Thomas-Fermi Description of Incoherent Light Scattering from an Atomic-Trap BEC

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Abstract

We present a Thomas-Fermi treatment of resonant incoherent scattering of low-intensity light by a dilute spatially confined Bose-Einstein condensate. The description gives simple analytical results and allows scattering data from finite-size condensates to be interpreted in terms of the properties of the homogeneous BEC-system. As an example, we show how the energy dispersion of the elementary excitations can be measured from scattering by a finite-size atomic-trap condensate. As a second example, we point out that a near-resonant scattering experiment can observe quasi-particle creation caused by particle annihilation.

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For most applications of the atomic-trap Bose-Einstein condensates [1] – [3], it will be beneficial to have a highly populated condensate, and recent experiments have succeeded in increasing the number of condensed atoms [4]. It is fortunate then, that in this limit (for bosons interacting through an inter-particle potential of positive scattering length) many quantities can be calculated analytically. The simplifying assumption is the Thomas-Fermi approximation [5]– [14], which presumes that the local behavior of the BEC is similar to that of a homogeneous BEC with a chemical potential equal to the local effective chemical potential \( \mu(r) = \mu_T - V(r) \), where \( \mu_T \) is the chemical potential of the trap system and \( V(r) \) the trapping potential.

In this letter, we describe incoherent light scattering in a Thomas-Fermi approximation. The result, with a proper understanding of its limits, offers insight that will be useful in interpreting experimental scattering spectra. The formalism describes the scattering event as scattering of light by quasi-particles and includes the effects of recoil and intermediate state energy neglected in the off-resonant limit reported by Javaneinen [15], [16].

The dynamical Thomas-Fermi approximation of this paper consists of describing the fluctuations that cause an incoherent scattering event to occur near a position \( r \), by the corresponding fluctuations in a homogeneous system of chemical potential \( \mu(r) \). In this picture, the differential cross section for incoherent scattering, \( \frac{d^2\sigma_{\text{inc}}}{d\Omega d\omega} \), where \( d\Omega \) represents the infinitesimal solid angle and \( d\omega \) the infinitesimal energy range (\( \hbar = 1 \), in our units) over which the scattered particles are detected, reduces to an integral over a cross section density :

\[
\frac{d^2\sigma_{\text{inc}}}{d\Omega d\omega} \approx \int d^3r \left[ \frac{d^2\sigma_{\text{inc, hom}}}{d\Omega d\omega} \right]_{\mu = \mu(r)},
\]

where \( \frac{d^2\sigma_{\text{inc, hom}}}{d\Omega d\omega} \) is the cross section for incoherent scattering from a macroscopic, homogeneous system of volume \( V \) and chemical potential \( \mu(r) \).

We use a second quantized representation in which \( c_k, c_k^\dagger \) denote the annihilation and creation of a single-atom state with the atom in the atomic ground state (the trapping state) and its center-of-mass in the plane wave state of wave vector \( k \); \( \tilde{c}_k, \tilde{c}_k^\dagger \) denote the correspond-
ing annihilation and creation operators for atoms in the excited (resonant) atomic state. In the resonant scattering process, the interaction with the electric field of the incident light, \( E_{\text{in}} \hat{\epsilon}_\text{in} \exp(\imath [k_{\text{in}} \cdot r - \omega_{\text{in}} t]) \), where \( E_{\text{in}} \) represents the intensity and \( \hat{\epsilon}_\text{in} \) the polarization of the incident light, promotes an atom to its excited atomic state. The excited atom subsequently deexcites, creating a photon of electric field \( E_0 \hat{\epsilon}_\text{out} \exp(-\imath [k_{\text{out}} \cdot r - \omega_{\text{out}} t]) \), where \( E_0 \) is the single-photon intensity (in Gaussian units, \( E_0 = \sqrt{2 \pi k c / V} \), where \( k \) is the resonant photon wave number). The corresponding absorption and emission Hamiltonian operators in the interaction picture are:

\[
\hat{H}^{(\text{abs})}(t) = -E_{\text{in}} (\mathbf{d} \cdot \hat{\epsilon}_\text{in}) \exp(-\imath \omega_{\text{in}} t) \sum_k \hat{c}_k^\dagger (t) c_k(t) ,
\]

\[
\hat{H}^{(\text{em})}(t) = -E_0 (\mathbf{d} \cdot \hat{\epsilon}_\text{out})^* \exp(\imath \omega_{\text{out}} t) \sum_{k'} \hat{c}_{k'}^\dagger (t) \hat{c}_{k'}(t),
\]

where \( \mathbf{d} \) denotes the resonant atomic dipole moment.

