Uncertainties in AGB evolution and nucleosynthesis

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Abstract. We summarise the evolution and nucleosynthesis in AGB and Super-AGB stars. We then examine the major sources of uncertainty, especially mass-loss.

1. Introduction

AGB stars are those that return to the giant branch after core helium exhaustion, and show thermal instabilities (or pulses) of the helium shell while ascending the second giant branch. They begin their lives with masses between about 0.8 and 8 $M_{\odot}$. The Super-AGB stars (hereafter SAGB) are the higher-mass cohort, from about 8 to 12 $M_{\odot}$, which burn carbon in their cores, then proceed to show thermal pulses. Their mass range is bounded above by the massive stars, which go on to more advanced nuclear burning. The shape of the initial mass function ensures that most stars above 0.8 $M_{\odot}$ experience an AGB phase. They are bright, and hence they dominate the light of many galaxies. Their copious mass-loss, and advanced nucleosynthesis, ensures that they are major producers of dust, possibly producing as much as 90% of the dust in our galaxy [1]. Their nucleosynthesis is now understood to be crucial for understanding the chemical evolution of the Universe. AGB stars are significant producers of carbon, nitrogen, fluorine and about half of the elements heavier than iron.

2. Summary of AGB evolution

In what follows we restrict our discussion to single stars. We will discuss pre-AGB evolution below, when we consider the uncertainties. The structure of an AGB star is well known. In the centre is the degenerate core composed mostly of C and O, surrounded by a He-burning shell. The next layer consists of the ashes of the H-burning shell, and is topped by said shell. There is then usually a small radiative buffer zone, and finally a deep convective envelope. The AGB phase follows immediately after the star exhausts its core He supply. While the He shell is becoming established, as a result of the core contracting and bringing He at the edge of the core to higher temperatures, there is a substantial energy output and the outer layers expand and cool. This is called the early AGB and the cooling outer layers cause convection to extend inwards in mass. For stars exceeding about 3 $M_{\odot}$ the H shell is extinguished and the convection may reach into the He-rich region, composed of the ashes of H burning. This is called the second dredge-up and it can produce substantial increases in the surface He content, up to about 38% by mass [2]. It is this event that produces the classic core-envelope structure required for stars to ascend the giant branch, and we will see later that the details of the timing and depth of the second dredge-up are vital in determining the final fate for SAGB stars [3].
Thermally pulsing evolution has been reviewed previously in the literature, and we refer the reader to [2,4,5] for details. Here we give only a summary, due to space limitations. The He shell is thermally unstable and experiences flashes or pulses every $10^3$ to $10^5$ years, depending on the core mass. During this “on” phase the He shell drives a convective region that extends from the He shell almost to the H shell, and is known as the *intershell* or *pulse-driven* convective zone. This distributes the products of He burning, mostly C, from the He shell almost to the bottom of the H shell. This region now comprises about 75% He and 25% C (by mass). After about 200 years the convection ends as the shell energy input decreases. The regions outside the He shell expand due to the energy input, causing the H shell to be pushed out to lower temperatures where it is extinguished (or nearly so). The opacity in these cooler regions increases and the bottom of the convective envelope extends inwards in mass. If the envelope reaches into the (top of the) region that was previously the intershell convective zone, then newly synthesised C in this region is mixed to the surface. This is known as the third dredge-up, and it can happen repeatedly, following (nearly) each thermal pulse. Note that a low-mass star may not experience a second dredge-up, but it may still experience many third dredge-up events. The nomenclature is well established, but not perfect. After the energy from the thermal pulse has diffused to the surface, the expanded envelope contracts and heats, and the H discontinuity becomes the new H-burning shell. The star begins the longest phase of the pulse cycle, the inter-pulse phase. At this stage the He shell is essentially inert and the star is powered by the H shell. This phase ends with the next thermal pulse of the He shell, some $10^3$ to $10^5$ years later, depending on the (core) mass of the star.

3. Summary of AGB nucleosynthesis
The above explains perhaps the most obvious consequence of pulses and dredge-up, namely the increase in the surface C content. But just as important is the synthesis of s-process elements, lithium, and the occurrence of hot bottom burning (hereafter HBB).

