Radiative Corrections

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Outline

1. Introduction and Motivation
   What is the role of radiative corrections in our endeavour to find the fundamental law(s) of nature, the final underlying theory, that determines the dynamics of matter?
   The various aspects and the importance of radiative corrections illustrated on the example of running couplings, anomalous magnetic moments, electroweak precision physics, specifically Z pole observables at LEPI/SLC, the $M_W - M_Z$ relation and $M_H$ prediction, and the $M_W$ measurement at LEPII.

2. QED – First Example of a QFT with direct experimental applications
   A brief discourse in Quantum Field Theory, i.e. the theoretical framework underlying elementary particle interactions.
   From the Lagrangian to Green’s functions to cross sections. The treatment of divergences and the removal of UV divergences via renormalization.

3. A specific example: Electroweak $O(\alpha)$ corrections to $W$ production at hadron colliders
1. Introduction and Motivation

Radiative corrections: higher order terms in the perturbative expansion of physical quantities that arise from the quantum structure of the underlying theory.

Why studying radiative corrections?

- 'analyse theory' (Electroweak Precision Physics, Beyond the SM, QCD)
  Performing tests of the Standard Model (SM) as a fully fledged Quantum Field Theory (QFT) and search for signals of physics beyond the SM.
  Examples: Running couplings, muon anomalous magnetic moment, precision observables at the Z pole, $M_W - M_Z$ relation (prediction for $M_W$ and $M_H$)

- 'analyse data' (QED, QCD, Monte Carlo programs)
  Extraction of precision observables from data, e.g., Bhabha scattering as luminosity monitor at LEP.
  Precise measurement of SM input parameters, e.g., $M_W$ at LEPII and the Tevatron.
  Realistic simulation of experiment to study effects of selection/analysis of data (systematic error).
Running couplings

The U(1), SU(2) and SU(3) gauge couplings are not constants but depend on the momentum transfer in the process under consideration due to higher-order corrections in perturbation theory.
World Summary of $\alpha_s(Q)$ (Sep. 2002)

$\alpha_s(Q)$

$\alpha_s(Q)$ vs. $Q$ [GeV]

Data
- Deep Inelastic Scattering
- $e^+e^-$ Annihilation
- Hadron Collisions
- Heavy Quarkonia

Theory
- NLO
- NNLO
- Lattice

$\Lambda^{(5)}_{\text{MS}}$ and $\alpha_s(M_Z)$

- QCD
- $O(\alpha_s^4)$

- $\Lambda^{(5)}_{\text{MS}}$
- $\alpha_s(M_Z)$

- $245$ MeV: $\alpha_s(M_Z) = 0.1210$
- $211$ MeV: $\alpha_s(M_Z) = 0.1183$
- $181$ MeV: $\alpha_s(M_Z) = 0.1156$

Ref: hep-ex/0211012

S. Bethke: $\alpha_s$ and QCD tests at hadron colliders
Collider Workshop, Santa Barbara, Jan. 2004
Gauge coupling unification?

\[
\alpha_1 = \frac{5}{3} \alpha_{\overline{\text{MS}}} / \cos^2 \theta_W^{\overline{\text{MS}}}, \quad \alpha_2 = \alpha_{\overline{\text{MS}}} / \sin \theta_W^{\overline{\text{MS}}}, \quad \alpha_3 = \alpha_s^{\overline{\text{MS}}}
\]

from W.de Boer, C.Sander, hep-ph/0307049
The anomalous magnetic moment of the muon: theory vs experiment

from the BNL muon $g - 2$ collaboration, hep-ex/0401008
\((g - 2)_\mu\) and \(a^{\text{had}}_\mu\)

\[ a_\mu = a^{\text{QED}}_\mu + a^{(1)}_{\text{had}} + a^{(2)}_{\text{had}} + a^{\text{weak}}_\mu + a^{(2)}_{\text{weak}} + a^{\text{lbl}}_\mu + a^{\text{new physics}}_\mu \]

**All kind of physics meets!** from LoopFest III, http://quark.phy.bnl.gov/loopfest3
What did we learn from **electroweak precision physics** so far?

Thanks to LEP/SLC and the Tevatron \((M_W, m_t)\), we can be confident that the SM, as a fully fledged Quantum Field Theory and based on the gauge principle, correctly describes electroweak interactions among fundamental particles at presently accessible energies.

**LEP/SLC succeeded in probing the electroweak sector of the SM at an unprecedented level of precision, with many observables being sensitive to genuine electroweak loop effects.**

Many LEP/SLC observables are sensitive to loop effects induced by either undiscovered sectors of the SM (Higgs sector) or new particles of physics beyond the SM.