The amplitude of a state \( |F\rangle \) of the scattering system, a time \( T \) after the system was in its initial state \( |in\rangle \), is, to lowest order in perturbation theory, equal to:

\[
\mathcal{A}(|in\rangle, |F\rangle; T) = - \int_{-T/2}^{T/2} dt \int_0^{T/2+t} d\tau \langle F|\hat{H}^{(\text{em})}(t)\hat{H}^{(\text{abs})}(t - \tau)|in\rangle ,
\]

where the photon absorption occurs a time \( \tau \) earlier than the photon emission \( |F\rangle \). In the limit of low-intensity incident light, each photon is scattered by a system of ground state atoms. Indicating the many-body vacuum of excited–state atoms, \( |0\rangle \), in the initial and final states, \( |in\rangle \rightarrow |in\rangle \ket{0}, |F\rangle \rightarrow |F\rangle \ket{0} \), we obtain

\[
\mathcal{A}(|in\rangle, |F\rangle; T) = -E_{\text{in}} E_0 (\mathbf{d} \cdot \hat{\epsilon}_\text{in}) (\mathbf{d} \cdot \hat{\epsilon}_\text{out})^* \int_{-T/2}^{T/2} dt \int_0^{T/2+t} d\tau \\
\sum_{k,k'} \langle F|\hat{c}_{k'}^\dagger (t) c_k(t - \tau)|in\rangle \langle \bar{0}|\hat{c}_{k'}(t)\hat{c}_{k+\epsilon_{\text{in}}}^\dagger (t - \tau)\ket{0} \exp(\imath [\omega_{\text{out}} t - \omega_{\text{in}} (t - \tau)]) .
\]

The vacuum expectation value in \( |\rangle \) is equal to \( \langle \bar{0}|\hat{c}_{k'}(t)\hat{c}_{k+\epsilon_{\text{in}}}^\dagger (t - \tau)\ket{0} = \delta_{k',k+\epsilon_{\text{in}}} \times \exp \left[-\imath \left(\omega_0 + \bar{E}_{k+\epsilon_{\text{in}}} - \gamma / 2\right) t\right] \), where we write the energy of the excited atom as the sum of the atomic excitation energy, \( \omega_0 \), and the kinetic energy, \( \bar{E}_{k+\epsilon_{\text{in}}} \), and where \( \gamma \) is the width of the excited atomic state, \( \gamma = (4/3)k^3d^2 \). The upper limit of the \( \tau \)-integration
interval, may be replaced by $\infty$ if $T/2+t \gg \gamma^{-1}$. Since we shall take the limit $T \to \infty$, we change the $\tau$-interval accordingly, and we introduce a scattering operator $\hat{S}_q(t)$:

$$A(|in\rangle, |F\rangle; T) = -E_{in} E_0 (d \cdot \hat{\epsilon}_{in})(d \cdot \hat{\epsilon}_{out})^* \int_{-T/2}^{T/2} dt \exp(i\omega t) \langle F|\hat{S}_q(t)|in\rangle,$$

where $\hat{S}_q(t) = \sum_k \int_0^\infty d\tau c_{k+q}^\dagger(t)c_k(t-\tau) \exp(-i[\omega_0 + \tilde{E}_{k_{in}} - \omega_{in} - i\gamma/2] \tau)$, (5)

where $q$ is the momentum transfer, $q = k_{in} - k_{out}$, and $\omega$ the energy transfer $\omega = \omega_{in} - \omega_{out}$.