3.1. Hot bottom burning
For the more massive stars, exceeding about 4 $M_\odot$ (for solar composition, and decreasing as [Fe/H] decreases) the bottom of the convective envelope reaches into the top of the H-burning shell during the interpulse phase. Proton captures can occur at these high temperatures, and this is called HBB. To follow it accurately requires simultaneous calculation of mixing and burning, usually done with a diffusive approximation although it is important to remember that convection is *advective* rather than *diffusive*. The major reactions in such a case are CNO cycles, which burn $^{12}\text{C}$ (and at extreme temperatures $^{16}\text{O}$) into $^{13}\text{C}$ and $^{14}\text{N}$. Often the $^{12}\text{C}$ is the same $^{12}\text{C}$ that was dredged-up following the previous thermal pulse. The result is that HBB can prevent the formation of a C-star, or even reduce the C/O ratio from above unity to below unity at the cost of increasing the (primary) $^{14}\text{N}$ content of the envelope. For higher temperatures, which means for more massive AGB stars or even SAGB stars, we find the activation of the Ne-Na chain and possibly the Mg-Al chain [6,7] or even beyond, for SAGB stars.

3.2. Lithium production
Lithium continues to cause headaches for astrophysics. It is very fragile and burns through pp chains at temperatures as low as 2.5 MK. The AGB stars contribute to the Li problems through being production sites for Li, but with an uncertain effect on Galactic Li content.

If the AGB star experiences HBB then it will produce Li through the Cameron-Fowler mechanism [8,9]. This seems to be verified by observations [10,11], which show that Li is present in AGB stars that are not C-stars, as expected when HBB operates. The problem is that the overall production or destruction of Li in these stars is unclear because of the uncertainties associated with mass loss [12,13]. Although Li can be efficiently produced, it is also destroyed by
proton captures in the hot bottom of the convective envelope. The production and destruction is a delicate balance, with production driven by the initial supply of $^3$He. When that is used up then the destruction will dominate. Hence the Li-rich phase is only temporary. Whether the overall yield of Li from such a star is positive or negative depends on when the majority of the mass-loss occurs. If it is after the destruction, as most models predict, then the stars are negligible producers of Li for the Galaxy. If, however, the mass-loss rate is high when Li is abundant then the stars can produce significant amounts of Li. Li is also a crucial guide to extra-mixing processes on the RGB, as we discuss below.

### 3.3. Producing s-process elements

The slow neutron capture process, or $s$-process, is responsible for producing about half of the elements heavier than Fe, and AGB stars are the main producers [14]. There are two ways for this to occur. The first is active in more massive AGB stars, and occurs in the intershell convective region during a thermal pulse. The H-shell transmutes essentially all CNO species into $^{14}$N. During the flash these $^{14}$N nuclei can capture two $\alpha$ particles (He nuclei) to produce $^{22}$Ne. If the temperature exceeds about 300 MK then one more $\alpha$ capture can occur, producing free neutrons via $^{22}$Ne($\alpha$,n)$^{25}$Mg and these neutrons are now available for capture on Fe and other heavy elements and the production follows the $s$-process path. The elements produced in this way are then mixed to the envelope at the next dredge-up episode.

The other way to produce neutrons is through the $^{13}$C source. This requires some form of partial mixing of protons below the formal bottom of the convective envelope at the end of dredge-up. There is much debate about the mechanism and how exactly this occurs (see discussion in [2]). But let us suppose it happens, as seems to be required by observations. Then these protons can be captured by the abundant $^{12}$C to produce $^{13}$C which can then capture an $\alpha$ particle to produce a free neutron (and a $^{16}$O nucleus). These neutrons produce $s$-processing as before, but at a lower neutron density than the $^{22}$Ne source, albeit for a longer time. We note that the formation and properties of the $^{13}$C pocket are major uncertainties in our understanding of the $s$-process.

