On the Relativistic Collapse of Dense Star Clusters

J. W. Moffat

Department of Physics, University of Toronto, Toronto, Ontario M5S 1A7, Canada

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Abstract

We investigate the fate of a relativistic star cluster with a dense core which is undergoing a gravothermal catastrophe and is far from thermodynamic equilibrium. Nonlinear cooperative contributions are included in the standard transport equations for the last stage of evolution of a highly dense core of stellar remnants. We find that the core redshift does not necessarily increase without limit as the core becomes increasingly dense, preventing collapse to a black hole. In particular, the redshift can remain less than the critical value for relativistic collapse, resulting in a stable, massive dark core with a Newtonian mantle and halo.

Subject headings: black hole physics - galaxies: nuclei - gravitation
According to numerical integrations of the full Einstein equations for the dynamical evolution of an arbitrary spherical, collisionless system in general relativity (Shapiro and Teukolsky 1985a,b,c, 1986), star clusters become relativistically unstable at sufficiently high central redshift, \( z_c > 0.5 \), confirming the original speculation by Zel’dovitch and Podurets (1965) and perturbative calculations performed by Ipser and Thorne (1968), Ipser (1969,1970,1980), Bisnovatyi-Kogan and Thorne (1970), Fackerell (1970) and Fackerell et al., (1969). However, more recent work by Rasio, Shapiro and Teukolsky (1988,1989), Bisnovatyi-Kogan, et al., (1993) and Merafina and Ruffini (1995), demonstrated that a family of cluster evolution models were relativistically stable for \( z_c \to \infty \). Merafina and Ruffini proposed three possible configurations of cluster equilibrium models in terms of central redshift versus temperature diagrams. They found that the stable relativistic equilibrium models, corresponding to the possibility of reaching infinite central redshift, were not of astrophysical interest, because they have ratios \( M_{\text{core}}/M \sim 10^{-24} \) and \( R_{\text{core}}/R \sim 10^{-25} \), where \( M_{\text{core}} \) and \( R_{\text{core}} \) denote the core values of the cluster mass and radius, respectively. We shall accept the point of view that astrophysically interesting clusters, which can evolve dense relativistic cores with masses \( 10^7 - 10^9 M_\odot \), will become unstable for \( z_c > 0.5 \) and collapse to black holes.

The question remains: \textit{do supermassive black holes form at the center of galaxies?} In a recent article (Moffat 1997), a model of a galaxy was constructed from a relativistic massive core and a Newtonian mantle and halo. The core consisted of stellar remnants such as neutron stars, brown dwarfs or planets with individual masses, \( m \sim 1 M_\odot \). This would not contradict the observations obtained by Lugger et al., (1992), which ruled out a change in mass-to-light ratio from \( \sim 2 \) at 1 arcsec to \( \geq 20 \) at 0.1 arcsec, yielding large broadband color gradients. Using the high-velocity maser emission data obtained for NGC 4258 \( (M \sim 4 \times 10^7 M_\odot) \) (Miyoshi et al., 1995; Maoz 1995), which impose constraints on the mass distribution of the dense stellar cluster, the evaporation time obtained was \( t_{\text{evap}} \sim 10 \) Gyr, a time scale big enough compared to the Hubble time.
Calculations have been performed of dense \((\geq 10^8 \text{ stars pc}^{-3})\) galactic nuclei composed initially of main-sequence stars (Gold, Axford, and Ray 1965; Spitzer and Stone 1967; Colgate 1967; Sanders 1970; Spitzer 1971). It was found that the evolution is mainly determined by coalescence and stellar collisions, and massive stars would undergo supernova explosions that can leave behind compact stellar remnants. The endpoint of this evolutionary phase is likely to be a cluster of neutron stars or other stellar remnants. Such a cluster could initially consist of \(10^8\) compact objects, within a region \(\leq 0.1\) pc in radius and moving with a velocity dispersion \(800 - 2000\) km s\(^{-1}\). Two epochs characterize the subsequent Newtonian evolution of the cluster of compact stars, (i) a low-redshift \((z_c < z_{\text{coll}} \approx 10^{-2})\) point-mass epoch during which the core undergoes collapse driven by the “gravothermal catastrophe” (Lynden-Bell and Wood 1968; Cohn 1980), (ii) a subsequent short, high-redshift \((z_c \geq z_{\text{coll}})\) epoch dominated by coalescence and collisions of compact stars in the cluster.

**II. BASIC NONLINEAR TRANSPORT EQUATIONS**

We must now investigate the evolution of the dense core following these two epochs. Shapiro and Teukolsky (1987), Goodman and Lee (1989), Quinlan and Shapiro (1989), Lee (1995), Quinlan (1996), van der Marel et al., (1997) have studied this epoch using transport equations for a cluster of compact stars (Lightman and Shapiro 1978). The number of core stars decreases and, at the same time, the central velocity dispersion and redshift increase ultimately leading to a relativistic redshift, \(z_c > z_{\text{crit}} \sim 0.5\), and therefore to gravitational collapse to a black hole.