The scattering rate is the ratio of the square of the amplitude (5) over $T$, summed over all $|F\rangle$-states and thermally averaged over the $|in\rangle$-states, in the limit of $T \to \infty$. The differential cross-section $d^2\sigma_{hom}/d\Omega d\omega$ is the product of the resulting rate with the ratio of the scattered particle final state density, $\rho_{out}$, over the incident particle flux, $J_{in}$. For photon scattering, $\rho_{out} = [V/(2\pi)^3] \times (k^2/c)$, and $J_{in} = (E^2_{in}/E_0^2) \times (c/V)$, so that

$$d^2\sigma_{hom}/d\Omega d\omega = \lim_{T \to \infty} \frac{1}{T} \left[ \sum_{|F>} |A(|in\rangle, |F\rangle; T)|^2 \right] \text{average} (|in\rangle) \times (\rho_{out}/J_{in})$$

$$= \left| (3\gamma/4k)(\hat{d} \cdot \hat{\epsilon}_{in})(\hat{d} \cdot \hat{\epsilon}_{out})^* \right|^2 \frac{1}{2\pi} \int_{-\infty}^{\infty} dt \exp(i\omega t) \langle \hat{S}_q(t)\hat{S}_q(0)\rangle.$$ (6)

where $\hat{d} = d/d$, and the $\langle \rangle$-brackets denote the thermal average over the $\langle in|in\rangle$-matrix elements.

To describe scattering from a dilute BEC of low depletion, $(N - N_0)/N \ll 1$, where $N$ is the total number of atoms and $N_0$ the number of condensed atoms, we work in the Bogoliubov approximation. In this scheme, we treat the zero-momentum operators as $c$-numbers, replacing them by $\sqrt{N_0}$, and we keep only terms proportional to $\sqrt{N_0}$:

$$\hat{S}_q(t) \approx \sqrt{N_0} \times \left[ \int_0^{\infty} d\tau c_{q}^\dagger(t) \exp(-i[\omega_0 + \tilde{E}_{k_{in}} - \omega_{in} - i\gamma/2] \tau) \right.$$\n
$$+ \int_0^{\infty} d\tau c_{-q}(t-\tau) \exp(-i[\omega_0 + \tilde{E}_{k_{in,q}} - \omega_{in} - i\gamma/2] \tau) \right]$$ \hspace{1cm} (7)

The first term on the right-hand side of (7) represents the scattering event in which an atom is taken out of the condensate, and the second term represents the event in which an atom is put into the condensate, each process transferring a momentum $q$ to the many-boson system. Since $k_{in} - q = k_{out}$, the excited atom momentum equals the resonant photon wave.
number \( k \) in both processes, and we absorb the recoil energy \( \tilde{E}_k \) into the definition of the detuning: \( \Delta = \omega_{\text{in}} - \omega_0 - \tilde{E}_k \).

Finally, we perform the Bogoliubov transformation to quasi-particle operators \( b, b^\dagger \):

\[
\begin{align*}
\hat{c}_q^\dagger(t) &= \cosh(\sigma_q) b_q^\dagger(t) - \sinh(\sigma_q) b_{-q}(t), \\
\hat{c}_{-q}(t - \tau) &= \cosh(\sigma_q) b_{-q}(t - \tau) - \sinh(\sigma_q) b_q^\dagger(t - \tau),
\end{align*}
\]

where the canonical nature of the boson \( b \)-operators is ensured by writing the coherence factors as \( \cosh \sigma \) and \( \sinh \sigma \). With the time-dependence of the creation and annihilation operators, \( b_{-q}(t - \tau) = b_{-q}(t) \exp(iE_q\tau), b_q^\dagger(t - \tau) = b_q^\dagger(t) \exp(-iE_q\tau) \), we obtain for (7):

\[
\hat{S}_q(t) \approx i N_0 \times \left( \left[ \cosh \frac{\sigma_q}{\Delta + i\gamma/2} - \frac{\sinh \sigma_q}{\Delta - E_q + i\gamma/2} \right] b_q^\dagger(t) + \left[ \cosh \frac{\sigma_q}{\Delta + E_q + i\gamma/2} - \frac{\sinh \sigma_q}{\Delta + i\gamma/2} \right] b_{-q}(t) \right),
\]

which describes the response of the BEC to the scattering as creating and annihilating quasi-particles. Each term of (9) represents a different process that creates or annihilates a quasi-particle. For example, the scattering can create a quasi-particle \( b_q^\dagger \) either by scattering an atom from the condensate into a state of momentum \( q \) \( \hat{c}_q^\dagger \), giving an amplitude \( \cosh \sigma_q \) for quasiparticle creation \( b_q^\dagger \), or by scattering an atom from a state of momentum \( -q \) \( \hat{c}_{-q} \) into the condensate, giving an amplitude \( \sinh \sigma_q \) for quasi-particle creation. The resonant denominators are the usual (complex) energy-differences between the initial and intermediate state: in the first process, the energy-difference is \( \omega_{\text{in}} - (\omega_0 + \tilde{E}_k - i\gamma/2) = \Delta + i\gamma/2 \), whereas in the second process, the intermediate state contains an extra quasi-particle and the energy difference is equal to \( \omega_{\text{in}} - (\omega_0 + \tilde{E}_k + E_q - i\gamma/2) = \Delta - E_q + i\gamma/2 \).

