Associated production of Higgs boson with a photon at electron-positron colliders

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A complete one-loop prediction for the single production of the neutral Higgs bosons in association with a photon in electron-positron collisions is presented in the framework MSSM, paying special attention to the individual contribution from each type of diagram. This process has no amplitude at tree level and is hence directly sensitive to one-loop impacts and the underlying dynamics of Higgs. In order to investigate the effect of the new physics, four different scenarios, which include a Higgs boson with mass and couplings consistent with those of the discovered Higgs boson and a considerable part of parameter space allowed by the bounds from the searches for additional Higgs bosons and sparticles, are chosen in the MSSM. The dependence of the cross section in both SM and MSSM on the center-of-mass energy is examined by considering the polarizations of the initial electron and positron beams. The effect of individual contributions from each type of one-loop diagrams on the total cross section is also investigated in detail. Furthermore, the total cross section of $e^-e^+ \rightarrow \gamma h^0$ as well as $e^-e^+ \rightarrow \gamma A^0$ are scanned over the plane $m_A - \tan \beta$ for each scenario. The full one-loop contributions are crucial for the analysis of BSM physics at a future electron-positron collider.

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I. INTRODUCTION

Despite its many success, the Standard Model (SM) leaves us with a lot of questions to be answered, such as the hierarchy problem, the origin of flavor, etc. Since the discovery of the Higgs boson at the LHC [1, 2] and until the date it has not yet been found any evidence of new physics beyond the SM (BSM). However, the observations of neutrino oscillations, matter-antimatter asymmetry, relic density of dark matter (DM), and so on, open the door to new physics BSM. Additionally, there are strong motivations to extend the scalar sector of the SM by introducing more than one Higgs doublet. Therefore, it seems compulsory the development of new attempts concentrated on the research of data which provide a hint about new physical degrees of freedom. This is the main goal of proposals at future $e^+e^-$ colliders such as the International Linear Collider (ILC) [3–5], Compact Linear Collider (CLIC) [5, 6], Circular Electron-Positron Collider (CEPC) [7] and Future Circular Collider (FCC) [8]. On the other hand, they are mainly designed to provide a high precision and complete picture of the Higgs boson and its couplings. The $e^+e^-$ colliders compared to the hadron colliders have a cleaner background, and hence the new physics signals are easily separated from the background. The ILC is one of the most developed linear collider planned to be a Higgs factory in the centre of mass energies of $\sqrt{s} = 250 - 500$ GeV (extendable up to a 1 TeV). The CLIC is a TeV-scale high luminosity linear collider planned to be operated at centre-of-mass energies of $\sqrt{s} = 380$ GeV, 1.5 TeV and 3 TeV. The CEPC collider with a circumference of 100 km is designed to operate at $\sqrt{s} = 240$ GeV. The potential for CEPC to probe a suite of loop-level corrections to Higgs and electroweak observables in supersymmetric models is comprehensively studied in [9].

Even at $\sqrt{s} = 250$ GeV with a total integrated luminosity of 2 ab$^{-1}$ for the electron-positron collider, there are some suggests to accurately determine the couplings of Higgs boson to gauge bosons, leptons and quarks [10, 11] with an accuracy of order one percent compared the 0.2% accuracy based on the SM predicted couplings in terms of $m_A$. That amplified precision may allow detecting the small deviations for BSM scenarios. A very precise prediction of Higgs boson production involving additional interactions which come from BSM scenarios can provide significant hints about new physics. There are many important motivations to choose the Minimal Supersymmetric Standard Model (MSSM) as BSM-scenario that could identify these new interactions.

The MSSM [12–15], one of the most attractive and widely considered extensions of SM, keeps the number of new fields and couplings to a minimum. It provides a solution for the hierarchy problem of the SM, offers a candidate for the DM postulated to explain astrophysical observations, and a prediction for the mass of the scalar resonance observed at the LHC. The MSSM has two Higgs doublets, which leads to a physical spectrum that include a couple of charged Higgs bosons $H^\pm$, a CP-odd Higgs boson $A^0$, and the light/heavy CP-even Higgs bosons $h^0/H^0$ in the CP conserving case. In Higgs sector of MSSM, all couplings and masses at tree-level can be described by only two parameters: the mass of pseudoscalar Higgs boson $m_{A^0}$, and the ratio of the vacuum expectation values of the two doublets, $\tan \beta$. The discovered Higgs with mass of around 125 GeV could be interpreted naturally as one of the two neutral CP-even Higgs bosons in the MSSM [16–18]. Moreover, many new particles in the MSSM are predicted such as scalar lep-

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The associated Higgs production with a photon, \( e^- e^+ \rightarrow γ h^0 \) is well suited to study the Higgs to neutral gauge boson couplings such as the \( hγγ \) and \( hZγ \) couplings. Future \( e^+ e^- \) colliders are optimal for studying \( e^- e^+ \rightarrow hγ \), where the cross section in the SM has a peak about \( \sqrt{s} = 250 \) GeV [21, 22]. Because the tree-level contribution of the process is highly suppressed by the electron mass and the process is protected by the electromagnetic gauge symmetry, it occurs at the one-loop level for the first time. Therefore, the visible size of cross section is an order of magnitude of \( 10^{-1} \) fb at \( \sqrt{s} = 250 \) GeV, which is rather small. However, since the signal is very clean, it can be observed at the future \( e^+ e^- \) colliders with the design luminosity. Furthermore, new physics contributions can considerably amplify the rate of production relative to the SM case; namely, the process is potentially very sensitive to new physics. There are few studies dedicated to the investigations of new physics effects on the process in the framework of an effective field theory or anomalous Higgs-boson couplings [23, 24] as well as the extended Higgs models (inert doublet/triplet model, and two Higgs doublet model (THDM)) [25, 26] and the MSSM [22, 27–29]. However, owing to the most recent constraints on the parameter space of the MSSM from Run 2 of the LHC, the size of the MSSM contributions to \( e^- e^+ \rightarrow hγ \) should be reevaluated in the allowed-parameter space.

In this work, the single production of the neutral Higgs bosons in association with a photon in electron-positron collisions is reinvestigated in the framework MSSM, paying a special attention to the individual contribution from each type of diagram. In this aim, it is examined that how and how much the individual contributions from each type of diagrams could amplify or lessen the \( h^0\) signal at a future \( e^- e^+ \) collider. For this aim, four different benchmark scenarios, which have a Higgs boson with mass and couplings consistent with those of the discovered Higgs boson and a considerable part of their parameter space is allowed by the bounds from the searches for additional Higgs bosons and supersymmetric particles, are chosen. These scenarios are named as \( m_h^{125} \), \( m_h^{125}(\text{light } \tilde{\tau}) \), \( m_h^{125}(\text{light } \tilde{χ}) \) and \( m_h^{125}(\text{alignment}) \) [30, 31]. Distributions for the total cross sections are computed as a function of the center-of-mass energy and the polarization of the incoming beams. Furthermore, the total cross section of both \( e^- e^+ \rightarrow γ h^0 \) and \( e^- e^+ \rightarrow γ A^0 \) are scanned over the plane \( m_A - \tan β \) for each scenario, and regions in which the cross section is large enough to be detectable at a future \( e^- e^+ \) collider, are determined in this paper.