### 4. Super-AGB stars

Recent years have seen a rise in the number of studies of SAGB stars. These are very demanding calculations and it is only recently that we have had the computer power to throw at this problem. SAGB stars ignite C in their cores, whereas the normal AGB stars do not. Subsequent evolution depends critically on the mass (and the details of how convection is calculated [15]). The C ignites off-centre in a small convective shell. This shell may burn all the C present in the convective region into Ne, and then a second shell ignites C, and so forth. These shells can be located further toward the centre or further outward, with the result that one eventually burns all of the core C in some cases, and in others the C burning may not reach the centre so we have a CO core and an ONe outer region. The crucial thing is that these stars then proceed to experience thermal pulses on the (S)AGB. They are quite separate beasts to “massive” stars, which go on to further nuclear burning stages after C burning. SAGB stars do not.

SAGB evolution is qualitatively the same as AGB evolution, but the quantitative differences are important. Firstly, the HBB occurs at high temperature [6,7,16] due to the deep convective envelopes. However, the intershell convective zone is very small in mass, typically $10^{-3}$ to $10^{-5}$ M$_\odot$ (compared to something like 0.01 M$_\odot$ for AGB stars). This means that the region undergoing neutron captures (from the $^{22}$Ne source) is tiny and when this region is diluted in the much deeper envelope (the core mass is perhaps 1 M$_\odot$, leaving an envelope mass of a few M$_\odot$) then the enhancements of $s$-process elements are not expected to be large. This may change near the end of the evolution, when the envelope mass has been dramatically reduced due to mass
loss. Further, SAGB stars have very small interpulse periods, more like 30–1000 yr as opposed to 1-100 kyr for AGB stars. Hence hundreds to thousands of pulses must be calculated.

5. Main uncertainties

We have indicated above some of the uncertainties in AGB evolution. We discuss these in more detail below. However, the AGB is the last phase of evolution for these stars, and hence the models begin with the uncertainties already accumulated over all of the earlier phases. Thus we need to briefly review these if we are to provide a realistic estimate of the confidence we should place in the models.

5.1. Extra-mixing on the first giant branch

It is now well established that the predictions for abundance changes resulting from first dredge-up are largely in agreement with observations. It is also well established that there is a second mixing event that changes these compositions, and it seems to begin at the position of the bump in the giant branch luminosity function (see [2] for a recent review, and [17] for a beautiful illustration using Li). The exact mechanism for this extra-mixing is not known, and early investigations focussed on the obvious candidate of meridional circulation in rotating stars [18], but modern models suggests that this does not match the observations [19,20]. Recent interest focusses on thermohaline mixing following the discovery of a molecular weight inversion that appears in RGB stars when they reach the bump [21]. Calculations show that this seems to match the observations reasonably well [22,23]. Debate exists concerning how to model this process, with 2D and 3D hydro calculations disagreeing with the typical 1D models used; see [24,25] and the extensive discussion in [2]. More work needs to be done before we can be confident in how to model this process, let alone verifying its role in the observed abundance patterns.

5.2. Extra-mixing on the asymptotic giant branch

If some process causes extra-mixing on the RGB, does it also operate on the AGB? This has been postulated by various authors as a possible explanation for O and Al isotope measurements in pre-solar grains [e.g. 26]. But there are also discrepancies with the C isotope ratios predicted for C stars. By the time the star has dredged-up sufficient $^{12}$C to produce $C/O > 1$ the ratio of $^{12}$C/$^{13}$C greatly exceeds the observed values [27]. There are also other problems that may be alleviated by some extra-mixing on the AGB. However, [28] showed that, at least for solar metallicities, the inclusion of the effects of extra-mixing on the RGB (usually ignored in the models) removed the problems on the AGB. The final word is yet to be written.

5.3. Core helium burning

The core helium-burning phase has a history of challenging our modelling skills. This is where semiconvection was first recognised in the 70s [29,30] and later the core-breathing pulses added more unwelcome complications [e.g. 31]. It is now well documented that small variations in numerical details of the determination of the convective boundaries can produce enormous differences in the size of the convective core, not to mention the semiconvection region, as discussed recently in [32]. The reality or otherwise of the core breathing pulses remains in debate [e.g. 33] and recent work [34] has tried to use asteroseismology as a probe of mixing in the cores of these stars. The problem is very difficult, with a core opacity source that is higher (in the regions rich in C and O) than in the outer He-rich region. This drives overshooting at the core edge. But the stellar conditions contrive to produce a local minimum in the ratio of the adiabatic to radiative temperature gradients. When this minimum reaches unity, the traditional value for convection according to the Schwarzschild criterion, then the correct way to calculate the behaviour of the outer edge of the core is far from clear. Further, when the central He content
drops below about 0.1 [35] then the cubic dependence of the triple-α energy production on the He content means that small perturbations on the He mass fraction can produce large changes in the energy output and hence drive larger convective cores – these are breathing pulses.