At zero temperature, \( \langle b_{-q}(t)b_{-q}(0) \rangle = 0 \), and \( \langle b_q^\dagger(t)b_q^\dagger(0) \rangle = \exp(-iE_qt) \), so that

\[
\frac{1}{2\pi} \int dt \exp(i\omega t) \langle \hat{S}_q^\dagger(t)\hat{S}_q(0) \rangle \approx N_0 \delta(\omega - E_q) \left| \cosh \frac{\sigma_q}{\Delta + i\gamma/2} - \frac{\sinh \sigma_q}{\Delta - E_q + i\gamma/2} \right|^2,
\]

giving a spectrum that consists of a single peak. The scattering processes neglected in the Bogoliubov approximation, give rise to an additional background in the spectrum. The ratio of the peak intensity to the integrated background intensity is roughly proportional to the
where condensate of atoms interacting through a potential of scattering length $a$, is approximation gives $\tanh 2\mu$.

We substitute (1), spherical symmetry reduces the expression to an integral over the radial distance $R$, and

$$\rho_R$$

manner by introducing temperature-dependent quasi-particle occupation numbers $\nu_q$, so that

$$\langle b_{-q}^*(t)b_{-q}(0) \rangle = \nu_q \exp(iE_q t)$$ etc... (see for example ref. [19]).

The value of $\sigma_q$ is determined by minimizing the free energy. At $T=0$, the Bogoliubov approximation gives $\tanh 2\mu = \mu/(q^2/2m+\mu)$, where $\mu$ is the chemical potential which, for a condensate of atoms interacting through a potential of scattering length $a$, is $\mu = (4\pi/m)a\rho_0$ where $\rho_0$ is the condensate density. Furthermore, $E_q = \sqrt{(q^2/2m + \mu)^2 - \mu^2}$, and with (3) and (10), we obtain the following expression for the cross section density in (1):

$$\left[ \frac{d^2\sigma_{\text{inc, hom}}/d\Omega d\omega}{V} \right] \propto \rho_0 \delta(\omega - E_q) \left| (3\gamma/4k)(\hat{d} \cdot \hat{\epsilon}_{in})(\hat{d} \cdot \hat{\epsilon}_{out}) \right|^2 \times \frac{1}{2} \left[ \frac{\sqrt{(q^2/2m + \mu + E_q)/E_q}}{\Delta + i\gamma/2} - \frac{\sqrt{(q^2/2m + \mu - E_q)/E_q}}{\Delta - E_q + i\gamma/2} \right],$$

(11)

where $E_q$ and $\mu$ are $r$-dependent. The energy-conservation factor, $\delta(\omega - E_q(r))$ implies that an energy interval $(\omega, \omega + d\omega)$ probes the condensate region $\omega < E_q(r) < \omega + d\omega$. In a magnetic trap, the excited atom experiences a potential energy due to its magnetic moment. We account for this effect by making the detuning position-dependent: $\Delta(r) = \Delta - \alpha V(r)$, where $\alpha$ is the ratio of the excited atom potential and the trapping potential. Finally, for the sake of simplicity, we specialize to a spherically symmetric harmonic oscillator trap, $V(R) = (\omega_T/2)(R/L)^2$, where $\omega_T$ is the trapping frequency and $L$ the extent of its single-particle ground-state, $L = 1/\sqrt{m\omega_T}$. In the Thomas-Fermi approximation, the condensate density is (10, 12) $\rho_0(R) = [\mu_T/4\pi a_m] \times [1 - (R/R_0)^2] \theta(R - R_0)$, where $R_0$ is the condensate radius, $R_0 = L(15aN/L)^{1/5}$ and $\mu_T = (\omega_T/2)(R_0/L)^2$. In performing the spatial integration (11), spherical symmetry reduces the expression to an integral over the radial distance $R$, and we substitute $R$ by the effective chemical potential $\mu$, $R = R_0 \sqrt{1 - \mu/\mu_T}$. With this substitution, $\rho_0(R) \rightarrow \mu/4\pi a_m$, $\Delta(R) \rightarrow \Delta - \alpha [\mu_T - \mu]$ and $\delta(\omega - E_q) \rightarrow \delta(\mu - \mu_q(\omega))$ $\left| \partial E_q/\partial \mu \right|^{-1}$, where $\mu_q(\omega)$ is the effective chemical potential at the positions where $E_q$ is equal to $\omega$,