The contents of the present work are the following: Section II provides the corresponding Feynman diagrams and the analytical expressions related to the process \( e^- e^+ \rightarrow hγ \). This section then gives information on how the numerical evaluation is done. Section III provides details of the considered benchmark scenarios. In Sec. IV, numerical results are presented and the some parameter dependencies of the cross section are discussed in detail. Finally, Section V presents the conclusions of the present study.

II. THEORETICAL FRAMEWORK

The associated production of single Higgs boson with a photon in an electron-positron collision is indicated by

\[
e^+(p_1)e^-(p_2) \rightarrow h(k_1)γ(k_2),
\]

(2.1)

where after each particle, as usual, its 4-momenta is written in parentheses. The Mandelstam variables can be written as

\[
s = (p_1 + p_2)^2, \quad t = (p_1 - k_1)^2, \quad u = (p_1 - k_2)^2.
\]

(2.2)

At tree level, the process occurs via \( t \)-channel electron-exchange diagram suppressed by the mass of electron. So the tree-level amplitude of the process is neglected, i.e., for the first time, the process is mediated by one-loop diagrams, and hence it is sensitive to all virtual particles inside the loop.

The total amplitude of one-loop level can be written as a linear sum of box, triangle, and bubble one-loop integrals. According to the type of loop-correction, the one-loop diagrams contributing to the process \( e^- e^+ \rightarrow hγ \) can be classified into four different types: the box-type, self-energy, quartic coupling-type, triangle-type diagrams. A complete set of one-loop Feynman diagrams and the corresponding amplitudes in the SM and MSSM are generated by the FeynArts [34]. For MSSM, the diagrams* are shown in Figure 1 to Figure 3. Since self-energy diagrams consist of loop corrections to the electron, and hence are highly suppressed by the electron mass, they are not explicitly shown here. Moreover, it is also possible another set of diagrams where particles in each loop are running counterclockwise. The bracket [...] represents that all possible combinations of the particles in the bracket can be written. In the Feynman diagrams, the label \( f_m \) (\( \tilde{f}_m \)) refers to fermions (scalar fermions) \( e_m, u_m, d_m, (\tilde{e}_m, \tilde{u}_m, \tilde{d}_m) \) and the label \( S^0 \) represents all neutral Higgs/Goldstone bosons \( h^0, H^0, A^0, G^0 \). The indexes \( m \) and \( x \) represent the generation of (scalar)quark and the scalar-quark mass eigenstates, respectively. In loop diagrams, scalar particles such as neutral

* The diagrams were drawn by using JaxoDraw [32, 33].
and charged Higgs/Goldstone bosons, sfermions, are denoted by dashed-lines, and γ, Z, W bosons are denoted by wavy-lines.

Figure 1 shows all possible box-type contributions, which have the loops of neutrino, electron, selectron, charginos \( \tilde{\chi}_{1,2}^{\pm} \), neutralinos \( \tilde{\chi}_{1,2,3,4}^{0} \), neutral Higgs bosons \( h^0, H^0, A^0, G^0 \), Z-boson, W-boson and charged Higgs/Goldstone bosons \( H^{\pm}, G^{\pm} \). Figure 2 shows all quartic coupling diagrams which consist of bubble \((q_1-7)\) attached to the initial state via an intermediate \( \gamma \) or Z or neutral Higgs bosons \( (h^0, H^0, A^0, G^0) \), and triangle loop \((q_8)\) of the neutrino \( \nu_e \) attached to the initial state via an intermediate \( \gamma \) or Z. Finally, Figure 3 shows all triangle-type contributions which consist of triangle-vertices \((t_1-7)\) attached to the initial state via an intermediate \( \gamma \) or Z or neutral Higgs bosons, and also include triangle-corrections to the top/bottom vertex. Feynman diagrams \( q_1-7 \) and \( t_1-6 \) are s-channel diagrams. Owing to the intermediate neutral Higgs bosons, the resonant effects could be observed at the triangle-type \((t_1-6)\) and bubble-type \((q_1-7)\) diagrams for some specific center of mass energy.

In this study, the process \( e^+e^- \rightarrow A^0\gamma \) is also examined. Due to the CP nature of the pseudoscalar Higgs boson \( A^0 \), the process \( e^+e^- \rightarrow A^0\gamma \) has no W and Z-contributions in the box-type diagrams, and no contribution from Z-boson, W-boson, \( f_m^2 \) and \( H^\pm \) in the triangle-type and bubble-type diagrams, compared to the process \( e^+e^- \rightarrow h^\pm \gamma \). Therefore, the process \( e^+e^- \rightarrow A^0\gamma \) occurs only via s-channel triangle-type diagrams which involve loops of fermions and charginos, as well as t-channel triangle-type and box-type diagrams which involve loops of sneutrino/chargino and selectron/neutralino.
The Higgs sector of MSSM at the tree level is described by two parameters, \( \tan \beta \) and a mixing angle \( \alpha \) in the CP-even Higgs sector. Furthermore, the angle \( \alpha \) could be also given in terms of \( m_A \) and \( \tan \beta \) as follows:

\[
\tan(2\alpha) = \tan(2\beta) \frac{m_A^2 + m_Z^2}{m_A^2 - m_Z^2} = \frac{\pi}{2} < \alpha < 0. \tag{2.3}
\]

Once \( m_A \) and \( \tan \beta \) are given and the leading radiative correction is involved in \( \alpha \), all the couplings of the Higgs bosons to gauge bosons, fermions, and Higgs bosons are fixed. Table 1 lists the couplings of the neutral Higgs boson to gauge bosons and Higgs bosons which are included in each type of diagrams for the process \( e^+ e^- \rightarrow h^0 \gamma \). The Feynman diagrams are dominated by triple couplings \( \lambda_{h^0G^+H^0} \), \( \lambda_{h^0H^0 - W^+} \), \( \lambda_{h^0G^+ - W^-} \), and \( \lambda_{h^0G^- - G^-} \) which are proportional to mixing angles \( \cos(\beta - \alpha) \) and \( \sin(\beta - \alpha) \). Some couplings of Higgs boson are given by

\[
\lambda_{h^0H^0}^{MSSM} = \frac{3igm_{W}}{2c_{W}} c_{2\alpha} s_{\beta + \alpha}, \tag{2.4}
\]

\[
\lambda_{h^0G^+H^0}^{MSSM} = \frac{igm_{W}}{2c_{W}} \left[ c_{2\alpha} c_{\alpha + \beta} - 2 s_{2\alpha} s_{\alpha + \beta} \right], \tag{2.5}
\]

\[
\lambda_{h^0h^0H^0}^{MSSM} = -\frac{igm_{W}}{2} \left[ \frac{c_{2\alpha} s_{\beta + \alpha}}{c_{W}^2} + 2 s_{\beta - \alpha} \right], \tag{2.6}
\]

where the gauge coupling constant \( g = e/s_{W} \) and \( m_W \) is the mass of \( W \) boson. All these couplings have a strong dependence on the mixing angles \( \alpha \) and \( \beta \). In this study,