These authors assumed that the transport equations were linear differential equations in the time variable \(t\). The arguments against dark clusters of compact stars rely mainly on the very high density of the core \((\geq 10^8 M_\odot \text{pc}^{-3})\). Treated as a fluid, the gravothermal catastrophe epoch is expected to be far from thermodynamic equilibrium (Lynden-Bell and Wood 1968).

There are many examples in physics in which subsystems cooperate with each other in a
self-organizing way (Haken 1975; Haken 1983; Bak, Tang and Wiesenfeld 1988; Bak 1996). The behavior of the total system can show characteristic changes which can be described as a transition from disorder to order, or a transition from one state to another. There are numerous examples of such behavior at both the macroscopic and quantum levels. Pronounced cooperative phenomena may occur in physical systems far from thermodynamic equilibrium. Such physical systems far from equilibrium can display ordered states created and maintained by an energy flux passing through the system. The choices of order parameters, well-known in phase-transition theory, play an important role. The order parameters represent the behavior of the system on a macroscopic scale and therefore describe macroscopic variables. Normally the order parameters satisfy simple differential equations with respect to the time variable, for the relaxation time of order parameters is normally much greater than those of the subsystems. The variables of the subsystem can be eliminated without increasing the degree of the time derivatives.

We shall generalize the differential equations for the dense core given by Lightman and Shapiro (1978) and Shapiro and Teukolsky (1985) to the following equations:

\[ \dot{E}_c = -\left(\alpha_1/t_r\right)E_c - \left(\beta_1/t_r^3\right)E_c^3 + \left(1/t_{\text{coll}}\right)E_c, \]  
\[ \dot{N}_c = -\left(\alpha_2/t_r\right)N_c - \left(\beta_2/t_r^3\right)N_c^3 - \left(1/t_{\text{coll}}\right)N_c, \]  
\[ \dot{m}_c = \left(1/t_{\text{coll}}\right)m_c, \]

where \( N_c(t) \) is the number of core stars, \( \dot{N}_c = \partial N_c/\partial t \), \( E_c(t) \) is the characteristic core binding energy and \( m_c(t) \) is the mean stellar mass. By using the virial theorem we get

\[ E_c \approx \frac{1}{2} N_c m_c v_c^2 \approx \frac{1}{4} (N_c m_c)^2 / r_c, \]  

and the relations

\[ n_c = N_c / (4\pi r_c^3 / 3) \approx 2 \times 10^9 \text{ pc}^{-3} (N_{c,8} v_{c,3}^6 m_{c,*}^{-3}), \]  
\[ r_c = 0.2 \text{ pc} (N_{c,9} v_{c,3}^{-2} m_{c,*}), \]

where \( N_{c,8} \equiv N_c / 10^8, v_{c,3} \equiv v_c / 10^3 \text{ km s}^{-1}, \) and \( m_{c,*} \equiv m_c/M_\odot. \) The central relaxation time is (Spitzer and Hart (1971)):
\[ t_r \approx v_c^3 \left[ \left( \frac{3}{2} \right)^{1/2} 4\pi m_c^2 n_c \ln(0.4N_c) \right] \approx 0.8 \times 10^8 \text{ yr} \left( N_{c,8} v_{c,3}^{-3} m_{c,8} \Lambda_8^{-1} \right), \]  

where \( \Lambda_8 \equiv \ln(0.4N_c)/\ln(0.4 \times 10^8) \). Moreover, the dynamical time scale \( t_d \ll t_r \) is

\[ t_d \approx r_c/v_c \approx 200 \text{ yr} \left( N_{c,8} v_{c,3}^{-3} m_{c,8} \right), \]  

and the two-body collision time scale is

\[ t_{\text{coll}} \approx \frac{1}{n_c \sigma_{\text{coll}} v_{\infty}} \approx 6 \times 10^{13} \text{ yr} \left( N_{c,8} v_{c,3}^{-5} m_{c,8} \right), \]  

where \( v_{\infty} \approx 2^{1/2} v_c \) is the asymptotic relative velocity, and \( \sigma_{\text{coll}} \) is the collision cross section.

The physical source of the nonlinear contributions in the transport equations for the order parameters \( E_c \) and \( N_c \) could be energy pumped into the core from binary gravitational radiation, heat dissipative effects caused e.g., by tidal interactions that can transfer large amounts of energy to the core in its final stage of evolution, or post-Newtonian contributions in the self-gravitating interactions of the point masses in the core.

### III. APPROXIMATE SOLUTION FOR THE REDSHIFT

By using the result for the redshift (Cohn 1980; Stuart and Teukolsky 1985):

\[ z_c \approx \phi_c \approx 14v_c^2/3 \approx 5 \times 10^{-5} v_{c,3}^2, \]  

where \( \phi_c \) is the central potential, we can replace Eq.(1a) by

\[ \dot{z}_c = \left( a_1/t_r \right) z_c - \left( b_1/t_r^2 \right) z_c^3 + \left( \beta_2/t_r^3 \right) N_c^2 z_c + \left( 1/t_{\text{coll}} \right) z_c, \]  

where \( a_1 = \alpha_2 - \alpha_1 \), \( b_1 = (3Ncm_c/28)^2 \beta_1 \) and the total core mass \( N_c m_c \) is constant.