$$\mu_q(\omega) = \frac{1}{2} \frac{\omega^2}{\omega_T^2/2m} - \frac{\sigma^2}{2m},$$

and

$$\frac{\partial E_q}{\partial \mu} = \frac{q^2/2m}{E_q},$$

we find

$$\frac{d^2\sigma_{\text{inc}}}{d\Omega d\omega} = \frac{1}{\omega_T} \left( \frac{R_0^3}{aL^2} \right) \frac{\mu_q(\omega)}{\mu_T} \sqrt{1 - \frac{\mu_q(\omega)}{\mu_T}} \frac{\omega}{q^2/2m} \left| (3\gamma/4k)(\hat{d} \cdot \hat{\epsilon}_{in})(\hat{d} \cdot \hat{\epsilon}_{out}) \right|^2 \times \mu_q(\omega)$$

(11)
\[
\begin{align*}
\frac{1}{4} \left| \frac{\sqrt{(q^2/2m + \mu_q(\omega) + \omega)/\omega} - \sqrt{(q^2/2m + \mu_q(\omega) - \omega)/\omega}}{\Delta - \alpha [\mu_T - \mu_q(\omega)] - \omega + i\gamma/2} \right|^2 \\
\quad \quad \text{if } 0 < \mu_q(\omega) \left( \frac{1}{2} \left[ \frac{\omega^2}{q^2/2m} - \frac{q^2}{2m} \right] \right) < \mu_T \\
\quad \quad = 0 \quad \text{otherwise}, \quad (12)
\end{align*}
\]

The single peak at \( \omega = E_q \) in the spectrum of the homogeneous BEC \((11)\) is broadened to a feature from \( \omega = q^2/2m \) to \( \omega = \sqrt{(q^2/2m + \mu_T)^2 - \mu_T^2} \). The region in parameter-space where the difference in intermediate state energies can be neglected, either because \(|\omega| \ll |\Delta - \alpha(\mu_T - \mu_q(\omega))|\), or \(|\omega| \ll \gamma/2\) is the ‘fast collision’ regime. As in the off-resonant limit \([13]\) \([16]\), which is part of the fast-collision regime, fast-collision resonant scattering data contain the same information as non-resonant scattering data \([17]\), giving a cross-section proportional the dynamical structure factor of the scattering system:

\[
\text{(in the fast collision regime ) } \frac{d^2\sigma}{d\Omega d\omega} \approx \left| \frac{(3\gamma/4k)(\hat{d} \cdot \hat{e}_{in})(\hat{d} \cdot \hat{e}_{out})^*}{\Delta - \alpha [\mu_T - \mu_q(\omega)] + i\gamma/2} \right|^2 S_{TF}(q, \omega),
\]

where \( S_{TF}(q, \omega) = \frac{1}{2\omega_T} \left( \frac{R_0^3}{aL^2} \right) \frac{\mu_q(\omega)}{\mu_T} \sqrt{1 - \frac{\mu_q(\omega)}{\mu_T}}, \quad 0 < \mu_q(\omega) < \mu_T, \quad (13)\)

is the dynamical structure factor of the condensate, calculated in the above Thomas-Fermi approximation. In Figure 1, we compare the cross-section \((12)\) and the fast-collision approximation \((13)\) for three different values of the detuning.