\[
\lambda_{h^0H^0 - H^0}^{MSSM} = \frac{igm_{W}}{2} \left[ \frac{c_{2\alpha} s_{\beta + \alpha}}{c_{W}^2} + 2 s_{\beta - \alpha} \right], \tag{2.7}
\]

\[
\lambda_{h^0G^+ - G^+}^{MSSM} = \frac{igm_{W}}{2} c_{2\beta s_{\alpha + \beta}}, \tag{2.8}
\]

\[
\lambda_{h^0H^- - W^+}^{MSSM} = \frac{igm_{W}}{2} c_{2\beta s_{\alpha + \beta}}, \tag{2.9}
\]

\[
\lambda_{h^0H^+ - W^-}^{MSSM} = -\frac{igm_{W}}{2} c_{2\beta s_{\alpha + \beta}}, \tag{2.10}
\]

\[
\lambda_{h^0H^0 - W^+}^{MSSM} = \frac{igm_{W}}{2} s_{\beta - \alpha}, \tag{2.11}
\]

\[
\lambda_{h^0H^0 - H^+}^{MSSM} = \frac{igm_{W}}{2} \left[ \frac{c_{2\alpha} s_{\beta + \alpha}}{c_{W}^2} - s_{2\alpha} s_{23} \right], \tag{2.12}
\]

\[
\lambda_{h^0H^- - W^-}^{MSSM} = \frac{igm_{W}}{2} s_{\beta - \alpha}, \tag{2.13}
\]

\[
\lambda_{h^0H^+ - W^-}^{MSSM} = \frac{igm_{W}}{2} s_{\beta - \alpha}. \tag{2.14}
\]
in particular, triple Higgs couplings and couplings of the scalar to gauge bosons are of interest.

Since the tree-level amplitudes of process $e^+e^- \rightarrow h^0\gamma$ suppressed by the electron mass are neglected and hence the process has only one-loop contributions as the lowest order, its one-loop amplitude can be easily obtained by summing all unrenormalized reducible and irreducible contributions. Consequently, the finite and gauge invariant results are obtained. Furthermore, the total amplitude is ultraviolet finite and this has been checked both numerically and analytically.

The corresponding total amplitude can be given in the form

$$\mathcal{M} = \mathcal{M}_\triangle + \mathcal{M}_\Box + \mathcal{M}_\diamond$$

as a sum over all contributions from triangle-, box- and quartic-type diagrams. The differential cross section of the process, summing over the polarization of the photon, can be calculated by

$$\frac{d\sigma}{d\cos\theta}(e^+e^- \rightarrow h^0\gamma) = \frac{s - m_h^2}{32\pi s^2} \sum_{\text{pol}} |\mathcal{M}|^2,$$

where $\sqrt{s}$ are the center-of-mass energy of $e^+e^-$ collisions and $\theta$ is the scattering angle between the photon and the electron in the centre-of-mass frame. The integrated cross section over all $\theta$ angles is given by

$$\sigma(e^+e^- \rightarrow h^0\gamma) = \int_{-1}^{+1} d\cos\theta \frac{d\sigma}{d\cos\theta}.$$  \hspace{1cm} (2.17)

At an $e^+e^-$ collider, the photon in association with the Higgs is produced as monochromatic with an energy of

$$E_{\gamma} = \frac{s - m_h^2}{2\sqrt{s}}.$$  \hspace{1cm} (2.18)

At $\sqrt{s} = 250$ GeV, this gives a “spectral-line” at $E_{\gamma} = 93.75$ GeV. The signal is easy to separate from the backgrounds.

With the help of FeynArts\cite{34} and FormCalc\cite{35} packages\footnote{Using the same tools, we have previously done a few more recent studies \cite{38-40} and achieved significant results.}, the numerical calculation is carried out in the 't Hooft-Feynman gauge using dimensional regularization. The corresponding amplitudes are generated by FeynArts. The analytical results of the squared amplitude are provided by FormCalc. The scalar integrals in

\begin{table}[h]
\centering
\begin{tabular}{|c|c|c|c|c|c|}
\hline
Couplings & Box-type & Triangle-type & Bubble-type & Quartic-type \\
\hline
$\lambda_{h^0\mu^+\mu^-}$ & $\checkmark$ & $\checkmark$ & $\checkmark$ & $\checkmark$ \\
$\lambda_{h^0\mu^+\mu^-}$ & $\checkmark$ & $\checkmark$ & $\checkmark$ & $\checkmark$ \\
$\lambda_{h^0\mu^+\mu^-}$ & $\checkmark$ & $\checkmark$ & $\checkmark$ & $\checkmark$ \\
$\lambda_{h^0\mu^+\mu^-}$ & $\checkmark$ & $\checkmark$ & $\checkmark$ & $\checkmark$ \\
$\lambda_{h^0\mu^+\mu^-}$ & $\checkmark$ & $\checkmark$ & $\checkmark$ & $\checkmark$ \\
$\lambda_{h^0\mu^+\mu^-}$ & $\checkmark$ & $\checkmark$ & $\checkmark$ & $\checkmark$ \\
$\lambda_{h^0\mu^+\mu^-}$ & $\checkmark$ & $\checkmark$ & $\checkmark$ & $\checkmark$ \\
$\lambda_{h^0\mu^+\mu^-}$ & $\checkmark$ & $\checkmark$ & $\checkmark$ & $\checkmark$ \\
$\lambda_{h^0\mu^+\mu^-}$ & $\checkmark$ & $\checkmark$ & $\checkmark$ & $\checkmark$ \\
$\lambda_{h^0\mu^+\mu^-}$ & $\checkmark$ & $\checkmark$ & $\checkmark$ & $\checkmark$ \\
$\lambda_{h^0\mu^+\mu^-}$ & $\checkmark$ & $\checkmark$ & $\checkmark$ & $\checkmark$ \\
\hline
\end{tabular}
\end{table}
loop amplitudes are evaluated by LoopTools [36]. The integration over phase space of $2 \to 2$ is numerically evaluated by using CUBA library. The properties of MSSM Higgs bosons are obtained by using FeynHiggs [37].

The polarisation effects are significant at electron-positron colliders and can be used to confer important advantages. In this study, the effect of beam polarisations on cross section is also analyzed. Since the electron and positron have only 2 spin-states, the cross section for general beam polarisations is defined by [11]

$$\sigma_{p_e-p_{e^+}} = \frac{1}{4} \left[(1 - P_{e^-})(1 - P_{e^+})\sigma_{LL} + (1 + P_{e^-})(1 + P_{e^+})\sigma_{RR} + (1 - P_{e^-})(1 + P_{e^+})\sigma_{LR} + (1 + P_{e^-})(1 - P_{e^+})\sigma_{RL}\right],$$

where $P_{e^-}/P_{e^+}$ indicates to the longitudinal polarisations of the initial electron/positron beam, equals to $-1 (+1)$ for completely polarised left(right)-handed beam. $\sigma_{LL}, \sigma_{RR}, \sigma_{LR}$ and $\sigma_{RL}$ indicate the cross sections with completely polarised beams of the 4 possible cases. At s-channel $e^-e^+$ annihilation processes, only $\sigma_{LR}$ and $\sigma_{RL}$ are nonzero under condition of helicity conservation. The intrinsic left-right asymmetry of the cross section can be calculated with

$$A_{LR} = \frac{\sigma_{LR} - \sigma_{RL}}{\sigma_{LR} + \sigma_{RL}}$$

for a given process.