**LEP/SLC collaborations together with CDF/D0 measurements of** \(m_t, M_W\) **succeeded in deriving an indirect upper limit on the SM Higgs boson mass through its presence in quantum loops contributing to electroweak precision observables:** \(M_H < 237 \text{ GeV} \) (95% C.L.). **LEPEWWG Winter 2004**

So far, we found no compelling direct or indirect signal of new physics but succeeded in strongly constraining the parameters of a number of new models (e.g., Technicolor, SUSY).
How did we learn it?

The ingredient

Sensitivity to quantum-loop effects through

**Theory:**
- predictive power of the SM beyond lowest order* in perturbation theory

**Experiment:**
- precision measurements at, e.g., $e^+ e^-$ and $pp/p\bar{p}$ colliders:
  - LEP, Tevatron, LHC, LC, . . .

* renormalizability (M. Veltman and G.’t Hooft): perturbative calculation of measurable quantities order by order in terms of only a few parameters

The method

Experiment:
20 precision (pseudo) observables have been extracted from the data collected at and around the Z-pole (SLD, LEP-I):

Z lineshape and leptonic forward-backward asymmetries: $M_Z, \Gamma_Z, \sigma_{had}^0, R_l, A_{f,b}^{0,l}$

Polarized lepton asymmetries: $A_l(P_T), A_l(SLD)$

Heavy flavor results: $R_b, R_c, A_{f,b}^{0,b}, A_{f,b}^{0,c}, A_b, A_c$, Hadronic charge asymmetry: $\sin^2 \theta_{eff}^l(Q_{fb})$
and from other experiments:

- W mass and width (LEP-II, Tevatron): $M_W, \Gamma_W$
- Top-quark mass (Tevatron): $m_t$
- Atomic parity violation (Caesium): $Q_{W(Cs)}$
- Hadronic vacuum polarization: $\Delta\alpha_{had}^{(5)}$
- NuTeV: $\sin^2 \theta_W (\rightarrow M_W)$

The experimental precision is at the per-mille level probing the SM down to distances of $10^{-17}$ m when comparing with at least equally precise theoretical predictions.
(Pseudo-)Observables around the Z resonance D.Bardin et al., hep-ph/9902452

Pseudo-observables are extracted from “real” observables (ROs) (cross sections, asymmetries) by de-convoluting them of QED and QCD radiation and by neglecting terms ($\mathcal{O}(\alpha \Gamma_Z / M_Z)$) that would spoil factorization ($\gamma, Z$ interference, $t$-dependent radiative corrections).

The $Zf\bar{f}$ vertex is parametrized as $\gamma_\mu (G_V^f + G_A^f \gamma_5)$ with formfactors $G_{V,A}^f$, so that the partial $Z$ width reads:

$$\Gamma_f = 4N_c \Gamma_0 (|G_V^f|^2 R_V^f + |G_A^f|^2 R_A^f) + \Delta_{EW/QCD}$$

$R_{V,A}^f$ describe QED, QCD radiation and $\Delta$ non-factorizable radiative corrections. Pseudo-observables are then defined as ($g_{V,A}^f = Re G_{V,A}^f$)

- $\sigma_{\nu}^0 = 12\pi \frac{\Gamma_e \Gamma_{h}}{M_Z^2 \Gamma_Z^2}, R_q,l = \Gamma_{q,h}/\Gamma_{h,l}$

- $A_{FB}^f = \frac{\sigma_F - \sigma_B}{\sigma_F + \sigma_B} \rightarrow A_{FB}^{f,0} = \frac{3}{4} A_e A_f, A_f = 2 \frac{g_V^f g_A^f}{(g_V^f)^2 + (g_A^f)^2}$

- $A_{LR} (SLD) = \frac{N_L - N_R}{N_L + N_R} \frac{1}{<p_e>} \rightarrow A_{LR}^{0} (SLD) = A_e$

and $4|Q_f| \sin^2 \theta_{eff}^f = 1 - \frac{g_V^f}{g_A^f}$ with $g_{V,A}^f$ being effective couplings including radiative corrections.
Theory:
To at least match (better: exceed) the experimental precision, theoretical predictions for the pseudo-observables need to go beyond the leading order and even beyond leading universal corrections (e.g., QED, running of $\alpha$).