Clearly the only way forward is multi-dimensional hydrodynamical simulations as discussed in [32]. Such calculations are demanding because the simulation must be performed for many turnover times. Indeed, determining the behaviour and timescale for semiconvection is one of the hoped-for outcomes of such simulations.

5.4. Convective boundaries

Thankfully the timescale for convective mixing is usually much smaller than the evolutionary timescale for the star. Usually one uses a diffusion equation to approximate mixing. Because mixing is usually very rapid, there is not a lot of dependence on the diffusion coefficient, as long as it is sufficiently large to produce rapid mixing, although there are some notable exceptions, such as HBB. As you can see from the previous discussion, the calculation of convective borders remains a serious problem for stellar models. Some overshoot must carry material beyond the naive Schwarzschild border. This is simply the result of conservation of momentum. The Schwarzschild criterion considers the buoyancy force, and places the border where that goes to zero. However material arrives at the neutral border with a finite momentum so it must penetrate the border – but by how much? It has become common to modify the diffusive mixing implementation by including an overshoot region where the diffusion coefficient is chosen to reproduce an exponential decay in velocity beyond the formal border [36]. This is qualitatively fine, but as always there is a parameter (determining the decay length) that is usually fixed by appealing to some observations. This procedure produces the partial mixing required to produce a $^{13}$C pocket for $s$-process nucleosynthesis, although of course the details depend on the overshoot procedure.

If one applies such a scheme to all convective borders, then interesting things happen. Of course, one must calibrate, or somehow choose, the decay length for each border, and there is no reason to think that overshoot inwards (to higher density) has a similar decay length to overshoot outwards into less dense material. Nevertheless, when applied to the intershell convective region, we find overshoot into the CO core with the result that the intershell becomes enriched in C and O. Models including this effect give better fits to observed abundances in H-deficient post-AGB stars [37] and possibly AGB stars as well [28].

5.5. Convection theory

Of course, the situation is even worse than described above. The Mixing-Length Theory (MLT) of convection has such a hold on stellar modelling that we forget that it is only one possible formulation. Another that has made substantial contributions is the Full Spectrum of Turbulence theory [38,39]. These two theories produce significantly different results [40] and it seems that this uncertainty is largely ignored in the literature. We note that researchers do continue to develop potential new convection models [e.g. 41] but these must be presented in a format that is easily implemented in an evolution code if they are to overcome the dominance of the MLT.

5.6. Opacities for varying envelope compositions

This is one area that has received a lot of attention recently, with the result that the current situation is very satisfactory. Thermal pulses increase the C (and possibly O, via overshoot, see above) content of the stellar envelope, and HBB can burn this C (and O) into N. After H and He, these can be the next most abundant species, and they are a very significant source of opacity. This is doubly so for stars with very low [Fe/H]. Such variations in the opacity have been ignored until recently, as tables for varying compositions were not available. This is no longer the case, with the AESOPUS tool now providing opacities for appropriate mixtures
The increase in opacity when the C content increases has also been shown to have a dramatic effect on the evolution of the AGB stars, increasing mass loss and hence terminating the evolution much sooner than in calculations that ignore this effect [44,45].

5.7. Envelope ejection?
In 1986 Wood and Faulkner [46] found convergence problems in a late AGB model which they described as due to the disappearance of hydrostatic solutions to the stellar structure for large cores. Further work on this problem was performed by [47] who confirmed that a super-Eddington luminosity developed at the bottom of the convective envelope. This was identified as being due to an opacity bump produced by Fe. An understanding of the subsequent behaviour of the star will require a hydrodynamical study. Of course this may be important for understanding the formation of planetary nebulae.