For the purpose of discussing the expected accuracy of future measurements, the centre of mass energies, integrated luminosities $\mathcal{L}$ and beam polarisations, proposed at future electron-positron colliders are presented in Table II.

### TABLE II. The centre of mass energies $\sqrt{s}$, integrated luminosities $\mathcal{L}$ and beam polarisations $P$, proposed at the future $e^-e^+$ colliders ILC [5], CLIC [6], CEPC [7] and FCC-ee [8].

| Collider | $\sqrt{s}$ [GeV] | $\mathcal{L}$ [ab$^{-1}$] | $(p_e^-,-p_{e^+})$ [%] |
|----------|------------------|----------------|------------------|
| ILC250   | 250              | 2.0            | (60%, ±30%)      |
| ILC500   | 250 + 500        | 2.0            | (60%, ±30%)      |
| CLIC380  | 380              | 1.0            | (60%, ±30%)      |
| CLIC1500 | 380 + 1500       | 2.5            | (60%, ±30%)      |
| CLIC5000 | 380 + 1500 + 3000| 5.0            | (60%, ±30%)      |
| CEPC     | 240              | 5.0            | (60%, ±30%)      |
| FCC-ee240| 240              | 5.0            | (60%, ±30%)      |
| FCC-ee385| 240 + 365        | 6.5            | (60%, ±30%)      |

### III. DEFINITION OF THE BENCHMARK SCENARIOS IN MSSM

This section provides details of the benchmark scenarios considered in the present study. The four different benchmark scenarios, which are called as $m_{h^{25}}$, $m_{h^{25}}^{(light}\tau$), $m_{h^{25}}^{(light}\tilde{\chi}$) and $m_{h^{25}}^{(alignment)}$ proposed in Refs. [30, 31], are used to illustrate the effect of the new physics based on MSSM. Note that all of the considered scenarios include a CP-even Higgs boson with mass about 125 GeV and couplings consistent with those of the discovered Higgs boson and a considerable part of their parameter space is allowed by the bounds from the researches for additional Higgs bosons and supersymmetric particles. In particular, they are consistent with the most recent experimental results from the LHC Run 2. The lighter CP-even Higgs boson $h^{0}$ of MSSM is SM-like in all scenario. In each scenario, two free parameters are left: $\tan\beta$ and $m_{A}$. Hence, for each scenario the cross section can be presented in plane of $m_{A} - \tan\beta$. The parameters $\tan\beta$ and $m_{A}$ are varying in the range of 100 GeV $\leq m_{A} \leq 2$ TeV and $0.5 \leq \tan\beta \leq 50$ for the first three scenarios, and 200 GeV $\leq m_{A} \leq 2$ TeV and $1 \leq \tan\beta \leq 20$ for the last scenario.

Considering the bounds placed on SUSY parameters from current experimental results, especially the direct SUSY research results from ATLAS [41, 42] and CMS experiments [43-45], a common soft SUSY-breaking parameter is fixed as $M_{\tilde{f}} = 2$ TeV for the 1st and 2nd-generations in the slepton and squark sector. The remaining parameters, which are the gaugino mass parameters $M_{1}$, $M_{2}$ and $M_{3}$; the Higgsino mass parameter $\mu$, the 3rd generation slepton mass parameters $M_{\tilde{f}}$, and $M_{\tilde{f}}^{\mu}$; the 3rd-generation squark mass parameters $M_{\tilde{q}_{1,2,3}}$; $M_{\tilde{u}_{3}}$ and $M_{\tilde{d}_{3}}$; the 3rd-generation the trilinear couplings $A_{t,b,\tau}$ are separately determined for each scenario. However, in the first three scenarios, the parameter $X_{t} = A_{t} - \mu \cot\beta$ is set as input parameter instead of the parameter $A_{t}$. In this study, for CP-conserving MSSM, all SUSY input parameters are chosen to be real and positive. We give a list of the input parameters for each scenario in Table III.

### TABLE III. The input parameters for each scenario, where all masses are given in TeV. The symbol $*$ means that $A_{t}$ is taken as $A_{t} = X_{t} + \mu \cot\beta$. In the last two rows, the values of $\tan\beta$ and $m_{A}$ are given for benchmark points corresponding to each scenario.

| $m_{h^{25}}$ | $m_{h^{25}}^{(light}\tau$ | $m_{h^{25}}^{(light}\tilde{\chi}$ | $m_{h^{25}}^{(alignment)}$ |
|-------------|----------------|----------------|----------------|
| $m_{1}$     | 1.5            | 1.5            | 2.5            |
| $m_{2}$     | 2.0            | 0.350          | 2.0            |
| $m_{3}$     | 1.0            | 0.180          | 0.180          |
| $A_{t}$     | 2.5            | 2.5            | 2.5            |
| $X_{t}$     | 2.8            | 2.8            | 2.5            |
| $A_{t}$     | 0.800          | 0.800          | 6.25           |

| $\tan\beta$ | $m_{A}$ |
|-------------|---------|
| 10          | 10      |
| 1.5         | 1.5     |
| 0.5         |         |

In the $m_{h^{25}}$ scenario, all sparticles are relatively heavy,
hence they have only mildly effect on productions and decays of the Higgs bosons. Therefore, the phenomenology of this scenario is similar to that of a type-2 of THDM with MSSM-inspired couplings of Higgs. The masses of the gluino and 3rd-generation squarks are allowed by the available bounds from direct researches at the LHC. In the $m_{h}^{125}(\text{light } \tilde{\tau})$ and $m_{h}^{125}(\text{light } \tilde{\chi})$ scenarios, the colorless sparticles (staus, and in one case, neutralinos and charginos) are relatively light, and the LSP is the lightest neutralino. The masses of gluino, sbottom and stop are the same as in the $m_{h}^{125}$ scenario, but trilinear interaction term for the staus and the stop mixing parameter $X_{i}$ are reduced in the $m_{h}^{125}(\text{light } \tilde{\tau})$ and $m_{h}^{125}(\text{light } \tilde{\chi})$ scenarios, respectively. These two scenarios can be a considerably effective on the Higgs phenomenology (see e.g. Refs. [39, 46]) via loop contributions to the couplings of Higgs boson to particles of SM, as well as via direct decays of the Higgs bosons into sparticles if kinematically possible [31]. At low values of $\tan \beta$, the $m_{h}^{125}(\text{align})$ scenario is defined by alignment without decoupling. In order to obtain an acceptable prediction for $m_{h}$ as well as to achieve alignment without decoupling, in the $m_{h}^{125}(\text{align})$ scenario the parameters determining the stop masses are remarkably larger values than in the other scenarios. For each scenario, taking both theory and experimental uncertainties into account, the impact on the parameter space of the available constraints from Higgs researches at LHC, Tevatron and LEP have been investigated in Ref. [31]. In this study, it is chosen four benchmark points (BPs) which are compatible with the most recent results of the LHC for the bounds on masses and couplings of new particles, and the Higgs-boson properties. These are checked by the HiggsBounds [47] and HiggsSignals [48] with results of 86 analyses.

