gsa^*} - \text{CS in } (\mathbb{P}, \tau_{N_m}) \). Therefore, \( f \) is \( N_{eu gsa^*} \) - irresolute.

**Theorem 5.10:** Let \( f : (\mathbb{P}, \tau_{N_m}) \to (\mathbb{Q}, \sigma_{N_m}) \) and \( g : (\mathbb{Q}, \sigma_{N_m}) \to (\mathbb{R}, \gamma_{N_m}) \) be totally \( N_{eu gsa^*} \) - continuous and \((\mathbb{Q}, \sigma_{N_m}) \) be \( N_{eu gsa^*} - T_{1\frac{1}{2}} \) space, then \( g \circ f : (\mathbb{P}, \tau_{N_m}) \to (\mathbb{R}, \gamma_{N_m}) \) is totally \( N_{eu gsa^*} \) - continuous.

**Proof:**

Let \( A \) be any \( N_{eu} - \text{CS in } (\mathbb{R}, \gamma_{N_m}) \). Since \( g \) is totally \( N_{eu gsa^*} \) - continuous, then \( g^{-1}(A) \) is both \( N_{eu gsa^*} - \text{OS and } N_{eu gsa^*} - \text{CS in } (\mathbb{Q}, \sigma_{N_m}) \). Since \((\mathbb{Q}, \sigma_{N_m}) \) is \( N_{eu gsa^*} - T_{1\frac{1}{2}} \) space, then \( g^{-1}(A) \) is \( N_{eu} - \text{CS in } (\mathbb{Q}, \sigma_{N_m}) \). Since \( f \) is totally \( N_{eu gsa^*} - \text{continuous, then } f^{-1}(g^{-1}(A)) = (g \circ f)^{-1}(A) \) is both \( N_{eu gsa^*} - \text{OS and } N_{eu gsa^*} - \text{CS in } (\mathbb{P}, \tau_{N_m}) \). Therefore, \( g \circ f \) is totally \( N_{eu gsa^*} \) - continuous.

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