We solve Eqs.(8), (11) and (10) subject to the constraint: \( \tau \leq H^{-1} = 2 \times 10^{10} \text{ yr} \), where \( \tau \) is the remaining time for the gravothermal catastrophe to drive secular core collapse to completion (\( N_c \rightarrow 0 \)). A solution for \( \alpha_1 \) and \( \alpha_2 \) (Cohn 1980; Stuart and Teukolsky 1985), using a matching to Fokker-Planck calculations for the advanced core collapse in the absence...
of collisions, yielded the result $\alpha_1/\alpha_2 = 0.701$ ($a_1 > 0$). It should be noted that the Fokker-Planck approximations used by Cohn (1980) eventually break down as the number of stellar remnants in the core decreases and large angle scattering and binary formation become important, so these calculations may not include nonlinear cooperative phenomena in the last highly dense stage of core evolution.

We shall assume that the parameter $a_1$ remains positive for the nonlinear equations, Eqs. (8) and (11), that $t_{\text{coll}} = \infty$, that the parameter $\beta_2 \approx 0$ and that in the time scale of the gravothermal catastrophe evolution $t_r \approx \text{const}$. Then, Eq. (8) can be written as

$$\dot{z}_c = az_c - bz_c^3,$$

where $a = a_1/t_r$ and $b = b_1/t_r^3$. This equation has an exact time dependent solution for $a > 0$ and $b > 0$ given by

$$z_c = \frac{a^{1/2}}{\{b + Ca \exp[-2a(t - t')]\}^{1/2}},$$

where $C$ is an integration constant. For $b = 0$ we get

$$z_c = C^{-1/2} \exp[a(t - t')],$$

and $z_c \rightarrow \infty$ as $t \rightarrow \infty$, which leads to the catastrophic collapse to a black hole. However, for $b > 0$ we get as $t \rightarrow \infty$:

$$z_c \approx (a/b)^{1/2},$$

and if $(a/b)^{1/2} < 0.5$, then the dark cluster can be relativistically stable if the adiabatic index $\Gamma_1 \approx 5/3$ and the cluster has a Newtonian mantle and halo with a mantle radius, $r_m \geq 0.1$ pc (Moffat 1997); the massive core will contain most of the mass of the galaxy.

IV. DISCUSSION

Observations of nuclear regions in the centers of galaxies have detected high velocities of revolution, and luminosity spikes suggesting masses between $10^7$ and several $10^9 M_\odot$ (Ford
et al., 1994; Miyoshi et al., 1995; Kormendy and Richstone, 1995; Kormendy et al., 1997). However, masses for more than 60 other nearby galaxies are of order $\sim 10^6 \, M_\odot$ (Kormendy 1987). For our Galactic center, the mass may be significantly less than $10^6 \, M_\odot$ (Allen and Sanders 1986; Kundt 1990; Sanders 1992). The black hole models of clusters have been used to explain AGNs, and other intense extragalactic radio sources powered by jets of plasma and magnetic fields produced by the compact massive black hole at the center of the galaxy (Begelman, Blanford, and Rees 1984). However, Kundt (1996) has formulated an alternative model to explain AGN observations, based on an application of standard accretion disk theory to the central parts of the galactic disk. A burning disk can possibly explain the phenomena commonly attributed to a supermassive black hole. It is not clear at present whether these models can successfully describe the complex phenomena of AGNs for all the observed galactic nuclei.

We have demonstrated that by using nonlinear transport equations for the order parameters $E_c$ and $N_c$, which take into account nonlinear cooperative effects of the subsystem of point masses, the inner core of a cluster may become relativistically stable, after having been formed by an epoch of gravothermal catastrophe. Treated as a fluid, the dense core is far from thermodynamic equilibrium and from known physical processes, we can expect cooperative phenomena to be important. The nonlinear cooperative contributions to the differential equation for the redshift can prevent it from achieving the critical value, $z_c \sim z_{\text{crit}} \sim 0.5$. In the linearized approximation to the transport equations, the standard result follows that as gravothermal catastrophe develops, the density of the core increases and the number of core stellar remnants decreases until $z_c > 0.5$, at which point the core inevitably undergoes gravitational collapse to a black hole.

We have used a heuristic model of the nonlinear cooperative effects in a highly dense core far from thermodynamic equilibrium, in order to obtain a picture of the qualitative behavior of the last epoch of core evolution. A more complete solution would require a numerical analysis of the problem. Equations such as Eq.(La) should include stochastic contributions, leading to non-zero correlation functions between two-particle interactions, and use should be
made of a Fokker-Planck equation and a related probability density distribution to analyze the dynamical evolution of the relativistic core. Moreover, it is unrealistic to ignore collisions, binary formation and gravitational radiation (Quinlan and Shapiro 1989) in the last stage of core evolution.

In spite of the shortcomings of our qualitative picture of the last stage of core evolution, it is clear that non-linear self-organizing, cooperative effects, associated with phase transition phenomena for the highly dense cluster cores, can dramatically change the collapse scenarios and possibly prevent the formation of supermassive black holes.

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