Moreover, the input parameters for SM are fixed as $m_{h} = 125.09$ GeV, $m_{W} = 80.385$ GeV, $m_{Z} = 91.1876$ GeV, $m_{\text{pole}} = 172.5$ GeV, $m_{h}(m_{h}) = 4.18$ GeV, $\alpha^{-1} = 137.036$, and $\alpha(m_{Z}^{-2})^{-1} = 127.934$ [20]. For all scenarios, the masses of Higgs bosons are computed to two-loop accuracy with help of the FeynHiggs. The theoretical uncertainty in the prediction of FeynHiggs is estimated as $\Delta m_{h}^{\text{theory}} = \pm 3$ GeV for the Higgs masses [49, 50]. The dependence of properties of the Higgs boson on other lepton and quark masses is not very pronounced, and the default values of FeynHiggs are considered. The values of the input flags of FeynHiggs are set such that the evaluation includes full next-to-leading logarithms (NLL) and partial next-to-NLL resummation of the logarithmic corrections.

IV. NUMERICAL RESULTS AND DISCUSSION

In this section, the numerical predictions of the single production of the neutral Higgs bosons in association with a photon in electron-positron collisions are discussed. The impact of the hadronic uncertainties into account, the impact on the parameter space of the available constraints from Higgs researches at LHC, Tevatron and LEP have been investigated in Ref. [31]. In this study, it is chosen four benchmark points (BPs) which are compatible with the most recent results of the LHC for the bounds on masses and couplings of new particles, and the Higgs-boson properties. These are checked by the HiggsBounds [47] and HiggsSignals [48] with results of 86 analyses.

Moreover, the input parameters for SM are fixed as $m_{h} = 125.09$ GeV, $m_{W} = 80.385$ GeV, $m_{Z} = 91.1876$ GeV, $m_{\text{pole}} = 172.5$ GeV, $m_{h}(m_{h}) = 4.18$ GeV, $\alpha^{-1} = 137.036$, and $\alpha(m_{Z}^{-2})^{-1} = 127.934$ [20]. For all scenarios, the masses of Higgs bosons are computed to two-loop accuracy with help of the FeynHiggs. The theoretical uncertainty in the prediction of FeynHiggs is estimated as $\Delta m_{h}^{\text{theory}} = \pm 3$ GeV for the Higgs masses [49, 50]. The dependence of properties of the Higgs boson on other lepton and quark masses is not very pronounced, and the default values of FeynHiggs are considered. The values of the input flags of FeynHiggs are set such that the evaluation includes full next-to-leading logarithms (NLL) and partial next-to-NLL resummation of the logarithmic corrections.

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A. The process $e^{-}e^{+} \rightarrow \gamma h^{0}$ in the SM

In Fig. 4, the SM cross section of $e^{+}e^{-} \rightarrow h\gamma$ is given as a function of center-of-mass energy in the range from 100 to 1500 GeV, focusing on individual contributions coming from different types of diagrams, and polarizations of initial beams. Note that the total cross section in-
creases quickly with the opening of the phase-space and then decreases near $\sqrt{s} \sim 2 \times m_t$ (2 times mass of top quark) close to the $t^+t^-$ threshold with increments of $\sqrt{s}$. It is seen that the cross section is very sensitive to the magnitudes of each amplitude and the relative-phases between them. At high center-of-mass energy, where the triangle- and the bubble-type contributions are suppressed, $\sigma^{\mathrm{SM}}(e^+e^- \rightarrow h\gamma)$ is dominated by the box-type contributions. However, the bubble-type contribution is larger than triangle-type contribution, and their interference (bub+tri) make a much smaller contribution from each of them in the region of $\sqrt{s} \leq 700$ GeV, since they nearly destroy each other. The (bub+tri) contribution is enhanced by the threshold effect when $\sqrt{s}$ is close to $2 \times m_t$, since the $t^+t^-$ threshold is seen at this energy. The top quark and W-boson contributions make destructive interference and the top contribution is maximal near the $t^+t^-$ threshold. After passing the threshold of $t^+t^-$, the cross section scales like $1/\sqrt{s}$ and hence drop steeply. Combining all the contributions, the total cross section ultimately has the first peak near $\sqrt{s} = 200$ GeV, and the second one near $\sqrt{s} = 500$ GeV. Note that the total cross section with completely polarised left-handed electron $e_L^-$ and right-handed positron $e_R^+$, $\sigma^{\mathrm{SM}}(e_L^- e_R^+ \rightarrow h\gamma)$, can be enhanced by about a factor between 2 and 4, compared with the unpolarized case. The longitudinal polarization of both the electron and positron beams is therefore significant to enhance the cross section. At $\sqrt{s} = 250$ GeV, $\sigma^{\mathrm{SM}}(e_L^- e_R^+ \rightarrow h\gamma)$ reaches to value of 0.88 fb. However, as expected, the cross sections for polarization cases of $e_L^+ e_L^-$ and $e_R^+ e_R^-$ are very small (see, the insert figure in Fig. 4(b)). The left-right asymmetry $A_{LR}$ has a peak at the region of $\sqrt{s} \leq 340$ GeV. After passing the $t^+t^-$ threshold, its value remains almost constant which is equal to 0.98 with increments of $\sqrt{s}$.

In Fig. 5(a)-(b), the cross section of process $e^-e^+ \rightarrow \gamma h^0$ in the SM is also presented in plane of $P_{e^-}$ and $P_{e^+}$ by varying from $-1$ to $+1$. Especially, the cross section reaches its sizable values in the region of $0 < P_{e^+} \leq +1$ and $-1 \leq P_{e^-} < 0$. The polarized cross section is maximum at point $(P_{e^-}, P_{e^+}) = (-1,+1)$, namely completely polarised left-handed electron and completely right-handed positron. The enhancement is raised up to a factor of 3.4 at the left top corner, compared with the unpolarized case. At $\sqrt{s} = 250$ GeV, the polarized cross section $\sigma^{\mathrm{SM}}(P_{e^-}, P_{e^+})$ reaches up to $\sigma^{\mathrm{SM}}(-0.8,+0.3) = 0.52$ fb and $\sigma^{\mathrm{SM}}(-0.8,+0.6) = 0.64$ fb. At $\sqrt{s} = 500$ GeV, the polarized cross section $\sigma^{\mathrm{SM}}(P_{e^-}, P_{e^+})$ reaches up to $\sigma^{\mathrm{SM}}(-0.8,+0.3) = 0.14$ fb and $\sigma^{\mathrm{SM}}(-0.8,+0.6) = 0.17$ fb.