Theory predictions for pseudo-observables are obtained with the Monte Carlo programs ZFITTER and TOPAZ0 including radiative corrections up to two loop (electroweak) and three loop (QED), using the following set of input parameters

$$\Delta \alpha_{had}^{(5)}, \alpha_s(M_Z), M_Z, m_t, M_H, G_\mu$$

Status:
Complete 2-loop electroweak corrections to $Z$ boson observables are not available, but highly desirable (LEP: $\sin^2 \theta_{eff}^{lept}$, GigaZ).
The outcome

By comparing measurements and theoretical predictions of electroweak precision observables

- the electroweak sector of the SM is probed at the quantum-loop level,
- the consistency of the SM is checked by comparing direct with indirect determinations of input parameters, e.g., $m_t$, $M_W$,
- the SM Higgs boson mass can be predicted, and
- the parameters of models beyond the SM can be constrained.
Illustration of the extraction of $Z$ lineshape parameters from the cross section to $e^+ e^- \rightarrow Z \rightarrow \text{hadrons}$:

\[ \sigma(s) \propto \frac{s \Gamma_Z^2}{(s - M_Z^2)^2 + (s \Gamma_Z)^2/M_Z^2} \]

Number of light neutrino species:

\[ N_\nu = 2.9841 \pm 0.0083 \]
The leptonic effective couplings to the Z boson

from LEPEWWG, Winter 2004, http://lepewwg.web.cern.ch/LEPEWWG
The global SM fit to all electroweak data:
from LEPEWWG

Two $\sim 3\sigma$ “anomaly”:
$A_{FB}^0, A_{FB}^b, \sin^2 \theta_W$ (NuTeV)

Possible sources:
statistical fluctuation
experimental systematics
theoretical uncertainties
non-standard physics (“tough”,
e.g., the MSSM does not help)
Precise W and top mass measurements and the Higgs mass:

$M_W - M_Z$ correlation:

$$M_W^2 \left( 1 - \frac{M_W^2}{M_Z^2} \right) = \frac{\pi \alpha(0)}{\sqrt{2} G_\mu (1 - \Delta r(M_W, m_t, M_H, \ldots))}$$
**Example:** Prediction for $M_W$ in the SM and the MSSM:

![Graph showing prediction for $M_W$ in the SM and the MSSM](image)

- **MSSM uncertainty:**
  - unknown masses of SUSY particles
- **SM uncertainty:**
  - unknown Higgs mass

*Heinemeyer, Weiglein '03*
Example: Prediction for $M_W$ in the SM and the MSSM:

MSSM uncertainty:
unknown masses of SUSY particles

SM uncertainty:
unknown Higgs mass

$M_W$, $m_t$ [GeV]

experimental errors 68% CL:
- LEP2/Tevatron (today)
- Tevatron/LHC
- LC+GigaZ

$M_H = 113$ GeV
$M_H = 400$ GeV

Heinemeyer, Weiglein '04
When only incl. universal leading ewk corr.

$\sigma_{WW}$ is about 2% higher $\Rightarrow$

experimental evidence for genuine, non-universal electroweak corrections in W-pair production
W mass measurement at a future Linear Collider

Sensitivity of the $W$-pair threshold scan to the $W$-mass:

The vertical axis shows the ratio of the cross section to the predicted cross section for $M_W = 80.39$ GeV. The error bars represent the expected experimental errors for the scan. from K.Moenig, hep-ph/0303023

$\Rightarrow \Delta M_W^{\text{experiment}} \approx 7$ MeV

Theory uncertainty:
$\Delta \sigma/\sigma \approx \pm 2\%$
$\Rightarrow \Delta M_W^{\text{theory}} \approx 20 - 30$ MeV
Technique well established

Partial results/special cases

012345678910

# legs

# loops

vacuum graphs

$\Delta \rho$

self-energies

$\Delta r$, masses

2$\rightarrow$2, 1$\rightarrow$3

Bhabha

1$\rightarrow$2 decays

$\sin^2 \theta_{\text{eff}}$

2$\rightarrow$3

ee$\rightarrow$4$f$ + $\gamma$

ee$\rightarrow$6$f$

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LoopFest III, Santa Barbara, April 2004

TASI 2004

Stefan Dittmaier (MPI Munich), The LoopVerein: (not only) European activities
Technique well established

Partial results/special cases

Required for LC physics

(□ = leading effects)

+ more?

vacuum graphs
Δρ

self-energies
Δr, masses

2 → 2, 1 → 3
Bhabha

1 → 2 decays
sin²θ_{eff}^{lept}

2 → 3

ee → 4f

ee → 6f

ee → 4f + γ