We also point out that the dependence on the energies of the intermediate states implies the interesting possibility of observing the creation of a quasi-particle caused by particle annihilation. The \( \Delta - \omega + i\gamma/2 \)-denominator corresponds to an intermediate state of higher energy than the \( \Delta + i\gamma/2 \)-intermediate state, indicating quasi-particle \textit{creation}, whereas the intermediate state was formed by the removal or \textit{annihilation} of a boson (by exciting it to a different atomic state). The observation of the difference in energy-denominators can be accomplished by varying \( \Delta \) while keeping \( \omega \) constant, and requires near-resonant detuning which, unfortunately, can make the condensate optically thick. Nevertheless, experimental techniques such as resonating on different atomic transitions and/or using the polarization of the photons (scattered by spatially oriented dipole moments), can reduce the optical
Finally, to understand in what sense the scattering data can be interpreted in a Thomas-Fermi manner, we need to investigate its limitations. A necessary condition for the validity of the dynamical Thomas-Fermi description, is the validity of the static Thomas-Fermi description. At $T = 0$, in a harmonic trap, a Thomas-Fermi condensate satisfies $R_0 >> L$, or equivalently, $\mu >> \hbar \omega_T$ (\cite{11}, \cite{12}, \cite{20}). Furthermore, the inability of our dynamical Thomas-Fermi approach to describe the discrete spectrum of a finite-size BEC, indicates that even for a Thomas-Fermi condensate, the dynamical Thomas-Fermi results should be interpreted within certain limits. For the scattering problem, we need to realize that only fluctuations confined to limited regions in space and time are well-described by a Thomas-Fermi approach. The first constraint pertains to the excited-atom propagation, which is only well described by means of a position dependent detuning if, during its lifetime, the excited atom experiences a change in potential small compared to $\hbar \gamma$, ($||F \cdot \mathbf{v}|/\gamma|| << \hbar \gamma$) where $F$ is the force and $\mathbf{v}$ the velocity of the excited atom. A second condition pertains to the Thomas-Fermi description of the BEC-fluctuations. Consequently, we require that $q > l_v^{-1}$, where $l_v$ indicates the scale on which $\mu(R)$ varies (for the harmonic oscillator trap we can choose $l_v = R_0/3$ ; $\mu_{\text{eff}}(R)$ varies by approximately 10 % from $R = 0$ to $R = R_0/3$). This condition is somewhat stricter than the condition to observe incoherent – as opposed to coherent – scattering, $q > R_0^{-1}$. Nevertheless, with a chemical potential $\mu_T \simeq 100(\omega_T/2)$ (realistic in present-day BEC technology) $R_0 \simeq 10L$ and $k_c \simeq 10/L$, where $k_c$ is the inverse of the local coherence length in the middle of the trap $k_c = 2\sqrt{\mu_T m}$. We find then that the condition $q > l_v^{-1} \simeq 3/R_0 \simeq 0.03k_c$, leaves almost all of the interesting part of the dispersion (expected to be phonon-like up to $k_c$) to be explored. As for the temporal constraint, the ‘locally homogeneous-like’ excitation picture breaks down on the time scale that it takes the BEC-response to be affected by its inhomogeneity. Assuming that an excitation in the middle of the trap propagates at the local speed of sound, $c = \sqrt{\mu_T/m} = \omega_T \times R_0/\sqrt{2}$, we can estimate the relevant time scale $t_v$ as $t_v \sim (l_v/R_0)\omega_T$. Reducing the frequency resolution in the scattering spectrum to $\Delta \omega \sim t_v^{-1} \sim \omega_T \times (R_0/l_v)$, restricts the scattering probe to short-
time \( (t < t_v) \) ‘homogeneous-like’ fluctuations. Thus, the Thomas-Fermi scattering spectrum should be interpreted as a ‘smooth’ version of the real spectrum and we should compare intensities integrated over frequency intervals larger than or equal to \( t_v^{-1} \) with experimental results. These estimates were made in the middle of the trap where the Thomas-Fermi description works best, and which is probed on the high–frequency side of the cross-section, whereas the low frequency-region, \( \omega \sim q^2/2m \), probes the edge of the condensate where the Thomas-Fermi results cannot be trusted.

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Figure 1. Differential cross section for resonant light scattering from a trap with the parameters shown in the figure caption. The plots show the cross section as a function of energy transfer, at fixed momentum transfer $q = \sqrt{m\mu_T}$, for three values of the detuning $\Delta$. The full line shows the Thomas-Fermi calculation and the dotted line shows the fast collision approximation result. As the detuning increases, the fast collision approximation becomes more and more accurate.
Energy Transfer (in units of $\gamma/2$)

Differential Scattering Cross Section

$\left( \mu_T = \gamma/2, \alpha=2, q=(m \mu_T)^{1/2} \right)$

Cross Section (arbitrary units)

Energy Transfer (in units of $\gamma/2$)