**B. The process $e^-e^+ \rightarrow \gamma h^0$ in the MSSM**

In Fig. 6, total cross section of process $e^-e^+ \rightarrow \gamma h^0$ in the MSSM is presented as a function of $\sqrt{s}$ from 150 GeV to 1.5 TeV for each benchmarks point scenario given in Table III. Additionally, to see deviations from the predictions of SM, the ratio of the total cross sections

$$\Delta R = \frac{(\sigma^{\text{MSSM}} - \sigma^{\text{SM}})}{\sigma^{\text{SM}}} \times 100 \quad (4.1)$$

is evaluated for each scenario, and this is presented in the lower panel of Fig. 6. The distribution of the cross section in MSSM has the same trend as the SM distribution. Due to the $t^+t^-$ threshold, a dip appears at $\sqrt{s} \approx 340$ GeV for all scenarios. However, the next dip at $\sqrt{s} \approx 500$ GeV is observed for only $m_h^{125}\chi^0\chi^0$ scenario. When the $\sqrt{s}$
to be detected. For scenarios where all sparticles are too heavy to be produced about 4 percent from the predictions of SM. For the scenarios, the unpolarized cross section reaches to deviations from the predictions of SM. The vertical solid-lines indicate to the proposed energy of each future lepton colliders. The lower panel shows the $\Delta R$ ratio, the deviations from the predictions of SM.

runs from 250 to 500 GeV, total cross section decreases from 0.27 to 0.064 fb in the $m_{h}^{125}$, $m_{h}^{125}(alignment)$ scenarios, while 0.33 to 0.036 fb in the $m_{h}^{125}(light \tilde{\chi})$ scenario. The remarkable deviations from the predictions of SM are seen at $m_{h}^{125}(light \tilde{\chi})$ scenario such that at $\sqrt{s} = 250$ GeV the production rate is enhanced by 25 percent whereas at $\sqrt{s} = 500$ GeV reduced by 44 percent. In other scenarios, i.e. $m_{h}^{125}$, $m_{h}^{125}(light \tilde{\tau})$ and $m_{h}^{125}(alignment)$, however, there is a deviation of about 4 percent from the predictions of SM. For the scenarios where all sparticles are too heavy to be produced directly at the selected center of mass energy, the MSSM contributions are small and will, therefore, be difficult to be detected. For $m_{h}^{125}(light \tilde{\chi})$ scenario, the unpolarized cross section is around 0.32 fb, 0.09 fb, 0.07 fb and 0.036 fb for the planned CEPC (at $\sqrt{s} = 240$ GeV), FCCee (at $\sqrt{s} = 350$ GeV), CLIC (at $\sqrt{s} = 380$ GeV) and ILC (at $\sqrt{s} = 500$ GeV) projects, respectively. For other scenarios, the unpolarized cross section reaches to values of 0.29 fb, 0.087 fb, 0.079 fb and 0.063 fb for the planned CEPC, FCCee, CLIC-380 and ILC-500 projects, respectively. The energy-dependent structure of the cross section is appeared at value of $\sqrt{s}$ which is close to the 2 times mass of some particles, i.e., threshold effects.

In Fig. 7, the polarized cross sections of $e^+e^- \rightarrow h\gamma$ are given as a function centre-of-mass energy in the range from 100 to 1500 GeV, for two benchmark points defined in the scenarios $m_{h}^{125}$ and $m_{h}^{125}(light \tilde{\chi})$. Note that for the other two scenarios, distributions of the polarized cross section are not shown here because they are similar to that of the $m_{h}^{25}$ scenario. The total cross section with completely polarised left-handed electron and right-handed positron, $\sigma_{\text{MSSM}}(e_L e_R \rightarrow h\gamma)$ for each benchmark points, can be enhanced by about a factor between 2.5 and 4, compared with the unpolarized case. The longitudinal polarization of both the positron and electron beams is, hence, significant to enhance the cross section. At $\sqrt{s} = 250$ GeV, $\sigma_{\text{MSSM}}(e_L e_R \rightarrow h\gamma)$ reaches to values of 0.93 fb and 1.16 fb for the benchmark points scenarios $m_{h}^{125}$ and $m_{h}^{125}(light \tilde{\chi})$, re-
respectively. For BP1($m_{h^0}^{25}$), the polarized cross section $\sigma_{\text{MSSM}}^{\gamma h}(P_{e^-}, P_{e^+})$ reaches up to $\sigma_{\text{MSSM}}^{\gamma h}(-0.8, +0.3) = 0.55$ fb and $\sigma_{\text{MSSM}}^{\gamma h}(-0.8, 0.0) = 0.43$ fb at $\sqrt{s} = 250$ GeV. On the other hand, for BP3($m_{h^0}^{25}(\chi)$), the polarized cross section $\sigma_{\text{MSSM}}^{\gamma h}(P_{e^-}, P_{e^+})$ reaches up to $\sigma_{\text{MSSM}}^{\gamma h}(-0.8, +0.3) = 0.68$ fb and $\sigma_{\text{MSSM}}^{\gamma h}(-0.8, 0.0) = 0.53$ fb. The left-right asymmetry $A_{LR}$ has a peak at the region of $\sqrt{s} \lesssim 340$ GeV. After passing the $t^+t^-$ threshold, its value remains almost constant which is equal to 0.98 with increments of $\sqrt{s}$. Furthermore, the cross sections are sorted according to various polarizations of initial beams as follows: $\sigma_{(LR)} > \sigma(-0.8, +0.3) > \sigma(-0.8, 0.0) > \sigma(\text{UU}) > \sigma(\text{RL})$ for each benchmark point. It is seen that the basic size of the total cross sections enhance by a factor of 4, depending on the polarizations of the initial $e^-$ and $e^+$ beams.

Figure 8 shows the effect of the individual contributions coming from different types of diagrams on total cross section of process $e^-e^+ \rightarrow \gamma h^0$ as a function of the center-of-mass energy ranging from 100 GeV to 3 TeV for each benchmarks point scenario. Here, the abbreviations “box”, “bub”, “qua”, “s-triang”, “t-triang” and “all” indicate to the contributions of box-type (diagrams $b_{1-12}$ in Fig. 1), bubble-type (diagrams $q_{1-7}$ in Fig. 2), quartic-type (diagram $q_{4}$ in Fig. 2), s-channel triangle-type (diagrams $t_{1-7}$ in Fig. 3) and t- and u-channel triangle-type (diagrams $t_{8-18}$ in Fig. 3) diagrams, and all of Feynman diagrams, respectively. The “channels” denotes total contribution from all t- and u-channels diagrams. The “box$\gamma h$” represent to contributions from the box-type diagrams with one or more W-bosons (diagrams $b_{3, 4, 7}$, and $b_{3}$ in Fig. 1). Also, the “bub+t-triang” is denoted to the contribution from interference between bubble-type and triangle-type diagrams.

It is clearly seen that the cross section is very sensitive to the magnitudes of each amplitude and the relative phases between them in all of the considered scenarios. The $\sigma_{\text{MSSM}}(e^-e^+ \rightarrow h\gamma)$ is dominated by the s-triangle- and bubble-type contributions at low center-of-mass energy. On the other hand, at high energies, $\sqrt{s} \gtrsim 1600$ GeV, these contributions are suppressed as $1/s$ and hence the box-type contributions become greater than them. However, the triangle- and bubble-type contributions are almost equal, and the interference among them (bub+t-triang) makes a much smaller contribution than any of them (by 2 orders of magnitude) because they make a destructive interference. On the other hand, the box-type contribution is larger by 1 order of magnitude than that of this interference. The t-channels contribution consist of box-type and t-triang diagrams, however the contribution of box-type diagrams is reduced by t-triang diagrams. The u-channels contribution is smaller than s-channels contributions (bubble-type and s-triang-type).