6. Final fate of AGB stars
An AGB (or SAGB) star ends its life when mass loss removes the envelope. Usually this produces a CO white dwarf. However the SAGB stars ignite core C and a number of outcomes become possible [3]. The C burning can ignite in a shell in the outer part of the CO core, and in many cases does not proceed further. This produces a CO(Ne) hybrid white dwarf. If the C burning proceeds to the core then we find the formation of an ONe white dwarf. The most massive SAGB stars experience “dredge-out” [48,49] where the convective C burning region meets with the convective envelope. This is a computationally demanding phase of the evolution and subject to all the uncertainties associated with time-dependent mixing, which are amplified by simultaneous rapid nuclear energy generation.

Following core He burning most SAGB stars have a core mass that easily exceeds the Chandrasekhar mass. We expect such a star to proceed through various nuclear burning stages and end life as a supernova. But the occurrence of second dredge-up in SAGB stars reduces the core mass below the critical value. The fate of such a star depends on the competition between core growth and mass loss. If the former dominates and the core reaches the Chandrasekhar mass then an electron-capture supernova will result. If mass loss terminates the evolution with the core mass below the critical value then the star ends as an ONe white dwarf [3].

7. Mass-loss
From the viewpoint of stellar models, what is required is a formula that specifies the mass-loss rate (MLR) in terms of known quantities. This mass loss is of course assumed to be steady and spherically symmetric, which we know is not always the case in reality. The mechanism believed to drive mass loss in AGB stars is the pulsation enhanced dust-driven wind scenario, where grains are driven outward by the photon wind. These are collisionally linked to the gas, and hence the gas is also removed. For stars with C/O > 1 we believe that amorphous C grains are involved. For O-rich stars it is presumably Mg and Fe silicate grains that are implicated. The difficulty is that these latter grains do not couple well with the gas unless they are very large [50]. It is only recently that such large grains were indeed observed [51].

There are many MLR expressions in the literature. We discuss here only the most commonly used formulae. The Reimers formula [52] was derived for giants and supergiants. Vassiliadis & Wood [53] instead fit the MLR to the pulsation period for red giants and AGB stars. The MLR was bounded by the radiation limit, and for massive stars the superwind phase was delayed to ensure that periods exceeding 500 d were obtained. The Blöcker formula [54] is one of many modifications to the Reimers rate, in this case motivated by dynamical pulsation models of Miras. The more recent (2005) Schröder & Cuntz formula [55] is a physically motivated, semi-empirical modification of the Reimers formula.
As one may expect, the MLR has a potentially enormous effect on the star’s evolution [6,40] and nucleosynthesis [6,56]. Increasing the MLR removes the envelope more rapidly, thus terminating the evolution and also terminating thermal pulses and all nucleosynthesis. The resultant yields are very dependent on the MLR used (and the free parameters chosen for those formulae with such parameters).

Various authors have performed tests of the MLRs. Mostly these are crude sanity checks, but some quantitative tests have also been performed. Schröder & Cuntz [57] looked at detailed models for some of the best studied galactic giants and supergiants, with considerable success. Another nice test was a critical examination of AGB luminosity functions [58] which again was very favourable to the Schröder & Cuntz MLR.

However, the Vassiliadis and Wood [53] formula also has been carefully tested. Detailed evolution and pulsation models for thermal pulsing stars in NGC419 and NGC1978 were compared with infrared data by [59] and an excellent agreement was found. The MLR in [53] also accurately predicted the magnitude of the tip of the AGB in these clusters. Another quantitative test of this rate with AGB stars in the SMC was performed by [60] and again the MLR produced a successful quantitative comparison.

In conclusion, one should carefully choose the MLR to be used, depending on the phase of evolution being investigated. The Schröder & Cuntz formula [55] seems suitable for giants and AGB stars, while the Vassiliadis and Wood [53] formula, tailored for AGB stars, does a very good job in that regime.

8. Conclusions

There remain many uncertainties in trying to model AGB and SAGB stars. As usual, these mostly centre on convection and its various manifestations. Recent work has led to substantial improvements in our understanding of mass loss and we now have formulae that seem to be quantitatively reliable, at least in a global sense. The use of the Reimers formula for AGB stars is not recommended.

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