Note that the s-triangle-type Feynman diagrams are dominated by triple couplings $\lambda_{hG+G\gamma}$, $\lambda_{hH+G\gamma}$, $\lambda_{hG+H\gamma}$ and $\lambda_{hG+H\gamma}$. The bubble-type contributions are mainly determined by triple couplings $\lambda_{hG+G\gamma}$ and $\lambda_{hH+G\gamma}$ as shown in Table I. They are proportional to mixing angles $\cos(\beta - \alpha)$ and $\sin(\beta - \alpha)$.

The box$\gamma h$ contribution ends up dominating over all the other ones because the masses of the sfermion and charged Higgs boson are fixed at the TeV scale. About 70% of the total contribution of box-type diagrams comes from box-type diagrams $b_{3, 4, 7}$ and $b_{3}$ with one or more W-bosons in the loop. Therefore, it is also possible to assess the contribution of box-type diagrams in terms of a single coupling (the higgs$\gamma$-$W$ coupling) given in Eq. (2.12) which is proportional to the term $\sin(\beta - \alpha)$.

Table IV shows numerical results over the scenarios for center-of-mass energies of the planned CEPC (at $\sqrt{s} = 240$ GeV), FCCee (at $\sqrt{s} = 350$ GeV) and ILC (at $\sqrt{s} = 500$ GeV) projects. Overall, the s-channel diagrams make a dominant contribution to the total cross section in all scenario. Therefore, the s-triangle- and bubble-type diagrams have a remarkable impact on production rate. Particularly, the total cross section of the production of $e^-e^+ \rightarrow h\gamma$ reaches a value of 0.325 fb at $\sqrt{s} = 240$ GeV and is more observable compared to others at the CEPC or ILC-250 for BP3. Additionally, the cross sections are sorted according to the BPs as follows $\sigma(\text{BP3}) > \sigma(\text{BP1}) > \sigma(\text{BP2}) > \sigma(\text{BP4})$ at low energies. Note that the basic size of the total cross-sections is not very sensitive according to the BPs.

| BPs | $\sqrt{s}$ (GeV) | box | s-triang | bubble | t-channels | All |
|-----|-----------------|-----|----------|---------|------------|-----|
| BP1 | 240             | 0.227 | 23.93    | 23.42   | 0.108      | 0.270 |
|     | 350             | 0.185 | 6.07     | 6.98    | 0.093      | 0.087 |
|     | 500             | 0.116 | 1.70     | 1.79    | 0.061      | 0.068 |
| BP2 | 240             | 0.228 | 23.54    | 23.03   | 0.107      | 0.269 |
|     | 350             | 0.185 | 5.98     | 6.87    | 0.093      | 0.087 |
|     | 500             | 0.117 | 1.67     | 1.76    | 0.061      | 0.064 |
| BP3 | 240             | 0.227 | 23.96    | 22.52   | 0.107      | 0.325 |
|     | 350             | 0.186 | 5.78     | 6.73    | 0.093      | 0.090 |
|     | 500             | 0.118 | 1.46     | 1.73    | 0.061      | 0.036 |
| BP4 | 240             | 0.227 | 26.40    | 25.89   | 0.107      | 0.268 |
|     | 350             | 0.186 | 6.79     | 7.75    | 0.093      | 0.087 |
|     | 500             | 0.118 | 1.89     | 1.99    | 0.061      | 0.063 |

It is well known that the total cross section of $e^-e^+ \rightarrow h\gamma$ depends on couplings of the Higgs to other particles and masses of corresponding particles. All the couplings of the Higgs bosons to gauge bosons, fermions, and Higgs bosons are determined by the parameters $m_A$ and $\tan \beta$. The regions of the parameter space where the enhancement of cross section is large enough to be detectable at a future collider can be found by the behavior with these parameters. In this context, total cross section of $e^-e^+ \rightarrow h\gamma$ is scanned over the plane of $m_A - \tan \beta$ at $\sqrt{s} = 250$ GeV as depicted in Fig. 9. The parameters $m_A$ and $\tan \beta$ are varying in the range of

\[10\]
100 GeV ≤ m_A ≤ 2 TeV and 0.5 ≤ tan β ≤ 50 for the m_h^{125}, m_h^{125}(light \bar{\tau}) and m_h^{125}(light \bar{\chi}) scenarios, and 200 GeV ≤ m_A ≤ 2 TeV and 1 ≤ tan β ≤ 20 for the m_h^{125}(alignment) scenario. Additionally, the predictions for the mass of light CP-even Higgs boson h, m_h = 122 GeV, 124 GeV, 125 GeV, 126 GeV and 127 GeV, are presented by the contour lines. The corresponding benchmark points are also marked by the red stars. It is clear that the total cross section decreases when both m_A and tan β increase for the all cases. In particular, the cross section reaches its maximum values at small values of tan β into the scan region.

In the scenario m_h^{125}, the predictions for the mass of light CP-even Higgs boson m_h are always below 126 GeV all over the plane of m_A − tan β, and at tan β < 6 remain outside the window 125.09 ± 3 GeV [as shown in Fig. 9(a)]. At low m_A, the decay and production rates of h have been obtained to be incompatible with the LHC results [31]. The total cross section of the production of e^−e^+ → γh^0 reaches about 0.27 fb for the region of tan β ≥ 6 and any values of m_A.

In the scenario m_h^{125}(light \bar{\tau}), the m_h predictions are smaller than 126 GeV all over the plane of m_A − tan β, and the smallest value of tan β allowed by the uncertainty Δm_{h}^{theory} = ±3 GeV is around 5 [as shown in Fig. 9(b)]. The total cross section of the production of e^−e^+ → γh^0 reaches about 0.28 fb for the region of tan β ≥ 5 and m_A ≥ 200. Additionally, at large tan β and low m_A, total cross section reaches its biggest values.

In the m_h^{125}(light \bar{\chi}) scenario, in spite of the decrement in X_t, the predictions of m_h display a mild increment with respect to the m_h^{125} scenario. However, these are m_h < 127 GeV, except for upper-left corner in the plane m_A − tan β. The smallest value of tan β allowed by Δm_{h}^{theory} = ±3 GeV is around 5 [as shown in Fig. 9(c)]. The total cross section of the production of e^−e^+ → γh^0 is about 0.34 fb for the region of 5 ≤ tan β ≤ 8 and...
FIG. 9. (color online). Total cross section of process $e^- e^+ \rightarrow h^0 \gamma$ as a 2D function of $m_A$ and $\tan \beta$ at $\sqrt{s} = 250$ GeV for a) $m_A^{125}$ scenario, b) $m_A^{125}$ (light $\tau$) scenario, c) $m_A^{125}$ (light $\chi$) scenario and d) $m_A^{125}$ (alignment) scenario. The colour heat map corresponds to the total cross section (in fb) in the scan region. The contour lines indicate predictions for the mass of the light CP-even higgs boson $h^0$. The red stars denote the corresponding benchmark points from the last two rows in Table III.

$m_A \geq 200$, and this value decreases with increasing value of $\tan \beta$.

In the scenario $m_A^{125}$ (alignment), the predictions $m_h$ which are compatible with the measured Higgs mass are placed in the region of $4 < \tan \beta < 13$ and any values of $m_A$ [as shown in Fig. 9(d)]. In the allowed parameter region, the cross section reaches about 0.27 fb.

Overall, for production of the CP-even Higgs boson $h^0$ in association with a photon, the total cross section could reach a level of $10^{-1}$ fb, depending on the model parameters. This renders the loop-induced process $e^- e^+ \rightarrow h^0 \gamma$ in principle observable at a future electron-positron collider. Particularly, FCCee produces high luminosity for Higgs, W, Z and top-quark researches, has multiple detectors, and could reach energies up to the top-pair threshold and beyond. As comparison with other linear $e^- e^+$ colliders such as ILC and CLIC, the expected-luminosity at FCCee is a factor of between 3 and 5 orders of magnitude larger than that proposed for a linear collider (as shown in Table II), at all energies from the Z-pole to the top-pair threshold, where precision measurements are to be made, hence where the collected statistics will be a key feature. For the expected high luminosity $\mathcal{L} = 10^4$ fb$^{-1}$, the FCCee can provide around two thousand events at $\sqrt{s} = 240$ GeV. Therefore, the FCCee can be expected to have a promising potential to detect the process $e^- e^+ \rightarrow \gamma h^0$. However, the basic size of the total cross section of $e^- e^+ \rightarrow \gamma h^0$ can be enhanced by a factor of 4, depending on the polarizations of the initial $e^-$ and $e^+$ beams. Therefore, the ILC and CLIC which will be operated with beam polarization, also have an essential role to detect the process $e^- e^+ \rightarrow \gamma h^0$.
C. The process $e^−e^+ \rightarrow \gamma A^0$ in the MSSM

In this study, the single production of the pseudoscalar Higgs boson in association with a photon in electron-positron collisions is also examined in the framework MSSM, considering full one-loop diagrams. Total cross section of $e^−e^+ \rightarrow \gamma A^0$ is scanned over the plane of $m_A - \tan \beta$ at $\sqrt{s} = 1$ TeV as depicted in Fig. 10. The parameters $m_A$ and $\tan \beta$ are varying in the range of $100$ GeV $\leq m_A \leq 900$ GeV and $0.5 \leq \tan \beta \leq 50$ for the $m_{h^0}^{125}$, $m_{h^0}^{125}$ (light $\tilde{\tau}$) and $m_{h^0}^{125}$ (light $\tilde{\chi}$) scenarios, and $200$ GeV $\leq m_A \leq 900$ GeV and $1 \leq \tan \beta \leq 20$ for the $m_{h^0}^{125}$ (alignment) scenario. The total cross section decreases with increments of $m_A$ in all scenarios. The size of the total cross section is at a visible level of $10^{-2}$ to $10^{-1}$ fb. In particular, the total cross section reaches its largest values at low $m_A$ and $\tan \beta$ into the scan region. In the scenario $m_{h^0}^{125}$, the total cross section of the production of $e^−e^+ \rightarrow \gamma A^0$ reaches about $1.22 \times 10^{-4}$ fb for the region of $\tan \beta \geq 6$ and value of $m_A = 400$ GeV. For the scenario $m_{h^0}^{125}(\tilde{\tau})$, the total cross section reaches about $5.9 \times 10^{-4}$ fb in the region of $\tan \beta \geq 5$ and $m_A = 200$ GeV. In the $m_{h^0}^{125}(\tilde{\chi})$ scenario, the total cross section is about $7.7 \times 10^{-4}$ fb for $\tan \beta = 5$ and $m_A = 200$ GeV, and this value decreases with increasing value of $\tan \beta$. In the $m_{h^0}^{125}(alignment)$ scenario, the total cross section is about $7.9 \times 10^{-4}$ fb for $\tan \beta = 8$ and $m_A = 400$ GeV, and this value decreases with increasing value of $\tan \beta$.

At the allowed parameter space of the considered scenarios, the size of total cross section of $e^−e^+ \rightarrow \gamma A^0$ for $\sqrt{s} = 1$ TeV is at a visible level of $10^{-3}$ fb, and rather small. This renders the process $e^−e^+ \rightarrow \gamma A^0$ at the border of observability. Consequently, the single production of the neutral Higgs bosons in association with a photon in electron-positron collisions appears to be observable for $\gamma h^0$, but it is very challenging for $\gamma A^0$ at a $e^−e^+$ collider.
V. CONCLUSION

In this study, the single production of the neutral Higgs bosons in association with a photon in electron-positron collisions has been analyzed in detail for both SM and MSSM, focusing on individual contributions from each type of one-loop diagrams, and polarizations of initial beams. This process has no amplitude at tree level and is hence directly sensitive to one-loop effects and the underlying Higgs dynamics. The four different benchmark scenarios $m_{h}^{125}$, $m_{h}^{125}$(light $\tau$), $m_{h}^{125}$(light $\chi$) and $m_{h}^{125}$(alignment), have been used to illustrate the effect of the new physics in the framework of the MSSM. Note that all of the considered scenarios include a CP-even Higgs boson with mass about 125 GeV and couplings that all of the considered scenarios include a CP-even Higgs boson with mass about 125 GeV and couplings consistent with those of the discovered Higgs boson and a considerable part of their parameter space is allowed consistent with those of the discovered Higgs boson and a considerable part of their parameter space is allowed by the bounds from the researches for additional Higgs bosons and supersymmetric particles.

A remarkable deviations from the predictions of SM is seen at $m_{h}^{125}$(light $\chi$) scenario where the production rate could be significantly enhanced. This scenario induces an enhancement in the production rate up to 25% of that predicted in the SM. In both SM and MSSM, the cross section is increased up to about 4 times by the longitudinal polarizations of the initial beams, compared with the unpolarized case. It is clear that the cross section is significantly dependent on the magnitudes of each one-loop amplitude and the relative phases between them in all of the considered scenarios. The $\sigma_{\text{MSSM}}(e^+e^- \rightarrow h\gamma)$ is dominated by the $s$-channel triangle- and the bubble-type contributions at low center of mass energy. Note that the $s$-triangle-type amplitude is mainly determined by triple couplings $\lambda_{hG^+W^-}$, $\lambda_{hW^+W^-}$, $\lambda_{hW^+H^-}$, and $\lambda_{hW^+G^-}$. The bubble-type amplitude is mostly determined by triple couplings $\lambda_{hG^+W^-}$ and $\lambda_{hW^+W^-}$, quartic couplings $\lambda_{hG^+W^-}$ and $\lambda_{hW^+H^-}$.

The total cross section of $e^+e^- \rightarrow \gamma A^0$ as well as $e^+e^\rightarrow \gamma A^0$ were measured both at the plane $m_A - \tan \beta$ for all scenarios. The regions of the parameter space where the enhancement of the cross section is large enough to be observable at a future collider have been presented.

It should be emphasized that precise and model-independent measurements for the single production of the neutral Higgs bosons in association with a photon would be possible at the future $e^+e^-$ colliders ILC, CLIC, CEPC and FCC, and therefore the results of this study will be useful in detecting new physics signals based on MSSM, and in providing more precise limits on the corresponding couplings and masses.

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